Nuclear spin-lattice relaxation time in TaP and the Knight shift of Weyl semimetals

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We first analyze the recent experimental data on the nuclear spin-lattice relaxation rate of the Weyl semimetal TaP. We argue that its non-monotonic temperature dependence is explained by the temperature dependent chemical potential of Weyl fermions. We also develop the theory of the Knight shift in Weyl semimetals, which contains two counteracting terms. The diamagnetic term follows \(-\ln[W/\max(|\mu|,k_BT)]\) with \(W\), \(\mu\) and \(T\) being the high energy cutoff, chemical potential and temperature, respectively, and is always negative. The paramagnetic term scales with \(\mu\) and changes sign depending on the doping level. Altogether, the Knight shift is predicted to vanish or even change sign upon changing the doping or the temperature, making it a sensitive tool to identify Weyl points. We also calculate the Korringa relation for Weyl semimetals which shows an unusual energy dependence rather than being constant as expected for a non-interacting Fermi system.

I. INTRODUCTION

With the advent of topological insulators, the observation of many fascinating phenomena became possible, including the magnetoelectric effect, axion electrodynamics, Majorana fermions. In their bulk, these materials resemble to a normal insulator, but their surfaces or edges host metallic states, which are protected by the underlying topology. In this respect, they are regarded as the descendant of quantum Hall states, which is manifested in e.g. the quantized spin-Hall conductivity in spin-Hall insulators.

The above story can further be twisted by designing materials whose bulk metallicity is protected by topology. A topological metal in 3D is incarnated in Weyl semimetals. The protection of metallic behaviour is best visualized in momentum space, where a Weyl point may be regarded as a magnetic monopole. These objects appear pairwise, and can only by annihilated by colliding two monopoles with opposite topological charge into each other. Due to the non-trivial topology, Weyl semimetals also feature a variety of extraordinary phenomena such as the chiral anomaly, i.e., the anomalous non-conservation of an otherwise conserved quantity, the chiral current in this case, or the anomalous Hall conductivity in spin-Hall insulators.

At the heart of the NMR lies the hyperfine coupling, describing the interaction between nuclear spins and the surrounding medium. In Ref. 17, we determined the hyperfine interaction for Weyl semimetals using an “ab-initio” treatment of the low energy effective Hamiltonian. This allowed to show that the spin-lattice relaxation rate is anomalous in Weyl semimetals and does not follow the behaviour expected from the density of states. Instead of a \(1/T\) scaling with \(E\) being the maximum of temperature \((k_BT)\) and chemical potential, the nuclear spin relaxation rate scales in a graphene-like manner as \(1/T\sim E^2\ln(E/\omega_0)\) with \(\omega_0\) the nuclear Larmor frequency. This allows us to analyze recent nuclear quadrupole relaxation data on TaP in Ref. 19 and demonstrate that by taking the temperature dependence of the chemical potential into account, we are able to describe the salient features of the experimental data. In addition, we provide a similar ab-initio evaluation of...
the Knight shift in Weyl semimetals as well, which reveals rich behaviour depending on the conspiracy of the chemical potential and temperature. Namely, it can cross over between diamagnetic and paramagnetic behaviour by tuning them, respectively. The Korringa relation of a Fermi liquid is not satisfied due to the strong spin-orbit coupling, which is essential to induce Weyl points. Our results are important in assessing the usefulness of Weyl semimetals in NMR quantum computers, where hyperfine effects can limit the device performance.

