THE EVOLUTION OF DUST IN THE EARLY UNIVERSE WITH APPLICATIONS TO THE GALAXY SDSS J1148+5251

ELI DWEK,1 FRÉDÉRIC GALLIANO,1 AND ANTHONY P. JONES2

Received 2006 October 26; accepted 2007 March 16

ABSTRACT

Dusty hyperluminous galaxies in the early universe provide unique environments for studying the role of massive stars in the formation and destruction of dust. At redshifts above ~6, when the universe was less than ~1 Gyr old, dust could have only condensed in the explosive ejecta of Type II supernovae (SNe), since most of the progenitors of the asymptotic giant branch stars, the major alternative source of interstellar dust, did not have time to evolve off the main sequence since the onset of star formation. In this paper we present analytical models for the evolution of the gas, dust, and metals in high-redshift galaxies, with a special application to SDSS J1148+5251 (hereafter J1148+5251), a hyperluminous quasar at $z = 6.4$. We find that an average SN must condense at least $1 \, M_\odot$ of dust to account for the observed dust mass in J1148+5251. Observationally, it is in excess of the largest dust yield of $\lesssim 0.02 \, M_\odot$ found thus far in the ejecta of any SN. If future observations find this to be a typical SN dust yield, then additional processes, such as accretion onto preexisting grains or condensation around the active galactic nucleus, will need to be invoked to account for the large amount of dust in this and similar objects. The galaxy’s star formation history is still uncertain, and current observations of the gas, metal, and dust contents of J1148+5251 can be reproduced by either an intensive and short burst of star formation ($\psi \gtrsim 10^3 \, M_\odot \, yr^{-1}$) with a duration of $\lesssim 10^8$ yr or a much lower star formation rate ($\psi \approx 100 \, M_\odot \, yr^{-1}$) occurring over the lifetime of the galaxy.

Subject headings: dust, extinction — galaxies: evolution — galaxies: formation — galaxies: high-redshift — galaxies: individual (SDSS J1148+5251) — galaxies: starburst — infrared: galaxies — infrared: general

1. INTRODUCTION

Quasars and galaxies detected at redshifts $z \gtrsim 6$ and observed at far-IR and submillimeter wavelengths (Hughes et al. 1998; Bertoldi et al. 2003a; Robson et al. 2004; Beelen et al. 2006) exhibit luminosities in excess of $\sim 10^{13} \, L_\odot$ and inferred dust masses and star formation rates (SFRs) in excess of $\sim 10^8 \, M_\odot$ and of $\sim 10^3 \, M_\odot \, yr^{-1}$, respectively (Bertoldi et al. 2003a). For comparison, the Milky Way contains only about $5 \times 10^7 \, M_\odot$ of dust, for an assumed gas mass of $5 \times 10^8 \, M_\odot$ and a dust-to-gas mass ratio of $\sim 0.01$. Some of these high-$z$ galaxies are younger than $\sim 1$ Gyr, strongly suggesting that the dust must have condensed in core-collapse supernovae (SNe) which inject their nucleosynthetic products and newly condensed dust into the interstellar medium (ISM) relatively promptly after their formation. In contrast, asymptotic giant branch (AGB) stars, the other main sources of interstellar dust, inject their dust products into the ISM only after their progenitor stars have evolved off the main sequence, resulting in a $\gtrsim 500$ Myr delay time since their formation (Dwek 1998). This delayed injection of AGB-condensed dust has been suggested by Dwek (2005) and Galliano et al. (2007) as a possible explanation for the observed trend of the IR emission from polycyclic aromatic hydrocarbon molecules with galaxies’ metallicity (Engelbracht et al. 2005; Madden et al. 2006).

SNe play a dual role in the evolution of interstellar dust. On the one hand, they are potentially the most important source of interstellar dust. If all the refractory elements precipitate with a 100% efficiency from the gas, a typical $25 \, M_\odot$ SN can form about $1 \, M_\odot$ of dust (Woosley & Weaver 1995; Nomoto et al. 2006). On the other hand, their expanding blast waves sweep up interstellar dust grains, destroying them by thermal sputtering and evaporative grain-grain collisions (Jones et al. 1996; Jones 2004). The presence of the large quantities of interstellar dust in these youthful galaxies therefore provides a unique opportunity for studying the dual role of SNe in the formation and destruction of dust.

The importance of SN condensates as important contributors to the abundance of interstellar dust was previously recognized by many authors (Dwek & Scalo 1980; Eales & Edmunds 1996; Dwek 1998; Tielens 1998; Edmunds 2001; Morgan & Edmunds 2003). In particular, Eales & Edmunds (1996), Morgan & Edmunds (2003), and Mailiano et al. (2004) recognized the importance of SNe for the production of the massive amounts of dust observed in early galaxies.

In this paper we focus on the evolution of dust in galaxies that are so youthful that AGB stars could not have contributed to the reservoir of dust in these objects. This places the burden of dust production solely on SNe, which in the remnant phase of their evolution are also efficient agents of grain destruction. The equations for the evolution of dust are therefore greatly simplified, since (1) they can be formulated using the instantaneous recycling approximation, which assumes that stars return their ejecta back into the ISM promptly after their formation; and (2) massive core-collapse SNe are the only sources of thermally condensed dust. For the purpose of this analysis we have ignored the possible accretion of refractory elements onto preexisting dust in the dense phases of the ISM. Consequently, the models developed here allow for the detailed analysis of the dependence of the evolution of dust mass on the dust yield in SNe and the grain destruction efficiency by their remnants in the ISM. Furthermore, since SNe play the dual role of producing and destroying dust, we also examine the dependence of dust evolution on the SFR.

---

1 Observational Cosmology Laboratory, NASA Goddard Space Flight Center, Greenbelt, MD 20771; eli.dwek@nasa.gov.

2 Institut d’Astrophysique Spatiale, F-91405 Orsay, France.
The paper is organized as follows. We first present the analytical models for the evolution of interstellar dust in high-redshift galaxies, assuming instantaneous recycling and neglecting the contribution of low-mass stars to their chemical evolution. We also present analytical expressions for the efficiency of grain destruction by SN remnants (SNRs) in the ISM (§ 2). The general properties of the equations, the evolution of gas mass, galactic metallicity, dust mass, the dust-to-gas mass ratio, and their dependence on SN yields and grain destruction efficiencies are discussed in detail in § 3. In § 4 we apply the general results to the quasar SDSS J1148+5251 (hereafter J1148+5251) at redshift \( z = 6.4 \), when the universe was a mere \( \sim 900 \) Myr old. We first summarize observations of this quasar, derive its far-IR luminosity and dust mass, discuss possible scenarios for the star formation history of this galaxy, and analyze its spectral energy distribution (SED), separating it into a starburst, an active galactic nucleus (AGN), and dust emission components. The results of our paper are summarized in § 5.

2. THE EVOLUTION OF GAS AND DUST AT HIGH REDSHIFTS

2.1. General Considerations

We define the stellar initial mass function (IMF), \( \phi(m) \), so that \( \phi(m) dm \) is the number of stars with masses between \( m \) and \( m + dm \), and normalize it to unity in the \([m_l, m_u]\) mass interval,

\[
\int_{m_l}^{m_u} \phi(m) dm = 1, \tag{1}
\]

where \( m_l \) and \( m_u \) are, respectively, the lower and upper mass limits of the IMF. We define the IMF-averaged stellar mass \( \langle m \rangle \) and the average mass of metals, \( Y_d \), and dust, \( Y_d \), returned by massive stars to the ISM as

\[
\langle m \rangle = \int_{m_l}^{m_u} m \phi(m) dm,
\]

\[
\hat{Y}_z = \int_{m_l}^{m_u} Y_z(m) \phi(m) dm / f_{\text{SN}}, \tag{2}
\]

\[
\hat{Y}_d = \int_{m_l}^{m_u} Y_d(m) \phi(m) dm / f_{\text{SN}},
\]

where \( m_l \) and \( m_u \) are, respectively, the lower and upper mass limits of stars that become core-collapse Type II SNe, taken to be equal to 8 and 40 \( M_\odot \), respectively (Heger et al. 2003). \( Y_z(m) \) is the total mass of metals in the gas and dust that is returned by a star of mass \( m \) to the ISM, \( Y_d(m) \) is the mass of dust produced by a star of mass \( m \), and \( f_{\text{SN}} \) is the fraction of stars that become Type II SNe, given by

\[
f_{\text{SN}} = \int_{m_l}^{m_u} \phi(m) dm < 1. \tag{3}
\]

We define the SFR, \( \psi(t) \), to be the mass of stars formed per unit time and the stellar birthrate, \( B(t) \), as the number of stars formed per unit time. For a constant IMF, the relation between the two is given by

\[
\psi(t) = B(t) \langle m \rangle. \tag{4}
\]

The Type II SN rate is given by

\[
R_{\text{SN}}(t) = B(t) \int_{m_l}^{m_u} \phi(m) dm = \frac{\psi(t)}{\langle m \rangle f_{\text{SN}}} = \frac{\psi(t)}{m_*}, \tag{5}
\]

where

\[
m_* \equiv \langle m \rangle / f_{\text{SN}} \tag{6}
\]

is the mass of all stars born per SN event.

