Coherent spin-wave processor of stored optical pulses

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A device being a pinnacle of development of an optical quantum memory should combine the capabilities of storage, inter-communication and processing of stored information. In particular, the ability to capture a train of optical pulses, interfere them in an arbitrary way and finally perform on-demand release would in a loose sense realize an optical analogue of a Turing Machine. Here we demonstrate the operation of an optical quantum memory being able to store optical pulses in the form of collective spin-wave excitations in a multi-dimensional wavevector space. During storage, we perform complex beamsplitter operations and demonstrate a variety of protocol implemented as the processing stage, including interfering a pair of spin-wave modes with 95% visibility. By engineering the phase-matching at the readout stage we realize the on-demand retrieval. The highly multimode structure of the presented quantum memory lends itself both to enhancing classical optical telecommunication as well as parallel processing of optical qubits at the single-photon level.

I. INTRODUCTION

As optical quantum memory technologies are becoming more mature, the range of applications increases. The basic memories operating in a single temporal and spatial mode can store only one optical pulse and interfere it with second pulse only during light-atom coupling [1]. Such memories, based either on Raman scattering or electromagnetically induced transparency (EIT), can achieve high efficiencies [2], but offer very limited capacity as multiplexing is limited by the number of atomic magnetic sublevels employed [2–5]. While a single atomic ensemble may be split into an array to offer parallel storage of light [6, 7], such scheme hinders manipulations within the memory as communication between memory cells must be inherently light-based. It is thus highly desirable to independently store many optical pulses within the same group of atoms. Such a multiplexing scheme may utilize either the spatial [8, 9] or temporal degree of freedom [10, 12]. In the latter case considered in the context of the atomic-ensemble based quantum memories the Gradient Echo Memory (GEM) [13,17] scheme stands out as an efficient way to engineer the phase-matching at readout stage to achieve mode-selective storage and retrieval. Similar feature is inherently offered by the atomic frequency comb (AFC) memories based on ensembles of ions in solids [10, 11, 18] thanks to their large inhomogeneous broadening. In the spatial degree of freedom atomic ensembles allow storage of light in many angular-emission modes through spin-wave wavevector multiplexing [8, 19]. These schemes allow storage of hundreds of optical modes, also when used with non-classical states of light.

Manipulation of stored optical pulses, however, remains a substantial challenge, both from technical and fundamental points of view. The AFC memory has been demonstrated to allow pre-programmed interference of two stored pulses with a single-output port [20, 21], and within the GEM scheme a beamsplitter operation between a pre-selected stored pulses and an input pulse has been realized [22, 23]. These schemes also allow basic spectral and temporal manipulations of stored light. More work is needed however to reach the regime of efficient and arbitrary manipulations of stored light. In particular, the ac-Stark shift caused by an additional light field has been proposed as a versatile way to realize the GEM scheme [24]. Recent theoretical proposals went beyond the simple gradient shape and suggested to engineer the stored spin-wave shape to realize Kapitsa-Dirac diffraction [25] or a quantum memory protected with a disordered password [26]. Finally, a recent experiment used the ac-Stark shift to realize a spin-wave beamsplitter at the single-excitation level demonstrating Hong-Ou-Mandel interference for stored light [27].

Here we present the first realization of ac-Stark-based spin-wave universal multiport interferometric processor (SUMIP) and join the advantages of the transverse-wavevector and temporal multiplexing to realize a variety of operations on the stored coherent states of light. The previously untackled regime of complex light patterns used to engineer spin waves is explored, which allows us to tap into the full three-dimensional potential of the wavevector-multiplexed optical quantum memory. We show that thanks to engineering of the spatial profile of ac-Stark modulation the stored pulses may be processed, interfered and conditionally retrieved. The scheme features both reprogrammable reordering and interference of pulses within the multiple-input, multiple-output paradigm, essential to realize true unitary operations. In the paper we first introduce the protocol by deriving its theoretical principles and realizing a scheme reminiscent of the Gradient Echo Memory [14]. Next, we realize a series of programmable beamsplitting experiments in spatial and temporal degrees of freedom. High-visibility interference of a pair of modes is demonstrated. Finally, we propose potential further applications and give technical details of the experiment and light-atom propagation simulations involved.

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II. OPERATION OF THE LIGHT-ATOM INTERFACE

The atomic optical memory based on an elongated ensemble of Rb-87 atoms employs a strong control field \( \mathcal{E}_C \) to map a weak signal field \( \mathcal{E}_s \) onto the atomic coherence \( \rho_{gh} \), between the two meta-stable ground states, for which we take \( |g\rangle \to F = 1, m_F = -1 \), and \( |h\rangle \to F = 2, m_F = 1 \) (see Fig. 1c for the atomic level scheme). In the experiment the atoms are first optically pumped to the \( |g\rangle \) state and control and signal fields operate with opposite circular polarizations. Typically we use 300 ns long pulses for storage and retrieval of atomic coherence.

