Flexural Behavior of Fiber-Reinforced SCC for Monolithic and Composite Beams

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Abstract

Reinforced beams with deteriorated concrete can be repaired by replacing the deteriorated part or the entire beam using new concrete. Either of the two scenarios is decided to be used based on the degree and distribution of deterioration along the beam. This paper compares the structural performance of composite and monolithic reinforced concrete beams which represents the two cases of partial and entire replacement of concrete, respectively. In total, 10 beams with dimensions of 3200×250×400 mm (L×W×D) were constructed. The reference monolithic beams were cast with either self-consolidating concrete (SCC) or fiber-reinforced self-consolidating concrete (FR-SCC). Composite beams were cast with conventional vibrated concrete (CVC) to a depth of 275 mm from the top then with either SCC or FR-SCC for the remaining third of the beam’s depth (125 mm) at the bottom. The composite beams were prepared to simulate beams repaired in the tension zone after the removal of the deteriorated concrete. The test variables were fiber inclusion, fiber type, and beam type. One hybrid, one steel, and two polypropylene fiber types were employed in the FR-SCC. All fiber types were added at 0.5% by volume. The beams were simply supported and were loaded in four-point bending. Test findings indicate that both composite and monolithic beams exhibited similar cracking patterns at failure. However, the crack width of composite beams was lower due to enhanced fiber orientation along the tension zone and concrete confinement during the casting process. The structural performance of the beams was found to be mainly governed by mechanical characteristics of the fibers in the case of monolithic beams and mainly by the fiber length in the case of composite beams where fibers take preferential orientation during casting in the repair zone.

1. Introduction

Repaired structural elements, either exposed to excessive loading or insufficiently engineered, may undergo cracking and degradation that can accelerate the ingress of deleterious materials that can reduce durability and service life of the repair and the structure. The selection of proper repair materials capable of enhancing the serviceability of structural elements and restoring the structural stiffness and load carrying capacity, as well as cracking resistance and durability is of prime importance. Repair materials should also be selected to secure adequate workability to facilitate placement and consolidation as repairs can involve the casting of concrete in restricted access and confined consolidation; for example, the repair of reinforced concrete beams in parking structures damaged by corrosion of the reinforcing steel. The repair materials would achieve adequate compatibility of mechanical and visco-elastic properties with the concrete substrate (Morgan 1996), and secure high durability and resistance to cracking, including that due to restrained shrinkage (Hwang and Khayat 2008).

Self-consolidating concrete (SCC) and fiber-reinforced SCC (FR-SCC) can develop many of these characteristics and have been successfully employed in repair applications. The findings of previous studies indicated that FR-SCC was able to flow under its own weight replacing the deteriorated concrete without showing any defects due to segregation and blockage (Arezoumandi et al. 2018; Kassimi et al. 2014). According to the ACI 237 (2007), SCC can be defined as “highly flowable, non-segregating concrete that can spread into place, fill the formwork, and encapsulate the reinforcement without any mechanical consolidation.” The use of SCC in repair can greatly facilitate construction operations, especially in confined sections and restricted areas. Properly design SCC can develop high durability with homogeneous quality and can reduce labor costs and eliminate consolidation noise at the jobsites (Khayat 1999). Due to these characteristics, the use of SCC has increased worldwide during the last decades, including the use in rehabilitation of concrete structures.

The use of fibers can extend the SCC’s possible field of applications (Grünewald and Walraven 2001). The use of fibers can enhance the resistance of the SCC to restrained shrinkage cracking, hence extending the service life of the repair (Kassimi and Khayat 2019). Com-
bination of SCC and fiber reinforcement resulting in FR-SCC can be used in the engineering applications, including repair of concrete infrastructures even for those where the presence of narrow sections is preponderant. The repair is generally carried out in loaded parts, mostly loaded in tension zones. In such sections, high fluidity and high resistance to cracking with adequate mechanical properties and ductility are required for successful repair. Properties of the new class of repair materials (SCC and FR-SCC) engender the singular properties of the individual components, leading hence to synergistic effect of the various constituent material properties.

The effect of the fiber inclusion and characteristics on fresh and hardened concrete has been investigated (Khayat and Roussel 2000; Khayat et al. 2014). For example, the workability of FR-SCC has been shown to closely depend on fiber volume ($V_f$), fiber length ($L_f$), fiber aspect ratio ($L_f/d_f$) where $d_f$ is fiber diameter, and fiber factor ($V_f^* L_f/d_f$) as well as shape, and surface roughness (Khayat et al. 2014; Nehdi and Ladancush 2004; Šahmaran et al. 2005). The increase in $L_f$, $V_f$, $L_f/d_f$ (slender or long and thin fibers), and $V_f^* L_f/d_f$ can hinder the filling ability and passing ability of the FR-SCC and increase the viscosity of the mixture (Groth and Nemegeer 1999). Fibers can change rheological properties of FR-SCC with interaction of fiber-aggregate (Khayat and Roussel 2000; Šahmaran et al. 2005). The $L_f$ and nominal size of aggregate and aggregate volume should be decreased to reduce the internal resistance to flow and increase workability (Khayat and Roussel 2000). However, an excessive reduction in $L_f$ can result in loss of pull-out resistance for fibers loaded in tension.

Incorporation of fibers affects the fresh air content and high-range water-reducing admixture (HRWRA) demand. The effect of fibers on workability also depends on the concrete consistency and water-to-cement ratio. In order to avoid blockage and segregation of the FR-SCC, a minimum reinforcing bar spacing should be used depending on $L_f/d_f$, $V_f$, and $L_f$ (Khayat et al. 2014). The segregation and blockage behind the reinforcement in FR-SCC made with relatively high $V_f$ can be remedied using appropriate mixture design. For example, Khayat et al. (2014) developed an approach for designing FR-SCC of different fiber types. The design requires adjustment in the fine and coarse aggregate contents and dosage of the HRWRA and air-entraining admixture (AEA). Mixture adjustment depends on key fiber characteristics ($L_f$, $V_f$, $d_f$, and fiber specific gravity) and coarse aggregate characteristics (specific gravity, grain size distribution, and content).

