Subleading contributions to the width of the $D_{s0}^*(2317)$

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Abstract

We construct the effective chiral Lagrangian involving the $D$–mesons and Goldstone bosons at next-to-leading order taking into account strong as well as electromagnetic interactions. This allows us to disentangle — to leading order in isospin violation — the electromagnetic and the strong contribution to the $D$–meson mass differences. In addition, we also apply the interaction to the decay $D_{s0}^*(2317) \to D_s \pi^0$ under the assumption that the $D_{s0}^*(2317)$ is a hadronic molecule. We find $(180 \pm 110) \text{ keV}$ for the decay width $\Gamma(D_{s0}^*(2317) \to D_s \pi^0)$ — consistent with currently existing experimental constraints as well as previous theoretical investigations. The result provides further evidence that this decay width can serve as a criterion for testing the nature of the $D_{s0}^*(2317)$.

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1 Introduction

In recent years, many heavy mesons with open or hidden charm were discovered, which contribute to the revival of hadron spectroscopy (for recent reviews, see [1]). One outstanding example among them is the $D_{s0}^*(2317)$ discovered by the BABAR Collaboration in the $D_s \pi$ final state [2]. The measured mass of the $D_{s0}^*(2317)$ which is 2317.8 ± 0.6 MeV [3] is much lower than that predicted in many quark models. An appealing alternative is that the $D_{s0}^*(2317)$ is a hadronic molecule, which means that it owes its existence to meson–meson dynamics [4, 5]. In this work we exploit further this idea. Note, in Refs. [6] it was argued that a molecular interpretation of the $D_{s0}^*(2317)$ (and its vector counter part) is at variance with heavy quark effective field theory. However, this conclusion is based on the assumption that the decay of a hadronic molecule is proportional to the molecular wave function at the origin — in Ref. [7] it is shown that this assumption is not justified for the decay of hadronic molecules.

The mass alone is not a signal for a molecule, as stressed, e.g., in Ref. [8]. A consistent treatment of the mass and various decays is required — see also Ref. [9]. The difference from the molecular

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state will presumably be revealed in the decay pattern into various channels. However, no branching ratio of the $D_{s0}^*(2317)$ has been reported accurately. The only experimental constraints are upper limits for the ratios of some other decay channels to the $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$. For instance [3],

$$\frac{\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+(2112)^+ \gamma)}{\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0)} < 0.059.$$  \hfill (1)

The $D_{s0}^*(2317)$ can be dynamically generated from Goldstone boson–$D$-meson scattering as a hadronic molecule using unitarized amplitudes from chiral perturbation theory [10, 11, 12, 14]. The width of the isospin violating decay $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ was estimated to be about 8.7 keV in Ref. [12] by considering only the $\pi^0$–$\eta$ mixing in the final state. However, Faessler et al. pointed out that the mass differences between neutral and charged kaons and $D$–mesons give an important contribution [13], which was confirmed later in Ref. [14]. In Refs. [11, 14] also subleading operators were studied. Note that isospin symmetry violation in hadronic physics has two different sources: one originates from the mass difference of the light $u$ and $d$ quarks, and the other one stems from the electromagnetic (e.m.) interaction. While the $\pi^0$–$\eta$ mixing and part of the meson mass differences account for the former one, the effect of the latter on the decay $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ will be investigated here for the first time.

The first step is to construct the interaction Lagrangian to next–to–leading order in the chiral expansion. Based on this we can disentangle the e.m. and the strong contribution to the $D$–meson mass difference. As will be demonstrated, the Lagrangian also links these mass differences directly to the isospin–violating Goldstone boson–$D$-meson scattering amplitudes, in full analogy to the case of $\pi N$ scattering and the proton–neutron mass difference [15, 16]. We can therefore calculate the decay width of the $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ within the molecular picture utilizing the chiral Lagrangian up to the next-to-leading order (NLO) $\mathcal{O}(p^2)$, where $p$ denotes a small parameter — with a small number of free parameters. Note, in this paper for the first time both the strong and the e.m. contributions to the decay are incorporated systematically.

