Research on the influence of straightness deviations on positioning precision of Cartesian industrial robots

A Luncanu and G Stan
“VasileAlecsandri” University of Bacau, Department of Industrial Engineering, CaleaMărășești Street, No. 157, 600115, Bacau, Romania
E-mail: andreiluncanu@yahoo.com

Abstract. Among the first generations of industrial robots used in various industries were the Cartesian industrial robots. The main advantages of these positioning systems are its flexibility and fast positioning capability. The reduced time of generating and programming of the industrial robots trajectory is a particular advantage of these systems. Over the years, a number of factors have been discovered that affect the positioning precision of Cartesian industrial robots. Presently among these factors that are affecting the positioning precision of a Cartesian industrial robot are the dimensional deviations and the geometrical deviations present in their structures. Among the geometric deviations, affecting the positioning accuracy of a Cartesian industrial robot there is the straightness deviation of the linear kinematic links. Due to the existence of straightness deviations in the linear kinematic links of the Cartesian industrial robot, it is necessary to use methods to minimize the positioning errors of industrial robots within the linear axes. In this article are presented methods of determining the values of rectilinear deviations and utilizing them in an optimized kinematic system. By choosing this method to minimize positioning error by designers in current generations of Cartesian industrial robots and systems using linear kinematic links, both positioning errors and trajectory generation times are diminished, resulting in the economic and technological efficiency of the products and services they offer.

1. Introduction
Industrial robots with linear kinematic links have a huge range of use in the following applications:
- Flexible manufacturing systems, as work ports for the machine tools supply systems;
- Automated mounting systems;
- Automatic welding systems;
- Automatic transport and storage of parts, tools and devices.

Cartesian industrial robots are the only industrial robots that use only linear kinematic links and the structure can be portal type or on the deck.

To carry out the above-mentioned activities, the industrial robot must achieve a high accuracy of positioning of the end-effector, which is the positioning precision of the manipulated object. All the kinematic and constructive factors of every linear kinematic link used in the Cartesian industrial robot’s structure affect the positioning precision of the end-effector. In the category of geometrical deviations that appear in the industrial robot’s structure, both deviations caused by radial and axial runout of couplings and reference position deviation affect the positioning precision.
The straightness deviation of the linear kinematic links is another important disruptive factor that affect the Cartesian industrial’s robot positioning precision. The automatic compensation of the straightness deviation of each linear kinematic link is an analytical method for increasing the positioning accuracy of the end-effector.

2. Determination of the kinematic link’s straightness deviation variation equations
The block structure of the linear kinematic link control system is shown in figure 1.

![Figure 1. The block diagram of the linear kinematic link control system.](image1)

The block structure shown in figure 1 is comprised of the numeric analog converter CNA, the actuator driver D, the servomotor SM, the transmission system T and the linear kinematic link’s slide S. In the block structure shown in figure 1 the reaction loops use the speed transducer TV and the positioning transducer TP. The feedback signal of the signal comparator C1 is e_p and for the velocity signal comparator C2 is e_v.

As can be seen in figure 1, the straightness deviation of the linear kinematic link’s slide S cannot be automatically compensated by the position loop, but by applying an automatic calculation method to the prescribed signal, would constitute a correction factor with an impact on positioning precision. Thus, knowing the deviation from straightness for each linear kinematic link based on an algorithm described is this article. The values automatically determined constitute correction signals for the other axes.

![Figure 2. Measurement method for straightness deviation.](image2)

In figure 2 are shown the movable element which is consistsed of the slide S and the linear motion guideline G. In order to correctly measure the straightness deviation of the linear kinematic link a measurement on both vertical plane v and horizontal plane h must be used. In order to measure the straightness deviation the laser interferometer prism or the mechanical comparator are mounted on the guideline L. In order to use the positioning precision minimization method of the Cartesian industrial robot, the straightness deviation variation equations are needed. For example, considering the Cartesian industrial robot’s linear kinematic links x, y, z where the x-axis is the one represented in figure 2, the values of the vertical deviation in the vertical plane, automatically determined by the variation equation, loop correction for the vertical axis z. The values of the deviation from the rectilinear plane in the horizontal plane will constitute correction signals on the horizontal axis y.
In many experiments, the straightness deviation has a linear characteristic, approximated by a one or two-degree mathematical function. In addition, most straightness deviation’s diagram may be divided into several regions along the linear kinematic link displacement[1].