In all our calculations we will use three different functional forms for the IMF, characterized by an \( m^{-\alpha} \) power law in the \([m_l, m_u]\) mass interval. Table 1 lists the parameters of the different IMFs considered in this paper. The first functional form is the Salpeter IMF, and the choice of the two additional functional forms is motivated by the possibility that at high redshifts the stellar IMF may have been more heavily weighted toward high-mass stars. In addition, the table lists the IMF-averaged values of some relevant quantities. In particular, the value of \( m_* \) is 147 \( M_\odot \) for a Salpeter IMF, dropping down to 50 \( M_\odot \) for a top-heavy IMF, which has a higher fraction of SN events.

2.2. The Evolution of the Gas

Let \( M_\text{g} \) be the mass of gas in a galaxy. For sufficiently young galaxies we can assume that stellar ejecta are instantaneously recycled back into the ISM. In this approximation, the rate of change in the ISM mass due to astration and infall and outflow of gas is given by

\[
\frac{dM_\text{g}}{dt} = -(1 - R) \psi(t) + \left( \frac{dM_\text{g}}{dt} \right)_{\text{in}} - \left( \frac{dM_\text{g}}{dt} \right)_{\text{out}} = -(1 - R) \psi(t) + \frac{M_{\text{in}}}{\tau_{\text{in}}} \exp \left( - \frac{t}{\tau_{\text{in}}} \right)
\]

\[
- \frac{M_{\text{out}}}{\tau_{\text{out}}} \exp \left( - \frac{t}{\tau_{\text{out}}} \right), \tag{7}
\]

where \( \psi(t) \) is the SFR and \( R \) is the fraction of the stellar mass that is returned to the ISM by either SN explosions or quiescent stellar winds. The last two terms are the net increase in the mass of the ISM to inflow and outflow, respectively. We assumed them to be exponential with e-folding times of \( \tau_{\text{in}} \) and \( \tau_{\text{out}} \), normalized so that the total mass of the infalling (outflowing) gas is given by \( M_{\text{in}} (M_{\text{out}}) \).

| IMF              | \( \alpha \) | \( m_{\text{low}} \) | \( m_{\text{up}} \) | \( \langle m \rangle \) | \( m_* \) | \( f_{\text{SN}} \) | \( \hat{Y}_z \) | \( \hat{Y}_d \) |
|------------------|-------------|-----------------|----------------|-----------------|--------|----------------|----------------|----------------|
| Salpeter         | 2.35        | 0.1             | 100            | 0.35            | 147    | 0.0024         | 1.4 (1.7)      | 0.5 (0.6)      |
| Mass-heavy       | 2.35        | 1.0             | 100            | 3.1             | 58     | 0.054          | 1.4 (1.7)      | 0.5 (0.6)      |
| Top-heavy        | 1.50        | 0.1             | 100            | 3.2             | 50     | 0.064          | 2.2 (2.7)      | 0.7 (0.9)      |

Notes.—See § 2.1 for the definition of all quantities. Masses and yields are in units of \( M_\odot \). Metallicity and dust yields were calculated for metallicities of 0.01 \( Z_\odot \) and \( Z_\odot \) (in parentheses).
We assume that the SFR is proportional to the gas mass to some power $k$,
\[
\psi(t) = \psi_0 \left( \frac{M_g(t)}{M_0} \right)^k = \psi_0 \mu_g(t)^k, \tag{8}
\]
where $M_0$ is the total mass of the system (stars + gas) at some arbitrary time $t_0$.

We will consider two scenarios for the chemical evolution of the system: (1) a closed-box model, in which a galaxy does not exchange any mass with its surrounding medium; that is, $M_{\text{inf}} = M_{\text{int}} = 0$; and (2) an infall model, in which a galaxy’s mass is initially zero and increases with time due to the infall of metal-free gas.

### 2.2.1. Closed-Box Model

In this model, the equation for $dM_g/dt$ can then be written as
\[
\frac{d\mu_g(t)}{dt} = -(1 - R) \left( \frac{\psi_0}{M_0} \right) \mu_g(t)^k, \tag{9}
\]
where $M_0 \equiv M_g(t_0 = 0)$ is the total mass of the system (which initially consists of only gas), $\psi_0 = \psi(t = 0)$, and $\mu_g(t) \equiv M_g(t)/M_0$ is the gas mass fraction at time $t$. Analytical solutions are available for arbitrary values of $k$,
\[
\mu_g(t) = \begin{cases} 
\exp \left[ -(1 - R) \left( \frac{\psi_0}{M_0} \right) t \right], & k = 1, \\
\left[ 1 - (1 - R)(1 - k) \left( \frac{\psi_0}{M_0} \right) t \right]^{1/(1 - k)}, & k \neq 1,
\end{cases} \tag{10}
\]
with the initial condition that $\mu_g = 1$ at time $t = 0$.

### 2.2.2. Infall Model

In this model, the equation for $dM_g/dt$ is
\[
\frac{dM_g}{dt} = -(1 - R)\psi(t) + \frac{M_{\text{inf}}}{\tau_{\text{inf}}} \exp \left( -\frac{t}{\tau_{\text{inf}}} \right). \tag{11}
\]
Since the initial mass of the system is zero, we define $M_0$ to be the total mass of the galaxy at some later time $t_0 > 0$, which, in the absence of outflows, is given by
\[
M_0 = \left( \frac{M_{\text{inf}}}{\tau_{\text{inf}}} \right) \int_0^{t_0} \exp \left( -\frac{t}{\tau_{\text{inf}}} \right) dt = M_{\text{inf}} \left[ 1 - \exp \left( -\frac{t_0}{\tau_{\text{inf}}} \right) \right]. \tag{12}
\]
Analytical solutions are available for $k = 1$ and constant values of $M_{\text{inf}}$ and $\tau_{\text{inf}}$ or for arbitrary values of $k$ provided that $(M_{\text{inf}}/\tau_{\text{inf}}) \propto \psi(t)$.

Defining $\mu_g(t) \equiv M_g(t)/M_0$, the solution for the evolution of $\mu_g(t)$ for $k = 1$ can be written as
\[
\mu_g(t) = \tilde{\mu} \left[ \exp \left( -\frac{t}{\tau_{\text{inf}}} \right) - \exp \left( -(1 - R) \frac{t}{\tau_0} \right) \right], \tag{13}
\]
where $\tau_0 \equiv M_0/\psi_0$ and
\[
\tilde{\mu} \equiv \frac{M_{\text{inf}}}{(1 - R)\psi_0\tau_{\text{inf}} - M_0} \left\{ 1 - \exp(-t_0/\tau_{\text{inf}}) \left[ (1 - R)(\tau_{\text{inf}}/\tau_0) - 1 \right] \right\}. \tag{14}
\]
For a given galactic mass, $M_0$, there exists only a limited range of values for $\psi_0$ that will produce a given gas mass fraction, $\mu_g(t_0)$, at time $t_0$, over a wide range of values for $\tau_{\text{inf}}$. The range of values for $\psi_0$ is given by
\[
\ln \left[ \frac{\mu_g(t_0)}{M_0} \right] \leq \psi_0 \leq \frac{x M_0}{1 - R}, \tag{15}
\]
where $x$ is the solution for $[1 - \exp(x)]/x = \mu_g(t_0)$. For values of $\tau_{\text{inf}}$ ranging from $\tau_{\text{inf}} \ll t_0$ to $\tau_{\text{inf}} \gg t_0$, $\psi_0$ ranges from $\sim 130$ to $270 M_{\odot} \, \text{yr}^{-1}$.

