Cosmic defects and CMB anisotropy

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Recent measurements of the cosmic microwave background (CMB) anisotropies by BOOMERANG and MAXIMA collaborations have tightened the observational constraints on theories of structure formation. They disagree with the predictions of conventional topological defect models. Considering the fact that topological defects are predicted by the majority of realistic particle physics models, the exact nature of the constraints imposed by the recent data on the population and the properties of the defects must be fully understood. We show that the predictions of current cosmic string models can be brought into a closer agreement with the observations by choosing a closed universe with \( \Omega = 1.3 \) and by including the effects of the small-scale structure and radiation products of the strings. These alone, however, are not sufficient for obtaining a good fit to the measured shape of the angular power spectrum. To fit the data cosmic strings would either have to be correlated on large (perhaps superhorizon) scales or would have to possess a higher degree of coherence, i.e. be more “time-correlated”.

I. INTRODUCTION

Observations of the Cosmic Microwave Background (CMB) Anisotropy have catalyzed the rapid growth in the field of Cosmology over the last decade. They have given theorists a rare opportunity to test their models against experiment. In particular, the CMB data can help identify the mechanism that led to the formation of large scale structure that we observe today.

The recent measurements of the CMB anisotropy by BOOMERANG \(^1\) and MAXIMA \(^2\) experiments have set a new precision standard in comparing the predictions of different theories. There are two main classes of models of structure formation - the inflationary models and models with topological defects. The experimental data has different implications for this two classes of models. If one assumes an inflationary scenario then the data can be fit quite well by choosing adiabatic initial conditions, a nearly flat universe with a cosmological constant and a relatively high baryon density \(^3\). Our goal is to understand the implications of the data for defect models.

It is widely believed that at sufficiently high energies all known forces of nature must be described by a single Grand Unified Theory (GUT) of fundamental interactions. It is also believed that the Universe evolved from a very hot state (with temperatures well above the GUT scale) and underwent a series of phase transitions as it cooled down due to the expansion. Topological defects would inevitably form during these phase transitions, hence their subsequent evolution and observational signatures must be understood. Defects provide the only alternative to inflation as a mechanism for generating density fluctuations. Despite its popularity, inflation still remains to be placed in a realistic particle physics context. Inflationary models can be carefully tailored to fit practically any observational data by a suitable choice of otherwise arbitrary parameters. In contrast, the defect models have the attractive feature that they have no parameter freedom as all the necessary information is in principle contained in the underlying particle physics model.

Calculating CMB anisotropies sourced by topological defects is a rather difficult task. In inflationary scenario the entire information about the seeds is contained in the initial conditions for the perturbations in the metric. In the case of cosmic defects, perturbations are continuously seeded over the period of time from the phase transition that had produced them until today. The exact determination of the resulting anisotropy requires, in principle, the knowledge of the energy-momentum tensor (or, if only two point functions are being calculated, the unequal time correlators \(^4\)) of the defect network and the products of its decay at all times. This information is simply not available! Instead, a number of clever simplifications, based on the expected properties of the defect networks (e.g. scaling), are used to calculate the source. The latest data from BOOMERANG and MAXIMA experiments clearly disagree with the predictions of these simple models of defects.

The approach that we would like to take is not to simply dismiss the defect models as candidates for structure formation but to find the properties that are responsible for the disagreement and try to formulate the constraints that data imposes on the population and the properties of topological defects.

As we will illustrate, some of the disagreements between the data and the predictions of the cosmic string models can be resolved by going to a marginally closed Universe \((\Omega = 1.3)\) and by including the effects of the string decay products and the small-scale scale structure on the long strings. These are still insufficient to obtain an acceptable fit to the BOOMERANG/MAXIMA...
data. The data can be fit by requiring a higher degree of coherence of the string network. None of the currently known string models has the required degree of coherence. It would have to be achieved artificially, e.g., by making the coherence time an arbitrary parameter. This is not the only way of adding an arbitrary parameter to fit the data. In mixed models of strings+inflation \(^1\) the ratio of the contributions from strings and inflation is an arbitrary parameter and in the models with a varying speed of light \(^5\) there is an arbitrariness in the choice of \(c_0\), the speed with which the scalar field interacts at the formation of the strings. Such models can be rather successful in fitting the observations at the cost of violating the causality in its conventional sense. Namely, in these models the strings are allowed to be correlated on superhorizon scales.

Even though these solutions do not have much of an appeal at the moment, the interest to defect models could be revived should significant non-gaussian signatures be detected in the CMB.

II. PREDICTIONS OF CURRENT COSMIC STRING MODELS

The CMB anisotropy from cosmic strings was calculated by a number of different research groups\(^1\) \(^{10–19}\).

