Cooperative beamforming with link-adaptive regenerative relays

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Abstract
This paper examines a new beamforming method, and detector for a multi-relay system is proposed, in which the signals from relays, and direct link are combined to achieve optimal performance. Relays employ link-adaptive regeneration relaying method to scale their decoded symbols before retransmitting them in order to alleviate the error propagation. The scaling is adapted with the signal-to-noise ratio of the source-relay, and the intended relay-destination links. In the proposed beamforming scheme, two-bit quantized phase information of the channel gain between the relay, and destination is used to retransmit decoded symbols at relays. It is analytically shown that the proposed method achieves full diversity order. Numerical results show a close match between simulation, and analytical derivations.

1 | INTRODUCTION

In wireless communication systems, diversity techniques are employed to enhance the performance under multipath fading. Cooperative diversity is a special case of spatial diversity where relay stations retransmit their decoded symbols to destination through independent channels [1–5]. Instead of using orthogonal channels, relays can retransmit their symbols simultaneously in the same band, and with proper detection techniques, the diversity can be achieved without any bandwidth or rate loss [6]. Distributed Space-Time coding [7–9] and selection of the best relay [10–12] are other ways that have been widely considered in the literature.

Amplify-and-forward (AF) and decode-and-forward (DF), are two primary policies used for retransmission at the relays in cooperative systems. In AF method, a multiple of the noisy received signal is retransmitted while in DF relaying, the symbols are first decoded after processing of the noisy received signal at the relay and then are transmitted to the destination. As a major setback for DF method, the erroneous detections at relay can lead to erroneous detections at destination. This issue which is referred to as error propagation, will deteriorate the bit error rate (BER) and diversity order of relaying systems. Cyclic redundancy codes (CRC) [13] and selection relaying [14–16] are two well-known approaches to alleviate error propagation in the source-relay link. In selection relaying, relays collaborate in retransmission whenever channel SNR in source-relay link goes beyond a certain threshold [17]. Another approach to mitigate error propagation is link-adaptive regeneration relaying [18]. In this method, the relays scale their detected symbols adaptively according to the SNR of the source-relay and the corresponding relay-destination links. Links with lower SNRs are scaled by smaller factors aimed at reducing their impact upon the final detected symbol at the destination.

With the presence of multiple relays, cooperative beamforming [19–24] is a smart approach for enhancing BER performance and obtaining higher diversity order in relay networks. In cooperative beamforming, the transmitted signals of relays are scaled in a way to improve the overall BER performance at the destination. Cooperative beamforming entails availability of relay-destination link information at relays. While Feedbacking this information is one of the major challenges of cooperative beamforming, quantized feedback beamforming [25] can be employed to reduce the amount of feedback information.

This paper proposes a novel cooperative beamforming method for regenerative relaying systems. The proposed beamforming approach considers constraints on maximum power of each relay node in a two-hop multi-relay cooperative network. The proposed beamforming algorithm requires only two-bit quantized phase of the channel gains of relay-destination links. It is shown that with two bit quantization, the proposed method achieves a BER performance very close to an ideal system that uses complete knowledge of the channel.
information. The proposed method utilizes link-adaptive regeneration relaying to reduce the error propagation effect. By investigating the asymptotic BER performance in high SNR regime analytically, it is shown that proposed joint cooperative beamforming with limited feedback and LAR relaying achieves full diversity order.

The rest of this paper is organized as follows. In Section 2, we introduce the system model for a multi-relay two-hop cooperative network and describe our proposed beamforming and LAR relaying in details. In Section 3, we analyze the asymptotic behaviour of the BER and the diversity order of the system for BPSK modulation. In Section 4, simulation results are presented and finally conclusions are drawn in Section 5.

2 | SYSTEM MODEL

Consider a multi-relay network with DF relaying as depicted in Figure 1. The symbols $S$, $R_m$ and $D$ stand for the source, $m$th relay and destination, respectively. Moreover, the number of relays is denoted by $L$. If the source node transmits the symbols $x_s \in \{\pm 1\}$ in the first hop, then the received signal at relays and destination can be expressed as

$$y_{sr_m} = h_{sr_m} \sqrt{P_s} x_s + z_{sr_m} \quad (1a)$$

$$y_{rd} = h_{rd} \sqrt{P_s} x_s + z_{rd} \quad (1b)$$

where $h_{ij}$ is a zero mean circularly symmetric complex Gaussian (ZMCSG) random variable representing the channel coefficient between node $i$ and node $j$. The channels between distinct pairs are considered to be statistically independent. We also assume $b_{sr_m} \sim CN(0, \sigma^2_{sr_m})$, $b_{rd} \sim CN(0, \sigma^2_{rd})$ for $m = 1, ..., L$, and $b_{sr_m} \sim CN(0, \sigma^2_{sr_m})$, $b_{rd} \sim CN(0, \sigma^2_{rd})$, $z_{sr_m}$ for $m = 1, ..., L$, and $z_{rd}$ denote the additive Gaussian noises at relays and destination with distribution $CN(0, N_0)$, respectively. The additive noises are independent at all nodes and the transmitted power of the source is denoted by $P_s$ while the total transmitted power by all relays is $P_r$.

