Real-Time Topology Detection and State Estimation in Distribution Systems Using Micro-PMU and Smart Meter Data

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Abstract—Topology detection and state estimation in real-time are critical for modern distribution systems management and control. However, the number of sensors in distribution networks is limited, and communication links between switch devices and distribution management systems are not well-established. In this regard, this article proposes mixed-integer quadratic programming formulations to determine the topology of distribution network and estimate distribution system states simultaneously using micro-phasor measurement units and smart meter data. Two approaches based on ac optimal power flow are proposed and analyzed: polar power-voltage (PPV) formulation and rectangular current-voltage (RIV) formulation. The proposed models can identify multiple simultaneous switching actions and different topology configurations including radial and meshed networks. Only measurement data at each time interval are needed to identify the topology and states of the system correctly. The proposed models are tested on a modified IEEE-33 bus system with realistic load data from the Pecan Street Inc. database and a real distribution feeder of an electric utility. The results confirm that both models can identify system topology and states with remarkable accuracy in real-time, whereas the RIV model outperforms the PPV model. Moreover, the impact of missing data on the performance of the model is evaluated.

Index Terms—AC optimal power flow, distribution system, micro-phasor measurement unit (micro-PMU), mixed-integer quadratic programming (MIQP), polar power-voltage (PPV) formulation, rectangular current-voltage (RIV) formulation, smart meter, state estimation, topology detection.

I. INTRODUCTION

The information of distribution network topology and system states are crucial for real-time (RT) operation and control of distribution systems, e.g., volt-Var control, especially for systems with high penetration of distributed energy resources (DERs). In the transmission network, most switches and breakers communicate their status (i.e., open or closed) with energy management system. Hence, the telemetered status of switches is considered as input measurements to topology error processing in the transmission system in order to identify any error in switches and breakers statuses [1]. However, communication links are not installed for the majority of switch devices in the distribution networks, which makes it difficult to maintain an updated network topology information in distribution management systems. Moreover, only limited numbers of sensors are installed in the distribution networks, which provide incomplete

NOMENCLATURE

Sets and Indices

\(\lambda_{a,c}\) Index for bus, \(a, c \in \psi\).
\(\Omega\) Index for micro-PMU, \(r \in \Omega\).
\(\gamma\) Index for measurement.
\(\Upsilon\) Index for switch, \(c \in \Upsilon\).

Parameters and Constants

\(g_{ac}\) Conductance of line connecting bus \(a\) and bus \(c\).
\(b_{ac}\) Susceptance of line connecting bus \(a\) and bus \(c\).

\(E^M_r\) Measured voltage magnitude by micro-PMU \(r\).
\(E^P_r\) Actual voltage magnitude at micro-PMU \(r\) location.
\(\varphi^M_r\) Measured voltage angle by micro-PMU \(r\).
\(\varphi^P_r\) Actual voltage angle at micro-PMU \(r\) location.
\(w_y\) Weight of measurement \(y\).
\(M_i\) Large positive number; \(i = 1, 2, 3, 4\).

Variables

\(P_{a,c}\) Active power flow from bus \(a\) to bus \(c\).
\(Q_{a,c}\) Reactive power flow from bus \(a\) to bus \(c\).
\(E_a\) Voltage magnitude at bus \(a\).
\(\varphi_a\) Voltage angle at bus \(a\).
\(P_{k,a}\) Active demand of load \(k\) at bus \(a\).
\(Q_{k,a}\) Reactive demand of load \(k\) at bus \(a\).
\(\psi^I_e\) Status of switch \(e\).
\(P_G^i\) Active power of generator \(i\) at bus \(a\).
\(Q_G^i\) Reactive power of generator \(i\) at bus \(a\).
\(I_{r,a}\) Real part of current from bus \(a\) to \(c\).
\(I_{c,a}\) Imaginary part of current from bus \(a\) to \(c\).
\(E^r_a\) Real part of voltage at bus \(a\).
\(E^i_a\) Imaginary part of voltage at bus \(a\).
\(I_{q,a}\) Real part of current injection at bus \(a\).
\(I_{q,a}\) Imaginary part of current injection at bus \(a\).
\(\lambda_{a,c}\) Auxiliary variable.
\(\chi_{a,c}\) Auxiliary variable.

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observability of the system for the distribution system operator. Also, integration of DERs may result in more reconfiguration and switching actions in the distribution system. Thus, an efficient distribution system topology processor and state estimation tool is critical for the success of distribution management systems. The purpose of topology processor is to construct the one-line diagram of the network based on the status of switches and breakers for performing state estimation in the system [2]. Topology processor converts a detailed bus section and switch model into an aggregated bus and branch model by identifying the status of switches and breakers in the system [2].

For enhanced reliability, modern distribution systems for urban areas are often designed with a loosely meshed or looped connection between feeders or substations. Even though the system may be operated radially, the loop provides more than one point of interconnection, improves efficiency and reliability, and prevents transmission fault currents from flowing across the distribution system and damaging equipment while reducing load shedding. Moreover, meshed distribution systems exist in many metropolitan areas. Also, networked microgrids are emerging within distribution systems. Recent research has evidenced that weakly-meshed operations may yield significant benefits including improvements in balancing power, losses, voltage profiles, and higher hosting capacity for distributed generation [3], [4].

