Unified solutions on axial bearing capacity of round-ended concrete-filled steel tube short columns with binding bars

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Abstract. Based on the unified strength theory, according to the characteristics of axial bearing capacity of round-ended concrete-filled steel tubular stub short columns with binding bars, the cross shaped section was divided into a rectangular section which has bars and two round-ended sections which have no bars. Considering intermediate principal stress, the ration of width to thickness of pipe to the bearing capacity of steel tube was in consideration too, the paper analyzed force mechanism. In the end, the mathematical formulation of bearing capacity for the round-ended concrete filled steel tube stub column with binding bars was obtained. The formula calculation values were in good agreement with the value of the relevant literature. Theoretical formulas are expected to be helpful for engineering applications.

1. Summary

With the continuous development of the application range of concrete-filled steel tube, the section forms of concrete-filled steel tube are also continuously evolving to the shapes of round, rectangular and multi-cavity abnormals to meet different requirements[1]. At the same time, because the transverse stiffness of the bridge structure is much lower than its longitudinal stiffness, many piers and arch ribs are designed as round-ended sections with large transverse stiffness, such as Shanghai Longquan Port Bridge, Changsha Baisha River Bridge[2]. References[3-9] have carried out experimental and simulation studies on round-ended concrete-filled steel tube columns, and the results show that the round-ended concrete-filled steel tube section has stronger constraints at both ends, while the constraints are weaker in the straight section.

CAI Jian et al. [10-12] conducted the axial compression tests research of square and rectangular concrete-filled steel tubular short columns with binding bars. The results show that the concrete-filled steel tubular stub short columns is strengthened by the binding bars, but at the same time the constitutive relation of concrete is more complex, and the constitutive relation proposed by using lateral equivalent effect can reflect the axial compressive property of concrete well. Zhao Jun Hai et al.[13-14] carried out theoretical analysis on cross-shaped and rectangular short concrete-filled steel tubular columns with binding bars. They proposed a simplified calculation formula for bearing capacity.

At present, there are few experimental and theoretical studies on axial compression of short concrete-filled steel tube with binding bar. Ren Zhigang et al. [15-16] proposed to add binding bars to the round-ended concrete-filled steel tube to improve the interface bonding performance of members, and they studied the axial compression working mechanism of the round-ended concrete-filled steel
tube members with binding bars. The results show that the round-ended section of concrete filled steel tube is a strong restraint zone, and the addition of binding bars in the rectangular area can strengthen the restraint effect of the straight section of steel tube on concrete, make the stress distribution of concrete more even, and effectively improve the ductility and ultimate bearing capacity of members.

In this paper, based on the unified strength theory, the round-ended section with binding bars is divided into 1 rectangular region and 2 semicircular regions (as shown in Figure 1). The constraint of binding bars and outer steel tube to concrete is equivalent to effective lateral stress. Meanwhile, the rectangular core concrete area is reasonably equivalent to the circular area. A formula for calculating the axial bearing capacity of round-ended concrete-filled steel tube short columns with binding bars is established, which are expected to be helpful for engineering applications.

2. Unified Strength Theory
In 1991, based on his twin shear strength theory, Yu Maohong established twin shear unified strength theory, which considering the impact of the intermediate principal stress \( \sigma_2 \) and the different effects of material between tensile and compressive[17].

For materials whose tensile strength and compressive strength are different, in the field of strength calculation standards, it need two material strength coefficients, namely the tensile ultimate strength \( f_t \) and the compressive strength limit \( f_c \). Then the formula is

\[
F = \sigma_1 \frac{\alpha}{1+b} (b\sigma_2 + \sigma_3) = \sigma_1, \quad \sigma_2 \leq \frac{\sigma_1 + \alpha \sigma_3}{1+\alpha} \quad (1a)
\]

\[
F' = \frac{1}{1+b} (\sigma_1 + b\sigma_2) - \alpha \sigma_3 = \sigma_1, \quad \sigma_2 \geq \frac{\sigma_1 + \alpha \sigma_3}{1+\alpha} \quad (1b)
\]

Where, \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the three principal stress; \( \alpha = f_t/f_c \) is material strength ratio of tension and pressure; \( b \) is not only a coefficient which can reflect the effect of intermediate principal stress and the corresponding surface of positive stress on material damage influence degree, but also a parameter which reflects the different theory of strength.
3. Ultimate Bearing Capacity Analysis

3.1. Bearing Capacity Mechanism
Previous studies have shown[15] that, under the action of axial pressure, steel tubes and binding bars in short round-ended concrete-filled steel tube columns with binding bars improve the mechanical properties of concrete. Binding bars can effectively improve the constraint effect of concrete in rectangular area, and steel tube begins to yield when the stress is close to the yield stress. When the axial strain of the binding bars reaches 0.015, part of the binding bars exceeds the yield strength and exits the work.

