Current status of the dynamical Casimir effect

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Abstract
This is a brief review of different aspects of the so-called dynamical Casimir effect and the proposals aimed at its possible experimental realizations. A rough classification of these proposals is given and important theoretical problems are pointed out.

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1. Introduction
The term Dynamical Casimir Effect (DCE), introduced apparently by Yablonovitch [1] and Schwinger [2], is frequently used nowadays for the plethora of phenomena connected with the photon generation from vacuum due to fast changes of the geometry (in particular, the positions of some boundaries) or material properties of electrically neutral macroscopic or mesoscopic objects
1. A rough qualitative explanation of such phenomena is the parametric amplification of quantum fluctuations of the electromagnetic (EM) field in systems with time-dependent parameters. The reference to vacuum fluctuations explains the appearance of Casimir’s name (by analogy with the famous static Casimir effect, which is also considered frequently as a manifestation of quantum vacuum fluctuations [3–5]), although Casimir himself did not write anything on this subject. In view of many different manifestations of the DCE considered until now, it seems reasonable to make some rough classification. It is worth remembering that the static Casimir effect has two main ingredients: quantum fluctuations and the presence of boundaries confining the EM field. Therefore, I shall use the abbreviation MI-DCE (mirror induced DCE) for those phenomena where the photons are created due to the movement of mirrors or changes of their material properties. An explicit connection between quantum fluctuations and the motion of boundaries was made in [6, 7], where the name ‘nonstationary Casimir effect’ was introduced, and

in [8, 9], where the names ‘mirror-induced radiation’ and ‘motion-induced radiation’ (with the same abbreviation MIR) were proposed. The phenomena where the photons can be created due to the parametric amplification of vacuum fluctuations in media without moving or changing boundaries will be referred to as PA-DCE (parametric DCE). Although these names contain an obvious tautology, I try to maintain the combination DCE because it has become generally accepted by now. I confine myself to the studies related to cavities, resonators or equivalent set-ups which could give the possibility of verifying the DCE experimentally, leaving aside many other special cases considered up to now, such as the single mirror examples or numerous situations where the amount of quanta created due to the DCE is obviously too small to be detected. Extensive lists of publications on the DCE and systems with moving boundaries can be found in the reviews [10–12]. The main theoretical ideas and predictions are briefly discussed in section 2. Concrete available experimental schemes are considered in section 3, where some important problems waiting for solution are also pointed out.

2. The main ideas and theoretical predictions
The most important requirements that must be fulfilled in order to observe the DCE can be easily understood if one remembers the main idea laying the foundation of the theory of EM field quantization, namely that (roughly speaking) the EM field behaves as a set of harmonic oscillators. Mathematically it is expressed by writing the Hamiltonian operator of the free field in some cavity as

\[ \hat{H}_0 = \sum_{n=1}^{\infty} \hbar \omega_n (\hat{a}_n^\dagger \hat{a}_n + 1/2) , \] (1)
where $\hat{a}_n$ is the annihilation bosonic operator for the field mode with frequency $\omega_n$. To change the energy of the field (in particular, to create quanta from the initial vacuum state) at the level of formulae one must add to $H_0$ some operator $\hat{H}_1(t)$ describing the interaction between the field and the ‘material’ world. Since the coupling between the field and ‘matter’ is usually weak, $\hat{H}_1(t)$ can be expanded in the series with respect to powers of the annihilation and creation operators. The linear terms of this expansion describe the field excitation by external currents and charges, which seems to be quite a classical effect. The second-order terms contain combinations of the operators $\hat{a}_n^\dagger \hat{a}_k$, $\hat{a}_n \hat{a}_k$ (and their Hermitian conjugated counterparts) with time-dependent coefficients. These terms describe, in particular, possible time-dependent changes of the eigenfrequencies and the squeezing effects [13]. Physically, such interactions arise due to changes of the cavity geometry or the EM properties of the medium filling in the cavity. Such changes of parameters also result in the field excitation due to the amplification of the initial fluctuations. In a sense, this is also a classical effect, except for the important special case of the initial ground state: there are no fluctuations in this state from the classical point of view, whereas ‘zero-point’ fluctuations are predicted by quantum physics. Therefore the photon creation from the initial vacuum state due to changes of parameters is considered usually as a quantum effect whose experimental observation could be interpreted as a ‘direct’ proof of the existence of vacuum fluctuations (in contradistinction to the ‘indirect’ manifestations through the static Casimir effect, Lamb shift and many other phenomena). This explains the attractiveness of the DCE for many researchers, both theoreticians and experimentalists.

Following this line of reasoning one may suppose that significant features of the phenomenon could be caught in the simplest example of a single harmonic quantum oscillator (representing some field mode) with a time-dependent frequency [14]. The theory of quantum nonstationary harmonic oscillators has been well developed since its foundation by Husimi in 1953 [15] (see e.g. [16] for the review and references). It appears that all dynamical properties of the quantum oscillator are determined by the fundamental set of solutions of the classical equation of motion

$$ \ddot{\varepsilon} + \omega^2(t) \varepsilon = 0. $$

In particular, if $\omega(t) = \omega_1$ for $t < 0$ and $\omega(t) = \omega_2$ for $t > t_1$, then the information on the state of the quantum oscillator at $t > t_1$ is encrypted in the complex coefficients $\rho_k$ of the asymptotic form of the solution $\varepsilon(t > t_1) = \omega_1^{-1/2} [\rho_+ e^{-i\omega_1 t} + \rho_- e^{+i\omega_1 t}]$ originated from $\varepsilon(t < 0) = \omega_2^{-1/2} e^{-i\omega_2 t}$. For the initial thermal state of temperature $\Theta$ the mean number of quanta at $t > t_1$ equals [15, 17, 18]

$$ N = \langle \hat{a}_n^\dagger \hat{a}_n \rangle = G \left( \frac{|\varepsilon|^2 + \omega_0^2 |\varepsilon|^2}{4\omega_0} - \frac{1}{2} \right) = G \frac{R}{T}, $$

where $G = \coth \left[ \hbar \omega_0 / (2k_B \Theta) \right]$. The quantities $R = |\rho_+|^2 / |\rho_-|^2$ and $T = 1 - R = |\rho_-|^2$ can be interpreted as energy reflection and transmission coefficients from an effective ‘potential barrier’ given by the function $\omega^2(t)$. 

