1. Introduction

Ultra-precision positioning platform is a typical structure of ultra-precision machinery, which is a complex composed of precision position detection technology, linear guidance technology, control technology, etc. (Lu et al., 2018; Liu et al., 2018; Peng et al., 2019; Wang et al., 2019). Ultra-precision positioning refers to the micro positioning on the micron or even nanometer scale (Wang et al., 2019). The recent paper (Lu et al., 2018) develops a nano rods system with rotational degrees of freedom which can meet the basic requirements of high-precision positioning of small-scale samples.

The positioning accuracy and free travel of the platform are the pivotal points of ultra-precision machining technology and the quality of the parts to be processed is ultimately affected by positioning accuracy (Liu et al., 2019). The working efficiency of equipment and its applicable range will be affected by the size of free travel (Peng et al., 2018). At present, electric drive mode plays a vital role in particular for ultra-precision positioning system due to the high efficiency of converting electric energy into kinetic energy. However, the electric drive positioning platform has a small free travel (Prakash et al., 2018). Large travel distance driven by motor and power loss of commutator during commutating (Zabihollah et al., 2017). Pneumatic system has a simple mechanical structure and uses the renewable energy as the driving source, so pneumatic drive is widely employed. Cylinder is a common pneumatic actuator that has been studied and many successful applications have also been publicized (D et al., 2018; Yao et al., 2018). For the cylinder servo system, (D et al., 2018) attempted a positioning system with two cylinders connected together and applied fuzzy control. Yet, the cylinder servo system has a large switching range of the control variables and high switching frequency. Based on the above questions, a cross coupling fuzzy PID controller based on feedback compensation input and load disturbance decoupling control method was established (Yao et al., 2018). Although good
control effect has been achieved, the positioning accuracy of cylinder itself is not high and the friction is large, it is difficult to achieve ultra-precise positioning.

Bellows are a new type of pneumatic actuator, which can undergo axial elastic deformation under pressure (Rehman et al., 2017). Compared to cylinder, pneumatic bellows have the advantage of low cost, excellent flexibility and low stiffness. Therefore, many scholars have studied pneumatic bellows actuators, and they are implemented in many occasions (Kang et al., 2006; Kawashima et al., 2010; Melingui et al., 2017). For example, Korean scientists Kang et al (Kang et al., 2006) applied pneumatic bellows to micro-modeling technology and made it possible to manufacture complex three-dimensional microstructures by using micro-stereoscopic lithography (MSTL) technology. Subsequently, Japanese scholars Kawashima et al (Kawashima et al., 2010) researched a single-degree-of-freedom coarse/fine bipolar platform with bellows as pneumatic actuator. At the same time, an adaptive control scheme for the manipulator composed of bellows was fabricated (Melingui et al., 2017). On account of the pneumatic system has plural type of unfavorable factors such as the low impact toughness, poor stability and uncertainty in the process of modeling, the development of high-performance pneumatic actuator is obstructed (Rehman et al., 2017). Consequently, the research on pneumatic bellows system has extremely high practical significance and theoretical value.

In recent years, control algorithms applied to pneumatic systems include PID algorithm (Al-Dhaifallah et al., 2018), adaptive control algorithm (Ren et al., 2016; Melingui et al., 2017), auto-disturbance rejection control algorithm (Almeida et al., 2019), and sliding mode control algorithm (Ai et al., 2018). Although several control methods are applied in the design of nonlinear control algorithm for pneumatic bellows system in these papers, the mathematical model of system is not strictly linear, and the unknown state variables are difficult to measure accurately. In the practical application of these algorithms, some parameters need to be adjusted, so the design of the controller is complicated, and the model parameters are not easy to determine.

The main innovations and contribution of this technical note are concluded as follows:
1. Based on the one-dimensional positioning system, a two-dimensional bellows-driven system is built, and the output displacement signal is collected by the grating ruler sensor.
2. According to kinematics equation of bellows actuator, mass flow equation and dynamic characteristic equation of electric pressure proportional valve, the mathematical model of pneumatic two-dimensional ultra-precision positioning system is established.
3. For the bellows actuator, the emphasis is on the design of the observer. NESO is designed to estimate the unknown state of the system which eliminates the need for large and expensive air pressure sensors. Design dynamic surface control algorithm, the controller design is simplified and applied to bellows servo system after state estimation.

Specifically, the rest of this context is organized as follows. In Section 2, model of pneumatic is established and several preliminaries are inferred. NESO is designed to estimate system unknown states in Section 3 and DSC control algorithms are shown in Section 4. Section 5 verified all signals in a closed-loop system are bounded by Lyapunov method. Section 6 illustrates the practical application to assess the dynamic tracking performance of the system and Section 7 summarizes the conclusion and outlook future directions for development.