3.2. Cross Section Division
The method of superposition after section division can be used to study short concrete filled steel tubular columns with binding bars. In this paper, the round-ended section is divided into a rectangular region with binding bars and two semi-circular regions without binding bars. It is assumed that the lateral stiffness at each cut surface is infinite, the normal displacement at the cut surface is zero, and the longitudinal and normal directions meet the condition of deformation coordination. Studies have shown that the stress mechanism of circular tubular concrete with semicircular section at both ends is similar to that of circular tubular concrete[15]. Therefore, the short section of round-ended concrete-filled steel tube with binding bars is decomposed into a round-ended concrete-filled steel tube and a rectangular section of concrete-filled steel tube with binding bars.

3.3. Steel Tube Bearing Capacity of Rectangular Region
Under the action of axial pressure, the outer steel tube of the concrete-filled rectangular steel tube is in the state of three-direction stress. However, because the radial stress is far less than the longitudinal and annular stress, the influence of radial stress is not considered[12].

The width-thickness ratio parameter of the long side of the rectangle is defined as follows:

$$R_1 = \frac{B - D}{t} \sqrt{\frac{12(1-\nu^2)}{4\pi^2}} \frac{f_{sy}}{E_a}$$  (2)

Where, $f_{sy}$ and $E_a$ are respectively the yield strength and elastic modulus of the steel pipe; $\nu$ is the Poisson's ratio of steel tube, take 0.3[18]; $t$ is the thickness of the steel tube; $B$ is the section length of round-ended concrete-filled steel tube; $D$ is the section width of round-ended concrete-filled steel tube.

The buckling mode of steel tube is mainly related to the width-thickness ratio parameter $R_1$ [19]. When $R_1 \geq 0.85$, local buckling occurred; when $R_1 < 0.85$, local buckling was not considered.

According to literature [14], the longitudinal strength of rectangular steel tubes can be reduced by the reduction coefficient $\phi_l$. The value of $\phi_l$ is defined as follows:

$$\phi_l = \begin{cases} 0.89 & R_1 \leq 0.85 \\ 0.89R_1^{0.37} & R_1 > 0.85 \end{cases}$$  (3)

When $R_1 > 0.85$, if $\phi_l > 0.89$, set $\phi_l = 0.89$.

Then the axial bearing capacity of the steel tube is:

$$R_1 = \phi_l A_{sl} f_{sy}$$  (4)

$$A_{sl} = 2(B - D)t$$  (5)

Where, $A_{sl}$ is the steel tube area in the rectangular region.

3.4. Bearing Capacity of Concrete in Rectangular Region
Round-ended concrete-filled steel tube short columns with binding bars is effective constraint area and weak constraint area. This paper will take different restraint in counting method for equivalent average, regardless of binding bars of core concrete constraints. And it will be uniform rectangular steel tube
equivalent for circular steel tube side pressure calculation [20].

According to literature [21], the calculation formula of core concrete bearing capacity of ordinary rectangular concrete-filled steel tubular column without binding bars is as follows:

$$N_c = A_c \left\{ f_c + \gamma_u k \left\{ \frac{2k_c f_y}{(B-D)^2(D/t-2)} \sqrt{D(B-D) + \sqrt{(D-2t)(B-D)}} \right\} \right\}$$

(6)

Where, $A_c$ is the cross section area of core concrete $A_c = (B-D)(D-2t)$; $f_c$ is the axial compressive strength of concrete; $k$ is the pressure measurement coefficient, $k = (1 + \sin \varphi)/(1 - \sin \varphi)$; $\varphi$ is the concrete internal friction angle, the specific value can be determined by the test; $\gamma_u$ is the reduction factor of concrete strength, $\gamma_u = D' - 0.112$; $D'$ is the outer diameter of the cylinder; $k_{ei}$ is the effective constraint coefficient of concrete cross-section[16].

$$D' = \sqrt{\frac{4D \times (B-D)}{\pi}}$$

(7)

$$k_{ei} = \left\{ 1 - \frac{2(B-D) \tan \theta}{3n_1(D-2t)} \right\} \left\{ 1 - \frac{b_s \tan \theta}{3(D-2t)} \right\}$$

(8)

In view of the additional constraint effect of the binding bars, on the basis of formula (6) multiply by an increasing coefficient $\varphi_c[22]$, $\varphi_c$ is mainly related to the diameter $d_s$ of the binding bars, the intensity $f_{y1}$, the horizontal spacing $a_s$, the longitudinal spacing $b_s$, etc.

