Collision Risk Analysis of Design for Traffic Signal Phasing-Sequence at Signalized Intersection

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Abstract. To improve the traffic safety at the two-phase intersection, collision risk value between the left-turn vehicle and the straight vehicle in the opposite direction could be used to judge their order. Based on the safety and risk, this method analyzed the process of conflict, modified the operating parameters and then collision risk analysis model was proposed. In the paper, one practical example was given. The result shows that the method can be applied to design the signal phasing-sequence of traffic at signalized intersection, which not only improve safety theory but also provide the theoretical basis for security information service.

1. Introduction

Signal controlling at intersections resolve the space conflict inside the intersection by time resources. Compared with other parameters of single control, phasing-sequence is a particular parameter which is closely related to intersection’s geometric condition, space design and so on. The unobstructed traffic flow at signalized intersection depends on the signal phasing-sequence, which can put traffic flow in order and reduce potential accident [1-2]. Previous research of phasing-sequence optimization confirms the phase release order according to conflict time difference based on single motion state. And vehicle speed is included in calculating time as a constant, which means that it moves in uniform motion. Also, researchers have proposed various methods of collision risk model, such as time different, overlapping interval, the difference in minimum, maximum speed and so on [3-7]. These risk indices determine the warning threshold with difficulty. And the collision risk model between left-turn and straight vehicle based on the probability analysis only considers two kinds of driver behaviour [8].

Summing up research on vehicle motion before stop-line at home and abroad [9-12], there are six motion states.

- Slow down first and then Speed up
- Drive at an even speed
- Speed up through the stop-line
- Slow down first and then drive at an even speed
- Stop by comfortable deceleration
- Brake with maximum deceleration

So in actual operation, vehicles that prepare to go into the intersection at low speed may be up while those with high speed may slow down, i.e. while they come to the intersection, the speed may be various and the speed changes occasionally [13-14]. Based on the vehicle motions above, the paper analyzes the process of conflict, modifies the operating parameters. Then the collision risk model
between the left-turn vehicle and the straight vehicle in the opposite direction is put forward from a safety perspective. On this basis, collision risk value between the left-turn vehicle and the straight vehicle in the opposite direction can be used to judge the order.

2. Parameters modification of left-turn and straight vehicle

Please straight or left-turn depend on not only phasing-sequence itself but also traffic track at the intersection during the setting of phasing-sequence. In this paper, the urban intersection is chosen as the research object. Conflict area between the left-turn vehicle and the straight vehicle at the two-phase intersection is shown in Figure 1. Related parameter formula is shown in Table 1.

![Figure 1. The left-turn and the straight vehicle motion process at intersection.](image)

| No. | Parameter                                      | No. | Parameter                                      |
|-----|------------------------------------------------|-----|------------------------------------------------|
| 1   | $R_2 = 2W_{lane} + D_2$                        | 5   | $QX = D_2 + W_e + W_{lane} + W_C$              |
| 2   | $R_1 = 3W_{lane} + D_2$                        | 6   | $D_0 = (v_F^2 - v_q^2)/2 \times a_F$           |
| 3   | $YO = \sqrt{R_1^2 - HY^2}$                     | 7   | $D_j = D_0 + D_i$                              |
| 4   | $HY = W_e + W_{lane} + D_2$                    | 8   | $D_i = 2D_2 + 4W_{lane}$                       |

- We is the distance from the right side of vehicle C to the edge of the right lane.
- WC is the width of vehicle C.
- D3 is the distance between vehicle C and the edge of entrance lane.
- Wlane is the width of the lane that vehicle C and vehicle F are running.
- D0 is vehicle F’s deceleration distance before turning.
- D1 is the distance that vehicle F moves with a constant speed.
- D2 is the distance between vehicle F and the edge of entrance lane before turning.
- Dj is the distance before vehicle F begins to turn.

The distance between right endpoint on the front of vehicle C and H point is

$$D_C^H = D_3 - D_2 + D_i - YO$$
The distance between right endpoint on the front of vehicle C and Q point is

\[ D_{CQ} = D_3 - D_2 + AQ = D_3 - D_2 + D_1 - \sqrt{R_2^2 - QX^2} \]  

(2)

The distance that vehicle C’s passing Q point is

\[ D_C = D_{CQ} + L_C \]  

(3)

Where

\[ L_C \] is the length of vehicle C.

The distance between right endpoint on the front of vehicle F and Q point is

\[ D_{KFQ} = D_f + Arc_{KFQ} = D_f + R_2 \cdot arcsin \left( \frac{R_2}{R_2} \cdot \frac{QX}{R_2} \right) \]  

(4)

The distance between right endpoint on the front of vehicle F and H point is

\[ D_{GH} = D_f + Arc_{GH} = D_f + R_f \cdot arcsin \left( \frac{R_f}{R_f} \cdot \frac{HY}{R_f} \right) \]  

(5)

3. Left-turn and straight vehicle collision risk model

As shown in Figure 1, conditions that vehicle C don’t collide with F is vehicle C has passed when vehicle F arrives, or vehicle F has left conflict area before C’s arrival. These conditions can be formulated as follows.

\[ \frac{D_C}{v_C} < \frac{D_{KFQ}}{v_F} \]  

(6)

\[ \frac{D_C}{v_C} > \frac{D_{GH} + L_F}{v_F} \]  

(7)

Where

\[ v_C \] is the speed of vehicle C; \( v_F \) is vehicle F’s; \( L_F \) is the length of vehicle F.

