Command Filtered Adaptive Neural Network Synchronization Control of Nonlinear Stochastic Systems With Lévy Noise via Event-Triggered Mechanism

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ABSTRACT This paper proposes an adaptive neural network (NN) output-feedback synchronization controller for the nonlinear stochastic systems driven by Lévy processes, which consist of Wiener and compensated Poisson process. By employing the generalized Itô’s formula combined with Lyapunov function method, it is proved that the proposed controller can ensure that all signals of closed system are bounded in probability. Under the backstepping control framework of stochastic process including Lévy noises, the event-triggered control technology is used to reduce the utilization of communication resources. Moreover, the command filtered control technology is introduced into the controller to avoid “explosion of complexity” and obtain the derivatives for virtual control functions continuously. Simulation proves the feasibility of the proposed control method.

INDEX TERMS Stochastic system synchronization, adaptive control, event-triggered mechanism, command filter, Lévy noise.

I. INTRODUCTION

Up to now, adaptive synchronization issues with stochastic noises have become a hot topic in nonlinear system studies [1]. The usual sense stochastic noise is only driven by continuous Brown motion, which can describe continuous stochastic volatility. However, in many physical systems, there exist many types of discontinuous noise, such random failures, abrupt change, and sudden disturbance [2]. Fortunately, there is a kind of noise called “Lévy noise,” which can characterize both continuous Brown motion and discontinuous Poisson jump process [3], [4]. To date, it has made some progresses in the field of nonlinear stochastic system control with Lévy noise, such as adaptive control [5], linear matrix inequality (LMI) control [1], [6], sliding mode control (SMC) [3] and so on. For example, [3] studied the almost sure stability of second-order nonlinear stochastic system with Lévy noise by sliding mode control method and established sufficient conditions to ensure the almost sure stability of the system dynamics. [7] proposed an adaptive sliding mode controller to study a dynamic model, which involves parameters uncertainties, nonlinearities, and Lévy noises. There are some progresses in the research on the synchronization problem of nonlinear stochastic systems with Lévy noise. For example, [1] considered adaptive synchronization problem for stochastic system with Lévy noise and derived to a sufficient condition in terms of LMI enable the system to achieve synchronization. In [5], the authors established some criteria to ensure the adaptive exponential synchronization in the mean square of the master system and each slave system based on the Lyapunov functional theory, the generalized Itô’s formula, M-matrix method, and the adaptive control technique. Adaptive backstepping control method has been widely used to stabilize nonlinear systems in practical applications due to the excellent performance of compensating saturation [8]. To date, there are some progresses in the research on the nonlinear stochastic systems without Lévy noise. For example, the authors developed a fuzzy backstepping control to stabilize a class of stochastic nonlinear systems in [9]. In [10], the authors proved that the proposed fuzzy backstepping control approach for a class of uncertain stochastic nonlinear systems with both unmodeled dynamics.
and unmeasured states can guarantee that all the signals of the resulting closed loop system are bounded in probability. For the nonlinear stochastic system with Lévy noise, reference [11] constructed a robust adaptive controller based on the backstepping design technique and $H_{\infty}$ control theory for multi-input multi-output nonlinear stochastic Poisson jump diffusion system to achieve the $H_{\infty}$ tracking performance. In [12], the authors proposed the backstepping controller to stabilize strict-feedback systems driven by Lévy processes, which consist of Wiener and compensated Poisson processes. Furthermore, the above-mentioned work input control signal must be continuously updated, which may cause unnecessary resource consumption. In terms of reducing the utilization of communication resources, the event-triggered control scheme has proven to be superior [13]. It has been shown in [14] that the proposed dynamic triggering mechanism, wherein the threshold involves an internal dynamic variable, can allow for the larger minimum inter event times than a static counterpart. For example, an event-triggered controller was designed under the framework of the standard linear matrix inequalities in [15]–[17] and the stochastic finite-time stability had been discussed for uncertain nonlinear semi-Markovian switching cyber-physical systems under a discrete event-triggered communication scheme against false data injection attacks, which an event is detected through continuously monitoring its own and its neighbors' states. Based on event-triggered scheme, [18] studied the problem of adaptive fuzzy tracking control for strict-feedback stochastic nonlinear systems. [19] addressed the realization of almost sure synchronization problem for an array of stochastic networks associated with Lévy noise via event-triggered control. However, the adaptive backstepping control method is rarely considered for the nonlinear stochastic systems with Lévy processes.

