Dynamo Action in Fully Convective Low-Mass Stars

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Abstract. Recent observations indicate that fully convective stars can effectively build magnetic fields without the aid of a tachocline of shear, that those fields can possess large-scale components, and that they may sense the effects of rotation. Motivated by these puzzles, we present global three-dimensional simulations of convection and dynamo action in the interiors of fully convective M-dwarfs of 0.3 solar masses. We use the Anelastic Spherical Harmonic (ASH) code, adopting a spherical computational domain that extends from 0.08-0.96 times the overall stellar radius. We find that such fully convective stars can generate magnetic fields of several kG strength, roughly in equipartition with the convective flows. Differential rotation is established in hydrodynamic progenitor calculations, but essentially eliminated in MHD simulations because of strong Maxwell stresses exerted by the magnetic fields. Despite the absence of interior angular velocity contrasts, the magnetic fields possess strong mean (axisymmetric) components, which we attribute partly to the very strong influence of rotation upon the slowly overturning flows.

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MOTIVATING QUESTIONS

Of all the many findings that have emerged from solar physics over the last two decades, the discovery of the tachocline of shear likely figures as the most significant for dynamo theory. Revealed by helioseismology (e.g., Gough & Toomre 1991), the tachocline is a narrow boundary layer situated near the base of the convection zone, in which the solar angular velocity transitions from differential rotation in the unstable envelope to nearly solid-body rotation beneath it. Its discovery motivated the now-prevalent “interface dynamo” paradigm, in which the tachocline plays a pivotal role – both because of its shear, which helps stretch and organize toroidal fields, and because of the stable stratification, which may allow fields to be greatly amplified before becoming susceptible to magnetic buoyancy (e.g., Ossendrijver 2003). Recently, simulations have helped affirm the likely importance of the tachocline in generating the Sun’s organized magnetism: global simulations that include a tachocline (Browning et al. 2006; Browning et al. 2007) give rise to magnetic fields that appear more “solar-like” than do simulations of the convection zone alone (Brun, Miesch & Toomre 2004). (See proceedings by Miesch et al. 2007, this volume.) And if the tachocline is vital in building the Sun’s orderly magnetism, then similar boundary layers are probably likewise central in the dynamo action of any star that possesses both a convective envelope and a radiative, stable core.

But not all stars are so configured. Moving down the main sequence to stars of lower mass, the size of the inner stable layer gradually decreases. By around spectral class M3, corresponding to a mass of about 0.35 solar masses, it has vanished entirely, and...
convection occurs throughout the interior. In a fully convective star, there is no radiative-convective interface, and so presumably no interface dynamo – implying that if fields are built in such stars, the dynamo responsible must operate rather differently than in the Sun. A natural conclusion is that such fully convective stars should show surface magnetic field behavior quite distinct from that realized in their more massive cousins (e.g., Durney, De Young & Roxburgh 1993; Chabrier & Kuker 2006).

Surprisingly, observations of low-mass stars have partly confounded even this rather basic expectation. In a recent breakthrough, Donati et al. (2006) used Zeeman Doppler imaging to constrain the surface magnetic field morphology of a very rapidly rotating fully convective M-dwarf. The kG-strength fields they detected were predominantly axisymmetric and large-scale, in contrast to the purely small-scale fields predicted by some models (e.g., Durney et al. 1993). Donati et al. (2006) also found that their target star did not exhibit any surface differential rotation. Separately, other observations have demonstrated that strong magnetic activity appears to be quite common in fully convective stars: indeed, the fraction of M-stars showing with detectable activity actually peaks at about spectral class M8 (West et al. 2004).

Further puzzles have come from study of the remarkable relation between stellar magnetic activity and rotation rate. In stars like the Sun, observations indicate that chromospheric and coronal activity increase with rotation rate, then “saturate” above a threshold velocity (e.g., Noyes et al. 1984; Delfosse et al. 1998; Pizzolato et al. 2003). Many authors have noted that this rotation-activity correlation is tightened considerably when rotation is expressed in terms of the Rossby number, estimated in these studies as just $P_{\text{rot}}/\tau_c$, with $\tau_c$ a typical convective overturning time; likewise, the threshold rotational period needed for “saturated” activity is apparently a function of stellar mass, varying from about a day in early-G stars to about ten days in early M-stars (Pizzolato et al. 2003). This rotation-activity connection appears to persist in some fashion into the mid-M stars: Mohanty & Basri (2003) argued that a sample of stars ranging from M0 to M5 showed a common “saturation-type” rotation-activity relationship, with observed activity roughly independent of rotation rate above a threshold value. Because measuring the rotation rates of the slowest rotators is difficult, it remains unclear whether magnetic activity in these stars increases gradually with rotation rate as in solar-like stars, or instead transitions more abruptly. In the late-M spectral classes and beyond, the rotation-activity relation does appear to break down (e.g., Reiners et al. 2007), but this may occur well beyond the point where stars become fully convective. In all, it appears that fully convective stars can certainly act as magnetic dynamos, can (in at least some cases) build large-scale magnetic fields (Donati et al. 2006), and may sense the effects of rotation in roughly the same fashion as more massive stars.

