Boundary Layer Circumplanetary Accretion: How Fast Could an Unmagnetized Planet Spin Up through Its Disk?

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Abstract

Gas giant planets are expected to accrete most of their mass via a circumplanetary disk. If the planet is unmagnetized and initially slowly rotating, it will accrete gas via a radially narrow boundary layer and rapidly spin up. Radial broadening of the boundary layer as the planet spins up reduces the specific angular momentum of accreted gas, allowing the planet to find a terminal rotation rate short of the breakup rate. Here, we use axisymmetric viscous hydrodynamic simulations to quantify the terminal rotation rate of planets accreting from their circumplanetary disks. For an isothermal planet-disk system with a disk scale height $h/r = 0.1$ near the planetary surface, spin-up switches to spin-down at between 70% and 80% of the planet’s breakup rate. In a qualitative difference from vertically averaged models—where spin-down can coexist with mass accretion—we observe decretion accompanying solutions where angular momentum is being lost. The critical spin rate depends upon the disk thickness near the planet. For a disk scale height of $h/r = 0.15$, the critical spin rate drops to between 60% and 70% of the planet’s breakup rate. In the disk outside the boundary layer, we identify meridional circulation flows, which are unsteady and instantaneously asymmetric across the midplane. The simulated flows are strong enough to vertically redistribute solid material in early stage satellite formation. We discuss how exoplanetary rotation measurements, when combined with spectroscopic and variability studies of protoplanets with circumplanetary disks, could determine the role of magnetic and nonmagnetic processes in setting planet spins.

Unified Astronomy Thesaurus concepts: Hydrodynamical simulations (767); Planet formation (1241); Accretion (14)

1. Introduction

Planetary spin is one of the fingerprints of planet formation. In the absence of planetary winds or strong tidal effects, planetary angular momentum is conserved until the end of planet formation, providing direct evidence by which we can compare theoretical models of giant planet formation against observations. In the solar system, Jupiter has a rotation period of $2\pi/\Omega_j = 9.9$ hr, while that of Saturn (which is harder to measure) is estimated to be $2\pi/\Omega_S = 10.7$ hr. Compared to the nominal breakup angular velocity, which for a planet of mass $M_p$ and radius $r_p$ is $\Omega_p = \sqrt{GM_p/r_p^3}$, $\Omega_j/\Omega_b = 0.30$, and $\Omega_S/\Omega_b = 0.39$. Extrasolar planetary spin measurements are in their infancy but can be inferred, in principle, from transit constraints on oblateness (Seager & Hui 2002; Zhu et al. 2014; Biersteker & Schlichting 2017), from combined photometric and spectroscopic data during transit (Akinsanmi et al. 2020), and from high resolution spectroscopy (Bryan et al. 2018). For five objects with masses at the upper end of the planetary mass range, Bryan et al. (2018) inferred rotation rates of 5%–30% of breakup, similar to that of a sample of brown dwarfs.

Gas giant planets are predicted to accrete most of their mass via a circumplanetary disk (Miki 1982; D’Angelo et al. 2003; Papaloizou & Nelson 2005; Machida et al. 2010; Szulágyi et al. 2014). If the planet is unmagnetized and initially slowly rotating, it will accrete gas from its circumplanetary disk via a radially narrow boundary layer with specific angular momentum roughly equal to the Keplerian value at the planetary surface (Pringle 1977). Rapid spin-up will ensue. Qualitatively different evolution occurs if the planet is sufficiently magnetized, with accretion occurring via a magnetosphere. Magnetic coupling between the planet and its circumplanetary disk can then regulate the spin to values $\Omega \lesssim \Omega_b$. Disk braking has been studied in the context of Classical T Tauri stars (Koenigl 1991; Edwards et al. 1993; Armitage & Clarke 1996), and would operate efficiently for Jupiter-mass planets given an ordered surface field of $B \sim 500$ G (Batygin 2018; Ginzburg & Chiang 2020). The strength of the dipole component of Jupiter’s current magnetic field is only about 4 G (Smith et al. 1974), though scaling arguments suggest that proto-Jupiters would generate much stronger fields (Christensen et al. 2009). By analogy with the stellar case, strong magnetic fields could be (relatively) directly detected through spectroscopic signatures of magnetospheric accretion (Edwards et al. 1994), or by observation of protoplanetary jets (Gressel et al. 2013).

Although a strong case can be made for magnetic regulation of planet spin, the strength and ubiquity of protoplanetary magnetic fields remain unknown. With that in mind, we focus on a simple question: what would be the terminal rotation rate of gas giant planets in the unmagnetized limit? One firm limit is the onset of bar instability, which occurs when the ratio of rotational kinetic energy to gravitational energy, $T/W \gtrsim 0.27$ (Durisen et al. 1986). A second limit arises from the hydrodynamics of boundary layer accretion. Popham & Narayan (1991), using steady one-dimensional models of the disk and boundary layer, showed that the specific angular momentum of accreted gas equals that of the central object at a
critical rotation speed that is below the breakup velocity. This behavior occurs because the boundary layer (defined as the region of the flow, where \( \Omega(r) \) is an increasing function of radius), first becomes broader with increasing spin, before ceasing to exist at the critical rotation speed. At and above the critical rotation speed, \( \Omega(r) \) is a monotonically decreasing function, and angular momentum can be lost from the central object and transported through the disk to large distances. Figure 1 illustrates this transition.

In this paper, we calculate the terminal planetary spin rate for a model system in which an unmagnetized giant planet interacts viscously with a circumplanetary disk via a boundary layer. Following Hertfelder & Kley (2017), who simulated compact object boundary layers, we extend the work of Popham & Narayan (1991) in two ways. First, we simulate the planet-disk interaction in two dimensions (assuming axisymmetry), rather than adopting a one-dimensional vertically averaged treatment. Axisymmetry allows for a better representation of the problem geometry, and captures meridional flows that can develop in viscous disks (Urpin 1984; Philippov & Rafikov 2017). Second, we evolve a time-dependent simulation toward a (quasi)-steady state, rather than solving directly for the time-independent hydrodynamic flow. This changes the mathematical character of the problem, allowing us in particular to explore the possibility that the solution changes from accretion to decretion (Pringle 1991) as \( \Omega_c \) increases.

