THE CHEMICAL COMPOSITION OF THE SUN FROM HELIOSEISMIC AND SOLAR NEUTRINO DATA

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RECEIVED 2013 December 16; accepted 2014 March 20; published 2014 April 29

ABSTRACT

We perform a quantitative analysis of the solar composition problem by using a statistical approach that allows us to combine the information provided by helioseismic and solar neutrino data in an effective way. We include in our analysis the helioseismic determinations of the surface helium abundance and of the depth of the convective envelope, the measurements of the $^7$Be and $^8$B neutrino fluxes, and the sound speed profile inferred from helioseismic frequencies. We provide all the ingredients to describe how these quantities depend on the solar surface composition, different from the initial and internal composition due to the effects of diffusion and nuclear reactions, and to evaluate the (correlated) uncertainties in solar model predictions. We include error sources that are not traditionally considered such as those from inversion of helioseismic data. We, then, apply the proposed approach to infer the chemical composition of the Sun. Our result is that the opacity profile of the Sun is well constrained by the solar observational properties. In the context of a two-parameter analysis in which elements are grouped as volatiles (i.e., C, N, O, and Ne) and refractories (i.e., Mg, Si, S, and Fe), the optimal surface composition is found by increasing the abundance of volatiles by $45 \pm 4\%$ and that of refractories by $19 \pm 3\%$ with respect to the values provided by Asplund et al. (2009, ARA&A, 47, 481). This corresponds to the abundances $e_0 = 8.85 \pm 0.01$ and $e_{\text{Fe}} = 7.52 \pm 0.01$, which are consistent at the $\sim 1\sigma$ level with those provided by Grevesse & Sauval (1998, SSRv, 85, 161). As an additional result of our analysis, we show that the best fit to the observational data is obtained with values of input parameters of the standard solar models (radiative opacities, gravitational settling rate, and the astrophysical factors $\delta_{54}$ and $\sigma_{17}$) that differ at the $\sim 1\sigma$ level from those presently adopted.

Key words: neutrinos – Sun: abundances – Sun: helioseismology – Sun: interior

Online-only material: color figures

1. INTRODUCTION

In the last three decades, there has been enormous progress in our understanding of stellar structure and evolution. Solar models have played a particularly important role, in large part because we have powerful diagnostics of the internal solar conditions from solar neutrino experiments and helioseismology. The deficit of the observed solar neutrino fluxes relative to solar model predictions, initially reported by Homestake (Davis et al. 1968) and then confirmed by GALLEX/GNO (Hampel et al. 1999; Altmann et al. 2005), SAGE (Abdurashitov et al. 1999), Kamiokande (Hirata et al. 1989) and Super-Kamiokande (SK; Cravens et al. 2008), SNO (Ahmad et al. 2002), and Borexino (Arpesella et al. 2008), gave rise to the solar neutrino problem: major changes were required in either the theory of stellar structure and evolution or neutrino physics. The development, refinement, and testing of the standard solar model (SSM) played an important role in its ultimate resolution in 2002. When the SNO experiment obtained direct evidence for flavor oscillations of solar neutrinos and confirmed the SSM prediction of the $^8$B neutrino flux with a precision that, according to the latest data, is equal to about 3%.

The Sun is a non-radial oscillator, and powerful insights have also emerged from the study of the solar frequency pattern (e.g., see Basu & Antia 2008). The sound speed as a function of depth can be reconstructed to a high precision, of the order of 0.1% except for the innermost region ($r \lesssim 0.05 R_\odot$). Abrupt changes in the solar thermal structure from ionization and the transition from radiative to convective energy transport induce acoustic glitches that can be precisely localized; we can therefore infer the depth of the convective envelope at the 0.2% level and the surface helium abundance at the 1.5% level. As a result, the solar structure is now well constrained and the Sun can be used as a solid benchmark for stellar evolution and as a laboratory for fundamental physics (see, e.g., Fiorentini et al. 2001; Ricci & Villante 2002; Bottino et al. 2002). In fact, it was the excellent agreement between solar models and helioseismic inferences on the solar structure (better than $1.5\sigma$ for all constraints) that gave a strong support to the idea that the root of the solar neutrino problem had to be found outside the realm of solar modeling (Bahcall et al. 2001), before evidence for neutrino oscillations was found.

All these important measurements acquire even more relevance when considering that, in recent years, a solar composition problem has emerged. The SSM treats the absolute and relative elemental abundances as an input, and the Grevesse & Sauval (1998, hereafter GS98) mixture yields concordance between the model and the data. Relative abundances of heavy elements can be precisely measured in meteorites (Lodders 2010), but the abundances of the important light CNO elements can only be measured in the photosphere. The Ne abundance is even less secure, as it is inferred from solar wind measurements. A systematic overhaul in solar model atmospheres, see Asplund et al. (2005) and Asplund et al. (2009, hereafter AGSS09net), has...
led to a downward revision in the inferred photospheric heavy element abundances (see Table 1) by up to 30%–40% for important species such as oxygen. The magnitude of the differences is model(er) dependent; independent measurements by Caffau et al. (2011; see also Lodders 2010) are intermediate between the GS98 and AGSS09met scales. The internal structure of SSMs using the lower solar surface metallicity of AGSS09met does not reproduce the helioseismic constraints; for example, the sound speed disagrees at the bottom of the convective envelope by about ~1% with the value inferred from helioseismology. In addition, the predicted surface helium abundance is lower by ~7%, and the radius of the convective envelope is larger by ~1.5% with respect to the helioseismic results. In synthesis, inferences from modern three-dimensional (3D) hydrodynamic models of the solar atmosphere lead to predictions for the solar interior that are in strong disagreement with observational constraints, well above the currently estimated errors.

The solar composition problem has been addressed by a number of independent interior calculations. Problems in reconciling helioseismic data with a low abundance scale became obvious early on (Basu & Antia 2004; Bahcall & Pinsonneault 2004; Turck-Chièze et al. 2004). Bahcall et al. (2006) performed a Monte Carlo analysis that included calculations of the rms deviations between the inferred solar sound speed profile and that predicted by models with the “high” and “low” abundances. Delahaye & Pinsonneault (2006) advocated an inversion of the problem, solving for the abundances consistent with the base of the surface convection zone and surface helium abundance. The ionization signature in the surface convection zone can also be used as an independent diagnostic suggesting high metallicity (see, however, discussion in Vorontsov et al. 2013; Basu & Antia 2008) summarized the initial results favoring a higher solar metallicity. In the context of the solar abundance problem, physical processes not included in the standard model have been considered as potential candidates to provide an explanation: low-Z accretion (Castro et al. 2007; Guzik & Mussack 2010; Serenelli et al. 2011), enhanced gravitational settling (Montalbán et al. 2004; Guzik et al. 2005), large mass loss rates (e.g., that the Sun has been more massive in the past; Guzik & Mussack 2010; Turck-Chièze et al. 2011), dynamical effects (derived from transport of angular momentum; Turck-Chièze et al. 2010). An intermediate solar metallicity using low-degree modes was reported by Houdek & Gough (2011); however, these authors note that their mixture produces a sound speed profile at variance with solar data. More recent work using solar envelope models even suggests a solar metallicity lower than the AGSS09met value (Vorontsov et al. 2014).

