Second quantization approach to composite hadron interactions
in quark models

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Abstract

Starting from the Fock space representation of hadron bound states in a quark model, a change of representation is implemented by a unitary transformation such that the composite hadrons are redescribed by elementary-particle field operators. Application of the unitary transformation to the microscopic quark Hamiltonian gives rise to effective hadron-hadron, hadron-quark, and quark-quark Hamiltonians. An effective baryon Hamiltonian is derived using a simple quark model. The baryon Hamiltonian is free of the post-prior discrepancy which usually plagues composite-particle effective interactions.
In the last 20 years the studies of the hadron-hadron interaction using quark models have been performed mainly by means of the traditional cluster techniques such as adiabatic methods, resonating group (RGM) or generator coordinate methods, and variational techniques [1]. In this paper we consider a different approach, which was developed independently by Girardeau [2] and Vorob’ev and Khomkin [3] to deal with problems in atomic physics where the internal degrees of freedom of atoms cannot validly be neglected. The method employs a second quantization formalism and shares some characteristics with Weinberg’s [4] quasi-particle approach and also with the “quark Born diagram” (QBD) approach recently introduced by Barnes and collaborators [5] [6]. Girardeau coined the name Fock-Tani (FT) representation for the formalism.

The use of a second quantization formalism offers several advantages over a first quantization one. Such advantages include the use of the known field theoretic techniques such as Feynman diagrams and Green’s functions which have proven to greatly simplify the discussion of many-body systems in different areas of physics. Since in a hadronic collision both the constituents of the hadrons and the hadrons themselves can participate in the intermediate states, one expects simplifications by describing the hadrons participating in the process in terms of macroscopic hadron field operators, instead of the microscopic constituent ones. The problem that arises is that composite hadron field operators in general do not satisfy canonical (anti)commutation relations and, therefore, the traditional field theoretic techniques cannot be directly applied. However, in the FT formalism one recovers the possibility of using these techniques by a change of representation in which the composite operators are redescribed by elementary-particle operators satisfying canonical (anti)commutation relations. In some sense this realizes the dual quark-hadron description of hadronic processes.

In this letter we present the extension of the original atomic physics formalism to a general class of quark models in which the baryons are described as bound states of three constituent quarks. In Fock space ($\mathcal{F}$) a one-baryon state of c.m. momentum $\mathbf{P}$, internal energy $\epsilon$, spin projection $M_S$, and isospin projection $M_T$ is denoted by $|\alpha\rangle \equiv |\mathbf{P},\epsilon,M_S,M_T\rangle = B^\dagger_\alpha|0\rangle$. 

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where $B^\dagger_\alpha$ is the baryon creation operator:
\[
B^\dagger_\alpha = \frac{1}{\sqrt{3!}} \Phi_{\mu_1 \mu_2 \mu_3}^{\mu_{\alpha}} q^{\dagger}_{\mu_1} q^{\dagger}_{\mu_2} q^{\dagger}_{\mu_3},
\]
and $|0\rangle$ is the vacuum state (no quarks). A summation over repeated indices is implied. The indices $\mu_i, \nu_i, \cdots$ denote the spatial, spin-flavor, and color coordinates of the i-th quark. $\Phi_{\mu_1 \mu_2 \mu_3}^{\alpha}$ is the baryon wave function, which is antisymmetric in the quark indices and orthonormalized. While the quark operators $q^{\dagger}_\mu$ and $q_\mu$ satisfy the canonical anticommutation relations, the baryon operators satisfy the following noncanonical anticommutation relations:
\[
\{ B_\alpha, B^\dagger_\beta \} = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \quad \{ B_\alpha, B_\beta \} = \{ B^\dagger_\alpha, B^\dagger_\beta \} = 0,
\]
where
\[
\Delta_{\alpha\beta} = 3 \Phi_{\alpha}^{\mu_1 \mu_2 \mu_3} \Phi^{\mu_1 \mu_2 \mu_3}_{\beta} q^{\dagger}_{\nu_3} q_{\mu_3} - \frac{3}{2} \Phi_{\alpha}^{\mu_1 \mu_2 \mu_3} \Phi^{\mu_1 \mu_2 \mu_3}_{\beta} q^{\dagger}_{\nu_3} q^{\dagger}_{\nu_2} q_{\mu_2}. \tag{3}
\]
The presence of the term $\Delta_{\alpha\beta}$ is the physical manifestation of the internal structure of the baryons. Because of this term, the usual field theoretic techniques, such as Wick’s theorem and Greens’s functions, cannot be directly applied to the baryon operators $B$ and $B^\dagger$. To circumvent these problems with the noncanonical nature of the baryon operators, a change of representation is implemented by means of a unitary transformation $U$. Of course, the effects induced by the term $\Delta$ will appear in the effective Hamiltonians describing the interactions among composites and constituents. The transformation $U$ is such that a single real-baryon state $|\alpha> = B^\dagger_\alpha |0\rangle$ is transformed into a single ideal-baryon state $|\alpha) = b^\dagger_\alpha |0\rangle \equiv U^{-1} |\alpha>$. The operator $U$ is of the general form [2,3]:
\[
U = \exp (-\pi/2 F), \quad F = O^{\dagger}_\alpha b_\alpha - b^\dagger_\alpha O_\alpha. \tag{4}
\]
The $b^\dagger_\alpha$ and $b_\alpha$ are the ideal-baryon creation and annihilation operators and $O^{\dagger}_\alpha$ and $O_\alpha$ are functionals of the $B^\dagger_\alpha$, $B_\alpha$ and $\Delta_{\alpha\beta}$ operators. The $b$’s and $O$’s satisfy canonical anticommutation relations:
\[
\{ b_\alpha, b^\dagger_\beta \} = \{ O_\alpha, O^{\dagger}_\beta \} = \delta_{\alpha\beta}, \quad \{ b_\alpha, b_\beta \} = \{ b^\dagger_\alpha, b^\dagger_\beta \} = \{ O_\alpha, O_\beta \} = \{ O^{\dagger}_\alpha, O^{\dagger}_\beta \} = 0, \tag{5}
\]

