Fuzzy Estimator Indirect Terminal Sliding Mode Control of Nonlinear Systems based on Adaptive Continuous Barrier Function

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ABSTRACT In this paper, the theory of control is considered on nonlinear systems. A closed-loop controller with a strong idea has been introduced to track system states and guarantee asymptotically stability. The proposed method is the indirect terminal sliding mode control technique based on adaptive and fuzzy rules, which has used the continuous barrier function as a new approach in its design to improve the performance of this controller. One of the significant challenges in the sliding mode control method is the chattering phenomenon due to the discontinuous sign function in the control law. In the proposed approach, the control law is obtained continuously and smoothly due to the mentioned continuous function and subsequently solves the chattering problem. Another feature of the proposed method is the asymptotical stability of the system dynamics within finite time, which is proved based on the Lyapunov function. The proposed Lyapunov function includes the function of fractional power of a sliding surface. On the other hand, the obtained control law using the sliding mode method is estimated using the fuzzy system. The adaptive approach adjusts the fuzzy law parameters and the unknown bound of external disturbance.

INDEX TERMS Finite-time stability, Continuous barrier function, Sliding mode, Fuzzy estimator.

I. INTRODUCTION

Since most systems are inherently nonlinear and have uncertainties, it is necessary to choose a control method capable of applying these types of systems. For example, the converters in grid drive power supply are made of semiconductor components and are also modeled on nonlinear systems practically. In order to investigate and control of the high-voltage insulation in these circuits, the nonlinear control algorithms such as primary and secondary side regulation controllers can be used, which cause high efficiency, low complexity in the control circuit, lower load voltage regulation, and capacitance stress [1]. There are several methods for controlling nonlinear systems, one of which is sliding mode control. The sliding mode control is based on a nonlinear structure, which is easily understood. It can also guarantee the stability and strength of a system. There are drawbacks to the standard sliding mode method, the most important of which is the chattering phenomenon caused by the use of the switching control law. In fact, in the ideal sliding mode method, the switching frequency is assumed to be unlimited. However, there is no ideal sliding mode in practice. In other words, switching time delays and small-time constants in actuators cause fluctuations in the vicinity of the sliding surface. This phenomenon may stimulate high-frequency unmodulated dynamics [2-4]. It is important to note that there is a direct relationship between chattering and maintaining stability in the sliding mode method. Combining sliding mode control with other control methods can create new approaches to control a system with uncertainty and reduce the chattering phenomenon. In recent years, the sliding mode control method has been used as a new control method for a wide range of systems, such as time-variant, nonlinear systems, and including uncertainties [5-8]. In the sliding mode method, when the system operates near the sliding surface, it causes the chattering phenomenon and leads to discontinuous control action. It is significant to note that there is a direct relationship between chattering and maintaining stability in the sliding mode method. When sliding mode control incorporates other control methods, it
can create new approaches to control a system with uncertainty and reduce the chattering phenomenon. These approaches include adaptive sliding mode [9, 10], fuzzy sliding mode [11, 12], adaptive fuzzy sliding mode [13, 14], neural fuzzy sliding mode [15, 16], and adaptive neural fuzzy sliding mode [17]. Recently, a lot of control research has been done with the sliding mode method. In [18], two MIMO structures of sliding mode control without model are proposed, confirmed by experimental results. In [19], an adaptive sliding mode control for nonlinear Markov jump systems with actuator disturbances is presented. Since the range of actuator disturbances is unknown, the research's primary purpose was to design an adaptive sliding mode controller to overcome these problems. Another adaptive sliding mode controller is provided in [20] for unspecified nonlinear systems based on the PID dynamic sliding mode controller. In mentioned research, the method of adaptation parameter adjustment has been used to estimate the distribution limits. Lyapunov's theory proves the stability of the controlling control system. Finally, the simulation results for an inverse pendulum system are shown. The sliding mode control has also been used to control nonlinear fractional-order systems [21]. In [22], an adaptive terminal sliding mode control (TSMC) technique is designed for nonlinear systems with disturbances. In [23], an adaptive second-order TSMC method for fixing a robot with two degrees of freedom is proposed. An adaptive dispersal attumment-based synchronization control on TSMC is designed to form the spacecraft [24]. A fast adaptive terminal sliding mode scheme with global sliding mode control is offered in [25] To track the control of nonlinear systems with uncertainty. Reference [26] An adaptive nonsingular TSMC is proposed for tracking a microelectromechanical system gyroscope in the presence of parameter changes and high bound disturbances. In [27], the TSMC is designed on an ultrasonic motor through an observer of disturbance. In [28], an adaptive continuous quick TSMC is proposed to track the place of robotic manipulators. Reference [29] The second-order sliding mode is proposed as an adaptive TSMC technique for near-space vehicles.

