Approximate seismic analysis of multi-story buildings with mass and stiffness irregularities

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Abstract

An approximate analysis is presented for multi-story setback buildings subjected to strong ground motions. Setback buildings with mass and stiffness discontinuities are common in modern architecture and quite often they are asymmetric in plan. Such buildings are classified by Eurocode 8 (EC8-2004) and codes from other countries as irregular structures, which specify a full 3-dimensional dynamic analysis. There are no recommendations of how the practicing engineer can assess the fundamental frequency by a simple formula or methodology and there are no provisions which allow the structural detailing by a pseudo-static design against an equivalent lateral load. Therefore, an approximate analysis which provides basic dynamic data (frequencies and peak values of base resultant forces) of setback buildings and furthermore an overview of their response during a ground excitation is a useful tool at the preliminary stage of a practical design. This methodology is based on Southwell’s formula and the concept of the equivalent single story system. This has been introduced by the authors in earlier papers for assessing the response of uniform along the height of buildings. At present, the accuracy of this procedure is examined in asymmetric tall buildings with a mass or stiffness irregularity. As basic data of the dynamic response of elastic multi-story building systems can be derived by analyzing simple (equivalent) single story systems, a structural layout of minimum elastic torsional response can easily be constructed. The behavior of such structural configurations, which is basically translational in the elastic phase, is also examined in the post elastic phase when the strength assignment of the various bents is stiffness proportional.

Keywords: Setback Buildings; Mass and Stiffness irregularity; Dynamic analysis
1. Introduction

The rotational response of building structures during strong ground motions has been proved to be the main cause of partial or total collapse. In recent years a number of investigations have been carried out to demonstrate the seismic vulnerability due to building asymmetry and mass or stiffness irregularity. Qualitative reviews have been published on this issue (e.g. [1,2]), but a few recent papers (e.g.[3]) have shown that the inadequate performance is not that severe as generally accepted.

Different types of lateral load resisting bents (structural walls, moment resisting frames, coupled walls systems, etc), and further more their asymmetric location, is the usual reason for having in-plan structural asymmetries in common multistory buildings, while a sudden change of the size of the floor dimensions above a certain level creates an in-elevation mass and stiffness irregularity. In such cases none of the modern building codes (e.g. EC8-2004) provides a simple formula for assessing the fundamental period and no recommendations are given of how to perform a structural design by means of a pseudo-static analysis, as in the case of buildings which satisfy the regularity criteria. The codes require a full 3D dynamic analysis, even for low height building. As the cost of a practical application is increasing, efforts have been made towards establishing an acceptable pseudo-static procedure of structural analysis. Suggestions of how the fundamental period of stepped buildings can be assessed have been presented in recent publications, based on one [4] or two [5] 'regularity indices'. The need of an accurate assessment of the fundamental period is crucial in a structural design which is based on the acceleration response spectrum, especially when it falls within the velocity region of the spectrum, where the spectral acceleration is sensitive to the fundamental period. Furthermore, as the asymmetric and irregular buildings are vulnerable to ground excitations, the issue of mitigating the torsional effects has also been raised. Different element strength distributions were studied by Aziminejad et al [6] and Aziminejad and Moghadam [7]. In these studies the problem of element strength distribution on the rotational response of the structure is studied by using a proper configuration of the centers of mass, strength and stiffness according to the findings obtained from single story systems with elements having strength dependant stiffness [8,9].

