Quadratic programming model for optimal decision making of supplier selection problem integrated with inventory control problem

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Abstract. In this paper, we propose a quadratic programming model to determine the optimal decision for integrated supplier selection and inventory control. The corresponding optimization is solved by using constrained quadratic optimization method i.e. Karush-Kuhn-Tucker Method performed in LINGO 17.0 to determine the optimal joint decisions which are the optimal supplier and the optimal inventory. A numerical experiment was performed to analyze the model with a multi-product, multi-supplier, multi-carrier and multi-period inventory system. From the results we have found, we can find the optimal supplier for each product and each period.

1. Introduction
In recent years, along with the development of technology and knowledge, the economic process is growing up where every businessman trying to develop their company so it can compete with other companies. For its development, company needs to get the maximum profit by optimizing the purchase of logistics to reduce the total cost. There are two processes in industrial company which can be optimized for cost minimizing which are supplier selection and inventory control. Supply of logistics for a company can be satisfied from multiple suppliers. By solving the supplier selection problem, the company can determine the number of logistics to be purchased from each supplier so that it minimizes the purchasing cost. If the company wants to minimize the purchasing cost for several time periods, the companies can save some logistics units on their warehouses so that the saved items can be used for the future/next period of time. Because of that, supplier selection and inventory control problem is needed to be solved in order for good plan determining and reaches the minimal total cost hence the company continues to grow up.

There are several articles that were discussed about supplier selection and inventory control problem. In [1] supplier selection problem solving considering contingency planning for failures on supplier was discussed whereas in [2] the integrating of DEA and MODE was discussed for sustainable supplier selection problem solving. Furthermore, references [3], [4] and [5] were discussed about the solution of the supplier selection problem considering quantity discount. In [6], it was described the fuzzy TOPSIS combined with MCGP model for supplier selection problem solving while in reference [7] it was compared about supplier selection problem solving by using fuzzy AHP and fuzzy TOPSIS. For inventory control, in [8], the joint replenishment inventory control with deterministic and stochastic model was discussed whereas in [9] the inventory control model for an incapacitated warehouse in a manufacturing facility under demand and lead time uncertainty was
supplied some product. In our model, it is assumed that the total value based on robust model predictive control for inventory control system considering random demand. Reference [12] discussed about the decision of the problem of selection of suppliers integrated inventory controls of inventory system considering purchase discount. In article [13], it was discussed about application of fuzzy optimization for solving of supplier selection problem combined with inventory control problem considering fuzzy demand by using fuzzy expected value based approach. Furthermore, in [14], it was discussed about inventory control model for split transport.

In this paper, we formulate a mathematical model in an integer quadratic optimization that will be used to solve supplier selection problem integrated with inventory control problem with several constraints. We conduct some numerical experiments to be used to describe and analyze the formulated model.

2. Mathematical model
Suppose a company will purchase some materials from \( S \) supplier alternatives. We have formulated a mathematical model that minimizes the total cost which is containing supplier selection and inventory control problem solving. In our model, it is assumed that in case of delay in delivery from some supplier, it will be sent in the following time period. To formulate the mathematical model, parameters and variables that will be used are defined as follows:

- \( T \): Optimization time set
- \( S \): Supplier set
- \( P \): Product set
- \( X_{tp} \): Product volume (unit) \( p \in P \) which should be supplied by supplier \( s \in S \) for time period \( t \in T \)
- \( UC_{tp} \): Product unit price of product \( p \in P \) which supplied by supplier \( s \in S \) for time period \( t \in T \)
- \( TC_{ts} \): Total transportation cost for all products supplied by supplier \( s \in S \) for time period \( t \in T \)
- \( Y_{ts} \): Binary variable which is 1 if supplier \( s \in S \) is chosen to supply some product for time period \( t \in T \) or 0 if supplier \( s \in S \) is not chosen at time period \( t \in T \)
- \( SC_{ts} \): Maximum capacity (unit) for supplier \( s \in S \) to supply product \( p \in P \) for time period \( t \in T \)
- \( D_{tp} \): Volume (unit) of the demand of product \( p \in P \) for time period \( t \in T \)
- \( UPC_{tp} \): Penalty cost per unit for unqualified delivered product (under quality standard) for product \( p \in P \) from supplier \( s \in S \) for time period \( t \in T \)
- \( UDC_{tp} \): Delay cost per unit for product \( p \in P \) that will be delivered from supplier \( s \in S \) at time period \( t \in T \)
- \( DLT_{tp} \): Delay rate for product \( p \in P \) that will be delivered from supplier \( s \in S \) for time period \( t \in T \)
- \( Q_{ts} \): Quality level requirement for product \( p \in P \) that will be supplied by supplier \( s \in S \) for time period \( t \in T \)
- \( Q_o \): Minimum standard quality percentage of all product set by the company
- \( H_p \): Holding Cost per unit for any product \( p \) at time period \( t \)
- \( M_p \): Capacity of storage/warehouse for product \( p \) at any time period
- \( I_{tp} \): Number of unit product \( p \in P \) that will be stored in the warehouse at time period \( t \)
$B_t$ : Budget value provided by the company for the procurement for whole optimization or decision making time period

$r_{tp}$ : Reference inventory level for control purposes at time period $t$ for product $p$ that it decided by the company.

This supplier selection problem considering inventory management can be modelled as follows:

$$\begin{align*}
\text{min} & \quad \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} X_{tsp}UC_{tsp} + \sum_{t=1}^{T} \sum_{s=1}^{S} T_{is} Y_{is} + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} (1 - Q_{tsp}) UPC_{tsp} X_{tsp} \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} UDC_{tsp} DLT_{tsp} X_{tsp} + \sum_{t=1}^{T} \sum_{p=1}^{P} H_{p} I_{tp} + \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{tp} - r_{tp} \right)^2 
\end{align*}$$

(1)

subject to:

$$X_{tsp} \leq SC_{tsp}, \forall t \in T, \forall s \in S, \forall p \in P$$

(2)

$$\sum_{t=1}^{T} X_{tsp} Y_{is} \geq \sum_{t=1}^{T} D_{tp}, \forall s \in S, \forall p \in P$$

(3)

$$Y_s = \begin{cases} 1 & \text{for } X_{tsp} > 0 \\ 0 & \text{for } X_{tsp} \leq 0 \end{cases} \forall s \in S$$

(4)

$$X_{tsp} \geq 0, \forall t \in T, \forall s \in S, \forall p \in P$$

(5)

$$\sum_{s=1}^{S} X_{tsp} - I_{tp} - \sum_{s=1}^{S} (1 - Q_{tsp}) X_{tsp} - \sum_{s=1}^{S} DLT_{tsp} X_{tsp} \geq D_{tp}, t = 1, \forall p \in P$$

(6)

$$I_{(t-1)p} + \sum_{s=1}^{S} DLT_{(t-1)sp} X_{(t-1)sp} + \sum_{s=1}^{S} X_{tsp} - \sum_{s=1}^{S} (1 - Q_{tsp}) X_{tsp} -$$

(7)

$$- \sum_{s=1}^{S} DLT_{tsp} X_{tsp} - I_{tp} \geq D_{tp}, t > 1, \forall p \in P$$

(8)

$$I_{tp} \leq M_{tp}, \forall t \in T, \forall p \in P$$

(9)

$$\left( \sum_{s=1}^{S} \sum_{p=1}^{P} X_{tsp} UC_{tsp} \right) + \left( \sum_{s=1}^{S} T_{is} Y_{is} \right) + \left( \sum_{s=1}^{S} \sum_{p=1}^{P} (1 - Q_{tsp}) UPC_{tsp} X_{tsp} \right)$$

(10)

$$+ \left( \sum_{s=1}^{S} \sum_{p=1}^{P} UDC_{tsp} DLT_{tsp} X_{tsp} \right) + \sum_{p=1}^{P} H_{p} \leq B_{s}, \forall t \in T, \forall s \in S, \forall p \in P$$

$$X_{tsp}, I_{tp} \geq 0 \text{ and integer.}$$

The above model consists of 10 mathematical expressions where expression (1) is the objective function that should be minimized that it is consisting of product purchasing cost, transportation cost, penalty cost for defected product, delay delivering product cost, holding cost and set point/reference tracking objective term. Expression (2) presents the supplier capacity constraint and expression (3) is demand constraint that must be satisfied for each time period. Expression (4) is the binary constraint for $Y$ while expression (5) is constraint so that $X$ must be nonnegative. Expression (6) is used for inventory constraint for period 1 while constraint (7) is inventory constraint for period 2 and the following period. Constraint (8) is presenting the storage capacity constraint whereas (9) is presenting the budget constraint for each time period. The last constraint, (10) is presenting the integer constraint.

