Fuzzy-Random Reliability Analysis of loess landslide Stability Under Earthquake

Peng Mu 1,*
1 College of Equipment Management and Support, Engineering University of the Chinese People’s Armed Police Force, Xi’an, Shanxi, 710086, China
*Corresponding author e-mail: 624778311@qq.com

Abstract. In this paper, the fuzziness of the failure events and the parameters of the random variables is considered. In the reliability analysis of the loess landslide stability, the combination of the randomness and fuzziness is proposed, and the fuzzy random reliability calculation method of the loess landslide stability under the earthquake action is proposed. The established model is applied to the seismic stability analysis of a loess landslide, and the evaluation results show that the loess landslide is in an under stable state under the seismic action.

1. Introduction

In the classical reliability theory, \( z = 0 \) is used as the limit to measure whether a landslide fails. On both sides of the zero point, the failure and safety state of a landslide are transformed in the form of catastrophe. That is, when \( z = 0 \), it is in the limit state; when \( z > 0 \), it is in the stable state; when \( Z < 0 \), it is in the unstable state. In this way, the two points of the limit small distance on the left and right sides of the zero point represent two different states, which is unreasonable. Generally speaking, loess landslides change gradually from stability to instability, and the boundary is fuzzy, that is to say, there is a fuzzy interval between stability and instability. Therefore, this paper considers the fuzziness of the loess landslide failure event and random variable parameters. In the reliability analysis of loess landslide stability, the combination of randomness and fuzziness is used to study the probability of loess landslide instability under the action of earthquake, and the fuzzy-random reliability calculation method of loess landslide stability under the action of earthquake is proposed.

2. Fuzzy-random variable and fuzzy-random processing of parameters

2.1. Fuzzy-random variable

Fuzzy random variable is a kind of uncertain variable which is used to reflect the fuzziness and randomness. The actual geotechnical engineering is often restricted by the objective conditions and the problem itself. The sampling and testing of the physical and mechanical parameters of the rock and soil are very limited. The measured values of the parameters obtained according to the statistics of the test samples are different from the objective true values of the material properties of the actual engineering points. This difference includes not only the spatial variation of the material properties of rock and soil, but also the errors in the process of sampling and testing. Therefore, the physical and mechanical parameters of rock and soil contain both random characteristics and fuzzy characteristics [1-5]. In this paper, cohesion \( c \), internal friction factor \( f \) ( \( f = \tan \varphi \) ) and seismic horizontal acceleration \( A \) of sliding zone soil are taken as random variables.
2.2. Fuzzy random processing of parameters

In practical landslide engineering, probability method is generally used to determine the probability distribution of mechanical parameters of rock and soil, and then to determine the probability model of mechanical parameters of rock and soil. However, the mechanical parameters of rock and soil have typical fuzzy characteristics, and the test samples are generally small samples, so it is difficult to determine the probability distribution of mechanical parameters of rock effectively. Therefore, this paper uses the fuzzy mathematics method to deal with the mean value and variance of the random variables of loess landslides more reasonably. The specific methods are as follows.

2.2.1. Calculation of fuzzy-random mean. Suppose that the sample value According to the principle that the actual event is the most likely to occur, the degree that the measured data belongs to the fuzzy subset of the sample as a whole should reach the maximum, thus the objective function can be established.

\[ J_1 = \sum_{i=1}^{n} \mu_A(x_i) = \text{max} \]

\[ \frac{dJ_1}{d\bar{x}} = 0 \]  

In order to determine the membership degree of all elements in a fuzzy set, a reference point must be selected. Obviously, the membership degree of core point is 1, which is the most appropriate choice. According to the nature of the problem discussed here, it is most appropriate to take the generalized distance between each element in the support set and the kernel point of the subset as the scale to consider its membership degree to the fuzzy subset \( A \). Because the generalized distance includes \( x_i \) and \( \bar{x} \), \( \mu_A(x_i, \bar{x}) \) can be expressed as follows.

\[ \mu_A(x_i, \bar{x}) = \nu_A(D_{1i}) \]  

In the formula, \( D_{1i} \) is the generalized distance between each element in the branch set of \( A \) and the core point; due to the change of independent variables, the corresponding functional relationship changes from \( \mu \) to \( \nu \). Substituting formula (1) and formula (3) into formula (2), we can get formula as follows.

