Peristaltic Transport and Heat and Mass Transfer on a Power-law Fluid Flow in an Elastic Tapered Tube with Variable Cross-Section Induced by Dilating Peristaltic Wave

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Abstract. Peristaltic transport and heat and mass transfer on a power-law fluid flow in an elastic tapered tube with variable cross-section induced by dilating peristaltic wave was illustrated in this paper. Problem was studied under the assumption of long wavelength ($δ ≪ 1$) and low Reynolds number ($Re ≪ 1$). The different emerging parameters effect are explained graphically for the exact solutions of the temperature distribution, heat transfer coefficient, rate of heat transfer and concentration of particles. The results showed that the temperature distribution and heat transfer coefficient are an increasing functions of the parameters power-law index $n$, dilation parameter $k$, amplitude ratio $ϕ$ and volume of flow rate $F$ whereas they are decreasing functions when the non-uniform parameter $b$ increases. It is noted an opposite behaviour for the rate of heat transfer and concentration distribution.

MATHEMATICA software is used to plot all figures.

Keywords: Peristaltic transport, Dilating peristaltic, Power-law fluid, Nusselt number.

Nomenclature

| Symbol | Meaning |
|--------|---------|
| $Z, z$ | dimension, dimensionless axial coordinate |
| $R, r$ | radius of the tube at the inlet |
| $a$ | dimension, dimensionless radial coordinate |
| $c$ | Wave speed |
| $b, k$ | dimension, dimensionless slope of the tube |
| $Φ, φ$ | dimension, dimensionless dilation parameter |
| $n$ | fluid behavior index |
| $m$ | consistency index |
| $ρ$ | density |
| $D_m$ | coefficient of mass diffusion |
| $κ$ | thermal conductivity |
| $K_T$ | thermal diffusion ratio |
| $T_m$ | mean temperature |
| $T_1, T_0$ | the wall and center temperature of the tube |
| $C_w, C_0$ | the wall and center concentration of the tube |
| $C_p$ | specific heat |
| $H, h$ | dimension, dimensionless wall of the tube |

$\bar{U}, \bar{W}$ | dimension components of velocity in fixed frame |
| $\bar{u}, \bar{w}$ | dimension components of velocity in wave frame |
| $\bar{p}, \bar{P}$ | dimension, dimensionless pressure |
| $T, \bar{T}$ | dimension, dimensionless temperature |
| $C, \bar{C}$ | dimension, dimensionless concentration |
| $\bar{t}_{ij}, \bar{τ}_{ij}$ | dimension, dimensionless shear stress |
| $S_{ij}$ | dimension, dimensionless shear strain |
| $Re$ | Reynolds number |
| $Br$ | Brinkman number |
| $Pr$ | Prandtl number |
| $Ec$ | Eckert number |
| $Sc$ | Schmidt number |
| $Sr$ | Soret number |
| $Θ$ | dimension, dimensionless stream function |
| $q, \bar{q}$ | dimension volume flow rate |
| $F$ | dimensionless volume flow rate |
1. Introduction
Peristalsis is a process which helps in moving of fluids in many engineering and physiological systems. Peristalsis is the mechanism of the fluids transport that occur by a sinusoidal wave of a rear contraction / expansion, which trains along the flexible walls of the tube. A peristaltic application can be found in locomotion of worms, urine transport from kidney to bladder and many others. Peristaltic behaviour can also exist in mathematical devices in the transportation of corrosive materials, open-heart sidestep heart-lung machines, dialysis, etc. [1, 2].

The form of energy is called heat. Heat transport which a cross a boundary due to the temperature difference is called heat transfer. Mass transfer is a process of transfer of mass as a result of species concentration difference in a system or mixture [3]. Many research have been developed to analysis the heat and mass transfer in peristaltic fluids flow. Pal [4] has examined heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. Ravikumar et al [5] explained the motion of a power-law fluid moving under peristaltic, through an asymmetric channel having permeable wall. Vajravelu et al. [6] analysed the flexible nature of the tube by taking the Herschel-Bulkley model to understand the flow of blood through arteries. Sucharitha et al. [7] studied the impact of slip and heat transfer on peristaltic flow of a Herschel-Bulkley fluid. Yang et al. [8] evaluated peristaltic transport of mixture flow of particles and power-law fluid in round tube. Murad and Abdualhadi [9] studied the peristaltic transport of power-law fluid in an elastic tapered tube with variable cross-section induced by dilating peristaltic wave.

In view of the above studies a mathematical model is considered to analyze the effect of heat and mass transfer on a power-law fluid in an elastic tapered tube with variable cross-section induced by dilating peristaltic wave. The exact expressions for axial velocity, radial velocity, stream function, pressure gradient, volume flow rate, temperature, heat transfer coefficient, rate of heat transfer and concentration distributions are obtained. The effect of all pertinent parameters on flow are explain through graphs.

