From the stress response function (back) to the sandpile ‘dip’

A.P.F. Atman♠, P. Brucet♣, J. Geng♠, G. Reydellet♠, P. Claudin♠, R.P. Behringer♣ and E. Clément♠.

♠ Laboratoire de Physique et Mécanique des Milieux Hétérogènes, ESPCI, 10 rue Vauquelin, 75231 Paris Cedex 05, France.
♣ Department of Physics & Center for Nonlinear and Complex Systems, Duke University, Durham NC, 27708-0305, USA.

March 23, 2022

Abstract. We relate the pressure ‘dip’ observed at the bottom of a sandpile prepared by successive avalanches to the stress profile obtained on sheared granular layers in response to a localized vertical overload. We show that, within a simple anisotropic elastic analysis, the skewness and the tilt of the response profile caused by shearing provide a qualitative agreement with the sandpile dip effect. We conclude that the texture anisotropy produced by the avalanches is in essence similar to that induced by a simple shearing – albeit tilted by the angle of repose of the pile. This work also shows that this response function technique could be very well adapted to probe the texture of static granular packing.

PACS. 45.70.-n Granular systems – 45.70.Cc Static sandpiles – 46.25.-y Static elasticity

The stress distribution below a pile of sand has been one of the problematic issues of the statics of granular materials in physics over the last few years [1]. In fact, experiments have shown that, when a granular pile is prepared from a point source, the bottom pressure profile has a clear local minimum – a ‘dip’ – below the apex [2;3;4]. The existence of this pressure dip has been strongly debated, and it is now well established that the presence or absence of this dip is closely related to the preparation history of the pile. This was demonstrated by Vanel et al. [4]. Using the same sand and experimental apparatus, these authors could generate the stress dip using a localized deposition technique or cause the dip to disappear by constructing a pile in successive horizontal layers. Similar conclusions were reached for two dimensional heaps with photo-elastic grains [5], and in numerical simulations [6;7;8].

This interesting effect has inspired the development of new models to describe how forces are transmitted in dense granular materials. Among them are those proposed by Bouchaud et al., initially developed in the context of the sand pile dip [9;10;11;12], and further extended to other geometries like that of the silo [12;13]. This approach is also intended to describe a collection of systems including dense colloids, granular matter or foams [14]. At the macroscopic level, these features are modelled by hyperbolic, partial differential equations (PDE) for the stress tensor. Although no explicit link was established, the characteristics of these hyperbolic equations were intuitively thought to be related to the mesoscopic ‘force chain’ network whose structure and orientation were shaped by the previous history of the granular assembly – see also [14;15].

Concerning force chains. Plasticity theories for granular deformations are also of hyperbolic type, although conceptually different than the previous cited models. From the classical, soil mechanics point of view, below the plastic threshold, granular material is thought to behave as an effective elastic material with PDE’s that belong to the elliptic class [17]. Finally, sound wave propagation techniques and numerical simulations of confined granular assemblies indicate that that asessment of effective elastic constitutive relations is still an open and difficult issue [18].

In order to distinguish between the very different mathematics of hyperbolic and elliptic PDEs a stress response experiment was proposed [19;19]. For instance, the pressure profile measured at the bottom of an isotropic elastic horizontal layer in response to a localized vertical overload at its top surface should be a single broad peak, while hyperbolic models would yield in 2D two thin peaks or a ring in 3D. Several experiments [20;21;22;23;24;25;26] and simulations [27;28;29;30;31;32;33;34;35;36] have recently addressed the issue of the stress response in a granular layer. Collectively, they have demonstrated two key points:

(i) the shape of the pressure response profiles is generally not in agreement with the predictions of the hyperbolic models. For the generic case of a disordered packing of frictional grains, the measured profiles show an elliptic-like behavior with one single peak broadening in proportion to the thickness/depth of the layer – except in [20].

