MULTI-VECTORS RETRIEVAL AS SPARSE ALIGNMENT

Yujie Qian∗, Jinhyuk Lee, Sai Meher Karthik Duddu, Zhuyun Dai, Siddhartha Brahma, Iftekhar Naim, Tao Lei, Vincent Y. Zhao‡∗
Google Research

ABSTRACT

Multi-vector retrieval models improve over single-vector dual encoders on many information retrieval tasks. In this paper, we cast the multi-vector retrieval problem as sparse alignment between query and document tokens. We propose ALIGNER, a novel multi-vector retrieval model that learns sparsified pairwise alignments between query and document tokens (e.g. ‘dog’ vs. ‘puppy’) and per-token unary saliences reflecting their relative importance for retrieval. We show that controlling the sparsity of pairwise token alignments often brings significant performance gains. While most factoid questions focusing on a specific part of a document require a smaller number of alignments, others requiring a broader understanding of a document favor a larger number of alignments. Unary saliences, on the other hand, decide whether a token ever needs to be aligned with others for retrieval (e.g. ‘kind’ from ‘what kind of currency is used in new zealand’). With sparsified unary saliences, we are able to prune a large number of query and document token vectors and improve the efficiency of multi-vector retrieval. We learn the sparse unary saliences with entropy-regularized linear programming, which outperforms other methods to achieve sparsity. In a zero-shot setting, ALIGNER scores 51.1 points nDCG@10, achieving a new retriever-only state-of-the-art on 13 tasks in the BEIR benchmark. In addition, adapting pairwise alignments with a few examples (≤ 8) further improves the performance up to 15.7 points nDCG@10 for argument retrieval tasks. The unary saliences of ALIGNER helps us to keep only 20% of the document token representations with minimal performance loss. We further show that our model often produces interpretable alignments and significantly improves its performance when initialized from larger language models.

1 INTRODUCTION

Neural information retrieval (IR) has become a promising research direction for improving traditional IR systems. The most-commonly adopted approach called the dual encoder operates by representing every query and document as a single dense vector. Given sufficient annotations, dual encoders directly learn task-driven similarity between vectors, and often surpass traditional IR systems on complex tasks such as question answering (Lee et al., 2019; Karpukhin et al., 2020; Ni et al., 2021). However, these models can struggle to generalize over out-of-domain datasets (Thakur et al., 2021) and/or entity-centric questions (Sciavolino et al., 2021) due to the limited representational capacity of single vectors. As a remedy, multi-vector retrieval models (Khattab & Zaharia, 2020; Luan et al., 2021; Gao et al., 2021) instead use multiple vectors, typically the contextualized token vectors, to represent the text. These models largely improve the model expressiveness, and exhibit much stronger performance and robustness compared to their single-vector counterparts.

Existing multi-vector retrieval models such as ColBERT (Khattab & Zaharia, 2020) computes query-document similarity by selecting the highest scoring document token for each query token and aggregating the scores. This sum-of-max method has two major limitations. First, restricting the selection to a single document token can be highly sub-optimal for some retrieval tasks. As we will show in our experiments, the retrieval performance can be improved by more than 16 points
nDCG@10 by relaxing this constraint. Second, the method also leads to a large search index and expensive computation. Specifically, the retrieval and storage cost scales linearly with the query and document length, making multi-vector retrieval models an inferior choice for efficiency-demanding applications. We directly tackle these challenges to build faster and more accurate models.

The representation learning problem of multi-vector retrieval can be formulated as optimizing token-level alignment. Specifically, we use a sparse alignment matrix to aggregate token-level similarities, where each element indicates the alignment of a pair of tokens. From this point of view, we are able to formulate different retrieval models in a unified manner (Figure 1) and discern the drawbacks of existing models.

Based on our formulation, we propose ALIGNER, a novel multi-vector retrieval model that consists of pairwise alignment and unary salience. Pairwise alignments form the basis of ALIGNER, where pairs of query and document tokens are sparsely aligned based on their contextual representations. It is discovered that changing the sparsity of alignment can significantly impact the performance on retrieval tasks. For instance, factoid questions often favor a small number of alignments since they often focus on a small part of a document. However, other queries for different tasks (e.g., argument retrieval and fact checking) require a larger number of alignments for a broader understanding of a document. Our findings also support the claim of Dai et al. (2022b) that retrieval tasks with different intents should be modeled differently.

ALIGNER also learns unary saliences, which decides whether each token ever needs to be aligned with any other token for retrieval. This corresponds to masking an entire row or column of the alignment matrix, rather than individual token alignments. To sparsify entire rows or columns, we introduce an algorithm that produces sparse token salience and is end-to-end differentiable based on a novel formulation of entropy-regularized linear programming. Sparsified unary saliences allow us to prune a large number of document and query token representations, making multi-vector retrieval a more efficient and affordable solution.

We evaluate ALIGNER on the BEIR benchmark (Thakur et al., 2021), which covers a diverse set of retrieval tasks in multiple domains. In a zero-shot setting, we show that simply scaling our model achieves the state-of-the-art performance, outperforming prior neural retrievers without contrastive pre-training, model-based hard negative mining, or distillation. By adapting the pairwise alignments with a few examples from the target task — similar to the setup of Dai et al. (2022b) — ALIGNER can be further improved by up to 15.7 points nDCG@10 on argument retrieval tasks. Meanwhile, pruning with our unary saliences can reduce 50% of query tokens for better run-time efficiency and 80% of document tokens for better storage footprint, with less than 1 point decrease of nDCG@10. The pairwise alignments and unary saliences are also highly interpretable so that they often serve as concise rationales for retrieval.

2 Multi-Vector Retrieval as Sparse Alignment

Given a query $Q$ and a collection of $N$ documents $\mathcal{C} = \{D^{(1)}, \ldots, D^{(N)}\}$, a key problem in retrieval is how to represent these textual inputs in order to facilitate efficient search. To this end, one approach is lexical retrieval using sparse bag-of-words representation of the text; the other approach is dense retrieval, which this work focuses on. Dense retrieval models learn a parameterized function that encodes the query and documents into query representation $q$ and document representations

1We will release our model checkpoints to encourage future research.
\{d^{(1)}, \ldots, d^{(N)}\}$ respectively. Typically, each representation is a single $d$-dimensional vector. For retrieval, the similarity function is often defined as $\text{sim}(Q, D^{(i)}) = q^\top d^{(i)}$, and documents having high similarity scores to the query are retrieved.

2.1 Multi-vector Retrieval

Instead of representing each query and document as a single fixed-length vector, multi-vector retrieval represents them with multiple token vectors, mainly to improve the limited capacity of fixed-length representations. Specifically, a query $Q = \{q_1, \ldots, q_n\}$ and a document $D = \{d_1, \ldots, d_m\}$ are encoded into a set of vectors $\{q_1, \ldots, q_n\}$ and $\{d_1, \ldots, d_m\}$. The similarity function between a query and a document is re-defined for multi-vector retrieval. For instance, ColBERT (Khattab & Zaharia, 2020) designs the similarity function as follows:

$$\text{sim}(Q, D) = \sum_{i=1}^{n} \max_{j=1 \ldots m} q_i^\top d_j.$$ 

For retrieval, instead of indexing $N$ document vectors, multi-vector retrieval pre-computes $N \times \bar{m}$ document token vectors where $\bar{m}$ is the average length of documents. Then, it retrieves $K$ document token vectors for each query token vector with Maximum Inner-Product Search (MIPS), resulting in $n \times K$ candidate document tokens. The retrieved tokens are used to trace back the original documents (Lee et al., 2021), often followed by a final refinement stage that scores the similarity $\text{sim}(Q, D)$ with all token representations of each document and the query (Khattab & Zaharia, 2020).

We adopt the same practice of ColBERT in our experiments.

2.2 Sparse Alignment Formulation

A key design question for retrieval models is defining the similarity function in a manner that balances model expressiveness and inference cost. To facilitate our discussion, we formalize the similarities used in previous methods into a class of sparse alignment functions. The formulation also leads to a principled extension over existing work, which we will describe in §.

We begin by defining a similarity matrix $S \in \mathbb{R}^{n \times m}$ computed from all pairs of query and document tokens, where $S_{i,j} = q_i^\top d_j$. Then, we use an alignment matrix $A \in [0, 1]^{n \times m}$ to compute the similarity between $Q$ and $D$ as follows:

$$\text{sim}(Q, D) = \frac{1}{Z} \sum_{i=1}^{n} \sum_{j=1}^{m} S_{i,j} A_{i,j}$$

(1)

where $Z$ is a normalization term defined as $Z = \sum_{i,j} A_{i,j}$. The alignment matrix $A$ can be directly derived from $S$ or computed as a function of $Q$ and $D$.

