Mobile Edge Computing for Cellular-Connected UAV: Computation Offloading and Trajectory Optimization

(Invited Paper)

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Abstract—This paper studies a new mobile edge computing (MEC) setup where an unmanned aerial vehicle (UAV) is served by cellular ground base stations (GBSs) for computation offloading. The UAV flies between a give pair of initial and final locations, during which it needs to accomplish certain computation tasks by offloading them to some selected GBSs along its trajectory for parallel execution. Under this setup, we aim to minimize the UAV’s mission completion time by optimizing its trajectory jointly with the computation offloading scheduling, subject to the maximum speed constraint of the UAV, and the computation capacity constraints at GBSs. The joint UAV trajectory and computation offloading optimization problem is, however, non-convex and thus difficult to be solved optimally. To tackle this problem, we propose an efficient algorithm to obtain a high-quality suboptimal solution. Numerical results show that the proposed design significantly reduces the UAV’s mission completion time, as compared to benchmark schemes.

I. INTRODUCTION

With recent technology advancement and manufacturing cost reduction, unmanned aerial vehicles (UAVs) have received growing interests in various applications such as cargo delivery, filming, rescue and search, etc [1]. To maintain the UAVs’ safe operation with real-time command/control and their distributed computing resources to improve the computation capability. Second, since the UAV has controllable mobility in the three-dimensional (3D) airspace, its trajectory can be jointly designed with its scheduling of computation offloading to the GBSs associated along the trajectory to optimize the performance. This is considerably different from prior studies on MEC with communication and computation resource allocation at a fixed terrestrial user and its associated GBS only (see, e.g., [5–8]), thus deserving a dedicated new investigation.

Specifically, this paper considers a practical scenario where a UAV is designated to fly from an initial location to a final location, during which it needs to accomplish certain computation tasks. We assume that the UAV can arbitrarily partition these tasks into smaller-size subtasks, and offload them to some selected GBSs along its trajectory for parallel execution. Under this setup, we aim to minimize the UAV’s mission completion time or total flight duration by jointly optimizing its trajectory and computation offloading scheduling, subject to the maximum speed and initial/final location constraints of the UAV, as well as the GBSs’ individual computation capacity.
Then we have ⃗u(0) = ⃗u_I and ⃗u(T) = ⃗u_F for the given initial and final locations, respectively. At time instant t, the distance between the UAV and GBS k is given by
\[ d_k(⃗u(t)) = \sqrt{H^2 + \|⃗u(t) - ⃗ν_k\|^2}, \] (1)
where \(\|\cdot\|\) denotes the Euclidean norm of a vector. Let \(V_{\text{max}} > 0\) denote the UAV’s maximum speed in m/s. Then we have
\[ \dot{x}(t) + \dot{y}(t) \leq V_{\text{max}}, \forall t \in [0, T], \]
in which \(\dot{x}(t)\) and \(\dot{y}(t)\) denote the first-derivatives of \(x(t)\) and \(y(t)\), respectively.

Normally, the air-to-ground channels from the UAV to GBSs are dominated by the LoS links, and hence we consider the free-space path-loss model similarly as in [2], [3]. At time instant t, the channel power gain from the UAV to GBS k is denoted as
\[ h_k(⃗u(t)) = \beta_0, \]
where \(\beta_0\) denotes the channel power gain at a reference distance of 1 m.

For ease of exposition, we discretize the mission duration T into N time slots each with a given duration \(δ_t\), i.e. \(T = N \cdot δ_t\), where \(δ_t\) is chosen to be sufficiently small such that the UAV’s location can be assumed to be approximately unchanged during each slot with \(δ_t \cdot V_{\text{max}} \ll H\), and N is thus a variable to be optimized. In this case, we denote the UAV’s horizontal location at time slot n as \(u[n] = \hat{u}(n \cdot δ_t), n \in \mathbb{N} \triangleq \{1, ..., N\}\), with \(u[0] = \hat{u}(0) = \hat{u}_I\) and \(u[N] = \hat{u}(T) = \hat{u}_F\).

