Hybrid Inflation with Quasi-canonical Supergravity

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Abstract

We construct a hybrid inflationary model associated with the superheavy scale $M_X \simeq 2 \times 10^{16} \text{GeV}$ of the minimal supersymmetric standard model which is based on the simplest superpotential for symmetry breaking and in which the inflaton potential along the inflationary trajectory is essentially provided by quasi-canonical supergravity. The resulting spectrum of adiabatic density perturbations is blue and the duration of inflation sufficient but rather limited.
Linde’s hybrid inflationary scenario \[1\] is certainly superior over all its predecessors \[2\] because it does not suffer from any serious generic naturalness problems but also because it succeeded, after a long time, in reconnecting inflation with phase transitions in grand unified theories (GUTs). This new inflationary model looks as a hybrid of chaotic inflation with a quadratic potential for the gauge singlet inflaton and the usual theory of spontaneous symmetry breaking involving a possibly gauge non-singlet field. During inflation the non-inflaton field finds itself trapped in a false vacuum state and the universe expands quasi-exponentially dominated by the almost constant false vacuum energy density. Inflation ends with a very rapid phase transition when the non-inflaton field rolls to its true vacuum state (“waterfall”).

Although the original hybrid model is non-supersymmetric it is so readily adaptable to supersymmetry (SUSY) that one would have easily thought that it was invented with SUSY in mind. The simplest and most commonly used superpotential for symmetry breaking and an inflaton mass of the order of 1 TeV, the SUSY breaking scale, gives rise to Linde’s hybrid model with an intermediate scale \((\sim 10^{11} - 10^{12} \text{ GeV})\) of symmetry breaking \[3\]. Moreover, the possibility of imposing R-symmetries in SUSY models in order to naturally forbid large self-couplings of the inflaton \[4\] should be regarded as an additional motivation for supersymmetry. In the context of supersymmetry it would certainly be desirable to associate hybrid inflation with the superheavy symmetry breaking scale \(M_X \simeq 2 \times 10^{16} \text{ GeV}\) which is consistent with the unification of the gauge coupling constants of the minimal supersymmetric standard model (MSSM). However, the electroweak mass of the inflaton provided by SUSY breaking proved too weak to account for the correct value of the observed temperature fluctuations \(\frac{\Delta T}{T}\) in the cosmic background radiation and soon the need for an appropriate inflaton potential became apparent. The first attempt in this direction was to employ radiative corrections \[4\] in the context of the simplest superpotential. This scenario, however, turned out to lead to scales smaller than the MSSM scale. Afterwards, a variation of the simplest model involving a non-renormalizable superpotential \[5\] was successful in obtaining the MSSM value of the scale.
Replacing global by local supersymmetry is a highly non-trivial extension because supergravity makes the potential very steep and typically forbids inflation through the generation of an inflaton mass larger than the Hubble constant \( H \). For the simplest superpotential, which during inflation could be regarded as consisting of just a linear term in the inflaton superfield, the disastrous generation of an inflaton mass-squared term is avoided provided the canonical form of the Kähler potential of \( N = 1 \) supergravity is employed \(^3\). However, even in this case supergravity is expected to affect global SUSY inflationary scenarios, especially if during inflation the inflaton takes values close to the supergravity scale \( M_P/\sqrt{8\pi} \approx 2.4355 \times 10^{18} \text{ GeV} \) \( (M_P \approx 1.221 \times 10^{19} \text{ GeV} \) is the Planck mass).

In order to illustrate in the clearest possible way the effects of supergravity on hybrid inflationary models we investigated \(^3\) the possibility that during inflation the inflaton potential is provided entirely by the terms generated when global supersymmetry is replaced by canonical supergravity and therefore canonical supergravity is primarily responsible for the generation of the inflationary density perturbations. Our supergravity dominated inflationary scenario turned out to have rather interesting and distinctive properties. Inflation has a very limited but still sufficient duration and the spectral index of the adiabatic density perturbations is considerably larger than unity (blue primordial spectra \(^4\)) and strongly varying. Although we succeeded in constructing a hybrid inflationary model associated with the superheavy scale of SUSY GUTs the MSSM value of the scale could be obtained naturally only in the case of a model involving two non-inflaton fields. In the context of the simplest superpotential the MSSM scale was obtained with a choice of very weak coupling. Larger scales could be obtained more naturally.

