Dynamics and reliability analysis of the D-knotter winding rope system

R S Na, Y Liu and S Li
School of Mechanical Engineering, Inner Mongolia University of Technology, 49 Aimin Street, Hohhot, China
Email: 1574437834@qq.com

Abstract. The knotter is the core component of the rectangular bale baler. In view of the short service life, low reliability, and easy fracture during the working process, this paper takes the D-knotter's winding rope system as the research object, lumped mass method and Lagrangain method were adopted to set up dynamic model and equations of winding rope system, Built a dynamic simulation block diagram. The dynamic response analysis of the winding rope system verified the accuracy of the dynamic differential equation and determined the fatigue analysis method. The Six Sigma reliability analysis module was used to analyze the reliability of the winding rope system at different rotational speeds was calculated, put forward some optimizing measures for fatigue life of winding rope system, the result indicated that the fatigue life of winding rope system after optimization were greatly improved.

Keywords: D-knotter, transient dynamics, fatigue reliability

1. Introduction
D-knotter is a mechanism that completely depends on mechanical transmission to realize the knotting function, it imitates the knotting movement of the human hand for movement, and realizes the mechanical automatic knotting. Influenced by factors such as the working environment and the uncertainty of working hours, during working time of D-knotter, it will wear out and break, so that decreases the efficiency of knotting and the safety and stability of the knotter. At present, the research of D-knotter: Zhang S et al. [1] according to D-knotter of operation requirement, the transmission scheme to implement sequentially the holding of the strand, the cutting of the strand, the pushing of the loop and the return of the knotter by transforming the rotation of the cam to that of the oscillating of the arm was determined, a detailed curve of the cam profile was obtained by programming in MATLAB and the force transfer characteristic was analyzed. S Xueke et al. [2] by analyzing the failure modes and failure mechanism of the knotter, the root cause of the knotter failure was determined to be the wear of five different-sided shaft holes on the frame body. By installing bronze bearings lubricated with lithium grease on the shaft holes of the frame body, the wear of the shaft hole can be greatly reduced, thereby greatly improving the reliability of the knotter. Aiming at the problem that the wear of gear disc affects the working stability of D-knotter. R Dezhi et al. [3] analyzed the stress, strain and fatigue of the driving gear disc of D-knotter by ANAYS platform, and obtained the stress concentration position and fatigue...
life of the driving gear disc of D-knotter. Xiong Ya et al. [4] based on rigid-flexible collision theory, a new improved design with elastic billhook shaft system was introduced. Marseille et al. [5] used fishbone diagrams to perform causal analysis of D-knotter, and applied contradiction analysis to obtain a solution to the trouble of disassembling and installing D-knotter.

In summary, the current research on D-knotter mainly focuses on the structural reconstruction and statics components. The traditional research methods of D-knotter lack the prediction of dynamics and fatigue life. In this paper, based on the lumped mass method and Lagrangain method, the dynamic model and dynamic equation of the D-knotter winding rope system are established. The modal theoretical calculation of the winding rope system of D-knotter is compared with the workbench modal analysis results to verify the accuracy of the dynamic equations. Using Workbench, the transient dynamics analysis of the D-knotter winding rope system was carried out. Using the Workbench n Code Design Life analysis module to analyze the fatigue life of the D-knotter winding rope system, obtained the minimum fatigue life and failure area of the winding rope system. Calculated the reliability of the winding rope system at different speeds, Using Six Sigma reliability analysis module to analyze the reliability of the winding rope system and optimize it reasonably, research result show that the fatigue life of the winding rope system is significantly improved after optimization.

2. D-knotter transmission and force analysis

2.1 Action analysis

One work cycle of D-knotter is 0.7s, during this period the knotting device needs to complete the actions of feeding rope, strapping, clamping rope, winding rope, cutting rope and tripping. The structure of D-knotter is shown in Figure 1.

![D-knotter structure diagram](image)

1-gear disc  2-frame  3-knotting pincer shaft bevel pinion  
4-knotting clamp shaft  5-knotting pincer mouth  6-cutter  
7-knife arm shaft  8-worm shaft bevel pinion  9-worm shaft  
10-worm  11-worm wheel  12-rope holder

**Figure 1. D-knotter structure diagram**

The action sequence of D-knotter as follows: before the knotting work begins, the primary binding rope is delivered to the hatch of knife arm shaft 7 through the rope removing pole and the knotting pincer mouth 5, the other binding rope threading through the baling needle moves upward following the baling needle, the two binding ropes closely attach to the surface of the knotting pincer mouth 5. When the baling needle continues to lift and reach above the rope holder 12, the incomplete tooth (inner contour) of the gear disc 1 will engage with the worm shaft bevel pinion 8, then the worm wheel 11 will rotate, driving the rope holder 12 also to rotate.

