Late evolution of cataclysmic variables:  
the loss of AM Her systems  

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Abstract  
The white dwarf in AM Her systems is strongly magnetic and keeps in synchronous rotation with the orbit by magnetic coupling to the secondary star. As the latter evolves through mass loss to a cool, degenerate brown dwarf it can no longer sustain its own magnetic field and coupling is lost. Angular momentum accreted then spins up the white dwarf and the system no longer appears as an AM Her system. Possible consequences are run-away mass transfer and mass ejection from the system. Some of the unusual cataclysmic variable systems at low orbital periods may be the outcome of this evolution.  

1. Introduction  
In cataclysmic variable systems the Roche lobe filling secondary star transfers mass to the primary white dwarf as the system loses angular momentum by gravitational wave emission and torques exerted from magnetized winds leaving the system. For low mass main sequence secondaries the mass - radius exponent $\zeta = d\ln R / d\ln M$ is about 0.8. Then stellar radius, i.e. Roche lobe, orbital separation, and orbital period decrease as mass loss proceeds. If however the secondary mass has dwindled to about 0.07 $M_\odot$ efficient nuclear energy production ceases and the secondary becomes a cool partially degenerate brown dwarf asymptotically reaching the mass-radius exponent $-1/3$ of an adiabatic gas sphere. As it passes through the value $+1/3$ the mean density reaches a maximum, the orbital period, which is proportional to its inverse square root, reaches its minimum value and increases again with further mass decrease [Paczynski and Sienkiewicz 1981, Rappaport et al. 1982, Ritter 1986].  

Two problems have appeared here. Firstly, stellar structure calculations yield a minimum period of 70 minutes while the observed minimum period is close to 80 minutes. Secondly, since all systems follow the same evolutionary path, they should accumulate in large number at the minimum period where the period becomes stationary in time. This is not observed (for a recent analysis see Kolb and Baraffe [1999]). An assumed increase of the orbital angular momentum loss rate to four times the gravitational wave value would raise the derived minimum period to the observed value, near 80 minutes [Kolb & Baraffe, 1999]. This may point to residual braking by the secondary stars magnetic field. Could this magnetic field differ from star to star and thereby spread out the individual period turning points thus explaining the missing accumulation of cataclysmic variables (CVs) at one unique minimum period?  

2. The secondary stars magnetic fields  
A solar type dynamo working between the outer convective envelope and the radiative interior is thought to cause the high angular momentum loss and strong mass transfer for systems at large orbital
periods. Its disappearance when the stars become fully convective is made responsible for the "period gap" between 3 and 2 hours [Spruit & Ritter 1983, Rappaport et al. 1982]. But, fully convective rotating late main sequence stars show chromospheric and coronal activity indicative of magnetic fields. Their fields have been appealed to for providing the friction of dwarf nova accretion discs in quiescence [Meyer & Meyer-Hofmeister 1999a]. A different dynamo working in fully convective stars may be responsible for these fields. Its long time average, however, would be the same for all secondaries of the same mass and thus also its magnetic braking, predicting a unique minimum period.

If however the magnetic fields of the secondaries are confined and held in the over-adiabatic structure of these fully convective stars in the same manner as suggested for Ap-stars [Meyer 1994], they are the preserved relics from an earlier dynamo phase and can contain flux to various degrees, as Ap-stars do. This would allow various degrees of magnetic braking and a spreading of the individual period turning points as desired. Both kinds of magnetic fields, dynamo produced and "fossil", require convection and over-adiabatic structure and these disappear when the stars evolve into degenerate adiabatic cool brown dwarfs.

3. Past the minimum period: dwarf novae versus AM Her system

When the magnetic fields of the secondary stars get lost as the systems pass through their minimum period any residual magnetic braking associated with it also disappears. We have suggested [Meyer & Meyer-Hofmeister 1999a] that the very low values for the $\alpha$-parameter describing friction in the quiescent accretion disks of WZ Sge systems is evidence for this loss of magnetic fields. With braking reduced to the gravitational wave value the mass transfer rate is likewise reduced. This can bring the dwarf novae systems close to and finally beyond the limit below which they never will burst out again (see the marginal case of WZ Sge, [Meyer-Hofmeister et al. 1998]) and make these systems inconspicuous, as has often been suggested. But it would possibly not suffice to explain the large number of "period bouncers" that are expected but not observed [Patterson 1998].

