Low-frequency band gaps in Y-type phononic crystals

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Abstract. In this paper, an Y-type of locally resonant phononic crystal (LRPC) material model is analyzed in detail. Some geometric parameters of the material model are investigated using FEM to assess the trends of band gaps (BGs). The BGs results show that the material model can open a low-frequency BG less than 200 Hz. When the lattice constant of the material model cell is determined, the geometrical parameters of scatterers are found to be the key to open BGs. The results show that the effect of the scatterer on BGs is studied by adjusting the two ratios (the ratio of the square bottom edge length to height of the scatterer composed of lead and the ratio of the square bottom edge length to height of the scatterer composed of aluminium), respectively. The changes of the two ratios have little effect on the upper edge of BGs. Similarly, the change of the ratio of the scatterer composed of aluminum has little effect on the lower edge of BGs. However, the variation of the ratio of the scatterer composed of lead can make the lower edge of BGs lower and increase BGs width. The vibration and noise reduction devices with different working frequencies can be designed by adjusting the geometric parameters of the scatterer.

1. Introduction

With the development of science and technology, the pollution sources of low-frequency vibration and noise are inevitable. In daily life, the frequency range of low-frequency vibration and noise that do harm to our health is about 20-250 Hz [1,2]. Low-frequency vibration and noise are always with us, doing harm to our health [3]. As a new kind of acoustic functional material, phononic crystals (PCs) can attenuate the propagation of elastic waves within a certain frequency range and effectively control the propagation of low-frequency noises [4]. Therefore, the existence of the band gap (BG) allows it to be applied in many aspects, such as filters, acoustic lenses, low-frequency sound insulation materials and so on [5,6].

PCs have good physical properties in controlling elastic wave propagation and can suppress noise or vibration diffusion in BG frequency range [7]. At the beginning, the Bragg scattering PC was proposed. Because the large size cell of Bragg scattering PC can obtain the low-frequency BG, it is very difficult to apply it in the low-frequency range [8]. A kind of locally resonant phonon crystals (LRPCs) that can obtain low frequency band gap by using small size cell is proposed by Liu et al. [9], so that LRPCs can be applied in low frequency range and draw wide attention. A 2D structure ring LRPCs is designed by Zhai et al. [10]. By effective optimization scheme, the frequency BG below 200Hz can be obtained. A new 3D structure PC slab consisting of piezoelectric inclusions is designed by Khelif [11] and found that the ratio of slab thickness to lattice period is the key parameter to determining the existence and width of band gap. Therefore, the frequency BG above 1000 Hz can be obtained. Hsu [12] proposed a three-dimensional (3D) structure LRPCs composed of an array of
stepped resonators and study the effect of different resonator geometries and the lattice symmetries of the resonator array on the BG by FEM. The BG can be opened below 150 Hz. A new elastic beam structure of 3D structure LRPCs is proposed by Zhang et al. [13], which obtained the low-frequency BG of 25-395Hz by introducing the stiffness array into the local resonance, and opened a new way for obtaining the ultra-low-frequency BG of 3D structure PGs.

At present, because 2D structure PCs are easy to obtain low frequency BG, most of the researches focus on 2D structure PCs. For 3D structure PCs, the vibration mode is more complex than that of 2D structures due to the z-direction vibration mode involved, and the band structure also has more coupling mechanism, which makes it difficult to obtain low-frequency BG [14,15]. For 2D structure, 3D structure has more practical value [16]. Therefore, a Y-type 3D structure LRPC model is proposed in this study, the BG characteristics are analyzed by using the FEM, and the variation mechanism of low-frequency BG is studied by adjusting some geometric parameters of the scatterer.