In the hardened state, the addition of fibers can significantly enhance many of the engineering responses of the cementitious materials, such as flexural and fatigue strengths, impact resistance, and toughness. The use of fibers can also enhance flexural stiffness, deformation (Swamy and Al-Ta’an 1981), shrinkage, shear resistance, and can partially or totally replace steel reinforcement in some applications (Ning et al. 2015). The most common role is reduction of cracking potential. These enhancements rely on the fiber properties. For example, the flexural toughness of the cementitious materials can be affected by the fiber type and form and can be enhanced by increasing $L_f$, $L_f/d_f$, and $V_f$ (ACI 544 2018; Gao et al. 1997; Zhang et al. 1997). The flexural strength is affected by the fiber factor, fiber form, and properties of the concrete matrix (Gao et al. 1997; Zhang et al. 1997). However, flexural strength is independent from $L_f$ (Dreux and Festa 1998). On the other hand, steel fibers enhance the flexural strength (Zhang et al. 2018), whereas the polypropylene (PP) fibers do not (Zhang et al. 1997). Flexural strength is also enhanced by $V_f$ that improves bond between the matrix and embedded fibers (Mueller and Holschemacher 2009). The bond quality depends on the mechanical and geometrical properties of fibers (Zhang et al. 2018).

Increasing $V_f$ can increase the capacity of pre-crack strain, load corresponding to first crack, post-crack load and behavior, ductility, and energy absorption, thus dissipating stress in tension through cracking of the concrete (ACI 544 2018; Khayat and Roussel 2000). Sufficient $V_f$ can enhance notably the cracking resistance and reduce crack width (Shah et al. 1998). With PP fibers, the crack load can be slightly influenced by $V_f$, and the crack width can be reduced with the increase in $L_f$, the crack load depends on the properties of the concrete matrix (Zhang et al. 1997).

It was established that high structural performance can be obtained using hooked-end, deformed-end, crimped, and straight-end fibers, in descending order (ACI 544 2018). The performance of FR-SCC depends also on the casting method, concrete composition, and rheological properties of the FR-SCC matrix. The latter is particularly important in narrow sections where concrete flow in relatively thin sections and resulting shear profile in confined sections can enhance fiber alignment and distribution, but also increase the risk of segregation and blockage, which have marked influence on concrete performance (Grünewald 2004).

In previous work (Kassimi et al. 2014), new technique for repairing reinforced concrete beams was validated. This technique was adapted to develop FR-SCC made with different fiber types. The structural performance of these beams in flexure was evaluated. The work reported here evaluates and compares the flexural performance of slender monolithic beams that can represent a total reconstruction of damaged structural elements, with composite beams that represent repaired sections. The composite beams represent the partial element reconstruction with conventional vibrated concrete (CVC) with the bottom sections cast using SCC or FR-SCC repair material. Understanding the behavior of beams cast totally using FR-SCC of different fiber types and those of composite beams made with the same fiber type and content is of particular interest in this study. This knowledge can contribute to selecting appropriate...
fiber types for construction and repair applications and determining the impact of each fiber type on key parameters of the fresh and hardened concrete.

2. Research significance

Depending on the degree of degradation, reinforced beams with deteriorated concrete are generally repaired by either replacing the deteriorated part or replacing the entire beam using adequate materials. These materials should have high mechanical properties and contribute to restore the initial load capacity. A combination of SCC and fibers can provide adequate solution for repairing and upgrading damaged concrete elements. This paper intends to present briefly the fresh and hardened properties of the used combined material of fiber reinforced SCC. The feasibility of using this combination for repairing reinforced concrete beams is assessed by investigating the structural behavior of such beams. The beams included two sets: composite and monolithic specimens representing the partial and full replacement of deteriorated concrete. The behavior of the two sets of beams is examined and compared.

3. Experimental program

3.1 Beam details

In total, 10 full-scale beam elements measuring 3200×250×400 mm (L×W×D) were cast. The test parameters included the concrete type (FR-SCC vs. SCC), fiber type (four fiber types), and beam type (monolithic vs. composite). The testing program included five monolithic beams made of plain SCC and FR-SCC with the four fiber types. Five corresponding composite beams were cast with CVC to a depth of 275 mm, and the remaining 125 mm of the section from the bottom was cast with the SCC or FR-SCC repair materials, as shown in Fig. 1.

The monolithic and composite beams were reinforced with two deformed reinforcing steel bars of No. 20M (d₀ = 19.5 mm) for tensile reinforcement and two deformed reinforcing steel bars of No. 10M (d₀ = 11.3 mm) as the top reinforcement in the compression zone. Double-legged stirrups of 8 mm in diameter spaced at 150 mm were used as shear reinforcement. The tested values of their yield strengths were 400, 450, and 400 MPa, respectively, and their elastic modulus was 200 GPa. The concrete cover on all sides of the beam cross section was fixed at 40 mm. The reinforcement ratio of all the tested beams was 0.7%. The beams were under reinforced (ρ/ρ₀ < 1) to highlight the effect of fibers on structural behavior.

3.2 Materials properties

Continuously-graded natural sand was used for all mixtures. Crushed limestone aggregate with a nominal maximum size of aggregate (MSA) of 10 mm was used for the SCC and FR-SCC mixtures. A similar aggregate with MSA of 20 mm was used for the upper layer of the composite beams that was cast using CVC. The particle-size distributions of the aggregates are in compliance with CSA A23.1 Standards (2014). Type GU (General Use) cement was used for the CVC. A ternary cement (CSA GUb-S/SF) containing approximately 70% Type GU cement, 25% granulated ground blast furnace slag, and 5% silica fume, by mass of binder, was used for the SCC and FR-SCC mixtures. A polycarboxylate (PCE)-based HRWRA and a compatible biogum-based polysaccharide viscosity-modifying admixture (VMA) were incorporated to enhance fluidity and stability, respec-

Fig. 1 Schematic of a composite beam specimen.
A synthetic resin AEA was used to secure the required air content of 5% to 9% approximately. Four fiber types were used in the FR-SCC mixtures, including monofilament polypropylene (MO) fibers, multifilament polypropylene (MU) fibers, hybrid (HY) steel-polypropylene fibers, and hooked-end steel (ST) fibers. The HY fibers is a pre-packed blend of 92% crimped steel fibers, and 8% micro-multifilament polypropylene fibers by mass. The physico-mechanical properties of the various materials are presented in Table 1.

