Scaling Functions and Superscaling in Medium and Heavy Nuclei

A.N. Antonov,1 M.V. Ivanov,1 M.K. Gaidarov,1 E. Moya de Guerra,2,3 P. Sarriguren,2 and J.M. Udias3

1Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
2Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain
3Departamento de Fisica Atómica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain

(Dated: July 6, 2018)

The scaling function \( f(\psi') \) for medium and heavy nuclei with \( Z \neq N \) for which the proton and neutron densities are not similar is constructed within the coherent density fluctuation model (CDFM) as a sum of the proton and neutron scaling functions. The latter are calculated in the cases of \( ^{62}\text{Ni} \), \( ^{82}\text{Kr} \), \( ^{118}\text{Sn} \), and \( ^{197}\text{Au} \) nuclei on the basis of the corresponding proton and neutron density distributions which are obtained in deformed self-consistent mean-field Skyrme HF+BCS method. The results are in a reasonable agreement with the empirical data from the inclusive electron scattering from nuclei showing superscaling for negative values of \( \psi' \), including those smaller than -1. This is an improvement over the relativistic Fermi gas (RFG) model predictions where \( f(\psi') \) becomes abruptly zero for \( \psi' \leq -1 \). It is also an improvement over the CDFM calculations made in the past for nuclei with \( Z \neq N \) assuming that the neutron density is equal to the proton one and using only the phenomenological charge density.

PACS numbers: 25.30.Fj, 21.60.-n, 21.10.Ft, 21.10.Gv

The studies of the scaling phenomenon which has been observed in inclusive electron scattering from nuclei make it possible to gain information about basic nuclear characteristics such as the local density \( \rho(r) \) and momentum distribution \( n(k) \) in nuclei. This concerns firstly the \( y \)-scaling (e.g., \( ^{4}\text{He} \), \( ^{6}\text{Li} \), \( ^{12}\text{C} \), \( ^{27}\text{Al} \), \( ^{56}\text{Fe} \)) in nuclei. As pointed out in [\( ^{11} \)–\( ^{13} \)], however, the actual nuclear dynamical content of the superscaling is more complex than that provided by the RFG model. It was observed that the experimental data have a superscaling behavior for large negative values of \( \psi' \) (up to \( \psi' \approx -2 \)), while the predictions of the RFG model are for \( f(\psi') = 0 \) at \( \psi' \leq -1 \). This imposes the consideration of the superscaling in realistic finite systems. Such works were performed in [\( ^{11} \)–\( ^{12} \)] in the CDFM, which is related to the \( \delta \)-function limit of the generator-coordinate method in [\( ^{11} \)–\( ^{14} \)]. The calculated CDFM scaling function \( f(\psi') \) agrees with the available experimental data from the inclusive electron scattering for \( ^{4}\text{He} \), \( ^{12}\text{C} \), \( ^{27}\text{Al} \), \( ^{56}\text{Fe} \) and, approximately, for \( ^{197}\text{Au} \) for various values of the transfer momentum \( q = 500, 1000, 1650 \text{ MeV}/c \) in [\( ^{11} \)] and \( 1560 \text{ MeV}/c \) in [\( ^{12} \)], showing superscaling for negative values of \( \psi' \) including also those smaller than -1 (in contrast to the RFG model result). It was shown in [\( ^{11} \)–\( ^{12} \)] that the superscaling in nuclei can be explained quantitatively on the basis of the similar behavior of the high-momentum components of the nucleon momentum distributions in light, medium and heavy nuclei. It is known that the latter is related to the effects of the short-range and tensor nucleon-nucleon correlations in nuclei (see, e.g., \( ^{13} \)). Our scaling function was obtained starting from that in the RFG model in two equivalent ways, on the basis of the local density distribution and of the nucleon momentum distribution. This gives a good opportunity to study simultaneously the role of the nucleon-nucleon correlations included in \( \rho(r) \) and \( n(k) \) in the case of the superscaling phenomenon.

