Solar-cycle variation of the rotational shear near the solar surface

A. Barekat¹, J. Schou¹, and L. Gizon¹²

¹ Max-Planck-Institut für Sonnensystemforschung, Justus-von-Liebig-Weg 3, 37077 Göttingen, Germany
e-mail: barekat@mps.mpg.de
² Institut für Astrophysik, Georg-August-Universität Göttingen, 37077 Göttingen, Germany

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ABSTRACT

Context. Helioseismology has revealed that the angular velocity of the Sun increases with depth in the outermost 35 Mm of the Sun. Recently, we have shown that the logarithmic radial gradient (dlnΩ/dln r) in the upper 10 Mm is close to −1 from the equator to 60° latitude. Aims. We aim to measure the temporal variation of the rotational shear over solar cycle 23 and the rising phase of cycle 24 (1996–2015). Methods. We used f mode frequency splitting data spanning 1996 to 2011 from the Michelson Doppler Imager (MDI) and 2010 to 2015 from the Helioseismic Magnetic Imager (HMI). In a first for such studies, the f mode frequency splitting data were obtained from 360-day time series. We used the same method as in our previous work for measuring dlnΩ/dln r from the equator to 80° latitude in the outer 13 Mm of the Sun. Then, we calculated the variation of the gradient at annual cadence relative to the average over 1996 to 2015. Results. We found the rotational shear at low latitudes (0° to 30°) to vary in-phase with the solar activity, varying by ∼±10% over the period 1996 to 2015. At high latitudes (60° to 80°), we found rotational shear to vary in anti-phase with the solar activity. By comparing the radial gradient obtained from the splittings of the 360-day and the corresponding 72-day time series of HMI and MDI data, we suggest that the splittings obtained from the 72-day HMI time series suffer from systematic errors. Conclusions. We provide a quantitative measurement of the temporal variation of the outer part of the near surface shear layer which may provide useful constraints on dynamo models and differential rotation theory.

Key words. Sun: helioseismology – Sun: interior – Sun: rotation

1. Introduction

One of the major challenges in solar physics is to understand the physics behind the 11-year solar cycle. In many dynamo models, which attempt to explain the solar cycle, the differential rotation of the Sun plays an important role (see the reviews by Brandenburg & Subramanian 2005; and Charbonneau 2010).

In an αΩ dynamo, rotational shear is responsible for the Ω-effect which generates toroidal magnetic field from a poloidal magnetic field. The time variation of the shear has a direct influence on the magnetic field generation in the Sun as it may provide non-linear feedback on the dynamo mechanism (Küker et al. 1999). Additionally, the radial shear in the near-surface shear layer is a potential explanation for the equatorward migration of the activity belt during the solar cycle (Brandenburg 2005). Hence, providing quantitative information about the radial gradient of the rotation close to the surface of the Sun is indispensable. Measurements of the radial shear can also deliver constraints on differential rotation models (e.g., Kitchatinov & Rüdiger 2005). Kitchatinov (2016) recently related the near-surface shear to the subsurface magnetic field. Therefore, the time variation of the shear with the solar cycle may also help estimate the strength of the magnetic field below the surface at different phases of the cycle.

The radial shear can be measured by several helioseismic techniques; see Thompson et al. (1996), Schou et al. (1998), and the latest reviews of global and local helioseismology by Howe (2009) and Gizon et al. (2010), respectively. Corbard & Thompson (2002) showed that the logarithmic radial gradient in the outer 16 Mm of the Sun is close to −1 up to 30° latitude and becomes positive above 55° latitude. However, Barekat et al. (2014, hereafter BSG), found no indication of a change of sign at this latitude.

Antia et al. (2008) studied the time variation of the radial and latitudinal shear during solar cycle 23. They used 12 years (1996–2007) of p mode and f mode frequency splitting data from the Michelson Doppler Imager (MDI; Scherrer et al. 1995) on board the Solar and Heliospheric Observatory (SOHO). They also used 13 years (1995–2007) of p mode frequency splitting data from the Global Oscillation Network Group (GONG). They applied a two-dimensional regularized least square method (Antia et al. 1998) for inferring the rotation rate. Then, they studied the time variation of both radial and latitudinal shears at several depths and latitudes. They found that the variation of the radial shear is about 20% of its average value at low latitudes at 14 Mm and below.

In this work, we investigate the solar cycle variation of the radial gradient of the rotation in the outer 13 Mm of the Sun using f modes. We use 19 consecutive years of frequency splitting data corresponding to the entire solar cycle 23 (1996–2010) and the rising phase of cycle 24 (2010–2015). These data are obtained from 360-day time series from the Medium-l program of MDI and from the Helioseismic and Magnetic Imager (HMI; Schou et al. 2012) on board the Solar Dynamics Observatory. These data are different from what we used in BSG in which the splittings were obtained from 72-day time series. Therefore,
2. Observational data

We consider only \( f \) modes. We denote mode frequency by \( \nu_{lm} \) where \( l \) and \( m \) are the spherical harmonic degree and for azimuthal order, respectively. We use 18 odd \( a \)-coefficients for each \( l \) (Schou et al. 1994) obtained from MDI and HMI data, which are defined by

\[ \nu_{lm} = \nu_l + \sum_{j=1}^{36} a_j \ell_j^{l}(m), \]

where \( \nu_l \) is the mean multiplet frequency, and \( \ell_j^{l}(m) \) are orthogonal polynomials of degree \( j \). We use two sets of data of each instrument; the \( a \)-coefficients which are obtained from 72-day and 360-day time series, resulting in four data sets:

- MDI360: 15 sets obtained from 360-day MDI (1996–2011);
- HMI360: 5 sets obtained from 360-day HMI (2010–2015);
- MDI72: 74 sets obtained from 72-day MDI (1996–2011);
- HMI72: 25 sets obtained from 72-day HMI (2010–2015).

