Chiral-odd pion generalized parton distributions beyond leading twist

B. Pire
CPhT, École Polytechnique, CNRS, 91128 Palaiseau, France
Email: pire@cpht.polytechnique.fr

L. Szymanowski
Theoretical Physics Division, National Centre for Nuclear Research (NCBJ), Hoża 69, 00-681 Warsaw, Poland
Email: Lech.Szymanowski@fuw.edu.pl

S. Wallon
Laboratoire de Physique Théorique, Bât. 210, Université Paris-Sud, CNRS, 91405 Orsay, France &
UPMC Univ. Paris 06, faculté de physique, 4 place Jussieu, 75252 Paris Cedex 05, France
Email: wallon@th.u-psud.fr

Abstract: We define in a systematic way, based on the light-cone collinear factorization method, the chiral-odd generalized parton distributions (GPDs) of a pseudoscalar hadron (such as the $\pi^0$) up to twist 5. For that, we introduce the relevant matrix elements for 2-parton non-local operators, as well as matrix elements for 3-parton non-local correlators. Their detailed parametrization is fixed based on parity, charge conjugation and time reversal invariance. This leads to the introduction of 20 real GPDs, whose symmetry properties are explicitly given. The reduction of these GPDs to a minimal set is performed by the use of constraints provided by QCD equations of motion and rotation on the light-cone. We show that these 20 GPDs can be expressed through 8 GPDs which satisfy 4 integral sum rules. A surprising outcome of this analysis is the fact that, when assuming the vanishing of 3-parton correlators, as in the so-called Wandzura-Wilczek approximation, the whole set of GPDs vanishes.
Contents

1. Introduction 1

2. The set of GPDs including two and three body correlators 2
   2.1 Kinematics 2
   2.2 LCCF factorization 3
   2.3 Parametrization of GPDs 6
   2.4 Symmetry properties 9

3. The minimal set of GPDs 9
   3.1 $n$–independence 9
      3.1.1 Arbitrariness of $p$ and $n$ 9
      3.1.2 Variation of a Wilson line 10
      3.1.3 Operator identity 12
      3.1.4 Application to matrix elements 12
   3.2 QCD equations of motion 14
   3.3 Toward a minimal set of GPD 18
      3.3.1 The twist 5 case 18
      3.3.2 The twist 4 limit 18
      3.3.3 The twist 3 limit 19
      3.3.4 The vanishing of GPDs in the Wandzura-Wilczek limit 19

4. Conclusion 19

A. Appendices 20
   A.1 Symmetry properties of GPDs under charge conjugation $C$ 20
   A.2 Symmetry properties of GPDs under the time invariance $T$ 21
1. Introduction

The understanding of exclusive reactions in the generalized Bjorken regime has made significant progresses in the recent years, thanks to the factorization properties of the leading twist amplitudes for deeply virtual Compton scattering [1, 2] and deep-inelastic exclusive meson production [3].

However, the transversally polarized $\rho^-$ meson production does not enter the leading twist controllable case [4, 5] but only the twist 3 more intricate part of the amplitude [6–9]. This is due to the fact that the leading twist distribution amplitude (DA) of a transversally polarized vector meson is chiral-odd, and hence decouples from hard amplitudes at the twist two level, even when another leading twist chiral-odd quantity is involved [4, 5], unless in reactions where more than two final hadrons are involved [10–12]. The corresponding DAs of $\rho^-$ meson have been discussed in great details in refs. [13–15].

In the case of the process $\gamma^* p \rightarrow \pi^0 p$, the inclusion of chiral-odd $\pi^0$ DAs beyond twist 2 has been investigated [16–19]. For consistency, this requires to investigate at the same level of twist expansion the proton chiral-odd GPDs. This is the ultimate motivation of our studies. But in the present paper, for simplicity and in order to present details of our method, we restrict ourselves to the classification of $\pi^0$ chiral-odd GPDs beyond leading twist, despite limited possibilities of their phenomenological application (see however ref. [20]). Note also that related form factors have been estimated on the lattice [21] and discussed in ref. [22]. An analysis of transverse momentum dependent distributions for the pion was performed in ref. [23].

In the literature there exist two approaches to the factorization of the scattering amplitudes in exclusive processes at leading and higher twists. The first approach [7, 8, 24–26], the light-cone collinear factorization (LCCF), being the generalization of the Ellis–Furmanski–Petronzio (EFP) method [27–33] to the exclusive processes, deals with the factorization in the momentum space around the dominant light-cone direction. On the other hand, the covariant collinear factorization (CCF) approach in coordinate space was successfully applied in refs. [13, 14] to a systematic description of DAs of hadrons carrying different twists. Although being quite different and using different DAs, both approaches can be applied to the description of the same processes. We investigated the relationship between these two approaches, and tested it for the case of high energy $\gamma^* p \rightarrow \rho T p$ electroproduction in ref. [25, 26]. This turned out to be a very powerful tool in order to obtain a consistent description of HERA data [34], including saturation effects after passing from momentum to coordinate space representation [35, 36].

The aim of the present exploratory paper is to use the LCCF method for the case of GPDs beyond leading twist. In particular, we want to implement the concept of $n$–independence at the correlator level, without dealing with the full scattering amplitude.

The focus on more phenomenological GPDs, namely chiral-even and chiral-odd nucleon GPDs will be the subject of forthcoming papers. Apart from complications related to
nucleon spinor degrees of freedom, the theoretical framework presented here can be applied without essential change.

The aim of our paper is to provide a classification of chiral-odd \( \pi^0 \) GPDs. We restrict ourselves to 2-parton and 3-parton correlators. Our analysis includes the whole tower of twist contributions from 2 to 5, but exclude any inclusion of pion mass effects, a question which has been addressed recently in refs. [37–40]. We perform our analysis within the LCCF method in momentum space and use the invariance of the non-local operators involved under rotation and dilatation of the light-cone vector \( n^\mu \). This allows us to reduce the obtained GPDs to a minimal set. In the so-called Wandzura-Wilczek limit, where the 3-parton correlators vanish, this reduction shows that each of the 2-parton GPDs vanishes.

The paper is organized as follows. In section 2 we present the classification of chiral-odd \( \pi^0 \) GPDs, based on 2-parton and 3-parton correlators including the whole twist expansion up to twist 5. In subsection 2.1 we present the kinematics used. In subsection 2.2 we discuss the general framework of the LCCF method, illustrated with processes of type \( A \pi^0 \rightarrow B\pi^0 \). In subsection 2.3, we present the parametrization of the matrix elements of relevant 2-parton and 3-parton correlators, and define a set of 20 chiral-odd GPDs for \( \pi^0 \). In subsection 2.4, we present the symmetry properties of these chiral-odd GPDs for \( \pi^0 \).

In section 3, we construct a minimal set of chiral-odd GPDs for \( \pi^0 \). In subsection 3.1 we derive a set of constraints on the 2-parton and 3-parton \( \pi^0 \) matrix elements based on the \( n \)-independence condition. In subsection 3.2 we derive the constraint on these matrix elements coming from the QCD equations of motion. In subsection 3.3 we use these constraints to perform a reduction to a minimal set of GPDs. A few appendices present the calculational details needed to complete the proofs.

2. The set of GPDs including two and three body correlators

2.1 Kinematics

The incoming (resp. outgoing) pion carries a momentum \( p_1 \) (resp. \( p_2 \)). We denote their mass by \( m \). We define

\[
P = \frac{p_1 + p_2}{2} \quad \text{and} \quad \Delta = p_2 - p_1 .
\] (2.1)

We use the Sudakov basis provided by \( p \) and \( n \) (with \( p^2 = n^2 = 0 \)), normalized such that \( p \cdot n = 1 \). Therefore, any vector is decomposed as

\[
k = (k \cdot n) p + (k \cdot p) n + k_\perp .
\] (2.2)

The skewness \( \xi \) is defined through the expansion

\[
\Delta = -2\xi p + (\Delta \cdot p)n + \Delta_\perp .
\] (2.3)

We choose the symmetric kinematics for \( p_1 \) and \( p_2 \) as

\[
p_1 = (1 + \xi) p + \frac{m^2 - \Delta^2}{2(1 + \xi)} n - \frac{\Delta_\perp}{2},
\]

\[
p_2 = (1 - \xi) p + \frac{m^2 - \Delta^2}{2(1 - \xi)} n + \frac{\Delta_\perp}{2},
\] (2.4)
such that $P$ is purely longitudinal, and reads

$$P = p + (P \cdot p) n = p + \frac{m^2 - \Delta^2}{1 - \xi^2} n. \quad (2.5)$$

From the point of view of twist counting, the Sudakov expansion in terms of $p, \perp, n$ components is more appropriate, as we discuss in the next subsection. However, from the point of view of constraints related to time invariance, we find the expansion in terms of $P, \perp, n$ components more natural. Note that $P \cdot n = p \cdot n = 1$.

### 2.2 LCCF factorization

Let us consider a hard exclusive process. For definiteness, we name as $Q$ the involved hard scale (e.g. the $\gamma^*$’s virtuality in the case of deeply virtual Compton scattering (DVCS)). We here recall the basics of the LCCF in order to deal with amplitude of exclusive processes beyond the leading $Q$ power contribution. For definiteness, in view of the next sections, we illustrate the key concepts for the hard process $A \pi^0 \rightarrow B \pi^0$ (where $A$ and $B$ denote generic initial and final states in kinematics where a hard scale allows for a partonic interpretation, for example $A = \gamma^*$ and $B = \pi \rho_T$ pair), written in the momentum representation and in $n \cdot A = 0$ axial gauge, as

$$A = \int d^4 \ell \; \text{tr} \left[ H(\ell) \Phi(\ell) \right] + \int d^4 \ell_1 d^4 \ell_2 \; \text{tr} \left[ H_{\mu}(\ell_1, \ell_2) \Phi^{\mu}(\ell_1, \ell_2) \right] + \ldots, \quad (2.6)$$

where $H$ and $H_{\mu}$ are the coefficient functions with two parton legs and three parton legs, respectively, as illustrated in figure 1.

