Analysis of Flow State of Inflatable Support Area in Aerostatic Guide Way under Condition of Rarefied Gas

Long Wei $^1$a Ning Fangwei $^2$b*

$^1$, $^2$School of Electromechanical Engineering, Kunming University of Science and Technology, Kunming
a daifor@163.com
b nfangwei@163.com

keywords: aerostatic guide way; velocity slip; Knudsen number; molecular collision; Newton friction

Abstract: Based on the molecular motion and collision law of the gas in the aerostatic guide way of inflatable support area, proposed the stratification theory of the gas film, the gas in inflatable support area was divided into near wall layer, thin layer, continuous flow layer along its height direction, proposed basis for dividing the thin layer and continuous flow layer, proposed physical model and the corresponding equation. Based on air film pressure decreasing in the aerostatic guide way and rate variation of gas flow, the gas in the longitudinal direction was divided into pressure-driven area and Newton friction area. Simulating flow patterns and calculating its velocity and pressure in gas film by LAMMPS and fluent, drew the relevant conclusions: the gas film in the longitudinal direction was divided into pressure-driven area and Newton friction area; and it also was divided into near wall layer, thin layer, continuous flow layer along its height direction; gas stratification present in the pressure-driven and Newton friction areas; the phenomenon of film layered became weaker with slower gas flow in pressure-driven area; Newton friction area remained stable stratification.

Introduction

In aerostatic guide way, the gas film is formed by supporting table and bearing plate which has an important influence on the precision and ultra-precision facilities of the aerostatic guide way.

Due to the thin gas in the film, velocity slip will be happened at the interface, which leads to the gas flow pattern changed in the lubricating film$^{[1]}$. Velocity boundary slip experienced the development of the without-slippage-velocity boundary condition$^{[2]}$, linear slip condition hypothesis$^{[3]}$ and the approximate constitutive equations for slip$^{[4]}$. No slip boundary conditions are no longer used in the micro scale flow analysis. When $Kn=0.1$(the ratio of the average free path of gas to the flow field characteristic length),it is the upper limit of the Navier-Stokes equation and the boundary conditions in the first order velocity slip boundary$^{[5-8]}$. When Knudsen number increases to 1, the two order model of Cercignani and Deissler is failed$^{[9,10]}$. Velocity distribution data obtained by fitting the DSMC slip factor to establish new slip model$^{[11]}$, which requires higher performance of the computer. The literature above to established the pressure gradient in the micro channel, and researched the flow pattern of the gas. However, the experimental study found that the gas flow velocity in the
air film is complicated\textsuperscript{[12]}. Not only the pressure flow field in the gas film existed in the air film, but also the flow field of the gas. The velocity slip in different field is not known, and this is also the problem to be faced in the study the pattern of gas film in air pressure guide rail.

In this paper, according to the gas flow and the changes of flow velocity in the gas film, the theory of film zoning and film Stratification are put forward, Relevant physical model and corresponding control equation are established. The theory of film zoning and film Stratification not only make up the deficiency of the traditional velocity slip theory, but also enrich the relevant theory of the air film flow, and provide theoretical guidance for the precision, stiffness, stability of aerostatic guide way.

![Fig. 1 Schematic diagram of supporting gas film layer](image)

**Theoretical Analysis**

**Air Film Zoning**

As shown in Figure 1, it is assumed that the work table and the bearing plate are all smooth enough. According to the Aerostatic lubrication principle, the high pressure gas supplied from the air supply hole into the gas chamber, and reflected by the bearing plate and entered into the gas film gap which is composed by working table and bearing plate. Gas flowed along radial of the bearing, and ultimately connected with the atmosphere. In this process, the gas pressure was throttled, so the pressure gradually reduced to the environment pressure, and accompanied by a series of changes of gas flow pattern in the gas film.

Assuming :

\[ M = \frac{\partial u}{\partial x} \]  

(1)

When \( M > 0 \), the local gas is in pressure-driven area, when \( M = 0 \),the ground gas is the Newton friction zone.

pressure-driving area

\( P_1 \) and \( P_2 \) are respectively the inlet and outlet pressure, \( y \) and \( x \) are respectively radial and axial.
coordinates, \( v(y) \) for the gas velocity distribution equation.

