Precision Predictions for Charged Higgs Boson Decays in the Real and Complex NMSSM

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We present the full next-to-leading order (NLO) supersymmetric (SUSY) electroweak and SUSY-QCD corrections to the decay widths of the charged Higgs boson decays into on-shell final states in the framework of the CP-conserving and CP-violating Next-to-Minimal Supersymmetric Model (NMSSM) of Ref. [1]. The newly calculated corrections have been implemented in the code \texttt{NMSSMCALCEW}. In these proceedings, we discuss the impact of the NLO corrections on the charged Higgs boson branching ratios in a wide range of the parameter space that is still compatible with the experimental constraints. We also investigate the effect of CP violation in these corrections.

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I. INTRODUCTION

The Standard Model (SM) has been established as a low-energy effective theory that describes electroweak symmetry breaking (EWSB). However, there exist several unsolved phenomena, such as neutrino oscillations, dark matter, baryon asymmetry of the Universe, and inflation. Since they are not explained within the framework of the SM, new physics is needed to address these puzzles. One among the models beyond the SM that solve the hierarchy problem is given by supersymmetry (SUSY) \cite{2-12}. Its simplest version is realized by the Minimal Supersymmetric SM (MSSM), where two Higgs doublet fields are required in order to ensure supersymmetry and to keep the theory anomaly-free. In this work, we focus on the Next-to minimal Supersymmetric SM (NMSSM) \cite{5, 13-27}. Besides the virtue of solving the so-called $\mu$ problem \cite{28}, the NMSSM disposes of an interesting Higgs boson phenomenology. Its Higgs sector contains a complex singlet field in addition to the two Higgs doublet fields which yields three CP-even Higgs bosons, two CP-odd Higgs bosons and two charged Higgs bosons after EWSB, implying interesting Higgs physics full of variety. Furthermore, the Higgs sector can be CP-violating already at the tree level unlike the MSSM.

So far the LHC experiments have not found any direct evidence of new particles beyond the SM, leading to lower bounds on the masses of new particles. In addition, measurements of the observables for the discovered Higgs boson (e.g. production cross sections, decay branching ratios and couplings) are consistent with the predictions of the SM within the current experimental uncertainties, meaning that the discovered Higgs boson behaves very SM-like. While the SUSY particles are required to be heavy enough to comply with the constraints from the direct searches they may still give significant contributions in the higher-order corrections to Higgs observables. In addition, the properties of the discovered Higgs boson will be measured very precisely in future collider experiments, such as the high-luminosity (HL-) LHC \cite{29}, the International Linear Collider (ILC) \cite{30}, the Future Circular Collider (FCC-ee) \cite{31}, or the Circular Electron Positron Collider (CEPC) \cite{32}. Hence, precise theoretical calculations are inevitable in order to be able to identify indirect signs of new physics. In the context of the NMSSM, higher-order corrections have been calculated to the NMSSM Higgs boson masses up to two-loop accuracy both in the CP-conserving and CP-violating NMSSM. For a recent overview, see \cite{33}. For the Higgs boson decays, full one-loop corrections to neutral Higgs boson decays into two gauge bosons and two fermions have been calculated in Ref. \cite{35} in the CP-violating NMSSM. The effects of Sudakov logarithms on fermionic decays of heavy Higgs bosons have been studied in Ref. \cite{36}. For Higgs-to-Higgs decays, not only the one-loop corrections \cite{37, 38}...
but also the two loop corrections with $O(\alpha_S \alpha_t)$ [39] have been studied. Furthermore, in Ref. [40], all on-shell two-body decays of neutral Higgs bosons have been evaluated at NLO electroweak (EW) and NLO SUSY-QCD order in the NMSSM with CP violation. In Ref. [41], the NLO EW corrections to charged Higgs boson decays into a $W$ boson and a neutral Higgs boson have been calculated and the gauge dependence, which arises from higher-order corrections to the mass and due to the resummed $Z$ factor of the external Higgs boson has been studied. This problem and possible strategies to preserve or restore gauge invariance has been further investigated in [42]. In Ref. [1], we completed the computation of the full NLO corrections to the on-shell two-body decays of the charged Higgs bosons in the NMSSM with CP violation. We evaluated the NLO SUSY-EW and SUSY-QCD corrections to the decays of the charged Higgs bosons into quarks, leptons, electroweakinos, squarks and sleptons for the first time.\footnote{In the previous implementation in the program \textsc{NMSSMCALC} [43], the charged Higgs decay widths included the state-of-the art QCD corrections to the decays into quarks and squarks as well as the resummed SUSY-QCD corrections and SUSY-EW corrections for the decays into quarks and leptons.} These new NLO corrections have been implemented in the new version of the program \textsc{NMSSMCalcew} [40]\footnote{The program is available at the URL: \url{https://www.itp.kit.edu/~maggie/NMSSMCalcew/}}. In these proceedings, we summarize our results obtained in Ref. [1], and discuss the sizes of the pure NLO corrections for the charged Higgs boson decays into quarks and electroweakinos. We furthermore show numerical results for the effects of CP violation on the loop-corrections to the decays into quarks.

