Actio causes reactio: Gravito-optical trapping of three-level atoms

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Abstract

We investigate an atomic three-level Λ-system which is exposed to two counterpropagating laser fields (inducing Raman transitions) and which is closed by a magnetic hyperfine field tuned to be in resonance with the transition between the two ground states. The influence of a homogeneous gravitational field is included in a full quantum treatment of the internal and external dynamics of the atom. It is shown that the combined influence of the gravitational field and the lasers lead for specific momentum values with a very high probability to a transition of the Landau-Zener type. This is accompanied by a momentum transfer resulting in an upward kick. For appropriate initial conditions a sequence of up and down motions is obtained. No mirror is needed. A gravito-optical trapping of atoms based on this effect seems to be realizable.

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1 Introduction

During the last decade new techniques in atom optics led to the possibility to produce atomic clouds and beams with very small velocity (see Ref. [1] and references therein). Under such circumstances the influence of gravity has to be taken into account. To see in detail how gravity changes the atomic center-of-mass motion in the presence of laser fields, atoms exposed simultaneously to a running laser wave and gravity were theoretically studied in Refs. [2, 3]. In many experiments gravity causes an unwanted change in the atomic momentum that must be accounted for in the design of an experiment (see, e.g., Ref. [4]). But there are several proposals and realizations in which gravity is used to slow or cool atoms. One example is the gravitational cavity [5, 6, 7] in which freely falling atoms are reflected by an atomic mirror at the bottom of the cavity. Here gravity is directly used to reverse the atomic upward motion. Another example is the gravitational Sisyphus cooling [8, 9] in which a magnetic field gradient used to cancel gravity produces a cooling force that is proportional to gravity. Another proposal [10] exploits the fact that the addition of the gravitational potential to a gradient force potential can result in new minima which may be used to form a trap.

The gravitational cavity designed to trap atoms which is discussed in the references given above is based on the multiple bouncing of atoms on a reflective surface. It essentially contains two elements: a lower mirror for atoms provided by an evanescent laser wave and the influence of gravity which results in a second upper "mirror" closing the cavity because it bends the atomic trajectories. By this a "trampoline" geometry is obtained. In the following we want to present a setup in which the lower mirror closing the trap at the bottom is replaced by a combined influence of the gravitational field and the laser fields. Gravitation as "actio" causes a free fall which together with the influence of the lasers leads to a "reactio" of the atoms turning them...
into an upward motion. A gravito-optically induced internal transition of the atoms is accompanied by a momentum transfer which causes an upward kick. Afterwards, the free fall of the atoms continues until at a certain downward momentum resonance is obtained again, the next kick happens, and so on. In this way a sequence of up and down motions is obtained without any reflection at a surface. The result obtained could be called "trampolining without trampoline".

In detail we study the free fall of a three-level Λ-system interacting with two laser fields and a magnetic field which is applied between the two lower levels, compare Fig. 1. In section 2 we perform the rotating-wave-approximation and switch to the interaction picture. In section 3 the excited state is adiabatically eliminated. It turns out that in momentum space the dynamical evolution of the three-level system in question is governed by the Hamiltonian

$$H := H_A + H_{c.m.} + H_{int}$$

where

$$H_A := E_e |e⟩⟨e| + E_+ |+⟩⟨+| + E_- |−⟩⟨−|$$

describes the internal energy levels $E_e > E_+ > E_-$ of the atom,

$$H_{c.m.} := \begin{bmatrix} \frac{\hbar^2}{2M} - \vec{M} \cdot \vec{x} \end{bmatrix}$$

is the center-of-mass part of the complete Hamiltonian, and

$$H_{int} := -\hbar \Omega \left\{ \cos[\omega_+ t - \vec{k}_+ \cdot \vec{x} - \varphi_-]|e⟩⟨e| + |+⟩⟨+| \right\}$$

$$+ \cos[\omega_- t - \vec{k}_- \cdot \vec{x} - \varphi_-]|−⟩⟨−|$$

$$+ \hbar \Omega_B \cos[\omega_B t - \varphi_B]|−⟩⟨−| + |−⟩⟨−|)$$

describes the influence of the two laser fields and the magnetic hyperfine field on the atom. $\vec{x}$ and $\vec{p}$ are the position and momentum operator of the atom’s center-of-mass motion and $M$ is its mass. The homogeneous gravitational acceleration is denoted by the vector $\vec{a}$ pointing towards the Earth. $\omega_\pm$ are the frequencies of the two Raman lasers which induce transitions between the upper state $|e⟩$ and the lower states $|+⟩$ and $|−⟩$, respectively, see Fig. 1. Their phases are given by $\varphi_\pm$, and $\vec{k}_\pm$ are their counterpropagating wave vectors. The Rabi frequency $\Omega$ is assumed to be equal for both laser fields. $\omega_B$, $\Omega_B$, and $\varphi_B$ denote the frequency, the Rabi frequency, and the phase of the magnetic field. $\mathbf{1}$ is the unit operator in the internal three dimensional space. No spontaneous emission is included in $H$ since we will work in a regime where it can be neglected.

