Inventory control model using exponential smoothing control chart

Mustafid¹, D Ispriyanti¹, Sugito¹ and D Safitri¹
¹ Department of Statistics, Diponegoro University
Jl. Prof. Soedharto, SH, Tembalang, Semarang 50275, Indonesia
E-mail: mustafid55@gmail.com

Abstract. Decision making to determine stock replenishment has an important role in inventory management, especially to anticipate unstable consumer demand. The research aims to design statistical inventory control model for stochastic lead time demand using exponential smoothing and normal distribution approach. The exponential smoothing approach used to design prediction of daily demand and prediction of lead time demand. The normal distribution approach used to design statistical control chart of lead time demand, and also to determine the position of the safety stock and reorder point. Empirical analysis for the research is provided based on case studies for stock replenishment analysis on apparel industry. The results of the research provide a basic concept for decision-making to determine the safety stock and reorder point as the basic parameters in inventory management. The main contribution of the research results provides the application of statistical control chart as statistical inventory control by using smoothing method for data of demand in a lead time as two random variables normal distributed.

1. Introduction

Recently, some researchers developed the statistical process control in inventory management for inventory position problems. Aini et al. [1,2] provides a control chart approach to monitor the performance of inventory through monitoring inventory position and out of stock. They use statistical process control chart to monitor demand flow with application for apparel industry. Costantino et al. [3] identified inventory control chart based statistical process control approach to handle supply chain dynamics, where the policy for inventory control charts depend on the application of individual control charts to control the position of inventory and placed orders adequately. Costantino et al. [4] developed a statistical control chart for implementing inventory decision support systems in real-time at the inventory and demand levels. Cheng and Chou [5] introducing an inventory decision system where an ARMA control chart is used to monitor market demand and inventory levels.

The inventory control model is designed as integrated control chart with decision rules is used to estimate demand adjusted inventory positions, so that can be estimate demand periodically. The control chart for lead time demand aims to control demand by adjusting variations in inventory positions by adjusting inventory position during all time of lead time [6]. The determination of limit of lead time demand is determined in a certain period based on a demand that is expected to restore the inventory position to increase demand or order.
Statistical control chart in inventory control model useful for monitor inventory positions by adjusting demand variability. The role of inventory chart model can be used to estimate the quantity of demand within all lead time based on information about the number of stock in accordance with the trend of the future [7]. The model is also useful for detecting changes in demand on the lead time, so that it can be used to design the inventory replenishment policy in rapidly changing conditions. In this case, the usefulness of the control chart can be monitor the performance of the inventory system that relates to the reorder-point through monitoring stock, demand and inventory renewal.

The research proposes inventory control model using statistical control charts based on the demand forecasting using exponential smoothing. Smoothing method also used for transformation in order to meet of normal distribution and stability as the assumption of data of demand [8]. Furthermore, the normal distribution assumption of demand and lead time is needed for design of control chart of demand and lead time demand. The inventory model is an integrated inventory control model that can monitor inventory positions and balanced demand in the supply chain up to retail [9]. This model framework uses a control chart that can establish a statistically valid zone, which is determined by the upper and lower control limits.

The inventory model uses two control charts that can be used to simultaneously monitor demand and inventory positions. First, the control chart is designed to monitor consumer demand variations so that changes of demand or orders can be detected at any time of major changes. If customer demand is stable or controlled, the order quantity will be the same as before. However, if the request control chart is out of control, for example, an out-of-control situation occurs, then the order quantity must be adjusted based on inventory stability.

The research aims to design inventory control chart based on predicted demand and lead time demand using probabilistic approach. Predictions for demand and lead time demand are done using exponential smoothing based actual demand and lead time. The exponential smoothing method is also used as a transformation so that the non normal distributed data becomes normally distributed. The inventory control chart aims to determine the quantity of products in the range of the upper and lower limits. The upper limit is also related to safety stock and reorder point for planning of product inventory renewal, so that it can manage the production and supply of products according to market needs or consumer demand. The case study was conducted in the apparel industry in Kudus, Central Java.

