Constraining Kerr–like black holes from Event Horizon Telescope results of Sgr A*

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ABSTRACT

The Event Horizon Telescope (EHT), recently released the image of supermassive black hole Sgr A* showing an angular shadow diameter $d_{sh} = 48.7 \pm 7\mu$as, with an inferred black hole mass $M = 4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$ and Schwarzschild shadow deviation $\delta = -0.08^{+0.09}_{-0.10}$ (VLTI), $-0.04^{+0.09}_{-0.10}$ (Keck). The EHT image of Sgr A* is consistent with a Kerr black hole’s expected appearance and the results directly prove a supermassive black hole in the center of the Milky Way. The Kerr hypothesis, a strong-field prediction of general relativity (GR), may not hold in the theories of gravity that admit Kerr-like black holes having an additional deviation parameter arising from the underlying theory. Here, we use the EHT observational results of Sgr A* to investigate the constraints on the deviation parameter whereby, such a rotating Kerr-like black hole can be an astrophysical black hole candidate, paying attention to three leading models. Modelling Kerr-like black holes as supermassive black hole Sgr A*, we observe that for it to be a viable astrophysical black hole candidate, the EHT results of Sgr A* put more stringent constraints on the parameter space than those put by the EHT results of M87*. However, a systematic bias analysis shows Kerr-like black hole shadows may capture Kerr black hole shadows over a good part of the constrained parameter space, making Kerr-like and Kerr black holes indistinguishable and one can’t rule out a possibility of potential modifications of the Kerr metric or GR.

Keywords: Astrophysical black holes (98); Black hole physics (159); Galactic center (565); Gravitation (661); Gravitational lensing (670)

1. INTRODUCTION

Black holes are one of the most profound predictions of Einstein’s theory of general relativity (GR) (Einstein 1915). A region outside the black hole event horizon does not have any closed photon orbits and all photons travelling in this region, must eventually cross the event horizon and fall into the singularity (Hawking & Ellis 1973): the result is a shadow imprinted by all black holes in the emission region originating in its vicinity (Bardeen 1973; Luminet 1979). Theoretically, the black hole shadow is appropriate for testing the strong-field features of gravity, which can deduce potential deviations from the Kerr geometry, and has resulted in a comprehensive literature addressing shadows in both GR (Falcke et al. 2000; de Vries 2000; Shen et al. 2005; Yumoto et al. 2012; Atamurotov et al. 2013; Abdurabbarov et al. 2015; Cunha & Herdeiro 2018; Kumar & Ghosh 2020a; Afrin & Ghosh 2022) and modified theories of gravity (MTGs) (Amarilla et al. 2010; Amarilla & Eiroa 2012, 2013; Amir et al. 2018; Singh & Ghosh 2018; Mizuno et al. 2018; Allahyari et al. 2020; Papnoi et al. 2014; Kumar et al. 2020b; Kumar & Ghosh 2020b; Kumar et al. 2020a; Ghosh et al. 2021; Afrin & Ghosh 2021; Vagnozzi et al. 2022; Vagnozzi & Visinelli 2019). The fact that the silhouette of black hole shadows encode in them, the strong-field properties of the spacetime, suggests that, we can use them for performing strong-field gravitational tests (Johannsen & Psaltis 2010; Cunha & Herdeiro 2018; Baker et al. 2015).

According to the Kerr hypothesis, the Kerr metric (1963) describes well, all the astrophysical black holes. Essentially, the idea originated from the uniqueness the-
ome that led to the GR prediction, encapsulated in the no-hair theorem, stating that the Kerr metric (1963) is the only stationary, vacuum, axisymmetric metric that solves Einstein’s field equations, does not have patholo-
gies outside the event horizon, and is asymptotically flat vacuum spacetime (Israel 1967, 1968; Carter 1971; Hawking 1972; Robinson 1975). But direct evidence of
outside this theorem is still inconclusive, and it may be difficult to rule out other Kerr-like black holes (Ryan 1995; Will 2006) admitted by alternative theories of gravity. GR is a well-tested standard gravity model that has withstood all experimental tests but its perceived shortcomings (e.g., singularities) have motivated several modifications of GR (Clifton et al. 2012) the so-called MTGs, i.e., extensions or generalizations of GR involving additional gravitational degrees of freedom, that have received signif-
icant attention to overcome the issues arising in GR. However, tests of GR have traditionally involved solar-system bodies (Will 2014), for which we can interpret precise measurements with minimal astrophysical comp-
lications. In GR, there remains inconclusive fundamental issues, e.g., whether event horizons cover curvature singularities according to cosmic censorship conjecture or can be naked, which is crucial for black holes with the strongest gravitational fields, possessing a curvature singularity at their center. It is a wide belief that the quantum theory could resolve curvature singularities, but, at the event horizon, it predicts inherent randomness for quantum particles, resulting in the black hole information loss paradox (Harlow 2016).

