Bidirectional quantitative force gradient microscopy

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Abstract
Dynamic operation modes of scanning force microscopy based on probe resonance frequency detection are very successful methods to study force-related properties of surfaces with high spatial resolution. There are well-recognized approaches to measure vertical force components as well as setups sensitive to lateral force components. Here, we report on a concept of bidirectional force gradient microscopy that enables a direct, fast, and quantitative real space mapping of force component derivatives in both the perpendicular and a lateral direction. It relies solely on multiple-mode flexural lever oscillations related to vertical probe excitation and vertical deflection sensing. Exploring this concept we present a cantilever-based sensor setup and corresponding quantitative measurements employing magnetostatic interactions with emphasis on the calculation of mode-dependent spring constants that are the foundation of quantitative force gradient studies.

1. Introduction

Dynamic scanning force microscopy (dSFM) is a versatile sub-nanometer resolution tool for examining the surface topography and many more force-related properties of a huge variety of materials [1, 2]. A basic dSFM setup requires a dSFM probe, which is typically a microstructured cantilever beam with a sharp tip. Also, it requires the means to excite and detect flexural oscillations of the cantilever, as probe–sample interactions result in changes of the resonance behavior of the probe. Appropriate measurement variables include the resonance frequency, the oscillation amplitude, and the phase shift of the cantilever oscillation. Scanning the probe above a sample’s surface allows for recording spatially resolved maps of the measurement variables. In many cases, dSFM can be sufficiently described by considering a simple harmonic motion of the dSFM probe at a single mechanical frequency. In more advanced approaches, however, the outcome of dSFM measurements can be significantly improved by either detecting additional frequency components of the dSFM signal or exciting the dSFM probe at more than one frequency. In this regard our paper presents an approach that enables either simultaneous or subsequent measurements of two orthogonal force gradients at two different resonant frequencies, while using only a one-dimensional excitation and detection scheme. Before describing this novel variant of dSFM, the advantages of multi-frequency versus mono-frequency measurements will be briefly discussed in the following.

1.1. The basic principles of dSFM

In mechanics, a free harmonic oscillator vibrates at its eigenfrequency, which is defined by its spring constant and its effective mass. The more complex oscillation of a damped harmonic oscillator driven by a time-dependent sinusoidal force is described by a superposition of two frequencies: the frequency of the driving force and the eigenfrequency of the free oscillator during the transient response. In the steady state, the amplitude of the latter is suppressed by damping, but nevertheless the oscillator’s movement is still heavily influenced by the eigenfrequency. The eigenfrequency affects both the amplitude and the oscillation’s phase with respect to the
periodic driving force. A single frequency model in the steady state can be applied to dSFM if a single frequency excitation is used and if anharmonic contributions to the probe–sample interaction potential are neglected.

In order to understand the basics of signal formation in dSFM the potential energy of both the cantilever deformation and the probe–sample interaction can be considered. The potential energy of a free cantilever oscillation corresponds to the deformation energy of the cantilever beam for the flexural oscillation mode in consideration. For a sufficiently small oscillation amplitude it can be approximated quite well by a harmonic potential \( U = cu^2/2 \). Here \( c \) is the effective spring constant, and \( u \) the corresponding deflection at a given point of reference. This potential when oscillating at the fundamental flexural mode deviates slightly from that of a static cantilever deflection induced by a point-like force acting on the cantilever’s free end. Therefore, the effective force constants for the static deformation and for the fundamental flexural deformation differ by a small amount. Mathematically, a description of this difference is given by the equivalence of the inverse static spring constant and the sum of all inverse dynamic spring constants [3]. The conservative part of the dSFM probe–sample interaction can be described by a local probe–sample potential. In case of small oscillation amplitudes the change of the probe’s eigenfrequency is proportional to the second derivative of the probe–sample potential with respect to the spatial coordinate pointing along the tip oscillation direction, i.e. the probe-sample force gradient. In general, at large probe oscillation amplitudes the probe-sample force gradient might not be constant along the tip trajectory. Here, a weighted average of the gradient can be used [4]. The dependence of the eigenfrequency on the probe–sample interaction is employed in frequency modulation dSFM [5].