2. Mathematical Formulation
We consider a peristalsis transport of an incompressible power-law fluid flow in an axisymmetric cylindrical, with variable cross sections, of an elastic tube with radius $H(z)$ and of length $L$ under the effect of heat and mass transfer. The peristaltic flow is generated by a sinusoidal wave travels with dilating amplitude on the tube walls travelling with constant speed $c$, as shown in figure 1.

In a cylindrical coordinate system ($\bar{R}, \varphi, \bar{Z}$), where $\bar{Z} – axis$ lies along the center line of the tube and $\bar{R}$ is the radius of the tube, at axial station $\bar{Z}$, the instantaneous radius of the tube is given by [10]

$$\bar{R} = H(\bar{Z}, t, k) = a + b\bar{Z} - \Phi e^{k^2 t^2 \cos^2 (\frac{\varphi}{\lambda})(\bar{Z} - ct)},$$

(1)

where $H, a, k, b, \lambda, \varphi$ and $\bar{Z}$ are the radial displacement of the wall from the centre line, radius of the tube in the absence of elasticity, dilation parameter, non-uniform parameter, time parameter, amplitude of the wave, wavelength and axial coordinate, respectively. Because of axisymmetric condition, the angle $\varphi$ in cylindrical coordinate is eliminated.

![Figure1. Geometry of the problem [14].](image-url)
3. Basic and Constitutive Equations

The basic governing equations of the present flow are [1,11]
equation of mass conservation
\[ \nabla \cdot \dot{V} = 0, \] (2)
motion equations (Navier-Stokes equations)
\[ \rho \frac{d\dot{V}}{dt} = \nabla \cdot \tau, \] (3)
energy equation
\[ \rho C_p \frac{d\ddot{T}}{dt} = k \nabla^2 \ddot{T} + \ddot{\tau}.L, \] (4)
and the concentration equation
\[ \frac{d\ddot{C}}{dt} = D_m \tau \ddot{C} + \frac{D_m K_F}{T_m} \nabla^2 \ddot{T}, \] (5)
in which \( \dot{V}, \rho, \frac{d\ddot{V}}{dt}, \tau, \ddot{T}, \kappa, \ddot{C}, D_m, K_F \) and \( T_m \) are velocity vector, density, material time
derivative, Cauchy stress tensor, Laplace operator, temperature, thermal conductivity, specific heat,
mass concentration, coefficient of mass diffusion, thermal diffusion ratio and mean temperature,
respectively. \( L = \text{grad} \ddot{V} \).

Let \( \ddot{V} = (\ddot{U}, \ddot{V}, \ddot{W}) \) be the velocity components in the fixed frame cylindrical coordinate system
\((\ddot{R}, \varphi, \ddot{Z})\). According to the assumption, the velocity components can be written as
\[ \ddot{V} = (\ddot{U}(\ddot{R}, \ddot{Z}, \ddot{t}), 0, \ddot{W}(\ddot{R}, \ddot{Z}, \ddot{t})), \] (6)
Ostwald-De Waele power-law fluid model is chosen and constitutive equations can be expressed as:[1,12]
\[ \ddot{\tau} = -\dddot{P} + \ddot{\tau}, \] (7)
\[ \dddot{T} = m(\ddot{\gamma})^{n-1}\ddot{S}, \] (8)
\[ \ddot{S} = 2D, \] (9)
where \( \ddot{\tau} \), \( \dddot{P} \), \( n \), \( \ddot{S} \) and \( m \) are the extra stress tensor, identity tensor, pressure, fluid behaviour index,
rates of deformation (the rate of strain) and consistency parameter (the consistency index of non-
Newtonian viscosity), respectively. \( D = \{ \frac{1}{2} [L + L^T] \} \) is the first Rivin-Ericksen tensor and \( \ddot{\gamma} \) is defined as:
\[ \ddot{\gamma} = \left( \sum_{i,j=1}^{2} \dddot{S}_{ij}\dddot{S}_{ij} \right)^{\frac{1}{2}}, \] (10)
where \( \dddot{S}_{ij} \) is the rate strain components \((l,j=1,2,3)\) in which the numbers 1, 2 and 3 are the coordinates
\( \ddot{R}, \varphi \) and \( \ddot{Z} \) respectively.