We summarize the number of modes found in each data set in Table 1, which was made from three non-consecutive 72 day time series (Larson & Schou 2015) because of problems with the SOHO spacecraft.

3. Method

Our method for measuring the radial gradient is identical to the one used by BSG. We explain our method here succinctly and refer the reader to BSG for detailed explanation. We model the rotation rate as changing linearly with depth

\[ \Omega(r, u) = \Omega_0(u) + (1-r)\Omega_1(u), \]

where \( r \) is the distance to the center of the Sun normalized by its photospheric radius (\( R_p \)), \( u \) is the cosine of co-latitude and, \( \Omega_0(u) \) and \( \Omega_1(u) \) are the rotation rate at the surface and the slope, respectively. Then, we perform a forward problem using the relation between the \( a \)-coefficients and \( \Omega \) which is given by

\[ 2\pi a_{l2+1} = \int_0^1 dr \int_{-1}^1 du K_{ls}(r,u)\Omega(r,u), \]

where \( K_{ls} \) are kernels. We obtain

\[ \bar{\Omega}_{ls} \equiv \frac{2\pi a_{l2+1}}{\beta_{ls}} = \langle \Omega_0 \rangle + (1 - \bar{r}_s)\langle \Omega_1 \rangle, \]

where the \( \beta_{ls} \) are the total integrals of the radial component of the kernels (see Eq. (4) in BSG) and \( \bar{r}_s \) is the central of gravity of the radial kernels. The \( \langle \rangle \) denotes latitudinal averages. Next, we perform an error-weighted least square fit of \( \Omega_{ls}/2\pi \) versus \((1 - \bar{r}_s)\) to determine \( \langle \Omega_0 \rangle \) and \( \langle \Omega_1 \rangle \), for each data set.

In the last step of our analysis, we apply the inversion method used by Schou (1999) to \( \langle \Omega_0 \rangle \) and \( \langle \Omega_1 \rangle \) to infer the rotation rate at each target latitude \( u_0 \) from this obtain \( d\Omega/d\ln{r} \).
In this section, we compare the radial gradient derived from split-
obtained from the same underlying data. Thus there is clear evidence that the splitting data suffer from systematic errors.

We applied the same comparison to sets HMI360 and HMI72. The five year time averages from using both all and only the common modes are shown in the bottom panel of Fig. 2. There is a significant discrepancy between the two results obtained from sets HMI360 and HMI72 above 60° latitude which does not disappear even when comparing the results obtained from the common modes. This shows that the HMI data are even more affected by systematic errors than the MDI data.

For HMI data, we carry out further analysis by comparing the results derived from common modes of each year. Except for the first and last year the difference between the results persists. The perfect agreement of the results in the last year encourages us to compare common modes between these two data sets. This comparison shows that the difference between \(a_3\) and \(a_5\) of those data sets is significant. In average, the values of \(a_3\) of HMI360 is larger and \(a_5\) is smaller than the corresponding HMI72 ones by about 3\(\sigma\). There are also clear systematic errors in those coefficients with larger discrepancies in the earlier than in the later years.

Unfortunately, these comparisons do not tell us what causes the systematic errors or how to correct them. Understanding this will require a more detailed analysis (Larson & Schou, in prep.). However, our results suggest that HMI72 suffer from systematic errors as the results obtained using HMI360 are not significantly different from the results of data sets MDI360 and MHD12. Moreover, we expect that the splittings obtained from longer time series have better quality as the peaks are better resolved (Larson & Schou 2015).

### 4.2. Solar cycle variation of the radial gradient

We measure the variation of \(\text{dln} \Omega/\text{dln} r\) relative to its time averaged value from 1996 to 2015 using data sets MDI360 and HMI360. We show the results in Fig. 4 together with the butterfly diagram. These measurements reveal two cyclic patterns; one at low latitudes from the equator to about 40° latitude and one above 60° latitude. There is no clear signal between about 40° and 60° latitude.

Below 40°, there exist bands where the rotation gradient is about 10% larger and smaller than the average. As illustrated by the butterfly diagram in Fig. 4, the band with steeper than average gradient (blue in Fig. 4) follows the activity belt quite closely. These bands are also similar to the torsional oscillation signal (see, e.g., Howe et al. 2006; Antia et al. 2008).

