Neutrino mixing and Leptogenesis with modular $S_3$ symmetry in the framework of type III seesaw

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Abstract

Discrete symmetries being preferred to explain the neutrino phenomenology, we chose the simplest $S_3$ group and explore the implication of its modular form on neutrino masses and mixing. Non-trivial transformations of Yukawa couplings under this symmetry, make the model phenomenologically interesting by reducing the requirement of multiple scalar fields. This symmetry imposes a specific flavor structure to the neutrino mass matrix within the framework of less frequented type III seesaw mechanism and helps to explore the neutrino mixing consistent with the current observation. Apart, we also explain the preferred scenario of leptogenesis to explain the baryon asymmetry of the universe by generating the lepton asymmetry from the decay of heavy fermion triplet at TeV scale.

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I. INTRODUCTION

The success of standard model (SM) is limited to accommodate certain experimental observations like neutrino masses, matter-anti matter asymmetry and existence of dark sector etc [1-4]. Therefore the extension of the SM particle spectrum is necessary to explain those limitations. Discrete symmetries are proven to be more fruitful in this direction, since imposition of these symmetries provides a specific flavor structure to the neutrino mass matrix and hence being widely used in neutrino phenomenology [5-11]. Few examples are $S_3$, $A_4$ and $S_4$ etc., which are found to be commonly used in the literature [12-15]. But these discrete groups always require the inclusion of multiple scalar fields with specific alignment of vacuum expectation values (VEV). Such complications can be avoided by the nontrivial transformation of Yukawa couplings under these symmetries and this idea has been achieved a decent attention and well explored in literature as modular symmetries [16-23]. Here, the couplings retain a modular form and can be expressed as a complex function of modulus $\tau$ [24-28]. Once the complex modulus $\tau$ acquires VEV, the symmetry becomes useful to study the neutrino masses and mixing. Hence unlike the usual discrete groups, the importance of scalar fields are somehow being replaced by the Yukawa couplings. Additionally, both fields and couplings transform under a modular group $\Gamma_N$. For various $\Gamma$s, one can infer the isomorphism of different discrete symmetries, for examples $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$ [29,31] and $\Gamma_4 \simeq S_4$ [32,34] and $\Gamma_5 \simeq A_5$ [35,36].

The well known permutation group $S_3$ is vastly used for model building purpose due to its simplistic representations [37-40]. But this needs atleast three Higgs doublets to explain the experimental results in quark and lepton sectors [41]. However the modular form can make it more simpler due to the introduction of less scalar fields, where the major role will be played by the Yukawa couplings and there exist very few literature in this direction [42]. Also the type III seesaw scenario within $S_3$ symmetry is less frequented as compared to the type I and type II mechanisms. Moreover there exist immense literature on the generic scenario of leptogenesis within the framework of type I and II seesaw but very few studies explore the same in type III case [43-52]. Therefore in the present work we explore neutrino masses and mixing within the framework of type III seesaw with implication of modular $S_3$ group. Here, the Yukawa couplings transform non-trivially under the $S_3$ symmetry and replace the need for multiple scalar fields. Along with the neutrino mass problem, matter-anti matter asymmetry of the universe remains an attractive question to be proven by experiments but the mystery is till yet unsolved. Thus we try to explore the the generation of lepton asymmetry from the decay of lightest heavy triplet to adequate the explanation of baryon asymmetric universe through leptogenesis phenomena. Unlike the type I seesaw, where the right handed neutrinos being gauge singlets do not have any gauge scattering
processes to affect the lepton number density. But the fermion triplets being charged under
gauge symmetry leads to gauge interactions, which is effectively contribute to the evolution
of their number densities.

The manuscript is structured as: The section II includes the detail description of model
and Lagrangian along with the charged lepton masses. The neutrino masses and mixing
within the framework of type III seesaw is discussed in section III. In section IV, we explore
the scenario of leptogenesis in detail followed by the solutions of Boltzmann equations. We
finally summarize the work in section V.

