SUGRA Grand Unification, LHC and dark matter

Pran Nath
Department of Physics, Northeastern University, Boston, MA 02115, USA
E-mail: nath@neu.edu

Abstract. A brief review is given of recent developments related to the Higgs signal and its implications for supersymmetry in the supergravity grand unification framework. The Higgs data indicates that the allowed parameter space largely lies on Focal Curves and Focal Surfaces of the Hyperbolic Branch of radiative breaking of the electroweak symmetry where TeV size scalars naturally arise. The high mass of the Higgs leads to a more precise prediction for the allowed range of the spin independent neutralino-proton cross-section which is encouraging for the detection of dark matter in future experiments with greater sensitivity. Also discussed is the status of grand unification and a natural solution to breaking the GUT group at one scale and resolving the doublet-triplet problem. It is shown that the cosmic coincidence can be compatible within a supersymmetric framework in a multicomponent dark matter picture with one component charged under $B-L$ while the other component is the conventional supersymmetric dark matter candidate, the neutralino.

1. Introduction
In December 2011 the ATLAS [1] and CMS [2] Collaborations reported the result of their updated search for the Higgs boson [3] which showed indications of a signal around the $3\sigma$ level. These results lead to considerable theoretical activity to extract the implications of the signal [4, 5, 6]. More recently the analyses of [1, 2] have received further support from the Tevatron analyses [7] and from the combined 7 TeV and 8 TeV CMS and ATLAS data [8]. Thus the CMS Collaboration indicates a signal for a boson with mass of $125.3 \pm 0.6$ GeV at $4.9\sigma$ and the ATLAS Collaboration indicates a signal for a boson with a mass $\sim 126.5$ GeV at $5.0\sigma$ in the combined 7 TeV and 8 TeV analysis [8] which confirm the previous indication of the boson signal by ATLAS [1] and CMS [2]. The properties of the boson still need to be fully established including its spin. Assuming the particle discovered is a spin 0 CP even boson, it is pertinent to ask if it is indeed the Higgs particle which enters in spontaneous breaking of the electroweak symmetry and gives mass to gauge bosons and to quarks and leptons [3]. Further, if indeed the discovered particle is the Higgs boson, it is of interest to determine its properties and any possible deviations from the standard model predictions which would be indicators of new physics beyond the standard model. Definitive answers to these questions will have to wait for much more data. However, partial answers already exist in the data from the Tevatron and from the CMS/ATLAS experiments. Thus there is evidence already that the observed particle emulates the most obvious property of the Higgs boson, i.e., that its coupling to heavier fermions is larger than its coupling to lighter fermions. Thus, if the Higgs boson coupled with equal strength to fermions one would have seen an equal abundance of $\mu^+\mu^-$ as of $b\bar{b}$. However, the Tevatron data suggests otherwise. While there is evidence of $b\bar{b}$ decay of the Higgs there is
no evidence of Higgs decaying into $\mu^+\mu^-$ [7].

2. Higgs and SUSY

An experimental determination of the Higgs couplings are essential in testing the standard model and to find any deviations from the standard model prediction which would be an indication of new physics. The relevant couplings to test are the Higgs boson couplings to fermion-anti-fermion pairs and to dibosons, i.e., $hf, hWW, hZZ, h\gamma\gamma, hZ\gamma$. Thus at the LHC with $\sqrt{s} = 14$ TeV, and $\mathcal{L} = 300$ fb$^{-1}$ up to 10% accuracy for couplings of $h^0$ with dibosons $WW, ZZ$ and up to $\sim 20\%$ for couplings to $b, \tau$ appear possible [9]. Additionally, of course, one needs to test the overall consistency of the total production cross-section with the SM prediction and here currently CMS has $\sigma/\sigma_{SM} = 0.80 \pm 0.22$ and ATLAS has $\sigma/\sigma_{SM} = 1.2 \pm 0.3$ [8]. Important clues to new physics can emerge by looking at deviations of the Higgs boson couplings from the standard model predictions. The possibility that CMS and ATLAS data may have an excess in the diphoton channel has already led to significant interest in the diphoton decay of the Higgs. In the SM the Higgs boson can decay into photons via $W^+W^-$ and $tt\bar{t}$ loops. The W-loop gives the larger contribution which is partially cancelled by the top-loop. To enhance the decay one could add extra charged particles in the loops and various works have appeared along these lines (see, e.g., [10]) More data is needed to confirm the diphoton excess as the result could be due to QCD uncertainties [11].