### Table 1 Physical and mechanical properties of materials.

| Fibers          | HY (steel, PP) | MO      | MU      | ST          |
|-----------------|----------------|---------|---------|-------------|
| Diameter $d_f$, mm | 1.2, 0.05     | 0.44    | 0.67    | 0.55        |
| Elastic modulus, GPa | 203 (steel portion) | 9.5    | 5       | 200         |
| Length $L_f$, mm | 42, 10         | 40      | 50      | 30          |
| Specific gravity | 7.8, 0.91      | 0.92    | 0.92    | 7.85        |
| Tensile strength, MPa | 966 to 1,242   | 620     | 575     | 1,200       |

### Table 2 Concrete mixture proportioning.

| Mixture | V作息/d发 | Water, kg/m³ | Cement, kg/m³ | Sand, kg/m³ | Coarse aggregate, kg/m³ | HRWRA, L/m³ | VMA, ml/m³ | AEA, ml/m³ |
|---------|----------|-------------|--------------|-------------|--------------------------|-------------|-----------|-----------|
| CVC^a   | 0        | 175         | 350^b        | 660         | 1070^c                   | —           | —         | 200       |
| SCC     | 0        | 45.5        | 200          | 781         | 792                      | 3.48        | 128       |
| MO      | 45.5    | 37.3        | 845          | 883         | 673                      | 4.5         |
| MU      | 37.3    | 23.8        | 802          | 845         | 715                      | 7.5         |
| HY      | 23.8    | 27.3        | 813          | 802         | 756                      | 4.65        |
| ST      | 27.3    | —           | 745          | 813         | 745                      | —           |

^a CVC used in the composite beams.
^b Type GU cement for base CVC and Ternary GUb-S/SF for remaining mixtures.
^c 20 mm-MSA type.

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### 3.3 Concrete mixtures

The mixture proportions of the CVC, SCC, and FR-SCC mixtures are reported in Table 2. The self-consolidating repair mixtures were designed to develop a 28-day compressive strength ($f'_c$) of 45 to 50 MPa. The FR-SCC mixtures were proportioned with reduced coarse aggregate content to maintain the same thickness of mortar layer covering the fiber and coarse aggregate, thus developing similar workability levels. This concept was inspired from Voigt et al. (2004) and further developed by Khayat et al. (2014) and validated on full-scale elements by Kassimi et al. (2014).

All the FR-SCC mixtures incorporated fibers at $V_f$ of 0.5%. This volume was optimized in previous investigations (Khayat et al. 2014) and is considered as an upper limit to ensure self-consolidating consistency for FR-SCC in relatively restricted repair sections.

The workability characteristics of the investigated SCC and FR-SCC mixtures were sufficient to ensure excellent filling ability, passing ability, filling capacity, and stability characteristics, which are essential for successful repair (Groth and Nemegeer 1999). The initial slump flow (ASTM C1611) at 10 min was 700 ± 20 mm. The maximum Visual Stability Index (VSI) (ASTM C1611) and surface settlement (Kassimi et al. 2014) at 10 min were 0.5 and 0.37%, respectively, which reflects adequate static stability for the tested SCC and FR-SCC mixtures. Despite the decrease in filling capacity due to fiber inclusion, the caisson filling capacity (Khayat and Roussel 2000) values ranged between 85% and 95% compared to 100% for the plain SCC, which is considered as excellent filling capacity. The detailed results on
the workability tests were presented in a previous work (Kassimi et al. 2014).

### 3.4 Casting procedure

The monolithic beams made with SCC and FR-SCC were cast without any mechanical consolidation and at a constant casting rate from the end A. The concrete flowed along the length of 3.2 m of the monolithic beams until the other end B (Fig. 1). In the composite beams, the CVC layer presenting the existing concrete was cast first. The repair SCC made with and without fibers was cast after a minimum age of 14 days that allowed gaining adequate mechanical strengths and experiencing a great portion of drying shrinkage. For the CVC layer in the composite beams, the cage of the reinforcing bars was inverted in the formwork with the tension reinforcement at the top in order to cast the CVC in a single lift. The CVC was consolidated using an internal vibrator. A set-retarding admixture was applied immediately on the top concrete surface and the beams were then covered with plastic sheet. After 24 hours, water jetting was used to remove the mortar at the treated concrete surface in order to expose some of the aggregate and secure a rough surface texture to enhance bond with the repair concrete. The beams were allowed to cure under moist conditions (under wet burlap covered with plastic sheet) for 2 weeks, then under the normal laboratory conditions until casting of the repair concrete. The beams were then inverted in the formwork to the normal position with the tension reinforcement at the bottom. The second layer representing the repair material (SCC and FR-SCC) was cast without any mechanical consolidation from an access hole located at the leading end A. The casting method and preparation for the flexural tests are described in detail in a previous work by the authors (Kassimi et al. 2014).

### 3.5 Testing procedure

The beams were tested under four-point bending after approximately 180 days of age. The simply supported clear span and the shear span were 2600 and 1050 mm, respectively. The beams were loaded to maintain a mid-span deflection rate of 1.2 mm/min. The loading procedure was stopped immediately after the appearance of the first two flexural cracks. The initial widths of the cracks were measured using a hand-held 50× microscope. Two linear variable displacement transducers (LVDTs) were then installed to continue measurements of the crack widths over the loading procedure using a data acquisition system connected to a computer and a 500 kN capacity closed-loop actuator. The deformation of the concrete in the compression zone and steel reinforcing bars in the tension zone at mid-span were monitored using strain gauges with electrical resistance of 120 Ohms. The strain gauges for concrete and steel reinforcement measured 70 mm and 6 mm, respectively. The deflection at the mid-span of beams and de-bonding at the interface between the CVC and repair concrete were monitored using LVDTs, as illustrated in Figs. 1 and 2.

