Rating Regulatory Mechanism Effect Promotion under the Environmental Issuance Effects: Based on the Incentive Difference Hotelling Model

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Abstract: Issuance effects are regarded as one of the most important aspects referring to the regulatory guidelines of green corporate bond ratings. This paper developed a new incentive difference Hotelling model, considering four major factors, i.e., the direct effect of issuance, the indirect effect of issuance, the reputation of rating agencies and the regulatory penalties. In this model, how the direct effect and the indirect effect impact the dual rating mechanism and the integrated rating mechanism was discussed. Numerical experiments were conducted to explore the regulatory effects on the two defined mechanisms in different situations. The results demonstrate that under each mechanism, the direct and indirect effects of issuance indirectly improve the effectiveness and efficiency of regulation by increasing the environmental benefit information content in the rating information, and the indirect effect has a greater impact. Moreover, it provides specific recommendations for the design of a regulatory regime.

Keywords: green corporate bond; direct issuance effect; indirect issuance effect; Hotelling model; rating mechanisms

JEL Classification: G18; G2

1. Introduction

While China’s green bonds market is a relative latecomer, it has grown rapidly since its inception. However, it is still at an early stage, and further improvements are needed to increase the issuance of green corporate bonds. One of the big obstacles to issuance of green corporate bonds in developing countries is lack of applicable regulations [1]. Compared with the international green bonds market, the two Standard documents, Green Bond Principle (GBP) and Climate Bonds Standard (CBS), are spontaneously formed and voluntarily participated by market entities. The initial formation is derived from the spontaneous organization of the market, which is “bottom-up” and can form an orderly competitive market. China’s green bond rating standard, mainly a series of guide files issued by the People’s Bank of China, the National Development and Reform Commission, securities regulatory commission, the stock exchange and relevant regulators, is the “top-down” led by the government. The short of green bond regulations, such as poor accountability mechanisms, inadequate incentive mechanisms and lacking penalties mechanism [2], may undermine the sustainable environmental benefits. According to the Chinese financial information network (2017), there are two suggestions of green corporate bond regulations discussed: the dual rating mechanism and the integrated rating mechanism. The dual rating mechanism involves two rating agencies, one rating agency providing the overall creditworthiness and the other (green certification agency) providing the green rating for the same green corporate bonds under the dual issuer–payment mode. The integrated
The integrated rating mechanism has only one agency providing both creditworthiness and green level. As for green corporate bonds, the differences and relations between the green certification and credit rating are: First, green certification is currently not a prerequisite while credit rating must be obtained before issuing green corporate bonds [3]. Second, the green certification agency focuses on the evaluation of the green level of the projects, while the credit rating agency mainly evaluates the solvency capability of the issuer [4]. The evaluation results directly reflect the default risk of the bonds and are closely related to green bond pricing. Third, the credit rating of bonds is quantitatively evaluated from four dimensions: green grade of the projects, use and management of raised funds, project evaluation and screening and information disclosure and report [5]. According to the content of green bond credit rating, there is a cross-relationship between green certification and credit rating. We describe the two mechanisms in Figures 1 and 2.

![Figure 1](attachment:image1.png)  
**Figure 1.** The dual rating mechanism of green corporate bonds.

![Figure 2](attachment:image2.png)  
**Figure 2.** The integrated rating mechanism of green corporate bonds.

At present, the global green bond rating system is dominated by the dual rating mechanism. That is, the environmental benefit assessment and default risk credit assessment are separated, and finally, green assessment score and credit rating grade are provided for
investors’ reference, such as in the United States. Meanwhile, some international organizations and banking institutions have tried to incorporate the environmental factors of green bonds into the credit rating indicator system to obtain an integrated rating for green bonds, such as the European Barclays Bank [6].

With the development of green finance, companies and projects tend to be rated as green enterprises and projects to acquire financing cost advantages. In 2019, the issuance scale of Chinese green corporate bonds was the world’s largest source of issuance. However, it has been shown that the issuance effects of green corporate bonds may not be consistent with their bond ratings. For green corporate bonds, the issuance effects consist of the direct effect and the indirect effect. The direct issuance effect is an environmental value, such as greenhouse gas reduction of green bond issuers. The indirect effect, including the environmental values and the economic values, is the external effect of one issued green corporate bond to promote more issued green corporate bonds in one region. The relationship between the direct effect, the indirect effect and ratings is shown in Figure 3.

![Diagram showing the relationship between direct effect, indirect effect, and ratings](image)

**Figure 3.** The relationship among the direct effect, the indirect effect and ratings.

The environmental issuance effects of green corporate bonds may not be consistent with their ratings. The environmental benefit of green financing projects is an important indicator to evaluate the green attribute of projects, but a standard evaluation system of direct and indirect environmental effects of green corporate bonds issuance has not been formed in China. The existing evaluation system of third-party green certification agencies is based on the evaluation of green financing projects’ contributions to environmental protection, resource utilization and environmental performance during construction and operation. However, most of these factors adopt secondary reference indicators, and there is a lack of quantitative evaluation of the effect of environmental issuance [7]. Therefore, the rating of green corporate bonds cannot effectively reflect the environmental issuance effects. We take two green corporate bonds as an example: China Longyuan Power Group Corporation Limited (CLPCL) and China Huadian Corporation Limited (CHCL) issued green corporate bonds in 2019. It has been found there are huge differences in both the direct issuance effect and the indirect issuance effect of these two corporations’ green bonds. As for the direct issuance effect, the greenhouse gas reduction was 915 million tons due to CLPC’s green corporate bond issuance, while it was 2304.37 million tons due to CHCL’s corporate bond issuance. This demonstrates that there are large differences in the direct issuance effect between Longyuan Power Limited and China Huadian Corporation. Furthermore, as for the indirect issuance effect, CLPLC issued green corporate bonds in the eastern and central regions of China, while CHCL issued in the western regions of China. It was shown that the increased number of green corporate bonds issued in the western regions was much higher than the increased number of green corporate bonds issued in the eastern and central regions. However, both of the green corporate bond ratings for the two corporations are AAA. Therefore, the issuance effects of green corporate bonds may not be consistent with their ratings. If the regulatory mechanism of the green bond rating does not consider the issuance effect, it will not be able to optimize the allocation of green bond resources, which will lead to the failure of the market allocation of environmental resources in the financial market and weaken the efficient allocation of environmental resources by financial means.

The issuance effects are important for the regulatory process in green corporate bond ratings. Without considering the issuance effects of green corporate bonds in rating
regulations, some green bonds have the “greenwashing” phenomenon [8]. According to China Credit Rating Co. Ltd. and Wind database, a total of 129 bonds were issued in the green bond market in 2020, of which only 59 were certified by third-party green certification agencies, accounting for 45.7% of newly issued green bonds, which was far less than 60.0% in 2019. The proportion has dropped significantly, which is lower than the average certification level in the international market. Meanwhile, China’s green bond market defaulted for the first time in 2019. Two green bonds “G17 Feng 1/G17 Feng Sheng 2” issued by Nanjing Construction Industry Group Co. Ltd. had a substantial default, involving an amount of CNY 2.5 billion. In the same year, Jiangsu Dongfang Shenghong Green Bond Fund was used to build a petrochemical refinery to increase crude oil production capacity. This shows that the traditional regulatory mechanisms are not considered the issuance effects of green corporate bonds, which leads to the occurrence of green bond “greenwashing”. We introduced the issuance effects to improve rating mechanisms for designing the dual rating mechanism and the integrated rating mechanism of green corporate bonds. We utilized the new rating regulatory mechanisms for high rating accuracy and good issuance effect.

Based on the phenomenon that the issuance effects of green bonds in China are significantly different but the bond ratings are consistent, this paper mainly discusses the following issues: (1) how to include the direct and indirect effects of green financing projects into the evaluation factors of rating agencies to improve the rating accuracy of green bonds; and (2) in the context of the rapid development of China’s green bond issuance market, how the government regulatory authorities should choose a rating mechanism based on the issuance effect to reduce the probability of “greenwashing” and inflated rating.

This paper considered the issuance effects (including direct effect and indirect effect) of green bond financing projects from the perspective of regulation, involving indicators such as reputation of rating agencies and regulatory penalties. Moreover, we developed an incentive difference Hotelling model based on the work of Shan et al. [9]. Then we discussed how the direct and indirect effects make the double rating mechanism and the comprehensive rating mechanism more effective when the issuance effect is taken into account. Then, to gain more insight, based on the current practical conditions of China’s green bond issuance market, numerical analysis and MATLAB numerical simulation were conducted to verify the regulatory effect in order to compare the applicable conditions of two regulatory mechanisms of rating agencies. The purpose of this study was to improve the design of the regulatory mechanism for green bond rating agencies; to prompt the rating agencies to consider issuance effects and then improve the accuracy of green bond ratings; and give full play to the function of financial means to assist in the efficient allocation of environmental protection resources.