Motivated by these puzzles, we have carried out global 3–D nonlinear simulations of convection and dynamo action in the interiors of fully convective M-dwarfs. In the sections that follow, we briefly describe our modeling (§2), its principal results (§3), and the implications of our work (§4). The simulations here are described in more detail in Browning (2007).
MODEL FORMULATION

Our simulations are intended to be simplified models of 0.3 solar mass M-stars rotating at the solar angular velocity ($\Omega = 2.6 \times 10^{-6}$ s$^{-1}$). We, like a host of others at this meeting, utilize the Anelastic Spherical Harmonic (ASH) code, which solves the 3-D Navier-Stokes equations with magnetism in the anelastic approximation (Clune et al. 1999; Miesch et al. 2000; Brun, Miesch & Toomre 2004). Our spherical computational domain extends from 0.08-0.96R, with R the overall stellar radius of $2.07 \times 10^{10}$ cm. We exclude the inner few percent of the star from our calculations both because the coordinate systems employed in ASH are singular there, and because the small numerical mesh sizes at the center of the star would require impractically small timesteps. Our computations also do not extend all the way to the stellar surface, because the very low densities in the outer few percent of the star favor the driving of fast, small-scale motions that we cannot resolve.

The initial stratifications of the mean density, energy generation rate, gravity, radiative diffusivity, and entropy gradient $dS/dr$ are adopted from a 1-D stellar model (I. Baraffe, private communication, after Baraffe & Chabrier 2003). We update these thermodynamic quantities throughout the course of the simulation, as the evolving convection modifies the spherically symmetric mean state. Variables are expanded in terms of spherical harmonic basis functions $Y^m_l(\theta, \phi)$ in the horizontal directions and Chebyshev polynomials $T_n(r)$ in the radial. In the simulations here, we have retained spherical harmonic degrees up to $l_{\text{max}} = 340$ (implying $N_\theta = 512$ and $N_\phi = 1024$) and retain $N_r = 192$ Chebyshev collocation points. As with all numerical simulations, we must employ eddy viscosities and diffusivities that are vastly greater than their counterparts in actual stars; here we have taken these to be constant in radius, and adopted a Prandtl number $\nu/\kappa = 0.25$ and a magnetic Prandtl number $Pm = \nu/\eta = 8$. We also conducted other simulations at varying $\nu, \eta$, and $Pm$, but have chosen to focus on one case in this paper for clarity; the others are briefly described in Browning (2007).

Only one prior large-scale numerical MHD simulation has modeled fully convective stars (Dobler, Stix & Brandenburg 2006, hereafter DSB06). Our approach differs from theirs in a few key ways. In order to keep thermal relaxation timescales small, DSB06 rescaled the stellar luminosity to a value many orders of magnitude greater than appropriate for an actual M dwarf; because this implies a commensurate increase in typical convective velocities, they also were forced to consider very rapid rotation rates in order to keep the Rossby number of their simulations realistic. This rescaling is needed in their simulations partly because they adopt the same thermal diffusivities for the mean temperature gradient and for the small-scale turbulent temperature fluctuations; in ASH, the mean temperature gradient is acted on by the thermal diffusivity $\kappa_{\text{r}}$ taken from a 1–D stellar model, whereas $\kappa$ for the turbulent temperature field is (as in DSB06) a sub-grid-scale eddy diffusivity. Thus we employ the actual stellar luminosity, rotation rate, etc. in our modeling. The method adopted in ASH allows to examine the radiative flux in the interior with reasonable fidelity, since $\kappa_{\text{r}}$ in our models is essentially set by the radiative opacities of the 1–D stellar model. Neither strategy is perfect: ours does not allow for adjustments to the thermal stratification that occur over the very long thermal timescale. Our simulations also differ from those of DSB06 in a few smaller ways. The overall density contrast between the inner and outer boundaries in our models is about 170,
consistent with the contrast between 0.1 and 0.96R in the 1–D stellar model we used for our initial conditions. In DSB06, the density varied by a factor of about 5 from center to surface; the larger density contrasts in our modeling have substantial impact on the morphology of the convective flows. The boundary conditions adopted in DSB06 also differ from ours; theirs is closer to a no-slip boundary condition than to the stress-free boundaries used here.