For well ordered packing, the measured response functions may produce rays diverging from the source, but this type of response is also compatible with anisotropic elasticity [30;37;38].
Fig. 1. Left: Sketch of the 2D shear box. The system is strained up to an angle $\phi$. Middle: Visualization of the force chains after shearing. The analysis of their orientations shows that a direction at $\tau = 45^\circ$ is preferred [25]. Right: Average response to a vertical overload after shearing with a strain angle $\phi = 5^\circ$. The stress maximum is oriented at $\beta = 22^\circ$ to the vertical.

(ii) the geometry of the response function is in fact simpler than that of the pile, and offers rich possibilities. Response function measurements directly detect any symmetry breaking due to texture originating from either a specific preparation or from any other external action on the pile. The layer can be loose, dense, compacted, sheared, avalanched, sedimented, ordered, vibrated, and so on. Nevertheless, there are still a number of unresolved questions. For instance in the limiting case of isostatic pilings, numerical simulations [39,40] and theoretical arguments [21,22,23,24,25] indicate that a hyperbolic equations should describe the stress propagation, although this is not in agreement with the work of Roux [35].

Experiments have shown that the response to a point force is very sensitive to the preparation of the system [24]. For instance, for a 2D system which has been subjected to strain in a 2D shear box, the response function is skewed in the direction of the shear [25] showing a strain-induced anisotropy. Here, we extend this result to 3D granular assemblies and propose a relation to the pressure ‘dip’ observed at the bottom of a sand pile when prepared by successive avalanches. Interestingly, anisotropy induced by preparation was suggested by Savage [16] to explain this phenomenon. Note that the occurrence of a stress solution with a dip can also be produced in a model pile composed of an elastic core and plastic wings [17].

The paper is organized as follows. We first describe experimental results obtained for the response of a sheared granular layer in 2D and in 3D (section 1) and for a layer prepared by successive avalanches. We then present a theoretical anisotropic elastic analysis of these experimental data in 2D and a numerical analysis in 3D (section 2). Thereafter, we numerically solve the case of a conical heap (section 3). We close with conclusions and suggested perspectives.

1 Experimental response function on sheared granular layers

The response function experiments of interest here, were carried out in shear cell geometries in two and three dimensions. In both cases, the cell consisted of vertical boundaries that could be tilted quasi-statically by an angle $\phi$ with respect to the vertical axis. This process deformed the samples from an original rectangular geometry to that of a parallelogram.

In 2D, the spacing between the horizontal boundaries was maintained strictly constant, so that the sample volume also remained constant. The particles were pentagons made of a photo-elastic material, which allows a direct measurement of the local force response and of the force chains. More details on these experiments can be found in Geng et al. [25]. As we can see from figure 1 the salient features of these series of experiments are (i) the force chain network is oriented at $45^\circ$ from the horizontal axis, which is the principal compressive direction and (ii) the response to a vertical force is tilted with an angle $\beta$ with respect of the vertical direction, which illustrates perfectly the symmetry breaking due to shearing. In a previous contribution we argue that this $\tau = 45^\circ$ angle is simply related to the principal axes of compression and dilation (respectively directions 1 and 2 on figure 1).

In 3D, we carry out two experiments to extend the study of the influence of shear on the texture of a granular assembly. First, we built an apparatus similar to the shear cell already used in 2D. Second, we deposited the material in a horizontal layer by the superposition of successive avalanches.

The shear cell – The shear cell is sketched in the left part of figure 2. In order to probe the mechanical properties and the symmetries associated with the induced structure, we mounted a capacitive stress probe at the bottom of the cell (labeled 4 in figure 2); the applied force on the top (labeled 1) is moved horizontally. Specifically, the force response measurements use a low frequency modu-
lation of the localized stress imposed at the top of the sample, and lock-in detection of the vertical pressure response on the bottom, as described by Reydellet et al. [21]. The lock-in detection provided a large signal/noise ratio, which allowed us to apply tiny forces, of the same order as the weight of few grains. This prevented any significant deformation of the packing which would have modified the fragile structure of contacts. A horizontal layer of grains (Fontainebleau sand, slightly polydisperse round grains, \( d \sim 300\mu m \) size) was initially prepared by pouring the material into shear box of horizontal size \( l = 25cm \). The lateral width of the box was \( w = 20cm \). The total height \( h \) of sand ranged between 4 and 15cm.