The first objective of this work is to present an approximate method for assessing basic dynamic data (periods, resultant base shears and torques) of multistory eccentric setback buildings. It is based on first author’s earlier papers [10,11,12] on uniform multi-story systems, where the aforementioned data can be found with reasonable accuracy by analyzing two equivalent single story modal systems. This methodology is now extended to irregular setback buildings with a mass and stiffness discontinuity. The method is based on the element frequencies, which for the full-height bents are evaluated from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure. For the bents which are curtailed at the level of the setback an approximate formula is proposed and the results of the method are presented and compared with the accurate data provided by the SAP2000 computer program for the case of 8-story buildings composed by frames, shear walls and coupled walls systems. The second objective of this work is to demonstrate that a structural configuration of minimum elastic torsion during a ground excitation preserves this practically translational response in the inelastic region when the strength assignment of its resisting bents is stiffness proportional. In other words, this response is obtained when the building is detailed as a planar structure under a code load [13,14]. This is attributed to the more or less concurrent yielding of all resisting elements, which preserves the translational response, attained at the end of the elastic phase, up to the post elastic one. At present this is investigated in asymmetric multistory structures with a mass and stiffness irregularity. The aforementioned 8-story setback buildings are examined under the characteristic ground motion of the Imperial Valley (1940), selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (http://peer.berkeley.edu) and scaled to a PGA=0.5g.

2. Equivalent single story systems of setback buildings

A typical setback building is shown in Fig. 1. The building consists of two uniform sub-structures: the base and the tower one with the corresponding heights as in Fig. 1. The masses, the radii of gyration and the number of floors are respectively equal to \(m_b, r_b, N_b\) and \(m_t, r_t, N_t\) for the two substructures. The centres of mass (CM) at each floor are assumed to lie on the same vertical line which is passing through the centroids of all decks. All bents within the perimeter of the tower structure (rigid frames, shear walls, etc) extend up to the top of the building, while those outside this area are assumed to be curtailed at the level of the setback.
The methodology to analyze elastic setback buildings, like that of Fig. 1, is outlined in first author’s earlier papers [15,16]. The backbone of this method is similar to that applied to uniform over the height systems [10,11,12], which are analyzed by two equivalent single story systems. For a ground excitation along, say, the y-direction, each of the equivalent systems has a mass equal to the n-mode effective mass, \( M_n^* \) (n=1,2) of the uncoupled multi-story structure in the same direction, a radius of gyration calculated as described in [15], and it is supported by elements (at the locations of the real bents) with a stiffness equal to the product of \( M_n^* \) with the first mode (when n=1) or second mode (when n=2) squared element frequencies of the corresponding real bents of the assumed multi-story structure. These frequencies, for the full height bents, are determined from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure. In the case of buildings composed by very dissimilar bents, the proposed method provides more accurate results, when the effective element frequencies are used [13]. For the curtailed bents, however the corresponding frequencies are calculated by means of an indirect method, which, in brief, may be described as follows: the frequency, \( \omega_n \), and the effective modal mass, \( nM_n^* \), of the uncoupled setback building of Fig. 1(a) with the symmetric plan configuration of Fig. 1(b), for the first two modes of vibration (n=1,2) along the y-direction, provide the stiffness of the corresponding single-degree-of-system, which is equal to

\[
k_n^* = \omega_n^2 M_n^*
\]  

(1)

where

\[
\omega_n^2 = \Phi_n^T K_{ov} \Phi_n = \Phi_n^T K_v \Phi_n + \Phi_n^T K_{cv} \Phi_n + \sum \Phi_n^T k_j \Phi_n + \Delta \omega_n^2
\]  

(2)

\( M \) is mass matrix of the assumed structure (as defined below), \( \Phi_n \) is the n-mode shape vector (n=1,2) in the y-direction (Fig. 1(d)) and \( K_{ov} \) is its lateral stiffness matrix in the same direction, which may be expressed by two parts: \( K_v \) and \( K_{cv} \), representing respectively the stiffness of the full height and the curtailed bents in the y-direction, as shown in the first of Eqs. (3).

\[
K_{ov} = K_v + K_{cv} = \Sigma k_j + 2k_{cj} , \text{ where } k_{cj} = \begin{bmatrix} k_{cj} & 0 \\ 0 & 0 \end{bmatrix}
\]  

(3)

