Analysis and improvement of Tian-Zhang-Li voting protocol based on controlled quantum teleportation

Kishore Thapliyal\textsuperscript{a}, Rishi Dutt Sharma\textsuperscript{b} and Anirban Pathak\textsuperscript{a}

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\textsuperscript{a}Jaypee Institute of Information Technology, A-10, Sector-62, Noida, UP-201307
\textsuperscript{b}National Institute Technology Patna, Ashok Rajhpath, Patna, Bihar 800005, India

Abstract

Recently Tian, Zhang and Li (TZL) have proposed a protocol for voting based on controlled quantum teleportation (Int. J. Theor. Phys. DOI 10.1007/s10773-015-2868-8). We have critically analyzed the protocol and have shown that it’s neither efficient nor secure. Further, it is shown that in the TZL protocol, the scrutineer Charlie does not have the required control over the voting process. Apart from showing the limitations of TZL protocol, two improved protocols for quantum voting along the line of TZL protocol are proposed here. One of the proposed protocols is designed using a standard scheme of controlled deterministic secure quantum communication, and the other one is designed using the idea of quantum cryptographic switch which uses a technique known as permutation of particles (PoP). A few possible alternative approaches to accomplish the same task have also been discussed. Further, the efficiencies of the proposed protocols are reported, and it is shown that the proposed protocols are free from the limitations of the TZL protocol, and they are more efficient than the TZL protocol.

Keywords: quantum voting, secure quantum communication, quantum cryptography, TZL protocol, controlled secure quantum communication

1 Introduction

Since the pioneering work of Bennett and Brassard on quantum key distribution (QKD) protocol \cite{BB84} (BB84 protocol), several protocols for secure quantum communication have been proposed (for a detail review see Chapter 8 of Ref. \cite{Ref}). Initial studies on secure quantum communication were restricted to single particle based QKD \cite{1,2}, entangled-state-based QKD \cite{3,4}, quantum secure direct communication (QSDC) \cite{5,6}, deterministic secure quantum communication (DSQC) \cite{7,8}, etc. However, in the last couple of years several innovative applications of secure quantum communication have been proposed \cite{17,20}. These recently proposed schemes of secure quantum communications (some of which are also hybrid in nature) have very interesting and important applications in our daily life. Specifically, we would like to note that in \cite{19}, it is shown that protocols of quantum dialogue can be modified to provide solution of the socialist millionaire problem; innovative quantum solutions have also been provided for auction \cite{17}, e-commerce \cite{18} and voting \cite{20,27}.

