Research on Parameters of the Trailing VLF Antenna Using Lumped-Mass Method

Tielin Ma¹,², *, a, Zhihua Wei³,5, b, Haibing Chen⁴ and Xiangsheng Wang³

¹ Institute of Unmanned System, Beihang University, Beijing, 100191, China
² Key Laboratory of Advanced Technology of Intelligent Unmanned Flight System of Ministry of Industry and Information Technology
³ School of Aeronautic Science and Engineering, Beihang University, Beijing, 100191, China
⁴ School of Transportation Science and Engineering, Beihang University, Beijing, 100191, China

*, a matielin@buaa.edu.cn; b wzhua@buaa.edu.cn.

Abstract. The verticality of Very Low Frequency (VLF) antenna is an important factor to decide the radiation intensity of airborne VLF communication system, and the close relation of which with the parameters of airplane, towed antenna, and drogue is an important issue. Considering from the aspect of system design, the flight speed and the drogue’s mass are primary design variables whereas the bank angle constrained by airplane’s maneuverability and cable’s basic parameters decided by design requirement can be chosen initially. After modelling the trailing system with lumped-mass method, sensibility analyzation about relations of the verticality of antenna with the flight speed and the drogue’s mass are carried on. The result is found that when a faster flight speed is chosen, it’s not easy to gain enough verticality required, and leads to a conclusion that, flight speed dominates the verticality which should be decided seriously whereas the drogue’s mass poses much less effect.

1. Introduction

As the sea is an electric conductor, electromagnetic wave decays quickly when spreading in it. To build communication between airplane and submarine, approaches, such as VLF signal, ELF (extremely low frequency) signal, blue-green laser, satellite delay communication, are developing. Among them, Airborne VLF submarine communication system is an important and developed communication relay approach, featured with maneuverability, anti-interference and survivability.

VLF antenna system is composed of airplane, antenna, and the drogue tied under the antenna for stabilization. For dynamic research, it’s regarded as a towing system which is highly nonlinear and complex. Modeling of antenna is the primary job of the simulation for this system. It can be divided into two ways, continuous model [1-3] presented by partial differential equation, and lumped-mass method including mass-spring model [4, 5] solved by ordinary differential equation, and lumped-mass rigid-rod model [6-7] solved by multi-body formulation. They are mature methods to simulate the towing system, among which the lumped-mass method is proved to be an efficient way to simulate complex cable system [4].

As for the trailing antenna, there are some researches have been done. Zheng Xiaohong et al. [8-10] solved the steady dynamic model by shooting technique and Newton-Raphson iteration respectively.
discuss some effect factors influencing the steady state of the cable and had provided some guidance for system designing. But the effect of the factors is discussed separately. As for the practical experience, some factors would dominate whereas some would be secondary. Because of this, the comparison of the effect from different factors is needed. Furthermore, they researched on the parameters too extensively. However, for a system towed by an airplane, the parameters are constrained by practical condition. For example, it’s hard to meet the multivalued solution, since the rotational velocity is small [7]. For a specific case, the range of the parameters should be smaller.

Therefore, this paper discussed about the working condition from the aspect of practical design, to get the appropriate range of parameters, and find out the factors which dominate the system and deserve more attention. The simulation of the system is done by using lumped-mass method which discretizes the towed antenna with mass-spring model. And the Runge-Kutta method is used to solve the nonlinear equations. The research on the sensibility of the verticality is carried on by building response surface.

2. The Lumped-Mass Method

2.1. Coordinate Frame

![Figure 1. Sketch of the coordinate frame](image)

![Figure 2. Sketch of the segment model](image)

The calculation is based on an inertial frame (figure 1), the Local Tangent Plane Coordinate System. The ground is assumed as a horizontal plane. The origin is set at the sealevel point of the rotation axis. The z axis points up. The x direction is aligned with an arbitrary direction in the horizontal plane, and the y axis can be decided by the right-hand principle.

2.2. Discretization

The antenna is discretized into a number of segments which is enough to gain satisfactory accuracy. By mass-spring model, each segment is modeled as damping spring connected with mass points (figure 2). Each mass point has 3 degrees of freedom and suffers from half of the gravity and aerodynamic force act on its adjacent segments.

2.3. Force on the Segments

2.3.1. Gravity. The gravity acceleration is assumed as constant with altitude.

\[
\vec{G} = m_0 \cdot \vec{g} \cdot l
\]  

where \( m_0 \) is the linear density of the antenna, \( l \) is the initial length of the segments.

2.3.2. Aerodynamic Force. The flight altitude is in the troposphere. So the variation of air density with the altitude is

\[
\rho = \left( \frac{288.15 - 0.0065h}{288.15} \right)^{4.25588} \cdot \rho_0
\]  

\( \rho_0 \)}}
Where \( \rho_0 \) is the air density at sealevel.