Multiple physical processes are important in boundary layers, and our model system, by construction, excludes some of them. In particular, the physical origin of angular momentum transport may differ between the boundary layer and the disk because the magnetorotational instability (Balbus & Hawley 1998) only operates where the angular velocity is a decreasing function of radius (Armitage 2002; Steinacker & Papaloizou 2002; Pessah & Chan 2012; Hertfelder & Kley 2015; Philippov et al. 2016; Belyaev & Quataert 2018). In the boundary layer, angular momentum transport may instead derive from the supersonic shear instability (Belyaev & Rafikov 2012; Belyaev et al. 2012), which has been extensively studied in Belyaev et al. (2013a, 2013b) and Hertfelder & Kley (2015). The supersonic shear between the planetary envelope rotation and the disk orbital rotation excites non-axisymmetric sonic modes that are trapped between the planetary envelope and a Lindblad resonance in the disk. The dissipation of the sonic modes develops weak shocks, and enables mass and angular momentum transport in the vicinity of the boundary layer. This mechanism is a global transport process that cannot be captured in axisymmetry, or reliably represented as a viscosity. Our approach, in which we adopt a constant kinematic viscosity whose value is indifferent to the sign of the angular velocity gradient, is thus at best a crude approximation to the actual physics. The value of the viscosity in our problem setup is determined given both physical considerations and numerical constraints (see Section 2 for more details). We also ignore thermal effects. Around a slowly rotating central object, half of the total accretion energy is released in the boundary layer region, yielding a large luminosity that can modify the structure of the boundary layer and inner disk (Kley & Lin 1996). Finally, our focus is on the consequences of boundary layer accretion for planetary spin evolution, and we do not study the impact of the accreted gas on the outer planetary envelope (Balsara et al. 2009).

The structure of the paper is as follows. In Section 2, we describe how the simulation parameters, including the disk aspect ratio, envelope sound speed, and kinematic viscosity, map to the physical parameters of circumplanetary systems. Section 3 details the problem setup within the Athena++ code (Stone et al. 2020). Our results are presented in Section 4. We discuss the implications of our findings, and avenues for further investigation, in Section 5.

2. Physical Parameters

Our simulation parameters, including the disk aspect ratio, envelope sound speed, and kinematic viscosity, are motivated by physical conditions of the circumplanetary system. In this section, we justify the use of these parameters.

The temperature of the inner circumplanetary disk and boundary layer are set by the mass accretion rate and the radial extent of the boundary layer. The mass accretion rate onto the planet through its boundary layer can be roughly estimated from the planet’s mass and the circumplanetary disk lifetime. Assuming that circumplanetary disks have a comparable lifetime to protoplanetary disks (i.e., a few Myr, Haisch et al. 2001), the mass accretion rate of the disk is \( 10^{-9} \text{ to } 10^{-8} M_\oplus \text{ yr}^{-1} \) for giant planets of a few Jupiter masses. Direct imaging of planets and their disks allows for limited observational constraints on the mass accretion rate. The H\( \alpha \) emission from the forming planet PDS 70b constrains that planet’s accretion rate to be approximately \( 10^{-8} M_\oplus \text{ yr}^{-1} \) (Wagner et al. 2018; Zhou et al. 2021). We assume an accretion rate of \( 10^{-8} M_\odot \text{ yr}^{-1} \) in our calculations. The protoplanetary radius can be estimated using theoretical models of early giant planet evolution. Fortney et al. (2011), for example,
find that Jupiter would have had a size of 1.6 \( R_{\text{Jup}} \) at an age of 1 Myr. With these values in hand, we can estimate the effective temperature of a steady-state circumplanetary disk \( T_{\text{eff}}(r) \) in the vicinity of the planet (e.g., Armitage 2007)

\[
T_{\text{eff}} = \frac{3GM_pM_{\text{BL}}}{8\pi \sigma r^3} \left(1 - \frac{r_p}{r}\right),
\]

where \( M_p \) is the planet mass, \( M_{\text{BL}} \) is the mass accretion rate onto the planet through the disk and boundary layer, \( \sigma \) is the Stefan–Boltzmann constant, \( r \) is the radial distance, and \( r_p \) is the planet size. For a one Jupiter-mass planet, with a radius of 1.6 \( R_{\text{Jup}} \), and an accretion rate of \( 10^{-8} M_{\odot} \text{ yr}^{-1} \), we obtain a disk temperature at \( r = 2 R_{\text{Jup}} \) of \( \sim 1600 \text{ K} \). The corresponding sound speed is

\[
c_s = \left(\frac{kT_{\text{eff}}}{\mu m_h}\right)^{1/2},
\]

where \( \mu \) is the mean molecular weight. Assuming \( \mu = 2.4 \), the ratio of the sound speed to the local Keplerian velocity is \( c_s / v_{\text{Kep}} \approx 0.1 \). For a circumplanetary disk in vertical hydrostatic equilibrium the disk aspect ratio is also \( h/r = c_s / v_{\text{Kep}} \approx 0.1 \).

Equation (1) is not valid in the boundary layer region, where the shear is non-Keplerian. In a general viscous (fluid) boundary layer/disk system, the dissipation rate is \( \propto (rd\Omega/dr)^2 \), and will be higher in the boundary layer than in the adjacent disk provided that the central object is slowly spinning and the boundary layer is narrow. It cannot be estimated accurately in advance of determining the boundary layer structure. Post facto, however, we have verified that the implied boundary layer dissipation (per unit surface area) in our solutions exceeds that of the disk for slowly spinning central objects, before becoming smaller as the critical spin rate is approached. We note that the actual boundary layer temperature depends additionally on radial and vertical radiative transfer effects—even in a simple fluid model—and requires more complete simulations to assess.