Now, from equations (2)-(5), the flow is governed by four coupled non-linear partial differential
equations (continuity, momentum, energy and the particles concentration equation) in the fixed frame
are given by
\[ \frac{1}{\ddot{R}} \frac{\partial (\ddot{R} \dddot{U})}{\partial \ddot{R}} + \frac{\partial \dddot{W}}{\partial \ddot{Z}} = 0, \] (11)
\[ \rho \left( \frac{\partial \dddot{U}}{\partial \ddot{R}} + \dddot{U} \frac{\partial \dddot{U}}{\partial \ddot{Z}} + \dddot{W} \frac{\partial \dddot{U}}{\partial \ddot{Z}} \right) = -\frac{\partial \dddot{P}}{\partial \ddot{R}} + \frac{1}{\ddot{R}} \frac{\partial (\ddot{R} \ddot{\dddot{U}})}{\partial \ddot{R}} + \frac{\partial \ddot{\dddot{C}}}{\partial \ddot{Z}} - \frac{\dddot{T}_{22}}{\ddot{R}}, \] (12)
\[ \rho \left( \frac{\partial \dddot{W}}{\partial \ddot{R}} + \dddot{U} \frac{\partial \dddot{W}}{\partial \ddot{Z}} + \dddot{W} \frac{\partial \dddot{W}}{\partial \ddot{Z}} \right) = -\frac{\partial \dddot{P}}{\partial \ddot{Z}} - \frac{1}{\ddot{R}} \frac{\partial (\ddot{R} \ddot{\dddot{W}})}{\partial \ddot{R}} + \frac{\partial \ddot{\dddot{C}}}{\partial \ddot{Z}}, \] (13)
\[ \rho C_p \left( \frac{\partial \dddot{T}}{\partial \ddot{R}} + \dddot{U} \frac{\partial \dddot{T}}{\partial \ddot{Z}} + \dddot{W} \frac{\partial \dddot{T}}{\partial \ddot{Z}} \right) = \kappa \left[ \frac{\partial^2 \ddot{T}}{\partial \ddot{R}^2} + \frac{\partial^2 \ddot{T}}{\partial \ddot{Z}^2} \right] + \frac{\partial \dddot{U}}{\partial \ddot{R}} \dddot{T}_{11} + \frac{\partial \dddot{W}}{\partial \ddot{R}} \dddot{T}_{13} + \frac{\partial \dddot{U}}{\partial \ddot{Z}} \dddot{T}_{31} + \frac{\partial \dddot{W}}{\partial \ddot{Z}} \dddot{T}_{33}, \] (14)
\[ \left( \frac{\partial \dddot{C}}{\partial \ddot{R}} + \dddot{U} \frac{\partial \dddot{C}}{\partial \ddot{Z}} + \dddot{W} \frac{\partial \dddot{C}}{\partial \ddot{Z}} \right) = D_m \left[ \frac{\partial^2 \ddot{C}}{\partial \ddot{R}^2} + \frac{\partial^2 \ddot{C}}{\partial \ddot{Z}^2} \right] + \frac{D_m K_F}{T_m} \left[ \frac{\partial^2 \dddot{T}}{\partial \ddot{R}^2} + \frac{\partial^2 \dddot{T}}{\partial \ddot{Z}^2} \right] + \frac{\partial \dddot{U}}{\partial \ddot{R}} \dddot{C}_{11} + \frac{\partial \dddot{W}}{\partial \ddot{Z}} \dddot{C}_{33}. \] (15)
The corresponding boundary conditions are
\[ \frac{\partial \dddot{W}}{\partial \ddot{Z}} = 0 \quad , \quad \dddot{U} = 0 \quad , \quad \frac{\partial \dddot{T}}{\partial \ddot{R}} = 0 \quad , \quad \frac{\partial \dddot{C}}{\partial \ddot{R}} = 0 \quad \text{at} \quad \ddot{R} = 0 \] (16)
\[ \dddot{W} = 0 \quad , \quad \dddot{T} = T_1 \quad , \quad \dddot{C} = C_1 \quad \text{at} \quad \ddot{R} = \dddot{H} \]
and from equation (9), the rate of strain tensor $\ddot{S}$ in the fixed frame has the following components [13]

$$
\ddot{S}_{11} = 2 \frac{\partial \ddot{u}}{\partial \overline{R}} \cdot \ddot{S}_{22} = 2 \frac{\ddot{u}}{K}, \quad \ddot{S}_{33} = 2 \frac{\partial \ddot{w}}{\partial \overline{X}}, \\
\ddot{S}_{13} = \ddot{S}_{31} = \frac{\partial \ddot{u}}{\partial \overline{Z}} + \frac{\partial \ddot{w}}{\partial \overline{R}}, \quad \ddot{S}_{12} = \ddot{S}_{21} = \ddot{S}_{23} = \ddot{S}_{32} = 0.
$$

(17)