Thus, the event horizon is accessible only for indi-
rect tests with strong field phenomena in the vicinity of black holes viz., with the black hole shadow. How-
ever, until recently, precision tests of gravity with black holes had not been possible; black holes have become a physical reality in 2019 with the release of the first horizon-scale image of the M87* black hole by the Event Horizon Telescope (EHT) collaboration (Akiyama et al. 2019a,b,c,d,e,f). Using a distance of \( d = 16.8 \) Mpc and estimated mass of M87* \( M = (6.5 \pm 0.7) \times 10^9 M_\odot \) (Akiyama et al. 2019a,d,b), bounds could be placed on the compact emission region size with angular diameter \( \theta_d = 42 \pm 3 \mu \text{as} \) along with the central flux depression with a factor of \( \gtrsim 10 \), which can be identified as the shadow. The observed image of the M87* black hole is consistent with the Kerr black hole as predicted by the GR. Nevertheless the current uncertainty in the measure-
ment of spin or angular momentum and the relative deviation of quadrupole moments do not elimi-
nate Kerr-like black holes arising in modified gravities (Akiyama et al. 2019a,d,b; Cardoso & Pani 2019). In turn, The EHT collaboration recently released the image of black hole Sgr A* in the Milkyway showing angular shadow diameter \( d_{sh} = 48.7 \pm 7 \mu \text{as} \); considering a black hole mass of \( M = 4.6^{+1.1}_{-0.6} \times 10^9 M_\odot \) and distance 8kpc from earth, the EHT demonstrate that the EHT images of Sgr A* are consistent with the expected appearance of a Kerr black holes (Akiyama et al. 2022a,b,c,d,e,f). Furthermore, when compared with the EHT results for M87*, it exhibits consistency with the predictions of GR stretching across three orders of magnitude in central mass (Akiyama et al. 2022e).

The EHT bounds on the astrophysical observables provide an excellent way to constrain the various black hole parameters and also in turn to test the underlying theories of gravity. While the EHT measure-
ments contain far more information related to the image of Sgr A*, for our purpose, we shall consider only the bounds on the shadow observables, angular shadow diameter \( d_{sh} \) and Schwarzschild shadow deviation \( \delta \) (Akiyama et al. 2019a), to put constraints on the deviation parameters, henceforth referred to as ”charges” (or hairs)–that the three well known rotating black hole metrics depend upon. We also invest-
igate whether the EHT bounds for Sgr A* can provide more stringent constraints on the black hole deviation parameters than previously obtained with the bounds for M87* observables (Kocherlakota et al. 2021; Psaltis et al. 2020; Afrin et al. 2021; Afrin & Ghosh 2021; Ghosh et al. 2021; Kumar & Ghosh 2020b). Thus, this paper aims to investigate if the images of Sgr A* can indeed help us probe better whether the three Kerr-
like black holes can be candidates for astrophysical black holes. Within the EHT constrained parameter space, we conduct a systematic bias analysis between the Kerr-like black hole shadows taken as models and Kerr shadows as injection, to determine the model parameter space \((a, g)\) where the Kerr-like black holes capture well the Kerr shadow and Sgr A* can thus be a Kerr-like black hole besides being a Kerr black hole as inferred by the EHT collaboration (Akiyama et al. 2022f). Outside this conforming parameter space, our analysis quantifies the distinguishability of GR from the theories of gravity in question, thus paving a way to test them.

The organization of the paper is as follows: in Sec-
tion 2, we analyze the influence of the deviation parameter on the photon motion and shadow characteristics, taking into consideration three Kerr-like black holes, and note the constraints on the deviation parameters from EHT observations of M87*. The Section 3 pertains to the shadow observables, and EHT’ deduced bounds on the Sgr A* image, wherefrom, we put constraints on the deviation parameters of the Kerr-like black hole models. In Section 4, we carry out a systematic bias analysis
to find out the model parameter regions consistent with the Kerr shadow characteristics. Finally, we conclude in Section 5.

We work with geometrized units $8\pi G = c = 1$ throughout this paper, unless units are specifically defined.

2. BLACK HOLE SHADOW: GENERAL ROTATING SPACETIMES

We begin by exploring analytically, the shadow features of general rotating metric, whose line element in Boyer-Lindquist coordinates $(t, r, \theta, \phi)$ reads (Abdujabbarov et al. 2016; Tsukamoto 2018; Kumar et al. 2020d)

$$ds^2 = -\left(1 - \frac{2m(r)r}{\Sigma}\right)dt^2 - \frac{4am(r)r}{\Sigma} \sin^2 \theta \, dt \, d\phi + \frac{\Sigma}{\Delta}dr^2 + \Sigma \, d\theta^2 + \left[r^2 + a^2 + \frac{2m(r)r^2a^2}{\Sigma} \sin^2 \theta \right] \sin^2 \theta \, d\phi^2,$$

and

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2m(r)r.$$  \hspace{1cm} (2)

where $m(r)$ is the mass function such that $\lim_{r \to \infty} m(r) = M$ with $M$ being the ADM mass of the rotating black hole and $a = J/M$ is the spin and $J$ is the angular momentum. Where $\Delta(r) = 0$, the metric (1) is singular, has one or more positive roots and that $r = r_+$ corresponds to event horizon. We assume that the $m(r)$ is well behaved when $r > r_+$. The metric (1) is Kerr (Kerr 1963) and Kerr-Newman (Newman et al. 1965) spacetimes respectively when $m(r) = M$ and $m(r) = M - \frac{Q^2}{r^2}$. In general the metric (1), depending on the choice of mass function $m(r)$, describes a wide variety of rotating black holes (Tsukamoto 2018; Kumar et al. 2020d; Kumar & Ghosh 2020a).

Interestingly, just as in case of the Kerr spacetime, the metric (1) has time translational and rotational invariance isometries which implies the presence of the Killing vectors $\chi_{(t)}^{\mu} = \delta_{(t)}^{\mu}$ and $\chi_{(\phi)}^{\mu} = \delta_{(\phi)}^{\mu}$ respectively. In what follows, we shall consider spacetimes with deviation parameters other than the black hole mass $M$, in order to constrain their possible range with the astrophysical observations such as that of the recent EHT image of Sgr A* and further go on to test their distinguishability from the Kerr black hole.

