Gluon polarisation in the proton

Steven D. Bass
Institute for Theoretical Physics, Universität Innsbruck, Technikerstrasse 25, Innsbruck, A 6020 Austria

Andrew Casey and Anthony W. Thomas
CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

We combine heavy-quark renormalisation group arguments with our understanding of the nucleon’s wavefunction to deduce a bound on the gluon polarisation $\Delta g$ in the proton. The bound is consistent with the values extracted from spin experiments at COMPASS and RHIC.

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Polarised deep inelastic scattering experiments have revealed a small value for the nucleon’s flavour-singlet axial-charge, $g_A^{(0)}|_{pDIS} \sim 0.3$, suggesting that the quarks’ intrinsic spin contributes little of the proton’s spin. The challenge to understand the spin structure of the proton has inspired a vast programme of theoretical activity and new experiments. Why is the quark spin content $g_A^{(0)}|_{pDIS}$ so small? How is the spin $\frac{1}{2}$ of the proton built up from the spin and orbital angular momentum of the quarks and gluons inside?

A major topic of investigation has been the role of polarised glue in the nucleon, both in terms of its contribution to the nucleon’s spin and possible supression of the nucleon’s singlet axial-vector charge through the QCD axial anomaly. Key experiments to measure gluon polarisation are COMPASS at CERN and PHENIX and STAR at RHIC. In this paper we investigate gluon polarisation via the charm-quark axial-charge, matching the results of heavy-quark renormalisation group with what we know of the proton’s wavefunction. We suggest a bound $|\Delta g(m_p^2)| \lesssim 0.3$, which is consistent with the results of the present experiments.

We start by recalling the $g_1$ spin-sum-rules, which are derived from the dispersion relation for polarised photon-nucleon scattering and, for deep inelastic scattering, the light-cone operator product expansion. At leading twist the first moment of the $g_1$ spin structure function measures a linear combination of the nucleon’s scale-invariant axial-charges $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}|_{inv}$ plus a possible subtraction constant $\beta_\infty$ in the dispersion relation $\Pi$:

$$\int_0^1 dx \, g_1(x, Q^2) = \left( \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) c_{NS}(\alpha_s(Q^2)) + \frac{1}{9} g_A^{(0)}|_{inv} \frac{1}{\alpha_s(Q^2)} + \beta_\infty. \tag{1}$$

Here $c_{NS}$ and $c_S$ are the non-singlet and singlet Wilson coefficients. In terms of the flavour dependent axial-charges $2M_s \Delta q = \mu \gamma \gamma \gamma \gamma s(\gamma \gamma \gamma \gamma s, s)$ the isovector, octet and singlet axial charges are $g_A^{(3)} = \Delta u - \Delta d$, $g_A^{(8)} = \Delta u + \Delta d - 2\Delta s$ and $g_A^{(0)}|_{inv}/E(\alpha_s) \equiv g_A^{(0)}(Q^2) = \Delta u + \Delta d + \Delta s$. Here $E(\alpha_s) = \exp \int_0^{\alpha_s} d\alpha_s (\alpha_s/G(\alpha_s))/\beta(\alpha_s)$ is a renormalisation group factor which corrects for the (two loop) non-zero anomalous dimension $\gamma(\alpha_s)$ of the singlet axial-vector current $\bar{q} \gamma \gamma \gamma \gamma \gamma s$. $J_{\mu \delta} = \bar{u}_\mu \gamma \gamma \gamma \gamma d + \bar{d}_\mu \gamma \gamma \gamma \gamma s$, which is close to one and which goes to one in the limit $Q^2 \to \infty$; $\beta(\alpha_s) = - \left( 11 - \frac{2}{3} f(\alpha_s/2\pi) + ... \right)$ is the QCD beta function and $\gamma(\alpha_s) = f(\alpha_s/\pi)^2 + ...$ where $f$ is the number of active flavours. The singlet axial charge, $g_A^{(0)}|_{inv}$, is independent of the renormalisation scale $\mu$ and corresponds to $g_A^{(0)}(Q^2)$ evaluated in the limit $Q^2 \to \infty$. The flavour non-singlet axial-charges are renormalisation group invariants.