2 Lagrangians at next-to-leading order

The scattering between the Goldstone bosons and $D$–mesons is similar to the case for pion-nucleon scattering (for reviews, see Refs. [17, 18]) because the $D$–mesons have heavy masses which do not vanish in the chiral limit\(^{#1}\). We count the $D$–meson masses ($\sim 1.9$ GeV) as order $\mathcal{O}(\Lambda^\chi)$, where $\Lambda^\chi \simeq 1$ GeV. Hence the leading order terms in the chiral Lagrangian are of $\mathcal{O}(p)$, and the NLO terms are of $\mathcal{O}(p^2)$.

The leading order Lagrangian is just the kinetic energy term of the heavy mesons [19]

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D^\mu D^\dagger - m_D^2 D D^\dagger$$ \hfill (2)

with $D = (D^0, D^+, D_s^+)$ denoting the $D$–mesons, and the covariant derivative being

$$\mathcal{D}_\mu = \partial_\mu + \Gamma_\mu,$$

$$\Gamma_\mu = \frac{1}{2} \left( u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right),$$ \hfill (3)

where

$$U = \exp \left( \frac{\sqrt{2} i \phi}{F_\pi} \right), \quad u^2 = U.$$ \hfill (4)

\(^{#1}\)In this work we consider the SU(3) chiral limit, $m_u, m_d, m_s \rightarrow 0$. 

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The Goldstone boson fields are collected in the matrix

$$\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^- \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^0 \\
-\frac{1}{\sqrt{6}} \eta
\end{pmatrix}. \quad (5)
$$

We now consider the NLO chiral Lagrangian describing the interactions of the pseudoscalar charm mesons with the Goldstone bosons. Considering the heavy mesons as matter fields, similarly to the pion-nucleon sector [16], the strong part is

$$\mathcal{L}^{(2)}_{\text{str.}} = D (-h_0 \langle \chi_+ \rangle - h_1 \bar{\chi}_+ + h_2 \langle u_\mu u_\mu \rangle - h_3 u_\mu u^\mu) \bar{D} + \mathcal{D}_\mu D \langle h_4 (u_\mu u^\mu) - h_5 \{u_\mu, u^\nu\} - h_6 [u_\mu, u^\nu]\rangle \mathcal{D}_\nu \bar{D}. \quad (6)$$

The chiral symmetry breaking terms, i.e. $h_0$ and $h_1$ terms, have been introduced before [20] [11]. The $h_2$ and $h_3$ terms were introduced in Ref. [11]. We stress that the contributions of the $h_5$ and $h_3$ terms to $s$–wave amplitudes differ only to order $\mathcal{O}(p/m_D)$. However, we still keep them in our covariant formalism for in this way we have an additional tool to estimate the theoretical uncertainty — see section 4.2. The electromagnetic part is

$$\mathcal{L}^{(2)}_{\text{e.m.}} = F_\pi^2 D \left[ g_0 (Q_+^2 - Q_-^2) + g_1 \langle Q_+^2 - Q_-^2 \rangle + g_2 Q_+ \langle Q_+ \rangle + g_3 \langle Q_+ \rangle^2 \right] \bar{D}, \quad (7)$$

where

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi u,$$

$$\bar{\chi}_+ = \chi_+ - \frac{1}{3} \langle \chi_+ \rangle,$$

$$u_\mu = i u^\dagger \mathcal{D}_\mu U u^\dagger,$$

$$Q_\pm = \frac{1}{2} \left( u^\dagger Qu \pm u Qu^\dagger \right). \quad (8)$$

The quark mass matrix and the $D$–meson charge matrix are diagonal

$$\chi = 2B \cdot \text{diag} \{ m_u, m_d, m_s \}, \quad Q = e \cdot \text{diag} \{ 0, 1, 1 \},$$