![Figure 3. Vertical straightness deviation diagram along the displacement of linear kinematic link x.](image)

In figure 3 is shows the vertical straightness deviation diagram along the displacement of linear kinematic link $x$, which is divided into three regions.

1. $e_{rl} = a \cdot x + b$, where $x \in [0, l_1]$ (1)

Where $a$ and $b$ are constants and they are determined by using equations (2) and (3):

\[
\begin{align*}
\begin{cases}
  x = 0 \\
  e_{rl} = 0 
\end{cases} & \rightarrow b = 0 \\
\begin{cases}
  x = l_1 \\
  e_{rl} = e_{rmax}
\end{cases} & \rightarrow e_{rmax} = a \cdot l_1 \rightarrow a = \frac{l_1}{e_{rmax}} 
\end{align*}
\]

(II) $e_{rII} = e_{rmax}$, where $x \in [l_1, l_2]$ (5)

(III) $e_{rIII} = a \cdot x + b$, where $x \in [l_2, l_3]$ (6)

\[
\begin{align*}
\begin{cases}
  x = 0 \\
  e_{rIII} = e_{max3}
\end{cases} & \rightarrow b = e_{max3} \\
\begin{cases}
  x = l_3 \\
  e_{rIII} = 0
\end{cases} & \rightarrow 0 = a \cdot l_3 + e_{max3} \rightarrow a = \frac{-e_{max3}}{l_3} 
\end{align*}
\]

If the (III) displacement is the continuous line, the straightness equation is:

\[
e_{rIII} = e_{max3} - \frac{e_{max3} \cdot x}{l_3}
\]

If the (III) displacement is the dotted line, the straightness equation is:

\[
\begin{align*}
\begin{cases}
  x = l_3 \\
  e_{rIII} = e_{max3}
\end{cases} & \rightarrow e_{max3} = a \cdot l_3 + e_{max1} \rightarrow a = \frac{-e_{max3} - e_{max1}}{l_3}
\end{align*}
\]
3. The experimental model for determining the influence of the straightness deviation on the positioning precision of the Cartesian industrial robot

In order to use the method of maximizing the positioning precision Cartesian industrial robots is necessary to measure and introduce the values of the straightness deviation in the mathematical model with homogeneous matrices of the Cartesian industrial robot [2, 3].

![Cartesian industrial robot kinematic link structure.](image)

The experimental model that is shown in figure 4, used in this article has the $d_1 = 300$ mm, $d_2 = 300$ mm and $d_3 = 300$ mm displacements. The homogeneous matrix of the Cartesian industrial robot is presented in equation (11) [4, 5].

$$T_{04} = D_1 \cdot D_2 \cdot D_3 \cdot T_{34}$$ (11)

For the application of the geometric method it is necessary to use the kinematic links matrices $D_j$ shown in the equation (12), the kinematic link matrices $D_2$ is shown in the equation (13), the kinematic link matrices $D_3$ is shown in the equation (14) and the homogeneous passage matrix of the distance between the kinematic coupler $T_{34}$ and the end effector is shown in the equation (15).

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & e_{ry1} \\ 0 & 0 & 1 & a_0 + e_{rz1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (12)

$$D_2 = \begin{bmatrix} 1 & 0 & 0 & e_{rx2} \\ 0 & 1 & 0 & e_{ry2} \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (13)

$$D_3 = \begin{bmatrix} 1 & 0 & 0 & e_{rx3} \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 1 & e_{rz3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (14)

$$T_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & e_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (15)
By applying the substitutions and making the matrices calculations the positioning equations of the Cartesian industrial robot (16-18).

\[
x_{04} = d_1 + e_{rx2} + e_{rx3} \\
y_{04} = d_3 + e_{ry1} + e_{ry2} \\
z_{04} = a_0 + a_4 + d_z + e_{rz1} + e_{rz3}
\]

(16)  
(17)  
(18)

As can be seen from the positioning equations of the Cartesian industrial robot (16-18), the straightness deviations are present in all linear kinematic links [6]. By applying the methods of determining the straightness deviations [7], the straightness deviations of the Cartesian industrial robot are shown in figures 5(a), 5(b), 6(a), 6(b), 7(a) and 7(b).