In the second hop, the $m$th relay scales and rotates its detected symbol by the complex coefficient $\beta_m = \alpha_m e^{j\theta_m}$. The value of $\alpha_m$ is adaptively adjusted by the SNRs of the corresponding source-relay and relay-destination channels to eliminate the error propagation effect. The phase $\theta_m$ is adjusted so that the signals add up constructively to boost SNR in the destination. Let $x_s \in \{\pm 1\}$; for $m = 1, ..., L$, denote the detected symbols at relays. The received signal at the destination transmitted by the relays can be expressed as

$$y_{rd} = \sqrt{P_r} \sum_{m=1}^{L} \beta_m |b_{rd}|^2 x_m + z_{rd}. \quad (2)$$

According to [18], the adaptation factor $\alpha_m$ is set as

$$\alpha_m \triangleq \min \left(\frac{y_{sr_m}}{y_{rd}}, \frac{y_{rmd}}{y_{rd}}\right). \quad (3)$$

In (3), the SNR values of the source-relay and relay-destination links are defined as $\gamma_{sr_m} \triangleq \frac{y_{sr_m}}{N_0}$, $\gamma_{rmd} \triangleq \frac{y_{rmd}}{N_0}$, and $\gamma_{rd} \triangleq \frac{y_{rd}}{N_0}$. If we define the equivalent source-relay-destination SNR for the $m$th relay as $\gamma_{eq_m} \triangleq \min(\gamma_{sr_m}, \gamma_{rmd})$ and the related phase of the equivalent relay-destination channel as $\epsilon_m \triangleq \angle b_{rd} + \theta_m$, (2) can be written as

$$y_{rd} = \sum_{m=1}^{L} \sqrt{\gamma_{eq_m} N_0} e^{j\epsilon_m} x_m + z_{rd}. \quad (4)$$

In our proposed beamforming scheme, we set $\theta_m$ for $m = 1, ..., L$ as follows. If all of the relay nodes detect the transmitted symbol correctly, according to (4), the SNR at the destination will be maximized if the transmitted signals from relays add up coherently, i.e. $\epsilon_m$ for $m = 1, ..., L$ are all the same and we call this ideal beamforming. If the relay-destination channel phase $\angle b_{rd}$ is available at the relays, ideal beamforming is possible. But with limitation of feedback from destination to relay nodes, we have to quantize the phase information before feedbacking them. This paper examines by rotating each relay-destination channel gain, we try placing all of them in the first quadrant of the Cartesian coordinate system as shown in Figure 2. This will make the received signals at the destination to add up constructively and will increase SNR. The rotations for this method can be expressed as

$$\theta_m \triangleq -\left\lfloor \frac{\angle b_{rd}}{\pi} \right\rfloor \times \frac{\pi}{2} \quad (5)$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to $x$ and (5) guarantees the condition $0 \leq \epsilon_m \leq \frac{\pi}{2}$ for all $m = 1, ..., L$.

In the next section, the asymptotic behaviour of the system BER is analyzed and diversity order of the proposed method is derived.
input of an MRC detector can be written as

\[ x_L = \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} + \sum_{n=s}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \]

3  ASYMPTOTIC DIVERSITY ORDER ANALYSIS

In this section, an analysis of the asymptotic behaviour of the BER is conducted to derive the diversity order of the proposed method. The well known maximal ratio combining (MRC) is employed for detection of the transmitted symbol at the destination node, using \( y_{\text{Id}} \) and \( f_{\text{Id}} \). According to (1) and (4), the output of an MRC detector can be written as

\[ y_{MRC} = \text{Real} \left\{ b_{\text{ld}}^* \sqrt{P_{\text{ld}}} y_{\text{ld}} + \left( \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right) \right\} \]

\[ = P_{\text{ld}} | b_{\text{ld}} |^2 x_{\text{ld}} + \text{Real} \left\{ \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right\} \]

\[ \times \left( \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right) \]

\[ + \text{Real} \left\{ b_{\text{ld}}^* \sqrt{P_{\text{ld}}} z_{\text{ld}} + \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right\} \].