\[ \sum_{i=1}^{n} \frac{d[\nu_A(D_{1i})]}{dD_{1i}} \cdot \frac{dD_{1i}}{d\bar{x}} = 0 \]

From equation (4), it can be inferred that \( D_{1i} \) is a quadratic polynomial of at least \( \bar{x} \). Both geometric distance and Ma distance meet the conditions and both make the mathematical expression of \( dD_{1i} / d\bar{x} \) the simplest. In fact, geometric distance is a special case of Ma distance \( x \), so choosing Ma distance is the most appropriate.

\[ D_{1i} = (x_i - \bar{x})^2 \cdot \omega_i \]  

Where \( \omega_i \) is called the weight. Considering that each element (i.e. each sample test observation value) only appears once in the test process, the weight of each sample observation value shall be equal.

\[ \omega_i = \text{const} = \omega_0 \]  

Substituting formula (5) and formula (6) into formula (4), we can get formula as follows.

\[ \bar{x} = \frac{\sum_{i=1}^{n} \frac{d[\nu_A(D_{1i})]}{dD_{1i}} \cdot x_i}{\sum_{i=1}^{n} \frac{d[\nu_A(D_{1i})]}{dD_{1i}}} \]  

According to formula (7), it should be satisfied that when fuzziness does not exist, stochastic fuzzy statistical formula can automatically degenerate into classical stochastic statistical formula, and the form of membership function can be inferred as follows.

\[ \nu_A(D_{1i}) = e^{-(D_{1i})} \]  

Combining equations (3), (5) and (8), we can get formula as follows.

\[ \mu_A(x_i) = \exp[-(x_i - \bar{x})^2 \cdot \omega_{10}] \]
Since the membership function is dimensionless, the dimension of \( \omega_i \) must be inversely related to that of \( \omega(x_i - \bar{x})^2 \). Considering data matching and core point taking into account.

\[
\omega_{10} = \frac{1}{(d_{1\text{max}} - d_{1\text{min}})/2},
\]
\[
d_{1i} = (x_i - \bar{x})^2 \quad (i = 1, 2, \ldots, n)
\]

In formula (10), \( d_{1\text{max}} \) and \( d_{1\text{min}} \) is the maximum and minimum value in \( d_{1i} \). The random fuzzy mean value of the sample observation value can be calculated by combining equations (5), (6), (7) and (10) as

\[
\bar{x} = \frac{\sum_{i=1}^{n} \exp\left[-2(x_i - \bar{x})^2 / (d_{1\text{max}} - d_{1\text{min}})\right] \cdot x_i}{\sum_{i=1}^{n} \exp\left[-2(x_i - \bar{x})^2 / (d_{1\text{max}} - d_{1\text{min}})\right]}
\]

2.2.2. Calculation of fuzzy-random variance. The sample value \((x_1, x_2, \ldots, x_n)\) with a variable capacity of \( n \) is measured, their random fuzzy average value is \( \bar{x} \) and random fuzzy variance is \( \sigma^2 \). In order to find \( \sigma^2 \), set up calculation sample \( \xi_i = (x_i - \bar{x})^2, (i = 1, 2, \ldots, n) \), the membership degree of the fuzzy subset \( B \) on the domain \( R = \{(x_i - \bar{x})^2 | i = 1, 2, \ldots, n\} \) of \( \xi_i = \mu_{B}(\xi_i) \). Transform \( x_i \) into \( \xi_i \), the principle is the same as the derivation of random fuzzy mean value.

\[
d_{2i} = [(x_i - \bar{x})^2 - \sigma^2]^2 \quad (i = 1, 2, \ldots, n)
\]
\[
\omega_{2n} = \frac{1}{(d_{2\text{max}} - d_{2\text{min}})/2}
\]
\[
\mu_{B}(\xi_i) = \exp\left[-2[(x_i - \bar{x})^2 - \sigma^2]^2 / (d_{2\text{min}} d_{2\text{max}})\right]
\]

In the formula, \( d_{2\text{max}} \) and \( d_{2\text{min}} \) is the maximum value and the minimum value in \( d_{2i} \) respectively.