Accordingly, the channel power gain from the UAV to GBS k is \(h_k(u[n])\) at slot n. Furthermore, let \(S_{\text{max}} = δ_t V_{\text{max}}\) denote the maximum UAV displacement during each time slot. Thus, the maximum UAV speed and initial/final location constraints are respectively re-expressed as
\[ \|u[n] - u[n-1]\|^2 \leq S_{\text{max}}^2, \forall n \in \mathbb{N}, \] (3)
\[ u[0] = \hat{u}_I, \ u[N] = \hat{u}_F. \] (4)

We consider the time-division-multiple-access (TDMA) protocol to implement the UAV’s computation offloading, by dividing each time slot \(n \in \mathbb{N}\) into K sub-slots each with duration \(τ_k[n] \geq 0\), where
\[ \sum_{k \in \mathbb{K}} τ_k[n] = δ_t, \ \forall n \in \mathbb{N}. \] (5)

In each sub-slot \(k \in \mathbb{K}\), the UAV offloads the respective task-input bits to GBS k. Suppose that the UAV adopts a constant transmit power \(P > 0\) for offloading. Then the achievable offloading rate from the UAV to GBS k in bits-per-second (bps) at slot n is expressed as
\[ R_k(u[n]) = B \log_2 \left( 1 + \frac{P h_k(u[n])}{\sigma^2} \right), \] (6)
where \(\sigma^2\) and B represent the noise power at the receiver of each GBS and the bandwidth, respectively, and \(\rho = \frac{P \beta_0}{\sigma^2}\) denotes the reference signal-to-noise ratio (SNR). In order for the UAV to offload all the L task-input bits to the K GBSs, we need to have
\[ \sum_{k \in \mathbb{K}} \sum_{n \in \mathbb{N}} τ_k[n] R_k(u[n]) \geq L. \] (7)

Next, we consider the remote task execution at each GBS k. Denoting \(f_k\) as the maximum CPU frequency at GBS k ∈ \(\mathbb{K}\),

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1Due to the SWAP limitations, the UAV usually has limited local computation resources. In this case, we consider that the UAV user does not perform any local computing, for the purpose of exposition.
in Hz, then we obtain the per-slot computation capacity of GBS \( k \) as \( f_k \delta_k \), which represents the maximum number of task-input bits that can be executed by GBS \( k \) over one slot. Note that as each task-input bit can be executed independently, each GBS can immediately start the execution as soon as the task-input bits are received. In other words, the offloaded task-input bits at each slot \( n - 1 \) are immediately executable at slot \( n \). Also note that at each GBS \( k \), all the offloaded task-input bits must be successfully executed before the mission completion time \( T \) (or \( N \)). Therefore, we have the following computation capacity constraints over time: for each GBS \( k \in \mathcal{K} \), the accumulative number of offloaded task-input bits over the last \( (N - n) \) slots must be no larger than the GBS’s accumulative computation capacity over the last \( (N - n) \) slots, \( \forall n \in \mathcal{N}, \) i.e.,

\[
\sum_{j=n}^{N-1} c_k \tau_k[j] R_k(u[j]) \leq (N - n) f_k \delta_k, \forall n \in \mathcal{N}. \tag{8}
\]

The computation capacity constraints in (8) can be understood intuitively as follows. First, for \( n = N \), we have \( c_k \tau_k[N] R_k(u[N]) = 0 \), which indicates that the UAV cannot offload any task in slot \( N \), as there is no time for each GBS to execute. Next, for \( n = N - 1 \), we have \( c_k \tau_k[N - 1] R_k(u[N - 1]) + c_k \tau_k[N] R_k(u[N]) \leq f_k \delta_k \). By combining this with \( c_k \tau_k[N] R_k(u[N]) = 0 \), we further have \( c_k \tau_k[N - 1] R_k(u[N - 1]) \leq f_k \delta_k \), which implies that the offloaded task-input bits in slot \( N - 1 \) cannot exceed the computation capacity in slot \( N \). Furthermore, by recursively considering time slots \( N - 2, N - 3, \ldots \), until the first slot, the constraints in (8) follow similarly.

