Black Hole and Neutron Star Mergers in Galactic Nuclei

Giacomo Fragione1,*, Evgeni Grishin2, Nathan W. C. Leigh3,4,5, Hagai. B. Perets2, Rosalba Perna3,6

1 Racah Institute for Physics, The Hebrew University, Jerusalem 91904, Israel
2 Physics Department, Technion - Israel institute of Technology, Haifa 3200002, Israel
3 Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794-3800, USA
4 Department of Astrophysics, American Museum of Natural History, New York, NY 10024, USA
5 Departamento de Astronomía, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Concepción, Chile
6 Center for Computational Astrophysics, Flatiron Institute, New York, NY 10010, USA

28 November 2018

ABSTRACT

Nuclear star clusters surrounding supermassive black holes (SMBHs) in galactic nuclei contain large numbers of stars, black holes (BHs) and neutron stars (NSs), a fraction of which are likely to form binaries. These binaries were suggested to form a triple system with the SMBH, which acts as a perturber and may enhance BH and NS mergers via the Lidov-Kozai mechanism. We follow-up previous studies, but for the first time perform an extensive statistical study of BH-BH, NS-NS and BH-NS binary mergers by means of direct high-precision regularized \textit{N}-body simulations, including Post-Newtonian (PN) terms up to order PN2.5. We consider different SMBH masses, slopes for the BH mass function, binary semi-major axis and eccentricity distributions, and different spatial distributions for the binaries. We find that the merger rates are a decreasing function of the SMBH mass and are in the ranges ∼0.17-0.52 Gpc$^{-3}$ yr$^{-1}$, ∼0.06-0.10 Gpc$^{-3}$ yr$^{-1}$ and ∼0.04-0.16 Gpc$^{-3}$ yr$^{-1}$ for BH-BH, BH-NS and NS-NS binaries, respectively. These rates are typically smaller than previous studies, where the supply rates of BHs and NSs were probably overestimated. Most of the mergers enter the LIGO band with very high eccentricities, approaching almost unity. We also compare our results to the secular approximation, and show that \textit{N}-body simulations generally predict a larger number of mergers. Finally, these events can also be observable via their electromagnetic counterparts, thus making these compact object mergers especially valuable for cosmological and astrophysical purposes.

Key words: Galaxy: centre – Galaxy: kinematics and dynamics – stars: black holes – stars: kinematics and dynamics – galaxies: star clusters: general

1 INTRODUCTION

Recently, the LIGO-Virgo\footnote{https://www.ligo.caltech.edu/} collaboration detected six sources of gravitational waves (GWs), five from merging BH-BH binaries (Abbott et al. 2016b,a, 2017a,h,c) and one from merging NS-NS binaries (Abbott et al. 2017d). With ongoing improvements to LIGO and upcoming instruments such as \textit{LISA}\footnote{http://www.et-gw.eu} and the Einstein Telescope\footnote{https://lisa.nasa.gov}, hundreds of BH-BH, BH-NS and NS-NS binary sources may be detected within a few years. Thus the modeling of the formation and evolution of BH and NS binaries is crucial for interpreting the signals from all the GW sources we expect to observe.

The origins of BH and NS mergers are actively debated. Several scenarios have been proposed, such as isolated binary evolution in the galactic field (Belczynski et al. 2016; Giacobbo & Mapelli 2018), gas-assisted mergers (Barlow et al. 2017; Stone et al. 2017a; Tagawa et al. 2018), stellar triple systems (Wen 2003; Antonini et al. 2014, 2017; Silsbee & Tremaine 2017; Liu & Lai 2018), or induced mergers of binaries in galactic nuclei (Antonini & Perets 2012; Prodan et al. 2015; Antonini & Rasio 2016; VanLandingham et al. 2016; Petrovich & Antonini 2017; Hamers et al. 2018; Hoang et al. 2018; Randall & Xianyu 2018) and other dense stellar systems (Askar et al. 2017; Banerjee 2018; Choksi et al. 2018; Fragione & Kocsis 2018; Rodriguez et al. 2018; Rodriguez & Loeb 2018; Samsing et al. 2018a). Each model predicts different rates (generally of the order of ∼ few Gpc$^{-3}$ yr$^{-1}$) and can in principle be distinguished from other channels.
using the observed mass, eccentricity, spin and redshift distributions (see e.g. O’Leary et al. 2016). For instance, dynamically assembled mergers are expected to have a non-negligible probability of appearing eccentric when observed (Antonini & Perets 2012; Samsing et al. 2018b; Zevin et al. 2018).

Most of the literature pertaining to dynamically-induced mergers focuses on BH and NS binaries forming in globular clusters, while only a few studies have paid attention to the formation of compact-object binaries in the vicinity of a super massive black hole (SMBH) and in nuclear star clusters (e.g. Hoang et al. 2018; Leigh et al. 2016, 2018). The pioneering work by Antonini & Perets (2012) showed that SMBHs can induce Lidov-Kozai (LK) oscillations on BH-BH and NS-NS binaries orbiting in its vicinity (Lidov 1962; Kozai 1962), thus enhancing the probability of merging compact binaries. In this scenario, the eccentricity of the BH/NS binary reaches large values (for a review on LK mechanisms see Naoz 2016), then GW emission drives the binary to merge. While Antonini & Perets (2012) adopted a secular treatment for the equations of motion at the quadruple level of approximation, Hoang et al. (2018) considered soft binaries and the importance of expansion up to the octuple order. These calculations adopt the secular approximation to study triples, which must satisfy hierarchical conditions (Naoz 2016). In some cases, the inner binary may undergo rapid oscillations in the angular momentum and eccentricity, thus the secular theory is not anymore an adequate description of the three-body equations of motion (Antonini & Perets 2012; Antognini, Shappee, Thompson & Amaro-Seoane 2014; Antonini, Murray & Mikkola 2014; Luo, Katz & Dong 2016; Liu & Lai 2018; Grishin, Perets & Fragione 2018). For these cases, direct precise N-body simulations, including regularization schemes and Post-Newtonian (PN) terms, are required to follow accurately the orbits of the objects up to the final merger (Grishin et al. 2018; Fragione & Leigh 2018a,b). Recently, VanLandingham et al. (2016) used N-body simulations of small (∼ 300–4000 stars) clusters surrounding $10^{-3}$–$10^{4}$ M_☉ black holes to study the effect of LK oscillations in dense environments, while Arca-Sedda & Gualandris (2018) used few-body simulations to check the merger rates of BH-BH binaries delivered by infalling star clusters at typical distances of ∼ few pc.

In this paper, we revisit the SMBH-induced mergers of compact binaries orbiting in its vicinity. We consider a three-body system consisting of an inner binary comprised of a BH-BH/NS-NS/BH-NS binary, and an outer binary comprised of the SMBH and the centre of mass of the inner binary. Figure 1 depicts the system we studied in the present paper. We denote the mass of the SMBH as $M_{\text{SMBH}}$ and the mass of the objects in the inner binary as $m_1$ and $m_2$, while the semimajor axis and eccentricity of the inner orbit are $a_{\text{in}}$ and $e_{\text{in}}$, respectively, and for the outer orbit, these are $a_{\text{out}}$ and $e_{\text{out}}$, respectively. While previous papers mainly adopted a secular approximation for the equations of motion, with a few direct N-body integrations in comparison to secular evolution (e.g. Antonini & Perets 2012; Hoang et al. 2018), here, we make the first systematic and statistical study of BH-BH, NS-NS and BH-NS mergers in the proximity of an SMBH by means of direct high-precision N-body simulations, including Post-Newtonian (PN) terms up to order PN2.5. Moreover, we consider how different masses of the SMBH affect the mergers of compact binaries, and adopt a mass spectrum for the BHs, while also studying different spatial distributions for the merging binaries. Finally, we discuss observational diagnostics that can help discriminate this compact object merger channel from other ones.

The paper is organized as follows. In Section 2, we discuss the properties and dynamics of BHs and NSs in galactic nuclei. In Section 3, we discuss the state-of-the-art secular approximations currently being used in the literature. In Section 4, we present our numerical methods to determine the rate of BH-BH, NS-NS and BH-NS mergers in galactic nuclei, for which we discuss the results. In Section 5, we discuss the predicted rate of compact object mergers in galactic nuclei and compare our results to secular approximations, while, in Section 6, we discuss the observational signatures of these events. Finally, in Section 7, we discuss the implications of our findings and draw our conclusions.

## 2 BLACK HOLE AND NEUTRON STAR BINARIES IN GALACTIC NUCLEI

The evidence in favour of the presence of close binaries composed of compact objects (COs) such as white dwarfs (WDs), and especially NSs and BHs, in dense stellar environments is rapidly growing. Recently, Hailey & Mori (2017) reported observations of a dozen quiescent X-ray binaries that form a central density cusp within ∼ 1 parsec of Sagittarius A*. The authors argue that the emission spectra they observe are inconsistent with a population of accreting WDs, suggesting that the X-ray binaries must contain mostly NSs and BHs. However, the relative numbers of these two types of COs is an open question. Six other X-ray transients are known to be present in the inner parsec of the Galactic Centre, which are also strongly indicative of binaries containing COs (e.g. Muno et al. 2005; Hailey & Mori 2017). Given the very dense stellar environments in galactic nuclei combined with the presence of a central SMBH, these CO binaries...
can undergo several fates. They can be hardened to shorter orbital periods, either by direct scattering interactions with single stars (Perets 2009a; Leigh et al. 2018), or by Kozai oscillations due to the SMBH combined with gravitational wave (GW) emission acting at pericentre (e.g. Antonini & Perets 2012; Prodan et al. 2015).

