Observing the Structure of the Interstellar Clouds

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Abstract.
We discuss some methods for characterizing the mass distribution in the interstellar clouds. This is done in the light of available observational techniques and of appropriate statistical tools capable to highlight the structural properties of the astronomical images. The potentialities of the optical and near-IR imaging of dark clouds as well as of their far-IR and millimetric mapping are also discussed with respect to the analysis in terms of statistical characterization. We briefly address the hypothesis that the origin of the structure observed in interstellar clouds is the turbulence which could be a natural consequence of the physical status of the interstellar medium. We finally discuss how the fractal geometry offers an interesting tool for characterizing the structure of the interstellar clouds introducing the so called multifractal spectrum. The physical interpretation of this spectrum can offer a further tool in discriminating among different possible kind of internal dynamics.

1. Introduction
The images of the interstellar clouds (hereinafter ISC), currently obtained in various spectral regions with both ground based and space telescopes, easily convince us that they are globally inhomogeneous and highly structured. This general statement seems to conflict with some instances in which, at a first glance, the ISC appear quite smooth and homogeneous as, e.g., in the case of the relatively small, almost round dark clouds known as Bok globules. However even in these cases, when the sensitivity of the observations is increased, a complex pattern emerges when their images are analyzed.

The physical interpretation of the observed structures is a major task of the modern astrophysics because it is a necessary step for improving our understanding of the star formation process that, taking place in molecular clouds, can be influenced (or even regulated) by the distribution of the mass in the ISC. Moreover, the structure of the ISC is also relevant for the intracloud chemistry because the penetration depth of the interstellar radiation field is clearly dependent on the way the matter is distributed.

We remind that, in studying the internal structure of the ISC, we deal with 2D images, i.e. the projection, on the sky plane, of the 3D clouds. Because of this, one should be careful in adopting appropriate tools in analyzing the observed images to extract the characteristics of the 3D physical objects. This is a difficult task because the de-projection 2D→3D is an inverse problem without a general solution. For this reason we release the requirement about the generality by assuming some prior knowledge of the object’s nature i.e., in our case, the knowledge of some characteristics of a realistic ISC model. In this respect we mention the growing interest we note in the literature for a turbulent model, that is implied by the values of the velocity, temperature and density typically observed in the ISC.
In Section 2 we discuss some observational techniques particularly suited to our aim of investigating the structure of the projected interstellar clouds. The Section 3 discusses useful tools for the statistical analysis of the images and in Section 4 the assumption of a turbulent regime is justified for the ISC. In the next Section 5 we briefly review some aspects of the fractal approach relevant to the analysis of the ISC images.

2. Observational techniques

There are evidences that interstellar gas and dust are well mixed (Bohlin et al. 1978, Gordon 1995, Kim & Martin 1996) at least on scales larger than 0.1 pc. This allows us to exploit the effects, both in absorption and in emission, of the two components to highlight the structure of the ISC. It is however possible that at smaller spatial scales the gas-dust mixing is not equally well assessed so that a more careful analysis can be required.

2.1. Dust extinction and star counts

Historically the detection of the ISC has been done by observing, in the optical band, the dark regions that appear projected on rich stellar backgrounds. When it was realized that these dark regions testify the presence of the interstellar dust the “darkness” of these region begun to be quantified by means of stellar counts. These counts are usually interpreted in terms of extinction suffered by the stellar light in traveling through the ISC interposed between the stars and the observer. The basic assumption is that of a homogeneous stellar background across the investigated region. Essentially this approach exploits the dependence of the observed stellar density, for a given limiting magnitude, on the dust column density projected in the line of sight.

In practice the stars are counted in the boxes of a spatial grid whose period is dictated by the need to reach statistical significance in counting also in regions where the stars are rare. This requirement, along with the sensitivity of the observations, limits the spatial resolution of the map so that the obtained extinction values should be regarded as a spatial average over the counting box.

Actually this technique gives us an extinction map which is relative to a reference region, chosen in the neighborhood of the investigated region and supposed to be unextinguished. This choice introduces the risk of a systematic underestimate of the absolute extinction in the case the reference region is itself affected by some extinction. Such a problem can be alleviated if the extinction of the reference region can be independently obtained.

2.2. Reddening of background stars

Another possibility to map the effect of dust is to exploit the reddening of the stellar light as an indirect measure of the extinction. This can be done by comparing the observed with the intrinsic colours of the stars, a task that can be done by means of two colour diagrams. In these plots the position of a reddened background object is compared with the locus of the unreddened stars, so that a colour excess is derived. This excess is finally related to the amount of extinction by means of the reddening law.

