Predictions of $SU(9)$ orbifold family unification

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November 2, 2015

Abstract

We study predictions of orbifold family unification models with $SU(9)$ gauge group on a six-dimensional space-time including the orbifold $T^2/Z_2$, and obtain relations among sfermion masses in the supersymmetric extension of models. The models have an excellent feature that just three families of the standard model fermions exist in a pair of Weyl fermions in the $84$ representation as four-dimensional zero modes, without accompanying any mirror particles.

1 Introduction

Gauge theories on a higher-dimensional space-time including an orbifold as an extra space possess suitable properties to realize a family unification [1–14]. Three families of the standard model (SM) fermions are embedded into a few multiplets of a large gauge group. Extra fermions including mirror particles can be eliminated and the SM fermions can survive as zero modes, through the orbifold breaking mechanism.

In our previous work, we have found a lot of possibilities that three families of the SM fermions appear as zero modes from a pair of Weyl fermions in $SU(N)$ gauge theories ($N = 9 \sim 13$) on the six-dimensional (6D) space-time $M^4 \times T^2/Z_M$ ($M = 2, 3, 4, 6$) [10]. The subjects left behind are to construct realistic models and to find out model-dependent predictions.

In this paper, we focus on the orbifold family unification in the minimal setup, because models tend to be more complex and less realistic by extending the structure of space-time and/or the ingredients of models such as gauge symmetries and representations of matters. We take orbifold family unification models based on $SU(9)$ gauge symmetry on $M^4 \times T^2/Z_2$ as a starting point, examine the reality of models, and find out some predictions. For the reality, we use the appearance of Yukawa interactions from interactions in the 6D bulk as a selection rule. For the predictions, we search

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specific relations among sfermion masses in the supersymmetric (SUSY) extension of models.

The contents of this paper are as follows. In Sec.2, we explain our setup and its properties. In Sec.3, we carry out the examination for the reality of models and the search for predictions. Sec.4 is devoted to conclusions and discussions.

2 SU(9) orbifold family unification

2.1 Setup

Our space-time is assumed to be the product of four-dimensional (4D) Minkowski space-time $M^4$ and two-dimensional (2D) orbifold $T^2/Z_2$. The $T^2/Z_2$ is obtained by dividing 2D lattice $T^2$ by the $Z_2$ transformation: $z \rightarrow -z$, where $z$ is a complex coordinate. Then, $z$ is identified with $(-1)^lz + me_1 + ne_2$, where $l$, $m$ and $n$ are integers, and $e_1$ and $e_2$ are basis vectors of $T^2$.

We impose the following boundary conditions (BCs) on a 6D field $\Psi(x, z)$,

$$\Psi(x, -z) = T_\Psi[P_0]\Psi(x, z), \quad (1)$$

$$\Psi(x, e_1 - z) = T_\Psi[P_1]\Psi(x, z), \quad (2)$$

$$\Psi(x, e_2 - z) = T_\Psi[P_2]\Psi(x, z), \quad (3)$$

where $T_\Psi[P_0]$, $T_\Psi[P_1]$ and $T_\Psi[P_2]$ represent the representation matrices, and $P_0$, $P_1$ and $P_2$ stand for the representation matrices of $Z_2$ transformations $z \rightarrow -z$, $z \rightarrow e_1 - z$ and $z \rightarrow e_2 - z$ for fields with the fundamental representation.

The eigenvalues of $T_\Psi[P_0]$, $T_\Psi[P_1]$ and $T_\Psi[P_2]$ are interpreted as the $Z_2$ parities on $T^2/Z_2$. The fields with even $Z_2$ parities have zero modes. Here, zero modes mean 4D massless fields surviving after compactification. Massive modes are called “Kaluza-Klein modes”, and they do not appear in the low-energy world because they have heavy masses of $O(1/L)$ where $L$ is the size of extra space. Fields including an odd $Z_2$ parity do not have zero modes. Hence, the reduction of symmetry occurs upon compactification, unless all components of multiplet have common $Z_2$ parities. This type of symmetry breaking mechanism is called the “orbifold breaking mechanism”.

We start with 6D $SU(9)$ gauge theories containing a pair of Weyl fermions $(\Psi_+, \Psi_-)$. These fermions own a same representation of $SU(9)$ but different chiralities and are represented as

$$\Psi_+ = \frac{1 + \Gamma_7}{2} \Psi = \left( \begin{array}{cc} \frac{1 - \gamma_5}{2} & 0 \\ 0 & \frac{1 + \gamma_5}{2} \end{array} \right) \left( \begin{array}{c} \psi_1^1 \\ \psi_2^2 \end{array} \right) = \left( \begin{array}{c} \psi_{1L}^1 \\ \psi_{2R}^2 \end{array} \right), \quad (4)$$

$$\Psi_- = \frac{1 - \Gamma_7}{2} \Psi = \left( \begin{array}{cc} \frac{1 + \gamma_5}{2} & 0 \\ 0 & \frac{1 - \gamma_5}{2} \end{array} \right) \left( \begin{array}{c} \psi_1^1 \\ \psi_2^2 \end{array} \right) = \left( \begin{array}{c} \psi_{1R}^1 \\ \psi_{2L}^2 \end{array} \right), \quad (5)$$