The effects of canonical supergravity on the model based on the simplest superpotential were also investigated in ref. \(^5\) in the context of a combined scenario involving large radiative corrections as well. Although the combined scenario employs natural values of the parameters the vacuum expectation value (vev) of the non-inflaton field is typically smaller than the MSSM scale. To obtain the MSSM value of the scale one has to make a choice of parameters leading to an unacceptably large spectral index.
Our purpose in the present paper is to extend our discussion of the supergravity dominated hybrid inflationary scenario \cite{6} to the case of a Kähler potential which deviates from the minimal one assuming, of course, that inflation is still allowed. We confine ourselves to the simplest superpotential for symmetry breaking and we parametrize the deviation from canonical supergravity with the size of the generated inflaton mass-squared term. By allowing deviations from canonical supergravity, apart from alleviating the fine-tuning that the minimal choice of the Kähler potential necessarily entails, we succeed in obtaining the MSSM value for the symmetry breaking scale in the context of the simplest superpotential and for various natural values of the parameters. The resulting scenario gives rise again to a limited number of e-foldings and to blue primordial spectra, although there is a tendency for smaller values of the spectral index. The effects of radiative corrections are taken into account as well.

We consider a SUSY GUT based on a (semi-simple) gauge group $G$ of rank $\geq 5$. $G$ breaks spontaneously directly to the standard model (SM) gauge group $G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ at a scale $M_X \sim 10^{16}$ GeV. The symmetry breaking of $G$ to $G_S$ is obtained through a superpotential which includes the terms

$$W = S(-\mu^2 + \lambda \Phi \bar{\Phi}).$$

Here $\Phi, \bar{\Phi}$ is a conjugate pair of left-handed SM singlet superfields which belong to non-trivial representations of $G$ and reduce its rank by their vevs, $S$ is a gauge singlet left-handed superfield, $\mu$ is a superheavy mass scale related to $M_X$ and $\lambda$ a real and positive coupling constant. The superpotential terms in eq. (1) are the dominant couplings involving the superfields $S, \Phi, \bar{\Phi}$ which are consistent with a continuous R-symmetry under which $W \rightarrow e^{i\gamma} W$, $S \rightarrow e^{i\gamma} S$, $\Phi \rightarrow \Phi$ and $\bar{\Phi} \rightarrow \bar{\Phi}$. Moreover, we assume that the presence of other SM singlets in the theory does not affect the superpotential in eq. (1). The potential obtained from $W$, in the supersymmetric limit, is

$$V = \left| -\mu^2 + \lambda \Phi \bar{\Phi} \right|^2 + \left| \lambda S \right|^2 \left( \left| \Phi \right|^2 + \left| \bar{\Phi} \right|^2 \right) + D - \text{terms},$$

where $D$ includes the D-term contributions.
where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. The SUSY vacuum

$$< S > = 0, < \Phi > < \bar{\Phi} > = \mu^2 / \lambda, \quad | < \Phi > | = | < \bar{\Phi} > |$$

lies on the D-flat direction $\Phi = \bar{\Phi}^*$. By appropriate gauge and R-transformation on this D-flat direction we can bring the complex $S$, $\Phi$, $\bar{\Phi}$ fields on the real axis, i.e. $S \equiv \frac{1}{\sqrt{2}} \sigma$, $\Phi = \bar{\Phi} \equiv \frac{1}{2} \phi$, where $\sigma$ and $\phi$ are real scalar fields. The potential in eq. (2) then becomes

$$V(\phi, \sigma) = \left( -\mu^2 + \frac{1}{4} \lambda \phi^2 \right)^2 + \frac{1}{4} \lambda^2 \sigma^2 \phi^2$$

and the supersymmetric vacuum corresponds to $| < \frac{\phi}{2} > | = \frac{\mu}{\sqrt{\lambda}} = \frac{M_X}{g}$ and $< \sigma > = 0$, where $M_X$ is the mass acquired by the gauge bosons and $g$ is the gauge coupling constant. For any fixed value of $\sigma > \sigma_c$, where $\sigma_c = \sqrt{2} \mu / \sqrt{\lambda} = \sqrt{2} | < \frac{\phi}{2} > |$, $V$ as a function of $\phi$ has a minimum lying at $\phi = 0$. The value of $V$ at this minimum for every value of $\sigma > \sigma_c$ is $\mu^4$.