The current state of affairs can be inferred by looking at the plots in Figures 1,2,3 and 4. Figures 1 and 2 are from \(^13\) in which the authors studied the effects of adding an extra fluid component describing the decay products of the long strings. They considered a class of models in which the equation of state for the extra fluid was parametrized by a single parameter \(w_X\). The choice of \(w_X = 1/3\) corresponds to the gravitational radiation. They assumed a flat Friedmann-Roberston-Walker (FRW) universe with cold dark matter and no cosmological constant. Figures 3 and 4 show the results of \(^17\) where the wiggly nature of long strings was taken into account. The plots in Figures 3 and 4 were obtained assuming a flat universe with cold dark matter and a cosmological constant \((\Omega_\Lambda = 0.7)\).

It is not hard to see that the agreement with the data is poor. The apparent problems are: 1) The main peak is too wide and there is no second peak. 2) The main peak in angular spectrum is in the wrong place. 3) The matter power spectrum needs a big bias factor, especially on large scales. How robust are these features? What can be done in order to improve the agreement with the data?

\*Considering the large amount of papers published on the subject our list of references cannot possibly be complete. We refer the reader the recent edition of the book by Vilenkin and Shellard \(^29\) for a more comprehensive review of the literature.

FIG. 1. A plot from \(^13\) of the CMB power spectrum for cosmic strings. A flat FRW geometry with no cosmological constant is assumed. The BOOMERANG and MAXIMA data were not available then. The higher curve corresponding to \(w_X = 1/3\) shows what happens if 5% of the energy goes into the radiation fluid.

FIG. 2. A plot from \(^13\). The CDM power spectrum for cosmic strings and the data points inferred by Peacock and Dodds from galaxy surveys \(^21\). The top \(2\) \(w_X = 1/3\) curves correspond to a 5% transfer into CDM, and a 20% transfer into baryons (top).
Let us start from the bottom of the list and work our way up.

The solution to the large bias problem in the matter power spectrum is partially tied to the solution of the other two problems on the list. Certain defect models, in particular, models with a cosmological constant \([12, 17]\), can fit the shape of the data well. There is still some uncertainty in the data itself as new and more precise measurement are eagerly awaited from the SLOAN collaboration \([22]\). Thus, we will postpone the detailed analysis of the matter power spectrum predictions.

### III. THE STRUCTURE OF DOPPLER PEAKS

The peak structure in the angular spectrum is determined by three main factors: the geometry of the universe, coherence and causality.

#### A. Strings in a closed universe

The curvature of the universe directly affects the paths of light rays coming to us from the surface of last scattering \([23]\). In a closed universe, because of the lensing effect induced by the positive curvature, the same physical distances between points on the sky would correspond to larger angular scales. As a result, the peak structure in the CMB angular power spectrum would shift to the larger angular scales or the smaller values of \(l\).

What are the current observational constraints on the value of \(\Omega_{\text{total}}\)? Excluding the CMB constraints on the cosmological parameters, since they are obtained using the adiabatic inflationary models, the current estimates of the matter content of the universe are (see \([24]\) and references therein)

\[
\Omega_{\text{Matter}} = 0.35 \pm 0.1 ,
\]

\[
\Omega_\Lambda \approx \frac{1}{3} (4\Omega_{\text{Matter}} + 1) + 0.15 = 0.8 \pm 0.15.
\]

Adding \(\Omega_{\text{Matter}}\) and \(\Omega_\Lambda\) with their uncertainties gives:

\[
\Omega_{\text{total}} \approx 1.15 \pm 0.25.
\]

Based on this estimate we can conclude that a closed universe with \(\Omega_{\text{total}} = 1.3\) is just as probable as a flat one.

In Figure 5 we plot the prediction of the cosmic string model of \([17]\) for \(\Omega_{\text{total}} = 1.3\). We used the modified version of CMBFAST \([26]\) with minor changes necessary in order to include active sources. The details of the string model can be found in \([17]\). The model was slightly altered by incorporating the recent improvements to the velocity dependent one-scale model \([15]\) which describes...
in the case of global strings \([10]\).

Ω wiggly string model of \([17]\) in a closed universe with not negligible they are sufficiently small for local strings

When doing calculations in closed geometry we only included the scalar contribution to the angular power spectrum. Although the vector and tensor contributions are included the scalar contribution to the angular power spectrum can be matched by and can be ignored at this point.

As can be seen from Figure 3 the position of the main peak in the angular power spectrum can be matched by choosing a reasonable value for \(\Omega_{\text{total}}\). This is in an approximate agreement with the estimate based on the simple formula for the shift of the peak due to the curvature effect alone given by S. Weinberg \([23]\). This formula, valid for significant \(\Omega_L\) and for \(\Omega_{\text{total}}\) close to 1, relates the position of the peak in the flat universe, \(l_0\), to the position of the peak in a curved case, \(l_1\), as \(l_1 = l_0 \Omega^{-1.58}\). There are additional important effects which affect the peak structure such as the changes in the evolution of the string network, which we included in the simulation.

