Social Welfare Maximization Auction in Edge Computing Resource Allocation for Mobile Blockchain

Yutao Jiao, Ping Wang, Dusit Niyato, and Zehui Xiong
School of Computer Science and Engineering, Nanyang Technological University, Singapore 639798

Abstract—Blockchain, an emerging decentralized security system, has been applied in many applications, such as bitcoin, smart grid, and Internet-of-Things. However, running the mining process may cost too much energy consumption and computing resource usage on handheld devices, which restricts the use of blockchain in mobile environments. In this paper, we consider deploying edge computing service to support the mobile blockchain. We propose an auction-based edge computing resource market of the edge computing service provider. Since there is competition among miners, the allocative externalities (positive and negative) are taken into account in the model. In our auction mechanism, we maximize the social welfare while guaranteeing the truthfulness, individual rationality and computational efficiency. Based on blockchain mining experiment results, we define a hash power function that characterizes the probability of successfully mining a block. Through extensive simulations, we evaluate the performance of our auction mechanism which shows that our edge computing resources market model can efficiently solve the social welfare maximization problem for the edge computing service provider.

Index Terms—mobile blockchain, auction, edge computing, pricing, resources allocation, proof of work

I. INTRODUCTION

Decentralized cryptocurrencies have witnessed explosive growth since the first decentralized distributed currency Bitcoin was launched in 2009 [1]. In contrast to traditional currencies, decentralized cryptocurrencies are traded among participants over a peer-to-peer (P2P) network without relying on trusted third parties like banks or financial regulatory authorities. As the backbone technology, blockchain protocol provides an effective consensus mechanism to successfully solve problems about incentive, tamper-resistance, trust and so on [2]. Recently, blockchain has heralded many applications in various fields, such as finance [3], Internet of Things [4], smart grid [5] and cognitive radio [6]. According to the market research firm Tractica, it is estimated that the annual revenue for enterprise applications of blockchain will increase to $19.9 billion by 2025 [7]. It is also worth noting that the access scope of the blockchain can be not only public as Bitcoin, but also private or consortium/community [8] where blockchain networks are established and usually managed by the blockchain owner.\footnote{A good example is R3 consortium (https://www.r3.com/) experimenting an Ethereum private blockchain on Microsoft Azure Blockchain Service.}

The security and reliability of blockchains depend on a distributed consensus mechanism. Specifically, a group of participants in the blockchain network, called miners, try to solve a computationally difficult problem, i.e., the proof of work (PoW) puzzle, where the process is called mining. First, each miner receives and selects certain number of transaction records from public. Once solving the puzzle, the miner will broadcast a block which combines the transaction records and relevant information in the blockchain network. Next, this block will be verified by the majority of other miners for consensus and then finally be added to the blockchain. The miner which successfully finishes the above steps will receive a fixed reward and certain transaction fees as incentives of mining.

However, blockchain applications in mobile environments are still seldom realized because solving the PoW puzzle needs high computing power and large amount of energy which mobile devices cannot satisfy. In this paper, we consider the edge computing services for mobile users to deploy their mining tasks and thus support the mobile blockchain applications. Specifically, we discuss the allocation and pricing issue for edge computing resource. We first propose an auction-based market model. The market consists of three entities, i.e., blockchain owner, edge computing service provider (ESP) and miners. Considering the competition among miners [2] and network effects of blockchain by nature [9], we then study the auction mechanism with allocative externalities to maximize the social welfare. Our social maximization mechanism is truthful (incentive compatible), individually rational and computationally efficient. Based on our real-world experiment of mobile blockchain, we analyze the probability of successfully mining a block and verify the probability function. Our simulation results show that the proposed auction market model can not only help the ESP to make practical sale strategies, but also incentivize the blockchain owner in adjusting the blockchain protocol. To the best of our knowledge, this is the first work that investigates resource management and pricing in the mobile blockchain with an auction model.

