RESEARCH ARTICLE

Time-Varying Heterogeneous Alternation Control for Synchronization of Delayed Neural Networks

FENGNA CHENG, SHAN TANG, XIN HAN, AND YUYAN ZHANG

1College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China
2Jiangsu Province Key Laboratory of Aerospace Power System, College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

Corresponding authors: Fengna Cheng (cfn1218@163.com) and Yuyan Zhang (yuyan_zhang@njfu.edu.cn)

This work was supported in part by the China Postdoctoral Science Foundation under Grant 2019M650127, in part by the Youth Science and Technology Innovation Foundation of Nanjing Forestry University under Grant CX2019006, and in part by the Innovation and Entrepreneurship Training Program for College Students in Jiangsu Province under Grant 202110298067Y.

ABSTRACT This paper proposes a time-varying heterogeneous alternation controller for the synchronization of delayed neural networks (DNNs). Different from the mixture of traditional impulse and intermittent control, we consider a time-varying intermittent controller which is more representative than traditional periodic or aperiodic intermittent controllers. To make the system under study more applicable, we consider the impact of time-varying delay. After doing a theoretical analysis, the sufficient condition of the synchronization of DNNs is obtained. Finally, two simulations are offered to verify the validity of our results.

INDEX TERMS Delayed neural networks, intermittent control, impulsive control, synchronization.

I. INTRODUCTION Synchronization has garnered substantial attention in theoretical analysis [1], [2], [3], [4], [5] and technical application [6], [7] as a key aspect of understanding neural network [8]. Actually, extra control is usually required to achieve this task for many systems due to the complexities between nodes and the node dynamics. To this end, a wide range of control strategies, including feedback control [9], adaptive control [10] and intermittent control [11], have been investigated and improved.

These controllers may often be split into continuous control and discrete control. Continuous control, such as feedback control and adaptive control [12], [13], [14], [15], aids the system until it reaches the desired trajectory by continuously applying the control signals. But this control technique requires continuous energy use as well. Consequently, discrete control is suggested as a solution to the aforementioned issue [16], [17], [18], [19], including intermittent control and impulsive control. It enables the controller to work at a certain discrete instant or for a certain amount of time [20]. Intermittent control, in particular, is based on the piecewise function technique which includes work and rest time [21]. Its usual benefit include cheap control costs and ease of deployment. For instance, [11] put forwards a periodically intermittent control with discrete-time state observations; [21] synchronized the delayed complex valued dynamic network by applying the aperiodic intermittent controller to some nodes of the network. In aperiodic intermittent control, a positive lower bound must be applied to the control width of each control period. To address this issue, [22] explored a time-varying intermittent control strategy.

Besides, impulsive control acts at certain discrete instants as a sampled-data-based method [23], [24], [25]. The two types of impulses—synchronous and asynchronous impulses—can be easily distinguished since not all impulse messages are beneficial [26], [27], [28], [29]. The authors [27] viewed asynchronous impulses as a disturbance and used feedback and synchronous impulse control, respectively, to synchronize their considered systems. Further, asynchronous and synchronous delay-free impulses, asynchronous and synchronous delayed impulses were all taken into consideration by [29]. To measure the impact of mixed impulses, the average delayed impulsive gain as an unique concept was introduced. Reference [30] dealt with the hybrid delay-dependent impulsive problem by

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/

VOLUME 10, 2022
establishing an extended Halanay-type inequality via mathematical induction.

The aforementioned studies favor the analysis of single-type control strategy. Nevertheless, the resilience of a system based on a single-type controller plainly lowers when the system is influenced by internal or external noise, which is an evident challenge. In [31], an intermittent impulsive controller was developed to achieve the consensus of multi-agent systems with time-varying delay. The authors in [32] built two-types impulsive controllers (effects) to force the delayed network with intermittent communications to a desired state.

In order to address the consensus difficulties of discrete systems with and without DoS attacks, the authors combined periodically intermittent control and impulsive control so as to ensure the stability and cost saving of the controller [20]. However, these intermittent-based considerations of intermittent impulsive controllers are also related to the lower bound of control width for each control period. Besides, it’s common to presume that the signal in an intermittent control alternates between fixed constants and zero for each interval. The aforementioned mixture control based such an intermittent controller is unavoidably conservative due to the two concerns, which motivates the current work.