While the interaction is well characterized by a set of coupled Maxwell-Bloch equations (see Appendix A), first we rather choose to describe the atom-light coupling qualitatively. In particular, Fig. 1b illustrates the geometry in which the coupling and signal fields co-propagate through an elongated atomic ensemble. Assuming that the coupling beam diameter is significantly larger than the transverse size of the ensemble, we may actually solve the coupled equations within the first order in the coupling strength and obtain a simple result by which a signal \( \mathcal{E}(k_x, k_y) \) couples to an atomic coherence

\[
\rho_{gh}(K_x, K_y, K_z) \propto \mathcal{E}_s(k_x = K_x, k_y = K_y) \exp(i\Delta_0 t)\delta(K_z - K_z),
\]

where \( K_z = \sqrt{\omega^2/c^2 - k_x^2 - k_y^2} - \omega_C/c, \delta \) is a Dirac delta function and \( \omega \) and \( \omega_C \) are frequencies of signal and coupling fields, respectively, and \( c \) is the speed of light. For \( k_x = k_y = 0 \) the longitudinal wavevector simplifies to a constant component \( K_z = c/\Delta_0 \approx 0.14 \text{ rad mm}^{-1} \), where \( \Delta_0 \approx 2\pi \times 6.8 \text{ GHz} \) is the nominal frequency splitting between levels \( |g\rangle \) and \( |h\rangle \). To exclude these trivial dependencies from further consideration we will define the stored spin-wave excitation as

\[
S(K_x, K_y, K_z) = \rho_{gh}(K_x, K_y, K_z) \exp(-i\Delta_0 t),
\]

where \( \mathcal{F} \) stands for the Fourier transform in the spatial domain, \( N(x, y, z) \) is the atom number density and * denotes convolution (here in the wavevector space). Importantly, after mapping the optical field we obtain a spin-wave excitation with \( K_z = 0 \) in terms of \( S \).

The process of reverse mapping or retrieval driven by the same coupling field occurs in a symmetric way. Essentially, an atomic spin-wave excitation will be mapped onto an optical field proportional to \( S \) in terms of the transverse wavevector dependence only if \( K_z = 0 \). This requirement arises due to the phase-matching condition. In particular, the allowed spread in the \( K_z \) space is inversely proportional to the atomic cloud length \( \sigma_z \) and most importantly spin waves with large \( K_z \) component \( (K_z \sigma_z \gg 1) \) will remain stored in the memory. This remains true unless we change the frame of reference significantly by selecting much different \( K_x, K_y \), as the actual phase matching is satisfied on an ellipsoid in a \( K \) space rather than a plane. Its curvature will depend on the particular geometrical configuration. Here we will remain within the regime where we may use the phase-matching planar approximation to consider which spin waves are retrievable.

III. SPIN-WAVE MANIPULATION WITH THE AC-STARK EFFECT

As discussed above, only a limited space, or more precisely a thin three-dimensional volume around \( K_z = 0 \) plane in the wavevector space may be populated by spin waves by means of Raman interaction. To manipulate the spin waves within and beyond this volume we use an additional far off-resonant beam [marked in Fig. 1(c)
as \(\text{acS}\) that induces an additional differential ac-Stark shift between levels \(|g\rangle\) and \(|h\rangle\) of \(\Delta_{\text{acS}}\) that adds to \(\Delta_0\). The ac-Stark beam propagating along the \(y\)-direction is \(z\)-polarized and red-detuned by approx. 1 GHz from the \(|h\rangle \rightarrow |e\rangle\) transition and is inducing \(\Delta_{\text{acS}} \sim 1\) MHz ac-Stark shift with \(\sim 100\) mW beam power. This shift causes the atomic coherence \(\rho_{gh}\), and thus the spin wave, to accumulate an additional phase \(\varphi_{\text{acS}} = \Delta_{\text{acS}} T\) over the interaction time \(T\). Typically we use ac-Stark pulses of approx. \(T \sim 2\) \(\mu\)s duration. By spatially shaping the ac-Stark beam intensity \(I_{\text{acS}}(x,z)\) we induce a spatially-dependent phase shift \(\varphi_{\text{acS}}(x,z) \propto I_{\text{acS}}(x,y)\), which due to the geometry of the experiment is limited to two dimensions [see Figure 1(b)]. Any spin wave is then re-shaped as:

\[
S(K_x,K_y,K_z) = \int \int (x,z) \mathcal{F}[\exp(i\varphi_{\text{acS}}(x,z))](k_x,k_z)
S(k_x + k_z,k_y,K_z + k_z)dk_xdk_z. \quad (1)
\]

A basic example is an ac-Stark analogue of the GEM, in which a phase shift linear in \(z\) \((\varphi_{\text{acS}} = \beta z)\) induced by a magnetic field gradient shifts the spin wave in the \(K_z\) direction by \(\beta\).

