Approximating RR Lyrae light curves using cubic polynomials

S. Reyner, 1* S. M. Kanbur, 2 C. Ngeow 3 and C. Morgan 1

1 Department of Mathematics, SUNY Oswego, Oswego, NY 13126, USA
2 Department of Physics, SUNY Oswego, Oswego, NY 13126, USA
3 Graduate Institute of Astronomy, National Central University, Jhongli City 32001, Taiwan, Republic of China

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ABSTRACT
In this paper, we use cubic polynomials to approximate RR Lyrae light curves and apply the method to Hubble Space Telescope data of RR Lyrae stars in the halo of M31. We compare our results to the standard method of Fourier decomposition and find that the method of cubic polynomials eliminates virtually all ringing effects and does so with significantly fewer parameters than the Fourier technique. Further, for RRc stars the parameters in the fit are all physical in the sense that they can, in principle, be related to pulsation physics. Our study also reveals a number of additional periodicities in this data not found previously: we find 23 RRc stars, 29 RRab stars and three multiperiodic stars.

Key words: stars: fundamental parameters – RR Lyrae.

1 INTRODUCTION

Even though a number of microlensing projects have yielded RR Lyrae light curves with excellent phase coverage (see e.g. Soszynski et al. 2009), phase coverage is more of a problem when studying RR Lyrae stars in external galaxies. In order to overcome this, a number of methods have been developed to use the observed data to get a smooth approximation to the actual RR Lyrae light curve that captures any real physical bumps or dips and eliminates any that are caused by numerics.

The traditional method applied has been Fourier decomposition wherein the observed data are fitted by an expression of the form

\[ m(t) = A_0 + \sum_{k=1}^{N} A_k \sin(k\omega t + \phi_k). \]  

Here \( N \) is the order of the fit, \( A_0 \) is the mean magnitude, \( \omega = 2\pi/P \), \( P \) is the pulsating period and \( t \) is the time of observation. Usually, a least-squares fitting procedure yields \( A_0, A_k \) and \( \phi_k \). A major problem with this technique is ringing: the Fourier curve given by equation (1) exhibits a series of unphysical bumps and dips. This can occur even when the original data are well distributed in phase but exhibit a large scatter. This is exacerbated when there are noticeable gaps in phase coverage. The solution to this is to reduce the order of the fit but then this loses real features of the light curve.

Another method is that of principal component analysis (PCA; see Kanbur & Mariani 2004; Deb & Singh 2009, for examples). Here instead of sine functions being the basis, the data itself determine these basis functions: the resulting light curve is expressed as a sum of these basis functions. PCA has the advantage that a very good approximation of the light curve can be realized with significantly fewer parameters than required by the Fourier method. However, while a stringent comparison of the two methods is beyond the scope of this paper, we can say that, at least in the case of RRc stars, the parameters of a fit using the methods developed here do have some physical interpretation.

Akerlof et al. (1994) studied the method of cubic splines in approximating variable star light curves. A cubic spline is a series of cubic polynomials pieced together such that the intersection of two such polynomials is required to be continuous up to the second derivative. Our method uses cubic polynomials but does not require continuity up to the second derivative. Looking at fig. 4 in Akerlof et al. (1994), the graphs for LCB12 and LCR12, we again see ringing as a result of having 16 parameters; as our results show, this is a clear reason why cubic polynomials as opposed to cubic splines are to be preferred in approximating RR Lyrae light curves.

One motivation for this study is that requiring continuity up to the second derivative seems too stringent. Pulsation shocks are dramatic events with sudden reversals. There is no reason to suppose that any fitted curve is continuously differentiable to the second degree. This is particularly true of fundamental mode RR Lyrae stars at phases close to minimum light where the star suddenly starts to get brighter. In this paper, we examine the use of cubic polynomials to fit the light curves of RR Lyrae stars by requiring continuity only up to the first derivative.

