Decision Support Model for Perishable Items Impacting Ramp Type Demand in a Discounted Retail Supply Chain Environment

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Abstract: A single item EOQ model has been developed considering demand as a two parameter ramp type function and deterioration as a Heaviside’s function. Both pre and post deterioration discounts are considered where the former helps in maintaining constancy in the demand rate and the latter one boosts the demand of decreased quality items. The starting time periods of pre and post deterioration discount have been determined. The effect of both types of discounts in optimising the profit is examined through numerical illustrations. Sensitivity analysis is also appended to find out the effect of various system parameters. From this study it is observed that it will be more advantageous for management to offer pre deterioration discount in enticing the profit.

Keywords: EOQ Model, Ramp Type Demand, Heaviside’s Function, Discounted Selling Price.

MSC: 90B05.

1. Introduction

Most of the inventory models are explored by considering the demand rate as constant, linearly increasing/decreasing or exponentially increasing/decreasing. But demand of all types of products may not follow these particular patterns over time. Demand for some products increases rapidly as they are introduced in the
market, but after a certain period of time it becomes constant. The ramp type function is used to represent such type of demand function. The following table gives a glance at research works undertaking different patterns of demand and deterioration.

| Authors & Year of Publication | Demand Pattern | Deterioration | Price Discount | Pre-Deterioration Discount | Post-Deterioration Discount | Both Pre & Post Deterioration Discount |
|-----------------------------|----------------|--------------|----------------|---------------------------|---------------------------|---------------------------------------|
| Shah et al. [13]            | Time dependent | Constant     | No             |                           |                           |                                       |
| Chatterji & Gothi [3]       | Time dependent | Weibull      | No             |                           |                           |                                       |
| Mishra et al. [7]           | Quadratic      | Weibull      | No             |                           |                           |                                       |
| Mishra et al. [14]          | Quadratic      | No           | No             |                           |                           |                                       |
| Tripathy & Pradhan [17]     | Weibull        | Time dependent | No             |                           |                           |                                       |
| Tripathy & Parvati [16]     | Weibull        | Time dependent | No             |                           |                           |                                       |
| Giri et al. [4]             | Ramp           | Weibull      | No             |                           |                           |                                       |
| Aggrawal & Singh [19]       | Ramp           | Time dependent | No             |                           |                           |                                       |
| Arya & Kumar [2]            | Ramp           | Weibull      | No             |                           |                           |                                       |
| Karmakar & Chaudhuri [6]    | Ramp           | Constant     | No             |                           |                           |                                       |
| Giri et al. [4]             | Ramp           | Weibull      | No             |                           |                           |                                       |
| Jain & Kumar [5]            | Ramp           | Weibull      | No             |                           |                           |                                       |
| Tripathy & Pradhan [18]     | Ramp           | Weibull      | No             |                           |                           |                                       |
| Panda et al. [10]           | Stock dependent | Heaviside's function | Yes | Yes | Yes | Yes | Yes |

The current study focuses on a certain kind of demand pattern which accelerates exponentially as the products are launched in the market, stabilizes with the passage of time, and ultimately declines and becomes asymptotic. Two parameter ramp type function is used to corroborate such type of demand pattern. The inventory deteriorates following a Heaviside’s function. Both pre and post deterioration discount are provided, where the former assists in maintaining the constancy in the demand and the latter enhances the demand of decreased quality items. The efficacy of the optimal result is attained by comparing the results obtained in three different scenarios. The sensitivity analysis is conducted to discern the effect of various system parameters in optimising the profit. The concavity of the total profit is also tested graphically.
2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

1. $C_0$: Set up cost.

2. $S$: Constant selling price of the product per unit.

3. $r_1$: Pre deterioration discount per unit.

4. $r_2$: Post deterioration discount per unit.

5. $h$: Holding cost per unit per unit time.

6. $d$: Disposal cost per unit.

7. $c$: Purchase cost of the product per unit.

8. $T_1$: The total cycle time.

9. $\mu$: The time period at which the pre deterioration discount is provided.

10. $\gamma$: The time period at which the deterioration starts.

11. $\pi$: Total profit of the system per unit time.

12. $I(t)$: The inventory level at time $t$.

13. $I(0) = Q_1$: The initial inventory level is $Q_1$.