The rest of this paper is organized as follows. Section II reviews related work, and the system model of edge computing resource market for mobile blockchain is introduced in Section III. Section IV formulates the social maximization problem and gives theoretical analysis. Section V presents
experimental results of mobile blockchain and performance analysis. Finally, Section VI concludes the paper.

II. RELATED WORK

Over the recent years, there are some studies on the economics and mining strategies in the blockchain network. As one of the pioneer papers, the authors in [10] modeled the mining process as a game played by miners. Each miner’s strategy is to choose which branch of blockchain to mine on. They proved the existence of a Nash equilibrium when all miners behave as expected by Bitcoin designer. Further, they explored the case that some miners may deviate from the expected behavior, which makes blockchain network unstable and vulnerable. The authors in [11] proposed a game model in which the occurrence of solving the PoW puzzle is modeled as a Poisson process. Miners have to decide the size of block to broadcast as their response. Analytical solutions to the Nash equilibrium in a two-miner case was given. In [12], the authors designed a cooperative game model to investigate the mining pool. In the pool, miners form a coalition to accumulate their computational power and have steady reward. However, these works only studied the internal mining scheme and paid little attention to the actual running of blockchain in more dynamic environments, i.e., mobile blockchain. Therefore, this motivates us to place the edge computing service as underlying technology for mobile blockchain network and build an edge computing resource market model. The blockchain technology can achieve more extensive and promising applications in the mobile information society. The auction design has been widely studied in other resource allocation problems, such as spectrum trading [13], [14] and data crowdsensing [15]. However, none of these works can be directly applied to edge computing applications for mobile blockchain, since they only focused on the specific properties constrained by the studied topics. For example, in the blockchain networks, the allocative externalities [16], [17] should be considered because of miners care about the computational power allocated to the others. The authors in [18] used deep learning to recover the classical optimal auction for revenue maximization and applied it in the edge computing resources allocation. However, it is less about the design of auction and cannot guarantee the incentive compatibility.

III. SYSTEM MODEL: MOBILE BLOCKCHAIN AND MARKET MODEL

Figure 1 shows the auction-based market model for trading edge computing resources. The blockchain owner launches a blockchain application and designs the protocol for blockchain network operation. The mobile users buy resources from the edge computing service provider (ESP) and become miners. In the miners network, they take part in the mining process to contribute new blocks to the blockchain.

A. Mobile Blockchain

Blockchain can be used to develop applications with mobile devices, as indicated in our earlier study [19]. To support the blockchain based service, there are a set of miners continuously running a consensus protocol [1] to confirm and secure distributed data or transactions at backend. According to the protocol, miners are required to finish the mining task, i.e., solving the PoW puzzle. The mining process is conducted in a tournament structure, and miners chase each other to obtain the solution. Specifically, the PoW algorithm involves finding a nonce value that, together with additional fields about all valid and received transactions, the previous block and the timestamp, the output satisfies a given condition. If the nonce is found, the miner will combine it and additional fields into a block and then broadcast the block to peers in the blockchain network for verification and reaching consensus. Finally, the new block can be linked to the existing accepted chain of blocks. However, for a mobile user, it is unrealistic to continuously run such a computationally difficult program which requires high computing power and consumes a large volume of energy and time. Because the outstanding characteristics of edge computing: low latency, mobility and wide-spread geographical distribution [20], we consider offloading the mining tasks to the edge servers.

