Research article

State of charge estimation of ultracapacitor based on forgetting factor recursive least square and extended Kalman filter algorithm at full temperature range

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A B S T R A C T

State of charge (SOC) of ultracapacitor plays an important role in the energy management optimization of hybrid energy storage system for electric vehicles. In addition to the perfection of the model and the SOC estimation algorithm, the parameter identification method and temperature factor should also be considered. In this paper, an ultracapacitor test platform is established, the characteristic parameters of ultracapacitor at full temperature range are obtained. This paper uses the forgetting factor recursive least squares algorithm (FFRLS) to identify the parameters of the second-order equivalent circuit model of ultracapacitor online. The extended Kalman filter (EKF) algorithm is used to estimate the SOC of ultracapacitor cell. The results show that (1) FFRLS algorithm can identify \( R_0, R_s, R_C, C_0 \), and \( C_s \) values of ultracapacitor at full temperature range. Under the hybrid pulse power characterization working condition, the average mean absolute error between the estimated voltage and the actual voltage is about 0.0132 V. (2) EKF algorithm has a good adaptability to estimate SOC of ultracapacitor under different temperatures and working conditions. The SOC estimation error under different working conditions is low. From the perspective of mean square error, the estimation error at \(-20^\circ C\) is the lowest. (3) FFRLS and EKF joint estimation algorithm with good robustness and reliability can be used to estimate the SOC of ultracapacitor under different temperatures and working conditions. This study can provide a useful guidance for the parameter identification and SOC estimation of ultracapacitor for electric vehicle at different temperatures.

1. Introduction

The two major problems of global environmental pollution and energy shortage have intensified in recent years [1]. The electric vehicle industry has developed rapidly due to its advantages of zero pollution and environmentally friendly processes, which promotes the full implementation of peaking carbon dioxide emissions and carbon neutralization [2, 3]. Power battery (hereafter called battery) as a kind of single-energy storage device that continues to demonstrate some problems in electric vehicles, such as low power density and short cycle life. Hybrid storage systems have attracted researchers' attention because they can compensate for limitations of a single-energy storage component and develop advantages of components of multiple-energy storage system. Hybrid storage systems include many types, such as fuel cell combined with battery [4], ultracapacitor combined with superconducting magnetic energy storage [5], battery combined with ultracapacitor [6], ultracapacitor combined with fuel cell [7], flywheel combined with battery [8], compressed air energy storage combined with battery [9], ultracapacitor combined with flywheel and battery [10], and battery combined with fuel cell and ultracapacitor [11]. Among them, ultracapacitor presents advantages of high power density and fast response speed, while battery shows high energy density but need a long time to response. Considering vehicle requirements, two energy storage components, battery and ultracapacitor, are combined to form a hybrid energy storage system (HESS) as the on-board energy source. The HESS can meet not only power and energy demands during vehicle driving...
[12, 13, 14], but also alleviate the degradation of performance and life which is caused by frequently use of high-rate charging and discharging [15].

The state of charge (SOC) of HESS plays an important role in optimizing electric vehicle energy management, improving battery energy utilization, preventing overcharge and over-discharge of battery, ensuring the safety of battery in use, and prolonging lifetime of battery. Although the SOC of battery has been extensively investigated, studies on ultracapacitor are limited. The accurate SOC estimation is an important basis for the design of efficient and safe ultracapacitor management systems. It is divided into three steps: model establishment, parameters identification, and SOC estimation. Algorithms for SOC estimation include open circuit voltage (OCV) method, ampere-hour (Ah) integration method, artificial neural network (ANN) method, and Kalman filter (KF) algorithm. These algorithms have their own advantages and disadvantages. Appropriate estimation algorithms are selected in different application scenarios, while the joint estimation of Ah integration method and the KF algorithm is widely used in engineering due to its ease of implementation and high precision [16]. In fact, the KF algorithm is suitable for linear system. The improved state estimation methods are urgently needed to improve its estimation accuracy given that the ultracapacitor is a nonlinear system.

