Leading term of the He-\(\bar{p}3\)He\(^+\) long-range interaction

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The long-range interaction between an antiprotonic helium atom \(\bar{p}3\)He\(^+\) and helium atom in its ground state is studied. We calculate the dispersion coefficients \(C_6\) using the complex coordinate rotation (CCR) formalism in order to comply with the resonant nature of metastable states of the antiprotonic helium. We present as well numerical data on static dipole polarizabilities of antiprotonic helium states. The obtained coefficients \(C_6\) may be used to estimate the collisional shift and broadening of transition lines in a low-density precision spectroscopy of the antiprotonic helium.

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I. INTRODUCTION

In 1991 in an experiment at KEK it was discovered that some fraction of antiprotons in a helium target survive for an unexpectedly long time [1]. Later in a series of experiments at CERN [2] it has been shown that such antiprotons form an exotic atom, antiprotonic helium, or \(\bar{p}3\)He\(^+\). An antiproton stopped in the target is then captured into some atomic state in a helium atom via a charge exchange reaction replacing one of the two electrons. The major part of such states disappears promptly due to annihilation of the antiproton on a nucleus, while a small fraction still survives, making up a set of metastable states, which decay predominantly via slow radiative transitions.

Further precision studies of these atoms by laser spectroscopy [3–5] reveal the possibility of investigating various properties of the antiprotonic helium atoms as well as inferring precise data on the antiprotons [6,7]. Particularly, it provides a stringent test of CPT invariance in a barion sector. Otherwise, assuming the validity of CPT invariance, one may extract the entangled test of CPT invariance in a barion sector. Otherwise, assuming the validity of CPT invariance, one may extract the

\[m_{\bar{p}}/m_e = 1.836.152.673(6)(23)[9]\]

which was carried out using two-photon Doppler reduced laser spectroscopy. This level of precision the collisional effects as well as Stark effects become important contributions to the total experimental error (see Table 2 in Ref. [9]).

The density shift and broadening were measured for antiprotonic helium spectral lines with \(T = 6\) K [5,6,11]. Good qualitative description of the experimental data were obtained in Ref. [18] using effective model potential of \(\bar{p}3\)He\(^+\)-He long-range attraction and short-range repulsion interactions. Quantitative agreement with the experimental data was achieved in Ref. [19] by direct \(ab\) initio calculation of an interatomic potential in the Born–Oppenheimer approximation.

Recently the ASACUSA Collaboration at CERN [20] announced that they want to slow down the pulsed antiproton beam up to 150 eV and cool down the experimental target cell to \(T \leq 1.5\) K. For precision spectroscopy beyond the 1-ppb level the long-range interaction between antiprotonic helium and ground-state helium atoms is of great importance for proper evaluation of collisional effects and their influence on the experimentally observed spectral lines.

In our work we intend to calculate static dipole polarizabilities for metastable states in \(\bar{p}3\)He\(^+\) and \(\bar{p}4\)He\(^+\) atoms. We use the complex coordinate rotation (CCR) formalism [21], which allows us to take into account in a proper way the resonant nature of the metastable states in the antiprotonic helium. Finally, the dispersion coefficients \(C_6\) will be evaluated numerically using the same CCR formalism. These coefficients determine the leading contribution to the long-range He-\(\bar{p}3\)He\(^+\) interaction. Here we assume that all helium atoms of the target are in the ground state.

Atomic units (\(\hbar = e = m_e = 1\)) are used throughout this paper.

II. THEORY

A. Wave functions of He and \(\bar{p}3\)He\(^+\)

Both atoms, the usual helium and antiprotonic helium, are three-body systems with Coulomb interaction and will be
considered using the same variational expansion. The strong interaction between $\bar{p}$ and the helium nucleus is strongly suppressed by the centrifugal barrier (the angular momentum of an antiproton orbital $l \approx 34$) and may be completely neglected.