B. Edge Computing Resources Trading

As shown in Fig. 1, we consider a scenario where there is one ESP, one blockchain owner and a community of mobile users \( N = \{ 1, \ldots, N \} \). Each mobile user wants to be a miner which runs a mobile blockchain application to record and verify the transactions or data sent to the community. Due to the computing limitation on their devices, mobile users want to offload the task of solving PoW to the nearby edge computing servers deployed by the ESP. In particular, the ESP launches an auction to sell its services. It first announces its service and relevant information to mobile users. Then, the mobile users submit their resource demand profile \( d = ( d_1, \ldots, d_N ) \) and corresponding bids \( b = ( b_1, \ldots, b_N ) \) which represents their valuations of the offered services. After receiving the demands and bids, the ESP selects the winners as the successful miners and notifies the mobile users the allocation \( x = ( x_1, \ldots, x_N ) \).
and the service price \( p = (p_1, \ldots, p_N) \). The setting \( x_i = 1 \) means user \( i \) is within the winner list and being allocated resources that it demands for while \( x_i = 0 \) is for no resource\(^2\). \( p_i \) is the sale price that user \( i \) is charged by the ESP\(^3\). At the end of the auction, the winners make the payment and access the edge computing service.

C. Blockchain Mining with Edge Computing Service

With the allocation \( x_i \) and demand \( d_i \), miner \( i \)'s hash power \( \gamma_i \) relative to other miners' allocated resources can be calculated by:

\[
\gamma_i(d, x) = \frac{d_i^\alpha x_i}{\sum_{j \in N} d_j^\alpha x_j}
\]

which is a fraction function that \( \sum_{i \in N} \gamma_i = 1 \). \( \alpha \) is the curve fitting parameter of the hash power function \( \gamma_i(d, x) \) verified by our real-world experiment, the detail of which will be presented in Section V.

In the mining tournament, miners compete to be the first to solve PoW with correct nonce value and propagate the block to reach consensus. The generation of new blocks follows a Poisson process with a constant rate \( \frac{1}{\lambda} \) throughout the whole blockchain network [21]. Before the tournament, miners collect unconfirmed transactions into their blocks. We represent the size of transactions of each miner by \( s = (s_1, \ldots, s_N) \). When miner \( i \) propagates its block to the mobile blockchain network for consensus, the time for verifying each transaction is affected by the size of transactions \( s_i \). The first miner which successfully has its block achieve consensus can get a reward \( R \). The reward is composed of a fixed bonus \( T \) for mining a new block and a flexible transaction fee \( t \) determined by the size of its collected transactions \( s \) and the transaction fee rate \( r \) [11]. Thus, miner \( i \)'s expected reward \( R_i \) can be expressed by:

\[
R_i = (T + rs_i)\mathbb{P}_i(\gamma_i(d, x), s_i),
\]

where \( \mathbb{P}_i(\gamma_i(d, x), s_i) \) is the probability that miner \( i \) receives the reward by contributing a block to the blockchain.

From the mining tournament above, winning the reward depends on the successful mining and instant propagation. The probability of mining a new block \( P^m_i \) is equal to miner \( i \)'s hash power \( \gamma_i \), i.e., \( P^m_i = \gamma_i \). However, the miner may even lose the tournament if its new block does not achieve consensus as the first. This kind of mined block that cannot be added on to the blockchain is called orphaned block [11]. Moreover, the block containing larger size of transactions has higher chance becoming orphaned. This is because a larger block needs more propagation time, thus causing higher delay for consensus. Here, we assume miner \( i \)'s block propagation time \( \tau_i \) is linear to the size of transactions in its block, i.e., \( \tau_i = \xi s_i \), \( \xi \) is a constant that reflects the impact of \( s_i \) on \( \tau_i \).

Since the arrival of new blocks follows a Poisson distribution, miner \( i \)'s orphaning probability can be approximated as follows [22]:

\[
P^O_i = 1 - \exp(-\frac{1}{\lambda}\tau_i).
\]

After substituting \( \tau_i \), we can express \( P_i \) as follows:

\[
P_i = P^m_i(1 - P^O_i) = \gamma_i e^{-\frac{1}{\lambda}\xi s_i}.
\]