The pouring procedure involved a sieve which was slowly raised in order to create a uniform rain of sand. Thereafter the sand layer was packed by pushing on the free-surface with a plate, and by simultaneously taping on the lateral edges of the box. Note that these conditions are similar to the so-called ‘dense preparation method’ proposed earlier by Serero et al. [22]. A vertical stress response was determined before and after shearing. During the shear deformation of the sample, a weight (~40kg) was imposed on the top surface and the lateral boundaries (labeled as 2 on the left of figure 2) were slowly tilted up to a final angle \( \phi \). The dense preparation as well as the large imposed load seemed necessary to avoid inhomogeneous deformations of the free-surface and help to hinder the formation of localized shear-bands. This result was confirmed by direct inspection of the displacement fields of the grains lying on the transparent side of the lateral boundaries. In order to monitor this displacement field, three vertical columns of colored grains were inserted next to the lateral boundaries prior to shearing (see right panel of figure 2). We note that the formation of a shear band was hindered only for limited values of aspect ratios (typically for ratios \( h/l \) from 0.3 to 1 used here) and for shear angles smaller than 5°. We also point out that due to the rather close-to-unit aspect ratio of the shear box, the response for large horizontal distances between the source and the stress probe was not measured so as to avoid the influence of lateral walls.

In figure 3, we show the experimental results for the vertical stress response \( \sigma_{zz} \) after shear for several shear angles \( \phi \) and layer depths \( h \). In this figure, the abscisa is the horizontal coordinate \( x \) normalized by the sand depth \( h \). We normalize the stresses \( \sigma_{zz}(x) \) by \( \sigma_{zz}(0) \), the value of the stress at \( x = 0 \) (i.e. immediately below the piston). This normalization is due to the finite lateral extension of the box since, contrary to previous situations [21,22], we cannot obtain a proper normalization of the stress by integration (i.e. assess the total force applied). For shear

![Fig. 2. Left: Sketch of the 3D shear box. 1 top lid, 2 tiltable lateral boundaries, 3 fix glass wall, 4 capacitive stress probe. Right: Visualization of the grain displacement due to shear. Coloured grains have been put along four vertical lines before shearing. These grains are located along inclined lines afterwards, so that the strain field of the layer looks reasonably homogeneous.](image)

![Fig. 3. Response profiles of a 3D sheared layer of Fontainebleau sand for various values of the layer thickness \( h \) and the strain angle \( \phi \). The maxima of the profiles correspond to a tilt angle \( \beta \) of \( 8° \pm 1° \). The collapse of the data is resonably fitted by the SEM (see section 2 for the definition of the SEM and the choice of the different parameters, e.g. Poisson ratios, ...) with \( t = 0.67 \) and \( u = 2.14 \) when \( \tau \) is set to 45°.](image)
angles $\phi$ varying from $2^\circ$ to $5^\circ$, the experiments clearly show, a skewing of the response function in association with a displacement of the maximum corresponding to an angle $\beta = 8^\circ \pm 1^\circ$ (deduced from the slope of the empirical relation between the height of sand and the shift). Thus, as in the 2D case, the response indicates that, due to shearing, the granular assembly is clearly anisotropic (at least, the vertical direction is no longer an axis of symmetry). It is worth mentioning that this tendency is robust, as it was observed for other types of grains (larger, slightly polydisperse, rough sand grains of diameter $\sim 1\text{mm}$, and smooth spherical beads of diameter $\sim 1.5\text{mm}$).

**The avalanche preparation** - Next we prepared a layer of sand structured by successive avalanches (see figure 4 for a sketch of the preparation procedure). The experimental results for two independent preparations are displayed in figure 5 and rescaled in a way similar to the case of the 3D shear box. Again, we observe a skewing of the response indicating an anisotropic texture induced by the avalanches. However, the tilt effect of the response is weaker, as it corresponds to an angle for the locus of the maxima $\beta = 3.5^\circ \pm 1^\circ$.

