Huygens’ synchronization experiment revisited: luck or skill?

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Abstract

In a letter to the Royal Society of London in February of 1665, Christiaan Huygens described ‘an odd kind of sympathy’ between two pendulums mounted side by side on a wooden beam, which inspired the modern studies of synchronization of coupled nonlinear oscillators. Despite the growth of synchronization studies in a variety of disciplines, the original phenomenon described by Huygens remains a puzzle to researchers. Here, by placing two mechanical metronomes on top of a freely moving plastic board, we revisit the synchronization experiment conducted by Huygens. Experimental results show that by introducing a small mismatch to the natural frequencies of the metronomes, the probability of generating the anti-phase synchronization (APS) state, i.e. the ‘odd sympathy’ described by Huygens, can clearly be increased. By numerical simulations of the system dynamics, we conduct a detailed analysis of the influence of frequency mismatch on APS. It is found that as the frequency mismatch increases from zero, the attracting basin of the APS state, i.e. the ‘odd sympathy’ described by Huygens, can clearly be increased. However, as the frequency mismatch exceeds some critical value, both the basins of APS and IPS suddenly disappear, resulting in the desynchronization states. The impacts of the friction coefficient and synchronization precision on APS are also studied, and it is found that with the increase of the friction coefficient and the precision requirement of APS, the critical frequency mismatch for desynchronization will decrease. Our study indicates that, instead of luck, Huygens might have
introduced, deliberately and elaborately, a small frequency mismatch to the pendulums in his experiment for generating the odd sympathy.

Keywords: synchronization, coupled oscillators, classroom demonstration

(Some figures may appear in colour only in the online journal)

1. Introduction

As a universal concept in nonlinear science, synchronization has been extensively studied by researchers in different fields [1–3]. Roughly, synchronization refers to the coherent motion between coupled nonlinear oscillators, which occurs only when the coupling strength between the oscillators is larger than some critical value. Depending on the specific form of the coherence, synchronization can be classified into different types [2], including complete synchronization, phase synchronization, generalized synchronization, etc. In complete synchronization, the trajectories of the oscillators are converged into a single one in the phase space, and the states of the oscillators are identical at any instant of the system evolution. In phase synchronization, only the phases of the oscillators are entrained, while the amplitudes of the oscillators remain uncorrelated; and in generalized synchronization, the states of the oscillators are constrained by a function, although the function might not be explicitly given. Among the different types of synchronizations, phase synchronization is special in that it describes the coherent motion in parallel to the trajectories (which, in many realistic situations, is more interested than complete and generalized synchronization) and, more importantly, can be realized in a variety of systems, e.g. systems made up of non-identical or different types of oscillators [1–3]. For all types of synchronizations, the general finding in previous studies is that by increasing the mismatch of the oscillator parameters, the propensity for synchronization will deteriorate [4–7].