Now, we shift the platform to a move frame of reference $(\overline{r}, \overline{z})$ moving with velocity $c$ from the fixed frame $(\overline{R}, \overline{Z})$ with respect to which the motion can be transformed as steady, using the following relations

$$
\overline{z} = \overline{Z} - ct, \quad \overline{r} = \overline{R}, \quad \overrightarrow{w}(\overline{r}, \overline{z}) = \overrightarrow{w}(\overline{R}, \overline{Z}, \overline{t}) - c, \quad \overrightarrow{u}(\overline{r}, \overline{z}) = \overrightarrow{U}(\overline{R}, \overline{Z}, \overline{t}),
$$

\( \overrightarrow{p}(\overline{r}, \overline{z}) = \overrightarrow{P}(\overline{R}, \overline{Z}, \overline{t}), \quad \overrightarrow{T}(\overline{r}, \overline{z}) = \overrightarrow{T}(\overline{R}, \overline{Z}, \overline{t}), \quad \overrightarrow{C}(\overline{r}, \overline{z}) = \overrightarrow{C}(\overline{R}, \overline{Z}, \overline{t}), \)

(18)

where $\overrightarrow{w}$ and $\overrightarrow{u}$ are the axial and radial velocity components in the moving coordinates.

We introduce the dimensionless quantities as:

$$
\overline{z} = \lambda z, \overline{r} = a r, \overrightarrow{w} = \frac{a c u}{\lambda}, \overrightarrow{w} = c w, \overrightarrow{t} = \frac{c^2 t}{\lambda}, \overrightarrow{p} = \frac{c^4 m \lambda}{a^{-n-2} \rho},
$$

$$
Re = \frac{a c u}{\lambda}, \overrightarrow{\phi} = a \phi, \overrightarrow{\delta} = \frac{a}{\lambda}, \overrightarrow{H} = a H, \overrightarrow{q} = c a^2 c F,
$$

$$
\overline{\gamma} = \frac{c}{a}, \overrightarrow{b} = \frac{a}{\lambda}, \overrightarrow{t} = \frac{c^4 m}{a^2} \overrightarrow{t}, \overrightarrow{f o r i \neq j}, \overrightarrow{q} = c a^2 \psi,
$$

$$
\overrightarrow{t} = \frac{c^4 m}{a^2} \overrightarrow{t}, \overrightarrow{f o r i = j}, \overrightarrow{t} = \frac{c}{a} \overrightarrow{t}, \overrightarrow{f o r i \neq j}, \overrightarrow{b} = \frac{k}{A},
$$

$$
\overrightarrow{S} = \frac{c^4 m}{a^2} \overrightarrow{S} j f o r i = j, \overrightarrow{S} = \frac{c}{a} \overrightarrow{S} j f o r i = j, \overrightarrow{b} = \frac{k}{A},
$$

$$
Pr = \frac{c^4 m}{a^2} \overrightarrow{S}, \overrightarrow{E} = \frac{c^2}{\overrightarrow{E}}, \overrightarrow{S} = \frac{a^2 m (T_1 - T_0) k F}{m T_m (c_1 - c_0)},
$$

(19)

in the above expression, $T_1$ and $T_0$ are the wall and centre temperature of the tube respectively whereas $C_1$ and $C_0$ denoted the wall and centre concentration of the tube respectively. $\overrightarrow{\psi}$, $\overrightarrow{\delta}$, $Re$, $Ec$, $Pr$, $Sr$, $Sc$, $\overrightarrow{\Omega}$ and $\overrightarrow{\theta}$ are the stream function, dimensionless wave number, Reynolds number ($Re \ll 1$), Eckert number, Prandtl number, Soret number, Schmidt number, non-dimensional concentration, temperature in the non-dimensional form, respectively.

Equations (1) and (11)-(17) in move frame of non-dimensional form are given by:

$$
\frac{1}{r} \theta \left( \frac{\partial u}{\partial r} + (w + 1) \frac{\partial u}{\partial z} \right) = 0,
$$

$$
Re \frac{\partial \theta}{\partial t} \left( u \frac{\partial u}{\partial r} + (w + 1) \frac{\partial u}{\partial z} \right) = \frac{\partial}{\partial r} \left[ \theta^2 \frac{\partial (r \theta)}{\partial r} + \frac{\partial^2 \theta}{\partial r^2} \right],
$$

$$
Re \theta \left( u \frac{\partial u}{\partial r} + (w + 1) \frac{\partial u}{\partial z} \right) = \frac{\partial}{\partial r} \left[ \theta \frac{\partial (r \theta)}{\partial r} + \frac{\partial^2 \theta}{\partial r^2} \right],
$$

$$
Re Pr \theta \left( u \frac{\partial u}{\partial t} + (w + 1) \frac{\partial u}{\partial z} \right) \left[ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right]
$$

