STATISTICS OF STRING VACUA

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We give an introduction to the statistical approach to studying vacua of string/M theory, and discuss recent results of Ashok and Douglas on counting supersymmetric flux vacua in type IIB Calabi-Yau compactification.
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1 Introduction

What is the most important problem of string phenomenology?
Most would probably say that it is to find a compactification which reproduces the Standard Model in all its details, and makes new predictions. More realistically, one could settle for qualitative agreement with the Standard Model, as long as one gets new predictions.

If one interprets “qualitative” loosely enough, this has been done, but the standard which has been met is not very high. While the general structure of supersymmetric grand unification was already realized in the early works, many of the important features, especially the hierarchy of scales and the small cosmological constant, not to mention specific parameter values, were controlled by non-perturbative effects, which were not at all understood at that time. Besides these classic problems, string and M theory constructions come with new problems of their own, which require solution or explanation: why our universe appears four-dimensional, what chooses a particular compactification or other extra dimensional structure, and so forth.

Building on the understanding of non-perturbative string/M theory achieved in the mid-late 1990’s, it is now possible to study compactifications which address these goals. Still, no model has been proposed which meets all of the requirements. Should we be worried?

One can argue that, to the contrary, this is good news for any hope for making predictions from string theory. The argument is simply that we have only explored a tiny fraction of the compactifications now believed to exist, and these seem to be a fairly random cross section chosen not because they are more likely to work, but more because of ease of analysis and historical
accident.

If we had already found satisfactory candidates in this small sample, it is likely that the full set would contain large numbers of theories which agree with the Standard Model in every detail. Even worse, different theories out of this set might lead to very different predictions. In a worst-case scenario, string theory might not be testable at all.

To bring home this point, let us look at the variety of string/M theory constructions, and ask how many might be candidates. Focus on Calabi-Yau compactification, where some numbers are known.

The number of distinct CY threefolds is believed to be $10^5$–$10^6$. All these constructions involve additional choices – a choice of bundle or brane, with comparable multiplicity, which directly affect the spectrum. Choices of flux or nonperturbative gauge theory vacuum probably bring in much larger multiplicities – numbers $\sim 10^{100}$ are often cited (more below).

In a weakly coupled construction, it is not hard to exclude models with exotic matter of various types. But most constructions contain sectors with $O(1)$ couplings. Many proposals exist for strongly coupled sectors (composite models, supersymmetric technicolor, the supersymmetry breaking sector, or just hidden sectors) and our current understanding of nonperturbative gauge theory tends to support these claims. Thus, one cannot throw out such models from the start. On the positive side, one is better able to check if a specific model works.

Even leaving out considerations of moduli fixing and couplings, numbers like $10^{10}$ qualitatively distinct models seem very plausible. So far, the number which have been considered in any detail is more like 100.

If one agrees with this argument, then one is led to the belief that constructing a model which agrees with the Standard Model in detail is not the primary goal we should be pursuing at this point:

- If it is possible within our present limitations, this is a negative result.
- If it is not possible within our present limitations, one cannot get a result.

While this conundrum oversimplifies the situation, it deserves consideration.

Of course there are ways around it. Before focusing on one, let us at least mention the two others we know of.

One is to look for predictions which cannot be matched by four dimensional effective field theory, for example short range modifications to gravity. This is important, and many speakers here are discussing it. On the other hand, we have no clear reason to think such effects must exist. Indeed, the “traditional” Kaluza-Klein scenario still seems to fit evidence such as unifi-
cation of coupling constants better than any of the more recently proposed variations, and must be taken seriously.

The other is to look for some a priori principle which selects among the possible vacuum configurations. Given such a principle, it is of course much more interesting to know if the vacuum it selects can reproduce the observations.

As an example to illustrate “Vacuum Selection Principles,” let me give you my best idea along these lines. One might imagine (see for example [9]) that our vacuum is in some sense “the most symmetric” among the various possibilities. Besides esthetics, many other candidate principles – for example, maximizing some natural wave function of the universe – seem likely to prefer such vacua.

