Primordial Perturbations From General Two-field Inflation

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Abstract: We consider primordial perturbations from general two-field inflation in interaction picture. We verify that normalized to the single-field case, the power spectrum of scalar perturbations in the two-field version is identical beyond any slow roll approximation, except with different scalar spectral index. We then report that the two bispectrums also coincide at the leading order of slow roll parameters, which divide only at the next-leading order. Combing the scalar spectral index and the tensor-to-scalar ratio, we finally show that two-field chaotic and natural inflation can be distinguished by current BK14/Planck and future CMB-S4 experiment respectively.
1 Introduction

Inflation [1–3], which attempts to resolve both horizon and flatness problem in the early Universe, is one of key ingredients in the standard ΛCDM cosmology. Current observations from Cosmic Microwave Background (CMB) experiments [4, 5] imply that power spectrum of primordial scalar perturbation(s) from inflation has to be both nearly scale invariant and Gaussian, and bispectrum of the primordial scalar perturbation(s) has a nonlinear parameter $f_{NL}$ with a magnitude smaller than $O(10)$. Future experiments such as Cosmic Microwave Background (CMB-S4) [6] as well as Large Scale Structure [7] are underway for higher precision measurements.

Similar to scattering amplitudes in collider physics based on quantum field theory, the technical difficulty in deriving high-order correlators such as the bispectrum of the scalar perturbation in interaction picture is due to a large number of “Feynman diagrams” from a complex action, and subsequently a large amount of complex integrals over conformal time. The technical issue can be addressed in certain circumstances where there are effective theoretical tools to handle it. The first well-known example is the single-field inflation [8, 9]. Since current data has not yet identified either the number of inflaton scalars or the shape of inflation potential, a comparison between a single-field and its multi-field version is of phenomenological interest.
Unlike earlier studies such as [10, 11], we consider the multi-field inflation where all of components contribute to the energy density during inflation. In this context, δN formalism [12, 13] enables us to calculate both the power spectrum [13–15] and the bispectrum [16–25] of the primordial perturbations. Although the δN formalism makes the aforementioned calculations plausible, they are however limited to special multi-field inflation. For possible generalization from other perspective, see e.g. [26–28]. Instead of δN formalism, in this study we will calculate the primordial perturbations in general two-field inflation in interaction picture. For earlier attempts, see [29, 30]. To serve this purpose, we have developed an analytic program.\footnote{It is available upon request.}

The rest of materials are organized as follows. In Sec.2, we introduce general two-field inflation, where two different gauges, suitable slow roll parameters, and dynamics of classical backgrounds are discussed. In order to establish the relationship between the perturbation variables in the two gauges, which is critical for the derivation of the bispectrum, we will use gauge-invariant variable developed by Bardeen [31]. Subsection.3.1 and Subsection.3.2 are devoted to discuss the scalar and tensor perturbation, respectively, where three important observables - scalar spectral index $n_\zeta$, tensor-to-scalar ratio $r$ and nonlinear parameter $f_{NL}$ are obtained. In Sec.4, we apply these theoretical results to the two-field version of natural [32] and chaotic [33] inflation, and estimate reaches of current and future experiments on them in well-motivated parameter region. Finally, we conclude in Sec.5.

## 2 General Two-field Inflation

In this section, we discuss general two-field inflation, with an emphasize on gauge choice to describe the primordial perturbations and suitable slow roll parameters to parametrize the interactions between the two different components.

We restrict to the two-field inflation with Lagrangian

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} \sum_{\alpha=1}^{N} g^{\mu\nu} \partial_\mu \Phi_\alpha \partial_\nu \Phi_\alpha - V(\Phi_\alpha) \right], \quad (2.1)$$

where $R$ is the curvature, $g_{\mu\nu}$ is the spacetime metric, $\Phi_\alpha$ with $\alpha = 1 - 2$ denote the scalar fields, and $V$ is the inflation potential which includes interactions between $\Phi_1$ and $\Phi_2$. The metric in the ADM framework reads as

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N_i dt)(dx^j + N_j dt), \quad (2.2)$$

in which $h_{ij}$ with $i, j = 1 - 3$ are the dynamical variables, while $N$ and $N_i$ are Lagrangian multipliers.
2.1 Gauges