By a sequence of unitary transformations we will now transform $H$ into a form which will allow us to read off the physical content more easily. First we perform the rotating wave approximation by using

$$U_1 := |e⟩⟨e| + |+⟩⟨+| e^{i\omega_+ t} + |−⟩⟨−| e^{-i\omega_- t}$$

(5)

to transform the original state vector $|ψ⟩$ to $|ψ_1⟩ = U_1^* |ψ⟩$. Throughout the paper we will use the convention that a unitary transformation $U$ acts always in the form $|ψ_{old}⟩ = U |ψ_{new}⟩$. Neglecting terms oscillating with frequency $2\omega_\pm$ and $2\omega_B$ we obtain

$$H_1 = E_e \mathbf{1} + E_{+} |+⟩⟨+| + E_{−} |−⟩⟨−| + H_{c.m.}$$

$$-\frac{\hbar \Omega}{2} \left\{ e^{i(\vec{k}_+ \cdot \vec{x} + \varphi_+)}|e⟩⟨e| + e^{i(-\vec{k}_- \cdot \vec{x} + \varphi_-)}|e⟩⟨−| + H.c. \right\}$$

$$+ \frac{\hbar \Omega_B}{2} \left\{ e^{i\varphi_B}|+⟩⟨−| + H.c. \right\}$$

(6)
Here we have set \( \vec{k} := \vec{k}_+ \) and have made use of the fact that the laser beams are counterpropagating and have about the same frequency so that \( \vec{k}_- \) is approximately equal to \( -\vec{k} \).

Note that we do not set \( \omega_+ = \omega_- \) in the calculations. This ambiguous treatment of frequency and wavelength of the lasers deserves a short comment. Setting \( \vec{k}_+ = -\vec{k}_- \) is of course an approximation which affects the momentum conservation. But since the corresponding error \( \delta \vec{k} := \vec{k}_+ + \vec{k}_- \) is very small compared to the vectors themselves this approximation is justified as far as momentum conservation is concerned and as long as we do not look at momenta which are of the order of \( \hbar \vec{k} \). But setting \( \omega_+ = \omega_- \) would violate the energy conservation by an amount of about \( E_+ - E_- \). Since also \( \hbar \omega_B \) and many other energy scales occurring in the system at hand lie in this range, this approximation would be unacceptable. The error

\[
\frac{\hbar^2 (\vec{k} + \delta \vec{k})^2}{2M} - \frac{\hbar^2 \vec{k}^2}{2M} \approx \frac{1}{M} \hbar^2 \vec{k} \cdot \delta \vec{k} = \frac{\hbar \omega_+ (\hbar \omega_+ - \hbar \omega_-)}{M c^2}
\]

in the kinetic energy caused by \( \delta \vec{k} \) is negligible, however.

To achieve the time independence of \( H_1 \) it was necessary to impose the condition

\[
\omega_B + \omega_+ - \omega_- = 0
\]

on the field frequencies. This also allows us to perform the rotating wave approximation simultaneously in \( \omega_\pm \) and \( \omega_B \). In the following we will restrict to setups where this is fulfilled. To facilitate the calculations we furthermore assume that the detunings \( \Delta_\pm := \omega_\pm - (E_+ - E_\pm) / \hbar \) of the two laser beams are equal: \( \Delta_+ = \Delta_- =: \Delta \). Eq. \( \[\] \) then implies that the magnetic field has to be in resonance with the hyperfine transition,

\[
\omega_B = \frac{E_+ - E_-}{\hbar}.
\]

It proves to be useful to perform a second unitary transformation with the operator

\[
U_2 = e^{-itE_+ / \hbar} \left\{ |e\rangle \langle e| + |+\rangle \langle +| + \left( e^{i\varphi_+} |+\rangle \langle -| + e^{-i\varphi_-} |\rangle \langle +| \right) \right\} \exp \left\[ iM \vec{a} \cdot \vec{x} / \hbar \right\].
\]

The first factor shifts the overall energy of the internal states, the second term removes the phase factors from the Raman transition matrix elements, and with the last term we switch to the interaction picture with respect to the gravitational potential. This leads to

\[
H_2 = \frac{1}{2M} \left\{ \frac{\vec{p}^2 + M \vec{a} t^2}{2M} + \hbar \Delta \left\{ |+\rangle \langle +| + |-\rangle \langle -| \right\} \right. 
- \frac{\hbar \Omega}{2} \left\{ e^{i\vec{k} \cdot \vec{x}} |\rangle \langle +| + e^{-i\vec{k} \cdot \vec{x}} |\rangle \langle -| + H.c. \right\} + \frac{\hbar \Omega_B}{2} \left\{ e^{i\varphi_+} |\rangle \langle +| + H.c. \right\}
\]

with \( \Delta \varphi := \varphi_B + \varphi_+ - \varphi_- \).

### 3 Reduction of the Schrödinger equation

To solve the Schrödinger equation it is advantageous to expand the wave function in momentum space according to

\[
|\psi_2\rangle = \int d^3q |\vec{q}\rangle \otimes \left\{ c_+ (\vec{q}) e^{i\varphi(t)} |e\rangle + c_+ (\vec{q}) e^{i(\varphi(t) - \Delta t)} |+\rangle + c_- (\vec{q}) e^{i(\varphi(t) - \Delta t)} |-\rangle \right\}
\]

with

\[
\varphi(t) := -\frac{1}{\hbar} \left\{ \frac{\vec{q}^2}{2M} t + \frac{1}{2} \vec{q} \cdot \vec{a} t^2 + \frac{M}{6} \vec{a}^2 t^3 \right\}
\]

where \( |\vec{q}\rangle \) are the eigenvectors of the untransformed momentum operator \( \vec{p} \) with \( \langle \vec{q} | \vec{p} |\vec{q}'\rangle = \vec{q} \cdot \vec{q}' \), and the \( c(\vec{q}) \) are functions of the time \( t \). Remember that we already have made two unitary transformations. The measured mean value of the momentum of the states \( |\vec{q}\rangle \) is therefore \( \langle \vec{q} | \vec{p}_2 |\vec{q}\rangle = \vec{q} + M \vec{a} t \). Consequently the center-of-mass part \( |\vec{q}\rangle \) of \( |\psi_2\rangle \) describes states already “falling” under the influence of gravity according to the
classical law of free fall which in this way has been separated in the subsequent calculation. \( \vec{q} \) is a momentum parameter which agrees with the measured momentum at \( t = 0 \).