$$
+ EcPr \left[ \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial r} \frac{\partial u}{\partial z} + \frac{\partial^2 \theta}{\partial r^2} \frac{\partial u}{\partial z} + \delta^2 \frac{\partial^2 \theta}{\partial z^2} \right]
$$

$$
Re \left( u \frac{\partial u}{\partial t} + (w + 1) \frac{\partial u}{\partial z} \right) = \frac{1}{Sc} \frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{Sr} \frac{\partial \theta}{\partial r} + a^2 \frac{\partial^2 \theta}{\partial z^2} + Sr \left[ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{a^2 \partial^2 \theta}{\partial z^2} \right],
$$

(24)

the dimensionless form of the corresponding boundary conditions in move frame are

$$
\frac{\partial u}{\partial r} = 0, \quad u = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad \frac{\partial \theta}{\partial r} = 0 \quad a t \ r = 0
$$

$$
\frac{\partial u}{\partial r} = 0, \quad u = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad \frac{\partial \theta}{\partial r} = 0 \quad a t \ r = H
$$

(26)

and the rate of strain tensor given by

$$
\frac{\partial w}{\partial r} = 0, \quad \frac{\partial w}{\partial r} = 0, \quad \frac{\partial w}{\partial r} = 0 \quad a t \ r = 0
$$

$$
\frac{\partial w}{\partial r} = 0, \quad \frac{\partial w}{\partial r} = 0, \quad \frac{\partial w}{\partial r} = 0 \quad a t \ r = H
$$

(27)
\[ S_{11} = 2 \frac{\partial w}{\partial r} , \quad S_{22} = 2 \frac{u}{r} , \quad S_{33} = 2 \frac{\partial^2 w}{\partial z^2} + \frac{\partial w}{\partial r} , \quad S_{12} = S_{21} = S_{23} = S_{32} = 0 \] \quad (27)

By applying \( \delta \ll 1 \) and \( Re \ll 1 \), then equations (22)-(25) reduces to
\[ \frac{\partial p}{\partial r} = 0 , \quad \frac{\partial p}{\partial z} = - \frac{1}{2} \frac{\partial (r \tau_{13})}{\partial r} , \quad \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) = Br \left[ - \frac{\partial w}{\partial r} \tau_{13} \right] , \quad \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) = -ScSr \left[ \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right] , \quad (30) \]
\[ \text{where, } Br = EcPr , \text{ is the Brinkman number.} \]

The local shear stress can be calculated from equation (8) as follows:

The dimensionless of equation (8), in view of equation (27), is given by
\[ \tau_{13} = - (\gamma)^{n-1} \left( \delta^2 \frac{\partial u}{\partial z^2} + \frac{\partial w}{\partial r} \right) , \quad (32) \]

since \( \bar{\gamma} = \frac{1}{\sqrt{2}} \sum_i \sum_j \bar{s}_{ij} \bar{s}_{ij} \) then
if \( i \neq j \) then \( \gamma = \frac{1}{\sqrt{2}} \sum_i \sum_j S_{ij} S_{ij} \), \quad (33)
and if \( i = j \) then \( \gamma = \frac{1}{\sqrt{2} \delta^2} \sum_i \sum_j S_{ij} S_{ij} \). \quad (34)

As mentioned in the previous \( \delta \to 0 \) that implies \( \gamma = 0 \) at \( i = j \), otherwise substituting equation (27) into equations (33), in view of \( \delta \to 0 \), to get \( \gamma = \frac{\partial w}{\partial r} \) and the shear rate has been assumed to be negative throughout the moving frame, thus: [8]
\[ \gamma = - \frac{\partial w}{\partial r} . \quad (35) \]

Now, by substituting equation (35) into equation (32), keep in our mind \( \delta \to 0 \), we have
\[ \tau_{13} = \left( - \frac{\partial w}{\partial r} \right)^n . \quad (36) \]

From equations (29) and (36), we get
\[ \frac{\partial p}{\partial z} = - \frac{1}{r} \frac{\partial}{\partial r} \left( (- \frac{\partial w}{\partial r})^n \right) . \quad (37) \]

4. Solution of the Problem

Rearrange and integrate equation (37) with respect to \( r \) (w.r.t. \( r \)), in view of equation (28), and apply boundary conditions \( \frac{\partial w}{\partial r} = 0 \) at \( r = 0 \) and \( w = -1 \) at \( r = H \) of equation (26), then the axial velocity, in view of equation (20), gives
\[ w = \frac{1}{2 \pi^2 \left( \frac{n+1}{n} \right) \left( \frac{n+1}{n} \right)^\frac{n+1}{n}} \left( \frac{n+2}{n} \right)^\frac{n+1}{n} r^{\frac{n+1}{n}} - 1 . \quad (38) \]