The isovector axial-charge is measured independently in neutron $\beta$-decays ($g_A^{(3)} = 1.270 \pm 0.003$ [5]) and the octet axial charge is commonly taken to be the value extracted from hyperon $\beta$-decays assuming a 2-parameter SU(3) fit ($g_A^{(8)} = 0.58 \pm 0.03$ [5]). The uncertainty quoted for $g_A^{(8)}$ has been a matter of some debate [10, 11]. Indeed, prompted by the work of Myhrer and Thomas [12], which showed that the effect of the one-gluon-exchange hyperfine interaction [13] and the pion cloud [14] of the nucleon was to reduce $g_A^{(0)}$ calculated in the cloudy bag model to near the experimental value, a re-evaluation of these effects on $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)}|_{inv}$ including kaon loops led to the value $g_A^{(8)} = 0.46 \pm 0.05$ [15]. Here the reduction from the SU(3) value came primarily from the pion cloud.

Assuming no twist-two subtraction constant, polarised deep inelastic scattering experiments have been interpreted in terms of a small value for the flavour-singlet axial-charge: $g_A^{(0)}|_{pDIS}, Q^2 \to \infty = 0.33 \pm 0.03$ (stat.) $\pm 0.05$ (syst.) [16] if one uses the SU(3) value for $g_A^{(8)}$. On the other hand, using the value $g_A^{(8)} = 0.46 \pm 0.05$ from SU(3) breaking, the corresponding experimental value of $g_A^{(0)}|_{pDIS}$ would increase to $g_A^{(0)}|_{pDIS} = 0.36 \pm 0.03 \pm 0.05$. In the naive parton model $g_A^{(0)}|_{pDIS}$ is interpreted as the fraction of the proton’s spin which is carried by the intrinsic spin of its quark and antiquark constituents.
Historically, the wish to understand the suppression of \( g_A^{(0)} \) relative to \( g_A^{(8)} \), led to considerable theoretical effort to understand the flavour-singlet axial-charge in QCD. QCD theoretical analysis leads to the formula [1,17,20]

\[
g_A^{(0)} = \left( \sum_q \Delta q - \frac{3\alpha}{2\pi} \Delta g \right)_{\text{partons}} + C_\infty. \tag{2}
\]

Here \( \Delta q_{\text{partons}} \) is the amount of spin carried by polarised gluons in the polarised proton \( (\alpha, \Delta g \sim \text{constant as } Q^2 \to \infty) \) and \( \Delta q_{\text{partons}} \) measures the spin carried by quarks and antiquarks carrying “soft” transverse momentum \( k_T^2 \sim P^2, m^2 \) where \( P \) is a typical gluon virtuality and \( m \) is the light quark mass. The polarised gluon term is associated with events in polarised deep inelastic scattering where the hard photon strikes a quark or antiquark generated from photon-gluon fusion with \( k_T^2 \sim Q^2 \) [19, 20]. It corresponds to the QCD axial anomaly in the flavour-singlet axial-vector current. \( C_\infty \) denotes a potential non-perturbative gluon topological contribution [1] associated with the possible subtraction constant in the dispersion relation for \( g_A \) and possible spin contributions at Bjorken \( x = 0 \), that is outside the range of polarised deep inelastic scattering experiments. The measured singlet axial-charge is \( g_A^{(0)} \mid_{\text{DIS}} = g_A^{(0)} - C_\infty \).

In the parton model \( \Delta q_{\text{partons}} \) is associated with the forward matrix of the partially conserved axial-vector measured singlet axial-charge is \( \Delta \). The different contributions involving experiments in semi-inclusive polarised deep inelastic scattering and polarised proton-proton collisions [2,23].

Heavy-quark axial-charges have been studied in the context of elastic neutrino-proton scattering [24,25] and heavy-quark contributions to \( g_1 \) at \( Q^2 \) values above the charm production threshold [26,29]. Charm production in polarised deep inelastic scattering is an important part of the COMPASS spin programme at CERN 30.

Following Eq.(2) we can write the charm-quark axial-charge contribution as

\[
\Delta c(Q^2) = \Delta c_{\text{partons}} = \left\{ \frac{\alpha}{2\pi} \Delta g \right\}_{Q^2, f=4} \tag{4}
\]

where \( \Delta c_{\text{partons}} \) corresponds to the forward matrix element of the plus component of the renormalisation group invariant charm-quark axial-current with just mass terms in the divergence (minus the QCD axial anomaly), viz. \( (\bar{c}c \gamma_5 c)_{\text{con}} = (\bar{c}c \gamma_5 c) - k_\mu \) with \( k_\mu \) the gluonic Chern-Simons current, and we neglect any topological contribution \(^1\). For scales \( Q^2 \gg m_c^2, \Delta c_{\text{partons}} \) corresponds to the polarised charm contribution one would find in the JET or AB factorisation schemes.