2. Review of the Literature

The direct effect and the indirect effect are different between regions and industries, especially the indirect effect. Dong and Wang [10] found that some regions with high environmental benefits easily transform their polluted industries to adjoining areas. The intensity of environmental regulations and the direction of incentive mechanisms are different between regions, which leads to interregional transformation of the polluted industries [11]. The punishments and restrictive control make the capital elements in polluted industries flow to highly productive enterprises and improve market shares [12]. Shao et al. [13] showed that green finance has a direct effect on carbon emissions and has an indirect effect on carbon emissions through energy intensity. The effect of environmental benefits on green economic efficiency is firstly promoting then inhibiting [14]. There are obvious differences in the driving effect of green bonds on the upgrading of industrial structure for corporates [15].

Because of the short-time development of the Chinese green corporate bond market, regulatory mechanisms are not perfect and the reputation effect of the rating market is low. These problems lead to poor rating quality, such as inflated ratings. Some researchers
consider introducing a reputation mechanism into the regulatory process. In the model of Bolton [16], the low reputation of rating agencies may lead to inflated ratings. Manso et al. [17] found that regulators and investors are dependent on ratings, which can encourage rating agencies to improve their reputation. Li [18] indicated that the government should propose some applicable reputation mechanisms for encouraging private enterprises to optimize their products and services. However, some researchers demonstrate that the reputation mechanisms cannot play important roles in rating quality to some extent. Becker [19] demonstrated that the reputation mechanism may cause irreversible damage for high-quality rating agencies in the fierce competition. Mathis et al. [20] suggested that reputation capital is effective when revenues of rating agencies are not paid by rating business.

Punishments and constraint mechanisms can improve rating accuracy. In the model of Stolper [21], he discussed the constraint mechanisms of collusion of rating agencies. Huang et al. [22] suggested that Chinese regulators should strengthen the inspection of rating inflation and should give strict punishments to prevent inflated ratings. Jin et al. [23] found that punishment can be a powerful method to achieve good guidance and control of enterprise environment. Zhou et al. [24] discussed the regulatory effects of asymmetric punishment and symmetric punishment in Chinese corporate bond ratings.

Hotelling model [25] is used to discuss the differential competition among different services or different products. Some researchers have improved this model to solve problems such as green financing incentives [9,26,27]. The model constructed by Zhang et al. [26] studied the incentive problem of green financing in the model constructed by Zhang et al. [26] studied the incentive problem of green financing but did not consider the environmental effect of green financing projects from the perspective of regulation. Carins and Robert used the Hotelling model to solve the problem of incentives and decision outcomes in green economy [28]. Tan et al. [27] utilized the Hotelling model to solve the problems of incentives of green finance. Shan [9] constructed a new Hotelling model to discuss the main factors and the differences between financial institutions. It can be seen that although the Hotelling model is applicable to describe the game competition process among different green economic subjects, as mentioned above, such applied research has not attracted enough attention and applied achievements are relatively lacking.

Overall, it was found that the direct and the indirect effects are different in regions and industries, especially the indirect effect. However, the ratings of green corporate bonds cannot reflect the effect of issuance. Therefore, we designed a new green bonds mechanism that considers both the direct and indirect issuance effects to urge rating agencies to provide more accurate ratings. In addition, many studies have shown that the reputation mechanisms and penalty mechanisms can improve the rating quality to a certain extent. Therefore, we also included the reputation and penalty mechanisms into the model. The embedded effect significantly affects the cost–benefit analysis of environmental policies, and it is necessary to eliminate the embedded effect before evaluating the environmental policies [29]. Few studies considered the embedded effect and divided the issuance effects of green corporate bonds into the direct and indirect effects. For the Hotelling model, most of the literature assumed that firms have homogeneous products, or have the same cost and marginal structure, or set the same prices. In fact, the green bond ratings after considering the direct and indirect effects have different situations in different stages, and thus they have different market characteristics. Therefore, they cannot be generalized and need to be discussed in stages.

In conclusion, this paper has several noteworthy features against the previous studies: (1) we divided the issuance effects of green corporate bonds into the direct effect and the indirect effect and then proposed a new model, namely the incentive difference Hotelling model, which considers the issuance effects, reputation and penalty influence of green corporate bonds; (2) in this model, we discussed how both the direct and indirect issuance effects improve the effectiveness and efficiency of the two green bonds mechanisms; and (3)
we compared it to the regulatory effect of the two mechanisms by numerical simulations to discuss the applicable conditions of the two mechanisms.

3. The Incentive Difference Hotelling Model

In the assumption of the traditional Hotelling model, the products produced by different enterprises are homogeneous, but the differences in the spatial locations of enterprises lead to different transportation costs for consumers in different locations. Such differences in transportation costs may lead to the fact that seemingly identical products are not completely substitutable [30]. In the Hotelling model proposed by Shan [9], there is a competition between the service of bank and the service of online finance. The external advantages of bank can increase the net utility $u_\lambda$. The total utilities of rating buyers in the service of bank and in the service of online finance are:

$$u_1 = u_0 + u_\lambda - p_1 - q_1 t_1$$  (1)
$$u_2 = u_0 - p_2 - q_2 t_2$$  (2)

When $u_1 = u_2$, there is no difference between the service of bank and the service of online finance purchased by customers. Based on $t_1 = t_2 = 1$ and $t_1 = t_2 = 1$, the demands of the service of bank $q_1$ and the service of online finance $q_2$ are:

$$q_1 = [1 + (\lambda - 1)\alpha - p_1 + p_2] / 2$$  (3)
$$q_2 = [1 + (\lambda - 1)\alpha + p_1 - p_2] / 2$$  (4)

The profits of the bank and the online finance in a different service are:

$$\pi_1 = (p_1 - c_1) (\alpha + q_1)$$  (5)
$$\pi_2 = (p_2 - c_2) q_2$$  (6)

Based on the model proposed by Shan (2016) and the traditional Hotelling model, we used the direct and indirect effects of green bond issuance as the differentiation characteristics of products and substituted the reputation and the regulatory punishment to develop a new incentive difference Hotelling model.

On this basis, we discuss how to construct differentiated competition under the dual rating mechanism and the integrated rating mechanism of green bonds, as well as the applicable conditions of direct effect and indirect effect under the two rating mechanisms.

3.1. Model Assumptions

There exist one rating regulator, some rating agencies and certified green authorities and many rating buyers (bond issuers) in the market. Rating agencies provide the evaluation of credit risk for green corporate bonds, while certified green authorities evaluate environment benefits of green corporate bonds. The rating service of the rating agencies and the certified green authorities have a strong relevant relationship, which causes differential competition between their rating prices and rating costs, respectively.

(1) We assume that the intrinsic utility of purchasing ratings is $u_0$. The total utilities of rating buyers in the two mechanisms are $u_1$ and $u_2$, respectively. The market share of rating agencies is $b_1$, and the market share of certified green authorities is $b_2$.

(2) Rating prices, rating costs, rating demands, reputation, rating accuracy and profits of the rating agencies and the certified green authorities in the dual rating mechanism and the integrated mechanism are $p_i$, $c_i$, $q_i$, $\rho_i$, $\epsilon_i$, and $\pi_i$ $(i = 1, 2)$ respectively.

(3) The loss of utility in the two mechanisms are $q_1 t_1$, $q_2 t_2$, respectively; $t_1$ is the unit loss of utility in the dual rating mechanism, such as the low rating demand caused by high rating prices; $t_2$ is the unit loss of utility in the integrated rating mechanism, such as the implicit cost caused by low quality of ratings. In reality, because of the perfective service of
rating agencies and the gradual maturity of green certified authorities, we suppose rating buyers can ignore the unit loss of utility, namely \( t_1 = t_2 = 1 \).

(4) In the two mechanisms, if both of the rating agencies and the certified green authorities provide high-quality ratings, the net utility of rating buyers will increase \( \tilde{u}_i = b_i \cdot \epsilon_i \) \((k = 1, 2 \text{ and } i = 1, 2)\).