We discuss now the implications of the 125 GeV Higgs for supersymmetry (see also [12]). There are cogent arguments for the 125 GeV Higgs being the first evidence for supersymmetry at the LHC. The argument assumes the following: (i) perturbative physics up to the grand unification scale; (ii) no large hierarchy problem. Item (i) disfavors composite Higgs models which require non-perturbative physics below the grand unification scale. Thus under constraint (i) the standard model is a possible candidate since the SM is a renormalizable theory and is valid up to the grand unification scale. However, the SM gives large loop corrections to the Higgs mass, so that at one loop one has $m_h^2 = m^2 + O(\Lambda^2)$. Since $\Lambda \sim O(10^{16})$ GeV a large fine tuning is involved, i.e., one part in $10^{11}$. Thus the SM while satisfying (i) does not satisfy (ii). On the other hand SUGRA models have all the merits of the SM, but are free of the large hierarchy problem and give perturbative physics up to the GUT scale. Here $\Lambda$ is essentially replaced by the stop masses.

The second strong hint that the newly discovered particle is a SUSY Higgs is its mass. In the standard model the upper limit on the Higgs mass could be much larger. Using the condition that the weak interactions not become strong one derives roughly the upper bound [13] $M_H < (8\pi\sqrt{2}/3G_F)^{1/2} \approx 1$ TeV. In SUSY, the Higgs quartic couplings are governed by gauge interactions and there is an upper limit which is much smaller, i.e., $M_H < 156$ GeV [14] and in well motivated models such as mSUGRA, GMSB, AMSB it is even smaller. Specifically in mSUGRA [15] $M_H \leq 130$ GeV [4, 16]. It is important to realize that SUSY could have been excluded or come under a shadow if the Higgs mass was found to be, say 200 GeV. Thus it is very meaningful that experimentally the Higgs mass ended up below the upper limit predicted in SUSY and more specifically below the upper limit predicted in SUGRA models. In SUGRA the dominant one loop contribution to the Higgs mass arises from the top/stop sector and is given by

$$\Delta m_H^2 = 3m_t^4/(2\pi^2v^2)\ln(M_{\tilde{t}}^2/m_t^2) + 3m\tilde{s}^2/(2\pi^2v^2)(X_t^2/M_{\tilde{s}}^2 - X_t^2/(12M_{\tilde{s}}^2)),$$

where $v = 246$ GeV, $M_{\tilde{s}}$ is an average stop mass, and $X_t$ is given by $X_t \equiv A_t - \mu\cot\beta$ (for a review see [17]). The loop correction is maximized when $X_t \sim \sqrt{6}M_{\tilde{s}}$. Thus the ‘large’ Higgs mass, i.e., 125 GeV points to the SUSY scale being rather high. Indeed most of the allowed parameter space consistent with all the experimental constraints appears to lie on the Hyperbolic Branch [18, 19, 20, 21], and specifically on Focal Curves and on Focal Surfaces [21] (see Sec.4) which is another indication.
of the overall SUSY scale being high. The 125 GeV Higgs boson also strongly constrains the
sparticle landscape [22] (for a review see [23]). The analysis of the sparticle spectra under the
constraints of the high Higgs boson mass allows for light sparticle masses, i.e., the neutralino,
the chargino, the gluino, the stau and the stop. The light gluino possibility has been extensively
discussed in the literature (see, e.g., [24, 25]). Thus a low lying gluino along with the neutralino,
the chargino, and the stop appear prime candidates for discovery at the LHC, although a more
quantitative analysis is needed correlated with the energy of the future runs at the LHC. The
discovery potential for SUSY in mSUGRA at LHC at $\sqrt{s} = 14$ TeV and at high luminosity with 300
(3000) fb$^{-1}$ of integrated luminosity is discussed in [26].

3. GUTS and SUGRA GUTS

Gauge symmetry based on $SO(10)$ provides a framework for unifying the $SU(3)_C \times SU(2)_L \times
U(1)_Y$ gauge groups and for unifying quarks and leptons in a single 16–plet spinor representation.
Additionally, the 16–plet also contains a right–handed singlet state, which is needed to give mass
to the neutrino via the seesaw mechanism. Supersymmetric $SO(10)$ models [27] have the added
attraction that they predict correctly the unification of gauge couplings, and solve the hierarchy
problem by virtue of supersymmetry. However, SUSY $SO(10)$ models, as usually constructed,
have two drawbacks, both related to the symmetry breaking sector. First, two different mass
scales are involved in breaking of the GUT symmetry, one to reduce the rank and the other to
reduce the symmetry all the way to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Thus typically three types of
Higgs fields are needed (for a review see [28]) e.g., $16 \oplus \overline{16}$ or $126 \oplus \overline{126}$ for rank reduction, and
a 45, 54 or 210 for breaking the symmetry down to the standard model symmetry, and a 10-plet
for electroweak symmetry breaking. Multiple step breaking requires additional assumptions re-
relating VEVs of different breakings to explain gauge coupling unification of the electroweak and
the strong interactions. A single step breaking does not require such an assumption and can be
achieved with $144 \oplus \overline{144}$ [29, 30] of Higgs fields. The reason for this is easily understood
by looking at the decomposition of 144 and $\overline{144}$ multiplets under $SU(5) \times U(1)$. Thus one has
$\overline{144} = (5(3) + 5(7) + 10(-1) + 15(7) + 24(-5) + 40(-1) + 75(3))$. Now the 24-plet carries a $U(1)$
quantum number and thus a VEV formation of it will reduce the rank of the group as well as
break $SU(5)$. Additionally, one can obtain a pair of light Higgs doublets needed for electroweak
symmetry breaking from the same irreducible $144 \oplus \overline{144}$ Higgs multiplet. Thus $SO(10)$ reduces
to $SU(3)_C \times U(1)_{em}$ with just one pair of $< 144 \oplus \overline{144} >$ multiplets.