There are two different gauges for explicit purpose

\[ \zeta \text{ gauge: } \Phi_\alpha = \phi_\alpha(t), \quad h_{ij} = a^2[(1 + 2\zeta)\delta_{ij} + \gamma_{ij}], \quad \partial_t \gamma_{ij} = 0, \quad \gamma_{ii} = 0, \]
\[ \varphi \text{ gauge: } \Phi_\alpha = \phi_\alpha(t) + \varphi_\alpha(t, x), \quad h_{ij} = a^2[\delta_{ij} + \gamma_{ij}], \quad \partial_t \gamma_{ij} = 0, \quad \gamma_{ii} = 0, \quad (2.3) \]

where \( \phi_\alpha \) and \( \varphi_\alpha \) refer to classical backgrounds and quantum fluctuations respectively, \( a(t) \) is the scale factor, \( \zeta \) is gauge-invariant curvature perturbation, and \( \gamma \) denotes tensor perturbation. In Eq.(2.3), the scalar perturbation is neatly described by \( \zeta \) in the \( \zeta \) gauge.

The scalar perturbations described by the perturbation freedoms \( \varphi_\alpha \) and \( \zeta \) in the two different gauges are connected to each other. Their relationship is important for analytic calculation of the bispectrum as below. In the single-field inflation, it is uncovered in terms of a suitable spacetime coordinate transformation which lets the perturbation freedom in the \( \varphi \) gauge vanishes [8]. However, it is not clear how to apply this simple method to general multi-field inflation. In the \( \delta N \) formalism, this issue is solved by imposing adiabatic condition on each component, under which \( \zeta \) is the sum of individual curvature perturbations \( \zeta_\alpha \) related to component field \( \varphi_\alpha \) [34].

In the interaction picture, one can resolve this issue in terms of gauge-invariant variable developed by Bardeen [31]

\[ \zeta = -iH \frac{k^2T^0_i}{\rho + P}, \quad (2.4) \]

where \( H \) is the Hubble rate, \( T^\mu_\nu \) is the energy-momentum tensor in the \( \varphi \) gauge, while \( \rho = -T^0_0 \) and \( P = T^i_i \) refer to the energy density and pressure respectively. \( \zeta \) in Eq.(2.4) can be easily extended to the multi-field case by a replacement

\[ T^\mu_\nu \rightarrow \sum_\alpha T^\mu_\nu_\alpha. \quad (2.5) \]

In our case, expanding \( T^\mu_\nu \) in power laws of \( \varphi_\alpha \), the details of which are shown in appendix A, yields

\[ \varphi_\alpha \approx -\frac{\dot{\varphi}_\alpha}{H} \zeta + O(\zeta^2). \quad (2.6) \]

The nonlinear terms in Eq.(2.6) do not alter either quadratic or cubic action for the curvature perturbation at leading order (LO).
2.2 Slow Roll Parameters

The equations of background fields are given as,

\[ 3\dot{\rho}^2 = \frac{1}{2} \sum_\alpha \dot{\phi}_\alpha^2 + V, \]  
\[ \ddot{\rho} = -\frac{1}{2} \sum_\alpha \dot{\phi}_\alpha^2, \]  
\[ 0 = \ddot{\phi}_1 + 3\dot{\phi}_1 + V_1, \]  
\[ 0 = \ddot{\phi}_2 + 3\dot{\phi}_2 + V_2, \]

where “dot” denotes derivative over time, \( V_\alpha = \partial V / \partial \phi_\alpha \), and \( \dot{\rho} = H \).

To ensure an exponential expansion during inflation, we introduce slow roll parameters similar to the single-field case

\[ \epsilon_\alpha \equiv \frac{1}{2} \left( \frac{V_\alpha}{V} \right)^2 \approx \frac{\dot{\phi}_\alpha^2}{2H^2}, \]  
\[ \eta_1 \equiv \frac{V_{11}}{V} \approx \epsilon - \frac{\dot{\phi}_1}{H\dot{\phi}_1} - \frac{V_{12}}{3H^2} \frac{\dot{\phi}_2}{\dot{\phi}_1}, \]  
\[ \eta_2 \equiv \frac{V_{22}}{V} \approx \epsilon - \frac{\dot{\phi}_2}{H\dot{\phi}_2} - \frac{V_{12}}{3H^2} \frac{\dot{\phi}_1}{\dot{\phi}_2}, \]

where \( V_{\alpha\beta} = \partial^2 V / \partial \phi_\alpha \partial \phi_\beta \), \( \epsilon = \sum_\alpha \epsilon_\alpha \), and the explicit form of \( V_{12} \) is shown in appendix.B.1. The smallness of \( \eta_\alpha \) can be satisfied by imposing \( |\dot{\phi}_\alpha| \ll 3H \dot{\phi}_\alpha \) for arbitrary \( \alpha \) and \( \beta \), which are “natural” for the multi-field inflation under consideration. Because both scalar masses should be light compared to \( H \), and for \( |\dot{\phi}_\alpha / \dot{\phi}_\beta| \) either far larger or smaller than unity the effective number of inflaton fields is actually reduced such as in quasi-single field inflation [35, 36].