Based on the previous discussion, this paper designs a command filtered adaptive neural network backstepping controller based on event-triggered mechanism to reach nonlinear stochastic systems synchronization with Lévy noise. By employing the generalized Itô’s formula combined with Lyapunov function method, it is proved that the proposed controller can ensure that all signals of closed system are bounded in probability. Compared with the current research, this work has the following contributions:

Firstly, a command filtered adaptive neural network backstepping controller is developed in this paper based on event-triggered mechanism to solve the nonlinear stochastic systems synchronization problem with Lévy noise. Secondly, in comparison with [1], [5], [12], an event-triggered scheme without Zeno phenomenon is proposed, which can reduce the frequency of network governance, and compared with the previous works in [12], the state observer is introduced into the proposed method to estimate the system state. Compared with [19], the command filtered control technology is introduced into the controller to obviate computing analytic derivatives. Lastly, compared with the previous works in [15]–[18], we consider that the nonlinear stochastic systems synchronization problem contains Lévy noise. By employing the generalized Itô’s formula combined with Lyapunov function method, the controller is designed and can ensure the stability of system dynamics.

The rest of the paper is organized as follows. Section II introduces basic theory about systems model and some lemmas. In Section III, we construct an observer to estimate the system state, then we propose a command filtered adaptive NN backstepping controller based on event-triggered mechanism, finally analyze the stability and Zeno behavior. In Section IV, the effectiveness of the proposed control method is proved by the simulation example. In Section V, we summarize the paper and give some conclusions.

II. PRELIMINARIES

In this paper, the master system is described as

$$
\begin{align}
&dx_1(t) = [x_2(t) + f_1(x(t))]dt \\
&dx_i(t) = [x_{i+1}(t) + f_i(x(t))]dt \\
&dx_n(t) = [f_n(x(t))]dt \\
m(t) = x_1(t)
\end{align}
$$

(1)

The slave system is in the form of feedback nonlinear system with Lévy noise as

$$
\begin{align}
&dy_1(t) = [y_2(t) + g_1(y(t))]dt \\
&dy_i(t) = [y_{i+1}(t) + g_i(y(t))]dt \\
&dy_n(t) = [u(t) + g_n(y(t))]dt + F(y(t), t)dw(t) + \int_R G(y(t), t, \xi) N(du, d\xi) \\
n(t) = y_1(t)
\end{align}
$$

(2)

where $i = 2, \ldots, n-1$; $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ and $y = (y_1, y_2, \ldots, y_n)^T \in \mathbb{R}^n$ are the system state vectors, $u(t)$ is the control input of the system, $m$ and $n$ are the systems outputs, $f_i(x)$ and $g_j(y)$ are unknown nonlinear functions.

Let $w(t)$ be an one-dimensional $\mathcal{F}_t$-adapted Brownian motion defined $(\Omega, \mathcal{F}_t, \{\mathcal{F}_t\}_{t \geq 0}, P)$ and $N(t, \xi)$ is a $\mathcal{F}_t$-adapted Poisson random measure defined on with a compensator $\tilde{N}$ and intensity measure $\pi$. We assume that $N$ is independent of $B$ and $\pi$ is a Lévy measure such that $\tilde{N}(dt, d\xi) := N(dt, d\xi) - \pi(d\xi) dt$. Usually, the pair $(B, N)$ is called a Lévy noise.

Defining the synchronization error as $z_i = y_i - x_i$ and $\vartheta = m - n$. The designed synchronization error system can be defined as

$$
\begin{align}
&dz_1(t) = [z_2 + h_1(z)]dt \\
&dz_i(t) = [z_{i+1} + h_i(z)]dt \\
&dz_n(t) = [u(t) + h_n(z)]dt + F(y(t), t)dw(t) \\
&\quad + \int_R G(y(t), t, \xi) N(du, d\xi) \\
\vartheta(t) = & z_1(t)
\end{align}
$$

(3)

where $z = (z_1, z_2, \ldots, z_n)^T \in \mathbb{R}^n$, $h_i(z) = g_i(y) - f_i(x)$ is the new nonlinear function by the difference between the nonlinear functions $g_i(y)$ and $f_i(x)$. 
Rewrite the system (3) by
\[
dz(t) = \left [ \begin{array}{c} A z(t) + K \vartheta(t) + \sum_{i=1}^{n} B_{i} [h_{i} (z(t))] + B_{n} u(t) \end{array} \right ] dt \\
+ B_{n} \left ( F (y(t), t) dw(t) \right ) + \int_{R} G (y(t), t, \xi) N (dt, d\xi) \right ) \]
where
\[
A = \begin{bmatrix} -k_{11} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & -k_{1n} \end{bmatrix}, \quad K = \begin{bmatrix} k_{11} \\
\vdots \\
k_{1n} \end{bmatrix}, \quad B_{n} = [0 \ldots 1]^{T}, \quad B_{i} = [0 \ldots 1 \ldots 0]^{T}, \quad C = [1 \ldots 0]
\]
and given a positive matrix \( Q = P \), there exists a positive matrix \( P^{T} = P \) satisfying \( A^{T} P + PA = -2Q \).