An accurate model is crucial in the SOC estimation of ultracapacitor. There are three kinds of ultracapacitor frameworks, namely, equivalent circuit model (ECM), ANN model, and electrochemical model. Wang et al. [17] dynamically identified characterization parameters of five different ECMS and revealed the optimal ultracapacitor model by comparing the accuracy of each model. Wu et al. [18] established a multilayer ANN model to determine the nonlinear dynamic relationship of ultracapacitors and adjust model parameters online. The results showed that the model can not only achieve system fidelity but also improve calculation efficiency. However, the mechanism of model parameter is unclear. Electrochemical models can not only achieve very high accuracy but also fully reflect electrochemical characteristics of ultracapacitors [19]. However, such models use a large number of parameters and partial differential equations to simulate the behavior of ultracapacitors. Complexity of partial differential equations leads to high computational cost and memory requirements that increase the difficulty in applying the electrochemical model in practice [20]. Zubiesta et al. [21] pointed out that the ECM only needs a few parameters to describe the charge and discharge process of ultracapacitors and presents high accuracy. Therefore, an ECM is established in this study.

The indirect SOC estimation of ultracapacitor depends on the accurate characterization parameter identification results [22]. Xie et al. [23] used the Levenberg-Marquardt method to identify model parameters. The simulation error of the nonlinear 3-branch ECM was smaller. The voltage error range of the linear 3-branch model was 1.5012 to 0.4961 V while the nonlinear 3-branch model was 0.8658 to 0.7490 V. Madhumitha et al. [24] proposed the marginalized Kalman filter (MKF) technique to estimate parameters. The MKF not only guaranteed stable convergence but also reduced the computational complexity yet achieving more accurate results. Although the traditional phased parameter identification method is simple and widely used, its model accuracy is low [22, 25, 26]. The parameter identification method of recursive least squares (RLS) must be recursive according to previous results. An unreliable initial value will lead to irreversible cumulative deviation [27, 28]. Wang et al. [29] presented the particle swarm optimization (PSO) based global optimization for online parameter identification with strict constraints. For the fractional-order model of the ultracapacitors, the mean absolute errors (MAEs) at 0 °C, 25 °C, and 45 °C were 26.4 mV, 37.3 mV, and 28.2 mV, respectively. The ANN method for parameter identification of ECM must train a large number of experimental data samples many times to achieve the optimal solution [18, 30]. The parameter identification method using Genetic Algorithm (GA) presents the disadvantage of low efficiency [17]. The experiment results showed that if the SOC is lower than 0.5, the proposed five ultracapacitor models are not suitable for further application because of the relatively poor accuracy and robustness. Zhao et al. [31] developed Segmentation Optimization (SO) method to identify parameters of ECM of ultracapacitor cell. The results of simulation and error analysis showed that SO is of higher recognition accuracy than circuit analysis method (CA) and circuit analysis-recursive least squares method (CA-RLS). Youssi et al. [32] constructed an improved metaheuristic approach of comprehensive learning marine predator algorithm (CLMPA) to identify the optimal parameters of the ultracapacitor ECM. And the CLMPA achieved a sum-squared error of 8.8757e−8. Tang et al. [33] proposed a migration-based framework for battery modeling, in which the effects of aging is treated as uncertainties at 5 °C, 25 °C, and 45 °C. The results indicate that the typical voltage prediction error can be limited within ±20 mV. For the cases of temperature change up to 40 °C, capacity degradation up to 20%. Wang et al. [34, 35] designed an improved feedforward-long short-term memory modeling method to realize an accurate whole-life-cycle SOC prediction by effectively considering the current, voltage, and temperature variations. The KF algorithm is often only applicable to dynamic models with few model parameters although it can realize accurate online identification of model parameters [36]. The extended (EKF) and unscented (UKF) Kalman filter algorithms are also applied despite their high complexity and computational cost [37, 38, 39]. Some researchers combine the two algorithms using a computer language to form an intelligent optimization algorithm, identify parameters of the battery model, solve the limitation of the linear relationship of the model, and improve the estimation accuracy of model parameters [40]. The accuracy of parameter identification results are insufficient because the simple RLS fails to track parameter changes effectively while other algorithms also demonstrate some limitations. Hence, the forgetting factor recursive least square (FFRLS) algorithm is designed in this study to identify ultracapacitor parameters. FFRLS can effectively solve the problem of unreliable initial value of the simple RLS method and remarkably reduce the calculation time because it presents advantages of simple and fast identification results [41].

