Coherence-based measure of quantumness in (non-) Markovian channels

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Abstract
We make a detailed analysis of quantumness for various quantum noise channels, both Markovian and non-Markovian. The noise channels considered include dephasing channels like random telegraph noise, non-Markovian dephasing and phase damping, as well as the non-dephasing channels such as generalized amplitude damping and Unruh channels. We make use of a recently introduced witness for quantumness based on the square $l_1$ norm of coherence. It is found that the increase in the degree of non-Markovianity increases the quantumness of the channel. This may be attributed to the fact that the non-Markovian dynamics involves the generation of entanglement between the system and environment degrees of freedom.

Keywords Channels · Quantumness · Non-Markovian dynamics

1 Introduction
Quantum coherence [1,2] is central to quantum mechanics, playing a fundamental role for the manifestations of quantum properties of a system. It is at the heart of the phenomena such as multi-particle interference and entanglement which are pivotal for carrying out various quantum information and communication tasks, viz. quantum key distribution [3,4] and teleportation [5]. An operational formulation of coherence as a resource theory was recently developed [6]. The notion of coherence [7] has its roots in quantum optics [8,9]. Recent developments have made use of coherence in superconducting systems [10], biological systems [11], non-Markovian phenomena [12], foundational issues [13,14] and subatomic physics [15,16].
Quantum channels are completely positive (CP) and trace-preserving maps between the spaces of operators and describe processes like transmission of classical as well as quantum information. Quantum information protocols are based on the fact that information is transmitted in the form of quantum states. This is achieved either by directly sending non-orthogonal states or by using pre-shared entanglement. The channels can reduce the degree of coherence and entanglement as the information flows from sender to receiver. Interestingly, it was shown in [17] that quantum channels can have cohering power and that a qubit unitary map has equal cohering and decohering power in any basis. In general, the extent to which the quantum features are affected depends on the underlying dynamics and the type of noise. Therefore, it is natural to ask to what extent is coherence preserved by a channel used to transmit quantum information.

The physical foundation of a large number of quantum channels relies on the Born and/or Markov approximations [18]. However, in a number of quantum communication tasks, the characteristic time scales of the system of interest become comparable with the reservoir correlation time. Therefore, a non-Markovian description for such scenarios becomes indispensable [19].

A quantum channel can be used to transport either classical or quantum information. The reliability of a quantum channel is tested by the probability that the output and input states are the same. A well-known measure to quantify the performance of a channel is the average fidelity [20–23]. The notion of fidelity of two quantum states provides a qualitative measure of their distinguishability [24]. Recently, a measure based on Fisher information has been introduced to quantify the invasiveness of quantum channels [25].

In [26], a witness of nonclassicality of a channel was introduced. This is based on average quantum coherence of the state space, using the square $l_1$ norm of coherence of qubit channels. It was shown that the extent to which quantum correlation is preserved under local action of the channel cannot exceed the quantumness of the underlying channel.

In this work, we use the definition of quantumness based on the average coherence and apply it to different channels, both Markovian and non-Markovian. Being a quantitative measure of the “closeness” of the output and input states, average channel fidelity is a useful figure of merit when considering channel transmission [27], particularly in the presence of noise. Accordingly, a corresponding study is made on these channels. The paper is organized as follows: In Sect. 2, we briefly review the definition of nonclassicality of quantum channels. Section 3 is devoted to analyzing the interplay of quantumness and average fidelity in various noise models. Results and their discussion are made in Sect. 4. We conclude in Sect. 5.

### 2 Quantum channels and the measure of quantumness

Here, we give a brief overview of quantum channels. This will be followed by a discussion on a coherence-based measure of quantumness of channels.
2.1 Quantum channel

A quantum channel in the Schrödinger picture is a completely positive and trace-preserving map $\Phi : \mathcal{T}(\mathcal{H}_A) \rightarrow \mathcal{T}(\mathcal{H}_B)$, where $\mathcal{T}(\mathcal{H}_A)$ and $\mathcal{T}(\mathcal{H}_B)$ denote the set of operators defined in the underlying Hilbert space $\mathcal{H}_A$ and $\mathcal{H}_B$, respectively. A corresponding description in the Heisenberg picture would invoke the dual channel [28].