Our objective is to minimize the UAV’s mission completion time \( N \) (or equivalently \( T \)) by optimizing the UAV trajectory \( \{u[n]\} \) and the time allocation for computation offloading \( \{\tau_k[n]\} \), subject to the maximum UAV speed constraint in (3), the initial/final UAV location constraints in (4), the TDMA constraints in (5), as well as the task execution constraints in (7) and (8). Therefore, the joint UAV trajectory and computation offloading optimization problem is formulated as

\[
\text{(P1): } \min_{\{u[n], \tau_k[n]\}, N \in \mathbb{Z}^+} N \tag{9}
\]

\[
\text{s.t. } \tau_k[n] \geq 0, \quad \forall k \in \mathcal{K}, \quad n \in \mathcal{N}
\]

where \( \mathbb{Z}^+ \) denotes the set of all strictly positive integers. Notice that (P1) is a non-convex optimization problem, as the optimization variable \( N \) is an integer, and constraints (7) and (8) are non-convex. Furthermore, as \( N \) is a \textit{a-priori} unknown, (P1) consists of an uncertain number of constraints in (8) and (9). Due to the above facts, (P1) is difficult to be solved optimally.

**III. PROPOSED SOLUTION TO (P1)**

In this section, we propose an efficient algorithm to solve (P1) sub-optimally.

First, we show that (P1) can be equivalently solved by first optimizing over \( \{u[n]\} \) and \( \{\tau_k[n]\} \) under any given \( N \), and then using a bisection search to find the optimal \( N \). In particular, under any given \( N \), (P1) becomes the following feasibility checking problem:

\[
\text{(P2): } \text{find } \{u[n]\} \text{ and } \{\tau_k[n]\} \tag{10}
\]

\[
\text{s.t. } (3), (4), (5), (7), \text{ and } (8). \tag{11}
\]

Suppose that the optimal solution of \( N \) to (P1) is \( N^* \). Then, consider (P2) under any given \( N \). If (P2) is feasible under \( N \), then it follows that \( N^* \leq N \); otherwise, we have \( N^* > N \). Therefore, we can solve (P1) by checking the feasibility of (P2) under any given \( N \) and using a bisection search over \( N \). As a result, we only need to consider (P2) under given \( N \).

Next, we show that solving (P2) is equivalent to solving the following problem (P3) to maximize the number of computation task-input bits under given \( N \).

\[
\text{(P3): } \max_{\{u[n]\}, \{\tau_k[n]\}, \tilde{L} \geq 0} \tilde{L}, \tag{12}
\]

\[
\text{s.t. } k \in \mathcal{K}, \quad n \in \mathcal{N}, \quad (3), (4), (5), (8), \text{ and } (9). \tag{13}
\]

Suppose that the optimal solution of \( \tilde{L} \) to (P3) is \( \tilde{L}^* \). Then it is evident that if \( \tilde{L}^* \geq L \), then (P2) is feasible; otherwise, (P2) is infeasible.

Now, it only remains to solve (P3). Note that (P3) is still non-convex, due to the non-convex constraints in (8) and (10). In the following, we propose an efficient algorithm to obtain a suboptimal solution to (P3) by optimizing the time allocation \( \{\tau_k[n]\} \) and the UAV trajectory \( \{u[n]\} \) in an alternating manner.

1) **Time Allocation for (P3) Under Given UAV Trajectory:** Under given \( \{u[n]\} \), (P3) is reduced to

\[
\text{(P3.1): } \max_{\{\tau_k[n]\}, \tilde{L} \geq 0} \tilde{L} \quad \text{s.t. } (3), (5), (8), \text{ and } (10). \tag{14}
\]

It is easy to show that (P3.1) is a linear program (LP), which can be solved by standard convex optimization techniques such as the interior point method [10]. We adopt the well-established optimization toolbox CVX [11] to solve (P3.1) optimally and efficiently.

2) **UAV Trajectory Optimization for (P2) Under Given Time Allocation:** Under given \( \{\tau_k[n]\} \), (P3) is reduced to

\[
\text{(P3.2): } \max_{\{u[n]\}, \tilde{L} \geq 0} \tilde{L} \quad \text{s.t. } (3), (4), (5), \text{ and } (10). \tag{15}
\]

Notice that (P3.2) is still non-convex, as constraints (8) and (10) are non-convex. To tackle this problem, we propose an iterative algorithm to obtain an efficient solution to (P3.2) by using the SCA technique. The idea is that under any given local point at each iteration, we approximate non-convex constraints (8) and (10) by their corresponding convex ones. By solving a series of approximate convex problems iteratively, we can attain an efficient suboptimal solution to (P3.2).

