MARFE FORMATION IN DIVERTED TOKAMAKS

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ABSTRACT. The equilibrium and stability of MARFEs are considered. It is demonstrated that important aspects of the two dimensional problem can be illustrated by a one dimensional treatment. A stability analysis indicates that MARFE formation requires low temperature, high impurity content and low cross-field transport. Additionally, MARFEs tend to form on flux surfaces that contain an X point. It is observed that the transition to a MARFE-like solution can result from a fluctuation induced jump from a neighbouring constant temperature solution.

1. INTRODUCTION

MARFEs are radiation condensation instabilities that appear in tokamaks as the density limit is approached [1-5]. They are manifested as dense, cool regions of radiating plasma that form a poloidally localized and toroidally symmetric ring usually located near an outer flux surface on the high field side of the torus. Recent observations indicate that MARFEs tend to locate themselves near to the X point [6, 7] in a diverted tokamak as the edge is cooled and densified.

A number of numerical and analytical investigations of this phenomenon have been performed [8-12]. The tokamak thermal equilibrium equations are two dimensional (2-D), i.e. in the perpendicular and along the field line co-ordinates (the perpendicular co-ordinate is predominantly the flux co-ordinate). McCarthy and Drake [10] replaced the cross-field heat flux by a constant heat source and solved the time dependent non-linear one dimensional (1-D) along-the-field-line equation. They demonstrated a transition from an unstable isothermal solution to a MARFE-like solution. The 2-D equations have also been studied [13, 14].

In this work we make a more correct (albeit still simplistic) approximation for the cross-field heat flux. We observe that the important behaviour is along the field line and that we can simplify the problem mathematically by differencing the cross-field diffusion term. This reduces the 2-D partial differential equation to a set of coupled non-linear 1-D ordinary differential equations. We can then solve the resulting non-linear along the field line equations for equilibrium. We consider first the properties of a single ordinary differential equation which model, respectively, the heat flow in the separatrix and scrape-off layer flux surfaces.

Examination of a single along-the-field-line equation reveals several stable solutions which may be divided into two classes: (a) those that exhibit a constant temperature along the field line and (b) those in which temperature varies along the field line, i.e. exhibit a MARFE-like character. The constant temperature equilibria will make a transition into a MARFE if they become linearly unstable. Additionally, temperature fluctuations can cause a stable constant temperature equilibrium to ‘jump’ into the MARFE-like solution when these two equilibria approach each other.

Formation of a stable MARFE-like solution can occur when the temperature and the cross-field transport are sufficiently low and the impurity content is sufficiently high. The model is consistent with the tendency of an X point to facilitate MARFE formation.

2. BASIC EQUATIONS

The theory of MARFEs is based on the fluid equations [15] for density $n$, temperature $T$ and velocity $u_i$. Assuming that $T_s = T_x = T$ we will examine the equilibria of the following equations:

$$\frac{\partial}{\partial s} \left( \kappa_{||} \frac{\partial T}{\partial s} \right) + \frac{\partial}{\partial x} \left( \kappa_x \frac{\partial T}{\partial x} \right) = n_e n_c L(T)$$

$$n_e T = p(x)$$

with $s$ and $x$ the respective along the field and cross-field directions, $\kappa_{||}$ and $\kappa_x$ the respective parallel and perpendicular thermal conductivities, and $L(T)$ the radiation function. We imagine that the field line length from
outside to inside the torus is \( L \sim q_s R \) with \( q_s \), the safety factor near the plasma edge) and impose the symmetry boundary condition

\[
\frac{\partial T(x, 0)}{\partial s} = \frac{\partial T(x, L)}{\partial s} = 0
\]  

(2)

If we consider cylindrical co-ordinates we can use \( r \) as the radial (flux) co-ordinate, \( \theta \) as the poloidal co-ordinate and \( \Phi \) as the toroidal co-ordinate. In this approximation \( B_\theta / B_\Phi = \tan \alpha \) with \( \alpha \) the field line pitch angle and \( L = \pi a \alpha / \sin \alpha \), with \( \kappa \) the flux tube ellipticity and \( \pi a \alpha \) approximately half of the flux tube circumference at the MARFE location. In the poloidal plane Eq. (1a) becomes

\[
\frac{1}{r^2} \frac{\partial}{\partial \theta} \left( r^2 (\kappa_1 \sin^2 \alpha + \kappa_\perp \cos^2 \alpha) \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial r} \left( \kappa_\perp \frac{\partial T}{\partial r} \right) = n_e n_c L(T)
\]