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Table 3 Mechanical properties results of investigated beams at 180 days.

| Concrete | SCC | SCC | MO | MO | MO | MO | MU | MU | MU | MU | ST | ST |
|----------|-----|-----|----|----|----|----|----|----|----|----|----|----|
| Use      | MB  | TO  | BO | MB | TO  | BO | MB | TO  | BO | MB | TO  | BO |
| $f'_{cc},$ MPa | 56  | 36  | 57 | 55 | 36  | 61 | 61 | 45  | 61 | 61 | 45  | 61 |
| Core     | 0.89 | 0.88 | 1.00 | 0.81 | —  | 1.01 | 0.97 | 0.98 | 0.97 | 0.79 | —  | 1.04 | 0.88 | —  | 1.06 |
| $f'_{cc}$, MPa | 4.8  | 3.9  | 4.6 | 5.6 | 4.9  | 5.7 | 6.4 | 4.9  | 5.7 | 6.4 | 4.9  | 5.7 |
| $E_c$, GPa | 30.0 | 29.5 | 31.1 | 28.0 | 29.5 | 28.0 | 32.0 | 33.5 | 32.0 | 28.0 | 32.0 | 28.0 |
| $f_{ss}$, MPa | 7.7  | —   | 7.8 | —  | 8.0  | —  | 10.0 | 8.1  | —  | 8.3  | —  |

* a MB: monolithic beam; TO: top made of CVC; BO: bottom made of SCC or FR-SCC in composite beams.
* b Compressive strength of cylinder.
* c 100×200-mm concrete cylindrical core samples.
* d $f'_{cc, A}/f'_{cc, cy}$ (ratio of $f'_{cc}$ of core at location A (first end of beam corresponding to concrete placement) to $f'_{cc}$ of cylinder).
* e $f'_{cc, B}/f'_{cc, cy}$ (ratio of $f'_{cc}$ of core at location B (second end of beam) to $f'_{cc}$ of cylinder).
* — means no available data.

Note: The values presented in Table 3 are the average values on three specimens for $f'_{cc}$ and two specimens for the rest of mechanical properties.
Concrete cylinders measuring 100×200 mm were prepared to evaluate $f'_{c}$ (ASTM C39), splitting tensile strength ($f'_{sp}$) (ASTM C496), and elastic modulus ($E_c$) (ASTM C469) at 180 days. Concrete prisms measuring 100×100×400 mm were prepared to evaluate flexural strength ($f_c$) (ASTM C78) at 180 days.

Prism samples used for $f_c$ testing were not consolidated and were demolded after 1 day and transferred to a 100% humidity chamber until the age of testing. The curing method for the $f'_{c}$, $f'_{sp}$, and $E_c$ tests is described by Kassimi et al. (2014). Table 3 depicts the test results on concretes used in the investigated beams in terms of mechanical strengths at the age of flexural tests on beams (180 d). The SCC and FR-SCC mixtures developed $f'_{c}$ values of 56-57 and 53-61 MPa, respectively. The corresponding $f'_{sp}$, $f_c$, and $E_c$ values were 4.6-4.8 and 5.6-6.8 MPa, 7.7 and 7.8-10.1 MPa, and 30-31.1 and 27.5-32 GPa, respectively.

In the case of full-scale composite beam elements, the second layer of repair SCC or FR-SCC was cast after gaining adequate strength of a minimum value of 85% $f'_{c}$, at approximately 14 days. The same curing regime was followed for the CVC specimens prepared during the casting of the base concrete layer. At the conclusion of the beam flexure testing, two cylindrical samples measuring 100 mm in diameter were cored from the A and B ends of the monolithic beams to determine in-situ $f'_{c}$ (ASTM C42). The core strengths were compared with $f'_{c}$ values obtained from cylinders prepared with the beams. For composite beams, additional samples were cored from holes at ends A and B filled with the repair concrete (Fig. 1). The coring was performed after the conclusion of the beam testing to avoid probable alteration of the beam’s body before testing. The core samples were taken from the uncracked concrete after the beam flexure testing, outside the region of the beam clear span (Fig. 1).

### 4. Test results and discussion

The evaluation of the structural performance of the investigated beams is treated in two parts. The first part compares the behavior of beams made with FR-SCC to those precast with SCC and no fibers, regardless of the beam type whether monolithic or composite (effect of fiber inclusion and fiber type). The second part compares the performance of composite beams to that of the monolithic beams (effect of beam type). The investigated structural performance responses included cracking load, yield load, ultimate load, mid-span deflection at service load (90 kN), ductility, crack width at 90 kN, stiffness, mid-span strain in reinforcement and top concrete surface, and cracking patterns at failure of the beams. Table 4 summarizes the structural response of the tested beams. It should be noted that the ultimate load represents the maximum (peak) load value obtained from the flexural test; however, failure corresponds to the beginning of the sudden drop in the load value. The load value at failure can correspond to the ultimate load value and can be lower; therefore, failure does not necessarily correspond to the ultimate load.

#### 4.1 FR-SCC vs. SCC

##### (1) Deflection characteristics

The load-deflection curves of the beams cast or repaired using SCC or FR-SCC are characterized by having tri-linear behavior as presented in Fig. 3. The first part of the load-deflection curves represents the pre-cracking behavior where a sharp slope exists between the loading initiation and the cracking load. Figure 3 shows similar pre-cracking deflection behavior for all beams as it depends on the gross moment of inertia of the concrete cross-section of uncracked beams. The second part of the load-deflection curves corresponds to the post cracking behavior up to the yield load. This part is characterized by progressive cracking and reduction in the moment of inertia and stiffness of the beams. The third part of the load-deflection curves is between yielding and failure of the beams and corresponds to loss of beam stiffness due to steel yielding and propagation and widening of cracks.

Compared to beams made or repaired using SCC, the use of FR-SCC reduced the mid-span deflection by up to 38%, as can be noticed from Table 4. This can be due to the increased stiffness and strength of the FR-SCC mixtures. The Behavior of beams made with FR-SCC showed a higher load-bearing capacity and a lower deflection compared to those made with SCC, indicating better performance in terms of stiffness and strength.

