Astrometric Detection of Terrestrial Planets in the Habitable Zones of Nearby Stars with SIM PlanetQuest

JOSEPH CATANZARITE, MICHAEL SHAO, ANGELLE TANNER, STEPHEN UNWIN, AND JEFFREY YU
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109-8099; jcat@s383.jpl.nasa.gov
Received 2005 October 21; accepted 2006 March 22; published 2006 September 5

ABSTRACT. SIM PlanetQuest (formerly the Space Interferometry Mission) is a space-borne Michelson interferometer for precision stellar astrometry, with a 9 m baseline, currently slated for launch in 2016. One of the principal science goals is the astrometric detection and orbital characterization of terrestrial planets in the habitable zones of nearby stars. Differential astrometry of the target star against a set of reference stars lying within $1^\circ$ will allow measurement of the target star’s reflex motion with astrometric accuracy of $1 \mu$as in a single measurement. The purpose of the present paper is to quantitatively assess SIM’s capability for detection (as opposed to characterization by orbital determination) of terrestrial planets in the habitable zones of nearby stars. Note that the orbital periods of these planets are generally shorter than the 5 year SIM mission. We formulate a “joint periodogram” as a tool for planet detection from astrometric data. For adequately sampled orbits (i.e., five or more observations per period over a sampling time span longer than the orbital period), we find that the joint periodogram is more sensitive than the $\chi^2$ test for the null hypothesis. In our analysis of the problem, we use Monte Carlo simulations of orbit detection, together with realistic observing scenarios, actual target and reference star lists, realistic estimates of SIM instrument performance, and plausible distributions of planetary system parameters.

1. INTRODUCTION

SIM PlanetQuest will be the first instrument to use astrometry to detect and characterize terrestrial planets. The ability to make these detections depends on the performance of the instrument, the details of the observing scenarios, and the analysis of the astrometric data. This paper presents detailed simulations of the planet detection process. It includes a realistic model of the instrument performance, realistic observations of target and reference stars, and plausible distributions of the astrophysical parameters defining the ensembles of planetary systems. SIM is now at the end of NASA’s phase B development. All significant technologies have been verified in laboratory test beds, and the project is on track to build the flight instrument for launch as early as 2013. The instrument model in this paper is based on results from these test beds.

Section 2 presents a brief description of SIM’s narrow-angle observing scenario. Section 3 contains a discussion of the habitable zone and shows how the astrometric signature of a planet in the habitable zone of a main-sequence star scales with stellar luminosity and distance. In § 4, we describe lists of SIM target stars and their characteristics, along with several possible survey modes that trade off number of stars versus number of observations per star. Section 5 provides a description of the joint periodogram technique for detection of periodicities in astrometric data. We describe the methodology of our study in § 6. The main results are presented in §§ 6 and 7, in which we quantitatively characterize SIM’s sensitivity for detection of terrestrial planets, the expected mass distribution and total number of terrestrial planets SIM will discover, and the completeness of detected planets as a function of planet mass. In § 8 we briefly consider how SIM’s discoveries can benefit the Terrestrial Planet Finder (TPF) mission.

2. NARROW-ANGLE OBSERVING SCENARIO

For narrow-angle astrometry, a target star should be surrounded by a group of reference stars located within a radius of about $1^\circ$. Differential delay measurements of the target and reference stars will be used to simultaneously estimate the position of the target star with respect to the reference frame and remove the linear field dependence in the delay measurements (J. Yu 2002, private communication; M. Milman 2002, private communication). The least-squares problem involves three parameters, requiring a minimum of three reference stars. Ideally, the target star is at the photocenter of the reference stars. For nonideal reference star positions, there is a penalty on the achievable position accuracy on the measurement of the target star with respect to the reference frame. It is best to have more than three reference stars per target star, since the more reference stars there are, the closer their photocenter is to the target star, and thus the lower the error penalty will be (assuming the distribution of reference star candidates is uniform on the sky near a target star). Selection of planet-search targets...
and their reference stars is discussed in § 4. We find that most SIM planet-search targets have eight or more bright K giant reference star candidates available within 1 kpc. According to our own simulations, we expect that four to six reference star candidates per target star will survive a ground-based radial velocity vetting program before SIM launches, and that of these, three or more will be astrometrically clean (i.e., will have astrometric reflex motion below SIM’s detection threshold).

For each visit to the target-reference group, the allocated integration time is divided into a sequence of “chopped” measurements, alternating between target and reference stars. In narrow-angle astrometry, one is interested in the motion of the target star with respect to the reference stars, rather than in the absolute motion of the target star. Thus, a basic narrow-angle measurement is always a difference between delay measurements of two stars. Differences between successive target and reference star measurements are, to first order, free of common-mode errors and linear temporal drift on the timescale of the chop. With an optimally selected integration time, chopping serves to mitigate systematic time-dependent drifts (primarily due to changes in thermal environment). A “target-reference chop” is a delay measurement on the target star, followed by a delay measurement on a reference star. A “chop cycle” is a complete set of target-reference chops. For five reference stars, the observing sequence for a chop cycle is $T \rightarrow R_1 \rightarrow T \rightarrow R_2 \rightarrow T \rightarrow R_3 \rightarrow T \rightarrow R_4 \rightarrow T \rightarrow R_5 \rightarrow$, where $T$ and $R_i$ refer to delay measurements of the target star and $i$th reference star, respectively, and arrows refer to a slew-settle-acquire sequence from either the target to reference star or reference to target star. The last arrow is a slew-settle-acquire shift back to the target star.

The astrometric precision obtained in this one-dimensional narrow-angle observing sequence is currently specified at $1.0 \mu$as for a $V = 7$ star with a group of $V = 10$ reference stars (Yu 2005). This is hereafter referred to as SIM’s “single-measurement accuracy.” This performance has been demonstrated in the microarcsecond metrology test bed at the Jet Propulsion Laboratory.
Laboratory and has been accepted by the SIM External Independent Review Team. At 1.0 \(\mu\)as single-measurement accuracy, a differential measurement with 780 s total integration time divided among a 7 mag target set and set of 10 mag reference stars has an astrometric accuracy of \(1.0 \times 10^{-3} \mu\)as. This includes photon noise, instrument noise, and a multiplier that accounts for the geometric distribution of the reference stars (Yu 2005).

A two-dimensional narrow-angle observation is a pair of visits (as described above) with the interferometer baseline oriented along quasi-orthogonal directions on the plane of the sky. For the first and subsequent odd-numbered visits to the target-reference group, the interferometer baseline is oriented parallel to a reference direction on a plane tangent to the spacecraft boresight direction. For the second and subsequent even-numbered visits to the target-reference group, the interferometer baseline is oriented along a direction in the tangent plane that is roughly orthogonal to the baseline orientation of the first observation. In this way, the two-dimensional motion of the target star on the plane of the sky is sampled. We assume even time sampling for the series of observations along each of the two baseline orientations, and that observation pairs are quasi-simultaneous, although the latter assumption is not strictly necessary.

In reality, scheduling constraints in the mission (including a solar exclusion zone) preclude even sampling. Sampling of SIM planet-search targets will be serendipitous, governed by their availability during repeated “orange peel” scans of the sky, spiraling toward and away from the solar exclusion zone (Boden et al. 1997). Yearly gaps in the sampling, ranging from several weeks to several months (for targets near the ecliptic), will occur during times when the target is in the solar exclusion zone.

Ford (2005) and Sozzetti et al. (2002) investigated planetary orbit detection with a number of sampling schemes. These include geometric, power-law, and periodic with random Gaussian perturbations, all with 24 two-dimensional observations. They found that all of these sampling schemes performed well as long as the minimum gap between observations did not deviate too much from the average sampling interval. The most promising observing schedules were “periodic with perturbations” of up to 40% of the average sampling interval (Ford 2005). Apart from annual sampling gaps due to the solar exclusion zone, we expect actual sampling to be quasi-even, comparable to the “periodic with perturbation” schemes investigated by Ford (2005). We have not investigated the impact of these solar exclusion gaps on planet detectability. However, previous studies (Ford 2005; Sozzetti et al. 2002) have shown that gaps much longer than the average sampling interval can attenuate detection at periods comparable to the survey length.