(5) Green bond rating accuracy \( \epsilon_i \) consists of two parts: rating accuracy \( \delta_i \) and green rating accuracy \( \sigma_i \) \((i = 1, 2)\). Due to the differences between the direct effect and the indirect effect, we introduce the direct issuance effect \( \mu_i \) and the indirect issuance effect \( \theta_i \) to improve rating accuracy of green bonds, \( \epsilon_i = \delta_i \cdot (\delta_i + \mu_i \cdot \sigma_i) \) \((\theta_i \geq 1, i = 1, 2)\).

(6) The total reputation \( \rho_i \) of rating agencies and certified green authorities in the two mechanisms can increase the net utility of rating buyers to \( \pi_i \). The net utility is a monotonic increasing function of market share \( b_i \), namely \( \pi_i = b_i \cdot \rho_i \) \((i = 1, 2)\).

(7) If green bonds have the phenomena of “greenwashing” or inflated ratings, the rating agencies and the certified green authorities will receive regulatory penalties \( c_s \).

3.2. Construction of the Incentive Difference Hotelling Model

The total utilities of rating buyers in the dual rating mechanism and in the integrated rating mechanism are:

\[
\begin{align*}
    u_1 &= u_0 + \tilde{u}_1 + \pi_1 - p_1 - q_1 \cdot t_1 \\
    u_2 &= u_0 + \tilde{u}_2 + \pi_2 - p_2 - q_2 \cdot t_2
\end{align*}
\]

When \( u_1 = u_2 \), there is no difference between the dual ratings and the integrated ratings purchased by rating buyers. The locations of buyers for the dual ratings and the integrated ratings are \( q_1 \) and \( q_2 \). Due to the distribution density of rating buyers being one, we let \( q_1 + q_2 = 1 \), \( t_1 = t_2 = 1 \), and the rating demands of the dual ratings \( q_1 \) and the integrated rating \( q_2 \) are:

\[
\begin{align*}
    q_1 &= \frac{1 + b_1 \cdot (\epsilon_1 + \rho_1) - b_2 \cdot (\epsilon_2 + \rho_2) + p_2 - p_1}{2} \\
    q_2 &= \frac{1 - b_1 \cdot (\epsilon_1 + \rho_1) + b_2 \cdot (\epsilon_2 + \rho_2) + p_1 - p_2}{2}
\end{align*}
\]

The profits of the dual ratings and the integrated rating in the two mechanisms are:

\[
\begin{align*}
    \pi_1 &= (p_1 - c_1) \cdot q_1 = \frac{(p_1 - c_1) \cdot [1 + b_1 \cdot (\epsilon_1 + \rho_1) - b_2 \cdot (\epsilon_2 + \rho_2) + p_2 - p_1]}{2} \\
    \pi_2 &= (p_2 - c_2) \cdot q_2 = \frac{(p_2 - c_2) \cdot [1 - b_1 \cdot (\epsilon_1 + \rho_1) + b_2 \cdot (\epsilon_2 + \rho_2) + p_1 - p_2]}{2}
\end{align*}
\]

4. Analysis of the Factors in the Incentive Difference Hotelling Model

We consider a two-stage game model and utilize the incentive difference Hotelling model to discuss the impact of rating accuracy and reputation of rating agencies and certified green agencies on rating prices, rating demands and rating profits.

4.1. The First Stage of Game Model

In the first stage, the regulator cannot observe the phenomena of “greenwashing” or inflated ratings. There are no regulatory penalties in this stage. Due to the indirect issuance effect not being obvious in the first stage, we suppose that the indirect effect is \( \theta_1 = \theta_2 = 1 \).

Based on the principle of the Hotelling model, when \( \frac{\partial \pi_1}{\partial p_1} = 0 \) and \( \frac{\partial \pi_2}{\partial p_2} = 0 \), we can obtain the equilibrium solutions of rating prices in the two mechanisms:

\[
\frac{\partial \pi_1}{\partial p_1} = \frac{1 + b_1(\epsilon_1 + \rho_1) - b_2(\epsilon_2 + \rho_2) + p_2 - 2p_1 + c_1}{2}
\]
\[
\frac{\partial \pi_2}{\partial p_2} = \frac{1 + b_2(\varepsilon_2 + \rho_2) - b_1(\varepsilon_1 + \rho_1) + p_1 - 2p_2 + c_2}{2}
\]
(14)

\[
p_1^* = \frac{3 - b_2 \cdot (\varepsilon_2 + \rho_2) + b_1 \cdot (\varepsilon_1 + \rho_1) + 2c_1 + c_2}{3}
\]
(15)

\[
p_2^* = \frac{3 - b_1 \cdot (\varepsilon_1 + \rho_1) + b_2 \cdot (\varepsilon_2 + \rho_2) + 2c_2 + c_1}{3}
\]
(16)

According to Equations (9) and (10), we can obtain equilibrium solutions of rating demands in the two mechanisms:

\[
q_1^* = \frac{3 + b_1 \cdot (\varepsilon_1 + \rho_1) - b_2 \cdot (\varepsilon_2 + \rho_2) + c_2 - c_1}{6}
\]
(17)

\[
q_2^* = \frac{3 - b_1 \cdot (\varepsilon_1 + \rho_1) + b_2 \cdot (\varepsilon_2 + \rho_2) + c_1 - c_2}{6}
\]
(18)

The profits of dual ratings and integrated rating in the two mechanisms are:

\[
\pi_1^* = \frac{[3 + b_1 \cdot (\varepsilon_1 + \rho_1) - b_2 \cdot (\varepsilon_2 + \rho_2) + 2c_1 + c_2] \cdot [3 - b_2 \cdot (\varepsilon_2 + \rho_2) + b_1 \cdot (\varepsilon_1 + \rho_1) + c_1 - c_2]}{18}
\]
(19)

\[
\pi_2^* = \frac{[3 - b_1 \cdot (\varepsilon_1 + \rho_1) + b_2 \cdot (\varepsilon_2 + \rho_2) + 2c_2 + c_1] \cdot [3 - b_1 \cdot (\varepsilon_1 + \rho_1) + b_2 \cdot (\varepsilon_2 + \rho_2) + c_1 - c_2]}{18}
\]
(20)

In this stage, the rating accuracy of the two mechanisms are:

\[
\varepsilon_1 = \delta_1 + \mu_1 \cdot \sigma_1
\]
(21)

\[
\varepsilon_2 = \delta_2 + \mu_2 \cdot \sigma_2
\]
(22)

**Situation 1.** When rating accuracy \(\varepsilon_1\) and \(\varepsilon_2\) satisfy \(\varepsilon_1 + \varepsilon_2 = 1\) and \(b_1 + b_2 = 1\), the rating accuracy is not reaching equilibrium.

\[
\pi_1^* - \pi_2^* = \frac{2b_2 \cdot (\varepsilon_2 + \rho_2) - 2b_1 \cdot (\varepsilon_1 + \rho_1) - c_1 + c_2}{3}
\]
(23)

When the direct issuance effect \(\mu_1 \in \left[\frac{1 - 2b_1}{2\delta_1', \frac{2 + 2p_2 - 2b_1 \cdot (1 + p_2 + p_2) - c_1 + c_2 - 2b_1}{2\delta_1'}\right]\) in the dual rating mechanism and the direct issuance effect \(\mu_2 \in \left[\frac{2 - 2p_1 - 2b_2 \cdot (1 + p_1 + p_2) + c_1 - c_2 - 2b_2}{2\delta_2'}, \frac{1 - 2b_2}{2\delta_2'}\right]\) in the integrated rating mechanism, we have \(\varepsilon_1 > \varepsilon_2\) and \(\pi_1^* > \pi_2^*\). This demonstrates that the regulatory effect of the dual rating mechanism is better in this situation.

When the direct issuance effect \(\mu_1 \in [0, \frac{1 - 2b_1}{2\delta_1'}]\) in the dual rating mechanism and the direct issuance effect \(\mu_2 \in (\frac{1 - 2b_2}{2\delta_2'}, 1]\) in the integrated rating mechanism, we have \(\varepsilon_1 < \varepsilon_2\) and \(\pi_1^* < \pi_2^*\). This demonstrates that the regulatory effect of the integrated rating mechanism is better in this situation.

**Situation 2.** When rating accuracy \(\varepsilon_1\) and \(\varepsilon_2\) satisfies \(\varepsilon_1 + \varepsilon_2 = 1\) and \(b_1 + b_2 = 1\), the rating accuracy is reaching equilibrium.