The existing protocols for quantum communication can be broadly categorized as: (A) insecure protocols (or protocols where security is not required, like teleportation), and (B) secure protocols (like QKD). Insecure protocols do not require any security measures while they may be used in designing secure protocols for which security analysis is considered. Teleportation, remote state preparation (RSP) and other variants of teleportation fall under the Category A, whereas protocols for QKD, QSDC and DSQC, etc., lie in Category B. The present paper is focused on quantum voting, which is a particular type of protocol for secure quantum communication. The reason for this specific choice is justified as among the above mentioned interesting applications of secure quantum communication, voting is of specific importance to our daily life. Specifically, major policies of a state (nation) are decided on the basis of voting, and in a democratic country policy makers are elected by voting. In voting, privacy of the voter is of major importance and as schemes of secure quantum communication can be made unconditionally secure, it is natural to exploit the findings of the secure quantum communication or cryptography for the purpose of designing schemes for quantum voting. In 2006, Hillery et al., proposed the first scheme of quantum voting \cite{20}. Almost immediately after that Vaccaro, Spring, and Chefles proposed an alternative scheme for quantum voting \cite{22}. Later
on in 2011, Hillery et al., proposed another scheme of quantum voting [25] in which they circumvented some of the limitations of their earlier scheme. Subsequently, several other protocols for quantum voting appeared ([26][27] and references therein). All these schemes were aimed to satisfy (i) security, (ii) verifiability, and (iii) privacy. The importance of voting in public life motivated us to analyze one of the most recent proposals for quantum voting. To be precise, in what follows, we critically analyze a quantum voting scheme recently proposed by Tian, Zhang and Li [21] using a scheme of controlled quantum teleportation. Our analysis reveals several limitations of the TZL protocol. In fact, it indicates that controlled quantum teleportation is not the correct choice for implementing a scheme of quantum voting, rather an analogous scheme based on controlled secure quantum communication can perform the task more appropriately and efficiently. To emphasize on this point, we have proposed two new protocols for quantum voting which are based on two schemes of controlled DSQC (CDSQC), and are free from the limitations of TZL protocol. The first protocol uses a standard scheme of CDSQC. However, the second protocol intrinsically uses a CDSQC scheme introduced recently [28] as a variant of quantum cryptographic switch [29]. It may be noted that the concept of quantum cryptographic switch was introduced by Srinatha et al., in 2014 [28], to describe a CDSQC scheme where the controller can control the information the receiver can extract to a continuously varying degree. In the cryptographic switch and its variants (various schemes of controlled communication discussed in Refs. [28][30]) a technique known as permutation of particles (PoP) plays a crucial role [31]. Specially, use of PoP reduces the requirement of quantum resources and provides security. This interesting technique which was introduced by Deng and Long in 2003 for QKD [31] is used here to design our second protocol which is more efficient than the TZL protocol and the other protocol proposed here using the standard CDSQC scheme. Further, it is shown that the proposed schemes are a bit general in nature, as CDSQC can be implemented in various ways and each of those different approaches can provide a scheme for quantum voting.

The remaining part of the paper is organized as follows. In Sec. 2 we briefly describe TZL scheme of quantum voting. In Sec. 3 limitations of TZL scheme are discussed. In Sec. 4 we propose two improved schemes for quantum voting which are free from the limitations of TZL protocol, and we also discuss a few alternative approaches that can be used to obtain schemes of quantum voting analogous to the TZL scheme. Finally, the paper is concluded in Sec. 5.

2 Controlled quantum teleportation and voting schemes of Tian, Zhang and Li

In this section, we briefly describe the controlled quantum operation teleportation scheme and the quantum voting protocol proposed in the recent paper of Tian, Zhang and Li [21].

2.1 TZL scheme of controlled quantum operation teleportation

The TZL controlled teleportation scheme uses four qubit entangled state which can be written as \( \frac{1}{\sqrt{2}} (|GHZ^0\rangle|0\rangle + |GHZ^{1+}\rangle|1\rangle) \), where \( |GHZ^{a\pm}\rangle = \frac{1}{\sqrt{2}} (|ijk\rangle \pm |\bar{i}\bar{j}\bar{k}\rangle) \) is a GHZ state with \( a \) as the decimal value of the binary number \( ijk \) for \( i, j, k \in \{0, 1\} \). The channel is shared between three parties Alice, Bob and Charlie as follows: Alice possesses the first two particles, whereas the third and last particles are with Bob and Charlie, respectively. Further, Alice owns a quantum state to be teleported. The scheme used in TZL quantum voting protocol can be summarized in the following steps:

Step 1: Alice performs a measurement in GHZ-basis on all the three qubits of her possession.

Step 2: Alice announces her measurement outcome.

Step 3: Charlie performs a measurement in the computational basis on his qubit and announces the outcome only when he wishes Alice and Bob to perform the teleportation.

Step 4: Using Alice’s and Charlie’s measurement outcomes, Bob performs a particular Pauli operation on his qubit using Table 1 (which is same as Table 1 of Ref. [21], except the fact that Columns 3 and 4 are added here to illustrate a limitation of TZL scheme) to reconstruct the quantum state sent by Alice.

It may be observed from Table 1 that if Alice announces a measurement outcome \( |GHZ^{0+}\rangle \) then for Charlie’s measurement outcome of \( |0\rangle \) (\( |1\rangle \)) Bob needs to apply \( I \) (\( X \)) operation. This was what proposed in TZL scheme. However, a closer look into the table (see Columns 3 and 4 of Table 1) reveals that Bob’s measurement outcome is independent of Charlie’s measurement outcome. Specifically, if we neglect the global phase, then even before
Voting 1: Between the most important phase of the scheme (i.e., voting scheme) happens in the following steps.