It is the same as the calculation method of fuzzy-random mean, the fuzzy-random variance of sample observations can be obtained by combining all kinds of data.

\[
\sigma^2 = \frac{1}{n-1} \left( \frac{\sum_{i=1}^{n} \exp\left[-2[(x_i - \bar{x})^2 - \sigma^2]^2 / (d_{2\text{max}} - d_{2\text{min}})\right] \cdot (x_i - \bar{x})^2}{\sum_{i=1}^{n} \exp\left[-2[(x_i - \bar{x})^2 - \sigma^2]^2 / (d_{2\text{max}} - d_{2\text{min}})\right]} \right)
\]

3. The establishment of function of loess landslide safety under earthquake force
Based on the unbalanced force transfer method, the corresponding function of loess landslide safety quantity is established.

\[
Z = R - S = \sum_{i=1}^{n} (R_i \prod_{j=1}^{n} \psi_j) + R_n - \sum_{i=1}^{n} (S \prod_{j=1}^{n} \psi_j) - S_n
\]
\[
= \sum_{i=1}^{n} \left[ W_i (\cos \alpha_i - A \sin \alpha_i) f + C_i L_i \prod_{j=1}^{n} \psi_j + W_n (\cos \alpha_n - A \sin \alpha_n) f + C_n L_n \right] - \sum_{i=1}^{n} W_i (\sin \alpha_i + A \cos \alpha_i) \prod_{j=1}^{n} \psi_j - W_n (\sin \alpha_n + A \cos \alpha_n)
\]

The parameter \( f = \tan \phi \); \( R \) is the anti sliding force of the landslide; \( S \) is the sliding force of the landslide; \( R_i \) is the anti sliding force of the block \( i \); \( S_i \) is the sliding force of the \( i \)-th block, \( W_i \) is the weight of the \( i \)-th block, \( \text{kN} / \text{m} \), \( C_i \) is the cohesion of the \( i \)-th sliding surface (belt), \( \text{kPa} \), \( L_i \) is the length of the \( i \)-th sliding surface (belt), \( \text{m} \); \( \phi_i \) is the internal friction angle of the \( i \)-th sliding surface (belt) (\(^\circ\)); \( \alpha_i \) is
the inclination angle of the i-th sliding surface (°); A is the horizontal acceleration of the earthquake, g; ψ is the transfer coefficient from the residual sliding force of the i-th block to the i+1 block. In the formula, c and f approximately follow the normal distribution, and the horizontal acceleration A follows the maximum type II distribution. The equivalent normal distribution is transformed into the normal distribution, so R and S follow the normal distribution, so Z also follow the normal distribution. Assuming that 

\[ EZ = a \text{ and } \mu(Z) = \delta \text{ are the mean and variance of the function, the probability density function of the function is as follows.} \]

\[ f(Z) = \frac{1}{\sqrt{2\pi}\delta} \exp \left[ -\frac{1}{2} \left( \frac{Z-a}{\delta} \right)^2 \right] \]  

(18)

4. Fuzzy-random limit state equation of loess landslide under earthquake

The basic state variable \( X_i \) is fuzzed into fuzzy random variable \( \tilde{X}_i \), and the safety function of loess landslide stability under earthquake is established.

\[ Z = g(\tilde{X}) = g(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N) \]  

(19)

The above formula takes the fuzzy random variable \( \tilde{X}_i \) as the independent variable and takes the value as the function of the fuzzy random variable, which is called the fuzzy-random safety function.

Generally speaking, loess landslides change gradually from stability to instability, and the boundary is fuzzy, that is to say, there is a fuzzy interval between stability and instability. Therefore, the fuzzy random limit state equation of loess landslide reliability under earthquake action is as follows.

\[ Z = g(\tilde{X}) = g(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N) \equiv \tilde{b} \]  

(20)

In the formula, \( \tilde{b} \) is the fuzzy random limit state value, when \( \tilde{b} = 0 \), it is transformed into the traditional reliable function. Three stability states of loess landslides are divided into: when \( Z = \tilde{b} \), the stability of loess landslides is in fuzzy state; When \( Z < \tilde{b} \), it is considered that the loess landslide is in an unstable state; When \( Z > \tilde{b} \), it is considered that the loess landslide is in a stable state.

5. Membership function of loess landslide safety under earthquake

According to the reliability theory, the reliability of loess landslide is as follows.

\[ P_s = P(Z > 0) = \int_0^{+\infty} f(Z)dZ \]  

(21)

Where \( P_s \) is reliability and \( f(Z) \) is probability density function. When considering the transition from "complete instability" to "complete stability", a membership function \( \mu(Z) \) is introduced to represent the stability of loess landslide.

