Multiplicity distributions in a thermodynamical model of hadron production in $e^+e^-$ collisions

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Abstract

Predictions of a thermodynamical model of hadron production for multiplicity distributions in $e^+e^-$ annihilations at LEP and PEP-PETRA centre of mass energies are shown. The production process is described as a two-step process in which primary hadrons emitted from the thermal source decay into final observable particles. The final charged tracks multiplicity distributions turn out to be of Negative Binomial type and are in quite good agreement with experimental observations. The average number of clans calculated from fitted Negative Binomial coincides with the average number of primary hadrons predicted by the thermodynamical model, suggesting that clans should be identified with primary hadrons.

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1 Introduction

Multiplicity Distributions (MDs) are fundamental observables in multiparticle production in $e^+e^-$ collisions. Theoretical investigations of their properties, in the QCD framework, are based on a comparison of approximate calculations at parton level with experimentally observed final particles distributions via the assumption of the Local Parton Hadron Duality [1] or its generalization [2]. In this context the important role of resonances and particles decay is not explicitly taken into account. This fact is particularly unsatisfactory because a large fraction of final particles are indeed decay products of heavier ones. A second approach is based on Monte Carlo models, like JETSET [3] and HERWIG [4], which account for the decay chain following the perturbative phase and hadronization. However, predictions of these models are obtained by using hadronization schemes requiring a large number of free parameters in order to reproduce experimental data.

In this paper we discuss the MDs obtained in a thermodynamical model of multiparticle production in $e^+e^-$ annihilations, introduced by one of the authors [5], which successfully reproduces the production rates of the various hadrons species both at PEP-PETRA and LEP centre of mass energies. The model is based on the identification of jets with thermalized hadron gas phases after the hadronization of primary quarks has taken place. As a first approximation it is assumed that only two phases with opposite momenta are generated in an $e^+e^\to q\bar{q}$ event, namely multi-jets events are neglected. The observed multi-particle production in hadronic events is the result of a two-step process: in the first one some primary hadrons (particles and resonances) emerge directly from the thermalized phases after having decoupled. These primary hadrons then decay according to known decay modes and branching ratios, giving rise to observable particles in the detector. Only three parameters describe those phases at the decoupling time: the temperature $T$, the volume $V$ and a strangeness chemical equilibrium suppression parameter $\gamma_s$. They are determined by fitting the calculated average production rates of each hadron to those measured both at LEP ($\sqrt{s}\approx 91$ GeV) and at PEP-PETRA ($\sqrt{s}= 29 \div 35$ GeV). A major role in the determination of the hadron rates is played by the conservation laws: the quantum numbers of the jet are related to those of the primary quark from which the jet itself originated. Accordingly, it is assumed that each jet keeps the charm and beauty of the parent quark while non-vanishing baryon number and strangeness are allowed provided that the baryon number and strangeness of the whole system are zero.

2 Thermal fluctuations and multiplicity distributions

Consider a hadron gas at temperature $T$ and volume $V$. If $T \approx \mathcal{O}(100)$ MeV, i.e. less than the mass of all hadrons but pions, the simple Boltzmann statistics holds for all hadrons except pions, which indeed obey Bose statistics [5]. Therefore, if the quantum numbers of the gas are not fixed, each species of hadron fluctuates independently according to a Poisson distribution, whereas pions fluctuate according to a different distribution that we define as the $\pi$-distribution $f_\pi$ and we derive in Appendix A. On the other hand, if the flavour quantum numbers of the gas are fixed, conservation laws generate correlations between different hadron species, so that fluctuations are no longer independent.

