Inelastic Tunneling Spectroscopy in a D-wave Superconductor.

A. V. Balatsky, Ar. Abanov, and Jian-Xin Zhu

Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Dated: October 29, 2018)

We propose to study the inelastic electron tunneling spectroscopy (IETS-STM) to detect a single spin in a d-wave superconductor and in a pseudogap state, based on a direct exchange coupling $J$ between the surface electrons and the local spin $S$ in a magnetic field. This coupling will produce a kink in a $dI/dV$ characteristic at Zeeman energy of the spin $\omega_0$. We find that for relevant values of parameters signal scales as $dI^2/dV^2 \approx (JN_0)^2\Theta(eV-\omega_0)$ and could be in the range of $10^{-2}$ of the bare density of states where $N_0$ is the density of states for surface electrons. Scattering in superconductor with the coherence peak at gap maximum $\Delta$ leads also to strong features at $\Delta + \omega_0$. This suggests a new technique for a detection of a local spin excitation with STM. We also consider a detection of a local vibrational mode as a simple extension of the spin case.

PACS numbers: 76.30.-v, 07.79.Cz, 75.75.+a

Inelastic electron tunneling STM spectroscopy (IETS-STM) is a well established technique that has been proven, starting with important experiments of Stipe et al. $^1$ In these experiments a step-like feature in tunneling current and local density of states have been observed. The physical explanation of the effect is straightforward: once energy of tunneling electrons exceeds the energy required to excite local vibrational mode, there is a new scattering process that contributes to the scattering of electrons due to inelastic excitation of the local mode $^2$. Similarly, in case of a single impurity spin $S$ in external field the localized spin state will be split with the Zeeman gap $\hbar \omega_0 = g\mu_B B$. If there is a local exchange coupling that allows conduction electrons to scatter inelastically off the local spin in an external magnetic field then one has a very similar situation of inelastic scattering off localized spin levels. Up to date the single spin inelastic tunneling spectroscopy in metals with STM has not been observed however. One reason that is often mentioned is a Kondo screening of a magnetic spin by conduction electrons that leads to a more complicated response $^3$.

Potential applications of the techniques that are sensitive to the single spin dynamics include studies of a single spin Kondo problem $^4$, studies of magnetic impurity states in superconductors $^5$ and single spin detection and manipulation in a context of a solid state quantum computing schemes $^4, 5$. Use of STM for spin detection is a promising approach that would allow one to combine a sub-Angstrom spatial resolution with the high electronic sensitivity.

We propose to study the inelastic electron tunneling spectroscopy of a single spin and of a localized vibrational mode with STM in a d-wave superconductor. In case of spin there is a crucial difference between metal and d-wave superconductor that might make the single spin observation in IETS with STM more feasible: vanishing DOS in d-wave superconductor drastically suppresses or even makes the Kondo temperature $T_K = 0$ for a single impurity spin. This leaves one with a simpler problem of a scattering off the single unscreened spin in a media with linearly vanishing density of states (DOS) $N(\omega) \sim \omega$.

The proposed approach to identify magnetic sites is based on the fact that the local density of states in the vicinity of a single impurity spin will have a kink-like feature with threshold at $eV = g\mu_B B$, $\omega_0$ is a Larmor frequency of a spin. This would be a natural, albeit not tried yet in correlated electron systems, extension of the single molecule vibrational spectroscopy and could allow a single spin detection. Our results can be summarized as follows: i) we find that spin produces the kink-like singularity in the density of states that as a function of position with respect to moment site at low $T \ll \omega_0$, $\omega \ll \Delta$ is:

$$
\delta N(r, \omega)/N_0 \approx 2\pi^2(\frac{\omega_0}{\Delta})^2 \ln(\frac{4\Delta}{\omega_0})^2 \times \frac{\omega_0}{\Delta} \Theta(\omega - \omega_0)(N_0 JS)^2 \Lambda(r),
$$

