Photon production through $A_1$ resonance in high energy heavy ion collisions

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Abstract
Electromagnetic radiation from excited hadronic matter is one of the best ways to study the properties of the matter. Considering various processes in the hadronic gas, Kapusta et al. have found a $\pi \rho \rightarrow \pi \gamma$ reaction with intermediate virtual pion or rho to be the main source of photons with energy greater than 0.7 GeV. However, at temperatures considered $T=100-200$ MeV, a $\pi \rho$ pair can easily form an $A_1(1260)$ resonance, and we show that this mechanism leads to the photon production rates exceeding those suggested previously.
1. Introduction

Theoretical and experimental studies of very dense and hot hadronic matter is one of the most active fields, recently created at the intercept of nuclear and high energy physics. One of the goals of this program is to find evidence for phase transitions into new form of matter – the quark-gluon plasma.

As suggested in [2], a promising way to detect these phenomena experimentally is to look at electromagnetic radiation of photons and lepton pairs, the ‘penetrating probes’ which do not suffer the final state interaction. A number of works [3, 4, 5, 6, 7] were devoted during the last decade to evaluation of the corresponding emission rates from quark-gluon plasma.

However, expanding hadronic matter spends most of its time in the form of rather cool hadronic gas, which also produces some electromagnetic radiation. Kapusta et al [1] have recently attempted to calculate the rate for photon emission, using a model Lagrangian describing interaction of $\pi, \rho$ mesons with photons. It was pointed out in this work, that out of all possible processes involved, the $\pi \rho \rightarrow \pi \gamma$ reaction plays the dominant role in yielding photons with energies larger than 0.7 GeV.

In a different context, one of us [8] has recently discussed a number of hadronic reactions in a hadron gas. It was similarly found, that the main effect of $\rho$ meson modification in hadronic gas is its scattering on pions. Moreover, comparing various contributions to this scattering, it was found that the one related with the $A_1$ resonance is much more important, than that with an intermediate pion. Since only the latter was included by Kapusta et al, the question was raised whether the $A_1$-related process can also produce an important (or even dominant) contribution to the photon production. As will be shown in the present work, it is indeed the case and the $\pi \rho \rightarrow A_1 \rightarrow \pi \gamma$ reaction (see Fig.(1)) does ‘outshine’ all others.

The paper is organized as follows. In section 2 we describe the possible form of $\pi \gamma A_1$ interaction. The corresponding coupling constant is estimated in the vector dominance model. Then we evaluate the partial width $\Gamma(A_1 \rightarrow \pi \gamma)$, and compare it with other estimates and experimental data. Another test of these estimates are provided by the pion polarizability, for which also some experimental and theoretical estimates are available in the
literature. We have found reasonable agreement with the theoretical estimates, while the experimental situation is extremely uncertain and even contradictory. In section 3 we evaluate photon production rate due to the mechanism considered, and compare it with results obtained by Kapusta et al.

2. The $\pi\gamma A_1$ interaction

The process we are going to discuss is $\pi\rho \rightarrow A_1 \rightarrow \pi\gamma$, see Fig.(1), and its total cross section in the region of $A_1$ resonance can be written as

$$\sigma(\sqrt{s}) = \frac{\pi}{p^2} \frac{\Gamma_{A_1 \rightarrow \pi\rho} \Gamma_{A_1 \rightarrow \pi\gamma}}{\left(\sqrt{s} - m_{A_1}\right)^2 + \Gamma_{A_1}^2/4}$$

(1)

where $p$ is the three momentum of the rho in the c.m frame. $\Gamma_{A_1}$ is the total width of $A_1$ and is approximately equal to $\Gamma_{A_1 \rightarrow \pi\rho}$.

The axial $A_1$ meson is a strong resonance known for about 3 decades, and (although its properties [9] measured in hadronic reactions and $\tau \rightarrow \nu_\tau + A_1$ [10] are slightly different) its total and partial width $\Gamma_{A_1 \rightarrow \pi\rho}$ are reasonably well known.

However, the radiative decay width $\Gamma_{A_1 \rightarrow \pi\gamma}$ is far from being firmly determined. Its value given by Particle Data Tables is based on one experiment only, and there seems to be some problems with it (see below). Therefore we try to estimate its magnitude theoretically, and also compare the results with as many previous theoretical and experimental on the subject as possible.