On the top of our formulation, the alignment matrix $A$ is constrained to be sparsely activated: $||A||_0 \leq \sigma$ where $||.||_0$ is the number of non-zero elements in a matrix. Sparse activation assumes that only a few query-document token matches are critical for retrieval, inspired by traditional retrieval methods. Indeed, most existing dense retrieval models already enforce the sparse alignment with their own heuristics. Figure illustrates how different models can be described under our formulation:

- **Dense passage retriever** (DPR; Karpukhin et al. 2020) uses a single [CLS] vector to represent each query and document. This is equivalent to setting $A_{1,1} = 1$ and 0 otherwise, resulting in $||A||_0 = 1$.
- **ME-BERT** (Luan et al. 2021) uses the first $k$ document token vectors for multi-vector representations of documents but a single vector for query. The similarity function is $\max_{j=1 \ldots k} q_i^\top d_j$, which is equivalent to setting $A_{1,j} = 1$ when $S_{i,j}$ is the maximum within $S_{i,1}$ to $S_{i,k}$, and 0 otherwise. The alignment sparsity is $||A||_0 = 1$.
- **ColBERT** uses the sum-of-max similarity function $\sum_{i=1}^{n} \max_{j=1 \ldots m} S_{i,j}$ that is equivalent to setting an alignment matrix to select the maximum element from each row of $S$, i.e., $A_{i,1} = 1$ when $S_{i,j}$ is the maximum within $S_{i,1 \ldots m}$. $||A||_0 = n$ in this case.
- **COIL** (Gao et al. 2021), similar to ColBERT, also selects the maximum element from each row of $S$, but requires a lexical exact match for a selected pair, i.e., $A_{i,j} = 1$ when $S_{i,j}$ is the maximum within $\{S_{i,1 \ldots m} | q_i = d_j\}$. $||A||_0 \leq n$ in this case.
Figure 2: ALIGNER factorizes the alignment matrix into pairwise alignments and unary saliences. Pairwise alignment focuses on the alignment of individual token pairs. Unary saliences are determined by per-token salience features.

The choice of similarity and sparsity can have a large impact on model capacity and efficiency. For instance, ColBERT is more expressive and robust than DPR (Thakur et al., 2021), but its retrieval and storage costs are much higher. Our work seeks to further advance expressiveness while retaining a strong efficiency. We describe our method in the next section.

3 ALIGNER

In this section, we present ALIGNER built upon the sparse alignment formulation. ALIGNER factorizes the alignment matrix into pairwise alignment and unary salience:

$$A = \tilde{A} \odot (u^q \otimes u^d)$$

(2)

where $\odot$ is the Hadamard product and $\otimes$ is the outer product of two vectors. Pairwise alignment $\tilde{A} \in \mathbb{R}^{n \times m}$ determines which pairs of query and document tokens should be aligned, with the sparsity constraints tailored for downstream tasks (§3.1). Unary salience $u^q \in \mathbb{R}^n$ and $u^d \in \mathbb{R}^m$ are sparse token weights deciding whether a token ever needs to be aligned (§3.2).

The factorization is introduced based on two critical hypotheses. First, the optimal sparsity of alignment can be task-dependent. Instead of imposing top-1 constraint as in ColBERT, activating more than one alignments for a query token can enhance retrieval performance for certain tasks. In our analyses for instance, we observe factoid questions that only concern a specific part of a document require a small number of alignments, while some other queries (such as fact checking) require more alignments for a broader understanding of the document. We explore different search spaces of the pairwise alignment matrix $\tilde{A}$ in order to achieve better retrieval performance for each downstream task. Second, alignment is only needed for very few tokens. For example, we analyzed 2000 most retrieved documents in our preliminary study, and found only 12.8% document tokens are retrieved by at least one query.\(^2\)

Intuitively, tokens that are uninformative do not need to be aligned and stored, corresponding to sparse activation over an entire row or column of $A$. ALIGNER directly learns the row and column sparsity as unary salience, and utilizes them to enhance retrieval efficiency.

3.1 ADAPTING PAIRWISE ALIGNMENT

Queries and documents can have varied distributions. For example, a query can be a single entity, a natural question, or a few sentences, and a document can range from a short paragraph to a long article. The search intent also changes from task to task (Dai et al., 2022b). These changes can lead to different optimal alignment strategies. We explore the following sparse alignment variants that go beyond the top-1 strategy commonly adopted in existing work:

- **Top-k.** Each query token is aligned with $k$ document tokens with highest similarity scores. Precisely, $\tilde{A}_{i,j} = 1$ when the $j$-th token is within top-$k$ of the row $S_i$. When $k = 1$, it is equivalent to ColBERT.
- **Top-p.** This strategy is similar to top-$k$, but instead of aligning each query token with exactly $k$ tokens, it makes the number of alignments proportional to the document length, i.e., each query token aligns with $\max(\lfloor p \cdot m \rfloor, 1)$ tokens where $m$ is the document length and $p \in [0, 1]$ is the alignment ratio.

\(^2\)The analysis was performed on MS MARCO (Nguyen et al., 2016) using our implementation of ColBERT.
Despite their simplicity, these variants can indeed enhance retrieval accuracy significantly on tasks such as argument retrieval. More importantly, while it is possible to train separate models for different alignment variants, we are interested in fast test-time adaptation using a single shared model as many important retrieval tasks lack sufficient training data [Thakur et al. 2021]. Specifically, we first train ALIGNER using a fixed alignment strategy such as top-1 in a source domain, and adapt the alignment strategy to each target task without changing the model parameters.3 We use the following few-shot alignment adaptation method. Given a corpus \{\{D^{(1)}, \ldots, D^{(N)}\}\}, and a few relevance-annotated query-document pairs from the target task \{(Q^1, D^1), \ldots, (Q^K, D^K)\}, we first retrieve candidate documents with the learned token representations, and decide the pairwise alignment strategy based on the ranking performance on the annotated data. This adaptation can be performed efficiently because the alignment only concerns the computation of similarity score (Eq. 1) in the refinement stage. In practice, for some tasks, we are able to find a well-suited alignment strategy and improve the retrieval performance with as few as 8 annotated examples.

### 3.2 Learning Unary Salience

ALIGNER predicts token saliences from their token representations. For brevity, we only present the formulation for document salience, and query salience is defined similarly. Specifically, the salience of the i-th document token \(u^d_i\) is defined as:

\[
u^d_i = \lambda^d_i \cdot f(W^d d_i + b^d)
\]

where \(W^d\) and \(b^d\) are learnable parameters. \(f\) is a non-linear activation function and we use ReLU such that salience is always non-negative. \(\lambda^d = \{\lambda^d_i\}\) are gating variables to control the overall sparsity of \(u^d\), which we will elaborate next.

For the document salience to be meaningful, we enforce salience sparsity as an inductive bias. ALIGNER jointly optimizes sparse salience with other parts of the model. Since tokens with zero salience do not contribute to computing similarity, our model will be encouraged to identify more salient parts of our model. The full derivation of this iterative algorithm is given in Appendix A.1.

In practice, it is sufficient to run only a few iterations and the final solution is given by \(\lambda_i = \exp(\frac{d_i^{\alpha} + b_i^\alpha}{\epsilon})\). These vector operations are differentiable so \(\Lambda\) can be end-to-end trained with other parts of our model. The full derivation of this iterative algorithm is given in Appendix A.1.

---

3We have also explored a differentiable alignment with sparsity constraints (Appendix A.2), but alignment adaptation is still necessary to achieve good performance on target tasks.

\(H(\lambda) = \sum_{i=1}^{m} -\lambda_i \log \lambda_i\)
Table 1: Comparison of different retrieval models. *: optionally used for alignment adaptation.

| Supervision | Hard Negatives | Distillation | Retriever | Per-domain | # Param. |
|-------------|----------------|--------------|-----------|------------|----------|
| Splade\textsubscript{v2} | MS MARCO | model-based | lexical | 110M |
| ColBERT\textsubscript{v2} | MS MARCO | model-based | multi-vector | 110M |
| GTR\textsubscript{xxl} | Pre-train + MS MARCO | fixed | single-vector | 6B |
| PROMPTAGATOR | Few (≤ 8) | single-vector | ≤ | 110M |
| ALIGNER\textsubscript{xxl} | MS MARCO + Few* (≤ 8) | fixed | multi-vector | 6B |

**Pruning Multi-vector Retrieval**

With the learned unary salience, we can naturally prune tokens for multi-vector retrieval. Pruning document tokens reduces the number of vectors in search index, and pruning query tokens reduces the number of searches. In our experiments, we control them using two pruning ratios $\beta_d$ and $\beta_q$ respectively. For each document, we obtain the token salience using Eq. (3) and only store the top $\beta_d$ percent of tokens in the index. Similarly we select the top $\beta_q$ percent query tokens to perform max inner-product search. Note that we vary these two ratios to control retrieval efficiency, and these ratios can be smaller than the sparsity ratio $\alpha_d$ and $\alpha_q$ which we use as constraints at training time. In the refinement stage, we still use the full model with all token vectors for scoring.

