Microscopic study of the compound nucleus formation in cold-fusion reactions

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The understanding of the fusion probability is of particular importance to reveal the mechanism of producing superheavy elements. We present a microscopic study of the compound nucleus formation by combining time-dependent density functional theory, coupled-channels approach, and dynamical diffusion models. The fusion probability and compound nucleus formation cross sections for cold-fusion reactions of projectiles with 48Ca, 208Pb, and 54Cr and targets with 208Pb and 209Bi are investigated and it is found that the deduced capture barriers, capture cross sections for these reactions are consistent with experimental data. Above the capture barrier, our calculations reproduce the measured fusion probability reasonably well. Our studies demonstrate that the restrictions from the microscopic dynamic theory improve the predictive power of the coupled-channels and diffusion calculations.

I. INTRODUCTION

The study on the superheavy elements (SHEs) is one of the most important topics in nuclear physics nowadays [1–4]. The cross section of fusion-evaporation reactions for producing superheavy nuclei (SHN) is extremely small, in the order of 10−36 cm² and strongly dependent on the combination of two colliding nuclei and the incident energy. Up to now, SHEs up to Z = 118 have been synthesized by employing two types of fusion reactions in the laboratory: the cold-fusion reactions with 48Ca and 208Pb and 209Bi as targets [1,5] and hot-fusion ones between 48Ca and actinide nuclei [6,7]. The former leads to the discoveries of new elements up to Z = 113 and the later for SHN with 113 ≤ Z ≤ 118 so far. Lots of efforts have been made to produce SHN with Z = 119 and Z = 120 [8–12], but there is no evidence for the synthesis of the new elements.

It is significant to understand the reaction dynamics to choose the appropriate combination of projectile and target nuclei to produce new SHN. Conceptually, the fusion-evaporation reaction can be divided into three steps, namely the capture process, the fusion process, and the de-excitation of the excited compound nucleus (CN) against fission and light particle emission. The capture process is generally treated as a one-dimensional quantum tunneling under a given ion-ion potential with considering the couplings to the excitations of projectile and target nuclei [13,14] and the results from semi-empirical systematic [15–17] and coupled-channels calculations [13] are well consistent with measurements [18–21]. For the de-excitation process, the survival probability of CN is well studied with the statistical models [16,22] and the dependence on reaction parameter is also well understood, although it is strongly influenced by the fission barrier height. Therefore, most of theoretical approaches used to estimate the evaporation-residue cross sections have similar conclusions on these two steps [15,16,23–33].

However, the fusion process is still not well understood up to now. The calculated fusion probabilities with empirical formulae, master equations, or diffusion models differ by several orders of magnitude [15,22,25,27,33–37]. Even more, the dependence of the fusion probability on the reaction entrance channel is not well established [35,36,38]. Experimentally, the fusion probability can be extracted from the measurement of fusion-evaporation residue cross sections [39,40], comparing the width of fragment mass distribution with the width expected in the case of pure fusion-fission [21,41,42], or the analysis of the fragment angular distribution [43–45]. Recent years, many efforts have been made to measure the fusion probability and lots of progresses have been achieved [46]. Very recently, experimentalists have extracted the fusion probabilities for both cold-fusion and hot-fusion reactions [21,35,45]. These measurements provide new constraints to theoretical investigations for the synthesis of SHN.

It has been shown that the macroscopic models can reproduce the cross section data, but a lot of adjustable parameters are involved. In addition, the lack of dynamical effect challenges its predictive power for reactions without available experimental data. Modern dynamical microscopic approaches can provide insight into the low-energy heavy-ion collisions [47–50]. The time-dependent Hartree-Fock (TDHF) approach has been successfully applied to study many aspects of low-energy heavy-ion collisions (see Refs. [51–55] and references therein). Based on a mass of TDHF simulations, the fusion probability has been estimated by using the sharp cut-off approximation for 48Ca+239,244Pu at several selected incident energies [56]. However, this method is restricted for a systematic study on fusion probabilities due to its computational cost. It has been shown that the TDHF simulations can provide the main ingredients of coupled-channels calculations for capture cross section [57,58] and diffusion processes [59]. Note that the TDHF approach is not capable for the whole process of fusion-evaporation reactions. The purpose of the present work is to combine TDHF with both the coupled-channels and dynamic diffusion approaches, aiming the study of fusion probability systematically. In this approach, one can eliminate the uncertainties of adjustable parameters for calculating capture and fusion cross sections under the restriction from microscopic TDHF theory, meanwhile the influences of the structures of reactants and dynamical effects can be taken into account. In this work, we use this method to study the systematics of the fusion probability of cold-fusion reactions. This article is organized as follows. In Sec. II, we show the main theoretical formulation to calculate capture cross sections and
fission probabilities. Section III presents the calculational details and the discussion of results. A summary and perspective is provided in Sec. IV.