1.2. Multi-frequency dSFM

To date, many force microscopy approaches have been developed that include at their heart the excitation or measurement of more than one frequency. Even if the cantilever is excited at its fundamental flexural eigenfrequency, higher harmonics contribute to the periodic motion if the probe–sample force includes nonlinear components. The amplitudes of these higher harmonics contain valuable information on the probe–sample interaction. These amplitudes can be used to reconstruct the probe–sample force field [6]. They are related to higher order force gradients that decay very fast. Therefore, the detection of the amplitudes of higher harmonics can be used to enhance the spatial resolution of dSFM [7]. Depending on the cantilever geometry specific higher harmonics of the cantilever’s fundamental flexural mode may coincide with a higher order flexural mode [8] leading to a resonant amplification of those higher harmonics. External excitation of two flexural eigenmodes of the cantilever, e.g. the fundamental mode and a higher order mode, allows for a simultaneous tracing of both the topography using the interaction response on one mode and compositional information contained in the higher order mode response [9, 10]. Transferred to magnetic force microscopy (MFM), a descendant of dSFM, such an approach based on the excitation of two flexural eigenmodes enables a parallel measurement of the sample topography and the long range magnetostatic interactions of the ferromagnetic tip with the sample stray field [11–13]. By this means some disadvantages of the widely used two-pass MFM technique can be avoided. A general overview of multi-frequency dSFM has been given by Garcia und Herruzo [14].

1.3. Bidirectional dSFM

Single-directional force or force gradient measurements that only probe a lateral rather than the usual vertical direction are well-established. Rotation of the force sensor by 90° leads to the pendulum or shear force configuration. Giessibl et al used this geometry to perform lateral force gradient and friction measurements at the atomic scale [15]. MFM based on the pendulum geometry is sensitive to lateral force derivatives of magnetostatic interactions [16, 17]. This setup, however, cannot easily be switched to perpendicular sensitivity. It is limited to in-plane measurements. In contrast, torsional resonance mode dSFM [18–20] that is sensitive to in-plane forces can be combined with perpendicular modes based on flexural oscillations.

Recently, we published a concept and a corresponding experimental implementation of a cantilever-based sensor in which the excitation of two different eigenmodes allowed for a successive or simultaneous resonant oscillation of the sensor’s tip along two orthogonal directions [21]. A high aspect ratio single-domain iron nanowire was employed to interact with the magnetic stray field to be measured. For practical reasons we used Fe nanowires contained in and stabilized by multiwalled carbon nanotubes (FeCNT). A selected FeCNT was attached to a cantilever beam via a spacer element. Exciting the cantilever’s fundamental flexural oscillation led to the obvious vertical oscillation of the free end of the probe’s magnetic nanowire. In contrast the free end oscillated along a horizontal direction, if the cantilever’s second flexural mode was excited. Such a bidirectional sensor, equipped with a magnetic monopole-like tip, is sensitive to magnetic field derivatives along these directions. This sensitivity is based on the measurement of two force derivatives, e.g. \( \partial F_x/\partial x \) and \( \partial F_z/\partial x \), if \( z \) denotes the vertical and \( x \) denotes the horizontal direction. Since then, Stirling has suggested a similar concept of a multipurpose vertical and lateral force microscopy sensor: here, a wire representing both the spacer and the
probe tip was proposed to be attached to the center of a two-side clamped quartz beam [22]. Again the fundamental and second mode flexural beam oscillations give rise to vertical and horizontal tip oscillations, respectively.

1.4. Outline of this work
After this introduction, we discuss the concept of bidirectional quantitative force gradient microscopy and present appropriate cantilever-based sensors. We build on our previous results [21] and generalize them in terms of force gradients. We calculate effective dynamic spring constants for two resonant oscillation modes of the cantilever, which correspond to two orthogonal oscillation directions of the probe tip. As an experimental force field we utilize the magnetostatic interaction between a magnetic tip and the well-defined magnetic stray field of a multilayer sample, so in essence this is an MFM experiment. Based on the calculated spring constants we convert resonance frequency shift measurements of probe–sample interactions for the fundamental flexural mode and the second order mode into perpendicular and in-plane force derivative maps, respectively. Then, a two-dimensional map of measured magnetostatic vertical force derivative data is used to calculate the corresponding in-plane force derivative; the latter is compared to the measured in-plane data. Finally, we confirm the capability to extract quantitative data from our bidirectional force gradient measurements.