### 2.3. The Evolution of the Dust

In very young galaxies with ages $\lesssim 800$ Myr, low-mass stars do not have time to evolve off the main sequence and enrich the ISM with their dusty ejecta. We will assume that any infalling gas is dust-free and that the dust only condenses in Type II SNe and does not grow by accretion of metals onto grains in the ISM. The equation for the evolution of the mass of dust, $M_d$, in the galaxy is then given by
\[
\frac{dM_d(t)}{dt} = -Z_d \psi(t) + \hat{Y}_d R_{\text{SN}}(t) - \frac{M_d(t)}{\tau_d}, \tag{16}
\]
where $Z_d \equiv M_d/M_g$ is the dust-to-gas mass ratio, $\hat{Y}_d$ is the average yield of dust in Type II SNe, $R_{\text{SN}}$ is the SN rate in the galaxy, and $\tau_d$ is the lifetime of the dust against destruction by SN blast waves. The lifetime for grain destruction is given by (Dwek & Scalo 1980; McKee 1989)
\[
\tau_d = \frac{M_d(t)}{(m_d) \, R_{\text{SN}}} = \frac{M_g(t)}{(m_{\text{ISM}}) \, R_{\text{SN}}}, \tag{17}
\]
where $(m_d)$ is the total mass of elements that are locked up in dust and returned by a single SNR back to the gas phase by either thermal sputtering or evaporative grain-grain collisions (Jones et al. 1996; Jones 2004), and $(m_{\text{ISM}}) \equiv (m_d)/Z_d$ is the effective ISM mass that is completely cleared of dust by a single SNR, more formally defined in § 2.6 below.

Using the expression for the SN rate given by equation (5), the equation for the evolution of dust can now be written as
\[
\frac{dM_d(t)}{dt} = - \left( \frac{\psi(t)}{M_g(t)} \right) \left[ 1 + \frac{(m_{\text{ISM}})}{m_*} \right] M_d(t) + \hat{Y}_d \frac{\psi(t)}{m_*}. \tag{18}
\]

Solutions for $M_d(t)$ depend on the assumed chemical evolution model.

#### 2.3.1. Closed-Box Model

In this model we can change variables from $t$ to $\mu_g$ and, using the expression for $d\mu_g/dt$ [eq. (9)], rewrite equation (18) in the form
\[
\frac{dM_d}{d\mu_g} = \left( \frac{\nu}{\mu_g} \right) M_d - \hat{Y}_d \frac{M_0}{(1 - R) m_*}, \tag{19}
\]
where
\[
\nu \equiv \frac{(m_{\text{ISM}}) + m_*}{(1 - R) m_*}. \tag{20}
\]
The solution to equation (19) gives the evolution of dust mass (or dust mass fraction) as a function of the fractional gas mass,

\[ M_d(\mu_g) = \hat{Y}_d \left( \frac{M_0}{\langle m_{\text{ISM}} \rangle + R_{m_\ast}} \right) \mu_g (1 - \mu_g^{-1}), \]

\[ \mu_d(\mu_g) = \left( \frac{\hat{Y}_d}{\langle m_{\text{ISM}} \rangle + R_{m_\ast}} \right) \mu_g (1 - \mu_g^{-1}), \tag{21} \]

where \( \mu_d \equiv M_d/M_0 \) and where we assumed that \( M_d = 0 \) at time \( t = 0 \), when \( \mu_g = 1 \). Equation (21) can be rewritten to give the dust yield required to obtain a given dust-to-gas mass ratio, \( Z_d \), at a given gas mass fraction \( \mu_g \),

\[ \hat{Y}_d = Z_d \left( \frac{\langle m_{\text{ISM}} \rangle + R_{m_\ast}}{1 - \mu_g^{-1}} \right). \tag{22} \]

A brief glance at equation (21) might suggest that the dust mass scales linearly with \( M_0 \), the total mass of the galaxy. However, the dependence of \( M_d \) on \( M_0 \) is more complex and depends on the amount of grain destruction in the galaxy. When grain destruction is efficient so that \( \langle m_{\text{ISM}} \rangle \gg R_{m_\ast} \), the values of \( \nu \ll 1 \) and \( \mu_g^{-1} \ll 1 \), and equation (21) approaches the asymptotic value of \( M_d(\mu_g) = \hat{Y}_d \frac{M_g}{\langle m_{\text{ISM}} \rangle} \),

which is independent of the initial mass of the galaxy. The dust yield required to obtain a given dust-to-gas mass ratio then becomes

\[ \hat{Y}_d = Z_d \langle m_{\text{ISM}} \rangle. \tag{24} \]

As we will see in § 4.3, these asymptotic limits are important when the total mass of the galaxy is very uncertain, but dust and gas masses are fairly well determined.

### 2.3.2. Infall Model

In the infall model, the solution for \( M_d(t) \), or equivalently \( \mu_d(t) \), cannot be simply expressed as a function of \( \mu_g(t) \). It has an explicit time dependence and is given by

\[ \mu_d(t) = \left( \frac{\hat{Y}_d}{m_\ast} \right) \hat{\mu} \left\{ A \exp \left( -\frac{t}{\tau_{\text{inf}}} \right) - B \right\} \times \exp \left\{ -(1 - R) \frac{t}{\tau_0} + (B - A) \exp \left( -\xi \frac{t}{\tau_0} \right) \right\}, \tag{25} \]

where \( \hat{\mu} \) is given by equation (14), \( \tau_0 \equiv M_0/\psi_0 \),

\[ \xi \equiv \langle m_{\text{ISM}} \rangle/m_\ast + 1, \]

\[ A \equiv (\xi - \tau_0/\tau_{\text{inf}})^{-1}, \]

\[ B \equiv (\xi - (1 - R))^{-1}. \tag{26} \]

### 2.4. The Evolution of the Metallicity

The evolution of the mass of metals (elements heavier than helium), \( M_z(t) \), can be formally obtained by substituting \( \hat{Y}_d \) instead of \( \hat{Y}_d \) and by letting \( \langle m_{\text{ISM}} \rangle \to 0 \) in equations (20) and (21) for \( \nu \) and \( M_d(t) \).

#### 2.4.1. Closed-Box Model

The solution for \( M_z(t) \) and \( \mu_z(\mu_g) \equiv M_z(t)/M_0 \) in this model is given by

\[ M_z(\mu_g) = \hat{Y}_z \left( \frac{M_0}{R_{m_\ast}} \right) \mu_g (1 - \mu_g^{-1}), \]

\[ \mu_z(\mu_g) = \left( \frac{\hat{Y}_z}{R_{m_\ast}} \right) (1 - \mu_g^{-1}). \tag{27} \]

where

\[ \nu = 1/(1 - R). \tag{28} \]

The metallicity of the gas, \( Z_g \), defined as the mass ratio of metals and the interstellar gas is then given by

\[ Z_g(\mu_g) \equiv \frac{M_z(\mu_g)}{M_0(\mu_g)} = \left( \frac{\hat{Y}_z}{R_{m_\ast}} \right) (1 - \mu_g^{-1}). \tag{29} \]

#### 2.4.2. Infall Model

Substituting \( \hat{Y}_d \) into \( \hat{Y}_z \) and setting \( \langle m_{\text{ISM}} \rangle \to 0 \) in the solutions of \( M_d(t) \) in the infall model gives the following equation for \( \mu_z(t) \):

\[ \mu_z(t) = \left( \frac{\hat{Y}_d}{m_\ast} \right) \hat{\mu} \left\{ A' \exp \left( -\frac{t}{\tau_{\text{inf}}} \right) - B' \right\} \times \exp \left\{ -(1 - R) \frac{t}{\tau_0} + (B' - A') \exp \left( -\xi \frac{t}{\tau_0} \right) \right\}, \tag{30} \]

where \( A' = \tau_{\text{inf}}/(|\tau_{\text{inf}} - \tau_0|) \) and \( B' = R^{-1} \).

The metallicity of the gas, \( Z_g(t) \), is given by the ratio \( \mu_z(t)/\mu_g(t) \), where \( \mu_g(t) \) is given by equation (13).

