Research Article

Soft Translations and Soft Extensions of BCI/BCK-Algebras

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The concept of soft translations of soft subalgebras and soft ideals over BCI/BCK-algebras is introduced and some related properties are studied. Notions of Soft extensions of soft subalgebras and soft ideals over BCI/BCK-algebras are also initiated. Relationships between soft translations and soft extensions are explored.

1. Introduction

Recently soft set theory has emerged as a new mathematical tool to deal with uncertainty. Due to its applications in various fields of study researchers and practitioners are showing keen interest in it. As enough number of parameters is available here, so it is free from the difficulties associated with other contemporary theories dealing with uncertainty. Prior to soft set theory, probability theory, fuzzy set theory, rough set theory, and interval mathematics were common mathematical tools for dealing with uncertainties, but all these theories have their own difficulties. These difficulties may be due to lack of parametrization tools [1, 2]. To overcome these difficulties, Molodtsov [2] introduced the concept of soft sets. A detailed overview of these difficulties can be seen in [1, 2]. As a new mathematical tool for dealing with uncertainties, Molodtsov has pointed out several directions for the applications of soft sets. Theoretical development of soft sets is due to contributions from many researchers. However in this regard initial work is done by Maji et al. in [1]. Later Ali et al. [3] introduced several new operations in soft set theory.

At present, work on the soft set theory is progressing rapidly. Maji et al. [4] described the application of soft set theory in decision making problems. Aktas and Cagman studied the concept of soft groups and derived their basic properties [5]. Chen et al. [6] proposed parametrization reduction of soft sets, and then Kong et al. [7] presented the normal parametrization reduction of soft sets. Feng and his colleagues studied roughness in soft sets [8, 9]. Relationship between soft sets, fuzzy sets, and rough sets is investigated in [10]. Park et al. [11] worked on notions of soft WS-algebras, soft subalgebras, and soft deductive system. Jun and Park [12] presented the notions of soft ideals, idealistic soft, and idealistic soft BCI/BCK-algebras. Further applications of soft sets can be seen in [13–25].

The study of BCI/BCK-algebras was initiated by Imai and Iseki [26] as the generalization of concept of set theoretic difference and propositional calculus. For the general development of BCI/BCK-algebras, the ideal theory and its fuzzification play an important role. Jun et al. [27–30] studied fuzzy trends of several notions in BCI/BCK-algebras. Application of soft sets in BCI/BCK is given in [12, 31].

Translations play a vital role in reducing the complexity of a problem. In geometry it is a common practice to translate a system to some new position to study its properties. In linear algebra translations help find solution to many practical problems. In this paper idea of translations is being extended to soft BCI/BCK algebras.

This paper is arranged as follows: in Section 2, some basic notions about BCI/BCK-algebra and soft sets are given. These notions are required in the later sections. Concept of translation is introduced in Section 3 and some properties of it are discussed here. Section 4 is devoted for the study of soft
2. Preliminaries

First of all some basic concepts about BCI/BCK-algebra are given. For a comprehensive study on BCI/BCK-algebras [32] is a very nice monograph by Meng and Jun. Then some notions about soft sets are presented here as well.

An algebra \((X, *, 0)\) is called a BCI-algebra if it satisfies the following conditions:

\[(1) (\forall x, y, z \in X) ((x * y) * (x * z)) * (z * y) = 0,\]
\[(2) (\forall x, y \in X) (x * (x * y)) * y = y,\]
\[(3) (\forall x \in X) (x * (x * y)) = 0,\]
\[(4) (\forall x, y \in X) (x * y = 0, y * x = 0 \Rightarrow x = y).\]

If a BCI-algebra satisfies the following identity:

\[(5) (\forall x \in X) (0 * x = 0),\]

then \(X\) is called a BCK-algebra. Any BCK-algebra satisfies the following axioms:

\[(i) (\forall x \in X) (x * 0 = x),\]
\[(ii) (\forall x, y, z \in X) (x * y = 0 \Rightarrow (x * z) * (y * z) = 0,\]
\[(iii) (\forall x, y, z \in X) ((x * y) * z = (x * z) * y),\]
\[(iv) (\forall x, y, z \in X) (((x * z) * (y * z)) * (x * y) = 0).\]

A subset \(S\) of a BCI/BCK-algebra \(X\) is called a subalgebra of \(X\) if \(x * y \in S\), for all \(x, y \in S\).