Intuition suggests that for the monotonic function $\omega(t)$ the effective reflection coefficient cannot exceed the value given by the Fresnel formula for the instantaneous jump of the frequency: $R \leq |(\omega_1 - \omega_2)/(\omega_1 + \omega_2)|^2$. This suggestion was proved rigorously in [19]. If the frequency varies due to the change of the cavity characteristic dimension $L$, then for small variation $\Delta L$ one obtains an estimation of the maximal possible number of created photons in a single mode $N_{\text{max}} \sim (\Delta L/L)^2$. Besides, the variations must be fast, because the number of quanta is the adiabatic invariant, so the photons cannot be created in slow processes. The duration of the fast process must be of the order of the period of the field oscillations (faster motions do not result in a significant increase of the number of photons, which is limited, after all, by the total change of frequency). This gives another estimation $N_{\text{max}} \sim (v/c)^2$, where $v$ is the characteristic velocity of the boundary and $c$ the speed of light. Consequently, the DCE is the relativistic effect of the second order and the expected number of created photons in a single mode is much less than unity for monotonic nonrelativistic motions.

This fact shuts down a possibility that the phenomenon of sonoluminescence (emission of bright short pulses of the visible light from air bubbles in the water when the bubbles pulsate due to the pressure oscillations in a strong standing acoustic wave [20, 21]) could be related to the DCE, although it was the starting point of Schwinger’s research [2]. Real hydrodynamical processes in the bubbles are too slow (even at the picosecond time scale) compared with fast oscillations of EM fields at the optical frequencies, so that the motion of the bubble’s surface should be considered as adiabatic from the electrodynamic point of view. Since the effective reflection coefficient is much smaller than unity for adiabatic processes [22], the mean number of photons created due to the bubble pulsations must be many orders of magnitude less than unity. (Besides, the actual change of the ‘vacuum energy’ due to the variations of the bubble size turns out to be ten orders of magnitude smaller than the initial Schwinger’s evaluations [23].)

However, it is well known that the effective reflection coefficient can be made as close to unity as desired in the case of periodic variations of parameters satisfying some resonance conditions (by analogy with periodic spatial structures). This is the basis of the proposals to use the parametric amplification effect in experiments on the DCE [9, 14, 18, 22, 24]. Earlier, this idea was put forward in [6, 25], but the evaluations of the effect were not correct. The possibility of significant amplification of the Casimir force under the resonance conditions was pointed out in [26] for the LC-contour and in [27] for the Fabry–Pérot cavity.

For harmonic variations of the frequency in the form $\omega(t) = \omega_0 [1 + 2k \cos(2\omega_0 t)]$ with $|k| \ll 1$, equation (2) can be solved approximately using, e.g., the method of averaging over fast oscillations or the method of slowly varying amplitudes [28]. Then equation (3) yields [17, 22]

$$ N = \sin^2(\omega_0 kt). $$

This formula does not take into account inevitable losses due to the dissipation in the cavity walls. For the cavity with a finite quality factor $Q = \omega_0/(2\gamma)$ (where $\gamma$ is the amplitude damping coefficient) one could suppose that formula (4) can be reliable for $\gamma t \ll 1$, so that the maximal number of
photon number $N_\text{max}$ can be roughly evaluated by putting $t = 1/\gamma$ in (4): $N_\text{max} \sim \sinh^2 (2Q\kappa)$, meaning that the necessary condition for the second statement is correct, while the estimation of $N_\text{max}$ is not. Indeed, the calculations made in the framework of the linear master equation with the standard dissipative superoperator [29, 30] gave the following asymptotical formula for the number of photons created from the initial thermal state for $2Q\kappa t > 1$ [30]: $N \approx (4\xi)^{-1} \exp(2Q\kappa \xi t)$, where $\xi = 1 - (2Q\kappa)^{-1}$. Consequently, exponential growth of the number of photons is possible if $2Q\kappa > 1$. Of course, such a fast growth cannot continue forever, since the linear approximation becomes invalid if $\omega_0 k^2 t \sim 1$. First estimations of the saturated value of the photon number due to nonlinear effects were made in [31], but this problem needs further investigations.

However, at the current moment the paramount task is to observe at least the beginning of the process of photon generation. Although formula (4) provides some insights, the real process is more complicated, in particular, due to the presence of infinitely many field modes in the cavity. How could this circumstance influence the rate of photon production in each mode and the total number of created quanta? A powerful tool for answering this question (although partially, neglecting the dissipation) is the method of effective Hamiltonians [13, 32]. It can be formulated as follows. Suppose that the set of Maxwell’s equations in a medium with time-independent parameters and boundaries can be reduced to an equation of the form $\hat{K}(\{L\})F_\alpha(\{r;\{L\}\}) = \omega_0^2(\{L\})F_\alpha(\{r;\{L\}\})$, where $\{L\}$ means a set of parameters (for example, the distance between the walls or the dielectric permittivity inside the cavity). $\omega_0(\{L\})$ is the eigenfrequency of the field mode labeled by the number (or a set of numbers) $\alpha$ and $F_\alpha(\{r;\{L\}\})$ is some vector function describing the EM field (e.g. the vector potential). In the simplest cases $\hat{K}(\{L\})$ is reduced to the Laplace operator. Usually, the operator $\hat{K}(\{L\})$ is self-adjoint, and the set of functions $\{F_\alpha(\{r;\{L\}\})\}$ is orthonormal and complete in some sense.

