Cross sections for semi-inclusive electroproduction of charged pions ($\pi^+$) from both proton and deuteron targets were measured for $0.2 < x < 0.5$, $2 < Q^2 < 4 \text{GeV}^2$, $0.3 < z < 1$, and $P_T^2 < 0.2 \text{GeV}^2$. We find the azimuthal dependence to be small and consistent with zero, for $P_T < 0.1 \text{GeV}$. In the context of a simple fit, the initial transverse momenta of $d$ quarks tends to be larger than for $u$ quarks, while the transverse momentum width of the favored fragmentation function is slightly larger than that of the unfavored function.

A central question in the understanding of nucleon structure is the orbital motion of partons. Much is known about the light-cone momentum fraction, $x$, and virtuality scale, $Q^2$, dependence of the up and down quark parton distribution functions (PDFs) in the nucleon. In contrast, very little is presently known about the dependence of these functions on their transverse momentum $k_T$. Simply based on the size of the nucleon in which the quarks are confined, one would expect characteristic transverse momenta of order a few hundred MeV, with larger values at small Bjorken $x$ where the sea quarks dominate, and smaller values at high $x$ where all of the quark momentum is longitudinal in the limit $x = 1$. Increasingly precise studies of the nucleon spin sum rule [1-4] strongly suggest that the net spin carried by quarks is relatively small, and therefore the net angular momentum must be significant. This in turn implies significant transverse momentum of quarks. Questions that naturally arise include: what is the flavor and helicity dependence of the

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transverse motion of quarks and gluons, and can these be modeled theoretically and measured experimentally? The process of semi-inclusive deep-inelastic lepton scattering (SIDIS), \(1N \rightarrow llX\) has been shown to factorize [5], in the high energy limit, into lepton-quark scattering followed by quark hadronization. Ideally, one could directly measure the quark transverse momentum dependence of the quark distribution functions \(q(x,k_t)\) by detecting all particles produced in the hadronization process. In the present experiment, we detect only a single hadronization product: a charged pion carrying an energy fraction \(z\) of the available energy. The probability of producing a pion with a transverse momentum \(P_t\) relative to the virtual photon \((\vec{q})\) direction is described by a convolution of the quark distribution functions and \(p_t\)-dependent fragmentation functions \(D^\pi(z,p_t)\) and \(D^-\pi(z,p_t)\), where \(p_t\) is the transverse momentum of the quark relative to the quark direction, with the imposed condition \(\vec{P}_t = z\vec{k}_t + \vec{p}_t\) (see Fig. 1). The “favored” and “unfavored” functions \(D^\pi(z,p_t)\) and \(D^-\pi(z,p_t)\) refer to the case where the produced pion contains the struck quark or not. Based on the semi-phenomenological string-breaking model [6], “soft” non-perturbative processes are expected to generate relatively small values of \(p_t\) with an approximately Gaussian distributions in \(p_t\). Hard QCD processes are expected to generate large non-Gaussian tails for \(p_t > 1\) GeV, and probably do not play a major role in the interpretation of the present experiment, for which the total transverse momentum \(P_t < 0.45\) GeV. The assumption that the fragmentation functions do not depend on quark flavor (for example \(D^\pi(z,p_t)\) applies equally well to \(u \rightarrow \pi^+\) and \(d \rightarrow \pi^-\)) in principle allows the \(k_t\) widths of up and down quarks to be distinguished. In the present experiment, the use of both proton and deuteron targets (the later with a higher \(d\) quark content than the former) and the detection of both \(\pi^+\) and \(\pi^-\) permits a first study of this problem.