A subset \(A\) of a BCI/BCK-algebra \(X\) is called an ideal of \(X\), denoted by \(A \triangleleft X\), if it satisfies:

\[(1) 0 \in A,\]
\[(2) (\forall x, y \in X) (x * y \in A, y \in A \Rightarrow x \in A).\]

Now we recall some basic notions in soft set theory. Let \(U\) be a universe and \(E\) be a set of parameters. Let \(P(U)\) denote the power set of \(U\) and let \(A, B\) be nonempty subsets of \(E\).

**Definition 1** (see [2]). A pair \((F, A)\) is called a soft set over \(U\), where \(F\) is a mapping given by \(F : A \rightarrow P(U)\).

**Definition 2** (see [3]). Let \(U\) be a universe, let \(E\) be the set of parameters, and let \(A \subseteq E\).

\[(a) (F, A) \text{ is called a relative null soft set (with respect to the parameters set } A\text{), denoted by } \theta_A, \text{ if } F(a) = \emptyset, \text{ for all } a \in A.\]
\[(b) (G, A) \text{ is called a relative whole soft set (with respect to the parameters set } A\text{), denoted by } U_A, \text{ if } G(e) = U, \text{ for all } e \in A.\]

**Definition 3** (see [3]). The complement of a soft set \((F, A)\) is denoted by \((F, A)^c\) and is defined by \((F, A)^c = (F^c, A)\), where \(F^c : A \rightarrow P(U)\) is a mapping given by \(F^c(a) = U - F(a)\), for all \(a \in A\). Clearly, \((F, A)^c)^c = (F, A)\).
Proof. Assume $F_U^T$ is a soft subalgebra of $X$ for some $U_1 \subseteq T$. Let $x, y \in X$, we have

$$F_A(x \ast y) \cup U_1 = F_U^T(x \ast y) \supseteq F_U^T(x) \cap F_U^T(y)$$

(5)

$$= (F_A(x) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= (F_A(x) \cap (y)) \cup U_1.$$  

Now by Lemma 6 we have

$$F_A(x \ast y) \supseteq F_A(x) \cap F_A(y),$$

(6)

for all $x, y \in X$. Hence $F_A$ is a soft subalgebra of $X$. \qed

From Propositions 7 and 8 we have the following.

**Theorem 9.** A soft set $F_A$ of $X$ is a soft subalgebra of $X$ if and only if $U_1$-translation $F_U^T$ of $F_A$ is a soft subalgebra of $X$ for some $U_1 \subseteq T$.

**Definition 10.** Let $F_A$ and $G_B$ be two soft sets over $X$. If $F_A(x) \subseteq G_B(x)$ for all $x \in X$, then we say that $G_B$ is a soft extension of $F_A$.

**Example 11.** Consider a BCI/BCK-algebra $X = \{0, 1, 2, 3\}$ presented as follows:

$$\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 \\
2 & 2 & 2 & 0 & 2 \\
3 & 3 & 3 & 3 & 0 \\
\end{array}$$

(7)

Define two soft sets $F_A$ and $G_B$ of $X$ as in Table 1.

Here $F_A(0) \subseteq G_B(0), F_A(1) \subseteq G_B(1), F_A(2) \subseteq G_B(2), \text{ and } F_A(3) \subseteq G_B(3)$, which implies that $G_B$ is a soft extension of $F_A$.

Next the concept of soft $S$-extension is being introduced.

**Definition 12.** Let $F_A$ and $G_B$ be two soft sets over $X$. Then $G_B$ is called a soft $S$-extension of $F_A$, if the following conditions hold:

1. $G_B$ is a soft extension of $F_A$.
2. If $F_A$ is a soft subalgebra of $X$, then $G_B$ is a soft subalgebra of $X$.

As we know $F_U^T(x) \supseteq F_A(x)$ for all $x \in X$. As a consequence of Definition 12 and Theorem 9, we have the following.

**Theorem 13.** Let $F_A$ be a soft subalgebra of $X$ and $U_1 \subseteq T$. Then the soft $U_1$-translation $F_U^T$ of $F_A$ is a soft $S$-extension of $F_A$.