The goal of this work is to perform a complete and quantitative analysis of the solar composition problem. In particular, we address the following questions: which is the chemical composition of the Sun that, by using the current input physics of solar models, can be inferred from helioseismic and solar neutrino data? How does different observational information combine in determining the optimal composition of the Sun? How does the obtained composition compare to the photospheric inferred values? Do the different observational data show tensions and/or inconsistencies that may point at some inadequacies in the SSM input parameters or assumptions? Even if the problem has been already considered in literature, a thorough self-consistent discussion is still missing. While a rigorous approach is not necessary for a qualitative assessment of the problem, it becomes essential for our goal, i.e., to use the helioseismic information in combination with the solar neutrino results to infer the properties of the Sun. In order to make a correct inference, one has to define an appropriate figure-of-merit (e.g., a $\chi^2$ statistic) that has to be non-biased and should combine the different pieces of the observational information with the correct relative weights.

In this respect, important progress has to be done at a methodological level. This paper starts addressing this problem. We propose to use a statistical approach, normally adopted in other areas of physics (e.g., in neutrino studies; see Fogli et al. 2002) in which all the relevant pieces of information can be combined in a correct and effective way. We discuss a strategy to include the observational information for the sound speed profile, the radius of the convective envelope, the surface helium abundance, and the $^7$Be and the $^8$B neutrino fluxes. We provide all the ingredients to describe how these quantities depend on the assumed chemical composition and to evaluate the (correlated) uncertainties in solar model predictions.

The plan of the paper is as follows. In Section 2, we review the status of SSM calculations and discuss in detail our treatment of uncertainties in theoretical predictions and also the observational constraints considered in the analysis, including sources of errors not traditionally considered such as those from inversion of helioseismic data. In Section 3, we describe the adopted statistical approach. In Section 4, we analyze the response of the Sun to variations of its surface composition. Section 5 contains the results of our analysis, i.e., the bounds on the chemical composition of the Sun that are inferred from helioseismic and solar neutrino data. Finally, we provide a summary and conclusions in Section 6.

### 2. Models and Data

Our theoretical working framework is the SSM, and in Section 2.1, we summarize the aspects most relevant to this work. More importantly, we present our treatment of errors, for which two qualitatively different sources must be identified. On one hand, errors in the input parameters $I$ for solar model construction induce theoretical uncertainties in the SSM predictions $Q$. These errors are fully correlated, as it is discussed in Section 2.2. On the other hand, the observational determinations $Q_{\text{obs}}$ of helioseismic quantities and solar neutrino fluxes are affected by observational errors. In this work, we treat these errors as uncorrelated, as it is discussed in Section 2.3.
Table 2
The Predictions of SSMs Implementing GS98 (Grevesse & Sauval 1998) and AGSS09met (Asplund et al. 2009) Admixtures

| Q      | AGSS09met | GS98 | Obs. | GS98rec |
|--------|-----------|------|------|---------|
| $Y_e$  | 0.2319 (1 ± 0.013) | 0.2429 (1 ± 0.013) | 0.2485 ± 0.0035 | 0.243 |
| $R_o / R_\odot$ | 0.7231 (1 ± 0.0033) | 0.7124 (1 ± 0.0033) | 0.713 ± 0.001 | 0.710 |
| $\Phi_{\text{N}}$ | 6.03 (1 ± 0.005) | 5.98 (1 ± 0.005) | 6.05 (1±0.003) | 5.98 |
| $\Phi_{\text{Fe}}$ | 4.56 (1 ± 0.06) | 5.00 (1 ± 0.06) | 4.82 (1±0.003) | 4.69 |
| $\Phi_{\text{Si}}$ | 4.59 (1 ± 0.11) | 5.58 (1 ± 0.11) | 5.00 (1 ± 0.03) | 5.49 |
| $\Phi_{\text{N}}$ | 2.17 (1 ± 0.08) | 2.96 (1 ± 0.08) | ≤6.7 | 2.89 |
| $\Phi_{\text{O}}$ | 1.56 (1 ± 0.10) | 2.23 (1 ± 0.10) | ≤3.2 | 2.15 |

Notes. The theoretical uncertainties (1σ) have been calculated as it is described in the text and do not include the contributions due to errors in the surface composition. In the third column, we show the observational values for helioseismic quantities (Basu & Antia 2004, 1997) and solar neutrino fluxes (Bellini et al. 2011). In the last column, we calculate the GS98 solar model predictions starting from the AGSS09met solar model by using the linear expansion given in Equation (22). The neutrino fluxes are given in the following units: 10^10 cm^-2 s^-1 (pp); 10^9 cm^-2 s^-1 (Be); 10^9 cm^-2 s^-1 (B); 10^9 cm^-2 s^-1 (N); and 10^9 cm^-2 s^-1 (O).

2.1. Standard Solar Models

The development of a new generation of stellar atmosphere models resulted in a downward revision of the solar photospheric abundances. This is shown, for example, in Table 1 where we give the abundances of C, N, O, Ne, Mg, Si, S, and Fe, which are the most relevant elements for solar model construction. By AGSS09met, we represent the (new) photospheric abundances from Asplund et al. (2009) for volatile elements (C, N, O, and Ne) combined with their recommended meteoritic abundances for refractory elements (Mg, Si, S, and Fe), where Si is used as the anchor of both scales. The choice of meteoritic abundances for refractories is rooted in their robustness and superior accuracy and precision over photospheric determinations, and it has traditionally been the preferred choice in SSM calculations (see Serenelli et al. 2009 for a discussion on this topic). By GS98, we represent the (old) heavy element admixture of Grevesse & Sauval (1998). The last column shows the fractional differences between the individual abundances (relative to hydrogen) in the two compilations.

The heavy element admixture determines to a large extent the opacity profile of the Sun and is a crucial input for solar model construction. This is seen in the first two columns of Table 2, where we report the results of two recent SSM calculations (Serenelli et al. 2011) that implement the AGSS09met (Asplund et al. 2009) and GS98 (Grevesse & Sauval 1998) surface compositions. The models have been computed with GARSTEC (Weiss & Schlatt 2008) using the input physics described in Serenelli et al. (2009). We utilize the nuclear reaction rates recommended in Adelberger et al. (2011) with weak screening (Salpeter 1954), and we employ the mixing length theory of convection (Cox 1968; Vitevne 1953). We use a gray atmosphere surface boundary condition; this has virtually no influence on the global seismic properties and the predicted neutrino fluxes of the solar model and bears no impact on results presented in this work (see Schlatt et al. 1997 for a comparison of solar models with different model atmospheres). Element diffusion in the solar interior is included according to Thoul et al. (1994). The models have been computed by using the radiative opacities from the Opacity Project (OP; Badnell et al. 2005), complemented at low temperatures with the opacities from Ferguson et al. (2005). Finally, we do not include additional mixing of any sort (rotational mixing, overshooting, or mixing by gravity waves) below the convection zone in the models that would be necessary to explain the solar lithium depletion (a factor of ~160; Asplund et al. 2009). We come back to this point in Section 6.