2. Sensor concept and spring constant calculation
Knowledge of the dynamic spring constants of a dSFM probe is vital to extract quantitative force gradient data from dSFM measurements. Therefore, in addition to presenting our novel sensor concept we provide a detailed description of the calculation of appropriate dynamic spring constants.

2.1. Sensor concept
Figure 1 shows the measurement principle of bidirectional force gradient microscopy, the corresponding sensor structure, and the coordinate system in use. The probe tip is attached to the cantilever via a spacer element at the nodal point of the cantilever’s second flexural oscillation mode. Of course any modifications on cantilevers may alter the location of nodal points. However, this can be neglected if the mass and the moments of inertia of the spacer element are much smaller than those of the cantilever. Furthermore, the spacer element needs to be stiff, i.e. its eigenfrequencies should exceed the frequencies used when operating the sensor. Exciting the cantilever at the fundamental or second order flexural mode leads to an oscillation of the probe tip along the z- or x-direction, respectively. In dSFM the resonance frequency shift is proportional to the weighted average of $\partial F_z / \partial s$. Here, $F_z$ is the projection of the interaction force on the tip oscillation direction and $s$ corresponds to the spatial coordinate along the mode dependent oscillation direction. If we neglect a spatial dependence of $\partial F_z / \partial s$, the frequency shift $\Delta f$ reads:
\[ \Delta f = -\frac{f_n}{2\pi \cdot \text{dyn}} \cdot \frac{\partial F}{\partial x}, \quad (1) \]

\( f_n \) denotes the resonance frequency of the cantilever and \( c_{\text{dyn}} \) the effective dynamic spring constant; \( f_n \) and \( c_{\text{dyn}} \) are mode-dependent. Thus, dSFM measurements according to figures 1(b) and 1(c) provide \( \Delta f \) data that are proportional to \( \partial F / \partial z \) and \( \partial F / \partial x \), respectively.

### 2.2. Static spring constant

In order to extract quantitative force gradients we need to know the dynamic spring constants of the probe with regard to the tip location and the particular vibration mode. Starting from the basic static spring constant of a cantilever beam we shall develop equations for the dynamic spring constants with respect to the probe tip position. All external force gradients are regarded as point-like and acting on the probe tip only. This condition is usually well satisfied in dSFM in general and with our long probe tip in particular.

To make the following calculations as simple as possible we use the special case of a constant cantilever cross section. We apply a dimensional approach [23] to calculate the static spring constant \( c_{L \text{stat}} \) of a cantilever with a point load acting on its free end:

\[ c_{L \text{stat}} = \frac{3EI}{L^3}, \quad (2) \]

with \( E \) being Young’s modulus, \( I \) the second moment of area of the cross section, and \( L \) the length of the cantilever beam. \( I \) can be calculated from the cantilever’s cross section geometry data which can be derived from scanning electron microscopy (SEM) measurements, for example. This approach, however, has the disadvantage of forwarding the uncertainties of \( E \) as well as \( I \) with the latter being proportional to the cube of the height of the cantilever \( h \). To circumvent this source of inaccuracy we use the measured resonance frequency \( f_1 \) of the fundamental flexural mode of the cantilever, which is related to \( EI \) by

\[ EI = \frac{4\pi^2 f_1^2 \rho A}{\beta_1^4}, \quad (3) \]

with \( \beta_1 \approx 1.875 / L \) being an eigenvalue of the characteristic equation of the Euler–Bernoulli beam theory [24], and \( \rho \) the cantilever’s mass density. Finally, we obtain

\[ c_{L \text{stat}} = \frac{12\pi^2 f_1^2 \rho A}{(\beta_1 L)^4} \approx \frac{12\pi^2 f_1^2 \rho A}{(1.875)^4} \quad (4) \]

with \( A \) being the cantilever’s cross sectional area. Now \( c_{L \text{stat}} \) has only a linear dependence on all geometrical quantities of the cantilever; no knowledge of \( E \) is required. Instead, we need data on \( \rho \), which are easily available.

