The Influence of Subgrade Subsidence on Train Track Dynamic Interaction

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Abstract

This paper reports results from a numerical model to calculate subgrade settlement in railway tracks due to repeated dynamic loading. The trains are modelled as rigid body 2-axle carriages on two layers of springs. The track is modelled as a beam supported on three layers of springs. Permanent deformation of the subgrade layer is calculated on the basis of a power law assumption. Patterns of dynamic forces are calculated on a smooth track and on a track subject to an initial subgrade 'pothole'. The latter is shown to grow with repeated loading and to change the mean patterns of applied force.

Keywords: dynamic, interaction, settlement, roughness, pothole.

1 Introduction

This paper considers the dynamic interaction between trains and track and how it influences and is influenced by track subgrade settlement. The process is modelled using a train/track dynamic interaction model, coupled with a simple model for subgrade settlement. Dynamic interaction causes patterns of dynamic force on the track which, after many repetitions, result in subgrade settlement. The settlement changes the excitation displacements applied to the train which, in turn, changes the pattern of applied dynamic force.

A great deal of literature has been published on the dynamic analysis of structures subjected to moving loads. Some detailed models have been developed that consider the coupling of vehicles and railway track [1-4]. These studies have focused on the dynamic interaction of the track and vehicle using non-linear Hertzian spring elements. Some studies, including this one, assume that each wheel is always in contact with the track using constraints at each contact point between wheels and rail [5-8].
In this paper, the vehicle is modelled as a 10 degree-of-freedom, lumped mass multi-body system comprising the vehicle body mass and its moment of inertia, the two bogie masses and their moments of inertia, and four wheel unsprung masses. The bogie mass is linked with the wheel unsprung mass through the primary suspension springs and linked with the vehicle body mass through secondary suspension springs. The railway track is divided into Euler-Bernoulli beam elements of finite length resting on three layers of elastic foundation. The three layers of discrete springs and dampers represent the elasticity and damping effects of the rail pads, the ballast, and the subgrade, respectively.

1.1 Modelling settlement due to cyclic loading of soils

The first major study of the response of soils to cyclic loading was undertaken by Seed [9-12]. Widespread interest in the area has resulted in a significant research effort. In a popular group of models, e.g., [13-20], a power law is applied to link the number of load cycles ($N$) and the accumulated strain ($\varepsilon_p$). Li & Selig [18] present a model which includes factors such as soil stress state, number of repeated loads, soil type and soil physical state:

$$\varepsilon_p = a(\sigma_d / \sigma_s)^m N^b$$

where $a$ and $m$ are material parameters, $\sigma_d$ is the cyclic deviator stress and $\sigma_s$ is the static soil strength. The model is used by Li & Selig [18] to predict the accumulated settlement in a track subgrade, and the predictions are shown to provide good agreement with experimental measurements. This approach is used here to predict the permanent deformation in the subgrade.

The sub-grade model analysed in this paper is a 1 m deep layer of high plasticity clay, divided into four layers, each 0.25 m deep. The input parameters for the subgrade material are based on values published in [18] for clay with a natural moisture content of 28.4%. On this basis, $\sigma_s = 193$ kPa, and $a$, $m$ and $b$ are 1.2, 2.4 and 0.18 respectively.

2 Modelling of the coupled vehicle and track system

2.1 Dynamic equations of the track

The finite element (FE) method is used to develop the equations of the railway track. The track used in this paper consists of three layers representing rail, pads, sleepers, ballast and subgrade. The rail is considered as a finite length Bernoulli-Euler beam. The sleepers and ballast are simplified as point masses and only vertical dynamic responses are taken into account. The rail clips and pads are treated as a spring/damper system with stiffness and damping. The ballast is represented as a
series of independent spring/dampers with stiffness and damping. Similarly, for short term response to load, the subgrade is represented as a series of spring/damper systems.

2.2 Coupling of the vehicle and the track

The vehicle-track interaction, illustrated in Figure 1, is analysed using an incremental procedure in the time domain. It is assumed that all displacements are known at time $t$, and that $\Delta t$ is a small time increment. The objective is to solve the equations at time $t + \Delta t$. The vehicle equations were written with partitions in the matrices for the contact and non-contact parts.

