On the temperature dependence of correlation functions in the space like direction in hot QCD

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Abstract

We study the temperature dependence of quark antiquark correlations in the space like direction. In particular, we predict the temperature dependence of space like Bethe-Salpeter amplitudes using recent Lattice gauge data for the space like string potential. We also investigate the effect of the space like string potential on the screening mass and discuss possible corrections which may arise when working with point sources.

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1 Introduction

It is generally believed that hadronic matter undergoes a transition to a new phase, the so-called quark-gluon-plasma (QGP), which is a gas of weakly interacting quarks and gluons. (For a general review see refs. [1, 2].) As far as thermodynamic properties are concerned this statement seems to be supported by lattice gauge calculations (LGC), which show a sharp rise of the energy density at the critical temperature $T_c$. Above $T_c$, the energy- and entropy density obtained in LGC are compatible with a perturbative gas of quarks and gluons. Moreover the quark condensate vanishes above $T_c$ indicating the restoration of chiral symmetry. Furthermore, above $T_c$ screening masses extracted from spatial point to point correlation functions assume the values of $M_{sc} = n\pi T$ [4, 5], $n$ being the number of quarks involved. The $(\pi, \sigma)$ correlators give screening masses smaller than the free value indicating considerable residual interactions in these channels above $T_c$. This is not too surprising as these modes can be viewed as long range fluctuations in the order parameter of a second order chiral restoration transition.

There exist, however, several phenomena which indicate a nontrivial structure even above $T_c$. Simple thermodynamic considerations [3], for instance, show that about 50% of the zero temperature gluon condensate still remains condensed above the transition. Moreover, while the large distance behavior of space like point to point correlators is consistent with free quarks, the measurement of Bethe-Salpeter amplitudes in the space like direction shows very strong correlations between quarks and antiquarks [7]. It has been suggested [8] and demonstrated explicitly [9] that these `observed' correlations can be related to the so-called space-like string-tension, which remains finite even above $T_c$ [10]. This space like string tension is extracted from the expectation value of a Wilson loop, which loops only in the spatial direction. Contrary to the time like Wilson loop, which measures the electric interactions and thus can be related to the heavy quark potential, the space like Wilson loop is sensitive to magnetic interactions. The associated string tension, therefore, does not relate to a heavy quark potential. It rather provides a measure for the strength of a current-current type interaction, analogous to that associated with Ampère's law in electrodynamics. In contradistinction to the time like string tension, which vanishes above $T_c$, the space like string tension remains finite at all temperatures [11, 12].

The effect of the spatial string tension on a quark antiquark pair can be best understood if one transforms to so called `funny space', where the euclidian time direction and one of the spatial directions, say the `z'-direction, are interchanged. At zero temperature, of course, real euclidian space and funny space are identical. At finite temperature, however, bosons and fermions follow periodic and antiperiodic boundary conditions in the temporal direction, respectively. In funny space these boundary conditions are moved to the $z_f$-direction with no boundary conditions in the $\tau_f$ direction. Consequently a sys-

\[1\] For a collection of the most recent LGC results see ref. [3].
tem at finite temperature in real space transforms to one at zero temperature in funny space where quarks and gluons form standing waves in the $z_f$-direction with momenta $p_q = (2n + 1)\pi T$ and $p_g = 2n\pi T$, respectively, where $n$ is an integer. Similarly, a spatial Wilson loop $W_{x,z}$ in real space, which involves, say the $x$ and $z$ direction, transforms into a time like Wilson loop $W^{f}_{x_f,z_f}$ in funny space. Hence, it can be associated with a heavy quark potential in funny space. The space-like Bethe-Salpeter amplitudes also obtain a ‘physical’ meaning in funny space. They can be understood as wave functions of an quark-antiquark pair.

