New ultraviolet operators in supersymmetric SO(10) GUT and consistent cosmology

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Abstract

We consider the minimal supersymmetric grand unified model (MSGUT) based on the group SO(10), and study conditions leading to possible domain wall (DW) formation. It has been shown earlier that the supersymmetry preserving vacuum expectation values (vev’s) get mapped to distinct but degenerate set of vev’s under action of $D$ parity, leading to formation of domain walls as topological pseudo-defects. The metastability of such walls can make them relatively long lived and contradict standard cosmology. We then consider the possibility of such domain walls being rendered unstable due to pressure difference induced by the effective potential evaluated in the two alternative vacua. We argue that the presence of higher order SO(10) preserving operators in the Higgs superpotential can not give rise to such pressure difference and thus does not avoid the danger of persistent domain walls. Thus we are led to consider adding an order four Planck scale suppressed nonrenormalisable operator, that breaks SO(10) symmetry but preserves Standard Model symmetry. This is shown to give rise to the required pressure difference to remove the domain walls without conflicting with consistent big bang nucleosynthesis (BBN) while avoiding gravitino overproduction.

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I. INTRODUCTION

Minimal Supersymmetric GUT (MSGUT) based models includes all the elegant features of both SO(10) GUT based models and supersymmetric models. It naturally accommodates a heavy right handed neutrino in order to fill the $\mathbf{16}$ representation of SO(10) with matter fermions. Thus it naturally allows implementation of the seesaw mechanism [1] which explains the small non-zero neutrino masses observed experimentally [9]. Supersymmetric models with left-right symmetric gauge groups ($\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$) naturally descend from SO(10) GUT. These models enjoy R-parity ($R = (-1)^{3(B-L)+2S}$) [2] conservation, and so predict a stable Lightest Supersymmetric Particle (LSP) which is a promising dark matter candidate. While no signatures of low energy supersymmetry have been found at the LHC so far, supersymmetry continues to be an appealing mechanism for a consistent UV completion for the Standard Model (SM), and justifies the considerations here which relate to the effects of this model in the very early universe.

MSGUT can be broken in several stages using various Higgs representations to finally descend to the Standard Model (SM). It gets broken via different routes depending on the Higgs representations used in the model and on the Higgs superpotential. One way to break the SO(10) group at intermediate energy scales is via Pati-Salam (PS) group times D-parity ($\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times D$) which manifests a discrete $\mathbb{Z}_2$ symmetry structure viz. the so called D-parity. The D-parity allows the left handed and right handed couplings to be same thereby maintaining left-right symmetric universe. The Pati-Salam group and D-parity get spontaneously broken at some lower energy scales to SM gauge group ($\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$). However not all the breaking paths of SO(10) along the PS route leads to D-parity violation [13].

So we see that starting from SO(10) MSGUT, there are natural scenarios that lead first to a left right symmetric universe which then eventually leads on to our familiar left handed SM universe. However there is one major hitch to this elegant theory. It is known that spontaneous breaking of a discrete symmetry leads to existence of network of domain walls which are in direct conflict with the observed Universe [3, 4]. Left-right symmetry and D-parity symmetry are examples of discrete symmetries that may get spontaneously broken along certain symmetry breaking paths of SO(10). Moreover in the context of the model studied here, it has been shown [11] that non-topological domain walls, dubbed “topological pseudo-defects” may still arise because of the energy barrier separating two sets of supersymmetry preserving vevs, related to each other by the discrete operation of D-parity. Therefore the domain walls, if created in the early universe, must go away quickly enough so as to be consistent with the current observed universe. Earlier works on the supersymmetric SO(10) GUT model studied here have not addressed this issue.

One way to remove these domain walls is to introduce Planck scale suppressed non-renormalizable operators [5, 6] that causes instability to domain walls. However this introduction of Planck suppressed operators may not work for all cases of SUSY and gauge symmetric models. In the Next to Minimal Supersymmetric Standard Model, the problem was found to persist [7] in the sense that the gauge hierarchy problem does not get addressed...
if the operators required to remove the domain walls are permitted. In the Supersymmetric
Left-Right symmetric Models (SUSYLR) with all Higgs carrying gauge charges, it is possible
to introduce Planck scale suppressed terms that are well regulated. One can then demand
that the new operators ensure sufficient pressure across the domain walls that the latter
disappear before BBN. This requirement has been discussed in detail in [8] in the context of
R-parity conserving SUSYLR models [10]. Similar analysis was shown to place constraints
also on R-parity violating SUSYLR models [12].

We begin by discussion of discrete set of vacua degenerate with that signalled by the
standard SM preserving vevs that are used in the first stage of breaking of SO(10) [19].
Applying D-parity to them gives us a new set of supersymmetry preserving SM preserving
vevs. The two sets of vevs are disconnected in the sense that they are separated by a
potential barrier [11]. This fact leads to the danger of domain walls formed as topological
pseudo-defects.

