Internal stability enforcement for data-driven control in the frequency domain

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Abstract

Stability enforcement is a challenge in data-driven control. In this article, this issue is addressed with an iterative approach. In particular from input/output data of a plant in the frequency domain, control specifications formulated as a reference model and an initial internally stabilizing controller an optimization problem is to improve matching between the reference model and the closed-loop, while maintaining the internal stability, is solved. From there, the obtained controller is also leading to internal stability and the process can be iterated. The proposed approach is illustrated on two numerical examples.

Keyword: Data-Driven Control, Internal Stability, optimization, Loewner Framework

1 Introduction

For many practical applications, a model cannot be derived from physical laws and input/output data may be the only accessible information concerning the system. In these cases, for control purposes, system data can be used to identify a model of the system. Then, based on closed-loop specifications, a controller can be designed applying some model-based techniques. However, in some context, the model of the system can be too complex or too costly to obtain. It may then be easier and faster to try to derive a controller by combining directly the system data and the specifications, as highlighted in [2]. These methods are known as Data Driven Control (DDC) [7].

Even if the traces of these methods go back to the 40s with Ziegler and Nichols PID-tuning-method [17], DDC methods have mostly been developed in the past 25 years. Among them, the Iterative Feedback Tuning (IFT) is an online method where and optimal structured controller is obtained through an iterative process consisting in minimizing the error between the output and the desired one.

The Correlation-based Tuning (CbT), is a time-domain method which consists in the minimization of the correlation between the reference signal and the error between the closed-loop output and the desired output, over some class of controller. The Virtual Reference Feedback Tuning (VRFT) is a one-shot off-line method. From one set of input/output experiment in time domain \((u(t_k),y(t_k))_{k=1...N}\), the virtual reference signal \(r_v(t_k)\), such that \(M_d(z) r_v(t_k) = y(t_k)\) is built, where \(M_d\) is the reference model. The difference between the experimental command \(u\) and the virtual command \(C(z,\theta)(r_v - y)\) is then minimized through the parameter \(\theta\) of the structured controller \(C(z,\theta)\). The Loewner Data-Driven Control (L-DDC) relies on frequency domain data for multivariable systems (MIMO). Considering frequency-domain input/output data of the system and control specifications given as a reference model, the ideal controller response is computed at the sample frequencies. The L-DDC then uses the Loewner framework to interpolate the ideal controller data to derive a realization of it. The obtained controller can then be reduced to fit low order controller.

While closed-loop stability can be enforced by selecting a specific controller structure with IFT, CbT or VRFT and by choosing a stable reference model with L-DDC, internal stability is way more difficult to assess. For instance, even if the closed-loop transfer is stable, the controller can lead to Right Half Plan (RHP) poles/zeros cancellations in the open-loop and thus to internal instabilities. To alleviate these issues, additional conditions on the controller have been taken into account in IFT and CbT. Concerning the L-DDC, an internal stability criterion has been developed in [10]. It consists, firstly, in an identification of the unstable poles and Non Minimum Phase (NMP) zeros of the plant. Then, the reference model is adapted to satisfy some interpolation conditions necessary for internal stability. A small-gain based condition is then derived to maintain internal stability while reducing the controller.

The main contributions of this paper consist in a process for structured controller synthesis, leading both to internal stability and closed-loop performances similar to a reference model, without any need of previous poles/zeros identification. This synthesis is formulated as an iterative solving of optimization sub-problems where the internal stability crite-
rion is based on the application of the small-gain theorem. To solve these sub-problems, a one-shot data-based estimation of the $\infty$-norm is suggested through the use of Loewner Framework.

This paper is organized as follows: in Section 2, the controller design is formulated an optimization problem. In Section 3 some technical notions are recalled. Section 4 contains the main contributions which lead to the MIS-LDDC algorithm. In Section 5 the application of MIS-LDDC algorithm on a simple example, the DC motor, from which the ideal controller (controller which lead to the reference model) is reachable, is used as a concept proof. Then, the algorithm is applied to a MIMO NMP aeronautic example to check the performances reached in the mismatch case (the reference is unreachable while ensuring internal stability and/or the chosen controller structure). The paper is concluded in Section 6 with some perspectives and remarks.