First LGC with pure glue [10] and more recent calculation involving dynamical fermions [13] have indicated that the space like string tension remains constant between zero temperature (where it is identical with the time like string tension) and temperatures slightly above the phase transition. In very recent work by Bali et al. [12] have studied the temperature dependence of the space like string tension in SU(2) gauge theory with high accuracy. For small temperatures, $T \leq 2T_c$, they confirm that the string tension is essentially independent of temperature. For temperatures $T \geq 2T_c$ they find that the string tension scales like $\sigma(T) = (g(T)T^2)^2\sigma_0$ where the T-dependence of the coupling $g(T)$ is given by the 2-loop $\beta$-function with a scale parameter $\Lambda = 0.076T_c$. This form of the temperature dependence is what one would expect from dimensional reduction arguments [14] and the resulting value for the string tension agrees within 10% with that of three dimensional SU(2) gauge theory [13].

With the knowledge of the space like string tension as a function of temperature, the resulting Bethe-Salpeter amplitudes can be calculated in the model proposed in ref. [1]. This will provide a prediction for the temperature dependence of these correlations which can be tested on the lattice in order to check the validity of this model. In this article we will present the wave functions for several temperatures. We will also discuss several possibilities to better probe the detailed structure of the quark-antiquark interaction and we will investigate the implications for observables such as the screening masses etc.

This paper is organized as follows: In the following section we briefly review the method we are going to use to calculate the Bethe-Salpeter amplitudes and present the resulting amplitudes for different temperatures. Then we investigate other possibilities to test the interaction more precisely. In section three we discuss the implications of the quark-antiquark interactions on the screening masses. In the final section we will compare our results with already existing LGC on the temperature dependence of the wavefunctions [10] and discuss possible discrepancies.
2 Wavefunctions

As mentioned in the introduction and explained in great detail in ref. \[9\] at high temperature the effect of the space like string tension on the Bethe-Salpeter amplitude is best studied in ‘funny space’. There the space like string tension acts just like a regular potential and the boundary conditions, originally in the temporal direction, lead to standing waves in the \(z_f\) direction. At high temperatures only the lowest momentum (Matsubara) modes contribute, i.e. \(p_z = \pm \pi T\) for the quarks/antiquarks, and the momenta of the standing waves \(|p_z|\) simply act like an effective mass, as far as the motion in the transverse direction \((x_f, y_f)\) is concerned. Under these assumptions the space like Bethe-Salpeter amplitude can be identified with the wavefunction of a quark-antiquark pair with masses \(m_q = \pi T\) in two dimensions (because the motion in the \(z_f\) direction leads to the effective mass) interacting via the space like string potential. At high temperature, finally, the effective quark mass becomes large so that a nonrelativistic approximation can be made.

Unless otherwise noted, from now on all variables and equations are understood to be defined in ‘funny space’. The subscript ‘f’, indicating funny space variables, will, therefore, be dropped.

The Schrödinger equation to be solved is \[9\]:

\[
-\frac{1}{\pi T} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) + \frac{l^2}{r^2} \right] \psi + V(r) \psi = E \psi \tag{1}
\]

where \(l\) is the angular momentum reflecting the azimuthal symmetry and \(V(r)\) is the potential which can be extracted from the space like Wilson loop. Before we will show the actual wavefunctions let us first discuss some general properties. For a stringlike potential \(V(r) = -g/r + \sigma r + \text{const}\) the large distance behavior of the wavefunction is governed by

\[
\left( -\frac{1}{\pi T} \frac{d^2}{dr^2} + \sigma r \right) \psi = 0 \tag{2}
\]

and consequently

\[
\psi \sim e^{\sqrt{\sigma \pi T} \frac{r^{3/2}}{2}} \quad r \rightarrow \infty \tag{3}
\]