2.2. Theoretical Uncertainties

We consider the following observable quantities: the surface helium abundance $Y_e$, the radius of the inner boundary of the convective envelope $R_c$, the neutrino fluxes $\Phi_q$, where the index $q$ = pp, Be, B, N, O refers to the neutrino producing reactions according to the usual convention, and the fractional sound speed difference $\delta c_i \equiv (c_{\text{obs},i} - c(r_i))/c(r_i)$ between the predicted sound speed $c(r)$ and the values $c_{\text{obs},i}$ inferred from helioseismic data. The latter is shown in Figure 1, where the black line refers the SSM model implementing the AGSS09met surface composition while the red line is obtained by using the GS98 admixture. The red band provides an estimate of the uncertainty in an inversion of the helioseismic data. The light blue band corresponds to the 1σ uncertainties in the theoretical predictions.

(A color version of this figure is available in the online journal.)

Figure 1. Fractional difference $\delta c_i \equiv (c_{\text{obs},i} - c(r_i))/c(r_i)$ between the sound speed $c(r)$ predicted by SSMs and the values $c_{\text{obs},i}$ inferred from the helioseismic data; the black line refers to the SSM model implementing the AGSS09met surface composition while the red line is obtained by using the GS98 admixture. The red band provides an estimate of the uncertainty in an inversion of the helioseismic data. The light blue band corresponds to the 1σ uncertainties in the theoretical predictions.

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In the following, we take the SSM implementing the AGSS09met composition as a starting point for our analysis, and we use the notation $Q(\tilde{T})$ to indicate the prediction (assumption) for the generic quantity $Q$ (input $I$) in this calculation.

2.2. Theoretical Uncertainties

The errors quoted in Table 2 and the light-blue band in Figure 1 correspond to 1σ uncertainties in theoretical predictions. They have been calculated by propagating the errors in
the following input parameters: the age of the Sun (age); the diffusion coefficients (diffu); the luminosity (lum); the opacity profile (opa) of the Sun; and the astrophysical factors \( S_{11}, S_{33}, S_{34}, S_{17}, S_{37}, \) and \( S_{114} \). As it is done, e.g., in Bahcall et al. (2000), albeit diffusion rates are computed individually for each elements, we assume the same fractional error for all, and we treat them as fully correlated, i.e., we rescale them by a common multiplicative factor. Differential variations of the diffusion coefficients within the present uncertainties (\( \approx 10\% \)) give negligible contributions to theoretical errors. Note that, as we are interested in establishing bounds on the solar chemical composition, we have explicitly omitted its contribution to theoretical uncertainties. Moreover, we have not considered the uncertainty of the solar radius because it provides subdominant contributions to the error budget of the considered observable quantities.\(^5\) Our test calculations show that the solar radius uncertainty corresponds to a fractional error at the level of 0.01% and 0.03% for \( ^{7}\text{Be} \) and \( ^{8}\text{B} \) neutrino fluxes; 0.01% and 0.004% for \( R_0 \) and \( Y_s \); 0.025% at most for the sound speed. The assumed value of \( R_0 \) also affects the determination of the sound speed by inversion of helioseismic frequencies. However, based on results by Basu et al. (2000), see their Figure 9), this uncertainty is of the order of 0.035% and plays a minor role in the full analysis (see the next section and the red band in Figure 1). By following the standard procedure, we have calculated the logarithmic derivatives

\[
B_{Q,I} = \frac{\partial \ln Q}{\partial \ln I}.
\]

\( B_{Q,I} \) values are available at Serenelli et al. (2013) for all observables except the sound speed \( c(r) \). To our knowledge, the logarithmic derivatives \( B_{c,I}(r) \) of the sound speed, defined as

\[
B_{c,I}(r) = \frac{\partial \ln c(r)}{\partial \ln I}
\]

have not been given elsewhere in the scientific literature and are shown in the left panel of Figure 2 as a function of the solar radius.

As mentioned before, the uncertainty \( \delta I \) in each input parameter \( I \) produces fully correlated errors on the SSM predictions of observables \( Q \). To emphasize this point, we use the symbol \( C_{Q,I} \) to indicate the fractional variation of \( Q \) when a fractional correction \( \delta I \) is applied to the input \( I \). The various error contributions \( C_{Q,I} \), shown in Table 3 and in the right panel of Figure 2, are calculated from the relation

\[
C_{Q,I} = B_{Q,I} \delta I
\]

with the \( \delta I \) values summarized in Serenelli et al. (2013). The only exception is the opacity, which we discuss below.

Opacity is not a single number but a complicated function of the properties of the solar plasma which can be modified in a non-trivial way. To take this into account, we use the opacity kernels derived in Villante (2010) by adopting the linearization procedure proposed in Villante & Ricci (2010). The kernels \( K_Q(r) \) represent the functional derivatives of the observable \( Q \) with respect to the opacity profile of the Sun and can be used to calculate the effects produced by an arbitrary opacity variation \( \delta \kappa(r) \) according to

\[
\delta Q = \int dr \ K_Q(r), \ \delta \kappa(r)
\]

where

\[
\delta Q \equiv \frac{Q}{Q} - 1,
\]

and

\[
\delta \kappa(r) \equiv \frac{\kappa(T(r), \rho(r), Y(r), Z_j(r))}{\kappa(T(r), \rho(r), Y(r), Z_j(r))} - 1.
\]

\(^5\) As a proxy for the solar radius uncertainty, we take half the difference between our reference \( R_S = 6.9598 \times 10^{10} \) cm and \( R_0 = 6.95508 \times 10^{10} \) cm (Brown & Christensen-Dalsgaard 1998), a value that has often been used in solar modeling and helioseismic studies.
In the above relation, the functions \( \kappa \) and \( \Phi \) are calculated along the density, temperature, and chemical composition profiles predicted by SSM.

By using the kernels \( K_Q(r) \), we propagate the uncertainty in the opacity profile of the Sun \( \delta \kappa_{\text{opa}}(r) \) according to

\[ C_{Q,\text{opa}} \equiv \int dr \, K_Q(r) \, \delta \kappa_{\text{opa}}(r). \]  

The standard prescription, e.g., as adopted in Serenelli et al. (2009), is to consider a 2.5% global rescaling factor that corresponds to take \( \delta \kappa_{\text{opa}}(r) \equiv 0.025 \). However, this is not realistic because, albeit different groups typically provide opacity values that differ by \( \sim \)few percent in the solar interior, there is a complicated dependence on the solar radius. Moreover, it was shown in Villante (2010) that the assumption of a global rescaling underestimates the uncertainties for the sound speed and for the depth of the convective envelope. The opacity kernels for these quantities are not positive everywhere, and a constant \( \delta \kappa \) produces effects in different regions of the Sun that partially compensate each other. For this reason, we take the difference between the OP (Badnell et al. 2005) and OPAL (Iglesias & Rogers 1996) tables as representative of uncertainties in opacity calculations, i.e., we assume

\[ \delta \kappa_{\text{opa}}(r) \equiv \frac{\kappa_{\text{OPAL}}(T(r), \rho(r), Y(r), Z_i(r)) - \kappa_{\text{OP}}(T(r), \rho(r), Y(r), Z_i(r))}{\kappa_{\text{OP}}(T(r), \rho(r), Y(r), Z_i(r))} - 1. \]  

The function \( \delta \kappa_{\text{opa}}(r) \) is shown in Figure 3. The sound speed errors obtained with this assumption are much larger that what is obtained by the standard approach, even if \( |\delta \kappa_{\text{opa}}(r)| \leq 0.025 \) almost everywhere.

