Numerical analysis on the three-dimensional flow and heat transfer of multiple nanofluids past a Riga plate

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Abstract. This examination is passed on to decide the properties of three-dimensional flow of $H_2O/NaC_6H_5O_7$ base liquid with $Fe_3O_4/Al_2O_3$ nanoparticles confined by a Riga plate. Mathematical model is detailed as PDEs and afterward transmuted into ODEs with the assistance of similarity transformations. The subsequent system is numerically dealt with the aid of the Runge-Kutta procedure bolstered by shooting technique. Highlights of the flow field and thermal field are exemplified quantitatively through plots. Results for the local skin friction coefficient and local Nusselt number are registered and examined tabularly. It is induced that the modified Hartmann number and stretching ratio parameter ameliorate the velocity profile. Additionally, it is likewise explained that $H_2O - Al_2O_3$ nanofluid has high skin friction values and the rate of heat transfer of $NaC_6H_5O_7 - Al_2O_3$ nanofluid is more desirable.

Keywords: Three-dimensional, Nanofluids, Nanoparticles, modified Hartmann number, Riga plate.

1. Introduction

Being dispersed throughout in various base fluids the nanoparticles can revise the characteristics of the normal heat transfer fluids. This creative composition was given a particular name, nanofluids by Choi \cite{choi2009enhancing}. The potency of metal as measured by the amount was three times the measure of strength needed to produce thermal conductivities by a normal fluid. This new context of microscopic metallic particles with the base fluids presented an incredible upgrade in the liquid’s thermophysical condition. All the way, nanofluids in different modern and procedures involving nano-tech, for example, avoiding electronic gadgets to become hot, controlling vehicle warm-up, atomic reactor and numerous others. Be that as it may, a couple of late examinations toward this path can be seen through the works Mahanthesh et al. \cite{mahanthesh2019}, Krishna et al. \cite{krishna2019}, Sheikholeslami and Rokni \cite{sheikholeslami2019}, Rahman \cite{rahman2019} and Hayat et al. \cite{hayat2019}.

Liquid flow affected by magnetohydrodynamics (MHD) assumes a significant job in different mechanical and building fields, for example, MHD generators, fluid cooling covers for combination reactors, stream meters, oil advancements, siphons, and so forth. Be that as it may, electrically directing streams can likewise be impacted by electromagnetic body powers that go about as operator to control the development of liquid particles in the limit layer. Liquids with high electrical conductivity are influenced by utilizing an external magnetic field...
(around 1 Tesla). A system of this sort is used to direct the standard MHD stream. The actuated current produced by an external magnetic field is lacking for pitifully leading liquids. In this way, an outside electric field is required to get a wall parallel Lorentz power for higher and proficient stream control.

The flow analysis over a Riga plate is a modern theory which is now attracting numerous researchers in the field of fluid dynamics. In traditional MHD, the magnetic field that is applied from the outer surface has a dominance on the flow of highly electric conducting fluids. Still, on weakly electric conducting fluids this magnetic field fail to produce significant amount of current (e.g. sea water). This lag can be equalized only by the formal application of the Lorentz force in wall parallel directions to regain the effectual flow control. Gailitis and Lielausis \[7\] structured a gadget called Riga-plate to create the Lorentz force toward the path which is parallel to the wall. Riga plate is an electromagnetic actuator which includes the electrodes and magnets that are alternately arranged into their appropriate relative positions above a completely flat surface. It can be useful in diminishing drag by putting a stop to boundary layer separation and decrease the production of turbulence. The ongoing examinations talked about on Riga plate can be found from the papers of Hakeem et al. \[8\], Ragupathi et al. \[9\], \[10\], Hakeem et al. \[11\], Nasir et al. \[12\], Shafiq et al. \[13\], Zaib et al. \[14\], Abbas et al. \[15\], Iqbal et al. \[16\], Nayak et al. \[17\], Anjum et al. \[18\] and Shaw et al. \[19\].