II. THEORETICAL FRAMEWORK

In the TDHF theory, the Hamiltonian $\hat{H}$ is a functional of densities and the dynamic process is described by the evolution of the one-body density $\hat{\rho}$, which is obtained by solving the TDHF equation

$$i\hbar \frac{\partial}{\partial t} \hat{\rho} = [\hat{H}[\hat{\rho}], \hat{\rho}]. \quad (1)$$

Since the TDHF theory describes the collective motion in a semiclassical way, the quantum tunneling of the many-body wave function is not included. Therefore when studying capture cross sections by using the TDHF theory, a commonly applied and very effective strategy is using the ion-ion potential obtained from frozen HF [60], density constrained (DC) TDHF [47,49,57,58,61–66], DC frozen HF [67], dissipative-dynamics TDHF [50], the Thomas-Fermi approximation [68], and the Woods-Saxon (WS) potential with the fitted parameters [57] as the input of the coupled-channels code CCFULL [69] to calculate the penetration probability. In our approach, for a given colliding system, we perform TDHF calculations to determine the capture barrier, which is minimum incident energy that the projectile can be captured by the target in TDHF simulation. Then the calculated densities in their ground states and the obtained capture barriers are used to fix the parameters of the WS potential [70]

$$V(R) = -\frac{V_0}{1 + \exp \left(\frac{R-r_{\text{diff}}A_p^{1/3} - r_{\text{diff}}A_T^{1/3}}{\alpha} \right)}, \quad (2)$$

with the depth $V_0$, the diffuseness parameter $\alpha$, radius parameter for target (projectile) $r_{\text{diff}} (r_{\text{diff}})$, and the mass number $A_T (A_p)$ of target (projectile). In this work, the diffuseness parameter is determined by

$$\alpha = \frac{1}{1.17 \left[1 + 0.53 \left(A_p^{1/3} + A_T^{1/3}\right)\right]} \text{ fm}, \quad (3)$$

taken from Ref. [17] and the determination of $V_0$, $r_{\text{diff}}$, and $r_{\text{diff}}$ will be introduced later. Considering the coupling to low-lying states of projectile and target nuclei, the capture cross sections are obtained by using the standard coupled-channels calculations [69] and read

$$\sigma_{\text{cap}}(E_{\text{c.m.}}) = \frac{\pi}{k^2} \sum_J (2J+1) T_J(E_{\text{c.m.}}), \quad (4)$$

where $k = \sqrt{2\mu E_{\text{c.m.}}} / \hbar^2$ with $\mu$ being the reduced mass in the entrance channel. $T_J(E_{\text{c.m.}})$, which is calculated by using the incoming wave boundary condition method [69], is the penetration probability for given incident energy $E_{\text{c.m.}}$ and angular momentum $J$. It should be noted that one can also use the DC-TDHF method to extract the microscopic internuclear potentials, but which needs much computational cost [64] for the reactions to produce SHN and the obtained potential depends on the incident energy. For a systematic study of fusion cross sections of cold-fusion reactions, the computational cost of the DC-TDHF is too high.

For the second stage of the fusion-evaporation reactions, the contact configuration of touching nuclei can be transformed to CN configuration by overcoming the inner barrier. In this process, before the formation of CN, quasifission happens and the heavy nuclear system is split into two fragments. Therefore, the formation of CN is strongly hindered by quasifission, which is dependent on the structure of the reactants and has also been studied by using the TDHF approach [56,64,71–74]. In the fusion-by-diffusion (FbD) model [15,29,75,76], the formation of CN is idealized as a one-dimensional diffusion over a parabolic barrier and the corresponding probability is totally determined by the distance between the surfaces of two colliding nuclei at the injection point. This quantity can be estimated from the TDHF evolution by choosing the moment when the collective kinetic energy is completely dissipated into the internal degrees of freedom in the over-damped regime for a given incident energy, see Ref. [59] for details. It should be mentioned that in Ref. [59] only $s$-wave scattering are considered and the penetration possibilities is obtained for one or two selected incident energies at above-barrier region. In our approach, the fusion probability $P_{\text{CN}}(E_{\text{c.m.}}, J)$ at each angular momentum and incident energy is obtained by using the formulas given in Ref. [29].

