Radiative $\pi^\pm\gamma$ transitions of excited light-quark mesons in the covariant oscillator quark model

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The COMPASS collaboration, as a part of their hadron spectroscopy program, is going to measure the radiative decay widths of light-quark mesons via the Primakoff reactions. In this letter we study the photon couplings of light-quark $q\bar{q}$ states in the covariant oscillator quark model and evaluate the transition rates for \{\rho(770), b_1(1235), a_1(1260), a_2(1320), \pi_2(1670), \rho_3(1690), \rho(1700)\}$^\pm \rightarrow \pi^\pm\gamma$, which are expected to be measured by the COMPASS. Such photon couplings could be useful not only for understanding the internal structures of observed light-quark mesons and their quark-model classification but also for the ongoing experimental studies by COMPASS.

Introduction. The COMPASS is a fixed-target experiment at the CERN SPS for investigating the structure and spectrum of hadrons. Concerning their hadron spectroscopy program, particular attention is paid to light-quark meson systems and one of them, the Primakoff production measurements, from which the radiative widths of light-quark mesons could be obtained, are going on [1–3].

Direct observation of the radiative decays, such as $X^\pm \rightarrow \pi^\pm\gamma$, is often difficult to carry out because of their very small rates, while their inverse reactions, called Primakoff reactions [4, 5], which are described as the scattering of a pion in the Coulomb field of atomic nuclei ($A, Z$)

$$\pi^\pm + (A, Z) \rightarrow \pi^\pm\gamma^* + (A, Z) \rightarrow X^\pm + (A, Z),$$

are relatively easy of access. Since the cross section for reaction (1) at very low-$q^2$ regions ($q$ being the four momentum transfer) is proportional to the radiative decay width $\Gamma(X^\pm \rightarrow \pi^\pm + \gamma)$, it is possible to determine them by measuring the Coulomb contribution of the absolute cross section [6]. While a number of new experimental measurements were performed in recent years [7], it should be noted that some earlier experiments, such as Ref. [8, 9], have been cited as the latest data. Thus there is no doubt that the high statistics data from COMPASS will play an important role in the progress of light meson spectroscopy.

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Theoretically it has been well known that radiative transitions probe the internal structures of hadrons and hence it offers a useful tool to investigate their nature. In the transitions of excited states of light-quark mesons, although the final mesons have large kinetic energy at the rest frame of initial ones, such an effect is neglected in the conventional treatment of naive nonrelativistic quark models (NRQM). In addition, respecting the actual situation that all physical observations are made through not quarks but hadrons, the relativistic treatment for the center-of-mass (CM) motion of hadrons is absolutely necessary.

The covariant oscillator quark model (COQM) is one of the possible covariant extension of NRQM, retaining the various success principally restricted to the static problem. The remarkable features of the COQM is that hadrons are treated in a manifestly covariant way and the conserved effective electromagnetic currents of hadrons are explicitly given in terms of hadron variables themselves.

In this work we shall apply the COQM to analyze the one photon couplings in the transitions \{\rho(770)(^3S_1), b_1(1235)(^1P_1), a_1(1260)(^3P_1), a_2(1320)(^3P_2), \pi_2(1670)(^1D_2), \rho_3(1690)(^3D_3), \rho(1700)(^3D_1)\} \pm \rightarrow \pi(1S_0)\pm \gamma$. The obtained radiative decay widths are compared with experiment and also other quark model predictions. The rates for excited $D$-wave states are newly predicted and these results could provide a useful clue to observe the radiative decays of excited light-quark mesons at COMPASS.

Basic framework of the COQM. Let us briefly summarize the framework of the COQM relevant to the present application. In the COQM the wave function (WF) of $u\bar{d}$ mesons is given by the bilocal bispinor field $\Psi(x_\mu, x_\mu)_{\alpha \beta} = \Psi(X_\mu, x_\mu)_{\alpha \beta}$, where $\alpha, \beta$ denote the Dirac spinor indices of respective constituents and $x_1, x_2$ represent their space-time coordinates, which are related with the CM and relative coordinates given by $X_\mu = (x_1\mu + x_2\mu)/2$ and $x_\mu = x_1\mu - x_2\mu$, respectively. The WF is assumed to satisfy the following basic equation [12]