Now suppose that parameters $L_1, L_2, \ldots, L_n$ become time dependent. If one can still satisfy automatically the boundary conditions, expanding the field $F(\{r,t\})$ over ‘instantaneous’ eigenfunctions $F(\{r,t\}) = \sum \alpha q_\alpha(\{t\}) F_\alpha(\{r;\{L(\{t\})\}\})$ (this is true, e.g. for the Dirichlet boundary conditions, which are equivalent in some cases to the TE polarization of the field modes), then the dynamics of the field is described completely by the generalized coordinates $q_\alpha(\{t\})$, whose equations of motion can be derived from the effective time-dependent Hamiltonian [13]

$$H = \frac{1}{2} \sum_\alpha \left[ \dot{q}_\alpha^2 + \omega_\alpha^2(\{L(\{t\})\})q_\alpha^2 \right] + \sum_{\substack{k=1}}^n \frac{L_k(\{t\})}{L_k(\{t\})} \sum_{\alpha \neq \beta} p_{\alpha k} m_{\alpha k}^{(k)} q_\beta,$$

$$m_{\alpha k}^{(k)} = -m_{\beta k}^{(k)} = L_k \int dV \frac{\partial F_\alpha(\{r;\{L\}\})}{\partial L_k} F_\beta(\{r;\{L\}\}).$$

Consequently, the field problem can be reduced to studying the dynamics of the infinite set of harmonic oscillators with time-dependent frequencies and bilinear specific (coordinate–momentum) time-dependent coupling. The preceding one-mode example shows that the most important (from the point of view of applications to the DCE) cases are those where the parameters $L_k(\{t\})$ vary in time periodically. In the case of small harmonic variations at the frequency close to the double unperturbed eigenfrequency of some mode $\omega_0$, the equations of motion resulting from Hamiltonian (5) can be solved approximately with the aid of the method of slowly varying amplitudes. If the difference $\omega_\alpha - \omega_\beta$ is not close to $2\omega_0$ for all those modes that have nonzero (or not very small) coupling coefficients $m_{\alpha k}$, then only the selected mode with label 0 can be excited in the long-time limit, and one can consider only single resonance mode [22]. However, the intermode coupling can be important in some cases, especially for large amplitudes of the frequency variation.

For example, the resonance coupling between two modes is possible in cubical cavities [33]. This case was studied in [33, 34]. It was shown that the number of photons in both the coupled modes grows exponentially with time in the ‘long-time’ limit $\omega_0 k^2 t \gg 1$, but the rate of photon generation (the argument of the exponential function) turns out to be twice smaller than the value of this rate in the absence of the resonance coupling. (Actually, this rate depends on the concrete values of the coupling coefficients $m_{\alpha k}$, but in any case it cannot exceed the ‘uncoupled’ values [34].) This example indicates that the resonance coupling between the modes should be avoided in order to achieve the maximal photon generation rate, at least in the case of TE modes. A detailed numerical study of this case for different sizes of the rectangular cavities was performed in [35]. The authors of a recent paper [36] used numerical methods taking into account the interaction between 50 lowest coupled modes in the rectangular cavity bisected by a ‘plasma sheet’ with a periodically varying number of free carriers. Some plots in that paper show that the intermode coupling can increase the number of photons in the modes of the EM field with the TM polarization. But the maximal number of created photons in those examples did not exceed 5 (in contrast with [35] where the limits of time integration were extended to much bigger values, permitting us to reach the regime of large numbers of created photons). Therefore more precise calculations for a larger time scale are necessary in the TM case and for other geometries.

In the most distinct form the ‘destructive’ role of coupling between the modes can be seen in the example of a one-dimensional cavity with an ideal moving boundary. The first calculations of the number of created photons in this case were made by Moore [37] 40 years ago, although the dynamics of classical EM fields in this geometry was studied by many authors since the 1920s [38–41]. One of the possible physical realizations of this model is the Fabry–Pérot resonator; another possibility is the TEM modes in a coaxial cylindrical cavity [42]. The specific feature of this model is the equidistant form of the spectrum of eigenfrequencies: $\omega_\alpha = c\pi n/\ell$. Namely, for this reason the Heisenberg equations of motion following from the Hamiltonian (5) can be reduced to a simple set of equations admitting analytical solutions [22, 43]. Numerical calculations made in [44, 45] confirmed the high accuracy of these analytical solutions. It was shown in [22, 43] that the number of photons created from vacuum in the $n$th (odd) mode $N_n$ depends on time
t linearly in the asymptotical regime \( \kappa \omega_1 t \gg 1 \), whereas the total number of photons in all the modes \( N_{tot} = \sum_{n=0}^{\infty} N_n \) grows with time quadratically:

\[
N_n \approx 8\kappa \omega_1 t / (\pi^2 n^2), \quad N_{tot} \approx 2(\kappa \omega_1 t)^2.
\]

The total energy \( E = \sum_{n=1}^{\infty} h\omega_n N_n \) increases exponentially, \( E = (\hbar \omega / 4) \sinh^2(2\kappa \omega_1 t) \), due to the exponential increase of the number of excited modes [43, 46].

On the other hand, just due to the mode coupling some interesting phenomena could be observed in cavities with equidistant spectra, such as, for example, the formation of narrow packets, both inside the cavity (where they bounce periodically between the walls [47–51]) and outside it [52, 53]. For this reason attempts to observe the DCE in such cavities are quite interesting, too.

