Bottom-up approach to high-temperature superconductivity

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Since the discovery of high-temperature superconductivity in the cuprates a theoretical understanding of their phase diagram has remained one of the major outstanding problems in condensed matter physics. Here we propose an effective low-energy Hamiltonian which produces both d-wave density wave (dDW) and d-wave superconducting (dSC) solutions within the BCS mean-field theory. This model predicts that (a) the observed pseudogap phase is a dDW state, (b) the superconducting phase is a d-wave BCS state, and (c) in the underdoped regime there is a gossamer superconducting state, i.e. dSC in coexistence with dDW. Moreover, this theory naturally explains the Uemura relation, the reduction of the quasiparticle density of states at the Fermi level, and the salient features in the tunneling conductivity measured in underdoped Bi2212.

1 Introduction

In 1986, the discovery of high-temperature superconductivity in the cuprates took the physics community by surprise\textsuperscript{[1]}. This was the starting point of a new era in condensed matter physics. Many practical applications of these new compounds were envisioned, and at the same time an intense debate arose regarding the origin and possible mechanisms leading to this new phenomenon. Enz recorded the often confusing discussions of the early days of high-Tc research in his beautiful textbook\textsuperscript{[2]}. One of the most influential contributions to the theory of these materials was provided by Anderson’s “dogmas”\textsuperscript{[3]}. He stated that the cuprate high-Tc phase diagram arises from an inherent competition between a Mott insulator phase and s-wave BCS superconductivity in these materials. In order to model high-Tc superconductivity, he proposed a two-dimensional one-band Hubbard model in combination with a resonant valence bond (RVB) wave function. A great portion of the theoretical community in the field has since embraced these dogmas. However, unfortunately we still remain without a clear vision as to where these dogmas are leading us\textsuperscript{[4]}. Around 1990, Scalapino and others\textsuperscript{[5]} pointed out that a perturbative analysis of the 2D Hubbard model in the weak-coupling limit produces d-wave superconductivity. Indeed, a d-wave superconducting order parameter was experimentally established around 1994 for single crystal samples of optimally doped Bi2212, YBCO and LSCO, using powerful angle resolved photoemission spectroscopy (ARPES)\textsuperscript{[6]} and elegant Josephson interferometry\textsuperscript{[7,8]}. These observations motivated us to investigate d-wave superconductivity within the BCS framework\textsuperscript{[9,10,11]}.

Before elaborating further, let us first examine the generic phase diagram of the hole doped high-Tc cuprate superconductors shown in Fig. 1. From the beginning, this phase diagram has been hotly debated. Around the year 2000, a few groups suggested that the pseudogap region can be described by a d-wave density wave (dDW) phase. Indeed, the giant Nernst effect observed in the underdoped Bi2212, YBCO and LSCO\textsuperscript{[12,13,14]} and the angle dependent magnetoresistance in Y_{0.68}Pr_{0.32}CuO\textsubscript{4}\textsuperscript{[15]} have been found to be fully consistent with dDW\textsuperscript{[21,22]}. In past work, we have shown that these are consequences of the Landau quantization of the quasiparticle spectrum in a magnetic field, analogous to earlier considerations...
Fig. 1 The phase diagram for the high-$T_c$ cuprates. $p$ denotes the hole doping concentration. PG is the pseudogap region.

by Nersesyan et al. Moreover, we note that the Fermi arcs (or pockets) in the ($\pi$, $\pi$) directions, observed by ARPES, follow directly from dDW. Furthermore, it is by now well established that the overdoped regions of the cuprates can also be described in terms of a d-wave BCS model. Recently, Laughlin pointed out that the Gutzwiller operator which is commonly used in the RVB wave function is not mathematically tractable, and proposed to replace it by a less constrained Jastrow operator. He named the resulting coexistence phase “gossamer superconductivity”, i.e. a condensate with a reduced superfluid density and reduced density of states near the Fermi surface. Such a reduction of the quasiparticle density of states has recently been observed by Tallon et al. by means of a thermodynamic analysis and the effect of Zn impurities in YBCO over a wide doping range. In section 4 we will return to the characterization of gossamer superconductivity. Note also that gossamer superconductivity emerges naturally from the phase diagram in Fig. 1 as a coexistent phase of d-wave superconductivity in the presence of a d-wave density wave. In the following we will explore this gossamer superconductivity phenomenon in detail.

