Suprathermal Electron Acceleration by a Quasi-perpendicular Shock: Simulations and Observations

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Abstract

The acceleration of suprathermal electrons in the solar wind is mainly associated with shocks driven by interplanetary coronal mass ejections (ICMEs). It is well known that the acceleration of electrons is much more efficient at quasi-perpendicular shocks than at quasi-parallel ones. Yang et al. studied the acceleration of suprathermal electrons with observations at a quasi-perpendicular ICME-driven shock event to claim the important role of shock-drift acceleration (SDA). Here, we perform test-particle simulations to study the acceleration of electrons in this event, by calculating the downstream electron intensity distribution for all energy channels assuming an initial distribution based on the average upstream intensities. Using simulations, we obtain the results similar to the observations from Yang et al. as follows. It is shown that the ratio of downstream to upstream intensities peaks at about 90° pitch angle. In addition, in each pitch angle direction the downstream electron energy spectral index is much larger than the theoretical index of diffusive shock acceleration. Furthermore, the estimated drift length is proportional to the electron energy but the drift time is almost energy independent. Finally, we use a theoretical model based on SDA to describe the drift length and drift time especially, to explain their energy dependence. These results indicate the importance of SDA in the acceleration of electrons by quasi-perpendicular shocks.

Unified Astronomy Thesaurus concepts: Solar energetic particles (1491); Solar particle emission (1517); Solar storm (1526); Interplanetary physics (827); Interplanetary shocks (829); Shock (806)

1. Introduction

Shock acceleration is a crucial source for energetic particles in the heliosphere and galaxy. Charged particles gain energy via different mechanisms at shocks. First-order Fermi acceleration (FFA) is due to the relative motion of scattering centers in the upstream and downstream regions. Shock-drift acceleration (SDA) is due to a gradient drift along the direction of convection electric field. Stochastic acceleration (second-order Fermi) is associated with the downstream turbulence. The well-known diffusive shock acceleration (DSA) theory (Axford et al. 1977; Krymsky 1977; Bell 1978; Blandford & Ostriker 1978), which successfully explains the power-law distribution of accelerated particles observed in the universe, is the combination of FFA and SDA.

Various heliospheric shocks, such as planetary bow shocks, coronal shocks, propagating interplanetary (IP) shocks, and the solar wind termination shock, are excellent acceleration sites of particles. It is assumed that there are two kinds of solar energetic particle (SEP) events observed at 1 au, one is impulsive events originating in solar flares (e.g., Cliver et al. 1982; Mason et al. 1984; Cane et al. 1986), the other is gradual events associated with IP shocks driven by coronal mass ejections (CMEs); e.g., Kahler et al. 1978, 1984; Reames 1999). Most space weather disturbances can be traced to large and long-duration gradual SEP events. Particle acceleration by CME-driven shocks has been widely studied (e.g., Reames et al. 1996; Zank et al. 2000; Li et al. 2003; Rice et al. 2003; Desai & Burgess 2008; Kong et al. 2017, 2019; Qin et al. 2018); however, some problems still remain unresolved concerning particle acceleration at propagating IP shocks. The geometry on the shock surface is variable when a CME-driven shock propagates from the Sun into the IP space. In addition, the turbulence level and other solar wind conditions are changeable. Therefore it is important to study the shock acceleration efficiency with varying obliquity angle, turbulence level, and other conditions (e.g., Giacalone 2005; Guo & Giacalone 2015; Qin et al. 2018). It is suggested by Qin et al. (2018) that particle acceleration processes with weak scatterings (weak turbulence) are generally FFA for parallel (quasi-parallel) shocks and SDA for perpendicular (quasi-perpendicular) shocks, and that SDA is more efficient than FFA.

Recently, some interesting phenomena are observed in the acceleration of electrons by shocks. Holman & Peses (1983) suggested that electrons are accelerated to high energies in the solar corona by SDA. Energetic electrons, which are observed as solar type II radio bursts, can also be generated via resonant interaction with whistler waves at quasi-perpendicular shocks in the solar corona (Miteva & Mann 2007). Furthermore, Li et al. (2013) and Kong et al. (2013) studied electron spectral hardening in solar flares with observations from Gamma-Ray Spectrometer onboard Solar Maximum Mission, to suggest that energetic electrons are produced by SDA.