In their Table 1, Generozov et al. (2018) combine the reported statistics from the literature to provide estimates for the numbers of BHs and NSs in the Galactic Centre. In short, the number of NS X-ray Binaries (XRBs) per stellar mass in the Galactic Centre is roughly three orders of magnitude higher than in the field, and comparable to the number expected to be in globular clusters. Similarly, the number of BH XRBs per stellar mass is roughly three orders of magnitude higher than in the field, and roughly an order of magnitude higher than in any known globular cluster (e.g. Strader et al. 2012; Leigh et al. 2014, 2016).

These large numbers of NS and BH binaries must come from somewhere. Yet, little is known about binaries in our Galactic Centre. A likely explanation is that they are the remnants of massive O/B stars, since hundreds of these are known to be present in the inner ∼ 1 pc of Sagittarius A*. In the Solar neighborhood, the massive-star binary fraction is very high (≥ 70%) and the most massive binaries have semi-major axes of up to a few AU (Sana et al. 2012). This alone is suggestive of a high rate of NS and BH formation in nuclear star clusters (e.g. Levin & Beloborodov 2003; Genzel et al. 2008). Interestingly, the discovery of even a single magnetar within ≤ 0.1 pc of Sgr A* would also argue in favour of a high rate of NS formation, given their short active lifetimes (Mori et al. 2013).

This population of CO binaries is continuously depleted through dynamical interactions with other stars and COs (evaporation) and GW mergers. A few mechanisms may replenish the CO population. The binaries may come from outside the central region near the SMBH, which thus serves as a continuous source term (Hopman 2009; Alexander 2017). In this scenario, CO binaries form far from the innermost region around the SMBH and gradually migrate on a 2-body relaxation timescale,

\[
T_{2b} = 1.6 \times 10^{10}\text{yr} \left(\frac{\sigma}{300\text{ km s}^{-1}}\right)^3 \left(\frac{m}{M_\odot}\right)^{-1} \times \left(\frac{\rho}{2.1 \times 10^6 M_\odot \text{ pc}^{-3}}\right)^{-1} \left(\frac{\ln \Lambda}{15}\right)^{-1},
\]

towards smaller distances, where they become active in the Lidov-Kozai regime. Here, \(\rho\) and \(\sigma\) are the 1-D density and velocity dispersion in the Galactic Centre, respectively, \(\ln \Lambda\) is the Coulomb logarithm and \(m\) is the average stellar mass. On the other hand, our Galactic Centre contains a large population of young massive O-type stars, many of which have been observed to reside in a stellar disk. Likely, most of them were born in-situ as a consequence of the fragmentation of a gaseous disk formed from an infalling gaseous clump (Genzel et al. 2010). An important question is which process can make their orbit, which are not observed closer than ∼ 0.05 pc, approach the SMBH, where efficient Lidov-Kozai oscillations take place. Both planet-like migration in the gaseous disc (Baruteau et al. 2011) and disc instability (Madigan et al. 2009) have been proposed to make the in situ binaries migrate, but to which innermost distance with respect to the SMBH is not known exactly. Other mechanisms include triple/quadruple disruptions, where a triple/quadruple is disrupted and the inner binary is left orbiting the SMBH (Perets 2009b; Ginsburg & Perets 2011; Fragione & Guandalini 2018; Fragione 2018), and infalls of star clusters (Antonini & Merritt 2012; Fragione, Capuzzo-Dolcetta & Kroupa 2017).

Outside our own Milky Way, Secunda et al. (2018) recently showed that CO binaries can form efficiently in Active Galactic Nucleus (AGN) disks. The COs migrate in the disk due to differential torques exerted by the gas, moving toward migration traps, where the torques actually cancel (e.g. Belovary et al. 2016). The COs drift toward the trap and get stuck there, waiting for the next CO to migrate toward it. Once close enough, the two COs can undergo a strong interaction and end up forming a bound binary due to the dissipative effects of the gas. COs can also accrete from the disk gas in this scenario, possibly growing substantially and in some cases even forming an IMBH (McKernan et al. 2012, 2014).

### 2.1 Outer semi-major axis and eccentricity

The numbers and spatial profiles of BHs and NSs are poorly known in galactic nuclei. In general, stars tend to form a power-law density cusp around an SMBH. The classical result by Bahcall & Wolf (1976) shows that a population of equal-mass objects forms a power-law density cusp around an SMBH, \(n(r) \propto r^{-\alpha}\), where \(\alpha = 7/4\). For multi-mass distributions, lighter and heavier objects develop shallower and steeper cusps, respectively (Hopman & Alexander 2006a; Freitag et al. 2006; Alexander & Hopman 2006b). If \(\alpha = 3\), a uniform density profile \((\rho)\) would also evolve in favour of a high rate of NS formation, given their short active lifetimes (Aharon & Perets 2015; Fragione & Sari 2018).

Recent observations of the Milky Way’s centre showed that the slope of the cusp appears to be shallower (\(\alpha \sim 5/4\); Gallego-Cano et al. 2018; Schödel et al. 2018). As BHs and NSs are heavier than average stars, they are expected to relax into steeper cusps, with BHs relaxing into steeper cusps than NSs (Freitag et al. 2006; Hopman & Alexander 2006b). If not all BHs have the same mass, but follow a mass distribution, only the more massive ones would have steeper slopes, while low-mass BHs should follow shallower slopes (Aharon & Perets 2016).

In the present study, we assume that the BH and NS number densities follow a cusp with \(\alpha = 2\). We also study the effects of the cusp slope, by considering a steeper cusp (\(\alpha = 3\)) and a uniform density profile (\(\alpha = 0\)) for BHs, and a shallower cusp (\(\alpha = 1.5\)) for NSs. For the maximum outer semi-major axis, we take \(a_{\text{out}} = 0.1\) pc following Hoang et al. (2018), which approximately corresponds to the value at which the eccentric Lidov-Kozai timescale is equal to the timescale over which accumulated fly-bys from single stars tend to unbind the binary (see Eqs. 6-11). As discussed, \(\text{in-situ}\) formation occurs in our Galactic Centre at larger distances and some mechanism that delivers such CO binaries closer to the SMBH has to be invoked. Finally, we sample the outer orbital eccentricity from a thermal distribution (Jeans 1919).
2.2 Inner semi-major axis and eccentricity

The inner binary (BH-BH/NS-NS/BH-NS) semi-major axis and eccentricity are not well known. Different models predict different distributions. Moreover, the dense environments characteristic of galactic nuclei should cause both distributions to diffuse over time, thus changing the relative distributions. Hopman (2009) made the only attempt to model binaries very close to the SMBH, even though this pioneering study accounted only for the 2-body relaxation process. Other relaxation processes, such as resonant relaxation (Rauch & Tremaine 1996), may affect the distribution as well (Hamers et al. 2018). For what concerns eccentricity, though it mostly depends on the natal-kick and the common-envelope phase, it also depends on the scattering of the CO binaries by local COs and stars. As a consequence, an in-situ formation would probably favour circular binaries, while the migration scenario would rather prefer a thermal distribution.

Antonini & Perets (2012, see Fig. 1) used the results of Belczynski et al. (2004) for the initial distribution of BH-BH binaries orbits, while, for NSs, they used the observed pulsar binary population as found in the ATNF pulsar catalog (Manchester et al. 2005). We note that their distributions referred to isolated binaries and used a simplified approach for to account for the softening and binary-destruction due to the crowded environments (following the approach in (Perets et al. 2007)). Inner eccentricities were sampled from a uniform distribution. Recently, Hoang et al. (2018) drew the inner semi-major axes from a log-uniform distribution in the range 0.1-50 AU, somewhat consistent with the observed distribution from Sana et al. (2012), which favors short period binaries, and the inner eccentricities from a uniform distribution (Raghavan et al. 2010). Note the caveat that this distribution corresponds to massive MS binaries, and not to CO binaries which already evolved and change their configurations.

2.3 Masses

The mass distribution of BHs is unknown, even in isolation. Moreover, in galactic nuclei, irrespective of the original mass function, mass-segregation can make the effective BH mass function even steeper in the inner-most regions (Aharon & Perets 2016). This would in turn also affect the outer orbit distribution, since more massive BHs would have steeper slopes (Aharon & Perets 2016).

In our models, we sample the masses of the BHs from

\[
\frac{dN}{dm} \propto M^{-\beta},
\]

in the mass range 5 M_\odot–100 M_\odot (O’Leary et al. 2016; Hoang et al. 2018). To check how the results depend on the slope of the BH mass function, we run models with \( \beta = 1, 2, 3, 4 \) for both the BH-BH and BH-NS binaries (O’Leary et al. 2016). For NSs, we fix the mass to 1.3 M_\odot (e.g. Fragione, Pavlík & Banerjee 2018).

2.4 Inclinations and relevant angles

We draw the initial mutual inclination \( i_\text{in} \) between the inner and outer orbit from an isotropic distribution (i.e. uniform
in \( \cos i \)). The other relevant angles, such as the arguments of pericentre, nodes and mean anomalies, are drawn randomly.

### 2.5 Timescales in galactic nuclei

In the dense stellar environment of a galactic nucleus, several dynamical processes other than 2-body relaxation (Eq. 1) can take place and affect the evolution of the stellar and compact object populations. On smaller timescales than \( T_{2h} \), resonant relaxation (Rauch & Tremaine 1996; Kocsis & Tremaine 2015) randomizes the direction and magnitude (hence eccentricity) of the outer orbit on a typical timescale

\[
T_{RR} = 9.2 \times 10^6 \text{ yr} \left( \frac{M_{\text{SBHB}}}{4 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{a_{\text{out}}}{0.1 \text{ pc}} \right)^{3/2} \left( \frac{m}{M_\odot} \right)^{-1}.
\]

On even shorter timescales, vector resonant relaxation changes the direction (hence the relative inclination) of the outer orbit angular momentum on a typical timescale

\[
T_{VRR} = 7.6 \times 10^6 \text{ yr} \left( \frac{M_{\text{SBHB}}}{4 \times 10^6 M_\odot} \right)^{1/2} \times \left( \frac{a_{\text{out}}}{0.1 \text{ pc}} \right)^{3/2} \left( \frac{m}{M_\odot} \right)^{-1} \left( \frac{N}{6000} \right)^{-1/2},
\]

where \( N \) is the number of stars within \( a_{\text{out}} \). In the context of Kozai-Lidov oscillations, vector resonant relaxation plays a role, since it may affect the initial inclination of the inner and outer orbit of the CO binary on timescales comparable to or even shorter than the Kozai-Lidov timescale (Hamers et al. 2018).

