Density fluctuations and a first-order chiral phase transition 
in non-equilibrium

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The thermodynamics of a first-order chiral phase transition is considered in the presence of spinodal phase separation using the Nambu-Jona-Lasinio model in the mean field approximation. We focus on the behavior of conserved charge fluctuations. We show that in non-equilibrium the specific heat and charge susceptibilities diverge as the system crosses the isothermal spinodal lines.

1. Introduction

The search for the critical end point (CEP) is one of the central issues in strongly interacting hot/dense QCD matter. It is of particular interest to identify the position of the CEP in the phase diagram and to study general properties of thermodynamic quantities in its vicinity. Modifications in the magnitude of fluctuations or the corresponding susceptibilities can be considered as a possible signal for deconfinement and chiral symmetry restoration. In this context, fluctuations related to conserved charges play an important role since they are directly accessible in experiments.

The enhancement of the baryon number fluctuations could be a clear indication for the existence of the CEP in the QCD phase diagram. However, finite density fluctuations along the first-order transition appear under the assumption that this transition happens in equilibrium. This is modified when there is a deviation from...
In this contribution we briefly show that the enhanced baryon number density fluctuations is a signal for the first-order phase transition in the presence of spinodal decomposition.

2. Quark number susceptibility and spinodal instabilities

In a non-equilibrium system, a first-order phase transition is intimately linked with the existence of a convex anomaly in the thermodynamic pressure. There is an interval of energy density or baryon number density where the derivative of the pressure, $\partial P/\partial V > 0$, is positive. This anomalous behavior characterizes a region of instability in the $(T, n_q)$-plane which is bounded by the spinodal lines, where the pressure derivative with respect to volume vanishes. The derivative taken at constant temperature and that taken at constant entropy, $\left(\frac{\partial P}{\partial V}\right)_T = 0$ and $\left(\frac{\partial P}{\partial V}\right)_S = 0$, define the isothermal and isentropic spinodal lines respectively.

If the first-order phase transition takes place in equilibrium, there is a coexistence region, which ends at the CEP. However, in a non-equilibrium first-order phase transition, the system supercools/superheats and, if driven sufficiently far from equilibrium, it becomes unstable due to the convex anomaly in the thermodynamic pressure. In other words, in the coexistence region there is a range of densities and temperatures, bounded by the spinodal lines, where the spatially uniform system is mechanically unstable. Spinodal decomposition is thought to play a dominant role in the dynamics of low energy nuclear collisions in the regime of the first-order nuclear liquid-gas transition. Furthermore, the consequences of spinodal decomposition in the connection with the chiral and deconfinement phase transitions in heavy ion collisions have been discussed.

In Fig. left we show the evolution of the net quark number fluctuations along a path of fixed $T = 50$ MeV calculated in the Nambu–Jona-Lasinio (NJL) model in the mean field approximation. When entering the coexistence region, there is a singularity in $\chi_q$ that appears when crossing the isothermal spinodal lines and where the fluctuations diverge and the susceptibility changes sign. In between the spinodal lines, the susceptibility is negative. Consequently, this implies instabilities in the baryon number fluctuations when crossing from a meta-stable to an unstable phase. The above behavior of $\chi_q$ is a direct consequence of the thermodynamics relation

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{n_q^2}{V} \frac{1}{\chi_q}.$$  (2)

Along the isothermal spinodals the pressure derivative in Eq. (2) vanishes. Thus, for non-vanishing density $n_q$, $\chi_q$ must diverge to satisfy (2). Furthermore, since the pressure derivative $\partial P/\partial V|_T$ changes sign when crossing the spinodal line, there must be a corresponding sign change in $\chi_q$, as seen in Fig. left.
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Fig. 1. (Left) The net quark number susceptibility at $T = 50$ MeV as a function of the quark number density across the first-order phase transition. (Right) The net quark number susceptibility in the stable and meta-stable regions.

In Fig. 1-right we show the evolution of the singularity at the spinodal lines when approaching the CEP. The critical exponent at the isothermal spinodal line is found to be $\gamma = 1/2$, with $\chi_q \sim (\mu - \mu_c)^{-\gamma}$, while $\gamma = 2/3$ at the CEP. Thus, the singularities at the two spinodal lines conspire to yield a somewhat stronger divergence as they join at the CEP. The critical region of enhanced susceptibility around the TCP/CEP is fairly small, while in the more realistic non-equilibrium system one expects fluctuations in a larger region of the phase diagram, i.e., over a broader range of beam energies, due to the spinodal instabilities.

The rate of change in entropy with respect to temperature at constant pressure gives the specific heat expressed as

$$C_P = T \left( \frac{\partial S}{\partial T} \right)_P = TV \left[ \chi_{TT} - \frac{2s}{n_q} \chi_{\mu T} + \left( \frac{s}{n_q} \right)^2 \chi_q \right].$$

The entropy $\chi_{TT}$ and mixed $\chi_{\mu T}$ susceptibilities exhibit the same behaviors as that of $\chi_q$ shown in Fig. 1-left. Thus $C_P$ also diverges on the isothermal spinodal lines and becomes negative in the mixed phase. It was argued that in low energy nuclear collisions the negative specific heat could be a signal of the liquid-gas phase transition. Its occurrence has recently been reported as the first experimental evidence for such an anomalous behavior.

3. Conclusions

We have shown that in the presence of spinodal instabilities the net quark number fluctuations diverge at the isothermal spinodal lines of the first-order chiral phase transition. As the system crosses this line, it becomes unstable with respect to spinodal decomposition. The unstable region is in principle reachable in non-equilibrium.

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$^a$ The specific heat with constant volume, on the other hand, continuously changes with $n_q$ and has no singularities on the mean-field level.
systems that is most likely created in heavy ion collisions. Consequently, large fluctuations of baryon and electric charge densities are expected not only at the CEP but also when system crosses a non-equilibrium first-order transition.

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