![Figure 1: 2- and 3-parton correlators attached to a hard scattering amplitude in the process $A \pi^0 \rightarrow B \pi^0$ (for example $\gamma^* \pi^0 \rightarrow \rho \pi^0 \pi^0$, i.e. $B = \rho \pi^0$).](image)

In (2.6), the soft parts are given by the Fourier-transformed two or three partons correlators which are matrix elements of non-local operators. In the present paper, we restrict ourselves to 2- and 3-parton correlators, and we perform the $1/Q$ twist expansion of the soft parts involved in these contributions. For consistency, this includes twist contributions varying from 2 to 5.

The amplitude (2.6) is not factorized yet because the hard and soft parts are related by the four-dimensional integration in the momentum space and by the summation over the Dirac indices.
To factorize the amplitude, we use the Sudakov expansion (2.2), the vector $p$ providing the dominant direction around which we intend to decompose our relevant momenta and we perform a Taylor expansion of the hard part. The loop momenta $\ell_i$ (for example in figure 1, $i = 1$ for the left diagram, and $i = 1, 2$ for the right diagram) are thus expanded as

$$\ell_i \mu = y_i p \mu + (\ell_i \cdot p) n \mu + \ell_i ^\perp \mu, \quad y_i = \ell_i \cdot n,$$

(2.7)

and we make the following replacement of the loop integrations in (2.6):

$$\int d^4 \ell_i \longrightarrow \int d^4 \ell_i \int dy_i \delta (y_i - \ell_i \cdot n).$$

(2.8)

We then expand the hard part $H(\ell)$ around the dominant $p$ direction:

$$H(\ell) = H(y p) + \frac{\partial H(\ell)}{\partial \ell^\alpha} \bigg|_{\ell = y p} (\ell - y p)^\alpha + \ldots,$$

(2.9)

where $(\ell - y p)^\alpha = \ell^\perp \alpha + (\ell \cdot p) n \alpha$ allows one to extract higher twist contributions. One can see that the above-mentioned steps (2.7)-(2.9) do not yet allow us to factorize collinearly the amplitude in the momentum space, since the presence of a $\ell^\perp$ dependence inside the hard part does not seem to fit with the standard collinear framework. To obtain a factorized amplitude, one performs an integration by parts to replace $\ell^\perp \alpha$ by $\partial^\perp \alpha$ acting on the soft correlator in coordinate space. This leads to new operators $O^\perp$ which contain transverse derivatives, such as $\bar{\psi} \partial^\perp \psi$, and thus to the necessity of considering additional non-perturbative correlators $\Phi^\perp(l)$. We note that since, when performing the integration over the Sudakov component $\ell_i \cdot p$, using residua method, one can always choose to close on the pole of $\ell_i^2 = 0$, so that when performing the Taylor expansion of the hard part, there is no non-trivial dynamics associated to Taylor coefficients along the $n$ direction (due to the mass-shell condition, one can always reorganize the expansion in terms of Taylor coefficients along the $\perp$ direction).

We now perform the loop momenta integrations over the Sudakov components $\ell_{i,i}$ and $\ell_i \cdot p$ (keeping in mind that the integration over $\ell_i \cdot p$ is done by residua, i.e. the denominators of $\ell_i$ propagators in $S$ are replaced by $-i\pi\delta(\ell_i^2)$). These integration over $\ell_{i,i}$ and $\ell_i \cdot p$ only affects the soft part since all the $\ell^\perp$ and $\ell_i \cdot p$ dependence of $H$ has been taken into account through Taylor expansion, and implies that the non-local correlators involve fields separated by a light-cone distance along the $n$ direction. The integration over the $\ell_i \cdot n$ component is straightforward, see eq. (2.8), and only the $y_i$ integrations remain, which connect the hard and soft parts.

Factorization in the Dirac space can be achieved by the Fierz decomposition. For example, in the case of two fermions, one should project out the Dirac matrix $\gamma_\alpha(0) \bar{\gamma}_\beta(z)$ which appears in the soft part of the amplitude on the relevant set of $\Gamma$ matrices.
After these two steps, the amplitude takes the simple factorized form

\[
A = \int_{-1}^{1} dy \text{tr} \left[ H_{qq}(y) \Gamma \right] \Phi_{qq}(y) + \int_{-1}^{1} dy \text{tr} \left[ H_{q\bar{q}}(y) \Gamma \right] \Phi_{q\bar{q}}(y)
\]

\[
\quad + \int_{-1}^{1} dy_1 dy_2 \text{tr} \left[ H_{q\bar{q}g}(y_1, y_2) \Gamma \right] \Phi_{q\bar{q}g}(y_1, y_2) + \cdots,
\]

(2.10)

in which the two first terms in the r.h.s correspond to the 2-parton contribution and the last one to the 3-parton contribution. As usual the antiquark contribution is interpreted as the \([-1, 0]\) part of this integral. The formula (2.10) is illustrated symbolically in the figure 2 for 2-parton contributions and in the figure 3 for 3-parton contributions.

**Figure 2:** Factorization of 2-parton contributions in the process $A \pi^0 \to B \pi^0$.

**Figure 3:** Factorization of 3-parton contributions in the process $A \pi^0 \to B \pi^0$.

Alternatively, combining the two last terms together in order to emphasize the fact they both originate from the Taylor expansion based on the covariant derivative, this
factorization can be written as

\[ A = \int_0^1 dy \, \text{tr} \left[ H(y) \Gamma \right] \Phi^\Gamma(y) + \int_0^1 dy_1 \, dy_2 \, \text{tr} \left[ H^\mu(y_1, y_2) \Gamma \right] \Phi^\Gamma(y_1, y_2). \quad (2.11) \]

For the process \( A \pi^0 \rightarrow B \pi^0 \), the soft parts of the amplitude read

\[ \Phi^\Gamma(y) = \int \frac{d(P \cdot z)}{2\pi} e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \psi(-z) | \pi^0(p_1) \rangle, \]

\[ \delta(x_g - x_2 + x_1) \Phi^\Gamma(x_1, x_2) = \int \frac{d(P \cdot z)}{2\pi} \frac{d(P \cdot y)}{2\pi} e^{-iP \cdot z(x_1+\xi)+iP \cdot y(x_g-iP \cdot z(x_2-\xi))} \]

\[ \times \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \bar{\rightarrow} D^T_\mu(y) \psi(-z) | \pi^0(p_1) \rangle, \quad (2.12) \]

where the covariant derivative is defined as

\[ \bar{\rightarrow} D_\mu = \bar{\rightarrow} \partial_\mu - igA_\mu(z). \quad (2.13) \]

Eq. (2.12) supplemented by the appropriate choice of the Fierz matrices defines the set of non-perturbative correlators relevant for the description of the \( \pi^0 \) GPDs, which we will discuss in the next subsection.

The LCCF is performed in a light-cone gauge, so that the only gluonic degrees of freedom which are exchanged between the hard and the soft part are transverse. In this LCCF framework, the non-local operators involving quark and antiquark fields supplemented by transverse gluon fields are closely related to non-local operators involving transverse derivative of quark and antiquark fields, both being the two parts of the Taylor coefficient along the \( \perp \) direction when expressed in terms of the covariant derivative.

### 2.3 Parametrization of GPDs

Following the discussion of the previous subsection, we now introduce the chiral-odd \( \pi^0 \) GPDs which parametrize the 2-parton and 3-parton correlators. This construction is done taking into account constraints based on charge invariance, time invariance and parity invariance.

The 2-partons correlators may be written as

\[ \langle \pi^0(p_2) | \bar{\psi}(z) \left[ \begin{array}{c} \sigma^{\alpha\beta} \\ 1 \\ i\gamma^5 \end{array} \right] \psi(-z) | \pi^0(p_2) \rangle = \int d\xi \, e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \times \]

\[ -\frac{i}{m_\pi} \left( P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha \right) H_T + i m_\pi (P^\alpha n^\beta - P^\beta n^\alpha) H_{T3} - i m_\pi \left( \Delta_\perp^\alpha n^\beta - \Delta_\perp^\beta n^\alpha \right) H_{T4} \]

\[ m_\pi H_S \]

\[ \begin{array}{ccc} \text{twist 2 & 4} & \text{twist 3} & \text{twist 4} \end{array} \]

where each GPD depends on the arguments \( x, \xi, t \), and we underlined their twist content.
We note that due to $\mathcal{P}$–parity invariance, there is no twist 3 GPD associated with the $\gamma^5$ structure.