Second, GUT theories typically have the doublet-triplet problem, i.e., one must do an extreme
fine–tuning at the level of one part in $10^{14}$ to get the Higgs doublets of MSSM light, while color-triplets remain superheavy. Some possible solutions to the doublet-triplet problem include (i) missing VEV mechanism where $SO(10)$ breaks in the B-L direction, (ii) flipped $SU(5) \times U(1)$, (iii) missing partner mechanism, (iv) orbifold GUTs. The flipped $SU(5) \times U(1)$, missing partner mechanism and the orbifold GUTs are rather compelling in that some doublets are forced to be massless. We will focus on the missing partner mechanism [31] and discuss how it works in $SU(5)$ and then discuss how one can extend to $SO(10)$. In $SU(5)$ to obtain Higgs doublets which are naturally light one uses an array of light and heavy Higgs
multiplets so that the heavy multiplets consist of 50, $\overline{50}$, 75 multiplets and the light multiplets consist of 5, 5, i.e., one has mass terms for 75, 50, 50 and no mass terms for 5 + 5. The 75-plet breaks the GUT symmetry to $SU(3) \times SU(2) \times U(1)$. The key element is that 50 + $\overline{50}$ have no doublet pairs (D) and only triplet/anti-triplet pairs (T). Thus 50 + $\overline{50}$ have 0D + 1T while 5 + 5 have 1D + 1T. The Higgs triplets/anti-triplets of 50 + $\overline{50}$ mix with the Higgs triplets/anti-triplets of 5 + 5 to become heavy. This is accomplished via the superpotential
$W(\text{higgs}) = W_0(75) + M_{50} \otimes \overline{50} + \lambda_1 50 \otimes 75 \otimes 5 + \lambda_2 50 \otimes 75 \otimes 5$. The doublets in 5 + 5 have nothing to pair up with and remain light. Next we extend the missing partner mechanism to
SO(10). Here one case has previously been worked out, i.e., the case with the heavy sector consisting of 126 ⊕ 126 and the light sector consisting of 10 ⊕ 120 [32]. In recent works several other cases have been found [33]. These models are anchored in 126 + 126 or 560 ⊕ 560 which play the role of 50 ⊕ 50. Analysis of these larger representations require special tools which utilize oscillator techniques [34, 35].

However, to connect the high scale physics to low energies we need to break supersymmetry. As is well known in global supersymmetry, it is difficult to achieve the breaking of supersymmetry in a phenomenologically acceptable fashion. To do so one must consider local supersymmetry which involves gravity [36, 37]. Thus one needs to construct supergravity grand unified models [15], coupled with radiative breaking of the electroweak symmetry (for a review see [38]) using renormalization group evolution of gauge, Yukawa and of soft terms [39]. A broad class of models fall under this rubric. These include mSUGRA (CMSSM), and SUGRA models with non-universalities in the Higgs sector and in the gaugino sector. mSUGRA has the parameter space $m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)$ where $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, $\tan \beta$ is the ratio $<H_2>/<H_1>$ where, $<H_2>$ gives mass to the up quarks and $<H_1>$ gives mass to the down quarks and leptons and $\mu$ is the Higgs mixing parameter. The mSUGRA model assumes a flavor independent Kahler potential. However, the nature of physics at the Planck scale is not fully understood. Thus it is useful to consider more general Kahler potentials, and indeed in string models and in D brane models one often encounters more general Kahler potentials (see, e.g., [40]). Such choices give rise to non-universal supergravity models which we may label as NUSUGRA models. These include models with non-universal gaugino masses [41] (see also [42]) where the gaugino sector is characterized by the three parameters $\tilde{m}_1, \tilde{m}_2, \tilde{m}_3$ which characterize the gaugino masses for the gauge groups $U(1)_A, SU(2)_L$ and $SU(3)_C$. Similarly, one could have non-universality in the Higgs boson sector [43] with different masses $m_{H_1}, m_{H_2}$ for the Higgses $H_1$ and $H_2$, as well as non-universalities in the scalar sector.

