One-loop Higgs mass finiteness in supersymmetric Kaluza-Klein theories

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Abstract

We analyze the one-loop ultraviolet sensitivity of the Higgs mass in a five-dimensional supersymmetric theory compactified on the orbifold $S^1/\mathbb{Z}_2$, with superpotential localized on a fixed-point brane. Four-dimensional supersymmetry is broken by Scherk-Schwarz boundary conditions. Kaluza-Klein interactions are regularized by means of a brane Gaussian distribution along the extra dimension with length $l_s \simeq \Lambda_s^{-1}$, where $\Lambda_s$ is the cutoff of the five-dimensional theory. The coupling of the $n$-mode, with mass $M^{(n)}$, acquires the $n$-dependent factor $\exp\left\{-\left(M^{(n)}/\Lambda_s\right)^2/2\right\}$, which makes it to decouple for $M^{(n)} \gg \Lambda_s$. The sensitivity of the Higgs mass on $\Lambda_s$ is strongly suppressed and quadratic divergences cancel by supersymmetry. The one-loop correction to the Higgs mass is finite and equals, for large values of $\Lambda_s$, the value obtained by the so-called KK-regularization.
One fundamental problem in particle physics is to understand the origin of electroweak symmetry breaking (EWSB) that leads to the pattern of vector boson and fermion masses in the Standard Model (SM). The only known perturbative mechanism (the Higgs mechanism) requires a fundamental scalar, the Higgs boson, which acquires a vacuum expectation value (VEV) and therefore breaks spontaneously the electroweak symmetry. In the SM, the radiative corrections to the squared Higgs mass are quadratically sensitive to the cutoff (which jeopardizes the consistency of the theory), while in its minimal supersymmetric extension (MMSM), the sensitivity is only logarithmic.

It is a common belief that in supersymmetric theories with one extra dimension radiative corrections to scalar masses are not sensitive (at least at one-loop) to the ultraviolet (UV) cutoff of the theory \[1-5\]. A similar result holds in (four-dimensional) theories at finite temperature, where the size of the extra dimension is given by the inverse temperature. Present calculations have been performed summing over all Kaluza-Klein (KK)-modes. This is known as KK-regularization and ignores that a five dimensional (5D) theory must be seen as an effective theory below the cutoff \(\Lambda_s\). This fact has recently been accounted for by imposing the sharp cutoff \(\Lambda_s\) on the momentum integration and truncating the summation in the KK-tower \[6\] to modes with \(M^{(n)} < \Lambda_s\). This leads to quadratic divergences because the sharp truncation of the KK-modes spoils the tower structure of the 5D theory.

In this letter we analyze a regularization where KK-modes are not truncated, but instead the brane is extended over the extra dimension with a finite length \(l_s \simeq 1/\Lambda_s\) by a Gaussian distribution. This regularization is suggested from string theories \[7\]. In particular we will study the UV sensitivity of the Higgs mass in the prototype model presented in Ref. \[5\], although the results are much more general and should also apply to other mass calculations and models. The model is based on a 5D \(N = 1\) theory whose massless modes constitute the usual four dimensional (4D) \(N = 1\) MSSM. Supersymmetry breaking is a bulk phenomenon induced by Scherk-Schwarz (SS) boundary conditions \[8,9\] and radiative breaking is triggered by the presence of a bulk top/stop hypermultiplet.

