Effects of the long-range neutrino-mediated force in atomic phenomena

Phillip Munro-Laylim, Vladimir Dzuba and Victor Flambaum

School of Physics, University of New South Wales, Sydney, Australia

ABSTRACT
As known, electron vacuum polarisation by nuclear Coulomb field produces the Uehling potential with the range $\frac{\hbar}{2m_e c}$. Similarly, neutrino vacuum polarisation by the $Z$ boson field produces a long-range potential $\sim \frac{G^2_F}{r^5}$ with a very large range $\frac{\hbar}{2m_\nu c}$. Measurements of macroscopic effects produced by the potential $\frac{G^2_{\text{eff}}}{r^5}$ give limits on the effective interaction constant $G_{\text{eff}}$, which exceed the Fermi constant $G_F$ by many orders of magnitude, while limits from the spectroscopy of simple atomic systems are approaching the Standard Model predictions. In this paper, we consider limits on $G_{\text{eff}}$ from hydrogen, muonium, positronium, deuteron and molecular hydrogen. Constraints are also obtained on fifth force parameterised by Yukawa-type potential mediated by a scalar particle.

1. Introduction
It has long been known that the exchange of a pair of (nearly) massless neutrinos between particles (see Figure 1) produces a long-range force $\sim \frac{G^2_F}{r^5}$, where $G_F$ is the Fermi constant $[3–5]$. However, due to a rapid decay with the distance $r$, the effects of this potential are about 20 orders of magnitude smaller than the sensitivity of the macroscopic experiments $[6–11]$. A recent paper by Stadnik $[11]$ introduced a new approach to obtaining constraints on this potential by considering spectra of atomic systems. In the Standard Model formulas for energy shifts produced by potential $\frac{G^2_F}{r^5}$, the Fermi constant $G_F$ has been replaced by an effective interaction constant $G_{\text{eff}}$. The $\frac{G^2_{\text{eff}}}{r^5}$ potential produces a small energy shift to atomic energy levels, and therefore it is possible to obtain constraints on $G^2_{\text{eff}}$ from differences between highly accurate QED calculations of...
energy levels and experimental results [12, 13]. The Stadnik paper has led to a breakthrough in sensitivity; constraints on the interaction constant $G_{\text{eff}}^2/r^5$ have improved by 18 orders of magnitude in comparison with constraints from the macroscopic experiments [6–11]. However, the highly singular potential $G_{\text{eff}}^2/r^5$ leads to divergent integrals in the matrix elements as $r$ approaches zero. This demonstrates the requirement of the correct extension of the potential for $r \to 0$; Ref. [11] used the Compton wavelength of the Z boson as the cut-off radius, $r_c = \hbar/M_Zc$, for positronium and muonium, and the nuclear radius $R$ for atoms with finite nuclei. As we will show below, this oversimplified treatment in Ref. [11] leads to limits which were overestimated by a factor of 6 in non-hadronic atoms and underestimated by 4–5 orders of magnitude in the case of deuteron binding energy. The aim of this paper is to provide more accurate estimates and also consider results of the measurements which have not been included in Ref. [11]. To avoid misunderstanding, we should note that this paper is not aimed to calculate all electroweak corrections to energy levels. This should be done by different methods.

We also consider fifth forces from beyond the Standard Model that are parameterised by a Yukawa-type potential. This fifth force would require the existence of a new scalar particle to mediate the interaction, thus constraints on the coupling strength of the interaction can be found for various scalar particle masses. Limits were previously obtained using precision hydrogen $1s–2s$ spectroscopy in Ref. [14], however, we improve upon them using more recent data and include additional hydrogen-like systems.