On the other hand, there exist low-energy electrons in IP space such as suprathermal electrons, which constitute important components of solar wind halo, strahl, and superhalo populations (e.g., Feldman et al. 1975; Lin 1998; Maksimovic et al. 2005), and can be accelerated by following the meandering magnetic field lines repeatedly across the shock (Jokipii & Giacalone 2007; Guo & Giacalone 2010). Kajdić et al. (2014) found 90° pitch angle enhancements of suprathermal electrons at IP shocks measured by Solar Wind Electron Analyzer onboard STEREO-A. Yang et al. (2018, hereafter YEA2018) investigated two strong electron flux enhancement events measured by electron electrostatic analyzers (EESA) in the 3DP instrument onboard Wind, one with a quasi-perpendicular shock, the other one with a quasi-parallel...
shock. They found that at energies of ~0.4–50 keV the ratios of the downstream to ambient electron intensities all peak at around 90° pitch angle. They also found that the energy spectrum in each pitch angle direction in the downstream region can be fit to a double power law with a spectral index much larger than that predicted by DSA. They suggested that SDA is the dominant acceleration mechanism in both shock events, and determined that the drift length is roughly proportional to the electron energy but the drift time almost does not vary with energy.

Kong & Qin (2019) performed numerical simulations to obtain 90° pitch angle enhancements for three sample energy channels in the range of 89–257 eV at the quasi-perpendicular shock on 2008 April 24 that was studied by Kajdić et al. (2014). In this paper, we study numerically the acceleration of suprathermal electrons in the range of ~0.3–40 keV at the quasi-perpendicular shock on 2000 February 11 that was studied by YEA2018. Pitch angle distributions (PADs) for all of the 12 energy channels, energy spectra for the parallel, perpendicular, and anti-parallel directions, and spectral indices for all pitch angles are obtained by solving the motion equation of electrons using a backward-in-time test-particle method. In Section 2 we briefly introduce the instruments onboard Wind used in the study, and we list the shock parameters for the event. We describe our physical and numerical models to accelerate electrons in Section 3. In Section 4, we first show the estimation of electron drift length and drift time from the data of distribution functions following YEA2008, then we provide a theoretical model of drift length and time, finally we show the calculation of electron drift length and time directly from simulations with a Monte-Carlo method. In Section 5 the simulation results and comparisons with the observations are presented. At last, we show in Section 6 the discussion and conclusions.

2. Observations

The Wind spacecraft was launched on 1994 November 1, and is located around the L1 Lagrangian point. The 3DP instrument onboard Wind is designed to measure the distribution of suprathermal electrons and ions in the solar wind. The EESA in the 3DP instrument measures the energy ranges of ~3–1000 eV and ~0.1–30 keV, respectively, for the low- and high-energy detectors. The 3DP instrument provides three-dimensional (3D) data with eight pitch angle channels, each covering 22°.5. For more details on the Wind 3DP instrument see Lin et al. (1995). The magnetic field data are measured by the magnetic field instrument (see, Farrell et al. 1995), and the plasma parameters in the solar wind such as bulk flow speed $V_{sw}$ and proton number density $n_p$ are provided by the Solar Wind Experiment instrument (Ogilvie et al. 1995).