On the face of it, this proposal is absurd. Higher dimensional models with more supersymmetry are obviously more symmetric. By rights, the “most symmetric” four dimensional theory, is the one with the most gauge symmetry. F theory examples are known with rank $10^5$ gauge groups. The Standard Model is not even in the running.

However, one might imagine that there exists some unstable nonsupersymmetric vacuum with a huge gauge group, which is preferred by Planck scale cosmology. It would then roll down to our physical vacuum. Then, the “right vacuum” would be near this preferred symmetric point.

This is as close to a plausible a priori principle as I have come, but using it still requires some fairly detailed knowledge about the set of possible vacua, and the configuration space which contains the vacua. Most such ideas require even more information. It does not seem reasonable to hope for a principle which will tell one in advance which string theory, Calabi-Yau, brane configuration etc. to look at.

Of course, it could be that the tests we already know of (or will know in ten years time) are already stringent enough to select out zero or one vacua. Maybe if we could work with them, an a priori principle would not be necessary.

And, we should keep in mind, that there is no guarantee that any a priori vacuum selection principle exists. We only have one sample, and the question of why we observe this one need not have any better answer than “because we are here.”

This is not an anthropic argument, which is a much more specific and predictive type of argument. It is just a conservative interpretation of the goal of science: to explain what we see, not why we see it.
2 Statistics of vacua

So what to do? There is a third approach, which has been advocated for some time by Dine [1], it is to look for “generic” predictions of string theory. Can we make this idea precise?

Any attempt to make generic predictions has to be founded on the idea that we know “all” string vacua or at least some representative set, in the sense that the distribution of a property of interest is the same in the representative set as in the whole. At present we can only hypothesize that the sets of vacua we can study concretely could be representative, and eventually check this by the results. For example, one can argue that the set of all type II compactifications on Calabi-Yau, to the extent that it can be studied at arbitrary volume and string coupling, could be a large fraction of the vacua, because the other known large classes of vacua are believed to be dual to these.

This at least gets us started, but it is clear that any representative class of vacua is far too complicated to study in any detail at present. We need to ask simpler questions, which might give us some picture of this huge “landscape” of theories.

To do this, we proposed in [5] to work as follows. The idea is to make a precise hypothesis for an approximate description of this set: the vacua are vacua of a specific ensemble of effective field theories, something like a list or set of theories which we believe can come out of string/M theory. While the ensemble should be precisely specified, we need not claim that it exactly represents the set of string/M theory vacua, only that it represents it well enough for our purposes.

We then can proceed in two directions:

• We can test whether our hypothesized ensemble is accurate, by comparing with actual string/M theory constructions.

• We can find out what fraction of vacua out of our ensemble meet a specified phenomenological test.

Let us give a very simple example to illustrate the point, by asking the question: Out of all the four dimensional vacua obtained by string/M theory compactification, how many of them are effective field theories with $SU(3) \times SU(2) \times U(1)$ gauge symmetry unbroken at low energy? If we define our terms, and if string/M theory has a precise definition, and if there are finitely many physically distinct vacua, then this question has a definite answer.

One can just as easily generalize the question to, out of all vacua, how many have low energy gauge group $G$? Let us denote this number by the
function
\[ d\mu(G), \]

While finding this function exactly is hard, perhaps it can be approximated in some simple and useful way.

For example, could it be that the rank \( r = \text{rk} G \) of the unbroken gauge group, roughly satisfies a power law distribution,
\[ d\mu[r] \sim N \times r^{-\alpha}. \]

If so, and if we could estimate \( N \) and \( \alpha \), we could get a rough estimate for how many vacua have a rank 4 gauge group, without much effort. One could go on to study the distribution \( d\mu[N_1, N_2, \ldots] \) of the ranks of the simple factors, etc. Although this may sound ambitious, given that the function \( d\mu[r] \) is well defined, why shouldn’t it have a simple approximate description?