D. Blockchain Management

The blockchain owner maintains the blockchain mining protocol that specifies the fixed bonus \( T \) for the contributing miner and the transaction fee rate \( r \). Through adjusting the difficulty of finding a new block, the blockchain owner keeps the average time \( \lambda \) at a reasonable constant value\(^4\). Additionally, a blockchain in PoW systems is only as secure as the amount of computing power dedicated to mining it [9]. This results in positive network effects: as more mobile users participate in mining and more computing resources are invested, the value of reward given to miners increases since the blockchain network is more stable and secure. Empirically, we define the network effects by a common S-shaped utility function [23]:

\[
w(d_N) = \frac{1 - e^{-\nu d_N}}{1 + \mu e^{-\nu d_N}},
\]

where \( d_N = \sum_{i \in N} d_i x_i \) is the total quantity of allocated resources and \( \mu, \nu \) are positive parameters. The monotonic increase of network effect function begins slowly from 0, then accelerates (convexly), and then eventually slows down (concavely) and converges asymptotically to 1.

IV. SOCIAL WELFARE MAXIMIZATION AUCTION FOR EDGE COMPUTING SERVICE

In this section, we propose an auction mechanism for the ESP to allocate edge computing resources efficiently. We focus on maximizing the social welfare while guaranteeing the truthfulness, individual rationality and computational efficiency.

A. Valuation of mobile users

To take part in the auction, a mobile user needs to give the bid representing its valuation to the auctioneer, i.e., the ESP. Since the mobile user \( i \) cannot know the number of winners and total supply of computing resources until auction ends, it can only give the bid \( b_i \) according to its expected reward \( R_i \) which is also called \textit{ex-ante} valuation \( v'_i \), i.e.,

\[
v'_i = R_i
\]

\(^2\)It is worth noting that the blockchain owner is not a central entity that controls the data storage and mining strategies of the miners. Similar to the bitcoin protocol [1], the blockchain owner in this paper only designs the protocol specifies the value of \( T, r \) and \( \lambda \), and does not affect the decentralization and security of blockchain.
After the auction result is released, user $i$ has an ex-post valuation $v''_i$ of the edge computing service considering network effects, which is defined by

$$v''_i = R_i w$$  \hspace{5cm} (7)

where $w$ is the network effect defined in (5).

After substituting (1), (4), (5) into (6) and (7), we have the specific expression of user $i$’s ex-ante and ex-post valuation:

$$v'_i = (T + rs_i)e^{-\frac{s}{2} \xi_i},$$  \hspace{5cm} (8)

$$v''_i = \frac{d^x_i x_i}{\sum_{j \in \mathcal{N}} d^x_j x_j} \frac{1}{1 + \mu e^{-\nu \sum_{i \in \mathcal{N}} d^x_i}(T + rs_i)} e^{-\frac{s}{2} \xi_i}$$  \hspace{5cm} (9)

### B. Auction Maximizing Social Welfare

Once receiving bids $b$ from all the mobile users, ESP will select winners and determine corresponding payments to maximize the social welfare. Let $c$ denote the unit cost of running the edge computing service. ESP’s total cost is $C(d_N) = cd_N$. Thus, designing such an auction becomes solving an optimization problem:

$$\max_{\mathbf{x}} \sum_{i \in \mathcal{N}} \frac{d^x_i x_i}{\sum_{j \in \mathcal{N}} d^x_j x_j} \frac{1}{1 + \mu e^{-\nu \sum_{i \in \mathcal{N}} d^x_i}(T + rs_i)} e^{-\frac{s}{2} \xi_i}$$

$$- \sum_{i \in \mathcal{N}} cx_i$$  \hspace{5cm} (10)

subject to:

$$\sum_{i \in \mathcal{N}} d_i x_i \leq D$$  \hspace{5cm} (11)

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N}$$  \hspace{5cm} (12)

where the objective function in (10) is the difference between the sum of all users’ ex-post valuations and ESP’s total cost. The constraint in (11) defines the maximum quantity of computing resources that ESP can offer denoted by $D$.