1. The notary organization CA (a trusted third party) issues quantum IDs to all eligible voters, scrutineer (controller) and tallyman. Finally, it ends with the counting phase where the controller decrypts

2. The TZL voting scheme has three phases: Initial phase, voting phase and counting phase.

3. The purpose of keeping the voting rule secret is two fold. Firstly, it provides controller/scrutineer Charlie a control over the time at which the votes will be counted. In other words, until Charlie discloses the voting rules, the tallyman Bob will not be able to count votes. Secondly, when voting happens sequentially, it restricts $i + 1$th voter from knowing the trend of voting happened until then (i.e., the trend up to the time $i$th voter has voted).

4. At the voter’s request Bob prepares a four qubit entangled state and sends first two qubits to the voter, while the last one to Charlie, keeping the third qubit with himself.

5. To cast her vote the $i$th voter Alice, will encode her vote (voting information) on the qubit sent by Bob. The encoding is to be done according to the voting rule assigned to her by Charlie in Voting 2. Then Alice

Table 1: The relationship between the measurement outcomes of Alice and Charlie with Bob’s operation in TZL scheme. Here, $GHZ^{0\pm} = \frac{(000\pm111)}{\sqrt{2}}$ and $GHZ^{3\pm} = \frac{(011\pm000)}{\sqrt{2}}$.

The application of Bob’s unitary operation, the quantum state of Bob’s qubit is the same for both the outcomes of Charlie. Further, it is worth noting that an application of $X$ gate on $\pm$ leaves the state unchanged up to the global phase. Hence, it can be concluded that Charlie’s measurement outcome is irrelevant for Bob and even before Charlie performs measurement on his qubit, Bob can reconstruct Alice’s teleported state only by knowing the outcome of Alice’s measurement. This fact can be mathematically verified by writing density matrix for the combined states of Charlie and Bob (after Alice’s measurement) and subsequently tracing out Charlie’s qubit, and measuring Bob’s qubit in $\{\pm\}$ basis. Similarly, for Alice’s measurement outcome $GHZ^{3\pm}$, $GHZ^{3\pm}$ and $GHZ^{0\pm}$ Bob can apply $I$, $Z$ and $Z$ operations, respectively, to obtain the state teleported by Alice, without bothering to know the measurement outcome of Charlie. Therefore, Charlie’s control is missing in the TZL controlled teleportation scheme if the sender wishes to teleport only $\pm$ state.

2.2 TZL scheme of quantum voting

The TZL voting scheme has three phases: Initial phase, voting phase and counting phase. The scheme begins with the initial phase in which the notary organization CA (a trusted third party) issues quantum IDs to all eligible voters, scrutineer (controller) and tallyman. Finally, it ends with the counting phase where the controller decrypts and announces the voting rules of each voter, using which the tallyman decides the number of votes in favor. In between the most important phase of the scheme (i.e., voting scheme) happens in the following steps.

Voting 1: The controller Charlie sets up a bulletin board, i.e., a classical channel to make the announcements.

Voting 2: The tallyman Bob authenticates identity of the voters with the help of the trusted third party CA. After that, Bob sends a qubit in $\pm = \frac{(000\pm111)}{\sqrt{2}}$ state to all the authenticated voters.

Voting 3: Charlie also authenticates the identity of the voters taking CA’s help. Further, Charlie sends the voting rule to $i$th voter Alice, using BB84 encryption after knowing her serial number. For example, Charlie can assign $i$th voter quantum operations $I$ and $Z$ gates to encode “yes” and “no”, respectively. The only other possibility of encoding scheme for Charlie will be to use $I$ ($Z$) gate for sending “no” (“yes”).

Here, it is important to note that the voting rule is known to $i$th voter and Charlie only.