In this expression, \( k_j \) is the NxN stiffness matrix of the j-full height bent aligned in the y-direction and \( k_{cj} \) is the corresponding matrix of the cj-curtailed bent. Note here that the latter matrix has zero elements below (beyond) the \( N_b \) row (column), as indicated in the second of Eqs. (3). That is, \( k_{cj}^e \) is the real, \( N_b x N_b \), lateral stiffness matrix of the cj-curtailed bent. The ratios \( \Phi_n^T k_j \Phi_n / \Phi_n^T M \Phi_n \) (j=1,2,...) of the last term of Eq. (2) may be approximated by the squared element frequencies of the j-full height bents, \( \omega_{jm} \), which are calculated under the assumption that each of these bents carries, as a planar frame, the mass of the complete structure. Therefore, replacing the shape vector \( \Phi_n \) with the corresponding vector \( \Phi_{jm} \) of the j-bent, each of the aforementioned ratios may be taken as the element squared frequency of the j-bent, which is given by the first of Eqs. (4). In the case of buildings composed by very
dissimilar bents, a better estimate of the aforementioned first mode ratios \((n=1)\) is given by the second of Eqs. (4), which represents the effective element frequency. In the latter equation \(M_{j1}^*\) is the effective first mode mass of the \(j\)-full height bent.

\[
\omega_{j2}^2 = (\Phi_j^T k_j \Phi_j) / (\Phi_j^T M \Phi_j), \quad \bar{\omega}_{j2}^2 = \omega_{j2}^2 (M_{j1}^* / M_n^*) , \quad n=1,2
\]

Inserting Eqs (4) into the Eq. (2), the overall contribution of the curtailed bents into the modal stiffness of the uncoupled structure (Eq. (1)) is determined as

\[
\Delta \omega_n^2 = \omega_n^2 - \Sigma \omega_{jn}^2 = (\Phi_n^T k \Phi_n / \Phi_n^T M \Phi_n) = \Sigma (\Phi_n^T k \Phi_n / \Phi_n^T M \Phi_n)
\]

The contribution of each of the curtailed bents may be evaluated by interpreting the ratios \(\Phi_n^T k \Phi_n / \Phi_n^T M \Phi_n\) in the expression above. Defining this ratio as the effective element square frequency of the \(cj\)-curtailed bent, i.e.:

\[
\omega_{jn}^2 = \frac{\Phi_j^T k \Phi_n}{\Phi_j^T M \Phi_n} = \frac{\Phi_j^T k \Phi_n}{\Phi_n^T M \Phi_n} = \frac{\Phi_j^T k \Phi_n}{\Phi_n^T M \Phi_n}
\]

where \(\rho_{cjjn}^2 = (\Phi_j^T k \Phi_n / \Phi_n^T M \Phi_n)\). a convenient interpretation may be given through the shapes of the mode vectors shown in Fig. 1(d). As \(\Phi_n\) \((n=1,2)\) indicates the \(n\)-mode shape vector (of order \(N x I\)) of the assumed uncoupled building, the sub-vector \(\Phi_{jn}\) (of order \(N x I\)) may be seen as its part which represents the deflections of the base structure. This sub-vector is diagrammatically shown by solid lines in Fig. 1(d), together with the sub-vector of the tower structure \(\Phi_n\), of order \(N x I\), shown by dotted lines (\(\Phi_n^T = \{ t M h, t M_n \}\)). It can be seen from the shape of \(\Phi_n\) that the ratio \(\rho_{cj}\) for the first mode of vibration \((n=1)\) may be approximated by the first frequency of the \(cj\)-curtailed bent when it is assumed to carry the mass of the base structure. Therefore, as the generalized coefficient \(\lambda_n\) is a common factor for all the curtailed bents, it is evident that the frequency difference of Eq. (5), when \(n=1\), which represents the total contribution of all the curtailed bents, should be distributed among them in proportion to the squares of their first frequencies, \(\omega_{cj}\). That is, the effective first mode frequencies of the \(cj\)-curtailed bents are taken equal to the first frequency of the \(cj\)-curtailed bent when it is assumed to carry the mass of the base structure. In this case \((n=2)\), \(\rho_{cj2}^2\), cannot be taken as a particular frequency. In setback buildings with a small tower structure, \(\rho_{cj2}\) may be considered as the second mode frequency, \(\omega_{cj2}\) of the \(cj\)-bent, computed again on the grounds of the assumption that it carries the mass of the base structure. On the other hand, in buildings with a short and stiff base structure, \(\rho_{cj2}\) may be considered as the first frequency, \(\omega_{cj1}\), of the \(cj\)-bent. At present, the overall frequency difference for the second mode of vibration is distributed among the curtailed bents by the second of Eqs. (7), which expresses their effective second mode square frequencies and provides their contribution to the second mode stiffness of the equivalent single story system. The effective stiffness of the elements aligned in the x-direction are computed in a similar way.