3. TERRESTRIAL PLANETS AND THE HABITABLE ZONE

Terrestrial planets are defined as those composed primarily of silicate rock. In the solar system, there are four (Mercury, Venus, Earth, and Mars), Earth being the most massive. Recent simulations of core accretion (Ida & Lin 2004) indicate that rocky planets form inwards of 3 AU from the parent star, and their masses can extend up to about 10 or 20 \(M_{\oplus}\). The upper mass limit results from the competition between core accretion and disk gas depletion. For the purpose of this study, we adopt a terrestrial planetary mass range of 1 to 10 \(M_{\oplus}\).

The habitable zone is the region around a star in which liquid water, considered essential for life, can exist. Although life on Earth exists in environments that are much more extreme than this definition allows, feasible future missions, such as the Terrestrial Planet Finder, will be limited to studying the macroscopic physical and chemical properties of planets. From arguments based on Stefan’s law, the Sun’s habitable zone is between 0.7 and 1.5 AU (Kasting et al. 1993). For the purpose of this study, we put the center of the Sun’s habitable zone at 1 AU. To remain inside the habitable zone, a planet with a semimajor axis of 1 AU should have eccentricity less than 0.35.

We define the occurrence rate \(\eta_{\text{terrestrial}}\) as the fraction of solar-type stars with terrestrial planets orbiting in their habitable zones. The mass distribution and occurrence rate of extrasolar terrestrial planets orbiting solar-type stars are at present unknown; only one candidate, at \(\sim 7.5 \, M_{\oplus}\) (Rivera et al. 2005), has been discovered to date. NASA’s Kepler mission,1 scheduled to launch in 2009, will survey 100,000 solar-type stars (F, G, and K dwarfs) over 4 yr for transits of planets with masses between 0.5 and 10 \(M_{\oplus}\). By the time of the SIM launch, data from NASA’s Kepler mission may have yielded much information about the statistics of terrestrial planets orbiting inward of ~1 AU. In any case, SIM will itself provide sufficient statistics to estimate the mass function and occurrence rate of terrestrial planets in habitable zones.

At the present time, the best approach is to extrapolate from the discoveries of radial velocity surveys. Masses of known extrasolar planets are roughly consistent with a power-law distribution \(dN/dM \propto M^{-1.1}\) (Tabachnik & Tremaine 2002; Marcy et al. 2005c), but they are generally tens to hundreds of times more massive than terrestrial planets. Nevertheless, the consistency of this power law with masses of solar system planets is evidence (albeit weak) that it may also apply to terrestrial planets (Tremaine & Zakamska 2003). Integrating the power laws for mass and period distributions (Tabachnik & Tremaine 2002) between 1 and 10 \(M_{\oplus}\) and periods corresponding to orbital radii between 0.7 and 1.5 AU, we obtain an estimate of \(\eta_{\text{terrestrial}} = 0.013\), or 1.3%, for the occurrence rate of terrestrial planets in the habitable zones of solar-type stars. In the remainder of this work, we adopt the Tabachnik-Tremaine power law for terrestrial planet masses, assuming that each star has one terrestrial planet at the midhabitable zone, with mass drawn from the \(1/M^{1.1}\) distribution. Results can then be scaled

1 See http://www.kepler.arc.nasa.gov.
to any value of $\eta_{\text{terrestrial}}$. In this work, we do not address the case of multiple-planet systems.

The habitable zone of a star scales with luminosity as

$$R_H = L_{\ast}^{0.5}, \quad (1)$$

where $R_H$ is the radius at the midhabitable zone in AU and $L_{\ast}$ is the stellar bolometric luminosity in units of solar bolometric luminosity $L_\odot$. The astrometric signature of a planet in the habitable zone is

$$\alpha'' = \frac{M_p R_H}{M_{\ast} D}, \quad (2)$$

where $\alpha''$ is the angular size of the semimajor axis of the stellar reflex motion (if the orbit were seen face-on) in arcseconds, $M_p$ and $M_{\ast}$ are the planetary and stellar masses, respectively, in solar units, and $D$ is the distance to the star in pc. Using equation (1) for $R_H$ in equation (2) gives

$$\alpha'' = \frac{M_p L_{\ast}^{0.5}}{M_{\ast} D}, \quad (3)$$

Most planet-search targets are main-sequence stars. A convenient form of the mass-luminosity relation for main-sequence stars with $M_{\ast} > 0.2$ is

$$L_{\ast} = M_{\ast}^{3.8} \quad (4)$$

(Cox 2000, p. 132). Thus, for main-sequence stars with $M_{\ast} > 0.2$, the radius of the habitable zone scales with stellar mass as

$$R_H = M_{\ast}^{1.9}. \quad (5)$$

More massive stars have larger habitable zones. One consequence for TPF-Coronograph (TPF-C) but not for SIM is that for a fixed planet size, a larger habitable zone lowers the contrast ratio between the planet’s reflected starlight and the star itself. Above a luminosity of $5.4 L_\odot$, corresponding to a habitable zone of radius 2.3 AU and a stellar mass of 1.6 $M_\odot$, the contrast ratio of a 10 $M_\oplus$ terrestrial planet falls below TPF-C’s contrast limit of $10^{-10}$. From equations (3) and (4), the astrometric signature of a planet in the habitable zone of a main-sequence star with $M_{\ast} > 0.2$ scales with stellar mass, planet mass, and distance as

$$\alpha'' = \frac{M_p M_{\ast}^{0.9}}{D}. \quad (6)$$

Evidently, at fixed stellar distances, planets of a given mass in the habitable zones of more massive stars have larger stellar reflex motion signatures. These are the best targets for the SIM mission; but some of them will be unsuitable for TPF-C, because of low contrast ratios. Targets for the TPF-C mission will be preferentially selected for large habitable zone angular size, subject to the contrast ratio constraint. In the next section (§ 4), we discuss hypothetical SIM target lists and their characteristics. In our simulations, we replaced equation (4) with a slightly more accurate mass-luminosity relation (Griffiths et al. 1988), given by

$$\log L_{\ast} = 4.20 \sin (\log M_{\ast} - 0.281) + 1.174 \quad (7)$$

for $-1 < \log M_{\ast} < 1.25$.

4. TARGET LISTS FOR SIM PLANET SURVEYS

Approximately 17% of SIM’s 5 yr mission time is designated for planet searching in narrow-angle mode. Within this allocation, we consider three hypothetical survey modes, each of which uses all of SIM’s planet-finding time to observe targets brighter than 7th magnitude:

1. Medium-deep survey.—240 target stars with 52 two-dimensional observations per target.

2. Deep survey.—120 target stars with 104 two-dimensional observations per target.

3. Ultradeep survey.—60 target stars with 208 two-dimensional observations per target.

Measurement errors in these surveys are scaled by target-star magnitude according to SIM’s nominal single-measurement accuracy of 1.0 $\mu$as for a $V = 7$ star and $V = 10$ reference stars. For this study, we conservatively assume $V = 10$ reference stars. Although actual reference stars have not yet been selected, our preliminary studies indicate that their magnitudes will likely be in the range $V = 9$ to 9.5. For each survey mode (except medium-deep, for which there are not enough TPF-C targets), we draw the stars from one of two target lists. The first is optimized for SIM, while the second is optimized for TPF-C.