The transformation of distribution systems from passive to active networks with DERs and meshed or weakly-meshed structures highlights the need for an efficient topology processor. In [5], a model is provided to optimize the sensor placement for topology identification. For a particular location of sensors, this model gives the confidence level of identifying changes in switches status. Residual error obtained from the state estimation is used to identify network topology [6]. A recursive Bayesian approach is employed in [7] to perform state estimation for all possible topologies and identify the topology with the highest probability as the correct network topology. However, the algorithm presented in [7] is not computationally efficient. The reason is that, for any possible topology configurations of a distribution network, this method performs state estimation and then chooses the topology with the highest probability. A topology error detection method based on state estimation is proposed in [1], where the circuit breakers statuses are considered as state variables and telemetered statuses of circuit breakers are incorporated into the model. However, the method proposed in [1] may not be applicable to the distribution networks with a limited number of telemetered switches. Moreover, data-driven approaches for topology processor have been proposed in [8]–[15]. Voltage correlation analysis is utilized in [8] to detect the distribution network topology using graph theory. A graph learning approach is proposed in [9] to reconstruct feeder topologies in distribution systems based on nodal voltage measurements. Smart meters and micro-phasor measurement units (micro-PMUs) have gained a reputation in monitoring power distribution systems [10]. Micro-PMUs provide synchronized measurements of voltage and current phasors [11]. Using the smart meter data for building voltage covariance, a maximum a posteriori probability method is proposed in [12] to identify the topology of a distribution system. Time-series signature verification method for identifying the topology of the distribution network based on measured voltages by micro-PMUs has been initially proposed in [13] and [14], which assumes the same resistance to reactance ratio for all electric grid lines. This method is further developed in [15], in which based on the prior information of switch statuses, a library of signatures is calculated to obtain possible topology configurations. Then, the change in the voltage time series measured by micro-PMUs is compared with the obtained library to detect the change in the topology of the distribution system. The main drawback of [13]–[15] is that the authors assume that the topology change may occur due to only one switching action at each time. Also, the prior information of switch statuses and voltage measured by micro-PMUs are needed to identify the network topology. In this regard, if the load variation is increased or the prior status of switches is obtained wrongly, the topology may not be identified correctly. Furthermore, this method is dependent on three parameter tunings. In [16], a single-shot mixed-integer quadratic programming (MIQP) problem is proposed based on dc power flow assumptions to obtain the circuit breaker statuses at substations. However, the assumptions of dc power flow model are not appropriate for the topology processor in the distribution networks. The distribution network topology processing and state estimation problem is a mixed-integer nonlinear programming (MINLP) problem due to binary variables associated with the status of switches and nonlinear ac power flow equations. A two-stage topology detection method is proposed in [17] based on smart meter data. In the first stage, an initial estimation of the network topology is obtained using linear regression. In the second stage, the Newton–Raphson method is utilized to increase the accuracy of the topology estimation iteratively. An ac power flow-based method is proposed in [18] to identify the radial distribution network topology and injection statistics of missing nodes. A data-driven method is proposed in [19] based on a neural network model for topology identification in a distribution network. A random-forest method is utilized for finding vital features extracted from the trained neural network. However, a large set of data is needed for training the neural network model in [19], and in the lack of enough dispersed dataset, the accuracy of the method may decrease.

A distribution system state estimation method is proposed based on the weighted least square algorithm in [20] considering neutral conductors. The voltage of nodes is considered as a state variable in both polar and rectangular forms. In order to enhance the scalability of the distribution system state estimation method, zero-injection buses and neutral nodes are eliminated in [20] to reduce the network size. However, the method in [20] is a nonlinear model, which may not converge and obtain the global optimal solution. In order to enhance the accuracy of the weighted least square based distribution system state estimation problem, an optimization method based on data-driven is developed in [21]. A neural network based on historical energy generation and load data is trained in [21] to obtain proper initialization points for the Gauss–Newton algorithm. A Bayesian state estimation is proposed in [22] using a deep learning method for an unobservable distribution network. However, data-driven methods in [21] and
The proposed MIQP approaches are based on two ac optimal power flow models, polar power-voltage (PPV) formulation and rectangular current-voltage (RIV) formulation, which are linearized using the iterative first-order approximation of the Taylor series. In order to eliminate nonlinearity due to the inclusion of binary variables associated with the status of switches, the big $M$ technique, which has been used in the authors’ prior work for transmission switching, is leveraged [25]–[29]. The proposed ac optimal power flow models include mixed-integer linear constraints and convex objective functions, which can obtain the global optimal solution via optimization solvers utilizing the branch and bound algorithm to solve MIQP problems.

3) The proposed approaches are able to identify multiple simultaneous switching actions at each time instant in RT without information of switch statuses in prior time intervals. Furthermore, the proposed models are single-shot optimization problems, i.e., they only require measurement data at each time snapshot to identify the topology of the system and estimate system states accurately. In other words, the proposed models do not require any information from previous time intervals (e.g., switch status, voltage measurements) or large historical diverse dataset.

4) The performance of the proposed MIQP-PPV-based model is compared with the method proposed in [12]–[15] in four case studies. It is shown that the proposed topology detection model has higher accuracy in all case studies compared to the method proposed in [12]–[15]. Moreover, the proposed MIQP-PPV- and MIQP-RIV-based topology detection and state estimation approaches are compared considering load’s variability, measurement noises, and simultaneous multiple switching actions using Monte Carlo simulation. It is shown in the result section that the proposed MIQP-RIV-based approach is more accurate and computationally efficient compared to the proposed MIQP-PPV-based approach.

5) A modified IEEE 33-bus distribution system [30] and a large real distribution feeder of a local electric utility in Arizona [31] are utilized to test the proposed models. The proposed MIQP-RIV-based model is suitable and scalable to a large distribution network with high accuracy and simulation time of 3 s. Also, the effect of missing data in the distribution feeder of a local electric utility on the proposed simultaneous topology detection and state estimation model is studied. A missing data estimation approach is also developed to increase the accuracy of the model.

The rest of this article is organized as follows. Sections II and III show PPV-based and RIV-based topology detection and state estimation formulation in the distribution network, respectively. The proposed iterative MIQP-based topology detection and state estimation model is explained in Section IV. In Section V, case studies and simulation results are provided. Finally, Section VI concludes this article.