and, by definition, the $b$ and $b^\dagger$ anticommute with the quark operators. $U$ acts on an enlarged Fock space $\mathcal{I}$, which is the graded direct product of the original Fock space $\mathcal{F}$ and an ideal state space $\mathcal{B}$ (the space spanned by the ideal baryons). The vacuum state of space $\mathcal{I}$, $|0\rangle$, is the direct product of the vacua of $\mathcal{F}$ and $\mathcal{B}$. In $\mathcal{I}$ the physical states, $|\psi\rangle$, constitute a subspace isomorphic to the original Fock space $\mathcal{F}$, and satisfy the constraint $b_\alpha |\psi\rangle = 0$. The unitary transformation acting on the physical states gives rise to the subspace $\mathcal{F}_\Phi = U^{-1} \mathcal{I}_0$, where the physical states are required to satisfy the transformed constraint: $U^{-1} b_\alpha |\psi\rangle = 0$, where $|\psi\rangle = U^{-1} |\psi\rangle$. In $\mathcal{F}_\Phi$ all operators satisfy canonical (anti)commutation relations and, therefore, the traditional field theoretical methods can be employed. In summary, the effect of the transformation on the real baryon states is to reexpress these by ideal ones, and since one simultaneously transforms the operators written in terms of the microscopic quark operators, such as the Hamiltonian, electroweak currents, etc, expectation values and matrix elements are preserved because the transformation is unitary. The advantage, as said above, is that all operators in the new representation are canonical, and the role played by the bound states in the processes is made explicit. A more detailed discussion of these and other formal issues can be found in Refs. [7–9].

$O_\alpha$ is constructed by an iterative procedure as a power series in the baryon wave functions $\Phi$: $O_\alpha = \sum_n O_\alpha^{(n)}$, where $n$ identifies the power of $\Phi$ in the expansion. The expansion starts at zeroth-order with $O_\alpha^{(0)} = B_\alpha$. The construction of the higher order terms $O_\alpha^{(n)}$, $n \geq 1$, involves addition of a series of counterterms such that anticommutation relations of $O$ and $O^\dagger$ are satisfied order by order. Since $\{O_\alpha^{(0)}, O_\beta^{(0)}\} = \delta_{\alpha\beta} - \Delta_{\alpha\beta}$, and $\Delta_{\alpha\beta}$ is of second order [see Eq. (3)], one has that $O_\alpha^{(1)} = 0$, and the next nonzero term is then of order $n = 2$. It is not difficult to show that the second order counterterm that has to be added to $O_\alpha^{(0)}$ to cancel the $\Delta_{\alpha\beta}$ in $\{O_\alpha^{(0)}, O_\beta^{(0)}\}$ is equal to $1/2 \Delta_{\alpha\beta} B_\beta$. Then, up to $n = 2$, $O_\alpha = B_\alpha + 1/2 \Delta_{\alpha\beta} B_\beta$ and one obtains $\{O_\alpha, O_\beta^\dagger\} = \delta_{\alpha\beta} - 1/2[\Delta_{\alpha\gamma}, B_\beta] B_\gamma - 1/2 B^\dagger [B_\alpha, \Delta_{\gamma\beta}] = \delta_{\alpha\beta} + \mathcal{O}(\Phi^3)$. A third order counterterm has to be added such that the $\mathcal{O}(\Phi^3)$ piece cancels, and so on to higher orders. However, for our purposes here one needs $O_\alpha$ up to $n = 3$ only:
\[ O_\alpha = B_\alpha + \frac{1}{2} \Delta_{\alpha \beta} B_\beta - \frac{1}{2} B^\dagger_\beta \left[ \Delta_{\beta \gamma}, B_\alpha \right] B_\gamma . \]  