2. Problem statement and preliminaries

This scientific research work concentrates on fabricating and detecting the availability of using a bellows-driven scheme. It is anticipated that such an approach will generate a comparatively large distance of travel and precise positioning. Then, for the sake of facilitating the design procedure, the system is qualitatively analyzed. Ultimately, such a design is, therefore, assembled and the mathematical model is established.

2.1 Bellows-driven positioning system

The two-dimensional ultra-precision positioning system uses "U" bellows as the actuator. Compressed gas is supplied by a small air source and the gas pressure is fixed at 0.4 mpa through the pressure regulating valve filter. The moving part of the working platform is composed of an air floating guide rail and a slide block above it. The grating ruler is used to detect the displacement signal of the system, and the encoder converts the displacement signal read by the reading head into digital pulse signal. SCM outputs voltage signal through D/A module to control the size of electrical pressure proportional valve which can adjust the bellows cavity pressure, make the bellows deformation, produce expansion, achieve the purpose of controlling the bellows displacement. The overall model of bellows servo
system is shown in Fig. 1, and the structural block diagram of part of the system is shown in Fig. 2.

Fig. 1 The model of bellows-driven system. 1.Carrier platform, 2. Floating guide rail, 3. Bellows

Fig. 2 Part of the system structure block diagram

2.2 Kinematic equation of bellows actuator

The flange of bellows was made of stainless acid-resistant steel, pressure class is 12Mpa, inner diameter is 130.00mm, and the inner diameter of the four bolt holes on the flange is 12.5mm. The bellows were also made of stainless acid-resistant steel, brand 1Crl8Ni9Ti. Take the Y-axis bellows as an example, the size parameters are: inner diameter 32.00mm, wall thickness 0.12mm, total length 340.00mm, and the inner diameter of bolt hole of the right fixed part is 10.00mm. Only the total length of the X-axis bellows is different from that of the Y-axis. All other dimensions are the same. The stroke $W$ of bellows is proportional to the number of waves in the bellows, that is, the maximum allowable stroke $W_{max}$ is equal to the allowable travel of a single wave times the wave number. According to the service manual, the maximum allowable axial stroke of single wave is 1.06mm, then $W_{max} = 1.06\text{mm} \times 65 = 68.90\text{mm}$. The material object of the bellows and the dimensioning drawing are shown in Fig. 3.

Fig. 3 Material object of the bellows and the dimensioning drawing

Fig. 4 Motion model of bellows actuator
The outer wall of bellows is a spiral rotating body (Tian et al., 2017; Jing et al., 2018). Its motion model is shown in Fig. 4. The force analysis of the entire bellows actuator is carried out within the framework of Newton’s law:

\[ m\ddot{x} = AP - C\dot{x} - kx, \]  

where, \( A \) is the equivalent stressed area of the bellows, \( C \) is the damping coefficient, \( k \) is the stiffness of the bellows, and the displacement of the bellows is \( x \); \( P \) is the gas pressure in the bellows' cavity.

### 2.3 Bellows mass flow equation

When the mass of the gas inside the bellows cavity changes, the volume of the bellows cavity changes, and the bellows actuator shifts. On the basis of the law of mass conservation, the amount of gas mass flow in the bellows sealed chamber is equal to the mass change rate of sealing cavity:

\[ Q_n = \frac{d(\rho V)}{dt}, \]  

where, \( V \) is the volume of the bellows tube cavity, \( \rho \) is the density of gas, and \( Q_n \) is the mass flow rate of gas.

Assume the ideal gas in the bellows tube cavity and favorable air tightness, in the light of the Clapeyron equation:

\[ \rho = \frac{P}{RT}, \]  

with \( R \) is the gas constant and \( T \) is the gas temperature. According to the first law of thermodynamics, the energy change equation can be obtained:

\[ \frac{dE}{dt} = c_p Q_n T - P \frac{dV}{dt} + \frac{dG}{dt}, \]  

where \( E = c_p PV / R \) are the total energy in the cavity, \( c_p \) is the specific heat capacity at constant pressure of the gas, and \( c_v \) is the specific heat capacity at constant volume of the gas. \( PdV / dt \) is the work done by the gas change in unit time, and \( dG / dt \) is the heat exchange caused by the temperature difference between the tube cavity and the outside world in unit time.