2. The long-range neutrino-mediated potential

The potential of the long-range neutrino-mediated force between two particles, presented in Ref. [11], is

$$V_\nu(r) = \frac{G_F^2}{4\pi^2 r^5} \left( a_1 a_2 - \frac{2}{3} b_1 b_2 \sigma_1 \cdot \sigma_2 - \frac{5}{6} b_1 b_2 [\sigma_1 \cdot \sigma_2 - 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r})] \right),$$  

where $\sigma_1$ and $\sigma_2$ are the Pauli spin matrix vectors of the two particles, and $a_i$ and $b_i$ represent the species-dependent parameters defined below. It is worth noting that the last term of Equation (1) is zero for $s$-orbitals which strongly dominate in the shifts of atomic energy levels.

A potential $\sim 1/r^5$ gives divergent integrals ($\int r_i d^3r/r^5 \approx 1/2r_i^2$) in the matrix elements for $s$-wave. Using the nuclear radius $R$ as a cut-off, $r_c = R$, would give incorrect results. A more accurate approach first requires the building of an effective potential for electron–quark interactions and then take into account the nucleon distribution $\rho(r)$ inside the nucleus. To include small distances, we present this potential for the finite size $R$ of the nucleus and cut-off for large momenta (small distances $r$) produced by the Z boson propagator $(1/(q^2 + M_Z^2))$ instead of $1/M_Z^2$, see Figure 2). To start, we replace $1/r^5$ in the potential Equation (1) with

$$F(r) = \frac{8m^4 e^4 I(r)}{3\hbar^4 r},$$  

where, for $z = M_Z/(2m)$,

$$I(r) = \int_1^\infty e^{-2x\hbar c/\hbar} \left( x^2 - \frac{1}{4} \right) \frac{\sqrt{x^2 - 1} x^4 dx}{(x^2 + z^2)^2}.$$

Here $m$ is the mass of the fermion in the loop on Figure 2. The function $F(r) \propto I(r)/r$ gives us the dependence of interaction between an electron and a quark (or an electron and other point-like fermions) on distance $r$ between them. For $\hbar/(M_Zc) \ll r \ll \hbar/(mc)$, we obtain $F(r) = 1/r^5$. In this area, there is no change for potential equation (1). For large $r \gg \hbar/(mc)$, we have $F(r) \propto \exp(-2mcr/\hbar)^{5/2}$. At small distance $r \ll r_c = \hbar/(M_Zc)$, function $F(r) \propto (\ln r)/r$ and has no divergence integrated with $d^3r$. Note that the behaviour of the neutrino-exchange potential at small distances has been investigated in Ref. [15], however, they do not study this potential in the Standard Model as they considered a new scalar particle instead of the Z boson.

The convergence of the integral in the matrix elements on the distance $r \sim r_c = \hbar/M_Zc$ indicates that this interaction in atoms may be treated as a contact interaction (see Figure 1). We can replace $F(r)$ by its contact limit,
same, the difference is proportional to and Majorana neutrino potentials are practically the Dirac neutrino potentials. At small distance, the Dirac neutrino loop producing potential with the range \( \hbar/(2mc) \).

![Figure 2](image)

\[ F(r) \rightarrow C\delta(r) \]

\[ C = \int_0^\infty F(r)d^3r = \frac{\pi M_Z^2 \hbar^2}{3}, \quad (4) \]

where we assume \( z = M_Z/(2m) \gg 1 \). Note that if we used the potential \( 1/r^3 \) with the cut-off \( r_c = \hbar/M_Z c \), the result would be six times bigger:

\[ C' = \int_r^{\infty} \frac{1}{r^3}d^3r = 2\pi \frac{M_Z^2 \hbar^2}{c^3}. \quad (5) \]

Using Equation (4), the potential in Equation (1) in the contact limit may be presented as, using natural units \( \hbar = c = 1 \),

\[ V^C_\nu (r) = \frac{G_F^2 M_Z^2 \delta(r)}{12\pi^2} \left( a_1 a_2 - \frac{2}{3} b_1 b_2 \sigma_1 \cdot \sigma_2 - \frac{5}{6} b_1 b_2 (\sigma_1 \cdot \sigma_2 - 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r})) \right) \equiv g^C \delta(r). \quad (6) \]

In Ref. [16], the potential was obtained for a Majorana neutrino loop instead of a Dirac neutrino loop. Using these results, we conclude that the neutrino-exchange potential for Majorana neutrinos requires the adjustment to \( I(r) \) as follows:

\[ I_2^{(M)}(r) = \int_1^\infty e^{-2mcr/h} (x^2 - 1)^{3/2} z^4 dx \left( x^2 + z^2 \right)^2. \quad (7) \]

This indicates that the nature of neutrinos may, in principle, be detected from the difference in Dirac and Majorana potentials. At small distance, the Dirac neutrino and Majorana neutrino potentials are practically the same, the difference is proportional to \( (m_{\nu,cr}/h)^2 \) and is very small. In the contact interaction limit, the relative difference is \( \sim (m_{\nu}/M_Z)^2 \). However, the asymptotic expression at large distance changes: for Majorana neutrinos we have \( I_2^{(M)}(r) \propto \exp(-2mcr/h)/r^{3/2} \), whereas \( I(r)/r \propto \exp(-2mc/r)/r^{5/2} \) for Dirac neutrinos. Therefore, the ratio of Dirac potential to Majorana potential is \( \sim m_{\nu,cr}/h \) [16]. Thus the difference between potentials is negligible at small distances and only becomes significant at large distances \( r \gtrsim h/m_{\nu,c} \).

Unfortunately, effects of the neutrino-exchange potential are many orders of magnitude smaller than sensitivity of current macroscopic experiments [6–10], motivating future experimental work.

At large distance a dominating contribution to the vacuum polarisation by the Z boson field is given by the lightest particles, which are neutrinos. However, at distance \( r \), all particles with the Compton wavelength \( \hbar/mc > r \) give a significant contribution. Following Ref. [11], we present interaction constants for the potential in Equations (1) and (6) in the following form:

\[ a_1 a_2 = a_1^{(1)} a_2^{(1)} + (N_{\text{eff}} - 1) a_1^{(2)} a_2^{(2)}, \quad (8) \]

\[ b_1 b_2 = b_1^{(1)} b_2^{(1)} + (N_{\text{eff}} - 1) b_1^{(2)} b_2^{(2)}, \quad (9) \]

where \( N_{\text{eff}} \) is the effective number of particles (normalised to one neutrino contribution) mediating the interaction in Figure 2. Contributions which are not proportional to \( N_{\text{eff}} \) appear due the diagrams with W boson; for example, such diagrams involve electron neutrinos for interactions between electrons and quarks [5].

In atoms, the dominating contribution comes from the distance \( r \sim h/M_Z c \). The summation of the contributions from \( \nu, e, \mu, \tau, u, d, s, c, b \) (all with mass \( m \ll M_Z \)) gives \( N_{\text{eff}} = 14.5 \) [11]. Consider an interaction between electron and nucleon with an exchange by electron neutrino, electrons have values \( a_\nu^{(1)} = 1/2 + 2\sin^2(\theta_W) \) and \( b_\nu^{(1)} = 1/2 \), while nucleons have values \( a_{n,n}^{(1)} = -1/2, a_p^{(1)} = 1/2 - 2\sin^2(\theta_W) \), \( b_n^{(1)} = -g_A/2 \) and \( b_p^{(1)} = g_A/2 \), where \( g_A \approx 1.27 \). For the contributions from the other neutrino species, there is no W boson contribution and we have values for charged leptons \( a_l^{(2)} = 2\sin^2(\theta_W) - 1/2, b_l^{(2)} = -1/2, a_N^{(2)} = a_N^{(1)} \) and \( b_N^{(2)} = b_N^{(1)} \). The value of the \( \sin^2(\theta_W) = 0.239 \) for a small momentum transfer [17], where \( \theta_W \) is the Weinberg angle.

### 3. Energy shift in hydrogen-like systems

Simple two-body systems provide the most accurate values of the difference between experimental result and result of QED calculation of the transition energies. Following Ref. [11], we use these differences to obtain limits on the effective interaction constant \( G_{\text{eff}} \). We consider hydrogen, muonium and positronium spectra and deuteron binding energy. A summary of our calculations is presented in Table 1.
3.1. Hydrogen spectroscopy

For a simple hydrogen-like system, the expectation value of a contact potential $g\delta(r)$ is

$$\langle \psi | g\delta(r) | \psi \rangle = \frac{gZ^3}{n^2\pi a_B^3},$$

(10)

where $Z$ is the atomic charge, $n$ is the principal quantum number and $a_B$ is the reduced Bohr radius. Therefore, we calculate the energy shift for $n^3S_1$ states in hydrogen using Equations (6) and (10),

$$\delta E_{n^3S_1} = -\frac{G_F^2 M_Z^2 Z^3}{12\pi^3 n^3 a_B^3} \left( a_e a_p - \frac{2}{3} b_e b_p \right).$$

(11)

The energy shift for hydrogen $1s-2s$ ($Z = 1$ and $a_B = a_B$) evaluates to

$$\delta E = 3.60 \times 10^{-16} \text{ eV}.$$  

(12)

The maximal energy shift to hydrogen $1s-2s$ spectroscopy from the neutrino-mediated potential is dominated by the theoretical uncertainty $E_{\text{thr}} = 6.6 \times 10^{-12} \text{ eV}$ [18]; experimental uncertainties are smaller ($E_{\text{exp}} = 4.5 \times 10^{-14} \text{ eV}$ [19]). We replace $G_F^2$ with the effective interaction constant $G_F^2$ to obtain constraints

$$\delta E \left( \frac{G_F^2}{G_F^2} \right) \leq E_{\text{exp}} - E_{\text{thr}},$$

(13)

$$G_F^2 \leq 1.8 \times 10^4 \ G_F^2.$$  

(14)

Similarly for hydrogen $1s-3s$, we find the energy shift to be

$$\delta E = 3.97 \times 10^{-16} \text{ eV}.$$  

(15)

Using the maximal energy difference is $E_{\text{exp}} - E_{\text{thr}} = 2.2 \times 10^{-11} \text{ eV}$ from Ref. [20], we then get the constraint

$$G_F^2 \leq 5.5 \times 10^4 \ G_F^2.$$  

(16)

Both of these constraints are strong and were not included in Ref. [11].

3.2. Muonium and positronium $1s-2s$

We calculate the energy shift for $n^3S_1$ states for muonium and positronium using Equations (6) and (10),

$$\delta E_{n^3S_1} = -\frac{G_F^2 M_Z^2 Z^3}{12\pi^3 n^3 a_B^3} \left( a_e^2 - \frac{2}{3} b_e^2 \right).$$

(17)

The energy shift for muonium $1s-2s$ ($Z = 1$ and $a_B = \tilde{a}_B$) evaluates to

$$\delta E = -2.00 \times 10^{-16} \text{ eV}.$$  

(18)

From Ref. [12], the maximal difference between experimental and theoretical muonium $1^3S_1 - 2^3S_1$ results is $E_{\text{exp}} - E_{\text{thr}} = -6.4 \times 10^{-8} \text{ eV}$. Replacing $G_F^2$ with $G_F^2$, we find the constraint on the neutrino-mediated potential in muonium $1s-2s$

$$G_F^2 \leq 3.2 \times 10^4 G_F^2.$$  

(19)

This constraint from muonium spectroscopy is a new result. It may be improved significantly in the near future with new experimental results from the ongoing experiment Mu-MASS [21], which aims at improving $1s-2s$ muonium spectroscopy by several orders of magnitude.

We also find that the energy shift for positronium $1s-2s$ ($Z = 1$ and $a_B = 2\tilde{a}_B$) is

$$\delta E = -2.50 \times 10^{-17} \text{ eV}.$$  

(20)

The maximal difference between experiment [22] and QED calculation [23] for the positronium $1s-2s$ energy shift is $E_{\text{exp}} - E_{\text{thr}} = -3.7 \times 10^{-8} \text{ eV}$. Therefore, we find the constraint on the effective interaction constant

$$G_F^2 \leq 1.5 \times 10^9 G_F^2.$$  

(21)

Compared to the positronium $1s-2s$ constraint of Ref. [11], our constraint is six times weaker due to a more accurate treatment of the potential at small distances.

3.3. Muonium and positronium ground-state hyperfine splitting

To find constraints from hyperfine splitting (HFS), we calculate the energy shift for muonium and positronium $n^1S_0$ states using Equations (6) and (10),

$$\delta E_{n^1S_0} = -\frac{G_F^2 M_Z^2 Z^3}{12\pi^3 n^3 a_B^3} \left( a_e^2 + 2 b_e^2 \right).$$

(22)

Using Equations (17) and (22), we can calculate the energy shift for muonium ground-state hyperfine splitting

$$\delta E = -1.33 \times 10^{-15} \text{ eV}.$$  

(23)

The maximal difference between experiment [24] and QED calculation [25, 26] of muonium ground-state
hyperfine splitting is \( E_{\text{exp}} - E_{\text{the}} = -1.5 \times 10^{-12} \text{ eV} \). Replacing \( G_F^2 \) with \( G_{\text{eff}}^2 \), we find the constraint
\[
G_{\text{eff}}^2 \leq 1.1 \times 10^3 G_F^2.
\]
(24)

We should add that the preprint [27] contains calculations of the electroweak corrections to the muonium hyperfine structure. Our aim is different: we aim to investigate effects of the neutrino-exchange potential in atomic systems and compare corresponding results with the results of the macroscopic measurements of this potential. Our method is certainly not the adequate one for the accurate calculations of all electroweak radiative corrections.

For positronium ground-state hyperfine splitting, we calculate the energy shift to be
\[
\delta E = -1.78 \times 10^{-16} \text{ eV}.
\]
(25)

The corresponding maximal difference between experiment [28] and QED calculation [23] is \( E_{\text{exp}} - E_{\text{the}} = -1.6 \times 10^{-8} \text{ eV} \). Therefore, we find the constraint on the effective interaction constant
\[
G_{\text{eff}}^2 \leq 9.0 \times 10^7 G_F^2.
\]
(26)

Both hyperfine splitting constraints are six times weaker than those obtained in Ref. [11] due to a more accurate treatment of the potential at small distances.

### 3.4. Deuteron binding energy

The wave function of deuteron may be found using the short range nature of the strong interaction and relatively small binding energy of the deuteron. Outside the interaction range, we use solution to the Schrödinger equation for zero potential. Within the interaction range \( r_0 = 1.2 \text{ fm} \), the wave function has a constant value for s orbital. Therefore, the wave function is given by
\[
\psi(r) = \begin{cases} \frac{Be^{-\kappa r}}{Br_0}, & \text{for } r > r_0, \\ \frac{B_j(0)}{r_0}, & \text{for } r < r_0, \end{cases}
\]
(27)

where the normalisation constant \( B \) is given by \( 4\pi B^2 = 2\kappa \) for \( \kappa = \sqrt{2m|E|} = 4.56 \times 10^7 \text{ eV} \) (reduced mass \( m = m_p/2 \) and binding energy \( |E| = 2.22 \text{ MeV} \)). The Jastrow factor, \( J(0) = 0.4 \) [29], is included to account for the nucleon repulsion at short distance. For a contact potential \( g\delta(r) \), perturbation theory is used to find
\[
\langle \psi | g\delta(r) | \psi \rangle = \frac{g\kappa J(0)^2}{2\pi r_0^2}.
\]
(28)

Substituting \( g \) for the neutrino-mediated potential in Equation (6), we obtain the energy shift for the deuteron binding energy
\[
\delta E = -\frac{G_F^2 M^2 e^2 J(0)^2}{24\pi^3 r_0^3} \left( a_p - \frac{2}{3} b_p b_p \right),
\]
(29)

which evaluates to
\[
\delta E = -1.10 \times 10^{-3} \text{ eV}.
\]
(30)

Following Ref. [11], we take difference between experimental [30] and theoretical [31] results as \( E_{\text{exp}} - E_{\text{the}} = -13.7 \text{ eV} \). This gives
\[
G_{\text{eff}}^2 \leq 1.2 \times 10^4 G_F^2.
\]
(31)

This constraint is 4 orders of magnitude stronger than previously calculated in Ref. [11]. This is mainly due to the Z boson propagator cut-off (Z boson Compton wavelength) instead of the nuclear radius cut-off in Ref. [11]. Formally, this looks like the second strongest constraint among two-body systems (the strongest constraint comes from muonium HFS). However, deuteron is a system with the strong interaction and this constraint is probably less reliable than the constraints from the lepton systems.

### 4. Energy shift in molecular hydrogen systems

We also examine the constraints obtained from molecular systems for the neutrino-exchange interaction. On the molecular scale, it is sufficient to use Equation (1) (only \( a_1 a_2 \) contribute) as the nuclei are separated by a distance at least \( a_B \). Additionally, this also means that only neutrinos contribute to the interaction, so we use \( N_{\text{eff}} = 3 \). We consider molecular hydrogen systems and thus present the potential
\[
V^{M}_V(r) = \frac{G_B^2 N_{\text{eff}} a_1^{(1)} a_2^{(2)}}{4\pi^3 r^5} \equiv \frac{g}{r^5},
\]
(32)

where the interacting particles are nucleons. Ref. [32] used precision molecular spectroscopy to obtain constraints with regard to gravity in extra dimensions, including a potential with \( 1/r^5 \) dependence. This was done similarly to our method in Section 3, where the difference between theoretical and experimental results was used to obtain constraints on the interaction strength.

We utilise the findings of Ref. [32] to obtain constraints from the systems \( \text{H}_2, \text{D}_2 \) and \( \text{HD}^+ \). Values for the chosen systems are \( a_p^{(1)} a_p^{(2)} = 1.5 \times 10^{-3}, a_n^{(1)} a_n^{(2)} = 0.75 \) and \( a_p^{(1)} a_n^{(2)} = -3.3 \times 10^{-2} \). Using the results of Ref. [32], we present constraints on the interaction strength of the neutrino-exchange interaction in Table 2.
Figure 3. Upper limits from atomic spectroscopy on the coupling strength of a Yukawa-type potential $\beta$ relative to the fine structure constant $\alpha$ as a function of the force-mediating scalar particle mass $m$.

Table 2. Summary of constraints $G_{\text{eff}}^2/G_F^2$ on neutrino-mediated potential (see Equation 32) in molecular systems.

| Case          | $g$ (GeV$^{-4}$) | $G_{\text{eff}}^2/G_F^2$ | Refs. |
|---------------|-----------------|------------------------|-------|
| $\text{H}_2$ (1-0) | $6 \times 10^{12}$ | $4 \times 10^{27}$      | [33, 34] |
| $\text{D}_2$ (2-0) | $2 \times 10^{12}$ | $1.2 \times 10^{27}$     | [35] |
| $\text{H}^+$ (2-0) | $1.7 \times 10^{12}$ | $2.5 \times 10^{24}$     | [36] |

Note: Values $(i \rightarrow j)$ correspond to vibrational transitions, $D_0$ corresponds to the dissociation limit. References correspond to experimental and theoretical works used to determine difference between experimental and theoretical results for the transition energies.

5. The Yukawa potential

In addition to constraints on the neutrino–exchange interaction, one can also use spectroscopy to place constraints on a fifth force from beyond the Standard Model. The force can be phenomenologically parameterised by a Yukawa-type potential

$$V_2 = \frac{\beta e^{-mcr/h}}{r},$$

where $\beta$ is the coupling strength and $m$ is the mass of a scalar particle mediating Yukawa-type interaction. Constraints were obtained using molecular hydrogen spectroscopy in Ref. [38] and atomic hydrogen $1s-2s$ spectroscopy in Ref. [14]. It was found that atomic systems provide stronger limits than molecular systems. We improve upon Ref. [14] by using more recent data and provide additional constraints from muonium $1s-2s$, positronium $1s-2s$, hydrogen $1s-3s$ and deuteron.

The energy shifts for $1s-2s$ and $1s-3s$ transitions are found from the expectation values of the Yukawa potential for hydrogen-like atoms (in units $\hbar = c = 1$)

$$\delta E_{1s-2s} = \frac{\beta}{4\tilde{a}_B} \left[ \frac{1 + 2\tilde{a}_B^2 m^2}{(1 + \tilde{a}_B m)^4} - \frac{16}{(2 + \tilde{a}_B m)^2} \right],$$

$$\delta E_{1s-3s} = \frac{4\beta}{9\tilde{a}_B} \left[ \frac{16 + 27\tilde{a}_B^2 m^2 (8 + 9\tilde{a}_B^2 m^2)}{(2 + 3\tilde{a}_B m)^6} - \frac{9}{(2 + \tilde{a}_B m)^2} \right],$$

where $\tilde{a}_B$ is the reduced Bohr radius. Note that in both transitions, in the large mass limit we observe $\delta E \propto \beta/m^2$. The energy shift for deuteron, using the wave function in Equation (27) is given by

$$\delta E = -2\beta \kappa \text{Ei}(- (m + 2\kappa)r_0),$$

where we have the exponential integral $\text{Ei}(x) = -\int_{-x}^{\infty} e^{-t}/t \, dt$. To obtain constraints we use limits on energy shifts for hydrogen, muonium, and positronium transitions and deuteron binding energy presented in Section 3. Note that our limits from positronium and muonium are in agreement with previous findings in Ref. [39].

For comparison, we include the previous constraint from Ref. [14], which used $|\delta E| = 2.1 \times 10^{-10}$ eV for hydrogen. Our results are presented in Figure 3. Limits from hydrogen $1s-2s$ spectroscopy give the strongest constraints at small masses $|\beta/\alpha| < 3.2 \times 10^{-13}$, while at large masses this constraint is $|\beta/\alpha| < 5.0 \times 10^{-21} (m/eV)^2$. 
6. Conclusion

Our work is motivated by the Stadnik paper [11] which demonstrated that the sensitivity of atomic spectral data to the neutrino mediated potential, introduced in Refs. [1–5], is up to 18 orders of magnitude better than the sensitivity of macroscopic experiments to this potential. However, an oversimplified cut-off treatment of this potential at small distances has led to inaccurate results in Ref. [11], especially in systems with particles of finite size. For example, a cut-off at the nuclear radius gave limits on the interaction strength which are 5 orders of magnitude weaker than the limits obtained in the present work. We argue that one should first build an effective interaction between point-like particles, like electrons and quarks. On the second step, this interaction should be integrated over the nuclear volume.

In this paper, we calculated energy shifts produced by neutrino-exchange potentials and extracted limits on the strength of this potential from atomic systems, namely hydrogen, muonium, positronium and deuteron. We also extracted constraints from spectra of H₂, D₂ and HD⁺ molecules. Following Ref. [11], we presented our results as constraints on the ratio of the effective strength of the neutrino-mediated potential $G_{\text{eff}}^2$ to the squared Fermi constant $G_F^2$. The best limit was obtained from the muonium hyperfine structure, $G_{\text{eff}}^2/G_F^2 < 10^3$. Our constraints are expected to be significantly enhanced in the near future, for example, the muonium 1s–2s measurement predicted to be improved by 3 orders of magnitude by the currently ongoing experiment Mu-MASS [21]. Constraints from atomic spectroscopy on the coupling strength and mass of a new scalar particle mediating a fifth force are also obtained and are an order of magnitude stronger compared to Ref. [14].

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ORCID

Phillip Munro-Laylim http://orcid.org/0000-0003-0043-3828
Vladimir Dzuba http://orcid.org/0000-0003-2758-5574
Victor Flambaum http://orcid.org/0000-0001-8643-7374

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