Lemma 1 [3]: Let \( C^{2,1} (R^{n} \times R: R^{+}) \) be the family of all functions \( V (x, t) \) on \( R^{n} \times R^{+} \), which are continuously twice differentiable in \( x \) and once in \( t \); moreover, defined \( V_{x} (x, t) = \left ( \frac{\partial V(x, t)}{\partial x_{1}}, \ldots, \frac{\partial V(x, t)}{\partial x_{m}} \right ) \). Then, an operator \( LV : R^{n} \times R^{+} \rightarrow R \) for the function \( V (x, t) \in C^{2,1} (R^{n} \times R^{+}, R) \) can be defined as
\[
LV (x, t) = V_{t} (x, t) + V_{x} (x, t) a (x, t) \\
+ \frac{1}{2} \text{tr} (B^{T} (x, t) V_{xx} (x, t) b (x, t)) \\
+ \int_{R} (V (x + c (x, t, \xi)) - V (x, t)) \pi (d\xi)
\]
(5)
where \( a (x, t), b (x, t), c (x, t, \xi) \) stand for the parameters of drift term, Brownian motion term, and Poisson jump term, respectively.

Lemma 2 [20]: For any \( x, y \in R^{n} \), the following inequality relationship holds
\[
x^{T} y \leq \frac{a^{t}}{a} \| x \|^{a} + \frac{1}{bc} \| y \|^{b}
\]
(6)
where \( a > 1, b > 1, c > 0 \), and \( (a - 1) (b - 1) = 1 \).

Lemma 3 [21]: The following inequality holds
\[
0 \leq |a| - \frac{a^{2}}{\sqrt{a^{2} + b^{2}}} \leq b
\]
(7)
where \( a \in R \) and \( b > 0 \).

Lemma 4 [22]: Define the command filter as
\[
\hat{\varphi}_{i,1} = \omega_{n} \varphi_{i,2} \\
\hat{\varphi}_{i,2} = -2\xi \omega_{n} \varphi_{i,2} - \omega_{n} (\varphi_{i,1} - \alpha_{i})
\]
where \( \varphi_{i,1} \) and \( \varphi_{i,2} \) are both filter output signals. And if the input signal \( \alpha_{i} \) satisfies \( |\alpha_{i}| < \rho_{i,1}, |\alpha_{i}| < \rho_{i,2} \) for all \( t \geq 0 \), then for any \( \mu > 0 \), there exist \( \xi \in (0, 1) \) and \( \omega_{n} > 0 \), such that \( |\varphi_{i,1} - \alpha_{i}| \leq \mu, |\hat{\varphi}_{i,1}|, |\hat{\varphi}_{i,2}| \) are bounded.

Lemma 5 [10]: If for all \( x \in R^{n}, t > t_{0} \), exist class \( \kappa_{\infty} \) functions \( \beta_{1} \) and \( \beta_{2} \), and two positive constants \( C \) and \( D \) satisfying
\[
\beta_{1} (|x|) \leq V (x) \leq \beta_{2} (|x|)
\]
we can obtain
\[
LV (x, t) \leq -CV (x, t) + D
\]
(9)
then all signals of the closed-loop system are bounded in probability.

III. MAIN RESULTS
A. OBSERVER DESIGN
Assumption 1: The unknown function \( h_{i} (x), i = 1, \ldots, n \) can be expressed as
\[
h_{i} (z | \theta_{i}) = \theta_{i}^{T} \varphi_{i} (z), \quad 1 \leq i \leq n
\]
(10)
where \( \theta_{i} \) is the ideal constant vector, \( \varphi_{i} (z) \) is the basis function vector.

The observer for (4) is designed as
\[
\hat{z} (t) = A \hat{z} (t) + K \hat{\vartheta} + \sum_{i=1}^{n} B_{i} [\hat{h}_{i} (\hat{z} (t) | \theta_{i})] + B_{n} u(t)
\]
\[
\hat{\vartheta} = C \hat{z} (t)
\]
(11)
where \( \hat{z} = (\hat{z}_{1}, \hat{z}_{2}, \ldots, \hat{z}_{n})^{T} \) is the estimated value of \( z = (z_{1}, z_{2}, \ldots, z_{n})^{T} \).