Without loss of generality, we suppose that the relays 1, 2, \( n \) detect the transmitted symbol correctly, i.e. \( x_1 = \cdots = x_n = x_i \), while the relays \( n+1, \ldots, L \) detect it incorrectly \( (x_{n+1} = \cdots = x_L \neq x_i) \). With this assumption, (6) can be rewritten as

\[ y_{MRC} = P_{\text{ld}} | b_{\text{ld}} |^2 x_{\text{ld}} + \sum_{n=1}^{n} y_{q_{\sigma_n} e^{j\theta_n}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \]

\[ - \left| \sum_{n=s+1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right|^2 \]

\[ + \text{Real} \left\{ b_{\text{ld}}^* \sqrt{P_{\text{ld}}} z_{\text{ld}} + \sum_{n=1}^{L} y_{q_{\sigma_n} e^{j\theta_n}} \right\} \].

(7)

Based on (7), the following conditional probability of error can be obtained

\[ Q_e \triangleq P( E | x_1 = \cdots = x_n = x_i, x_{n+1} = \cdots = x_L \neq x_i ) \]

\[ = Q \left( \frac{2 \sum_{n=1}^{n} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 - \left| \sum_{n=s+1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right|}{\sqrt{2} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 + \left| \sum_{n=s+1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right|} \right) \]

(8)

By defining auxiliary variables \( Z_{c,n} = y_{q_{\sigma_n} e^{j\theta_n}} \) and \( Z_{c,n} = \left| \sum_{n=1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right| \), and using the inequality \( (\sum_{n=1}^{L} | y_{q_{\sigma_n} e^{j\theta_n}} |^2)^2 \leq 2 \sum_{n=1}^{L} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 + 2 \left| \sum_{n=s+1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right| \), we can derive a simple upper bound for the conditional probability of error as

\[ Q_e \leq Q \left( \frac{\sqrt{2} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 + \sum_{n=1}^{n} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 - \left| \sum_{n=s+1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right|}{\sqrt{2} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 + \left| \sum_{n=s+1}^{L} \sqrt{P_{q_{\sigma_n}}} | y_{q_{\sigma_n} e^{j\theta_n}} |^2 \right|} \right) \]

(9)

Averaging the conditional BER in (9) over the error pattern yields the instantaneous overall BER as

\[ P_e = \sum_{n=1}^{L} \left\{ P( E | x_1 = \cdots = x_n = x_i, x_{n+1} = \cdots = x_L \neq x_i ) \times P( x_1 = \cdots = x_n = x_i, x_{n+1} = \cdots = x_L = x_i ) \right\} \]

(10)

Considering BPSK demodulation at the relays, the probability of error at each relay can be expressed as \( P(x_n \neq x_i) = Q(\sqrt{2} \sigma_{e,n}) \) and therefore we can obtain the error probability as

\[ P(x_1 = \cdots = x_n = x_i, x_{n+1} = \cdots = x_L \neq x_i ) \]

\[ = \prod_{n=1}^{n} \left( 1 - Q(\sqrt{2} \sigma_{e,n}) \right) \prod_{n=s+1}^{L} Q(\sqrt{2} \sigma_{e,n}) \].

(11)

Using the inequality \( 1 - Q(\sqrt{2} \sigma_{e,n}) \leq 1 \) and according to (10), the derived upper bound for BER can be expressed as

\[ P_e \leq \sum_{n=0}^{L} \left\{ \left( \frac{Z_{c,n} - Z_{c,n}}{Z_{c,n} + Z_{c,n}} \right) \prod_{n=s+1}^{L} Q(\sqrt{2} \sigma_{e,n}) \right\} \]

(12)

In order to obtain the average BER, we take the statistical average of (12) over the instantaneous SNR of links.
\( \{y_{\sigma_m}, y_{\tau_n}\}_{m=1}^L \). To simplify the above equation, the Q-function upper bound \( Q(x) \leq \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left(- \frac{x^2}{2}\right); & x \geq 0 \\ 1; & x < 0 \end{cases} \) is employed yielding,

\[
\prod_{m=s+1}^L Q\left(2\sqrt{y_{\sigma_m}}\right) \leq \frac{1}{2^{L-s}} \exp\left(- \sum_{m=s+1}^L y_{\sigma_m}\right). \tag{13}
\]

Denoting \( \sqrt{\sum_{m=1}^Q y_{\sigma_m}} \) as \( X_m + JY_m \) results in \( y_{\sigma_m} = X_m^2 + Y_m^2 \). Thus based on the fact that \( y_{\varphi_m} < y_{\sigma_n} \), we can derive a lower bound for \( \sum_{m=s+1}^L y_{\varphi_m} \) as

\[
Z_{\varphi,n} = \left( \sum_{m=s+1}^L X_m \right)^2 + \left( \sum_{m=s+1}^L Y_m \right)^2 \leq (L - n) \sum_{m=s+1}^L X_m^2 + Y_m^2 \leq (L - n) \sum_{m=s+1}^L y_{\varphi_m}. \tag{14}
\]

