Modulation of acoustic waves by a broadband metagrating

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Metasurface has recently attracted a lot of attentions for controlling wave fields. Based on the diffraction effects of phase gratings, we demonstrate a broadband acoustic metagrating which can concentrate the diffracted waves in the first (±1) orders and achieve multifunctional wave steering such as broadband anomalous diffraction. In the acoustic metagrating, the subwavelength rectangular waveguides (SRWs) function as the periodic elements to replace the fences in ordinary gratings. Thus, we can achieve a group of phase delay from 0 to 2π independently with frequency just by reconfiguring the relative locations of the effective apertures. With the iterative algorithm, the acoustic metagrating can be used to record the phase profile and then control the output waveform. We further demonstrate that the broadband metagrating can be used to achieve the acoustic Gaussian beam. By rotating the periodic elements into a two-dimensional structure, the Bessel beam is further obtained.

Acoustic metasurface is composed of periodic subwavelength elements which can exhibit untraditional manipulation of local and far-field sound pressure distributions. When acoustic waves reach and react with metasurface, phase and amplitude should be modulated, and then the trace of incident waves can be manipulated artificially. By properly designing the positions of periodic elements, metasurface can achieve multifunctional steering of acoustic waves, i.e., acoustic focusing¹–⁴, acoustic carpet cloaking⁷–⁹, asymmetric acoustic transmission¹⁰,¹¹, acoustic trapping¹²,¹³, acoustic holography¹⁴–¹⁶, sound vortices¹⁷ and Mie resonance¹⁷. However, most of the reported metasurface have certain limiting factors, i.e., the narrow work frequency band. For example, in order to control wave trace, the metasurface must provide corresponding phase profile in different positions. This phase profile is usually derived from the generalized laws of diffraction¹⁸, leading to the inherent dependence on the working frequency. Thus, the phase aberrations make the metasurface only work in a narrow frequency band. Recently, as an emerging kind of metasurface, the artificial metagratings have received much attentions due to their more combination functions and more advantage performance over ordinary phase gratings, such as the enhanced acoustic transmission through a rigid plate¹⁹, directional beam forming²⁰, and asymmetric acoustic transmission²¹. In fact, a new broadband optical metagrating has been proposed to achieve anomalous wave steering by engineering diffraction optical gratings²². Based on the grating equation, the frequency-dependent specially can be easily eliminated in the metagratings.

In this paper, we demonstrate a broadband acoustic metagrating by reconfiguring the typical diffraction effects. The metagrating is composed of periodic structured elements consisted by subwavelength rectangular waveguides (SRWs) to achieve the steering of acoustic waves. It is found that the metagrating can convert the normally incident waves into two symmetrical directions in a wide frequency band and provide a phase delay independently with frequency. We further determine the multifunction of the metagrating in wave steering, such as achievement of acoustic Gaussian and Bessel beams.

Results

The diffraction intensity characteristics of grating. First, we start from the transmitted intensity through the phase grating. Figure 1(a) shows the schematic diagram of a regular phase grating. When plane waves are normally incident onto the grating, the angles of diffracted waves satisfy the grating equation of \( \sin \theta_m = m \lambda / d \), where \( m = 0, \pm 1, \pm 2, \ldots \) represents the diffraction order, \( \theta_m \) is the diffraction angle of the \( m \)th order diffraction, \( \lambda \) is the wavelength of incident waves and \( d \) is the grating constant. Here, the grating constant \( d \) is fixed as twice of the aperture size \( a \), i.e., \( d = 2a \). Based on the aperture angular spectrum theory²³, when plane waves of unit amplitude are normally incident to the phase grating, the angular spectrum of diffraction pressure just crossing the grating is equal to the Fourier transform of the grating’s transmissivity function \( t \), and then the angular
The spectrum of diffraction intensity can be the square of Fourier transform $F(t)$. The grating's transmissivity $t(x)$ can be obtained as a combination of that for the fences and apertures. When plane waves cross the phase grating, the fences and apertures should result in different phase delay $\varphi = 2\pi n_H/\lambda$ and $\varphi = 2\pi n_a/\lambda$, respectively. Thus, as shown in Fig. 1(b), the transmissivity function $t(x)$ of the grating can be expressed as

$$t(x) = \begin{cases} e^{2\pi n_H/\lambda}, & nd < x < nd + a \\ e^{2\pi n_a/\lambda}, & nd + a < x < (n + 1)d \end{cases} \quad (n = 0, 1, 2, \ldots)$$