Identified with their viscosity conduct as a component of shear rate, stress, deformation rate, and so forth., the particular idea of liquids is portrayed as Newtonian or non-Newtonian. The thickness of Newtonian liquids will keep on existing as a steady, in any case, how quick they are compelled to move through a funnel or channel. The condition of being strikingly not quite the same as Newtonian liquids, non-Newtonian liquids put in a conspicuous spot that either a non-straight connection between shear pressure and shear rate, have a yielding pressure, or consistency that depends on schedule or deformation rate. Truth be told most liquids are non-Newtonian, which implies that their thickness depends upon shear rate. Such examinations on rheological liquids are stimulating for their upgrading significance in science and building. Regardless, a relatively few non-Newtonian fluid stream issues in fluid mechanics got thought, because of their novel test to physicists, architects and mathematicians. As a piece of this numerous specialists contributed their work as in \[20\]- \[24\]. Additionally, the Casson model, structured explicitly to assess these non-Newtonian liquids has pulled in numerous specialists. For example, Durgaprasad et al. \[25\], Raju et al. \[26\], Zaigham Zia et al. \[27\] and Prashu \[28\] have examined their ongoing explores on Casson model.

![Figure 1. Flow configuration.](image)

Our inspiration for this exploration is to examine the three-dimensional flow with $Fe_3O_4$ and
Al$_2$O$_3$ nanoparticles. A writing assessment shows that no work has been done to delineate the three-dimensional flow using H$_2$O/NaC$_6$H$_9$O$_7$ based nanofluids because of a Riga plate. For the numerical arrangement, we utilize the Runge-Kutta calculation of the fourth request through the shooting iteration plan. The effect of different administering parameters is concentrated graphically and depicted in detail.

### Table 1. Thermophysical properties [23]

|        | $\rho$ ($Kg/m^3$) | $C_p$ ($J/kgK^{-1}$) | $k$ ($W/mK^{-1}$) | $\nu_f$ ($m^2/s^{-1}$) | $Pr$ |
|--------|-------------------|---------------------|------------------|-----------------|------|
| H$_2$O | 997               | 4179                | 0.613            | 8.90 x 10$^{-4}$ | 6.2  |
| NaC$_6$H$_9$O$_7$ | 989              | 4175                | 0.6376           | -               | 6.5  |
| Fe$_3$O$_4$ | 5180             | 670                 | 9.7              | -               | -    |
| Al$_2$O$_3$ | 3970             | 765                 | 40               | -              | -    |

2. Mathematical Formulation

We consider a laminar, steady, three-dimensional, incompressible H$_2$O/NaC$_6$H$_9$O$_7$ based nanofluids flow over a Riga plate. Fe$_3$O$_4$ and Al$_2$O$_3$ are chosen to be the nanoparticles. The flow is caused by the Riga plate which is put in a position $z = 0$. The $xy$–plane bares the plate where $z$ equals zero ($z = 0$) and the domain where $z > 0$ is chosen to assume that the flow occurs as shown in Fig. 1. Let the $x$–direction stretching velocity be $u = U_w(x) = ax$ and the $y$–direction stretching velocity be $v = V_w(y) = by$ of the Riga plate, respectively. Further, no attention is paid towards the viscous dissipation effects. Likewise, it is accepted that the base liquids and the nanoparticles are in thermal balance and no slip happens between them. Likewise, we expect that the rheological of state for an incompressible Casson liquid can be composed with respect to (more subtleties see Abbas et al. [15])

$$\tau = \tau_0 + \mu \gamma^*.$$  

or

$$\tau_{ij} = \begin{cases} 2 \left( \frac{\sqrt{2\Pi B} + py}{\sqrt{2\Pi}} \right) e_{ij}, & \Pi_c < \Pi \\ 2 \left( \frac{\sqrt{2\Pi B} + py}{\sqrt{2\Pi_c}} \right) e_{ij}, & \Pi_c > \Pi \end{cases}$$

The governing boundary layer equations of momentum and energy for three-dimensional flow can be written as Ganesh Kumar et al. [31]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu_f (\beta^{-1} + 1) \frac{\partial^2 u}{\partial z^2} + \frac{\pi j_0 M_0}{8 \rho_{nf} a_1} e \left( -\frac{\pi z}{a_1} \right).$$
\[
\begin{align*}
    u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \nu // f (\beta^{-1} + 1) \frac{\partial^2 v}{\partial z^2} \\
    u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} &= \alpha // f (\beta^{-1} + 1) \frac{\partial^2 T}{\partial z^2}
\end{align*}
\]

where \( u, v \) and \( w \) are the \( x, y \) and \( z \) segments of velocity, \( \beta \) means the Casson parameter, \( j_0 \) means the current density exerted to the electrodes, \( M_0 \) means the magnetic property of the perpetual magnets that are sorted out on the plate surface, \( a_1 \) means the distance across of the magnets situated in the interim isolating the electrodes.

**Table 2. Nanofluids properties**

| Properties                | Formulation                                                                 |
|---------------------------|-----------------------------------------------------------------------------|
| Density                   | \( \rho // nf = (1 - \phi) \rho // f + \phi \rho // s \)                   |
| Thermal diffusivity       | \( \alpha // nf = \frac{k // nf}{\rho // nf (C_p) // nf} \)               |
| Heat capacity             | \( (\rho C_p) // nf = (1 - \phi) (\rho C_p) // f + \phi (\rho C_p) // s \) |
| Kinematic viscosity       | \( \nu // nf = \frac{\mu // nf}{\rho // nf} \)                            |
| Dynamic viscosity         | \( \mu // nf = \frac{(1 - \phi)^2 \mu // f}{k // s + 2k // f - 2\phi (k // f - k // s)} \) |
| Thermal conductivity      | \( \frac{k // nf}{k // f} = k // s + 2k // f + \phi (k // f - k // s) \)   |

The boundary conditions for the present flow analysis are:

\[
\begin{align*}
    u &= U_w(x), \quad v = V_w(y), \quad w = 0, \quad T = T_w, \quad \text{at} \quad z = 0, \\
    u &\to 0, \quad v &\to 0, \quad T &\to T_{\infty} \quad \text{as} \quad z &\to \infty
\end{align*}
\]

The fluid temperature of the wall is \( T_w \).

\[
\begin{align*}
    u &= ax f'(\eta), \quad v = ay g'(\eta), \quad w = -\sqrt{ax//f} (f(\eta) + g(\eta)), \\
    \theta(\eta) &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \eta = z \sqrt{\frac{\alpha}{\nu // f}}
\end{align*}
\]

### 3. Similarity Transformation and Non-dimensionalization

The continuity equation Eq. (3) is automatically satisfied by assisting the similarity transformations in Eq. (8) whereas the transformations change the Eqs. (4), (5) and (6) as

\[
\begin{align*}
    (\beta^{-1} + 1) \varphi_1 f'''' + (f + g) f'' - f'^2 + Q \varphi_2 e^{-n A} &= 0 \\
    (\beta^{-1} + 1) \varphi_1 g'''' + (f + g) g'' - g'^2 &= 0 \\
    \left( \frac{\varphi_1}{\varphi_3} \right) \left( \frac{1}{Pr} \right) \theta'' + (f + g) \theta' &= 0
\end{align*}
\]
The transformed boundary conditions are

\[ f = 0, \quad g = 0, \quad f' = 1, \quad g' = \alpha, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \]

\[ f' \to 0, \quad g' \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \]

(12)

where, \( \phi \) means the nanoparticle of volume fraction and \( \kappa_0 = 1 \quad (1 - \phi)^{2.5} \), \( \kappa_1 = 1 \quad (1 - \phi) \), \( \kappa_2 = \frac{1}{1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right)} \), \( \kappa_3 = \frac{1}{1 - \phi + \phi \left( \frac{\rho C_p}{\rho C_p} \right)} \), \( \kappa_4 = \frac{k_{nf}}{k_f} \), \( \alpha = \frac{b}{a} \) is the stretching ratio parameter, \( Pr = \frac{\mu_f (C_p)_f}{k_f} \) is the Prandtl number, \( Q = \frac{\pi j_0 M_0 x}{8 \rho_f U_w^2} \) is the modified Hartmann number, \( A = \frac{\pi}{a_1} (\frac{a}{\nu})^{-1/2} \) is the dimensionless parameter.

### Table 3. Estimation of similarities with Wang [29], Hayat et al. [30] and Ganesh Kumar et al. [31] for individual values of \( \alpha \)

| \( \alpha \) | Wang [29] | Hayat et al. [30] | Ganesh Kumar et al. [31] | Present Result |
|---|---|---|---|---|
| \( f''(0) \) | \( g''(0) \) | \( f''(0) \) | \( g''(0) \) | \( f''(0) \) | \( g''(0) \) |
| 0 | -1.0488 | -0.1945 | -1.048810 | -0.19457 | -1.04906 | -0.19457 | -1.04881108 | -0.19456383 |
| 0.25 | -1.0930 | -0.4652 | -1.093095 | -0.465205 | -1.09324 | -0.46532 | -1.09309502 | -0.46520485 |
| 0.5 | -1.1344 | -0.7946 | -1.134500 | -0.794620 | -1.13458 | -0.79470 | -1.13448575 | -0.79461826 |
| 0.75 | -1.1737 | -1.1737 | -1.173721 | -1.173721 | -1.17378 | -1.17378 | -1.17372074 | -1.17372074 |

### 4. Physical Quantities of Interest

If \( C_{fx} \) and \( C_{fy} \) are the local skin-drag coefficients and \( Nu_x \) is the local Nusselt number, at that point we have

\[
C_{fx} = \frac{\tau_w x}{\rho_f U_w^2}, \quad C_{fy} = \frac{\tau_w y}{\rho_f V_w^2}, \quad Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}
\]

(13)

where \( \tau_w \) and \( q_w \) are the wall shear stress and the wall heat flux, respectively. The above equation in dimensionless form can be written as

\[
Re_x^{1/2} C_{fx} = \kappa_0 (\beta^{-1} + 1) f''(0),
\]

\[
Re_y^{1/2} C_{fy} = \alpha^{-3/2} \kappa_0 (\beta^{-1} + 1) g''(0)
\]

(14)

\[
Re_x^{-1/2} Nu_x = -\kappa_4 \theta'(0)
\]

where the meaning of Reynold’s number is

\[
Re_x = \frac{U_w(x)x}{\nu_f} \quad \text{and} \quad Re_y = \frac{V_w(y)y}{\nu_f}.
\]
5. Numerical Solution

Equations (9)-(11) are highly non-linear and it is very tough to find solutions to them analytically. So, an alternate way is to assist the numerical methods. Here we opt to use the Runge-Kutta method, a classical method. In order to assist this method we first need to transform the BVP into an IVP, this is done with the shooting technique. The transformed IVP is

\[ \begin{align*}
\zeta_1' &= \zeta_2 \\
\zeta_2' &= \zeta_3 \\
\zeta_3' &= (\beta^{-1} + 1) \left\{ \left( - (\zeta_1 + \zeta_4) \zeta_3 - Q \lambda_2 e^{-\eta A} + \zeta_2^2 \right) / \lambda_1 \right\} \\
\zeta_4' &= \zeta_5 \\
\zeta_5' &= \zeta_6 \\
\zeta_6' &= (\beta^{-1} + 1) \left\{ \left( - (\zeta_1 + \zeta_4) \zeta_6 + \zeta_5^2 \right) / \lambda_1 \right\} \\
\zeta_7' &= \zeta_8 \\
\zeta_8' &= -\left( \frac{\lambda_5}{\lambda_4} \right) Pr (\zeta_1 + \zeta_4) \zeta_8
\end{align*} \]

(15)

(16)

(17)

with the boundary conditions

\[ \zeta_1(0) = 0, \quad \zeta_4(0) = 0, \quad \zeta_2(0) = 1, \quad \zeta_5(0) = \alpha, \quad \text{and} \quad \zeta_7(0) = 1 \]

(18)

The guess for the initial values are made to \( \zeta_3(0) \) i.e., \( f''(0) \), \( \zeta_6(0) \) i.e., \( g''(0) \) and \( \zeta_8(0) \) i.e., \( \theta'(0) \). Then the results are checked with the boundary conditions and the convergence criterion, if it is satisfied no further modification in the guess values is required. If not the guess values are again modified until it matches the boundary conditions (18) and the convergence criterion of \( 10^{-8} \).

\[ \text{Figure 2. } Q \text{ influencing } f''(\eta). \]

6. Results and Discussion

The numerical examination of this issue is gained by the fourth order RK strategy with the shooting plan. Graphical portrayal underpins the examination of important parameters which
Figure 3. $A$ influencing $f'(\eta)$.

Figure 4. $\beta$ influencing $f'(\eta)$ & $g'(\eta)$.

are related with this issue. Accepting $\beta \to \infty$ we switch over to the Newtonian base liquid ($H_2O$) case. It is through Table 1 the thermophysical properties of the base liquids and its nanoparticles are listed point by point and the relating formulation in Table 2. The precision of the numerical arrangement has been cultivated by contrasting and crafted by Wang et al. [29], Hayat et al. [30] and Ganesh Kumar et al. [31]. Following this Table 3 assumes the liability of speaking to the legitimacy of the current outcomes.

The assortment of changed Hartmann number $Q$ on the velocity assignment is portrayed in Figs. 2(a) and 2(b) for the two geometries. The higher $Q$ brings about an intensification of velocity appointment and the limit layer degree. In the way that higher estimations of $Q$ relate to the force of the outside electric field loosening up over the run of the typical level, heading up in the reduction of wall parallel Lorentz power. In this manner, velocity appropriation improves.

Figs. 3(a) and 3(b) uncovers the lead of the velocity appropriation being passed on by dimensionless parameter $A$. It is essential that velocity allocation must show diminishing
behavior for colossal estimations of dimensionless parameter $A$ for picked cases. This is an eventual outcome of shrinkage in the limit layer degree. Further, it is seen that nanoparticles have an overwhelming effect on the velocity apportionment with the base fluids than nanoparticles.

![Figure 5. $\alpha$ influencing $f'(\eta)$ & $g'(\eta)$.](image)

![Figure 6. $\phi$ influencing $\theta(\eta)$.](image)

Fig. 4 is planned to show the essential features of Casson parameter $\beta$ on the $x-$ and $y-$direction velocities. The Casson parameter is the tie-up between the viscosity and yield stress of the fluid, so an elevation in $\beta$ leads to upsurge of the fluid viscosity level. Thus it results in a thinner momentum boundary layer and lower fluid velocity.

Fig. 5(a) displays the velocity appropriation of $H_2O$ based nanofluid for extending values of stretching ratio parameter $\alpha$. From this figure, it is seen that, development in $\alpha$ prompts to
Table 4. Estimations of local skin-drag coefficients for $H_2O$ base fluid with $Fe_3O_4$ and $Al_2O_3$ nanoparticles

| Parameter | Values | $\kappa_0f''(0)$ | $\alpha^{-3/2}\kappa_0g''(0)$ |
|-----------|--------|------------------|---------------------|
|           | $Fe_3O_4$ | $Al_2O_3$ | $Fe_3O_4$ | $Al_2O_3$ |
| $A$       | 0.5    | -1.427717       | -1.360665           | -1.740473 | -1.664911 |
|           | 1.0    | -1.457401       | -1.390755           | -1.733119 | -1.657570 |
|           | 1.5    | -1.469859       | -1.403414           | -1.730608 | -1.655076 |
| $\alpha$  | 0.5    | -1.426662       | -1.360878           | -1.797962 | -1.719701 |
|           | 1.0    | -1.534191       | -1.463697           | -1.600103 | -1.530340 |
|           | 1.5    | -1.632579       | -1.557776           | -1.526964 | -1.460084 |
| $\phi$    | 0.05   | -1.242022       | -1.208473           | -1.498083 | -1.460182 |
|           | 0.10   | -1.449106       | -1.382339           | -1.735027 | -1.659470 |
|           | 0.15   | -1.676313       | -1.575351           | -1.995290 | -1.880873 |
| $Q$       | 0.5    | -1.220785       | -1.150545           | -1.763742 | -1.687919 |
|           | 1.0    | -0.950529       | -0.875863           | -1.794484 | -1.718410 |
|           | 1.5    | -0.691828       | -0.612744           | -1.821544 | -1.745259 |

diminish the velocity appropriation along $x$–direction while a backward propensity can be seen for velocity appropriation along the $y$–direction. The rising estimations of $\alpha = b/a$, starts an addition in $b$ or crumbling in $a$. In this manner, alongside the $y$–direction the speed increments, also, downturns along the $x$–direction. Practically this is identical with the case of $NaC_6H_9O_7$ based nanofluid as shown in Fig. 5(b).

![Figure 7](image)

**Figure 7.** $\alpha$ influencing $\theta(\eta)$.

The reaction of the temperature profile for the variation in nanoparticle volume fraction $\phi$ appears in Figs. 6(a) and 6(b), respectively. These figures demonstrate that in the whole domain the temperature profile is stimulated with the help of $\phi$. 
Table 5. Estimations of local skin-drag coefficients for NaC₆H₉O₇ base fluid with Fe₃O₄ and Al₂O₃ nanoparticles

| Parameter Values | (β⁻¹ + 1) z₀f''(0) | (β⁻¹ + 1) z₀α⁻³/₂g''(0) |
|------------------|---------------------|-------------------------|
| Fe₃O₄            | Al₂O₃               | Fe₃O₄                  | Al₂O₃                  |
| 0.5              | -2.519166           | -2.402825               | -3.008328              | -2.876584              |
| 1.0              | -2.556223           | -2.440477               | -3.000808              | -2.869116              |
| 1.5              | -2.571677           | -2.456201               | -2.998532              | -2.866868              |
| α                | 0.5                | -2.506483               | -2.392291              | -3.110445              | -2.974031              |
| 1.0              | -2.695424           | -2.572951               | -2.771440              | -2.649815              |
| 1.5              | -2.867975           | -2.737937               | -2.645579              | -2.529443              |
| φ                | 0.05               | -2.185372               | -2.127128              | -2.590103              | -2.524001              |
| 0.10             | -2.545956           | -2.430034               | -3.002634              | -2.870924              |
| 0.15             | -2.941585           | -2.766279               | -3.455611              | -3.256245              |
| Q                | 0.5                | -2.265696               | -2.145896              | -3.029555              | -2.897437              |
| 1.0              | -1.921922           | -1.797233               | -3.061205              | -2.928625              |
| 1.5              | -1.584385           | -1.454781               | -3.091008              | -2.958006              |
| β                | 0.5                | -2.545956               | -2.430034              | -3.002634              | -2.870924              |
| 1.0              | -2.070096           | -1.975304               | -2.453366              | -2.345813              |
| 1.5              | -1.885839           | -1.799245               | -2.240436              | -2.142249              |

Table 6. Estimations of local Nusselt number for H₂O/NaC₆H₉O₇ base fluids with Fe₃O₄ and Al₂O₃ nanoparticles

| Parameter Values | H₂O | NaC₆H₉O₇ |
|------------------|-----|---------|
|                 | Fe₃O₄ | Al₂O₃ | Fe₃O₄ | Al₂O₃ |
| A                | 0.5  | 2.504241 | 2.542082 | 2.701428 | 2.739622 |
| 1.0              | 2.498923 | 2.536575 | 2.699616 | 2.737724 |
| 1.5              | 2.496769 | 2.534340 | 2.698870 | 2.736942 |
| α                | 0.5  | 2.419627 | 2.456172 | 2.613276 | 2.650217 |
| 1.0              | 2.796269 | 2.838324 | 3.019979 | 3.062561 |
| 1.5              | 3.122216 | 3.169168 | 3.374295 | 3.421880 |
| φ                | 0.05 | 2.390292 | 2.409610 | 2.569315 | 2.588567 |
| 0.10             | 2.500394 | 2.538100 | 2.700118 | 2.738250 |
| 0.15             | 2.615243 | 2.670876 | 2.835428 | 2.892574 |
| Q                | 0.5  | 2.530489 | 2.569339 | 2.711367 | 2.749995 |
| 1.0              | 2.564015 | 2.604173 | 2.724904 | 2.764129 |
| 1.5              | 2.594379 | 2.635734 | 2.737933 | 2.777733 |
| β                | 0.5  | -      | 2.700118 | 2.738250 |
| 1.0              | -      | 2.659452 | 2.698080 |
| 1.5              | -      | 2.638181 | 2.677073 |
Fig. 7(a) and 7(b) reveal the attributes of stretching ratio parameter \( \alpha \) on the temperature appropriation. The power of warmth present near the plate comparatively equals the heat present in the ambient region with the rising values of \( \alpha \). This response is due to the expansion of the thermal boundary region. Separate tables, Table 4 and Table 5 are given to show the effect of significant parameters on the skin-drag coefficient for both base liquids. We watched raise in the local skin-drag coefficient for expanded \( Q \) and \( \beta \). Further, Table 6 puts in plain view the local Nusselt number qualities expressing that it is expanded for \( Q \), \( \alpha \) and \( \phi \).

7. Conclusion

A point by point examination and investigation on the momentum and energy transfer of attributes of three-dimensional flow of \( H_2O/NaC_6H_9O_7 \) based nanofluids with \( Fe_3O_4/Al_2O_3 \) nanoparticles over a Riga plate is finished. Different non-dimensional controlling parameters that impact the basic profiles and the physical amounts of intrigue are talked about and introduced through plots and tables. The ends are as per the following:

- Stretching ratio parameter and modified Hartmann number ameliorate the velocity profile.
- The nanoparticle volume fraction parameter benefits the temperature profile with its hiking values. Also, the velocity profile grows less due to increasing \( \beta \).
- The modified Hartmann number organizes the numerical estimations of skin-drag factor to become more intense.
- Giving special importance to the relevant parameters make it more clear that \( H_2O - Al_2O_3 \) nanofluid has high skin friction values and the rate of heat transfer of \( NaC_6H_9O_7 - Al_2O_3 \) nanofluid is more desirable.

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**NOMENCLATURE**

| Symbol | Definition |
|--------|------------|
| A      | Material constants |
| a, b   | Constants |
| a1     | The width of the magnets between the electrodes |
| C       | Specific heat coefficient [J/kg.K] |
| Cf, Cg  | Skin friction coefficients |
| f, g    | Dimensionless stream functions |
| j0      | Applied current density in electrodes |
| k       | Thermal conductivity |
| M0      | Magnetization of the permanent magnets |
| Nu       | Local Nusselt number |
| Pr   | Prandtl number of base fluid |
| Q       | Modified Hartmann number |
| qw      | Wall heat flux [W/m^2] |
| Re x, Re y | Local Reynolds number |
| T       | Local fluid temperature [K] |
| Tw      | Temperature at the surface of the plate [K] |
| T∞      | Free stream temperature [K] |
| uw, vw  | Stretching velocities |
| u, v, w | Components of velocity [m/s] |
| x, y, z | Coordinates [m] |

**Greek Letters**

| Symbol | Definition |
|--------|------------|
| α      | Stretching ratio parameter |
| αnf    | Thermal diffusivity of the nanofluid [m^2/s] |
| β      | Casson parameter |
| ϕ      | Volume fraction of nanoparticle |
| η      | Similarity variable |
| μ      | Absolute viscosity [Ns/m^2] |
| ν      | Kinematic viscosity [m^2/s] |
| ρ      | Density [kg/m^3] |
| ρcp    | Heat capacity [kg/m^3K] |
| θ      | Dimensionless temperature |
| Ψ      | Stream function |
| τw      | Viscous stress at the surface of the plate [Nm^-2] |

**Subscripts**


\[ n_f \quad \text{Nanofluid} \]
\[ f \quad \text{Base fluid} \]
\[ s \quad \text{Solid nanoparticles} \]
\[ \infty \quad \text{Boundary layer edge} \]

\textbf{Superscript}

\[ r \quad \text{Differentiation with respect to } \eta \]