Accelerated Non-Coaxial Rotating Flow of MHD Viscous Fluid with Heat and Mass Transfer

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Abstract. An exact analysis on heat and mass transfer of mixed convection MHD viscous fluid flow due to accelerated non-coaxial rotation is analyzed. This analysis is considered through a porous medium. The rotation and buoyancy forces involved in this problem give effects on the fluid motion. Dimensional momentum, energy and concentration equations are presented in partial differential equations with initial and boundary conditions which then are reduced into non-dimensional form by introducing non-dimensional variables. This problem is analytically solved by using Laplace transform method and the exact solution obtained for velocity, temperature and concentration profile are graphically plotted in order to investigate the impact of Grashof number \( G_r \), modified Grashof number \( G_m \), Prandtl number \( Pr \) and Schmidt number \( Sc \) as well as accelerated, magnetic and porosity parameter on its behavior. The results show that the increasing acceleration parameter raises the velocity profile while increasing of Prandtl number and Schmidt number leads to a fall in velocity profile. A verification of obtained solution is conducted by comparing present results with published results and it was found to be in a good agreement.

1. Introduction
The fluid flow related to the heat and mass transfer problems on mixed convection due to the rotating frame has drowned significant attention from many researchers because of their great applications of rotating machinery in aeronautical, marine and industrial. Providing that the rotating flow design and model has provided great capabilities in jet engine, gas turbine, vacuum cleaner, pumps, blowers and etc. The influence of non-coaxial rotation in analyzing the fluid flow problem has grabbed special interest and an exact solution for non-coaxial rotating flow of viscous fluid at infinity is obtained by Erdogan [1]. This work is further continued by Erdogan [2] in solving the time-dependent Navier Stokes equations for the flow under the non-coaxial rotations of a disk which executed oscillations at its own plane. The similar problem is carried out by Hayat et al. [3] by considering the non-coaxial rotating of porous disk with effect of superimposed injection or suction. Ersoy [4] investigated the unsteady flow that affected by the opposite direction of the disk oscillations at their own planes while rotating eccentrically. After that, Ersoy [5] expanded the work in periodic flow caused by two porous disks with non-torsional oscillation and rotate with same angular velocity about non-coincident axes.
Then, same problem of non-coaxial rotation Newtonian fluid flow is studied by Ersoy [6] without implementing the effect of porosity.

In recent years, the effect of magnetic field in numerous applications are explored from different areas such as automotive and electronic sensor, rotating transformers and medical imaging devices. The non-coaxial rotation of electrically conducting flow with the presence of magnetic field for an oscillatory disk is examined by Hayat et al. [7]. Next, Asghar et al. [8] studied non-Newtonian flow with a moving porous disk at a uniform acceleration and Hayat et al. [9] analyzed the non-Newtonian flow generated by an oscillating porous disk. In both studies, the impact of magnetic field is taken into account by using second-grade fluid at infinity. Apart from that, the combination of Hall current and slip condition effects on the unsteady MHD flow over non-coaxial rotations disk embedded in porous medium is discussed by Guria et al. [10]. In this study, Laplace transform technique is used to solve for an exact solution. The same technique is also utilized by Hayat et al. [11] in obtaining the exact solution of the velocity profile for MHD non-coaxial rotating flow in the presence of Hall current, rotation and suction or blowing effect. An exact solution for non-coaxial rotating free convection flow with an oscillating plate is obtained by Mohamad et al. [12]. Then, the previous work is extended by Mohamad et al. [13] in analyzing the heat and mass transfer effect and Mohamad et al. [14] by discussing the double convection of non-coaxial rotating MHD flow in a porous medium.

