Evaluation of the resonance enhancement effect in neutrinoless double-electron capture in $^{152}$Gd, $^{164}$Er and $^{180}$W atoms

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We study the resonant neutrinoless double-electron capture ($0\nu$ECEC) in $^{152}$Gd, $^{164}$Er and $^{180}$W atoms, associated with the ground-state to ground-state nuclear transitions. The corresponding matrix elements are calculated within the deformed QRPA using the realistic Bonn-CD nucleon-nucleon interaction. The half-lives are estimated with the use of the most recent precision data on the $Q$-values of these processes. Perspectives of experimental search for the $0\nu$ECEC with the isotopes $^{152}$Gd, $^{164}$Er and $^{180}$W are discussed.

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I. INTRODUCTION

One of the unsolved mysteries of today’s particle physics and cosmology is the question of whether neutrinos are Dirac or Majorana particles. In the first case neutrinos and antineutrinos are fundamentally different, whereas in the second case neutrinos and antineutrinos are identical. Theoretical arguments in favor of Majorana neutrinos exist for decades in connection with the smallness of neutrino masses. Some grand unified theories explain smallness of the masses, e.g., in a seesaw scenario with heavy Majorana leptons [1].

Neutrinoless double-beta decay ($0\nu\beta\beta$),

$$\begin{align*}
(A, Z) &\rightarrow (A, Z + 2) + e^- + e^-, \quad (1)
\end{align*}$$

is being considered as the unique practical tool for determining the nature of neutrinos (for a review, see [2]). The most favorable decays for the experimental search are those with high mass difference between the ground state neutral atoms, i.e. $Q$-values, in which the parent nuclei decay to ground states of the daughter nuclei.

The mere observation of neutrinoless double-electron capture ($0\nu$ECEC),

$$\begin{align*}
e^- + e^- + (A, Z) &\rightarrow (A, Z - 2)^{**}, \quad (2)
\end{align*}$$

could also prove the Majorana nature of neutrinos as well as the violation of the total lepton number conservation. A double asterisk in Eq. (2) means that, in general, the final atom $(A, Z - 2)$ is excited with respect to both the electron shell, due to formation of two vacancies for the electrons, and the nucleus. This process, as noted long time ago by Bernabéu, De Rujula and Jarlskog [3], may have a resonant character under the condition of degeneracy of the masses of initial and intermediate atoms. In contrast to the $0\nu\beta\beta$ decays, in the $0\nu$ECEC capture small $Q$-values are favorable. The capture rate is a sensitive measure of the neutrino mass.

The resonance enhancement can increase the probability of capture by many orders of magnitude. In searches for Majorana neutrinos, the neutrinoless double-electron capture can compete with the neutrinoless double-$\beta$ decay provided the resonance condition is satisfied within a few tens of electron-volts. So far, however, there was no way to identify promising isotopes for experimental search of $0\nu$ECEC, because of poor experimental accuracy of measurement of $Q$-values which until recently were known with uncertainties of 1 - 10 keV only [4]. Progress in precision measurement of atomic masses with Penning traps [5,6] has revived the interest in the old idea on the resonance $0\nu$ECEC capture.

Sujecki and Wycech [7] and Lukaszuk et al. [8] analyzed the $0\nu$ECEC process for nuclear $0^+ \rightarrow 0^+$ transitions accompanied by a photon emission in the resonance and non-resonance modes. The physical background for the process was calculated. A new theoretical approach developed by Šimkovic and Krivoruchenko [9,10] and Krivoruchenko et al. [11] allowed a unified description of the oscillations of stable and quasistationary atoms, which take place with violation of the total lepton number conservation and are followed by de-excitation with emission of photons. Based on the most recent data and realistic evaluation of the decay half-lives, a complete list of the most perspective isotopes for which the $0\nu$ECEC capture may have the resonance enhancement was provided in Ref. [11] for further experimental study.
Some isotopes such as $^{156}$Dy have several closely-lying resonance levels. A more accurate measurement of $Q$-value of atoms $^{156}$Dy and $^{156}$Gd confirmed the existence of overlapping $0\nu$ECEC resonance levels [12]. Assuming an effective mass for the Majorana neutrino of 50 meV and an appropriate value of nuclear matrix element, half-lives of some of the isotopes were found to be as low as 10$^{25}$ years in the unitary limit, which is one order of magnitude shorter than the $0\nu\beta\beta$ half-life of $^{76}$Ge for the same mass of Majorana neutrino [13].