The experiment (E00-108) used the Short Orbit (SOS) and High Momentum (HMS) spectrometers in Hall C at Jefferson Lab to detect final state electrons and pions, respectively. An electron beam with energy of 5.5 GeV and currents ranging between 20 and 60 \(\mu\)A was provided by the CEBAF accelerator. Incident electrons were scattered from 4-cm-long liquid hydrogen or deuterium targets. The experiment consisted of three parts: i) at a fixed electron kinematics of \((x,Q^2) = (0.32, 2.30\) GeV\(^2\)), \(z\) was varied from 0.3 to 1, with nearly uniform coverage in the pion azimuthal angle, \(\phi\), around the virtual photon direction, but at a small average \(P_t\) of 0.05 GeV; ii) for \(z = 0.55\), \(x\) was varied from 0.2 to 0.5 (with a corresponding variation in \(Q^2\), from 1.5 to 4.2 GeV\(^2\)), keeping the pion centered on the virtual photon direction; iii) for \((x,Q^2) = (0.32, 2.30\) GeV\(^2\)), \(z\) near 0.55, \(P_t\) was scanned from 0 to 0.4 GeV by increasing the HMS angle (with average \(\phi\) near 180 degrees). The virtual photon-nucleon \((\gamma^*N)\) invariant mass \(W\), is always larger than 2.1 GeV (typically 2.4 GeV), corresponding to the traditional deep inelastic region for inclusive scattering.

At lower virtual photon energy and/or mass scales, the factorization ansatz is expected to break down, due to the effects of final state interactions, resonant nucleon excitations, and higher twist contributions [7]. In particular, in the present experiment the residual invariant mass \(M_z\) of the undetected particles (see Fig. 1) ranges from about 1 to 2 GeV (inversely correlated with \(z\)), spanning the mass region traditionally associated with significant baryon resonance excitation. The extent to which this situation leads to a break-down of factorization was studied in our previous paper [8]. It was found that good agreement with expectations based on higher energy data was achieved for \(z < 0.7\), approximately corresponding to \(M_z > 1.5\) GeV. The ratio of total up to down quark distributions \(u(x)/d(x)\) extracted from ratios of cross sections, as well as the ratio of valence only up to down ratios \(u_v(x)/d_v(x)\), were also found to be reasonably compatible with higher energy extractions, provided \(z < 0.7\). Finally, the ratio of unfavored to favored fragmentation functions \(D^-\pi(z)/D^\pi\pi(z)\) (from the \(\pi^-/\pi^+\) ratios on the deuteron) was found to be consistent with extractions from other experiments. All of these studies were done with the \(z\)-scan and \(x\)-scan data, for which the average \(P_t\) was small (< 0.1 GeV), and the average value of \(\cos(\phi)\) was close to zero.

In this paper, we focus on the \(P_t\) dependence, with the goal of searching for a possible flavor dependence to the quark distribution functions and/or fragmentation functions. Since the average value of \(\cos(\phi)\) in the present experiment is correlated with \(P_t\) (approaching -1 for the largest \(P_t\) value of 0.45 GeV), we first study the limited data available from this experiment on the \(\phi\) dependence, which must be an even function since neither the beam nor the target were polarized. We parameterize [9] the data for each target and pion flavor according to:

\[
\frac{d\sigma_{\pi^+\pi^-}}{d\sigma_{ee\pi}} = \frac{dN}{dz} b \exp(-bP_t^2) \frac{1 + A\cos\phi + B\cos(2\phi)}{2\pi}
\]

(1)

where the parameters \(A(x, Q^2, z, P_t)\) and \(B(x, Q^2, z, P_t)\) are a measure of the relative importance of the interference terms \(\sigma_{LT}\) and \(\sigma_{TT}\), respectively. The assumed Gaussian \(P_t^2\) dependence (with slopes \(b\) for each case) is an effective parameterization that seems to describe the data adequately for use in making radiative and bin-centering corrections. We use this model for studying the \(\phi\) dependence, then return to a more detailed study of the \(P_t\) dependence in the context of a simple model.

For each kinematic point in the \(x\) and \(z\) scans, we extracted \(A\) and \(B\) and found no statistically significant difference between the results for \(\pi^+\) or \(\pi^-\), or proton or deuteron targets. We therefore combined all four cases together, and present the results in Fig. 2. Systematic
effects are proportional to curves on the figures and Levelt-Mulders [13]. These
the expectations based on kinematic shifts due to parton motion as described by Cahn [12] (shown as the solid
curves on the figures) and Levelt-Mulders [13]. These
effects are proportional to $P_t$ for $A$ and $P_t^2$ for $B$ respectively
[12–15], so are suppressed at low $P_t$. Other possible
higher twist contributions will also be proportional
to powers of $P_t/\sqrt{Q^2}$ [16,17]. One contribution that is
not a priori small at small $P_t$ and moderate to large $Q^2$
is the twist-2 Boer-Mulders [18] contribution to $B$, since
it includes a term proportional to 1/$P_t$. However, the
numerical values in the models of Ref. [18,19] are small
for our average kinematic variables and consistent with
our results.