**2 Elastic calculations**

We next turn to modeling the experimental results using elasticity theory. We show here that a simple anisotropic description is sufficient to capture all the salient features of the data. Isotropic elasticity, in both 2D and 3D, has only two constitutive parameters for a given material, namely the Young modulus $E$ and the Poisson ratio $\nu$. When anisotropy is introduced, however, the number of independent parameters increases significantly. For example, five parameters are required for the case of simple 2D orthotropy that we use below. In 3D, this number is even higher. Fortunately, for the static equations, only combinations of these parameters enter (e.g. the parameters $r$ and $t$, below). Still, a meaningful fit of the response profiles to an anisotropic elastic model is a formidable task. Although such an approach is very instructive, it is not an essential point for the present purposes. We will present the results of this type of approach elsewhere.

**2.1 Elastic response in 2D**

Let us first focus on the simpler two dimensional case. What is indicated by the 2D shearing experiment described in the previous section, is that the sheared force chain network clearly has a preferred direction at an angle of $45^\circ$ with respect to the initial axis of the box. Contacts between the grains are gained or reinforced in this strong direction, whereas they are lost or weakened in the perpendicular direction. It is then expected that an effective elastic medium model for this situation would have a stiff direction (direction 1) which makes a fixed angle $\tau = 45^\circ$ relative to the vertical axis $z$ ($x$ denotes the horizontal coordinate) characterized by a Young modulus $E_1$. Perpendicular to direction 1 is the softer direction (direction 2) that is characterized by a Young modulus $E_2 < E_1$. This orthotropic elastic description is completed by three additional parameters: two Poisson coefficients $\nu_{12}$ and $\nu_{21}$ and the shear modulus $G$. Then, the strain-stress relation, expressed in these tilted axes, can be written in the following matrix form:

\[
\begin{pmatrix}
u_{11} \\ u_{22} \\ u_{12}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\
-\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G}
\end{pmatrix} \begin{pmatrix}
\sigma_{11} \\ \sigma_{22} \\ \sigma_{12}
\end{pmatrix},
\] (1)
Because the matrix involved in this relation must be symmetric, the Young’s moduli and Poisson ratios are not independent, but satisfy the relation $E_2/E_1 = \nu_{12}/\nu_{11}$. The elastic free energy is well defined if
\begin{align*}
E_1, E_2, G &> 0, \\
1 - \nu_{12}\nu_{21} &> 0.
\end{align*}

The isotropic case is recovered for $E_1 = E_2 = E$, $\nu_{12} = \nu_{21} = \nu$ and $G = \frac{E}{2(1+\nu)}$.

In reference \[22,48\], Otto et al. have analytically computed the stress tensor components for such an anisotropic elastic material in the case of a semi-infinite medium ($z > 0$) loaded with a unitary point force at the origin $x = z = 0$. They have shown that, for a given anisotropy angle $\tau$, the stress response profiles only depend on two quantities that involve a combination of the elastic coefficients:
\begin{align*}
t & = \frac{E_2}{E_1} = \frac{\nu_{21}}{\nu_{11}}, \\
r & = \frac{1}{2} \frac{E_2}{E_1} \left( \frac{1}{G} - \frac{\nu_{12}}{E_1} - \frac{\nu_{21}}{E_2} \right).
\end{align*}

It is notable that for other geometries, the stresses could depend on the $E$’s, $\nu$’s or $G$ independently of $t$ and $r$ due to the effect of different boundary conditions, for example at the bottom of a finite thickness slab. By contrast, the corresponding familiar isotropic solution is completely independent of $E$ and $\nu$, although some (weak) variations of the stress profiles with $\nu$ are found for an isotropic layer of finite thickness \[22,15\]. Depending on the values of $\tau$, and in particular on the sign of $r$, and of $(r^2 - t)$, different response shapes result, including single or double peaked responses, and symmetrical or skewed profiles (see numerous figures in \[22\]).

Although it is natural to fix the angle $\tau$ and to use the ratio $E_2/E_1$ as a control parameter for the amplitude of the anisotropy created by the shearing, it is rather difficult to have any kind of intuition about the $\nu$’s and the shear modulus $G$, i.e. on $r$. Figure 6 shows the locus of values of $r$ and $t$ which give a response profile for a semi-infinite medium with a deflection angle of $\beta = 22^\circ$, the experimental value. Interestingly, we see the tendency for a higher value of $r$ to significantly broaden the response function. This would correspond, for example, to small values of the shear modulus $G$. Note also, that the width at half-height of the response is a quantity that is easily accessible experimentally, and hence provides a good method to test this type of model.