The setup of the model is as follows. The 5D space-time is compactified on \(\mathcal{M}_4 \times S^1/\mathbb{Z}_2\). The orbifold \(\mathcal{M}_4 \times S^1/\mathbb{Z}_2\) has two fixed points at \(y = 0, \ell\) where the two 3-branes are located (\(\ell \equiv \pi R\) is the length of the segment). There are two types of fields: those living in the bulk of the extra dimensions, similar to the untwisted states in the heterotic string language (\(U\)-states), and those living on the branes localized at the fixed points, similar to the heterotic string twisted states (\(T\)-states). We will assume that gauge fields are \(U\)-states organized in \(N = 2\) gauge multiplets \(\mathbb{V} = (V_\mu, \lambda_1; \Sigma + iV_5, \lambda_2)\). Even and odd components under \(\mathbb{Z}_2\) are separated by a semicolon, as \((\text{even}; \text{odd})\). Notice that only the even fields have zero modes. Matter fields can either be \(U\)- or \(T\)-states. In order to have Yukawa interactions to generate (after EWSB) fermion masses only two possibilities for the localized superpotential are allowed \[5\]: they are of the form \(UUU\) or \(UTT\). We will deal only with the latter \footnote{The case with \(UUU\) was recently studied in Refs. \[3,4\].}, as the former one is suppressed by a relative factor \((\ell \Lambda_s)^{-1}\). So we will consider the model where all \(SU(2)_L\) doublets, \(H_{1,2}, Q, L\), and \(L\), are localized in the boundary \(y = 0\), and the singlets \(\mathbb{U} = (U, \Psi_U; U', \Psi_U'), \mathbb{D} = (D, \Psi_D; D', \Psi_D')\) and \(\mathbb{E} = (E, \Psi_E; E', \Psi_E')\) are in the bulk. Both, even and odd components, form independent chiral multiplets.
Supersymmetry breaking is done by compactification using the SS-mechanism\textsuperscript{3}, with parameter $\omega$ and mass spectrum as follows (for details, see Ref. \textsuperscript{5}):

- KK-modes of gauge bosons and right-handed matter fermions have masses $M^{(n)} = n\pi/\ell$. There are massless states corresponding to $n = 0$.
- KK modes of gauginos and scalar partners of right-handed matter fermions get masses $M^{(n)} = (n + \omega)\pi/\ell$. Zero modes acquire soft masses $\omega/R$.
- Left-handed fermions and sfermions, and the Higgs sector remain massless at tree-level.

We will introduce a superpotential with Yukawa couplings that, along with the gauge couplings, will induce radiative masses to brane scalars. We will regularize the interactions with localized states by assuming that the brane has a finite extension of length $l_s \sim 1/\Lambda_s$ along the fifth dimension with a Gaussian distribution

$$f_G(y; l_s) = \frac{1}{\sqrt{2\pi l_s}} e^{-\frac{y^2}{2l_s^2}}. \quad (1)$$

The rapid fall-off of the Gaussian will produce an exponential suppression of the coupling of KK-modes with masses $M^{(n)} \gtrsim \Lambda_s$ which leads to an effective cutoff of these modes without spoiling the tower structure of the 5D theory. Notice that the so-called KK-regularization corresponds to the distribution (1) in the limit $l_s \to 0$, according to the limit $\delta(y) = \lim_{l_s \to 0} f_G(y; l_s)$.

We then use the superpotential

$$W = [h_U Q H_2 U + h_D Q H_1 D + h_E L H_1 E] f_G(y; l_s), \quad (2)$$

where we denote with the same symbols both the supermultiplets and their scalar components. The use of the Gaussian distribution in (2) will change the couplings between the Higgs and matter fields, with respect to the common formalism where a $\delta$-distribution is used (KK-regularization). We will calculate these coupling using the off-shell formalism\textsuperscript{10}.

The Lagrangian for the fields involving $h_U$-couplings is:

$$\mathcal{L}_Y = \int dy \left\{ |F_U|^2 + f_G(y; l_s) \left[ |F_Q|^2 + |F_{H_2}|^2 + \left( \frac{\partial W}{\partial U} (F_U - \partial_y U') + \frac{\partial W}{\partial Q} F_Q + \frac{\partial W}{\partial H_2} F_{H_2} + \frac{\partial^2 W}{\partial Q \partial U} \Psi_Q \Psi_U + h.c. \right) \right] \right\}, \quad (3)$$

where $F_U$ ($F_Q$) is the $F$-component of the $U$ ($Q$) superfield, $\Psi_U$ ($\Psi_Q$) its fermionic component, and $U'$ is the odd scalar of the $U$ hypermultiplet, which couples to the brane through its derivative with respect to the extra dimension.