A new nonsingular quick finite-time sliding mode method has also been investigated in [30] for programming the direction of the terminal angle limit to intercept the maneuvering target by reducing the chattering in the conduction law. In [31], a nonlinear super twisting finite-time sliding mode technique is programmed to track the quadcopter angle. In [32], in the presence of nonlinear disturbances, an adaptive terminal sliding mode technique is suggested for ultrasonic flying vehicles. Another adaptive TSMC method with a programming user in [33] is recommended to control the tracking of the hybrid energy storage system. An adaptive finite-time sliding state approach based on nonlinear distribution has been proposed in [34] to regulate the vehicle angle. In [35], a high-order adaptive TSMC with delay time estimation for the robot manipulator in the presence of reactive waste is proposed. A nonsingular adaptive fractional-order super twisting finite-time sliding mode is programmed for the robot cable arms based on a delay estimate of [36]. In [37], A nonsingular adaptive fractional-order super twisting finite-time sliding mode with delay time estimation is planned for high-accurate tracking of cable arms despite integrated uncertainty. In [38], a strong nonsingular adaptive TSMC according to dynamic inversion is designed to track the position and angle of a practical quadcopter. Reference [39] proposes a combination TSMC relied on the adaptive observer to stabilize indeterminate nonlinear dynamic systems. In addition, an adaptive nonsingular fast TSMC technique based on delay estimation [40] is proposed to satisfy the robot's axial cable arm tracking. In [41], an adaptive integrated TSMC method for robust monitoring of quite active mechanical systems is proposed using a neural system. In [42], an adaptive fast TSMC based on adapted backstepping integral controls the quadcopter finite-time tracking. In [43], a continuous adaptive control approach for a dual integrator with continuous Lipschitz distribution is proposed, ensuring states' convergence to the origin in a finite time. Two output feedback controllers are designed based on the constant twisting algorithm in [44], in which the suggested mode observers are based on first- and exact second-order derivatives. In [45], a continuous integral-based super twisting sliding mode control approach is proposed for linear and nonlinear systems with synchronous disturbances, which replaces the discontinuous part of the feedback controller with a super twisting rule. Thus, none of [43-45] studies have focused on the TSMC approach based on the adaptive barrier function. The proposed method in this paper, is a developed method from approaches that have already been designed and implemented on robotic arm and multi-agent systems. For example, an adaptive terminal sliding mode control method with a continuous barrier function is proposed to control the robot arm despite external disturbances [46]. Also, a particular class of Lyapunov function based on continuous barrier function has been used to control multi-agent systems to eliminate errors and ensure system stability [47]. Another study proposes a feedback control approach using a time-varying barrier function for optimal and robust performance of controller and multi-agent systems [48]. By using controllers based on continuous barrier function, in addition to eliminating system state error, finite-time stability can also be guaranteed in multi-agent systems [49]. Also, the assumptions of the problem indicate that the external disturbances are unknown, but a range of them is clear. As far as the author knows, very little effort has been made to propose a fuzzy adaptive TSMC approach.

In this paper, in contrast to the above, the sliding mode control law, which is divided into two parts, equivalent control and switching control, is approximated using a fuzzy system. This fuzzy system operates so that its parameters are regulated online by an adaption law. Thus, in the end, the system's output follows the desired value asymptotically. The expression of indirect sliding mode control based on the proposed fuzzy adaptive method is simpler and independent of the unknown bound characteristics of the model and

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disturbances. The stability of the closed-loop system equipped with indirect sliding mode control based on the fuzzy adaptive method has also been proven. Since the ordinary sliding mode controllers have a discontinuous sign function, they encounter the chattering phenomenon’s fundamental problem. The proposed scheme of a Lyapunov candidate function presents a fractional-order, including the sliding surface, which is based on the adaptive continuous barrier function so that the control law designed on nonlinear, continuous, and smooth systems is obtained. Consequently, the chattering phenomenon is eliminated and the unknown bound of external disturbances is estimated using an adaptive barrier function and fuzzy estimator. Also, this improved control method satisfies the convergence of states around the switching surface in a finite time. Finally, the simulation results obtained from the controller design on the reverse pendulum system in the presence of the desired signal of the time variable and unknown disturbance are evaluated.