The operator sum representation of a channel is given as

$$\Phi[\rho] = \sum_{\mu} M_{\mu} \rho M_{\mu}^\dagger,$$  \hfill (1)

such that the operators $M_{\mu}$, called as Kraus operators, obey the completeness condition, $\sum_{\mu} M_{\mu}^\dagger M_{\mu} = I$. Here, $I$ is the identity operator. Note that $\rho$, in Eq. (1), need not be a pure state. A linear map given in Eq. (1) is called a quantum channel or superoperator (as it maps operators to operators) or completely positive trace-preserving (CPTP) map. A quantum channel is characterized by the following properties:

1. They are linearity transformations, that is, for states $\rho_1$ and $\rho_2$, we have

$$\Phi[\alpha \rho_1 + \beta \rho_2] = \alpha \Phi(\rho_1) + \beta \Phi(\rho_2),$$

where $\alpha$ and $\beta$ are complex numbers.

2. They represent hermicity-preserving maps, that is,

$$\rho = \rho^\dagger \implies \Phi[\rho] = \Phi[\rho]^\dagger.$$

3. They preserve positivity, that is,

$$\rho \geq 0 \implies \Phi[\rho] \geq 0.$$

4. They are trace preserving, that is,

$$\text{Tr}(\Phi[\rho]) = \text{Tr}(\rho).$$

2.2 A coherence-based measure of quantumness of channels

A coherence-based measure was introduced in [26]

$$Q_C(\Phi) = N_C \int C(\Phi(\rho)) d\mu(\rho).$$  \hfill (2)

Here, $\Phi$ is the channel under consideration, $C$ denotes the chosen measure of coherence, $N_C$ is a normalization constant and $\mu(\rho)$ is the Haar measure. To proceed, we analyze the effect of a qubit channel on a state $\rho = \frac{1}{2}(I + \xi\sigma)$. This turns out to be an affine transformation on a Bloch sphere, such that the Bloch vector $\xi$ transforms as
Here, $\xi' = A\xi + B$, such that the matrices $A_{3\times3}$ and $B_{3\times1}$ depend on the channel parameters. By choosing the square $l_1$-norm as the measure of coherence, we compute the coherence with respect to an arbitrary orthonormal basis. The $l_1$-norm of coherence $C_{l_1}$ is often used as a coherence measure, since it is easy to compute and algebraically easy to manipulate [29]. Further, $C_{l_1}$ links different coherence and entanglement measures. For example, $C_{l_1}$ is the upper bound for another important coherence measure called robustness of coherence for all qubit states [30]. It is also useful in studying Deutsch–Jozsa algorithm [31,32] and Grover algorithm [33,34]. It plays an important role in quantifying the cohering and decohering powers of quantum operations [35] and also corresponds to the maximum entanglement generated by incoherent operations acting on the system and an incoherent ancilla [36].

To make the quantumness witness a basis independent quantity, one performs optimization over all orthonormal basis, leading to a closed expression for the quantumness witness

$$Q_{C^2_{l_1}}(\Phi) = \lambda_2 + \lambda_3.$$ (4)

Here, $\lambda_1 \geq \lambda_2 \geq \lambda_3$ are eigenvalues of matrix $\mathcal{L} = \frac{1}{2}(AA^T + 5BB^T)$, with $T$ denoting the transpose operation. Thus, Eq. (4) gives an operational definition of the quantumness of a channel. In what follows, we will drop the subscript $C^2_{l_1}$ and call the quantumness of a map $\Phi$ just as $Q(\Phi)$. It is worth mentioning here that for the unital channels, which map identity to identity, i.e., $\Phi(I) = I$, the above definition of quantumness coincides with the geometric discord [26].

### 3 Specific channels

In this section, we give a brief account of various quantum channels [27,37] used in this work. The dephasing class includes random telegraph noise (RTN) [38–40], non-Markovian dephasing (NMD) [41] and phase damping (PD) [42] channels while in the non-dephasing class, we consider generalized amplitude damping (GAD) [43,44] and Unruh channels [45].