Based on the above system model, we first consider a simpler case where all mobile users submit various bids to compete for a fixed quantity of resources. Without loss of generality, we set $d_i = 1, \forall i \in \mathcal{N}$. Then, the optimization problem can be expressed as follows:

$$\max_{\mathbf{x}} \sum_{i \in \mathcal{N}} x_i \frac{1}{\sum_{j \in \mathcal{N}} x_j} \frac{1}{1 + \mu e^{-\nu \sum_{i \in \mathcal{N}} x_i}(T + rs_i)} e^{-\frac{s}{2} \xi_i}$$

$$- \sum_{i \in \mathcal{N}} cx_i$$  \hspace{5cm} (13)

subject to:

$$\sum_{i \in \mathcal{N}} x_i \leq D$$  \hspace{5cm} (14)

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N}$$  \hspace{5cm} (15)

We aim to solve this integer programming efficiently while making the auction process truthful and individually rational. The proposed auction is based on the Myerson’s well-known characterization [24] as described in Theorem 1.

**Theorem 1.** (\cite[Theorem 13.6]{myerson1981optimal}) An auction is truthful if and only if it satisfies the following two properties:

1) **Monotonicity of winner selection rule:** If user $i$ wins the auction with bid $b_i$, then it will also win with any higher bid $b'_i > b_i$.

2) **Critical payment:** The payment by a winner is the smallest value needed in order to win the auction.

By using Theorem 1, our auction mechanism is illustrated in Algorithm 1. In Lines 5-12, the winner selection process is conducted with a greedy scheme. We define a winner set $\mathcal{W}$.

**Proposition 2.** The resource allocation $\mathbf{x}$ output by Algorithm 1 is globally optimal to the social welfare maximization problem given in (13)-(15).

**Proof:** With proof by contradiction, this result follows from Claim 3.

**Claim 3.** Let $\mathcal{W}_A$ be the solution output by Algorithm 1 on input $b$, and $\mathcal{W}_O$ the optimal solution. If $\mathcal{W}_A \neq \mathcal{W}_O$, then we can construct another solution $\mathcal{W}'_O$ the social welfare of which $S(\mathcal{W}_O')$ is even larger than $\mathcal{W}_O$.

**Proof:** Without loss of generality, we assume $b_1 \geq \cdots \geq b_N$ and $\mathcal{W}_A \neq \mathcal{W}_O$. Let $m$ be the first element (while-loop Lines 6-12) where $m \notin \mathcal{W}_O$. Since $(b_m, m)$ is maximal, we must have $1, \ldots, m-1 \in \mathcal{W}_O$ and in particular, the set of corresponding bid $b_{\mathcal{W}_O}$ has the form $b_{\mathcal{W}_O} = \{b_1, b_2, \ldots, b_{m-1}, b'_m, b'_{m+1}, \ldots, b'_{\mathcal{W}_O}\}$, where the bids $b_1, \ldots, b'_{\mathcal{W}_O}$ are listed in the descending order. Meanwhile, Algorithm 1 chooses $b_{\mathcal{W}_A} = \{b_1, b_2, \ldots, b_{m-1}, b_m, b_{m+1}, \ldots, b_{\mathcal{W}_O}\}$ and there must be $b_m > b'_m$ for all $j \geq m$. In particular, we have $b_m > b'_m$. Hence, we define $b_{\mathcal{W}'_O} = b_{\mathcal{W}_O} \cup \{b_m\} \setminus \{b'_m\}$, i.e., we obtain $b_{\mathcal{W}'_O}$ by deleting the $m$th bid in $b_{\mathcal{W}_O}$ and adding $b_m$. Now we have the social welfare of $b_{\mathcal{W}'_O}$:

$$S(\mathcal{W}_O') = S(\mathcal{W}_O) + \frac{1}{|\mathcal{W}_O|} \frac{1}{1 + \mu e^{-\nu |\mathcal{W}_O|}} (b_m - b'_m).$$
Since \( b_m - b_m' > 0 \) and \( |\mathcal{W}_O'| = |\mathcal{W}_O| \), \( S(\mathcal{W}_O') \) is strictly larger than \( S(\mathcal{W}_O) \), which is in contradiction to that \( \mathcal{W}_O \) is the optimal solution. This proves the claim.