4 Experiments

4.1 Experimental Setup

ALIGNER uses shared transformer encoder initialized from T5 version 1.1 (Raffel et al., 2020). We project token embeddings to 128 dimension and apply L2 normalization. Following GTR (Ni et al., 2021), we finetune ALIGNER on MS MARCO with hard negatives released by RocketQA (Qu et al., 2021). The models are trained with a batch size of 256 for 25k steps, using query sequence length of 64 and document sequence length of 256. We train ALIGNER with top-1 pairwise alignment.

For retrieval, we pre-compute the token encodings of all the documents in the corpus, and use ScaNN (Guo et al., 2020) to index and perform max inner-product search (MIPS). We retrieve 4,000 nearest neighbors for each query token, and return the top-1,000 after the refinement stage. We evaluate ALIGNER on the BEIR benchmark (Thakur et al., 2021) and compare with state-of-the-art retrieval models shown in Table 1. Note that ALIGNER does not rely on contrastive model pre-training (Izacard et al., 2022; Ni et al., 2021), model-based hard negative mining (Santhanam et al., 2021), or distillation (Santhanam et al., 2021). We intentionally decide this simple recipe and focus on studying the impact of pairwise alignment and unary salience.

For few-shot alignment adaptation of ALIGNER (§3.1), we split the test data into multiple folds such that each fold contains 8 examples. Then we find the best alignment strategy that maximizes nDCG@10 on each fold with $k \in \{1, 2, 4, 6, 8\}$ for top-$k$ and $p \in \{0.5\%, 1\%, 1.5\%, 2\%\}$ for top-$p$. Based on the best alignment strategy from each fold, we measure the retrieval performance on the remaining test examples with the best strategy. We report the average (± std.) of these test scores where the number of test scores equals the number of folds. The average of few-shot adaptation indicates the expected performance of using few examples to choose the best alignment strategy.

4.2 Retrieval Accuracy

Table 2 shows the document retrieval performance of ALIGNER on both MS MARCO and the BEIR benchmark. For this experiment, we do not prune any query or document tokens with unary saliences, but show their effects in §4.3 instead. ALIGNER\textsubscript{xxl} outperforms all baselines on MS-MARCO, showing how multi-vector retrieval models can benefit from large pretrained language models. ALIGNER\textsubscript{xxl} also outperforms GTR\textsubscript{xxl} on 9 out of 13 BEIR datasets and advances the retriever-only state-of-the-art (CoIBERT\textsubscript{v2}) by 1.2 points nDCG@10 on average. Figure 3 shows that our multi-vector retriever model scales better than single-vector dual encoder GTR.

---

5 We have trained models with other top-$k$ and top-$p$ pairwise alignments, but the MS MARCO training data favors top-1 alignment (see Appendix A.4 for details).

6 Unlike CoIBERT, ALIGNER does not use pad token embeddings for retrieval. Hence, retrieving 4,000 neighbors per query token results in a similar number of retrieved candidates to CoIBERT.
Steering towards optimal task-specific adaptation.

In the rightmost column of Table 2, we show the effect of adapting pairwise alignment with ALIGNER on the BEIR benchmark. With only 8 examples for finding the proper alignment sparsity, its expected performance reaches 52.6 nDCG@10 on average. Alignment-adapted ALIGNER also benefits from scaling up, and consistently outperforms its non-adapted counterparts, as shown in Figure 3. The gains are further explained in Table 3 where we show individual task’s performance under various alignment strategies. Although ALIGNER is trained with top-1 alignment, top-1 is not always the best strategy at inference time. Specifically, for ArguAna, we observe 16 points improvement by adjusting the number of alignments proportional to the document length with \( p = 1.5\% \). Other tasks such as Touché-2020 also prefer other alignment strategies, which shows that different tasks might require different sparsity. In general, keeping the sparsity low enough is preferable and supports our hypothesis that pairwise alignments should be sparse.

We further check whether this observation holds when ALIGNER is trained with other pairwise alignment strategies. Figure 4 shows ALIGNER variants trained on four alternative strategies. We evaluate their performance with training-time alignment strategy (default) and the optimal alignment strategy selected per dataset (oracle). While these models perform differently with their default alignments, they perform similarly after oracle alignment adaptation.

Figure 5 shows the effectiveness of few-shot alignment adaptation — dynamically selecting task-specific alignment strategy based on a few examples. When the default alignment (\( k = 1 \)) is not optimal, we can identify a good alignment strategy using only 8 examples, which significantly improves model performance on argument retrieval tasks. Using 16 examples further improves the average score and reduces the variance. However, when the default alignment is already optimal, it only benefits from scaling up, and consistently outperforms its non-adapted counterpart. For other tasks such as Touché-2020, other alignment strategies might be preferred.

Table 2: Results on MS MARCO (top; MRR@10) and the BEIR benchmark (bottom; nDCG@10). Best scores before adaptation are denoted in boldface. *: trained with distillation. \( R \): Promptagator (Dai et al., 2022b). †: uses few examples (\( \leq 8 \)) for task-specific adaptation.

Alignment Adaptation In the rightmost column of Table 2 we show the effect of adapting pairwise alignment with ALIGNER on the BEIR benchmark. With only 8 examples for finding the proper alignment sparsity, its expected performance reaches 52.6 nDCG@10 on average. Alignment-adapted ALIGNER also benefits from scaling up, and consistently outperforms its non-adapted counterparts, as shown in Figure 3. The gains are further explained in Table 3 where we show individual task’s performance under various alignment strategies. Although ALIGNER is trained with top-1 alignment, top-1 is not always the best strategy at inference time. Specifically, for ArguAna, we observe 16 points improvement by adjusting the number of alignments proportional to the document length with \( p = 1.5\% \). Other tasks such as Touché-2020 also prefer other alignment strategies, which shows that different tasks might require different sparsity. In general, keeping the sparsity low enough is preferable and supports our hypothesis that pairwise alignments should be sparse.

We further check whether this observation holds when ALIGNER is trained with other pairwise alignment strategies. Figure 4 shows ALIGNER variants trained on four alternative strategies. We evaluate their performance with training-time alignment strategy (default) and the optimal alignment strategy selected per dataset (oracle). While these models perform differently with their default alignments, they perform similarly after oracle alignment adaptation.

Figure 5 shows the effectiveness of few-shot alignment adaptation — dynamically selecting task-specific alignment strategy based on a few examples. When the default alignment (\( k = 1 \)) is not optimal, we can identify a good alignment strategy using only 8 examples, which significantly improves model performance on argument retrieval tasks. Using 16 examples further improves the average score and reduces the variance. However, when the default alignment is already optimal.
(top-k=1 is optimal for QA tasks), few-shot alignment adaptation hurts performance due to the variance of our few-shot method. Nevertheless, ALIGNER outperforms Promptagator (Dai et al., 2022b), another few-shot retrieval baseline, in 6 out of 11 datasets.

### 4.3 Retrieval Efficiency

The next experiment shows how ALIGNER’s unary salience impacts retrieval efficiency. We train ALIGNERbase with salience sparsity ratios $\alpha^u = 50\%$ and $\alpha^d = 40\%$ based on empirical performance. The gating variables are optimized with $\epsilon = 0.002$. At retrieval time, we prune query and document tokens with ratios $\beta^u$ and $\beta^d$ ($\S$3.2).

Figure 6 shows the ALIGNER performance on MS MARCO with various pruning ratios. When pruned at the same ratio as training ($\beta^u = 50\%$ and $\beta^d = 40\%$), the model is close to a full ALIGNER model without pruning (MRR@10 38.1 vs. 38.8), but greatly saves the computation cost. We can further prune tokens by adjusting $\beta^u$ and $\beta^d$. The model achieves 37.3 MRR@10 with is $\beta^d = 10\%$, i.e. it remains accurate with only 10% of the original index size. Decreasing the query pruning ratio $\beta^u$ to 30% does not sacrifice performance too much, although decreasing $\beta^u$ to 10% leads to worse performance. Figure 6 also compares ALIGNER’s entropy-regularized linear program (Eq. 4) with alternative methods. With just a ReLU gate and no sparsity constraints (‘ReLU’ in Figure 6), the model performance retains good when $\beta^u = 40\%$, but drops significantly for smaller $\beta^d$. Removing the entropy regularization in Eq. 4 leads to simply selecting the hard top-k tokens with the highest predicted salience (‘Hard’ in Figure 6). The hard top-k solution has worse performance for all $\beta^d$.