In high-$Z$ atoms, the electrons in inner shells are moving with relativistic velocities. Effects associated with the relativistic structure of the electron shells reduce the $0\nu$ECEC half-lives by almost one order of magnitude. In contrast to the non-relativistic theory, the capture of electrons from the $np_{1/2}$ states is only moderately suppressed in comparison with the capture from the $ns_{1/2}$ states. In the relativistic formalism, selection rules appear to require that nuclear transitions with a change in the nuclear spin $\Delta J \geq 2$ are strongly suppressed. The relativistic effects also enhance the violation of parity in the the $0\nu$ECEC process, as a result of which nuclear transitions $0^+ \rightarrow 0^\pm, 1^\pm$ become all attainable for a mixed capture of $s$- and $p$-wave electrons [11]. A similar effect occurs due to weak right-hand currents as discussed by Vergados [14].

Recently there has been fast progress in the measurement of atomic masses with the help of Penning traps. The accuracy of $Q$-values at around 100 eV was achieved [12, 15–23], which has already allowed to exclude a number of isotopes from the list of the most promising candidates for searching the neutrinoless double-electron capture. The best candidate is currently $^{152}$Gd, which although does not reach the unitary limit, however, undergoes a significant increase in the capture rate due to the proximity to the resonance level [20]. Further precise measurements of masses of prospective isotopes are vigorously encouraged to continue.

Neutrinoless double-electron capture has a number of important advantages with respect to experimental signatures and background conditions. In recent years, experimental searches for the capture process were continued [24–30]. New upper limits of about $10^{19} - 10^{21}$ years for the half-lives of $^{74}$Se, $^{106}$Cd and $^{112}$Sn have been obtained [24, 25, 27, 28, 30]. To make further progress new experimental data on excited states of finite-state nuclei (excitation energy, angular momentum, parity) and precise calculations of transition nuclear matrix elements (NMEs) are required. Among the promising isotopes, $^{152}$Gd, $^{164}$Er and $^{180}$W have likely resonance transitions to the $0^+$ ground states of the final nuclei. [1]

In this paper, accurate calculations of the $0\nu$ECEC half-lives of $^{152}$Gd, $^{164}$Er and $^{180}$W are performed. The electron wave functions in the atoms are treated in the relativistic Dirac-Hartree-Fock approximation [32]. The nuclear matrix elements are calculated within the proton-neutron deformed quasiparticle random-phase approximation (deformed QRPA) with a realistic residual interaction [33, 36].

## II. RESONANT NEUTRINOLESS DOUBLE-ELECTRON CAPTURE

The $0\nu$ECEC leads to a violation of conservation of total lepton number by two units. If the process is due to the Majorana neutrino exchange mechanism, the capture rate is determined by the effective Majorana neutrino mass

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 m_i.$$  

(3)

Here, $U_{ei}$ is the element of the Pontecorvo-Maki- Nakagawa-Sakata neutrino mixing matrix [37, 38] and $m_i$ are the diagonal Majorana neutrino masses.

The half-life of the $0\nu$ECEC has the form

$$T_{1/2}^{0\nu\text{ECEC}} = \frac{\ln 2}{\Gamma_{0\nu\text{ECEC}}},$$  

(4)

where the decay width is given by

$$\Gamma_{ab}^{0\nu\text{ECEC}} = \frac{|V_{ab}|^2}{\Delta^2 + 4\Gamma_{ab}^2} \Gamma_{ab},$$  

(5)

with

$$\Delta = M_{A,Z} - M_{A,Z-2}^* = Q - B_{ab}$$  

(6)

being the difference of masses of the initial and final excited atoms with masses $M_{A,Z}$ and $M_{A,Z-2}^*$, respectively. $B_{ab} = E_a + E_b + E_C$ is the energy of two electron holes, whose quantum numbers $(n, j, l)$ are denoted by indices $a$ and $b$, $E_C$ is the interaction energy of the two holes, while $\Gamma_{ab}$ is the width of the excited final state with the electron holes. Since we consider the transitions to ground states of the final nuclei, the single asterisk for $M_{A,Z-2}^*$ refers to the excitation of electron shells only.

