Quantum mechanics and the continuum problem
(with referee reports)

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Abstract

It is shown that Feynman’s formulation of quantum mechanics can be
reproduced as a description of the set of intermediate cardinality. Prop-
erties of the set follow directly from the independence of the continuum
hypothesis.

Six referee reports of Physical Review Letters, Europhysics Letters,
and Journal of Physics A are enclosed.

The concept of discrete space is always regarded as a unique alternative to
continuous space. Nevertheless, there is one more possibility originated from
the continuum problem: since discrete space is a countable set, the set of inter-
mediate cardinality represents the golden mean between the opposing concepts.

But although the continuum problem has been solved [1], the status of the
set of intermediate cardinality is still unclear. The commonly held view is that,
according to the independence of the continuum hypothesis (CH), we can neither
prove nor refute existence of the intermediate set and, therefore, CH or its
negation must be taken as an additional axiom of standard Zermelo-Fraenkel set
theory (ZF). Thus we get two alternative set theories: with and without the set
of intermediate cardinality. The choice is a matter of experimental verification
analogously to the choice of true geometry caused by the independence of fifth
Euclid’s postulate. In other words, the independence of CH converts existence of
the intermediate set into a physical problem of determination of the set-theoretic
structure of space-time.

Assume that the set of intermediate cardinality exists and consider the maps
of the intermediate set $I$ to the sets of real numbers $R$ and natural numbers $N$.

Let the map $I \rightarrow N$ decompose $I$ into the countable set of mutually disjoint
infinite subsets: $\cup I_n = I$ ($n \in N$). Let $I_n$ be called a unit set. All members of
$I_n$ have the same countable coordinate $n$.

Consider the map $I \rightarrow R$. By definition, continuum $R$ contains a subset $M$
equivalent to $I$, i.e., there exists a bijection

$$f : I \rightarrow M \subset R.$$ (1)
This bijection reduces to separation of the intermediate subset $M$ from continuum. Since any separation procedure is a proof of existence of the intermediate set and, therefore, contradicts the independence of the continuum hypothesis, we, in principle, do not have any algorithm for assigning a real number to an arbitrary point $s$ of the intermediate set. Hence, any bijection can take the point only to a random (arbitrarily chosen) real number. Thus we have the probability $P(r)dr$ of finding the point $s \in I$ about $r$.

This does not mean that the intermediate set consist of random numbers or that the members of the set are in any other sense random. Each member of the set of intermediate cardinality equally corresponds to all real numbers untill the mapping has performed operationally. After the mapping, a concrete point gets random real number as its coordinate (image) in continuum. It is clear that we can get images of only finite number of points.

We see that the independence of CH may be understood as inseparability of the subset of intermediate cardinality from continuum. Recall that the intermediate set should be a subset of continuum by definition.

Thus the point of the intermediate set has two coordinates: a definite natural number and a random real number:

$$s : (n, r_{\text{random}}).$$

(2)

Only the natural number coordinate gives reliable information about relative positions of the points of the set and, consequently, about size of an interval. But the points of a unit set are indistinguishable. It is clear that the probability $P(r)$ depends on the natural number coordinate of the corresponding point.

For two real numbers $a$ and $b$ the probability $P_{a\cup b}dr$ of finding $s$ in the union of the neighborhoods $(dr)_a \cup (dr)_b$:

$$P_{a\cup b}dr \neq [P(a) + P(b)]dr$$

(3)

because $s$ corresponds to both (all) points at the same time (the elemental events are not mutually exclusive). In other words, the probability is inevitably non-additive. In order to overcome this obstacle, it is most natural to introduce a function $\psi(r)$ such that $P(r) = \mathcal{P}[\psi(r)]$ and $\psi_{a\cup b} = \psi(a) + \psi(b)$. The idea is to compute the non-additive probability from some additive object by a simple rule. It is quite clear that this rule should be non-linear. Indeed,

$$P_{a\cup b} = \mathcal{P}(\psi_{a\cup b}) = \mathcal{P}[\psi(a) + \psi(b)] \neq \mathcal{P}[\psi(a)] + \mathcal{P}[\psi(b)],$$