II. PPV-BASED TOPOLOGY DETECTION AND STATE ESTIMATION MODEL IN DISTRIBUTION NETWORK

This section discusses the PPV-based simultaneous topology detection and state estimation model in a distribution system using micro-PMUs and smart meters data. First, the nonlinear PPV-based model is explained. Second, the formulation of the proposed MIQP-PPV-model is presented.
A. PPV-Based Topology Detection and State Estimation Formulation

The nonlinear ac power flow equations can be formulated in various forms including the PPV or RIV models. In this section, the PPV-based topology detection and state estimation problem in distribution networks is formulated, which is valid for meshed, looped, or radial topology structures. Assume a distribution network with set of buses \( \psi = \{1, 2, \ldots, O\} \) and set of lines \( \Phi = \{1, 2, \ldots, P\} \). Set of micro-PMUs is represented by \( \Omega = \{1, 2, \ldots, N\} \). For the line \( p \in \Phi \), which connects bus \( a \in \psi \) to bus \( c \in \psi \) and is not equipped with a switch (such line can be energized or nonenergized), the nonlinear active and reactive ac power flow equations are defined using (1) and (2) [32], [33].

\[
P^L_{a,c} = E_a^2 g_{ac} - E_a E_c g_{ac} \cos(\varphi_a - \varphi_c)
- E_a E_c b_{ac} \sin(\varphi_a - \varphi_c) \quad \forall (a, c) \in \Phi \tag{1}
\]

\[
Q^L_{a,c} = - E_a^2 b_{ac} + E_a E_c b_{ac} \cos(\varphi_a - \varphi_c)
- E_a E_c g_{ac} \sin(\varphi_a - \varphi_c) \quad \forall (a, c) \in \Phi \tag{2}
\]

Let \( \Upsilon = \{1, 2, \ldots, E\} \) be the set of switches in a distribution system. The active and reactive power flow in the line \( p \in \Phi \) equipped with a switch device \( e \in \Upsilon \) is modeled by including binary variable \( \psi^e \) in (1) and (2) as follows:

\[
P^L_{a,c} = \psi^e L (E_a^2 g_{ac} - E_a E_c g_{ac} \cos(\varphi_a - \varphi_c))
- E_a E_c b_{ac} \sin(\varphi_a - \varphi_c) \quad \forall (a, c) \in \Phi \tag{3}
\]

\[
Q^L_{a,c} = \psi^e L (E_a^2 b_{ac} + E_a E_c b_{ac} \cos(\varphi_a - \varphi_c)
- E_a E_c g_{ac} \sin(\varphi_a - \varphi_c)) \quad \forall (a, c) \in \Phi \tag{4}
\]

where \( \psi^e \) = 0 indicates the switch is open and the line is disconnected, whereas \( \psi^e = 1 \) implies the switch is closed and the line is energized.

The active and reactive power balance constraints at bus \( a \in \psi \) in a distribution network are given by:

\[
\sum_{\forall c \in \psi (a)} P_{1,a,c} = \sum_{\forall c \in \psi (a), c \neq a} P_{a,c} + \sum_{\forall k \in K(a)} P_{k,a} \forall a \in \psi \tag{5}
\]

\[
\sum_{\forall c \in \psi (a)} Q_{1,a,c} = \sum_{\forall c \in \psi (a), c \neq a} Q_{a,c} + \sum_{\forall k \in K(a)} Q_{k,a} \forall a \in \psi \tag{6}
\]

The synchronized voltage magnitude and phase angle measurements provided by the micro-PMUs not only improve the RT monitoring of distribution system, but also provide direct measurement of system states [34]. However, the number of micro-PMUs is limited to only few in distribution systems. To evaluate micro-PMU measurement noise, the total vector error (TVE) index is used [15]. TVE is expressed as a normalized value of the difference between actual and measured phasor values. The micro-PMU voltage phasor measurement \( r \in \Omega \) can be modeled by (7) and (8):

\[
E_r^M = E_r^P + \mu_r \tag{7}
\]

\[
\varphi_r^M = \varphi_r^P + \gamma_r \tag{8}
\]

where \( \mu_r \) and \( \gamma_r \) are Gaussian noises with respect to the TVE index. The PPV-based topology detection and state estimation formulation in the distribution system is proposed as follows:

\[
\min \sum_{y=1}^{Y} w_y (h_y(f) - z_y)^2 \text{subject to (1) – (6)} \tag{9}
\]

where \( z_y \) is measurement value \( y \), \( f \) is a vector of the system states including \( E \) and \( \varphi \), and \( h_y(f) \) is a nonlinear function of system states related to the measurements in a distribution network, which include substation, smart meter, and micro-PMU measurements. Since most switch devices do not communicate their status with the distribution management system in the distribution network, it is assumed that there are no available switch status measurements. The vector \( \Lambda = \{\vartheta_1^l, \vartheta_2^l, \ldots, \vartheta_E^l\} \) represents the network topology.

B. Proposed MIQP-PPV-Based Topology Detection and State Estimation Formulation

The PPV-based distribution network topology detection and state estimation problem in (9) is a MINLP problem. The nonlinear terms are the product of binary variable \( \vartheta_{e,a}^l \) and continuous variables as well as the nonlinear active and reactive ac power flow equations. Such problems can be solved using nonlinear algorithms, which may diverge or obtain local optimal solutions. A MIQP model based on dc power flow is proposed in [16] to determine the breaker statuses at substations. However, the dc power flow model is not suitable for the topology processor in the distribution networks. To cope with such challenges, a MIQP formulation based on a linearized PPV (MIQP-PPV-based) ac power flow model is proposed in this article to determine the topology and states of a distribution system using micro-PMUs and smart meters measurements. To this end, first, the linear approximations of nonlinear active and reactive ac power flow constraints in (3) and (4) are proposed using the iterative first-order approximation of the Taylor series, which are defined in (10) and (11) at the bottom of the next page, where \( E_{a, it-1}, E_{c, it-1}, \) and \( \varphi_{ac, it-1} \) are updated in each iteration of the proposed MIQP-PPV-based model based on the solution of the previous iteration. However, (10) and (11) are still nonlinear as a result of the multiplication of \( \vartheta_{e}^l \) and continuous variables \( E_{a}, E_{c}, \varphi_{a}, \) and \( \varphi_{c} \). To eliminate such nonlinearity, the big \( M \) technique, which has been used in authors’ prior work for topology control, is leveraged [25]–[29]. The nonlinear equations (10) and (11) are linearized using the big \( M \) method as follows:

\[
-M_1 (1 - \psi_e^l) \leq \lambda_{a, it-1}^L - P^L_{a,c} \leq M_1 (1 - \psi_e^l) \tag{12}
\]

\[
-M_1 \psi_e^l \leq P^L_{a,c} \leq M_1 \psi_e^l \tag{13}
\]

\[
-M_2 (1 - \psi_e^l) \leq \lambda_{a, it-1}^L - Q^L_{a,c} \leq M_2 (1 - \psi_e^l) \tag{14}
\]

\[
-M_2 \psi_e^l \leq Q^L_{a,c} \leq M_2 \psi_e^l \tag{15}
\]
where $\chi_{a,c}^L$ and $\lambda_{a,c}^L$ are the linear parts of (10) and (11), i.e., right-hand side of (10) and (11) without $\vartheta^i_e$ multiplier. Based on the proposed linear constraints, if $\vartheta^i_e$ is equal to zero, $P^L_{a,c}$ and $Q^L_{a,c}$ will become zero, and constraints (12) and (14) will not be binding.

The proposed joint PPV-based topology detection and state estimation formulation in the radial, looped, and meshed distribution system is formulated as follows:

$$\min \sum_{y=1}^{N_w} w_y (h_y(f) - z_y)^2 \text{ subject to } (1) - (6), (10) - (15)$$

(16)

where $h_y(f)$ is the linear function of system states associated with the measurements. For nonswitchable lines, multiplier $\vartheta^l_e$ is eliminated from (10) and (11), i.e., $\vartheta^l_e = 1$. For switchable lines, (12)--(15) are considered. The proposed model of (16) is MIQP with convex objective function and mixed-integer linear constraints.

### III. PROPOSED MIQP-RIV-BASED TOPOLOGY DETECTION AND STATE ESTIMATION IN DISTRIBUTION NETWORK

In this section, the linearized RIV-based distribution network topology detection and state estimation problem is proposed based on MIQP formulation. The current flow of nonswitchable line $p \in \Phi$, which connects bus $a \in \psi$ to bus $c \in \psi$, can be obtained using linear constraints (17) and (18) [35].

$$I^r_{a,c} = g_{ac} (E^r_a - E^r_c) - b_{ac} (E^{im}_{a} - E^{im}_{c}) \ \forall (a,c) \in \Phi$$

(17)

$$I^{im}_{a,c} = b_{ac} (E^r_a - E^r_c) + g_{ac} (E^{im}_{a} - E^{im}_{c}) \ \forall (a,c) \in \Phi$$

(18)

If there is a switch device $e \in Y$ on the line $p \in \Phi$, (17) and (18) are modified by considering binary variable $\vartheta^l_e$ associated with the status of the switch as follows:

$$I^r_{a,c} = \vartheta^l_e (g_{ac} (E^r_a - E^r_c) - b_{ac} (E^{im}_{a} - E^{im}_{c})) \ \forall (a,c) \in \Phi$$

(19)

$$I^{im}_{a,c} = \vartheta^l_e (g_{ac} (E^r_a - E^r_c) + b_{ac} (E^{im}_{a} - E^{im}_{c})) \ \forall (a,c) \in \Phi$$

(20)

Constraints (19) and (20) are nonlinear due to the product of binary variable $\vartheta^l_e$ with continuous variables. In order to eliminate such nonlinearity, the big $M$ method is utilized in this article to linearize constraints (19) and (20) as follows:

$$-M_3 (1 - \vartheta^l_e) \leq I^r_{a,c} - \left[ g_{ac} (E^r_a - E^r_c) - b_{ac} (E^{im}_{a} - E^{im}_{c}) \right] \leq M_3 (1 - \vartheta^l_e)$$

(21)

$$I^r_{a,c} - \left[ g_{ac} (E^r_a - E^r_c) - b_{ac} (E^{im}_{a} - E^{im}_{c}) \right] \leq M_3 (1 - \vartheta^l_e)$$

(22)

$$-M_3 \vartheta^l_e \leq I^r_{a,c} \leq M_3 \vartheta^l_e$$

(23)

$$-M_4 (1 - \vartheta^l_e) \leq I^{im}_{a,c} - \left[ b_{ac} (E^r_a - E^r_c) + g_{ac} (E^{im}_{a} - E^{im}_{c}) \right] \leq M_4 (1 - \vartheta^l_e)$$

(24)

$$I^{im}_{a,c} - \left[ b_{ac} (E^r_a - E^r_c) + g_{ac} (E^{im}_{a} - E^{im}_{c}) \right] \leq M_4 (1 - \vartheta^l_e)$$

(25)

$$-M_4 \vartheta^l_e \leq I^{im}_{a,c} \leq M_4 \vartheta^l_e$$

(26)

The current injection constraints at bus $a \in \psi$ of a distribution system are formulated as (27) and (28).

$$I^r_{a} = \sum_{c \in \delta(a)} I^r_{a,c} \ \forall a \in \psi$$

(27)

$$I^{im}_{a} = \sum_{c \in \delta(a)} I^{im}_{a,c} \ \forall a \in \psi$$

(28)

The nonlinear active and reactive power injection constraints at bus $a \in \psi$ of the system are expressed in (29) and (30).

$$\sum_{\forall i \in I(a)} P^G_{i,a} - \sum_{\forall k \in K(a)} P^D_{k,a} = E^r_{a} I^r_{a} + E^{im}_{a} I^{im}_{a} \ \forall a \in \psi$$