Since virtually all likely SIM targets are known stars, it is unnecessary to use synthetic stellar populations. Our hypothetical SIM-optimized target list is derived from an initial list of 2350 stars taken from the Hipparcos catalog, with distances of less than 30 pc (Turnbull & Tarter 2003). We excluded stars with luminosity greater than 25 times solar, thereby eliminating giants from our sample. To eliminate the possibility of fringe contamination from a binary companion, we applied the following: stars with a companion closer than 0'4 were excluded; for stars with a companion that was separated by 0'4 to 1.5, both components were included as target-star candidates if the magnitude difference was greater than 1; otherwise, both components were excluded. If the target-star candidate had a wide binary companion that was separated by more than 1.5, the companion was added to the list of target-star candidates. Sur-
viving candidates were ranked ordered by effective mass sensitivity at the midhabitable zone, as described in §§ 6.2–6.4.

For the hypothetical TPF-C targets, we used a list of 384 stars (C. Burrows 2005, private communication). This list was derived from the Hipparcos database (Turnbull & Tarter 2003). It comprises all single F, G, or K main-sequence stars brighter than 7th magnitude, closer than 30 pc, and with \( B - V \) colors in the range 0.3 to 1.4. Two further constraints are imposed: the midhabitable zone must be outside of TPF-C’s inner working angle of 62 mas, and the luminosity of the star must be less than \( 5.4 L_\odot \), so that the contrast ratio exceeds \( 10^{-10} \) for a terrestrial planet of \( 10 M_\oplus \). Of the 384 stars on the list, about 10 survived application of these constraints, so for the TPF-optimized list, there is no medium-deep survey. These stars were then ranked in descending order of star-planet angular separation for a planet at the midhabitable zone. For the ultra-deep and deep TPF-C surveys, we chose the best 60 and 120 stars, respectively, from this final list. We note that these are not necessarily the best targets for the TPF-Interferometer mission (TPF-I). TPF-I has a smaller inner working angle and could potentially detect planets in the much smaller habitable zones of M stars.

Histograms of \( V \) magnitude and orbital period at the midhabitable zone for the best 120 TPF-C and 240 SIM targets are shown in Figures 2 and 3. Note the wider range of midhabitable zone periods in the SIM target list, which has been selected for large habitable zones of M stars.

Reference star candidates are K giants selected from the Tycho-2 and Two Micron All Sky Survey (2MASS) catalogs (A. Tanner et al. 2006, in preparation). The following criteria were used:

1. They should be K giant stars, which are luminous and therefore distant, in order to minimize gravitational perturbations due to planetary companions. Reduced proper motion, defined as \( \text{RPM} = K_\mu + 5 \log \mu \), serves as a proxy for distance. We require \( \text{RPM} < 1 \).
2. They should have an IR color range \( 0.5 < J - K_s < 1.0 \).
3. They should have an optical color range \( 1.0 < B - V < 1.5 \).
4. They should lie within a 1\(^{\circ}\)25 radius of the target star.
5. They should be bright \( (V < 10) \), to minimize observing time.
6. They should have favorable geometry (we choose four candidate reference stars in each quadrant around the target if possible).

Our planet-search target and reference stars have not been screened for photometric variability, which may be an indicator of starspot activity. Sozzetti (2005) and Hatzes (2002) have modeled photocenter shifts due to starspots. Their results show that for typical planet-search targets, large spots causing a flux change of 1% can induce photocenter shifts of the same order as astrometric reflex motion due to terrestrial planets in the habitable zone. Sozzetti (2005) notes that since the effect correlates strongly with photometric variability, photometric screening and/or monitoring of planet-search targets should be considered. An average sunspot group contains about 10 sunspots, each with an area of about 0.04% of the Sun’s disk (Cox 2000). The presence of simultaneous multiple starspots tends to randomize the photocenter shift. For a solar-type target star at 10 pc, we find, in agreement with Sozzetti (2005), that 0.1% flux variations due to a single starspot introduce a photocenter shift with an amplitude of \( \sim 0.3 \mu \text{as} \). While larger starspots may be possible, we believe that since the photometric shift due to starspots is color dependent, whereas reflex motion is not, spectral information provided by SIM observations will serve to break the degeneracy.
We also consider the problem of photocenter shifts in reference stars. Henry et al. (2000) performed a survey of 187 F, G, and K giants for photometric variability. They found that photometric variability exhibits a strong color dependence; for giants meeting our selection criterion of $1.0 < B - V < 1.5$, most exhibit maximum flux variations of well under 1%. They concluded that for giants cooler than K2 III, the observed photometric variation is most likely due to radial stellar pulsations (which cause no photocenter shift) rather than starspots. If, however, we assume that the observed photometric variation is due entirely to starspots, a simple starspot model shows that the photocenter shift of K giants at 1 kpc is expected to be well under 0.4 μas. Our conclusion is that starspots are not a major concern for reference stars.

Reference stars for the planet-search targets will ultimately be vetted by a radial velocity observing campaign with a precision of 20 m s$^{-1}$ over a 6 yr time baseline, before SIM PlanetQuest launches. Stars with measurable radial velocity jitter due to stellar, substellar, or planetary companions will be eliminated.

5. THE JOINT PERIODOGRAM: A TOOL FOR ASTROMETRIC DETECTION OF PLANETS

The standard method of detecting periodicity in one-dimensional time-series data is the Lomb-Scargle periodogram (Scargle 1982). This periodogram and its variants are widely used in the detection of planets in radial velocity data (Cumming 2004; Cumming et al. 1999; Nelson & Angel 1998; Walker et al. 1995). A discussion of the application of the periodogram to the detection of planets in astrometric data is found in Black & Scargle (1982), and Sozzetti et al. (2003) have investigated using the periodogram to detect multiple planets in astrometric data.

The Lomb-Scargle periodogram is readily extended to the detection of periodicity in two-dimensional time-series astrometric data. We define the joint periodogram as the sum of the Lomb-Scargle periodogram power of the astrometric signal in the two independent channels associated with the orthogonal baseline orientations of the SIM interferometer. There is no requirement on the maximum time interval between a measurement in one channel and the corresponding measurement in the other. The detection of a planet is registered when its joint power exceeds a detection threshold that is set according to the desired false-alarm level.

Measurements in each channel are assumed to be uniformly sampled in time. A comparison of many sampling schemes shows that apart from the aliasing problem, even sampling is best for the detection of orbits with periods that are shorter than the length of the survey (Ford 2005). In our use of the periodogram, we have sidestepped aliasing effects by counting as a detection any signal that exceeds the detection threshold, regardless of whether the detected and actual periods match. Thus, our analysis should yield detection efficiencies comparable to those found using other sampling schemes that are optimized to reduce aliasing (Ford 2005; Sozzetti et al. 2002). Our use of even sampling in the simulations is mainly for convenience in the analysis.

The joint periodogram uses all the information in both channels of the astrometric data for planet detection and period estimation. It is ideally suited for the detection of well-sampled circular orbits with periods that are shorter than the time baseline of the observations. For elliptical orbits, detection efficiency is reduced for two reasons: power leaks into the overtones of the orbit frequency, and for certain orbital geometries, the full astrometric signature of the stellar reflex motion is not observed. However, we find that these effects are not significant for the relatively small eccentricities ($e < 0.35$) we consider.
Our simulations show that the joint periodogram detects Keplerian orbits with higher efficiency than using separate periodograms on the two channels.

A necessary preliminary step in any detection scheme is to establish the false-alarm probability (FAP) corresponding to a range of detection thresholds. The FAP corresponding to a given detection threshold (periodogram power) is the likelihood that pure Gaussian noise would produce a periodogram peak whose power exceeds the detection threshold. Lowering the detection threshold raises the FAP. In our simulations, we determined detection thresholds corresponding to various FAPs, using 100,000 Monte Carlo realizations of the no-planet case, in which the simulated observations consist purely of Gaussian measurement noise. For the purpose of discovering planets in radial velocity data, a 1% FAP is commonly required (Marcy et al. 2005b). The detection of a signal exceeding the threshold corresponding to a 1% FAP is said to be at a 99% significance level.