(4)

i.e., the dependence $\mathcal{P}[\psi(r)]$ is non-linear. We may choose the dependence arbitrarily but the simplest option is always preferable. The simplest non-linear dependence is the square dependence:

$$\mathcal{P}[\psi(r)] = |\psi(r)|^2.$$

(5)

We shall not discuss uniqueness of the chosen options. The aim of this paper is to show that quantum mechanics is, at least, one of the simplest and most natural descriptions of the set of intermediate cardinality.
The probability $P(r)$ is not probability density because of its non-additivity. This fact is very important. Actually, the concept of probability should be modified, since the additivity law is one of the axioms of the conventional probability theory (the sample space should consist of the mutually exclusive elemental events). But we shall not alter the concept of probability because it is not altered in quantum mechanics. This means that we shall regard $P(r) = |\psi(r)|^2$ as probability density, i.e., we accept Born postulate.

The function $\psi$, necessarily, depends on $n$: $\psi(r) \rightarrow \psi(n, r)$. Since $n$ is accurate up to a constant (shift) and the function $\psi$ is defined up to the factor $e^{i\text{const}}$, we have

$$\psi(n + \text{const}, r) = e^{i\text{const}} \psi(n, r).$$

Hence, the function $\psi$ is of the following form:

$$\psi(r, n) = A(r)e^{2\pi in}.$$  

Thus the point of the intermediate set corresponds to the function Eq. (6) in continuum. We can specify the point by the function $\psi(n, r)$ before the mapping and by the random real number and the natural number when the mapping has performed. In other words, the function $\psi(n, r)$ may be regarded as the image of $s$ in $R$ between mappings.

Consider probability $P(b, a)$ of finding the point $s$ at $b$ after finding it at $a$. Let us use a continuous parameter $t$ for correlation between continuous and countable coordinates of the point $s$ (simultaneity) and in order to distinguish between the different mappings (events ordering):

$$r(t_a), n(t_a) \rightarrow \psi(t) \rightarrow r(t_b), n(t_b),$$

where $t_a < t < t_b$ and $\psi(t) = \psi[n(t), r(t)]$. For simplicity, we shall identify the parameter with time without further discussion. Note that we cannot use the direct dependence $n = n(r)$. Since $r = r(n)$ is a random number, the inverse function is meaningless.

Assume that for each $t \in (t_a, t_b)$ there exists the image of the point in continuum $R$.

Partition interval $(t_a, t_b)$ into $k$ equal parts $\varepsilon$:

$$k\varepsilon = t_b - t_a, \quad \varepsilon = t_i - t_{i-1}, \quad t_a = t_0, \ t_b = t_k,$$

$$a = r(t_a) = r_0, \ b = r(t_k) = r_k.$$  

The conditional probability of of finding the point $s$ at $r(t_i)$ after $r(t_{i-1})$ is given by

$$P(r_{i-1}, r_i) = \frac{P(r_i)}{P(r_{i-1})},$$

i.e.,

$$P(r_{i-1}, r_i) = \left| \frac{A_i}{A_{i-1}} e^{2\pi i\Delta n_i} \right|^2.$$  

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where $\Delta n_i = |n(t_i) - n(t_{i-1})|$. 

The probability of the sequence of the transitions

$$r_0, \ldots, r_i, \ldots, r_k$$

is given by

$$P(r_0, \ldots, r_i, \ldots, r_k) = \prod_{i=1}^{k} P(r_{i-1}, r_i) = \left| \frac{A_k}{A_0} \exp 2\pi i \sum_{i=1}^{k} \Delta n_i \right|^2. \quad (13)$$