2.2. Assumptions

1. Replenishment rate is infinite.

2. The deterioration rate is assumed as a Heaviside’s function.

$$\bar{\theta} = \theta H(t - \gamma).$$

Where $t$ is the time measured from the instant arrivals of a fresh replenishment indicating that the deterioration of the items begins after a time $\gamma$ from the instant of the arrival in stock. $\theta$ is a constant ($0 < \theta < 1$) and $H(t - \gamma)$ is the well known Heaviside’s function defined as

$$H(t - \gamma) = \begin{cases} 1, & \text{if } t \geq \gamma \\ 0, & \text{otherwise.} \end{cases}$$
3. Demand rate is a two parameter ramp type function defined as

\[ D(t) = ae^{b(t-(t-\mu)H(t-\mu)-(t-\gamma)H(t-\gamma))}, 0 < \mu < \gamma, a > 0, b > 0, \]

where

\[ H(t-\mu) = \begin{cases} 1, & t \geq \mu \\ 0, & t < \mu \end{cases} \]

and

\[ H(t-\gamma) = \begin{cases} 1, & t \geq \gamma \\ 0, & t < \gamma. \end{cases} \]

So

\[ D(t) = \begin{cases} ae^{bt}, & 0 \leq t < \mu \\ ae^{b\mu}, & \mu \leq t < \gamma \\ ae^{b(\mu+\gamma)}e^{-bt}, & t \geq \gamma. \end{cases} \]

4. \( r_1(0 \leq r_1 \leq 1) \) is the percentage pre deterioration discount offer on unit selling price. \( \alpha_1 = (1 - r_1)^{-n_1}, n_1 \in \mathbb{R} \) is the effect of pre deterioration discount on demand. \( r_2(0 \leq r_2 \leq 1) \) is the percentage post deterioration discount offer on unit selling price. \( \alpha_2 = (1 - r_2)^{-n_2}, n_2 \in \mathbb{R} \) is the effect of post deterioration discount on demand.

3. MATHEMATICAL MODEL AND ANALYSIS

Let \( Q_1 \) be the inventory level at the beginning of the cycle. The depletion in the inventory occurs due to demand up to time \( \gamma \). After time \( \gamma \), the inventory declines due to demand and deterioration. Ultimately, inventory reaches zero level at time \( T_1 \). Before the starting of deterioration i.e., from \( \mu \) to \( \gamma \), \( r_1 \% \) discount on unit selling price of the product is imposed in order to maintain constancy in the demand rate. After starting of deterioration, \( r_2 \% \) discount on unit selling price is provided to enhance the demand of decreased quality items. This discount is continued for the rest of the replenishment cycle. Then the behavior of the inventory level is governed by the following differential equations

\[
\frac{dI(t)}{dt} = -ae^{bt}, \quad 0 \leq t \leq \mu. \quad (1)
\]

\[
\frac{dI(t)}{dt} = -\alpha_1 ae^{b\mu}, \quad \mu \leq t \leq \gamma. \quad (2)
\]

\[
\frac{dI(t)}{dt} + \beta I(t) = -\alpha_2 ae^{b(\mu+\gamma)}e^{bt}, \quad t \geq \gamma \quad (3)
\]
with the initial boundary conditions $I(0) = Q_1$ and $I(T_1) = 0$. For the condition
$I(0) = Q_1$, the solution of equation (1) yields
\[ I_1(t) = \frac{a}{b}(1 - e^{bt}) + Q_1. \]
At the point $t = \mu$, the inventory level is
\[ I_1(\mu) = \frac{a}{b}(1 - e^{b\mu}) + Q_1. \]
With the condition $I_1(\mu) = I_2(\mu)$, solution of equation (2) yields
\[ I_2(t) = \alpha_1 e^{b\mu}(\mu - t) + I_1(\mu). \]
At the point $t = \gamma$, the inventory level is
\[ I_2(\gamma) = \alpha_1 e^{b\mu}(\mu - \gamma) + I_1(\mu). \]
(4)
With condition $I_2(\gamma) = I_3(\gamma)$, the solution of equation (3) yields
\[ I_3(t) = -\alpha_2 a e^{b(\mu+\gamma)} \frac{e^{-bt}}{(\theta - b)} + I_2(\gamma) + \alpha_2 a \frac{e^{b\mu}}{(\theta - b)} e^{\theta(\gamma-t)}. \]
The boundary condition $I_3(T_1) = 0$ yields
\[ I_2(\gamma) = \frac{\alpha_2 a}{(\theta - b)} e^{b\mu} e^{(b-\theta)(\gamma-T_1)-1}. \]
(5)
Equations (4) and (5) generate,
\[ I_1(\mu) = I_2(\gamma) - \alpha_1 e^{b\mu}(\mu - \gamma). \]
(6)
So, equation (6) yields
\[ Q_1 = I_1(\mu) - \frac{a}{b}(1 - e^{b\mu}). \]
Holding cost and disposal cost of inventories in the cycle is
\[ HC + DC = h \int_0^\mu I_1(t)dt + h \int_\mu^\gamma I_2(t)dt + (h + \theta d) \int_\gamma^{T_1} I_3(t)dt. \]
Purchase cost of the cycle is given by
\[ PC = cQ_1. \]
Total sales revenue in the order cycle is
\[ SR = S \int_0^\mu D_1(t)dt + S\alpha_1(1 - r_1) \int_\mu^\gamma D_2(t)dt + S\alpha_2(1 - r_2) \int_\gamma^{T_1} D_3(t)dt. \]
The total profit per unit time of the system is
\[ \pi = \frac{1}{T_1} [SR - PC - HC - DC - C_0]. \] (7)