The gear disc 1, through its rotating, engages with the knotting pincer shaft bevel pinion 3 through the incomplete tooth of the outer contour, which drives the knotting pincer mouth 5 to rotate. The two binding ropes inside the knotting pincer mouth 5 are being led into between upper claw and knotting pincer mouth 5, and are knotted along with rotating of the knotting pincer mouth 5.

The knife arm shaft 7 under the knotting pincer mouth 5 swings forward due to the rotation of the
gear disc 1, and the two binding ropes are tightened. At this time, the cutter 6 fixed to the knife arm shaft 7 cuts off the two binding ropes under the rope holder 12 and the two binding ropes wrapped around the knotting pincer mouth 5 falls.

2.2 Force analysis

The winding rope system of D-knotter power input is provided by meshing of gear disc 1 and worm shaft bevel pinion 8, the knotting system of D-knotter power input is provided by meshing of gear disc 1 and knotting pincer shaft bevel pinion 3. Take the winding rope system as an example (The force diagram of the winding rope system is shown in Figure 2), the forces experienced by the worm shaft 9 during operation include: The circumferential force $F_{a1}$, radial force $F_{r1}$ and tangential force $F_{t1}$ generated when the gear disc 1 meshes with the worm shaft bevel pinion 8, the lower end of the worm shaft 9 will be subjected to axial force $F_{a3}$, circumferential force $F_{r3}$ and radial force $F_{r3}$ generated by the worm 10 and the worm wheel 11, the torque $M_1$ generated by the rotation of the gear disc 1.

The forces on the worm shaft 9 are:

$$F_{a1} = \frac{F_{r1}}{\cos \beta_m} \left( \tan \alpha \cdot \sin \delta_1 - \sin \beta_m \cdot \cos \delta_1 \right)$$

(2)

$$F_{a2} = \frac{F_{r1}}{\cos \beta_m} \left( \tan \alpha \cdot \sin \delta_2 + \sin \beta_m \cdot \cos \delta_2 \right)$$

(3)

$$F_{r1} = \frac{F_{r1}}{\cos \beta_m} \left( \tan \alpha \cdot \cos \delta_1 + \sin \beta_m \cdot \sin \delta_1 \right)$$

(4)

$$F_{r2} = \frac{F_{r1}}{\cos \beta_m} \left( \tan \alpha \cdot \cos \delta_2 \cdot \sin \beta_m \cdot \sin \delta_2 \right)$$

(5)

$$F_{r3} = -F_{r4} = \frac{2000T_3}{d_3}$$

(6)

$$F_{a3} = -F_{r4} = \frac{2000T_4}{d_4 + 2x_4 \cdot m}$$

(7)

$$F_{r3} = -F_{r4} \approx -F_{r4} \cdot \tan \alpha$$

(8)

$$T_1 = i \cdot T_3 \cdot \eta$$

(9)

Where, $T_1$: torque transmitted by the bevel gear of the gear disc, $d_{ml}$: reference diameter of gear disc inner contour bevel gear, $\alpha$: pressure angle, $T_1 = 200N \cdot m$, $d_{ml} = d_1 (1 - 0.5\phi R)$, $\phi R = 0.3$, $\beta_m$:...
helix angle of helical bevel gear, $\delta_1$: reference cone angle of gear disc inner contour bevel gear, $\delta_2$: reference cone angle of worm shaft bevel pinion, $T_3$: worm shaft torque, $d_3$: reference diameter of worm gear, $\chi_4$: modification coefficient, $m$: normal modulus of worm and worm gear, $\alpha_5$: normal pressure angle, $\eta$: worm drive efficiency, the specific parameters are shown in Tables 1 and 2.

| Parameter name          | gear disc inner contour bevel gear | worm shaft bevel pinion |
|-------------------------|-----------------------------------|-------------------------|
| Modulus m/mm            | 4                                 | 4                       |
| Theory of teeth Z       | 38                                | 8                       |
| Reference cone angle $\delta$/° | 85.88                             | 12.12                   |
| Pressure angle $\alpha$/°| 20                                | 20                      |
| Helix angle $\beta$/°   | 15                                | 15                      |
| Width B/mm              | 15                                | 15                      |
| Equivalent number of teeth $Z_v$ | 587 | 9.1 |

| Parameter name          | worm                  | Worm gear          |
|-------------------------|-----------------------|--------------------|
| Teeth Z                 | 2                     | 8                  |
| Modulus m/mm            | 3.5                   | 3.5                |
| Lead angle $\gamma$ / Helix angle $\beta$/° | 15 | 15 |