When bellows are working, the charging and discharging processes are independent of each other. According to the laws of thermodynamics, we can get:

\[ \frac{dP}{dt} = \frac{c_p}{c_v} \frac{TR}{V} Q_n - \frac{P}{Vc_v} \frac{dV}{dt} + \frac{R}{Vc_v} \frac{dG}{dt}. \]  

Heat exchange \( dG / dt \) is determined by the temperature difference between the inside and outside of the bellows tube cavity. Due to the good heat dissipation performance of the bellows, it can be considered as complete heat dissipation. The gas temperature in the bellows tube cavity is constant, which can be seen as an isothermal process. Therefore \( (dG / dt) = 0 \)

\[ \frac{dV}{dt} = \frac{Adx}{dt}. \]  

Substitute equation (6) into equation (5), and the model of bellows' inflation process is:

\[ \frac{dP}{dt} = \frac{c_p}{c_v} \frac{TR}{V} Q_n - \frac{AP}{Vc_v} \frac{dx}{dt}. \]  

### 2.4 Pressure-flow characteristics of electric proportional valve

The electric pressure proportional valve is an electronically controlled gas pressure valve, which is composed of the valve body and the oil collecting coil. The flow of the valve body is regulated by the force on the oil collecting coil, and the force on the oil collecting coil is controlled by the current on the oil collecting coil (Ren et al., 2017). The displacement of bellows is not only determined by the gas pressure in the pipe cavity, but also related to the mass flow...
of the electric proportional valve into the pipe cavity. By changing the valve control voltage, the output gas pressure of the proportional valve is adjusted to achieve the purpose of displacement control. The electric proportional valve is through the voltage control opening area, and then control the gas mass flow. (Rodrigo et al., 2017). When the control voltage of the valve remains stable, the proportional valve is not affected by the air source pressure and can keep the output gas pressure unchanged (Laski et al., 2017). It is difficult to obtain the accurate mathematical model because of the influence of the dynamic shrinkage of air and other factors. Therefore, the characteristics of large frequency width of electric pressure proportional valve can be applied in practice, and it can be treated in accordance with a proportional link. Sanvil. F.E formula is adopted to obtain:

\[
Q_n = c_q A_q P_t \sqrt{\frac{2}{RT}} f \left( \frac{P}{P_r} \right),
\]

with,

\[
f(P/P_r) = \begin{cases} 
\frac{2}{r+1} \sqrt{\frac{r}{r+1}}, & 0 \leq P/P_r \leq 0.528 \\
\frac{r}{r-1} \left[ \left( \frac{P}{P_r} \right)^{\frac{r}{r-1}} - \left( \frac{P}{P_r} \right)^{\frac{r}{r-1}} \right], & 0.528 \leq P/P_r \leq 1
\end{cases}
\]

where, \( c_q \) is the flow coefficient, \( f(P/P_r) \) is the gas velocity, \( A_q \) is the effective opening area, \( r \) is the isentropic coefficient, \( P_r \) is the proportional valve outlet pressure.

In order to simplify its mathematical model, according to the proportional valve dynamic characteristics equation:

\[
X_r = K_v u,
\]

\[
A_q P_t = w X_r,
\]

where, \( K_v \) is the displacement of valve core, \( X_r \) is the proportional coefficient, \( w \) is the gradient of opening area, and \( u \) is the controlled voltage. By substituting equation (11) into equation (10), we can get:

\[
A_q = \frac{wK_v}{P_r} u,
\]

That is, electric pressure valve is approximately equivalent to a linear relationship. Equation (8) --(12) is simplified to get:

\[
Q_n = K_\omega (t) u,
\]

\[
K_\omega (t) = wK_q c_q \sqrt{\frac{2}{RT}} f \left( \frac{P}{P_r} \right),
\]

where, \( K_\omega (t) \) is the direct proportionality coefficient.

2.5 Mathematic model

Based on the above analysis, the mathematical model of two-dimensional ultra-precision positioning system can be considered as:

\[
x_a^{(1)} = f_a \left( x_a, \dot{x}_a, \ddot{x}_a \right) + bu,
\]

where, \( x_a \in R \) is the output of the system, \( f_a \left( x_a, \dot{x}_a, \ddot{x}_a \right) \) is the unknown nonlinear function of the system, \( u \) is
the control signal. Since \( X, Y \) degree of freedom has the same control mode and mutually independent, the state variable \( y_1, y_2, y_3 \) of the system is defined by taking the \( Y \) axis as an example and \( y_1 \) is selected as the displacement, that is \( y_1 = x, y_2 = \frac{dx}{dt} = y_1, y_3 = P \).