We let \(\frac{\partial \pi_1^*}{\partial \varepsilon_1} = 0\) and \(\frac{\partial \pi_1^*}{\partial \varepsilon_2} = 0\) in the two mechanisms, and the equilibrium solutions of rating accuracy are:

\[
\frac{\partial \pi_1^*}{\partial \varepsilon_1} = \frac{b_1 \cdot b_2 \cdot (\varepsilon_2 + \rho_2) - 3b_1 - b_2^2 \cdot (\varepsilon_1 + \rho_1) + b_1 \cdot (2c_2 - c_1)}{6}
\]
(24)

\[
\frac{\partial \pi_2^*}{\partial \varepsilon_2} = \frac{b_1 \cdot b_2 \cdot (\varepsilon_1 + \rho_1) - 3b_2 - b_2^2 \cdot (\varepsilon_2 + \rho_2) - b_2 \cdot (2c_2 - c_1)}{6}
\]
(25)

\[
\varepsilon_1^* = 4 - b_1 \cdot (1 + \rho_1) + b_2 \cdot \rho_2 + 2c_2 - c_1
\]
(26)

\[
\varepsilon_2^* = 4 + b_1 \cdot (1 + \rho_1) - b_2 \cdot \rho_2 - 2c_2 + c_1
\]
(27)
\[ \mu_1^e = \frac{4 - b_1 \cdot (1 + \rho_1) + b_2 \cdot \rho_2 + 2c_2 - c_1 - \delta_1}{c_1} \]  
\[ \mu_2^e = \frac{4 + b_1 \cdot (1 + \rho_1) - b_2 \cdot \rho_2 - 2c_2 + c_1 - \delta_1}{c_2} \]  

When \( b_2 \cdot \rho_2 + 2c_2 > b_1 \cdot (1 + \rho_1) + c_1 \), we have \( \varepsilon_1^e > \varepsilon_2^e \) and \( \pi_1^e > \pi_2^e \). This shows that the regulatory effect of the dual rating mechanism is better. When \( b_2 \cdot \rho_2 + 2c_2 < b_1 \cdot (1 + \rho_1) + c_1 \), we have \( \varepsilon_1^e < \varepsilon_2^e \) and \( \pi_1^e < \pi_2^e \). This indicates that the regulatory effect of the integrated rating mechanism is better.

According to the analysis of the first stage of the game model, we can conclude the following:

1. When the rating accuracy does not reach equilibrium, the conditions of usage of the dual rating mechanism are the direct issuance effect \( \mu_1 \in [\frac{1-2b_1}{3\sigma_1}, \frac{2+2b_2-2b_1(1+\rho_1+\rho_2)-c_1-c_2-2\delta_1}{3\sigma_1}] \) in the dual rating mechanism and the direct issuance effect \( \mu_2 \in (\frac{2+2b_2-2b_1(1+\rho_1+\rho_2)+c_1-c_2-2\delta_2}{3\sigma_2}, \frac{1-2b_1}{2\sigma_2}] \) in the integrated rating mechanism.  

2. When the rating accuracy does not reach equilibrium, the conditions of usage of the integrated rating mechanism are the direct issuance effect \( \mu_1 \in [0, \frac{1-2b_1}{3\sigma_1}] \) in the dual rating mechanism and the direct issuance effect \( \mu_2 \in (\frac{1-2b_2}{3\sigma_2}, 1) \).  

3. When the rating accuracy reaches equilibrium, the conditions of usage of the dual rating mechanism are \( b_2 \cdot \rho_2 + 2c_2 > b_1 \cdot (1 + \rho_1) + c_1 \), while the conditions of usage of the integrated rating mechanism are \( b_2 \cdot \rho_2 + 2c_2 < b_1 \cdot (1 + \rho_1) + c_1 \).

The Hotelling model analysis in the first stage shows that under the circumstance that the indirect effect has no influence on the competition of rating agencies, the market share, rating cost and reputation have influence on the rating accuracy, thus affecting the applicable conditions of dual rating and integrated rating. Based on the above conclusions, we find that the applicable conditions of the dual rating mechanism and the integrated rating mechanism mostly depend on the accuracy of credit rating and green rating. Specifically, when the accuracy \( \delta, \sigma \) increases, the regulatory authorities are more inclined to choose the dual rating mechanism. According to the direct effect of green financing projects, the selection of appropriate regulatory mechanism of rating agencies can effectively solve the problem of overrating green bonds.

4.2. The Second Stage of Game Model

In the second stage, the regulator can observe the phenomena of greenwashing and inflated ratings and give regulatory penalties \( c_\varepsilon \). Due to the obvious indirect issuance effect in the second stage, we suppose that the indirect effect in the dual mechanism and the indirect effect in the integrated mechanism are different \( \theta_1 \neq \theta_2 > 1 \).

The profits of dual ratings and integrated rating in the two mechanisms are:

\[ \pi_1 = (p_1 - c_1 - c_\varepsilon) \cdot q_1 = \frac{(\mu_1 - c_1 - c_\varepsilon) \cdot [1 + b_1 \cdot (c_1 + \rho_1) - b_2 \cdot (c_2 + \rho_2) + p_2 - p_1]}{2} \]  
\[ \pi_2 = (p_2 - c_2 - c_\varepsilon) \cdot q_2 = \frac{(\mu_2 - c_2 - c_\varepsilon) \cdot [1 - b_1 \cdot (c_1 + \rho_1) + b_2 \cdot (c_2 + \rho_2) + p_1 - p_2]}{2} \]

Based on the principle of the Hotelling model, when \( \frac{\partial \pi_1}{\partial p_1} = 0 \) and \( \frac{\partial \pi_2}{\partial p_2} = 0 \), we can obtain the equilibrium solutions of rating prices in the two mechanisms:

\[ \frac{\partial \pi_1}{\partial p_1} = \frac{1 + b_1 (\varepsilon_1 + \rho_1) - b_2 (\varepsilon_2 + \rho_2) + p_2 - 2p_1 + c_1 - c_\varepsilon}{2} \]  
\[ \frac{\partial \pi_2}{\partial p_2} = \frac{1 + b_2 (\varepsilon_2 + \rho_2) - b_1 (\varepsilon_1 + \rho_1) + p_1 - 2p_2 + c_2 - c_\varepsilon}{2} \]
\[ p_1^* = \frac{3 - b_2 \cdot (c_2 + c_3) + b_1 \cdot (c_1 + c_2) + 2c_1 + c_2 + c_3}{3} \]  \\
\[ p_2^* = \frac{3 - b_1 \cdot (c_1 + c_3) + b_2 \cdot (c_2 + c_3) + 2c_2 + c_1 + c_3}{3} \]  \\

According to Equations (9) and (10), we can obtain equilibrium solutions of rating demands in the two mechanisms in the second stage:

\[ \sigma_1^* = \frac{3 + b_1 \cdot (c_1 + c_3) - b_2 \cdot (c_2 + c_3) + c_1 - 3c_2 + c_3}{6} \]  \\
\[ \sigma_2^* = \frac{3 - b_1 \cdot (c_1 + c_3) + b_2 \cdot (c_2 + c_3) - c_1 + 3c_2 + c_3}{6} \]  \\

The profits of dual ratings and integrated rating in the two mechanisms are:

\[ \pi_1^* = \frac{3 + b_1 \cdot (c_1 + c_3) - b_2 \cdot (c_2 + c_3) + 2c_1 + c_2 + c_3 \cdot [3 - b_2 \cdot (c_2 + c_3) + b_1 \cdot (c_1 + c_3) + c_2 - c_1 + c_3]}{18} \]  \\
\[ \pi_2^* = \frac{3 - b_1 \cdot (c_1 + c_3) + b_2 \cdot (c_2 + c_3) + c_1 + 2c_2 + c_3 \cdot [3 - b_1 \cdot (c_1 + c_3) + b_2 \cdot (c_2 + c_3) + c_1 - c_2 + c_3]}{18} \]  \\

In the second stage, the rating accuracy of the two mechanisms is:

\[ \tau_1 = \theta_1 \cdot (\delta_1 + \mu_1 \cdot \sigma_1) \]  \\
\[ \tau_2 = \theta_2 \cdot (\delta_2 + \mu_2 \cdot \sigma_2) \]  \\

**Situation 3.** When rating accuracy \( \tau_1 \) and \( \tau_2 \) satisfies \( \tau_1 + \tau_2 = 1 \) and \( b_1 + b_2 = 1 \), the rating accuracy is not reaching equilibrium.

\[ \pi_1^* - \pi_2^* = \frac{2b_2 \cdot (c_2 + c_3) - 2b_1 \cdot (c_1 + c_3) - c_1 + c_2 + c_3}{3} \]  \\