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1 The $Z_2$ orbifolding was used in superstring theory [15] and heterotic $M$-theory [16, 17]. In field theoretical models, it was applied to the reduction of global SUSY [18, 19], which is an orbifold version of Scherk-Schwarz mechanism [20, 21], and then to the reduction of gauge symmetry [22].
where $\Psi_+$ and $\Psi_-$ are fermions with positive and negative chirality, respectively, and $\Gamma_7$ and $\gamma_5$ are the chirality operators for 6D fermions and 4D fermions, respectively. We use the following representation for the $8 \times 8$ gamma matrices $\Gamma^M (M = 0, 1, 2, 3, 5, 6)$,

$$\Gamma^\mu = \gamma^\mu \otimes \sigma^3, \quad \Gamma^5 = I_{4 \times 4} \otimes i\sigma^1, \quad \Gamma^6 = I_{4 \times 4} \otimes i\sigma^2,$$

where $\gamma^\mu (\mu = 0, 1, 2, 3, 4)$, $I_{4 \times 4}$ and $\sigma^i (i = 1, 2, 3)$ are the $4 \times 4$ gamma matrices, the $4 \times 4$ unit matrix and Pauli matrices, respectively. In the chiral representation, $\gamma^\mu$ are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix},$$

where $\sigma^\mu = (I, \sigma)$ and $\overline{\sigma}^\mu = (I, -\sigma)$. Here, $I$ is the $2 \times 2$ unit matrix. The $\Gamma^M$ satisfy the Clifford algebra $\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}$. Note that the theories are free of chiral anomalies, as a result of cancellations between contributions from $\Psi_+$ and those from $\Psi_-$. When we take the representation matrices

$$P_0 = \text{diag}([+1]_{p_1}, [+1]_{p_2}, [+1]_{p_3}, [+1]_{p_4}, [-1]_{p_5}, [-1]_{p_6}, [-1]_{p_7}, [-1]_{p_8}),$$
$$P_1 = \text{diag}([+1]_{p_1}, [+1]_{p_2}, [-1]_{p_3}, [+1]_{p_4}, [+1]_{p_5}, [-1]_{p_6}, [-1]_{p_7}, [-1]_{p_8}),$$
$$P_2 = \text{diag}([-1]_{p_1}, [+1]_{p_2}, [-1]_{p_3}, [+1]_{p_4}, [+1]_{p_5}, [-1]_{p_6}, [+1]_{p_7}, [-1]_{p_8}),$$

the breakdown of $SU(9)$ gauge symmetry occurs as

$$SU(9) \to SU(p_1) \times SU(p_2) \times \cdots \times SU(p_8) \times U(1)^{7-m},$$

where $[+1]_{p_i}$ represents $\pm 1$ for all $p_i$ elements, $p_1 + p_2 + \cdots + p_8 = 9$, and $m$ is a sum of the number of $SU(1)$ and $SU(0)$. Here, $SU(1)$ unconventionally stand for $U(1)$ and $SU(0)$ means nothing. Then, 6D fields with the rank-$k$ completely asymmetric tensor representation $\left(\begin{smallmatrix} 9 \\ k \end{smallmatrix}\right)$ are decomposed as

$$\left(\begin{smallmatrix} 9 \\ k \end{smallmatrix}\right) = \sum_{l_1=0}^{k} \sum_{l_2=0}^{k-l_1} \cdots \sum_{l_7=0}^{k-l_1-\cdots-l_6} \left(\begin{pmatrix} p_1 \\ l_1 \end{pmatrix}, \begin{pmatrix} p_2 \\ l_2 \end{pmatrix}, \cdots, \begin{pmatrix} p_7 \\ l_7 \end{pmatrix}, \begin{pmatrix} p_8 \\ l_8 \end{pmatrix}\right),$$

where $\left(\begin{smallmatrix} 9 \\ k \end{smallmatrix}\right)$ stands for the representation whose dimension is the combinatorial number, $l_8 = k - l_1 - l_2 - \cdots - l_7$ and $p_8 = 9 - p_1 - p_2 - \cdots - p_7$.

In case that $\Psi_+$ and $\Psi_-$ have $\left(\begin{smallmatrix} 9 \\ k \end{smallmatrix}\right)$ of $SU(9)$, we denote the intrinsic $Z_2$ parities of $\psi^1_L$ and $\psi^2_L$ as $\left(\eta^0_{k+}, \eta^1_{k+}, \eta^2_{k+}\right)$ and $\left(\eta^0_{k-}, \eta^1_{k-}, \eta^2_{k-}\right)$, respectively. Then, those of $\psi^1_R$ and $\psi^2_R$ are fixed as $\left(-\eta^0_{k+}, -\eta^1_{k+}, -\eta^2_{k+}\right)$ and $\left(-\eta^0_{k-}, -\eta^1_{k-}, -\eta^2_{k-}\right)$ from the $Z_2$ invariance of kinetic terms and the transformation properties of the covariant derivatives $Z_2$ : $D_x \to -D_x$ and $D_\xi \to -D_\xi$. On the breakdown of $SU(9)$ due to $\left(\begin{smallmatrix} 9 \\ 0 \end{smallmatrix}\right)$, the $Z_2$ parities of the component with the representation $\left(\begin{smallmatrix} p_1 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \end{smallmatrix}\right)$ are given by

$$\mathcal{P}_{0\pm} = (-1)^{l_5 + l_6 + l_7 + \cdots + l_8} \eta^0_{k\pm} = (-1)^{k - l_1 - l_2 - l_3 - l_4} \eta^0_{k\pm},$$

where $\eta^0_{k\pm}$ are determined.
In the first and second columns, particles are denoted by using the symbols $U_8$ and they belong to the following chiral fermions, $Z_{p_a}$ subgroup of $SU_2$. They are classified into two cases based on the pattern of gauge symmetry breaking such that $SU(9) \rightarrow SU(3)_C \times SU(2)_L \times SU(3)_F \times U(1)^3$ and $SU(9) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_F \times U(1)^4$. We study how well the three families of fermions are embedded into $\Psi_+ \text{ and } \Psi_-$, in the following.