### 2.2 Elastic response in 3D

It is not difficult to generalize this approach to 3D. However, in this case, two axes (directions 2 and 3) orthogonal to direction 1 must be specified. We keep the notation $x$ for the horizontal axis along which the shearing is applied, and the stress response measured. Direction 2 is in the vertical plane $(z, x)$, and direction 3 $(y)$ is perpendicular to these two directions.
only vary three parameters that we believe are crucial
namely, the stiffness of the direction of anisotropy $E_1$, the
shear modulus $G$ and $\tau$ the direction of the orthotropic
axis with respect to the vertical direction. More precisely,
we consider the same value of $G$ for all directions, i.e.
$G_1 = G_2 = G_3 = G$. The average Young’s modulus is
taken constant: $E = (E_1 + E_2 + E_3)/3 = 150$MPa. We
also take $E_2 = E_3$. Finally, the three Poisson coefficients
$\nu_{12}$, $\nu_{13}$ and $\nu_{23}$ are also kept constant and bear the val-
ues $\nu = 0.3$. We checked that the value of this parameter
is not very sensitive. The remaining $\nu_{ij}$ are such that
the strain-stress matrix is symmetrical (see above). In the
spirit of the 2D analytical results, we use two dimension-
less parameters to present our data, i.e. the stiffness ratio
t = $E_2/E_1$, and the shear ratio $u = E_2/G$. $u$ is analogous
to the parameter $r$ in 2D – see equation 4.

The shear cell – An essential result of the shear box
experiment was that the tilt angle of the stress response
profiles is around $8^\circ$ (figure 7). As suggested in the 2D
space, it is natural that the orthotropic direction is tilted
with an angle $\tau = 45^\circ$. In figure 8 we thus present the
curve $t(u)$ corresponding to a tilt angle $\beta = 8^\circ$ and for
$\tau = 45^\circ$ as computed by CASTEM using the SEM. We see
that similarly to the 2D situation, moving on this curve
from lower to higher values of $u$ significantly enlarges the
width of the response function (see inset of figure 8). This
is indeed an experimental observable that can be used to
discriminate in parameter space. In the framework of
the SEM, good agreement with the experimental data on
the shear box can be obtained for values $u_0 = 2.14$ and
t_0 = 0.67. The curve corresponding to ($u_0$, $t_0$) is presented
in figure 8.

The avalanche preparation – Now, we seek an inter-
pretation of the response function for the granular slab
prepared by successive avalanches. First we note that dur-
ing the avalanching process, the flowing layer experiences
a strong shear along the avalanche angle $\varphi$. An interesting
outcome of the shear box experiment is that for all
finite shear angle values $\phi$ that we tested, the shape of
the response function as well as tilt angle $\beta$ did not vary
significantly. We argue that each deposited layer retains
a memory of the shear due to the avalanching process,
which induces an anisotropic texture. In first approxima-
tion, we may assume that, the avalanche acts like a shear
box whose bottom is tilted at an angle $\varphi \sim 30^\circ$ with respect
to the horizontal direction. We propose to use the param-
eters ($u_0$, $t_0$) which compares reasonably to the shear box
data to interpretate the avalanche deposition experiment.
Of course now, we need to tilt the orthotropic angle $\tau$ from
45$^\circ$ to a value $\tau = 45^\circ - \varphi \sim 15^\circ$. In figure 8, we presen-
the results of CASTEM calculations on the SEM using these
parameters and indeed we find an angle $\beta = 3.0^\circ \pm 0.5^\circ$,
close to the experimental value $\beta = 3.5^\circ \pm 1^\circ$.