The aim of this paper is to study whether this degeneracy can be lifted using new op-
erators in the superpotential. Since generic dimension 3 terms have all been used up in
the superpotential, it is natural to search for non-renormalisable ones. We next argue that
adding such SO(10) preserving terms to the superpotential will not lead to a pressure dif-
ference across any domain wall that can arise, and thus, cannot contribute to the required
instability. This forces us to go beyond the cherished principle of exact SO(10) gauge sym-
metry, in particular sacrifice some of its features related to ultraviolet (UV) completion. The
mildest such violation could be considered along the lines of Gell-Mann [14] and Okubo [15]
and add SO(10) breaking but SM preserving terms to the superpotential. In order to not
touch the relevant and marginal operators, the leading new term in the superpotential could
be of mass dimension 4, suppressed by one power of the cutoff scale of the theory which we
take to be the Planck scale $M_{Pl}$. We implement our idea by finding a coefficient matrix for
the dimension four term drawn from the SO(10) group that commutes with SM gauge group
embedded into SO(10). The systematically developed Pati-Salam embedding[20] proves to
be a good computational aid for this purpose. This leads to a novel ultraviolet operator
that can potentially cause sufficient instability and remove any domain wall that may arise.

Towards consistency check for such a scenario, we demand that the effects remain confined
to UV limit approaching the Planck scale, and naturalness of such deviation from full SO(10)
is ensured by the coefficients of such terms remaining small. This naturalness test is carried
out by considering the implications of such pressure difference to cosmology, specifically
to ensuring that the DW disappear without conflicting with standard cosmology. This is
done by testing these terms for three different scenarios of domain wall dynamics and their
eventual removal during three possible kind of cosmological epochs, viz. radiation dominated
era, early stage of matter dominated era and late stage of matter dominated era following
a period of weak inflation. Domain wall removal during the same cosmological periods
for the cases of left right symmetric supersymmetric models was earlier studied in [8] and
[12]. We calculate the conditions for successful domain wall removal during the three stated
cosmological periods in our extended MSGUT model, leading to constraints on the scale
of right hand symmetry breaking characterised by mass scale $M_R$. Generically it results in
upper bounds on $M_R$, in fact forcing it to remain small compared to generic GUT scale.

The paper is organised as follows. In Sec. II we show the existence of $D$-flipped degenerate vacua, and the inadequacy of $SO(10)$ invariant terms in lifting this degeneracy. In sec. III we propose our strategy for overcoming the problem along the lines of octet dominance in old hadron flavour physics, and identify the required $SO(10)$ matrix. Section IV computes the effective potential terms arising from the new nonrenormalisable operator, with details in appendix A. In its subsection IV A we impose the required conditions on the pressure difference terms to ensure consistent cosmology and obtain constraints on the scale of right handed symmetry breaking $M_R$. The final section V contains conclusions.

II. MSGUT SUPERPOTENTIAL AND THE DEGENERATE MSSM VACUA

The heavy Higgs part of the renormalisable superpotential of $SO(10)$ MSGUT is as follows [19]:

$$W_{\text{ren}} = \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnij} + \frac{M}{3!} \Sigma_{ijklm} \Sigma_{ijklm} + \frac{\eta}{4!} \Phi_{ijkl} \Sigma_{ijmno} \Sigma_{ijmno},$$

(1)

where $\Phi$ is the 4-index antisymmetric representation $210$, $\Sigma$ is the 5-index self-dual antisymmetric representation $126$, and $\Sigma$ is the 5-index anti-self-dual antisymmetric representation $\overline{126}$ of $SO(10)$. In the above expression, all indices range independently from 1 to 10. Below, for brevity of notation, we shall use 0 to denote the index 10 in our expressions.

In order to break $SO(10)$ while preserving supersymmetry, we consider the following vevs. These vevs are uniquely determined by the requirement that they be singlet under the SM gauge group embedded into $SO(10)$ via Pati-Salam and satisfy electric charge conservation [19].

$$\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a, \quad \Phi_{1278} = \Phi_{12910} = \Phi_{3478} = \Phi_{3490} = \Phi_{5678} = \Phi_{5690} = \omega, \quad \Phi_{7890} = p,$$

$$\Sigma_{a+1,b+3,c+5,d+7,e+9} = i^{a+b+c-d-e} \frac{a}{2^{5/2}}, \quad a, b, c, d, e \in \{0, 1\},$$

$$\Sigma_{a+1,b+3,c+5,d+7,e+9} = i^{-a-b-c+d+e} \frac{a}{2^{5/2}}, \quad a, b, c, d, e \in \{0, 1\}.$$  

(2)

Remaining coordinates are set to zero in the vevs.