As already mentioned above, we focus on the acceleration of electrons by the quasi-perpendicular shock on 2000 February 11. The shock arrived at Wind at 23:34 UT according to Wind IP shock list https://www.cfa.harvard.edu/shocks/wi_data/, with a shock-normal angle of $\theta_{bn} \sim 89^\circ$, a shock speed of $V_{sh} \sim 682 \text{ km s}^{-1}$, and a compression ratio $s \sim 2.87$ from calculations of YEA2018. The upstream magnetic field, solar wind bulk speed, and proton number density are set to be $B_{01} = 7.0 \text{ nT}$, $V_{sw} = 434 \text{ km s}^{-1}$, and $n_p = 5.19 \text{ cm}^{-3}$, respectively, by averaging the observational data over the time range of 23:20–23:30 (data used from the website https://cdaweb. sci.gsfc.nasa.gov). The upstream speed is set to be $U_{1} = V_{sh} - V_{sw} \sim 248 \text{ km s}^{-1}$ for simplicity. We obtain the upstream Alfvén speed $V_{A1} = 67 \text{ km s}^{-1}$, and Alfvén Mach number $M_{A1} = 3.70$.

3. IP Shock Models to Accelerate Electrons

We study here the acceleration of electrons at an IP shock by solving numerically the equation of motion of test particles, and such method has been used in previous studies (Decker & Vlahos 1986a, 1986b; Giacalone 2005; Giacalone & Jokipii 2009; Kong et al. 2017, 2019; Qin et al. 2018).

3.1. Physical Model

For simplicity, we consider a planar shock with the geometry shown in the cartoon of Figure 1. The shock is located at $z = 0$ with a thickness $L_{th}$. The plasma flows in the positive $z$-direction with the upstream and downstream speeds, $U_1$, $U_2$, respectively, in the shock frame. In this work, we usually use subscripts 1 and 2 to indicate the upstream and downstream, respectively. In the shock transition the plasma speed is assumed to be in the form of

\[ U(z) = \frac{U_1}{2s} \left\{ (s + 1) \tanh \left[ \tan \left( \frac{\pi z}{L_{th}} \right) \right] \right\}, \]  

(1)

Figure 1. Shock geometry used in the simulations.
where $s$ is the shock compression ratio. The motion equation of test particles is given by

$$\frac{dp}{dt} = q[E(r, t) + v \times B(r, t)],$$

(2)

where $p$ is the particle momentum, $v$ is the particle velocity, $q$ is the electron charge, and $t$ is time. The electric field $E$ is the convection electric field $E = -U \times B$. The total magnetic field consists of the background magnetic field and turbulent magnetic field, and is given by

$$B(x', y', z') = B_0 + b(x', y', z').$$

(3)

Note that the background magnetic field $B_0$ is in the $x$-$z$ plane. The input parameters for the shock are shown in Table 1. The value of shock thickness $L_{th}$ is set as $2 \times 10^{-6}$ au according to YEA2018.

The turbulent magnetic field is given by

$$b(x', y', z') = b_{slab}(z') + b_{2D}(x', y'),$$

(4)

where $b$ is a turbulent magnetic field perpendicular to $B_0$ with zero mean, and $(x', y', z')$ is the coordinate system with $z'$ in the direction of $B_0$. The turbulent magnetic field $b$ is composed of slab and 2D components with the energy density ratio assumed to be $E_{slab}: E_{2D} = 20:80$ (“two-component” model, Matthaeus et al. 1990; Zank & Matthaeus 1992; Bieber et al. 1996; Gray et al. 1996; Zank et al. 2006). We assume the slab correlation length $\lambda = 0.02$ au at 1 au, and the 2D correlation length $\lambda_2 = \lambda/2.6$ on the basis of previous studies (Osman & Horbury 2007; Weygand et al. 2009, 2011; Dosch et al. 2013). A dissipation range in which low-energy electrons resonate is included in the slab turbulence, as applied in the work of Qin et al. (2018). The break wavenumber $k_b$ from the inertial range to the dissipation range is assumed to be $k_b = 10^{-6}$ m$^{-1}$ based on the observational investigations in Leamon et al. (1999).

The values of spectral indices of the inertial and dissipation ranges ($\beta_i = 5/3$, $\beta_d = 2.7$) are set as the same as those in Qin et al. (2018). A periodic turbulence box with sizes $10\lambda \times 10\lambda$ and $25\lambda$ for the 2D and slab components, respectively, is adopted in the simulations. The turbulence levels of the upstream and downstream regions are taken to be $(b/B_0)^2 = 0.25$ and 0.36, respectively. The input parameters for the turbulence are shown in Table 2.