3. Experimental proposals for observing the DCE

3.1. Difficulties with real moving boundaries

According to formula (4) (confirmed by several groups using different analytical [18, 33, 42] and numerical [35] approaches), the possibility of experimental verification of the DCE depends on the amplitude of the frequency variation \( \Delta \omega = 2\kappa \omega_0 \). The main difficulty is due to the very high frequency \( 2\omega_0 \). The most exciting dream is to observe the ‘Casimir light’ [2] in the visible part of the EM spectrum. But it seems to be very improbable, at least in the ‘pure’ form, using real mirrors oscillating at a frequency of about \( 10^{15} \) Hz. Indeed, let us consider a suspended metallic plate of density \( \rho \), area \( S \) and thickness \( b \), illuminated by the laser beam of frequency \( \omega_0 \) and average intensity \( I \). The radiation pressure force depends on time as \( F(t) = 2JS[1 + \cos(2\omega_0 t)]/c \) (for the uniform illumination), so it can cause the forced oscillations of the plate exactly at twice the frequency \( 2\omega_0 \). Since the optical frequency \( \omega_0 \) is many orders of magnitude higher than the mechanical frequency of the suspension, the amplitude of displacements of the plate from the mean position equals \( \Delta L = F_{max} \omega_0 / [m(2\omega_0)^2] = \pi / (2c\rho b\omega_0^2) \). It results in the amplitude of the frequency variation \( \Delta \omega = \xi \omega_0 / (\sqrt{L}/c) \approx \xi(\lambda/L)\sqrt{4\pi \rho c / \omega_0^2} \), where \( \lambda = 2\pi c / \omega_0 \) is the wavelength in vacuum corresponding to frequency \( \omega_0 \). \( L \) is the average value of the variable length of the cavity and \( \xi \) is the numerical factor of the order of unity (\( \xi = 1 \) for the Fabry–Pérot cavity modeled by two infinite parallel plates). Taking \( \lambda/L \sim 1 \), \( \rho \sim 3 \times 10^3 \) kg m\(^{-3}\) (Al) and \( b \sim 1 \mu m \), one can see that the frequency variation amplitude \( \Delta \omega \approx 1 \times 10^7 \) s\(^{-1}\) can be achieved for the laser intensity \( I \sim 3 \times 10^{14} \) W m\(^{-2}\). According to formula (4) the product \( \Delta \omega t \) should not be smaller than unity to generate more than one photon in the mode. Consequently, the total laser energy per unit area of the plate should be not less than \( 3 \times 10^{15} \) J m\(^{-2}\), which is obviously unrealistic. This estimation shows the impossibility of exciting and maintaining the high-frequency oscillations of the suspended plate with a large amplitude and for a long time. On the contrary, the low-frequency oscillations at the mechanical frequency of the suspension can be excited. They result, in particular, in the Kerr-like back-action effect on the field [54]. This is a very interesting area, including the generation of the so-called ‘nonclassical states’ (e.g. quantum superpositions) of the field and the mirror (considered as a quantum object) [55–57], the mirror–field entanglement [58–61], cooling mirrors by the radiation pressure [62–66], etc, but it is totally distinct from the DCE.

It seems that the only possible way to realize the real motion of material boundaries at high frequencies is not to move the whole mirror, but to cause its surface to perform harmonic vibrations with the aid of some mechanism, e.g. using the piezo-effect [67]. The amplitude of such vibrations \( \Delta L \) is connected with the maximal relative deformation \( \delta \) in a standing acoustic wave inside the wall as \( \delta = \omega_0 \Delta L / v_0 \), where \( v_0 \approx 5 \times 10^7 \) m s\(^{-1}\) is the sound velocity. Since usual materials cannot bear deformations exceeding the value \( \delta_{max} \sim 10^{-2} \), the velocity of the boundary cannot exceed the value \( v_{max} \sim \delta_{max} v_0 \sim 50 \) m s\(^{-1}\) (independent of the frequency). The maximal possible frequency variation amplitude \( \Delta \omega \) can be evaluated as \( \Delta \omega = \xi v_0 \delta_{max} (2L/L) \). For the optical frequencies \( 2L > 1 \mu m \) and \( \Delta \omega < 5 \times 10^{-1} \) s\(^{-1}\), whereas for the microwave frequencies (in the GHz band) \( L < 1 \) cm and \( \Delta \omega < 10^{-2} \) s\(^{-1}\). Since the time of excitation \( t \) must be bigger than \( 1/\Delta \omega \), the quality factor of the cavity \( Q \) must be not less than \( Q_{min} \approx \omega_0 / (\Delta \omega) \approx (L/L) / (4\pi c / \omega_0 ) \sim 10^6 (L/L) \). Consequently, there are two main challenges: how to excite high-frequency surface oscillations and how to maintain the high quality factor in the regime of strong surface vibrations. The excitation of high-amplitude surface vibrations at the optical frequencies seems very problematic. Therefore hardly the ‘Casimir light’ in the visible region can be generated in systems with really moving boundaries. However, this seems to be possible in other schemes, where changes of some parameters can be interpreted as variations of an ‘effective length’ of the cavity; see section 3.5.

The GHz frequency band seems more promising. In such a case, the dimensions of cavities must be of the order of a few centimeters. Superconducting cavities with the quality factors exceeding \( 10^{10} \) in the frequency band from 1 to 50 GHz are available for a long time [68–70]. Therefore the most difficult problem is to excite the surface vibrations. At lower frequencies it was solved long ago. For example, the excitation of vibrations of the mirror at the frequency of 60 MHz with the aid of a quartz transducer was reported in [71]. The calculated values of the peak displacement and velocity were \( 1.4 \times 10^{-8} \) cm and \( 5.3 \) cm s\(^{-1}\). Recently, significant progress was achieved in the fabrication of the so-called ‘film bulk acoustic resonators’ (FBARs): piezoelectric devices working at the frequencies from 1 to 3 GHz [72]. They consist of an aluminum nitride (AlN) film of thickness corresponding to one half of the acoustic wavelength, sandwiched between two electrodes. It was suggested [73] to use such a kind of devices to excite the surface vibrations of cavities in order to observe the DCE. However, the problem is very difficult, and no experimental results in this direction have been reported as yet.

