Preheating following the inflationary phase in models of open inflation is considered. The most significant difference from a preheating scenario in flat space is that supercurvature modes can be populated and amplified in the open universe. In certain models, such modes can dominate the usual resonant particle production, thus altering the ensuing thermal history within an open universe. The accuracy to which the masses and couplings need to be tuned to produce such supercurvature modes, however, makes any large deviation from the flat space cases unlikely.

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1. INTRODUCTION

There has been much activity lately in two exciting areas of inflationary cosmology: the construction of “open inflation” models, and the development of a new theory of post-inflationary reheating. To date, these two developments have been treated separately; in this paper, we explore some qualitatively new behavior for the reheating epoch in an open universe. The open inflation models [1–3] exploit the fact, first noted by Coleman and De Luccia and independently by Gott and Statler [4], that the metric on the interior of a bubble nucleated in a sea of de-Sitter-like false vacuum would be that of an open Friedmann-Robertson-Walker spacetime. By arranging for an epoch of “slow-roll” inflation to occur within the nucleated bubble, and arranging for the number of e-folds of expansion during this second inflationary phase not to exceed $N \sim 65$, it is possible to produce an observable universe whose mass density today falls below the critical mass density required for a flat universe: $\Omega_0 \sim 0.2 - 0.3$, rather than the standard inflationary prediction, $\Omega_0 = 1.0$.

Independently, several groups have re-examined the process of particle production at the close of an inflationary phase, and have found resonances and exponential instabilities in the decay of an inflaton field into bosons, which had been overlooked in the older two-stage picture: the highly efficient, often resonant, decay of the oscillating inflaton field into boson decay products (which might include inflaton bosons themselves, due to the inflaton’s self-interaction), followed by the decay of these decay products (along the methods originally developed in [5]), followed by the thermalization of these particles and the establishment of the radiation-dominated era. Kofman, Linde, and Starobinsky term this first, resonant stage the “preheating” stage.

As demonstrated in this paper, under certain conditions the resonant decay of the inflaton in an open universe can populate “supercurvature” modes, which have no analogue in the flat space case, and no particle-like interpretation. In some open inflation models the amplification of such supercurvature modes can even dominate the usual resonant creation of particles at the time of reheating. Thus, models of open inflation can display dramatically different thermal histories following the end of inflation from inflationary scenarios in a spatially flat universe.

A Friedmann-Robertson-Walker universe with nonvanishing spatial curvature has a physical curvature scale given by $a(t)/|K|$, where $a(t)$ is the cosmic scale factor, and $K = -1$ or $+1$ for an open or closed universe, respectively. With $K = -1$, the comoving curvature scale is thus simply $+1$. From the Friedmann equation it can further be shown that when $K = -1$ the physical curvature scale will always be larger than the Hubble distance, $H^{-1}$, for any cosmological epoch. Eigenfunctions of the Laplacian operator can be found for this background spacetime with eigenvalue $-(k/a)^2$, where $(k/a)$ is the inverse of a physical length: $k/a = 2\pi/\lambda_{ph}$, with $0 < k^2 < \infty$. Modes with comoving wavenumber $k^2 > 1$ thus vary on scales shorter than the comoving curvature scale, and are labelled “subcurvature” modes, whereas modes with $0 < k^2 < 1$ correspond to “supercurvature” modes. (See, e.g., [6,7].) (Note that tensor modes can have different values from scalar modes, such as $k^2 = -3$ for gravitational waves. [8,9]) In a flat universe, the comoving curvature scale runs off to infinity, so there are never supercurvature modes. It is this difference from the flat space case that we examine here, for the epoch of preheating: even though $\Omega \to 1$ at the end of the second round of inflation, there will still exist a comoving curvature length scale at $+1$, the existence of which may change the usual physics at the time of preheating.