(29)

$$\sum_{\forall i \in I(a)} Q^G_{i,a} - \sum_{\forall k \in K(a)} Q^D_{k,a} = E^{im}_{a} I^r_{a} + E^r_{a} I^{im}_{a} \ \forall a \in \psi$$

(30)
The nonlinear active and reactive power injection constraints for bus \( a \in \psi \) of the system are formulated as linear constraints (31) and (32) using the iterative first-order approximation of the Taylor series, respectively:

\[
\begin{align*}
\sum_{i \in (a)} P^G_{a,i} - \sum_{k \in K(a)} P^K_{k,a} &= E^r_{a,t-1} + \sum_{i \in (a)} E^{im}_{a,it-1} 1^m_{a,im} \\
E^r_{a,t-1} &= I^r_{a,t-1} + \sum_{i \in (a)} I^{im}_{a,im}, \quad \forall a \in \psi
\end{align*}
\]

where \( E^r_{a,t-1}, I^r_{a,t-1}, I^{im}_{a,im} \), and \( I^{im}_{a,im} \) are updated in each iteration of the proposed RIV-based model based on the solution of the previous iteration. The proposed MIQP problem based on the linearized RIV (MIQP-RIV-based) model is formulated in the following equation to identify the topology and states of a distribution network:

\[
\begin{align*}
\text{Min } & \sum_{y=1}^Y w_y (h_y(f) - z_y)^2 \\
\text{subject to } & (17) - (18), (21)-(28), (31)-(32)
\end{align*}
\]

where \( f \) is a vector of the system states including \( E^r_a \) and \( E^{im}_{a,im} \) and \( h_y(f) \) is the linear function of system states associated with the measurements. The proposed RIV-based model comprises convex objective function and mixed-integer linear constraints.

IV. PROPOSED ITERATIVE MIQP-BASED TOPOLOGY DETECTION AND STATE ESTIMATION

Fig. 1 shows the flowchart of the simulation procedure for the proposed iterative MIQP-PPV-based and MIQP-RIV-based topology processor and state estimation models in distribution systems. The input to the proposed models is RT measurements of the sensors in the system, such as micro-PMUs, smart meters, and substation measurements. In the first iteration of the two models, a flat start point is considered for linearization parameters of the Taylor series. Hence, in the MIQP-PPV-based model, \( E_{a,it-1}, E_{c,it-1}, \) and \( \varphi_{ac,it-1} \) are considered for all buses in the first iteration equal to 1, 1, and 0, respectively. In the MIQP-RIV-based formulation, the real and imaginary parts of voltage are, respectively, considered 1 and 0 in the first iteration for all buses. Then, the proposed PPV-MIQP-based (given in (16)) and RIV-MIQP-based (given in (33)) models are solved using branch and bound algorithm in CPLEX optimization solver [36] to identify topology and states of the distribution system. The optimality gap is considered equal to zero in CPLEX, which results in the global optimal solution. It is worth noting that the accuracy of the proposed linearized models based on the first-order approximation of the Taylor series is enhanced by solving them iteratively. In the iterative process, the values of \( E_{a,it-1}, E_{c,it-1}, \) and \( \varphi_{ac,it-1} \) in the proposed PPV model and \( E^r_{a,it-1}, E^{im}_{a,im}, I^r_{a,it-1}, \) and \( I^{im}_{a,im} \) in the proposed RIV model are updated using the solution from the previous iteration. The simulations for the proposed iterative MIQP-PPV-based and MIQP-RIV-based topology processor and state estimation models are conducted until their associated stop criteria are met.

V. SIMULATION RESULTS

The performances of the proposed MIQP-PPV-based and MIQP-RIV-based topology detection and state estimation methods are demonstrated using a modified IEEE 33-bus distribution system [30] and a real distribution feeder of a local electric utility in Arizona [31].

A. Results of IEEE Test System

The modified IEEE 33-bus distribution system depicted in Fig. 2 includes both radial and meshed topology configurations based on switching actions. The smart meter data are assembled from residential load data of the Pecan Street Inc. database to model the variability of load based on real-world data [37]. For each bus, a random number of houses are selected such that the aggregated load profile of residences follows the nominal value.
in the IEEE test system. The location and number of micro-
PMUs are extracted from [13] and [15] and shown in Fig. 2. In
order to calculate actual voltages for various network topologies,
nonlinear ac power flow is solved via MATPOWER toolbox in
MATLAB [38]. Measurement noise of sensors in the system,
including micro-PMUs and smart meters, is considered due to
device error or communication issues in RT. The measurement
noise of micro-PMUs is modeled as a Gaussian distribution
function with \( TVE \leq 0.05\% \), i.e., maximum voltage magnitude
error of 0.05\% and maximum voltage angle error of 0.0286° [15],
[39]. The substation injected active and reactive power measure-
ments are also considered, where it is modeled as an ideal voltage
source [13]. Smart meters and substation measurements errors
are modeled as Gaussian distribution functions with 10\% and
1\% errors, respectively [10]. To model load’s variability, the
topology detection and state estimation problem is simulated
in 1000-s time window with measurement frequency equal to
0.1 s in RT. Accordingly, the proposed model is tested for 101
time instants in the RT. At each time instant, RT measurements
of micro-PMU, smart meters, and substation sensor are input to the
proposed simultaneous topology detection and state estimation
model. The proposed model is solved using CPLEX [36] on
an Intel Core i7 CPU @ 3.10 GHz computer with 16 GB of
RAM.