2 Problem formulation

Well known performance limitations are imposed by the unstable poles and NMP zeros of the plant [5] thus making some reference model unreachable while ensuring internal stability. As a consequence, the controller leading the closed-loop to the reference model, would not lead to internal stability. Therefore, a challenge is to find a controller that closed-loop to the reference model, would not lead to internal stability and/or the chosen controller structure. Finally, to avoid any implementation issues, one may focus on unquantified loss of performances, one may consider the optimization problem

$$\begin{align*}
\min_{K \in \mathcal{K}} & \quad || M_d - M(K) || \\
\text{s.t.} & \quad \text{Closed-loop system internally stable,} \\
& \quad K \text{ Stable}
\end{align*}$$

(1)

where $M_d$ is the reference model, $M(K)$ is the closed-loop transfer obtained with the controller $K$, $||.||$ is some norm to be specified and $\mathcal{K}$ is a subspace of the controller space which only considers controllers with a chosen structure.

The objective of the next sections is to specify the internal stability criterion, the chosen objective norm and the chosen controller structure to provide a implementable version of the problem $P$ (1). To make the problem easier to solve, it has been preferred to consider only sufficient criterion for internal stability based on the knowledge of an initial internally stabilizing controller. Nevertheless, since the obtained controller leads to internal stability as well, one may consider the opportunity to use it as a new initial controller and iterate the process to enlarge the research space.

3 Technical preliminaries

3.1 Loewner interpolation Framework

The Loewner interpolation process exposed in [12] is an interpolation method based on tangential Lagrangian interpolation to obtain a rational representation of a function interpolating $N$ data points $(\nu_k, \Phi_k)_{k=1}^{N}$ where $\nu_k \in \mathbb{C}$ and $\Phi_k \in \mathbb{C}^{n_a \times n_i}$.

In order to construct such a representation, let us divide the sample points into left $q$ interpolation points $(\mu_i, \Psi_i)_{i=1}^{q} \in \mathbb{C} \times \mathbb{C}^{n_a \times n_i}$ and $k$ right interpolation points $(\lambda_j, \Xi_j)_{j=1}^{k} \in \mathbb{C} \times \mathbb{C}^{n_a \times n_i}$.

Then, let us consider $q$ left tangential directions $(l_i)_{i=1..q} \in \mathbb{R}^{1 \times n_o}$ and $k$ right tangential directions $(r_j)_{j=1..k} \in \mathbb{R}^{n_i \times 1}$. From these, Loewner and shifted-Loewner matrices can be built as

$$
\begin{align*}
[L]_{i,j} &= \frac{l_i^T \Psi_{ij} - l_i^T \Xi_{ij} r_j}{\mu_i - \lambda_j}, \\
[L_{\sigma}]_{i,j} &= \frac{\mu_i l_i^T \Psi_{ij} - \lambda_i l_i^T \Xi_{ij} r_j}{\mu_i - \lambda_j}.
\end{align*}
$$

(2)

Then, a Singular Value Decomposition (SVD) is proceeded on the Loewner pencil $(L, L_{\sigma})$ such that, 

$$
[L, L_{\sigma}] = Y_1 \Sigma_1 X_1^H, \quad \begin{bmatrix} L \\ L_{\sigma} \end{bmatrix} = Y_2 \Sigma_2 X_2^H,
$$

(3)

where $^H$ represents the conjugate transpose. During this process, the McMillan order $r$ of the interpolation model is computed such that $\Sigma_1 \in \mathbb{R}^{r \times r}$ and $\Sigma_2 \in \mathbb{R}^{r \times r}$, more detailed are available in [12].

Finally, a realization of the interpolation model is provided via

$$
E = -Y_1^H L X_2, \quad A = -Y_1^H L \omega X_2, \quad B = Y_1^H V, \quad C = W X_2
$$

(4)

where $V = l_i^T \Psi_i$ and $W = \Xi_j r_j$.

As proven in chapter 4 of [1], if the samples are extracted from a rational model and more numerous than the order of the aforementioned model, the obtained realization is a realization of the entire system.