The CN formation cross section is

$$\sigma_{\text{fus}}(E_{\text{c.m.}}) = \frac{\pi}{k^2} \sum_J (2J+1) T_J(E_{\text{c.m.}}) P_{\text{CN}}(E_{\text{c.m.}}, J). \quad (5)$$

Thus, one can study the systematics of the fusion probability by comparing the effective fusion probabilities $P_{\text{fus}}$ defined as $\sigma_{\text{fus}}/\sigma_{\text{cap}}$ with measured ones.

III. RESULTS AND DISCUSSIONS

In a recent experiment [21], the upper limits of fusion probabilities for cold-fusion reactions of $^{48}$Ca, $^{50}$Ti, and $^{54}$Cr with $^{208}$Pb have been extracted systematically. Therefore it is quiet interesting to examine whether our approach is valid for these reactions. The time evolution of each reaction system is obtained by using the modified version of the Sky3D code [77], which was also used to perform calculations in Refs. [56,58,65,66,78–82]. Recently, the density functional SLy5 [83] has been adopted in many investigations [47,56,58,65,66,81,82,84] and also used in present work. The ground states of $^{48}$Ca, $^{50}$Ti, $^{54}$Cr, and $^{208}$Pb are obtained by solving the static HF equation on a three-dimensional grid $28 \times 28 \times 28$ fm$^3$. To get a spherical ground state, the proton orbital $1f_{7/2}$ is fully filled with the equal weight for $^{50}$Ti and the proton orbital $1f_{7/2}$ and neutron orbital $2p_{3/2}$ are equally occupied for $^{54}$Cr. We mention that $^{50}$Ti is spherical due to the magic number $N = 28$ and $^{54}$Cr has a prolate shape with
quadrupole deformation parameter of 0.21 in static HF calculations. Thus this filling approximation is suitable for $^{50}$Ti. Since in this work we disregard the orientation effects of the projectile, $^{54}$Cr is also set to be spherical by the same strategy used in Ref. [59]. The dynamic evolution for these three reactions are performed in a three-dimensional grid with the size of $60 \times 40 \times 40$ fm$^3$. The grid spacing in each direction is taken to be 1 fm and the time step is 0.2 fm/c. All the numerical conditions have been checked for achieving a good numerical accuracy for all the cases studied here.

To get the Coulomb barrier of the each reaction system, we perform TDHF simulations of central collisions at different incident energies and around the barrier the step of the incident energy is taken to be 0.2 MeV. In Fig. 1, we show the time evolution of the separation distance between the fragments for $^{48}$Ca+$^{208}$Pb central collisions at several selected incident energies. This distance is determined by using the standard TDHF approach of finding left and right dividing planes and is the separation distance between the centers of the density in these two halves [47,77]. It should be mentioned that the time period 1300 fm/c is enough for these systems to come over the Coulomb barrier and to form the contact configuration, i.e., the projectile captured by the target. If one wants to get the fusion threshold energy or to judge quasifission, a much longer time TDHF simulation is needed [85]. It is clear that when $E_{\text{c.m.}} \geq 173.3$ MeV, capture happens for $^{48}$Ca+$^{208}$Pb, in which the collective kinetic energy can be entirely converted into the internal excitation of the contacting system. Thus, the capture threshold energy $V^\text{TDHF}_b$ for this reaction is 173.3 MeV. The first moment corresponding to a zero conjugate momentum of $R$ at each reaction energy is represented by the open triangles in Fig. 1. We also perform similar calculations for $^{50}$Ti+$^{208}$Pb and $^{54}$Cr+$^{208}$Pb and the corresponding threshold energies are 191.4 MeV and 209.4 MeV, respectively. The Coulomb barrier for these three reactions obtained from such TDHF calculations reasonably agree with those deduced from experimental capture cross sections by using a classical barrier-passing model for fissionlike cross sections, 173.4 $\pm$ 0.1, 192.6 $\pm$ 0.1, and 207.3 $\pm$ 0.3 MeV for $^{48}$Ca, $^{50}$Ti, and $^{54}$Cr, respectively [21].

