New scheme of the microwave signal formation for quantum frequency standard on the atoms of caesium-133

A A Petrov and V V Davydov
Peter the Great Saint-Petersburg Polytechnic University, Saint-Petersburg, Russia
Alexandrpetrov.spb@yandex.ru

Abstract. In present work several directions of quantum frequency standard modernization are considered. A new implementation of a frequency synthesizer and a magnetic field control unit are presented. Experimental study of a frequency synthesizer showed improvement parameters of a microwave-excitation signal, such as step frequency tuning, time frequency tuning, range of generating frequencies and spectral characteristics. Magnetic field control unit eliminates one of the most important perturbing factors affecting the long-term frequency stability. Daily frequency stability of quantum frequency standard improved on 15%.

1. Introduction
The accurate measurement of time and frequency is vital to the success of many fields of science and technology. Examples from atomic physics are atom-photon interactions, atomic collisions, and atomic interactions with static and dynamic electromagnetic fields. Geodesy, radio-astronomy (very long baseline interferometry) and pulsar astronomy rely on the availability of stable local frequency standards and uniform timescales. The same is valid for operation of satellite-based navigation systems.

Quantum frequency standards on the atoms of caesium are used as a clock generators in the communications equipment and in a data transmission devices, applied in the satellite navigation systems GLONASS and GPS as a clock generators and in the various metrological services. Also these standards perform a role of the reference signals with high precision and stability in radio equipment.

Considering the high importance of precision atomic clocks in science and technology and a vast area of their application, modernization of existing and development new quantum frequency standards are the urgent tasks.

The process of frequency standards modernization includes various directions: reducing energy consumption, weight and dimensions, improving metrological characteristics. And for frequency standards characterized by the fact that modernization may not be for the whole construction and may be for individual units or blocks. And in present work one of the directions of modernization of the caesium atomic clock is considered.

The article begins with the theoretical foundation of a caesium atomic clock operation and the role of frequency synthesizer and magnetic field control unit. Then we calculate range of generated frequency, step frequency tuning, frequency hopping and suppression of lateral amplitude components. Then we present a new implementation of a digital frequency synthesizer. After that, we describe how new synthesizer construction can be used for elimination one of the contributions to the
long-term frequency instability. Finally, several results of actual experiments are given, and advantages of the new schemes which lead to improvement metrological characteristic of frequency standard are discussed.

2. Principles of quantum frequency standard on atoms of caesium-133 operation.

The work of a caesium atomic clock is based on the principle of adjustment a highly stable crystal oscillator frequency to quantum transition frequency of caesium-133 atoms [1-3]. Figure 1 shows a block diagram of a caesium atomic clock.

![Block diagram of a caesium atomic clock](image)

**Figure 1.** Block diagram of a caesium atomic clock. 1 - a source of caesium atoms, 2 - magnet polarizer, 3 - magnetic field, 4 - magnetic shield, 5 - Ramsey resonator, 6 - the waveguide, 7 - frequency converter, 8 - crystal oscillator, 9 - automatic frequency control system, 10 - magnet analyzer, 11 – detector.

The output signal frequency 5MHz of the crystal oscillator 8 is supplied to the frequency converter 7. Frequency converter consists of the frequency synthesizer, the mixer signals and the multiplier signals. In the frequency synthesizer input signal frequency 5MHz is converted to the signal frequency 12,631772 MHz and supplied to the input of mixer signals. In the multiplier signals input signal frequency 5 MHz is multiplied to the frequency 270 MHz and then to frequency 9180 MHz. This signal frequency 9180 MHz is also supplied to the input of mixer signals. As a result, the output signal of the frequency converter is the signal of ultrahigh frequency 9192,631772 MHz. This signal is supplied in the waveguide 6 and then on the input of atomic beam tube.

In caesium atomic clock with the help of magnet polarizer 2 the atoms are prepared such that they are either in the \(|F=4, m_F=0>\) or in \(|F=3, m_F=0>\) state. Afterwards the atoms interact with an electromagnetic field that induces transitions into the former unoccupied state.