Having factorized the electron shell structure and the nuclear matrix element, the lepton number violating transition amplitude can be represented as [11]

$$V_{ab} = m_{\beta\beta} G_F^2 G_A^2 \sigma \frac{\alpha}{4\pi R} (F_{ab}) M^{0\nu}.$$  

(7)

Here, $G_F = G_F \cos \theta_C$, where $\theta_C$ is the Cabibbo angle, $g_A$ is the axial-vector coupling constant, $R$ is the nuclear radius, $(F_{ab})$ is a combination of averaged upper and lower bispinor components of the atomic electron wave functions defined in Ref. [11]. $M^{0\nu}$ is the nuclear matrix element of $0^+$ ground state to $0^+$ ground state transition,
which is a sum of the Fermi (F), Gamow-Teller (GT), and tensor (T) contributions \[ M^{0\nu} = -\frac{M_F}{\gamma_A^2} + M_{GT} + M_T. \] (8)

In comparison with the corresponding $0\nu\beta\beta$ decays, the isospin operators $\tau^+$ of nucleons entering the NMEs are replaced by $\tau^-$. 

### III. CALCULATION OF NMEs

Nuclei participating in the $0\nu$ECEC ground state to ground state nuclear transitions $^{152}$Gd $\rightarrow$ $^{152}$Sm, $^{164}$Er $\rightarrow$ $^{164}$Dy and $^{180}$W $\rightarrow$ $^{180}$Hf are deformed. The deformation parameter $\beta = \sqrt{\pi/5Q_p/(Zr^2)}$ can be deduced from the intrinsic quadrupole moment $Q_p$ of the first $2^+$ state measured by the Coulomb excitation reorientation technique ($r_c$ is the root mean square charge radius).

Unfortunately, the electric quadrupole moment of $^{152}$Gd has not been measured yet by this method \[41\]. Alternatively, the deformation parameter $\beta$ can be extracted from the values of measured E2 transition probability, $\langle Q_{\beta\beta} \rangle = \sqrt{16\pi B(E2)/5e^2}$, the sign cannot be extracted \[42\]. From Table I one can see that the deformation parameters determined in both ways agree quite well with each other.

NMEs of the considered $0\nu$ECEC transitions are calculated within the deformed QRPA with a realistic nucleon-nucleon interaction \[33, 30\]. The details of the formalism for the $0\nu\beta\beta$-decay NME are given in Refs. \[33, 30\]. The generalized expression of the basic equations of Refs. \[33, 30\] to the case of $0\nu$ECEC NME is straightforwardly achieved by replacing $\tau^+$ with $\tau^-$ operator.

We note that in the calculation of $M^{0\nu}$ within the deformed QRPA the tensor contribution has been neglected till now. Within the spherical QRPA this component of $M^{0\nu}$ reduces its total value by less than 10 \% \[40, 44, 45\].

The single-particle states are those of the axially symmetric Woods-Saxon mean field and are expressed in the basis of an axially-deformed harmonic oscillator states.

The parametrization of the mean field is adopted from the spherical calculations of Refs. \[40, 44, 45\]. The single-particle model space consists of $4 - 6\hbar\omega$ shells in the spherical limit. Only the quadrupole deformation is taken into account in the calculation. The fitted values of the parameter $\beta_2$ of the deformed Woods-Saxon mean field, which allow us to reproduce the experimental $\beta(E2)$, are shown in Table I. The spherical limit (i.e., $\beta_2 = 0$) is considered as well, to compare with the earlier results \[43\] obtained in the spherical QRPA.

We use the nuclear Brueckner G-matrix, obtained by a solution of the Bethe-Goldstone equation with the Bonn CD one boson exchange nucleon-nucleon potential, as a residual interaction. The BCS equations are solved to obtain occupation amplitudes and quasiparticle energies, constituents of the nuclear Hamiltonian. The pairing interactions are adjusted to fit the empirical pairing gaps for protons and neutrons. The gap parameters are determined phenomenologically from the odd-even mass differences through a symmetric five-term formula involving the experimental binding energies. The values obtained from this procedure for the nuclei under consideration can be seen in Table I. The calculated BCS overlap factors of the initial and final quasiparticle mean fields are given in the last column of Table I. This factor represents a possible suppression of the $0\nu$ECEC NME due to different deformations of initial and final nuclei \[43\].