3.2. Effective moving boundaries: MIR experiment with semiconductor mirrors

In view of the difficulties in the excitation of oscillating motion of real boundaries, ideas concerning the imitation of this movement have attracted more and more attention in
the course of time. The first concrete suggestion was made two decades ago by Yablonovich [1], who proposed to use a medium with a rapidly decreasing in time refractive index (‘plasma window’) to simulate the so-called Unruh effect. Also, he pointed out that fast changes of dielectric properties can be achieved in semiconductors illuminated by subpicosecond optical pulses and supposed that ‘the moving plasma front can act as a moving mirror exceeding the speed of light’. Similar ideas and different possible schemes based on fast changes of the carrier concentration in semiconductors illuminated by laser pulses were discussed in [74–76]. Yablonovich [1, 74] put emphasis on the excitation of \textit{virtual electron–hole pairs} by optical radiation tuned to the transparent region just below the band gap in a semiconductor photodiode. He showed that big changes of the \textit{real part} of the dielectric permittivity could be achieved in this way.

The key idea of the experiment named ‘MIR’, which is under preparation in the University of Padua [77, 78], is to imitate the motion of a boundary, using an effective ‘plasma mirror’ formed by \textit{real electron–hole pairs} in a thin film near the surface of a semiconductor slab, illuminated by a periodical sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the slab. This can be interpreted as periodical displacements of the boundary. The basic physical idea was nicely explained in [78]: ‘…this effective motion is much more convenient than a mechanical motion, since in a metal mirror only the conduction electrons reflect the electromagnetic waves, whereas a great amount of power would be wasted in the acceleration of the much heavier nuclei.’

The main advantage of the semiconductor mirror is a great increase of the maximal frequency shift, compared with the case of vibrating surface. This shift is determined mainly by the thickness of the semiconductor slab. Using the slabs of a few millimeters thickness, one can easily obtain the frequency variation amplitude $\Delta\omega \sim 10^3 \text{s}^{-1}$ or even bigger in the GHz range of the cavity resonance frequencies. Then the total excitation time can be reduced to less than 1 $\mu$s and the cavity quality factor can be lowered to the easily achievable values of the order of $10^3$ or even $10^4$. There are proposals [36] to put the semiconductor slab in the middle of the rectangular cavity. In this case the frequency shift attains the maximal value. However, it is not quite clear whether the strong intermode coupling will not diminish the final number of photons. Besides, in this configuration one whether the strong intermode coupling will not diminish the number of photons. Besides, in this configuration one

The thickness of the photo-excited conducting layer near the surface of the semiconductor slab is determined mainly by the absorption coefficient of the laser radiation, so it is about a few micrometers or less (depending on the laser wavelength), being much smaller than the thickness of the slab itself. Therefore laser pulses with a surface energy density of about a few $\mu J \text{cm}^{-2}$ can create a highly conducting layer with the carrier concentration exceeding $10^{11} \text{cm}^{-2}$, which gives rise to an almost maximal possible change of the cavity eigenfrequency for the given geometry [77, 84]. It is worth noting that although the thickness of the conducting layer is less than the skin depth, it gives the same frequency shift as the conductor filling in all the slab. This interesting fact was explained and verified experimentally in [85].

Note also that the laser wavelength $\lambda_{\text{las}}$ is of the order of 1 $\mu$m, while the wavelength $\lambda_{\text{cw}}$ of the fundamental cavity mode that is supposed to be excited due to the DCE is about 10 cm. Consequently, if an antenna put somewhere beside the cavity and tuned to the resonance cavity frequency will register a strong signal after the set of laser pulses, one can be sure that the quanta of EM field in the fundamental mode were created due to the DCE and they do not belong to some ‘tail’ of the laser pulse, just due to the difference by five orders of magnitude between $\lambda_{\text{las}}$ and $\lambda_{\text{cw}}$.

However, using the semiconductor mirror in the DCE experiments, one has to overcome several serious difficulties, resulting from the fact that laser pulses create pairs of \textit{real} carriers, which change mainly the \textit{imaginary part} $\epsilon_i \equiv 4\pi\sigma(\omega)/\omega$ of the complex dielectric permittivity $\epsilon = \epsilon_i + i\epsilon_r$. Here $\sigma(\omega)$ is the real conductivity at frequency $\omega$ (in the CGS system of units). For example, let us use the simple Drude model formula $\epsilon(\omega) = \epsilon_a + [4\pi\sigma_0/\omega(1 - i\omega\tau)]$, where a real constant $\epsilon_a$ describes the contribution of bounded electrons and ions, $\sigma_0 = ne^2/\tau m$ is the static (zero-frequency) conductivity, $n$ is the concentration of free carriers (created by laser pulses) with charge $e$ and effective mass $m$, and $\tau$ is the relaxation time. The imaginary part of $\epsilon$ can be neglected under the condition $\omega\tau \gg 1$, which means that the low-frequency mobility $b = |e|\tau/m$ (related to the low-frequency conductivity $\sigma_0$ as $\sigma_0 = n|e|b$) must be much bigger than $b_a(\omega) = |e|/(\omega a)$. For the optical frequencies $\omega \sim 3 \times 10^{15} \text{s}^{-1}$ and for $m \sim m_e$ (the mass of free electron) one has $b_a(\omega) \sim 5 \times 10^{-5} \text{m}^2 \text{V}^{-1} \text{s}^{-1}$, so that the condition $b \gg b_a$ can be easily fulfilled, meaning that one can use the \textit{real-valued} function $\epsilon(\omega)$. Namely this special case was considered by several authors [36, 81, 86] who studied quantum effects caused by the periodical variations of properties of thin \textit{ideal} dielectric slabs or infinitely thin \textit{ideal} conducting films (described by means of time-dependent $\delta$-potentials in the framework of the ‘plasma sheet’ model) put inside the resonance cavities. Unfortunately, the results of those studies, being interesting by themselves, cannot be applied to the MIR experiment, where the resonance frequency is about 2.3 GHz ($\omega \approx 1.4 \times 10^{10} \text{s}^{-1}$). For this frequency one obtains $b_a(\omega) \sim 10 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$, whereas the reported values of the mobility in the highly doped GaAs samples used in
this experiment are of the order of $1 \text{m}^2\text{V}^{-1}\text{s}^{-1}$ [88, 89], and hardly this mobility can be increased by two orders of magnitude (maintaining the necessary very small recombination time) to satisfy the condition $b \gg b_*$. Therefore a much more reliable approximation of the complex dielectric function $\epsilon(\omega)$ that should be used in the analysis of realistic DCE experiments with semiconductor time-dependent mirrors is $\epsilon(\omega) \approx \epsilon_0 + 4\pi|\sigma_0|/\omega$. As a consequence, the ‘instantaneous’ time-dependent resonance frequency becomes complex-valued function $\Omega = \omega - i\gamma$. The calculations made in [11, 12, 84] gave the following formula for the time-dependent damping coefficient $\gamma(t)$ and the frequency variation $\chi(t) = \omega(t) - \omega_0$:

$$\chi(t) = \frac{|\chi_m| A^2(t)}{A^2(t) + 1}, \quad \gamma(t) = \frac{|\chi_m| |A(t)|}{A^2(t) + 1} \quad (8)$$

where $\chi_m$ is the maximal possible frequency shift attained when the slab becomes a perfect conductor. The function $A(t)$ is proportional to the time-dependent integral of the free carriers concentration across the slab. For ultrashort pulses and negligible surface recombination and diffusion coefficients, $A(t) = A_0 \exp(-t/T_r)$, where $T_r$ is the recombination time and $A_0$ is proportional to the product of the total energy of the laser pulse by the mobility of carriers. Equation (8) clearly shows that although the damping coefficient $\gamma(t)$ can be safely neglected if $\epsilon_2 \ll 1$ and $\Delta \ll 1$ (an almost ideal dielectric) or $\epsilon_2 \gg 1$ and $A \gg 1$ (an almost ideal conductor), it becomes very important in the intermediate regime, when the high concentration of carriers achieved after the action of a short laser pulse returns continuously to the initial value. Even if $A_0 \gg 1$, during some time interval one has $A(t) \sim 1$ and $\gamma(t) \sim |\chi_m|/2 \sim |\chi(t)|$. Therefore, the influence of dissipation is predominant at the final stages of the recombination process. These observations show that without taking into account inevitable losses inside the semiconductor slab during the excitation–recombination process, one cannot predict the results of the realistic DCE experiments for microwaves even qualitatively.

A simple model taking into account the dissipation was developed in [11, 12, 84, 90]. It was assumed that the dynamics of a single nonstationary quantum oscillator (representing the resonance mode of the field) with a time-dependent linear damping can be described in the framework of the Heisenberg–Langevin equations with two noncommuting and delta-correlated time-dependent noise operators. One of the results is the following formula for the maximal mean number of photons that could be generated from the initial thermal field state after $n \gg 1$ pulses of periodicity $T$:

$$N_\omega \approx \frac{G_1(v - \Lambda) + G_w \Lambda}{4(v - \Lambda)} e^{2n(v - \Lambda)}. \quad (9)$$

Here $v = \int_0^T \chi(t) \exp(-2i\omega_0t) dt$ and $\Lambda = \int_0^T \gamma(t) dt$. Note that the initial temperatures and corresponding amplification coefficients of the field mode $G_1$ and the cavity walls $G_w$ can be different. The exact value of $T$ must be close to the half-period of the excited field mode but not coincide exactly with this half-period, in order to fulfill the resonance conditions. Numerical calculations show that the difference $v - \Lambda$ can be positive if only the energy of laser pulses exceeds some critical value [11, 12, 84]. The existence of this critical value takes its origin in the different behaviors of the real and imaginary parts of the frequency shift in the semiconductor with real free carriers: for small concentrations of created carriers (i.e. for low pulse energies) the imaginary part increases linearly as a function of energy, whereas the real part increases quadratically, as can be seen, in particular, in equation (8). Numerically, this critical value turns out rather high: different estimations give the values from 1 to $10 \mu J$ or even $100 \mu J$, depending on the cavity geometry, recombination time and mobility of carriers. But the decisive factor is the energy gap of the semiconductor of an order of $1 \text{eV}$.

For $A_0 \gg 1$ one can obtain [90] simple approximate formulas $\Lambda \approx \pi |\chi_m| |\omega_0| T_r/2$ and

$$2v/|\chi_m| \approx \left| 1 - \frac{\pi \omega_0 T_r}{\sinh(\pi |\omega_0| T_r)} \right|^2. \quad (10)$$

They show that the photon generation can be achieved only for short recombination times $T_r < 0.5|\omega_0|^{-1}$. This requires hard work in preparing the semiconductor samples satisfying contradictory requirements: a short recombination time (less than 20 ps) but a high mobility. Nonetheless it seems that these problems can be resolved [88, 89], and the first experiment on the MI-DCE will be done soon. It is expected that 1000–2000 laser pulses will be sufficient to generate several thousand microwave photons, much more than the measured sensitivity level of about 100 photons [91].