The Schrödinger equation is then reduced to
\[
\begin{align*}
    i\dot{c}_+ (\vec{q}) &= -\frac{\Omega}{2} e^{-i\Delta t} e^{-i\delta_n t} \left\{ c_+ (\vec{q} - \hbar \vec{k}) e^{i((\vec{k} t/M + \vec{k} \cdot \vec{a} t^2/2)} + c_- (\vec{q} + \hbar \vec{k}) e^{-i((\vec{k} t/M + \vec{k} \cdot \vec{a} t^2/2)} \right\} \quad (14) \\
    i\dot{c}_+ (\vec{q}) &= -\frac{\Omega}{2} c_+ (\vec{q} + \hbar \vec{k}) e^{i\Delta t} e^{-i((\vec{k} t/M + \vec{k} \cdot \vec{a} t^2/2)} + \frac{\Omega_B}{2} e^{i\varphi} c_- (\vec{q}) \quad (15) \\
    i\dot{c}_- (\vec{q}) &= -\frac{\Omega}{2} c_- (\vec{q} - \hbar \vec{k}) e^{i\Delta t} e^{i((\vec{k} t/M - \vec{k} \cdot \vec{a} t^2/2)} + \frac{\Omega_B}{2} e^{-i\varphi} c_+ (\vec{q}) \quad (16)
\end{align*}
\]

\( \delta_R := \hbar \vec{k}^2 / (2M) \) is the recoil shift of the atom.

This form is particularly suited for the adiabatic elimination of the excited state. We obtain an effective two-level system with internal states \(|+\rangle \) and \(|-\rangle \). This can be performed if the detuning \( \Delta \) is much larger than all other frequencies appearing in Eqs. (14) to (16). Proceeding as in Ref. [1], we assume that the time dependence of the r.h.s. of Eq. (14) is governed by the factor of \( \exp[-i\Delta t] \). Integrating Eq. (14) by neglecting any other time dependence of the r.h.s. then yields an algebraical expression for \( c_\pm (\vec{q}) \) in terms of \( c_\pm (\vec{q}) \) which can be used to eliminate it in Eqs. (15) and (16). The resulting differential equations are
\[
\begin{align*}
    i\dot{\tilde{c}}_+ (\vec{q}) &= \frac{\Omega_{eff}}{2} \tilde{c}_- (\vec{q} + 2\hbar \vec{k}) e^{-i(\vec{k} \cdot \vec{a} t^2 + 2\vec{q} \cdot \vec{k} t + M4\delta_n t)} + \frac{\Omega_B}{2} e^{i\varphi} \tilde{c}_- (\vec{q}) \\
    i\dot{\tilde{c}}_- (\vec{q}) &= \frac{i\Omega_{eff}}{2} \tilde{c}_+ (\vec{q} - 2\hbar \vec{k}) e^{i(\vec{k} \cdot \vec{a} t^2 + 2\vec{q} \cdot \vec{k} t + M4\delta_n t)} + \frac{\Omega_B}{2} e^{-i\varphi} \tilde{c}_+ (\vec{q}) \quad (17)
\end{align*}
\]

with \( \tilde{c}_\pm := \exp[i\Omega_{eff}/2]c_\pm \). The quantity \( \Omega_{eff} := \Omega^2 / (2\Delta) \) denotes as usual the effective Rabi frequency of the Raman transition caused by the lasers.

The algebraical expression for \( c_\pm \) which we have not written down above also shows that \( c_\pm \) is suppressed by a factor of \( \Omega / \Delta \) as compared to \( c_\pm \). This implies that because of the very small amount of excited atoms spontaneous emission from \(|e\rangle \) to \(|\pm\rangle \) can be neglected for not too long interaction times. To check how long the neglection of spontaneous emission is possible we have numerically calculated the eigenvalues of the Liouville operator \( \mathcal{L} \) defined by
\[
    i\hbar \partial_t \rho = [\hat{H}, \rho] + \Gamma \rho =: \mathcal{L} \rho \quad (18)
\]
where \( \rho \) is the atomic density matrix, \( \hat{H} \) is given by Eq. (3) if the center-of-mass degrees of freedom are completely removed by setting \( \vec{x} = \vec{p} = 0 \), and the operator \( \Gamma \) acts on \( \rho \) by
\[
    \Gamma \rho := i\hbar \gamma \rho_{ee} (-2|e\rangle \langle e| + |+\rangle \langle +| + |+\rangle \langle +| - |\mp\rangle \langle \mp| - i\hbar \gamma (\rho_{ee}|e\rangle \langle +| + \rho_{ee} |e\rangle \langle -| + H.c.) \quad (19)
\]
where \( 2\gamma \) is the decay rate. Adopting the experimental data of Ref. [12] (\( \Omega = \gamma = 10^7 \text{ Hz}, \Delta = 2.5 \cdot 10^9 \text{ Hz} \)) and by setting \( \varphi_\pm = \varphi_B = 0 \) and \( \Omega_B = 10^5 \text{ Hz} \) we have found that the coherences between \(|+\rangle \) and \(|-\rangle \) have a lifetime of about 1/80 second. This will turn out to be enough for our purposes.