2 PRELIMINARIES

We define \( t \mod P \) (for any real number \( t \) and positive real \( P \)) to be that positive number \( x \) satisfying both \( 0 \leq x < P \) and \( (t - x)/P \) is an integer. We are interested in approximating data points \((t_1, y_1), \ldots, (t_n, y_n)\) by a periodic function \( y = f(t) \) of period \( P \). The residuals from our fit are \( r_i = y_i - f(t_i) \). We define \( PD \) to be the
proportion of the period when the luminosity is decreasing (which, when magnitudes are used, results in the proportion of the period when the $y_i$ are increasing). Let $\bar{y}$ be the average of the $y_i$. Define the total sum of squares, $SST$, to be

$$SST = \sum_{i=1}^{n} [\bar{y} - f(t_i)]^2,$$

and the error sum of squares, $SSE$, as

$$SSE = \sum_{i=1}^{n} r_i^2.$$

The quantity $R^2$ is defined as

$$R^2 = \frac{(SST - SSE)}{SST}$$

and is the portion of the variation of the $y_i$’s explained by our model $f(t)$. The adjusted $R^2$ (adj), denoted by $RA$, is

$$R^2(adj) = 1 - \frac{(n - 1)(1 - R^2)/(n - r - 1)}{SST},$$

where there are $r$ parameters. Then the best fit is from that combination of parameters such that $SSE$ is a minimum.

For any function which is twice differentiable except for a finite set of points, we define the total bending (TB) as follows: divide the domain $(0, P)$ if periodic with period $P$ into intervals so that on each interval, the function is only concave up or down. On each interval, the bending is the angle between the tangent lines at the two endpoints (not through the vertical) and at each non-differentiable point, the angle between the left- and right-sided tangent lines. TB is the sum of all these angles. This provides a good measure of “ringing”.

We use an $F$ test (Weisberg 1980) to test for the significance of having a more or less complex model. This $F$ statistic is

$$F = \frac{[SSE(N) - SSE(A)]/[d.o.f.(N) - d.o.f(A)]}{SSE(A)/d.o.f.(A)},$$

where $SSE$ is the error sum of squares and $N$ and $A$ stand for null hypothesis (less complex model) and alternative hypothesis (more complex model), respectively. The expression d.o.f. stands for degrees of freedom which is the number of data points minus the number of parameters.

### 3 OUR APPROXIMATION

Our goal is to obtain good approximations which reflect the true shape of the light curve, yet are simple and without ringing or other anomalies. Our method is based on the observation that an increasing or decreasing portion of the $y(t)$ function can be remarkably well approximated by a cubic polynomial. We also note that a cubic polynomial is uniquely determined by the $y$ coordinate and the slope at two points.

RR Lyrae light curves come in basically two varieties: types $ab$ and $c$ corresponding to fundamental and overtone modes, respectively. For the overtone $c$ type, both increasing and decreasing portions are roughly half the period of a sine curve (of different periods) and each can be approximated by a cubic. For the fundamental $ab$ type, the increasing portion is roughly half of a period of a sine curve, while the decreasing part is similar to the bottom half of the decreasing half of a sine curve, though near the minimum, there is a noticeable dip before it starts to increase again.

In this paragraph, we outline the method in general terms and describe the details in the following paragraphs. Essentially approx-imate RR Lyrae light curves by either two cubics or three cubics. When we use two cubics, for example when trying to model RRc curves, the parameters of our fit are the period, shift (phase point at which the first observation occurs), the maximum and minimum, the proportion of the curve that is decreasing and the slope at maximum, that is a total of six parameters. In this case, it is clear that these parameters have a physical meaning. We choose these parameters in order to minimize the $SSE$. In this case, the fitted curve is continuous as is its derivative except perhaps at maximum. When we use three cubics, we choose four points with the understanding that the first and last points are the same and three corresponding time intervals. Note the sum of these three time intervals is the period. We have the $x$ and $y$ values at these points and the slopes. This is a total of nine parameters together with the shift as defined in the case of using two cubics. We choose these 10 parameters to minimize the $SSE$ as before.