Voting 4: At the voter’s request Bob prepares a four qubit entangled state and sends first two qubits to the voter, using BB84 encryption after knowing her serial number. For example, Charlie can assign $i$th voter quantum operations $I$ and $Z$ gates to encode “yes” and “no”, respectively. The only other possibility of encoding scheme for Charlie will be to use $I$ ($Z$) gate for sending “no” (“yes”).

Here, it is important to note that the voting rule is known to $i$th voter and Charlie only.

Voting 5: To cast her vote the $i$th voter Alice, will encode her vote (voting information) on the qubit sent by Bob. The encoding is to be done according to the voting rule assigned to her by Charlie in Voting 2. Then Alice

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1. The purpose of keeping the voting rule secret is two fold. Firstly, it provides controller/scrutineer Charlie a control over the time at which the votes will be counted. In other words, until Charlie discloses the voting rules, the tallyman Bob will not be able to count votes. Secondly, when voting happens sequentially, it restricts $i + 1$th voter from knowing the trend of voting happened until then (i.e., the trend up to the time $i$th voter has voted).
performs a measurement in GHZ-state basis on the encoded qubit and two qubits of the quantum channel set by Bob in Voting 4. This is followed by an announcement of the measurement outcome on the bulletin board prepared in Voting 1 and Charlie’s measurement of his qubit of the shared channel (in Voting 4) in computational basis. Finally, Charlie also announces the measurement outcome.

**Voting 6:** Using the measurement outcomes of Alice and Charlie, Bob can perform suitable Pauli operator (from Table 1) to obtain the teleported quantum state. Finally, the reconstructed state is measured in diagonal basis and the result is broadcasted classically.

**Voting 7:** In a similar approach, Bob performs the voting procedure for all the voters.

In Voting 5 and Voting 6 the scheme of controlled teleportation described in Sec. 2.1 has been used and as a consequence, this scheme of quantum voting may be viewed as a quantum voting scheme based on controlled quantum teleportation and in the TZL paper it was referred in this manner.

### 3 Limitations of the TZL protocol

A careful look into the TZL protocol for quantum operation teleportation reveals that Alice actually encodes a single bit of information and transmits that to Bob. Say, we consider encoding by operation of identity is “0” and encoding by operation of $Z$ as “1”. Secure controlled quantum communication of 1 bit of classical information can be performed in a large number of ways and controlled teleportation is neither essential nor a good choice. This is not a good choice because, teleportation or controlled teleportation in its original form is not secure, and standard controlled teleportation schemes require $n$-partite entanglement ($n \geq 3$) and at least 2 bits of classical communication to communicate one bit of voting information. There are many alternative ways to perform the same task and controlled teleportation is just one of the ways and it’s not secure in its original form. Further, in TZL protocol, Alice always encodes her message on the quantum state $|φ_a⟩ = \frac{1}{\sqrt{2}} (|0⟩ + |1⟩) = |+⟩$, so it is not required that Bob prepares the state. Alice herself can prepare the state. In fact, when Bob prepares this quantum state and transmits that to Alice via a quantum channel that just reduces the quantum efficiency of the protocol as it increases the amount of quantum communication. Further, in practical situations (i.e., in a noisy environment) this additional communication would reduce the fidelity of the final state. We can further note that after encoding operation of Alice, she knows which state is to be teleported (it is either $|+⟩$ or $|−⟩$ depending upon the operator used by her) as she knows what operation she has performed on the initial state $|+⟩$. Teleportation of a known state is technically referred to as the remote state preparation, and thus the problem here is of controlled remote state preparation. It is well known that quantum channel that can perform (controlled) teleportation can always perform (controlled) remote state preparation. However, for RSP we don’t need to prepare an additional qubit in $|+⟩$ state. Further, controlled quantum teleportation and controlled RSP of one qubit can be performed using 3 qubit states only. 4 qubit state is not required. For example, in Ref. 32, 33, we have already shown how controlled teleportation can be performed using GHZ-like states. We note this point just to show that use of 4-qubit state in TZL protocol was unnecessary. However, we are not going to elaborate a 3-qubit controlled teleportation based version of TZL quantum voting scheme as the other limitations of the TZL protocol will remain present in that, too.