2. Research methods

This research is development of inventory control model to estimate inventory position based on demand forecasting with exponential smoothing method and probabilistic approach with normal distribution. The research uses statistical inventory control chart to analyze inventory position so that it can determine safety stock and reorder point based on stochastic daily demand forecasting and lead time [10]. The exponential smoothing approach is used as a transformation method for normal distribution and stability process. The basic framework of the inventory control chart uses information flow model in the form of demand and lead time, and the flow of product material to be distributed to consumers [9]. The application of inventory control chart for inventory management is used as basis for design of inventory control chart to determine inventory position. Based on the inventory control chart are also designed to design the estimation of safety stock and reorder point based on daily demand prediction.

The hypothesis testing with normal distribution is used to state that data of the daily demand and lead time are meet the assumption of normal distribution. Because the data is continuous, we use the hypothesis using the Kolmogorov-Smirnov test. The normal distribution test of Kolmogorov Smirnov is based on the cumulative empirical distribution function from the observation data as sample data. The hypothesis test decision is carried out using the critical area, that is that the hypothesis of the sample data is normally distributed if $\alpha \leq p$-value for significance level $\alpha$.

The design of inventory control chart using a variable daily demand and lead times, with case study in the apparel industry. Daily demand data is used as an empirical data in the form of sales or orders from
consumers at a number of retailers. Data of lead time is determined based on variations of daily demand. The sample data used observation for 105 days from April to June 2016, and grouped into eight lead times. The profile of actual daily demand is shown in figure 1.

The characteristics of daily demand and lead time are the basis for designing inventory control chart. The position of inventory and safety stock is determined by lead time demand which is determined based on daily demand and lead time. The theory of normal distribution is used to determine the distribution of lead time demand. In this case, the lead time demand is a random variable, so that the distribution is determined by the combination of the distribution of daily demand and distribution lead time.

3. Exponential smoothing for Lead time demand
In this section, we design daily demand and lead time demand prediction using exponential smoothing methods. Suppose that the daily demand $D_t$ is stochastic process at time $t$ in a periodic lead time $L$. The stochastic processes daily demand ($D_t$) and lead time ($L$) are assumed to be mutually independent and distributed identically, and $D_t$ is also assumed to be independent with $L$.

Lead time demand $X$ at lead time $L$ is expressed by random variable [11]:

$$X = D_1 + D_1 + \cdots + D_L$$  

(1)

Predicted daily demand $\hat{D}_{t+1}$ for the next time $t$ (figure 2) is determined based on average of daily demand on the previous time period $T$ (Figure 3) using exponential smoothing with the formula [12]:

$$\hat{D}_{t+1} = \alpha D_t + (1 - \alpha) \sum_{i=t-T}^{t-1} D_t / T$$  

(2)
where $\tilde{D}_{t+1}$ is the demand prediction for time $t+1$ and $\alpha$ is the parameter which is the weight of the previous demand. The weight value of $\alpha$ is determined in the condition that the smoothed data is normally distributed with a low error.

The above procedure is repeated to determine the demand prediction for period $t$ to lead time $L$ (figure 3). By using predictive formula for demand (2) during lead time $L_j$, then can be determined the predicted of daily demand number during lead time $L_j$, by formula:

$$\text{Predicted Daily Demand}_{L_j} = \sum_{i=t}^{t+L_j} \tilde{D}_t$$

Furthermore, using formula (3), the number of all daily demand prediction with $k$ of lead time, predicted of lead time demand is expressed by formula:

$$\text{Total Predicted Demand} = \sum_{k=1}^{K} \text{Predicted Daily Demand}_{L_j}$$

Based on the actual demand, the profile of predicted daily demand using exponential smoothing is given in figure 1. Figure 1 shows that daily demand and lead time are probabilistic. The prediction accuracy for predicted daily demand from actual daily demand can be determined by the MAPE (Mean Absolute Percentage Error) method, with the formula:

$$\text{MAPE} = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{D_t - \tilde{D}_t}{D_t} \right| \times 100\%$$

where $N$ is the number of demand for the model prediction.