3 Primordial Perturbations

3.1 Scalar Perturbation

With a gauge choice in Eq.(2.3), the Lagrangian multipliers \( N \) and \( N_i \) can be solved in terms of the equations \( \partial \mathcal{L} / \partial N = 0 \) and \( \partial \mathcal{L} / \partial N_i = 0 \). Following the notation in ref.[8], we expand the Lagrangian multipliers as follows,

\[ N = 1 + \omega, \quad N_i = \partial_i \tilde{\omega}, \]  

where \( \omega \) and \( \tilde{\omega} \) are linear functions of the scalar perturbations, as keeping up to the linear order is sufficient [8] to derive both the quadratic and cubic action of the curvature perturbation.
In the $\varphi$ gauge, the explicit form of Lagrangian multipliers are given by,

$$\omega = \frac{1}{2H} \sum_{\alpha} \dot{\varphi}_\alpha \varphi_\alpha,$$

$$\partial^2 \tilde{\omega} = -\frac{a^2}{2H} \sum_{\alpha} (\ddot{\varphi}_\alpha \varphi_\alpha - H \epsilon_\alpha \dot{\varphi}_\alpha \varphi_\alpha - \ddot{\varphi}_\alpha \dot{\varphi}_\alpha) + \frac{a^2}{2} \sum_{\alpha \neq \beta} \epsilon_\alpha \dot{\varphi}_\beta \varphi_\beta, \quad (3.2)$$

where $\partial^2 = \partial^i \partial_i$. Eq.(3.2) reduces to the result of single-field inflation [37] under the single-field limit.

Substituting Eq.(3.2) into Eq.(2.1) and replacing $\varphi_\alpha$ in terms of $\zeta$ in Eq.(2.6) give rise to the quadratic action for the curvature perturbation

$$S_2 = \int d^4x \epsilon \left[ a^3 \dot{\zeta}^2 - a (\partial_i \zeta)^2 \right], \quad (3.3)$$

where $\epsilon = \sum_\alpha (\dot{\varphi}_\alpha^2 / 2H^2)$. Here, Eq.(3.3) is exact in the sense that no slow roll approximations have been required in the derivation of Eq.(3.3). To verify this point, one can recalculate $S_2$ in the $\zeta$ gauge, in which the Lagrangian multipliers read as

$$\omega = \frac{\dot{\zeta}}{H},$$

$$\tilde{\omega} = -\frac{\zeta}{H} + a^2 \epsilon \partial^{-2} \dot{\zeta}. \quad (3.4)$$

Substituting Eq.(3.4) into Eq.(2.1), we reproduce Eq.(3.3).

From Eq.(3.3) it is straightforward to derive the equation of motion

$$-\frac{d}{dt} (\epsilon a^3 \dot{\zeta}) + \epsilon a \partial^2 \zeta = 0, \quad (3.5)$$

to which the classical solution is given by

$$\zeta_k = \frac{1}{\sqrt{2\epsilon_*}} \frac{H_*}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau}, \quad (3.6)$$

with $\zeta_k$ the Fourier transformation of $\zeta(t, x)$, $\tau$ the conformal time coordinate, and the subscript “*” referring to the value at the time of horizon crossing. Substituting Eq.(3.6) into the two-point correlation function

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta^3(k + k') \frac{2\pi^2}{k^3} P_\zeta(k) \quad (3.7)$$

yields the power spectrum

$$P_\zeta(k) = \frac{1}{8\pi^2 \epsilon_* M_P^2} \frac{H_*^2}{k^3}, \quad (3.8)$$

and the scalar spectral index

$$n_\zeta - 1 = \frac{d \ln P_\zeta(k)}{d \ln k} = -4 + \frac{2}{\epsilon} [\epsilon_1 (\epsilon_1 - \eta_1) + \epsilon_2 (\epsilon_2 - \eta_2)]. \quad (3.9)$$
A couple of comments are in order regarding the observables in Eqs. (3.8) and (3.9).