We begin the study following the nucleation of a single bubble in the midst of a sea of de Sitter space false vacuum. Inside the nucleated bubble, there is a nonvanishing scalar field potential and a single relevant scalar field to drive a second round of inflation within the bubble; both the single-field models of open inflation [10] and Linde’s two-field “hybrid” open inflation model [11]...
fit such a description. Inside the bubble, the Lagrangian density can thus be written:
\[
\mathcal{L} = \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} g^{\mu
u} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right],
\] (1)
and the background spacetime takes the form
\[
ds^2 = -dt^2 + a^2(t) \left[ d\chi^2 + \sinh^2 \chi (\partial t^2 + \sin^2 \theta d\phi^2) \right].
\] (2)

If we make the familiar decomposition,
\[
\varphi(t, \chi, \theta, \phi) = \varphi_o(t) + \delta \varphi(t, \chi, \theta, \phi),
\] (3)
then the equation for the quantum fluctuations \( \delta \varphi \) becomes
\[
\square (\delta \varphi) - \left( \frac{\partial^2 V}{\partial \varphi^2} \right)_{\varphi_o} (\delta \varphi) = 0.
\] (4)

Switching to conformal time, \( d\eta = a^{-1} dt \), and using the expression for the d’Alembertian, \( \square \varphi = (-g)^{-1/2} \partial^\mu \left[ (-g)^{1/2} g^{\mu\nu} \partial_\nu \varphi \right] \), then inside the bubble equation (4) takes the form:
\[
\left[ \frac{\partial^2}{\partial \eta^2} + 2 \frac{a'}{a} \frac{\partial}{\partial \eta} - \bar{L}^2 + a^2 \left( \frac{\partial^2 V}{\partial \varphi^2} \right)_{\varphi_o} \right] \delta \varphi = 0,
\] (5)
where primes denote derivatives with respect to conformal time, \( \eta \), and \( \bar{L}^2 \) is the comoving Laplacian operator.

Eigenvectors of \( \bar{L}^2 \) in the metric of equation (3) have been studied by many people (see, e.g., [14, 15]). Denoting these harmonics as \( Y_{k\ell m}(\chi, \theta, \phi) \), with \( \bar{L}^2 Y_{k\ell m} = -k^2 Y_{k\ell m} \), then we may parametrize the fluctuations in terms of mode functions as:
\[
\delta \varphi_k(\eta, \chi, \theta, \phi) = \frac{1}{a(\eta)} \psi(\eta) Y_{k\ell m}(\chi, \theta, \phi).
\] (6)

(We will not need the explicit form of the \( Y_{k\ell m} \)’s here, which can be written in closed form in terms of associated Legendre polynomials. [16]) Finally, if we write \( (\partial^2 V/\partial \varphi^2)_{\varphi_o} = M^2(\varphi_o) \), then the equation for the inflaton fluctuations becomes:
\[
\psi''_k - \frac{a''}{a} \psi_k + k^2 \psi_k + a^2 M^2 \psi_k = 0.
\] (7)

(We have neglected the metric perturbations which couple to \( \psi_k \), which should be a good approximation during the reheating epoch; for more on such metric perturbations, see [17, 18].) In this paper, we will be concerned with the behavior of solutions of equation (7) after the end of inflation, as the inflaton field \( \varphi_o \) oscillates near the minimum of its potential.

In Section II, we briefly review the conditions under which the oscillating inflaton may decay resonantly into bosons. In an open universe, with the possibility of populating supercurvature modes during the inflaton’s parametric resonance, we must pay closer attention than is usually done to the conditions under which the inflaton’s decay may be resonant: the harmonic relations between the frequency of the inflaton’s oscillation and the decay products’ momenta are specified here for all resonance bands within the narrow resonance regime. In Section III, these resonance conditions are studied for the simplest model of an inflaton decaying into inflaton bosons, due to a \( \varphi^4 \) self-coupling. As will be shown, in this case no supercurvature modes may be populated during preheating, and there should be no deviation from the usual flat space case. The model is generalized in Section IV to the case of an inflaton decaying into a distinct species of boson, via Yukawa and quartic couplings, and in these cases supercurvature modes can be populated, if the masses and couplings of the model satisfy certain relationships to very high accuracy. None of the resonant behavior will affect inflaton decay into fermions, because of Fermi-Dirac statistics, and we will ignore such couplings here. The results are discussed in Section V.