**FLOWS AND MAGNETIC FIELDS REALIZED**

**Morphology of the Flows**

The convective flows realized in our simulation possess structure on many scales. Near the surface, there is a marked asymmetry between upflows and downflows: the former tend to be broad and weak, whereas the latter are strong and narrow. This asymmetry is a generic feature of turbulent compressible convection (e.g., Brummell et al. 2002), and arises mainly because of the strong density stratification. At depth, the flows are weaker and of larger physical scale: motions can span large fractions of a hemisphere and extend radially for great distances. Motions at the two depths are linked, with small downflow plumes near the surface coalescing as they descend to form the broader flows in the deep interior. The contrast between flows at the two depths is apparent in Figure 1a,d, which show the radial velocity on spherical surfaces at \( r = 0.88R \) and \( r = 0.24R \) respectively. The flow amplitudes also vary appreciably with depth, with typical rms velocities declining by a factor of about ten in going from the surface to the center; because the typical pattern scale of the convection is also greater at depth, the convective overturning timescale varies by a factor of about 20 across the domain.

The variation in flow amplitude is linked to both the density stratification and to variations with radius in the amount of energy that must be transported by convection. Although the star is unstably stratified everywhere, the radiative flux actually carries a majority of the energy at small radii. Together with the overall increase of the total luminosity with radius (out to the radius where nuclear energy generation stops), this implies that the total luminosity carried by convection peaks at large radii (around \( r = 0.80R \)). Thus the convective velocity is also appreciably greater near the surface than at depth.

**Dynamo Action Achieved**

The flows act as a magnetic dynamo, amplifying a tiny seed field by many orders of magnitude and sustaining it against Ohmic decay. The magnetic energy (ME) grows exponentially until it is approximately in equipartition with the flows. Over the last 200 days of the simulation, a period during which no sustained growth or decay of the various energy densities was evident, ME was approximately 120% of the total kinetic energy KE (relative to the rotating frame) and about 140% of the convective (non-axisymmetric) kinetic energy (CKE).
As the fields grow, they react back on the flows through the Lorentz force. Thus KE begins to decline once ME reaches a threshold value of about 5% of KE; here this decline is associated mainly with a decrease in the energy of differential rotation DRKE, whereas CKE remains largely unaffected by the growing fields. In the kinematic phase, DRKE is approximately 6 times CKE; after saturation of the dynamo, DRKE/CKE is only about 0.2-0.4.

**Morphology of the Fields**

Like the flows that build them, the magnetic fields possess both intricate small-scale structure as well as substantial large-scale components. The typical length scale of the field increases with depth, partly tracking the radial variation in the size of typical convective flows. A sampling of such behavior is provided by Figure 1, which shows the radial field $B_r$ and azimuthal field $B_\phi$ on surfaces at two depths.

By decomposing the magnetism into its azimuthal mean (TME), and fluctuations around that mean (FME), we can gain a coarse estimate of the typical size of field structures: if the field is predominantly on small scales, only a small signal will survive.

**FIGURE 1.** Radial velocity and magnetic fields at a single instant, shown on spherical surfaces at two depths.
this azimuthal averaging. In these simulations, TME accounts for about 20% of the total magnetic energy in the bulk of the interior; it is smallest near the surface (where TME \( \approx 5\% \) ME), and largest (as a fraction of ME) at depth.

It is striking that the axisymmetric mean fields account for a fairly large fraction of the total magnetic energy. In simulations of the bulk of the solar convective envelope, TME was typically only about 3% of ME (Brun et al. 2004); in simulations including a tachocline of shear, similar TME/ME ratios to those reported here were attained only within the stably stratified tachocline itself (Browning et al. 2006). Similarly, Brun, Browning & Toomre (2005) found that TME/ME \( \approx 0.05 \) within most of the convective cores of A-type stars, with higher values achieved only within a shear layer at the boundary of that core. Power spectra constructed for the simulation here confirm the impressions above: at depth, the magnetic field is dominated by the largest-scale components, while near the surface it is more broadly distributed in \( \ell \). Note that the field strength is not given simply by equipartition at each spherical harmonic degree; rather, ME can substantially exceed on some scales (see Browning 2007).

