Summary: The first author [Proc. A, R. Soc. Lond. 471, No. 2176, Article ID 20140963, 19 p. (2015; Zbl 1371.35219)] introduced a variational framework for stochastically parametrising unresolved scales of hydrodynamic motion. This variational framework preserves fundamental features of fluid dynamics, such as Kelvin’s circulation theorem, while also allowing for dispersive nonlinear wave propagation, both within a stratified fluid and at its free surface. The present paper combines asymptotic expansions and vertical averaging with the stochastic variational framework to formulate a new approach for developing stochastic parametrisation schemes for nonlinear waves in fluid dynamics. The approach is applied to two sequences of shallow water models which descend from Euler’s three-dimensional fluid equations with rotation and stratification under approximation by asymptotic expansions and vertical averaging. In the entire family of nonlinear stochastic wave-current interaction equations derived here using this approach, Kelvin’s circulation theorem reveals a barotropic mechanism for wave generation of horizontal circulation or convection (cyclogenesis) which is activated whenever the gradients of wave elevation and/or topography are not aligned with the gradient of the vertically averaged buoyancy.

MSC:

- 76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction
- 76M35 Stochastic analysis applied to problems in fluid mechanics
- 76M45 Asymptotic methods, singular perturbations applied to problems in fluid mechanics
- 80A19 Diffusive and convective heat and mass transfer, heat flow

Keywords:

stochastic variational formulation; Kelvin circulation theorem; Euler three-dimensional equations; dispersive nonlinear wave wave; asymptotic expansion; vertical averaging

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