2.2 $SU(9) \rightarrow SU(3)_C \times SU(2)_L \times SU(3)_F \times U(1)^3$

For the case that $p_1 = 3, p_2 = 2$, either of $p_3, p_4, p_5$ or $p_6$ is 3 and either of $p_7$ or $p_8$ is 1, $SU(9)$ is broken down as

$$SU(9) \rightarrow SU(3)_C \times SU(2)_L \times SU(3)_F \times U(1)_1 \times U(1)_2 \times U(1)_3,$$

where $SU(3)_F$ is the gauge group concerning the family of fermions, $U(1)_1$ belongs to a subgroup of $SU(5)$ and is identified with $U(1)_Y$ in the SM, and others are originated from $SU(9)$ and $SU(4)$ as

$$SU(9) \supset SU(5) \times SU(4) \times U(1)_2,$$

$$SU(4) \supset SU(3) \times U(1)_3.$$

Let us illustrate the survival of three families in the SM, using two typical BCs.

(BC1) $(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8) = (3, 2, 3, 0, 0, 0, 0, 1)$

In this case, 84 is decomposed into particles with the SM gauge quantum numbers and its opposite ones, and their $U(1)$ charges and $Z_2$ parities are listed in Table II. In the first and second columns, particles are denoted by using the symbols in the SM, and those with primes are regarded as mirror particles. Here, mirror particles are particles with opposite quantum numbers under the SM gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. The $U(1)$ charges are given up to the normalization. The $Z_2$ parities of $\psi^{(1,2)}_L$ are given by omitting the subscript $k(=3)$ in the last column. The $Z_2$ parities of $\psi^{(1,2)}_R$ are opposite to those of $\psi^{(1,2)}_L$.

When we assign the intrinsic $Z_2$ parities of $\psi^1_L$ and $\psi^2_L$ as

$$(\eta^0_L, \eta^+_L, \eta^-_L) = (+1, -1, +1), \quad (\eta^0_L, \eta^-_L, \eta^2_L) = (+1, -1, -1),$$

all mirror particles have an odd $Z_2$ parity and disappear in the low-energy world. Then, just three sets of SM fermions ($q^i_L, (u^i_R)^c, (d^i_R)^c, l^i_L, (e^i_R)^c$) survive as zero modes and they belong to the following chiral fermions,

$$\psi^1_L \supset (u^i_R)^c, (e^i_R)^c, (\nu_R)^c, \quad \psi^2_L \supset d^i_R, \quad \psi^1_R \supset (l^i_L)^c, \quad \psi^2_R \supset q^i_L.$$
Table 1: Decomposition of 84 for \((p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8) = (3, 2, 3, 0, 0, 0, 0, 1)\).

| \( \psi_{1(2)}^L \) | \( \psi_{1(2)}^R \) | \( SU(3)_C \times SU(2)_L \times SU(3)_R \) | \( U(1)_1 \) | \( U(1)_2 \) | \( U(1)_3 \) | \( (\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2) \) |
|-----------------|-----------------|---------------------------------|--------|--------|--------|-----------------|
| \( e'_R \) | \( e_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 1, 1) \) | -6 | 12 | 0 | \((+\eta^0, +\eta^1, +\eta^2)\) |
| \( q'_L \) | \( q_L \) | \((\frac{3}{2}, \frac{3}{0}, \frac{3}{0})\) | \( (\overline{3}, 2, 1) \) | -1 | 12 | 0 | \((+\eta^0, +\eta^1, -\eta^2)\) |
| \( u'_R \) | \( u_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (3, 1, 1) \) | 4 | 12 | 0 | \((+\eta^0, +\eta^1, +\eta^2)\) |
| \( u_R \) | \( u'_R \) | \((\frac{3}{2}, \frac{3}{0}, \frac{3}{0})\) | \( (\overline{3}, 1, 1) \) | -4 | 3 | 1 | \((+\eta^0, -\eta^1, +\eta^2)\) |
| \( q_L \) | \( q'_L \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (3, 2, 3) \) | 1 | 3 | 1 | \((+\eta^0, -\eta^1, -\eta^2)\) |
| \( e'_R \) | \( e_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 1, 1) \) | 6 | 3 | 1 | \((+\eta^0, -\eta^1, +\eta^2)\) |
| \( e'_R \) | \( e_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (3, 1, \overline{3}) \) | -2 | -6 | 2 | \((+\eta_0, +\eta_1, +\eta_2)\) |
| \( d'_R \) | \( d_R \) | \((\frac{3}{2}, \frac{3}{0}, \frac{3}{0})\) | \( (3, 1, 1) \) | -6 | -6 | -2 | \((+\eta^0, +\eta^1, -\eta^2)\) |
| \( d'_R \) | \( d_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (\overline{3}, 1, 1) \) | 3 | -6 | 2 | \((+\eta^0, +\eta^1, -\eta^2)\) |
| \( l'_L \) | \( l_L \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 2, \overline{3}) \) | 3 | -6 | -2 | \((+\eta^0, +\eta^1, +\eta^2)\) |
| \( l'_L \) | \( l_L \) | \((\frac{3}{2}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 2, 3) \) | 3 | -6 | -2 | \((+\eta^0, +\eta^1, +\eta^2)\) |
| \( (\nu_R) \) | \( \nu_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 1, 1) \) | 0 | -15 | 3 | \((+\eta^0, -\eta^1, +\eta^2)\) |
| \( (\nu_R) \) | \( \nu_R \) | \((\frac{3}{3}, \frac{3}{0}, \frac{3}{0})\) | \( (1, 1, \overline{3}) \) | 0 | -15 | -1 | \((+\eta^0, -\eta^1, -\eta^2)\) |
where \(i(=1,2,3)\) stands for the family index. By exchanging \(\eta_L^a\) for \(\eta_R^a\), \(\psi_L^1\) and \(\psi_R^2\) are exchanged for \(\psi_R^2\) and \(\psi_L^1\), respectively. Note that a right-handed neutrino \((\nu_R)^c\)

appears alone. We obtain the same result (19) by assigning the intrinsic \(Z_2\) parities suitably, in case with \(p_4, p_5\) or \(p_6 = 3\) in place of \(p_3 = 3\).