(1)

where $\Lambda(r)$ is a combination of Greens function of an electron in real space (assumed 2D), describing Friedel oscillation as a function of $r$ and reflecting four-fold anisotropy due to superconducting gap. The singularity at $\omega = \omega_0(B)$ changes as a function of applied external magnetic field since $\omega_0 = g\mu_B B$, $\mu_B$ being the Bohr magneton and $g$ the gyromagnetic ratio; We also find a strong feature at energies $\Delta + \omega_0$ as a result of sharp coherence peak in DOS of a superconductor, $\omega \approx -\Delta$:

$$
\delta N(r, \omega)/N_0 \approx \ln^2(\frac{4\Delta}{|\omega - \Delta|}) \ln(\frac{4\Delta}{\omega - \omega_0 + \Delta}) \Theta(\omega - \omega_0)(N_0 JS)^2 \Lambda(r).
$$

(2)

(see Eq. 7); these kink-like singularities in $dI/dV$ characteristic of a STM tunneling in a vicinity of magnetic site lead to a step in $d^2I/dV^2$; ii) the strength of the effect is of second order in a dimensionless coupling $N_0J$. If we take typical values of $J \sim 1-0.1 eV$ and $N_0 = 1/eV$ we find that the magnitude of the correction to DOS is on the order of $10^{-2}$; iii) The proposed effect can also be trivially expanded to be tried for a molecular spectroscopy of a local vibrational mode. One would have...
to assume $\omega_0$ be the eigenfrequency of a local mode, independent of the field, and replace $JS$ by the coupling constant to the local vibrational mode in Eq. (3).

Important potential application of the proposed technique is the study of the induced magnetic moment near Zn and Ni impurities in the high-Tc compounds. It has been argued that the Zn, Ni and Li impurities in the Cu-O plane generate uncompensated spin. A Zn$\text{i}^{2+}$ that substitutes Cu$^{2+}$ has a closed shell and is nonmagnetic and induced moment near Zn has to be a collective response of the neighbor sites. Claims also have been made about evidence of the Kondo effect [5]. Direct and independent test of magnetism induced by Zn, Ni, and Li impurities would be important for our understanding of the physics of strong correlations in Cu-O planes in high-Tc compounds. The IETS STM technique would allow the direct and alternative approach to distinguish between magnetic and nonmagnetic sites in a d-wave superconductor.

Assume that we have localized magnetic atom with spin S on a surface of a d-wave superconductor. Electrons in a superconductor interact with the localized spin via point-like exchange interaction at one site $JS \cdot \sigma$:

$$H = \sum_k c_{k\sigma}^\dagger \epsilon(k)c_{k\sigma} + \sum_k (\Delta(k) c_{k\uparrow}^\dagger c_{-k\downarrow} + h.c.) + \sum_{k,k',\sigma,\sigma'} JS \cdot c_{k\sigma}^\dagger \sigma \sigma' c_{k'\sigma'} \mu_B S \cdot B,$$  

(3)

where $c_{k\sigma}$ is annihilation operator for the conduction electron of spin $\sigma$, $\epsilon(k)$ is the energy of the electrons, $\Delta(k) = \Delta (\cos k_x - \cos k_y)$ is the d-wave superconducting gap of magnitude $\Delta \approx 30 \text{meV}$ in typical high-Tc materials. The local spin $S$ is a $|S| = 1/2$. We focus here on the effect of the Zeeman splitting of the otherwise degenerate local spin state in the external magnetic field $B$ with splitting energy $\omega_0 \equiv \omega_L = \mu_B B$. Below we use a mean field description of superconducting state at low temperatures $T \ll T_c$. Assuming field $B \ll H_{c2}$ we will ignore the orbital and Zeeman effect of the field on the conduction electrons [3].

We are interested in a local effect of inelastic scattering of electrons. Thus only local properties will determine the conduction electron self-energy. Results we obtain will also hold for a normal state with linearly vanishing DOS, such as a pseudogap state of high-Tc superconductors. In the case of a normal state one would model normal pseudogap state with a single particle Hamiltonian $H_0 = \sum_k \epsilon(k)c_{k\sigma}^\dagger c_{k\sigma}$, with $N(\omega) \sim \omega$.