When the indirect effect \( \theta_1 \in \left( \frac{1}{2(\sigma_1 + \mu_1 \cdot \sigma_1)}, \frac{2 + 2\sigma_2 - 2\sigma_1 - (1 + \mu_1 \cdot \sigma_1) - c_1 + c_2 + c_3}{2(\sigma_1 + \mu_1 \cdot \sigma_1)} \right) \) in the dual rating mechanism and the indirect effect \( \theta_2 \in \left( \frac{1}{2(\sigma_2 + \mu_2 \cdot \sigma_2)}, \frac{2 + 2\sigma_1 - 2\sigma_2 - (1 + \mu_2 \cdot \sigma_2) - c_1 + c_2 + c_3}{2(\sigma_2 + \mu_2 \cdot \sigma_2)} \right) \) in the integrated rating mechanism, we have \( \sigma_1 > \sigma_2 \) and \( \pi_1^* > \pi_2^* \). According to the first stage of the game model, we consider the results of the first stage and can obtain \( \theta_1 \in \left( \frac{2 + 2\sigma_2 - 2\sigma_1 - (1 + \mu_1 \cdot \sigma_1) - c_1 + c_2 + c_3}{2(\sigma_1 + \mu_1 \cdot \sigma_1)}, +\infty \right) \) and \( \theta_2 \in \left( \frac{2 + 2\sigma_1 - 2\sigma_2 - (1 + \mu_2 \cdot \sigma_2) - c_1 + c_2 + c_3}{2(\sigma_2 + \mu_2 \cdot \sigma_2)}, +\infty \right) \). This demonstrates that the regulatory effect of the dual rating mechanism is better in this situation.

When the indirect effect \( \theta_1 \in \left( \frac{2 + 2\sigma_2 - 2\sigma_1 - (1 + \mu_1 \cdot \sigma_1) - c_1 + c_2 + c_3}{2(\sigma_1 + \mu_1 \cdot \sigma_1)}, +\infty \right) \) in the dual rating mechanism and the indirect effect \( \theta_2 \in \left( \frac{2 + 2\sigma_1 - 2\sigma_2 - (1 + \mu_2 \cdot \sigma_2) - c_1 + c_2 + c_3}{2(\sigma_2 + \mu_2 \cdot \sigma_2)}, +\infty \right) \) in the integrated rating mechanism, we have \( \sigma_1 < \sigma_2 \) and \( \pi_1^* < \pi_2^* \). We consider the results of the first stage and can obtain \( \theta_1 \in \left( \frac{2 + 2\sigma_2 - 2\sigma_1 - (1 + \mu_1 \cdot \sigma_1) - c_1 + c_2 + c_3}{2(\sigma_1 + \mu_1 \cdot \sigma_1)}, +\infty \right) \) and \( \theta_2 \in \left( \frac{2 + 2\sigma_1 - 2\sigma_2 - (1 + \mu_2 \cdot \sigma_2) - c_1 + c_2 + c_3}{2(\sigma_2 + \mu_2 \cdot \sigma_2)}, +\infty \right) \). This demonstrates that the regulatory effect of the integrated rating mechanism is better in this situation.

**Situation 4.** When rating accuracy \( \bar{\tau}_1 \) and \( \bar{\tau}_2 \) satisfies \( \bar{\tau}_1 + \bar{\tau}_2 = 1 \) and \( b_1 + b_2 = 1 \), the rating accuracy is reaching equilibrium.

We let \( \frac{\partial \pi_1^*}{\partial \bar{\tau}_1} = 0 \) and \( \frac{\partial \pi_2^*}{\partial \bar{\tau}_2} = 0 \) in the two mechanisms, and the equilibrium solutions of rating accuracy are:

\[ \frac{\partial \pi_1^*}{\partial \bar{\tau}_1} = \frac{b_1 \cdot b_2 \cdot (c_2 + c_3) - 3b_1 - b_2^2 \cdot (c_1 + c_3) + b_1 \cdot (2c_2 - c_1 + c_3)}{6} \]  \\
\[ \frac{\partial \pi_2^*}{\partial \bar{\tau}_2} = \frac{b_1 \cdot b_2 \cdot (c_1 + c_3) - 3b_2 - b_2^2 \cdot (c_2 + c_3) - b_2 \cdot (2c_2 - c_1 - c_3)}{6} \]
\[
\begin{align*}
\mathcal{E}_1^* &= 4 - b_1 \cdot (1 + \rho_1) + b_2 \cdot \rho_2 + 2c_2 - c_1 + c_s \\
\mathcal{E}_2^* &= 4 + b_1 \cdot (1 + \rho_1) - b_2 \cdot \rho_2 - 2c_2 + c_1 - c_s \\
\mathcal{B}_1 &= \frac{4 - b_1 \cdot (1 + \rho_1) + b_2 \cdot \rho_2 + 2c_2 - c_1 + c_s}{\delta_1 + \mu_1 \cdot \sigma_1} \\
\mathcal{B}_2 &= \frac{4 + b_1 \cdot (1 + \rho_1) - b_2 \cdot \rho_2 - 2c_2 + c_1 + c_s}{\delta_2 + \mu_2 \cdot \sigma_2}
\end{align*}
\]

Based on the results of the direct effect in the first stage, we have:

\[
\begin{align*}
\mathcal{B}_1^* &= 1 + \frac{c_s}{4 - b_1 \cdot (1 + \rho_1) + b_2 \cdot \rho_2 + 2c_2 - c_1} \\
\mathcal{B}_2^* &= 1 + \frac{c_s}{4 + b_1 \cdot (1 + \rho_1) - b_2 \cdot \rho_2 - 2c_2 + c_1}
\end{align*}
\]

When \(b_2 \cdot \rho_2 + 2c_2 + c_s > b_1 \cdot (1 + \rho_1) + c_1\), we have \(\mathcal{E}_1^* > \mathcal{E}_2^*\) and \(\mathcal{B}_1^* > \mathcal{B}_2^*\). This shows that the regulatory effect of the dual rating mechanism is better. When \(b_2 \cdot \rho_2 + 2c_2 + c_s < b_1 \cdot (1 + \rho_1) + c_1\), we have \(\mathcal{E}_1^* < \mathcal{E}_2^*\) and \(\mathcal{B}_1^* < \mathcal{B}_2^*\). This indicates that the regulatory effect of the integrated rating mechanism is better.

Compared with the first stage, when the regulatory penalty conditions were added in the second stage, the indirect effect of the issuance of green bonds and the rating accuracy of green bonds were also affected by the regulatory penalty conditions. According to the analysis of the second stage of the game model, we can conclude the following:

(1) When the rating accuracy does not reach equilibrium, the conditions of usage of the dual rating mechanism are the indirect issuance effect \(\theta_1 \in (1, \frac{2 + 2b_2 - 2b_1 (1 + \rho_1 + \rho_2) - c_1 - c_2 + c_s}{2(\delta_1 + \rho_2)}, +\infty)\) in the dual rating mechanism and the indirect issuance effect \(\theta_2 \in \left(\frac{2 + 2b_1 - 2b_2 (1 + \rho_1 + \rho_2) - c_1 - c_2 + c_s}{2(\delta_2 + \rho_1)}, +\infty\right)\) in the integrated rating mechanism.

(2) When the rating accuracy does not reach equilibrium, the conditions of usage of the integrated rating mechanism are the indirect issuance effect \(\theta_1 \in (1, 1 + \frac{2c_s}{2(\delta_1 + \rho_2)}, +\infty)\) in the dual rating mechanism and the indirect issuance effect \(\theta_2 \in \left(1, 1 + \frac{2c_s}{2(\delta_2 + \rho_1)}, +\infty\right)\).

(3) When the rating accuracy reaches equilibrium, the conditions of usage of the dual rating mechanism are \(b_2 \cdot \rho_2 + 2c_2 + c_s > b_1 \cdot (1 + \rho_1) + c_1\), while the conditions of usage of the dual rating mechanism are \(b_2 \cdot \rho_2 + 2c_2 + c_s < b_1 \cdot (1 + \rho_1) + c_1\).

Overall, we propose three propositions based on the analysis of this model:

**Proposition 1.** When the rating accuracy does not reach equilibrium, the conditions of usage of the dual rating mechanism are the direct issuance effect \(\mu_1 \in \left(\frac{1 - 2c_1}{2(\delta_1)}, \frac{2 + 2b_2 - 2b_1 (1 + \rho_1 + \rho_2) - c_1 - c_2 + 2c_1}{2(\delta_1)}\right)\) in the dual rating mechanism, the direct issuance effect \(\mu_2 \in \left(\frac{2 + 2b_1 - 2b_2 (1 + \rho_1 + \rho_2) - c_1 - c_2 + c_s}{2(\delta_2)}, \frac{1 - 2c_2}{2(\delta_2)}\right)\) in the integrated rating mechanism, the indirect issuance effect \(\theta_1 \in \left(1, \frac{2 + 2b_2 - 2b_1 (1 + \rho_1 + \rho_2) - c_1 - c_2 + c_s}{2(\delta_1 + \rho_2)}, +\infty\right)\) in the dual rating mechanism and the indirect issuance effect \(\theta_2 \in \left(\frac{2 + 2b_1 - 2b_2 (1 + \rho_1 + \rho_2) - c_1 - c_2 + c_s}{2(\delta_2 + \rho_1)}, +\infty\right)\) in the integrated rating mechanism.