Adopting this method we both avoid any reference to a supposedly unreddened region in our images and derive the extinction in each line of sight toward a background star. Note that the spatial resolution in this case is very high but the image is poorly sampled because the number of useful stars is typically much lower than the spatial resolution elements of the image. There are however two drawbacks in this case: the first is the need to observe at least in three spectral bands to obtain the two stellar colours required to plot the background objects in a col-col diagram. The second difficulty is that only a subset of the detected stars, which depends on the colours we choose, is useful to derive unambiguously the reddening. This is illustrated in Figure 1 that shows the case of a [V-I] vs [B-V] diagram in which the useful stars
Figure 1. Two colour diagram. The locus of the unreddened stars (continuous line) is reported along with the position occupied by the stars extinguished of $A_V = 3$ mag (dashed line). Are those located in the upper right. For these objects the corresponding reddening line crosses the unreddened locus in a well-defined point.

2.2.1. Dust and gas emission

The most direct way to observe the ISC is to detect their emission that, because of their quite low typical temperatures, peaks in the spectral region between the far IR and the millimetric wavelengths. Both dust and gas contribute to this emission with a continuum and a line spectrum, respectively. The observational products in these spectral regions are mainly maps of the dust continuum emission in the FIR and sub-mm as well as intensity or velocity-channel maps obtained in typical spectral lines emitted by the molecular constituents (notably CO and its isotopes, but also HCN and CS tracing the higher density gas) of the ISC.

These maps, whose typical spatial resolution is of the order of a few tens of arcsec, depending on the telescope beamsize, are usually obtained by sequentially pointing the telescope itself on a grid of positions sampling the target ISC. Another advantage of these observations is that the clouds can be detected also in directions far from the galactic disc where the poor stellar background prevents the use of methods involving the extinction/reddening of the stellar light.

While the physical interpretation of the dust continuum emission maps is quite straightforward because the observed intensity can be directly related to the dust column density, in the case of the line maps this is made more difficult because the observed intensity also depends on the excitation and depletion conditions of the emitting gas. In general these conditions are a function of the temperature and density along the line of sight. On the other side, this difficulty is compensated by the possibility offered by the line emission maps to obtain further information on the velocity field of the ISC, an important clue for clarifying the dynamical processes influencing the cloud structure.

3. Structure extraction

There are many mathematical tools that can be used to investigate the internal structure of our objects. These are mainly based on a statistical analysis of the observed maps and, strictly speaking, they are designed to operate on 2D signals, the 3D de-projection remaining a matter of theoretical modeling.
Among the classical tools for structure analysis of the signals we mention:
- the structure function of order $p$
  \[ S_p(\Delta r) = \langle |A(\mathbf{r}) - A(\mathbf{r} + \Delta \mathbf{r})|^p \rangle \] (1)
- the autocorrelation function
  \[ C(\Delta r) = \langle A(\mathbf{r}) A(\mathbf{r} + \Delta \mathbf{r}) \rangle \] (2)
- the power spectrum
  \[ P(k) = \hat{A}(k) \cdot \hat{A}^\ast(k) \] (3)

In these expressions $A$, $\hat{A}$ and $\hat{A}^\ast$ are the function (the signal) to be analyzed, the corresponding Fourier transform and its complex conjugate, respectively. Note that $\mathbf{r}$ identifies a position in the map, $\Delta \mathbf{r}$ is an increment whose module $\Delta r$ is the investigated spatial scale. In these expressions the brackets $\langle \rangle$ operate an ensemble average over all possible values of $\mathbf{r}$. In the case of our maps the variables $\mathbf{r}$ and $k$ represent the position and the spatial frequency, respectively.

Each of these expressions highlights some special characteristic as the presence of gradients, of spatial correlation, or the particular signal decomposition in terms of periodic functions. It is noteworthy that all these methods are interrelated so that, in the limit of infinitely extended signals (e.g. infinite size of the maps), they can be equally well used to obtain the power spectrum.

However, because we never deal with infinite images of the ISC, we have to adopt the most appropriate tool whose choice can depend not only on the map size but also on other characteristics as the noise, the signal dynamics, etc. Because of this, dealing with real images, many approaches have been proposed and used in the analysis of the astronomical data. In Table 1 we briefly list these methods along with a recent reference illustrating their corresponding application in the astronomical context.

