From Super QCD to QCD

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Abstract

We present a “toy” model for breaking supersymmetric gauge theories at the effective Lagrangian level. We show that it is possible to achieve the decoupling of gluinos and squarks, below a given supersymmetry breaking scale \( m \), in the fundamental theory for super QCD once a suitable choice of supersymmetry breaking terms is made. A key feature of the model is the description of the ordinary QCD degrees of freedom via the auxiliary fields of the supersymmetric effective Lagrangian. Once the anomaly induced effective QCD meson potential is deduced we also suggest a decoupling procedure, when a flavored quark becomes massive, which mimics the one employed by Seiberg for supersymmetric theories. It is seen that, after quark decoupling, the QCD potential naturally converts to the one with one less flavor. Finally we investigate the \( N_c \) and \( N_f \) dependence of the \( \eta' \) mass.

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GENERAL STRATEGY

In the last few years there has been an enormous progress in understanding supersymmetric gauge theories via effective Lagrangians. Such a progress is partially due to some papers of Seiberg [1] and Seiberg and Witten [2] in which a number of “exact results” were obtained. There are already several review articles [3–5].

It is natural to expect that information obtained from the more highly constrained supersymmetric gauge theories can be used to learn more about ordinary gauge theories. Here we illustrate the general strategy behind a “toy” model presented in [6] for breaking supersymmetric gauge theories at the effective Lagrangian level.

Let us consider an “exact” effective super potential \( W \) which can be constructed for a given, confining, supersymmetric gauge theory. The superpotential, by construction, correctly saturates all the supersymmetric quantum anomalies. The contribution to the bosonic part of the potential, contained in the superpotential, before imposing the equation of motion for the auxiliary fields, is:

\[
- V_0 (\mathcal{F}, \phi) = \int d^2 \theta \ W (\mathcal{S}, \mathcal{T}) + \text{H.c.} ,
\]

where the chiral superfields \( \mathcal{S} \) and \( \mathcal{T} \) schematically describe gauge invariant supersymmetric composite operators whose bosonic components \( \phi \) respectively contain gluino-ball and squark-antisquark mesons. \( \mathcal{F} \) are the set of auxiliary fields associated with the chiral superfields \( \mathcal{S} \) and \( \mathcal{T} \). We note that the previous potential term, due to supersymmetry, is holomorphic in the fields, i.e. \( V_0 (\mathcal{F}, \phi) = \chi (\mathcal{F}, \phi) + \chi^\dagger (\mathcal{F}^*, \phi^*) \) and \( \chi \) is a function of the complex fields \( \mathcal{F} \) and \( \phi \). The composite operators \( \mathcal{F} \) are seen to describe the ordinary glue-ball and mesonic objects (see for example Eq. (0.8)).

Let us imagine to add SUSY breaking terms in the fundamental Lagrangian whose order parameters (i.e. gluino mass and squark mass terms) can be schematically represented by \( \mathcal{M} \). This will induce in the low energy effective theory a SUSY breaking potential \( V_B (\mathcal{M}, \phi) \) which should be added to the one in Eq. (0.1). The full potential for finite \( \mathcal{M} \) can then be written as

\[
V (\mathcal{F}, \phi, \mathcal{M}) = V_0 (\mathcal{F}, \phi) + V_B (\mathcal{M}, \phi) .
\]

If \( \mathcal{M} \ll \Lambda_S \), where \( \Lambda_S \) is the SUSY invariant scale of the theory we are close to the supersymmetric limit and \( \mathcal{F} \) must be eliminated via its equation of motion

\[
\frac{\partial V}{\partial \mathcal{F}} = 0 \quad \text{for} \quad \mathcal{M} \ll \Lambda_S .
\]
In the absence of the Kähler term, the previous equation simply reproduces the supersymmetric vacuum solution for the bosonic fields \( \phi \). To recover the “soft” SUSY breaking effects [3], beside modelling \( V_B \) via supersymmetric spurions, one must consider a model for the Kähler term which, in turn, should be invariant under the anomalous transformations. It is amusing to note that Kähler terms in the effective theory can be regarded as higher order in a derivative expansion with respect to the invariant scale of the theory and that \( \mathcal{M}/\Lambda_S \) corrections arise when also Kähler terms are present.