From equation (38), we obtains
\[ - \frac{\partial w}{\partial r} = \frac{1}{2 \pi^2} \left( \frac{\partial p}{\partial z} \right)^{\frac{1}{n}} r^{\frac{1}{n}} . \quad (39) \]

Putting equations (36) and (39) into the temperature equation (equation (30)), one obtains
\[ \frac{\partial}{\partial r} \left( r \cdot \frac{\partial \theta}{\partial r} \right) = Br \frac{1}{2 \pi^2 \frac{n+1}{n}} \left( - \frac{\partial p}{\partial z} \right)^{\frac{1}{n}} \frac{1}{r^{\frac{n+1}{n+2}}}. \]
By integrating w.r.t. \( r \) and applying \( \frac{\partial \theta}{\partial r} = 0 \) at \( r = 0 \) of equation (26), (implies \( c_1 = 0 \) ), and then divided the result by \( r \), one gets

\[
\frac{\partial \theta}{\partial r} = Br \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( - \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} r^{\frac{1}{n+2}}.
\] (40)

Again integrate equation (40) w.r.t. \( r \) and applying \( \theta = 1 \) at \( r = H \) of equation (26), one gets the temperature profile as

\[
\theta = Br \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( - \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} \left( r^{\frac{1}{n+3}} - H^{\frac{1}{n+3}} \right) + 1.
\] (41)

Variance between temperature of fluid and temperature on the wall made the heat transfer coefficient \( Z_1 \) occurs. \( Z_1 \) at the wall is defined as

\[
Z_1 = \frac{\partial \theta}{\partial z} \frac{\partial \theta}{\partial r}.
\] (42)

above equation gives \( Z_1 \) a function of \( z \).

Now from equation (20) , we have

\[
\frac{\partial \Omega}{\partial r} = b - \phi k e^{kz} \cos^2 \pi z + \pi \phi e^{kz} \sin 2\pi z.
\] (43)

Substituting equations (40) and (43) into equation (42), we have

\[
Z_1 = Br \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( - \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} \left( b - \phi k e^{kz} \cos^2 \pi z + \pi \phi e^{kz} \sin 2\pi z \right) r^{\frac{1}{n+2}}.
\] (44)

Also, we can measures the rate of heat transfer from Nusselt number, which defined as ( \( Nu = -\frac{\partial \theta}{\partial r} \) at \( r = H \) ) and from equation (40), we get

\[
Nu = -Br \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( - \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} \left( 1 + bz - \phi k e^{kz} \cos^2 \pi z \right)^{\frac{1}{n+2}}.
\] (45)

Putting equation (40) into equation (31), we obtains

\[
\frac{\partial}{\partial r} \left( r \frac{\partial \Omega}{\partial r} \right) = -ScSrBr \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( \left( \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} \frac{\partial}{\partial r} \right) r^{\frac{1}{n+2}}.
\]

by integrating above equation w.r.t. \( r \) and applying \( \frac{\partial \Omega}{\partial r} = 0 \) at \( r = 0 \) of equation (26), (implies \( c_1 = 0 \) ), and then divided the result by \( r \), one gets

\[
\frac{\partial \Omega}{\partial r} = -ScSrBr \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} r^{\frac{1}{n+2}},
\]

integrating the result w.r.t. \( r \) and applying \( \Omega = 1 \) at \( r = H \) of equation (26), one gets the concentration profile as

\[
\Omega = -ScSrBr \frac{1}{2 \pi^{1+\frac{1}{n+1}}} \left( \frac{\partial p}{\partial z} \right)^{\frac{1}{n+1}} \left( r^{\frac{1}{n+3}} - H^{\frac{1}{n+3}} \right) + 1.
\] (46)

At any cross-section, volume flow rate \( \bar{q} \) is given by

\[
\bar{q} = 2\pi \int_0^H \bar{w} \bar{r} d\bar{r},
\]

the non-dimensional volume flux \( F \) is given by

\[
F = \frac{1}{2 \pi^{\frac{1}{n+1}}} \left( \frac{\partial p}{\partial z} \right)^{\frac{1}{2}} \left( 1 + bz - \phi k e^{kz} \cos^2 \pi z \right)^{\frac{1}{n+2}} - \left( 1 + bz - \phi k e^{kz} \cos^2 \pi z \right)^{\frac{1}{n+3}}.
\] (47)
Solving equation (47) for $\frac{\partial p}{\partial z}$, yields the pressure gradient as

$$\left(-\frac{\partial p}{\partial z}\right)^\frac{1}{n} = \frac{2\pi \left(\frac{1}{n}+3\right) \left(F+(1+bz-\phi e^{kz\cos^2\pi z})^2\right)}{(1+bz-\phi e^{kz\cos^2\pi z})^{\frac{1}{n}+3}},$$

simplifying

$$\frac{\partial p}{\partial z} = -2 \left(\frac{1}{n}+3\right) \left(F+(1+bz-\phi e^{kz\cos^2\pi z})^2\right)^n \left(1+bz-\phi e^{kz\cos^2\pi z}\right)^{-\frac{1}{n}},$$