A magnetic field is used to separate energetically the otherwise degenerate magnetic sub-levels in order to allow the excitation of the clock transition \(|F = 3, m_F = 0> \rightarrow |F = 4, m_F = 0>\) isolated from the other transitions. By convention such a field is referred to as the C field as it is applied between the fields of the polariser and the analyser which historically were called the A field and the B field, respectively. The magnitude of the C field is chosen as a compromise between two conflicting requirements. First, it has to be large enough to separate the otherwise overlapping resonances. Second, the C field shifts the resonance frequency quadratically which has to be corrected. However, in a larger field the frequency of the clock is influenced to a larger extent by fluctuations of the magnetic field. In the scheme of a commercial Cs clock the C field is often generated by a coil with windings in the paper plane wound around the Ramsey resonator and hence, points perpendicularly to
that plane. Owing to the dependence of the frequency of the clock transition from the magnetic field, efficient magnetic shielding has to be provided in order to attenuate the ambient magnetic field and the magnitude of the associated fluctuations.

The atoms in the former unoccupied state are detected and allow one to determine the frequency of the interrogating field where the transition probability has a maximum. The observed transition frequency is corrected for all known frequency offsets that would shift the transition frequency from the unperturbed transition and is used to produce a standard frequency or pulse per second every 9192631772 cycles [1-3].

Scanning the frequency $\nu$ of the atomic resonance leads to a detector current. The signal shows the Ramsey resonance structure on a broader, so-called, Rabi pedestal.

The central feature with the maximum at the transition frequency $\nu_0$ is used to stabilize the frequency of the crystal oscillator to the atomic transition frequency. To this end, the frequency from the synthesizer is modulated across the central peak. The signal from the detector is phase-sensitively detected in the automatic frequency control system, integrated and this signal is used for stabilising the frequency of the crystal oscillator.

Improvement the accuracy of the signal frequency 12,631770 MHz results in better accuracy of the resonant frequency of the atomic transition.

The main characteristic of the frequency synthesizer is its ability to impact the characteristic of frequency stability of the quantum frequency standard output signal. Frequency instability introduced by the synthesizer is determined by the lateral discrete spectrum components of the signal that occurs in dividing, multiplying, mixing frequency signals, the accuracy of the generated frequency, and the impact on the signal of natural and technical noise.

Experimental study showed up that the present method of generating the output signal of the frequency synthesizer needs to increase the accuracy. The large resolution of step frequency is necessary. New scheme of the frequency synthesizer is designed using direct digital synthesis (DDS - Direct Digital Synthesis) [4]. This method allows to generate the output signal of the synthesizer with accuracy more than $10^{-6}$ Hz. Step frequency tuning $\Delta F_{\text{out}}$ calculated by the formula below:

$$\Delta F_{\text{out}} = \frac{F_{\text{clk}}}{2^N},$$

where $F_{\text{clk}}$ is the clock frequency, $N$ is the capacity of accumulator.

In our scheme the clock frequency is equal $F_{\text{clk}}=20$ MHz, the capacity of accumulator is equal $N=40$. Step frequency tuning is equal $\Delta F_{\text{out}} = 1.8 \times 10^{-5}$ Hz.

The application of direct digital synthesis gave the possibility of obtaining the generated frequencies in a wide range (0 - 3 MHz), in contrast to previous schemes, where this feature was absent. Range of generated output frequencies may be calculated by the formula below:

$$F_{\text{out}} = \frac{M \cdot F_{\text{clk}}}{2^N},$$

where $M$ is the frequency code in decimal, $F_{\text{clk}}$ is the clock frequency, $N$ is the capacity of accumulator.

The implementation of our proposed method enables us to control the frequency of the output signal frequency synthesizer in real time. Time hopping $t_h$ is the sum of the time adjustment the digital part of the system and the delay time of the low-pass filter (LPF). Time adjustment digital part in synthesizer is three clock cycles. The delay time of the LPF is inversely proportional to the low-pass filter bandwidth. If the cut-off frequency is equal $F_{\text{cut}} = F_{\text{clk}}/4$ then time delay is about four clock periods. It turns out that time hopping is equal $t_h = 7/F_{\text{clk}}$. The clock frequency $F_{\text{clk}}$ is equal 20 MHz, then the time hopping is equal $t_h = 0.35 \, \mu s$. This ensures a high rate of frequency tuning, which makes it possible to more efficiently adjust the crystal oscillator frequency in contrast the previously used scheme.
To meet the requirements for spectral purity of output signal 10 bit DAC was used. It is possible to obtain the suppression of lateral amplitude components in the spectrum of the output signal is not worse than -60 dB. In figure 2 new design of the frequency converter is presented.