For reasons of computational simplicity, we assume that the outer envelope of the planet, the boundary layer, and the circumplanetary disk are all isothermal. The structure of the boundary layer is expected to depend most strongly on the aspect ratio of the adjacent disk, so we set the sound speed so that \( h/r \) would equal 0.1 if the disk extended to the planetary surface. The disk flares as \( h/r \propto r^{1/2} \) at larger radii (e.g., \( h/r = 0.1 \) at \( r = 1 \); \( h/r = 0.11 \) at \( r = 1.2 \); and \( h/r = 0.14 \) at \( r = 2 \)). The assumed sound speed implies a high but not grossly unreasonable temperature and pressure scale height in the outer envelope of the planet.

The physical origin of angular momentum transport in circumplanetary disks is not yet well understood. Several mechanisms are possible, including the magnetorotational instability (Balbus & Hawley 1998) and spiral density waves (Zhu et al. 2016). We model angular momentum transport in the fluid system using a fixed kinematic viscosity \( \nu \). The value of \( \nu \) is set based on physical considerations (primarily estimates derived from modeling of dwarf nova systems; King et al. 2007), together with numerical constraints from the need to reach a time-independent accretion state throughout at least the inner disk. Setting \( \nu = 10^{-3} \), which corresponds to a Shakura–Sunyaev \( \alpha \sim 0.1 \) in the inner disk, satisfies these requirements.

We note that angular momentum transport mechanisms likely differ between the circumplanetary disk and the boundary layer, as discussed in Section 1. The values of \( \alpha \), to the extent that boundary layer transport can be represented as a viscosity at all, are likely to be different in these two regions. The modeling of dwarf nova can constrain the value of \( \alpha \) in the bulk of the disk but not in the boundary layer. We adopt the same \( \alpha \) value for these two regions so that we can use a single model for both low planetary spin cases where there is a boundary layer, and high planetary spin cases where there is no longer a boundary layer.

3. Numerical Setup

We perform two-dimensional, axisymmetric viscous hydrodynamic simulations to study boundary layer circumplanetary accretion at different planetary spin rates using the Athena++ (Stone et al. 2020) grid-based code. The equations of hydrodynamics solved in the code for our problem are

\[
\frac{\partial \rho}{\partial t} + \mathbf{\nabla} \cdot (\rho \mathbf{v}) = 0,
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \mathbf{\nabla} \cdot (\rho \mathbf{vv} + P \mathbf{I} - \mathbf{T}) = -\rho \mathbf{\nabla} \phi_p,
\]

where \( \rho \) is the gas density, \( \mathbf{v} \) is the velocity vector, \( P \) is the gas pressure written as \( P = c_s^2 \rho / 2 \) for an isothermal equation of state, \( I \) is the identity tensor, \( \mathbf{T} \) is the viscous stress tensor, and \( \phi_p \) is a static gravitational potential from the planet. The mass of the planet exterior to the inner boundary is small, and so it suffices to treat the gravitational potential of the planet as a point mass at the origin. Since we restrict consideration to an isothermal and unmagnetized regime, the energy density and magnetic field equations are not evolved. We adopt a spherical-polar coordinate system \( (r, \theta, \phi) \) to achieve an axisymmetric setup in the \( \phi \) direction. The fiducial simulation domain spans \( r \in [0.9, 10] \) and \( \theta \in [0, \pi] \), with \( N_r \times N_\theta = 2048 \times 2048 \). We run one simulation in a larger domain \( r \in [0.9, 20] \), with identical spatial resolution, to test whether the flow features of interest are influenced by the outer boundary condition. In the \( r \) direction, the planet is initialized at radii between 0.9 and 1.1, while the disk is initialized at radii between 1.1 and 10. We adopt a logarithmic grid setup with a ratio of 1.001176 (i.e., \( r_{i+1} = 1.001176 \times r_i \)). The setup allows us to resolve the planetary envelope and the boundary layer at high resolution (about 10 grid points per pressure scale height \( h_{\text{planet}} \) of the planet, where \( h_{\text{planet}} = c_s^2 / g \) with an isothermal sound speed of 0.1), and resolve the outer disk region where the scale height is much larger (\( h_{\text{disk}} = c_s / \sqrt{\Omega} \)) at lower resolution to reduce computational cost. In the \( \theta \) direction we use a uniform grid spacing. We use the HLLE Riemann solver with second order reconstruction for hydrodynamics.\(^6\) The number of ghost cells is set to be 4. Default Athena++ configurations are used if not otherwise stated.

The planetary envelope is setup in a nonrotating hydrostatic equilibrium (satisfying \( c_s^2 \frac{dp}{dr} = -\rho GM_p / r^2 \)) with the following

\[\text{Meat the Riemann solver leads to numerical divergences using our simulation setup. Compared to the HLLE Riemann solver, the Roe solver is more accurate and less diffusive, but also less robust. The sharp density transition from the planetary envelope to the low density region near the pole may cause the numerical issue with the Roe solver.}\]
The polar-wedge boundary condition is similar to the reflecting boundary condition, but switches the sign of azimuthal velocity across the pole.

The density profile as the initial condition:

\[ \rho(r) = \rho_p \exp\left[ \beta (\rho_p / r - 1) \right], \]

where \( \beta = GM_p/c_s^2 \rho_p \) and \( r \in [0.9, 1.1] \). At \( r = r_p \), \( \rho = \rho_p \).

With an isothermal equation of state, the planet does not have a sharply defined surface, but the boundary layer–planet system evolves to a steady state in which it is reasonable to approximately identify \( r_p \) with the planetary “surface.” We set \( GM_p, r_p, \) and \( \rho_p \) to unity. An isothermal sound speed \( c_s = 0.1 \) is used, as justified in Section 2.

The circumplanetary disk is initialized in vertical hydrostatic equilibrium. The density profile of the disk in cylindrical coordinates with \( R = r \sin \theta \) and \( z = r \cos \theta \) is

\[ \rho(R, z) = \rho_{\text{disk}} \exp\left[ -z^2 / 2h_{\text{disk}}^2 \right], \]

where \( \rho_{\text{disk}} \) is the midplane density and \( h_{\text{disk}} \) is the vertical disk scale height. We set \( \rho_{\text{disk}} = 0.1 \) as a constant throughout the disk. The disk scale height, \( h_{\text{disk}} = c_s/\Omega = c_s r^{3/2} \), is an increasing function of the radial distance. We apply a density floor \( \rho_{\text{floor}} = 10^{-6} \) to avoid cells approaching zero density at large \( z \). The disk initially rotates at the Keplerian angular velocity and we have \( v_{\phi} = R^{-1/2} \).

