1. Introduction

Associated petroleum gas, mixtures of natural gas with hydrogen or other hydrocarbons, for example, petrochemical waste application is an advanced trend in heat power engineering. The substitute of natural gas with alternative fuels creates a number of problems, one of which is the optimization of the combustion regime. In work [1] an experimental water heating installation was presented, in which a mixture of methane with hydrogen or propane-butane fuel was used. Mathematical model of the optimization process is presented in articles: [2, 3, 4]. It is linear and applicable in case of small changes (up to 10%) in the combustion value. The main disadvantage of this model is that the concentrations of the components that make up the fuel are present not obviously. According to this model, the optimal combustion regimes were calculated in a low-power water heating unit using APG of variable composition. However, the calculation results have not yet been confirmed experimentally. The transition regularities from the combustion of methane to the combustion of mixtures of methane with other gases have not been studied, under the optimal combustion conditions and the initial heating capacity of boilers.

The purpose of this work is to determine the conditions for which, at minimum consumption and complete fuel combustion, the heat release rate for the mixtures under consideration is equal

2. Mathematical model

Consider a water heating unit consisting of a combustion chamber, a heat exchange device located in the flow of combustion products, inside which a heat carrier moves at a constant speed [1]. The heat carrier warming occurs due to the fact that part of the heat $Q_c$ released per unit time as a result of fuel
Combustion is transferred to the heat carrier, i.e. \( Q_w = k_{w,c} Q_c \). For a steady process the heat carrier warming with a mass flow rate \( G_w \), the thermodynamic relation is complete:

\[
Q_w = c_p G_w \Delta t_w
\]

where \( c_p \) is the heat capacity of the heat carrier, \( \Delta t_w \) is the temperature difference of the heat carrier at the outlet and inlet to the heat exchanger.

In the case of complete combustion of fuel with a calorific value, a unit of volume equal to \( q \) and a volumetric flow rate \( V_f \), a volumetric air flow \( V_a \) corresponding to the stoichiometric ratio of fuel and air, the efficiency of the installation \( k_{w,c} \) has a maximum value. Such a mode of operation of the water heating unit, when the specified difference between the outlet and inlet temperatures of the coolant is achieved as a result of complete combustion of the minimum possible volume of fuel, i.e., at the maximum efficiency of the installation, we will call it optimal. In this case, it follows from relation (1):

\[
\Delta t = \frac{(k_{w,c} Q_c)}{(c_p G_w)} = k_{w,c} q V_f
\]

In work [35], an analysis was made of the influence on the optimal combustion mode of a change in the heat of combustion of hydrocarbon fuel \( q_0 \) by the value \( |q - q_0| = \Delta q \). It was found that after a decrease in the heat of fuel combustion by the indicated value, the optimal combustion mode is restored with an increase in the initial fuel consumption \( V_{f,0} \) by a relative value:

\[
\Theta_f = \frac{V_f - V_{f,0}}{V_{f,0}} = \Theta_q/(1 - \Theta_q), \Theta_q = \Delta q/q_0
\]

In the case of an increase in the heat of fuel combustion by the value \( \Delta q \), the condition for the optimal combustion mode has the form:

\[
\Theta_f = - (\Theta_q/(1 + \Theta_q))
\]

i.e., it is necessary to reduce fuel consumption.

The analysis showed that in both cases considered there is no need to change the air flow, i.e.:

\[
\Theta_a = (V_a - V_{a,0})/V_{a,0} = 0
\]

Gaseous fuel is a mixture of gases, the heat of combustion of which depends on the concentrations of its components and their heat of combustion. In this case, it is necessary to establish the dependence of the conditions of the optimal combustion in the heating installation on the concentration of gases contained in the fuel.

Consider the combustion of a mixture of the main fuel - methane, to which hydrogen or propane-butane fuel is added. In this case \( q_0 \) is the heat of combustion of 1 m³ of methane. The heat of combustion of 1 m³ of a mixture of methane with other gases will depend on the concentration \( \eta \) of these gases (Fig. 1.).