in terms of $B = |\langle 0 | \bar{q} q | 0 \rangle|/F_\pi^2$ and the elementary charge $e$. Further, $F_\pi$ is the pion decay constant. The unknown coefficients $h_i \ (i = 0, 1, ..., 6)$ and $g_i \ (i = 0, 1, 2, 3)$ in Eqs. (6,7) are the so-called low energy constants (LECs). As we will show in the next section, $h_1$ and a linear combination of $g_0$ and $g_2$, namely $g_0 + 2g_2$, can be determined from the mass differences among the $D$–mesons. Since $\langle Q_+ \rangle = 2e$, the $g_3$ term in $\mathcal{L}^{(2)}_{\text{e.m.}}$ only gives an overall e.m. mass shift of the $D$–mesons, and hence can be absorbed in the bare masses. The $g_4$ term contains only isospin symmetric e.m. interaction and is irrelevant here. Terms with one more flavor trace in the strong interaction Lagrangian are suppressed in the large $N_C$ limit of QCD [21]. We therefore follow Ref. [14] and drop the $h_0$, $h_2$ and $h_4$. Formally the $h_6$ term is of $\mathcal{O}(p^2)$. However, due to the commutator structure, it is suppressed by one order, see Appendix A. We are therefore left with only two free, active parameters, both isospin conserving, namely $h_3$ and $h_5$. We will investigate their effect on the isospin violating decay of the $D_{s0}^*$ below.

One should note that the $\pi^0$ and $\eta$ in Eq. (5) are not mass eigenstates because of the $\pi^0$–$\eta$ mixing. The mass eigenstates are defined as

$$\tilde{\pi}^0 = \pi^0 \cos \epsilon_{\pi^0 \eta} + \eta \sin \epsilon_{\pi^0 \eta},$$

$$\tilde{\eta} = -\pi^0 \sin \epsilon_{\pi^0 \eta} + \eta \cos \epsilon_{\pi^0 \eta}, \quad (10)$$
where $\epsilon_{\pi^0\eta}$ is the well-known $\pi^0$–$\eta$ mixing angle, which reads to leading order
\begin{equation}
\epsilon_{\pi^0\eta} = \sqrt{3} \frac{m_d - m_u}{4} \frac{m_s - \hat{m}}{m_s - m_d}.
\end{equation}
with $\hat{m} = (m_u + m_d)/2$ the average mass of the $u$ and $d$ quarks.

3 $D$–meson mass differences

The terms which can contribute to the mass differences among the $D^+$, $D^0$ and $D_{s}^+$ mesons come from the Lagrangian of order $O(p^2)$, Eqs. (11,12). Only three among all the terms contribute. We find
\begin{align}
m_{D^0}^2 - m_{D^+}^2 &= \tilde{h}\lambda + \tilde{g},
m_{D^+}^2 - m_{D_{s}^+}^2 &= \tilde{h} \left(1 - \frac{\lambda}{2}\right),
\end{align}
where we define
\begin{equation}
\tilde{h} = 4Bh_1(m_s - \hat{m}), \quad \tilde{g} = F^2_\pi e^2(g_0 + 2g_2).
\end{equation}
The strength of isospin violation due to quark mass effects is encoded in the parameter $\lambda$ that is connected to $\epsilon_{\pi^0\eta}$ through
\begin{equation}
\lambda = \frac{m_d - m_u}{m_s - \hat{m}} = \frac{4}{\sqrt{3}} \epsilon_{\pi^0\eta}.
\end{equation}
A recent analysis of $\rho$–$\omega$ mixing in chiral perturbation theory gives $1/\lambda = 42 \pm 4$ [22], correspondingly we have $\lambda = 0.024 \pm 0.002$. Using the masses for the $D$–mesons [3], $m_{D^0} = 1864.84 \pm 0.17$ MeV, $m_{D^+} = 1869.62 \pm 0.20$ MeV, and $m_{D_{s}^+} = 1968.49 \pm 0.34$ MeV, we find
\begin{equation}
\tilde{h} = (384.1 \pm 2.5) \times 10^3 \, \text{MeV}^2, \quad \tilde{g} = (4 \pm 1) \times 10^3 \, \text{MeV}^2,
\end{equation}
where the largest uncertainty comes from the masses of the $D$–mesons. Note that here only the uncertainties from the experimental inputs are considered. For a discussion of the theoretical uncertainty, see Section [12] Then the dimensionless LECs $h_1$ and $g_0 + 2g_2$ can be determined as
\begin{equation}
h_1 = 0.42 \pm 0.00, \quad g_0 + 2g_2 = 11 \pm 3,
\end{equation}
where we use $B(m_s - \hat{m}) = (M_{K^0}^2 + M_{K^+}^2)/2 - M_{\pi^0}^2$. The basic assumption in setting up an effective field theory is the naturalness of the low energy constants — especially the dimensionless coefficient $h_1$ should be of order one. This is indeed the case for we find $h_1 = 0.42$. A naturalness estimate for $g_0$ and $g_2$ comes from requiring that the contribution of the corresponding operators to the $D$–meson mass shift should be of the order of a typical virtual photon loop [23], thus
\begin{equation}
e^2 F^2_\pi g_i \sim \left(\frac{e}{4\pi}\right)^2 m^2_D.
\end{equation}
This leads to $g_i \sim 4$ as a natural estimate of the order of magnitude, compatible with the value determined for $g_0 + 2g_2$.