II. THE LAGRANGIAN OF THE NMSSM

We briefly describe the Higgs potential of the NMSSM and the electroweakino sector in order to fix our conventions and input parameters. We work in the framework of the scale-invariant NMSSM with $Z_3$ symmetry. The tree-level Higgs potential obtained from the $F$- and $D$-terms of the supersymmetric Lagrangian and the soft SUSY-breaking Lagrangian is given by

$$V_H = |\lambda S|^2 \left( H_u^\dagger H_u + H_d^\dagger H_d \right) + |\epsilon_{ij} \lambda (H_u^i H_u^j) + \kappa S|^2$$

$$+ \frac{1}{8} g_2^2 \left( H_u^\dagger H_u \right)^2 + \frac{1}{8} \left( g_1^2 + g_2^2 \right) \left( H_u^\dagger H_u - H_d^\dagger H_d \right)^2$$

$$+ m_H^2 H_u^\dagger H_u + m_H^2 H_d^\dagger H_d + m_S^2 |S|^2 + \left[ -\epsilon_{ij} \lambda A_\lambda (H_d^i H_u^j) S + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.} \right],$$

where $\lambda$ and $\kappa$ are complex dimensionless parameters defined in the superpotential, and the corresponding terms proportional to $A_\lambda$ and $A_\kappa$ arise from the soft SUSY-breaking terms. The gauge couplings of the $U(1)_Y$ and $SU(2)_L$ symmetry are denoted by $g_1$ and $g_2$, respectively. The Higgs doublet fields $H_u$ and $H_d$ and the complex singlet field $S$ are expressed in terms of the component
fields and vacuum expectation values (VEVs) as

\[ H_u = e^{i\varphi_u} \left( \begin{array}{c} h^+_u \\ \frac{1}{\sqrt{2}}(v_u + h_u + ia_u) \end{array} \right), \quad H_d = \left( \begin{array}{c} \frac{1}{\sqrt{2}}(v_d + h_d + ia_d) \\ h^-_d \end{array} \right), \quad S = \frac{1}{\sqrt{2}}e^{i\varphi_s}(v_s + h_s + ia_s), \]

where \( v_u, v_d \) and \( v_s \) are the VEVs of \( H_u, H_d, \) and \( S \), respectively. The EW VEV is given by \( v^2 = v_u^2 + v_d^2 \approx (246 \text{ GeV})^2 \). The two CP-violating phases \( \varphi_u \) and \( \varphi_s \) denote the phase differences between the VEVs. The neutral component fields are transformed into the mass eigenstates through the orthogonal rotation matrix \( \mathcal{R} \),