The above literature review shows that few attempts have been devoted to discussing the effects of the full temperature range on the state estimation of ultracapacitor. Furthermore, the research on the SOC estimation of ultracapacitor mostly focuses on the improvement of the model and the optimization of the SOC estimation algorithm, and there is less research on the parameter identification algorithm. Therefore, an ultracapacitor test platform is built in this paper to study the ultracapacitor cell in full temperature range which are −20 °C, 0 °C, 25 °C and 45 °C [42]. Moreover, FFRLS algorithm is used to identify the parameters of the second-order ECM of ultracapacitor cell dynamically. The model is verified with the experimental data under different working conditions. Finally, the EKF algorithm is used to estimate the SOC of ultracapacitor cell [43]. Using FFRLS algorithm can significantly improve the accuracy of identification compared with using RLS algorithm. Analyzing the changes of ultracapacitor SOC at different temperatures lays a foundation for further research and design of temperature-based state estimation algorithm. Also, this work has great reference value for the design of vehicle energy management of HESS for electric vehicle at different temperatures.

The main research contents of this study are shown in Fig. 1. Module A which represents the test part is introduced in section 2. Module B which represents the identification part is given in section 3. Module C which represents the SOC estimation part is discussed in section 4. The result analysis is presented in section 5. Finally, the conclusion is drawn in section 6.

2. Model and experiment

2.1. Second-order equivalent circuit model of ultracapacitor

ECMs of the ultracapacitor include Rint, first-order resistance capacity (RC, also known as the Thevenin model), second-order RC, and
third-order RC models [30, 44]. The second-order RC model is selected for the experimental ultracapacitor given the computing power of its vehicle controller and its comprehensive consideration of dynamic and static characteristics of the ultracapacitor. The second-order RC model of the ultracapacitor simulates static ultracapacitor characteristics through equivalent series resistance as well as electrochemical and concentration polarizations during the dynamic response of the ultracapacitor via two equivalent parallel circuits [45]. The circuit structure of ECM is shown in Fig. 2. $R_0$ is the equivalent internal resistance, $U_0$ is the voltage in $R_0$, $U_{OC}$ is the open circuit voltage, $R_1$ and $R_2$ are internal resistance values of the branch, $C_1$ and $C_2$ are capacitance values of the branch, $U_1$ and $U_2$ are voltages of the branch, $I$ is the total current, and $U_L$ is the load terminal voltage. The discharge process is negative whereas the charge process is positive.

2.2. Test bench of ultracapacitor

The ultracapacitor cell produced by Maxwell is the research object in this study, whose farad capacity is 3000 F, rated voltage is 2.7 V, and rated capacity is 2.25 Ah. Table 1 lists the basic parameters of the ultracapacitor cell.

The ultracapacitor test platform is built in this study, which includes a thermotank (−40°C–150°C and humidity setting of 85%), a set of charge and discharge equipment (NEWARE CT-4008T-5V6A-S1-80 channel high-performance battery detection system), and a host computer to control operation and record data. The structure is shown in module A of Fig. 1. Given that the key point of this study is the SOC estimation of ultracapacitor at full temperature range, four temperatures of −20°C, 0°C, 25°C, and 45°C are selected for the experiment. The experiment includes the following steps. Firstly, set the experimental temperature and maintain at that temperature for 1 hour to allow the ultracapacitor to reach the stable state. Secondly, perform a cycle experiment on the ultracapacitor cell at the set temperature with five tests of static capacity (CAP), OCV-SOC, hybrid pulse power characterization (HPPC), typical China urban bus driving cycle (CUDC) working condition, and urban dynamometer driving schedule (UDDS) working condition. Finally, change the temperature of the thermotank after a cycle experiment and then carry out the same group of cycle tests until experimental tests under four different temperatures are completed. Specific steps of the experiment are further designed according to the FreedomCAR Ultracapacitor Test Manual.

![Fig. 1. Research framework.](image)

![Fig. 2. Second-order ECM of ultracapacitor cell.](image)

| Name                        | Numerical value |
|-----------------------------|-----------------|
| Rated capacitance           | 3000 F          |
| Rated voltage               | 2.7 V           |
| DC internal resistance      | 0.29 mΩ         |
| Maximum peak current        | 1900 A          |
| Maximum leakage current (25°C) | 5.4 mA         |
| Working temperature         | −40−65°C        |
| Mass                        | 510 g           |
| Length                      | 138 mm          |
| Diameter                    | 60.1 mm         |
| Long life                   | ≥5 years        |
The ultracapacitor cell capacity is clearly affected by temperature. Therefore, the capacity of ultracapacitor cell at full temperature range is obtained using the CAP test results. The ultracapacitor cell capacity can be calculated as equation (1) [38]:

\[ C = \frac{U_{\text{max}} \cdot C_{\text{max}}}{3600} \]  

Where \( C \) is the ampere-hour capacity of the ultracapacitor, \( U_{\text{max}} \) is the maximum voltage, and \( C_{\text{max}} \) is the maximum farad capacity of the ultracapacitor.

