Intrinsic Spin Torque Without Spin-Orbit Coupling

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For non-uniform magnetic textures, we derive an intrinsic contribution to the non-adiabatic spin torque that may be the dominant contribution. It differs from previously considered contributions in several ways. It does not depend on the change in occupation of the electron states due to the current flow but rather is due to the perturbation of the electronic states when an electric field is applied. Therefore it should be viewed as electric-field-induced rather than current-induced. Unlike previously reported non-adiabatic spin torques, it does not originate from extrinsic relaxation mechanisms nor spin-orbit coupling. This intrinsic non-adiabatic spin torque is related by a chiral connection to the intrinsic spin-orbit torque that has been calculated from the Berry phase for Rashba systems.

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Electrical manipulation of magnetization is a promising technique for enabling a new generation of magneto-electronic devices. Spin-transfer torque [1–4] is an efficient way to implement the electrical control of magnetization, as has been demonstrated for various magnetic nanostructures such as spin valves, magnetic tunnel junctions, and magnetic nanowires. In the standard picture of spin-transfer torque, an external electric field generates a spin-polarized electrical current, which in turn gives rise to current-induced spin-transfer torque. In magnetic nanowires with continuously varying magnetic textures, this picture leads to two components of current-induced spin torque, which are known as adiabatic spin torque [1, 5] and non-adiabatic spin torque [6, 7]. The adiabatic spin torque arises from spin angular momentum conservation when conduction electron spins adiabatically follow the local magnetization direction.

The non-adiabatic spin torque, which is perpendicular to the adiabatic spin torque, arises from a variety of mechanisms and is a crucial factor for efficient electrical manipulation of magnetization. One mechanism for non-adiabatic spin torques occurs only for very short length scale variations in the magnetic texture [2, 3, 4], when the spins cannot adiabatically follow the magnetization texture. In slowly varying magnetic textures, all previously considered mechanisms for non-adiabatic spin torques require either spin relaxation [6] or spin-orbit coupling [10] related to magnetic damping [11]. Here, we describe an intrinsic contribution to the non-adiabatic spin torque that arises in the slowly varying limit even in the absence of spin-orbit coupling. It is distinguished from other contributions in that it is electric-field-induced rather than current-induced.

Electric-field-induced spin-transfer torques differ from current-induced spin-transfer torques in that they do not originate from the electron occupation change giving rise to current flow. Instead, they originate from the perturbation of the electronic states by an external electric field. In general, electric-field-induced effects depend on the modification of the electron states summed over the whole Fermi sea, much as densities involve the sum over all occupied states, while current-induced effects depend on properties only at the Fermi surface, much as electrical currents do. Examples of electric-field-induced effects include voltage-induced magnetic anisotropy changes [12, 13], the intrinsic spin Hall effect [14], and the intrinsic spin-orbit torque [15]. Electric-field-induced torques are promising for significantly enhancing electrical manipulation efficiencies [12, 13, 15, 16]. Unfortunately their mechanisms are less well understood than current-induced spin-transfer torques.

In this paper, we examine continuously varying magnetic textures in the absence of spin-orbit coupling and demonstrate the existence of an electric-field-induced spin torque. The result is intrinsic in the sense that its mechanism is not due to impurity scattering. For this particular model, we find that this electric-field-induced torque has the same form as the non-adiabatic spin torque but does not originate from extrinsic relaxation mechanisms, spin-orbit coupling, nor rapidly varying textures. Moreover, we demonstrate that it is significantly larger than other contributions to the non-adiabatic spin torque in some models, making it potentially important for optimizing the manipulation of magnetic structures such as magnetic domain walls and Skyrmions.

The intrinsic non-adiabatic spin torque that we report here is closely related to the intrinsic spin-orbit torque [15] calculated from a Berry phase. Previously, we reported [17] that spin-orbit coupling generates chirality in magnetic properties and that many properties
of a system acquire chiral counterparts upon the introduction of spin-orbit coupling. We demonstrate below that the intrinsic spin-orbit torque is the chiral counterpart of the intrinsic non-adiabatic spin torque that we report here. This connection indicates the common origin of the two, which can be computed through a variety of techniques including a Berry phase as done earlier or time-dependent perturbation theory like we do here.

We use a free electron model with exchange splitting for illustration, but the result can be easily generalized for arbitrary dispersions. As is the case for the spin Hall effect in the closely related system with Rashba spin-orbit coupling, the intrinsic non-adiabatic spin torque is exactly canceled by vertex corrections due to spin-independent scattering. However, we demonstrate in the Supplementary Information (VI) that such exact cancellation only occurs for non-magnetic scatterers and this particular free-electron model.

We consider the Hamiltonian

$$H = \frac{p^2}{2m_e} + J\sigma \cdot m(r, t),$$

where $p$ is the electron momentum operator, $m_e$ is the effective electron mass, $\sigma$ is the spin Pauli matrix, $m$ is the direction of local magnetization, and $J$ is the exchange energy. Below, we show that in the slowly varying limit, the system can be described by the locally defined eigenstates which are denoted by $|k, \pm\rangle_{(0)}$. Here $k$ corresponds to the electron momentum and $\pm$ is for minority and majority states. The subscript $(0)$ refers to the eigenstates unperturbed by an electric field. The eigenstates have spins aligned with the magnetization but with small deviations as discussed in Refs. and illustrated in Fig. (a). The local spin expectation value is

$$\sigma_{k, \pm}^{(0)} = \pm m \mp \frac{\hbar}{2J} m \times (v_k \cdot \nabla)m,$$

where $v_k = \hbar k / m_e$ is the velocity of the $|k, \pm\rangle_{(0)}$ state. In equilibrium, the deviations cancel on summing up over all occupied states. However with non-equilibrium electron distributions, they give rise to the current-induced adiabatic spin torque. If an electron relaxation mechanism is present, it relaxes the net deviations, giving the current-induced non-adiabatic spin torque.