Even with the main peak in the right place the agreement with the data is far from satisfactory. The peak is significantly wider than that in the data and there is no sign of a rise in power at \(l \approx 600\). As can be seen from Figs. 1 and 3 the sharpness and the height of the main peak in the angular spectrum can be enhanced by including the effects of the gravitational radiation and wiggles. More precise high-resolution numerical simulations of string networks in realistic cosmologies with a large contribution from \(\Omega_L\) are needed to determine the

\[ \text{FIG. 5. The CMB power spectrum produced by the wiggly string model of [13] in a closed universe with } \Omega_{\text{total}} = 1.3, \Omega_{\text{baryon}} = 0.05, \Omega_{\text{CDM}} = 0.35, \Omega_L = 0.9, \quad H_0 = 65 \text{km s}^{-1} \text{Mpc}^{-1}. \]


\[ ^{\dagger} \text{The contribution of vector modes is much more significant in the case of global strings [10].} \]
tions in the metric and the densities of different particle species. If one assumes that the defects are formed by a causal mechanism in an smooth universe then the correct initial condition are obtained by setting the components of the stress-energy pseudo-tensor $\tau_{\mu\nu}$ to zero \[21\], \[8\]. These are the same as the isocurvature initial conditions \[9\]. A generic prediction of isocurvature models (assuming perfect coherence) is that the first Doppler is almost completely hidden. The main peak is the second Doppler peak and in flat geometries it appears at $l \approx 300 - 400$. This is due to the fact that after entering the horizon a given Fourier mode of the source perturbation requires time to induce perturbations in the photon density.

The causality also manifests itself through the fact that no superhorizon correlations in the string energy density are allowed. The correlation length of a “realistic” string network, is normally between 0.1 and 0.4 of the horizon size.

A interesting study was performed by Magueijo et al \[7\] where they have constructed a toy model of defects with two parameters: the coherence length and the coherence time. The coherence length was taken to be the scale at which the energy density power spectrum of the strings turns from a power law decay for large values of $k$ into a white noise at low $k$. This is essentially the scale corresponding to the correlation length of the string network. The coherence time was defined in the sense described in the beginning of this section, in particular, as the time difference needed for the unequal time-correlators to vanish. Their study showed (see Figure 6) that by accepting any value for one of the parameters and varying the other (within the constraints imposed by causality) one can reproduce the oscillations in the CMB power spectrum. Unfortunately for cosmic strings, at least as we know them today, they fall into the parameter range corresponding to the upper right corner in Figure 6.

In order to fit the observations, the cosmic strings must either be more coherent or they have to be stretched over larger distances, which is another way of making them more coherent. To understand this imagine that there was just one long straight string stretching across the universe moving with some velocity. The evolution of this string would be linear and the induced perturbations in the photon density would be coherent. By increasing the correlation length of the string network we would move closer to this limiting case of just one long straight string and so the coherence would be enhanced.

We have tried experimenting with our string model by artificially making it more coherent. This can be achieved, for instance, by limiting the range over which the random phases of the time oscillations of the source are allowed to vary. The results are shown in Fig. 7, where we have also chosen $\Omega_{\text{total}} = 1.3$. The agreement with the data is somewhat better than that in Figures 1-5 but there is an additional small peak at $l \approx 80$ (the so-called isocurvature peak) which is not present in the data. Different
The question of whether the defects can result in a pattern of the CMB power spectrum similar to the Doppler peaks produced by the adiabatic inflationary models was repeatedly addressed in the literature [28, 27, 11, 12]. In particular, it was shown [27, 11] that one can construct a causal model of active seeds which for certain values of parameters can reproduce the oscillations in the CMB spectrum. The problem with the current models of cosmic strings is that they fall out of the parameter range that is needed to fit the observations. At the moment, the models with varying speed of light [8] and the hybrid models of strings+inflation [15] are the only models involving topological defects that can match the observations. These models require a violation of causality and contain an arbitrary parameter. Fixing one parameter may not sound so terrible considering the degree of arbitrariness involved in inflationary models. But, of course, the defect models would lose their most attractive feature, namely, being defined only by the symmetry breaking sequence that led to the particle physics we see in accelerators.

If causality is preserved, then incoherence is the main serious problem faced by the string models. Introducing an arbitrary parameter, the coherence time, and assuming a marginally closed universe may be sufficient to fit the data if the string network has a somewhat large correlation length. Can we think of a credible theory that would result in a more “time-organized” string network? Currently, we do not know of any physical mechanism that would make strings more coherent.

In summary, unless some additional new physics is discovered or postulated, the CMB data excludes the topological defects as the primary source of the structure formation. This does not imply that defects were not involved at all. Measurements of the non-gaussian features of the CMB anisotropy will further test the predictions of simple inflationary models. Certain inflationary models can produce a significant non-gaussian component with a $\chi^2$ distribution. A different distribution would mean that a mechanism other than inflation took part in seeding the perturbations. Future CMB measurements covering a bigger range of angular scales will also tell us if there is really a second and higher order peaks in the angular power spectrum. The results from the SLOAN collaboration will soon provide us with reliable data for the matter power spectrum. With improving computers and modeling techniques more complete high-resolution simulations of the formation and evolution of topological defects may point to some new, yet unknown properties.

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