Suppose that \( \{u^{(i)}[n]\} \) denotes the local point at the \( i \)-th iteration, \( i \geq 0 \). Then, we approximate constraints (8) and (10) in the following, respectively. First, consider constraint (8). Notice that by checking the first-order Taylor expansion of the convex term \( H^2 + \|u[n] - \nu_k\|^2 \) with respect to \( u[n] \) at the local point \( u^{(i)}[n] \), we have

\[
H^2 + \|u[n] - \nu_k\|^2 \geq g^{(i)}[n] + 2(\omega^{(i)}[n])^T u[n], \tag{16}
\]

where \( \omega^{(i)}[n] \) is an iterative correction term.
with \( \omega^{(i)}[n] = u^{(i)}[n] - \nu_k \) and \( q_k^{(i)}[n] = H^2 + \| u^{(i)}[n] - \nu_k \|^2 - 2(\omega^{(i)}[n])^T u^{(i)}[n] \), where \((\cdot)^T\) indicates the transpose. Based on \((12)\), we obtain an upper bound of \( R_k(u[n]) \) as
\[
R_k(u[n]) \leq B \log_2 \left( 1 + \frac{\rho}{q_k^{(i)}[n] + 2(\omega^{(i)}[n])^T u[n]} \right)
\]
denoted by \( R_k^{(i)}(u[n]) \), where \( R_k^{(i)}(u[n]) \) is convex with respect to \( u[n] \). Replacing \( R_k(u[n]) \) in \((8)\) as \( R_k^{(i)}(u[n]) \), we have the approximated convex constraints as
\[
\sum_{j=n}^{\infty} c_k \kappa_k R_k^{(i)}(u[n]) \leq (N - n) f k \delta_n, \quad \forall n \in N.
\]
Next, consider constraint \((10)\). Notice that \( R_k(u[n]) \) is a convex function with respect to the term \( \| u[n] - \nu_k \|^2 \). Then by taking the first-order Taylor expression of \( R_k(u[n]) \) at local point \( u^{(i)}[n] \) as follows.
\[
R_k(u[n]) \geq R_k(u^{(i)}[n]) - b_k^{(i)}[n](\| u[n] - \nu_k \|^2 - \| u^{(i)}[n] - \nu_k \|^2),
\]
where \( b_k^{(i)}[n] = B \rho/\left( \ln 2d_k^n(u^{(i)}[n]) \left( \rho + d_k^n(u^{(i)}[n]) \right) \right) \). Here, \( R_k^{(i)}(u[n]) \) is a concave function with respect to \( u[n] \). By replacing \( R_k(u[n]) \) in constraint \((11)\) as \( R_k^{(i)}(u[n]) \), we have the approximated convex constraints as
\[
\sum_{k \in K, n \in N} \kappa_k R_k^{(i)}(u[n]) \geq \bar{L}.
\]
Finally, with \((13)\) and \((15)\) at hand, \((P3.2)\) is approximated as the following convex optimization problem \((P3.3)\) at local point \( \{u^{(i)}[n]\} \), which can be solved optimally via convex optimization techniques such as CVX.

\[
(P3.3) : \max_{\{u[n]\}, \bar{L} \geq 0} \bar{L} \text{ s.t. } (3), (4), (13), \text{ and } (15).
\]
Let \( \{u^{(i)}[n]\} \) denote the optimal UAV trajectory solution to \((P3.3)\) at local point \( \{u^{(i)}[n]\} \). Then, we can obtain an efficient iterative algorithm to solve \((P3.2)\) as follows. In each iteration \( i \geq 1 \), the UAV trajectory is updated as \( \{u^{(i+1)}[n]\} \) by solving \((P3.3)\) at local point \( \{u^{(i)}[n]\} \). i.e. \( u^{(i+1)}[n] = u^{(i)}[n], \forall n \in N \), where \( \{u^{(0)}[n]\} \) denotes the initial UAV trajectory. In summary, the proposed algorithm is presented in Table I as Algorithm 1.