The solution existence analysis of the model can be explained as follows. The first five terms in the objective function are linear functions where the last term is a one dimensional quadratic function.
with positive coefficient. Hence, the objective function is a convex function. Furthermore, if the feasible set is not empty, then it guarantees that the solution is exist.

3. Numerical experiment
A numerical experiment was performed to illustrate and analyze the model. This experiment was considered with five suppliers which are S1, S2, S3, S4 and S5, and four product types which are P1, P2, P3 and P4, and we were optimized for 12 time periods. The parameter values for the performed experiment were randomly generated given in table 1 to table 10 and the reference point value for inventory level target point for all product for all time periods is 50 units.

Table 1. Unit price $UC_{isp}$ for all $t$

| Supplier | Product | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 |
|----------|---------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| S1       | P1      | 23 | 27 | 35 | 21 | 25 | 17 | 27 | 19 | 15 | 23  | 17  | 23  |
|          | P2      | 17 | 18 | 25 | 31 | 28 | 30 | 21 | 21 | 28 | 24  | 21  | 17  |
|          | P3      | 18 | 29 | 29 | 20 | 26 | 33 | 31 | 15 | 35 | 29  | 28  | 19  |
|          | P4      | 15 | 24 | 21 | 16 | 18 | 25 | 22 | 19 | 21 | 19  | 18  | 17  |
| S2       | P1      | 20 | 31 | 32 | 29 | 31 | 19 | 30 | 16 | 28 | 32  | 15  | 27  |
|          | P2      | 19 | 34 | 15 | 17 | 15 | 32 | 23 | 21 | 17 | 22  | 21  | 19  |
|          | P3      | 15 | 20 | 24 | 15 | 28 | 20 | 23 | 19 | 24 | 29  | 29  | 19  |
|          | P4      | 31 | 18 | 31 | 18 | 30 | 16 | 31 | 25 | 30 | 21  | 18  | 19  |
| S3       | P1      | 29 | 16 | 26 | 24 | 16 | 29 | 18 | 19 | 15 | 27  | 34  | 30  |
|          | P2      | 19 | 32 | 19 | 30 | 31 | 19 | 34 | 30 | 23 | 21  | 17  | 22  |
|          | P3      | 25 | 21 | 35 | 19 | 30 | 32 | 21 | 15 | 30 | 16  | 35  | 29  |
|          | P4      | 26 | 19 | 29 | 18 | 29 | 23 | 32 | 21 | 31 | 35  | 19  | 24  |
| S4       | P1      | 28 | 20 | 16 | 29 | 35 | 25 | 21 | 19 | 35 | 31  | 18  | 22  |
|          | P2      | 19 | 26 | 27 | 22 | 30 | 31 | 32 | 24 | 21 | 20  | 30  | 20  |
|          | P3      | 23 | 29 | 32 | 16 | 19 | 24 | 24 | 20 | 19 | 16  | 19  | 25  |
|          | P4      | 29 | 19 | 16 | 20 | 21 | 35 | 17 | 32 | 28 | 21  | 15  | 17  |
| S5       | P1      | 16 | 20 | 30 | 21 | 35 | 30 | 20 | 34 | 27 | 34  | 19  | 29  |
|          | P2      | 30 | 35 | 24 | 30 | 17 | 24 | 34 | 35 | 21 | 28  | 35  | 31  |
|          | P3      | 33 | 22 | 31 | 34 | 35 | 23 | 25 | 18 | 31 | 24  | 35  | 24  |
|          | P4      | 30 | 35 | 29 | 23 | 30 | 17 | 33 | 28 | 29 | 16  | 31  | 21  |