Here we work with optical modes characterized by small wavevector (angular) spread and therefore the spin waves are well-localized in the wavevector space. Discrete mode transformation in such a space are most conveniently performed by a spatially periodic ac-Stark modulation. Taking a spatial period \(2\pi/k_{\text{acS}}\) of the modulation in the form \(\varphi_{\text{acS}}(x,z) = \varphi_{\text{periodic}}(k_{\text{acS}}') \cdot (x,z)\) we may express the spin-wave transformation using Fourier series as:

\[
S(K) \varphi_{\text{acS}}(x,z) \sum_{n=-\infty}^{+\infty} c_n S(K + nk_{\text{acS}}), \quad (2)
\]

with Fourier coefficients \(c_n\) defined as:

\[
c_n = \frac{1}{2\pi} \int_0^{2\pi} \exp(i\varphi_{\text{periodic}}(\xi) - in\xi)d\xi. \quad (3)
\]

Figure 1(a) presents a simplified simulation (assuming perfect write-in and read-out) of a protocol operation within this paradigm. In this exemplary protocol three pulses \(\mathcal{E}_0, \mathcal{E}_{-k}, \mathcal{E}_k\) with transverse optical wavevector component \(k_x\) equal \(0, -k\) and \(k\), respectively, are stored, processed and released from the memory. When the first pulse is mapped to the ensemble, the created spin wave is phase-modulated using a sawtooth-shaped \(\mathcal{M}\) (periodic) modulation \(\varphi_{\text{acS}}(x,z) = \varphi_{\text{acS}}(k_{\text{acS}}')\). Such a modulation with amplitude equal \(2\pi\) shifts the spin wave in the \(K_z\) direction by \(k_{\text{acS}}'\), making the spin wave unreadable, as \(k_{\text{acS}}'z \gg 1\). Then, two pulses with \(k_z\) components separated by \(2k\) arriving at the same time are written to the ensemble. Next, square-shaped \(\mathcal{M}\) modulation \(\varphi_{\text{acS}}(x,z) = \varphi_{\text{acS}}(k_{\text{acS}}')\) with \(k_{\text{acS}}' = 2k\) is applied. The amplitude and phase of this modulation are chosen so the first three Fourier coefficients are in following relations \(c_0 = 0 = -c_2\). This way the two pulses with non-zero \(k_x\) component are combined in a way reminiscent of a two mode beam-splitter transformation [27] resulting in constructive interference in the mode with \(k_x = -k\).

Note that the spin wave component corresponding to the first pulse is transformed as well (split), but because of shifted \(K_x\) and most importantly \(K_z\) component it does not take part in the interference. At this stage the first readout is performed during which all readable (zero \(K_z\)) spin wave components are converted to light pulses. To read out the first pulse, all the previous transformations are undone by applying versions of previous modulations shifted by half a period in reversed order. At the very end of the protocol the first pulse is retrieved.

IV. RECONFIGURABLE AC-STARK ECHO MEMORY

For the experimental demonstration we begin by moving the spin waves outside the zero \(K_z\) to allow storage of subsequent incoming optical pulses. This configuration, most reminiscent of the GEM, here operates best with a triangle-shaped \(\mathcal{M}\) grating (pattern A), which can be conventionally written in a closed form:

\[
\varphi_{\text{acS}}(\xi) = A^\mathcal{M} \cdot \delta \left( \frac{\xi}{2\pi} - \left( \frac{\xi}{2\pi} + \frac{1}{2} \right) \right), \quad (4)
\]

with \(k_{\text{acS}}' = 9.6\text{mm}^{-1}\) for which most essentially the zeroth order \(c_0^\mathcal{M} \propto |\text{sinc}(A/2)|\) disappears periodically with modulation strength \(A^\mathcal{M}\) (with period equal \(2\pi\)) except for \(A^\mathcal{M} = 0\). With this scheme we may thus apply a grating with \(A^\mathcal{M} = 2\pi\) and remove the pulse from the \(K_z = 0\) plane. Due to the periodicity of \(c_0^\mathcal{M}\) in the modulation strength \(A^\mathcal{M}\), if a subsequent pulse is stored, the first and any previous pulse remains phase-mismatched at consecutive grating operations with amplitude \(A^\mathcal{M} = 2\pi\). To retrieve the pulses we apply a pattern with the same amplitude shifted by a half of period in the spatial domain (pattern B), that restores the spin waves to the \(K_z = 0\) plane.

The scheme lends itself to both first-in, first-out (FIFO) and last-in, first-out (LIFO) operation, as shown in Fig. 2. For the FIFO operation on two pulses, after storage of a second pulse we apply a shifted pattern B to simultaneously transfer the first pulse back to the \(K_z = 0\) plane and phase-mismatch the second pulse. After first retrieval operation the phase matching is restored for the second pulse with pattern B.

The efficiency of our memory is currently limited by the optical depth of the ensemble as well as available coupling power. By comparing the intensity of light at the input and output of the memory we obtain write-in efficiency for the first pulse of about 39% and 44% for the second
Figure 2. Operation of the ac-Stark Echo Memory. Panel (a): ac-Stark triangle-wave (\(\mathbb{W}\)) modulation patterns for storage (pattern A) retrieval (pattern B) of subsequent pulses. Pattern B is a shifted by a half of a period version of pattern A with the same amplitude. Storage and retrieval of two coherent optical pulses in the last-in, first-out (LIFO) as well as first-in, first-out (FIFO) configurations [panels (b) and (c), respectively] and the LIFO configuration for storage of three pulses [panel (d)].

pulse. For immediate retrieval (as for the second pulse in LIFO scheme) we achieve 35% efficiency of retrieval, while net storage and retrieval efficiency is equal 44% \(\times\) 35% = 15%. For the pulses that are manipulated the efficiency is diminished by dephasing due to the ac-Stark light intensity inhomogenities [28] (see Appendix B for details).