### Table 4 Structural response of monolithic and composite beams.

| Response                  | Monolithic beams | Composite beams |
|---------------------------|------------------|-----------------|
| Cracking load, kN         | SCC MO MU HY ST  | SCC MO MU HY ST |
| 55                        | 58              | 70              | 58               | 60               | 50              | 58              | 59               | 57               | 65               |
| Yield load, kN            | 150             | 167             | 187             | 174              | 192              | 148             | 151              | 171              | 152              | 176              |
| Ultimate load, kN         | 219             | 252             | 242             | 249              | 248              | 203             | 213              | 224              | 204              | 227              |
| Deflection, mm            |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.8              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.4              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 2.8              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.1              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 2.3              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 4.5              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.8              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.6              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 4.3              |
|                           |                 |                 |                 |                  |                 |                 |                  |                  |                  | 3.2              |
| Reinforcement strain μm/m | 1612            | 16521           | 16168           | 16016           | 16327           | 20876           | 16015           | 16520           | 16015           | 16520           |
| Concrete strain μm/m      | -3358           | -3810           | -4212           | -3587           | -3777           | -1605           | -3674           | -2796           | -2704           | -2557           |
| Stiffness, kN/mm          | 8               | 9               | 17              | 9                | 10               | 6               | 9                | 17               | 9               | 7               |
| Crack width mm/μm         | 0.243           | 0.294           | 0.338           | 0.205           | 0.093           | 0.391           | 0.153           | 0.130           | 0.268           | 0.122           |
| Deflection, mm            | 57.4            | 72.0            | 149.6           | 70.3             | 130             | 56.0            | 79.7            | 167.3           | 70.6            | 78.7            |
| Ductility*                | 8               | 9               | 17              | 9                | 10               | 6               | 9                | 17               | 9               | 7               |
| Ductility*                | 7.2             | 7.7             | 8.9             | 7.9              | 7.3             | 9.0             | 8.9              | 9.8              | 9.6              | 9.0              |
| Ductility*                | 57.4            | 72.0            | 149.6           | 70.3             | 130             | 56.0            | 79.7            | 167.3           | 70.6            | 78.7            |
| Stress ratio*             | 7.2             | 7.7             | 8.9             | 7.9              | 7.3             | 9.0             | 8.9              | 9.8              | 9.6              | 9.0              |
| Stress ratio*             | 57.4            | 72.0            | 149.6           | 70.3             | 130             | 56.0            | 79.7            | 167.3           | 70.6            | 78.7            |

* At beam’s mid-span at service load (90 kN). † At yield. ‡ At failure. * Ratio of deflection at failure to deflection at yield.

* Measurements not completed.

Fig. 1 shows similar initiation and the cracking load. The load-deflection curves represent the pre-cracking behavior where a sharp slope exists between the loading initiation and the cracking load. Figure 3 shows similar pre-cracking deflection behavior for all beams as it depends on the gross moment of inertia of the concrete cross-section of uncracked beams. The second part of the load-deflection curves corresponds to the post cracking behavior up to the yield load. This part is characterized by progressive cracking and reduction in the moment of inertia and stiffness of the beams. The third part of the load-deflection curves is between yielding and failure of the beams and corresponds to loss of beam stiffness due to steel yielding and propagation and widening of cracks.

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to the enhanced bond created between the steel reinforcement and the concrete matrix by the confining action and reduced deformations applied by the addition of fibers, as reported by Ning et al. (2015). Table 4 also gives the values of flexural stiffness of the tested beams. These values were calculated based on the slope of the second part of the load-deflection curves of Fig. 3. The stiffness calculation was done in the elastic part of the load-deflection curve comprised between the cracking and yield loads. Figure 3 and Table 4 indicate that the beams cast with steel fibers exhibited stiffer performance and lower deflection compared with the beams cast with plain SCC or with those cast with other fiber types. This is attributed to the higher modulus of elasticity of the steel fibers.

(2) Cracking behavior and mode of failure

Cracks were initiated at the bottom tensile zone for all beams when the cracking moment was attained in the constant moment zone between the two concentrated loads where flexural tensile stresses are higher than the tensile strength of concrete. These cracks were vertical and perpendicular to the direction of the maximum principal tensile stress induced by the pure bending moment. With the increase in loading, the cracks widened, and new cracks started to open up in the region of the shear span. The cracks developed in the shear span became more inclined and propagated towards the loading point due to the dominance of shear stresses. No new cracks appeared after reaching the yielding load; however, the cracks continued to widen until failure. All beams failed in flexure by crushing of the concrete after yielding of the tensile reinforcement. Figure 4 illustrates the typical failure mode while Fig. 5 shows schematically the cracking patterns at failure for typical tested beams.

Figure 6 plots the variation of measured crack width with the applied load for the tested beams. Generally, the use of fibers significantly increased the resistance to cracking and reduced crack width. The crack widths of beams made or repaired with FR-SCC was reduced by up to 69% compared to those made or repaired with SCC (Table 4). This is due to the crack bridging provided by the fibers. Regardless of monolithic or composite, beams cast with steel fibers exhibited the best crack controlling behavior with the narrowest crack widths compared with the beams cast with plain SCC or with those cast with other fiber types. The steel fibers used in this study are characterized by hooked ends that can increase the pulling-out resistance, thus reducing crack widths.

(3) Cracking, yield, and ultimate load capacities

Table 4 reports the values of the cracking, yield, and ultimate loads of the tested beams. All FR-SCC beams showed higher cracking load compared to nonfibrous SCC beams. The increase in cracking load was in the range of 5% to 30% (mean value of 16%), regardless of the beam type (monolithic or composite) and fiber type. The FR-SCC beams also exhibited increases in the yield loads up to 28% over those of nonfibrous SCC beams. The composite beams MO and HY repaired with monofilament polypropylene fibers and hybrid steel-polypropylene fibers, respectively, exhibited increases in the yield load less than 5% than that of the control beam with nonfibrous SCC. This may be attributed to the relatively lower fiber length of these two types of fibers. Table 4 also shows that the ultimate load capacity of the FR-SCC beams was higher than that of the nonfibrous SCC beams. The increase in the ultimate capacity was in the range of 11-15% for the monolithic beams and up to 12% for the composite beams.