How will AM Her systems fare? When the secondaries lose their magnetic field the white dwarf’s magnetic field remains strong at the location of the secondary. But all its coupling mechanisms are gone: dipole-dipole interaction, conductive tying between reconnected field lines of primary and secondary, convective mixing in of primary field into secondary envelope [Campbell 1985, 1986, 1989, Lamb 1985, Lamb and Melia 1988]. Electrical conductivity in the very cool photosphere of such brown dwarfs [Allard et al. 1990] becomes poor. Primary fields which ohmically diffuse into the secondary then can not support currents and thus not transfer coupling stresses between secondary and primary. On the irradiated side facing the primary temperatures are higher, conductivity is good but also prevents diffusive penetration of the primary’s field into the secondary. On losing its coupling the primary white dwarf must spin up driven by the angular momentum accreted with the transferred mass.

4. Spin-up of the white dwarf

The loss of coupling and the beginning of spin-up has a strong effect on the mass transfer rate because the angular momentum that spins up the white dwarf reduces the angular momentum of the orbit. This increases the mass transfer rate from the secondary. Then the time scale of mass loss of the secondary becomes small (compared to its thermal time scale) and the stellar response to mass loss becomes nearly adiabatic, $\zeta$ changes from about 1/3 to -1/3, i.e. the star expands as its mass keeps decreasing. Altogether this significantly raises the mass transfer rate.

The equations describing this and the effect of a possible later mass loss from the system follow in a standard way by linking polytropic structure [Emden 1907], Roche geometry [Paczynski 1971], mass transfer through the inner Lagrange point $L_1$ (see Kolb and Ritter [1990] and references there), and
angular momentum loss by gravitational waves (e.g. Misner et al. [1973]). They are

\[
\frac{d\dot{M}}{dt} = CM^{2/3} \left( \dot{M}_{GW} - X\dot{M} \right),
\]

\[
C = 10^{21.43} \left( \frac{M_2}{0.07M_\odot} \right)^{-2/3} P_8^{-1/3} \frac{g^{-2/3}s^{-1/3}}{s},
\]

\[
\dot{M}_{GW} = 10^{14.79} q^{2/3}(1-q)^{1/3} \left( \frac{M_2}{0.07M_\odot} \right)^{8/3} P_8^{-8/3} \frac{g}{s},
\]

\[
X = \zeta - \frac{1+\beta q}{3(1+q)} - 2f \sqrt{(1+q)\left( \frac{r_{LS}}{a} \right)} + 2(1-\beta q) - (1-\beta) \frac{q}{1+q}.
\]

Here \(\dot{M}\) is the mass transfer rate from the secondary, \(q\) the mass ratio \(M_2/M_1\), \(M_1\) and \(M_2\) the masses of primary and secondary star, \(k(q)\) a factor from the Roche geometry, \(k(q) = 5.97\) for \(q = 0.1\) (c.f. Meyer & Meyer-Hofmeister [1983]). \(P_8\) is the orbital period in units of 80 minutes. It enters into the polytropic constant by the relation between orbital period and mean density [Frank et al. 1985], for a Roche lobe filling secondary and into the gravitational wave term driving the mass transfer \(\dot{M}_{GW}\). \(\beta\) is the fraction of the transferred mass that is accreted on the white dwarf, the fraction \(1-\beta\) is lost from the system. During spin-up \(\beta = 1\). The factor \(f\) is the specific angular momentum lost from the orbit by the matter flow (either accreted on the white dwarf or expelled from the system) measured in units of the angular momentum arriving with the accretion stream. The latter has been calculated by Lubow & Shu [1975]. \(r_{LS}\) is the radius of the Kepler orbit around the white dwarf with this angular momentum and \(a\) is the binary separation. \(f = 1\) for the spin-up phase.

For a positive value of \(X\) the solution of equation (1) tends to the stable fixpoint \(\dot{M} = \dot{M}_{GW}/X\). As described above this value increases with the onset of spin-up when the response function \(X\) becomes smaller with the change of \(\zeta\) and the switch-on of the \(f\)-term. Had the effective driving term \(\dot{M}_{GW}\) been four times higher by inclusion of some residual magnetic braking before loss of coupling and had dropped to its value as given above, the change in \(X\) would still have increased \(\dot{M}\) by about a factor of three with the onset of spin-up.

We assume that the matter falling towards the white dwarf in the rotating magnetosphere behaves diamagnetically and experiences a braking force from the relative motion between field and fluid [King 1993, Wynn & King 1995]. The strongest interaction occurs at the point where the free fall path would reach closest approach and the magnetic field is largest. If the speed of the rotating magnetic field becomes faster than that of the material there a new phase involving expulsion of matter can set in.