The parameter $\tilde{h}$, fixed from the amount of $SU(3)$ violation encoded in the mass difference between the $D_{s}^+$ and the $D^+$ (see Eq. (12)), controls also the strong part of the $D^0$ and $D^+$ mass difference. Therefore, the electromagnetic contribution to this mass difference, which is given by
the $\bar{g}$ term, can be extracted from data. This is different to the case of, e.g., nucleons, where the operator structure is more complicated. We therefore get

\begin{align}
(m_{D^+} - m_{D^0})_{\text{str.}} &= (2.5 \pm 0.2) \text{ MeV}, \\
(m_{D^+} - m_{D^0})_{\text{e.m.}} &= (2.3 \pm 0.6) \text{ MeV},
\end{align}

where the first equation refers to the strong contribution to the mass difference and the second to its e.m. counterpart. Note, contrary to what is common for nucleons as well as kaons, here the electromagnetic and the strong effects enter with the same sign. This is a direct consequence of the different quark content of the states.

The strong and e.m. mass differences of $m_{D^+} - m_{D^0}$ are consistent with those determined long time ago by Gasser and Leutwyler using a simple quark model ansatz \cite{24}, which are $3.3 \pm 0.9$ MeV and $1.7 \pm 0.5$ MeV, respectively.

\section{Width of the $D_s^*(2317)$}

For studying isospin violating decays, it is better to work in the particle basis. For the scalar charm-strange sector, there are four channels involving a $D$–meson and a Goldstone boson: $D^0 K^+$, $D^+ K^0$, $D_s^+ \eta$ and $D_s^+ \pi^0$. One can expect that the isospin violating contributions to the mass of the $D_{s0}^*(2317)$ is negligible, therefore we only consider them for calculating the isospin violating decay width. The strong contribution to this decay is given in terms of the known $\pi^0 - \eta$ mixing angle and the meson mass differences. The only non-vanishing e.m. contribution is from the transition $D^0 K^+ \to D^+_s \pi^0$

\begin{equation}
V_{D^0 K^+ \to D^+_s \pi^0}^{\text{e.m.}} = -\frac{\sqrt{2}}{8} (g_0 + 2g_2)c^2.
\end{equation}

Especially, the amplitude for $D_s^+ \eta \to D_s^+ \pi^0$ vanishes. The linear combination of LECs ($g_0 + 2g_2$) has been determined from the $D$–meson mass differences in the previous section. Thus, all relevant isospin violating interactions are fixed from data.

\subsection{Unitarization of the scattering amplitudes at next-to-leading order}

A unitarization procedure was proposed in Ref. \cite{25} which can be used for any finite order in the chiral expansion. Similar to the case for pion-nucleon scattering, in our case, up to NLO there is no loop contribution. Hence we obtain the following $T$–matrix equation after matching to the chiral expansion at NLO

\begin{equation}
T(s) = V(s) [1 - G(s) \cdot V(s)]^{-1},
\end{equation}

with $V(s) = V_{\text{LO}}(s) + V_{\text{NLO}}(s)$ the sum of the S–wave scattering amplitudes of the LO and NLO orders \cite{25}. $G(s)$ is a diagonal matrix with the diagonal element given by the two-meson loop integral \cite{25, 26}