![Image](image.png)

**Figure 2.** A new block of the microwave generator circuit.

3. **Experimental research of synthesizer’s output signal.**

As experimental researches showed, new scheme of the frequency synthesizer has great advantages in comparison with previously used scheme. In figure 3, 4, 5 and 6, as an example, spectrums of the output signal measured in the different bands for new scheme and previously used scheme respectively are presented.

![Image](image1.png)

**Figure 3.** Spectrum of the new synthesizer scheme measured in the band of 8 kHz.

![Image](image2.png)

**Figure 4.** Spectrum of the previously used scheme measured in the band of 8 kHz.
Suppression of lateral discrete components in the spectrum of microwave-excitation is improved on 25 dB in the both bands [5, 6].

The shift of the peak of the line resonance atomic transition $\Delta f$ is arising due to there are lateral discrete components in the microwave-excitation signal spectrum. The shift is defined by the following expression:

$$\Delta f = \frac{A}{I} \frac{(\delta f)^2}{f(f-f_s)}$$

(3)

where $A$ is the amplitude of the lateral components, $I$ is the amplitude of the carrier, $\delta f$ is the width of the spectral line, $(f - f_s)$ is the detuning of the lateral components relative to peaks of the spectral line.

In the developed scheme of the frequency synthesizer the suppression of lateral components is not less than -88 dB, spectral line width is equal 500 Hz and the band of the lateral components is equal 1 kHz. Then relative frequency shift of the atomic transition is equal $\frac{\Delta f}{f} = 1.08 \times 10^{-14}$. Previously obtained result is equal $\frac{\Delta f}{f} = 4.83 \times 10^{-13}$ Hz.

By reducing the frequency shift of the atomic transition $\Delta f$ the more fine-tuning on the center of the resonance line is occur. This leads to a more accurate determination of the value of the nominal output frequency of frequency standard and, consequently, improves the long-term frequency stability of a cesium atomic clock [5].

4. Eliminate frequency shift.

How we noted earlier, improved spectral characteristics directly affect on long-term frequency stability. In addition the new design of the frequency synthesizer allows to eliminate one of the most important perturbing factors affecting the long-term frequency stability too.

The stable isotope $^{133}$Cs has a two hyperfine states $F = 4$ and $F = 3$ which are split in the magnetic field into 16 components. In accordance with the selection rules seven transitions between the components of hyperfine sublevels are possible. These are represented in figure 7.
Figure 7. Microwave resonances in atomic beam tube.

The central resonance $|F=3, m_F=0\rangle \rightarrow |F=4, m_F=0\rangle$ due to the Zeeman effect expose a quadratic frequency shift. With the help of formula (5) we can calculate a frequency shift.

$$\Delta f_{B^2} \approx 4.2745 \times 10^{-2} \text{ Hz} \times \left(\frac{6 \times 10^{-6} \text{ T}}{\text{uT}}\right)^2 = 1.5388 \text{ Hz}. \quad (4)$$

For a typical value of the C field near 8 uT the frequency shift is 2.7 Hz corresponding to a relative frequency shift of $3 \times 10^{-10}$. The accuracy of the output signal quantum frequency standard is dependent on the shift of the central resonance. It should be noted that not only the central resonance is exposed the frequency shift, but also all six transitions $(3, m_F) \leftrightarrow (4, m_F)$, which $\Delta m_F = 0$. To express these changes as a function of $B$ and atomic constants use the equation Bright Rabi:

$$E(F, m_F) = -\frac{\hbar v}{2(2I+1)} - g_I \mu_B B m_F + \frac{\hbar v}{2} \left(1 + \frac{4m_F}{2I+1}x + x^2\right)^{1/2}, \quad (5)$$

where $E(F, m_F)$ is the change energy of atoms in the ground state; $I$ is the quantum number of nuclear spin; $g_I$ is the factor Lande for electron; $\mu_B$ is the Bohr magneton; $B$ is the magnetic field; $m_F$ is the magnetic quantum number; $x = \left(\frac{g_I + g_e}{\mu_B B}\right) \frac{\hbar v}{2}.$

This formula can be used for calculation the frequency shift of any transition between two hyperfine sublevels, depending on the magnetic field. Revealing this expression, we find that the first member is proportional to the magnetic field $B$. For cesium beam primary frequency standards we must consider the quadratic member of this expression [1, 5].