### 3.2. Numerical Model

Based on the Wind/3DP observations, we simulate PADs of 12 energy channels with central energies ~0.266, 0.428, 0.691, 1.116, 1.952, 2.849, 4.161, 6.076, 8.875, 12.96, 27.32, and 39.50 keV. A backward-in-time test-particle method is used to simulate the PAD of a given energy channel $E_i$ ($i = 1, 2, 3, \ldots, 12$) downstream of the shock. A total number of 30,000 electrons with an energy $E_i$ and a pitch angle $\mu_j$ are put into the downstream range $[z_0, z_1]$ at the initial time $t = 0$, where $z_0 = L_{th}/2$ and $z_1 = V_{th} \Delta t \approx 2.7 \times 10^{-3}$ au with $\Delta t = 10$ minutes. We take the spatial domain size in the $x$, $y$, $z$ directions to be $x_{box} = y_{box} = 10^3 \lambda$ and $z_{box} = 10^3 \lambda$. The trajectory of each electron is followed using an adaptive step fourth-order Runge–Kutta method with a normalized accuracy to $10^{-9}$ until the simulation time $t_{acc} = 10$ minutes. After the numerical calculations, a few electrons whose energy is less than $0.1E_i$ are discarded. The downstream PAD, $f_{dn}(E_i, \mu_j)$, for $E_i$ channel can be obtained as

$$f_{dn}(E_i, \mu_j) = \frac{1}{N_i} \sum_{k=1}^{N_i} f_k(E_{ik}, \mu_{jk}),$$

(5)

where $N_{ij}$ is the number of test particles in the statistics, $f_0(E_{ik}, \mu_{jk})$ is the initial distribution, and $E_{ik}$ and $\mu_{jk}$ are the $k$th particle energy and pitch angle, respectively, when it is traced back to the initial time. The initial distribution is constructed by averaging the 3DP data in the time period of 23:20–23:30 UT before the shock arrival. Note that we employ linear interpolation in log–log space between the adjacent particle energies to calculate the value of $f_k(E_{ik}, \mu_{jk})$. In addition, we do not consider wave excitation by the accelerated particles in this work in order to use test-particle method.

### 4. Drift Length and Time

#### 4.1. Estimation of Electron Drift Length and Time from Distribution Functions

We can apply Liouville’s theorem to consider electron acceleration following YEA2018. It is assumed that the electron phase space density is conserved after they are accelerated from the upstream to downstream of the shock if the downstream electrons are due to SDA mechanism, i.e.,

$$f_2(p_2) = f_1(p_1),$$

(6)

where $p$ and $f$ are the electron momentum and phase space density, respectively. Here, the subscripts 1 and 2 mean the upstream and downstream of the shock, respectively. The energy gain $\Delta E$ after the acceleration of upstream electrons with a momentum $p_1$ can be obtained through Equation (6).
The electron drift length is then written as

\[ L_{\text{drift}} = \frac{\Delta E}{q|E|}, \quad (7) \]

where \( E \) is the convection electric field.

According to Jokipii (1982), the gradient drift velocity at the shock front is

\[ \mathbf{v}_{\text{drift}} = \hat{z} \cdot \frac{p v}{3q} \left( \frac{B_{z1}}{B_1^2} - \frac{B_{z2}}{B_2^2} \right) \delta(z), \quad (8) \]

where \( B_1 \) and \( B_2 \) are the background magnetic fields with their \( x \)-components \( B_{x1} \) and \( B_{x2} \) upstream and downstream of the shock, respectively. We can integrate the above equation for \( z \) from \(-L_{\text{th}}/2\) to \( L_{\text{th}}/2\) to obtain the average gradient drift velocity

\[ \mathbf{v}_{\text{drift}} = \hat{z} \cdot \frac{p v}{3qL_{\text{th}}} \left( \frac{B_{z1}}{B_1^2} - \frac{B_{z2}}{B_2^2} \right). \quad (9) \]

In addition, the drift time, \( T_{\text{drift}} \), can be obtained by

\[ T_{\text{drift}} = \frac{L_{\text{drift}}}{\mathbf{v}_{\text{drift}}}. \quad (10) \]

Liouville’s theorem, with the phase space density of particles, is used to obtain the electron drift length and drift time. The approach of using observed phase space densities both upstream and downstream of the shock from YEAP2018 yields the results that are denoted as being from observations. If simulated downstream phase space densities are used instead, the electron drift length and drift time are denoted as being derived from simulations.