We now consider correlators involving the 3-parton and 2-parton (with transverse derivative). For the $\sigma^{\alpha\beta}$ structure, they read

$$
\langle \pi^0(p_2) | \bar{\psi}(z) \sigma^{\alpha\beta} \left( \frac{i}{g A^\gamma(y)} \right) \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l} 
\frac{1}{i} \int_{-1}^{1} dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_{1,2}, g] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_2 + iP \cdot z(x_2-\xi)} 
\end{array} \right. 
$$

\begin{align*}
\times \left[ i m_\pi \left( P^\alpha g_\perp^{\beta \gamma} - P^\beta g_\perp^{\alpha \gamma} \right) \left\{ T_i^T \right\} + i m_\pi \left( P^\alpha \Delta_\perp^{\beta} - P^\beta \Delta_\perp^{\alpha} \right) \left\{ T_2^T \right\} \right] 
twist 3 & \& 5 \quad (2.15)
\end{align*}

\begin{align*}
+i m_\pi \left( \Delta_\perp^{\alpha} g_\perp^{\beta \gamma} - \Delta_\perp^{\beta \gamma} g_\perp^{\alpha} \right) \left\{ T_3^T \right\} + i m_\pi \left( P^\alpha n^\beta - P^\beta n^\alpha \right) \left\{ T_4^T \right\} 
twist 4 \quad (2.16)
\end{align*}

\begin{align*}
+i m_\pi^2 \left( n^\alpha g_\perp^{\beta \gamma} - n^\beta g_\perp^{\alpha \gamma} \right) \left\{ T_5^T \right\} + i m_\pi \left( n^\alpha \Delta_\perp^{\beta} - n^\beta \Delta_\perp^{\alpha} \right) \left\{ T_6^T \right\} , 
twist 5 \quad (2.17)
\end{align*}

where

$$
\int d^3[x_{1,2}, g] \equiv \int_{-1+\xi}^{1+\xi} dx g \int_{-1}^{1} dx_1 \int_{-1}^{1} \delta(x_g - x_2 + x_1) , \quad (2.18)
$$

and $\partial_\perp^{\gamma} \equiv \frac{1}{2} \left( \partial_\perp^{\gamma} \right. - \left. \partial_\perp^{\gamma} \right)$. The functions $T_i^T \ (i = 1, \cdots ; 6)$ should be understood as $T_i^T(x, \xi, t)$, while $T_i \ (i = 1, \cdots ; 6)$ denotes $T_i(x_1, x_2, \xi, t)$.

For the $\bar{1}$ structure, the correlators are defined as

$$
\langle \pi^0(p_2) | \bar{\psi}(z) \bar{\psi}(z) \left( \frac{i}{g A^\gamma(y)} \right) \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l} 
\frac{1}{i} \int_{-1}^{1} dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_{1,2}, g] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_2 + iP \cdot z(x_2-\xi)} 
\end{array} \right. 
$$

\begin{align*}
\times m_\pi \left\{ \frac{H_{T_{4}}^{T_4}}{T_S} \right\} . \quad (2.19)
\end{align*}

For the $i\gamma^5$ structure, the correlators read

$$
\langle \pi^0(p_2) | \bar{\psi}(z) i\gamma^5 \left( \frac{i}{g A^\gamma(y)} \right) \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l} 
\frac{1}{i} \int_{-1}^{1} dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_{1,2}, g] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_2 + iP \cdot z(x_2-\xi)} 
\end{array} \right. 
$$

\begin{align*}
\times m_\pi e^{\gamma n^\alpha P \Delta_\perp^{\alpha}} \left\{ \frac{H_{T_{P}}^{T_P}}{T_p} \right\} . \quad (2.20)
\end{align*}

The various factors of $i$ are introduced in order to ensure that the above GPDs are real. This can be easily checked based on time-reversal and complex conjugation.

Altogether, the 2- and 3-parton correlators lead to the introduction of 20 different GPDs. They are not independent, and the reduction to an independent set is postponed to the section 3.
| GPD               | $\mathcal{C}$                      | $\mathcal{T}$                      |
|-------------------|-----------------------------------|-----------------------------------|
| $H_T(x,\xi,t)$    | $-H_T(-x,\xi,t)$                  | $H_T(x,-\xi,t)$                   |
| $H_{T3}(x,\xi,t)$ | $-H_{T3}(-x,\xi,t)$               | $-H_{T3}(x,-\xi,t)$               |
| $H_{T4}(x,\xi,t)$ | $-H_{T4}(-x,\xi,t)$               | $H_{T4}(x,-\xi,t)$                |
| $H_S(x,\xi,t)$    | $H_S(-x,\xi,t)$                   | $H_S(x,-\xi,t)$                   |
| $T_1^T (x,\xi,t)$ | $-T_1^T (-x,\xi,t)$               | $-T_1^T (x,-\xi,t)$               |
| $T_2^T (x,\xi,t)$ | $-T_2^T (-x,\xi,t)$               | $-T_2^T (x,-\xi,t)$               |
| $T_3^T (x,\xi,t)$ | $-T_3^T (-x,\xi,t)$               | $T_3^T (x,-\xi,t)$                |
| $T_4^T (x,\xi,t)$ | $-T_4^T (-x,\xi,t)$               | $T_4^T (x,-\xi,t)$                |
| $T_5^T (x,\xi,t)$ | $-T_5^T (-x,\xi,t)$               | $T_5^T (x,-\xi,t)$                |
| $T_6^T (x,\xi,t)$ | $-T_6^T (-x,\xi,t)$               | $T_6^T (x,-\xi,t)$                |
| $T_7^T (x,\xi,t)$ | $-T_7^T (-x,\xi,t)$               | $T_7^T (x,-\xi,t)$                |
| $T_8^T (x,\xi,t)$ | $-T_8^T (-x,\xi,t)$               | $T_8^T (x,-\xi,t)$                |

**Table 1:** Symmetry properties of the 20 chiral-odd $\pi^0$ GPDs under the charge conjugation $\mathcal{C}$ and the time reversal $\mathcal{T}$.
Passing from the above definitions to the twist expansion is done using the following identities

\[ P^\alpha \Delta^\beta_1 - P^\beta \Delta^\alpha_1 = p^\alpha \Delta^\beta_1 - p^\beta \Delta^\alpha_1 + (P \cdot p)(n^\alpha \Delta^\beta_1 - n^\beta \Delta^\alpha_1), \]  
(2.21)

\[ P^\alpha n^\beta - P^\beta n^\alpha = p^\alpha n^\beta - p^\beta n^\alpha, \]  
(2.22)

and

\[ P^\alpha g^\beta \gamma - P^\beta g^\alpha \gamma = p^\alpha g^\beta \gamma - p^\beta g^\alpha \gamma + (P \cdot p) \left(n^\alpha g^\beta \gamma - n^\beta g^\alpha \gamma\right). \]  
(2.23)

Using eqs. (2.21) and (2.14), one sees that \( H_T \) contributes to twist 2 and twist 4, while eq. (2.22) shows that \( H_{T3} \) is purely of twist 3. Similarly, combining eq. (2.15) with eqs. (2.23, 2.21), one sees that \( T_T^1, T_T^2, T_1, T_2 \) contribute to twist 3 and 5. Combining eq. (2.22) with eq. (2.16) shows that \( T_T^4, T_T^6 \) are purely of twist 4. Finally, direct inspection of eq. (2.16) shows that \( T_4, T_5 \) are purely of twist 4 while eq. (2.17) leads to the conclusion that \( T_5^T, T_6^T, T_5, T_6 \) are purely of twist 5.

2.4 Symmetry properties

The fact that \( \pi^0 \) is an eigenstate under \( C \)–conjugation leads to simple symmetry properties, as displayed in table 1. Details of the proof are given in appendix A.1.

The time invariance also leads to symmetry properties for the \( \pi^0 \) GPDs, as shown in table 1.

3. The minimal set of GPDs

3.1 \( n \)–independence

3.1.1 Arbitrariness of \( p \) and \( n \)

The light-cone vector \( n \) which appears in the above decomposition is arbitrary, as soon as the constraint \( n \cdot p = n \cdot P = 1 \) is satisfied. Starting from an arbitrary choice of \( n \) denoted by \( n^{(0)} \), any other choice can be represented as

\[ n = n^{(0)} - \frac{n^2}{2} p + n_\perp. \]  
(3.1)

One can now also notice that \( p \) is not fixed by the kinematics (contrarily to \( P \)), so that one can vary at the same time \( n \) and \( p \). Indeed, starting from an initial choice for \( p \) and \( n \), denoted as \( p^{(0)} \) and \( n^{(0)} \), one can write

\[ n = \alpha n^{(0)} - \frac{n^2}{2\alpha} p^{(0)} + n_\perp, \]  
(3.2)

\[ p = \beta p^{(0)} - \frac{p^2}{2\beta} n^{(0)} + p_\perp, \]  
(3.3)

which satisfy the constraint \( p \cdot n = 1 \) provided that

\[ \alpha \beta + \frac{n^2 p^2}{4\alpha \beta} + n_\perp \cdot p_\perp = 1. \]  
(3.4)
The generators of these transformations can be easily extracted by first noting that using a rotation about an axis orthogonal to the plane provided by the $z$ and the $p_\perp$ axis, supplemented by a boost along the $z-$axis, the $p$ vector can be transformed to $\beta p^{(0)}$. These two transformations, after acting on $n$, lead to an expression similar to Eq. (3.2), after proper redefinition of $n_\perp$, and the constraint (3.4) now simply turns out to be $\alpha \beta = 1$, therefore completely fixing the vector $p$ as soon as $n$ is known. Thus, without loss of generality, since the above transformations we have used just reflect the global Lorentz invariance of the physical system, the transformation we want to extract are completely characterized by Eq. (3.2), and the three generators of these transformations are given by the scaling of $n^{(0)}$ and the two translations in $\perp$ space.

The arbitrariness in the choice of the vector $n$ can be used to further reduce the number of GPDs. We used this principle at the level of the amplitude of the process $\gamma^* p \to p \rho$ in refs. [25, 26], in the form
\[ \frac{d}{dn_\perp} A = 0. \] (3.5)
This relation leads to two relations between the various involved DAs, after proper use of Ward identities, which allow one to factorize out the hard parts involved in the amplitude $A$. In ref. [25], we have obtained the same equations after implementing this arbitrariness at the level of the matrix elements of the non-local operators involved in the definition of DAs. We now rely here on this idea for the chiral-odd pion GPDs.

### 3.1.2 Variation of a Wilson line

Consider a Wilson line $[y, x]_C$ between $x$ and $y$ along an arbitrary path $C$, defined as
\[ [y, x]_C \equiv P_C \exp ig \int_x^y dx_\mu A^\mu (x), \] (3.6)
in accordance with the covariant derivative normalized as in eq. 2.13.

