An integrated study on the optimal shape design of cruciform specimen used in equibiaxial tensile test of a hyperelastic material

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Abstract. To conduct a finite element analysis of a material exhibiting large deformations (hyperelastic material), hyperelastic Ogden model is commonly used. However, hyperelastic Ogden model requires several tensile tests data including equibiaxial tensile test during the parameter identification. In equibiaxial tensile test, although cruciform-shaped specimen is often adopted as the test specimen, high stress concentration at the corner area causes an early rupture in the specimen. Therefore, it is difficult to obtain the mechanical response of the material under large equibiaxial strain condition. In present study, an experimental study using equibiaxial tensile test device developed previously by authors and a numerical study using finite element analysis were conducted to investigate the stress distribution and the stress-strain response of cruciform specimen under an equibiaxial tensile loading. The optimal shape design of the corner area of the cruciform specimen was then discussed according to the results.

1. Introduction
Hyperelastic Ogden model [1] is commonly selected to model the mechanical behaviour of material exhibiting large deformation (hyperelastic material), e.g. rubber, gel and human soft tissue. In order to identify the parameter value of hyperelastic Ogden model, the use of several tests data obtained from uniaxial, planar and equibiaxial tensile tests during the curve fitting process was recommended [2]. Although uniaxial and planar tensile tests can be performed easily using a universal testing machine equipped with proper chucks, equibiaxial tensile test required a special testing machine equipped with multiple actuators [3]. Therefore, several studies in the development of a simple low-cost test device for biaxial tensile test [4, 5] have recently been actively pursued. Authors have also developed an equibiaxial tensile test device targeting on hyperelastic material [6]. A low-cost equibiaxial tensile test could be delivered by attaching the test device to a single column universal testing machine. An equibiaxial tensile test using this test device requires cruciform specimen. Although a cruciform specimen with specific dimensions has been proposed for the test, it was confirmed that high stress concentration occurred at the corner area of the cruciform specimen [6]. The stress concentration caused an early rupture of the corner, led to a difficulty in acquiring the mechanical behaviour of test material at high strain region. In order to obtain the mechanical behaviour of the test material, it is
necessary to reduce the stress concentration occurred at the corner area of the cruciform specimen. Modification of the shape and dimensions of the cruciform specimen, especially at the corner area is considered to give an advantageous effect on the mitigation of the stress concentration. In previous studies, shape optimization of cruciform specimen and investigation of the effect of shape on the mechanical parameters were frequently performed on metallic or composite materials [7-10]. Although the results of the studies were also considered as valuable references for designing cruciform specimen of hyperelastic materials, the shape and dimensions of the cruciform specimens varied widely and were different from the cruciform specimen used in the study conducted by authors [6]. Therefore, an independent study should be conduct in order to confirm the optimal shape design of the cruciform specimen designated for the equibiaxial tensile test device.

In this study, several shape designs of the corner area of cruciform specimen were proposed. The effect of the corner shapes on the stress distribution of a hyperelastic material specimen under an equibiaxial tensile loading was then investigated using finite element analysis. Furthermore, equibiaxial tensile tests on the target specimens were also conducted in order to validate the analysis results. The optimal corner shape design was then discussed based on the results of the finite element analysis and the tensile test.

2. Analysis of stress distribution on the cruciform specimens

Finite element analysis was considered to be an effective tool for investigating the stress distribution of the cruciform specimen. Therefore, finite element models of the target specimens were constructed and were then subjected to an equibiaxial tensile loading in simulations. Distributions of von Mises stress in the specimen model were obtained from the analysis results. The details of the finite element analysis are described below.

2.1. Finite element models of cruciform specimens

Finite element models of cruciform specimens and their dimensions are shown in figure 1. In this study, three types of corner shapes (type R, type RD and type RR) as shown in figure 2 were devised. The specimen models were constructed using hexahedral solid elements. The numbers of elements and nodes of each model were listed in table 1. In order to simulate the large deformation behaviour of hyperelastic material, Ogden rubber model was adopted as the material model. Since the base material of the specimen was assumed to be a RTV silicone rubber (KE-12, Shin-Etsu Chemical Co. Ltd., Tokyo, Japan), the parameter values of Ogden rubber model used in this study were same to those in the previous study [6].

![Figure 1. Finite element model of the cruciform specimen and the simulation conditions.](image-url)
Figure 2. Shapes of the corner area of the cruciform specimen. (a) Type R; (b) Type RR; (c) Type RD.