### 2.3. Dynamic spring constants

As indicated in the introduction, the momentary deformation shape, or mode shape, of an oscillating cantilever and the related potential energy is different compared to the case of static bending. The dynamic spring constants of the fundamental and the second mode flexural oscillation, \( c_{L,1}^{\text{dyn}} \) and \( c_{L,2}^{\text{dyn}} \), respectively, are related to their static counterpart in the following way [3]:

\[ \frac{c_{L,1}^{\text{dyn}}}{c_{L,1}^{\text{stat}}} = 1.03; \quad \frac{c_{L,2}^{\text{dyn}}}{c_{L,2}^{\text{stat}}} = 40.2. \quad (5) \]

Our sensor design requires that the probe tip is not positioned at the end of the cantilever \( x = L \), but at the position of the nodal point of the second oscillation mode instead: \( x = x_{\text{tip}} \approx 0.783L \). The whole energy of the system can be expressed by the elastic energy of the beam at the turning point of its motion \( U_{\text{max}}^{\text{elastic}} \). In analogy to the potential energy stored in a spring, it can be described by the product of the squared peak oscillation amplitude of any point of reference and a corresponding dynamic spring constant \( c_{x,n}^{\text{dyn}} \):

\[ U_{\text{max}}^{\text{elastic}} = \frac{1}{2} c_{x,n}(x_n) \quad (6) \]

with \( n \) being the number of the vibration mode and \( x_n(x) \) the vertical displacement amplitude of the cantilever at a position \( x \) along the cantilever, i.e. the shape of mode \( n \) according to the Euler–Bernoulli beam theory. Now we apply equation (6) to both \( x = x_{\text{tip}} \) and \( x = L \) and use the fact that the respective maximum elastic energy \( U_{\text{max}}^{\text{elastic}} \) does not depend on the point of reference \( x \), i.e. it is identical in both cases. Hence, we are able to calculate \( c_{x,n}^{\text{dyn}} \).
Using the fundamental natural mode of an Euler–Bernoulli beam we obtain:

\[
e_{\text{dyn},1} = \left( \frac{u_1(L)}{u_1(x_{\text{tip}})} \right)^2 c_{L,1}^2.
\] (7)

Next we calculate the dynamic spring constant of the second mode corresponding to an in-plane oscillation of the probe tip. Assuming a rigid spacer element, the horizontal displacement \( \Delta x \) of the tip in the second mode is proportional to the slope of the cantilever at the position of the spacer element in a small angle approximation:

\[
\Delta x \approx l_{\text{dist}} \cdot \frac{du_2(x = x_{\text{tip}})}{dx}.
\] (10)

\( l_{\text{dist}} \) denotes the distance between the probe tip and the neutral fiber of the cantilever. Equation (6) can be adapted for the horizontal displacement \( \Delta x \) with the same argumentation as given above:

\[
U_{\text{max}}^{\text{elastic}} = \frac{1}{2} e_{\text{dyn},2}^2 (\Delta x)^2.
\] (11)

By combining equations (6), (10) and (11) this results in

\[
e_{\text{dyn},2} = \left( \frac{u_2(L)}{l_{\text{dist}} \cdot du_2(x = x_{\text{tip}})/dx} \right)^2 c_{L,2}^2,
\] (12)

or after calculating the second mode’s cantilever displacement and its derivative:

\[
e_{\text{dyn},2} \approx 0.0576 \left( \frac{L}{l_{\text{dist}}} \right)^2 e_{L,2}^2,
\] (13)

or

\[
e_{\text{dyn},2} \approx 2.31 \left( \frac{L}{l_{\text{dist}}} \right) c_{L,2}^2,
\] (14)

Now with the knowledge of the mode-dependent spring constants, bidirectional dSFM \( \Delta f \) data can be translated directly into tip–sample interaction force gradient data \( \partial F_{zz}/\partial z \) and \( \partial F_{xx}/\partial x \) using equation (1) or more elaborate deconvolution methods [4, 25].

