Photon propagator, monopoles and the thermal phase transition in 3D compact QED

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We investigate the gauge boson propagator in three dimensional compact Abelian gauge model in the Landau gauge at finite temperature. The presence of the monopole plasma in the confinement phase leads to appearance of an anomalous dimension in the momentum dependence of the propagator. The anomalous dimension as well as an appropriate ratio of photon wave function renormalization constants with and without monopoles are observed to be order parameters for the deconfinement phase transition. We discuss the relation between our results and the confining properties of the gluon propagator in non–Abelian gauge theories.

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Three–dimensional compact electrodynamics (cQED$_3$) shares two outstanding features of QCD. confinement [1] and chiral symmetry breaking [2]. With some care, it might be helpful for the understanding of certain non–perturbative aspects of QCD to study them within cQED$_3$. The non–perturbative properties of cQED$_3$ deserve interest by themselves because this model was shown to describe some features of Josephson junctions [3] and high–T$_c$ superconductors [4].

Here, we want to elaborate on cQED$_3$ as a toy model of confinement. Indeed, this has been the first non–trivial case in which confinement of electrically charged particles was understood analytically [1]. Confinement is caused here by a plasma of monopoles which emerge due to the compactness of the gauge field. Other common features of the two theories are the existence of a mass gap and of a confinement–deconfinement phase transition at some non–zero temperature. According to universality arguments [5] the phase transition of cQED$_3$ is expected to be of Kosterlitz-Thouless type [6].

In QCD$_4$, the deconfinement phase transition is widely believed to be caused by loss of monopole condensation (for a review see Ref. [7]) within the effective dual superconductor approach [8]. Studying the dynamics of the monopole current inside gluodynamics, monopole de–condensation at the critical temperature is appearing as de–percolation, i.e. the decay of the infrared, percolating monopole cluster into short monopole loops [9]. This change of vacuum structure has a dimensionally reduced analog in the 3D monopole–antimonopole pair binding which has been observed in cQED$_3$ [10, 11].

At present, the gluon propagator in QCD$_4$ is under intensive study. The analogies mentioned before encouraged us to study the similarities between the gauge boson propagators in both theories. In order to fix the role of the monopole plasma in cQED$_3$, not just for confinement of external charges but also for the non-perturbative modification of the gauge boson propagator, we consider it in the confinement and the deconfined phases. On the other hand, on the lattice at any temperature we are able to separate the monopole contribution to the propagator by means of eq. (2) below.

We have chosen the Landau gauge since it has been adopted in most of the investigations of the gauge boson propagators in QCD [12, 13] and QED [14, 15]. In order to avoid the problem of Gribov copies [16], the alternative Laplacian gauge has been used recently [17]. The Coulomb gauge, augmented by a suitable global gauge transformation is enhanced at intermediate momenta which can be characterized by an anomalous dimension [12] (see the last reference in [12] for a comparison of different model functions).

The numerical lattice results for gluodynamics show that the propagator for all these gauges in momentum space is less singular than $p^{-2}$ in the immediate vicinity of $p^2 = 0$. Moreover, the results for the propagator at zero momentum are ranging from a finite [17] (Laplacian gauge) to a strictly vanishing [16, 18, 19] (Coulomb gauge) value. Recent investigations in the Landau gauge show that, beside the suppression at $p \to 0$, the propagator is enhanced at intermediate momenta which can be described by a Debye mass and by an anomalous dimension which both vanish at the deconfinement transition. This mechanism can be clearly attributed to magnetic monopoles. The plasma contribution is relatively easy
to exhibit by explicit calculation and can be eliminated by monopole subtraction on the level of the gauge fields. The results of a study of the propagator in SU(2) gluodynamics have been interpreted \([13]\) in a similar spirit, where \(P\)-vortices appearing in the maximal center gauge were shown to be essential for the enhancement of the Landau gauge propagator at intermediate momenta.

For our lattice study we have adopted the Wilson action, \(S[\theta] = \beta \sum_p (1 - \cos \theta_p)\), where \(\theta_p\) is the \(U(1)\) field strength tensor represented by the plaquette curl of the compact link field \(\theta_l\), and \(\beta\) is the lattice coupling constant related to the lattice spacing \(a\) and the continuum coupling constant \(g_3\) of the 3D theory, \(\beta = 1/(a g_3^2)\). We focus here on the difference between confined and deconfined phase. All results presented have been obtained on lattices of size \(32^2 \times 8\). The finite temperature phase transition is known to take place \([14, 21]\) at \(\beta_c \approx 2.35\).

The Landau gauge fixing is defined by maximizing the functional \(\sum_x \cos \theta_l^0\) over all gauge transformations \(G\). For details of the Monte Carlo algorithm we refer to \([14]\). A more complete presentation of our studies, including also a thorough analysis of the propagator in the zero temperature case is in preparation \([21]\). Details on the implementation of Landau gauge fixing, including the elimination of zero momentum modes and the careful control of double Dirac strings can be found in Ref. \([15, 22]\).