- Normalized to the single-field case with $\epsilon = \sum \epsilon_\alpha$, the power spectrum of the two-field inflation is the same\(^2\) as that of the single-field inflation. It mainly follows the fact that the quadratic action of the scalar perturbations only depends on the slow roll parameters $\epsilon_\alpha$.

- The differences between single-field inflation and its two-field version can be exposed in terms of the scalar spectral index. Under the single-field limit, Eq. (3.9) reduces to the well-known result $n_\zeta - 1 \rightarrow 2\eta - 6\epsilon$. In Eq. (3.9) the derivation in $n_\zeta$ from the single-field prediction can be of the same order as the slow roll parameters, which allows them to play a role in future precision measurement on $n_\zeta$ [6].

To derive the bispectrum, one has to evolve the action for the curvature perturbation to the third order of $\zeta$. Unlike the quadratic action, where it is straightforward to work within the $\zeta$ gauge, there are certain uncertainties in labelling $\zeta$ in this gauge. This obscure can be eliminated in the $\varphi$ gauge.

In the $\varphi$ gauge, the cubic action is separated into two parts

$$S_3 = \sum_\alpha (\text{Maldacena})_\alpha + S_{\text{mix}}, \quad (3.10)$$

where the first part is just two copies of the single-field result [8] which involves in the individual field, while the second part $S_{\text{mix}}$ contains mixing terms such as $V_{\alpha\beta}$ with $\alpha \neq \beta$ which involve in the two different fields. Substituting Eq. (2.6) into Eq. (3.3) does not contribute to the LO cubic action. On the contrary, substituting Eq. (2.6) into Eq. (3.10) gives rise to respectively,

$$\sum_\alpha (\text{Maldacena})_\alpha \rightarrow \sum_\alpha (\text{Maldacena}^{\text{LO}})_\alpha + \cdots \quad (3.11)$$

and

$$S_{\text{mix}} \rightarrow \int d^4 x \sum_{\alpha \neq \beta} \left[ 2a^3 \epsilon_\alpha \epsilon_\beta \zeta \dot{\zeta}^2 + 2a \epsilon_\alpha \epsilon_\beta \zeta (\partial_i \zeta)^2 - 4a^3 \epsilon_\alpha \epsilon_\beta \zeta (\partial_i \zeta)(\partial_i \partial^2 \zeta) \right]^{\text{LO}} + \cdots, \quad (3.12)$$

where Maldacena\(^{\text{LO}}\) denotes the LO terms in Eq. (3.9) of ref [8], and the sub-leading terms have been neglected. To obtain Eqs. (3.11) and (3.12) we have imposed the equation of motion for $\zeta$, which follows the similarity that the equations of motion for external particles in Feynman diagrams are imposed to evaluate scattering amplitudes.

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\(^2\)See also [29], with $X$ therein of canonical form.
After a combination of the LO terms in Eqs. (3.11) and (3.12), we arrive at the final form of the cubic action

$$S_3 = \int d^4x \; \epsilon^2 \left[ a^3 \dot{\zeta}^2 + a \dot{\zeta} (\partial_i \zeta)^2 - 2a^3 \dot{\zeta} (\partial_i \zeta) \partial_i \partial^{-2} \dot{\zeta} \right]^{\text{LO}} + \cdots ,$$

(3.13)

where the sub-leading terms have been neglected. Although the sub-leading terms include contributions arising from $V_{12}$, $V_{112}$ and $V_{122}$ etc, whose explicit forms are shown in appendix B.1, they are at least the third orders of the slow roll parameters. This implies that under the same normalization, the LO cubic action in Eq. (3.13) coincides with that of the single-field inflation.

As a result of coincidence in the LO bispectrum in Eq. (3.13), the deviation in observable nonlinear parameter related to the bispectrum is

$$\Delta f_{NL} = f_{NL} - f_{NL}^\ast \sim O(\epsilon^2, \eta^2, \epsilon \eta),$$

(3.14)

where $f_{NL}^\ast$ represents single-field reference value. Current Planck data [4] has constrained $|\Delta f_{NL}| \sim O(10)$. Even though the future CMB-S4 [6] or Large Synoptic Survey Telescope (LSST) [38] experiment can reduce this error bar by a few times, the uncertainties in $f_{NL}$ are still too large to identify the two-field inflation.

