Numerical solution of the integral equations of the electromagnetic field in geosteering problems

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Abstract. Extra-deep azimuthal resistivity measurements improve the depth of investigation up to 30 m from the wellbore. Interpretation of electromagnetic logging data in the neighbourhood of a well becomes an important technical problem. We present an efficient parallel method for computation of induction tool responses with multiple transmitter–receiver configurations in 2D pixel-based anisotropic model crossed by an arbitrary well trajectory. The cornerstone of the approach is volume integral equation method. We consider the conductivity distribution as a sum of background and anomalous conductivities. Background conductivity is 1D-layered. Anomalous conductivity has arbitrary 2D distribution. Electromagnetic fields are the superposition of background and anomalous fields. Background fields are calculated exactly using rigorous analytical solution for 1D-layered background model. With this approach, the 2D pixel-based model is treated as an extension of the 1D-layered model. The anomalous fields are required only in pixels with conductivity different from the conductivity of the 1D-layered model. Anomalous fields are calculated using convergent series of integral operators [1]. The approach takes into account conductivity anisotropy and allows obtaining both the exact solution and the fast approximate one. The convergence of approximate solution is investigated on some synthetic examples.

1. Introduction
The goal of this work is to develop a method for approximate calculation of electromagnetic tool response in 2D pixel-based environment model. A pixel model of the medium is the result of discretization of the conductivity distribution of any 2D geoelectric model. We choose approximate approach to speed up forward 2D modeling. The cornerstone of this method is volume integral equation. The equation uses 1D-layered model as a background. We consider conductivity distribution ($\sigma$) as a sum of background ($\sigma_b$) and anomalous conductivities ($\sigma_a$), where background conductivity is layered and one-dimensional:

$$\sigma = \sigma_a + \sigma_b \quad (1)$$

Electromagnetic fields ($\vec{E}$, $\vec{H}$) are a superposition of background ($\vec{E}_b$, $\vec{H}_b$) and anomalous fields ($\vec{E}_a$, $\vec{H}_a$).

$$\vec{E} = \vec{E}_a + \vec{E}_b \quad (2)$$

$$\vec{H} = \vec{H}_a + \vec{H}_b \quad (3)$$
Background fields are calculated exactly using rigorous analytical solution for 1D-layered background model. Anomalous fields are calculated using method of successive iterations of integral operator [1-3]. The approach has the following advantages:

- takes into account conductivity anisotropy;
- allows for obtaining both the exact solution and the fast approximate one;
- uses rigorous analytical solution for background field calculation. Therefore, all 1D effects (such as ‘horn effect’ [4] etc.) are calculated with high level of accuracy. Accurate modelling of complex 1D case with high number of layers and high conductivity contrasts is a challenge for majority of the 2D solvers. Presented method simulates this kind of model rigorously.

1.1. Simulation of 2D geoelectrical models

Geoelectrical model can be treated as two-dimensional, if conductivity distribution does not change along one axis (Fig. 1). This axis is the strike axis represented by Y. Tool trajectory is an arbitrary 3D curve. Majority of effective methods for 2D electromagnetic modeling are based on the same technique: Fourier transform is made along the Y axis [5]. This transformation turns our problem into a set of two-dimensional problems. Each of 2D problems corresponds to its spatial harmonics. This technique allows for calculating faster and using less memory than direct 3D simulation; moreover the resulting set of 2D problems can be effectively parallelized.

![Figure 1. Example of 2D geoelectrical model.](image)

2. Integral equations method for 2D modelling of EM fields

In electromagnetic field calculation, the quasi-analytical series method described in [1-3] is used. This method was adapted for two-dimensional modeling by Fourier transform along the strike axis Y. Fourier transformation along Y axis is represented by a wave above the notation of a variable \( \tilde{\phi} \).

Initial step of this method is to split the electromagnetic fields in the spectral domain \( (x, k_y, z) \) into anomalous fields \( (\tilde{E}_a, \tilde{H}_a) \) and background fields \( (\tilde{E}_b, \tilde{H}_b) \). The background fields are rigorous solution for the background 1D-layered model. To calculate anomalous fields, it is necessary to solve the following integral equations:

\[
\tilde{E}_a(\tilde{\rho}) = \sqrt{2\pi} \int_S \tilde{G}^{EIJ}(\tilde{\rho}, \tilde{\rho}')\sigma_a(\tilde{\rho}') \left( \tilde{E}_a(\tilde{\rho}') + \tilde{E}_b(\tilde{\rho}') \right) dS', \quad (4)
\]

\[
\tilde{H}_a(\tilde{\rho}) = \sqrt{2\pi} \int_S \tilde{G}^{HJ}(\tilde{\rho}, \tilde{\rho}')\sigma_a(\tilde{\rho}') \left( \tilde{E}_a(\tilde{\rho}') + \tilde{E}_b(\tilde{\rho}') \right) dS', \quad (5)
\]

where \( \tilde{G}^{EIJ} \) and \( \tilde{G}^{HJ} \) are electric and magnetic Green’s tensors for current as a source, calculated for 1D-layered background model and \( \tilde{\rho} \) is the coordinate vector on the model plane (XZ).
\[ \vec{\beta} = (x, z). \]  

To solve the integral equation (4), we first calculate the anomalous electric field distribution in quasi-analytical approximation \( \vec{E}_a^0 \) [1].

\[ \vec{E}_a^0 = \frac{g}{1-g} \vec{E}_b, \]  

\[ g(\vec{\rho}) = \frac{\sqrt{2\pi} \int \vec{G}(\vec{\rho}, \vec{\rho}') \sigma_a(\vec{\rho}') \delta_a(\vec{\rho}) d\vec{s}', \delta_a(\vec{\rho}) \vec{E}_b(\vec{\rho})}{\vec{E}_b(\vec{\rho}) \cdot \vec{E}_b(\vec{\rho})}, \]  

where \( g \) is scalar depolarization coefficient, * is complex conjugation.

