Symmetry Remnants: Rationale for Having Two Higgs Doublets

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Abstract
There is a good reason why the standard electroweak SU(2) × U(1) gauge model may be supplemented by two Higgs scalar doublets. They may be remnants of the spontaneous breaking of an SU(2) × SU(2) × U(1) gauge symmetry at a much higher energy scale. In one case, the two-doublet Higgs potential has a custodial SU(2) symmetry and implies an observable scalar triplet. In another, a light neutral scalar becomes possible.
In the standard SU(2) × U(1) electroweak gauge model, only one Higgs scalar doublet is needed for the spontaneous generation of all particle masses. Yet there are numerous research papers dealing with the possibility of having two (or more) doublets. A good reason is of course supersymmetry, but in that case, there should be many other particles as well. Nevertheless, a general two-doublet extension of the standard electroweak model without supersymmetry is routinely studied with little theoretical justification other than the obvious fact that it is not known to be wrong. To remedy this situation, we will show in the following that if the standard SU(2) × U(1) electroweak gauge group is the remnant of a larger symmetry, then the appearance of two (or more) doublets at the electroweak energy scale is actually required in some cases and the special form of the corresponding Higgs potential may even be indicative of what the larger theory is.

Consider the following Higgs potential for two doublets:

\[
V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_1)^2, \tag{1}
\]

where

\[
\Phi_{1,2} = \begin{pmatrix} \phi_{1,2}^+ \\ \phi_{1,2}^0 \end{pmatrix}
\]

and \(\mu_{12}^2\) has been chosen real by virtue of the arbitrary phase between \(\Phi_1\) and \(\Phi_2\). This \(V\) is invariant under a \(Z_2\) discrete symmetry where \(\Phi_1\) (\(\Phi_2\)) may be considered even (odd) except for the \(\mu_{12}^2\) term, but which breaks it only softly. Consequently, it allows for the natural suppression of flavor-changing neutral currents as long as each fermion gets its mass from only one scalar vacuum expectation value, \(i.e.\) either \(< \phi_1^0 >\) or \(< \phi_2^0 >\) but not both.

Such two-doublet extensions of the standard electroweak model have been studied extensively for their phenomenological implications. However, a more fundamental question
to be considered is why they should be studied at all. In supersymmetry, two scalar doublets are necessary because each is accompanied by a fermionic partner having a nonzero contribution to the axial-vector triangle anomaly but their sum is zero. The requirement of supersymmetry also constrains the parameters of $V$ as follows:

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2, \quad \lambda_4 = -\frac{1}{2}g_2^2, \quad \lambda_5 = 0,$$

(3)

where $g_1$ and $g_2$ are the U(1) and SU(2) gauge couplings of the standard model respectively. The soft terms, *i.e.* $\mu_1^2$, $\mu_2^2$, and $\mu_{12}^2$, are considered arbitrary because they are allowed to break the supersymmetry. Discovery of scalar particles with a mass spectrum conforming to such a Higgs potential would certainly be a strong indication of supersymmetry.

Consider now a different rationale for the existence of two Higgs doublets. They may be remnants of the spontaneous breaking of a larger gauge symmetry at some higher energy scale. Take for example the gauge group $SU(2)_1 \times SU(2)_2 \times U(1)$. Let the scalar sector consist of two doublets $\Phi_{1,2}$ and one self-dual bidoublet $\eta$ transforming as $(2,1,1/2)$, $(1,2,1/2)$, and $(2,2,0)$ respectively:

$$\Phi_{1,2} = \begin{pmatrix} \phi^0_{1,2} \\ \phi^+_{1,2} \end{pmatrix}, \quad \eta = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta^0 \\ \eta^+ \\ -\eta^- \\ \eta^0 \end{pmatrix}.$$

(4)