Adding to $V$ a mass-squared term for $\sigma$ we essentially obtain Linde’s potential. When $\sigma > \sigma_c$ the universe is dominated by the false vacuum energy density $\mu^4$ and expands quasi-exponentially. When $\sigma$ falls below $\sigma_c$ the mass-squared term of $\phi$ becomes negative, the false vacuum state at $\phi = 0$ becomes unstable and $\phi$ rolls rapidly to its true vacuum thereby terminating inflation.

Let us now replace global supersymmetry by $N = 1$ canonical supergravity. From now on we will use the units in which $\frac{M_P}{\sqrt{8\pi}} = 1$. Then, the potential $V(\phi, \sigma)$ becomes

$$V(\phi, \sigma) = \left[ (-\mu^2 + \frac{1}{4} \lambda \phi^2)^2 (1 - \frac{\sigma^2}{2} + \frac{\sigma^4}{4}) + \frac{1}{4} \lambda^2 \sigma^2 \phi^2 (1 - \frac{\mu^2}{\lambda} + \frac{1}{4} \phi^2)^2 \right] e^{\frac{1}{2} (\sigma^2 + \phi^2)}.$$

$V$ still has a minimum with $V = 0$ at $| \frac{\phi}{2} | = \frac{\mu}{\sqrt{\lambda}}$ and $\sigma = 0$ and a critical value $\sigma_c$ of $\sigma$ which remains essentially unaltered. The important difference lies in the expression of $V(\sigma)$ for $\sigma > \sigma_c$ and $\phi = 0$

$$V(\sigma) = \mu^4 (1 - \frac{\sigma^2}{2} + \frac{\sigma^4}{4}) e^{\frac{\sigma^2}{2}},$$

which now is $\sigma$-dependent. Obviously the inflaton potential $V(\sigma)$ during inflation is obtainable from the simple linear superpotential
\[ W = -\mu^2 S, \]  

(7)

with the choice

\[ K = | S |^2 \]  

(8)

for the Kähler potential. Expanding \( V(\sigma) \) in powers of \( \sigma^2 \) and keeping the first non-constant term only we obtain

\[ V(\sigma) \simeq \mu^4 + \frac{1}{8}\mu^4\sigma^4 \quad (\sigma^2 << 1). \]  

(9)

We see that no mass-squared term for \( \sigma \) is generated \[ \text{[3]}. \]

Allowing deviations from the canonical form of the Kähler potential \( K \) of eq. (8) which respect the R-symmetry we are led to a Kähler potential

\[ K = | S |^2 - \frac{\beta}{4} | S |^4 + \ldots \]  

(10)

By an appropriate choice of the omitted terms in the expansion of \( K \) in eq. (10) we can arrange for a potential whose expansion in powers of \( \sigma^2 \) (keeping the first two non-constant terms only) is

\[ V(\sigma) \simeq \mu^4 + \frac{1}{2}\beta\mu^4\sigma^2 + \frac{1}{8}\mu^4\sigma^4 \quad (\sigma^2 << 1). \]  

(11)

The model now resembles the original hybrid inflationary model with a quadratic \( \frac{1}{2}m^2\sigma^2 \) term and an additional quartic \( \frac{1}{4}\kappa\sigma^4 \) term, where \( m^2 = \beta\mu^4 \) and \( \kappa = \frac{1}{7}\mu^4 \). The derivative of \( V(\sigma) \) with respect to \( \sigma \) is

\[ V'(\sigma) \simeq \frac{\mu^4}{2\sigma}(2\beta\sigma^2 + \sigma^4) \quad (\sigma^2 << 1). \]  

(12)

In the following we are going to study inflation in the context of the simple model of eq. (1) with an almost-canonical Kähler potential \( K \) identifying the properly normalized inflaton field with \( \sigma \) (thus neglecting the effect of the non-canonical kinetic term) and approximating the inflaton potential with the constant term only

\[ V(\sigma) \simeq \mu^4. \]  

(13)
For the derivative $V'(\sigma)$ of $V(\sigma)$ we are going to use the expression

$$V'(\sigma) \simeq \frac{\mu^4}{2\sigma} \left[ \left( \frac{\lambda}{2\pi} \right)^2 + 2\beta \sigma^2 + \sigma^4 \right].$$

(14)