### 3.2 Tensor Perturbation

Apart from the quadratic action of the scalar perturbation, substituting Eq. (3.2) into Eq. (2.1) yields that of the tensor perturbation as well,

$$\mathcal{S} = \frac{1}{8} \int d^4x \; a^3 \dot{\gamma}_{ij} \dot{\gamma}_{ij} - a \partial_k \dot{\gamma}_{ij} \partial_k \dot{\gamma}_{ij} .$$

(3.15)

which does not depend on the slow roll parameters [13]. This verifies the well-known result that the tensor perturbation decouples from the scalar perturbations. Rewrite $\gamma$ with the polarization mode $\gamma_k^s$ via

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s = \pm} \epsilon_{ij}^s \gamma_k^s(t) e^{ik \cdot x} ,$$

(3.16)

one obtains the well-known gravitational wave spectrum

$$\langle \gamma_k^s \gamma_{k'}^{s'} \rangle = (2\pi)^3 \delta^3 (k + k') \delta_{ss'} \frac{1}{2k^3} \frac{2H_s^2}{M_P^2} .$$

(3.17)

Combing Eq. (3.17) and Eq. (3.8) gives rise to the tensor-to-scalar ratio

$$r = 16\epsilon .$$

(3.18)
Figure 1. The largest deviation of $n_\zeta$ (in orange) in the two-field natural inflation with $f = 5M_P$ from the single-field case (in black), where the combination of BICEP2/Keck Array experiments and Planck [39] (in blue) and the expected CMB-S4 sensitivity [6] (in red) are simultaneously shown for comparison. While current BK14/Planck data cannot distinguish them, they are potentially identified by CMB-S4.

4 Numerical Analysis

So far, we have derived the observables $n_\zeta$, $f_{NL}$ and $r$ in general two-field inflation. Now, we estimate the sensitivities of these parameters to current and future high precision experiments on the primordial perturbations. As noted above, the differences of $f_{NL}$ between the single-field and two-field cases are beyond the reaches of either CMB-S4 or LSST experiments. We will use only $n_\zeta$ and $r$ for numerical analysis.

How to uncover the dependence of $n_\zeta$ in Eq.(3.9) on $r$ in Eq.(3.18) in general two-field inflation? In a specific inflation scenario, the explicit values of $n_\zeta$ and $r$ for CMB relevant mode depend on the values of the slow roll parameters at horizon crossing for the mode, which are determined by the values of the scalar field(s) at this moment. In the single-field case, after one eliminates the slow roll parameters, the one-to-one correspondence between $n_\zeta$ and $r$ is easily established, whereas the correspondence in the two-field case is complexed by the $\eta$
terms in Eq. (3.9) especially when they also vary independently. The complexity is reduced in certain circumstances under which the $\eta$ terms are either determined in terms of the $\epsilon$ terms or simply fixed.

For illustration, we firstly consider the two-field version of natural inflation \cite{33} with $V = \sum_\alpha V_\alpha [1 + \cos(\phi_\alpha / f)] + m^2 \phi_1 \phi_2$, where $V_\alpha$ are constants, scale $f$ is related to the Planck mass, and mass term to characterize the interaction\footnote{It is more obvious through a field redefinition, under which the mixing term disappears but the interaction effect occurs in the cosine terms.} between the two different components, respectively. In this model, the $\eta$ parameters are fixed as $\eta_1 = \eta_2 \approx -M_P^2 / 2f^2$. With an explicit value of $f$, the correspondence between $n_\zeta$ and $r$ is obtained, where we rewrite the $\epsilon$ terms as $\epsilon_1 = \epsilon \sin^2 \theta$ and $\epsilon_2 = \epsilon \cos^2 \theta$. Under the single-field limit $\tan \theta \to 0$, while in the two-field situation $\tan \beta \to 1$ in the well-motivated region. Fig.1 shows the largest deviation (in orange) of $n_\zeta$ in the two-field natural inflation with $f = 5M_P$ compared to the single-field case (in black), where the current best constraint from a combination of BICEP2/Keck Array experiments and Planck \cite{39} (in blue) and the expected CMB-S4 sensitivity \cite{6} (in red) are
simultaneously shown for comparison. Even though current BK14/Planck data is unlikely to distinguish them, they are potentially identified by CMB-S4.