2 Effective Hamiltonian

In this section, we construct an effective low-energy Hamiltonian that constitutes the basis of the bottom-up approach. This approach should be viewed as an alternative to the common top-down approaches originating from higher-energy Hamiltonians, such as the t-J and Hubbard models. Considering the energy scales of these models, e.g. typically a Hubbard $U$ of the order of $10^6$K, it turns out to be a rather difficult task to arrive at superconducting phenomena that exist at scales of $T_c \sim 10^3$K. Based on the renormalization group analysis of 2D electron systems we understand that the normal state is a Fermi liquid, i.e. not a Luttinger liquid or bosonic liquid. Here we define the Fermi liquid via a quasiparticle Green function which has simple poles, a definition that is consistent with Shankar and Landau. Furthermore, we know that at sufficiently low temperatures the normal state becomes unstable against infrared divergences in the 2-particle and/or 2-hole channels, implying superconductivity, or unstable against divergences in the particle-hole channel, implying density wave phases.
The effective low-energy Hamiltonian for such a system is given by\[14, 32\]

\[ H = \sum_{k,\sigma} (\epsilon_k - \mu) c_{k,\sigma}^\dagger c_{k,\sigma} - \sum_{k,\sigma} \left( \Delta_1(k) c_{k+Q,\sigma}^\dagger c_{k,\sigma} + \Delta_1^*(k) c_{k,\sigma}^\dagger c_{k+Q,\sigma} \right) - \sum_k \left( \Delta_2(k) c_{k,\sigma}^\dagger c_{-k,\sigma} + \Delta_2^*(k) c_{-k,\sigma}^\dagger c_{k,\sigma} \right) - g_1^{-1} |\Delta_1(k)|^2 - g_2^{-1} |\Delta_2(k)|^2, \]

(1)

where the amplitude and angular parts of the order parameters separate via \( \Delta_1(k) = \Delta_1 f(k) \) and \( \Delta_2(k) = \Delta_2 f(k) \), and \( Q \sim (\pi, \pi) \) is the nesting vector. Two self-consistent gap equations follow directly from this Hamiltonian:

\[ \Delta_1^* = \frac{g_1}{(f^2(k))} \sum k,\sigma f(k) \langle c_{k+Q,\sigma}^\dagger c_{k,\sigma} \rangle, \]

(2)

\[ \Delta_2^* = \frac{g_2}{(f^2(k))} \sum k,\sigma f(k) \langle c_{k,\sigma}^\dagger c_{-k,\sigma} \rangle. \]

(3)

Here \( \Delta_1 \) and \( \Delta_2 \) are the order parameters of dDW and dSC respectively. In the following, we use \( f(k) = \cos(2\phi) \) as the angular dependence. A similar Hamiltonian has been considered in related work by Thalmeier.\[32\] However this study was limited to conventional DW and conventional SC, and to the case of vanishing chemical potential \( \mu \). As we shall see here, \( \mu \) is an important control parameter in the present model.\[14\] Moreover, when both DW and SC are conventional, there is little room for their coexistence; instead phase separation is the rule.\[32\] On the other hand, when both order parameters are unconventional, there is ample opportunity for their coexistence.\[33\] This fact will be exploited in the following.

The Nambu-Gorkov Green function\[34\] of this model is given by

\[ G^{-1}(k, \omega_n) = i\omega_n - \epsilon_k \rho_3 \sigma_3 + \mu \sigma_3 + |\Delta_1| \exp(-i\phi_1 \rho_3) f(k) \rho_1 \sigma_3 + |\Delta_2| \exp(-i\phi_2 \sigma_3) f(k) \sigma_1, \]

(4)

and the corresponding spinor field is

\[ \Psi_k = \left( c_{k,\sigma}^\dagger, c_{-k,-\sigma}^\dagger, c_{k+Q,\sigma}^\dagger, c_{-k-Q,-\sigma} \right). \]

(5)

One notes that unlike in Ref.\[14\], the present \( G^{-1}(k, \omega_n) \) possesses two Abelian gauge transformations associated with \( \phi_1 \) (sliding motion of dDW) and \( \phi_2 \) (supercurrent in dSC). The determinant of \( G^{-1}(k, \omega_n) \) is given by

\[ D = \det |G^{-1}(k, \omega_n)| = \left( \omega_n^2 + \epsilon_k^2 + \mu^2 + |\Delta_1(k)|^2 + |\Delta_2(k)|^2 \right)^2 - 4\mu^2 \left( \epsilon_k^2 + |\Delta_1(k)|^2 \right). \]

(6)