(6)

When the unitary transformation is applied to the microscopic quark Hamiltonian, one obtains effective Hamiltonians which describe all possible processes involving hadrons and their constituents. We discuss this for a microscopic quark Hamiltonian in which quarks interact by two-body forces. Such a Hamiltonian can always be written as:

\[ H = T(\mu)q_\mu^\dagger q_\mu + \frac{1}{2} V(\mu \nu; \sigma \rho) q_\mu^\dagger q_\nu^\dagger q_\rho q_\sigma , \]  

(7)

where \( T \) is the kinetic energy and \( V_{qq} \) is the quark-quark interaction. In free space, \( \Phi \) satisfies the equation of motion:

\[ H(\mu \nu; \sigma \rho) \Phi^{\sigma \rho \lambda}_{\mu} = 3 \left[ \delta_{[\mu} |\sigma \delta_{\nu \rho]} T([\mu]) + V_{qq}(\mu \nu; \sigma \rho) \right] \Phi^{\sigma \rho \lambda}_{\mu} = E_{[\alpha} \Phi^{\mu \nu \lambda}_{[\alpha}] , \]  

(8)

where we are using the convention that there is no sum over repeated indices inside square brackets, and \( E_\alpha \) is the total energy (center-of-mass energy plus internal energy) of the baryon. The transformation of the Hamiltonian is made by transforming initially the quark operators \( q \) and \( q^\dagger \). Since the \( O \) operators are given by a power series, the transformed quark operators are also obtained as a power series: \( U^{-1} q_\mu U = \sum_n q_\mu^{(n)} \). The \( q_\mu^{(n)} \) can be obtained by expanding the exponential in Eq. (8) to the desired order or, equivalently, by means of the “equation of motion” technique \[7\]. Up to third order, one obtains:

\[
\begin{align*}
q_\mu^{(1)} &= -\sqrt{3} \Phi^{\mu \nu \beta \gamma}_{\alpha} q_\mu^\dagger q_\nu^\dagger q_\beta q_\gamma (b_\alpha + B_\beta), \\
q_\mu^{(2)} &= \frac{3}{2} \Phi^{\mu \nu \beta \gamma}_{\alpha} \hat{\Phi}^{\beta \gamma}_{\alpha} \left( B_\alpha B_\beta + B_\alpha q_\beta q_\gamma + 2 B_\alpha q_\beta q_\gamma \right) \\
q_\mu^{(3)} &= -\frac{27}{8} \Phi^{\mu \nu \beta \gamma}_{\alpha} \left[ \frac{1}{3} q_\mu^\dagger q_\nu^\dagger q_\beta q_\gamma \Delta_{\alpha \beta} (2 B_\beta + b_\beta) - \Phi^{\mu \nu \beta \gamma}_{\alpha} \Phi^{\mu \nu \beta \gamma}_{\gamma} \right] \\
&\quad \times B_\beta q_\nu q_\gamma B_\beta q_\nu q_\gamma + \left( 2 \Phi^{\mu \nu \beta \gamma}_{\alpha} \Phi^{\mu \nu \beta \gamma}_{\gamma} - \Phi^{\mu \nu \beta \gamma}_{\alpha} \Phi^{\mu \nu \beta \gamma}_{\gamma} \right) \\
&\quad \times \left( b_\alpha B_\beta q_\nu q_\gamma b_\beta + B_\alpha q_\nu q_\gamma B_\beta B_\beta + B_\alpha q_\nu q_\gamma B_\beta B_\gamma + B_\alpha q_\nu q_\gamma B_\beta B_\gamma + B_\alpha q_\nu q_\gamma B_\beta B_\gamma \right) \\
&\quad - 2 \Phi^{\mu \nu \beta \gamma}_{\alpha} \Phi^{\mu \nu \beta \gamma}_{\gamma} \left( b_\alpha q_\nu q_\gamma B_\beta B_\beta + B_\alpha q_\nu q_\gamma B_\beta B_\gamma + B_\alpha q_\nu q_\gamma B_\beta B_\gamma + B_\alpha q_\nu q_\gamma B_\beta B_\gamma + B_\alpha q_\nu q_\gamma B_\beta B_\gamma \right) \\
&\quad + \left( B_\alpha q_\nu q_\gamma q_\beta q_\gamma B_\beta B_\gamma \right). 
\end{align*}
\]  