The study of synchronization can be traced back to the discovery of the Dutch scientist Christiaan Huygens in the 17th century. In his letter to the Royal Society of London in February of 1665 [8], Huygens described that two mechanical clocks hanging from a common beam always end up with ‘an odd kind of sympathy’ in which the clocks are swinging in exactly the same frequency but are 180° out of phase. Although Huygens’ experiment failed in solving the longitude problem for navigation, the intriguing phenomenon he discovered, now known as anti-phase synchronization (APS), has inspired the study of synchronization of coupled nonlinear oscillators in recent years [1–3]. Even now, the experiment designed by Huygens, or its variants, is still employed as an economic yet effective setup for exploring the collective dynamics of coupled oscillators, as well as for classroom demonstrations of synchronization [9–26]. Besides experiments, studies have been also carried out in recent years on the necessary conditions for generating APS in coupled pendulums [9–15, 21]. With high-precision modern equipment, Bennett et al re-examined Huygens’ experiment and found that, to generate APS, the frequencies of the pendulums should be very close, with a precision that could not be achieved in the 17th century [9]. It is thus suggested that Huygens’ observation of odd sympathy depended somewhat on luck. Kanunnikov and Lamper studied the impact of the nonlinear interaction between the beam and pendulums on APS, and found that the pendulums cannot be exactly out of phase by 180° [11]. A similar phenomenon has also been observed by Czolczynski et al in coupled pendulums of different masses [13]. A variant of Huygens’ experiment has been proposed by Pantaleone, in which two metronomes are placed on a freely moving base [10]. It was found that the metronomes are typically developed to in-
phase synchronization (IPS), with APS only observable under special conditions. By the theoretical model proposed by Pantaleone, Ulrichs et al numerically studied the synchronization of coupled metronomes and pointed out that APS is always unstable and exists only in the transient processes [12]. Wu et al studied the attracting basin of APS in the phase space and found that, by slightly adjusting the friction between the base and its support, the basin can be significantly enlarged, making APS observable for the general initial conditions [15, 21]. Despite the extensive studies on pendulum synchronization, the odd sympathy discovered by Huygens remains a puzzle to researchers: did Huygens generate APS by luck?

As one of the greatest scientists of the 17th century, Huygens is famous for his rigorous design of various instruments and devices. The designs were usually accompanied with hand-drawn sketches, which, in most cases, were elaborate and accurate. In describing the experiment that generates the odd sympathy, Huygens also drew a sketch in his laboratory notebook, as shown in figure 1(a) [8]. To provide more details about the experiment, Huygens separately drew another sketch for the pendulums (figure 1(b)). Apparently, the two pendulums are of different lengths. In particular, a weight is attached at the bottom of pendulum A (to stabilize the case), while this weight is absent for pendulum B. This ‘careless’ drawing is contrary to the way Huygens usually acts, say, for example, the sketch of the verge pendulum clock drawn in his book Horologium Oscillatorium (figure 1(c)). By the time Huygens did the synchronization experiment, he had already derived the formula for the
period of the physical pendulum, so it is hard to believe that the difference between the pendulums was made by Huygens casually. Intrigued by this serendipity, in the present work we revisit Huygens’ experiment by introducing a small mismatch to the natural frequencies of the pendulums, and study the impact of the frequency mismatch on APS. We are able to demonstrate experimentally and argue theoretically that below a critical frequency mismatch, the attracting basin of APS in the phase space is gradually enlarged as the frequency mismatch increases. Our study therefore suggests that the masses of the pendulums might be deliberately set differently by Huygens in order to generate APS.

2. Experimental study

Our experimental setup is presented in figure 2(a). Following the design of Pantaleone [10], we placed two mechanical metronomes on top of a freely moving plastic board. The plastic
board is supported by two empty soda cans. The metronomes (Cherub WSM-330) are of nearly identical parameters (parameter mismatch less than 1%), with the energy supplied by a hand-wound spring. The frequency of the metronome can be adjusted by changing the position of the mass on the pendulum bob, ranging from 40 ticks per minute (largo) to 208 ticks per minute (prestissimo) in a discrete fashion. Meanwhile, a paper clip is attached to each pendulum bob, which will be used to slightly adjust the metronome frequency. Two small paper markers are sticked on each metronome, with one sticked on the top of the pendulum bob, and the other one sticked at the pivot center. The whole setup is then placed on top of the desk which is horizontal to the ground. (As the plastic board is rigid and supported by identical cans, it is also horizontal to the ground.) This experimental setup of coupled metronomes is easy to assemble and can generate robust IPS and APS within a few minutes, making it an excellent experiment for classroom demonstration.