From this asymptotic form it is evident that the wavefunctions should become narrower with increasing temperature even if the string tension does not increase! This is simply due to the increase of the effective quark mass \(m_q = \pi T\) in ‘funny space’ or, equivalently, to the decrease of the de Broglie wavelength of the quarks. An increase in the string tension would lead to an even stronger fall off of the wavefunctions.
In fig. 2a we show the ground state wave functions for temperatures $T = .5, \ldots, 8 T_c$ using the temperature dependent potentials from ref. [12], which are plotted in fig. 1. For comparison in fig. 2b the wavefunctions have been calculated with a potential fixed at $T = 0.5 T_c$. In both cases the wave functions become narrower with increasing temperature, somewhat more in the first case where the string tension increases with temperature as well. The difference between the wavefunctions in (a) and (b) for a given temperature is not very large, however. As suggested by eq. (3) a better way to illustrate the effect of the temperature dependent potential is to plot the wavefunctions as function of $r^{3/2} T^{1/2}$. This is done in fig. 3. The wavefunction in the constant potential (fig. 3b) nicely follow the above scaling law, eq. (3), (in case of $T = 8 T_c$ one only can observe the concave shape which would eventually lead to a slope parallel to the others.) In fig. 3a, on the other hand, the effect of the string potential is clearly seen. Although in principle the string tension can be read off from the asymptotic slope, it requires the knowledge of the Bethe-Salpeter amplitudes over several orders of magnitude and thus may not be feasible in practical LGC. As can be seen from fig. 1, over the range of the de Broglie wavelength $d \sim 1/T$, where the bulk of the wavefunction is expected to be located, the effect of temperature on the potential is small. Thus, in order to efficiently probe the long range part of the interaction, one has to find ways to move the wavefunction to larger distances.

One such possibility is to project on states with finite orbital angular momentum ($l \neq 0$ in eq. (4)). These states ‘feel’ a centrifugal barrier $\Delta V = l^2/r^2$ and, hence, are pushed outside to larger distances as can be seen in fig. 4, which shows the wavefunctions for $l=0,1,2,3$ for a temperature $T = 2 T_c$. The increased sensitivity on the potential is demonstrated in fig. 5. There the ratio of the average radii for wavefunctions in the temperature dependent and temperature independent potential ($<r(T-\text{dep})>/<r(T-\text{indep})>$) with

$$<r> = \int d^2r r |\psi(r)|^2$$

is plotted as a function of the temperature for orbital angular momentum $l = 0$ (open squares) and (l=3) full squares. In case of $l = 0$ the average radius is almost insensitive to the temperature dependence of the potential. Even for the highest temperature, where the string tension has increased considerably, the difference in the mean radii of the wave functions is only $\sim 10\%$. Thus, due to the increase of the quark effective mass, the ($l = 0$) wavefunctions only ‘see’ the short distance part of the potential, where the temperature dependence is small (see fig. (4)). Only the tails of the wavefunctions are sensitive to the linear rising part of the potential as demonstrated in figs. 2 and 3. These of course affect the mean radius only very little. In case of the $l = 3$ wave functions, on the other hand, the centrifugal potential $\Delta V = l^2/r^2$ dominates at short distances and pushes the wavefunctions out into the region where it becomes sensitive to the string potential. At temperatures $T = T_c$ and $T = 1.33 T_c$, where the string tension differs very little from the
$T = 0.5T_c$ value, an effect in the ratio of the mean radii can already be observed. At high temperatures, the ‘squeezing’ of the state due to the increased string potential becomes very clear.

In summary, the bulk of the s-wave ($l = 0$) wave functions is mainly dominated by the effective quark mass and information about the strength of the string tension can be extracted only from very precise LGC. The sensitivity can be increased, however, by projecting on states with finite angular momentum. Thus sources and sinks in LGC should be designed in a way that they produce states of good angular momentum in the two dimensional transverse (x,y) plane. Similar projections have already been carried out in the study of the glueball spectrum.

### 3 Screening masses

In principle one would expect the temperature dependence of the space like string potential to reflect on the screening masses. The eigenvalues $E_{Schr}$ of the Schrödinger equation are directly related to the screening mass by

$$m_{scr} = 2\pi T + E_{Schr}$$

and an increase of the potential results in an increase of the eigenvalue. But the eigenvalue is also affected by the T-dependence of the effective quark mass, because with increasing mass the wavefunction can sit deeper in the potential well. This dropping of the eigenvalue as a result of the increasing quark mass is demonstrated by the dashed line in fig. (5), which gives the eigenvalue as a function of temperature for the constant potential. If the temperature dependence of the potential is taken into account as well (full line) the eigenvalue does not change with temperature for $T \geq 2T_c$. Obviously the decrease of the eigenvalue due to the effective mass and the increase as a result of the potential cancel each other with a surprising accuracy.