(9)
Substituting these in Eq. (7), one obtains that the general structure of the transformed Hamiltonian is:

\[ U^{-1}HU = H_q + H_b + H_{bq}, \]  

(10)

where the subindices identify the operator content of each term. \( H_q \) describes true quark-quark scattering only, i.e., it is unable to bind the quarks into the bound state baryons; the bound states appear in \( H_b \). This feature leads [10] to the same effect of curing the bound state divergences of the Born series as in Weinberg’s quasi-particle method [4]. \( H_{bq} \) describes quark-baryon processes as baryon breakup into three quarks and three quarks recombining into a baryon. In models where quarks are confined, these terms contribute to free-space baryon-baryon processes as intermediate states only. However, in high temperature and/or density systems hadrons and quarks can coexist and the breakup and recombination processes can play important role. Work is in progress where the explicit form and applications involving the terms \( H_q \) and \( H_{bq} \) are discussed. Next we discuss the baryon Hamiltonian \( H_b \).

Using the transformed quark operators given in Eqs. (9), one obtains the effective baryon Hamiltonian:

\[ H_b = \Phi_{\alpha}^{* \mu \nu \lambda} H(\mu \nu ; \sigma \rho) \Phi_{\beta}^{\sigma \rho \lambda} b_{\alpha}^\dagger b_{\beta} + \frac{1}{2} V_{bb}(\alpha \beta ; \delta \gamma) b_{\alpha}^\dagger b_{\beta}^\dagger b_{\gamma} b_{\delta} , \]  

(11)

where \( V_{bb} \equiv V^{dir} + V^{exc} + V^{int} \) is an effective baryon-baryon potential, which we divide for later convenience into direct, exchange, and intra-exchange parts:

\[ V^{dir}_{bb}(\alpha \beta ; \delta \gamma) = 9 V_{qq}(\mu \nu ; \sigma \rho) \Phi_{\alpha}^{* \mu \nu \mu \nu} \Phi_{\beta}^{\sigma \rho \nu \nu} \Phi_{\gamma}^{\sigma \rho \nu \nu} \Phi_{\delta}^{\sigma \rho \nu \nu} , \]  

(12)

\[ V^{exc}_{bb}(\alpha \beta ; \delta \gamma) = 9 V_{qq}(\mu \nu ; \sigma \rho) \left\{ 2 \Phi_{\alpha}^{* \mu \nu \mu \nu} \Phi_{\beta}^{\sigma \rho \nu \nu} \Phi_{\gamma}^{\sigma \rho \nu \nu} \Phi_{\delta}^{\sigma \rho \nu \nu} - \Phi_{\alpha}^{* \mu \nu \mu \nu} \right\} , \]  

(13)

\[ V^{int}_{bb}(\alpha \beta ; \delta \gamma) = -3 H(\mu \nu ; \sigma \rho) \left( \Phi_{\alpha}^{* \mu \nu \mu \nu} \Phi_{\beta}^{\sigma \rho \nu \nu} \Phi_{\gamma}^{\sigma \rho \nu \nu} \Phi_{\delta}^{\sigma \rho \nu \nu} \right. \]  

\[ + \Phi_{\alpha}^{* \mu \nu \mu \nu} \Phi_{\beta}^{\sigma \rho \nu \nu} \Phi_{\gamma}^{\sigma \rho \nu \nu} \Phi_{\delta}^{\sigma \rho \nu \nu} + \Phi_{\alpha}^{* \mu \nu \mu \nu} \Phi_{\beta}^{\sigma \rho \nu \nu} \Phi_{\gamma}^{\sigma \rho \nu \nu} \Phi_{\delta}^{\sigma \rho \nu \nu} . \]  

(14)

