Abstract

We use a simple model to study the long time fluctuations induced by random pinning on the motion of driven non-interacting vortices. We find that vortex motion seen from the co-moving frame is diffusive and anisotropic, with velocity dependent diffusion constants. Longitudinal and transverse diffusion constants cross at a characteristic velocity where diffusion is isotropic. The diffusion front is elongated in the direction of the drive at low velocities and elongated in the transverse direction at large velocities. We find that the mobility in the driven direction is always larger than the transverse mobility, and becomes isotropic only in the large velocity limit.

Key words: Random Pinning, Diffusion, Mobility
PACS: 74.25.Qt

1. INTRODUCTION

Understanding the non-equilibrium statistical properties of driven particles in disordered media is a challenging question relevant to many experimental situations. A prominent example are the moving phases of driven vortex lattices in superconductors [1]. A key feature of these systems is that the disorder induces anisotropic response and fluctuations which are strongly controlled by the velocity [1,2,3].

In spite of its relevance to understand situations of incoherent or plastic vortex flow the simple case of an isolated vortex driven in a d-dimensional random potential with $d > 1$ has been tackled analytically only by perturbation theory [2], valid at high velocities, or by mean field theory [4], valid for $d \gg 1$. In this paper we propose a simple model whose long time behaviour can be computed analytically at $d = 2$ for any finite velocity.

2. MODEL

Let us consider the equation of motion, in two dimensions, of a driven isolated vortex at zero temperature,

$$\eta \mathbf{v}(t) = \mathbf{F} + \sum_i f_p(\mathbf{r} - \mathbf{r}_i)$$ (1)

where $\mathbf{v} = d\mathbf{r}/dt$ is the instantaneous velocity of the vortex located at $\mathbf{r}(t)$, $\mathbf{f}$ is the driving force and $\eta$ the friction coefficient. We model the disorder as a random arrangement of hard disks with center $\mathbf{r}_i$ and radius $\xi$. Outside the disks the vortex has a free motion and inside it feels a pinning force,

$$f_p(\mathbf{r}) = -\frac{A_p}{\xi^2} \mathbf{r} \Theta(1 - r^2/\xi^2)$$ (2)

where $A_p$ is the amplitude, and $\Theta$ the step function. This disorder models a diluted distribution of pinning centers separated at a distance $d > 2\xi$. In
the following we use adimensionalized variables: ξ is the length unit, $A_p$, the energy unit, and $A_p/\xi^2\eta$ the time unity.

Above the depinning transition the motion of the vortex consists in straight segments of free motion interrupted by the collisions with the different pinning centers. At each collision the vortex is delayed and deflected with respect to the free motion. The equation describing the motion of the vortex inside the trap centered at $r_i = 0$ is,

$$\frac{dr}{dt} = -r + F$$  \hspace{1cm} (3)

The collision starts with the vortex at some initial position $r(0) = r_0$ on the border of the trap, $r_0^2 = 1$. The solution of Eq.(3) is

$$r(t) = (r_0 - F) e^{-t} + F$$  \hspace{1cm} (4)

After a time interval $\delta t$ the vortex will exit from the trap, therefore $r^2(\delta t) = 1$, $\delta t > 0$. Using this condition in equation 4 we obtain the following expression for $\delta t$,

$$e^{-\delta t} = \frac{f_0^2F - \sqrt{[f_0^2F]^2 - (F^2 - 1)[f_0^2]^2}}{(f_0^2)^2}$$  \hspace{1cm} (5)

where $f_0^2 = f_0(r_0)$. The displacement $\delta r$ induced by the collision is then given by,

$$\delta r \equiv r(\delta t) - r_0 = (r_0 - F)(e^{-\delta t} - 1)$$  \hspace{1cm} (6)

Due to the random distribution of pinning centers, the motion can be considered as a random walk in the long time limit. The fluctuations of $r(t)$ are thus induced by the uncorrelated sequence of collisions. Assuming identical pinning centers, the randomness comes exclusively from the random initial conditions of each collision, described by Eq.(4).