**Random Telegraph Noise** This channel characterizes the dynamics when the system is subjected to a bi-fluctuating classical noise, generating RTN with pure dephasing. The dynamical map acts as follows:

$$\Phi_{\text{RTN}}(\rho) = \mathcal{R}_0 \rho \mathcal{R}_0^\dagger + \mathcal{R}_1 \rho \mathcal{R}_1^\dagger,$$ (5)

where the two Kraus operators are given by

$$\mathcal{R}_0 = \sqrt{\frac{1 + \Lambda(t)}{2}} I, \quad \mathcal{R}_1 = \sqrt{\frac{1 - \Lambda(t)}{2}} \sigma_z.$$ (6)
Here, $\Lambda(t)$ is the memory kernel

$$
\Lambda(t) = e^{-\gamma t} \left[ \cos \left( \sqrt{\frac{2b}{\gamma}}^2 - 1 \right) \gamma t \right] + \sin \left( \sqrt{\frac{2b}{\gamma}}^2 - 1 \right) \gamma t, \tag{7}
$$

where $b$ quantifies the system–environment coupling strength and $\gamma$ is proportional to the fluctuation rate of the RTN. Also, $I$ and $\sigma_z$ are the identity and Pauli spin matrices, respectively. The completeness condition reads $\mathcal{R}_0 \mathcal{R}_0^\dagger + \mathcal{R}_1 \mathcal{R}_1^\dagger = I$. The dynamics is Markovian [non-Markovian] if $(4b \tau)^2 > 1$ [$(4b \tau)^2 < 1$], where $\tau = 1/(2\gamma)$. Starting with the state $\rho = \frac{1}{2}(I + \xi \sigma)$, the new Bloch vector is given by $\xi' = [\xi_x \Lambda(t), \xi_y \Lambda(t), \xi_z]^T$. This implies $A = \text{diag}.[A(t), A(t), 1]$ and $B = 0$, and consequently, $L = \text{diag}.[\frac{1}{2}[A(t)]^2, \frac{1}{2}[A(t)]^2, \frac{1}{2}]$. Since, $-1 \leq \Lambda(t) \leq 1$, we identify both the small eigenvalues as $\frac{1}{2}[\Lambda(t)]^2$, leading to

$$
Q(\phi^{\text{RTN}}) = |\Lambda(t)|^2. \tag{8}
$$

We next compute the fidelity for the states $\rho$ and $\rho'$, the initial and final states, respectively. This will be followed by a study of its interplay with quantumness. The fidelity between qubit states $\rho$ and $\rho'$ is [24]

$$
F(\rho, \rho') = \text{Tr}[\rho \rho'] + 2\sqrt{\text{Det}[\rho] \text{Det}[\rho']}. \tag{9}
$$

Using a general qubit parametrization

$$
\rho = \begin{bmatrix}
\cos^2(\theta/2) & \frac{1}{2}e^{-i\phi} \sin(\theta) \\
\frac{1}{2}e^{i\phi} \sin(\theta) & \sin^2(\theta/2)
\end{bmatrix}, \tag{10}
$$

the fidelity for RTN model turns out to be

$$
F_{\text{RTN}} = \frac{1}{4} \left[ 3 + \cos(2\theta) + 2 \sin^2(\theta) \Lambda(t) \right]. \tag{11}
$$

In order to make this quantity state independent, we calculate the average fidelity $\mathcal{F} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi F(\sin(\theta), d\theta) d\phi$. We have

$$
\mathcal{F}_{\text{RTN}} = \frac{1}{3} [2 + \Lambda(t)]. \tag{12}
$$

Since $-1 \leq \Lambda(t) \leq 1$, the average fidelity is symmetric about its classical value 2/3.