1) MIQP-PPV-Based Topology Detection and State Estima-
tion: In this section, the performance of the proposed MIQP-
PPV-based algorithm in identifying the topology of radial and
meshed networks is demonstrated by considering the measure-
ment noise of micro-PMUs. Seven micro-PMUs are considered
according to the micro-PMU placement study in [13] and [15].
Five switches are considered in the test system, which results in
\( 2^5 = 32 \) different topologies, including radial and meshed
configurations. At \( x = 440 \) s, the network topology changes from
a radial system with \( \Lambda = \{0,0,0,0,0\} \) to a meshed system
with \( \Lambda = \{1,1,1,0,0\} \), whereas the status of three switches
is changed simultaneously. The simulation is conducted for
each time interval, and the identified status of switches for the
simulated time window is shown in Fig. 3. The results in Fig. 3
confirm that the proposed MIQP-PPV-based topology detection
method accurately identifies the radial and meshed topology in
time intervals even while considering load’s variability, mea-
surement noise, and multiple simultaneous switching actions. It
is worth noting that the proposed MIQP-PPV-based model de-
tects radial and meshed network topology without knowing the
status of switches, micro-PMUs, and smart meter measurements
in prior time intervals. Furthermore, the proposed MIQP-PPV-
based topology detection model can simultaneously estimate
system states. The state estimation results and the corresponding
actual system state values before the topology change, i.e., radial
configuration, and after the topology change, i.e., meshed config-
uration, are compared in Figs. 4 and 5. These figures confirm that
the estimated voltage magnitude and angle closely follow the
real voltage profiles in both radial and meshed networks. Also,
the absolute error (AE) values of voltage magnitude and error
values of voltage angle at each bus are depicted in Figs. 4 and 5.
As these figures show, the AE of voltage magnitudes and error
of voltage angles for both radial and meshed networks are small.
In order to statistically evaluate the performance of the proposed
MIQP-PPV-based state estimation model for all time intervals,
three indices, namely, root-mean-square error (RMSE), mean
absolute error (MAE), and maximum absolute error (ME) are
used. The obtained values of indices for voltage magnitude and
the angle at each bus over the simulated time window are shown
in Figs. 6 and 7. The small values of RMSE, MAE, and ME for
all buses confirm that the proposed MIQP-PPV-based model is
able to estimate system states with remarkable accuracy. Also,
in order to evaluate the accuracy of the proposed method with

Fig. 3. Status of switches during the simulated time window.

Fig. 4. Estimated, real, and AE of voltage magnitude at \( x = 430 \) and \( x = 440 \).

Fig. 5. Estimated, real, and error of voltage angle at \( x = 430 \) and \( x = 440 \).
respective to the number of micro-PMUs, a sensitivity analysis is conducted. Table I illustrates the accuracy of the proposed topology detection and state estimation model considering different numbers of micro-PMUs, load’s variability, measurement noise, and multiple simultaneous switching actions. In this table, topology detection accuracy is reported for all time intervals. It can be seen in Table I that the proposed topology detection model is able to identify the system topology with high accuracy even with one micro-PMU. Also, the maximum RMSE and MAE indices for voltage magnitude and angle among all buses are presented in Table I for different micro-PMU numbers. It should be noted that RMSE, MAE, and ME indices for voltage magnitude and the angle at each bus are shown in Figs. 6 and 7 for the case of seven micro-PMUs. It can be observed in Table I that the proposed state estimation model is robust and accurate even with fewer micro-PMUs. Hence, in the case of missing data of micro-PMUs shown in Fig. 2 due to communication issues or micro-PMUs losing GPS signal, the proposed model can still identify the topology and states of the system with only one micro-PMU data with high accuracy.

2) Method Comparison: In this section, the performance of the proposed MIQP-PPV-based topology processor algorithm is evaluated by comparing it with a data-driven method proposed in [13]–[15]. It should be noted that the work in [15] is an extension of the method proposed in [13] and [14]. In [15], using the prior information of switch statuses, a library of possible topology configurations based on the change in status of only one switch in the system is determined. Then, if the difference between the voltage measured by micro-PMUs at time $x$ (i.e., $E_{r,x}$) and time $x-\tau$ (i.e., $E_{r,x-\tau}$) is larger than a predefined parameter (i.e., min_norm in [15]), it will be projected onto the obtained library of possible system topologies. Finally, the topology with the highest projection value, larger than a predefined parameter (i.e., min_proj in [15]), is selected as the correct system configuration and topology change time is reported. For the sake of comparison, 100 scenarios are generated based on Monte Carlo simulation while only considering noise for micro-PMUs measurement data. In each scenario, the time interval of topology change within 1000-s time window, the status of one switch in the system, and the measurement noise of micro-PMUs are randomly selected. Four cases are considered for comparing the two methods. In cases 1–3, the smart meter data are collected based on the nominal values of loads provided in the IEEE 33-bus test system with different standard deviations (SDs) of the load change between different time intervals. The SD of the load change between different time intervals is considered 2.22%, 3%, and 4% for cases 1–3, respectively. It should be noted that the nominal values of loads provided in IEEE 33-bus test system are only for one snapshot (i.e., time interval) of the system. The nominal values of loads provided in the IEEE 33-bus test system are considered for the first time interval. To model load’s variability for the rest of time intervals in 1000-s simulated time window (as mentioned in Section V-A, there are 101 time intervals), different SDs of change of the load between different time intervals are considered based on the method presented in [15] for cases 1–3. In case 4, the smart meter data are collected from residential load data of the Pecan Street Inc. database. In this case, instead of considering SD of change of load to model load’s variability, real-world load data are considered to model different load values at each time interval. The 10 s load data of the Pecan Street Inc. database are mapped to each load bus of the IEEE test system as explained in Section V-A. Tables II–V compare the accuracy of the proposed MIQP-PPV-based topology processor method with the one proposed in [15] by considering three different parameter tunings for the three parameters (i.e., min_norm, min_proj, and $\tau$), which are used in [15]. According to Table II, the accuracy of the proposed MIQP-PPV-based method among all 100 scenarios is equal to 100%, whereas the accuracy of the model proposed
Comparing Accuracy of the Proposed MIQP-PPV-Based Method With the Method Proposed in [15] With SD of 2.22%

| Smart meter data | SD  | Proposed model | [15]     |
|------------------|-----|----------------|---------|
|                  |     | Accuracy | min_norm | min_proj | \( \tau \) | Accuracy |
| IEEE 33-bus      | 2.22| 100%     | 0.004    | 0.8      | 5         | 97%     |
|                  |     | 0.006    | 0.9      | 5         | 92%     |
|                  |     | 0.006    | 0.8      | 4         | 91%     |