\[
(\text{BC2}) \quad (p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8) = (3, 2, 3, 0, 0, 0, 1, 0)
\]

In this case, 84 is decomposed into particles with the same gauge quantum numbers but slightly different \(Z_2\) parities from those of (BC1). Concretely, the third \(Z_2\) parity \(\mathcal{P}_2\) of fields with \(l_I = 1\) is opposite to that with \(l_I = 1\), i.e., \(\mathcal{P}_2\) of \((\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}), \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix})\), \((\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}), \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix})\), \((\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}), \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix})\) and \((\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}), \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix})\) is given by \(+\eta^2, -\eta^2, +\eta^2, -\eta^2\) and \(+\eta^2\), respectively.

Under the same assignment of the intrinsic \(Z_2\) parities as (17), all mirror particles have an odd \(Z_2\) parity and disappear in the low-energy world. Then, just three sets of SM fermions survive as zero modes such that

\[
psi^1_L \supset (u^c_R)^c, (e^c_R)^c, (\nu_R)^c, \quad psi^2_R \supset (l^c_L)^c, \quad psi^1_R \supset d^c_R, \quad psi^2_L \supset q^c_L.
\]

Note that \((u^c_R)^c\) and \(d^c_R\) are embedded into \(\psi^2_R\) and \(\psi^1_R\), respectively, different from the case of (BC1). We obtain the same result (19) by assigning the intrinsic \(Z_2\) parities suitably, in case with \(p_4, p_5\) or \(p_6 = 3\) in place of \(p_3 = 3\).

We summarize fermions with zero modes and those gauge quantum numbers in Table 2. Here, \(G_{323} = SU(3)_C \times SU(2)_L \times SU(3)_F\), \(l_a\) is a number appearing in a representation \(p^a\) of \(SU(3)_F\) for \(a = 3, 4, 5, 6\), and, in the 7-th and 8-th columns, the way of embeddings for the SM species are shown for \(p_8 = 1\) and \(p_7 = 1\), respectively.

### Table 2: Gauge quantum numbers of fermions with even \(Z_2\) parities for \(SU(9) \to G_{323} \times U(1)_1 \times U(1)_2 \times U(1)_3\).

| species | \(G_{323}\) | \((l_1, l_2, l_a)\) | \(U(1)_1\) | \(U(1)_2\) | \(U(1)_3\) | \(p_8 = 1\) | \(p_7 = 1\) |
|---------|-------------|------------------|-------------|-------------|-------------|-------------|-------------|
| \(q^c_L\) | (3, 2, 3)   | (1, 1, 1)        | 1           | 3           | 1           | \(\psi^2_R\) | \(\psi^2_L\) |
| \((u^c_R)^c\) | (3, 1, 3)   | (2, 0, 1)        | -4          | 3           | 1           | \(\psi^1_R\) | \(\psi^1_L\) |
| \(d^c_R\) | (3, 1, 3)   | (1, 0, 1)        | -2          | -6          | -2          | \(\psi^2_R\) | \(\psi^2_L\) |
| \((l^c_L)^c\) | (1, 2, 3)   | (0, 1, 1)        | 3           | -6          | -2          | \(\psi^1_R\) | \(\psi^1_L\) |
| \((e^c_R)^c\) | (1, 1, 3)   | (0, 2, 1)        | 6           | 3           | 1           | \(\psi^1_R\) | \(\psi^1_L\) |
| \((\nu_R)^c\) | (1, 1, 1)   | (0, 0, 3)        | 0           | -15         | 3           | \(\psi^1_R\) | \(\psi^1_L\) |

### 2.3 \(SU(9) \to SU(3)_C \times SU(2)_L \times SU(2)_F \times U(1)^4\)

For the case that \(p_1 = 3, p_2 = 2\), either of \((p_3, p_4)\) or \((p_5, p_6)\) is \((2, 1)\) or \((1, 2)\) and either of \(p_7\) or \(p_8\) is 1, \(SU(9)\) is broken down as

\[
SU(9) \to SU(3)_C \times SU(2)_L \times SU(2)_F \times U(1)_1 \times U(1)_2 \times U(1)_3 \times U(1)_4,
\]

(20)
where \( U(1)_1 \) belongs to a subgroup of \( SU(5) \) and is identified with \( U(1)_Y \) in the SM, and others are originated from \( SU(9), SU(4) \) and \( SU(3) \) as

\[
SU(9) \supset SU(5) \times SU(4) \times U(1)_2, \quad (21)
\]
\[
SU(4) \supset SU(3) \times U(1)_3, \quad (22)
\]
\[
SU(3) \supset SU(2) \times U(1)_4. \quad (23)
\]

The embedding of species are classified into two types, according to \( p_8 = 1 \) or \( p_7 = 1 \). For the case with \( p_8 = 1 \), just three sets of SM fermions survive as zero modes such that

\[
\psi_L^{1(2)} \supset (u_R^i)^c, (e_R^i)^c, q_L, \quad \psi_R^{2(1)} \supset d_R^i, (l_L)^c, \\
\psi_R^{1(2)} \supset d_R^i, (l_L^i)^c, \quad \psi_L^{2(1)} \supset (u_R)^c, (e_R)^c, q_L^i, (\nu_R)^c,
\]

where \( i = 1, 2 \). For the case with \( p_7 = 1 \), just three sets of SM fermions survive as zero modes such that

\[
\psi_L^{1(2)} \supset (u_R^i)^c, (e_R^i)^c, q_L, \quad \psi_R^{2(1)} \supset d_R^i, (l_L)^c, \\
\psi_R^{1(2)} \supset d_R^i, (l_L^i)^c, \quad \psi_L^{2(1)} \supset (u_R)^c, (e_R)^c, q_L^i, (\nu_R)^c,
\]

where \( i = 1, 2 \).