![Figure 4: Variation of a Wilson line from path $C$ to path $C'$.](image)

The following equation for the deformation of this Wilson line $[y, x]_C$, illustrated in figure 4, can be derived in a gauge-invariant form as [41]
\[ \delta [y, x]_C = -i g \int_0^1 [y, x[\sigma]]_C G_{\nu\gamma}(x[\sigma]) \frac{dx^\gamma[\sigma]}{d\sigma} [x[\sigma], x]_C d\sigma \]
\[ + i g A(y) \cdot \delta x[1] [y, x]_C - i g [y, x]_C A(x) \cdot \delta x[0], \] (3.7)
where $\sigma$ is a parameter encoding the mapping between the arbitrarily chosen domain $[0, 1]$ and the path $C$, with the boundaries $x[0] = x$ and $x[1] = y$.

One can now apply the result (3.7) to the case of a path joining the points $-z$ and $z$, with $z$ along the light cone defined by the vector $n$. For simplicity, although this path along the light-cone can be completely arbitrary, we choose it to be a straight line from $-z$ to $z$. It is thus parameterized as $x[\tau] = \tau z$ with $\tau \in [-1, 1]$.

According to the discussion of section 3.1.1, the arbitrariness in the choice of the vector $n$ should be encoded by either considering the rescaling of $n$, i.e. of $z$, or the translations of $n$ by $n_\perp$, i.e. of $z$ by $z_\perp$.

Let us first consider a dilation of $z$ by a factor $\lambda$. We thus have

$$\delta z^\gamma = \tau^\gamma \delta \lambda.$$  \hfill (3.8)

For the considered path encoded by $x[\tau] = \tau z$ the corresponding variation reads

$$\delta x^\gamma[\tau] = \tau z^\gamma \delta \lambda = \tau \delta z^\gamma,$$  \hfill (3.9)

while

$$\frac{dx^\nu}{d\tau}[\tau] = z^\nu,$$  \hfill (3.10)

so that

$$\frac{dx^\nu}{d\tau}[\tau] \delta x^\gamma[\tau] = z^\nu \tau \delta z^\gamma.$$  \hfill (3.11)

Thus, the variation of the considered Wilson line reads

$$\delta[z, -z] = -ig \int_{-1}^{1} [z, x[\tau]] G_{\nu\gamma}(x[\tau]) \delta x^\gamma[\tau] \frac{dx^\nu}{d\tau}[\tau] [x[\tau], x] d\tau$$
$$+ ig A(y) \cdot \delta x[1] [y, x] C - ig [y, x] A(x) \cdot \delta x[-1]$$
$$= -ig \int_{-1}^{1} \tau d\tau [z, x[\tau]] z^\nu G_{\nu\gamma}(x[\tau]) [x[\tau], x] \delta z^\gamma$$
$$+ i g A_\gamma(y) \delta z^\gamma [-z, z] + i g [-z, z] A_\gamma(x) \delta z^\gamma,$$  \hfill (3.12)

where we have used the fact that $\delta x^\gamma[1] = \delta z^\gamma$ and $\delta x^\gamma[-1] = -\delta z^\gamma$.

Second, we consider a translation of $z$ by $\delta z_\perp$. We thus have

$$\delta z_\perp^\gamma = \delta z_\perp^\gamma,$$  \hfill (3.13)

and

$$\delta x^\gamma[\tau] = \tau \delta z_\perp^\gamma,$$  \hfill (3.14)

while

$$\frac{dx^\nu}{d\tau}[\tau] = z^\nu,$$  \hfill (3.15)

so that

$$\frac{dx^\nu}{d\tau}[\tau] \delta x^\gamma[\tau] = z^\nu \tau \delta z_\perp^\gamma.$$  \hfill (3.16)

Thus, the obtained variation for the Wilson line is again given by eq. (3.12).
3.1.3 Operator identity

Consider now an arbitrary Dirac \(\Gamma\) matrix. From (3.12), the following operator identity can be immediately derived [42]

\[
\frac{\partial}{\partial z^\gamma} \left[ \bar{\psi}(z) \Gamma[z, -z] \psi(-z) \right] = (3.17)
\]

\[-\bar{\psi}(z) \Gamma[z, -z] \overrightarrow{D}_\gamma \psi(-z) + \bar{\psi}(z) \overleftarrow{D}_\gamma \Gamma[z, -z] \psi(-z) + ig \int_{-1}^{1} dv v \bar{\psi}(z)[z, vz] z^\nu G_{\nu\gamma}(vz) \Gamma[vz, -z] \psi(-z),
\]

where the covariant derivative is defined as in eq. (2.13) so that \(\overrightarrow{D}_\alpha = \partial_\alpha - igA_\alpha(-z)\) and \(\overleftarrow{D}_\alpha = \partial_\alpha + igA_\alpha(z)\), and the derivative along \(z_\gamma\) should be understood as acting either along the \(n\) direction or along the \(\perp\) direction, in accordance to the above discussion.

3.1.4 Application to matrix elements

The next stage is to use this operator identity at the level of a matrix element. Thus, we consider the various 2-partons correlators (2.14) and write

\[
\frac{\partial}{\partial z^\gamma} \left[ \langle \pi^0(p_2)|\bar{\psi}(z) \Gamma[z, -z] \psi(-z)|\pi^0(p_1) \rangle \right] = (3.18)
\]

\[-\langle \pi^0(p_2)|\bar{\psi}(z) \Gamma[z, -z] \overrightarrow{D}_\gamma \psi(-z) + \bar{\psi}(z) \overleftarrow{D}_\gamma \Gamma[z, -z] \psi(-z)|\pi^0(p_1) \rangle + ig \int_{-1}^{1} dv v \langle \pi^0(p_2)|\bar{\psi}(z)[z, vz] z^\nu G_{\nu\gamma}(vz) \Gamma[vz, -z] \psi(-z)|\pi^0(p_1) \rangle,
\]

(3.20)

Up to now, the above identities are valid in an arbitrary gauge. Eqs. (3.17) and (3.18) are valid for \(\delta z_\gamma\) in the \(n\) or \(\perp\) directions. In the case of a variation \(\delta z_\gamma\) along \(n\), \(z^\nu G_{\nu\gamma}\) vanishes, since \(n^\nu n^\gamma G_{\nu\gamma} = 0\). In order to deal with the case of a variation in the \(\perp\) direction, we now restrict ourselves to the light-like gauge

\[
\begin{align*}
\text{In this gauge, the needed field-strength matrix element reduces to}
\end{align*}
\]

\[
z^\nu G_{\nu\gamma} = z^\nu \partial_\nu A_\gamma
\]

since \(z\) is along \(n\). Note that in eq. (3.22), the index \(\gamma\) can be \(\perp\) or along \(p\), although in the later case both side of this relation vanishes (the left-hand-side because of antisymmetry of \(G_{\nu\gamma}\), and the right-hand-side because of light-like gauge (3.21)).

The fact that only the \(\gamma_\perp\) index contributes in eq. (3.22) in a non-trivial way is consistent with our LCCF framework in which only matrix elements involving the \(\perp\) components of the field \(A_\gamma\) are introduced, see eqs. (2.15, 2.16, 2.17), (2.19) and (2.20).
Let us now consider the part (3.20) of eq. (3.18). We thus have, in light-like gauge,

\[-ig \int_{-1}^{1} dv \, v \langle \pi^0(p_2) | \bar{\psi}(z) z^\beta G_{\nu\gamma}(vz) \Gamma \psi(-z) | \pi^0(p_1) \rangle\]

\[= -ig \int_{-1}^{1} dv \, v \langle \pi^0(p_2) | \bar{\psi}(z) z^\beta [\partial_\beta A_\gamma(vz)] \Gamma \psi(-z) | \pi^0(p_1) \rangle\]

\[= -ig \int_{-1}^{1} dv \, v \langle \pi^0(p_2) | \bar{\psi}(z) v z^\beta \frac{\partial}{\partial(vz)_\beta} A_\gamma(vz) \Gamma \psi(-z) | \pi^0(p_1) \rangle\]

\[= +ig \int_{-1}^{1} dv \, \langle \pi^0(p_2) | \bar{\psi}(z) A_\gamma(vz) \Gamma \psi(-z) | \pi^0(p_1) \rangle - i g \langle \pi^0(p_2) | \bar{\psi}(z) A_\gamma(z) \Gamma \psi(-z) | \pi^0(p_1) \rangle - i g \langle \pi^0(p_2) | \bar{\psi}(z) A_\gamma(-z) \Gamma \psi(-z) | \pi^0(p_1) \rangle\]

(3.23)

where we used \( v z^\beta \frac{\partial}{\partial(vz)_\beta} A_{\gamma\perp}(vz) = v \frac{\partial}{\partial v} A_{\gamma\perp}(vz) \) and then integrated by part to pass from 3rd to 4th line. Now, combining (3.23) with the part of eq. (3.19) which involves the field \( A_\gamma \), we get

\[\frac{\partial}{\partial z^\gamma} \left[ \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \psi(-z) | \pi^0(p_1) \rangle \right] =\]

(3.24)

\[-\langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \partial_\gamma \psi(-z) + \bar{\psi}(z) \Gamma \partial_\gamma \psi(-z) | \pi^0(p_1) \rangle\]

(3.25)

\[+ i g \int_{-1}^{1} dv \, \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma A_\gamma(vz) \Gamma \psi(-z) | \pi^0(p_1) \rangle .\]

(3.26)

For the index \( \gamma \) along \( n \), since \( A \cdot n = 0 \), this relation reduces trivially to

\[\frac{\partial}{\partial z^\gamma} \left[ \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \psi(-z) | \pi^0(p_1) \rangle \right] = -\langle \pi^0(p_2) | \bar{\psi}(z) \Gamma \partial_\gamma \psi(-z) + \bar{\psi}(z) \Gamma \partial_\gamma \psi(-z) | \pi^0(p_1) \rangle ,\]

(3.27)

which is obviously satisfied.