Finally, binaries may evaporate due to dynamical interactions with field stars in the dense environment of a galactic nucleus when

\[
\frac{E_b}{(m_1 + m_2) \sigma^2} \lesssim 1,
\]

where \( E_b \) is the binary internal orbital energy and \( \sigma \) is the velocity dispersion. This happens on an evaporation timescale (Binney & Tremaine 1987)

\[
T_{EV} = 3.2 \times 10^7 \text{ yr} \left( \frac{m_1 + m_2}{2 M_\odot} \right) \left( \frac{\sigma}{300 \text{ kms}^{-1}} \right) \left( \frac{m}{M_\odot} \right)^{-1} \times \left( \frac{a_{\text{in}}}{1 \text{ AU}} \right)^{-1} \left( \frac{\rho}{2.1 \times 10^6 M_\odot \text{ pc}^{-3}} \right)^{-1} \left( \frac{\ln A}{15} \right)^{-1}.
\]

### 3 Secular Averaging Techniques

The merger time of an isolated binary of component masses \( m_1, m_2 \), semimajor axis \( a \) and eccentricity \( e \) emitting GWs is (Peters 1964)

\[
T_{\text{GW}}(a, e) = \frac{5}{256} \frac{c^3 a^4}{G^3 m_1 m_2 (m_1 + m_2)} (1 - e^2)^{7/2}.
\]

For a triple system made up of an inner binary that is orbited by an outer companion, the inner eccentricity can be pumped by the tidal potential of a distant body via the Lidov-Kozai (LK) mechanism (Lidov 1962; Kozai 1962). The LK oscillations occur on a secular timescale (Antognini 2015)

\[
t_{\text{sec}} = \frac{8 \pi}{15} \frac{m_{\text{tot}}}{m_{\text{out}}} \frac{P_{\text{out}}^2}{P_{\text{in}}} (1 - e_{\text{out}}^2)^{3/2},
\]

where \( m_{\text{out}} = M_{\text{SBHB}} \) and \( m_{\text{tot}} = M_{\text{SBHB}} + m_{\text{in}} \approx M_{\text{SBHB}} \). \( P_{\text{in}} \) and \( P_{\text{out}} \) are the orbital periods of the inner and outer binary, respectively. The large values attained by the inner eccentricity make the overall merger time of the inner binary shorter since it efficiently dissipates energy when \( e \sim e_{\text{max}} \) (e.g., see Antonini & Perets 2012). The LK-induced merger time is (Antonini & Perets 2012; Randall & Xianyu 2018; Liu & Lai 2018)

\[
T_{\text{LK}}^{GW}(a, e_{\text{max}}) \approx T_{\text{GW}}(a, e_{\text{max}})/\sqrt{1 - e_{\text{out}}^2}.
\]

The maximal eccentricity is a function mostly of the initial mutual inclination, \( i_0 \), and is usually evaluated by the secular approximation, which relies on double-averaging of both the inner and the outer orbits (Randall & Xianyu 2018). In the leading, quadrupole order, the system is integrable and has been widely studied (see recent review by Naoz 2016, and references therein), where coupled oscillations between the eccentricity and inclination of the inner binary are excited for sufficiently large initial mutual inclinations. The inner binary eccentricity approaches almost unity as \( i_0 \) approaches \( \sim 90 \) deg.

When the outer orbit is eccentric and the inner binary has an extreme mass ratio, octupole-level perturbations turn the system from integrable to chaotic (Li et al. 2014), and can potentially induce extreme orbital eccentricities, orbital flips and even direct collisions (Katz et al. 2011; Lithwick & Naoz 2011). The strength of the octupole perturbation is encapsulated in the octupole parameter as

\[
\epsilon_{\text{oct}} \equiv \frac{m_1 - m_2}{m_1 + m_2} \frac{e_{\text{oct}}}{1 - e_{\text{oct}}^2}.
\]

Generally, increasing \( \epsilon_{\text{oct}} \) will increase the parameter space corresponding to orbital flips and very large eccentricities. The typical timescale for an orbital flip is (Antognini 2015)

\[
t_{\text{flip}} = \frac{8}{\pi} \sqrt{\frac{10}{\epsilon_{\text{oct}}}} t_{\text{sec}}.
\]

The secular approximation assumes that the triple system is hierarchical, namely that \( P_\text{in}/P_\text{out} \propto (a_\text{in}/a_\text{out})^{2/3} \ll 1 \), thus ignoring short-term variations \( (t \sim P_\text{out}) \) of the oscillating elements, whose typical strength can be parameterized by the so-called 'single-averaging parameter' (Luo et al. 2016)

\[
\epsilon_{\text{SA}} \equiv \left( \frac{a_\text{in}}{a_\text{out}} \right)^{3/2} \left( \frac{M_{\text{SBHB}}}{m_\text{in}} \right)^{1/2} = \frac{P_\text{out}}{2\pi t_{\text{sec}}}.
\]

Also, Luo et al. (2016) point out that in addition to the fluctuating terms, additional secular evolution can take place. Consequently, the resulting fate of the system could be different, since additional extra apsidal and nodal precession changes the structure of the LK resonance. Grishin et al. (2017) showed that extra apsidal precession shifts the critical inclination for the LK resonance and affects the Hill stability limit of irregular satellites.

Recently, Grishin et al. (2018) used Luo et al. (2016)'s result to find an analytic formula for the maximal eccentric-
ity that can be reached due to LK oscillations
\[ \epsilon_{\text{max}} = \epsilon_{\text{max}} + \delta \epsilon, \]
\[ \epsilon_{\text{max}} = \sqrt{1 - \frac{5}{3} \cos^2 i_0 \frac{1 + \frac{9}{8} \epsilon_{SA} \cos i_0}{1 - \frac{9}{8} \epsilon_{SA} \cos i_0}}, \]
\[ \delta \epsilon = \frac{35 \epsilon_{SA}}{128 \epsilon_{\text{max}}^2 \epsilon_{SA}} \left[ \frac{16}{9} \sqrt{\frac{2}{3} \sqrt{1 - \epsilon_{\text{max}}^2} + \epsilon_{SA} - 2 \epsilon_{SA}^2 \epsilon_{\text{max}}^2} \right]. \] (13)

If the inner binary is too compact or the tertiary is too far away, the maximal eccentricity could be quenched, e.g. by GR (general relativistic) precession, and the average maximal eccentricity is given by solving (Grishin et al. 2018)
\[ \tilde{A}(1 - \epsilon_{\text{max}}^2) = 8 \frac{\epsilon_{GR}}{\epsilon_{\text{max}}^2} \sqrt{1 - \epsilon_{\text{max}}^2} + 15 j z^2 \left( 1 + \frac{9}{8} \epsilon_{SA} \tilde{j}_z \right), \]
\[ \tilde{A}(j z, \epsilon_{\text{max}}) \equiv 9 - \epsilon_{SA} \frac{81 - 8 j z + 8 \epsilon_{GR}}{\epsilon_{\text{max}}}, \] (14)

where \( j_z = \sqrt{(1 - \epsilon_{\text{max}}^2) \cos i_0} \) is the initial normalized angular momenta of the inner binary and
\[ \epsilon_{GR} \equiv \frac{3 m_{\text{min}} (1 - \epsilon_{\text{out}}^2)^{3/2}}{m_{\text{out}}} \left( \frac{a_{\text{out}}}{a} \right)^3 G m_{\text{min}} \frac{a c^2}{a}. \] (15)

measures the ratio between the apsidal precession rates induced by Lidov-Kozai and GR perturbations. The above formulae are valid in the limit \( \epsilon_{\text{out}} = 0 \).

In some configurations the inner binary may undergo rapid oscillations in the angular momentum and eccentricity, thus the secular theory is no longer an adequate description of the three-body equations of motion (Antonini & Perets 2012; Antognini et al. 2014). For instance, this can happen when the typical time-scale for the angular momentum of the inner orbit to change by order of magnitude becomes comparable to (or even shorter than) the outer or inner orbital periods (Antonini et al. 2014). Thus, in this case, the secular approximation can fail to predict both the correct maximum eccentricity and merger time. Although computationally expensive (in particular in the case of a third very massive companion as in this paper), direct N-body simulations including Post-Newtonian (PN) terms represent the most reliable option for accurately studying the effects of the tertiary companion in reducing the GW merger time of the inner binary.