From all the above reviews, the implementation of accelerated boundary condition together with mass transfer effect have not been emphasized in the non-coaxial rotation flow. Chaudhary and Jain [15] utilized Laplace transform method to get an exact solution for MHD convection flow over an accelerated vertical plate embedded in porous medium. Then, Narahari and Debnath [16] continued the similar work in the presence of heat flux and heat generation or absorption. Rajesh [17] and Hussanan et al. [18] investigated the unsteady MHD flow over an accelerated plate in the existence of magnetic field and porosity. Mohamad et al. [19] analyzed the heat transfer on free convection of second grade fluid flowing past an accelerated plate. Most recently, Mohamad et al. [20] obtained the exact solution for MHD mixed convection flow of viscous fluid due to the accelerated non-coaxial rotation with the effect of porosity. Motivated with the above literature, the aim for present study is to investigate mixed convection of viscous fluid flow induced by accelerated non-coaxial rotation embedded in porous medium with heat and mass transfer analysis. The exact solution for velocity, temperature and concentration profile are obtained by using Laplace transform method. The impact of related parameters is graphically plotted coupled with its discussion.

2. Mathematical Formulation
Considered a Cartesian coordinate of unsteady flow of incompressible viscous fluid occupying semi finite space \( z > 0 \) with \( z \)-axis is normal to rigid disk and \( x \)-axis is taken in upward direction along the disk. The rotation axes for the disk and fluid are assumed to be in the plane \( x = 0 \) and the distance between rotation axes is \( t \). A uniform strength of transverse magnetic field \( B_0 \) is applied perpendicular to the direction of flow of the fluid and the disk passing through porous medium. Initially, at time \( t = 0 \), the disk and the fluid at infinity are rotating with the same angular velocity \( \Omega \) about \( z' \)-axis with temperature \( T_0 \). After time \( t > 0 \), the disk suddenly starts to accelerate rotate with same angular velocity \( \Omega \) about \( z \)-axis while the fluid at infinity continues to rotate with the same angular velocity \( \Omega \) about the \( z' \)-axis as that of the disk. The temperature of the disk raises to \( T_w \) respectively. The model of problem is represented as in Figure 1. Regarding to these assumptions, the solutions for velocity, temperature and concentration profiles are established by using the governing equations as below

\[
\frac{\partial F}{\partial t} + dF = \frac{\partial^2 F}{\partial z^2} + d\Omega \ell + g\beta_f (T - T_0) + g\beta_c (C - C_0)
\]

\[
\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}
\]
\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}
\]  

(3)

together with initial and boundary conditions

\begin{align*}
F(z,0) &= \Omega t \quad \forall z > 0, \quad F(0,t) = A t & F(\infty,t) &= \Omega t \quad \forall t > 0 \\
T(z,0) &= T_\infty \quad \forall z > 0, \quad T(0,t) = T_w & T(\infty, t) &= T_\infty \quad \forall t > 0 \\
C(z,0) &= C_\infty \quad \forall z > 0, \quad C(0,t) = C_w & C(\infty, t) &= C_\infty \quad \forall t > 0
\end{align*}

(4)

(5)

(6)

where \( d = \Omega i + \frac{\sigma B^2}{\rho} + \frac{v}{k_1} \).

Figure 1: Schematic diagram of the problem

Here, the solutions are presented in term of complex velocity function, \( F = f + ig \), where \( f \) and \( g \) are the primary velocity (real part) and secondary velocity (imaginary part) respectively. Then, \( v \) is the kinematic viscosity, \( g \) is the gravity, \( \beta_T \) is the volumetric coefficient of thermal expansion for temperature, \( \beta_C \) is the volumetric coefficient of mass transfer, \( T \) is the temperature function, \( \rho \) is the density of rotation fluid, \( C_p \) is the specific heat capacity, \( k \) is the thermal conductivity, \( A \) is the acceleration of the disk and fluid, \( \sigma \) is an electrical conductivity and \( k_1 \) is the permeability of the porous medium.