3 Back to the sandpile pressure profile

Several experimental determinations of the stress distribu-
tion below a sand pile prepared from a point source were
obtained under various experimental conditions. First, we
note that all stress data can be rescaled so that it is pos-
sible to compare the prediction of the elastic model with
experiments made with different pile sizes. For a conical
pile of height $h$, radius at the base $R = h/\tan \varphi$, and pile
slope angle $\varphi$, the relevant parameters are the rescaled
vertical stress $\sigma_{zz}/\rho gh$ and the rescaled horizontal posi-
tion $\xi = r/R$ ($\rho$ is the density of the granular packing and
g the gravity constant).

In figure 7, we show the experimental data obtained
by three groups, Smid and Novosad [2], Brockbank et al.
[3] and Vane et al. [4]. The underlying idea behind the
use of an anisotropic elasticity is to include the frozen
texture caused by shearing that ensues during avalanche
deposition. From the response function experiments un-
der shear (shear box and avalanche) displayed before,
it is natural to propose a modelling of the sand pile pre-
bred by successive avalanche using an orthotropic elastic
model like SEM. Of course, we see here the limitations of
such a modelling, since in the SEM version of the CASTEM
computation we only have three parameters to vary. Fur-
thermore, as discussed previously, in the case where the
sand pile repose angle is $\varphi = 30^\circ$, the orthotropic direc-
tion should be fixed at $\tau = 15^\circ$. We show in figure 8 the
results of a CASTEM computation for a conical sand pile
using the ‘optimal’ parameters ($u_0$, $t_0$) that yielded a rea-
sonable agreement to the shear box and the avalanche de-
position experiments. Note that we also checked that the
stress state is always below the Coulomb criterion (with
an internal friction angle of $30^\circ$), i.e. that failure never
The repose angles of the material used by these authors are $\phi$ for the prediction of the seming direction. This choice corresponds to the direction of the data of Šmíd and Novosad [2], the stars are from Brockbank Fig. 8. Normal stress below a conical pile. Filled symbols are simply taken at an angle and a shear modulus that can be varied independently of the features are the presence of a stiff axis in the orthotropic model used in this paper is relatively simple, since its main principal compression and is consistent with our recent experimental findings in 2D [24]. We also present new sets of experiments on a 3D sand packing in a shear box, where shear induced anisotropy is present. The second experiment is a sheared granular slab constructed by successive avalanches. The agreement with the orthotropic model is quantitatively correct, as the tilt of the response function can be reproduced by elastic modelling in both experiments using the same set of parameters.

In a second series of analysis, we use the best fit parameters obtained from comparison with response function experiments to see whether such a relatively simple modelling is likely to explain the dip below a sand pile, as suggested initially by Savage [10]. Indeed from a finite element calculation of a conical pile, we obtain a good qualitative agreement with the available experimental data but we fail to obtain really quantitative agreement, as the dip amplitude and its width seem both underestimated by a factor of almost 30% in the central part of the pile. It is not yet clear whether the elasticity model we use is oversimplified since it contains only two parameters which are sensitive to shear, or whether the preparation procedure under avalanches is not accounted for correctly in the context of the 3D sand-pile. We note here the interesting suggestion of Jenkins [50] that the upward moving ‘stoppage’ waves produced when the avalanche hits the ground could modify the main compression axis so that its direction could be further away from the vertical. This effect would indeed enlarge the size of the dip.

The tendency for deposition history or external action like shear or a biaxial compression to modify the constitutive structure of a material was noted in experiments by Oda et al. [51] and in numerical simulations by Radjai et al. [52] and is at present time still a very open and difficult question. This paper calls for more extensive systematic studies both experimentally and numerically (possibly analytically). The sand pile pressure dip, as interesting as it seems, appears a bit too complex to analyse for the moment. For a better understanding of this crucial issue, this work suggests a systematic use of response function techniques – stress responses as here, as well as displacement responses as in [53,54,55] – so as to extract several effective constitutive parameters of the material along a given stress-strain history. This is a promising systematic approach, and a possible alternative to sound propagation techniques, which can precisely identify internal structural changes due to external action a granular material [34]. This is the scope of ongoing projects in our laboratories.