V. PROGRAMMABLE BEAMSPLETTING OF STORED OPTICAL PULSES

To demonstrate the beamsplitting capability for pulses arriving at different times we use again the triangle-wave modulation in the \(z\)-direction, with \(k_\omega = 22 \text{ mm}^{-1}\). After subsequent storage of two pulses (which is done the same way as in FIFO and LIFO demonstration using pattern A) we apply the shifted pattern B for a half of period \(T\), modulating the spin wave with amplitude \(\pi\) instead of \(2\pi\). This way the two pulses are combined and \(K_z = 0\) component of resulting spin wave becomes the first output port of the temporal-mode beamsplitter. Then, after the first readout, we modulate the unread part again with pattern B with amplitude \(A \approx 2.25\pi\) to transfer a part of the second port to readable \(K_z = 0\) plane, then the second readout is performed. Note that it is crucial to always perform the first readout, as otherwise the unread spin wave will interfere and spoil the operation of the second output port. It is thus necessary to simulate the operation of this scheme to a full extent, including possibly imperfect first readout which can affect the second output port.

To characterize the two pulses interference we change the relative phase between the pulses by changing the two-photon detuning \(\delta\) to observe intensity fringes. Essentially, the phase difference between the two interfering spin waves is the product of the two-photon detuning \(\delta\) and the time between two first pulses \(\tau\). Furthermore, as we move outside the two-photon resonance the interaction becomes inefficient. This behavior constitutes the Ramsey interference. In Fig. 3(b) we plot the total number of photons collected after the first (port 1) and second (port 2) readout as a function of the two-photon detuning \(\delta\). The observed behavior is properly predicted by the simulation described in detail in Appendix A.

The relative phase between the pulses can be also modified within the spin-wave domain. To demonstrate this, we implement another interference protocol; instead of splitting the first pulse into many orders we simply shift its \(K_z\) component by \(k_\omega\) using sawtooth wave \(\mathbb{V}\) modulation

\[
\varphi_\mathbb{acS}(\xi) = A^\mathbb{acS}(\frac{\xi}{2\pi} - \frac{\xi}{2\pi})
\]

in the \(z\)-direction. Then, the second pulse is written to the memory and the resulting spin wave is modulated using a triangle-shaped grating of depth \(A^\mathbb{acS} \approx 1.16\pi\) satisfying the equation \(|c_0^\mathbb{acS}| = |c_1^\mathbb{acS}| = |c_{-1}^\mathbb{acS}|\). The spatial period of the \(\mathbb{V}\) modulation is chosen to satisfy \(k_\mathbb{acS} = k_\omega = 22 \text{ mm}^{-1}\), thus the pulses are combined in such a manner that the zeroth order of the first pulse overlaps with first diffraction order of the second pulse and conversely. The first interferometer port is again a resulting \(K_z = 0\) spin wave component so it can be com-
VI. TRANSVERSE SPACE INTERFERENCE AND MANIPULATION

To go beyond a single transverse mode we now add the $K_x$ dimension to the scheme. In a simple yet highly robust scenario we map two equally bright pulses arriving at the same time yet into two different spin waves with $K_x = \pm k$, where $k = 75.4$ mm$^{-1}$ using a pattern presented in Fig. 5(a). We then apply a sinusoidal grating modulation $\gamma_{\text{periodic}}(\xi - \zeta)$ the complex amplitudes of subsequent orders change as $c_n \sim e^{in\zeta}$. We directly witness this behavior by shifting the sawtooth $\mathcal{W}$ grating portrayed in Fig 4(a) in the $z$-direction and measuring interference fringes in the total energy of the released pulses. In Fig. 4(c) we plot the resulting interference pattern, accompanied with a proper simulation, showing the interference in wavevector space [Fig. 4(b)].
We scan the phase using a piezo-actuated mirror and thus write the (lossy) beamsplitter transformation as:

\[
\begin{pmatrix}
    c_{\text{out}}^+ \\
    c_{\text{out}}^-
\end{pmatrix} = C \begin{pmatrix}
    1 & e^{i\zeta} \\
    e^{-i\zeta} & 1
\end{pmatrix} \begin{pmatrix}
    c_{\text{in}}^+ \\
    c_{\text{in}}^-
\end{pmatrix}
\]