The non trivial case arises when the index \( \gamma \) is along the \( \perp \) direction, because of the term (3.26). We write symbolically this contribution as

\[i g \int_{-1}^{1} dv \, \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma A_{\gamma\perp}(vz) \Gamma \psi(-z) | \pi^0(p_1) \rangle = i g \int_{-1}^{1} dv \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1+\xi} dx_g \]

\[\times \delta(x_1 - x_2 + x_g) e^{iP.z(x_1 + \xi) - iP.z(x_2 - x_1) + iP.z(x_2 - \xi)} g(i, m_\pi) T^{\gamma_1 \cdots \gamma_\ell} f(x_1, x_2, x_g) ,\]

(3.28)

relying on the parametrizations of 3-parton correlators introduced in eqs. (2.15, 2.16, 2.17), (2.19) and (2.20). The factors of \( i \) and \( m_\pi \) are included in \( g(i, m_\pi) \), while \( f(x_1, x_2, x_g) \) denotes the various GPDs introduced there (we do not introduce the variables \( \xi \) and \( t \) for
simplicity of notations), with the corresponding tensor structures $T^{\gamma\cdots}$. Integrating over $v$ gives

$$
ig \int_{-1}^{1} dv \langle \pi^0(p_2)|\tilde{\psi}(z) \Gamma A_\gamma(vz) \Gamma \psi(-z)|\pi^0(p_1)\rangle = ig \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1+\xi} \frac{dx_g}{x_2 - x_1} 
$$

$$
\times \delta(x_1 - x_2 + x_g) \frac{i}{P \cdot z (x_2 - x_1)} (e^{2iP \cdot z x_1} - e^{2iP \cdot z x_2}) g(i, m_\pi) T^{\gamma\cdots} f(x_1, x_2, x_g)
$$

$$= -2 g(i, m_\pi) T^{\gamma\cdots} \int_{-1}^{1} d\eta \int_{-1}^{1+\xi} dx_g \left[ \int_{-1}^{\eta} dx_1 - \int_{-1}^{\eta} dx_2 \int_{-1}^{1} dx_1 \right] \delta(x_1 - x_2 + x_g) 
$$

$$\times \frac{1}{x_2 - x_1} e^{2iP \cdot z \eta} f(x_1, x_2, x_g), \quad (3.29)
$$

where we have used the representation

$$
e^{2iP \cdot z x_1} - e^{2iP \cdot z x_2} = 2i(P \cdot z) \int_{x_2}^{x_1} d\eta e^{2iP \cdot z \eta}, \quad (3.30)
$$

and we have interchanged the order of integrals.

The derivative of $\langle \pi^0(p_2)|\tilde{\psi}(z) \Gamma \psi(-z)|\pi^0(p_1)\rangle$ with respect to $z_1$ vanishes, as can be seen from the right-hand-side of eq. (2.14) where the dependence with respect to $z$ only involves its $n$ component through $P \cdot z$.

Thus, eq. (3.24) implies that the opposite of line (3.25), parametrized by eqs. (2.15, 2.16, 2.17) should equal line (3.26), given by eq. (3.29). This finally leads, after identification of $\eta$ with $x$, to 8 equations:

$$
\begin{align*}
\begin{cases}
T^T_i(x, \xi, t) \\
H^T_{S_i}(x, \xi, t) \\
H^T_{P}(x, \xi, t)
\end{cases}
= \int_{-1+\xi}^{1+\xi} dx_g \left[ \int_{-1}^{x} dx_2 \int_{-1}^{1} dx_1 - \int_{-1}^{1} dx_2 \int_{-1}^{x} dx_1 \right] \frac{\delta(x_1 - x_2 + x_g)}{x_2 - x_1} \begin{pmatrix} T_1(x, \xi, t) \\ T_2(x, \xi, t) \\ T_P(x, \xi, t) \end{pmatrix},
\end{align*}
$$

where $i = 1, \cdots, 6$ in the first line, which corresponds to the structure $\Gamma = \sigma^{\alpha\beta}$. The second line is related to $\Gamma = \mathbb{1}$, and the last line to $\Gamma = i\gamma^5$.

### 3.2 QCD equations of motion

In this subsection, we derive the QCD equations of motion satisfied by the pion chiral-odd GPDs.

We start with the Dirac equation

$$
0 = \langle \pi^0(p_2)| (i\not{D}\psi)_\alpha(-z) \tilde{\psi}_\beta(z)|\pi^0(p_1)\rangle 
$$

$$= \langle \pi^0(p_2)| [(i\not{D}\psi)_\alpha(-z) + (i\not{D}L\psi)_\alpha(-z) + g(A\psi)_\alpha(-z)] \tilde{\psi}_\beta(z)|\pi^0(p_1)\rangle 
$$

$$= (a) + (b) + (c). \quad (3.32)
$$
Let us first derive an algebraic identity which is based on translational invariance of the considered correlator. One has

$$\langle \pi^0(p_2) | \psi_\alpha(z_2) \bar{\psi}_\beta(z_1) | \pi^0(p_1) \rangle = e^{i \frac{z_2 + z_1}{2}} \Delta \langle \pi^0(p_2) | \psi_\alpha \left( \frac{z_2 - z_1}{2} \right) \bar{\psi}_\beta \left( - \frac{z_2 - z_1}{2} \right) | \pi^0(p_1) \rangle$$

(3.33)

so that

$$\langle \pi^0(p_2) | [ \partial_\mu \psi_\alpha(z_2) ] \bar{\psi}_\beta(z_1) | \pi^0(p_1) \rangle = e^{i \frac{z_2 + z_1}{2}} \Delta \left[ \frac{i}{2} \Delta_\mu + \frac{\partial}{\partial z_2^\mu} \right] \langle \pi^0(p_2) | \psi_\alpha \left( \frac{z_2 - z_1}{2} \right) \bar{\psi}_\beta \left( - \frac{z_2 - z_1}{2} \right) | \pi^0(p_1) \rangle.$$  

(3.34)

(3.35)

Taking $z_2 = - z$ and $z_1 = z$, we get

$$\langle \pi^0(p_2) | [ \partial_\mu \psi_\alpha( - z) ] \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle = \left[ \frac{i}{2} \Delta_\mu - \frac{1}{2} \frac{\partial}{\partial z^\mu} \right] \langle \pi^0(p_2) | \psi_\alpha ( - z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle.$$  

(3.36)

Let us first use this trick to deal with term (a) of eq. (3.32). We have

$$\int d(P \cdot z) e^{-i(x - \xi)^P - i(x + \xi)^P \cdot z} \langle \pi^0(p_2) | (i \gamma^\alpha \psi_\alpha( - z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle$$

(3.37)

$$= \int d(P \cdot z) e^{-i(x - \xi)^P - i(x + \xi)^P \cdot z} < \langle \pi^0(p_2) | \psi_\alpha'( - z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle$$

$$= - \frac{1}{4} \int d(P \cdot z) e^{-i(x - \xi)^P - i(x + \xi)^P \cdot z} \left[ \frac{1}{2} (\gamma^\beta)_{\alpha \beta} \langle \pi^0(p_2) | \bar{\psi}(z) \gamma^\rho \psi(- z) | \pi^0(p_1) \rangle$$

$$+ \gamma^\alpha \langle \pi^0(p_2) | \bar{\psi}(z) 1 \psi(- z) | \pi^0(p_1) \rangle - i (\gamma^5)_{\alpha \beta} \langle \pi^0(p_2) | \bar{\psi}(z) i \gamma^5 \psi(- z) | \pi^0(p_1) \rangle \right],$$

(3.38)

where we have used eq. (3.36) and performed an integration by part, assuming the vanishing of fields at infinity to get the second line, and performed Fierz decomposition to get the last line. Thus, we obtain after expanding into a basis of Dirac structures,

$$\int d(P \cdot z) e^{-i(x - \xi)^P - i(x + \xi)^P \cdot z} \langle \pi^0(p_2) | (i \gamma^\alpha \psi_\alpha( - z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle$$

(3.39)

$$= - \frac{2\pi}{4} (x + \xi) m_\pi \left\{ - \frac{i}{m_\pi^2} P \sigma_{P \Delta H_T(x)} + i P \sigma_{P H_T(x)} - i P \sigma_{\Delta H_T(x)} + P H_S(x) \right\}$$

$$= \frac{-2\pi}{4} (x + \xi) \left\{ m_\pi \Delta_\perp \left( \frac{P^2}{m_\pi^2} H_T(x) - H_T(x) \right) + P \left( m_\pi H_T(x) + m_\pi H_S(x) \right)$$

$$- \gamma^\alpha \gamma^\beta P \Delta_\perp P^\alpha \gamma^\beta \gamma_\mu H_T(x) \right\}.$$  

(3.40)

We now consider the contribution (b). Again, we use eq. (3.36), in the form

$$\langle \pi^0(p_2) | [ \partial_\mu \psi_\alpha( - z) ] \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle = \frac{i}{2} \Delta^\perp_\mu \langle \pi^0(p_2) | \psi_\alpha( - z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle$$

$$- \frac{1}{2} \langle \pi^0(p_2) | \psi_\alpha( - z) \left[ \partial^\perp_\mu - \partial^\perp_\mu \right] \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle.$$  

