Design and strength estimation of composite multi-cavity pressure vessels

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Abstract. The article discusses a layer-by-layer method for calculating of composite products, and the "thread analogy" is provided for pressure vessels calculating. Stress fields at the multi-cavity cylindrical and spherical pressure vessels are calculated using FEM. Gas injection system is designed.

1. Introduction
Composites are widely used in various industries, for example, in aircraft and automobile construction, in military and space technology, and in civil engineering. In particular, the problem of increasing the efficiency of using composites in multi-cavity pressure vessels is urgent.

Purpose of the study: to carry out the strength calculation of composite gas tanks and substantiate their advantages over steel counterparts.

The following tasks were set:
1. Conduct a design calculation of composite pressure vessels based on the simplest "thread analogy and using the layer-by-layer calculation method for monolayers and for pairs of layers."
2. Calculate using the FEM stress fields in a multi-cavity pressure vessel of cylindrical and spherical shape.
3. Design a system for pumping gas into a multi-cavity gas tank.

2. Thread analogy for calculating pressure vessels
A simpler scheme for calculating the winding gas tanks is to use three assumptions, the so-called ‘thread analogy’ [3]:

1. The fibers work only on tension condition and carry the entire load, and the matrix is not loaded at all.
2. All fibers are equally stressed.
3. Failure of the structure occurs as a result of reaching in all fibers (simultaneously - according to the second assumption) the ultimate stress $\sigma_0$.

Assumption 2 is very “strong”, it limits the considered winding systems only to equally stressed ones: there are no extra fibers, everyone works at their limit.
As an example, consider the scheme of reinforcement of the cylindrical part of a gas tank by two families of fibers: 1 – with orientation $\pm \alpha_1$ and with layer thickness $h_1$ and 2 – with orientation $\pm \alpha_2$, with thickness $h_2$. The total wall thickness $h = h_1 + h_2$.

The average axial $\sigma_z$ and circumferential $\sigma_\theta$ stresses

$$
\sigma_z = \frac{pR}{2h}, \quad \sigma_\theta = \frac{pR}{h}
$$

in a critical state can be expressed through the limiting stress along the fibers $\sigma_0$, summing up the forces created in the two families of layers,

$$
\frac{A}{2} = h_1 \cos^2 \alpha_1 + h_2 \cos^2 \alpha_2;
$$

$$
A = h_1 \sin^2 \alpha_1 + h_2 \sin^2 \alpha_2.
$$

Two equations (2) contain four design parameters, two of which can be found after arbitrary specifying the other two. The question arises, is it really possible to set two winding angles arbitrarily, i.e. can any angle values be specified? Let us illustrate this with a few examples. Let's pretend that $h_2 = 0$. Two parameters remain, which are found from (2): $h = h_1 = \frac{3A}{2}$; $\alpha_1 = \alpha^* = 54.4^\circ$; ($\tan^2 \alpha^* = 2$).

Choosing angles $\alpha_1$, $\alpha_2$ equal to ($\pm 30/90$), ($\pm 45/90$), (0/±60), we will make sure from the table above that the total wall thickness remains the same $3A/2$ (table 1).

| $h_0$ | $h_{90}$ | $\alpha$ | $h_{\alpha}$ | $h$ |
|-------|----------|--------|-------------|-----|
| 0     | 0        | 54     | $\frac{3}{2}A$ | $\frac{3}{2}A$ |
| 0     | $\frac{5}{6}$ | 30     | $\frac{2}{3}A$ | $\frac{3}{2}A$ |
| 0     | $\frac{1}{2}$ | 45     | $A$ | $\frac{3}{2}A$ |
| $\frac{1}{2}$A | $A$ | 0      | 0 | $\frac{3}{2}A$ |
| 0     | $-\frac{1}{2}$ | 60     | $2A$ | $\frac{3}{2}A$ |
| $\frac{1}{2}$A | 0      | 60     | $\frac{4}{3}A$ | $\frac{3}{2}A$ |
| $\frac{5}{2}$A | 0      | 30     | $4A$ | $\frac{3}{2}A$ |

The same will remain for the algebraic sum of the thicknesses and for a combination of angles ($\pm 60/90$), (0/±45) or (0/±30), only one of the thicknesses will be negative, which means that it is impossible to provide equilibria in two stresses if two families of fibers have an orientation angle both
smaller or both larger than $\alpha^\ast$. More precisely, it is not the thickness that turns out to be negative, but the stresses in one of the families must be compressive in order to ensure uniform stress - equal absolute values of stresses [2].

