Collisional excitation of $\text{HC}_3\text{N}$ by para- and ortho-$\text{H}_2$

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ABSTRACT

New calculations for rotational excitation of cyanoacetylene by collisions with hydrogen molecules are performed to include the lowest 38 rotational levels of $\text{HC}_3\text{N}$ and kinetic temperatures to 300 K. Calculations are based on the interaction potential of Wernli et al. *A& A*, 464, 1147 (2007) whose accuracy is checked against spectroscopic measurements of the $\text{HC}_3\text{N}$–$\text{H}_2$ complex. The quantum coupled-channel approach is employed and complemented by quasi-classical trajectory calculations. Rate coefficients for ortho-$\text{H}_2$ are provided for the first time. Hyperfine resolved rate coefficients are also deduced. Collisional propensity rules are discussed and comparisons between quantum and classical rate coefficients are presented. This collisional data should prove useful in interpreting $\text{HC}_3\text{N}$ observations in the cold and warm ISM, as well as in protoplanetary disks.

Key words: ISM: molecules, molecular data, molecular processes, scattering.

1 INTRODUCTION

Cyanopolyne molecules, with general formula $\text{HC}_{3n+1}\text{N}$ ($n \geq 1$), have been detected in a great variety of astronomical environments, from the solar system to extragalactic sources. The simplest one, $\text{HC}_3\text{N}$ (cyanoacetylene) is the most abundant of the family. It has been first detected towards the giant molecular cloud Sgr B2 by Turner (1971) and in comet Hale-Bopp by Bockelée-Morvan et al. (2004). Because of its low rotational constant and large dipole moment, $\text{HC}_3\text{N}$ is considered as a very good thermometer and barometer in the ISM. It has been detected in the ground level and in excited vibrational levels, thanks to the presence of low-lying bending modes (below 1000 cm$^{-1}$). Cyanopolyne is also an abundant nitrogen bearing species and it has been proposed as a precursor of prebiotic molecules such as cytosine, one of the four nitrogen bases found in DNA and RNA (e.g. Orge 2002).

In most astrophysical regions where $\text{HC}_3\text{N}$ is observed, the low frequency of collisions (due to low density) cannot maintain local thermodynamical equilibrium (LTE). This is best exemplified by the natural occurrence of the maser phenomenon (microwave amplification by stimulated emission of radiation) in the $1 \rightarrow 0$ transition at 9.1 GHz, as first observed towards Sgr B2 by Morris et al. (1976). The interpretation of $\text{HC}_3\text{N}$ emission spectra, in terms of density, temperature and column density, thus requires to solve the equations of statistical equilibrium, which in turn necessitates a good knowledge of the collisional rate coefficients. In the cold and dense ISM, the most abundant colliders are hydrogen molecules and helium atoms. In warmer regions, such as photodissociation regions (PDRs) or comets, electron collisions may also play an important role, due to the large $\text{HC}_3\text{N}$ dipole (3.7 D) (see e.g. Gratier et al. 2013), as well as hydrogen atoms.

The collisional excitation of $\text{HC}_3\text{N}$ by He and $\text{H}_2$ was first studied by Morris et al. (1976) using the “hard-ellipsoidal” classical approximation. This pioneering work was soon followed by the first determination of the $\text{HC}_3\text{N}$–He potential energy surface (PES), using an approximate “electron gas” model, combined with Monte-Carlo quasi-classical trajectory (QCT) calculations (Green & Chapman 1975). Classical mechanics was employed instead of quantum mechanics due to the difficulty of expanding the PES in terms of Legendre polynomials. Indeed, for such extremely anisotropic interactions ($\text{HC}_3\text{N}$ is ~ nine bohr long), the Legendre polynomial expansion is very slowly convergent and subject to severe numerical problems, as first discussed by Chapman & Green (1975). These problems were circumvented by a novel approach proposed by Wernli et al. (2007a) who derived the first PES for $\text{HC}_3\text{N}$–$\text{H}_2$ (hereafter denoted as W07). These authors were thus able to perform both classical and quantum calculations for $\text{HC}_3\text{N}$ colliding with para-$\text{H}_2(j_2 = 0$, where $j_2$ is the $\text{H}_2$ angular momen-
tum). They provided rate coefficients for the lowest 51 rotational levels of HC$_3$N ($j_1 = 0 - 50$, where $j_1$ is the HC$_3$N angular momentum) and kinetic temperatures in the range 10-100 K. Quantum results were however restricted to the lowest 16 levels, due to the high computational cost of these calculations. In addition, numerical errors in the implementation of the PES routine were found to introduce small inaccuracies ($\lesssim 20\%$) in the quantum calculations, as reported by Wernli et al. (2007b).

In the present work, the W07 PES is first checked against recent high-resolution spectroscopic measurements of the HC$_3$N–H$_2$ complex. It is then employed to derive new rate coefficients for HC$_3$N colliding with para-H$_2$($j_2 = 0$) and, for the first time, with ortho-H$_2$($j_2 = 1$). The quantum coupled-channel approach is employed for the lowest 31 rotational levels of HC$_3$N ($j_1 = 0 - 30$) while the QCT approach is employed for higher levels, up to $j_1 = 37$ which is the highest level below the first excited bending mode at 222 cm$^{-1}$. Hyperfine selective collisional cross sections are also deduced using the almost exact (but computationally demanding) recoupling method and the infinite-order-sudden (IOS) approximation. In Section 2, the spectra of the HC$_3$N–H$_2$ complex is computed and it is compared to experimental data. In Section 3, scattering calculations are described and the procedures used to derive cross sections and rate coefficients are briefly introduced. Results are presented in Section 4. We discuss in particular the propensity rules and a comparison between quantum and classical rate coefficients is made. Conclusions are summarized in Section 5.

2 INTERACTION POTENTIAL AND BOUND-STATES

All bound-state and scattering calculations presented in the following sections were performed with the W07 PES. This PES is briefly described below. The lowest bound-state rovibrational energy levels supported by the PES are computed and the corresponding HC$_3$N–H$_2$ transitions frequencies are compared to the recent experimental results of Michaud, Topic & Jäger (2011).

2.1 Potential energy surface

The W07 PES was computed at the coupled-cluster with single, double, and perturbative triple excitations [CCSD(T)] level of theory with an augmented correlation-consistent triple zeta [aug-cc-pVTZ] basis set for HC$_3$N and quadruple zeta [aug-cc-pVQZ] for H$_2$, as described in Wernli et al. (2007a). The molecules were both treated as rigid rotors. Owing to the computational cost of the calculations (HC$_3$N is a 26 electron system), the angular sampling of the PES was performed by using only five independent H$_2$ orientations. The resulting five PES were interpolated using a bicubic spline function and the full HC$_3$N–H$_2$ interaction potential was reconstructed analytically (Wernli 2008). Due to the steric hindrance caused by the HC$_3$N rod, a standard spherical harmonics expansion of the full PES, as required by quantum calculations, was not possible (see Chapman & Green (1977)). This problem was solved by Wernli et al. (2007a) using a novel approach called “regularization”. This technique consists in scaling the PES when it is higher than a prescribed threshold so that the potential smoothly saturates to a limiting value in the highly repulsive regions of the PES. The “regularized” PES thus retain only the low energy content of the original PES but it is accurately adapted to low collisional energies, in practice lower than $\sim 1000$ cm$^{-1}$ here. The final spherical harmonics fit includes terms up to $l_1 = 24$ and $l_2 = 2$, resulting in 97 angular functions. Full details can be found in Wernli (2006) and Wernli et al. (2007a).

The equilibrium structure of the HC$_3$N–H$_2$ complex is linear with the nitrogen of HC$_3$N pointing towards H$_2$ at an intermolecular separation of 9.58 bohr. The corresponding well depth is -193.7 cm$^{-1}$, which compares well with the PES of Michaud et al. (2011) (hereafter denoted as M11), also computed at the CCSD(T) level, whose grid minimum is at -188.8 cm$^{-1}$. When averaged over H$_2$ orientation (corresponding to para-H$_2$ in its ground state $j_2 = 0$), the W07 PES has a shallower well of -111.2 cm$^{-1}$, also in good agreement with the M11 averaged PES minimum of -112.6 cm$^{-1}$. It should be noted, however, that the calculations of Michaud et al. (2011) were performed for only three orientations of the hydrogen molecule within the complex and no global fit of the PES was provided by the authors.

2.2 Bound-states

A spectroscopic and theoretical study of the HC$_3$N–H$_2$ dimer was reported by Michaud et al. (2011). The bound-state energies of the complex were computed by these authors using scaled and unscaled versions of their PES. These calculations were performed in two-dimensions (2D) using the Lanczos iterative procedure applied to the averaged M11 PES for para-H$_2$ and to the three PES (corresponding to the three hydrogen orientation) separately for ortho-H$_2$. The results were compared with high-resolution microwave spectroscopy measurements. Theoretical and experimental transition frequencies were shown to be in good agreement, within a few percent or better (see below).

We have also computed the bound-state energies supported by the W07 PES using the coupled-channel approach, as implemented in the BOUND program (Hutson 1994). The full spherical harmonics expansion of the W07 PES was employed with 97 functions. The coupled equations were solved using the diabatic modified log-derivative method. These calculations were performed in four-dimensions (4D) for both para- and ortho-H$_2$. Both molecules were taken as rigid rotors with the rotational constants $B_0=59.322$ cm$^{-1}$ for H$_2$ and $B_0=0.151739$ cm$^{-1}$ for HC$_3$N. A total of 46 rotational states (i.e. up to $j_1 = 45$) were included in the HC$_3$N basis set while the two lowest rotational states of para-H$_2$ ($j_2 = 0, 2$) and ortho-H$_2$ ($j_2 = 1, 3$) were considered. The calculations were performed with a propagator step size of 0.01 bohr and the other propagation parameters were taken as the default BOUND values.

The transition frequencies in the HC$_3$N–H$_2$ complex were deduced from the computed bound-state energies. They are compared to the spectroscopic and theoretical data of Michaud et al. (2011) in Tables 1 and 2 below. The assignment of the (approximate) quantum numbers $J_{K_a, K_c}$ was performed by Michaud et al. (2011). These authors reported a set of experimental line frequencies including the splitting into several hyperfine components due to the nuclear spin of...
the nitrogen atom \((J = 1)\). In the Tables below, the theoretical line frequencies are compared to the unsplit experimental frequencies of Michaud et al. (2011). The agreement between the present values and the experimental data is within 0.5\% (i.e. better than 0.01 cm\(^{-1}\)) for HC\(_3\)N–para-H\(_2\) and within a few percent (better than 0.1 cm\(^{-1}\)) for HC\(_3\)N–ortho-H\(_2\). The corresponding average differences are 0.32\% and 4.2\%, respectively. This very good agreement is similar, although slightly better, than the (nonscaled) theoretical results of Michaud et al. (2011). This was expected since the level of \textit{ab initio} theory is similar and the improvement is likely due to the 4D approach. We note that Michaud et al. (2011) were able to obtain an even better agreement by applying simple scaling techniques (radial shifts) to the M11 PES.

In summary, as observed previously for smaller systems such as CO–H\(_2\) (Jankowski, McKellar & Szalewicz 2012) and HCN–H\(_2\) (Denis-Alpizar et al. 2013), the comparison between spectroscopic and theoretical data for HC\(_3\)N–H\(_2\) confirms that van der Waals rigid-rotor PES computed at the CCSD(T) level with large basis sets can be used with confidence for analyzing van der Waals complex spectra.

## 3 SCATTERING CALCULATIONS

Scattering calculations for HC\(_3\)N–H\(_2\) were performed using both quantum and classical approaches. The objective was to obtain cross sections for all transitions among the first 38 rotational levels of HC\(_3\)N, i.e. up to \(J = 37\) which lies 213.3 cm\(^{-1}\) above \(J = 0\) and just below the first excited bending mode \(\nu_2\) which opens at only 222 cm\(^{-1}\). In practice, for levels higher than \(J = 30\), the basis set necessary for converged quantum calculations would require an excessive number of (open and closed) states, including vibrationally excited states. In addition to the substantial CPU cost, such calculations would require the determination of a flexible, nonrigid-rotor, HC\(_3\)N–H\(_2\) PES, which is beyond the scope of this work. For the high-lying levels close to the vibrational threshold (\(J = 31 – 37\)), the approximate but efficient classical approach was therefore employed. Details on these scattering calculations are given below.

### 3.1 Quantum calculations

The scattering quantum calculations were conducted at the full coupled-channel level using the MOLSCAT program (Hutsen & Green 1995). The spherical harmonics expansion of the W07 PES, including 97 angular functions, was employed and the coupled differential equations were solved using the hybrid modified log-derivative Airy propagator. Total energies up to 2120 cm\(^{-1}\) were considered using a fine grid below 200 cm\(^{-1}\) (with increments as small as 0.01 cm\(^{-1}\)) for the correct assessment of resonances close to thresholds. The total number of collision energies was 324 for para-H\(_2\)(\(J = 0\) and 344 for ortho-H\(_2\)(\(J = 1\)). The highest rotational level of HC\(_3\)N in the basis set was \(J = 44\) while one single rotational level of H\(_2\) was included for both the para (\(J = 0\)) and ortho (\(J = 1\)) modifications. The level \(J = 2\) of para-H\(_2\) was neglected because it was found to affect the cross sections by less than 10–20\% on average (see also Wernli et al. 2007a). The CPU cost was increased by large factors. Thus, at a total energy of 100 cm\(^{-1}\), the number of channels was increased by a factor of ~6 (exceeding 4,000 channels) corresponding to a factor of ~200 in CPU time. We note that we performed a few high-energy calculations with a basis set \(J = 2, 0\) in order to compare the cross sections for H\(_2\)(\(J = 2\)) with those for H\(_2\)(\(J = 1\)), see Section 4.3 below. The neglect of HC\(_3\)N levels \(J > 44\) was found to change cross sections by less than a few percent for all transitions among the first 31 levels of HC\(_3\)N. The maximum value of the total angular momentum \(J\) used in the calculations was \(J = 130\) at the highest collisional energy. The rotational constants were taken as \(B_1 = 0.853\) cm\(^{-1}\) for H\(_2\) \(B_2 = 0.151739\) cm\(^{-1}\) for HC\(_3\)N, as in Wernli et al. (2007a). Finally, convergence was checked as a function of the propagator step size (parameter STEPS) and the other propagation parameters were taken as the default MOLSCAT values.

Rate coefficients were obtained for kinetic temperatures in the range 10–300 K by integrating the product of the cross section by the velocity over the Maxwell-Boltzmann distribution of velocities at each temperature. Cross sections and rate coefficients are presented in Section 4 below.

### 3.2 Classical calculations

The objective of the classical calculations was to provide rate coefficients for all (classically allowed) transitions among the levels \(J_1 = 31 – 37\). Kinetic temperatures were restricted to the range 100–300 K since rate coefficients at lower temperatures are available in Wernli et al. (2007a), where full details on the QCT approach can be found. Briefly, it consists in solving the classical Hamilton equations of motion instead of the Schrödinger equation. In the Monte-Carlo QCT approach, batches of trajectories are sampled with random (Monte-Carlo) initial conditions and they are analyzed through statistical methods. In the canonical formalism employed here, the initial collision energies are selected according to the Maxwell-Boltzmann distribution of velocities and rate coefficients, instead of cross sections, are directly obtained (see Eqs. (3) and (4)) in Wernli et al. (2007a). State-to-state rate coefficients are obtained by use of the correspondence principle combined with the bin histogram method for extracting the final \(J_1\) quantum number.

Preliminary calculations had shown that QCT calculations for para-H\(_2\)(\(J = 0\)) and ortho-H\(_2\)(\(J = 1\)) give very similar cross sections (Wernli 2006). As in Wernli et al. (2007a), we therefore computed QCT cross sections for para-H\(_2\)(\(J = 0\)) only, using the W07 PES averaged over H\(_2\) orientation and fitted with a bicubic spline function. The classical equations of motion were numerically solved using an extrapolation Bulirsh-Stoer algorithm and numerical derivatives for the potential. The total energy and total angular momentum were checked and conserved up to seven digits, i.e. within 0.01 cm\(^{-1}\) for the energy. The maximum impact parameter \(b_{\text{max}}\) was found to range between 20 and 21 bohr for temperatures in the range 100-300 K. Batches of 10,000 trajectories were run for each initial level \(J_1 = 31 – 37\) and 1 This value was used for consistency with our previous calculations. Using \(B_0 = 0.932\) cm\(^{-1}\) instead of \(B_0\) would not modify the cross sections since a single rotational level of H\(_2\) was included.
each temperature, resulting in a total of 770,000 trajectories. It should be noted that a particular advantage of the QCT method is that the computational time decreases with increasing collision energy, in contrast to quantum methods. QCT calculations however ignores purely quantum effects such as interference and tunneling, as will be shown below.

### 3.3 Hyperfine excitation

Due to the presence of a nitrogen atom, the HC$_3$N molecule possesses a hyperfine structure. Indeed, the coupling between the nuclear spin (I = 1) of the nitrogen atom and the molecular rotation results in a splitting of each rotational level into 3 hyperfine levels (except for the $j_1 = 0$ level). Each hyperfine level is designated by a quantum number $F_1$ ($F_1 = I + j_1$) varying between $|I - j_1|$ and $I + j_1$. As already discussed (e.g. Faure & Lique 2012, Lanza & Lique 2014, and references therein), if the hyperfine levels are assumed to be degenerate, it is possible to simplify considerably the hyperfine scattering problem. Almost exact hyperfine resolved cross sections can then be obtained from nuclear spin-free S-matrices using the so-called recoupling approach. This approach requires to store the S-matrices and to compute hyperfine resolved S-matrices by properly combining the nuclear spin-free S-matrices. However, in the case of HC$_3$N–H$_2$ collisions, the recoupling approach becomes rapidly prohibitive in terms of both memory and CPU time because of the small rotational constant of the target molecule, i.e. the large number of channels to include.

Alternatively, hyperfine resolved rate coefficients can be directly obtained from rotational rate coefficients using the scaled-infinite-order-sudden limit (S-IOS) method first introduced by Neufeld & Green (1994) in the case of HCl–He collisions and then extended to the hyperfine excitation of linear molecules by para- and ortho-H$_2$ by Lanza & Lique (2014). This adiabatic method is expected to be reliable at high collision energies and/or for target molecules with small rotational constants so that the rotational period is small compared to the collision time scale. In practice, the hyperfine rate coefficients (or cross sections) are obtained by scaling the coupled-channel rotational rate coefficients by the ratio of hyperfine and rotational IOS rate coefficients as follows:

$$k_{S-IOS}^{S-IOS} = \frac{k_{IOS}^{IOS}}{k_{IOS}^{IOS} F_1 F_2 \rightarrow F_1' F_2'}$$

where $k_{IOS}^{F_1 F_2 \rightarrow F_1' F_2'}$ and $k_{IOS}^{F_1 F_2 \rightarrow F_1' F_2'}$ can be found in Eqs. (6) and (7) of Lanza & Lique (2014). We note that for quasi-elastic transitions (those with $j'_1 = j_1$) no scaling is applied, as explained in Faure & Lique (2012). We note also that Eq. (1) guarantees that the summed hyperfine rate coefficients are identical to the coupled-channel pure rotational rate coefficients.

The S-IOS approach was applied recently to the HCl–H$_2$ collisional system Lanza & Lique (2014). Despite the large rotational constant of HCl, the S-IOS method was found to be accurate to within a factor of 2-3 in the case of collisions with para-H$_2$ ($j_2 = 0$) at intermediate and high kinetic energies (> 200 cm$^{-1}$). It was however found to fail by factors larger than 3 in the case of ortho-H$_2$ ($j_2 = 1$). As HC$_3$N is much heavier than HCl and hence more adapted to IOS type methods, one can expect the accuracy of the S-IOS approach to be substantially better.
4 RESULTS

4.1 Collisional propensity rules

In Fig. 1 cross sections for the rotational excitations \( j_1 = 0 \) to \( j'_1 = 1, 2, 3, 4 \) are plotted as function of collision energy for the colliders para-H\(_2\)(\( j_2 = 0 \)) and ortho-H\(_2\)(\( j_2 = 1 \)). It is first noticed that these cross sections show prominent resonances at low energies, especially in the case of para-H\(_2\)(\( j_2 = 0 \)). These resonances, of both shape and Feshbach types, arise from purely quantum effects. They have been observed experimentally only recently (see e.g. Chefdeville et al. 2012, in the case of CO–H\(_2\)). We also observe in the case of para-H\(_2\)(\( j_2 = 0 \)) that transitions with even \( \Delta j_2 \) (i.e. \( \Delta j_2 = 2, 4 \)) have the largest cross sections, with \( \Delta j_2 = 4 \) being even dominant in the energy range \( \sim 15-45 \text{ cm}^{-1} \). This result was interpreted by Wernli et al. (2007a) as originating from the shape of the HC\(_3\)N–para-H\(_2\) PES, being nearly a prolate ellipsoid. It should be noted that these results are unchanged when the basis set on H\(_2\) include the \( j_2 = 2 \) level (see Fig. 1 in Wernli et al. 2007b). In contrast, in the case of ortho-H\(_2\), cross sections follow an energy-gap law behaviour with \( \Delta j_2 = 1 > \Delta j_1 = 2 > \Delta j_1 = 3 \), etc. We note, still, that above \( \sim 400 \text{ cm}^{-1} \), the transition \( 0 \to 2 \) dominates.

These results illustrate clearly the difference between para- and ortho-H\(_2\). It was previously observed for many other systems where the target has a large dipole, e.g. for HCN–H\(_2\) (Vera et al. 2014). In the case of formaldehyde (H\(_2\)CO), it was even used to indirectly constrain the ortho-to-para ratio of H\(_2\) (Truscompt et al. 2009). Indeed, the general distinction between para- and ortho-H\(_2\) is attributable to the permanent quadrupole moment of H\(_2\), which vanishes for \( j_2 = 0 \) but not for \( j_2 > 0 \). When the dipole of the target is significant, the long-range dipole-quadrupole interaction term is large and the difference between para-H\(_2\)(\( j_2 = 0 \)) and ortho-H\(_2\)(\( j_2 = 1 \)) is substantial (at the quantum level).

4.2 Comparison between classical and quantum rates

A comparison between quantum and classical rate coefficients is presented in Fig. 2 for HC\(_3\)N initially in level \( j'_1 = 15 \). The kinetic temperature is fixed at 100 K and the rate coefficients are plotted as function of the final HC\(_3\)N level \( j_1 \). We first notice that the quantum results for para-H\(_2\)(\( j_2 = 0 \)) show a pronounced even \( \Delta j_1 \) (“near homonuclear”) propensity, as expected from the cross sections reported above. This propensity is not observed at the QCT level whereas the PES are identical, demonstrating that this result indeed arises from a purely quantum effect. In fact, this propensity was explained semi-classically by McCurdy & Miller (1977) in terms of an interference effect related to the even anisotropy of the PES. Interferences are of course ignored in QCT calculations so that the “zig-zags” are totally absent. Now when ortho-H\(_2\)(\( j = 1 \)) is the projectile, the zig-zags are almost entirely suppressed and, interestingly, the quantum results are in excellent agreement with the QCT calculations, within error bars (except for the largest \( \Delta j_1 = 14 \) transfer). This demonstrates that the quadrupole moment of H\(_2\), which vanishes for H\(_2\)(\( j_2 = 0 \)), plays a crucial role by breaking the even symmetry of the PES and suppressing the interference effect. But it does not modify the magnitude of the cross sections, which are well reproduced by purely classical mechanics with para-H\(_2\)(\( j_2 = 0 \)). This result also suggests that the scattering process is dominated by the short-range part of the PES, i.e. by the ellipsoidal shape of the potential. It should be noted that the interference structure is extremely sensitive to the PES anisotropy and rotationally state-selected experiments which resolve this structure would provide a critical test of theory (see e.g. Carty et al. 2004, in the case of CO+He).

These results show that QCT calculations can be em-
employed with confidence to mimic collisions with rotationally excited \( \text{H}_2 \) \((j_2 \geq 1)\). They should be reliable to within 10-30% for rate coefficients larger than \( \sim \times 10^{-12} \text{ cm}^3\text{s}^{-1} \). Smaller rate coefficients have much larger uncertainties because they correspond to rare or “classically forbidden” transitions with small probabilities.

4.3 Thermalized rate coefficients

The ortho-to-para ratio of \( \text{H}_2 \) can be out of thermal equilibrium in the ISM (e.g. Faure et al. 2013 and references therein). It is thus crucial to consider the two nuclear spin species of \( \text{H}_2 \) as two separate colliders. We can however generally assume that each nuclear spin species has a thermal distribution of rotational levels. At temperatures below \( \sim 80 \text{ K} \), only the ground states are significantly populated so that in practice only para-\( \text{H}_2 \)(\( j_2 = 0 \)) and ortho-\( \text{H}_2 \)(\( j_2 = 1 \)) must be considered. At higher temperatures, however, the levels \( j_2 = 2, 3 \) and 4 also play an important role because their (thermal) populations become larger than 1% above \( \sim 80 \text{ K} \), \( \sim 160 \text{ K} \), and \( \sim 260 \text{ K} \), respectively. Due to the prohibitive CPU cost, we did not perform extensive scattering calculations for these excited levels but a few energy points were computed. We have thus found that rotational cross sections for \( \text{H}_2 \) initially in \( j_2 = 2 \) differ by less than \( \sim 20-30\% \) with those for \( \text{H}_2 \) in \( j_2 = 1 \). As a result, it can be assumed that all cross section, and by extension all rate coefficients, for \( \text{H}_2(j_2 > 1) \) are identical to those for \( \text{H}_2(j_2 = 1) \). This result was observed previously for many other systems before, as a rather general rule (e.g. Daniel et al. 2011, and references therein). We note that at higher temperatures where the \( \text{H}_2 \) levels can be (de)excited, this rule holds for “effective” rate coefficients which are summed over the final \( \text{H}_2 \) levels.

For para-\( \text{H}_2 \), we have thus computed the “thermalized” rate coefficients by weighting the para-\( \text{H}_2(j_2 = 0) \) rate coefficients, \( k_{j_1,j_2=0} (T) \) [3], by the thermal distribution of the \( j_2 = 0 \) level and the ortho-\( \text{H}_2(j_2 = 1) \) rate coefficients, \( k_{j_1,j_2=1} (T) \), by the thermal distribution of all para levels with \( j_2 > 0 \):

\[
k_{j_1,\text{pH}_2 \rightarrow j'_1} (T) = \rho_0 k_{j_1,j_2=0 \rightarrow j'_1} (T) + (1-\rho_0) k_{j_1,j_2=1 \rightarrow j'_1} (T),
\]

where

\[
\rho_0 = \frac{1}{\sum_{j_2=0,2,4,...} (2j_2 + 1) \exp(-E_{0,j_2}/k_BT)}
\]

is the thermal population of \( j_2 = 0 \).

For ortho-\( \text{H}_2 \), all \( \text{H}_2 \) levels were assumed to have the same rate coefficients so that the “thermalized” rate coefficients were simply obtained as:

\[
k_{j_1,\text{oH}_2 \rightarrow j'_1} (T) = k_{j_1,j_2=1 \rightarrow j'_1} (T).
\]

In Fig. 3, the thermalized rate coefficients for para-\( \text{H}_2 \) and ortho-\( \text{H}_2 \) are plotted as function of temperature for the ground-state transition \( j_1 = 1 \rightarrow 0 \) at 9.1 GHz. The contribution of \( j_2 = 2 \) is significant above \( \sim 80 \text{ K} \), as expected, and it is here amplified by the fact that the corresponding rate coefficient (set equal to \( k_{j_1,j_2=1} (T) \)) is about a factor of 10 larger than \( k_{j_1,j_2=0} (T) \) (see Eq. 2). At 300 K, the contribution of \( j_2 = 2 \) (and to much a lesser extent \( j_2 = 4 \)) increases the rate coefficient of para-\( \text{H}_2 \) by a factor of \( \sim 6 \). It is therefore of crucial importance at these temperatures.

The whole set of thermalized rate coefficients for the lowest 38 rotational levels of HC\( \text{N}_2 \) and kinetic temperatures in the range 10-300 K are available at the LAMDA\textsuperscript{3} (Schöier et al. 2005) and BASECOL\textsuperscript{4} (Dubernet et al. 2013) data bases. We note that for HC\( \text{N}_2 \) levels between \( j_1 = 31 \) and \( j_1 = 37 \), only QCT rate coefficients for para-\( \text{H}_2(j_2 = 0) \) are available and no thermal averaging was applied to this set which is employed for both para-\( \text{H}_2 \) and ortho-\( \text{H}_2 \).

4.4 Hyperfine cross sections and rate coefficients

In order to evaluate the accuracy of the S-IOS method (see Section 3.3) in the case of HC\( \text{N}_2 - \text{H}_2 \) collisions, we have computed recoupling and S-IOS hyperfine cross sections at two selected collisional energies (10 and 50 cm\(^{-1}\)). In the above Eq. (1), cross sections were employed instead of rate coefficients. Fig. 4 shows a comparison between recoupling and S-IOS hyperfine cross sections for all the de-excitation transitions from the initial level \( j_1 = 5 \), including quasi-elastic transitions (those with \( j_1 = j'_1 \) and \( F_1 \neq F'_1 \)), for collisions with para-\( \text{H}_2(j_2 = 0) \) and ortho-\( \text{H}_2(j_2 = 1) \). We note that the S-IOS approach imposes selection rules (through Wigner 6j symbols) and some cross sections (not plotted) are strictly zero. The corresponding transitions are those between the \( (j_1 = 1, F_1 = 0) \) level and levels with \( F_1 = j_1 \), e.g. \((1,0) \rightarrow (1,1)\). At the close-coupling level, these transitions have indeed cross sections lower than \( \sim 1 \text{ Å}^2 \).

\[\text{Figure 3. Rate coefficients for the HC}_3\text{N deexcitation } j_1 = 1 \rightarrow 0 \text{ by para-} \text{H}_2 \text{ and ortho-} \text{H}_2 \text{ in the temperature range 10-300 K. The thermalized rate coefficient for para-} \text{H}_2(j_2 = 0, 2, 4, ...) \text{ is compared to the contribution of the ground state } j_2 = 0.\]

\[\text{Figure 4. Comparison of recoupling and S-IOS cross sections for the de-excitation from } j_1 = 5 \text{ to } j_1 = 1 \text{ by para-} \text{H}_2(j_2 = 0) \text{ and ortho-} \text{H}_2(j_2 = 1).\]

\[\text{3 http://www.strw.leidenuniv.nl/~moldata}\]

\[\text{4 http://basecol.obspm.fr}\]

\[\hspace{2cm}\text{© 2016 RAS, MNRAS}\]
For collisions with para-H$_2$(j$_2$ = 0), the agreement between recoupling and S-IOS calculations is very satisfactory. At low energies, the differences between the two sets of data are less than 20-30% (especially for the dominant cross sections). At kinetic energies greater than 50 cm$^{-1}$, the differences between the two sets decrease and the average difference is lower than a factor of two.

To summarize, the S-IOS method can provide hyperfine resolved rate coefficients with an average accuracy better than 20-30% in the case of collisions with para-H$_2$(j$_2$ = 0) and within a factor of 2-3 in the case of collisions with ortho-H$_2$(j$_2$ = 1). With respect to the memory and CPU cost of full recoupling calculations, the S-IOS approximation therefore represents a suitable (and low-cost) alternative for this system. Finally, in terms of radiative transfer application, it should be noted that at moderate and high opacities, where the relative hyperfine populations can significantly depart from the statistical weights, the S-IOS method is notably better than the statistical approach (see Faure & Lique 2012).

In practice, hyperfine resolved rate coefficients were obtained for the lowest 61 hyperfine levels of HC$_3$N, i.e. up to (j$_1$ = 20, F$_1$ = 20) which lies 63.73 cm$^{-1}$ above (0, 1), and for kinetic temperatures in the range 10-100 K. This set of data is available at the LAMDA and BASECOL data bases.

**5 CONCLUSION**

We have reported in this paper rate coefficients for the rotational excitation of HC$_3$N by para- and ortho-H$_2$. The lowest 38 rotational levels of HC$_3$N were included and kinetic temperatures up to 300 K were considered. The scattering calculations were performed at the quasi-classical and quantum coupled-channel level using the interaction potential of Wernli et al. (2007a). This potential was also employed to compute the bound-states of the complex in order to make comparisons with the spectroscopy measurements of Michaud et al. (2011). Theory and experiment were found to agree within 0.5% for para-H$_2$(j$_2$ = 0) and within a few percent for ortho-H$_2$(j$_2$ = 1), demonstrating the high accuracy of the potential. It appears from these comparisons that the calculated state-to-state rotational rate coefficients are likely to be accurate to about 20-30%. Hyperfine resolved rate coefficients were also deduced using the S-IOS approximation, with a somewhat lower accuracy. The whole set of data represent a significant improvement and extension over the previous data of Green & Chapman (1978) and Wernli et al. (2007a).

The next step is to determine a flexible potential energy surface in order to treat the ro-vibrational excitation of HC$_3$N. The lowest vibrational state is the $\nu_7$ bending mode which lies at 222 cm$^{-1}$ above the ground vibrational...
Ro-vibrational excitation due to collisions is therefore expected to play a role above \(\sim 300 \text{ K}\). Such calculations are highly challenging due to excessively large number of channels involved. This problem has been however tackled recently at the quantum coupled-channel level using the rigid-bender approximation \cite{Stoecklin13,Stoecklin15}. This latter approximation was also employed previously in quasi-classical trajectories \cite{Faure05} which offer an (economical) alternative to quantum computations, as shown in the present work.

In summary, the present set of data, possibly complemented by electron-impact rate coefficients (as given in \cite{Gratier13}), should help in modelling non-LTE HC\(_3\)N spectra in cold to warm regions of the ISM. We note in particular that HC\(_3\)N has recently been detected in protoplanetary disks \cite{Chapillon12,Oberg15}. The collisional data provided here should prove very useful in interpreting such observations.

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