2. The unit cell model and band structures of the Y-type 3D LRPC structures

![Figure 1. Schematics of (a) a square lattice unit of the 3D structures, (b) the front view of Y-type LRPC structure, (c) the stereograms of the bottom of Y-type LRPC structure, (d) the XY plan at the bottom of the Y-type LRPC structure.](image)

In this paper, the novel Y-type of LRPC materials structure is showed. Figure 1(a) shows the square lattice unit cell model of 3D stereogram. It can be seen that the Y-type 3D structure is composed of three parts: the top, middle and bottom, and the top and bottom are symmetrical about the middle, which is just a cuboid. From figures 1(a) and 1(b), it can be noted that the boundary surface marked red indicates the Bloch boundary and the BG is calculated by applying the Bloch boundary conditions using the Comsol FEM software. Figures 1(b) and 1(c) are the front view and the stereograms of the bottom of Y-type LRPC structure, respectively. The bottom of this Y-type structure is composed of four elastic beams in the Y shape, a cuboid in the middle and a rectangular external frame. Therefore,
for the top and bottom parts of Y-type LRPC, their external frame is the host medium, and cuboid in the middle and Y-type elastic beam are scatterers. The lattice constant of this cell is 40 mm. Moreover, for the middle part of Y-type LRPC structure, the middle part is a cuboid with length and width of \( i \) and height of \( H \), and the middle part also belong to the scatterer.

Figure 1(d) shows the XY plan at the bottom of the Y-type LRPC structure. In this diagram, \( a \) is the lattice constant, \( b \) is the thickness of the outer frame, \( c \) is the square bottom edge length of the scatterer in the middle, \( d \) and \( e \) respectively represent the width and length of the part of Y-type elastic beam connected with the intermediate scatterer, \( f \), \( g \), \( h \) are the dimensions of each part of the XY plane of the Y-type elastic beam. In addition, the height of the host medium of the square outer frame and the scatterer of the four Y-shaped elastic beams is \( h_1 \), and the height of the scatterer composed of aluminum in the middle is \( h_2 \). For the top part of Y-type LRPC, the materials and structural geometric parameters of the top and bottom are the same because the top and bottom are symmetrical about the middle.

The cuboid scatterer material in the middle part of the Y-shaped LRPC is lead. In figure 1(c), A, B and C represent different material components, respectively, for the top and bottom parts of the Y-shaped LRPC, part A is composed of silicon rubber, part B is composed of PMMA, and part C is composed of aluminum. The material properties that make up the structure are shown in table 1.

| Materials       | Density(\( \rho \))(Kg/m\(^3\)) | Young’s modules(E) Pa | Poisson’s ratio(\( \nu \)) |
|-----------------|----------------------------------|-----------------------|---------------------------|
| Silicon rubber  | 1300                             | 0.1175\( \times 10^6 \) | 0.4688                    |
| Aluminum        | 2700                             | 7\( \times 10^10 \)   | 0.3000                    |
| PMMA            | 1142                             | 0.2\( \times 10^10 \) | 0.3890                    |
| Lead            | 11600                            | 4.08\( \times 10^10 \) | 0.3690                    |

Table 1. Materials parameters.

In this paper, the Comsol FEM software is used to calculate the band structures of the Y-type LRPC structure. The geometric parameters of the structure calculated by Comsol are as follows: \( a = 40\times 10^{-3} \) m, \( b = 2.5\times 10^{-3} \) m, \( c = 10\times 10^{-3} \) m, \( d = 2\times 10^{-3} \) m, \( e = 5\times 10^{-3} \) m, \( f = 7.5\times 10^{-3} \) m, \( g = 2\times 10^{-3} \) m, \( h = 13\times 10^{-3} \) m, \( i = 35\times 10^{-3} \) m, \( h_1 = 10\times 10^{-3} \) m, \( h_2 = 15\times 10^{-3} \) m, \( H = 10\times 10^{-3} \) m.

Figure 2 shows the band structure of Y-type LRPC calculated by Comsol, a FEM software. We can find 10 lines in the band structure, and there are two BGs, BG1 and BG2. BG1 (81.5 to 101.5 Hz) is between the upper edge of the sixth line and the lower edge of the seventh line, while BG2 (106.9 to 158.7 Hz) is between the upper edge of the eighth line and the lower edge of the ninth line.

![Figure 2. The band structures of the Y-type LRPC structure.](image-url)
From the band structure, it can be seen that the Y-type LRPC material can generate low-frequency BGs with a frequency below 200 Hz, and the starting frequency of BG1 can be lower than 85 Hz, which can meet the requirements of low-frequency in engineering vibration reduction. In addition, the BG range can be adjusted by adjusting some geometric parameters of the structure, so as to meet the requirements of designing vibration and noise reduction devices with different working frequencies.

3. The influence of geometric parameters on BGs
In order to facilitate and guide the design of Y-type LRPC composites, this paper studies the influence of two geometric parameters on the low-frequency BGs. The two geometric parameters are the ratio of the square bottom edge length \(i\) to height \(H\) of the scatterer composed of lead and the ratio of the square bottom edge length \(c\) to height \(h_2\) of the scatterer composed of aluminum. For the convenience of description, \(S_1\) and \(S_2\) are used to represent these two ratios, respectively, and the expressions of \(S_1\) and \(S_2\) are described as follows:

\[
S_1 = \frac{i}{H} \quad (1)
\]

\[
S_2 = \frac{c}{h_2} \quad (2)
\]

in which, \(i\) and \(H\) denote the square bottom edge length and height of the scatterer composed of lead, respectively, and \(c\) and \(h_2\) denote the square bottom edge length and height of the scatterer composed of aluminum, respectively.

![Figure 3](image-url)

**Figure 3.** Schematics of (a) the influence of the ratio \(S_1\) on geometric model, (b) the influence of the ratio \(S_2\) on geometric model, (c) the influence of the ratio \(S_1\) on BGs, (d) the influence of the ratio \(S_2\) on BGs.
From figures 3(a) to 3(b), it clearly noted that the influence of the ratio S1 (fixed height H is the initial 10x10^-3 m) and S2 (fixed the height h_2 is the initial 15x10^-3 m) on geometric model and the corresponding geometric model changes, respectively. Figures 3(c) and 3(d) show the influence of the ratio S1 and S2 on BGs, respectively. Based on the structure in the previous section, Comsol, a FEM software, calculates the different square bottom edge length l and c of the Y-type LRPC, respectively, and other geometric parameters remain the same as those in the Section 2 in the calculation.

Figure 3(c) shows the influence of the ratio S1 on BGs, and the four curves in the figure represent the lower boundary of the BG1, respectively, the upper edge of the BG1, the lower edge of the BG2 and the upper edge of the BG2. The ratio S1 calculated ranges from 2.5 to 3.7, and figure 3(a) shows the corresponding geometric model changes. As can be seen from the figure, BG2 is gradually widening with the increase of the ratio S1. For BG1, BG1 does not appear until the ratio S1 is 3, and then gradually widens (from 3 to 3.7). Therefore, it can be concluded from the variation trend of the BGs that the upper edge of the BG2 is always lower than 180Hz, and the lower edge of the BG1 starts to be lower than 80Hz as the BGs widths, which can meet the demand for low-frequency vibration and noise reduction. In figure 3(d), the four curves in the figure successively represent the lower edge of BG1, the upper edge of BG1, the lower edge of BG2, and the upper edge of BG2. It can be concluded from the figure that when the height h_2 of the scatterer composed of aluminum is fixed, the BG1 and BG2 hardly changes with the increase of the ratio S2. Therefore, it can be concluded that the ratio S2 has little effect on BGs in a certain range (from 5/15 to 1).

4. Conclusions
In this paper, a novel LRPC with Y-type structure for the vibration and noise reduction in engineering field is proposed, and its band structure is calculated by defining the Bloch boundary condition using the Comsol FEM software. The structure can produce low frequency BGs below 200 Hz, and the BGs ranges can be adjusted by changing two geometric parameters (the ratio S1 of the scatterer composed of lead and the ratio S2 of the scatterer composed of aluminum), so as to meet engineering requirements and guide the design of vibration and noise reduction devices with different BGs ranges.

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