Let us consider the exchange of the $SU(2)_L$ and $SU(2)_R$ subgroups in this embedding. It amounts to the following element of $SO(10)$ which achieves the desirable left-right (LR) flipping. [20, Equations 28,29]:

$$0 \neq i \neq j \quad (L \leftrightarrow R)_{ij} = -1 \quad i = j = 0 \quad 1 \ i = j = 1, \ldots, 9.$$  

(3)

This flipping can be effectively obtained by utilising the $D$-parity operator defined to be,

$$D = \exp(-i\pi J_{23}) \exp(i\pi J_{67})$$  

(4)
In the present embedding, it amounts to the following element of \( \text{SO}(10) \) [20, Equation 42]:

\[
D_{ij} =
\begin{cases} 
0 & i \neq j \\
-1 & i = j = 2, 3, 6, 7 \\
1 & i = j = 1, 4, 5, 8, 9, 0.
\end{cases}
\]

To see the detail, we note that the D-parity flipped vevs are as follows:

\[
\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a, \\
\Phi_{1278} = -\Phi_{1290} = \Phi_{3478} = -\Phi_{3490} = \Phi_{5678} = -\Phi_{5690} = \omega, \\
\Phi_{7890} = -p,
\]

\[
\Sigma_{a+b+c+5,d+7,e+9} = i^{a-b-c+d-e} \frac{\sigma}{25/2}, & \quad a, b, c, d, e \in \{0, 1\}, \\
\overline{\Sigma}_{a+b+c+5,d+7,e+9} = i^{a+b-c-d+e} \frac{\overline{\sigma}}{25/2}, & \quad a, b, c, d, e \in \{0, 1\}.
\]

On the other hand the LR-parity flipped vevs are:

\[
\Phi_{1234} = \Phi_{1256} = \Phi_{3456} = a, \\
\Phi_{1278} = \Phi_{3478} = \Phi_{5678} = -\Phi_{1290} = -\Phi_{3490} = -\Phi_{5690} = \omega, \\
\Phi_{7890} = -p,
\]

\[
\Sigma_{a+b+3,c+5,d+7,e+9} = i^{a-b-c+d-e} \frac{\sigma}{25/2}, & \quad a, b, c, d, e \in \{0, 1\}, \\
\overline{\Sigma}_{a+b+3,c+5,d+7,e+9} = i^{a+b-c-d+e} \frac{\overline{\sigma}}{25/2}, & \quad a, b, c, d, e \in \{0, 1\}.
\]

Observe that

\[
D(\langle 210 \rangle) = (L \leftrightarrow R)(\langle 210 \rangle), D(\langle 126 \rangle) = \frac{\sigma}{\overline{\sigma}}(L \leftrightarrow R)(\langle 126 \rangle), D(\langle \overline{126} \rangle) = \frac{\overline{\sigma}}{\sigma}(L \leftrightarrow R)(\langle \overline{126} \rangle).
\]

Thus D-parity mimics left right flipping in SO(10) MSGUT.

While this discrete symmetry is securely embedded in a compact group, the engineering of the superpotential required to obtain the MSSM encodes an accidental symmetry into the \( F \) flatness conditions according to which for every choice of vev’s resulting in MSSM, there exists a set of \( D \)-flipped vev’s which satisfy the same \( F \) flatness conditions. Further, as argued in [11], there exist no flat directions connecting these mutually flipped set of vev’s. On the other hand a one parameter curve \( U(1)_D \) generated by \( D \) necessarily crosses a barrier in connecting the two sets of vev’s. Thus the vacuum manifold possesses accidental discrete symmetry, which can give rise to pseudo-topological defects due to causal structure of the evolving early Universe. Due to rapid cooling of the universe, such walls can remain metastable until destroyed by fluctuation and tunnelling processes. The situation becomes analogous to transient DW arising from breaking of LR symmetry in certain SUSYLR models [8], although the walls are only metastable. In the LR case it was possible to assume Planck scale suppressed terms that break the discrete LR symmetry since gravity does not respect global symmetries. For a compact group \( Spin(10) \) there is no consistent way to generate an energy imbalance between sectors related by a discrete symmetry operation which belongs to
the group. This is why we shall need to investigate other methods for domain wall removal under the action of D-parity in SO(10) MSGUT.

We start this investigation by computing the $F$-terms evaluated at the original vevs as well as at the flipped vevs. D-flatness at the original and flipped vevs is ensured by utilising the condition $|\sigma| = |\bar{\sigma}|$ as in [19]. The evaluations at the original vevs can also be found in [19, Equations 6-9] and [21, Equations 20-23]. Now we note that exactly the same values of $a$, $\omega$, $p$ ensure $F$-flatness at the original vevs as well as at the $D$-flipped vevs. The detail is shown in Table I. From Table I we see that the $F$-terms evaluated at the original and $D$-parity flipped vevs are either the same, if the field value remains same after shifting, or negated if the field value gets negated after flipping. This is not a coincidence, but rather a consequence of the $SO(10)$-invariance of the superpotential where each index occurs exactly twice in a term. This is true even if we add higher degree $SO(10)$ invariant non-renormalisable terms to the superpotential. So if $|\sigma| = |\bar{\sigma}|$, the scalar potential is the same whether evaluated at the original or the $D$-parity flipped vevs. Thus, a $SO(10)$-invariant superpotential will never give a pressure difference. It becomes necessary to consider other alternatives.