The predicted demand $\tilde{D}_j$ and lead time $L_j$ are can be estimated using predicted observation data using the average value of predicted demand and lead time respectively. Let the mean and variance of predicted daily demand $\tilde{D}_j$ and lead time $L_j$ are expressed by:

$$E(\tilde{D}_j) = \mu_{\tilde{D}_j} \quad \text{and} \quad \text{Var}(\tilde{D}_j) = \sigma_{\tilde{D}_j}^2$$

and mean and variance of predicted of lead time demand are expressed by:

$$E(\tilde{X}) = \mu_{\tilde{X}} \quad \text{and} \quad \text{Var}(\tilde{X}) = \sigma_{\tilde{X}}^2$$

The mean values of predicted daily demand (5) can be estimated using the average and variance of daily demand by formula:

$$\tilde{D}_j = \frac{1}{L_j} \sum_{i=t}^{t+L_j} \tilde{D}_t$$

$$S^2_{\tilde{D}_j} = \frac{1}{L_j-1} \sum_{i=t}^{t+L_j} (\tilde{D}_t - \tilde{D}_j)^2$$

The mean and variance of lead time $L$ are expressed by formula:

$$E(L) = \mu_L \quad \text{and} \quad \text{Var}(L) = \sigma_L^2$$

Similarly, the mean and variance of predicted lead time demand $\tilde{X}$ (6) can be estimated by formula:

$$\tilde{X} = \frac{1}{k} \sum_{j=1}^{k} \tilde{D}_j$$

$$S^2_{\tilde{X}} = \frac{1}{k-1} \sum_{j=1}^{k} (\tilde{D}_j - \tilde{X})^2$$

Furthermore, mean and variance of the total lead time $L$ can be estimated by formula:

$$\bar{L} = \frac{1}{k} \sum_{j=1}^{k} L_j$$

$$S^2_{\bar{L}} = \frac{1}{k-1} \sum_{j=1}^{k} (L_j - \bar{L})^2$$

In statistical theory, the predicted lead time demand $\tilde{X}$ as in equation (4) becomes a compound random variable. Using the properties of compound random variables, mean and variance of $\tilde{X}$ are expressed by:

$$E(\tilde{X}) = \mu_{\tilde{X}} \quad \text{and} \quad \text{Var}(\tilde{X}) = \sigma_{\tilde{X}}^2$$

$$\text{Var}(\tilde{X}) = E\left[\sum_{i=1}^{k} \tilde{D}_i \right] = \mu_L \mu_{\tilde{X}}$$

$$= \mu_L \sigma_{\tilde{X}}^2 + \sigma_L^2 \mu_{\tilde{X}}^2$$
Using the central limit theorem for predicted lead time demand $\hat{X}$, if the predicted daily demand $\hat{D}$ and lead time $L$ have normal distribution, then distribution of $\hat{X}$ can be determined as a normal distribution with mean $\mu_L \mu_K$ and variance $\mu_L \sigma_X^2 + \sigma_L^2 \mu_X^2$. 

4 Inventory position control chart

Inventory control chart is designed as an inventory position control chart on the supply chain within the supply chain information system with daily demand and lead time as input data of system. Daily demand for each lead time is used to make predictions for planning the products and inventory position. The role of inventory control charts is expected to assist management in determining maximum profits through optimal inventory that is determined based on the level of demand or order from consumers. The important parameters in the inventory position control chart are safety stock (SS) and reorder point (ROP).

The inventory control chart has an upper control limit (UCL) and a lower control limit (LCL), and a center line (CL) [13]. The control limit functions to determine the position of each observation data, so that it can identify whether the observation data is at the control limit or outside the control limit. The data used is observation data from one sample, so that the control chart used to determine the position of demand and inventory is an individual control chart. The statistical process control in this research uses an average control chart $\bar{X}$ which is used to detect data value plots between the upper and lower limits with the center line of average value, or vice versa whether the data value is outside the upper and lower limits.. If the value of the data is between the upper and lower limits, it is said that the process is under controlled conditions, whereas if the value of the data is outside the upper and lower limits it is said that the process is uncontrolled.

The basic theory of inventory control chart is used probability approach with normal distribution to design upper control limit, lower control limit, and center line. The inventory control chart is designed with input data of daily demand $D_t$ and lead time demand (4) as stochastic processes. We use the input data of daily demand and lead time based on data of the observational data derived from the paret industry as case study in the research. The exponential smoothing model for demand (2) can be used for the smoothing model as a transformation model of normal distribution and as a prediction model. The probability theory is used to find the normal distribution of lead time demand variable.