Because of the vanishing DOS in a d-wave superconducting state Kondo singlet formation occurs only for a coupling constant exceeding some critical value $J_c$ [10]. For a particle-hole symmetric spectrum Kondo singlet is not formed for arbitrarily large values of $J$. Another situation where Kondo effect is irrelevant is the case of ferromagnetic coupling $J$. This allows us, quite generally, to consider a single spin in a d-wave superconductor that is not screened and we ignore the Kondo effect.

In the presence of magnetic field $B|\hat{z}$ spin degeneracy is lifted and components of the spin $S|\hat{z}$ and $S \perp B$ will have different propagators. It is obvious that only transverse components of the spin will contain information about level splitting at $\omega_0 = \omega_L$. We have therefore focused on $S^\perp, S^z$ components only. The propagator in imaginary time $\tau$ is $\chi(\tau) = (T, S^z(\tau) S^z(0))$ with Fourier transform and continuing to real frequency $\chi(\omega) = G(\omega) = \omega/(\omega^2 - (\omega - \omega_L)^2)$. For free spin we have $\langle S_z \rangle = \tanh(\omega_0/2T)/2$. For more general case of magnetic anisotropy this does not have to be the case. To be general we will keep $\langle S_z \rangle$.

We begin with evaluation of the DOS correction due to coupling to localized spin. Self-energy correction is:

$$\Sigma(\omega) = J^2 T \sum_{k,n} G(k, \omega - \Omega_n) \chi^+(\Omega_n),$$  

(4)

where $G^0(k, \omega_n) = [i \omega_n - \epsilon(k)][(i \omega_n)^2 - \epsilon^2(k) - \Delta^2(k)]^{-1}$ is the particle Green’s function in d-wave superconductor, $G^{-1} = G^0(0)^{-1} - \Sigma, F_0^0(k, \omega_n) = [\Delta(k)][(i \omega_n)^2 - \epsilon^2(k) - \Delta^2(k)]^{-1}$; $\Omega_l = 2\pi l T$ is the bosonic Matsubara frequency and $\omega_L = (2l + 1)\pi T, l = 0, 1, 2,...$ is the fermionic frequency. Using spectral representation and analytical continuation onto real axis $i \omega_n \rightarrow \omega + i\delta$ we find for imaginary part of self energy $\Sigma(\omega)$:

$$\text{Im}\Sigma(\omega) = -J^2 \langle S_z \rangle \text{Im}G(\omega - \omega_0)[n_F(\omega - \omega_0) - n_B(\omega_0) - 1],$$  

(5)