The radius parameters of the WS potential can be obtained from the ground-state density in static HF calculations. Taking $^{208}$Pb as an example, the radius from the center of nucleus to the isosurface with half of the saturation density ($\rho_0 = 0.16$ fm$^{-3}$) is 6.672 fm. Then its radius parameter is $R_0 = 6.67 \times 208^{1/3}$ fm = 1.126 fm. Similarly, we have $R_{0Pb} = 1.61$ fm, 1.088 fm, and 1.687 fm for $^{48}$Ca, $^{50}$Ti, and $^{54}$Cr, respectively. The depth $V_0$ can be adjusted for reproducing the capture thresholds from TDHF calculations and they are 165.42 MeV, 251.998 MeV, and 225.07 MeV for reactions $^{48}$Ca+$^{208}$Pb, $^{50}$Ti+$^{208}$Pb, and $^{54}$Cr+$^{208}$Pb, respectively. It should be noted that the low-lying collective vibrations can also be estimated by the TDHF evolution with external fields. But for the systems in question, the low-lying spectra of these nuclei are well known. Therefore, we use the fitted WS potentials together with the experimental data of excitation energies and deformations taken to be the same as those provided in the supplement of Ref. [21], to calculate the capture cross section with the code CCFULL. Finally, the calculated capture cross sections $\sigma_{\text{cap}}$ are shown in Fig. 2 and compared with experimental data and the results from the ECC model [17]. As we can see, for each reaction, our calculations are in agreement with the ECC results and reproduce the experimental data reasonably well.

In the recent work [21], the fissionlike cross sections are measured and also shown in Fig. 2. The corresponding capture cross sections are obtained by using the CCFULL calculations with the capture barriers from the classical barrier-passing model and experimental low-lying excitation information. It is shown that the capture cross sections are larger than the measured fissionlike cross sections and after scaling with suppression factors $S$, good agreements are achieved. This suppression might be due to the energy dissipation before reaching the Coulomb barrier [87,88]. Therefore, we also multiply capture cross sections in our calculations by the same suppression factors used in Ref. [21] and find scaled cross sections are more consistent with the measurements, shown in Fig. 4. In conclusion, by deducing the parameters involved in coupled-channels calculations from TDHF simulations, the capture cross sections can be well determined for both below- and above-barrier regions. These results demonstrate the effectiveness of our model.

For the calculations of $P_{CN}$, the only input parameter in the FBD model is the injection parameter, which can be estimated by TDHF evolution [59] and is defined as

$$s_{\text{inj}} = R_{\text{min}} - R_P - R_T,$$

where $R_{\text{min}}$ is the distance between two fragments at the injection point. In this work, we choose the first moment with a zero canonical momentum in the TDHF evolution as the injection point. In Fig. 1, we show the injection points labelled by open triangles for $^{48}$Ca+$^{208}$Pb for several selected incident energies. In this way, $R_{\text{min}}$ is determined. The radii of colliding nuclei are estimated by using the empirical for-
mula \( R_i = r_0 A_i^{1/3} \) (\( i = P, T \)) or taken to be the same as the root-mean-square matter radii in static HF calculation, which are 3.56 fm, 3.59 fm, 3.67 fm, and 5.55 fm for \(^{48}\text{Ca}^+, ^{50}\text{Ti}^+, ^{54}\text{Cr}^+, \) and \(^{208}\text{Pb}^+\), respectively. The calculated injection parameters corresponding to the empirical formula of radius are labelled as \( s_{\text{inj}}^I \) and \( s_{\text{inj}}^II \) for those with radii from static HF results. The values of \( s_{\text{inj}} \) from the empirical formula used in the FdD model \([29,89]\) are also shown for comparison. When \( E_{\text{c.m}} - V_B^{\text{TDHF}} > 4.59 \text{ MeV} \), \( s_{\text{inj}} \) from the empirical formula given in Ref. \([89]\) is taken to be zero. The shaded area in Fig. 3 means a derivation of \( \pm 1 \text{ fm} \) for the formula in Ref. \([89]\).