The converse of Theorem 13 is not true in general as seen in the following example.

| $X$ | 0 | 1 | 2 | 3 |
|-----|---|---|---|---|
| $F_A$ | [0] | [0, 1] | [0, 2] | [1, 2] |
| $G_B$ | [0] | [0, 1, 2] | [0, 2] | [0, 1, 2] |

**Example 14.** Consider a BCI/BCK-algebra $X = \{0, 1, 2, 3\}$ given as follows:

$$\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 \\
2 & 2 & 2 & 0 & 2 \\
3 & 3 & 3 & 3 & 0 \\
\end{array}$$

(8)

Define a soft set $F_A$ of $X$ by Table 2.

Then $F_A$ is a soft subalgebra of $X$. For soft set $F_A, T = \{3\}$.

Let $G_B$ be a soft set over $X$ given by Table 3.

Then $G_B$ is a soft $S$-extension of $X$. But it is not a soft $U_1$-translation of $F_A$ for any nonempty $U_1 \subseteq T$.

For a soft set $F_A$ of $X, U_1 \subseteq T$ and $U_2 \in P(X)$ with $U_2 \supseteq U_1$, let

$$U_{U_1}(F_A; U_2) := \{ x \in X \mid F_A(x) \supseteq U_2 - U_1 \}.$$  

(9)

If $F_A$ is a soft subalgebra of $X$, then it is clear that $U_{U_1}(F_A; U_2)$ is a subalgebra of $X$ for all $U_2 \in P(X)$ with $U_2 \supseteq U_1$. But, if we do not give condition that $F_A$ is a soft subalgebra of $X$, then $U_{U_1}(F_A; U_2)$ may not be a subalgebra of $X$ as seen in the following example.

**Example 15.** Let $X = \{0, 1, 2, 3, 4\}$ be a BCI/BCK-algebra presented as follows:

$$\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
2 & 2 & 1 & 0 & 0 \\
3 & 3 & 3 & 1 & 0 \\
4 & 4 & 3 & 3 & 1 \\
\end{array}$$

(10)

Define a soft subset $F_A$ of $X$ by Table 4.

Then $F_A$ is not a soft subalgebra of $X$ with $T = \{1\}$. Since $F_A(3 \ast 4) = \{0\} \supseteq \{0, 4\} = F_A(3) \cap F_A(4)$ for $U_2 = \{1, 4\}$ and $U_1 = \{1\}$, we obtain $U_{U_1}(F_A; U_2) = \{3, 4\}$ which is not a subalgebra of $X$ since $3 \ast 3 = 0 \notin U_{U_1}(F_A; U_2)$.

In the following theorem, relationship between $U_1$-translations and $U_{U_1}(F_A; U_2)$ is studied in case of soft subalgebra.

**Theorem 16.** Let $F_A$ be a soft set over $X$ and $U_1 \subseteq T$. Then the soft $U_1$-translation $F_U^T$ of $F_A$ is a soft subalgebra of $X$ if and only if $U_{U_1}(F_A; U_2)$ is a subalgebra of $X$ for all $U_2 \in P(U)$ with $U_2 \supseteq U_1$.

**Proof.** Assume that the soft $U_1$-translation $F_U^T$ of $F_A$ is a soft subalgebra of $X$. Then by Theorem 9, $F_A$ is a soft subalgebra.
Theorem 17. Let $F_A$ be a soft subalgebra of $X$ and let $U_1, U_2 \subseteq T$. If $U_1 \supseteq U_2$, then the soft $U_1$-translation $F_{U_1}^T$ of $F_A$ is a soft $S$-extension of the soft $U_2$-translation $F_{U_2}^T$ of $F_A$.

Proof. Since $U_1 \supseteq U_2$, this implies $F_{U_1}^T(x) \supseteq F_{U_2}^T(x)$, for all $x \in X$. So $U_1$-translation is an extension of $U_2$-translation, and from Theorem 9, $F_{U_1}^T$ and $F_{U_2}^T$ are soft subalgebras of $F_A$. Hence soft $U_1$-translation $F_{U_1}^T$ of $F_A$ is a soft $S$-extension of the soft $U_2$-translation $F_{U_2}^T$ of $F_A$.