When an electric field $E$ is applied, it perturbs the eigenstates and generates an additional deviation in the spin direction. With the perturbed eigenstates, $\sigma_{k, \pm} = \sigma_{k, \pm}^{(0)} + \Delta \sigma_{k, \pm}$ where

$$\Delta \sigma_{k, \pm} = \pm \frac{\hbar^2 e}{4m_e J^2} (E \cdot \nabla)m.$$

Here $e > 0$ is the electron charge. We demonstrate below that this deviation in the spin direction gives an intrinsic contribution to the non-adiabatic spin torque. Equation is electric-field-induced and is a main result of this paper. The perturbation due to the electric field has a characteristic length $\Delta r = \hbar^2 eE/4m_e J^2$. In Fig. (b), we show that one way to understand Eq. is to imagine that the electric field shifts the spins spatially by an amount $\Delta r$ as in

$$\sigma_{k, \pm} [m(r, t)] = \sigma_{k, \pm}^{(0)} [m(r + \Delta r, t)].$$

Expanding the functional on the right hand side to lowest order in $E$ gives Eq. and Eq. . We now derive the deviations and in the slowly varying limit, by keeping only terms up to first order in derivatives of magnetization. In this limit, it is useful to transform the coordinate system in spin space to make the magnetic texture uniform along $z$. We use a unitary transformation of the wavefunction $\psi$ to $U^\dagger \psi$.
with $U^1 = e^{i\theta \sigma_y / 2} e^{i\phi \sigma_z / 2}$, where $\theta(r, t)$ and $\phi(r, t)$ are defined by $m(r, t) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. After the transformation, Eq. (1) becomes

$$H' = \frac{p^2}{2m_e} + J \sigma_z - \sum_{i=x,y,z} \frac{p_i}{m_e} \sigma \cdot A_i - \sigma \cdot A_1. \quad (5)$$

The magnetic texture becomes uniform and the effect of the original non-uniform texture is contained in $A_1$, which is defined as $A_1 = \text{ith} \sigma \cdot \partial U / \partial U$ ($\mu = x, y, z, t$). Note that $A_1$ and $A_2$ account for spatial and temporal variation of $m$ respectively. The third term in Eq. (5) acts as an effective spin-orbit coupling, allowing us to apply the theory of intrinsic spin-orbit torque [15]. After some algebra, Eqs. (2) and (3) lead to

$$E_{\text{tot}}(\beta, B) = E_{\text{kin}} + J_{\text{int}} \beta + J_{\text{ex}} \beta,$$

where $E_{\text{kin}}$ and $E_{\text{int}}$ are the kinetic and internal energy, respectively.

The effective spin-orbit coupling in Eq. (5) induces interband transitions between majority $|k, +\rangle_{(0)}$ and minority states $|k, +\rangle_{(0)}$. For a small $E$, time-dependent perturbation theory (see Supplementary Information (II)) gives modified wavefunctions $|k, \pm\rangle_{(0)}$ and a modified local spin expectation value $\sigma_{k, \pm}(r) = |k, \pm| U^1 \sigma U |k, \pm\rangle_{(0)}$, giving Eq. (2). An electric field perturbs the electronic states. The perturbation is found by replacing $p$ by $p + eE(r)$, after which the effective spin-orbit coupling in Eq. (5) induces interband transitions between majority $|k, +\rangle_{(0)}$ and minority states $|k, +\rangle_{(0)}$. For a small $E$, time-dependent perturbation theory (see Supplementary Information (II)) gives modified wavefunctions $|k, \pm\rangle_{(0)}$ and a modified local spin expectation value $\sigma_{k, \pm}(r) = |k, \pm| U^1 \sigma U |k, \pm\rangle_{(0)}$, giving Eq. (3). Equation (3) can also be obtained from the Kubo formula [13,18].

The equation of motion for the magnetization, in the absence of damping, is $d\mathbf{m} / dt = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \mathbf{T}$, where $\mathbf{H}_{\text{eff}}$ is the effective magnetic field. The spin torque $\mathbf{T}$ is calculated from $\mathbf{T} = (\gamma / M_s) \sum_{k, s} \mathbf{m} \times \sigma_{k, s} f_{k, s}$, where $\gamma$ is the gyromagnetic ratio, $M_s$ is the saturation magnetization, and $f_{k, s}$ is the electron distribution function. After some algebra, Eqs. (2) and (3) lead to

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \frac{\mu_B \mathbf{m}}{\epsilon M_s} \mathbf{m} \times (\mathbf{E} \cdot \nabla) \mathbf{m},$$

where $\mu_B$ is the Bohr magneton, $j_b = e \sum_{k, s} s v_{k, s} f_{k, s}$ is the spin-polarized electrical current density, and $n_s = -\sum_{k, s} s f_{k, s}$ is the spin-polarized density [22]. The last two terms are the spin torques that result when an electric field is applied. The first of these terms, the adiabatic spin torque, comes from the changes in the occupation of the electron states removing the cancellation of terms from Eq. (2). Note that it is proportional to $j_b$. The last term in Eq. (6), which is proportional to $E$ but not to current, is the electric-field-induced spin torque and arises from $\Delta \sigma_{k, \pm}$. This is the central result of this paper. That this contribution is electric-field-induced rather than current-induced can be seen through the fact that summing $\Delta \sigma_{k, \pm}$ over the equilibrium Fermi sea is finite. The occupation changes associated with a finite charge current only make higher order corrections to the result. While we demonstrate this torque for quadratic dispersion, we show in the Supplementary Information (II) that the calculation proceeds in a similar way for an arbitrary dispersion $\varepsilon(k)$. In more general cases, Eq. (2) remains the same but with a generalized velocity $v_k = (1/h) \nabla \varepsilon_k$, and Eq. (5) becomes $\Delta \sigma_{k, \pm} = \pm \hbar^2 e / 4 J^2 \sum_{ij} E_i (M_k)_{ij} \partial_j \mathbf{m}$ based on the generalized mass tensor $(M_k)_{ij} = (1/h^2) \partial^2 \varepsilon_k / \partial k_i \partial k_j$.