(3)

where \( \kappa_\perp \) is the perpendicular thermal conductivity within the flux tube. Transport perpendicular to the field occurs both across \( (\kappa_\perp) \) and within the flux tubes \( (\kappa_\perp) \). Generally \( \kappa_1 \sin^2 \alpha \gg \kappa_\perp \cos^2 \alpha \) and we can ignore the \( \kappa_\perp \) term. In the vicinity of an X point, however, \( \sin \alpha \sim 0 \) and the \( \kappa_\perp \) term cannot be ignored.

### 3.1. Equilibrium

Consider first radial equilibrium. If initially the equilibrium plasma is MARFE free \( (\partial T / \partial \theta \sim 0) \) there are two classes of radial equilibria: (a) those in which the edge is sufficiently hot that low \( Z \) impurities (carbon, oxygen, etc.) are fully stripped and do not radiate or (b) those in which low \( Z \) impurity radiation dominates the edge power balance and prevents the edge temperature from rising above the level required to strip these impurities. Analysis of Eqs (1a, b) without the parallel term in fact yields these two solutions [8, 11, 12]. In general, a second equation for \( p(x) \) must be solved consistently with Eq. (1a).

A MARFE may occur near the plasma edge in a layer of width \( 2 \delta \) bounded by flux tubes with mean radii respectively of \( r_a \) and \( r_b \), i.e. \( r_b - r_a \sim 2 \delta \). The flux tubes at \( r_a \) and \( r_b \) are assumed to be at the respective temperatures \( T_a \) and \( T_b \). For \( \delta \ll r_a, r_b \) we can simplify Eqs (1a, b) by differencing the cross-field conduction term as follows:

\[
\frac{\delta^2 T}{\delta x^2} = \frac{T_a + T_b - 2 T}{\delta^2} \sim \frac{T_a - 2 T}{\delta^2}
\]

(4)

where for simplicity it is assumed that \( T_b \ll T_a \). We will first consider a single ordinary differential equation and afterwards consider two coupled ordinary differential equations. Equation (4) indicates that if the plasma within the layer of interest (in which a MARFE can occur) heats up, the cross-field power source into the layer decreases and vice versa. Furthermore, in a clean plasma without significant radiation cooling, the equilibrium is initially described by the non-radiating edge solution which requires \( \kappa_\perp \delta^2 T / \delta x^2 \sim 0 \) and Eq. (4) permits this result (when \( T \sim T_b / 2 \)). We will therefore initially consider the following equation in our studies:

\[
(\kappa_1 T_a)_a = -\kappa_\perp \frac{T_a - 2 T}{\delta^2} + n_e n_c L(T)
\]

(5a)

\[
T_a(0) = T_a(L) = 0
\]

(5b)

and \( n_e T = p_0 = \text{constant} \). The subscripts indicate partial derivatives. We note that \( \kappa_1 \) is generally a function of temperature (classically, \( \kappa_1 = \kappa_1 T^{5/2} \)). Following Ref. [10] we will approximate the radiation function by

\[
L(T) = L_0 \frac{T^3}{T^3 + T_{\text{rad}}^3}
\]

(6)

\( T_{\text{rad}} \) represents the temperature at the peak of the radiation curve, i.e. for carbon in local thermodynamic equilibrium \( T_{\text{rad}} = 8 \text{ eV} \). The details of this approximation do not affect the results and a more accurate form of the radiation function can easily be included. We will take

\[
L_0 = 1.2 \times 10^{-31} \text{ W} \cdot \text{m}^3
\]

\[
k_1 = 1.29 \times 10^{22} \text{ m}^{-1} \cdot \text{s}^{-1} \cdot \text{eV}^{-5/2}
\]

and

\[
\kappa_\perp = 3 n_e \text{ m}^{-1} \cdot \text{s}^{-1}
\]

### 3.1.1. Solution by method of pseudo-potential

Equation (5a) is a non-linear ordinary differential equation. It can easily be solved numerically and possesses multiple solutions. Valuable insight into the number and character of the solutions can be gained through the method of pseudo-potentials.