### 2.5. The Dust-to-Metal Mass Ratio

An important quantity is the fraction of the mass of metals in the ISM that is locked up in dust, \( f_d \), defined as

\[ f_d \equiv M_d/M_z. \tag{31} \]

For the closed-box model, \( f_d \) is given by

\[ f_d = \frac{\hat{Y}_d}{\hat{Y}_z} \left( \frac{R_{m_\ast}}{\langle m_{\text{ISM}} \rangle + R_{m_\ast}} \right) \frac{1 - \mu_g^{-1}}{1 - \mu_{d_\ast}^{-1}}. \tag{32} \]

For the infall model, \( f_d \) is given by the ratio of \( \mu_d(t) \) to \( \mu_z(t) \), which are given by equations (25) and (30), respectively.

An upper limit to this quantity can be obtained by assuming that the dust mass is not altered in the ISM, either by grain destruction in SNRs or by accretion in molecular clouds. The value of \( f_d \) is then identical in the infall and closed-box models and, for no grain destruction or accretion in the ISM, is given by

\[ f_d = \hat{Y}_d/\hat{Y}_z, \tag{33} \]

which is the mass fraction of metals in the SN ejecta that condensed into dust grains. Table 2 gives the values of \( Y_z, Y_d \), for
different dust compositions, the total dust yield \( Y_d \), and the mass fraction of metals that form dust in SN ejecta as a function of stellar mass for two different initial metallicities of the progenitor stars. Stellar yields were taken from Woosley & Weaver (1995). The results are also shown in Figure 1 for an initial stellar metallicity of \( Z = Z_\odot \). The calculations assume that all refractory elements form dust grains with a condensation efficiency of unity.

To estimate the amount of oxygen that gets incorporated into the dust, we assumed that the silicate dust grains are of the form SiO\(_2\), TiO\(_2\), and CaO. The results show that the mass fraction of metals that can form dust actually decreases with stellar mass, as more massive stars form larger amounts of oxygen, which is less efficiently incorporated into dust. The IMF-averaged mass fraction of metals that can be locked up in dust is less than \(~0.40\) (see Table 1). This is a strong upper limit, since these results ignore the effect of grain destruction. An observed dust mass fraction of this magnitude in a galaxy in which grain destruction is important must therefore imply that the dust mass must be enhanced by additional processes, such as the accretion of metals onto grains in the ISM.

### 2.6. The Lifetime of Interstellar Dust Grains

#### 2.6.1. Homogeneous ISM

An important input parameter governing the evolution of the dust is its lifetime against destruction by SNRs. The mass of ISM gas which has been cleared of dust by a single SNR is a measure of the efficiency of this process and is given by

\[
\langle m_{\text{ISM}} \rangle = \int_{v_1}^{v_2} \zeta_d(v_1) \left| \frac{dM}{dv} \right| dv,
\]

where \( \zeta_d(v_1) \) is the fraction of the mass of dust that is destroyed in an encounter with a shock wave with a velocity \( v_1\). \( (dM/dv)dv \) is the ISM mass that is swept up by shocks in the \([v_1, v_2 + dv]\) velocity range, and \( v_0 \) and \( v_f \) are the initial and final velocities of the SNR.

Cioffi et al. (1988) presented analytical solutions for the evolution of a SNR expanding into a uniform ISM. Initially, the remnant expands adiabatically and its evolution is described by the Sedov-Taylor solution. When the cooling time of the shocked gas becomes comparable to its dynamical timescale, the remnant evolves as a pressure-driven snowplow (PDS). For the sake of our analysis we cast the standard solutions in the form of the mass of the swept-up ISM as a function of shock velocity. The evolution of the remnant mass, \( M_{\text{SNR}} \), in a medium with solar metallicity can then be written as

\[
M_{\text{SNR}}(v_1) = 400 \left( \frac{v_1}{28} \right)^{0.86} \left( \frac{v_1}{v_s} \right)^{0.28} e^{-\alpha},
\]

\[
\alpha = \begin{cases} 
-2.0, & v_s \geq 1.0, \\
-1.28, & v_s \leq 1.0, 
\end{cases}
\]

\[
\frac{dM}{dv} = -\alpha \frac{M_{\text{SNR}}}{v_s}.
\]
where $E_{51}$ is the energy of the explosion in units of $10^{51}$ ergs, $n_0$ the density of the ISM in cm$^{-3}$, and $v_s = v_0/v_{PDS}$, where $v_{PDS}$ is the velocity of the remnant when it transitions from the Sedov-Taylor to the PDS phase of its evolution. For a gas of solar abundances and a He/H number ratio of 0.10, $v_{PDS}$ is given by (Cioffi et al. 1988)

$$v_{PDS} = 413 n_0^{1/7} E_{51}^{1/14} \text{ km s}^{-1}. \quad (36)$$

Figure 2 depicts the velocity dependence of $\zeta_d(v_s)$ and the value of $\langle m_{\text{ISM}} \rangle$ as a function of ISM density. For values of $n_0 \approx 0.1 - 1.0$ cm$^{-3}$, corresponding to the average density of the Galactic ISM, we get that $\langle m_{\text{ISM}} \rangle \approx 1100 - 1300 M_\odot$ for an equal mix of silicate and graphite grains. For the Galactic ISM mass of $M_g = 5 \times 10^8 M_\odot$ and SN rate of 0.01 yr$^{-1}$, we get an average dust lifetime $\tau_d \approx 4 \times 10^6$ yr, in good agreement with the value of $=4 \times 10^6$ yr ($=6 \times 10^6$ yr) derived by Jones et al. (1996) and Jones (2004) for the average lifetime of silicate (carbon) dust in the Milky Way.

### 2.6.2. Inhomogeneous ISM

The ISM in galaxies is inhomogeneous, and when the SFR is sufficiently high, as the case is for J1418+5251, it is dominated by a hot and low-density gas created by the expanding SN blast waves. The multiphase ISM is then characterized by hot (h), warm (w), and cold (c) phases with volume filling factors and densities of $f_i$ and $\rho_i$, with $\sum f_i = 1$ ($i = \{h, w, c\}$). SN blast waves propagate predominantly through the low-density intercloud medium. The shocked phases are in rough pressure equilibrium so that the velocity of the shock propagating through, say, the warm phase is related to its velocity in the hot phase by $\rho_b v_b^2 \approx \rho_w v_w^2$. The value of $\langle m_{\text{ISM}} \rangle$ for a three-phase ISM can be written as

$$\langle m_{\text{ISM}} \rangle = \sum_{i=h,w,c} f_i \int_{v_b}^{v_s} f_a \left( \frac{v_s}{\chi_i} \right) \left| \frac{dM}{dv_a} \right| dv_a, \quad (37)$$

where $\chi$ is the density contrast between the phase “i” and the dominant hot phase into which the remnant is expanding. Since the density contrast between the warm or cold phases and that of the hot ISM can be quite large, the shocks propagating through these clouds are quite ineffective at destroying the dust (Jones 2004). For a density of $n_b \approx 3 \times 10^{-3}$ cm$^{-3}$ and a density contrast $\chi \geq 10^3$ ($\geq 10^6$) between the warm (cold) and hot phases, grain destruction in these phases is negligible.

However, the warm and cold phases are ultimately cycled through the hot phase of the ISM by cloud evaporation, cloud crushing by shocks, or cloud disruption by star formation. Injected into a hot ($\approx 10^6$ K) ISM, a dust grain of radius $a$ will be destroyed by thermal sputtering on a timescale of (Dwek et al. 1996; Jones 2004)

$$\Delta t_{\text{sput}} \approx 10^6 \frac{d(\mu m)}{m_{\text{dust}}(\text{cm}^2)} \text{ yr.} \quad (38)$$

A grain of radius $a = 0.1 \mu m$ will therefore survive for a period of $\Delta t_{\text{sput}} \approx 3 \times 10^7$ yr. In high-redshift galaxies with total gas masses $\geq 10^{10} M_\odot$ and SFRs in excess of $10^3 M_\odot$ yr$^{-1}$, the timescale for the disruption of the cold molecular clouds by star formation, $\approx M_g/\psi \approx 10^7$ yr, is comparable to $\Delta t_{\text{sput}}$. The effective lifetime of the dust in the three-phase ISM of these objects is therefore determined by $\Delta t_{\text{sput}}$. Using equation (17), this lifetime can be expressed in terms of $\langle m_{\text{ISM}} \rangle$, giving $\langle m_{\text{ISM}} \rangle \approx 50 M_\odot$ for $M_g = 10^{10} M_\odot$, $\psi = 10^7 M_\odot$ yr$^{-1}$, and $m_* = 150 M_\odot$.