To solve the deformed QRPA equations, one has to fix the particle-hole $g_{ph}$ and particle-particle $g_{pp}$ renormalization factors of the residual interaction of the nuclear Hamiltonian. Since experimental information about the position of the Gamow-Teller giant resonance for $^{152}$Gd, $^{164}$Er and $^{180}$W is not available we consider $g_{ph} = 0.9$ like in previous calculation of the $0\nu$ECEC NME of $^{150}$Nd and $^{160}$Gd. In \[44\] it was proposed to adjust the particle-particle strength parameter $g_{pp}$ to the measured $2\nu\beta\beta$-decay half-lives, i.e., to reproduce the experimental value of the matrix element $M_{GT}^{2\nu}$. This procedure makes the $0\nu\beta\beta$-decay NMEs essentially independent of the size of the single-particle basis and the nuclear structure input. Due to a small $Q$-value, the half-life of the double-electron capture with emission of two neutrinos ($2\nu$ECEC) of $^{152}$Gd, $^{164}$Er and $^{180}$W is too large to be measurable. Therefore, the parameter $g_{pp}$ of the QRPA is fixed by the assumption that the matrix element $M_{GT}^{2\nu}$ of the $2\nu$ECEC process lies within the range $(0, 0.10)$ MeV$^{-1}$. Recall that $M_{GT}^{2\nu}$ for double $\beta$-decaying nuclei from the region $^{128, 130}$Te, $^{136}$Xe and $^{150}$Nd does

### Table I: Phenomenological paring gaps for protons $\Delta_p$ and neutrons $\Delta_n$ and deformation parameters $\beta$ deduced from intrinsic quadrupole moments measured by the Coulomb excitation reorientation technique ($\beta_{Q_p}$, sign is given explicitly if known) \[41\] and $B(E2)$ values ($\beta_{B(E2)}$) \[42\] in $^{152}$Gd, $^{164}$Er and $^{180}$W. $\beta_2$ is the deformation parameter of Woods-Saxon mean field and are expressed in the experimental quadrupole moments. \(\langle BCS_1|BCS_2\rangle\) is the BCS overlap between the initial and final BCS vacua \[43\].

| nucleus          | $\Delta_p$ (MeV) | $\Delta_n$ (MeV) | $\beta_{Q_p}$ | $\beta_{B(E2)}$ | $\beta_2$ | $\langle BCS_1|BCS_2\rangle$ |
|------------------|------------------|------------------|---------------|-----------------|-----------|--------------------------|
| $^{152}$Gd \(^{152}\)Sm | 1.478 (1.117) 1.179 (1.192) | (0.29) 0.212 (0.306) 0.166 (0.256) | 0.44 |
| $^{164}$Er \(^{164}\)Dy | 1.025 (0.879) 1.020 (0.825) 0.36 (0.302) 0.289 (0.302) | 0.73 |
| $^{180}$W \(^{180}\)Hf | 0.927 (0.832) 0.788 (0.713) 0.27 (0.272) 0.252 (0.273) 0.237 (0.244) 0.75 |
not exceed the above range by assuming the weak-axial coupling constant $g_A$ to be unquenched ($g_A = 1.269$) or quenched ($g_A = 1.0$). As it will be shown below this procedure of fixing $g_{pp}$ is not a significant source of uncertainty in the calculated $0\nu$ECEC half-lives.

### TABLE II

The matrix element $M_{0\nu}$ for the $2\nu$ECEC of $^{152}$Gd, $^{164}$Er and $^{180}$W calculated within the spherical and deformed QRPA with realistic nucleon-nucleon interaction (Bonn-CD potential) [41].

| Nucleus | $M_{GT}^{2\nu}$ | $M_{GT}^{0\nu}$ | $M_{0\nu}$ |
|---------|-----------------|-----------------|-------------|
|         | [MeV$^{-1}$]    | def.            | def. QRPA   |
| $^{152}$Gd | 0.10            | 7.59            | 7.50        | 3.23        |
|          | 0.00            | 7.21            | 2.67        |
| $^{164}$Er | 0.10            | 6.12            | 7.20        | 2.64        |
|          | 0.00            | 5.94            | 2.27        |
| $^{180}$W | 0.10            | 5.79            | 6.22        | 2.05        |
|          | 0.00            | 5.56            | 1.79        |