\[ (h_1, h_2, h_3, h_4, h_5, G^0) = \mathcal{R}(h_d, h_u, h_s, a_d, a_u, a_s)^T, \]

where \( G^0 \) is the neutral Nambu-Goldstone (NG) boson. Similarly, the mass eigenstates for the charged Higgs bosons are obtained by,

\[ \left( \begin{array}{c} G^+ \\ H^+ \end{array} \right) = \left( \begin{array}{cc} -c_\beta & s_\beta \\ s_\beta & c_\beta \end{array} \right) \left( \begin{array}{c} h^+_d \\ h^+_u \end{array} \right), \]

where the mixing angle \( \beta \) is defined by \( \tan \beta = v_u/v_d \).

The tree-level Higgs sector of the CP-violating NMSSM is described by eighteen independent input parameters which we choose as

\[ m^2_{H_d}, m^2_{H_u}, m^2_S, M^2_W, M^2_Z, e, \tan \beta, v_s, \varphi_s, \varphi_u, |\lambda|, |\varphi_\lambda|, |\kappa|, \varphi_\kappa, \text{Re}A_\lambda, \text{Im}A_\lambda, \text{Re}A_\kappa, \text{Im}A_\kappa. \]

Here the three Lagrangian parameters \( g_1, g_2 \) and \( v \) have been replaced by the three physical observables \( M_W, M_Z \) and the electric coupling \( e \). The effective \( \mu \) parameter can be expressed as

\[ \mu_{\text{eff}} = |\mu_{\text{eff}}|e^{i\varphi_\mu} = \frac{|\lambda|v_s}{\sqrt{2}}e^{i(\varphi_\lambda + \varphi_s)}. \]

The real part of \( A_\lambda, \text{Re}A_\lambda \), can be replaced by the charged Higgs boson mass \( M^2_{H^\pm} \) through

\[ M^2_{H^\pm} = M^2_W + \frac{\lambda v_S}{\sin(2\beta)} \left( \sqrt{2} \text{Re}A_\lambda + \kappa v_S \right) - \frac{\lambda^2 v^2}{2}. \]

The soft SUSY-breaking parameters \( m^2_{H_d}, m^2_{H_u}, m^2_S, \text{Im}A_\lambda, \text{Im}A_\kappa \) are fixed by the tadpole conditions for the five neutral Higgs bosons.

Adding the superchiral singlet field \( \hat{S} \) introduces an additional degree of freedom, the singlino \( \tilde{S} \), in the electroweakino sector. This results in five neutralino mass eigenstates, which are related to the gauge eigenstates via the 5×5 unitary matrix \( N \),

\[ \chi^0 = N\psi^0, \]
with \( \psi^0 = (\tilde{B}, \tilde{W}^3, \tilde{H}_d, \tilde{H}_u, \tilde{S})^T \), where \( \tilde{B} \), \( \tilde{W}^3 \) stand for neutral gaugino states, \( \tilde{S} \) for the singlino state and \( \tilde{H}_d \), \( \tilde{H}_u \) are the Higgsino states. The chargino mass eigenstates in the basis of the Weyl spinors for the charginos, \( \tilde{\chi}^+_{L} = (\tilde{\chi}^+_{L1}, \tilde{\chi}^+_{L2})^T \), \( \tilde{\chi}^-_{R} = (\tilde{\chi}^-_{R1}, \tilde{\chi}^-_{R2})^T \), are obtained by rotating the spinors in gauge basis, \( \psi^-_{R} = (\tilde{W}^-, \tilde{H}^-_d)^T \), \( \psi^+_{L} = (\tilde{W}^+, \tilde{H}^+_u)^T \), as

\[
\tilde{\chi}^+_{L} = V \psi^+_{L}, \quad \tilde{\chi}^-_{R} = U \psi^-_{R},
\]

with the \( 2 \times 2 \) unitary matrices \( U, V \). The specific expressions for the mass matrices of the neutralinos and charginos in terms of the input parameters can be found in Eqs. (17) and (21) of Ref. [1]. By definition, they are diagonalized by the unitary matrices \( N, U \) and \( V \).

