Matching orthogonal code symbols and modulation methods

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Abstract. The objective of the article consists in a development and a research of the offered by the author orthogonal coding as a way of noise immunity’s increase using the example of the information communication systems with a phase-shift keying. It is shown that the orthogonal coding allows to provide the required quality of communication with the smaller energy cost. The energy gain in signal-to-noise ratio (up to 4.5 dB in the channels with the AWGN without error correcting) is provided by more effective use of energy of transmitted signals without increase in complexity and cost of transmitting/receiving devices. The main attention is paid to a development of the procedure of harmonization of symbols of an orthogonal code with phase-shift keying.

1. Introduction

In modern ways of noise immunity’s increase a channel is fixed as a rule. Thus, transitional probabilities of output signals (with the fixed probabilities of input signals) don’t change [1]. As a result, the number of phases in modulation in such systems doesn’t change also [2]. The feature of signals’ processing in receivers in telecommunication systems is the basis of the offered orthogonal coding: the transmitted signals are chosen according to the decision of developers, and processing of the transmitted signals and noise is made with use of special approach [3]. If in this case orthogonal codes are applied, increase of transmitted signals and decrease of noise are provided [4]. This characteristic exists not only in the channels with additive noise, but also in the fading channels [5-6].

2. Synthesis of encoding and decoding matrices

For formation of orthogonal codes, it is required to implement the synthesis of square matrices so that their product is the unitary matrix multiplied by the monomial characterizing the correcting ability of the code.

Previously, indicated properties of matrices were developed due to combinatorial methods, as a result of which only a few orthogonal codes were obtained. Therefore, the task arose to develop a regular matrix synthesis algorithm for constructing orthogonal codes. The indicated problem was solved in [3] with the condition that the elements of the applied matrices are polynomials of the degree 1. As a result, the class of the matrices allowing to solve practical problems of noise immunity’s increase was synthesized.

The encoding $G(D)$ and decoding $H(D)$ matrices of the delay variable $D$ shall satisfy the relation
where $I$ is the identity matrix. The multiplier $\rho \cdot D^x$ indicates the amplitude increase of the input signal at $\rho$ times and that the symbols in the receiver are obtained with a delay of $x$ time units.

In the process of joint application of orthogonal coding and differential phase-shift keying (DPSK) we will use matrices $H(D)$ with polynomials in variable $D$ of degree 1.

According to the proposed algorithm [7], at the first step of the synthesis of a matrix $H(D)$ of order $n$ the first $z = 2k$ elements of the main diagonal get values $1 + D$, $z \leq n$, the integer $z$ will be called the depth of the matrix. The following elements of the main diagonal assigned value 1. In the last step, elements outside the main diagonal get the following values: elements of odd rows at the right and odd columns downwards from the main diagonal are equal to $1 - D$; elements of even rows at the right and even columns downwards from the main diagonal are equal to $1 + D$.

When reducing the depth of a decoding matrix $H(D)$, the number of the corrected errors should increase, but the maximum element in an encoding matrix $G(D)$, that is, the range of symbols received at the output of the encoder, should also increase [8].

As an example, let’s consider the decoding matrix of order 4 and depth 2

$$H(D) = \begin{pmatrix} 1 + D & 1 - D & 1 - D & 1 - D \\ 1 - D & 1 + D & 1 + D & 1 + D \\ 1 - D & 1 + D & 1 & 1 - D \\ 1 - D & 1 + D & 1 - D & 1 \end{pmatrix}$$

and the corresponding encoding matrix $G(D)$

$$G(D) = \begin{pmatrix} 3 + 3D & -3 + 3D & 0 & 0 \\ -3 + 3D & -5 + 3D & 4 & 4 \\ 0 & 4 & 4 & -8 \\ 0 & 4 & -8 & 4 \end{pmatrix}.$$ 

Multiplication of the matrices $G(D)$ and $H(D)$ gives

$$G(D) \cdot H(D) = \begin{pmatrix} 12D & 0 & 0 & 0 \\ 0 & 12D & 0 & 0 \\ 0 & 0 & 12D & 0 \\ 0 & 0 & 0 & 12D \end{pmatrix}.$$ 

3. Determination of the angle between the received vectors

Consider the question of finding the angle between the received vectors when performing demodulation on the receiving side of the communication system using the example of the encoding matrix of size $(4 \times 4)$ and the decoding matrix $H(D)$ of the same size. $G(D)$ and $H(D)$ can be represented as encoding and decoding matrices $G$ and $H$ of size $(4 \times 8)$ and $(8 \times 4)$ respectively:
Note that orthogonal codes consist of sequences of integers with different signs. Therefore, there is a need for matching code symbols and modulation methods [9]. To implement orthogonal coding, it is necessary to use DPSK of very high multiplicity. So, in this example, the possible number of phase shifts is 45 (doubled maximum amount of one column absolute values of the encoding matrix $G$ plus one).