where $n_1$ and $n_2$ are the diffraction indexes of the fence and aperture, respectively, and $H$ is the height of the fence. Based on Fourier expansion for $t(x)$, we can get

$$t(x) = \sum_{m=-\infty}^{\infty} c_m e^{-im\pi x/a}. \quad (2)$$

Here, $c_m = \frac{1}{2} \int_0^{2a} t(x)e^{-im\pi x/a}dx$. Then the angular spectrum of the $m$th diffraction intensity is

$$I(f_x) = |F(t(x))|^2 = \left| \sum_{m=-\infty}^{\infty} c_m e^{im\pi x/a} \right|^2 = \left| \sum_{m=-\infty}^{\infty} c_m e^{imKx} \right|^2$$

$$= \sum_{m=-\infty}^{\infty} c_m^2 \left| e^{2\pi mKx/2\pi} \right|^2 = \sum_{m=-\infty}^{\infty} c_m^2 \left| f_x - mK/2\pi \right|^2$$

$$= \sum_{m=-\infty}^{\infty} c_m^2 \left| f_x - \frac{mK}{2\pi} \right|^2$$

Here, $f_x = \frac{\sin \theta}{\lambda}$ and $K = \frac{2\pi}{\lambda} = \frac{x}{a}$ represents the magnitude of the grating vector. Then it can be obtained by combining the grating equation.
It is obvious that plane waves with different wavelengths diffract in different directions. To be specific, trapezoidal, triangular, rectangular or other shapes of waveguides can be used to replace the fences of grating as long as we choose proper parameters of waveguides to make the effective diffraction index of metagrating and the ambient medium consistent with that of the 1st order diffraction. We further use the effective diffraction index theory to calculate the effective diffraction index of the metagrating. As shown in Fig. 1(c), the metagrating can be constructed by implementing artificial reconfiguration of diffraction intensity based on above functions. Here, each fence of the grating shown in Fig. 1(a) is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs).

From Eq. (3), the intensities of 0th and 1st orders are $I_0 = \cos^2 \left( \frac{\pi H}{\lambda} (n_1 - n_2) \right)$ and $I_1 = \frac{4}{\pi} \sin \left( \frac{\pi H}{\lambda} (n_1 - n_2) \right) \frac{\pi H}{\lambda} (n_1 - n_2)$, respectively. Note that the intensities of the other order diffractions $(m = 2, 3, \ldots)$ can also be determined by Eq. (3). According to the diffraction characteristics of an ordinary grating, the intensity of $m$th order diffraction decreases with the increases of diffraction order $m$. From Eq. 3, we know that the intensity on the even diffraction order is 0 and the intensity on the odd diffraction order is $I_m = \frac{4}{\pi} \sin \left( \frac{\pi H}{\lambda} (n_1 - n_2) \right) \frac{\pi H}{\lambda} (n_1 - n_2)$, i.e., $I_m = 0$. It is obvious that the intensity on the 3rd diffraction order is small and negligible. The intensity on other orders is less than 3rd diffraction order, then the intensity on the order $m > 1$ can be neglected. Obviously, the energies of the zero and first order diffractions take up majority energy of the diffracted waves. When the grating’s parameters meet certain conditions resulting in $I_0 = 0$, i.e.,

$$n_1 - n_2 = \left( k + \frac{1}{2} \right) \lambda / H, \quad (k = 0, \pm 1, \pm 2, \ldots)$$

the 0th order diffraction is totally suppressed and the diffracted waves should mainly distribute in the 1st orders.

**Design of the acoustic metagrating.** As shown in Fig. 1(c), the metagrating can be constructed by implementing artificial reconfiguration of diffraction intensity based on above functions. Here, each fence of the grating shown in Fig. 1(a) is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs). When the effective area of metagrating is replaced by three subwavelength rectangular waveguides (SRWs).