2.1. Black Hole Shadow

The no-hair theorem in GR is essential in the tests of gravity (Israel 1967; Carter 1968). Testing the theorem requires the general spacetimes that introduce deviations in the Kerr metric. Here, we aim to use observational results of Sgr A* (Akiyama et al. 2022e,f) to put bounds on the deviation parameters. We use inferred angular shadow diameter and Schwarzschild shadow deviation observables to put constraints on the parameters of the rotating metric that deviates from Kerr. The shadow outline is the locus of photon trajectories on the observer’s celestial plane. The photon motion in the spacetime (1) is governed by the Hamilton-Jacobi equation (Carter 1968),

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2}g_{\alpha \beta} \frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta},$$

where $\tau$ is the affine parameter along the geodesics, and $S$ is the Jacobi action, which reads as

$$S = -E t + L \phi + S_r(r) + S_\theta(\theta),$$

where $S_r(r)$ and $S_\theta(\theta)$, respectively, are functions of coordinates $r$ and $\theta$ only.

Due to the translational and rotational invariance, the spacetime (1) has conserved quantities, energy $E = -p_t$ and
and axial angular momentum $L_z = p_z$, where $p_{p}$ is the photon’s four-momentum. Further, the Petrov Type D metric (1) ensures the presence of Carter’s constant $\mathcal{K}$ (Carter 1968), eventually leading to the following complete set of null geodesics in the first-order differential form (Carter 1968; Chandrasekhar 1985):

$$\sum \frac{d \ell}{d \lambda} = \Delta \left[ E(v^2 + a^2) - a(L_z) - a(aE \sin^2 \theta - L_z), \right]$$

(5)

$$\sum \frac{d r}{d \lambda} = \pm \sqrt{\mathcal{R}(r)},$$

(6)

$$\sum \frac{d \theta}{d \lambda} = \pm \sqrt{\Theta(\theta)},$$

(7)

$$\sum \frac{d \phi}{d \lambda} = \frac{a}{\Delta} [E(r^2 + a^2) - a(L_z)] - \left( aE - \frac{L_z}{\sin^2 \theta} \right),$$

(8)

where $\mathcal{R}(r)$ and $\Theta(\theta)$, respectively, correspond to the radial and polar effective potentials and are given by

$$\mathcal{R}(r) = \left( (r^2 + a^2)E - a(L_z) \right)^2 - \Delta(\mathcal{K} + (aE - L_z)^2),$$

(9)

$$\Theta(\theta) = \mathcal{K} - \left( \frac{L_z^2}{\sin^2 \theta} - a^2E^2 \right) \cos^2 \theta.$$

(10)

The separability constant $\mathcal{K} = \mathcal{Q} - (aE - L_z)^2$ is related to the Carter constant (1968) which is essentially related to the metric symmetries through a quadratic Yano–Killing tensor (Hioki & Miyamoto 2008). The $\mathcal{Q}$ is related to the $\theta$-velocity of the photon and for $\mathcal{Q} = 0$ the photon motion is restricted to the equatorial plane; while the $L_z$ controls the $\phi$-motion (Teo 2021). The black hole shadow silhouette is formed by the spherical photons (SPOs) that move on constant radii $r_p > r_+$ obtained by solving (Chandrasekhar 1985)

$$\mathcal{R}(r_{p=r_p}) = \frac{\partial \mathcal{R}}{\partial r} \bigg|_{(r=r_p)} = 0 \text{ and } \frac{\partial^2 \mathcal{R}}{\partial r^2} \bigg|_{(r=r_p)} > 0.$$  

(11)

Introducing dimensionless quantities: $\xi \equiv \mathcal{L}/\mathcal{E}$, $\eta \equiv \mathcal{K}/\mathcal{E}^2$, in Eq. (9) and solving for Eq. (11) yields the critical impact parameters $(\xi_c, \eta_c)$ for the SPOs, (Tsukamoto 2018; Kumar & Ghosh 2020a)

$$\xi_c = \frac{[a^2 - 3r_p^2 m(r_p) + r_p [a^2 + r_p^2] (1 + m'(r_p))]}{a[m(r_p) + r_p[-1 + m'(r_p)]]},$$

$$\eta_c = \frac{-r_p^3}{a^2 [m(r_p) + r_p[-1 + m'(r_p)]^2} \left[ r_p^3 + 9r_p m(r_p)^2 + 2[2a^2 + r_p^2 + r_p^2 m'(r_p)] \right.$$

$$\times r_p m'(r_p) - 2m(r_p) [2a^2 + 3r_p^2 + 3r_p^2 m'(r_p)] \right],$$

(12)

where $'$ stands for the derivative with respect to $r$ and taking $m(r) = M$, Eq. (12) reduces to the Kerr impact parameters. The photons with $\eta_c = 0$ form circular orbits in the equatorial plane, whereas, $\eta_c > 0$ leads to the spherical photon orbits (Chandrasekhar 1985; Teo 2021).

The gravitationally lensed image of the photon sphere around the black hole yield the apparent black hole shadow. For an asymptotically faraway observer ($r_o \rightarrow \infty$), making an inclination angle $\theta_0$ with the spin axis, the black hole shadow is a dark region in the celestial sky outlined by a bright ring (Johannsen 2016; Johnson et al. 2020) with Cartesian coordinates, (Afrin & Ghosh 2021; Kumar et al. 2020d; Bardeen 1973)

$$\{X, Y\} = \{-\xi_c \csc \theta_0, \pm \sqrt{\eta_c + a^2 \cos^2 \theta_0 - \xi_c^2 \cot^2 \theta_0}\}.$$  

(13)

The shadow coordinates $\{X, Y\}$ for the Kerr-like black holes have the same form as that for a Kerr black hole with the $M$ replaced by $m(r)$.