If \( \mathcal{M} \gg \Lambda_S \) the light degrees of freedoms are now the ordinary fields (quarks, gluons, etc.). These seems to be contained in the auxiliary fields of the effective Lagrangian description. Hence we expect \( \mathcal{F} \) to remain, while \( \phi \), which describes the colorless objects made of gluinos and squarks, to decouple. This can be obtained by assuming, as proposed in [3], the following equation of motion

\[
\frac{\partial V}{\partial \phi} = 0 \quad \text{for} \quad \mathcal{M} \gg \Lambda_S . \tag{0.4}
\]

This provides the relation

\[
\phi = \phi (\mathcal{F}, \mathcal{M}, \Lambda_S) , \tag{0.5}
\]

By substituting the previous expression in Eq. (0.2) we obtain the potential:

\[
V = V (\mathcal{F}, \mathcal{M}, \Lambda_S) . \tag{0.6}
\]

A smooth decoupling is achieved if, at least at one loop in the underlying theories, the scales \( \mathcal{M} \) and \( \Lambda_S \) combine in the unique scale \( \Lambda \) associated with the ordinary gauge theory.

Of course the knowledge of \( V_B \) is essential. We partially fix the breaking potential [3] by requiring the anomalies (trace anomaly as well as global anomalies) to match at the one loop level and assuming holomorphy for the breaking potential. The latter assumption is partially supported by the holomorphic behavior of the one–loop QCD coupling constant.

**FROM SUPER YANG MILLS TO YANG MILLS**

In this section we illustrate, in some detail, how the previous strategy works in the SUSY Yang Mills case [3]. The effective Lagrangian for Super Yang Mills was given [7] by Veneziano and Yankielowicz (VY) and is described by the Lagrangian

\[
\mathcal{L} = \frac{9}{\alpha} \int d^2 \bar{\theta} d^2 \theta \left( \mathcal{S} \mathcal{S}^\dagger \right)^{\frac{1}{3}} + \left\{ \int d^2 \theta \mathcal{S} \left[ \ln \left( \frac{S}{\Lambda_{SYM}^2} \right)^{N_c} - N_c \right] + \text{H.c.} \right\} , \tag{0.7}
\]
where $\Lambda_{SYM}$ is the super $SU(N_c)$ Yang Mills invariant scale and the chiral superfield $S$ stands for the composite object $S = \frac{g^2}{32\pi^2}W_\alpha^a W_a$. Here $g$ is the gauge coupling constant and $W_\alpha^a$ is the supersymmetric field strength. At the component level with $S(y) = \phi(y) + \sqrt{2}\theta \psi(y) + \theta^2 F(y)$ we have

$$
\phi \approx \lambda^2, \quad \psi \approx \sigma^{mn} \lambda_a F_{mn,a} \quad \text{and} \quad F \approx -\frac{1}{2} F_{mn}^a F^{mn,a} - \frac{i}{4} \epsilon_{mnrs} F_{mn}^a F_{rs}^a . \quad (0.8)
$$

$\lambda_a$ is the gluino field, $F_{mn,a}$ the gauge field strength.

We interpret the complex field $\phi$ as representing scalar and pseudoscalar gluino balls while $\psi$ is their fermionic partner. The auxiliary field $F$ is seen to contain scalar and pseudoscalar glueball type objects.