Finally, the total theoretical error for each observable \( Q \) is calculated by combining in quadrature all the error contributions, i.e.,

\[ \sigma^2_{Q,\text{theo}} = \sum_{i} C_{Q,i}^2, \]  

where the sum extends over the 10 parameters listed in the beginning of this section.

### 2.3. Observational Constraints

In the third column of Table 2, we give the observational values \( Q_{\text{obs}} \) for helioseismic and solar neutrino observables. The solar neutrino fluxes are obtained by performing a fit to all available solar neutrino data (Bellini et al. 2011). Among the various components, the \( ^7\text{Be} \) and \( ^8\text{B} \) neutrino fluxes are essentially determined by SK (Cravens et al. 2008) and SNO (Ahmad et al. 2002) and by Borexino (Arpesella et al. 2008), respectively, i.e., by two independent sets of experimental data. We thus include the two values

\[ \Phi_{\text{Be,obs}} = 4.82 (1^{+0.05}_{-0.04}) \times 10^9 \text{ cm}^{-2} \text{ s}^{-1} \]
\[ \Phi_{\text{B,obs}} = 5.00 (1 \pm 0.03) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \]

and assume their errors are uncorrelated. Generically denoting with \( U_Q \) an uncorrelated fractional error in the observable \( Q \), we adopt

\[ U_{\text{Be}} = 0.045 \]
\[ U_{\text{B}} = 0.03 \]  

for \( \Phi_{\text{Be}} \) and \( \Phi_{\text{B}} \), respectively. We note that the observational errors are smaller than the uncertainties in theoretical predictions.

The surface helium abundance and the inner radius of the convective envelope are obtained by inversion of helioseismic frequencies. We adopt the values

\[ Y_s,\text{obs} = 0.2485 \pm 0.0035 \]
\[ R_b,\text{obs} = 0.713 \pm 0.001, \]

which are obtained in Basu & Antia (2004) and Basu & Antia (1997), respectively, and we indicate with

\[ U_Y = 0.015 \]
\[ U_R = 0.0014 \]  

the fractional errors that we assume, as it is usually done (see, e.g., Delahaye & Pinsonneault 2006), not being significantly correlated.

The sound speed data points \( \delta \kappa \) reported in Figure 1 have been obtained in Basu et al. (2009) with the set of frequencies of solar low-degree \( p \)-modes from the BiSON network by using the subtractive optimally localized averages.
A method combining the SOLA and the RLS techniques was used. The displayed error bars $U_{i,\text{exp}}$ are calculated by propagating the observational uncertainties of the measured frequencies. It is well known, however, that larger errors arise from the choice of the parameters in the inversion procedure and from the assumed starting model for the inversion. An extensive investigation of uncertainties in helioseismic determinations of sound speed has been performed in Degl’Innocenti et al. (1997). The red band in Figure 1 corresponds to the so-called “statistical” uncertainty $U_{\text{stat}}(r)$ that is obtained in Degl’Innocenti et al. (1997) by combining in quadrature all the relevant error contributions. In our analysis, we define total (fractional) observational errors for the sound speed by combining in quadrature experimental and “statistical” errors according to

$$U_{c,i} = \sqrt{U_{i,\text{exp}}^2 + U_{\text{stat}}^2(r_i)},$$

(14)

where the index $i$ indicates the considered data point. Clearly, sound speed determinations at different radii are expected to have a certain degree of correlation because uncertainties are mainly related to the inversion procedure. Unfortunately, the information provided in the scientific literature does not allow us to quantify these correlations. For this reason, we include the sound speed errors as being uncorrelated. The correct quantification of helioseismic errors is an important ingredient, and it would be desirable that the complete information were provided in future investigations.

As it is well known, solar models implementing the AGSS09met admixture do not correctly reproduce the helioseismic constraints. By combining in quadrature theoretical and observational errors, we see that the predictions for $Y$, $R_b$, and $R_s$ deviate from observational data at $\sim 3.6\sigma$ and $\sim 3.9\sigma$, respectively. The sound speed at $r \sim 0.65 R_\odot$ differs from the results of helioseismic inversion by $\sim 4.6\sigma$. Helioseismic data are much better fitted by solar models implementing the GS98 surface composition, as can be seen from Table 2 and Figure 1. A reasonable agreement exists between predicted and reconstructed solar neutrino fluxes both for the AGSS09met and the GS98 solar models.

3. THE STATISTICAL APPROACH

Our goal is to build a $\chi^2$ function that can be used as a figure-of-merit for SSMs with different compositions. Let us consider a generic observable quantity with its associated observational value $Q_{\text{obs}}$ and theoretical prediction $Q$. We indicate the fractional difference between the observational value and the theoretical result with

$$\delta Q_{\text{obs}} = \frac{Q_{\text{obs}}}{Q} - 1.$$  (15)

In this work, we consider a set of $N = 34$ differences $\delta Q$ given by

$$\{\delta Q_{\text{obs}}\} = \{\delta \Phi_B, \delta \Phi_{\text{Be}}, \delta Y_s, \delta R_b, \delta c_1, \delta c_2, \ldots, \delta c_30\},$$

(16)

where we include the sound speed determinations $c_{i,\text{obs}}$ of Basu et al. (2009) that are localized at $r \lesssim 0.8 R_\odot$.

The differences $\delta Q_{\text{obs}}$ are affected by the uncorrelated errors $U_Q$ (e.g., from neutrino experiments and helioseismic data) and by a set of systematic correlated errors $C_{Q,i}$ induced by $K = 10$ independent sources that, in our approach, are

$$\{I\} = \{\text{opa}; \text{age}; \text{diffu}; \text{lum}; S_{11}, S_{33}, S_{34}, S_{17}, S_{11}, S_{14}\}.  \quad (17)$$

Following Fogli et al. (2002), we define $\chi^2$ as

$$\chi^2 = \min_{\{\xi\}} \left[ \sum_{Q} \left( \frac{\delta Q_{\text{obs}} - \sum_{i} \xi_i C_{Q,i}}{U_Q} \right)^2 + \sum_{i} \xi_i^2 \right],$$

(18)

This definition describes the effects of systematic correlated errors $C_{Q,i}$ by introducing the shifts $-\xi_i C_{Q,i}$, where $\xi_i$ is a univariate Gaussian random variable. Expressing $\chi^2$ in this way is completely equivalent to the standard covariance matrix approach (for a formal proof refer to Fogli et al. 2002). However, it offers some relevant advantages: (1) it is more easily implemented numerically and; (2) it allows for tracking the individual contributions to the $\chi^2$. Denoting with $\tilde{\xi}_i$ the values that minimize the $\chi^2$, one obtains

$$\chi^2 = \chi^2_{\text{obs}} + \chi^2_{\text{syst}} = \sum_{Q} \tilde{X}_Q^2 + \sum_{i} \tilde{\xi}_i^2,$$

(19)

where

$$\tilde{X}_Q = \frac{\delta Q_{\text{obs}} - \sum_{i} \tilde{\xi}_i C_{Q,i}}{U_Q}$$

(20)

are the so-called pulls of observational quantities. The values $\tilde{\xi}_i$ are instead referred to as the “pulls” of systematical error sources that, in our analysis, coincides with input parameters in solar model construction. The distribution of the $\tilde{\xi}_i$ can thus be used to highlight tensions in SSM assumptions. The optimal composition of the Sun is found by minimizing the $\chi^2$, and the obtained value $\chi^2_{\text{min}}$ provides information on the goodness of the fit. The allowed regions are determined by cutting at prescribed values of the variable $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$.