The nonrelativistic Hamiltonian of a three-body system is taken in a form

$$\mathbf{H} = -\frac{1}{2\mu_1} \nabla^2 r_1 + \frac{1}{2\mu_2} \nabla^2 r_2 - \frac{1}{M} \nabla \cdot \mathbf{V}_r - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_1 r_2} + \mathbf{r}_1 \cdot \mathbf{r}_2.$$  \hfill (1)

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors for two negative particles, $\mathbf{r}_1 = \mathbf{r}_1 - \mathbf{r}_2$, $\mu_1 = Mm_1/(M + m_1)$ and $\mu_2 = Mm_2/(M + m_2)$ are reduced masses, $M$ is a mass of helium nucleus, and the nucleon charge is $Z = 2$. We assume that in case of helium atom $m_1 = m_2 = 1$ and masses of electrons, while for the antiprotonic helium we set $m_1 = m_{\bar{p}}$ and $m_2 = 1$, where $m_{\bar{p}}$ is a mass of an antiproton.

The helium atoms are in their ground state. Antiprotonic helium is a more complicated object. It presents a quasidiabatic system with a heavy antiproton orbiting over helium nucleus with a velocity about 40 times slower than a remaining electron. Using atomic terminology, the electron occupies its state. Complex parameters $L_{L,L}$ are some prime numbers, and $[A_1, A_2] = \{a_{L-1} + a_L + a_{L+1}\}$.

In the CCR approach the coordinates of the dynamical system are rotated to some angle $\varphi$, parameter of the complex rotation $r_{ij} \rightarrow r_{ij}e^{i\varphi}$. Under this transformation the Hamiltonian changes as a function of $\varphi$

$$H_{\varphi} = Te^{-2i\varphi} + Ve^{-i\varphi},$$  \hfill (4)

where $T$ and $V$ are the kinetic energy and Coulomb potential operators. The continuum spectrum of $H_{\varphi}$ is rotated on the complex plane around branch points (thresholds) to uncover resonant poles situated on the unphysical sheet of the Riemann surface. The resonance energy is then determined by solving the complex eigenvalue problem for the rotated Hamiltonian

$$(H_{\varphi} - E)\Psi_\varphi = 0.$$  \hfill (5)

The eigenfunction $\Psi_\varphi$ obtained from Eq. (5), is square integrable and the corresponding complex eigenvalue $E = E_r - i\Gamma/2$ defines the energy $E_r$ and the width of the resonance, $\Gamma$, the latter is being related to the Auger rate as $\lambda_A = \Gamma/\hbar$.

### B. Static dipole polarizability

The static dipole polarizability tensor operator, which is a tensor of rank 2, on a subspace of fixed total angular momentum $L$ can be represented [26] by a scalar, $\alpha_s$, and irreducible tensor, $\alpha_t$, operators:

$$\hat{\alpha}_d^{L}(n,l) = \alpha_s(n,l) \delta_{ij} + \alpha_t(n,l) \left[ \hat{L}_i \hat{L}_j + \hat{L}_j \hat{L}_i - \frac{2}{3} \delta_{ij} \hat{L}_i \hat{L}_j \right].$$  \hfill (6)

We use notation $\hat{\delta}$ to distinguish between operators and $c$-numbers. The coefficients $\alpha_s$ and $\alpha_t$ then may be expressed in terms of three contributions corresponding to the possible values of the angular momentum of intermediate state, $L' = L, L \pm 1$ (see Ref. [27] for details),

$$\alpha_s = \frac{1}{3} (a_{L-1} + a_L + a_{L+1}),$$  \hfill (7a)

$$\alpha_t = - \frac{a_{L-1}}{2L(L-1)} + \frac{a_L}{2L(L+1)} - \frac{a_{L+1}}{2(L+1)(2L+3)},$$  \hfill (7b)

and $a_{L'}$ can be calculated by summing up the oscillator strengths as follows:

$$a_{L'} = 3 \sum_{n' \frac{1}{2}} f_{n(n')}^{(L',L)} \left( E_{n'} - E_{n} \right)^2,$$  \hfill (8)

where the $2'$-pole averaged oscillator strength is defined in terms of reduced matrix elements by the expression

$$f_{n(n')}^{(L',L)} = \frac{8\pi}{(2L+1)^2(2L+1)} \left| \langle \Psi_{n'} | \sum_i r_i Z_i \nabla_i \Psi_n \rangle \right|^2.$$  \hfill (9)

For the CCR calculations we use a modified version of the perturbation theory provided by the theorem [28].