We wish to highlight the function of the paper clips in our experiment. The clips are of the same length (29 mm ± 1%) and weight (0.5 g ± 1%), and are placed just above the masses on the pendulum bobs. Compared to the masses (which are about 20 g), the clips are much lighter and, by moving them along the bob, provide a slight change to the natural frequency of the metronomes. Specifically, as the clip lifts up, the frequency of the metronome will gradually decrease, with a precision difficult to achieve by moving the mass. In our experiment, we fix the position of the clip on metronome 1, while lifting the clip on metronome 2 to enlarge the frequency mismatch between the metronomes. Throughout our experimental study, we fix the positions of the masses at presto for both metronomes (184 ticks per minute).

The motions of the pendulums are recorded with a smartphone camera (50 frames per second), and the video is analyzed by the MATLAB software. In analyzing the video, the markers are used as reference points for acquiring the instant phases, \( \phi_{1,2}(t) \), of the metronomes (see the appendix for a detailed description of the acquisition of instant phases). In doing the experiment, we shifted the bobs to random (different) initial phases and then released them from the static states. For each experiment, the video is started after a transient period of 60 s, which lasts for 180 s. If during the recorded period the phase difference between the metronomes, \( \Delta \phi = |\phi_1 - \phi_2| \), is always smaller to a predefined threshold \( \Delta \phi = 0.3 \text{ rad} \), the system is judged as having reached IPS; if \( |\Delta \phi - \pi| < \Delta \phi \), we regard the system as having reached APS; otherwise, the system is regarded as being in the non-synchronization state.

We start by placing the clips at the same position (just above the masses), and measure the natural frequencies, \( f_{1,2} \), of the isolated metronomes by putting them on the desk. The results are averaged over three realizations. Data analysis shows that \( f_1 \approx 1.493 \text{ Hz} \) and \( f_2 \approx 1.496 \text{ Hz} \). That is, the frequency mismatch between the metronomes is \( \Delta f \equiv f_2 - f_1 \approx 3.0 \times 10^{-3} \text{ Hz} \). We then put the metronomes on the plastic board, and repeat the same experiment for 100 realizations, with the initial phases of the metronomes being randomly chosen (within the range of −0.5 to 0.5) in each realization. The results are summarized in table 1, which shows that the system reaches IPS and APS with probabilities of 51% and 49%, respectively, and no non-synchronization state is observed. Figures 2(b) and (c) show, respectively, the typical IPS and APS states observed in the experiment.

We next lift up the clip on metronome 2 about 1 cm. This leads to the decreased natural frequency \( f_2 \approx 1.489 \text{ Hz} \), and the frequency mismatch is changed to \( \Delta f \approx -4.0 \times 10^{-3} \text{ Hz} \). Having determined \( f_2 \), we put both metronomes on the board and investigate again the probabilities of generating IPS and APS. As shown in table 1 (the second row), the
probabilities of generating IPS and APS are 26% and 74%, respectively. Lifting up the clip on metronome 2 about 2 cm, we have $f_2 \approx 1.4787 \text{ Hz}$ and $\Delta f \approx -14.3 \times 10^{-3} \text{ Hz}$. The probabilities of generating IPS and APS are 2% and 98%, respectively, as shown in table 1 (the third row). Like the case of $\Delta f \approx 3 \times 10^{-3} \text{ Hz}$, no non-synchronization state is observed for $\Delta f \approx -4 \times 10^{-3} \text{ Hz}$ and $\Delta f \approx -14.3 \times 10^{-3} \text{ Hz}$ either. Clearly, the probabilities for observing IPS and APS are dependent of the frequency mismatch. More specifically, with the increase of the frequency mismatch, $f_D$, the probability for generating APS is increased, while it is decreased for IPS.