2.1 Distribution of quiver gauge theories

Let me turn to another problem along these lines, for which I can even suggest a simple approximate description (details can be found in [3]). We consider \( U(N) \) quiver gauge theories, i.e. with gauge group \( \prod_i U(N_i) \), and a spectrum of purely bifundamental matter. Such theories arise on the world-volumes of D-branes in type II string theory, embedded in the 3+1 observable dimensions, and constructing these is the first step in making “brane world” realizations of the Standard Model, as discussed in many talks here.

It would be quite interesting to know the distribution of gauge groups and matter content for the theories which come out of string compactification. Let us focus on part of this information: the difference between the number of multiplets \((\tilde{N}_i, N_j)\), and the number \((\tilde{N}_j, N_i)\). For a theory with \( K \) factors in the gauge group, which would arise by wrapping branes on \( K \) distinct cycles, these are \( K(K-1)/2 \) a priori independent numbers; we can summarize them in an antisymmetric matrix, the “intersection matrix” \( I_{ij} \).

These numbers counts chiral matter multiplets which cannot be lifted by mass terms, and thus generalize the “number of generations” in the Standard Model. In explicit brane constructions, they are entirely determined by the topology of the branes \( B_i \) and \( B_j \) carrying the \( U(N_i) \) and \( U(N_j) \) gauge groups. The simplest example of this is to consider D6-branes in Calabi-Yau compactification of IIa string theory: in this case, the intersection numbers are literally topological intersection numbers between the three-dimensional world-volumes of the branes in the CY. Similar formulas are known for the other types of branes, in all cases topological.
Let us grant that the totality of type II compactifications on CY, with subsequent choices, leads to a finite set of vacua, each with a quiver gauge theory realized on D-brane world-volumes. If this set is finite, it defines a distribution $d\mu[N_i; I_{ij}]$, the number of theories realizing each possible choice of the $N_i$ and $I_{ij}$.

In [8], we give arguments that as a matrix element $I_{ij}$ becomes large (but not too large), this distribution goes as

$$d\mu[N_i; I_{ij}] \sim \frac{dI_{ij}}{|I_{ij}|},$$

(with a cutoff $I_{\text{max}} = \min(N_1, N_2)$, determined in string theory compactification to be $I_{\text{max}} \sim 100$ by tadpole cancellation.)

The basic argument for this is that any given $U(N_1) \times U(N_2)$ theory leads to a distribution $I \sim k, k^2, k^3, \ldots$ with this power-like falloff, and the “total” distribution obtained by adding such distributions will also have this power-like falloff.

We then generalize to many gauge groups, by taking the distributions of bifundamentals for each pair of gauge groups to be independent. This ensemble has the great virtue of simplicity, and should not be dismissed out of hand. However, it is probably too simple, as it ignores the fact that branes tend to wrap groups of cycles which intersect among themselves, and do not intersect between groups.

A better candidate ensemble of theories can be obtained by considering $K \times K$ matrices which can be decomposed into blocks of size $K_1 \times K_1$, $K_2 \times K_2$, and so on, and using (1) to describe the expected distribution of matter content in each block. This is also very simple, and not obviously wrong.

Of course, we are not claiming that this is an exact description of the list of gauge theories coming out of string theory compactification, only that it models some features of the true list. One could go further and assert that, at the moment, this is the best simple candidate description of the range of matter spectra coming from brane constructions; it would be interesting to test it by checking it against the families of models which have already been concretely developed. In any case, it is a precise ansatz which one can use to study the fraction of models with a specified matter content.

For example, one can obtain the Standard Model by taking the gauge group $U(3) \times U(2) \times U(1) \times U(1)$, and the intersection matrix

$$
\begin{pmatrix}
0 & -3 & 3 & 3 \\
3 & 0 & -1 & 2 \\
-3 & 1 & 0 & 0 \\
-3 & -2 & 0 & 0
\end{pmatrix}.
$$
and applying a subsequent orientifold projection.