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -\frac{K}{m} y_1 - \frac{C}{m} y_2 + \frac{A}{m} y_3, \\
\dot{y}_3 &= \frac{c_p TR}{C_v} K_m (t) u - \frac{A (R + c_v)}{C_v} y_2 y_3.
\end{align*}
\]  

(16)

Carry out coordinate transformation on the above equation, let \( \bar{y}_1 = y_1, \bar{y}_2 = y_2, \bar{y}_3 = -(k/m)y_1 - (C/m)y_2 + (A/m)y_3 \). Then the mathematical model can be written as follows:

\[
\begin{align*}
\dot{\bar{y}}_1 &= \bar{y}_2 \\
\dot{\bar{y}}_2 &= \bar{y}_3 \\
\dot{\bar{y}}_3 &= a(\bar{y}) + bu,
\end{align*}
\]

(17)

where,

\[
a(\bar{y}) = -\left[ A (R + c_v) / C_v \right] \bar{y}_1 \bar{y}_3 - \left[ A K (R + c_v) / mC_v \right] \bar{y}_2 \bar{y}_3 + \left[ A C (R + c_v) / mC_v \right] \bar{y}_1 \bar{y}_2 - \left( K / m \right) \bar{y}_1 - \left( C / m \right) \bar{y}_3,
\]

\[b = c_p AK_m(t)RT/c_mV\,.
\]

Let \( \bar{y}_4 = a(y) + (b - b_0)u \) be the expanding state of the system, assume \( |\bar{y}_4| \leq H, H \) be constants greater than zero. Suppose \( b = b_0 + \Delta b, b_0 \) is the best adjustable estimate of \( b \), and \( \Delta b \) is the uncertainty associated with it. The system model can be further rewritten:

\[
\begin{align*}
\dot{\bar{y}}_1 &= \bar{y}_2 \\
\dot{\bar{y}}_2 &= \bar{y}_3 \\
\dot{\bar{y}}_3 &= \bar{y}_4 + b_0u, \\
\dot{\bar{y}}_4 &= h.
\end{align*}
\]

(18)

3. Design of NESO

Since the system is not strictly linear, a NESO is designed in this chapter to observe the unknown state of the system.

Assuming that \( \hat{y}_i (i = 1, 2, 3, 4) \) is the estimation of system state variable \( y_i (i = 1, 2, 3, 4) \), the form of the NESO is given as follows:

\[
\begin{align*}
\dot{\hat{y}}_1 &= \hat{y}_2 - \beta_1 g (e_1) \\
\dot{\hat{y}}_2 &= \hat{y}_3 - \beta_2 g (e_1) \\
\dot{\hat{y}}_3 &= \hat{y}_4 - \beta_3 g (e_1) + b_0u, \\
\dot{\hat{y}}_4 &= -\beta_4 g (e_1)
\end{align*}
\]

(19)

where \( \beta = [\beta_1, \beta_2, \beta_3, \beta_4]^T \) is the gain of the NESO, \( e_1 = \hat{y}_1 - y_1 \) is the observation error of the NESO, and the nonlinear feedback term is selected as \( \beta_4 g (e_1) \). Professor Han J proposed the expression of the nonlinear function in reference (Han., 2009):

\[
g (e_1) = \text{sat} (e_1, \chi, \sigma) = \begin{cases} 
\frac{|e_1|}{\chi} \text{sign} (e_1), & |e_1| > \sigma \\
\frac{e_1}{(e_1)^{\sigma-\chi} + |e_1|}, & |e_1| \leq \sigma
\end{cases}
\]

(20)
$\chi_i, \sigma$ is the design parameter, where $\sigma > 0$ represents the error boundary; $0 < \chi_i (i = 1, 2, 3, 4) < 1$. When the error is large, $g \left( \right)$ produces high gain; when the error is small, $g \left( \right)$ produces a low gain.

Define the observation error of the NESO:

\[
\begin{align*}
    e_1 &= y_1 - \hat{y}_1 \\
    e_2 &= y_2 - \hat{y}_2 \\
    e_3 &= y_3 - \hat{y}_3 \\
    e_4 &= y_4 - \hat{y}_4
\end{align*}
\]  

(21)

Take the derivative of the above equation:

\[
\begin{align*}
    \dot{e}_1 &= e_2 - \beta_1 g \left( e_1 \right) \\
    \dot{e}_2 &= e_3 - \beta_2 g \left( e_1 \right) \\
    \dot{e}_3 &= e_4 - \beta_3 g \left( e_1 \right) \\
    \dot{e}_4 &= -\beta_4 g \left( e_1 \right) - h
\end{align*}
\]  

(22)

**Hypothesis 1**: For $e = [e_1, e_2, e_3, e_4]^T \in \mathbb{R}$, there is a normal number $\kappa_{11}, \kappa_{12}, \kappa_{13}, \kappa_{14}, \lambda_i$ and continuous differentiable function $V_{si}, F_i$, so that:

\[
\begin{align*}
    \kappa_{11} \| e \|^2 &\leq V_{si} \left( e \right) \leq \kappa_{12} \| e \|^2 \\
    \kappa_{13} \| e \|^2 &\leq F_i \left( e \right) \leq \kappa_{14} \| e \|^2
\end{align*}
\]  