### 2.2. Kerr-like black holes

We examine here three Kerr-like (or non-Kerr) black holes viz. Kerr-Newman (Newman et al. 1965), Kerr-like black hole in Horndeski theory of gravity (Afrin & Ghosh 2021; Kumar et al. 2021) (Henceforth rotating Horndeski black holes) and rotating hairy Kerr-like black holes obtained by gravitational decoupling approach (Afrin & Ghosh 2022; Contreras et al. 2021). These black holes are defined by the metric (1) with appropriate mass function $m(r)$.

**Kerr-Newman black hole**—The Kerr-Newman spacetime is the most general stationary, asymptotically flat solution to the Einstein-Maxwell field equations obtained by complexification of the Reissner-Nördstrom metric (Newman et al. 1965). The Kerr–Newman is a proxy for any non-Kerr or Kerr-like metric that permits a well-defined and straightforward demonstration of the impact of deviations from Kerr on the photon-ring properties (Broderick et al. 2022). The mass function of the Kerr-Newman metric is given by,

$$m(r) = M - \frac{Q^2}{2r},$$  

(14)

where $Q$ is the electric charge of the black hole. The Kerr-Newman black holes encompass Kerr ($Q = 0$), Reissner-Nördstrom ($a = 0$) and Schwarzschild ($Q = 0 = a$) as special cases. We can analyse null-geodesics to explore shadow of Kerr-Newman black hole. The apparent size and shape of the KNBH shadow depend on $a$ and $Q$ (de Vries 2000). Like Kerr black hole shadow (Bardeen 1973), the Kerr-Newman black hole shadows are distorted and the distortion reduces as the observer
progresses towards axis of black hole symmetry from the equatorial plane and no distortion for inclination angle $\theta_0 = 0, \pi$ (Fig. 1). Modelling the Kerr-Newman black holes as M87*, puts constraints on the parameters $Q/M \in (0, 0.95]$ (Kocherlakota et al. 2021).

Though astrophysical black holes are supposed to be neutral – for black holes with a mass smaller than $\sim 10^8 M_\odot$ the charge is spontaneously lost to the environment and thus maximum charge of the black hole is thus prohibited (Gibbons 1975) – still, there is a good amount of motivation to constrain the electric charge of black holes which has seen a multitude of effort in this direction (Takahashi 2005; Kocherlakota et al. 2021; Akiyama et al. 2022f). The charge of Sgr A* has also been theoretically constrained to be $Q \leq 3.1 \times 10^8 C$ using Chandra X-ray data (Zajaček et al. 2018; Karouzos 2018). We intend here, to put better constrains on the charge of Sgr A* using electromagnetic observations of supermassive black holes viz. with the EHT data. It is also provides a method to establish or discard the charge-neutrality of the astrophysical black holes, observationally, by this analysis.

Rotating Horndeski black hole—The Horndeski black hole is an exact solution to a class of quartic Horndeski gravity which is asymptotically flat (Bergliaffa et al. 2021). The rotating Horndeski black hole (Afrin & Ghosh 2021; Kumar et al. 2021) is described by the metric (1) with

![Figure 2. Shadow angular diameter $d_{sh}$ modeling Sgr A* as Kerr-Newman black hole (top left), rotating Horndeski black hole (top right) and rotating hairy black hole (bottom). The light gray and black solid curves correspond to 41.7\(\mu\)as and 55.7\(\mu\)as respectively. The mass and distance of Sgr A* used are $M = 4 \times 10^6 M_\odot$ and $d = 8kpc$. The white region corresponds to forbidden parameter space.](image)
Table 1. Constrained on deviation parameters from EHT observation of SgrA*

| Spacetimes            | Constrains from EHT observations |
|-----------------------|----------------------------------|
|                       | $d_{sh}$                      | $\delta$             |
| Kerr-Newman           | $Q/M \in (0,0.915]$             | VLTI: $Q/M \in (0,0.8915]$ |
|                       | Keck: $Q/M \in (0,0.8328]$      |                        |
| rotating Horndeski    | $h/M \in [-0.3804,0)$           | VLTI: $h/M \in (-0.04754,0)$ |
|                       | Keck: $h/M \in [-0.2325,0)$      |                        |
| rotating hairy        | $l_0/M \in [0,1)$               | Keck: $l_0/M \in [0.0696,1)$ |

the mass function

$$m(r) = M - \frac{h}{2} \ln \left( \frac{r}{2M} \right).$$

(15)

where the deviation charge parameter is coming from the Horndeski theory (Bergliaffa et al. 2021). We observe distortion in the rotating Horndeski black hole shadows are also because of the parameter $h$ apart from the spin $a$ (c.f Fig. 1). Modelling the rotating Horndeski black hole as M87*, put constraints on the parameters viz., $0.0077 M \leq a \leq 0.9353 M$, $-0.7564 M \leq h < 0$ at $\theta_o = 90^\circ$ and $0.0048 M \leq a \leq 0.9090 M$, $-0.7920 M \leq h < 0$ at $\theta_o = 17^\circ$ (Afrin & Ghosh 2021). Also, within the constrained space $(a,h)$, the Horndeski gravity can model M87* within the present observational uncertainties (Afrin & Ghosh 2021).