But more importantly, the screening mass is dominated by the ‘thermal’ mass term $\sim 2\pi T$. The energy eigenvalue $E_{Schr}$ is only a small correction to this term and the effect of the temperature dependence of the potential is even smaller. This is demonstrated in fig. (5) (lower curves), where the ratio $M_{scr}/2\pi T$ is plotted. For all temperatures both

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2In principle the wavefunctions of the radially excited states with $l = 0$ also extend further out (see fig. 4) and, hence, should be more sensitive to the string potential. However, it seems very difficult to device sources and sinks which are sensitive to one particular radial excitation only.

3The nonrelativistic approximation, which led to the Schrödinger equation (1), implies, of course, that the mass term is much larger than the nonrelativistic energy eigenvalue.
temperature dependent and temperature independent potential lead to the same screening mass. Consequently, any effects from the T-dependence of the string potential on the screening mass, although possibly present, are not accessible on lattices of the present size. For example for $T = 4 T_c$, $\Delta E_{Schr} \simeq 0.5 T_c$ and $M_{scr} \simeq 25 T_c$. Hence, in order to be sensitive to the temperature dependence of the potential a lattice which is about 200 times as large in the spatial direction than in the time direction would be required!

4 Point sources

Usually screening masses are defined as the logarithmic derivative of point to point correlators taken at large distances. On the lattice, however, the maximum separation between source and sink of the correlator is limited for practical reasons. In this case the signal may be ‘contaminated’ by an admixture of excited states, because point source have an overlap with all radial excitations of the quark antiquark-system and the limited separation between sink and source may not be enough to screen the excited states sufficiently. As we will show, in our case, the presence of excited states hardly affects the results for the screening masses but leads to considerable corrections for the Bethe-Salpeter Amplitudes. Assuming the validity of the above potential model, the point to point correlation function in funny space can be written as (for details see appendix)

$$C(\tau) = \frac{1}{V_4} \sum_n |\psi_n(0)|^2 e^{-M_n \tau}$$

(6)

with $M_n = 2\pi T + E_{Schr}$ and $V_4$ is the volume of the four-dimensional euclidian box. Here the contribution of states with higher Matsubara frequencies has been neglected (see appendix). Because of the factor $|\psi(0)|^2$ only those states with angular momentum $l = 0$ contribute to the sum in eq. (6). Similarly, for the Bethe-Salpeter amplitudes one has (see appendix)

$$\Psi(\tau, r_\perp) = \frac{1}{V_4} \sum_n \psi^*(0)\psi(r_\perp)e^{-M_n \tau}$$

(7)

As already mentioned, the ground state is projected out by going to large distances $\tau$. Usually this distance is assumed to be reached once the screening mass

$$M_{scr} = \frac{d}{d\tau} \ln(C(\tau))$$

(8)