4.2. A Theoretical Model for Electron Drift Length and Time

Next, we provide a theoretical model based on SDA to describe the electron drift length and drift time. Since an electron’s gyroradius is much smaller than the shock thickness \( L_{\text{th}} \), it is assumed that an electron can be accelerated by SDA when it is in the quasi-perpendicular shock transition range. When particles are in the shock transition range they would easily move downstream through solar wind convection, so the electron drift time \( T_{\text{drift}} \) can be written as

\[ T_{\text{drift}} = \frac{L_{\text{th}}}{2U_i} + \frac{L_{\text{th}}}{2U_2}, \quad (11) \]

where \( U_i \) indicates the fluid speed upstream and downstream of the shock with \( i = 1 \) and \( i = 2 \), respectively. Since the shock thickness \( L_{\text{th}} \) and fluid convection speed \( U_i \) can be considered constant, the drift time \( T_{\text{drift}} \) is constant too, i.e., independent of the electron kinetic energy \( E \), and thus can be expressed as

\[ T_{\text{drift}} \propto E^0. \quad (12) \]

Furthermore, according to Equations (9), (10), and (11), the electron drift length \( L_{\text{drift}} \) can be calculated as

\[ L_{\text{drift}} = T_{\text{drift}} \mathbf{v}_{\text{drift}} = \frac{p v}{6q} \left( \frac{1}{U_1} + \frac{1}{U_2} \right) \left( \frac{B_{z1}}{B_1^2} - \frac{B_{z2}}{B_2^2} \right). \quad (13) \]

It is found that the electron drift length \( L_{\text{drift}} \) is proportional to the electron kinetic energy \( E \) if the relativistic effects are not considered, i.e.,

\[ L_{\text{drift}} \propto E. \quad (14) \]

4.3. Calculation of Electron Drift Length and Time Directly from Simulations with the Monte-Carlo Method

Using numerical simulations for the trajectory of each test particle, we can calculate the total displacement in the drift direction and drift time when the gyro-orbits of particles are in the shock transition region. We average over the upstream intensities of all test particles in the simulation to obtain the average drift length and drift time directly. These results are denoted as resulting from simulations based on a Monte-Carlo method.

5. Simulation Results and the Comparisons with Observations

Figure 2 shows the electron intensity versus pitch angle in the energy channels ranging from 0.266 to 39.50 keV for the shock on 2000 February 11. The 10 minute average (23:20 UT–23:30 UT) upstream electron intensities for the observations (blue diamonds) show an anisotropic distribution with higher values in the parallel (\( PA = 0^\circ \)) and anti-parallel (\( PA = 180^\circ \)) magnetic field directions and lower values in the perpendicular (\( PA = 90^\circ \)) direction except in the highest energy channel of 39.50 keV (Figure 2(l)), where the intensity in the anti-parallel direction is not higher than that in the perpendicular direction. The upstream average electron intensities also show that the anisotropy decreases as the electron energy increases. The downstream simulation results (black circles) are obtained using an initial distribution based on the 10 minute average upstream intensities from the observations. It is shown that in all energy channels ranging from 0.266 to 39.50 keV, the simulated downstream intensities increase much more compared with the upstream intensities around 90° pitch angle. We also plotted the 10 minute average (23:34 UT–23:44 UT) downstream electron intensities for the observations (red diamonds), which generally show an anti-parallel beam in the anti-parallel direction, except for the energy channels of 12.96 and 27.32 keV (Figures 2(j)–(k)). We note that there is poor agreement between the downstream simulations and the observations, especially at energies below 8.875 keV, which can be attributed to the occurrence of anti-parallel beams for the observations and weak shock acceleration efficiency for the simulations.