Although relatively low content of fiber used in this study (0.5%), the beams made with fibers exhibited higher loading capacities than those of nonfibrous beams. This was attributed to the increase in tensile strength of concrete due to inclusion of fibers. This increased tensile strength enhanced the cracking load. It was also added to tensile strength enabled by the tensile steel bars enhance the yield and ultimate load capacities of the beams. It can be also observed that the beneficial

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**Note:** To easily identify beams, curves of each monolithic beam and corresponding composite beam in Figs. 3, 6, and 7 are plotted with the same color but with continuous and discontinuous lines, respectively. Curves of beams made with SCC without fibers and with the MO, MU, HY, and ST fibers were plotted with black, pink, red, green, and blue colors, respectively, regardless of the beam set (monolithic or composite).

**Fig. 3 Load-deflection response.**

**Fig. 4 Typical flexural failure mode (monolithic beam HY).**
The effect of fibers was more noticeable for cracking and yield loads of the beams in comparison to the ultimate load. This is because after yielding of the reinforcing steel, the cracks dramatically widen causing pulling out of the fibers. Once the fibers are pulled out, they stop contributing to the ultimate capacity.

Based on the three characteristic loads (cracking, yield, and ultimate) that are reported in Table 4, the beams made or repaired with multifilament polypropylene (MU) and steel fibers (ST) generally led to the highest performance. For the multifilament polypropylene fibers, the improvement in performance was most likely due to the length of the fibers, rough surface texture, and forced orientation in the direction of tensile stresses along the flow direction in the thin repair layer, which enhanced the pull-out resistance of the fibers. First, the effect of fiber orientation is more remarkable in SCC than in conventional concrete and with long fibers than short ones (Grünewald, 2004). Second, the smaller the cross-section, the more restricted the possibilities for free orientation of the fibers (Grünewald 2004). Third, the fibers were oriented into the flow direction (Grünewald 2004). For the ST fiber, the improved performance can be due to their highest mechanical properties (modulus and tensile strength) and the hooked ends that can enhance bond and pull-out resistance.

Failure by crushing of concrete in the compressed zone occurred after yielding of the steel reinforcement since all beams were under-reinforced with $\rho/\rho_b$ ratio lower than 1. At the post-cracking stage, the reinforcing steel bars continued reversible elongation until the yield limit, followed by an irreversible elongation.

(4) Strain in reinforcement and concrete
The variation of the strains developed in the steel bars and top concrete surface at mid span of the beams with the applied load are plotted in Fig. 7. The maximum...

![Fig. 5 Cracking pattern at failure of typical monolithic and composite beams.](image)

![Fig. 6 Load-crack width response.](image)
steel elongation was 16500 µstrains. Similar strain behavior can be seen for fibrous and nonfibrous beams. The strain curves were trilinear following the same behavior of the deflection curves shown in Fig. 3. The mean strain values of steel reinforcement and concrete for fibrous beams at failure were 5% and 49%, respectively, over the nonfibrous beams. This means that the incorporation of fibers increased strains of steel reinforcement and concrete and allowed additional time before failure. It should be noted that the measured rebar strain depends on the relationship between the gauge and the crack location. The maximum reinforcement and concrete strains also depend on the gluing condition of the strain gauge.

(5) Ductility
The ductility was calculated as the ratio of the deflection at failure to the deflection at yield. As indicated in Table 4, the ductility of the fibrous beams increased by 17-172% over those of the nonfibrous beams. The ductility was consistent with the strains in concrete and steel reinforcement.

4.2 Monolithic beams vs. composite beams

(1) Deflection characteristics

From Fig. 3, the load-deflection curves of the monolithic and composite beams were trilinear. The three curve parts are discussed earlier. Table 4 shows that the monolithic beams exhibited post-cracking stiffness values of 20% to 44% higher than those of the corresponding composite beams, regardless of the fiber inclusion and type. Figure 3 and Table 4 indicate that the monolithic beams exhibited 11% to 28% (average value of 27%) lower mid-span deflection at the service load compared to their corresponding composite beams. The mid-span deflection values of the service load level for the monolithic and composite beams were 2.3 mm to 3.8 mm and 3.2 mm to 4.5 mm, respectively, as can be seen from Table 4. The maximum permissible computed immediate deflection according to the CSA A23.3 (2014) is limited to L/360, where L is the clear span (L = 2600 mm). Thus, the permissible deflection value for the tested beams is 7.2 mm, which is quite higher than the experimental deflections for all investigated beams.

(2) Cracking behavior

Figure 5 indicates that both monolithic and composite beams experienced similar cracking patterns characteristics. Except the monolithic beam made with SCC that developed three major flexural cracks, all other beams developed one or two major flexural cracks at the region of constant moment, regardless of the beam set. However, all beams failed in the zone of constant moment after crushing of the concrete, which is represented by the shaded area in Fig. 5.

Figure 6 shows that both monolithic and composite beams have similar variation of crack opening under applied load. Actually, the crack opening can be used for a direct comparison between the two sets of beams. This is because the cracking occurs in the tension zone at the bottom of the beam and the crack width is measured at this zone. Both sets of beams have fibers in this bottom zone and the measured crack opening is virtually not affected by the fibers above the tension zone. The comparison at the service load level indicates that the composite beams generally experienced narrower crack widths as can be seen from Table 4. The cracks of the composite beams were 33%, on average, narrower than those of their corresponding monolithic beams. This can be attributed to the confinement effect experienced during casting the bottom 125 mm repair layer. This layer was confined by the formwork and the existing CVC. This obtained result is of a practical impact because if the controlling of crack opening is the main concern during the repair process, no need to replace the entire section as it is adequate to replace only the bottom tension zone using FR-SCC.

(3) Cracking, yield, and ultimate load capacities

The data reported in Table 4 indicate that the monolithic beams exhibited slightly higher cracking load (average increase of 5%) over the corresponding composite beams. This appears to be consistent with the measured tensile strength $f_{\text{up}}$ of the beams as reported in Table 3. The similar level of $f_{\text{up}}$ determined in the tension zone of maximum tensile stresses in monolithic and composite beams (4.8-6.4 MPa vs. 4.6-6.8 MPa, respectively) resulted in similar cracking load values.