**Non-Markovian dephasing** This channel is an extension of the dephasing channel to non-Markovian class. The non-Markovianity is identified with the appearance of a not completely positive (NCP) intermediate map [41]. The Kraus operators are given by

$$
\mathcal{N}_0 = \sqrt{(1 - \alpha p)(1 - p)} \ I, \quad \text{and} \quad \mathcal{N}_1 = \sqrt{p + \alpha p(1 - p)} \ \sigma_z. \tag{13}
$$

Here, the parameter $\alpha$ quantifies the degree of non-Markovianity of the channel, such that $\alpha = 0$ corresponds to conventional dephasing, while as $\alpha$ increases, the non-Markovian behavior correspondingly increases. Further, $p$ is a time-like parameter such that $0 \leq p \leq 1/2$. In this case, the quantumness parameter turns out to be

$$
Q(\phi^{\text{NMD}}) = \Omega^2(p), \tag{14}
$$

where $\Omega = 1 - 2p - 2\alpha p(1 - p)$. The average fidelity, in this case, is given by

$$
\mathcal{F}_{\text{NMD}} = \frac{1}{3} [2 + \Omega(p)]. \tag{15}
$$
We use the parametrization $p = \frac{1}{2}(1 - e^{-\kappa t})$, such that as $t : 0 \to \infty$, $p : 0 \to 1/2$.

**Phase damping (PD) channel** PD channel models the phenomena where decoherence occurs without dissipation (loss of energy). The dynamical map in this case has the Kraus representation

$$\mathcal{P}_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-S} \end{bmatrix}, \quad \mathcal{P}_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{S} \end{bmatrix}. \quad (16)$$

The parameter $S$ can be modeled by the following time dependence $S = 1 - \cos^2(\chi t)$, for $0 \leq \chi t \leq \pi/2$. The quantumness parameter, in this case, is given by

$$Q(\Phi_{PD}) = 1 - S = \cos^2(\chi t). \quad (17)$$

The average fidelity turns out to be

$$F_{PD} = \frac{1}{3}[2 + \cos(\chi t)]. \quad (18)$$

**Generalized Amplitude Damping (GAD) channel** GAD is a generalization of the AD channel to finite temperatures [19]. The later models processes like spontaneous emission from an atom and is pertinent to the problem of quantum erasure [46]. The dynamics, in this case, is governed by the following Kraus operators

$$\mathcal{A}_0 = \begin{bmatrix} \sqrt{\Theta} & 0 \\ 0 & \sqrt{s(1-\Theta)} \end{bmatrix}, \quad \mathcal{A}_1 = \begin{bmatrix} 0 & \sqrt{\Theta} \\ 0 & 0 \end{bmatrix},$$

$$\mathcal{A}_2 = \begin{bmatrix} 0 & 0 \\ \sqrt{s(1-\Theta)} & 0 \end{bmatrix}, \quad \mathcal{A}_3 = \begin{bmatrix} 0 & 0 \\ \sqrt{\Theta(1-\Theta)} & 0 \end{bmatrix}. \quad (19)$$

Here, $\Theta = \frac{n+1}{\beta t+1}$, and $s = \exp[-\frac{\gamma t}{2}(2n+1)]$. Also, $n$ is the mean number of excitations in the bath and $\gamma$ represents the spontaneous emission rate. In the zero temperature limit, $n = 0$, implying $\Theta = 1$, thereby recovering the AD channel. The quantumness parameter for GAD channel comes out to be

$$Q(\Phi_{GAD}) = \begin{cases} \frac{1}{2}s + \tilde{s} & \text{for } t \leq \tau, \\ s & \text{for } t > \tau. \end{cases} \quad (20)$$

with

$$\tilde{s} = \frac{5}{2}(2\Theta - 1)^2(1-s)^2,$$

$$\tau = -\frac{2}{\gamma(2n+1)} \ln \left[ \frac{5}{6 + 4n + n^2} \right].$$

The average fidelity in this case is given by

$$F_{GAD} = \frac{1}{6}[3 + \sqrt{s} + s]. \quad (21)$$

Here $s = \exp[-\frac{\gamma t}{2}(2n+1)]$.