The heavy-quark contributions to the non-singlet neutral current axial-charge measured in elastic neutrino-proton scattering have been calculated to NLO in Ref. 24. For charm quarks, the relevant electroweak doublet contribution (at LO) is

\[
(\Delta c - \Delta s)_{\text{inv}}^{(f=4)} = -\frac{6}{27\pi} \alpha_s^{(3)}(m_c^2) g_A^{(0)} \mid_{\text{inv}}^{(f=3)} - \Delta s_{\text{inv}}^{(f=3)} + O(1/m_c^2). \tag{5}
\]

For the LO contribution this is made up from

\[
\Delta s_{\text{inv}}^{(f=3)} = \Delta s_{\text{inv}}^{(f=3)} + \frac{6}{27\pi} \alpha_s^{(3)}(m_c^2) g_A^{(0)} \mid_{\text{inv}}^{(f=3)} + O(1/m_c^2),
\]

\[
\Delta c_{\text{inv}}^{(f=4)} = -\frac{6}{27\pi} \alpha_s^{(3)}(m_c^2) g_A^{(0)} \mid_{\text{inv}}^{(f=3)} + O(1/m_c^2). \tag{6}
\]

Here \( \Delta c_{\text{inv}}^{(f=4)} = \Delta c(Q^2) \) evaluated in the limit \( Q^2 \to \infty \), where the charm-quark axial-charge contribution is \( 2M_s \Delta c = (p, s c) \). The change in \( \Delta s_{\text{inv}}^{(f=3)} \) between the 4 and 3 flavour theories in Eq.(6) comes from the different number of flavours in \( E(\alpha_s) \) for the 4 and 3 flavour theories.

Eq.(6) contains vital information about \( \left\{ \frac{\alpha}{2\pi} \Delta g \right\}_{\infty} \) in Eq.(4) if we know the RG invariant quantity \( \Delta c_{\text{partons}} \). Indeed, if the latter were zero and if we ignore the NLO evolution associated with the two-loop anomalous dimension \( \gamma(\alpha_s) \), then Eq. (6) would imply (at NLO):

\[
\Delta g^{(f=4)}(Q^2) = \frac{12}{25} \frac{\alpha_s^{(f=3)}(m_c^2) g_A^{(0)} \mid_{\text{inv}}^{(f=3)}}{(f=4) Q^2}. \tag{7}
\]

\(^1\) Any topological contribution will be associated with some of \( \Delta c_{\text{partons}} \) being shifted to Bjorken \( x = 0 \). In general, topological contributions are suppressed by powers of \( 1/m_c^2 \) for heavy-quark matrix elements [31].
or $\Delta q \sim 0.23$ when $\alpha_s(Q^2) \sim 0.3$. The following discussion is aimed at assessing the possible size of $\Delta c_{\text{partons}}$ plus the NLO evolution associated with $\gamma(\alpha_s)$, and hence the error on this value.

Canonical (anomaly free) heavy-quark contributions to the proton wavefunction are, in general, suppressed by powers of $1/m_c^2$, so we expect $\Delta c_{\text{partons}} \sim O(1/m_c^2)$. The RG invariant quantity $\Delta c_{\text{partons}}$ takes the same value at all momentum scales. We may evaluate it using quark-hadron duality in a hadronic basis with meson cloud methods [32]. Experimental studies of the strange quark content of the nucleon over the last decade have given us considerable confidence that both the matrix elements of the vector and scalar charm quark currents (which are anomaly free) in the proton are quite small [33, 34]. This gives us confidence in estimating the polarised charm contribution through its suppression relative to the corresponding polarised strangeness $\sim 0.01$ [15] by the factor $\sim (m_s - m_N + m_K)^2/4m_c^2 < 0.1$ so that $|\Delta c_{\text{partons}}| < 0.001$. QCD 4-flavour evolution and Eq.(6) then enables an estimate of $\Delta q$ at scales relevant to experiments at COMPASS and RHIC.

