The Universal Property of the Entropy Sum of Black Holes in All Dimensions

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Abstract

It is proposed by Cvetic et al. [1] that the product of all horizon areas for general rotating multi-change black holes has universal expressions depending only on the quantized charges, angular momenta and the coupling constants of the gravitational theory, while not depending on the mass. When we consider the product of all horizon entropies, however, the mass will be present in some cases, e.g. in the Gauss-Bonnet gravity, while another new universal property [2] is preserved, which says that the sum of all horizon entropies depends only on the coupling constants of the theory and the topology of the black hole. We investigate this conjectured universality in arbitrary dimensions by studying the maximally symmetric black holes in Lovelock gravity and $f(R)$ gravity and Kerr-(anti-)de-Sitter black holes in Einstein gravity and prove that the sum of all horizon entropies only depends on the coupling constants and the topology of the black hole.

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1 Introduction

Studying the black hole entropy has been an attracting work after the establishment of black hole thermodynamics, but it is still a challenge to explain the black hole entropy at the microscopic level. Recently, the microscopic entropy of extreme rotating solutions has drawn some attention, as well as the detailed microscopic origin of the entropy of non-extremal rotating charged black holes. There has been some promising progress and results \[5,6\]. The further study of the properties of black hole entropy may give us a deeper understanding of black holes. The product of all horizon areas for a general rotating multi-charge black hole has been studied by Cvetic et al \[1\], both in asymptotically flat and asymptotically anti-de Sitter spacetimes in four and higher dimensions, showing that the area product of the black hole does not depend on its mass \( M \), but depends only on its charges \( Q_i \) and angular momenta \( J_i \). In Einstein gravity, the entropy and the horizon area of the black hole are simply related by \( S = \frac{A}{4} \), so the area product is proportional to the entropy product. Recently, a new work \[4\] studies the entropy product and another entropy relation in the Einstein-Maxwell theory and \( f(R)\) (Maxwell) gravity. However, in (for example) the Gauss-Bonnet gravity where the horizon area and entropy do not satisfy the relation \( S = \frac{A}{4} \) and the entropy seems to have more physical meaning than the horizon area, the mass will be present in the entropy product (see the next section). In fact, Ref. \[27\] has studied the entropy product by introducing a number of possible higher curvature corrections to the gravitational action, showing that the universality of this property fails in general.

Recently, it is found by Meng et al \[2\] that the sum of all horizon entropies including "virtual" horizons has a universal property that it depends on the coupling
constants of the theory and the topology of the black hole, but does not depend on the mass and the conserved charges such as the angular momenta $J_i$ and charges $Q_i$. The conjectural property has only been discussed in limited dimensions. In this paper, we prove a useful formula that makes it possible for us to investigate the universal property in all dimensions. Based on this formula, we discuss the entropy sum of general maximally symmetric black holes in the Lovelock gravity, $f(R)$ gravity and Kerr-(anti-)de-Sitter (Kerr-(A)dS) black holes in the Einstein gravity, and prove that the entropy sum depends only on the coupling constants of the theory and the topology of the black holes.

This paper is organized as follows. In the next section, we will discuss the Gauss-Bonnet case, and then we will express the formula and give a brief proof. In the sections 4 and 5, we will use the formula to calculate the entropy sum of (A)dS black holes in the Einstein-Maxwell theory and the Lovelock gravity in all dimensions. In the section 6, we will study rotating black holes to calculate the entropy sum of Kerr-(A)dS metrics in arbitrary dimensions. In the section 7, we will discuss the $f(R)$ gravity where the universal property also holds. At last, we give the conclusion and brief discussion.

2 (A)dS black holes in the Gauss-Bonnet gravity

The action of the Einstein-Gauss-Bonnet-Maxwell in $d$ dimensions is

$$I = \frac{1}{16\pi G} \int d^d x \sqrt{-g} \left[ R - 2\Lambda + \alpha (R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2) - F_{\mu\nu} F^{\mu\nu} \right]$$