Here we would like to emphasize, however, that in [\( ^{11} \)–\( ^{12} \)] we encountered some difficulties to describe within the CDFM the superscaling in the case of \( ^{197}\text{Au} \) which was the most heavy nucleus considered. We related this to the particular A-dependence of \( n(k) \) in the model that does not lead to realistic high-momentum components of the momentum distribution in the heaviest nuclei. We followed in [\( ^{11} \)–\( ^{12} \)] a somewhat artificial way to “improve” the high-momentum tail of \( n(k) \) in \( ^{197}\text{Au} \) by taking the value of the diffuseness parameter \( b \) in the Fermi-type charge density distribution of this nucleus to be \( b = 1 \text{ fm} \) instead of the value \( b = 0.449 \text{ fm} \) (as obtained from electron elastic scattering experiments, see \( ^{17} \)). In such a case the high-momentum tail of \( n(k) \) for \( ^{197}\text{Au} \) in CDFM becomes similar to those of \( ^{4}\text{He} \), \( ^{12}\text{C} \), \( ^{27}\text{Al} \), and \( ^{56}\text{Fe} \) nuclei and this leads to a good agreement of the scaling function \( f(\psi') \) with the data also for \( ^{197}\text{Au} \). Discussing this in [\( ^{11} \)–\( ^{12} \)] we pointed out, however, that all the nucleons may contribute to \( f(\psi') \) for the transverse electron scattering and this could reflect on the diffuseness of the matter density for a nucleus like \( ^{197}\text{Au} \) whose value can be different from that of the charge density used in our previous works [\( ^{11} \)–\( ^{12} \)].

The aim of the present work is to apply the CDFM by using both proton and neutron densities for medium and heavy nuclei (for which \( Z \neq N \)) in contrast to our
previous approach, in which we assumed that the neutron density was equal to that of protons and we used only
the phenomenological charge density \([17]\). In our work
now the total scaling function \(f(\psi')\) will be a sum of two
scaling functions, those for protons and neutrons.

In [12] the CDFM scaling function \(f(\psi')\) was given
in two equivalent ways, firstly, by means of the density distribution

\[
\frac{\alpha}{(k_F |\psi'|)} f(\psi') = \int_0^\infty dR |F(R)|^2 f_{\text{RFG}}(R, \psi'),
\]

(1)

where

\[
|F(R)|^2 = -\frac{1}{\rho_0(R)} \frac{dp(r)}{dr} \bigg|_{r=R},
\]

(2)

\[
\rho_0(R) = \frac{3A}{4\pi R^3}, \quad \alpha = \left(\frac{9\pi A}{8}\right)^{1/3} \approx 1.52A^{1/3},
\]

(3)

\[
f_{\text{RFG}}(R, \psi') = \frac{3}{4} \left(1 - \left(\frac{k_F |\psi'|}{\alpha}\right)^2\right) \left\{1 + \left(\frac{Rm_N}{\alpha}\right)^2\right\}^2 \times \left(\frac{k_F |\psi'|}{\alpha}\right)^2 \left[2 + \left(\frac{\alpha}{Rm_N}\right)^2 - 2\sqrt{1 + \left(\frac{\alpha}{Rm_N}\right)^2}\right],
\]

(4)

\((m_N\text{ being the nucleon mass}),\) and secondly, by means of the momentum distribution

\[
f(\psi') = \int_{k_F |\psi'|}^\infty d\kappa_F |G(\kappa_F)|^2 f_{\text{RFG}}(\kappa_F, \psi'),
\]

(5)

where

\[
|G(\kappa_F)|^2 = -\frac{1}{n_0(\kappa_F)} \frac{dn(\rho)}{d\rho} \bigg|_{\rho=\kappa_F},
\]

(6)

and

\[
n_0(\kappa_F) = \frac{3A}{4\pi \kappa_F^3}.
\]

(7)

In Eq. (5) the RFG scaling function \(f_{\text{RFG}}(\kappa_F, \psi')\) can be
obtained from \(f_{\text{RFG}}(R, \psi')\) (Eq. (1)) by changing there
\(\alpha/R\) by \(k_F\). In Eqs. (11), (12), and (13) the Fermi momentum
\(k_F\) is not a free fitting parameter for different nuclei
as in the RFG model, but it is calculated in the CDFM
for each nucleus using the corresponding expressions:

\[
k_F = \int_0^\infty dR k_F(R) |F(R)|^2 = \int_0^\infty dR \frac{\alpha}{R} |F(R)|^2 = \frac{4\pi(9\pi)^{1/3}}{3A^{2/3}} \int_0^\infty dR \rho(R) R
\]

(8)

when

\[
\lim_{R \to \infty} \left[\rho(R) R^2\right] = 0
\]

(9)

is fulfilled and

\[
k_F = \frac{16\pi}{3A} \int_0^\infty d \kappa_F n(\kappa_F) \kappa_F^3
\]

(10)

when

\[
\lim_{\kappa_F \to \infty} \left[n(\kappa_F) \kappa_F^4\right] = 0
\]

(11)

is fulfilled.