ALIGNER’s salience estimation also generalizes to other retrieval datasets. As shown in Figure 7, pruning with $\beta^u = 10\%$ with $\beta^d = 50\%$ causes minimal performance decrease for a majority of BEIR datasets. We even observe performance increase for Touché-2020, as the model can only retrieve salient tokens after pruning. Besides, we show that alignment adaptation can be combined with pruning, resulting in an effective yet efficient retrieval model.
what happens when stop drinking alcohol

alcohol : symptoms of alcohol withdrawal may begin from 4 to 12 hours after you cut down or stop drinking or as long as several days after the last drink, and can last a few days. they can range from mild to life-threatening. 1 mild withdrawal symptoms may include: 2 intense worry, 3 nausea or vomiting.

where is the heart in the human body

heart the heart is a muscular organ in most animals, which pumps blood through the blood vessels of the circulatory system. blood provides the body with oxygen and nutrients, as well as assists in the removal of metabolic waste s. in humans, the heart is located between the lungs, in the middle compartment of the chest.

Table 4: Examples of the pairwise alignment and unary salience learned by ALIGNER. Three most salient query tokens and their top-1 pairwise-aligned document tokens are indicated with the same color. We highlight top 50% query tokens and 20% document tokens according to their salience.

4.4 INTERPRETABILITY

Table 4 shows examples of the pairwise alignment and unary salience learned by ALIGNER. The model aligns query tokens to contextually similar tokens, but not necessarily identical tokens. The salience features are also highlighted in Table 4. Important noun phrases and verbs are usually assigned higher salience, which is consistent with human intuition. We show more examples of alignments for different tasks in the Appendix A.3. In general, we observe question answering tasks usually require fewer alignments for each query token, while other tasks that require a broad understanding of the document favor larger number of alignments.

5 RELATED WORK

Recent research on information retrieval often improves the retrieval accuracy with contrastive pre-training (Ni et al., 2021, Izacard et al., 2022, Oguz et al., 2022), model-based hard negative mining (Xiong et al., 2020, Lu et al., 2021, Qu et al., 2021), and knowledge distillation (Santhanam et al., 2021, Zhang et al., 2022, Reddi et al., 2021). Retrieval efficiency is improved via quantization (Santhanam et al., 2021) or lower-dimensional vectors (Hofstatter et al., 2022). These improvements are orthogonal to this work.

Term importance and salience have a long history in information retrieval: from term frequency (tf) and inverse document frequency (idf), to recent BERT-based importance measures such as DeepCT (Dai & Callan, 2020), SPARTA (Zhao et al., 2021), and Splade (Formal et al., 2021a,b). These works mostly focus on sparse lexical retrieval and learn term weights for sparse bag-of-words representations. Term importance in multi-vector dense retrieval is less explored. Our work is probably most related to a recent work from Hofstatter et al. (2022), which prunes ColBERT by predicting salience scores from a word’s embedding with a ReLU gate and L1-norm regularization.

Recently, Promptagator (Dai et al., 2022b) points out the importance of using a few annotated examples to adapt to a new retrieval task. Promptagator achieves few-shot task adaptation via query generation (Ma et al., 2021, Lee et al., 2021a, Dai et al., 2022a) using large language models (Sanh et al., 2022, Brown et al., 2020, Wei et al., 2022), which has high inference cost. ALIGNER is more versatile and can be fast adapted to a new task via few-shot alignment adaptation.

6 CONCLUSION

In this paper, we introduce ALIGNER, a novel sparse alignment method for multi-vector document retrieval. We first formulate different retrieval models with token-level sparse alignments and propose ALIGNER to tackle the limitations of existing models. Specifically, ALIGNER uses pairwise alignments and unary saliences that allow us to adapt to different tasks and prune unimportant tokens, respectively. As a result, we achieve strong performance on both zero-shot and few-shot document retrieval tasks while drastically improving the run-time and storage complexity of multi-vector retrieval. With its interpretable alignments and better performance with large language models, we envision that our multi-vector retrieval model can serve as a strong standalone retriever in the future.
7 Author Contributions

Yujie Qian and Vincent Y. Zhao are the leading authors of this work who designed the model and experiments. All the authors have contributed to the code, experiments, and paper writing. Sai Meher Karthik Duddu conducted the interpretability analysis. Jinhyuk Lee, Zhuyun Dai, Iftekhar Naim, and Tao Lei advised on the research direction. Tao Lei and Siddhartha Brahma proposed the algorithm for entropy-regularized linear programming. Vincent initiated the project and proposed the modeling framework. Vincent and Tao were the co-hosts of Yujie’s internship.
REFERENCES

Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel Ziegler, Jeffrey Wu, Clemens Winter, Chris Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language models are few-shot learners. In H. Larochelle, M. Ranzato, R. Hadsell, M.F. Balcan, and H. Lin (eds.), Advances in Neural Information Processing Systems, volume 33, pp. 1877–1901. Curran Associates, Inc., 2020.

Marco Cuturi. Sinkhorn distances: Lightspeed computation of optimal transport. Advances in neural information processing systems, 26, 2013.

Zhuyun Dai and Jamie Callan. Context-aware term weighting for first-stage passage retrieval. In Proceedings of the 42nd International ACM SIGIR Conference on Research and Development in Information Retrieval, 2020.

Zhuyun Dai, Arun Tejasvi Chaganty, Vincent Y. Zhao, Aida Amini, Qazi Mamunur Rashid, Mike Green, and Kelvin Guu. Dialog inpainting: Turning documents into dialogs. In Kamalika Chaudhuri, Stefanie Jegelka, Le Song, Csaba Szepesvári, Gang Niu, and Sivan Sabato (eds.), International Conference on Machine Learning, ICML 2022, 17-23 July 2022, Baltimore, Maryland, USA, volume 162 of Proceedings of Machine Learning Research, pp. 4558–4586. PMLR, 2022a.

Zhuyun Dai, Vincent Y. Zhao, Ji Ma, Yi Luan, Jianmo Ni, Jing Lu, Anton Bakalov, Kelvin Guu, Keith B. Hall, and Ming-Wei Chang. Promptagator: Few-shot dense retrieval from 8 examples. arXiv preprint arXiv:2209.11755, 2022b.

Thibault Formal, Carlos Lassance, Benjamin Piwowarski, and Stéphane Clinchant. SPLADE v2: Sparse lexical and expansion model for information retrieval. CoRR, abs/2109.10086, 2021a.

Thibault Formal, Benjamin Piwowarski, and Stéphane Clinchant. SPLADE: sparse lexical and expansion model for first stage ranking. In Fernando Diaz, Chirag Shah, Torsten Suel, Pablo Castells, Rosie Jones, and Tetsuya Sakai (eds.), SIGIR ’21: The 44th International ACM SIGIR Conference on Research and Development in Information Retrieval, Virtual Event, Canada, July 11-15, 2021, pp. 2288–2292. ACM, 2021b. doi: 10.1145/3404835.3463098.

Luyu Gao, Zhuyun Dai, and Jamie Callan. Coil: Revisit exact lexical match in information retrieval with contextualized inverted list. In Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, 2021.

Ruiqi Guo, Philip Sun, Erik Lindgren, Quan Geng, David Simcha, Felix Chern, and Sanjiv Kumar. Accelerating large-scale inference with anisotropic vector quantization. In International Conference on Machine Learning, pp. 3887–3896. PMLR, 2020.

Sebastian Hofstätter, Omar Khattab, Sophia Althammer, Mete Sertkan, and Allan Hanbury. Introducing neural bag of whole-words with colberter: Contextualized late interactions using enhanced reduction. arXiv preprint arXiv:2203.13088, 2022.

Gautier Izacard, Mathilde Caron, Lucas Hosseini, Sebastian Riedel, Piotr Bojanowski, Armand Joulin, and Edouard Grave. Unsupervised dense information retrieval with contrastive learning. Transactions on Machine Learning Research, 2022.

Vladimir Karpukhin, Barlas Oguz, Sewon Min, Patrick Lewis, Ledell Wu, Sergey Edunov, Danqi Chen, and Wen-tau Yih. Dense passage retrieval for open-domain question answering. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP), pp. 6769–6781, 2020.

Omar Khattab and Matei Zaharia. Colbert: Efficient and effective passage search via contextualized late interaction over bert. In Proceedings of the 43rd International ACM SIGIR conference on research and development in Information Retrieval, pp. 39–48, 2020.