II. PREHEATING AND RESONANT DECAYS

Near the minimum of its potential, the inflaton will begin to oscillate, and this can set up a parametric resonance in the inflaton’s decay to bosons. [14] Assuming that the frequency of the inflaton’s oscillations is greater than the Hubble parameter at this time, we may neglect Hubble expansion during the course of the resonant decays. Then the inflaton field oscillates as
\[
\varphi_o(\eta) \simeq \varphi_o \cos(\omega_{osc} \eta),
\] where \( \varphi_o \) is slowly decreasing on the oscillation time-scale \( \omega_{osc}^{-1} = (a_e H_e)^{-1} \). Here \( a_e \) is a constant giving the value of the cosmic scale factor at the end of inflation. Note that \( (a_e H_e)^{-1} < 1 \), as will be true for any cosmological epoch in an open universe. Hence, our assumption about the frequency of the inflaton’s oscillation translates to \( (a_e m)^{-1} < (a_e H_e)^{-1} < 1 \), or \( a_e m \gg 1 \). The \( M^2(\varphi_o) \) term then contains two types of terms, oscillating and non-oscillating, and may be written:
\[
M^2(\varphi_o) = A + B \cos(q \omega_{osc} \eta),
\] (8)
where \( A, B, \) and \( q \) are determined by a given model’s particular couplings. The equation of motion for the inflaton fluctuations, equation (7), then becomes:
\[
\psi''_k + \omega_k^2 [1 + g \cos(q \omega_{osc} \eta)] \psi_k \simeq 0,
\]
\[
\omega_k^2 = k^2 + a^2 A, \quad g \equiv a^2 B / \omega_k^2.
\] (9)
This is now in the form of the Mathieu equation, solutions of which reveal exponential instabilities within var-
The narrow resonance regime of equation (9), with $g < 1$, can be studied perturbatively. The analytic analysis in [6,8,9] demonstrates that in this case the solutions are exponentially unstable in narrow resonance bands $nq\omega_{osc} = 2\omega_k + \frac{1}{2}|\epsilon_n|$, where $|\epsilon_n| < \omega_k$, and $n$ a positive integer. Each resonance band with $n > 1$ corresponds to keeping terms of $O(g^n)$ in the solution to equation (9), and the width of each successive resonance band in the narrow resonance regime shrinks as [13]:

$$|\epsilon_n| = \frac{n^{2n-3} g^n \omega_k}{2^{3(n-1)} [(n-1)!]} \equiv b_n g^n \omega_k.$$  

(10)

Near the center of each resonance band, solutions take the form

$$\psi_{\pm k,n}^{(n)}(\eta) \simeq \frac{\exp(\pm s_k^{(n)}(\eta))}{\sqrt{2\omega_k}},$$

(11)

with $s_k^{(n)}$ real and positive. Modes with a given $k^2$ slide out of the resonance band due to the slow decrease of $\varphi^2_0$, and hence of $g$, due to scatterings and back-reaction effects. Given such a time-dependence of $g$, modes will only become resonantly amplified if they initially satisfy

$$nq\omega_{osc} = 2\omega_k + \frac{1}{2}|\epsilon_n|;$$

(12)

modes initially satisfying $nq\omega_{osc} = 2\omega_k - \frac{1}{2}|\epsilon_n|$, on the bottom edge of the resonance band, will not remain within the band long enough to become amplified. We will thus study equation (12) for various models to determine which areas of parameter space allow for the production and amplification of supercurvature modes. The subcurvature modes should behave as in the flat space case, and hence we will not treat them further here.

As demonstrated explicitly in [6], including a nonzero Hubble expansion during the period over which the inflaton oscillates does not change the qualitative form of these solutions, but merely narrows the width of the resonance bands. Similarly, including back-reaction and rescattering effects should further decrease the width of each resonance band. [6,9,16] We may accommodate these effects by inserting a phenomenological parameter $\alpha(\eta)$, with $0 < \alpha(\eta) < 1$, into the band width, writing the width of each resonance band as $|\epsilon_n| = b_n \alpha^n(\eta) g^n \omega_k$, though we will set $\alpha = 1$ in the following.

The broad resonance regime of equation (9) ($g \gg 1$), identified in [6] has been studied analytically in [10], and also leads to solutions of the form $\psi_{k,n}^{(n)} \propto \exp(\pm s_k^{(n)}(\eta))$, with $s_k^{(n)}$ real and positive (and $s_k^{(n)} \gg s_k^{(n)}$). However, such a broad resonance regime is ruled out for most quartic couplings [10], and even for models with Yukawa couplings between the inflaton and a distinct species of boson, the coupling strength (and hence $g$) may not be made arbitrarily large, because such large couplings would interrupt the usual inflationary dynamics prior to the preheating epoch. Furthermore, the particles produced during a broad resonance have much higher momenta than particles produced in the narrow resonance regime, and hence no supercurvature modes are expected to be populated from a broad resonance. For these reasons, we will restrict attention here to the narrow resonance regime for each of the models studied below.