**Theorem.** Let $H$ be a three-body Hamiltonian with Coulomb pairwise interaction and $W(\theta)$ be a dilatation analytic small perturbation of a complex parameter $\theta$ (for the CCR we choose $\theta = i\varphi$). Let $E_0$ be an isolated simple resonance energy...
Table I. Test of convergence of the CCR calculations for the dipole scalar, \( \alpha_s \), and tensor, \( \alpha_t \), polarizability. The (31,30) state of \(^{4}\text{He}^{+} \) is studied. The last line is the Feshbach closed-channel calculation. Here for simplicity we use the same number of basis functions \( N \) for the initial and all \((L' = L, L \pm 1)\) intermediate states.

| \( N \) | \( \alpha_i \) | \( \alpha_t \times 10^3 \) |
|------|-------|------------|
| 2200 | 2.00311 + 0.000531 | 0.176280 - 0.000441 |
| 3400 | 2.003011 + 0.000424 | 0.176341 - 0.000342 |
| 4400 | 2.003030 + 0.000455 | 0.176332 - 0.000384 |
| 5400 | 2.003033 + 0.000453 | 0.176330 - 0.000382 |
| \( \infty \) | 2.003033(1) + 0.000453 | 0.176330(1) - 0.000382 |
| 1000 | 2.0031(4) | 0.1762(4) |

(denote eigenvalue of \( H(\theta) \)). Then for \( \beta \) small, there is exactly one eigenstate of \( H(\theta) + \beta W(\theta) \) near \( E_0 \) and

\[
E(\beta) = E_0 + a_1 \beta + a_2 \beta^2 + \cdots \tag{10}
\]

is analytic near \( \beta = 0 \). In particular,

\[
\begin{align*}
a_1 &= \langle \Psi_0(\theta)|W(\theta)|\Psi_0(\theta)\rangle, \\
a_2 &= \sum_{n \neq 0} \frac{\langle \Psi_0(\theta)|W(\theta)|\Psi_n(\theta)\rangle \langle \Psi_n(\theta)|W(\theta)|\Psi_0(\theta)\rangle}{E_n - E_0(\theta)}, \tag{11}
\end{align*}
\]

where the sum is carried out over the states of discrete and continuum spectra of the rotated Hamiltonian \( H(\theta) \).

It is assumed that the wave functions are normalized as \( \langle \Psi^*(\theta), \Psi(\theta) \rangle = 1 \). Coefficients \( a_1, a_2, \) etc. do not depend on \( \theta \) if only rotated branches of the continuum spectrum of \( H(\theta) \) uncover \( E_0 \) and its vicinity on the complex plane. These coefficients are complex and the imaginary part contributes to the width of the resonance, to the imaginary part of the complex energy of Eq. (5) \( E = E_r - i \Gamma/2 \), as follows from Eq. (10).

In Table I convergence of the (31,30) state of \(^{4}\text{He}^{+} \) is studied. This state is of much importance as a daughter state for the two-photon precision spectroscopy of the (33,32) \rightarrow (31,30) transition.

C. Dispersion coefficients

The long-range interaction between two neutral atoms can be expanded in terms of a series of inverse powers of the separation distance \( R \)

\[
U(R) = -\frac{C_6}{R^6} - \frac{C_8}{R^8} + \cdots , \tag{12}
\]

where \( C_6, C_8, \) etc., are the dispersion coefficients. For two like atoms that are not both in their ground states, the perturbation theory of the dispersion coefficients has been well discussed in Refs. [31,32]. In the case of two different neutral atoms, the dispersion coefficients may be derived similarly in the frame of the perturbation theory.