In Table II the NME $M_{0\nu}$ for $^{152}$Gd, $^{164}$Er and $^{180}$W calculated within the spherical and deformed QRPA are presented. There is a qualitative agreement between the results of the spherical QRPA and the spherical limit of the deformed QRPA ($\beta_2 = 0$). Since the deformed nuclei are described within the adiabatic Bohr-Mottelson approximation, the spherical limit of the deformed QRPA should be taken with caution. The differences can be attributed to the fact that within the spherical QRPA one-body and two-body matrix elements entering the expressions for $M_{GT}^{2\nu}$ and $M_{GT}^{0\nu}$, respectively, are calculated with single-particle wave functions approximated by the spherical harmonic oscillator ones, while realistic Woods-Saxon single-particle wave functions are used in the deformed QRPA [34, 35]. In addition, $M_{0\nu}$ obtained within the spherical QRPA contains also $M_T$ contribution, which can reduce its value by up to 10%. The results in Table II indicate that the nuclear deformation decreases the value of $M_{0\nu}$ by more than factor of 2-3. We note that the largest suppression of $M_{0\nu}$ due to deformation is realized for $A = 180$ nuclear system, where deformations of initial and final nuclei are comparable (same sign is assumed, see Table I). It means that the suppression of $M_{0\nu}$ can be associated with the large deformation of initial and final nuclei and large value of $A$. Before the effect of deformation on $M_{0\nu}$ for nuclei with smaller $A$ was associated with difference in deformations of the initial and final nuclei [42, 46, 47].

### IV. THE $0\nu$ECEC HALF-LIVES OF $^{152}$Gd, $^{164}$Er AND $^{180}$W

Equations (4) - (7) imply that the half-lives are determined by the following properties of the initial and final atoms:

i) The mass difference $\Delta M$ determines the proximity to the resonance condition and ultimately the magnitude of the effect. It depends on the $Q$-value and the energy of two electron vacancies in the final atom. The selection rule of Ref. [11] imply that it suffices to consider the capture of $ns_{1/2}$ and $np_{1/2}$ electrons.

### TABLE III

Upper and lower average components of the Dirac bispinors $\langle f \rangle$ and $\langle g \rangle$ in $^{158}$Gd and $^{166}$Er for $1s_{1/2}$, $2s_{1/2}$, $3s_{1/2}$, and $2p_{1/2}$ electron shells (in keV$^3/2$). The upper lines give solutions based on the Dirac equation in the Coulomb field [11], the lower lines give the same solutions based on the Dirac-Hartree-Fock method [32].

| Average | $^{158}$Gd | $^{166}$Er |
|---------|------------|------------|
| $\langle f(1s_{1/2}) \rangle$ | $1.33 \times 10^4$ | $1.57 \times 10^4$ |
| $\langle f(2s_{1/2}) \rangle$ | $1.27 \times 10^4$ | $1.50 \times 10^4$ |
| $\langle f(3s_{1/2}) \rangle$ | $5.20 \times 10^3$ | $6.20 \times 10^3$ |
| $\langle f(2p_{1/2}) \rangle$ | $2.84 \times 10^3$ | $3.39 \times 10^3$ |
| $\langle g(2s_{1/2}) \rangle$ | $2.15 \times 10^3$ | $2.60 \times 10^3$ |
| $\langle g(2p_{1/2}) \rangle$ | $-1.12 \times 10^4$ | $-1.43 \times 10^4$ |
| $\langle g(3p_{1/2}) \rangle$ | $-9.53 \times 10^2$ | $-1.22 \times 10^3$ |

Recently, Q-values in the $0\nu$ECEC transitions of $^{152}$Gd [20], $^{164}$Er [22], and $^{180}$W [23] have been remeasured using Penning-trap mass spectrometry with an uncertainty of 180 eV, 120 eV, and 270 eV, respectively.

In atomic physics, the electron binding energies are usually measured to within a few eV. We used the binding energies of single electron holes $E_n$ from Ref. [48]. Noticeable corrections arise from the Coulomb interaction between two holes $E_C$. Coulomb interaction energy is calculated on the basis of the Dirac equation with account of the screening of the nuclear charge, which gives an accuracy comparable to the present experimental errors in Q-values. More accurate estimates can be obtained by averaging the Fermi-Breit potential over the states of the two vacancies.

ii) In the unitary limit the capture rate is inversely proportional to the width of the decay of excited atoms in the final state. The radiative width of the electron shell with two holes is estimated as $\Gamma_{ab} = \Gamma_a + \Gamma_b$ on the basis of the measured and recommended values of radiative widths $\Gamma_a$ of the single vacancies [49]. The radiative width also determines the desired accuracy in measuring Q-values.

iii) In nuclei with large values of Z, electrons in the lower shells are moving at a speed close to the speed of light. The wave functions of electrons in the Coulomb field plus the self-consistent field of neighboring electrons.
Table IV: The calculated 0νECEC half-lives of 152Gd, 164Er, and 180W for $m_{\beta\beta} = 50$ meV. The second and third columns show the quantum numbers of the electron holes. Here, $n$ is the principal quantum number, $j$ is the total angular momentum, and $l$ is the orbital momentum. Shown in the columns four, five and six are the hole energies and their Coulomb interaction energy (in units of keV). The column seven shows the radiative widths of the excited electron shells. The column eight shows the mass difference of the initial and final atoms. The last two columns show the minimum and maximum half-lives of the 0νECEC transitions (in years). The masses and the energies are in keV.