We summarize fermions with zero modes and those gauge quantum numbers in Table 3. Here, \( G_{322} = SU(3)_C \times SU(2)_L \times SU(2)_R \).

**Table 3:** Gauge quantum numbers of fermions with even \( Z_2 \) parities for \( SU(9) \to G_{322} \times U(1)_1 \times U(1)_2 \times U(1)_3 \times U(1)_4 \).

| species          | \( G_{322} \) | \( U(1)_1 \) | \( U(1)_2 \) | \( U(1)_3 \) | \( U(1)_4 \) | \( p_8 = 1 \) | \( p_7 = 1 \) |
|------------------|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \((u_R^i)^c\), \((e_R^i)^c\) | \((3, 1, 2)\) | -4           | 3            | 1            | 1            | \(\psi_L^{1(2)}\) | \(\psi_L^{1(2)}\) |
| \((u_R)^c\)     | \((3, 1, 1)\) | -4           | 3            | 1            | -2           | \(\psi_L^{2(1)}\) | \(\psi_L^{2(1)}\) |
| \(q_L^1, q_L^2\) | \((3, 2, 2)\) | 1            | 3            | 1            | 1            | \(\psi_L^{2(1)}\) | \(\psi_L^{2(1)}\) |
| \((e_R^i)^c\), \((l_L^i)^c\) | \((1, 1, 2)\) | 6            | 3            | 1            | 1            | \(\psi_L^{2(1)}\) | \(\psi_L^{2(1)}\) |
| \((e_R)^c\)     | \((1, 1, 1)\) | 6            | 3            | 1            | -2           | \(\psi_L^{2(1)}\) | \(\psi_L^{2(1)}\) |
| \(d_R^1, d_R^2\) | \((3, 1, 2)\) | -2           | -6           | -2           | 1            | \(\psi_R^{2(1)}\) | \(\psi_R^{2(1)}\) |
| \(d_R\)         | \((3, 1, 1)\) | -2           | -6           | -2           | -2           | \(\psi_R^{2(1)}\) | \(\psi_R^{2(1)}\) |
| \((l_L^i)^c\), \((l_L^i)^c\) | \((1, 2, 2)\) | 3            | -6           | -2           | 1            | \(\psi_R^{2(1)}\) | \(\psi_R^{2(1)}\) |
| \((\nu_L)^c\)   | \((1, 2, 1)\) | 3            | -6           | -2           | 1            | \(\psi_R^{2(1)}\) | \(\psi_R^{2(1)}\) |
| \((\nu_L)^c\)   | \((1, 1, 1)\) | 0            | -15          | 3            | 0            | \(\psi_L^{2(1)}\) | \(\psi_L^{2(1)}\) |
3 Predictions

3.1 Yukawa interactions

We examine whether four types of $SU(9)$ orbifold family unification models, where the embedding of the SM fermions are realized as (18), (19), (24) and (25), are realistic or not, by adopting the appearance of Yukawa interactions from interactions in the 6D bulk as a selection rule. This rule is not almighty to select models, because Yukawa interactions can be constructed on the fixed points of $T^2/Z_2$. Here, we carry out the analysis under the assumption that such brane interactions are small compared with the bulk ones in the absence of SUSY.

We assume that the Yukawa interactions in the SM come from interaction terms containing fermions in the bilinear form and products of scalar fields in the 6D bulk. From the Lorentz, gauge and $Z_2$ invariance, the Lagrangian density containing interactions among a pair of Weyl fermions ($\Psi_+, \Psi_-$) and scalar fields $\Phi^I$ on 6D space-time is, in general, written as

$$L_{\text{int}} = \sum_{a, \ldots, f} \tilde{\Psi}_+^{a b c} \Phi^{d e f} F_{a b e}^{d e} (\Phi^I) + \sum_{a, \ldots, f} \Psi_+^{T a b c} E^{d e f} G_{a b c d} (\Phi^I) + \text{h.c.}$$

$$= \sum \left( \tilde{\Psi}_L \tilde{\Psi}_R^2 + \tilde{\Psi}_R \tilde{\Psi}_L^2 \right) F^I (\Phi^I) + \sum \left( (\tilde{\Psi}_L^1)^c (\tilde{\Psi}_L^2) + (\tilde{\Psi}_R^1)^c \tilde{\Psi}_R^2 \right) G^I (\Phi^I) + \text{h.c.}, \quad (26)$$

where $\tilde{\Psi}_L \equiv \Psi_+ \Gamma^0$, $\tilde{\Psi}_L^{(2)} = \Psi_+^{(2)} \gamma^0$, and $(\tilde{\Psi}_L^{(2)})^c = i \gamma^0 \gamma^2 \psi_L^{(2)*}$. In the final expression of (26), we omit indices of $SU(9)$ such as $a, b, \ldots, f$ designating the components to avoid complications. The $F (\Phi^I)$ and $G (\Phi^I)$ are some polynomials of $\Phi^I$, e.g., $F (\Phi^I)$ is expressed by