As boundary conditions, we apply the polar-wedge boundary condition for the \( \theta \) boundaries and periodic boundary conditions for the \( \phi \) boundaries. For the inner \( r \) boundary, we extrapolate the hydrostatic equilibrium equation of the planet (Equation (5)) to the ghost cells and use a reflecting radial momentum boundary condition. For the outer \( r \) boundary, we use an outflow boundary condition and set the radial velocity component of ghost cells to zero if it points inward to avoid infall.

Generating a numerically stable model of a rapidly rotating planet, as an effective initial condition for our simulations, requires some care. After experimentation, we have found it best to run the simulations in two stages, of which the first is driven spin-up of initially nonrotating planet models. Figure 2 shows the scheme. In the left panel of Figure 2, we show the initial condition of the system with both the planet and the disk in hydrostatic equilibrium and the disk rotating at Keplerian angular velocity. The planet starts with zero angular velocity. During Stage 1 of our simulations, we gradually spin up the planet by adding a small amount of angular momentum to the planetary envelope at each time step until it reaches the desired spin rate. The kinematic viscosity is set to zero at this stage.

Since we are only interested in the interaction of the outer planetary envelope with the disk through the boundary layer, and spinning up the inner planetary envelope tends to numerically destabilize the inner radial boundary, we design an angular velocity profile that keeps the inner planetary envelope rotation at zero while the outer envelope rotates at the desired angular velocity. The angular velocity profile follows a logistic function, written as

\[ \Omega(r, \theta) = \frac{\Omega_p \sin \theta}{1 + \exp[-400(r - 0.93)]}, \]

where \( \Omega_p \) is the targeted angular velocity. We add an extra \( \sin \theta \) term to the angular velocity profile to avoid planetary rotation near the \( \theta \) boundaries that would otherwise lead to instability. Since the disk scale height is relatively small at the boundary layer (\( h_{\text{disk}} \sim 0.1 \) at \( r = 1 \)), the \( \sin \theta \) term does not significantly reduce the planetary rotation at latitudes where the planet and the disk interact. As the planet gradually spins up, its density structure adjusts such that the pressure gradient and rotation jointly balance gravity. Therefore, for higher rotation rates, shallower pressure gradients are obtained. We add a small amount of angular momentum at each time step such that over

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9 The polar-wedge boundary condition is similar to the reflecting boundary condition, but switches the sign of azimuthal velocity across the pole.

8 The logistic function follows a form of \( f(x) = a/(1 + \exp[-b(x - x_0)]) \), where \( a, b, \) and \( x_0 \) are constants.
the time interval of 100 orbits at $r = 1$, the angular velocity is increased by $\sim 0.1$. We only modify the planetary angular velocity at $r < 0.96$ to make sure that we do not influence the physical behavior of the boundary layer. To avoid angular velocity overshooting, we also include a damping term that reduces the angular velocity by 0.0015 per time step (i.e., corresponding to a damping timescale of 100 orbits at $r = 1$) if it is above the designed profile. We run the first stage for 800 orbits at $r = 1$ to make sure that the planetary envelope spins up to the expected angular velocity and the planet and circumplanetary system reach a steady state. Five sets of simulations are performed with five different planetary spin rates, $\Omega_0 \in [0, 0.3, 0.5, 0.7, 0.8]$. A snapshot of one of the circumplanetary systems after the initial 800-orbit integration is shown in the middle panel of Figure 2. The density structure in the outer disk region modifies notably from the initial condition. This modification is likely caused by the outer radial boundary condition. In the outer disk region, the $h/r$ is quite large (e.g., $h/r \approx 0.3$) and the disk structure could adjust in a few local dynamical timescales, leading to significant changes from the initial condition. However, we do not expect the changes on the outer disk structure to affect the inner disk mass and angular momentum transport.

For Stage 2 of our simulations, we use the output profiles from Stage 1 as the initial conditions. We now add viscosity to the system at radii $r > 0.96$. We set the kinematic viscosity $\nu = \alpha c_s^2 / \Omega = 10^{-3}$ as a constant. This results in a radially dependent value of the effective $\alpha$ parameter (Shakura & Sunyaev 1973). Near the boundary layer, the effective $\alpha$ is about 0.1. Since the interaction between the planetary envelope and the disk modifies the rotation rate of the planet, we maintain the forcing of the spin rate of the inner planetary envelope (i.e., $r < 0.96$) during this stage to keep it at the desired value. We find that for the high spin-rate model, we have to add angular momentum more aggressively (by a factor of 25 stronger) to maintain steady planetary envelope rotation. We simulate this second stage for another 800 orbits defined at $r = 1$. The final circumplanetary system density profile is shown in the right panel of Figure 2.

4. Results

We simulate and analyze the circumplanetary systems at different spin rates of the planetary envelope. In Section 4.1, we calculate the time-averaged radial mass and angular momentum fluxes for different model and demonstrate the existence of a critical spin rate above which mass and angular momentum accretion are prohibited. In Section 4.2, we present the hydrodynamic flow patterns of the systems. In Section 4.3, we discuss the time-dependent accretion. In Section 4.4, we present results for a higher sound speed and demonstrate the dependence of the critical spin rate on this parameter. In Section 4.5, we discuss how the results would change if we varied the value of the effective $\alpha$.