Another major limitation of the TZL voting scheme lies in the fact that in Voting 4 step, Bob prepares the quantum state to be used for controlled teleportation. Here, Bob can always prepare a quantum state such that Charlie’s state is separable from the state shared by Alice and Bob, and this would help him to circumvent Charlie’s control without being detected. To be precise, consider that Bob randomly prepares one of the states: $|GHZ^{0+}\rangle_{A_1A_2B} |0⟩_C = \frac{1}{\sqrt{2}} (|000⟩ + |111⟩)_{A_1A_2B} |0⟩_C$ or, $|GHZ^{0+}\rangle_{A_1A_2B} |1⟩_C = \frac{1}{\sqrt{2}} (|000⟩ + |111⟩)_{A_1A_2B} |1⟩_C$ and shares the state as described in the step Voting 4. In this case, Charlie’s measurement outcome will be randomly $|0⟩$ and $|1⟩$, but it will not have any control over the voting scheme. Thus, the fact that in TZL protocol controller Charlie does not have the desired control over the voting process is now established in two different ways.

It is easy to circumvent the problem arising due to the fact that Bob has been assigned the responsibility to prepare the quantum channel. Specifically, this problem can be circumvented by allotting the state preparation task to Charlie. However, even this modification of TZL protocol would not solve the problem completely (i.e. would not provide the required control to scrutineer). Specifically, as mentioned in Sec. 2, Charlie’s control in the protocol will be missing as Bob can reconstruct the state even before Charlie’s measurement and announcement of corresponding outcome.

Further, in the TZL protocol, it is intrinsically assumed that the sender sends a qubit to the receiver and there is no eavesdropper who can attempt to replace the original qubit with an auxiliary one and finally change the vote.
with her choice. This is not desired. In fact, the voters would come to know about this attack when their votes are announced on the bulletin board. It is reasonable to think that it would be hard to claim unfairness as the voter would have to disclose her original choice in that case. This can be circumvented by using decoy qubits while sharing the qubits and quantum channel. Using a proper eavesdropping checking scheme this kind of an attack can be circumvented.

4 Improved protocols

We have already shown that in the TZL protocol, the task performed is essentially communication of one bit of information by controlled RSP of a qubit. This observation may motivate one to modify the original TZL protocol by replacing the controlled teleportation scheme used in TZL protocol by a scheme of controlled RSP. However, such a modification will not succeed to circumvent the limitations of the original TZL scheme. In fact, in such a modified scheme, Alice’s vote will be leaked as soon as she will disclose the outcome of her measurement. Technically, here the ignorance of tallyman Bob or an eavesdropper is of one bit (they only need to know whether the state to be prepared remotely is a $|+\rangle$ or a $|−\rangle$ state), and Alice’s disclosure removes this ignorance (in other words, provides required one bit information to both Bob and eavesdropper). Thus, we note that to obtain an improved protocol for quantum voting, it would be apt to use a scheme of secure controlled communication (say a scheme of CDSQC) over an insecure quantum communication protocol (say a scheme of controlled teleportation or controlled RSP).

Keeping these facts in mind, let us now introduce our first scheme of quantum voting based on CDSQC.

4.1 Protocol 1: Quantum voting based on controlled-deterministic secure quantum communication (CDSQC)

The TZL quantum voting protocol can be visualized as a protocol for transmitting one bit of classical information by teleporting a qubit from the voter (sender) to a tallyman (receiver) under the supervision of controller/scrutineer. As discussed above, the TZL protocol in its original form has several limitations. However, the controlled communication task performed by TZL protocol can be performed in a secure manner by using various alternative approaches. To be precise, there exist various protocols for controlled deterministic secure quantum communication (CDSQC), which are controlled version of DSQC or QSDC protocols [28]. In what follows, we will show that any CDSQC protocol can be reduced to a protocol of binary quantum voting. To elaborate this point, let us first explicitly provide an e-voting scheme based on a particular CDSQC protocol. The scheme proposed below is designed along the line of TZL protocol, and the initial phase and the counting phase of the TZL protocol remain unchanged in our protocol. Only the voting phase is modified. Specifically, the voting phase of the improved scheme can be summarized as follows:

CDSQC 1: Same as Voting 1.