4 N-BODY SIMULATIONS: BLACK HOLE AND NEUTRON STAR MERGERS IN GALACTIC NUCLEI

In this section, we use N-body simulations to study the fate of BH-BH, NS-NS and BH-NS binaries in galactic nuclei that host an SMBH. We consider three different SMBH masses, i.e. \( M_{\text{SMBH}} = 4 \times 10^6 M_\odot \) for a Milky-Way-like nucleus (Models MW), \( M_{\text{SMBH}} = 10^7 M_\odot \) for a M31-like nucleus (Models GN) and \( M_{\text{SMBH}} = 10^9 M_\odot \) for more massive host galaxies (Models GN2). For the inner semi-major axis and eccentricities, we follow the prescriptions given by Hoang et al. (2018), but also run some models with the sampling suggested by Antonini & Perets (2012) to check how the initial conditions affect the final rates. Given the set of initial parameters as described in Sect. 2, we draw the main parameters of the three-body system and require that the inner binary does not cross the Roche limit of the SMBH at its orbital pericentre distance
\[ \frac{a_{\text{out}}}{a_{\text{in}}} > \eta \frac{1 + \epsilon_{\text{in}}}{(1 - \epsilon_{\text{out}})} \left( \frac{3 M_{\text{SMBH}}}{m_1 + m_2} \right)^{1/3}. \] (16)

Following Antonini & Perets (2012), we set \( \eta = 4 \) since at shorter distances the inner binary is unstable. We then integrate the triple SMBH-CO-CO differential equations of motion
\[ \ddot{r}_i = -G \sum_{j \neq i} \frac{m_j (r_i - r_j)}{|r_i - r_j|^3}, \] (17)

with \( i = 1, 2, 3 \). The integrations are performed using the ARCHAIN code (Mikkola & Merritt 2006, 2008). This code is fully regularized and is able to model the evolution of objects of arbitrary mass ratios and eccentricities with extreme accuracy, even over long periods of time. We include PN corrections up to order PN2.5.

For each set of parameters in Tab. 1, we run \( \sim 1500 \) simulations up to a maximum integration time of \( T = 1 \) Myr, for a total of \( \sim 35000 \) simulations. From the computational
Figure 3. Cumulative distributions of the inner semi-major axis $a_{in}$ (left) and of the outer semi-major axis $a_{out}$ (right) for BH-BH (top), BH-NS (centre), NS-NS (bottom) binaries that merge in all models with $f(a_{in})$ from Hoang et al. (2018).

from Hoang et al. (2018).

In our simulations the CO binary has three possible

point of view, this limit represents a good compromise between the numerical effort (large mass ratios and GW effects slow down the code) and the size of the statistical sample we want to take into account. From the physical point of view, we note that our total integration time is smaller than the typical timescale for vector resonant relaxation to operate ($\sim$ few Myr, see Eq. 4; Rauch & Tremaine 1996; Kocsis & Tremaine 2015), which reorients the binary centre-of-mass orbital plane with respect to the SMBH, thus affecting the relative inclination of the inner and outer orbits and the relative LK dynamics, rendering the 3-body approximation insufficient (Hamers et al. 2018). We also note that our total integration time is smaller than the typical evaporation time of the CO binaries in galactic nuclei (see Eq. 6). We consider in total 23 different models, which take into account different COs in the inner binary (BH-BH, BH-NS and NS-NS), different masses of the SMBHs, different slopes of the BH/NS mass functions, different spatial distributions of the CO binaries, and different inner semi-major axis and eccentricity distributions. Table 1 summarizes all the models considered in this work.
that successfully undergo a merger event originally orbit in a plane highly inclined with respect to the outer orbital plane (Fragione & Leigh 2018b). In these binaries, the LK mechanism influences the dynamics of the system and induces oscillations both in eccentricity and inclination, whenever not suppressed by GR precession. Figure 2 also shows that some systems that merge have inclinations far from $\sim 90^\circ$, in particular when the total binary mass is large and the inner semi-major axis is small. Also, these systems typically have relatively high initial inner eccentricities.

### 4.2 Inner and outer orbital parameter distributions

We present in Fig. 3 the cumulative distribution of $a_{in}$ (left panel) for BH-BH (top), BH-NS (centre), NS-NS (bottom) binaries for all models with $f(a_{in})$ from Hoang et al. (2018). Both the SMBH mass and the slope $\alpha$ of the CO binary spatial distribution around the SMBH significantly affect the inner and outer semi-major axes of merging binaries, as a consequence of their Hill stability (Grishin et al. 2017). Larger SMBH masses imply that, on average, smaller values of $a_{in}$ are needed to avoid tidal disruption of the CO binaries after only a few orbits about the SMBH. For the same reason, large SMBH masses typically produce mergers at larger distances from the SMBH. Obviously, steep binary distributions (i.e., large $\alpha$’s) imply that the simulated CO binaries will, on average, be closer to the SMBH when they merge. Hence, smaller inner semi-major axes are needed to avoid tidal dissociation by the SMBH. Finally, the total mass of the CO binary plays some role in shaping the final outer orbital semi-major axis distribution. CO binaries with smaller total masses ($m_1 + m_2$) typically merge at larger distances from the SMBH, since their binding energy is easily overcome by the gravitational pull of the SMBH, which tends to break the binaries at smaller distances. For BHs, this translates into steeper mass functions typically producing mergers at farther distances from the SMBH.
4.3 Mass distribution

Figure 4 shows the distribution of the total \((m_1 + m_2)\) BH-BH mass (left) and the primary BH mass \((m_1)\) in BH-NS binaries (right) that merge in all models with \(f(a_{\text{in}})\) from Hoang et al. (2018). The resulting mass distribution is barely affected by the slope of the binary spatial distribution around the SMBH, with a roughly constant shape in the range \(\sim 25 M_\odot - 125 M_\odot\) and a tail extending up to \(\sim 180 M_\odot\) for BH-BH binaries. Also, the mass of the central SMBH does not significantly affect the mass distribution. As expected, the parameter that governs the resulting shape of the mass distribution is the slope \(\beta\) of the BH mass function: the shallower the BH mass function, the larger the typical total mass of merging BH-BH binaries. In the case \(\beta = 1\), we find that \(\sim 95\%\) of the mergers have \(m_1 + m_2 \lesssim 150 M_\odot\), while roughly all the mergers have total masses \(\lesssim 100 M_\odot\), \(\lesssim 50 M_\odot\), and \(\lesssim 25 M_\odot\) for \(\beta = 2\), \(\beta = 3\), and \(\beta = 4\), respectively. Similar results also hold for BH-NS binaries.

The slope of the BH mass function is unknown. We can use the results of our simulations along with the BH-BH merger events observed by LIGO (see Tab. 2; Abbott et al. 2016b,a, 2017a,b,c) to constrain the BH mass function, assuming these mergers took place in a galactic nucleus. We show in Fig. 5 density maps for the masses of the two merging BHs \((m_1 > m_2)\), along with data from the LIGO-observed BH merger events. It is clear that a steep mass function \((\beta > 1)\) is disfavored by the current data, which suggest a shallow BH mass function. We also note that, although the mass distribution is only slightly affected by the SMBH mass, the data seem to prefer more massive nuclei than the Milky-Way. Note, however, that mass-segregation processes, that can give rise to much steeper effective mass-functions of BHs in galactic nuclei (Aharon & Perets 2016), only operate in small-SMBH nuclei where relaxation (and mass-segregation) times are short.

5 MERGER TIME DISTRIBUTIONS AND RATES

As discussed, the SMBH plays a fundamental role in reducing the merger timescale from the nominal value in Eq. (7) (Antonini & Perets 2012; Hoang et al. 2018; Fragione & Leigh 2018a). In Fig. 6, we present for all models the cumulative distribution of merger times \((t_{\text{merge}};\) left panel) for BH-BH (top), BH-NS (centre) and NS-NS (bottom) binaries. The merger time distribution is nearly independent of our assumptions for the BH mass function slope \(\beta\) and the CO binary spatial distribution slope \(\alpha\). It depends only on the SMBH mass. Larger SMBH masses imply shorter merger times due to more intense perturbations from the SMBH. In the right panel of Fig. 6, we show \(t_{\text{merge}}\) as a function of the nominal (Peters 1964) GW merger time-scale \(T_{\text{GW}}\), for all binaries that merge in our simulations. Due to oscillations in the orbital elements, the CO binaries merge much faster than predicted by Eq. 7, by several orders of magnitude.

For the systems that merge in our simulations, we compute a proxy for the GW frequency of the merging binaries which we take to be the frequency corresponding to the harmonic that gives the maximal emission of GWs (Wen 2003)

\[
f_{\text{GW}} = \frac{\sqrt{G(m_1 + m_2)}}{\pi} \frac{(1 + e_{\text{in}})^{1.1954}}{a_{\text{in}}(1 - e_{\text{in}})^{1.5}}.
\]

We find that almost all mergers in our simulations enter the LIGO band \((10 \text{ Hz})\) with very high eccentricities approaching almost unity. The very high eccentricity in the LIGO band could offer a potential means of distinguishing this mechanism for merging CO binaries from others proposed in the literature, such as their dynamical assembly in star clusters or isolated field binaries. We note that somewhat high eccentricities are also predicted for field triple BHs (Antonini et al. 2017; Rodriguez & Antonini 2018).

5.1 Comparing secular techniques and N-body simulations

In order to compare the distribution of merger times with different prescriptions, we take the initial conditions and calculate the merger time from Eq. (9). We find the maximal eccentricity by solving Eq. (14) (Grishin et al. 2018). For the secular case, we use \(e_{\text{SA}} = 0\), which also implies \(\delta e = 0\) from Eq. (13), while for the corrected case, \(e_{\text{SA}} > 0\) is set from the system’s initial conditions. If the merger time is shorter than the secular LK timescale that is required to reach the maximal eccentricity, we use the secular LK time. We then filter out the merger times longer than 1 Myr and compare to the simulated merger times.

We use four different models: three for the NS-NS case in the MW, GN and GN2 environments, and one for the BH-BH case in the MW environment. In the latter case, we set \(\beta = 1\) for BH-BH binaries, and assume \(\alpha = 2\) for all cases considered here.

Figure 7 shows the cumulative distribution of the merger times for the different cases, where each panel corresponds to a different model. We see that there are more mergers for more massive central SMBHs, since the typical LK timescales are shorter. Overall, the total number of mergers predicted by the corrected (Grishin et al. 2018, hereafter GPF18) model is larger, since larger eccentricities are involved, which is compatible with the numerical results. The secular model is less accurate with decreasing merger timescales; this is because the eccentricities involved are extreme, therefore deviations from the secular regime are more severe.