4. Natural TeV Size Scalars

Radiative breaking of the electroweak symmetry naturally allows TeV size scalars without the necessity of excessive fine tuning. To illustrate this phenomenon we consider the generic form of the radiative breaking of the electroweak symmetry (REWSB) which can be written in the form $\mu^2 + \frac{1}{2} M_Z^2 = m_0^2 C_1 + A_0^2 C_2 + m_1^2 C_3 + m_2^2 A_0 C_4 + \Delta \mu^2_{\text{loop}}$. With a redefinition of parameters one can write this equation in the form $\mu^2 = m_0^2 C_1 + \tilde{A}_0^2 C_2 + m_1^2 / 2 \tilde{C}_3 + \Delta \mu^2_{\text{loop}}$ where $\tilde{A}_0 \equiv A_0 + (C_4/2C_2)m_{1/2}$ and $\tilde{C}_3 = C_3 - C_2^2 / (4C_2)$. To simplify matters one can go the renormalization group (RG) scale $Q$ where the loop contribution is minimized (and ideally vanishes). In this case we can use the tree part of the REWSB reliably. Typically $C_1, C_2, \tilde{C}_3$ are all positive which implies that the soft parameters sit on the surface of an ellipsoid given by the REWSB equation. Thus for fixed $\mu$ none of the soft parameters can get too large. One may call this the Ellipsoidal Branch [15]. Suppose now that for certain choices of the inputs such as the top Yukawa, $\tan \beta$, the gauge couplings etc., that the co-efficient $C_1$ turns negative. In this case large value of $m_0$ can occur due to the possibility of cancellation of the $C_1$ term with the rest of the terms in REWSB. This is the Hyperbolic Branch [18, 19, 20, 21] which contains the Focal Point (HB/FP), Focal Curves (HB/FC), and Focal Surfaces (HB/FS). The Focal Point corresponds to the case when $C_1$ vanishes [19], Focal Curves are when two parameters get large, i.e., $m_0, A_0$ (HB/FC1) or $m_0, m_{1/2}$ (HB/FC2), and Focal Surfaces (HB/FS) occur when all the three soft parameters $m_0, m_{1/2}, A_0$ get large while $\mu$ remains fixed and small. In all these cases one finds that large scalar masses exist. Using the LHC constraints the HB/FP region is seen to be depleted [21] while most of the remaining parameter space appears to lie on Focal Curves.
and Focal Surfaces [21, 44].

5. Dark Matter

In supergravity models the neutralino is the lightest R parity odd particle over most of the parameter space of the model and with R parity it is a possible candidate for dark matter. The annihilation of dark matter proceeds via the Z and the Higgs poles ($h^0, H^0, A^0$) in the s-channel and via the sfermion exchange in the t-channel. Thus the relic density is sensitive to the Higgs masses as well as to the masses of the sfermions. Similarly, the spin independent neutralino-proton cross-section arises from the scattering of the neutralino from quarks in a nucleus and the scattering proceeds via squark poles in the s-channel and via the $Z, h^0, H^0, A^0$ poles in the t-channel. Consequently again the spin independent neutralino-proton cross-section is sensitive to the scale of squark masses and to the Higgs masses. Thus one might ask as to the impact of the lightest Higgs mass at 125 GeV on dark matter. As discussed above, the direct implication of a 125 GeV Higgs mass is to raise the scale of supersymmetry. The larger scale leads to heavier sfermion masses which implies that the relic density will be governed mostly by the s-channel $Z$ and Higgs poles and by coannihilation. Analogously the spin independent neutralino-proton cross-section will be largely governed by the t-channel $Z$ and Higgs pole exchanges. These constraints lead to further limits on the allowed parameter space of the model consistent with WMAP [45]. In this context we mention that often the upper limit WMAP constraint is imposed so as to allow for the possibility of multicomponent dark matter [46]. An analysis of the Higgs constraint on the spin independent neutralino-proton cross-section was given in [4, 47] (see also [44]) and it is found that most of the allowed parameter space of mSUGRA model lies between the current sensitivity of the XENON100 experiment [48] and the expected sensitivity in the XENON-1T [49] and SuperCDMS [50]. Thus the prospects for the discovery of dark matter look bright if the proposed sensitivities in the new generation of dark matter experiments can be reached.