### TABLE I
ALGORITHM I FOR SOLVING PROBLEM (P3.2)

1. Initialization: Given the UAV trajectory \( \{u^{(0)}[n]\} \); let \( i = 0 \).
2. Repeat:
   i. Solve problem \((P3.3)\) under given \( \{u^{(i)}[n]\} \) to obtain the optimal solution as \( \{u^{(i+1)}[n]\} \).
   ii. Update \( u^{(i+1)}[n] = u^{(i)}[n], \forall n \in N \).
   iii. Update \( i = i + 1 \).
3. Until the optimal value converges within a given threshold or a maximum number of iterations is reached.

Notice that after each iteration in Algorithm 1, the objective value of \((P3.2)\) is monotonically non-decreasing. As the optimal value of \((P3.2)\) is upper-bounded, Algorithm 1 should converge to (at least) a locally optimal solution to \((P3.2)\).
is constrained by the flying distance between the initial and final locations. When $L = 200$ Mbits, the UAV trajectory is observed to deviate from the straight line by flying closer towards GBSs 1, 4, and 5, in order to exploit better wireless channels for computation offloading towards them. When $L$ further increases to 500 Mbits, the UAV is observed to reach and hover above all the five GBSs and even fly back and forth between GBSs 4 and 5. In this case, the mission completion time is mainly constrained by the computation task execution, and thus the UAV trajectory is designed for most efficient computation offloading.

Next, we validate the performance of our proposed design as compared to two benchmark schemes, namely the above straight flight trajectory and the following heuristic design.

- **Successive hover-and-fly**: the UAV flies to successfully reach at the top of the $K$ GBSs at the maximum speed $V_{\text{max}}$, and hovers above each of them for efficient computation offloading. The visiting order is determined by solving the Traveling Salesman Problem (TSP) \cite{12} to minimize the flying distance. Under such a UAV trajectory design, the mission completion time minimization problem can be solved similarly as in the straight flight scheme, while the only difference is that during checking the computation feasibility under any given $T$ (or $N$), we need to optimize the hovering durations above these GBSs jointly with the time allocation while flying.

Fig. 3 shows the mission completion time $T = N\delta_t$ versus the number of task-input bits $L$. It is observed that as $L$ becomes larger, the mission completion time increases for all the three schemes, while the proposed design performs best among the three schemes over all $L$ values. When $L$ is small (e.g., $L = 100$ Mbits), the straight-flight scheme is observed to achieve the same mission completion time as the proposed design, and outperforms the successive-hover-and-fly scheme. This is due to the fact that in this regime, the mission completion time is constrained by the flying distance, and the successive-hover-and-fly scheme leads to longer flying distance as the UAV needs to visit all GBSs. When $L$ is larger than 200 Mbits, it is observed that the straight-flight scheme performs worse than the successive-hover-and-fly scheme and the proposed design. This is due to the fact that in this regime, the mission completion time is constrained by the computation task execution, and the latter two schemes can more efficiently explore the UAV trajectory design for computation offloading. In addition, the successive-hover-and-fly scheme is observed to perform close to the proposed design when $L = 300$ Mbits, but the performance gap increases when $L$ further increases. This is due to the fact that when $L$ becomes larger, in the proposed design the UAV can fly back and forth among different GBSs (see Fig. 2 for $L = 500$ Mbits) in order to explore multiple GBSs’ distributed computation resources more efficiently by time sharing.

V. CONCLUSION

This paper investigates a new MEC application scenario where a cellular-connected UAV offloads its computation tasks to multiple GBSs along its trajectory. The UAV trajectory is jointly designed with the computation offloading scheduling, to minimize the mission completion time, subject to the UAV’s maximum speed and initial/final location constraints, as well as the GBSs’ individual computation capacity constraints. By exploiting alternating optimization and SCA techniques, an efficient algorithm is proposed to solve the formulated problem sub-optimally. Numerical results show a significant performance gain of our proposed design over the benchmark schemes.

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