4.1. Mass and Angular Momentum Flux

We study the properties of the circumplanetary systems at five different spin rates of the central planet ($\Omega_p \in [0, 0.3, 0.5, 0.7, 0.8]$ at $r = 0.96$). In Figure 3, we present the density profiles integrated over the $\theta$ direction (upper panel), and the midplane angular velocity profiles (lower panel), at the end of the Stage 2 simulations when the inner disk has reached a steady state. The initial conditions of the surface density and angular velocity are shown in gray dashed lines, and are the same for all simulations. As shown in Figure 3, the density profiles adjust from the initial setup to balance the planet and disk rotation. The structural adjustment of the outer planetary envelope is most prominent for the high spin-rate cases, $\Omega_p = 0.7$ or 0.8. The planetary rotation in these cases is strong enough to provide significant support against gravity, and the

![Figure 3](image-url)
pressure and thus density gradients become correspondingly shallower. For the low-spin cases (Ω_p = 0, 0.3, or 0.5), the final angular velocity profiles are altered by at most a modest amount. In the outer disk region, the gas rotates at nearly Keplerian velocity until it approaches the planetary envelope. By continuity, we expect a turning point where dΩ/dr = 0 as the near-Keplerian disk matches on to the more slowly rotating planetary envelope, and this turning point is usually defined as the outer radial range of the boundary layer. Throughout the boundary layer, the gas gradually brakes until it matches the rotation rate of the planetary envelope, which is usually defined as the inner radial range of the boundary layer. The gas is then accreted onto the planet. We note that at Ω_p = 0, 0.3, or 0.5, the disk rotates at slightly super-Keplerian velocity from r ~ 1.1–1.4 because that region has a positive density gradient. For the high-spin cases (Ω_p = 0.7 or 0.8), the simulated angular velocity profiles are very different from the classical picture. It is no longer obvious how to define the boundary layer. In the Ω_p = 0.7 case, we find a flat angular velocity profile and in the Ω_p = 0.8 case, we find a monotonically decreasing profile.

To understand how the change of the angular velocity profiles affects the accretion, we calculate the time-averaged radial mass and angular momentum flux. The radial mass flux M can be expressed as

$$M = \iiint \rho v_r r^2 \sin \theta d\phi d\theta,$$

where ρ and v_r are the density and radial velocity, respectively. In our two-dimensional, axisymmetric setup, we compute the radial mass flux at cell (r_i, θ_j) as

$$M(r_i, \theta_j) = 2\pi \rho v_r(r_i, \theta_j) r_i^2 (\cos \theta_{j-1/2} - \cos \theta_{j+1/2}).$$

A negative M indicates gas accretion onto the planet from the disk, whereas a positive M indicates gas decretion from the planet to the disk. To calculate the radial angular momentum transport, we write down the angular momentum equation (Equation 4 in Kley et al. 1993),

$$\frac{\partial (\rho v_\phi \sin \theta)}{\partial t} + \nabla \cdot (\rho v_\phi \sin \theta v_r - r \sin \theta \rho v_r) = 0,$$ (10)

where t_φ is the φ component of the viscous stress tensor, written as t_φ = (t_φr, t_φθ, and t_φφ). Specifically, the tensor component in the r direction is t_φr = ρv_r sin θ (∂Ω/∂r), where Ω is the angular velocity. We expect zero net angular momentum flux in the θ direction because our simulation grid extends from θ = 0 to θ = π. Plugging in the tensor component t_φr, we may rewrite Equation (10) as

$$\frac{\partial (\rho v_\phi \sin \theta)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho r^3 v_\phi \sin \theta - \rho v_r^2 \sin^2 \theta \frac{\partial \Omega}{\partial r} \right) = 0,$$ (11)

where the first term on the left-hand side is the rate of angular momentum change per unit volume, the first term in the parentheses describes the radial angular momentum change due to the advection, and the second term in the parentheses describes the viscous torque. In a steady state, the parenthesized terms should be a constant as a function of the radial distance. To calculate the radial angular momentum flux J, we integrate the parenthesized terms by ∫ sin θ dφ dθ,

$$J = \iint \rho r^3 v_\phi \sin \theta d\phi d\theta + \iint -\rho v_r^2 \sin^2 \theta \frac{\partial \Omega}{\partial r} d\phi d\theta.$$ (12)
To apply the equation to our simulation grid, we approximate \( J \) at cell \((r_i, \theta_j)\) as

\[
J(r_i, \theta_j) = 2\pi \rho r_i^3 v_r v_\phi \left[ \frac{1}{2} (\theta_{j+1/2} - \theta_{j-1/2}) \right. \\
- \frac{1}{4} (\sin 2\theta_{j+1/2} - \sin 2\theta_{j-1/2}) \\
- 2\pi \rho r_i^4 \frac{\Omega_{i+1,j} - \Omega_{i,j}}{r_{i+1} - r_i} \left( \cos \theta_{j-1/2} - \cos \theta_{j+1/2} \right) \\
- \frac{1}{3} (\cos^3 \theta_{j-1/2} - \cos^3 \theta_{j+1/2}) \right],
\]  
(13)

where \( \rho, v_r, \) and \( v_\phi \) are all functions of \((r_i, \theta_j)\).

In Figure 4, we present the radial mass flux (upper panel) and radial angular momentum flux (lower panel) averaged over 100 orbits at \( r = 1 \) at the end of Stage 2 simulations. The inner circumplanetary systems have reached a steady state, illustrated by the constant fluxes beyond \( r = 1.1 \). The flat profiles extend up to \( r \approx 2.5 \). For \( \Omega_p = 0, 0.3, 0.5, \) or even 0.7, the mass and angular momentum fluxes are negative and the central planet is accreting mass and angular momentum to grow and spin up through its disk. We find the fluxes have similar values for these spin rates, implying that there is no strong dependence on planetary spin rates as long as accretion occurs. The system at these spin rates is controlled by the supply of mass and angular momentum from the circumplanetary disk, and the precise nature of the boundary condition the planet presents at the inner edge of the boundary layer is unimportant. At \( \Omega_p = 0.8 \), conversely, both the mass and the angular momentum flux become positive. At this spin rate the planet is decreasing mass and losing angular momentum, leading to long-term mass loss and spin-down. From these results, we infer that for this sound speed there exists a critical spin rate in the region between \( \Omega_p = 0.7 \) and 0.8. Beyond the critical spin rate, the planet would no longer grow and spin up through its disk, but will instead lose mass and spin down. Comparing with the angular velocity profiles in Figure 3, the critical spin rate corresponds to where the flat angular velocity profile switches to a monotonically decreasing angular velocity profile.