By using the vector dominance model (VDM) [11], we can approximately relate the radiative decay width to the width to $\pi\rho$:

$$\Gamma_{A_1 \rightarrow \pi\gamma} = \Gamma_{A_1 \rightarrow \pi\rho} \left(\frac{e}{f_\rho}\right)^2$$

(2)

where $e$ is the electron charge and $f_\rho$ is the well known coupling constant of $\rho N N$ and $\rho\pi\pi$ (from the $\rho \rightarrow \pi\pi$ width, we find it be $\frac{f_\rho^2}{4\pi} = 2.9$).

Of course, Eq.(2) is a rough estimate, and one may try to improve it by taking into account the ratio of the phase space in both reactions. Also since $A_1$ is a wide resonance, the ratio of the matrix elements is modified when the total invariant mass of the process $\sqrt{s}$ is shifted away from the centroid of
the resonance. Particularly in our thermal production problem, the region
near the threshold, at $\sqrt{s} < m_{A_1}$, is especially important, since it is strongly
favored by the thermal distribution. To evaluate those corrections, one has
to study the $A_1$ physics within some Lagrangian model.

In constructing the $\pi\gamma A_1$ Lagrangian, we note that it is a subject for two
general constraints: (i) gauge invariance with respect to the photon field; (ii)
the interaction of soft pions should be proportional to either their momenta
or quark masses, since the pion is a Goldstone boson related with chiral
symmetry breaking. Let the fields for $A_1$, photon, and pion be denoted as
$a^\mu$, $A^\mu$, and $\phi$ respectively; then we can write the Lagrangian as

$$L_{\pi\gamma A_1} = G_\gamma a^\mu \Gamma_{\mu\nu} A^{\nu} \phi$$
$$= G_\gamma a^\mu (g_{\mu\nu} p_\pi \cdot p_\gamma - p_{\gamma\mu} p_{\pi\nu}) A^{\nu} \phi.$$  \hspace{1cm} (3)

In the above $p_\pi$ and $p_\gamma$ are four momenta carried by the pion and the photon.
With the above two constraints, Eq.(3) is the only possible structure of the
Lagrangian with minimal number of derivatives.

The effective coupling strength $G_\gamma$ (which bears the dimension of inverse
mass) can be estimated as above, by using the VDM \[11\]. This implies that
the $\pi\rho A_1$ Lagrangian has similar form as that of $\pi\gamma A_1$, namely

$$L_{\pi\rho A_1} = G_\rho \rho^\mu (g_{\mu\nu} p_\pi \cdot p_\rho - p_{\rho\mu} p_{\pi\nu}) A^{\nu} \phi$$ \hspace{1cm} (5)

where $\rho^\mu$ is the field for the rho and $p_\rho$ is its four-momentum.

The $\pi\rho A_1$ Lagrangian has been studied before in the chiral gauge model,
first by Schwinger \[12\], and further by others \[13, 14\]. Note that their La-
grangian looks as the first term in Eq.(3), but in this case, when rho is
substituted by gamma, it violates the gauge invariance and therefore cannot
be used for our VDM-type estimates. \[\]

The real accurate Lagrangian can only be made if the quantitative measurements of
s- and d-wave contributions be made. To construct the accurate Lagrangian requires the
experimental measurement of the angular distribution of the $A_1$ decay. By choosing $\theta$
to be the angle between the $A_1$ polarization and the moving rho in the $A_1$ rest frame, our
model yields

$$f(\theta) \sim 1 + 0.946 \cos^2 \theta$$
The $A_1 \to \pi \rho$ decay width is then found to be

$$\Gamma_{A_1 \to \pi \rho} = \frac{G_\rho^2 |\mathbf{p}|}{24\pi m_{A_1}^3} \left[ 2(p_\pi \cdot p_\rho)^2 + m_\rho^2 (m_\pi^2 + \mathbf{p}^2) \right]$$

(6)

where $\mathbf{p}$ is the pion momentum in the rest frame of $A_1$. From the above equation we determine $G_\rho = 14.8 \text{ GeV}^{-1}$, using the averaged experimental value for the total width $\Gamma_{A_1} = 0.4 \text{ GeV}$.