With this in mind, we define two different piecewise-defined functions and their periodic extensions. $S_1(t)$ is defined on $0 \leq t \leq T_1$ to be cubic which passes through the points $(0, M)$, and $(T_1, m)$, with derivative equal to $D$ at $t = 0$ and with zero derivative at $T_1$. Here $M$ and $m$ are the maximum and minimum, respectively, of the curve to be fitted. Furthermore, $S_1(t)$ is defined on $T_1 \leq t \leq P$ to be that cubic which passes through the points $(T_1, m)$ and $(P, M)$, with derivative equal to zero at $t = T_1$ and $P$. The periodic extension $S_1(t \mod P)$ is said to be of type $2C$. This is continuous since $S_1(0) = S_1(P)$. Note that for sinusoidal curves, $D$ will equal 0 ($2C$ will be differentiable) while for fundamental $ab$-type curves, $D$ will be positive and $2C$ will not be differentiable. In all that follows, when we approximate a type $c$ by $2C$, we require $D = 0$ and denote this by $2C - 0$. Our parameters here are period, shift (how far into the period the first data point is), $D, m, M$ and $PD$. Each, again of these parameters has some physical meaning. Similarly $S_2(t)$ is piecewise defined by dividing $[0, P]$ into three intervals, using a cubic on each and requiring continuity and differentiability where any two cubics meet. As before, its periodic extension is of type $3C$. Our parameters here are period, shift, the two time values where continuity is required between two cubics, three $y$ values and three slopes. Because of the simplicity of both $2C - 0$ and $3C$, there is no ringing. Consequently, no noise is added when obtaining residuals. Furthermore, having fewer parameters improves the power of an $F$ test.

In practice, we compute the mean and standard deviation for the original data. Points beyond 2.5 standard deviations away from the mean are considered outliers and omitted. We need good initial guesses to best approximate out data. For $2C$, our initial approximation comes from the Fourier series of seven terms and $D = 0$. We use $T_1 = PD \times P$ and replace each data point $(t_i, y_i)$ by $(t_i - \text{shift, } y_i)$. We are then in a position to minimize $SSE$. From the Fourier fit, we find the maximum as the point (shift, $M$), followed by the minimum as the point (shift + $T_1$, $m$). For $3C$, our initial approximation uses $2C$ and we define $T_0$ to be between 60 and 90 per cent of $T_1$ (see later) and use the intervals $0 \leq t \leq T_0, T_0 \leq t \leq T_1$ and $T_1 \leq t \leq P$ for our three cubics. $P$ is obtained by maximizing the power function. We obtain both the $y$ coordinate and slope at $0, T_0, T_1$ from $2C$ from which we obtain $3C$. For $3C$, we define $T_0$ as 60, 70, 80 and then 90 per cent of $T_1$. For each case, we find the minimum $SSE$ and keep the best (smallest $SSE$) set of parameters. We continue our removal of outliers. For the residual, removing outliers based on the Fourier series is not appropriate because of ringing. We look at residuals using both $2C$ and $3C$. For each, we compute the mean and standard deviation. We remove any points more than 3 standard deviations out using both criteria. When
comparing various approximations, we compare all on this same final data set.

We minimize $SSE$ by looping through the parameters with successively smaller step sizes, $s$. For a fixed parameter, in addition to having the current value of $SSE$, we evaluate $SSE$ at this parameter plus $s$ and at this parameter minus $s$. Finally we fit these three points by a quadratic polynomial, find its minimum and evaluate $SSE$ at this point. Of the current four estimates of our parameter, we select the one giving the smallest $SSE$ as the parameter’s new value. We continue this until we have a good approximation of a relative minimum.