**Unruh channel** To an observer undergoing acceleration $a$, the Minkowski vacuum appears as a warm gas emitting black-body radiation at temperature given by $T = \frac{h a}{2\pi c \kappa_B}$, called the Unruh temperature, and the effect is known as the Unruh effect. The Unruh effect has been described as a noisy quantum channel with the following Kraus operators

$$\mathcal{U}_0 = \begin{bmatrix} \cos(r) & 0 \\ 0 & \sin(r) \end{bmatrix} \quad \text{and} \quad \mathcal{U}_1 = \begin{bmatrix} 0 & 0 \\ -\sin(r) & 0 \end{bmatrix}. \quad (22)$$

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Here, $\cos(r) = [1 + \exp(-2\pi c\omega/a)]^{-1/2}$. The quantumness parameter for the Unruh channel turns out to be

$$Q(\Phi^{\text{Unruh}}) = \cos^2(r).$$

(23)

The average fidelity here is

$$\mathcal{F}_{\text{Unruh}} = \frac{1}{12} (4\cos(r) + \cos(2r) + 7).$$

(24)

### 4 Results and discussion

The quantumness of noisy channels is quantified by the coherence measure given by Eq. (2). For specific case of a two-level system (qubit), using square $l_1$ norm as a measure of coherence, one obtains a simple working rule for computing the quantumness of a channel, given in Eq. (4).

For RTN channel, the quantumness measure turns out to be the square of the memory kernel $\Lambda(t)$, defined in Eq. (7). In the non-Markovian regime, both the quantumness and fidelity are seen to sustain

![Fig. 1 RTN channel: the quantumness $Q(\Phi^{\text{RTN}})$ Eq. (8) and average fidelity $\mathcal{F}_{\text{RTN}}$ Eq. (12), plotted with respect to time $t$, for a qubit subjected to RTN. The solid (blue) and dashed (red) curves correspond to non-Markovian ($b = 0.05, \gamma = 0.001$) and Markovian ($b = 0.07, \gamma = 1$) cases, respectively. The fidelity oscillates symmetrically about $2/3$ in non-Markovian case, while in Markovian case, it decreases monotonically and saturates to this value (Color figure online).](image)
\[ \alpha = 0 \quad \alpha = 0.9 \quad \alpha = 0.5 \]

**Fig. 2** NMD channel: the quantumness \( Q(\Phi^{NMD}) \) Eq. (14) and average fidelity \( \mathcal{F}_{NMD} \) Eq. (15), plotted with respect to the dimensionless parameter \( \kappa t \), for a qubit subjected to NMD, for different values of parameter \( \alpha \)

\[ \chi = 0 \quad \chi = 0.9 \quad \chi = 0.5 \]

**Fig. 3** PD channel: the quantumness \( Q(\Phi^{PD}) \) Eq. (17) and average fidelity \( \mathcal{F}_{PD} \) Eq. (18), plotted with respect to the dimensionless quantity \( \chi t \), for a qubit subjected to PD noise
much longer in time as compared with the Markovian case, as shown in Fig. 1. In the limit $t \to \infty$, $\Lambda(t) \to 0$, consequently, we have

$$Q(\Phi^{\text{RTN}}) = \Lambda^2(t) \to 0, \quad \text{and} \quad F^{\text{RTN}} = \frac{1}{3}(2 + \Lambda(t)) \to \frac{2}{3}. \quad (25)$$

This is consistent with our notion of fidelity less than or equal to $2/3$ for a processes that can be simulated by a classical theory. The NMD channel shows nonzero quantumness within the allowed range, i.e., $[0, 1/2]$, of time-like parameter $p$, for $0 < \alpha \leq 1$. In this case, the parameter $\alpha$ quantifies the degree of non-Markovianity, which increases as $\alpha$ goes from 0 to 1. At $p = 1/2$, i.e., for $t \to \infty$, $\Omega(p) = -\alpha/2$, we have

$$Q(\Phi^{\text{NMD}}) = \alpha^2/4, \quad \text{and} \quad F^{\text{NMD}} = \frac{2}{3}(1 - \alpha/2). \quad (26)$$

that is, the quantumness parameter is always positive, but the average fidelity goes below its classical limit. This is consistent with [26] that a nonzero value of the coherence-based measure of quantumness is a necessary but not sufficient criterion for quantum advantage in teleportation fidelity. This is also consonant with the use of fidelity as a tool to assess quantumness [47]. However, in the Markovian limit, i.e., $\alpha \to 0$, 