Comparing Accuracy of the Proposed MIQP-PPV-Based Method With the Method Proposed in [15] With SD of 3%

| Smart meter data | SD  | Proposed model | [15]     |
|------------------|-----|----------------|---------|
|                  |     | Accuracy | min_norm | min_proj | \( \tau \) | Accuracy |
| IEEE 33-bus      | 3   | 99%      | 0.004    | 0.8      | 5         | 82%     |
|                  |     | 0.006    | 0.8      | 5         | 90%     |
|                  |     | 0.008    | 0.9      | 5         | 74%     |

Comparing Accuracy of the Proposed MIQP-PPV-Based Method With the Method Proposed in [15] With SD of 4%

| Smart meter data | SD  | Proposed model | [15]     |
|------------------|-----|----------------|---------|
|                  |     | Accuracy | min_norm | min_proj | \( \tau \) | Accuracy |
| IEEE 33-bus      | 4   | 97%      | 0.004    | 0.8      | 5         | 55%     |
|                  |     | 0.007    | 0.8      | 5         | 79%     |
|                  |     | 0.008    | 0.9      | 5         | 67%     |

Comparing Accuracy of the Proposed MIQP-PPV-Based Method With the Method Proposed in [15] Using Pecan Street Database

| Smart meter data | Proposed model | [15]     |
|------------------|----------------|---------|
| Pecan Street     | Accuracy | min_norm | min_proj | \( \tau \) | Accuracy |
|                  | 100%     | 0.006    | 0.8      | 5         | 49%     |
|                  |          | 0.007    | 0.8      | 5         | 58%     |
|                  |          | 0.008    | 0.9      | 5         | 57%     |

In [15] is dependent on three parameter tunings and at best is equal to 97%. As it can be observed from Tables III and IV, by increasing the SD of change of the load, the accuracy of the proposed MIQP-PPV-based method is significantly higher in comparison with the accuracy of the method of [15] with different parameter tunings. In case 4, since the SD of change of the load is high for residential load data of the Pecan Street Inc. database, the accuracy of the topology detection method proposed in [15] is remarkably low. In contrast, the proposed MIQP-PPV-based topology processor algorithm identifies the topology with 100% accuracy, as illustrated in Table V. The reason is that higher load variations, i.e., high SD of the load change, makes the voltage difference in time series data of micro-PMU measurements to be larger than min_norm parameter. This change in voltage measurements is projected onto the library of possible system topologies. Therefore, the data-driven method of [15] wrongfully considers the change in the measured voltage time series, caused by the load change, as the change in the network topology. However, the proposed MIQP-PPV-based model is a single-shot optimization problem, i.e., it only requires measurement data at each time snapshot to identify the topology of the system and estimate system states accurately. Therefore, the load’s variability does not affect the accuracy of the proposed MIQP-PPV-based model, whereas the method proposed in [15] requires the information of switch statuses and measured voltage values by micro-PMUs in prior time intervals to identify network topology at current time intervals. In this regard, if prior statuses of switches are wrong, the topology may not be identified correctly. Furthermore, the data-driven method in [15] is dependent on three parameter tunings, which limits the application of the method in RT. Since the data-driven method in [15] assumes that the topology change may occur due to only one switching action at each time interval, the status of only one random switch in the system is changed at topology transition time in each scenario of Monte Carlo simulation. However, as it is shown in Section V-A1, the proposed MIQP-PPV-based topology processor model can handle identifying multiple simultaneous switching actions at each time interval without information of switch statuses, micro-PMUs, and smart meters measurements in prior time intervals. Also, the proposed model is able to identify topology and system states simultaneously, as shown in Section V-A1.

3) Comparing Performance of the Proposed MIQP-PPV-Based Model With the Proposed MIQP-RIV-Based Model: In this section, the performances of the proposed MIQP-PPV-based and MIQP-RIV-based topology processor and state estimation models are compared by simultaneous modeling of micro-PMUs, smart meters, and substation measurements noise. The simulation is conducted for 100 scenarios, which are generated using Monte Carlo simulation. In each scenario, switches operation time during 1000-s time window, the status of five switches, and measurement noise of all measurement data are randomly chosen. The accuracy of the proposed MIQP-PPV-based and MIQP-RIV-based methods among all 100 scenarios with 101 time intervals for the topology identification is 99.83% and 99.84%, respectively.

Since the proposed models are also able to estimate power system states in the distribution system, the obtained voltage magnitude and angle values from the two models are evaluated for each bus and scenario using RMSE and MAE indices, as shown in Figs. 8 and 9. The figures confirm that the error in estimating system states is small with analogous voltage magnitude errors between the two methods. However, the MIQP-RIV-based model performs more accurately in terms of estimating voltage angles. The proposed MIQP-RIV-based model outperforms the proposed MIQP-PPV-based model in terms of topology processor and state estimation accuracy. The reason is that in the MIQP-RIV-based model, the current flow constraints on the distribution lines are inherently linear, and the only nonlinearity
due to the inclusion of binary variable associated with the status of switches is linearized using the big $M$ technique. However, in the MIQP-PPV-based model, the ac power flow constraints are linearized in addition to linearization of nonlinearity as a result of adding binary variables associated with the status of switches. The average computational time for each snapshot is equal to 0.05 and 0.03 s using the proposed MIQP-PPV-based model and the proposed MIQP-RIV-based model, respectively, which illustrates the proposed models are computationally efficient for RT applications.