Here $G$ is the Newton constant in $d$ dimensions, $\alpha$ is the Gauss-Bonnet coupling constant, and $\Lambda = \pm \frac{(d-1)(d-2)}{2l^2}$ is the cosmological constant. Varying this action with respect to the metric tensor gives equations of motion, which admits the $d$-
dimensional static charged Gauss-Bonnet-(A)dS black hole solution [7–10, 14]

\[ ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2d\Omega_{d-2}^2 \]  

where \( d\Omega_{d-2}^2 \) represents the line element of a \((d-2)\)-dimensional maximal symmetric Einstein space with constant curvature \((d-2)(d-3)k\), and \( k = -1, 0 \) and \( 1 \), corresponding to the hyperbolic, planar and spherical topology of the black hole horizon, respectively. The function \( V(r) \) in the metric (2) is given by

\[ V(r) = k + \frac{r^2}{2\tilde{\alpha}}(1 - \sqrt{1 + \frac{64\pi\tilde{\alpha}M}{(d-2)r^{d-1}} - \frac{2\tilde{\alpha}Q^2}{(d-2)(d-3)r^{2d-4}} + \frac{8\tilde{\alpha}\Lambda}{(d-1)(d-2)}) = 0, \]  

where \( \tilde{\alpha} = (d-3)(d-4)\alpha \), \( M \) and \( Q \) are the black hole mass and black hole charge respectively. The entropy is

\[ S = \frac{\Omega_{d-2}r^{d-2}}{4}(1 + \frac{2(d-2)k\tilde{\alpha}}{(d-4)r^2}), \]  

where \( \Omega_{d-2} = 2\pi^{(d-1)/2}/\Gamma^2(\frac{d-1}{2}) \). The area of the horizon is

\[ A = \frac{\Omega_{d-2}r^{d-2}}{4}. \]  

When we consider the five dimensional charged black hole, the function (3) can be written into

\[ 2\Lambda r^6 - 12kr^4 + (64\pi M - 12k^2\tilde{\alpha})r^2 - Q^2 = 0. \]  

Then, we can calculate the product of the areas by using Vieta’s theorem and (6)

\[ \prod_{i=1}^{6} A_i = \left( \frac{\Omega_3^3}{4} \right)^6 \prod_{i=1}^{6} r_i^3 = \left( \frac{\Omega_3^3}{4} \right)^6 \left( -\frac{Q^2}{2\Lambda} \right)^3. \]  

The result does not include the mass \( M \), preserving the property revealed in Ref. [1].
As we have mentioned in the Introduction, the entropy seems to have more physical meaning than the horizon area in the case that the horizon area and entropy are not proportional to each other. In five dimensions, the entropy product has been calculated when $\Lambda = 0$ [27]. Here we will give the explicit result with a nonvanishing cosmological constant $\Lambda$. The product of the entropies is

$$\prod_{i=1}^{6} S_i = \left(\frac{\Omega}{4}\right)^6 \prod_{i=1}^{6} (r_i^3 + k_\alpha r_i) = -\left(\frac{\Omega}{4}\right)^6 \frac{Q^2}{4\Lambda^2} [Q^2 + (64\pi M - 12k^2 \tilde{\alpha})k\tilde{\alpha} + 12k^3 \tilde{\alpha}^2 + 2\Lambda k^3 \tilde{\alpha}^3]$$

and the result depends on the mass.

However, it seems that the sum of all entropies including non-physical entropies proposed by [2] has a better performance, which depends only on the coupling constants of the theory and the topology of the black holes. We find that the Gauss-Bonnet case, which is included in the Lovelock gravity, obeys the property in all dimensions, and we will give the proof later.

To study the sum of all entropies in the Kerr-(A)dS black hole, Lovelock gravity and $f(R)$ gravity in all dimensions, we will use a formula to perform the calculation. For a polynomial, Vieta’s theorem says the fundamental symmetric polynomials of the roots can be expressed by its coefficients. It is well-known that a general symmetric polynomial of the roots can be expressed by the fundamental symmetric polynomials of the roots.

### 3 A useful formula

In this section, we will prove a formula, which is useful in the following sections. With regard to the polynomial as follows:

$$a_m r^m + a_{m-1} r^{m-1} + \cdots + a_0 r^0 = 0,$$
we denote the roots as \( r_i, i = 1, 2 \cdots m \), and denote \( s_n = \sum_{i=1}^{m} r_i^n \), then we get

\[
s_n = \frac{-1}{a_m} \sum_{i=0}^{m-1} s_{n-m+i} a_i.
\]  

(9)

if \( n - m + i < 0 \), \( s_{n-m+i} = 0 \), if \( n - m + i = 0 \), \( s_{n-m+i} = n \).