Diffraction characteristics of the metagrating. The far-field intensity distribution exhibits that the diffracted waves mainly distribute in two symmetrical directions, and almost no wave is found in the directions of other orders, as shown in Fig. 2(a). In the simulations, the parameters of SRW are set as $S = 0.3$ mm, $W = 0.1$ mm, $H = 0.6$ mm, and $d = 3$ mm, respectively, and the incident wavelengths are in the range of 1.4 ~ 1.8 mm. Figure 2(b) shows the transmissivity of different orders. The transmissivity of negative orders is limited to the wavelength range of 1.5 mm ~1.6 mm. To sum up, the metagrating can concentrate the transmitted waves in the 1st orders under a wide work frequency range. Thus, by setting proper parameters of the SRW (e.g., $W, H, S$), the metagrating can achieve broadband anomalous diffraction, and then can be used to work as a beam splitter.
Implementation of Gaussian beam. Expect for anomalous diffraction, the metagrating can also be used to achieve the acoustic Gaussian beam. As shown in Fig. 3(a), six SRWs function as the effective aperture. In the metagrating, the displacement of effective aperture leads to the variation of acoustic path difference and thus causes the phase delay $\varphi_1$. In the momentum space, as shown in Fig. 3(b), different displacements of effective apertures result in the same transverse momentum, i.e., $\varphi + \varphi_1 + \varphi = Dd_1$. Here $D$ represents twice the grating constant, i.e., $D = 2d$, $\varphi = 4\pi$ represents the phase delay caused by the path difference $D\sin\theta_1$. $d_1$ represents the displacement of effective aperture. Then we can get the relation of phase delay and displacement $d_1$, i.e.,

$$\varphi_1 = \frac{d_1}{D}$$

It is noted a group of phase delay $\varphi_1$ ranging from 0 to $2\pi$ can be obtained by altering $d_1$, and $\varphi_1$ is independent with frequency. The simulated results in Fig. 3(c) show that the phase delay covers from 0 to $2\pi$ and varies linearly with the distance variation $d_1$. Thus, the metagrating can record the phase information of designed waveform. The required phase profile of metagrating is obtained by the Gerchberg-Saxton phase-retrieval with iterative angular spectrum algorithm. In this algorithm when the waveform is known, the corresponding phase distribution in the metagrating can be obtained through several iterations. With the phase distribution, the displacement $d_1$ in different positions is also available from Eq. (5). By arranging the effective apertures according to $d_1$, the metagrating can be constructed. When the plane waves are normally incident onto the metagrating, the wanted waveform pattern should be obtained in the output side.

Based on the above method of phase control, the plane waves can be converted to the Gaussian beam. Figure 4 shows the simulated results when the incident wavelengths are 1.5 mm, 1.8 mm, 2.1 mm, respectively. In the metagrating, thirty pairs of SRWs are used to implement the Gaussian beam. The arrangement of periodic elements should be obtained by the above iterative algorithm. The Gaussian beam can be viewed as get a one-dimensional image of the gaussian at the position of its beam waist. Here, we set the width of the waist position of the Gaussian beam at the image plane to be about four times the wavelength at $\lambda = 1.5$ mm. The image plane is perpendicular to the direction of the 1st order diffraction and its length is equal to the projection of the entire metagrating in the diffraction direction. Figure 3(d) shows the theoretical phase distribution of metagrating to implement the Gaussian beam. Then the displacement $d_1$ of each element in different positions is also available from Eq. (5). By

![Figure 2](image-url)
arranging the effective apertures according to $d_1$, the metagrating can be constructed and its details is shown in the black dashed box in Fig. 4. In arrangement, if two adjacent effective apertures overlap, only the waveguide in the second effective aperture will be retained. Then large errors will be caused in achieving the Gaussian beam if one or two SRWs are used to replace each fence of the grating. The right panels in Fig. 4(a–c) show the intensity distribution at the beam waist position and the waist position is indicated by a white dashed line. The length of these white dashed lines are the same, and the length is fixed at the width of image plane when the incident wavelength is 1.5 mm. It is found that the Gaussian beam is obtained in different directions when the incident wavelength changes, but the beam remains approximately the same width at the waist position. Thus, the metagrating is broadband to achieve one dimensional images.

Implementation of Bessel beam. Finally, we rotate the elements to get a two-dimensional structure, which can create the “non-diffracting” Bessel beam and the Bessel beam can be used to achieve microparticle manipulations. An ideal 0th order Bessel beam can be described by

$$ P(r, y) = P_{00}(k, r)e^{ik_0y} $$

Where $P(r, y)$ is amplitude in the radial distance $r$ and the axial distance $y$, $P_{00}(k, r)$ is the zero order Bessel function, and $k_0 = \sqrt{k_x^2 + k_y^2}$, $k_x$, $k_y$ are radial and axial wave vectors, respectively. Then we can get the intensity at the position $(r, y)$