4. DESCRIBING THE ROLE OF METALS

We study the response of the Sun to changes in the heavy element admixture $[z_j]$, expressed in terms of the quantities

$$z_j \equiv Z_{j,b}/X_b,$$

(21)

where $Z_{j,b}$ is the surface abundance of the $j$-element, $X_b$ is that of hydrogen, and the index $j$ runs over all relevant metals (see Table 4). We determine the dependence of the observables on the surface composition by constructing solar models in which each

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8 Degl’Innocenti et al. (1997) adopted two different inversion methods, depending on the value of the radial coordinate. For $r/R_\odot \gtrsim 0.1$, the regularized least square (RLS) method was used. In the inner region, a hybrid method combining the SOLA and the RLS techniques was used.

9 These discrepancies are slightly different from what was found by Bahcall et al. (2006). This is due to the fact that we use a different prescription for the opacity uncertainty; we do not include composition uncertainties in the error budget. Bahcall et al. (2006) use the surface composition of Asplund et al. (2005).

10 In this work, correlated systematic errors incidentally coincide with theoretical uncertainties, while uncorrelated errors coincide with observational uncertainties. However, this is not necessarily the case. For example, were we to use $\Phi_{\text{Be}}$ as an observable in our analysis, there would exist a correlation between its experimental error and that of $\Phi_{\text{Be}}$ due to the luminosity constraint usually adopted in the analysis of solar neutrino data; see, e.g., Gonzalez-Garcia et al. (2010).
The logarithmic derivatives of the sound speed with respect to the surface abundances \( z_j \). Right panel: logarithmic derivatives of the sound speed with respect to the total CNO, Ne, and meteoritic elements surface abundances.

(A color version of this figure is available in the online journal.)

Table 4

| \( Q \) | C | N | O | Ne | Mg | Si | S | Fe | CNO | Met |
|--------|---|---|---|----|----|----|---|----|------|-----|
| \( Y_\odot \) | -0.003 | 0.001 | 0.025 | 0.030 | 0.032 | 0.063 | 0.043 | 0.086 | 0.023 | 0.223 |
| \( R_\odot \) | -0.005 | -0.003 | -0.027 | -0.011 | -0.004 | 0.002 | 0.004 | -0.009 | -0.035 | -0.007 |
| \( \Phi_{NP} \) | 0.004 | 0.002 | 0.052 | 0.046 | 0.048 | 0.103 | 0.073 | 0.204 | 0.058 | 0.429 |
| \( \Phi_{Be} \) | 0.026 | 0.007 | 0.112 | 0.088 | 0.089 | 0.191 | 0.134 | 0.501 | 0.145 | 0.916 |
| \( \Phi_N \) | 0.874 | 0.147 | 0.057 | 0.042 | 0.044 | 0.102 | 0.072 | 0.263 | 1.078 | 0.480 |
| \( \Phi_O \) | 0.827 | 0.206 | 0.084 | 0.062 | 0.065 | 0.145 | 0.102 | 0.382 | 1.177 | 0.694 |

Note. The last two columns describe the effects obtained by grouping CNO and meteoritic elements (i.e., Mg + Si + S + Fe) and forcing them to vary by a common multiplicative factor; see the text for details.

\( z_j \) is varied individually. We observe that the effects produced by a change of composition \( \{ \delta z_j \} \) are well described by a linear relation

\[
\delta Q = \sum_j B_{Q,j} \delta z_j ,
\]

where \( \delta z_j \) is the fractional variation of \( z_j \)

\[
\delta z_j = \frac{z_j}{z_j} - 1
\]

with respect to the AGSS09met value \( z_j \). In this assumption, the coefficients \( B_{Q,j} \) represent the logarithmic derivatives of \( Q \) with respect to the \( j \)-element surface abundance, i.e.,

\[
B_{Q,j} = \frac{\partial \ln Q}{\partial \ln z_j} .
\]

The values obtained for the \( B_{Q,j} \) coefficients are reported in Table 4. Our results can be compared with the coefficients presented in Serenelli et al. (2009). The small differences arise from the fact that we considered relatively large variations for the various abundances, in order to check the adequacy of relation (22) over the ranges of compositions required by our analysis. The logarithmic derivatives \( B_{c,j}(r) \) of the sound speed with respect to the surface composition have not been shown elsewhere in scientific literature and are given in the left panel of Figure 4.

The accuracy of Equation (22) can be tested by using it to reproduce the predictions of the GS98 solar model starting from those obtained by implementing the AGSS09met surface composition. The results of this exercise, the reconstructed GS98 (GS98rec) model, are shown in the last column of Table 2 and in Figure 5. The differences between GS98 and AGSS09met are reproduced with \( \sim 20\% \) accuracy (in the worst cases) and the errors introduced by the use of the linear approximation are always smaller or comparable to theoretical and observational uncertainties. The use of relation (22) greatly simplifies the numerical problem of scanning over the range of possible compositions. In this assumption, indeed, the \( \chi^2 \) is expressed as a quadratic function of the various \( \delta z_j \) that can be effectively minimized. We obtain

\[
\chi^2 = \min_{\{\xi_i\}} \left[ \sum_Q \left( \frac{\delta Q - \sum_j \delta z_j B_{Q,j} - \sum_i \xi_i C_{Q,i}}{U_Q} \right)^2 + \sum_i \xi_i^2 \right] ,
\]
and are reported in Table 4. The functions $B_{\text{CNO}}(r)$, $B_{\text{Ne}}(r)$, and $B_{\text{met}}(r)$ describing the effects of each group of elements on the sound speed profile of the Sun are shown in the right panel of Figure 4.

5. INFERRING THE SOLAR COMPOSITION

5.1. Volatiles and Refractories: A Two-parameter Analysis

As a first application, we consider a scenario in which the neon-to-oxygen ratio is fixed to the value prescribed by the AGSS09met compilation, i.e., we further constrain the possible variations of the heavy element admixture by assuming

$$\delta z_{\text{CNO}} = \delta z_{\text{Ne}}.$$ 

In this hypothesis, the $\chi^2$ is defined in terms of two independent parameters ($\delta z_{\text{CNO}}$, $\delta z_{\text{met}}$) that are varied to fit helioseismic and solar neutrino constraints. A similar exercise was performed in Delahaye & Pinsonneault (2006), where, however, only the determinations of the surface helium abundance and of the convective radius were considered. Here, we include the information provided by the sound speed profile and the neutrino fluxes in a global quantitative analysis. The redundancy of the different pieces of experimental information allows us to obtain more solid constraints on the solar composition and, even more relevant, to verify that a coherent picture emerges from the data and/or to highlight tensions in SSM assumptions.