**III. THE CHARGED HIGGS BOSON DECAY WIDTHS INCLUDING HIGHER-ORDER CORRECTIONS**

In Ref. [1], we evaluated the NLO SUSY-EW corrections and SUSY-QCD corrections to the two-body on-shell decay widths of the charged Higgs bosons, i.e., \( H^+ \to t\bar{b} \), \( H^+ \to \nu\bar{\tau} \), \( H^+ \to \chi^+_i \chi^0_j \) \((i = 1, 2, j = 1, \ldots, 5)\), \( H^+ \to \tilde{t} \tilde{b} \) and \( H^+ \to \tilde{\tau} \tilde{\nu} \). In order to get UV finite results, we used a mixed OS and \( \overline{\text{DR}} \) renormalization scheme for the Higgs sector and the OS scheme for the gauge sector and the SM fermion sector. For the renormalization of the electroweakino, the squark and the slepton sectors, both the OS scheme and the \( \overline{\text{DR}} \) scheme were utilized. In this section, we give schematic formulae for the partial decay widths into \( t\bar{b} \) and electroweakinos, including the higher-order corrections.

For the decay \( H^+ \to t\bar{b} \), the loop-corrected partial decay width can schematically be written as

\[
\Gamma(H^+ \to t\bar{b})^{\text{NLO}} = \Gamma_{H^+ \to t\bar{b}}^{\text{LO imp}} + \Gamma_{H^+ \to t\bar{b}}^{\text{SUSYQCD}} + \Gamma_{H^+ \to t\bar{b}}^{\text{SUSYEW}}.
\]

The improved leading-order (LO) contribution \( \Gamma_{H^+ \to t\bar{b}}^{\text{LO imp}} \), which already includes the state-of-art QCD corrections and the resummed SUSY-QCD and SUSY-EW corrections, is given in terms of \( \mu_t = m_t^2/M_{H^\pm}^2 \) and \( \mu_b = m_b^2/M_{H^\pm}^2 \) by

\[
\Gamma_{H^+ \to t\bar{b}}^{\text{LO imp}} = \frac{3G_F M_{H^\pm}^2}{4\sqrt{2\pi}} |V_{tb}|^2 \beta^{1/2} \left[ (1 + 4 \frac{\alpha_s}{3 \pi}) \delta_{tb}^+ \right] \left( 1 - \mu_t - \mu_b \right) \left[ \mu_t \tan^2 \beta \left( 1 + 4 \frac{\alpha_s}{3 \pi} \delta_{tb}^+ \right) \right] + \mu_t \mu_b \tan^2 \beta R^2 \left( 1 + 4 \frac{\alpha_s}{3 \pi} \delta_{tb}^- \right) - 4 \mu_t \mu_b R \left( 1 + 4 \frac{\alpha_s}{3 \pi} \delta_{tb}^- \right),
\]

with

\[
\beta^{1/2}(x, y) = (1 - x - y)^2 - 4xy,
\]