Harold W Kuhn and Albert W Tucker. Nonlinear programming. In Traces and emergence of nonlinear programming, pp. 247–258. Springer, 2014.
Jinhyuk Lee, Alexander Wettig, and Danqi Chen. Phrase retrieval learns passage retrieval, too. In *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pp. 3661–3672, 2021a.

Kenton Lee, Ming-Wei Chang, and Kristina Toutanova. Latent retrieval for weakly supervised open domain question answering. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pp. 6086–6096, 2019.

Kenton Lee, Kelvin Guu, Luheng He, Tim Dozat, and Hyung Won Chung. Neural data augmentation via example extrapolation. *CoRR*, abs/2102.01335, 2021b.

Jing Lu, Gustavo Hernandez Abrego, Ji Ma, Jianmo Ni, and Yinfei Yang. Multi-stage training with improved negative contrast for neural passage retrieval. In *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pp. 6091–6103, Online and Punta Cana, Dominican Republic, November 2021. Association for Computational Linguistics. doi: 10.18653/v1/2021.emnlp-main.492.

Yi Luan, Jacob Eisenstein, Kristina Toutanova, and Michael Collins. Sparse, dense, and attentional representations for text retrieval. *Transactions of the Association for Computational Linguistics*, 9:329–345, 2021.

Ji Ma, Ivan Korotkov, Yinfei Yang, Keith B. Hall, and Ryan T. McDonald. Zero-shot neural passage retrieval via domain-targeted synthetic question generation. In Paola Merlo, Jörg Tiedemann, and Reut Tsarfaty (eds.), *Proceedings of the 16th Conference of the European Chapter of the Association for Computational Linguistics: Main Volume*, EACL 2021, Online, April 19 - 23, 2021, pp. 1075–1088. Association for Computational Linguistics, 2021. doi: 10.18653/v1/2021.eacl-main.92.

Tri Nguyen, Mir Rosenberg, Xia Song, Jianfeng Gao, Saurabh Tiwary, Rangan Majumder, and Li Deng. MS MARCO: A human generated machine reading comprehension dataset. In Tarek Richard Besold, Antoine Bordes, Artur S. d’Avila Garcez, and Greg Wayne (eds.), *Proceedings of the Workshop on Cognitive Computation: Integrating neural and symbolic approaches 2016 co-located with the 30th Annual Conference on Neural Information Processing Systems (NIPS 2016)*, Barcelona, Spain, December 9, 2016, volume 1773 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2016. URL http://ceur-ws.org/Vol-1773/CoCoNIPS_2016_paper9.pdf.

Jianmo Ni, Chen Qu, Jing Lu, Zhuyun Dai, Gustavo Hernández Ábrego, Ji Ma, Vincent Y Zhao, Yi Luan, Keith B Hall, Ming-Wei Chang, et al. Large dual encoders are generalizable retrievers. arXiv preprint arXiv:2112.07899, 2021.

Barlas Oguz, Kushal Lakhota, Anchit Gupta, Patrick Lewis, Vladimir Karpukhin, Aleksandra Piktus, Xilun Chen, Sebastian Riedel, Scott Yih, Sonal Gupta, and Yashar Mehdad. Domain-matched pre-training tasks for dense retrieval. In *Findings of the Association for Computational Linguistics: NAACL 2022*, pp. 1524–1534, Seattle, United States, July 2022. Association for Computational Linguistics. doi: 10.18653/v1/2022.findings-naacl.114.

Yingqi Qu, Yuchen Ding, Jing Liu, Kai Liu, Ruiyang Ren, Wayne Xin Zhao, Daxiang Dong, Hua Wu, and Haifeng Wang. Rocketqa: An optimized training approach to dense passage retrieval for open-domain question answering. In *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pp. 5835–5847, 2021.

Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J Liu. Exploring the limits of transfer learning with a unified text-to-text transformer. *Journal of Machine Learning Research*, 21:1–67, 2020.

Sashank J. Reddi, Rama Kumar Pasumarthi, Aditya Krishna Menon, Ankit Singh Rawat, Felix X. Yu, Seunghyeon Kim, Andreas Veit, and Sanjiv Kumar. Rankdistil: Knowledge distillation for ranking. In *AISTATS*, pp. 2368–2376, 2021. URL http://proceedings.mlr.press/v130/reddi21a.html.
Victor Sanh, Albert Webson, Colin Raffel, Stephen Bach, Lintang Sutawika, Zaid Alyafeai, Antoine Chaffin, Arnaud Stiegler, Arun Raja, Manan Dey, M Saiful Bari, Canwen Xu, Urmish Thakker, Shanya Sharma Sharma, Eliza Szczechla, Taewoon Kim, Gunjan Chhablani, Nihal V. Nayak, Debajyoti Datta, Jonathan Chang, Mike Tian-Jian Jiang, Han Wang, Matteo Manica, Sheng Shen, Zheng Xin Yong, Harshit Pandey, Rachel Bawden, Thomas Wang, Trishala Neeraj, Jos Rozen, Abheesht Sharma, Andrea Santilli, Thibault Févry, Jason Alan Fries, Ryan Teehan, Teven Le Scao, Stella Biderman, Leo Gao, Thomas Wolf, and Alexander M. Rush. Multitask prompted training enables zero-shot task generalization. In The Tenth International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29, 2022. OpenReview.net, 2022. URL https://openreview.net/forum?id=9Vrb9D0WI4

Keshav Santhanam, Omar Khattab, Jon Saad-Falcon, Christopher Potts, and Matei Zaharia. Colbertv2: Effective and efficient retrieval via lightweight late interaction. arXiv preprint arXiv:2112.01488, 2021.

Christopher Sciavolino, Zexuan Zhong, Jinhuk Lee, and Danqi Chen. Simple entity-centric questions challenge dense retrievers. In Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing, pp. 6138–6148, 2021.

Nandan Thakur, Nils Reimers, Andreas Rücklé, Abhishek Srivastava, and Iryna Gurevych. Beir: A heterogeneous benchmark for zero-shot evaluation of information retrieval models. In Thirty-fifth Conference on Neural Information Processing Systems Datasets and Benchmarks Track (Round 2), 2021.

Jason Wei, Maarten Paul Bosma, Vincent Zhao, Kelvin Guu, Adams Wei Yu, Brian Lester, Nan Du, Andrew Mingbo Dai, and Quoc V. Le. Finetuned language models are zero-shot learners. In International Conference on Learning Representations, 2022.

Stephen J Wright. Coordinate descent algorithms. Mathematical Programming, 151(1):3–34, 2015.

Lee Xiong, Chenyan Xiong, Ye Li, Kwok-Fung Tang, Jialin Liu, Paul N Bennett, Junaid Ahmed, and Arnold Overwijk. Approximate nearest neighbor negative contrastive learning for dense text retrieval. In International Conference on Learning Representations, 2020.

Hang Zhang, Yeyun Gong, Yelong Shen, Jiancheng Lv, Nan Duan, and Weizhu Chen. Adversarial retriever-ranker for dense text retrieval. In International Conference on Learning Representations, 2022.

Tiancheng Zhao, Xiaopeng Lu, and Kyusong Lee. SPARTA: efficient open-domain question answering via sparse transformer matching retrieval. In Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, NAACL-HLT, pp. 565–575, 2021.
AAPPENDIX

A.1 DERIVATION OF THE ITERATIVE UPDATES

We present the derivation of Eq. 5 for solving optimization problem (4) in Section 3.2. The maximization problem (4) can be written as an equivalent minimization problem:

$$\max_{\lambda} \ s^\top \lambda + \epsilon H(\lambda)$$

$$\iff \min_{\lambda} \ -s^\top \lambda - \epsilon H(\lambda)$$

$$\iff \min_{\lambda} \ -s^\top \lambda - \epsilon H(\lambda) - \epsilon 1^\top \lambda$$

s.t. $1^\top \lambda = k$, $\lambda_i \in [0, 1], \ i = 1, \ldots, m.$

Note the term $\epsilon 1^\top \lambda$ will be a constant $\epsilon \times k$, but we include it in the minimization object to make our derivation simpler later.