Note that Eq. (6) differs from the usual Landau-Lifshitz-Gilbert equation because at the present level of approximation there is no damping. Including relaxation mechanisms [6,10] in the model gives two additional terms in Eq. (6): Gilbert damping $\alpha \mathbf{m} \times \partial_t \mathbf{m}$ and the current-induced non-adiabatic spin torque $-\beta (\mu_B / e M_s) \mathbf{m} \times (j_b \cdot \nabla) \mathbf{m}$. Here $\alpha$ and $\beta$ are respectively the Gilbert damping constant and the non-adiabatic parameter. Since $E$ and $j_b$ are parallel, the form of the electric-field-induced spin torque is equivalent to $\mathbf{m} \times (j_b \cdot \nabla) \mathbf{m}$, making it a non-adiabatic spin torque. Hence the electric-field-induced spin torque is as important for domain wall motion as current-induced non-adiabatic spin torques. In the rest of this paper, we use “intrinsic” to denote the electric-field-induced non-adiabatic spin torque, and “extrinsic” to denote the current-induced contribution.

To compare the magnitudes of the intrinsic and extrinsic non-adiabatic spin torques, we cast the expression for the intrinsic non-adiabatic spin torque into a current-induced form by using $j_b = n_e e^2 \tau^* / m_e$ in the Drude model. Here $j_b$ is the current charge, $n_e e^2 \tau^* / m_e$ is the charge conductivity, $n_e$ is the electron density, and $\tau$ is the momentum relaxation time. Assuming the current polarization is approximately given by the electron polarization gives $j_b = (n_e / n_h) j_s$. We obtain the form of non-adiabatic spin torque $-\beta_{\text{int}} (\mu_B / e M_s) \mathbf{m} \times (j_s \cdot \nabla) \mathbf{m}$ with the intrinsic non-adiabaticity $\beta_{\text{int}}$ given by

$$\beta_{\text{int}} = \frac{\hbar}{2J_s} \tau^*.$$ 

We compare $\beta_{\text{int}}$ to $\beta$ in a similar model due to spin-flip scattering [9], for which $\beta$ is very similar to Eq. (7). There, $\beta = \hbar / 2J_{\tau_s}$ where $\tau_s$ is the spin relaxation time rather than the momentum relaxation time. Note that $\beta$ is generally significantly smaller than $\tau_s$. For typical parameters, $\tau = 10^{-15}$ s to $10^{-14}$ s and $J = 1$ eV, one obtains $\beta_{\text{int}} = 0.03$ to 0.33, which is significantly larger than commonly reported values of $\beta \sim 0.01$. 
This comparison is a crude estimate of the order of magnitude because \( \beta_{\text{int}} \) is sensitive to vertex corrections. Vertex corrections [19] are known to cancel intrinsic effects in certain cases. For instance, the intrinsic spin Hall conductivity for a two-dimensional Rashba model [15] is exactly canceled by vertex corrections from nonmagnetic impurities [19, 24–27]. Even when magnetization is introduced, the intrinsic anomalous Hall conductivity for the Rashba model [21] also suffers an exact cancellation. Moreover even for the Rashba model, the existence of magnetic impurities changes the situation drastically and vertex corrections may even enhance the intrinsic spin Hall conductivity and intrinsic anomalous Hall conductivity [21, 30–34]. The situation is similar for intrinsic spin torques. A recent experiment [15] on (Ga,Mn)As confirms the robust existence of the intrinsic spin-orbit torque in real materials whose dispersion deviates from a quadratic dispersion in the Rashba model. Moreover for \( H \) in Eq. (1), we show in the Supplementary Information (VI) that the intrinsic non-adiabatic spin torque does not vanish due to vertex corrections unless all impurities are perfectly nonmagnetic [35] and that \( \beta_{\text{int}} \) can be even enhanced by vertex corrections.

The enhancement of \( \beta \) due to \( \beta_{\text{int}} \) leads to significantly faster motion of magnetic domain walls [6, 7] and Rashba model. Moreover for \( \text{dispersion deviates from a quadratic dispersion in the intrinsic spin-orbit coupling in real materials whose anomalous Hall conductivity [20, 30–34]. The situation enhances the intrinsic spin Hall conductivity and intrinsic intrinsic spin-orbit coupling such as Dresselhaus spin-orbit coupling [50] and Weyl spin-orbit coupling [51]. We explicitly demonstrate in the Supplementary Information (V) that Rashba spin-orbit coupling and Dresselhaus spin-orbit coupling are just two particular cases.

The existence of the intrinsic non-adiabatic spin torque implies that there is an additional contribution to the spin motive force \( E^{\text{SMF}} \) since they are related by an Onsager relation. According to the Onsager relation, the intrinsic non-adiabatic spin torque implies an intrinsic charge current \( j^{\text{SMF}} \) induced by the magnetization dynamics where \( \beta_{\text{int}}/\alpha \) can be significantly larger than one. We remark that while many experiments find the ratio \( \beta/\alpha \) to be close to one, some experiments [40] report large values for this ratio.

We have shown earlier [17] that there is a one-to-one correspondence between effects due to spatial variation of \( \mathbf{m} \) and those due to Rashba spin-orbit coupling, \( (\alpha_{\text{R}}/\hbar) \sigma \cdot (\mathbf{p} \times \hat{z}) \), where \( \alpha_{\text{R}} \) is the Rashba parameter and \( \hat{z} \) is the surface normal direction. Rashba spin-orbit coupling effects can be obtained by simply replacing conventional derivatives \( \partial_i \mathbf{m} \) by chiral derivatives \( \partial_i \mathbf{m} = \partial_i \mathbf{m} + k_R (\hat{z} \times \hat{x}_i) \times \mathbf{m} \) in the equation of motion, where \( k_R = 2 \alpha_{\text{R}} m_e / \hbar^2 \) and \( \hat{x}_i \) is the unit vector along \( i \) direction. For example, the interface Dzyaloshinskii-Moriya interaction [41, 42] is obtained from the microscopic exchange energy in this way. Out of equilibrium, current-induced field-like spin-orbit torques [43, 45] and damping-like spin-orbit torques [46, 48] can be obtained from current-induced adiabatic and nonadiabatic spin torques, respectively. For the intrinsic non-adiabatic spin torque in Eq. (5), replacing \( \mathbf{m} \times (\mathbf{E} \cdot \nabla) \mathbf{m} \) by the chiral derivative \( \mathbf{m} \times (\mathbf{E} \cdot \nabla) \mathbf{m} \) generates the original term and an additional torque term,