(48)

5. Results and Discussion

Graphical results are introduced in this section to see that the influence of different parameters, like power-law index $n$, non-uniform parameter $b$, dilation parameter $k$, amplitude ratio $\phi$, volume flow rate $F$, Brinkman number $Br$, Soret number $Sr$ and Schmidt number $Sc$ on the temperature $\theta$, mass concentration $\Omega$, heat transfer coefficient $Z_1$ and the rate of heat transfer $Nu$. All graphs have been plotted by means of MATHEMATICA software.

5.1. Temperature Distribution $\theta$

The purpose of this subsection is to analyse the impact of all parameters on $\theta$. The graphs of $\theta$ against the radial axis $r$ are plotted in figures 2-7. The influence of the power-law index $n$ on $\theta$ is illustrated in figure 2. It is noted that $\theta$ increases with an increase in power-law index $n$. Figure 3 explains that the increases in $b$ reduce the temperature. This result is found to be in good agreement with result of Sankan and Patil [1]. As the dilation parameter $k$ increases, $\theta$ also increases as shown in figure 4. From figure 5 it can be inferred that the temperature is rapidly increases with an increase in amplitude ratio $\phi$. Figure 6 shown that the temperature increases with an increase in Brinkman number $Br$. In fact an increase in $Br$ means that viscous dissipation gets stronger and that implies enhance fluid temperature. It is concluded from figure 7 that temperature increases when the flux $F$ increased. Sankan and Patil [1] and Hayat et al. [11] have also analysed that $\theta$ increases with $Br$.

**Figure 2.** Effect of power-law index $n$ on the temperature $\theta$ at $Br = 0.3$, $k = 0.1$, $b = 0.05$, $\phi = 0.2$, $F = 2$, $z = 0.1$.

**Figure 3.** Effect of $b$ on the temperature $\theta$ at $Br = 0.3$, $n = 1.1$, $k = 0.1$, $\phi = 0.2$, $F = 2$, $z = 0.1$. 
5.2. Heat Transfer Coefficient $Z_1$

The figures 8-13 shown an oscillatory behaviour of $Z_1$ as a result of the propagation of peristaltic wave. Figure 8 discuss the impact of power-law index $n$ on heat transfer coefficient $Z_1$. It is shown that the absolute value of $Z_1$ increases with an increase in power-law index $n$. Figure 9 shows that with increase in non-uniform parameter $b$ the absolute value of heat transfer coefficient $Z_1$ decreases. This result concurs with the result of Sankan and Patil [1]. For increasing dilation parameter $k$, the absolute value of heat transfer coefficient $Z_1$ also increase as illustrated in figure 10. The effect of amplitude ratio $\phi$, Brinkman number $Br$ and volume flow rate $F$ are discussed through figures 11-13. It is observed that $Z_1$ increases with an increase of the parameters $\phi$, $Br$ and $F$. 

**Figure 4.** Effect of dilation parameter $k$ on the temperature $\theta$ at $Br = 0.3, n = 1.1$, $b = 0.05, \phi = 0.2, F = 2, z = 0.1$.

**Figure 5.** Effect of amplitude ratio $\phi$ on the temperature $\theta$ at $Br = 0.3, n = 1.1$, $k = 0.1, b = 0.05, F = 2, z = 0.1$.

**Figure 6.** Effect of Brinkman number $Br$ on the temperature $\theta$ at $n = 1.1, k = 0.1$, $b = 0.05, \phi = 0.2, F = 2, z = 0.1$.

**Figure 7.** Effect of flux $F$ on the temperature $\theta$ at $Br = 0.3, n = 1.1, k = 0.1, b = 0.05$, $\phi = 0.2, z = 0.1$. 

5.3. Rate of Heat Transfer $N_u$

The heat transfer rate is quantified by Nusselt number $N_u$. Figures 14-19 are plotted to explain the impact of $n, b, k, \phi, Br$ and $F$ on $N_u$. It is noted that the impact of all parameters on $N_u$ is opposite behavior as to the influence on $Z_1$. The variation in $N_u$ with an increase of $n$ is discussed in figure 14. It is noted that $N_u$ decreases. The increases of non-uniform parameter $b$ is explain in figure 15. It is noticed that $N_u$ increases. From figures 16-19 it is seen that enhancing the values $k, \phi, Br$ and $F$
implies $Nu$ reduces. Sankan and Patil [1] is also of the similar opinion of the results of Brinkman number $Br$ and non-uniform parameter $b$.