In a long time interval $t$ the vortex collides with a large number $N_c(t)$ of pinning centers. Since the core covers an area $\sim Vt$, where $V$ is the mean velocity, we get,

$$N_c(t) \approx n_p \xi Vt.$$  \hspace{1cm} (7)

By symmetry, the long time displacement is along the direction of $F$, $\Delta r(t) = \Delta r_\parallel(t)\hat{F}$, and

$$\Delta r_\parallel(t) \equiv Vt \approx N_c(t)\langle\delta r_\parallel \rangle + F(t - N_c(t)\langle\delta t \rangle)$$ \hspace{1cm} (8)

where $\langle\ldots\rangle$ denotes an average over a random distribution of initial conditions $r_0$ in equations 5 and 6. From Eq. (8) we get the mean velocity,

$$V = \frac{F}{1 - n_p \xi [\langle\delta r_\parallel \rangle - F\langle\delta t \rangle]}$$  \hspace{1cm} (9)

We can now define longitudinal $D_\parallel$ and transverse $D_\perp$ diffusion constants,

$$D_\parallel \equiv \langle[\Delta r_\parallel(t) - V\delta t]^2/t \rangle$$  \hspace{1cm} (10)

$$D_\perp \equiv \langle[\Delta r_\perp(t)]/t \rangle = \frac{N_c(t)}{t} \langle\delta r_\perp^2 \rangle$$ \hspace{1cm} (11)

To calculate the longitudinal diffusion constant in terms of the single collision displacement we use that $d \equiv V(F\delta t + d - \delta r_\parallel)/F$ is the random longitudinal displacement with respect to the average longitudinal motion in a single collision, with $d = n_p^{-1/2}$ the longitudinal mean distance between pinning centers of two given consecutive collisions. We thus get,

$$D_\parallel = \frac{N_c(t)}{t} \left[ d - \frac{V}{F}(F\delta t + d - \delta r_\parallel) \right]^2$$ \hspace{1cm} (12)

We also define the longitudinal mobility as $\mu_\parallel \equiv [dV/df_\parallel]$. In terms of single collision quantities we get,

$$\mu_\parallel = \frac{V}{F} \left[ 1 - n_p \xi \left( \langle\delta t \rangle + F \frac{d\langle\delta t \rangle}{df_\parallel} - \frac{d\langle\delta r_\parallel \rangle}{df_\parallel} \right) \right]$$ \hspace{1cm} (13)

To define the transverse mobility we need to introduce a small perturbative force $f_\perp$. The velocity induced by this force is,

$$v_\perp = Vn_p \xi [\langle\delta r_\perp \rangle - f_\perp \langle\delta t \rangle] + f_\perp$$ \hspace{1cm} (14)

and thus we can define the transverse mobility $\mu_\perp \equiv [df_\perp/df_\parallel]_{f_\parallel \rightarrow 0}$. In terms of single collision quantities we get,

$$\mu_\perp = 1 + Vn_p \xi \left( \frac{d\langle\delta r_\perp \rangle}{df_\perp} - \langle\delta t \rangle \right)_{f_\parallel \rightarrow 0}$$ \hspace{1cm} (15)

Finally we can define effective temperatures using generalized Einstein relations,

$$T_\perp \equiv \frac{1}{2 \mu_\perp}$$  \hspace{1cm} (16)

$$T_\parallel \equiv \frac{1}{2 \mu_\parallel}$$ \hspace{1cm} (17)

In order to calculate the transport properties defined above ($V$, $D_\perp$, $D_\parallel$, $\mu_\perp$, $\mu_\parallel$, $T_\perp$, and $T_\parallel$)
we need to calculate the first moments of the distribution of $\delta t$, $d(\delta t)/dF$, $\delta r$, $\delta r_\|\delta t$, $d(\delta x_\perp)/df_\perp$, $d(\delta x_\parallel)/dF$ by performing simple integrals.