For sufficiently small values, the FAP is proportional to the number of independent frequencies scanned in the periodogram (Horne & Baliunas 1986; Press et al. 1992). For \( N \) evenly spaced observations in the time series, the range of detectable frequencies is spanned by the \( N/2 \) independent frequencies \( \{1/T, 2/T, \ldots, N/2T\} \), where \( T \) is the time between the first and last measurement (nominally, 5 yr for the SIM mission), and \( N/2T \) is the Nyquist frequency. But to adequately sample a peak, the periodogram must be scanned at a finer frequency interval. Our Monte Carlo experiments show, in agreement with Horne & Baliunas (1986) and Press et al. (1992), that the effective number of independent frequencies is \( \sim N \), rather than \( N/2 \), because of this oversampling. Evidently, limiting the range of frequencies over which the periodogram is scanned increases the detection efficiency at a given confidence level. For the results presented in this paper, we chose to sample the periodogram down to periods as low as 0.2 yr (semimajor axis of 0.34 AU for a planet orbiting a solar-mass star), corresponding to the Nyquist frequency for 50 samples evenly spaced in time over a 5 yr mission. Figure 4 is a plot of the FAP versus detection threshold from a Monte Carlo ensemble of 100,000 realizations of a series of 50 two-dimensional observations that are evenly spaced over 5 yr of Gaussian noise, with a single-measurement error of 1 \( \mu \)as. The 1% FAP threshold corresponds to an astrometric signal of 1.36 \( \mu \)as.

6. DETECTION OF TERRESTRIAL PLANETS IN THE HABITABLE ZONES OF NEARBY STARS WITH SIM

In this section, we determine SIM’s sensitivity for the detection of terrestrial planets in the habitable zone for hypothetical SIM and TPF-C target lists for each target list and survey strategy.

We generated Keplerian reflex motion orbits for a Monte Carlo sample of solar-mass stars at 10 pc, each with a single terrestrial planet (i.e., a planet with mass in the range of 1 to 10 \( M_{\oplus} \)). In § 6.1, we present results for the detection efficiency as a function of planet mass. In § 6.2, we extend this result, deriving a universal detection efficiency curve for a star of arbitrary mass, luminosity, and distance, and observed with an arbitrary number of two-dimensional measurements and differential measurement error. We define the key concept of effective mass sensitivity, which is essentially a one-parameter characterization of the detection efficiency curve. From the universal detection efficiency curve, we develop a semianalytical formula for SIM’s effective mass sensitivity for the detection of terrestrial planets in the habitable zone. In § 6.3, we discuss how to correct for the effects of parallax and proper motion. In § 6.4, we present detailed results for effective mass sensitivity. For each survey strategy, we determine the effective mass sensitivity for each target star.

These results provide the best metric of the capability of the SIM instrument for detecting terrestrial-mass planets orbiting in the habitable zones of nearby stars. This metric depends only on the performance of the SIM instrument as captured in its astrometric error budget, and on the known characteristics of the target stars, and is independent of assumptions concerning the occurrence rate and mass distribution of terrestrial planets. Finally, in § 6.5 we compare our results with those of previous studies.

6.1. Monte Carlo Sample

The starting point is a Monte Carlo simulation of the astrometric detection of planets in the habitable zone of a solar-mass star at a distance of 10 pc in order to determine the detection efficiency as a function of planet mass. Detection efficiency at a given planet mass is defined as the likelihood that a planet of that mass will be detected in the presence of
measurement noise. For each planet mass in the range of 0 to 10 $M_{\oplus}$, at intervals of 0.5 $M_{\oplus}$, we generate 10,000 realizations of Keplerian orbits with a 1 yr period around a solar-mass star at 10 pc. Eccentricity is uniformly distributed between zero and a maximum of 0.35, consistent with orbits lying entirely within the habitable zone. We randomly draw other orbital parameters (inclination, longitude of ascending node, longitude and time of periastron) from their allowed domains. For each realization of an orbit, we generate a time series of 50 two-dimensional astrometric observations of the star’s position, evenly spaced in time over a 5 yr time period. Each observation is perturbed with a Gaussian measurement error of 1 $\mu$as. In these initial simulations, we did not account for the effects of parallax and proper motion.

We employ the joint periodogram to detect periodic stellar reflex motion indicating the presence of a planet. Detection efficiency at a given mass and FAP is defined as the fraction of the ensemble of Keplerian orbit realizations for which the joint periodogram power exceeds the detection threshold associated with this FAP.

At moderate signal, a detection threshold corresponding to the peak of the periodogram power distribution corresponds roughly to 50% detection efficiency—noise is equally likely to add to the signal, raising it above the threshold, or subtract from the signal, pushing it below the threshold (since the distribution of periodogram power is nearly symmetric). Increasing or decreasing the detection threshold by a small increment results in roughly equal and opposite changes in the number of detected planets (see Fig. 5).

Figure 6 shows SIM’s detection efficiency (for detection thresholds corresponding to FAPs of 1% and 5%) as a function of planet mass for the fiducial case of terrestrial planets in 1 yr orbits around a solar-mass star at a distance of 10 pc. Note that as required, in the limit as the signal approaches zero, the detection efficiency approaches the FAP.

6.2. SIM Planet Detection Sensitivity

If we assume that detection efficiency is independent of orbital period, then it should depend only on the signal-to-noise ratio (S/N). This is expected to hold when there are sufficiently many observations, the orbital period is shorter than the observation time baseline, and the data do not include the effects of parallax and proper motion. We define the S/N of the astrometric data as

$$S/N = \frac{\alpha \sqrt{\nu_{\text{obs}}}}{\sqrt{2} \sigma},$$

where $\alpha$ is the angular size of the semimajor axis of the orbit in $\mu$as if it were viewed in a face-on orientation, $\nu_{\text{obs}}$ is the number of two-dimensional observations of the star, and $\sigma$ is the single-measurement accuracy in $\mu$as. A factor of $\sqrt{2}$ occurs in the denominator because narrow-angle measurements are differential (§ 2). The SIM technology test beds have demonstrated that the S/N improves with $\nu_{\text{obs}}$ according to equation (8), up to $\nu_{\text{obs}} \geq 100$. Substituting for $\alpha$ by using $\alpha'$ from equation (3) (remembering to convert from arcseconds to microarcseconds), converting planet mass from solar to Earth mass units, and using the mass-luminosity relation of equa-
S/N are indeed identical.

In the foregoing arguments, we implicitly assume that detection probability is attenuated for orbital periods approaching and exceeding the 5 yr observation window (due to confusion of the orbital trajectory with parallax). To characterize the effect, we generated sets of Monte Carlo simulations for the same range of planet masses as reflected in the range 0 < e < 0.35, observed 50 times along each of two orthogonal baseline directions, with a single-measurement error of 1 μas. We have confirmed via additional Monte Carlo simulations for cases of 24, 100, and 200 two-dimensional observations over 5 yr, that the curves of detection efficiency versus S/N are indeed identical.

We find that the joint periodogram is more sensitive than the χ² test of the null hypothesis for the detection of planets in Keplerian orbits (see Fig. 7). Our simulations indicate that this result is generally valid for well-sampled data when the orbital period is shorter than the survey duration. One reason may be that since Gaussian noise has a flat frequency spectrum, the χ² test is sensitive to noise at all frequencies. By contrast, periodogram power includes only noise at the natural frequencies near the detected peak.

SIM planet detection sensitivity for a target can conveniently be expressed in terms of effective mass sensitivity. For the purpose of discovering planets in radial velocity data, a 1% FAP is commonly required (Marcy et al. 2005c). For each target star, we define effective mass sensitivity to be the mass of a planet that is orbiting at the mid-habitable zone and is detectable at 1% FAP with a 50% detection efficiency. For each target star, the effective mass sensitivity serves as a one-parameter characterization of the detection efficiency curve.