The second part of this paper presents a comprehensive model for n-order nonlinear systems, which includes unknown disturbances. The third section presents the details of the terminal indirect sliding control strategy based on the adaptive Barrier continuous function and its application in nonlinear systems. In section four, the simulation results from the proposed controller design using simulation in MATLAB-Simulink environment are analyzed. Finally, the fifth section is devoted to the conclusion.

II. NOTATIONS AND DEFINITIONS

The structure of the nonlinear system is expressed as equation (1).

\[ x^{(n)} = f(x, t) + u(t) + d(t) \]

In equation (1), \( x \in \mathbb{R}^n \) is the vector of the system state variables, \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) is the input and output, respectively; \( f(x, t) \) represents the nonlinear function, and \( d(t) \) defines the input disturbance to the system. According to the vector of state variables \( x \), equation (1) can also be rewritten and checked in the structure of equation (2):

\[ \dot{x}(t) = Ax + F(x, t) + Bu(t) + d(t) \]  

In equation (2), \( A \) and \( B \) are matrices with fixed components, \( F \) is the vector for the nonlinear function of the system, and \( d(t) \) is the vector for the input disturbance.

The control strategy in this nonlinear system is to converge the state variables \( x \) to the desired values \( x_d = (x_d, \dot{x}_d, ..., x_d^{(n−1)})^T \) despite the disturbance. In this way, the system state error vector is also defined as equation (3):

\[ \hat{x} = x - x_d = (\dot{x}, \ddot{x}, ..., \dot{x}^{(n−1)})^T \]

The sliding surface equation can also be defined as a linear combination of system state errors as shown in equation (4):

\[ s(\hat{x}) = (c_1 \dot{x} + c_2 \ddot{x} + ... + c_{n−1} \dot{x}^{(n−2)} + \dot{x}^{(n−1)}) = C\dot{x}(t) \]  

In equation (4), \( C = [c_1, c_2, ..., c_{n−1}, 1] \) is a vector of the coefficients \( c_1 \) to \( c_{n−1} \), which must first represent positive parameters. And then prove the convergence of states to the sliding surface in a finite time and asymptotically stable closed-loop.

Lemma 1 [50]: According to \( x \in \Omega \subset \mathbb{R}^n \), a continuous function \( \dot{x} = \mathcal{S}(x) \) maps \( \mathbb{R}^n \) to \( \mathbb{R}^n \) in the region \( \Omega \) as a function Lipschitz is local at \( \Omega(0) \) and \( \mathcal{S}(0) = 0 \). Also, despite a continuous function \( V: \Omega \to \mathbb{R} \) as assumed (a) \( V \) is positive-definite; (b) \( V \) is definite by \( \Omega(0) \) negative; (c) There are real values of \( m \) and \( 0 < \alpha < 1 \), and a neighborhood \( N \subset \Omega \) of the origin where inequality (5) holds at \( N \backslash \{0\} \).

\[ \dot{V} + MV^a \leq 0 \]  

Therefore, the origin is limited-time stable for system \( \dot{x} = \mathcal{S}(x) \).

In this way, the system states also converge from the origin to the initial time \( t_0 \) in a limited time \( t_b \). equation (6) expresses the finite time equation \( t_b \):

\[ t_b = t_0 + \frac{V^{1−a}(t_0)}{c(1 − a)} \]  

III. PROBLEM DESCRIPTION

According to the design steps of the sliding mode controller, as the first stage, the convergence of the states to the sliding surface is examined. By defining equation (4) as the sliding surface, the equations \( s = 0 \) and \( \dot{s} = 0 \) are examined to calculate the equivalent control input in sliding mode control. Therefore, a law similar to the assumption \( \mathcal{C} = I \) is obtained as equation (7).

\[ u(t) = −\mathcal{C} \ddot{x} - \frac{\mu_1}{\eta_m} s_1(t)^{b+1−\frac{m}{\pi}} \]