### III. Inflaton Decay into Inflaton Bosons

In this section, we consider a model in which the oscillating inflaton field decays into inflaton bosons, due to a $\varphi^4$ self-coupling. Near the minimum of its potential, any model of open inflation should assume a general chaotic inflation form [22]:

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{1}{4} \lambda \varphi^4.$$  

(13)

In this case, the parameters in equation (9) become

$$\omega_k^2 = k^2 + a^2_m m^2 + \frac{3}{2} a^2_m \varphi^2_0, \quad g = \frac{3}{2} a^2_m \varphi^2_0 / \omega_k^2,$$

(14)

or, combining terms,

$$\omega_k^2 = \frac{(k^2 + a^2_m m^2)}{(1 - g)}.$$  

(15)

Note from the form of $g$ in equation (14) that $g < 1$ always in this model: in the case of inflaton decay into inflaton bosons, the only possible resonances lie in the narrow resonance regime, and hence the explicit expressions in section II may be used. Finally, for the potential of equation (13), the oscillating term in equation (9) oscillates at frequency $2\omega_{osc} = 2a_m m$, or $q = 2$, after use has been made of a double-angle formula for $\cos^2(\omega_{osc} \eta)$.

Given that $a_m \ll 1$, we need only consider wavenumbers in the range $0 < k \ll a_m$ in our search for any supercurvature modes which may be resonantly amplified during preheating. Defining

$$\ell \equiv \frac{k}{a_m},$$

(16)

then equation (12) becomes:

$$2 \left( n \sqrt{1 - g} - 1 - \frac{1}{4} b_n g^n \right) = \left[ \ell^2 - \frac{1}{4} \ell^4 + O(\ell^6) \right] \left( 1 + \frac{1}{4} b_n g^n \right).$$

(17)
Keeping only the lowest-order terms in $\ell$, this gives
\[ k_{\text{res}}^2 \simeq 2a_e^2 m^2 \left[ \frac{4n\sqrt{1 - g - 4 - b_n g^n}}{4 + b_n g^n} \right]. \tag{18} \]

When $n = 1$, the quantity on the righthand side of equation (18) is negative, revealing that neither supercurvature nor subcurvature modes will be amplified in the lowest resonance band when $k < a_e m$. For $n > 1$, the approximation $\ell \ll 1$ is violated, and thus equation (18) cannot be used to study resonant subcurvature modes. Instead, it can easily be demonstrated that when $\ell > 1$ (and hence for subcurvature modes only), resonant modes correspond to
\[ k_{\text{res}} \simeq \frac{n\sqrt{1 - g}}{2\gamma} a_e m \left[ 1 \pm \sqrt{1 - \frac{2\gamma^2}{n^2(1 - g)}} \right] + O(\ell^{-2}), \tag{19} \]
\[ \gamma \equiv 1 + \frac{1}{4} b_n g^n \geq 1, \]
again revealing that no consistent solution (with $k_{\text{res}}$ real and positive) may be found for $n = 1$. Thus, for the potential of equation (13), only subcurvature modes may be amplified during preheating, and only in the narrow resonance regime with $n \geq 2$.

For this particular potential, a limit on $g$ may be set by the independent constraints on $m$ and $\lambda$ from observed microwave background anisotropies: $\lambda \lesssim 10^{-14}$, and $m/M_{pl} \leq 10^{-6}$.\footnote{An upper bound may be placed on $\varphi_0$ by equating this with the value of $\varphi_0$ once inflation ends, that is, once the slow-roll conditions are violated, at $\partial^2 V/\partial \varphi^2 \simeq 24\pi M_{pl}^{-2} V$. This gives $\varphi_0 \lesssim 0.16 M_{pl}$ (using the constraints on $\lambda$ and $m$), or $\lambda \varphi_0^2 / m^2 \lesssim 2.6 \times 10^{-4}$, guaranteeing that $g \leq 4 \times 10^{-4}$. With so small a $g$, resonance bands with $n > 2$ are unlikely to produce large occupation numbers of inflaton bosons, and the resonant decay of the inflaton may be rather inefficient in transferring the energy density of the decaying inflaton field into boson decay products. Such tight constraints on the potential’s parameters are absent in the case of inflaton decay into a distinct species of boson, assuming that the second boson field is unimportant during the inflationary phase (when the primordial density perturbation spectrum is determined). It is to these models that we now turn.}