To apply this method we define the pseudo-potential by

\[
\phi(T) = -\int T \left( n_e n_c L(T) - \frac{p_0 X_I}{\delta^2} (T_a / T_b - 2) \right) dT^
\]

(7)

where \( n_e X_I = \kappa_1 \). To include a temperature dependence, \( \kappa_1 \propto T^{5/2} \), we could perform a change variables \( \tilde{T} = T^{7/2} \) to eliminate the temperature dependence from the \( \kappa_1 \) term. This change of variables would distort the pseudo-potential without changing the relative positions of the maxima which determines the character of the solutions.
For the present we ignore the temperature dependence of \( \kappa_1 \) and obtain from Eq. (5)
\[
\kappa_1 \frac{\partial^2 T}{\partial s^2} = - \frac{d\phi}{dT}
\]  
(8)

If we consider \( s \) as the pseudo-time co-ordinate and \( T \) as the pseudo-spatial co-ordinate, Eq. (8) describes the motion of a ball of mass \( \kappa_1 \) on a potential hill described by \( \phi(T) \). (The speed of the ball is \( \partial T/\partial s = T_s \).) The ball will begin at some position \( T_{\text{init}} \) with zero speed and come back to a stop at time \( L \) at position \( T_{\text{final}} \). Each extremum of the pseudo-potential, \( \phi \), gives rise to a constant temperature (along-the-field-line) solution. We will see that a result of the unusual boundary conditions is that the constant temperature equilibrium associated with locating the ball at a maximum is stable and the equilibrium associated with a potential minimum is unstable.

Figure 1 displays the pseudo-potentials that correspond to low and moderate radiation levels. The three extrema indicate: (i) a constant (along-the-field-line) solution at a warm edge temperature \( T = T_3 \sim T_0/2 \); (ii) a radiation burnout solution \( T = T_1 \leq T_{\text{rad}} \) which corresponds to a detached plasma state; (iii) a solution having \( T = T_2 \gtrsim T_{\text{rad}} \).

Consider the higher radiation pseudo-potential curve displayed in Fig. 1. In addition to the three constant temperature solutions just pointed out there will be a second class of solutions, the MARFE-like solutions such as those shown in Fig. 2. For these solutions the pseudo-position varies with pseudo-time, or in other words \( T \) varies with \( s \). An acceptable solution requires that a ball set at position \( T = T_{\text{init}} \) to the left of and close to \( T_3 \), will roll down the potential hill and stop in pseudo-time \( t = L \) at \( T = T_{\text{final}} \). This solution is shown in Fig. 2. Higher order solutions will also exist for a trajectory that will execute a finite number of passes through the position \( T_2 \) and have starting and ending positions \( T_{\text{final}} < T < T_{\text{init}} \).

If the plasma is initially at the stable warm edge equilibrium solution \( T = T_3 \) the system will make a transition to a new equilibrium if this solution is lost. Referring to Fig. 1, the warm edge temperature solution \( T = T_3 \) will evolve into the detached solution \( T = T_1 \) if the maximum at \( T = T_3 \) disappears owing to increased radiation or reduced edge temperature. A second possibility is a jump from the \( T = T_3 \) equilibrium to a neighbouring MARFE-like equilibrium brought about by edge temperature fluctuations [16-19]. The closest neighbouring equilibrium will always be the lowest order MARFE.

Let us examine this possibility more closely. The maximum temperature point for the MARFE-like solution will approach \( T_3 \) as the field line length \( (L \sim q_s R) \) increases. The constant temperature and MARFE solutions will also approach each other as the warm temperature boundary condition \( T = T_0 \sim 2T_3 \) decreases, either owing to a decrease of heating power into the core of the discharge or an increase of edge ion density.

In the vicinity of the X point \( L \rightarrow \infty \) and the high temperature end of the MARFE solution approaches the constant temperature solution, \( T = T_3 \), provided that \( \phi(T_1) \gtrsim \phi(T_3) \). Therefore a transition can occur relatively easily at an X point. (Near the X point, however, the cross-field transport within the flux tube will dominate.
the along-the-field-line transport and therefore the thermal transport within the flux tube will not disappear as the X point is approached.)