II. THE MODEL

In the current section, we introduce the particle spectrum and corresponding group
charges of the model. The SM particle content is extended with the inclusion of three
fermion triplets ($\Sigma_{1,2,3}$) and one scalar singlet ($\rho$). First two fermion triplets are combined
to transform as a doublet under the $S_3$ symmetry, where the third triplet transform as a
singlet. All the fermion triplets are assigned with a modular weight $-1$. However, the new
scalar transform as a singlet under $S_3$ symmetry with assignment of modular weight $-2$.
Similarly the first two lepton generations ($L_{1,2}$) transform as a doublets under the $S_3$ sym-
metry and the third family ($L_3$) remains singlet. The anti-particle states of these leptons
are assigned with modular weight of $-1$. The right handed lepton generations transform in
a similar manner and their modular weights (1) are adjusted to make the Lagrangian invari-
ant. Three Yukawa couplings are introduced to transform as doublet ($y_1(\tau)$, $y_2(\tau)$) under
$S_3$ modular group with a modular weight of 2 and one Yukawa ($y_3(\tau)$) transform as a singlet
with modular weight 4. All the particles and their charges under $\text{SU}(2)_L \otimes \text{U}(1)_Y \otimes S_3$ are
provided in Table I and II.

|                                           | Fermions                        | Scalars |
|-------------------------------------------|---------------------------------|---------|
|                                           | $(E_{1R}, E_{2R})$ | $E_{3R}$ | $(L_{1}^c, L_{2}^c)$ | $L_{3}^c$ | $(\Sigma_{1R}, \Sigma_{2R})$ | $\Sigma_{3R}$ | $H$ | $\rho$ |
| $\text{SU}(2)_L$                         | 1                               | 1       | 2                   | 2         | 3                     | 3               | 2    | 1       |
| $\text{U}(1)_Y$                         | $-2$                           | $-2$    | 1                   | 1         | 0                     | 0               | 1    | 0       |
| $S_3$                                   | 2                               | 1       | 2                   | 1         | 2                     | 1               | 1    | 1       |
| $k_I$                                   | 1                               | 1       | $-1$                | $-1$      | $-1$                  | 0               | $-2$ |

TABLE I: Particle content of the model and their charges under $\text{SU}(2)_L \otimes \text{U}(1)_Y \otimes S_3$, where $k_I$
denotes the modular weight.
TABLE II: Modular weight of the Yukawa and quartic couplings and their transformation under $S_3$ symmetry.

| Couplings | $A_4$ | $k_1$ |
|-----------|-------|-------|
| $(y_1(\tau), y_2(\tau))$ | 2 | 2 |
| $y_3(\tau)$ | 1 | 4 |
| $\lambda_p$ | 1 | 8 |

A. Modular Transformation

A set of linear fractional transformation operates on complex modulus $\tau$ in the upper-half complex plane, forms a modular group $\tilde{\Gamma}$ [16, 53]. The transformation leads as following

$$\tau \rightarrow \gamma \tau = \frac{a\tau + b}{c\tau + d}, \text{ where } a, b, c, d \in \mathbb{Z} \text{ and } ad - bc = 1, \text{ Im}[\tau] > 0, \quad (1)$$

This is isomorphic to $PSL(2, \mathbb{Z}) = SL(2, \mathbb{Z})/\{I, -I\}$. The modular transformations in $S$ and $T$ diagonal basis are defined as

$$S: \tau \rightarrow -\frac{1}{\tau}, \quad T: \tau \rightarrow \tau + 1, \quad (2)$$

We can have a set of modular groups: $\Gamma(N) \ (N = 1, 2, 3, \ldots)$, which are denoted as follows

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ (\text{mod} N) \right\}. \quad (3)$$