In modern quantum frequency standards the magnetic field is maintained by the active stabilization system. For this purpose the neighboring transition $|F=3, m_F=1\rangle \leftrightarrow |F=4, m_F=1\rangle$ is used. The method of C-field adjustment is similar to the method of frequency adjustment to the main maximum. For this purpose, the average value of the sampling frequency $v_i$ is changed from the value of the $v_{Cs}$ to $v_{Cs} + \Delta v$, where $v_{Cs}$ is the frequency of the main transition of the cesium atom, $\Delta v$ - difference between the transitions for a preset value of the magnetic field. Then the value of magnetic field is adjusted in a such way that the frequency of transition $|F=3, m_F=1\rangle \leftrightarrow |F=4, m_F=1\rangle$ match the preset frequency. This adjustment set up automatically several times per minute. The value of the applied field is automatically maintained at a predetermined level. In this case, excluded effects associated with changes in the magnetic field (for example, long-term drift of the current source, temperature dependence, effect of external magnetic field, etc.)

This feature gave us the opportunities to close the ring-locked loop frequency oscillator on the neighboring resonance transition.
5. Experimental research of quantum frequency standard’s output signal.

Experimental research of quantum frequency standard’s output signal consisted of measuring the output frequency and calculation the Allan variance, which allows to evaluate the frequency instability.

The formula for calculation the Allan variance is as follows:

\[
\sigma_f^2 = \frac{\sum \sigma_{\alpha}^2}{n-1},
\]

where \( \sigma_{\alpha} = \frac{f_{1\alpha} - f_{1\beta}}{f} \) i-th relative frequency variation, \( n \)- number of variations.

After processing the obtained values of relative frequency variations were plotted Allan variance via time for the new and the previous design of frequency synthesizer. (Figure 8).

Figure 8. The Allan variance via time for new (2) and the previous (1) design of frequency synthesizer.

From these results it is clear that the use of new design of the frequency synthesizer makes it possible to obtain a better frequency stability of quantum frequency standard output signal. It is achieved through an improved spectrum of the output signal of the frequency synthesizer and by compensating the frequency offset central transition.

6. Conclusion.

Due to a new algorithm of formation microwave - excitation output signal, parameters such as the step frequency tuning, time frequency tuning, range of generated frequencies and spectral characteristics have been improved.

The range of generated output frequencies is expanded, the possibility of detuning the neighboring resonance frequency of spectral line that makes it possible to adjust the C-field in quantum frequency standard is implemented.

Metrological characteristics (daily frequency stability) of the quantum frequency standard on the atoms of caesium - 133 with new scheme of the microwave – excitation signal improved on 15%.

References
[1] Riechle F 2004 Frequency Standards. Basics and Applications (Weinheim: WILEY-VCH Verlag GmbH Co.KGaA)
[2] Oduan K and Gino B 2002 Chronometry and Basics of GPS (Moscow: Technosphere)
[3] Dudkin V I and Pachomov L N 2002 Quantum Electronics. Devices and Their Applications (Moscow: Technosphere)
[4] Ridiko L I 2005 Components and Technology vol 7 p 83
[5] Claude A, Noel D, Vincent D and Jacques V 2003 Physical origin of the frequency shifts in cesium beam frequency standards: related environmental sensitivity *IEEE Trans. on UFFC* vol 39 pp 412–421
[6] Petrov A A and Davydov V V 2015 Improvement frequency stability of caesium atomic clock for satellite communication system *Proc. 15th International Conference NEW2AN 2015 and 8th Conference ruSMART 2015* pp 739-744