We further discuss the two-field version of chaotic inflation \cite{32} with $V = \sum_i \lambda_i \phi_i^p$. Since the chaotic inflation with integer $p = 2 - 4$ has been disfavored \cite{40}, we restrict to the case $p = 1$. In this model, one finds $\eta_1 = \eta_2 = 0$ and establishes the correspondence between $n_\zeta$ and $r$ by rewriting the $\epsilon$ terms as above, with the single-field case given by $\tan \theta = 0$. Fig.2 shows that the largest deviation of $n_\zeta$ (in orange) in this two-field chaotic inflation, compared to the single-field case (in black). It suggests that unlike the single-field case, the two-field version in the well-motivated parameter region has been disfavored by current BK14/Planck data at 95% CL.

5 Conclusion

Motivated by current constraints on the primordial perturbations from inflation, in this study we have considered general two-field inflation. To achieve this goal, we have developed a program in the interaction picture instead of the $\delta N$ formalism as adopted in the literature. In terms of suitable gauges necessary to calculate the bispectrum of the scalar perturbations and suitable slow roll parameters to describe the inflation potential, we have derived the analytic expressions about the scalar spectrum index $n_\zeta$, the tensor-to-scalar ratio $r$ and the nonlinear parameter $f_{NL}$ in general two-field inflation. They are valid for general multi-field inflation with slow roll potential.

Combing the theoretic results about $n_\zeta$ and $r$, we approached the important phenomenological question - whether the constraints on single-field inflation change in its two-field situation? We answer this question in two illustrative examples. For the natural inflation, while current BK14/Planck data cannot distinguish the two cases, they are potentially identified by CMB-S4 in the well-motivated parameter region. For the chaotic inflation with $p = 1$, current BK14/Planck data has already disfavored the two-field version in the well-motivated parameter region, whereas both two cases can be excluded by CMB-S4. Along this line, more two-field examples can be similarly treated.

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A Energy-Momentum Tensor

To obtain the relationship between $\zeta$ and $\varphi_\alpha$, we expand the energy-momentum respect to the two-field Lagrangian in Eq.(2.1) in the power laws of the perturbations $\varphi_\alpha$. In the ADM framework, the explicit forms of its components $\rho = -T^0_0$, $P = T^i_i$, and $T^i_0$ read as respectively

\[
\begin{align*}
\rho^{(0)} &= \sum_\alpha \frac{1}{2} \dot{\varphi}_\alpha^2 + V, \\
\rho^{(1)} &= \sum_\alpha [-\omega \dot{\varphi}_\alpha^2 + \dot{\varphi}_\alpha \dot{\varphi}_\alpha + V_\alpha], \\
P^{(0)} &= \sum_\alpha \frac{1}{2} \dot{\varphi}_\alpha^2 - V, \\
P^{(1)} &= \sum_\alpha [-\omega \dot{\varphi}_\alpha^2 + \dot{\phi}_\alpha \dot{\varphi}_\alpha - V_\alpha], \\
T^0_0^{(0)} &= 0, \\
T^0_0^{(1)} &= -\sum_\alpha \dot{\phi}_\alpha \partial_i \varphi_\alpha, \\
T^0_i^{(2)} &= \sum_\alpha [2 \omega \dot{\phi}_\alpha \partial_i \varphi_\alpha - \dot{\varphi}_\alpha \partial_i \varphi_\alpha].
\end{align*}
\]  

(A.1)

where the superscript "(i)" means the $i$-th order of $\varphi_\alpha$.

B Derivatives of Potential

In the $\varphi$ gauge $V$ can be expanded in power laws of $\varphi_\alpha$, with the derivatives of $V$ over the classical fields $\phi_\alpha$ as coefficients. These derivatives can be organized in the power laws of the slow roll parameters. The first order derivatives $V_1$ and $V_2$ are directly read from the equations of motion in Eqs.(2.9) - (2.10). The second order derivatives such as $V_{11}$ and $V_{22}$ follow the definitions of $\eta_\alpha$ in Eqs.(2.12)-(2.13), whereas $V_{12}$ is determined by imposing the field derivative $\phi_2$ ($\phi_1$) on both sides of Eq.(2.9) ((2.10)). Similarly, higher order derivatives in Eq.(B.1) can be obtained by imposing higher order time and field derivatives on both sides of Eqs.(2.9)-(2.10). To summarize, they are given by,

\[
\begin{align*}
V_\alpha &\approx -\frac{6H^2}{\sqrt{2}} \sqrt{\epsilon_\alpha}, \\
V_{\alpha\alpha} &\approx 3H^2 \eta_\alpha, \\
V_{12} &\approx 3H^2 \sqrt{\epsilon_1 \epsilon_2}, \\
V_{112} &\approx \frac{3H^2}{\sqrt{2}} \sqrt{\epsilon_2 (\epsilon_1 - \eta_1)}, \\
V_{122} &\approx \frac{3H^2}{\sqrt{2}} \sqrt{\epsilon_1 (\epsilon_2 - \eta_2)},
\end{align*}
\]  