3. Theoretical study

How could frequency mismatch facilitate APS while deteriorating IPS? To have a deeper understanding of the impact of frequency mismatch on synchronization, we proceed to investigate the dynamics of coupled metronomes by a theoretical model. Following [12, 15, 16, 21], we model the experimental setup shown in figure 2(a) by the sketch plotted in figure 3(a). In figure 3(a), the rigid beam and the point pendulums represent, respectively, the plastic board and metronomes in the experimental setup. The mass of the beam is denoted by $M$. $x$ denotes the displacement of the beam from its equilibrium point ($x = 0$), $k$, represents the stiffness coefficient of the spring (so as to keep the beam oscillating around the equilibrium point), and $c_x$ denotes the friction coefficient between the beam and the support (the empty cans). The metronomes are of the same mass, $m$, but with different lengths, $l_1 \neq l_2$. The instant phases of the pendulums are denoted by $\phi_{1,2}(t)$. To mimic the energy input of the mechanical metronome, we drive each pendulum by the torque $D_{1,2}$ when $|\phi_{1,2}|$ is smaller to a small angle $\gamma_N$. Specifically, $D_{1,2} = D$ when $0 < \phi_{1,2} < \gamma_N$ and $\dot{\phi}_{1,2} > 0$, $D_{1,2} = -D$ when $-\gamma_N < \phi_{1,2} < 0$ and $\dot{\phi}_{1,2} < 0$, otherwise $D_{1,2} = 0$.

The Lagrangian of the theoretical model reads

$$L = \frac{1}{2} [M \dot{x}^2 + m v_1^2 + m v_2^2] + mg (l_1 \cos \phi_1 + l_2 \cos \phi_2) - \frac{1}{2} k_x x^2,$$

with

$$v_{1,2}^2 = \left[ \frac{d}{dt} (x + l_{1,2} \sin \phi_{1,2}) \right]^2 + \left[ \frac{d}{dt} (l_{1,2} \sin \phi_{1,2}) \right]^2$$

and g the gravity constant. Taking into account the friction and driving torque, we can obtain from the Lagrangian the following equations of the system dynamics:

**Table 1.** By tuning the frequency mismatch, $\Delta f$ (Hz), between the metronomes, the probabilities of generating different states in 100 realizations.

| $\Delta f$ (Hz) | IPS   | APS   | Non-synchronization |
|-----------------|-------|-------|---------------------|
| $3.0 \times 10^{-3}$ | 51/100 | 49/100 | 0                   |
| $-4.0 \times 10^{-3}$ | 26/100 | 74/100 | 0                   |
| $-14.3 \times 10^{-3}$ | 2/100  | 98/100 | 0                   |
Here $c_\phi$ represents the damping coefficient of the pendulums. In our theoretical studies, the system parameters are chosen as: $M = 10.0 \text{ kg}$, $m = 1.0 \text{ kg}$, $c_\phi = 0.01 \text{ Ns} \text{ m}^{-1}$, $c_x = 5.0 \text{ Ns} \text{ m}^{-1}$, $g = 9.81 \text{ m s}^{-2}$, $k_x = 1.0 \text{ N m}^{-1}$, $D = 0.075 \text{ N m}$, and $\gamma_N = \pi \times 36^\circ$. It is noted that this set of parameters are just for the purpose of illustration, and are not derived from the experimental setup directly. As for the lengths of the pendulums, we fix $l_1 = 0.124 \text{ m}$ ($f_1 \approx 1.42 \text{ Hz}$), while changing $l_2$ to adjust the frequency mismatch. (The larger $l$, the smaller the natural frequency of the pendulum is.) The beam initially stays at the equilibrium point with 0 velocity, i.e. $x = 0 \text{ m}$ and $\dot{x} = 0 \text{ m s}^{-1}$. As in our experiments, the pendulums are released from the static states, and the initial phases of the pendulums are randomly chosen within the range of $-0.5$ to $0.5$. Equation (2) is solved numerically by the fourth-order Runge–Kutta method, with the time step $\delta t = 1 \times 10^{-3} \text{ s}$. After a transient period of $T_\text{tr} = 1 \times 10^3 \text{ s}$, we compare the phase difference, $\delta \phi = |\phi_1 - \phi_2|$, between the pendulums for a period of
The criteria for IPS and APS are the same as that of the experimental study: the system is regarded as reaching IPS if $\delta f < \Delta f$ during the period of $T$, reaching APS if $|\delta \phi - \pi| < \Delta \phi$, and reaching non-synchronization otherwise. Still, the detection threshold for synchronization is chosen as $\Delta f = 0.3$ rad.