II. HYPERFINE INTERACTION IN WEYL SEMIMETALS

By focusing on the low energy excitations, the Hamiltonian of Weyl semimetals is written as

$$H = \varepsilon_F(p_x\sigma_x + p_y\sigma_y + p_z\sigma_z),$$

(1)

where $\sigma$’s are spin-1/2 Pauli matrices, corresponding to the physical spin, $\varepsilon_F$ is their Fermi velocity, typically of the order of $10^5 - 10^6$ m/s. Its dispersion relation is also linear in momentum, as is usual for zero mass Weyl fermions in arbitrary dimension (e.g. for graphene as well) as

$$\varepsilon_\pm(k) = \lambda v_F |k|$$

(2)

with $\lambda = \pm$ and $k = |k|$ for the length of the 3D momentum. The spinor eigenfunctions are written as

$$|k, +\rangle = \begin{bmatrix} \cos \left(\frac{\varphi_k}{2}\right) \\ \sin \left(\frac{\varphi_k}{2}\right) \exp(i\varphi_k) \end{bmatrix},$$

$$|k, -\rangle = \begin{bmatrix} \sin \left(\frac{\varphi_k}{2}\right) \\ -\cos \left(\frac{\varphi_k}{2}\right) \exp(i\varphi_k) \end{bmatrix}. $$

(3a)

(3b)

These correspond to positive and negative eigenenergies, respectively, and spherical coordinates were used such that $\varphi_k$ is the azimuthal angle in the $(k_x,k_y)$ plane and $\varphi_k$ is the polar angle made from the $k_z$ axis.

In Ref. [17], the standard route outlined in Refs. [13] and [18] was followed to obtain the hyperfine interaction. After representing the nuclear spin as a dipole with dipole moment $m = \hbar\gamma I$, its vector potential is

$$A = \frac{\mu_0}{4\pi} m \times r = \frac{\mu_0}{4\pi} \hbar\gamma I \times r = \frac{\mu_0}{4\pi} \nabla \times \left(\frac{m}{r}\right).$$

(4)

Here $\mu_0$ is the vacuum permeability and $\gamma$ is the gyromagnetic ratio of the studied nucleus. This vector potential enters into the Hamiltonian through the Peierls substitution as $p \rightarrow p - eA$, and its magnetic field, $\nabla \times A$ through the Zeeman term.

Using this "ab-initio" treatment of the nuclear spin within the low energy effective Hamiltonian of Eq. (1), the hyperfine interaction between a localized nucleus and the surrounding Weyl fermions after some lengthy calculation reads as

$$H_{HF} = \frac{\mu_0}{q^2}\gamma I \left[ iev_F (\sigma \times q) - \frac{g\mu_B}{2} (q \times (q \times \sigma)) \right],$$

(5)

where the momentum transfer between the incoming ($k$) and outgoing ($k'$) electron, which gets scattered off the localized spin, is $q = k - k'$. The first and second term are the orbital and the spin part of the hyperfine interaction. For small momentum scattering, the $H_{HF}$ diverges as $ev_F/q$ for $q \rightarrow 0$ in the orbital part of the hyperfine coupling. The terms containing $\mu_B$ remain finite in the same small $q$ limit, since $1/q^2$ prefactor in Eq. (5) is compensated in the numerator.

III. NUCLEAR SPIN RELAXATION IN THE WEYL SEMIMETAL TAP

In Ref. [17], we derived the spin-lattice relaxation rate of Weyl-fermions from an effective low energy description of the fermionic excitations. Surprisingly, the dominant contribution at low $T$ and $\mu$ comes from the orbital part of the hyperfine interaction, which usually gives a small contribution in normal metals. The relaxation time is found to be

$$\frac{1}{T_1} = \frac{\pi \mu_0^2 \gamma^2}{4v_F(2\pi)^6} \int_{-\infty}^{\infty} dk \frac{(kev_F)^2}{\cosh^2 \left((\hbar v_F k - \mu)/2k_BT\right)},$$

(6)

where $k_0 = \omega_0/v_F$ is the Larmor wavenumber, $\omega_0 = B\gamma$ is the nuclear Larmor frequency, which is the smallest energy scale of the problem due to the heavy mass of the nucleus, $\gamma$ is the gyromagnetic ratio of the studied nucleus and the dimensionless functions $F(x \rightarrow 0) \approx 52.7\ln(2x)$. From Ref. [23], the numerical constant 52.7 is $(4\pi)^2/3$.