For every soft subalgebra $F_A$ of $X$ and $U_2 \subseteq T$, the soft $U_2$-translation $F_{U_2}^T$ of $F_A$ is a soft subalgebra of $X$. If $G_B$ is a soft $S$-extension of $F_{U_2}^T$ and then there exists $U_1 \subseteq T$ such that $U_1 \supseteq U_2$ and $G_B(x) \supseteq F_{U_2}^T(x)$, for all $x \in X$. Then by Theorem 17, $G_B$ is a soft $S$-extension of $F_U^T$ of $F_A$.

**Definition 19.** A soft $S$-extension $G_B$ of a soft subalgebra $F_A$ of $X$ is said to be normalized if there exists $x_0 \in X$ such that $G_B(x_0) = X$.

**Definition 20.** Let $F_A$ be a soft subalgebra of $X$. A soft set $G_B$ of $X$ is called a maximal soft $S$-extension of $F_A$ if it satisfies the following conditions:

1. $G_B$ is a soft $S$-extension of $F_A$,
2. there does not exist another soft subalgebra of $X$ which is a soft extension of $G_B$.

**Example 21** (see [33]). Let $Z^+$ be a set of positive integers and let $*$ be a binary operation on $Z^+$ defined by $x * y = \frac{x}{(x, y)}$, (13)

$\forall x, y \in Z^+$, where $(x, y)$ is the greatest common divisor of $x$ and $y$. Then $(Z^+, *, 1)$ is a BCK-algebra. Let $F_A$ and $G_B$ be soft sets of $Z^+$ which are defined by $F_A(x) = \{1, 2, 3\}$ and $G_B(x) = \{0, 1, 2\}$ for all $x \in Z^+$. Clearly, $F_A$ and $G_B$ are soft subalgebras of $Z^+$. By using definition of maximal soft $S$-extension, then it is easy to see that $G_B$ is a maximal soft $S$-extension of $F_A$.

**Proposition 22.** If a soft set $G_B$ of $X$ is a normalized soft $S$-extension of a soft subalgebra $F_A$ of $X$, then $G_B(0) = X$.

**Proof.** Assume that $G_B$ is a normalized soft $S$-extension of a soft subalgebra $F_A$ of $X$ then there exists $x_0 \in X$ such that $G_B(x_0) = X$, for some $x_0 \in X$. Consider $G_B(0) = G_B (x_0 * x_0) \supseteq G_B (x_0) \cap G_B (x_0) = X$. (14)

This implies $G_B(0) = X$.

**Theorem 23.** Let $F_A$ be a soft subalgebra of $X$. Then every maximal soft $S$-extension of $F_A$ is normalized.

**Proof.** This follows from the definitions of the maximal and normalized soft $S$-extensions.

4. Soft Translations of Soft Ideals in Soft BCI/BCK-Algebras

Now concept of translation of a soft ideal of a BCI/BCK-algebra is introduced.

**Definition 24.** A soft subset $F_A$ of a BCI/BCK-algebra is called a soft ideal of $X$, denoted by $F_A \triangleleft_S X$, if it satisfies:

1. $(\forall x \in X) (F_A(0) \supseteq F_A(x))$,
2. $(\forall x, y \in X) (F(x) \supseteq F_A(x * y) \cap F_A(y))$.

**Theorem 25.** If $F_A$ is a soft subset of $X$, then $F_A$ is a soft ideal of $X$ if and only if soft $U_1$-translation $F_{U_1}^T$ of $F_A$ is a soft ideal of $X$ for all $U_1 \subseteq T$. 

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**Table 2**

| $X$ | 0   | 1   | 2   | 3   |
|-----|-----|-----|-----|-----|
| $F_A$ | (0,1,2) | (0,1) | (0,2) | (1,2) |

**Table 3**

| $X$ | 0   | 1   | 2   | 3   |
|-----|-----|-----|-----|-----|
| $G_B$ | (0,1,2) | (0,1,2) | (0,2) | (1,2) |

**Table 4**

| $X$ | 0   | 1   | 2   | 3   |
|-----|-----|-----|-----|-----|
| $F_A$ | (0) | (0,2) | (0,2,3) | (0,3,4,0,4) |
Proof. Assume that $F_A \triangleleft_X X$ and let $U_1 \subseteq T$. Then $F_{U_1}^T(0) \subseteq F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1 = F_{U_1}(x)$ and

$$F_{U_1}^T(x) = F_A(x) \cup U_1 \supseteq (F_A(x \ast y) \cap F_A(y)) \cup U_1$$

$$= (F_A(x \ast y) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= F_{U_1}^T(x \ast y) \cap F_{U_1}(y) \quad \forall x, y \in X. \quad (15)$$

Hence $F_{U_1}^T \triangleleft_X X$.

