Optimization of mechatronic drives motion laws for automated equipment

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Abstract. At present, when designing mechatronic cyclic drives the developer as a rule chooses the law of motion, specifying as input data, the cycle time and the amount of movement and often introducing restrictions, for example, on maximum acceleration in case of developing high-speed drives. However, this approach leads to an overestimation of the maximum drive power and, as a result, a significant increase in energy consumption. In this paper it is shown that by choosing a rational law of motion of the drive at the stage of equipment design, it is possible to achieve a sufficiently effective optimization according to various criteria. The efficiency of optimization of different motion laws of cycle drives according to various criteria is analyzed. The dependences between the maximum instantaneous power, the amount of movement, the speed and energy consumption are established and graphically presented confirming the effectiveness of the rational choice of the law of motion parameters under an inertial load. At the same time, it is shown that when synthesizing the law of motion, it is necessary to take into account the maximum possible number of parameters so that the improvement of some parameters does not lead to a decrease in other parameters.

1. Introduction

When designing robot and process equipment drives the designer is guided by a lot of various criteria. Modern achievements in the field of mechatronics allow us to implement virtually any law of motion of the process operating device providing optimization according to the selected criterion [1, 2, 3, 4, 5].

In general, the motion cycle has the following set of parameters: the amount of movement, the time of movement, the maximum instantaneous power, the energy consumed by the drive, the maximum speed and acceleration, as well as the size of the trajectory length on which the movement is carried out at a constant speed. Depending on the task, the law of motion can be selected according to one or more different criteria. However, in practice in robots and technological machines with cycle drives the laws of motion are usually chosen based on the cycle time and the amount of movement. In the case of designing high-speed drives or in the case there are any special features of technological processes, a limitation for the maximum acceleration is introduced. This approach to the selection of the law of motion usually leads to an unjustified overestimation of the maximum drive power and an increase in energy consumption [6, 7, 8, 9, 10]. Often a reduction in the maximum power and energy consumption and an increase in productivity can be achieved using machines in operation without any modernization only by adjusting the program that controls the drive [11, 12]. Even more efficient
optimization of the law of motion can be achieved with a rational choice of the law of motion of the drives at early stages of equipment development. This may include a reduction in the size of the motor, gearbox, electrical converters, battery, etc.

Thus, the problem of optimizing the law of motion of the cyclic drive is quite relevant. However, first, it is necessary to quantify the effectiveness of such an optimization per each possible criterion.

2. Materials and methods
As an example of an algorithm for studying the efficiency of optimizing the law of motion according to the criteria that correspond to the cycle parameters mentioned above, the rectangular law of acceleration variation $\dot{\rho}(\alpha, \zeta)$ is selected as the most commonly used in practice. In this case, the expressions for the piecewise linear dimensionless functions of acceleration $\dot{\rho}(\alpha, \zeta)$, speed $\dot{\rho}(\alpha, \zeta)$, and displacement $\rho(\alpha, \zeta)$ have the following form:

$$\dot{\rho}(\alpha, \zeta) = \begin{cases} \frac{1}{\zeta(1-\zeta)}, & 0 \leq \alpha \leq \zeta \\ 0, & 0 < \alpha \leq 1 - \zeta \\ \frac{1}{\zeta(1-\zeta)}, & 1 - \zeta < \alpha \leq 1 \end{cases}$$

(1)

$$\dot{\rho}(\alpha, \zeta) = \begin{cases} \frac{1}{1-\zeta}, & 0 \leq \alpha \leq \zeta \\ \frac{1-\alpha}{\zeta(1-\zeta)}, & 1 - \zeta < \alpha \leq 1 \end{cases}$$

(2)

$$\rho(\alpha, \zeta) = \begin{cases} \frac{a^2}{2\zeta(1-\zeta)}, & n \leq \alpha \leq \zeta \\ \frac{\zeta-2\alpha}{2(\zeta-1)}, & \zeta < \alpha \leq 1 - \zeta \\ \frac{1-a^2}{2\zeta(1-\zeta)+1}, & 1 - \zeta < \alpha \leq 1 \end{cases}$$

(3)

where $\alpha = t/T$ is the dimensionless current time; $t$ is the current time; $T$ is the cycle time; $\zeta = \tau/T$ is the dimensionless acceleration and deceleration time (here they are assumed to be the same); $\tau$ is the acceleration time.