| F-term | Original vev | $D$-Flipped vev |
|--------|--------------|-----------------|
| $F_{\Phi_{1234}} = F_{\Phi_{1256}} = F_{\Phi_{3456}}$ | $2ma + 2\lambda(a^2 + 2\omega^2) + \eta\sigma\bar{\sigma}$ | $2ma + 2\lambda(a^2 + 2\omega^2) + \eta\sigma\bar{\sigma}$ |
| $F_{\Phi_{1278}} = F_{\Phi_{3479}} = F_{\Phi_{5678}}$ | $2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$ | $2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$ |
| $F_{\Phi_{1290}} = F_{\Phi_{3490}} = F_{\Phi_{5690}}$ | $2m\omega + 2\lambda(2a + p)\omega - \eta\sigma\bar{\sigma}$ | $-2m\omega - 2\lambda(2a + p)\omega + \eta\sigma\bar{\sigma}$ |
| $F_{\Phi_{7890}}$ | $2mp + 6\lambda\omega^2 + \eta\sigma\bar{\sigma}$ | $-2mp - 6\lambda\omega^2 - \eta\sigma\bar{\sigma}$ |
| Other $F_{\Phi_{ijkl}}$ | 0 | 0 |

TABLE I. Property of various $F$-terms under $D$-parity flip
III. NON-RENORMALISABLE SUPERPOTENTIAL TERM

We are thus led to consider adding terms that depart from $SO(10)$ invariance, but clearly this must be done cautiously and without sacrificing the desirable features of low energy effective renormalisable theory. Such an alternative could affect the UV end of the theory without serious damage to the low energy theory. If the new terms are suppressed by Planck scale, one may later seek an explanation in the effects of quantum gravity, but more likely within a superstring type framework since gravity by itself is capable of violating global charges due to horizons, but unlikely to interfere with local gauge invariance. A minimal $SO(10)$ violating scenario can be constructed as a generalisation of the Gell-Mann-Okubo “octet dominance hypothesis” from the eightfold way of flavour $SU(3)$. Indeed we take a more drastic step of adapting this to a true gauge symmetry. We seek $SO(10)$ breaking but SM preserving non-renormalisable terms.

A. Gell-Mann Okubo Formalism

We briefly recall the formalism of Gell-Mann [14] and Okubo [15] who independently gave an explanation for the mass splittings within several of the multiplets of flavour $SU(3)$, viz., the pseudoscalar mesons octet, baryon octet and baryon decuplet. (For the later more systematic developments see [16–18]). Suppose one starts with exact flavour symmetry amongst the $u$, $d$ and $s$ quarks. In other words, we consider the global symmetry group $SU(3)$. The pseudoscalar mesons are made up of these three quarks and can be arranged to form an octet $\Pi$ in the adjoint representation of $SU(3)$. The Hamiltonian describing them at rest can be written as $H_0 = \mu^2 \text{Tr}[\Pi^\dagger \Pi]$, where $\mu^2$ denotes their squared mass. This Hamiltonian is $SU(3)$ symmetric and predicts that all the pseudoscalar mesons will have exactly the same mass $\mu$. However, this is not the case. This deviation from universal masses can be explained by postulating a perturbed Hamiltonian $H = H_0 + H'$, where the perturbation $H'$ is given by $H' = \frac{\alpha}{2} \text{Tr}[\Phi^T M \Phi]$. The $3 \times 3$ 'coefficient matrix' $M$ is chosen so as to break $SU(3)$ symmetry but preserve the lower energy isospin $SU(2)$ symmetry. This is done by taking $M$ from the Lie algebra $su(3)$ and requiring it to commute with $su(2)$ embedded into $su(3)$. This requirement fixes $M$ uniquely to be the Gell-Mann $\lambda_8$.

B. Choice of $SO(10)$ breaking matrix

Along similar lines we now consider the following extended superpotential:

$$W = W_{\text{ren}} + \frac{b}{M_{\text{Pl}}} W_{nr},$$

where

$$W_{nr} = \frac{1}{(4!)^4} (\Phi^T M' \Phi)^2.$$
Above $M'$ is a linear operator acting on the column vector $\Phi$. We shall take $M'$ to be the representation of a carefully chosen group element $M$ of SO(10). With a matrix $M$ in the notation of Sec. II, $M' = M \otimes 4$. We require that $M$ commute with all group elements $N \in$ SO(10) that arise as images of the embedding of the SM group $SU(3)_c \times SU(2)_L \times U(1)_Y$ into SO(10). It is then clear that $W_{nr}$ is SM-invariant but not necessarily SO(10) invariant.