Rotating hairy black holes—The rotating hairy black hole is generated via gravitational decoupling approach (Contreras et al. 2021). It modifies the Kerr black hole solution due to surrounding fluid like dark energy or matter with conserved energy momentum tensor satisfying SEC resulting into hairy rotating black holes. It is also a Kerr-like black holes described by metric (1) with mass function (Contreras et al. 2021)

$$m(r) = M - \frac{\alpha}{2} e^{-\frac{r}{(M-l_0)}}$$

(16)

where the parameter $l_0 \leq 2M$ is the deviation parameter which determines asymptotic flatness of metric (1) with (16). The $\alpha = 0$ corresponds to the absence of surrounding matter sources and thus to the Kerr metric. Besides, in the limit $l_0 \rightarrow l_k = 2M$, one recovers the Kerr black hole. The Rotating hairy black holes represent a family of BHs described by the parameters $(M,a,l_0)$, where $l_0 = \alpha l$ represents a charge associated with the deviation parameter. Modelling the rotating hairy black holes as M87*, yields constraints on the parameters $l_0/M \in [0.7122,1)$ (Afrin et al. 2021).

3. CONSTRAINTS FROM EHT RESULTS OF SAGITTARIUS A*

The black hole shadow shape serves as a direct probe of strong field gravity as it is the most direct manifestation of the background spacetime notwithstanding multifarious astrophysical phenomena like accretion flow, emission phenomena etc. For Kerr-like black holes, the distortion in circular shadow shape is found to be incurred by the deviation parameter in addition to spin (Cunha et al. 2015, 2019; Kumar et al. 2020c; Afrin et al. 2021; Afrin & Ghosh 2021, 2022) for a given inclination angle. A Horizon-scale image of a supermassive black hole provides a new avenue for testing the theory of GR. Photons that originate in the deep gravitational fields of black holes form these images and therefore carry imprints of the spacetime properties in the strong-field regime (Jaroszynski & Kurpiewski 1997; Falcke et al. 2000). The EHT collaboration has unveiled the first horizon-scale images of Sgr A*, the supermassive black hole at the centre of our Galaxy (Akiyama et al. 2022e). This section presents constraints on the potential deviations parameter associated with the three Kerr-like black holes imposed by these images.

The observed EHT image of SgrA* has an angular shadow diameter $d_{sh} = 48.7 \pm 7 \mu$as and Schwarzschild shadow deviation $\delta = -0.08^{+0.09}_{-0.09}$ (VLTI), $-0.04^{+0.1}_{-0.10}$ (Keck) (Akiyama et al. 2022e,f). The EHT image of Sgr A* is consistent with a Kerr black hole’s expected appearance with an inferred black hole mass $M = 4.0^{+1.1}_{-0.6} \times 10^6 M_{\odot}$ (Akiyama et al. 2022e) and distance $d = 8kpc$ from earth (Akiyama et al. 2022e,f) while no stringent comment on the inclination angle has been made. The predicted size of its shadow of EHT observation is in accordance with Kerr black holes, and hence there is no evidence for any violations of GR. Our goal is to place constraints on deviations parameters of the three underlying black hole metrics. The three metrics in question are prototype non-Kerr with an additional deviation parameter. They revert to the Kerr metric when the deviation parameter is zero but remain free of pathologies for a wide range of parameter values. While the EHT observations contain far more information related to the image of Sgr A*, to put constraints on the deviation parameters, we shall use only the EHT bounds on the two observables, angular shadow diameter $d_{sh}$
Figure 3. Shadow diameter deviation $\delta$ modeling Sgr A* as Kerr-Newman black hole (top left), rotating Horndeski black hole (top right) and rotating hairy black hole (bottom). The light gray and black solid curves correspond to respectively to lower bounds -0.14 (Keck) and -0.17 (VLTI). The mass and distance of Sgr A* used are $M = 4 \times 10^6 M_\odot$ and $d = 8$ kpc. The white region corresponds to forbidden parameter space.

and Schwarzschild shadow deviation $\delta$ (Akiyama et al. 2019a).

For estimating angular shadow diameter $d_{sh}$ for the Kerr-like black holes, we define the area enclosed within the shadow silhouette as (Kumar & Ghosh 2020a),

$$A = 2 \int Y(r_p) dX(r_p)$$

where $r_p^\pm$ are the prograde and retrograde SPO radii given respectively by the smallest and largest roots of:

$$\eta_c = 0, \ \xi_c(r_p^\pm) \geq 0 \ (\text{Teo} \ 2021).$$

We consider first, the shadow angular diameter which for a distance $d$ from the black hole is defined as (Bambi et al. 2019; Kumar & Ghosh 2020b; Afrin et al. 2021)

$$d_{sh} = 2 \frac{R_a}{d}, \ R_a = \sqrt{A/\pi}. \ (18)$$

$R_a$ being the areal shadow radius.

Next, we calculate the angular diameter of the shadow, which, apart from other black hole parameters, depends on mass $M$ and the distance $d$ of the black hole. Using EHT, inferred mass of the black hole $M = 4.0^{+1.4}_{-0.6} \times 10^6 M_\odot$ and taking $d = 8$ kpc from earth for the three Kerr-like black holes, the angular diameter of the shadows are calculated and is depicted in Figure 2. Thus, modeling Sgr A* as the three Kerr-like black holes
we show that $41.7\mu as \leq d_{sh} \leq 55.7\mu as$, i.e., within the 1σ region of the SgrA* shadow angular diameter, is satisfied when $0 < Q \lesssim 0.9150M$ for the Kerr-Newman black holes, $-0.3804M \lesssim h < 0$ for the rotating Horndeski black holes whereas the entire parameter space is consistent at $\alpha = 0.5, 1.0$ in case of the rotating hairy black holes (see Figure 2). For these constrained parameter range the model shadows are consistent with the shadow of Sgr A*.