For $N = 2$, $\tilde{\Gamma}(2) \equiv \Gamma(2)/\{I, -I\}$, where as for $N > 2$ one can define $\tilde{\Gamma}(N) = \Gamma(N)$. Quotient groups, which come from the finite modular group are defined as $\Gamma_N \equiv \tilde{\Gamma}/\tilde{\Gamma}(N)$. The modular groups $\Gamma_N$ with $N = 2, 3, 4, 5$ are isomorphic to $S_3, A_4, S_4$ and $A_5$ respectively [54]. The level $N$ modular forms are holomorphic functions $f(\tau)$ and this transforms as following

$$f(\gamma \tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \Gamma(N), \quad (4)$$

here, $k$ is known to be the modular weight. Since in the present context, we will discuss the modular $S_3$ symmetric group with $N = 2$, any field $\phi^{(I)}$ transforms Eq.$(1)$

$$\phi^{(I)} \rightarrow (c\tau + d)^{-k_I} \zeta^{(I)}(\gamma)\phi^{(I)}, \quad (5)$$

here, $\zeta^{(I)}(\gamma)$ denotes the unitary representation matrix of $\gamma \in \Gamma(2)$. The kinetic term for the scalar field is defined as

$$\sum_I \frac{|\partial_\mu \phi^{(I)}|^2}{(-i\tau + i\bar{\tau})^{k_I}}, \quad (6)$$
Since the Yukawa couplings transform non-trivially under the $S_3$ symmetry and assigned with finite modular weights, they can be expressed in terms of Dedekind eta functions ($\eta$) and their derivatives ($\eta'$) as following [35]

$$y_1^{(2)}(\tau) = \frac{i \eta'(\tau/2)}{4\pi \eta(\tau/2)} + \frac{\eta'((\tau + 1)/2)}{\eta((\tau + 2)/2)} - \frac{8\eta'(2\tau)}{\eta(2\tau)},$$  

(7)

$$y_2^{(2)}(\tau) = \frac{\sqrt{3}i}{4\pi} \left( \frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'((\tau + 1)/2)}{\eta((\tau + 2)/2)} \right),$$  

(8)

$$y_3^{(4)}(\tau) = [(y_1(\tau), y_2(\tau)) \otimes (y_1(\tau), y_2(\tau))]_1 = y_1^2(\tau) + y_2^2(\tau).$$  

(9)

B. Scalar potential

Since we have one SM Higgs doublet and a singlet scalar $\rho$ with modular weight 0 and $-2$ respectively, we can write the interaction potential with required Yukawa and quartic coupling along with the free parameters $\alpha''$ and $\beta''$ as following

$$V = \mu^2_H H^\dagger H + \lambda_H (H^\dagger H)^2 + y_3 \mu_3^2 (\rho^3 \rho) + \alpha'' \lambda_\rho (\rho^4 \rho)^2 + \beta'' y_3 (H^\dagger H)(\rho^3 \rho).$$  

(10)

The vacuum expectation values of the scalars can be written as $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ and $\langle \rho \rangle = \frac{v_\rho}{\sqrt{2}}$.

C. Charged lepton masses

As per the symmetric and modular weight assigned to the fermion doublets, one can write the charged lepton invariant Lagrangian as following

$$\mathcal{L}_l = -y_\ell \left[ \bar{L}_1 H E_{1R} + \bar{L}_2 H E_{2R} \right] - y_\tau \left[ \bar{L}_3 H E_{3R} \right] - y_{SB} \left[ \bar{L}_1 H E_{2R} + \bar{L}_2 H E_{1R} \right].$$  

(11)

Since the transformation of first two generation leptons as doublet under the $S_3$ symmetry, which leads to degenerate masses for them. Therefore the soft symmetry breaking term $y_{SB}$ is introduced to explain the correct charged lepton masses by finetuning. The mass matrix for the charged lepton can be structured as

$$M_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_\ell & y_{SB} & 0 \\ y_{SB} & y_\ell & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. $$  

(12)

Therefore it is straightforward to obtain the mixing matrix that diagonalizes the charged lepton masses, which can be expressed as following

$$U_{el} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. $$  

(13)
Hence, we can have the mass eigenvalues \( m_\mu = \frac{v}{\sqrt{2}}(y_e + y_{SB}) \), \( m_e = \frac{v}{\sqrt{2}}(y_e - y_{SB}) \) and \( m_\tau = \frac{y_\tau v}{\sqrt{2}} \). The Yukawa couplings and the soft breaking parameter can be adjusted to obtain observed masses of charged leptons.