We turn back to the coupled differential equations (17). One can read off directly that only states with the same momentum or states which in momentum space are separated by \( \pm 2\hbar \vec{k} \) are coupled. The first coupling is proportional to \( \Omega_B \) and is not related to a momentum transfer. The second is proportional to \( \Omega_{eff} \) and is related to a momentum transfer \( \pm 2\hbar \vec{k} \), respectively. Because of this, Eqs. (17) couple in fact a total ladder of states separated by multiples of \( 2\hbar \vec{k} \) in momentum space, see Fig. 2. Because only differences are fixed, it is evident that there is not only one ladder but a total family of ladders. We will parametrize different ladders by an arbitrary momentum parameter \( \vec{q}_0 \) that later will denote the measured momentum at \( t = 0 \).

To make this structure more transparent, and especially to clarify the physical nature of the transition between states \(|+\rangle \) and \(|-\rangle \) of different momentum, we change from \( \tilde{c}_\pm \) to the new coefficients \( u_\pm \) by
\[
\begin{align*}
    \tilde{c}_+ (\vec{q}_0 + 2n\hbar \vec{k}) &= e^{2iD_0 n t} e^{i4\delta_n t^2} e^{i(n+1)\Delta \varphi} e^{i(n\vec{k} \cdot \vec{a} t^2} u_+ (n) \\
    \tilde{c}_- (\vec{q}_0 + 2n\hbar \vec{k}) &= e^{2iD_0 n t} e^{i4\delta_n t^2} e^{i(n\vec{k} \cdot \vec{a} t^2} u_- (n) \quad (20)
\end{align*}
\]

\( D_0 := \vec{q}_0 \cdot \vec{k} / M \) denotes the Doppler shift associated with \( \vec{q}_0 \) and \( + \) and \( - \) refer to the internal states \(|+\rangle \) and \(|-\rangle \). \( n = 0, \pm 1, \pm 2, \ldots \) indicates for a state the difference \( 2n\hbar \vec{k} \) in momentum parameter relative to \( \vec{q}_0 \). This
leads to the final form of the dynamical equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} u_+(n) \\ u_-(n) \\ u_+(n-1) \\ u_-(n-1) \\ \vdots \end{pmatrix} = \begin{pmatrix} d_n(t) & \Omega_B/2 & 0 & \cdots & 0 \\ \Omega_B/2 & d_n(t) & \Omega_{eff}/2 & \cdots & 0 \\ 0 & \Omega_{eff}/2 & d_{n-1}(t) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & d_{n-1}(t) \end{pmatrix} \begin{pmatrix} u_+(n) \\ u_-(n) \\ u_+(n-1) \\ u_-(n-1) \\ \vdots \end{pmatrix}$$  \tag{21}

with

$$d_n(t) := 2n\{D_0 + 2n\delta_R + \vec{k} \cdot \vec{a}t\}$$ \tag{22}

$$= \frac{(\vec{q}_0 + 2n\hbar\vec{k} + M\vec{a}t)^2}{2M\hbar} - \frac{(\vec{q}_0 + M\vec{a}t)^2}{2M\hbar}. \tag{23}$$

The difference between the diagonal elements $d_n(t)$ and $d_{n-1}(t)$

$$d_n(t) - d_{n-1}(t) = 2[D_0 + 2\delta_R(2n - 1) + \vec{k} \cdot \vec{a}t]$$  \tag{24}

varies linearly with $t$ with a factor independent of $n$. It is at this point where the influence of the homogeneous gravitational field appears. Several overlapping $2 \times 2$ matrices can be recognized on the r.h.s. of Eq. (21). Based on this we can read off from Eq. (21) the following types of transitions: If the lasers are switched off there are no Raman transitions ($\Omega_{eff} = 0$). It remains the Rabi flopping between the states $|+\rangle$ and $|-\rangle$ caused by the magnetic field ($\Omega_B \neq 0$). There is no accompanying change of momentum and accordingly no coupling to states with different $n$. If on the other hand the magnetic field is switched off ($\Omega_B = 0$) there are transitions between $|+\rangle$ and $|-\rangle$ with momentum transfer $\pm 2\hbar\vec{k}$. It is essential for the following to note that these transitions are, because of the time dependence in Eq. (24), of the Landau-Zener type if gravitation is present ($\vec{a} \neq 0$). We will come back to this below. If in addition $\vec{a}$ is switched off we have the Raman transitions between $|+\rangle$ and $|-\rangle$ with momentum transfer leading to a sort of vibration of the center-of-mass motion. Leaving apart for the moment the effectivity of transitions and the question when transitions take place we obtain for nonvanishing $\vec{a}$, $\Omega_{eff}$, and $\Omega_B$ a ladder of combined influences a section of which is depicted schematically in Fig. 2. Let us now turn to the details of the processes involved.

## 4 Landau-Zener transitions

A Landau-Zener transition between two states happens if in the $2 \times 2$ matrix of the responsible Hamiltonian the difference between the diagonal terms changes linearly in time while the non-diagonal terms remain constant. It has been discussed for a harmonically oscillating two-level system. Above in Eq. (21) it is a transition between the states $|+\rangle$ and $|-\rangle$, below in Eq. (23) it will be a transition between dressed states. In both cases the non-diagonal elements are proportional to $\Omega_{eff}$ and the difference between the diagonal elements is of the form $2\vec{k} \cdot \vec{a} \Delta t$ with appropriate $\Delta t$. Because of its ingredients the L.Z. transitions represent in our case a typical gravito-optical effect. The dynamics of the L.Z. transition can be solved analytically. It results in a certain efficiency of a population inversion. In our case it is the following: If the atom is prepared in an initial state $u_-$ or $u_+$ the probability to find it in this state after the transition has occurred is

$$P_{\text{stay}} = \exp \left\{ -\pi \frac{\Omega_{eff}^2}{16|\vec{k} \cdot \vec{a}|} \right\}. \tag{25}$$