Using this result, the quasiparticle energy is found to be

\[ E = \pm \sqrt{\left( \epsilon_k^2 + |\Delta_1(k)|^2 + \mu \right)^2 + |\Delta_2(k)|^2}, \]

(7)

which agrees with earlier results.\[12, 30\] Finally, the quasiparticle density of states is given by

\[ \frac{N(E)}{N_0} = \left| \text{Re} \left( \frac{E}{\sqrt{\left( \epsilon_k^2 - \Delta_2^2 f(k)^2 + \mu \right)^2 - \Delta_2^4 f(k)^2}} \left( 1 \mp \frac{\mu}{\sqrt{E^2 - \Delta_2^2 f(k)^2}} \right) \right) \right|, \]

(8)

where + and - stand for the positive-energy and negative-energy solutions respectively.

In Fig. 2 the quasiparticle density of states is shown for a particular set of parameters. For these parameters, we observe clear dips in the vicinity of \( E=0 \), as well as a quasi-linear dependence on energy. In the regime \( \Delta_1 - \mu < \Delta_2 \) one of the peaks splits into two peaks. Although such a split peak has not yet been observed experimentally, except in the presence of Ni impurities,\[35\] the present result is consistent with recent measurements on underdoped Bi2212.\[36\] Moreover, this result is rather different from earlier work by Zeyher and Greco\[37\] who assumed \( \mu = 0 \).
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Fig. 2 Quasiparticle density of states in a gossamer superconductor with the energy scale set by the superconducting energy gap $\Delta_2 = 1$.

3 D-Wave Density Wave Phase

Equations 2 and 3 can be transformed to

$$\lambda_1^{-1} = 4\pi T \sum_n \text{Re} \langle f(k)^2 d^{-1} \rangle$$

$$\lambda_2^{-1} = 4\pi T \sum_n \text{Re} \langle f(k)^2 \text{Re} \left( \left( 1 - \frac{i\mu}{\sqrt{\omega_n^2 + \Delta_2^2 f(k)^2}} \right) d^{-1} \right) \rangle,$$

where $d = [\left( \sqrt{\omega_n^2 + \Delta_2^2 f(k)^2} - \mu \right]^2 + \Delta_1^2 f(k)^2]^{1/2}$. Here $\lambda_1 = g_1 N_0$ and $\lambda_2 = g_2 N_0$ are dimensionless coupling constants, and $N_0 = N(0)$ is the quasiparticle density of states in the normal state.

First, the phase diagram of the pure dDW states is easily obtained by setting $\Delta_2 = 0$. In this case, Eq. 10 reduces in the limit $\Delta_1 \rightarrow 0$ to

$$-\ln \left( \frac{T_{c1}}{T_{c10}} \right) = \text{Re} \psi \left( \frac{1}{2} - \frac{i\mu}{2\pi T_{c1}} \right) - \psi \left( \frac{1}{2} \right),$$

where $T_{c1}$ is the transition temperature for dDW and $\psi(z)$ is the di-gamma function. Using $T_{c10}=800K$, one arrives at the phase diagram shown in Fig. 3. Note that Eq. 13 is the same for s-wave and d-wave superconductors in the limit when the Pauli term dominates over the orbital term.

It is observed that $T_{c1}$ bends backwards in the region $\mu/\Delta_1 \leq 0.558$. A similar diagram has also been found in Ref. $[37]$. However, if we additionally allow a spatial variation of the dDW order parameter $\Delta_1$, we obtain

$$-\ln \left( \frac{T_{c1}}{T_{c10}} \right) = \text{Re} \left( (1 \pm \cos(4\phi)) \psi \left( \frac{1}{2} - \frac{i\mu(1 - p \cos(\phi))}{2\pi T_{c1}} \right) \right) - \psi \left( \frac{1}{2} \right),$$

where $p = v|q|/2\mu$. This yields the extended portion shown in Fig. 3, analogous to the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in d-wave superconductors. There is a further transition
when the q-vector is rotated from the [100] to the [110] direction. Finally, the dDW regime terminates when $\mu/\Delta_{10} = 0.824$. We call these periodic dW phases dDWII and dDWIII respectively.