Therefore, the probability to observe $n_1 \pi^+$, $n_2 \pi^0$, $n_3 \pi^-$, $n_4$ hadrons of kind 4,..., $n_K$ hadrons of kind $K$ in a hadron gas system is given by:
\[ P(n_1, \ldots, n_K) = \frac{1}{Z(Q)} f_\pi(n_1, n_2, n_3) \prod_{i=4}^{K} z_{n_i}^{n_i} e^{-z_i} \delta_{Q, \sum_{i=1}^{K} n_i q_i}, \]

where

\[ z_i = (2J_i + 1) \frac{V}{(2\pi)^3} \int d^3p \ e^{-\frac{\sqrt{p^2+m_i^2}}{T}}, \]

\[ Q = (N, S, C, B) \]

is a four dimensional vector with integer components representing baryon number, strangeness, charm and beauty of the gas; \( q_i, J_i, m_i \) are the quantum numbers vector, the spin and the mass of the \( i^{th} \) particle and \( f_\pi \) is the \( \pi \)-distribution. The factor \( \delta_{Q, \sum_{i=1}^{K} n_i q_i} \) accounts for the conservation of the mentioned quantum numbers. \( Z(Q) \) is the partition function of the system, namely:

\[ Z(Q) = \sum_{n_1=0}^{\infty} \ldots \sum_{n_K=0}^{\infty} f_\pi(n_1, n_2, n_3) \prod_{i=4}^{K} z_{n_i}^{n_i} e^{-z_i} \delta_{Q, \sum_{i=1}^{K} n_i q_i}. \]

With the substitution

\[ \delta_{Q, \sum_{i=1}^{K} n_i q_i} = \frac{1}{(2\pi)^4} \int d^4\phi \ \exp \left\{ i(Q - \sum_{i} n_i q_i) \cdot \phi \right\} , \]

where \( \phi = (\phi_1, \phi_2, \phi_3, \phi_4) \), Eq. (3) becomes:

\[ Z(Q) = \frac{F_\pi}{(2\pi)^3} \int d^4\phi \ e^{iQ\phi} \exp \left\{ \sum z_i e^{-iz_i} \cdot \phi \right\} , \]

which is the expression obtained for the hadron gas partition function in [6] starting from the general formulae of partition functions of thermodynamical systems with internal symmetry [3].

The function \( F_\pi \) turns out to be:

\[ F_\pi = \exp \left\{ -\sum_{i=1}^{3} \frac{V}{(2\pi)^3} \int d^3p \ \log \left( 1 - e^{-\frac{\sqrt{p^2+m_i^2}}{T}} \right) \right\} ; \]

where the sum runs over the three pion states.

From Eq. (1) one can build up the expression of the joint probability to observe a \( K \)-uple \( n = (n_1, \ldots, n_K) \) of numbers of primary hadrons in the first jet and a \( K \)-uple \( m = (m_1, \ldots, m_K) \) in the second jet

\[ P(n, m) = \frac{1}{Z} f_\pi(n_1, n_2, n_3) f_\pi(m_1, m_2, m_3) \prod_{i=4}^{K} z_{n_i}^{n_i} e^{-z_i} \prod_{i=4}^{K} z_{m_i}^{m_i} e^{-z_i} \delta_{0, \sum_i n_i q_i + m_i q_i} ; \]

with \( \hat{Z} \) given by

\[ \hat{Z} = \sum_{n_1=0}^{\infty} \ldots \sum_{n_K=0}^{\infty} \sum_{m_1=0}^{\infty} \ldots \sum_{m_K=0}^{\infty} f_\pi(n_1, n_2, n_3) f_\pi(m_1, m_2, m_3) \cdot \prod_{i=4}^{K} z_{n_i}^{n_i} e^{-z_i} \prod_{i=4}^{K} z_{m_i}^{m_i} e^{-z_i} \delta_{0, \sum_i n_i q_i + m_i q_i} . \]

It should be pointed out that the partition function of the two-jet system is further modified by other constraints.
These constraints can be easily implemented by multiplying the right-hand side of Eq. (7) by additional suitable $\delta$'s and by recalculating accordingly the $\bar{Z}$ in the Eq. (8). Furthermore, the probability of production of primary hadrons containing $n$ strange valence quarks should be multiplied by a suppression factor $\gamma_s^n$.

The Eq. (7) suggests a simple procedure for calculating the charged tracks MDs in $e^+e^- \rightarrow q\bar{q}$ events with a Monte-Carlo method. The first step of the procedure is to pick up two $K$-uples $((n_1, \ldots, n_K), (m_1, \ldots, m_K))$ randomly according to Poisson or $\pi$-distribution and to accept the event only if the conditions imposed by the $\delta$'s in Eq. (7) and by the other requirements are fulfilled. The second step is to perform all decays of the generated primary hadrons according to known branching ratios, until $\pi$, $K$, $K_L^0$, $\mu$ or stable particles are reached, in order to match the MDs measured by $e^+e^-$ colliders experiments, including all decay products of particles with $c\tau < 10$ cm.