CDSQC 2: Bob only authenticates the identity of the voters with the help of the trusted third party CA.

CDSQC 3: Same as Voting 3.

CDSQC 4: After receiving the voter’s request, Charlie prepares a three qubit entangled state (GHZ-like state) and sends first and second qubits to the voter and tallyman, respectively. He keeps the third qubit with himself.

Here, it is important that the controller (Charlie) prepares the quantum state and distribute it among the users, unlike Voting 4 where the same task was done by Bob. Thus, the quantum channel prepared by Charlie is

$$|\psi\rangle_{\text{GHZ-like}} = \frac{|\psi_1\rangle|0\rangle + |\psi_2\rangle|1\rangle}{\sqrt{2}},$$

where $|\psi_i\rangle \in \{|\psi^+\rangle, |\psi^-\rangle, |\phi^+\rangle, |\phi^-\rangle\}$. Here, $|\psi^\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$ and $|\phi^\pm\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$ are Bell states. Further, a different choice of controller’s qubit in $\{|+, -\}\}$ basis would give us a GHZ state. Therefore, GHZ states are also suitable for the proposed scheme.

In fact, an arbitrary choice of controller’s qubit in $\{|a\rangle, |b\rangle\}$ basis will work, where the measurement outcome in this basis will reveal the information about the quantum channel shared by Alice and Bob. In brief, any quantum state of the form $|\psi\rangle_{\text{GHZ-like}} = \frac{|\psi_1\rangle|a\rangle + |\psi_2\rangle|b\rangle}{\sqrt{2}}$ can be used to implement this protocol.
CDSQC 5: After receiving the first particle of the quantum channel Alice operates \( I \) or one of the Pauli gates \((X, iY, \text{ or } Z)\) to cast her vote as “yes” or “no” as the voting rule set by the controller in CDSQC 3. Then she sends the encoded qubit to Bob. This is followed by Charlie’s measurement of the third qubit in computational basis and announcement of his measurement outcome.

It would be worth mentioning here that dense coding is not exploited here. A scheme exploiting quantum dense coding can be designed where the voters can encode 2 bits of information.

CDSQC 6: Depending upon the measurement outcomes of Charlie, Bob can perform Bell state measurement on the first two qubits and broadcast the measurement outcome on the bulletin board.

CDSQC 7: In a similar approach, Bob performs the voting procedure for all the voters.

For sharing the entangled channel in CDSQC 4 or sending the encoded qubit in CDSQC 5 suitable decoy qubits are prepared and only after going through a successful eavesdropping checking the next step is followed. More detail of decoy qubit assisted eavesdropping checking schemes and suitable choices of decoy qubits can be found in Ref. [34].