Fig. 3 shows the capacity changes of the ultracapacitor calculated from the experimental results under four different temperatures. The available capacity is 2.26 Ah at -20 °C, 2.3 Ah at 0 °C, 2.4 Ah at 25 °C, and 2.25 Ah at 45 °C. The available capacity at 0 °C is slightly higher than that at 45 °C and -20 °C. The available capacity of the ultracapacitor cell reaches the maximum at 25 °C. The cubic polynomial (2) is fitted using MATLAB to determine the relationship between ultracapacitor capacity and temperature, where \( p_1 = -4.615e-6, p_2 = 6.752e-5, p_3 = 0.005197, p_4 = 2.3 \). The fitting curve in red is illustrated in Fig. 3.

\[ C(x) = p_1 x^3 + p_2 x^2 + p_3 x + p_4 \]  

Where \( x \) represents the temperature of ultracapacitor.

Furthermore, the relationship between ultracapacitor SOC-OCV is obtained via experiments. Output equation (3) shows that if the current is disconnected and the ultracapacitor is still in the process of discharge, then \( U_0, U_1, \) and \( U_2 \) gradually approach zero at the end of discharge and the terminal voltage \( (U_L) \) rises and gradually approaches the open circuit voltage \( (U_{OC}) \) during ultracapacitor discharge. The measured \( U_L \) at this time can be used as the value of \( U_{OC} \). Because the SOC value of ultracapacitor can only be measured indirectly, the principle of Ah integration method is used for calculation. The SOC value obtained via this method is utilized as the comparison between actual and estimated SOCs. The terminal voltage \( (U_L) \) can be expressed as follows:

\[ U_L = U_{OC} - U_0 - U_1 - U_2 \]  

Fig. 4 shows the OCV-SOC relationship of ultracapacitor discharging at different temperatures. The SOC interval is \([0, 2]\) in OCV-SOC test. The enlarged part is the OCV value at different temperatures when SOC is 0.8. The experimental results show slight differences between the discharge start and end of SOC interval at various temperatures and in the same overall interval.

### 3. Parameter identification

#### 3.1. Mathematical model of ultracapacitor based on FFRLS

The relation between the model terminal voltage and capacitance resistance of the second-order ECM is expressed as equation (4):

\[ U_{OC} = [R_1(R_1C_1S + 1)^{-1} + R_2(C_2S + 1)^{-1} + R_3] \cdot I + U_1 \]  

Where \( S \) is the frequency domain.

The results of capacitor resistors \( R_0, R_1, R_2, C_1, \) and \( C_2 \) can be obtained through equation (5) discrete processing in the identification process:

\[
\begin{align*}
R_0 & = k_1 k_2^{-1} \\
R_1 & = (\tau_1 c + \tau_2 R_0 - d_k \tau_1 - \tau_2)^{-1} \\
R_2 & = c - R_1 - R_0 \\
C_1 & = \tau_1 R_1^{-1} \\
C_2 & = \tau_2 R_2^{-1}
\end{align*}
\]

Where \( \tau_1 \) and \( \tau_2 \) represent polarization time constants \( \tau_1 = R_1 C_1, \tau_2 = R_2 C_2, a, b, c, \) and \( d \) are intermediate parameters. \( k_1, k_2, k_3, k_4, \) and \( k_5 \) represent intermediate parameters as equation (6):

\[
\begin{align*}
k_1 & = (b \tau - 2a)(T^2 + b \tau + a)^{-1} \\
k_2 & = (2a)(T^2 + b \tau + a)^{-1} \\
k_3 & = (cT^2 + d \tau + a R_0)(T^2 + b \tau + a)^{-1} \\
k_4 & = (-d \tau - 2a R_0)(T^2 + b \tau + a)^{-1} \\
k_5 & = (a R_0)(T^2 + b \tau + a)^{-1}
\end{align*}
\]