(23)

For any $\varepsilon > 0$, there is $t_1, t_2$, the derivative of $\bar{y}_4$, the extended state variable of the system, with respect to time $t$ on the interval $[t_1, t_2]$ satisfies:

\[
\| \dot{y} \| = \| \bar{y}_3 \frac{\partial f}{\partial y_3} + \bar{y}_1 \frac{\partial f}{\partial y_1} + \left( \bar{y}_4 + b_5 u \right) \frac{\partial f}{\partial y_5} \| \leq M_0,
\]  

(24)

where $M_0$ is the normal number, then the derivative of $V_{si} \left( e \right)$ with respect to time $t$ on the range $[t_1, t_2]$ satisfies:

\[
\begin{align*}
    \frac{dV_{si} \left( e \right)}{dt} &= \frac{1}{e} \left\{ \sum_{i=1}^{4} \left( e_i - g \left( e_i \right) \right) \frac{\partial V_{si}}{\partial e_i} \right\} + \left( e h - g \left( e_i \right) \right) \frac{\partial V_{si}}{\partial e_i} \\
    &\leq -\frac{\kappa_{13}}{e} \| e \|^2 + M_0 \lambda_i \| e \|^2 \\
    &\leq -\frac{\kappa_{12}}{\kappa_{11} e} V_{si} \left( e \right) + M_0 \lambda_i \frac{V_{si} \left( e \right)}{\kappa_{11}}
\end{align*}
\]  

(25)

Due to:

\[
\frac{dV_{si} \left( e \right)}{dt} = \frac{d}{dt} \left[ \sqrt{V_{si} \left( e \right)} \right]^2 = 2 \sqrt{V_{si} \left( e \right)} \frac{dV_{si} \left( e \right)}{dt}.
\]  

(26)

Inequality (25) can be further simplified to:

\[
\frac{dV_{si} \left( e \right)}{dt} \leq -\frac{\kappa_{13}}{2 \kappa_{12} e} \sqrt{V_{si} \left( e \right)} + \frac{M_0 \lambda_i}{2 \sqrt{\kappa_{11}}}.
\]  

(27)

According to hypothesis 1:
\[ \|e\| \leq \frac{\sqrt{V_{i_1}(e)}}{K_{11}} \leq e \left( \frac{\kappa_1}{\tau_{i_1} e} \right) \left( \sqrt{V_{i_1}(e(0))} - \frac{M_0 \hat{\lambda}_i K_{12} e}{K_{11} K_{13}} \right) + \frac{M_0 \hat{\lambda}_i K_{12} e}{K_{11} K_{13}}. \]  

(28)

When \( e \to 0 \), the right term of inequality (28) converges to zero on the interval \([t_1, t_2]\), so \( M_{0\hat{\lambda}_i} \{\sqrt{V_{i_1}(e)}\}/K_{11} \) is bounded. Therefore, \( (dV_{i_1}(e)/dt) \leq 0 \), the nonlinear extended state observer can well track the state variables of the two-dimensional pneumatic servo system, and can be asymptotically stable at \( t \to \infty \).

4. Design of dynamic surface controller

**Control objectives:** For the pneumatic bellows system (18), the traditional backstepping is combined with the dynamic surface. In the first two steps, a first-order low-pass filter is introduced to obtain the differential signal of the virtual controller, the appropriate dynamic surface control law \( u \) is designed in the third step with the purpose of making the system output gradual track the desired trajectory and the output tracking error converges to a neighborhood near the origin. The close loop control block diagram of dynamic surface based on NESO is shown in Fig. 5.

![Dynamic surface control diagram](image)

**Fig. 5 Close loop control block diagram of dynamic surface based on NESO**

The dynamic surface error of system (18) is defined as:

\[
\begin{align*}
\eta_1 &= \hat{y}_1 - r \\
\eta_2 &= \hat{y}_2 - \hat{y}_d \\
\eta_3 &= \hat{y}_3 - \hat{y}_{2d}
\end{align*}
\]

(29)

where \( r \) is the expected trajectory of \( \hat{y}_i \), \( \hat{y}_{id} \ (i = 1, 2, 3) \) is the output of virtual control signal \( \alpha_i (i = 1, 2, 3) \) through first-order low-pass filter \( (1/(\tau_i s + 1)) (i = 1, 2, 3) \), with the filtering constant \( \tau_i > 0 \), and satisfies:

\[
\begin{align*}
\tau_i \hat{y}_{id} + \hat{y}_{id} &= \alpha_i \\
\hat{y}_{id}(0) &= \alpha_i(0)
\end{align*}
\]