Table 2. Maximum supplier capacity

| Supplier | Product | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 |
|----------|---------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| S1       | P1      | 520| 670| 500| 1200| 1150| 800| 670| 540| 680| 1000| 890| 950 |
|          | P2      | 680| 720| 820| 590 | 790 | 900| 750| 950| 1120| 1200| 850| 700 |
|          | P3      | 1200| 1130| 930| 1000| 750 | 670| 740| 780| 500 | 590 | 680| 900 |
|          | P4      | 900| 650| 590| 910 | 880 | 650| 1120| 1000| 670 | 900 | 560| 670 |
| S2       | P1      | 780| 800| 1200| 1130| 1000| 950| 1150| 980| 650| 790 | 550| 870 |
|          | P2      | 870| 1200| 1180| 780 | 590 | 760| 890| 650| 870 | 590 | 900| 760 |
Table 2. (Cont.)

| Supplier | Product | Time Period |
|----------|---------|-------------|
|          | P1      | T1 1000 740 1100 990 870 970 740 690 590 600 680 |
|          | P2      | T2 900 950 890 650 790 520 590 660 760 850 1000 940 |
|          | P3      | T3 1000 970 910 750 650 550 860 1200 970 890 650 1170 |
|          | P4      | T4 570 680 500 540 700 750 900 1120 870 510 1000 1200 |
| S3       | P1      | T5 870 1200 750 650 690 1120 980 1100 980 790 990 970 |
|          | P2      | T6 930 530 990 1000 710 940 870 760 540 650 910 990 |
|          | P3      | T7 500 1200 900 820 1180 660 900 790 1120 730 860 880 |
|          | P4      | T8 660 890 600 520 890 750 1190 650 910 940 870 1200 |
| S4       | P1      | T9 700 1150 750 650 1050 1200 950 600 900 700 950 1000 |
|          | P2      | T10 500 950 700 600 650 850 900 800 850 550 850 |
|          | P3      | T11 650 800 650 1150 900 750 1100 550 850 950 |
|          | P4      | T12 1100 1050 900 850 650 600 800 900 650 900 800 550 |