The terminal voltage, terminal current, and previous voltage and current are known. The recurrence formula according to the theorem is expressed as equation (7):

\[
\begin{align*}
\dot{\theta}(k + 1) & = \dot{\theta}(k) + K(k + 1)[y(k + 1) - \varphi'(k + 1)\dot{\theta}(k)] \\
K(k + 1) & = P(k)\varphi(k + 1) + \lambda + \varphi'(k + 1)P(k)\varphi(k + 1)^{-1} \\
P(k + 1) & = \lambda^{-1}[I - K(k + 1)\varphi(k + 1)]P(k)
\end{align*}
\]

Where \( \varphi \) is the data vector, \( \theta \) is the parameter vector to be estimated, \( P(0) = \alpha I, \dot{\theta}(0) = \epsilon, \epsilon \) is an infinitesimal zero vector or a positive real vector, \( k \) is the discrete time, \( K(k) = P(k)\varphi(k), \) and \( \lambda \) is the forgetting factor. The parameters in equation (8) must be estimated:

\[ \theta = [k_1, k_2, k_3, k_4, k_5]^T \]
Where \( U \)

Therefore, experimental

3.2. Experimental data of parameter identification

The parameter identification of the ultracapacitor cell depends on experimental data of the HPPC working condition. An inadequate number of terms will lead to insufficient accuracy whereas an excessive number of terms will increase the calculation time of the computer. Therefore, the following eight-term formula is used in this study to fit the OCV-SOC relationship as equation (9):

\[
U_{OC} = f(SOC) = a_1 \cdot \kappa_{SOC} + a_2 \cdot \delta_{SOC} + a_3 \cdot \lambda_{SOC} + a_4 \cdot \zeta_{SOC} + a_5 \cdot \gamma_{SOC} + a_6 \cdot \delta_{SOC} + a_7 \cdot \lambda_{SOC} + a_8
\]

Where \( \zeta_{SOC} \) is the value of SOC, \( a_i \sim a_8 \) is the fitting coefficient.
Fig. 6. Parameter identification at different temperatures.

3.3. Parameter identification results

FFRLS is used to identify parameters of the second-order ECM, and values of $R_0$, $R_1$, $R_2$, $C_1$, and $C_2$ of important parameters are obtained. The estimated voltage value is compared with the actual voltage value, and the identification error is calculated. Tables 2, 3, 4, and 5 present the characteristic parameter values of the second-order ECM identified via the FFRLS at different temperatures.

Fig. 6 shows the identification results of $R_0$, $R_1$, $R_2$, $C_1$, and $C_2$ at different temperatures. The analysis of Fig. 6(a) demonstrates that $R_0$ values are similar at 25 °C and 45 °C but slightly decrease with the in-
increase of SOC. $R_0$ values are similar at $-20 \, ^\circ C$ and $0 \, ^\circ C$, and show an upward trend with the increase of SOC with the minimum value occurring at SOC=0.3. The analysis of Fig. 6(b) presents that $R_1$ is higher at $25 \, ^\circ C$ than at other temperatures when the SOC interval is [0.2,0.6]. The analysis of Fig. 6(c) demonstrates that $R_2$ has little overall change at $0 \, ^\circ C$, increases first and then decreases at $-20 \, ^\circ C$, and increases at 25 $^\circ C$ and 45 $^\circ C$ in general. $R_2$ is higher at $-20 \, ^\circ C$ than at other temperatures when the SOC interval is [0.2,0.8]. The analysis of Fig. 6(d) exhibits that $C_1$ is higher at $-20 \, ^\circ C$ than at other temperatures when the SOC interval is [0.2,1]. The analysis of Fig. 6(e) shows that $C_2$ is higher at $45 \, ^\circ C$ than at other temperatures when the SOC interval is [0.2,0.9]. The identification results of $R_{0}$, $R_1$, $R_2$, $C_1$, and $C_2$ are evidently different at full temperature range but the overall trend of each parameter with SOC is similar. Hence, the strong reliability of identification results of the ultracapacitor at different temperatures is demonstrated.

Since the capacity of ultracapacitor is mainly affected by the change of temperature and internal concentration, the RC parameter value of ultracapacitor will also be affected by the joint action of temperature and internal concentration during charge and discharge. The change of $R_1$ at $25 \, ^\circ C$ is more definite. It is speculated that the reason may be that the temperature has a weak influence on the parameter value of ultracapacitor at $25 \, ^\circ C$, mainly because the change of internal concentration affects the parameter value of ultracapacitor. $R_1$ and $R_2$ represent the parameter changes of static branch and dynamic branch inside the ultracapacitor, respectively. $R_2$ mainly modifies the static branch, so it has little correlation with temperature.