Equation (0.7) describes the vacuum of the theory and saturates the anomalous Ward identities at tree level. These anomalies arise in the axial current of the gluino field, the trace of the energy momentum tensor and in the special superconformal current. In supersymmetry these three anomalies belong to the same supermultiplet and hence are not independent. For example

$$
\theta_m^m = 3 N_c (F + F^*) = -\frac{3N_c g^2}{32\pi^2} F_{mn,a}^m F_{mn,a} , \quad (0.9)
$$

$$
\bar{\sigma}^m J^5_m = 2i N_c (F - F^*) = \frac{N_c g^2}{32\pi^2} \epsilon_{mnrs} F_{mn}^a F_{rs}^a . \quad (0.10)
$$

where $J_5^m = \bar{\lambda}_a \bar{\sigma}_m \lambda_a$ is the axial current. The effective Lagrangian yields \cite{7} the gluino condensation of the form $\langle \phi \rangle = -\frac{g^2}{32\pi^2} (\lambda^2) = \Lambda^3 e^{\frac{2\pi i k}{N_c}}$ where $k = 0, 1, 2, \cdots, (N_c - 1)$.

Masiero and Veneziano investigated the “soft” supersymmetry breaking regime \cite{8} by introducing a “gluino mass term” in the Lagrangian

$$
\mathcal{L} = \cdots + m \langle \phi + \phi^* \rangle , \quad (0.11)
$$

with the softness restriction $m \ll \Lambda_{SYM}$. The results of this model \cite{8} indicate that the theory is “trying” to approach the ordinary Yang Mills case.

It seems very desirable to extend this model to the case of large $m(\gg \Lambda_{SYM})$ in which the superparticles decouple from the theory and the theory gets reexpressed in terms of ordinary glueball fields. In Ref. \cite{6} we proposed a toy model which accomplishes these goals. Our approach is based on the general strategy presented in the previous paragraph which we specialize, here, for the super Yang Mills case:

i) We shall concentrate completely on the superpotential. This contains all the information on the anomaly structure and seems to be the least model dependent part of the effective Lagrangian.
ii) We will show that the generalisation of the supersymmetry breaking term Eq. (0.11) to

\[ V_B = -m^\delta \phi^\gamma + \text{H.c.} , \]  

(0.12)

where \( \delta = 4 - 3\gamma \) and \( \gamma = \frac{12}{11} \) automatically accomplishes the decoupling of the underlying gluino degree of freedom at the scale \( m \). The deviation of the exponent \( \gamma \) from unity is being thought of as an effective description of the evolution of the symmetry breaker Eq. (0.11) for large \( m \).

iii) Since the Yang Mills fields of interest are contained in \( F \) the heavy gluino ball field \( \phi \) is eliminated by its equation of motion \( \frac{\partial V}{\partial \phi} = 0 \) (see Eq. (0.4)).

The potential of our model

\[ V(F, \phi) = -F \ln \left( \frac{\phi}{A_{\text{SYM}}^3} \right) - m^\delta \phi^\gamma + \text{H.c.} , \]  

(0.13)

provides the equation of motion, \( \frac{\partial V}{\partial \phi} = 0 \) for eliminating \( \phi \): \( \phi^\gamma = -\frac{N_c F}{\gamma m^\delta} \). Our physical requirement is that the presence of the symmetry breaker Eq. (0.12) should convert the anomalous quantity \( \theta_m^a \) into the appropriate one for the ordinary Yang Mills theory. This is in the same spirit as the well known [9] criterion for decoupling a heavy flavor (at the one loop level) in QCD. We compute \( \theta_m^a \) at tree level obtaining

\[ \theta_m^a = \frac{4N_c}{\gamma} (F + F^*) = -\frac{4N_c}{\gamma} \left( \frac{g^2}{32\pi^2} F_{mn} F_{mn, a} \right) . \]  

(0.14)

Now the 1–loop anomaly in the underlying theory is given by

\[ \theta_m^a = -b \frac{g^2}{32\pi^2} F_{mn} F_{mn, a} , \]  

(0.15)

where \( b = 3N_c \) for supersymmetric Yang Mills and \( b = \frac{14}{3}N_c \) for ordinary Yang Mills. In order that Eq. (0.14) match Eq. (0.13) for ordinary Yang Mills we evidently require \( \gamma = \frac{12}{11} \) as mentioned above. With \( \phi \) eliminated in terms of \( F \) the potential becomes

\[ V(F) = -\frac{11N_c}{12} F \ln \left( \frac{-11N_c F}{12m^\frac{8}{3} A_{\text{SYM}}^3} \right) - 1 \]  + \text{H.c.} . \]  