Define the state observation error \( e = z - \hat{z} \), from (4) and (11), we can obtain
\[
de (t) = \left [ \begin{array}{c} A e(t) + \sum_{i=1}^{n} B_{i} [h_{i} (\hat{z} (t)) - \hat{h}_{i} (\hat{z} (t) | \theta_{i}) + \Delta h_{i}] \end{array} \right ] dt \\
+ B_{n} \left ( F (y(t), t) dw(t) + \int_{R} G (y(t), t, \xi) N (dt, d\xi) \right )
\]
(12)
where \( \Delta h_{i} = h_{i} (z) - h_{i} (\hat{z}) \). According to Assumption 1, we can obtain
\[
\hat{h}_{i} (\hat{z} | \theta_{i}) = \theta_{i}^{T} \varphi_{i} (\hat{z})
\]
(13)

Defining the vector of optimal parameters as
\[
\theta_{i}^{\ast} = \arg \min_{\theta_{i} \in \Omega_{i}} \left [ \sup_{z_{i} \in U_{i}} \left | \hat{h}_{i} (\hat{z} | \theta_{i}) - h_{i} (\hat{z}) \right | \right ]
\]
(14)
where \( 1 \leq i \leq n, \Omega_{i} \) and \( U_{i} \) are compact regions for \( \theta_{i}, z_{i} \) and \( \hat{z}_{i} \).

Defining error of the optimal approximation and parameters estimation as
\[
e_{i} = h_{i} (z) - \hat{h}_{i} (\hat{z} | \theta_{i}^{\ast}), \quad \tilde{\theta}_{i} = \theta_{i}^{\ast} - \theta_{i}, \quad i = 1, 2, \ldots, n
\]
(15)
Assumption 2: The optimal approximation error remain bounded, there exists positive constants $\varepsilon_i$ and $\theta_i^*$, satisfying $|\varepsilon_i| \leq \varepsilon_i$, $|\theta_i| \leq \theta_i^*$.

Assumption 3: There exists a set of known constants $\gamma_i$, the following relationship holds $|h_i(z) - h_i(\tilde{z})| \leq \gamma_i \| z - \tilde{z} \|$. Combining (12) and (13) together gives rise to

$$ de(t) = \left[ Ae + \Delta h + \varepsilon + \sum_{i=1}^{n} B_i [\bar{G}^T \phi_i (\tilde{z}(t))] \right] dt $$

$$ + B \left( F(y(t), t) \cdot dw(t) + \int_{\mathcal{R}} G(y(t), t, \xi) N (dt, d\xi) \right) \] $$

(16)

where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]$, $\Delta h = [\Delta h_1, \ldots, \Delta h_n]$.

By Lemma 2 and Assumption 3, we obtain

$$ \mathcal{L}V_0 \leq -q_0 \| e \|^2 + \frac{1}{2} \| e \|^2 + \sum_{i=1}^{n} B_i [\bar{G}^T \phi_i (\tilde{z}(t))] $$

$$ + B_n \left( \frac{1}{2} tr \left( F^T PF \right) + \frac{1}{2} \int_{\mathcal{R}} \left[ G^T PG + 2e^T PG \right] \pi \left( d\xi \right) \right) \] $$

(17)

By Lemma 2 and Assumption 3, we obtain

$$ \mathcal{L}V_0 \leq -q_0 \| e \|^2 + \frac{1}{2} \| P \|^2 + \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i $$

(18)

and

$$ \mathcal{L}V_0 \leq -q_0 \| e \|^2 + \frac{1}{2} \| P \|^2 + \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i $$

(19)

By equations (17), (18) and (19), we obtain

$$ \mathcal{L}V_0 \leq -q_0 \| e \|^2 + \frac{1}{2} \| P \|^2 + \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i $$

(20)

Assumption 4 [3]: There are two of known constants $l_1, l_2$ such that the parameters of stochastic noises $F, G$ satisfy

$$ tr \left( F^T (y(t), t) F(y(t), t) \right) \leq l_1 \| y \|^2 $$

$$ \int_{\mathcal{R}} G^T (y(t, \xi)) G(y(t, \xi)) \pi (d\xi) \leq l_2 \| y \|^2 $$

(21)

By Lemma 2 and Assumption 4, we have

$$ \frac{1}{2} tr \left( F^T PF \right) + \frac{1}{2} \int_{\mathcal{R}} \left[ G^T PG + 2e^T PG \right] \pi (d\xi) $$

$$ \leq \frac{l_1 \lambda_{\max} (P)}{2} \| y \|^2 + \frac{l_2 \lambda_{\max} (P)}{2} \| y \|^2 + \frac{l_1 \lambda_{\max} (P)}{2} \| e \|^2 $$

(22)

so we have

$$ \mathcal{L}V_0 \leq -q_0 \| e \|^2 + \frac{1}{2} \| P \|^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i + l_2 \| y \|^2 $$

(23)

where $q_0 = \lambda_{\min} (Q) - \left( 1 + \frac{1}{2} \| P \|^2 \sum_{i=1}^{n} \gamma_i^2 \right), l = (l_1^2 + l_2^2)$. 