$$\left(\sum_{i=1}^{2} \frac{-1}{2m} \frac{\partial^2}{\partial x_{i\mu}^2} + \frac{K}{2}(x_{1\mu} - x_{2\mu})^2\right)\Psi(x_1, x_2)_{\alpha \beta} = 0,$$

which is equivalently rewritten as

$$\left( - \frac{\partial^2}{\partial X_\mu^2} + M^2(x, \frac{\partial}{\partial x})\right)\Psi(X, x)_{\alpha \beta} = 0, \quad M^2 = d \left( - \frac{1}{2\mu} \frac{\partial^2}{\partial x_\mu^2} + \frac{K}{2} x_\mu^2\right),$$

where $M^2$ represents the spin-independent squared-mass operator in the pure confining force limit, $d = 4m$, $\mu = m/2$ ($m$ being the effective quark mass) and $K$ is the spring constant. In order to freeze the redundant freedom of relative time, we adopt the definite-metric type of subsidiary condition for the four dimensional harmonic oscillator (HO), leading to the

\footnote{The COQM has a long history of development. It had been applied to investigate the radiative decays of light-quark meson systems [10] and heavy quarkonium systems [11] with considerable success.}

\footnote{In the following we restrict ourselves to the case of $u\bar{d}$ mesons.}

\footnote{The WF satisfying this condition is normalizable and gives the desirable asymptotic behavior of electromagnetic form factors of hadrons [13].}
eigenvalue solutions of the squared-mass operator as
\[ M_N^2 = M_0^2 + N\Omega, \quad N = 2N_r + L, \] (4)
where \( N_r \) and \( L \) are the radial and orbital quantum numbers respectively and \( \Omega \) is given by
\[ \Omega = d\sqrt{K/\mu} = \sqrt{32mK}. \] The relation (4) is in accord with the well-known linear rising Regge trajectory concerning the squared mass spectra, which is particularly evident in light-quark hadron sectors.

The WF describing mesons with the CM four momentum \( P_\mu \) can be written as
\[ \Psi(x_1, x_2)^{(\pm)\beta}_\alpha = \frac{1}{\sqrt{2P_0}} e^{\pm P_\mu X_\mu} \Phi(v, x)_\alpha^{\beta(\pm)}, \] (5)
where \( v_\mu = P_\mu/M \) is the four velocity (\( M \) being the meson masses). The internal WF \( \Phi(v, x) \) is taken as a form of the “LS-coupling” product in an analogous fashion to NRQM,
\[ \Phi(v, x)^{(\pm)\beta}_\alpha = f(v, x)^{(nL)}_{\mu\nu\ldots} \otimes \left( W^{(\pm)\beta}_\alpha(v) \right)_{\mu\nu\ldots}, \] (6)
where \( f(v, x)^{(nL)}_{\mu\nu\ldots} \) with \( n = N + 1 \) are the definite-metric-type wave functions of the four dimensional HO for the space-time part and \( W^{(\pm)\beta}_\alpha(v) \) are the Bargmann-Wigner spinor functions\(^4\) for the spin part. Decomposing the \( W^{(\pm)}_\alpha(v) \) into irreducible components with the definite \( JPC \), the detailed expressions for meson states relevant to the present study are obtained as follows:
\[ \Phi(v, x)^{(\pm)} = f^{(1S)}(v, x) W^{(\pm)}(v) = f_0(v, x) \left( \frac{1 + iv_\rho \gamma_\rho}{2\sqrt{2}} (-\gamma_5 + i\gamma_\mu\epsilon_\mu(P)) \right) \] (7a)
for the \( S \)-wave states,
\[ \Phi(v, x)^{(\pm)} = f^{(1P)}(v, x)_\nu W^{(\pm)}(v)_\nu = \sqrt{2\beta^2} x_\mu f_0(v, x) \left( \frac{1 + iv_\rho \gamma_\rho}{2\sqrt{2}} (-\gamma_5 \epsilon_\nu(P) + i\gamma_\mu\epsilon_{\mu\nu}(P)) \right) \] (7b)
for the \( P \)-wave states and
\[ \Phi(v, x)^{(\pm)} = f^{(1D)}(v, x)_{\nu\lambda} W^{(\pm)}(v)_{\nu\lambda} = 2\beta^2 x_\mu x_\lambda f_0(v, x) \left( \frac{1 + iv_\rho \gamma_\rho}{2\sqrt{2}} (-\gamma_5 \epsilon_{\nu\lambda}(P) + i\gamma_\mu\epsilon_{\mu\nu\lambda}(P)) \right) \] (7c)
for the \( D \)-wave states, where \( f_0(v, x) \) is the ground-state WF of the four dimensional HO given by
\[ f_0(v, x) = \left( \frac{\beta^2}{\pi} \right) \exp \left( -\frac{\beta^2}{2} \left( x_\sigma^2 + 2(v_\sigma x_\sigma)^2 \right) \right) \] (8)
with the parameter \( \beta^2 = \sqrt{\mu K} \) and \( \epsilon_{\mu\ldots} \) are the polarization tensors for respective meson states.