Moreover, based on (13) and (14), we can also derive an upper bound for the last term in (12) as

\[
\prod_{m=s+1}^L Q\left(2\sqrt{y_{\varphi_m}}\right) \leq \frac{1}{2^{L-s}} \exp\left(- \frac{Z_{\varphi,n}}{L - n}\right). \tag{15}
\]

Once again employing the Q-function upper bound, another simplification for (12) can be obtained as

\[
Q\left(\frac{Z_{\varphi,n} - Z_{\sigma,n}}{Z_{\varphi,n} + Z_{\sigma,n}}\right) \leq \begin{cases} \frac{1}{2} \exp\left(- \frac{Z_{\varphi,n} + Z_{\sigma,n}}{2}\right) + \frac{Z_{\varphi,n}Z_{\sigma,n}}{Z_{\varphi,n} + Z_{\sigma,n}}; & Z_{\varphi,n} \geq Z_{\sigma,n} \\ 1; & Z_{\varphi,n} < Z_{\sigma,n} \end{cases}. \tag{16}
\]

Utilizing the trivial inequality \( \frac{Z_{\varphi,n}Z_{\sigma,n}}{Z_{\varphi,n} + Z_{\sigma,n}} < Z_{\varphi,n} \), yields

\[
Q\left(\frac{Z_{\varphi,n} - Z_{\sigma,n}}{Z_{\varphi,n} + Z_{\sigma,n}}\right) \leq \frac{1}{2} \exp\left(\frac{3Z_{\varphi,n} - Z_{\sigma,n}}{2}\right); \quad Z_{\varphi,n} \geq Z_{\sigma,n}. \tag{17}
\]

Substituting (15) and (17) in (12) leads to

\[
P_s \leq \sum_{w=1}^L \frac{1}{2^{L-s-w}} \exp\left(- \frac{Z_{\varphi,w}}{L - n}\right) \times \begin{cases} \frac{1}{2} \exp\left(\frac{3Z_{\varphi,w} - Z_{\sigma,w}}{2}\right); & Z_{\varphi,w} \geq Z_{\sigma,w} \\ 1; & Z_{\varphi,w} < Z_{\sigma,w} \end{cases}. \tag{18}
\]

In (18), we observe that the upper bound of BER depends only on the auxiliary variables \( \{Z_{\varphi,w}, Z_{\sigma,w}\}_{w=1}^L \), and if we derive their PDFs, the average BER can be obtained as

\[
\overline{P_s} = \frac{1}{L} \sum_{w=1}^L \frac{1}{2} \int_0^\infty f_{\sqrt{Z_{\sigma,w}}} (y) dy \times \left(\frac{1}{2} \int_0^\infty \int_0^{\frac{y_{\sigma,w}}{Z_{\sigma,w}}} f_{\sqrt{Z_{\varphi,w}}} (y) dy + \int_0^\infty \int_{\frac{y_{\sigma,w}}{Z_{\sigma,w}}}^\infty f_{\sqrt{Z_{\varphi,w}}} (y) dy \right). \tag{19}
\]

According to the proposed beamforming method represented in (5), in which the channel coefficients are rotated by \( \{0, \pm\frac{\pi}{2}\} \), it can be simply demonstrated that the equivalent link phase \( \varphi_m \) has uniform distribution in the interval of \( [0, \frac{\pi}{2}] \).

By representing \( y_{\varphi_m} = Z_{\varphi,m} \) as the absolute value of \( y_{\varphi_m} \), by factorizing (22) in terms of \( x \) and \( y \), we can derive the PDF of \( X_m \) and \( Y_m \) separately. Moreover, since \( X_m \) and \( Y_m \) are independent, we can prove that their marginal PDFs are

\[
f_{X_m}(x) = f_{Y_m}(y) = \frac{2}{\sqrt{\pi y_{eq}}} e^\frac{-x^2}{2y_{eq}} u(x) u(y). \tag{22}
\]

According to [26], the Taylor expansion of the PDF can be employed for positive random variables in order to analyze
the asymptotic behaviour of the BER and extract the diversity order of the system. For this purpose, we will obtain asymptotic behaviour of PDFs for \( \{Z_{s,s}^*, Z_{r,s}^*, \}_{s=1}^L \):

\[
f_{X_\alpha}(\xi) = f_{Y_\alpha}(\xi) = \frac{2}{\sqrt{\pi} \gamma_{\alpha}}; \quad \xi \approx 0
\]

which yields

\[
\alpha_{\alpha} = \frac{2}{\sqrt{\pi} \gamma_{\alpha}}; \quad t_n = 0.
\]