As an example of how detection efficiency and FAP are related, consider Figure 7. At a S/N of ~5.4, the detection efficiency is 50% at 1% FAP for planets with eccentricities uniformly distributed in the range 0 < e < 0.35. According to equation (9), a S/N of ~5.4 corresponds to an effective mass sensitivity of 3.6 M⊕ for 50 two-dimensional observations of a solar-mass star at 10 pc with single-measurement precision of 1 μas. Figure 6 shows this graphically: 50% detection efficiency occurs for a planet mass of 3.6 M⊕. More generally, equation (9) evaluated at S/N = 5.4 relates the effective mass sensitivity to the number of observations, single-measurement precision, distance, and luminosity of the star. A comparison with the case of 50 observations at 1 μas single-measurement precision for a solar-mass star at 10 pc yields a useful semi-analytical scaling law for the effective mass sensitivity M_eff of a terrestrial planet in the habitable zone of a main-sequence star:

\[ M_{\text{eff}} = 3.6 \left( \frac{\sigma}{1 \, \mu\text{as}} \right) \sqrt{\frac{50}{N_{\text{obs}}}} \frac{D}{10 \, \text{pc}} \left( \frac{1}{L_{\star}^{0.24}} \right) M_{\odot}, \]  

where \( N_{\text{obs}} \) is the number of two-dimensional observations, \( \sigma \) is the single-measurement accuracy in microarcseconds, \( D \) is the distance to the star in parsecs, and \( L_{\star} \) is the bolometric stellar luminosity in solar units.

### 6.3. Correction for Effects of Parallax and Proper Motion

In the foregoing arguments, we implicitly assume that detection efficiency is independent of orbital period. But when the effects of parallax and proper motion are accounted for, detection probability is attenuated for orbital periods approaching 1 yr (due to confusion of the orbital trajectory with parallax) and also for orbital periods approaching and exceeding the 5 yr observation window (due to confusion of the orbital trajectory with proper motion). For discussions and studies of these effects, see Black & Scargle (1982), Lattanzi et al. (2000), Sozzetti et al. (2002), and Ford (2005). We need to revise our approach in order to account for sensitivity to orbital period. To characterize the effect, we generated sets of Monte Carlo ensembles for the same range of planet masses as reflected in Figure 6. Instead of putting every star at 10 pc and every planet at an orbital period of 1 yr as before, we generated ensembles of planets, with each ensemble having a fixed orbital period chosen from a range of values between 0.2 and 7.5 yr; for each period, we adjusted the distance to keep the astrometric signal (the angular size of the orbit if it were viewed in a face-on
Fig. 8.—Sensitivity of periodogram detection to period for several values of S/N. Each data point represents an ensemble of 10,000 Monte Carlo orbits. The loss in sensitivity near orbital periods of 1 yr is due to confusion of stellar reflex motion with parallax; the attenuation at longer orbital periods is due to confusion of stellar reflex motion with proper motion. The detection threshold corresponds to a 1% FAP.

The marked reduction in detection efficiency for periods in the range of 0.8 to 1.2 yr is due to confusion of stellar reflex motion with parallax. Confusion of reflex motion with proper motion causes falloff in detection efficiency with increasing period. To fully account for sensitivity to period, we used the results of our Monte Carlo simulations to construct a lookup table for detection efficiency as a function of S/N and orbital period. For any target star, given the number of observations, the measurement noise, and the orbital period at its midhabitable zone, a detection efficiency curve (similar to Fig. 6 but corrected for the effects of parallax and proper motion) is obtained by interpolation in this table.

Figure 9 shows a histogram of the corrections to effective mass sensitivity for the medium-deep survey. Note that although for some stars the correction can be quite large, the corrections for most stars are $<0.25 M_{\odot}$.

From the detection efficiency curve for any target star (now corrected for the effects of parallax and proper motion), we can determine the effective mass sensitivity of a terrestrial planet at the midhabitable zone of that star. For each survey, we obtained the effective mass sensitivity for all target-star candidates using the corrected detection efficiency curves. Then we ranked the stars by effective mass sensitivity and chose the best 240, 120, or 60 stars for the medium-deep, deep, and ultradeep planet surveys, respectively. In all plots and tables that follow, effective mass sensitivity has been corrected for the effects of parallax and proper motion.

6.4. Results for Effective Mass Sensitivity

By the time SIM launches in 2016, the ubiquity of terrestrial planets orbiting solar-type stars may already be known from...
Fig. 11.—Same as Fig. 10, but for a deep planet survey including best 120 stars for SIM (left) and TPF (right). For both plots, there are 104 two-dimensional measurements per star.

discoveries of the Kepler mission. For a fixed amount of SIM mission time, the optimum number of stars to survey for planets depends on the abundance of Earth-mass planets in the local Galactic environment. For example, if terrestrial planets turn out to be relatively rare, a reasonable strategy for SIM is to survey a larger number of stars with correspondingly fewer observations per star. To explore this trade-off, we consider the medium-deep, deep, and ultradeep survey modes described in § 4, which use the same amount of SIM mission time to observe different numbers of targets.

For the hypothetical SIM target list, Figure 10 and the left panels of Figures 11 and 12 show histograms of the effective mass sensitivity for the medium-deep, deep, and ultradeep surveys of the best SIM targets. The right panels of Figures 11 and 12 show distributions of effective mass sensitivity for the deep and ultradeep surveys for the best TPF-C targets. There is no medium-deep survey for the TPF-C target list, since there are only ~120 stars meeting the requirements that the habitable zone lie outside the inner working angle of 62 mas and close enough to the star so that the contrast exceeds $10^{-10}$.

The main results of this study are presented in Figures 13 and 14. These figures characterize SIM’s planet detection capability, comparing effective mass sensitivity for surveys of the best TPF-C and SIM targets, respectively. These results,
together with results for a 10 yr survey of the best 120 SIM targets with 208 two-dimensional observations each, are summarized in Tables 1 and 2. Figures 15 (left) and 16 show effective mass sensitivity versus stellar distance. Figures 15 (right) and 17 show effective mass sensitivity versus star-planet separation at the midhabitable zone. We find that for a 5 yr mission, SIM can probe the best 60, 120, and 240 planet-search targets down to median effective mass sensitivities of 1.4, 2.5, and 4.5 $M_{\text{Jup}}$, respectively. SIM’s effective mass sensitivities for the stars in these surveys range between 0.2 and 1.4 $M_{\text{Jup}}$, 0.3 and 2.5 $M_{\text{Jup}}$, and 0.4 and 5.8 $M_{\text{Jup}}$, respectively. For a 10 yr mission, SIM can probe the best 120 planet-search targets with effective mass sensitivities ranging from 0.2 to 2.2 $M_{\text{Jup}}$, and a median of 1.8 $M_{\text{Jup}}$. Recent simulations of the giant planet formation process (Benz et al. 2006) produce large populations of low-mass planets whose growth was halted before they could become giant planets. These planets orbit beyond 1 AU of the parent star. The results indicate that for every currently known exoplanet, there should be 20 to 30 times as many of these “failed” giant planet cores with masses smaller than $\sim 5 M_{\text{Jup}}$. Accordingly, every solar-type star should have one or two of these low-mass planets. SIM’s mass sensitivity will enable it to probe a significant portion of this expected population of low-mass planets, making possible important checks on current models of planet formation. It is important to note that these results depend only on assumptions about the SIM instrument and the known characteristics of planet-search target stars. They do not depend on the astrophysics of planetary mass distribution and frequency of occurrence.