The EHT image of SgrA* exhibits a bright thick ring of emission with a diameter of $51.8 \pm 2.3 \mu as$ surrounding a brightness depression of diameter, namely the black hole shadow (Akiyama et al. 2022e,f). The diameter of the shadow, $d_{\text{metric}}$, can measure the properties associated with the black hole metric and determine its agreement with the Kerr solution of GR for a black hole of a given angular size $\theta_\text{g}$ (Akiyama et al. 2022e,f).

Schwarzschild shadow deviation ($\delta$) quantifies the deviation of the model shadow diameter ($d_{\text{metric}}$) from the Schwarzschild shadow diameter $6\sqrt{3}M$ and is given by (Akiyama et al. 2022e,f),

$$\delta = \frac{d_{\text{metric}}}{6\sqrt{3}} - 1. \quad (19)$$

We take $d_{\text{metric}} = 2R_a$ where $R_a$ is given by Eq. (18). The Kerr shadow diameter differs from the Schwarzschild diameter by 7.5% as spin varies from 0 to $M$ and inclination from 0 to $\pi/2$; thus $\delta \in [-0.075, 0]$ implies consistency of a model with Kerr predictions while values outside the range would clearly show discord (Akiyama et al. 2022f). Interestingly, the EHT has inferred bounds : $-0.17 \lesssim \delta \lesssim 0.01$ (VLTI) and $-0.14 \lesssim \delta \lesssim 0.05$ (Keck), using the VLTI and Keck priors on mass-to-distance ratio of Sgr A* (Akiyama et al. 2022e,f) which comprises allowed values $\delta < -0.075 \cup \delta > 0$ well outside the Kerr accordant range $\delta \in [-0.075, 0]$. This opens possibilities of testing alternate theories of gravity predicting shadows, both smaller than ($\delta < -0.075$) and larger than ($\delta > 0$) Kerr. All the models that we consider casting shadows smaller or larger than the corresponding Kerr shadows – are thus candidates for Sgr A*. Modeling Sgr A* as Kerr-Newman, rotating Horndeski and rotating hairy black holes we get constraints $0 < Q \lesssim 0.8915M$ (VLTI), $0 < Q \lesssim 0.8328M$ (Keck), $-0.04754M \lesssim h < 0$ (VLTI), $-0.2325M \lesssim h < 0$ (Keck) and $0.0696M \lesssim l_0 < 1$ respectively (see Fig. 3); the bounds on $\delta$ however do not constrain the $l_0$ parameter at $\alpha = 0.5$ (see Fig. 3) and the entire parameter space is consistent with observations.

We tabulate the EHT constrained parameter range for the different models considered, in Table 1, from which the inferred constraints considering both the bounds on $d_{sh}$ and $\delta$ are : (i) $Q/M \in (0, 0.8328]$ for the Kerr-Newman black holes (ii) $h/M \in [-0.04754, 0]$ for the rotating Horndeski black holes (iii) $l_0/M \in [0.0696, 1]$ at $\alpha = 1.0$ for the rotating hairy black holes. The entire parameter space for the rotating hairy black holes are consistent with the EHT results for Sgr A*. Thus with the EHT results for Sgr A*, we are able to put more stringent constrains on both Kerr-Newman and rotating Horndeski black holes, than previously obtained from M87* shadow observations with EHT.

4. CHI-SQUARE ANALYSIS

The Kerr-like metric (1) has an additional deviation parameter, apart from those of the Kerr metric, because of the underlying theory, and it encompasses the Kerr metric as a particular case when this deviation parameter vanishes. If EHT observations of Sgr A* favour near-zero values of the deviation parameters, then the Kerr hypothesis holds true within the present observational uncertainties. A strong bias for non-zero deviation parameters would suggest scope for potential modifications of the Kerr metric. Besides, the various deviation parameters may change the shadow characteristics in ways similar to the changes caused by the spin parameter in the Kerr metric, thus causing degeneracy in the shadows, which can be investigated to quantify the agreement between the underlying theory of gravity and GR. This forms the basis of our formalism for testing the Kerr-like metric. While the theories of gravity have been tested using varied methods (Zhou et al. 2018; Chen et al. 2021; Will & Yunes 2004), the black hole shadow observations accord one of the most robust means, which can be repeated over several epochs (Akiyama et al. 2019a) and thus can give increased accuracy of results over several observations for the same source. Taking the metric (1) with (14)-(15) as models for Sgr A* and the Kerr metric as injection, we carry out a systematic bias analysis between the shadows of the models and the injection, within the EHT constrained parameter space summarized in Table 1. We utilize two shadow observables $d_{sh}$ and $\delta$ given in Eqs. (2) and (19), to form a cost function, the reduced $\chi^2$, that is minimized over the model parameter space to determine the detectability of any deviations from GR. We consider the maximum possible deviation of the model from GR by fixing $\theta_0 = 90^\circ$. We compute the bias, reduced $\chi^2$ given by (Ayzenberg & Yunes 2018; Kumar et al. 2020d; Afrin & Ghosh 2021),