Inspired by the above discussions, we propose a time-varying heterogeneous alternation controller for the synchronization of delayed neural networks. Instead of considering aperiodic or aperiodic intermittent control, here we combine time-varying intermittent control with impulse control to explore a general form of control mixture strategy. The impacts of synchronous and asynchronous impulses are specifically taken into account. Besides, we introduce a time-varying delay term to broaden the scope of the studied systems. Sufficient criteria for delayed network synchronization under the proposed control are obtained by theoretical analysis. Finally, the effectiveness of the proposed control is then confirmed using experimental instances.

The following is how this paper is structured. Section II describes the model of delayed neural networks in details and introduces our proposed time-varying heterogeneous alternation controller. Then Section III gives the theoretical analysis of the proposed strategy. The obtained results is validated by experimental simulation in Section IV. At last, Section V concludes with a summary.

Notation: Let \( \mathbb{R}^n \) be the \( n \)-dimensional Euclidean space. \( \mathbb{R}^{m \times n} \) means the set of all \( n \times m \) real matrices. \( I_n \) is the \( n \)-dimensional identity matrix. \( \lambda_{\text{max}}(\cdot)/\lambda_{\text{min}}(\cdot) \) signifies the maximum/minimum eigenvalue of matrix. The superscript \( T \) indicates the transpose of a matrix or a vector. For any vector \( v = (v_1, v_2, \ldots, v_n)^T \in \mathbb{R}^n \), \( \| v \| = (v_1^2 + v_2^2 + \ldots + v_n^2)^{1/2} \) denotes the Euclidean norm.

II. MODEL DESCRIPTION

The dynamics of delayed neural networks (DNNs) are given by:

\[
\dot{v}(t) = Av(t) + Bf(v(t)) + Cg(v(t - \tau(t))),
\]

with the initial state \( v(t) = \chi_0(t), \quad -\tau_0 \leq t < 0 \), where \( v(t) \in \mathbb{R}^n \) denotes the neuron states of DNNs, \( A \in \mathbb{R}^{n \times n} \) is the state coefficient matrix, \( B \in \mathbb{R}^{n \times m} \) and \( C \in \mathbb{R}^{m \times n} \) mean the connection weight matrices, both \( f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n \) and \( g(\cdot) : \mathbb{R}^n \to \mathbb{R}^n \) are nonlinear functions satisfying the following Lipschitz condition, i.e., \( \| f(x) - f(y) \| \leq s_1 \| x - y \| \) and \( \| g(x) - g(y) \| \leq s_2 \| x - y \| \) with two positive Lipschitz constant \( s_1 \) and \( s_2 \) for any vectors \( x, y \in \mathbb{R}^n \), the time delay is bounded, i.e., \( 0 < \tau(t) < \tau_0 \). Here the system (1) is set as the master DNNs, which corresponds to the slave DNNs listed below:

\[
\dot{u}(t) = Au(t) + Bf(u(t)) + Cg(u(t - \tau(t))) + w(t),
\]

with the initial state \( u(t) = \gamma_0(t), \quad -\tau_0 \leq t < 0 \), where \( u(t) \in \mathbb{R}^n \) denotes the neuron states of DNNs, \( w(t) \) is a control input to be designed.

Remark 1: In real-world applications, the effects of time delay cannot be disregarded due to the limited speed of sending and switching signals [32], [33], [34], [35], [36]. The delay taken into account here may be either constant or time-varying. Consequently, the delay discussed in this study is more broad-based compared to [22], [35], and [36]. Specifically, this work just requires the delay to be bounded, whereas [22], [35], and [36] require the delay to be a constant or the derivative of the delay to be bounded.

Let \( z(t) = u(t) - v(t) \) be the system error. Then the states of the system error are characterized by

\[
\dot{z}(t) = Az(t) + Bf(z(t)) + Cg(z(t - \tau(t))) + w(t),
\]

where \( f(z(t)) = f(u(t)) - f(v(t)) \) and \( g(z(t - \tau(t))) = g(u(t - \tau(t))) - g(v(t - \tau(t))) \). To achieve the synchronization of master-slave DNNs, a time-varying heterogeneous alternation control is designed:

\[
w(t) = m(t)z(t) + u(t, u(\ell^k_0), v(t)),
\]