The most general Higgs potential $V$ is then given by

$$V = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + m_3^2 Tr(\eta^\dagger \eta)$$

$$+ \frac{1}{2} f_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} f_2 (\Phi_2^\dagger \Phi_2)^2 + \frac{1}{2} f_3 (Tr(\eta^\dagger \eta))^2$$

$$+ f_4 (\Phi_1^\dagger \Phi_1) Tr(\eta^\dagger \eta) + f_5 (\Phi_2^\dagger \Phi_2) Tr(\eta^\dagger \eta) + f_6 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2)$$

$$+ t (\Phi_1^\dagger \eta \Phi_2 + \Phi_2^\dagger \eta^\dagger \Phi_1),$$

(5)

where $t$ has been chosen real by virtue of the arbitrary relative phase between $\Phi_1$ and $\Phi_2$. Note that because

$$\Phi_1^\dagger \eta \Phi_2 + \Phi_2^\dagger \eta^\dagger \Phi_1 = Tr \left( \begin{pmatrix} \phi^0_1 & -\phi^+_1 \\ \phi^-_1 & \phi^0_1 \end{pmatrix} \right) \frac{1}{\sqrt{2}} \begin{pmatrix} \eta^0 \\ -\eta^- \\ \eta^0 \end{pmatrix} \begin{pmatrix} \phi^+_2 & \phi^0_2 \\ \phi^-_2 & \phi^0_2 \end{pmatrix},$$

(6)
this $V$ has automatically an extra global SU(2) symmetry.\[3\] As the first step of symmetry breaking, consider only $\langle \eta^0 \rangle = \langle \eta^0 \rangle = u \neq 0$, then our $SU(2)_1 \times SU(2)_2 \times U(1)$ breaks down to the standard $SU(2)_L \times U(1)_Y$, resulting in a massive vector-boson triplet $(g_1 W_{1 \pm 0}^\pm - g_2 W_{2 \pm 0}^0) / \sqrt{g_1^2 + g_2^2}$ and preserving the extra global SU(2) symmetry. The reduced $V$ now has the form of Eq. (1) but with the important restriction that $\lambda_4 = \lambda_5 = 0$. [The other parameters are $\mu_2^2 = m_2^2 + f_4 u^2$, $\mu_2^2 = m_2^2 + f_5 u^2$, $\mu_{12}^2 = tu / \sqrt{2}$, $\lambda_1 = f_1$, $\lambda_2 = f_2$, and $\lambda_3 = f_6$.]

Both $\Phi_1$ and $\Phi_2$ now transform as doublets under the standard $SU(2)_L \times U(1)_Y$ gauge group, as well as the extra global SU(2). As $\phi_1^0$ and $\phi_2^0$ acquire vacuum expectation values $v_1$ and $v_2$, the gauge symmetry $SU(2)_L \times U(1)_Y$ breaks down to electromagnetic $U(1)_Q$, but a custodial SU(2) symmetry remains, in exact analogy to the well-known case of the standard model with only one Higgs doublet. Consequently, of the 5 physical scalar bosons, 3 are organized into a triplet

$$H_3^\pm = -\sin \beta \phi_1^\mp + \cos \beta \phi_2^\pm,$$

$$H_3^0 = \sqrt{2}(-\sin \beta Im \phi_1^0 + \cos \beta Im \phi_2^0),$$

where $\tan \beta \equiv v_2/v_1$, with a common mass given by

$$m_{H_3}^2 = -\frac{2\mu_{12}^2}{\sin 2\beta}.$$  

The other 2 are singlets

$$H_1 = \sqrt{2}(\cos \beta Re \phi_1^0 + \sin \beta Re \phi_2^0),$$

$$H_2 = \sqrt{2}(-\sin \beta Re \phi_1^0 + \cos \beta Re \phi_2^0),$$

with mass-squared matrix given by

$$\mathcal{M}^2 = \begin{pmatrix}
2(c^2\lambda_1 v_1^2 + s^2\lambda_2 v_2^2 + 2sc\lambda_3 v_1 v_2) & 2sc((-\lambda_1 + \lambda_3)v_1^2 + (\lambda_2 - \lambda_3)v_2^2) \\
2sc((-\lambda_1 + \lambda_3)v_1^2 + (\lambda_2 - \lambda_3)v_2^2) & m_{H_3}^2 + 2sc(\lambda_1 + \lambda_2 - 2\lambda_3)v_1 v_2
\end{pmatrix},$$

where $s = \sin \beta$, $c = \cos \beta$. Since $\lambda_1 + \lambda_2 > 2|\lambda_3|$ is required for $V$ to be bounded from
below, the above matrix shows that at least one of the singlet scalars must be heavier than the triplet.