By performing the remaining integral, we eventually obtain

$$\frac{\hbar}{T_1k_BT} = \frac{52.7\pi \mu_0^2 \gamma^2 e^2}{(2\pi)^6 v_F^7} \times \left\{ \begin{array}{l} \left(\frac{k_BT}{\hbar}\right)^2 \pi^2 \frac{\ln\left(\frac{4k_BT}{\hbar\omega_0}\right)}{3} \, , \, \mu \ll k_BT \\ \left(\frac{\mu}{\hbar}\right) \frac{\ln\left(\frac{2\mu}{\hbar\omega_0}\right)}{3} \, , \, \mu \gg k_BT. \end{array} \right.$$
a constant, $T$ independent behavior at low temperatures, which crosses over to a $T^2$ increase with increasing temperature. This agrees with our analytical results in Eq. 7. However, to account for the fine details of the experimental data, we have to take into account the temperature dependence of the chemical potential. The experiment was performed at a fixed number of electrons which did not vary with the temperature, which amounts to consider $\mu(T)$ chemical potential. As we show below, this explains quantitatively all features of the experiment.

\begin{align*}
N(T) &= \int d\varepsilon \frac{g(\varepsilon)}{\exp[(\varepsilon - \mu(T))/k_B T] + 1},
\end{align*}

where $g(\varepsilon) = \varepsilon^2 v_F^2 / 2\pi^2 \hbar^3 \gamma^2$ is the density of states in Weyl semimetals and $V$ is the volume of the sample. Using particle number conservation, $N(T \neq 0) - N(T = 0) = 0$, we get

\begin{align*}
\int_{-\infty}^{\infty} \varepsilon^2 d\varepsilon \left( \frac{1}{\exp[(\varepsilon - \mu(T))/k_B T] + 1} - \Theta(\mu(0) - \varepsilon) \right) &= 0,
\end{align*}

where $\Theta(x)$ is the Heaviside function and $\mu(0)$ is the chemical potential at $T = 0$. Upon evaluating Eq. 9, we obtain

\begin{align*}
\mu^3(T) - \mu^3(0) + \pi^2 (k_B T)^2 \mu(T) &= 0.
\end{align*}

This equation has two complex roots, which are irrelevant for our current study, and its real root reads as

\begin{align*}
\mu(T) &= \frac{E(T)}{6} - \frac{2\pi^2 (k_B T)^2}{E(T)}.
\end{align*}

where

\begin{align*}
E(T) &= (108 \mu^3(0) + 12\sqrt{12\pi^6 (k_B T)^6 + 81 \mu^6(0)})^{1/3}.
\end{align*}

This yields

\begin{align*}
\mu(T) &= \frac{\mu(0)}{1 + c(k_B T/\mu(0))^2},
\end{align*}

The $T^2$ initial decrease of the chemical potential is identical to that in a normal Fermi gas, with the Fermi energy replacing the chemical potential in the denominator. In that case, however, the typical Fermi energy scale is $10^4$ K, thus the $T$ dependence of the chemical potential is negligible at the typical energy scales of condensed matter. On the other hand, for the present case, upon small doping, the temperature dependence of the chemical potential is important and cannot be neglected, since as we show below, $\mu(0)$ can be of the order of 10-100 K and even the $k_B T \gg \mu(0)$ region can easily be reached.

Eq. 11 arises from an ideal Weyl-fermionic band structure, where the linearly dispersing bands extend to arbitrary energies. For any real system, this is clearly not the case as bands usually terminate at some cut-off energy and also display deviations from Eq. 12 at higher energies, which requires the explicit knowledge of the full band structure. This, in turn, is expected to alter the temperature dependence of the chemical potential. We model this effect by a phenomenological $\mu(T)$ function, which still preserves the overall features found in the above calculations. To be explicit, we use