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\(^4\) They are defined by the direct tensor-product of respective constituent Dirac spinors with the four velocity of mesons as \( W^{(\pm)\beta}_\alpha(v) \sim u_\alpha(v)\bar{\psi}_\beta(v) \), \( W^{(-)\beta}_\alpha(v) \sim v_\alpha(v)\bar{\psi}\beta(v) \),
Electromagnetic Meson Current in the COQM. Next, we introduce the single photon couplings of \( q \bar{q} \) meson systems. The relevant decay amplitude is described by

\[
\int d^4x \langle f | \mathcal{H}_{\text{int}} | i \rangle = - \int d^4x \langle f | J_\mu (x) A_\mu (X) | i \rangle = \sqrt{\frac{1}{8 P_{\gamma \rho} P_{\rho 0} q_0}} \delta^4 (P_{\gamma} - P_{\rho} - q_\mu) M_{fi},
\]

where \( P_{\gamma}, P_{\rho} \) and \( q_\mu = (P_{\gamma} - P_{\rho})_\mu \) are the four momenta of initial and final mesons and emitted photon, respectively. In the COQM, as is the case in NRQM, we consider that the one-photon emission proceeds through a single quark transition in which the respective constituents couple with a photon. In order to obtain the electromagnetic current of mesons, we perform the minimal substitutions \( [14, 15] \) \( \partial / \partial x_\mu \rightarrow \partial / \partial x_\mu - i e Q_\mu A_\mu (x_i) \) (\( i = 1, 2 \)) in Eq.(2) and then obtain

\[
j_\mu (x_1, x_2) = j_\mu^{(\text{convec})} (x_1, x_2) + j_\mu^{(\text{spin})} (x_1, x_2)
\]

with

\[
\begin{align*}
\tilde{j}_\mu^{(\text{convec})} (x_1, x_2) &= - \frac{i e Q_\mu}{2 m} \langle \Psi (x_1, x_2) \left( \frac{\partial}{\partial x_\mu} \right) \Psi (x_1, x_2) \rangle, \\
\tilde{j}_\mu^{(\text{spin})} (x_1, x_2) &= - \frac{i e Q_\mu}{2 m} \langle \Psi (x_1, x_2) \left( i g_M (\sigma_{\mu \nu} \frac{\partial}{\partial x_\nu} + \frac{\partial}{\partial x_\mu}) \right) \Psi (x_1, x_2) \rangle,
\end{align*}
\]

where \( \langle \cdots \rangle \) means taking trace concerning the Dirac indices, \( \tilde{\Psi} \equiv - \gamma_4 \Psi^\dagger \gamma_4 \), \( Q_\mu \) represent quark charges in units of \( e \) \( (Q_1 = Q_u = 2/3 \) and \( Q_2 = Q_d = -1/3 \) for the present application) and \( g_M^{(1)} = g_M^{(2)} \equiv g_M \) are the parameters concerning the anomalous magnetic moment of quarks. It is worth mentioning that there exist two kinds of current, \( j_\mu^{(\text{convec})} \) and \( j_\mu^{(\text{spin})} \) (denoting the convection and spin currents respectively), which are conserved independently.