If using the above asymptotic analysis the PDF of a positive R.V. \( Z_i \) is written as

\[
f_{Z_i}(\xi) = \alpha_i \xi^{|t_i|} + O(\xi^{|t_i|+\epsilon_{\xi}}).
\]

then the moment generating function (MGF) of \( Z_i \) can be written as

\[
\Phi_{Z_i}(\beta) = \frac{\alpha_i \Gamma(|t_i|+1)}{\beta^{|t_i|+1}} + O\left(\beta^{-1(|t_i|+\epsilon_{\beta})}\right).
\]

For instance, the asymptotic behaviour of the sum of \( k \) independent R.Vs \( \sum_{i=1}^k X_i \) can be expressed as

\[
\Phi_{\sum_{i=1}^k X_i}(\beta) = \prod_{i=1}^k \frac{\alpha_i \Gamma(|t_i|+1)}{\beta^{|t_i|+1}} + O\left(\beta^{-1(|\sum_{i=1}^k t_i|+k-k-\epsilon_{\beta})}\right)
\]

which gives

\[
f_{\sum_{i=1}^k X_i}(\xi) = \prod_{i=1}^k \frac{\alpha_i \Gamma(|t_i|+1)}{\Gamma\left(\sum_{i=1}^k t_i+k\right)} \xi^{\sum_{i=1}^k t_i+k-1} + O\left(\xi^{-1(|\sum_{i=1}^k t_i|+k-k-\epsilon_{\beta})}\right).
\]

To obtain the asymptotic behaviour of \( Z_{s,s}^* \) and \( Z_{r,s}^* \), we define two auxiliary variables: \( \Sigma X_k \triangleq X_1 + \cdots + X_k \) and \( \Sigma Y_k \triangleq Y_1 + \cdots + Y_k \). According to (29) we can write

\[
f_{\Sigma X_k}(\gamma) = f_{\Sigma Y_k}(\gamma) = \left(\frac{4}{\pi \gamma_{\alpha}}\right)^{\frac{k}{2}} \gamma^{\frac{1}{2}(|k-1|)(|k-1|+1)} + O(\gamma^{-1|k-1|+\epsilon_{\gamma}}).
\]

\[
f_{\Sigma Y_k}(\gamma) = f_{\Sigma Y_k^2}(\gamma) = \frac{1}{2 \sqrt{\pi} \gamma_{\alpha}} \xi \gamma \sqrt{\gamma} = \left(\frac{4}{\pi \gamma_{\alpha}}\right)^{\frac{k}{2}} \gamma^{\frac{1}{2}(|k-1|)(|k-1|+1)} + O(\gamma^{-1|k-1|+\epsilon_{\gamma}}).
\]

\[
f_{\Sigma Y_k^2 + \Sigma Y_k^2}(\gamma) = \left(\frac{4}{\pi \gamma_{\alpha}}\right)^{k} \frac{\Gamma\left(\frac{k}{2}\right)^{2}}{4(|k-1|)!^2} \gamma^{k-1} + O(\gamma^{-1|k-1|+\epsilon_{\gamma}}).
\]

Since \( \gamma_{id} \triangleq \gamma_{i} |b_{id}|^2 \) and \( b_{id} \sim \mathcal{CN}(0, \sigma_{id}^2) \) we can write

\[
f_{\gamma}(\xi) = \frac{1}{\gamma \sigma_{id}^2} e^{-\frac{\xi}{\gamma \sigma_{id}^2}} \gamma \sigma_{id}^2 = \frac{1}{\gamma \sigma_{id}^2} \gamma \sigma_{id}^2 \xi \approx 0
\]

Finally, the asymptotic behaviour of \( Z_{s,s}^* \) and \( Z_{r,s}^* \) can be written as

\[
f_{Z_{s,s}^*}(\xi) \approx \left(\frac{4}{\pi \gamma_{\alpha}}\right)^{\frac{k}{2}} \frac{\Gamma\left(\frac{k}{2}\right)^{2}}{8 \gamma \sigma_{id}^2} \gamma \sigma_{id}^2 \xi = \frac{\beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta}}{\gamma^{|k} + \epsilon_{\beta}}
\]

(34a)

\[
f_{Z_{r,s}^*}(\xi) \approx \left(\frac{4}{\pi \gamma_{\alpha}}\right)^{\frac{k}{2}} \frac{\Gamma\left(\frac{k}{2}\right)^{2}}{8 \gamma \sigma_{id}^2} \gamma \sigma_{id}^2 \xi = \frac{\beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta}}{\gamma^{|k} + \epsilon_{\beta}}
\]

(34b)