$$\chi^2(a, g, a^*) = \frac{1}{2} \sum_{i=1}^{2} \left[ \frac{W_i(a, g) - \tilde{W}_i(a^*)}{\sigma_i} \right]^2, \quad (20)$$
Figure 4. Reduced $\chi^2$ between injected Kerr shadow and shadow of Sgr A* modelled as Kerr-Newman black hole (top), rotating Horndeski black hole (middle) and rotating hairy black hole (bottom) in the EHT constrained parameter space. The white region corresponds to forbidden parameter space.
where $W^i = \{d_{\phi h}, \delta\}$ are the black hole shadow observables, $g = \{Q, h, l_0\}$ are the model deviation parameters and the measurement error $\sigma_i = \sqrt{\frac{\partial W^i}{\partial a}}$ is assumed to be 10% of the range of $W^i$ (Akiyama et al. 2022e,f; Afrin & Ghosh 2021); the injected spin is denoted by $a^\ast$. We utilize 150 sample points $(a, g)$ for the bias analysis. For fixed extrinsic parameters $\{r_0, \theta_0\}$, the injection depends solely on spin $a^\ast/M$, whereas, the models depend on both spin $a/M$ and deviation parameter $g/M$. $\chi^2$ can be adopted as a measure of distinguishability between the model and injected shadows since $\chi^2 \leq 1$ would signify that the model shadows are degenerate with the injected shadows and the underlying theory of gravity is indistinguishable from GR at current observational precision, whereas, $\chi^2 > 1$ implies the distinguishability of the model and injected shadows and thus open possibility of testing GR against the underlying theory of gravity.

The Fig. 4 shows a contour map of the $\chi^2(a, g, a^\ast)$ within the EHT constrained model parameter space (see Fig. 2 and 3). Interestingly, for the Kerr-Newman and rotating hairy models, the $\chi^2 \leq 1$ contour bounds parameter region centered around non-zero high values of the respective deviation parameters $Q$ and $l_0$, whereas, for the rotating Horndeski model the $\chi^2 \leq 1$ is centered around very small values of deviation parameter $h$. In this region, the underlying theory of gravity in the vicinity of Sgr A* cannot be ascertained within the present observational confidence. However, outside the $\chi^2 = 1$ contour, i.e., in a substantial parameter region, the Kerr-like black holes are discernible from the Kerr black holes via the shadow observations. Furthermore, with the increase in the injected spin $a^\ast$, the $\chi^2 = 1$ contour shifts to a higher value of deviation parameter for Kerr-Newman model but to a lower value of deviation parameter in case of rotating hairy black hole model; for the rotating Horndeski model, the shift is to a higher spin value. This indicates a one-to-one correspondence between the injected and model spins only in the rotating Horndeski case, whereas, for the other two models there is evidently a correlation between the deviation parameter and Kerr spin. Interestingly, for all the three models, there is a sizeable parameter space with $\chi^2 > 1$ (see Fig. 4), where the Kerr-like black holes are distinguishable from the Kerr black holes via the EHT shadow observations within the current limits of resolution. This entails the exciting possibility of testing the theories of gravity, admitting Kerr-like black holes, against GR.

5. CONCLUSION

The EHT observations of the black hole shadow of supermassive black holes Sgr A* at the center of our galaxy Milky Way is an ideal and natural laboratory for testing the properties of black holes and the nature of strong-field gravity. The Kerr hypothesis holds in GR, but there are some modified gravity theories it is violated, which do not admit Kerr black holes. Using the EHT observation of the black hole shadow in SgrA*, we place constraints on deviation parameters associated with three well-motivated Kerr-like or non-Kerr black holes viz. Kerr-Newman, rotating Horndeski and rotating hairy black holes. Interestingly, our analysis suggests a degeneracy in the shadow characteristics of Kerr black holes and Kerr-like black holes, which results from the fact that the deviation parameters of Kerr-like black holes, similar to the spin parameter, incur distortions in shadow shapes. We have also explored the possibility of placing constraints on the deviation parameters of the Kerr-like using EHT observations results of SgrA*. Modelling Sgr A* as Kerr-like black holes, we impose EHT inferred bounds on its shadow observables, viz. $d_{\phi h}$, and $\delta$ to constrain the model parameter space. We find the constraints obtained for both the Kerr-Newman and rotating Horndeski black holes with Sgr A* results are more stringent than previously obtained with EHT results for M87* (Kocherlakota et al. 2021; Afrin & Ghosh 2021). We show, as a first, that the EHT observations of Sgr A* can place more stringent constraints on the deviation parameter, than current constraints placed with EHT observation of M87*.

The Sgr A* can be both a Kerr-like and a Kerr black hole within the constrained parameter region. We conduct a chi-square test to assess the regions where the shadows in GR are indistinguishable (or distinguishable) from that in the other Kerr-like black holes in question. We find that in regions bounded within $\chi^2 \leq 1$, the minimum $\chi^2$ contour is centred around non-zero deviation parameters for the Kerr-Newman and rotating hairy models, which shows that consistency with Kerr observations favours non-zero higher values of deviation. However, for the rotating Horndeski model, the minimum $\chi^2$ contour is centred around higher $a$ and tiny value of $h$, showing that the consistency with Kerr shadow causes near-zero values of the deviation parameter. For all the three models considered, there is a substantial parameter space within the EHT constrained space with $\chi^2 > 1$ where the Kerr-like shadows are distinguishable from the Kerr shadows subject to the current observational uncertainties of the EHT measurements.

Our primary goal in this work has been to test the possibility of Kerr-like black holes with an additional deviation parameter of being a candidate for an astrophysical black hole, with EHT observations of Sgr A*. In the
substantial part of the constrained parameter space, we find that, Kerr-like black holes can capture Kerr black hole’s shadow, which is a background spacetime for astrophysical black holes. Thus, the Kerr-like black holes can be suitable background spacetime for an astrophysical black hole. Our results now warrant a more detailed investigation by considering some other Kerr-like black holes of well-motivated modified gravity. We will explore these in a forthcoming article.