\begin{align*}
\mu(T) &= \frac{\mu(0)}{1 + c(k_B T/\mu(0))^2}.
\end{align*}

The experimental data is fitted by plugging Eq. 13 into Eq. 6 using $c$, $\mu(0)$ and the overall scale of $1/T_1 T$ as free parameters. The experimental data determines roughly $\mu(0)$, which then fixes the scale factor, thus the only free fitting parameter is $c$. Other functions than Eq. 13 with similar asymptotics work equally well. The result, together with the $\mu(T)$ curve from Eq. 11 is shown in Fig. 2. This gives a convincing agreement between experiment and theory. Note that that phenomenological $\mu(T)$ follows qualitatively the behaviour of Eq. 11.

### IV. KNIGHT SHIFT

The conduction electrons induce an average static magnetic field through the hyperfine interaction at the position of the nucleus, which is associated by the Knight shift.\cite{12,29}. As a result, the nuclear Zeeman energy is given by $h\gamma_n B(1 + K)$ with $K$ the Knight shift. A static magnetic field in the $z$ direction cannot depend on the $z$ coordinate, thus its spatial Fourier transform depends only on $q_{x,y}$. This follows from that fact that $B = [0, 0, B(x, y)]$ has to satisfy $\nabla B = \partial_z B_z = 0$, so its Fourier transform $B_q$ is independent of $q_z$. 

\begin{align*}
\text{FIG. 2. The experimental spin-lattice relaxation rate on TaP from Ref. 14 (red squares), together with the theoretical $T_1$ of Eq. 11 using the chemical potential from Eq. 11 (green dashed line) and also the approximate expression from Eq. 12 (blue line) with $\mu(0)/k_B = 75$ K and $\hbar\omega_0/k_B = 0.0013$ K. Inset: Temperature dependence of $\mu(T)$ from Eq. 11 (green dashed line) and of the approximate function (blue) with $c = 12$.}
\end{align*}
The external magnetic field appears in Eq. (1) through the vector potential and the Zeeman term. These give rise to an additional perturbation as

$$H' = -e v_F \sigma A - \frac{g \mu_B}{2} B \sigma_z. \quad (14)$$

Then, the basic question is how this external magnetic field in the vector potential and the Zeeman term in the Hamiltonian of Weyl semimetals influences the nuclear spin through the hyperfine interaction in Eq. (1). The effective magnetic field felt by the nuclear spin is given by

$$\Delta E^o = \mu_0 \gamma_n \hbar v_F \frac{i I_q}{q^2} (q_y \langle \sigma_x \rangle - q_x \langle \sigma_y \rangle). \quad (15)$$

Here only the $z$ component of the nuclear spin is relevant since the magnetic field point in the $z$ direction. In a similar fashion, the spin part of the hyperfine coupling gives rise to an energy shift as

$$\Delta E^s = \mu_0 \gamma_n \hbar \frac{g \mu_B}{2} I_z \langle \sigma_z \rangle. \quad (16)$$

In order to obtain the Knight shift, we calculate within linear response theory in the external magnetic field the quantity $\langle \sigma_{x,y,z} \rangle$ from $H'$ in Eq. (14). Since we need the expectation value of the spin operators and both external perturbations, i.e. the vector potential $A$ and the Zeeman term $B$ couple to the physical spin of Weyl fermions in Eq. (14), we need the spin-spin correlation function between $\sigma_a$ and $\sigma_b$, denoted by $\Pi^{ab}(\omega = 0, \mathbf{q})$, to determine $\Delta E^o_3$ and $\Delta E^o_4$ from the Kubo formula, respectively. This is given by

$$\Pi^{ab}(\mathbf{q}) = -\sum_k \sum_{\lambda,\lambda'} \frac{f(\varepsilon_\lambda(\mathbf{k})) - f(\varepsilon_{\lambda'}(\mathbf{k} - \mathbf{q}))}{\varepsilon_\lambda(\mathbf{k}) - \varepsilon_{\lambda'}(\mathbf{k} - \mathbf{q})} \times \langle \mathbf{k}, \lambda | \sigma_a | \mathbf{k} - \mathbf{q}, \lambda' \rangle \langle \mathbf{k} - \mathbf{q}, \lambda' | \sigma_b | \mathbf{k}, \lambda \rangle,$$  

(17)

where $f$ is the Fermi function and the $\omega = 0$ limit has already been taken. This expression is complex in general due to the complex matrix elements using Eqs. (3).