(30)

Define filtering error:

\[
\begin{align*}
\delta_1 &= \hat{y}_{id} - \alpha_i \\
\delta_2 &= \hat{y}_{2d} - \alpha_2 \\
\delta_3 &= \hat{y}_{3d} - \alpha_3
\end{align*}
\]

(31)

then:

\[
\hat{y}_{id} = \frac{\alpha_i - \hat{y}_{id}}{\tau_i} = -\frac{\delta_i}{\tau_i},
\]

(32)

the derivative of filtering error is:

\[
\dot{\delta}_i = -\frac{\delta_i}{\tau_i} - \dot{\alpha}_i,
\]

(33)
\[
\begin{aligned}
\dot{\delta}_i + \frac{\delta_i}{r_i} &\leq J_i \left( \eta_i, \dot{\eta}_i, \delta_i, \dot{\delta}_i, y_{id}, \dot{y}_{id}, \ddot{y}_{id} \right), \\
\end{aligned}
\]  
(34)

where \( J_i \left( \eta_i, \dot{\eta}_i, \delta_i, \dot{\delta}_i, y_{id}, \dot{y}_{id}, \ddot{y}_{id} \right) \) is a non-negative continuous function.

From equations (33) and (34):

\[
\dot{\delta}_i \leq -\frac{\delta_i^2}{r_i} + \left| \delta_i \right| J_i \leq \frac{\delta_i^2}{r_i} + \delta_i^2 + \frac{J_i^2}{4},
\]  
(35)

select the appropriate virtual control signal:

\[
\begin{align*}
\alpha_1 &= -\gamma_1 \eta_i + \dot{\eta}_i, \\
\alpha_2 &= -\gamma_2 \eta_i + \dot{\eta}_d, \\
\alpha_3 &= -\gamma_3 \eta_i + \dot{\eta}_d,
\end{align*}
\]  
(36)

where \( \gamma_i \) is the design parameter and greater than zero.

Select appropriate actual control law to compensate the system disturbance:

\[
u = \frac{\alpha_3 - \tilde{\eta}_d}{b_0}.
\]  
(37)

5. Stability analysis

The main work of this paper can be summarized by the following theorem:

**Remark 1**: Consider a closed-loop system composed of pneumatic two-dimensional servo positioning system (18), NESO (19), virtual control signal \( \alpha_1, \alpha_2, \alpha_3 \), and control rate \( u \). On the premise of the initial conditions for bounded, for any given number of normal \( \omega \), there is a design parameters \( \gamma_i, \rho, \tau_i \), if \( V(0) \leq \omega \) is satisfied, the whole closed loop system is semi-global uniformly bounded stability. All state signals and tracking errors of the whole closed loop system can finally converge to a bounded closed set by selecting appropriate design parameters and arbitrary initial bounded closed set.

From equations (29) and (31), we can get:

\[
\begin{aligned}
\dot{\tilde{y}}_2 &= \eta_2 + \bar{y}_{2d} = \eta_2 + \delta_i + \alpha_1, \\
\dot{\tilde{y}}_3 &= \eta_3 + \bar{y}_{3d} = \eta_3 + \delta_i + \alpha_2.
\end{aligned}
\]  
(38)

(39)

The candidate functional form of Lyapunov is selected as follows:

\[
\begin{align*}
V_1 &= \frac{1}{2} \eta_i^2, \\
V_2 &= \frac{1}{2} \eta_i^2, \\
V_3 &= \frac{1}{2} \eta_i^2.
\end{align*}
\]  
(40)

Take the derivative of \( V_1, V_2, V_3 \) and substitute into equations (36) --(39) to get:
$$\begin{align*}
\dot{V}_t &= \eta_3 \left[ \hat{\delta}_3 - \beta_3 g(e) - \tau \right] \\
&= \eta_3 \left[ \hat{\delta}_3 - \beta_3 g(e) - \tau \right]
\dot{V}_s &= \eta_4 \left[ \hat{\delta}_4 - \beta_4 g(e) - \tau \right] \\
&= \eta_4 \left[ \hat{\delta}_4 - \beta_4 g(e) - \tau \right]
\dot{V}_i &= \eta_i \left[ \hat{\delta}_i - \beta_i g(e) - \tau \right] \\
&= \eta_i \left[ \hat{\delta}_i - \beta_i g(e) - \tau \right]
\end{align*}$$

(41)

Select the candidate functional form of Lyapunov again:

$$V = \sum_{i=1}^{n} V_i = \frac{1}{2} \sum_{i=1}^{n} \delta_i^2 = \frac{1}{2} \eta_1^2 + \frac{1}{2} \eta_2^2 + \frac{1}{2} \eta_3^2 + \frac{1}{2} \delta_4^2 + \frac{1}{2} \delta_5^2 + \frac{1}{2} \delta_i^2 .$$