Collision can be avoided while satisfying either of these inequalities. Otherwise, collision accident will happen. According to straight risk study, we can build collision risk formula between the left-turn and the straight vehicle.

\[ P(R) = 1 - P\left( \frac{D_C}{v_C} < \frac{D_{KFQ}}{v_F} \right) \cup P\left( \frac{D_C}{v_C} > \frac{D_{GH} + L_F}{v_F} \right) \]  

(8)

As vehicle C has passed when vehicle F arrives and vehicle F has left conflict area before C’s arrival can’t happen at the same time, i.e. they are mutually exclusive events. So the expression above can be changed into

\[ P(R) = 1 - P\left( \frac{D_C}{v_C} < \frac{D_{KFQ}}{v_F} \right) - P\left( \frac{D_C}{v_C} > \frac{D_{GH} + L_F}{v_F} \right) \]  

(9)

Where

\[ P\left( \frac{D_C}{v_C} < \frac{D_{KFQ}}{v_F} \right) = P(v_F < K_1 \cdot v_C) \]

\[ = P\left( (v_C, v_F) \in AR_1 \right) \]

\[ = \int_{AR_1} f(v_C, v_F) \, dx \, dy = \frac{1}{K_1} \cdot S_{AR} \cdot f(v_C) \cdot f(v_F) \]  

(10)

\[ K_1 = \frac{D_{KFQ}}{D_C} \]  

(11)

\( f(v_C) \) is \( v_C \)’s density function, while \( f(v_F) \) is \( v_F \)’s. Variables are independent. \( S_{AR} \) is the area that
satisfies \( v_C \) and \( v_F \)'s change, \( v_F < K_1 v_C \). When \( v_C \) and \( v_F \) are evenly distributed.

\[
\begin{align*}
 f(v_C) &= \begin{cases} 
       1/(V_{C\text{max}} - V_{C\text{min}}), & \quad V_{C\text{min}} \leq v_C \leq V_{C\text{max}}, \\
       0, & \quad \text{other} 
\end{cases} \\
 f(v_F) &= \begin{cases} 
       1/(V_{F\text{max}} - v_q), & \quad v_q \leq v_F \leq V_{F\text{max}}, \\
       0, & \quad \text{other} 
\end{cases}
\end{align*}
\]

So we can get the expressions of vehicle F and C's maximum and minimum velocity, which can be calculated by the formulas in [4]. Due to the different motions between left-turn and straight vehicle, the left-turn vehicle has an obvious deceleration process while approaching the intersection, so turning velocity can’t exceed the specified value. Similarly,

\[
P\left(\frac{D_C^H}{v_C} > \frac{D_{gh} + L_F}{v_F}\right)
= P(v_F > K_2 v_C) = P\{v_F \in E_2\} \int_{E_2} f(v_C, v_F) \, dx \, dy
\]

Where

\[
K_2 = \frac{D_{gh} + L_F}{D_C^H}
\]

By solving \( P(v_F < K_1 v_C) \), we can get \( P(v_F > K_2 v_C) \). Finally, we get the value of \( P(R) \).

![Figure 2.](image)

Figure 2. Relationship between risk and safety crossing time.

With the formulas above, we can move on to the simulation. The trend of left-turn and straight vehicle’s collision risk value and safety crossing time is drawn by simulation, as shown in Figure 2. It corresponds with straight collision risk rule, which means the change of risk index is similar to actual vehicle time interval. Safety crossing time makes sure the last vehicle of current phase can pass through the conflict point when the first vehicle of another is running. The safety cross time between motor vehicles is 5 seconds [15-16]. Then the collision risk rule matching with 5 seconds is chosen as the standard safety value.

4. Example analysis

Figure 3 is the four-phase signal intersection design between Chaowang Road and Hushu Road in Hangzhou. Related parameter formula is shown in Table 2. Vehicle flow mainly consists of passenger cars. Assume that, straight vehicle PVC and left-turn vehicle PVF are the same style; the vehicle is
4.841×1.846×1.468 meters, and the intersection is designed according to relevant specifications, where the speed limit is 50 km/h, and maximum left-turning velocity can’t exceed 30 km/h.

![Figure 3. Intersection between Chaowang Road and Hushu Road.](image)

**Table 2.** Parameters modification of the intersection.

| No. | Parameter | No. | Parameter          |
|-----|-----------|-----|--------------------|
| 1   | \( R_2 = 2W_{lane} + D_2 \) | 5   | \( QX = D_2 + W_e + W_{PVc} \) |
| 2   | \( R_1 = 3W_{lane} + D_2 \) | 6   | \( D_6 = (v_{PVc}^2 - v_q^2)/2 \times a_f \) |
| 3   | \( YO = \sqrt{R_1^2 - HY^2} \) | 7   | \( D_j = D_0 + D_j \) |
| 4   | \( HY = W_e + D_2 \) | 8   | \( D_j = 2D_2 + SW_{lane} \) |

The collision risk value (i.e. standard safety value) in different phase sequence design at the intersection can be calculated as follows.

When straight vehicles are released first, the collision risk value is 

\[ P(R)_{straight} = 0.3722 \]

When left-turn vehicles are released first, the collision risk value is 

\[ P(R)_{left-turn} = 0.3227 \]

Because of 

\[ P(I)_{straight} = 0.3722 > P(I)_{left-turn} = 0.3227 \]

left-turn vehicles should be released first and then straight.

5. **Conclusion**

Traffic signal phasing-sequence has a close relationship with intersection geometry design at signalized intersection. Unreasonable traffic signal phasing-sequence can lead to the potential accident. In the actual cases, vehicle speed usually changes and distributes randomly in the process of approaching intersection. The collision risk model between the left-turn vehicle and the straight...
vehicle is established to reflect this feature and overcomes the deficiency of single motion state based on traffic track at the intersection. Also, collision risk value of different phasing-sequence is analyzed by the simulation software called VISSIM, and it provides a basis for phasing-sequence design. In the follow-up studies, we hope for a further experiment in the actual road to validate the effectiveness of the model.

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