where \( m(t) \) is an intermittent gain with the positive lower bound \( m_0 > 0 \), that is, \( \bar{m} = \frac{1}{k-k_0-1} \int_{k_0}^{\infty} m(s)ds \geq m_0 \) for \( k = 1, 2, \ldots, m(t, u(t_{\ell_0}^k), v(t)) \) denotes the impulsive controller given next. Specifically, the derivative of the system error is re-expressed below:

\[
\begin{cases}
\dot{z}(t) = Az(t) + Bf(z(t)) + Cg(z(t - \tau(t)))
+m(t)z(t) + u(t, u(\ell^k_0), v(t)), t \geq t_0, \\
u(t, u(\ell^k_0), v(t)) = \sum_{k=1}^{\infty} \kappa_k [u(t^k_0 - \tau(t)) - v(t) - \delta(t - t_k)]
\end{cases}
\]

where \( \kappa_k \) is the impulsive control gain, the time sequence \( \{t_k\}_{k \geq 1} \) satisfies \( t_k < t_{k+1} \) and \( \lim_{k \to \infty} t_k = \infty \). \( z(t^k_0) \) and \( z(t^k_0) \) are the left-limit and right-limit of the state \( z(t) \) at impulsive instant \( t_k \), respectively. \( \delta(\cdot) \) is the Dirac delta function, i.e., \( \delta(t) = 0 \) for \( t \neq 0 \) and \( \int_{-\infty}^{\infty} \delta(t)dt = 1 \). One of the fundamental properties is \( \int_{-\infty}^{t} \sigma(\tau)\delta(t - \alpha)d\tau = \sigma(\alpha) \) for \( \epsilon \neq 0 \) and all continuous compactly supported function
\(\sigma(t)\) [37], [38]. It is generally utilized to model a tall narrow spike function, i.e., an impulse [38].

The following definitions and lemmas, which will be applied to the main results, are introduced before moving on.

**Definition 1:** The DNNs (1) and (2) are said to be globally exponential synchronization if the scalars \(\ell > 0\) and \(\lambda > 0\) exist such that

\[
\|v(t) - u(t)\| \leq \ell e^{-\lambda t}, \quad t \geq 0.
\]

**Lemma 1:** [39] The following inequality holds if a positive definite matrix \(P \in \mathbb{R}^{n \times n}\) exists:

\[
2v^T u \leq v^TP v + u^TP^{-1} u.
\]

with any vector \(v, u \in \mathbb{R}^n\).

**Lemma 2:** [40] Assuming \(Q \in \mathbb{R}^{n \times n}\) be a symmetric matrix, one has

\[
\lambda_{\text{min}}(Q)v^T v \leq v^T Q v \leq \lambda_{\text{max}}(Q)v^T v.
\]

with any \(v \in \mathbb{R}^n\).

### III. MAIN RESULTS

The key results are presented here together with rigorous proof. To demonstrate the wide-ranging applications of the results, additional discussions are provided.

The Eq. (5) is rewritten as follows to make the understanding of the following deduction easier:

\[
\dot{z}(t) = Az(t) + Bf(z(t)) + Cg(z(t) - \tau(t)) + m(t)z(t) + \sum_{k=1}^{\infty} \kappa_k(u(t_k^-) - v(t))\delta(t - t_k).
\]

Integrating the two sides of the above equation obtains [38, 41]:

\[
z(t_k + \varepsilon) - z(t_k - \varepsilon) = \int_{t_k - \varepsilon}^{t_k + \varepsilon} [Az(t) + Bf(z(t)) + Cg(z(t) - \tau(t)) + m(t)z(t) + \sum_{k=1}^{\infty} \kappa_k(u(t_k^-) - v(t))\delta(t - t_k)] dt.
\]

where \(\varepsilon > 0\) is sufficiently small. As \(\varepsilon \to 0^+\), it reduces to

\[
z(t) = (1 + \kappa_k)z(t_k^-), \quad t = t_k, k \in \mathbb{Z}^+.
\]

Then the error system is further expressed as:

\[
\begin{cases}
\dot{z}(t) = Az(t) + Bf(z(t)) + Cg(z(t) - \tau(t)) + m(t)z(t), \quad t \neq t_k, t \geq 0, \\
z(t) = (1 + \kappa_k)z(t_k^-), \quad t = t_k, k \in \mathbb{Z}^+.
\end{cases}
\]

**Remark 2:** The strength of the impulse determines whether it is harmful or beneficial to the system [28], [32]. For this reason, we consider the effects of two cases, that is, when the condition \(|1 + \kappa_k| > 1\) is met, the impulse will destroy the synchronization of complex systems. In this case, it is usually described as internal or external noise. When the condition is opposite, i.e., \(|1 + \kappa_k| < 1\), it will help to realize system synchronization.