6.5. Comparison with Earlier Studies

Several previous studies (Sozzetti et al. 2002, 2003; Ford & Tremaine 2003) have addressed SIM’s detection and orbital characterization capabilities. These studies employed the $\chi^2$ test rather than the periodogram for planet detection. A detection is registered when the $\chi^2$ test warrants a rejection of the null (no-planet) hypothesis. The studies of Sozzetti et al. adopt a significance level of 95% when determining whether a planet has been detected. They define a scaled signal $S$ as the ratio of the astrometric amplitude $\alpha$ and the single-measurement astrometric accuracy, $a$. They determine that $S$ must be greater than 2.2 to detect a planet with 95% probability, and they assume that SIM will make only 24 two-dimensional observations per star surveyed. Other points of difference between their work and our own are that (1) they assume that planetary orbits span the full range of eccentricities, while we focus only on planets in the habitable zone, and (2) they assume a single-measurement accuracy of $2\,\mu$as (and higher for faint stars).

Ford & Tremaine (2003) generally accept the Sozzetti et al. (2002) conclusions regarding SIM’s detection efficiency. They further investigate the issue via similar Monte Carlo simulations for planets around stars of spectral types F, G, K, and M within 100 pc and up to a $V$ magnitude of 10.5. They consider a range of single-measurement accuracies of 1, 1.4, and 2 $\mu$as associated with samples of 120, 240, and 480 stars. They also consider a two-tier observing strategy, with the first tier of stars measured to $1\,\mu$as accuracy and the second tier measured to $4\,\mu$as accuracy, and they consider both 5 and 10 yr missions. As in this work, they adopt the power-law planetary mass distribution (Tabachnik & Tremaine 2002) that is consistent with currently known planets discovered by the radial velocity method.

We believe that the results of the present study represent a
better estimate of SIM’s likely science return in this field, for two reasons. First, we have realistically modeled the likely SIM performance and observing scenario, taking total observation time into consideration. Second, the joint periodogram represents a more effective method of extracting planetary signals from astrometric data than the χ² test of the null hypothesis.

### 7. DISCOVERY OF TERRESTRIAL PLANETS BY SIM

In this section, we address two further questions: What is the mass distribution and number of planets SIM will discover? And what is SIM’s completeness for the detection of terrestrial planets in the habitable zone; i.e., what fraction of the terrestrial planet discovery space does SIM probe?

To answer these questions, additional (astrophysical) assumptions are needed. We first extrapolate the mass distribution of currently known planets (Tabachnik & Tremaine 2002) to terrestrial planet masses. Next, for each target list and survey strategy, we use the universal detection efficiency curve to determine the detection efficiency versus planet mass for each target star. These two relations allow us to determine the expected mass distribution of terrestrial planets SIM will detect. In each mass bin, the completeness is defined as the ratio of the number of planets detected to those that are expected. With an assumed value of \( \eta_{\text{terrestrial}} \) (defined as the fraction of F, G, and K stars having terrestrial planets in their habitable zones), we can also estimate the expected number of terrestrial planets SIM will discover in each mass bin. Detailed results in graphic and tabular form are presented in the next two subsections.

#### 7.1. What Is the Mass Distribution and Number of Planets SIM Will Discover?

Our starting point is the universal detection efficiency versus S/N curve for each star in the hypothetical target lists, as discussed in the previous section. From equation (3), \( \alpha \) depends on distance, stellar mass, planet mass, and luminosity (through the size of the habitable zone). Given these parameters, together with values for \( N_{\text{det}} \) and \( \sigma \), we can derive a relation between detection efficiency and planet mass for each star in the target list. To proceed further, we evaluate the hypothetical case in which each star has one terrestrial planet at its midhabitable zone, with a mass ranging from 1 to 10 \( M_{\oplus} \) that is distributed as \( dN/dM \propto M^{-1.1} \) (Tabachnik & Tremaine 2002; Marcy et al. 2005a).

The expected probability distribution function for a detected planet at a given mass is the product of detection efficiency and \( dN/dM \) at that mass. This probability distribution function has a peak, since it is the product of the monotonically decreasing planetary mass distribution and the monotonically increasing, \( S \)-shaped detection efficiency versus mass curve.

Summing the expected probability distribution functions for all the stars in the target list gives the distribution of the number of terrestrial planets discovered per unit-mass interval. Integrating over unit mass bins from 1 to 10 \( M_{\oplus} \) gives a histogram of the number of planets discovered in each mass bin. Results for the three survey modes for the SIM target list are shown in the left panels of Figures 18, 19, and 20; corresponding results for surveys involving the TPF-C target list are shown in the left panels of Figures 21 and 22. The

#### Table 1: Effective Mass Sensitivity for SIM Detection of Terrestrial Planets

| Survey          | Duration (yr) | Median \( M_{\text{eff}} \) (\( M_{\oplus} \)) | Minimum \( M_{\text{eff}} \) (\( M_{\oplus} \)) | Maximum \( M_{\text{eff}} \) (\( M_{\oplus} \)) |
|-----------------|---------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Best 60 for SIM | 5             | 1.4                                          | 0.2                                          | 1.8                                          |
| Best 120 for SIM| 5             | 2.5                                          | 0.3                                          | 3.2                                          |
| Best 240 for SIM| 10            | 1.8                                          | 0.2                                          | 2.2                                          |
| Best 60 for TPF | 5             | 4.5                                          | 0.4                                          | 5.8                                          |
| Best 120 for TPF| 5             | 2.0                                          | 0.2                                          | 2.8                                          |
| Best 240 for TPF| 5             | 3.9                                          | 0.3                                          | 5.2                                          |

*Note.*—For a 50% detection efficiency and a 1% FAP.

#### Table 2: Number of Stars vs. Effective Mass Sensitivity \( M_{\text{eff}} \)

| Survey          | Duration (yr) | \( M_{\text{eff}} \leq 1 \) | \( M_{\text{eff}} \leq 2 \) | \( M_{\text{eff}} \leq 3 \) | \( M_{\text{eff}} \leq 4 \) | \( M_{\text{eff}} \leq 5 \) | \( M_{\text{eff}} \leq 6 \) |
|-----------------|---------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Best 60 for SIM | 5             | 9                           | 60                          | 60                          | 60                          | 60                          | 60                          |
| Best 120 for SIM| 5             | 6                           | 30                          | 97                          | 120                         | 120                         | 120                         |
| Best 240 for SIM| 10            | 10                          | 86                          | 120                         | 120                         | 120                         | 120                         |
| Best 60 for TPF | 5             | 4                           | 9                           | 35                          | 80                          | 160                         | 240                         |
| Best 120 for TPF| 5             | 4                           | 32                          | 60                          | 60                          | 60                          | 60                          |
| Best 240 for TPF| 5             | 2                           | 11                          | 34                          | 68                          | 113                         | 120                         |

*Note.*—For a 50% detection efficiency and a 1% FAP.
results are summarized in Table 3. These results are for the case of $\eta_{\text{terrestrial}} = 1$ and are easily scaled to any other value of $\eta_{\text{terrestrial}}$.

Integrating the distribution of the number of terrestrial planets discovered per unit mass interval up to mass $M$ gives the cumulative distribution for the total number of planets discovered up to mass $M$. Figures 23 and 24 show results for surveys of the best TPF-C and SIM targets, respectively. These plots reveal the relative merits of the three survey strategies. For both target lists, the ultradepth survey nets more low-mass terrestrial planets, but fewer total discoveries. For the SIM target list, the medium-deep survey finds the most planets, but with the distribution skewed toward higher masses.

7.2. What Fraction of the Terrestrial Planet Discovery Space Does SIM Probe?

For each mass bin, we determine completeness, which is the ratio of planets detected to those that are expected. We also determine cumulative completeness, the ratio of the number of
planets below mass $M$ that are detected to those that are expected. Completeness depends on our assumption that the probability distribution function (pdf) of terrestrial planet masses is $\propto M^{-1.1}$; however, no assumption about $\eta_{\text{terrestrial}}$ is needed.