The coupling constants $G_\gamma$ and $G_\rho$ relate to each other in the VDM simply by

$$G_\gamma = G_\rho \frac{e}{f_\rho} = 0.743 \text{ GeV}^{-1}.$$  

(7)

With this we predict the radiative decay width

$$\Gamma_{A_1 \to \pi \gamma} = \frac{G_\gamma^2 |\mathbf{p}|}{12\pi m_{A_1}^2} (p_\pi \cdot p_\gamma)^2 = 1.42 \text{ MeV}.$$  

(8)

Now we are going to check whether this value produces results consistent with the experimental data and other theoretical estimates. The theoretical evaluation of the $A_1$ radiative width has been carried out in the framework of the non-relativistic SU(6) quark model in [15]. Their result

$$\Gamma_{A_1 \to \pi \gamma} = 1.0 \sim 1.6 \text{ MeV}.$$  

(9)

is quite consistent with our estimate.

The experimental evidence for the radiative decay of $A_1$ was found through its reverse process, i.e., by studying $A_1$ production in pion-nucleus collisions. With two different targets, lead and copper, the experiments were performed and yield the partial width [10]

$$\Gamma_{A_1 \to \pi \gamma} = 0.640 \pm 0.246 \text{ MeV},$$  

(10)

assuming the reaction is purely electromagnetic. However, in this case the cross section of the $A_1$ production should be proportional to $Z^2$, with $Z$ the charge of the target. This dependence is not clearly observed in the
data, which implies that strong interaction somewhat interferes with the electromagnetic one and can produce some systematic errors.

The pion polarizability sheds some light on $A_1\pi\rho$ interaction as well. Theoretically it was studied through the low energy Compton scattering on the pion [17, 18]. The forward scattering amplitude relates with the electric and the magnetic polarizabilities $\alpha_E, \beta_M$ through [19]

$$f(0) = \frac{1}{4\pi} (\epsilon \cdot \epsilon' w w' \alpha_E + \epsilon \times q \cdot \epsilon' \times q' \beta_M)$$

where $(w, q), \epsilon$ and $(w', q'), \epsilon'$ are the four momenta and polarizations of the incident and the outgoing photon respectively. In the chiral limit, it has been shown [17] that the pion electric polarizability from exchanging the vector current cancels the part from the pion formfactor, and therefore the process of intermediate $A_1$ is believed to dominate the pion polarizability.

With our estimated parameters, the pion electric polarizability from $A_1$ contribution is found to be

$$\alpha_E = \frac{G_2^2 m_\pi}{2m_{A_1}^2} = 1.8 \times 10^{-4} \text{fm}^3. \quad (12)$$

This value should be compared to the former QCD-based calculation [20] which yields $2.8 \times 10^{-4}$ fm$^3$, and the result of the current-algebra calculation [17], $2.6 \times 10^{-4}$ fm$^3$.

The experiments give two conflicting values for the pion electric polarizability. Both are larger than the theoretical results:

$$\alpha_E \approx (6.8 \pm 1.4) \times 10^{-4} \text{fm}^3 \quad (13)$$

from radiative pion scattering [21] and

$$\alpha_E \approx (20 \pm 12) \times 10^{-4} \text{fm}^3 \quad (14)$$

from the pion photoproduction [22]. The discrepancy of the theoretical predictions and the experimental measurements remains unsolved.

3. Photon Production from Hadronic Gas
Photon production from the hot hadronic phase was studied recently by Kapusta et al [1], who have calculated the photon production from the light ($\pi, \rho$) meson interactions. They have found that the pion-rho collision channel dominates for photons with energy greater than 0.7 GeV.

Including $A_1$ as an intermediate state, one can generally add many diagrams to photon production processes, but from kinematic arguments it is clear that only the $s$ channel $\pi\rho$ interaction is significant, since in the hadronic gas of $T = 100 - 200$ MeV the total energy of a pion and a rho combined is right at the $A_1$ resonance.

The photon production rate from a hadronic gas with temperature $T$ can then be calculated from

$$E_\gamma \frac{dR}{d^3p_\gamma} = \mathcal{N} \int \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2} f(E_1)f(E_2) \sum_{\lambda,\sigma} |\mathcal{M}^{\lambda,\sigma}|^2 \frac{d^3p_3}{(2\pi)^6 E_3} (1 + f(E_3)). \quad (15)$$