We can now utilise the Pati-Salam embedding of SM group into SO(10) as given in [20, Section 2.2]. It is easy to see that SO(10) matrices of the form

$$M = \begin{pmatrix} I_{6 \times 6} & 0_{6 \times 4} \\ 0_{4 \times 6} & K_{4 \times 4} \end{pmatrix}$$

(10)

commute with all SO(10) matrices $N$ that arise from the embedding of the SM group into SO(10). Above $K$ is an SO(4) matrix of the form

$$K = \exp(2\theta J),$$

(11)

where

$$J = j_1 J_1^+ + j_2 J_2^+ + j_3 J_3^+ \in so(4),$$

$\theta$ is a real number between 0 and 2$\pi$, $j_1$, $j_2$, $j_3$ are real numbers satisfying $j_1^2 + j_2^2 + j_3^2 = 1$, and

$$J_1^+ = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, J_2^+ = -\frac{1}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, J_3^+ = \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

(12)

are the three self-dual generators of so(4). Recall that the self-dual elements of so(4) form the embedding of su(2)$_R$ into so(4). It can now be shown that $K$ is the image of the matrix $\exp(i\theta j_1 \tau_1 + j_2 \tau_2 + j_3 \tau_3)$ $\in$ SU(2)$_R$ when embedded into SO(4).

The reason behind $M$ commuting with SM is as follows. Recall that SU(3)$_c$ of SM gets embedded into the SO(6) part on the top left. The generator of $u(1)_{B-L}$ then maps to the so(10) matrix

$$B - L = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(13)

The su(2)$_L$ of SM maps into the anti-self-dual generators and su(2)$_R$ maps into the self-dual generators of so(4) in the bottom right. Since the self-dual generators of so(4) commute with the anti-self-dual generators, any linear combination of the three self-dual generators of so(4) will commute with su(2)$_L$. We fix one such linear combination $J = j_1 J_1^+ + j_2 J_2^+ + j_3 J_3^+ \in so(4)$, and map $u(1)_R$ to it. This $J$ commutes with the embedding of $SU(3)_c \times u(1)_{B-L} \times SU(2)_L \times u(1)_R$ into so(10) and thus with the embedding of SM into so(10).
We shall take $\theta = \pi/2$, $j_1 = j_3 = \frac{1}{\sqrt{2}}$, $j_2 = 0$ to get

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{pmatrix}. \quad (14)$$

This gives us

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \mathbb{1}_{6 \times 6} & 0_{6 \times 4} \\ 0_{4 \times 6} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{pmatrix} \end{pmatrix}. \quad (15)$$

Observe that the element $M \in \text{SO}(10)$ does not commute with D-parity. In other words, the D-parity generator viz.

$$J_{23} + J_{67} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (16)$$

is a broken symmetry generator in the non-renormalisable superpotential. This allows us the possibility of breaking the degeneracy between the two sets of vevs by means of the non-renormalisable term $W_{nr}$ in the superpotential. If there exist basic reasons for the existence of such a term then there could be more terms of the same type of higher mass dimensions. However this leading term should suffice for our purpose.

**IV. THE EFFECTIVE POTENTIAL AND DOMAIN WALL REMOVAL**

We next proceed to compute the effective potential in the two quasi symmetric vacua now split by the new non-renormalisable term. Observe that

$$\frac{\partial W_{nr}}{\partial \Phi_{ijkl}} = \frac{2(\Phi^T M^{\otimes 4} \Phi) \partial(\Phi^T M^{\otimes 4} \Phi)}{(4!)^4 \partial \Phi_{ijkl}}, \quad (17)$$

To keep the computation simple we note that the vev’s themselves will shift by $O(1/M_{Pl})$, and thus the contribution to the effective potential from the change in the vev’s is $O(1/M_{Pl}^4)$.
and can be ignored compared to the difference of \( O(1/M_{Pl}^2) \) arising from the leading Planck suppressed terms. Thus we can use the vev’s already determined by Bajc et al. [19]. In appendix A we have shown the evaluation of the resulting effective potential in this approximation. Since the scalar potentials evaluated at the \( D \)-flipped vevs are not the same, we get a pressure difference across the domain wall as follows

\[
V|_{\text{flipped vevs}} - V|_{\text{original vevs}} = \frac{b^2}{M_{Pl}^2} (4^2(3\alpha^2 + 6\omega^2 + p^2)((3\alpha^2 + 6\omega^2 + p^2)^2 - (3\alpha^2 + p^2)^2)).
\]

(18)

From the above expression for the pressure difference, it is clear that \( \omega \) is the controlling parameter for the difference in the scalar potential between the original and flipped vevs. For most ‘interesting’ regions of the parameter space, \( \max\{a, p\} \gg \omega \) [19, 21, 22]. For example, consider some ‘representative’ values

\[
a = -0.67 \frac{m}{\lambda}, \quad \omega = -0.21 \frac{m}{\lambda}, \quad p = -0.27 \frac{m}{\lambda}, \quad \sigma = \sigma = 0.51 \frac{m}{\sqrt{\eta\lambda}}, \quad \lambda = 0.12, \quad \eta = 0.21,
\]