VII. SIMULTANEOUS SPIN-WAVE PROCESSING IN TWO DIMENSIONS

Finally we combine the longitudinal and transverse manipulations to exhibit the time-space interference of two sequentially stored pulses. To access the time-space beamsplitting we design the square-wave grating \( \varphi_{\text{acS}}^m(\xi) = A^m \left( 2 \left| \frac{\xi}{2\pi} \right| - \left| \frac{\xi}{2\pi} \right| + 1 \right) \) in the \( x-z \) direction \( \varphi_{\text{acS}}^m(k^m_w \cdot (x,y)) \), where \( k^m_w = 12 \text{ mm}^{-1} \hat{e}_x + 5 \text{ mm}^{-1} \hat{e}_z \). The periodic collapse-revival behavior of a \( c_{\text{in}}^m \) allows us to use this very grating for both subsequent storage and interference of two coherent pulses. By applying the square-wave modulation (\( \mu_m \), pattern A in Fig. 6b) of amplitude \( A^m = \pi \) after arrival of the first pulse and \( A^m = \pi/2 \) after the second pulse is stored we combine the pulses in \( t-x \) space. As in previous cases, the zero-\( K_z \) component of the processed spin wave becomes the first port of the Mach-Zehnder interferometer. To sample the second port (which is spread into successive diffraction orders \( c_{\text{in}}^m \neq 0 \)) we use sawtooth grating in the \( z \)-direction \( \varphi_{\text{acS}}^m(k^m_{\text{acS}} z) \) with \( k^m_{\text{acS}} \hat{e}_z = 5 \text{ mm}^{-1} \) (pattern B in 6b) to transfer the 1-st order to readable \( K_z = 0 \) space, and perform the readout. The simulation reveals that in this protocol the two output ports turn out to not be in perfectly opposite phases. Same behavior is observed in the experiment, as demonstrated in Fig. 6d. We attribute this effect to imperfect retrieval of the first port which in turn is partially leaks to the second read-out operation. We envisage that further simulations will facilitate a more elaborate scheme that could yield two output ports that are perfectly in opposite phases, as in experiments described in Sec. V and VI.

VIII. DISCUSSION

We have demonstrated a reprogrammable device that processes atomic spin waves through interference. Starting with the first demonstration of an ac-Stark controlled atomic memory for light we have extended the concept of ac-Stark control to enable interference of coherent spin-wave states stored in the memory. In particular, the processing is performed simultaneously in two dimensions of the wavevector space. With this, we simultaneously exploit temporal and spatial multiplexing. We show how to perform spin-wave interference between light pulses stored both at different times, as well as sent to the memory at different angles. By switching only a pair of patterns we achieve a substantial degree of reprogrammability and control, which paves the way towards creating complex unitary quantum networks through spin-wave interference.

The demonstrated optical processor lends itself to many critical schemes in quantum and classical telecommunication, including the quantum memory-enabled superadditive communication [29, 32] or implementation...
Figure 6. Processing and interference in the two-dimensional space. (a) phase modulation patterns used to split spin waves in the tilted wavevector-space direction (pattern A) and shift them in the $K_z$ direction (pattern B). (b) Simulated time trace of the scheme demonstrating spin-wave processing in two wavevector-space dimensions. (c) Simulated wavevector-space spin wave densities at different time instants of the protocol, with subsequent numbers corresponding to marked positions in (a). In the protocol the spin wave with amplitude $\alpha$ is first created by storing a single coherent pulse (1). Next, the spin wave is split using pattern A (2) and another pulse is stored to create spin wave with amplitude $\beta$ (3). Pattern A is then applied again to cause interference of the two stored pulses (4). Readout is then performed to first observe the output port corresponding roughly to $\alpha + \beta$, and after shifting the spin wave in the $K_z$ direction with pattern B the output port corresponding to the $\alpha - \beta$ component. Panel (d) portrays intensities registered in the two output ports as a function of relative phase between pulses. Solid lines correspond to the simulated result.

of a receiver operating with an error rate below the standard quantum limit [33] as well as quantum metrology through collective measurements on many optical pulses [34, 35]. The ability of programming interference of stored states provides a robust tool for probing fundamental properties of quantum systems. Recently, a tunable beamsplitter transformation has been used to demonstrate Hong-Ou-Mandel interference between two microwave quantum memories [36]. The techniques presented here pave the way towards programmable complex interference experiments which can be used to reveal fundamental properties of given quantum system.

The ac-Stark control owes its versatility to the possibly very high speed of switching and operation, as compared with magnetic field gradients. This feature makes it applicable to recently developed short-lived quantum memories that operate in the ladder atomic scheme in warm atomic vapors and achieve very low noise levels [37, 38]. The high speed of the ac-Stark control also facilitates real-time feedback processing that could lead to realization of an even broader class of operations, including enhanced single-photon generation through multiplexing [8, 39, 41]. Here, such a scheme could also include engineering of photonic spatial and temporal mode. This could be taken even further with techniques used in stationary-light experiments, where amplitude of the stored spin-wave is non-destructively reshaped using a multi-laser field [42, 43].

Furthermore, note that here we did not use the ac-Stark shift during write and read operations of the optical memory, and thus the two-photon absorption line is not broadened. Thus, the Gradient Echo Memory advantage of avoiding reemission of stored is not yet exploited. Combined with larger optical densities this could significantly improve efficiency of the presented memory [16].