The error between estimated and actual voltage values under the HPPC working condition at four temperatures is presented in Table 6. MAX is the maximum error value obtained by comparing the absolute error value of each time. The MAE and MSE can be expressed as equations (10)–(11):

$$
\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i| 
$$

(10)

$$
\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 
$$

(11)

Where $\hat{y}_i$ and $y_i$ are the estimated value and experimental data, respectively. $n$ is the number of results.

It can be seen that the minimum MAX is 0.0434 V at 25 $^\circ C$, the minimum MAE is 0.0050 V at 45 $^\circ C$, and the minimum MSE is 0.0003 V at 25 $^\circ C$. All errors reach the maximum at 0 $^\circ C$. The results show that the average MAE is approximately 0.0132 V at different temperatures.

4. SOC estimation

4.1. Mathematical model of SOC estimation based on the EKF

The KF algorithm is an optimal state estimation method that can be applied to dynamic systems with random disturbances. This study uses the EKF algorithm with Taylor series expansion for recursive state estimation of nonlinear dynamic systems. First, the condition is initialized as follows: set the covariance to 0 and input the initial state values of SOC, $U_{1}(k)$, and $U_{2}(k)$, which are obtained from the experiment. The state equation of the EKF algorithm and the observation equation are expressed as equations (12)–(13):

$$
x(k + 1) = f[k, X(k)] + \nu(k) 
$$

(12)

$$
z(k) = h[k, X(k)] + \nu(k) 
$$

(13)

Where $f$ is the nonlinear function of the state equation, $x(k+1)$ represents the generalized estimated state value and $z(k)$ represents the generalized measured state value, $\nu(k)$ is the input noise that obeys the Gaussian distribution and corresponds to the noise of each component in $x(k+1)$. $h$ is the transformation matrix from the state variable to the observation. $\nu(k)$ is the observation noise that obeys the Gaussian distribution.

Second, the EKF algorithm is predicted and updated. Prediction evaluates the state of the current time $k$ according to the posteriori estimate of the previous $k-1$ and obtains the priori estimate of time $k$. Update uses the measured value at the current time to correct the estimated value at the prediction stage to obtain the estimated posteriori value at the current time. Time and status updates are expressed as equations (14)–(15):

$$
\begin{align*}
\hat{X}(k+1|k) &= f(\hat{X}(k|k)) \\
\hat{P}(k+1|k) &= A(k+1|k)\hat{P}(k|k)A^T(k+1|k) + W(k+1) \\
K(k+1) &= \frac{P(k+1|k)H^T(k+1)}{H(k+1)...P(k+1|k)H^T(k+1) + V(k+1)} \\
\hat{X}(k+1|k+1) &= \hat{X}(k+1|k) + K(k+1)...[Z(k+1) - h(\hat{X}(k+1|k))] \\
\hat{P}(k+1|k+1) &= [I - K(k+1)H(k+1)]P(k+1|k+1) \\
\end{align*}
$$

(14)

(15)

Where $P$ is the covariance of the state variable, $K$ is the Kalman gain, $W$ is the covariance matrix of the state noise, and $V$ is the covariance matrix of the observation noise.

Finally, the state equation of ultracapacitor ECM under the EKF algorithm and the observation equation can be obtained by combining relationship equations (3) and (4) of the second-order ECM of ultracapacitor as equations (16)–(17):

$$
\begin{align*}
\xi_{SOC}(k) &= 1 - \frac{\xi_{SOC}(k-1) \Delta t}{C} \\
U_{1}(k) &= U_{OC}(k) - I(k-1) - U_{2}(k) - i(k)R_{0}(k) + \nu(k) \\
U_{1}(k) &= \frac{1}{\nu_1(k)} - \frac{\nu_2(k)}{\nu_1(k)} \frac{\nu_1(k)}{I(k-1) \Delta t} \\
\end{align*}
$$

(16)

(17)

Where $\xi_{SOC}$ represents the state of charge of the ultracapacitor, $U_{OC}(k)$ is the open circuit voltage at the time of $k$ that demonstrates a function relationship with SOC-OCV, and $\eta$ is the coulomb efficiency.