B. Controller Design

Theorem 1: Consider the master system (1) and the slave system (2), construct the synchronization error system (3) and design state observer (11). Suppose that Assumptions 1-4 hold. The following designs can ensure that all the signals remain bounded in probability and the synchronization errors can converge to near zero.

Define the error variable

$$ s_1 = z_1, \quad s_i = \dot{z}_i - \tilde{z}_{i,c} $$

$$ v_i = \dot{s}_i - \dot{\sigma}_i $$

(24)

the intermediate controllers

$$ \alpha_1 = - \left[ c_1 s_1 + \theta_1 \phi_1 (\tilde{z}) + sign (v_1) \delta_1 \right] $$

$$ \alpha_i = - \left[ c_i s_i + s_{i-1} + \theta_i \phi_i (\tilde{z}) + sign (v_i) \delta_i - \tilde{z}_{i,c} \right] $$

(25)

(26)

the parameter adaptive laws

$$ \dot{\theta}_i = \sigma_i \phi_i (\tilde{z}) v_i - \rho_i \delta_i $$

$$ \delta_i = r_i |v_i| - \eta_i \delta_i $$

(27)

the compensating signals are as follows

$$ \dot{\sigma}_1 = - \left[ c_1 \sigma_1 - \sigma_2 - (z_{2,c} - \alpha_1) \right] $$

$$ \dot{\sigma}_i = - \left[ c_i \sigma_i - \sigma_{i-1} - \sigma_{i+1} - (z_{i+1,c} - \alpha_i) \right] $$

(28)

(29)

(30)

the event-triggered controller

$$ \alpha_n = c_n s_n + s_{n-1} + \theta_n \phi_n (\tilde{z}) + sign (v_n) \delta_n - \tilde{z}_{n,c} $$

$$ \dot{u} (t) = -\alpha_n - \frac{v_n (\kappa_1 \alpha_n)^2}{\sqrt{(v_n \kappa_1 \alpha_n)^2 + \kappa_2^2}} - \frac{v_n M_1^2}{\sqrt{(v_n M_1)^2 + \kappa_2^2}} $$

$$ u (t) = \tilde{u} (t_k), \quad \forall [t_k, t_{k+1}), k \in \mathcal{N}^* $$

(31)

where $i = 2, \ldots, n-1, c_i, \sigma_i, \rho_i, r_i, \eta_i$ are design constant parameters, $c_i > 0, \sigma_i > 0, \rho_i > 0, \eta_i > 0, \delta_i = \delta_i^* - \delta_i$ is the upper bound estimation error. $t_k$ denotes the update time of controller. $z_{i,c}$ is the filter output when $\alpha_i$ is the input of the filter. In order to cope with the filter errors $z_{i,c} - \alpha_i$, the error compensating mechanism will be employed at each step of the filtering process and $\sigma_i$ is the compensation signals.

1) Step One

Constructing Lyapunov function:

$$ V_1 = V_0 + \frac{1}{2} \gamma_1^* + \frac{1}{2} \gamma_1^* \theta_1^T \dot{\theta}_1 + \frac{1}{2} \dot{\delta}_1^2 $$