Let us consider a dimer system composed of He in its ground state \((L_{16e} = 0, M_{16e} = 0)\) and \(^{4}\text{He}^{+} \) in its excited state \((n, L, M)\). The electric interaction potential between He and \(^{4}\text{He}^{+} \) at large separation \( R \) can be expressed as a multipole harmonic expansion [31–33]

\[
V(R; 1,2) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{V_{l+1}}{R^{l+1}} , \tag{13}
\]

where

\[
V(R; 1,2) = \sum_{l,j=1}^{3} \frac{Z_{1l}Z_{2j}}{|(\mathbf{r}_{2j} + \mathbf{R}) - \mathbf{r}_{1l}|} , \tag{14}
\]

and \( \mathbf{r}_{1l} \) and \( \mathbf{r}_{2j} \) are the center-of-mass position vectors of the three particles for the helium and antiprotonic helium atoms, respectively, and \( Z_{1l} \) and \( Z_{2j} \) are the charges of particles for corresponding atoms.

If the \( z \) axis of frames of both atoms is taken along \( \mathbf{R} \), the expression for \( V_{l+1} \) may be written explicitly,

\[
V_{l+1} = \sum_{m} \frac{(-1)^l (4\pi)(l+1)!}{\sqrt{(2l+1)(2l+1)(2l+1)}} \frac{1}{l(l+1)!} \mathcal{M}^{[1m]}_{l+1} \mathcal{M}^{[2-l]}_{l+1} , \tag{15}
\]

where \( \mathcal{M}^{[m]}_{l} \) are the multipole moments of an atom:

\[
\mathcal{M}^{[m]}_{l} = \sum_{i} Z_{il} r_{il}^{m} \mathbf{l}_{il}^{m} , \tag{16}
\]

In Eq. (15) and in what follows, superscripts in square brackets denote a particular subsystem (or atom), namely, 1 stands for the helium atom and 2 is for the antiprotonic helium.

Let \( \Psi(1,2) \) be an eigenfunction of the interacting system:

\[
[H_0 + V(1,2)]\Psi(1,2) = E \Psi(1,2) . \tag{17}
\]

Then assuming that \( V(1,2) \) is small, one may use the following expansion:

\[
\Psi(1,2) = \sum_{n=0}^{\infty} \Psi_n(1,2) , \quad E(1,2) = \sum_{n=0}^{\infty} E_n(1,2) , \tag{18}
\]

where the zeroth-order wave function can be written as a product of two individual atomic wave functions

\[
\Psi_0(1,2) = \Psi_0^{[1]} \Psi_0^{[2]} , \tag{19}
\]

and the associated state energy is \( E_0 = E_0^{[1]} + E_0^{[2]} \).
Substituting expansions (18) into Eq. (17), one obtains a set of equations:

\[
\begin{align*}
(H_0 - E_0)\Psi_1 + (V(1,2) - E_1)\Psi_0 &= 0, \\
(H_0 - E_0)\Psi_2 + (V(1,2) - E_2)\Psi_1 - E_2\Psi_0 &= 0, \\
&\quad \ldots \quad .
\end{align*}
\]

Equation (20) can be simplified by writing

\[
\Psi_1 = \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} \frac{\Omega_{l_1l_2}}{R^{1+l_1+l_2+1}},
\]

where \( L \) is the total orbital angular momentum of the antiprotonic helium state, while index \( i \) runs over the \( P \) states of the helium atom.