**Table 1.**

| Method                        | Reference                  |
|-------------------------------|----------------------------|
| $\Delta$-variance             | (Stutzki et al. 1998)      |
| Principal Component Analysis  | (Brunt 2003)               |
| Independent Component Analysis| (Maino et al. 2002)        |
| Spectral Correlation Function | (Rosolowsky et al. 1999)   |
| Wavelet transform             | (Langer et al. 1993)       |
| Multifractal spectrum         | (Chappel & Scalo 2001)     |

Among these we shall briefly discuss only the $\Delta$-variance method because it is both easily applied in the spatial domain and particularly suited for the analysis of relatively small maps. These are the cases in which the Fourier analysis can be severely biased by the the limited size and non periodicity of the real ISC images.

### 3.1. $\Delta$-variance method

The essentials of this methods are captured by considering a 1D signal $s(x)$ scanned by a filter function as shown in Figure 2. Note that the filter function is defined so that the total underlying area is zero. In this way any constant component of the signal will be zeroed and the application of this filter will correspond to evaluate a difference (the $\Delta$) between the local signal (in the
Figure 2. Schematic of the $\Delta$-variance. In the 1D case (left) the signal is convolved with a zero-mean down-up-down filter function whose characteristic size is $L$. The variance of the result is calculated and plotted for different filter sizes. In the 2D case (right) the appropriate filter becomes a down-up-down cylinder ("French hat") which is scanned over the 2D image.

central part of the filter) and the signal in the immediate neighborhood. By scanning this filter we obtain an output signal that fluctuates around the zero. It is just the variance $\sigma^2$ of these fluctuations that bears information on the power spectrum of the original signal.

It is now straightforward, by repeating the same analysis for different filter sizes, to obtain a plot of the variance $\sigma^2(L)$ as a function of the filter size $L$.

It has been shown (Stutzki et al. 1998) that, when the power spectrum of the analyzed signal is a power law $P(f) \propto f^{-\beta}$, it happens that the relationship between the $\Delta$-variance and the spatial scale is

$$\Delta \sigma^2(L) \propto L^{(\beta-2)}$$

so that, evaluating the slope of the logarithmic plot of the $\Delta$-variance vs filter size, we can obtain the slope $\beta$ of the power spectrum without the need to Fourier transform the data. This fact, along with the capability to distinguish the small scale (high frequency) noise in the maps, makes this method useful even for sizes as small as $30 \times 30$ pixel (Bensch et al. 2001).

Two examples illustrating the results obtained by applying the $\Delta$-variance method to both a real extinction map and a synthetic (fBm) map are shown in Figure 2 of Campeggio et al. (2005).

The reason why there is often some emphasis in searching for a possible power law behaviour in the power spectrum of the ISC maps is related to the expectation that a fractal structure is associated to such a power spectrum because this is characteristic of the objects showing the same structure at all the scales (scale-free). Such a behaviour is of particular interest because there is an increasing number of observations suggesting that the ISC are driven by turbulence and then the corresponding internal structure should show the signature of the corresponding self-similarity.

4. Turbulence and IS clouds

In fluid dynamics the turbulent regime is empirically associated to large values of the Reynolds number ($Re > 100$) which is defined as:

$$Re = \frac{\rho L U}{\mu}$$
where \( \rho \), \( L \), and \( U \) are the density, size, and velocity of the fluid, respectively. The variable \( \mu \) represents the dynamical viscosity and is related to the details of the interaction among the fluid particles. It was von Weizsäcker (1951) that, in a pioneering work, considered the value of this number for the interstellar medium and outlined the modern view we have of the interstellar matter: cloudy objects with a hierarchy of structures produced by interacting shock waves generated by supersonic turbulence. This turbulence is feed, on the largest scales, by the galactic rotation and is dissipated, on the smallest scales, by atomic viscosity.

Actually we now know that the situation is certainly more complex because the energy is also injected at intermediate scales by protostellar outflows, HII region and supernova shells. Despite of this the general framework outlined by von Weizsäcker remains well founded.