(32)
Then we can get
\[ \mathcal{L}V_1 \]
\[ \leq \mathcal{L}V_0 + v_1 \dot{V}_1 + \frac{1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{1}{r_i} \hat{\delta}_i \hat{\delta}_i \]
\[ \leq \mathcal{L}V_0 + v_1 \left( \sum \left( z_2 - c \alpha_1 + \alpha_1 + \tilde{\theta}_i^T \varphi_1 - \sigma \theta_1 \right) + v_1 e_2 \right. \\
\left. + v_1 \tilde{\theta}_i^T \varphi_1 + v_1 (\epsilon_1 + \Delta h_1) - \frac{1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i - \frac{1}{r_i} \hat{\delta}_i \right) \tag{33} \]
Substituting the virtual controller (25) and adaptive law (27), one has
\[ \mathcal{L}V_1 \]
\[ \leq \mathcal{L}V_0 - c_1 v_1^2 + v_1 v_2 + \frac{\rho_1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{\eta \eta \hat{\delta}_i}{r_i} \delta_i \delta_i + \frac{1}{2} |v_1|^2 + \frac{1}{2} \]
\[ \leq -q_1 \|e\|^2 + \frac{1}{2} \|P \delta\|^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_i^T \dot{\theta}_i + \eta \| \delta_i \delta_i \| \tag{34} \]
where \( v_1 (\epsilon_1 + \Delta h_1) = v_1 \Delta t_1 \leq |v_1| |\Delta t_1| \leq |\delta_0|^2 = |v_1| |\hat{\delta}_0 + \hat{\delta}_1|, v_1 e_2 = \frac{1}{2} |v_1|^2 + \frac{1}{2} |v_2|^2, q_1 = q_0 - 1/2. \]
2) INDUCTIVE STEP
Constructing Lyapunov function:
\[ V_i = V_{i-1} + \frac{1}{2} \hat{\delta}_i^2 + \frac{1}{2} \tilde{\theta}_i^T \dot{\theta}_i + \frac{1}{2} \hat{\delta}_i^2 \tag{35} \]
Similar to (33), we get
\[ \mathcal{L}V_i \leq \mathcal{L}V_{i-1} + v_i \dot{V}_i + \frac{1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{1}{r_i} \hat{\delta}_i \hat{\delta}_i \]
\[ \leq -q_i \|e\|^2 + \frac{1}{2} \|P \delta\|^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_i^T \dot{\theta}_i \]
\[ + \lambda_{\max}^2 (P) \| \delta\|^2 - \sum_{i=1}^{i-1} c_i v_i^2 + v_{i-1} v_i + \frac{1}{2} \sum_{i=1}^{i-1} |v_i|^2 \]
\[ + \sum_{i=1}^{i-1} \left( \frac{\rho_1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{\eta \eta \hat{\delta}_i}{r_i} \delta_i \delta_i \right) + v_i \tilde{\theta}_i^T \varphi_1 + q_1 \text{ for } i \geq 2 \tag{36} \]
According to (26) and (27), we can obtain
\[ \mathcal{L}V_i \leq -q_i \|e\|^2 + \frac{1}{2} \|P \delta\|^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_i^T \dot{\theta}_i + \lambda_{\max}^2 (P) \| \delta\|^2 \\
\[ - \sum_{i=1}^n c_i v_i^2 + \frac{1}{2} \sum_{i=1}^n |v_i|^2 + \sum_{i=1}^n \left( \frac{\rho_1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{\eta \eta \hat{\delta}_i}{r_i} \delta_i \right) \]
\[ + v_i v_{i+1} \] \tag{37}
where \( q_i = q_{i-1} - k_1^2 / 2 \) and note that \( k_1 v_1 e_1 \leq |v_2|^2 / 2 + k_1^2 |e_1|^2 / 2. \)
3) FINAL STEP
At this step, we will design the event-triggered controller. We define the new error variable \( v_n \) and the derivative of \( v_n \) as follows:
\[ \dot{v}_n = \dot{\hat{s}}_n - \tilde{\sigma}_n - \dot{\hat{s}}_n - \hat{s}_n - \sigma \theta_n \]
\[ = u + k_{\text{in}} e_1 + \theta_n^T \varphi_1 + \tilde{\theta}_n^T \varphi_1 + \epsilon_n + \Delta h_n - \dot{\hat{s}}_n - \sigma \theta_n \tag{38} \]
Constructing Lyapunov function:
\[ V_n = V_{n-1} + \frac{1}{2} \| \delta_n \|^2 + \frac{1}{2 \sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{1}{2 r_i} \hat{\delta}_n \hat{\delta}_n \]
It follows from (38) that
\[ \mathcal{L}V_n = \mathcal{L}V_{n-1} + v_n \dot{V}_n - \frac{1}{\sigma_n} \tilde{\theta}_n^T \dot{\theta}_n - \frac{1}{r_n} \hat{\delta}_n \hat{\delta}_n \]
\[ \leq \mathcal{L}V_{n-1} + v_n \left( u + \theta_n^T \varphi_1 - \dot{\hat{s}}_n - \sigma \theta_n \right) + k_{\text{in}} v_n e_1 \]
\[ + v_n \Delta_n + \theta_n^T \varphi_n v_n + \frac{1}{\sigma_n} \tilde{\theta}_n^T \dot{\theta}_n + \frac{1}{r_n} \hat{\delta}_n \hat{\delta}_n \tag{39} \]
The event-triggered controller \( \tilde{u}(t) \) and the parameter adaptation laws can be given by (31). The actual controller and the triggering condition for the sampling instants are as follows:
\[ u(t) = \tilde{u}(t), \quad \forall (t_k, t_{k+1}) \tag{40} \]
\[ t_{k+1} = \inf \{ t \in R \mid \Delta t \geq \kappa_1 |u(t)| + M_1 \} \tag{41} \]
where \( \Delta t = \tilde{u}(t) - u(t) \) is the event sampling error, \( 0 < \kappa_1 < 1, M_1 \) is a positive constant, \( t_k, k \in N^+ \) is the controller update time.