4.2. SOC estimation results

Fig. 7 shows the comparison between estimated and actual SOC values under the HPPC working condition at different temperatures. The consistency between SOC estimated and actual values at four temperatures indicates the acceptable adaptability of the EKF algorithm. The accuracy of SOC estimation results at 0 $^\circ C$ is lower than that at other temperatures and the estimation error shows an increasing trend with the decrease of SOC under the HPPC working condition.

Fig. 8 shows the SOC estimation errors at different temperatures. It can be seen from the figure that the estimation results of EKF algorithm have good followability on the whole. In the enlarged area of Fig. 8, it can be seen that the error changes significantly in the interval of frequent charge and discharge. The larger the charge and discharge current is, the greater the estimation error increases, indicating that the random error value of EKF algorithm is also large in the obvious current change interval.
5. Verification and discussion

5.1. Identification and estimation results under the CUDC working condition

Fig. 9 shows the current variation under the CUDC working condition.

Fig. 10 shows the verification of parameter identification results under the CUDC working condition at different temperatures. The voltage is high while the SOC is large. And the estimated voltage based on identified parameters basically coincides with the actual voltage at the beginning of discharge. However, the voltage and the SOC value decreases and the error increases slightly with further discharge. The estimated voltage obtained according to the identification results basically coincides with the actual voltage and the error is low in the entire discharge process.
Fig. 9. Current variation under the CUDC working condition.

Fig. 10. Estimated voltage results under the CUDC working condition.

Fig. 11 presents the verification of SOC estimation results under the CUDC working condition. Basically, the SOC estimation results of the entire discharge process coincide with actual SOC values with small error. Among the results, the estimation at −20 °C is the most accurate.

5.2. Identification and estimation results under the UDDS working condition

Experimental steps under the UDDS working condition are the same as those under the CUDC working condition. Fig. 12 shows the current variation under the UDDS working condition at different temperatures.
Fig. 11. SOC estimated results under the CUDC working condition.

Fig. 12. Current variation under the UDDS working condition.
Fig. 13 presents the verification of parameter identification results under the UDDS working condition. The estimated voltage according to the parameter identification results basically coincides with the actual voltage in the entire discharge process.

Fig. 14 illustrates the verification of SOC estimation results under the UDDS working condition. The SOC estimation results of the entire discharge process basically coincide with actual SOC values with small error. The SOC estimation result at −20 °C is the most accurate.

5.3. Errors under different conditions

Figs. 15(a) and 15(b) show the parameter identification errors under the CUDC and UDDS working conditions, respectively. Figs. 15(c) and 15(d) present the estimation errors under the CUDC and UDDS working conditions, respectively. Parameter identification and estimation errors reach the maximum at 0 °C. Parameter identification errors at −20 °C and 45 °C are basically the same, but the estimation error of 45 °C is significantly greater than that of −20 °C. Overall, optimal estimation results are achieved at −20 °C.

Parameter identification and SOC estimation errors under the CUDC and UDDS working conditions are analyzed according to the obtained parameter identification results of ultracapacitor at different temperatures. Table 7 presents the parameter identification error at different temperatures. The minimum value of MAX is 0.0065 V at 25 °C and the maximum value of MAX is 0.1545 V at 0 °C. The minimum value of MAE is 0.0058 V at 45 °C and the maximum value of MAE is 0.0497 V at 25 °C. The minimum value of MSE is 0.0009 V at −20 °C and the maximum value of MSE is 0.0042 V at 0 °C. The results show that the average MAE is approximately 0.0201 V under different working conditions at full temperature range.

The SOC estimation error of the ultracapacitor is listed in Table 8. The minimum value of MAX is 0.0330 at −20 °C and the maximum value of MAX is 0.0823 at 0 °C. The minimum value of MAE is 0.0088 at −20 °C and the maximum value of MAE is 0.0357 at 0 °C. The minimum value of MSE is 0.0002 at −20 °C and the maximum value of MSE is 0.0014 at 0 °C. The results show that the average of MAE is approximately 0.0233 under different working conditions at full temperature range.