For determination average axial and circumferential stresses in cylindrical part of the cylinder were applied formulas (1). For example, with $p = 5$ MPa, average radius $R = 50$ mm and wall thickness $h = 1.6$ mm.

$$\sigma_0 = 5 \cdot \frac{0.05}{0.0016} \approx 156 \text{ MPa}.$$  

Accordingly,

$$\sigma_z = 5 \cdot \frac{0.05}{2 \cdot 0.0016} \approx 78 \text{ MPa}.$$  

3. Layer-by-layer method for calculating composite products

Layer-by-layer method - one of the main methods of computer calculation stress states and limit loads for layered composite elements (laminates): plates and shells manufactured from unidirectional prepregs. The problem is set as follows: a design or a part consisting of a set unidirectional monolayers (prepregs) [3] (prepregs) is given.

As a first approximation, the previously popular "thread analogy", according to which only fibers bear the entire load, is considered. The requirement for the fibers to be evenly stressed makes it possible to design rational reinforcement structures and to estimate the possible weight of an "ideal" (optimal) balloon in which all families of fibers are equally loaded.

Calculation of a balloon reinforced with symmetrical pairs of layers.

The following technical material constants are specified:

\[ E_1 = 45 \text{ GPa}, \quad E_2 = E_3 = 10 \text{ GPa}, \]
\[ \mu_{12} = \mu_{13} = 0.3, \quad \mu_{21} = 0.067, \]
\[ G_{12} = G_{13} = 5 \text{ GPa}, \]
\[ G_{12} \approx 3.8 \text{ GPa}. \]

1. For the first reinforcement structure: $\alpha_1 = \pm 45^\circ$; $\alpha_2 = 90^\circ$.

\[ p = 100 \text{ atm} = 10 \text{ MPa}, \]
\[ R = 150 \text{ mm}, \quad h = 3 \text{ mm}. \]

\[ \sigma_z = \frac{pR}{2h} = 250 \text{ MPa}, \quad \sigma_\theta = \frac{pR}{h} = 500 \text{ MPa}. \]

Number layers with angles orientations $\pm 45 - 5; \ 90 - 10$.

2. For comparison was performed calculation stresses for angles of reinforcement $\pm 30 - 5; \ 90 - 10$.

| Table 2. Results of the layer-by-layer calculation method for pairs of layers. |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                  | $\varepsilon_z,\%$ | $\varepsilon_\theta,\%$ | $\sigma_z(\alpha), \text{ MPa}$ | $\sigma_\theta(\alpha), \text{ MPa}$ | $\sigma_z(90^\circ), \text{ MPa}$ | $\sigma_\theta(90^\circ), \text{ MPa}$ |
| $(\pm 45^\circ / 90^\circ)$ | 1.1 | 1.4 | 395 | 432 | 194 | 907 |
| $(\pm 30^\circ / 90^\circ)$ | 0.8 | 1.6 | 357 | 254 | 170 | 959 |

An example of a layer-by-layer calculation method for monolayers
Initial data:

- $\alpha$ is the angle of reinforcement; $h$ is the wall thickness;
- $n = 29$ is the number of layers; $R$ is the radius of the pipe.

- $p = 2.5$ MPa; $R = 150$ mm; $h = 8$ mm.

- $E(0) = 14$ GPa;
- $E(90) = 0.9$ GPa;
- $G_{12} = 0.7$ GPa;
- $E_{12} = 2.5$ GPa.

Figure 1. Cylindrical pressure vessel with designations for fiber placement angles and coordinate axes.

1. Calculation of average axial and circumferential stresses.

$$\sigma_\varepsilon = \frac{pR}{2h} = \frac{2.5 \cdot 150}{16} \approx 23.5 \text{ MPa};$$

$$\sigma_\theta = \frac{pR}{h} = \frac{2.5 \cdot 150}{8} \approx 47 \text{ MPa}.$$

2. Calculation of the average Young's moduli for a package of monolayers.

$$\bar{E}_\varepsilon = \frac{1}{n} \sum_{i=1}^{n} E(0) \cos^4 \alpha + E(90) \sin^4 \alpha + (2E_{12} + 4G_{12}) s^2 c^2 = 5.4 \text{ GPa};$$

$$\bar{E}_\theta = \frac{1}{n} \sum_{i=1}^{n} E(0) \sin^4 \alpha + E(90) \cos^4 \alpha + (2E_{12} + 4G_{12}) s^2 c^2 = 6.5 \text{ GPa}.$$