For our standard case \(M_1 = 0.7M_\odot, M_2 = 0.07M_\odot, P_{\text{orbital}} = 80\) minutes, and using the value 0.2 for the square of the radius of gyration of the white dwarf (H. Ritter, private communication), the amount of matter required to reach this critical rotation of the white dwarf is \(\Delta M = 0.002M_\odot\), accumulated in about 6.10^6ys.

This idealized case assumes the free fall orbit of Lubow & Shu’s (1975) calculation. The true orbit will already be affected by the magnetic interaction and not pass as close to the white dwarf. The critical rotation speed will correspondingly be smaller.

Figure 1 shows how the mass transfer rate and the white dwarf spin develop when magnetic coupling between secondary star and white dwarf is lost. A driving term for orbital angular momentum loss of four times the gravitational wave value was assumed. For comparison the decrease of mass transfer in dwarf nova systems on loss of residual magnetic braking is also shown.
5. The propeller phase

The falling matter enters the magnetic field of the rotating dipole. For an inclined dipole the braking and acceleration force experienced by the matter depends on the rotational phase of the dipole. Thus one part of the matter reaches the strong interaction a bit farther away from the primary and the other comes closer in. The latter would experience braking and will finally be accreted. The former can be accelerated outward and flung out of the system. A propeller effect was already discussed by Illarionov & Sunyaev [1975]. The interaction of the magnetic field and the matter stream is complex. We adapt here results obtained from a model for AE Aqr by Wynn et al. [1997].

The intermediate polar AE Aqr is observed to spin down at a rate of $P_{\text{spin}} = 5.64 \times 10^{-14}$. In the model of Wynn et al. [1997] the angular momentum given off by the white dwarf adds to the angular momentum of the stream itself to expell nearly all of the matter transferred from the secondary. The interpretation of the observed Doppler tomogram supports this expulsion of matter. Along these lines we now estimate the orbital angular momentum loss $J_{\text{propeller}}$ involved in the expulsion of matter in our case. We obtain an estimate for the fraction of matter $1-\beta$ expelled from the system by the following consideration. The matter arriving at the distance of closest approach $r_{\text{min}}$ needs additional angular
momentum to be accelerated above escape speed. This is provided by the angular momentum of the accreted fraction $\beta$.

\[
(1 - \beta) \left[ \sqrt{(1 + b^2)} \sqrt{2GM_1r_{\text{min}}} - \sqrt{GM_1r_{\text{LS}}} \right] = \beta \sqrt{GM_1r_{\text{LS}}}
\]

(2)

We take for $r_{\text{min}}$ the value determined by Lubow & Shu [1975] $r_{\text{min}} = a \cdot \tilde{\omega}_{\text{min}}$. Assuming that the matter arrives at infinity with velocity one half of the escape speed at distance $r_{\text{min}}$ from the white dwarf, $b = v_{\infty}/v_{\text{escape}}= 1/2$ one obtains the estimate

\[
\beta = 1 - \frac{1}{\sqrt{2}r_{\text{min}}/r_{\text{LS}}} = 0.2.
\]

(3)

The numerical value results for $q=0.1$.

The orbital angular momentum loss in the propeller phase has to be taken with respect to the binary’s center of gravity. The forces that produce the roughly 90° swing around the primary before ejection retard the primary’s orbital motion and thereby extract orbital angular momentum. We use the computed trajectories from Lubow & Shu, interpolated for $q=0.1$ and write the estimate for $J_{\text{propeller}}$

\[
\dot{J}_{\text{propeller}} = (1 - \beta) \dot{M} \left[ \sqrt{v_{\text{escape}}^2 + v_{\infty}^2} \cdot r_{\text{min}} + v_{\infty} \frac{q}{1 + q} \right],
\]

(4)

where we assume that the expelled matter keeps its angular momentum with respect to the primary as it rapidly climbs out of the gravitational potential. The last term on the right side accounts for the distance between the primary and the binary’s center of mass. This yields

\[
\dot{J}_{\text{propeller}} = \dot{M} f \sqrt{GM_1r_{\text{LS}}}
\]

(5)

with

\[
f = (1 - \beta) \sqrt{\frac{2r_{\text{min}}}{r_{\text{LS}}}} \cdot \left( 1 + \frac{q}{q + 1} \frac{b}{1 + b^2} \frac{1}{r_{\text{min}}/a} \right).
\]

(6)

For $q=0.1$ and $\beta=0.2$ one obtains $f=1.3$. A graphical evaluation of trajectories calculated by Wynn et al. [1997] for AE Aqr taking into account the smaller mass ratio leads to $f=1.5$. We emphasize that these estimates are rough. With such values of $f$ the increase of orbital angular momentum loss compared to $\dot{J}_{\text{spin-up}}$ changes the sign of $X$ and then leads to an accelerated growth of the mass transfer rate [Eq. (1)]. Whether this occurs depends on the detailed process during the swing around the white dwarf. For $f=1.3$ and our standard case the time to reach arbitrarily large $\dot{M}$ is only $10^{4.6}$ys. The rate cannot grow indefinitely, finally the magnetic field becomes unable to handle the ever growing mass transfer. The efficiency of acceleration diminishes and angular momentum loss from the system gets limited.