3. Experimental verification of the quantitative sensor concept using magnetostatic force gradients

In the following we experimentally validate our sensor model as introduced in the previous section. We measure and simulate force derivatives resulting from magnetostatic interactions of a ferromagnetic nanowire probe and a ferromagnetic multilayer sample, i.e. we perform MFM-type experiments in order to confirm the more general measurement principle of bidirectional force gradient microscopy by comparing measured to calculated results.

3.1. Probe preparation

The basis of our sensor is a tipless silicon cantilever. We remove the triangular free end by focused ion beam (FIB) milling in order to obtain an approximately constant cross section along the length of the cantilever. A constant cross section is not mandatory but it simplifies the nodal point calculation of the second flexural vibration mode. At the calculated position of this nodal point \( x_{\text{tip}} \) a pillar-like spacer element is grown onto the cantilever by FIB-assisted deposition of carbon. Usual dSFM or MFM instruments require the cantilever to be introduced at a certain angle deviating from parallel to the sample plane. This causes the oscillation direction to be slightly off-axis. For the second mode such a deviation can be taken into account when defining the orientation of the spacer element to make sure the tip oscillation is precisely parallel to the sample surface. The FeCNTs used as magnetostatic interaction tips of our probes were grown by chemical vapor deposition [26]. The iron fillings are high aspect ratio nanowires having lengths of several microns and diameters in the range.
of 15 nm to 50 nm. It has been shown that the iron nanowires have a magnetic single domain configuration with the easy axis parallel to the long wire axis. With the help of a micro-manipulator, a single FeCNT is attached to the end of the spacer element by electron beam induced carbon deposition. Finally, we apply electron beam assisted etching in water vapor environment to remove unfilled parts of the FeCNT at its free end [27, 28]. Figure 2 shows SEM images of the bidirectional probe that has been used for the measurements described below. Its properties include the cantilever length $L = (217.8 \pm 3.3) \mu m$, the cantilever’s cross sectional area $A = (75.9 \pm 1.4) \mu m^2$, the distance between the FeCNT end and the neutral fiber of the cantilever $l_{\text{dist}} = (30.10 \pm 0.45) \mu m$. The probe’s eigenfrequencies of the first and the second mode are $f_1 = (79.39 \pm 0.01) kHz$ and $f_2 = (497.64 \pm 0.01) kHz$, respectively. This results in spring constants of $k_{\text{xy}} = (4.87 \pm 0.26) N m^{-1}$ and $k_{\text{xy}} = (282.51 \pm 13.8) N m^{-1}$. The errors of the spring constants may be somewhat underestimated, since some sources of uncertainty are difficult to quantify, e.g. thickness variations of the cantilever along its length. Furthermore, the diameter of the Fe nanowire is $d_{\text{Fe}} = (32 \pm 5) nm$, and the thickness of the carbon cap $t_{\text{cap}} = (40 \pm 5) nm$. Not visible in figure 2 is the $10^\circ$ inclination angle of the spacer element with respect to the cantilever’s surface normal to ensure that the tip movement in the second mode is parallel to the sample surface. As mentioned before, this is necessary to compensate for the cantilever tilting that is required by our MFM instrument. Please note such compensation has not been considered in the probe shown in the SEM images of figure 1.

3.2. Ferromagnetic test sample

Our perpendicular anisotropy test samples, which provide well defined magnetic stray fields are Co/Pt multilayer thin films with the following architecture: Pt (5nm)/[Pt (0.9 nm)Co (0.4 nm)]_{100}/Pt (2 nm). In the zero field state, where the sample shows a band domain configuration the domain structure, as shown in figure 3(a), it is usually not affected by MFM probe stray-fields and therefore constitutes an ideal reference as shown for similar multilayers in previous work [29]. Vibrating sample magnetometry measurements revealed an uniaxial anisotropy constant of $K_u = 517 kJ m^{-3}$ and a saturation magnetization of $M_s = 554 kA m^{-1}$. Hence the ratio of $K_u$ to the shape anisotropy constant $K_3 = \frac{1}{2} \mu_0 \cdot M_s^2 = 192 kJ m^{-3}$ is 2.7, in agreement with a magnetization perpendicular to the film plane within each domain.