Then we get probability of the corresponding continuous sequence of the transitions $r(t)$:

$$P[r(t)] = \lim_{\varepsilon \to 0} P(r_0, \ldots, r_1, \ldots, r_k) = \left| \frac{A_k}{A_0} e^{2\pi im} \right|^2, \quad (14)$$

where

$$m = \lim_{\varepsilon \to 0} \sum_{i=1}^{k} \Delta n_i. \quad (15)$$

Since at any time $t_a < t < t_b$ the point $s$ corresponds to all points of $R$, it also corresponds to all continuous random sequences of mappings $r(t)$ simultaneously, i.e., probability $P[r(t)]$ of finding the point at any time $t_a \leq t \leq t_b$ on $r(t)$ is non-additive too. Therefore, we introduce an additive functional $\phi[r(t)]$. In the same way as above, we get

$$P[r(t)] = |\phi[r(t)]|^2. \quad (16)$$

Taking into account Eq.(14), we can put

$$\phi[r(t)] = \frac{A_N}{A_0} e^{2\pi im} = \text{const} e^{2\pi im}. \quad (17)$$

Thus we have

$$P(b, a) = \sum_{a \in r(t)} \text{const} e^{2\pi im} |^2, \quad (18)$$

i.e., the probability $P(a, b)$ of finding the point $s$ at $b$ after finding it at $a$ satisfies the conditions of Feynman’s approach (section 2-2 of [2]) for $S/h = 2\pi m$. Therefore,

$$P(b, a) = |K(b, a)|^2; \quad (19)$$

where $K(a, b)$ is the path integral (2-25) of [2]:

$$K(b, a) = \int_a^b e^{2\pi im} Dr(t). \quad (20)$$

Since Feynman does not essentially use in Chap.2 that $S/h$ is just action, the identification of $2\pi m$ and $S/h$ may be postponed.
In section 2-3 of \[2\] Feynman explains how the principle of least action follows from the dependence

$$P(b, a) = \left| \sum_{\text{all } r(t)} \text{const } e^{(i/\hbar)S[r(t)]} \right|^2. \quad (21)$$

We can apply the same reasoning to Eq.(18) and, for very large \(m\), get “the principle of least \(m\)”. This also means that for large \(m\) the point \(s\) has a definite stationary path and, consequently, a definite continuous coordinate. In other words, the corresponding interval of the intermediate set is sufficiently close to continuum (let the interval be called macroscopic), i.e., cardinality of the intermediate set depends on its size. Recall that we can measure the size of an interval of the set only in the unit sets (some portions of points).

Since large \(m\) may be considered as continuous variable, we have

$$m = \lim_{\varepsilon \to 0} \sum_{i=1}^{k} \Delta n_i = \int_{t_a}^{t_b} dn(t) = \min.$$ 

(22)

The function \(n(t)\) may be regarded as some function of \(r(t)\): \(n(t) = \eta[r(t)]\). It is important that \(r(t)\) is not random in the case of large \(m\). Therefore,

$$\int_{t_a}^{t_b} dn(t) = \int_{t_a}^{t_b} \frac{dn}{dr} \dot{r} \; dt = \min,$$

(23)

where \(\frac{dn}{dr} \dot{r}\) is some function of \(r, \dot{r}\), and \(t\). This is a formulation of the principle of least action (note absence of higher time derivatives than \(\dot{r}\)), i.e., large \(m\) can be identified with action.

Since the value of action depends on units of measurement, we need a parameter \(\hbar\) depending on units only such that

$$hm = \int_{t_a}^{t_b} L(r, \dot{r}, t) \; dt = S.$$ 

(24)

Finally, we may substitute \(S/\hbar\) for \(2\pi m\) in Eq.(20) and consider Feynman’s formulation of quantum mechanics as a natural description of the set of intermediate cardinality.

Of course, the large intermediate interval is not exact continuum. The principle of least action is an indication of intermediate cardinality of the interval. Exact continuum does not have the principle as its inherent property.

We see that if a point has sufficiently long countable path, then it has a stable continuous coordinate. In other words, sufficiently long, in the sense of the countable coordinate, interval of intermediate cardinality has stable length, unlike unstable length of a small interval, i.e., length of an intermediate interval becomes stabilized with increase in its cardinality.

Since length is establishment of equipotence \((r_1, r_2) \leftrightarrow R\), if we reduce size of a large intermediate interval \((r_1, r_2)\), its length becomes unstable and then
collapses when cardinality of the interval becomes sufficiently different from cardinality of continuum, i.e., we have three basic kinds of the interval:

Macroscopic interval. This interval is large enough to be regarded as continuous. It has stable non-zero length.