### 3.3. MI-DCE with illuminated superconducting boundaries

Some of the problems mentioned in the preceding subsection can be softened if laser pulses illuminate not the semiconductor but superconductor surfaces. In this case, the changes of dielectric properties happen due to the transition from the superconducting to the normal conducting phase caused by the local heating of the surface. Since the energy gap in superconductors is several orders of magnitude smaller than the energy gap in semiconductors, the energy of laser pulses can be made several orders of magnitude smaller than in the case of semiconductor mirrors. The frequency modulation of the superconducting microwave resonator by laser irradiation was reported in [92]. The authors of that paper used the parallel-plate resonator consisting of two superconducting $\text{YBa}_2\text{Cu}_3\text{O}_7-\gamma$ films of 300 nm thickness with dimensions $10 \times 10 \text{mm}^2$, separated by a sapphire spacer of thickness 0.3 mm. The temperature varied from 30 K up to $T_c \approx 91 \text{ K}$. They illuminated one plate by the cw Ar laser with the diameter of the illuminated spot of about 1 mm and demonstrated the downwards shift of the cavity resonance curve by 0.5 MHz from the initial value of 5400 MHz without a visible change in the width of the curve. This suggests that the imaginary part of the resonance frequency change can be neglected. Therefore the schemes based on changing the electrodynamic properties of superconducting mirrors seem to be rather promising.
3.4. Various PA-DCE schemes

A concrete proposal relating DCE with the laser-illuminated superconductors was made in [93], where the superconducting stripline resonator (a ring having a radius of 6.39 nm and width 347 µm composed of an NbN film of 8 nm thickness deposited on a sapphire wafer [94]) was considered as a promising candidate for the photon generation from vacuum in the range from 2 to 8 GHz. Strictly speaking, it is difficult to connect the change of the frequency shift of this resonator with an effective motion of some boundary. But if one assumes the definition of the DCE as the phenomenon of photon creation from vacuum due to the change of some parameters of a system [14], then this scheme fits perfectly to the PA-DCE family. The advantages of proposals based on the periodic illumination of superconductors consist of the easy modulation of the resonance frequency, the big amplitude of its variations and a low necessary energy of laser pulses. For example, a parabolic dependence of frequency shift on pulse energy was reported in [95]. The 70 ps pulses of energy 3 nJ resulted in a 20 MHz shift at the temperature of 20 K and almost 100 MHz at 80 K (for the YBa$_2$Cu$_3$O$_7$$_x$ strips). The NbN films demonstrated [93] an almost 40 MHz frequency shift (at the liquid helium temperature), caused by pulses of the infrared laser (1550 nm wavelength) modulated at twice the resonator eigenfrequency 7.74 GHz. The reported laser power was 27 nW.

Different PA-DCE schemes were proposed recently in the framework of the so-called circuit QED (which uses superconducting qubits as ‘artificial atoms’ coupled to microwave resonators [96–99]). The idea to use quantum resonant oscillatory contours or Josephson junctions with time-dependent parameters (capacitance, inductance, magnetic flux, critical current, etc) to observe the DCE was put forward many years ago [14, 100]. But concrete schemes were proposed only recently. One of them was reported in [101]. Its principal part is a double rf-SQUID system whose Josephson critical current can be controlled by an external time-dependent magnetic flux. Another scheme was proposed in [102]. It uses the coplanar waveguide terminated by a tunable (also by an external magnetic flux) superconducting quantum interference device (SQUID), which is equivalent to a short-circuited transmission line with a tunable length simulating a tunable mirror. It was suggested to detect the flux of microwave radiation going along the transmission line outwards. The evaluations give the photon production rate $10^5$ photons s$^{-1}$ in the 100 MHz bandwidth around the central frequency 9 GHz. The necessary temperature should be below 70 mK. However, both the proposals did not contain any information concerning one crucial detail: how to change the parameters at the time scale shorter than the period of the EM field mode (for example, faster than 100 ps in the case of [102])? The possibility of fast tuning the field in the microwave resonator was demonstrated experimentally in [103], where the device consisted of a quarter wavelength coplanar waveguide resonator terminated to ground via SQUIDs in series. The SQUID inductance was varied by applying an external magnetic field. It was shown that the resonance frequency of an order of 4.8 GHz can be changed by about 740 MHz by applying magnetic flux pulses whose duration is of the order of 10 ns. However, although this time is smaller than the photon lifetime in the cavity with the quality factor $10^4$, it is still two orders of magnitude bigger than the duration of pulses necessary for the observation of the DCE.

Many effects of the circuit QED can be understood in the framework of a simple model with the interaction Hamiltonian $H_{int} = h g(t) (\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_x + \hat{\sigma}_z)$ [104] (where $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are the raising and lowering operators describing atomic (electron) excitations) or its generalizations [96]. Since this Hamiltonian is formally linear with respect to the photon creation and annihilation operators $\hat{a}$ and $\hat{a}^\dagger$, one can expect, in principle, a higher photon generation rate than in the case of the frequency variation (where the interaction Hamiltonian is quadratic with respect to $\hat{a}$ and $\hat{a}^\dagger$), provided a strong modulation of the effective Rabi frequency $g(t)$ can be achieved. It seems that such a possibility exists: see the next subsection.

The photon generation due to the time-dependent variations of properties of dielectric transparent media was considered by many authors [17, 29, 105–111]. This kind of phenomenon also belongs to the PA-DCE class. However, realistic experimental schemes were proposed only recently. They are described in the next subsection.