The mean (axisymmetric) fields realized in the simulation are remarkably strong and stable. Mean toroidal field strengths can exceed 10 kG in some locations; some prominent field structures persist for thousands of days. The overall field polarity has flipped only once in roughly 30 years of simulated evolution: this, too, is in sharp contrast to simulations of solar convection without a tachocline (Brun et al. 2004), in which the field polarity flipped at irregular intervals of less than 600 days.

**Establishment and Quenching of Differential Rotation**

Our hydrodynamical simulation began in a state of uniform rotation, but convection quickly established interior rotation profiles that varied with radius and latitude. The resulting differential rotation, displayed in Figure 2 (contour plot and panel \( b \)), was similar to that observed at the solar surface, in that the equator rotated more rapidly than the poles; unlike the solar convective envelope, our simulation also exhibited strong radial angular velocity contrasts, with the outer regions rotating more rapidly than the interior. The interior angular velocity profile was largely constant on cylindrical lines parallel to the rotation axis, in keeping with the strong Taylor-Proudman constraint. The overall angular velocity contrast between the equator and 60 degrees latitude was about 90 nHz, implying \( \Delta \Omega / \Omega \approx 22\% \).

The interior rotation profiles are quite different in the presence of strong dynamo-generated magnetic fields. In our MHD simulation, the magnetic fields react back strongly upon the flows, acting to essentially eliminate the differential rotation. This behavior is assessed in Figure 8\( a \), which shows the angular velocity \( \hat{\Omega} \) as a function of radius along cuts at various latitudes. The interior is in nearly solid body rotation; the angular velocity contrasts realized in the progenitor hydrodynamic case have been almost entirely eliminated. This results largely from the action of strong Maxwell stresses that effectively oppose the transport of angular momentum by Reynolds stresses.
FIGURE 2. Interior angular velocity established in the hydrodynamic progenitor (contour plot and panel $b$), and in the MHD simulation (panel $a$). Shown as a contour plot is the longitudinal velocity $V_\phi$, averaged in time and in longitude; the right panels show the angular velocity $\Omega$ as a function of radius along latitudinal cuts at 0, 30, and 45 degrees. The MHD simulation rotates essentially as a solid body.

CONCLUSIONS AND PERSPECTIVES

The simulation presented here is, we believe, the most faithful description yet achieved of dynamo action in a fully convective M-dwarf. Although we have made many simplifications in our modeling, some of the main findings may turn out to be robust. We summarize some of these findings and their implications here.

The strong stratification plays a major role in setting convective flow amplitudes and pattern scales, and thus indirectly magnetic field strengths and morphologies. Although our model star is unstably stratified everywhere, there are still two conceptually distinct regions: one near the surface where convection is quite vigorous and small-scale, and a more quiescent deep interior.

Convection establishes differential rotation in hydrodynamic cases, but the angular velocity contrasts are essentially eliminated in MHD simulations. The strong Maxwell stresses ultimately yield an interior in nearly solid-body rotation.

The magnetic fields realized here possess structure on many spatial scales. The axisymmetric mean component accounts for a surprisingly high percentage of the total magnetic energy ($\sim 20\%$), despite the absence of persistent angular velocity contrasts. The polarity of mean fields is remarkably stable, having flipped only once in the $\sim 30$ years we have so far evolved the simulation.

We attribute some of these effects to the strong influence of rotation upon the slowly overturning flows. Although these simulations rotate at the solar angular velocity, the influence of rotation is much stronger than in the Sun because the luminosity, and hence the rms convective velocity, is much lower. Comparison to prior simulations of the Sun and A-type stars (Browning et al. 2004; Brun et al. 2005; Brun et al. 2004; Browning et al. 2006) suggests that there are plausibly three different regimes of behavior, separated primarily by the Rossby number. When rotation is weak, magnetic energies of less than $\sim$...
30% KE are typically realized, and do not greatly modify the differential rotation established in hydrodynamic cases. When rotation is somewhat stronger, yielding ME greater than 30% KE, cyclical variations of magnetism and differential rotation are possible; when it is stronger still (as in the simulation here, and the most rapidly rotating cases of Brun, Browning & Toomre 2005), ME can exceed KE without the aid of differential rotation, and any persistent angular velocity contrasts are strongly quenched. These conclusions are also (tentatively) in keeping with calculations of magnetism in rapidly rotating Sun-like stars (Brown et al., this volume). These ideas lead to some straightforward observational predictions – namely that magnetic, rapidly rotating M-stars should seldom exhibit differential rotation, that less rapidly rotating M-stars should generally exhibit weaker fields and (for very slow rotation) may show differential rotation, and that the axisymmetric component of the field should generally increase with rotation rate.

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