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Metaheuristic optimization approach and computational study on advanced mathematical modeling of solar cell

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ABSTRACT
A solar cell current usually depends on bandgap, carrier lifetime, diffusion length, diffusion constant, ideality factor, and so on. This article proposes a model that shows direct dependency of bandgap in the solar cell current equation. It is based on foundation principles satisfactory of a single junction being extended to multi-junction solar cell applications with different bandgaps. Different proven optimization techniques such as the Gauss–Newton optimization, Levenberg–Marquardt optimization, differential evolution algorithm, and whale optimization algorithm (WOA) were used for parameter extraction and optimization. To demonstrate the better optimization of model data with experimental data, further analysis of results was carried out on the basis of percentage deviation and error comparison. Complete comprehensive and comparative analysis of results of error and percentage deviation from different standard iterative optimization techniques proves the WOA as one of the best optimization techniques among all. As a result, the WOA offers a better optimal solution of model parameters with the best convergence of model data with the experimental data that shows the smallest possible value of error and minimum percentage deviation.

I. INTRODUCTION
Mathematical modeling of the solar cell is imperative to analyze and comprehend the lumped circuit model parameters such as open circuit voltage ($V_{oc}$), short circuit current ($I_{sc}$), efficiency, maximum power point (Pmax), and fill factor (FF). Validating the design and evaluation of the photovoltaic (PV) array involves several steps of iterative calculations for parameter extraction. Solving the implicit form of the solar cell equation to achieve the best degree of correctness is still a challenge. Hence, several numerical models are proposed to prevail over the cumbrous analytical process involved in peak power calculation, parameter extraction, and the methodology of biasing point analysis.

Under the influence of the solar radiation current–voltage (I–V) relationship, a function of series and shunt resistances in the form of a single exponential model (SEM) is given by

$$I = I_{ph} - I_0 \left[ \exp \left( \frac{V + I \cdot R_s}{\eta \cdot V_t} \right) - 1 \right] - \left[ \frac{V + I \cdot R_s}{R_{sh}} \right]. \quad (1)$$

Here, $R_s$ and $R_{sh}$ represent the unit area parasitic series and shunt resistance, respectively, $\eta$ represents the ideality factor, $V_t$ corresponds to the thermal voltage, $I_0$ represents the reverse saturation current, and $I_{ph}$ is denoted as the generated current by incident radiation. The electrical equivalent circuit of the SEM of a solar cell is shown in Fig. 1.

Different I–V models are put into practice to facilitate the tough process involved in iterative calculations of implicit functions for the computation of peak power point analysis and the fill factor. The proposed I–V model by Mehmet Akbaba and Aiattawi is expressed as

$$I = \frac{V_{oc} - V}{[A + B \cdot V^2 - C \cdot V]}. \quad (2)$$

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where A, B, and C are the three model parameters, and \( V_{oc} \) corresponds to the open circuit voltage.

A model is developed, and parameter extraction is followed by biasing point analysis, where the I–V model is proposed as

\[
J = J_{sc}(1 - kV^m) - g_dV.
\]

Here, \( m \), \( g_d \), and \( k \) are the model parameters used for analysis of the fill factor and peak power point analysis.

Our previous research work was based on the conventional process for model development which prevailed over perplexity in model parameter calculation. As reported, the I–V model for a solar cell is expressed as

\[
I = \left( \frac{I_{sc}}{e^{A E_{gi}} - 1} + B \cdot V \right) \cdot (e^{A V_{oc}} - e^{A V}).
\]

Here A and B are the two model parameters, whereas \( V_{oc} \) and \( I_{sc} \) correspond to the open circuit voltage and short circuit current of the solar cell, respectively.

Mathematical modeling using the Lambert W function and Green’s function was found to be challenging and complex in approach. Several analytical methodologies for parameter extraction involve numerical solutions to the transcendental I–V equation of the solar cell, which includes approximation and low precision of model parameters.

This article depicts the numerical modeling of a solar cell unified with bandgap through empirical analysis of two model parameters, as shown in Sec. II. Extraction of model parameters and the use of several optimization techniques such as the whale optimization algorithm (WOA), Levenberg–Marquardt optimization (LMO), differential evolution algorithm (DEA), and Gauss–Newton optimization (GNO) are presented in Sec. III. These rigorous iterative optimization techniques are verified for parameter extraction and model optimization in which the best convergence of model data with experimental data from different literature works is selected. These optimization processes are developed in a MATLAB environment. Further analysis of results was carried out with respect to % deviation, error analysis, and model parameter values, as given in Sec. IV.

II. PROPOSED MODEL

Semiconductors having a higher bandgap absorb high-energy photons compared to those having a lower bandgap. Hence, in device modeling, alteration of the bandgap is indispensable for achieving desired electrical and optical properties. In all these developed models, the solar cell current depends on physical parameters such as shunt resistance, series resistance, thermal voltage, ideality factor, conductance, open circuit voltage, and some model parameters or constants. Hardly any bandgap dependent empirical model in terms of I–V equation validating both single junction and multi-function solar cells are discussed, which facilitates the study of solar cell current variation in terms of the material bandgap.