$$F (\Phi^I) = \sum_{I_1} f_{I_1} \Phi^{I_1} + \sum_{I_1, I_2} f_{I_1 I_2} \Phi^{I_1} \Phi^{I_2} + \ldots = \sum_{n} \sum_{I_1, \ldots, I_n} f_{I_1 \ldots I_n} \Phi^{I_1} \ldots \Phi^{I_n}, \quad (27)$$

where $f_{I_1 \ldots I_n}$ are coupling constants. Note that mass terms of $\Psi_\pm$ such as $m_D \tilde{\Psi}_+ \tilde{\Psi}_-$ and $m_M \tilde{\Phi}_+ \tilde{\Phi}_-$ are forbidden at the tree level, in case that $\Psi_+$ and $\Psi_-$ have different intrinsic $Z_2$ parities. Using the representation given by (6) and (7), $E$ is written as

$$E \equiv \Gamma^1 \Gamma^2 \Gamma^6 = \begin{pmatrix}
0 & 0 & i\sigma^2 & 0 \\
0 & 0 & 0 & i\sigma^2 \\
-i\sigma^2 & 0 & 0 & 0 \\
0 & -i\sigma^2 & 0 & 0
\end{pmatrix}, \quad (28)$$

where $\sigma^2$ is the second element of Pauli matrices. It is shown that $L_{\text{int}}$ is invariant under the 6D Lorentz transformation, $\Psi_+ \rightarrow \exp \left[ -i \frac{1}{2} \omega_{MN} \Sigma^{MN} \right] \Psi_+$, where $\Sigma^{MN} = \frac{i}{2} \left[ \Gamma^M, \Gamma^N \right]$ and $\omega_{MN}$ are parameters relating 6D Lorentz boosts and rotations.

After the dimensional reduction occurs and some components acquire the vacuum expectation values (VEVs) generating the breakdown of extra gauge symmetries, the
linear terms of the Higgs doublet $\phi_h$ and its charge conjugated one $\bar{\phi}_h$ can appear in $F(\Phi^l)$ and $G(\Phi^l)$ and then the Yukawa interactions are derived. For instance, the linear term $\bar{f} \phi_h$ appears from $F(\Phi^l) = f \Phi_1 \Phi_3 \Phi_5$ where $\Phi_m$ are scalar fields whose representations are $(\frac{9}{m})$, after some SM singlets in $\Phi_3$ and $\Phi_5$ acquire the VEVs.

From the above observations, we impose the selection rule that Yukawa interactions $f^i_{ij} \bar{q}_L^i u_R^j \phi_h$, $f^d_{ij} \bar{\tau}_L^i d_R^j \phi_h$ and $f^e_{ij} \bar{\tau}_L^i e_R^j \phi_h$ in the SM can be derived from $\mathcal{L}_{\text{int}}$ on orbifold family unification models.

For (BC1), the following Lagrangian density is derived at the compactification scale $M_C$,

$$\mathcal{L}_{(BC1)} = \sum_{i,j=1}^{3} \bar{d}_R^i q_L^j \tilde{F}_{1ij}^{(1)}(\phi) + \sum_{i,j=1}^{3} \bar{\tau}_L^i e_R^j \tilde{F}_{2ij}^{(1)}(\phi) + \sum_{i,j=1}^{3} \bar{\tau}_R^i q_L^j \tilde{G}_{ij}^{(1)}(\phi) + \text{h.c.}, \quad (29)$$

using (18), and Yukawa interactions in the SM can be obtained, after some SM singlet scalar fields in the polynomials $\tilde{F}_{1ij}^{(1)}(\phi)$, $\tilde{F}_{2ij}^{(1)}(\phi)$ and $\tilde{G}_{ij}^{(1)}(\phi)$ acquire the VEVs. Because all gauge quantum numbers of the operator $\bar{\tau}_L^i d_R^j$ are same as those of $\bar{\tau}_L^i e_R^j$, there is a possibility that $\tilde{F}_{1ij}^{(1)}(\phi)$ is identical with $\tilde{F}_{2ij}^{(1)}(\phi)$ as a simple case. In this case, we have the relations $f^i_{ij} = f^i_{ji}$ at the extra gauge symmetry breaking scale.

For (BC2), the following Lagrangian density is derived,

$$\mathcal{L}_{(BC2)} = \sum_{i,j=1}^{3} \bar{\tau}_R^i q_L^j \tilde{G}_{ij}^{(2)}(\phi) + \text{h.c.}, \quad (30)$$

using (19). In this case, down-type quark and charged lepton masses cannot be obtained from $\mathcal{L}_{\text{int}}$ at the tree level at $M_C$.

For (BC3), the following Lagrangian density is derived,

$$\mathcal{L}_{(BC3)} = \sum_{i,j=1}^{2} \bar{d}_R^i q_L^j \tilde{F}_{1ij}^{(3)}(\phi) + \sum_{i,j=1}^{2} \bar{\tau}_L^i e_R^j \tilde{F}_{2ij}^{(3)}(\phi) + \sum_{i,j=1}^{2} \bar{\tau}_R^i q_L^j \tilde{G}_{ij}^{(3)}(\phi) + \text{h.c.}$$

$$+ \sum_{i,j=1}^{2} \bar{\tau}_R^i q_L^j \tilde{G}_{1ij}^{(3)}(\phi) + \bar{\tau}_L^i u_R \tilde{G}_{2}^{(3)}(\phi) + \text{h.c.}, \quad (31)$$

using (24). For (BC4), the following Lagrangian density is derived,

$$\mathcal{L}_{(BC4)} = \sum_{i=1}^{2} \left( \bar{d}_R^i q_L^i \tilde{F}_{1ii}^{(4)}(\phi) + \bar{\tau}_L^i d_R^i \tilde{F}_{2ii}^{(4)}(\phi) + \bar{\tau}_R^i e_R^i \tilde{F}_{3ii}^{(4)}(\phi) + \bar{\tau}_R^i u_R \tilde{G}_{4ii}^{(4)}(\phi) \right) + \text{h.c.}$$

$$+ \sum_{i,j=1}^{2} \bar{\tau}_R^i q_L^j \tilde{G}_{1ij}^{(4)}(\phi) + \bar{\tau}_L^i u_R \tilde{G}_{2}^{(4)}(\phi) + \text{h.c.}, \quad (32)$$

using (25). In both cases, the full flavor mixing cannot be realized at the tree level at $M_C$.