where $n_F(\omega) = 1/[1 + \exp(\beta\omega)], n_B(\omega) = 1/[\exp(\beta\omega) - 1]$ are Fermi and Bose distribution functions. This local self-energy leads to the modifications of the DOS. In this solution we treat the self-energy effects in G to all orders, i.e. $G$ in Eq. (4) is full Green’s function $G^{-1} = G_0^{-1} - \Sigma(\omega)$ and solution for $\Sigma$ is found self-consistently for a local vibrational mode. The modifications of the superconducting order parameter and bosonic propagator were ignored in this calculation. Results are presented in Fig. 4. To proceed with analytic treatment, unless stated otherwise, we limit ourselves below to second order scattering in $\Sigma$. Difference between self-consistent solution and second order calculation are only quantitative and small for small coupling. Corrections to the Green’s function $G(\mathbf{r}, \mathbf{r}', \omega) = G^0(\mathbf{r}, \mathbf{r}', \omega) + G^0(\mathbf{r}, 0, \omega)\Sigma(\omega)G^0(\mathbf{r}, 0, \omega) + F_0^0(\mathbf{r}, 0, \omega)\Sigma(\omega)F_0^0(0, \mathbf{r}, \omega)$. For simplicity we define $K(T, \omega, \omega_0) = -[n_F(\omega - \omega_0) - n_B(\omega_0) - 1 \approx \Theta(\omega - \omega_0)]$ which becomes a step function at low $T \ll \omega_0$, the limit we will focus on hereafter. Correction to the local density of states as a function of position comes from the correction to the bare Green’s function $G^0 : \delta N(\mathbf{r}, \omega) = 1/\pi \text{Im}G^0(\mathbf{r}, 0, \omega)\Sigma(\omega)G^0(0, \mathbf{r}, \omega) + F_0^0(\mathbf{r}, 0, \omega)\Sigma(\omega)F_0^0(0, \mathbf{r}, \omega)$, where keeping it general, the plus sign corresponds to the coupling to the local vibrational mode and minus – to the spin scattering respectively. The strongest effect will be at the impurity
site. For on-site density of states we have:
\[
\frac{\delta N(r=0, \omega)}{N_0} = \frac{\pi^2}{2} (JSN_0)^2 \frac{\omega - \omega_0}{\Delta} K(T, \omega, \omega_0)
\times \left( \frac{2\omega}{\Delta} \ln \left( \frac{\Delta}{\omega} \right) \right)^2, \quad \omega \ll \Delta,
\]
(6)
\[
\frac{\delta N(r=0, \omega)}{N_0} = 2\pi^2 (JSN_0)^2 K(T, \omega, \omega_0) \ln^2 \left( \frac{|\omega - \Delta|}{4\Delta} \right)
\times \ln \left( \frac{4\Delta}{|\omega + \omega_0 - \Delta|} \right) + (\omega_0 \rightarrow -\omega_0), \quad \omega \simeq |\Delta|,
\]
(7)
where we used for on-site Green’s function \(G^0(0,0,\omega) = N_0 \left( \frac{2\omega}{\Delta} \ln \left( \frac{\Delta}{\omega} \right) + i\pi \frac{\omega}{\Delta} \right) \), for \(\omega \ll \Delta\) and we retained only dominant real part of \(G^0\). In opposite limit \(\omega \simeq \Delta\) we retained only dominant imaginary part of \(G^0(0,0,\omega) = -2iN_0 \ln \left( \frac{\omega - \Delta}{\Delta} \right) \). At \(r = 0\) we have \(F^0(0,0,\omega, \omega_0) = 0\). Complete DOS \(N(\omega)\) and derivative \(\frac{\delta N(\omega)}{\delta \omega}\) are shown on Fig. 1. For arbitrary position \(N(r, \omega)\) we would have to add a Friedel oscillation factor \(\Lambda(r) = \left\{ |G^0(r, \omega)|^2 \pm |F^0(r, \omega)|^2 \right\} \sim \frac{\sin(kFr)}{(kF r)^{1/2}}\) that describes the real space dependence of the Green’s function on distance for small \(\omega \ll \Delta\). Here \(F_{\perp} = \langle \mathbf{F}_{\perp} \rangle\) is the component of \(r_{\perp} = (r_{\perp}, r_{\parallel})\) that is along the Fermi surface near the nodal point of the gap and \(r_{\parallel} \parallel \mathbf{k}_{F\parallel}\) is the component perpendicular to the Fermi surface at nodal point. Existence of the nodes in d-wave case results in the power law decay of \(\Lambda(r)\) in all directions and it has a four fold modulation due to gap anisotropy (See detailed discussions in PRB ’97 reference in [11]). The final result is our Eqs. (11, 12).

It follows immediately that
\[
\delta \frac{dI}{dV} \sim \delta N(r=0, \omega)/N_0 \sim (JSN_0)^2 \frac{V - \omega_0}{\Delta} \Theta(V - \omega_0),
\]
\[
\delta \frac{d^2I}{dV^2} \sim (JSN_0)^2 \Theta(V - \omega_0).
\]
(8)
Here we have used the fact that the derivative of \((\omega - \omega_0)\Theta(\omega - \omega_0)\) with respect to \(\omega\) yields \(\Theta(\omega - \omega_0)\). Thus in a d-wave superconductor and in a metal with vanishing DOS \(N(\omega) = N_0 \frac{2\omega}{\Delta}\) one should expect a step discontinuity in \(d^2I/dV^2\) at the energy of a local mode with the strength \(J^2N_0^2\) (see Fig. 1). This result is qualitatively different from the case of conventional metal. For metal with energy independent DOS we have from Eq. (6) for \(T \ll \omega_0\)
\[
d\frac{dI}{dV} \sim \delta N(r=0, \omega) \sim J^2N_0^2 \Theta(V - \omega_0),
\]
(9)
and the second derivative will reveal a delta function \(d^2I/dV^2 \sim J^2N_0^2 \delta(\omega - \omega_0)\) The effect in d-wave superconductor is clearly smaller than correction to DOS in a normal metal with the same coupling strength.