does not vary with $\tau$ anymore. We will demonstrate that this method may have practical problems, if one wants to extract the ground state wavefunctions as well. The argument
is, that, due to the dominance of the ‘free, thermal’ contribution $\sim \pi T$ to the screening mass, the screening mass of the ground state and of the first excited state differ only by a few percent. Therefore, the screening mass appears to have assumed its asymptotic value even if a nonnegligible admixture of excited states is still present. While these excited states obviously do not alter the value of the screening mass, they lead to substantial corrections for the Bethe-Salpeter amplitudes. The reason is, that at intermediate transverse distances $r_{\perp}$ the contributions to the ‘wavefunction’ of the ground- and first excited state ($n = 1, l = 0$) are comparable in magnitude but opposite in sign. Thus, the resulting wavefunction appears narrower than that of the ground state$^4$. This is demonstrated in fig. 7, where in part (a) we have plotted the relative difference of the local screening mass $M_{\text{scr}}(\tau)$ (eq. 4.3) to that of the true ground state $M_{\text{scr}}(\infty)$ as a function of $\tau$ for temperatures $T = T_c$ and $T = 1.33T_c$. (For the results presented here, the four lowest states ($n = 0, 1, 2, 3$) have been taken into account.) At $\tau = 1/T_c$ the resulting screening mass already differs by less than 3% from that of the ground state$^5$. In (b) the same ratio is plotted for the average radii of the wavefunction as defined in eq. (4). At the same distance $\tau = 1/T_c$ the difference is still of the order of 20%. This difference in the wavefunction is further illustrated in fig. 8 where we have plotted the radial density distributions $r_{\perp} |\psi(r_{\perp})|^2$ for the true ground state wavefunction (thick lines) and for the wavefunction as seen at a distance $\tau = 1/T_c$. The narrowing of the wavefunction can clearly be seen. This narrowing actually has been observed in the work of ref. $^{16}$ where the wavefunction as obtained from a LGC, using point sources, is compared with the ground state wave function of the potential model for temperatures $T = 0.66, 1.0, 1.33T_c$. The lattice used has a spatial extent of exactly $2/T_c$, resulting in a maximum distance of $\tau_f = T_c$ in funny space, because of periodicity. Thus, the conditions are identical to those considered here. For the highest temperature, where the approximations of our potential model are best justified, agreement could only be found if one assumed a quark mass which is twice as big as given in the model$^{16}$. Although a fairly large current quark mass $m_q = 200$ MeV has been used, this cannot account for doubling the effective quark mass

$^4$The narrowing of the wavefunction due to the admixture of excited state is simply a remnant of the initial $\delta$-function, which corresponds to a sum over all excited states, due to completeness. With increasing $\tau$ this $\delta$-function disperses and eventually assumes the shape of the ground state.

$^5$We have selected these comparatively low temperatures, because later on we want to discuss the results of $^{16}$, where the same temperatures have been chosen. The choice of temperature does not affect our argument and we find the same results at higher temperatures.

$^6$Usually in LGC periodic boundary conditions are imposed in the real-space spatial directions, which implies that only half the lattice can be used to relax the wavefunction in funny space. Therefore, distances $\tau$ much larger the $1/T_c$ are rarely realized.

$^7$In ref. $^{16}$ the calculation in the potential model accidentally has been made using the full quark mass in the Schrödinger equation instead of the reduced mass $^{17}$. Thus, the agreement the authors find for the highest temperature in effect means that one has to double the quark mass.
of \( \pi T \) at all. The fact that a large effective quark mass is needed in order to reproduce the lattice result implies, that the lattice wavefunction is narrower than the one given by the potential model. This difference, of course, is exactly what we would predict if an admixture of the excited states is still present. For comparison in fig. 8 we have also plotted the ground state wave function obtained after doubling the quark mass (dashed-dotted line). We find that it agrees fairly well with the effective wavefunction extracted at a distance \( \tau = 1/T_c \) (thin dashed line).

At this point we should note, that the narrowing of the apparent wavefunction due to the superposition of excited states may also be relevant for the measurement of Bethe-Salpeter amplitudes at zero temperature. Monitoring the screening mass in order to assure sufficient screening of the excited states is only successful if the energy eigenvalue of the ground state is smaller than the excitation energy of the first radial excitation (see appendix for a more formal argument). This implies that the measurement of the Bethe-Salpeter amplitudes for the pion using this method should be fine while in case of the \( \rho \) the monitoring of the wavefunction itself is advised.