The suitable non-dimensional variables are introduced as follows

\[
F^* = \frac{F}{\Omega \ell}, \quad z^* = \frac{z}{t}, \quad t^* = \Omega t, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad C^* = \frac{C - C_w}{C_w - C_\infty}.
\]

(7)

By using (7), the dimensional governing equations (1), (2), (3) with the initial and boundary conditions (4), (5), (6) are converted into non-dimensional form which then can be written as

\[
\frac{\partial F}{\partial t} + d_i F = \frac{\partial^2 F}{\partial z^2} + GrT + GmC
\]

(8)

\[
\frac{\partial T}{\partial t} = \frac{1}{Pr} \frac{\partial^2 T}{\partial z^2}
\]

(9)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2}
\]

(10)

corresponding to non-dimensional initial and boundary conditions

\begin{align*}
F(z,0) &= 0 \quad \forall z > 0, \quad F(0,t) = \frac{A t}{\Omega \ell} - 1 & F(\infty, t) &= 0 \quad \forall t > 0 \\
T(z,0) &= 0 \quad \forall z > 0, \quad T(0,t) = 1 & T(\infty, t) &= 0 \quad \forall t > 0 \\
C(z,0) &= 0 \quad \forall z > 0, \quad C(0,t) = 1 & C(\infty, t) &= 0 \quad \forall t > 0
\end{align*}

(11)

(12)

(13)
where \( d_1 = i + M^2 + \frac{1}{K} \), 
\( M = \frac{\sigma B_0^2}{\rho \Omega} \), 
\( 1 = \frac{\nu}{\Omega k_1} \), 
\( A = \frac{A_1}{\Omega \ell} \), 
\( Gr = \frac{g \beta r \Delta T}{\Omega^2 \ell} \), 
\( Gm = \frac{g \beta c (C_m - C_a)}{\Omega^2 \ell} \), 
\( Pr = \frac{\mu C_p}{k} \), and \( Sc = \frac{\nu}{D} \).

Here, \( d_1 \) is a constant parameter, \( M \) is magnetic field parameter, \( K \) is permeability parameter, \( A \) is acceleration parameter, \( Gr \) is Grashof number, \( Gm \) is modified Grashof number, \( Pr \) is Prandtl number and \( Sc \) is Schmidt number.

3. Solution of Problem

After that, Laplace transform method is applied onto non-dimensional governing equations, Eqs. (8), (9), (10) as well as non-dimensional initial and boundary conditions, Eqs. (11), (12), (13). Hence, it transforms to

\[
\frac{\partial^2}{\partial z^2} F(z,q) - (q + d_1) F(z,q) = -Gr \frac{1}{q} \exp{-z\sqrt{Pr q}} - Gm \frac{1}{q} \exp{-z\sqrt{Sc q}} \tag{14}
\]

\[
\bar{F}(0,q) = \frac{A t}{\Omega \ell} \frac{1}{q^2} - \frac{1}{q}, \quad \bar{F}(\infty,q) = 0 \tag{15}
\]

\[
\frac{\partial^2}{\partial z^2} \bar{T}(z,q) - Pr q \bar{T}(z,q) = 0 \tag{16}
\]

\[
\bar{T}(0,q) = \frac{1}{q}, \quad \bar{T}(\infty,q) = 0 \tag{17}
\]

\[
\frac{\partial^2}{\partial z^2} \bar{C}(z,q) - Sc q \bar{C}(z,q) = 0 \tag{18}
\]

\[
\bar{C}(0,q) = \frac{1}{q}, \quad \bar{C}(\infty,q) = 0 \tag{19}
\]