Our results are presented in Figure 6, where we use the astronomical scale for logarithmic abundances $\epsilon_j$ in order to facilitate the comparison with observational data. The conversion from $\delta z_j$ to $\epsilon_j$ is obtained by using the relation

$$\epsilon_j = \bar{\epsilon}_j + \log (1 + \delta z_j)$$

with the AGSS09met abundances $\tau_j$ given in Table 1. The colored lines are obtained by cutting at $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}} = 2.3, 6.2, 11.8$ that correspond to 1, 2, 3 $\sigma$ confidence levels for a $\chi^2$ variable with 2 d.o.f. The data points show the observational values and errors for the oxygen and iron abundances in the AGSS09met and GS98 compilations. In order to show how different observational information combine in determining the optimal composition, we present separately the bounds obtained by using the following: the helioseismic constraints on the surface helium abundance and the convective radius (upper left panel); the $^7$Be and $^8$B neutrino flux determinations (upper right panel); the 30 sound speed data points $\epsilon_{r,\text{obs}}$ from Basu et al. (2009) that are localized at $r \leq 0.8 R_\odot$ (lower left panel); and all the observational data simultaneously (lower right panel). The main conclusions of our analysis are discussed in the following.

1. The SSM implementing AGSS09met composition is excluded at a high confidence being $\chi^2/\text{d.o.f.} = 176.7/32$ when all the available observational constraints are considered. This result essentially arises from helioseismic observables that are in severe disagreement with AGSS09met predictions. The $^7$Be and $^8$B solar neutrino flux determinations do not discriminate among different compositions with the sufficient level of accuracy. This is mainly due to theoretical uncertainties which are dominated by the contributions from $S_{34}$ and $S_{17}$, respectively, but also it is due to a very mild dependence on CNO abundances.

2. There is a reasonable agreement between the information provided by the various observational constraints, as it can be seen by comparing the different panels of Figure 6. The best fit to the observational data is obtained for

$$\delta z_{\text{CNO}} = 0.45 \pm 0.04$$

$$\delta z_{\text{met}} = 0.19 \pm 0.03,$$

which corresponds to $\epsilon_O = 8.85 \pm 0.01$ and $\epsilon_Fe = 7.52 \pm 0.01$. These values are consistent at $\sim 1\sigma$ level with those quoted in the GS98 compilation. They are close to the results of Delahaye & Pinsonneault (2006) but have considerably smaller uncertainties. The quality of the fit is quite good being the $\chi^2_{\text{min}}/\text{d.o.f.} = 39.6/32$ when all the observational constraints are considered. The errors on the inferred abundances $\epsilon_O$ and $\epsilon_{Fe}$ are smaller than what is obtained by observational determinations. One caveat is, however, that we are considering a simplified scenario in
which different elemental abundances are grouped together and forced to vary by the same multiplicative factors.

3. The observational and systematic contribution to \( \chi^2 \) are given by \( \chi^2_{\text{obs}} = 35.1 \) and \( \chi^2_{\text{syst}} = 4.5 \), respectively, with the distribution of systematic pulls \( \xi_I \) at the best-fit point reported in Table 5. The effects of systematic pulls (that correspond to correlated error sources) are relevant and cannot be neglected. This is seen, e.g., in the upper left panel of Figure 6 where we see that the error ellipse axes do not coincide with the lines \( Y_s = Y_s, \text{obs} \) and \( R_b = R_b, \text{obs} \), as it would be expected if error correlations were negligible. It is also evident from the red dots and the red line in Figure 7 that show the predictions \( Q \) of solar models implementing the best-fit composition calculated by using the linear expansion (22). These predictions deviate from the observational constraints by amounts that are larger than the uncorrelated observational errors. However, this does not imply that the quality of the fit is bad since one has the possibility to change the SSM inputs within their range of uncertainty as it is described in Equation (25). The blue dots and the blue line in Figure 7 show the quantities \( \tilde{Q} = Q \left[ 1 + \sum_I C_{Q,I} \xi_I \right] \) (29) that include the effects due to the pulls of systematic errors. The quantities \( \tilde{Q} \) differ quite substantially from the corresponding \( Q \) and agree quite well with the observational constraints. The dominant contributions to systematic shifts are shown by the black arrows in Figure 7 and are provided by the following: opacity for the sound speed \( c(r) \) (see discussion in the next paragraph); diffusion coefficients which are decreased by \( \sim 11\% \) in order to improve consistency between \( Y_s \) and \( R_b \), and the sound speed profile \( c(r) \); and the astrophysical factors \( S_{34} \) and \( S_{17} \) that are decreased by \( \sim 5\% \) and \( \sim 9\% \), respectively in order to improve agreement with \( \Phi_B \) and \( \Phi_{\text{Be}} \) measurements.

4. The large systematic shift of the sound speed due to opacities, \( \xi_{\text{opa}} = 1.07 \), indicates that there is tension between observational data and SSMs implementing OP opacity tables. Indeed, if we restrict our analysis to OP opacities, i.e., we choose \( \xi_{\text{opa}} \equiv 0 \) in our approach, the quality of the fit is considerably decreased. The best fit is obtained for \( \delta z_{\text{CNO}} = 0.33 \pm 0.03 \) and \( \delta z_{\text{met}} = 0.19 \pm 0.03 \) with \( \chi^2_{\text{min}} / \text{d.o.f.} = 66.9 / 32 \) when all observational constraints are considered. Solar models implementing OP opacities are disfavored because they provide a less satisfactory fit of the sound speed in the region \( 0.3 < r / R_\odot < 0.6 \), as it can be seen from Figure 8. It is interesting to note that, when \( \xi_{\text{opa}} \) is allowed to vary, the best fit is obtained with \( \xi_{\text{opa}} \sim 1 \) which means that observational data are better described when using OPAL opacity tables.\(^{11}\) The

\(^{11}\) We remind the readers that we used the fractional difference between OPAL and OP opacities to define the opacity profile uncertainty \( \delta k_{\text{opa}}(r) \); see Equation (8).
Figure 7. Helioseismic and solar neutrino observables predicted by the AGSS09met solar model (black) and by the solar model providing the best fit to all observational constraints with (blue) and without (red) taking into account the pulls of systematical errors.

(A color version of this figure is available in the online journal.)

Figure 8. Same as Figure 7, but assuming $\xi_{\text{opa}} \equiv 0$.

(A color version of this figure is available in the online journal.)

Table 5

| $\tilde{\xi}_I$ | $\tilde{\xi}_{I\delta I}$ | $\tilde{\xi}_{I\delta I}$ |
|-----------------|-----------------|-----------------|
| Opa             | Age             | Lum             | Diffu           | $S_{11}$ | $S_{13}$ | $S_{17}$ | $S_{14}$ | $S_{34}$ | $S_{7}$ |
| 1.07            | 0.03            | −0.41           | −0.74           | ∼0       | 0.46    | −0.97    | 0.32     | −1.20    | ∼0     |
| $\tilde{\xi}_{I\delta I}$ | $\tilde{\xi}_{I\delta I}$ | $\tilde{\xi}_{I\delta I}$ |
| 1.07 ($\kappa_{\text{OPAL}}/\kappa_{\text{OP}} - 1$) | ∼0 | −0.0016 | −0.11 | ∼0 | 0.024 | −0.05 | 0.007 | −0.09 | ∼0 |

Notes. All the available observational information are simultaneously fitted. The entries with $\sim$0 are smaller (in magnitude) than $10^{-2}$ in the first line and $10^{-3}$ in the second line.