Hence, we developed a model that entails numerical modeling of a solar cell incorporated with bandgap through empirical analysis that uses two model parameters A and B. The model can be used to compare proper tuning of bandgap of different applications either in a multi-junction or single junction solar cell along with computation of cell performance and characterization.

The proposed numerical model of the solar cell incorporated with bandgap is given by

\[
J = \sum_{i=1}^{n} \left( \frac{I_{sc}}{e^{A E_{gi}} - 1} + B \cdot V \right) \cdot (e^{A V_{oc}} - e^{A V}).
\]

Here, the sum of terms is \( f(E_{gn}) \), where \( E_{g1}, E_{g2}, \ldots, E_{gn} \) are defined for single junction, double junction, . . . n-junction solar cells, respectively. A and B are the two model parameters, and \( I_{sc} \) and \( V_{oc} \) are the short-circuit current density and open circuit voltage, respectively.

In multi-junction solar cells, the photons with a higher energy are absorbed by the first cell as its bandgap \( E_{g1} \) superimposes on the second cell, with a lower bandgap \( E_{g2} \), which absorbs low-energy photons. Hence, this model can be better used for high density multi-junction solar cells for fine tuning of the desired bandgaps that drive maximum efficiency.

III. PARAMETER EXTRACTION AND OPTIMIZATION

Parameter extraction and model optimization were carried out by the Whale Optimization Algorithm (WOA), Levenberg–Marquardt optimization (LMO), Differential Evolution Algorithm (DEA), and Gauss–Newton optimization (GNO), where different bandgap values are considered from the corresponding literature. Comparison and analysis of different analytical models and extraction of model parameter values using different optimization processes are shown in Table I.

A. Gauss–Newton optimization

Gauss–Newton optimization (GNO) is used for solving nonlinear least square problems. It is based on applying its solution to the minimum of a function. GNO assumes residuals to be zero, and, it converges faster than other optimization techniques to find a best solution. GNO is found to have better optimization results with least number iterations. Robustness of data has been adopted to obtain better convergence and proper tuning of model parameters. The goodness of optimization is well understood from the R-square value as it approaches 1 and is found to be \( n > 0.99 \) for most of the solar cell structures. Figure 2 demonstrates well that the optimization process carried out for the solar cell structures InGaP/InGaAs/Ge.
solution of the nonlinear quadratic function to reduce the sum of squared errors and yields better optimal solutions when the model data are less convergent with the experimental data. As shown in Table II, Levenberg–Marquardt optimization exhibits a minimal error and the smallest possible % deviation in comparison to Gauss–Newton optimization. Robustness of optimization is adopted to obtain proper convergence of model data with experimental data. Good convergence of model data with experimental data yields an R-square value close to 0.99 for most of the cell structures.

Figure 3 is well comprehensible in demonstrating good optimization of model data with its experimental data set for InGaP/InGaAs/Ge, GaInP/GaAs/InGaAsP, GaInP/GaAs, CdS/CdTe, and GaAs solar cell structures.

C. Differential evolution algorithm

Use of the differential evolution algorithm (DEA) yields better computational efficiency, exploring search space, and better suitable convergence of data; the accuracy of finding the optimum with the lowest possible value of percentage deviation is found to be 1.64% in a GaAs solar cell. The DEA consists of four process flows such as primary initialization, mutation, crossover, and selection. Primary initialization includes determination of population size to obtain a complete set of solution vectors in each iteration process step. Generation process steps include the random selection of solutions such as primary initialization, mutation, crossover, and selection. The selection process includes comparison between the trial vector by a crossover rate, creating a new trial vector. The process step includes the interchange of a new or donor vector with the target vector and reserves one of them. Failure in termination criteria reiterates the process steps until the optimum solution is reached. The computational efficiency in terms of time is estimated to be O(n^1.7), where n denotes the population size ∈ [0, 500], and O represents the order of computation. Figure 4 illustrates the best optimization result for model data with experimental data for InGaP/InGaAs/Ge, GaInP/GaAs/InGaAsP, GaInP/GaAs, CdS/CdTe, and GaAs solar cell structures.
### Table II: Extracted model parameter values along with the analysis of average error and average % deviation.

| Different cells | Gauss–Newton optimization | Levenberg–Marquardt optimization | Differential evolution algorithm | Whale optimization algorithm |
|-----------------|----------------------------|----------------------------------|-------------------------------|-----------------------------|
|                 | A                          | B                                | A                             | B                           |
|                 | Average error (mA/cm²)     | Average % deviation              | Average error (mA/cm²)        | Average % deviation         |
| InGaP/InGaAs/Ge | 11.01 × 10⁻¹²              | 0.0682                            | 1.61 × 10⁻¹²                   | 0.0425                      |
| GaInP/GaAs/InGaAsP | 9.72 × 10⁻¹⁴              | 0.1563                            | 2.25 × 10⁻⁰¹                   | 0.0305                      |
| GaInP/GaAs     | 14.57 × 10⁻¹⁰              | 0.0544                            | 3.22 × 10⁻¹⁰                   | 0.024                       |
| CdS/CdTe       | 21.02 × 10⁻⁰¹              | 0.0837                            | 2.14 × 10⁻¹⁰                   | 0.0325                      |
| GaAs           | 21.02 × 10⁻₁⁰              | 0.0837                            | 2.14 × 10⁻¹⁰                   | 0.0325                      |

#### FIG. 3: Optimization plot of experimental data (circles) and model data (solid line) of J–V characteristics of different solar cells using the Levenberg–Marquardt optimization method.