\textsuperscript{3}We are using the $N = 2 SU(2)_R$ global symmetry (or, more specifically, its $U(1)_R$ subgroup preserved upon orbifold action) as the generator of supersymmetry breaking. Thus, only fields transforming under $SU(2)_R$ will get a mass.
Taking into account the non-trivial twist of the bosonic fields due to the SS boundary conditions, the Fourier expansion of the fields is given by

\[
F_U = \sum_n \cos \left[ \frac{(n+\omega)\pi y}{\ell} \right] F_U^{(n)}
\]

\[
U = \sum_n \cos \left[ \frac{(n+\omega)\pi y}{\ell} \right] U^{(n)}
\]

\[
U' = \sum_n \sin \left[ \frac{(n+\omega)\pi y}{\ell} \right] U^{(n)}
\]

\[
\Psi_U = \sum_n \cos \left[ \frac{n\pi y}{\ell} \right] \Psi_U^{(n)}.
\] (4)

Using the identity

\[
\int_{-\infty}^{\infty} dy \cos \left[ \frac{(n+\omega)\pi y}{\ell} \right] f_G(y; l_s) = e^{-\frac{(n+\omega)^2\pi^2}{2(\ell\Lambda_s)^2}}
\] (5)

and integrating out the auxiliary fields we end up with the following Lagrangian for Yukawa interactions:

\[
\mathcal{L}_Y = \sum_{n=-\infty}^{\infty} \left[ h_t^2 e^{-\frac{(n+\omega)^2\pi^2}{2(\ell\Lambda_s)^2}} \left\{ |H_2|^2 |U^{(n)}|^2 + |H_2|^2 |Q|^2 + |Q|^2 |U^{(n)}|^2 \right. \right.
\]

\[
\left. - \left( \frac{(n+\omega)\pi}{\ell} \right) U^{(n)} H_2 Q + h.c. \right] \right\} + h_t e^{-\frac{(n+\omega)^2\pi^2}{2(\ell\Lambda_s)^2}} H_2 \Psi \Psi_U^{(n)} + h.c. \right].
\] (6)

Notice that the Lagrangian (6) can be interpreted as one where the couplings of heavy KK-modes \(U^{(n)}\) are model dependent and suppressed as:

\[
h_t^{(n)} = h_t \exp \left\{ -\frac{1}{2} \left( \frac{M^{(n)}}{\Lambda_s} \right)^2 \right\}.
\] (7)

In this way the decoupling of heavy KK-modes occurs without spoiling the tower structure of the theory.

We can now calculate the contribution at one-loop to the Higgs mass as [2]:

\[
m_H^2 = \Delta m^2(\omega) - \Delta m^2(0),
\] (8)

where

\[
\Delta m^2(\omega) = 2h_t^2 N_c \ell^2 \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4 \ell^2} e^{-\frac{(n+\omega)^2\pi^2}{2(\ell\Lambda_s)^2}}
\] (9)

\(N_c\) being the number of colours.

\[\text{Notice that, strictly speaking, the integral in (9) should be performed over the orbifold length. However since the gaussian distribution decays exponentially fast we are allowed to extend the interval of integration over the whole real axis. This provides a good enough approximation and is not changing our finite final result.}\]
Using the Schwinger representation for the propagators and performing the $p$ integration over $0 \leq |p| \leq \Lambda_s$ one gets for $\Delta m^2(\omega)$ the expression

$$\Delta m^2(\omega) = \frac{h_t^2 N_c}{8\pi^2 \ell^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} ds \ e^{-\frac{(n+\omega)^2\pi^2 + 1}{(\ell \Lambda_s)^2}} \left[ 1 - e^{-s\ell^2\Lambda_s^2 (1 + s\ell^2\Lambda_s^2)} \right]. \quad (10)$$

Notice that the apparent divergence of the integrand at $s \to 0$ (UV limit) cancels because of the presence of the cutoff $\Lambda_s$.

The integral over $s$ can then be performed and yields

$$\Delta m^2(\omega) = \frac{h_t^2 N_c}{8\pi^2 \ell^2} \sum_{n=-\infty}^{\infty} \left[ (\ell \Lambda_s)^2 - (n + \omega)^2 \pi^2 \log \frac{(n + \omega)^2\pi^2 + (\ell \Lambda_s)^2}{(n + \omega)^2\pi^2} \right] e^{-\frac{(n + \omega)^2\pi^2}{(\ell \Lambda_s)^2}}. \quad (11)$$