For completeness we also have calculated the effect of inelastic scattering in a metal with the more general DOS
\[
N(\omega) = 1/\pi \text{Im} G^0(0,0,\omega) = (\omega/\Delta)^{\gamma} N_0 \quad \text{with power } \gamma > 0,
\]
that is determined by the microscopic properties of the material. Then, from Eqs. (6, 8) we have for \(\omega \ll \Delta\)
\[
\delta \frac{dI}{dV} \sim \delta N(r=0, \omega)/N_0 \sim (V - \omega_0)\gamma \Theta(V - \omega_0),
\]
\[
\delta \frac{d^2I}{dV^2} \sim (V - \omega_0)^{\gamma - 1} \Theta(V - \omega_0).
\]
(10)
Depending on the value, we get divergent singularity at \(\omega_0\) for \(\gamma < 1\), or a power law rise for \(\gamma \geq 1\). In case of \(\gamma = 1\) we recover the result for d-wave superconductor and for a pseudogap normal state.

Quite generally one can express the results in terms of the spectrum of superconductor. We can write \(\text{Im} \Sigma(\omega)\) using spectral representation for \(G(r, \omega)\). In superconducting case, using Bogoliubov \(u_\alpha(r), v_\alpha(r)\) for eigenstate \(\alpha\), we have \(G(r, \omega) = \sum_\alpha \left[ |u_\alpha(r)|^2 + |v_\alpha(r)|^2 \right]\). Taking imaginary part of \(G(r, \omega)\) we arrive for \(T \ll \omega_0\) at:
\[
\text{Im} \Sigma(\omega) = \frac{\pi J^2}{2\omega_0} (S_2) |u_\alpha(r=0)|^2 \delta(\omega - \omega_0 - E_\alpha)
\]
\[+ |v_\alpha(r=0)|^2 \delta(\omega - \omega_0 + E_\alpha) \quad \text{, } \omega > 0. \]
(11)
At negative \( \omega < 0 \) one has to replace \( \omega_0 \to -\omega_0 \) in Eq.\(^{[11]}\). For example, consider a magnetic impurity resonance in d-wave superconductor at energy \( \omega_{\text{imp}} \), such as a Ni induced resonance \(^{[11,12]}\). Then only the term with resonance level \( E_0 = E_{\text{imp}} \) will dominate the sum over eigenstates \( \alpha \) in the vicinity of impurity site. Inelastic scattering off this impurity induced resonance will produce additional satellite \textit{split away from the impurity level by} \( \omega_0 \), see Fig.\(^2\). Sharp coherence peaks will also produce split satellites. Again, for a local phonon mode one gets a similar splitting of impurity level with \( \omega_0 \) now being the phonon energy.

Our results suggest the possibility of \textit{single spin detection} as one monitors the feature in \( d^2I/dV^2 \) as a function of position and external magnetic field. If we take experimentally seen DOS \( N_0 \approx 1/eV \) with \( JN_0 \approx 0.14, \Delta = 30meV \)\(^{[12]}\) and assuming the field of \( \sim 10T \) we have \( \omega_0 = 1meV \) (corresponding to the Zeeman splitting of \( \sim 1meV \) in a magnetic field, we have from Eqs.\(^{[11,12]}\)

\[
\delta N(r = 0, \omega)/N_0 \approx 10^{-2} \frac{\omega - \omega_0}{\Delta} \Theta(\omega - \omega_0). \tag{12}
\]

We point out here that result is expressed in terms of the relative change of DOS of a metal \( N_0 \). For observation of this effect one would have to sample DOS in the vicinity of \( eV = \omega_0 \propto B \). Assuming \( \omega - \omega_0 = \omega_0 \) we have from Eq.\(^{[12]}\) \( \delta dI/dV / dV \sim 10^{-2} \). Expressed as a relative change of DOS of a superconductor \( N(\omega) = N_0/\omega/\Delta \) effect is: \( \delta dI/dV / dV \sim \delta N(r = 0, \omega)/N(\omega_0) \sim 10^{-2} \frac{\omega_0}{\omega} \Theta(\omega - \omega_0). \) It is of the same order of magnitude as the observed vibrational modes of localized molecules in inelastic electron tunneling spectroscopy STM, IETS-STM \(^{[1]}\). The satellites at \( \Delta + \omega_0 \) produce the effect on the scale of unity and clearly seen even for small coupling. The important difference is that for localized spin the kink in DOS is \textit{tunable} with magnetic field and this should make its detection easier.