(B.1)

where higher orders of the slow roll parameters are neglected.
References

[1] A. H. Guth, “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,” Phys. Rev. D23, 347 (1981).

[2] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems,” Phys. Lett. B 108, 389 (1982).

[3] A. Albrecht and P. J. Steinhardt, “Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking,” Phys. Rev. Lett. 48, 1220 (1982).

[4] P. A. R. Ade et al. [Planck], “Planck 2015 results. XVII. Constraints on primordial non-Gaussianity,” Astron. Astrophys. 594, A17 (2016), [arXiv:1502.01592 [astro-ph.CO]].

[5] G. Hinshaw et al. [WMAP], “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results,” Astrophys. J. Suppl. 208, 19 (2013), [arXiv:1212.5226 [astro-ph.CO]].

[6] K. N. Abazajian et al. [CMB-S4], “CMB-S4 Science Book, First Edition,” [arXiv:1610.02743 [astro-ph.CO]].

[7] M. Alvarez, et al., “Testing Inflation with Large Scale Structure: Connecting Hopes with Reality,” [arXiv:1412.4671 [astro-ph.CO]].

[8] J. M. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” JHEP 05, 013 (2003), [arXiv:astro-ph/0210603 [astro-ph]].

[9] X. Chen, M. x. Huang, S. Kachru and G. Shiu, “Observational signatures and non-Gaussianities of general single field inflation,” JCAP 01, 002 (2007), [arXiv:hep-th/0605045 [hep-th]].

[10] F. Bernardeau and J. P. Uzan, “NonGaussianity in multifield inflation,” Phys. Rev. D 66 (2002), 103506, [arXiv:hep-ph/0207295 [hep-ph]].

[11] K. Enqvist and A. Vaihkonen, “Non-Gaussian perturbations in hybrid inflation,” JCAP 09 (2004), 006, [arXiv:hep-ph/0405103 [hep-ph]].

[12] A. A. Starobinsky, “Multicomponent de Sitter (Inflationary) Stages and the Generation of Perturbations,” JETP Lett. 42, 152-155 (1985).

[13] M. Sasaki and E. D. Stewart, “A General analytic formula for the spectral index of the density perturbations produced during inflation,” Prog. Theor. Phys. 95, 71-78 (1996), [arXiv:astro-ph/9507001 [astro-ph]].

[14] J. Garcia-Bellido and D. Wands, “Metric perturbations in two field inflation,” Phys. Rev. D 53 (1996), 5437-5445, [arXiv:astro-ph/9511029 [astro-ph]].

[15] D. H. Lyth and Y. Rodriguez, “The Inflationary prediction for primordial non-Gaussianity,” Phys. Rev. Lett. 95, 121302 (2005), [arXiv:astro-ph/0504045 [astro-ph]].

[16] D. Seery and J. E. Lidsey, “Primordial non-Gaussianities from multiple-field inflation,” JCAP 09 (2005), 011, [arXiv:astro-ph/0506056 [astro-ph]].

[17] L. E. Allen, S. Gupta and D. Wands, “Non-gaussian perturbations from multi-field inflation,” JCAP 01, 006 (2006), [arXiv:astro-ph/0509719 [astro-ph]].
[18] F. Vernizzi and D. Wands, “Non-gaussianities in two-field inflation,” JCAP 05, 019 (2006), [arXiv:astro-ph/0603799 [astro-ph]].

[19] T. Battefeld and R. Easther, “Non-Gaussianities in Multi-field Inflation,” JCAP 03, 020 (2007), [arXiv:astro-ph/0610296 [astro-ph]].

[20] K. Y. Choi, L. M. H. Hall and C. van de Bruck, “Spectral Running and Non-Gaussianity from Slow-Roll Inflation in Generalised Two-Field Models,” JCAP 02, 029 (2007), [arXiv:astro-ph/0701247 [astro-ph]].