| Nucleus (n2jl)α (n2jl)β | $E_e$ | $E_h$ | $E_C$ | $\Gamma_{ab}$ (keV) | $\Delta$ (keV) | $T_{1/2}^{\text{min}}$ (y) | $T_{1/2}^{\text{max}}$ (y) |
|------------------------|------|------|-------|-------------------|----------------|-----------------|-----------------|
| 152Gd                  | 110  | 210  | 46.83 | 7.74 0.34 2.3 × 10^{-2} | −0.83 ± 0.18 4.7 × 10^{28} 4.8 × 10^{29} |
|                        | 110  | 211  | 46.83 | 7.31 0.32 2.3 × 10^{-2} | −1.27 ± 0.18 4.2 × 10^{31} 1.1 × 10^{32} |
| 164Er                  | 110  | 310  | 46.83 | 1.72 0.11 3.2 × 10^{-2} | −7.07 ± 0.18 9.4 × 10^{31} 1.1 × 10^{32} |
| 180W                   | 110  | 310  | 9.05  | 6.58 0.23 8.3 × 10^{-3} | −6.82 ± 0.12 7.5 × 10^{12} 8.4 × 10^{13} |

must be considered in a relativistic approach. The upper and lower radial functions of the Dirac bispinors are averaged over the volume of the nucleus. Relativistic effects lead to an increase in the probability of 0νECEC process. Estimates of the average values of the radial component of the Dirac wave functions inside the nucleus can be obtained using well-known wave functions of electrons in the Coulomb field, or by solving many-body problem using the Dirac-Hartree-Fock method. The $n_{s1/2}$ electron capture is proportional to the mean value of the upper bispinor radial component, while the capture from the $np_{1/2}$ states is determined by the mean value of the lower radial components of the Dirac wave function. In Table IV the average values of the upper and lower radial components of Ref. [11] are compared with the values obtained in the Dirac-Hartree-Fock method [32] for 158Gd and 166Er. The agreement is quite good.

iv) The probability of capture is proportional to the square of the nuclear matrix element, which we discussed in the previous section in detail.

In Table IV the maximum and minimum values of the 0νECEC half-lives (in years) of 152Gd, 164Er and 180W for $m_{\beta\beta} = 50$ meV assuming the capture of most favored atomic electrons, the associated values of the binding energies and the Coulomb interaction energy of the two holes, and the mass difference $\Delta$ of initial and final excited atoms are shown. The estimates for $T_{1/2}^{\text{min}}$ are taken for the minimum mass difference deviating by not more than three standard errors from the experimental mean value unlike in [13], where the unitary limit was considered. Among the transitions, the favoured one is the capture of electrons from K and L shells in the case of 152Gd, which results in the half-life in the range $4.7 \times 10^{28} \div 4.8 \times 10^{29}$ years. This transition is still rather far from the resonant level. The half-life appears thereby 2 - 3 orders of magnitude greater as compared to the half-life of 0νββ decay of 76Ge [4].

V. CONCLUSION

As shown in Ref. [11], the 0νECEC half-lives of a dozen of isotopes are comparable to the shortest half-lives of the 0νββ decays of nuclei provided the resonance condition is matched with an accuracy of tens of electronvolts. Among the promising isotopes 152Gd, 164Er, and 180W were found to be associated with the transitions between ground states of the nuclei. The estimates of the 0νECEC half-lives were recently improved by more accurate measurements of Q-values for 152Gd, 164Er, and 180W in a Penning trap.

In this paper, we have made a further step to refine the estimates of the half-lives by going beyond the spherical approximation in the calculation of nuclear matrix elements. We found within the deformed QRPA that the deformation of the nuclei leads to suppression of the NMEs by the factor of 2-3 as compared with the spherical limit. The suppression of NMEs depends not only on the relative deformation of the initial and final nuclei, but also on their absolute values.

We conclude that the 0νECEC half-life of 152Gd is 2-3 orders of magnitude longer than the half-life of 0νββ decay of 76Ge corresponding to the same value of the Majorana neutrino mass. Our calculation excludes 164Er and 180W from the list of prospective isotopes to search for the neutrinoless double-electron capture.

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