In TDHF calculations, the contact configuration can only be reached in above-barrier reactions while the elastic scattering happens in below-barrier ones. This leads to a sudden change of \( s_{\text{inj}} \). It is found that for below-barrier region, most values of \( s_{\text{inj}} \) are located in the shaded area while \( s_{\text{inj}}^I \) gives a much larger distance at the injection points compared with the empirical formula in FbD model. Above the barrier, the values of \( s_{\text{inj}}^I \) are negative and most of them are well located at the shaded area, indicating that the overlap of the densities for two colliding nuclei is very large and resulting in a relatively large \( P_{CN} \). For \( s_{\text{inj}}^I \), all of them are positive, indicating a small overlap of the densities and smaller \( P_{CN} \) compared with these for \( s_{\text{inj}}^I \). Further more, \( s_{\text{inj}} \) extracted from TDHF calculations and the excess of incident energy also hold a linear relation in below-barrier region, which is consistent with that fitted to experimental data shown in FbD model \([15,29,30,32,89,90]\).

In above-barrier region, most values of \( s_{\text{inj}}^I \) and \( s_{\text{inj}}^II \) are close to \( -0.6 \text{ fm} \) and \( 1 \text{ fm} \), respectively, while the closest distance is usually taken to be 0 in FdD calculations. In our method, \( s_{\text{inj}} \) is no longer an adjustable parameter in FbD model, thus eliminating the uncertainties of the fusion cross section originated from \( s_{\text{inj}} \). Generally, the energy dependence behavior of \( s_{\text{inj}} \) becomes weaker in above-barrier collisions compared with those in below-barrier region, i.e., the linear relation holding in below-barrier region disappears in above-barrier collisions. This energy dependence behavior and the relevance with entrance channel need to be explored further.

![FIG. 2. Calculated capture cross sections for \(^{48}\text{Ca}^+\text{Pb}\) (a), \(^{50}\text{Ti}^+\text{Pb}\) (b), and \(^{54}\text{Cr}^+\text{Pb}\) (c). The calculated values with the ECC model \([17]\) and the experimental data taken from Bock 1982: \([86]\), Prokhorova 2008: \([20]\), Pacheco 1992: \([19]\), Clerc 1984: \([18]\), Naik 2007: \([35]\), and Banerjee 2019: \([21]\) are shown for comparison. The capture thresholds are labelled by the up arrows for each reaction. The scaled cross sections are shown by the red dashed lines.](image)

![FIG. 3. The injection parameters \( s_{\text{inj}}^I \) (stars) and \( s_{\text{inj}}^II \) (squares) for reactions \(^{48}\text{Ca}^+\text{Pb}\) (green), \(^{50}\text{Ti}^+\text{Pb}\) (red), and \(^{54}\text{Cr}^+\text{Pb}\) (black) obtained from TDHF simulations. \( s_{\text{inj}} \) from the empirical formulae given in Refs. \([89]\) and \([29]\) are labelled by black solid and dashed lines, respectively. The shaded area represents an error corridor of \( \pm 1 \text{ fm} \) for the formulæ given in Ref. \([89]\) and is drawn to guide the eye.](image)

After obtaining the injection points, by using formulæ given in Ref. \([29]\) with the nuclear data tables in Ref. \([91]\) as inputs, we calculate the fusion probabilities \( P_{CN}(E_{\text{c.m}}, J) \) and fusion cross sections \( \sigma_{\text{fus}} \). In order to compare with the upper limits of measured fusion probabilities \( P_{\text{sym}} \) \([21]\), we calculate the effective fusion probabilities \( P_{\text{fus}} \), defined as the ratio between fusion cross sections and capture cross sections in our model, which is independent on angular momenta. The upper panel of Fig. 4 shows the comparison between calcu-
lateral $P_{\text{fus}}$ and $P_{\text{sym}}$ taken from Ref. [21]. For $^{50}\text{Ti}+^{208}\text{Pb}$, the measurements given in Ref. [35] are presented by black triangles and shown in Fig. 4(b). The discontinuity of calculated $P_{\text{fus}}$ around the barrier is due to the sudden change of $s_{\text{inj}}$. Compared with experimental data in below-barrier region, our calculated results are smaller than $P_{\text{sym}}$ obviously. It is found that our calculations with $s_{\text{inj}}$ overestimate the experimental data while the results with $s_{\text{inj}}^{II}$ well reproduce the measurements in above-barrier region. The differences of $P_{\text{fus}}$ between the calculations with $s_{\text{inj}}$ and $s_{\text{inj}}^{II}$ become larger from $^{48}\text{Ca}$ to $^{54}\text{Cr}$, because the height of the inner barrier is more sensitive to the injection parameter with the increase of the charge number of CN [15]. Our calculations demonstrate that a precise determination of $s_{\text{inj}}$ is necessary for achieving a reasonable description of fusion probability and our method can well reproduce the data of above-barrier collisions.