**Remark 3:** Different from traditional impulse intermittent systems [20], [31], [32], the intermittent technique taken into consideration here is time-varying, which is more broad than periodic or nonperiodic intermittent methods and may cover a wider range of situations. Note that the proposed controller degenerates to the traditional impulse intermittent control [20], [31], [32] if the control gain switches between fixed constants and zero in one period. The impulse intermittent paradigm may also be applied to other systems, such as the systems with intermittent communication or with impulse disturbances, even though it is primarily considered from control level.

The following notation is defined before proof. \(\lambda_{\text{max}}(\cdot)\) is the maximum matrix eigenvalue. \(\kappa_{\text{desyn}}^*\) and \(\kappa_{\text{syn}}^*\) denote desynchronizing and synchronizing impulse strength, respectively. \(T_{\text{min}} = \min_{1 \leq k \leq 1} \{t_k - t_{k-1}\}\) means the minimum desynchronizing impulse interval, and \(T_{\text{max}} = \max_{1 \leq k \leq 1} \{t_k - t_{k-1}\}\) denotes maximum synchronizing impulse interval.

**Theorem 1:** The globally exponential synchronization of master-slave DNNs is achieved under the time-varying heterogeneous alternation control (4) if the following condition is met:

\[
\frac{\ln (1 + \kappa_{\text{desyn}}^*)}{T_{\text{min}}} + \frac{\ln (1 + \kappa_{\text{syn}}^*)}{T_{\text{max}}} + \zeta < 0.
\]

where \(\zeta = \lambda_{\text{max}}(A) + s_1 \|B\| - m_0 + s_2 \|C\| e^{\tau_0}\).

**Proof:** Construct the following quadratic equation:

\[
W(t) = \frac{1}{2} z^T(t)z(t).
\]

For \(t \in [t_{k-1}, t_k]\), the derivative of Eq. (14) gives

\[
\dot{W}(t) = z^T(t)\dot{z}(t) = z^T(t)A\dot{z}(t) + z^T(t)Bf(v(t)) - f(u(t))
\]

\[
+ z^T(t)Cg(v(t) - \tau(t)) - g(u(t) - \tau(t))
\]

\[
- m(t)z^T(t)z(t).
\]

Using the given Lipschitz condition of \(f(x)\) and \(g(x)\), Lemma 1 and Lemma 2, one has

\[
\dot{W}(t) \leq \lambda_{\text{max}}(A) + s_1 \|B\| - m(t)z^T(t)z(t)
\]

\[
+ s_2 \|C\| \|z(t)\| \|z(t) - \tau(t)\|.
\]

Assume \(P(t) = \sqrt{W(t)}\), and then we have \(\|z(t)\| = \sqrt{2P(t)}\) and \(\|z(t) - \tau(t)\| = \sqrt{2P(t) - \tau(t)}\). As for \(W(t) \neq 0\), one has \(\dot{W}(t) = 2P(t)\dot{P}(t)\). The inequality (16) can be further expressed as:

\[
\dot{P}(t) \leq (\lambda_{\text{max}}(A) + s_1 \|B\| - m(t))P(t)
\]

\[
+ s_2 \|C\| P(t) - \tau(t))
\]

(17)
As there is a variable \( m(t) \) in Eq. (17), a new quadratic function is constructed:

\[
Q(t) = \varphi(t)P(t),
\]

where \( \varphi(t) = e^{\int_0^t (\ell(w) - m_0)dw} \). It is not hard to obtain that \( Q(t) \) is continuous and differentiable. Consequently, its derivative is computed below:

\[
\dot{Q}(t) = (m(t) - m_0)\varphi(t)P(t) + \varphi(t)\dot{P}(t) \\
\leq (\lambda_{\text{max}}(A) + s_1 \| B \| - m_0)Q(t) + s_2 \| C \| \varphi(t)P(t - \tau(t))
\]

Motivated by [42] and [22], one has

\[
\ell(t) \leq e^{\int_{t_0}^t (m(w) - m_0)dw} \leq e^{t_0\nu},
\]

where \( \ell(t) = \sup (\varphi(t) - \varphi(t - s)) \) and \( \nu = \sup |m(t) - m_0| \).