The aerodynamic force act on the segments can be calculated by

\[
\vec{D}_t = 0.5 \cdot \rho \cdot |\vec{v}| |\vec{v}| \cdot S_t \cdot C_{dt}
\]

(3)

\[
\vec{D}_n = 0.5 \cdot \rho \cdot |\vec{v}| |\vec{v}| \cdot S_n \cdot C_{dn}
\]

(4)

\[
S_t = \pi \cdot d^2 (l + dl)
\]

(5)

\[
S_n = d^2 (l + dl)
\]

(6)

Where \( \vec{D}_t \) is the tangential friction, \( \vec{D}_n \) is the normal drag, \( C_{dt} \) is the coefficient of tangential friction., \( C_{dn} \) is the coefficient of normal drag, \( \vec{v} \) is the airspeed of the segments, \( \vec{v}_t \) and \( \vec{v}_n \) is the tangential and normal component of \( \vec{v} \) along the segments respectively, \( S_t \) is the superficial area of the segments, while \( S_n \) is the area of longitudinal section.

2.3.3. Tension and Damping Force. Each segment is considered as a linear damping spring. Thus the tension \( \vec{T} \) and the damping force \( \vec{T}_{\text{m}} \) can be calculated by

\[
\vec{T} = k \cdot \vec{a} \cdot d\vec{l} / l
\]

(7)

\[
\vec{T}_{\text{m}} = k_{m} \cdot A \cdot \frac{d\vec{l}}{dt}
\]

(8)

\[
\vec{T} = \vec{T}_{\text{m}} = \vec{\alpha} \quad (dl < 0)
\]

(9)

Where \( k \) is the Young’s modulus, \( d\vec{l} \) is the deformation quantity of the segments, \( l \) is the original length of the segments, \( k_{m} \) is the damping coefficient, \( A \) is the area of cross section. The change of the diameter is neglected.

2.3.4. Dynamics Equations for the Mass Points.

\[
\vec{F} = m \cdot \vec{a}
\]

(10)

\[
(\vec{T}_{\text{m1}} + \vec{T}_{\text{m2}}) + (\vec{T}_1 + \vec{T}_2) + (\vec{G}_1 + \vec{G}_2) / 2 +
\]

\[
(\vec{D}_t + \vec{D}_n) / 2 + (\vec{D}_{n1} + \vec{D}_{n2}) / 2
\]

\[
= (m_{t1} + m_{t2}) \cdot l / 2 \cdot \vec{\alpha}
\]

(11)

For each mass point, forces with the subscript 1 and 2 represent the forces act on the two segments connected with it.

2.4. Modelling of the airplane

In keeping with the assumption of the steady-state model, a constant bank angle, circular orbit was assumed, with the effect from the antenna neglected. The orbiting radius and angular velocity is calculated by bank angle

\[
r = \frac{V^2}{g \sqrt{(1 / \cos \phi)^2 - 1}}
\]

(12)

\[
\omega = V / r
\]

(13)

Where \( V \) is the flight speed of the plane, \( \phi \) is the bank angle.

2.5. Conclusion of the method

Up to now, the simplest towing system, composed of airplane and cable, has been modeled. As for the trailing antenna system, the boundary condition, drogue in this case, should also be concerned. Its aerodynamic force, gravity and mass are inherited by the first mass point.

As shown by figure 3, the result of simulating the TACAMO system is aligned with that given by the AD report [11] using second order accurate central differencing techniques. Therefore, the model’s accuracy is ensured.
3. Research on the parameters

3.1. Discussion of the primary parameters

The primary parameters of airborne VLF communication system include the length, diameter, aerodynamic characteristics of the antenna, the flight speed, altitude, bank angle of the airplane, the mass, aerodynamic characteristics of the drogue. But it’s not necessary to discuss all the parameters listed here.

Some parameters can be decided by working condition. The length of antenna related to the wave length of the signal ranges from 5km to 10km. The diameter of antenna is normally 4mm to 6mm. Airplane used is usually medium transport or communication relay aircraft, such as E-6A used in TACAMO system. The flight altitude is set to match the length of antenna. Additionally, there are trustable results about the aerodynamic characteristics of the antenna and the drogue [11]. As for normal cases, with bigger bank angle, lower flight speed, heavier drogue, the verticality increases. In fact, the bank angle, which should be almost 40°, is constrained by the airplane’s maneuverability. Therefore, the design space of the flight speed and drogue mass is greater, which deserves more researches.

The working condition decided initially for a practical case is specified that, the antenna’s length is 6km, its diameter is 5mm, the flight altitude is 6km, and the bank angle is 40°. The requirement is that the drogue’s mass and the airplane’s speed should be designed to gain a verticality of 75%. To compensate for some simplification, such as steady atmosphere, the verticality required in simulation result is set to 80%.