\[
T^{\text{int}}_R = k_R \frac{n_e \mu_B \hbar e}{2 m_e J M_s} \mathbf{m} \times [\mathbf{m} \times (\hat{z} \times \mathbf{E})],
\]

which is exactly the intrinsic spin-orbit torque reported in Ref. [17] and which was calculated by a Berry phase. This connection shows that both the intrinsic non-adiabatic spin torque in Eq. (6) and the intrinsic spin-orbit torque can be calculated in a variety of methods including via a Berry phase. This can be directly verified by observing the relation between the Kubo formula and the Berry phase [49]. In a similar way, when combined with the intrinsic non-adiabatic spin torque, a proper generalization of the chiral derivative provides an easy way to obtain a Berry phase spin-orbit torque from other types of linear spin-orbit coupling such as Dresselhaus spin-orbit coupling [50] and Weyl spin-orbit coupling [51]. We explicitly demonstrate in the Supplementary Information (V) that Rashba spin-orbit coupling and Dresselhaus spin-orbit coupling are just two particular cases.

The left expression is the current predicted from the Onsager relation, and the right expression is the spin-dependent electric field giving \( E^{\text{SMF}} \) within the Drude model. Note that \( j^{\text{SMF}} \) does not explicitly depend on the charge conductivity, unlike the other contributions to the spin motive force. In the Supplementary Information (IV), we verify for a time-dependent spin spiral configuration of \( \mathbf{m} \) that magnetization dynamics indeed generates such charge current. We emphasize that for this calculation, it is crucial to take into account the inter-band transition contribution from \( A_t \) [57] no matter how small \( \partial_t \mathbf{m} \) is. This is conceptually consistent with the fact that the intrinsic non-adiabatic spin torque originates from inter-band transitions due to an electric field. Equation (9) is of the same form as the non-adiabatic spin motive force [52, 56] but can be larger since \( \beta_{\text{int}} \) is can be larger than extrinsic contributions to \( \beta \). Moreover, the chiral counterpart of Eq. (9) also implies a non-adiabatic Rashba spin motive force similar to what we
reported previously \cite{17} but intrinsic in origin and potentially larger.

In conclusion, electric-field-induced changes in electronic states make an intrinsic contribution to the non-adiabatic spin torque. This contribution arises from modifications to the states over the whole Fermi sea and is independent of changes in the occupancy of the electron states. Thus it should be regarded as an electric-field-induced contribution rather than one that is current-induced. This effect, which occurs in the absence of spin-orbit coupling, can be derived from a Berry phase due to the motion of the electron spins through a spatially varying magnetization. Through a chiral connection, it is closely related to the intrinsic spin-orbit torque that has been calculated from a Berry phase in a uniformly magnetized system with Rashba spin-orbit coupling. While the magnitude of the intrinsic contribution is sensitive to vertex corrections, we estimate that it is larger than other contributions to the non-adiabatic spin torque at least in some systems. Thus, it may play an important role in efficient electrical manipulation of domain walls and Skyrmions.

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Supplementary Information for "Intrinsic Spin Torque Without Spin-Orbit Coupling"

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I. DERIVATION OF THE INTRINSIC NON-ADIABATIC SPIN TORQUE

A. Unitary transformation

In the main text, we start from the Hamiltonian

\[ H_0 = \frac{p^2}{2m_e} + J\mathbf{\sigma} \cdot \mathbf{m}(r,t), \]  \hfill (S1)

and take a unitary transformation defined by

\[ U^\dagger = e^{i\hbar\sigma_y/2}e^{i\hbar\sigma_z/2}, \]  \hfill (S2)

where \( \mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \). Here we put the subscript 0 to emphasize the absence of electric field. The transformed wavefunction is \( \psi' = U^\dagger \psi \). The time-dependent Schrödinger equation \( i\hbar \partial_t \psi = H_0 \psi \) is transformed to \( i\hbar \partial_t \psi' = H_0' \psi \) where \( H_0' = U^\dagger H_0 U - i\hbar U^\dagger \partial_t U \).

\[ H_0' = \frac{1}{2m_e}(p - i\hbar U^\dagger \nabla U)^2 + J\sigma_z - i\hbar U^\dagger \partial_t U. \]  \hfill (S3)

Here, explicitly,

\[ i\hbar U^\dagger \partial_t U = \frac{\hbar}{2}[(\sigma_z \cos \theta - \sigma_x \sin \theta)\partial_\mu \phi + \sigma_y \partial_\mu \theta] \equiv \mathbf{\sigma} \cdot \mathbf{A}_\mu, \]  \hfill (S4)

where \( \mu = x, y, z, t \). Assuming a slowly varying magnetization texture, one can neglect terms like \( O(\partial_\mu \mathbf{m} \cdot \partial_\nu \mathbf{m}) \) and \( O(\partial_\mu \partial_\nu \mathbf{m}) \). Then,

\[ H_0' = \frac{p^2}{2m_e} - \frac{i\hbar}{m_e} \mathbf{p} \cdot (U^\dagger \nabla U) + J\sigma_z - i\hbar U^\dagger \partial_t U, \]  \hfill (S5)

which is Eq. (5) in the main text. As mentioned in the main text, we neglect the variation of \( \mathbf{A}_\mu \) since its variation comes from \( \partial_\mu \partial_\nu \mathbf{m} \) or \( \partial_\mu \mathbf{m} \cdot \partial_\nu \mathbf{m} \). Then, \( \mathbf{k} = \mathbf{p}/\hbar \) is a good quantum number and the locally and instantaneously defined eigenstates are readily given by \( |\mathbf{k}, \pm \rangle_{(0)} \).

B. Time-dependent perturbation theory

We start from Eq. (5) in the main text. In this section, we discard \( \partial_\mu \mathbf{m} \) and \( \partial_\nu \mathbf{m} \) for simplicity of presentation, since restoring their effects is straightforward [see the final result, Eq. (S23)]. Then, \( \mathbf{A}_y \) and \( \mathbf{A}_z \) disappear from Eq. (S5) and only \( \mathbf{A}_x \) and \( \mathbf{A}_t \) remain. When a constant external electric field is applied, the vector potential \( \mathbf{A}_E = -t\mathbf{E} \) starts to contribute to the momentum as \( \mathbf{p} \rightarrow \mathbf{p} + e\mathbf{A}_E \). We use adiabatic turning on technique by putting \( \mathbf{A}_E = -t\mathbf{E}e^{\delta t} \), obtaining the eigenstates at \( t = 0 \) calculated from \( t = -\infty \), and taking \( \delta \rightarrow 0 \) limit. The Hamiltonian in the presence of electric field is

\[ H' = \frac{(\mathbf{p} - et\mathbf{E}e^{\delta t})^2}{2m_e} + \mathbf{\sigma} \cdot \left[J\mathbf{\hat{z}} - \frac{(p_x - eEt e^{\delta t})}{m_e} \mathbf{A}_x - \mathbf{A}_t \right] \]
The denominator is 1 since \( H'_0 \) is given by

\[
H'_0 = \frac{p^2}{2m_e} + J\sigma \cdot \hat{z} - \frac{p_e}{m_e}\sigma \cdot A_i - \sigma \cdot A_t = \frac{p^2}{2m_e} + J_{\text{eff,k}}\sigma \cdot \hat{n},
\]

(S7)

\[
H'_1 = -\frac{et}{m_e}e^{\delta t}E \cdot p + \frac{et\delta t}{m_e}E_x\sigma \cdot A_x.
\]

(S8)

In Eq. (S7), \( J_{\text{eff}} \) and \( \hat{n} \) are defined by collecting all spin dependent terms in \( H'_0 \). Explicitly, \( J_{\text{eff,k}} = |J\hat{z} - (\hbar k_x/m_e)A_i - \hat{A}_t|/J_{\text{eff}} \).

The time evolution operator from \( t = -\infty \) to \( t = 0 \) in the interaction picture is given by

\[
\mathcal{U}^{(I)} = -\frac{i}{\hbar} \int_{-\infty}^{0} dt H_1^{(I)}(t),
\]

(S9)

where

\[
H_1^{(I)}(t) = e^{iH'_0 t/\hbar}e^{-iH'_0 t/\hbar}.
\]

(S10)

\[
\mathcal{U}^{(I)} = -\frac{ie}{m_e\hbar^2}E \cdot p + \frac{ieE_x}{m_e\hbar}\sigma \cdot \left[ \frac{A_x \cdot \sigma}{\delta^2} - \left( \frac{1}{\delta^2} + \frac{1}{2(2J/h + i\delta)^2} + \frac{1}{2(2J/h - i\delta)^2} \right) (\sigma \times \hat{z}) \cdot (A_x \times \hat{z}) \right.
\]

\[
-\left. i\left( \frac{1}{2(2J/h + i\delta)^2} - \frac{1}{2(2J/h - i\delta)^2} \right) \sigma \cdot (A_x \times \hat{z}) \right] + \mathcal{O}(\delta^3, \delta^4).
\]

(S11)

\[
C. \ \text{Spin expectation value in the rotated frame}
\]

The effective eigenstate \(|k, \pm\rangle\) in the presence of the electric field is given by

\[
|k, \pm\rangle = \frac{1}{\sqrt{(k, \pm\langle 0)|1 + \mathcal{U}^{(I)}(1 + \mathcal{U}^{(I)})|k, \pm\rangle_{(0)}}} + \mathcal{O}(E^2).
\]

(S12)

Now the spin expectation value \( \langle \sigma \rangle'_{k,\pm} = \langle k, \pm|\sigma|k, \pm\rangle \) is calculated by

\[
\langle \sigma \rangle'_{k,\pm} = \frac{\langle k, \pm|0\rangle(1 + \mathcal{U}^{(I)})\sigma(1 + \mathcal{U}^{(I)})|k, \pm\rangle_{(0)}}{\langle k, \pm|0\rangle(1 + \mathcal{U}^{(I)})|1 + \mathcal{U}^{(I)}|k, \pm\rangle_{(0)}}
\]

\[
= \frac{\pm \hat{n} + 2 \text{Re}[(k, \pm|0\rangle\sigma\mathcal{U}^{(I)}|0\rangle k, \pm\rangle_{(0)}]}{\pm \hat{n} + 2 \text{Re}[(k, \pm|0\rangle\mathcal{U}^{(I)}|k, \pm\rangle_{(0)}]} + \mathcal{O}(E^2).
\]

(S13)

The denominator is 1 since \( \langle k, \pm|0\rangle\mathcal{U}^{(I)}|k, \pm\rangle_{(0)} \) is imaginary. When calculating the numerator, it is convenient to use \( \langle k, \pm|0\rangle\sigma|k, \pm\rangle_{(0)} = \pm \hat{n} \), and \( \sigma(a \cdot \sigma) = a + i(a \times \sigma) \)

\[
\sigma \mathcal{U}^{(I)} = -\frac{ie}{m_e\hbar^2}E \cdot p + \frac{ieE_x}{m_e\hbar}\sigma \cdot \left[ \frac{A_x \cdot \sigma}{\delta^2} - \left( \frac{1}{\delta^2} + \frac{1}{2(2J/h + i\delta)^2} + \frac{1}{2(2J/h - i\delta)^2} \right) \sigma \cdot (A_x \times \hat{z}) \right.
\]

\[
+ \left. \left( \frac{1}{2(2J/h + i\delta)^2} - \frac{1}{2(2J/h - i\delta)^2} \right) \sigma \cdot (A_x \times \hat{z}) \right] + \mathcal{O}(E^2)
\]

(S14)
Thus,

$$2 \text{Re}[\langle k, \pm | 0 \rangle \sigma U^{(f)} | k, \pm \rangle_{(0)}] = \pm \frac{\hbar E_x}{2m_e J^2} (A_x \times \hat{z}),$$

(S15)

as $\delta \to 0$. Finally one obtains the nonequilibrium spin expectation value as

$$\langle \sigma \rangle_{k, \pm} = \pm \frac{J \hat{z} - (\hbar k_x / m_e) A_x - A_t}{J_{\text{eff, } k}} \mp \frac{\hbar E_x}{2m_e J^2} (\hat{z} \times A_x).$$

(S16)

D. Back to the lab frame

The nonequilibrium spin expectation value in Eq. (S16) is calculated in the rotated frame in which the magnetization direction is along $\hat{z}$. In the lab frame, the eigenstates are $U_k | k, \pm \rangle$. In this frame the spin expectation value is, after simple algebra,

$$\langle \sigma \rangle_{k, \pm} = \langle k, \pm | U^\dagger U | k, \pm \rangle = \begin{pmatrix} \cos \theta \cos \phi & -\sin \phi & \sin \theta \cos \phi \\ \cos \theta \sin \phi & \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \langle \sigma \rangle_{k, \pm}.'$$

(S17)

This $\langle \sigma \rangle_{k, \pm}$ amounts to $\sigma_{k, \pm}$ in the main text. By the following relations,

$$\begin{pmatrix} \cos \theta \cos \phi & -\sin \phi & \sin \theta \cos \phi \\ \cos \theta \sin \phi & \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \hat{z} = m,$$

(S18)

$$\begin{pmatrix} \cos \theta \cos \phi & -\sin \phi & \sin \theta \cos \phi \\ \cos \theta \sin \phi & \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} A_\mu = \frac{\hbar}{2} (m \times \partial_\mu m + \cos \theta \partial_\mu \phi m),$$

(S19)

$$\begin{pmatrix} \cos \theta \cos \phi & -\sin \phi & \sin \theta \cos \phi \\ \cos \theta \sin \phi & \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} (\hat{z} \times A_\mu) = -\frac{\hbar}{2} \partial_\mu m,$$

(S20)

We finally obtain, in more general form to consider $\partial_y m$ and $\partial_z m$,

$$\langle \sigma \rangle_{k, \pm} = \pm \frac{J \hat{m} - (\hbar / 2) \cos \theta D_t \phi m - (\hbar / 2) m \times D_t m + \frac{\hbar^2 e}{4m_e J^2} E \cdot \nabla m}{J_{\text{eff, } k}}$$

$$= \pm m \mp \frac{\hbar}{2J} m \times D_t m \pm \frac{\hbar^2 e}{4m_e J^2} (E \cdot \nabla)m,$$

(S21)

(S22)

where $D_t = \partial_t + v_\text{eff} \cdot \nabla$ is the convective derivative. Neglecting time dependence, this is exactly the same as the result [Eq. (2)+Eq. (3)] in the main text. Here, we keep the convective derivative to illustrate that the effect of $A_t$ is indeed negligible.

E. Landau-Lifshitz-Gilbert equation

With the same procedure as the main text (including relaxation), one obtains the Landau-Lifshitz-Gilbert equation including $A_t$ as

$$\frac{\partial m}{\partial t} = -\gamma' m \times H_{\text{eff}} + \alpha' m \times \frac{\partial m}{\partial t} + \frac{\mu_B'}{eM_s} (j_x \cdot \nabla)m - \frac{\beta' \mu_B}{eM_s} m \times (j_x \cdot \nabla)m - \frac{n_s \mu_B' h e}{2m_e J M_s} m \times (E \cdot \nabla)m,$$

(S23)

where $\gamma' = \gamma(1 + n_s \gamma_0 h / 2M_s)$ and $\alpha' = \alpha(1 + n_s \gamma_0 h / 2M_s)$ are respectively renormalized gyromagnetic ratio and Gilbert damping parameter, $\mu_B' = \gamma' h / 2$ is the renormalized Bohr magneton. Note that taking into account $A_t$ does not change the form of the Landau-Lifshitz-Gilbert equation, but only renormalizes several parameters. As demonstrated in Ref. 3, the renormalization is negligible, justifying neglecting $A_t$ in the main text.
II. THE INTRINSIC NON-ADIABATIC SPIN TORQUE FOR A GENERAL DISPERSION

Let us assume a general form of the kinetic part.

\[ H_0 = H_K(p) + J\sigma \cdot m. \]  (S24)

\[ \varepsilon(k) \] in the main text amounts to \( H_K(hk) \). After unitary transformation, one obtains

\[ H'_0 = H_K(p - ihU^\dagger \nabla U) + J\sigma_z - ihU^\dagger \partial_t U = H_K(p) - ih(\nabla_p H_K)U^\dagger \nabla U + J\sigma_z - ihU^\dagger \partial_t U + O(\partial_p \partial_\sigma). \]  (S25)

When second order derivatives of \( m \) are ignored, \( k = p/h \) is a good quantum number.

\[ H'_0(k) = H_K(hk) - \sum_i \frac{\partial H_K}{\partial p_i} \bigg|_{p=hk} \sigma \cdot A_i + J\sigma_z - \sigma \cdot A_t. \]  (S26)

Thus, the equilibrium spin expectation value is

\[ \langle k, \pm | \sigma | k, \pm \rangle = \frac{J\hat{z} - \sum_i \frac{\partial H_K}{\partial p_i} A_i - A_t}{|J\hat{z} - \sum_i \frac{\partial H_K}{\partial p_i} A_i - A_t|} = \hat{n}. \]  (S27)

Turning on an electric field adiabatically, \( H' = H'_0 + H'_1 \) where

\[ H'_0 = H_K(p) - \sum_i \frac{\partial H_K}{\partial p_i} \sigma \cdot A_i + J\sigma_z - \sigma \cdot A_t \equiv H_K(p) + J_{\text{eff}, k} \sigma \cdot \hat{n}, \]  (S28)

\[ H'_1 = -e\theta e^{i\theta} \sum_i E_i \frac{\partial H_K}{\partial p_i} + e\theta e^{i\theta} \sum_{ij} E_i \frac{\partial^2 H_K}{\partial p_i \partial p_j} \sigma \cdot A_j \]

\[ = -e\theta e^{i\theta} \sum_i E_i v_{k,i} + e\theta e^{i\theta} \sum_{ij} E_i (M^{-1}_k)_{ij} \sigma \cdot A_j, \]  (S29)

where \( v_{k,i} = \frac{\partial H_K}{\partial p_i} \) is the velocity operator for a general dispersion and \((M^{-1}_k)_{ij} = \frac{\partial^2 H_K}{\partial p_i \partial p_j} \) is the inverse mass tensor, which are dependent on \( k \) in general. Now, the time evolution operator from \( t = -\infty \) to \( t = 0 \) in the interaction picture is given by

\[ U(t) = -(i/h) \int_{-\infty}^0 dt H'_1(t), \]

where

\[ H'_1(t) = e^{iH'_0 t/h} H'_0 e^{-iH'_0 t/h} = -e\theta e^{i\theta} \sum_i E_i v_{k,i} + e\theta e^{i\theta} \sum_{ij} E_i (M^{-1}_k)_{ij} e^{i\sigma \cdot \hat{n} J_{\text{eff}, k}/h} \sigma \cdot A_j e^{-i\sigma \cdot \hat{n} J_{\text{eff}, k}/h} \]  (S30)