### 3 Results

We calculated the multiplicity distributions by the Monte-Carlo method described in the previous section at LEP and PEP-PETRA centre of mass energies. We used the same values of the parameters $T, V$ and $\gamma_s$ determined in [1] (see Table 1) by fitting the calculated average production rates of various hadron species to the measured ones [3]. Two additional parameters are introduced in order to take into account the possibility of different values of $V$ as a function of the primary quark mass, i.e. two dimensionless variables $x_c, x_b \in [0, 1]$ such that $x_c V$ is the volume in $e^+e^- \rightarrow c\bar{c}$ and $x_b V$ in $e^+e^- \rightarrow b\bar{b}$ events, whereas $V(1-x_c R_c-x_b R_b)/(R_u+R_d+R_s)$ is the volume for $e^+e^- \rightarrow q\bar{q}$ where $q$ is a light quark and $R_q = \sigma(e^+e^- \rightarrow q\bar{q})/\sigma(e^+e^- \rightarrow \text{hadrons})$. The factor $x_c$ ($x_b$) has been determined constraining the difference $\delta_{cq}$ ($\delta_{bq}$) between the average charged multiplicity in $e^+e^- \rightarrow c\bar{c}$ ($e^+e^- \rightarrow b\bar{b}$) and the average charged multiplicity in $e^+e^- \rightarrow q\bar{q}$, with $q = u, d, s$ for LEP, $q = u, d, s, c, b$ for PEP-PETRA to be equal to the measured ones [3,10]. The measured values of $\delta_{cq}$ and $\delta_{bq}$ have been averaged according to the procedure described in [11].

We generated 10000 events for each centre of mass energy in order to get the MDs of primary hadrons and of charged tracks both in single hemisphere (i.e. single jet according to the thermodynamical model) and in full phase space.

The calculated MDs are fitted with a Negative Binomial (NB) distribution with Maximum Likelihood method (see Fig. 1a, 1b and 2a, 2b). Average charged tracks multiplicity $\bar{\pi}$ and parameter $k$ (linked to dispersion $D$ by the relation $D^2 = \bar{\pi} + \bar{\pi}^2/k$) obtained from the fit are quoted in Table 2 together with corresponding values from analogous NB fits to experimental data [12,13]. Two kinds of error affect the determination of the NB parameters: the first one is the fit error, due to the limited statistics of 10000 events; the second one is related to the
uncertainty on the parameters of the thermodynamical model $T, V, \gamma_s, x_c, x_b$ used as input in the event generation. This latter error, a systematic one, has been estimated by varying the values of parameters by their error, as quoted in Table 1, generating a new sample of events and repeating the NB fit. The covariance matrix $M$ of $\pi$ and $k$ has been determined with the usual formula

$$M = J C J^T$$

where $C$ is the covariance matrix of $T, V, \gamma_s, x_c, x_b$ and $J$ is the jacobian matrix relating $(\pi, k)$ to $(T, V, \gamma_s, x_c, x_b)$.

The obtained systematic errors dominate over statistical ones, as shown in Table 2. Since the $\chi^2$ test on the NB fit consistency is also affected by those systematic effects, we estimated the uncertainty on $\chi^2$ according to the same procedure used for $\pi$ and $k$. Taking this error into account, the $\chi^2$ test indicates a clear compatibility with the NB distribution within the uncertainty of the input parameters.

The agreement between the parameters obtained by the model and those obtained from NB fits to experimental data is remarkable (see Table 2). However, discrepancies arise in the direct comparison between calculated distributions and measured ones [13,14], as shown in Fig. 3. The overestimation of the probability distribution in the low-multiplicity bins at PETRA energies and the consequent increase of the dispersion $D$, can be attributed to violations of the statistical framework of the canonical ensemble at low centre of mass energies. It is expected indeed that the probability of producing a low number of primary hadrons (for instance 0 or 1) should be strongly suppressed in comparison to the predictions of the canonical distribution, due to the exact energy conservation. This deviation from the canonical behaviour is much less visible at LEP energies, where the average number of primary hadrons is higher.