Next, a switching control input that called \( u_{sw} \), should be added to the control law to reduce chattering. For this purpose, the Discontinuous sign function is used in conventional methods. But in proposed, a continuous function is used to improve the controller’s performance and obtain a smooth and continuous control law. Thus, despite \( \eta_l \) and \( \mu_1 \), two fixed positive parameters for regulating the rate of convergence of states to the sliding surface and \( m \) and \( n \) positive integers with \( m > n \) and \( b \) being a positive odd number, a Lyapunov function is defined in equation (8). This function is introduced to analyze the stability of nonlinear system (1).

\[ V_l(t) = \eta_l s_1(t)^{\frac{m}{\pi}} \]

\[ = \eta_l s_1(t)^{\frac{m}{\pi}} \text{sgn}(s_1(t)^{\frac{m}{\pi}}) \]

\[ = \eta_l s_1(t)^{\frac{m}{\pi}} \text{sgn}(s_1(t)) \]  

Following, according to inequality (9), to prove the stability of the system (1), \( \dot{V_l}(t) \) must be negative.
\( \dot{V}_1(t) = \eta \frac{m}{n} - s_1(t) \frac{m-1}{n} \dot{s}_1(t) sgn(s_1(t)) \)

\( \dot{V}_1(t) < 0 \) \hspace{1cm} (9)

On the other hand, equation (10) also can be defined for \( \dot{V}_1(t) \).

\[ \dot{V}_1(t) = -\mu_i s_1(t) b \]

\[ = -\mu_i s_1(t) b \text{sgn}(s_1(t)) < 0 \] \hspace{1cm} (10)

Now, according to the two equations (9) and (10), the changes in the sliding surface can be written as equation (11).

\[ \dot{s}_1(t) = -\frac{\mu_i n}{\eta \mu} s_1(t) b \frac{+1 - m}{n} \] \hspace{1cm} (11)

According to the equation (2), (4), and (11) of the control law (7) are calculated and then by substituting equation (7) in (9) The next step has been taken to improve the sliding mode control rule to eliminate chattering. And thus, equation (12) is obtained.

\[ \dot{V}_1(t) = \eta \frac{m}{n} s_1(t) \frac{m-1}{n} \text{sgn}(s_1(t)) \]

\[ \times (C d(t) - \frac{\mu_i n}{\eta \mu} s_1(t) b \frac{+1 - m}{n}) \leq 0 \] \hspace{1cm} (12)

In other words, inequality (14) can be expressed to guarantee the stability of the nonlinear system (2).

\[ \mu_i \geq \frac{\eta m D}{n} s_1(t) \frac{m-1}{n} b \] \hspace{1cm} (14)

As can be seen from Equation (14), the amplitude of the \( \mu_i \) changes directly related to the amplitude of the sliding surface changes. So, it can be concluded that the boundary of changes \( \mu_i \) is defined as the relation \( \mu_i \geq \Lambda s_1(t) \kappa \), which includes the constant-coefficient \( \Lambda \) and the fractional power \( \kappa \) of the sliding surface changes.

Equation (14), guarantees the stability of nonlinear systems (2).

Since, one of the important challenges in control theory is approximation of unknown upper bound of external disturbance, so to solve this problem, two controllers are planned in two theorem templates.

**Theorem 1**: Although the external disturbance bound \( d_i(t) \) is unknown, it nevertheless has a definite bound in the range \( |d_i(t)| < D \). Thus \( \hat{D}_i \) can be an estimate of \( D_i \) for the nonlinear system (2) and the sliding surface (4) as calculated from Equation (15).

\[ \hat{D}_i = \psi_i |s_i(t)| \] \hspace{1cm} (15)

In equation (15), \( \psi_i > 0 \), and according to \( \hat{D}_i \), the control law that ensures the error convergence of system states to the sliding surface is proposed as equation (16).

\[ u(t) = -CAx - CF(x, t) + CBu(t) + \hat{D}_i(t) \text{sgn}(s_1(t)) \] \hspace{1cm} (16)

Next, the behavior of control law (16) is proved by defining \( \bar{d}_i = \hat{D}_i - D_i \) as the error of estimating disturbance and \( \mu_i \) a scalar number in the range \( 0 < \mu_i < \psi_i^{-1} \).