In contrast, the longitudinal-transverse and transverse-
transverse coefficients $A$ and $B$ are much larger in exclusive
pion production ($M_x = M$, where $M$ is the nucleon mass) than those predicted for SIDIS. This is evidenced by our extracted average values for exclusive $\pi^\pm$
electroproduction on deuterion and for $\pi^+$ on proton, shown as the open symbols near $z = 0.98$ in Fig. 2. This underlines the importance of accounting for the radiative tail from exclusive production, which in our analysis was done using the computer code EXCLURAD [20] together with a reasonable model of exclusive pion electroproduction. The corrections were checked with the Hall C simulation package SIMC [21], which treats radiative corrections in the energy and angle peaking approximation.

Having established that the $\phi$ dependence is small for the kinematics of the $P_t$ scan ($z = 0.55$), and most importantly independent of target and pion charge, we next proceed to extract the $P_t^2$-dependences of the experimental cross-sections for $\pi^\pm$ electroproduction on proton and deuterion, using the $P_t$ scan data. In this analysis, we fixed $A = B = 0$ in all four cases, and used the cross section model from our previous paper [8] to describe the $Q^2$ dependence (needed because $P_t$ and $Q^2$
are somewhat correlated). The resulting cross sections are shown in Fig. 3 and listed in Table I. The dashed lines illustrate exponential fits to the data in each case, and the corresponding values of $b$ in Eq. 1 are indicated.

The statistical errors on the slopes range from 0.16 to 0.38, with an estimated systematic error of 0.4, which is
highly correlated for the four cases, so that the differences remain almost unchanged. The systematic error was estimated using the Cahn prescription [12] for the $P_t$ dependence of $A$ and $B$, instead of $A = B = 0$. We did not consider any other models for $A$ and $B$, some of which could introduce a difference between targets or pion charge. Averaged over targets and pion charge, we find an averaged slope $b = 4.1 \pm 0.1 \pm 0.4$ $\text{GeV}^{-2}$, which is in reasonable agreement with the value of 4.67 \pm 0.02 found by the HERMES collaboration [22] at $\langle Q^2 \rangle = 2.5$
$\text{GeV}^2$ and $W^2 > 10$ $\text{GeV}^2$.

Closer examination shows that the slopes for $\pi^+$ and $\pi^-$ are very similar to each other for each target, but that the slopes for the deuteron target are about three standard deviations smaller than those for the proton. For a more quantitative understanding of the possible implications, we study the data in the context of a simple model in which the $P_t$ dependence is described in terms of two Gaussians for each case, rather than a single $b$
parameter. Following Refs. [10,11], we assume that the
widths of quark and fragmentation functions are Gaussians in $k_t$ and $p_t$, respectively, and that the convolution of these distributions combines quadratically. The widths of the up and down distributions are given by $\mu_u$ and $\mu_d$, respectively, and the favored (unfavored) fragmentation widths are given by $\mu_u (\mu_d)$. We assume that only the energy fraction $z$ (0.55 for our data) of the quark momentum contributes to the final pion momentum, and further that sea quarks are negligible (typical global fits show less than 10% contributions at $x = 0.3$). The simple model can then be written as:

$$
\begin{align*}
\sigma^+_{\pi^+} &= C[4e^{-b_{u+}P_t^2} + (d/u)(D^-/D^+)e^{-b_{u-}P_t^2}] \\
\sigma^-_{\pi^+} &= C[4(D^-/D^+)e^{-b_{d+}P_t^2} + (d/u)e^{-b_{d-}P_t^2}] \\
\sigma^+_{\pi^-} &= C[4(d/u)e^{-b_{d+}P_t^2} + (D^-/D^+)e^{-b_{d-}P_t^2}] \\
\sigma^-_{\pi^-} &= C[4(d/u)(D^-/D^+)e^{-b_{u+}P_t^2} + e^{-b_{u-}P_t^2}] 
\end{align*}
$$