Next, the system of equations stated in Eqs. (14), (16), (18) are solved using the boundary conditions in (15), (17), (19). The inverse Laplace is taken on the resulting solutions to obtain the exact solution for velocity, temperature and concentration profile. Then, the solution forms as

\[
F(z,t) = F_1(z,t) - F_2(z,t) - F_3(z,t) + F_4(z,t) - F_5(z,t) + F_6(z,t) + F_7(z,t) + F_8(z,t) - F_9(z,t) \tag{20}
\]

\[
T(z,t) = \text{erfc} \left( \frac{z}{2 \sqrt{Pr t}} \right) \tag{21}
\]

\[
C(z,t) = \text{erfc} \left( \frac{z}{2 \sqrt{Sc t}} \right) \tag{22}
\]

where

\[
F_1(z,t) = \frac{A \ell}{2} \int_0^t \exp\left(-z\sqrt{d_1 s}\right) \text{erfc}\left(\frac{z}{2 \sqrt{s}} - \sqrt{d_1 s}\right) ds,
\]

\[
F_2(z,t) = \frac{1}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{1}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_3(z,t) = \frac{Gr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Gr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_4(z,t) = \frac{Gm}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Gm}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_5(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_6(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_7(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_8(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_9(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]

\[
F_10(z,t) = \frac{Pr}{2} \exp(z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} + \sqrt{d_1 t}\right) + \frac{Pr}{2} \exp(-z\sqrt{d_1}) \text{erfc}\left(\frac{z}{2 \sqrt{d_1}} - \sqrt{d_1 t}\right),
\]
\[ F_4(z,t) = \frac{G_r}{2} \exp \left( d_4 t + z \sqrt{d_1 + d_4} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} \pm \sqrt{(d_1 + d_4) t} \right) + \]

\[ \frac{G_r}{2} \exp \left( d_4 t - z \sqrt{d_1 + d_4} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} - \sqrt{(d_1 + d_4) t} \right), \]

\[ F_5(z,t) = \frac{G_m}{2} \exp \left( z \sqrt{d_1} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} + \sqrt{d_1 t} \right) + \frac{G_m}{2} \exp \left( -z \sqrt{d_1} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} - \sqrt{d_1 t} \right), \]

\[ F_6(z,t) = \frac{G_m}{2} \exp \left( d_6 t + z \sqrt{d_1 + d_6} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} + \sqrt{(d_1 + d_6) t} \right) + \]

\[ \frac{G_m}{2} \exp \left( d_6 t - z \sqrt{d_1 + d_6} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} - \sqrt{(d_1 + d_6) t} \right), \]

\[ F_8(z,t) = \frac{G_r}{2} \exp \left( d_8 t + z \sqrt{Pr d_3} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} + \sqrt{Pr d_3 t} \right) + \]

\[ \frac{G_r}{2} \exp \left( d_8 t - z \sqrt{Pr d_3} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} - \sqrt{Pr d_3 t} \right), \]

\[ F_{10}(z,t) = \frac{G_m}{2} \exp \left( d_{10} t + z \sqrt{Sc d_5} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} + \sqrt{Sc d_5 t} \right) + \frac{G_m}{2} \exp \left( d_{10} t - z \sqrt{Sc d_5} \right) \text{erfc} \left( \frac{z}{2 \sqrt{t}} - \sqrt{Sc d_5 t} \right). \]

4. Results and Discussion

The best understanding on the fluid flow in this problem is assisted by the pictorial discussion of parameter acceleration \( A \), magnetic \( M \) and porosity \( K \) as well as Prandtl number \( Pr \), Schmidt number \( Sc \), Grashof number \( Gr \) and modified Grashof number \( Gm \) from Figures 2 to 11.

As the result, the impact of \( A \) is displayed on Figure 2 where velocity increases by increasing the values of \( A \). This effect is due to the imposing of accelerated disk which also acts as the external force that enhances the movement of rotation flow. The reverse effect is noticed in Figure 3 when the values of \( M \) increase, the velocity decreases. This is because the implementation of higher \( M \) cause the drag force also known as Lorentz force acting on fluid to increase and thus reduces the velocity as the fluid flows through a high resistance. Different to porosity effect, when the values of \( K \) increase, the fluid able to flow with less frictional force due the less pressure is acted on the medium. This results in the decreasing of velocity as clearly depicted in Figure 4.