4.2. Gas Flow Streamlines

An advantage of our two-dimensional simulations is that we are able to capture gas flow patterns at different disk latitudes. In Figure 5, we show the time-averaged gas streamlines at representative spin rates. The three spin rates we choose represent an accreting planet with a boundary layer (the \( \Omega_p = 0.5 \) case), an accreting planet with a flat angular velocity profile (the \( \Omega_p = 0.7 \) case), and a decreasing planet with a monotonically decreasing angular velocity profile (the \( \Omega_p = 0.8 \) case). The flow patterns are similar for all the low-spin cases, so we only present the \( \Omega_p = 0.5 \) case for illustration. At all spin rates, we observe a meridional circulation pattern, which can be described as a radial outflow at and near the midplane with a radial inflow at high latitudes. Such a pattern was predicted for an isothermal, viscous accretion disk in previous work (e.g., Urpin 1984; Philippov & Rafikov 2017). Although the gas is transported outward at the midplane, the boundary layer and the inner disk can still have a net mass inflow if more mass can be transported inward from high latitudes. At \( \Omega_p = 0.5 \) and 0.7, we find a radial inflow at the midplane in the inner disk region extending from \( r = 1-2 \); whereas at \( \Omega_p = 0.8 \), we find a radial outflow at the midplane even for the inner disk region. The radial flow direction we observed is consistent with the prediction of Philippov & Rafikov (2017). For the \( \Omega_p = 0.7 \) case, the circulation patterns are slightly asymmetric about the midplane. Since the outer disk has not yet reached a steady state during our simulation timescale, the asymmetry pattern is not too surprising.

Our high resolution simulations allow us to resolve flow patterns near the boundary layer. In Figure 6, we present the detailed flow patterns in the outer planetary envelope and the boundary layer. At \( \Omega_p = 0.5 \), gas from all disk latitudes flows inward and accretes onto the planetary envelope first at the
The midplane radial velocity $|v_r|$ increases sharply to a maximum of $\sim 40\%$ of the sound speed as the gas approaches to the boundary layer region and then decreases sharply in the boundary layer to accrete onto the planetary surface. The radial inflow at the midplane changes direction near $r = 2$ to radial outflow. At $\Omega_p = 0.7$, the disk flow pattern is similar to the $\Omega_p = 0.5$ case, but gas can only accrete onto part of the planetary envelope due to the fast rotation of the planet. Finally, at $\Omega_p = 0.8$, gas flows outward from the planetary envelope to the disk. The meridional circulation is a large-scale flow structure that extends throughout the simulated circumplanetary disk. To confirm that the circulation pattern in the inner disk, near the boundary layer, is not an artificial feature introduced by the outer boundary conditions, we ran an extended simulation of the $\Omega_p = 0.8$ case with an extended radial range out to $r = 20$. We keep the spatial resolution and other parameters the same. As shown in Figure 7, the meridional circulation pattern is again observed in the large disk simulation (Cs01A08L) and is located at a similar location and latitude to the regular disk simulation (Cs01A08). This indicates that the observed meridional flow features in the inner disk are not introduced by the boundary condition.

Lastly, we note that an isotropic effective viscosity is assumed in this work. Physical disk angular momentum transport mechanisms, including the magnetorotational instability and the vertical shear instability, could introduce an anisotropic effective stress and modify the flow patterns (e.g., Jacquet 2013; Stoll et al. 2017). The thermal properties of the disk could also influence the meridional circulation (Philippov & Rafikov 2017). The study of the robustness of the flow patterns that we have found, and how those patterns are affected by the anisotropic nature of the effective viscosity and the thermal properties of the circumplanetary disk, are deferred to future work.

4.3. Time-dependent Variations

We presented the time-averaged radial mass and angular momentum fluxes, and the averaged gas flow streamlines, in the previous subsections. These properties also display time-dependent features introduced by the variability of the meridional flows. To demonstrate the variability, we plot the radial mass fluxes as a function of time at different radial distances in Figure 8. The data is taken from Stage 2 of our simulation (i.e., after adding in the kinematic viscosity) for the $c_s = 0.1$, $\Omega_p = 0.3$ case. We present the $\Omega_p = 0.3$ case because the variation of the meridional circulation pattern is most obvious for the $\Omega_p = 0$ and 0.3 cases. The radial mass flux $M$ is calculated every $t_0$, which is defined as one orbit at $r = 1$. As shown in Figure 8, the radial mass flux varies periodically, both in the inner disk (e.g., at $r = 3$) and the outer disk (e.g., at $r \geq 5$). The amplitude of the radial flow variation is greater in the outer disk, indicating a stronger meridional flow variation. Figure 8 also illustrates the response of the circumplanetary system to the addition of disk viscosity. The inner disk region ($r < 2.5$) has reached a flat $M$ profile after $\sim 600$ orbits and has a constant $M$ over that radial range (see, e.g., the upper panel of Figure 4 and the overlapping of the orange and yellow curves). The outer disk has not yet reached a steady state during our simulation timescale (800 orbits at $r = 1$) and its instantaneous and time-averaged $M$ deviate from the inner disk value.

The variation of the meridional flows can also be visualized via streamline plots. In Figure 9, we compare a snapshot of the gas flow streamlines (the left panel) to the time-averaged gas flow streamlines (the right panel), again for the $c_s = 0.1$, $\Omega_p = 0.3$ case. The time-averaged flow pattern is similar to the one of the $c_s = 0.1$, $\Omega_p = 0.5$ case (shown in the left panel of Figure 5). As shown in the left panel of Figure 9, several regional and small-scale meridional circulation patterns are found. These circulation patterns are unstable and appear at different locations at different times. However, when we integrate the system for a long timescale, the turbulent features average out and we identify the steady streamline pattern shown in the right panel of Figure 9.

To better characterize the flow perturbations, we calculate the flow patterns after subtracting the mean velocity field at different times, as shown in Figures 10 and 11. The variation in the flow is periodic, and repeats approximately every 31 orbits (at $r = 1$) for the $\Omega = 0.3$ case and approximately every 27 orbits for the $\Omega = 0.5$ case. In Figure 12, we show the vertical velocity perturbations $\Delta v$, at the midplane over one oscillation cycle (i.e., 31 orbits) for the $\Omega = 0.3$ case. A wave is
with one vertical node; Figure 11) oscillations. Since \( k_r \) varies at different \( r \), it is difficult to precisely determine its value. However, in general, we find a close match between the frequencies we see in our simulation and the analytic solution of Lubow & Pringle (1993). The differences can probably be attributed to the global nature of our simulations.