In perturbative QCD the LO contribution to heavy-quark production through polarised photon-gluon fusion yields

$$\int_0^1 dx g_1(\gamma^g \to h\bar{h}) \sim 0, \quad Q^2 \gg m_h^2 \quad (8)$$

where $m_h$ is the heavy-quark mass ($h = c, b, t$). The anomalous $-\alpha_s/2\pi$ term is cancelled against the canonical term when $m_h^2 \gg P^2$ (the typical gluon virtuality in the proton) [19]. If the gluon polarisation were large so that $-\alpha_s/2\pi \Delta g$ made a large contribution to the suppression of $g_A^{(0)}$, at this order one would find also a compensating large canonical polarised charm contribution in the proton. To understand this more deeply, we note that the result in Eq.(8) follows from the complete expression [20]

$$\int_0^1 dx \ g_1(\gamma^g) = -\frac{\alpha_s}{2\pi} \left[ 1 + \frac{2m_c^2}{P^2} \ln \left( \frac{1 + \frac{4m_c^2}{P^2}}{1 + \frac{4m_c^2}{P^2} - 1} \right) \right].$$

Here $m$ is the mass of the struck quark and $P^2$ is the gluon virtuality. We next focus on charm production. The first term in Eq.(9) is the QCD anomaly and the second, mass-dependent, canonical term gives $\Delta c_{\text{partons}}^{(\text{gluon})}$ for a gluon “target” with virtuality $P^2$. Evaluating Eq.(9) for $m_c^2 \gg P^2$ gives the leading term $\int_0^1 dx g_1(\gamma^g) \sim -\frac{\alpha_s}{2\pi} \frac{P^2}{m_c^2},$ hence the result in Eq.(8).

It is interesting to understand Eqs.(8-9) in terms of deriving the QCD axial-anomaly via Pauli-Villars regularisation (instead of the usual dimensional regularisation derivation used in [19]). The anomaly corresponds to the heavy Pauli-Villars “quark”, which will cancel against the heavy charm-quark for a charm-quark mass much bigger than gluon virtualities in the problem (there are no other mass terms to set the scale). When the axial-vector amplitude is evaluated at two-loop level there will be gluon loop momenta between $m_c$ and the ultraviolet cut-off scale generating a small scale dependence so that the cancellation between canonical heavy-quark and anomalous polarised glue terms is not exact in full QCD. This scale dependence corresponds to the two-loop anomalous dimension $\gamma(\alpha_s)$ in $E(\alpha_s)$. The result in Eq.(8) was previously discussed in Refs. [10, 27] in the context of the phenomenology that would follow if there were large gluon polarisation in the proton. Non-perturbative evaluation of $\Delta c_{\text{partons}}$ allows us to constrain the size of $\Delta q$ given what we know about charm and strangeness in the nucleon’s wavefunction.

There is a further issue that the derivation of Eq.(6) involves matching conditions where the spin contributions are continuous between the 3 and 4 flavour theories at the threshold scale $m_c$ modulo $O(1/m_c^2)$ corrections, which determine a theoretical error for the method. Using QCD evolution with the renormalisation group factor $E(\alpha_s)$, the results in Eq.(6) are equivalent to the leading twist term $\Delta c_{\text{partons}}^{(m_c^2)}$ vanishing at the threshold scale $m_c$ modulo $O(1/m_c^2)$ corrections, viz. $\Delta c_{\text{partons}}^{(m_c^2)} = O(1/m_c^2)$ [10]. The leading $O(1/m_c^2)$ term is estimated using effective field theory in Refs. [10, 22, 27]. For polarised photon-gluon fusion, this is the $-\alpha_s/2\pi \Delta g$ leading term in the heavy-quark limit of Eq.(9). The heavy charm-quark is integrated out at threshold to give the matrix element of a gauge invariant gluon operator with dimension 5 and the same quantum numbers as the axial-vector current, viz. $\Delta c_{\text{partons}}^{(m_c^2)} \sim O\left(\frac{\alpha_s(m_c^2)}{4\pi} \frac{m_c^2}{2\pi}\right)$ [22] or $\Delta c_{\text{partons}}^{(m_c^2)} \sim O(\alpha_s(m_c^2)\Lambda^2_{\text{QCD}}/m_c^2) \sim 0.017$ [10] [2]. Taking this as an estimate of the theoretical error gives $\Delta c_{\text{partons}}^{(m_c^2)} = 0 \pm 0.017$.