To show that the proposed scheme is more efficient we use the quantitative measure of efficiency as discussed in Refs. [28, 35], i.e., \( \eta = \frac{c}{q} \), where \( c \) denotes total classical bits transmitted using the total number of qubits \( q \), while \( b \) is the classical communication involved in decoding of information (classical communication required for eavesdropping check is not included). Usually, \( \eta \) is referred to as qubit efficiency and is frequently used to compare the protocols of secure quantum communication. In the proposed protocol based on CDSQC, for each vote (a bit of classical information) to be transmitted a 3 qubit state is used. Additionally, 3 decoy qubits are required to ensure security. Charlie informs Alice the voting rule with 1 bit of classical communication which he announces in the counting phase with another 1 bit of communication; and he also announces his measurement outcome at the end with 1 bit of information. Thus, \( q = 6, b = 3 \) and \( c = 1 \), and as a consequence qubit efficiency of the proposed protocol is \( \eta = 11.11\% \). This is not high, but if we compare it with the original TZL protocol we can recognize that qubit efficiency of the protocol proposed here is greater than that of TZL protocol. This point can be clearly understood if we note that for the original TZL protocol, \( q = 5 \), as a 4-qubit cluster state is used and as Bob transmits another qubit on which voting is to be done. Further, \( b = 6 \), as the announcement of measurement outcomes of Alice and Charlie require transmission of classical information of 3 bits and 1 bit, respectively, in addition, disclosure of voting rules involves transmission of 2 bits (as it is done twice once from Charlie to Alice and once from Charlie to Bob). Thus, for the original TZL protocol qubit efficiency \( \eta = \frac{1}{15} = 0.07\% \). Here it would be apt to note that the original TZL protocol is insecure, and since no qubit is used as decoy qubit to check eavesdropping \( q \) is relatively low. However, if in the original TZL protocol Bob desires to check that the quantum states prepared by him in Voting 2 and Voting 4 states reach the desired users without eavesdropping he has to use 4 decoy qubits for each vote, and that would make \( q = 9 \). In that case, \( \eta \) for TZL would be \( \frac{1}{15} = 6.67\% \), which only 60% of the efficiency of the protocol proposed here.

The qubit efficiency can be increased either by increasing the number transmitted classical bits \( c \) (however, we have to discard this possibility as in case of binary voting described here \( c \) is always 1) and/or decreasing \( b \) and/or \( q \). Increase in the amount of transmitted classical bits \( c \) using the same resource will allow us to construct new schemes of voting (where more than two choices exist, say there are four candidates and the voter has to choose one), which is beyond the realm of binary voting discussed in the original TZL protocol and the protocol proposed here. So in this paper, we don’t discuss the possibilities associated with the increase in \( c \). We will discuss this possibility elsewhere. In the protocol proposed here \( q \) and \( b \) are already reduced with respect to TZL protocol. In the next section, we will propose another scheme of quantum voting protocol with further reduced values of \( q \) and \( b \). Before we describe the new protocol, we would like to note that \( b \) can be reduced by avoiding different voting rules for each voter and using same voting rule for all voters (as identity in favor and one of the 3 Pauli operators for against). This would reduce the classical communication associated with the protocol by 2 bits.

4.2 Protocol 2: Quantum voting based on controlled-deterministic secure quantum communication (CDSQC) solely using Bell states

In the previous section, we have proposed a scheme for quantum voting using a GHZ-like state, which is three qubit maximally entangled state. This reduced the requirement of quantum resources with respect to the original TZL protocol, where 4-qubit entanglement was used. However, tripartite entanglement is not essential. In this section, we will provide a scheme for quantum voting using bipartite entanglement (Bell states). This would increase efficiency of the voting protocol and reduce the difficulties associated with the generation and management of multi-partite entanglement. Keeping the initial phase unchanged a modified scheme for quantum voting can be described as follows:
CDSQC2 1: Same as CDSQC 1.

CDSQC2 2: Same as CDSQC 2.

CDSQC2 3: Charlie prepares $n$ copies of one of the Bell states. He sends first qubit of the $i$th Bell state to the $i$th voter and prepares an $n$-qubit string with the second qubits of all the Bell states. Then he applies a permutation operator on the string formed using the second qubits and sends that to Bob. In this step, security of transmitted qubits are ensured by random insertion of an equal number of decoy qubits (in other words by testing half of the travel qubits using BB84 subroutine or GV subroutine). If the errors obtained by a standard eavesdropping checking process is found below the tolerable limit they proceed to the next step.

CDSQC2 4: Each voter can encode her vote on the qubit received by her using the known voting rule. The predecided voting rule can be to operate $I$ for “yes” and one of the Pauli gates ($X$, $iY$, or $Z$) for “no”. She sends the encoded qubit to Bob afterward. Exploiting dense coding capacity of the Bell states Alice could have sent 2 bits of information. This can add some more features to the voting process, which will be explored elsewhere. Again in this step a decoy qubit based security subroutine should be employed.