(3.41)
This leads to
\[
\int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} (\pi^0(p_2)) \langle (i\slashed{\partial}_\perp \psi)_\alpha (-z) \bar{\psi}_\beta (z) \mid \pi^0(p_1) \rangle
\]
\[
= -\frac{1}{4} \int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} \left[ -\frac{1}{4} (\Delta_\perp \sigma_{\rho\sigma})_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \sigma^{\rho\sigma} \psi(-z) \mid \pi^0(p_1) \rangle
\right.
\]
\[
- \frac{1}{2} (\gamma_\mu \sigma_{\rho\sigma})_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) i \gamma^\rho \gamma^5 \psi(-z) \mid \pi^0(p_1) \rangle,
\]
\[
+ \frac{1}{2} (\gamma_\mu \sigma_{\rho\sigma})_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \sigma^{\rho\sigma} i \partial_\perp^\mu \psi(-z) \mid \pi^0(p_1) \rangle,
\]
\[
+ (\gamma_\mu)_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) i \partial_\perp^\mu \psi(-z) \mid \pi^0(p_1) \rangle.
\]
which involves the 2-partons GPDs without transverse derivative defined in eqs. (2.14) and with transverse derivative defined in eqs. (2.15, 2.16, 2.17, 2.19, 2.20). Substituting these parametrizations and performing basic algebra, we obtain
\[
\int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} (\pi^0(p_2)) \langle (i\slashed{\partial}_\perp \psi)_\alpha (-z) \bar{\psi}_\beta (z) \mid \pi^0(p_1) \rangle = -\frac{2\pi}{4} \quad (3.43)
\]
\[
\times \left\{ \frac{\Delta_\perp^2}{2m_\pi} H_T(x) + 2m_\pi T_1^T(x) + \frac{\Delta_\perp^2}{m_\pi} T_2^T(x) \right\}
\]
\[
+ m_\pi \Delta_\perp \left( -\frac{1}{2} H_S(x) + T_3^T(x) + H_5^T(x) \right)
\]
\[
+ i m_\pi (\gamma_\mu \gamma^5) \langle \pi^0(p_2) \mid \bar{\psi}(z) A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle
\]
\[
+ \frac{1}{2} (\gamma_\mu \gamma^5) \langle \pi^0(p_2) \mid \bar{\psi}(z) \gamma^5 A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle.
\]

We now consider the contribution (c). After Fierz transform, one gets
\[
\int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} (\pi^0(p_2)) \langle (i\slashed{\partial}_\perp \psi)_\alpha (-z) \bar{\psi}_\beta (z) \mid \pi^0(p_1) \rangle
\]
\[
= -\frac{1}{4} \int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} \left[ \frac{1}{2} \langle \gamma^\mu \sigma_{\rho\sigma} \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \sigma^{\rho\sigma} A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle,
\right.
\]
\[
+ \langle \gamma_\mu \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle
\]
\[
+ \langle \gamma_\mu \gamma^5 \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \gamma^5 A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle
\]
\[
\left. \right].
\]
This relation only involves \(\pi^0\) GPDs defined in eqs. (2.15, 2.16, 2.17, 2.19, 2.20). After simple algebra, one gets
\[
\int d(P \cdot z) e^{-i(x-\xi)P \cdot z - i(x+\xi)P \cdot z} (\pi^0(p_2)) g(A \psi)_\alpha (-z) \bar{\psi}_\beta (z) \mid \pi^0(p_1) \rangle
\]
\[
= \frac{2\pi}{4} \quad (3.46)
\]
\[
\times \left\{ \frac{1}{2} \langle \gamma^\mu \sigma_{\rho\sigma} \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \sigma^{\rho\sigma} A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle,
\right.
\]
\[
- m_\pi \Delta_\perp \langle T_3(y,x) + T_5(y,x) \rangle - \langle \gamma_\mu \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle
\]
\[
- i m_\pi \epsilon_{\perp} \langle \gamma_\mu \rangle_{\alpha\beta} \langle \pi^0(p_2) \mid \bar{\psi}(z) \gamma^5 A_\mu(-z) \psi(-z) \mid \pi^0(p_1) \rangle \right\}.
\]
Summing-up contributions (a)+(b)+(c) as given from eqs. (3.39, 3.43, 3.46), and demanding the vanishing of the contributions multiplying the four independent structures $P_{\alpha\beta}$, $\Delta_{\parallel\alpha\beta}$, $\phi_{\alpha\beta}$ and $i e A_{\tau}^\mu (\gamma^\tau\gamma^\mu)_{\alpha\beta}$, we obtain the following four equations

\[(x + \xi)(m_\pi H_{T3} + m_\pi H_S) + \frac{\Delta_\parallel^2}{2m_\pi} H_T + 2m_\pi T_1^T + \frac{\Delta_\perpendicular^2}{m_\pi} T_2^T\]

\[+ \int d x_g \int d y \delta(x_g - x + y) \left(2m_\pi T_1(y, x) + \frac{\Delta_\parallel^2}{m_\pi} T_2(y, x)\right) = 0, \quad (3.48)\]

\[(x + \xi) \left(\frac{P^2}{m_\pi} H_T - m_\pi H_{T4}\right) + m_\pi \left(-\frac{1}{2} H_S + T_3^T + H_{S}^{T4}\right)\]

\[+ \int d x_g \int d y \delta(x_g - x + y) \left(T_3(y, x) + T_S(y, x)\right) = 0, \quad (3.49)\]

\[(x + \xi) m_\pi P^2 H_{T3}(x) + \frac{m_\pi \Delta_\parallel^2}{2} H_{T4}(x) - 2m_\pi T_5^T(x) - m_\pi \Delta_\perpendicular^2 T_6^T(x)\]

\[\int d x_g \int d y \delta(x_g + y - x) \left(2m_\pi T_5(y, x) + m_\pi \Delta_\parallel^2 T_6(y, x)\right) = 0, \quad (3.50)\]

and

\[(x + \xi) H_{T4}(x) - \frac{1}{2} H_{T3}(x) + T_4^T(x) + H_P^T(x)\]

\[\int d x_g \int d y \delta(x_g + y - x) \left(T_4(y, x) + T_P(y, x)\right) = 0. \quad (3.51)\]

A second set of four equations is obtained in an analogous way by considering the various correlators involved in the following equation

\[0 = \langle \pi^0(p_2)|\psi_\alpha(-z) (i\mathcal{D}\bar{\psi})_\beta(z) |\pi^0(p_1)\rangle. \quad (3.52)\]

These equations read

\[(x - \xi)(-m_\pi H_{T3} + m_\pi H_S) + \frac{\Delta_\parallel^2}{2m_\pi} H_T - 2m_\pi T_1^T - \frac{\Delta_\perpendicular^2}{m_\pi} T_2^T\]

\[\int d x_g \int d y \delta(x_g - y + x) \left(2m_\pi T_1(x, y) + \frac{\Delta_\parallel^2}{m_\pi} T_2(x, y)\right) = 0, \quad (3.53)\]

\[(x - \xi) \left(\frac{P^2}{m_\pi} H_T + m_\pi H_{T4}\right) + m_\pi \left(-\frac{1}{2} H_S - T_3^T + H_{S}^{T4}\right)\]

\[\int d x_g \int d y \delta(x_g - y + x) \left(T_3(x, y) - T_S(x, y)\right) = 0, \quad (3.54)\]
\[ (x - \xi) m_\pi P^2 H_{T3}(x) - \frac{m_\pi \Delta^2}{2} H_{T4}(x) - 2m_\pi^3 T^T_S(x) - m_\pi \Delta^2 \Delta^2 T^T_6(x) \]
\[ - \int_{-1+\xi}^{1+\xi} dx_g \int_{-1}^{1} dy \delta(x_g - y + x) (2m_\pi^3 T_5(x, y) + m_\pi \Delta^2 T_6(x, y)) = 0, \quad (3.55) \]

and
\[ (x - \xi) H_{T4}(x) + \frac{1}{2} H_{T3}(x) + T^T_4(x) - H^T_4(x) \]
\[ - \int_{-1+\xi}^{1+\xi} dx_g \int_{-1}^{1} dy \delta(x_g - y + x) (-T_4(x, y) + T_4(x, y)) = 0. \quad (3.56) \]

Note that the two sets of equations (3.48, 3.49, 3.50, 3.51) and (3.53, 3.54, 3.55, 3.56) are related by charge conjugation, as can be checked explicitly using table 1.

3.3 Toward a minimal set of GPD

3.3.1 The twist 5 case

In section 2.3, we introduced a set of 20 chiral-odd \( \pi^0 \) GPDs. They are related by \( n \)-independence constraints, see section 3.1, and by QCD equations of motion, discussed in section 3.2. The \( n \)-independence constraints lead to the 8 equations (3.31) while the QCD equation of motions gives 8 other equations (3.48, 3.49, 3.50, 3.51, 3.53, 3.54, 3.55, 3.56). The reduction to a minimal set of GPDs is not straightforward, and not unique. We now show that a particular reduction procedure results in expressing the set of 20 GPDs in terms of the 8 GPDs \( T_i \) (\( i = 1, \ldots, 6 \)), \( T_P \), \( T_S \), which are related by 4 integral sum rules.

First, using the \( n \)-independence constraints (3.31), one can express \( T^T_i \) in terms of \( T_i \) (\( i = 1, \ldots, 6 \)), as well as \( H^T_S \) in term of \( T_S \), and \( H^T_P \) in term of \( T_P \), thus reducing the set of 20 GPDs to 12.

Second, adding and substracting eq. (3.50) and eq. (3.55), one can express \( H_{T3} \) and \( H_{T4} \) as functions of the combination \( 2m_\pi^3 T_5(y, x) + m_\pi \Delta^2 T_6(y, x) \).

Third, adding and substracting eq. (3.49) and eq. (3.54), one can express \( H_T \) and \( H_S \) as two functions of \( T_3 \), \( T_S \) and \( 2m_\pi^3 T_5(y, x) + m_\pi \Delta^2 T_6(y, x) \).

Fourth, inserting the already obtained expressions for \( H_{T3} \) and \( H_{T4} \) in eq. (3.51) and eq. (3.56), one gets two integral sum-rules, relating \( T_4 \) \( T_P \), and \( 2m_\pi^3 T_5(y, x) + m_\pi \Delta^2 T_6(y, x) \), and are related by charge conjugation.