B. Results of Actual Test System

In this section, the performance of the proposed MIQP-RIV-based model is illustrated on an actual primary distribution feeder of a local electric utility in Arizona with DERs. This test system includes 2100 buses, 1790 lines, four three-phase capacitor banks, and 371 distribution transformers [31]. The total numbers of aggregated loads and DERs including rooftop PV units at the secondary of the distribution transformers are 342 and 250, respectively. There are 859 switches in the feeder comprising 23 switch cabinets with 157 switches. Only one pseudo-micro-PMU measurement is considered at the substation, noise of which is modeled as a Gaussian distribution function with $TVE \leq 0.05\%$ [15, 39]. Also, measurements noise of smart meters data, PV output power sensors, capacitor banks sensors, and substation power measurements are modeled using Gaussian distribution function with errors of 10%, 10%, 1%, and 1%, respectively [10]. All practical details of the test system including no-load loss of distribution transformers, different phase configuration of lines and laterals including three-phase and single-phase, and switch cabinets are modeled. Since mutual couplings between phases are negligible in the test system compared to self-coupling, they are ignored. Table VI illustrates the accuracy of the proposed topology detection model for the actual distribution network. It can be observed in Table VI that the proposed model detects the topology of a large distribution system with DERs with remarkable accuracy. It should be noted that the small error in topology detection accuracy is due to the error in identifying the status of switches that are connected to the no-load area of the network. These switches are located at the end of laterals, where they have been expanded by the local electric utility for supplying future loads. However, no distribution network devices or loads have been connected to these parts. Therefore, the feeder control and connectivity are not affected by the error in the status of these switches. Without considering the status of these switches not connected to anything, the accuracy of the proposed topology detection method is 100%. Also, results of the state estimation are presented in Table VI using RMSE and MAE indices for all buses and phases in the network. It can be seen in Table VI that the system states are estimated with high accuracy. The average simulation time for the simultaneous topology detection and state estimation model is 3 s. Therefore, it can be concluded that the proposed
model is accurate and suitable for RT implementation in actual distribution systems.

1) Missing Data: The impact of missing data on the performance of the proposed simultaneous topology detection and state estimation model is discussed in this section. Missing data can be caused due to measurements devices failure or communication issues. In the studied distribution feeder, about six to eight smart meters are connected to the secondary of each distribution transformer. In order to study the impact of the missing data, it is assumed that the smart meters data of half of the load buses (i.e., 171 load buses) are missed. The number of missing smart meters data of each load bus is randomly selected such that the maximum missed smart meters data is 50%. The accuracy of the topology detection and state estimation algorithm is presented in Table VII. By comparing Tables VI and VII, it can be seen that the topology detection model is robust against missing data. Also, by comparing RMSE and MAE indices for the state estimation results in Table VII with Table VI, it can be seen that although the state estimation error is increased with missing data, the model can estimate states of the system with acceptable high accuracy.

Also, in order to increase the accuracy of the proposed model in the case of missing data, an approach is proposed in this article. Hence, the missing smart meter data are estimated based on the measurement data of the previous time instant. Table VIII presents the accuracy of the topology detection and state estimation with estimating missing smart meter data. Comparison of Table VII with Table VIII demonstrates that the RMSE and MAE of estimating voltage magnitude and voltage angle of all buses and phases are decreased with replacing missed smart meter data based on measurements of previous time instant. The accuracy of topology detection is 98.60%. It can be concluded that the proposed topology detection and state estimation method is robust and accurate with having missing measurement data, whereas by implementing the approach for replacing the missing data, the model’s accuracy is further enhanced.

### VI. Conclusion

In this article, a simultaneous topology processor and state estimation method is proposed using two MIQP formulations, which utilize micro-PMUs and smart meters data. The proposed MIQP approaches are proposed based on two ac optimal power flow models: PPV formulation and RIV formulation. The results confirm that the proposed MIQP-PPV-based and MIQP-RIV-based models are computationally efficient for RT application and able to identify different topology configurations including radial and meshed distribution networks. The proposed models are able to detect multiple simultaneous switching actions at each time instant without knowledge of the status of switches in prior time intervals. Also, each of the proposed models is a single-shot optimization problem and only requires measurement data at each time snapshot to obtain the topology and states of the system. Monte Carlo simulation is conducted to generate different scenarios of topology and switching actions, switches operation time, and measurement noise. Simulation results illustrate that the proposed models can perform topology identification of a distribution network with high accuracy under load’s variability and measurement noises. Moreover, the performance of the proposed MIQP-based state estimation models is examined using statistical indices. The indices confirm that the proposed methods estimate distribution system states with remarkable accuracy. However, the proposed MIQP-RIV-based model outperforms the proposed MIQP-PV-based model in terms of accuracy and speed for topology detection and state estimation. The proposed MIQP-RIV-based model is also tested on a large distribution feeder of a local electric utility in Arizona. It is illustrated that the proposed model can perform simultaneous topology detection and state estimation with significantly high accuracy even with considering missing data in RT with an average simulation time of 3 s. Also, by utilizing a replacement approach for missing data, the accuracy is further increased.

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### Table VII

| Accuracy of the Proposed Topology Detection and State Estimation Model for Missing Data Case |
| --- |
| Topology detection accuracy (%) | 98.60 |
| RMSE magnitude (pu) | 6.90E-04 |
| MAE magnitude (pu) | 4.95E-04 |
| RMSE angle (radian) | 5.37E-04 |
| MAE angle (radian) | 4.63E-04 |

### Table VIII

| Accuracy of the Proposed Topology Detection and State Estimation Model With Estimating Missing Data |
| --- |
| Topology detection accuracy (%) | 98.60 |
| RMSE magnitude (pu) | 2.09E-04 |
| MAE magnitude (pu) | 1.75E-04 |
| RMSE angle (radian) | 3.60E-04 |
| MAE angle (radian) | 3.07E-04 |
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