4.1. Observational grounds
There are in fact good reasons, based on good observations, suggesting a supersonic turbulent regime in the realm of ISC. In the following we give a list, which is not intended to be exhaustive, of the observational evidences that have been invoked to infer the presence of turbulence. For some of them we refer the interested reader to specific papers:

- a the relative velocities implied by the width of the spectral lines observed toward the ISC almost invariably show that the the speeds in these clouds are largely suprathermal;
- b the line width observed increases with increasing telescope beamsize (see, e.g., Schneider et al. (1996));
- c the observed molecular lines generally do not show flat topped profiles indicating absence of saturation effects, even for the relatively abundant and radiatively efficient \(^{12}\text{CO}\) that should be optically thick in many circumstances;
- d the line profiles often show non-gaussian wings, suggesting a non-thermal distribution of the gas velocities (see, e.g., Falgarone & Phillips (1990), Decamp & Le Bourlot (2002));
- e the line intensity ratio \( \text{CO}(J=2-1)/\text{CO}(J=1-0) \) is observed constant across cloud images, again indicating the absence of saturation effects (Falgarone & Phillips 1996);
- f the morphology of the clouds appears quite structured and clearly clumpy (as noted since 1978 by Burton & Gordon (1978))
- g the extinctions measured on background stars show an increasing dispersion with increasing mean extinction, a behaviour expected in a clumpy, highly structured medium (see, e.g., Lada et al. (1994), Campeggio et al. (2004)).

Some of these evidences are of spectral nature, other are morphological, other statistical, but taken as a whole they can be coherently understood in the framework of the interstellar turbulence.

4.2. Comparison with observations
Turbulence in fluids is a complex phenomenon whose characteristics are not yet fully understood. A model successfully proposed in the context of incompressible fluids is the Kolmogorov (1941) model in which the input energy, given to the fluid at a large spatial scale, is transferred to smaller scales by turbulent motions until the dissipative scale is reached and the energy transfer is stopped. The resulting energy spectrum of the fluid in this case is a power law \( E(k) \propto k^{-5/3} \) with \( k \) representing the spatial frequencies.

Due to the presence of compressibility, supersonic velocity, and magnetic field, the case of the interstellar fluid is clearly more complicated and requires a numerical treatment. Recent progresses have been done [see, e.g., Padoan et al. (2000)] so that the observations can be compared with more realistic models. In Campeggio et al. (2005) (Figure 3) the behaviour of the structure functions obtained for the extinction map of the dark globule CB 107 is compared
with the expectations in the Kolmogorov (1941) case as well as in two other cases taking into account multifractal scaling and supersonic compressibility (Boldyrev et al. 2002).

5. Fractals and clouds structure
It is expected that turbulence naturally give rise to fractal structures because it is intrinsically a cascading process repeating itself from larger to smaller scales without a characteristic lengthscale. In this respect, if we look at the structures of the ISC we note that in a large interval of scales (100 ÷ 0.01 pc) they lack of a characteristic length. It seems then reasonable to try a description of the interstellar structures by adopting the tools of the fractal geometry. In this respect it can be shown [see Stutzki et al. 1998] that the images with a power law power spectrum correspond to a fractal with a well-defined fractal dimension given by 

\[ D = \frac{8 - \beta}{2}, \]

being \( \beta \) the exponent of the power spectrum.

In this sense, when the \( \Delta \)-variance analysis allows the identification of a linear behaviour in a certain range of scales (see Figure 2 in Campeggio et al. (2005)), then the image is represented by a power-law power spectrum (see Equation 4) and then at these scales we can speak of a fractal object.

Note, however, that only rarely natural objects are pure fractals, so that a more general approach has been proposed by Hulsey et al. (1986) and applied to ISC for the first time by Chappel & Scalo (2001).

These authors have shown that the infrared (IRAS) maps of different ISCs are characterized by different multifractal spectra so that it would be now interesting to compare these spectra with those produced by the analysis of numerical simulations of realistic (compressible, supersonic, magnetic) IS turbulence. In particular it is noteworthy that the IRAS maps considered in their analysis have shown clear differences in the slopes of the multifractal spectrum, suggesting that this feature could be used to characterize the ISC. The physical meaning of such a feature is however still to be investigated.

6. Conclusion
The interest for the study of the structural properties of the ISC is mainly due to the connections with the starformation mechanism and with the intracloud chemistry. This interest is also growing thanks to the improved observational capabilities that allow us to obtain large and sensitive maps of the ISC, exploiting both their extinction and emission properties. Because the ISC lack of a well-defined lengthscale it is useful, in analyzing these maps, to choose appropriate statistical tools that allow us to characterize the observed structure and compare it with fluidodynamical models. At present only a few objects have been investigated from the structural point of view so that it is difficult to give a general answer to the question if the ISC are or not the same everywhere in the Galaxy. However, the observed ISC structure can be reasonably understood in a framework in which turbulence is the driving mechanism for the mass distribution. Coherently with this approach, a promising perspective is offered by the multifractal analysis of the ISC structure.
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