Microscopic interval. This interval may not be regarded as continuous. In other words, it has no length, i.e., its continuous image is exactly a point.

Submicroscopic interval. It is an intermediate kind of the interval with unstable random length. Submicroscopic intervals make the region of quantum mechanics.

Thus the intermediate set is a set of variable infinite cardinality. Taking into account that any infinite set should be equivalent to its proper subset, we get that the set should have constant cardinality ranges, i.e., intermediate cardinality changes stepwise. Addition of only large enough “portion” of points changes cardinality of an intermediate subset to the next level. It is reasonable to identify these portions with the unit sets.

Note that only sufficiently small intermediate interval manifests explicit features of intermediate cardinality (the instability of length). Such an interval may be called microscopic. Existence of the ranges of constant cardinality makes possible equivalence relations (symmetries) inside these ranges. This fact determines applicability of symmetry groups which is important for microscopic subsets because of absence of stable metrics and geometric structure. Outside a subset of certain cardinality its internal symmetry should be broken.

From macroscopic point of view, there are two kinds of points: the true points and the composite points. A composite point (the microscopic interval) consist of an infinite number of the true points. It is uniquely determined by the natural number of unit sets. Cardinality of the proper microscopic interval may be regarded as some qualitative property of the point (charge). If the interval is destroyed (decay of the corresponding point), this property vanishes and turns to the properties (cardinalities) of the output intervals.

Thus the description of the proper microscopic intervals reduces to the description of transmutation of expanded (non-local) but, at the same time, point-like objects and their properties.

We see that the complete description of the set of intermediate cardinality falls into a chain of three theories: classical mechanics, quantum mechanics, and the description of the proper microscopic (point-like) intervals whose properties are analogous to particle phenomenology (note that we do not import any structure or law into the intermediate set).

At present, all the descriptions are imbedded in the continuous space of classical mechanics. As a result, the dimensions and the directions of the submicroscopic and microscopic descriptions are lost.

The total number of space time dimensions of three 3D descriptions is ten. The same number of dimensions appear in string theories. But the extra dimensions of the intermediate set are essentially microscopic and do not require compactification.

The directions of the submicroscopic and microscopic descriptions are replaced with spin. Reliable separation of the descriptions needs careful exami-
nation but it may be preliminarily stated that integer spin is the direction of the microscopic description and half integer spin is the direction of the submicroscopic one. Since the submicroscopic interval is the (unstable) continuous interval, its direction is associated with the direction of the macroscopic continuous interval. Therefore, the submicroscopic direction (half integer spin) is not a vector of full value but only spinor.

Since microscopic intervals are substantially non-equivalent, the proper microscopic description should in turn split into several “asymmetric” parts (different theories) with additional extra dimensions down to the single unit set.

Fermions and bosons obey different statistics because they belong to the different descriptions. The Pauli exclusion principle is a condition for keeping inside the submicroscopic description (in other words, this is just a condition of conservation of submicroscopic cardinality): if two points at a submicroscopic distance come close enough, in the sense of the countable coordinates, they form the proper microscopic interval and go over to the proper microscopic description. In this case, some macroscopic and submicroscopic properties of the points of the interval may be lost.

Each microscopic scale should have analogous condition of conservation of its cardinality, i.e., the law of conservation of some qualitative property (charge). Violation of this law means conversion of initial cardinality into cardinality of another proper microscopic scale.

The physical description of nature falls into a collection of different theories steadily resisting unification. The complete description of the intermediate set exhibits the same tendency. This is a consequence of the inherent structural nonuniformity of the set. Thus instead of expected new fundamental principles, we get a new fundamental object. The different fundamental theories are the legitimate component parts of its complete description.

References

[1] Cohen P., Set theory and the continuum hypothesis, New York: W. A. Benjamin, 1966.
[2] Feynman R. P., Hibbs A. R., Quantum mechanics and Path Integrals, McGraw-Hill Book Company, New York, 1965.
[3] Landau L. D., Lifshitz E. M., Mechanics, Oxford; New York: Pergamon Press, 1976.