Now, let $a \in \mathbb{R}$ and $b \in \mathbb{R}^m$ be the Lagrangian variables corresponding to the linear constraints $1^\top \lambda = k$ and $\lambda_i \leq 1 \ \forall i$. The minimization problem is equivalent to its Lagrangian expression:

$$\min_{\lambda \in \mathbb{R}^m} \max_{a \in \mathbb{R}, b \leq 0} \ -s^\top \lambda - \epsilon H(\lambda) - \epsilon 1^\top \lambda + a(k - 1^\top \lambda) + b^\top (1 - \lambda) \quad (7)$$

The objective function (7) is strongly convex and the solution space of $\lambda$ is a convex set. As a result, strong duality holds and we can instead solve the dual problem that exchanges the min and max operators in (7):

$$\max_{a \in \mathbb{R}, b \leq 0} \min_{\lambda \in \mathbb{R}^m} \ -s^\top \lambda - \epsilon H(\lambda) - \epsilon 1^\top \lambda + a(k - 1^\top \lambda) + b^\top (1 - \lambda) \quad (8)$$

The optimal solution $(a, b, \lambda)$ must have the Karush-Kuhn-Tucker (KKT) conditions hold (Kuhn & Tucker, 2014), namely

$$\frac{\partial \left( -s^\top \lambda - \epsilon H(\lambda) + \epsilon 1^\top \lambda + a(k - 1^\top \lambda) + b^\top (1 - \lambda) \right)}{\partial \lambda} = 0$$

$$\iff \lambda = \exp \left( \frac{s + a + b}{\epsilon} \right) \iff \lambda_i = \exp \left( \frac{s_i + a_i + b_i}{\epsilon} \right) \ \forall i = 1, \ldots, m$$

Substituting $\lambda$ using the above equation in (8), the dual problem now has a simple form:

$$\max_{a \in \mathbb{R}, b \leq 0} k \cdot a + 1^\top b - 1^\top \exp \left( \frac{s + a + b}{\epsilon} \right)$$

We can solve this problem using coordinate descent (Wright, 2015) by successively maximizing the function with either $a$ or $b$ fixed. This leads to the iterative updates (Eq. 5) described in Section 3.2

$$a_i' = \epsilon \ln(k) - \epsilon \ln \left( \sum_i \exp \left( \frac{s_i + b_i}{\epsilon} \right) \right)$$

$$b_i' = \min(-s_i - a_i', 0)$$

Discussion In short, we solve the dual problem of optimization (4) by performing coordinate descent of the dual variables $a$ and $b$. That is, we find the optimal $a$ that maximizes the dual objective given a fixed $b$, and vice versa.

This iterative algorithm is also closely related to the Sinkhorn algorithm of Optimal Transport (OT). In fact, Sinkhorn algorithm solves the entropy-regularized version of Optimal Transport (Cuturi, 2013). However, our work concerns a different optimization instance. While OT solves a transportation problem where the solution space is defined with the marginal constraints over the rows and columns of a transportation matrix, our optimization problem is constrained with a total budget ($\sum \lambda_i = k$) and upper bounds ($\lambda_i \leq 1 \ \forall i$). This leads to different iterative updates.$^7$ $\lambda_i \geq 0 \ \forall i$ is already implied by the entropy term $H(\lambda)$ in the objective.
A.2 DIFFERENTIABLE ALIGNMENT WITH SPARSITY CONSTRAINTS

Besides the Top-k and Top-p alignments in §3.1, we also explore a differentiable pairwise alignment with sparsity constraints (DA). Both Top-k and Top-p are doing hard selection of alignments, i.e., \( A_{i,j} \) is either 1 or 0. We relax it by introducing soft sparsity constraints. Similar to our formulation for unary salience (§3.2), we determine the alignment \( \tilde{A} \) by the following optimization problem:

\[
\max_{A} \langle S, A \rangle + \varepsilon H(A)
\]

s.t. \( \sum_j A_{i,j} = k, \ i = 1, \ldots, n \)

\[
A_{i,j} \in [0,1], \ i = 1, \ldots, n, \ j = 1, \ldots, m
\]

where \( H(\cdot) \) is the elementwise entropy function and \( \varepsilon > 0 \) is a small constant. We constrain the sum of each row of \( \tilde{A} \) to equal \( k \). When \( \varepsilon = 0 \), the solution of Eq. 9 is the same as Top-k. When \( \varepsilon > 0 \), the entropy term makes the optimization problem strongly concave, which can be solved by the same algorithm in Appendix A.1. The solution is differentiable, thus can be trained end-to-end in our model.

A.3 QUALITATIVE ANALYSIS

| Dataset       | Query                                                                 | Gold Document                                                                 |
|---------------|----------------------------------------------------------------------|--------------------------------------------------------------------------------|
| Quora         | what is the best birthday gift for a friend?                         | what is a good birthday gift for a friend?                                     |
| MS MARCO (dev)| when would you use a fathom measurement                              | a fathom is a unit of length in the imperial and the u.s. customary systems equal to 6 feet (1.8288 m), used especially for measuring the depth of water. there are two yards (6 feet) in an imperial fathom. |
| Touché-2020   | should animals be used for scientific or commercial testing?         | animal testing should not be allowed. . . [truncated] . . . albeit the non-precocious mistakes of scientists, also. . . [truncated] . . . skeptic of the scientist in question’s abilities . . . [truncated] . . . continuous use if animals for clinical and basic research. . . [truncated] . . . majority of the scientific community thinks on this issue, . . . |

Table 5: Examples of pairwise alignment with the top-k value up to 4 for the Quora, MS MARCO, and Touché-2020 datasets. Query tokens being aligned are shown in blue, and corresponding aligned document tokens are shown in red. The superscript on the document token (k) indicates top-k alignment. We notice that the top-1 alignment quality is generally good across all three tasks. However, larger value of k results in spurious irrelevant alignments for Quora and MS MARCO, while remains fairly useful for Touché-2020.

Table 5 shows examples of top-k pairwise alignments of a query token (highlighted in blue) to the corresponding document tokens for several different tasks. For question-answering (e.g., MS MARCO) and duplicate question retrieval (Quora), fewer alignments seem to be preferable, and as k increases, we start to see spurious alignments to unrelated documents tokens. For argument retrieval tasks such as Touché-2020, on the other hand, larger value of k tends to provide useful semantically relevant alignments (e.g., scientific vs clinical). These qualitative examples provide intuitive insights regarding why different alignment strategies are helpful for different tasks, and why alignment adaptation is necessary.

A.4 RESULTS ON MS MARCO

Table 6 shows the retrieval performance of ALIGNER and previous models on the MS MARCO dev set. We deliberately kept the training configuration of ALIGNER relatively simpler (e.g., no distillation or model-based hard negatives). However, ALIGNER still achieves the best MRR@10 simply because of scaling to larger pretrained language models. We have also trained ALIGNER with other alignment strategies, such as top-4, top-1%, and DA (Appendix A.2). However, the results suggest top-1 is favorable in MS MARCO.
| Model        | MRR@10 | Recall@1000 |
|-------------|--------|-------------|
| BM25        | 18.7   | 85.7        |
| SPLADE\textsubscript{v2} | 36.8   | 97.9        |
| DPR         | 31.1   | 95.2        |
| GTR\textsubscript{base}   | 36.6   | 98.3        |
| GTR\textsubscript{large}  | 37.9   | 99.1        |
| GTR\textsubscript{xl}    | 38.5   | 98.9        |
| GTR\textsubscript{xxl}   | 38.8   | 99.0        |
| ColBERT     | 36.0   | 96.8        |
| ColBERT\textsubscript{v2}| 39.7   | 98.4        |
| COIL        | 35.5   | 96.3        |
| ME-BERT     | 33.4   | –           |

| ALIGNER\textsubscript{base} | 38.8 | 97.8 |
| ALIGNER\textsubscript{large} | 39.4 | 98.3 |
| ALIGNER\textsubscript{xl}   | 39.9 | 98.4 |
| ALIGNER\textsubscript{xxl}  | 40.3 | 98.7 |

| ALIGNER\textsubscript{base} (top-4) | 37.1 | 97.5 |
| ALIGNER\textsubscript{base} (top-1%) | 38.8 | 97.6 |
| ALIGNER\textsubscript{base} (DA)    | 37.8 | 97.4 |

Table 6: Retrieval performance on MS MARCO. The top half shows baselines from previous work. The bottom half shows different ALIGNER models. DA: differential alignment. See Appendix A.2

A.5 Full Result Tables on BEIR

Table\textsuperscript{7} to \textsuperscript{10} presents complete results of ALIGNER’s performance on the BEIR datasets initialized from T5 base, large, XL, and XXL checkpoints. We set $k = 1$ during training, and show results across different inference-time alignment strategies (both top-$k$ and top-$p$). As expected, model accuracy improves as we scale to larger models. Moreover, we observe similar benefits of alignment adaptation across all the different model sizes.