The actual acceleration and speed can be determined by multiplying the dimensionless acceleration and the dimensionless speed by the coefficients $K' = s/T^2$ and $K'' = s/T$. Here $s$ is the amount of movement.

The dependences of the acceleration, speed and movement on time at different values of $\zeta$ are graphically presented in Figure 1.

![Figure 1](image-url). The graphs of dependence of the acceleration, speed and movement on time at different values of $\zeta$. 

(a) For $\zeta = 0.3$  
(b) For $\zeta = 0.5
In the case of the prevailing inertial load which is inherent in many robot mechanisms and technological machines, the maximum power value \( N \) for the rectangular law of acceleration variation takes place at the end of the acceleration region and equals:

\[
N = K' \dot{\rho} (\zeta, \zeta) K' \ddot{\rho} (\zeta, \zeta)m = \eta(\zeta, \zeta)K_\eta = ms^2/(\tau(T-\tau)^2)
\]  
(4)

where \( m \) is the reduced mass; \( \eta(\alpha, \zeta) \) is the dimensionless power; \( K_\eta = ms^2/T^3 \) is the dimensionless power factor.

Solving this equation with respect to \( T \), we can obtain the following expression:

\[
T(N, \tau, s) = s\sqrt{m/N\tau} + \tau.
\]  
(5)

The dependences of the cycle time on the acceleration time at different power values are graphically shown in Figure 2.

Figure 2. The graphs of dependence of the cycle time on the acceleration time at different power and travel values

(a) For \( s = 1 \) m and \( m = 1 \) kg at different power values \( N = 2 \) Wt, \( N = 4 \) Wt, \( N = 6 \) Wt
(b) For \( N = 3 \) Wt and \( m = 1 \) kg at different travel values \( s = 1 \) m, \( s = 2 \) m, \( s = 4 \) m

The analysis of the graphs presented above shows the following. Firstly, an increase in the power at a constant mass (an increase in the specific power) leads to a decrease in the cycle time and a decrease in the optimal acceleration time. Secondly, an increase in the amount of movement at a constant cycle time (an increase in the average speed) leads to an increase in the value of the optimal acceleration time and the disappearance of the well-defined extremum.

Since in the latter expression the first term decreases and the second term increases as the acceleration time increases, we can assume that the function has an extremum. The necessary condition for the existence of an extremum has the following form:

\[
dT(N, \tau, s)/d\tau = 1 - s/2\sqrt{m/Ns^3} = 0.
\]  
(6)

The second derivative of this expression which is equal to zero for any positive values of the parameters indicates the presence of the minimum for the function under consideration:

\[
\tau_{\text{min}} = s/2\sqrt{m/N}.
\]  
(7)

With this acceleration time, the minimum cycle time is defined as:

\[
T(N, \tau_{\text{min}}, s) = s\sqrt{3/m} + T/3.
\]  
(8)

To evaluate the efficiency of optimizing the acceleration time by the criterion of high-speed response at the given displacement and power, the following formula is used:
By substituting \( N \) expressed in terms of the dimensionless power, we can deduce that:

\[
\delta T(N, \tau, s) = \frac{T(N, \tau, s) - T(N, \tau_{\text{min}}, s)}{T(N, \tau_{\text{min}}, s)} \times 100\% = \frac{s}{\sqrt{\frac{m}{N\tau}}} \left(1 - \frac{3\sqrt{\frac{m}{2N}}}{\sqrt{4N}}\right) \times 100\%.
\] (9)

The given formula allows one to determine by how many percent the cycle time increases in comparison with the minimum possible value for the acceleration time arbitrary selected. Similarly, we can formulate the problem of ensuring maximum travel at the given power and time values.