For reasons that will become clear in the discussion below, in Eqs. (11-14) we are not supposing that the \( \Phi \)'s are eigenstates of the microscopic quark Hamiltonian, as in Eq. (8).
The higher order terms which are neglected in Eq. (6) give rise to effective many-baryon (higher than two-baryon) forces, and also introduce orthogonality corrections. These orthogonality corrections are similar to the “wave function renormalization” of RGM calculations \(^1\) and have the effect of weakening the “intra-exchange” interactions, i.e., interactions where the microscopic quark-quark interaction occurs within a single composite \(^7\) \(^9\). When the \(\Phi\)’s are eigenstates of the microscopic quark Hamiltonian, the lowest order orthogonalization cancels the term \(V_{bb}^{\text{int}}\). The weakening of the intra-exchange terms is the major effect of the orthogonalization; higher order corrections start at \(O(\Phi^8)\) and vanish asymptotically with increasing baryon energy.

Eqs. (6,9,11-14) embody the main results of the present paper; these are valid for any quark model which describes the nucleon as a three quark system, and quarks interacting by two-body forces. The \(\Phi\)’s are not restricted to ground states, they describe the entire baryon spectrum derived from the microscopic quark Hamiltonian.

An important feature of the effective baryon Hamiltonian is that it gives rise to scattering amplitudes that are symmetric under exchange of initial and final channels. The lack of this symmetry is known as the post-prior discrepancy \(^11\). The discrepancy is catastrophic for processes with initial and final states with different masses. It is not difficult to convince oneself of this property by exchanging \(\alpha\) and \(\beta\) with \(\gamma\) and \(\delta\) in the expressions above. In the example that follows the symmetry is evident since the effective potential is symmetric under exchange of initial and final momenta \((p \leftrightarrow p')\) [see Eq. (18) below].

To finalize, we present the result of a derivation \(^8\) of an effective nucleon-nucleon interaction following the model of Barnes et al. \(^6\). These authors restrict the \(\Phi\)’s to nonrelativistic s-wave gaussians and use for the microscopic quark-quark interaction the spin-spin part derived from the nonrelativistic reduction of the one-gluon exchange. In second-quantized notation this interaction can be written as:

\[
V_{qq} = \frac{1}{2} \frac{8\pi \alpha_s}{3m_q^2} \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^3} \delta(p_1 + p_2 - p_3 - p_4) \delta_{t_1 t_3} \delta_{t_2 t_4} \\
\times \frac{1}{4} \lambda^a_{c_1 c_3} \lambda^a_{c_2 c_4} \frac{1}{4} \alpha_{s_1 s_3} \cdot \alpha_{s_2 s_4} q_{s_1 t_1}^{c_1} (p_1) q_{s_2 t_2}^{c_2} (p_2) q_{s_3 t_3}^{c_3} (p_3) q_{s_4 t_4}^{c_4} (p_4),
\] (15)
The restriction to this part of the full quark-quark interaction Hamiltonian which in principle should include confinement and other spin-dependent components, is because previous calculations have shown that the spin-spin component provides the dominant contribution to s-wave NN scattering.

Supposing that $\Phi$ is an eigenstate of the full quark Hamiltonian, one is left to lowest order with the $V^{exc}$ part only for the NN effective interaction. Because the single-quark wave functions are gaussians and the quark-quark interaction is a constant (in momentum space), the 12-dimensional integral can be evaluated analytically. The sum over the quark color-spin-flavor indices can be done in closed form using the elegant technique of Ref. [12].

The final result for the effective NN potential is given by:

$$V_{NN} = \frac{1}{2} \int d^3P d^3P' d^3p d^3p' \delta(P' - P) \langle \lambda_1 \lambda_2 | V_{NN}(\omega, \phi, p, p') | \lambda_3 \lambda_4 \rangle \times b^\dagger_{\lambda_1}(p' + P'/2) b_{\lambda_2}(p' - P'/2) b_{\lambda_3}(p - P/2) b_{\lambda_4}(p + P/2),$$

where $\lambda = (M_S, M_T)$ and $V_{NN}(\omega, \phi, p, p') = \kappa_{ss} \sum_{i=1}^5 O_i(\omega, \phi) v_i(p, p'),$ where $\kappa_{ss} = 8\pi \alpha_s/3m_q^2 (2\pi)^3$, $O_1 = 0,$ and:

$$O_2 = \frac{1}{12} \left[ \left( 1 + \frac{1}{9} \phi_N^1 \phi_N^2 \right) + \frac{1}{3} \left( 1 + \frac{1}{9} \phi_N^1 \phi_N^2 \right) \alpha_N^1 \alpha_N^2 \right],$$