3. Dynamic analysis
D-knotter is an elastic vibration system with multiple degrees of freedom, when an elastic vibration system with multiple degrees of freedom is subjected to an external force, the response of the system is formed by the superposition of each mode. The equation of the vibration system is:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [F]$$

(10)

Where, $[M]$: structural mass matrix; $[C]$: structural damping matrix; $[K]$: structural stiffness matrix; $\{u\}$: node acceleration vector; $\{\dot{u}\}$: node speed vector; $\{u\}$: node displacement vector; $[F]$: load vector under external loading.

3.1. Kinetic model
The power transmission route of the winding rope system is: gear disc- worm shaft bevel pinion- worm- worm wheel- rope holder. The dynamic model of the winding rope system is shown Figure 3. Where, $k_1$, $c_1$: the meshing rigidity and damping between main shaft and gear disc; $k_2$, $c_2$: the meshing rigidity and damping between the gear disc and the worm shaft bevel pinion; $k_3$, $c_3$: the meshing rigidity and damping between the worm shaft bevel pinion and the worm shaft; $k_4$, $c_4$: the meshing rigidity and damping between worm and worm wheel; $k_5$, $c_5$: the meshing stiffness and damping between worm gear and rope holder.
Figure 3. Dynamic model of the winding rope system

The differential equation of the winding rope system is:

\[ m_1 \cdot \ddot{x}_1 + k_1 \cdot x_1 + k_2 \cdot (x_1 - x_2) + c_1 \cdot \dot{x}_1 + c_2 \cdot (\dot{x}_1 - \dot{x}_2) = F_{12}(t) \]
\[ m_2 \cdot \ddot{x}_2 + k_2 \cdot (x_2 - x_1) - k_2 \cdot (x_2 - x_3) + c_2 \cdot (\dot{x}_2 - \dot{x}_1) - c_2 \cdot (\dot{x}_2 - \dot{x}_3) = F_2(t) \]
\[ m_3 \cdot \ddot{x}_3 - k_3 \cdot (x_3 - x_2) + k_3 \cdot (x_3 - x_4) - c_3 \cdot (\dot{x}_3 - \dot{x}_2) + c_4 \cdot (\dot{x}_3 - \dot{x}_4) = F_3(t) \]
\[ m_3 \cdot \ddot{x}_4 + k_3 \cdot (x_4 - x_3) - k_4 \cdot (x_4 - x_5) - c_4 \cdot (\dot{x}_4 - \dot{x}_3) + c_5 \cdot (\dot{x}_4 - \dot{x}_5) = F_4(t) \]
\[ m_5 \cdot \ddot{x}_5 - k_5 \cdot (x_5 - x_4) - c_5 \cdot (\dot{x}_5 - \dot{x}_4) = F_5(t) \]  \(^{(11)}\)

Mass matrix \( M_1 \), stiffness matrix \( K_1 \), damping matrix \( C_1 \) as follows:

\[
M_1 = \begin{bmatrix}
  m_1 & 0 & 0 & 0 & 0 \\
  0 & m_2 & 0 & 0 & 0 \\
  0 & 0 & m_3 & 0 & 0 \\
  0 & 0 & 0 & m_4 & 0 \\
  0 & 0 & 0 & 0 & m_5 \\
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
  k_1 + k_2 & -k_2 & 0 & 0 & 0 \\
  -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\
  0 & -k_3 & k_3 + k_4 & -k_4 & 0 \\
  0 & 0 & -k_4 & k_4 + k_5 & -k_5 \\
  0 & 0 & 0 & -k_5 & k_5 \\
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
  c_1 + c_2 & -c_2 & 0 & 0 & 0 \\
  -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\
  0 & -c_3 & c_3 + c_4 & -c_4 & 0 \\
  0 & 0 & -c_4 & c_4 + c_5 & -c_5 \\
  0 & 0 & 0 & -c_5 & c_5 \\
\end{bmatrix}
\]

\(^{(1)}\)Parameter solution

The torsional stiffness of the shaft is:\(^{(6)}\):

\[ k = G \frac{J}{l} \]  \(^{(12)}\)

Where, \( G \) : shear modulus of elasticity; \( G = \frac{E}{2(1 + \mu)} \), \( J \) : polar moment of inertia, \( J = \frac{\pi d^4}{32} \) ; \( d \) indicates the diameter of the shaft; \( l \) : torsion length of shaft. Bring the parameters of the main shaft, worm shaft and knotting clamp shaft into the formula \((12)\): \( k_1 = 3101N\cdot m/rad \), \( k_3 = 1064N\cdot m/rad \), \( k_5 = 4825N\cdot m/rad \).