(42)

According to Young’s inequality:

$$\begin{align*}
\eta_1 \eta_{i+1} &\leq \frac{1}{2} \eta_i^2 + \frac{1}{2} \eta_{i+1}^2 \\
\eta_3 \delta_i &\leq \frac{1}{4} \eta_i^2 + \frac{1}{4} \delta_i^2 \\
\eta_4 \beta_i g(e) &\leq \frac{1}{4} \eta_i^2 + \frac{1}{4} \beta_i^2 g^2(e)
\end{align*}$$

(43)

where, \( \rho_{i1}, \rho_{i2} \) is a positive design parameter.

Take the derivative of \( V \), and substitute (35), (41) and (43) into it to get:

$$\dot{V} \leq \sum_{i=1}^{n} \left[ -\gamma_i - \frac{1}{\rho_{i2}} \eta_i^2 + \left( 1 - \frac{1}{\tau_1} \right) \delta_i^2 - \frac{\rho_{i1}}{4} \beta_i^2 g^2(e) + \frac{J_i^2}{4} \right] + \sum_{i=1}^{n} \left[ \frac{1}{2} + \frac{1}{\rho_{i2}} \eta_i^2 + \frac{1}{2} \eta_{i+1}^2 + \frac{\rho_{i1}}{4} \delta_i^2 \right].$$

(44)

Simplify the above equation again to:

$$\dot{V} \leq -\mu V + l,$$

(45)

where

$$\mu = \min \left\{ \frac{\rho_{i1}}{4} \beta_i^2 g^2(e) + \frac{J_i^2}{4} \right\},$$

(46)

$$l = \sum_{i=1}^{n} \left[ -\frac{\rho_{i1}}{4} \beta_i^2 g^2(e) + \frac{J_i^2}{4} \right].$$

(47)

As can be seen from the above equation, \( l \) is negative definite. Only by selecting appropriate controller parameters, \( \mu > 0 \) can ensure that \( \dot{V} \leq 0 \), all signals of the whole closed-loop system are bounded.

6. Practical implementations

In this chapter, in order to illustrate the applicability of the developed control algorithm, the pneumatic
two-dimensional ultra-precision positioning system is applied as an experimental platform, which consists of bellows, air floating guide, grating ruler, Mercury3000 grating ruler encoder, M10 encoder, grating ruler reading head, STM32 microcontroller, ITV2050-212BL electrical pressure proportional valve, AW20-01 pressure regulating valve filter, and small air source (as shown in Fig. 6).

Fig. 6 Physical connection diagram

6.1 Initial load characteristic experiment

The grating ruler is fixed on the air floating guide while the reading head is fixed on the objective table and the direction is opposite to the grating ruler. When position of the controlled object is moved, the reading head transmits the displacement signal detected by the grating ruler to the encoder, above the encoder is a 15 pin high-density outside screw connection, the pin4&5 and pin9&10 have a A, B two phase digital pulse signals. Take the A-phase pulse as an example, for each rising edge, it represents the bellows moving the load by 1 um. As the controller of the whole system, the SMC converts the digital pulse signal formed after the processing of the encoder into voltage signal through the D/A module to control the opening area of the proportional valve, and then controls the displacement of bellows. Experimental data between bellows displacement and proportional valve input voltage are shown in Table 1.

| Voltage(v) | Displacement(um) | Voltage(v) | Displacement(um) |
|-----------|-----------------|-----------|-----------------|
| 0.000     | 0000            | 0.541     | 0948            |
| 0.034     | 0028            | 0.605     | 1075            |
| 0.066     | 0094            | 0.668     | 1160            |
| 0.092     | 0156            | 0.721     | 1251            |
| 0.140     | 0221            | 0.787     | 1372            |
| 0.206     | 0330            | 0.839     | 1467            |
| 0.252     | 0418            | 0.893     | 1554            |
| 0.314     | 0502            | 0.950     | 1610            |
| 0.367     | 0603            | 1.012     | 1712            |
| 0.429     | 0725            | 1.064     | 1806            |
| 0.483     | 0837            | 1.120     | 1921            |

6.2 Location tracking experiment

The whole pneumatic bellows positioning platform is driven by small air supply and electric pressure proportional valve regulating the pressure of intake chamber in bellows ensure that the bellows actuator can reciprocate. With Y axis as the research object, the mathematical model parameters of the system are shown in Table 2.
Table 2 Model parameters