The dependence of the amount of movement on the acceleration time at the given power and movement time can be obtained from the formula (4):

\[
s(\tau) = (T - \tau) \sqrt{N\tau/m}.
\] (11)

The graphs of the dependence of the displacement on the acceleration time at different power values are shown in Figure 3.

From the curves presented in Figure 3 taking into account the formula (11), the following conclusions can be drawn. Firstly, when changing the acceleration time of the reduced mass of moving parts and the drive power, the optimal acceleration time remains constant and equals one-third of the period. Secondly, with a proportional change in mass and power, the specific power and the travel remain constant.

(a) For \( m = 1 \text{ kg} \) at different power values
(b) For \( N = 6 \text{ Wt} \) at different masses
\( N = 2 \text{ Wt}, N = 4 \text{ Wt}, N = 6 \text{ Wt} \)
\( m = 1 \text{ kg}, m = 2 \text{ kg}, m = 4 \text{ kg} \)

**Figure 3.** The graphs of dependence of the displacement on the acceleration time at different power values

The function under consideration has a maximum because the first factor decreases and the second factor increases as the argument increases.

The necessary condition for the existence of a maximum has the following form:

\[
ds(\tau)/d\tau = (T - 3\tau)/2\sqrt{N\tau/m} = 0.
\] (12)

Solving this equation with respect to \( \tau \), we obtain the following expression:

\[
\tau = T/3 \quad \text{or} \quad \xi_{\text{max}} = 1/3.
\] (13)

With this value of the acceleration and deceleration time, the movement has the maximum value of:

\[
s(T/3) = 2T\sqrt{NT/27m}.
\] (14)
To evaluate the efficiency of optimizing the acceleration time according to the maximum displacement criterion, the relative decrease of the maximum displacement is used:

$$\delta s(\zeta) = \frac{\rho(\zeta_{\text{max}}) - \rho(\zeta)}{\rho(\zeta_{\text{max}})} \times 100\% = \left[ 1 - \sqrt{2\pi \zeta \left( \frac{1-\zeta}{2} \right)} \right] \times 100\%.$$  \hspace{1cm} (15)

The value of $\delta s(\zeta)$ shows by how many percent the travel value will decrease when the optimal acceleration time $\zeta_{\text{max}}$ is replaced by an arbitrary value of $\zeta$.

According to the formula (4), the value of the dimensionless power under the prevailing inertial load is determined by the expression:

$$\eta(\alpha, \zeta) = \dot{p}(\alpha, \zeta) / \dot{p}(\alpha, \zeta).$$ \hspace{1cm} (16)

The maximum power value in the cycle equals the following:

$$\eta_m(\zeta) = \dot{p}(0, \zeta) / \dot{p}(\zeta, \zeta).$$ \hspace{1cm} (17)

The dependence of the maximum power in the cycle on the acceleration time is graphically presented in Figure 4.

The minimum power under the inertial load for the rectangular law of motion occurs at the acceleration time equal to one-third of the period at the end of the acceleration region:

$$\zeta_{\text{min}} = 1/3.$$ \hspace{1cm} (18)

The value of the minimum power in this case is defined by:

$$\eta(\zeta_{\text{min}}) = 27/4.$$ \hspace{1cm} (19)

The relative efficiency of optimizing the acceleration time according to the power minimization criterion for a given cycle time and displacement is estimated by the formula:

$$\delta\eta(\zeta) = \frac{\eta(\zeta) - \eta(\zeta_{\text{min}})}{\eta(\zeta_{\text{min}})} \times 100\% = \left[ 1 / (\zeta (1 - \zeta)^2) \right] \times 100\%.$$ \hspace{1cm} (20)

At the stage of acceleration, the power increases according to the linear law and time dependence is a right triangle.