Another discrepancy is related to the so-called ”shoulder effect”: the experimental charged tracks MD deviates from the NB distribution due to a shoulder structure in the high multiplicity region, which is interpreted as the effect of the superposition of low-multiplicities 2-jets events with multi-jets events yielding higher multiplicities. On the other hand, it has been shown that good NB fits are obtained with MDs in selected 2, 3 or 4 jets event samples as well as with individual jets MDs [15]. Since the thermodynamical model is based on a 2-jet scheme and neglects multi-jets events, an analogous disagreement with data is expected, while the agreement with NB confirms its validity in reproducing MDs in 2-jets events. It should be mentioned also that a comparison between our predictions and a 2-jet events data sample is not possible because the parameters of the model have been tuned to reproduce the production rates of the hadron species in the overall $e^+e^- \rightarrow q\bar{q}$ data sample.

4 Comparison with the clan model

The clan model has been introduced in order to interpret the wide occurrence of the NB regularity of charged tracks MDs in all high energy reactions [16]. This model is based on the assumption that initial independent, i.e. Poissonian, production of primary objects called clans, is followed in the second step by their decay into final particles according to a logarithmic distribution. The average number of clans is determined in terms of the $\pi$ and $k$ parameters of the observed Negative Binomial distribution according to the formula:

$$\overline{N}_c = k \log \left(1 + \pi/k\right).$$

Since the thermodynamical model also contains a two step structure of the hadron production process consisting in an initial generation of a number of primary hadrons from the thermal
source which then decay into final observable particles, one can ask whether clans can be identified with the primary hadrons of the thermodynamical model. The average number of primaries, in this case, is determined independently by using the parameters of the thermodynamical model determined as mentioned in Sect. 3.

Results of this comparative analysis are summarized in Table 3. The agreement between the average number of primaries as resulting from the two models is striking. It should be noticed that in the clan model primaries are distributed according to an exact Poisson distribution, while in the thermodynamical model the distribution of primaries can be fitted by a NB distribution with a quite large $k$ value, which can be well approximated by a Poisson distribution (see Fig. 1c, 2c). This small deviation from an exact Poissonian behaviour is a consequence of the presence of conservation laws as specified in the $\delta$’s of Eq. 8, and of correlations contained in the $\pi$-distribution for primary pions. It should be noticed, however, that the $\pi$-distribution, with the actual values of $T$ and $V$, is very close to a Poisson distribution (see Table 4).

5 Conclusions

The thermodynamical model of hadron production in $e^+e^-$ collisions has been applied to the study of the charged tracks multiplicity distributions. Predictions of the model reproduce the gross features of experimental data both at LEP and PEP-PETRA centre of mass energies. The observed discrepancy with experimental measured MD at LEP energy is due to the presence of multi-jets events which are neglected in the model (shoulder effect). A natural violation of the canonical distribution, expected at low energy, is responsible for the observed deviation from the experimental measured MD at PETRA in the low multiplicity region. Also, the predicted charged tracks MDs are shown to be in very good agreement with the long-standing observed NB regularity and both at LEP and PEP-PETRA centre of mass energies. The primary hadrons MD is also well fitted by a NB distribution with a quite large value of $k$ parameter, close to the Poissonian limit. The average number of primary hadrons, independently determined in the thermodynamical model, is approximately equal to the average number of clans calculated from the parameters of the NB fit to the final charged tracks MDs. This identity suggests identifying clans with primary hadrons and confirms the intuition that the occurrence of Negative Binomial distribution is deeply related to a two-step process [16].