3.2 A sufficient condition for internal stability

As the plant is known only through some input/output data, internal stability cannot be assessed using the gang of four [15]. Alternative approaches may however be considered as in [9].
Contributions

By applying the small-gain theorem between $K_1 - K_0$ and $T_0$, a sufficient condition on the controller $K_1$ for being internally stabilizing is that

$$\|K_1 - K_0\|_\infty < \gamma_0^{-1},$$

where $\gamma_0 = \|T_0\|_\infty$.

As the model of the plant $P$ is unknown, the determination of the $\infty$-norm of $T_0$ cannot be performed in a classic way. Data-based estimation of the $\infty$-norm are available in [14] or via expert advice in [13]. Another approach based on Loewner interpolation is suggested in Section 4.

4 Contributions

Considering some plant samples $(\omega_k, \Phi_k)_{k=1}^N$ and a controller $K$, the closed-loop transfer can be computed at each sample point

$$M(K, \omega_k) = (I + \Phi_k K(\omega_k))^{-1} \Phi_k K(\omega_k).$$

4.1 Numerical formulation of the optimization problem

The problem $P$ [1] presented Section 3 is not implementable. It is necessary to choose a way to quantify the distance between the reference model $M_d$ and the closed-loop transfer which has been done here as

$$\|M_d - M(K)\| = \frac{1}{N} \sum_{k=1}^N \|M_d(\omega_k) - M(K, j\omega_k)\|^2,$$

where $\|\cdot\|_F$ is the Frobenius norm. This norm quantifies the quadratic error between the reference model and the closed-loop at the frequency sample. Should the points be uniformly distributed and their amount tending to $\infty$, equation (8) would tend to $\|M_d - M(K)\|_F^2$.

Using the internal stability criterion formulated in Section 3.2, if a controller $K_0$ leading to internal stability is known, a controller $K$ such that

$$K \in \mathcal{RH}_\infty, \quad \|K - K_0\|_\infty < \gamma_0^{-1},$$

would also lead to internal stability.

Problem $P$ [1] can thus be reformulated as

$$\mathcal{D} : \min_{K \in \mathcal{RH}_\infty} \sum_{k=1}^N \|M_d(\omega_k) - M(K, \omega_k)\|^2_F \quad \text{s.t.} \quad \|K - K_0\|_\infty < \gamma_0^{-1},$$

(10)

The controller is structured by a real vector so that the constraint, $K \in \mathcal{RH}_\infty$ is easily satisfied. To do so, a ZPK-inspired structure allows to monitor easily the poles and, in the SISO case, the zeros of the controller. In addition, this choice permits easily to consider a specific controller order or a specific controller form, PI, PID, Lead-Lag, etc. The structure is

$$K(s) = \frac{1}{d(s)} N(s),$$

(11)

where $d(s) \in \mathbb{C}$ and $N(s) \in \mathbb{C}^{n_1 \times n_o}$ are polynomial in $s$. Then, the polynomial $d$ is parameterized by the real vector $\beta \in \mathbb{R}^{n_p}$ such that

$$d(\beta, s) = \left( \prod_{l=1}^{n_p} \left( s^2 + \beta_{2l-1}s + \beta_{2l} \right) \right) f(s + \beta_{n_p}),$$

(12)

where,

$$f(s + \beta_{n_p}) = \begin{cases} 
    s + \beta_{n_p} & \text{if } n_p \text{ is odd} \\
    1 & \text{if } n_p \text{ is even}
\end{cases},$$

(13)

not to lose any generality. In a similar way, each coefficient of the polynomial matrix $N_{i,j}(s)$ is structured with a vector $\alpha^{i,j}$ and the scalar $k^{i,j}$ such that, for all $1 \leq i \leq n_i$ and $1 \leq j \leq n_o$,

$$N_{i,j}(\alpha^{i,j}, s) = k^{i,j} \left( \prod_{l=1}^{n_p} \left( s^2 + \alpha^{i,j}_{2l-1}s + \alpha^{i,j}_{2l} \right) \right) f(s + \alpha^{i,j}_{n_p}),$$