Next step is to calculate anomalous electric field distribution \( \vec{E}_a^n \) by successive iterations [3] shown in (9) starting with quasi-analytical approximation \( \vec{E}_a^0 \) as initial distribution.

\[ \vec{E}_a^n = \frac{1}{a} \left( \vec{G}_m[\beta a \vec{E}_a^{n-1}] + \vec{G}_m[\beta a \vec{E}_b] - \beta a \vec{E}_b \right), \]  

where \( a \) and \( \beta \) are functions that depend on background and anomalous conductivity distribution, \( \vec{G}_m \) is modified Green operator that has to be constructed based on electric Green’s tensor.

\[ a = \frac{2 \text{Re}(\sigma_b) + \sigma_a}{2 \text{Re}(\sigma_b)}, \]  

\[ \beta = \frac{2 \text{Re}(\sigma_b) + \sigma_a}{2 \text{Re}(\sigma_b)}, \]  

\[ \vec{G}_m[F] = F(\vec{\rho}) + 2 \sqrt{\text{Re}(\sigma_b(\vec{\rho}'))} \sqrt{2\pi} \int \vec{G}_m(\vec{\rho}, \vec{\rho}') \sqrt{\text{Re}(\sigma_b(\vec{\rho}'))} \text{F}(\vec{\rho}') dS'. \]  

The successive iterations (9) always converge for any lossy medium because operator \( \vec{G}_m \) L_2 norm is less than one [3]. The following expression can be used to evaluate the accuracy (\( \epsilon_n \)) of an anomalous electric field calculation at n\text{th} iteration \( \vec{E}_a^n \) [1]:

\[ \epsilon_n = \left\| a \vec{E}_a - a \vec{E}_a^n \right\| \leq \frac{\|\beta\|_\infty}{1 - \|\beta\|_\infty} \left\| a \vec{E}_a - a \vec{E}_a^{n-1} \right\|. \]  

When the required accuracy in anomalous electric field is achieved, the anomalous magnetic field in receivers is calculated using integral equation (5). After this, we calculate full magnetic field in receivers by inverse Fourier transform of anomalous magnetic field and add the background magnetic field to the result.

3. Method implementation and performance

Based on the described method, an electromagnetic 2D solver was developed on C++ using Intel®MKL [6] and OpenMP [7] technology. The working name of the code is PACK2D described as Pixel-based Approximate Computational Kernel for 2D modeling. The discretization of the continuous equations (4-13) is carried out by the Galerkin method on a rectilinear grid in the model plane (XZ).

The performance of the method mainly depends on the area in the model plane occupied by a region with nonzero anomalous conductivity. A region with anomalous conductivity equal to zero is naturally excluded from the equations (4-13) and from discretized representations of these equations. Commonly encountered 2D situations (fault, unconformity, scour, pinching-out, and anticline) are the extension of the 1D-layered model. In these cases, the anomalous conductivity is equal to zero in the larger part of the model. Therefore, this approach becomes computationally efficient for geosteering problems.

As an example let us consider case of extra-deep azimuthal and bulk resistivity measurements by EDAR tool [8] in the model with angular unconformity (Fig. 2). The model parameters are shown in Table 1.
Table 1. Parameters of angular unconformity model.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( \rho_1 \) | 10 Ωm | d | 7 m |
| \( \rho_2 \) | 40 Ωm | h | 3 m |
| \( \rho_3 \) | 20 Ωm | dip | 90 deg. |
| \( \rho_4 \) | 1 Ωm | azimuth | 45 deg. |

Azimuth and dip in the Table 1 are angles describing the tool trajectory. Azimuth is the angle between tool trajectory and axis Y and dip is the angle between tool trajectory projection on plane XZ and axis Z.

Simulation was performed for the trajectory interval of lengths 20 m with 21 measure points. The calculation results of the EDAR tool signals for the model are shown below. Coaxial measurement of the ZZ component using one transmitter and two receivers (attenuation and phase difference) are shown in Figures 4 and 5. Azimuthal measurement of the XZ component using transmitter and receiver coil (imaginary and real part of voltage) are shown in figures 6 and 7. Verification of PACK2D calculations was carried out using in-house code based on surface integral equations method Pie2d [9, 10]. The results of the calculations of both solvers coincide with high accuracy.
**Figure 6.** Imaginary part of voltage at frequency 20 and 50 kHz. Transmitter moment assumed equals 1 A·m$^2$ and specific receiver moment assumed equals 1 m$^2$.

**Figure 7.** Real part of voltage at frequency 20 and 50 kHz. Transmitter moment assumed equals 1 A·m$^2$ and specific receiver moment assumed equals 1 m$^2$.

The calculation time for 20 meters interval was 10 minutes for all EDAR signals using Xeon E5-1620 V3 processor (performance at single precision is 448 GFLOPs).

**Conclusions**

An efficient parallel method for calculating induction tool responses in 2D anisotropic geoelectric model crossed by an arbitrary trajectory is presented. With this approach, the 2D model is treated as an extension of the 1D-layered model what increases the accuracy and speed of calculations in common 2D situations in geosteering. The achieved calculation speed is a few minutes per 20 meter trajectory interval using standard mid-level desktop hardware.

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