Research of piezoelectric vibratory angular rate sensors with improved metrological characteristics

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Abstract. Piezoelectric vibratory angular rate sensors are widely used in different applications. The current article presents and discusses a theoretical research of scale factor temperature drift of the piezoelectric gyroscope. Numerical simulation of scale factor temperature sensitivity using ANSYS finite element software package is also considered. The theoretical results are compared with the experimentally obtained values.

1. Introduction

In recent years areas related to the development of compact, reliable and cost-effective navigation and control systems for moving objects have been actively developing. These are: spatial orientation control of wells; orientation, navigation and control systems for unmanned aerial vehicles, autonomous underwater vehicles and mobile robots; stabilization systems for railroad transport. These systems use angular rate sensors (gyroscopes) to measure angular displacements of objects.

Analysis of publications and research work on the subject [1-3] has shown that despite of the fact that MEMS gyroscopes receive more and more wide application in the systems of navigation, stabilization and orientation of various objects, the problems concerning research and design of piezoelectric angular rate sensors are still actual. On a number of parameters (cost, power consumption) piezoelectric vibrating gyroscopes overcome micromechanical gyroscopes that make them more suitable for mobile, autonomous instrumentation [1].

Various designs and principles of operation of the piezoelectric vibrating gyroscopes were previously described in articles. However, above publications contain no detailed description of the dynamics and errors of the piezoelectric vibrating gyroscopes. Methods of calculation of sensitivity, zero offset and temperature drift are also not well presented.

Thus it is of great relevance to research and develop reliable and efficient modeling and design methods of the piezoelectric gyroscopes. As well as development of adequate models, providing a detailed study of the physical processes occurring in piezoelectric gyroscopes, and search for new design solutions.

One of the main problems of vibratory gyroscopes development is increasing of the gyroscope sensitivity to angular velocity. It is known, that the highest sensitivity of a vibratory gyroscope is achieved at coincidence of resonance frequencies of primary and secondary vibrations. That is why designs of piezoelectric vibratory gyroscopes based on axisymmetrical resonators look promising since rotational symmetry allows to achieve equality of frequencies of primary and secondary vibrations. Another important advantage of such gyroscopes is monolithic design of their resonators. Monolithic design of the resonator allows to minimize using of diverse materials and thus to increase
stability of gyroscope characteristics in comparison with the gyroscopes based on conventional prismatic and bimorph resonators.

The design of such vibratory angular rate sensor containing thin-walled cylinder (resonator) made of piezoelectric ceramics is presented on figure 1.

Figure 1. Tubular piezoelectric angular rate sensor.

Resonator has two pairs of electrodes on external surface, and a common electrode on inner surface. The principle of action of the piezoelectric vibratory gyroscope is based on transformation of energy of the primary vibrations excited by means of the first pair of electrodes in plane $XZ$ to energy of secondary vibrations in $YZ$-plane under action of Coriolis forces exerted by rotation of a gyroscope around $Z$-axis. The second pair of electrodes serves for detecting of vibrations in $YZ$-plane. An oscillator is connected between the driving electrodes for self-oscillating driving of resonator. When the excitation signal is applied to driving electrodes, resonator makes bending vibrations in $XZ$-plane. The signal due to the Coriolis force is synchronously detected by the synchronous detection circuit, low-pass filtered and then amplified by DC amplifier.

Simple 1-D analytical model of the vibratory piezoelectric gyroscope based on classical beam theory is considered in [4]. Disadvantage of this model is that shearing strain and rotational inertia are not taken into account when considering bending vibration of the resonator. This results in a higher predicted eigenfrequencies for a given set of boundary conditions. To increase the prediction accuracy Timoshenko beam model was used for the description of the gyroscope dynamics. Timoshenko beam theory considers effects of a shear strain and rotational inertia and thus can be used for describing the behavior of short beams (it’s a better approximation of the real resonators). The obtained differential equations system was solved using MATLAB software.

Using the obtained modified model numerical estimations of the instability of the gyroscope scaling factor and shift of the resonator working frequencies in various temperature conditions were performed. It was found that the main factors affecting sensitivity of piezoelectric tubular vibratory gyroscope (figure 2) are temperature variations of dielectric permittivity $\varepsilon_{33}$ and mechanical quality $Q_m$ (37% and 16% respectively). The temperature variations of piezoelectric modulus $d_{31}$ and compliance $s_{11}$ have much less influence on sensitivity (10 % and 2 % respectively).