6. $g_\mu - 2$ constraint

The anomalous magnetic moment of the muon $g_\mu - 2$ is a sensitive test of new physics. In supersymmetric models $g_\mu - 2$ can receive important contributions from supersymmetric loops. In the standard model electroweak contributions arise from the exchange of the W and Z boson. Analogously in supersymmetric models one has contributions arising from the exchange of charginos and sneutrinos, and from the exchange of neutralinos and smuons [51]. If the particle spectrum is light, one finds that these contributions can be comparable to or even exceed the standard model contribution. The experimental determination of the correction to $g_\mu$ or to $a_\mu$, defined by $a_\mu = (g_\mu - 2)/2$, i.e., $\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$ is sensitively dependent on the hadronic corrections in the standard model. Thus there are two main estimations of the hadronic error: one using the $e^+ e^-$ annihilation and the other using $\tau$ decay. The analysis using $e^+ e^-$ estimation of hadronic corrections gives [52] $\delta a_\mu = (28.7 \pm 8.0) \times 10^{-10}$ which is a 3.6σ deviation from the standard model, while the analysis using $\tau$ decay gives [52] $\delta a_\mu = (19.5 \pm 8.3) \times 10^{-10}$ (2.4σ) which is 2.4σ deviation from the standard model result. As discussed above the high Higgs mass leads to a high scale for the sfermions and with universal boundary conditions on soft parameters they lead to a rather small correction to $a_\mu$. Thus there is a tension between the $g_\mu - 2$ experimental result and the high Higgs mass. Assuming the $g_\mu - 2$ result does not undergo further oscillations, this tension could be relieved if one assumes that part of the Higgs mass arises from sources other than from MSSM. Thus extra matter could give additional contribution to the Higgs mass [53] effectively lowering the component of the Higgs mass arising from MSSM which would allow an $a_\mu$ correction consistent with experiment. Similarly, the presence of an extra gauge group under which the Higgs is charged would lead to a contribution from the D term to the Higgs mass again effectively lowering the MSSM Higgs mass. Yet another possibility
is that \( g_\mu - 2 \) contribution arises from new physics other than SUSY. Thus, for example, a \( Z' \) which couples to the muon will produce a contribution to \( a_\mu \). For, instance a \( Z' \) arising from a gauged \( L_\mu - L_\tau \) can have a significantly lower mass than the current experimental limits on a generic \( Z' \) and can generate a correction to \( g_\mu - 2 \) of the right size [46] (see the discussion in Sec.7).

7. Cosmic coincidence

An interesting cosmic coincidences is the fact that the ratio of dark matter to baryonic matter is roughly 5, or more precisely \( \Omega_{DM}/\Omega_B = 4.99 \pm 0.20 \) [45]. The fact that they are about the same size points to a possible common origin for them. The so called asymmetric dark matter (AsyDM) idea proposes that dark matter originated from baryonic matter in the early universe by the transfer of a net \( B - L \) charge from the visible sector to the dark sector [54] (For a review see [55]). The implementation of this idea has two basic ingredients: first is to find a mechanism for the transfer of \( B - L \) from the visible sector to the dark sector, and second to find a mechanism to deplete the symmetric component of dark matter produced by thermal processes. The standard procedure for the transfer of \( B - L \) from the visible sector to the dark sector is to consider an interactions of the type [56] \( M_a^n O_{DM}^a O_{asy}^S \) which is operative at temperatures \( T_{int} \) so that \( T_{int} > (M_a^2 / M_{Pl}^1) \frac{1}{e^{\mu/T}} \), \( M_{Pl} = 2.4 \times 10^{18} \) GeV where \( O_{asy}^S \) is constituted of only standard model particles and \( O_{DM} \) is constituted of dark matter fields and they have opposite \( B - L \) charges so that overall the interaction is \( B - L \) charge neutral. Concerning the depletion of the symmetric component of dark matter, one needs a mechanism for an efficient annihilation of the thermally produced dark matter.