The energy consumption $p(\zeta)$ in the absence of recuperation is numerically equal to the area of this triangle:

$$p(\zeta) = \frac{\zeta}{2} \dot{p}(\zeta, \zeta) \dot{p}(\zeta, \zeta) = 1/(2(1 - \zeta)^2).$$ \hspace{1cm} (21)

The dependence of the energy consumption of one cycle on the acceleration time is graphically shown in Figure 5.

**Figure 4.** The graph of dependence of the maximum power in the cycle on the acceleration time

**Figure 5.** The graph of dependence of the energy consumption of one cycle on the acceleration time
In contrast to the power, when the acceleration time tends to zero, the work (in other words, the energy consumption) is finite in its magnitude. This is due to the fact that the infinitely high power acts on the infinitely small time interval.

This function is monotonically increasing from 1 to 4 in the interval of $[0; 0.5]$.

The efficiency of the optimization of the law of motion according to the criterion of the minimum energy consumption for a given amount of movement and cycle time is estimated by the formula:

$$\delta p(\zeta) = \frac{p(\zeta) - p(\zeta_{\text{min}})}{p(\zeta_{\text{min}})} 100\% = \zeta(2 - \zeta)/(1 - \zeta)^2 100\%.$$  \hspace{1cm} (22)

The maximum speed of movement depends on the acceleration time and is determined according to the following formula:

$$v(\zeta) = \dot{\rho}(\zeta, \zeta).$$  \hspace{1cm} (23)

The curve of dependence of the maximum speed on the acceleration time is graphically presented in Figure 6.

The minimum speed at the execution of the cycle for the rectangular law of motion is observed when the acceleration time $\zeta_{\text{min}}$ tends to zero and the time and displacement are specified and is determined by the following equation:

$$\dot{\rho}(\zeta_{\text{min}}, \zeta_{\text{min}}) = 1.$$  \hspace{1cm} (24)

In this case, the efficiency of optimization by the speed minimization criterion is estimated by the formula as follows:

$$\delta v(\zeta) = \frac{v(\zeta) - v(\zeta_{\text{min}})}{v(\zeta_{\text{min}})} 100\% = \zeta/(1 - \zeta) 100\%.$$  \hspace{1cm} (25)

The maximum acceleration is determined by the expression:

$$a(\zeta) = \dot{\rho}(0, \zeta).$$  \hspace{1cm} (26)

The dependence of the maximum acceleration on the acceleration time is graphically presented in Figure 7 below.

The minimum acceleration for the rectangular law of motion is observed when the acceleration time is being the half of the period and equals:

$$\dot{\rho}(\zeta_{\text{min}}, \zeta_{\text{min}}) = 4.$$  \hspace{1cm} (27)

The efficiency of optimizing the acceleration time of the law of motion according to the acceleration minimization criterion is determined by the following expression:
\[ \delta a(\zeta) = \frac{a(\zeta) - a(\zeta_{min})}{a(\zeta_{min})} \cdot 100\% = \frac{(2\zeta - 1)^2}{4\zeta(1-\zeta)} \cdot 100\%. \tag{28} \]

Figure 8 shows the dependence curves for the efficiency of optimizing the acceleration time according to the criteria proposed in the text.

**Figure 8.** The dependence curves of the optimizing efficiency of the acceleration time according to the criteria of the minimum speed, cycle time, acceleration, power, energy consumption and according to the criteria of the maximum travel and the length of the region of movement with a constant speed.

3. Results and Discussion
The analysis of the presented dependencies showed the following.
1. The efficiency of optimization according to the criteria of the minimum speed and the maximum of the section of the movement trajectory with a constant speed is completely identical.
2. The efficiency of optimization the speed and the length of the section of the movement trajectory with a constant speed reaches 100%.
3. The efficiency of optimization according to the criterion of maximum travel at a given time reaches 60%.
4. The efficiency of optimization according to other criteria significantly exceeds 100%.

4. Conclusion
Based on the presented results, it is established that the optimization of the laws of motion is not only possible, but also quite effective. However, the improvement of some parameters leads to a significant decrease in other parameters. The optimal result in the synthesis of the law of motion can be achieved only with all the parameters taken into account.

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