Then, further derivation of Eq. (19) leads to:

\[
\dot{Q}(t) \leq (\lambda_{\text{max}}(A) + s_1 \| B \| - m_0)Q(t) + s_2 \| C \| e^{t_0\nu}\dot{Q}(t),
\]

where \( \dot{Q}(t) = \sup_{s \in [t_0, t]} Q(s) \). Using the Gronwall-Bellman inequality yields

\[
Q(t) \leq Q(t_0) e^{\int_{t_0}^t \lambda_{\text{max}}(A) + s_1 \| B \| - m_0} + s_2 \| C \| e^{t_0\nu}.
\]

When \( t = t_k \),

\[
W(t_k) = (1 + \kappa_k^*)W(t_k^-).
\]

The following formula can be obtained by taking the root sign from both sides of the above inequality:

\[
P(t_k) = |1 + \kappa_k| P(t_k^-).
\]

Multiplying both sides of Eq. (24) by \( \varphi(t) \) obtains

\[
Q(t) = |1 + \kappa_k| Q(t_k^-)
\]

The following inequality can easily be obtained using a straightforward deduction [27]:

\[
Q(t) \leq \prod_{i=1}^k |1 + \kappa_i| Q(t_0) e^{\int_{t_0}^t \lambda_{\text{max}}(A) + s_1 \| B \| - m_0} + s_2 \| C \| e^{t_0\nu}
\]

for any \( \forall t \in [t_{i-1}, t_i] \), where \( k_1 \) and \( k_2 \) mean the number of desynchronizing and synchronizing impulse times at interval \( (t_0, t) \), respectively.

The aforementioned derivation allows us to eventually get

\[
Q(t) \leq Q(t_0) e^{\int_{t_0}^t \lambda_{\text{max}}(A) + s_1 \| B \| - m_0} + s_2 \| C \| e^{t_0\nu} + \kappa(t - t_0)
\]

where \( \varphi = \frac{\ln(1 + \kappa^*_d)}{T_{\min}} + \zeta \) and \( \dot{Q}(t_0) = 1 + \kappa^*_d \| Q(t_0) \|.

From the sufficient condition \( \frac{\ln(1 + \kappa^*_d)}{T_{\min}} + \zeta < 0 \) given in Theorem 1, we know that the master and slave DNNs achieve globally exponential synchronization. This completes the proof.

If system (1) suffers from the impulse disturbances, we apply the time-varying intermittent control technique to force the network to the synchronous trajectory (2) while also preventing the impact of disturbances. The time-varying intermittent control is represented as:

\[
w(t) = m(t)z(t).
\]

These are the conclusions that we can reach.

**Theorem 2:** The globally exponential synchronization of master-slave DNNs is achieved under the time-varying intermittent control (28) if the following condition is met:

\[
\frac{\ln(1 + \kappa^*_d)}{T_{\min}} + \zeta < 0.
\]

**Proof:** Similar to the proof of Theorem 1, one also has the following deduction as given in Eq. (26):

\[
Q(t) \leq \prod_{i=1}^k \left( 1 + \kappa_i \right) Q(t_0) e^{\int_{t_i}^t \lambda_{\text{max}}(A) + s_1 \| B \| - m_0} + s_2 \| C \| e^{t_0\nu}
\]

for any \( \forall t \in [t_{i-1}, t_i] \). We can arrive to the following derivation since the considered impulses are asynchronous:

\[
Q(t) \leq Q(t_0) e^{\int_{t_i}^t \lambda_{\text{max}}(A) + s_1 \| B \| - m_0} + s_2 \| C \| e^{t_0\nu} + \kappa(t - t_0)
\]

where \( \dot{Q}(t_0) = 1 + \kappa^*_d \| Q(t_0) \|.

Based on the sufficient condition \( \frac{\ln(1 + \kappa^*_d)}{T_{\min}} + \zeta < 0 \) given in Theorem 2, one has that the master-slave DNNs achieve globally exponential synchronization. This completes the proof.