When calculating the strength of core concrete, the above formula has taken into account the influence of the ratio of width to thickness of core concrete, which is reflected by the angle of constraint boundary parabola, $\theta = 45^\circ$; $n_1$ is the total number of parabolas on the long side of the rectangular section, which is related to the number of binding bars. In Fig.1, $n_1 = 6[16]$.

Based on literature [14], define binding bars’ constraint coefficient of $\zeta_1 = f_{y1} A_{s1}/(f_c a_s b_s)$, then the core concrete of the rectangular concrete-filled steel tube column strength increase coefficient expression is:

$$\varphi_c = 1.033 + 9.233 \times \zeta_1$$

(9)

According to Equations (6) and (9), the bearing capacity of core concrete in the rectangular area can be uniformly solved as follows:

$$N_2 = \varphi_c N_c$$

(10)

3.5. The Bearing Capacity of Circular Concrete-filled Steel Tube

According to literature [23], the unified solution of bearing capacity of circular concrete-filled steel tube short column is as follows:

$$N_3 = \frac{\pi D^2 f_{sy}}{4} + \frac{\pi D t f_{sy}}{1 + 2b}$$

(11)

Where, $f_{sy}$ is the uniaxial compressive strength of the cylinder specimen. When the standard cube specimen is used in the test, $f_{sy} = 0.8 f_{cu}[24]$ is taken. The $b$ value is also uncertain because the shear yield limit strength $s$ and tensile yield limit strengths cannot be obtained at the time of the test.

3.6 The Axial Bearing Capacity Formula of the Round-ended Concrete-filled Steel Tube Short Columns with Binding Bars

The axial bearing capacity ($N$) of the round-ended concrete-filled steel tube short columns with binding bars is the sum of the bearing capacity of region "1" to region "3", that is:

$$N = N_1 + N_2 + N_3$$

(12)
4. Calculation of Bearing Capacity

The pressure measurement coefficient $k$ is related to the lateral pressure of concrete and the strength of concrete itself. When the concrete strength is certain, the value $k$ decreases with the increase of the lateral pressure; when the lateral pressure is certain, the value $K$ increases with the increase of the strength of concrete. The specific value can be measured by experiments, and generally $k = 1.5 \sim 7$. When $\tau_s/\sigma_s=0.5$, $b=0$, that is corresponding to the Tresca criterion, namely the lower yield criterion of metals. When $\tau_s/\sigma_s=0.577$, $b=0.364$, corresponding to the Mises criterion for the linear approximation. Where $\tau_s$ is shearing yield limit and $\sigma_s$ is tensile yield strength limit of the material. Unable to get the $\tau_s$ and $\sigma_s$ and $\phi$ by the real test, this paper let $b=0.364$, $k=1.5$, use Eq.12 to calculate the literature data[15], results are listed in Table 1.