This fixes for a cloud of atoms the redistribution of the population on the two levels. It is an exact result which cannot be derived within perturbation theory because of its nonanalytical dependence on $\vec{k} \cdot \vec{a}$. In addition, it is known for L.Z. transitions that they essentially happen at the time when the difference of the diagonal elements vanishes. This is in our case according to Eq. (24) at the times

$$t_n = -\frac{D_0 + 2\delta_R(2n - 1)}{\vec{k} \cdot \vec{a}}.$$
Two subsequent L.Z. transition can happen after a time difference
\[ \Delta t := t_{n+1} - t_n = -\frac{\delta R}{k \cdot \vec{a}} = -\text{sign}(\vec{k} \cdot \vec{a}) \frac{2|\vec{k}|}{M|\vec{a}|}. \]  \hspace{1cm} (27)

This is just the time during which the atom’s momentum is changed by the amount \(2|\vec{k}|\) through the influence of the Earth’s acceleration. The order of magnitude of the effective duration of the inversion process is
\[ t_{\text{L.Z.}} := \frac{\Omega_{\text{eff}}^2}{|k \cdot a|^{3/2}}. \]  \hspace{1cm} (28)

We are now able to answer the following question: For a given initial situation, at what time and between which states do L.Z. transitions happen? In a first step we analyze the equations obtained and add then below a more intuitive picture so that the underlying physics becomes even more transparent. For simplicity we use vertically oriented lasers: \(-\vec{k} \cdot \vec{a} = ka > 0\) with \(\vec{a} = -a\vec{e}_z\) whereby \(\vec{e}_z\) is pointing upwards. Let us assume that at \(t = 0\) the atom is prepared with initial momentum \(\vec{q}_0\) in one of the states \(|+\rangle\) and \(|-\rangle\) corresponding to \(u_+(n = 0)\) or \(u_-(n = 0)\), respectively. The magnetic field will then immediately induce Rabi oscillations between these states so that both of them can be the starting point for a L.Z. transition, compare Fig. 2. Possible candidates are \(n = 0 \to n = -1\) at the time \(t_0\) and \(n = 0 \to n = 1\) at the time \(t_1\). In any case, a necessary condition is that the transition time \(t_n\) is positive.

We distinguish three different domains for the initial momentum: (i) For \(\vec{k} \cdot \vec{q}_0 > h\vec{k}^2\) Eq. (26) shows that at the time \(t_0\) a L.Z. transition is possible. Fig. 2 demonstrates in detail that it must be the transition \(u_-(n = 0) \to u_+(n = -1)\). From states with \(n = -1\) L.Z. transitions seem to be possible to \(u_-(n = 0)\) and \(u_+(n = -2)\). But the corresponding times are \(t_0\) and \(t_1\) which are both not later than \(t_0\). Accordingly the first transition at \(t_0\) remains the only one. Note that because of the free fall we have for the measured momentum \(\vec{p} = \vec{q}_0 - M\vec{a}\dot{t}\). Accordingly, \(t_0\) turns out to be the time after which gravitation has decelerated the atom so that the z-component of its momentum is \(+h\vec{k}\). After the L.Z. transition and the corresponding momentum transfer \(-2h\vec{k}\) it is \(-h\vec{k}\) and the atom carries on falling without further L.Z. transitions. This happens, too, in our next case: (ii) \(\vec{k} \cdot \vec{q}_0 < -h\vec{k}^2\). From an analysis based on Eq. (26) it follows that there is only the free fall without any L.Z. transition. (iii) Finally, for \(-h\vec{k}^2 < \vec{k} \cdot \vec{q}_0 < h\vec{k}^2\), Eq. (24) shows that a first L.Z. transition happens for \(t = t_1\). Because of \(\Delta n = +1\) we read off from Fig. 2 that it must be \(u_+(n = 0) \to u_-(n = 1)\). Analysing again Eq. (26) we see that in this case successively at \(t_2 > t_1\) and then at \(t_3 > t_2\) and so on on L.Z. transitions from \(|+\rangle\) to \(|-\rangle\) take place. Referring to the z-component of the measured momentum and taking the free fall into account it is easy to see that \(t_1\) is the time at which the z-component of the measured momentum reaches \(-h\vec{k}\). The L.Z. transition transfers to a momentum value \(+h\vec{k}\), the atom starts falling again until \(-h\vec{k}\) is reached and so on.

For an intuitive understanding it may be helpful to illustrate the analytical results derived above in a diagram. Let us restrict for simplicity to a one-dimensional problem in assuming the vertical initial momentum \(\vec{q}_0 = q_0\vec{e}_z\) at \(t = 0\). The momentum parameter is \(\vec{q} = q\vec{e}_z\). Then the measured momentum \(\vec{p} = p\vec{e}_z\) is because of the free fall \(p = q - M\vec{a}\dot{t}\). The total energy \(E_{\text{tot}}\) of the atom is the sum of the kinetic energy and the internal energy \(E_\perp\) of the respective state. \(E_{\text{tot}}\) is shown in Fig. 3 as function of the measured momentum \(p\). The rapid Rabi oscillations caused by the magnetic field are vertical transitions between the two curves (no momentum transfer, the dashed arrows are examples). A L.Z. transition is a transition from the \(|+\rangle\) curve to the \(|-\rangle\) curve with momentum transfer \(+2h\vec{k}\) or from the \(|-\rangle\) curve to the \(|+\rangle\) curve with transfer \(-2h\vec{k}\) (solid diagonal arrows). We know from the calculation that for the accelerated atom the transitions caused by the combined influence of the gravitational field and the lasers are of the L.Z. type. They are transitions from the \(|+\rangle\) curve to the \(|-\rangle\) curve with momentum transfer \(+2h\vec{k}\) or from the \(|-\rangle\) curve to the \(|+\rangle\) curve with transfer \(-2h\vec{k}\). But for such a transition to be possible we have in addition to fulfill the energy condition
\[ E_{+}^{\text{tot}} + h\omega_+ - h\omega_- = E_{-}^{\text{tot}} \]  \hspace{1cm} (29)

or with the specifications assumed in Eqs. (8) and (3)
\[ E_{+}^{\text{tot}} - E_{-}^{\text{tot}} = E_+ - E_- \]  \hspace{1cm} (30)