According to the theory of fuzzy mathematics, the reliability of loess landslide is as follows.

\[ P_s = \int_{-\infty}^{+\infty} \mu(Z)f(Z)dZ \]  

(22)

Where \( \mu(Z) \) is the membership function of the safety quantity. According to the classical probability theory, for the random fuzzy variable \( N(a, \delta^2) \) of normal distribution, its probability of change in \( a \pm 3\delta \) is 0.999, Available value is \( \mu(a \pm 3\delta) = 1 \), \( \mu(a - 3\delta) = 0 \), \( \mu(a) = 0.5 \).

The membership function of sine curve is selected, and its expression is as follows.

\[ \mu(Z) = \begin{cases} 
0 & \text{if } Z < -3\delta \\
\frac{1}{2} + \frac{1}{2} \sin \left( \frac{Z\pi}{6\delta} \right) & \text{if } -3\delta \leq Z < 3\delta \\
1 & \text{if } Z \geq 3\delta 
\end{cases} \]  

(23)

This kind of membership function is widely used in reliability research. In this paper, the membership function of loess landslide safety under earthquake is analyzed in this form.
6. Calculation method of fuzzy-random reliability of loess landslide under earthquake

In the function of formula (17), c and f approximately obey the normal distribution, and the horizontal acceleration A of earthquake obeys the maximum II distribution. The equivalent normal distribution is treated as normal distribution, so R and S obey the normal distribution, so Z also obeys the normal distribution. Let $E(Z) = a$ and $\mu(Z) = \delta$ be the mean and variance of the function respectively, then the probability density function of the function is as follows.

$$f(Z) = \frac{1}{\sqrt{2\pi}\delta} \exp\left(-\frac{1}{2}\left(\frac{Z-a}{\delta}\right)^2\right)$$

(24)

By introducing the membership function $\mu_Z(Z)$ which can represent the safety of loess landslide under earthquake, the fuzzy random reliability of landslide can be obtained as follows.

$$P_s = \int_{-\infty}^{\infty} \mu_Z \frac{1}{\sqrt{2\pi}\delta} \exp\left(-\frac{1}{2}\left(\frac{Z-a}{\delta}\right)^2\right) dZ$$

(25)

The fuzzy-random reliability index is as follows.

$$\beta = \phi^{-1}(P_s)$$

(26)

The fuzzy-random failure probability is as follows.

$$P_f = 1 - P_s$$

(27)

7. Fuzzy-random reliability analysis of the stability of a loess landslide under earthquake

The calculation section of fuzzy random reliability of a loess landslide is shown in Figure 1.

![Figure 1. Stability calculation section of loess landslide.](image)

The parameters are fuzzy treated by the methods of formula 12 and formula 16. According to formula 25 and formula 27, formula 17 is used as the function of safety quantity of loess landslide, and formula 23 is used as the subordinate function of safety quantity. The results show that the probability of fuzzy random instability of a loess landslide is 38.5%, which indicates that a loess landslide is in an unstable state under earthquake.

8. conclusion

This paper combines randomness with fuzziness in the reliability analysis of loess landslide stability, and puts forward the fuzzy random reliability calculation method of loess landslide stability under earthquake. The established model is applied to the seismic stability analysis of a loess landslide, and the evaluation results show that the loess landslide is in an under stable state under the seismic action.

9. Acknowledgments

This work was financially supported by Basic research fund project of Engineering University of the Chinese People’s Armed Police Force (WJY201921).
References
[1] S.Y.Wang, H. Zhang, Fuzzy random reliability analysis of bedding rock slope stability, Hydroelectric power generation. 44 (2018) 31-34.
[2] C.Liu, Study on random and fuzzy selection of rock mechanical parameters in slope engineering, Geotechnical Mechanics. 25 (2004) 1327-1329.
[3] P.Wang, Determination of rock shear parameters based on Stochastic Fuzzy Theory, Journal of rock mechanics engineering. 24 (2005) 547-552.
[4] H.S.Li, Random fuzzy estimation method for probability distribution of rock mechanical parameters, Journal of solid mechanics. 14 (1993) 347-351.
[5] H.S.Li, Concept and method of random fuzzy reliability in geotechnical engineering, Geotechnical Mechanics. 14 (1993) 25-34.