Precision tests with $K_{\ell 3}$ and $K_{\ell 2}$ decays

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Abstract.

In 2000, the analysis indicated that the unitarity relation, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, might be broken at the 2.3$\sigma$ level. At that time, however, the $|V_{us}|$ was inferred from the old experimental data. Since then, a great experimental and theoretical effort has been invested to understand the source of that deviation. Thanks to the new and improved measurements by BNL-E865, KLOE, KTeV, ISTRA+ and NA48 the old $K_{\ell 3}$ decay rate got shifted so that the new $|V_{us}|$ is now consistent with unitarity. On the theory side, much progress in the lattice QCD community has been made in order to tame the systematic uncertainties related to the computation of the $K_{\ell 3}$ form factors.

This joint progress allowed to assess the validity of the CKM unitarity relation at the level less than 1%. Undergoing lattice studies will corroborate and improve this scenario. The key challenge will be to simulate lighter pions in the region in which ChPT predictions apply. Also interesting is the recent progress in accurately computing the kaon and pion decay constants on the lattice, which then give us access to $|V_{us}|$ and $|V_{ud}|$ from the corresponding leptonic decays.

In addition, we argue that the $K_{\ell 3}$ and $K_{\ell 2}$ decays offer the possibility to test various scenarios of physics beyond Standard Model.

1. Introduction

From the experimental information on down- to up- quark transitions (such as $d \to u$, $s \to u$ and $b \to u$), we access the effective dimension-six operators of the form, $D_1^{\dagger} \Gamma_1 U_\ell \Gamma_2 \nu$, with $D$ (U) being a generic "down" ("up") flavor, and $\ell \in \{e, \mu, \tau\}$. Their effective coupling can be parametrized as the Standard Model (SM) contribution $G_F^2 |V_{UD}|^2$, plus a possible new physics terms, $G_F^2 \epsilon_{NP}$. Since the dimension-six operators are not protected by gauge invariance the possible effects of non-decoupling are proportional to $(1 + M_W^2 / \Lambda_{NP}^2)$. The effects of these non-standard contributions cannot be very large, but possibly detectable in high-precision experiments.

A convenient strategy to measure these effects against the SM parameters, $G_F^2$ and $|V_{UD}|$, rely on the Cabibbo universality hypothesis (or unitarity constraint):

$$G_{CKM}^2 = G_{\mu}^2, \quad \text{[or]} \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \quad \text{and} \quad G_F \equiv G_{\mu}$$

where $G_{CKM}^2 = G_{\mu}^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)$, and $G_{\mu} = 1.166371(6) \times 10^{-5}$ GeV$^{-2}$, as extracted from the accurate measurement of the muon lifetime [2].

We report on the progress related to the verification of the unitarity relation (1), particularly emphasizing the progress in taming the hadronic uncertainties. As we shall see the current
accuracy of the CKM unitarity relation (1), is at the 1 % level (and even less), which therefore becomes an important constraint to the model builders of the beyond SM physics scenarios. For example, in SO(10) grand unification theories, the CKM unitarity relation (1) can be used to set the bound on the mass of $Z'$, namely $m_{Z'} > 1.2$ TeV, which is more than competitive with the one set through the direct collider searches, $m_{Z'} > 720$ GeV [3]. The unitarity also provides a useful constraint in various supersymmetry breaking scenarios [4].

In what follows, I discuss the present status of $|V_{us}|$, as obtained from the studies of semileptonic ($K_{l3}$) and leptonic ($K_{l2}$) decays. I will mainly concentrate on the theoretical progress. For the experimental novelties the reader is encouraged to consult the other contributions of the Flavienet Kaon Working group [1, 5, 6]. I will also present some prospects of making the new physics searches from $K_{l2}$ decays.

2. $|V_{us}|$ and the CKM test: $K_{l3}$

In the SM, we deal with the following master formulas for $K_{l3}$ and $K_{l2}$ decay rates:

$$\Gamma(K_{\ell3}(\gamma)) = \frac{G_F^2 M_K^5}{128\pi^3} C_K S_{\text{ew}} |V_{us}|^2 f_+(0)^2 I_K^\prime(\lambda_{+0}) \left(1 + \delta_{SU(2)}^{\ell} + \delta_{em}^{\ell}\right)^2,$$

(2)

$$\frac{\Gamma(K_{\ell2}^\pm)}{\Gamma(\pi_{\ell2}^\pm)} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(1 - \frac{m_\pi^2/m_K^2}{1 - m_\pi^2/m_\pi^2}\right) \times (1 + \delta_{em})$$

(3)

where $C_K = 1 (1/2)$ for the neutral (charged) kaon decay. $I_K^\prime(\lambda_{+0})$ is the phase space integral which also includes the integration of the shape of the form factors parameterised by $\lambda_{+0}$. The universal short-distance electromagnetic correction, $S_{\text{ew}} = 1.0232(3)$, has been computed at $\mu = M_\rho$ in ref. [7], while the long-distance electromagnetic corrections, $\delta_{em} = 0.9930(35)$ and $\delta_{em}^{\ell}$, as well as the isospin-breaking ones, $\delta_{SU(2)}^{\ell}$, have been recently computed in ref. [8]. The remaining quantities, $f_+(0)$, the vector form factor at zero momentum transfer $[q^2 = (p_K - p_\ell)^2 = 0]$, and $f_K/f_\pi$, the ratio of the kaon and pion decay constants encode the non-perturbative QCD information on the flavour SU(3) breaking effects arising in the relevant hadronic matrix element.