On longer merger timescales, \(t_{\text{merge}} \gtrsim 10^4 \text{ yr}\), if the typical LK timescales are short enough (e.g. GN and GN2 cases; top right and bottom left, respectively), the maximal eccentricity attained, \(e_{\max}\), is usually larger than the predicted one from 1PN theory alone (see their Fig. 5 of Grishin et al. 2018). Therefore, the mergers occur of faster than expected and the CDF is underestimated. On the other hand, for short merger times, the maximal eccentricity of the GPF18 model unbound, and the merger occurs on the secular timescale. If, however, \(t_{\text{LK}} \sqrt{1 - e_{\max}^2} \lesssim P_{\text{in}}\), the GPF18 model also breaks down and cannot describe the system (Antonini et al. 2017), while the value of \(e_{\max}\) is stochastic. This is because the fraction of time the inner orbit spends near \(e_{\max}\) (i.e. \(\sim t_{\text{LK}} \sqrt{1 - e_{\max}^2}\)) is too short for the inner orbit to complete one revolution and reach pericentre. Thus it takes longer time (at least a few secular times) to merge and the CDF is overestimated.
In the bottom right panel, we also plot for comparison the BH-BH case for the MW environment. Here, the masses are drawn from a distribution ($\propto M^{-1}$) and generally not equal. In this case there are even more mergers than expected in the NS-NS case on shorter timescales. One possible explanation is that the octupole term leads to more mergers on the longer octupole timescale, which increases the slope of the CDF. Remarkably, it fits well our predicted CDF, even though our modeling in Grishin et al. (2018) is not strictly valid for non-equal masses with outer eccentric orbits. We argue that the abundance of mergers both on short timescales from the 1PN breakdown discussed earlier and on long timescales due to chaotic octupole evolution may compensate each other and may 'even out' in the CDF plot.

In order to get a better sense of the corrected prescription, we perform two sided Kolmogorov-Smirnov (KS) tests comparing the cumulative distribution function (CDF) from the simulated results versus each one of the secular merger time distributions. Table 3 shows the total number of sim-
Figure 6. Left panel: cumulative merger time ($t_{\text{merge}}$) distribution for BH-BH (top), BH-NS (centre) and NS-NS (bottom) binaries, for all models with $f(a_{\text{in}})$ from Hoang et al. (2018). Right panel: $t_{\text{merge}}$ as a function of the nominal Peters (1964) GW merger time-scale $T_{GW}$, for the same models as shown in the left panel.

Table 3 shows the number of mergers from simulations, the secular and corrected GPF18 model, and the resulting D values and p values upon comparing the $N$-body simulations with the secular and the corrected GPF18 model. Overall, the number of events is underpredicted by an order of magnitude for the secular model, and by a factor of $\sim 2$-3 for the G18 model. The possible origins of these discrepancies are discussed above.

For the KS statistics, the D values are better for the G18 model, since the distance between the simulated and G18 CDFs is smaller. The p values are comparable and small. Besides the aforementioned discrepancies, a possible statistical artifact could be the low number of predicted mergers, namely in the MW case. Thus, in the GN2 case, where there are more mergers, the p value is the smallest, suggesting neither of the models is comparable with the simulated distribution.

To summarize, the simulated distribution cannot be
Figure 7. Cumulative Distribution of merger times for merged orbits. In each panel we compare the results from ARCHAIN (red) with the prescription for the merger times with secular (blue) and corrected eccentricity found in Grishin et al. (2018) (green).

Table 3. Total number of merged systems in the simulation ($N_{\text{sim}}$), as predicted from secular theory ($N_{\text{sec}}$) and corrected averaging ($N_{\text{GPF18}}$), and the $D$ and $p$ values of two sided KS statistics for each distribution.

|          | NS-NS MW | NS-NS GN | NS-NS GN2 | BH-BH MW |
|----------|----------|----------|-----------|----------|
| $N_{\text{sim}}$ | 67       | 97       | 214       | 59       |
| $N_{\text{sec}}$ | 7        | 16       | 34        | 7        |
| $N_{\text{GPF18}}$ | 25       | 42       | 101       | 16       |
| $D_{\text{sec}}$ | 0.29     | 0.38     | 0.4       | 0.44     |
| $p_{\text{sec}}$ | 0.58     | 0.028    | $8.7 \cdot 10^{-5}$ | 0.13     |
| $D_{\text{GPF18}}$ | 0.18     | 0.29     | 0.28      | 0.31     |
| $p_{\text{GPF18}}$ | 0.5      | 0.0115   | $3.25 \cdot 10^{-5}$ | 0.14     |

fully described by the GPF18 model in a statistical sense, but the overall trend of larger merger fractions on shorter merger timescales is consistent with the simulations. Thus, further improvements in the analytic understanding both in the GPF18 model and in the Peters (1964) formulæ are highly desired and deserve future work.

5.2 Merger Rates
Although we explore only a limited number of SMBH masses, we note that the range is relatively representative of what is expected for galactic nuclei. With the results of our simulations in hand, we can derive the expected merger rates of BH-BH, BH-NS and NS-NS binaries. With this, we can infer the dependence of the rate on the distribution of SMBH masses in the nearby Universe. To date, this rate remains poorly constrained in the literature.

Following Hamers et al. (2018), we calculate the merger rate for CO binaries as

$$\Gamma(M_{\text{SMBH}}) = n_{\text{gal}} \alpha_{\text{SMBH}} \Gamma_{\text{CO}} \alpha_{\text{bin}} \alpha_{\text{merge}},$$  \hspace{1cm} (19)

where $n_{\text{gal}}$ is the galaxy density, $\alpha_{\text{SMBH}} \approx 0.5$ is the fraction
of galaxies containing an SMBH (Antonini et al. 2015a,b), \( \Gamma_{\text{sup}}^{\text{CO}} \) is the compact object supply rate, \( f_{\text{bin}} \) is the fraction of stars forming compact object binaries, and \( f_{\text{merge}} \) is the fraction of mergers we find in our simulations. For galaxies, we assume that the SMBH number density scales as \( \Phi(M_{\text{SMBH}}) \propto 1/M_{\text{SMBH}} \) (Aller & Richstone 2002), hence the integrated number density of galaxies scales as

\[
n_{\text{gal}} \propto \int \Phi(M_{\text{SMBH}})dM_{\text{SMBH}} \propto \log(M_{\text{SMBH}}) \, . \tag{20}
\]

As in Hamers et al. (2018), we neglect the weak dependence on the SMBH mass and fix \( n_{\text{gal}} = 0.02 \) Mpc\(^{-3}\) (Conselice, Blackburne & Papovich 2005).

The fraction of CO binaries in the GC strongly depends on the assumptions regarding their origins. Several possibilities have been discussed in Antonini & Perets (2012); here we focus on two: ex-situ and in-situ origins. In the ex-situ scenario, stars form outside the nuclear cluster and then diffuse inwards. In the in-situ formation scenario, stars are formed in-situ close to the SMBH. We use simplified assumptions to estimate the supply rate in both cases. For a relaxed nuclear cluster, Fokker-Planck, Monte-Carlo and N-body simulations suggest that the fractions of BHs and NSs in the central 0.1 pc are of the order of \( \gamma_{\text{CO}} = 0.06, 0.01 \) for BHs and NSs, respectively (the higher BH fractions are due to mass-segregation), assuming the background stellar population has a continuous star-formation rate (Hopman & Alexander 2006a). Following Antonini & Perets (2012) we take initial binary fractions of \( f_{\text{bin}} = 0.1, 0.07 \) for BHs and NSs, respectively. For the compact object formation rate, we assume that the compact objects are supplied to the galactic nucleus by 2-body relaxation and mass segregation

\[
\Gamma_{\text{CO}}^{\text{sup}} = \frac{\gamma_{\text{CO}} N_\star(0.1 \text{ pc})}{t_{\text{seg}}(0.1 \text{ pc})} \propto M_{\text{SMBH}}^{(3-\beta)/\beta} \, , \tag{21}
\]

where \( \gamma_{\text{CO}} \) is the fractional number of compact objects; \( t_{\text{seg}} = T_{\text{2b}}(m_{\text{bin}}/M_\odot) \) is the timescale for mass segregation for binaries with mass \( m_{\text{bin}} \); and we assume \( M_{\text{SMBH}} \propto \sigma^4 \) (Merritt & Ferrarese 2001).

Normalizing the rates to the Milky Way’s Galactic Centre

\[
\Gamma_{\text{BH}}^{\text{sup}} = 2.5 \times 10^{-6} \left( \frac{4 \times 10^6 M_\odot}{M_{\text{SMBH}}} \right)^{1/4} \text{ yr}^{-1} \, . \tag{22}
\]

\[
\Gamma_{\text{NS}}^{\text{sup}} = 2.3 \times 10^{-8} \left( \frac{4 \times 10^6 M_\odot}{M_{\text{SMBH}}} \right)^{1/4} \text{ yr}^{-1} \, . \tag{23}
\]

The final expression for our rate becomes

\[
\Gamma_{\text{BH}}(M_{\text{SMBH}}) = 3.5 f_{\text{merge}} \text{ Gpc}^{-3} \text{ yr}^{-1} \times \left( \frac{4 \times 10^6 M_\odot}{M_{\text{SMBH}}} \right)^{1/4} , \tag{24}
\]

\[
\Gamma_{\text{NS}}(M_{\text{SMBH}}) = 3.2 \times 10^{-2} f_{\text{merge}} \text{ Gpc}^{-3} \text{ yr}^{-1} \times \left( \frac{4 \times 10^6 M_\odot}{M_{\text{SMBH}}} \right)^{1/4} , \tag{25}
\]

which is weakly dependent on the SMBH mass, and where the merger fraction \( f_{\text{merge}} \), which is typically a few up to a few tens of percents for the various models we considered, can be found in Tab. 1. We note that in \( f_{\text{merge}} \) we have also included the CO binaries that would merge by emission of GWs within \( \sim 1 \) Myr, without the assistance of LK oscillations. These are typically a few percent of the total mergers in the case of BH-NS and NS-NS binaries, while \( \sim 20-50\% \) in the case of BH-BH binaries.