\[
\frac{1}{\tan^2 \beta} (1 + 4 \frac{\alpha_s}{3 \pi} \delta_{tb}^+) = 1 - \mu_t - \mu_b.
\]
and where $G_F$ denotes the Fermi constant, $\alpha_s$ the strong coupling constant, and $V_{tb}$ the top-bottom CKM matrix element. Explicit expressions for the QCD correction factors $\delta_{tb}^+, \delta_{bt}^+$ and $\delta_{\bar{t}b}^-$ can be found in Ref. [44]. The universal SUSY-QCD and SUSY-EW corrections that are enhanced in the large $\tan \beta$ regime are resummed into effective bottom Yukawa couplings. These $\Delta_b$ corrections [44–52] are included in the decay width through the $R$ factor, $R = \frac{1}{1 + \Delta_b} \left[ 1 - \frac{\Delta_b}{\tan^2 \beta} \right]$. The explicit formula for $\Delta_b$ is given in Ref. [43]. The contributions $\Gamma_{H^+ \to t\bar{b}\gamma}^{\text{SUSY-QCD}}$ and $\Gamma_{H^+ \to t\bar{b}\gamma}^{\text{SUSY-EW}}$ correspond to one-loop SUSY-QCD and SUSY-EW corrections, respectively, where subtraction terms are added to avoid double counting arising from the $\Delta_b$ corrections. The SUSY-EW corrections contain, besides the pure vertex correction, the external leg corrections from the mixing between the charged Higgs boson and the charged Goldstone and the $W$ boson, respectively, and the real photon emission $\Gamma_{H^+ \to t\bar{b}\gamma}$. Specific expressions for these NLO corrections are given in Ref. [1].

The partial decay widths for the decays into electroweakinos are written in the same notation as

$$\Gamma(H^+ \to \chi^+_{i_1} \chi^0_{j_1})^{\text{NLO}} = \Gamma_{H^+ \to \chi^+_{i_1} \chi^0_{j_1}}^{\text{LO}} + \Gamma_{H^+ \to \chi^+_{i_1} \chi^0_{j_1}}^{\text{SUSY-EW}},$$

(13)

where $i = 1, 2$, $j = 1, \ldots, 5$. The LO decay width is given by

$$\Gamma_{H^+ \to \chi^0_{i_1} \chi^+_{j_1}}^{\text{LO}} = \frac{3^{1/2}}{16\pi} \left( \frac{\beta_{i_1} \beta_{j_1}}{M_{H^+}} \right)^{1/2} \left( 1 - \frac{\beta_{i_1} \beta_{j_1}}{M_{H^+}} \right) \left[ |g_{H^+ \chi^0_{i_1} \chi^+_{j_1}}^{\text{tree}}|^2 + |g_{H^+ \chi^0_{i_1} \chi^-_{j_1}}^{\text{tree}}|^2 \right],$$

(14)

with $\mu_i = M_{\chi^0_{i_1}}^2 / M_{H^+}^2$ and $\mu_j = M_{\chi^+_{j_1}}^2 / M_{H^+}^2$, where the tree-level couplings are given by

$$g_{H^+ \chi^0_{i_1} \chi^+_{j_1}}^{\text{Ltree}} = \frac{1}{\sqrt{2}} V_{j_1} \left( (g_1 N_{i_1} + g_2 N_{i_2}) c_\beta e^{i \phi_u} + \sqrt{2} \lambda^* N_{i_5} s_\beta \right) - g_2 e^{i \phi_u} V_{j_1} N_{i_4} c_\beta, \quad \text{and}$$

$$g_{H^+ \chi^0_{i_1} \chi^-_{j_1}}^{\text{Rtree}} = \frac{1}{\sqrt{2}} V_{j_2} \left( -g_1 N_{i_1} s_\beta - g_2 N_{i_2}^* s_\beta + \sqrt{2} \lambda^* N_{i_5} c_\beta \right) - g_2 U_{j_2} N_{i_3}^* c_\beta.$$

(15)

There are no NLO SUSY-QCD corrections, and the NLO SUSY-EW corrections comprise besides the vertex corrections the mixing between $H^+$ and the charged Goldstone and $W$ bosons, respectively, and the real photon emission $\Gamma_{H^+ \to \chi^+_{i_1} \chi^0_{j_1} \gamma}$. The concrete formulae are also given in Ref. [1].