We next combine this number for $\Delta c_{\text{partons}}^{(m_c^2)}$ with our estimate of the canonical charm contribution $|\Delta c_{\text{partons}}^{(\text{gluon})}| < 0.001$ in quadrature to obtain a bound including theoretical error on the size of the polarised gluon contribution: $| -\frac{\alpha_s}{2\pi} \Delta g(m_c^2)| \lesssim 0.017$ or

$$|\Delta g(m_c^2)| \lesssim 0.3 \quad (10)$$

with $\alpha_s(m_c^2) = 0.4$. Values at other values of $Q^2$ are readily obtained with Eq.(3) or $\alpha_s \Delta g \sim \text{constant}$ for large values of $Q^2$.

2 These $O(1/m_c^2)$ terms associated with the full $\Delta c$ are manifest in polarised photon-gluon fusion through the heavy-quark limit of Eq.(9), $\int_0^1 dx g_1(\gamma^g) \sim -\frac{\alpha_s}{4\pi} \frac{P^2}{m_c^2}$, and are to be distinguished from the model evaluation of $\Delta c_{\text{partons}}$. 

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### Footnotes

2 These $O(1/m_c^2)$ terms associated with the full $\Delta c$ are manifest in polarised photon-gluon fusion through the heavy-quark limit of Eq.(9), $\int_0^1 dx g_1(\gamma^g) \sim -\frac{\alpha_s}{4\pi} \frac{P^2}{m_c^2}$, and are to be distinguished from the model evaluation of $\Delta c_{\text{partons}}$. 

It is interesting to extend this analysis to full 6-flavour QCD. The values of $\Delta c^{(f=6)}$, $\Delta f^{(\ell=6)}$ and $\Delta \pi^{(\ell=6)}$ were derived in Ref. [23] to NLO in the heavy-quark expansion. Taking just the leading-order contribution plus the heavy-quark power correction according to the recipe [10, 24] described above gives $\Delta c^{(f=5)}(m_h^2) = -0.006 \pm 0.017$ and $\Delta c^{(f=6)}(m_h^2) = -0.009 \pm 0.017$ for polarised charm. For the bottom and top quarks one obtains $\Delta f^{(\ell=5)}(m_h^2) = 0 \pm 0.001 \pm 0.017$, $\Delta f^{(\ell=6)}(m_h^2) = -0.003 \pm 0.001 \pm 0.017$ and $\Delta t(m_h^2) = 0 \pm 2 \times 10^{-7} \pm 0.017$. Here the first error comes from the $O(1/m_h^2)$ mass correction for the heaviest quark of $c, b, t$. The second error comes from the other heavy-quarks with lesser mass as we evaluate these heavy-quark contributions in terms of the measured value of the light-quark quantity $g^{(0)}_{\pi \alpha}$. These numbers overlap with a zero value for $\frac{\Delta x}{2 \pi} \Delta g$ in the relevant $f$-flavour theories. The QCD scale dependence of $\frac{\Delta x}{2 \pi} \Delta g$ starts with NLO evolution induced by Kodaira’s two-loop anomalous dimension $\gamma_\alpha$. The combination $\frac{\Delta x}{2 \pi} \Delta g$ is scale invariant at LO. This means that if we work just to LO and $\Delta g$ vanishes at one scale, it will vanish at all scales (in LO approximation). The LO QCD evolution equation for gluon orbital angular momentum in the proton [22] then simplifies so that $L_\alpha(\infty) = -\frac{1}{4} (16/(16+3f))$. In practice, the two-loop anomalous dimension generates slow evolution of $\frac{\Delta x}{2 \pi} \Delta g$. Dividing the finite value of this combination at large scales by the small value of $\alpha_s$ gives a finite value for the gluon polarisation $\Delta g$, which can readily be the same order of magnitude as the gluon total angular momentum (or larger with cancellation against a correspondingly larger gluon orbital angular momentum contribution).

It is interesting that the value of $\Delta g$ deduced from present experiments COMPASS at CERN and PHENIX and STAR at RHIC typically give $|\Delta g| < 0.4$ with $\alpha_s \sim 0.3$ corresponding to $-3 \frac{\Delta x}{2 \pi} \Delta g < 0.06$ [23]. This experimental value is extracted from direct measurements of gluon polarisation at COMPASS in the region around $x_{\text{gluon}} \sim 0.1$, NLO QCD motivated fits to inclusive $g_1$ data taken in the region $x > 0.006$, and RHIC Spin data in the region $0.02 < x_{\text{gluon}} < 0.4$. The theoretical bound, Eq.(10), is consistent with this experimental result.