Substituting (34) in (19), we can analyze the asymptotic behaviour of BER in the high SNR regime, and thus we can obtain the diversity order of our proposed scheme. After substitution, we can write

\[
\int_0^\infty e^{\frac{\xi}{\gamma}} f_{Z_{s,s}^*}(\xi) d\xi \\
\approx \int_0^\infty e^{\frac{\xi}{\gamma}} \beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta} d\xi \\
= \frac{\beta_{\epsilon_{\gamma}}}{\gamma^{|k} + \epsilon_{\beta}} \left(\frac{1}{\gamma} \right) \Gamma\left(\gamma; n - \left(\frac{1}{\gamma} - n\right)\right)
\]

(35)

\[
\int_0^\infty e^{-\frac{\xi}{\gamma}} f_{Z_{r,s}^*}(\xi) d\xi \\
\approx \int_0^\infty e^{-\frac{\xi}{\gamma}} \beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta} d\xi \\
= \frac{\beta_{\epsilon_{\gamma}}}{\gamma^{|k} + \epsilon_{\beta}} \left(\frac{1}{\gamma} \right) \Gamma\left(\gamma; n - \left(\frac{1}{\gamma} - n\right)\right)
\]

(36)

Equations (35) and (36) are obtained using [27, page.346, section 3.381]. We can rewrite (19) as

\[
\mathcal{P}_e \approx \sum_{s=1}^L \mathcal{P}_{e,s} = \frac{1}{2 \gamma^{|k} + \epsilon_{\beta}} \beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta} + \frac{1}{2 \gamma^{|k} + \epsilon_{\beta}} \beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta}
\]

(37)

\[
= \sum_{s=1}^L \left(\frac{1}{2 \gamma^{|k} + \epsilon_{\beta}} \beta_{\epsilon_{\gamma}} \xi^{|k} + \epsilon_{\beta}\right)
\]
\[
\times \left( \int_0^\infty \frac{e^{-\frac{u}{L-n}}}{u^\delta} \gamma_1 \left( L - n, \frac{1}{L-n} - \frac{3}{2}u \right) du \right)_{L_1} \\
+ \left( \frac{1}{L-n} \right)^{L+n} \left( \int_0^\infty \frac{u^{\gamma_2}}{u^\delta} \gamma_2 \left( L - n, \frac{1}{L-n} - \frac{3}{2}u \right) du \right)_{L_2} \right), \quad (37)
\]

We can obtain the integrals \( L_1 \) and \( L_2 \) in terms of the hypergeometric functions as \cite[page 657, section 6.455]{27}.

In (37), \( \gamma_1 \) and \( \gamma_2 \) are defined as

\[
\gamma_1 = \left( \frac{1}{L-n} - \frac{3}{2} \right) \Gamma\left( L + 1 \right) \left( L - n \right) \left( \frac{1}{L-n} - 1 \right)^{L+1} 2^{-1} \\
\times F_1 \left( 1, L + 1; L - n + 1; \frac{1.5(L - n) - 1}{L - n - 1} \right) \\
\gamma_2 = \left( \frac{1}{L-n} \right)^{L-n} \Gamma\left( L + 1 \right) \left( n + 1 \right) \left( \frac{1}{L-n} \right)^{L+1} 2 F_1 \left( 1, L + 1; n + 2; 0 \right). \quad (38a)
\]

Now, the asymptotic behaviour of the system BER can be represented in a simple form as

\[
\bar{P}_i \approx \sum_{n=0}^{L} \left( \frac{1}{2} \right)^{\gamma_i} \beta_{\alpha,n} \gamma_i \left( L + 1 \right) \frac{1}{2^{L-n} \gamma_i^{L+1}} \left( \frac{1}{2} \alpha_1 \left( L, n \right) + \alpha_2 \left( L, n \right) \right) \\
= \frac{\alpha_L}{\gamma_i^{L+1}}. \quad (39)
\]

In (39), \( \alpha_1 \left( L, n \right), \alpha_2 \left( L, n \right), \beta_{\alpha,n}, \beta_{\gamma,n} \), and \( \alpha_L \) are constants in terms of \( n \) and \( L \). In (39), it can be easily verified from the power of \( \gamma_i \) in the denominator, that the full diversity order, i.e. \( L+1 \), is achieved. In the next section, we present the simulation results that confirm our analysis.