| ALIGNER\textsubscript{base} | Top-$k$ | Top-$p$ |
|----------------------------|---------|---------|
|                            | 1*      | 2       | 4       | 6       | 8       | 0.5%  | 1%    | 1.5%  | 2%    |
| A.R.                       |         |         |         |         |         |       |       |       |       |
| ArguAna                    | 28.8    | 24.4    | 18.3    | 14.5    | 11.4    | 33.3  | 45.5  | 48.1  | 46.9  |
| Touché-2020                | 34.8    | 30.0    | 25.1    | 21.1    | 17.1    | 31.3  | 43.2  | 46.3  | 44.8  |
| F.C.                       |         |         |         |         |         |       |       |       |       |
| FEVER                      | 72.4    | 75.0    | 68.3    | 57.6    | 49.0    | 72.5  | 55.0  | 44.9  | 29.9  |
| Climate-FEVER              | 18.1    | 20.8    | 23.0    | 23.0    | 22.7    | 18.2  | 13.7  | 13.8  | 10.8  |
| SciFact                    | 70.4    | 68.8    | 65.0    | 60.9    | 55.6    | 71.1  | 69.4  | 67.2  | 62.7  |
| Q.A.                       |         |         |         |         |         |       |       |       |       |
| NQ                         | 52.2    | 48.3    | 36.3    | 26.6    | 19.9    | 52.2  | 49.2  | 43.5  | 36.3  |
| HotpotQA                   | 61.7    | 58.6    | 36.0    | 21.9    | 13.9    | 61.7  | 60.1  | 54.3  | 47.3  |
| FiQA                       | 33.4    | 30.8    | 23.8    | 19.5    | 15.5    | 33.4  | 28.5  | 24.6  | 19.0  |
| BioASQ                     | 49.6    | 45.8    | 37.4    | 30.8    | 24.5    | 47.4  | 37.7  | 31.9  | 25.0  |
| NFCorpus                   | 34.0    | 33.2    | 32.0    | 29.9    | 27.8    | 33.6  | 31.7  | 30.5  | 28.6  |
| MISC.                      |         |         |         |         |         |       |       |       |       |
| TREC-COVID                 | 68.3    | 74.0    | 73.2    | 67.2    | 61.7    | 66.8  | 64.4  | 56.7  | 46.7  |
| SCIDOCS                    | 14.1    | 14.3    | 13.1    | 11.4    | 9.7     | 14.4  | 14.9  | 14.9  | 14.8  |
| DBPedia                    | 41.6    | 39.4    | 29.6    | 20.2    | 14.2    | 41.6  | 41.7  | 39.9  | 36.6  |
| Quora                      | 82.3    | 64.9    | 30.6    | 13.3    | 6.3     | 82.3  | 82.3  | 82.3  | 82.3  |
| Average                    | 47.3    | 46.3    | 38.4    | 31.9    | 27.0    | 47.1  | 44.2  | 40.9  | 35.9  |

Table 7: nDCG@10 on the BEIR benchmark with different $k$ and $p$ in ALIGNER\textsubscript{base}. \textsuperscript{*}: alignment strategy during training ($k = 1$).
### Table 8: nDCG@10 on the BEIR benchmark with different $k$ and $p$ in ALIGNER\textsubscript{large}. $^\ast$: alignment strategy during training ($k = 1$).

| ALIGNER\textsubscript{large} | Top-$k$ | Top-$p$ |
|-------------------------------|---------|---------|
|                               | 1$^\ast$ | 2 | 4 | 6 | 8 | 0.5% | 1% | 1.5% | 2% |
| A.R. ArguAna                  | 29.5    | 25.2 | 19.5 | 16.0 | 13.6 | 33.2 | 44.5 | 47.9 | 46.8 |
| Touché-2020                   | 36.7    | 47.5 | 53.0 | 53.4 | 52.0 | 32.5 | 25.4 | 20.7 | 16.3 |
| F.C. FEVER                    | 72.9    | 75.2 | 69.6 | 61.8 | 53.6 | 72.8 | 52.8 | 43.0 | 29.3 |
| Climate-FEVER                 | 18.6    | 20.7 | 23.3 | 23.4 | 23.5 | 18.6 | 13.1 | 14.2 | 11.4 |
| SciFact                       | 71.5    | 70.7 | 69.5 | 67.7 | 64.1 | 72.2 | 70.6 | 69.6 | 66.8 |
| Q.A. NQ                       | 57.2    | 52.8 | 43.4 | 36.3 | 31.2 | 57.2 | 53.7 | 47.5 | 41.0 |
| HotpotQA                      | 63.6    | 62.6 | 44.8 | 32.1 | 24.4 | 63.6 | 61.8 | 55.6 | 48.5 |
| FiQA                          | 39.4    | 35.6 | 30.4 | 26.2 | 23.4 | 39.4 | 33.3 | 28.4 | 22.8 |
| BioASQ                        | 53.3    | 50.7 | 43.1 | 36.8 | 31.6 | 49.2 | 38.2 | 33.2 | 27.4 |
| NFCorpus                      | 35.5    | 33.9 | 32.5 | 31.2 | 29.6 | 34.6 | 32.1 | 30.8 | 29.6 |
| MISC. TREC-COVID               | 71.9    | 79.4 | 77.3 | 74.4 | 69.3 | 70.0 | 66.1 | 59.7 | 52.4 |
| SCIDOCS                       | 15.3    | 15.5 | 14.9 | 13.6 | 12.5 | 15.4 | 15.8 | 16.0 | 15.8 |
| DBPedia                       | 43.5    | 41.9 | 34.7 | 29.0 | 24.3 | 43.5 | 43.5 | 41.5 | 37.4 |
| Quora                         | 84.5    | 75.5 | 46.4 | 20.9 | 8.4  | 84.5 | 84.5 | 84.5 | 84.5 |
| Average                       | 49.5    | 49.1 | 43.0 | 37.3 | 33.0 | 49.0 | 45.4 | 42.3 | 37.9 |

### Table 9: nDCG@10 on the BEIR benchmark with different $k$ and $p$ in ALIGNER\textsubscript{xl}. $^\ast$: alignment strategy during training ($k = 1$).

| ALIGNER\textsubscript{xl} | Top-$k$ | Top-$p$ |
|----------------------------|---------|---------|
|                               | 1$^\ast$ | 2 | 4 | 6 | 8 | 0.5% | 1% | 1.5% | 2% |
| A.R. ArguAna                  | 32.4    | 28.3 | 22.5 | 18.9 | 16.3 | 35.3 | 44.9 | 47.2 | 47.0 |
| Touché-2020                   | 36.2    | 46.4 | 53.2 | 51.0 | 50.5 | 33.0 | 26.1 | 21.9 | 18.2 |
| F.C. FEVER                    | 72.9    | 75.6 | 70.9 | 63.2 | 56.0 | 72.9 | 56.9 | 48.2 | 34.1 |
| Climate-FEVER                 | 18.7    | 21.2 | 23.2 | 23.9 | 24.2 | 18.8 | 15.2 | 16.6 | 14.3 |
| SciFact                       | 71.5    | 70.8 | 69.7 | 67.6 | 65.6 | 73.0 | 71.6 | 69.9 | 69.5 |
| Q.A. NQ                       | 58.8    | 54.9 | 46.0 | 39.5 | 34.1 | 58.8 | 55.5 | 50.1 | 43.8 |
| HotpotQA                      | 63.9    | 62.6 | 45.5 | 32.9 | 25.0 | 63.9 | 62.5 | 57.5 | 51.4 |
| FiQA                          | 40.8    | 37.4 | 32.2 | 28.9 | 25.2 | 40.8 | 35.0 | 31.2 | 25.7 |
| BioASQ                        | 53.6    | 50.4 | 43.0 | 36.6 | 32.2 | 50.2 | 39.7 | 34.4 | 28.9 |
| NFCorpus                      | 35.4    | 34.3 | 33.0 | 31.5 | 29.7 | 35.0 | 33.1 | 31.6 | 30.0 |
| MISC. TREC-COVID               | 75.1    | 80.6 | 80.5 | 78.1 | 73.5 | 72.5 | 69.4 | 62.0 | 54.0 |
| SCIDOCS                       | 15.4    | 16.0 | 15.5 | 14.3 | 13.3 | 15.6 | 16.2 | 16.5 | 16.4 |
| DBPedia                       | 43.6    | 42.2 | 36.2 | 30.6 | 26.5 | 43.6 | 43.6 | 42.4 | 39.4 |
| Quora                         | 85.3    | 85.3 | 85.3 | 85.3 | 85.3 | 85.3 | 85.3 | 85.3 | 85.3 |
| Average                       | 50.3    | 49.8 | 44.4 | 38.9 | 34.7 | 49.9 | 46.8 | 43.9 | 39.8 |
| ALIGNER<sub>xxl</sub> | Top-<var>k</var> | Top-<var>p</var> |
|----------------------|----------------|----------------|
|                      | 1<sup>*</sup> | 2  | 4 | 6 | 8 | 0.5% | 1% | 1.5% | 2% |
| A.R. ArguAna         | 33.8          | 29.6 | 24.1 | 20.4 | 18.2 | 36.6 | 46.9 | 49.8 | 49.7 |
|                      | 34.5          | 47.4 | 49.8 | 51.1 | 50.6 | 30.1 | 21.8 | 16.3 | 11.7 |
| F.C. Climate-FEVER   | 74.2          | 77.0 | 73.9 | 67.9 | 62.2 | 75.2 | 47.1 | 36.2 | 24.1 |
|                      | 19.7          | 21.6 | 23.7 | 23.2 | 24.3 | 19.7 | 12.0 | 12.3 | 9.3  |
| SciFact              | 73.1          | 71.2 | 71.3 | 69.1 | 67.0 | 74.4 | 71.5 | 69.9 | 69.5 |
|                      |               | 5%  | 1% | 1.5% | 2% |
| Q.A.                 |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
|                       |               |     |     |     |     |     |     |     |     |
| Average              | 51.3          | 50.9 | 46.2 | 41.4 | 37.7 | 50.8 | 45.1 | 40.9 | 36.4 |