3.3. Magnetostatic force gradient measurements

The measurements of the fundamental and second mode cantilever resonance frequency were conducted under high vacuum conditions in a NanoScan AG hr-MFM employing frequency modulation. Tip oscillation amplitudes of around 10 nm were used in both modes. All measurements were performed at a tip–sample distance of $z_{\text{scan}} = 80 nm$. Figures 3(a) and (b) show MFM force gradient images corresponding to the perpendicular and the in-plane tip oscillation direction, respectively. The primary MFM signal, i.e. $\Delta f$, is already converted into force gradient data using the calculated spring constants and equation (1). The software WSxM
was used to apply a plane subtraction to the data and, in order to take small $x$–$y$ drifts into consideration, extract overlapping image parts.

### 3.4. Magnetostatic force gradient calculation

The force gradient as displayed in figures 3(a) and (b) is caused by magnetostatic interactions between the stray field of the sample and the magnetization of the FeCNT. Using two-dimensional Fourier transforms of the magnetic field distribution, $\mathbf{H}(k_x, k_y, z)$, where the spatial in-plane coordinates $(x, y)$ are replaced by the spatial frequencies $(k_x, k_y)$, enables the calculation of the $x$ and $y$ components of the magnetic stray field from the $z$-component in the $x$–$y$ plane, except for the average value corresponding to the point $(k_x = 0, k_y = 0)$ \[15\]:

$$\mathbf{H}(k_x, k_y, z) = -\frac{V}{k} \mathbf{H}_z(k_x, k_y, z). \quad (15)$$

$k$ denotes the absolute value of the in-plane wave vector ($k = \sqrt{k_x^2 + k_y^2}$). The tilde sign ($\sim$) denotes the Fourier transform of the corresponding quantity. The nabla operator in Fourier space is given by $\nabla = (ik_x, ik_y, -k)$.

Equation (15) leads to a relation between the $x$-component and the $z$-component of the field gradient:

$$\frac{\partial H_x}{\partial x} = -\frac{k_x^2}{k^2} \cdot \frac{\partial H_z}{\partial z}. \quad (16)$$

Taking the probe–sample interaction into account, this equation can be converted into a corresponding force gradient relation. In the simple case of a magnetic point charge tip, a multiplication of equation (16) by the tip charge $-q$ and by $\mu_0$ leads to:

$$\frac{\partial F_x}{\partial x} = -\frac{k_x^2}{k^2} \cdot \frac{\partial F_z}{\partial z}. \quad (17)$$

For arbitrarily shaped MFM tips the force gradient in Fourier space results from the multiplication of the Fourier transform of $\partial H_z/\partial z$ with a so called tip transfer function (TTF) $(TTF)(k_x, k_y)$, which can be fully determined by a calibration measurement of an appropriate reference sample [29, 31]. Although the TTF now depends on the spatial frequencies $(k_x, k_y)$, it is independent of the oscillation direction of the sensing tip, as long as the oscillation amplitude is smaller than the length scale of force gradient changes, which ensures an amplitude independent force gradient. The condition cited in connection with equation (1) is thus sufficient to ensure that equation (17) applies for an arbitrary MFM sensor.

After applying equation (17) to the experimental $\partial F_z/\partial z$ data, the inverse Fourier transform provides calculated $\partial F_x/\partial x$ data, as shown in figure 3(c), which can be compared with the corresponding experimental lateral force gradient $\partial F_x/\partial x$ data shown in figure 3(b). The excellent quantitative agreement is further illustrated.
by two line sections presented in figures 3(d) and (e). This result is thus a manifestation of equation (17) and, even more importantly, also of the correctly calculated ratio between the fundamental and second mode spring constants, which can be found in equations (9) and (14).