We study the gauge boson propagator, \(\langle \theta_p(x) \theta_l(0) \rangle\), in the momentum space. The propagator is a function of the lattice momentum, \(p_\mu = 2 \sin(\pi k_\mu/L_\mu)\), where \(k_\mu = 0, \ldots, L_\mu/2\) is an integer. We discuss here the finite temperature case and focus on the temporal component of the propagator,

\[
D_{33}(p^2, 0) = \frac{1}{L_x L_y L_z} \langle \theta_3(p, 0) \theta_3(-p, 0) \rangle
\]

as function of the spatial momentum, \(p^2 = \sum_{\mu=1}^3 p_\mu^2\). We remind that at finite temperature the confining properties of static electrically charged particles are encoded in the temporal component of the gauge boson field, \(\theta_3\).

In order to pin down the effect of monopoles we have divided the gauge field \(\theta_l\) into a regular (photon) and a singular (monopole) part which can be done following Ref. \([22]\). In the notation of lattice forms this is written:

\[
\theta = \theta^{\text{phot}} + \theta^{\text{mon}}, \quad \theta^{\text{mon}} = 2\pi \Delta^{-1} \delta p[j],
\]

where \(\Delta^{-1}\) is the inverse lattice Laplacian and the 0-form \(j \in \mathbb{Z}\) is vanishing on the sites of the dual lattice occupied by the monopoles. The 1-form \(*p[j]\) corresponds to the Dirac strings (living on the links of the dual lattice) which connect monopoles with anti–monopoles, \(\delta^*p[j] = *j\). For a Monte Carlo configuration, we have fixed the gauge, then located the Dirac strings, \(p[j] \neq 0\), and constructed the monopole part \(\theta^{\text{mon}}\) of the gauge field according to the last equation in \([3]\). The photon field is just the complement to the monopole part according to the first equation of \([3]\).

The photon and monopole parts of the gauge field contribute to the propagator, \(D = D^{\text{phot}} + D^{\text{mon}} + D^{\text{mix}}\), where \(D^{\text{mix}}\) represents the mixed contribution from regular and singular fields. We show the propagator for \(p = (p, 0)\) together with the separate contributions, multiplied by \(p^2\) and averaged over the same \(p^2\) values, in Figure \(1\) for coupling constant \(\beta = 1.8\).

The regular part of the propagator has perfectly the free field form

\[
D^{\text{phot}}_{33} = \frac{1}{\beta} \frac{Z^{\text{phot}}}{p^2},
\]

at all available \(\beta\). The perturbative propagator defined in terms of \(\theta_l\) is obviously proportional to \(g_3^2\), which is taken into account by the factor \(1/\beta\) in eq. \([3]\). The fits of the photon part of the propagator by the above expression give the parameter \(Z^{\text{phot}}\) as a function of lattice coupling (dash-dotted line in Figure \(1\) for \(\beta = 1.8\)).

The singular contribution to the gauge boson propagator shows a maximum in \(p^2 D^{\text{mon}}_{33}\) at some momentum (Figure \(1\), moving with increasing \(\beta\) nearer to \(|p| a = 0\). The mixed component gives a negative contribution to \(p^2 D^{\text{mix}}_{33}\), growing with decreasing momentum. The central point of our paper is that all these contributions together do not sum up to a simple massive Yukawa propagator. To quantify the difference between a Yukawa–type and the actual behavior we use the the following four–parameter model function for \(D_{33}(p^2, 0)\),

\[
D_{33}(p^2, 0) = \frac{Z}{\beta} \frac{m^2}{p^{2(1+\alpha)} + m^{2(1+\alpha)} + C},
\]

FIG. 1: Different contributions to the full \(D_{33}\) propagator (multiplied by \(p^2\)) vs spatial lattice momentum squared and fits as described in the text for \(\beta = 1.8\) on a \(32^2 \times 8\) lattice.
where $Z$, $\alpha$, $m$ and $C$ are the fitting parameters. This model is similar to some of Refs. \[13\] \[14\] where the propagator in gluodynamics has been studied.

The first part of the function (4) implies that the photon acquires a Debye mass $m$ (due to screening \[13\]) together with the anomalous dimension $\alpha$. The (squared) photon wave function renormalization constant $Z$ describes the renormalization of the photon wave function due to quantum corrections. The second part of (4) represents a $\delta$–function–like interaction in coordinate space.

Before fitting we average the propagator over all lattice momenta at same $p^2$ to improve rotational invariance. Thus the errors entering the fits include both the variance among the averages for individual momenta and the individual errors. The fits were performed using standard Mathematica packages combined with a search for the global minimum in $\chi^2$/d.o.f. To check the stability of the fits, we studied several possibilities of averaging and thinning out the data sets, a procedure which will be discussed elsewhere \[21\].