6
It can directly be read off from Fig. 3 that there are only two transitions which fulfill the conditions of energy transfer and momentum transfer simultaneously. They are indicated by the solid line arrows.

We now discuss different initial conditions with the help of Fig. 3. Let us start at $t = 0$ with an initial momentum $q_0$ between $-\hbar k$ and $+\hbar k$. We know that the Rabi transitions caused by the magnetic field are vertical transitions between the $|+\rangle$ and the $|-\rangle$ curve which are permanently happening (dashed arrows). To obtain the diagram with the dashed arrows to the left in the diagram until $p = -\hbar k$ is reached. Then the transition from $|+\rangle$ to $|-\rangle$ and to $p = +\hbar k$ is in resonance and can happen (solid line arrows). The respective probability is the one of a L.Z. transition given by $1 - P_{\text{stay}}$ of Eq. (28). The influence of gravity continues. The atoms which have not made the L.Z. transition continue to move to the left in Fig. 3 without having another possibility for a L.Z. transition. The atoms which have made the L.Z. transition fall freely starting with $p > \hbar k$ and that there will be no L.Z. transition at all if one starts with $p < -\hbar k$.

It is important to check that the time (27) between two subsequent Landau-Zener transition is long enough so that each transition can be completed. This means that the time difference (27) must be large compared to $t_{L,Z}$. To check whether this condition is fulfilled for optical transitions we adopt $\Omega = 10^7$ Hz. The momentum $q_0$ should be of the order of $\hbar k$ so that $D_0$ is of the order of $\hbar k$. A typical value for $|k|$ is $10^5$ m$^{-1}$ so that for light atoms ($M \approx 10^{-26}$ kg) $\delta R$ and $D_0$ are of the order of $10^6$ Hz. For $\Omega_B$ we will assume that it can be made large enough to be one order above the effective Rabi frequency $\Omega_{eff} \approx 2 \cdot 10^4$ Hz. The last time scale is introduced by $\sqrt{|\vec{k} \cdot \vec{a}|}$ which is of the order of $10^4$ Hz. Inserting this into Eqs. (28) and (27) we see that the time between two transition is about 0.01 seconds whereas $t_{L,Z}$ is of the order of $10^{-4}$ seconds so that the transitions have enough time to be completed.

5 Complete solution in terms of dressed states

Especially the discussion of Fig. 3 given above revealed that the transitions due to the magnetic field are permanently present and that they are necessary to enable a chain of L.Z. transitions. It is therefore reasonable to switch to a description of the process which incorporates these facts from the beginning. This can be done in referring not to $|\pm\rangle$ but to dressed states of the atom with respect to the magnetic field. They are eigenstates of the atomic energy and the interaction energy of the magnetic field taken together. In this case we expect pure L.Z. transitions only.

To reduce Eq. (21) to a Landau-Zener problem we introduce the functions

$$w_\pm(n) := \frac{1}{\sqrt{2}}(u_+(n) \pm u_-(n))e^{\pm i\Omega_B t/2}$$

where $\Psi(t)$ is a time dependent phase. It is easy to show that these wave vectors are eigenvectors of the Hamiltonian (11) if $\Omega = 0$. Hence they are dressed states in the sense that the effect of the (classical) magnetic field is already taken into account.

For $w_\pm(n)$ the Schrödinger equation (21) becomes

$$i\dot{w}_+(n) = d_n w_+(n) + \frac{\Omega_{eff}}{4}\left\{w_+(n+1) + w_+(n-1) + e^{i\Omega_B t}w_-(n-1) - w_-(n+1)\right\}$$

$$i\dot{w}_-(n) = d_n w_-(n) - \frac{\Omega_{eff}}{4}\left\{w_-(n+1) + w_-(n-1) + e^{-i\Omega_B t}w_+(n-1) - w_+(n+1)\right\}$$

Assuming that $\Omega_B$ is large compared to $\Omega_{eff}$ we see that the terms proportional to $\exp(\pm i\Omega_B t)$ oscillate rapidly compared to the other coupling terms. Hence we can perform a second rotating wave approximation.
in neglecting these rapidly oscillating terms. This step decouples the $w_+$ functions from the $w_-$ functions. The new evolution equation for the $w_+(n)$ can be written in the suggestive way as follows

\[
\begin{align*}
\frac{i\hbar}{2} & \begin{pmatrix}
w_+(n+1) \\
\vdots \\
w_+(n) \\
\vdots \\
w_+(n-1)
\end{pmatrix} = \begin{pmatrix}
\ddots \\
\ddots \\
d_n(t) + 2\bar{k} \cdot \bar{a}(t - t_{n+1}) \\
\Omega_{eff}/4 \\
\Omega_{eff}/4 - \bar{k} \cdot \bar{a}(t - t_n) \\
\ddots \\
\ddots \\
\end{pmatrix} \begin{pmatrix}
w_+(n+1) \\
\vdots \\
w_+(n) \\
\vdots \\
w_+(n-1)
\end{pmatrix}
\end{align*}
\]

(34)

Here one can see that at the time $t \approx t_n$ the system $w_+(n)$ and $w_+(n-1)$ indeed performs a L.Z. transition. The coupling to other states is negligible because of the smallness of $\Omega_{eff}/(\bar{k} \cdot \bar{a} \Delta t)$.