The pre deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product i.e., \( S(1 - r_1) - c > 0 \). Similarly, \( S(1 - r_2) - c > 0 \). Applying these constraints on the unit total profit function, we have the following maximization problem

Maximize \( \pi(\mu, \gamma) \)
Subject to \( r_1, r_2 < 1 - \frac{c}{S} \); (8)
\( r_1, r_2, \mu, \gamma \geq 0 \).

The optimum values of \( \mu \) and \( \gamma \), which minimize the unit profit, can be obtained by solving the equations
\[ \frac{\delta \pi}{\delta \mu} = 0 \text{ and } \frac{\delta \pi}{\delta \gamma} = 0. \] (9)

The values satisfy the sufficient conditions
\[ \frac{\delta^2 \pi}{\delta \mu^2} < 0, \quad \frac{\delta^2 \pi}{\delta \gamma^2} < 0 \]
and \( \frac{\delta^2 \pi}{\delta \mu^2} \frac{\delta^2 \pi}{\delta \gamma^2} - \frac{\delta^2 \pi}{\delta \mu \gamma} < 0 \). (10)

### 3.1. Model for Pre Deterioration Discount

In this case the discount is provided before starting of deterioration. So, there is no post deterioration discount and hence \( r_2 = 0 \). Thus, the total profit per unit time of the system is
\[ \pi = \frac{1}{T_1} [SR - PC - HC - DC - C_0]. \] (11)

The maximization problem in this case is

Maximize \( \pi(\mu, \gamma) \)
Subject to \( r_1 < 1 - \frac{c}{S} \); (12)
\( r_1, \mu, \gamma \geq 0 \).

The optimum values of \( \mu \) and \( \gamma \) are obtained by using equation (9). These values satisfy the conditions in equation (10).
3.2. Model for Post Deterioration Discount

In this case the discount is provided only after starting of deterioration. So, there is no pre deterioration discount and hence \( r_1 = 0 \). Thus, the total profit per unit time of the system is

\[
\pi = \frac{1}{T_1}[SR - PC - HC - DC - C_0].
\]  

(13)

The maximization problem in this case is

Maximize \( \pi(\mu, \gamma) \)

Subject to \( r_2 < 1 - \frac{c}{S} \);  

(14)

\[ r_2, \mu, \gamma \geq 0. \]

The optimum values of \( \mu \) and \( \gamma \) are obtained by using equation (9). These values satisfy the conditions in the equation (10).

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Example 1.

The values of the system parameters are 

\[ a = 90, b = 0.35, h = 0.3, d = 3, S = 15, C_0 = 80, c = 5, \theta = 0.06, n_1 = n_2 = 2, r_1 = 0.15, r_2 = 0.35, \alpha_1 = 1.18, \alpha_2 = 2.37, T_1 = 3. \]

Scenario-I: Both type of discounts

\[ \mu = 1.45533, \gamma = 1.64047, \pi = 1271.39 \text{ and } Q = 813.936. \]

Scenario-II: Only pre deterioration discount

\[ \mu = 1.57963, \gamma = 2.15897, \pi = 1497.42 \text{ and } Q = 709.205. \]

Scenario-III: Only post deterioration discount

\[ \mu = 1.52388, \gamma = 1.65557, \pi = 1299.6 \text{ and } Q = 820.783. \]

The following figures represent the concavity of total profit per unit time with respect to the pre and post deterioration discount starting time.

Figure 1: Concavity of total profit per unit time in Scenario-I
Figure 2: Concavity of total profit per unit time in Scenario-II

Figure 3: Concavity of total profit per unit time in Scenario-III
Table 2: Sensitivity Analysis for scenario-I

| Parameter | % change | Parameter | % change |
|-----------|----------|-----------|----------|
| a | -60% | 1.43582 | 4.31937 | 452.419 | 365.360 |
| | -40% | 1.44547 | 6.38989 | 712.074 | 448.104 |
| | -20% | 1.45599 | 8.35955 | 1011.73 | 651.021 |
| | +20% | 1.45559 | 6.40998 | 1331.02 | 976.852 |
| | +40% | 1.45562 | 6.41074 | 1700.71 | 1309.77 |
| | +60% | 1.45577 | 6.41245 | 2090.38 | 1502.99 |
| b | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.56145 | 6.43599 | 1344.09 | 833.572 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |
| c | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.52259 | 6.41826 | 1498.82 | 943.113 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |
| d | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.54309 | 6.4383 | 1287.50 | 811.900 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |
| e | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.58197 | 6.47134 | 1230.70 | 723.499 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |
| r1 | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.54951 | 6.4467 | 1236.41 | 729.235 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |
| r2 | -60% | - | - | - | - |
| | -40% | - | - | - | - |
| | -20% | - | - | - | - |
| | +20% | 1.55203 | 6.5540 | 1287.50 | 811.900 |
| | +40% | - | - | - | - |
| | +60% | - | - | - | - |

Example 2.