\section*{IV. Inflaton Decay into a Distinct Boson Species}

In the case of inflaton decays into a distinct species of boson, $\chi$, we may study a potential of the form:
\[ V(\varphi, \chi) = \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} m^2 \chi^2 + \mu \varphi \chi^2 + \lambda^2 \varphi^2 \chi^2. \tag{20} \]

In supersymmetric models, these couplings will satisfy $\mu = 2\lambda m$.\footnote{Writing $\chi = a^{-1}(\eta) f(\eta) Y_{plm}(\vec{x})$, the equation for $f(\eta)$ will assume the same form as equation (6), with $M^2 \rightarrow \partial^2 V/\partial \varphi^2 = m^2 + 2\mu \varphi + 2\lambda^2 \varphi^2$. Again taking $\omega_{\text{res}} = (a_e m)^{-1} \ll (a_e H)^{-1}$, the $\chi$-modes will obey:
\[ f_k'' + [\omega_{\text{res}}^2 (1 + h_1 \cos(2\omega_{\text{res}} \eta) + h_2 \cos(\omega_{\text{res}} \eta))] f_k \simeq 0, \]
\[ \omega_{\chi}^2 = k^2 + a_e^2 m^2 + a_e^2 \lambda \varphi_0^2, \]
\[ h_1 \equiv a_e^2 \lambda_2 \varphi_0^2 / \omega_{\chi}, \quad h_2 \equiv 2 a_e^2 \mu / \omega_{\chi}. \tag{21} \]

Note from the form of $h_1$ that $h_1 \ll 1$ always, so only the narrow resonance regime is viable for the quartic coupling. If we first consider $h_2 \rightarrow 0$, and if the mass difference between the inflaton and the $\chi$ boson is small, then modes with $k \ll a_e m_\chi$ will become exponentially amplified in bands centered on:
\[ k_{\text{res}}^2 \simeq 2a_e^2 m_\chi^2 \left[ \frac{4n(m/m_\chi)^2 \sqrt{1 - h_1 - 4 - b_n h_1^2}}{4 + b_n h_1^n} \right]. \tag{22} \]

The next order term is of $O(k^2 a_e^2 m_\chi^{-2})$, which will be completely negligible if $0 < k < 1$ and $a_e m_\chi \gg 1$. Though the form of equation (22) for $k_{\text{res}}^2$ looks quite similar to the corresponding expression for $\varphi \rightarrow \varphi$ decays, equation (18), there are important differences. First, there are now two free parameters, $(m/m_\chi)$ and $h_1$, rather than only the single free parameter $g$ in equation (18). Second, the coupling constant $\lambda_\chi$ (and hence $h_1$) is not limited directly by today’s observed microwave background spectrum, because it is assumed that the $\chi$ field plays no role during inflation, so $h_1$ need not be constrained to $h_1 \sim O(10^{-4})$ as $g$ is. Finally, the ratio $(m/m_\chi)$ may be either greater or less than unity: as noted in \footnote{Any lightness of the inflaton tends to be arranged for such conditions to be met (and the nearly-exact cancellation of combinations of parameters is not in itself foreign to particle physics). Furthermore, if $n_\ast = 1$ and $h_1 \ll 1$, then all higher (subcurvature) modes could fall outside of resonance bands, leaving only the supercurvature modes to be produced and amplified at the preheating stage. Note that if $m_\chi \ll m$, then equation (23) is inappropriate, and only subcurvature modes will be amplified at preheating. Other resonant subcurvature}, it is possible for the inflaton to decay into a particle heavier than itself during the resonant preheating stage (a decay which is kinematically forbidden according to non-resonant Born theory). Thus, supercurvature modes will be produced and resonantly amplified in a resonance band $n_\ast$ which satisfies
\[ 0 < \frac{8a_e(m/m_\chi) \sqrt{1 - h_1 - 8 - 2b_n h_1^{n_\ast}}}{4 + b_n h_1^n} \lesssim (a_e m_\chi)^{-2}. \tag{23} \]
modes, with \( k > a_e m_\chi \), may be found analogously to equation (19), and will not be pursued here.