The authors of ref. [16] also find that the wavefunctions obtained on the lattice for the three different temperatures \((T = 0.66, 1, 1.33T_c)\) are almost identical. In contrast our model would predict a narrowing of the wavefunctions due to the increase of the effective quark mass \( \sim \pi T \) (see fig. 8). (The changes in the potential are minimal over the temperature interval considered here.) Also for the more realistic wavefunctions, where the presence of excited states due to point sources is taken into account, the narrowing as a function of the temperature persists. The ratio of the average radii between wavefunctions of different temperatures is about the same for the true ground state wavefunction and that taken at a distance \( \tau = 1/T_c \) (see also fig. 4b). If we assume that the qualitative behavior of the space like string potential is the same in SU(3) and in SU(2) gauge theory, the independence of the wavefunction on the temperature as observed in ref. [16] cannot be understood within our model. Of course the temperatures considered \( T \leq 1.33T_c \) are somewhat low for the dimensional reductions arguments of our model to be good. Furthermore, at the lower two temperature points, \( T = 0.66T_c \) and \( T = T_c \), the constituent quark mass may still be of considerable magnitude, which also would affect the wavefunction (the authors give a screening mass for the \( \rho \) at \( T = 0.66T_c \) of about \( \sim 5.7T_c \) which should correspond to roughly twice the constituent quark mass). The results of ref. [16], therefore, may be at or below the lower limit of applicability of our model. However, it would be very interesting to carry out LGC calculations at larger temperatures \( T \geq 2T_c \) in order to see if the wavefunctions eventually follow the prediction of our model or if the independence of the temperature, as found in ref. [16] persist even at temperatures where one would expect dimensional reduction to be valid.
5 Conclusions

In this article we have studied the effect of the temperature dependence of the space-like Wilson loop on the Bethe-Salpeter amplitudes in the space-like direction. The temperature dependence of the wavefunctions is controlled not only by the temperature dependence of the potential but also by the increase of the effective quark mass with temperature in funny space ($m_q = \pi T$). Because of that ‘trivial’ effect, only high accuracy LGC will be able to distinguish between ground state wavefunctions from temperature dependent and temperature independent string potential at high temperatures. We, therefore, propose to project instead on states with finite angular momentum in the transverse directions. As a result of the centrifugal potential, these states are pushed towards larger transverse distances and are thus much more sensitive to the string potential.

We have further demonstrated, that screening masses are extremely insensitive to the temperature dependence of the potential. They are essentially dominated by the ‘free thermal’ value $\sim 2\pi T$ and deviations due to the string potential are very small. We also have shown that the presence of excited states in the correlation function due to point sources minimally changes the result for the screening mass. The Bethe-Salpeter amplitudes, on the other hand, come out much more narrow if excited states are still present. As a consequence, monitoring the screening mass in order to decide if a state at a given distance has settled into the ground state is not sufficient and may lead to wrong conclusions about the actual size of the Bethe-Salpeter amplitudes.

Finally, we have presented unique predictions for the temperature dependence of the space like Bethe-Salpeter amplitudes based on the potential extracted from high accuracy SU(2) LGC. In our model these amplitudes are understood as Schrödinger wavefunctions of quarks with mass $\pi T$ confined by the space like string potential. A measurement of these amplitudes on the lattice would, therefore, provides an important test for the validity of our understanding of these correlations.

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6 Appendix

In this appendix we will derive how point to point correlation functions and Bethe-Salpeter amplitudes are expressed in terms of the eigenstates and eigenvalues of the Schrödinger equation (1). We will also give an estimate on how the presence of excited states affects the results for screening masses and Bethe-Salpeter amplitudes. In the entire appendix we will work in funny space i.e. we have periodic/antiperiodic boundary conditions in z-direction and none in the τ-direction.

6.1 Point to point correlation functions

The screening masses are determined from the exponential fall off in the τ direction of the point to point correlation functions. The correlation function is given by:

\[ C(\tau, \vec{r}) = \langle 0 | j(\tau, \vec{r}) j(0) | 0 \rangle \]  \hspace{1cm} (9)

where \( \tau \geq 0 \) and \( r_3 = z \geq 0 \) is assumed. Our main assumption is that this correlation function can be saturated by the eigenstates of the Schrödinger eq. (1). Therefore, we can insert a complete set of states

\[ 1 = \sum_{n,p} \frac{|n, \vec{p}> < n, \vec{p}|}{2E_n(p)} \]  \hspace{1cm} (10)

where \( \vec{p} \) refers to the momentum of the center of mass motion and \( n \) to the quantum numbers of the internal motion, i.e. to that of the Schrödinger equation. The correlation function can then be written as

\[ C(\tau, \vec{r}) = \sum_{n,p} \frac{1}{2E_n(p)} < 0 | j(\tau, \vec{r}) | n, \vec{p} > < n, \vec{p} | j(0) | 0 > \]  \hspace{1cm} (11)