Supplement

All the papers on this subject
(Interpretation of quantum mechanics and the structure of physical space 1975),
Quantum phenomenology and the continuum problem,
Quantum mechanics and the continuum problem,
Quantum mechanics without interpretation,
The status of the set of intermediate cardinality)
were rejected by
Voprosy filosofii,
Kratkie so’obschenia po fizike,
Physical Review Letters,
Journal of Physics A,
Europhysics Letters,
Physics Letters A,
Foundation of Physics Letters,
The European Physical Journal C,
International Journal of Mathematics and Mathematical Sciences
Journal of Experimental and Theoretical Physics Letters
(Pis’ma v Zhurnal Eksperimental’noi i Teoreticheskoi Fiziki).
Available referee reports (Physical Review Letters, Europhysics Letters,
Journal of Physics A) are given below. Note that the e-print versions of the
papers differ from the more moderate versions sent to the journals. My remarks
I put into square brackets.

Physical Review Letters
Second report of Referee A \[after resubmission.- O.Y./\]
Fri, 31 Mar 2000
Report on “Quantum mechanics and the continuum problem”,
by O. Yaremchuk, LB7485.
Although the paper still contains aspects and assumptions that should be
clarified, which is somehow unavoidable for such an original proposal, I nevertheless noticed the author’s considerable effort in improving both the content and the presentation. Overall I consider this paper based on an underlying idea that, perhaps with some modification, deserves some attention as it relates several basic aspects in a rather new perspective. Thus, considered also the preliminary nature of the investigation, I recommend it for publication in PRL.

Referee C
Referee’s report:
The referenced manuscript raises the question: Is there a relationship between the continuum hypothesis and quantum mechanics? The continuum hypothesis was originally stated by George Cantor in the mid 19th Century \[in 1878 - O.Y./\]. It hypothesizes the non-existence of sets of intermediate cardinality between the cardinality of the natural numbers and the cardinality of the real numbers (the continuum). In 1963, the American mathematician Paul Cohen established the independence of the continuum hypothesis from the other axioms of Zermelo-Fraenkel set theory. As it is independent, the negation of the continuum hypothesis is consistent with Zermelo-Fraenkel set theory. It is the resulting set of ”intermediate” cardinality that the author wishes to connect to quantum mechanics.
This is an intriguing idea. It raises the possibility of two outcomes:

* The intermediate set can be used to inform us of quantum mechanics.
* Quantum mechanics can be used to give us a novel set theory.

It seems that the former is the possibility that the author tries to address. If he had done so, such a paper would warrant publication in Physical Review. Addressing the latter possibility would produce the paper on the philosophy of mathematics (not a physics paper).

It is my sad duty that the author does not reach his desired goal; i.e., he does not show that the existence of an intermediate set provides any insight (new or old) to quantum mechanics. His paper fails on at least five counts:

1. There is no discussion of the continuum hypothesis nor Zermelo-Fraenkel set theory. I have provided more in the first paragraph than this author manages in nine pages.
2. He does not establish what the intermediate set tells us about quantum mechanics; there is no insight given about quantum mechanics.
3. His reasoning is, at best, non-rigorous. [The rigor of the paper quite conforms to that of Feynman’s ‘Quantum mechanics and Path Integrals.’-O.Y.]
4. He uses unexplained novel methods.
5. He gets some things plain wrong.

I now provide examples of each of the five failures listed above:

As to the first count: At a minimum the author should have sited references for the relevant ideas in set theory. The two references he cites are both physics texts. [I added the well-known Cohen’s book.- O.Y.]

As to counts two and three: The closest that the author comes to providing insight is the following passage on page two:

"Then some subset cannot be separated from continuum if each point of the subset does not have its own peculiar properties but only combines properties of the members of the countable set and continuum.

At first sight, this seems to be meaningless. But the content of the requirement coincides with the content of wave-particle duality: quantum particle combines properties of a wave (continuum) and a point-like particle (the countable set)."

This statement comes out of the blue. Instead of telling us something about quantum mechanics he seems to be saying: "Quantum mechanics is weird; the intermediate set is weird; ergo, one must be related to the other." If anything, the author is trying to use quantum mechanics to provide insight to sets of intermediate cardinality, rather than the other way round. [I think that the analogy between wave-particle duality and intermediate cardinality is obvious.-O.Y.]