Similarly, if \( h_1 \to 0 \) and the Yukawa coupling in equation (24) is active \( (h_2 \neq 0) \), then supercurvature modes will be produced resonantly in a resonance band \( n_* \) if

\[
0 < \left[ \frac{4n_* (m/m_\chi) - 8 - 2b_n h_2^{n_*}}{4 + b_n h_2^{n_*}} \right] < (a_e m_\chi)^{-2}
\]

in the narrow resonance regime, up to terms of \( O(k^4 a_e^{-2} m_\chi^{-2}) \). Again, the pair of parameters \((m/m_\chi)\) and \( \mu \) may be tuned to satisfy this condition, and if \( n_* = 1 \) and \( h_2 < 1 \), then the production and amplification of supercurvature modes will dominate the preheating phase. As for the \( h_1 \) case, if \( m_\chi < m \) or if \( k > a_e m_\chi \), then only subcurvature modes will be produced at preheating.

Care must thus be taken to identify the (model-dependent) dominant decay channel of the inflaton, especially if the two couplings in equation (24) are in competition with the inflaton self-coupling in equation (13). The conditions under which the supercurvature modes may become amplified and dominate the production of subcurvature modes during the preheating phase depends sensitively upon the couplings and mass scales of the decaying inflaton and its decay-product bosons. Note that although \( k^2_{\text{res}} \to 0 \) for these supercurvature modes, there is no infrared catastrophe, since every resonance band in the narrow resonance regime has a finite band width.

\[\text{V. DISCUSSION}\]

What do these exponentially amplified supercurvature modes represent? In the usual flat space preheating scenario, narrow resonance modes simply correspond to the production of low-momentum (small \( k \)) particles, whose occupation numbers may be estimated with the aid of a Bogolyubov transformation. Yet in the open universe, such small-\( k \) modes can stretch beyond the curvature scale (and hence beyond the horizon), and no particle interpretation may be given to them. (This is a specific example of how tenuous the notion of “particle” can become in general curved spacetimes.) Instead, they will contribute to (very) long-wavelength curvature perturbations, akin to supercurvature modes produced during inflation, though their evolution will be complicated by the highly non-adiabatic nature of the reheating epoch. The detailed evolution of these modes may be tracked using the methods of 13, though it is unlikely that they will contribute greatly to any observed microwave background anisotropies today, given their very long correlation lengths; this is a subject of further study.

Rather, their most dramatic role could be in changing the thermal history immediately following reheating: the new reheating scenario depends upon the explosive production of particles, far from thermal equilibrium; these decay products then thermalize via interactions on a (potentially) quite different time-scale. When the occupation numbers of out-of-equilibrium decay-product particles becomes quite large, this opens up the possibility for such microphysical processes as non-thermal phase transitions, preheating-induced supersymmetry breaking, and GUT-scale baryogenesis. Yet the foregoing analysis reveals that in an open universe, (resonant) preheating production of particles may be either totally absent or strongly diluted, due to some portion of the inflaton’s energy density being ‘siphoned off’ into supercurvature modes. For models in which the resonant production of subcurvature modes is subdominant, the inflaton will still decay into particles and populate the universe following the end of the resonance regime, according to the processes of the older theory of reheating. If this resonant production of supercurvature modes and non-resonant production of particles becomes the dominant effect, then these other microphysical processes in the post-preheating epoch may need to be reconsidered.

Given the exponentially tiny supercurvature resonance regimes in equations (22) and (23), it is rather unlikely (though still possible) that the supercurvature modes from the resonant \( \varphi \to \chi \) decays would dominate the preheating epoch. And such modes are completely absent from a strictly \( \varphi \to \varphi \) preheating epoch. Thus, large deviations from preheating scenarios in flat space are unlikely to appear for models of open inflation.

Finally, as noted in the conclusion in 9, for models with non-minimally coupled scalar fields which recover the Einstein-Hilbert gravitational action near the end of inflation (e.g. models of the form studied in 27), the analysis of reheating carries over unchanged. Thus, the recent open inflation model based on induced-gravity theory 28 should display the same resonance features and supercurvature modes as analyzed above.