The model function (4) works perfectly for all $p^2$ and couplings $\beta$. For $\beta \geq 2.37$ the best fit for mass parameter $m$ and anomalous dimension $\alpha$ are both consistent with zero. Therefore we set $m = 0$ and $\alpha = 0$ for these values of $\beta$ to improve the quality of the fit of $Z$ and $C$.

It turns out that the inclusion of a constant term, $C$, in the model function (4) is crucial for obtaining good fits in the confinement phase, despite the fact that it is very small (as function of $\beta$ the parameter $C$ decreases from $C(1.0) = 0.18(4)$ to $C(2.2) = 0.009(2)$, it rapidly vanishes in the deconfined phase). Similarly to $m$ and $\alpha$ parameters we set $C$ to zero for $\beta \geq 2.45$, where $C$ becomes smaller than $10^{-4}$.

An example of the best fit of the full propagator for $\beta = 1.8$ is shown in Figure 2 by the solid line (with $C = 0.033(5)$). The parameter $Z$ distinguishes clearly between the two phases (Figure 3). It coincides with the photon part $Z^{\text{phot}}$ (defined without monopoles) in the deconfined phase while it is much larger in the confined phase. This indicates that the photon wave function gets strongly renormalized by the monopole plasma. In contrast, the factor $Z^{\text{phot}}$ smoothly changes crossing the deconfinement transition at $\beta_c \approx 2.35$.

The anomalous dimension $\alpha$ also distinguishes the two phases (Figure 3): it is equal to zero in the deconfinement phase (perturbative behaviour) while in the confinement phase the monopole plasma causes the anomalous dimension growing to $\alpha \approx 0.25 \ldots 0.3$.

To characterize the properties of $Z$ and $\alpha$ approaching the phase transition we fit the excess of the ratio of $Z$’s over unity,

$$R_Z(\beta) = \frac{Z(\beta)}{Z^{\text{phot}}(\beta)} - 1,$$  \hspace{1cm} (5)

and the anomalous dimension $\alpha$ in the following form:

$$f_i(\beta) = h_i(\beta^{(i)}_c - \beta)^{\gamma_i}, \quad \beta < \beta^{(i)}_c, \quad (i = \alpha, Z).$$  \hspace{1cm} (6)

where $i = Z, \alpha$. The $\beta^{(\alpha, Z)}_c$ are the pseudo–critical couplings which might differ on finite lattices.

The best fits $f_\alpha$ and $f_Z$ are shown in Figures 3 and 4 respectively. The solid lines in both plots extend over

![Figure 2: Coefficients $Z$ of fit (4) for full propagator and $Z^{\text{phot}}$ for photon contribution (3) vs $\beta$.](image)

![Figure 3: Anomalous dimension $\alpha$ vs $\beta$ and its best fit near $\beta_c$ using function (4).](image)

![Figure 4: Same as in Figure 3 for ratio $R_Z$, eq. (5).](image)

the fitting region. The corresponding parameters are presented in Table I. The pseudo–critical couplings $\beta^{(\alpha, Z)}_c$
and $\beta_i^{(Z)}$ are in agreement with previous numerical studies giving $\beta_i = 2.346(2)$. Note that the critical exponents $\gamma_i$ are close to 1/2, both for the anomalous dimension $\alpha$ and for $R_Z$ expressing the ratio of photon field renormalization constants.

Finally, the $\beta$–dependence of the mass parameter, $m$, is presented in Figure 5. As expected, the mass scale generated is non–vanishing in the confinement phase due to presence of the monopole plasma. It vanishes at the deconfinement transition point when the very dilute remaining monopoles and anti–monopoles form dipoles.

Summarizing, we have shown that the presence of the monopole plasma leads to the appearance of a non–vanishing anomalous dimension $\alpha > 0$ in the boson propagator of cQED$_3$ in the confinement phase. We would hope that our observation stimulates an analytical explanation.

At this stage of studying cQED$_3$ as a model of confinement we conjecture that in the case of QCD the Abelian monopoles defined within the Abelian projection may be responsible for the anomalous dimension of the gluon propagator observed in Refs. [12, 23]. If true, a monopole subtraction procedure analogous to that employed here would be able to demonstrate this. We found that the anomalous dimension $\alpha$ and the ratio of the photon wave function renormalization constants with and without monopoles, $R_Z$ (5), represent alternative, also non–local order parameters characterizing the confinement phase.

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![FIG. 5: The mass $m$ vs $\beta$.](image-url)

| $i$ | $h_i$ | $\beta_i^{(Z)}$ | $\gamma_i$ |
|-----|-------|----------------|-----------|
| $\alpha$ | 0.250(9) | 2.368(3) | 0.50(2) |
| $Z$ | 2.63(7) | 2.363(3) | 0.48(3) |

TABLE I: Best parameters for the fits (1).