In Figure 3, we plot the ratio of the downstream to upstream average intensities for both the simulations (black circles) and observations (red diamonds). The simulated intensity ratio in all energy channels of 0.266–39.50 keV shows a peak at \( \sim 80^\circ–100^\circ \) pitch angles. The observed intensity ratio also has a peak around 90° pitch angle with the exception of 39.50 keV energy channel. This indicates that at quasi-perpendicular shocks, the strongest acceleration occurs at pitch angles around 90°.

For each pitch angle direction, the integral energy intensity \( I_{\text{th}} \) and \( I_{\text{sh}} \) in the regions upstream and downstream of the shock, respectively, are obtained by integrating the differential intensity over the energy range of 1.116–39.50 keV to pay attention to the more accelerated particles. In the upper panel of Figure 4, blue and red diamonds indicate the upstream and downstream observational integral energy intensities, respectively, and black circles indicate the simulated downstream intensity. From the figure it is shown that, for the observations,
the upstream integral energy intensity has the lowest value in around 90° pitch angle. However, the downstream integral energy intensity is much higher compared to the upstream in around 90° pitch angle, and relatively higher in the anti-parallel direction than in the parallel direction. In addition, the downstream integral energy intensity obtained from the simulations is higher in around 90° pitch angle and lower in the parallel directions. The lower panel of Figure 4 shows the ratio of the downstream to upstream integral energy intensities. It is shown that from both the observations (red diamonds) and simulations (black circles) the ratio reaches its peak around the perpendicular direction. This indicates that the shock acceleration efficiency is the strongest in the perpendicular direction according to the observations and simulations. We find that the ratio for the observations is larger than that from the simulations. The reason might be that there are anisotropic beams in the anti-sunward-traveling (anti-parallel) direction downstream of the shock (YEA2018).

Figure 2. Electron differential intensity vs. pitch angle in the energy channels of 0.266–39.50 keV marked with (a)–(l). The blue and red diamonds correspond to 10 minute average upstream (23:20 UT–23:30 UT) and downstream (23:34 UT–23:44 UT) intensities from observations, respectively, and the black circles are downstream results from simulations using an initial distribution based on the observed upstream intensities.

In Figure 5 we compare the energy spectra of electrons in the directions parallel (a), perpendicular (b), and anti-parallel (c) to the magnetic field. The blue and red diamonds denote 10 minute average electron intensities in the upstream and downstream of the shock, respectively. The black circles indicate the simulation results downstream of the shock. From the figure we can see that the observed downstream intensities are several times the upstream intensities in the perpendicular and anti-parallel directions, but they are similar in the parallel direction. In addition, the simulated downstream intensities increase significantly relative to the initial upstream intensities, which is set as the upstream observations, in the perpendicular...
direction, but they stay almost unchanged in the parallel and anti-parallel directions. It is shown that a more prominent intensity enhancement occurs in the perpendicular direction for both the observations and simulations, especially in the energy channel of 0.266 keV where the perpendicular intensity increases $\sim 30$ times for the observations and $\sim 5$ times for the simulations. We fit the electron spectrum in each direction as a power law with spectral indices $\alpha_{1,o}$, $\alpha_{2,o}$, and $\alpha_{2,s}$, where subscripts 1 and 2 denote upstream and downstream, and o and s denote the observations and simulations, respectively. The dashed line in each direction indicates a power-law fit to the simulation results downstream of the shock. We can see that the values of $\alpha_{1,o}$, $\alpha_{2,o}$, and $\alpha_{2,s}$ are larger than $\alpha_1 = 1.30$, which is predicted by DSA.

$$\alpha_1 = \frac{s + 2}{2s - 2}. \quad (15)$$

It is shown that the direction perpendicular to the magnetic field at the quasi-perpendicular shock front plays an important role in the shock acceleration of particles.