![Figure 14](image1.png)

**Figure 14.** Effect of power-law index $n$ on $Nu$ at $Br = 0.3, b = 0.05, k = 0.1, \phi = 0.2, F = 2$.

![Figure 15](image2.png)

**Figure 15.** Effect of non-uniform parameter $b$ on $Nu$ at $Br = 0.3, n = 1.1, k = 0.1, \phi = 0.2, F = 2$.

![Figure 16](image3.png)

**Figure 16.** Effect of dilation parameter $k$ on $Nu$ at $Br = 0.3, n = 1.1, b = 0.05, \phi = 0.2, F = 2$.

![Figure 17](image4.png)

**Figure 17.** Effect of amplitude ratio $\phi$ on $Nu$ at $Br = 0.3, n = 1.1, b = 0.05, k = 0.1, F = 2$.

![Figure 18](image5.png)

**Figure 18.** Effect of Brinkman number $Br$ on $Nu$ at $n = 1.1, b = 0.05, k = 0.1, \phi = 0.2, F = 2$.

![Figure 19](image6.png)

**Figure 19.** Effect of flux $F$ on $Nu$ at $Br = 0.3, n = 1.1, b = 0.05, k = 0.1, \phi = 0.2$. 
5.4. Concentration Distribution $\Omega$

The concentration of particles for various values of embedded parameters is explained through figures 20-27. It is clear that the concentration distribution $\Omega$ is almost parabola in nature. Figure 20 explains the impact of increasing of the value in power-law index $n$ on the concentration distribution. It is observed that the concentration distribution $\Omega$ decreases. The features of non-uniform parameter $b$ on $\Omega$ is shown in figure 21. It is noted that $\Omega$ increases when $b$ increase. Figure 22 deduced that $\Omega$ decreases with an increase of $k$. Figure 23 explained the impact of amplitude ratio $\phi$ on concentration distribution $\Omega$. It is noted that an increase in $\phi$ leads to strongly reduced in $\Omega$. It is concluded from figures 24-27 that an increasing in the volume flow rate $F$, the Brinkman number $Br$, the Soret number $Sr$, and Schmidt number $Sc$ decreases the concentration distribution $\Omega$. The results obtained for $Br$ and $Sc$ agree fully with those of Hayat et al. [11].

**Figure 20.** Effect of power-law index $n$ on the concentration $\Omega$ at $Br = 0.3, Sc = 0.1, Sr = 0.1, b = 0.05, k = 0.1, \phi = 0.2, F = 2, z = 0.1$.

**Figure 21.** Effect of non-uniform parameter $b$ on the concentration $\Omega$ at $Br = 0.3, Sc = 0.1, Sr = 0.1, n = 1.1, k = 0.1, \phi = 0.2, F = 2, z = 0.1$.

**Figure 22.** Effect of dilation parameter $k$ on the concentration $\Omega$ at $Br = 0.3, Sc = 0.1, Sr = 0.1, n = 1.1, b = 0.05, \phi = 0.2, F = 2, z = 0.1$.

**Figure 23.** Effect of amplitude ratio $\phi$ on the concentration $\Omega$ at $Br = 0.3, Sc = 0.1, Sr = 0.1, n = 1.1, b = 0.05, k = 0.1, F = 2, z = 0.1$. 
6. Conclusions

In this study, we given a theoretical approach to explain the Peristaltic transport and heat and mass transfer on a power-law fluid flow in an elastic tapered tube with variable cross-section induced by dilating peristaltic wave. The effect of all parameters on the temperature, heat transfer coefficient, rate of heat transfer and concentration are illustrated graphically. We can conclude the following observations:

- Temperature is an increasing function of the parameters $n, k, \phi, Br$ and $F$.
- Temperature is decreasing function of the non-uniform parameter $b$.
- Absolute value of heat transfer coefficient is increases with an increase of the parameters $n, k, \phi, Br$ and $F$, while it is decreases when $b$ increase.
- Nusselt number $Nu$ decreases for the increasing values of $n, k, \phi, Br$ and $F$ whereas increases in $Nu$ is noted in case of increases $b$.
- Concentration increases in the presence of $b$.
- Concentration is decreasing function of the parameters $n, k, \phi, Br, Sr, Sc$ and $F$.
- Concentration distribution is almost parabola in nature.
Opposite behavior for temperature distribution is noticed compared to concentration distribution.

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