![Figure 4. Upper and lower limit for inventory position control chart](image)

To state lead time demand satisfy the assumption of normal distribution will be used Kolmogorov-Smirnov test. We use the normal distribution test of Kolmogorov-Smirnov for $\alpha = 0.05$ to prove that input data of daily demand and lead time are normal distribution. First, we show that the lead time is a random variable having a normal distribution with p-value = 0.200. However, for actual daily demand data is not normally distributed, it is shown that the hypothesis test results give a p-value = 0.026. Therefore, it is necessary to transform with exponential smoothing method using formula (2). The profile of daily demand after use smoothing with exponential smoothing is given in Figure 1, and based on test of Kolmogorov-Smirnov gives p-value = 0.200. Thus accepting the hypothesis that the data of smoothing predicted for daily demand is normally distributed. Based on the test hypothesis of normally distributed, the exponential smoothing transformation for daily demand and lead time variables have normally distributed, then the predicted lead time demand $\hat{X}$ as in equation (4) is normal distribution with mean $\mu_L \mu_K$ and
variance $\mu_\tau \sigma_\tau^2 + \sigma_\tau^2 \mu_\tau^2$ as given in (14) and (15). The computational results of MAPE show that for the prediction lead time demand obtained MAPE = 6% (high accuracy).

Based on the normal curve distribution in Figure 4, the center line (CL), upper control limit (UCL) and lower control limit (LCL) for lead time demand $\tilde{X}$ are expressed by:

$$\text{CL}(\tilde{X}) = E(\tilde{X}) = \mu_\tau = \mu_\tau \mu_\tau$$ (16)

$$\text{UCL}(\tilde{X}) = \text{CL}(\tilde{X}) + Z_{a/2} \sqrt{\text{Var}(\tilde{X})}$$

$$= \mu_\tau \mu_\tau + Z_{a/2} \sqrt{\mu_\tau_\tau \sigma_\tau^2 + \sigma_\tau^2 \mu_\tau^2}$$ (17)

$$\text{UCL}(\tilde{X}) = \text{CL}(\tilde{X}) - Z_{a/2} \sqrt{\text{Var}(\tilde{X})}$$

$$= \mu_\tau \mu_\tau - Z_{a/2} \sqrt{\mu_\tau_\tau \sigma_\tau^2 + \sigma_\tau^2 \mu_\tau^2}$$ (18)

The center limit CL(\tilde{X}) states the central limit of predicted lead time demand $\tilde{X}$, the value of CL(\tilde{X}) is estimation of $\tilde{X}$ (16) which determined by average with formula (10). The UCL(\tilde{X}) and UCL(\tilde{X}) are the upper control limit and lower control limit which have value can be determined by (17) and (18) value of parameters which can be estimated by average and variance of predicted lead time demand (10), (11), and estimated by average and variance of lead time (12) and (13). $Z_{a/2}$ is the percentage point normal standard distribution such that $P(z \geq Z_{a/2}) = a/2$.

With the same procedure, we have design inventory control chart based on predicted daily demand $\tilde{D}_j$ on each lead time $L_j$ as expressed by:

$$\text{CL}(\tilde{D}_j) = E(\tilde{D}_j) = \mu_\tilde{D}_j$$

$$\text{UCL}(\tilde{D}_j) = \text{CL}(\tilde{D}_j) + Z_{a/2} \sqrt{\text{Var}(\tilde{D}_j)}$$

$$= \mu_\tilde{D}_j + Z_{a/2} \sigma_\tilde{D}_j$$

$$\text{UCL}(\tilde{D}_j) = \text{CL}(\tilde{D}_j) - Z_{a/2} \sqrt{\text{Var}(\tilde{D}_j)}$$

$$= \mu_\tilde{D}_j - Z_{a/2} \sigma_\tilde{D}_j$$

The parameter's value can be determined by value of parameters which can be estimated by average and variance of daily demand (7) and (8).