3.2. Stability of MARFE equations

Returning to Eq. (8) we now examine the stability of the MARFE-like solutions for \( T(s) \). If we perturb Eq. (8) \( T \rightarrow T + \delta T \) we obtain for \( \delta T \)

\[
\gamma \delta T = \frac{\partial}{\partial s} \left( \kappa_1 \frac{\partial \delta T}{\partial s} \right) + \frac{\partial^2 \phi}{\partial T^2} \delta T
\]

(9)

with \( \gamma \) the growth rate. The boundary conditions on \( \delta T \) are \( \delta T_s(0) = \delta T_s(L) = 0 \) with \( \delta T_s = \delta T/\delta s \). We can obtain a quadratic form by multiplying by \( \delta T \) and integrating along the length of the field line,

\[
\gamma \int_0^L \delta T^2 ds = - \int_0^L \kappa_1 \delta T_s^2 ds + \int_0^L \phi_{TT}(T) \delta T^2 ds
\]

(10)

Equation (10) can be shown to be maximizing and therefore if a trial function is found that yields \( \gamma > 0 \) this provides a sufficient condition for instability. Notice that the term containing \( \kappa_1 \) is always stabilizing whereas the term containing \( \phi_{TT} \) can either be stabilizing or destabilizing. Recalling the form of the pseudo-potential (Fig. 1) the potential has negative \( \phi_{TT} \) (stabilizing) regions in the vicinity of \( T_1 \) and \( T_2 \) and positive \( \phi_{TT} \) in the vicinity of \( T_3 \).

Consider first the constant temperature solutions that occur at the extrema of the pseudo-potential. Since \( \phi_{TT}(T) \) is constant for these solutions it can be removed from the integral in Eq. (10). At the maxima \( \phi_{TT} < 0 \), and therefore these solutions (\( T = T_1 \) or \( T_3 \) in Fig. 1) are stable. Consider next a potential minimum point (\( T = T_2 \) in Fig. 1). If we take as a trial function \( \delta T(s) = \) const the stabilizing \( \kappa_1 \) term is eliminated and since \( \phi_{TT} > 0 \) we obtain an instability. Therefore we find the counterintuitive result that constant temperature solutions at the maxima of the pseudo-potential are stable while those at the minima are unstable.

Consider next MARFE-like equilibria. We again consider a trial function \( \delta T(s) = \) const to eliminate the stabilizing \( \kappa_1 \) term. We then obtain a sufficient condition for instability,

\[
\int_0^L \phi_{TT} ds > 0
\]

(11a)

while a necessary condition for stability will be

\[
\int_0^L \phi_{TT} ds < 0
\]

(11b)

Referring to the potential in Fig. 1 we observe, for example, that a MARFE-like solution that sits entirely in the region where \( \phi_{TT} > 0 \) would be unstable. The spatial dependence of \( \phi_{TT}(T) \), however, depends on the equilibrium solution for \( T(s) \). Furthermore, since \( -\phi_{TT} \) is the temperature derivative of the cross-field and radiation function (RHS of Eq. (5a) we see that stability requires that the field line average of the slope of the cross-field and radiation function be positive.

From the pseudo-potential picture in which \( T \) corresponds to pseudo-position and \( s \) to pseudo-time, a ball started off near a potential maximum will spend a lot of time in the \( \phi_{TT} < 0 \) region. The edge temperature, \( T_e \) (\( T_1 \sim T_2/2 \)), must be sufficiently low for the ball to stop at time \( t = L \). Thus a stable MARFE-like solution requires that the high temperature point be close to the maximum at \( T = T_3 \) so that a large region of the solution lies in the stable \( \phi_{TT} \) region near \( T_3 \).

A solution with \( T \leq T_3 \) also requires that the pseudo-potential maximum satisfy the condition \( \phi(T_1) > \phi(T_3) \). This imposes a necessary condition on the radiation term, \( \int_{T_1}^{T_3} n_e n_c L(T) dT > \int_{T_1}^{T_3} \frac{2n_e c L}{\delta^2} (T_3 - T) dT \)

(12)

Thus a stable MARFE-like equilibrium with \( T \leq T_3 \) can occur as the impurity content, \( n_c \), is increased or as the edge temperature is reduced. The MARFE formation is also inhibited by an increase in \( \chi_L \).

Further insight into the stability of the equilibria can be gained if we notice that \( \phi_{TT} \) has a maximum in the vicinity of \( T_2 \). We can then obtain a second necessary condition for stability,

\[
\int_0^L \kappa_1 \delta T_s^2 ds > \phi_{TT}^x \int_0^L \delta T^2 ds
\]

(13)

If we consider, for example, a trial function that is localized in the destabilizing region (in which \( \phi_{TT} > 0 \)) of width \( \Delta \) \( \delta T_s \sim \delta T/\Delta \) Eq. (13) approximately gives \( \kappa_1 > \phi_{TT}^x \Delta^2 \).