In addition, we plot the energy spectral index as a function of electron pitch angle in Figure 6. Blue and red diamonds show the results from the upstream and downstream observations, respectively. We also show the spectral indices downstream of the shock from the simulations with black circles. It is shown that the downstream spectral indices in the perpendicular direction from both the observations and simulations are larger than the upstream one. It is assumed that there is more effective shock acceleration for lower energy particles, and strong shock acceleration causes much higher flux enhancement in the lower energy range for energetic particles, generating softer downstream particle spectrum. Accordingly, the much softer downstream energy spectrum from both the observations and simulations relative to the upstream observations in the quasi-perpendicular direction indicates stronger shock acceleration in this direction. However, the observed spectral indices
downstream of the shock are higher than those upstream of the shock in the anti-parallel direction, the reason might also be the anti-sunward-traveling beams downstream of the shock as mentioned above.

The fact that the acceleration of electrons in the perpendicular direction is more efficient reveals the importance of SDA process at quasi-perpendicular shocks. Next, we calculate the electron drift length and drift time. As shown in Figure 7, the distribution functions, $f_1$ (blue dashed line) and $f_2$ (black dashed line), are obtained by a linear fit in log–log space to the data points from the observations upstream of the shock (blue diamonds) and the simulations downstream of the shock (black circles), respectively. We are able to obtain the energy gain $\Delta E$ after the acceleration of upstream electrons with a momentum $p_1$ with Equation (6), considering the same phase space density with Liouville’s theorem. The electron drift length and drift time are obtained from the simulations with Liouville’s theorem through Equations (7) and (10). In addition, following YEA2018, in the above method for the downstream distribution function $f_2$ using observational data we can obtain the electron drift length and drift time from the observations with Liouville’s theorem. Furthermore, when using numerical simulations for the test particles, we can directly calculate the drift length and drift time from simulations with the Monte-Carlo method.

Figures 8(a) and (b) show the electron drift length $L_{\text{drift}}$ and drift time $T_{\text{drift}}$ as a function of energy, respectively. The results from simulations with Liouville’s theorem and Monte-Carlo method are indicated by black and green circles (i.e., SIMU LT and SIMU MC), respectively. The results from observations with Liouville’s theorem in YEA2018 are indicated by red diamonds (i.e., OBSEr LT). It is shown in Figure 8(a) that the drift length obtained from simulations increases linearly with the electron energy in log–log space with a slope of $\sim 1.1$ for SIMU LT and a slope of $\sim 1.1$ for SIMU MC, which compare well with that from the observations with Liouville’s theorem.
with a slope of \( \sim 1.0 \). In Figure 8, a linear fit to the estimated drift time and electron energy in log–log space from the simulations yields a slope of 0.1 for SIMULT and a slope of 0.1 for SIMU MC, which agree approximately with that from the observations with Liouville’s theorem (OBSER LT) with a slope of 0.0. In other words, the drift time almost does not vary with the energy according to both the observations and simulations. We note that the estimated drift length and drift time from simulations with Monte-Carlo method (SIMU MC) agree well with that from observations with

![Figure 5](image1.png)

**Figure 5.** Electron energy spectra in the directions parallel (a), perpendicular (b), and anti-parallel (c) to the magnetic field. The blue (red) diamonds denote 10 minute average electron intensities upstream (downstream) of the shock. The black circles correspond to the downstream intensities obtained from simulations. The values of \( \alpha_{1,o}, \alpha_{2,o}, \) and \( \alpha_{2,s} \) correspond to the energy spectral indices of power-law fits to observational data in the upstream and downstream regions, and simulations downstream of the shock, respectively. Dashed lines indicate power-law fits to the simulation results downstream of the shock. Also denoted is the theoretical spectral index, \( \alpha_t \).

![Figure 6](image2.png)

**Figure 6.** Energy spectral index by a power-law fit to the energy spectrum in each pitch angle direction in Figure 2. The blue and red diamonds indicate the results from the observations upstream and downstream of the shock, respectively. The black circles indicate the results from simulations downstream of the shock.
Liouville’s theorem (OBSER LT), but they are larger than that from simulations with Liouville’s theorem (SIMU LT). The reason may be the insufficient accuracy and less efficient acceleration of electrons in our numerical model.