The calculations were carried out according to the generally accepted method [5].
According to the results of calculations, the dependences of the mixtures combustion heat on the concentration of impurities to the main fuel are linear. For methane-hydrogen fuel we have:

\[ q(\eta) = q_0 (1 - \lambda_h \eta), \quad \lambda_h = 0.69 \]  
(6)

For a mixture of methane with propane-butane fuel, we get:

\[ q(\eta) = q_0 (1 + \lambda_p \eta), \quad \lambda_p = 1.57 \]  
(7)

The decrease in the heat of methane mixture combustion with hydrogen relative to the value for methane is: \( \Delta q = \lambda_h \eta q_0, \quad \Theta_q = \lambda_h \eta \). Substituting the relative value \( \Theta_q \) into relation (3), we obtain:

\[ \Theta_{f,h}(\eta) = \frac{\lambda_h \eta}{1 - \lambda_h \eta} \]  
(8)

For a mixture of methane with propane-butane fuel, the increase in the heat of combustion is: \( \Delta q = \lambda_p \eta q_0, \quad \Theta_q = \lambda_p \eta \). Then from expression (4) for the relative change in fuel consumption it follows:

\[ \Theta_{f,p}(\eta) = - \frac{\lambda_p \eta}{1 + \lambda_p \eta} \]  
(9)

Relationships (8), (9) determine the optimal methane mixture consumption with hydrogen or propane-butane fuel with a given concentration of gases – impurities, if the fuel consumption corresponding to the optimal combustion of one methane (\( \eta = 0 \)), the corresponding air consumption included in the condition (5).

If the impurities are waste from petrochemical industries, it becomes possible to reduce the consumption of the traditional main fuel - natural gas, which is almost entirely methane. From relation

Figure 1. Change in the specific heat of combustion of fuel with an increase in the concentration of added gases in a mixture with methane.
(9) it follows that the presence of hydrocarbons in the mixture, which are "heavier" than methane, reduces the total fuel consumption, and, consequently, the consumption of methane entering the fuel.

If methane is mixed with hydrogen, the total fuel consumption increases according to (8). Suppose the optimal fuel consumption when burning only methane \( V_f = V_{m,0} \). Assume that the concentration of hydrogen in the methane mixture is equal to \( \eta \). In the case of optimal combustion according to (1), the total fuel consumption is \( V_f = V_{m,0}(1 + \Theta_{f,h}) \). As \( V_f = V_m + V_h \), we have \( V_m = V_f(1 - \eta) = V_{m,0}(1 + \Theta_{f,h})(1 - \eta) \).

After substitution of relation (4) we obtain:

\[
\Theta_m = \frac{(V_m - V_{m,0})}{V_{m,0}} = -\eta(1 - \lambda_h)/(1 - \lambda_h \eta)
\]

It was previously shown that \( \lambda_h < 1 \). Hence, \( \Theta_m < 0 \), i.e., methane consumption is also reduced if it is burned together with hydrogen.

The conditions for optimal steady-state combustion were calculated for various concentrations of methane with hydrogen or propane-butane fuel. The relative changes in fuel consumption corresponding to optimal combustion were determined, depending on the concentration of gases mixed with methane (Fig. 2).

![Figure 2](image)

**Figure 2.** Dependence of the relative value of fuel consumption on the concentration of the added mixture

| Table 1. The chemical composition of natural gas. |
|-------------------------------------------------|
| Gas composition | CH\(_4\) | C\(_2\)H\(_6\) | H\(_2\)S | O\(_2\) |
| %                | 94,85   | 0,32       | 3,63    | 1,2    |

| Table 2. The chemical composition of propane-butane fuel. |
|----------------------------------------------------------|
| Gas composition | C\(_3\)H\(_8\) | C\(_4\)H\(_{10}\) (butane gas) | C\(_4\)H\(_{10}\) (isobutane) | H\(_2\)S | O\(_2\) |
| %                | 74,978   | 6,9        | 8,77    | 4,13   | 5,2    |

| Table 3. The chemical composition of hydrogen. |
|------------------------------------------------|
| Gas composition | H\(_2\) |
| %                | 100     |
3. Conclusion
A mathematical model of the optimal combustion of mixtures of methane with hydrogen or propane-butane fuel has been developed. Depending on the concentration of impurities to methane, changes in fuel consumption were determined, for which the rate of heat release is the same as after combustion of only methane. Air consumption is constant and does not depend on the fuel composition.

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