5. The CNO neutrino fluxes are expected to be $\sim$50% larger than those predicted by SSMs implementing AGSS09met composition. Indeed, solar models providing a good fit to the observational data give $\Phi_N \simeq 3.4 \times 10^8$ cm$^{-2}$ s$^{-1}$ and $\Phi_O \simeq 2.5 \times 10^8$ cm$^{-2}$ s$^{-1}$ as a combined effect of the changes in composition and, to a minor extent, of the systematic shifts in the input parameters. These values are even larger than predictions obtained by assuming GS98 surface composition. However, this result depends on the assumed heavy element grouping. The CNO neutrino fluxes, in fact, are essentially determined by the carbon abundance, see Table 4, while the observational data included in our analysis are basically sensitive to the oxygen content of the Sun, since this element provides a large contribution to the solar opacity.

In Figure 6, we also include for reference the solar oxygen and iron abundances determined by Caffau et al. (2011) shown by the blue points labeled “CO$^9$BOLD.” Unfortunately, Caffau et al. (2011) have not determined the solar photospheric Si abundance, so a direct comparison of their iron abundance with AGSS09met or GS98 cannot be made because it is not possible to construct a meteoritic solar abundance scale. It is, however, interesting to point out that, albeit based on 3D solar atmosphere models with very similar structure (Beeck et al. 2012), the solar abundances they derive are closer to our best-fit values than AGSS09met, particularly for oxygen.

5.2. Three-parameter Analysis

It is important to discuss how the above results change when the neon-to-oxygen ratio is allowed to vary, since Ne lacks photospheric features and the Ne/O ratio has to be inferred indirectly from solar wind measurements. In this
assumption, the $\chi^2$ is described as a function of three parameters ($\delta z_{\text{CNO}}, \delta z_{\text{Ne}}, \delta z_{\text{met}}$) that can be adjusted independently to reproduce helioseismic and solar neutrino constraints. In order to prevent unphysical results, we add a penalty function to the $\chi^2$ given by

$$\chi^2_{\text{pen}} = \left[ \frac{\delta z_{\text{Ne}} - \delta z_{\text{CNO}}}{\Delta (1 + \delta z_{\text{CNO}})} \right]^2,$$

where $\Delta = 0.3$, that forces the neon-to-oxygen ratio to the value prescribed by AGSS09met compilation with a 1σ accuracy equal to 30%, as has been observed by Bahcall et al. (2005). The bounds obtained by considering all the available observational constraints are shown in Figure 9. The best-fit composition is

$$\begin{align*}
\delta z_{\text{CNO}} &= 0.37 \pm 0.07 \\
\delta z_{\text{Ne}} &= 0.80 \pm 0.26 \\
\delta z_{\text{met}} &= 0.13 \pm 0.05,
\end{align*}$$

which corresponds to $\varepsilon_0 = 8.83 \pm 0.02$, $\varepsilon_{\text{Ne}} = 8.19 \pm 0.06$, and $\varepsilon_{\text{Fe}} = 7.50 \pm 0.02$. These values are still consistent at $\sim 1\sigma$ with those obtained in the GS98 compilation. However, the errors in the inferred abundances are larger than before. We note, in particular, that the neon abundance is bounded at the level of accuracy prescribed by the function (30) indicating that the observational data are not effective in constraining it. The neon-to-oxygen ratio is increased by about $\sim 30\%$ with respect to the AGSS09met value. The quality of the fit, however, is not significantly improved being $\chi^2_{\text{min}}$/d.o.f. = 37.8/31 and the assumption $1 + \delta z_{\text{Ne}} = 1 + \delta z_{\text{CNO}}$ is allowed at 1σ.

The consequence of leaving neon as a free parameter is to introduce degeneracies between the various $\delta z_j$, as it is understood from inspection of Figure 9 and, in particular, by comparison of the left panel in Figure 9 and the lower left panel of Figure 6. It exists, in fact, a combination of $\delta z_{\text{CNO}}, \delta z_{\text{Ne}},$ and $\delta z_{\text{met}}$ that, taking also into account the effects of systematic pulls $\xi_i$, leaves substantially unchanged the observational properties of the Sun. An increase of the neon-to-oxygen ratio can be compensated for by a slight reduction of CNO and/or Heavy elements according to simple approximate formula

$$\begin{align*}
\delta z_{\text{CNO}} &= 0.45 - 0.19 \Delta z_{\text{CNO}} \\
\delta z_{\text{met}} &= 0.19 - 0.14 \Delta z_{\text{CNO}},
\end{align*}$$

where $\Delta z_{\text{CNO}} = \delta z_{\text{Ne}} - \delta z_{\text{CNO}},$ together with a re-adjustment of the systematic pulls:12

$$\begin{align*}
\xi_{\text{opa}} &= 1.07 + 0.76 \Delta z_{\text{CNO}} \\
\xi_{\text{diff}} &= -0.74 - 0.41 \Delta z_{\text{CNO}} \\
\xi_{\text{h1}} &= 0.46 - 0.28 \Delta z_{\text{CNO}} \\
\xi_{\text{h3}} &= -0.97 + 0.58 \Delta z_{\text{CNO}},
\end{align*}$$

From the above relations, we see that a 30% uncertainty in the neon-to-oxygen ratio roughly corresponds to $\sim 6\%$ and $\sim 4\%$ errors in the inferred CNO and heavy element abundances.

This degeneracy can be discussed at a more basic level by considering that the main effect produced by a change of the solar composition is the modification of the opacity profile of the Sun. The source term $\delta \kappa(r)$ that drives the modification of the solar properties and that is probed by observational data can be written as the sum of two contributions (Villante 2010):

$$\delta \kappa(r) = \delta \kappa_1(r) + \delta \kappa_2(r).$$

The intrinsic opacity change, $\delta \kappa_1(r)$, represents the fractional variation of the opacity along the SSM profile and it is given, in our approach, by $\delta \kappa_1(r) = \xi_{\text{opa}} \delta \kappa_{\text{opa}}(r)$. The composition opacity change $\delta \kappa_2(r)$ can be approximately calculated as

$$\delta \kappa_2(r) \simeq \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \delta z_j$$

by using the logarithmic derivatives $\partial \ln \kappa/\partial \ln Z_j$ that are presented in the left panel of Figure 10. Taking advantage of relation (34), we calculate the effective opacity change $\delta \kappa(r)$ that corresponds to the models that provide a good fit to observational data. We see that $\delta \kappa(r)$ is well constrained by the available observational information. Opacity should be increased by $\sim$ few percent at the center of the Sun and by $\sim 25\%$ at the bottom of the convective envelope, as it was calculated by Villante (2010). The moderate increase at the solar center improves the agreement with $Y_s, \text{obs}$ without affecting the solar neutrino fluxes.

12 For simplicity, we report here only the systematic pulls that are sensitive the neon-to-oxygen ratio.
The increasing trend of $\delta \kappa (r)$ is required to fit the convective radius $R_\text{c}$ and sound speed profile $\delta c_1$ (see Villante 2010). The wavy behavior at intermediate radii improves consistency with inferred sound speed values in the region $0.3 < r/R_\odot < 0.6$. The general features of $\delta \kappa (r)$ are essentially independent on the assumptions about the opacity uncertainty. In this respect, the increase of the CNO and/or Ne content is interpreted as providing the “tilt” to $\delta \kappa (r)$ and is a solid conclusion of our analysis.