Another illustration of the lack of insight and rigor comes on page three in the following passage:

"The simplest non-linear dependence is a square dependence:

\[ P[\psi(r)] = |\psi(r)|^2. \] (25)"
What is his justification for saying this? I submit it is precisely because it is the mathematical choice that gives us quantum mechanics. Here I see no insight being provided in either direction; there is only a bald assertion that leads to the author desired result.

[Unfortunately, referee C does not inform us of his discovery of the simpler non-linear dependence. In fact, I did not make even this choice. This is a choice of the founders of quantum mechanics justified by experiment. I only show that their (perhaps arbitrary, perhaps uniquely determined) sequence of actions is suitable and quite natural in order to reproduce quantum mechanics as a description of the intermediate set. I have to prove that quantum mechanics describes the set of intermediate cardinality but it is not necessary to insist that this is a unique or the best way to describe the set.-O.Y.]

As to count four: A prime example is found on page three:

“Hence, any bijection can take a point of the intermediate set only to a random real number. If we do not have preferable real numbers, then we have the equiprobable mapping. This already conforms to the quantum free particle.”

I have no idea what is meant by any mapping (much less a bijection) to a “random real number.” I can describe the class of random number by algorithmic methods but I have no idea how to set up a mapping into this class.

As a mapping must involve a definite rule, I see no way to get what the author desires here. For him to then discuss an image under this mapping as having the form (natural number, random real) simply leaves me speechless. Maybe he has something in mind here; if so, he needs to explain it carefully and rigorously.

[Referee C seems to look for some subtle sense. In fact, the meaning is rather primitive. If one takes three balls out of the urn which contains ten balls (and if one is interested only in the number of members in the separated subset which is equivalent to absence of any selection rule), one establishes a bijection:]

$$(1, 2, 3) \rightarrow \text{(three random balls)} \subset \text{(ten balls)}.$$

If our urn contains ten black balls and ten white balls, we can take three white balls, that is, we get random balls with the non-random property. Analogously, we can control, theoretically, the natural number coordinate and cannot the real number one.-O.Y.]

As to count five: A prime illustration occurs on page eight:

“The total number of space time dimensions of three 3D descriptions is ten.

The number of spatial (not space-time) dimensions is ten. This is the most intelligible statement in the last two pages and he gets it wrong.

[This is a quotation of John H. Schwarz’s “Introduction to Superstring Theory,” hep-th/0008017 (p.7): “…that there are five distinct consistent string theorems, and that each of them requires spacetime supersymmetry in the ten dimensions (nine spatial dimensions plus time).” Such a mistake of referee and editor (George Basbas) of such a respectable journal seems to me inexplicable.-O.Y.]

This paper in no way qualifies for publication anywhere. I can not imagine any possible revision making this a publishable piece of work. This is a true
pity, as I think the author has the kernel of an interesting idea.

Europhysics Letters
G11661
"QUANTUM MECHANICS AND THE CONTINUUM PROBLEM"

REPORT A

The author assumes the existence of a set of intermediate cardinality, between continuous and discrete spaces. Then he derives a connection between this set and many features of quantum mechanics. In particular, he shows how a wave function can be introduced, and how it is related to a probability density. Moreover, some properties of the functional integral formulation of quantum mechanics are given the interpretation in this particular framework. The essential content of the paper is highly speculative, and it is not clear how the proposed framework could be useful in some concrete problem. However, the paper contains new ideas, quite stimulating. It could be interesting to let them be known to the scientific community. Therefore, I am inclined to recommend publication of this paper on Europhysics Letters, on the basis of its interesting and stimulating original conceptual content. On the other hand, due to the speculative character, this is a case where the decision involves directly the editorial policy of the Journal.

[It is interesting that discreteness and continuity are not considered as speculative in contrast to the mean concept.-O.Y.]