The first term in the above expression is the contribution of radiative corrections along the inflationary trajectory. We assume that $\frac{1}{3} > \beta > \frac{\lambda}{2\pi}$. The inequality $\beta < \frac{1}{3}$ is necessary in order that $\frac{\lambda^2}{3\pi} < 1$ ($H = \frac{\mu}{\sqrt{3}}$ is the Hubble constant during inflation). The assumption $\beta > \frac{\lambda}{2\pi}$ is made since we are primarily interested in a scenario in which the deviation from canonical supergravity is as large as possible and the effect of radiative corrections as small as possible. The coupling $\lambda$ is determined by requiring that the vev of the non-inflaton field takes the MSSM value $|< \hat{\phi}_2 | = \frac{M_X}{g} \simeq 0.011731$ ($M_X \simeq 2 \times 10^{16}$ GeV, $g \simeq 0.7$):

$$\lambda = \left( g \frac{\mu}{M_X} \right)^2.$$  

(15)

The critical value $\sigma_c$ is fixed by the relation

$$\sigma_c = \sqrt{2} \frac{M_X}{g} \simeq 0.01659$$

(16)

(or $\sigma_c \simeq 4.0406 \times 10^{16}$ GeV) which holds in our simple model. Inflation ends through the “waterfall” mechanism at $\sigma_c$ provided $\sigma_c^2 \gtrsim \frac{\lambda^2}{8\pi^2}$, i.e. $\lambda \lesssim 0.147$ or equivalently $\frac{\mu}{M_X} \lesssim 0.548$.

Assuming, as it turns out to be the case, that $(\Delta_T)^2 / (\Delta_T)^2_S << 1$, where $(\Delta_T)^2$ and $(\Delta_T)^2_S$ are the tensor and scalar components of the quadrupole anisotropy $\Delta_T$, respectively, we identify $\Delta_T$ with $(\Delta_T)^2_S$ and obtain

$$\frac{\Delta T}{T} \simeq \frac{1}{4\pi\sqrt{45}} \left( \frac{V^3/2}{V'} \right)_{\sigma_H} = \frac{\mu^2 \sigma_H}{2\pi\sqrt{45}} \left[ \left( \frac{\lambda}{2\pi} \right)^2 + 2\beta \sigma_H^2 + \sigma_H^4 \right]^{-1}.$$  

(17)

Here $\sigma_H$ is the value that the inflaton field had when the scale $\ell_H$, corresponding to the present horizon, crossed outside the inflationary horizon.

Let us define

$$N(\sigma) \equiv \frac{1}{2\sqrt{\beta^2 - \left( \frac{\lambda}{2\pi} \right)^2}} \ln \left[ 1 + \frac{2\sqrt{\beta^2 - \left( \frac{\lambda}{2\pi} \right)^2}}{\sigma^2 + \beta - \sqrt{\beta^2 - \left( \frac{\lambda}{2\pi} \right)^2}} \right].$$  

(18)
Then the number of e-foldings $\Delta N(\sigma_{in}, \sigma_f)$ for the time period that $\sigma$ varies between the values $\sigma_{in}$ and $\sigma_f$ ($\sigma_{in} > \sigma_f$) is given, in the slow roll approximation, by

$$\Delta N(\sigma_{in}, \sigma_f) = -\int_{\sigma_{in}}^{\sigma_f} \frac{V}{V'} d\sigma = N(\sigma_f) - N(\sigma_{in}).$$ \hspace{1cm} (19)$$

Let us denote by $\ell_H$ the scale corresponding to our present horizon and by $\ell_o$ another length scale. Also let $\sigma_o$ be the value that the inflaton field had when $\ell_o$ crossed outside the inflationary horizon. We define the average spectral index $n(\ell_o)$ for scales from $\ell_o$ to $\ell_H$ as

$$n(\ell_o) \equiv 1 + 2 \ln[(\frac{\delta \rho}{\rho})_{\ell_o}/(\frac{\delta \rho}{\rho})_{\ell_H}]/\ln(\ell_H/\ell_o) = 1 + 2 \ln[\frac{V^{3/2}}{V'}]_{\sigma_o}/(\frac{V^{3/2}}{V'}_{\sigma_H})/\Delta N(\sigma_H, \sigma_o).$$ \hspace{1cm} (20)$$

Here $(\delta \rho/\rho)_\ell$ is the amplitude of the energy density fluctuations on the length scale $\ell$ as this scale crosses inside the postinflationary horizon and $\Delta N(\sigma_H, \sigma_o) = N(\sigma_o) - N(\sigma_H) = \ln(\ell_H/\ell_o)$.