In the ensemble (1) with $K = 4$, the fraction of brane models which realizes this spectrum is

$$d\mu(-3)d\mu(3)d\mu(3)d\mu(-1)d\mu(2)d\mu(0) \sim 10^{-6}.$$  

If a given compactification has more than 4 gauge groups, we need to enumerate subsets of 4 and compare them with the SM. The fraction which work depends on whether we allow exotic matter charged under the SM gauge groups, which would live in the off-diagonal terms in the following block decomposition:

$$I = \begin{pmatrix} I_{SM} & I_{exotic} \\ -I_{exotic}^t & \ddots \end{pmatrix}$$

If we do not allow exotic matter, since most of the distribution has $I_{ij} \neq 0$, models in which the SM is realized in a block with $K_i = 4$ are very much favored.

While these ensembles are rather oversimplified, we believe that a description of the true ensemble of brane gauge theories, which suffices for this purpose, need not be too much more complicated. One can refine our estimate by formulating more detailed ensembles, and comparing them with actual string theory constructions. We suspect this will lead to similar results, say

$$10^{-16} < \frac{N_{SM}}{N_{all \ G,R}} < 1.$$  

In any case, we have formulated a quantitative sense in which the Standard Model matter content is “generic.” It is not to say that most models have this spectrum; indeed the fraction which do is small. But it might be large, when compared with other numbers.

3 Flux vacua

Perhaps the most straightforward class of vacua to which to apply these ideas is the set of “flux vacua” obtained by turning on gauge field strengths in the compact dimensions. These have been the focus of much recent work, in which it has been shown that their contributions to the vacuum energy can stabilize moduli.

Flux compactifications can even realize the observed small positive vacuum energy, and in this sense solve the cosmological constant problem. A particularly simple proposal of this type was made by Bousso and Polchinski.
They argued that, in compactification on a Calabi-Yau with $K$ cycles, the number of vacua with small cosmological constant could go as $c^K$, growing exponentially with the number of cycles. Since typical Calabi-Yau’s have $K \sim 100 - 500$ cycles, this suggests that string theory could have a huge multiplicity of vacua, $N_{\text{vac}} \sim 10^{100} - 10^{500}$. Furthermore, these vacua realize a spectrum or “discretuum” of values for the cosmological constant, which is roughly uniform near zero. Even if the distribution has no special properties near zero, this makes it quite likely that vacua with the small observed cosmological constant could exist just on statistical grounds.

In more detail, let $F$ be a gauge field strength. The equations of motion $\nabla F = 0$ force it to be harmonic, so determined by its integral over non-trivial homology cycles $\Sigma_\alpha$. Let

$$N^\alpha = \int_{\Sigma_\alpha} F$$

be the quantized number of $F$ fluxes on the cycle $\Sigma_\alpha$, and $K$ be the number of cycles.

A qualitative description of the total energy is

$$E = E_0 + \frac{1}{l^4} \sum_{i=1}^{K} q_i(z)^2 N_i^2$$

where $E_0$ is a flux-independent contribution, $q_i$ is a “charge” (determined by kinetic terms) and $l$ is the length scale of the internal space.

Suppose $E_0 < 0$ and $q_i \sim 1$, then the number of flux vacua with given $\Lambda = E(N)$ is roughly

$$d\mu_{\text{vac}}(\Lambda) \sim \int d^K N \, \delta(\Lambda - E) \sim (\Lambda - (E_0 l^4))^{K/2-1}.$$  \hspace{1cm} (2)

$$d\mu_{\text{vac}}(\Lambda \sim 0) \sim \epsilon L^{K/2}$$

with $L = E_0 l^2$, substantiating the claims.

While we see the exponential emerge, and the large number of vacua, this argument raises many questions: for example, what determines the crucial parameters $E_0$ and $l$.