The $V$ of Eq. (1) is in general not invariant under an extra global SU(2) symmetry, hence the presence of two Higgs doublets is expected to contribute significantly to the radiative correction which makes the electroweak parameter $\rho$ different from one. Experimentally, there is no evidence of any deviation which cannot be accounted for with a $t$-quark mass of about 150 GeV or so. Hence such a custodial symmetry is desirable for $V$, but that would require $\lambda_4 = \lambda_5$ which cannot be maintained naturally in the context of the standard model because infinite radiative corrections are unavoidable. In our case, the restriction $\lambda_4 = \lambda_5 = 0$ is obtained from the reduction of a larger theory and it can easily be shown that $\lambda_4$ and $\lambda_5$ have finite radiative corrections which go to zero as $u$ goes to infinity.

The reason for both $\Phi_1$ and $\Phi_2$ to be present in the reduced Higgs potential has to do with the original SU(2)$_1 \times$ SU(2)$_2 \times$ U(1) theory. If some of the fermions couple to SU(2)$_1 \times$ U(1) and others to SU(2)$_2 \times$ U(1), then both $\Phi_1$ and $\Phi_2$ are required to allow all fermions to acquire mass. At the $10^2$ GeV energy scale, all fermions couple to the standard SU(2) $\times$ U(1) in the usual way and the only clue to their original difference is the two Higgs doublets with $\lambda_4 = \lambda_5 = 0$ in $V$. Discovery of the scalar triplet $H^{\pm,0}_3$ would certainly be indicative of such a possibility.

As a second example, consider again the gauge group SU(2)$_1 \times$ SU(2)$_2 \times$ U(1) but with an unconventional assignment of fermions. An exotic quark $h$ of electric charge $-1/3$ is added so that $(u, d)_L$ transforms as $(2,1,1/6), (u, h)_R$ as $(1,2,1/6)$, whereas both $d_R$ and $h_L$ are singlets $(1,1,-1/3)$. There are again the two Higgs doublets $\Phi_{1,2}$ but now the bidoublet is not self-dual, i.e.

$$\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ -\eta_1^- & \eta_2^0 \end{pmatrix}.$$

(13)

Assume also a $Z_4$ discrete symmetry under which $\Phi_1 \to \Phi_1, \Phi_2 \to i\Phi_2, \eta \to i\eta, (u, d)_L \to
has many interesting and remarkable phenomenological consequen ces.

λ(14) does not have an extra global SU(2) symmetry. Hence only term in Eq. (14) involving 3 different neutral scalar fields is

tµ means that < ηL standard SU(2)

SU(2)λ Eq. (1) but with

is given by

The most general Higgs potential V invariant under the assumed Z4 discrete symmetry is given by

V = m2Φ†1Φ1 + m2Φ†2Φ2 + m2Tr(η†η) + 1/2f1(Φ†1Φ1)2 + 1/2f2(Φ†2Φ2)2 + 1/2f5(Tr(η†η))2 + 1/4f4Tr(η†η)Tr(η†η) + 1/8f5(Tr(η†η))2 + 1/8f5(Tr(η†η))2 + f6(Φ†1Φ1)Tr(η†η) + f7(Φ†2Φ2)Tr(η†η) + f8(ηη†Φ1) + f9(Φ†2η†ηΦ2) + f10(Φ†1Φ1)(Φ†2Φ2) + t(Φ†1ηΦ2 + Φ†2η†Φ1),