where $C$ is an arbitrary normalization factor, and $b_{u\pm} = (z^2\mu_{u\pm}^2 + \mu_{u\pm}^2)^{-1}$, $b_{d\pm} = (z^2\mu_{d\pm}^2 + \mu_{d\pm}^2)^{-1}$, and we assume $\sigma_d = \sigma_{\pi^+} = \sigma_{\pi^-}$. We can then fit for the four widths ($\mu_u, \mu_d, \mu_+$, and $\mu_-$), $C$, and the ratios $D^-/D^+$ and $d/u$, where the fragmentation ratio is understood to represent the data-
averaged value at $z = 0.55$, and the quark distribution ratio is understood to represent the average value at $x = 0.3$. The fit describes the data reasonably well ($\chi^2 = 78$ for 73 degrees of freedom), and finds the reasonable results $d/u = 0.30 \pm 0.02$ and $D^-/D^+ = 0.42 \pm 0.01$, largely uncorrelated with other fit parameters. Since the data are at fixed $z$, it is not possible to distinguish large fragmentation widths from large quark widths, leading to a significant inverse correlation between the two most important ones, $\mu_u$ and $\mu_+$, as shown in Fig. 4a. To illustrate the $1\sigma$ contours in this figure, we used a Monte Carlo method, and plotted all fit results with a $\chi^2$ values less than 2 units above the minimum value.

The fit tends to favor a larger $k_t$ width for $d$ quarks than for $u$ quarks, as illustrated in Fig. 4b, although the error on the $d$ quark width is very large and asymmetric (i.e., values of infinity are not excluded by the fit). A slightly larger width for $d$ is consistent with a diquark model [23] in which the $d$ quarks are only found in an axial diquark, while the $u$ quarks are predominantly found in a scalar diquark. If the axial and scalar diquarks have
different masses, for example 0.9 and 0.6 GeV, then the $d$ quark distribution falls off more slowly with $k_t$ than the $u$ quark distribution. In this model, the distributions show considerable deviation from an exponential falloff, but if we take the slope between $k_t^2 = 0.2$ and 1.0 GeV$^2$, we obtain the “effective” values of $\mu_u^2 = 0.31$ and $\mu_d^2 = 0.36$ GeV$^2$, as shown in Fig. 4b. The magnitude of both widths is moderately sensitive to the choice of the model parameter $\lambda_0$ (we used 0.6 GeV), although the difference in widths is largely driven by the difference in axial and scalar diquark masses. For example, $\mu_u^2 = 0.28$ and $\mu_d^2 = 0.32$ GeV$^2$ for $\lambda_0 = 0.5$ GeV. We cannot find any reasonable choice of parameters in the diquark model that would give $\mu_d^2 > 0.5$ GeV$^2$, so only the low edge of the 1$\sigma$ contour of our fit can be considered in agreement with this model.

We find that the fragmentation widths $\mu_+$ and $\mu_-$ are highly correlated, as illustrated in Fig. 4c. While neither width is determined well due to the strong correlation with the quark widths, the difference in widths approximatively $0.04 \pm 0.02$. This is a 2-$\sigma$ difference from the widths being identical as has been assumed in the Lund string model [6] and also in Refs. [10,11]. This is an interesting result, because one might have guessed that unfavored fragmentation would involve an extra string breaking, and therefore be associated with larger average $p_t$ than favored fragmentation. However, our results show the opposite trend, with the favored width larger than the unfavored one. As a systematic check, we re-did the fits fixing $\mu_+ = \mu_-$, and find that the solutions for $\mu_d$ versus $\mu_u$ are not changed significantly: by eye they look very similar to those shown in Fig. 4b.

Another possible explanation for the difference in proton and deuteron $b$ slopes as shown in Fig. 3 could be a difference in $A$ near $P_t^2 = 0.2$ GeV$^2$, since the average value of $\phi$ is near 180 degrees for the largest values of $P_t$ of this experiment. However, the difference in $A$ between the proton and neutron would have to be of order 0.5, which is very large compared to the values we find at lower $P_t$ and also very large compared to theoretical expectations [12,18,19].