Note that the evaporation time of NS binaries (see Eq. 6) could become comparable to the segregation time, and therefore the rates of NS-NS mergers could be even lower, if supplied from outside the central region of the nuclear cluster. If more massive stellar-BHs exist, the most massive ones will dominate the inner regions due to strong mass-segregation (Alexander & Hopman 2009b; Akaron & Perets 2016), and can be resupplied into the inner regions faster, enhancing the rates by up to a factor of a few. The larger numbers and faster supply will therefore bias the mass-function of merging BHs through this process to higher masses. Also, the relaxation and mass-segregation times in non-cuspy nuclear clusters, or when no nuclear cluster exists (e.g. for SMBHs more massive than \( \sim 10^6 M_\odot \), could be so long that the stellar density around the SMBHs is low. As a consequence, the resupply of stars close to the SMBH cannot be efficiently attained through 2-body relaxation processes, but is more likely to depend on the gas-inflow and in-situ star formation close to the SMBH (Antonini 2013, 2014).

The star-formation rate close to non-resolved regions around SMBHs is difficult to estimate theoretically. Here, we try to use an empirical estimate based on our own resolved Galactic Centre (see e.g. Bartko et al. 2009). Approximately \( \sim 200 \) O-stars (likely to later form stellar black holes) are observed and inferred to have formed over the last 10 Myrs in the young stellar disk close \((\sim 0.05 - 0.5 \text{ pc})\) to the SMBH. The number of lower-mass B-stars in the same environment suggests that similar continuous star-formation has not occurred over the last 100 Myr. Based on these observations we may consider an in-situ formation rate of BHs of \( \sim 200/10^5 = 2 \times 10^{-6} \) yr\(^{-1}\), i.e. comparable to the estimated supply rate of \( 2.5 \times 10^{-6} \) from mass-segregation of BHs from outside the central regions. The comparable formation rate of NSs, however, would increase their resupply rates to the same level as BHs, i.e. much higher than the resupply from NSs migrating in from the outside (\( \sim 2 \times 10^{-6} \) yr\(^{-1}\)) and thereby

\[
\Gamma_{\text{NS}}(M_{\text{SMBH}}) = 2.8 f_{\text{merge}} \text{ Gpc}^{-3} \text{ yr}^{-1} \times \left( \frac{4 \times 10^6 M_\odot}{M_{\text{SMBH}}} \right)^{1/4} . \tag{26}
\]
Table 4 reports the resulting rates as a function of the SMBH mass. Upon using the semi-major axis and eccentricity distributions following the prescriptions of Antonini & Perets (2012), we typically get a merger fraction $\sim 2\text{ to }5$ times larger than in the case of adopting the initial conditions from Hoang et al. (2018). This is probably related to the fact that the semi-major axes of CO binaries are typically smaller in the former case. BH-NS binaries should have mass segregation times similar to BH-BH binaries, hence we use Eq. 24. For all SMBH masses considered in this study, the rates are in the range $\sim 0.17-0.52 \text{ Gpc}^{-3} \text{ yr}^{-1}$, $\sim 0.06-0.10 \text{ Gpc}^{-3} \text{ yr}^{-1}$ and $\sim 0.41-1.71 \times 10^{-3} \text{ Gpc}^{-3} \text{ yr}^{-1}$ for BH-BH, BH-NS and NS-NS binaries, respectively. In the star-formation channel, the NS-NS rate may be as high as $\sim 0.04-0.16 \text{ Gpc}^{-3} \text{ yr}^{-1}$. We note that the merger rate is a decreasing function of the SMBH mass, even though the relative fraction of merger events is typically larger for more massive SMBHs (see Tab. 1). On the other hand, more massive SMBHs imply longer relaxation times, that contribute to a reduction in the merger fraction and make the relative rates smaller.

5.3 Comparison of merger rates to previous studies

We find rates comparable with Antonini & Perets (2012), but lower with respect to other works that explored the role of the SMBH in reducing the merger timescale of binaries due to Lidov-Kozai oscillations (Petrovich & Antonini 2017; Hoang et al. 2018). Other merger channels typically predict larger rates. Typical values for globular clusters are $\sim 2-10 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Askar et al. 2017; Fragione & Kocsis 2018; Rodriguez et al. 2018) and for nuclear star clusters are $\sim 1-15 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Antonini & Rasio 2016). For reference, the BH-BH merger rate inferred by LIGO is $\sim 12-213 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Abbott et al. 2017a).

We note that previous studies that used a similar approach made use of much larger supply rates, $\sim 10-40$ times higher than considered here. The differences arise for several reasons. Petrovich & Antonini (2017), Hamers et al. (2018) and Hoang et al. (2018) considered a resupply rate of BHs of $10^{-5}-10^{-3} \text{ yr}^{-1}$ from star-formation, where the latter rate is derived assuming a top-heavy initial mass function from (Maness et al. 2007). However, this gives rise to several difficulties: (1) This rate is based on the formation rates derived by Lückmann & Baumgardt (2009) and Lückmann et al. (2009) for both NSs and BHs lumped together, while the formation rate for NSs is actually 8 times higher than that of BHs for regular (e.g. Kroupa or Miller-Scalo) IMFs, and becomes comparable only for top-heavy IMFs; one can therefore not discuss the number of BHs taking these numbers at face value, but consider the division between BHs and NSs, and its dependence on the assumed IMF. (2) The higher rate estimates are based on a top-heavy IMF, which in-turn is based on results from (Maness et al. 2007). However, this top-heavy IMF was only derived from observations of old low-mass stars and the results are therefore highly problematic for the use in this context, as they are based on a large extrapolation from the low-mass regime up to that of NS and BH progenitors not probed at all by Maness et al. (2007). Also note that low-mass stars could already dynamically evolve through mass-segregation and their observed distribution in the GC does not necessarily reflect the actual IMF (e.g. see (Aharon & Perets 2015)). Moreover, direct observations of massive stars in the GC today, though suggestive of a somewhat top-heavy IMF (Lu et al. 2013), find a much shallower power-law of $-1.7 \pm 0.2$ compared with $-0.8$ in the Maness et al. study of low-mass stars (and compared with $-2.3$ for Salpeter or Kroupa IMFs in the relevant mass-range). (3) Even more important, all of these estimates considered star formation throughout the nuclear cluster, rather than the innermost regions, where the induced mergers actually take place (especially in the cases considered by Hoang et al. (2018)), i.e. the COs would have to migrate inwards over long timescales, and one should then refer to the ex-situ resupply rate discussed above. The overall numbers of COs they derive are therefore at least 10 times higher than actually expected in the central parts, and, in fact, are at least 10 times higher than what one can infer from the observed young massive stars in the GC (Bartko et al. 2009, 2010), or those inferred from X-ray sources (Muñoz et al. 2005).

In summary, we believe the overall estimated rates near SMBHs have probably been overestimated by a factor of $\sim 10-40$ in previous studies, regardless of the specific secular and quasi-secular dynamical evolution considered.

6 ELECTROMAGNETIC COUNTERPARTS AND OBSERVATIONAL SIGNATURES OF SMBH-INDUCED CO MERGERS

As we noted previously (cfr Sec. 4) the very high (close to unity) eccentricity, with which the GW signal enters the LIGO band in the scenario explored potentially provide an important observational diagnostic of CO mergers induced by LK oscillations. In the following, we discuss further observational diagnostics of this merger channel in relation to possible EM counterparts to the mergers. In particular, mergers of compact object binaries are expected to be associated with a strong release of electromagnetic radiation, if the right conditions arise to power an energetic outflow.

In the case of a NS-NS merger, tidal disruption during the inspiral phase leaves behind an accretion torus surrounding the merged object (either a NS or a BH), unless the two NSs have identical masses (Shibata et al. 2006; Rezzolla et al. 2010; Giacomazzo et al. 2013; Hotokezaka et al. 2013; Kiuchi et al. 2014; Ruiz et al. 2016; Radice et al. 2016). An energetic engine can be driven by rapid accretion onto to the remnant object and/or by dipole radiation losses if the remnant is an hypermassive or stable NS (Giacomazzo & Perna 2013; Ciolfi et al. 2017). Growth and collimation of magnetic fields during the merger, as well as neutrino losses, are then believed to power a relativistic outflow. Dissipation within the expanding flow, and later interaction of the flow with the interstellar medium, gives rise to radiation that spans a wide window in the electromagnetic spectrum, from high-energy $\gamma$-rays down to the radio. This basic scenario has been observationally confirmed with the recent event GW170817/GRB170817A (Abbott et al. 2017).

Mergers of BH-NS binaries (always resulting in a BH as the resulting compact remnant) are expected to be accompanied by the formation of an hyperaccreting disk only if the mass ratio between the BH and NS does not exceed the value $\sim 3-5$, with the precise value depending on the equa-
tion of state of the NS and the BH spin (Pannarale et al. 2011; Foucart 2012; Foucart et al. 2018). For larger mass ratios, the tidal disruption radius of the NS is smaller than the radius of the innermost stable circular orbit, and no disk will form, resulting in a direct plunge into the BH (see e.g. Bartos et al. 2013 for a review). If a rapidly accreting disk forms, then the resulting EM phenomenology is expected to be similar to that of the NS-NS case, at least in so far as the bulk properties are concerned. For the initial conditions explored in this work, \( \sim 10-20\% \) of mergers is expected to have mass ratios \( \lesssim 5 \), and hence possibly giving rise to an accretion-powered EM counterpart.