The cruise speed is usually about 550km/h, the minimum speed can be 300km/h. So the range of the flight speed discussed is set from 300km/h to 550km/s. As reference to the AD report [11], the small drogue’s mass can work, of which the range is set from 30kg to 90kg.

3.2. Analyzation

First, to analyze the effect from the two parameters on the verticality, the response surface of the flight speed, the drogue’s mass and the verticality is built. As shown in the 3D response surface (figure 4), under the practical situation that needs to be discussed, the effect from the airplane’s speed and the drogue’s mass on the verticality is aligned with the normal cases, in which the lower speed and the heavier drogue lead to a larger verticality. It’s easy to get that the flight speed plays a dominant role. Furthermore, when a high flight speed is chosen, such as 150m/s, it’s hard to meet the requirement even with a heavier drogue. So, the airplane’s airspeed should be decided first.

Then, to distinctly observe the effect shown above, the steady states under different conditions are calculated and visualized. As shown by figure 5, the steady state changes little with different mass of drogue under both low and high flight speeds. And it’s obvious that, a lower speed can lead to a
visibly higher verticality. More steady states under different speeds (with a 40-kg-drogue) are shown by figure 6 and figure 7. The verticality decreases dramatically from 79% to 22% and the drogue’s orbiting radius increases from 190m to 1832m when the flight speed changing from 400km/h to 500km/h.

At last, based on the requirement specified above, a flight speed of 400km/h, and a 40-kg-drogue is chosen, under which the verticality can reach 79%. The steady state of this case can be seen in figure 6.

3.3. Estimation of the flight speed
Taking into consideration that the antenna’s length and the bank angle constraint may differ to a big extent in different cases, the flight speed chosen to reach 75% verticality under the antenna mentioned above with different length and bank angle constraint is calculated and shown by figure 8. Apparently, the flight speed for different cases differs considerably. For bigger bank angle and longer antenna, the verticality required is easier to reach with higher speed. It’s obviously that the points meeting requirement can be fitted linearly. A law can be drawn roughly is that, when the antenna’s length is 1km longer, the flight speed can be about 7m/s higher with the same bank angle. What is more, the bank angle should be 1 degree bigger when the flight speed increase by 1.5 m/s with the same antenna’s length. By this law, when another working condition required, the approximately correct flight speed can be estimated rapidly based on the known case, so that it’s more efficient to decide the exact parameters.

The factors which set constant, such as antenna’s diameter, drogue’s mass, and the aerodynamic characteristic of them, exert much less effect on the verticality [12]. Because of this, although this law
is gained by simulating specific antenna with solely different length, it could work for an extensive range of trailing antenna system.

![Flight speed chosen for different working conditions](image)

**Figure 8.** Flight speed chosen for different working conditions

4. Conclusion

(1) The lumped-mass method is used to simulate the trailing antenna system under constant circular motion, of which the accuracy is ensured.

(2) Considering the primary system design procedure, the flight speed and the drogue’s mass is distinguished as the primary design variables, of which the former is found to dominate the verticality of antenna whereas the latter influences little. A higher flight speed would make the verticality hard to meet the requirement.

(3) Taking into account that the appropriate flight speed differs for variable bank angle constraint and the antenna’s length, a law to estimate flight speed is given, and is suitable for an extensive range of trailing antenna system.

References

[1] S A Crist, J G Eisley 1970 *J.Spacraft* 7 p 1352
[2] F Zhu, C D Rahn 1998 *J. Sound Vib.* 217 p 435-452
[3] Ma Dongli, Wang Shaoqi, Yang Muqing, Dong Yongpan 2016 *Chin. J. Aeronaut.* 29 pp 1484-1495
[4] S. A. Crist, J. G. Eisley 1969 *J.Spacecraft* 6 p 819-824
[5] Williams P, Lapthorne P, and Trivailo P 2006 *AIAA Modeling and Simulation Technologies Conference and Exhibit*
[6] González F, Prada A D L, Luaces A. and González M 2017 *Ocean Engineering* 131 pp 295-307
[7] Paul Williams 2010 *Journal of Guidance Control & Dynamics* 33 pp 1251-63.
[8] Zheng Xiaohong, Hou Zhiqiang and Li Jixin 2011 *Journal of Naval Aeronautical & Astronautical University* 26 pp 628-632
[9] Zheng Xiaohong, Han Wei, JIA Zhonghu and Guo Weigang 2013 *Flight Dynamics* 31 pp 234-238.
[10] Zheng Xiaohong, Han Wei, JIA Zhonghu 2013 *Acta Aeronautica ET Astronautica Sinica* 31 pp 234-238.
[11] James M C. 1992 *ADA256450*
[12] Zhu Yanjuan and Luo Dan 2018 *Journal of Tongji University (natural science)* 46 pp 81-86