Assuming that the air flow is Newton fluid, according to Newton's law of shear, \( v(y) \) can be expressed as

\[
v(y) = -\frac{\Delta p}{L} \frac{y^2}{4\mu} + \frac{C_1}{\mu} \ln y + C_2
\]

(2)

Where \( \Delta p = p_1 - p_2 \), \( \mu \) is the dynamic viscosity of the fluid, and \( L \) is the length of the film. \( C_1 \) and \( C_2 \) are the undetermined constants, which can be solved by boundary conditions, that is, when \( y = 0 \), the shear stress is 0, and on the wall of the boundary, \( v(h/2) = \Delta v \). So \( v(y) \) can be expressed as

\[
v(y) = \frac{\Delta p}{4\mu L} \left( -y^2 + \left( \frac{h}{2} \right)^2 \right) + \Delta v
\]

(3)

By formula (3) can be obtained, in different pressure, the different position of the velocity distribution is different, the gas film flow velocity slip is not the same.

Newton friction area

In this region, the interaction force between gas molecules in the moving process is mainly based on the Newton internal friction, and the velocity and the law of the velocity are basically followed the boundary layer theory in the process of gas viscosity. When the inlet pressure and the outlet pressure of the air film is equal, that is, \( \Delta p = p_1 - p_2 = 0 \), the formula (3) is

\[
v(y) = \Delta v
\]

(4)

Obviously, \( v(y) \) is a constant, that is, the gas flows at a constant speed.

**Film Stratification**

As shown in Figure 1, the laminar flow in the gas film, according to the flow state there, the air flow should be divided into three layers. The first layer, named as the near wall layer, which is near the counter top and the bearing plate; The third layer, named as the continuous layer, acrossed the center line of the film in the middle area. The second layer, named as the thin
layer, which is between the near wall layer and the continuous layer.

In the thin layer, flow of rarefied gas, the thickness of gas film is sub micron, by estimating
the available local $0.01 < Kn < 0.1$, namely the gas in the slipstream field\textsuperscript{[13]}, Fluid at low Reynolds number, i.e., $1 < Re < 10$, can be considered as a layer of complete development of gas flow
in the gas film.

Definition :

$$N_{fw} = \frac{K_n}{K_e} = \frac{v d l}{\lambda v}$$

(5)

where: $R_e$ is the Reynolds number; $\lambda$ is the average free path of the molecule; $L$ is the length of
the fluid characteristic.

When $N_{fw} > 1$, local gas belongs to the continuous layer. while $N_{fw} < 1$, the local gas be-

Near Wall Layer

The thickness of the near wall layer is assumed as the width of average free path of the molecule
(under the working conditions, a gas molecule in the average distance for two times by the colli-
sion). Molecular motion in the layer is in accordance with the boundary conditions of fully diffuse reflection\textsuperscript{[14]}. The relationship of quality and normal momentum between molecular and solid sur-
face are established.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (V^i + V) m_i f_i^s(V) d^3c =$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{m_{is}}{m_{ik}} f_i^s(V) d^3c +$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{m_{is}}{m_{ik}} f_i^w(V) d^3c$$

(6)

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (V^i + V)^2 m_i f_i^s(V) d^3c =$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{m_{is}}{m_{ik}} f_i^s(V) d^3c +$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{m_{is}}{m_{ik}} f_i^w(V) d^3c$$

(7)

Formula (6) and formula (7) expresses the mass flux and the flux of this layer respectively.
Where: $m_i$ and $m_{ir}$ indicate the group of the quality of the net flux and the incident mass flux;

$m_{ir}$ is the diffuse reflection and the diffuse emission mass flux. $V^i$ and $V$ respectively for gas ma-

Thin Layer
VHS model (variable hard sphere) was used as typical collision member, its physical principle that can be expressed as Boltzmann equation.

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + F \frac{\partial f}{\partial t} = \int_{-\infty}^{\infty} \int_{0}^{\infty} (f f'_i - f f_i) g \sigma (g, r) d\Omega dv_i
\]

(8)

\[
f = f (r, v, t) \quad f_i = f (r, v, t), \quad f' = f (r, v, t) \quad f'_i = f (r, v'_i, t)
\]

Where: \( f \) is the times of \( t \) range of \( r \) to \( r + dr \) micro volume element of gas molecule number in velocity range of \( v \) to \( v + dv \); \( v \) and \( v_i \) respectively for the speed of the two molecules before the collision; \( v' \) and \( v'_i \) respectively for the speed of the two molecules after the collision; \( \Omega \) in post collision relative velocity in the direction of \( g' \); \( \sigma \) is differential collision cross section; \( F \) is the force of the unit mass molecule.