According to GB10062.1-2003, the meshing rigidity of the bevel gear is:

\[ C_y = C_{yo} \cdot C_F \cdot C_b \]  \(^{(13)}\)

Where, \( C_{yo} \) : gear tooth stiffness under average conditions, is \( 20N/mm\cdot \mu m \); \( C_F \) : Correction factor; \( b_e \) : effective tooth width. Substitute the parameters of Table 1 into equation \((13)\), and \( C_{r2} = 20N/mm\cdot \mu m \), then the meshing rigidity of the gear is \( k_2 = 4.6 \times 10^8N/m \).
The meshing rigidity of the worm and worm gear:

$$k = \frac{4}{3} R^{0.5} E'$$  \hspace{1cm} (14)$$

Where,  

$$E' = 1/(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2})$$,  

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$,  

$$R_1, R_2$$: worm and worm gear radius;  

$$\mu_1, \mu_2$$: poisons’ ratio of worm and worm gear material;  

$$E_1, E_2$$: elastic modulus of worm and worm gear material,  

$$k = 1.0 \times 10^9 \text{ N/m}^{0.5}$$.  

The torsional damping of the shaft is:

$$c = 2\xi \sqrt{\frac{k}{J_1 + J_2}}$$  \hspace{1cm} (15)$$

Where,  

$$\xi$$: torsional damping coefficient of shaft, is 0.01;  

$$J_1, J_2$$: torque at both ends of the shaft;  

$$k$$: torsional stiffness of shaft. The torsional damping of the main shaft, worm shaft and rope holder shaft is obtained as:  

$$c_1 = 57\text{ N/(m/s)}$$,  

$$c_2 = 12.3\text{ N/(m/s)}$$,  

$$c_3 = 14.4\text{ N/(m/s)}$$.  

The meshing damping of the gear is:

$$c_m = 2\xi \sqrt{\frac{k_m}{\frac{1}{m_1} + \frac{1}{m_2}}}$$  \hspace{1cm} (16)$$

Where,  

$$r_1, r_2$$: radius of driving wheel and driven wheel;  

$$m_1, m_2$$: weight of driving wheel and driven wheel;  

$$\xi$$: gear damping ratio, is 0.1;  

$$k_m$$: the average meshing stiffness of the gear. Gear disc inner contour bevel gear and worm shaft bevel gear belong to helical bevel gear transmission system,  

According to GB/T 3480-1997, and  

$$k_{m2} = 4.62 \times 10^9 \text{ N/m}$$ . Substitute the calculation result into equation (16), the meshing damping is:  

$$c_2 = 1402\text{ N/(m/s)}$$ .  

The meshing damping of worm and worm gear is $750\text{ N/(m/s)}$.

3.2. Dynamic solution

The simulation calculation frame of the D-knotter winding rope system built in Simulink is shown in Figure 4. Choose ode45 solver, the error value is 1e-3, Choose the initial condition as:  

$$x_i = 0.2$$,  

$$\dot{x}_i(0) = 0$$,  

$$x_i(0) = 0$$,  

$$\dot{x}_i = 0$$ ($i = 1, 2, ...$).

Figure 4. Block diagram of the winding rope system

Figure 5 shows the displacement changes with time of main shaft, the displacement changes with
time of worm shaft, and the displacement changes with time of rope holder shaft, affected by their respective damping, corresponding displacement decay with time, after completing the wrap rope, system displacement tends to zero.

(a)Scope response  
(b)Scope1 response  
(c)Scope2 response  

**Figure 5.** Scope response of the winding rope system

4. Dynamic response analysis

4.1. Modal analysis

The modal numerical equation of the winding rope system is:

\[
\begin{align*}
    m_1 \cdot \ddot{x}_1 + k_1 \cdot x_1 + k_3 \cdot (x_1 - x_3) &= 0 \\
    m_3 \cdot \ddot{x}_3 - k_3 \cdot (x_1 - x_3) + k_5 \cdot (x_3 - x_5) &= 0 \\
    m_5 \cdot \ddot{x}_5 - k_5 \cdot (x_3 - x_5) &= 0
\end{align*}
\]

Solve with MATLAB, the first order frequency of the winding rope system is 46.26Hz, the second order frequency is 79.72Hz, the third order frequency is 134.11Hz. The corresponding vibration modes are shown in Figure 6.