IV. NUMERICAL RESULTS

A. Branching ratios for $H^+$ at LO

We show the branching ratios of the charged Higgs bosons at LO in Fig. 1 to illustrate the behavior as a function of the charged Higgs boson mass. In the plot, pure NLO corrections are switched
off for all decay processes. The input parameters for the Higgs sector and the electroweakino sector are fixed as

\[
\begin{align*}
\tan \beta &= 10.14, & |\lambda| &= 0.093, & |\kappa| &= -0.0821, \\
M_{H^+} &= 1500 \text{ GeV}, & \mu_{\text{eff}} &= -891 \text{ GeV}, & \Re A_\kappa &= -1.6 \text{ TeV}, \\
|M_1| &= 752 \text{ GeV}, & |M_2| &= 806 \text{ GeV}.
\end{align*}
\]

The soft SUSY-breaking parameters for the sfermions and the gluino, and the complex phases are fixed as

\[
\begin{align*}
|M_3| &= 2334 \text{ GeV}, & |A_t| &= 3.7 \text{ TeV}, & |A_b| &= 2 \text{ TeV}, & |A_\tau| &= 2 \text{ TeV}, \\
m_{\tilde{Q}_3} &= 1.43 \text{ TeV}, & m_{\tilde{U}_R} &= 1.93 \text{ TeV}, & m_{\tilde{D}_R} &= 2.16 \text{ TeV}, & m_{\tilde{L}_3} &= 1.22 \text{ TeV}, \\
m_{\tilde{\tau}_R} &= 1.21 \text{ TeV}, & m_{\tilde{t}_R, \tilde{c}_R} &= m_{\tilde{d}_R, \tilde{s}_R} = m_{\tilde{Q}_{1,2}} = m_{\tilde{L}_{1,2}} = m_{\tilde{e}_R, \tilde{\mu}_R} &= 3 \text{ TeV}, \\
\varphi_{M_1, M_2, M_3} &= \varphi_{A_t, A_b, A_\tau} = \varphi_{\mu} = \varphi_{\kappa} = 0.
\end{align*}
\]
In this plot, the decays into the neutral Higgs bosons plus the $W$ boson and the electroweakinos, respectively, are summed with respect to the final states as
\[
\text{BR}(H^+ \rightarrow W^+ H) \equiv \sum_{i=1}^{3} \text{BR}(H^+ \rightarrow W^+ H_i) + \sum_{j=1}^{2} \text{BR}(H^+ \rightarrow W^+ A_j),
\]
\[
\text{BR}(H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^0) \equiv \sum_{i=1}^{5} \text{BR}(H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^0).
\] (18)

Similarly, the decays into the slepton final states are summed up. The mass spectrum for this benchmark point is given by
\[
M_{H_1} = 123.33 \text{ GeV}, \quad M_{H_2} = 1.09 \text{ TeV}, \quad M_{H_3} = M_{A_1} = 1.50 \text{ TeV},
\]
\[
M_{A_2} = 1.93 \text{ TeV}, \quad M_{\chi_1^+} = 817.87 \text{ GeV}, \quad M_{\chi_2^+} = 927.87 \text{ GeV},
\]
\[
M_{\chi_1^0} = 747.69 \text{ GeV}, \quad M_{\chi_2^0} = 820.07 \text{ GeV}, \quad M_{\chi_3^0} = 894.20 \text{ GeV},
\]
\[
M_{\chi_4^0} = 929.24 \text{ GeV}, \quad M_{\chi_5^0} = 1.56 \text{ TeV}, \quad M_{t_1} = 1.38 \text{ TeV},
\]
\[
M_{b_1} = 1.43 \text{ TeV}, \quad M_{\tilde{t}_1} = M_{\tilde{b}_2} = M_{\tilde{\nu}_e} = 1.2 \text{ TeV}.
\] (19)

As can be inferred from the plot, for $M_{H^+} \lesssim 1.9$ TeV, the decay into the top-bottom final state is the main decay mode because of the $\tan \beta$ enhancement for the bottom Yukawa coupling. Most of the decay channels into the electroweakinos are kinematically open for $1.9 \lesssim M_{H^+}$ TeV, so that the branching ratios into electroweakinos become more important than those into top-bottom.