3. Calculation of the average axial and circumferential deformations.

$$\bar{\varepsilon}_\varepsilon = \frac{\sigma_\varepsilon}{\bar{E}_\varepsilon}; \quad \bar{\varepsilon}_\varepsilon = \frac{23.5}{5381} = 0.00435;$$

$$\bar{\varepsilon}_\theta = \frac{\sigma_\theta}{\bar{E}_\theta}; \quad \bar{\varepsilon}_\theta = \frac{46.9}{6490} = 0.00722.$$
4. Checking the calculation for the average axial and circumferential stresses.
\[ \bar{\sigma}_z = E_z \cdot \varepsilon_z = 5381 \cdot 0.00435 \approx 23.5 \text{ MPa}; \]
\[ \bar{\sigma}_\theta = E_\theta \cdot \varepsilon_\theta = 6490 \cdot 0.00722 \approx 47 \text{ MPa}. \]

5. Application of strength criteria for composite pipes.
Applying one or another strength criterion, the load is determined at the first destruction of one of the layers. Sometimes this load is taken as critical for the entire package, but such a calculation for the first failure provides too much safety margin. A more accurate result is given by taking into account the sequential accumulation of destruction.

For this, the problem of the destruction of the layers is consistently solved. If the fibers break, the layer (element) is turned off from work, if the matrix is destroyed, then the shear and Young moduli across the fibers are zeroed in this element, and then the calculation is carried out again up to a complete loss of bearing capacity.

The calculated axial and circumferential stresses are shown as a point in figure 2. In the same figure 2, the limiting surface (ellipse, rectangle) is constructed for a given pair of layers. If the point lies inside the limiting surface, then there is no destruction, and it is possible to determine how many times the pressure must be increased so that this point, moving along the beam with the same stress ratio, reaches the limiting surface. This number of "times" will be the safety factor, which is different for different pairs of layers with different laying angles. The layer in which the safety factor is lower will be the first to collapse. And in the "optimal" structure, one must strive for the safety factor for all pairs of layers to be the same, as in the thread analogy.

![Figure 2. Limiting surfaces in stress space for a cylinder.](image)

4. Design model
In the software complex ANSYS WB, a computational model of the cylindrical and spherical parts of the balloon was created. In figure 3 and figure 4 shows the calculation of axial and circumferential stresses in spherical and cylindrical shells [1].
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Figure 3. Results of calculating stresses in a spherical pressure vessel.

Figure 4. Results of calculating stresses in a cylindrical pressure vessel.

The performed analytical calculation of stresses in a multi-cavity gas tank was verified by the finite element method. In each shell, a pressure was set, which was calculated analytically. The circumferential stresses in each spherical shell turned out to be approximately equal to 300 MPa, which corresponded to the estimates by formulas (1).

5. FEM design cases
When calculating the cylindrical gas tank, internal pressure was applied and stresses, deformations, and safety factor were calculated. The pressure varied from 2.7 MPa to 9.9 MPa, since the working pressure of the cylinder is 2.7 MPa, and according to the current standards, the safety factor should be about three.

In figure 5 shows displacements in a cylindrical container, and figure 6 - safety factor.

Figure 5. Deformations.

Figure 6. Safety factor.

In figure 7 and 8 show the results of calculating normal stresses and equivalent stresses according to von Mises.

Figure 7. Normal stress.

Figure 8. Equivalent von Mises stresses.
6. Designing a gas injection unit

A metal-composite multi-cavity gas tank (MCB) includes spherical shells mounted with a gap one into the other and forming cavities for gas placement. The shells are reinforced with pipelines with the possibility of gas flow. The cylinder is equipped with a filling device - a gas inlet/outlet connection that provides the required, different pressures in the cavities.

As a result of using a multi-cavity gas tank, a larger amount of gas can be pumped in the same external dimensions.

The advantages of composite gas tanks:
• weight efficiency;
• safety (MCB is destroyed without fragments);
• reliability and durability (not afraid of scratches and shocks during operation).

![2D model of MCB.](image)

Figure 9. 2D model of MCB.

Conclusions
1. A variant of the design of the gas injection unit for a metal-composite multi-cavity spherical pressure vessel is proposed.
2. FEM-calculation of stress fields in the cylindrical and bottom parts of the balloon, and in the zone of the pole hole.
3. The applied layer-by-layer method calculation of composite products, taking into account the successive destruction of monolayers, makes it possible to estimate the critical pressure in the cylinder and the safety factor.

References
[1] Sklemina O Yu, Tatus N A and Polilov A N 2020 Analytical and finite element analysis of composite multi-cavity compressed gas cylinders *Materials of the Jubilee LXX Open Int. Student Scientific Conf. (SNK-2020) of the Moscow Poly* pp 700-4
[2] Polilov A N, Tatus N A and Sklemina O Yu 2019 Three etude problems on composite fuel gas tank *IOP Conf. Series: Materials Science and Engineering*
[3] Polilov A N 2016 *Experimental Mechanics of Composites 2nd ed* (Moscow: Publishing house of MSTU im. N.E. Bauman) p 376