The meridional flow induces vertical velocities that are 1%–2% of the sound speed. The time-dependent nature of these flows, and the fact that there is a nonzero instantaneous velocity at \( z = 0 \), means that they would oppose dust settling if present in real circumplanetary disks. We consider solid particles with material density \( \rho_m \) and radius \( s \), interacting aerodynamically with gas of density \( \rho \) and thermal speed \( v_{th} \). The settling velocity in the Epstein drag regime (appropriate to most circumplanetary disk conditions) is (Armitage 2010)

\[
\nu_{sett} = \frac{\rho_m s}{\rho v_{th}} \Omega_s^2 z. \tag{15}
\]

The condition for time-dependent meridional flows with characteristic velocity \( \epsilon c_s \) to fully mix particles can be estimated by requiring that

\[
\nu_{sett} < \epsilon c_s \tag{16}
\]

at \( z = h \). Dropping numerical factors that are of the order of unity, this condition can be written as

\[
\frac{\rho_m s}{\Sigma} \lesssim \epsilon, \tag{17}
\]

where \( \Sigma \) is the circumplanetary disk gas surface density. For millimeter-sized icy particles (\( \rho_m \approx 1 \text{ g cm}^{-3} \)), and \( \epsilon \approx 0.01 \), we then estimate that meridional flows would be strong enough to oppose settling in disks with \( \Sigma \lesssim 10 \text{ g cm}^{-2} \). This is a relatively relaxed condition. It implies that settling would not occur for most observationally interesting particle sizes (\( s \sim a \) millimeter, or smaller) provided that the circumplanetary disk is at least moderately optically thick (assuming standard opacities and gas-to-dust ratios).

### 4.4. The Dependence on Isothermal Sound Speed

We now examine the dependence of the critical spin rate on the isothermal sound speed. In previous simulations, we adopted an isothermal sound speed of 0.1 and found that the critical spin rate was between 0.7 and 0.8 of the breakup angular velocity. Here, we set an isothermal sound speed of 0.15 and evaluate the resulting critical spin rate. Given the experience from previous simulations, we run simulations at two spin rates, \( \Omega_p = 0.6 \) and 0.7. In Figure 13, we show their density and angular velocity profiles. In Figure 14, we show their radial mass and angular momentum fluxes. At \( \Omega_p = 0.6 \), the system has a flat angular velocity profile and both the radial mass flux and the angular momentum fluxes are negative, whereas at \( \Omega_p = 0.7 \), the system has a monotonically decreasing angular velocity profile and both fluxes are positive. Our simulations thus indicate a critical spin rate between 0.6 and 0.7 of the planet’s breakup angular velocity for \( c_s = 0.15 \).

We find a dependence of critical spin rate on the isothermal sound speed of the circumplanetary system. The critical spin rate decreases as the isothermal sound speed increases. Since the width of boundary layer roughly scales as \( c_s^2 \) (Lynden-Bell & Pringle 1974; Pringle 1977), the higher sound speed leads to a wider boundary layer. The critical spin rate corresponds to the

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**Figure 7.** Streamline plot of the large disk simulation (Cs01A08L, in orange) overplotted with the regular disk simulation (Cs01A08, in black). The large disk simulation reproduces the meridional circulation flow at a similar radial distance as the one from the regular disk simulation, indicating that the meridional circulation patterns found in the regular disk simulation are not introduced by the outer boundary condition in the \( r \) direction.
condition where the boundary layer no longer exists, i.e., an infinitely wide boundary layer. The critical condition is easier to achieve at a higher sound speed, and therefore we find a lower critical spin rate for a higher isothermal sound speed system.

4.5. The Dependence on Effective $\alpha$

We adopt a fixed kinematic viscosity $\nu = 10^{-3}$ (i.e., $\alpha \sim 0.1$) for all production simulations given the physical considerations, and numerical constraints related to the need to reach a time-independent accretion state throughout at least the inner disk. To check the importance of this choice, we have also run a single experimental simulation with $\nu = 10^{-4}$. As expected based on general considerations, and previous studies (e.g., Popham & Narayan 1995), lower values of the viscosity reduce the values of $v_r$ approximately linearly. We find, however, that the low viscosity run has an angular velocity profile similar to that of the $\nu = 10^{-3}$ case. Since it is the angular velocity profile that determines the critical spin rate, we think that at leading order our results for the terminal rotation rate still stand for different (plausible) values of $\alpha$.
5. Discussion and Conclusion

In this work we have made an estimate of the terminal rotation rate of unmagnetized planets, which accrete gas from their circumplanetary disks via an equatorial boundary layer. As prior work on boundary layers (Popham & Narayan 1991) makes clear, the limiting spin in this mode of accretion—although rapid—falls short of breakup. Rather, an equilibrium state can be reached in which the accreted specific angular momentum equals that of the central object, and spin-up ceases. In our model system, where the planet, boundary layer, and disk are taken to be a viscous isothermal fluid, with axisymmetry assumed, we find that the equilibrium spin rate is between 70% and 80% of breakup if the disk aspect ratio near the planet is $h/r = 0.1$ (Figures 3 and 4). The equilibrium spin drops to between 60% and 70% for a thicker disk, with $h/r = 0.15$ (Figure 13 and 14; see Table 1 for a summary). The boundary layer and circumplanetary disk are found to be variable, on timescales of the order of 10 inner orbital periods, due to the development of meridional flows within the circumplanetary disk (e.g., Figure 5, 6, and 8; Urpin 1984; Kley & Lin 1992; Fromang et al. 2011; Philippov & Rafikov 2017). The variability arises because of time dependence introduced by the meridional flow itself, which over time loses reflection symmetry across the disk equator (Figure 9). Popham & Narayan (1991), in the context of compact object accretion, demonstrated that boundary layer accretion leads to sub-breakup central object spin.