In Lines 13-24, for each iteration, we exclude one winner from the user set and rerun the winner selection process to calculate the payment for the winner. The payment calculation is based on the well-known Vickrey–Clarke–Groves (VCG) mechanism [26].

Algorithm 1 Social Welfare Maximization Auction

**Input:** Mobile users’ bid profile \( \mathbf{b} = (b_1, \ldots, b_N) \).

**Output:** Resource allocation profile \( \mathbf{x} = (x_1, \ldots, x_N) \) and payment profile \( \mathbf{p} = (p_1, \ldots, p_N) \).

1. begin
2. for each \( i \in \mathcal{N} \) do
3. \( x_i \leftarrow 0, p_i \leftarrow 0 \)
4. end for
5. \( \mathcal{W} \leftarrow \emptyset, \mathcal{W}_t \leftarrow \emptyset, S \leftarrow 0, S_t \leftarrow 0 \)
6. \( \mathcal{W}_t \leftarrow \mathcal{W} \cup \{j\}, S \leftarrow S_t \)
7. \( S_t \leftarrow \frac{1}{|\mathcal{W}_t|} \sum_{i \in \mathcal{W}_t} b_i - c |\mathcal{W}_t| \)
8. end while
9. for each \( j \in \mathcal{W} \) do
10. \( x_j \leftarrow 1 \)
11. \( S' \leftarrow S \leftarrow \emptyset \)
12. while \( S' \leq S \) do
13. \( \mathcal{W}_j \leftarrow \mathcal{W} \setminus \{j\}, \mathcal{W}' \leftarrow \mathcal{W}' \setminus \{j\} \)
14. \( \mathcal{W}_j \leftarrow \emptyset, \mathcal{W}_j \leftarrow \emptyset, S_j \leftarrow 0, S_j' \leftarrow 0 \)
15. \( \mathcal{W}_j \leftarrow \mathcal{W}_j \cup \{k\}, S_j \leftarrow S_j' \)
16. \( S_j' \leftarrow \frac{1}{|\mathcal{W}_j|} \sum_{i \in \mathcal{W}_j} b_i - c |\mathcal{W}_j| \)
17. \( S_j \leftarrow \frac{1}{|\mathcal{W}_j|} \sum_{i \in \mathcal{W}_j} b_i - c |\mathcal{W}_j| \)
18. end while
19. \( \mathbf{p}_j = S_j - \frac{1}{|\mathcal{W}_j|} \sum_{i \in \mathcal{W}_j} b_i - c |\mathcal{W}_j| \)
20. end for
21. end while
22. end

Proposition 4. The Social Welfare Maximization Auction (Algorithm 1) is truthful.

Proof: Since the calculation of payment by the algorithm relies on VCG mechanism, it directly satisfies the second condition in Theorem 1. For the first condition about monotonicity, we only need to show that if a winner \( i \) raises his bid from \( b_i \) to \( b_i^+ \) where \( b_i^+ > b_i \), it still stays in the set of winners. We denote the original set of winners as \( \mathcal{W} \) and the new set of winners \( \mathcal{W}^+ \) after winner \( i \) changes its bid to \( b_i^+ \). The original bid set is \( \mathbf{b} = \{b_1, \ldots, b_N\} \) \((i \leq |\mathcal{W}|)\) sorted in the descending order. In addition, we define \( S(\mathbf{b}_m') = S(\mathcal{U}), \forall \mathcal{U} \subseteq \mathcal{N} \) which means the social welfare of a set of bids is equal to the set of its corresponding users. We discuss the monotonicity in two cases:

1) Case 1: \( b_{i-1} \geq b_i^+ \geq b_i \geq b_{i+1} \). The new set of ordered bids is \( \mathbf{b}^+ = \{b_1, \ldots, b_{i-1}, b_i^+, b_{i+1}, \ldots, b_N\} \). We have

\[
S(\{b_1, \ldots, b_i\}^+) = \frac{1 - e^{-\nu i}}{1 + \mu e^{-\nu i}} \left( \sum_{j=1}^{i-1} b_j + b_i^+ \right) - ci
\]

\[
> S(\{b_1, \ldots, b_i\}) = \sum_{j=1}^{i} \frac{1 - e^{-\nu i}}{1 + \mu e^{-\nu i}} b_j - ci.
\]

(17)

The social welfare of new set of bids \( \{b_1, \ldots, b_i^+\} \) is larger than that of original set of bids \( \{b_1, \ldots, b_i\} \), which guarantees \( b_i^+ \) being in the set of winning bids.

2) Case 2: \( b_{k-1} \geq b_k \geq b_k \geq \ldots \geq b_i \). \( 1 < k < i \). The new set of ordered bids is \( \mathbf{b}^+ = \{b_1, \ldots, b_{k-1}, b_k^+, b_{k+1}, b_{i+1}, \ldots, b_N\} \). We have

\[
S(\{b_1, \ldots, b_{k-1}, b_k\}^+) = \frac{1 - e^{-\nu k}}{1 + \mu e^{-\nu k}} \sum_{j=1}^{k-1} b_j + b_k^+ - ck,
\]

\[
S(\{b_1, \ldots, b_{k-1}, b_k\}) = \frac{1 - e^{-\nu (k-1)}}{1 + \mu e^{-\nu (k-1)}} \sum_{j=1}^{k-1} b_j - ck - ck.
\]

(18)

\[
S(\{b_1, \ldots, b_{k-1}, b_k\}^+) = \frac{1 - e^{-\nu (k-1)}}{1 + \mu e^{-\nu (k-1)}} \sum_{j=1}^{k-1} b_j - c(k-1).
\]

(20)

Since in the original set of bids \( \mathbf{b}, \{b_1, \ldots, b_{k-1}, b_k, \ldots, b_i\} \) are all selected as winning bids, \( S(\{b_1, \ldots, b_{k-1}, b_k\}) > S(\{b_1, \ldots, b_{k-1}\}) \). Because the inequality in (16), we have

\[
S(\{b_1, \ldots, b_{k-1}, b_k^+\}) > S(\{b_1, \ldots, b_{k-1}\}),
\]

which implies that \( b_i^+ \) still wins the auction. This concludes the proof.

Proposition 5. The Social Welfare Maximization Auction (Algorithm 1) is computationally efficient and individually rational.

Proof: Since the time complexity of finding the maximum miner’s bid is \( O(N) \) and the number of winners is at most \( N \), the time complexity of the winner selection process (while-loop, Lines 6-12) is \( O(N^2) \). In each iteration of payment calculation process (Lines 13-24), a similar winner selection process is executed. Therefore, the whole auction process can be performed in polynomial time with the time complexity of \( O(N^3) \) which is efficient. According to Proposition 2 and the properties of the VCG mechanism [26], the payment scheme in Algorithm 1 guarantees the individual rationality.

V. EXPERIMENT RESULTS AND PERFORMANCE ANALYSIS

In this section, we provide simulation results of the proposed auction, from which we can further obtain useful decision making strategies for ESP and the blockchain owner.
\[ S(b_1, \ldots, b_{k-1}, b_k) = \frac{1 - e^{-\nu k}}{(1 + \mu e^{-\nu k})^k} \left( \sum_{j=1}^{k-1} b_j + b_k \right) < \frac{1 - e^{-\nu k}}{(1 + \mu e^{-\nu k})^k} \left( \sum_{j=1}^{k-1} b_j + b_i^+ \right) = S(b_1, \ldots, b_{k-1}, b_i^+) \] (16)