These results confirm previous successful predictions of the thermodynamical model, opening a new perspective in the understanding of the hadronization process at high energy. An extension of this model to include multi-jets events is going to be pursued in order to reproduce also fine features of experimental multiplicity distributions.
Appendix

A Determination of the $\pi$-distribution

In a boson gas at temperature $T$ and volume $V$ the probability to observe $n_1$ particles in the kinematical state 1 of energy $\varepsilon_1$, $n_2$ particles in the kinematical state 2 of energy $\varepsilon_2$, ... is:

$$P(n_1, \ldots, n_M) = \prod_i (1 - e^{-\varepsilon_i/T}) e^{-n_i\varepsilon_i/T}.$$  \hfill (11)

The probability that the overall population is $N$ can be written as:

$$P(N) = \sum_{n_1=0}^{\infty} \ldots \sum_{n_M=0}^{\infty} \prod_i (1 - e^{-\varepsilon_i/T}) e^{-n_i\varepsilon_i/T} \delta_{N, \sum_i n_i}.$$  \hfill (12)

Since:

$$\delta_{N, \sum_i n_i} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp\{-i(N - \sum_i n_i)\phi\},$$  \hfill (13)

Eq. (12) becomes:

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iN\phi} \prod_i \frac{1 - e^{-\varepsilon_i/T}}{1 - e^{-\varepsilon_i/T+i\phi}}.$$  \hfill (14)

In the limit of a continuum of energy levels Eq. (14) can be rewritten as:

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \ e^{-iN\phi} \exp\{(2J+1)\frac{V}{(2\pi)^3} \int d^3p \ \log \frac{1 - e^{-\varepsilon/T}}{1 - e^{-\varepsilon/T+i\phi}}\}.$$  \hfill (15)

where $\varepsilon = \sqrt{p^2 + m^2}$ and $J$ is the spin of the particle.

Eq. (15) in this form can be transformed into an integral over the unitary circle in the complex plane. Let $w = \exp\{i\phi\}$ and

$$I = \exp\{(2J+1)\frac{V}{(2\pi)^3} \int d^3p \ \log (1 - e^{-\varepsilon/T})\}.$$  \hfill (16)

Then:

$$P(N) = \frac{I}{2\pi i} \int_{w=0} \frac{dw}{w^{N+1}} \exp\{- (2J+1)\frac{V}{(2\pi)^3} \int d^3p \ \log (1 - e^{-\varepsilon/T}w)\}.$$  \hfill (17)

The residuals theorem can be used to get the final expression of $P(N)$:

$$P(N) = \frac{I}{N!} \lim_{w \to 0} \frac{d^N}{dw^N} \exp\{- (2J+1)\frac{V}{(2\pi)^3} \int d^3p \ \log (1 - e^{-\varepsilon/T}w)\}.$$  \hfill (18)

It turns out that the derivatives in $w = 0$ can be expressed as a function of the functions $z_{(n)}$ defined as:

$$z_{(n)} = \frac{V}{(2\pi)^3} \int d^3p \ e^{-n\sqrt{p^2 + m^2}/T} = (2J+1)\frac{VT}{2\pi^2 n} m^2 K_2\left(\frac{nm}{T}\right)$$  \hfill (19)

where $K_2$ is the modified Bessel function of order 2.

Finally, by using the above expression of $P(N)$ one gets the $\pi$-distribution $f_{\pi}(n_1, n_2, n_3) =$
$P_{+}(n_1)P_{0}(n_2)P_{-}(n_3)$. In Table 4 we show probabilities $P_{+}(N)$ of producing $N\ \pi^+$ up to $N = 6$ at $T = 163$ MeV and $V = 20$ Fm$^3$. The probability values are compared with those obtained from a Poisson distribution with the same average value $N$. 
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Figure captions

Figure 1 Multiplicity distributions calculated in the framework of the thermodynamical model at \( \sqrt{s} = 91.2 \) GeV (dots) and corresponding Negative Binomial fits (solid lines): a) charged tracks in full phase space, b) charged tracks in a single hemisphere, c) primary hadrons.

Figure 2 Multiplicity distributions calculated in the framework of the thermodynamical model at \( \sqrt{s} = 29 \div 35 \) GeV (dots) and corresponding Negative Binomial fits (solid lines): a) charged tracks in full phase space, b) charged tracks in a single hemisphere, c) primary hadrons.