6. Further evolution and conclusions

What will be the further evolution? If the system stabilizes at a high mass transfer rate the secondary may lose all its mass in a short time and a single fast rotating magnetic white dwarf like RE J0317-853 [Barsow et al. 1995] will remain. Alternatively, and depending on the field strength of the primary, a disc may form which would return the accreted angular momentum to the orbit. This would result in a rapid decrease of the mass transfer rate. But the strong magnetic pressure of the white dwarf would remove such a disc before the mass transfer rate had dropped to the stationary value of Eq. (1) possibly resulting in a cyclic behaviour with phases of high mass transfer rates and rapid depletion of the secondary. Perhaps most intriguingly, if expelled matter would form a circumbinary disc significant (even unlimited) angular
momentum could be extracted from the orbit. First considerations indicate that mass transfer rates would
decrease from initially high values to decreasingly lower values on increasingly longer time scale as the
circumbinary disk evolves. There, even some supersoft sources (c.f. van Teeseling & Kim [1998]) and
systems like ER Uma with unusually high mass transfer rates [Osaki 1996] might find their evolutionary
place. But this must be left to future investigations.

This account closely follows a paper to be published in A&A Letters [Meyer & Meyer-Hofmeister 1999b].

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References
Allard F., Hauschildt P.H., Baraffe I., Chabrier G., 1996, ApJ 465, L123.
Barsow M., Jordan S., O'Donoghue D., Burleigh M.R., Napiwotzki R., Harrop-Allin M.K., 1995, MNRAS 277, 971
Campbell C.G., 1985, MNRAS 215, 509
Campbell C.G., 1986, MNRAS 219, 589
Campbell C.G., 1989, MNRAS 236, 475
Emden R., 1907, Gaskugeln, Teubner Verlag Leipzig
Frank J., King A.R., Raine D.J., 1985, Accretion Power in Astrophysics, Cambridge Univ. Press, Cambridge
Illarionov A.F., Sunyaev R.A., 1975, A&A 39, 185
King A.R., 1993, MNRAS 261, 144
Kolb U., Baraffe I., 1999, in: Proc. Anapolis Workshop on Magnetic Cataclysmic Variables, ASP Conf. Series
Vol. 157, eds. Hellier C. & Mukai A., p. 273
Kolb U., Ritter H., 1990, A&A 236, 385
Lamb D.Q., Melia F., 1988, in: Polarized Radiation of Circumstellar Origin, eds. Mason K.O. et al., Springer
Verlag, Berlin, p.113
Lubow S.H., Shu F.H., 1975, ApJ 198, 383
Meyer F., 1994, in: Cosmic Magnetism, ed. Lynden-Bell D., Kluwer, Dordrecht, p. 67
Meyer F., Meyer-Hofmeister E., 1983, A&A 121, 29
Meyer F., Meyer-Hofmeister E., 1999a, A&A 341, L23
Meyer F., Meyer-Hofmeister E., 1999b, A&A Letters, in press, MPA report 1177
Meyer-Hofmeister E., Meyer F., Liu B.F., 1998, A&A 339, 507
Misner C.W., Thorne K.S., Wheeler J.A., 1973, Gravitation, W.H. Freeman and Co., San Francisco
Osaki Y., 1996, PASP 108, 39
Paczynski B., 1971, ARA&A 9, 183
Paczynski B., Sienkiewicz R., 1981, ApJ 248, L27
Patterson J., 1998, PASP 110, 1132
Rappaport S., Joss P.C., Webbink R.F., 1982, ApJ 254, 616
Ritter H., 1996, in: The Evolution of Galactic X-ray Binaries, eds. Trümper J. et al. Reidel, Dordrecht, p. 271
Spruit H.C., Ritter H., 1983, A&A 124, 267
van Teeseling A., King A., 1998, A&A 129, 83
Wynn G.A., King A.R., 1995, MNRAS 275, 9
Wynn G.A., King A.R., Horne K., 1997, MNRAS 286, 436