In [11, 12] we used the charge density distributions to
determine the weight function \(|F(R)|^2\) in calculations of
\(f(\psi')\) from Eqs. (11–13) and (15). In the present work we
assume that the reason why the CDFM does not work
properly in the case of \(^{197}\text{Au}\) is that we use in [11, 12]
only the charge density, while this nucleus has many more
neutrons than protons \((N = 118 \text{ and } Z = 79)\), and therefore
proton and neutron densities may differ considerably.
In this case the proton \(f_p(\psi')\) and neutron \(f_n(\psi')\) scaling functions will be given by the contributions of the proton
and neutron densities \(\rho_p(r)\) and \(\rho_n(r)\), correspondingly:

\[
f_{p(n)}(\psi') = \int_0^\infty dR |F_{p(n)}(R)|^2 f_{RFG}(R, \psi'),
\]

(12)

where the proton and neutron weight functions are obtained from the corresponding proton and neutron densities

\[
|F_{p(n)}(R)|^2 = -\frac{4\pi R^3}{3Z(N)} \frac{d\rho_{p(n)}(\rho)}{d\rho} \bigg|_{\rho=R},
\]

(13)

\[
\alpha_{p(n)} = \left(\frac{9\pi Z(N)}{4}\right)^{1/3},
\]

(14)

\[
\int_0^\infty \rho_{p(n)}(\overline{r}) d\overline{r} = Z(N),
\]

(15)

and the Fermi momentum for the protons and neutrons is given by

\[
k_{p(n)} = \alpha_{p(n)} \int_0^\infty dR \frac{1}{R} |F_{p(n)}(R)|^2.
\]

(16)

The RFG proton and neutron scaling functions
\(f_{RFG}(R, \psi')\) have the form of Eq. (11), where \(\alpha\) and \(k_F\) are changed by \(\alpha_{p(n)}\) from Eq. (14) and \(k_{p(n)}\) from Eq. (16),
correspondingly. The normalizations of the functions are as follows:

\[ \int_0^\infty |F_p(n)(R)|^2 dR = 1, \quad (17) \]

\[ \int f_p(n)(\psi')d\psi' = 1. \quad (18) \]

Then the total scaling function can be expressed by means of both proton and neutron scaling functions:

\[ f(\psi') = \frac{1}{A} (Z f_p(\psi') + N f_n(\psi')) \quad (19) \]

and is normalized to unity.

The same consideration can be performed equivalently on the basis of the nucleon momentum distributions for protons \( n^p(k) \) and neutrons \( n^n(k) \) presenting \( f(\psi') \) by the sum of proton and neutron scaling functions calculated similarly to Eqs. (12)–(19) and (to Eqs. (10), (14), (15), and (11)):

\[ f_p(n)(\psi') = \int_{k_p^{p(n)}} d\vec{k}_F|G_p(n)(\vec{k}_F)|^2 f_{pFG}(\vec{k}_F, \psi'), \quad (20) \]

where

\[ |G_p(n)(\vec{k}_F)|^2 = -\frac{4\pi k_F^3}{3Z(N)} \left. \frac{dn_p^{p(n)}(p)}{dp} \right|_{p=\vec{k}_F} \quad (21) \]

with \( f_{pFG}(\vec{k}_F, \psi') \) containing \( \alpha_p(n) \) from Eq. (12) and \( k_p^{p(n)} \) calculated by

\[ k_p^{p(n)} = \int_0^\infty d\vec{k}_F \vec{k}_F|G_p(n)(\vec{k}_F)|^2. \quad (22) \]

We calculate the scaling function for several examples, for the medium stable nuclei \(^{62}\text{Ni}\) and \(^{82}\text{Kr}\) and for the heavy nuclei \(^{118}\text{Sn}\) and \(^{197}\text{Au}\) following Eqs. (12)–(19) using the corresponding proton and neutron densities obtained in deformed self-consistent mean-field (HF+BCS) calculations with density-dependent Skyrme effective interaction (SG2) using a large harmonic-oscillator basis with 11 major shells.

The results of the calculations of \( f(\psi') \) for \(^{62}\text{Ni}\), \(^{82}\text{Kr}\), \(^{118}\text{Sn}\), and \(^{197}\text{Au}\) for \( q = 1000 \text{MeV}/c \) are shown in Fig. 1, which clearly illustrates the different tails of the proton and neutron scaling functions when \( \psi' \leq -1 \). As an example the proton and neutron scaling functions for \(^{62}\text{Ni}\) are also given in Fig. 1 which shows the agreement with the experimental charge densities. At the same time we note also the improvement in comparison with the RFG model result in which \( f(\psi') = 0 \) for \( \psi' \leq -1 \). As an example the proton and neutron scaling functions for \(^{62}\text{Ni}\) are also given in Fig. 1 which clearly illustrates the different tails of the proton and neutron scaling functions when \( Z \neq N \), as well as their role in building up the observed scaling function.