(0.16)

We now check that this is consistent with a physical picture in which the gauge coupling constant evolves according to the super Yang Mills beta–function above scale \( m \) and according to the Yang Mills beta–function below scale \( m \). Since the coupling constant at scale \( \mu \) is given by
\[
\left( \frac{\Lambda_{YM}}{\mu} \right)^b = \exp \left( \frac{8\pi^2}{g^2(\mu)} \right), \text{ the matching at } \mu = m \text{ requires } \left( \frac{\Lambda_{YM}}{m} \right)^b = \left( \frac{\Lambda_{YM}}{m} \right)^{b_{YM}}, \text{ which yields } \\
\Lambda_{YM}^4 = m^2 \Lambda_{SYM}^{36}, \text{ in agreement with the combination appearing in Eq. (1.16). The Lagrangian in Eq. (1.16) manifestly depends only on quantities associated with the Yang Mills theory, the gluino degree of freedom having been consistently decoupled. Equation (1.16) thus seems to be a reasonable candidate for the potential term of a model describing the trace anomaly in Yang Mills theory.}
\]

The model is seen to contain a scalar glueball field Re\( F \) and a pseudoscalar glueball field Im\( F \). In order to relate our present results to previous investigations we also eliminate Im\( F \) by its equation of motion which yields Im\( F = 0 \). Substituting this back into Eq. (1.16) and using the notation \( H = \frac{11N_c}{3} \frac{g^2}{32\pi^2} F_{mn} F_{mn,a} \), leads to the potential function
\[
V(H) = \frac{H}{4} \ln \left( \frac{H}{8e\Lambda_{YM}^4} \right) .
\]
This may be considered as a zeroth order model for Yang Mills theory in which the only field present is a scalar glueball. \( V(H) \) has a minimum at \( \langle H \rangle = 8\Lambda_{YM}^4 \), at which point \( \langle V \rangle = -2\Lambda_{YM}^4 \). This corresponds to a magnetic–type condensation of the glueball field \( H \).

A number of phenomenological questions have been discussed using toy models based on Eq. (0.17) [12–14]. It can also be shown that \( m_\psi \to \infty \) in the case \( m \to \infty \); thus, as expected, \( \psi \) decouples [6].

FROM SUPER QCD TO QCD AND QUARK DECOUPLING

The more complicated case of adding matter fields with number of flavors \( N_f \) less than number of colors \( N_c \) with \( N_c \neq 2 \) has been analyzed in [8,15]. Here we summarize some of the relevant results. The needed “mesonic” composite superfield is the complex \( N_f \times N_f \) matrix \( T_{ij} = Q_i \tilde{Q}_j = t_{ij} + \sqrt{2}\theta \psi_{T_{ij}} + \theta^2 M_{ij}, \) where \( i \) and \( j \) are flavor indices. \( Q \) and \( \tilde{Q} \) are the quark anti-quark chiral superfields. It can be seen that the mesonic auxiliary field \( M \approx -\psi_Q \psi_{\tilde{Q}} \) contains the ordinary quark-antiquark meson field while \( t = \phi_Q \phi_{\tilde{Q}} \) describes the squark anti-squark composite operator. In Ref. [14] a straightforward generalization of the supersymmetric potential presented in Eq. (1.7) for \( N_f < N_c \) was derived. By a suitable decoupling of the squark as well as gluino degrees of freedom (see Ref. [3] for more details) the following potential for ordinary QCD can be deduced

\*

See discussion in Ref. [3]
\[ V (M) = - C (N_c, N_f) \left[ \frac{ \Lambda_{QCD}^{N_c-N_f}}{\det M} \right]^{\frac{12}{N_c-N_f}}, \quad (0.18) \]

where \( \Lambda_{QCD} \) is the invariant QCD scale and \( C (N_c, N_f) \) is a definite positive quantity which cannot be fixed by requiring the potential to satisfy the anomalies. We also eliminated the glue-ball degrees of freedom via their equation of motion (see Ref. [15]).