It should be clear that all quantities characterizing inflation in our scenario, such as $\sigma_H$, the (average) spectral index $n$ and the number of e-foldings $N_H \equiv \Delta N(\sigma_H, \sigma_c)$ for the time period that $\sigma$ varies between $\sigma_H$ and $\sigma_c$, depend on just two parameters, namely $\mu$ and $\beta$ or equivalently the Hubble constant $H = \frac{\mu}{\sqrt{3}}$ and the ratio $\frac{m}{H} = \sqrt{3}\beta$ of the inflaton “mass” $m$ to $H$. Therefore for each value of $\mu$ we can determine $\beta$ by requiring that $N_H$ takes the appropriate value. For $0.1 \lesssim \frac{\mu}{M_X} \lesssim 1$ we choose $N_H \simeq 55$.

Table 1 gives the values of $\beta$, $\lambda$, $\sigma_H$, $n \equiv n(\ell_1)$ and $n_{COBE} \equiv n(\ell_2)$, where $\ell_1$ ($\ell_2$) is the scale that corresponds to 1 $Mpc$ (2000 $Mpc$) today, for different values of $\mu$ together with the chosen value of $N_H$ assuming that the present horizon size is 12000 $Mpc$ and $\Delta T = 6.6 \times 10^{-6}$. We see that for a wide range of values of the mass scale $\mu$ our simple model is able to accomodate the MSSM scale with $\beta > \frac{\lambda}{2\pi}$ (and even with $\beta > \lambda$ provided $\mu \lesssim 5 \times 10^{15}$ $GeV$). Also for $2 \times 10^{15}$ $GeV \lesssim \mu \lesssim 8 \times 10^{15}$ $GeV$ we have $0.021 \lesssim \beta \lesssim 0.031$ and consequently $0.25 \lesssim \frac{m}{H} \lesssim 0.31$. Comparing with the case of canonical supergravity we believe that the improvement concerning naturalness is quite impressive. The spectrum of density perturbations is blue, like in the case of canonical supergravity, with an apparent tendency, however, for lower values of the spectral index. As a result of this lowering values
of $\mu$ almost as high as $9 \times 10^{15} \text{ GeV}$ are now consistent with the COBE data. Moreover, there is also a tendency for lower values of $\sigma_H$ due to the contribution of the inflaton mass-squared term. This effect becomes more important as $\mu$ decreases.

One can understand the drastic lowering of $\sigma_H$ for “small” $\mu$ by observing that the quadratic term in the bracket of eq. (17) dominates for $\frac{\lambda}{2\pi \sqrt{2\beta}} \lesssim \sigma_H \lesssim \sqrt{2\beta}$. Also we see that $\sigma_H$ decreases with $\mu$ provided $\sigma_H^2 \gg \left(\frac{\lambda}{2\pi \sqrt{2\beta}}\right)^2 \frac{1}{\sqrt{2\beta}}$. Therefore as $\mu$ decreases $\sigma_H$ approaches $\sqrt{2\beta}$ and the serious contribution of the quadratic term results in a drastic lowering of $\sigma_H$ relative to the canonical supergravity scenario.

One expects that with increasing $\lambda$ radiative corrections will start playing an increasingly important role towards the end of inflation. For $\sigma_c \gtrsim \frac{\lambda}{2\pi \sqrt{2\beta}}$, or $\mu \lesssim 4 \times 10^{15} \text{ GeV}$, the radiative correction term is never dominant in $V' (\sigma)$ and its effect on inflation is expected to be rather limited. For $\mu \gtrsim 5 \times 10^{15} \text{ GeV}$ we see that $\lambda > \beta$ and $\beta$ starts falling slowly with $\lambda$ increasing. The peak of $\beta$ around $\mu \simeq 5 \times 10^{15} \text{ GeV}$ is actually a result of radiative corrections. To understand better their role in our scenario we repeated all calculations neglecting radiative corrections altogether. Table 2 contains the results of this investigation for some values of $\mu$. Comparing the corresponding values listed in Table 1 and Table 2 we see that the suppression of $\beta$ with increasing $\mu$ is clearly due to the contribution of radiative corrections. Apart from their negative contribution to naturalness radiative corrections have also a minor effect on the spectral index and a somewhat larger one on $\sigma_H$. Both these effects, however, are due to the suppression of $\beta$ since radiative corrections, which become more important for $\sigma \lesssim \frac{\lambda}{2\pi \sqrt{2\beta}}$, could not directly affect $\sigma_H$ or the spectral index. We conclude that the blue perturbation spectra and the success of our scenario in obtaining the MSSM scale in the context of the simplest model should be attributed primarily to supergravity.