Figure 3 Charged tracks multiplicity distributions in full phase space; comparison between predictions of the thermodynamical model (histogram) and data (dots) [13,14] at a) \( \sqrt{s} = 91.2 \) GeV and b) \( \sqrt{s} = 29 \div 35 \) GeV.
Tables

| Parameters | $\sqrt{s} = 91.2$ GeV | $\sqrt{s} = 29 \div 35$ GeV |
|------------|-----------------------|-----------------------------|
| Temperature(MeV) | 162.9±2.1 | 169.3±3.5 |
| Volume(Fm$^3$) | 21.4±1.9 | 9.3±1.4 |
| $\gamma_s$ | 0.696±0.027 | 0.811±0.046 |
| $x_c$ | 0.884±0.029 | 0.80±0.12 |
| $x_b$ | 0.695±0.016 | 0.47±0.15 |

Table 1: Values of the parameters of the thermodynamical model [4].

| Parameters | $\sqrt{s} = 91.2$ GeV | $\sqrt{s} = 34$ GeV |
|------------|-----------------------|-----------------------------|
| $\bar{\pi}$ | Calculated | Measured | Calculated | Measured |
| $k$ | 28.34 ± 1.43 ± 1.41 | 24.33 ± 0.71 | 13.03 ± 0.58 ± 0.66 | 52.63 ± 5.6 |
| $D$ | 6.14 ± 0.07 ± 0.17 | 6.28 ± 0.43 | 5.04 ± 0.06 ± 0.50 | 4.14 ± 0.39 |
| $\chi^2$/dof | 1.81 ± 0.60 | 2.35 | 10.9 ± 5.53 | 2.38 |

| Parameters | $\sqrt{s} = 91.2$ GeV | $\sqrt{s} = 34$ GeV |
|------------|-----------------------|-----------------------------|
| $\bar{\pi}$ | Calculated | Measured | Calculated | Measured |
| $k$ | 16.42 ± 0.65 ± 0.45 | 15.06 ± 0.39 | 9.14 ± 0.35 ± 0.78 | 50.0 ± 5.0 |
| $D$ | 4.21 ± 0.035 ± 0.056 | 4.19 ± 0.32 | 3.30 ± 0.03 ± 0.23 | 2.78 ± 0.22 |
| $\chi^2$/dof | 1.76 ± 0.30 | 2.87 | 8.83 ± 2.8 | 4.68 |

Table 2: Results of Negative Binomial fits to calculated charged tracks multiplicity distributions compared to results of analogous fits to experimental data [12, 13]. The first quoted error is the fit error, the second one is due to the uncertainty on parameters of the thermodynamical model. The dispersion $D$ is determined with the formula $D^2 = \bar{\pi} + \bar{\pi}^2/k$. 
Table 3: Average number of clans $N_c$ extracted from the Negative Binomial fits to charged tracks multiplicity distributions compared with the average number of primary hadrons $N_p$. Also shown are the results of a Negative Binomial fit to the calculated primary hadron distribution ($k$ and $\chi^2$). The first quoted error is the fit error, the second one is due to the uncertainty on parameters of the thermodynamical model.

|         | $\sqrt{s} = 91.2$ GeV | $\sqrt{s} = 29 \div 35$ GeV |
|---------|------------------------|-----------------------------|
| $N_c$   | 15.97 ± 0.20 ± 0.49    | 8.91 ± 0.12 ± 0.78          |
| $N_p$   | 17.10 ± 0.07 ± 0.55    | 9.39 ± 0.05 ± 0.36          |
| $k$     | 41.42 ± 3.09 ± 9.45    | 32.18 ± 3.11 ± 26.0         |
| $\chi^2$/dof | 1.63 ± 0.60          | 2.04 ± 0.99                 |

Table 4: Probability of producing $N \pi^+$ at $T = 163$ MeV and $V = 20$ Fm$^3$ compared to a Poisson distribution with the same average value.

| $N$ | $P_\pi(N)$ | Poisson |
|-----|------------|---------|
| 0   | 0.311      | 0.293   |
| 1   | 0.346      | 0.359   |
| 2   | 0.208      | 0.221   |
| 3   | 0.0903     | 0.0904  |
| 4   | 0.0320     | 0.0278  |
| 5   | 0.00998    | 0.00682 |
| 6   | 0.00287    | 0.00139 |
Figure 1:

(a) Full phase space

(b) Single hemisphere

(c) Primary hadrons
Figure 2:

a) Full phase space

b) Single hemisphere

c) Primary hadrons
Figure 3: