Quark Chromoelectric Dipole Moment Contribution to the Neutron Electric Dipole Moment

Tanmoy Bhattacharya\textsuperscript{a,b}
Vincenzo Cirigliano\textsuperscript{a} Rajan Gupta\textsuperscript{a}
Boram Yoon\textsuperscript{a}

\textsuperscript{a}Los Alamos National Laboratory

\textsuperscript{b}Santa Fe Institute

July 25, 2016
Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

  Dar arXiv:hep-ph/0008248.

- CP-violating mass term and effective $\Theta G \tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

  Crewther et al., Phys. Lett. B88 (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM
Introduction
Form Factors

Vector form-factors

Dirac $F_1$, Pauli $F_2$, Electric dipole $F_3$, and Anapole $F_A$

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

\[
\langle N|V_{\mu}(q)|N\rangle = \bar{u}_N \left[ \gamma_{\mu} F_1(q^2) + i \frac{[\gamma_{\mu}, \gamma_{\nu}]}{2} q_{\nu} \frac{F_2(q^2)}{2m_N} \right. \\
+ \left. (2i m_N \gamma_5 q_{\mu} - \gamma_{\mu} \gamma_5 q^2) \frac{F_A(q^2)}{m_N^2} \right. \\
+ \left. \frac{[\gamma_{\mu}, \gamma_{\nu}]}{2} q_{\nu} \gamma_5 \frac{F_3(q^2)}{2m_N} \right] u_N 
\]

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- $F_A$ and $F_3$ violate P; $F_3$ violates CP.
Introduction
Effective Field Theory

- Energy
- TeV
- QCD
- nuclear
- atomic

fundamental CP–odd phases

- $d_e$
- $C_{qe}, C_{qq}$
- $\theta, d q, \tilde{d} q, w$
- $C_{S,P,T}$
- $g_{\pi NN}$
- neutron EDM

EDMs of paramagnetic atoms (Tl)
EDMs of diamagnetic atoms (Hg)

Pospelov and Ritz, *Ann. Phys.* 318 (2005) 119.
Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.

- Suppressed by $v_{EW}/M_{BSM}^2$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.

- Suppressed by $1/M_{BSM}^2$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.
The quark chromo-EDM operator is a quark bilinear. **Schwinger source method:** Add it to the Dirac operator in the propagator inversion routine:

\[
\begin{align*}
\mathcal{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu} &\rightarrow \mathcal{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu}) \\
\end{align*}
\]

The fermion determinant gives a ‘reweighting factor’

\[
\frac{\det(\mathcal{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu}))}{\det(\mathcal{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})}
= \exp \text{Tr} \ln \left[ 1 + i\epsilon \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\mathcal{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right]
\approx \exp \left[ i\epsilon \text{Tr} \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\mathcal{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right].
\]
The chromoEDM operator is dimension 5.
Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{QCD} \sim 1$.
Need to check linearity.
Two point functions

Tests on two MILC ensembles.

- $a \approx 0.12$ fm, $M_\pi \approx 310$ MeV,
  $\kappa \approx 0.1272103$, $c_{SW} = 1.05094$, $u_P^{HYP} = 0.9358574(29)$.
  400 Configurations, 64 LP + 4 HP calculations/configuration.

- $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV,
  $\kappa \approx 0.1266265$, $c_{SW} = 1.04243$, $u_P^{HYP} = 0.9461130(10)$.
  270 Configurations, 64 LP + 4 HP calculations/configuration.

Use two CP violating operators that mix under renormalization.

- CEDM: $a^2 \bar{\psi} \tilde{G} \cdot \Sigma \psi$
- P: $\bar{\psi} \gamma_5 \psi$
Two point functions
Neutron Propagator

![Graphs showing two point functions and neutron propagator](image)

Preliminary; Connected Diagrams Only
Two point functions

Linearity

Preliminary; Connected Diagrams Only

Use $\epsilon \approx \frac{a}{30\text{fm}} \approx 6.6\text{MeV}$ $a \approx 0.36\text{ma}$ for experiments.
Two point functions

Connected $\gamma_5$

\[ a(\not{D} + m) + i\epsilon \gamma_5 = e^{\frac{i}{2}\alpha_q \gamma_5} (a\not{D} + am\epsilon) e^{\frac{i}{2}\alpha_q \gamma_5} \]

where $\alpha_q \equiv \tan^{-1}\left(\frac{\epsilon}{am}\right)$

and $am\epsilon \equiv \sqrt{(am)^2 + \epsilon^2}$
### Lattice Calculation

**Two point functions**

- \( a m^0 \equiv \frac{1}{2\kappa} - 4 \)
- \( a m_{cr} \equiv \frac{1}{2\kappa_c} - 4 \)
- \( a m \equiv a m^0 - a m_{cr} \)
- \( \epsilon \)
- \( a m_{\epsilon} \)

|            | a12m310 | a09m310 |
|------------|---------|---------|
| \( a m^0 \) | -0.0695 | -0.05138 |
| \( a m_{cr} \) | -0.08058 | -0.05943 |
| \( a m \)   | 0.01108 | 0.00805 |
| \( \epsilon \) | 0.004 | 0.003 |
| \( a m_{\epsilon} \) | 0.01178 | 0.00859 |

**Ensembles**

- Neutron Propagator
- Linearity
- Connected \( \gamma_5 \)
- \( \alpha \ N \)

### Connected \( \gamma_5 \)

\( a m_0 \equiv \frac{1}{2\kappa} - 4 \)

|            |           |           |
|------------|-----------|-----------|
| \( M_\pi^0 \) | 0.1900(4) | 0.1404(3) |
| \( M_{\pi CEDM} \) | 0.1906(4) | 0.1407(3) |
| \( M_\pi^{\gamma_5} \) | 0.1961(4) | 0.1450(3) |

\( M_\pi^0 \times \sqrt{m_\epsilon} m \)

|            |           |           |
|------------|-----------|-----------|
| \( M_\pi^0 \times \sqrt{m_\epsilon} m \) | 0.1959(4) | 0.1450(3) |
Two point functions

$\alpha_N$
The three point function we calculate is

\[ N \equiv \bar{d} c \gamma_5 \frac{1 + \gamma_4}{2} u d \]

\[ \langle \Omega | N(\vec{0}, 0) V_\mu (\vec{q}, t) N^\dagger (\vec{p}, T) | \Omega \rangle = u_N e^{-m_N t} \langle N | V_\mu (q) | N' \rangle e^{-E_{N'} (T-t)} \bar{u}_N \]

We project onto only one component of the neutron spinor with

\[ \mathcal{P} = \frac{1}{2} (1 + \gamma_4)(1 + i\gamma_5 \gamma_3) \]

Noting that in presence of CP violation \( u_N \bar{u}_N = e^{i\alpha_N \gamma_5 (ip + m_N)} e^{i\alpha_N \gamma_5} \)

and assuming \( N' = N \), we can extract:

\[ \text{Tr} \mathcal{P} \langle \Omega | N V_3 N^\dagger | \Omega \rangle \propto \]

\[ im_N q_3 G_E \]

\[ + \alpha_N m_N (E_N - m_N) F_1 + \alpha_N [m_N (E_N - m_N) + \frac{q_3^2}{2}] F_2 \]

\[ - 2i (q_1^2 + q_2^2) F_A - \frac{q_3^2}{2} F_3 \]
\[ \epsilon = 0.004, \ a \approx 0.12 \text{ fm}. \]
$\epsilon = 0.003$, $a \approx 0.09$ fm.
Three point functions

$F_3$ Form factor from $\gamma_5$

$\epsilon = 0.004, \ a \approx 0.12 \text{ fm}.$
\( \epsilon = 0.003, \ a \approx 0.09 \text{ fm} \).
Three point functions

$F_3(\gamma_5)$ and $F_3(\text{CEDM})$

\[ a(\mathcal{D} + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha_q\gamma_5} a(\mathcal{D} + m\epsilon) e^{\frac{i}{2}\alpha_q\gamma_5} \]

\[ \rightarrow a(\mathcal{D}_L + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha\gamma_5} e^{-\frac{i\phi}{2}\gamma_5 a(\mathcal{D} + m\epsilon)} a(\mathcal{D} + m\epsilon) e^{-\frac{i\phi}{2}\gamma_5 a(\mathcal{D} + m\epsilon)} e^{\frac{i}{2}\alpha\gamma_5} + O(a^3) \]

where

\[
\begin{align*}
\mathcal{D}_L &= \mathcal{D} + aD^2 - \frac{\kappa c_{SW}}{2} a\Sigma^{\mu\nu} G_{\mu\nu}; \\
\mathcal{D} &= \mathcal{D} + \zeta aD^2 - \chi a\Sigma^{\mu\nu} G_{\mu\nu} e^{i\xi\gamma_5} \\
\end{align*}
\]

\[
\begin{align*}
m\epsilon a &= \sqrt{m^2 a^2 + \epsilon^2}, & \phi &= \frac{\epsilon}{m\epsilon a}, & \xi &= \frac{2\phi}{\kappa c_{SW}}, & \chi &= \frac{\kappa c_{SW}}{2} \sqrt{1 + \left(\frac{2\phi}{\kappa c_{SW}}\right)^2}, \\
\zeta &= \frac{m}{m\epsilon}, & \alpha &= \tan^{-1} \left(\frac{\epsilon}{ma} + 2\epsilon\right) \\
\end{align*}
\]

\[ e^{-\frac{i\phi}{2}\gamma_5 a(\mathcal{D} + m\epsilon)} \text{ does not contribute on shell.} \]
Introduction
Lattice Calculation
Two point functions
Three point functions
Conclusions

Projection
$F_3$ Form factor from CEDM
$F_3$ Form factor from $\gamma_5$
$F_3(\gamma_5)$ and $F_3(\text{CEDM})$

$nEDM$ from $qCEDM$
Introduction
Lattice Calculation
Two point functions
Three point functions
Conclusions

Projection
$F_3$ Form factor from CEDM
$F_3$ Form factor from $\gamma_5$
$F_3(\gamma_5)$ and $F_3$(CEDM)

Tanmoy Bhattacharya
nEDM from qCEDM
Conclusions

Future

- Signal in the connected diagram for $a = 0.12$ and $a = 0.09$ fm and $M_\pi = 310$ MeV.
- Mixing with lower dimensional operator not a problem.
- Need to calculate renormalization and mixing coefficients non-perturbatively.
- Fermions with better chiral symmetry does not avoid mixing.