Finally, by bringing the presented techniques to spin-waves that involve a Rydberg state [44, 48], the attainable range of operations between storage modes could be enriched with nonlinear interactions in order to realize efficient and deterministic quantum gates for photonic states. This could be particularly advantageous in engineering complex correlations within the spatial domain of a Rydberg atomic ensemble [49].

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**APPENDIX A: LIGHT-ATOM COUPLING SIMULATIONS**

To correctly predict efficiencies during storage and retrieval as well as non-trivial shapes of spin waves created in the atomic ensemble, we choose to describe the system within the three-level model described by an interaction picture Hamiltonian within the rotating wave approximation, which is subsequently reduced using the typical adiabatic elimination approach by setting the time derivatives of all excited-state coherences and populations to zero. The coupled equations are then most conveniently expressed within the frame co-moving with the pulses (t → t - z/c) and in the frame co-moving with the pulses (t → t - z/c)

The equations take the following form (see Refs. [50–53]):

\[
\frac{d\Omega}{dz} = -i g N^{-1/2} S^+ \Omega_C + \Omega / (2\Delta + i\Gamma) \tag{8a}
\]

\[
\frac{dS}{dt} = \frac{1}{2} N^{1/2} \Omega_C^* \Omega + \left( \frac{-2\Gamma\delta - 2\gamma\Delta + i\Gamma\gamma N^2\Omega_C^2 - 4i\delta\Delta}{2(2\Delta - i\Gamma)} \right) S + i\Delta_{acS} S \tag{8b}
\]

where we have also introduced \( \Gamma = 2\pi \times 6 \text{ MHz} \) as the excited state \( |e\) decay rate and \( \gamma \approx 2\pi \times 10 \text{ kHz} \) as the intrinsic spin-wave decoherence rate, dominated by motional dephasing. The one-photon \( \Delta = 2\pi \times 20 \text{ MHz} \) and two-photon \( \delta \) detunings are defined as in Fig. 1(b). In the first equation the two terms in the nominator correspond to the two-photon and one-photon processes, respectively. In the second equation the first term corresponds to the Raman interaction, while the second term is the free evolution under the ac-Stark shift Hamiltonian due to the coupling light, which includes both the additional phase acquired as well as the power broadening. Even though the ac-Stark modulation is applied only during dark periods of the memory, for completeness we include its influence as an additional term in Eq. 8b given by \( i\Delta_{acS} S \). Note that in all cases the atom number density \( N \) is implicitly \( z \)-dependent, and so is the coupling constant \( g \). In the simulation we model this dependence as a Gaussian function in the \( z \) dimension with a width of 5 mm. Finally, we also add a small imaginary component due to inhomogeneous ac-Stark light intensity.

We determine the coupling constant \( g \) by observing single-photon off-resonant absorption. This allows us to experimentally determine its peak value as \( g_0 \approx 200 \text{ cm}^{-1} \mu\text{s}^{-1} \), which corresponds to the optical depth OD \( \approx 70 \). For the coupling field we take short pulses with smooth slopes (modeling \( \approx 100 \text{ ns experimental rise times} \) and peak \( \Omega_C \approx 2\pi \times 9 \text{ MHz} = 1.5\Gamma \). Typical signal field intensities correspond to peak \( \Omega \approx 2\pi \times 50 \text{ kHz} \). The evolution is simulated using the XMDS package [54] on a two-dimensional \( z-t \) grid, or three dimensional \( x-z-t \) grid for the results in Section VI B. For this case we also include a diffraction term in Eq. 8a although the diffraction effects prove to be negligible for the plane-wave modes we work with.

**APPENDIX B: PATTERN PREPARATION AND IMAGING**

The ac-Stark laser is frequency-stabilized using an offset beat-note lock [53]. It is then spatially filtered using a single-mode fiber and amplified using a tapered amplifier.

Figure 7. Schematic illustrating the rapidly-reprogrammable double pattern imaging system (HWP - half-wave plate, QWP - quarter-wave plate, PZT - piezoelectric transducer, AOM - acousto-optic modulator). The AOM diffracts the input beam onto disparate regions of the SLM, which are then simultaneously imaged onto the MOT after being combined on the polarizing beamsplitter (PBS). The final PBS projects both light fields onto the \( z \) polarization.
of the SLM is used for fine adjustment of grating position and the final PBS. An additional mirror placed in the far-field in the current configuration we loose half of the power at the pattern on a different path. The two paths are joined then imaged onto a D-shape mirror which sends each pattern in two disparate regions. The SLM surface is synthesizer (DDS). On the SLM matrix we display two acousto-optical modulator (AOM) situated in the far-field of the elongated atomic ensemble. Simultaneously, an reshaped using a cylindrical lens to better fit the shape of stored dual-channel spin-wave excitations in a single tripod system, Quantum interference qubits in a spatially-multiplexed cold atomic ensemble, \textit{Nature Communications} \textbf{9}, 363 (2018)