**Proof**: A Lyapunov candidate function is considered as equation (17).

\[ V_i(t) = 0.5\mu_i \hat{D}_i(t)^2 + 0.5s_i(t)^2 \] \hspace{1cm} (17)

In equation (17), \( \hat{D}_i(t) = \hat{D}_i(t) - D_i \) and \( \mu_i \) is a scalar number in the range of \( 0 < \mu_i < \psi_i^{-1} \). Next, \( \dot{V}_i(t) \) is calculated using equations (4) and (15) as equation (18).

\[ \dot{V}_i(t) = \mu_i \hat{D}_i(t) s_i(t) \]

\[ + s_i(t)(CAx(t) + C F(x, t) + CBu(t) + \hat{D}_i(t) - C \dot{x}_d(t)) \] \hspace{1cm} (19)

In the next step, using equation (3) and substituting the dynamic model (2), \( \dot{V}_i(t) \) is extended as equation (19).

\[ \dot{V}_i(t) = \mu_i \psi_i \hat{D}_i(t) |s_i(t)| \]

\[ + |s_i(t)| \] \hspace{1cm} (19)

\[ + s_i(t)(CAx(t) + C F(x, t) + CBu(t) + \hat{D}_i(t) - C \dot{x}_d(t)) \]

\[ + s_i(t)(CDd(t) - C x_d(t)) \]

\[ \dot{V}_i(t) \leq \mu_i \psi_i |s_i(t)| \]

\[ + |s_i(t)| \] \hspace{1cm} (20)

\[ + s_i(t)(CDd(t) - C x_d(t)) \]

\[ \leq (1 - \mu_i \psi_i) |s_i(t)| \] \hspace{1cm} (20)

\[ \leq -|\hat{D}_i(t)| s_i(t) \]

\[ \leq -D_i s_i(t) \]

As mentioned, \( |d_i(t)| < D \) and, on the other hand, \( \mu_i \psi_i < 1 \) thus equation (21) can be defined.
\[ V_i(t) \leq \sqrt{2}(D_i - |C_d(t)|) \frac{|s_i(t)|}{\sqrt{2}} \]
\[ - \sqrt{\frac{2}{\mu_i}} (1) \]
\[ - \mu_i|s_i(t)| \frac{\mu_i}{2} D_i \]
\[ \leq -\min \left\{ \sqrt{2}(D_i - |C_d(t)|), \frac{2}{\mu_i} (1) \right\} \]

Clearly, in equation (21), \( \Xi_i \) will be equal to Equation (22).

\[ \Xi_i = \min \left\{ \sqrt{2}(D_i - |C_d(t)|), \frac{2}{\mu_i} (1) \right\} \]

As a result, it should be noted that, according to Lemma 1, the control law (16) causes the state’s error of the system to converge to the sliding surface in a finite time.

**Theorem 2:** Although the external disturbance bound \( d(t) \) is unknown, a range of it is available as \( D > 0 \). Thus \( \hat{D} \) can be an estimation of \( D \) for the nonlinear system (2) and the sliding surface (4) as calculated from equation (23).

\[ \hat{D} = \psi_i|s_i(t)|^{\frac{m}{n}} \]

In equation (23), \( \psi_i > 0 \) and the control law can be calculated according to equation (24). This control input converges the state errors to the sliding surface in finite time.

\[ u(t) = -CAx - CF(x, t) - \hat{D} + C \hat{x}_d \]
\[ - \frac{\mu_i}{\eta_i m} s_i(t)^{b+1} \frac{m}{n} \]

Next, the behavior of the control law (24) is proved by defining \( \tilde{D} = D_i - \hat{D} \) as the error of estimating disturbance and numerical \( \ell_i \) in the range \( 0 < \ell_i < \frac{\eta_i m}{\psi_i m} \).

**Proof:** First, a Lyapunov function is defined as equation (25).

\[ V_i(t) = \eta_i s_i(t)^{\frac{m}{n}} sgn(s_i(t)) + 0.5\ell_i \tilde{D}_i^2 \]

In Eq. (25), \( \tilde{D}_i(t) = \tilde{D}_i(t) - D_i \) and \( \ell_i \) is in the range \( 0 < \ell_i < \frac{\eta_i m}{\psi_i m} \). Next, \( \dot{V}_i(t) \) is obtained by Equations (4) and (23) as Equation (26).

\[ \dot{V}_i(t) = \ell_i \psi_i \tilde{D}_i(t) s_i(t)^{\frac{m}{n}} \]
\[ + \eta_i s_i(t)^{\frac{m}{n}} sgn(s_i(t)) \]
\[ + \dot{\tilde{D}}_i(t) \]
\[ = \eta_i \frac{m}{n} s_i(t)^{\frac{m}{n}} (C \tilde{x}(t)) sgn(s_i(t)) \]
\[ + \ell_i \psi_i \tilde{D}_i(t) s_i(t)^{\frac{m}{n}} \]