### III. TYPE III SEESAW NEUTRINO MASSES

The fermion triplets are defined in SU(2) basis and is given by [56, 57]

\[
\Sigma_i = \left( \frac{\Sigma_0^i}{\sqrt{2}} \Sigma_i^\dagger - \frac{\Sigma_0^i}{\sqrt{2}} \right).
\] (14)

The interaction Lagrangian invariant under the \( SU(2) \times U(1)_Y \times S_3 \) symmetries, which involves the fermion triplets, scalars and lepton doublets can be written as following

\[
\mathcal{L}_\nu = -y_1(\tau) \left[ \bar{L}_1 \Sigma_{2R} \hat{H} + \bar{L}_2 \Sigma_{1R} \hat{H} \right] \alpha - y_2(\tau) \left[ \bar{L}_1 \Sigma_{1R} \hat{H} - \bar{L}_2 \Sigma_{2R} \hat{H} \right] \alpha 
- y_1(\tau) \left[ \bar{L}_1 \Sigma_{3R} \hat{H} \right] \gamma - y_2(\tau) \left[ \bar{L}_2 \Sigma_{3R} \hat{H} \right] \gamma - y_1(\tau) \left[ L_3 \Sigma_{1R} \hat{H} \right] \beta 
- y_2(\tau) \left[ L_3 \Sigma_{2R} \hat{H} \right] \beta - y_3(\tau) \left[ L_3 \Sigma_{3R} \hat{H} \rho \right] \alpha' + \text{H.c.}
\] (15)

Here, \( \alpha, \beta, \gamma \) and \( \alpha' \) are the free parameters. Instead of writing \( y_i(\tau) \), we use the notation \( y_i \) for the following expressions to avoid lengthy conventions. Now we can construct the flavor structure of Dirac mass matrix for neutrinos as follows

\[
M_D = \frac{v}{\sqrt{2}} \begin{pmatrix}
\alpha y_2 & \alpha y_1 & \gamma y_1 \\
\alpha y_1 & -\alpha y_2 & \gamma y_2 \\
\beta y_1 & \beta y_2 & \alpha' y_3 \frac{v}{\Lambda}
\end{pmatrix}.
\] (16)

The Lagrangian for the fermion triplet involves the kinetic and mass terms is given by

\[
\mathcal{L}_\Sigma = -i \text{Tr} \left[ \Sigma_{iR} \gamma^\mu D_\mu \Sigma_{iR} \right] - \frac{1}{2} \text{Tr} \left[ y_3 \Sigma_{3R}^c \Sigma_{3R} \rho \right] M'_0 
- \frac{1}{2} \text{Tr} \left[ y_1 (\Sigma_{1R}^c \Sigma_{2R} + \Sigma_{2R}^c \Sigma_{1R}) + y_2 (\Sigma_{1R}^c \Sigma_{1R} - \Sigma_{2R}^c \Sigma_{2R}) \right] M_0.
\] (17)

Here, \( M_0 \) and \( M'_0 \) are the free mass parameters. Thus the mass matrix for the fermion triplets can be constructed as following

\[
M_\Sigma = \begin{pmatrix}
M_0 y_2 & M_0 y_1 & 0 \\
M_0 y_1 & -M_0 y_2 & 0 \\
0 & 0 & M'_0 y_3 \frac{v}{\Lambda}
\end{pmatrix}.
\] (18)
The small Majorana mass matrix for the neutrinos can be written as following

$$\mathcal{M}_\nu = M_D M_\Sigma^{-1} M_D^T$$

$$= \frac{v^2}{2} \begin{pmatrix} y_1 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & y_2 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & y_1 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) \\ y_1 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & y_2 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & y_2 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) \\ y_1 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & y_2 \left( \frac{\sigma_y^2}{M_0} + \frac{\gamma^2}{M_0^2} \right) & -\beta^2 y_2 (y_2 - 3y_2^2) \end{pmatrix}. \tag{19}$$