In this calculation, one can utilize the result in the previous sections actively. Since the expression of the spin expectation value and \( H'_1(t) \) are linearly related, one immediately obtains

\[ \langle \sigma \rangle_{k, \pm} = \pm m \mp \frac{\hbar}{2J} m \times D_t \pm \frac{\hbar^2 e}{4J^2} \sum_{ij} E_i (M^{-1}_k)_{ij} \partial_j m, \]  (S31)

where \( D_t = \partial_t + v_k \cdot \nabla \).

III. SPIN EXPECTATION VALUE FOR SPIN SPIRALS

A. Drifting spin spiral

The model is \( m = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) where

\[ \theta(x, t) = px + \omega t, \ \phi(x, t) = 0. \]  (S32)

Then, one immediately obtains from Eq. (S21)

\[ \langle \sigma \rangle_{k, \pm} = \pm \frac{J m - \left( \frac{\hbar^2 k p}{2m_e} + \frac{\hbar \omega}{2} \right) \hat{y}}{\sqrt{J^2 + \left( \frac{\hbar^2 k p}{2m_e} + \frac{\hbar \omega}{2} \right)^2}} \pm \frac{\hbar^2 e}{4m_e J^2} E_x \partial_x m. \]  (S33)
where \( p \) comes from \( A_x \) and \( \omega \) comes from \( A_t \). It is illustrative to consider a few spacial cases.

**Case (i)** \([\omega = 0 \text{ and } E_x = 0]\).

\[
\langle \sigma \rangle_{k,\pm} = \pm \frac{J m - \left( \frac{\hbar^2 k_x p}{2m_e} \right) \hat{y}}{\sqrt{J^2 + \left( \frac{\hbar^2 k_x p}{2m_e} \right)^2}} = \pm (\cos \alpha_k m - \sin \alpha_k \hat{y}),
\]

(S34)

where

\[
\sin \alpha_k = \frac{k_x p}{\sqrt{k_x^2 p^2 + k_B^2}} \frac{\hbar^2 k_B^2}{2m_e} = J.
\]

(S35)

This result agrees exactly with the result Eq. (28) in Ref. [2]. The physical implication of \( \alpha_k \) (or \( A_x \)) is well discussed in the reference. \( \alpha_k \) is shown in Fig. 1(a) in the main text.

**Case (ii)** \([\omega \neq 0 \text{ and } E_x = 0]\).

\[
\langle \sigma \rangle_{k,\pm} = \pm \frac{J m - \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right) \hat{y}}{\sqrt{J^2 + \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right)^2}} = \pm (\cos(\alpha_k + \varphi) m - \sin(\alpha_k + \varphi) \hat{y}),
\]

(S36)

where

\[
\sin \varphi = \frac{\hbar \omega / 2}{\sqrt{(\hbar \omega / 2)^2 + J^2}}.
\]

(S37)

There is an additional tilting towards \( \hat{y} \) direction by \( \varphi \). One finds a physical origin of \( \varphi \) from inter-band transitions due to \( \partial_t \mathbf{m} \). Within the adiabatic approximation, the electronic states can be approximated by the instantaneous eigenstates \( |\Psi \rangle \sim |\psi_0 \rangle \) up to a phase factor. If considering the first order inter-band transition, it reads [3]

\[
|\Psi \rangle \approx e^{i \gamma_j(t)} e^{-i / h} \int d t' E_0(t') \left[ |\psi_0 \rangle + i h \sum_{j \neq 0} |\psi_j \rangle \frac{\langle \psi_j | \partial_t |\psi_0 \rangle}{E_j - E_0} \right],
\]

(S38)

with a Berry’s phase \( \gamma_j(t) = i \int_0^t dt' \langle \psi_j | \partial_t |\psi_0 \rangle \). One can show that the spin expectation value from Eq. (S38) is nothing but Eq. (S39), implying that \( A_t \) captures inter-band transitions during magnetization dynamics.

**Case (iii)** \([\omega \neq 0 \text{ and } E_x \neq 0]\).

\[
\langle \sigma \rangle_{k,\pm} = \pm \frac{J m - \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right) \hat{y}}{\sqrt{J^2 + \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right)^2}} \pm \frac{\hbar^2 e}{4m_e J^2} E_x \partial_x \mathbf{m} = \pm \frac{J m(x + \Delta x, t) - \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right) \hat{y}}{\sqrt{J^2 + \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right)^2}},
\]

(S39)

where \( \Delta x = \hbar^2 e E_x / 4m_e J^2 \). Note that Eq. (S39) differs from Eq. (S36) by changing the argument \( x \) of \( \mathbf{m} \) to \( x + \Delta x \). This is the spin shift discussed in the main text.

**B. Rotating spin spiral**

The model is \( \mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) where

\[
\theta(x,t) = px, \ \phi(x,t) = \omega t.
\]

(S40)

Then, one immediately obtains from Eq. (S21)

\[
\langle \sigma \rangle_{k,\pm} = \pm \frac{J m(x + \Delta x, t) - \left( \frac{\hbar^2 k_x p}{2m_e} + \frac{\hbar \omega}{2} \right) \hat{y}}{\sqrt{J^2 - J \hbar \omega \cos px + \frac{\hbar^2 \omega^2}{4} + \frac{\hbar^2 k_x^2 p^2}{4m_e}}},
\]

(S41)