Based on the characteristics of the gas molecule collision relaxation, the BGK model is used in this paper. The collision term of the Boltzmann equation is replaced by a simple operator, and its expression (without considering the external force) can be written as

\[
\frac{\partial f}{\partial t} + \nabla \frac{\partial f}{\partial r} = \nu (f_m - f)
\]

(9)

Where: \( \nu \) is the collision frequency; the \( \sigma \) is the constant, which is independent of the speed \( f \) of the gas molecule, \( f_m \) is the local Maxwell equilibrium state velocity distribution function, the value is

\[
f_m = \frac{n}{(2\pi RT)^{3/2}} \exp \left( -\frac{C^2}{2RT} \right)
\]

(10)

Continuous Layer

Air flow in this layer is considered as a two dimensional flow. In the coordinate system as shown in Figure 1, the mass force is neglected, and the Stokes hypothesis is applied, and the corresponding two-dimensional steady incompressible Navier-Stokes equation is obtained (\( x \) is the direction of the air flow).

\[
p \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

(11)

\[
p \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

(12)
The two order velocity slip condition is

\[ u_{w} = 1.1466 \frac{\partial u}{\partial y} - 0.9757 \lambda^2 \frac{\partial^2 u}{\partial^2 y} \]

(13)

Where: \( p \) is the pressure, \( \rho \) as the gas density, \( \mu \) for gas dynamic viscosity coefficient, \( u \) for fluid particle velocity along the direction of the \( x \) component, and \( v \) is along the \( y \) direction of the component.

**Simulation Analysis**

**Air film zoning**

Simulating Aerostatic Guide Way working conditions, supply pressure \( P_s \) is 0.6MPa, 0.8MPa, 1.0MPa, Outlet is communicating with the atmosphere, which \( P_a \) is 0.1MPa, gas membrane height is 1\( \mu \)m, there is room temperature.

Figure 2 is velocity vector of gas which is from the supply holes into the gas chamber and then into air film support area.

Fig. 2 velocity vector of gas in gas film

As shown in figure 2, according to the changes of gas flow rate, it can be drawn that air film is divided into two different areas, there are larger flow rate changes in the height and length direction of gas film; On the contrary, there are smaller flow rate changes in the height and length direction of gas film.

Fig. 3 pressure in the radial direction of gas film

Fig. 4 velocity in the radial direction of gas film

Further analysis and calculating in gas flow rate in the gas film, as shown in Figure 3 and Figure 4. the gas from the stomata to the gas chamber whose speed is rapid changed, when the supply pressure is 0.8MPa, which can be obtained in Figure 4. Gas flow rate from 4.2m/s was reduced to
1.5 m/s, that is pressure-driven area; near the exit area, gas flow rate to maintain 1.5 m/s unchanged, that is Newton friction area.

**Verified LAMMPS Simulation Method**

When the film's long high ratio is 100, $Kn$=1.5, gas flow velocity $Ma$=1, the calculation of the $y/H=0$ at the bottom of the wall, $y/H=1$ for the upper wall position. LAMMPS and DSMC were used to calculate the pressure along the height of the film, as shown in Figure 5. From the results of LAMMPS and DSMC, the closer to the upper and lower wall, the smaller the pressure. Also, the obvious change of the pressure gradient near the wall, and the pressure change is not obvious in the middle of the supporting gas film. Compared with the results of DSMC, the numerical error is small, which can be seen from the results of LAMMPS, and it is also more advanced than DSMC.

**Fig. 5 Comparison of results of DSMC and LAMMPS**

**Stratification in Newton Friction and Pressure-driven Areas**

**Stratification in Newton Friction Area**

Calculating gas velocity at different locations along the height direction in pressure-driven area. Supply pressure $Ps$ is 0.8 MPa, Outlet is communicating with the atmosphere, which $Pa$ is 0.1 MPa, gas membrane height is 1 μm, there is room temperature. As shown in figure 6, thin layer occurs at

( $0 < y < 4 \text{nm}$ and $996 \text{nm} < y < 1000 \text{nm}$ ) and Continuous Flow Layer occurs at( $4 \text{nm} < y < 996 \text{nm}$ ). With decreasing of gas flow velocity, gas flow rate is stabler, and velocity difference decreases in thin layer, stratification is weaker in pressure-driven area.