The above mass matrix is diagonalized numerically and the corresponding eigenvector matrix \((U_\nu)\) is obtained and the mixing parameters are discussed in detail with the standard convention of neutrino mixing matrix \(U_{\text{PMNS}} = U^\dagger U_\nu\), in the numerical analysis section.

Now, focusing on the diagonalization of mass matrix for fermion triplets in Eq.\((18)\), we can have the eigenvector matrix as following

$$U_R = \begin{pmatrix} \frac{u_-}{\sqrt{N_-}} & \frac{u_+}{\sqrt{N_+}} & 0 \\ \frac{1}{\sqrt{N_-}} & \frac{1}{\sqrt{N_+}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{20}$$

here,

$$u_\pm = \left[ \frac{y_2}{y_1} \pm \sqrt{1 + \left( \frac{y_2}{y_1} \right)^2} \right], \quad N_\pm = 1 + (u_\pm)^2. \tag{21}$$

The mass eigenvalues are obtained upon diagonalization \(M_\Sigma^D = U_R M_\Sigma U_R^T\) and are given by

$$|M_{\Sigma_1}| = |M_{\Sigma_2}| = \left| \left( \sqrt{y_1^2 + y_2^2} \right) M_0 \right|, \quad |M_{\Sigma_3}| = \sqrt{y_3 M_0 y_3^2} \Lambda. \tag{22}$$

A. Numerical Analysis

To explore the numerical analysis for this model, we have considered the following 3σ observed limit of neutrino oscillation parameters \([58]\)

\[
\text{NO} : \Delta m_{\text{atm}}^2 = [2.431, 2.622] \times 10^{-3} \text{eV}^2, \quad \Delta m_{\text{sol}}^2 = [6.79, 8.01] \times 10^{-5} \text{eV}^2,
\]

\[
\sin^2 \theta_{13} = [0.02044, 0.02437], \quad \sin^2 \theta_{23} = [0.428, 0.624], \quad \sin^2 \theta_{12} = [0.275, 0.350].
\]

We randomly vary the model parameters within the following range and constrained them from the experimentally observed 3σ limit of neutrino oscillation data.

\[
\text{Re}[\tau], \quad \text{Im}[\tau] \in [1, 2], \quad \alpha, \gamma \in [0.005, 0.01], \quad \beta \in [0.02, 0.06], \quad \alpha' \in [0.1, 1],
\]

\[
M_0 \in [10^2, 5 \times 10^4], \quad M'_0 \in [5 \times 10^2, 10^6], \quad \frac{y_3}{\Lambda} = 0.1.
\]
The input values for the model parameters are randomly varied within the above mentioned ranges and their allowed regions are obtained by constraining from the $3\sigma$ observed values of the neutrino oscillation parameters and also the observed neutrino mass bound ($\sum m_{\nu_i} < 0.12 \text{ eV}$) [59]. We consider the complex modulus $\tau$ to vary $1 \lesssim \text{Re}[\tau] \lesssim 2$ and $1 \lesssim \text{Im}[\tau] \lesssim 2$ for normal ordering of neutrino masses. Thus we found the modular Yukawa couplings, which depend on $\tau$ by the relation defined in Eq. (7) to (9), vary within the region $0.12 \lesssim y_1(\tau) \lesssim 0.14$, $0 \lesssim y_2(\tau) \lesssim 0.08$ and $0.01 \lesssim y_3(\tau) \lesssim 0.03$. The variation of these couplings with the real and imaginary part of the complex modulus are represented in the left and right panel of Fig. 1 respectively. In the top left and right panels of Fig. 2 we have represented the variation of mixing angle $\theta_{13}$ with $\theta_{12}$ and $\theta_{23}$ respectively, within their $3\sigma$ observed values. In the down panel, we have shown the correlation of rephasing invariant (JCP) with the Dirac CP violating phase, which is found to lie within the range $\delta_{CP} \in [-0.06, 0.06]$ and $[\pm 2.6, \pm 3.14]$ rad. The correlation of modular Yukawa couplings $y_1$ with $y_2$ and $y_2$ with $y_3$ are displayed in top left and right panel of Fig. 3 respectively, however in the down left and right panel, we have shown the correlation of $y_1$ with $y_3$ and the variation of Majorana mass for the fermion triplet $M_{\Sigma_1}$ with the lightest heavy triplet mass $M_{\Sigma_3}$.