**Figure 6.** Vibration diagram of the winding rope system

Import the winding rope system into Modal of Workbench, set the grid size to 3mm, adopt automatic
division method, the total number of nodes is 18279, and the total number of elements is 9391. The modal analysis results of the winding rope system are shown in Figure 7.

![1st order mode shape](image1)

(a) 1st order mode shape

![2nd order mode shape](image2)

(b) 2nd order mode shape

![3rd order mode shape](image3)

(c) 3rd order mode shape

**Figure 7.** The first 3 modes of the winding rope mode

According to Figure 7, the first order frequency of the winding rope system is 52.988Hz, the overall performance is that gear disc up and down swing and displacement, the second order frequency is 89.752Hz, the overall performance is that gear disc left and right swing and displacement, the third order frequency is 109.25Hz, the overall performance is that gear disc front and back the swing and displacement.

Comparing the modal numerical calculation results of equation (17) with the winding rope system and the finite element results, the first order frequency error is 13%, the second order frequency error is 11%, and the third order frequency error is 23%. The theoretical calculation results of the winding rope system deviate from the modal analysis results, and the difference is within 30%, which meets the requirements of the modal analysis. Therefore, the dynamic differential equations of the winding system rope can reflect its dynamic characteristics.

4.2. Transient dynamic analysis

The D-knotter needs to complete the action in 0.7s. Short term flipping will cause impact on the rotating shaft and affect the structural stability. In addition, the load that changes with time during the work process, variable stress is the main reason for the failure of the knotter. Therefore, the transient dynamic analysis of the winding rope system is required, and it also provides the prerequisites for the fatigue analysis of the winding rope system.

Import the geometric model of the winding rope system into Workbench and divide the grid. The automatic gridding method is adopted. The grid size is 3mm. Set the corresponding material properties and enter the periodic load through the list, set the contact mode to frictional contact, and friction coefficient is 0.2, set the initial step as 20, set the minimum step as 30, and set maximum step of the winding rope system as 200, because the D-knotter completed a knotting time 0.7s, the analysis duration
is set to 0.7s. The equivalent stress obtained from the transient dynamic analysis is shown in Figure 8, and the equivalent stress change curve is shown in Figure 9.

![Stress distribution diagram of the winding rope system](image1)

**Figure 8.** Stress distribution diagram of the winding rope system

![Time history of stress variation of the winding rope system](image2)

**Figure 9.** Time history of stress variation of the winding rope system

From the stress change curve of Figure 9, it can be obtained that the maximum stress occurring in the winding rope system in one cycle is 116 MPa. As can be seen from Figure 8, the maximum equivalent stress of the winding rope system is generated in the lower end area of the frame and the worm shaft. The material of the knotter frame is gray cast iron, and the tensile strength of the material is 277MPa, the maximum equivalent stress $\sigma_{\text{max}}$ is less than the ultimate tensile strength of the material. Although the maximum value of the equivalent stress is less than the ultimate tensile strength, due to the long term alternating load of system, it is easy to cause fatigue damage, cracks or even fracture, so the fatigue life analysis of the winding rope system has more practical value.

5. Fatigue reliability analysis

5.1. Fatigue analysis

In the actual working process, the working environment of the D-knotter is harsh, and it has been subjected to complex and variable loads for a long time. There are many failures, and even serious damages such as cracks and fractures occur. The stress when the D-knotter is damaged is usually less than the tensile limit of the material, and the time and location of fatigue failure are often difficult to predict and have suddenness. Therefore, fatigue reliability analysis and research on the D-knotter have important engineering significance.