Remembering that $P_{stay}$ gives the probability that no transition occurs one sees that the transition probability can be very close to one if the exponent is of the order of ten or larger. Since $\Omega_{eff}^2$ grows like $\Omega^4$ the laser power needs not to be much larger than in Ref. [12] to achieve this goal. Given that this is the case it is clear that the new states $\hat{w}_+$ (or $\hat{w}_-$) perform (for appropriate initial conditions) exactly that sequence of Landau-Zener transitions which we heuristically described in the last section. The introduction of the dressed states made us getting rid of the $\Omega_B$ coupling in Eq. (21) that could disturb the effectiveness of the Landau-Zener transitions.

6 A trap for atoms with drops falling out

To elucidate the practical significance of the considerations above let us refer to a cloud of atoms and consider the following initial condition: At $t = 0$ atoms are prepared with a definite momentum $\vec{q}_0$ with $|\vec{q}_0| < \hbar \bar{k}$ in the dressed state corresponding to $w_+(n = 0)$. Around $t_1$ the first L.Z. transition happens and with the probability

\[
q := 1 - \exp \left\{ -2\pi \left| \frac{\Omega_{eff}}{32 \bar{k} \cdot \bar{a}} \right| \right\}
\]

the atoms are kicked upwards with momentum $\hbar \bar{k}$. In other words, the fraction $(1 - q)$ keeps falling and forms the first drop while the fraction $q$ is returned. After this all atoms are accelerated by gravity until at the time $t_2$ atoms which have been sent upwards at $t = t_1$ fall down with momentum $-\hbar \bar{k}$. The next L.Z. transition may happen and the fraction $(1 - q)$ of these atoms keeps on falling forming the second drop while a fraction $q$ is kicked upwards, and so on. Accordingly we have obtained a trap with atoms moving upwards at $t_n$ with momentum $\hbar \bar{k}$, being decelerated by gravity to $-\hbar \bar{k}$ at $t_{n+1}$ when a fraction $q$ is "reflected" to $+\hbar \bar{k}$ and moves upwards again. The trap is leaking. Drops of atoms are falling out at the times $t_n$ with vertical momentum $-\hbar \bar{k}$ (pointing downwards). The fraction of atoms which is still within the trap at time $t$ with $t_n < t < t_{n+1}$ is

\[
P_{\text{trap}} = q^n.
\]

(36)

The fraction which is contained in the drop number $l$ is

\[
P_{\text{drop}}(l) = q^{l-1}(1 - q).
\]

(37)

With regard to experimental realizations it is interesting to note that the time when the fraction $e^{-1}$ of the atoms is still remaining in the trap ("lifetime") is $\Delta t / \ln(1/q)$ with $\Delta t$ of Eq. (27). It is about 36 ms for the experimental data given above. A slight increase of the laser power to $\Omega = 1.4 \cdot 10^7$ Hz and therefore $\Omega_{eff} = 4 \cdot 10^4$ Hz changes the lifetime to 0.5 s, however. An idea about the extension in space of the cloud of trapped atoms can be obtained in the following way: It is sufficient to consider the position classically. An atom which just performed a L.Z. transition moves upwards with velocity $\hbar \bar{k}/M$ until after a time $\Delta t/2$ it is stopped by gravity and begins to fall. The height difference between the two positions is classically given by $a\Delta t^2/8 = (\hbar \bar{k}/M)^2/(2a)$. Putting in the numbers given above this height difference turns out to be about 0.5 mm.
Above we have assumed a well defined initial momentum \( \vec{q}_0 \). For different values of \( \vec{q}_0 \) the respective times \( t_1 \) (and all \( t_n \)) differ. Lifetime and extension of the trap are not changed. The trapped atoms have always vertical momenta between \( +\hbar \vec{k} \) and \( -\hbar \vec{k} \) and atoms leak out continuously with \( -\hbar \vec{k} \).

On the basis of this picture we can easily complete our mathematical discussion. Taking again \( \vec{q}_0 \) as initial momentum and \( |w_+(n=0)|^2 = 1 \) at \( t = 0 \) we find that the quantum state at later times is decomposed with regard to the states \([2] \) with coefficients obeying at the time \( t_n < t < t_{n+1} \).

\[
|w_+(i)|^2 = \begin{cases} 
(1−q)q^j & \text{for } 0 < i < n-1 \\
q^n & \text{for } i = n \\
0 & \text{for } i < 0 \text{ and } i > n 
\end{cases}
\]  

Based on this the mean momentum at the time \( t_n < t < t_{n+1} \) turns out to be

\[
\langle \psi | \vec{p} | \psi \rangle (t_n) = M \vec{a} t_n + \vec{q}_0 + 2\hbar \vec{k} \sum_{j=0}^{n} j |w_+(j)|^2 
\]

\[
= \vec{q}_0 + \hbar \vec{k} \left\{ -2n + 1 + 2q \frac{1-q^n}{1-q} \right\}
\]

where \( \vec{q}_0 \) is that part of \( \vec{q}_0 \) which is orthogonal to \( \vec{k} \). These equations show how the undisturbed free fall is decelerated.