3.5. Determination of the magnetic moment of the tip

For a verification of the absolute force gradient data we make use of the fact that in the measurements both the magnetic sample and the magnetic sensor element can be characterized by independent means. We start by using the $\frac{\partial F_{zz}}{\partial z}$ measurement of the well-characterized multilayer reference sample to calculate the magnetic behavior of the tip. The latter will be compared to those that are expected from the simple nanowire geometry of the FeCNT that acts as the tip.

Going back to the formulation of magnetic force gradients in Fourier space,

$$\frac{\partial F_z}{\partial z} = \nabla \cdot \mathbf{H},$$

this can be rephrased as [29]:

$$\frac{\partial F_z}{\partial z} = \sigma^* \cdot \mathbf{H}(k_x, k_y),$$

$\sigma^*$ is the effective surface charge pattern of the studied sample. For the utilized Co/Pt multilayer this pattern can be constructed from a qualitative MFM measurement and independent knowledge of the saturation polarization $\mu_0 M_s$, film thickness and domain wall width [29]. A deconvolution of the measured perpendicular force gradient $\frac{\partial F_{zz}}{\partial z}$ with the constructed effective surface charge pattern by means of a Wiener filter leads to the spatially resolved stray field gradient of the tip $\frac{\partial H_z}{\partial z}(\text{tip})(x, y)$ in a distance of $z_{\text{scan}} = 80$ nm below the tip apex.

Figure 4 shows a line scan through the obtained symmetrical tip field gradient. It is compared to the stray field characteristics of an ideal point charge or monopole-like tip, which has been demonstrated to be a good description for high aspect ratio single-domain nanowire MFM tips [32, 33]. The stray field gradient of a monopole $q$ along a line at a distance $z_{\text{scan}} + \delta$ is given by [34, p 28]:

$$\frac{\partial H_z(x)}{\partial z} = \frac{q}{4\pi} \left( \frac{x^2 - 2(z_{\text{scan}} + \delta)^2}{x^2 + (z_{\text{scan}} + \delta)^2} \right)^{3/2},$$

where $\delta$ accounts for the distance of the monopole away from the carbon nanotube end and $z_{\text{scan}}$ for the scan height in the MFM experiment, i.e. the average distance of the carbon nanotube end to the sample surface. If we use $q$ and $\delta$ as free parameters, the tip’s real stray field gradient can be fitted with equation (20). Small deviations might be explained by the nonvanishing lateral extension of the nanowire. The resulting fit parameters are: $q = (1.2 \pm 0.25) \times 10^{-9}$ A m and $\delta = (37 \pm 0.5)$ nm.

Alternatively, the FeCNT tip parameters can be directly deduced from the SEM image shown in figure 2(b). The measured diameter of the iron nanowire is $d_{\text{Fe}} = (32 \pm 5)$ nm and the carbon cap thickness at the free end...
of the FeCNT is \( f_{\text{cap}} = (40 \pm 5) \) nm. With the saturation magnetization of iron \( M_s = 1.71 \times 10^6 \) A m\(^{-1}\) the resulting monopole moment is given by \( q_{\text{geom}} = M_s \pi d^2 / 4 = (1.4 \pm 0.4) \times 10^{-9} \) A m. Taking the relatively large error intervals caused by the SEM-based diameter measurement into account, this corresponds very well to the parameters obtained by fitting the measurement-based probe stray field gradient with the monopole model. Thus the absolute force gradient data are verified.

4. Conclusions

In this paper, we introduced a new measurement principle of quantitative bidirectional force gradient microscopy. Quantitative force gradient measurements rely on the knowledge of the corresponding dynamic spring constants. We calculated the latter for the case of a constant cantilever cross section. Force gradient measurements exploiting magnetostatic interactions of a ferromagnetic nanowire tip with a well-characterized magnetic multilayer film agreed very well with calculations confirming the presented quantitative approach. These results prove the applicability of the bidirectional force gradient microscopy for a direct, fast and quantitative real space analysis of force gradients in two dimensions. Our presented measurement principle has a strong practical advantage, because it relies solely on flexural vibrations, i.e. all microscopy modes are based on vertical probe excitation and vertical deflection measurement only.

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