On the other hand, the average increases 9% and 13% in both yield and ultimate loads, respectively, of the monolithic beams over those of the repair beams appeared to be more obvious than the increase in the cracking load. This is because the cracking starts to occur at the bottom extreme fibers of the beams, where the tensile stresses are highest, propagating upwards and this zone has fibers in both types of beams. This explains why the two types of beams showed approximately similar cracking loads. The situation is different in the other cases of yield and ultimate loads. In the...
monolithic beams, the fibers are distributed over the whole depth of the cross section which makes the fibers in the whole tension zone contribute to the resisting moment of the beams at yield and at ultimate loads. In the repair beams, however, the fibers are distributed over a part of the tension zone only (lower 125 mm) which limited their contribution to the yield and ultimate loads compared with the monolithic beams.

(4) Strain in reinforcing steel and concrete
Figure 7 shows that the load-strain relationships for steel and concrete at failure were approximately similar for the two sets of beams and followed the same trilinear relationship characterizing the load-deflection behavior of the beams. The steel strains of the monolithic beams were slightly lower (mean decrease of 6%) over those of the composite beams and the concrete strains were, however, higher with a mean increase of 49%.

(5) Ductility
The ductility values of the monolithic beams were up to 27% higher than those of their corresponding composite beams. The highest spread in ductility was obtained between the nonfibrous monolithic and composite beams. The presence of fibers in the tension zone in the two sets of fibrous beams, however, decreased this spread.

(6) Interfacial de-bonding between repair layer and existing concrete
In the composite beams and regardless of the presence of fibers, no interfacial bond failure (horizontal cracks) occurred at the interface between the repair and existing concrete layers. The LVDT readings for detecting any slip between the two layers were constantly close to zero. The bonding between the two layers was secured by the steel stirrups and preparation of a rough interface. It was also enhanced by using silica fume that can densify the microstructure of the cementsitious repair matrix and enhance bond to the substrate (Wu et al. 2016).

(7) Confinement and flow velocity
The concrete flow in the bottom layer of composite beams was faster and more restricted than in the monolithically cast beams. The time for the concrete to flow and fill the repair section was 20-25 s, which included flow through the 400-mm deep vertical access, horizontal flow along the 3.2-m long beam section, and upward movement through the ventilation holes (Fig. 1), which translates into a path of 4 m in length. The total volume of concrete to be cast in the repair section including the four holes was 0.12 m³. To fill an equal volume in the monolithic beams (3.2×0.25×0.15 m (L×W×D)) with the similar flow rate used to cast the repair concrete, the estimated time was 20-30 s. This difference in the flow velocity was due to the confinement effect (concrete flowed in 250×125-mm cross section) as well as the overhead pressure by the funnel used at the leading end A for casting.

The concrete matrix flowed through clear spacings in the 250×125-mm repair cross section in the composite beams, illustrated on the right side of Fig. 1. These clear spacings represented the clear spacing comprised between the steel reinforcement and formwork (cover of 40 mm), clear spacing between the steel reinforcement and repair interface (57.5 mm), and clear spacing between the two parallel longitudinal steel bars of No. 20M (115 mm). The confinement effect was manifested by the obtained results on core compressive strengths exposed in the following.

The in-place $f'_{c}$ was used to evaluate the effectiveness of the SCC and FR-SCC to achieve self-consolidation. Based on the results reported in Table 3, the average ratio values of $f'_{c}$ of core to $f'_{c}$ of cylinder ($f'_{c \text{ core}}/f'_{c \text{ cy}}$) were 0.98 and 0.85 for the composite and monolithic beams, respectively, regardless of the location of the core samples from the leading or end of the beam sections (ends A or B, respectively). In the case of composite beams, the mean $f'_{c \text{ core}}/f'_{c \text{ cy}}$ was 1.02 at the casting point (location A), which was higher than the mean ratio of $f'_{c}$ of core at location B (second end of beam) to $f'_{c}$ of cylinder ($f'_{c \text{ core}}/f'_{c \text{ cy}}$) of 0.95. This is due to the higher consolidation energy at the leading location A near the casting point. The confinement had a positive effect on crack width of composite beams over the corresponding monolithic beams, as presented earlier.

5. Comparison of experimental and predicted results
5.1 Cracking load
For the test setup used in the study, the cracking load $P_{cr}$ can be calculated according to the following equation:

$$P_{cr} = 2 \times M_{cr} / a'$$  \hspace{1cm} (1)

where $M_{cr}$ can be determined as follows:

$$M_{cr} = I \times f'_{cr} / Y$$  \hspace{1cm} (2)

The ACI 318 (2018), CSA A23.3 (2014), and ACI 544 (2018) codes were used to determine the cracking strength of concrete $f'_{cr}$. The cracking strength according to ACI 318 code is given by the following equation.

$$f'_{cr} = 0.62 (f'_{c})^{0.5} \text{ (MPa)}$$  \hspace{1cm} (3)
The CSA A23.3 standard recommends the following equation for determining $f_{cr}$:

$$f_{cr} = 0.6 \left( f'c \right)^{0.5} \text{ (MPa)} \tag{4}$$

On the other hand, ACI 544 considers the contribution of fibers in determining the cracking load, as indicated in Eq. (5).

$$f_{cr} = 0.843 f_t (1 - V_f) + 616.39 \times 10^{-7} V_f L_f / d_f \text{ (MPa)} \tag{5}$$

The prediction results of the crack loads are presented in Table 5. The predictability analysis is divided into three parts: the entire set of 10 investigated beams, the composite vs. monolithic beams regardless of the use of the fibers, and the FR-SCC vs. SCC regardless of the beam set.

### 5.2 Moment capacity

The theoretical ultimate capacity of the investigated beams was calculated considering the fundamentals of linear strain distribution over the depth of the cross section. Both tensile strengths of the steel reinforcement and fibers are considered in calculation of the flexural moment capacity, as given by Eq. (6) (ACI 544 2018).

$$M_e = A_f \left[ \left( d - d_f \right) + A_f (a) \left( \frac{a}{2} - d_f \right) \right] + \sigma_f \left( b - e \right) \left( \frac{h}{2} + \frac{e - a}{2} \right) \tag{6}$$

The tensile stress of the fibrous concretes can be given by Eq. (7) (ACI 544 2018).

$$\sigma_f = 0.00772 \frac{L_f}{d_f} V_f F_{we} \tag{7}$$

The correlation results between the theoretical and experimental ultimate loads are presented in Table 5. The predictions of the ultimate load for both sets of beams are consistent and conservative. The mean theoretical-to-experimental ultimate load ratio and its corresponding coefficient of variation for the monolithic beams were 0.68 and 6%, respectively, for the composite beams. It can be seen that the correlation for the composite beams was slightly better than that obtained for the monolithic beams (0.68 vs. 0.63). The correlation ratio for all beams was 0.66 with a coefficient of variation of 6%.