A. Verification for Hash Power Function

An earlier real-world mobile blockchain mining experiment has been done in [19], [27]. In the experiment, we designed a mobile blockchain client application in the Android platform and implemented it on three mobile devices (miners). Each of the three client applications generates transactions and then starts mining with one CPU core. The miners’ CPU utilization rate is managed and measured on the Docker platform [28]. Each mobile device mines the block under Go-Ethereum [29] blockchain framework. To verify the hash power function (1), we vary one miner’s service demand while fixing the other two miners’ service demand (CPU utilization) at 40 and 60. Besides, we set the number of transactions in each mined block to be 10 for all miners. Figure 2 shows the change of the hash power, i.e., the probability of successfully mining a block with different amount of computing resources. We note that the hash power function defined in (1) can well fit the actual experimental data. From these results, we choose the hash power function with parameter \( \alpha = 1.2 \) in the rest of this section.

![Fig. 2. Estimation of the hash power function \( \gamma(d) \).](image)

B. Simulation Results

We vary the number of mobiles users \( N \) from 100 to 1000, the mining bonus \( T \) from 0 to 5, and the transaction fee rate \( r \) from 0.001 to 0.009. We set \( \mu = 0.5, \nu = 0.005, \xi = 1 \) and \( c = 0.02 \). The transaction size \( s \) of each user is uniformly distributed over [0, 1000]. Since the blockchain owner can adjust the average time of mining a block, we also varied the average time of mining a block \( \lambda \) from 100 to 1800 with increment of 212.5. Each measurement is averaged over 100 instances.

1) Impact of the number of mobile users \( N \): Figure 3 shows the impact of the total number of mobile users \( N \) on the social welfare \( S \) and the number of participating users \(|W|\). We fix \( T = 2.5, r = 0.007 \) and \( \lambda = 600 \). We observe that \(|W| \) and \( S \) increase at diminishing rate as the base of mobile users becomes larger. Naturally, the ESP can select more winners as miners to increase the social welfare with more mobiles users. However, at the same time, the negative effects from the competition among a larger number of miners are apparent, which slows down the rise of the social welfare as well as the number of winners.

2) Impact of the fixed bonus \( T \) for mining a block and the

![Fig. 3. Impact of the number of mobile users on social welfare \( S \) and number of winners \(|W|\).](image)

![Fig. 4. Impact of fixed bonus \( T \).](image)

![Fig. 5. Impact of the transaction fee rate \( r \).](image)
transaction fee rate $r$: We set $N = 600$ and $\lambda = 600$. By fixing $r = 0.007$ and $T = 2.5$, we consider the impact of varied fixed bonus and transaction fee rate on the social welfare and the number of selected miners. From Figs. 4 and 5, we note that as if the blockchain owner raises the bonus or the transaction fee rate, more social welfare will be generated in nearly proportion. However, the number of winners increases and tends to be stable. This is because there will be fierce competition if too many miners participate in the blockchain network, which causes the loss of social welfare.

3) Impact of the average time $\lambda$ for successfully mining a block: In Fig. 6, we fix $N = 600$, $T = 2.5$ and $r = 0.007$. When the blockchain owner raises the difficulty of mining a block, represented by $\lambda$, the social welfare increases while the number of winners initially increases and then declines. Note that the user’s expected reward $R$, i.e., the valuation for edge computing service, grows with increasing $\lambda$. When the difficulty $\lambda$ is small and each user’s valuation is also small, the ESP has to accept more users, i.e., more winners, to maximize the social welfare. However, if the difficulty of mining a block becomes high and each user values the service more, the ESP can reduce the number of winning users while achieving the optimal social welfare. Another reason for the decreasing number of winners is the increasingly intense competition among them.

VI. CONCLUSIONS

In this paper, we have investigated the edge computing services that enable mobile blockchain. To efficiently allocate computing resources, we have proposed an auction-based market model to maximize the social welfare. In the auction design, we have considered allocative externalities, including the competition among the miners as well as the network effects in the blockchain network. By theoretical analysis and simulation, we have proved that the auction mechanism is truthful, individually rational and computationally efficient and solves the social welfare maximization problem. For the future work, we will consider variable demands and corresponding bids of mobile users.