A. Chemical potential dependence

We expand Eq. (17) in Taylor series in $\mathbf{q}$ up to second order. After some tedious though straightforward algebra, the spin correlation function is evaluated in this small $\mathbf{q}$ limit at $T = 0$ as

$$\Pi^{ab}(\mathbf{q}) = \left( g^{ab} \right) \ln \left( \frac{|W|}{|\mu|} \right) - \frac{14}{15} + i e a b c q^c \mu \frac{4 \pi^2 (\hbar v_F)^2}{4 \pi^2 (\hbar v_F)^2}, \quad (18)$$

where $(a, b, c)$ denotes the spatial direction $(x, y, z)$, $a \neq b$ and $W$ is a sharp high energy cutoff regularizing the theory and $\epsilon^{abc}$ is the Levi-Civita symbol. We note that while the logarithmic cutoff dependence is expected in the real part of $\Pi^{ab}(\mathbf{q})$ for any kind of cutoff, i.e. sharp, exponential, gaussian etc., the numerical constant, $-14/15$ is not universal but is expected to be an order one constant for all cutoff schemes. We also evaluated Eq. (17) numerically and found perfect agreement with Eq. (18).

Starting with $\Delta E^o_A$, the Fourier transform of vector potential for a magnetic field in the $z$ direction is represented in different gauges as

$$\mathbf{A}(\mathbf{q}) = \left( 0, -\frac{B_q}{q_x}, 0 \right)$$

using the two distinct gauges. Substituting it into Eq. (15), we get

$$\Delta E^o_A = -\mu_0 \gamma_n (\hbar v_F)^2 \frac{i I_q B_q}{q^2} \left( q_y \Pi^{yx} + q_x \Pi^{x y} \right). \quad (21)$$

A similar calculation is carried out the consider the effect of the electronic Zeeman term on the spin expectation values, yielding

$$\Delta E^o_B = \mu_0 \gamma_n \hbar v_F g \mu_B \frac{i I_q B_q}{q^2} \left( q_y \Pi^{yz} - q_x \Pi^{zy} \right). \quad (22)$$

The spin part of the hyperfine interaction is mostly affected by the magnetic vector potential part of the Weyl Hamiltonian. This gives

$$\Delta E^s = \mu_0 \gamma_n \hbar \frac{g \mu_B}{2} I_z \Pi^{zz} \frac{B_q}{i q_y} \quad (23)$$

Finally, an additional contribution from the spin part of the hyperfine interaction is in principle possible from the Zeeman term in Eq. (1), involving the $\chi_{zz}(\mathbf{q}) = 0$ spin susceptibility. In accord with Ref. 30 and 31, this can in principle yield a non-universal constant term, independent of both $T$ and $\mu$, which arises entirely by the high energy part of the spectrum, not taken into account by Eq. (1). This constant term can be merged with the chemical shift.$^{18}$

Using the spin correlation function in Eq. (18) for Eqs. (21) and (22) and also the fact that $q_z = 0$ for a magnetic field in the $z$ direction, we finally obtain the zero temperature Knight shift as

$$K = \frac{\mu_0 e}{4 \pi^2 \hbar} \left( \frac{g \mu_B}{\hbar v_F} - \frac{e v_F}{3} \ln \left( \frac{|W|}{|\mu|} \right) - \frac{14}{15} \right). \quad (24)$$
Here the first term stems from the electronic Zeeman term and is the paramagnetic contribution, while the second terms arise due to the electronic orbital contribution, and represents the diamagnetic term. The logarithmic term, dominating the diamagnetic term, is always negative since \( W/\mu \gg 1 \). However, the sign of the first, paramagnetic term can change sign depending on whether the system is electron or hole doped. These agree qualitatively with Ref. [32]. This means that already the para-