**Figure 5.** Straightness deviation grafics for X axis.

**Figure 6.** Straightness deviation grafic for Z axis.

**Figure 7.** Straightness deviation grafic for Y axis.
By using the measured straightness deviations values shown in the figures 5-7 we obtain the straightness deviations variation equations presented in equations (19-24).

\[ e_{ry1} = 0.00018 \cdot d_1 \]  \hspace{1cm} (19)  
\[ e_{rz1} = -0.00034 \cdot d_1 \]  \hspace{1cm} (20)  
\[ e_{rx2} = 0.00009 \cdot d_2 \]  \hspace{1cm} (21)  
\[ e_{ry2} = 0.00011 \cdot d_2 \]  \hspace{1cm} (22)  
\[ e_{rx3} = -0.00012 \cdot d_3 \]  \hspace{1cm} (23)  
\[ e_{ry3} = 0.00021 \cdot d_3 \]  \hspace{1cm} (24)

In (19) and (20) are presented the equations of straightness deviation \( e_{ry1} \) and \( e_{rz1} \) for the kinematic coupling \( C_1 \), in (21) and (22) are presented the equations of straightness deviations \( e_{rx2} \) and \( e_{ry2} \) for kinematic couple \( C_2 \) and in (23) and (24) the equations of straightness deviations \( e_{rx3} \) and \( e_{ry3} \) of the kinematic couple \( C_3 \).

**Table 1. Comparative values of the Cartesian industrial robots positioning.**

| With straightness deviation values | Without straightness deviation values |
|-----------------------------------|--------------------------------------|
| \( D_i \) (mm) | 30 | 60 | 120 | 120 | 200 | 160 | 220 | 200 | 120 | 120 | 200 | 160 | 220 | 200 |
| X [mm] | 30.213 | 120.245 | 220.147 | 30.413 | 120.445 | 220.247 |
| Y [mm] | 60.254 | 200.357 | 220.285 | 60.454 | 200.457 | 220.485 |
| Z [mm] | 120.634 | 160.618 | 200.165 | 120.734 | 160.818 | 200.565 |

In order to more clearly observe the influences caused by the rectilinear deviations [8], table 1 is created. In table 1 the displacements on the three kinematic links are introduced and the positions with the straightness deviations in the left side of the table are obtained and without the straightness deviations on the right side of the table.

4. Conclusions

Through the research presented in the article, we can see the degree of influence of the straightness deviation of a kinematic translation couple. The precision of positioning of the Cartesian industrial robot is improved by using the method presented in the article. Manufacturers of industrial robots may use the research presented in the article to improve positioning specifications, implicitly trajectory specifications.

5. References

[1] Jakubiec W 2009 Analytical estimation of uncertainty of coordinate measurements of geometric deviations. Models based on distance between point and straight line Advances in manufacturing science and technology 33(2)

[2] Nicolescu AF, Ilie FM and Alexandru TG 2015 Forward and inverse kinematics study of industrial robots taking into account constructive and functional parameter’s modeling Proceedings in Manufacturing Systems 10 pp 157-164
[3] Melchiorri C Kinematic Kinematic Model of Robot Manipulators Dipartimento di Ingegneriadell’Energia Elettrica e dell’Informazione Università di Bologna http://www-lar.deis.unibo.it/people/cmelchiorri/Files_Robotica/FIR_04_Kinem.pdf Accessed on: 14.01.2019

[4] Singh T P, Suresh Dr P and Chandan Dr S 2017 Forward and inverse kinematic analysis of robotic manipulators International Research Journal of Engineering and Technology (IRJET) 4 1459

[5] Gouasmi M, Ouali M, Fernini B and Meghatria M 2012 Kinematic Modelling and Simulation of a 2-R Robot Using SolidWorks and Verification by MATLAB/Simulink International Journal of Advanced Robotic Systems 1 78-93

[6] Das M T and Dulger L C 2005 Mathematical modelling, simulation and experimental verification of a scara robot Simulation Modelling Practice and Theory 13 257–271

[7] Siciliano B, Sciavicco L, Villani G and Oriolo G 2009 Robotics: Modelling, Planning and Control (London: Springer)

[8] Zlajpah L 2008 Simulation in robotics Mathematics and Computers in Simulation 79(4) 879-897