### III. CALCULATION AND RESULTS

Using Eqs. (7) and (8), one can get the static dipole polarizability (scalar, \( \alpha_s \), and tensor part, \( \alpha_t \)) for metastable states of \( \bar{p}^4\text{He}^+ \). In order to check the validity of our calculations, we use the generalized Thomas-Reiche-Kuhn (TRK) sum rule for the oscillator strengths developed by Yan and coworkers [34]. For \( \bar{p}^4\text{He}^+ \), the left-hand side \( \mathcal{L} \) and right-hand side \( \mathcal{R} \) of the TRK equality are expressed as follows:

\[
\mathcal{L} = \sum_{n_i} \beta_n^{(1)}, \quad \mathcal{R} = \frac{4}{m_{\bar{p}^4\text{He}^+}} + \frac{1}{m_p} + 1.
\]

A test of convergence of \( \mathcal{L} \) and a comparison with the exact value, \( \mathcal{R} \), for the (36,35) state of \( \bar{p}^4\text{He}^+ \) are listed in Table II, which demonstrates reliability of our calculations. In the numerical results of this section the CODATA10 [10] recommended values were adopted.

Tables II and III provide tests of convergence of \( \alpha_s \) and \( C_6(M) = 0 \) for the two kind of states: the metastable (36,35) state \( \bar{p}^4\text{He}^+ \) decaying via slow radiative transition and the (32,31) state \( \bar{p}^4\text{He}^+ \), where the Auger rate becomes dominant. Appearance of the imaginary part in the data and the physical meaning of it has been clarified in Sec. IIIB right after the theorem. As is seen from Table III, the imaginary part is substantially large in the case of polarizability of the (31,30) state. That may be explained by strong correlation with broad short-lived states having excited electronic configuration and lying in a vicinity of the (31,30) state on the Reimann surface of complex energy (see Ref. [2], Sec. 4.8 and discussion below).

It is also important to note that the dispersion coefficients, \( C_6(M) \), are much less affected by this phenomenon and the imaginary part may be ignored. Since the finite mass of a helium nucleus produce a visible effect on \( C_6(M) \) we explicitly compare the \( C_6(M) \) for \( ^4\text{He} - \bar{p}^4\text{He}^+ \) and \( ^6\text{He} - \bar{p}^4\text{He}^+ \) in the tables. A choice of a finite or infinite mass for the helium atom causes changes in the fourth decimal place in the \( C_6 \) coefficient. That is the essential effect and in our final calculations of the dispersion coefficients we use the wave functions for the helium atom obtained with the finite mass of a nucleus.

Numerical calculations of the dipole polarizabilities (\( \alpha_s \) and \( \alpha_t \)) for metastable states in the antiprotonic helium are presented in Table IV. Results were obtained with the use of the CCR method, and thus the final values have an imaginary part. Some of the states, particularly the (37,33) state in \( \bar{p}^4\text{He}^+ \), have anomalously large polarizability, which makes the states unstable against collisions. Such anomalous behavior is again connected with the excited electron Rydberg states, which strongly affect the overall polarizability of the atom.

Yet another, maybe less obvious example, is the (32,31) state in \( \bar{p}^4\text{He}^+ \) atom. This state, as was observed in experiment [35], at large densities become unstable. The most apparent explanation of this phenomena is a considerable (by a factor

\[
(H_0 - E_0)\Omega_{l_1l_2} + (V_{l_1l_2} - \epsilon^{(1)}_{l_1l_2})\Psi_0 = 0,
\]

with

\[
\epsilon^{(1)}_{l_1l_2} = \langle \Psi_0 | V_{l_1l_2} | \Psi_0 \rangle.
\]

For neutral atoms, of which one is in the ground \( S \) state, the first-order energy vanishes, \( E_1 = 0 \) [32]. The second-order term is then expressed as

\[
E_2(1,2) = \sum_{l_1=1}^{\infty} \sum_{l_2=1}^{\infty} \frac{\epsilon^{(2)}_{l_1l_2}}{R^{2(l_1+l_2+1)}}
\]

and

\[
\epsilon^{(2)}_{l_1l_2} = \langle \Psi_0 | V_{l_1l_2} | \Omega_{l_1l_2} \rangle.
\]