Substituting Eq.(5) and \( A_\mu (x_i) = (1/\sqrt{2 q_0}) e^* (q) e^{-i q_\mu x_i} \) into

\[
- \int d^4x_1 \int d^4x_2 \langle f | j_\mu (x_1, x_2) A_\mu (x_1) | i \rangle + (1 \leftrightarrow 2)
\]

and equating it with the matrix element Eq.(9), we obtain a formula to calculate the decay amplitudes as

\[
M_{fi} = -e Q_1 \int d^4x \langle \tilde{\Phi}_F (-) (v_F, x) \rangle (P_{\gamma} + P_{\rho})
\]

\[
- \frac{d}{2m} i \frac{\partial}{\partial x_\mu} + \frac{d}{2m} g_M \sigma_{\mu \nu} i q_\nu \Phi_\mu (v_F, x) ) e^{-i q_\mu x_i} e^* (q) + (1 \leftrightarrow 2),
\]

where \( e_\mu (q) \) is the polarization vector of the photon. By using this formula, we can derive the covariant expressions of invariant amplitudes for the respective radiative transitions summarized in Table 1.

Numerical Predictions. Taking the following values of parameters,

- \( M_0 = 0.75 \) GeV and \( \Omega = 1.11 \) GeV\(^2\) [16], which give \( M_1 = 1.29 \) GeV and \( M_2 = 1.67 \) GeV
- \( g_M = 0.82 \), determined from the experimental width of \( \rho^+ \rightarrow \pi^+ \gamma \)

we calculate numerical values of the radiative decay widths. The results are shown in Table 2 in comparison with experiment and other quark-model predictions. From this table we can see that our results are in fairly agreement with experimental values.
Table 1  Invariant amplitudes and formulas of decay width in the relevant transitions. Here $\epsilon_{\mu}$- and $|q_\gamma|$ denote the polarization tensors for initial mesons and physically emitted photon momentum at the rest frame of initial mesons, respectively, $\omega = -v_{1\mu}v_{F\mu} = (M_1^2 + M_F^2)/(2M_1 M_F)$, and $F$ is defined in the text.

| $j^{(\text{spin})}$ Process | $M_{fi}$ | $\Gamma$ |
|-----------------------------|---------|---------|
| $3S_1 \rightarrow 1S_0 \gamma$ | $-eg_{\rho}\epsilon_{\mu\rho\sigma\lambda}e^\mu_\sigma(q)\epsilon_\rho(P_1)P_{1\alpha}$ | $\frac{4\alpha}{3}|q_\gamma|^2 g_\rho^2$ |
| $3P_{1,2} \rightarrow 1S_0 \gamma$ | $ieg_{\rho}\epsilon_{\mu\rho\sigma\lambda}e^\mu_\sigma(q)\epsilon_\rho(P_1)q_{\alpha}$ | $\frac{2(7\pi+1)}{27}\pi |q_\gamma|^2 g_\rho^2$ |
| $3D_j \rightarrow 1S_0 \gamma$ | $eg_{\rho\sigma}\epsilon_{\mu\rho\sigma\lambda}(P_1)q_{\sigma}q_{\lambda}P_{1\alpha}e^\mu_\sigma(q)$ | $C_{J\alpha}|q_\gamma|^2 g_\rho^2$ |

Coupling parameters:

$g_\rho = g_M(Q_u + Q_d)\left(\frac{1}{2M_1} + \frac{1}{2M_F}\right)F, \quad g_\sigma = \frac{1}{\sqrt{2\pi}}\frac{Q_u + Q_d}{M_1^2 - M_1^2} \frac{1}{2\pi}F, \quad g_{\rho\sigma} = \frac{1}{\sqrt{2\pi}}\frac{Q_u - Q_d}{M_1^2 - M_1^2} \frac{1}{2\pi}M_1 F, \quad g_{\rho\sigma} = \frac{1}{\sqrt{2\pi}}\frac{Q_u - Q_d}{M_1^2 - M_1^2} \frac{1}{2\pi}M_1 F$.