To keep the model results most general, we will consider the grain destruction efficiency as an unknown and adopt $\langle m_{\text{ISM}} \rangle$ as a free parameter of the model ranging from $\langle m_{\text{ISM}} \rangle = 0$ (no grain destruction) to a value of $\langle m_{\text{ISM}} \rangle = 1000 M_\odot$.

### 3. General Results

Figures 3–6 depict the evolution of various quantities as a function of fractional gas mass, $\mu_g$, for closed-box and infall models as a function of time. Results are presented for different values of $\langle m_{\text{ISM}} \rangle$, ranging from 0 to $10^3 M_\odot$, corresponding to the range of uncertainty in the lifetime of the interstellar dust grains. The value of $R$, the IMF-averaged fraction of the stellar mass that is returned to the ISM over the stellar lifetime, is taken to be 0.50. Two different functional forms were used for the IMF, a Salpeter and a top-heavy IMF. The IMF parameters and the values of relevant IMF-averaged quantities are given in Table 1. The values of the IMF-averaged dust and gas yields, $Y_d$ and $Y_z$, respectively, are given in Table 1.

#### 3.1. The Evolution of the Gas

Figure 3 depicts the evolution of the gas mass fraction as a function of time for the closed-box and infall models. In the closed-box model (Fig. 3, left), the initial gas mass fraction is equal to 1, decreasing as the ISM gas is converted into stars. The calculations were performed for a SFR law $\psi(t) = \psi_0 \left[ M_g(t)/M_0 \right]^\lambda$. 

---

**Fig. 2.—Left:** Fraction of the mass of dust that returned to the gas phase after being swept up by a shock of velocity $v_s$ as a function of shock velocity (after Jones et al. [1996]). **Right:** Mass of the ISM gas that is cleared of dust, $\langle m_{\text{ISM}} \rangle$, plotted against the density of the homogeneous ISM into which the SNR is expanding. The figure depicts the value of $\langle m_{\text{ISM}} \rangle$ for silicate (dotted curve) and carbon (solid curve) grains. The relation between grain lifetime and $\langle m_{\text{ISM}} \rangle$ is given in eq. (17).
with $M_0 = 5 \times 10^{10} M_\odot$ and $k = 1.5$. The curves are labeled by the value of $\psi(t_0)$ at time $t_0 = 400$ Myr, which is our adopted age of J1148+5251 (see § 4.1).

In the infall model (Fig. 3, right) described by equation (13), the initial gas mass fraction is zero. It first increases in time as the galaxy accretes mass from its surrounding, but decreases later on, when star formation consumes gas at a higher rate than its rate of replenishment by infall. The curves are labeled by $\psi_0$, which is related to the current SFR by $\psi_0 = \mu_z f(t)/\psi(t)$. For each value of $\psi_0$, equation (13) was solved for the value of $\mu_z$ that produced the adopted values of $M_0$ and $\psi_0$ at the epoch of $t_0 = 400$ Myr.

Figure 3 is important for reconstructing the star formation history of a galaxy from current observations of the fractional gas mass and the SFR. The figure is used in § 4.5 to construct possible star formation scenarios for J1148+5251.

### 3.2. The Evolution of the Dust and Metals

Figure 4 depicts the evolution of the dust mass, $M_d(\mu_d)$, and mass of metals, $M_z$, normalized to the initial mass, $M_0$, for various values of $(m_{\text{ISM}})$, the mass of ISM gas that is cleared of dust by a single SNR, as a function of $\mu_d$. Initially, $\mu_d = 1$, but decreases as the gas is converted into stars. Calculations are presented for two different stellar IMFs: a Salpeter IMF (Fig. 4, left) and a top-heavy IMF (Fig. 4, right). Initially, $M_d = 0$ and rises as the ISM is enriched by SN-produced dust. However, eventually the gas and dust in the ISM are incorporated into stars, and the mass of interstellar dust decreases.

Figure 5 presents the same quantities for the infall model. Both figures show the maximum values of $\mu_d$ and $\mu_z$ attainable with each IMF. Larger values of $\mu_d$ and $\mu_z$ are obtained with a top-heavy IMF. The figures also show that without any grain destruction, the mass of dust is simply proportional to the mass of metals, but decreases more rapidly than the mass of metals when grain destruction is taken into account.

### 3.3. The Evolution of the Dust-to-Metals and Dust-to-Gas Mass Ratios

The effect of grain destruction is to decrease the fraction of condensable elements in the solid phase of the ISM. This point is illustrated in Figure 6, which depicts the evolution of the mass fraction of metals locked up in dust, $f_d = M_d/M_0$, versus $\mu_z$ for the closed-box model. When $(m_{\text{ISM}}) = 0$, $f_d$ is constant and equal to $Y_d/Y_z \approx 0.35$, the fraction of the metals in the SN ejecta that condensed and formed dust. As the figure illustrates, this fraction...
decreases with /C22 g as the grain destruction efficiency increases. Similar quantitative results can be obtained for the infall model. Figure 7 shows that when grain destruction is ignored, i.e., h/ISM = 0, the dust-to-gas mass ratio, Zd, continues to rise, since both the gas and dust are incorporated into stars, but the ISM is continuously enriched by dust formed in SN ejecta. When grain destruction is taken into account, Zd reaches a steady state at values of /C22 g which become increasingly smaller as the grain destruction efficiency, which is related to the value of h/ISM, increases.

3.4. The SN Dust Yields Needed to Produce an Observed Dust-to-Gas Mass Ratio

Figure 8 shows how much dust an average SN must produce in order to give rise to a given dust-to-gas mass ratio, for various grain destruction efficiencies. The value of Yd was calculated when /d reaches a value of 0.60, the adopted gas mass fraction of J1148+5251 at 400 Myr. The figure shows that, for example, to produce a value of Zd = 0.0067 at /d = 0.60, a SN must produce about 0.4 (1.2) M☉ of dust for a top-heavy (Salpeter) IMF, provided the dust is not destroyed in the ISM. Even with a modest amount of grain destruction, <mISM> = 100 M☉, the required SN dust yield is dramatically increased to about 1–2 M☉, depending on the IMF. The horizontal line in the figure corresponds to a value of Yd = 0.02 M☉, the largest amount of dust directly observed in the ejecta of a SN (Sugerman et al. 2006). The figure shows that even without grain destruction, the largest observed yield can only give rise to a gas-to-dust mass ratio of ~3 × 10⁻⁴.

4. APPLICATION TO J1148+5251

The results of our chemical evolution model can be readily applied to any galaxy sufficiently young so that AGB stars are only minor contributors to the dust abundance in the ISM. Here we concentrate on the quasar J1148+5251.

4.1. Observational Properties

Table 3 summarizes the observed properties of J1148+5251. At redshift z = 6.4 the age of the universe is 890 Myr for a ΛCDM universe with Ωm = 0.27, ΩΛ = 0.73, and a Hubble constant H₀ = 70 km s⁻¹ Mpc⁻¹. Figure 9 depicts the observed far-IR and submillimeter fluxes at the observed wavelengths. The different curves are spectral fits to these fluxes for different dust compositions. The optical properties for the silicate and graphite grains were taken from Draine & Lee.
Dust masses vary from $10^8$ to $5 \times 10^8 M_\odot$, corresponding to a formation time of about equal to the total mass of condensable elements produced in a SN to date (Sugerman et al. 2006). The horizontal dashed line near the bottom of the figure corresponds to a value of $\mu_d = 0.0067$ with $\tau_d = 400$ Myr. Finally, we adopted a far-IR luminosity of $2 \times 10^{13} L_\odot$. Table 5 summarizes the various derived and adopted properties of J1148+5251.

4.2. The Dust Mass and the Required Dust Yield in Core-Collapse SNe

Figures 4 and 5 give an upper limit on the mass of dust that can be produced in 400 Myr by SNe when grain destruction is ignored, that is, when $\langle m_{\text{ISM}} \rangle = 0$. The maximum dust mass is $\sim 10^6 M_\odot$ ($\mu_d \approx 2 \times 10^{-3}$) for a Salpeter IMF and $\sim 5 \times 10^8 M_\odot$ ($\mu_d \approx 10^{-2}$) for a top-heavy IMF. The observed dust mass of $2 \times 10^8 M_\odot$ therefore requires a top-heavy IMF, which can produce this much dust even with a grain destruction efficiency corresponding to a value of $\langle m_{\text{ISM}} \rangle = 100 M_\odot$. This value of $\langle m_{\text{ISM}} \rangle$ corresponds to a grain lifetime of

$$
\tau_d = \frac{M_d}{\langle m_{\text{ISM}} \rangle R_{SN}} = \frac{3 \times 10^{10}}{100 \times 7.4} = 27 \text{ Myr},
$$

where the SN rate was calculated for a SFR of $10^3 M_\odot \text{ yr}^{-1}$ and a Salpeter IMF for which $\langle m_{\text{SN}} \rangle = 147 M_\odot$.