**B. Temperature dependence**

The knowledge of the finite temperature spin-spin correlation function in Eq. (18) is required to obtain the temperature dependence of the Knight shift. Since it is calculated from the Kubo formula for non-interacting electrons in Eq. (17), it depends linearly on the Fermi-Dirac distribution function. We then use the trick of Ref. [32] for the Fermi function \( f(\varepsilon; \mu; T) \) as

\[
f(\varepsilon; \mu; T) = \int_{-\infty}^{\infty} d\mu' \left( -\frac{df(\mu; \mu'; T)}{d\mu} \right) \Theta(\mu' - \varepsilon), \tag{25}
\]

where \( f(\varepsilon; \mu; T) = 1/(\exp(\varepsilon - \mu/k_BT) + 1) \) and its \( T = 0 \) limit is the Heaviside function as \( \Theta(\mu - \varepsilon) \). Although the expression in Eq. (17) is valid for any temperature, only its zero temperature limit is evaluated in Eq. (18). Nevertheless, using the transformation in Eq. (24), the zero temperature response is transformed to finite \( T \) by an integral over the chemical potential as

\[
\Pi^{ab}(\mu, T) = \int_{-\infty}^{\infty} d\mu' \left( -\frac{df(\mu; \mu'; T)}{d\mu} \right) \Pi^{ab}(\mu', T = 0). \tag{26}
\]

Using Eq. (18) in Eq. (26) to get the finite \( T \) spin correlator, its imaginary part remains unchanged and only its real part is influenced by finite temperatures. For \( \mu = 0 \), it reads as

\[
\text{Re} \Pi^{ab}(q, T) = \frac{q^a q^b}{12\pi^2 \hbar v_F} \left( \ln \left( \frac{2\gamma W}{\pi k_B T} \right) - \frac{14}{15} \right), \tag{27}
\]

where \( \gamma \approx 0.577 \) is the Euler-Mascheroni constant. Thus, the temperature dependent Knight shift for undoped Weyl semimetals is

\[
K = \frac{\mu_0 e}{4\pi^2 e} \left( \frac{g\mu_B}{\hbar v_F} \mu - \frac{e v_F}{3} \ln \left( \frac{W_{\text{max}}}{\text{max}(|\mu|, k_B T)} \right) \right), \tag{29}
\]

and the chemical potential itself is temperature dependent and vanishes gradually with temperature as in Eq. (23). The first term is interpreted in terms of the Knight shift in normal metals [33,34], where \( K \approx A_{hf}(\mu)g(\mu) \) with \( A_{hf} \) the hyperfine coupling, which is usually energy independent and \( g(\mu) \) is the density of states. For Weyl semimetals, \( g(\mu) \sim \mu^2 \), thus an energy dependent hyperfine coupling is required to satisfy this relation as \( A_{hf} \sim 1/\mu \). The effective hyperfine coupling diverges upon approaching the Weyl point and changes sign depending on the doping level. This is in accord with the analysis of the relaxation time.

Depending on the temperature and the doping level, it can either be dominated by the diamagnetic term with the logarithmic temperature and chemical potential dependence, or by the paramagnetic term which can still change sign depending on the electron or hole doping level, respectively. In a typical NMR experiments, the temperature dependence of the relaxation time and the Knight shift is measured, because tuning the temperature is an easier task than tuning the chemical potential. In Fig. 3, we show typical behaviours of Knight shift with different zero temperature chemical potentials.