CDSQC2 5: After all the voters have sent their qubits of Bell states to Bob with their voting information he notifies Charlie.

This is followed by the counting phase in which Charlie announces information regarding permutation operator and the initial state. This allows Bob to find out which qubit of the sequence received by him directly from Charlie was initially entangled to which qubit sent by a voter. Thus, after the announcement of Charlie, Bob can perform Bell measurement on partner particles to obtain the vote cast by each voter as he already knows the initial Bell states. Finally, he counts all the votes and announces the total number of votes in favor.

Similar to the protocol proposed in the last subsection quantitative measure of efficiency can be calculated. In this scheme, for each vote (a bit of classical information) to be transmitted 2 qubits are used directly. In addition 3 decoy qubits are also required to ensure security. Further, Charlie has to reveal an additional 1 bit of information for decoding the votes. Thus, $c = 1$, $q = 5$, $b = 1$ and as a consequence, the efficiency of this PoP-based protocol of quantum voting is $\eta = 16.67\%$, which is higher than the previous proposal and the original TZL protocol.

4.3 Possible alternative approaches that can lead to schemes of quantum voting

Clearly, the quantum voting schemes described above are based on schemes of CDSQC. As mentioned in Subsection there are various ways to implement schemes of CDSQC. Specifically, CDSQC can be implemented using a large set of entangled states (e.g., W state, GHZ state, GHZ-like state, $Q_4$ state, $Q_5$ state, cluster state, $|\Omega\rangle$ state, Brown state, etc.) . Though to exploit the dense coding capacity of these states it would be preferable to use these states for sending more information.

The quantum voting task can also be achieved using entanglement swapping based CDSQC scheme. In this scheme a large number of copies of a 3-qubit entangled state is prepared by the controller. The qubits are shared between the voters and the tallyman, in such a way that the first two qubits of a 3-qubit entangled state are sent to one of the voters, while a sequence prepared by all the last qubits is sent to the tallyman Bob after applying a permutation operator on the sequence. Subsequently, the voters can follow entanglement swapping based DSQC scheme proposed by Shukla et al., . Finally, the tallyman will require an additional information of the permutation operator from the controller to retrieve the votes. It is worth noting here that in this protocol the encoded qubits do not travel through the channel.

It may be noted that except the voting part where security is ensured by the security proofs of the CDSQC protocol used here to design the voting protocols, the rest of the scheme is the same as the original TZL protocol and hence the verifiability and privacy established in TZL paper is applicable to our protocols, too. Keeping this in mind, we have not elaborately discussed these two important aspects of voting protocols in this paper.

Thus, possibilities of eavesdropping are checked using standard decoy qubit based techniques, like BB84 subroutine or GV subroutine (for detail of these eavesdropping checking techniques see our earlier works). A suitable choice of decoy qubits and the subroutine can be opted depending on the interaction with the environment or the nature of noise present in the channel (for a detail discussion see Ref.).
5 Conclusion

The TZL quantum voting scheme has been critically analyzed here. Specifically, it has been established that the TZL protocol is neither secure nor efficient, and the scrutineer Charlie does not have the required control over the voting process. Specifically, we noted that a quantum communication scheme (controlled teleportation) where security measures are not involved directly had been used to construct a protocol for performing a highly classified job (voting). Clearly, it would have been more appropriate to investigate the possibility of modifying a scheme of secure quantum communication to build a scheme for quantum voting. This approach is followed in the present paper to propose two new schemes for secure and efficient quantum voting based on CDSQC. The proposed schemes are shown to be free from the limitations of the TZL protocol. Further, a few modified alternative approaches suitable for the same task have also been discussed. In brief, limitations of the TZL protocol have been explored and a set of alternatives has been provided for the realization of binary quantum voting. We conclude the paper with a hope that in the near future the experimentalists will find it interesting to implement one or more of the CDSQC based schemes for binary quantum voting discussed in this paper.

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