In the above the indices 1, 2, 3, and $\gamma$ are for incident pion, incident rho, outgoing pion, and outgoing photon respectively; $\mathcal{N}$ is the isospin degeneracy of the process; The matrix element for the process is

$$\mathcal{M}^{\lambda,\sigma} = G_\rho G_\gamma e_\rho^{\mu,\lambda} \Gamma_{\mu\nu} D^{\alpha\beta} \Gamma_{\nu\sigma} e_\gamma^{\nu,\sigma} \quad (16)$$

where $e_\rho, e_\gamma$ are the polarization of the incident rho and outgoing photon; $\lambda, \sigma$ are their spin indices; The vertex structure function $\Gamma_{\mu\nu}$ is defined by Eq.(3); $D^{\alpha\beta}$ is the propagator for $A_1$

$$D^{\alpha\beta}(p) = \left( g^{\alpha\beta} - p^\alpha p^\beta / m_{A_1}^2 \right) \frac{1}{p^2 - m_{A_1}^2 - i m_{A_1} \Gamma_{A_1}}; \quad (17)$$

$f(E) = 1/(\exp(E/T) - 1)$ is the Bose-Einstein distribution function. We replace it by the Boltzmann distribution, since we are interested in photons with energy much greater than the temperature. With this approximation, the right-hand side of Eq.(15) can be simplified as a three-dimensional integral. We numerically computed the integral and get the photon spectra. With the matrix elements of [1] due to exchanging intermediate pions and
rhos, we have also reproduced their spectra, which are shown in Fig.(2) for comparison (the dashed curves). Our results for photon production through the $A_1$ resonance are shown in Fig.(2) by solid curves. We have calculated the spectra for three different temperatures $T = 200, 150, \text{ and } 100 \text{ Mev}$, and for all curves, the combination of the thermal distribution and the matrix element results in the photon production peaking at $0.5 \sim 0.7 \text{ GeV}$. The pion-rho scattering processes dominate the photon production for $E_\gamma > 0.7 \text{ GeV}$; while photons with smaller energy come from other sources.

One can see from Fig.(2), that in the pion-rho dominated region, the photon production from the $A_1$ resonance is consistently greater than that from the diagrams with virtual pions and rhos. This is true even at the lowest temperatures considered, and with increasing T the effect is naturally more pronounced. At the same time, as one approaches the critical temperature of chiral restoration, the calculation becomes more uncertain because of possible strong modification of mesons, especially masses of mesons in matter. It has been proposed \[23\] that meson masses, other than the mass of the pion, decrease with temperature, going essentially to zero at the temperature for chiral restoration $T_c = T_{\chi SR}$. This temperature is now estimated to be $T_c \sim 140 \text{ MeV}$ in lattice gauge calculations \[24\]. The number of $\rho$ mesons will be greatly increased if $m^*_\rho$ drops, since Boltzmann factors will be larger, and this would strongly increase the process $\pi \rho \rightarrow \pi \gamma$, while the increase should bias at low energy photons more than at high energy photons. If one also takes into account the expected decrease in the $A_1$ mass, the photon production at high T becomes even larger.

Let us also note that two mechanisms considered above, those due to virtual $\pi, \rho$ and $A_1$ mesons, have some overlap in terms of partial waves, so after integration over angles some interference effect should appear. It may result in some small changes in the photon production rate. We have not calculated such details because due to uncertainties involved it does not make much sense.

Recently there are some efforts devoted to evaluation of photon production using the realistic space-time picture of the collision, especially using the hydrodynamic model \[25\]. Photons are accumulated at each temperature...
stage of the expansion of the fireball formed after the heavy-ion collisions. For the convenience of these calculations, we parametrized the rate of photon production \( \text{Eq.(15)} \) by a simple analytic function. Following \[26\] we suggest the parametrization to be

\[
E_\gamma \frac{dR}{d^3 p_\gamma} = 2.4 \times T^{2.15} \exp[-1/(1.35 T E_\gamma)^{0.77} - E_\gamma/T] \text{ (fm}^{-4}\text{GeV}^{-2}) \tag{18}
\]

here \( E_\gamma \) and \( T \) should be in units of GeV. We marked the parametrization by the solid squares in Fig.(2) for different temperatures. When comparing with the exact values given by the solid curves, we see that Eq.(18) is an excellent fit for the production rate.

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Figure Captions

**Figure 1:** Feynman diagram of $\pi \rho \rightarrow \pi \gamma$ through $A_1$ resonance.

**Figure 2:** Energy differentiated rate of photon production from $\pi \rho \rightarrow \pi \gamma$ processes in hot hadronic gas of temperature $T= 200, 150,$ and $100$ MeV (top, middle, and bottom). The solid curves are contribution from mechanism of exchanging $A_1$; the solid squares are from the suggested parametrization which fits the solid curves; the dashed curves are photon production from exchanging virtual pions and rhos.