## 4 SIMULATION RESULTS

In this section we compare the proposed approach with a relay selection method in DF networks that uses LAR relays, like section IV of \cite{28}. In the Monte Carlo simulations the average SNRs of all of links are equal, i.e. \( \bar{\gamma}_{rk} = \bar{\gamma}_{rd} = \bar{\gamma} \). The simulation results for non-symmetric scenarios, i.e. \( \bar{\gamma}_{rk} - 10dB = \bar{\gamma}_{rd} = \bar{\gamma} \) and \( \bar{\gamma}_{rk} = \bar{\gamma}_{rd} - 10dB = \bar{\gamma}_{id} = \bar{\gamma} \) are also provided. Simulation results for different number of relays including the cases \( L = 2, 3, 4, 5 \) are considered. The allocated power to source node and relay nodes are equal, i.e. \( P_s = P_r = P \) in all of the simulations. In the relay selection algorithm, only one of the relay nodes cooperates, while in the proposed scenario, all of the relays participate in the relaying step. Therefore, to provide a fair comparison between these two scenarios, an equal power is allocated to each scenario. Briefly, we can express the power of the nodes in the discussed scenarios as

\[
\left( L + 1 \right) P_{rs-BF} = \left( L + 1 \right) P_{rs-RS} = 2P_{rs-RS} = 2P_{rs-RS} = P_{r}. \quad (40)
\]

In (40), the BF subscript refers to the proposed beamforming method, and the RS points to the relay selection method mentioned in \cite{28}. According to (40), it can be observed that the proposed scenario requires lower peak power at relays. This is a major advantage in cases with restrictions on the peak power of transmission in this step.

First, the simulation results of the proposed method in the symmetric scenario are investigated for different numbers of relays. We consider both BPSK and QPSK modulations in the simulations. Figure 3 shows different curve slopes in high SNR for different numbers of relays and it clearly shows the full diversity order of the proposed method. There are no significant differences between the BPSK and QPSK curves, as we expect that the diversity order is not dependent on the modulation type.

In Figure 4 simulation results for a non-symmetric network with 10dB gain in source-relay channel is illustrated. In this case, improvement of the source-relay channel gain, reduces the probability of erroneous detection at the relays and leads to a better BER performance in comparison to the symmetric networks. Figure 4 shows a remarkable performance gap, but with the same BER curve slopes for the same number of relays, and reveals that the diversity order remains unchanged.

Simulation results for another non-symmetric case, with a 10dB gain in relay-destination channels are shown in Figure 5 for QPSK modulation. In this case, source-relay SNRs remain constant, thus, the BER at the relays, that are an important factor in the overall performance, are not changed. Therefore, a smaller performance deterioration in this scenario is observed. According to Figure 5 and using the fact that \( \gamma_{rd} \Delta \min \text{ max} \gamma_{rd,} \gamma_{rd} \) is a major element in LR relaying, only increasing the relay-destination gain does not significantly affect the system performance, due to the fact that \( \gamma_{rd} \) remains unchanged.

Finally, we compare the proposed method with the relay selection method in DF networks that uses LAR relays, as mentioned in Section IV of \cite{28}. In addition, we use the assumption expressed in (40). Figure 6 shows the superiority of the relay selection method. However, according to (40), the applied power allocation conditions in the relay selection method, cause more extra power at source-relay and relay-destination channels. Therefore, taking this consideration into account, we have to perform the comparison once again. For example, for \( L = 5 \) relays, we apply \( \frac{L+1}{2} = 3 \equiv 4.77dB \) power gain in relay selection mode, while we obtain a gain of only about 2dB. Thus, on aggregate -2.7dB loss in relay selection method is expected. Consequently, the proposed method outperforms the relay selection while requires less peak power.
**Figure 3** BER of BPSK and QPSK for a symmetric network

**Figure 4** BER performance of symmetric and non-symmetric relay networks. In non-symmetric case source-relay link is 10dB stronger
**FIGURE 5** BER performance of symmetric and non-symmetric relay networks. In non-symmetric case relay-destination link is 10dB stronger.

**FIGURE 6** Comparison of LAR beamforming and best relay selection with LAR relays.
5 | CONCLUSIONS

This paper has proposed a joint LAR relaying and beamforming approach in a multi-relay cooperative communication network. The proposed method operates with less feedback for beamforming from destination to relays (2 bits per each relay). Employing LAR relaying surmounts the source-relay channel error propagation problem, in which relays adapt the transmission power according to source-relay and relay-destination channels quality. It was shown analytically that the proposed method achieves full diversity order and simulation results also corroborated BER performance and diversity order results. Moreover, the proposed beamforming method has less peak power requirement compared to the best relay selection scenario.