We first study the synchronization behavior of two identical pendulums, i.e. setting $l_2 = l_1$ in equation (2). To search for all the possible states, we scan over the region $(-0.5$ to $0.5)$ in the 2D phase space $(\phi_1, \phi_2)$ by the resolution of $1 \times 10^{-2}$ rad, and in figure 4(a) plot the attracting basins of the different states. In figure 4(a), the black squares and red disks denote, respectively, the initial conditions that lead to IPS and APS. Statistically, the larger the basin of a specific state is, the higher the probability is for this state to be generated by a randomly chosen initial condition. Denote $p_{\text{ips}}, p_{\text{aps}},$ and $p_{\text{nsy}}$ as the fractions of the initial conditions inside the basins of the IPS, APS, and non-synchronization states, respectively, we have $p_{\text{ips,aps,nsy}} \approx \rho_{\text{ips,aps,nsy}}$, with $\rho_{\text{ips,aps,nsy}}$ the probability of generating each state. For the

\[ T = 5 \times 10^3 \text{s.} \]
numerical results shown in figure 4(a), we have $p_{ips} \approx 0.52$, $p_{aps} \approx 0.48$, and $p_{ays} \approx 0$. The typical IPS and APS states observed in the simulations are shown in figures 3(b) and (c), respectively.

Next, we next tune the natural frequency of the second pendulum by increasing $l_2$. Increasing $l_2$ to 0.13 m, we have $f_2 \approx 1.39$ Hz and $\Delta f = -3.0 \times 10^{-2}$ Hz. The attracting basins of the asymptotic states are plotted in figure 4(b). We have $p_{ips} \approx 0.41$, $p_{aps} \approx 0.59$ and $p_{ays} \approx 0$. Compared to the case of the identical pendulums (see figure 4(a)), it is seen that the basin of APS is clearly enlarged. Increasing $l_2$ further to 0.134 m, we have $f_2 \approx 1.36$ Hz and $\Delta f = -6.0 \times 10^{-2}$ Hz. The basins of the asymptotic states are plotted in figure 4(c). It can be seen that all initial conditions in the scanned region lead to APS, i.e. $p_{aps} \approx 1$. To systematically investigate the impact of frequency mismatch on the attracting basins, in figure 4(d) we plot the variations of $p_{ips}$ and $p_{aps}$ with respect to $|\Delta f|$. Figure 4(d) shows that as $|\Delta f|$ increases from 0, the basin of IPS (APS) gradually decreases (increases), and, at about $\Delta f_a = 3.7 \times 10^{-2}$ Hz, we have $p_{ips} \approx 0$ and $p_{aps} \approx 1$. The value of $p_{aps}$ remains at 1 in the range of $|\Delta f| \in [\Delta f_a, \Delta f_b]$, with $\Delta f_b \approx 0.58$ Hz. After that, $p_{aps}$ suddenly decreases from 1 to 0, and remains at 0 as $|\Delta f|$ increases further. As $p_{ips} = p_{aps} = 0$, the pendulums always reach the non-synchronization state for $|\Delta f| > \Delta f_b$. Basin analysis thus indicated that, as observed in the experiments, the probability of generating APS is indeed influenced by the frequency mismatch.