**IV. NUMERICAL SIMULATIONS**

Two examples in this part serve to validate the effectiveness of the theoretical analysis presented above.

Case 1. The first example is a small-scale delayed neural network with seven nodes with the neuron state [43] described as follows:

\[
\dot{x}(t) = Ax(t) + Bf(x(t)) + Cg(x(t - \tau(t))),
\]

where \( x(t) = (v_1(t)^T, v_2(t)^T, \ldots, v_7(t)^T)^T \) and \( v(t) \in \mathbb{R}^3 \),

\[
g(x(t - \tau(t))) = -(1/2b(\ell_1 - \ell_2)(|v_1| + 1) - |v_1(t) - 1|),
\]

\[
(n x(t - \tau(t)) = -(1/2b(\ell_1 - \ell_2)(|v_1| + 1) - |v_1(t) - 1|)
\]

where \( A = \begin{bmatrix} -0.1 & 0.5 \\ 0.5 & -0.1 \\ 0 & 0.5 \\ 0.5 & 0 & -0.1 \\ 0.1 & 0 & 0.5 \\ 0.5 & 0.1 & 0 \\ 0.5 & 0.1 & 0.5 \end{bmatrix} \), \( B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \), \( C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \), and \( \tau(t) = |v_1(t) - 1| + 1 \).
0, 0)^T + (0, 0, −am \sin ρ(\nu_1(t − τ(t))))^T, f(\nu(t)) = \nu(t), \quad A = I_N \otimes \hat{A}, \hat{A} = \begin{pmatrix} −b(1 + \ell_2) & b & 0 \\ 1 & −1 & 1 \\ 0 & −a & γ \end{pmatrix}, B = k \hat{B} \otimes I_3, \\
k = 0.2 \text{ denotes the coupling strength, } C = I_N \otimes I_3, \\
b = 10, \quad a = 19.52, \quad γ = 0.1636, \quad \ell_1 = −1.4325, \quad \ell_2 = −0.7831, \quad ρ = 0.5, \quad m = 0.2, \quad τ(t) = 0.1. \quad \text{The Lipschitz condition can be verified by a straightforward derivation of the nonlinear terms} [43].

\[ \hat{B} = \begin{bmatrix} −1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & −1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & −1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & −1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & −1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & −1 \end{bmatrix} \]

The control gain is set as \( c(t) = 5 + 2 \sin(2πt) \). \( t_k − t_{k−1} = 0.5 \). \( t_k \) is the time of intermittent gains is shown in Figure 1. Then one has \( m_0 = 1 \int_{t_{k−1}}^{t_k} m(s)ds = 5 \). In this case, the effects of asynchronous impulses are considered. Let \( \kappa_{dyn}^a = 0.25 \), and it is clear that Theorem 1’s sufficient condition is satisfied by the parameters listed above.

Figure 2 shows how the error of master-slave systems steadily decreases to zero when using the proposed controller. It should be emphasized that when the system is only exposed to external impulse noise, the proposed strategy can also deal with this situation well. This reflects that the obtained conclusion can be simply extended to other systems.

Based on the above system, we make a comparative simulation experiment with the controller in [31], [32], and [20] to show the benefits of the control strategy, as shown in Figure 3. Note that the control gain is 5 (the average value of the time-varying intermittent control gain in this example), the period of intermittent control is 1, and the working time ratio is 0.5. It is not difficult to know from the figure that the convergence rate of method is obviously faster than that in [31], [32], and [20]. It is due to that the controller we used in this case is continuous as opposed to intermittent, in addition to the different setting of control parameters.

Remark 4: The same system under different controllers will have different trajectories, as shown in Figure 3. We would like to emphasize once again that the strategy proposed in this paper covers two cases. In other words, it can degenerate into the traditional impulse intermittent control [20], [31], [32], which demonstrates the broad applicability of our strategy.