3. Buildings studied

To illustrate the application and accuracy of the proposed method, the setback systems (Example buildings 1 and 2) shown in Fig. 2 were analyzed. Both systems are 8-story mono-symmetric buildings, which are divided in two substructures: the base structure represents a uniform building system composed by floors of 22x15m and, the tower substructure which is composed by floors of reduced dimensions 15x10m. The full height lateral load resisting bents, within the perimeter of the tower section, are two structural walls (Wa and Wb) and a moment resisting frame (FR) which are aligned along the y-direction and a pair of coupled-wall bents (CW) which is oriented along the x-axis of symmetry. The structural walls Wa and Wb are of cross sections 30x500cm, the moment resisting frame FR consists of two 75x75cm columns, 5m apart, connected by beams of a cross section 40x70cm, while the CW bents are composed two 30x300cm walls, 5m apart, connected by lintel beams of a cross section 25x90cm at the floor levels. The latter bents are located symmetrically to CM at the edges of the floors of the tower.
structure. The curtailed bents are a moment resisting frame FRcy and a shear wall Wcy along the y-direction and a pair of shear walls Wcx along the x-direction, which are located in a symmetrical configuration at the edges of the base floor. The member dimensions of FRcy are the same as those of FR, while the curtailed walls Wcy and Wcx are of a cross-section 30x300cm. The mass and the radius of gyration about CM per floor are: \( m_b = 264 \text{kN} \cdot \text{s}^2/\text{m}, \quad r_b = 7.687 \text{m} \) and \( m_t = 120 \text{kN} \cdot \text{s}^2/\text{m}, \quad r_t = 5.204 \text{m} \). The story height is 3.5m and the modulus of elasticity \( E = 20 \times 10^6 \text{kN/m}^2 \).

![Diagram](image-url)

Fig. 2. (a,b) Example setback buildings; (c) Eurocode 8 acceleration design spectrum

The centres of mass of the floor slabs lie on a same vertical line, which passes through the centroids of all the orthogonal floor plans of the example structure. In Example building 1, the curtailed frame FRcy is located on the right of CM at the edge of the base structure, at \( x = 11 \text{m} \), while the curtailed wall Wcy is located on the other side of CM at \( x = -11 \text{m} \). In Example building 2 the aforementioned curtailed bents are located in a reversed order as shown in Fig. 2(b). In both example structures, the wall Wa and frame FR are assumed to be located at fixed positions, the first on the left of CM in a distance equal to 4m and the second on the right of CM at a distance of 6.5m, while the second wall Wb is taking all the possible locations along the x-axis within the limits of the tower section. Three different models are examined for each of the example buildings described above. In the first model of Example building 1 (T2-B6:m1) the tower structures consists of two floors, in the second model (T4-B4:m1) of four floors and, finally, the third model (T6-B2:m1) has a tower structure consisting of six floors. The same models of Example building 2 (T2-B6:m2, T4-B4:m2, T6-B2:m2) are formed in a similar way.