Table 10: nDCG@10 on the BEIR benchmark with different <var>k</var> and <var>p</var> in ALIGNER<sub>xxl</sub>. <sup>*</sup>: alignment strategy during training (<var>k</var> = 1).

| ALIGNER<sub>base</sub> | Top-<var>k</var> | Top-<var>p</var> |
|------------------------|----------------|----------------|
|                        | 1  | 2  | 4<sup>*</sup> | 6 | 8 | 0.5% | 1% | 1.5% | 2% |
| A.R. ArguAna           | 26.5 | 23.6 | 18.8 | 15.9 | 13.6 | 30.7 | 42.8 | 45.7 | 46.2 |
|                      | 25.2 | 32.4 | 43.9 | 48.2 | 49.5 | 19.7 | 14.6 | 11.9 | 9.5  |
| F.C. Climate-FEVER     | 59.0 | 67.0 | 72.8 | 73.6 | 73.0 | 59.0 | 34.3 | 25.4 | 18.4 |
|                      | 14.7 | 17.5 | 20.9 | 22.8 | 23.7 | 14.7 | 7.8  | 5.1  | 5.1  |
| SciFact               | 70.1 | 70.6 | 70.4 | 69.2 | 68.0 | 69.6 | 66.7 | 65.6 | 64.9 |
| Q.A.                  |     |     |     |     |     |     |     |     |     |
|                       |     |     |     |     |     |     |     |     |     |
|                       |     |     |     |     |     |     |     |     |     |
|                       |     |     |     |     |     |     |     |     |     |
|                       |     |     |     |     |     |     |     |     |     |
| Average               | 42.0 | 45.4 | 46.9 | 45.5 | 43.1 | 41.1 | 36.3 | 33.4 | 30.5 |

Table 11: nDCG@10 on the BEIR benchmark with different <var>k</var> and <var>p</var> in ALIGNER<sub>base</sub>. <sup>*</sup>: alignment strategy during training (<var>k</var> = 4).
### Table 12: nDCG@10 on the BEIR benchmark with different $k$ and $p$ in ALIGNER$_{base}$. ∗: alignment strategy during training ($p = 1%$).

| ALIGNER$_{base}$ | Top-$k$ | Top-$p$ |
|------------------|---------|---------|
|                  | 1 | 2 | 4 | 6 | 8 | 0.5% | 1% | 1.5% | 2% |
| A.R. ArguAna     | 28.4 | 23.7 | 17.9 | 14.2 | 11.1 | 32.9 | 45.3 | 48.0 | 47.7 |
| Touché-2020      | 38.1 | 51.3 | 53.0 | 51.3 | 50.0 | 35.0 | 26.6 | 22.6 | 17.1 |
| F.C. FEVER       | 72.1 | 74.6 | 67.7 | 57.5 | 49.2 | 72.2 | 56.1 | 46.2 | 31.2 |
| Climate-FEVER    | 17.8 | 20.3 | 22.0 | 22.1 | 21.8 | 17.8 | 13.5 | 14.4 | 11.3 |
| SciFact          | 69.0 | 67.9 | 64.8 | 61.5 | 57.2 | 70.5 | 70.3 | 68.1 | 63.9 |
| Q.A. NQ          | 52.6 | 48.2 | 36.4 | 26.4 | 20.2 | 52.5 | 50.4 | 44.7 | 38.2 |
| HotpotQA         | 60.6 | 57.7 | 35.1 | 21.1 | 13.1 | 60.7 | 59.6 | 53.9 | 47.6 |
| FiQA             | 33.7 | 29.4 | 23.5 | 18.8 | 15.8 | 33.6 | 29.8 | 26.2 | 20.6 |
| BioASQ           | 40.3 | 48.4 | 39.9 | 33.1 | 26.4 | 50.0 | 40.3 | 35.3 | 28.8 |
| NFCorpus         | 34.7 | 33.7 | 32.1 | 30.2 | 27.8 | 34.3 | 32.7 | 31.6 | 29.6 |
| MISC. TREC-COVID  | 67.6 | 73.4 | 73.1 | 67.7 | 62.5 | 67.2 | 65.7 | 60.6 | 52.0 |
| SCIDOCS          | 13.9 | 14.2 | 13.1 | 11.5 | 9.9  | 14.1 | 14.6 | 14.8 | 14.9 |
| DBPedia          | 41.3 | 40.3 | 29.5 | 21.2 | 14.0 | 41.2 | 41.3 | 40.3 | 37.1 |
| Quora            | 83.6 | 60.8 | 22.8 | 8.9  | 4.3  | 83.6 | 83.6 | 83.6 | 83.6 |
| Average          | 43.6 | 42.9 | 35.4 | 29.7 | 25.5 | 44.4 | 42.0 | 39.3 | 34.9 |

### Table 13: nDCG@10 on the BEIR benchmark with ALIGNER$_{base}$ and differentiable alignment (Appendix A.2). ∗: alignment strategy during training ($k = 1$).

| ALIGNER$_{base}$ | $k$ | $\epsilon$ |
|------------------|-----|-------------|
|                  | 1* | 2 | 4 | 6 | 8 | 0.01 | 0.02 | 0.04 | 0.06 | 0.1 |
| A.R. ArguAna     | 30.8 | 26.3 | 20.1 | 15.8 | 12.4 | 30.8 | 33.2 | 38.8 | 43.5 | 50.1 |
| Touché-2020      | 37.4 | 50.1 | 52.0 | 51.1 | 49.0 | 37.4 | 35.0 | 29.8 | 26.9 | 19.4 |
| F.C. FEVER       | 68.8 | 71.4 | 63.7 | 46.1 | 44.3 | 68.8 | 68.0 | 66.4 | 64.5 | 56.2 |
| Climate-FEVER    | 16.7 | 19.4 | 21.2 | 21.6 | 21.3 | 16.7 | 16.2 | 15.6 | 15.6 | 15.4 |
| SciFact          | 69.8 | 68.2 | 64.7 | 61.8 | 57.0 | 69.8 | 69.5 | 70.3 | 71.0 | 70.1 |
| Q.A. NQ          | 52.0 | 47.4 | 35.0 | 25.8 | 18.5 | 52.0 | 51.6 | 50.8 | 50.0 | 46.0 |
| HotpotQA         | 59.6 | 56.1 | 33.4 | 20.6 | 12.6 | 59.6 | 59.5 | 60.0 | 61.0 | 59.8 |
| FiQA             | 32.9 | 29.6 | 23.0 | 18.6 | 14.5 | 32.9 | 32.7 | 32.9 | 32.4 | 30.9 |
| BioASQ           | 50.4 | 47.2 | 38.0 | 31.4 | 24.5 | 50.4 | 50.4 | 50.8 | 50.4 | 43.7 |
| NFCorpus         | 34.1 | 33.6 | 31.6 | 29.6 | 27.4 | 34.1 | 34.4 | 34.2 | 34.2 | 33.5 |
| MISC. TREC-COVID  | 63.9 | 73.2 | 72.5 | 67.7 | 62.8 | 63.9 | 65.5 | 63.6 | 63.8 | 54.2 |
| SCIDOCS          | 14.0 | 14.4 | 13.4 | 11.6 | 9.6  | 14.0 | 14.1 | 14.2 | 14.5 | 15.4 |
| DBPedia          | 40.9 | 38.7 | 28.7 | 19.9 | 13.2 | 40.9 | 40.5 | 39.9 | 39.7 | 38.6 |
| Quora            | 82.8 | 61.2 | 21.5 | 7.4  | 3.5  | 82.8 | 83.1 | 83.5 | 84.0 | 85.0 |
| Average          | 43.6 | 42.4 | 34.6 | 28.6 | 24.7 | 43.6 | 43.6 | 43.4 | 43.4 | 41.2 |