The early universe can be viewed as a weakly interacting plasma in which each particle carries a chemical potential \( \mu_i \). In such a plasma the particle-anti-particle asymmetries are given by \( n_i - \bar{n}_i \simeq (g_i \beta T^3 / 6)(\mu_i(\text{fermi}), 2 \mu_i(\text{bose})) \) where \( g_i \) is the degrees of freedom, and \( \beta = 1 / T \). The chemical potentials are constrained by (i) sphaleron interactions, (ii) conservation of charge and hypercharge, (iii) Yukawa and gauge interactions. When the transfer interaction is in equilibrium (see, e.g., [57]), one can solve for the ratio \( \Omega_{DM}/\Omega_B = (X/B)(m_{DM}/m_B) \simeq 5 \), where \( X \) is the dark matter number density and \( B \) is the baryon number density. One can generate a variety of models such as models where the visible sector is taken to be the standard model, the two Higgs doublet model, or the MSSM [58]. For the MSSM many variations are possible depending on the scale of sparticle masses. For each model there are various interactions that allow a transfer of the \( B - L \) asymmetry from the standard model sector to the dark matter sector. The relic density in AsyDM will have two components: a thermal and a non-thermal component. Thus one may write the total as a sum \( \Omega_{DM} = \Omega_{DM}^{\text{sym}} + \Omega_{DM}^{\text{asy}} \). For AsyDM to work we need \( \Omega_{DM}^{\text{asy}} \ll \Omega_{DM}^{\text{sym}} \). Thus we need an efficient mechanism for the annihilation of thermally produced dark matter. We accomplish this via the exchange of a gauge field using the Stueckelberg formalism where the gauge field couples to \( L_\mu - L_\tau \) [58]. In the unitary gauge the massive vector boson field will be called \( Z' \) and its interaction with fermions is given by \( L_{int} = Q^f g_C \bar{\psi} \gamma^\mu \psi Z'_\mu + Q^f g_C \bar{f} \gamma^\mu f Z'_\mu \), \( f = \mu, \tau \), \( \mu, \tau \) runs over \( \mu, \mu \) and \( \tau \) families and \( Q^f \). The LEP constraints on the \( M_{Z'} \) mass are not valid since \( Z' \) does not couple with the first generation leptons. This result also holds at the loop level to a good approximation. The strongest constraint comes from \( g_\mu - 2 \). Thus the correction to \( g_\mu - 2 \) arising from the exchange of \( Z' \) boson associated with the \( L_\mu - L_\tau \) symmetry is given by \( \Delta (g_\mu - 2) = (\frac{g_C Q^f \sum m_i^2}{6 \mu^2 M_{Z'}^2}) \). Imposing the constraints \( \Delta \mu = \Delta (g_\mu - 2)/2 \leq 3 \times 10^{-9} \) one finds the restriction \( M_{Z'}/(g_C Q^f) \geq 90 \) GeV. The above constraint allows for a low lying \( Z' \) which couples only to muons and taus and allows for a rapid annihilation of symmetric dark matter via the \( Z' \) pole.
To obtain relic densities at current temperatures for $\psi$ and $\bar{\psi}$ one must solve the Boltzmann equations in the presence of asymmetries. The Boltzmann equations obeyed by $f_\psi$ and $f_{\bar{\psi}}$ take the form $df_\psi/dx = \alpha(\sigma v)(f_\psi f_{\bar{\psi}} - f^{eq}_\psi f^{eq}_{\bar{\psi}})$, $df_{\bar{\psi}}/dx = \alpha(\sigma v)(f_{\bar{\psi}} f_\psi - f^{eq}_{\bar{\psi}} f^{eq}_\psi)$, where $x = k_BT/m_\psi$ and $f_\psi = n_\psi/hT^3$, $f_{\bar{\psi}} = n_{\bar{\psi}}/hT^3$ where $h$ is the entropy degrees of freedom. One finds that $\gamma = f_\psi - f_{\bar{\psi}}$, is a constant independent of temperature. The relic densities for $\psi$ and $\bar{\psi}$ are then given by $\Omega_\psi h^2_0/(\Omega_{\bar{\psi}} h^2_0)_{\xi=0} \simeq \xi J(x_f)/(1 - \exp(-\xi J(x_f))) \rightarrow 1$ as $\xi \rightarrow 0$, where $\xi = \gamma C$ and $C$ is a numerical constant and $J(x_f) \equiv \int_x^{x_f} (\sigma v) dx$. Similarly, $\Omega_{\bar{\psi}} h^2_0/(\Omega_\psi h^2_0) \simeq \exp(-\xi J(x_f)) \rightarrow 1$ as $\xi \rightarrow 0$. We need to show that $\Omega_\psi h^2_0 << (\Omega h^2_0)_{\text{WMAP}}$ and that $\Omega_{\bar{\psi}} h^2_0$ is the major component of WMAP. This has been exhibited explicitly in [58].

We discuss now collider implications of the model. In a muon collider there would be final states with muons and taus and their neutrinos but no $e^+e^-$ final states providing a smoking gun signature for the model [58]. The analysis is done including one loop corrections arising from the first and the second generation leptons in the loop. One can carry out a direct extension of AsyDM to the supersymmetric case. The basic interaction responsible for the asymmetry has the form $W_{\text{asy}} = M_{r}^{-n}O_{DM}O^{\text{mssm}}_{\text{sym}}$. In general there are many possibilities for the operators $O^{\text{mssm}}_{\text{sym}}$ such as $LH_2, LE_2, QLD_2, UCD_2, DCD_2$ or any product thereof. Obviously $O_{DM}$ will carry the opposite quantum numbers to those of $O_{\text{asy}}$. In this case there are two dark matter particles, i.e., $\psi$ and the neutralino $\chi^0$ and here the total relic density is $\Omega DM = \Omega DM^{\text{asy}} + \Omega DM^{\text{ym}} + \Omega \chi^0$, where $\Omega \chi^0$ is the relic density from the neutralino. One must show that the neutralino contribution is subdominant, i.e., it is no more than 10% of the WMAP value. An interesting question is if a subdominant neutralino is detectable. This appears to be the case as demonstrated in [58].