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[1] S. D. Bass, Rev. Mod. Phys. 77 (2005) 1257.
[2] S. D. Bass, The Spin structure of the proton (World Scientific, 2008).
[3] S. D. Bass, Mod. Phys. Lett. A24 (2009) 1087.
[4] A. W. Thomas, Prog. Part. Nucl. Phys. 61 (2008) 219.
[5] A. W. Thomas, Phys. Rev. Lett. 101 (2008) 102003.
[6] M. Anselmino, A. Efremov and E. Leader, Phys. Rep. 261 (1995) 1; H-Y. Cheng, Int. J. Mod. Phys. A11 (1996) 5109; J. Ellis and M. Karliner, Phys. Rev. D47 (1993) 2962; G. Altarelli, R. D. Ball, S. Forte and G. Ridolfi, Acta Phys. Pol. B 29 (1998) 1145; G. M. Shore, B. Lampe and E. Reya, Phys. Rep. 332 (2000) 1; B. W. Filipponne and X. Ji, Adv. Nucl. Phys. 26 (2001) 1; S. J. Brodsky, Int. J. Mod. Phys. A18 (2003) 1531; S.E. Kuhn, J.-P. Chen and E. Leader, Prog. Part. Nucl. Phys. 63 (2009) 1.
[7] J. Kodaira, Nucl. Phys. B165 (1980) 129.
[8] Particle Data Group: C. Amsler et al., Phys. Lett. B667 (2008) 1.
[9] F. E. Close and R. G. Roberts, Phys. Lett. B316 (1993) 165.
[10] R. L. Jaffe and A. Manohar, Nucl. Phys. B337 (1990) 509.
[11] P. G. Ratcliffe, Czech J. Phys. 54 (2004) B11.
[12] F. Myhrer, A. W. Thomas, Phys. Lett. B663 (2008) 302.
[13] F. Myhrer, A. W. Thomas, Phys. Rev. D38 (1988) 1633.
[14] A. W. Schreiber, A. W. Thomas, Phys. Lett. B215 (1988) 141.
[15] S. D. Bass, A. W. Thomas, Phys. Lett. B684 (2010) 216.
[16] COMPASS Collab. (V. Yu. Alexakhin et al.), Phys. Lett. B647 (2007) 8.
[17] G. Altarelli and G. G. Ross, Phys. Lett. B212 (1988) 391.
[18] A. V. Efremov and O. Teryaev, JINR Report No. E2-88-287.
[19] R. D. Carlitz, J. C. Collins and A. Mueller, Phys. Lett. B214 (1988) 229.
[20] S. D. Bass, B. L. Ioffe, N. N. Nikolaev and A. W. Thomas, J. Moscow Phys. Soc. 1 (1991) 317.
[21] E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D58 (1998) 114028; R. D. Ball, S. Forte and G. Ridolfi, Phys. Lett. B378 (1996) 255.
[22] X. Ji, J. Tang and P. Hoodbhoy, Phys. Rev. Lett. 76 (1996) 740.
[23] G. K. Mallot, hep-ex/0612055.
[24] S. D. Bass, R. J. Cre werth, F. M. Steffens and A. W. Thomas, Phys. Rev. D66 (2002) 031901 (R).
[25] D. B. Kaplan and A. V. Manohar, Nucl. Phys. B310 (1988) 527.
[26] G. Altarelli and B. Lampe, Z. Phys. C47 (1990) 315.
[27] A. V. Manohar, Phys. Lett. B242 (1990) 94.
[28] S. D. Bass and A. W. Thomas, Phys. Lett. B293 (1992) 457.
[29] S. D. Bass, S. J. Brodsky and I. Schmidt, Phys. Rev. D60 (1999) 034010.
[30] COMPASS Collab. (M. Alekseev et al.), Phys. Lett. B676 (2009) 31.
[31] E. V. Shuryak, Phys. Rept. 115 (1984) 151.
[32] W. Melnitchouk and A. W. Thomas, Phys. Lett. B414 (1997) 134.
[33] R. D. Young, A. W. Thomas, Phys. Rev. D81 (2010) 014503.
[34] R. D. Young, J. Roche, R. D. Carlini and A. W. Thomas, Phys. Rev. Lett. 97 (2006) 102002.