### B. Impact of the NLO Corrections on the Charged Higgs Branching Ratios

In this section, we discuss the impact of the NLO corrections on the branching ratios of the charged Higgs boson for the decays into the top and bottom quark pair and the electroweakino final states. In order to describe the size of the NLO corrections for the branching ratios as well as the partial widths of the decays $H^+ \rightarrow XY$, we introduce the following quantities,

\[
\Delta_{\text{BR}}(H^+ XY) = \frac{\text{BR}^{\text{NLO}}(H^+ \rightarrow XY) - \text{BR}^{\text{LO}}(H^+ \rightarrow XY)}{\max(\text{BR}^{\text{NLO}}(H^+ \rightarrow XY), \text{BR}^{\text{LO}}(H^+ \rightarrow XY))},
\] (20)

\[
\delta_{\Gamma}(H^+ XY) = \frac{\Gamma^{\text{NLO}}(H^+ \rightarrow XY)}{\Gamma^{\text{LO}}(H^+ \rightarrow XY)} - 1.
\] (21)

The relative change $\Delta_{\text{BR}}$ is normalized to the maximum of the branching ratio at NLO and LO, respectively, to avoid an enhancement due to the smallness of $\text{BR}^{\text{LO}}$. The notation ‘LO’ basically means that we refer to the old implementation in NMSSMCALC without the genuine SUSY-EW and
SUSY-QCD vertex corrections. This means that the LO quantities are calculated with ‘Higgs effective tree-level couplings’, meaning that the Higgs tree-level rotation matrix elements have been replaced by the loop-corrected ones. Furthermore, these ‘LO’ quantities also include the QCD corrections and resummed SUSY-EW and SUSY-QCD corrections in effective quark couplings as already implemented in the first release of NMSSMcalc [43]. Hence, the relative changes $\Delta_{\text{BR}}$ and $\delta_{\Gamma}$ quantify the effect of the pure NLO corrections that were newly evaluated in Ref. [1]. For the following scatter plots, we scanned the NMSSM input parameters in the ranges given in Table 1 together with the parameter settings in Eqs. (128) and (129) of Ref. [1] to understand the typical size of the NLO corrections in a wide range of the parameter space. We only retained points that are compatible with the LHC Higgs data. For details, see Ref. [1]. For the calculation of the NLO corrections, we used the OS1 renormalization scheme for the electroweakino sector, the DR scheme for the stop and sbottom sector, and the OS scheme for the stau sector. The definition of each scheme can be found in Ref. [40]

In the left panel of Fig. 2, we show the relative change $\Delta_{\text{BR}}$ for $H^+ \rightarrow t\bar{b}$ as a function of the branching ratio at NLO. The color code denotes the relative change of the decay width. As written in Eq. (10), the total NLO corrections for the partial width are determined by the sum of the SUSY-QCD and the SUSY-EW corrections. The latter gives negative corrections ranging between
FIG. 3. CP-violating phase dependence of the partial decay width for $H^+ \to t\bar{b}$ at LO (dashed lines) and NLO (solid lines) for the CP-violating phases of $\mu_{\text{eff}}$ (red), $A_t$ (blue) and $M_2$ (green). The lower panel displays the dependence of the relative change of the partial decay width $\delta \Gamma$.

$-15\%$ and $-2\%$, and the former remains between $-15\%$ and $7\%$ so that the relative change $\delta \Gamma$ is negative in most of the parameter region and the maximum size $|\delta \Gamma|$ is $29\%$. For this decay channel, the relative change of the branching ratio $\Delta_{\text{BR}}$ is basically correlated with that of the partial decay width.