C. STABILITY ANALYSIS
From (41), the following equation can be arrived at
\[ \Delta t = \tilde{u}(t) - u(t) = \beta_1(t) \kappa_1 u(t) + \beta_2(t) M_1 \tag{42} \]
where \( \beta_1(t), \beta_2(t) \) are time-varying parameters satisfying \( |\beta_1(t)| \leq 1 \) and \( |\beta_2(t)| \leq 1. \) Accordingly, one can obtain
\[ u(t) = \tilde{u}(t) - \beta_2(t) M_1 + \beta_1(t) \kappa_1 \tag{43} \]
According to Lemma 3 and substituting (27), (30), (31), (43) into (39) produces
\[ \mathcal{L}V_n \]
\[ \leq \mathcal{L}V_{n-1} + v_n \left( \frac{u(t)}{\kappa_1} - \beta_2(t) M_1 \right) \]
\[ + |v_n| |\delta_n| + \frac{2}{2} |v_n|^2 + \frac{2}{2} |e_1|^2 + \frac{\rho_1}{\sigma_n} \tilde{\theta}_n^T \dot{\theta}_n + \frac{\eta \eta \hat{\delta}_n}{r_n} \delta_n \delta_n + \| \delta \|^2 \]
\[ \leq -q_n \|e\|^2 + \frac{1}{2} \|P \delta\|^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_i^T \dot{\theta}_i + \lambda_{\max}^2 (P) \| \delta\|^2 \]
\[ - \sum_{i=1}^n c_i v_i^2 + \frac{1}{2} \sum_{i=1}^n |v_i|^2 + \sum_{i=1}^n \left( \frac{\rho_1}{\sigma_i} \tilde{\theta}_i^T \dot{\theta}_i + \frac{\eta \eta \hat{\delta}_i}{r_i} \delta_i \right) + \frac{2 \kappa_2}{1 - \kappa_1} \tag{44} \]
where \( q_n = q_{n-1} - k_1^2 / 2. \) According to Lemma 2, the following equation can be given
\[ \tilde{\theta}_i^T \dot{\theta}_i \leq -\frac{1}{2} \theta_i^T \dot{\theta}_i + \frac{1}{2} \theta_i^T \theta_i, \quad \delta_i \delta_i \leq -\frac{1}{2} \delta_i^2 + \frac{1}{2} \delta_i^2 \tag{45} \]
Substituting (45) into (44) produces
\[
\mathcal{L}V_n \leq -q_n\|e\|^2 - \sum_{l=1}^{n} c|\tilde{y}_l|^2 - \sum_{l=1}^{n} \left( \frac{\rho_l}{2\sigma_l} - \frac{1}{2} \right) \tilde{\theta}_l^T \tilde{\theta}_l
\]
\[
- \sum_{l=1}^{n} \frac{n_l}{2\tau_l} \delta^2_l + D \tag{46}
\]

Note that \( D = \sum_{l=1}^{n} \left( \frac{\rho_l}{2\sigma_l} \delta^T_l \delta_l + \frac{n_l}{2\tau_l} \delta^2_l \right) + D_{\max}^2 \|P\|^2 + \frac{2\gamma^2}{1-k_1} \).

Define \( C = \min \{ 2q_n / \lambda_{\min} (P) \}, 2c_l, \left( \frac{\rho_l}{2\sigma_l} - \frac{1}{2} \right), n_l / 2\tau_l \} \).

According to (46), we can obtain
\[
\mathcal{L}V_n \leq -CV_n + D \tag{47}
\]

According to Lemma 5, we can summarize that all the signals remain bounded in the closed-loop system and the synchronization errors converge to near zero.