The potential for the meson variables in Eq. (0.18) is similar to the effective Affleck-Dine-Seiberg (ADS) [17] superpotential for massless super QCD theory with \( N_f < N_c \)

\[ W_{ADS} (T) = - (N_c - N_f) \left[ \frac{\Lambda_S^{3N_c-N_f}}{\det T} \right]^{\frac{1}{N_c-N_f}}, \quad (0.19) \]

where \( \Lambda_S \) is the invariant scale of SQCD.

An intriguing feature of the potential in Eq. (0.18) is that it presents a fall to the origin rather than a run-away vacuum associated with the ADS superpotential. The fall to the origin can be fixed by adding an anomalous free non holomorphic term in the manner outlined in [3], which in turn requires spontaneous chiral symmetry breaking. It is worth noticing [14] that one can directly derive Eq. (0.18) from QCD, if one assumes, besides the correct anomalous transformations, also one–loop holomorphicity in the QCD coupling constant [15,19].

As for the SUSY case [1] we can partially deduce the \( N_f \) and \( N_c \) dependence of \( C \) by defining a decoupling procedure for quarks. In Ref. [15] it has been shown that by adding the following, generalized quark mass operator,

\[ V_m = - m^\Delta M_{N_f}^{N_f} + H.c., \quad (0.20) \]

to the potential in Eq. (0.18), with \( \Delta = 4 - 3\Gamma \), is possible to obtain a complete decoupling when a flavored quark becomes massive. This procedures mimics the one employed by Sieberg for supersymmetric gauge theories. It is seen that, after decoupling, the QCD potential naturally converts to the one with one less flavor provided that \( \Gamma = 12/11 \) and the coefficient \( C \) has the following functional form

\[ C (N_c, N_f) = (N_c - N_f) D (N_c) \left[ \frac{1}{N_c-N_f} \right]. \quad (0.21) \]

\( D (N_c) \) is an unknown \( N_c \) dependent function. It is interesting to contrast the coefficient of the “holomorphic” part of the QCD potential Eq. (0.18) with Seiberg’s result [1] \( C (N_c, N_f) = N_c - N_f \) for the coefficient of the ADS superpotential (Eq. (0.19)). Clearly, in the SUSY case the analog of \( D (N_c) \) is just a constant. This feature arises in the SUSY case
because of the existence of squark fields which can break the gauge and flavor symmetries by the Higgs mechanism. The possibility of a non-constant $D(N_c)$ factor can thus be taken as an indication that there is no Higgs mechanism present in QCD-like theories.

Finally the knowledge about $C(N_c,N_f)$ can be used to suggest that the well known \[18\] large $N_c$ behaviour of the $\eta'$ (pseudoscalar singlet) meson mass should also include an $N_f$ dependence of the form:

$$M_{\eta'}^2 \propto \frac{N_f}{N_c - N_f} \Lambda^2 \quad (N_f < N_c). \quad (0.22)$$

It is amusing to observe that when $N_f$ is close to $N_c$ the resulting pole in Eq. (0.22) suggests a possible enhancement mechanism for the $\eta'$ mass. This would explain the unusually large value of this quantity in the realistic three flavor case.

In future we would like to understand the very important and yet elusive case $N_f = N_c$, where as argued in Ref. \[15\] we expect non holomorphic terms to be relevant, since the coefficient $C(N_c,N_f)$, in analogy with the SUSY case, of the anomalous potential vanishes for $N_f = N_c$.

In Ref. \[20\], we computed the one–loop effective action in the specific case of $N_f = N_c+1$ and $N_c = 2$ while keeping only the auxiliary fields on the external legs, and in the presence of supersymmetry breaking terms. This procedure amounts to integrate out order by order, in a loop expansion, the scalar fields. It was shown how a non-trivial kinetic term for the auxiliary field naturally emerges, reinforcing our assumption that the latter can be associated with a physical field once the supersymmetric particles decouple.

It is also very interesting to explore the $N_f > N_c$ case which might shed some light on the zero temperature chiral restoration and a possible relation with the conformal window \[21\].

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