Now the dispersion coefficient \( C_6 \) may be written explicitly as

\[
C_6 = \epsilon^{(2)}_{11} = \sum_{i,j} \frac{|(0)|V(1)_{ij}|^2}{E_{ij} - E_{00}} = \frac{1}{3} \left( \frac{4\pi}{3} \right)^2 \sum_{i,n} (0)M^{[1]}_{1i} |i|^2 (0)L|A^{[1]}_{11}|n(L+1)|^2 (L + 1)(5L + 6) - 3M^2 \]

\[
(L + 1)(2L + 1)(2L + 3)
\]

\[
+ \sum_{i,n} (0)M^{[1]}_{1i} |i|^2 (0)L|A^{[2]}_{11}|n(L)|^2 (L + 1) + 3M^2 \]

\[
(L + 1)(2L + 1)(2L + 3)
\]

\[
+ \sum_{i,n} (0)M^{[1]}_{1i} |i|^2 (0)L|A^{[2]}_{11}|n(L - 1)|^2 (L + 1) + 3M^2 \]

\[
(L + 1)(2L - 1)(2L + 1)
\]

\[
(25)
\]
TABLE II. Test of convergence for the TRK sum rule, $\mathcal{E}$, the static scalar dipole polarizability $\alpha_s$, and dispersion coefficient $C_d(M = 0)$ for the $(36,35)$ state of $^3\text{He}^+$ atom for a finite and infinite nuclear mass of He atom. $N_{L'}$ is the number of basis functions for the intermediate states with the total angular momentum $L'$. 

| $N_{L-1}$ | $N_L$ | $N_{L+1}$ | $\mathcal{E}$ | $\alpha_s$ | $^3\text{He}^-\bar{p}^3\text{He}^+$ | $^4\text{He}^-\bar{p}^4\text{He}^+$ |
|-----------|-------|-----------|--------------|-----------|----------------|----------------|
| 140       | 50    | 130       | 1.008 927 6047 | 0.899 415 52 | 1.311 880 85 | 1.312 410 57 |
| 300       | 100   | 300       | 1.001 467 4291 | 0.923 357 23 | 1.317 432 70 | 1.317 964 83 |
| 500       | 220   | 500       | 1.001 096 8815 | 0.924 043 64 | 1.317 652 12 | 1.318 184 36 |
| 700       | 400   | 700       | 1.001 093 2793 | 0.924 050 35 | 1.317 652 18 | 1.318 184 41 |
| 900       | 600   | 900       | 1.001 093 0402 | 0.924 050 34 | 1.317 654 27 | 1.318 186 51 |
| 1100      | 800   | 1100      | 1.001 092 9902 | 0.924 050 76 | 1.317 654 23 | 1.318 186 47 |
| Convergent values | | | | 0.924 051(2) | 1.317 654(1) | 1.318 186(2) |

TABLE III. Test of convergence for the static scalar dipole polarizability $\alpha_s$ and dispersion coefficients $C_d(M = 0)$ for the the $(32,31)$ state of $^3\text{He}^+$ atom, for a finite and infinite nuclear mass of He atom. $N_{L'}$ is the number of basis functions for the intermediate states with the total angular momentum $L'$. 

| $N_{L-1}$ | $N_L$ | $N_{L+1}$ | $\alpha_s$ | $^3\text{He}^-\bar{p}^3\text{He}^+$ | $^4\text{He}^-\bar{p}^4\text{He}^+$ |
|-----------|-------|-----------|-----------|----------------|----------------|
| 1000      | 1000  | 1000      | 0.362 24 + i 0.013 25 | 1.617 238 + i 0.000 012 | 1.617 892 + i 0.000 012 |
| 1500      | 1500  | 1500      | 0.352 81 + i 0.011 30 | 1.617 232 + i 0.000 008 | 1.617 886 + i 0.000 008 |
| 2400      | 2400  | 2400      | 0.352 56 + i 0.011 76 | 1.617 233 + i 0.000 007 | 1.617 887 + i 0.000 007 |
| 3300      | 3300  | 3300      | 0.352 61 + i 0.011 72 | 1.617 233 + i 0.000 007 | 1.617 887 + i 0.000 007 |
| Convergent values | | | 0.352 6(1)+i 0.011 8 | 1.617 233(1)+i 0.000 007 | 1.617 887(1)+i 0.000 007 |