Case 2. Here we consider a large-scale delayed neural network with 200 nodes.

\[ \dot{v}(t) = \tilde{A}v(t) + \tilde{B}f(v(t)) + \tilde{C}g(v(t − τ(t))), \quad (33) \]

where \( v(t) = (v_1(t)^T, v_2(t)^T, \ldots, v_{200}(t)^T)^T \) and \( v_i(t) \in \mathbb{R}^3 \), \( f(v(t)) = \tanh(v(t)) \) and \( g(v(t − τ(t))) = \tanh(v(t − τ(t))) \),
The signal of time-varying intermittent gains is depicted in Figure 4. The signal of impulse gains is shown in Figure 5. The control gain is set below:

\[
m(t) = \begin{cases} 
5 + 1.3 \sin(3\pi t / 0.1) & t \in [kT, kT + \delta_0 T) \\
6 - 1.2 \sin(3\pi t / 0.2) & t \in [kT + \delta_0 T, kT + \delta_1 T) \\
0 & \text{else}
\end{cases}
\]

In this setting, \(\delta_0 = 0.2\), \(\delta_1 = 0.4\) and \(T = 0.6\). The signal of time-varying intermittent gain is depicted in Figure 4. Given the sufficient condition in Theorem 1, the synchronous impulses are set as \(\kappa^s = -0.8\) with the gain of synchronous impulses \(\kappa^s = 0.1\). The fluctuation of synchronous and asynchronous impulses is shown in Figure 5. Meanwhile, the trajectories of the error systems are described in Figure 6, which shows that the obtained results are easily extended to the control of the large-scale networks. Overall, the above simulations validate the effectiveness of our method.

V. CONCLUSION

This paper puts forward a time-varying heterogeneous alternation controller for the synchronization of delayed neural networks. We integrate more comprehensive impulse control with time-varying intermittent control for the first time. Further, the impulse contains two types: synchronous impulse and asynchronous impulse. Additionally, in order to make this study more universal, we incorporate time-varying delay into the considered systems. The sufficient condition for network synchronization is obtained by Lyapunov stability analysis. Finally, two examples are given to verify the effectiveness of our results.

Theoretical analysis and discussion presented above demonstrate the benefits of our proposed controller. However, because the controller in this study is used by each network node, it will consume a large amount of energy. So, in the upcoming work, we will take into account other energy-saving control methods like pinning control and event-trigger control [17].

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] J. Ruths and D. Ruths, “Control profiles of complex networks,” Science, vol. 343, no. 6177, pp. 1373–1376, Mar. 2014.
[2] B. Liu, W. Lu, and T. Chen, “Synchronization in complex networks with stochastically switching coupling structures,” IEEE Trans. Autom. Control, vol. 57, no. 3, pp. 754–760, Mar. 2012.
[3] Z. Gu, P. Shi, D. Yue, S. Yan, and X. Xie, “Fault estimation and fault-tolerant control for networked systems based on an adaptive memory-based event-triggered mechanism,” IEEE Trans. Netw. Sci. Eng., vol. 8, no. 4, pp. 3233–3241, Oct. 2021.
[4] M. S. Anwar and D. Ghosh, “Intralayer and interlayer synchronization in multiplex network with higher-order interactions,” *Chaos: Interdiscipl. J. Nonlinear Sci.*, vol. 32, no. 3, Mar. 2022, Art. no. 033125.

[5] A. Arbi and N. Tahri, “Stability analysis of inertial neural networks: A case of almost anti-periodic environment,” *Math. Methods Appl. Sci.*, 2022, doi: 10.1002/mma.8379.

[6] W. Chen, S. Luo, and W. X. Zheng, “Impulsive synchronization of reaction–diffusion neural networks with mixed delays and its application to image encryption,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 12, pp. 2696–2710, Dec. 2016.

[7] D. Ouyang, J. Shao, H. Jiang, S. K. Nguang, and H. T. Shen, “Impulsive synchronization of coupled delayed neural networks with actuator saturation and its application to image encryption,” *Neural Netw.*, vol. 128, pp. 158–171, Aug. 2020.

[8] A. Arbi, “Novel traveling waves solutions for nonlinear delayed dynamical neural networks with leakage term,” *Chaos, Solitons Fractals*, vol. 152, Nov. 2021, Art. no. 114136.

[9] S.-S. Zhou, H. Jahan, A., Q. Din, S. Bekiros, R. Alcaraz, M. O. Alassafi, F. E. Alassadi, and Y.-M. Chu, “Discrete-time macroeconomic system: Bifurcation analysis and synchronization using fuzzy-based activation feedback control,” *Chaos, Solitons Fractals*, vol. 142, Jan. 2021, Art. no. 110378.