![Diagram](image1.png)

**FIG. 1:** Left(Right) panel represents the variation of modular Yukawa couplings with real (imaginary) component of complex modulus $\tau$.

IV. LEPTOGENESIS

The well known phenomena of leptogenesis is found to be widely used in the literature due to its simplest formalism. Instead of generating baryon asymmetry directly, one can generate the asymmetry in the lepton sector, which can partially be stored in to the baryon sector during the sphaleron transition [60–62]. The Davidson Ibara bound on right-handed neutrino mass in case of type I seesaw to be greater than $10^9$ GeV, which is very difficult to
be tested in colliders. Thus bringing down the scale of leptogenesis as low as TeV through resonance enhancement, is proven to be an attractive scenario, which may opens up exciting options in the future experiments [63–66]. Obtaining a finite CP asymmetry from the decay of right-handed neutrinos within the simplistic framework of type I seesaw is well explored, however leptogenesis with type III seesaw is less frequented in the literature [52, 67, 68]. Thus in the present context, we focus on the generation of asymmetry from the decay of lightest heavy fermion triplet at TeV scale. Since in this model, two of the fermion triplets belongs to the doublet representation of $S_3$ have exactly same masses ($|M_{\Sigma_1}| = |M_{\Sigma_2}|$), we consider the resonance enhancement in the self energy, provided by the condition $|M_{\Sigma_3}| \simeq |M_{\Sigma_1}|$. Since the component of the triplet are having same masses and equal decay width, the CP asymmetry reduces three times than type I case. The tree level decay width for the fermion triplet is given by [69]

$$\Gamma_{\Sigma} = \Gamma(\Sigma^0 \rightarrow LH) + \Gamma(\Sigma^0 \rightarrow \bar{L}\bar{H}) = \frac{1}{8\pi} M_{\Sigma_i} (\tilde{Y}_\Sigma^i \tilde{Y}_\Sigma^i)_{ii}. \quad (25)$$
Here,

\[
Y_\Sigma = \begin{pmatrix}
\alpha y_2 & \alpha y_1 & \gamma y_1 \\
\alpha y_1 & -\alpha y_2 & \gamma y_2 \\
\beta y_1 & \beta y_2 & \alpha' y_3 \frac{\rho}{\Lambda}
\end{pmatrix}
\quad \text{and} \quad \bar{Y}_\Sigma = Y_\Sigma U_{el} U_{R}.
\]

(26)

The decay width for the charged components of the triplets can be written in a similar way.

\[
\Sigma_3 \rightarrow \ell L H, \Sigma_3 \rightarrow \ell L \Sigma_1, \Sigma_2 \rightarrow \ell L H, \Sigma_3 \rightarrow \ell L \Sigma_1, \Sigma_2.
\]

FIG. 4: Tree and one loop Feynman diagrams for the decay of heavy fermion triplet.