taken from Equation 4 of Aulakh et al. [22] and Equation 57 of Aulakh and Girdhar [21]. This corresponds to \( x = 0.21 \) and \( \frac{\Lambda_{Msusy}}{\hat{\eta}M} = 1.03 \) in Equation 4 of [22]. The mass \( m \) is set to the mass \( M_X \) which is the lightest superheavy vector particle mediating proton decay, as mentioned in page 291 of [21]. For the above parameters, we take \( m = 10^{14} \text{ GeV}/c^2 \) using Fig. 4 and Equation 55 of Aulakh and Girdhar [21]. This gives us in units of GeV/c^2

\[
a \sim -10^{15}, \quad \omega \sim -10^{14}, \quad p \sim -10^{14}, \quad \sigma = \sigma \sim 10^{15},
\]

leading to \( a \gg p \geq \omega \). With these vevs, SO(10) MSGUT breaks down directly to MSSM. The assumption \( \max\{a, p\} \gg \omega \) occurs for many other regions of parameter space breaking SO(10) MSGUT directly to MSSM, as well as for the case of one or two intermediate scales of symmetry breaking with both Pati-Salam and left-right symmetric MSSM as the intermediate gauge groups [19, Section V].

In the following, we shall take \( \max\{a, p\} \sim M_X \) and \( \omega \sim M_R \), where \( M_R \) is the (lower) energy scale at which a scalar field acquires a non-zero vev also setting the scale of domain wall tension. With this, the pressure difference becomes

\[
|V|_{\text{flipped vevs}} - |V|_{\text{original vevs}}| \sim \frac{b^2}{M_{Pl}^2} M_X^4 M_R^2.
\]

(19)

A. Constraint from cosmology

We now derive the constraint the parameter \( b \) must satisfy if the non-renormalisable term were to be able to remove any domain walls that may have arisen in the early universe. For this, we equate the left hand side of Equation (19) to the pressure difference \( \delta \rho \) across the domain wall required to cause instability during various eras in the evolution of the universe. We take the energy scales in units of GeV/c^2. Following Mishra and Yajnik [8], we consider
the following three eras for domain wall appearance and removal. In the radiation dominated era, the pressure difference $\delta \rho$ must satisfy

$$\delta \rho_{\text{RD}} \geq \frac{M_R^6}{M_{\text{Pl}}^2}, \quad (20)$$

In the matter dominated era,

$$\delta \rho_{\text{MD}} \geq \frac{M_R^{11/2}}{M_{\text{Pl}}^{3/2}}. \quad (21)$$

In the case of weak inflation,

$$\delta \rho_{\text{WI}} \geq \frac{T_{D}^{12} M_{\text{Pl}}^3}{M_R^{11}}, \quad (22)$$

where $T_D$ is the (unknown) temperature scale at which the walls begin to experience instability.

For successful domain wall removal in the radiation dominated era, combining Equations (19) and (20) gives

$$b^2 \geq \frac{M_R^4}{M_X^4}. \quad (23)$$

For the matter dominated era, Equations (19) and (21) give

$$b^2 \geq \frac{M_R^{7/2} M_{\text{Pl}}^{1/2}}{M_X^4}. \quad (24)$$

For the case of weak inflation, Equations (19) and (22) lead to

$$|b|^2 \geq \frac{T_{D}^{12} M_{\text{Pl}}^5}{M_R^{13} M_X}. \quad (25)$$

We now tabulate the constraints arising from the above inequalities, assuming $M_X = 10^{16}$ viz. of the order of the GUT scale, and $M_{\text{Pl}} = 10^{19}$. We consider three candidate values for $M_R$: a ‘low’ value of $10^7$ corresponding to non-thermal or resonant leptogenesis, an ‘intermediate’ value of $10^9$ consistent with thermal leptogenesis without gravitino overabundance, and a ‘large’ value of $10^{13}$. For the radiation dominated and matter dominated eras, we obtain constraints on $|b|$ in order to ensure successful domain wall removal. For the case of weak inflation, we assume $|b| \leq O(1)$ and instead calculate constraints on the ratio $T_D/M_R$ in order to ensure successful domain wall removal. This is because the energy scale $T_D$ where the walls first experience instability is unknown to us.

From Table II, we see that our model is easily capable of ensuring domain wall removal in the radiation dominated and matter dominated eras without conflicting with existing experimental data. This is because the constraints on $|b|$ are very easy to meet; the miniscule symmetry breaking term is enough to cause instability in domain walls formed during the two eras. For the case of weak inflation, assuming $|b| \leq O(1)$, we get constraints on the ratio $T_D/M_R$. The ratio is required to be less than $10^{-3}$, making this case marginal for our model.
\begin{tabular}{|c|c|c|}
\hline
Era & \( M_R \) & \( b \geq \) \\
\hline
RD & \( 10^7 \) & \( 10^{-18} \) \\
 & \( 10^9 \) & \( 10^{-14} \) \\
 & \( 10^{13} \) & \( 10^{-6} \) \\
 & \( 10^7 \) & \( 10^{-15} \) \\
MD & \( 10^9 \) & \( 10^{-11.5} \) \\
 & \( 10^{13} \) & \( 10^{-4.5} \) \\
\hline
\end{tabular}

TABLE II. Constraints arising from different values of \( M_R \) in different cosmological eras

V. CONCLUSION

In this paper, we take the SO(10) based MSGUT studied extensively in earlier works and examine it from a cosmological angle. Namely, we observe that applying the D-parity operator to the SUSY preserving vevs breaking SO(10) MSGUT down to left handed MSSM gives a new set of SUSY preserving vevs. These two sets of vevs are disconnected in the sense that there is no SUSY preserving path connecting them in parameter space. We then remark that this leads to a danger of domain wall formation in the early universe depending on the precise symmetry breaking pattern. A similar conclusion has recently been made in [11].