Figure 10, right panel, also compares the effective variations of opacity $\delta \kappa (r)$ obtained in the two and three parameter analysis. In particular, the black dashed line corresponds to the solar model with the composition given by Equation (31) and the value $\tilde{\xi}_\text{opa} = 1.40$ calculated from Equation (33). The red dotted line represents the effective opacity variation obtained with parameters given by Equation (28) and $\tilde{\xi}_\text{opa} = 1.07$. We see that the two lines coincide at the 2% level or better. From this, we infer that the reconstructed opacity profile does not depend on the assumed heavy element grouping. Moreover, we understand that the compositions (28) and (31) cannot be discriminated by the adopted observational constraints. More generally, they cannot be distinguished by any conceivable observational test that is dominated by the opacity profile in the radiative region of the Sun: the 2% difference is indeed smaller than the accuracy to which the opacity of the solar plasma is known. In summary, the neon-to-oxygen ratio cannot be effectively constrained with current data.

6. CONCLUSIONS AND PERSPECTIVES

In this work, we have investigated the properties of the Sun by using a statistical approach, normally adopted in other area of physics, in which the information provided by solar neutrino and helioseismic data can be combined in a quantitative and effective way. Namely, we have inferred the chemical composition of the Sun by using the helioseismic determinations of the surface helium abundance and of the depth of the convective envelope; the measurements of $^7$Be and $^8$B neutrino fluxes; and the solar sound speed profile inferred from helioseismic frequencies.

A consistent picture emerges from the combination of the different pieces of observational information which can be summarized as discussed in the following.

1. The surface composition prescribed by AGSS09met is excluded at a high confidence level, being the $\chi^2$/d.o.f. = 176.7/32 when all observational constraints are considered, unless the SSM’s chemical evolution paradigm is not correct and/or the opacity calculations are wrong.

2. A satisfactory fit to the available observational data ($\chi^2$/d.o.f. = 39.6/32) is obtained in the context of a two parameter analysis in which volatile (i.e., C, N, O, and Ne) and refractory elements (i.e., Mg, Si, S, and Fe) are grouped together and forced to vary by the same multiplicative factors. The abundance of volatile elements should be increased by $(45 \pm 4)$% while that of refractory elements should be increased by $(19 \pm 3)$% with respect to AGSS09met values.

3. If the neon-to-oxygen ratio is allowed to vary within the currently allowed range (i.e., $\pm 30\%$ at $1\sigma$), the best-fit composition is obtained by increasing by $(37 \pm 7)$% the CNO elements; by $(80 \pm 26)$% the neon; and by $(13 \pm 5)$% the refractory elements. The quality of the fit is, however, not significantly improved with respect to the two parameter analysis, being $\chi^2$/d.o.f. = 37.8/31.

By taking advantage of the adopted statistical approach, we were able to obtain few additional conclusions concerning the properties of the Sun which are discussed in the following.

4. Under the two and three parameter analyses, the CNO neutrino fluxes are expected to be substantially larger than those predicted by SSM implementing the AGSS09met surface composition, although the exact value cannot be predicted in a model independent way since it depends on the assumed heavy elements grouping. In particular, this stems from assuming a the same fractional variation between C, N, and O, a constraint that should be lifted when CNO neutrino fluxes are finally determined experimentally.

5. The sound speed in the region $0.3 < r/R_\odot < 0.6$ is better fitted by using the old OPAL opacity tables rather than the
more recent OP opacity table. Indeed, when we restrict our analysis to OP opacities, the quality of the fit is considerably decreased giving $\chi^2/d.o.f. = 66.9/32$.

6. The observational data prefer values for the input parameters of the SSMs that are slightly different from those presently adopted. Namely, the best fit is obtained by decreasing the diffusion coefficients by $\sim 10\%$ and the astrophysical factors $S_{\text{S}}$ and $S_{\text{T}}$ by $\sim 5\%$ and $\sim 9\%$, respectively, when all observational constraints are considered.

Our calculations are done in the context of the SSM in which chemical evolution is driven by nuclear reaction and microscopic diffusion only. It is known, however, that solar lithium is depleted relative to the meteoritic value by a factor of $\sim 160$ (Asplund et al. 2009), a result that is not reproduced within the standard assumptions. Lithium is easily destroyed in stellar interiors, so the most likely explanation is mild envelope mixing, probably tied to rotation (Pinsonneault 1997). The modeling of angular momentum transport and of associated mixing represents a complex problem in stellar physics that is not yet solved and that is affected by significant uncertainties (see Denissenkov et al. 2010 and Ogletorre & Garaud 2013 for recent work). However, the impact on the solar sound speed profile can be approximated by a modest reduction in the effective diffusion coefficient (see Richard et al. 1996; Bahcall et al. 2001; Turck-Chièze et al. 2010), an approach adopted by Delahaye & Pinsonneault (2006). In this paper, we do not include such an effect, but we naturally obtain a best-fit model with a reduction in the diffusion coefficients similar to that expected (between $\sim 15\%$ and $25\%$; see, e.g., Bahcall et al. 2001) in models with rotational mixing that reproduce Li depletion. Our results are therefore consistent with the existence of mild envelope mixing sufficient to reconcile the solar light element abundances with theoretical expectations.

Results presented in this work are obtained by using a simplified approach in which elements are lumped together in two or three groups, and they essentially follow from the fact that the opacity profile of the solar radiative region is well constrained by the combination of the different observational data, as it is shown by the gray band in the right panel of Figure 10. The current observational constraints do not allow for a finer discrimination of elements. A substantial improvement with respect to the present analysis could be provided by observational constraints where the degeneracy between opacities and composition is lifted. One such constraint has already been explored before and is linked to the sensitivity of the acoustic $p$-modes to the adiabatic index $\Gamma_1 = (\partial P/\partial \rho)_{\text{ad}}$. Results available in the literature are contradictory. Lin et al. (2007) concludes the metallicity of the solar envelope is comparable to that of GS98. However, using different techniques for constructing the solar envelope models and the inversion procedures, and also a different equation of state, Voronov et al. (2013) find a solar metallicity that is even lower than AGSS09met values. It is important to mention that $\Gamma_1 = (\partial P/\partial \rho)_{\text{ad}}$, while independent of radiative opacities, depends crucially on the details of the equation of state.

A second possibility is offered by the neutrino fluxes from the CN-cycle. While a detailed quantitative analysis will be presented elsewhere (F. L. Villante & A. M. Serenelli 2013, in preparation), it is important to stress at least qualitatively the importance of a CNO neutrino measurement. Even a low accuracy measurement, providing a direct determination of the metallicity of the solar core, permits to remove the degeneracy between opacity and composition effects. Let us imagine, e.g., to measure the CNO flux at the 20% level. If the detected fluxes were consistent with the expectations from our analysis (i.e., about 50% larger than the reference predictions), this would be sufficient to conclude that the AGSS09met surface abundances are wrong and/or the chemical evolution paradigm of the SSM is not correct. There would be no possibility to explain the observed results by assuming that opacity (or, more in general energy transport in the Sun) is not correctly described. On the contrary, if the detected fluxes were consistent with those predicted by solar models using AGSS09met admixture, then this would imply a tension with other observational constraints. This tension could be only explained by assuming that opacity calculations are wrong by a factor much larger than the presently estimated uncertainties. Both these results would have enormous implications for stellar evolution.

A.S. is supported by the MICINN grant AYA2011-24704 and by the ESP EUROCORES Program EuroGENESIS (MICINN grant EU11-2009-04170).

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