According to the Miner damage criterion, if the effective stress level is $\Delta \sigma$ given, the relationship between the number of cycles $n_1$ and the cycle life $N_{\delta}$ at this time is as follows[8]:

$$D_1 = \frac{n_1}{N_{\delta1}}$$  \hspace{1cm} (18)

Where, $D_1$: fatigue damage cumulative. Similarly, if under the action of stress $\Delta \sigma_2$, damage accumulation at this time if $D_2 = 0$, then $n_2 = 0$, so there is no damage. If $D_2 = 1$, then $n_2 = N_{\delta2}$ at this point the fatigue fracture begins.
When a two-stage loading unit occurs, there is a linear accumulation of damage as follows:

$$\frac{n_1}{N_{\delta_1}} + \frac{n_2}{N_{\delta_2}} = 1$$  \hspace{1cm} (19)

According to formula (19), if the sum of damage under each stress is equal to 1, fatigue fracture will occur. This accumulative law can be extended to cases where there are any \( \infty \) stages:

$$\sum_{i=1}^{\infty} \frac{n_i}{N_{\delta_i}} = 1$$  \hspace{1cm} (20)

Where, \( n_i \): the number of cycles of mechanical parts or materials under stress \( \delta_i \); \( N_{\delta_i} \): life of mechanical parts or materials under stress \( \delta_i \). Failure will occur if the total damage reaches 1.

The ANSYS transient dynamics analysis results and load spectrum files are imported into n Code Design Life, and set the material parameters of the winding rope system. According to the processing specification of the D-knotter, the surface treatment method and the application of the load, the S-N curve is corrected by the Gerber method:

$$\frac{S_u}{S_e(R = -1)} + \left( \frac{S_m}{UTS} \right)^2 = 1$$  \hspace{1cm} (21)

Where, \( S_u \): the stress amplitude under actual working conditions of the material; \( S_e(R = -1) \): the stress amplitude of the material is equal to -1; \( S_m \): the average stress under the actual working condition of the material; \( UTS \): the materials tensile ultimate.

Establish a flow chart of the module in n Code for analysis, and obtain the damage result of the rope winding system as shown in Figure 10:

![Figure 10](image)

**Figure 10.** Damage distribution diagram of the winding rope system

In the fatigue analysis module, the Damage in the property is changed to Life, and the life of the winding rope system can be obtained. In the n Code Design Life software, the unit for calculating the life is Repeats, and the calculated unit is the number of cycles, and the fatigue life is shown in Figure 11:

![Figure 11](image)

**Figure 11.** Life distribution diagram of the rope winding system

It can be seen from Figures 10 and 11 that the maximum damage of the winding rope system is \( 2.8 \times 10^{-6} \), the corresponding lifetime is \( 3.15 \times 10^8 \). Most damage ranges are \( 1.417 \times 10^{12} - 2.24 \times 10^8 \),
the corresponding lifetime ranges are $3.96 \times 10^6 - 6.29 \times 10^6$. The area with a lower life value in the life distribution diagram corresponds to the area with greater fatigue damage, and the minimum life appears in the area where the lower end of the worm shaft cooperates with the frame.

5.2. Reliability analysis

According to the fatigue life results of the winding rope system, the reliability parameter model of the winding rope system is established. By setting the input and output variable parameters, the Six Sigma module in the workbench of the finite element analysis software is used to analyse the reliability of the winding rope system, and the sensitivity and probability density functions to evaluate the reliability of the winding rope system.

According to the analysis results of the fatigue life of the winding rope system, the position that has a greater impact on the winding rope system is selected: the diameter of the worm shaft, the height of the frame support and the width of the frame support are used as input parameters for the analysis [9], as shown in Table 3.

| Input variables | Variable symbol | Distribution type | Mean/mm | Standard deviation |
|-----------------|-----------------|------------------|---------|-------------------|
| diameter        | P2-plane9.D4    | Normal distribution | 14.5    | 0.725             |
| height          | P4-Extrude17.FD1 | Normal distribution | 80.5    | 4.025             |
| width           | P7-Extrude3.FD1  | Normal distribution | 35      | 1.75              |

Choose a small sample of random input variables to obey the law of normal distribution. The random distribution function graphs of each input variable are shown in Figures 12, 13, and 14:

**Figure 12.** Diameter distribution function of worm shaft

**Figure 13.** Distribution function of frame support height

**Figure 14.** Distribution function of frame support width

The minimum fatigue life, the maximum average stress value in transient dynamics and the maximum strain are selected as the output variables to evaluate the reliability of the rope winding system,
as shown in Table 4. The parameter settings of the input and output variables in Six Sigma are shown in Figure 15.

### Table 4. Output variable parameter settings

| Variable name          | Variable symbol |
|------------------------|-----------------|
| Fatigue life           | P1              |
| Maximum average stress  | P5              |
| Maximum strain         | P6              |

#### Figure 15. Six Sigma input and output variable settings

5.2.1. Analysis of the influencing factors

Based on the central composite design method, the influence of random input variables on output variables is analysed, and the degree of influence of each variable on the reliability of the winding rope system is obtained through the analysis results, which provides a theoretical basis for the optimization of the winding rope system.