Since \( |n, \vec{p}> \) are eigenstates of the momentum operator – with discrete eigenvalues for \( \hat{p}_z \) – we can write

\[ C(\tau, \vec{r}) = \sum_{n,p} | < 0 | j(0) | n, \vec{p} >|^2 \exp(-E_n(p)\tau + i\vec{p}\vec{r}) \]  \hspace{1cm} (12)

By integrating over the spatial directions we project on states with zero transverse momentum and zero Matsubara frequency \( p_z \) of the quark antiquark pair.

\[ C(\tau) = \sum_n | < 0 | j(0) | n, \vec{p} = 0 >|^2 \exp(-E_n\tau) \]  \hspace{1cm} (13)
The sum still includes all pairs of quark Matsubara frequencies which add to zero, i.e. \((2n + 1)\pi T; -(2n + 1)\pi T\). We will, however, neglect the contribution of the states with \(|n| > 1\), because they have a screening mass of \(M_{scr} \geq 4\pi T\) and, thus, are screened very efficiently.

Finally the matrix element \(<0|j(0)|n, \vec{p}>\) relates to the Schrödinger wavefunction by

\[<0|j(0)|n, \vec{p}> = \sqrt{2E_n(p)}\Psi_{n,p}(0)\] (14)

with

\[\Psi(\vec{r}) = \frac{1}{\sqrt{\beta}}\Phi_{n,p}(\vec{r})\psi_n(0)\] (15)

Here \(\Phi(\vec{r})\) denotes the wavefunction of the center of mass motion and \(\psi_n(0)\) is the wavefunction of eigenstates of the Schrödinger equation describing the relative motion in the transverse \((x, y)\) direction. \(\beta = 1/T\) gives the box size in the z-direction in funny space. The normalization factor \(1/\sqrt{\beta}\) is the remnant of the standing waves \(\sim \exp(-ipz/\beta)\) for the motion in the z-direction in the relative coordinate. Therefore, with \(|\Phi_n(0)|^2 = 1/V_3\) the correlation function is given by \((V_4 = V_3\beta)\)

\[C(\tau) = \frac{1}{V_4} \sum_n |\psi_n(0)|^2 \exp(-E_n\tau)\] (16)

with \(E_n = 2\pi T + E_{Schr}\), where \(E_{Schr}\) is the eigenvalue of the Schrödinger equation \((\Pi)\). As explained above, only the lowest Matsubara frequency of the individual quarks are taken into account.

### 6.2 Bethe-Salpeter amplitudes

The Bethe-Salpeter amplitude is defined as

\[B(\tau, \vec{r}, \vec{r}_\perp) = <0\left(\bar{\psi}(\tau, r - \frac{r_\perp}{2})\Gamma\psi(\tau, r + \frac{r_\perp}{2})\right) | j(0) >\] (17)

where \(\Gamma\) denotes the appropriate spin-isospin matrix, which we will, for simplicity, ignore from now on. Again we can write the product

\[<n, \vec{p}|\bar{\psi}(\tau, r - \frac{r_\perp}{2})\Gamma\psi(\tau, r + \frac{r_\perp}{2})|0 > = \frac{\sqrt{2E_n(p)}}{\sqrt{\beta}}\Phi_{n,p}(\vec{r})\psi(r_\perp)\] (18)

(Note, that \(r_\perp\) is in the transverse \((x, y)\) direction only.) By inserting a complete set of states, we can factor out the c.m. motion and obtain

\[B(\tau, \vec{r}, \vec{r}_\perp) = \sum_{n,p} <n, \vec{p}|j(0)|0 > \frac{\exp(-E_n\tau + i\vec{p}\vec{r})}{\sqrt{2E_n}}\psi_n(r_\perp)\] (19)
and after integrating over the spatial coordinates the states $|n, \vec{p} = 0>$ with vanishing c.m. momentum are projected out. Using eq. (14) we finally have

$$B(\tau, r_\perp) = \frac{1}{V_4} \sum_n \psi_n^*(0) \psi_n(r_\perp) \exp(-E_n \tau)$$