(14)
where $f$ is defined as in (13). The controller denoted $K(\theta)$ in the sequence is thus parameterized by

$$\theta = [\beta, \alpha^{1,1} \ldots \alpha^{n_p,n_o}, k^{1,1}, k^{1,2}, \ldots k^{n_p,n_o}] .$$

One may notice that such a structure is general since all real controller can be reached with it. To ensure $K(\theta) \in RH_\infty$ the following constraint must be satisfied,

$$\begin{align*}
\beta_l & > 0 \\
n_p & > n_{1,1} \quad 1 \leq l \leq n_p \\
n_{i,j} & > 1 \quad 1 \leq i \leq n_1, 1 \leq j \leq n_o .
\end{align*}$$

The problem $P_0$ (10) can be reformulated as

$$P_0 : \begin{cases}
\min_{\theta \in \mathbb{R}^m} & \sum_{k=1}^N \|M_d(\omega_k) - M(K(\theta), \omega_k)\|_F^2 \\
\text{s.t.} & \|K(\theta) - K_0\|_\infty < \gamma_0^{-1} \\
& A\theta < 0
\end{cases}$$

where the matrix $A$ is such that

$$A = [ -I_{n_p} \quad 0_{n_p \times (m-n_p)} ] ,$$

and translate the constraints of (16).

### 4.2 Adaptation of the method for PI and PID controller

As popular control structure such as PI and PID $\not\in RH_\infty$, the method must be adapted to account for these cases.

Let us suppose that the desired controller form does not belong to $RH_\infty$. Let us also suppose that a controller with the desired form, leading to internal stability $K_0$ is available. Then it is possible to find a transfer $F$ such that

$$\begin{align*}
\tilde{K}_0 & = FK_0 \\
K(\theta) & = FK(\theta) ,
\end{align*}$$

where $\tilde{K}_0 \in RH_\infty$ and $K(\theta) \in RH_\infty$.

Then the interconnection in Figure 2 can be replaced by the interconnection in Figure 3. It is then possible to merge the transfer $F^{-1}$ and $P$ to compute a new plant data $\Phi_k$ such that

$$\tilde{\Phi}_k = \Phi_k F(\omega_k)^{-1} \quad 1 \leq k \leq N .$$

Then, the same optimization problem $P_1$ (17) can be formulated with $K_0$ instead of $\tilde{K}_0$, $K(\theta)$ instead of $K(\theta)$, $\tilde{\Phi}_k$ instead of $\Phi_k$ and $\gamma_0$ instead of $\gamma_0$.

For instance, for a PID controller, $K(s) = k_p + \frac{k_i}{s} + \frac{k_o}{1+\tau s}$, $F$ can be chosen as $F(s) = \frac{s^2 + (k_p + k_i) + k_o}{(s+a)(1+\tau s)}$, with $a$ a real positive number to avoid instabilities. With this transfer,

$$\tilde{K}(s) = \frac{(k_p r + k_i) + k_o}{(s+a)(1+\tau s)} \in RH_\infty$$

and

$$\tilde{\Phi}_k = \Phi_k F(\omega_k)^{-1} \quad 1 \leq k \leq N$$

### 4.3 One-shot data-based process for the estimation of $\infty$-norm

The data-based determination of $\gamma_0 = \|T_0\|_\infty$ is critical element to solve the problem $P_0$ (17). In [14] and [13] methods are suggested to compute this norm. However, these methods need to proceed to several experiments on the system. Then, the determination of the $\infty$-norm is asymptotic and needs an infinite amount of experiment to be exact.

In one-shot data-driven context, such a methods are discounted. Therefore, it is needed to find another method to determine an approximation of $\gamma_0$. The following approach is based on Loewner interpolation.

Considering the provided $N$ plant samples $(\omega_k, \Phi_k)_{k=1}^N$, $T_0$ is computed at the sample frequencies $\omega_k$, $1 \leq k \leq N$ such that

$$T_0(\omega_k) = (I + \Phi_k K_0(\omega_k))^{-1} \Phi_k K_0(\omega_k).$$

The Loewner interpolation is then performed on the new sample set and a transfer function $\tilde{T}_0$ is obtained. As mentioned Section 3.1 if enough sample points are provided, $\tilde{T}_0 = T_0$.