**Proposition 2.** When the rating accuracy does not reach equilibrium, the conditions of usage of the integrated rating mechanism are the direct issuance effect \(\mu_1 \in \left[0, \frac{1 - 2c_1}{2(\delta_1)}\right]\) in the dual rating mechanism, the direct issuance effect \(\mu_2 \in \left(\frac{1 - 2c_2}{2(\delta_2)}, 1\right)\), the indirect issuance effect \(\theta_1 \in \left(1, \frac{2c_2}{2(\delta_1 + \rho_2)}, +\infty\right)\) in the dual rating mechanism and the indirect issuance effect \(\theta_2 \in (1, 1 + \frac{2c_2}{2(\delta_2 + \rho_1)}, +\infty)\).

**Proposition 3.** When the rating accuracy reaches equilibrium, the conditions of usage of the dual rating mechanism are \(b_2 \cdot \rho_2 + 2c_2 > b_1 \cdot (1 + \rho_1) + c_1\), while the conditions of usage of the
integrated rating mechanism are $b_2 \cdot \rho_2 + 2c_2 < b_1 \cdot (1 + \rho_1) + c_1$ without considering the indirect effect. When the rating accuracy reaches equilibrium, the conditions of usage of the dual rating mechanism are $b_2 \cdot \rho_2 + 2c_2 + c_s > b_1 \cdot (1 + \rho_1) + c_1$, while the conditions of usage of the dual rating mechanism are $b_2 \cdot \rho_2 + 2c_2 + c_s < b_1 \cdot (1 + \rho_1) + c_1$ with considering the indirect effect.

Propositions 1–3 indicate that the direct and indirect effects of green financing projects can effectively regulate the rating accuracy of green bonds according to the improved Hotelling model of difference, and compared with the direct effect, the indirect effect has more impact on rating regulatory. The value range of direct and indirect effects of green financing projects directly determines the regulatory effect under the two rating mechanisms. Regulators can choose different green bond rating mechanisms according to the actual situation to improve the quality of China’s green bond rating.

5. Numerical Analysis and Simulations

Shi et al. [31] used data simulation to verify the influence of dual reputation incentives on the quality of major construction projects, and Zhou [32] used data simulation to analyze the regulatory effect of the collusion incentive and restraint mechanism of rating agencies. For testing the three propositions in the model, we refer to the research methods of Shi et al. and Zhou et al. and utilize the numerical analysis and simulations with MATLAB to examine the regulatory effect of the two mechanisms without enough data.

5.1. Data Assignment

We obtain the conditions of the direct effect and the indirect effect of green corporate bonds in the model. Furthermore, we provide some random values of these indicators. The random values are shown in Table 1.

| Indicators                                      | Values |
|------------------------------------------------|--------|
| Market share of rating agencies $b_1$           | 0.5    |
| Market share of certified green authorities $b_2$ | 0.3    |
| Rating accuracy in the dual rating mechanism $\delta_1$ | 0.3    |
| Rating accuracy in the integrated rating mechanism $\delta_2$ | 0.5    |
| Green rating accuracy in the dual rating mechanism $\sigma_1$ | 0.5    |
| Green rating accuracy in the integrated rating mechanism $\sigma_2$ | 0.4    |
| Rating costs of rating agencies $c_1$           | 0.3    |
| Rating costs of certified green authorities $c_2$ | 0.6    |
| The total reputation of rating agencies and certified green authorities in the dual rating mechanism $\rho_1$ | 0.5    |
| The total reputation of rating agencies and certified green authorities in the integrated rating mechanism $\rho_2$ | 0.2    |
| Regulatory punishments $c_s$                    | 0.5    |

We simulate the regulatory effect of the two mechanisms. Based on the changes in rating accuracy and profits, we can describe the regulatory effect. Higher rating accuracy and profits in one of the mechanisms demonstrate that rating agencies are willing to provide accurate rating for high revenues and this mechanism has a better regulatory effect.

5.2. Simulation of Rating Regulatory Mechanism Effect under Proposition 1

Based on Table 1, we can evaluate the values of direct issuance effect $\mu_1$, $\mu_2$ and indirect issuance effect $\theta_1$, $\theta_2$ in Proposition 1, which are shown in Table 2.
The results indicate that the dual rating mechanism has a better regulatory effect when the rating accuracy does not reach equilibrium, and only the impact of direct issuance on the rating accuracy and profit is considered in the interval presented by Proposition 1.

| Indicators                                      | The First Stage | The Second Stage |
|------------------------------------------------|-----------------|------------------|
| The direct effect in the dual rating mechanism $\mu_1$ | $[0.1, 1.6]$   | $[0.1, 1.6]$    |
| The direct effect in the integrated rating mechanism $\mu_2$ | $[-1.025, -0.125]$ | $[-0.82, -0.07]$ |
| The indirect effect in the dual rating mechanism $\theta_1$ | 1               | $[1.4286, 4.4286]$ |
| The indirect effect in the integrated rating mechanism $\theta_2$ | 1               | $[1.25, 2.75]$   |

When the rating accuracy does not reach equilibrium, we only consider the impact of direct issuance effect on the rating accuracy and profits in Figure 4. As shown in Figure 4, X-axis is the direct issuance effect of green bonds in the dual rating mechanism. Y-axis is the direct issuance effect in the integrated rating mechanism. Z-axis is the rating accuracy. The left side of this figure is the comparison of rating accuracy in the two mechanisms. In this situation, the rating accuracy in the dual rating mechanism $\epsilon_1$ is higher than the rating accuracy in the integrated rating mechanism $\epsilon_2$. Meanwhile, the right side of this figure is the comparison of profits in the two mechanisms. The profit of rating agencies and the certified green authorities in the dual rating mechanism $\pi_1$ is higher than the profit in the integrated rating mechanism $\pi_2$. The results indicate that the dual rating mechanism has a better regulatory effect when the rating accuracy does not reach equilibrium, and only the impact of direct issuance on the rating accuracy and profit is considered in the interval presented by Proposition 1.

(a) The rating accuracy of the two mechanisms

(b) The profits of the two mechanisms

Figure 4. The impact of direct effect on the regulatory effect of the two mechanisms in Proposition 1. Data Source: Tables 1 and 2.

Furthermore, we consider the impact of direct issuance effect and indirect issuance effect on rating accuracy and profits when the rating accuracy does not reach equilibrium in Figure 5. When introducing the indirect issuance effect, there are big differences in regulatory effect between the dual rating mechanism and the integrated rating mechanism. In this situation, the rating accuracy in the dual rating mechanism $\epsilon_1$ is much higher than the rating accuracy in the integrated rating mechanism $\epsilon_2$. Meanwhile, the profit in the dual rating mechanism $\pi_1$ is much higher than the profit in the integrated rating mechanism $\pi_2$. 

Table 2. Values of direct issuance effect and indirect issuance effect in Proposition 1.

| Indicators                                      | The First Stage | The Second Stage |
|------------------------------------------------|-----------------|------------------|
| The direct effect in the dual rating mechanism $\mu_1$ | $[0.1, 1.6]$   | $[0.1, 1.6]$    |
| The direct effect in the integrated rating mechanism $\mu_2$ | $[-1.025, -0.125]$ | $[-0.82, -0.07]$ |
| The indirect effect in the dual rating mechanism $\theta_1$ | 1               | $[1.4286, 4.4286]$ |
| The indirect effect in the integrated rating mechanism $\theta_2$ | 1               | $[1.25, 2.75]$   |
The results indicate that the dual rating mechanism has a better regulatory effect when the rating accuracy does not reach equilibrium, and the direct issuance effect and the indirect issuance effect are both considered in the interval presented by Proposition 1.

![Figure 4](image_url)

**Figure 4.** The impact of direct effect on the regulatory effect of the two mechanisms in Proposition 1. Data Source: Table 1 and 2.