Figure 5. Measurement of safety stock and reorder point

The quantity of safety stock and reorder point for product within supply chain can be determined using the properties of the normal probability for lead time demand as random variable. In the same way, to design a confidence interval for inventory position control chart, the expecting of lead time demand...
prediction for each lead time on the inventory is less than reorder point \((\bar{X} \leq ROP)\) can be determined the formula of probability model with the least probability of \(1 - \alpha\) (one tail), as given in Figure 5. As shown in curve of normal distribution as in Figure 5, the probability for predicted lead time demand is less than ROP is expressed by:

\[
P(\bar{X} \leq ROP) \geq 1 - \alpha.
\]

Using the same way as to determine inventory position control chart, from curve of normal distribution curve (Figure 6), upper control limit as the one side as the reorder point (ROP) is expressed by formula:

\[
ROP = E(\bar{X}) + Z_{\alpha} \sqrt{\text{Var}(\bar{X})}
\]

\[
= \mu_\bar{X} + Z_{\alpha} \sqrt{\frac{\sigma^2}{n} + \frac{\sigma^2 n^2}{\mu^2}}
\]

The distance between ROP and mean is called safety stock:

\[
\text{Safety stock} = Z_{\alpha} \sqrt{\frac{\sigma^2}{n} + \frac{\sigma^2 n^2}{\mu^2}}
\]

Based on the case study in apparel industry, the result of statistical computing for daily demand control chart, CL, UCL and LCL with \(\alpha = 0.05\), \(Z_{\alpha/2} = 1.96\) are given in Table 1. The profile of daily demand control chart for lead time \(L_8\) is shown in Figure 7. Furthermore, the result computing for inventory position control chart, safety stock and reorder point with \(\alpha = 0.05\), \(Z_{\alpha} = 1.65\) are given in table 2.

Table 1. Parameters of Daily demand prediction control chart

| Lead time (days) | LTD actual (total) | Total | Average | Deviation | Standard | CL | UCL | LCL |
|-----------------|--------------------|-------|---------|-----------|----------|----|-----|-----|
| L_1             | 14                 | 1054  |         |           |          |    |     |     |
| L_2             | 16                 | 1264  | 1372    | 86        | 9.67     | 86 | 115 | 57  |
| L_3             | 14                 | 1178  | 1185    | 85        | 9.54     | 85 | 113 | 56  |
| L_4             | 17                 | 1258  | 1351    | 79        | 9.18     | 79 | 107 | 52  |
| L_5             | 11                 | 1141  | 988     | 90        | 9.94     | 90 | 120 | 60  |
| L_6             | 10                 | 1105  | 1031    | 103       | 10.16    | 103| 134 | 73  |
| L_7             | 10                 | 1073  | 1088    | 109       | 10.99    | 109| 142 | 76  |
| L_8             | 13                 | 1261  | 1318    | 101       | 9.38     | 101| 130 | 73  |

Table 2. Inventory position control chart

| Lead time (days) | LTD prediction | Inventory position control chart | Inventory position |
|-----------------|----------------|----------------------------------|--------------------|
|                 |                | CL | UCL | LCL | SS | ROP |
| L_1             | 14             | 1183 | 1695 | 671 | 431 | 1631 |
| L_2             | 16             | 1372 | 1185 |    |    |      |
| L_3             | 14             | 1351 | 1031 | 431 |    |      |
| L_4             | 17             | 988  | 1088 |    |    |      |
| L_5             | 11             | 988  | 1031 |    |    |      |
| L_6             | 10             | 1088 | 1318 |    |    |      |
| L_7             | 10             | 1088 | 1318 |    |    |      |
| L_8             | 13             | 1318 | 1318 |    |    |      |

Table 1 shows all the values of the parameters for daily demand prediction based on data of actual daily demand from the eight lead times. Using the model of exponential smoothing as the prediction
model obtained total lead time predictions for each lead time period starting from the second lead time to the 8th lead time, by giving the numbers of total, average, deviation standards, center line CL, upper control limit UCL, and lower control limit LCL in each lead time. Examples for daily demand prediction profiles for the eighth lead time are given in Figure 7.

Table 2 shows the parameter values for inventory position control chart for lead time demand \( \bar{X} \) for seven lead times based on prediction of actual lead time demand, by giving the numbers of center line CL, upper control limit UCL, lower control limit LCL, safety stock SS and reorder point ROP for all eight lead time. The value of safety stock is the distance from ROP with CL. The number of ROP is the same as UCL for one-side of normal distribution curve, so the number of ROP is greater than UCL.