Table 2  Calculated widths in comparison with experiment and other models.

| Process | $|q_\gamma|/\text{GeV}$ | This work | Experiment [7] | Ref. [17] | Ref. [18] |
|---------|----------------|----------|----------------|---------|---------|
| $\rho^\pm \rightarrow \pi^\pm \gamma$ | 0.375 | 68 (input) | 68 ± 7 | 68 (input) | - |
| $a_1(1260)^\pm \rightarrow \pi^\pm \gamma$ | 0.608 | 278 | 640 ± 246 [8] | 314 | (1.0-1.6)·10^3 |
| $a_2(1320)^\pm \rightarrow \pi^\pm \gamma$ | 0.652 | 237 | 287 ± 30 | 302 (input) | 375 ± 50 |
| $\rho_3(1690)^\pm \rightarrow \pi^\pm \gamma$ | 0.839 | 21 | - | - | - |
| $\rho(1700)^\pm \rightarrow \pi^\pm \gamma$ | 0.854 | 14 | - | - | - |
| $b_1(1235)^\pm \rightarrow \pi^\pm \gamma$ | 0.607 | 57.8 (Model A) | 230 ± 60 [9] | 397 | 184 ± 30 |
| $\pi_2(1670)^\pm \rightarrow \pi^\pm \gamma$ | 0.829 | 335 (Model A) | 116 (Model B) | - | - |
| $\pi_2(1670)^\pm \rightarrow \pi^\pm \gamma$ | 521 (Model B) | - | - | - | - |

Discussion.  Let us discuss in detail about obtained results. At first we would like to remark that a relativistic effect of the transition form factor $F$ commonly contained in all decay amplitudes, plays an important role throughout this work. It is given by the overlapping integral of 4-dimensional HO functions

$$F = \int d^4 x f_0(v_F, x)f_0(v_t, x)e^{-i\frac{2v_{1\mu}v_{F\mu}}{\omega}} = \frac{1}{\omega} \exp \left( -\frac{1}{16\beta^2} \frac{2v_{1\mu}q_{\mu}v_{F\mu}q_{\nu}}{\omega} \right) P_{\mu = 0} \frac{M_F}{(P_F)^0} \exp \left( \frac{-|q|^2}{16\beta^2} \frac{2(1 + |P_F|)}{(P_F)^0} \right).$$

(15)

Resulting ratio is $\Gamma(a_2(1320)^\pm \rightarrow \pi^\pm \gamma)/\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma) = 3.49$, which is independent from choice of the parameter $g_M$, in satisfactory agreement with data: 4.22 ± 0.876. Thus it turns out that our form factor yields a desirable damping effect.
On the other hand, concerning the \( a_1(1260) \) \( (b_1(1235)) \rightarrow \pi\gamma \) process, it seems that our results are poorly-fitted to the experiments [8, 9], being both only measurements quoted in Ref. [7]. However, it was pointed out that in Ref. [19], no evidence of the \( a_1 \) was found in the another charge-exchange photo-production experiment [20], while a clear \( a_2(1320) \) signal was observed. This results suggest that total width of \( a_1(1260) \) meson is extremely large or radiative \( \pi\gamma \) width is rather small. ⁵ In any case, a high-statistics confirmation of these process by the COMPASS would be desirable.

Concerning the \( \gamma^{(\text{convec.})} \) process, there are two possible ways for evaluating numerically the factor \( M_I^2 - M_F^2 \) in the couplings \( g_b \) and \( g_{\pi\gamma} \): In Table 2, we apply \( M_I^2 - M_F^2 = N\Omega \) in the Model A by using Eq.(4), while \( M_I^2 - M_F^2 = 2M_I|q_\gamma| \) in the Model B. In both cases, it is shown that the radiative decay widths of \( \pi_2(1670) \) into \( \pi\gamma \) have large fraction, predicted⁶ as \( \text{Br}(\pi_2(1670)^\pm \rightarrow \pi^\pm \gamma) = \Gamma(\pi_2(1670)^\pm \rightarrow \pi^\pm \gamma)/\Gamma_{\text{tot}}(\pi_2(1670)) = (2.0 \pm 0.069) \times 10^{-3} \) for the Model A, while \((1.3 \pm 0.045) \times 10^{-3} \) for the Model B, respectively. Thus it must have been detected at the COMPASS Primakoff measurements with enough statistics.

In summary we have investigated the radiative \( \pi\gamma \) decays of the excited light-quark mesons in the COQM. Radiative decay widths of \( D \)-wave excited mesons are predicted. We expect that forthcoming experiments at COMPASS will make these predictions to verify.

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⁵ Related works treating these mesons from another viewpoint have been reported [21–24].

⁶ Here we use \( \Gamma_{\text{tot}}(\pi_2(1670)) = 260 \pm 9 \) MeV taken from Ref. [7].