By using the experimental data of fissionlike cross section $\sigma_{\text{fus}}$ and the upper limit of fusion probability $P_{\text{sym}}$ given in Ref. [21], the experimental fusion cross section can be estimated as $\sigma_{\text{fus}}/P_{\text{sym}}/S$, where $S$ is the suppression factor used in Ref. [21]. We compare our results of fusion cross section $\sigma_{\text{fus}}$ (cf. Eq. 5) with those deduced from measurements [21] in the bottom panel of Fig. 4, although they are not fully equivalent. For the region $E_{\text{c.m.}} < V_B^{\text{TDHF}} + 2$ MeV, the calculations with both $s_{\text{inj}}$ and $s_{\text{inj}}^{II}$ are smaller than the data about an order of magnitude while for other region, a good agreement with is achieved by the calculations with $s_{\text{inj}}^{II}$. Generally speaking, $P_{\text{fus}}$ and $\sigma_{\text{fus}}$ of below-barrier collisions are not well reproduced but for above-barrier ones, our calculations with $s_{\text{inj}}^{II}$ are well consistent with data. Among these three reaction systems, the differences between experimental data and our calculations with $s_{\text{inj}}^{II}$ are largest for $^{54}\text{Cr}+^{208}\text{Pb}$. This might be due to that static deformation effects and dynamic pairing are not included in the present investigation. In addition, during the TDHF evolution, the dynamic changes of the shapes for the colliding nuclei influence the radii $R_T$ and $R_P$ at the injection points, leading to changes of injection parameters that affect fusion probabilities further. In this work, we use the fitted WS potentials that assumes frozen shapes, its combination with the threshold energy deduced from TDHF might also be a source of discrepancy.

![Figure 4](image_url)

**FIG. 4.** The effective fusion probabilities (the upper panel) and the fusion cross sections (the bottom panel) for $^{48}\text{Ca}+^{208}\text{Pb}$ (a, d), $^{50}\text{Ti}+^{208}\text{Pb}$ (b, c), and $^{54}\text{Cr}+^{208}\text{Pb}$ (e, f). Red circles (blue triangles) represent the calculated results with $s_{\text{inj}}^{I}$ ($s_{\text{inj}}^{II}$). The capture thresholds are labelled by the up arrows for each reaction. Experimental data taken from Banerjee 2019: [21] are shown by solid points with error bars. For $^{50}\text{Ti}+^{208}\text{Pb}$, the measurements from Naik 2007: [35] are presented by black triangles.

IV. SUMMARY AND PERSPECTIVE

We present a microscopic calculations of the fusion probability and the CN formation cross section by combining the TDHF with the coupled-channels approach and the FbD model. The capture cross section is obtained by performing coupled-channels calculations, in which the involved parameters of the WS potential are fixed by using the ground-states properties from the static HF calculations and capture thresholds determined from TDHF simulations. The fusion probability is given by using the FbD model with the only one
input parameter, the injection-point distance, which is estimated by the time evolution of two colliding nuclei with the TDHF approach. We apply our model to the cold-fusion reactions $^{48}$Ca+$^{208}$Pb, $^{50}$Ti+$^{208}$Pb, and $^{54}$Cr+$^{208}$Pb. The dynamic evolution of the central collisions in both below- and above-barrier regions are obtained with the effective interaction SLy5. It is found that the capture thresholds from TDHF calculations are in line with those extracted from measurements and the capture cross sections can well reproduce the experimental data. By estimating the injection points with the TDHF approach, the fusion probabilities and resulted CN formation cross sections agree with experiments reasonably.

In the present work, the ground states of the target and projectile nuclei are all spherical or restricted to be spherical. For hot-fusion reactions, it is necessary to take static deformation into account since most actinide nuclei are deformed in their ground states and the orientation in the entrance channel also affects the TDHF dynamic evolution, the capture cross section, and the injection point. The orientation effects can be taken into account in both capture and fusion processes by rotating the initial wave function of reactants in TDHF simulation. This generalization is our next step. Additionally, it should be mentioned that the dynamic process of colliding nuclei is very complicated and the formation of CN from contact configuration undergoes complex evolution of shape degrees of freedom. This challenges the definition of the surfaces for two colliding nuclei. Finally, we hope that our microscopic approach can provide new and reliable supports for choosing the optimal combination of target and projectile nuclei for the synthesis of the SHEs with $Z = 119$ and $Z = 120$ in the future.

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