4 TEST DATA

We first test our method on a known function, where the known function is taken from the RR Lyrae light-curve templates developed by Layden (1998). We draw a random number of points from this function and independently add Gaussian noise to each phase point. Specifically, we add noise normally distributed with mean zero and standard deviation 0.1 to the synthetic data. In our tests, these synthetic data are not a ‘train’ of data but just cover approximately one period. We then try to reproduce the original curve using our cubic polynomial method and the traditional Fourier technique. Fig. 1 presents our results. Here the known function (chosen to resemble a typical RRab-type light curve) is the dashed curve. The open squares with error bars are the points drawn randomly from this function and to which Gaussian noise has been added independently. The solid dark lines represent the Fourier and cubic polynomial fits (top and bottom panels, respectively) to these open square points. The curves are plotted as a function of phase, going from 0 to 1. However, we allow the two methods to ‘rediscover’ this periodicity. The Fourier fit of the order of 6 yielded considerable ringing, a period of 1.14 and had an $SSE$ of 0.027. We emphasize that by a period of 1.14, we do not mean a period of 1.14 d, but that this signifies a change in the period as reported by the Fourier method when compared to the period of the original template curve from which the data were drawn. A reported period of 1 signifies no difference between the estimated and original periods. Approximating by a pair of cubics (2C), we obtained a slightly different period of 0.997. $D = 2.93$, $PD = 0.905$ and $SSE = 0.050$. These values of $D$ and $PD$ both indicate Bailey-type $ab$. Using a 3C (differentiable) approximation, we obtained a period of 1.000 04 and $SSE = 0.04$. We see that our method does a very good job at mimicking the known function and has little to no ringing. In contrast, a Fourier fit to the same points produces noticeable and significant ringing. Further, the period obtained by the cubic polynomial method matches exactly that of the original curve.

Next, we tested our method on real data. The data were taken from Brown et al. (2004) and consist of Hubble Space Telescope (HST) observations of RR Lyrae stars in the Andromeda halo – along the south-east minor axis of M31, about 51 arcmin from the nucleus. The data are available at two wavelengths, $F606W$ and $F814W$. In what follows, we report results based on both bands ($F606W$ and $F814W$ are referred to as the first and second bands, respectively). Brown et al. (2004) used a fast algorithm based on the Lomb–Scargle periodogram (Scargle 1982) to search for periodicities in their time series data after data reduction and photometry. Brown et al. (2004) analysed these data and found 169 variables of which 55 were clearly RR Lyrae stars. Of these 55 stars, Brown et al. (2004) classified 29 as RRab, 25 as RRc and one as RRd. It is the data for these 55 stars which we discuss in this paper. Note that we start with the photometry for these 55 stars as published in Brown et al. (2004). As pointed out by the referee, intrinsic precision in this data set is about 0.03 and 0.04 in $V$ and $I$, respectively. This measurement error is not accounted for in our fits but for the purposes of this paper it is appropriate since we are presenting a differential comparison between our method and that of Fourier series. Our data are not corrected for reddening. Brown et al. (2004) found a ratio of RRc to RRabc of 0.46 and mean periods of RRc and RRab stars of 0.316 and 0.594 d, respectively. In Figs 2–4, the label ‘normalized magnitude’ just refers to the fact that the magnitudes are scaled to lie between 0 and 1.

We note that these are HST data and as such somewhat immune to the 1-d aliasing problems arising when RR Lyrae stars are observed from the ground. The sampling rate was 250 exposures over a 41-d

**Figure 1.** Results of fitting to data (open squares) drawn from a known (dashed curves) function using Fourier (top panel, solid curve) and cubic polynomial (bottom panel, solid curve) methods.

**Figure 2.** Results for RRc star V100: open squares are data, thin and thick dashed curves represent 2C and Fourier, respectively, and the solid curve represents 3C. The y-axis is scaled such that the range of magnitudes increases from 0 to 1.
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5 RESULTS

A major finding of our work is that when using the cubic polynomials method on the data set mentioned, we find 23 RRc stars with a mean period of 0.312 and 29 RRab stars with a mean period of 0.594. This leads to a ratio of RRc to RRabc stars very similar to previous work: 0.442. This ratio is lower than that reported in Brown et al. (2004) because there were two RRc stars which were reclassified as RRd in our work. In almost all cases, the periods discovered by our method is very close to those published by Brown et al. (2004). An intriguing result is that our method reveals significantly more multimode stars than previously discovered, and we discuss this later in this section.