### Table 5 Prediction of cracking and ultimate loads of monolithic and composite beams.

| Response            | Monolithic beams | Composite beams |
|---------------------|------------------|-----------------|
|                     | SCC MO MU HY ST  | SCC MO MU HY ST |
| Cracking load ACI 31.8 | 1.09 1.01 0.88 1.02 0.90 | 1.12 1.06 0.89 1.03 0.96 |
|                     | 0.98 / 0.07 / 8% | --              |
|                     | 1.00 / 0.09 / 9% | 1.01 / 0.07 / 7% |
| Cracking load CSA A23.3 | 1.06 0.98 0.85 0.98 0.87 | 1.09 1.03 0.87 0.99 0.93 |
|                     | 0.92 / 0.7 / 8% | --              |
|                     | 0.95 / 0.09 / 9% | 0.98 / 0.9 / 9% |
| Cracking load ACI 544 | 1.58 1.72 1.41 1.40 1.64 | 1.73 1.72 2.06 1.42 1.52 |
|                     | 1.55 / 0.16 / 11% | --              |
|                     | 1.68 / 0.28 / 17% | 1.69 / 0.24 / 14% |
| Ultimate load ACI 31.8 | 0.66 0.63 0.64 0.61 0.62 | 0.71 0.71 0.73 0.66 |
|                     | 0.62 / 0.01 / 2% | --              |
|                     | 0.63 / 0.02 / 3% | 0.68 / 0.04 / 6% |
| Ultimate load CSA A23.3 | 0.66 / 0.04 / 6% |
|                     | 0.62 / 0.01 / 2% | --              |
|                     | 0.63 / 0.02 / 3% | 0.68 / 0.04 / 6% |

*Theoretical-to-experimental ratio value

*Average value / Standard deviation / Coefficient of variation.
6. Conclusions

Based on the results presented in this study, the following conclusions can be made:

(1) The incorporation of fibers in beams made with SCC and exposed to flexure can enhance their cracking and yield loads and stiffness by up to 30%, ductility by up to 170%, deflection by up to 40%, crack width up to 70%, and concrete strain by up to 130%, compared to beams made of nonfibrous SCC, regardless of the beam set (monolithic or composite).

(2) Fiber distribution along the depth of the beam (case of monolithic beams) did not considerably influence the cracking load (spread of -10% to 20%) compared to composite beams. However, the difference was clear in the yield and ultimate loads (up to 15% and 20%, respectively), and deflection (10% to 30%), thanks to the tensile stresses offered by fibers. The two beam sets gave similar cracking patterns at failure. The repair beams did not display any debonding between the two material layers.

(3) The composite beams exhibited better performance than that of the monolithic beams in terms of controlling the crack opening. All composite beams showed lower crack widths compared to the monolithic beams by an average value of 35%. This may be attributed to the effect of forced fiber orientation by casting velocity and pressure of concrete during casting of the repair layer. This finding is of a practical impact because if the controlling of crack opening is the main concern during the repair process, no need to replace the entire section as it is adequate to replace only the bottom tension zone using FR-SCC.

(4) The steel fiber with high mechanical and geometrical properties (elastic modulus, tensile strength, and hooked ends) offered the best performance in case of monolithic beams. In the case of repair beams where fibers experience preferential orientation through the tension zone, the best performance was obtained using long multifilament polypropylene fibers (50 mm vs. 10 mm to 42 mm).

(5) The flexural capacity of the tested reinforced concrete beams was conservatively and consistently predicted after accounting for the contribution of the tensile properties of the fibers to the resisting moment of the beams.

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**List of symbols and nomenclatures**

- $a$: Depth of rectangular stress block
- $a'$: Shear span
- $A_t$ and $A'_c$: Areas of tensile and compressive steel reinforcement, respectively
- $b$: Overall width of beam member
- $d$ and $d'$: Distances from extreme compression fiber to centroid of tensile and compressive steel reinforcement, respectively
- $d_f$: Fiber diameter
- $e$: Distance from extreme compression fiber to the top of fibrous concrete
- $E_c$: Elastic modulus of concrete
- $f'_{ce}$: Compressive strength in the tensioned part of the beam
- $f'_{sp}$: Splitting tensile strength of concrete
- $F_{bc}$: Bond efficiency of the fiber (Henager and Doherty 1976)
- $f_{cr}$: Cracking strength in the tensioned part of the beam
- $f_r$: Rupture modulus of concrete
- $f_t$ and $f_y$: Yield stress of tensile and compressive steel reinforcement, respectively
- $h$: Overall height of the beam member
- $I$: Moment of inertia
- $L_f$: Fiber length
- $L_f/d_f$: Fiber aspect ratio
- $M_{cr}$: Cracking moment
- $M_u$: Ultimate flexural moment
- $P_{cr}$: Cracking load
- $V_f$: Fiber volume (%)
- $V_f 	imes L_f/d_f$: Fiber factor
- $Y$: Distance from the centroidal axis of section to the extreme fiber in tension
- $\alpha$ and $\beta$: Factors ($\alpha = 0.85 - 0.0015 \times f'_{c} \geq 0.67$ and $\beta = 0.97 - 0.0025 \times f'_{c} \geq 0.67$)
- $\varepsilon_{cu}$: Ultimate strain of compressive concrete ($\varepsilon_{cu} = 3\%_c$)
- $\varepsilon_{fu}$: Ultimate strain of steel reinforcement ($\varepsilon_{fu} = E_s \times \sigma_{fu}$)
- $\rho$: Reinforcement ratio ($\rho = A_t / A_c = A_t / (b \times d)$)
- $\rho_b$: Balanced reinforcement ratio ($\rho_b = \beta \times \alpha \times f_{t} \times \varepsilon_{cu} \times 100 / f_{y} \times (\varepsilon_{cu} + \varepsilon_{fu})$)
- $\sigma_t$: Tensile stress of fibrous concrete