$$O_3 = \frac{3}{4} \left[ \left( 1 + \frac{1}{9} \phi_N^1 \phi_N^2 \right) - \frac{1}{27} \left( 1 + \frac{25}{9} \phi_N^1 \phi_N^2 \right) \alpha_N^1 \alpha_N^2 \right],$$

$$O_4 = O_5 = \frac{1}{4} \left[ \left( 1 - \frac{1}{9} \phi_N^1 \phi_N^2 \right) - \frac{1}{9} \left( 1 - \frac{5}{9} \phi_N^1 \phi_N^2 \right) \alpha_N^1 \alpha_N^2 \right].$$

(17)

$\alpha_N$ and $\phi_N$ are nucleon spin and isospin operators, and

$$v_1(p, p') = v_3(p, p') = \exp \left[ -\frac{a^2}{3} (p - p')^2 \right],$$

$$v_2(p, p') = \left( \frac{3}{4} \right)^{1/2} \exp \left[ -\frac{a^2}{6} (p^2 + p'^2) \right],$$

$$v_4(p, p') = v_5(p', p) = \left( \frac{12}{11} \right)^{1/2} \exp \left[ -\frac{2a^2}{11} (p - p')^2 - \frac{a^2}{33} (p^2 + 7p'^2) \right],$$

and $a$ is the r.m.s. radius of the nucleon. $O_1 = 0$ because of color: there is no one gluon exchange between colorless baryons. This term corresponds to the direct part $V^{dir}$ of the baryon-baryon interaction.
From the expressions for the \( v_i(p, p') \), it is clear that the interaction is symmetric under \( p \leftrightarrow p' \) and, therefore, free from the post-prior discrepancy. The effective baryon-baryon interaction \( V_{NN} \) is similar to the one derived by Barnes et al. \cite{6} using the quark Born diagram technique. When calculating the on-shell \( (p^2 = p'^2) \) Born-order T-matrix with the above effective interaction, we arrive at their expressions in \cite{6}. Barnes et al. calculated phase shifts by appropriately defining a local NN potential and obtained results qualitatively similar to the RGM ones. We have also calculated phase shifts, however, we solved numerically the Lippman-Schwinger equation and used the full non-local NN interaction above. Using the parameter set of Barnes et al. \cite{6}, \( \alpha_s/m_q^2 = 0.6/(330)^2 \) MeV\(^{-2} \), \( a = 0.5 \) fm and \( m_q = 330 \) MeV, we obtain for the s-wave phase shifts \(^1S_0\) and \(^3S_1\) the results shown in the figure below.

![Figure](image)

Figure 1. S-wave phase-shifts from the \( V_{NN} \) of Eqs. (16-18). Solid line is for \( S=0, I=1 \) and dashed is for \( S=1, I=0 \). Parameter set: \( \alpha_s/m_q^2 = 0.6/(330)^2 \) MeV\(^{-2} \), \( a = 0.5 \) fm and \( m_q = 330 \) MeV.

The nature of the repulsive core of the NN interaction is clearly seen in this figure. The magnitude of the phase shifts are quantitatively similar to the RGM ones. Moreover, when a purely phenomenological long-range attractive quark-quark interaction is added to the spin-spin one-gluon exchange interaction, we can fit the experimental low-energy s-wave phase shifts by adjusting the parameters of the attractive interaction. We have also checked the effect of the post-prior discrepancy in asymmetrical systems. We found that the lack of the initial-final state symmetry induces large errors in scattering cross sections.

Similar techniques to the one discussed here, mainly developed in the context of nuclear structure, have recently been adapted to the hadronic physics problems \cite{13}. However their emphasis and aim are different from ours. In particular, references \cite{13} deal with the problem of constructing a transformation such that many real-hadron states are mapped to many ideal-hadron states, whereas here the transformation is constructed such that a single
real-hadron state is mapped into a single ideal-hadron state.

In this paper we have applied the formalism to a simple example, however, the formalism is sufficiently powerful and practical to be used in more realistic studies of composite hadrons interactions. In particular, the formalism should be useful for studying hadronic interactions using relativistic light-cone quark models. The Fock space structure of the hadronic states in such models is similar to the ones considered in this paper. Also, the study of the high temperature and/or density regime of hadronic matter, where hadrons and quarks can coexist, by using standard many-body techniques with the effective Hamiltonians derived with the present formalism is particularly interesting.

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