Numerical solution of the inverse coefficient problem of filtration in a multilayer reservoir

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Abstract.
This work proposes a computational algorithm based on the regularization method for determining the hydraulic conductivity coefficient of a layered oil reservoir. The results of hydrodynamic investigations of vertical wells, opening a layered reservoir, are used as initial information.

1. Introduction
The majority of oil reservoirs has a layered structure due to features of the sedimentation process. If the ratio of the permeability coefficients of two neighboring layers is less than $10^{-3}$ then the Myatiev-Girinskii scheme is applicable [1].

One of the field hydrodynamic methods for determining the reservoir properties of a multilayer reservoir is the method of steady-state selection [2, 3]. The problem of determining the coefficient of hydraulic conductivity of a multilayered oil reservoir is among the class of inverse problems of underground hydromechanics. This work proposes a computational algorithm based on the regularization method for determining the hydraulic conductivity coefficient of a layered oil reservoir. In [4, 5] the problems of determining the filtration properties of a multilayer reservoir were considered.

2. Statement and solution of the coefficient inverse problem
According to the Myatiev-Girinskii scheme the problem of determining the pressure fields $p_1 = p_1(x, y)$, $p_2 = p_2(x, y)$, $\ldots$, $p_n = p_n(x, y)$ in a reservoir with nonpermeable roof and floor, separated by a weakly permeable layers under simultaneous separate exploitation, is solving a system of partial differential equations in a multiconnected domain $D$ with boundaries $\partial D = \Gamma + \Gamma_1$, $\Gamma_1$ is circle of radius $r_c = 0.1m$.

\begin{align*}
L_1 p_1 + \omega_1 (p_1 - p_2) &= 0, \\
L_2 p_2 + \omega_1 (p_2 - p_1) + \omega_2 (p_2 - p_3) &= 0,
\end{align*}
\[
L_n p_n + \omega_{n-1} (p_n - p_{n-1}) = 0
\]  

(1)

with boundary conditions

\[
\int_{\Gamma_1} \sigma_{2k-1} \frac{\partial p_k}{\partial n} dS = q_k
\]  

(2)

\[
\frac{\partial p_k}{\partial \tau} \big|_{\Gamma_1} = 0, p_k \big|_{\Gamma} = 0, k = 1, 2, \ldots, n,
\]  

(3)

where \( L_k p_k \equiv -\text{div}(\sigma_{2k-1} \text{grad} p_k) \), \( \sigma_{2k-1}, H_{2k-1} \) \((k = 1, 2, \ldots, n)\) is the hydraulic conductivity coefficient and thickness of well permeable layers; \( \omega_k = \frac{\sigma_{2k}}{H_{2k}}, \sigma_{2k}, H_{2k} \) is the hydraulic conductivity coefficient and thickness of weakly permeable layers.

The inverse problem is to find the hydraulic conductivity coefficient \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{2n-1}) \) of a multilayered oil reservoir. Its initial data are the flow rates \( q_k \), the pressure functions on the boundary of the filtration domain, approximate values of the hydraulic conductivity coefficient, known from the technical documentation of the reservoir. Additionally, bottomhole pressures measured at a well are considered known.

\[
p_k^{(z)} = p_k \big|_{\Gamma_1}, k = 1, 2, \ldots, n
\]  

(4)

The solution of the inverse coefficient problem (1) - (4) is sought in the class of piecewise constant functions \( \sigma_k = \text{const}, c_1 \leq \sigma_k \leq c_2, c_1, c_2 \) are positive constants. This inverse problem generates a certain implicitly given nonlinear operator

\[
A \sigma = P^*
\]  

(5)

where \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{2n-1}) \) , \( P^* = \{p_k^{(z)}\}_{k=1}^{n} \) is bottomhole pressure vector. The vector \( P^* \) is ordinarily known inexacty: \( \|P^* - P^*_\delta\| \leq \delta \) , \( \|\cdot\| \) is the norm in the Euclidean space \( \mathbb{R}^n \) , \( \delta \) is the error in measurement. Solution of the operator equation (5) with the approximate right side is realized on the basis of minimizing the smoothing functional A. N. Tikhonov [6,7]

\[
M^\alpha (\sigma) = \|A \sigma - P^*_\delta\|^2 + \alpha \Omega (\sigma)
\]  

