Experimental Constraints on Polarizability Corrections to Hydrogen Hyperfine Structure

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We present a state-of-the-art evaluation of the polarizability corrections—the inelastic nucleon corrections—to the hydrogen ground-state hyperfine splitting using analytic fits to the most recent data. We find a value $\Delta_{\text{pol}} = 1.3 \pm 0.3$ ppm. This is 1–2 ppm smaller than the value of $\Delta_{\text{pol}}$ deduced using hyperfine splitting data and elastic nucleon corrections obtained from modern form factor fits.

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Hyperfine splitting in the hydrogen ground state is measured to 13 significant figures [1],

$$E_{\text{hfs}}(e^- p) = 1420.4057517667(9) \text{ MHz}. \tag{1}$$

Theoretical understanding of hydrogen hyperfine splitting is far less accurate, being at about the part-per-million (ppm) level. The main theoretical uncertainty lies in proton structure corrections, which are not presently calculable from fundamental theory. However, proton structure corrections can be calculated as functionals of quantities measurable in other experiments, specifically as integrals [2] over proton form factors measured in elastic electron-proton scattering plus integrals [3–7] over structure functions measured in inelastic polarized electron-proton scattering. The quality of the data for the latter has improved greatly in recent times, especially in the lower momentum transfer region which is important for proton hyperfine corrections. In this article, we present a state-of-the-art evaluation of the polarizability corrections using analytic fits to the most recent data, in particular, using lower momentum transfer data [8–10] from Jefferson Lab.

The proton structure corrections, $\Delta S$, below, can be isolated by taking the experimental values for the hyperfine splitting (hfs) and subtracting the other corrections, which include QED corrections $\Delta_{\text{QED}}$, recoil corrections $\Delta R$, as well as some smaller terms due to hadronic vacuum polarization $\Delta_{\text{hvp}}$, muonic vacuum polarization $\Delta_{\text{muvp}}$, and weak interactions $\Delta_{\text{weak}}$. One has [11–13]

$$E_{\text{hfs}}(e^- p) = (1 + \Delta_{\text{QED}} + \Delta R + \Delta_{\text{hvp}} + \Delta_{\text{muvp}} + \Delta_{\text{weak}} + \Delta S)E_F. \tag{2}$$

where the scale is set by the Fermi energy, given by

$$E_F = \frac{8\pi}{3\alpha^3} \mu_B \mu_p \frac{m_e^3 m_p^3}{(m_p + m_e)^3}, \tag{3}$$

for an electron of mass $m_e$ bound to a proton of mass $m_p$, magnetic moment $\mu_p = (g_p/2)(e/2m_p)$, Landé $g$-factor $g_p$, and fine structure constant $\alpha$. By convention, the exact magnetic moment $\mu_p$ is used for the proton, but only the lowest order term, the Bohr magneton $\mu_p$, is inserted for the $e^-$. The proton structure correction inferred from atomic hyperfine splitting data by removing other corrections is

$$\Delta_S = \Delta_Z + \Delta_{\text{pol}} = -38.58(16) \text{ ppm}. \tag{4}$$

The uncertainty comes mainly from $\Delta_{Z}^p$; we used $5.84 \pm 0.15$ ppm. The central value [12] uses the CODATA [14] charge radius and the dipole magnetic radius. The uncertainty encompasses a spread due to other choices of these radii. A discussion is in [15]. Other quantities were taken from [12]. Eliminating the large QED corrections by using both hydrogen and muonium hfs [15] leads to a similar result, with an uncertainty limit 0.18 ppm.

The structure-dependent correction $\Delta_S$ is conventionally split into two terms, $\Delta Z$ and $\Delta_{\text{pol}}$. The bulk of the first term was calculated by Zemach [2] long ago, and the modern expression is

$$\Delta_S = -2\alpha m_e r_Z(1 + \delta_Z^\text{rad}), \tag{5}$$

where the Zemach radius is

$$r_Z = -\frac{4}{\pi} \int_0^\infty dq \frac{dQ^{(2)}}{Q^2} \left[ G_E(Q^2) \frac{G_M(Q^2)}{1 + \kappa_p} - 1 \right]; \tag{6}$$

$G_E$ and $G_M$ are the electric and magnetic form factors of the proton, normalized so that $G_E(0) = G_M(0) / (1 + \kappa_p) = 1$, $\kappa_p = (g_p - 2)/2$. The radiative correction $\delta_Z^\text{rad}$ is estimated in [11] and calculated in [16] for form factors that are represented by dipole forms: $\delta_Z^\text{rad} = (\alpha/3\pi) \times [2\ln(\Lambda^2/m_e^2) - 4111/420]$. For $\Lambda^2 = 0.71$ GeV$^2$, one finds $\delta_Z^\text{rad} = 0.0153$; for a $\Lambda^2$ corresponding to the relatively large charge radius in [17], one would instead get 0.0150, or a 0.01 ppm change in the hyperfine splitting.

The second term, $\Delta_{\text{pol}}$, involves contributions where the proton is excited [3–7]. In the limit where the electron mass is neglected except for one overall factor,

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2), \tag{7}$$

(the prefactor is 0.2265 ppm) with
\[ \Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2(Q^2) + \frac{8m_p^2}{Q^2} B_1(Q^2) \right\}, \]
\[ \Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2). \]

Here \( F_2 \) is the Pauli form factor of the proton,
\[ B_1 = \int_0^{\infty} dx \beta(x) g_1(x, Q^2), \]
\[ B_2 = \int_0^{\infty} dx \beta(x) g_2(x, Q^2). \]

\[ \beta(x) = \frac{4}{3} \left[ -3\tau + 2\tau^2 + 2(2 - \tau) \sqrt{\tau(\tau + 1)} \right] \]

and \( \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)} \),

in which \( \tau = \frac{\nu^2}{Q^2}, \nu \) is the lab-frame energy transfer, \( Q^2 \) is the squared four-momentum transfer, \( x_{th} = Q^2/(2m_p m_\pi + m_\pi^2 + Q^2) \), \( m_\pi \) is the pion mass, and \( g_1 \) and \( g_2 \) are the spin-dependent structure functions, measured in doubly polarized electron-proton inelastic scattering.

Reference [7] was to our knowledge the first to use \( g_{1,2} \) data to obtain results not consistent with zero for the polarizability corrections. However, \( \Delta_1 \) and \( \Delta_2 \) are sensitive to the behavior of the structure functions at low \( Q^2 \). Hence with new low \( Q^2 \) data available [8,10], there is a significant possibility that the numerical results for \( \Delta_{pol} \) could require noticeable revision.

The integrands of Eq. (8) converge because there are factors that cancel the poles at \( Q^2 = 0 \). The function \( \beta_2 \) has limiting behavior
\[ \beta_2(\tau) = \begin{cases} 1/4\tau = Q^2/4\nu^2 & \tau \to \infty, Q^2 \to 0 \\ 1 & \tau \to 0, Q^2 \to \infty \end{cases}, \]

Given that \( \nu \) is never zero for inelastic processes, even for \( Q^2 \to 0 \), and noting that the width of the integration region for the \( B_i \) is proportional to \( Q^2 \) for small \( Q^2 \), one sees that the integral for \( \Delta_2 \) is well behaved for finite \( g_2 \).

For the integral \( \Delta_1 \) to be finite, given
\[ \beta(\tau) = \begin{cases} 1 - 5/18\tau & \tau \to \infty \\ 0 & \tau \to 0 \end{cases}, \]

one needs a cancellation that follows from the Gerasimov-Drell-Hearn (GDH) [18,19] sum rule,
\[ \lim_{Q^2 \to 0} \frac{8m_p^2}{Q^2} \int_0^{x_{th}} dx g_1(x, Q^2) = -\kappa_p^2; \]

\( \kappa_p = F_2(0) \) is the proton anomalous magnetic moment.

Further, regarding \( \Delta_2 \), there is little \( g_2 \) data for the proton—there is some from SLAC E155 at higher \( Q^2 \) [20] and there is some Jefferson Lab Hall C data at \( Q^2 = 1.3 \) GeV\(^2\) under analysis [21]. Hence, the \( g_2 \) results rely on models. However, the \( g_2 \) contributions to the polarizability corrections are small because the weighting factor \( \beta_2 \) in the \( \Delta_2 \) integral is generally small within the integral. The weighting factor \( \beta \) by contrast is on average close to 1.

Our main results for \( \Delta_{pol} \) are detailed in Table I. We evaluated \( \Delta_{pol} \) using two different fits to \( g_1 \) and \( g_2 \) and several different parameterizations of \( F_2(Q^2) \).

The current best and most up-to-date parameterization is the one developed by CLAS EGI [8,9]. This fit begins with the most recent published data [8], which has \( Q^2 \cong 0.15 \) GeV\(^2\), and is based on AO [22] and MAID [23] parameterizations of the resonances, the E155 fit [24] in the deep-inelastic scattering (DIS) region, and the Wandzura-Wilczek [25] form \( g_{2\text{WW}} = -\bar{g} + \int_x^1 g(y)dy/y \) for \( g_2 \) in the DIS region. This fit also gives a good account of the new data [10], which has \( Q^2 \) down to 0.05 GeV\(^2\). The other structure function fit is that of Simula et al. [26], which is based on data available through the year 2001. Our results using this fit are shown in the last column of Table I, for one \( F_2(Q^2) \).

For \( F_2 \), we show results using the parameterizations of Kelly [27] and of Sick [17], and include the dipole fit \( F_2(Q^2) \) as a benchmark. Although the dipole fit is a traditional standard, it does not fit modern data well and results obtained from it are not reliable. The Kelly parameterization fits form factor data well overall, and for \( g_E \) elects to fit the polarization transfer results [28]. (There has been a discrepancy between the Rosenbluth and polarization transfer determinations of the proton \( F_2 \). Theoretical analysis [29] is resolving this by suggesting that two-photon corrections to the Rosenbluth determination at non-zero \( Q^2 \) will give agreement with the polarization method.)

The continued fraction parameterization of [17] concentrates on the low \( Q^2 \) data, and is valid for \( 0 \leq Q^2 \leq 0.62 \) GeV\(^2\). Beyond this, the \( F_2(Q^2) \) contributions to the integrals are small and we substituted the dipole form. This procedure was also used by Friar and Sick [30] in their analysis of the Zemach radius. Substituting the Kelly parameterization instead made no difference on a scale set by our uncertainty limits.

We show our numerical results in Table I. We have split the \( Q^2 \) integration into segments [0–0.05] GeV\(^2\) (where no data exist), [0.05–20] GeV\(^2\), and >20 GeV\(^2\), and show contributions from these regions separately. We also separate, except for the lowest \( Q^2 \) region for \( \Delta_1 \), the contributions from \( F_2, g_1, \) and \( g_2 \).

Three columns in Table I use the CLAS EGI model [8] for \( g_1 \) and \( g_2 \). Errors were assigned to the \( F_2 \) contribution to \( \Delta_1 \) using the parameter uncertainties that coherently gave the largest error. For \( Q^2 < 0.05 \) GeV\(^2\) this error was added in quadrature with a 10% overall systematic uncertainty as a guess about the absolute accuracy of the data at low \( Q^2 \). The contribution from \( g_1 \) was given a 10% error for the deep-inelastic region (\( Q^2 > 20 \) GeV\(^2\)), and a 50% error (0.05 < \( Q^2 \) < 20) dominated by the uncertainties in the preliminary CLAS data near \( Q^2 = 0.05 \) GeV\(^2\). The
We find that $\Gamma_2^{(N)} = -N \Gamma_1^{(N)}/(N + 1)$. Therefore, if we naively extrapolate this relation to low $Q^2$, we get $\Delta_1[0, 0.05] = -0.40 \pm 0.05$ using $y_0$ from above. From Ref. [31], keeping terms to $O(Q^4)$, we find that $\Delta_1[0, Q^2]$ = $3m^2_p Q^2 \gamma_0 - \delta_{LT}/2x$. Using the MAID $\pi$-channel estimate [31] $\delta_{LT} = 1.35 \times 10^{-4} \text{ fm}^{-2}$, we obtain $\Delta_2[0, 0.05] = -1.4$. Both of these values would lower the final $\Delta_{\text{pol}}$. We quote the CLAS EG1 model ($-0.24 \pm 0.24$) in Table I.

For the low $Q^2$ contributions in the Simula et al. [26] fit, we straightforwardly integrated the analytic form. The numerical differences from the EG1 result arise because a Taylor expansion of $\Gamma_1^{(N)}(Q^2)$ from Ref. [26] leads to a small curvature parameter $c_1$.

For all $Q^2$, the $g_1$ and $g_2$ fits of [26] have available error bands for the $g_i$. To evaluate uncertainties in the polarizability corrections due to uncertainties in the $g_i$, we recalculated the $\Delta_1$ using consistently the largest $g_1$ and smallest $g_i$. This gives an uncertainty estimate for $\Delta_1$ from $g_2$ of $\pm 0.37$ units, as quoted in Table I. The uncertainty estimate for $\Delta_1$ due to $g_1$ and $F_2$ is $\pm 2.48$ units.

The uncertainty limits involving $g_2$ on the proton may appear remarkable considering the amount of existing data. However, one expects that much of $g_2$ is due to the Wilezek-Wandzura term, which is gotten directly from $g_1$. This can be verified from the much larger body of data for $g_2$ on the neutron (using polarized $^3\text{He}$ targets).

Table II shows $\Delta_\text{pol}$ and the Zemach radius calculated from the form factor sets used above. Table II also gives $\Delta_3(\text{data}) - \Delta_2(\text{calc})$, i.e., the value of $\Delta_{\text{pol}}$ “desired” for consistency between measurement and proton structure corrections calculated using a given form factor set.

In summary, we quote our best value as

$$\Delta_{\text{pol}} = (1.3 \pm 0.3) \text{ ppm}.$$  

The earlier value of Faustov and Martynenko [7] was

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Term & $Q^2$ (GeV$^2$) & From & CLAS EG1 & Using Simula et al. $g_1, g_2$ fit \\
\hline
\hline
$\Delta_1$ & $[0, 0.05]$ & $F_2$ and $g_1$ & 0.45 $\pm$ 0.30 & 0.49 $\pm$ 0.30 & 0.60 $\pm$ 0.28 & -1.78 $\pm$ 0.67 \\
& $[0.05, 20]$ & $F_2$ & 7.01 $\pm$ 0.22 & 6.86 $\pm$ 0.27 & 7.12 & 7.01 $\pm$ 0.22 \\
& $g_1$ & & -1.10 $\pm$ 0.55 & -1.10 $\pm$ 0.55 & -1.10 $\pm$ 0.55 & -1.78 $\pm$ 1.86 \\
& $[20, \infty]$ & $F_2$ & 0.00 & 0.00 & 0.00 & 0.00 \\
& $g_1$ & & 0.12 $\pm$ 0.01 & 0.12 $\pm$ 0.01 & 0.12 $\pm$ 0.01 & 0.10 $\pm$ 0.01 \\
total $\Delta_1$ & & & 6.48 $\pm$ 0.89 & 6.38 $\pm$ 0.92 & 6.74 $\pm$ 0.84 & 3.55 $\pm$ 2.48 \\
$\Delta_2$ & $[0, 0.05]$ & $g_2$ & -0.24 $\pm$ 0.24 & -0.24 $\pm$ 0.24 & -0.24 $\pm$ 0.24 & -0.72 $\pm$ 0.14 \\
& $[0.05, 20]$ & $g_2$ & -0.33 $\pm$ 0.33 & -0.33 $\pm$ 0.33 & -0.33 $\pm$ 0.33 & -1.14 $\pm$ 0.23 \\
total $\Delta_2$ & & & -0.57 $\pm$ 0.57 & -0.57 $\pm$ 0.57 & -0.57 $\pm$ 0.57 & -1.86 $\pm$ 0.37 \\
$\Delta_1 + \Delta_2$ & & & 5.91 $\pm$ 1.06 & 5.81 $\pm$ 1.08 & 6.18 $\pm$ 1.02 & 1.69 $\pm$ 2.51 \\
$\Delta_{\text{pol}}$ & & & 1.34 $\pm$ 0.24 ppm & 1.32 $\pm$ 0.24 ppm & 1.40 $\pm$ 0.23 ppm & 0.38 $\pm$ 0.57 ppm \\
\hline
\end{tabular}
\caption{Contributions to $\Delta_{\text{pol}}$ using various models.}
\end{table}
It is remarkable that this value, based on few data, agrees with the determination using fits to the extensive CLAS data set. We thus corroborate extractions of hadronic quantities from data as done in [12,13].

We should now focus on modern form factor parameterizations represented by [17] or [27], which fit low $Q^2$ data well. Accepting these and Table II could lead one to desire fits that could give a more suitable Zemach radius.

We cannot, in our opinion, anticipate that new proton $g_1$ or $g_2$ data will change the evaluation of $\Delta_{\text{pol}}$ by enough to reconcile the proton structure corrections with the measured hydrogen hyperfine splittings. For example, to get the requisite $\Delta_{\text{pol}}$ by changing the curvature parameter we called $c_1$, would require making it about 5 times larger than the value we used. This is essentially unthinkable given data already available. A further look at elastic form factor fits that could give a more suitable Zemach radius $r_2$ could well be warranted.

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