Noting that \( \Delta < L/2 \), we conclude that formation of a stable MARFE requires a sufficiently large parallel thermal conductivity, \( \kappa_1 \).

Finally we note that an 'inverse MARFE' equilibrium can be a stable solution in the low radiation limit \( \phi(T_1) < \phi(T_3) \). Referring to Fig. 1; for this solution we would start the ball off close to and to the right of \( T = T_1 \). The resulting solution for temperature would be close to \( T_1 \) along a large part of the field line length and only rise rapidly close at the other end of the field line 

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(at the outside of the torus) to form a small hot spot. Such equilibria would have a radiating region of much larger extent than an ordinary MARFE. Observations of these structures have not been reported in the literature.

Consider the startup sequence of a tokamak for which the plasma edge is initially warm, \( T = T_3 \). If the edge is then cooled (owing to the injection of gas or impurities) there will be a point when a stable MARFE-like solution with \( T < T_3 \) will become possible and edge temperature fluctuations can cause the edge to ‘jump’ into the MARFE equilibrium. Sufficient cooling can eliminate the pseudo-potential maxima at \( T = T_3 \) and leave only the radiation collapse solution, \( T = T_1 \), which would be manifested as a detached plasma.

3.3. Two layer model

To consider the interaction between the flux layer near the X point and the adjoining scrape-off layer we consider two coupled equations for \( T_a \) and \( T_b \) which model these two respective flux layers:

\[
\frac{\partial}{\partial s} \left( \kappa_T \frac{\partial T_a}{\partial s} \right) = -\kappa_T \frac{\partial T_a + T_b - 2T_a}{\delta^2} + n_e n_C L(T_a) \tag{14a}
\]

\[
\frac{\partial}{\partial s} \left( \kappa_T \frac{\partial T_b}{\partial s} \right) = -\kappa_T \frac{\partial T_a + T_{edge} - 2T_b}{\delta^2} + n_e n_C L(T_b) + P_{recomb} + n_p T_b \frac{d\nu_l}{ds} \tag{14b}
\]

with boundary conditions

\[
\delta T_a / \delta s (s = 0) = \delta T_b / \delta s (s = 0) = \delta T_a / \delta s (s = L) = 0
\]

In the X point layer (Eq. (14a)) \( \xi \) takes account of the relatively long field line length in the vicinity of the X point (which effectively reduces parallel thermal conduction). The scrape-off layer model equation (14b) contains a term for recombination and a flow term. These equations can easily be solved.

From the single layer problem discussed above we can assume that in each layer individually a radiation collapse is facilitated by a high impurity content, low temperature and low \( \lambda_L \). In the layer containing the X point it is also facilitated by a reduced thermal conductivity along the field line. The two layers interact and a thermal collapse in the inner layer will reduce the cross-field heat flow into the second layer. Thus the thermal collapse will tend to spread towards the edge. If the outer layers do not contain much pressure the observed radiation (which is proportional to \( p^2 \)) will appear to decrease in the low pressure layer. A typical solution is shown in Fig. 3.

Recombination of the hydrogenic species becomes important when the temperature falls below 1 eV. It can be included as an additional term in the radiation function.

3.3.1. Stability of 2-D equilibrium

We can rewrite Eq. (1a) as

\[
(\kappa_T T_s) + (\kappa_L (x) T_s)_x = f_{imp} n_2(x) L(T) = p^2 R(T) \tag{15}
\]

with \( f_{imp} \) the impurity fraction and \( n_2(x) = p(x)/T \), i.e. pressure is constant along a field line and varies across flux surfaces. Equation (15) will yield solutions similar to those discussed in the previous section. The stability of the solutions can be obtained from the perturbed form of Eq. (15),

\[
\gamma \delta T = (\kappa_T \delta T)_s + (\kappa_L (x) \delta T)_x - p^2 (x) R \delta T \tag{16}
\]

with boundary conditions \( \delta T(a) = \delta T(b) = 0 \) at the radial boundaries of the flux layers and \( \delta T_s(L) = \delta T_s(0) = 0 \) at either end of the field lines. Integrating Eq. (16) in \( s \) and \( x \) we obtain