Technological imperfections, which are always present during the fabrication process of piezoelectric gyroscope, are of primary importance since they significantly reduce the measurement accuracy. Such technological imperfections include deviations from the ideal mass, stiffness and damping distributions. Resonators with imperfections show splitting of natural frequencies, location of the two mode shapes and different damping factors associated with each vibration mode. Another reason for the errors of piezoelectric gyroscopes is the presence of temperature-induced errors due to the temperature sensitivity of mechanical and electrical material properties of piezoelectric ceramics.
The change of material properties leads to imbalance and a change in the dynamic characteristics of the resonator.

![Graph showing parameters of the material](image)

**Figure 2.** The influence of material properties variations on gyroscope sensitivity [5].

To evaluate the effect of technological temperature-induced errors on the output signal of the gyroscope, the finite elements method in three-dimensional formulation was used. Practical implementation of the method is executed using the package finite element simulation ANSYS v11. Numerical calculations for imperfect thin-walled cylindrical piezoelectric resonator (axial misalignment) were performed.

The transient analysis was performed to calculate the trajectories of the gyroscope resonator motion (figure 3, figure 4). These graphs show that the wall thickness variation leads to imbalance of the resonator. Thus, the displacement along the Y axis of the resonator under the influence of the Coriolis force is distorted by the projection of the drive on the same axis, which leads to an error in the measurement of angular velocity. As a result, at the output of the piezoelectric angular rate sensor the spurious output signal in the absence of angular speed \( \Omega \) will be observed.

![Graph showing trajectory of ideal resonator motion](image)

**Figure 3.** The trajectory of the ideal resonator motion (\( \Omega = 0 \) rad/s).

![Graph showing trajectory of imperfect resonator motion](image)

**Figure 4.** The trajectory of the imperfect resonator motion (\( \Omega = 0 \) rad/s).
Temperature-induced position shift of the resonator vibration mode planes leads to the instability of the scale factor of an angular rate sensor [6]. The corresponding plot for two values of splitting of the natural frequencies of the resonator (0.3 Hz and 0.5 Hz) is shown in figure 5.

![Figure 5. Temperature dependence of the scale factor of an angular rate sensor.](image)

2. Results and discussion

Simulation results were verified experimentally. Characterization of the resonance frequency was conducted in laboratory conditions at room temperature. The resonators were excited by applying an AC voltage with amplitude not exceeding 100 mV in order to avoid resonators dissipative heating.

For the validation of the developed analytical model, results were compared with experiment and finite-element calculations (table 1). The closest match to the measured values of the resonant frequency provided numerical modeling. Also from the presented data it is visible that the proposed analytical model gives larger deviation of resonant frequency than numerical calculation but still provides a smaller error than the known analytical model [4].

| Models to compare          | Natural frequency, Hz | Deviation from the experimental results, % |
|---------------------------|-----------------------|-------------------------------------------|
| Known analytical model [4]| 13220                 | 32.46                                     |
| Proposed analytical model | 12288                 | 23.1                                      |
| Proposed numerical model  | 10648                 | 6.69                                      |
| Experimental results      | 9980                  | –                                         |

Temperature-frequency characteristic of the resonator for the main flexural mode was measured over a -20 ... +80 °C temperature range and compared to the analytical and numerical results (figure 6). From figure 6 it is visible that a quadratic parabola with a slope coefficient $9 \times 10^{-3} (°C)^{-2}$ provides good agreement with the experimentally obtained curve. The difference between experimental and calculated data obtained within analytical and numerical models is 23.6% and 7% respectively.
Also it was found that the average value of the scale factor (magnitude of the drive signal is in the range 5 – 15 V) is 0.35 – 1.04 mV/deg/s (figure 7). Preliminary experiments showed that the value of zero drift is 0.8 deg/s. Obtained characteristics are in qualitative agreement with the results of analytical and finite-element modeling.

Figure 6. Temperature-frequency response of the piezoelectric resonator.

Figure 7. Rate response of the angular rate sensor.

Thus, experimental studies of the piezoelectric vibratory gyroscope prototype confirmed the accuracy of the theoretical studies and efficiency of the developed gyroscope as an angular rate sensor. The results obtained in this work can be used to develop error models of piezoelectric angular rate sensors of similar design, and provide an opportunity to improve metrological characteristics of position and deformation control instruments.

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