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REFERENCES

Abdujabbarov, A., Amir, M., Ahmedov, B., & Ghosh, S. G. 2016, Phys. Rev. D, 93, 104004, doi: 10.1103/PhysRevD.93.104004

Abdujabbarov, A. A., Rezzolla, L., & Ahmedov, B. J. 2015, Mon. Not. Roy. Astron. Soc., 454, 2423, doi: 10.1093/mnras/stv2079

Afrin, M., & Ghosh, S. G. 2021. https://arxiv.org/abs/2110.05258

—. 2022, Universe, 8, 52, doi: 10.3390/universe8010052

Afrin, M., Kumar, R., & Ghosh, S. G. 2021, Mon. Not. Roy. Astron. Soc., 504, 5927, doi: 10.1093/mnras/stab1260

Akiyama, K., et al. 2019a, Astrophys. J. Lett., 875, L1, doi: 10.3847/2041-8213/ab0ec7

—. 2019b, Astrophys. J. Lett., 875, L6, doi: 10.3847/2041-8213/ab1141

—. 2019c, Astrophys. J. Lett., 875, L3, doi: 10.3847/2041-8213/ab0e57

—. 2019d, Astrophys. J. Lett., 875, L5, doi: 10.3847/2041-8213/ab0f43

—. 2019e, Astrophys. J. Lett., 875, L4, doi: 10.3847/2041-8213/ab0e85

—. 2019f, Astrophys. J. Lett., 875, L2, doi: 10.3847/2041-8213/ab0c96

—. 2022a, Astrophys. J. Lett., 930, L15, doi: 10.3847/2041-8213/ac6736

—. 2022b, Astrophys. J. Lett., 930, L16, doi: 10.3847/2041-8213/ac6672

—. 2022c, Astrophys. J. Lett., 930, L13, doi: 10.3847/2041-8213/ac6675

—. 2022d, Astrophys. J. Lett., 930, L14, doi: 10.3847/2041-8213/ac6429

—. 2022e, Astrophys. J. Lett., 930, L12, doi: 10.3847/2041-8213/ac6674

—. 2022f, Astrophys. J. Lett., 930, L17, doi: 10.3847/2041-8213/ac6576

Allahyari, A., Khodadi, M., Vagnozzi, S., & Mota, D. F. 2020, JCAP, 02, 003, doi: 10.1088/1475-7516/2020/02/003

Amarilla, L., & Eiroa, E. F. 2012, Phys. Rev. D, 85, 064019, doi: 10.1103/PhysRevD.85.064019

—. 2013, Phys. Rev. D, 87, 044057, doi: 10.1103/PhysRevD.87.044057

Amarilla, L., Eiroa, E. F., & Giribet, G. 2010, Phys. Rev. D, 81, 124045, doi: 10.1103/PhysRevD.81.124045

Amir, M., Singh, B. P., & Ghosh, S. G. 2018, Eur. Phys. J. C, 78, 399, doi: 10.1140/epjc/s10052-018-5872-3

Atamurotov, F., Abdujabbarov, A., & Ahmedov, B. 2013, Phys. Rev. D, 88, 064004, doi: 10.1103/PhysRevD.88.064004

Ayzenberg, D., & Yunes, N. 2018, Class. Quant. Grav., 35, 235002, doi: 10.1088/1361-6382/aae87b

Baker, T., Psaltis, D., & Skordis, C. 2015, Astrophys. J., 802, 63, doi: 10.1088/0004-637X/802/1/63

Bambi, C., Freese, K., Vagnozzi, S., & Visinelli, L. 2019, Phys. Rev. D, 100, 044057, doi: 10.1103/PhysRevD.100.044057

Bardeen, J. M. 1973, in Les Houches Summer School of Theoretical Physics: Black Holes

Bergliaff, S. E. P., Maier, R., & Silvano, N. d. O. 2021. https://arxiv.org/abs/2107.07839

Broderick, A. E., Tiede, P., Pesce, D. W., & Gold, R. 2022, Astrophys. J., 927, 6, doi: 10.3847/1538-4357/ac4970

Cardoso, V., & Pani, P. 2019, Living Rev. Rel., 22, 4, doi: 10.1007/s41114-019-0020-4

Carter, B. 1968, Phys. Rev., 174, 1559, doi: 10.1103/PhysRev.174.1559

—. 1971, Phys. Rev. Lett., 26, 331, doi: 10.1103/PhysRevLett.26.331

Chandrasekhar, S. 1985, The mathematical theory of black holes (New York: Oxford Univ. Press)

Chen, S., Wang, Z., & Jing, J. 2021, JCAP, 06, 043, doi: 10.1088/1475-7516/2021/06/043

Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. 2012, Phys. Rept., 513, 1, doi: 10.1016/j.physrep.2012.01.001

Contreras, E., Ovalle, J., & Casadio, R. 2021, Phys. Rev. D, 103, 044020, doi: 10.1103/PhysRevD.103.044020
Zajaček, M., Tursunov, A., Eckart, A., & Britzen, S. 2018, Mon. Not. Roy. Astron. Soc., 480, 4408, doi: 10.1093/mnras/sty2182

Zhou, M., Cao, Z., Abdikamalov, A., et al. 2018, Phys. Rev. D, 98, 024007, doi: 10.1103/PhysRevD.98.024007