Then, a bisection algorithm can be applied on $\tilde{T}_0$ to obtain its $\infty$-norm.

### 4.4 MIS-LDDC procedure

As mentioned in Section 2, the solving process can be iterated using the obtained controller at step $i$ as a new initial controller for the step $i + 1$. More generally, the optimization problem to solve at a step $i$ is

$$P_i : \begin{cases}
\min_{\theta_{i+1} \in \mathbb{R}^m} & \sum_{k=1}^N \|M_d(\omega_k) - M(K(\theta_{i+1}), \omega_k)\|_F^2 \\
\text{s.t.} & \|K(\theta_{i+1}) - K_i\|_\infty < \frac{1}{\gamma_i} \\
& A\theta_{i+1} - \theta_i < 0
\end{cases}$$

where $K_i$ is the optimal controller obtained at step $i - 1$ and $\gamma_i$ is the $\infty$-norm estimated by the method described in Section 4.3 with $K_i$ as controller.

This iterative procedure is denoted MIS Iterative Structured Loewner Data-Driven Control MIS-LDDC procedure and is given in Algorithm 1.

One may notice the possibility to change the structure of the controller through the MIS-LDDC iterations. However,
5 Numerical Example

This Section focuses on the application of the MIS-LDDC procedure on two examples. Firstly, a conceptual example, the DC motor, from which the considered class of controller makes it possible to reach the exact reference model while leading to internal stability. The second example is a multi-variable system, the yaw and roll angle control of a F16 air-fighter from which internal limitations make impossible to reach the reference model while ensuring internal stability.

5.1 An ideal case: the DC motor

Let us firstly consider a stable and minimum-phase system: a DC motor. In this case, the dry friction and other potential non-linearities are neglected. In addition, it is supposed that no resisting moment is applied on the motor. Let us suppose that a user is providing a reference model

\[
M_d(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\xi \frac{s}{\omega_0} + 1},
\]

where \(\omega_0 = 10\) rad/s and \(\xi = 1\) to avoid overshoot. One may consider that the frequency of interest are between \(10^{-2}\) rad/s and \(10^2\) rad/s. Let us suppose that 50 plant samples, distributed in a logarithmic way in the frequency range, are available. For a simulation purpose, the sample have been obtained from the following model

\[
\frac{\Omega(s)}{U(s)} = \frac{P(s) = \frac{K}{JR+K^2}}{\frac{L+JR}{JR+K^2}s + 1}
\]

where, \(\Omega\) is the angular velocity, \(U\) is the input voltage

- electromagnetic coefficient \(K = 0.021\) Nms/rad
- fluid friction coefficient \(f = 0.0182\) Nms/rad
- electrical resistance \(R = 0.56\) Ohms
- inductance \(L = 5.3\) mH
- moment of inertia \(J = 5 \cdot 10^{-4}\) kg.m²

Thus, the controller leading exactly to the reference model is

\[
K_{id}(s) = 12.618 \frac{s^2 + 36.51s + 4.011}{s(s + 20)}.
\]

In order to reach the exact reference model, the chosen controller structure is

\[
K(\theta, s) = \theta_0 \frac{s^2 + \theta_3 s + \theta_4}{s^2 + \theta_1 s + \theta_2},
\]

as described in Section 4.1. As \(K_{id} \notin \mathcal{RH}_\infty\), the goal of the MIS-LDDC procedure is to reach asymptotically the ideal controller.

Remark 1 In a real DDC context the ideal controller is unknown, consequently, is is not possible to determine a priori the structure of controller which can lead to the reference model. Such a case is described more in details in Section 5.2.

From a stabilizing controller, the MIS-LDDC procedure is applied with a tolerance \(\epsilon = 10^{-6}\). The resulting closed-loop is plotted in Figure 4. In this reachable case, all the curves overlap both in the frequency range and beyond, thus showing the capacity of the MIS-LDDC to make closed-loop transfer equal to a reference model when it is reachable.

5.2 Roll and Yaw angles control on a F16 air-fighter

Here is considered the problem of yaw and roll angle control on a F16 air-fighter. The derived model is described in [15]. The considered plant is modeled for an air speed of 62.5 m/s, an altitude of 0 and an angle of attack of 18.8 deg. From these information, it is possible to derive a linearized model for small yaw and roll angles from the aileron and rudder angle. This model is NMP since it has a zero at 0.5.