More to the point, this is only a heuristic argument, which ignored the fact the fluxes back react on the metric. In the above formula, this was expressed in the dependence $q_i(z)$ of the “charges” on moduli. The actual
energy is found by minimizing $z$, and this is what determines the distribution
of vacua in the moduli space.

Could it be that for many fluxes, there is no minimum apart from the
infinite volume limit? Then, back reaction would eliminate most of this sup-
posed large number of vacua? Or, could most of them be dual realizations
of the same vacua?

3.1 Counting flux vacua

In [1], with Sujay Ashok, we answer this question by giving the first precise
estimate for the number of vacua in a family of compactifications. Without
going into excessive detail, we work with the IIb compactifications developed
by Giddings, Kachru and Polchinski [10] in which the problem of finding vacua
in the full ten dimensional theory can be shown (in the large volume limit) to
precisely reduce to a problem in an $\mathcal{N} = 1$ effective supergravity theory.

This effective theory has the following chiral superfields:

$$
\tau = C^{(0)} + i e^{-D}
$$

the axion-dilaton,

$$
z^i
$$

the complex structure moduli of $M$, and

$$
\rho^i
$$

the Kähler moduli of $M$. Their Kähler potential is the same (up to truncating fields) as
in the related $\mathcal{N} = 2$ supersymmetric compactification with no flux,

$$
K(z, \bar{z}) = -\log \text{Im} \, \bar{z}^i \frac{\partial F(z)}{\partial z^i} - \log \tau - 3 \log \text{Im} \, \rho.
$$

Besides a choice of CY and orientifolding, a flux compactification sector
is characterized by a quantized flux. In IIb theory these are described by two
integers for each three-cycle of $M$,

$$
N^\alpha \equiv N^\alpha_{RR} + \tau N^\alpha_{NS} = \int_{\Sigma_\alpha} F^{(3)}_{RR} + \tau H^{(3)}_{NS}.
$$

The potential can then be computed exactly at large volume, using special
geometry and the superpotential [11]

$$
W(z) = \int_M (F^{(3)}_{RR} + \tau H^{(3)}_{NS}) \wedge \Omega(z) \equiv \int_M G \wedge \Omega(z);
$$

This $W$ and $K$ can be computed explicitly, using techniques developed in the
study of mirror symmetry.

This superpotential depends on dilaton and complex structure moduli in
a fairly complicated way, and indeed stabilizes all of these moduli. On the
other hand, it does not depend on Kähler moduli, and as is well known this
$K$ exhibits “no scale” structure and does not stabilize these moduli at all.
Let us completely neglect the Kähler moduli for the time being and treat the others, returning to this point below.

In this problem, it turns out that the allowed “amount of flux,” the number $L$ of the previous argument, is constrained by tadpole cancellation:

$$\int F \wedge H = N(O3 \text{ planes}) - N(D3 \text{ branes}) \equiv L$$  \hspace{1cm} (4)

Thus, the precise question we ask, is to count the number of supersymmetric vacua, \textit{i.e.} solutions of

$$D_i W(z, \tau) = 0,$$

satisfying this bound, as a function of $L$, up to duality equivalences. The duality group in this context (large volume Calabi-Yau) is $SL(2, \mathbb{Z})$ acting on the dilaton-axion, times a subgroup of $Sp(b_3, \mathbb{Z})$, which acts simultaneously on the choice of flux, and on the complex structure. A simple way to get one representative of each duality class is to sum over all choices of flux, but only count vacua which are stabilized at $(z, \tau)$ in a fundamental region (of both duality groups) in moduli space.

In we compute the large $L$ asymptotics for an “index” which counts supersymmetric vacua with signs, and is thus a lower bound for the total number of supersymmetric vacua. The general formula for this is

$$I_{\text{vac}}(L \leq L_{\text{max}}) = \frac{(2\pi L)^{b_3}}{\pi^{n+1}b_3!} \int_{\mathcal{F} \times \mathcal{H}} \det(-R - \omega \cdot 1),$$  \hspace{1cm} (5)

where $\mathcal{F}$ is a fundamental region in the complex structure moduli space, and $\mathcal{H}$ is the fundamental region of $SL(2, \mathbb{Z})$ in dilaton-axion moduli space. This formula will have corrections in a series in $1/L$, but these should be small if $L >> K$.