REPORT B

REPORT ON THE PAPER "QUANTUM MECHANICS AND THE CONTINUUM PROBLEM"

The paper proposes an approach to quantum mechanics based on the independence of the continuum hypothesis. The independence of the continuum hypothesis means that the statement "there does not exist a subset of the reals whose cardinality is intermediate between that of the integers and that of the reals" is not deducible from the axioms of set theory. One of the proofs consists in constructing a model of real numbers in which sets of intermediate cardinality exist (see for example Yu. Manin "A course in mathematical logic" Ch. III). My impression is that the author of the paper is not sufficiently familiar with the continuum problem.

[Not ‘one of the proofs’ but the only proof of the independence of CH consist in constructing models of ZF (not real numbers) with (Cohen) and without (Goedel) the set of intermediate cardinality. Yu. Manin only explains Cohen’s idea by model of real numbers (the language of real numbers is simpler than that of ZF). Unlike Goedel model, which is really ‘smallest set theory’ consisting only of constructible sets (constructible universe), the model with the ‘intermediate set’ is an unnatural extension of set theory. Of course, the real set of intermediate cardinality was not constructed contrary to the false conclusion of the referee. The incomprehension stated below also follows from this misunderstanding.-O.Y.]
The crucial sentence after eq. (1)
'Since any separation rule is a proof of existence of the intermediate set and, therefore, contradicts the independence of the continuum hypothesis, we, in principle do not have a rule for assigning a definite real number to an arbitrary point of the intermediate set.'
is wrong for the first part and meaningless for the second. All subsequent developments look to me confused and arbitrary and I am not able to see the pretended connection between quantum mechanics and the continuum problem.
The paper does not meet acceptable standards of scientific communication and I recommend rejection.

Journal of Physics A: Mathematical and General
FIRST REFEREE REPORT
A/113435/PAP
Quantum mechanics without interpretation
O Yaremchuk

1 As the configuration space for quantum mechanics, the author proposes the existence of a unique set of intermediate cardinality between the set of integers and the continuous space. Because of the independent of continuum hypothesis (CH), the negation of CH can be taken as an axiom. [The set of intermediate cardinality is not a configuration space.-O.Y.]

2 Because of independent of CH, the intermediate set can not be described by some formula. Therefore the author argues that the set as a subset of continuous space must have combined properties of the members of countable set and continuum, which coincide the content of wave particle duality in quantum mechanics.

3 The author gives an argument with which the properties of the intermediate set can be described by Feynman’s path integral method.

4 However the referee thinks that the configuration space of quantum mechanics should be constructive and can be described mathematical formula. Philosophically the referee can not accept the author’s approach.

[Note that the reason for rejection of the paper is philosophical.-O.Y.]

SECOND REFEREE REPORT
Quantum mechanics without interpretation
O Yaremchuk
A/113435/PAP

The author suggests that the quantum wave-particle duality arises when a system has a phase space of cardinality intermediate between \( \aleph_0 \) (the cardinality of the integers) and \( C \) (the cardinality of the real numbers). He calls such sets “intermediate sets.” [This is very peculiar understanding which I cannot confirm. The intermediate set is not a phase space. Duality means that the intermediate set combines properties of the continuous and countable sets and have not any exclusive property, therefore, any description (and its phase space)
of any intermediate set contains only continuous and discrete (quantized) quantities. The non-mathematical concept of wave-particle duality was introduced into quantum theory because of absence of adequate mathematical notion. I try to explain this duality in terms of intermediate cardinality in order to establish the complete coincidence between the the intermediate set and the mathematical object described by quantum mechanics.-O.Y.

He implicitly proposes that Bohr undecidability is a consequence of Goedel undecidability. This proposal has often been made, but has rarely resulted in printable papers. [One reference would be enough.-O.Y.] Yaremchuk carries it further than anyone else has, so far as I know. [Anonymous referee makes anonymous references. No one else ever stated that quantum mechanics described the set of intermediate cardinality. Moreover, my first attempt to discuss relationship between quantum mechanics and the continuum problem with mathematicians in sci.math (deja news) caused the following response: extremely silly...keep your idiocy off my mail box...-O.Y.]