Next, according to equation (3) and using the dynamic model (2), \( \dot{V}_i(t) \) is rewritten as equation (27).

\[ \dot{V}_i(t) = \ell_i \psi_i (\tilde{D}_i(t) - D_i) s_i(t)^{\frac{m}{n}} \]
\[ + \eta_i \frac{m}{n} s_i(t)^{\frac{m}{n}} (C \tilde{x}(t)) sgn(s_i(t)) \]
\[ + \eta_i \frac{m}{n} C_d(t) s_i(t)^{\frac{m}{n}} \]

Then, by applying the control law (24) into (27), \( \dot{V}_i(t) \) is obtained as equation (28).

\[ \dot{V}_i(t) = \ell_i \psi_i (\tilde{D}_i(t) - D_i) s_i(t)^{\frac{m}{n}} \]
\[ - \mu_i s_i(t)^{b} - \eta_i \frac{m}{n} s_i(t)^{\frac{m}{n}} \]
\[ + \eta_i \frac{m}{n} C_d(t) s_i(t)^{\frac{m}{n}} \]

Equation (28) can be analyzed as Equation (29) in the next step.

\[ \dot{V}_i(t) \leq \ell_i \psi_i (\tilde{D}_i(t) - D_i) s_i(t)^{\frac{m}{n}} \]
\[ - \eta_i \frac{m}{n} \tilde{D}_i(t) s_i(t)^{\frac{m}{n}} \]
\[ + \eta_i \frac{m}{n} (\tilde{D}_i(t) - D_i) s_i(t)^{\frac{m}{n}} \]
\[ \leq - \eta_i \frac{m}{n} (D_i - |C_d(t)|) s_i(t)^{\frac{m}{n}} \]

Finally, with \( |d(t)| < D \) and \( \ell_i \psi_i \leq \frac{\eta_i m}{\psi_i m} \), equation (30) can be defined.
\(V_i(t)\)
\[
\leq \frac{m\sqrt{n_i}}{n} (D_i - C_i \epsilon_i) - |C_d (t)| |S_i(t)|^{r - 1} \left( \sqrt{n_i} |S_i(t)|^{m/n} \right)
- 2 \frac{\epsilon_i}{\eta_i} (\eta_i)^{m/n} |S_i(t)|^{m/n - 1}
- \epsilon_i \phi_i |S_i(t)|^{m/n - 1} \left( \frac{\epsilon_i}{\sqrt{2}} D_i(t) \right)
\]
\[
\leq -\min \left\{ \frac{m\sqrt{n_i}}{n} (D_i - C_i \epsilon_i), - |C_d (t)| |S_i(t)|^{r - 1} \left( \sqrt{n_i} |S_i(t)|^{m/n} \right), - \frac{2 \epsilon_i}{\eta_i} (\eta_i)^{m/n} |S_i(t)|^{m/n - 1}, - \epsilon_i \phi_i |S_i(t)|^{m/n - 1} \left( \frac{\epsilon_i}{\sqrt{2}} D_i(t) \right) \right\}
\]
(30)

A closer look at Equation (30) shows that the parameter \(Z_i\) is expressed as a relation (31).
\[
Z_i = \min \left\{ \frac{m\sqrt{n_i}}{n} (D_i - C_i \epsilon_i), - |C_d (t)| |S_i(t)|^{r - 1} \left( \sqrt{n_i} |S_i(t)|^{m/n} \right), - \frac{2 \epsilon_i}{\eta_i} (\eta_i)^{m/n} |S_i(t)|^{m/n - 1}, - \epsilon_i \phi_i |S_i(t)|^{m/n - 1} \left( \frac{\epsilon_i}{\sqrt{2}} D_i(t) \right) \right\} > 0
\]
(31)

Thus, according to Lemma 1, it is proved that the control law (24) converges the state's error of the system to the sliding surface in a finite time.

Next, the disturbance estimate is then developed using the continuous barrier function, according to which the proposed controller solves the chattering problem.