This result is also illustrated in Figure 8, which shows that the yield of dust per SN required to produce a dust-to-gas mass ratio of 0.0067 with $\langle m_{\text{ISM}} \rangle = 100 M_\odot$ is about 1 $M_\odot$ per SN. This is about equal to the total mass of condensable elements produced

---

**Table 3**

| Observed Property of the QSO J1148+5152 |
|----------------------------------------|
| R.A. (J2000) | 11 48 16.6 | Fan et al. 2003 |
| Decl. (J2000) | +52 51 50 | Fan et al. 2003 |
| $\theta$ | 0.2" | Walter et al. 2004 |
| $z$(Ly$\alpha$) | 6.37 ± 0.03 | White et al. 2003 |
| $z$([Mg II]) | 6.403 ± 0.005 | Iwamuro et al. 2004 |
| $z$(CO) | 6.419 ± 0.001 | Bertoldi et al. 2003b |
| M(CO)(3-2) | $\sim 1.6 \times 10^{10} M_\odot$ | Walter et al. 2004 |
| M(CO)(7-6), (CO(6-5)) | $\sim 2 \times 10^{10} M_\odot$ | Bertoldi et al. 2003b |
| $M_{\text{syn}}$ | (5.0 ± 2.5) $\times 10^9 M_\odot$ | Walter et al. 2004 |
| $M_{\text{BH}}$ | $3 \times 10^9 M_\odot$ | Willott et al. 2003 |

**Note:** Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.
in a SN with a 25 $M_\odot$ progenitor star (Woosley & Weaver 1995; Nomoto et al. 2006). Such average dust yield is theoretically only attainable with a top-heavy IMF, provided that the dust condensation efficiency in the SN ejecta is about 100%. Observationally, this yield is significantly higher than the $\sim 0.02 M_\odot$ of dust in SN II 2003gd in the galaxy NGC 628, inferred from the analysis of the IR emission and the internal extinction in the SN ejecta (Sugerman et al. 2006). If this low yield is typical, then SNe cannot be the dominant source of dust in these young galaxies. Alternative mechanisms that can produce the observed dust mass, such as accretion in molecular clouds or formation around the AGN (Elvis et al. 2002), need then to be included in the dust evolution model.

4.3. The Dependence of the Required SN Dust Production Yield on the Total Mass of the Galaxy

The dust yield required to produce a given dust-to-gas mass ratio at time $t_0$ depends on the total mass of the galaxy, which we took to be the dynamical mass of J1148+5251. Since this quantity is uncertain, we explore the dependence of the dust mass produced in the model on the adopted total mass of the system. For the closed-box model, the relation between these two quantities, $Y_d$ and $M_0$, is given by equation (21).

The total mass of the galaxy must exceed its gas mass, which we took to be $3 \times 10^{10} M_\odot$, so we varied $M_0$ from $4 \times 10^{10}$ to $8 \times 10^{10} M_\odot$, which spans the dispersion in the observed dynamical mass of the galaxy. Figure 10 depicts the results of our calculations for the two different IMFs and the three different values of $\langle m_{\text{ISM}} \rangle$ used in Figure 8. In the absence of grain destruction, $Y_d$ decreases by a factor of $\sim 4-5$ over the entire mass range of $M_0$. However, when grain destruction is taken into account, $Y_d$ becomes increasingly independent of the total galactic mass, approaching the asymptotic behavior for $\langle m_{\text{ISM}} \rangle \gg R_m$, presented in equation (24). For example, for $\langle m_{\text{ISM}} \rangle \approx 300 M_\odot$ and a dust-to-gas mass ratio of $Z_d \approx 0.003$, we get that $Y_d \gtrsim 1 M_\odot$, significantly larger than any dust mass observed in any SN ejecta.

4.4. The SFR in J1148+5251

From the observed luminosity and gas mass we can derive the current SFR using the relations between the SFR and the far-IR luminosity of the galaxy by

$$\psi(M_\odot \text{ yr}^{-1}) = 1.7 \times 10^{-10} L(L_\odot),$$

which for a value of $L_{\text{IR}} = 2 \times 10^{13} L_\odot$ gives a SFR of $\sim 3400 M_\odot \text{ yr}^{-1}$. A high SFR of $\sim 3000 M_\odot \text{ yr}^{-1}$ was also derived by Maiolino et al. (2005) from the luminosity in the [C II] 158 $\mu$m line detected in this quasar.

The SFR can also be derived from the gas mass using the empirical relation between the SFR per unit area, $\Sigma_{\text{SFR}}$, and the

![Graph showing the dependence of the dust yield on the total mass of the galaxy](image)

**Table 4**

| Type       | $T_{\text{dust}}$ (K) | $M_{\text{dust}}$ ($M_\odot$) | $L_{\text{IR}}$ ($L_\odot$) |
|------------|-----------------------|-------------------------------|-----------------------------|
| Graphite   | 49                    | $2.7 \times 10^8$             | $1.9 \times 10^{13}$        |
| Silicate   | 47                    | $4.9 \times 10^8$             | $2.0 \times 10^{13}$        |
| Carbon BE  | 64                    | $9.7 \times 10^7$             | $2.4 \times 10^{13}$        |
| Carbon AC  | 74                    | $9.3 \times 10^7$             | $2.9 \times 10^{13}$        |

Note.—Graphite and silicate optical properties were taken from Draine & Lee (1984), and the optical properties of the carbon dust were taken from Rouleau & Martin (1991).
gas mass surface density, $\Sigma_{g}$, in star-forming galaxies (R. C. Kennicutt et al. 2007, in preparation),

$$\Sigma_{\text{SFR}}(M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}) = \left(4.5^{+1.0}_{-0.4}\right) \times 10^{-5} \left(\frac{\Sigma_{g}}{M_{\odot} \text{ pc}^{-2}}\right)^{1.56 \pm 0.04}. \quad (41)$$

The CO maps of J1148+5251 suggest that it consists of two blobs of comparable size with a diameter of about 1 kpc. For an adopted gas mass of $3 \times 10^{10} M_{\odot}$, we get a total SFR $\psi \approx 300 M_{\odot} \text{ yr}^{-1}$.

The two different tracers give very different SFRs. Both estimates are highly uncertain. The first assumes that the dust reradiates all the starburst’s luminosity and ignores any possible contribution of an AGN to the heating of the dust. The second assumes that the CO observation traces the densest gas regions in the object, as inferred from HCN-CO correlations, and is therefore a good measure of the total gas mass in galaxies (Gao & Solomon 2004). However, HCO$^+$ observations which probe denser regions of molecular gas have cast doubts on the reliability of HCN (and hence CO) as an unbiased tracer of dense molecular gas in ultraluminous IR galaxies (Graciá-Carpio et al. 2006).

Another major source of uncertainty is the stellar IMF. For example, the SFR derived from the IR luminosity using a Salpeter IMF will decrease from a value of $3400 M_{\odot} \text{ yr}^{-1}$ to a rate of about $380 M_{\odot} \text{ yr}^{-1}$ for a mass- or top-heavy IMF. Because of all these uncertainties, the SFR of J1148+5251 cannot be uniquely determined. In what follows, we discuss the dependence of its star formation history on the assumed current SFR.

4.5. The Star Formation History of J1148+5251

In the closed-box model, the evolution of the dust and metals could be presented as a function of the gas mass fraction $\mu_{g}$ (Figs. 4–7). The translation of the dependence of these quantities from $\mu_{g}$ to time requires knowledge of the star formation history of J1148+5251, which in turns depends on the assumed SFR and the initial gas mass of the object. For example, given an initial gas mass of $M_{0} = 5 \times 10^{10} M_{\odot}$, a SFR of 3000 $M_{\odot} \text{ yr}^{-1}$ will exhaust almost all the available gas in less than $\sim 100$ Myr. This suggests one or more of the following possibilities: (1) the onset of star formation occurred relatively shortly before the observations; (2) the galaxy started with an initially larger reservoir of mass; or (3) rapid infall, comparable to the SFR, kept the reservoir of gas sufficiently high.