We move on to investigate the impacts of the synchronization criterion, $\Delta \phi$, and friction coefficient, $c_\omega$, on the generation of APS. With the same parameters used in figure 4, in figure 5(a) we plot the variation of $p_{aps}$ with respect to $|\Delta f|$ for different synchronization criteria. It is seen that as $\Delta \phi$ decreases, the desynchronization point, $\Delta \phi_b$, moves to the left gradually, whereas before this point $p_{aps}$ is independent of $\Delta \phi$. The numerical results thus suggest that by changing the synchronization criterion, it is only the upper limit of the attracting basin that is affected; before the desynchronization point $|\Delta \phi_b|$, the basin of APS always monotonically increases with the frequency mismatch. However, by decreasing $\Delta \phi$, the range over which the basin of APS is enlarged with $|\Delta f|$ is narrowed. (In the extreme case of $\Delta \phi = 0$, we have $\Delta \phi_b = 0$, which means that the introduction of any frequency mismatch will result in desynchronization.) Figure 5(b) shows the impact of $c_\omega$ on the attracting basin of APS. It can be seen that by increasing $c_\omega$, the basin of APS gradually increases, especially for small $\Delta f$. (Of course, when $c_\omega$ is too large, the basin of APS disappears. In such a case, neither APS nor IPS will be observed.) The phenomenon depicted in figure 5(b) is consistent with the findings in [15, 21], where it is shown that the basin of APS is enlarged by increasing $c_\omega$ slightly.

Figure 5(a) suggests that the basin of APS increases at the cost of decreased synchronization precision. For more details on the trade-off between the probability of the generation and synchronization precision of APS, we finally investigate the dependence of synchronization precision on frequency mismatch. When $\Delta f = 0$ and without noise perturbations, the pendulums can be perfectly synchronized, i.e. $d\phi(t) = \phi_1(t) - \phi_2(t) = 0$ (IPS) or $d\phi(t) = 0$ (APS) in the asymptotic states. When $\Delta f \neq 0$, the states $\phi_1(t) = \phi_2(t)$ and $\phi_1(t) = \phi_2(t) + \pi$ are no longer solutions of the system dynamics. For the latter, the IPS and APS states can only be loosely defined, i.e. $|d\phi(t)| < \Delta \phi$ (IPS) or $|d\phi(t) - \pi| < \Delta \phi$ (APS), with $\Delta \phi$ the synchronization criterion (threshold or precision). Figure 6(a) shows the time evolution of $d\phi$ of the APS state for different values of $\Delta f$. It is seen that after the transient period, $d\phi(t)$ is stabilized into a constant value which, as $|\Delta f|$ increases, gradually deviates from $\pi$. To obtain a global picture of the impact of $\Delta f$ on synchronization precision, in figure 6(b) we plot the variation of $d\phi$ with respect to $|\Delta f|$ over a large range (the phases are not locked when $|\Delta f| > 0.14$). It is seen that for both the IPS and APS states, $d\phi$ monotonically increases with
Figure 6 shows that the introduction of frequency mismatch facilitates APS rather than IPS, as in IPS the phases are unlocked at $f_{0.03} \Delta f \approx 0$, while in APS the phases are unlocked at $f_{0.14} \Delta f \approx 0.14$. This observation is consistent with the results depicted in figure 4(d), where IPS disappeared at $|\Delta f| \approx 3.7 \times 10^{-2}$ and only APS is observed within the range of $|\Delta f| \in (\Delta f_c, \Delta f'_c)$.

4. Discussion and conclusion

In the study of oscillator synchronization, a well-known feature is that the introduction of parameter mismatch will deteriorate synchronization [4–7]. For example, in coupled phase oscillators described by the Kuramoto model [5], the onset of synchronization will be postponed to larger coupling strength as the distribution of the natural frequency is broadened; and in coupled chaotic oscillators, around the critical point the synchronization error monotonically increases with the increase of parameter mismatch [6, 7]. Yet, in realistic situations, parameter mismatch is always unavoidable. Facing this reality, an active topic in synchronization studies has been developing new strategies to reduce the destructive role of
parameter mismatch, e.g. by rearranging the oscillators or updating the coupling schemes [27]. Our present work reveals, from a different viewpoint, the constructive role of parameter mismatch on synchronization. That is, parameter mismatch facilitates APS. This finding provides new perspectives on the collective behavior of coupled oscillators, and might have implications on the functionality and operation of many realistic systems, e.g. the critical role of APS in realizing the working memory of the human brain [28], the emergence of APS in influenza propagation [29], and the APS pattern appearing in restoring the oscillation of coupled electrochemical systems from amplitude death [30]. (Large scale IPS, on the other hand, usually implies dysfunctions, e.g. epilepsy seizures [31], and disasters, e.g. bridge collapses [32], in realistic situations.)