\[
\int \int \gamma \delta T^2 \, ds \, dx = - \int \int ds \, dx (\kappa_T \delta T_s^2 + \kappa_L \delta T_x^2 + p^2 R \delta T^2) \tag{17}
\]
As is well known [20, 21] the negative slope region of the radiation function can drive instability while thermal conductivity is stabilizing. \( R_T(T) \) is a function of space through the equilibrium relation. Equation (17) is consistent with a result of the simpler 1-D stability criterion (Eq. (11)): if we choose a perturbation which zeros the parallel transport, stability is determined by a competition between cross-field transport and radiation from the unstable region of the radiation function.

An interesting special case occurs when pressure is assumed to be constant across the flux tubes, i.e. \( p(x) = p_0 \) and \( \kappa_x \) is independent of \( x \). (We are already assuming that pressure is constant along field lines.) Taking this limit and assuming that \( T_x \) does not change sign \( (T_x < 0) \) we can take the \( x \) derivative of Eq. (15) to obtain \( R_T \) and substitute \( R_T \) back into Eq. (17) to obtain

\[
\int \int \gamma \delta T^2 \, ds \, dx = - \int \int ds \, dx \left[ \kappa_1 \left( \delta T_x - \frac{T_{sx} \delta T_x}{T_x} \right)^2 \right] + \kappa_1 \left( \delta T_x - \frac{T_{sx} \delta T_x}{T_x} \right)^2 \]  

(18)

Thus when pressure is constant and the radial temperature gradient does not change sign all equilibria are stable. In general the radial dependences of \( \kappa_x \) \( (\kappa_x = n_e(x) \chi_x) \) and \( p(x) \) add additional terms to Eq. (18), and Eq. (17) must be considered to evaluate stability. The more stable nature of 2-D equilibria indicates that an unstable 1-D equilibrium may be stabilized when it is radially coupled to other more stable regions.

4. CONCLUSIONS

By differencing the cross-field conduction term we have reduced the thermal conductivity equation to one dimension (along the field line) in a manner that still contains information about cross-field transport. We find that a stable MARFE-like equilibrium can form when the edge is cooled or when the edge impurity content increases sufficiently.

An increase in \( \chi_x \) will inhibit MARFE formation. When both the constant temperature and the MARFE solutions are thermally stable, temperature fluctuations can cause a transition from one to the other. Furthermore, a necessary condition for the stability of the MARFE (Eq. (11b)) requires that the temperature remain close to the constant warm temperature equilibrium solution throughout most of the field line length and then fall quickly into the strongly radiating temperature range. This condition eliminates a solution with a gradual temperature drop which would be desirable for spreading out the radiation.

Two types of MARFE solutions are observed: in the low impurity concentration case \( \phi(T_1) > \phi(T_2) \) and the MARFE forms in the cool end of the temperature range with \( T_1 \leq T < T_2 \). In the high impurity concentration case \( \phi(T_1) < \phi(T_2) \) the MARFE forms at the warm end of the temperature range with \( T_2 < T < T_1 \). The latter case represents the MARFE solution that can most easily make a transition from the warm constant temperature solution.

We have also approximated the 2-D thermal diffusion equation with coupled 1-D equations. An examination of the stability of the 2-D fluid equations indicates that perpendicular coupling can enhance stability. In the special case of constant pressure and when the cross flux temperature gradient does not change sign we have seen that all equilibria are stable.

The approach taken in this study illustrates the qualitative features of edge/scrape-off layer phenomena. More detailed calculations can be obtained from 2-D codes [22-24]. However, because multiple solutions exist simulation codes can miss the correct solution. Additionally, although time dependent codes can follow the evolution of unstable to stable equilibria, they cannot predict jumps between neighbouring stable solutions.

Although in principle MARFEs could be avoided in sufficiently pure hydrogenic plasmas only small fractions of low Z impurities are required for their formation. It therefore appears unlikely that a strong cooling of the plasma edge (i.e. to form a radiative divertor) can occur without the formation of a MARFE. In principle a tokamak reactor could take advantage of this tendency by using an X point MARFE to radiate a significant fraction of the power leaving the plasma. In a double null (up/down symmetric) divertor it may be possible to create two MARFEs (above and below the plasma) which would increase the power radiated and distribute it more evenly on the reactor first wall.

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