In conclusion, we point out that the scaling function \( f(\psi') \) for nuclei with \( Z \neq N \) for which the proton and neutron densities are not similar has to be expressed by the sum of the proton and neutron scaling functions. The latter can be calculated within the CDFM on the basis of the knowledge (obtained theoretically and/or experimentally) of the corresponding proton and neutron local density distributions or momentum distributions. We should also point out that the agreement with experiment is quite reasonable given that no adjustable parameter at all has been used in the present calculations.

Acknowledgments

Three of the authors (A.N.A., M.V.I. and M.K.G.) are thankful to the Bulgarian National Science Foundation for partial support under Contracts No.Φ-1416 and Φ-1501. This work was partly supported by funds provided by DGI of MCyT (Spain) under Contract Nos. FIS 2005-00640, BFM 2000-0600, and BFM-04147-C02-01 and by the Agreement (2004 BG2004) between the CSIC (Spain) and the Bulgarian Academy of Sciences. One of us (E.M.G.) is indebted to Prof. T.W. Donnelly for valuable comments and suggestions.
[1] G. West, Phys. Rep. 18, 263 (1975).
[2] I. Sick. D. Day, and J.S. McCarthy, Phys. Rev. Lett. 45, 871 (1980).
[3] E. Pace and G. Salme, Phys. Lett. B 110, 411 (1982).
[4] C. Ciofi degli Atti, E. Pace, and G. Salme, Phys. Lett. B 127, 303 (1983); Phys. Rev. C 43, 1155 (1991); C. Ciofi degli Atti and G.B. West, Phys. Lett. B 458, 447 (1999).
[5] D. Day, J.S. McCarthy, T.W. Donnelly, and I. Sick, Ann. Rev. Nucl. Part. Sci. 40, 357 (1990).
[6] W.M. Alberico, A. Molinari, T.W. Donnelly, E.L. Kronenberg, and J.W. Van Orden, Phys. Rev. C 38, 1801 (1988); M.B. Barbaro, R. Cenni, A. De Pace, T.W. Donnelly, and A. Molinari, Nucl. Phys. A643, 137 (1998).
[7] T.W. Donnelly and I. Sick, Phys. Rev. Lett. 82, 3212 (1999).
[8] T.W. Donnelly and I. Sick, Phys. Rev. C 60, 065502 (1999).
[9] C. Maieron, T.W. Donnelly, and Ingo Sick, Phys. Rev. C 65, 025502 (2002).
[10] M.B. Barbaro, J.A. Caballero, T.W. Donnelly, and C. Maieron, Phys. Rev. C 69, 035502 (2004).
[11] A.N. Antonov, M.K. Gaidarov, D.N. Kadrev, M.V. Ivanov, E. Moya de Guerra, and J.M. Udias, Phys. Rev. C 69, 044321 (2004).
[12] A.N. Antonov, M.K. Gaidarov, M.V. Ivanov, D.N. Kadrev, E. Moya de Guerra, P. Sarriguren, and J.M. Udias, Phys. Rev. C 71, 014317 (2005).
[13] A.N. Antonov, P.E. Hodgson, and I.Zh. Petkov, “Nucleon Momentum and Density Distributions in Nuclei” (Clarendon Press, Oxford, 1988); “Nucleon Correlations in Nuclei” (Springer-Verlag, Berlin-Heidelberg-New York, 1993).
[14] A.N. Antonov, V.A. Nikolaev, and I.Zh. Petkov, Bulg. J. Phys. 6, 151 (1979); Z. Phys. A297, 257 (1980); A304, 239 (1982); Nuovo Cimento A 86, 23 (1985).
[15] A.N. Antonov, E.N. Nikolov, I.Zh. Petkov, Chr.V. Cristov, and P.E. Hodgson, Nuovo Cimento A 102, 1701 (1989); A.N. Antonov, D.N. Kadrev, and P.E. Hodgson, Phys. Rev. C 50, 164 (1994).
[16] J.J. Griffin and J.A. Wheeler, Phys. Rev. 108, 311 (1957).
[17] J.D. Patterson and R.J. Peterson, Nucl. Phys. A717, 235 (2003).
[18] E. Moya de Guerra, P. Sarriguren, J.A. Caballero, M. Casas, and D.W.L. Sprung, Nucl. Phys. A529, 68 (1991).
[19] D. Vautherin, Phys. Rev. C 7, 296 (1973).