Table 3. Penalty cost per unit

| UPC | Product | Time period |
|-----|---------|-------------|
|     | P1      | T1 3.0 4.0 1.5 2.5 2.5 2.0 3.0 1.5 2.0 1.5 3.0 4.0 |
| S1  | P2      | T2 2.5 3.5 2.5 3.0 3.5 1.5 2.5 3.0 2.5 3.0 1.5 2.5 |
|     | P3      | T3 1.5 1.5 3.0 4.0 1.5 3.0 1.5 2.5 4.0 3.5 2.0 3.0 |
|     | P4      | T4 3.0 2.5 3.5 1.5 1.5 2.5 3.0 1.5 2.0 1.5 2.5 1.5 |
|     | P1      | T5 3.5 4.0 2.0 3.0 4.0 2.0 2.5 3.5 1.5 4.0 1.5 4.0 |
| S2  | P2      | T6 1.5 3.0 2.5 2.5 3.0 3.5 2.0 3.0 1.5 3.0 1.5 2.5 |
|     | P3      | T7 1.5 2.0 4.0 3.0 2.0 1.5 2.5 1.5 2.5 3.5 4.0 2.0 |
|     | P4      | T8 2.5 3.5 2.0 1.5 3.5 4.0 3.0 2.0 1.5 1.5 3.5 1.5 |
|     | P1      | T9 3.0 2.0 1.5 3.5 2.5 3.0 3.5 3.0 3.5 3.0 2.5 4.0 |
| S3  | P2      | T10 2.5 1.5 1.5 3.0 1.5 2.5 2.5 1.5 1.5 4.0 1.5 3.5 |
|     | P3      | T11 1.5 3.0 2.0 4.0 2.5 1.5 2.0 3.0 4.0 2.5 3.5 2.5 |
|     | P4      | T12 3.0 3.5 1.5 2.5 3.5 2.5 3.5 1.5 3.5 2.0 4.0 2.5 |
|     | P1      | T13 4.0 1.5 4.0 2.0 2.5 1.5 2.0 1.5 3.0 1.5 3.5 3.5 |
| S4  | P2      | T14 3.5 1.5 2.0 3.0 1.5 2.5 3.0 3.5 1.5 2.0 2.5 1.5 |
|     | P3      | T15 1.5 2.5 4.0 2.5 1.5 3.0 2.0 1.5 4.0 2.5 3.5 1.5 |
|     | P4      | T16 2.5 3.5 3.0 2.5 3.5 3.0 2.5 3.5 3.0 3.5 2.0 2.5 |
|     | P1      | T17 3.0 2.0 4.0 1.5 2.0 3.0 2.5 4.0 2.0 2.5 1.5 3.0 |
| S5  | P2      | T18 1.5 2.5 3.0 2.0 2.5 3.5 3.0 1.5 1.5 4.0 3.5 3.5 |
|     | P3      | T19 4.0 1.5 2.0 2.5 3.0 2.5 4.0 2.5 4.0 3.5 2.0 1.5 |
| Supplier | Product | Time Period |
|----------|---------|-------------|
| S1       | P1      | 0.00 0.05 0.03 0.05 0.02 0.01 0.04 0.02 0.04 0.02 0.01 0.03 |
|          | P2      | 0.02 0.04 0.02 0.04 0.04 0.02 0.05 0.04 0.00 0.05 0.04 0.01 |
|          | P3      | 0.04 0.02 0.05 0.00 0.03 0.01 0.04 0.05 0.04 0.02 0.03 0.04 |
|          | P4      | 0.03 0.05 0.01 0.04 0.04 0.03 0.02 0.05 0.02 0.05 0.03 0.05 |
| S2       | P1      | 0.01 0.03 0.05 0.04 0.02 0.04 0.04 0.01 0.04 0.00 0.04 0.00 |
|          | P2      | 0.05 0.04 0.01 0.04 0.05 0.01 0.05 0.02 0.02 0.04 0.03 0.05 |
|          | P3      | 0.04 0.03 0.04 0.02 0.04 0.03 0.04 0.03 0.00 0.03 0.01 0.05 |
|          | P4      | 0.01 0.04 0.01 0.04 0.05 0.04 0.01 0.05 0.04 0.05 0.03 0.02 |
| S3       | P1      | 0.03 0.05 0.04 0.04 0.02 0.03 0.05 0.04 0.03 0.02 0.04 0.05 |
|          | P2      | 0.05 0.04 0.02 0.02 0.00 0.05 0.00 0.05 0.01 0.05 0.03 0.00 |
|          | P3      | 0.05 0.01 0.04 0.04 0.05 0.04 0.05 0.03 0.04 0.03 0.04 0.05 |
|          | P4      | 0.00 0.04 0.05 0.03 0.01 0.04 0.01 0.04 0.02 0.00 0.04 0.03 |
| S4       | P1      | 0.02 0.04 0.00 0.04 0.02 0.05 0.04 0.03 0.04 0.04 0.05 0.02 |
|          | P2      | 0.05 0.04 0.05 0.02 0.04 0.01 0.04 0.03 0.01 0.05 0.01 0.04 |
|          | P3      | 0.04 0.02 0.00 0.04 0.05 0.04 0.05 0.00 0.05 0.03 0.04 0.05 |
|          | P4      | 0.01 0.05 0.05 0.04 0.01 0.05 0.02 0.05 0.04 0.04 0.02 0.01 |
| S5       | P1      | 0.02 0.01 0.04 0.02 0.05 0.01 0.00 0.02 0.03 0.00 0.01 0.02 |
|          | P2      | 0.04 0.04 0.05 0.00 0.04 0.03 0.01 0.03 0.05 0.03 0.04 0.01 |
|          | P3      | 0.03 0.02 0.03 0.05 0.01 0.02 0.04 0.00 0.02 0.02 0.01 0.05 |
|          | P4      | 0.01 0.03 0.02 0.04 0.04 0.03 0.02 0.02 0.01 0.04 0.05 0.00 |