As shown in Figure 16 (a), when the diameter of the worm shaft is in the range of 12.26mm–14.5mm, as the diameter increases, the fatigue life of the worm shaft gradually decreases; when the diameter of the worm shaft is in the range of 14.5mm–16.74mm, the fatigue life of the worm shaft is proportional to the diameter. According to Figure 16 (b) and (c), when the diameter of the worm shaft ranges from 12.26mm–14.5mm, the average stress and strain of the worm shaft are proportional to the diameter of the worm shaft; when the diameter of the worm shaft is in the range of 14.5mm–16.74mm, the average stress and strain of the worm shaft are inversely proportional to the diameter of the worm shaft. Therefore, in order to improve the fatigue life of the winding rope system, reduce the average stress, and if consider the problem of weight reduction, reduce the size of the worm shaft diameter.

(a) Effect of diameter on fatigue life  
(b) Effect of diameter on average stress
Figure 16. The effect of worm shaft diameter on output variables

As shown in Figure 17 (a), when the support height of the frame in the mating area is in the range of 68.062mm–80.5mm, as the height increases, the fatigue life of the winding rope system gradually decreases; when the height of the frame is in the range of 80.5mm–92.938mm, the fatigue life of the winding rope system increases with the height. According to Figure 17 (b), when the height of the frame is in the range of 68.062mm–77mm, the average stress of the rope winding system increases with the height; when the height of the frame is in the range of 77mm–80.5mm, the average stress on the winding rope system increases with the height; when the height of the frame is in the range of 80.5–87mm, the average stress value on the winding rope system decreases with the increase of the height; When the height of the frame is in the range of 87mm–92.938mm, the average stress received by the winding rope system is proportional to the height. According to Figure 17 (c), when the height of the frame is in the range of 68.062mm–85mm, the average stress on the winding system rope decreases with the increase of the height; when the height of the frame is in the range of 85mm–92.938mm, the average stress of the winding rope system will increase with height. Therefore, when optimizing the winding rope system, the height of the winding rope system is 85mm.

Figure 17. The effect of the support height of the frame on the output variables

As shown in Figure 18 (a), when the width of the frame is in the range of 29.592mm–35mm, as the size increases, the fatigue life of the winding rope system gradually decreases; when the width of the frame is in the range of 35mm–40.405mm, the fatigue life of the winding system rope is proportional to the width. According to Figure 18 (b) and (c), when the width of the frame is in the range of
29.592mm~33mm, the average stress and strain of the winding rope system are proportional to the width of the frame; When the support width of the frame is 33mm~40.408mm, the average stress and strain of the winding rope system are inversely proportional to the support width. In order to improve the fatigue life of the winding rope system, reduce the average stress received, and at the same time consider the problem of light weight, the width of the frame body in the matching area should be reasonably reduced.

In summary, the structural dimensions of the winding rope system will have varying degrees of influence on its reliability, and a critical value for controlling reliability to reach the optimal value can be found in the impact distribution diagram, reasonable design of the structure size of the winding rope system is the key to the optimal design of the reliability of the winding rope system.

5.2.2. Sensitivity analysis

In the field of mechanical engineering, the sensitivity of the structure reflects the role of random variables that have an important impact on the reliability of the structure.

It is known from Figure 19 that the diameter of the worm shaft has the greatest influence on the fatigue life of the winding rope system and is positively related to the fatigue life of the winding rope system; the average stress and strain of the winding rope system are negatively correlated. The support height of the frame has a greater influence on the average stress and strain, and has a smaller influence on the fatigue life of the winding rope system. The support height of the frame body is inversely related to the three output variables. The support width of the frame body is similar to the support height. Therefore, when optimizing the winding rope system, the distribution of sensitivity should be considered, and the design parameters of the winding rope system should be selected reasonably.

Figure 18. Effect of the support width of the frame on the output variables

Figure 19. Sensitivity of each input variable to the output variable
In Six Sigma, the reliability of the winding rope system can be viewed according to the cumulative distribution function. The corresponding failure rate value can be obtained by setting the probability level, and then the reliability of the winding rope system can be evaluated. The fitting curve of the cumulative distribution function of fatigue life, maximum average stress and strain are shown in Figures 20, 21 and 22. As can be seen from Figure 20, according to the cumulative distribution function of the fatigue life of the winding rope system, when the fatigue life of the winding rope system is $10^4$ times, its reliability is 67%. It can be seen from Figure 21 that the maximum average stress of the winding rope system is mainly in the range of 125MPa~130MPa, and the reliability can reach 92% in this range. According to Figure 22, the strain variable of the winding rope system is mainly concentrated at 0.0012mm, and the reliability can reach 99%.