3. H. Wang, S. Li, Z. Xu, X. Zhao, L. Zhang, J. Li, Y. Wu, C. Xie, K. Peng, and M. Xiao, Quantum interference of stored dual-channel spin-wave excitations in a single tripod system, \textit{Physical Review A} \textbf{83}, 043815 (2011)

4. M.-J. Lee, J. Ruseckas, C.-Y. Lee, V. Kudrišas, K.-F. Chang, H.-W. Cho, G. Juzelianas, and I. A. Yu, Experimental demonstration of spinor slow light, \textit{Nature Communications} \textbf{5}, 5542 (2014)

5. Z. Xu, Y. Wu, L. Tian, L. Chen, Z. Zhang, Z. Yan, S. Li, H. Wang, C. Xie, and K. Peng, Long Lifetime and High-Fidelity Quantum Memory of Photonic Polarization Qubit by Lifting Zeeman Degeneracy, \textit{Physical Review Letters} \textbf{111}, 240503 (2013)

6. S.-Y. Lan, A. G. Radnaev, O. A. Collins, D. N. Matsukevich, T. A. Kennedy, and A. Kuzmich, A multiplexed quantum memory, \textit{Optics Express} \textbf{17}, 13639 (2009)

7. Y.-F. Pu, N. Jiang, W. Chang, H.-X. Yang, C. Li, and L.-M. Duan, Experimental realization of a multiplexed quantum memory with 225 individually accessible memory cells, \textit{Nature Communications} \textbf{8}, 15359 (2017)

8. M. Parniak, M. Dabrowski, M. Mazelaniak, A. Leszczyński, M. Lipka, and W. Wasilewski, Wavevector multiplexed atomic quantum memory via spatially-resolved single-photon detection, \textit{Nature Communications} \textbf{8}, 2140 (2017)

9. D.-S. Ding, Z.-Y. Zhou, B.-S. Shi, G.-C. Guo, and S. E. Harris, Single-photon-level quantum image memory based on cold atomic ensembles, \textit{Nature Communications} \textbf{4}, 183601 (2013)

10. M. Gündogan, M. Mazzera, P. M. Ledingham, M. Cristiani, and H. de Riedmatten, Coherent storage of temporally multimode light using a spin-wave atomic frequency comb memory, \textit{New Journal of Physics} \textbf{15}, 045012 (2013)

11. K. Kuthner, M. Mazzera, and H. de Riedmatten, Solid-State Source of Nonclassical Photon Pairs with Embedded Multimode Quantum Memory, \textit{Physical Review Letters} \textbf{118}, 210502 (2017)

12. A. Tiranov, P. C. Strassmann, J. Lavoie, N. Brunner, M. Huber, V. B. Verma, S. W. Nam, R. P. Mirin, A. E. Lita, F. Marsili, M. Afzelius, F. Bussières, and N. Gisin, Temporal Multimode Storage of Entangled Photon Pairs, \textit{Physical Review Letters} \textbf{117}, 240506 (2016)

13. J. Nunn, K. Reim, K. C. Lee, V. O. Lorenz, B. J. Sussman, I. A. Walmsley, and D. Jaksch, Multimode Memories in Atomic Ensembles, \textit{Physical Review Letters} \textbf{101}, 260502 (2008)

14. M. Hosseini, B. M. Sparkes, G. Hétel, J. J. Longdell, P. K. Lam, and B. C. Buchler, Coherent optical pulse sequencer for quantum applications, \textit{Nature} \textbf{461}, 241–5 (2009)

15. M. Hosseini, B. M. Sparkes, G. Campbell, P. K. Lam, and B. C. Buchler, High efficiency coherent optical memory with warm rubidium vapour, \textit{Nature Communications} \textbf{2}, 174 (2011).

16. B. M. Sparkes, J. Bernu, M. Hosseini, J. Geng, Q. Glorieux, P. A. Altim, P. K. Lam, N. P. Robins, and B. C. Buchler, Gradient echo memory in an ultra-high optical depth cold atomic ensemble, \textit{New Journal of Physics} \textbf{15}, 085027 (2013)

17. B. Albrecht, P. Ferrara, G. Heinze, M. Cristiani, and H. De Riedmatten, Controlled Rephasing of Single Collective Spin Excitations in a Cold Atomic Quantum Memory, \textit{Physical Review Letters} \textbf{115}, 160501 (2015)

18. N. Sinclair, E. Saglamyurek, H. Mallahzadeh, J. A. Slater, M. George, R. Ricken, M. P. Hedges, D. Ohablak, C. Simon, W. Sohler, and W. Tittel, Spectral Multiplexing for Scalable Quantum Photonics using an Atomic Frequency Comb Quantum Memory and Feed-Forward Con-
trol, Physical Review Letters 113, 053603 (2014)

[19] H.-N. Dai, H. Zhang, S.-J. Yang, T.-M. Zhao, J. Rui, Y.-J. Deng, L. Li, N.-L. Liu, S. Chen, X.-H. Bao, X.-M. Jin, B. Zhao, and J.-W. Pan, Holographic storage of biphoton entanglement, Physical Review Letters 108, 210501 (2012).