Exactly at the Weyl point, the Knight shift displays strong diamagnetic behaviour and diverges with decreasing temperature as \( -\ln(W/k_BT) \). At \( T = 0 \), Eq. (24) applies and the sign of the Knight shift is determined by the conspiracy of the paramagnetic and diamagnetic contributions, but for \( \mu < 0 \), it is always negative. Upon increasing the temperature, two things kick in: first, the chemical potential starts to decrease and the paramagnetic term slowly vanishes as predicted in Eq. (11) and visualized in the inset of Fig. 2. Second, the temperature starts to compete with the chemical potential in the diamagnetic term and for \( k_B T > \mu \), it reduces the contribution of the diamagnetic term. Therefore, at high temperatures \( k_B T \gg \mu(0) \), the sign of the Knight shift is most probably negative as the paramagnetic term vanishes due to the vanishing of the chemical potential and only the diamagnetic contribution remains as \( \sim -\ln(W/k_BT) \). These features are visualized in Fig. 3.

**V. KORRINGA RELATION**

The calculation of the relaxation time \( T_1 \) and the Knight shift allows to test the validity of the Korringa relation, i.e. whether \( 1/T_1 T K^2 = \text{const} \) holds. In general, the Korringa relation is valid for a Fermi liquid. In particular for a non-interacting Fermi-gas \( \frac{1}{T_1 T K^2} = \frac{4\pi k_B^2}{\hbar} \left( \frac{\hbar}{g \mu_B} \right)^2 \).
while deviations from this usually indicate certain instabilities, strong correlation effects or transitions.

Since our Weyl fermions are non-interacting, it is interesting to investigate to what extent this Korringa relation holds. From our results in Eqs. (7) and (29), we infer that while $T_1$ shows rather smooth behaviour and increases roughly with the temperature, the Knight shift exhibits more intricate behaviour and can even vanish in certain cases, as exemplified in Fig. 3. This means that $(T_1TK^2)^{-1}$ can change significantly with both temperature and chemical potential, and can even diverge when the Knight shift changes sign.

Therefore, it is much more instructive to focus on the $T = 0$ behaviour and assume significant doping away from the Weyl point. In this limit, there is a well developed and large Fermi surface, similar to that in normal metals. In this case, by neglecting the logarithmic terms both in the relaxation time and the Knight shift, we deduce

$$
\frac{1}{T_1TK^2} \approx \frac{4\pi k_B}{3h} \left( \frac{h^2}{g\mu_B} \right)^2,
$$

which is three times smaller than what is expected in a normal Fermi gas.

Finally, by tuning the system to the close vicinity of the Weyl point with $\mu(T) = 0$ or by moving to high temperatures with $k_BT \gg \mu(0)$, it acquires a strong temperature dependence as

$$
\frac{1}{T_1TK^2} \approx \frac{4\pi k_B}{h} \left( \frac{\hbar\gamma_n}{g\mu_B} \right)^2 \left( \frac{g\mu_B\pi^{3/2}k_BT}{\hbar e^2F} \right)^2.
$$

VI. CONCLUSIONS

The purpose of this work is twofold: first, we focused on the spin relaxation time of Weyl fermions in TaP. We took into account the temperature dependence of the chemical potential, whose characteristic energy scale, separating the high and low temperature behaviour in $\mu(T)$, is the zero temperature chemical potential, i.e. the Fermi energy of the system, measured from the Weyl point. Unlike in normal metals, this scale can be of the order of 10-100 K for weakly doped Weyl systems, and the temperature dependence of the chemical potential is essential to understand quantitatively the observed relaxation time.

We also investigated carefully the other characteristics of nuclear magnetic resonance, the Knight shift, which determines the position of the resonance for nuclear spins. It exhibits rich behaviour as a function of temperature and doping and can even vanish and change sign as a function of these parameters. Close to absolute zero, it is diamagnetic for small doping, but can become either positive or negative with increasing doping depending on the doping level (i.e. electron or hole doping). At high temperature, on the other hand, it is always dominated by the diamagnetic term and decays very slowly as $-\ln(W/k_BT)$ with increasing temperatures. These unique features, in our opinion, can be used to identify signatures of Weyl points in the band structure even at significantly large doping level.

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