$$ I(r, y) = |P_{00}(k, r)|^2 $$

For Bessel function, the cross-section intensity $I(r, y)$ is changeless along the axial direction ($y$-direction) and composed of a set of concentric annulus. Indeed, the Bessel beam can be seen as a cluster of plane waves going through a cone with taper angle $\theta = \tan^{-1}\left(\frac{k_y}{k_x}\right)$. Thus the beam is just like an interference field composed of a cluster of iso-amplitude plane waves with the different azimuth angles but identical intersection angle with the axial direction. Then we can use the above-mentioned metagrating to get Bessel beam by rotating the waveguides to obtain a disc metagrating composed of a set of toroidal waveguides. The metagrating can divide the normally incident waves into two symmetrical directions centered on the $y$ axis. The symmetrical transmitted waves will intervene in the intersection region and then form the Bessel beam, as shown in Fig. 5(a). The inset of Fig. 5(a)
shows the phase distribution derived from the generalized laws diffraction of theoretical metagrating (red line) and the discretized phases of disc metagrating (blue circle). Note that the metagratings can provide the phase distribution to implement the Bessel beam. Figure 5(b) shows the intensity profile of the vertical section at $z = 0$ when the incident wavelengths are 1.5 mm, 1.8 mm, 2.1 mm, respectively. It is found that the smaller the wavelength is, the farther it propagates. The area in rhombus exhibits the waves which concentrate in a small area and propagate a long distance with shape changeless. The radius of the plane waves is about $8 \lambda$ while the diffracted waves can propagate non-diffractively over $20 \lambda$ long when the wavelength is 1.5 mm. The top panel of Fig. 5(c) takes the incident wavelength of 1.5 mm as an example, which shows the intensity distribution at the cross section at $y = 5.5d$. The diffracted waves in the cross section exhibit water-ripples-like shape, which are composed of a center peak and a set of side-lobes in concentric circles. The envelope line of the projection is just like the 0th order Bessel function curve in Fig. 5(a). The bottom panel in Fig. 5(c) shows the intensity distribution of the blue dashed line in Fig. 5(b) at $y = 5.5d, 4d, 3d$ when the incident wavelengths are 1.5 mm, 1.8 mm, 2.1 mm, respectively. Although the intensity at the center position varies with the incident wavelength, the contour of the curves always conform to the Bessel curve, which confirm that the disc metagrating can convert the incident waves to the Bessel beam in a wide frequency work band.
Discussion
We have proposed a broadband acoustic metagrating. By reconfiguring the diffraction effects through the distributed SRWs, the acoustic metagrating can achieve broadband multifunctional wave steering such as anomalous diffraction in a wide frequency band. Besides, by arranging the SRWS according to the diffraction effects of irregular gratings, the metagratings can also provide a group of phase delay from 0 to $2\pi$, which is only related to the grating constant and the displacement of effective apertures. Thus, the metagratings can modulate the wave trace in a wide frequency band, and convert the plane waves to the Gaussian beams or Bessel beams at different incident wavelengths. The proposed metagrating may provide a possibility to the broadband acoustic devices.

Methods
To demonstrate the versatility of the metagrating, the full wave simulations are performed with the COMSOL Multiphysics based on the finite element analysis (FEA) method. In the simulation, water ($c_w=1500$ m/s, $\rho_w=1\times10^3$ kg/m$^3$, $n=0.23$) and rubber ($c_r=336$ m/s, $\rho_r=0.89\times10^3$ kg/m$^3$) are used as the ambient medium and the material of subwavelength rectangular waveguides, respectively. When calculating the far-field intensity distribution in Fig. 2(a), the SRWs with one period is selected and the periodic boundary conditions are applied on left and right sides. In order to get the transmission coefficient of different orders in Fig. 2(b), twenty periods of SRWs are used in simulation. By combining the grating equation and integrating the intensity around the structure, the total intensity and the intensity of different diffraction angles can be obtained. Then the transmission coefficient can be calculated.

Data Availability
The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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## Acknowledgements

This work was supported by the National Basic Research Program of China (2017YFA0303702), NSFC (Grant Nos. 11834008, 11874125, 11574172 and 11574148), Jiangsu Provincial NSF (BK20160018) and the Fundamental Research Funds for the Central Universities (02041438001 and 020414380134).

## Author Contributions

Y.W. and Y.C. performed the theoretical analysis and numerical simulations. Y.W., Y.C. and X.L. prepared the manuscript. All authors reviewed the manuscript.

## Additional Information

### Competing Interests

The authors declare no competing interests.

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