Nonetheless, all of the above fit results can only be considered as suggestive at best, due to the limited kinematic range covered, and the very simple leading order assumptions made. Many of these limitations could be removed with future experiments covering a wide range of $Q^2$ (to resolve higher twist and gluon radiation effects), full coverage in $\phi$, a larger range of $P_t$, a wide range in $z$ (to distinguish quark width, which is weighted by $z^2$, from fragmentation widths, which likely vary slowly with $z$), and including the $\pi^0$ final state for an additional consistency check.

In summary, we have measured semi-inclusive electroproduction of charged pions ($\pi^{\pm}$) from both proton and deuteron targets, using 5.5 GeV energy electrons at Jefferson Lab. We find the azimuthal dependence to be small and consistent with zero, for the limited data set where we have full angular coverage. In the context of a simple fit, we find that the $k_t$ width of $d$ quarks tends to be larger than for $u$ quarks, while the $p_t$ width of the favored fragmentation function is slightly larger than that of the unfavored function.

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FIG. 1. Schematic diagram of semi-inclusive pion electroproduction within a factorized QCD parton model at lowest order in $\alpha_s$. Final transverse momenta of the detected pion $\vec{P}_t$ arises from convoluting the struck quark transverse momenta $\vec{k}_t$ with the transverse momentum generated during fragmentation process $\vec{p}_t$.

FIG. 2. The parameters $A$ and $B$ [the relative coefficients of the $\cos \phi$ ($\sigma_{LT}$) and $\cos 2\phi$ ($\sigma_{TT}$) terms] averaged over $\pi^+$ and $\pi^-$ detected from proton and deuteron targets, as a function of $x$ at $\langle z\rangle=0.55$ (left), and as a function of $z$ at $\langle x\rangle=0.32$ (right). The average value of transverse momentum ($\langle |\vec{P}_t|\rangle$) is $\sim 0.05$ GeV. The dashed lines indicate the weighted averages for $z < 0.7$, which are also enumerated in each panel. Errors indicated include only statistical contributions. Systematic errors are highly correlated from point to point, and are estimated at 0.03 on both $A$ and $B$. The open symbols are from exclusive pion production (see text). The solid lines are theoretical predictions [12].

FIG. 3. The $P_t^2$ dependence of $\phi$-averaged differential cross-sections per nucleus for $\pi^+$ production on hydrogen (H) and deuterium (D) targets at $\langle z\rangle=0.55$ and $\langle x\rangle=0.32$. The dashed lines are exponential fits to data, and the solid lines show the result of the seven-parameter fit described in the text. The error bars are statistical only.

TABLE I. Differential cross-sections per nucleus for $\pi^+$ production on hydrogen and deuterium versus $P_t^2$.

| $P_t^2$ (GeV$^2$) | $\sigma_{H}^{\pi^+}$ (nb/sr GeV$^3$) | $\sigma_{D}^{\pi^+}$ (nb/sr GeV$^3$) | $\sigma_{H}^{\pi^-}$ (nb/sr GeV$^3$) | $\sigma_{D}^{\pi^-}$ (nb/sr GeV$^3$) |
|------------------|----------------------|----------------------|----------------------|----------------------|
| 0.008            | 2.177±0.075          | 0.956±0.021          | 2.912±0.038          | 1.796±0.030          |
| 0.018            | 2.058±0.077          | 0.951±0.024          | 2.824±0.040          | 1.800±0.037          |
| 0.028            | 1.885±0.082          | 0.885±0.030          | 2.689±0.045          | 1.690±0.045          |
| 0.038            | 1.834±0.089          | 0.863±0.035          | 2.602±0.051          | 1.599±0.052          |
| 0.048            | 1.815±0.094          | 0.825±0.038          | 2.504±0.055          | 1.567±0.056          |
FIG. 4. Results from the seven-parameter fit to the data shown in Fig. 3: a) $u$ quark width squared $\mu_u^2$ versus favored fragmentation width squared $\mu_f^2$; b) $\mu_u^2$ versus $\mu_d^2$; c) $\mu_\nu^2$ vs $\mu_\nu^2$. The small open circles indicate correlated fit parameters which fit within a 1σ contour. The large dot near the bottom of panel b) is from a diquark model [23]. The dashed line in panels b and c indicate $\mu_u^2 = \mu_d^2$ and $\mu_\nu^2 = \mu_\nu^2$, respectively.