(14)

where

The couplings f5 and t have been chosen real by virtue of the arbitrary relative phases among Φ1,2 and η. As the first step of symmetry breaking, consider now only < φ2 > = ν2 ≠ 0, then SU(2)2 × U(1) breaks down to U(1)Y, whereas SU(2)1 remains unbroken and is in fact the standard SU(2)L. Eliminating the heavy Φ2 and η2 scalar bosons from V, we again obtain Eq. (1) but with λ5 = 0 and Φ2 replaced by η1. [The other parameters are µ2 = m2 + f10ν22, µ2 = m2 + f7ν22, µ12 = tv2, λ1 = f1, λ2 = f3, λ3 = f6 + f8, and λ4 = −f8.] Note that the only term in Eq. (14) involving 3 different neutral scalar fields is t(φ2η1φ02 + φ02η1φ01) which means that < η2 > = 0 is allowed. Note also that because η is not self-dual, the V of Eq. (14) does not have an extra global SU(2) symmetry. Hence λ4 ≠ λ5 and H†3 and H3 have
different masses. However, because $\lambda_5 = 0$, the mass of $H_3^0$ is still given by Eq. (9), whereas

$$m_{H_3^\pm}^2 = m_{H_3^0}^2 - \lambda_4 (v_1^2 + u_1^2).$$

(16)

Since $H_3^0$ is now the only scalar boson with a mass-squared proportional to $\mu_{12}^2$, it may in fact be light. [If $\mu_{12}^2$ were zero as well as $\lambda_5$, then $V$ has an extra global $U(1)$ symmetry, the spontaneous breaking of which would result in a massless $H_3^0$.] The decay $Z^0 \to H_3^0 H_3^0$ is absolutely forbidden by angular-momentum conservation and Bose statistics, whereas $Z^0 \to H_{1,2}^0 H_3^0$ and $W^\pm \to H_3^\pm H_3^0$ may be forbidden kinematically because $H_{1,2}^0$ and $H_3^\pm$ are heavy. However, since $H_{1,2}^0$ couple to $H_3^0 H_3^0$ through $V$, the decay $Z^0 \to H_3^0 H_3^0 H_3^0$ may be possible, although the branching fraction is expected to be very much suppressed. [9] Note that $\lambda_5 = 0$ also in supersymmetry, but there it may be argued that $\mu_{12}^2$ should not be small.

In the Yukawa sector, since $d_R$ only couples to $\Phi_1$ and $u_R$ only to $\eta_1$, the usual $Z_2$ discrete symmetry assumed for the natural suppression of flavor-changing neutral currents is also realized.

It has been shown in the above that the scalar sector accompanying the standard model at the electroweak energy scale may very well consist of two doublets, obeying the Higgs potential of Eq. (1), but with the important restriction that $\lambda_4 = \lambda_5 = 0$ in the first case, and $\lambda_5 = 0$ in the second. These have interesting phenomenological consequences because of the existence of an unbroken custodial SU(2) symmetry in the former, and a softly broken U(1) symmetry in the latter. A scalar triplet $H_3^{\pm,0}$ with a common mass is then predicted in the first case, and a possibly light $H_3^0$ in the second. Both can be experimentally tested with future high-energy accelerators such as the Superconducting Super Collider (SSC) and the Large Hadron Collider (LHC).

In closing, we should point out that with the fermionic content of our second example, it is actually possible to have the same reduced $V$ as in our first example, i.e. with $\lambda_4 = \lambda_5 = 0$, but a rather ad hoc assumption is then required. Let us choose the bidoublet $\eta$ to be self-dual,
which means that we cannot impose any additional symmetry to distinguish $\eta$ from $\bar{\eta}$ as in our second example. The mass matrix linking $(d_L, \bar{d}_L)$ to $(d_R, h_R)$ is no longer restricted to be diagonal. In particular, there is a $\bar{d}_L d_R$ term. However, if we make the ad hoc assumption that this term is small compared to the $\bar{h}_L h_R$ term which comes from $<\phi^0_2>$, then again the heavy particles will decouple and we obtain the $V$ of our first example. Another way to achieve this result is to forbid the $\bar{h}_L d_R$ term with a discrete symmetry by adding a second $\Phi_2$, the existence of which is of course not very well motivated.

The same $V$ for two Higgs doublets may come from very different models at a much higher energy scale. However, their couplings to the quarks and leptons will generally not be the same. We have not considered these here because they are highly model-dependent. If two Higgs doublets are discovered in the future, detailed experimental determination of their properties will likely point to a larger gauge theory at some higher energy scale.

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