This contribution to the Higgs mass contains a quadratically divergent term. However, this quadratic divergence is canceled by supersymmetry. In fact, using the Poisson re-summation formula

$$\sum_{n=-\infty}^{\infty} g(n + \omega) = \sum_{n=-\infty}^{\infty} e^{-2\pi i n \omega} \int_{-\infty}^{\infty} dz \ e^{-2\pi i n z} g(z) \quad (12)$$

which, for the case of the Gaussian simply gives

$$\sum_{n=-\infty}^{\infty} e^{-\frac{\Lambda_s^2}{\ell^2}(n+\omega)^2} = \frac{\ell \Lambda_s}{\sqrt{\pi}} \sum_{n=-\infty}^{\infty} e^{-2\pi i n \omega} e^{-n^2(\ell \Lambda_s)^2}, \quad (13)$$

we find for the first term of $(11)$

$$\frac{h_t^2 N_c}{8\pi^2 \ell^2} (\ell \Lambda_s)^3 \sum_{n=-\infty}^{\infty} e^{-2\pi i n \omega} e^{-(\ell \Lambda_s)^2 n^2}. \quad (14)$$

The corresponding contribution to $m_{H_2}^2$ can then be written (after including supersymmetric terms) as

$$\frac{h_t^2 N_c}{8\pi^2 \ell^2} (\ell \Lambda_s)^3 \sum_{n=-\infty}^{\infty} \left( e^{-2\pi i n \omega} - 1 \right) e^{-(\ell \Lambda_s)^2 n^2}, \quad (15)$$

whose $n = 0$ term cancels from supersymmetry and whose leading contribution in the limit $\ell \Lambda_s \to \infty$ is provided by the $n = 1$ mode. This one behaves like $(\ell \Lambda_s)^3 e^{-(\ell \Lambda_s)^2}$ and clearly vanishes when $\ell \Lambda_s \to \infty$.

If we neglect the contribution $(15)$ we can write $m_{H_2}^2$ as:

$$m_{H_2}^2 = -\frac{h_t^2 N_c}{8\pi^2 \ell^2} \sum_{n=-\infty}^{\infty} \left\{ \pi^2 (n + \omega)^2 \log \frac{(n + \omega)^2\pi^2 + (\ell \Lambda_s)^2}{(n + \omega)^2\pi^2} e^{-\frac{(n + \omega)^2\pi^2}{(\ell \Lambda_s)^2}} 
- n^2 \log \frac{n^2\pi^2 + (\ell \Lambda_s)^2}{n^2\pi^2} e^{-(\ell \Lambda_s)^2 n^2} \right\}. \quad (16)$$
Using again the Poisson resummation formula we can cast this expression into the form
\[ m^2_{H_2} = \frac{h_t^2 N_c}{8\pi^3 \ell^2} \langle \ell \Lambda_s \rangle^3 \sum_{n=-\infty}^{\infty} \left( e^{-2\pi i n\omega} - 1 \right) \tilde{g}(2n\ell\Lambda_s), \] (17)

where the function \( \tilde{g}(p) \) is the Fourier transform of
\[ g(y) = y^2 e^{-y^2} \left[ \log(1 + y^2) - \log(y^2) \right]. \] (18)

The dependence of \( \tilde{g}(p) \) for large Fourier modes \( p \) is obtained from the behaviour of the function \( g(y) \) at the non-analytic point \( y = 0 \). We find that the leading term of \( \tilde{g}(p) \), in the limit \( |p| \to \infty \), is
\[ -\frac{4\pi}{|p|} + O\left(\frac{1}{|p|}\right). \]

Thus, in the limit \( \ell\Lambda_s \to \infty \), \( m^2_{H_2} \) tends to
\[ m^2_{H_2}(\infty) = \frac{h_t^2 N_c}{16\pi^2 \ell^2} \left[ Li_3(e^{-2i\pi\omega}) + Li_3(e^{2i\pi\omega}) - 2\zeta(3) \right] + O\left(\frac{1}{\ell\Lambda_s}\right), \] (19)

which agrees with the expression obtained using the KK-regularization \footnote{Again the \( n = 0 \) term is canceled by supersymmetry since \( \tilde{g}(0) \) is finite.}. Moreover, the convergence of \( m^2_{H_2} \) to \( m^2_{H_2}(\infty) \) is very fast, as can be seen from Fig. 1, where \( \ell^2 [m^2_{H_2}(\infty) - m^2_{H_2}] \) is plotted versus \( \ell\Lambda_s \) for different values of \( \omega \).