(6)

where \( \Omega (\sigma) = \sum_{i=1}^{2n-1} (\sigma_i - \sigma_i^0)^2 \), \( \alpha = \alpha (\delta) \) is the regularization parameter that agrees with the observation error, \( \sigma_i^0 \) is approximate values of the hydraulic conductivity coefficients.

Successive approximations \( \sigma^n \) are constructed in this manner: in the neighborhood of \( \sigma^n \) for a fixed value of the regularization parameter \( \alpha = \alpha_n \) a nonlinear operator \( A \sigma \) in the formula (6) is represented in the form

\[
A \sigma = A \sigma^n + A'_{\sigma} (\sigma^n) (\sigma - \sigma^n) + o\|\sigma - \sigma^n\|
\]

where \( A'_{\sigma} (\sigma^n) (\sigma - \sigma^n) \) is the Frechet differential in \( \sigma^n \). To find \( \sigma \) with a fixed \( \alpha \), the Gauss-Newton procedure is applied [7]. The functional (6) takes the form:

\[
M^{\alpha_n} (\sigma) = \|A \sigma + A'_{\sigma} (\sigma^n) (\sigma - \sigma^n) - P^*_\delta\|^2 + \alpha_n \Omega (\sigma)
\]
The Frechet differential is calculated on the basis of perturbation theory and has the form

\[ A' (\sigma) (\sigma - \hat{\sigma}) = (A_1, A_2, \ldots, A_n) \]

where \( A_i = (\delta \sigma_i \text{grad} \hat{p}_i, \text{grad} \hat{p}_i) + \cdots + (\delta \sigma_{2n-1} \text{grad} \hat{p}_n, \text{grad} \hat{p}_n) + \sum_{l=1}^{n-1} \frac{\delta \sigma_{2l-2}}{\delta \psi_{2l-2}} \left( \hat{p}_l - \hat{p}_{l+1}, \hat{p}_l \right) + (\hat{p}_{l+1} - \hat{p}_l, \hat{p}_{l+1}) \), \((a, b) = \int_D a(x, y) b(x, y) \, dx \, dy\) is the scalar product, \( \hat{p} = (\hat{p}_1, \hat{p}_2, \hat{p}_3, \ldots, \hat{p}_n) \) is solution of problem (1) - (4), when the hydraulic conductivity coefficient equals \( \hat{\sigma} \), \( \hat{p}_i \) are solutions conjugate problems:

\[
\tilde{L}_1 \hat{p}_1 + \hat{\omega}_1 (\hat{p}_1 - \hat{p}_2) = 0, \\
\tilde{L}_2 \hat{p}_2 + \hat{\omega}_1 (\hat{p}_2 - \hat{p}_1) + \hat{\omega}_2 (\hat{p}_2 - \hat{p}_3) = 0, \\
\cdots \\
\tilde{L}_n \hat{p}_n + \hat{\omega}_{n-1} (\hat{p}_n - \hat{p}_{n-1}) = 0
\]

with boundary conditions

\[
\int_{\Gamma} \sigma_{2k-1} \frac{\partial \hat{p}_k}{\partial n} \, dS = \delta_{ki}, \quad \frac{\partial \hat{p}_k}{\partial \tau} \bigg|_{\Gamma_i} = 0, \quad \hat{p}_k \bigg|_{\Gamma} = 0, \quad i, k = 1, 2, \ldots, n,
\]

where \( \tilde{L}_k \hat{p}_k \equiv \text{div} (\sigma_{2k-1} \text{grad} \hat{p}_k) \), \( \delta_{ki} \) is Kronecker symbol.