The right panels of Figures 18, 19, and 20 show completeness for medium-deep, deep, and ultradeep surveys of the best SIM targets. For these surveys, SIM detects 90% of the planets above 7, 4, and $2.5 M_{\oplus}$, respectively; for the deep survey, SIM finds all of the planets above $5 M_{\oplus}$. The right panels of Figures 21 and 22 show completeness for deep and ultradeep surveys of the best TPF-C targets. For these surveys, SIM finds 90% of the planets more massive than 6 and $3.5 M_{\oplus}$, respectively.

One important conclusion of this study is that SIM will find essentially all Neptune-mass planets residing in the habitable zones of target stars. This may be a significant population of planets; simulations by Ida & Lin (2004) support a maximum terrestrial planet mass of up to $20 M_{\oplus}$ for core accretion models. Recently discovered Neptune-mass planets (Butler et al. 2004; McArthur et al. 2004) may be the first indication of that population, since they both reside inside the orbits of gas giants (A. Boss 2005, private communication).

Cumulative completeness for surveys of the best TPF-C and SIM targets is shown in Figures 25 and 26, respectively. We find that SIM will detect 38%, 62%, and 83% of terrestrial...
8. HOW SIM DISCOVERIES WILL BENEFIT TPF

The Terrestrial Planet Finder mission is being designed with the capability of directly detecting terrestrial planets in the habitable zones of nearby stars. In particular, the coronagraph mission TPF-C will be able to detect a potentially habitable planet around at least 100 nearby stars (TPF and SIM synergy white paper, 2005). While TPF-C can do this in the absence of a priori information, achievement of its scientific priorities will undoubtedly be furthered by knowledge of planetary statistics from Kepler and detections of terrestrial planets by SIM.

SIM’s observations of TPF-C targets provide potentially valuable information for the TPF mission. SIM’s effective mass sensitivity for a target provides a rough lower limit for a detected planet’s mass, while the orbital period provides the star-planet separation. SIM will also yield information about the

---

2 See http://planetquest.jpl.nasa.gov/documents/synergy_finalNew.pdf.
terrestrial planets with SIM PlanetQuest

Fig. 21.—Same as Fig. 18, but for a deep planet survey of the best 120 TPF targets (104 two-dimensional measurements per star).

Fig. 22.—Same as Fig. 18, but for an ultradeep planet survey of the best 60 TPF targets (208 two-dimensional measurements per star).

"confidence in a detection" (the probability that a planet is present in the case of a positive detection) and the "confidence in a nondetection" (the probability that a planet is absent in the case of a nondetection). It is straightforward to apply Bayes’ rule to determine these quantities. Results (averaged over the best 120 TPF-C targets observed in a deep survey) are shown in Figure 27. (See the Appendix for a discussion.)

If SIM detects a planet at a given candidate TPF-C target star, SIM’s effective mass sensitivity, and the star-planet separation, together with the degree of confidence that a terrestrial planet is present, allows the TPF mission to assign a quantitative priority for observing this potential target. On the other hand, if no planet is detected by SIM, then the degree of confidence that no terrestrial planet exists, together with SIM and TPF’s effective mass sensitivities, again can help prioritize this target for TPF. For example, for a given target, if the confidence that no terrestrial planet exists is high, and if TPF’s effective mass sensitivity is higher than SIM’s, the target would be given a very low priority.

For many stars with terrestrial planets, SIM will also characterize the orbits, providing estimates of inclination, eccentricity, and semimajor axis, allowing specification of optimal times and mirror orientations for TPF observations. Because SIM detects planets dynamically, it can unambiguously measure a planet’s mass. SIM’s potential to characterize astrometric orbits and to determine planet masses has been comprehen-
TABLE 3

SIM Terrestrial Planet Discoveries

| Survey          | Duration (yr) | No. Planets $\leq 3$ $M_\oplus$ | No. Planets $\leq 10$ $M_\oplus$ | Mean Mass $\langle M_\oplus \rangle$ |
|-----------------|---------------|----------------------------------|-----------------------------------|--------------------------------------|
| Best 60 for SIM | 5             | 21                               | 50                                | 4.2                                  |
| Best 120 for SIM| 5             | 18                               | 74                                | 5.0                                  |
| Best 120 for SIM| 10            | 34                               | 93                                | 4.4                                  |
| Best 240 for SIM| 5             | 10                               | 91                                | 6.0                                  |
| Best 60 for TPF | 5             | 14                               | 43                                | 4.6                                  |
| Best 120 for TPF| 5             | 8                                | 55                                | 5.6                                  |

Note.—Assuming each target has a terrestrial planet, with masses distributed as $M^{-1.1}$ (Tabachnik & Tremaine 2002).

Fig. 23.—SIM planet discoveries: cumulative distribution of detected planet masses for surveys of the best TPF targets. The single-measurement precision is 1.0 $\mu$as for a $V = 7$ star, and the detection threshold corresponds to a 1% FAP. It is assumed that every target star has one terrestrial planet and that the planet masses are distributed as $M^{-1.1}$.

Fig. 24.—Same as Fig. 23, but for surveys of the best SIM targets.

9. SUMMARY AND CONCLUSIONS

We then introduced the joint periodogram and showed that it is more sensitive than the $\chi^2$ test for the null hypothesis for planet detection. We derived a semiempirical relation for an effective mass sensitivity in terms of stellar distance and luminosity, number of measurements over 5 years, and instrument noise. We showed how this relation can be corrected for the effects of parallax and proper motion. Using actual SIM target lists, we determined SIM’s sensitivity for the detection of terrestrial planets in the habitable zones of nearby stars. For the medium-deep, deep, and ultradeep surveys of the best SIM target stars within 30 pc, we achieved the following:

1. Evaluated the median effective mass sensitivity for a planet at the midhabitable zone. For 5 yr surveys of the best 240, 120, and 60 SIM target stars, we determine median effective mass sensitivities of 4.5, 2.5, and 1.4 $M_\oplus$, respectively. For a 10 yr survey of the best 120 SIM targets, the median effective mass sensitivity is 1.8 $M_\oplus$.

2. Determined (subject to our assumptions about mass distribution, range, and occurrence frequency of terrestrial planets) the expected mass distribution and total number of terrestrial planets that SIM will discover. If each target star has a terrestrial planet orbiting at its midhabitable zone, we find that for surveys of the best 240, 120, and 60 SIM target stars, SIM will discover 91, 74, and 50 terrestrial planets, with mean mass of 6.0, 5.0, and 4.2 $M_\oplus$, respectively; of these, 10, 18, and 21 planets, respectively, will have masses below 3 $M_\oplus$.

3. Determined the completeness of SIM’s terrestrial planet discoveries (i.e., the ratio of detected planets to expected planets) as a function of planet mass. For the three specified surveys of the best SIM targets, we find that SIM detects 38%, 62%, and 83% of terrestrial planets below 10 $M_\oplus$ and 8%, 25%, and 68% of terrestrial planets below 3 $M_\oplus$, respectively.

Finally, we discussed the confidence in SIM detections and nondetections and described how information from SIM’s...
Fig. 25.—Cumulative completeness of *SIM* planet discoveries for surveys of the best *TPF* targets. Cumulative completeness is the ratio of planets with masses <<M> that are detected to those that are expected. The single-measurement precision is 1.0 μas for a *V* = 7 star, and the detection threshold corresponds to a 1% FAP. It is assumed that every target star has one terrestrial planet and that the planet masses are distributed as *M*<sup>-1.1</sup>.

Fig. 26.—Same as Fig. 25, but for surveys of the best *SIM* targets.

Fig. 27.—*Left:* Confidence in the presence of a terrestrial planet, given a detection. *Right:* Confidence in the absence of a terrestrial planet, given a nondetection. Both plots are for deep planet survey of the best 120 *TPF* targets (104 two-dimensional measurements per star at a single-measurement precision of 1.0 μas for a *V* = 7 star, and showing results for detection thresholds corresponding to false-alarm probabilities of 0.5%, 1%, and 5%). Planetary mass distribution is assumed as *M*<sup>-1.1</sup>. The abscissa is the occurrence rate of terrestrial planets in the habitable zone. Results shown are averaged over all the stars in the survey.