The finding that frequency mismatch could promote APS will be helpful to classroom demonstrations. In classroom demonstrations of metronome synchronization, it is much more difficult to generate APS than IPS. This difficulty results in many instructors choosing to only demonstrate IPS in the classroom, whereas APS is only briefly mentioned or interpreted with experimental photos. In [15], the authors proposed enlarging the basin of APS by increasing
the friction coefficient, $c_f$, which is realized by taping paper tissues between the empty cans and the desk. While this is able to increase the probability of generating APS, it has the drawbacks of inconvenient operation and non-controllable parameters. In particular, the friction coefficient cannot be precisely measured and, more seriously, the friction coefficient gradually decreases as the cans roll on the tissues. These drawbacks are overcome in our current scheme. First, when the position of the clip is fixed, the natural frequency of the metronome will be determined (time independent), and so is the frequency mismatch between the metronomes. Second, the positions of the clips can be adjusted continuously along the pendulum bob, making it possible to freely control the frequency mismatch. Finally, the frequency mismatch can be precisely measured, making it possible to evaluate the impact of frequency mismatch on synchronization in a quantitative manner. We therefore hope that, equipped with the new scheme, the demonstration of APS will not be a 'headache', but a 'showtime' for instructors.

In summary, we have studied, experimentally and theoretically, the synchronization of two coupled mechanical metronomes, and found that by increasing the frequency mismatch between the metronomes, the probability of generating APS from the random initial conditions can clearly be increased. The study sheds lights on the collective behavior of coupled oscillators, and will be helpful to classroom demonstrations of synchronization. Recalling the odd sympathy discovered by Huygens, we have reason to speculate that the pendulums in Huygens’ sketch might have been deliberately drawn as different, so as to emphasize the role of frequency mismatch on generating APS.

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Appendix. Method for acquiring the instant phases of the metronomes

The instant phases of the metronomes, $\phi_{1,2}(t)$, are acquired from the recorded video by software, as follows. First, we treat each frame of the video as a 2D picture, while setting the lower-left corner of the picture as the origin of the coordinates. For the camera used in our experiment, the resolution of the picture is $1920 \times 1080$ pixels. We then use $(x, y)$, with $x \in [1,1920]$ and $y \in [1,1080]$, to locate the pixels, and assign each pixel its color value (RGB triplet). For the blue-colored markers sticked on the metronomes, the associated RGB triplet is 00255. By scanning over the picture, we are able to identify all the blue pixels in the picture. Second, according to the position of the markers, we group the blue pixels into four clusters, with $(x_i, y_i)$ the coordinates of the $i$th pixel in cluster $l$. For the $l$th cluster, we calculate the averaged coordinates $x_i^c = \frac{\sum_{i=1}^{n_l} x_i}{n_l}$ and $y_i^c = \frac{\sum_{i=1}^{n_l} y_i}{n_l}$ ($n_l$ is the number of pixels in cluster $l$), and define $(x_c^l, y_c^l)$ as the center of this cluster. Third, we pair the centers according to the metronomes, and define the instant phase of the metronome as

$$\phi = \arctan \frac{y_c^l - y_c^l'}{x_c^l - x_c^l'}$$

with $(x_c, y_c)$ and $(x_c', y_c')$ the upper and lower centers associated with the same metronome. Finally, we analyze the video frame by frame by the same process, and obtain the time

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evolution of the instant phases depicted in figures 2(b) and (c). We would like to note that this analysis can be realized by any software, which in our study was MATLAB.

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