First, overtone stars all have periods less than 0.39 while fundamental mode stars have periods greater than 0.44. A nice result is that the type c stars all have $0.51 < PD < 0.76$ while all type ab stars have $PD > 0.76$. We observe three other imperfect tests for the Bailey type. Type c has $D$ (first band) less than 0.76 except for V76 and V58. $PD$ (second band) is less than 0.76 except for V120 and $D$ (second band) is less than 0.25 except for V40 and V120. Also, type ab has $D$ (first band) greater than 0.25 except for V78, $PD$ (second band) greater than 0.76 except for V122 and $D$ (second band) is greater than 0.25 except for V66, V71 and V82. In every case, both bands have $PD$ greater than 0.51 with two exceptions, both of which are in the second band where there is generally more noise. For V76, $PD = 0.36$ while for V157, $PD = 0.48$. Generally, the $PD$s of the two bands correlate nicely, as do the $Ds$. Further, type c all have $D < 0.2$ while all type ab except V78 have $D > 0.5$. This provides a good way to distinguish between types ab and c. Type c can be approximated about equally well by a Fourier series of the order of 2–4, or 2C or 3C. The advantages of using 2C are its simplicity, minimal TB and using only parameters that have physical meaning. Further, we can generate $PD$ and $D$ whose importance has already been established.

The 23 type RRc data sets can be summarized on average as follows. We denote the average RA for 2C and 3C as $R_{2c}$ and $R_{3c}$, respectively. A similar quantity for an order 2 Fourier series is labelled as $R_{f2}$. Likewise, $TB$ for 2C is named as $TB_{2c}$ and so on. We have $R_{2c} = 0.906$, $R_{3c} = 0.907$ and $R_{f2} = 0.902$ while $TB_{2c} = 5.2$, $TB_{3c} = 8.2$ and $TB_{f2} = 5.2$. While 3C may give slightly better approximations, the extra bending strongly suggests that it is not worth the effort. Fourier series give somewhat worse approximations than 2C with no reduction in bending. Finally, 2C is simplest and only involves parameters with physical meaning which clearly makes it superior. Fig. 2 displays a typical example of type c using star V100. For this star, $R_{2c} = 0.951$, $R_{3c} = 0.951$, $R_{f2} = 0.950$, $TB_{2c} = 5.5$, $TB_{3c} = 10.5$ and $TB_{f2} = 5.5$.

The 29 type ab stars can be summarized on average as follows. Using similar nomenclature to that specified for the RRc stars above,
we have $R_{2ab} = 0.954$, $R_{3ab} = 0.962$ and $R_{8ab} = 0.962$ while $TB_{2ab} = 5.4$, $TB_{3ab} = 8.4$ and $TB_{8} = 16.4$. Since $C$ gives about as good an approximation as Fourier series with eight terms but with much less bending and fewer parameters, we prefer $3C$. Comparing $R_{2ab}$ to $R_{3ab}$ we see that $RA$ has gone from $0.954$ to $0.962$, which means the ratio of errors $(1 - R_{2ab})/(1 - R_{3ab}) = 1.21$, a 21 per cent reduction in error, so $3C$ is clearly superior. The increase in bending simply means that we are twisting more to fit the data, as can be seen especially well in Figs 1–3. Figs 1–3 are typical examples of the sort of approximations possible with cubic polynomials. Further, Fig. 3 presents typical examples of type $ab$ using V57 and V136. V57 (left-hand panel of Fig. 3) is an example (of six or seven stars) where the decreasing portion seems to momentarily increase before a final dip to the minimum, while V136 (right-hand panel of Fig. 3) does not show this behaviour. For V57 we have $R2 = 0.953$, $R3 = 0.969$, $R8 = 0.970$, $TB2 = 5.8$, $TB3 = 10.4$ and $TB8 = 16.2$ and for V136 we have $R2 = 0.933$, $R3 = 0.937$, $R8 = 0.938$, $TB2 = 4.7$, $TB3 = 7.1$ and $TB8 = 13.8$.

The proportion decreasing using $3C$ is not the same as $PD$ (using $2C$). However, it is usually within 0.001. The periods obtained by $2C$ and $3C$ are usually within 0.0001, while they differ from the period using Fourier series by perhaps 0.001. Worse yet, the optimal period for a Fourier series depends on the order of the series.