TABLE IV. The static dipole polarizability, $\alpha_s$, and $\alpha_s \times 10^3$ for metastable states $(n,L)$ of $^3\text{He}^+$ and $^4\text{He}^+$. 

| $(n,L)$ | $\alpha_s$ | $\alpha_s \times 10^3$ | $\alpha_s$ | $\alpha_s \times 10^3$ |
|--------|---------|----------------|---------|----------------|
| (31,30) | 1.78619 − i 0.00081 | 0.21466 + i 0.00028 | 2.00303 + i 0.00045 | 0.17633 − i 0.00038 |
| (32,31) | 1.56805 | 0.23644 | 0.35261 + i 0.01172 | 1.13825 − i 0.00951 |
| (33,32) | 1.11874 + i 0.17764 | 2.16493 − i 0.13634 | 1.34456 + i 0.00059 | 0.28779 − i 0.00045 |
| (34,32) | 1.34708 | 0.26348 | 1.57429 | 0.22059 |
| (35,32) | 1.10917 + i 0.00007 | 0.34856 − i 0.00005 | 1.3.6108 | 0.24470 |
| (35,33) | 0.89521 + i 0.00735 | 0.44815 − i 0.00546 | 2.1.9437 + i 0.06382 | −0.22096 − i 0.04261 |
| (35,34) | 0.87871 | 0.39529 | 1.12525 | 0.32360 |
| (36,32) | 0.89434 | 0.33300 | 1.14520 | 0.27278 |
| (36,33) | 1.37921 + i 0.07366 | 1.03868 − i 0.04512 | 1.94467 + i 0.17316 | −0.07879 − i 0.07193 |
| (36,34) | 0.70924 + i 0.00289 | 0.48140 − i 0.00197 | 0.90508 + i 0.00179 | 0.42010 − i 0.00126 |
| (37,33) | 0.64656 | 0.44771 | 0.90739 | 0.36234 |
| (37,34) | 1.06192 + i 0.00487 | 0.24605 − i 0.00344 | 52.0183 + i 12.5824 | −31.8571 − i 8.3973 |
| (37,35) | 0.40244 | 0.59725 | 0.67456 | 0.47854 |
| (38,33) | 0.40318 | 0.51175 | 0.68389 | 0.40841 |
| (38,34) | 1.33938 + i 0.01240 | 0.14569 − i 0.00780 | 0.31099 − i 0.11508 | 0.91666 − i 0.01162 |
| (38,35) | 0.16689 + i 0.00029 | 0.78694 − i 0.00019 | 0.45394 + i 0.00207 | 0.62294 − i 0.00138 |
| (39,34) | 0.15570 | 0.68149 | 0.44716 | 0.54035 |
| (39,35) | 0.02278 − i 0.07504 | 0.98716 + i 0.04701 | 0.24206 + i 0.00157 | 0.80977 − i 0.00995 |
| (40,35) | −0.30463 | 1.16374 | −0.51572 + i 0.11763 | 1.29819 − i 0.05401 |
| (40,36) | −0.02227 | 0.80180 | 0.21561 | 0.70833 |
| (41,35) | −1.92803 + i 0.00669 | 2.20173 − i 0.00416 |
TABLE V. Dispersion coefficients $C_6(M)$ for long-range interaction between $\tilde{\text{He}}$ and $\tilde{\text{pHe}}$.