In conclusion, we propose the extension of the inelastic tunneling spectroscopy on the strongly correlated electrons states, such as a d-wave superconductor and pseudogap normal state. The DOS in these systems has a nontrivial energy dependence of general form \( N(\omega) \sim \omega^2, \gamma > 0 \). This technique could allow for a Zeeman level spectroscopy of a single magnetic center, thus, in principle, allowing a single spin detection. We find the feature in \( dI/dV \sim (\omega - \omega_0)^{\gamma-1} \Theta(\omega - \omega_0) \) near the threshold energy \( \omega_0 \). We also find strong satellite features near the gap edge due to coherence peak for a superconducting case. The singularity is a power law and qualitatively different from the results for a simple metallic DOS \(^{[1]}\). For the relevant values of parameters for high-Tc the feature is on the order of several percents and makes the feature observable in these materials. Similar predictions are also applicable to the local vibrational modes, where \( \omega_0 \) becomes a vibrational mode frequency.

Acknowledgments: This work was supported by the US Department of Energy. We are grateful to S. Davis, conversations with whom initiated this research. We are grateful to A. Chubukov, E. Hudson, Y. Manassen and D. Scalapino for useful discussions. Ar. A. was supported by LDRD 200153, AVB and JXZ were supported by LDRD X1WX at Los Alamos.



\[\text{References}\]