Unlike the type II case, there is no asymmetry in particle and antiparticle since it does not have several decay modes rather it decays only through Yukawa interaction. The general
expression for CP asymmetry can be written as following [63, 70, 71]

$$\epsilon_{CP} = -\sum_j \frac{3}{2} \frac{\Gamma_{\Sigma_i}}{M_{\Sigma_j}} \frac{V - 2S}{3} \frac{\text{Im} \left( \tilde{Y}_{\Sigma_i} \tilde{Y}_{\Sigma_j}^\dagger \right)^2_{ij}}{\left( \tilde{Y}_{\Sigma_i} \tilde{Y}_{\Sigma_j}^\dagger \right)_{ii} \left( \tilde{Y}_{\Sigma_i} \tilde{Y}_{\Sigma_j}^\dagger \right)_{jj}}. \quad (27)$$

Here V and S are the vertex and self energy contribution respectively. These are expressed as follows

$$S = \frac{M_{\Sigma_j}^2 \Delta M_{ij}^2}{(\Delta M_{ij}^2)^2 + M_{\Sigma_i}^2 \Gamma_{\Sigma_j}^2}, \quad \text{with } \Delta M_{ij} = M_{\Sigma_j} - M_{\Sigma_i}, \quad (28)$$

$$V = \frac{2M_{\Sigma_i}^2}{M_{\Sigma_i}^2} \left[ \left( 1 + \frac{M_{\Sigma_j}^2}{M_{\Sigma_i}^2} \right) \log \left( 1 + \frac{M_{\Sigma_j}^2}{M_{\Sigma_i}^2} \right) - 1 \right]. \quad (29)$$

FIG. 5: Variation of modular Yukawa couplings $y_1$ and $y_3$ with the CP asymmetry parameter is represented in the left and middle panel. The extreme right panel shows the correlation of CP violating phase with the CP asymmetry.

The Feynman diagrams, those finitely contribute to the CP asymmetry are provided in Fig.4. Since for a TeV scale heavy particle, the CP asymmetry comes out to be small and not enough to generate the required asymmetry, the scenario of resonant leptogenesis is preferred. If we consider the mass difference between the heavy states to be comparable with the decay width, one can clearly infer from Eq.(29) a resonantly enhanced self energy contribution, i.e $M_{\Sigma_j} - M_{\Sigma_i} \approx \frac{\Gamma_{\Sigma_j}}{2}$, which leads an enhancement of S value upto $1/2$ with an almost similar order Yukawa couplings. Therefore we can safely neglect the contribution from the vertex diagram. We vary the modular Yukawa couplings $(y_1, y_3)$, which satisfy the neutrino oscillation constraints, with the CP asymmetry parameter and found its value to be order $\approx O(10^{-3})$. The variation of CP violating phase with the CP parameter is displayed in the right panel of Fig.5.
A. Boltzmann Equations

The evolution of particle number densities are governed by the dynamics of relevant Boltzmann equations. Sakharov conditions [72] demands the decaying fermion to remain out of equilibrium to generate the asymmetry in lepton sector. One need to compare the Hubble expansion with the decay rate to satisfy this condition, which is given follows.

\[ K_{\Sigma_i} = \frac{\Gamma_{\Sigma_i}}{H(T = M_{\Sigma_i})}. \]  

(30)

Here, \( H = \frac{1.67 \sqrt{T^2}}{M_{\text{Pl}}} \), with \( g_* = 106.75 \), \( M_{\text{Pl}} = 1.22 \times 10^{19} \text{ GeV} \). Coupling strength of the triplet fermion with the leptons of order \( 10^{-7} \) gives \( K_{\Sigma_i} \sim 1 \), which confirms the inverse decay not to come into thermal equilibrium. The Boltzmann equations for the evolution of triplet fermion and lepton number densities can be written in terms of yield parameter (i.e the ratio of number density to entropy density), which are provided by [73–75]

\[ sHz \frac{dY_{\Sigma}}{dz} = -\left( \frac{Y_{\Sigma}}{Y_{\Sigma}^{\text{eq}}} - 1 \right) \gamma_D - 2 \left( \frac{Y_{\Sigma}^2}{(Y_{\Sigma}^{\text{eq}})^2} - 1 \right) \gamma_A, \]

\[ sHz \frac{dY_L}{dz} = -\gamma_D \left( \frac{Y_{\Sigma}}{Y_{\Sigma}^{\text{eq}}} - 1 \right) \epsilon_{\text{CP}} - \frac{Y_L}{Y_{\text{eq}}^{\text{eq}}} \gamma_D \left( \frac{\gamma_D}{2} + 2\gamma_W \right), \]  