| $(n, L)$ | $\text{He} - \tilde{\text{pHe}}$ | $\text{He} - \tilde{\text{pHe}}^+$ |
|----------|-----------------|-----------------|
|          | $M = 0$         | $M = \pm L$     | $M = 0$         | $M = \pm L$     |
| (31,30)  | 1.612 77        | 1.071 49        | 1.071 49        | 1.070 12        |
| (32,31)  | 1.525 611       | 1.052 632       | 1.061 89        | 1.062 28        |
| (33,31)  | 1.613 7         | 1.049 8        | 1.053 20        | 1.051 27        |
| (33,32)  | 1.445 058       | 1.043 646       | 1.052 723       | 1.052 045       |
| (34,32)  | 1.387 246       | 1.034 281       | 1.045 162       | 1.042 249       |
| (34,33)  | 1.372 491       | 1.034 852       | 1.044 535       | 1.043 956       |
| (35,32)  | 1.359 370       | 1.026 273       | 1.043 56        | 1.043 956       |
| (35,33)  | 1.335 233       | 1.026 855       | 1.039 356       | 1.038 092       |
| (35,34)  | 1.311 625       | 1.026 614       | 1.037 966       | 1.035 603       |
| (36,32)  | 1.467           | 1.028           | 1.045           | 1.032           |
| (36,33)  | 1.331 77        | 1.020 45        | 1.036 841       | 1.027 199       |
| (36,34)  | 1.298 994       | 1.019 786       | 1.033 492       | 1.023 340       |
| (36,35)  | 1.264 137       | 1.018 617       | 1.031 817       | 1.027 250       |
| (37,33)  | 1.361 13        | 1.013 439       | 2.135           | 1.698           |
| (37,34)  | 1.324 189       | 1.013 346       | 1.331 131       | 1.212 822       |
| (37,35)  | 1.281 479       | 1.013 709       | 1.301 665       | 1.206 030       |
| (37,36)  | 1.234 243       | 1.012 883       | 1.269 911       | 1.198 418       |
| (38,33)  | 1.443 6         | 1.015 42        | 1.431           | 1.212           |
| (38,34)  | 1.390 551       | 1.013 412       | 1.357 03        | 1.172 43        |
| (38,35)  | 1.341 39        | 1.011 32        | 1.320 232       | 1.160 541       |
| (39,34)  | 1.500 6         | 1.013 78        | 1.421 71        | 1.154 37        |
| (39,35)  | 1.446 4         | 1.011 15        | 1.376 708       | 1.136 844       |
| (40,35)  | 1.598 173       | 1.011 394       | 1.480 3         | 1.139 5        |

Thus only numerical values for cases $M = 0$ and $M = \pm L$ are displayed in the table. One may find that dependence on $M$ of $C_6(M)$ increases with $n$ and $L$. That may be explained by growing asymmetry of the state: The antiproton spends more time outside of the electronic cloud. The data in the table demonstrates quite a regular behavior of $C_6$ coefficients with one exception: the $(37,35)$ state in the $\tilde{\text{pHe}}^+$ atom has larger values of $C_6$. That means that atoms in this state experience stronger attraction while interacting with the helium atoms of the target, and eventually experience stronger collisional disintegration of the atom and subsequent prompt annihilation of $\tilde{p}$.

Using the numerical values of $C_6(M)$ one can get a rough estimate for the collisional shift for transitions between metastable states of the antiprotonic helium. When we apply the simplest static approximation, the frequency shift can be expressed as

$$\Delta\omega = \frac{C_6 - C_f}{R_{av}^6},$$

where $C_i$ and $C_f$ are the dispersion coefficients $C_6(M)$ for the initial and final states, respectively, and $R_{av}$ is an averaged separation distance between atoms. Assuming the separation distance $R \approx 19$ a.u. that approximately corresponds to a number density of $10^{21} \text{ cm}^{-3}$, the spectroscopy shift for the $(33,32) \rightarrow (32,31)$ transition would be 12 GHz. More sophisticated models of the collisional shift and broadening may be found in Refs. [36–38].

In conclusion, the static dipole polarizability and the leading order van der Waals coefficient $C_6$ have been evaluated for metastable states of $\tilde{\text{pHe}}^+$ and $\tilde{\text{pHe}}^+$ atoms for a wide range of metastable states of practical interest. These data may be used for estimating the collisional shift and broadening of the transition frequencies of the antiprotonic helium at low temperature.

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