Table 1. Numbers of nodes and elements of the models and the maximum von Mises stress at the corner area.

| Specimen | Nodes  | Elements | Maximum von Mises stress [MPa] |
|----------|--------|----------|--------------------------------|
| Type R   | R5     | 63,965   | 50,080                         | 0.793 |
|          | R10    | 43,594   | 34,144                         | 0.696 |
|          | R10R3  | 44,245   | 34,464                         | 0.908 |
| Type RR  | R10R4  | 47,705   | 37,264                         | 0.641 |
|          | R10R5  | 42,405   | 32,944                         | 0.754 |
| Type RD  | R10D2  | 44,805   | 34,848                         | 1.233 |
|          | R10D3  | 46,207   | 36,016                         | 1.224 |

2.2. Simulation of equibiaxial tensile test
As shown in figure 1, grip areas were defined as the 30 mm length areas from the edges of the four arms to the center of the specimen model. The inferior grip area was completely fixed during the analysis. In order to simulate an equibiaxial tensile test, an upward displacement was imposed on the superior grip area, while 45 degree obliquely upward displacements were also imposed on the right and left grip areas respectively. The displacements were imposed at a constant velocity of 0.1 m/s until the upward displacement of the superior grip area reached 10 mm. The finite element analysis was performed using a commercial software PAM-CRASH v2012 (ESI Group, Paris, France).

2.3. Maximum von Mises stress at the corner area
Distributions of von Mises stress were determined from the analysis results. Comparison of maximum values of von Mises stress at the corner area was shown in table 1. The R10R4 model showed the lowest value, while the R10D2 model showed the highest value of von Mises stress. Overall, the RD-typed models indicated a tendency to have higher von Mises stress rather than the R-typed models and the RR-typed models. An addition of a R4 notch on a R10 corner (R10R4) caused a decrease of maximum von Mises stress at the corner area. However, the decrease was considered to be relatively small.
3. Equibiaxial tensile test for validating the analysis result

In order to validate the result of the finite element analysis, equibiaxial tensile test on silicone rubber specimens which have similar shapes to the specimen finite element models was carried out. The equibiaxial tensile test was conducted considering also the variations that may occur due to the different defects in each specimen. Corner shape design that could provide more data at larger strain region was evaluated from the stress-strain responses. The details of the equibiaxial tensile test are described below.

3.1. Cruciform specimens

Some specimens that showed lower maximum von Mises stress were selected among the specimens used in the finite element analysis. R-type specimens (R5 and R10) as well as R10D3 specimen and R10R4 specimen were finally selected and tested in order to investigate the differences in the stress-strain response and rupture point under an equibiaxial tensile loading. The test specimens were manufactured from RTV silicone rubber (KE-12, Shin-Etsu Chemical Co. Ltd., Tokyo, Japan) by vacuum casting method. The test specimens with different corner shapes are shown in figure 3.

![Test specimens with different corner shapes](image)

Figure 3. Test specimens with different corner shapes. (a) R5; (b) R10; (c) R10D3; (d) R10R4.

3.2. Equibiaxial tensile test

Equibiaxial tensile tests were conducted using a compact universal testing machine (EZ-LX, Shimadzu Corp., Kyoto, Japan) and an equibiaxial tensile test device developed by authors [6]. Loading rate was 100 mm/min. Number of specimens was six specimens for each specimen type. In total, 24 tests were carried out. A preloading of 1 N was applied to the specimens prior to the tests. Load was measured using a load cell (LUR-A-SA1, Kyowa Electronic Instruments Co. Ltd., Tokyo, Japan) attached to the test device. Elongation was determined from the video images taken using a video camera (HDR-S11, Sony Corp., Tokyo, Japan).

3.3. Comparison of the mechanical responses

Figure 5 shows the comparison of stress-strain curves obtained from the equibiaxial tensile tests. Although specimen variations could be seen from the result, the repeatability of the result was considered to be sufficiently high. The R10D3 specimen showed the lowest rupture strain, while the R10R4 specimen showed the highest rupture strain. This finding was well accorded with the finding obtained from the finite element analysis. In contrast, the R10 specimen and the R5 specimen indicated an opposite result to the result obtained from the finite element analysis. However, the
difference was confirmed to be statistically insignificant. Furthermore, according to the analysis and test results, the R10 specimen demonstrated higher stress response compared to other specimen types. The constraint conditions might have a contribution on this behaviour.

![Figure 5. Comparison of stress-strain curves among four types of specimen. Solid lines indicate the average values. Dash lines indicate the average values ± standard deviation.](image)

4. Conclusion
An integrated study combining finite element analysis and mechanical test was proposed to investigate the optimal shape design of the corner area of a cruciform specimen used in the equibiaxial tensile test. As a result, an addition of a small circle notch on the round corner area of the cruciform specimen could reduce the maximum von Mises stress at the area, indicating a possibility to obtain the mechanical behaviour of a material at larger strain region. In addition, the results of the equibiaxial tensile tests also supported the finding obtained from the finite element analysis. The integrated method used in this study can save time and cost by reducing the number of tests that should be conducted. However, several limitations such as maximum stroke of the test device and defect in the specimens may provide different result to that obtained from the finite element analysis. Furthermore, introduction of an optimization method into the analysis should be done in the future.

5. References
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