The proof is as follows:

\[
\frac{-1}{a_m} (a_{m-1} s_{n-1} + a_{m-2} s_{n-2}) = (r_1 + \cdots + r_m)(r_1^{n-1} + \cdots + r_m^{n-1}) - (\sum_{1 < i < m} r_i r_j)(r_1^{n-2} + \cdots + r_m^{n-2})
\]

\[
= (r_1^n + \cdots + r_m^n) - \sum_{i=1}^{m} [r_i^{n-2}\left(\sum_{0 < j_1 < j_2 < m+1, j_1, j_2 \neq i} r_{j_1} r_{j_2}\right)].
\]

\[
\frac{-1}{a_m} (a_{m-1} s_{n-1} + a_{m-2} s_{n-2} + a_{m-3} s_{n-3}) = (r_1^n + \cdots + r_m^n) + \sum_{i=1}^{m} [r_i^{n-3}\left(\sum_{0 < j_1 < j_2 < m+1, j_1, j_2 \neq i} r_{j_1} r_{j_2} r_{j_3}\right)].
\]

\[
\vdots
\]

If \( m \geq n \),

\[
\frac{-1}{a_m} (a_{m-1} s_{n-1} + a_{m-2} s_{n-2} + \cdots + a_m s_1) = (r_1^n + \cdots + r_m^n)
\]

\[
+ (-1)^n \sum_{i=1}^{m} [r_i(\sum_{0 < j_1 < \cdots < j_{n-1} < m+1, j_1, \cdots, j_{n-1} \neq i} r_{j_1} \cdots r_{j_{n-1}})]
\]

\[
= (r_1^n + \cdots + r_m^n) + (-1)^n \sum_{0 < j_1 < \cdots < j_n < m+1} r_{j_1} \cdots r_{j_n},
\]

so if we set \( s_0 = n \), then

\[
\frac{-1}{a_m} (a_{m-1} s_{n-1} + a_{m-2} s_{n-2} + \cdots + a_m s_1 + a_n s_0)
\]

\[
= (r_1^n + \cdots + r_m^n) + (-1)^n n \sum_{0 < j_1 < \cdots < j_n < m+1} r_{j_1} \cdots r_{j_n} + (-1)^{n+1} n \sum_{0 < j_1 < \cdots < j_n < m+1} r_{j_1} \cdots r_{j_n}
\]

\[
= r_1^n + \cdots + r_m^n.
\]
If \( m < n \), we continue the process until \( a_{m-l} = a_0 \), one can find that

\[
\frac{-1}{a_m} \sum_{i=0}^{m-1} s_{n-m+i} a_i = r^n_1 + \cdots + r^n_m = s_n.
\]

Also we obtain

\[
\sum_{1 \leq i < j \leq m} r^n_i r^n_j = \left[ (\sum_{i=1}^{m} r^n_i)^2 - \sum_{i=1}^{m} r^{2n}_i \right]/2
\]

\[
= \left[ s_n^2 - s_{2n} \right]/2
\]

and

\[
\sum_{1 \leq i < j < k \leq m} r^n_i r^n_j r^n_k = \left[ (\sum_{i=1}^{m} r^n_i)^3 - 3 \sum_{i=1}^{m} r^{2n}_i \sum_{i=1}^{m} r^n_i + 2 \sum_{i=1}^{m} r^{3n}_i \right]/6
\]

\[
= \left[ s_n^3 - 3s_{2n}s_n + 2s_{3n} \right]/6.
\]

### 4 \((A)dS\) black holes in the Einstein-Maxwell theory

The Einstein-Maxwell action in \( d \) dimensions is

\[
I = \frac{1}{16\pi G} \int d^d x \sqrt{-g} \left[ R - F_{\mu\nu} F^{\mu\nu} - 2\Lambda \right]. \tag{10}
\]

In the maximally symmetric case, solving the equation of motion from the above action gives the RN-(A)dS solution, which is of the form (2). The horizons are located at the roots of the function \( V(r) \) \[11\]-\[13\]

\[
V(r) = k - \frac{2M}{r^{d-3}} + \frac{Q^2}{r^{2(d-3)}} - \frac{2\Lambda}{(d-1)(d-2)} r^2 = 0. \tag{11}
\]

The entropy of horizon is given by

\[
S_i = \frac{A_i}{4} = \frac{\pi^{(d-1)/2}}{2\Gamma \left( \frac{d-1}{2} \right)} r_i^{d-2}. \tag{12}
\]
In odd dimensions, just as [2] has showed, the radial metric function is a function of $r^2$ and the entropy $S_i$ is a function of $r_i$ with odd order. The pairs of roots $r_i$ and $-r_i$ vanish the entropy sum, i.e. $\sum_i S_i = 0$.