8. R parity and Proton Stability

R parity is needed to get rid of baryon and lepton number violating dimension 4 operators which can generate unacceptably fast proton decay. Within MSSM R parity is ad hoc. Further, R parity as a global symmetry is not desirable since it can be broken by wormhole effects. This problem can be evaded if a model possesses a gauge symmetry so that R parity arises as a discrete remnant of this gauge symmetry (see, e.g., [59]). Since $R = (-1)^{2S + 3(B - L)}$ the obvious extended symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. In this case the $U(1)_{B-L}$ gauge symmetry will forbid R parity violating interactions such as $QLd\nu^c, LE_c\nu^c, \nu^c d^c d^c, LH$. Now an unbroken $U(1)_{B-L}$ gauge symmetry is undesirable since there would be associated with it a massless gauge boson which can generate an unacceptable long range force and thus one must generate a mass for the $B - L$ gauge boson. If the $B - L$ gauge symmetry is broken spontaneously, R parity is no longer guaranteed. Specifically R parity is protected if $3(B - L)$ is an even integer but is not protected if $3(B - L)$ is an odd integer. To illustrate this point in a specific example one may consider an extension of MSSM with a $U(1)_{B-L}$ symmetry which includes three right handed neutrinos fields $\nu^c$ for anomaly cancellation. The extended superpotential in this case is $W = \mathcal{W}_{\text{MSSM}} + h_\nu LH_2 \nu^c + h_\nu \nu^c \nu^c \Phi + \mu_\Phi \Phi \Phi$ where the new fields are assigned the $B - L$ quantum numbers as $(\nu^c, \Phi, \Phi) : (-1, -2, 2)$. Since the $B - L$ of the $\Phi$ field is even, a VEV growth for it will not violate R parity. However, the VEV growth for $\nu^c$ for which $B - L$ is odd, will violate R parity. It turns out that the latter possibility can be realized in radiative breaking. This is so because the beta functions due to the coupling of the $\Phi$ and $\nu^c$ can turn the mass of $\nu^c$ tachyonic which leads to a VEV growth for $\nu^c$ and a violation of R parity [60, 61].

A violation of R parity by the $\nu^c$ VEV growth can be avoided if one uses the Stueckelberg mechanism [62, 63] (see also [64]) for the mass growth of the $B - L$ gauge boson rather than spontaneous breaking. Specifically let us assume a Stueckelberg mechanism for the $B - L$ gauge boson mass growth with the additional assumptions $\langle \tilde{q} \rangle = 0, \langle \tilde{e}_L \rangle = 0 = \langle \tilde{\nu} \rangle$. 


which are just the conditions for charge conservation. Since $\tilde{\nu}_L$ and $\tilde{\nu}_L$ belong to the same $SU(2)_L$ multiplet, the vanishing of $\langle \tilde{\nu}_L \rangle$ also implies the vanishing of $< \tilde{\nu}_L >$, i.e., $< \tilde{\nu}_L >= 0$ since the RG evolution of $M_{\tilde{\nu}_L}$ and of $M_{\tilde{\nu}_L}$ are very similar. It then follows after integrating over the remaining Stueckelberg fields that the scalar potential takes the form $V_{\nu} = M_{\tilde{\nu}_L}^2 \tilde{\nu}^\dagger \tilde{\nu} + (g_{BL} M_{BL})^2 / [2(M_{BL} + M_{BL})]^2|\langle \tilde{\nu}^\dagger \tilde{\nu} \rangle|^2$. In this case there are no beta functions to turn $M_{\tilde{\nu}_L}^2$ negative in the renormalization group evolution. As a consequence the potential cannot support spontaneous breaking to generate a VEV of $\tilde{\nu}$ and $\langle \tilde{\nu}^\dagger \rangle = 0$. Thus with the Stueckelberg mechanism the $B - L$ gauge boson gains a mass but R parity remains unbroken.

We discuss now briefly proton decay. While R parity can eliminate baryon and lepton number violating dimension 4 operators, baryon and lepton number violating dimension five and dimension six operators do exist in unified models. The baryon and lepton number violating dimension five operators arise from color higgsino exchange while the baryon and lepton number violating dimension six operators arise from the exchange of lepto-quarks. The baryon and lepton number violating dimension six operators lead to the dominant proton decay mode $p \rightarrow e^+ \pi^0$ and estimates give a lifetime which could be in the $10^{35}$ yrs range (For a recent review see [65]). The current experimental limit from SuperKamiokande for this mode is $\tau(p \rightarrow e^+ \pi^0) > 1.4 \times 10^{34}$ yrs. It is expected that at Hyper-Kamiokande [66] one will reach the limit $\tau(p \rightarrow e^+ \pi^0) > 1 \times 10^{35}$ yrs, and thus there is a chance for the observation of the $e^+ \pi^0$ mode. Proton decay from baryon and lepton number violating dimension five operators is more model dependent. Thus the baryon and lepton number violating dimension five operators must be dressed by chargino, gluino, and neutralino exchanges to produce baryon and lepton number violating dimension six operators responsible for proton decay. Since the sparticle spectrum enters in the dressing loops, the proton lifetime is dependent on the nature of the sparticle spectrum as well on CP phases (for a review see [67]) and thus proton decay provides a test of models of strings and branes via proton decay branching ratios (see, e.g., [68],[28]). Specifically a light sparticle spectrum will lead to a shorter proton decay while a heavier spectrum will lead to a longer proton decay lifetime. The current experimental limit on p decay from SuperKamiokande gives $\tau(p \rightarrow \mu K^+) > 4 \times 10^{33}$ yrs which puts severe constraints on supersymmetric models. These constraints can be relieved either by GUT models which invoke a cancellation mechanism for the baryon and lepton number violating dimension five operators or a heavy sparticle spectrum. The data from the large hadron collider which indicates that the scale of SUSY breaking for the scalar sector may be large helps stabilize the proton against too fast a decay from dimension five operators. It is expected that Hyper-Kamiokande [66] will reach a sensitivity of $2 \times 10^{34}$ yrs for the $\tau(p \rightarrow \mu K^+)$ mode. In any case proton decay from dimension five operators could become visible even with modest increase in sensitivity for the detection of this mode in the future.