An important point about the result is that it gives not just the number, but the actual distribution of points where the flux vacua stabilize the moduli. The term $\det(-R - \omega \cdot 1)$ which appears under the integral sign, is a $2n + 2$ form, derived from the Kähler form $\omega$, and the matrix of curvature two-forms $R$. If we integrate this form over a subregion in moduli space, we obtain the number of vacua which sit in this region. These results can be used to compare the numbers of vacua in different regimes of coupling and field space, as we will discuss in 5.

While mathematical techniques exist to do this type of integral, the actual result is known at present only for compactification on $T^6/\mathbb{Z}_2$, for which $K = b_3 = 20$ and $L = 32$: one finds

$$I_{\text{vac}} = \frac{7 \cdot (2\pi L)^{20}}{4 \cdot 181440 \cdot 12 \cdot 20!} \sim 4 \cdot 10^{21}. $$
To complete the discussion, we need to discuss stabilization of Kähler moduli. Now no-scale structure is generically spoiled by $\alpha'$ and non-perturbative corrections, say

$$W_{NP} = e^{iN\rho} + \ldots.$$  

which, it has been suggested \cite{12}, can be arranged to arise from some brane world-volume theory. Very generally, a solution of $DW(z) = 0$ for the complex structure moduli, with $e^{K|W|^2} << M_{pl}^4$, will become a stable supersymmetric AdS vacuum once these are taken into account. \cite{12,8} For example, 

$$0 = D_\rho W = iNe^{iN\rho} - \frac{3}{\rho - \rho} W_{rest}$$

has a solution for

$$\frac{2N}{3}(\text{Im} \rho)e^{iN\rho} = W_{rest}.$$ 

The function on the l.h.s. can take any value up to $2/3e \sim 1$, and one expects an exact nonperturbative $W(\rho)$ to behave similarly. Thus, any vacuum with $W_{rest}$ not too large, can be stabilized.

Thus, we need to know the distribution of AdS cosmological constants. This distribution can be computed in the same way as above; details will appear in \cite{5}. One finds that the distribution is “uniform near zero,” in other words the number of vacua with small $\Lambda = \epsilon << L$ goes as $N_{vac} \times \epsilon b_3 / L$.

There is a simple intuitive argument for this result.\cite{12} The AdS cosmological constant can be regarded as the length squared of a “vector in flux space” defined by the values of the various periods of cycles entering \cite{1}. If we assume that this vector is totally uncorrelated with the specific choice of flux, all vacua with flux roughly orthogonal to this vector will have small cosmological constant, leading to this result. Now this assumption is not innocent and is false in very similar examples (such as flux in the heterotic string); however the actual computation in type II does produce this result.

Thus, taking $\epsilon \sim 10^{-3}$, we obtain a lower bound $4 \cdot 10^{18}$ on the number of flux vacua on $T^6/\mathbb{Z}_2$. One might be even more conservative and cut out the strong coupling regime, where the analysis is not presently under control. If we do this by insisting on $\text{Im } \tau > 40$, we find that $10^{17}$ vacua satisfy these constraints.

4 Conclusions

We discussed the statistical approach to string phenomenology, and gave the first computation of the number of flux vacua in a specific model. This result...
confirmed the suggestion of Bousso and Polchinski that this number should
grow as $L^K$, exponentially in the number of cycles $K$, and is a first step in
getting a solid idea of just how many string theory vacua there are.