In the case of a BH-BH binary merger, there is no tidally disrupted material which can readily supply the accretion power for a relativistic outflow\(^5\). However, following the tentative detection of a \( \gamma \)-ray counterpart by the \textit{Fermi} satellite to the event GW150914 (Connaughton et al. 2016), several ideas were proposed for providing the merged BH with a baryonic remnant to accrete from (Perna et al. 2016; Loeb 2016; Woosley et al. 2016; Stone et al. 2017; Bartos et al. 2017; Kimura et al. 2017; Janiuk et al. 2017; de Mink & King 2017). Within the context of this study, the scenario proposed by Bartos et al. (2017) is of particular relevance; they note that BH-BH binaries merging within an AGN disk can accrete a significant amount of gas from the disk, well above the Eddington rate, and possibly give rise to high-energy EM emission.

Electromagnetic counterparts to binary mergers provide crucial information on the production mechanism of the binaries, since they can potentially allow a much better localization compared to GWs alone. A distinctive signature of binary mergers enhanced by LK oscillations in the vicinity of SMBHs is their relatively short merger timescale compared to that of other formation channels. For example, the classical channel of isolated binary evolution predicts merger times \( \sim 100 \) Myr-15 Gyr (Belczynski et al. 2006). The short lifetimes of the binaries, coupled with their production in the galactic centers, lead to correspondingly short distances traveled prior to mergers. We find that these distances are typically \( \lesssim 0.1 \) pc, which makes these merger events practically occurring within the close nuclear region. This constitutes a major difference with respect to the standard isolated binary evolution scenario (Perna & Belczynski 2002; Belczynski et al. 2006; O’Shaughnessy et al. 2017; Perna et al. 2018): whether it is a small or a large galaxy, the bulk of the merger events occurs at projected distances (from the galaxy center) \( \gtrsim 10 \) pc\(^6\). Localization via EM counterparts hence becomes an especially useful discriminant.

Short GRBs associated with NS-NS mergers, and BH-NS mergers with a small enough mass ratio to allow tidal disruption, are expected to be followed by broadband radiation called afterglow, resulting from the dissipation of a relativistic shock propagating in the interstellar medium (Sari, Piran & Narayan 1998). The maximum flux intensity (at any wavelength) is given by

\[
F_{\nu, \text{max}} = 110 n_1^{-1/2} \xi_B^{-1/2} E_{52} D_{28}^{-2} (1+z) \text{ mJy},
\]

where \( E_{52} \) is the explosion energy in units of \( 10^{52} \) erg, \( D_{28} \) the luminosity distance in units of \( 10^{28} \) cm, \( z \) is the redshift, \( n_1 \) the number density of the interstellar medium in \( \text{cm}^{-3} \), and it is assumed that the magnetic field energy density in the shock rest frame is a fraction \( \xi_B \) of the equipartition value.

The broadband spectrum evolves with time, and we compute it numerically using the formalism of Sari et al. (1998). For an energy \( E = 10^{50} \) erg as typical of short GRBs, standard assumptions for the shock parameters and medium ambient density \( \sim \) a few \( \text{cm}^{-3} \) (more typical of the inner regions of a galaxy), the afterglow luminosity in some representative bands (X-rays and radio) at some typical observation times is found to be \( L_{[2-10] \text{keV}} \sim 5 \times 10^{45} \) erg s\(^{-1} \) at \( t_{\text{obs}} = 1 \) hr and \( L_{\text{GHz}} \sim 6 \times 10^{39} \) erg s\(^{-1} \) Hz\(^{-1} \) at \( t_{\text{obs}} = 7 \) days. In the X-rays, a representative flux threshold is the \textit{Swift}/XRT flux sensitivity of \( F_{\text{lim}} = 2.5 \times 10^{-13} \) erg s\(^{-1} \) cm\(^{-2} \) Hz\(^{-1} \), while in the radio, a 1hr integration with the VLA leads to a flux threshold for detection of \( F_{\text{lim}} \approx 50 \mu \text{Jy} \). In both these bands, detection would be possible up to a redshift of \( \sim 2 \), considerably larger than the LIGO horizon\(^7\). The detection distances are larger than in the isolated binary evolution scenario, for which the large traveled distances lead to a sizable fraction of mergers to occur in low-density environments, where the afterglow luminosity is considerably dimmer.

Additionally note that, independently of the post-merger EM signal, a fraction on the order of a few \( \times 10^{-3} \) of the GW sources is expected to be accompanied by an SN-type precursor (Michaely & Perets 2018). This is due to the fact that the distribution of the delay time between the last SN explosion and the binary merger has a non-negligible tail of ultra-short times, on the order of 1-100 yr (see also (Dominik et al. 2012)).

Detection of an EM counterpart to a GW event generally allows a redshift measurement. The redshift distribution of the channel studied here would be one that follows the star-formation rate, since the merger times are shorter or at most comparable to the lifetimes of the most massive stars (as a reference, the lifetime of a \( \sim 100 M_\odot \) star is about 1 Myr). This would hence constitute another observational diagnostic.

The relatively easier prospects for detecting EM counterparts from CO mergers in galactic nuclei makes this channel especially useful for extraction of astrophysical and cosmological information from combined GW/EM detections. This includes, among other, measurements of the Hubble constant, new tests of the Lorentz invariance, constraints on the speed of GWs, probes of the physics of mergers and jet

\(^5\) Note however that alternative scenarios, involving pure electromagnetic energy, have also been invoked as alternatives to accretion to produce energy (Zhang 2016; Liebling & Palenzuela 2016).

\(^6\) Note that, even if the isolated binary evolution scenario does predict a fraction of tight binaries with sub-Myr lifetimes (Belczynski et al. 2006), and even ultra-short merger times (Michaely & Perets 2018), but the merger sites are still dominated by large scales since isolated binaries are born throughout the galactic disk.

\(^7\) It should however be noted that in the X-rays, when the shock is still moving at relativistic speeds, relativistic beaming of the emission will lead to a reduced luminosity for jets which are not observed on-axis. The fraction of on-axis jets is expected to be on the order of 1/20 by taking the jet size of \( \sim 16 \) deg inferred for short GRBs (Fong et al. 2015).
formation, constraints on the equation of state of neutron stars (see Abbott et al. 2017, and references therein).

7 DISCUSSION AND SUMMARY

In this paper, we have revisited the SMBH-induced mergers of compact binaries orbiting within its sphere of influence. While previous studies in the literature adopted the secular approximation for the equations of motion (Antonini & Perets 2012; Hamers et al. 2018; Hoang et al. 2018), here we have performed an extensive statistical study of BH-BH, NS-NS and BH-NS binary mergers by means of ~ 35000 direct high-precision regularized N-body simulations, including Post-Newtonian (PN) terms up to order PN2.5.

We have shown that the secular approach breaks down for systems with mild and extreme hierarchies. We used the recent corrections to the maximal eccentricity $e_{\text{max}}$ and merger times in the quasi-secular regime (Grishin et al. 2018, GPF18) and tested it against N-body population synthesis integrations. The total number of mergers is under-predicted by a factor of $\sim 6 - 10$ in the secular approach, and by a factor of $\sim 2 - 3$ by the corrected GPF18 model. The CDF of merger times fail to fit either of the distributions, although the D-value distance between the simulated CDF and the GPF18 model is closer than for the secular approach. The difference can be attributed to the original underestimate of $e_{\text{max}}$ in the latter model, which leads to faster and more frequent mergers.

In our numerical simulations, we have considered different SMBH masses, different slopes for the BH mass function and the binary spatial distributions, and different CO binary semi-major axis and eccentricity distributions. We find that the majority of binary mergers happen when the mutual inclination of the binary orbit and its center of mass $i_0$ is $90^\circ$, as a consequence of the Lidov-Kozai mechanism. Most of these mergers enter the LIGO band with eccentricities almost approaching unity. We have also shown that the distributions of the inner and outer semi-major axes of the merging binaries depend mainly on the mass of the SMBH and on the slope $\alpha$ of the binary spatial distribution around the SMBH. On the other hand, the shape of the resulting CO mass distributions depend on the slope $\beta$ of the BH mass function. BH mergers observed by LIGO seem to favour $\beta \sim 1$, if those mergers were to happen around SMBHs.

We have also calculated the resulting rates as a function of the SMBH mass. We find that the merger rates are a decreasing function of the SMBH mass and are in the ranges $\sim 0.17 - 0.52 \text{ Gpc}^{-3} \text{ yr}^{-1}$, $\sim 0.06 - 0.10 \text{ Gpc}^{-3} \text{ yr}^{-1}$ and $\sim 0.41 - 1.71 \times 10^{-3} \text{ Gpc}^{-3} \text{ yr}^{-1}$ for BH-BH, BH-NS and NS-NS binaries, respectively. In the star-formation channel, the NS-NS rate may be as high as $\sim 0.04 - 0.16 \text{ Gpc}^{-3} \text{ yr}^{-1}$. We find rates consistent with Antonini & Perets (2012), but lower with respect to previous works (Petrovich & Antonini 2017; Hamers et al. 2018; Hoang et al. 2018), which may have overestimated the amount of CO binaries supplied to galactic nuclei through star-formation.

We have also discussed the possible EM counterparts of these events. Due to their locations, these mergers may have higher probabilities of being detected also via their EM counterparts, hence making these CO mergers especially valuable for cosmological and astrophysical purposes.