We thank C. Goldenberg, I. Goldhirsch, J. Jenkins, J. Lanuza, S. Luding and J. Snoeijer for fruitful discussions. A.P.F. Atman’s present address is Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, C.P. 702, 30123-970, Belo Horizonte, MG, Brazil. P. Brunet’s present address is Royal Institute of Technology - Department of Mechanics. Teknikringen 8, 10044 Stockholm, Sweden. The LPMMH is UMR 7636 of the CNRS.
References

1. For a broad perspective on granular materials, see The focus issue on the physics of granular media of the Comptes-Rendus de l’Académie des Sciences, Physique 3, pp 129-245 (2002); the focus issue on granular materials of Chaos 9, pp 509-696 (1999); Physics of Dry Granular Media, H.J. Herrmann, J.-P. Hovi, and S. Luding editors, NATO ASI Series, Kluwer, (1997); Powders and Grains 97, R.P. Behringer and J.T. Jenkins editors, Balkema, (1997); H.M. Jaeger, S.R. Nagel, and R.P. Behringer, Rev. Mod. Phys. 68, 1259 (1996).

2. J. Šmíd, and J. Novosad, Proc. Powtech. Conference 1981, Ind. Chem. Eng. Symp. 63, D3V 1 (1981).

3. R. Brockbank , J.M. Huntley, and R.C. Ball, J. Phys. (France) II 7, 1521 (1997).

4. L. Vanel, D.W. Howell, D. Clark, R.P. Behringer and E. Clémont, Phys. Rev. E 60, R540 (1999).

5. J. Geng, E. Longhi, R.P. Behringer and D.W. Howell, Phys. Rev. E 64, 060301(R) (2001).

6. K. Lifman, D.Y.C. Chan and B.D. Hughes, Powder Technology 72, 255 (1992).

7. S. Luding, Phys. Rev. E 55, 4720 (1997).

8. H.-G. Matuttis, Granular Matter 1, 83 (1998).

9. J.-P. Bouchaud, M.E. Cates and P. Claudin, J. Phys. (France) I 5, 639 (1995).

10. J.P. Wittmer, P. Claudin, M.E. Cates and J.-P. Bouchaud, Nature, 382, 336 (1996).

11. J.P. Wittmer, M.E. Cates and P. Claudin, J. Phys. (France) I 7, 39 (1997).

12. P. Claudin, Ph.D. thesis La physique des tas de sable, Annales de Physique 24, n°2, 1 (1999).

13. L. Vanel, P. Claudin, J.-P. Bouchaud, M.E. Cates, E. Clément and J.P. Wittmer, Phys. Rev. Lett. 84, 1439 (2000).

14. J.-P. Bouchaud, P. Claudin, D. Levine and M. Otto, Eur. Phys. J. E 4, 451 (2001); J.E.S. Socolar, D.G. Schaeffer and P. Claudin, Eur. Phys. J. E 7, 353 (2002) + erratum EPJE 8, 453 (2002); Y. Roichman, D. Levine and I. Yavneh, Phys. Rev. E 70, 061301 (2004).

15. R. Blumenfeld, Phys. Rev. Lett. 93, 108301 (2004).

16. M.E. Cates, J.P. Wittmer, J.-P. Bouchaud and P. Claudin, Phys. Rev. Lett. 81, 1841 (1998); Phil. Trans. R. Soc. Lond. A 356, pp 2535-2560 (1998); Chaos 9, 511 (1999).

17. D.M. Wood, Soil Behaviour and Critical State Soil Mechanics, Cambridge University Press, Cambridge (1990).

18. H. Makse, N. Gland, D.L. Johnson and L.M. Schwartz, Phys. Rev. Lett. 83, 50705073 (1999).

19. P.-G. de Gennes, Physica A 261, 267 (1998); Rev. Mod. Phys. 71, S374 (1999).

20. M. da Silva and J. Rajchenbach, Nature 406, 708 (2000).

21. G. Reydellet and E. Clément, Phys. Rev. Lett. 86, 3308 (2001)

22. D. Serero, G. Reydellet, P. Claudin, E. Clément and D. Levine, Eur. Phys. J. E 6, 169 (2001).

23. J. Geng, D. Howell, E. Longhi, R.P. Behringer, G. Reydellet, L. Vanel, E. Clément and S. Luding, Phys. Rev. Lett. 87, 035506 (2001).