The disturbance estimation is then developed using the continuous barrier function, according to which the chattering problem will be solved in response by the proposed controller. Thus, for disturbance estimation to the neighborhood of the sliding surface, the two-equation can be defined according to equation (32).
\[
\bar{D}_i(t) = \begin{cases} 
D_i(t), & \text{if } 0 < t \leq \bar{t} \\
\bar{D}_{psd}(t), & \text{if } t > \bar{t}
\end{cases}
\]
(32)

In equation (32), \(\bar{t}\) defines the duration of convergence of state errors to neighborhood \(\bar{\epsilon}\) from the sliding surface, and \(\bar{D}_{psd}(t)\) represents the continuous barrier function proposed for the design of the controller on the nonlinear system (2) and is given according to equation (33).
\[
\bar{D}_{psd}(t) = \frac{|S_i(t)|}{\epsilon_i - |S_i(t)|}
\]
(33)

Thus, the law of control for times \(t > \bar{t}\) to eliminate chattering is defined by the adaptation law of \(\bar{D}_{psd}(t)\) as relation (34).
\[
\dot{u}(t) = -CAx - CF(x, t) + \bar{C}x_d - \bar{D}_{psd}(t) sgn(S_i(t))
\]
(34)

Thus, in general, the dynamics of the nonlinear systems defined in equation (2) converge asymptotically in finite time. The chattering problem is also solved using the continuous barrier function, which calculated according to the adaptive law. This control law will be estimated based on fuzzy rules in the next section.

According to the reference [51], the obtained control law can be estimated based on fuzzy rules using the Singleton and mean-centralized fuzzy method. The membership functions of the fuzzy system are considered as equation (35).
\[
y(x) = \frac{\sum_{j=1}^{m} y_j (\prod_{i=1}^{n} \Phi_{A_{ij}}(x_i))}{\sum_{j=1}^{m} (\prod_{i=1}^{n} \Phi_{A_{ij}}(x_i))}
\]
(35)

In view of membership function \(\Phi_{A_{ij}}(x_i)\), \(x_i\) and \(y_j\) are the variables instead of which \(\Phi_{B_{ij}}(y_j)\) reach its maximum value. Next, equation (35) can be rewritten as equation (36) based on the vector of the fuzzy base function \(\xi(x)\).
\[
y(x) = \theta^T \xi(x)
\]
(36)

In equation (36), the vectors \(\theta = [y^1, ..., y^m]^T\) and \(\xi(x) = [\xi^1(x), ..., \xi^m(x)]^T\) are the parameters that make up the fuzzy system mentioned. Despite the fuzzy system (36), the nonlinear part of the control law (34) is estimated according to the fuzzy laws as equation (37).
\[
\dot{u}(t) = CAx + \bar{f}(x|\theta_f) + \bar{D} + \bar{C}x_d - \bar{D}_{psd}(t)s gn(S_i(t))
\]
(37)

\(\dot{f}(x|\theta_f)\) in equation (37), is obtained according to the fuzzy law as (38).
\[
\dot{f}(x|\theta_f) = \theta_f^T \xi(x)
\]
(38)

Hence, assuming that the parameters \(\xi(x)\) are constant, again using the adaption law, the parameters \(\theta_f^T\) is estimated.

The estimation of the mentioned parameters based on the adaptive method is done according to equation (39).
\[
\dot{\theta}_f = r s \xi(x)
\]
(39)

In equation (39), \(r\) is a fixed and positive number that can be defined according to the minimum estimation error (40). One of the optimal methods to choose \(r\) is the utilization of
a suitable Lyapunov function to eliminate the estimation error \( \omega \) [51].

\[
\omega = CF(x, t) - \hat{f}(x|\theta_f)
\]  
(40)

The last step of the controller design is completed by estimating the nonlinear part of the control law and adjusting its parameters online by fuzzy adaptive rules. To better understand the proposed controller structure, design and implementation steps on nonlinear systems are shown in Figure 1.

\[
\dot{\theta} = x_2 \\
\dot{x}_2 = g \sin x_1 - mlx_2^2 \cos x_1 \sin x_1/(m_c + m) \\
\quad + l(\frac{4}{3} - m \cos^2 x_1)/(m_c + m)
\]  
(41)

In equation (41), \( x_1 \) and \( x_2 \) express the angular position and its rate of change, respectively. Among the system, parameters \( g \) is gravitational acceleration, \( m_c \) is the mass of the pendulum conveyor, \( m \) denotes the mass of its rotation pole, and \( l \) illustrates its length. The parameters values and other familiar characteristics are specified in Table 1.