The influence of \( Pr \) and \( Sc \) on velocity profile are shown in Figures 5 and 6 while temperature and concentration profiles presented in Figures 7 and 8. Both velocity profiles show a decelerating trend as the values of \( Pr \) and \( Sc \) increase. This trend is due to the domination of momentum diffusivity as well as increase the fluid viscosity and the velocity is reduced. These effects are followed by declining in temperature and concentration profiles as well as accompanied by decreasing of thermal boundary layer and mass transfer boundary layer. However, the opposite effect is observed for \( Gr \) and \( Gm \) where increase in \( Gr \) and \( Gm \) will increase the velocity as depicted in Figure 9 and 10. This effect is possible as the buoyancy force and inertia force dominated the flow. The fluid become less viscous and thus increase the fluid velocity. An excellent agreement is observed as in Figure 11 and Table 1 when present results are compared with published results from Mohamad et al. [20] by letting \( Gm = 0 \) and \( Sc = 0 \).

| \( z \) | \( A \) | \( M \) | \( K \) | \( Pr \) | \( Gr \) |
|-------|-----|-----|-----|-----|-----|
|       | Primary | Secondary | Primary | Secondary |
| Present results, Eq. (20) | Mohamad et al. (2020), Eq. (15) |

Table 1. Comparison of present results and published results by Mohamad et al. (2020)
| 0 | 2.0| 2.0| 2.0| 6.2| 5.0| 1.000| 0 | 1.000| 0 |
|---|----|----|----|----|----|-------|---|-------|---|
| 0.2| 2.0| 2.0| 2.0| 6.2| 5.0| 0.7478| 0.0313| 0.7478| 0.0313 |
| 0.4| 2.0| 2.0| 2.0| 6.2| 5.0| 0.5420| 0.0404| 0.5420| 0.0404 |
| 0.6| 2.0| 2.0| 2.0| 6.2| 5.0| 0.3779| 0.0385| 0.3779| 0.0385 |
| 0.8| 2.0| 2.0| 2.0| 6.2| 5.0| 0.2528| 0.0319| 0.2528| 0.0319 |
| 1.0| 2.0| 2.0| 2.0| 6.2| 5.0| 0.1623| 0.0243| 0.1623| 0.0243 |

**Figure 2.** The effect of $A$ on primary (a) and secondary (b) velocity profile

**Figure 3.** The effect of $M$ on primary (a) and secondary (b) velocity profile

**Figure 4.** The effect of $K$ on primary (a) and secondary (b) velocity profile
Figure 5. The effect of $Pr$ on primary (a) and secondary (b) velocity profile

$Pr = 5, 6.2, 7.2$

Figure 6. The effect of $Sc$ on primary (a) and secondary (b) velocity profile

$Sc = 0.62, 0.78, 2.0$

Figure 7. The effect of $Pr$ on temperature profile

$Pr = 5, 6.2, 7.2$

Figure 8. The effect of $Sc$ on concentration profile

$Sc = 0.62, 0.78, 2.0$

Figure 9. The effect of $Gr$ on primary (a) and secondary (b) velocity profile

$Gr = 5, 6, 7$
5. Conclusion
The unsteady non-coaxial rotating convection flow of MHD viscous fluid over an accelerated plate saturated in a porous medium is investigated analytically using Laplace transform method. The impact of pertinent parameters is graphically plotted with a comprehensive discussion. The validity of obtained results is verified by having an excellent agreement with the previous published result. In this study, the main results are concluded as follow

- The increasing values of $A, K, Gr$ and $Gm$ cause both primary and secondary velocities increase.
- The increasing values of $M, Pr$ and $Sc$ cause both primary and secondary velocities decrease.
- Temperature decreases with increasing values of $Pr$ and thermal boundary layer thinner.
- Concentration decrease with increasing values of $Sc$ and mass transfer boundary layer thinner.

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