In this way, we find that the model based on the embedding (18) is a possible candidate to realize the fermion mass hierarchy and flavor mixing, in case that radiative
corrections are too small to generate mixing terms with suitable size for (BC2), (BC3) and (BC4). In any case, we have no powerful principle to determine the polynomials of scalar fields, and hence we obtain no useful predictions from the fermion sector.

### 3.2 Sfermion masses

The SUSY grand unified theories on an orbifold have a desirable feature that the triplet-doublet splitting of Higgs multiplets is elegantly realized [23, 24]. Hence, it would be interesting to construct a SUSY extension of orbifold family unification models.

In the presence of SUSY, the model with (BC1) does not obtain advantages of fermion sector over that with (BC2), (BC3) or (BC4), because any interactions other than gauge interactions are not allowed in the bulk and Yukawa interactions must appear from brane interactions. In SUSY models, complex scalar fields \( \Phi^+ \), \( \Phi^- \) are introduced as superpartners of \( \Psi^+ \), \( \Psi^- \), and they consist of two sets of complex scalar fields \( \Phi^+_1, \Phi^+_2 \) and \( \Phi^-_1, \Phi^-_2 \), respectively. Here, we pay attention to superpartners of the SM fermions called sfermions and study predictions of models.

Based on the assignment (18) for (BC1), sfermions are embedded into scalar fields as follows,

\[
\phi^+_1 \supset \tilde{u}^*_R, \quad \tilde{e}^*_R, \quad \phi^+_2 \supset \tilde{d}^*_R, \quad \phi^-_1 \supset \tilde{l}^*_L, \quad \phi^-_2 \supset \tilde{q}^*_L. \tag{33}
\]

Gauge quantum numbers for sfermions are given in Table 4. Here, the charge conjugation is performed for scalar fields \( \tilde{d}^*_R \) and \( \tilde{l}^*_L \) corresponding to the right-handed fermions, and \( G_{323} = SU(3)_C \times SU(2)_L \times SU(3)_F \). Note that \((l_1, l_2, l_3)\) is untouched by change as a mark of the place of origin in 84.

| species | \( G_{323} \) | \((l_1, l_2, l_3)\) | \( U(1)_1 \) | \( U(1)_2 \) | \( U(1)_3 \) |
|---------|---------------|-----------------|-----------|-----------|-----------|
| \( \tilde{q}^*_L \) | \((3, 2, 3)\) | \((0, 1, 1)\) | 0 | \(-15\) | 3 |
| \( \tilde{u}^*_R \) | \((3, 1, 3)\) | \((0, 2, 1)\) | 6 | 3 | 1 |
| \( \tilde{d}^*_R \) | \((3, 1, 3)\) | \((0, 1, 1)\) | 2 | 6 | 2 |
| \( \tilde{l}^*_L \) | \((1, 2, 3)\) | \((0, 1, 1)\) | \(-3\) | 6 | 2 |
| \( \tilde{e}^*_R \) | \((1, 1, 3)\) | \((0, 2, 1)\) | 6 | 3 | 1 |
| \( \tilde{\nu}^*_R \) | \((1, 1, 1)\) | \((0, 0, 3)\) | 0 | \(-15\) | 3 |

We study the sfermion masses based on the following two assumptions.

1) The SUSY is broken down by some mechanism and sfermions acquire the soft SUSY breaking masses respecting \( SU(9) \) gauge symmetry. Then, \( \tilde{u}^*_R, \tilde{e}^*_R, \tilde{\nu}^*_R \) and \( \tilde{d}^*_R \) get a common mass \( m_+ \), and \( \tilde{q}^*_L \) and \( \tilde{l}^*_L \) get a common mass \( m_- \) at some scale \( M_S \).

2) Extra gauge symmetries \( SU(3)_F \times U(1)_2 \times U(1)_3 \) are broken down by the VEVs
of some scalar fields at $M_S$. Then, the $D$-term contributions to the scalar masses can appear as a dominant source of mass splitting.

The $D$-term contributions, in general, originate from $D$-terms related to broken gauge symmetries when the soft SUSY breaking parameters possess non-universal structure and the rank of gauge group decreases after the breakdown of gauge symmetry $[25] [28]$. The contributions for scalar fields specifying by $(l_1, l_2, l_a)$ are given by

$$m^2_{D(l_1, l_2, l_a)} = (-1)^{l_1 + l_2} [Q_1 D_{F1} + Q_2 D_{F2} + \{9(l_1 + l_2) - 15\} D_2 + \{4l_a - 3(3 - l_1 - l_2)\} D_3],$$

where $Q_1$ and $Q_2$ are the diagonal charges (up to normalization) of $SU(3)_F$ for the triplet, i.e., $(Q_1, Q_2) = (1, 1), (-1, 1)$ and $(0, -2)$. $D_{F1}, D_{F2}, D_{2}$ and $D_{3}$ are parameters including $D$-term condensations for broken symmetries.