REFERENCES

1. Laneman, J.N.: Cooperative diversity in wireless networks: Algorithms and architectures. Massachusetts Institute of Technology (2002)
2. Sendonaris, A., et al.: User cooperation diversity. Part i. System description. Commun., IEEE Trans. 51(11), 1927–1938 (2003)
3. Laneman, J.N., et al.: Cooperative diversity in wireless networks: Efficient protocols and outage behavior. Information Theory, IEEE Trans. 50(12), 3062–3080 (2004)
4. Sendonaris, A., et al. User cooperation diversity. Part ii. Implementation aspects and performance analysis. Commun., IEEE Trans. 51(11), 1939–1948 (2003)
5. Fikadu, M.K., et al.: Error rate and power allocation analysis of regenerative networks over generalized fading channels. IEEE Trans. Commun. 64(4), 1751–1768 (2016)
6. Beaulieu, N.C., Hu, J: A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar rayleigh fading channels. Commun. Lett., IEEE 10(12), 813–815 (2006)
7. Laneman, J.N., Wornell, G.W.: Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. Information Theory, IEEE Trans. 49(10), 2415–2425 (2003)
8. Shirzadian,Gilan, M., Olfat, A.: New beamforming and distributed space-time coding with selection relaying over nakagami-m fading channels. Trans. Emerging Telecommunications Technologies 29(12), e3528 (2018)
9. Gilan, M.S., Olfat, A.: On the performance of distributed space-time coding with selection relaying and beamforming for two-path successive relay network. In: 2017 IEEE International Black Sea Conference on Communications and Networking (BlackSeaCom). IEEE, (2017) 1–5
10. Ibrahim, A.S., et al.: Cooperative communications with relay-selection: when to cooperate and whom to cooperate with?. Wireless Commun., IEEE Trans. 7(7), 2814–2827 (2008)
11. Bletsas, A., et al.: A simple distributed method for relay selection in cooperative diversity wireless networks, based on reciprocity and channel measurements. In: Vehicular Technology Conference, 2005. VTC 2005-Spring. 2005 IEEE 61st. IEEE, 3, 1484–1488 (2005)
12. Zhang, Y., Li, Y: Performance of multiple relay selection system based on a novel selection scheme combined precoding design. In: 2018 24th Asia-Pacific Conference on Communications (APCC). IEEE, 535–540 (2018)
13. Su, W., et al.: Ser performance analysis and optimum power allocation for decode-and-forward cooperation protocol in wireless networks. In: Wireless Communications and Networking Conference, 2005 IEEE. IEEE, 2, 984–989 (2005)
14. Moharrer, H., Olfat, A.: Joint relay selection and cooperative beamforming in two-hop multi-relay decode-and-forward networks. IET Communications 8(18), 3245–3253 (2014)
15. Vu, T.X., et al.: On the diversity of network-coded cooperation with decode-and-forward relay selection. IEEE Trans Wireless Commun. 14(8), 4369–4378 (2015)
16. Zainaldin, A., et al.: Joint resource allocation and relay selection in hetnetworks using hybrid co-operative relaying and network coding. IEEE Trans. on Wireless Commun. 15(6), 4348–4361 (2016)
17. Onat, F.A., et al.: Threshold selection for snr-based selective digital relaying in cooperative wireless networks. Wireless Commun., IEEE Trans. 7(11), 4226–4237 (2008)
18. Wang, T., et al.: Smart regenerative relays for link-adaptive cooperative communications. Commun., IEEE Trans. 56(11), 1950–1960 (2008)
19. Jing, Y., Jafarkhani, H.: Network beamforming using relays with perfect channel information. Information Theory, IEEE Trans. 55(6), 2499–2517 (2009)
20. Moharrer, H., et al.: Cooperative beamforming for two-hop multi-relay decode-and-forward networks. IEEE Trans. Commun. 63(9), 3143–3156 (2015)
21. Badarneh, O.S., Mesleh, R.: Cooperative dual-hop wireless communication systems with beamforming over \( \eta - \mu \) fading channels. IEEE Trans. Vehicular Technology 65(1), 37–46 (2016)
22. Mohammad, A.M.A., Olfat, A., Beaulieu, N.C.: Novel beamforming scheme for multicasting in cooperative wireless networks with a multiple antenna relay. Wireless Commun., IEEE Trans. 51(11), 1939–1948 (2015)
23. Kong, H.B., et al.: Joint mmse receiver designs for mimo af relaying systems with direct link. IEEE Trans. Wireless Commun. 16(6), 3547–3560 (2017)
24. Huang, Y., Clerckx, B.: Relaying strategies for wireless-powered mimo relay networks. IEEE Trans. Wireless Commun. 15(9), 6033–6047 (2016)
25. Koyuncu, E., et al.: Distributed beamforming in wireless relay networks with quantized feedback. Selected Areas in Communications, IEEE Journal 26(8), 1429–1439 (2008)
26. Wang, Z., Giannakis, G.B.: A simple and general parameterization quantifying performance in fading channels. Commun., IEEE Trans. 51(8), 1389–1398 (2003)
27. Jeffrey, A., Zwillinger, D.: Table of integrals, series, and products. Academic Press, Massachusetts, (2007)
28. Yi, Z., Kim, I.M.: Diversity order analysis of the decode-and-forward regenerative relays. IET Commun., 2021;15:580–588. https://doi.org/10.1049/cmu2.12089