Conversely, assume that $F_{U_1}^T$ is a soft ideal of $X$ for some $U_1 \subseteq T$. Let $x, y \in X$. Then

$$F_{U_1}^T(0) \supseteq F_{U_1}^T(x) \implies F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1$$

$$\implies F_A(0) \supseteq F_A(x) \quad \text{by Lemma 6,}$$

and so $F_A(0) \supseteq F_A(x)$. Next

$$F_A(x) \cup U_1 \supseteq F_{U_1}^T(x)$$

$$\supseteq F_{U_1}^T(x \ast y) \cap F_{U_1}(y) \quad (17)$$

$$= (F_A(x \ast y) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= (F_A(x \ast y) \cap F_A(y)) \cup U_1,$$

which implies that $F_A(x) \supseteq F_A(x \ast y) \cap F_A(y)$ (by Lemma 6). Hence $F_A$ is a soft ideal of $X$. \qed

5. Soft Extensions and Soft Ideal Extensions of Soft Subalgebras

In this section concept of soft ideal extension is being introduced and some of its properties are studied.

Definition 26. Let $F_A$ and $G_B$ be the soft subsets of $X$. Then $G_B$ is called the soft ideal extension of $F_A$, if the following conditions hold:

1. $G_B$ is a soft extension of $F_A$.
2. $F_A \triangleleft_X X \implies G_B \triangleleft_X X$.

For a soft subset $F_A$ of $X$, $U_1 \subseteq T$ and $U_2 \subseteq P(X)$ with $U_2 \supseteq U_1$, define $E_{U_2}(F_A; U_2) := \{x \in X \mid F_A(x) \cup U_2 \supseteq U_2 \geq U_1\}$.

It is clear that if $F_A \triangleleft_X X$, then $E_{U_1}(F_A; U_2) \triangleleft X$ for all $U_2 \supseteq U_1$.

Theorem 27. For $U_1 \subseteq T$, let $F_{U_1}^T$ be the soft $U_1$-translation of $F_A$. Then the following are equivalent:

1. $F_{U_1}^T \triangleleft_X X$.
2. $(U_2 \in P(U)) \cup U_1 \implies E_{U_2}(F_A; U_2) \triangleleft X).$

Proof. (1) $\implies$ (2) Consider $F_{U_1}^T \triangleleft_X X$ and let $U_2 \in P(U)$ be such that $U_2 \supseteq U_1$. Since $F_{U_1}^T(0) \supseteq F_{U_1}^T(x)$ for all $x \in X$, we have

$$F_A(0) \cup U_1 \supseteq F_{U_1}^T(0) \supseteq U_2 \supseteq F_A(x) \cup U_1 \geq U_2, \quad (18)$$

for $x \in E_{U_1}(F_A; U_2)$.

Hence $0 \in E_{U_1}(F_A; U_2)$. \quad (19)

Let $x, y \in X$ be such that $x \ast y \in E_{U_1}(F_A; U_2)$ and $y \in E_{U_1}(F_A; U_2)$. Then $F_A(x \ast y) \cup U_1 \supseteq U_2$ and $F_A(y) \cup U_1 \supseteq U_2$, that is, $F_{U_1}^T(x \ast y) = F_A(x \ast y) \cup U_1 \supseteq U_2$ and $F_{U_1}^T(y) = F_A(y) \cup U_1 \supseteq U_2$. Therefore $F_{U_1}^T \triangleleft_X X$.

$$(F_A(x \ast y) \cup U_1 \supseteq U_2 \supseteq F_A(y) \cup U_1 \supseteq U_2) \implies F_{U_1}^T(x \ast y) \cap F_{U_1}^T(y) \cap U_1 \supseteq U_2.$$

Now, let $F_{U_1}^T$ is a soft ideal of $X$, then $E_{U_1}(F_A; U_2) \triangleleft X$ for all $x \in X$. Therefore $F_{U_1}^T \triangleleft_X X$.