### 5.3. Simulation of Rating Regulatory Mechanism Effect under Proposition 2

The same as Table 2, we can evaluate the values of direct issuance effect $\mu_1$, $\mu_2$ and indirect issuance effect $\theta_1$, $\theta_2$ in Proposition 2, which are shown in Table 3.

**Table 3.** Values of direct issuance effect and indirect issuance effect in Proposition 2.

| Indicators                                      | The First Stage | The Second Stage |
|------------------------------------------------|-----------------|------------------|
| The direct effect in the dual rating mechanism $\mu_1$ | [0, 0.75]       | [0, 0.75]        |
| The direct effect in the integrated rating mechanism $\mu_2$ | [0, 1]         | [0, 1]          |
| The indirect effect in the dual rating mechanism $\theta_1$ | 1              | (1, 1.35]       |
| The indirect effect in the integrated rating mechanism $\theta_2$ | 1              | (1, 6.25]       |

In Figure 6, when the rating accuracy does not reach equilibrium, we only consider the impact of direct issuance effect on the rating accuracy and profits based on Proposition 2. The left side of this figure is the comparison of rating accuracy in the two mechanisms. In this situation, the rating accuracy in the integrated rating mechanism $\epsilon_2$ is higher than the rating accuracy in the dual rating mechanism $\epsilon_1$. Meanwhile, the right side of this figure is the comparison of profits in the two mechanisms. The profit of rating agencies in the integrated rating mechanism $\pi_2$ is higher than the profit of rating agencies and the certified green authorities in the dual rating mechanism $\pi_1$. The results show that under the condition of Proposition 2, the integrated rating mechanism has a better regulatory effect when the direct issuance effect is in certain conditions.
Meanwhile, the right side of this section presents the results. The results show that under

\[ \text{Proposition 2.} \]

The impact of direct effect and indirect effect on the two mechanisms in Proposition 2. Data Source: Tables 1 and 3.

\[ \text{Figure 6.} \]

(a) The rating accuracy of the two mechanisms

(b) The profits of the two mechanisms

Furthermore, we consider the impact of direct issuance effect and indirect issuance effect on rating accuracy and profits when the rating accuracy does not reach equilibrium. The simulations are shown in Figure 7. When introducing the indirect issuance effect, there are big differences in regulatory effect between the dual rating mechanism and the integrated rating mechanism. In this situation, the rating accuracy in the integrated rating mechanism \( \varepsilon_2 \) is much higher than the dual rating mechanism \( \varepsilon_1 \). Meanwhile, the profit in the integrated rating mechanism \( \pi_2 \) is much higher than the profit in the dual rating mechanism \( \pi_1 \). The results indicate that the integrated rating mechanism has a better regulatory effect when the rating accuracy does not reach equilibrium, and the direct issuance effect and the indirect issuance effect are both considered in the interval presented by Proposition 2.

\[ \text{Figure 7.} \]

(a) The rating accuracy of the two mechanisms

(b) The profits of the two mechanisms

The impact of direct effect and indirect effect on the two mechanisms in Proposition 2. Data Source: Tables 1 and 3.
5.4. Numerical Analysis of Rating Regulatory Mechanism Effect under Proposition 3

When the rating accuracy reaches equilibrium, the direct effect and the indirect effect are in certain conditions. We can obtain the equilibrium solutions of rating accuracy and profits with the data from Table 1. The equilibrium solutions of rating accuracy and profits are shown in Tables 4 and 5.

### Table 4.
The equilibrium solutions of rating accuracy and profits in Proposition 3.

| Indicators                                      | The First Stage | The Second Stage |
|------------------------------------------------|----------------|-----------------|
| The direct effect in the dual rating mechanism $\mu_1$ | 7.82           | 7.82            |
| The direct effect in the integrated rating mechanism $\mu_2$ | 1.5            | 1.5             |
| The indirect effect in the dual rating mechanism $\theta_1$ | 1              | 1.1188          |
| The indirect effect in the integrated rating mechanism $\theta_2$ | 1              | 3.1319          |
| Rating accuracy in the dual rating mechanism $\varepsilon_1^*$ | 4.21           | 4.7100          |
| Rating accuracy in the integrated rating mechanism $\varepsilon_2^*$ | 1.10           | 1.3596          |
| Profit in the dual mechanism $\pi_1^*$ | 0.9392         | 1.0503          |
| Profit in the integrated mechanism $\pi_2^*$ | 0.0670         | 0.1846          |

### Table 5.
The equilibrium solutions of rating accuracy and profits in Proposition 3 (2).

| Indicators                                      | The First Stage | The Second Stage |
|------------------------------------------------|----------------|-----------------|
| The direct effect in the dual rating mechanism $\mu_1$ | 3.5870         | 3.5870          |
| The direct effect in the integrated rating mechanism $\mu_2$ | 8.7250         | 8.7250          |
| The indirect effect in the dual rating mechanism $\theta_1$ | 1              | 1.1200          |
| The indirect effect in the integrated rating mechanism $\theta_2$ | 1              | 1.1319          |
| Rating accuracy in the dual rating mechanism $\varepsilon_1^*$ | 2.0935         | 2.3447          |
| Rating accuracy in the integrated rating mechanism $\varepsilon_2^*$ | 3.9900         | 4.5164          |
| Profit in the dual rating mechanism $\pi_1^*$ | 0.2499         | 0.2713          |
| Profit in the integrated rating mechanism $\pi_2^*$ | 0.3667         | 0.2786          |

Comparing with Tables 4 and 5, it is different between the regulatory effect of the dual rating mechanism and that of the integrated rating mechanism when the rating accuracy reaches equilibrium. Comparing $\mu_1$ and $\mu_2$ in Tables 4 and 5 shows that when the direct effect reaches a large value, the regulatory effect of the dual rating mechanism is much better; when the direct effect reaches a small value, the regulatory effect of the integrated rating mechanism is much better. The comparison of $\theta_1$ and $\theta_2$ at the second stage in Tables 4 and 5 also shows that under the equilibrium condition, when the indirect effect of green bonds is large enough and reaches a certain value, the profit under the dual rating mechanism is larger; otherwise, the regulatory effect of the integrated rating mechanism is better. These results show that when the direct effect and the indirect effect both reach a large value, the dual rating mechanism has a better regulatory effect. In the meantime,
when both the direct effect and the indirect effect reach one of small values, the regulatory effect of the integrated rating mechanism is better.

6. Discussion
From the model and the simulations, the results show that the direct effect and the indirect effect of green corporate bonds have large differences in the regulatory effect of the dual rating mechanism and the integrated rating mechanism. There is no applicable condition of the results, which are universal. In this article, we developed a new incentive difference Hotelling model to divide the issuance effects of green corporate bonds into the direct effect and the indirect effect. We discussed the applicable conditions of the two mechanisms in this model and used simulation analysis to verify the regulatory effect. In Table 6, the conditions of direct and indirect effects in the two mechanisms are respectively given by combining the game results and simulation results.

Table 6. The conditions of the dual rating mechanism and the integrated rating mechanism.

| Situation          | The Direct Issuance Effect | The Indirect Issuance Effect | The Choice of Mechanisms |
|--------------------|----------------------------|-----------------------------|--------------------------|
| Non-equilibrium    | $\mu_1 \in [\frac{2\theta_1^2 - 2\theta_1 (1 + \theta_1 + \theta_2)}{\theta_1^2 - 2\theta_1 + 1}, \frac{2\theta_1^2 - 2\theta_1 (1 + \theta_1 + \theta_2)}{\theta_1^2 - 2\theta_1 + 1}]$ | $\theta_1 \in (1, \frac{2\theta_1^2 - 2\theta_1 (1 + \theta_1 + \theta_2)}{\theta_1^2 - 2\theta_1 + 1}]$ | The dual rating mechanism |
|                    | $\mu_2 \in (\frac{2\theta_1^2 - 2\theta_1 (1 + \theta_1 + \theta_2)}{\theta_1^2 - 2\theta_1 + 1}, 1]$ | $\theta_2 \in (1, +\infty)$ | The integrated rating mechanism |
| Equilibrium        | $\mu_1^* = 4 + \frac{4 - \theta_1 (1 + \theta_1) - \theta_2 (\theta_2 - 2\theta_1 - \theta_1 - \delta)}{\theta_1} - \delta$ | $\mu_2^* = 4 + \frac{4 - \theta_1 (1 + \theta_1) - \theta_2 (\theta_2 - 2\theta_1 - \theta_1 - \delta)}{\theta_2} - \delta$ | $\theta_1^* = 1 + \frac{1}{4 + \frac{4 - \theta_1 (1 + \theta_1) - \theta_1 (\theta_1 - 2\theta_1 - \theta_1 - \delta)}{\theta_1} - \delta}$ | The dual rating mechanism |
|                    | $\theta_2^* = 1 + \frac{1}{4 + \frac{4 - \theta_1 (1 + \theta_1) - \theta_1 (\theta_1 - 2\theta_1 - \theta_1 - \delta)}{\theta_2} - \delta}$ | $\mu_1^* = 1 + \frac{1}{4 + \frac{4 - \theta_1 (1 + \theta_1) - \theta_1 (\theta_1 - 2\theta_1 - \theta_1 - \delta)}{\theta_1} - \delta}$ | The integrated rating mechanism |

According to Table 6, we can define two research conclusions based on the non-equilibrium state and the equilibrium state: (1) Under the non-equilibrium state, when both the direct effect and the indirect effect are big, the regulatory effect of the dual rating mechanism is better. When both the direct effect and the indirect effect are small, the regulatory effect of the integrated rating mechanism is better. (2) Under the equilibrium state, when both the direct effect and the indirect effect reach one of big values, the regulatory effect of the dual rating mechanism is better. When both the direct effect and the indirect effect reach one of small values, the regulatory effect of the integrated rating mechanism is better.