Table 4. Delay lead time

| Supplier | Product | Time Period |
|----------|---------|-------------|
| S1       | P1      | 2.5 3.0 3.5 2.0 5.0 4.0 3.0 2.5 3.5 4.0 2.0 3.5 |
|          | P2      | 3.0 4.5 3.0 2.5 3.0 4.5 2.0 4.5 5.0 2.5 3.5 2.5 |
|          | P3      | 4.0 2.0 2.5 4.0 4.5 5.0 4.5 4.0 2.0 4.5 5.0 4.5 |
|          | P4      | 3.5 4.5 3.5 4.5 2.0 2.0 3.5 4.5 2.0 3.0 3.5 5.0 |
| S2       | P1      | 4.0 5.0 4.0 5.0 4.5 3.5 2.5 3.5 3.0 4.5 5.0 4.5 |
|          | P2      | 2.0 3.0 4.5 2.0 5.0 4.5 4.0 4.5 5.0 2.5 3.0 2.5 |
|          | P3      | 4.5 2.5 5.0 2.5 4.0 2.5 4.5 3.0 3.5 4.5 4.5 3.5 |
|          | P4      | 4.0 2.0 4.0 4.5 3.0 4.5 2.5 4.5 2.5 3.0 2.5 5.0 |
| S3       | P1      | 4.5 2.5 4.5 2.5 4.5 4.0 2.0 3.5 5.0 2.5 4.5 4.0 |
|          | P2      | 3.5 5.0 3.0 4.0 5.0 3.5 4.5 3.0 4.5 4.0 3.0 4.5 |
|          | P3      | 2.0 3.5 4.5 2.0 4.5 4.0 3.5 4.5 3.5 4.0 4.5 2.5 |
|          | P4      | 2.5 4.5 5.0 2.5 3.5 2.5 3.0 5.0 2.0 4.5 2.5 4.0 |

Table 5. Unit delay cost

| Supplier | Product | Time Period |
|----------|---------|-------------|
| S1       | P1      | 2.5 3.0 3.5 2.0 5.0 4.0 3.0 2.5 3.5 4.0 2.0 3.5 |
|          | P2      | 3.0 4.5 3.0 2.5 3.0 4.5 2.0 4.5 5.0 2.5 3.5 2.5 |
|          | P3      | 4.0 2.0 2.5 4.0 4.5 5.0 4.5 4.0 2.0 4.5 5.0 4.5 |
|          | P4      | 3.5 4.5 3.5 4.5 2.0 2.0 3.5 4.5 2.0 3.0 3.5 5.0 |
| S2       | P1      | 4.0 5.0 4.0 5.0 4.5 3.5 2.5 3.5 3.0 4.5 5.0 4.5 |
|          | P2      | 2.0 3.0 4.5 2.0 5.0 4.5 4.0 4.5 5.0 2.5 3.0 2.5 |
|          | P3      | 4.5 2.5 5.0 2.5 4.0 2.5 4.5 3.0 3.5 4.5 4.5 3.5 |
|          | P4      | 4.0 2.0 4.0 4.5 3.0 4.5 2.5 4.5 2.5 3.0 2.5 5.0 |
| S3       | P1      | 4.5 2.5 4.5 2.5 4.5 4.0 2.0 3.5 5.0 2.5 4.5 4.0 |
|          | P2      | 3.5 5.0 3.0 4.0 5.0 3.5 4.5 3.0 4.5 4.0 3.0 4.5 |
|          | P3      | 2.0 3.5 4.5 2.0 4.5 4.0 3.5 4.5 3.5 4.0 4.5 2.5 |
|          | P4      | 2.5 4.5 5.0 2.5 3.5 2.5 3.0 5.0 2.0 4.5 2.5 4.0 |