24. N.W. Mueggenburg, H.M. Jaeger and S.R. Nagel, Phys. Rev. E 66, 031304 (2002).

25. J. Geng, G. Reydellet, E. Clément and R.P. Behringer, Physica D 182, 274 (2003).

26. M.J. Spannuth, N.W. Mueggenburg, H.M. Jaeger and S.R. Nagel, cond-mat/0308580.

27. C. Eloy and E. Clément, J. Phys. I 7, 1541 (1997).

28. J.-J. Moreau, in the proceedings of the colloque ‘Physique et mécanique des matériaux granulaires’, Champs-sur-Marne (France), 199 (2000).

29. L. Breton, P. Claudin, E. Clément and J.-D. Zucker, Europhys. Lett. 60, 813 (2002).

30. C. Goldenberg and I. Goldhirsch, Phys. Rev. Lett. 89, 084302 (2002).

31. C. Goldenberg and I. Goldhirsch, Granular Matter 6, 87 (2001).

32. R. da Silveira, G. Vitalenc and C. Gay, cond-mat/0208214.

33. S. Ostojic and D. Panja, cond-mat/0403321, cond-mat/0409160.

34. A.P.F. Atman, P. Brunet, J. Geng, G. Reydellet, G. Combe, P. Claudin, R.P. Behringer and E. Clément, to appear in J. Phys. Cond. Mat. special issue on Granular Materials (M. Nicodemi Editor), cond-mat/0411734.

35. N. Gland, P. Wung and H.A. Makse, preprint (2004).

36. C. Goldenberg and I. Goldhirsch, to appear in Nature (2005).

37. C. Goldenberg and I. Goldhirsch, Eur. Phys. J. E 9, 245 (2002).

38. M. Otto, J.-P. Bouchaud, P. Claudin and J.E.S. Socolar, Phys. Rev. E 67, 031302 (2003).

39. D.A. Head, A.V. Tkachenko and T.A. Witten, Eur. Phys. J. E 6, 99 (2001); see also the comment of J.-N. Roux, Eur. Phys. J. E 7, 297 (2002).

40. A. Kasahara and H. Nakanishi, cond-mat/0405169.

41. S.F. Edwards and D.V. Grinev, Phys. Rev. Lett. 82, 5397 (1999).

42. A.V. Tkachenko and T.A. Witten, Phys. Rev. E 60, 687 (1999).

43. R.C. Ball and R. Blumenfeld, Phys. Rev. Lett. 88, 115505 (2002).

44. C.F. Moukarzel, J. Phys. Condens. Matter 14, 2379 (2002).

45. J.-N. Roux, Phys. Rev. E 61, 6802 (2000).

46. S.B. Savage, in ‘Physics of Dry Granular Media’, H.J. Herrmann, J.P. Hovi and S. Luding editors, NATO ASI series, Kluver Amsterdam, 25 (1998).

47. A.K. Didwania, F. Cantelaube and J.D. Goddard, Proc. R. Soc. Lond. A 456 2569 (2000).

48. J. Garnier, Tassement et contraintes. Influence de la rigidité de la fondation et de l’anisotropie du massif, PhD thesis, Université de Grenoble (1973).

49. with CAST3M, see http://www.castem.org:8001.

50. J. Jenkins, private comm.

51. M. Oda, S. Nemat-Nasser and J. Konishi, Oils and Fundations 25, 85 (1985).

52. F. Radjai, D. Wolf, M. Jean and J.-J. Moreau, Phys. Rev. Lett. 80, 61 (1998).

53. E. Kolb, J. Cvikliinski, J. Lanzuza, P. Claudin and E. Clément, Phys. Rev. E 69, 031306 (2004).

54. F. Leonforte, A. Tanguy, J.P. Wittmer and J.-J. Moreau, Phys. Rev. B 70, 014203 (2004).

55. C.F. Moukarzel, H. Pacheco-Martinez, J.C. Ruiz-Suarez and A.M. Vidales, Granular matter 6, 61 (2004).