In addition, we use Equations (13) and (11) to calculate the theoretical results of electron drift length $L_{\text{drift}}$ and drift time $T_{\text{drift}}$, respectively. With the theories, it is found that the electron drift length is proportional to energy $E$ and drift time is independent of energy. We plot the theoretical results of drift length and drift time as blue solid lines in Figures 8(a) and (b), respectively. It can be seen that the theoretical results agree well with the observational ones with Liouville’s theorem.

6. Discussion and Conclusions

We have used test-particle numerical simulations of a backward-in-time test-particle method to study the acceleration of suprathermal electrons in the energy range of $\sim$0.3–40 keV at a quasi-perpendicular shock event on 2000 February 11 to compare with the observational study in YEA2018. We obtain electron PADs from simulations for all 12 energy channels, and find that the ratio of the downstream to upstream differential intensities peaks at about $\sim$90° pitch angle. These results are in good agreement with the spacecraft observations in YEA2018.

In addition, it is found that the observed and simulated electron energy spectral index for each pitch angle direction downstream of the shock is significantly larger than the theoretical index of DSA. The results indicate that SDA plays an important role in the acceleration of electrons at quasi-perpendicular shocks as suggested by YEA2018.

Furthermore, with Liouville’s theorem, considering SDA, YEA2018 used observational data to show that the electron drift length $L_{\text{drift}}$ is approximately proportional to the electron energy $E$. In addition, it is suggested that the drift time $T_{\text{drift}}$ is almost independent of the electron energy. We obtain the similar energy dependence of the drift length and drift time in simulations with Liouville’s theorem. Furthermore, we get similar results in simulations directly with the Monte-Carlo method. Next, we provide a theoretical model based on SDA to describe the electron drift length and drift time, which agree well with the observational results. This suggests that the proposed theoretical model can be used to explain the energy
dependence of electron drift length and drift time found by the observations and simulations.

It is shown that the work of this paper mainly confirms the results for electron acceleration at the quasi-perpendicular shock from YEA2018. However, our work is still beneficial. YEA2018 paid more attention to the observational data. It is objective to get scientific conclusions directly from the observations. However, data analysis alone lacks some details of the physical mechanism, so numerical simulations to follow energetic particles are useful for the study of shock acceleration. For example, it is found from the spacecraft observations that in the anti-parallel (PA = 180°) direction, the downstream energy intensities are much larger than the upstream energy intensities, so YEA2018 claimed that the reason might be the anisotropic beams in the anti-sunward-traveling (anti-parallel) direction downstream of the shock. In our results of numerical simulations for quasi-perpendicular shock acceleration without such kind of anisotropic beams, we do not get the enhanced downstream energy intensities in the anti-parallel direction, so the hypothesis of YEA2018 is supported. In addition, in order to verify YEA2018’s study of electrons drift length and drift time which uses observations with Liouville’s theorem, we use numerical simulations with two independent methods, i.e., Liouville’s theorem and Monte-Carlo methods. Furthermore, in this work, the formulae we provide to describe the electron drift length and drift time are also new.

The reason why the simulations show less acceleration and poor agreement with the observations is that we use a simple local shock acceleration model but the real condition can be complicated, such as the complex shock geometry and magnetic field. In addition, in the observations there are energetic particles with other sources, e.g., there may exist an anti-sunward-traveling beam of energetic electrons downstream of the shock in the observations, which does not appear in our shock acceleration model.

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Figure 8. The drift length $L_{\text{drift}}$ and drift time $T_{\text{drift}}$ as a function of electron energy in (a) and (b), respectively. Red diamonds are results from observations with Liouville’s theorem in YEA2018. Black and green circles are simulation results using Liouville’s theorem and Monte-Carlo methods, respectively. Blue dotted, black dashed-dotted, and green dashed lines indicate linear fits to the data with red diamonds, black circles, and green circles, respectively. Blue solid lines indicate the theoretical results.
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