Theorem 28. Let $F_A \triangleleft_X X$ and $U_1, U_2 \subseteq T$. If $U_1 \supseteq U_2$, then the soft $U_1$-translation $F_{U_1}^T$ of $F_A$ is a soft ideal extension of the soft $U_2$-translation $F_{U_2}^T$ of $F_A$.

Proof. Since

$$F_{U_1}^T(x) = F_A(x) \cup U_1, \quad F_{U_2}^T(x) = F_A(x) \cup U_2, \quad (21)$$

$U_1 \supseteq U_2$, this implies that $(F_{U_1}^T(0) \supseteq F_{U_2}^T(0)) \quad \forall x \in X$. This shows that $F_{U_1}^T$ is a soft extension of $F_{U_2}^T$.

Now, let $F_{U_1}^T$ is a soft ideal of $X$, then $F_{U_1}^T(0) = F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1 \supseteq F_{U_1}^T(x) \quad \forall x \in X$, so we have $(F_{U_1}^T(0) \supseteq F_{U_1}^T(x)).$

Consider

$$F_{U_1}^T(x) = F_A(x) \cup U_1$$

$$\supseteq (F_A(x \ast y) \cap F_A(y)) \cup U_1$$

$$= (F_A(x \ast y) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= F_{U_1}^T(x \ast y) \cap F_{U_1}^T(y), \quad \forall x, y \in X \quad (22)$$

That is $(F_{U_1}^T(x) \supseteq F_{U_1}^T(x \ast y) \cap F_{U_1}^T(y)) \quad \forall x, y \in X$, so $F_{U_1}^T$ is a soft ideal of $X$. Hence $F_{U_1}^T$ is a soft ideal extension of $F_{U_2}^T$. \qed

6. Conclusion

Soft set theory is a mathematical tool to deal with uncertainties. Translation and extension are very useful concepts in mathematics to reduce the complexity of a problem. These concepts are frequently employed in geometry and algebra. In this paper, we presented some new notions such as soft translations and soft extensions for BCI/BCK-algebras. We
also examined some relationships between soft translations and soft extensions. Moreover, soft ideal extensions and translations have been introduced and investigated as well. It is hoped that these results may be helpful in other soft structures as well.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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**References**

[1] P. K. Maji, R. Biswas, and A. R. Roy, “Soft set theory,” Computers & Mathematics with Applications, vol. 45, no. 4–5, pp. 555–562, 2003.

[2] D. Molodtsov, “Soft set theory: first results,” Computers & Mathematics with Applications, vol. 37, no. 4–5, pp. 19–31, 1999.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, “On some new operations in soft set theory,” Computers & Mathematics with Applications, vol. 57, no. 9, pp. 1547–1553, 2009.

[4] P. K. Maji, A. R. Roy, and R. Biswas, “An application of soft sets in a decision making problem,” Computers & Mathematics with Applications, vol. 44, no. 8–9, pp. 1077–1083, 2002.

[5] H. Aktaş and N. Çağman, “Soft sets and soft groups,” Information Sciences, vol. 177, no. 13, pp. 2726–2735, 2007.

[6] D. Chen, E. C. C. Tsang, D. S. Yeung, and X. Wang, “The parameterization reduction of soft sets and its applications,” Computers & Mathematics with Applications, vol. 49, no. 5–6, pp. 757–763, 2005.

[7] Z. Kong, L. Gao, and L. Wang, “Comment on “a fuzzy soft set theoretic approach to decision making problems”, Journal of Computational and Applied Mathematics, vol. 223, no. 2, pp. 540–542, 2009.

[8] F. Feng, C. Li, B. Davvaz, and M. I. Ali, “Soft sets combined with fuzzy sets and rough sets: a tentative approach,” Soft Computing, vol. 14, no. 9, pp. 899–911, 2010.

[9] F. Feng, X. Liu, and V. Leoreanu-Fotea, “Soft sets and soft rough sets,” Information Sciences, vol. 181, no. 6, pp. 1125–1137, 2011.

[10] M. I. Ali, “A note on soft sets, rough soft sets and fuzzy soft sets,” Applied Soft Computing Journal, vol. 11, no. 4, pp. 3329–3332, 2011.

[11] C. H. Park, Y. B. Jun, and M. Öztürk, “Soft WS-algebras,” Communications of the Korean Mathematical Society, vol. 23, no. 3, pp. 313–324, 2008.