Based on these conclusions, the regulatory authority can select the appropriate green bond rating regulatory mechanism based on the specific value of the environmental issuance effect. In this way, the rating accuracy of green bonds can be improved and the default probability of green bonds can be reduced.

7. Conclusions
Under the current China’s green corporate bond issuance supervision, this paper constructs a Hotelling model based on incentives and constraints between the dual rating mechanism and the integrated rating mechanism. How the rating accuracy of the two rating mechanisms is influenced by the direct and indirect issuance effects of green financing projects is discussed here. According to the research conclusion of this paper, there are some managerial insights in this research: (1) Introducing the direct effect and the indirect effect in regulatory rating mechanisms of green corporate bonds can effectively improve regulatory effects. (2) The indirect effect has more impact on green bond rating supervision. (3) The regulator can utilize different rating mechanisms based on the direct and indirect effects. The mechanism of rating significantly influences the effectiveness of green corporate issuance effects. Through the analysis of the green bonds market, social funds can be guided to invest in projects with significant positive environmental benefits, and financial means can be utilized to assist the efficient allocation mechanism of environmental resources.
This research can be extended in several aspects in future work. Due to lack of enough data, we chose some random values of indicators in the model for numerical analysis and simulations. The direct and indirect effects of issuance listed in this paper can also be quantitatively measured by the econometrics method using China’s real data to provide more sufficient empirical evidence for practical application. In future research, we will collect relevant data and replace the current data with it. In addition, as the influencing factors to be considered in green bond rating are more complex in reality, there may be cross-interaction between various influencing factors. In this paper, only representative influencing factors are abstracted. Thus, the influence of cross-interaction among various factors and influence of other factors could be further discussed.

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References
1. Banga, J. The green bond market: A potential source of climate finance for developing countries. *J. Sustain. Financ. Invest.* **2018**, *9*, 17–32. [CrossRef]
2. Trompeter, L. Green is good: How green bonds cultivated into Wall Street’s environmental paradox. *Sustain. Dev. Law Policy* **2017**, *17*, 30–43.
3. Li, Y. Green Bond Series Study—Evaluation and Certification of Green Bond; China Bond Rating Co., Ltd.: Shanghai, China, 2020; pp. 6–21.
4. Wang, Y. Problems and Optimization of the Indicator System of the Third-Party Evaluation Method for Green Bonds; Northwest Normal University: Chongqing, China, 2018. (In Chinese)
5. Gu, M. Research on the Third-Party Certification Method and Function in Green Bond Issuance; Nankai University: Nanjing, China, 2020. (In Chinese)
6. Climate Bonds Initiative. Green Bond Market SummaryQ3 2020. *Clim. Bonds* **2020**, *11*, 1–2.
7. Huang, C. Analysis of the impact of third-party certification on the issuance cost of green bonds. *Bond* **2019**, *4*, 67–71. (In Chinese)
8. Ehlers, T.; Packer, F. Green bond finance and certification. *BIS Q. Rev.* **2017**, *1*, 89–104.
9. Shan, C. Games between e-finance and traditional banking: Based on a three-stage difference Hotelling model under double oligarch monopolization. *Syst. Eng.* **2016**, *34*, 1–9.
10. Dong, Z.Q.; Wang, H. The effect of environmental regulation on green technology progress of “local-neighborhood”. *China Ind. Econ.* **2019**, *1*, 100–118.
11. Shen, S.R.; Jin, G.; Fang, X. Does environmental regulation cause pollution to be transferred nearby? *China Ind. Econ.* **2017**, *5*, 46–61.
12. Han, C.; Zhang, W.G.; Feng, Z.B. How to “remove” resource mismatch in environmental regulation. *China Ind. Econ.* **2017**, *4*, 115–134.
13. Shao, S.; Zhang, K.; Dou, J.M. Energy saving and emission reduction effects of economic agglomeration: Theory and experience of China. *Manag. World* **2019**, *1*, 36–60.
14. Lin, B.Q.; Tan, R.P. China’s economic agglomeration and green economic efficiency. *Econ. Res. J.* **2019**, *2*, 119–132.
15. Yi, M.; Li, G.; Peng, J.C. Study on the spatial-temporal differentiation of green TFP in the Yangtze River Economic Belt. *Manag. World* **2018**, *34*, 178–179.
16. Bolton, P.; Freixas, X.; Shapiro, J. The credit ratings game. *J. Financ.* **2012**, *67*, 85–111. [CrossRef]
17. Manso, G. Feedback effects of credit ratings. *J. Financ. Econ.* **2013**, *109*, 535–548. [CrossRef]
18. Li, X.L. Game Analysis on the evolution of public private partnership project supervision considering reputation. *J. Syst. Eng.* **2013**, *32*, 199–206.
19. Becker, B.; Milbourn, T. How did increased competition affect credit ratings? *J. Financ. Econ.* **2011**, *101*, 93–514. [CrossRef]
20. Mathis, J.; Mc Andrews, J.; Rochet, J.C. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *J. Monet. Econ.* **2009**, *56*, 657–674. [CrossRef]
21. Stolper, A. Regulation of credit rating agencies. *J. Bank. Financ.* **2009**, *33*, 1266–1273. [CrossRef]
22. Huang, X.L.; Zhu, S.; Chen, G. The impact of bond default on credit ratings agencies: Analysis based on bond default in Chinese bond market. *Financ. Res.* **2017**, *441*, 130–144.
23. Jin, S.; Zhang, Y.; Meng, Q.F. Dynamic analysis on the evolution of enterprise environment compliance behavior under dynamic punishment mechanism. *J. Syst. Manag.* 2017, 26, 1122–1130.

24. Zhou, X.; Tian, Y. Incentive and constraint regulations of rating inflation in collusion over the separation of economic cycles—Markov rating shopping dual reputation model. *PLoS ONE* 2018, 13, e0205415. [CrossRef] [PubMed]

25. Hotelling, H. Stability in competition. *Econ. J.* 1929, 39, 41–57. [CrossRef]

26. Zhang, T.; Huo, Y.; Zhang, X. Endogenous third-degree price discrimination in Hotelling model with elastic demand. *J. Econ.* 2018, 2, 125–145. [CrossRef]

27. Tan, X.; Chen, Z.; Yang, C. Green innovative strategy of logistics enterprises based on Hotelling expansion model. *J. Interdiscip. Math.* 2018, 21, 907–916. [CrossRef]

28. Cairns, D.R. Economic accounting in the simple Hotelling model. *Resour. Energy Econ.* 2018, 51, 18–27. [CrossRef]

29. Quan, S.W.; Huang, B. Embeddedness effect in environmental policy benefit evaluation: Based on a case of haze and dust control policies in Beijing. *China Ind. Econ.* 2016, 8, 25–41.

30. Hu, E. *Research on Substitution Sequential Auction and Information Disclosure with Hotelling Difference*; Huazhong University of Science and Technology: Wuhan, China, 2019. (In Chinese)

31. Shi, Q.; Zhu, J.; Sheng, Z. Dynamic Incentive Mechanism of Factory Prefabrication Based on Double Reputation. *J. Syst. Manag.* 2017, 26, 338–345. (In Chinese)

32. Zhou, X.Y.; Tian, Y.X. Research on incentive and constraint mechanism of rating agencies collusion under economic cycle separation. *J. Syst. Eng.* 2020, 2, 210–221. (In Chinese)