4 Gossamer Superconductivity

Now we can ask how dSC appears on top of this dDW background. In the following, we shall limit ourselves to the region $\Delta_1 \gg \Delta_2, \mu$. Then we can deduce in the vicinity of the superconducting transition temperature, $T_{c2}$, the quasiparticle density of states at the Fermi surface which is given by

$$g(\mu, 0) = \langle g(0, k) \rangle = \frac{2}{\pi} x K(x),$$

(13)

with $x = \mu/\Delta_1(\mu)$ and $K(z)$ is the complete elliptic function of the first kind. $g(\mu, 0)$ resembles the quasiparticle density of states deduced from the analysis of Zn-impurities[28]. The corresponding low-temperature entropy is given by[29]

$$\frac{S}{T} = \frac{27\zeta(3)}{2\pi^2} \frac{\gamma_N g(\mu)}{\Delta_2(\mu)},$$

(14)

where $\gamma_N = \pi^2 N_0/3$. On the other hand, the superconducting transition temperature and free energy are controlled by

$$g_1(\mu, 0) = 2\langle \cos^2(2\phi)g(0, k) \rangle = \frac{4x}{\pi} (K(x) - E(x)),$$

(15)

as

$$T_{c}(\mu) = 1.136\Delta_1(\mu) \exp[-(\lambda^{-1} - \lambda_2^{-1})g^{-1}(\mu, 0)],$$

(16)

$$U_0 = -\frac{1}{4} N_0 g_1(\mu)[\Delta_2(\mu, 0)]^2.$$

(17)
Eq. (16) appears to somewhat overestimate $T_c(\mu)$, and hence a more detailed treatment needs to be developed in order to be more realistic. The functions $\Delta_1(\mu)/\Delta_{10}$, $g(\mu, 0)$ and $g_1(\mu, 0)$ are shown in Fig. 4. The dependence of the functions $g(\mu, 0)$ and $g_1(\mu, 0)$ on $\mu/\Delta_{10}$ is in agreement with available experimental data.\[28\ \[29\]

![Fig. 4](image_url)

Fig. 4 Functions $\Delta_1(\mu)/\Delta_{10}$, $g(\mu, 0)$ and $g_1(\mu, 0)$, controlling the low-doping region of the high-Tc superconductors.

By solving the coupled gap equations in the regime $\Delta_2, \mu \ll \Delta_1$ we find\[16\]

$$\rho_s(0) = \simeq \frac{\Delta_2^2(\mu, 0)}{\Delta^2(0)}, \quad (18)$$

$$T_{c2} = \frac{1}{2 \ln 2} \frac{\Delta_2^2(\mu, 0)}{\Delta^2(0)}, \quad (19)$$

$$\lambda^{-2}(\mu, 0) = 4 \pi e^2 \frac{m^*}{\rho_s(0)}. \quad (20)$$

These are essentially the Uemura relations.\[35\] Therefore, if we limit ourselves to the deeply underdoped region, many experimentally observed features of gossamer superconductivity follow naturally from the present model.

## 5 Concluding Remarks

The model treated here is based on an effective low-energy Hamiltonian which describes dDW and dSC states with the chemical potential as a control parameter. This theory accounts for the following principle features of high-temperature cuprate superconductors: (a) the normal state is a Fermi liquid, (b) the pseudogap phase is a dDW (more recently a d-wave spin density wave has also been suggested\[46\ \[47\]), (c) the superconductivity in the optimal to overdoped regime has a BCS d-wave order parameter, and (d) in the underdoped regime there is gossamer superconductivity, i.e. dSC coexisting with dDW.

Recent related studies on heavy fermion compounds, such as CeCoIn$_5$ under pressure\[48\], and organic conductors, such as $\beta$'-((BEDT-TTF)$_4$(N$_3$O)M(C$_2$O$_4$)C$_5$H$_7$N with M = Ga and Cr\[49\ \[50\], have revealed...
many parallels between the high-Tc cuprates and these systems. These include (a) a layered structure or quasi-two-dimensionality, (b) d-wave superconductivity\cite{51, 52}, and (c) d-wave density wave phases\cite{47, 53, 54}. In the heavy fermion and organic conductors the horizontal axis in Fig. 3 needs to be replaced by the external pressure P, but otherwise their phase diagrams look very similar. This suggest strongly that the present model is rather universal and applies to many strongly correlated electron systems.

Let us finally note that gossamer superconductivity is not necessarily restricted to dSC and dDW. For example, recent experiments on the Bechgaard salts (TMTSF)$_2$PF$_6$ at ambient pressure\cite{55, 56, 57} and measurements of the angle dependent magnetothermal conductivity in URu$_2$Si$_2$\cite{58} suggest that there are other kinds of gossamer superconductivity, e.g. f-wave superconductivity coexisting with a d-wave spin density wave. Hence it will not be surprising if similar coexistence states will soon be discovered in related strongly correlated electron systems.

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