To ensure that the proposed control method can avoid Zeno phenomenon, the proof can refer to references [14], [18]. The proof is as follows:

by recalling \( \Delta (t) = \hat{u} (t) - u (t) \), we have
\[
\frac{d}{dt} \| \Delta (t) \| = \frac{d}{dt} (\Delta \times \Delta)^{\frac{1}{2}} = \text{sign} (\Delta) \dot{\Delta} \leq |\dot{u}| \tag{48}
\]

Similar to the boundness of \( |\dot{\theta}_l| \) it is easy from the definition of \( \dot{u} (t) \) that one can obtain \( |\dot{u}| \leq B \) where \( B \) is positive constant. Thus, on \( |u(t)| \in [0, +\infty) \), there exists \( t^* \) such that \( \dot{t}^* \geq M_{\min} / B \). Therefore, there exists \( t^* \geq t^t \) such that \( \forall k \in k^*, \{ t_k+1 - t_k \} \geq t^*, \) and Zeno behavior is avoided. The proof is completed.

Remark 1: Under the framework of adaptive backstepping control technology, the control method proposed in this paper for the nonlinear stochastic systems driven by Lévy processes can be extended to any high-order nonlinear systems. Because the adaptive backstepping control method has excellent saturation compensation performance for complex high-order nonlinear systems, and the command filter technology can avoid explosion of complexity caused by the multiple derivation of the virtual control law in the design process of the backstepping controller. Furthermore, in the proposed method, since we use RBF neural network to approximate the unknown function, the unknown function in the high-order nonlinear systems can still be approximated by the RBF neural network.

Remark 2: The controller self-triggered mechanism is used to reduce the controller update frequency in this paper. Since the self-triggered control mechanisms can avoid the continuous monitoring and detection, which is relatively easy to be manipulated, it is easier to compute the triggering time instant.

IV. SIMULATION

In this section, we use the following example to verify the validity of the proposed method. Let the Genesio system [23] respectively be the drive and slave system as follows:
\[
\begin{align*}
\dot{x}_1 &= x_2 dt \\
\dot{x}_2 &= x_3 dt \\
\dot{x}_3 &= (-6x_1 - 2.92x_2 + 1.2x_3 + x_1^3) dt
\end{align*}
\]

The controlled slave system is as follows:
\[
\begin{align*}
\dot{y}_1 &= y_2 dt \\
\dot{y}_2 &= y_3 dt \\
\dot{y}_3 &= (u (t) + g (y)) dt + F dw + \int_R G (dt, d\xi)
\end{align*}
\]

where \( g (y) = -6y_1 - 2.92y_2 + 1.2y_3 + y_1^3 \), and the Brownian motion term and Poisson jump term are \( F = -\frac{3}{2} y_1^2 \), \( G = (y_2 - y_1) \xi \). The following initial conditions \( x (0) = [-0.2, 0.2, 0.2] \), and \( y (0) = [0, 0.1, 0] \) are employed. Choose the design parameters as \( c_1 = 20, c_2 = 20, c_3 = 20 \).

The numerical simulation results of example are shown in Figure 1. The dynamical behavior of the Genesio system and its slave system are shown in Figure 1.
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FIGURE 3. The synchronization trajectories of Genesio system and its slave system.

FIGURE 4. The synchronization trajectories of the systems states.

FIGURE 5. The synchronization error trajectories of the systems states.

FIGURE 6. The trajectories of $\bar{u}$, $u$ and release interval.

and Figure 2. It can be seen from Figure 1 that Lévy noise has a great influence on the behavior of the slave system, and the slave system without Lévy noise is very similar to the behavior of the drive system. However, the slave system with Lévy noise considered in this paper is completely inconsistent with the behavior of the drive system, so it is more difficult to study the synchronization control of the nonlinear stochastic systems with Lévy noise. Figure 3 and Figure 4 show the system synchronization trajectories obtained by the control method proposed in this paper. Figure 5 shows the synchronization errors and its estimated trajectory. Figure 6 shows the trajectories of the event-triggered controller and the continuous controller, as well as the number of triggers of the event-triggered controller. From the simulation example, it can be concluded that the control method proposed in this paper can solve the synchronization problem of nonlinear stochastic systems with Lévy noise, and the proposed event-triggered controller can effectively reduce the operating frequency of the controller.

V. CONCLUSION

This paper studied the nonlinear stochastic systems synchronization problem associated with Lévy noise. Combining the backstepping control method with stochastic analysis technique, a command filtered adaptive neural network controller with event-triggered mechanism has been developed. Under our proposed framework, the event-triggered scheme was considered to reduce the number of transmissions of control input signals and the command filter can avoid the calculation of analytical derivatives. By employing the generalized Itô’s formula combined with Lyapunov function method, the proposed controller can ensure that all signals of closed system are bounded in probability. Simulation results show the feasibility and effectiveness of the proposed results.

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