Determination of relative phase permeabilities in stochastic model of pore channel distribution by diameter

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Abstract. The problem of the pore space description and the calculation of relative phase permeabilities (RPP) for two-phase filtration is considered. A technique for constructing a pore-network structure for constant and variable channel diameters is proposed. A description of the design model of RPP based on the capillary pressure curves is presented taking into account the variability of diameters along the length of pore channels. By the example of the calculation analysis for the core samples of the Urnenskoye and Verkhnechonskoye deposits, the possibilities of calculating RPP are shown when using the stochastic distribution of pores by diameters and medium-flow diameters.

1. Introduction

Modern methods of forecasting oil production are based on solving the equations of multiphase filtration [1, 2 et al]. A significant uncertainty in the calculation of oil flow rates is the calculation of the relative phase permeabilities (RPP) for oil and water, depending on the structure of the pore space, saturation and properties of the fluids.

Russian and foreign literature [3-14] provides results of ongoing studies of methods for describing the structure of pore space, as well as methods for calculating pressure losses in pore-network structures, with the help of which the filtration characteristics of the formation can be found and generalized (absolute permeability, RPP and others). Stationary and non-stationary experimental methods are widely used for determining RPP on a core scale. Numerous versions of empirical and approximate analytical methods have been developed. At the initial stage of development there are methods based on solving the inverse problems of underground hydrodynamics with the use of field data.

2. Experimental research

The most promising methods for determining RPP include calculation and experimental methods [3-11, 13, 14 et al]. These methods include two stages: construction of a geometric model of pore space and simulation of a multiphase flow in a certain pore space. Models of a pore space are constructed on the basis of curves of capillary pressure, analysis of thin slices, tomography of core samples, etc. [3-6, 9-14 et al]. Methods for simulating multiphase flows include: semi-empirical channel methods (non-overlapping channels) [9, 10, 11]; quasi-one-dimensional modeling in channels with a network structure [3, 4, 5, 6, 13]; 2D and 3D modeling in spatial structures [7, 8, 14 et al].
The authors’ approach is based on the representation of pore space as unconnected clusters, each of which contains one "large" capillary and $\beta = 4 - 12$ "small" capillary channels connected by bridges to a "large" channel. Diameters of "large" and "small" channels are assumed to be different for different sections in accordance with the capillary pressure curve known from the experiment. The sequence of locations of sections with different diameters along the flow is specified multiple times randomly. In calculations, the value of local resistances is corrected by the value of local resistances at the boundary values of water and oil saturation [4]. It turns out that in "large" capillaries in the "train flow" mode, oil and water move, and in small capillaries and crosspieces only water moves (Fig. 1, 2).

Figure 1. Capillary cluster. [4, 5, 6]
Sc – cluster section, P1, P2 – pressures at the entrance and exit from the cluster, L – cluster length equal to the core sample length.

Figure 2. Pattern of fluid flow within the cluster [5, 6].
$L$ — cluster length, $d$ — medium-rate diameters of peripheral (small, crosspieces) capillaries, $D$ — diameters of central (large) capillaries; $d', D', D''$ — diameters in the histogram of pore distribution corresponding to the beginning of the range, the boundary value, the end of the range (see Fig. 3); $P_{in}$, $P_{out}$ — pressures at the entrance and exit from the cluster; $p$, $w$ — oil and water; $\Omega_{in}$, $\Omega_{pw}$, $P_{in}$, $P_{out}$ — cluster wall, oil-water interface, input and output cross sections of the cluster.

Thus, within a single capillary, the diameter distribution corresponds to the distribution along the diameters extracted from the capillary pressure curve (Figure 3).

**Figure 3.** Histogram of pore distribution by diameters for sandstone deposits of Western Siberia [5, 6].

($d'$, $D'$, $D''$ — diameters corresponding to the beginning of the range, the boundary value, the end of the range for "small" and "large" channels, $d$, $D$ - medium-rate diameters of peripheral (small, crosspieces) and central (large) capillaries.

An alternative deterministic approach [4, 5, 6] is the introduction of "small" and "large" channels for calculating the average diameters:

$$
d = \left( \frac{1}{N_M} \sum_{j=min}^{N_{INT}} N_j d_j^4 \right)^{0.25},
$$

$$
D = \left( \frac{1}{N_B} \sum_{j=INT+1}^{N_{INT}+1} N_j D_j^4 \right)^{0.25},
$$

$$
\sum_{j=min}^{N_{INT}} N_j \leq N_M \leq \sum_{j=min}^{N_{INT}+1} N_j.
$$

(1)

where $N_M$, $N_B$ — the number of "small" and "large" channels in the cluster, $\frac{N_M}{N_H} = \beta$