A physical analysis of the system can lead to frequency of interest between \(10^{-2}\) rad/s and \(10^2\) rad/s. It is supposed that 200 plant samples, distributed in a logarithmic way, are available in the frequency range of interest.
The system behaviour is plotted in black in Figure 5, where, r is the roll angle, y the yaw angle, \( \delta_a \) the aileron angle and \( \delta_r \) the rudder angle. One may consider a reference model such that the system is decoupled and such that the peaks on the diagonal transfers are limited without proceeding to high change of bandwidth. Such a reference model can be formulated as

\[
M_d(s) = \begin{bmatrix}
\frac{5}{s+5} & 0 \\
0 & \frac{0.8}{s+0.8}
\end{bmatrix}.
\]  

(30)

Note that \( M_d \) is minimum-phase, thus the system cannot reach the reference model while being internally stable [15]. A mismatch is thus to be expected.

Let us suppose that, due to cost purpose, one may only consider a MIMO controller of order 2 where all transfer are bi-proper. With the controller structure explained in 4.1, the denominator \( d(s) \) and each coefficient of the matrix \( N(s) \) are in the form

\[
d(s) = s^2 + \beta_1 s + \beta_2 \\
N_{i,j}(s) = k_{i,j} \left(s^2 + \alpha_{1,j} s + \alpha_{2,j}\right).
\]  

(31)

\( \theta \) is consequently a vector of 14 components.

The MIS-LDDC procedure is then applied from a stabilizing controller. The obtained result is plotted in red in Figure 5. One may notice that, in the frequency range, the closed-loop system has a similar behaviour than the reference model. Actually, the anti-diagonal transfer are maximized by \(-20 \) dBs which correspond to an attenuation of at least \(90\%\).

In addition, the diagonal transfers are similar to the reference model in the frequency range. However, out of the frequency range, the closed-loop is quite far from the reference model. This can mostly be seen on the upper left transfer in Figure 5. This can be explained by the objective function of the MIS-LDDC procedure which does not take into account the closed-loop behaviour out of the frequency range. Since the reference model is not reachable, due to its MP aspect, there is no reason for the closed-loop to act similarly to the reference model out of the frequency range.

In order to analyse, on this example, the efficiency of the MIS-LDDC procedure, one may have a look at the evolution of the objective function through the iterations. This result has been plotted in Figure 6. As planned by the theory, one may notice that the objective function is only decreasing. In addition, one may highlight that the decrease is strong in the first iterations. However, after approximately 100 iterations, the decrease is slower. A huge amount of iterations is needed to obtain a noticeable improvement of the objective function. This can be explained by the method used to solve the problem \( P_i \) [24]. Effectively, one may highlight that the optimization problem to solve is non-smooth, due to the \( \infty \)-norm constraint. Nevertheless, methods used for smooth optimization problem can still be used to reduce the objective function. It is actually not necessary to find a local/global optimum of the problem \( P_i \) to move to step \( i+1 \). But, when the frontier of the non-smooth feasible space are reached, it leads to a slow convergence.

6 Conclusions

In this paper, the MIS-LDDC procedure, a MIMO, one-shot, DDC method, relying in the frequency domain, to ensure internal stability, has been proposed. The specifications, provided to this methods, need to be in a reference model form. The MIS-LDDC procedure consists, iteratively, in optimizing the matching between the closed-loop and the reference model, under a small-gain constraint to ensure internal stability.

Compared to some other DDC procedures, the reference model does not need to be adapted to become reachable. This advantage is considerable since the modification of the refer-
ence model needs to proceed to a deeper analysis of the plant to find how to change it. Furthermore, the structured controller approach is a key advantage for practical applications where the controller form is not a tunable parameter. While, the iterative aspect of the procedure leads to higher computation time, it also makes the final closed-loop system less dependant on the initial stabilizing controller.

A more efficient resolution of the sub-problems, based for instance, on convex relaxation and a possible weighting of the objective function with respect to the considered frequencies, is currently under investigation.

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