(20)

### 6.3 Effect of excited states on screening mass and Bethe-Salpeter amplitudes

Finally let us give an estimate on how the presence of excited states affects the screening mass and the Bethe-Salpeter amplitude. Let us, for simplicity, consider only 2-states. Then the correlation function is given by (up to a constant)

$$C(\tau) = \exp(-E_0 \tau) + \alpha^2 \exp(-E_1 \tau)$$

(21)

with

$$\alpha^2 = \left| \frac{\psi_0(0)}{\psi_1(0)} \right|^2$$

(22)

The screening mass at given distance $\tau$ follows from the logarithmic derivative of the correlation function

$$M_{scr} = -\frac{d}{d\tau} \ln C(\tau) = E_0 (1 + \Delta)$$

(23)

with

$$\Delta = \alpha^2 \frac{E_1 - E_0}{E_0} \exp(-(E_1 - E_0) \tau)$$

(24)

The screening mass is, thus, unaffected by the presence of the excited state if $|\Delta| \ll 1$. This is achieved by going to a sufficiently large distance $\tau$.

For the Bethe-Salpeter amplitude or wavefunction, on the other hand, one has

$$\psi(r_\perp) = \psi_0(r_\perp) \exp(-E_0 \tau) + \alpha \psi_1(r_\perp) \exp(-E_1 \tau)$$

$$\alpha = \pm \left| \frac{\psi_0(0)}{\psi_1(0)} \right|$$

(25)

which can be related to the accuracy of the screening mass $\Delta$

$$\psi(r_\perp) = \psi_0(r_\perp) \exp(-E_0 \tau) \left( 1 \pm \frac{\Delta}{\alpha^2 E_1 - E_0} \frac{E_0}{E_1 - E_0} \right)$$

(26)
Typically $\alpha^2 \leq 1$ (see e.g. fig. [4]) and hence the accuracy of the Bethe-Salpeter amplitude $\Delta_{B.S.}$ is

$$\Delta_{B.S.} \geq \Delta_{S.M.} \frac{E_0}{E_1 - E_0} \quad (27)$$

Therefore, the error in the wavefunction may be considerably larger than that in the screening mass if the energy eigenvalue of the ground state is much larger than the excitation energy of the first excited state, i.e. $E_0 \gg E_1 - E_0$.

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Figure captions

Figure 1: Space like string potential from ref. [12]

Figure 2: Space like Bethe-Salpeter amplitude for ground state for temperature dependent potential (a) and for potential fixed at $T = 0.5 T_c$ (b).

Figure 3: Space like Bethe-Salpeter amplitude for ground state for temperature dependent potential (a) and for potential fixed at $T = 0.5 T_c$ (b). Here x-axis has been rescaled to demonstrate the asymptotic behavior.
Figure 4: Space like Bethe-Salpeter amplitude for ground state \((n=0,l=0)\), the first radial excitation \((n=1,l=0)\), and for finite angular momentum \((n=0,l=1,2,3)\). The temperature is \(T = 2T_c\).

Figure 5: Ratio of average radii between temperature dependent and temperature independent potential for states with \(l=0\) (full line) and \(l=3\) (dashed line).

Figure 6: Eigenvalues of the ground state of the Schrödinger equation and associated screening masses for temperature dependent (solid line) and temperature independent (dashed line). The dotted line gives the screening mass of the first radial excitation.

Figure 7: Apparent screening mass \(a\) and average radii of the apparent wavefunctions as a function of separation of source and sink \(\tau\).

Figure 8: Probability density distribution for the true ground state (thin lines) and for the effective wavefunction as obtained at a distance of \(\tau = 1/T_c\) (thick lines). The dashed-dotted line corresponds to the wavefunction obtained with twice the quark mass for \(T = 1.33T_c\).
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