In even dimensions, according to equations (9) and (11), we obtain

$$\sum_{i=1}^{2(d-2)} r_i^{d-2} = 2 \left( \frac{(d-1)(d-2)k}{2\Lambda} \right)^{(d-2)/2}. \quad (13)$$

The reason is as follows. According to equations (9) and (11), we have

$$s_{d-2} = \sum_{i=1}^{2(d-2)} r_i^{d-2} = \frac{-a_{2d-6}}{a_{2d-4}} s_{d-4} = \cdots = \left( \frac{-a_{2d-6}}{a_{2d-4}} \right)^{d-2} s_2$$

so

$$s_2 = \frac{-a_{2d-6}}{a_{2d-4}} s_0 = \frac{2}{a_{2d-4}},$$

Then we get

$$\sum_i S_i = \sum_i A_i = \frac{\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)} \frac{(d-1)(d-2)k}{2\Lambda} \left(\frac{d-2}{2}\right)^{(d-2)/2} \quad (14)$$

which depends only on the cosmological constant $\Lambda$ and the horizon topology $k$.

To summarize briefly, considering all the horizons including the un-physical "virtual" horizons, we find out the general expression of the entropy sum, which depends on the cosmological constant and the topology of the horizon, does not depend on the conserved charge $Q$ and mass $M$, and the entropy sum vanishes in the odd dimensions.

5 Black holes in the Lovelock gravity

In this section, we will discuss the case of Lovelock gravity. The action of
general Lovelock gravity can be written as [15]

\[ I = \int d^d x \left( \frac{\sqrt{-g}}{16\pi G} \sum_{k=0}^{m} \alpha_k L_k + L_{\text{matter}} \right) \]  

(15)

with \( \alpha_k \) the coupling constants and

\[ L_k = 2^{-k} \delta_{c_1 \ldots c_d}^{a_1 \ldots a_k b_1 \ldots b_k} R^{c_1 \ldots c_k}_{a_1 \ldots a_k} \ldots R^{c_d}_{a_k b_k} \]  

(16)

where \( \delta^{a_1 \ldots a_d}_{c_1 \ldots c_d} \) is the generalized delta symbol which is totally antisymmetric in both sets of indices. It can be shown that

\[ L_0 = 1, L_1 = R, L_2 = R^2 - 4 R_{\mu \nu} R^{\mu \nu} + R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma}. \]

If only keeping \( \alpha_0 = -2\Lambda \) and \( \alpha_1 = 1 \) nonvanishing, we obtain the Einstein gravity, while keeping \( \alpha_2 \) nonvanishing as well, we get the Gauss-Bonnet gravity.

Varying the above action with respect to the metric tensor and then solving the resultant equation of motion [16–19] by assuming that the metric has the form (2), one can find that the function \( V(r) \) is given by

\[ V(r) = \frac{d - 2}{16\pi} \Omega_{d-2} r^{d-4} \sum_{k=0}^{N} \tilde{\alpha}_k \left( \frac{1}{r^2} \right)^k - M r^{d-3} + \frac{Q^2 (d-2) \Omega_{d-2}}{16\pi} = 0, \]  

(17)

where

\[ N = \left\lfloor \frac{d}{2} \right\rfloor, \tilde{\alpha}_0 = \frac{\alpha_0}{(d-1)(d-2)}, \tilde{\alpha}_1 = \alpha_1, \tilde{\alpha}_{k>1} = \alpha_k \prod_{j=3}^{2k} (d-j). \]

The entropy of horizon is given by

\[ S = \frac{d - 2}{4} \Omega_{d-2} r^{d-2} \sum_{k=1}^{N} \tilde{\alpha}_k k \left( \frac{1}{r^2} \right)^{k-1}. \]