Using $m_+, m_-$ and $m^2_{D(l_1, l_2, l_a)}$, we derive the following formulae of mass square for each species at $M_S$. \[3\]

$$m^2_{\tilde{u}_R} = m^2_+ + D_{F1} + D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{c}_R} = m^2_+ - D_{F1} + D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{t}_R} = m^2_+ - 2D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{t}^3_R} = m^2_+ + D_{F1} + D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{t}^2_R} = m^2_+ - D_{F1} + D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{t}^1_R} = m^2_+ = 2D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{d}^3_R} = m^2_+ - D_{F1} - D_{F2} + 6D_2 + 2D_3,$$

$$m^2_{\tilde{d}^2_R} = m^2_+ + D_{F1} - D_{F2} + 6D_2 + 2D_3,$$

$$m^2_{\tilde{d}^1_R} = m^2_+ = 2D_{F2} + 6D_2 + 2D_3,$$

$$m^2_{\tilde{q}_1} = m^2_+ + D_{F1} + D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{q}_2} = m^2_+ + D_{F1} - D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{q}_3} = m^2_+ = 2D_{F2} + 3D_2 + D_3,$$

$$m^2_{\tilde{f}_1} = m^2_+ - D_{F1} - D_{F2} + 6D_2 + 2D_3,$$

$$m^2_{\tilde{f}_2} = m^2_+ + D_{F1} + D_{F2} + 6D_2 + 2D_3,$$

$$m^2_{\tilde{f}_3} = m^2_+ = 2D_{F2} + 6D_2 + 2D_3.$$

By eliminating unknown parameters such as $m^2_+, m^2_-, D_{F1}, D_{F2}, D_{2}$ and $D_{3}$, we
obtain 15 kinds of relations:\footnote{Sum rules among sfermion masses have also been derived using the orbifold family unification models on five-dimensional (5D) space-time\cite{20,31}.

\[ m_{\tilde{u}_{R}}^2 = m_{\tilde{u}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{L}}^2 = m_{\tilde{d}_{L}}^2, \quad m_{\tilde{u}_{R}}^2 = m_{\tilde{u}_{R}}^2, \]
\[ m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{L}}^2 - m_{\tilde{d}_{L}}^2 = m_{\tilde{d}_{L}}^2 - m_{\tilde{d}_{L}}^2, \]
\[ = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{L}}^2 - m_{\tilde{d}_{L}}^2. \] \]

They are compactly rewritten as
\[ m_{\tilde{u}_{R}}^2 = m_{\tilde{u}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \]
\[ m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = -m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = -m_{\tilde{d}_{L}}^2 + m_{\tilde{d}_{L}}^2, \]
where \( i, j = 1, 2, 3 \).

Furthermore, we obtain the specific relations,
\[ m_{\tilde{u}_{R}}^2 - m_{\tilde{u}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \]
\[ m_{\tilde{u}_{R}}^2 + m_{\tilde{u}_{R}}^2 = m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2, \]

for (BC3) and
\[ m_{\tilde{u}_{R}}^2 - m_{\tilde{u}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 - m_{\tilde{d}_{R}}^2, \]
\[ m_{\tilde{u}_{R}}^2 + m_{\tilde{u}_{R}}^2 = m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2, \quad m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2 = m_{\tilde{d}_{R}}^2 + m_{\tilde{d}_{R}}^2, \]

for (BC4). Here, \( i, j = 1, 2, 3 \) and we denote \( \tilde{u}_{R}, \tilde{d}_{R}, \tilde{d}_{L}, \tilde{l}_{R} \) and \( \tilde{q}_{L} \) as \( \tilde{u}_{R}^3, \tilde{e}_{R}^3, \tilde{d}_{R}^3, \tilde{l}_{L}^3 \) and \( \tilde{q}_{L}^3 \). The relations for (BC4) are obtained by exchanging \( m_{\tilde{d}_{R}}^2 \) for \( m_{\tilde{d}_{L}}^2 \) in those for (BC3).

The above relations become predictions to probe models because they are specific to models, in case that the extra gauge symmetry breaking scale is near \( M_S \).
4 Conclusions and discussions

We have taken orbifold family unification models based on $SU(9)$ gauge symmetry on $M^4 \times T^2/Z_2$ as a starting point and have examined the reality of models, by adopting the appearance of Yukawa interactions from the interactions in the 6D bulk as a selection rule. We have picked out a candidate of model compatible with the observed fermion masses and flavor mixing. The model has an feature that just three families of fermions in the SM exist as zero modes and any mirror particles do not appear in the low-energy world after the breakdown of gauge symmetry $SU(9) \rightarrow SU(3)_C \times SU(2)_L \times SU(3)_F \times U(1)^3$ by orbifolding. Depending on the assignment of intrinsic $Z_2$ parities, $((u_R^c)^c, (e_R^c)^c, d_R^c)$ and $((l_R^c)^c, q_R^c)$ belong to $\Psi_\pm$ and $\Psi_\mp$ with $84$ of $SU(9)$, respectively. We have found out specific relations among sfermion masses as model-dependent predictions in the SUSY extension of models.

The mass degeneracy for each squark and slepton species in the first two families is favorable for suppressing flavor-changing neutral current (FCNC) processes. The $D$-term contributions relating $SU(3)_F$, however, can spoil the mass degeneracy. Such dangerous situations induce sizable FCNC processes can be avoided if the sfermion masses in the first two families are rather large, the fermion and its superpartner mass matrices are aligned, or the $D$-term contributions to lift the degeneracy are small enough.

As a future work, we need to answer the question whether the fermion mass spectrum and flavor mixing are successfully achieved at the low energy scale, in our orbifold family unification model.

Acknowledgments

This work is supported in part by funding from Nagano Society for The Promotion of Science (Y. Goto).

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