| No. | $B$ (mm) | $D$ (mm) | $t$ (mm) | $n_b$ | $b_s$ (mm) | $d_s$ (mm) | $d_s$ (mm) | $f_{sk}$ (MPa) | $N_{ue}$ (kN) | $N_c$ (kN) | $N_c/N_{ue}$ |
|-----|----------|----------|----------|-------|------------|------------|------------|---------------|--------------|-------------|---------------|
| C1  | 400      | 200      | 4        |       | 0          | 40         | 40         | 3989          | 3738         | 0.94        |
| C2  | 400      | 200      | 4        | 2     | 75         | 8          | 75         | 40            | 4290         | 4797        | 1.12          |
| C3  | 400      | 200      | 4        | 2     | 100        | 8          | 75         | 40            | 4240         | 4536        | 1.07          |
| C4  | 400      | 200      | 4        | 2     | 150        | 8          | 75         | 40            | 4156         | 4275        | 1.03          |
| C5  | 400      | 200      | 4        | 1     | 100        | 14         | 140        | 40            | 4272         | 4937        | 1.16          |
| C6  | 400      | 200      | 4        | 2     | 100        | 10         | 75         | 40            | 4595         | 4972        | 1.08          |
| C7  | 400      | 200      | 4        | 3     | 100        | 8          | 50         | 40            | 4643         | 4925        | 1.06          |
| C8  | 600      | 200      | 4        |       | 0          | 4          | 5694       | 5200          | 5789         | 1.00        |
| C9  | 600      | 200      | 4        | 1     | 100        | 8          | 200        | 40            | 5802         | 5789        | 1.00          |
| C10 | 600      | 200      | 4        | 2     | 100        | 8          | 150        | 40            | 5889         | 5983        | 1.02          |
| C11 | 600      | 200      | 4        | 2     | 100        | 10         | 150        | 40            | 5982         | 6414        | 1.07          |
| C12 | 600      | 200      | 4        | 2     | 100        | 12         | 150        | 40            | 6139         | 6840        | 1.11          |
| C13 | 600      | 200      | 4        | 3     | 100        | 8          | 100        | 40            | 6058         | 6368        | 1.05          |
| C14 | 600      | 200      | 6        | 2     | 100        | 8          | 150        | 40            | 6998         | 6903        | 0.99          |
| C15 | 600      | 200      | 8        | 2     | 100        | 8          | 150        | 40            | 8528         | 7888        | 0.92          |
| C16 | 800      | 200      | 4        |       | 0          | 4          | 7355       | 6628          | 0.90         |            |
| C17 | 800      | 200      | 4        | 1     | 100        | 8          | 300        | 40            | 7578         | 7212        | 0.95          |
| C18 | 800      | 200      | 4        | 2     | 100        | 8          | 200        | 40            | 7712         | 7503        | 0.97          |
| C19 | 800      | 200      | 4        | 3     | 100        | 8          | 150        | 40            | 7659         | 7792        | 1.02          |
| C20 | 400      | 200      | 4        |       | 0          | 60         | 4944       | 4954          | 1.00         |            |
| C21 | 400      | 200      | 4        | 2     | 75         | 8          | 75         | 60            | 5200         | 6048        | 1.16          |
| C22 | 400      | 200      | 4        | 2     | 100        | 8          | 75         | 60            | 5182         | 5779        | 1.12          |
| C23 | 400      | 200      | 4        | 2     | 150        | 8          | 75         | 60            | 5082         | 5509        | 1.08          |
| C24 | 400      | 200      | 4        | 1     | 100        | 14         | 100        | 60            | 5174         | 6015        | 1.16          |
| C25 | 400      | 200      | 4        | 2     | 100        | 10         | 75         | 60            | 5506         | 6229        | 1.13          |
| C26 | 400      | 200      | 4        | 3     | 100        | 8          | 50         | 60            | 5288         | 6181        | 1.17          |
| C27 | 600      | 200      | 4        |       | 0          | 60         | 7190       | 7129          | 0.99         |            |
| C28 | 600      | 200      | 4        | 1     | 100        | 8          | 200        | 60            | 7304         | 7740        | 1.06          |
| C29 | 600      | 200      | 4        | 2     | 100        | 8          | 150        | 60            | 7414         | 7942        | 1.07          |
| C30 | 600      | 200      | 4        | 2     | 100        | 10         | 150        | 60            | 7482         | 8389        | 1.12          |
The comparison results show that the calculation results in this paper are in good agreement with the results in the literature. And the average ratio of the test results to the theoretical calculation results is 1.072, and the maximum error is not more than 16%, so the error is small. It shows that the section of the round-ended concrete-filled steel tube columns with binding bars is divided according to the method presented in this paper, and the lateral constraint of the core concrete is equivalent to the uniform lateral pressure. Based on the unified strength theory, the bearing capacity of each part is calculated separately, and the ultimate superposition of the bearing capacity of the column is feasible. Moreover, when binding bars aren’t set, the error between the bearing capacity of the test and the calculated value of the bearing capacity formula in this paper is still within 10%. That indicates that the calculation formula of bearing capacity in this paper is still applicable when there are no binding bars.

5. Conclusions
(1) Based on the unified strength theory, the section of the round-ended concrete-filled steel tube short columns with binding bars can be divided into three areas. The mechanical mechanism of each area is analyzed and the formula for calculating the axial bearing capacity of the round-ended concrete-filled steel tube short columns with binding bars is derived. The formulas obtained are of high precision and
simple expression, and have good engineering applicability.

(2) The influence of intermediate principal stress is considered in this paper. $k$ reflects the internal friction angle of core concrete. When $k$ is not the same, bearing capacity values of different accuracy can be obtain.

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