This year, all values for the branching ratios of both neutral and charged $K_{l3}$ decay modes from the new kaon experiments became available [9]. When translated into the uncertainty in $|V_{us}|f_+(0)$, it is only 0.4% for the charged modes and 0.1% for the neutral ones [5]. In average, the uncertainty on $|V_{us}|f_+(0) = 0.21668(45)$ is estimated to be about 0.2%.

For the time being, such a highly precise measurement could not be translated to a similar error on the $|V_{us}|$ determination and therefore to the test of CKM unitarity. The obstacle is obviously the difficulty to keep the theoretical uncertainties in $f_+(0)$ at the per-mil level. Many theoretical approaches have been attempted over the years [14]. Recent progress in lattice QCD gives us more optimism as far as the prospects of reducing the error on $f_+(0)$ to well below 1% are concerned [15]. An important step to resolving this issue has been recently made by the UKQCD-RBC collaboration [12]. Their preliminary result $f_+(0) = 0.961(5)$ is obtained from the unquenched study with $N_F = 2 + 1$ flavors of the quarks which have a good chiral properties on the lattice with finite lattice spacing (so called, Domain Wall quarks), and their pions are about 300 MeV, which is much lighter than what is reported in the previous lattice QCD studies. Their overall error is estimated to be 0.6%, which is very encouraging.

If the RBC-UKQCD estimate of $f_+(0) = 0.961(5)$ is combined with the experimental average of $|V_{us}|f_+(0) = 0.21668(45)$ [1], one gets that the CKM unitarity is confirmed to a precision well below 1% (see fig. 1), which is a new result.

A complementaly research to provide the accurate estimate of $|V_{us}|$ is made through the ratio $\Gamma(K^+ \rightarrow \mu^+\nu)/(\Gamma(\pi^+ \rightarrow \mu^+\nu)$ in eq. (3). As we can see in eq. (3), the QCD uncertainty enters with $f_K/f_\pi$. The novelty are the new lattice results with $N_F = 2 + 1$ dynamical quarks and
Figure 1. Summary of CKM unitarity test. To see the impact of high-precision lattice data, we use $f_+(0) = 0.961(4)$ [12] and $f_K/f_\pi = 1.189(7)$ [17]. As described, all lattice groups are contributing to improve present knowledge.

$pions as light as 280 \text{ MeV}$ [16, 17], obtained by using the so-called staggered quarks. Further improvement is expected soon. PACS-CS Collaboration [19] has recently presented preliminary results for $N_F = 2 + 1$ clover quarks with pion masses of about 200 MeV (with $m_\pi L \sim 2.9$).

From the present knowledge of $f_K/f_\pi$, we see in fig. 1 that $|V_{us}|$ from $K\ell^+\ell^-$ is in agreement with unitarity too. Although this latter value seems to be lower than the $|V_{us}|$ extracted from the $K\ell^3$ decays, the current accuracy does not allow us to draw any conclusion on that issue. Note however that in some models the new-physics indeed affects differently $K\ell^3$ and $K\ell^2$ [20, 21]. Therefore, a deviation of $|V_{us}|_{K\ell^3}/|V_{us}|_{K\ell^2}$ from unity might be a signal of the new physics effects, without breaking the unitarity relation.

3. Future perspectives
Here we briefly summarise the probable perspectives of the $K\ell^3$ and $K\ell^2$ studies.

The lepton-universality searches in $R_K = \Gamma(K \rightarrow \mu\nu)/\Gamma(K \rightarrow e\nu)$, could give us some precious hints on new physics scenarios. On one hand, $R_K$ can be predicted to 0.04 \% accuracy in the SM, while on the other hand the Higgs contributions $[\propto (s_{RUL})(e_{R\nu L})]$, arising from lepton violating couplings at large $\tan\beta$, can give an effect $\sim 1\%$ [22]. Recent measurements of this ratio by NA48 [23] and KLOE [24] reached a percent level of accuracy which makes it very exciting to see what the NA60 [23] experiment will bring as they aim at lowering the uncertainty to the per-mil level.

A possible deviation of $|V_{us}|_{K\ell^3}/|V_{us}|_{K\ell^2}$ from unity, as argued in some models of physics beyond SM, represents another exciting avenue for the future searches [20, 21]. Notice that in this case the hadronic uncertainties enter through $(f_K/f_\pi)/f_+(0)$, which will hopefully be reduced by the future lattice QCD studies with ever lighter pions, as mentioned in the previous section. Moreover, it would be particularly interesting to compute directly $(f_K/f_\pi)/f_+(0)$ on the lattice, i.e., in the same set of simulations.

4. Acknowledgments
FlaviaNet is a Marie Curie Research Training Network supported by the European Union Sixth Framework Programme under contract MTRN-CT-2006-035482.
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