Some of these possibilities were presented more qualitatively in Figure 3, which depicts the evolution of the mass fraction of gas as a function of time for the closed-box and infall models. For the closed-box model (Fig. 3, left), the calculations assume an initial gas mass of $5 \times 10^{10} M_{\odot}$ and the Kennicutt law for the relation between the SFR and the gas mass, $\psi(t) \propto M_{\text{gas}}(t)^{k}$, with $k = 1.5$ [see eq. (7)]. The solid curve depicts the evolution of $\mu_{g}(t)$ for an initial SFR $\psi_{0} = 150 M_{\odot} \text{ yr}^{-1}$, corresponding to a value of $\psi = 70 M_{\odot} \text{ yr}^{-1}$ at $t = 400$ Myr, which reproduces the adopted value of $\mu_{g} = 0.60$ at that epoch. The additional curves depict the evolution of $\mu_{g}(t)$ for initial SFRs of 650, 2.2 $\times 10^{3}$, and $6.5 \times 10^{3} M_{\odot} \text{ yr}^{-1}$, corresponding to values of $\psi_{0}$ of 300, 1 $\times 10^{3}$, and $3 \times 10^{3} M_{\odot} \text{ yr}^{-1}$, at $t_{0} = 400$ Myr. The corresponding gas mass fractions at that epoch are 0.19, 0.036, and 0.0052, respectively. The figure shows that a low observed SFR of only $70 M_{\odot} \text{ yr}^{-1}$ is required to fit the observations of J1148+5251, given the initial conditions and assumptions summarized in Table 5. A current SFR of 3000 $M_{\odot} \text{ yr}^{-1}$ requires changes in the initial conditions and model assumptions. If the galaxy is indeed 400 Myr old, then the value of $\mu_{g}$ is 0.0052, requiring the initial gas mass of J1148+5251 to be $5 \times 10^{10} / 0.0052 \approx 1 \times 10^{13} M_{\odot}$. A more plausible scenario is that $\mu_{g}$ is 0.60, but that the age of the starburst is only about 10$^{7}$ yr.

Similar conclusions are reached for the infall model. The right panel of Figure 3 depicts the evolution of the gas mass, constrained to fit the adopted values of $M_{0}$ and $\psi_{0}$ at time $t_{0} = 400$ Myr. The fit requires the values of the product $\psi_{0} t_{0}$ to be $\sim (5-11) \times 10^{10} M_{\odot} \text{ yr}^{-1}$. If star formation has been an ongoing process over a period of 400 Myr, then the current SFR must be between $\sim 125$ and 285 $M_{\odot} \text{ yr}^{-1}$. To accommodate a much larger SFR, say of $3000 M_{\odot} \text{ yr}^{-1}$, requires the age of the starburst to be about $3 \times 10^{7}$ yr.

4.6. The Spectral Energy Distribution of J1148+5251

The SED of J1148+5251 offers very few clues regarding the relative starburst or AGN contribution to the thermal dust emission from this galaxy. Figure 11 depicts the galaxy’s SED from UV to submillimeter wavelengths. Data and references are listed in Table 6. The UV and optical parts of the spectrum are most likely dominated by escaping starlight, and the far-IR by reradiated thermal emission from dust. The dash-dotted gray curve in the figure represents the intrinsic stellar radiation field synthesized with PÉGASE (Fioc & Rocca-Volmerange 1997) for a continuous SFR of age $t = 400$ Myr, with a top-heavy IMF and a SFR of $2500 M_{\odot} \text{ yr}^{-1}$. The total intrinsic stellar luminosity is $1.0 \times 10^{12} L_{\odot}$. Part of this stellar energy is absorbed by dust and reradiated at IR wavelengths. We used a simple screen model with a Galactic extinction law (Zubko et al. 2004) to calculate the spectrum of the escaping stellar radiation, depicted by the dotted curve in the figure. The magnitude of the extinction was chosen so that the total energy absorbed by the dust, shown as a gray shaded area in the figure, is equal to the total reradiated far-IR emission. The total luminosity radiated by the starburst-heated dust is $4.6 \times 10^{11} L_{\odot}$. The composition of this dust was taken to consist of a mixture of silicate and graphite dust with
mass fractions of $\frac{x}{3}$ and $\frac{1}{4}$, respectively, and a $7^{-6}$ distribution of dust temperatures ranging from 40 to 150 K.

The thick solid curve represents the sum of all emission components and represents the best $\chi^2$ fit of select model parameters (the intensity of the starburst and the slope of the power law describing the AGN spectrum) to the observations. The model described above is only a plausible one and definitely not unique. The possible existence of many distinct emission components described above is only a plausible one and definitely not unique. The origin of the near- to mid-IR (NMIR) emission is more uncertain. The rest-frame $\sim 0.5 - 1 \mu m$ fluxes are in excess of the stellar emission that can be produced by a young starburst. It also cannot be produced by dust, since it requires the grains to radiate at temperatures above their sublimation point of $\sim 1500$ K. We therefore fit the NMIR emission with two components: an AGN represented by a $L^{-1.3}$ power law and a hot dust component represented by a $1:9$ mix (by weight) of silicate and graphite grains with a $7^{-6}$ distribution of dust temperatures ranging from 150 to 1500 K. The relatively low silicate-to-graphite mass ratio was chosen to avoid the production of a mid-IR excess due to the $9.7 \mu m$ silicate emission feature. The power law is in good agreement with the average value found in the sample of Infrared Space Observatory Palomar-Green quasi-stellar objects studied by Haas et al. (2003). The total intrinsic luminosity of the AGN is about $7 \times 10^{13} L_{\odot}$, and the luminosity radiated by the AGN-heated dust is $1.3 \times 10^{13} L_{\odot}$.

If the black hole (BH) radiates at the Eddington luminosity,

$$L_{\text{E}}(L_{\odot}) \approx 3 \times 10^{4} \left( \frac{M_{\text{BH}}}{M_{\odot}} \right),$$

then the BH mass required to produce the AGN luminosity is $\sim 2 \times 10^{9} M_{\odot}$, comparable to the mass estimate derived by Willott et al. (2003) from the width of the Mg ii line. Mechanisms for the formation of seed black holes that enable their rapid growth to masses in excess of $\sim 10^{9} M_{\odot}$ have been discussed by Lodato & Natarajan (2006).

### 5. SUMMARY AND DISCUSSION

The early universe is a unique environment for studying the role of massive stars in the formation and destruction of dust. In this paper we developed analytical models describing the evolution of the gas, dust, and metallicity in high-redshift galaxies. The equations describing their chemical evolution can be greatly simplified by using the instantaneous recycling approximation and by neglecting the delayed contribution of low-mass stars to the metal and dust abundances of the ISM. Neglecting any accretion of metals onto dust in the ISM, the evolution of the dust is then completely driven by the condensation of refractory elements in the ejecta of Type II SNe and the destruction by SN blast waves in the ISM. The solutions for the evolution of the mass of gas, dust, and metals are presented in § 2 for closed-box and infall models for the chemical evolution of the galaxy and for different functional forms for the stellar IMF. The results of our paper can be briefly summarized as follows.

1. The maximum attainable dust-to-metal mass ratio in any system is equal to the IMF-averaged mass fraction of metals that are refractory and able to condense onto grains in SN ejecta, which is about 0.35 (see Table 2).

2. This mass fraction is significantly reduced when grain destruction is taken into account (Fig. 6). An observed dust-to-metals mass fraction $\gtrsim 0.4$ will therefore imply that accretion of ices onto interstellar grains in the ISM may be important in determining the dust mass in the galaxy.

3. Grain destruction plays an important role in the evolution of dust. However, its efficiency depends on the morphology of the ISM and is therefore highly uncertain (§ 2.6). We therefore present all our results for different values of $\langle m_{\text{ISM}} \rangle$, the effective mass of ISM gas that is completely cleared of dust by a single SNR.

4. In § 3 we present the general results of our models, describing the evolution of the gas, the dust, and the metals for both the closed-box and infall models.

5. We applied our general results to J1148+5251, a dusty, hyperluminous quasar at redshift $z = 6.4$. The observed and adopted quantities of J1148+5251 are summarized in Tables 3, 4, and 5.