4. Model frequencies and observed linear seismic response

The first four periods of vibration of the model setback structures, computed by the proposed method on the grounds of the effective element frequencies (red lines) for different locations of the Wb (indicated by the normalized coordinate \( \bar{x} = x/r_b \)), are shown in Fig. 3, together with the accurate SAP2000 computer values (black lines). In the computer analyses, the out of plane stiffness of the bents was neglected and in the wide column analogy used to simulate the CW bents the clear span of the coupling beams was increased by the depth of the beams [17]. The above estimates are quite satisfactory for practical applications and it is reminded here that unsafe spectral acceleration values may be derived by overestimating the periods of a given structure.

Normalized base shears (in the y-direction) and torques, derived by the proposed approximate procedure for the case of the EC8-2004 acceleration response spectrum (Fig. 2(c)), are shown in Fig. 4 for all models. They have been normalized in respect to the total shear, along the y-direction, \( V_o \), of the corresponding uncoupled structure, while the base torques are also divided by the radius of gyration of the base structure \( r_b \). All the aforesaid data are shown in red lines and in the same figure are also shown the accurate data (black lines) given by the computer program SAP2000-V11 on the basis of the first 12 peak modal values combined according to the CQC rule (the damping ratio in each mode of vibration was taken equal to 5%). The prediction of the base shears is quite reasonable, but the approximate base torques are not of the same accuracy. In general, the proposed approximate method underestimates the base torques, but their variation for the different locations of Wb (with the exception of model T2-B6:m1) follows the trend of the accurate values and the location of Wb of minimum torque is well predicted.
The inelastic response of the assumed model structures was investigated under the ground motion of Imperial Valley 1940 (component 180), scaled to a PGA=0.5g (unidirectional excitation along the y-axis). All the nonlinear response history analyses were performed by means of the program SAP2000-V11, using inelastic link elements at the assumed locations of plastic hinges. The bent strength assignment was based on a planar static analysis under a set of floor forces determined from Equation (4.11) of EC8-2004 and summing to a base (design) shear, $V_d$, equal to 20% of the total weight of the assumed model structure. More specifically, plastic hinges are allowed at the bases of
walls and the frames FR and FRcy are detailed according to the strong column-weak beam philosophy (that is, allowing plastic hinges at the ends of the beams and at the foot of the ground floor columns).

Two response parameters, obtained by time history analyses assuming a 5% damping ratio, are shown in Fig. 5: normalized base shears and base torques. Shears are normalized in respect to \( V_d \), while the base torques are also divided by the radius of gyration of the base structure \( r_b \). The red lines represent the peak elastic response and the black lines represent the peak inelastic behaviour. Envisaging this figure it can be seen that the base torques in the inelastic models with a rather low base structure (T4-B4 and T6-B2) are not very different from those of the corresponding elastic systems. This is not the case however, in the models with a tall base structure (T2-B6), where the inelastic torques are lower than the elastic ones and have a trend similar to that observed in uniform over the height structures [13, 14]. It is notable that minimum values of base torques are observed when the location of wall Wb receives values close to those predicted by the proposed method (as shown in Fig. 4).

5. Conclusions

An approximate method is presented for the analysis of multi-story asymmetric setback buildings. Basic dynamic data (periods and base shears) can be estimated with reasonable accuracy and, to some extent, base torques. The proposed method is based on the analysis of two equivalent, single-story asymmetric modal systems, the masses of which are determined from the first two vibration modes of the uncoupled multi-story structure and the radius of gyration is computed as a Rayleigh quotient as described in an earlier paper. The stiffness of the supporting elements, at the locations of the real bents, when they represent full-height resisting bents, are determined from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure, but an indirect procedure is used for the curtailed bents.

The method may be found useful at the stage of the preliminary design, where the decisions about the structural layout have to be taken prior to a full 3D dynamic analysis. Besides, the method predicts the structural configuration of minimum torsion, which implies that the building elastic response during a ground motion is more or less translational. This response is preserved in the inelastic phase, when the strength assignment of the lateral load resisting bents is derived from a planar static analysis, as a consequence of the almost concurrent yielding of these bents. This is demonstrated in common 8-story setback buildings under a characteristic ground motion.
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