Finally, we note that we have adopted 1 Myr for the maximum integration time in our simulations, since this limit sets a good compromise between the computational effort and the size of the statistical sample we generate. This choice is further justified by noting that the typical timescale for vector resonant relaxation to operate is $\sim 1 - 10$ Myr, over which the mutual orbital inclination is reoriented by interactions with other background objects, and renders the 3-body approximation insufficient (Hamers et al. 2018). Also, we have neglected the possible precession of the CO binaries’ motion induced by continual weak interactions with other stars and COs in the stellar cusp surrounding the SMBH (Alexander 2017). A comprehensive N-body study over long integration timescales that includes both the SMBH-induced Lidov-Kozai oscillations and the detailed effects of the background stars surrounding the CO binaries deserves consideration in future work.

ACKNOWLEDGEMENTS

GF is supported by the Foreign Postdoctoral Fellowship Program of the Israel Academy of Sciences and Humanities. GF also acknowledges support from an Arskin postdoctoral fellowship at the Hebrew University of Jerusalem. EG acknowledges support from the Technion Irwin and Joan Jacobs Excellence Fellowship for outstanding graduate students. EG and HBP acknowledge support by Israel Science Foundation I-CORE grant 1829/12. NL and RP acknowledge support by NSF award AST-1616157. GF thanks Seppo Mikkola for helpful discussions on the use of the code ARCHAIN. Simulations were run on the Astric cluster at the Hebrew University of Jerusalem. The Center for Computational Astrophysics at the Flatiron Institute is supported by the Simons Foundation.

REFERENCES

Abbott B. P., Abbott R., Abbott T. D., Acernese F., Ackley K., Adams C., Adams T., Addesso P., Adhikari R. X., Adya V. B., et al. 2017, ApJL, 848, L13
Abbott B. P., et al., 2016a, Physical Review Letters, 116, 241103
Abbott B. P., et al., 2016b, Physical Review Letters, 116, 061102
Abbott B. P., et al., 2017a, Physical Review Letters, 118, 221101
Abbott B. P., et al., 2017b, ApJ Lett, 851
Abbott B. P., et al., 2017c, Physical Review Letters, 119, 141101
Abbott B. P., et al., 2017d, Physical Review Letters, 119, 161101
Aharon D., Perets H. B., 2015, ApJ, 799, 185
Aharon D., Perets H. B., 2016, ApJLett, 830, L1
Alexander T., 2017, Ann Rev Astron Astrop, 55, 17
Alexander T., Hopman C., 2009a, ApJ, 697, 1861
Alexander T., Hopman C., 2009b, ApJ, 697, 1861
Aller M. C., Richstone D., 2002, AJ, 124, 3035
Antognini J. M., Shappee B. J., Thompson T. A., Amaro-Seoane P., 2014, MNRAS, 439, 1079
Antognini J. M. O., 2015, MNRAS, 452, 3610
Antonini F., 2013, ApJ, 763, 62
Antonini F., 2014, ApJ, 794, 106
Antonini F., Barausse E., Silk J., 2015a, ApJ, 812, 72
Antonini F., Barausse E., Silk J., 2015b, ApJ Lett, 806, L8
Antonini F., Merritt D., 2012, ApJ, 745, 83
Antonini F., Murray N., Mikkola S., 2014, ApJ, 781, 45
Antonini F., Perets H. B., 2012, ApJ, 757, 27
Antonini F., Rasio F. A., 2016, ApJ, 831, 187
Antonini F., Toonen S., Hamers A. S., 2017, ApJ, 841, 77
Arca-Sedda M., Guandalini A., 2018, MNRAS, 477, 4423
Asker A., Szukularek M., Gondek-Rosińska D., Giersz M., Bulik T., 2017, MNRAS, 464, L36
Bachall J. N., Wolf R. A., 1976, ApJ, 209, 214
Banerjee S., 2018, MNRAS, 473, 909
Bartko H., et al., 2010, ApJ, 708, 834
Bartko H., et al., 2009, ApJ, 697, 1741
Bartos I., Brady P., Márka S., 2013, Classical and Quantum Gravity, 30, 123001
Bartos I., Kocsis B., Haiman Z., Márai S., 2017, ApJ, 835, 165
Bartos I., Kocsis B., Haiman Z., Márai S., 2017, ApJ, 835, 165
Baruteau C., Cuadra J., Lin D. N. C., 2011, ApJ, 726, 28
Baumgardt H., Amaro-Seoane P., Schödel R., 2018, A&A, 609, A28
Belczynski K., Holz D. E., Bulik T., O’Shaughnessy R., 2016, Nature, 534, 512
Belczynski K., Perna R., Bulik T., Kalogera V., Ivanova N., Lamb D. Q., 2006, ApJ, 648, 1110
Belczynski K., Sadowski A., Rasio F. A., 2014, ApJ, 711, 1068
Bellovary J. M., Mac Low M.-M., McKernan B., Ford K. E. S., 2016, ApJL, 819, L17
Binney J., Tremaine S., 1987, Galactic dynamics
Choksi N., Volonteri M., Colpi M., Gnedin O. Y., Li H., 2018, arXiv:1809.01164
Ciolfi R., Kastaun W., Giacomazzo B., Endrizzi A., Siegel D. M., Perna R., 2017, Phys. Rev. D, 95, 063016
Connaughton V., et al., 2016, ApJ Lett, 826, L6
Conseil C. J., Blackburne J. A., Papovich C., 2005, ApJ, 620, 564
de Mink S. E., King A., 2017, ApJL, 839, L7
Dominik M., Belczynski K., Fryer C., Holz D. E., Berti E., Bulik T., Mandel I., O’Shaughnessy R., 2012, ApJ, 759, 52
Fong W., Berger E., Margutti R., Zauderer B. A., 2015, ApJ, 815, 102
Foucart F., 2012, Phys. Rev. D, 86, 124007
Foucart F., Hinderer T., Nissanka S., 2018, ArXiv e-prints
Fragione G., 2018, MNRAS, 479, 2615
Fragione G., Capuzzo-Dolcetta R., Kroupa P., 2017, MNRAS, 475, 4986
Fragione G., Kocsis B., 2018, Phys Rev Lett, 121, 161103
Fragione G., Leigh N., 2018a, MNRAS
Fragione G., Leigh N., 2018b, MNRAS, 479, 3181
Fragione G., Pavlík V., Banerjee S., 2018, MNRAS, 480, 4955
Fragione G., Sari R., 2018, ApJ, 852, 51

BH and NS mergers in Galactic Nuclei

Freitag M., Amaro-Seoane P., Kalogera V., 2006, ApJ, 649, 91
Gallego-Cano E., et al., 2018, A&A, 609, A26
Genzerov A., Stone N. C., Metzger B. D., Ostriker J. P., 2018, MNRAS, 478, 4030
Genzel R., Eisenhauer F., Gillessen S., 2010, Rev Mod Phys, 82, 3121
Genzel R., et al., 2008, ApJ, 687, 59
Giacobbo N., Mapelli M., 2018, MNRAS, 480, 2011
Giacomazzo B., Perna R., 2013, ApJL, 771, L26
Giacomazzo B., Perna R., Rezzolla L., Troja E., Lazzati D., 2013, ApJL, 762, L18
Ginsburg I., Perets H. B., 2011, ArXiv e-prints
Grishin E., Perets H. B., Fragione G., 2018, MNRAS, 481, 4907
Grishin E., Perets H. B., Menou K., 2017, MNRAS, 466, 276
Hailey C. J., Mori K., 2017, in AAS/High Energy Astrophysics Division #16 Vol. 16 of AAS/High Energy Astrophysics Division, NuSTAR observations of black hole binary candidates in the Galactic Center and its environs.

p. 109.12
Hamers A. S., Bar-Or B., Petrovich C., Antonini F., 2018, arXiv:1805.10313
Hoang B.-M., Naoz S., Kocsis B., Rasio F. A., Dosopoulou F., 2018, ApJ, 856, 140
Hopman C., 2009, ApJ, 700, 1933
Hopman C., Alexander T., 2006a, ApJL, 645, L133
Hopman C., Alexander T., 2006b, ApJL, 645, L133
Hotokezaka K., Kiuchi K., Kyutoku K., Okawa H., Sekiguchi Y.-i., Shibata M., Taniguchi K., 2013, Phys. Rev. D, 87, 024001
Janiuk A., Beijger M., Charzyński S., Sukova P., 2017, New A, 51, 7
Jeans J. H., 1919, MNRAS, 79, 408
Katz B., Dong S., Malhotra R., 2011, Physical Review Letters, 107, 181101
Kimura S. M., Murase K., Mészáros P., 2017, ArXiv e-prints
Kiuchi K., Kyutoku K., Sekiguchi Y., Shibata M., Wada T., 2014, Phys. Rev. D, 90, 041502
Kocsis B., Tremaine S., 2015, MNRAS, 448, 3265
Kozai Y., 1962, AJ, 67, 591
Leigh N. W. C., Antonini F., Stone N. C., Shara M. M., Merritt D., 2016, MNRAS, 463, 1605
Leigh N. W. C., Geller A. M., McKernan B., Ford K. E. S., Mac Low M.-M., Bellovary J., Haiman Z., Lyra W., Sampson J., O’Dowd M., Kocsis B., Endlich S., 2018, MNRAS, 474, 5672
Leigh N. W. C., Lützgendorf N., Geller M. A., Maccarone T. J., Heinke C., Sesana A., 2014, MNRAS, 444, 29
Levin Y., Beloborodov A. M., 2003, ApJL, 590, L33
Li G., Naoz S., Holman M., Loeb A., 2014, ApJ, 791, 86
Lidov M. L., 1962, P& SS, 9, 719
Liebling S. L., Palenzuela C., 2016, Phys. Rev. D, 94, 064046
Lithwick Y., Naoz S., 2011, ApJ, 742, 94
Liu B., Lai D., 2018, ApJ, 863, 68
Lückmann U., Baumgardt H., 2009, MNRAS, 394, 1481
Lückmann U., Baumgardt H., Kroupa P., 2009, MNRAS, 398, 429
Loeb A., 2016, ApJL, 819, L21