In the right panel of Fig. 2, the relative change $\Delta_{\text{BR}}$ for the decay into the charged Wino and the Bino final state, $H^+ \to \tilde{W}^+ \tilde{B}$, is shown as a function of the corresponding branching ratio at NLO. We select the charged Wino-like chargino and Bino-like neutralino states from the electroweakino mass eigenstates. They are identified by the square of the mixing matrix elements, i.e. $|U_{i1}|^2$ and $|N_{j1}|^2$, which are required to exceed $0.5$ for $\tilde{W}^+$ and $\tilde{B}$, respectively. In the plot, we applied cuts for the left tree-level coupling of Eq. (16) as well as for the mass difference among the electroweakinos, because we found that artificially enhanced NLO corrections can appear without such a cut, which are discussed in Sec. 5.5 of Ref. [1]. As can be inferred from the plot, the relative change of the branching ratio is $|\Delta_{\text{BR}}| < 50\%$ for the bulk of the parameter points. On the other hand, if the size of the branching ratio at NLO becomes smaller than about $5 \times 10^{-2}$, outliers appear and the relative change $\Delta_{\text{BR}}$ can reach almost $-100\%$. These large corrections arise from large contributions due to the wave function renormalization constants (WFRCs) for electroweakinos, which are involved in $\Gamma_{\text{SUSYEW}}^{H^+ \to \chi_i^+ \chi_j^0}$ in Eq. (13). The WFRCs contains mixing contributions between another electroweakino state and the one in question, and they can be significant if the other electroweakino has a large coupling with the charged Higgs boson.
Before we close this section, we discuss how CP-violating phases can affect the NLO corrections to the decays of the charged Higgs boson. We just examine the theoretical behavior of the relative change of the partial width when the phase parameters are varied, and do not take into account experimental constraints by the measurement of the electric dipole moment (EDM) in this analysis. In order to illustrate the effect of CP violation, we show the partial decay width for $H^+ \rightarrow t\bar{b}$ as a function of complex phases for $\mu_{\text{eff}}$, $A_t$ and $M_2$ in Fig. 3. In the plot, the benchmark scenario for the CP-conserving case presented in Sec. 5.7 of Ref. [1] is used, and the three phases are varied individually. Note that we set the phase $\varphi_\lambda$ such that we do not encounter CP violation at tree level. The nevertheless observed subtle phase dependence on $\varphi_\mu$ and $\varphi_{A_t}$, respectively, originates from SUSY loop contributions that are resummed in the $\Delta_b$ corrections. Once the pure NLO corrections are included, we see a stronger dependence on the CP-violating phases. The relative change in the partial decay width can reach e.g. around $-10\%$ at $\varphi_\mu \simeq -0.46$ while it is around $-20\%$ at $\varphi_\mu = 0$.

V. CONCLUSION

We have summarised the results for the NLO SUSY-QCD and NLO SUSY-EW corrections for various on-shell two body decays of the charged Higgs boson, presented in Ref. [1]. The newly evaluated corrections are included in the latest version of NMSSMCALCEW. The NLO corrections are evaluated by using a mixed OS and $\overline{\text{DR}}$ renormalization scheme in the Higgs sector. For the results presented here, in the electroweakino sector the OS1 scheme is used while the $\overline{\text{DR}}$ (OS) scheme is chosen for the squarks (slepton) sector. We analysed the relative change of the branching ratios and the partial decay widths, which are defined in Eqs. (20) and (21), for $H^+ \rightarrow t\bar{b}$ and $H^+ \rightarrow \tilde{W}^+\tilde{B}$ in a wide range of the parameter space. We found that the NLO corrections to $\text{BR}(H^+ \rightarrow t\bar{b})$ are moderate and can reach almost $-30\%$ at $\text{BR}^{\text{NLO}} \simeq 6 \times 10^{-2}$. On the other hand, we found that for $H^+ \rightarrow \tilde{W}^+\tilde{B}$, negative large corrections can appear in a corner of the parameter space where the size of the branching ratio at NLO becomes small. We also presented the effects of CP-violating phases on the NLO corrections to $H^+ \rightarrow t\bar{b}$. We found that the changes of the CP-violating phases can significantly affect the NLO corrections to the decays of the charged Higgs boson.

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