3. RESULTS

In Fig. 1(a) we show the Velocity - Force characteristics (VF) of our model, calculated from Eq.(9). The critical depinning force is $F_c = 1$. At zero temperature, for $F < F_c$, the particle is trapped after a transient, and thus $V = 0$. At low velocities, $F \rightarrow F_c^+$, the VF curve is strongly nonlinear with $V \sim \left[n_p \log((F - F_c)/2F_c)\right]^{-1}$ [6] and at large velocities free flux flow is approached with corrections that scale as $F - V \sim V^{-1}$, as shown in Fig.1(b). In Fig. 2(a) we show the longitudinal and transverse diffusion constants. $D_\parallel$ and $D_\perp$ are both non-monotonous functions of the velocity $V$. At small velocity diffusion constants grow linearly with $V$ while at large velocity $D_\parallel \sim V^{-3}$ and $D_\perp \sim V^{-1}$. Let us note that $D_\parallel$ and $D_\perp$ cross at a characteristic velocity $V_o$. This crossing means that the long time diffusion front changes aspect ratio at $V_o$. For $V < V_o$ the diffusion front is elongated in the driven direction, while for $V < V_o$ is elongated in the transverse direction. At $V_o$ diffusion is isotropic. Interestingly, the same behavior is observed in numerical simulations of interacting vortices in two dimensions [3].

In Fig. 2(b) we show the longitudinal and transverse mobilities. We observe that both are velocity dependent and approach the free flux response, $\mu \sim 1$, at large velocities. Since the $\mu_\parallel$ is the differential resistance, $dV/dF$, the divergence observed at small velocity is a signature of the depinning transition. $\mu_\parallel$ near depinning is high since any small force can reduce strongly the waiting time $\delta t \sim 1/V$ inside the trap. On the contrary, $\mu_\perp \sim V$ at small velocity. A small transverse force has a small effect in the trap time $\delta t$ compared with the linear $V$ dependence (Eq.(7)) of the number of collisions per unit time.

In Fig. 3 we show the effective temperatures in the longitudinal and transverse directions. Since at large velocities the mobilities saturate, the velocity dependence of the effective temperatures are domin-
inated by the diffusion constants, and thus \( T_{\perp}^\text{eff} \sim 1/V \) and \( T_{\parallel}^\text{eff} \sim 1/V^3 \). At low velocities the effective temperatures are instead strongly determined by the velocity dependence of the mobilities. \( T_{\parallel}^\text{eff} \) reaches a maximum at an intermediate velocity and decreases quickly to zero with decreasing \( V \). On the other hand, \( T_{\perp}^\text{eff} \) saturates at a finite value as the velocity vanishes, due to the linear dependence with velocity of both the diffusion constant and the mobility.

It is worth noting that only at large velocities and in the transverse direction, the effective temperature is found to be identical, except from numerical factors, to the shaking temperature \[2\]. This confirms that the shaking temperature is equivalent to an effective temperature defined from a generalized fluctuation relation only in the limit of large velocity of non–interacting vortices \[3\]. It is important to point out here that the thermodynamic nature of the effective temperatures of this model is still unclear, since the system is non–interacting and strongly driven \[5\].

4. CONCLUSIONS

We have studied the pinning–induced anisotropic diffusion of driven non–interacting vortices. We find that the diffusion front is elongated in the direction of the driving force at low velocities and in the transverse direction at large velocities. This implies the existence of isotropic diffusion at a characteristic velocity \( V_0 \). The analysis of the anisotropic low frequency voltage noise in superconductors could be a possible experimental probe of this result, since diffusion constants and velocity fluctuations are related by generalized Green-Kubo relations \[3\].

Even if the depinning transition does depend on the peculiarities of our model, we find that the main features of the long time fluctuations we obtain at intermediate and large velocities are in agreement with perturbation theory predictions \[2\] and with numerical simulations of non interacting vortices for the non–simplified model \[6\]. Furthermore, it is easy to show that our model can be solved at any dimension \( d > 1 \) and generalized to more complicated short-range random potentials \[6\].

We acknowledge discussions with A. Rosso, A. Iucci, T. Giamarchi, and D. Domínguez. This work was supported in part by the Swiss National Fund under Division II.

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