When DPSK information is embedded in a sequence of phase differences of the carrier wave, which can take a finite number of values: $\Delta \varphi_1, \Delta \varphi_2, \ldots, \Delta \varphi_q$. The corresponding discrete information transmission system is called a $q$-position system with DPSK. As a rule, in modern discrete information transmission systems, the number $q$ is equal to the integer power of two, that is, $q = 2^t$. Such systems are called $t$-fold DPSK systems.

In accordance with general demodulation algorithms, the transmitted binary symbols are determined during DPSK through cosines and sines of the phase differences of the received signal [10].

The following procedure is proposed for matching the symbols of the orthogonal code with the DPSK and digital processing of these signals at the receiving side to obtain an estimate of the error probability in the channel with the additive white Gaussian noise (AWGN).

We perform a uniform sampling at $N$ points of each signal $s_i(t)$. Thus, with DPSK, the symbol with index $i$ is determined by two vectors. Denote them as $X^{(i-1)} = (x_1^{(i-1)}, x_2^{(i-1)}, \ldots, x_N^{(i-1)})$, $X^{(i)} = (x_1^{(i)}, x_2^{(i)}, \ldots, x_N^{(i)})$.

Let’s determine the angle between the received vectors. To estimate the angle, we calculate the values of the sine and cosine of the phase differences between the received vectors:

$$\cos \Delta \varphi = \cos \left( X^{(i)}, X^{(i-1)} \right) = \frac{\left( X^{(i)} \cdot X^{(i-1)} \right)}{\left\| X^{(i)} \right\| \left\| X^{(i-1)} \right\|},$$

$$\sin \Delta \varphi = \sin \left( X^{(i)}, X^{(i-1)} \right) = \frac{\left( X^{(i)} \cdot X^{(i-1)} \right)}{\left\| X^{(i)} \right\| \left\| X^{(i-1)} \right\|},$$

where $\left( X^{(i)} \cdot X^{(i-1)} \right)$, $\left( X^{(i)} \cdot X^{(i-1)} \right)$, $\left\| X^{(i)} \right\|$, $\left\| X^{(i-1)} \right\|$ are scalar products and norms of the corresponding vectors, $\left( X^{(i)} \right)^*$ is the vector obtained from the signal $s_i(t)$ using the Hilbert transform. The vector corresponding to the signal converted according to Hilbert is obtained from the vector $X^{(i)}$ by a cyclic shift by the number of samples corresponding to $\pi / 2$. According to the
obtained estimates of the sine and cosine values, we calculate the value of the angle between the vectors \( X^{(i)} \) and \( X^{(i-1)} \), thereby find the number of the decisive region.

In accordance with equations (1) and (2), the demodulation procedure is determined by the method of calculating the scalar products of vectors, \( X^{(i)} \), \( [X^{(i)}]^* \) and \( X^{(i-1)} \), which can be represented as integral convolutions

\[
\left( X^{(i)} \cdot X^{(i-1)} \right) \approx \int_{(i-1)T}^{iT} s(t) s(t - T) \, dt ,
\]

\[
\left( [X^{(i)}]^* \cdot X^{(i-1)} \right) \approx \int_{(i-1)T}^{iT} s^*(t) s(t - T) \, dt .
\]

Consider the channel with AWGN. Let the space of the received signals be two-dimensional, and some signal are received by the demodulator. In the general case, the conditional probability density of the output signal \( u = (x, y) \) when transmitting a signal with a phase has the form

\[
\frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x - \sqrt{E} \cos \phi)^2}{N_0}} \cdot \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y - \sqrt{E} \sin \phi)^2}{N_0}} .
\]

Due to the symmetrical arrangement of the signal points on the circle, we assume that the signal with the number \( i = 0 \) is received. Then the conditional probability density of the output signal has the form

\[
\frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x - \sqrt{E})^2}{N_0}} \cdot \frac{1}{\sqrt{\pi N_0}} e^{-\frac{y^2}{N_0}} .
\]

In this case, the probability of the received signal falling into the \( i \)-th decisive region for

\[
i \in \left[ 0, \frac{q - 1}{4} \right) \], \quad \text{where} \quad q \text{ is odd}, \quad \text{or for} \quad i \in \left[ 0, \frac{q}{4} \right) , \quad \text{where} \quad q \text{ is even}, \quad \text{can be estimated as}
\]

\[
P(i) = \int_{x \cdot tg \left( \frac{(2i+1)\pi}{q} \right)}^{\infty} \int_{x \cdot tg \left( \frac{(2i-1)\pi}{q} \right)}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x - \sqrt{E})^2}{N_0}} \cdot \frac{1}{\sqrt{\pi N_0}} e^{-\frac{y^2}{N_0}} \, dy \, dx .
\]

The probability of the received signal falling into the \( i \)-th decisive region for \( i \in \left[ \frac{q-1}{4}, \frac{q-1}{2} \right] \), where \( q \) is odd, or for \( i \in \left[ \frac{q}{4}, \frac{q}{2} \right) \), where \( q \) is even, can be estimated as
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The probability of the received signal falling into the $i$-th decisive region for $i = \frac{q-1}{4}$, where $q$ is odd, or for $i = \frac{q}{4}$, where $q$ is even, can be estimated as

$$P(i) = \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{y^2}{N_0}} dy \cdot \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x-\sqrt{E})^2}{N_0}} dx. \quad (8)$$

which, in turn, allows us to estimate the probability of error as $1 - P(i)$.