We then investigate conditions under which domain walls, if formed, can go away early enough so as not to conflict with standard cosmology. We first observe that domain walls cannot go away even with dimension four operators in the superpotential as long as the theory is SO(10) symmetric. We then postulate a non-renormalisable term that breaks of SO(10) symmetry while still preserving SM symmetry. Armed with this, we investigate domain wall removal in three scenarios of the early universe viz. radiation dominated era, matter dominated era which is generic to string theory inspired models, and a weak inflation era following matter domination.

Our main conclusions are as follows:

1. If domain walls are formed in the radiation dominated era, they can easily disappear in that era itself in our model. This is true for a wide range of energy scales of domain wall formation. A similar conclusion was obtained by [8] for two left-right symmetric supersymmetric (LRSUSY) models called ABMRS and BM models in their paper.

2. If domain walls are formed early in the matter dominated era, they can easily disappear in that era itself in our model. The ABMRS model studied in [8] was significantly constrained in this scenario and required a low value of the domain wall energy scale of \( 10^7 \) GeV/\( c^2 \) which further requires non-thermal leptogenesis. The BM model studied in [8] was however not constrained in this scenario. Our model is also not constrained for a wide range of values of the the domain wall energy scale.

3. If domain walls are formed late in the matter dominated era and continue on to a following weak inflation phase, then domain walls can eventually completely disap-
pear only under a certain condition. The condition required by our model is that the temperature at which the walls become unstable be at least three to four orders of magnitude smaller than the energy density at which the walls first appear. This makes our model marginal for this scenario. Our model is less constrained than the ABMRS and BM models for this scenario \[^8\], both of which required some careful parameter setting in order to get rid of domain walls while being consistent with thermal leptogenesis without gravitino overabundance.

Thus this work invites further careful analysis of other cosmological implications of SO(10) MSGUT.

**Appendix A**

We begin by evaluating the new term, expressing it in terms of components,

\[
\frac{1}{(4!)^2} \Phi^T M^{\otimes 4} \Phi = \sum_{i<j<k<l \leq 6} \Phi_{ijkl}^2 + \sum_{i<j \leq 6} \left( \frac{\Phi_{ij78}^2}{2} + \Phi_{ij78} \Phi_{ij70} + \Phi_{ij78} \Phi_{ij89} - \Phi_{ij78} \Phi_{ij90} + 2 \Phi_{ij79} \Phi_{ij80} \\
+ \Phi_{ij70}^2 - \Phi_{ij70} \Phi_{ij89} + \Phi_{ij70} \Phi_{ij90} + \Phi_{ij89}^2 + \Phi_{ij89} \Phi_{ij90} + \Phi_{ij90}^2 \right) + \Phi_{7890}^2. \tag{A1}
\]

Evaluating \(W_{nr}\) at the original vevs, we get

\[
W_{nr}|_{\text{orig. vevs}} = \left( 3a^2 + 3 \left( \frac{\omega^2}{2} - \omega^2 + \frac{\omega^2}{2} \right) + p^2 \right)^2 = (3a^2 + p^2)^2. \tag{A2}
\]

Evaluating \(W_{nr}\) at the flipped vevs, we get

\[
W_{nr}|_{\text{flipped vevs}} = \left( 3a^2 + 3 \left( \frac{\omega^2}{2} + \omega^2 + \frac{\omega^2}{2} \right) + p^2 \right)^2 = (3a^2 + 6\omega^2 + p^2)^2. \tag{A3}
\]

We now evaluate the partial derivatives of \(\frac{1}{(4!)^2} \Phi^T M^{\otimes 4} \Phi\) with respect to the various fields.

\[
\frac{1}{(4!)^2} \frac{\partial (\Phi^T M^{\otimes 4} \Phi)}{\partial \Phi_{ijkl}} = \begin{cases} 
2\Phi_{ijkl} & 1 \leq i < j < k < l \leq 6, \\
\Phi_{ij78} + \Phi_{ij70} + \Phi_{ij89} - \Phi_{ij90} & 1 \leq i < j \leq 6, kl = 78, \\
2\Phi_{ij80} & 1 \leq i < j \leq 6, kl = 79, \\
\Phi_{ij78} + \Phi_{ij70} - \Phi_{ij89} + \Phi_{ij90} & 1 \leq i < j \leq 6, kl = 70, \\
2\Phi_{ij79} & 1 \leq i < j \leq 6, kl = 89, \\
-\Phi_{ij78} + \Phi_{ij70} + \Phi_{ij89} + \Phi_{ij90} & 1 \leq i < j \leq 6, kl = 90, \\
2\Phi_{7890} & \text{i} \text{ijkl} = 7890, \\
0 & \text{otherwise}.
\end{cases} \tag{A4}
\]
Evaluating the partial derivatives at vevs determined in [19], we get