3. The results of the numerical experiment

Two-layer oil reservoir \( D = \{0 \leq x, y \leq 1000m\} \) is exploited by one well, pressure on the boundary \( \Gamma \) is zero. Reservoir parameters: fluid viscosity \( \mu = 10mPa \cdot s \), permeability coefficients and thickness of layers \( k_1 = 0.7 \mu m^2 \), \( H_1 = 10m \), \( k_2 = 0.0001 \mu m^2 \), \( H_2 = 1m \), \( k_3 = 0.35 \mu m^2 \), \( H_3 = 8m \), \( \sigma_i = \frac{k_i H_i}{\mu}, \quad i = 1, 3 \).

Problem (1) - (3) and conjugate problems were solved by the finite difference method. In this case, the well is considered a point source with a power equal to the flow rate of the well \( \alpha = 200 \). Initial permeability coefficient approximations: \( k_1 = 0.5 \mu m^2 \), \( k_2 = 0.0001 \mu m^2 \), \( k_3 = 0.5 \mu m^2 \). At each step of the iterative process, the residuals were computed:

\[
\| \Delta p^n \|^2 = \sum_{i=1}^{2} \sum_{j=1}^{3} (p_{ij}^{(N)} - p_{ij}^{(V)})^2, \quad \| \sigma - \sigma^n \|^2 = \sum_{j=1}^{3} (\sigma_i - \sigma_i^n)^2, \quad i = 1, 2, \ldots
\]

The calculations showed that the rate of convergence of the iterative process depends on the choice of the initial approximation of the permeability coefficient of the weakly permeable layer. Figure 1 and Figure 2 show the results of convergence of the iterative process with different regularization parameters.

The results of the calculations showed that the iterative process converges faster with the value of the regularization parameter \( \alpha = 200 \) and this regularization parameter is close to optimal according to the residual criterion.

4. Well testing N2046 of the Republic of Tatarstan

The well opened terrigenous Tula (in the range of 1101-1105 m.) and Bobrikov (in the range of 1112.5-1123 m) reservoir. Flow rate well \( q = 13.9 m^3/day \). Hydrodynamic investigations were carried out during simultaneous drainage of two layers, and during their separate exploitation.
The separation of the layers was carried out using the installation for simultaneous-separate exploitation.

An indicator card was constructed based on the results of field hydrodynamic investigations of a well operating only the Tula horizon. The estimates of the productivity coefficient $\eta_T = 0.027 \text{m}^3/\text{MPa} \cdot \text{day}$ and the flow rate $q_T \approx 11.1 \text{m}^3/\text{day}$, which gives the Tula horizon during the simultaneous exploitation of two layers, are obtained. The Table 1 shows the results of calculations. In the calculations, we used the scheme of “isolated” layers (1 line) and the Myatiev-Girinsky scheme (line 2).

### Table 1. Calculation results.

|          | $P_{pB}, \text{MPa}$ | $P_{pT}, \text{MPa}$ | $P_{B}, \text{MPa}$ | $P_{T}, \text{MPa}$ | $q_B, \text{m}^3/\text{day}$ | $q_T, \text{m}^3/\text{day}$ | $k_B, \mu m^2$ | $k_T, \mu m^2$ |
|----------|----------------------|----------------------|---------------------|---------------------|-----------------------------|-----------------------------|----------------|----------------|
| Line 1   | 6.123                | 6.193                | 2.339               | 2.433               | 2.8                         | 11.1                        | 0.018          | 0.213          |
| Line 2   | 6.156                | 6.156                | 2.339               | 2.433               | 2.8                         | 11.1                        | 0.025          | 0.254          |

Here $P_{pB}, P_{pT}$ are reservoir pressure, $P_B, P_T$ are bottomhole pressure, $q_B, q_T$ are flow rates, $k_B, k_T$ are permeability coefficients of Bobrikov and Tula horizons, respectively.

Calculations carried out both on model problems and on real data show that the proposed computational algorithm allows determining filtration properties of oil reservoirs.

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