Figure 1 shows the example illustrating the application of equation (7) for getting the received signal into the decisive region with the number $i = 2$ at $q = 16$.

4. Message transfer example

The transmission system operates as follows. The coding operation is a multiplication of the $\pm 1$ information sequence of unlimited length and the semi-infinite encoding matrix. As a result, we get the semi-infinite codeword, each symbol of which takes the value from the set.
\{-22, -21, \ldots, 0, \ldots, 21, 22\}. This sequence is fed to a modulator, which creates phase modulation with 45 positions. At the receiving side, the magnitude of the phase shift between adjacent symbols of the received message is estimated. At the output of the demodulator we also get a sequence of numbers from the same set \{-22, -21, \ldots, 0, \ldots, 21, 22\}. Next, the decoder calculates the scalar product of the obtained sequence and the decoding matrix. As a result, we obtain estimates of the transmitted symbols, which should be ±12 in the absence of noise. Decisions about transmitted symbols are based on a comparison with the zero threshold.

Let the message contain eight symbols and is following:

\(1, -1, 1, -1, -1, 1, 1\).

Then the codeword supplied to the modulator is represented by the following vector:

\((6, 2, 8, -16, 0, 8, 8, -16, -6, -6, 0, 0)\).

It follows from the form of the encoding matrix that the possible values of the phases at the output of the modulator are 0, ±1, ±2, \ldots, ±22, multiplied by \(\frac{2\pi}{45}\). In our example, phases with the following numbers will be obtained in the modulator:

\((6, 8, 16, 0, 0, 8, 16, 0, -6, -12, -12, -12)\).

Thus, in accordance with the received codeword, the sequence of phases is obtained:

\(\left(\frac{12\pi}{45}, \frac{16\pi}{45}, \frac{32\pi}{45}, 0, 0, \frac{16\pi}{45}, \frac{32\pi}{45}, 0, -\frac{12\pi}{45}, -\frac{24\pi}{45}, -\frac{24\pi}{45}, -\frac{24\pi}{45}\right)\).

The sequence of signals with such phases is transmitted through the channel. At the receiving side, the phase differences of the received oscillations are estimated, and the received vector is determined from their values, the components of which, in the absence of noise in the channel, also take values from the set \{-22, -21, \ldots, 0, \ldots, 21, 22\}:

\((6, 2, 8, -16, 0, 8, 8, -16, -6, -6, 0, 0)\).

Further, multiplying by the decoding matrix, we obtain estimates of the transmitted symbols, which take values ±12:

\((12, -12, 12, -12, -12, -12, 12, -12, -12, -12, -12, 12)\).

Decisions about transmitted symbols are based on a comparison with the zero threshold.

5. Results of analytical and simulation modeling

On the basis of the received relations the detailed analysis of efficiency of joint application of orthogonal coding and DPSK was made [11, 12]. Information symbols were chosen from \{+1, -1\}. Estimations of quantities of coding gain in the AWGN channel (e.g. from 3.0 dB to 4.5 dB at bit error rate 10^{-6}) are shown in figure 2. For example, by use of orthogonal coding OC-32 bit error rate 10^{-4} is assured by the signal-to-noise ratio \(E_b/N_0 = 6.14\) dB, which is 3.1 dB less, than in case of BDPSK without coding. By use of orthogonal coding OC-32 bit error rate 10^{-6} is assured by the signal-to-noise ratio \(E_b/N_0 = 6.71\) dB, which is 4.5 dB less, than in case of BDPSK without coding [13, 14].
6. Conclusions

Technical implementation of orthogonal coding is characterized by low complexity: decoding is reduced to calculation of several dot products and comparison with the zero threshold. For this reason, the offered way of coding and design of receiving and transmitting devices can be implemented in various communication systems. The proposed way of orthogonal coding is a variety of reception of M-ary DPSK signals with an optimum choice of a modulation code that matches the binary combinations of the source with the phase shift of the DPSK signal. The reason for this optimization is to average the error probability over all bits of the M-ary code [15].

The procedure of symbols matching of orthogonal codes with high multiplicity DPSK differs from existing methods in the rules for assigning a set of phases and in the way of digital processing in a demodulator. It is quite simple and reduces to calculating the phase increment angle equal to $2\pi/q$, where $q$ is the doubled maximum sum of the absolute values of one column of the encoding matrix $G$ plus 1. Using the example of an orthogonal code based on a pair of encoding and decoding matrices, the dependence of the error probability on the signal-to-noise ratio in the channel is considered with AWGN for binary DPSK and for orthogonal encoding scheme.

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