5.2.3. Reliability analysis at different speeds

According to the fatigue life of the winding rope system, the reliability can be calculated when the fatigue life reaches $10^4$ repeats. In this paper, the normal probability density function is used to analyse the fatigue life reliability of the winding rope system.

The probability density curve of the standard normal distribution is a curve symmetrical about the y-axis. Through the normal distribution table, different y values can be obtained, that is, unreliability, reliability can be expressed as $R = 1 - y$. After taking the logarithm of fatigue life $n$, $\lg n$ follows a normal distribution. So the probability function to $n$ is[10]:

\[ F(n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\ln n} e^{-\frac{y^2}{2}} dy \]
\[ y = \frac{1}{n\sigma_y\sqrt{2\pi}} e^{-\frac{(\lg n - \mu)^2}{2\sigma_y^2}} \]  

(22)

Substituting \( z = \left( \frac{\lg n - \mu}{\sigma_y} \right) \) into (22), get the standard normal density distribution function:

\[ y = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \]  

(23)

The reliability can be calculated by substituting (22), (23):

\[ R = 1 - y = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz \]  

(24)

According to the fatigue life analysis results of the winding rope system, the corresponding fatigue life at different speeds is shown in Table 5.

| Speed \( \omega \) (rad/s) | Fatigue life \( n \) (repeats) |
|---------------------------|-----------------|
| 31.4                      | \( 5.52 \times 10^4 \) |
| 39.3                      | \( 3.73 \times 10^4 \) |
| 45                        | \( 3.15 \times 10^4 \) |
| 63                        | \( 2.18 \times 10^4 \) |

The reliability of the rope winding system can be calculated:

\[ \mu = \frac{1}{m} \sum_{i=1}^{m} \lg n \]  

(25)

\[ \sigma_y = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (\lg n - \mu)^2} \]  

(26)

Where, \( m: 4; n: \) fatigue life. \( y = \left( \frac{\lg n - \mu}{\sigma_y} \right) = \left( \frac{\lg 10^4 - 4.54}{0.85} \right) = -0.64 \), Reliability: \( R = 1 - y = 0.7389 \).

By calculation, the reliability of the rope winding system at \( 10^4 \) repeats is 73.89%.

5.3. Optimized design

According to the fatigue life analysis and reliability analysis results of the winding rope system, a reasonable size structure of the winding rope system is designed. That is, the diameter of the worm shaft is 16mm, the support height of the frame is 85mm, and the width of the frame is 40mm. The fatigue life after optimization is \( 4.79 \times 10^4 \) repeats, which effectively improves the fatigue life of the winding rope system.

6. Conclusion

Combining transient dynamics and n Code Design Life, the fatigue reliability analysis of the winding rope system was carried out, and the following conclusions were obtained:

(1) According to the movement process of the winding rope system, based on the Jeffcott rotor model, the lumped mass method and the Lagrangain method were used to establish the dynamic model
and dynamic equation, and the dynamic simulation block diagram was established using Simulink to obtain the displacement response of the system, provided data support for dynamic response analysis.

(2) Comparing the workbench modal analysis results with the theoretical numerical calculation results, the results show that the system error is within 30%, which verifies the accuracy of the dynamic model. The transient dynamic analysis of the winding rope system was carried out, and the maximum stress of the winding rope system in one cycle was less than the allowable stress of the material. It was determined that the fatigue analysis engine should choose the stress fatigue mode to calculate the winding rope for subsequent analysis, provided a prerequisite for subsequent analysis and calculation of the fatigue reliability of the winding rope system.

(3) The fatigue reliability analysis of the winding rope system was carried out. The workbench's n Code Design Life module was used to perform fatigue analysis on the winding rope system, and the fatigue life distribution of the winding rope was obtained. The Six Sigma module of Workbench was used to analyse the structural reliability of the winding rope system, and the probability density distribution functions of fatigue life, stress and strain were obtained. The reliability of the winding rope system was calculated at different speeds, and the structure of the winding rope system was optimized. The results showed that the optimized life of the winding rope system is significantly improved, which can provide a certain reference for the structural optimization of the winding rope system.

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