Hydraulic losses in the $i$ section of a large capillary of the cluster are described on the basis of a system of equations derived from the generalized Bernoulli equation [4, 5, 6]:

$$
|\Delta P|_B^i = Q_{1WB} C_{BW}^i + Q_{1P} C_{BP}^i + \bar{F}_{MP}(S^k) \cdot |\Delta P|_B^i,
$$

$$
C_{BP}^i = \frac{\kappa_1 12 B t_k \psi}{\pi D_i^4}, \psi = W, P.
$$

(2)
Here $Q_{iWB}$, $Q_{iP}$ – flow of water, oil, m³/s, $l_i^i$ – length of the i capillary section, m, $\bar{P}_{MF}(S^k) = \frac{\Delta P_{MF}^{i}(S^k)}{|\Delta P_{B}^{i}(S^k)|}$ – $\bar{P}_{MF}(S^k)C[0;1]$ – dimensionless ratio of losses at interfacial interactions to total losses, $W$ – water, $P$ – oil, $K_p$, $K_W$, - scale coefficients showing how many times the pressure drop across the core sample at the water-saturation boundary points in the RPP measurement experiment is greater than the corresponding design pressure drops due to friction on the cluster.

Pressure drops in the $i$ section of a small capillary, crosspiece:

$$Q_{ix}^i = \frac{\pi d_i^i |\Delta P_{x}^i|}{K_{W12} \rho_i^i \mu_W} = |\Delta P_{x}^i| \frac{c_i^i}{c_x^i}$$

where index $\chi=M$ – corresponds to small capillaries, $\chi=PER$ – crosspieces.

The system of equations (2-3) contains $3n_c$ linear equations, where $n_c$ – the number of closed hydraulic circuits in the diagram in Fig. 4 (without taking into account the pump circuit).

Fig. 4 denotes: $C_{BW}^i$, $C_{BP}^i$, $C_{PER}^i$ – coefficients that combine friction losses and "local" for water in the i section of a large capillary, a small capillary, a crosspiece; $C_{BP}^i$ – coefficient that combines friction losses and "local" for oil in the i section of a large capillary; $Q_{iWB}$, $Q_{iM}$, $Q_{iPER}$; $Q_{iP}$ – flow of water and oil; $\bar{P}_{MF}$ – interphase losses referred to the total; $Q, N_c$ – total liquid flow through the core sample and the number of clusters in the core sample.
Setting the total flows of fluids $Q_{p0}(S^I), Q_{w0}(S^I)$ as a result of the solution, let us obtain fluid flows in all the components of the cluster, as well as the pressure drop across the cluster. RPPs are determined by the formulae:

$$
\begin{align*}
K_{RP}(S^I) &= \frac{Q_{p0}(S^I) \mu_{pL}}{\Delta P_{H}(S^I) F_{K} F_{PM}}, \\
K_{RW}(S^I) &= \frac{Q_{w0}(S^I) \mu_{WL}}{\Delta P_{H}(S^I) F_{K} F_{PM}},
\end{align*}
$$

(3)

where $F_{PM}$ – absolute normalization permeability of the core sample for oil at the point of bound water, m$^2$; $F_K$ – sample section area, m$^2$; $\mu_W, \mu_p$ – viscosity of water, oil, Pa*s.

Bell-shaped function $\bar{F}_{MF}(S^k)$ has two parameters for which the empirical dependences [5] have been developed on the basis of experimental data on three deposits in Western Siberia using the correlation analysis apparatus by dimensionless criteria $L_7 = L \alpha \beta / a \gamma$, $2b = c_4 / d$, where $L$ – absolute permeability, m$^2$, $a$ – porosity, $d$ – interfacial tension coefficient, $c_4$ – rate of oil filtration at the point of bound water.

For six core samples from Verkhnechonskoye (1.1-1.6) and five core samples from Urnenskoye (2.1-2.5) deposits, a calculated analysis of the dependence of the normalized difference in RPP at the interior points of the water saturation range (between bound water $S_{wc}$ and residual oil $S_{or}$) was carried out using two algorithms. Fig. 5 shows the dependence of the relative difference in RPT of oil obtained on the basis of flow calculation with variable diameters of capillary sections $\bar{K}_{RP}$ and with average (1) diameters $K_{RP}, \Delta K_{RP} = \frac{\bar{K}_{RP} - K_{RP}}{K_{RP}}$.

![Figure 5](image)

**Figure 5.** Error in determining the OPP for oil under the assumption of the constancy of diameters with the correction of local losses by the experimental data. (unit fractions, Water saturation)

From the calculation results, presented in Fig. 5, it follows that the error associated with the replacement of diameter distribution obtained from the capillary pressure curves for average diameters depends on the saturation and on the conditions considered $\Delta K_{RP} \leq 0.1$.

### 3. Conclusions

The results of the research are as follows.

A model of the network structure of pore channels with variable length diameters corresponding to the pore distribution found from the capillary pressure curves is proposed. An algorithm for the stochastic description of variable channel diameters has been developed.
With respect to the generated network structure of pore space, based on the known model of pressure losses in capillary channels including pressure losses due to friction, local losses and losses due to interfacial interaction, relative phase permeabilities are determined when the water saturation is changed.

It has been established that replacement of variable diameters of "large" and "small" channels by medium-rate diameters with local losses corrected by the experimental data leads to design errors within ±10% for oil RPP.

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