Fifth, inserting the already obtained expressions for \( H_{T3} \), \( H_S \) and \( H_T \) in eq. (3.48) and eq. (3.53), one gets two integral sum-rules related by charge conjugation, involving \( T_3 \), \( T_S \), \( 2m_\pi T_1 + \Delta^2 T_2 \) and \( 2m_\pi^3 T_5(y, x) + m_\pi \Delta^2 T_6(y, x) \).

3.3.2 The twist 4 limit

In the limit where we only consider the twist contributions up to 4, 16 chiral-odd \( \pi^0 \) GPDs should be introduced. The \( n \)-independence constraints lead to the 6 equations (3.31) while the QCD equation of motions gives 8 other equations (3.48, 3.49, 3.50, 3.51, 3.53, 3.54, 3.55, 3.56).
Following the same reduction procedure as in section 3.3.2, the set of 16 GPDs can be expressed in terms of the 6 GPDs $T_i$ ($i = 1, \cdots, 4$), $T_P$, $T_S$, which are related by 4 integral sum rules.

Indeed, first, using the $n-$independence constraints (3.31), one can express $T_i^T$ in terms of $T_i$ ($i = 1, \cdots, 4$), as well as $H_S^T$ in term of $T_S$, and $H_P^T$ in term of $T_P$, thus reducing the set of 16 GPDs to 10.

Second, adding and subtracting eq. (3.50) and eq. (3.55) shows that $H_T^T = H_T = 0$.

Third, adding and subtracting eq. (3.49) and eq. (3.54), one can express $H_T$ and $H_S$ as two functions of $T_3$, $T_S$.

Fourth, eq. (3.51) and eq. (3.56) turns to be two integral sum-rules, relating $T_4$ and $T_P$, which are related by charge conjugation.

Fifth, inserting the already obtained expressions for $H_S$ and $H_T$ in eq. (3.48) and eq. (3.53), one gets two integral sum-rules related by charge conjugation, involving $T_3$, $T_S$ and $2m_\pi T_1 + \Delta^2 T_2$.

### 3.3.3 The twist 3 limit

In the limit where we restrict ourselves to the twist contributions up to 3, 7 chiral-odd $\pi^0$ GPDs should be introduced. The $n-$independence constraints lead to the 2 equations (3.31) while the QCD equation of motions gives 5 other equations (3.48, 3.49, 3.50, 3.53, 3.54).

Following the same reduction procedure as in section 3.3.2, the set of 7 GPDs can be expressed in terms of the 2 GPDs $T_1$ and $T_2$, which are related by 2 integral sum rules.

Indeed, first, using the $n-$independence constraints (3.31), one can express $T_1^T$ and $T_2^T$ in terms of $T_1$ and $T_2$ respectively, thus reducing the set of 7 GPDs to 5.

Second, eq. (3.50) shows that $H_T^T = H_T = 0$.

Third, adding and subtracting eq. (3.49) and eq. (3.54), show that $H_T = H_S = 0$.

Fourth, eq. (3.48) and eq. (3.53) provide two integral sum-rules, involving $2m_\pi T_1 + \frac{\Delta^2}{m_\pi} T_2$, which are related by charge conjugation.

### 3.3.4 The vanishing of GPDs in the Wandzura-Wilczek limit

To conclude this section, we consider the Wandzura-Wilczek limit, i.e. assuming that the 3-parton correlators vanish, $T_i = 0$ ($i = 1, \cdots, 6$) and $T_S = T_P = 0$. In this limit, the discussion of section 3.3.1 shows that all GPDs actually vanish. Thus, in this limit we get the important and somewhat surprising result that the amplitude of any process involving the chiral-odd $\pi^0$ GPDs simply vanish.

### 4. Conclusion

We have analyzed in a systematic way, using the LCCF framework, the classification of chiral-odd GPDs for the $\pi^0$. For that, we introduced the relevant matrix elements for 2-parton non-local operator with and without transverse derivative, as well as matrix elements for 3-parton non-local correlators. Their detailed parametrization has been fixed.
using parity, charge conjugation and time reversal invariance. This leads to the introduction of 20 real GPDs, whose symmetry properties are explicitly given. The reduction of these GPDs to a minimal set is performed with the help of QCD equations of motion and $n$–independence, which is discussed at length at the operator level. We show that these 20 GPDs can be expressed through 8 GPDs which satisfy 4 sum rules. An important outcome of this analysis is the fact that when assuming the vanishing of 3-parton correlators, in the so-called Wandzura-Wilczek approximation, the whole set of GPDs vanishes.

In future papers, we plan to investigate using the same method the structure of nucleon GPDs for chiral-even and chiral-odd sectors.

For these nucleon GPDs, important phenomenological progress may come from real or virtual photon–photon collisions, which may be accessible either at electron–positron colliders or in ultraperipheral collisions at hadronic colliders [43, 44].

Acknowledgments

We acknowledge Igor Anikin for discussions during a longstanding collaboration, which inspired the present work.

This work is partly supported by the Polish Grant NCN No. DEC-2011/01/B/ST2/03915, the French-Polish collaboration agreement Polonium, the ANR “PARTONS”, the PEPS-PTI “PHENO-DIFF”, the Joint Research Activity Study of Strongly Interacting Matter (acronym HadronPhysics3, Grant Agreement n.283286) under the Seventh Framework Programme of the European Community and by the COPIN-IN2P3 Agreement.

A. Appendices

A.1 Symmetry properties of GPDs under charge conjugation $C$

We present here basic steps which lead to symmetry properties of GPDs shown in the second column of table 1, based on charge conjugation. As an example we consider first the 2-parton GPD related to the matrix element $\langle \pi^0(p_2)|\bar{\psi}(-z)\sigma^{\alpha\beta}\psi(z)|\pi^0(p_1) \rangle$, in which we omit the Wilson line. Substitution of $1 = C^\dagger C$ into the correlator of the above matrix element, where $C$ is the charge conjugation operator satisfying

$$C\psi(z)C^\dagger = \eta_c C \bar{\psi}^T(z), \quad (A.1)$$

with $C = -C^T = -C^\dagger$, we obtain the relation

$$\langle \pi^0(p_2)|\bar{\psi}(-z)\sigma^{\alpha\beta}\psi(z)|\pi^0(p_1) \rangle = -\langle \pi^0(p_2)|\bar{\psi}(z)\sigma^{\alpha\beta}\psi(-z)|\pi^0(p_1) \rangle. \quad (A.2)$$

The introduction of parametrization of this GPD leads to the symmetry property

$$H_T(x, \xi, t) = -H_T(-x, \xi, t), \quad (A.3)$$

shown in the table 1.
Let us now consider, as an example, the 3-parton GPD defined by the matrix element 
\[ \langle \pi^0(p_2) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(y) \psi(z) | \pi^0(p_1) \rangle \]. To obtain the analog of eq. (A.2) we use apart of (A.1) the transformation property of the gluonic field \( A_a(z) = A^a_t t^a \) (where \( t^a \) are colour group generators) under the generalized charge conjugation

\[ CA^\gamma(z) C^\dagger = -A^\gamma(z)^T. \] (A.4)

In this way we obtain that

\[ \langle \pi^0(p_2) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(y) \psi(z) | \pi^0(p_1) \rangle = e^{-2i\xi(P-z)} \langle \pi^0(p_2) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(y-z) \psi(-z) | \pi^0(p_1) \rangle, \] (A.5)

from which, using the parametrization of GPDs, it follows the properties

\[ T_i(x_1, x_2, \xi, t) = T_i(-x_2, -x_1, \xi, t), \quad i = 1, 2, \] (A.6)

shown in the table 1. Similar procedure is applied to all other 2- and 3-parton GPDs.

**A.2 Symmetry properties of GPDs under the time invariance \( T \)**

We present here the basic steps which lead to symmetry properties of GPDs shown in the third column of table 1, based on time-reversal. The quark field transforms under time-reversal \( T \) as [45]

\[ T \psi(t, x) T^\dagger = \eta_T A \psi(-t, x) = \eta_T A \psi(-\bar{x}) \] (A.7)

with

\[ \bar{x}^\mu \equiv (t, -x) \quad \text{and} \quad A = -i\gamma^5 C. \] (A.8)

Based on the transformation of creation and annihilation operators

\[ T a(k) T^\dagger = \eta_T a(\bar{k}), \]

\[ T a^\dagger(k) T^\dagger = \eta_T^* a^\dagger(\bar{k}), \] (A.9)

the \( \pi^0(p) \) state transforms under \( T \) as

\[ |T \pi^0(p)\rangle = \eta_T^* |\pi^0(\bar{p})\rangle \quad \text{and} \quad \langle T \pi^0(p) | = \langle \pi^0(\bar{p}) | \eta_T. \] (A.10)

As a first example we consider first the 2-parton GPD entering the parametrization of the matrix element \( \langle \pi^0(p_2) | \bar{\psi}(z) \sigma^{\alpha \beta} q(-z) | \pi^0(p_1) \rangle \), in which we omit the Wilson line. Substitution of \( 1 = T^\dagger T \) into the correlator of the above matrix element leads to

\[ \langle \pi^0(p_2) | \bar{\psi}(z) \sigma^{\alpha \beta} \psi(-z) | \pi^0(p_1) \rangle = -\langle \pi^0(p_2) | \bar{\psi}(\bar{z}) \sigma_{\alpha \beta} \psi(-\bar{z}) | \pi^0(p_2) \rangle \] (A.11)

and thus to the symmetry property

\[ H_T(x, \xi, t) = H_T(x, -\xi, t). \] (A.12)

Second, we consider the 3-parton correlator \( \langle \pi^0(p_2) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(y) \psi(z) | \pi^0(p_1) \rangle \). The field \( A^\gamma \) transforms under \( T \) as

\[ TA^\gamma(y) T = A^\gamma(\bar{y}). \] (A.13)
We thus have
\[
\langle \pi^0(p_2) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(y) \psi(z) | \pi^0(p_1) \rangle = - e^{i \bar{z} \cdot \bar{P}(-2\xi)} \langle \pi^0(\bar{p}_1) | \bar{\psi}(0) \sigma^{\alpha \beta} A^\gamma(-\bar{y} + \bar{z}) \psi(\bar{z}) | \pi^0(\bar{p}_2) \rangle,
\] (A.14)

which implies that
\[
T_i(x_1, x_2, \xi) = -T_i(x_2, x_1, -\xi), \quad i = 1, 2.
\] (A.15)
References

[1] X.-D. Ji and J. Osborne, One-loop corrections and all order factorization in deeply virtual Compton scattering, Phys. Rev. D58 (1998) 094018, [hep-ph/9801260].