![Figure 1: Coupling of vehicle and track structure (after [21]).](image)

A rail element that is directly under a wheel load is considered to be an interaction element. Thus, at time $t + \Delta t$, only the 4 elements of the track that are directly acted upon by the wheel, are taken into account. The remaining parts of the track that are not in direct contact with the loads, are modelled with the original track element. To apply the Condensation Equation for the vehicle element, the vehicle degrees of freedom are eliminated.

3 Rail contact forces

Vehicle properties for the Chinese high speed train (CHR3) are used in these simulations [21,22]. To decrease the boundary effects, a total length of 68.4 m is chosen for the computation with 150 generalized beam elements, each 0.57m long to
match the sleeper spacing. The rail is taken to be 60 kg/m and continuously welded. It is first assumed to be perfectly smooth.

Figure 2 shows the deflection at one instant in time for the first 50 m of track, when the vehicle is travelling at a speed of 120 km/h. At each time, four points on the track, corresponding to the wheel positions, show maximum deflections, as can be seen in the figure. Figure 3 shows the variation in contact force with wheel location on the track. As expected, contact force peaks shortly after the wheel passes the relatively stiff supports provided by the sleepers. A second lesser peak occurs between sleepers due to an interference effect. The dynamic increment is modest relative to the static weight of 68.64 kN.

![Figure 2: Deflection of smooth rail at one instant in time.](image)

4 Effect of settlement

Contact force development is implemented using a feedback loop in the subgrade model, i.e., the pattern of forces cause permanent deformation in the subgrade layer, which leads to changes in the pattern of applied forces. Over many cycles of load, the pattern of forces changes and the deformations increase. Deformations are small so the model is only updated after every 1000 carriages.
A ‘pothole’ in the subgrade, i.e., local subsidence, is illustrated in Figure 4. The equation for the Gaussian (Normal) statistical distribution is used to generate a bell shaped depression. The maximum depth is 5 mm and the width (standard deviation) is 250 mm. This is simulated in the model using a distribution of initial displacements at the ballast layer that vary according to the Gaussian function. The resulting pattern of displacement for the other layers is also illustrated in the figure. Using initial displacements to generate the subgrade subsidence in this way causes considerably less pronounced settlement profiles in the other layers, including the rail (Figure 4). This is significant as rail settlement influences the pattern of applied forces.

Figure 5 illustrates the influence of the Gaussian distributed subgrade subsidence at $x = 61.8$ m on the contact forces. The velocity is 120 km/h and, apart from the subsidence, the rail is assumed to be perfectly smooth. A perfectly smooth track subgrade will settle due to repeated load. In this case, the pattern of forces is calculated from $x = 0$. Permanent deformation (settlement) is only calculated in the range from $x = 51$ m to $x = 75$ m. There is a clear boundary effect near these points as the carriage experiences the effect of a sudden change in subgrade elevation (after a number of passes, the carriages will experience a fall at $x = 51$ and a rise again at $x = 75$).
Figure 4: Displacement in each layer due to subgrade subsidence.

Figure 5: Contact forces due to 5 mm initial subgrade subsidence.
It can be seen that the forces near the pothole increase but not by much. The pothole grows with increasing numbers of carriages but not significantly more than adjacent points. The local variations in force due to the sleepers at 0.57 m intervals are more significant than the increase in forces adjacent to the pothole due to 30 000 carriages. The effect appears to be local – there are no significant increases in force over time at points remote from the initial pothole. Longer runs show that the rate of growth in the pothole tends to decrease over time.

4 Conclusions

This paper presents a numerical model for vehicle-track dynamic interaction using the dynamic condensation technique. The dynamic interaction model is coupled with a model of subgrade permanent deformation under repeated loading. The combined model provides unique insights into the nature of subgrade settlement in response to repeated rail dynamic loading. It is shown that subgrade settlement tends to grow but locally and modestly and at a decreasing rate over time.

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