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| C (kg/s)   | 380   | X (mm)     | 1 x 10^{-3} |
| k (N/m)    | 3.2 x 10^5 | R          | 287.1 |
| A (m^2)    | 2.24 x 10^{-2} | T        | 280 |
| V (m^3)    | 7.63 x 10^{-3} | P (Mpa)  | 0.4 |
| Qm (kg/V·h)| 0.2 ± 0.1 | m (kg) | 100 |

The gain of the observer is greatly affected by the bandwidth \( \omega \), select the gain of observer reasonably and ensure \( \left\{ \frac{dV_{ei}(e)}{dt} \right\} \leq 0 \). The selection of control signal feedback gain \( \gamma_i (i = 1, 2, 3) \) satisfies \( \gamma_3 > \gamma_2 > \gamma_1 > 0 \), and \( \mu > 0 \), \( l \) is bounded in equation (44), specific control algorithm parameters are shown in Table 3.

Table 3 Parameters value of control algorithm

| Parameters | Value |
|------------|-------|
| \( \alpha \) | 100 |
| \( \beta_1 \) | 3 x 10^2 |
| \( \beta_2 \) | 6 x 10^4 |
| \( \beta_3 \) | 4 x 10^5 |
| \( \beta_4 \) | 1 x 10^6 |
| \( \gamma_1 \) | 31 |
| \( \gamma_2 \) | 82 |
| \( \gamma_3 \) | 97 |

To test the effect of DSC controller, the PID controller is compared with it when a reference rectangular pulse signal, trigonometric signal which is selected as \( f(t) = 0.2(\sin 2\pi t + \cos (\pi t/2)) \). The sampling time interval is set to 0.001 seconds with the sampling frequency of 10000. The location tracking effect and error are shown in Fig. 7(a) and Fig. 7(b). From the results, it is apparent that DSC controller has less overshoot, faster response speed and the static error is reduced effectively.

Fig. 7 Tracking renderings of two reference signals

For further comparison, the amplitude of the trig signal varies from 0.2 to 0.05 in Fig. 8. The PID controller appears obvious phase lag and more overshoot while DSC controller still can achieve the tracking task.

Fig. 8 Tracking effect after changing amplitude
Apply step disturbance signal with amplitude of 0.1 to bellows system when \( t = 4.5 \) seconds. Afterwards, observe response variation of two controller. From Fig. 9, the DSC controller has slight fluctuation, shorter adjusting time and the transient performance is still well. From what has been discussed above, DSC controller has stronger abilities to against external disturbance and uncertainties.

![Fig. 9 Tracking effect after adding disturbance](image)

Compared with ESO to verify the observation effect of NESO designed, continue to track the rectangular pulse model. The feedback gain of ESO is selected as \( \beta_1 = 1.8 \times 10^2 \), \( \beta_2 = 2.16 \times 10^4 \), \( \beta_3 = 8.64 \times 10^5 \), \( \beta_4 = 1.296 \times 10^7 \).

![Fig. 10 Tracking effect of observer](image)

In Fig. 10, it can be seen that NESO successfully estimate the displacement signal of the system with better filtering performance and does not fluctuate with the increase of the order of observation state.

In order to further verify the practicability of DSC controller, high-precision half-arc curve is tracked. The curve equation is:

\[
\begin{align*}
\mathbf{\hat{x}}(t) &= \begin{pmatrix} \sin(\pi t / 4) \\ \cos(\pi t / 2) \end{pmatrix} \\
\mathbf{\hat{y}}(t) &= \begin{pmatrix} 10^{-7} \times \sin(\pi t / 4) \\ 10^{-7} \times \cos(\pi t / 2) \end{pmatrix}.
\end{align*}
\]

(48)

The sampling time interval is set to 0.001 seconds with the sampling frequency of 200000, the trace effect is shown in Fig. 11(a) and \( X' Y' \) axis error curve in Fig. 11(b). The output signal of the system can accurately track the reference signal, with less fluctuation and faster response. The difference between the two degrees of freedom is due to the difference in load quality and reference signal frequency.
7. Conclusions

In this brief, we have designed pneumatic two-dimensional platform with a novel bellows-driven and proposed the DSC method based on NESO. Through a series of tracking and positioning experiments, it is shown that the DSC method designed in this paper has faster response, stronger robustness and well tracking performance. NESO can well estimate the system state, and will not fluctuate with the increase of order, thus realizing the requirement of ultra-precision of large stroke with nanometer motion stroke and micron error. In recent years, many scholars have designed three-degree-of-freedom micro-nano positioning platform, which is driven by a stepping motor (Tian et al., 2016) or piezoelectric ceramics (Wang et al., 2016) to achieve ultra-precise positioning effect. Therefore, bellows, as a new type of pneumatic actuator, should have more practical value in the positioning platform of three degrees of freedom.

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