$$\frac{1}{(4!)^2} \frac{\partial (\Phi^T M^{\otimes 4} \Phi)}{\partial \Phi_{ijkl}} \bigg|_{\text{orig. vevs}} = \begin{cases} 2a & ijkl = 1234, 1256, 3456, \\ 2\omega & ijkl = 1270, 3470, 5670, 1289, 3489, 5689, \\ 2p & ijkl = 7890, \\ 0 & \text{otherwise}. \end{cases} \quad (A5)$$

Evaluating the partial derivatives at the flipped vevs, we get

$$\frac{1}{(4!)^2} \frac{\partial (\Phi^T M^{\otimes 4} \Phi)}{\partial \Phi_{ijkl}} \bigg|_{\text{flipped vevs}} = \begin{cases} 2a & ijkl = 1234, 1256, 3456, \\ 2\omega & ijkl = 1270, 3470, 5670, 1289, 3489, 5689, \\ 2p & ijkl = 7890, \\ 0 & \text{otherwise}. \end{cases} \quad (A6)$$

We can now compute the contribution to the F-terms arising from $W_{nr}$ for both types of vevs.

$$(F_{nr})_{ijkl} \big|_{\text{orig. vevs}} = \begin{cases} 4(3a^2 + p^2) a & ijkl = 1234, 1256, 3456, \\ 4(3a^2 + p^2) \omega & ijkl = 1270, 3470, 5670, 1289, 3489, 5689, \\ 4(3a^2 + p^2) p & ijkl = 7890, \\ 0 & \text{otherwise}. \end{cases} \quad (A7)$$

$$(F_{nr})_{ijkl} \big|_{\text{flipped vevs}} = \begin{cases} 4(3a^2 + 6\omega^2 + p^2) a & ijkl = 1234, 1256, 3456, \\ 4(3a^2 + 6\omega^2 + p^2) \omega & ijkl = 1270, 3470, 5670, 1289, 3489, 5689, \\ -4(3a^2 + 6\omega^2 + p^2) p & ijkl = 7890, \\ 0 & \text{otherwise}. \end{cases} \quad (A8)$$

Since both the original vevs as well as the flipped vevs are F-flat at the renormalisable level for suitably chosen values of $a$, $\omega$, $p$, the above expressions are the complete values, including renormalisable and non-renormalisable contributions, of the respective F-terms. In other words,

$$F_{\Phi_{ijkl}} \big|_{\text{orig. vevs}} = \frac{b}{M_p} (F_{nr})_{ijkl} \big|_{\text{orig. vevs}},$$

$$F_{\Phi_{ijkl}} \big|_{\text{flipped vevs}} = \frac{b}{M_p} (F_{nr})_{ijkl} \big|_{\text{flipped vevs}}, \quad (A9)$$

$$F_{\Sigma_{ijklm}} \big|_{\text{orig. vevs}} = F_{\Sigma_{ijklm}} \big|_{\text{flipped vevs}} = 0.$$ 

The F-term contribution to the scalar potential evaluated at the original and flipped vevs becomes

$$V_F \big|_{\text{orig. vevs}} = \frac{k^2}{M_p} \sum_k |(F_{nr})_{fk}|^2 \big|_{\text{orig. vevs}},$$

$$V_F \big|_{\text{flipped vevs}} = \frac{k^2}{M_p} \sum_k |(F_{nr})_{fk}|^2 \big|_{\text{flipped vevs}}. \quad (A10)$$
This gives us the following expressions for F-term contributions to the scalar potential.

\[ V_F|_{\text{orig. vevs}} = \frac{\nu^2}{M_{Pl}^2} (3(3a^2 + p^2)a^2 + 6(3a^2 + p^2)\omega^2 + (4(3a^2 + p^2)p)^2), \]

\[ = \frac{\nu^2}{M_{Pl}^2} (4^2(3a^2 + p^2)^2(3a^2 + 6\omega^2 + p^2)), \]

\[ V_F|_{\text{flipped vevs}} = \frac{\nu^2}{M_{Pl}^2} (3(4(3a^2 + 6\omega^2 + p^2)a^2 + 6(4(3a^2 + 6\omega^2 + p^2)\omega^2 + (4(3a^2 + 6\omega^2 + p^2)p)^2), \]

\[ = \frac{\nu^2}{M_{Pl}^2} (4^2(3a^2 + 6\omega^2 + p^2)^2). \]

Since we take \(|\sigma| = |\bar{\sigma}|\), both original and flipped vevs are D-flat. Thus, the scalar potentials evaluated at the two types of vevs arise solely from F-term contributions.

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