(18)

In odd dimensions, \( \sum_i S_i = 0 \) with the same reason as before.
For the even dimensions, according to (9) and (17), when we calculate $\sum_{j=1}^{2d-4} r_j^{d-2}$,

$$s_{d-2} = \sum_{j=1}^{2d-4} r_j^{d-2} = -\frac{a_{2d-5}}{a_{2d-4}} s_{d-3} + \cdots + -\frac{-a_{d-2}}{a_{2d-4}} s_0,$$

we only use the coefficient of $r$ whose order is not smaller than $d - 2$, so the mass $M$ and the charge $Q$ will not be present for they belong to the coefficients $a_{d-3}$ and $a_0$ respectively. When we calculate the sum of the entropy (18), the sum of the highest order of roots is $\sum_{j=1}^{2d-4} r_j^{d-2}$, so the mass $M$ and the charge $Q$ will disappear in the sum of the other order of roots according to (9). It is suggested that the sum of the entropies is independent of mass and charge, just depends on the coupling constants of the theory and the topology constants of the horizon.

6 Kerr-(anti-)de-Sitter black holes

Thus far we have only considered the maximally symmetric black holes. It is of great interest to investigate the entropy sum of rotating black holes, albeit in the Einstein gravity. In this section, we will discuss the sum of the entropies in Kerr-de Sitter metrics of all dimensions [20–22]. It is necessary to deal with the case of even dimensions and that of odd dimensions separately.

6.1 odd dimensions

In odd spacetime dimensions, $d = 2n + 1$, the equation that determines the horizons can be written as

$$\frac{1}{r^2}(1 - \Lambda r^2) \prod_{i=1}^{n} (r^2 + a_i^2) - 2M = 0 \quad (19)$$
where $\Lambda$ is the cosmological constant. The area of the horizon is given by

$$A_j = \frac{A_{2n-1}}{r_j} \prod_{i=1}^{n} \frac{r_j^2 + a_i^2}{1 + \Lambda a_i^2}$$  \hspace{1cm} (20)$$

where

$$A_m = \frac{2\pi^{(m+1)/2}}{\Gamma((m+1)/2)}.$$  \hspace{1cm} (21)$$

The entropy is $S_i = \frac{A_i}{4}$. The sum of the area (20) can be divided into two parts:

$$\sum_{j=1}^{2n+2} [A_j - \frac{A_{2n-1}}{r_j} \prod_{i=1}^{n} \frac{a_i^2}{1 + \Lambda a_i^2}]$$

and

$$\sum_{j=1}^{2n+2} [\frac{A_{2n-1}}{r_j} \prod_{i=1}^{n} \frac{a_i^2}{1 + \Lambda a_i^2}]$$.

The first part is a function of $r$ with odd order. The horizon function (19) is a function of $r^2$, which results in roots $r_i$ and $-r_i$ in pair and vanishes the first part. In the second part,

$$\sum_{j=1}^{2n+2} [\frac{A_{2n-1}}{r_j} \prod_{i=1}^{n} \frac{a_i^2}{1 + \Lambda a_i^2}] = \frac{A_{2n-1}}{r_1 r_2 \ldots r_{2n+2}} \sum_{0<i_1<i_2<\cdots<i_{2n+1}<2n+3} r_{i_1} r_{i_2} \ldots r_{i_{2n+1}}$$

so it also vanishes because we can find $\sum_{0<i_1<i_2<\cdots<i_{2n+1}<2n+3} r_{i_1} r_{i_2} \ldots r_{i_{2n+1}}$ vanishes from (19). Therefore, the sum of entropies vanishes, i.e. $\sum_i S_i = 0$.

### 6.2 even dimensions

In even dimensions, $d = 2n$, the equation that determines the horizons can be written as

$$\frac{1}{r} (1 - \Lambda r^2) \prod_{i=1}^{n-1} (r^2 + a_i^2) - 2M = 0.$$  \hspace{1cm} (22)$$
The area of the horizon is given by
\[ A_j = \mathcal{A}_{2n-2} \prod_{i=1}^{n-1} \frac{r_j^2 + a_i^2}{1 + \Lambda a_i^2}. \quad (23) \]

Using the equation (22) and the formula of (9), we obtain the sum of (23)
\[ \sum_{j=1}^{2n} A_j = \mathcal{A}_{2n-2} \frac{2}{\Lambda^{n-1}}. \]

Therefore the sum of entropies is \[ \sum_{j=1}^{2n} S_j = \frac{\mathcal{A}_{2n-2}}{2\Lambda^{n-1}}, \] which depends only on \( \Lambda \).