[2] J. C. Collins and A. Freund, Proof of factorization for deeply virtual Compton scattering in QCD, Phys. Rev. D59 (1999) 074009, [hep-ph/9801262].

[3] J. C. Collins, L. Frankfurt, and M. Strikman, Factorization for hard exclusive electroproduction of mesons in QCD, Phys. Rev. D56 (1997) 2982–3006, [hep-ph/9611433].

[4] M. Diehl, T. Gousset, and B. Pire, Exclusive electroproduction of vector mesons and transversity distributions, Phys. Rev. D59 (1999) 034023, [hep-ph/9808479].

[5] J. C. Collins and M. Diehl, Transversity distribution does not contribute to hard exclusive electroproduction of mesons, Phys. Rev. D61 (2000) 114015, [hep-ph/9907498].

[6] L. Mankiewicz and G. Piller, Comments on exclusive electroproduction of transversely polarized vector mesons, Phys. Rev. D61 (2000) 074013, [hep-ph/9905287].

[7] I. V. Anikin and O. V. Teryaev, Wandzura-Wilczek approximation from generalized rotational invariance, Phys. Lett. B509 (2001) 95–105, [hep-ph/0102209].

[8] I. V. Anikin and O. V. Teryaev, Genuine twist 3 in exclusive electroproduction of transversely polarized vector mesons, Phys. Lett. B554 (2003) 51–63, [hep-ph/0211028].

[9] I. V. Anikin and O. V. Teryaev, Non-factorized genuine twist 3 in exclusive electroproduction of vector mesons, Nucl. Phys. A711 (2002) 199–202, [hep-ph/0208126].

[10] D. Y. Ivanov, B. Pire, L. Szymanowski, and O. V. Teryaev, Probing chiral-odd GPD’s in diffractive electroproduction of two vector mesons, Phys. Lett. B550 (2002) 65–76, [hep-ph/0209300].

[11] R. Enberg, B. Pire, and L. Szymanowski, Transversity GPD in photo- and electroproduction of two vector mesons, Eur. Phys. J. C47 (2006) 87–94, [hep-ph/0601138].

[12] M. El Beiyad, B. Pire, M. Segond, L. Szymanowski, and S. Wallon, Photoproduction of a πrho T pair with a large invariant mass and transversity generalized parton distribution, Phys. Lett. B688 (2010) 154–167, [arXiv:1001.4491].

[13] P. Ball and V. M. Braun, The ρ Meson Light-Cone Distribution Amplitudes of Leading Twist Revisited, Phys. Rev. D54 (1996) 2182–2193, [hep-ph/9602232].

[14] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Higher twist distribution amplitudes of vector mesons in QCD: Formalism and twist three distributions, Nucl. Phys. B529 (1998) 323–382, [hep-ph/9802299].

[15] P. Ball and V. M. Braun, Higher twist distribution amplitudes of vector mesons in QCD: Twist-4 distributions and meson mass corrections, Nucl. Phys. B543 (1999) 201–238, [hep-ph/9810475].

[16] S. Goloskokov and P. Kroll, An Attempt to understand exclusive π+ electroproduction, Eur. Phys. J. C65 (2010) 137–151, [arXiv:0906.0460].

[17] G. R. Goldstein and S. Liuti, Pseudoscalar Meson Electroproduction and Transversity, arXiv:1009.0582.

[18] G. R. Goldstein and S. Liuti, Exclusive meson leptoproduction and spin dependent generalized parton distributions, PoS DIS2010 (2010) 226, [arXiv:1008.2004].
[19] S. Goloskokov and P. Kroll, *Transversity in hard exclusive electroproduction of pseudoscalar mesons*, Eur.Phys.J. A47 (2011) 112, [arXiv:1106.4897].

[20] D. Amrath, M. Diehl, and J.-P. Lansberg, *Deeply virtual Compton scattering on a virtual pion target*, Eur. Phys. J. C58 (2008) 179, [arXiv:0807.4474].

[21] QCDSF and UKQCD Collaboration, D. Brommel et. al., *The Spin structure of the pion*, Phys.Rev.Lett. 101 (2008) 122001, [arXiv:0708.2249].

[22] M. Diehl and L. Szymanowski, *The transverse spin structure of the pion at short distances*, Phys. Lett. B690 (2010) 149, [arXiv:1003.4171].

[23] S. Meissner, A. Metz, M. Schlegel, and K. Goeke, *Generalized parton correlation functions for a spin-0 hadron*, JHEP 0808 (2008) 038, [arXiv:0805.3165].

[24] I. V. Anikin, B. Pire, and O. V. Teryaev, *On the gauge invariance of the DVCS amplitude*, Phys. Rev. D62 (2000) 071501, [hep-ph/0003203].

[25] I. V. Anikin, D. Y. Ivanov, B. Pire, L. Szymanowski, and S. Wallon, *On the description of exclusive processes beyond the leading twist approximation*, Phys. Lett. B682 (2010) 413–418, [arXiv:0903.4797].

[26] I. V. Anikin, D. Y. Ivanov, B. Pire, L. Szymanowski, and S. Wallon, *QCD factorization of exclusive processes beyond leading twist: $\gamma^*_T \rightarrow \rho^*_T$ impact factor with twist three accuracy*, Nucl. Phys. B828 (2010) 1–68, [arXiv:0909.4090].

[27] A. V. Efremov and O. V. Teryaev, *On spin effects in Quantum Chromodynamics*, Sov. J. Nucl. Phys. 36 (1982) 140.

[28] E. V. Shuryak and A. I. Vainshtein, *Theory of Power Corrections to Deep Inelastic Scattering in Quantum Chromodynamics. 1. $Q^2$ Effects*, Nucl. Phys. B199 (1982) 451.

[29] E. V. Shuryak and A. I. Vainshtein, *Theory of Power Corrections to Deep Inelastic Scattering in Quantum Chromodynamics. 2. $Q^4$ Effects: Polarized Target*, Nucl. Phys. B201 (1982) 141.

[30] R. K. Ellis, W. Furmanski, and R. Petronzio, *Unraveling Higher Twists*, Nucl. Phys. B212 (1983) 29.

[31] A. V. Efremov and O. V. Teryaev, *The transversal polarization in Quantum Chromodynamics*, Sov. J. Nucl. Phys. 39 (1984) 962.

[32] O. V. Teryaev, *Twist - three in proton nucleon single spin asymmetries*, hep-ph/0102296.

[33] A. V. Radyushkin and C. Weiss, *Kinematical twist-3 effects in DVCS as a quark spin rotation*, Phys. Rev. D64 (2001) 097504, [hep-ph/0106059].

[34] I. V. Anikin, A. Besse, D. Y. Ivanov, B. Pire, L. Szymanowski, and S. Wallon, *A phenomenological study of helicity amplitudes of high energy exclusive leptonproduction of the $\rho$ meson*, Phys. Rev. D84 (2011) 054004, [arXiv:1105.1761].

[35] A. Besse, L. Szymanowski, and S. Wallon, *The dipole representation of vector meson electroproduction beyond leading twist*, Nucl. Phys. B867 (2013) 19–60, [arXiv:1204.2281].

[36] A. Besse, L. Szymanowski, and S. Wallon, *Saturation effects in exclusive rhoT, rhoL meson electroproduction*, arXiv:1302.1766.

[37] V. Braun and A. Manashov, *Kinematic power corrections in off-forward hard reactions*, Phys.Rev.Lett. 107 (2011) 202001, [arXiv:1108.2394].
[38] V. Braun and A. Manashov, *Operator product expansion in QCD in off-forward kinematics: Separation of kinematic and dynamical contributions*, *JHEP* 1201 (2012) 085, [arXiv:1111.6765].

[39] V. Braun, A. Manashov, and B. Pirnay, *Finite-t and target mass corrections to DVCS on a scalar target*, *Phys.Rev.* D86 (2012) 014003, [arXiv:1205.3332].

[40] V. Braun, A. Manashov, and B. Pirnay, *Finite-t and target mass corrections to deeply virtual Compton scattering*, *Phys.Rev.Lett.* 109 (2012) 242001, [arXiv:1209.2559].

[41] L. Durand and E. Mendel, *Functional equations for path dependent phase factors in Yang-Mills theories*, *Phys.Lett.* B85 (1979) 241.

[42] I. I. Balitsky and V. M. Braun, *Evolution Equations for QCD String Operators*, *Nucl. Phys.* B311 (1989) 541–584.

[43] LHeC Study Group Collaboration, J. Abelleira Fernandez et. al., *A Large Hadron Electron Collider at CERN: Report on the Physics and Design Concepts for Machine and Detector*, *J.Phys.* G39 (2012) 075001, [arXiv:1206.2913].

[44] D. Boer, M. Diehl, R. Milner, R. Venugopalan, W. Vogelsang, et. al., *Gluons and the quark sea at high energies: Distributions, polarization, tomography*, [arXiv:1108.1713].

[45] C. Itzykson and J. Zuber, *Quantum field theory*. McGraw-Hill New York, 1985.