\[1\] B. C. Stipe, M. A. Rezaei, and W. Ho, Science \textbf{280}, 1732 (1998); J. R. Hahn and W. Ho, Phys. Rev. Lett. \textbf{87}, 166102 (2001) and referrences therein.
\[2\] R.C.Jaklevich and J. Lambe, Phys. Rev. Lett., \textbf{17}, 1139, (1966); D.J. Scalapino and S. M. Marcus, Phys. Rev. Lett., \textbf{18}, 459, (1967).
\[3\] J. Appelbaum, Phys. Rev. \textbf{154}, 633, (1967); J. Appelbaum, Phys. Rev. Lett. \textbf{17}, 91, (1966).
\[4\] H. C. Manoharan, C. P. Lutz, and D. M. Eigler, Nature \textbf{403}, 512 (2000). M. F. Crommie, C. P. Lutz and D. M. Eigler, Science \textbf{262}, 218, (1993); V. Madhavan, W. Chen, T. Jameela, M. F. Crommie, and N. S. Wingreen, Science \textbf{280}, 567, (1998); M. Bode, M. Getzlafl, and R. Wiesendanger Phys. Rev. Lett. \textbf{81}, 4256 (1998).
\[5\] A. Yazdani, B. A. Jones, C. P. Lutz, M. F. Crommie, and D. M. Eigler, Science \textbf{275}, 1767 (1997).
\[6\] B. E. Kane, Nature \textbf{393} 133 (1998).
\[7\] D. Loss and D. P. DiVincenzo, Phys. Rev. A \textbf{57}, 120 (1998).
\[8\] J. R. Hahn and W. Ho, Science \textbf{280}, 1732 (1998); J. R. Hahn and W. Ho, Phys. Rev. Lett. \textbf{87}, 166102 (2001) and referrences therein.
\[9\] R.C.Jaklevich and J. Lambe, Phys. Rev. Lett., \textbf{17}, 1139, (1966); D.J. Scalapino and S. M. Marcus, Phys. Rev. Lett., \textbf{18}, 459, (1967).
\[10\] J. Appelbaum, Phys. Rev. \textbf{154}, 633, (1967); J. Appelbaum, Phys. Rev. Lett. \textbf{17}, 91, (1966).
\[11\] H. C. Manoharan, C. P. Lutz, and D. M. Eigler, Nature \textbf{403}, 512 (2000). M. F. Crommie, C. P. Lutz and D. M. Eigler, Science \textbf{262}, 218, (1993); V. Madhavan, W. Chen, T. Jameela, M. F. Crommie, and N. S. Wingreen, Science \textbf{280}, 567, (1998); M. Bode, M. Getzlafl, and R. Wiesendanger Phys. Rev. Lett. \textbf{81}, 4256 (1998).
\[12\] A. Yazdani, B. A. Jones, C. P. Lutz, M. F. Crommie, and D. M. Eigler, Science \textbf{275}, 1767 (1997).
\[13\] B. E. Kane, Nature \textbf{393} 133 (1998).
\[14\] D. Loss and D. P. DiVincenzo, Phys. Rev. A \textbf{57}, 120 (1998).
\[15\] J. R. Hahn and W. Ho, Science \textbf{280}, 1732 (1998); J. R. Hahn and W. Ho, Phys. Rev. Lett. \textbf{87}, 166102 (2001) and referrences therein.
\[16\] R.C.Jaklevich and J. Lambe, Phys. Rev. Lett., \textbf{17}, 1139, (1966); D.J. Scalapino and S. M. Marcus, Phys. Rev. Lett., \textbf{18}, 459, (1967).
\[17\] J. Appelbaum, Phys. Rev. \textbf{154}, 633, (1967); J. Appelbaum, Phys. Rev. Lett. \textbf{17}, 91, (1966).
\[18\] H. C. Manoharan, C. P. Lutz, and D. M. Eigler, Nature \textbf{403}, 512 (2000). M. F. Crommie, C. P. Lutz and D. M. Eigler, Science \textbf{262}, 218, (1993); V. Madhavan, W. Chen, T. Jameela, M. F. Crommie, and N. S. Wingreen, Science \textbf{280}, 567, (1998); M. Bode, M. Getzlafl, and R. Wiesendanger Phys. Rev. Lett. \textbf{81}, 4256 (1998).
\[19\] A. Yazdani, B. A. Jones, C. P. Lutz, M. F. Crommie, and D. M. Eigler, Science \textbf{275}, 1767 (1997).
\[20\] B. E. Kane, Nature \textbf{393} 133 (1998).
\[21\] D. Loss and D. P. DiVincenzo, Phys. Rev. A \textbf{57}, 120 (1998).
\[22\] J. R. Hahn and W. Ho, Science \textbf{280}, 1732 (1998); J. R. Hahn and W. Ho, Phys. Rev. Lett. \textbf{87}, 166102 (2001) and referrences therein.
Phys. Rev. Lett. 83, 4381 (1999); A. Polkovnikov, S. Sachdev, and M. Vojta, Phys. Rev. Lett. 86, 296 (2001).

[9] To minimize the orbital effect of magnetic field one can apply it parallel to the surface of superconductor. The magnetic field is penetrating the surface sheath on the scale of penetration depth so that its effect on the conduction electrons for d-wave SC is small.

[10] D. Withoff and E. Fradkin, Phys. Rev. Lett. 64, 1835, (1990); C.R. Casanello and E. Fradkin, Phys. Rev. B 53, 15079, (1996); C. Gonzales-Buxton and K. Ingersent, Phys. Rev. B 57, 14254, (1998); M. Vojta and R. Bulla, Phys. Rev. Lett. B 65, 014511, (2002); K. Ingersent and Q. Si, Phys. Rev. Lett, to be published.

[11] A.V. Balatsky, M.I. Salkola, and A. Rosengren, Phys. Rev. B 51, 15547 (1995); M.I. Salkola, A.V. Balatsky, and J.R. Schrieffer, Phys. Rev. B 55, 12648 (1997); M. E. Flatté, Phys. Rev. B 61, R14920 (2000); H. Tsuchiura, Y. Tanaka, M. Ogata and S. Kashiwaya, Phys. Rev. Lett. 84, 3165 (2000).

[12] E. Hudson et al., Nature 411, 920 (2001).