[20] N. Sinclair, D. Oblak, C. Thiel, R. Cone, and W. Tittel, Properties of a Rare-Earth-Ion-Doped Waveguide at Sub-Kelvin Temperatures for Quantum Signal Processing, Physical Review Letters 118, 100504 (2017).

[21] E. Saglamyurek, N. Sinclair, J. A. Slater, K. Heshami, D. Oblak, and W. Tittel, An integrated processor for photonic quantum states using a broadband light-matter interface, New Journal of Physics 16, 065019 (2014).

[22] G. Campbell, M. Hosseini, B. M. Sparkes, P. K. Lam, and B. C. Buchler, Time- and frequency-domain polarization interference, New Journal of Physics 14, 033022 (2012).

[23] G. Campbell, O. Pinel, M. Hosseini, T. Ralph, B. Buchler, and P. Lam, Configurable Unitary Transformations and Linear Logic Gates Using Quantum Memories, Physical Review Letters 113, 063601 (2014).

[24] B. M. Sparkes, M. Hosseini, G. Hétet, P. K. Lam, and B. C. Buchler, ac Stark gradient echo memory in cold atoms, Physical Review A 82, 043847 (2010).

[25] G. Hétet and D. Guéry-Odelin, Spin wave diffraction control and read-out with a quantum memory for light, New Journal of Physics 17, 075003 (2015).

[26] S.-W. Su, S.-C. Gou, L. Y. Chew, Y.-Y. Chang, I. A. M. Jarzyna, V. Lipińska, A. Klimek, K. Banaszek, and R. Demkowicz-Dobrzański, Efficient quantum information processing, Quantum-memory-assisted multi-photon generation for efficient quantum information processing, Optica 4, 1034 (2017).

[27] M. Parniak, D. Pęczak, and W. Wasilewski, Correlation steering in the angularly multimode Raman atomic memory, Optics Express 24, 21995 (2016).

[28] J. Nunn, N. K. Langford, W. S. Kolthammer, T. F. M. Champion, M. R. Sprague, P. S. Michelberger, X. M. Jin, D. G. England, and I. A. Walmsley, Enhancing multiphoton rates with quantum memories, Physical Review Letters 110, 133601 (2013).

[29] E. Distante, P. Xu, J. Chapman, and P. G. Kwiat, Quantum-memory-assisted multi-photon generation for efficient quantum information processing, Optica 4, 1034 (2017).

[30] I. Mirgorodskiy, F. Christaller, C. Braun, A. Paris-Mandoki, C. Tresp, and S. Hofferberth, Electromagnetically induced transparency of ultra-long-range Rydberg molecules, Physical Review A 96, 011402 (2017).

[31] E. Distante, P. Farrera, A. Padrón-Brito, D. Paredes-Barato, G. Heinze, and H. de Riedmatten, Storage Enhanced Nonlinearities in a Cold Atomic Rydberg Ensemble, Physical Review Letters 117, 113001 (2016).

[32] I. Misogrodzki, F. Christaller, C. Braun, A. Paris-Mandoki, C. Tresp, and S. Hofferberth, Electromagnetically induced transparency of ultra-long-range Rydberg molecules, Physical Review A 96, 011402 (2017).

[33] E. Distante, P. Farrera, A. Padrón-Brito, D. Paredes-Barato, G. Heinze, and H. de Riedmatten, Storing single photons emitted by a quantum memory on a highly excited Rydberg state, Nature Communications 8, 14072 (2017).

[34] H. Busche, P. Huillery, S. W. Ball, T. Ilieva, M. P. A. Jones, and C. S. Adams, Contactless nonlinear optics mediated by long-range Rydberg interactions, Nature Physics 13, 655 (2017).

[35] M. Parniak, D. Pęczak, and W. Wasilewski, Multimode Raman light-atom interface in warm atomic ensemble as multiple three-mode quantum operations, Journal of Modern Optics 63, 2039 (2016).
[51] W. Wasilewski and M. Raymer, *Pairwise entanglement and readout of atomic-ensemble and optical wave-packet modes in traveling-wave Raman interactions*, Physical Review A **73**, 063816 (2006).

[52] J. Kolodynski, J. Chwedenczuk, and W. Wasilewski, *Eigenmode description of Raman scattering in atomic vapors in the presence of decoherence*, Physical Review A **86**, 013818 (2012).

[53] Y.-W. Cho, G. T. Campbell, J. L. Everett, J. Bernu, D. B. Higginbottom, M. T. Cao, J. Geng, N. P. Robins, P. K. Lam, and B. C. Buchler, *Highly efficient optical quantum memory with long coherence time in cold atoms*, Optica **3**, 100 (2016).

[54] G. R. Dennis, J. J. Hope, and M. T. Johnsson, *XMDS2: Fast, scalable simulation of coupled stochastic partial differential equations*, Computer Physics Communications **184**, 201 (2013).

[55] M. Lipka, M. Parniak, and W. Wasilewski, *Optical frequency locked loop for long-term stabilization of broadline DFB laser frequency difference*, Applied Physics B **123**, 238 (2017).