From Fig. 6 (a), it is obvious that the change of internal resistance $R_i$ is large at −20 °C and 0 °C, while the change of $R_0$ is relatively gentle at 25 °C and 45 °C. And there is a mutation about $R_0$ when SOC equals 0.2 at 0 °C. From Fig. 7 and Fig. 11, the estimation error at 0 °C is large. The reason is that $R_0$ changes greatly and the capacitance reaches the cut-off voltage quickly when a large current occurs at 0 °C. At the same time, the algorithm does not compensate it. Therefore, the error at 0 °C is relatively large. Fig. 6 (b) and (c) show that $R_i$
Fig. 14. SOC estimation results under the UDDS working condition.

Table 7. Identification error of Voltage under different working conditions.

| Temperature/Test | Error of Voltage | −20 °C | 0 °C | 25 °C | 45 °C |
|------------------|-----------------|--------|------|-------|------|
| MAX(V)           |                 | 0.0661 | 0.1545 | 0.0077 | 0.0639 |
| CUDC             | MAE(V)          | 0.0099 | 0.0135 | 0.0497 | 0.0058 |
|                  | MSE(V)          | 0.0010 | 0.0042 | 0.0034 | 0.0011 |
| MAX(V)           |                 | 0.0437 | 0.1492 | 0.0065 | 0.0649 |
| UDDS             | MAE(V)          | 0.0139 | 0.0131 | 0.0472 | 0.0080 |
|                  | MSE(V)          | 0.0009 | 0.0042 | 0.0030 | 0.0011 |

Table 8. SOC calculation error under different working conditions.

| Temperature/Test | Error of SOC | −20 °C | 0 °C | 25 °C | 45 °C |
|------------------|--------------|--------|------|-------|------|
| MAX              |              | 0.0330 | 0.0823 | 0.0545 | 0.0676 |
| CUDC             | MAE          | 0.0088 | 0.0357 | 0.0212 | 0.0260 |
|                  | MSE          | 0.0002 | 0.0020 | 0.0006 | 0.0011 |
| MAX              |              | 0.0406 | 0.0713 | 0.0547 | 0.0684 |
| UDDS             | MAE          | 0.0177 | 0.0270 | 0.0204 | 0.0292 |
|                  | MSE          | 0.0004 | 0.0014 | 0.0006 | 0.0012 |

Table 7. Identification error under different working conditions.

Table 8. SOC calculation error under different working conditions.

and $R_2$ change gently at 0 °C. It can be seen that the change of internal resistance has a greater impact on the accuracy of estimation than branch resistance. It is concluded that the algorithm is not the only factor affecting the estimation results, and temperature is also an important factor.

6. Conclusions

The joint estimation of FFRLS and EKF algorithm is proposed in this paper to estimate SOC of the ultracapacitor cell. Firstly, an ultracapacitor test platform is built. Charge and discharge characteristics are investigated at different temperatures. Secondly, dynamic parameters of the ultracapacitor cell are identified by using FFRLS algorithm, and corresponding parameters of resistance and capacitance of the second-order ECM at different temperatures are obtained. Finally, the EKF algorithm is used to estimate the SOC of ultracapacitor which is verified under CUDC and UDDS working conditions. The following conclusions can be drawn from this study:

1. The FFRLS algorithm shows satisfactory robustness, which can identify $R_0$, $R_1$, $R_2$, and $C_1$ values of the ultracapacitor cell at different temperatures. The average MAE is approximately 0.0132 V under the HPPC working condition.

2. The EKF algorithm demonstrates satisfactory adaptability. SOC estimation errors at different temperatures and under different working conditions are small. The comparison of MSE values showed that the estimation error at −20 °C is the minimum.
(3) The proposed FFRLS and EKF algorithm present acceptable robustness and are suitable for different temperatures and working conditions. What's more, not only the algorithm is a cause that affects the estimation results of state of charge of ultracapacitor, but the temperature is also an important factor.

In the future, the further optimization of the algorithm will be investigated. It is expected to design a temperature-based parameter identification algorithm for ultracapacitor cell. Thus, the SOC of ultracapacitor cell in the full temperature range can be estimated more accurately, which can be used as the basis for designing energy management of HESS for electric vehicle.

**Declarations**

**Author contribution statement**

Jing Ren: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper. Yonghong Xu: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data. Hongguang Zhang, Fubin Yang: Contributed reagents, materials, analysis tools or data. Yifang Yang, Xu Wang, Peng Jin, Denggao Huang: Performed the experiments.

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**Data availability statement**

Data included in article/supp.material/referenced in article.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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