The results we obtain in this section is unchanged when the cosmological constant is negative.

7 (A)dS black holes in the \( f(R) \) gravity

To explain the cosmic acceleration, one approach is to modify general relativity by adding higher powers of the scalar curvature \( R \), and \( f(R) \) theories\[23, 25\] is one of the approaches. In this paper, we will study the entropy sum in \( f(R) \) gravity. We consider the action of \( R + f(R) \) gravity in \( d \)-dimensional spacetime coupled to a conformally invariant Maxwell field\[3\]
\[ I = \int d^d x \sqrt{-g} [R + f(R) - (F_{\mu\nu}F^{\mu\nu})^p] \quad (24) \]
where \( f(R) \) is an arbitrary function of scalar curvature \( R \). Solving the corresponding equation of motion in the maximally symmetric case again gives a solution of the form \( (2) \), where the function \( V(r) \) is given by
\[ V(r) = k - \frac{2M}{r^{d-3}} + \frac{Q^2}{r^{d-2}} \frac{(-2Q^2)^{(d-4)/4}}{1 + f'(R_0)} - \frac{2\Lambda_f}{(d-1)(d-2)} r^2 = 0 \quad (25) \]
with \( f'(R_0) = \left. \frac{\partial f(R)}{\partial R} \right|_{R=R_0}, R_0 = \frac{2d}{d-2} \Lambda_f, \Lambda_f \) is the cosmological constant.
The entropy of horizon is given by

\[ S_i = \frac{A_i}{4} (1 + f'(R_0)), \tag{26} \]

and the area of the horizon is given by

\[ A_i = \frac{2\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)} r_i^{d-2}. \tag{27} \]

According to equations (9) and (25), in odd dimensions, considering \( s_1 = \sum_{i=1}^{d} r_i = 0 \), we obtain

\[ s_{d-2} = \sum_{i=1}^{d} r_i^{d-2} = \frac{-a_{d-2}}{a_d} s_{d-4} = \cdots = \left(\frac{-a_{d-2}}{a_d}\right)^{d-2} s_1 = 0 \tag{28} \]

So the sum of entropies vanishes, i.e. \( \sum_i S_i = 0 \).

In even dimensions,

\[ s_{d-2} = \sum_{i=1}^{d} r_i^{d-2} = \frac{-a_{d-2}}{a_d} s_{d-4} = \cdots = \left(\frac{-a_{d-2}}{a_d}\right)^{d-2} s_0 = 2 \left(\frac{(d-1)(d-2)k}{2\Lambda_f}\right)^{d-2}. \tag{29} \]

So the entropy sum is

\[ \sum_i S_i = \frac{\pi^{(d-1)/2}}{\Gamma\left(\frac{d-1}{2}\right)} (1 + f'(R_0)) \left(\frac{(d-1)(d-2)k}{2\Lambda_f}\right)^{(d-2)/2}, \tag{30} \]

which does not depend on the mass \( M \) and the conserved charge \( Q \).

\section{Conclusion and discussion}

In order to investigate the property of entropy sum in all dimensions, we find that the formula (9) is very useful for the calculation. By studying the maximally symmetric black holes in Lovelock gravity and \( f(R) \) gravity and Kerr-(anti)de-Sitter black holes in Einstein gravity, we prove that the sum of all horizons indeed
only depends on the coupling constants of the theory and the topology of the black hole, and does not depend on the conserved charges $J_i, Q_i$ and mass $M$, therefore we can believe that it is a real universal property in all dimensions.

In this paper, we have just discussed some special black hole solutions in several gravitational theories. It is important to verify this universal property in more general settings, i.e. black holes with less symmetry in more general gravitational theories with various matter contents. The rotating black holes in the Gauss-Bonnet (or even Lovelock) gravity are of special interest, whose exact analytical form for general parameters is not yet known. However, some approximate forms (e.g. in the slowly rotating case [26]) are known, which can be used to investigate the universal property of the entropy sum. We will explore these aspects in future works.

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