Extrapolating the results listed in Table 1 for $\mu \ll 10^{15} \text{ GeV}$ we reach the weak coupling canonical supergravity scenario of ref. [6] in the context of the simplest model. An extrapolation for $\mu \gtrsim 10^{16} \text{ GeV}$ shows how one could obtain the MSSM scale in the context of the scenario of ref. [8] with large radiative corrections and canonical supergravity. Notice
that in both scenarios for each value of $\mu$ the only parameter left, namely $\lambda$, is fixed once the exact value of $N_H$ is chosen. Consequently, in order to obtain the MSSM scale with canonical supergravity and in the context to the simplest model, one is left only with the choice of $\mu$ and with the very general choice of weak or strong coupling or equivalently the choice of the scenario.

In the above discussion we only considered a one-parameter deviation of the Kähler potential from its canonical form. It is understood that different choices of the omitted terms in eq. (10) could possibly lead to a further improvement concerning naturalness.

The evolution of the universe after the “waterfall” cannot be addressed on general terms since it depends crucially on the details of the specific particle physics model incorporating the present inflationary scenario. In the context of a concrete model one should discuss the numerous difficult issues that the subsequent evolution involves like the “reheat”, the gravitino problem, the generation of the baryon asymmetry, the existence of suitable dark matter candidates, the formation of topological defects etc. A detailed discussion of these issues is clearly beyond the scope of the present paper.

We conclude by summarizing our results. We considered a supergravity hybrid inflationary scenario in the context of the simplest superpotential giving rise to symmetry breaking. By allowing deviations from the minimal Kähler potential we succeeded in obtaining the MSSM value of the symmetry breaking scale for several natural values of the parameters and with an inflaton “mass” only three to four times smaller than the Hubble constant. The spectrum of adiabatic density perturbations is blue and the duration of inflation rather limited. We believe that our quasi-canonical supergravity scenario is the most natural realization of Linde’s hybrid inflation in the context of supersymmetry.

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| $\mu/10^{15}GeV$ | $N_H$ | $\beta$ | $\lambda$ | $\sigma_H/10^{17}GeV$ | $n$ | $n_{COBE}$ |
|------------------|-------|--------|-------|---------------------|----|-----------|
| 1                | 56.12 | 0.00987 | 0.0012 | 0.7149              | 1.022 | 1.022    |
| 2                | 55.63 | 0.0205  | 0.0049 | 1.3355              | 1.048 | 1.049    |
| 3                | 55.29 | 0.0273  | 0.0110 | 2.1221              | 1.071 | 1.076    |
| 4                | 55.16 | 0.0307  | 0.0196 | 3.0422              | 1.094 | 1.104    |
| 5                | 54.99 | 0.0312  | 0.0306 | 4.0556              | 1.118 | 1.138    |
| 6                | 54.74 | 0.0294  | 0.0441 | 5.1111              | 1.145 | 1.178    |
| 7                | 54.68 | 0.0258  | 0.0600 | 6.1704              | 1.173 | 1.224    |
| 8                | 54.49 | 0.0212  | 0.0784 | 7.1910              | 1.203 | 1.275    |
| 9                | 54.43 | 0.0158  | 0.0992 | 8.1697              | 1.233 | 1.328    |

Table 1. The values of $N_H$, $\beta$, $\lambda$, $\sigma_H$, $n$ and $n_{COBE}$ as a function of $\mu$.

| $\mu/10^{15}GeV$ | $N_H$ | $\beta$ | $\sigma_H/10^{17}GeV$ | $n$ | $n_{COBE}$ |
|------------------|-------|--------|----------------------|----|-----------|
| 3                | 55.30 | 0.0285 | 2.0687               | 1.073 | 1.077    |
| 5                | 54.92 | 0.0380 | 3.7165               | 1.122 | 1.140    |
| 7                | 54.73 | 0.0432 | 5.3622               | 1.173 | 1.216    |
| 9                | 54.48 | 0.0464 | 6.9042               | 1.223 | 1.301    |

Table 2. The values of $N_H$, $\beta$, $\sigma_H$, $n$ and $n_{COBE}$ as a function of $\mu$, ignoring radiative corrections.