CY’s are known with $K \sim 500$ and $L \sim 10^4$, so there is some danger that
the number of vacua is large enough to spoil predictivity. We can roughly
quantify the number at which we should start to worry, as follows.\footnote{8} Consider
the twenty dimensional parameter space of the Standard Model couplings,
along with the cosmological constant. Now, compute the volume in this space
consistent with present-day observations, where we measure each coupling in
Planck units (so, a coupling $\lambda$ of dimension $n$ has measure $M^{-n} \int d\lambda$). For the
Standard Model, this comes out to about $10^{-240}$, where $10^{-120}$ of this comes
from matching the cosmological constant, and the rest from the Higgs mass
and other couplings.

Now, suppose we consider string models which match the spectrum of
the Standard Model at low energies. As we discussed, the fraction of four
dimensional models which do this is surely much greater than $10^{-100}$, and
$10^{-10}$ would seem to be a reasonable guess.

Take these models, and plot their couplings in this twenty dimensional
space. The computation we discussed is also a first step towards doing this, as
it gives the distribution of vacua in moduli space, and these moduli will control
the couplings in the resulting low energy theory. To some approximation,
the result\footnote{6} says that models are roughly uniformly distributed in moduli
space; this suggests that the dimensionless couplings could come out roughly
uniformly distributed as well. This is less clear for dimensionful couplings, and
discussing these requires taking supersymmetry and its breaking into account
in more detail. Let us continue however to illustrate the idea, and return to
this point.

Then, we can distinguish various cases. If the number of models is less
than $10^{120}$, we do not expect to match the cosmological constant, and need
to find mechanisms which produce the observed small value, to reasonably
expect string theory to reproduce the data. If it is between $10^{120}$ and $10^{240}$,
we expect models which reproduce the cosmological constant to exist, and can
in principle test string theory by checking that the observed Standard Model
couplings are possible in some compactification – just on statistical grounds,
this would be unlikely.

On the other hand, suppose there were many more than $10^{240}$ models, say
$10^{1000}$ for definiteness. In this case, one would expect a vast number of models
to reproduce the Standard Model in every detail, just on statistical grounds.
If it furthermore turned out that they led to many different predictions at
higher energies, one would clearly have grounds to worry that string theory
was not testable.

Of course supersymmetry changes these numbers. With low scale breaking (say 10 TeV), the corresponding “volume in coupling space” becomes about $10^{-120}$. Thus, unless the ratio of nonsupersymmetric candidates to these candidates is greater than $10^{120}$, these considerations might be regarded as favoring supersymmetry, as discussed at more length in 2.

There are some important points to be made here. First, it is essential to have some estimate of the a priori numbers, here the ratio between the numbers of nonsupersymmetric and supersymmetric vacua, to make this claim. It is true that one does not need a very accurate estimate of this ratio (here any accuracy better than $10^{100}$ would suffice), but one does need some estimate, for this argument to have any content beyond the standard argument for supersymmetry from naturalness.

Second, it should be realized that “favoring” one mechanism over another is interesting but not in itself decisive, if in fact both types of vacuum exist. We only observe one vacuum, and it might be of either type. On the other hand, suppose we estimated the number of supersymmetric vacua which pass all tests as $10^{40}$, while the number of non-supersymmetric vacua was estimated as $10^{-40}$. This could come out of the type of approximate estimates we are discussing, and would mean that we need a coincidence which tunes parameters at the $10^{40}$ level to get a candidate nonsupersymmetric vacuum. If our estimates were reliable, then the most reasonable interpretation of such evidence would be that there are in fact no candidate nonsupersymmetric vacua, and we would get a strong prediction from the statistics of vacua. It is this goal which motivates trying to get controlled estimates for these numbers.

As a final comment, it is interesting that the numbers which are coming out, of order $10^{100}$, are of the general order we need to solve the cosmological constant problem, while not obviously spoiling predictivity. Thus, the picture we just outlined need not be depressing – there are many further conditions the correct vacuum must satisfy, and it may be that further work along these lines will demonstrate that only one or a few vacua work. In any case, our main point is that these are questions about string theory which, if the theory has a precise definition, have definite answers, and that are now becoming accessible to investigation.

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