Figure 10. Flow perturbations at different phases after subtracting the mean velocity field for the $\Omega = 0.3$ case. The flow variation is periodic and repeats approximately every 31 orbits, where one orbit is defined as one orbital period at $r = 1$. We observe the zero order ($n = 0$) vertical mode.
We confirm their key finding, but note that our results differ from theirs at a qualitative level. Specifically, whereas Popham & Narayan (1991) obtained solutions in which the central object accreted mass while spinning down, our spin-down solutions are accompanied by decretion. This difference is likely to be a consequence of contrasting formulations of the problem. Popham & Narayan (1991) solved directly for the steady-state solution of one-dimensional, vertically averaged, disk equations. This approach leads to a boundary value problem, in which the specific angular momentum of the flow $j$ is recovered for a specified $M > 0$ as an eigenvalue. By dropping the vertical averaging and moving to axisymmetry, we instead solve for the quasi-steady state of an initial value problem, applying boundary conditions deep in the planetary interior, and far out in the disk, that are intended to be minimally coercive to the boundary layer properties. With this approach, we find that the transition between spin-up and spin-down occurs at the same (or very similar) spin rate as that between accretion and decretion. Our results agree with those of Hertfelder & Kley (2017), who also found decretion at high spin rates in radiation hydrodynamics simulations of compact object boundary layers.

Our results suggest that, if some subset of young giant planets lack strong dipolar magnetic fields, they would spin up during the accretion phase to between 60% and 80% of their breakup speed. Subsequent spin-up, occurring at constant angular momentum as the planets contract, is strongly mass

![Figure 11](image)

Figure 11. Flow perturbations at different phases after subtracting the mean velocity field for the $\Omega = 0.5$ case. The flow variation is periodic and repeats approximately every 27 orbits, where one orbit is defined as one orbital period at $r = 1$. We observe the first order ($n = 1$) vertical mode.
dependent (Fortney et al. 2011). Planets with masses between those of Saturn and Jupiter contract substantially, but on a timescale that is shorter at higher masses (accordingly, any early braking process works better for Jupiter than for Saturn). Ice giant contraction is much less significant. At late times, exoplanets that were experienced boundary layer accretion during their growth phase would then be expected to spin with a minimum of 60% of their breakup speed. Such planets might be oblate enough to identify from transit data (Seager & Hui 2002), or from future spectroscopic observations. Boundary layer and magnetospheric accretion (Batygin 2018) also differ in their predictions for protoplanetary properties. Magnetospheric accretion results in accretion shocks and (typically) localized hot spots on the planetary surface, leading
decreted from the planetary envelope at which we plot the mass and angular momentum are accreted onto the planetary envelope at 100 orbits at different planetary spin rates, \( \Omega_p \in [0.6, 0.7] \) at \( r = 0.96 \), for the \( c_s = 0.15 \) setup. The gray filled region, \( r \in [0.9, 0.96) \), is the nonphysical region in which we force the planetary envelope to keep it rotating at the desired value. Mass and angular momentum are accreted onto the planetary envelope at \( \Omega_p = 0.6 \), but decreed from the planetary envelope at \( \Omega_p = 0.7 \). This indicates a critical spin rate between \( \Omega_p = 0.6 \) and 0.7 for an isothermal sound speed of 0.15. For comparison, we plot the \( \Omega_p = 0.7 \) curve in Figure 4 for the \( c_s = 0.1 \) setup, which has a critical spin rate between 0.7 and 0.8.

to photometric modulation on the planetary spin period and strong emission in lines such as H\( \alpha \). A boundary layer is not expected to be a strong source of line emission, and will only produce a thermal component that is distinguishable from that of the disk if the planet is slowly spinning. Non-axisymmetric boundary layer instabilities—which are not captured in our simulations—may lead to variability with a timescale comparable to that of the Keplerian orbital period at the surface of the central object (Belyaev et al. 2012). Variability on these timescales that may be associated to the boundary layer is observed in dwarf nova systems (Warner 2004). We note that if cirumplanetary disk accretion is strongly episodic, as suggested by Brittain et al. (2020), then boundary layer accretion during high accretion rate phases could coexist with magnetospheric accretion during quiescence.

Future work will need to address two obvious limitations of the present study. First, by adopting an isothermal equation of state, we do not capture the often-substantial release of energy in the boundary layer. The dynamical effects of that energy release—in the form of a thickening and radial broadening of the boundary layer region—can be modeled using viscous radiation hydrodynamics simulations, as was already done for protostellar systems by Kley & Lin (1996), and more recently, for compact objects by Hertzfelder & Kley (2017). We cannot exclude the possibility that such thermal effects could broaden protoplanetary boundary layers substantially, thereby reducing the predicted terminal spin rate. Second, our assumption of axisymmetry means that we cannot attempt to represent physical mechanisms of angular momentum transport within either the disk or the boundary layer. Three-dimensional simulations (e.g., Philippov et al. 2016; Belyaev & Quataert 2018) that model the transport processes properly are needed to predict the detailed structure of protoplanetary boundary layers, along with potentially observable properties such as their intrinsic variability.

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Figure 14. Time-averaged radial mass fluxes (upper panel) and radial angular momentum fluxes (lower panel) taken from the end of Stage 2 simulations averaged over 100 orbits at different planetary spin rates, \( \Omega_p \in [0.6, 0.7] \) at \( r = 0.96 \), for the \( c_s = 0.15 \) setup. The gray filled region, \( r \in [0.9, 0.96) \), is the nonphysical region in which we force the planetary envelope to keep it rotating at the desired value. Mass and angular momentum are accreted onto the planetary envelope at \( \Omega_p = 0.6 \), but decreed from the planetary envelope at \( \Omega_p = 0.7 \). This indicates a critical spin rate between \( \Omega_p = 0.6 \) and 0.7 for an isothermal sound speed of 0.15. For comparison, we plot the \( \Omega_p = 0.7 \) curve in Figure 4 for the \( c_s = 0.1 \) setup, which has a critical spin rate between 0.7 and 0.8.