\begin{align}
G(s)_{ii} &= \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2 + ie) [(P - q)^2 - m_2^2 + ie]} \\
&= \frac{1}{16\pi^2} \left\{ a(\mu) + \ln \frac{m_2^2}{\mu^2} + \ln \frac{m_1^2 - m_2^2 + s}{2s} + \ln \frac{m_1^2 - m_2^2 + \sigma}{2s} \right\} \left[ \ln(s - m_1^2 + m_2^2 + \sigma) \\
&\quad - \ln(-s + m_1^2 - m_2^2 + \sigma) + \ln(s + m_1^2 - m_2^2 + \sigma) - \ln(-s - m_1^2 + m_2^2 + \sigma) \right],
\end{align}

where $a(\mu)$ is the subtraction constant, $\mu$ denotes the scale of the dimensional regularization, and $\sigma = \left\{ [s - (m_1 + m_2)^2][s - (m_1 - m_2)^2] \right\}^{1/2}$. In our analysis we use the somewhat lengthy
expression given above, for it allows for a straightforward analytic continuation into the complex plain, contrary to more compact representations that are applicable in a particular parameter space only.

The subtraction constant \(a(\mu)\) is determined by fitting the mass of the \(D^\ast_{s0}(2317)\) using the LO Lagrangian. It turns out to be \(a(\mu = 1 \text{ GeV}) = -1.846\) for reproducing \(m_{D^\ast_{s0}} = 2317.8\text{ MeV}\) [3]. Probably by accident, it coincides exactly with that obtained from matching the value of the loop function at the threshold of the \(D\) and \(K\) calculated by Eq. (22) with that calculated by using a 3-momentum cut-off \(q_{\text{max}} = m_\rho\) [12].

4.2 Results

In calculations, we take physical values of all the meson masses and the pion decay constant, as listed in the following [3]:

\[
\begin{align*}
F_\pi &= 92.42\text{ MeV}, \quad M_{\pi^0} = 134.98\text{ MeV}, \quad M_{\pi^+} = 139.57\text{ MeV}, \\
M_{K^0} &= 497.65 \pm 0.02\text{ MeV}, \quad M_{K^+} = 493.68 \pm 0.02\text{ MeV}, \\
m_{D^0} &= 1864.84 \pm 0.17\text{ MeV}, \quad m_{D^+} = 1869.62 \pm 0.20\text{ MeV}, \\
m_{D_s^+} &= 1968.49 \pm 0.34\text{ MeV}, \quad M_\eta = 547.51 \pm 0.18\text{ MeV}.
\end{align*}
\] (23)

After unitarization, the hadronic molecule \(D^\ast_{s0}(2317)\) appears as a pole in the second Riemann sheet at \((m_{D^\ast_{s0}} - i\Gamma(D^\ast_{s0} \to D_s\pi^0)/2)\). Denoting the three-momentum of one particle in the center-of-mass frame of channel \(i\) by \(k_i\), the second Riemann sheet is specified by \(\text{Im}k_{D^s_{i} \pi^0} < 0\) and \(\text{Im}k_i > 0\) \((i = D^0K^+, D^+K^0, \ D^s_s \eta)\). First, let us focus on the results considering the LO amplitudes only. There are two different kinds of contributions to the isospin violating decay width at leading order. One is from the \(\pi^0 - \eta\) mixing and the other one is from the mass differences between charged and neutral kaons and \(D\)-mesons, which predominantly enters through an isospin violating contribution to the loop function of Eq. (22). However, \(\epsilon_{\pi^0\eta} = 1/\sqrt{3}B(m_d - m_u)/(M_\eta^2 - M_{\pi^0}^2)\) is suppressed by \(M_\pi^2/M_\eta^2\) — in SU(2) chiral perturbation theory, where the strange quark is also viewed as heavy, this operator appears only at NNLO, even an order below those given in Eqs. (6) and (7). Therefore, one can expect that the mass differences give a larger contribution. The results confirm this expectation as shown in the second and third column in Table 1, corresponding to the widths considering only the \(\pi^0 - \eta\) mixing and meson mass differences, respectively. Furthermore, similar to Refs. [13] [14], in our calculation the interference between these two kinds of contributions are constructive, giving rise to a width of about 150 keV — see the first column of Table 1. However, when it comes to a quantitative comparison, the result for the width to leading order of Ref. [14] is smaller by a factor of two — a direct comparison with the more phenomenological work of Ref. [13] is not possible. The difference can be traced to differences in the input parameters and a different method to fix the subtraction constant \(a(\mu)\) of Eq. (22). Those differences should be of higher order. Thus the spread in the reported results calls for a calculation to next-to-leading order in the chiral expansion, c.f. Ref. [14], together with an analysis of the uncertainties.