7 Discussion

We have investigated a three level \( \Lambda \)-system falling in a homogeneous gravitational field. To reverse the free fall of the atom it is exposed to two counterpropagating laser fields with wave vector \( \pm \vec{k} \), which induce Raman transitions, and to a magnetic hyperfine field tuned to be in resonance with the transition between the two ground states \( |±\rangle \) and \( |−\rangle \). A full quantum treatment of the coupled external and internal degrees of freedom is given. For sufficiently large detuning the excited state can be eliminated adiabatically. There remains a dynamical development which shows a.) Rabi flopping between the states \( |±\rangle \) and \( |−\rangle \) caused by the magnetic field (no accompanying change of momentum), and b.) transitions between these states with momentum transfer \( \pm 2\hbar \vec{k} \) happening for certain values of the center-of-mass momentum. These transitions turn out to be of the Landau-Zener type. We have also given a complete description of the process with reference to dressed states (interaction energy of the magnetic field included) which is theoretically more elegant. In this description it becomes even more evident that the fundamental transitions are the Landau-Zener transitions caused by the combined influence of the lasers and the gravitational field. They disappear if one of these influences is switched off. Accordingly they represent a true gravito-optical effect.

The resulting center-of-mass motion is the following: If the atom starts with momentum \( |\vec{p}| < \hbar \vec{k} \) it falls until \( \vec{p} = -\hbar \vec{k} \) is reached and the transition takes place. Afterwards it has obtained \( \vec{p} = +\hbar \vec{k} \), falls again until \( \vec{p} = -\hbar \vec{k} \) and so on. Periodically the gravitationally caused fall (actio) induces in combination with the influence of the lasers an internal transition (reactio) with momentum transfer which turns the atom into an upward motion. Obviously such a process can among other applications be used to construct a coherent atom trap.

It is instructive to interpret the result also in the light of the equivalence principle. This method has already been used in Refs. [16, 3] to explain the gravitational influence of atoms in an interferometer and in a running laser wave. In our case the equivalence principle states that the atom moving in the Earth’s gravitational field is equivalent to a free atom moving in an uniformly accelerated reference frame to which the sources of the electromagnetic fields are fixed. In this reference frame the potential term \( \vec{M} \vec{a} \cdot \vec{x} \) is absent, but the Doppler effect causes the laser frequencies to change according to

\[
\omega'_\pm = \omega_\pm \mp \vec{k} \cdot (\vec{v} + \vec{a}t) .
\]  

The absolute change of the magnetic field frequency \( \omega_B \) is negligible for our argument. Let the atom be initially at rest and in resonance with the lasers. If the atom now makes a transition from \( |+\rangle \) to \( |−\rangle \) it gets the momentum \( 2\hbar \vec{k} \). Consequently, due to the Doppler effect, the atom will be out of resonance. As can be
seen in Eq. (10) an acceleration \( \vec{a} \) which is anti-parallel to \( \vec{k} \) will decrease the momentum until the atom is in resonance again. This is the time when the next Landau-Zener transition can happen. Consequently the atom is periodically tuned into and out of resonance with the laser fields.

Let us finally ask the question, what are the physical processes which abort the sequence of Landau-Zener transitions even for high laser powers? The most stringent restriction comes from the breakdown of the adiabatic elimination procedure for the excited state. As we have seen above this will happen after 1/80 second. But this is about the time which lies between two subsequent transitions. Hence with contemporary experimental devices we only can hope to see the onset of the transition sequence. Another process which weakens the efficiency of the sequence is the coupling between \( \tilde{\omega}_+ \) and \( \tilde{\omega}_- \) which is neglected here after the rotating wave approximation with respect to \( \Omega_B \) in Eq. (33). Since it is likely that an \( \Omega_B \) exceeding \( \Omega_{eff} \) by several powers of ten cannot be produced with the present technology this effect can be relevant. Numerical investigations indicate that for \( \Omega_B \approx 10\Omega_{eff} \) the population transfer from \( \tilde{\omega}_+ \) to \( \tilde{\omega}_- \) is in the order of 10 \%. We therefore expect that this process will interrupt the Landau-Zener sequence after a few transitions.

Note added: After the submission of this paper we became aware of a preprint [17] reporting the measurement of Bloch oscillations of atoms. The connection between these oscillations and the present work will be considered elsewhere [18].

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Figure captions

Figure 1: A three-level atom (Λ-system) is simultaneously exposed to two counterpropagating running laser beams, a magnetic hyperfine field causing transitions between the two ground states, and the gravitational field of the Earth.

Figure 2: Pictorial representation of the ladder of influences in Eq. (21). The Landau-Zener-like Raman transitions (L.Z.) are connected with a momentum transfer of $2\hbar \vec{k}$ to the atom (diagonal arrows) whereas the magnetic hyperfine transition ($\Omega_B$) induces an interaction between states of equal momentum (horizontal arrows). Both transitions together couple only a discrete set of momentum states labeled by $u_\pm(n)$.

Figure 3: Total energy $E_{tot}$ of the two ground states $|+\rangle$ and $|-\rangle$ of a Λ-system as a function of its momentum $\vec{p}$. The Landau-Zener transition between $|+\rangle$ and $|-\rangle$ induced by the laser fields is connected with an energy transfer $\pm(E_+ - E_-)$ and a momentum transfer $\pm2\hbar \vec{k}$. Because of the associated change in the kinetic energy such a transition can only be in resonance if the atom has the momentum $\vec{p} = +\hbar \vec{k}$ or $\vec{p} = -\hbar \vec{k}$ (solid line arrow). In addition, there are for all values of $\vec{p}$ Rabi oscillations caused by the magnetic field (vertical dashed arrows).