Figure 1: Plot of the difference \( [m^2_{H_2}(\infty) - m^2_{H_2}] / 4h_t^2 N_c \) in units of \( \ell \) as a function of \( \ell\Lambda_s \), for \( \omega = 1/2 \) (solid) and \( \omega = 1/4 \) (dashed).

Even if the Gaussian distribution \footnote{Even if the Gaussian distribution (1) we have assumed for the localization of the brane along the extra dimension is a physical one, and strongly motivated by string theories, we would like to comment about the generality of our results with respect to the distribution choice. Since the aim of a general distribution \( f(y; \ell_s) \) is to regularize the \( \delta \)-function, a clear requirement the function \( f(y; \ell_s) \) must satisfy is that \( \lim_{l_s \to 0} f(y; \ell_s) = \delta(y) \). A simple example satisfying that requirement is provided by the distribution
\[ f(y; \ell_s) = \frac{1}{\arctan(1/\ell_s)} \frac{\ell_s}{y^2 + \ell_s^2}. \] (20)

Its Fourier transform over the orbifold length, \( \tilde{f}(p; \ell_s) \), defines the \( n \)-dependent couplings in (6) as
\[ h^{(n)}_{ti} = h_t \tilde{f}(\pi(n + \omega)/\ell; \ell_s) \] (21)

Again the \( n = 0 \) term is canceled by supersymmetry since \( \tilde{g}(0) \) is finite.} we have assumed for the localization of the brane along the extra dimension is a physical one, and strongly motivated by string theories, we would like to comment about the generality of our results with respect to the distribution choice. Since the aim of a general distribution \( f(y; \ell_s) \) is to regularize the \( \delta \)-function, a clear requirement the function \( f(y; \ell_s) \) must satisfy is that \( \lim_{l_s \to 0} f(y; \ell_s) = \delta(y) \). A simple example satisfying that requirement is provided by the distribution
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Its Fourier transform over the orbifold length, \( \tilde{f}(p; \ell_s) \), defines the \( n \)-dependent couplings in (6) as
\[ h^{(n)}_{ti} = h_t \tilde{f}(\pi(n + \omega)/\ell; \ell_s) \] (21)
which again provides an exponential decoupling of heavy modes. In fact, for $l_s \ll \ell$ the function $\tilde{f}$ behaves as

$$
\tilde{f}(p; l_s) \simeq e^{-|p| l_s}.
$$

(22)

while, unlike $e^{-|p| l_s}$, it is analytic at $p = 0$.

Carrying out similar steps as in the Gaussian case leads to Eq. (19) with different subleading, $O(1/\ell \Lambda_s)$, corrections. We have found that analyticity of $\tilde{f}$ is an essential ingredient for the UV insensitivity of the Higgs mass in a supersymmetric theory. Moreover, any distribution with well defined moments, and satisfying the property that $\lim_{l_s \to 0} f(y; l_s) = \delta(y)$, should lead to a suppressed UV sensitivity for the one-loop Higgs mass. We have checked this point by explicit calculations.

To conclude, we have proven that in the case of a Gaussian distribution the sensitivity on the cutoff of the Higgs mass is suppressed at one loop and no quadratic divergences appear in a supersymmetric theory. It is therefore fully justified to exchange the (infinite) summation and the (infinite) integral as done in the KK-regularization. Notice also that a similar calculation can be done for the gauge interactions, and its contributions to the Higgs mass, leading to a mode dependent gauge coupling $g^{(n)} = g \exp\{-\frac{1}{2} \left( \frac{M^{(n)}}{\Lambda_s} \right)^2 \}$ and a finite correction. Moreover radiative corrections to the mass of other massless scalars (squarks, sleptons) localized on the brane also lead to finite results. We have also shown that the same conclusions hold for any well-defined distribution of the brane along the extra dimension.

Let us finally notice that this result is also supported by explicit string calculations [11], where the squared Higgs mass is obtained to be $\sim M_s^2$ (the string scale, $M_s$, playing the role of the UV cutoff $\Lambda_s$) in the region $\ell M_s \lesssim 1$ (stringy region), and $\sim 1/\ell^2$ in the region $\ell M_s \gg 1$ (field theory limit) and given by the expression (19), in agreement with our present results.

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