**Fig. 6 rate changes at different locations in pressure-driven area**

**Stratification in Pressure-driven Area**

Gas flow rate remains stable in Newton friction area as shown in Figure 7, thin layer occurs at

( $0 < y < 5 \text{nm}$ and $995 \text{nm} < y < 1000 \text{nm}$ ) and Continuous Flow Layer occurs at($5 \text{nm} < y < 995 \text{nm}$). the
thickness of continuous flow layer and thin layer remain constant, the speed of slippage remains unchanged.

Fig. 7 different locations of gas flow rate variation diagram in Newton friction area

As shown in Figure 8, in the case of speed of these two regions, gas stratification present in the pressure-driven and Newton friction areas, there is significant speed slip in pressure-driven area, because of larger gas flow rate. In Newton friction zone, the gas flow rate is relatively small, the speed of slippage is not obvious.

Fig. 8 different locations of gas flow rate variation diagram in Newton friction and pressure-driven areas

Conclusions

(1) Proposed the stratification theory of the gas film, the gas in inflatable support area was divided into the near wall layer, the thin layer, and the continuous flow layer, established corresponding equation. Verified the validity of stratification theory by the calculation of LAMMPS.

(2) Proposed theory of film zoning. Based on rate variation of gas flow, the gas in the longitudinal direction was divided into pressure-driven area and Newton friction area, and established corresponding equation. Concluded that gas stratification present in the pressure-driven and Newton friction areas, the Phenomenon of film Layered became weaker with slower gas flow in pressure-driven area; Newton friction area remained stable stratification.

(3) LAMMPS simulation result is reliable compare with DSMC. Further proof the film area is divided into the near wall layer, the thin layer, and the continuous layer is feasible.

Reference

[1] ZHOU . JF , GU . BQ , SHAO . CL , Boundary Velocity Slip of Pressure Driven Liquid Flow in a Micron Pipe. Chinese Science Bulletin (2095-9273). May 2011. Vol.56:1603-1610
[2] Bernoulli, D. Specimen theoriea novae de mensura sortis. Commentarii Academiae Scientiarum Imperialis Petropolitanae. 1738.5:175-192

[3] Navier, C. L. M. H., Memoire sur les lois du movement des floudes. Mem Lacad Roy Sci Inst France. 1823,6:289-440

[4] Maxwell, J. C., On stresses in rarefied gasses arising from in-equalities of temperature. Philosophical Transactions of the Royal Society. 1879,170:231-256

[5] Murthy, C. S., Singer, K., Klein, M. L., et al., Pairwise additive effitive potential for nitrogen. Mol. Phys. 1980.41(6):1378-1399

[6] Klein, M., Levesque, D., Weis, J. J., Molecular-dynamics study of solid □-O2. Phys. Rev. B. 1980,21(12):5785-5792

[7] Lide, D. R., CRC, Handbook of Chemistry and Physics. 81st ed. CRC:Boca Raton. FL. 2000

[8] Grujicic, M., Cao, G., Roy, W. N., Suitability of boron-nitride single-welled nanotubes as fluid-flow conduits in nano-valve applications. Applied Surface Science. 2005,246(1-3):149-158

[9] XIE, C., Fan, Jing, the Test of Second Order Velocity Slip Boundary Condition in Navier-Stokes Equations. Chinese Journal of Theoretical and Applied Mechanics. 2007,39(1):1-6.

[10] Jiali Liu, Jiye Zhang, Weihua Zhang, the Test of Velocity Slip Boundary Condition in MicroGas Flow Based on Lattice Boltzmann Method. Chinese Journal of Vacuum Science and Technology. 2013,33(7): 647-653

[11] Jianqing Long, Bingbing Chen, Sanlong Zheng, Wei Zhang, Slip Coefficient of Micro Channel Flows Based on DSMC Method (in Chinese). Journal of Zhejiang University of Technology. 2010,38(4):392-395

[12] Wei Long, Gang Bao, Analysis of supersonic flow of the throttle thrust bearing (in Chinese). Journal of Hydraulics Pneumatics & Seals. 2008,4:64-67

[13] Cercignani, C., Mathematical Methods in Kinetic. Theory. Plenum Press, 1969

[14] Haijun Zhang, Changsheng Zhu, Qing Yang, et al, Generalized maxwell velocity slip boundary model (in Chinese). Sin-Phys Mech Astron. 2013,43:662-669. doi:10.1360/132011-827

Found by the National Natural Science Foundation of China, project of “Effect of micro and rarefied gas flow on aerostatic guide way” No. 51305185