We take \([-1, 1]\) as a natural range for the dimensionless parameter \(h_5' \equiv h_5/m_{\pi^0}^2\) — note that for \(h_5' = \pm 1\) the contribution of the \(h_5\) term to the \(D^0K^+ \to D^0K^+\) scattering amplitude is of the

| LO \(\pi^0 - \eta\) mixing | Mass differences |
|---------------------------|------------------|
| 149.4                     | 69.7             |

Table 1: Decay widths of the \(D^\ast_{s0}(2317) \to D_s\pi^0\) with LO amplitudes. All units of the decay widths are in keV.
same order as the leading one. The subtraction constant is kept fixed to $a(1\,\text{GeV}) = -1.846$. The value of $h_3$ is then determined from fitting the pole position in the second Riemann sheet to the mass of the scalar charm meson $m_{D_s^0} = 2317.8 \pm 0.6\,\text{MeV}$ in the isospin symmetric case. For each value of $h_5'$, there is a corresponding $h_3$, as shown in Fig. 1.

For each value of $h_5'$ the resulting widths are plotted in Fig. 2. In the figure, the solid line is the result for producing the mass of the $D_{s0}^*(2317)$ at 2317.8 MeV with both the strong and e.m. isospin violating contributions and using central values for all the parameters. The dashed curve represents the result without e.m. contributions. The uncertainties from the experimental inputs are reflected in the shaded band. To estimate the theoretical uncertainty observe that, as stressed above, the effect of the $h_3$ and the $h_5$ term differ at $O(p/m_{D_s})$. Therefore, since one of the two is fixed already from the mass of the $D_{s0}$, the dependence of the width on $h_5$ is a measure of (some) NNLO effects. We may read off the figure directly a spread of about 50 keV around the central value of 180 keV induced by the variation of $h_5$. To be on the safe side we take as the theoretical uncertainty of our calculation twice this spread. Another way to estimate the theoretical uncertainty is to use $2(M_K/L_\chi)^2 \approx 50\%$, since we include contributions to the amplitude up to next–to–leading order in $(M_K/L_\chi)$ in our calculation. The factor of 2 appears since the width is proportional to the square of the amplitude. It is reassuring that both methods lead to essentially similar numbers for the uncertainty. Our final result therefore reads

$$\Gamma(D_{s0}^*(2317)^+ \to D_s^+\pi^0) = (180 \pm 40 \pm 100)\,\text{keV},$$

where the first error is from experimental inputs and the second reflects the theoretical uncertainty.

Amongst the former uncertainties, the largest ones are from the uncertainties of the $D$–meson masses and the $\pi^0$–$\eta$ mixing parameter $\lambda = 0.024 \pm 0.002$. Schematically, let us consider the case corresponding to $h_5' = 1$. The central result for the width is 233 keV. When the central values of all the meson masses are taken, the width can change from 219 keV for $\lambda = 0.022$ to 248 keV for...
\( \lambda = 0.026 \). When we take the central value \( \lambda = 0.024 \) and the central values of the masses of all the mesons except \( D^0 \) and \( D^+ \), the width can change from 233 keV for taking the central values of \( m_{D^0} \) and \( m_{D^+} \) to 249 keV for taking \( m_{D^0} = 1864.65 \) MeV and \( m_{D^+} = 1869.82 \) MeV. Among all the others, the uncertainties caused by \( g_0 + 2g_2 \), see Eq. (16), and the masses of kaons are the largest, and they amount to an uncertainty of 3 keV and 1 keV at most, respectively. All other terms give negligible contributions to the uncertainty.

Within the molecular picture, different calculations gave the width of the radiative decay \( D^*_{s0}(2317) \to D_s^* \gamma \) in the range from 1 – 6 keV [27, 13, 14]. Combining with the experimental result in Eq. (11), the lower limit for the width of the decay \( D^*_{s0}(2317) \to D_s \pi^0 \) is of the order of 100 keV. Our result is compatible with this extracted lower limit.