6. The formation of about $2 \times 10^{8} M_{\odot}$ of dust in this galaxy requires an average SN to produce about $1 M_{\odot}$ of dust (Fig. 8). Theoretically, such a large amount of dust can be produced if stars are formed with a top-heavy IMF and with a moderate amount of grain destruction ($\langle m_{\text{ISM}} \rangle \approx 100 M_{\odot}$). A Salpeter IMF fails to produce this amount of dust even in the absence of any grain destruction. Observationally, the required dust yield is in excess of the largest amount of dust ($\sim 0.02 M_{\odot}$) observed so far to have formed in a SN. This suggests that accretion in the ISM may play an important role in the growth of dust mass.

7. Figure 11 depicts the galaxy’s SED from UV to far-IR wavelengths. The SED includes emission from the starburst, the AGN, and hot and cold dust components, radiating at mid- and far-IR wavelengths, respectively.

8. Uncertainties in the fraction of the IR luminosity that is powered by the starburst and in the stellar IMF prevent any accurate determination of the current SFR in the galaxy or the unique determination of its star formation history (see § 3.4 and Fig. 3).

9. Simple decomposition of the galaxy’s SED into its emission components suggests that the intrinsic starburst luminosity is about $1 \times 10^{14} L_{\odot}, 4.6 \times 10^{13} L_{\odot}$ of which is absorbed and re-radiated by dust at far-IR wavelengths.

### TABLE 6

| $\lambda_{\text{obs}}$ (\mu m) | $\lambda_{\text{rest}}$ (\mu m) | $F_{\text{obs}}$ (mJy) | References |
|---|---|---|---|
| 0.77 | 0.10 | 0.0017 ± 0.0002 | Fan et al. 2003 |
| 0.91 | 0.12 | 0.0325 ± 0.0033 | Fan et al. 2003 |
| 1.08 | 0.14 | 0.0087 ± 0.0089 | Fan et al. 2003 |
| 1.22 | 0.16 | 0.0073 ± 0.0069 | Willott et al. 2003 |
| 1.63 | 0.22 | 0.0024 ± 0.0009 | Willott et al. 2003 |
| 2.19 | 0.30 | 0.103 ± 0.010 | Willott et al. 2003 |
| 3.66 | 0.49 | 0.124 ± 0.002 | Jiang et al. 2006 |
| 4.51 | 0.61 | 0.140 ± 0.003 | Jiang et al. 2006 |
| 5.81 | 0.78 | 0.133 ± 0.010 | Jiang et al. 2006 |
| 8.01 | 1.08 | 0.241 ± 0.016 | Jiang et al. 2006 |
| 16.01 | 2.16 | 0.51 ± 0.25 | Charmandaris et al. 2004 |
| 22.01 | 2.96 | 0.74 ± 0.37 | Charmandaris et al. 2004 |
| 24.01 | 3.23 | 1.52 ± 0.13 | Jiang et al. 2006 |
| 70.01 | 9.43 | ≤10 | Jiang et al. 2006 |
| 350.01 | 47.2 | 21 ± 8.1 | Beelen et al. 2006 |
| 450.01 | 60.7 | 24.7 ± 8.6 | Robson et al. 2004 |
| 850.01 | 115 | 7.8 ± 7 | Robson et al. 2004 |
| 1200.01 | 162 | 5.0 ± 1.1 | Bertoldi et al. 2003a |
| 3000.01 | 404 | 0.5 ± 0.52 | Bertoldi et al. 2003b |
| 2.1 × 10^4 | 2.9 × 10^4 | 0.055 ± 0.012 | Carilli et al. 2004 |
10. The $\sim 3$ $\mu$m IR emission from the galaxy can be produced by neither starlight nor hot dust. It therefore must be emission from the AGN, and we estimate the AGN luminosity to be about $7 \times 10^{13} L_\odot$, $1.3 \times 10^{13} L_\odot$ of which is assumed to be absorbed and reradiated by dust at mid-IR wavelengths.

11. The AGN luminosity requires the formation of a black hole of a mass $\approx 2 \times 10^9 M_\odot$ at this redshift.

We thank Brad Gibson and the anonymous referee for comments that have contributed to the improvement of the manuscript. E. D. acknowledges the support of NASA’s LTSA 03-0000-065. The work of F. G. was supported by Research Associateship awards from the National Research Council (NRC) and from the Oak Ridge Associated Universities (ORAU) at NASA Goddard Space Flight Center.

REFERENCES

Beelen, A., et al. 2006, ApJ, 642, 694
Bertoldi, F., et al. 2003a, A&A, 406, L55
———. 2003b, A&A, 409, L47
Carilli, C. L., et al. 2004, AJ, 128, 997
Charmandaris, V., et al. 2004, ApJS, 154, 142
Cioffi, D. F., McKee, C. F., & Bertchinger, E. 1988, ApJ, 334, 252
Draine, B. T., & Lee, H. M. 1984, ApJ, 285, 89
Dwek, E. 1998, ApJ, 501, 643
———. 2005, in AIP Conf. Proc. 761, The Spectral Energy Distributions of
Gas-Rich Galaxies: Confronting Models with Data, ed. C. C. Popescu &
R. J. Tuffs (New York: AIP), 103
Dwek, E., Foster, S. M., & Vancura, O. 1996, ApJ, 457, 244
Dwek, E., & Scalo, J. M. 1980, ApJ, 239, 193
Eales, S. A., & Edmunds, M. G. 1996, MNRAS, 280, 1167
Edmunds, M. G. 2001, MNRAS, 328, 223
Elvis, M., Marengo, M., & Karovska, M. 2002, ApJ, 567, L107
Engelbracht, C. W., et al. 2005, ApJ, 628, L29
Fan, X., et al. 2003, AJ, 125, 1649
Fioc, M., & Rocca-Volmerange, B. 1997, A&A, 326, 950
Galliano, F., Dwek, E., & Chianal, P. 2007, ApJ, submitted
Gao, Y., & Solomon, P. M. 2004, ApJ, 606, 271
Graci´-Carpio, J., Garcia-Burillo, S., Planesas, P., & Colina, L. 2006, ApJ, 640, L135
Haas, M., et al. 2003, A&A, 402, 87
Heger, A., Fryer, C. L., Woosley, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288
Hughes, D. H., et al. 1998, Nature, 394, 241
Iwamoto, F., et al. 2004, ApJ, 614, 69
Jiang, L., et al. 2006, AJ, 132, 2127
Jones, A. P. 2004, in ASP Conf. Ser. 309, Astrophysics of Dust, ed. A. N. Witt,
G. C. Clayton, & B. T. Draine (San Francisco: ASP), 347
Jones, A. P., Tielens, A. G. G. M., & Hollenbach, D. J. 1996, ApJ, 469, 740
Kennicutt, R. C., Jr. 1998a, ARA&A, 36, 189
———. 1998b, ApJ, 498, 541
Lodato, G., & Natarajan, P. 2006, MNRAS, 371, 1813
Madden, S. C., Galliano, F., Jones, A. P., & Sauvage, M. 2006, A&A, 446, 877
Maiolino, R., et al. 2004, Nature, 431, 533
———. 2005, A&A, 440, L51
McKee, C. F. 1989, in IAU Symp. 135, Interstellar Dust, ed. L. J. Allamandola
& A. G. G. M. Tielens (Dordrecht: Kluwer), 431
Morgan, H. L., & Edmunds, M. G. 2003, MNRAS, 343, 427
Nomoto, K., Tominaga, N., Umeda, H., Kobayashi, C., & Maeda, K. 2006,
Nucl. Phys. A, 777, 424
Robson, I., Priddey, R. S., Isaak, K. G., & McMahon, R. G. 2004, MNRAS, 351, L29
Rouleau, F., & Martin, P. G. 1991, ApJ, 377, 526
Sugerman, B. E. K., et al. 2006, Science, 313, 196
Tielens, A. G. G. M. 1998, ApJ, 499, 267
Walter, F., et al. 2004, ApJ, 615, L17
White, R. L., Becker, R. H., Fan, X., & Strauss, M. A. 2003, AJ, 126, 1
Willott, C. J., McLure, R. J., & Jarvis, M. J. 2003, ApJ, 587, L15
Woosley, S. E., & Weaver, T. A. 1995, ApJS, 101, 181
Zubko, V., Dwek, E., & Arendt, R. G. 2004, ApJS, 152, 211