[21] C. T. Byrnes and G. Tasinato, “Non-Gaussianity beyond slow roll in multi-field inflation,” JCAP 08, 016 (2009) [arXiv:0906.0767 [astro-ph.CO]].

[22] D. Battefeld and T. Battefeld, “On Non-Gaussianities in Multi-Field Inflation (N fields): Bi and Tri-spectra beyond Slow-Roll,” JCAP 11, 010 (2009), [arXiv:0908.4269 [hep-th]].

[23] N. S. Sugiyama, E. Komatsu and T. Futamase, “Non-Gaussianity Consistency Relation for Multi-field Inflation,” Phys. Rev. Lett. 106, 251301 (2011), [arXiv:1101.3636 [gr-qc]].

[24] J. Frazer and A. R. Liddle, “Multi-field inflation with random potentials: field dimension, feature scale and non-Gaussianity,” JCAP 02, 039 (2012), [arXiv:1111.6646 [astro-ph.CO]].

[25] Z. Kenton and D. J. Mulryne, “The squeezed limit of the bispectrum in multi-field inflation,” JCAP 10 (2015), 018, [arXiv:1507.08629 [astro-ph.CO]].

[26] C. M. Peterson and M. Tegmark, “Testing multifield inflation: A geometric approach,” Phys. Rev. D 87, no.10, 103507 (2013), [arXiv:1111.0927 [astro-ph.CO]].

[27] C. M. Peterson and M. Tegmark, “Testing Two-Field Inflation,” Phys. Rev. D 83 (2011), 023522, [arXiv:1005.4056 [astro-ph.CO]].

[28] Q. G. Huang, “A Geometric description of the non-Gaussianity generated at the end of multi-field inflation,” JCAP 06, 035 (2009), [arXiv:0904.2649 [hep-th]].

[29] D. Langlois and S. Renaux-Petel, “Perturbations in generalized multi-field inflation,” JCAP 04, 017 (2008), [arXiv:0801.1085 [hep-th]].

[30] A. Avgoustidis, S. Cremonini, A. C. Davis, R. H. Ribeiro, K. Turzynski and S. Watson, “The Importance of Slow-roll Corrections During Multi-field Inflation,” JCAP 02 (2012), 038, [arXiv:1110.4081 [astro-ph.CO]].

[31] J. M. Bardeen, “Gauge Invariant Cosmological Perturbations,” Phys. Rev. D 22, 1882-1905 (1980).

[32] A. D. Linde, “Chaotic Inflation,” Phys. Lett. B 129, 177 (1983).

[33] K. Freese, J. A. Frieman and A. V. Olinto, “Natural inflation with pseudo - Nambu-Goldstone bosons,” Phys. Rev. Lett. 65, 3233 (1990).

[34] D. H. Lyth, K. A. Malik and M. Sasaki, “A General proof of the conservation of the curvature perturbation,” JCAP 05 (2005), 004, [arXiv:astro-ph/0411220 [astro-ph]].

[35] X. Chen and Y. Wang, “Quasi-Single Field Inflation and Non-Gaussianities,” JCAP 04, 027 (2010), [arXiv:0911.3380 [hep-th]].

[36] T. Noumi, M. Yamaguchi and D. Yokoyama, “Effective field theory approach to quasi-single field inflation and effects of heavy fields,” JHEP 06, 051 (2013), [arXiv:1211.1624 [hep-th]].
[37] Y. Wang, “Inflation, Cosmic Perturbations and Non-Gaussianities,” Commun. Theor. Phys. 62 (2014), 109-166, [arXiv:1303.1523 [hep-th]].

[38] P. A. Abell et al. [LSST Science and LSST Project], “LSST Science Book, Version 2.0,” [arXiv:0912.0201 [astro-ph.IM]].

[39] P. A. R. Ade et al. [BICEP2 and Keck Array], “Improved Constraints on Cosmology and Foregrounds from BICEP2 and Keck Array Cosmic Microwave Background Data with Inclusion of 95 GHz Band,” Phys. Rev. Lett. 116, 031302 (2016), [arXiv:1510.09217 [astro-ph.CO]].

[40] P. A. R. Ade et al. [BICEP2 and Planck], “Joint Analysis of BICEP2/KeckArray and Planck Data,” Phys. Rev. Lett. 114 (2015), 101301, [arXiv:1502.00612 [astro-ph.CO]].