### 5 Summary and Outlook

In this paper we investigate the isospin violating decay \( D^*_{s0}(2317) \to D_s \pi \) up to NLO in the chiral expansion, assuming that the \( D^*_{s0}(2317) \) is a hadronic molecule. We take into account electromagnetic contributions systematically for the first time. Up to order \( \mathcal{O}(p^2) \), we obtain both the strong and e.m. mass differences of the \( D \)-mesons. We confirm that the mass differences between charged and neutral mesons in the same isospin multiplets play a significant role in the decay width. The decay width of the \( D^*_{s0}(2317) \to D_s \pi \) calculated to next–to–leading order is found to be \( 180 \pm 110 \) keV, where the uncertainties are added in quadrature. The uncertainty is dominated by the theoretical one. The resulting width is consistent with the present experimental constraint.

Our results for the hadronic decay width of the \( D^*_{s0}(2317) \) within uncertainty are consistent with...
previous analyses of the $D_{s0}^*(2317)$ within a molecular picture considering both the $\pi^0$–$\eta$ mixing and the meson mass differences: Ref. [13] gives 79.3 ± 32.6 keV, while Ref. [14] gives 76 keV (140 keV) as the result at leading (next–to–leading) order. Assigning the $D_{s0}^*(2317)$ to be a $c\bar{s}$ meson, its hadronic width was estimated within quark models with typical values of the order of 10 keV [28], although larger values were reported — see collection of results in table II of Ref. [13], which is consistent with the analysis utilizing heavy quark effective field theory [6]. Within the tetraquark picture a similar width as in the $c\bar{s}$ picture was found [29], which is about one order of magnitude smaller than our predictions in the molecular picture. We therefore conclude that the decay width of the $D_{s0}^*(2317)$ → $D_s\pi^0$ can be a good criterion for testing the nature of the $D_{s0}^*(2317)$. A simultaneous study of radiative decays within the various scenarios is also necessary, as advocated for the light scalar mesons in Ref. [30]. To expose the nature of the $D_{s0}^*(2317)$, experimental efforts are highly appreciated to improve the quantitative understanding of the $D_{s0}^*(2317)$ decays.

In this paper, we only considered the pseudoscalar $D$–mesons and the Goldstone bosons. The effects of all the higher states are incorporated in the LECs. However, the mass difference between $D^*$ and $D$ is only about 140 MeV, which is approximately equal to $(m_{\Delta} - m_N)/2$. As it is sometimes important to include the $\Delta(1232)$ in the chiral effective field theory for baryons (for a recent review, see Ref. [18]), it would be interesting to check what would happen if we include the vector charm mesons explicitly in the effective Lagrangian, as in Ref. [14]. This extension of the scheme will be investigated in the future.

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A Suppression of the $h_6$ terms

Due to the commutator structure in Eq. (3), all the $h_6$ terms in amplitudes are proportional to

$$ (p_1 \cdot p_2)(p_3 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3) ,$$

where $p_1$ ($p_2$) and $p_3$ ($p_4$) are the momenta of the heavy mesons (Goldstone bosons) in the initial and final state, respectively. Let $v$ denote the velocity of a heavy meson, we separate the momenta of the heavy mesons into two parts as

$$ p_1 = M_1 v + k_1,$$
$$ p_3 = M_3 v + k_3,$$

where $M_1$ and $M_3$ are the masses of the heavy mesons, and $k_1$ and $k_3$ are small residual momenta which are of order $\mathcal{O}(p)$. Let $p_3 = p_1 + \Delta p$ with $\Delta p = (M_3 - M_1) v + k_3 - k_1$, we have

$$ (p_1 \cdot p_2)(p_3 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3) = \Delta p \cdot [(p_1 \cdot p_2) p_4 - (p_1 \cdot p_4) p_2] .$$

Because $|M_3 - M_1| \simeq 100$ MeV at most, $\Delta p$ should be counted as $\mathcal{O}(p)$. Thus the above equation should be counted as $\mathcal{O}(p^3)$, and hence is suppressed by one more order.
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