[12] Y. B. Jun and C. H. Park, “Applications of soft sets in ideal theory of BCK/BCI-algebras,” Information Sciences, vol. 178, no. 11, pp. 2466–2475, 2008.

[13] A. Aygünnoğlu and H. Aygün, “Introduction to fuzzy soft groups,” Computers & Mathematics with Applications, vol. 58, no. 6, pp. 1279–1286, 2009.

[14] N. Çağman and S. Enginoğlu, “Soft matrix theory and its decision making,” Computers & Mathematics with Applications, vol. 59, no. 10, pp. 3308–3314, 2010.

[15] N. Çağman and S. Enginoğlu, “Soft set theory and uni-int decision making,” European Journal of Operational Research, vol. 207, pp. 848–855, 2010.

[16] F. Feng, Y. Li, and N. Çağman, “Generalized uni-int decision making schemes based on choice value soft sets,” European Journal of Operational Research, vol. 220, no. 1, pp. 162–170, 2012.

[17] F. Feng and Y. Li, “Soft subsets and soft product operations,” Information Sciences, vol. 232, pp. 44–57, 2013.

[18] F. Feng, H. Fujita, Y. B. Jun, and M. Khan, “Decomposition of fuzzy soft sets with finite value spaces,” The Scientific World Journal, vol. 2014, Article ID 902687, 10 pages, 2014.

[19] X. Liu, F. Feng, and Y. B. Jun, “A note on generalized soft equal relations,” Computers & Mathematics with Applications, vol. 64, no. 4, pp. 572–578, 2012.

[20] X. Liu, F. Feng, and H. Zhang, “On some nonclassical algebraic properties of interval-valued fuzzy soft sets,” The Scientific World Journal, vol. 2014, Article ID 192957, 11 pages, 2014.

[21] X. Ma and H. S. Kim, “(M:N)-soft intersection BL-algebras and their congruences,” The Scientific World Journal, vol. 2014, Article ID 461060, 6 pages, 2014.

[22] G. Muhiuddin, F. Feng, and Y. B. Jun, “Subalgebras of BCK/BCI-algebras based on cubic soft sets,” The Scientific World Journal, vol. 2014, Article ID 458638, 9 pages, 2014.

[23] A. S. Sezer, “A new view to ring theory via soft union rings, ideals and bi-ideals,” Knowledge-Based Systems, vol. 36, pp. 300–314, 2012.

[24] A. S. Sezer, A. O. Atagün, and N. Çağman, “Soft intersection near-rings with its applications,” Neural Computing and Applications, vol. 21, no. 1, pp. 221–229, 2012.

[25] X. Xin and W. Li, “Soft congruence relations over rings,” The Scientific World Journal, vol. 2014, Article ID 541630, 9 pages, 2014.

[26] Y. Imai and K. Iseki, “On axiom systems of propositional calculi, XIV,” Proceedings of the Japan Academy, vol. 42, no. 1, pp. 19–22, 1966.

[27] Y. B. Jun and S. Z. Song, “Fuzzy set theory applied to implicative ideals in BCK-algebras,” Bulletin of the Korean Mathematical Society, vol. 43, no. 3, pp. 461–470, 2006.

[28] J. B. Jun and X. L. Xin, “Involutory and invertible fuzzy BCK-algebras,” Fuzzy Sets and Systems, vol. 117, no. 3, pp. 463–469, 2001.

[29] Y. B. Jun and J. Meng, “Fuzzy commutative ideals in BCI-algebras,” Communications of the Korean Mathematical Society, vol. 9, no. 1, pp. 19–25, 1994.

[30] J. Meng, Y. B. Jun, and H. S. Kim, “Fuzzy implicative ideals of BCK-algebras,” Fuzzy Sets and Systems, vol. 89, no. 2, pp. 243–248, 1997.

[31] Y. B. Jun, “Soft BCI/BCI-algebras,” Computers & Mathematics with Applications, vol. 56, no. 5, pp. 1408–1413, 2008.

[32] J. Meng and Y. B. Jun, BCK-Algebras, Kyung Moon, Seoul, South Korea, 1994.

[33] K. J. Lee, Y. B. Jun, and M. I. Doh, “Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras,” Communications of the Korean Mathematical Society, vol. 24, no. 3, pp. 353–360, 2009.