We conclude that *SIM PlanetQuest* will be capable of probing the populations of terrestrial and ice giant planets that may reside in the habitable zones of a large sample of stars within 30 pc. *SIM PlanetQuest*’s scientific discoveries will potentially reveal the erstwhile hidden regime of rocky planets, allowing the first thorough checks of predictions of theories of planet formation.

We are grateful to Serge Dubovitsky, Debra Fischer, Chris Gelino, Andy Gould, Geoff Marcy, Chris McCarthy, Bijan Nemati, and Stuart Shaklan for helpful discussions and other contributions to this work. We wish to thank an anonymous referee for a most thorough critical review, with many recommendations that greatly improved the paper. This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA.
APPENDIX

CONFIDENCE IN A DETECTION OR NONDETECTION

Given the occurrence rate $\eta_{\text{terrestrial}}$ for terrestrial planets, the false-alarm probability $F$ at the detection threshold, and the probability $p_{\text{det}}$ that an existing terrestrial planet will be detected, we can use Bayes’ rule to determine the confidence in a detection (i.e., the probability that a detection is not a false positive) for a given target star.

Formally, Bayes’ rule for the confidence in a detection is

$$p(\text{TP exists} | \text{TP detected}) = \frac{p(\text{TP detected} | \text{TP exists})p(\text{TP exists})}{p(\text{TP detected})}, \quad \text{(A1)}$$

where TP stands for terrestrial planet and (in accordance with standard usage) the vertical bar means “conditioned on” or “given.” The denominator on the right side of eq. (A1) can be expanded as $p(\text{TP detected}) = p(\text{TP detected} | \text{TP exists})p(\text{TP exists}) + p(\text{TP detected} | \text{TP exists})p(\text{TP exists})$. Recognizing that $p(\text{TP detected} | \text{TP exists}) \equiv F$, $p(\text{TP detected} | \text{TP exists}) \equiv p_{\text{det}}$, $p(\text{TP exists}) \equiv \eta_{\text{terrestrial}}$, and $p(\text{TP exists}) \equiv 1 - \eta_{\text{terrestrial}}$, we rewrite the denominator on the right-hand side of equation (A1) as $p(\text{TP detected}) = p_{\text{det}}\eta_{\text{terrestrial}} + F(1 - \eta_{\text{terrestrial}})$ and the numerator as $p_{\text{det}}\eta_{\text{terrestrial}}$. The confidence in a detection is therefore

$$p(\text{TP exists} | \text{TP detected}) = \frac{p_{\text{det}}\eta_{\text{terrestrial}}}{p_{\text{det}}\eta_{\text{terrestrial}} + F(1 - \eta_{\text{terrestrial}})} \quad \text{(A2)}$$

Similarly, the confidence in a nondetection (i.e., the probability that a nondetection is not a missed detection) at a given target star can also be expressed in terms of Bayes’ rule:

$$p(\text{TP exists} | \text{TP detected}) = \frac{p(\text{TP detected} | \text{TP exists})p(\text{TP exists})}{p(\text{TP detected})}.$$

The denominator on the right side of equation (A3) can be expanded as $p(\text{TP detected}) = p(\text{TP detected} | \text{TP exists})p(\text{TP exists}) + p(\text{TP detected} | \text{TP exists})p(\text{TP exists})$. Finally, since $p(\text{TP detected} | \text{TP exists}) \equiv 1 - F$ and $p(\text{TP detected} | \text{TP exists}) \equiv 1 - p_{\text{det}}$, the confidence in a nondetection becomes

$$p(\text{TP exists} | \text{TP detected}) = \frac{(1 - F)(1 - \eta_{\text{terrestrial}})}{(1 - F)(1 - \eta_{\text{terrestrial}}) + (1 - p_{\text{det}})\eta_{\text{terrestrial}}} \quad \text{(A4)}$$

In practice, the confidences would be computed on a star-by-star basis. Figure 27 shows confidences in a detection and in a nondetection, averaged over all targets, for a deep survey of the best 120 TPF-C targets. If $\eta_{\text{terrestrial}}$ is 0.1, then for a detection threshold corresponding to a 1% false-alarm probability, the average confidence in a detection is 82%, and the average confidence in a nondetection is 94%. The former result means that we are 82% certain, on average, that a detection is really a terrestrial planet in the habitable zone and is not a false positive. The latter result means that on average, we are 94% certain that a nondetection rules out the existence of terrestrial planet in the habitable zone.

REFERENCES

Benz, W., Alibert, Y., Mordasini, C., & Naif, D. 2006, in IAU Colloq. 200, Direct Imaging of Exoplanets: Science and Techniques, ed. C. Aime & F. Vakili (Cambridge: Cambridge Univ. Press), 1
Black, D. C., & Scargle, J. D. 1982, ApJ, 263, 854
Boden, A., Unwin, S., & Shao, M. 1997, in Hipparcos-Venice '97, ed. B. Battrick (ESA SP-402: Noordwijk: ESA), 789
Butler, R. P., et al. 2004, ApJ, 617, 580
Cox, A. N., ed. 2000, Allen’s Astrophysical Quantities (4th ed.; New York: Springer)
Cumming, A. 2004, MNRAS, 354, 1165
Cumming, A., Marcy, G. M., & Butler, R. P. 1999, ApJ, 526, 890
Ford, E. B. 2005, AJ, 129, 1706
Ford, E. B., & Tremaine, S. 2003, PASP, 115, 1171
Griffiths, S. C., Hicks, R. B., & Milone, E. F. 1988, JRASC, 82, 1
Hatzes, A. P. 2002, Astron. Nachr., 323, 392
Henry, G. W., et al. 2000, ApJS, 130, 201
Horne, J. H., & Baliunas, S. L. 1986, ApJ, 302, 757
Ida, S., & Lin, D. N. C. 2004, ApJ, 604, 338
Kasting, J. F., Whitmire, D. P., & Reynolds, R. T. 1993, Icarus, 101, 108
Lattanzi, M. G., Spagna, A., Sozzetti, A., & Casertano, S. 2000, MNRAS, 317, 211
Marcy, G. W., Fischer, D. A., McCarthy, C., & Ford, E. B. 2005a, in ASP Conf. Ser. 338, Astrometry in the Age of the Next Generation of Large Telescopes, ed. P. K. Seidelmann & A. K. B. Monet (San Francisco: ASP)
Marcy, G. W., et al. 2005b, ApJ, 619, 570
———. 2005c, Prog. Theor. Phys. Suppl., No. 158, 24
McArthur, B. E., et al. 2004, ApJ, 614, L81
Nelson, A. F., & Angel, J. R. 1998, ApJ, 500, 940
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in C (2nd ed.; Cambridge: Cambridge Univ. Press)
Rivera, E., et al. 2005, ApJ, 634, 625
Scargle, J. D. 1982, ApJ, 263, 835
Sozzetti, A. 2005, PASP, 117, 1021

2006 PASP, 118:1319–1339
Sozzetti, A., Casertano, S., Brown, R. A., & Lattanzi, M. G. 2002, PASP, 114, 1173
———. 2003, PASP, 115, 1072
Tabachnik, S., & Tremaine, S. 2002, MNRAS, 335, 151
Tremaine, S., & Zakamska, N. 2003, in AIP Conf. Proc. 713, The Search for Other Worlds: 14th Astrophys. Conf., ed. S. S. Holt & D. Deming (College Park: AIP)

Turnbull, M. C., & Tarter, J. C. 2003, ApJS, 149, 423
Walker, G. A., Walker, A. R., Irwin, A. W., Larson, A. M., Yang, S. L., & Richardson, D. C. 1995, Icarus, 116, 359
Yu, J. 2005, SIM PlanetQuest Astrometric Error Budget