Figure 6 shows that for daily demand in lead time \( L_8 \) have the center line CL = 101 pcs, upper control limit UCL = 130 pcs, and lower control limit LCL= 73 pcs. Some point of daily demand are above UCL and below LCL, so the daily demand in lead time \( L_8 \) shows an unstable state. Whereas in figure 7 shows that for all lead time demand have center line CL=1183 pcs, UCL= 1647 pcs and LCL= 719 pcs, and all points of lead time demand are under UCL and above LCL, so the lead time demand prediction shows a stable state.

In this research focuses on design of inventory control chart using a probability and statistical approach to design statistical inventory position control chart. The assumptions in designing inventory control are normal distribution. As an input for inventory position control chart is stochastic daily demand and lead time as random variables. The combined of both variables is lead time demand as a compound variable. In this case, the assumption of variable daily demand and lead time has a normal distribution, so that the lead time demand meets the assumption of normal distribution.

The results of the research provide new contributions to the development of probability theory and statistical control charts for the inventory position control chart on integrated systems in the supply chain within the framework of information systems. An important contribution is primarily on design of statistical inventory position control chart for combined variables of stochastic daily demand and lead time as two random variables with normal distribution. The application of the inventory position control chart can be used to determine the safety stock and reorder point as a basis for determining the replenishment order. As an input in the inventory position control chart are daily demand or order from the consumer, so that the number of products and inventory position can be designed.

5. Conclusion

Inventory position control chart is designed within the supply chain control system framework using lead time demand as compound variable which formed by stochastic process of daily demand and random variable of lead time. Model of inventory control chart can be used to determine the number of safety stock and reorder point as inventory parameters used to update product stocks, so as to regulate the production and supply of products according to market needs or consumer demand. Design of inventory
control chart using the statistics and probability theory with a normal distribution curve properties. Using properties of normal distribution curve, we can determinate the parameters of inventory position control, i.e. center line, upper control limit, lower control limit, safety stock, reorder point and quantity of demand with significance level $1-\alpha$. The usefulness of inventory control charts is to determine the minimum number of estimates for stock renewal.

An important part of the design of the inventory position control chart model is the use of probability and statistical theories. With the statistical hypothesis theory, the assumption of normal distribution is tested by the central limit theorem, and in practice using normal distribution of Kolmogorov-Smirnov test. An important part in using the exponential smoothing method is determining the smoothing weight. In the research, the exponential smoothing approach is used as a transformation method so that actual data of demand that are not normally distributed can be transformed into normal distributed. Determination of the $\alpha$ weight in exponential smoothing based on the condition that the refined data has a low average error, which can be proven by MAPE. Furthermore, the smoothing model functions as a prediction model to determine the number of demand for the future.

References

[1] Aini N, Mustafid and Kusumaningrum R, 2017. Int Conf on Information Technology Systems and Innovation (ICITSI) IEEE Xplore 134

[2] Sabila A D, Mustafid and Suryono S 2018 E3S Web of Conferences 31 11015.

[3] Costantino F, Di Gravio G, Shaban A and Tronci M 2014 Int. J. of Engineering and Technology 6 418.

[4] Costantino F, Di Gravio G, Shaban A and Tronci, M 2014 Expert Systems with Applications 42 1665.

[5] Cheng J C and Chou C Y 2008 Expert Systems with Applications 35 755.

[6] Efrilianda D A, Mustafid and Rizal I 2018 Int. Conf. on Information and Communications Technology (ICOIACT) IEEE Explore 844

[7] Prak D, Teunter R and Syntetos A 2016 European J. of Operational Research 256 454.

[8] Snyder R D, Koehler A B, Hyndman R J and Ord J K 2014 European J. of Operational Research 158 444.

[9] Mustafid, Karimariza A S and Jie F 2018 Int. J. of Agile Systems and Management 11 1.

[10] Roldán R F, Basagoiti R and Coelho L C 2017 J. Appl. Log 24 Part A 15.

[11] Cobb B R, Johnson A W, Rumí R and Salmerón A 2015 International J. of Production Economics 163 124.

[12] Rawat M and Altik T 2008 Int. J. of Production Research 40 1.

[13] Montgomery D 2009 Introduction to Statistical Quality Control 6th ed (New Jersey: John Wiley & Sons Inc).