[10] G. Rajchakit, A. Prapat, R. Raja, J. Cao, J. Alzabut, and C. Huang, “Hybrid control scheme for projective lag synchronization of Riemann–Liouville sense fractional order memristive BAM NeuralNetworks with mixed delays,” *Mathematics*, vol. 7, no. 8, p. 759, 2019.

[11] Y. Wu, J. Zhu, and W. Li, “Intermittent discrete observation control for synchronization of stochastic neural networks,” *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2414–2424, Jun. 2020.

[12] S. Jiang, X. Lu, C. Xie, and S. Cai, “Adaptive finite-time control for overlapping cluster synchronization in coupled complex networks,” *Neurocomputing*, vol. 296, pp. 188–195, Nov. 2017.

[13] A. Prapat, R. Raja, J. Cao, G. Rajchakit, and F. E. Alassadi, “Further synchronization in finite time analysis for time-varying delayed fractional memristor competitive neural networks with leakage delay,” *Neurocomputing*, vol. 317, pp. 110–126, Nov. 2018.

[14] A. Prapat, R. Raja, C. Sowmiya, O. Bagdasar, J. Cao, and G. Rajchakit, “Global projective lag synchronization of fractional order memristor based BAM neural networks with mixed time-varying delays,” *Asian J. Control*, vol. 22, no. 1, pp. 570–583, Jan. 2020.

[15] T. Xiong and Z. Gu, “Observer-based adaptive fixed-time formation control for multi-agent systems with unknown uncertainties,” *Neurocomputing*, vol. 423, pp. 506–517, Jan. 2021.

[16] F. Yang, S. Yan, and Z. Gu, “Derivative-based event-triggered control of switched nonlinear cyber-physical systems with actuator saturation,” *Int. J. Control, Autom. Syst.*, vol. 20, no. 8, pp. 2474–2482, Aug. 2022.

[17] Z. Gu, S. Yan, J. H. Park, and X. Xie, “Event-triggered synchronization of chaotic Lu’s systems via memory-based triggering approach,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 3, pp. 1427–1431, Mar. 2022.

[18] X. Han, F. Cheng, S. Tang, Y. Zhang, Y. Fu, W. Cheng, and L. Xu, “Synchronization analysis of fractional-order neural networks with adaptive intermittent-active control,” *IEEE Access*, vol. 10, pp. 75097–75104, 2022.

[19] F. Liu, C. Liu, H. Rao, Y. Xu, and T. Huang, “Reliable impulsive synchronization for fuzzy neural networks with mixed controllers,” *Neural Netw.*, vol. 143, pp. 759–766, Nov. 2021.

[20] J. Zhuang, S. Peng, and Y. Wang, “Exponential consensus of stochastic discrete multi-agent systems under DoS attacks via periodically intermittent control: An impulsive framework,” *Appl. Math. Comput.*, vol. 433, Nov. 2022, Art. no. 127389.

[21] P. Zhou, J. Shi, and S. Cai, “Pinning synchronization of directed networks with delayed complex-valued dynamical nodes and mixed coupling via intermittent control,” *J. Franklin Inst.*, vol. 357, no. 17, pp. 12840–12869, Nov. 2020.

[22] Q. Jia, E. S. Mwanandiywe, and W. K. Tang, “Master–Slave synchronization of delayed neural networks with time-varying control,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 5, pp. 2292–2298, May 2021.

[23] J. Lu, D. W. C. Ho, and J. Cao, “A unified synchronization criterion for impulsive dynamical networks,” *Automatica*, vol. 46, no. 7, pp. 1215–1221, Jul. 2010.
FENGNA CHENG received the Ph.D. degree from the College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, China, in 2018. Currently, she is with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing. Her research interests include neural networks, image processing, and heat transfer.

SHAN TANG is currently pursuing the B.S. degree with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing, China. Her research interests include fractional-order neural networks and nonlinear system and its synchronization.

XIN HAN is currently pursuing the M.S. degree with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing, China. His research interests include fractional-order neural networks, adaptive control, and nonlinear systems.

YUYAN ZHANG received the Ph.D. degree from the School of Mechanical Engineering, Beijing Institute of Technology, Beijing, China, in 2016. Currently, she is an Associate Professor with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing, China. Her research interests include synchronization analysis of complex systems, mechanical problems in MEMS design and manufacturing technology, and the rotor dynamics analysis of high speed bearing.