The temporal variation of \(\text{dln} \Omega/\text{dln} r\) at high latitudes is more than 10% of its average value and has the opposite behavior to that at low latitudes. However, as we pointed out earlier the measured values of the gradient above 50° latitude are not reliable, so any results here have to be interpreted with caution.

The statistical significance of these signals is shown in Fig. 5 and the standard deviation in time and the time averaged errors in Fig. 6. The measured signals are statistically significant at low and high latitudes as they are at the 3 to 8\(\sigma\) level, while they are indeed not significant between 40° and 60° latitude.

We note here that the results obtained from MDI360 and HMI360 are only different by about 1% when using modes with \(I \geq 120\), corresponding roughly to the range used by the 72-day analysis and over which the rotation rate changes linearly with depth.

It is well known that the phase and amplitude of the solar cycle variations of the rotation rate vary with depth and latitude.

![Figure 3](image-url)  
**Fig. 3.** \(\tilde{\Omega}_0/2\pi\) versus \((1-\tau_0)\) obtained from the data set starting 10 April 2000 of MDI360. The black line is the error weighted linear least square fit. We avoid plotting the error bars as they are in similar size of the symbols.

![Figure 4](image-url)  
**Fig. 4.** Time variation of \(\text{dln} \Omega/\text{dln} r\) relative to its 19 year time average. The thin stripe in the plot shows the result obtained from data set 15 overplotted on data set 16 as there is 288 days overlap between these two data sets. The contours show the two hemisphere averaged butterfly diagram of the sunspot area of 5 per millionths of a hemisphere (courtesy of D. Hathaway; see [http://solarscience.msfc.nasa.gov/greenwch.shtml](http://solarscience.msfc.nasa.gov/greenwch.shtml)).

![Figure 5](image-url)  
**Fig. 5.** Statistical significance in change of \(\text{dln} \Omega/\text{dln} r\) relative to its average values at different latitudes and time. \(\sigma\) is the error on \(\text{dln} \Omega/\text{dln} r\) of each year.
We also compare the radial gradient obtained from common modes of two different data sets of each instrument. These comparisons reveal that the measured values of $\frac{\text{d} \ln \Omega}{\text{d} \ln r}$ above 50° latitude are not reliable. Another important finding is that there are considerable systematic errors in HMI data that needs further investigation.

By measuring the variation of rotational shear relative to its 19 year time averaged value we find two cyclic patterns at low (0° to 30°) and at high (60° to 80°) latitudes with similar period of the solar cycle. Both patterns show bands of larger and smaller than average shear moving toward the equator and poles at low and high latitudes, respectively. The relative change in the shear is about 10% at low latitudes and 20% at high latitudes. Although the values of $\frac{\text{d} \ln \Omega}{\text{d} \ln r}$ above 50° are not reliable, the temporal variation of $\frac{\text{d} \ln \Omega}{\text{d} \ln r}$ is significant above 60° latitudes. This finding may have important implications for dynamo models as this variation is considerable compared to the torsional oscillation (Antia et al. 2008).

The cyclic behavior of the shear at low latitudes agrees with the recent theoretical work by Kitchatinov (2016) who showed that the strength of the shear increases because of the presence of the strong magnetic field. Therefore accurate measurements of the shear might be a way of determining the sub-surface magnetic field.

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Fig. 6. Comparison between the standard deviation of the time variation of the radial shear relative to its time averaged value (dashed line) and the time averaged error of the shear (solid line) obtained from data sets MDI360 and HMI360.

(Vorontsov et al. 2002; Basu & Antia 2003; Howe et al. 2005; Antia et al. 2008), but the temporal variation of the gradient has not been previously reported over the same depth range as used in this work.

Antia et al. (2008) found a similar pattern with similar amplitude of the temporal variation of the radial gradient at 0.98 $R_\odot$ as ours. They used the first eight odd $\alpha$-coefficients obtained from MDI $p$ and $f$ modes and GONG $p$ modes spanning 1995 to 2007. Despite the similarity in pattern and amplitude, the sign of the change in the gradient of their results is opposite to ours. They saw that sunspots occurred where the absolute value of the gradient is smaller than the average value which is the opposite of what we see. This difference might come from the fact that we are measuring the temporal variation around 0.99 $R_\odot$ while they measured it at 0.98 $R_\odot$. We also note that Antia et al. (2008) used the earlier version of the MDI data (see Larson & Schou 2015 and BSG) which might explain the discrepancy that Antia et al. (2008) saw between GONG and MDI data at 0.98 $R_\odot$ and shallower layers.

5. Conclusion

We make measurements of the radial gradient over 19 years (1996–2015) corresponding to solar cycle 23 and the rising phase of cycle 24 in the outer 13 Mm of the Sun. We use recently available $f$ mode frequency splittings data obtained from 360-day time series of MDI spanning 1996 to 2011 and HMI spanning 2010 to 2015. The values of the radial gradient derived from MDI360 and HMI360 fluctuate between $-0.97$ and $-0.9$ up to 50° latitude. These values are a few percent larger than measured values by BSG which are obtained from MDIT2 and HMI72. It turns out that this difference comes from the fact that the angular velocity does not change linearly with depth to deeper than about 10 Mm below the surface.