His title is exactly the opposite of his paper. What he proposes is exactly what those who speak of “interpretations” of quantum mechanics mean. [Interpretation is necessary in order to explain what is really described by quantum mechanics. I start with the definite mathematical object which is sufficiently primitive to be regarded as real (in contrast to Hilbert space or operator algebra).-O.Y.] Undoubtedly the title responds to a recent paper of the same title by Peres et al. in Physics Today. [I tried to find the paper ‘of the same title,’ that is, “Quantum mechanics without interpretation.” I found only “Quantum mechanics needs no ‘interpretation’ ” by Fuchs and Peres. This paper presents very radical (‘empty’) interpretation.-O.Y.] But the point of Peres is that works like this paper are not necessary in order to make quantum theory satisfactory. I agree with Peres.

I do not share the paper’s goal or philosophy. In my opinion the author is misled by a superficial resemblance between the two kinds of undecidability. Goedel undecidability is a limit to the power of the postulational method, which generates fewer theorems than decidability requires. Bohr undecidability is a revision of the operator algebra. The author is simply clinging the old postulates — selective operation must commute, complete description must exist — in the face of new experience. The paper is of the same general class as papers showing that in spite of special relativity there might still be an absolute time, or paper showing that in spite of general relativity there might still be absolute acceleration. Such theories generally accomplish their goals by introducing quantities that allegedly “exist but are unobservable.” Here the unobservable is the point of the intermediate set. [I did not introduce the concepts of “observable” or “unobservable.” This blame may be rather laid on the founders of quantum mechanics. The “observable” principle of least action and quite “observable” quantum-mechanical formalism, which describes the behavior of the point, are the indications of intermediate cardinality of space to which the point belongs.-O.Y.]

Certainly the present paper does not succeed in deriving the basic principles of quantum kinematics from the hypothesis of the intermediate set. The key
formula (5) still has to be postulated, as it is in most presentations of quantum theory. In quantum theory (5) actually does not need to be postulated separately but follows from assumed simplicity of the operator algebra. The problems and peculiarities of quantum-mechanical way of description, which I must reproduce, and the problems with the intermediate set as a new object are regarded as weak points of my paper. In the new context of intermediate cardinality, it becomes obvious that postulate is not the best way to introduce the probability density. But this is not my idea. The problem needs careful analysis.-O.Y./

The author identifies physical processes like measurements with mathematical mappings like deductions. No evidence is offered for a structural similarity between measurements and deductions. Mathematical mapping is a general concept relating to any special case. Since any additional condition is not required, our reasoning is valid for any realization of mapping including measurement and natural phenomena without observers.-O.Y./

The author proceeds as if the wave-function ψ were a property of the system like coordinate or an electric field. In quantum theory it is not, but describes a process by which a system can be prepared or registered.

The identification (in the text between (3) and (4)) of the union of neighbourhoods with the addition of wave-functions ψ is a simple blunder. It is consistent with the author’s own understanding that the wave-function is only determined up to phase. [Note that no one of the other referees has paid attention to this “blunder.”]-O.Y./

The infinities introduced by this hypothesis make it unlikely that a convergent physical theory will result. It is possible that all the thermal infinities that quantum theory avoids will plague this theory.

I recommend that the paper be declined as not sufficiently correct. I would encourage the author to pursue his investigation of the physics of the intermediate sets until he produces a more rigorous study, with more attention to the roles and interpretations of the elements of quantum theory. His hypothesis is intriguing and will continue to arise until a definitive work settles it one way or the other. He has gone further with it than anyone else so far.

A minor points: The writing needs improvement. The articles “a” and “the” seem to occur rather randomly and impede the reading. The author is carried away by enthusiasm in the later pages of the paper, presenting his hopes as if they were facts. They should either be supported by proofs or presented as conjectures. The main difficulty is that intermediate cardinality seems intuitively unclear pure mathematical concept. Therefore, in the latter pages, I try to show that the intermediate set gives convincing informal picture and can generate the same peculiar concepts that have been introduced into distinct quantum theories for consistency with experiments (wave-particle, spin, two kinds of spin, exclusion principle, charges of particles, strings, extra dimensions, confinement, etc) and clarify physical meanings of all the concepts the only explanation of which is ‘absence of classical analog’. Detailed coincidence of informal pictures cannot be accidental, i.e., all the quantum phenomena unambiguously point to the set of intermediate cardinality.-O.Y./