#### D. Whale optimization algorithm

The whale optimization algorithm (WOA) is a part of metaheuristic optimization, which depicts behavioral analysis of humpback whales.\(^\text{19–21}\) The whale optimization algorithm achieves better optimization results than GNO, LMO, and the DEA as it is found to be better at exploring population information and convergence with the global optimal solution with the lowest possible value of percentage deviation of \(\sim 1.24\%\) for a GaAs solar cell, as shown in Table II. The whale optimization algorithm is implemented by some basic behavioral analysis of humpback whales such as encircling the prey, the exploitation phase as in the bubble net attacking method, and the exploration phase as in search for a prey. In the process flow of the encircling the prey method, humpback whales know the
location of the prey and encircle them. The target prey is considered to be the current best optimal solution in the search space. In the whale optimization algorithm, distinctive search agents are prone to update their positions toward the best candidate solution until the best search agent is found. In the bubble net attacking method, the exploitation phase is trailed by a spiral updating position method and a shrinking encircling mechanism. The spiral updating position method finds the best suitable distance between the prey location and whale location by calculating the humpback whale movement in a helical shape. The shrinking encircling mechanism locates the position of the search agent anywhere between the location of the original agent or the current best agent. In the exploration phase, the search agents (humpback whales) search for the best solution, usually the prey, instead of searching for the optimum search agent. This process updates the positions of new search agents with randomly selected search agents. Figure 5 shows the best optimization result for model data with experimental data for InGaP/InGaAs/Ge, GaInP/GaAs/InGaAsP, GaInP/GaAs, CdS/CdTe, and GaAs solar cell structures.

IV. RESULT ANALYSIS

Detailed analysis of results is carried out by error comparison and percentage deviation analysis followed by model optimization using the Gauss–Newton optimization (GNO), Levenberg–Marquardt optimization (LMO), differential evolution algorithm (DEA), and whale optimization algorithm (WOA) for experimental data of five different solar cell structures extracted from different literature works. Table II summarizes the model data deviation in comparison to experimental data in terms of error and percentage deviation. In this section, analysis of results is performed for InGaP/InGaAs/Ge, GaInP/GaAs/InGaAsP, GaInP/GaAs, CdS/CdTe, and GaAs solar cell structures. Figures 6–10 show error and percentage deviation results for five different cell structures using the following optimization techniques: GNO, LMO, DEA, and WOA.
A. Error and % deviation analysis

In this section, it is well comprehensible to analyze the result as shown in Table II, which shows that the whale optimization algorithm flags ahead of all other optimization techniques. It shows the smallest possible value of error and minimum percentage deviation. Maximum error variation and percentage deviation limits are within ±5%. The optimization figure of the WOA shows better convergence of model data with experimental data. After the WOA, the DEA proves to be the best optimization technique over LMO and GNO. The DEA possesses good convergence and faster computational efficiency; thus, it is found to have few errors and less percentage deviation in comparison to LMO and GNO. LMO yields better results in comparison to GNO because LMO optimizes the data more like the gradient descent method when desired roots and searched roots are found to be far from each other. Hence, during the optimization of J–V characteristics, LMO has an edge over GNO as the slope starts to increase. GNO shows a higher error and percentage deviation than other optimization techniques because when desired roots and searched roots are far from each other, it fails to converge properly, and it is more convenient when the slope is increasing.

V. CONCLUSION

In this study, bandgap dependent numerical modeling of the solar cell is demonstrated. The major importance of this model is to achieve desired optical and electrical properties, computation of cell performance, and characterization with proper tuning of bandgap values along with the comparative analysis of model parameters.
Parameter extraction and model optimization are carried out by using some standard iterative optimization techniques, where different bandgap values are considered from the corresponding literature. Further analysis of results shows the whale optimization algorithm (WOA) to be the best one with a better optimum solution for model parameters and with the best convergence of model data with experimental data. Applications involving numerical modeling and the use of various optimization techniques have an edge over the analytical process involved in peak power calculation, parameter extraction using differential approaches, and the methodology of biasing point analysis for an implicit J–V relationship in a practical solar cell. Further analysis of results is carried out on the basis of percentage deviation and error analysis, where the WOA found to be the best optimization technique among all with the smallest possible value of error of $\sim 5.55 \times 10^{-5}$ and a minimum percentage deviation of $\sim 1.2\%$.

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