3.5. Emission of Casimir radiation outside a cavity

The photon flux radiated outside the one-dimensional Fabry–Pérot cavity with harmonically oscillating semitransparent boundaries was calculated in [9]. Using the spectral approach, it was shown that the radiation can be essentially enhanced under the resonance conditions, comparing with the case of a single oscillating mirror. It was suggested recently [112] that the motion of the boundary could be imitated by putting inside the cavity a thin nonlinear crystal (thickness 0.1 µm and nonlinear susceptibility $\chi^{(2)} \sim 10^{-11}$ mV$^{-1}$), pumped by an optical beam of frequency $f = \Omega/2\pi \approx 3 \times 10^{14}$ Hz ($\lambda = c/f = 1 \mu m$) and power about 1 W, focalized over an area $A = 10^{-4}$ m$^2$ (the ‘Casimir’ photons can be distinguished from the pump ones due to the orthogonal polarizations). The evaluations were made in the framework of the one-dimensional model of the cavity (because the equidistant spectrum of eigenfrequencies was used explicitly). Consequently, the cavity must be very small: its length $L$ must obey the inequality $L \ll \sqrt{A} = 10 \mu m$. Actually, this inequality together with the resonance conditions can be satisfied in the case involved only for $L = \lambda$. These values of parameters result in the amplitude of variation of the effective cavity length $\Delta L \sim 10^{-12}$ m and the relative maximum velocity of the equivalent moving boundary $\beta = v/c \sim 10^{-5}$. Then the formulae derived in [9] give the following average total flux of ‘Casimir’ photons leaving the cavity with finesse $F = 10^4$ after time $t$: $(N^{out})/t = \beta^2 F \Omega^3/(3\pi) \sim 10^7$ photons per second (accidentally the same number as for the microwave photons in the scheme of [102]). As a matter of fact this is not a big number, because it means that photons are emitted with intervals of about $10^{-5}$ s, so it is necessary to wait for 100 µs in order to register about ten photons. This is explained by the low stationary mean number of photons inside the cavity: according to [9] $(N^{in}) = 2(\beta F)^3/(3\pi^2) \sim 10^{-5}$, and this evaluation follows also from formula (7), if one identifies $t$ with the relaxation time of the leaking cavity $t \sim F/\omega_0$ and puts $\kappa = \Delta L/(2L) = 5 \times 10^{-7}$.
For comparison, in the MIR experiment (discussed in subsection 3.2) it is expected to generate from $10^3$ to $10^4$ microwave quanta after 1000–2000 laser pulses of total duration 0.2–0.4 µs and total energy about 10–20 mJ. To emit the same amount of photons from the Fabry–Pérot cavity under consideration, one needs from 10 to 100 mJ in the pumping laser beam.

The possibility of emission of infrared photons from semiconductor microcavities with a time-modulated vacuum Rabi frequency was studied theoretically in [113, 114]. The authors considered a planar Fabry–Pérot resonator embedding a sequence of many identical quantum wells doped with a two-dimensional electron gas. It was shown that such a system permits one to obtain an ultrastrong light–matter coupling (namely, the ratio of the Rabi frequency to the frequency of the intersubband transition can be of the order of 0.1), which can be easily tuned by applying to the metallic mirrors a bias voltage (which changes the density of the two-dimensional electron gas). As was demonstrated in [115], the Rabi frequency can be changed on a timescale shorter than the cycle of emitted light. The cavity contained 50 identical undoped GaAs quantum wells. The effective thickness of the structure corresponded to $\lambda/2$ for the intersubband absorption line $\lambda = 11 \mu$m (the period $T_0 = 37$ fs). The electronic transitions from the valence band into the conduction subband were activated by near-infrared 12 fs control pulses with photon energy 1.55 eV and intensity up to 0.1 mJ cm$^{-2}$. The authors wrote in [115] that the number of vacuum photons released per pulse could be of the order of $10^3$. However, this number seems to be exaggerated, because it strongly depends on the modulation amplitude of the Rabi frequency. For example, figure 3 of [114] shows that for an extremely big modulation amplitude 20% (which can hardly be achieved, although no evaluations of the realistic values of this important parameter were given) the rate of emitted photons $dN/d\tau$ does not exceed $10^{-7} \omega_{12}$, where $\omega_{12} = 2\pi/\tau_0$. For $T_0 = 37$ fs, one obtains $dN/d\tau \sim 2 \times 10^{12}$ photons s$^{-1}$ or 0.07 photons pulse$^{-1}$. For the cavity quality factor $10^6$ this would finally result in about $10^3$–$10^4$ infrared photons, i.e. more or less the same number as expected for other schemes. Therefore this scheme also seems to be prospective.

3.6. Detection and photon statistics

The distribution function of photons generated via the DCE is non-thermal [22, 116]. The probability $f(m)$ to generate exactly $m$ photons in the single mode case is given by the formula [12, 90] $f_{DCE}(m) \approx (2\pi Nm)^{-1/2} \exp[-m/(2N)]$, where $N$ is the mean number of photons (this simple result holds for $m \gg 1$ and $N \gg 1$). On the other hand, the thermal distribution has the form $f_{th}(m) \approx N^{-1} \exp[-m/N]$ if $N \gg 1$. This example shows the difference between the DCE and the so-called Unruh effect [117–119].

But how to detect the quanta of the EM field generated due to the DCE? One of the possibilities (quite standard for the cavity QED experiments [120]) could be to pass a beam of Rydberg atoms through the cavity [22, 67]. To achieve better sensitivity, it was proposed [73, 121] to send an ensemble of population-inverted atoms, using the effect of superradiance. The electron beams were proposed for this purpose in [122].

The simplest theoretical models of the detector are the two-level systems (whose interaction with the resonance field mode is described by means of the Jaynes–Cummings model with time-dependent parameters [22, 67, 123–125]) or harmonic oscillators [22, 67]. It was shown that the field–detector interaction can change significantly both the photon generation rate and the photon distribution function [22, 67]. But this subject needs more thorough investigations using more realistic models. Another important problem is related to the statistics of counts by detectors (it can be quite different from the photon statistics in the field mode due to the effects of counting efficiency, dead times, etc). This problem (with respect to the DCE experiments) was not considered at all until now.

4. Conclusion

This brief review of the most important results obtained by different groups of theoreticians and experimentalists for the past few years shows that experimental observations of different manifestations of the dynamical Casimir effect are quite possible and can be expected in the nearest future.

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