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Fig. 4 GAD channel: the quantumness $Q(\Phi^{\text{GAD}})$ Eq. (20) and fidelity $F^{\text{GAD}}$ Eq. (21), plotted with respect to time $t$, for a qubit subjected to GAD noise. With $\gamma = 1$, the top and bottom panels correspond to the cases when $n = 50$ and 0, respectively. Here, $\tau \approx 0.1246$ and 0.3646 in the former and later case, respectively. The $n = 0$ case corresponds to the zero temperature limit, such that GAD reduces to AD noise.
Fig. 5 Unruh channel: the behavior of quantumness and average fidelity depicted with respect to the acceleration $a$ (in units $\hbar = c = 1$)

$\Omega \rightarrow 1 - 2p$. Since $p \in [0, 1/2]$, it implies $\Omega \in [0, 1]$. Therefore, $Q(\Phi^\text{NMD}) = (1 - 2p)^2$ and $F_N^\text{NMD} = \frac{1}{2}(2 + (1 - 2p))$; both the quantities lead to similar predictions in this limit. These features are depicted in Fig. 2.

One of the purely quantum noise channels is the PD channel which characterizes the processes accompanied with the loss of coherence without loss of energy. The behavior of quantumness and average fidelity, in this case, is depicted in Fig 3. The parameter $Q(\Phi^\text{PD})$ becomes zero as $F^\text{PD}$ reaches $2/3$.

Next we analyzed non-dephasing models such as GAD and Unruh channels. From the GAD channel, one can recover the AD channel in the zero temperature limit, i.e., when $n = 0$, see Eq. (19). In this case, $\Theta = 1$ and the quantumness parameter, with $\xi = 1 - s$, becomes

$$Q(\Phi^\text{AD}) = \begin{cases} \frac{1}{2}[6\xi^2 - 3\xi + 2] & \text{for } \xi \leq 1/6, \\ 1 - \xi & \text{for } \xi > 1/6. \end{cases} \quad (27)$$

This is consistent with the results given in [26]. In the case of GAD channel, the quantumness parameter is nonzero even though the average fidelity goes below its classical limit $2/3$. This reiterates the statement made earlier regarding quantumness and average teleportation fidelity, as shown in Fig 4. In the high-temperature regime, both the measures, i.e., quantumness as well as average fidelity, seem to lead to similar predictions at the same time. For Unruh channel, the quantumness and average fidelity are studied with respect to the acceleration $a$. Both the measures show a saturation at values which are well above their classical limits, as shown in Fig 5. This is in consonance with [45], where it was shown that the Unruh channel, though structurally similar to the AD channel, is different from it.

### 5 Conclusion

We studied the quantumness and average fidelity of various channels, both Markovian and non-Markovian. Specifically, we considered the dephasing channels like RTN, NMD and PD channels and non-dephasing channels such as GAD and Unruh channels. The non-Markovian dynamics (exhibited by RTN and NMD channels in this case) is found to favor the nonclassicality. This is explicitly seen from the fact that a nonzero value of parameters controlling the degree of non-Markovianity takes the quantumness beyond the classical value. The non-Markovian assisted enhancement of nonclassicality can be of profound importance in carrying out quantum information tasks. This can be realized by effectively engineering the system–reservoir models. The quantumness measure and average fidelity exhibit similar predictions for the Unruh channel. Similar behavior is observed for the dephasing channels, albeit, in the Markovian regime. This can be seen in RTN and NMD channels. In contrast, in the non-dephasing Lindbladian channel, considered here, the quantumness witness and average fidelity show qualitatively similar results.
Such a study of the interplay between nonclassicality of the quantum channels with the underlying dynamics can be useful from the quantum information point of view and also brings out the effectiveness of the measure of quantumness under different types of dynamics.

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