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[1] M. Bucher, A. Goldhaber, and N. Turok, Phys. Rev. D 52, 3314 (1995).
[2] K. Yamamoto, M. Sasaki, and T. Tanaka, Astrophys. J 455, 412 (1995).
[3] A. Linde, Phys. Lett. B 351, 99 (1995); A. Linde and A. Mezhlumian, Phys. Rev. D 52, 6789 (1995).
[4] S. Coleman and F. De Luccia, Phys. Rev. D 21, 3305 (1980); J. Gott, Nature 295, 304 (1982); J. Gott and T. Statler, Phys. Lett. B 136, 157 (1984).
[5] L. Kofman, A. Linde, and A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994).
[6] Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D 51, 5438 (1995).
[7] D. Boyanovsky, M. D’Attanasio, H. de Vega, R. Holman, D.-S. Lee, and A. Singh, Phys. Rev. D 52, 6805 (1995).
[8] M. Yoshimura, Prog. Theo. Phys. 94, 873 (1995).
[9] D. Kaiser, Phys. Rev. D 53, 1776 (1996).
[10] H. Fujisaki, K. Kumekawa, M. Yamaguchi, and M. Yoshimura, Phys. Rev. D 53, 6805 (1996).

Recent reviews of reheating include A. Linde, “Recent Progress in Inflationary Cosmology,” Preprint astro-ph/9601004; and L. Kofman, “The Origin of Matter in the Universe: Reheating after Inflation,” Preprint astro-ph/9605154. Linde’s article also reviews the new models of open inflation.

[11] A. Dolgov and A. Linde, Phys. Lett. B 116, 329 (1982); L. Abbott, E. Fahri, and M. Wise, Phys. Lett. B 117, 29 (1982).
[12] D. Lyth and A. Woszczyna, Phys. Rev. D 52, 3338 (1995); J. García-Bellido, A. Liddle, D. Lyth, and D. Wands, “The open universe Grischuk-Zel’dovich effect,” Preprint astro-ph/9508003.
[13] J. Garriga, “Bubble fluctuations in \( \Omega < 1 \) inflation,” Preprint gr-qc/9602023.
[14] J. García-Bellido, “Metric Perturbations from Quantum Tunneling in Open Inflation,” Preprint astro-ph/9510026.

[15] K. Yamamoto, M. Sasaki, and T. Tanaka, “Quantum fluctuations and CMB anisotropies in one-bubble open inflation models,” Preprint astro-ph/9605106.
[16] E. Lifshitz, J. Phys. (Moscow) 10, 116 (1946); E. Lifshitz and I. Khalatnikov, Adv. Phys. 12, 185 (1963); M. Bender and C. Itzykson, Rev. Mod. Phys. 38, 346 (1966); E. Harrison, Rev. Mod. Phys. 39, 862 (1967).
[17] V. Mukhanov, H. Feldman, and R. Brandenberger, Phys. Rep. 215, 203 (1992).
[18] L. Landau and E. Lifshitz, Mechanics (Pergamon Press, New York, 1960).
[19] S. Khlebnikov and I. Tkachev, Phys. Rev. Lett. 77, 219 (1996).
[20] R. Allahverdi and B. Campbell, “Cosmological Reheating and Self-Interacting Final State Bosons,” Preprint hep-ph/9606468.

[21] A. Linde, Phys. Lett. B 129, 177 (1983).
[22] N. Birrell and P. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, 1982); R. Wald, Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (University of Chicago Press, Chicago, 1994).
[23] L. Kofman, A. Linde, and A. Starobinsky, Phys. Rev. Lett. 76, 1011 (1996); I. Tkachev, “Phase Transitions at Preheating,” Preprint hep-th/9510140.
[24] G. Anderson, A. Linde, and A. Riotto, “Preheating, Supersymmetry Breaking and Baryogenesis,” Preprint hep-ph/9606416.
[25] E. Kolb, A. Linde, and A. Riotto, “GUT Baryogenesis after Preheating,” Preprint hep-ph/9606200; M. Yoshimura, “Baryogenesis and thermal history after inflation,” Preprint hep-ph/9605246.
[26] D. Kaiser, Phys. Rev. D 52, 4295 (1995).
[27] A. Green and A. Liddle, “Open inflationary universes in the induced gravity theory,” Preprint astro-ph/9607166.