(31)

where \( s \) denotes the entropy density, \( z = M_{\Sigma_i}/T \), \( Y_L = Y_{\ell} - Y_{\bar{\ell}} \) and the equilibrium number densities are given by

\[ Y_{\Sigma}^{\text{eq}} = \frac{45g_{\Sigma}}{4\pi^4 g_*} z^2 K_2(z), \quad Y_{\ell}^{\text{eq}} = \frac{3}{4} \frac{45\zeta(3) g_{\ell}}{2\pi^4 g_*}. \]  

(32)

Here, \( K_{1,2} \) denote the modified Bessel functions of type 1 and 2, \( g_{\ell} = 2 \) and \( g_{\Sigma} = 2 \) are the degrees of freedom of lepton and fermion triplets respectively. The reaction rate for the decay (\( \gamma_D \)) and gauge annihilation processes (\( \gamma_A \)) are given by [71]

\[ \gamma_D = sY_{\Sigma}^{\text{eq}} \Gamma_{\Sigma} \frac{K_1(z)}{K_2(z)}, \quad \gamma_A = \frac{M_{\Sigma}^3 T^3}{32\pi^3} e^{-2z} \left[ \frac{111g_4^4}{8\pi} + \frac{3}{2z} \left( \frac{111g_4^4}{8\pi} + \frac{51g_4^4}{16\pi} \right) \right] + O(1/z)^2. \]  

(33)

The gauge annihilation processes of the decaying fermion includes \( \Sigma \bar{\Sigma} \rightarrow f \bar{f}, G \bar{G}, H^* \bar{H} \), where \( G \) stands for the gauge boson and \( g \) is the usual gauge coupling. \( \gamma_w \) are the lepton number violating washout processes (\( \ell H \rightarrow \ell \bar{H} \)), which are suppressed due to the small coupling and can be safely neglected. We obtained a lepton asymmetry of order \( O(10^{-10}) \) by solving the Boltzmann equations, represented in the left panel of Fig.6 and the right panel clearly signifies the impact of decay, inverse decay and gauge scattering rates on the evolution of number densities with decrease in temperature.
V. SUMMARY

We explore the impact of modular $S_3$ symmetry on neutrino mixing and leptogenesis within the framework of type III seesaw. Since the usual $S_3$ symmetry requires number of scalar doublets, which leads to certain complications in explaining FCNCs and VEV alignments. We prefer the modular form of $S_3$, where the couplings transform non-trivially under the symmetry and replaces the requirement of multiple scalars. Since the scenario of type III seesaw is less frequented as compared to type I or type II, we explored a detailed analysis of neutrino mixing consistent with the $3\sigma$ observation. Numerical diagonalization of the flavored neutrino mass matrix provides an explanation of neutrino masses and mixing parameters in terms of the Yukawa couplings and free parameters. Thus we constrained all the model parameters from the neutrino oscillation data and the observed sum of active neutrino masses to obtain the correct ranges for the model predicted mixing angles and CP phase. We found the reactor mixing angle and Dirac CP phase to lie within the experimental limit. Apart, we also discussed the scenario of resonant leptogenesis by generating the lepton asymmetry from the decay of lightest fermion triplet to the final state lepton and Higgs. We solved the coupled Boltzmann equations to obtain the evolution of lepton asymmetric number density of required order ($Y_L \approx \mathcal{O}(10^{-10})$), which is adequate to generate an observed baryon asymmetry of order $Y_B \approx \mathcal{O}(10^{-11})$.

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