Table 5. Cont.
### Table 6. Quality level of product

| Supplier | Product | Time Period |
|----------|---------|-------------|
| **S4**   |         |             |
|          | P1      | 4.5 2.0 4.5 3.0 5.0 4.5 5.0 3.5 4.5 3.0 3.5 2.0 |
|          | P2      | 5.0 4.5 3.0 3.5 4.5 3.0 4.5 3.0 3.5 5.0 4.0 3.5 |
|          | P3      | 3.5 5.0 3.5 4.5 5.0 2.0 3.5 4.5 3.0 4.5 2.0 2.5 |
|          | P4      | 3.0 4.5 2.0 5.0 3.0 2.5 3.5 2.0 2.5 4.0 3.5 5.0 |
| **S5**   |         |             |
|          | P1      | 2.0 4.0 4.5 2.5 4.0 2.5 3.0 4.5 5.0 3.0 2.5 3.0 |
|          | P2      | 2.5 3.5 3.5 4.0 2.0 4.0 5.0 3.5 5.0 5.0 2.5 2.0 |
|          | P3      | 4.5 5.0 3.0 3.5 5.0 4.5 2.5 2.0 3.5 5.0 4.5 5.0 |
|          | P4      | 3.0 2.5 3.5 5.0 4.0 5.0 4.5 5.0 5.0 3.0 2.5 2.0 |

### Table 7. Volume of demand

| Product | Time Period |
|---------|-------------|
| **P1**  | 1200 1800 2100 1300 1500 2200 1900 1500 1900 1700 1500 1700 |
| **P2**  | 1400 1900 2300 1600 1300 1400 2300 1600 2500 2100 2200 2000 |

| Time Period |
|-------------|
| P1 1200 1800 2100 1300 1500 2200 1900 1500 1900 1700 1500 1700 |
| P2 1400 1900 2300 1600 1300 1400 2300 1600 2500 2100 2200 2000 |
Table 8. Total transportation cost

| Supplier | Time Period |
|----------|-------------|
|          | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 |
| S1       | 350| 400| 450| 550| 600| 700| 950| 1000| 1200| 1250| 1350| 1600|
| S2       | 300| 400| 500| 550| 650| 750| 1000| 1100| 1150| 1200| 1400| 1450|
| S3       | 250| 300| 350| 450| 550| 700| 800| 900| 1050| 1250| 1300| 1350|
| S4       | 400| 450| 550| 600| 650| 800| 900| 950| 1200| 1400| 1450| 1600|
| S5       | 300| 350| 450| 600| 700| 900| 1150| 1200| 1300| 1350| 1450| 1550|

Table 9. Budget provided per period

| Time Period | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 |
|--------------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| Budget (×1000) | 350| 450| 500| 375| 300| 400| 425| 375| 400| 350| 475| 500 |

Table 10. Holding cost and capacity storage

| Product | H | M |
|---------|---|---|
| P1      | 4 | 75|
| P2      | 2 | 75|
| P3      | 3 | 75|
| P4      | 1 | 75|

The following figures are the results of the optimization process performed in LINGO 17.0 that was used quadratic optimization solver. Figure 1 to figure 3 show the optimal decision which is the optimal product volume for all time periods for all products.
Figure 2. Optimal product volume per time period 5 to period 8

Figure 3. Optimal product volume per time period 9 to period 12

From figure 1 to figure 3 above, it can be seen that at time period 1, 16 units of product P1 should be purchased from S1, 780 units of product P1 should be purchased from S2, 1 units of product P1 should be purchased from S3, and 700 units of product P1 should be purchased from S4. From the amount of product 1 that purchased from each supplier, as many as 33 units of product 1 must be stored. Whereas for product 2 on period 1, 680 units of product 2 should be purchased from S1, 870 units of product 2 should be purchased from S2, and 199 units of product 2 should be purchased from S4. From the amount of product 2 that purchased from each supplier, as many as 36 units of product 2 must be stored. The solution for the following time periods can be derived analogously. The iteration of optimization process is 243.372 steps that was performed in LINGO 17.0 software and the minimum total cost was 1,917,964. From figure 4, it can be derived the optimal product volume stored in the inventory for each time period and it can be seen that the inventory level for each time period for each product was sufficiently closed to the reference point.

4. Conclusions

In this paper, an integrated dynamic supplier selection with inventory level control problem was solved by using the formulated model in the form of integer quadratic programming model. From the obtained model, the optimal decision in determining of the number of product purchased from each supplier was derived. From the conducted numerical experiment, the optimal decision was obtained and the inventory level was followed and located near the reference level.
Figure 4. Inventory level for each time period for each product

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