The values of the system parameters are

\[
a = 90, b = 0.9, h = 0.3, d = 10, S = 17, C_0 = 90, c = 8, \theta = 0.003, n_1 = n_2 = 2, r_1 = 0.20, r_2 = 0.30, \alpha_1 = 1.5625, \alpha_2 = 2.04082, T_1 = 2.8.
\]
Scenario-I: Both type of discounts
$\mu = 1.32064, \gamma = 1.35691, \pi = 1070.84$ and $Q = 612.8.$

Scenario-II: Only pre deterioration discount
$\mu = 1.40145, \gamma = 1.80495, \pi = 1410.46$ and $Q = 541.645.$

Scenario-III: Only post deterioration discount
$\mu = 1.32151, \gamma = 1.42422, \pi = 1111.3$ and $Q = 613.757$

The following figures represent the concavity of total profit per unit time with respect to the pre and post deterioration discount starting time.

Figure 4: Concavity of total profit per unit time in Scenario-I

Figure 5: Concavity of total profit per unit time in Scenario-II

Figure 6: Concavity of total profit per unit time in Scenario-III
Table 3: Sensitivity Analysis for scenario-I

| Parameter | µ | γ | π | Q |
|-----------|---|---|---|---|
| a | -60 % | 1.31523 | 1.35261 | 609.91 | 244.645 |
|  | -40 % | 1.31651 | 1.35393 | 428.91 | 267.384 |
|  | -20 % | 1.32028 | 1.35615 | 849.91 | 267.384 |
|  | +20 % | 1.32098 | 1.35742 | 1240.81 | 375.52 |
|  | +40 % | 1.32108 | 1.35776 | 1512.78 | 458.235 |
|  | +60 % | 1.32115 | 1.35805 | 1733.75 | 580.957 |
| h | -60 % | 1.32023 | 1.35742 | 1075.70 | 614.545 |
|  | -40 % | 1.32054 | 1.35742 | 1075.70 | 614.545 |
|  | -20 % | 1.32229 | 1.35742 | 1075.70 | 614.545 |
|  | +20 % | 1.32265 | 1.35742 | 1075.70 | 614.545 |
|  | +40 % | 1.32281 | 1.35742 | 1075.70 | 614.545 |
|  | +60 % | 1.32299 | 1.35742 | 1075.70 | 614.545 |
| b | -60 % | 1.32023 | 1.35742 | 1075.70 | 614.545 |
|  | -40 % | 1.32054 | 1.35742 | 1075.70 | 614.545 |
|  | -20 % | 1.32229 | 1.35742 | 1075.70 | 614.545 |
|  | +20 % | 1.32265 | 1.35742 | 1075.70 | 614.545 |
|  | +40 % | 1.32281 | 1.35742 | 1075.70 | 614.545 |
|  | +60 % | 1.32299 | 1.35742 | 1075.70 | 614.545 |

5. DISCUSSIONS

The present paper develops an inventory model for perishable items considering price discount. Here, two types of price discount are considered in three different
scenarios. Firstly, both pre and post deterioration discounts are provided. Secondly, only pre deterioration discount, and finally, only post deterioration discount is provided. The efficacy of discounted selling price on optimising the total profit per unit time is studied by stacking up the results obtained in the given scenarios. The results clarify that the maximum profit can be attained in this inventory system only if the pre deterioration discount is provided. The post deterioration discount acquires less profit followed by the case of offering both types of discounts. Furthermore, the sensitivity analysis of the model reveals that the total average profit bumps up for increase in the values of the selling price, total cycle time, and the constants \(a, b, n_1\) and \(n_2\). It declines for increase in the values of disposal cost, deterioration rate, purchase cost, pre and post deterioration discount. The results of sensitivity analysis can act as the guide for managing the aforesaid inventory system.

6. CONCLUSION

Offering of price discount is the way of enticing the customers’ preference for the product. It acts as promotional aid for the seller and becomes essential for the short life span products or the products which get deteriorated over time. Most of the business organisations prefer post deterioration discount, but this paper suggests that, under the prevailing circumstances, pre deterioration discount is more beneficial for the decision makers. The management accordingly may embark up on studying the timing and quantity of price discount in pre deterioration period in order to minimise the pre deterioration cost. The model considered here is more suitable for the decoratively perishable items displayed to attract customers.

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