9. Conclusion
We have given a brief summary of the current status of supergravity grand unification. A very significant constraint on the model arises from the recent discovery of the Higgs boson by the ATLAS and CMS detectors and the determination that the mass of the Higgs is high, i.e., around 125 GeV. It has been known for some time that in mSUGRA the mass of the Higgs must lie below 130 GeV, and thus it is interesting that the mass of the Higgs as determined by experiment lies below the predicted limit. However, the high mass of the Higgs implies typically that the average mass of the stops must be significantly large and further that the trilinear coupling $A_0$ typically must be large. These results imply that the part of the parameter space of mSUGRA which gives rise to the large Higgs mass would lie on the Hyperbolic Branch of radiative breaking of the electroweak symmetry and more specifically on Focal Curves or Focal Surfaces. A theoretical analysis using the Higgs mass as a constraint also implies that there is a significant region of the parameter space consistent with all experimental constraints where
the gaugino masses are relatively light. Additionally in some restricted regions of the parameter space on may also have relatively light stops and staus. Thus several light sparticles appear to be prime candidates for discovery at the LHC. These include the neutralino, the chargino, the gluino, and the stop. Another implication of the heavy Higgs mass pertains to neutralino dark matter. The Higgs mass constraint significantly restricts the allowed range of the spin independent neutralino-proton cross section. Interestingly most of the allowed range lies between the current sensitivity of the XENON100 and the expected sensitivity of XENON-1T and SuperCDMS experiments. This result is very encouraging for the discovery of supersymmetric dark matter. The issue of the $g_\mu - 2$ constraint was also discussed. The excess of $g_\mu - 2$ over the standard model result is dependent on the hadronic corrections. The $e^+e^-$ annihilation data and the $\tau$ decay data give somewhat different results for this quantity leading to either $3.6\sigma$ or $2.4\sigma$ discrepancy. If the deviation from the standard model stays there would be tension between the $g_\mu - 2$ result and the high Higgs mass in models with universal boundary conditions on the soft parameters. This tension can be relieved in several ways, such as by having part of the Higgs mass arise from extra matter at the loop level, or from D terms if there is an extra gauge group under which the Higgs is charged. Alternately flavored supergravity models with non-universalities in the flavor sector could account for the discrepancy.

Also discussed was the status of grand unification. Currently $SO(10)$ is the favored group for the unification of the electroweak and the strong interactions. $SO(10)$ has the advantage over $SU(5)$ in that it unifies a full generation of quarks and leptons in a single 16-plet representation. However, the current models based on $SO(10)$ suffer from a couple of drawbacks. The first one concerns the breaking of the GUT group. Here in conventional models more than one scale enters in the breaking of the group. Thus one scale enters in reducing the rank and the other to break the symmetry all the way down to the standard model gauge group. This drawback can be overcome if one considers $144 + 144$ of Higgs whose VEV formation can break the group down to the standard model gauge group in one step. A second draw back which is a generic problem in GUT models concerns the doublet-triple splitting problem. A new class of $SO(10)$ models using $560 + 560$ of Higgs fields not only break the gauge symmetry at one scale they also solve the doublet-triplet problem by the missing partner mechanism. Further, work using these models is thus desirable. The baryon and lepton number violating dimension five and dimension six operators which give rise to proton decay and the prospects for the discovery of proton decay in future experiments were discussed. Finally we discussed the so called cosmic coincidence which pertains to the fact that the dark matter and the baryonic matter are in the ratio $\sim 5 : 1$. The realization of this possibility in the context of supersymmetry was discussed in a multicomponent dark matter framework consisting of a Dirac fermion (carrying $B - L$ charge) and a Majorana fermion (the neutralino).

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