Crack width evaluation of Reinforced Concrete beam in flexure

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Abstract. Crack width evaluation of flexural RC member is evaluated analytically by two significant formulas given by researchers and four standard code i.e. IS 456-2000, BSEN 1992-1-1-2004, ECP 203-2007, ACI 318-95 and compared. Parameters such as clear cover, the effect of diameter, variation of depth, number of bars with the same cross-section and increasing external moment are considered for computing the crack width. From the study of results of this study, it can be concluded that clear covers estimated based on IS 456 2000 are highest and those estimated based on EGYPTIAN CODE; 203-2007 are lowest as compared to other codes and mathematical models considered in this study. It can also be concluded that crack width increases with increase in clear cover or external moment while it decreases with increase in reinforcement diameter, depth of the beam or number of bars.

Keywords: Crack width, Crack Spacing, Flexural RC member, Steel Stress.

1. Introduction
Generally, crack width of cracks that develop during the loading of a member (flexural) should satisfy the serviceability criteria. The crack width of beams is evaluated when loading occurs at the tension or compression face of an RC beam. Due to low tensile strength, cracks occur in the RC beam. Control of cracking is important for obtaining long-term durability for concrete structures, especially for those that are subjected to aggressive environments. When the crack width is increasing, the service life of the structure will be decreasing by allowing more rapid penetration of chlorides to reach the reinforcement causing corrosion. Various factors such as high humidity, repeated loads, and gases with chemicals may cause corrosion of reinforcement and spalling of concrete. Cracking in reinforced concrete structures affects structural performance including stiffness, energy absorption, capacity etc., Consequently, there is an interest in the control of cracking by building codes and scientific organizations. Anyways, in some exceptional cases, we may check the crack width by some theoretical expressions. In some cases, shrinkage crack also may occur, we can reduce those cracks by adding extra reinforcement in reinforced concrete.

2. Objective of the work
The effect of a various parameter such as clear cover, the effect of diameter, variation of depth, number of bars with same cross-section and increasing external moment analytically determined crack width is...
studied for RC beam. The main objective of the paper is to compare the crack width of reinforced concrete beam as calculated by various codes of practices (i.e. IS: 456-2000[15], BSEN 1992-1-1-2004 [11], ECP 203-2007 [12], ACI 318-95 [13]) and two most commonly adopted formula proposed by literature.

3. An analytical method for crack width computation

3.1. Crack width prediction equations according to some building code provision

3.1.1. IS: 456-2000 [15] (See figure 1)

The crack width ($W_{cr}$) can be determined by the following formula

$$W_{cr}=\frac{3a_{cr}c_{min}}{4(2a_{cr}-c_{min})}$$

$$\varepsilon_m=\varepsilon_1+\frac{b(h-x)(a-x)}{3E_SA_s(d-x)}$$

$$I_C=\frac{bx^3}{3}+m A_{st}(d-x)^2$$

Where,

- $I_C$ = Moment of Inertia of the cracked section
- $W_{cr}$ = Crack width
- $a_{cr}$ = Distance considered from the point of the surface to the nearest longitudinal bar
- $\varepsilon_m$ = Average steel strain
- $c_{min}$ = Minimum cover of the longitudinal bar
- $h(or)D$ = Overall depth of the member [2]
- $x$ = Depth of the neutral axis [2]
- $a$ = Distance from the compression face to point at which crack width is estimated [2]
- $E_S$ = Modulus of elasticity of steel [2]
- $A_S(or)A_{st}$ = Area of tension reinforcement
- $\varepsilon_1$ = Strain at the level considered, calculation by ignoring concrete in the tension zone
- $b$ = Breadth of beam
- $d$ = Effective depth
- $m$ = Modular ratio

![Cross-section of the beam.](image)

3.1.2 BSEN 1992-1-1-2004 [11].

The equation for crack width as per EURO code

$$W_k=S_{r,\text{max}}(\varepsilon_{sm}-\varepsilon_{cm})$$

$$\frac{f_{ct,\text{eff}}(1+n_{\text{eff}})}{E_S} \geq 0.6 \frac{f_S}{E_S}$$
\[ n = \frac{E_s}{E_c} \]
\[ \rho = \frac{A_s}{A_{e,eff}} \]
\[ S_{r,\text{max}} = 3.4C + 0.425 k_1 k_2 \frac{\Phi}{\rho_{e,ff}} \]

Where,
\( W_k \) = Design crack width as per BS EN1992-1-1-2004 [11]
\( \varepsilon_{sm} \) = Mean steel strain mostly under the effect of tension stiffening or shrinkage
\( \varepsilon_{om} \) = Mean strain in concrete
\( f_s \) = Stress in tension reinforcement
\( k_1 \) = Factor that expresses the duration of loading
\( f_{ct,eff} \) = Mean value of tensile strength of the concrete
\( n \) = Modular ratio
\( A_{e,eff} \) = Effective tension area
\( k_2 \) = Coefficient which considers the bond properties of bonded reinforcement
\( k_3 \) = Coefficient of strain distribution
\( \beta \) = Coefficient which is related to average crack width
\( \beta_2 \) = Coefficient for bar bond characteristics
\( \beta_2 \) = Coefficient which accounts load duration
\( f_{scr} \) = Stress in tension longitudinal reinforcement that causes under first crack stress in the tension reinforcement
\( f_s \) = Stress in tension reinforcement
\( \Phi \) = Dia of bar
\( \rho_{e,ff} \) = Effective reinforcement ratio
\( d_c \) = Effective cover
\( M \) = Applied movement
\( m \) = Modular ratio

3.1.3 EGYPTIAN CODE; 203-2007 [12].

The formula for crack width as per Egyptian code is,
\[ W_k = \beta \varepsilon_{sm} S_{rm} \]
\[ \varepsilon_{sm} = \frac{f_s}{E_s} \left( 1 - \beta_1 \beta_2 \left( \frac{f_{scr}}{f_s} \right)^2 \right) \]
\[ S_{rm} = 50 + 0.25 k_1 k_2 \frac{\Phi \rho_{e,ff}}{f_{cr}} \]

Where,
\( W_k \) = Design crack width as per EGYPTIAN CODE; 203-2007 [12]
\( E_s \) = Modulus of elasticity for steel
\( S_{rm} \) = Average stabilized crack spacing [3]
\( \varepsilon_{sm} \) = Mean steel strain mostly under the effect of tension stiffening or shrinkage
0
\( k_1 \) = Coefficient which considers the bond properties of bonded reinforcement
\( k_2 \) = Coefficient of strain distribution
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\( \Phi \) = Dia of bar
\( \rho_{e,ff} \) = Effective reinforcement ratio
\( d_c \) = Effective cover
\( M \) = Applied movement
\( m \) = Modular ratio
Moment of inertia for cracked section

3.1.4 ACI 318-95 [13].
The equation for crack width as per ACI code is,

\[ W_{\text{max}} = 0.011 \beta f_s^3 \left( \frac{d c}{A_0} \right)^{10^{-3}} \text{mm}, \]

Where,
\[ W_{\text{max}} = \text{Design crack width as per ACI 318-95 [13]} \]
\[ h = \text{Overall depth} \]
\[ x = \text{Depth of neutral axis} \]
\[ \beta = \text{Coefficient relating the average crack width to the design value [3]} \]
\[ d_c = \text{Effective cover} \]
\[ A_0 = \text{Area of concrete surrounding each reinforcing bar, } (A_0 = \frac{2d_c b}{3}) \]
\[ f_s = \text{Stress in tension reinforcement} \]
\[ d = \text{Effective depth} \]

3.2 Other important formulas are given by researchers for crack width

3.2.1 GERGELY AND LUTZ (1968) [9].
As per Gergely, the crack width

\[ W_s = 0.011 \sqrt{\left( c + \frac{\phi}{2} \right) A_0 (f_s - 34.45) \times 10^{-3}} \]

Where,
\[ W_s = \text{Design crack width as per GERGELY AND LUTZ (1968) [9]} \]
\[ c_m_i_n = \text{Minimum cover to the longitudinal bar [2]} \]
\[ f_s = \text{Stress in tension reinforcement} \]
\[ \phi = \text{Dia of bar} \]
\[ A_0 = \text{Area of concrete surrounding each reinforcing bar [2]} \]

3.2.2 FROSCH (1999) [10].
As per Frosch, the equation of crack width,

\[ W_{\text{max}} = 2 f_s d \beta \]

Where,
\[ W_{\text{max}} = \text{Design crack width as per FROSCH (1999) [10]} \]
\[ E_s = \text{Modulus of elasticity for steel} \]
\[ f_s = \text{Stress in tension reinforcement} \]
\[ d = \text{Effective depth} \]
\[ \beta = \text{Coefficient which is related to average crack width} \]

4. Scope of the work
The limitation of crack width as per IS 456 2000 [15] for Normal and Aggressive environmental conditions are 0.1 mm and 0.3 mm respectively. The crack width as calculated based on expressions presented in section 3 for various clear cover, the effect of diameter, variation of depth, number of bars with the same cross-section and increasing external moment are presented in figure 2 to figure 6. Other details related to this parametric comparison are

Case 1: Effect of the clear cover
In this case, the clear cover is varied from 25 to 50mm with an incremental order of 5mm. However, the width is (b) 250mm, depth is (D) 400mm, area of reinforcement is (A_0) 3 numbers of 28mm diameter and moment is (M) 100kNm as considered for the comparative study based on different code provisions as well as models from literature.

Case 2: Effect of reinforcement diameter
In this case the diameter of reinforcement considered in increasing order as 20mm, 22mm, 25mm, 32mm, 36mm and 40mm with the geometrical properties like width (b) 250mm, depth (D) 400mm, clear
cover of 25mm and also moment (M) 100kNm taken for the comparative study based on different code provisions as well as models from literature [9][13].

Case 3: Effect of depth variations for RC beam
In this case, the variation of depth is considered in ascending order as 400mm, 450mm 500mm, 550mm, 600mm, 650mm with width (b) 250mm, the clear cover of 25mm, area of Reinforcement (A_{st}) 3 numbers of 28mm diameter and moment (M) 100kNm are taken for the comparative study based on different code provisions as well as models from literature.

Case 4: Effect of increasing the number of bars for constant width
In this case the variation of the number of reinforcement bars considered in ascending order as 3 to 8 with an incremental order of 1. However, width (b) 250mm, the clear cover of 25mm, the diameter of reinforcement 28mm diameter and moment(M) 100kNm are taken for the comparative study based on different code provision as well as models from literature.

Case 5: Effect of an externally applied moment concerning the constant geometrical properties
In this case the variation of moment considered in ascending order as 100 to 600kNm with an incremental order of 100kNm. However, width (b) 250mm, clear cover of 25mm, the diameter of reinforcement 28mm diameter are taken for the comparative study based on different code provisions as well as models from literature.

5. Results and Discussion
From the results, for the above 5 cases (figure) the following observation can be made [15].

1. The cases that are up to 0.1mm acceptable in Aggressive environment as per IS: 456-2000.
2. The cases that in between 0.1mm and 0.3mm are acceptable in Normal environment as per IS: 4562000.

Case 1: Effect of the clear cover
The effect of clear cover considering the crack width evaluation by various codes and models from literature in figure 2. From the figure 2, it is observed that the EGYPTIAN CODE; 203-2007 [12] gives the lowest value 0.1 mm, whereas the other codes give crack width in between 0.1 - 0.2 mm and are normal according to IS 456 2000 [15]. From figure 2 it is found that all the clear covers considered, IS 456-2000[15] shows higher crack width and EGYPTIAN CODE; 203-2007 [12] shows lowest as compared to other codes and formula consider in this study. Further, it is also observed from figure 2 that, when the clear cover increases the crack width also increases [4].

![Figure 2. Crack Width Values for Effect of the clear cover](image)

Case 2: Effect of reinforcement diameter
The effect of diameter of the reinforcement on the crack width evaluation analysis is presented in figure 3 [11]. From figure 3, it is observed that the crack width as per EGYPTIAN CODE; 203-2007 [12]
offered for 20mm diameter applicable for the normal and aggressive environment. Whereas for 22mm
diameter the value of crack width for Egyptian code is less when compared to remaining codes and
researchers. For 25mm, 32mm diameter the values of crack width for BS EN 1992-1-1-2004 [11] [14],
Egyptian Code; ECP 203-2007 [12] are less when compared to remaining codes and researchers. For
40mm diameter, the values of crack width for Egyptian code is giving less value among the remaining
codes and researchers. However, it is also observed from figure 3 that, when the reinforcement diameter
increases the crack width reduced.

![Figure 3. Crack Width Values for Effect of reinforcement diameter]

**Case 3: Effect of depth variations**
The effect of depth variations consider for the crack width evaluation analysis is presented in figure 4.
From the figure 4, it is observed that the EGYPTIAN CODE; 203-2007 [12] offered as aggressive in
nature, whereas the other considered code prevising are given normal according to IS 456-2000 [15].
From the graph it is found that all the depth variation considered, Frosch (1999) [10] shown higher crack
width and ACI 318-95 [13] shown lowest at a maximum depth of 650 mm as compared to other code
consider in this study. However, it is also observed from figure 4 that, when the depth of the beam
increases the crack width decreased.

![Figure 4. Crack Width Values for Effect of depth variation]
Case 4: Effect of increasing the no. of bars to the constant width

The effect of increasing the no. of bars to constant width consider for the crack width evaluation analysis is presented in figure 5. From the figure 5, it is observed that the EGYPTIAN CODE; 203-2007 [12] offered as aggressive in nature, whereas the other considered code prevising are given normal according to IS 456 2000 [15]. From the graph it is found that all the increase in the no. of bars considered, IS 456 2000 [15] shown higher crack width and EGYPTIAN CODE; 203-2007 [12] shown lowest as compared to other code consider in this study. However, it is also observed from figure 5 that, when the number of bars increases the crack width decreased.

Figure 5. Crack Width Values for Effect of increasing the no. of bars to the constant width

Case 5: Effect of the externally applied moment to the constant geometrical properties

The effect of moment consider for the crack width evaluation analysis is presented in figure 6. From the figure 6, it is observed that the EGYPTIAN CODE; 203-2007 [12] offered as normal in nature, whereas the other considered code prevising are given normal according to IS 456 2000 [15]. Form graph it is found that all the moments considered, all the codes and researchers shown higher values. However, it is also observed from figure 6 that, when the moment increases the crack width also increased.

Figure 6. Crack Width Values Effect of the externally applied moment to the constant geometrical properties

6. Conclusion

The crack width evaluation of RC beam based on equations given by two researchers and standard codes of different countries for different cases like the effect of clear cover, reinforcement diameter, depth variation, increasing the number of bars and externally applied moment for the same cross-section are presented in this paper. From the study of results of this study, it can be concluded that clear covers
estimated based on IS 456 2000 [15] are highest and those estimated based on EGYPTIAN CODE; 203-2007 [12] are lowest as compared to other codes and mathematical models considered in this study. It can also be concluded that crack width increases with increase in clear cover or external moment while it decreases with increase in reinforcement diameter, depth of the beam or number of bars.

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