There are three stars, listed in Table 1, which have two periods. These were analysed as follows. We removed outliers in the data as before. We then fit by $2C$ or $3C$ or $0$ depending on type and removed outliers based on their residuals. This gave an initial period equal to $pr1$, $RA = RA1$ and $SSE = SSE1$. We then obtained the next period and its power as $pwr$. We subsequently refit by $2C - 0$ if necessary, and then we fit the residuals by another $2C - 0$ and finally obtained a combined best fit (with both $D = 0$). This gives $pr2$, $RA2$ and $SSE2$, from which we calculated an $F$ statistic. These are all listed in Table 1. For each star, the first and second lines correspond to the first ($F606W$) and second ($F814W$) bands, respectively. The $F$ statistic tells us that we are more than 99.95 per cent certain that the second $2C - 0$ is significant and so the second period is significant. A different approach is based on Scargle’s analysis. Using his equation (18), if it is possible to select $N$ possible periods a priori, then in order to conclude that, with greater than 99 per cent certainty, the best one is valid, the power for more than one must exceed the threshold $-\ln [1 - 0.99^{1/N}]$. However, there is no way of knowing in advance what the true period is. If we assume that it lies between 0.250 and 0.800 and round to three decimal places, there are 551 possibilities which give a threshold of 10.9. Each of the powers listed in Table 1 is above this except for V95 in the second band. If a second period is present, it should be present in each band. We conducted an extensive search in the first band which has higher amplitude and then checked some periods near the predicted period – the second power was conclusive. For a number of stars, using only one band, we discovered that we could obtain a much better approximation using two periods differing by only about 0.001. We assume that these are anomalies and ignore them though more data might lead to different results. Several stars had significant evidence of a second period in one band but not in the other and these were ignored. These stringent criteria leave the three stars in Table 1 with two periods. In each case, the period ratio is 0.75. Fig. 4 displays a graph of $V1$ over four primary periods which equal three secondary periods. This also presents the interaction of the two periods. These three stars have primary periods between 0.353 and 0.383.

### 6. Maximum and Minimum Light Colours

Recent work has focused on the properties of RR Lyrae stars at minimum light that have a possible way to estimate reddening (Preston 1959; Lub 1977; Clementini et al. 1995; Kanbur & Fernando 2005). The theoretical basis for this has been established by Simon, Kanbur & Mihalas (1993), Kanbur (1995) and Kanbur & Phillips (1996). These authors showed the importance of period–colour (PC) relations at maximum light. Cepheids have flatter PC relations at maximum light and definite relations at minimum light such that higher amplitude Cepheids are driven to cooler and hence redder colours. In the case of RR Lyrae stars, this is reversed with a flat PC relation at minimum light and a discernable relation at maximum light. Fig. 5 presents PC relations at maximum and minimum light for the Brown et al. (2004) data calculated using both Fourier series (open circles) and cubic polynomials (solid squares) to approximate the data. First, we see broad support for the contention that PC relations at minimum light are much flatter than those at maximum light.

![Figure 5. PC results at maximum (top) and minimum (bottom) light using a sixth-order Fourier fit (open circles) and cubic polynomials (solid squares).](image-url)
maximum light. Secondly, as pointed out by the referee, we note that somewhat tighter and flatter relations are present for the PC relation at minimum light when using a cubic polynomial fit.

7 CONCLUSIONS

We have found a new way to approximate the light curve of an RR Lyrae star by fitting cubic polynomials to the data. This method can fit the data with fewer parameters than Fourier series and suffers virtually no ringing. It can also estimate periodicities in the data. When we apply this method to RR Lyrae data in the Andromeda halo, we find, in addition to the multiperiodic star V90 reported by Brown et al. (2004), an additional two other multiperiodic stars (V1 and V95, previously classified as type RRc) in the data sample; here we require this multiperiodicity to be present in both bands. Then the ratio of the number of RRc stars to the number of RRabc stars is 0.442, as opposed to Brown et al. (2004) who found a ratio of 0.462. In this ratio, Brown et al. (2004) do not count RRd stars in either the numerator or the denominator. The ratio of the number of RRc to the number of RRab where we include RRd stars together with the RRc stars is in this case 0.473.

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