| \( l \) | 0.5 m | \( x(0) \) | \([ -\pi / 60, 0 ]^T \) |
|---|---|---|---|
| \( g \) | 9.8 m/s^2 | \( C \) | \([ 30, 1 ]^T \) |
| \( m \) | 0.1 kg | \( y_d(t) \) | 0.1 sin(t) |
| \( m_c \) | 1 kg | \( d(t) \) | 10 sin(t) |

The parameter \( \theta_f \) in the fuzzy estimator is a vector of \( 1 \times 3 \), with the initial values from each of its components being 0.1. The membership functions used in the fuzzy estimator for \( i = 1, 2, 3 \) are considered according to equations (42), which are in terms of the system’s state.

\[
f_{A_i}(x_i) = \exp\left(-\frac{x_i - x_i^{-1}}{\pi/24}\right)
\]  
(42)

In equation (42), \( x_i^{-1} \) are equal to \( -\pi/6, -\pi/12, \pi/12 \), and also \( A_i \) are a fuzzy set consisting of NB, ZO, NS, Ps, and PB that \( l \) th belongs to. In order to evaluate the simulation results accurately, the proposed method is compared with similar methods in terms of system response and control law. In references [52-54] are examined a terminal sliding mode controller on an inverted pendulum system attempted to improve the controller performance with different approaches. Figure 2 shows the angular position of the system concerning the desired value. Clearly, in the proposed controller, the system tracking speed was much higher, and consequently, the convergence time of the system state error was improved. Table 2 shows the system state error’s convergence time in the proposed and similar controllers.

| Finite-time sliding mode controller | convergences time (s) |
|---|---|
| Proposed controller | 1.29 |
| designed Controller in [52] | 1.32 |
| designed Controller in [53] | 1.84 |
| designed Controller in [54] | 1.79 |

The essential point in evaluating the proposed controller is to examine the control law. As mentioned, due to the use of the continuous adaptive barrier function in the proposed method, the chattering problem is extremely reduced and causes the control input to be obtained as a smooth and continuous signal. This can be found in Figure 3 and comparison with the performance of similar controllers.
According to the comparison made in Figure 3, it can be seen that the proposed controller, in addition to eliminating chattering, performs better than similar controllers in terms of performance, and its fluctuation range is much less than other control rules. But, one of the most important parts of assessing a sliding mode controller is the evaluation of the sliding surface changes. As shown in Figure 4, it can be seen that the sliding surface in the proposed controller is obtained without chattering and with a suitable time response.

According to Figure 4, the amplitude of the sliding surface changes in the system is approximately equal to $|s_i(t)| = 6$ Thus the interval of changes $\mu_i$ will also be equal to $\mu_i \geq \Lambda \times 6^6$. Another important part of the assessment of the response of this controller is to examine the parameter $\bar{D}$, which is calculated based on the adaptive continuous barrier function. Changes in this parameter are observed in the system control period according to Figure 5. In Figure 5, the suitable time response of the controller is also well observed.

The proposed method can be generalized to all nonlinear systems, and on the other hand, it indirectly designs and implements advanced control methods. The fuzzy control method is one of the most widely used methods that reduce the complexity of problems. Also, it has high flexibility as the control system parameters adjust online using adaptive rules to improve the system response.

**VI. CONCLUSION AND FUTURE WORKS**

The control of nonlinear systems theory is always important according to complexity, uncertainty, and the effect of external disturbances on nonlinear systems. The sliding mode control method is one of the most practical nonlinear control methods developing but has always faced a significant problem called the Chattering phenomenon. The primary purpose of this paper is to solve this challenge in nonlinear control. The sliding mode controller is designed based on an adaptive barrier function that replaces the discontinuous sign function in conventional methods to solve this problem. Also, the asymptotic stability and convergence of the system states to the desired value in a finite time is ensured by considering a Lyapunov function, which includes the fractional power of the sliding surface. In addition, to reduce the complexity of the control law, its nonlinear term is estimated online using the fuzzy system. The improvement of the results in the proposed method is clearly shown in the form of a section. As future activities, some topics such as using an observer to estimate system states and evaluate the controller in its presence, using a variety of evolutionary optimization algorithms to find the optimal amount of controlling interest, and using higher-order sliding equations for the proposed system model can be suggested which have a good effect on improving the performance of the controller.

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