For \( \omega = 0 \) and \( E_x = 0 \), the result is clearly consistent with Ref. [2] as demonstrated in [Case (i)] for a drifting spin spiral. In [Case (ii)] for a drifting spin spiral, for non-zero \( \omega \), inter-band transitions give rise to an additional tilting angle \( \varphi \). However, in this case the inter-band transitions do not give rise to an additional tilting defined by a single value because \( \partial_x \mathbf{m} \) and \( \partial_t \mathbf{m} \) are not parallel. One can still observe that a finite \( \omega \) gives rise to an additional tilting along \( \hat{z} \) direction by the \( -(\hbar \omega / 2) \hat{z} \) term. Also, it is still clear that a spin shift with the same amount exists when an electric field \( E_x \) is applied as in [Case (iii)] as for a drifting spin spiral.
IV. ONSAGER RECIPROCITY OF THE INTRINSIC NON-ADIABATIC SPIN TORQUE FOR A SPIN SPIRAL

As we mention in the main text, to obtain the Onsager counterpart of the intrinsic non-adiabatic spin torque, it is important to take into account inter-band transitions due to $\partial_t \mathbf{m}$. Here we demonstrate this with a drifting spin spiral for which $\partial_t \mathbf{m} \cdot \partial_t \mathbf{m}$ is non-zero. Since we provide an effective spin direction for each $k$ state in Eq. (S36), one just needs to calculate the current expectation value for each $k$ state with the help of wave functions described in Refs. [2, 4]. However, in order to emphasize roles of inter-band transitions, we adopt an alternative approach, which is outlined in Ref. [3]. The electrical current density $j_e$ due to inter-band transitions is given by

$$j_e = -\frac{e\hbar^2}{2\pi m_e} \int dk f_k \left( 1 - f_{k+} \right) \left[ \langle \partial_t \psi_k \psi_k^+ \rangle \langle \psi_k^+ | \psi_k \rangle \langle \psi_k | \partial_t \psi_k^- \rangle + \langle \partial_t \psi_k \psi_k^- \rangle \langle \psi_k^- | \psi_k \rangle \langle \psi_k | \partial_t \psi_k^+ \rangle \right],$$  \hspace{1cm} (S42)

where $\psi_{ks}$ represents the instantaneous eigenstate without considering $\partial_t \mathbf{m}$ and $s = \pm$ correspond to minority/majority bands. Here $k$ is a scalar since the system is one-dimensional. $\partial_t \psi_{ks}$ and $\partial_t \psi_{ks}$ respectively come from current operator and $\partial_t \mathbf{m}$. Using the eigenstates presented in Refs. [2, 4], after some algebra one obtains within Eq. (S32)

$$\langle \psi_k^+ | \partial_t \mu | \psi_k^- \rangle = -\langle \partial_t \mu | \psi_k^- \rangle = -i \frac{\hbar}{2} \partial_{\mu} \theta \cos \alpha_k.$$  \hspace{1cm} (S43)

Keeping lowest order terms in derivatives, one can use $E_-(k) - E_+(k) = -\hbar^2 k_B^2 / m_e$ and $\cos \alpha_k = 1$. Finally, using

$$\int dk f_k (1 - f_{k+}) = 2(\sqrt{k_F^2 + k_B^2} - \sqrt{k_F^2 - k_B^2}),$$  \hspace{1cm} (S44)

one obtains

$$j_e = e n_e \frac{n_+ - n_-}{2} \frac{\hbar^2}{2m_e J} \partial_x \theta \partial_y \theta,$$  \hspace{1cm} (S45)

where $n_+ = \sqrt{k_F^2 + k_B^2} / \pi$ is the minority/majority electron density. This expression is equivalent to Eq. (9) in the main text. As we see in the previous section, inter-band transitions are captured by considering $\mathbf{A}_t$ in our language. Thus, for an Onsager counterpart, one should take into account $\mathbf{A}_t$ even though it gives negligible effects for spin torques.

V. CHIRAL CONNECTION TO SPIN-ORBIT COUPLING SYSTEMS

In this section, we show that the Rashba and Dresselhaus spin-orbit couplings are nothing but two particular cases of our theory within the first order approximation. Here, one should note that it shows a mathematical equivalence but not a physical equivalence of each systems.

A. Rashba model as a particular case

Consider a extremely slowly varying magnetic structure $\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ as

$$\theta = \frac{\pi}{2} + px, \quad \phi = py,$$  \hspace{1cm} (S46)

where the small parameter $p$ satisfies $pL \ll 1$ for the system size $L$. Then, one obtains up to $O(p)$

$$\mathbf{A}_x = \frac{p \hbar}{2} \hat{y}, \quad \mathbf{A}_y = -\frac{p \hbar}{2} \hat{x}.$$  \hspace{1cm} (S47)

Then, the effective Hamiltonian within our theory reads

$$H'_0(k) = \frac{\hbar^2 k^2}{2m_e} + \frac{p \hbar^2}{2m_e} (\sigma_x k_y - \sigma_y k_x) + J \sigma_z,$$  \hspace{1cm} (S48)

which is nothing but a Rashba model $H_{SO} = \alpha_R \mathbf{\sigma} \cdot (\mathbf{k} \times \hat{z})$ for $\alpha_R = p \hbar^2 / 2m_e$. 

B. Dresselhaus model as a particular case

Let

\[ \theta = \frac{\pi}{2} + py, \quad \phi = px, \]  \hspace{1cm} (S49)

for the same condition. Then, one obtains

\[ A_x = -\frac{p\hbar}{2} \hat{x}, \quad A_y = \frac{p\hbar}{2} \hat{y}. \]  \hspace{1cm} (S50)

Now, the effective Hamiltonian within our theory reads

\[ H_0'(k) = \hbar^2 k^2 / 2m_e + \frac{p\hbar^2}{2m_e} (\sigma_x k_x - \sigma_y k_y) + J\sigma_z, \]  \hspace{1cm} (S51)

which is nothing but a Dresselhaus model \( H_{SO} = \alpha_{D} (\sigma_x k_x - \sigma_y k_y) \) for \( \alpha_{D} = \frac{p\hbar^2}{2m_e} \).

VI. VERTEX CORRECTION FOR MAGNETIC IMPURITY

We examine the effect of vertex correction for magnetic random impurities. Instead of a general discussion, we take a Rashba model as an example. As we described in Sec. V A, a Rashba model is one of examples of our theory.

The form of magnetic impurities is assumed by \( V_{m} = \sum_i \int d^2 r \delta(r-R_i) (\sigma_x S_x + \sigma_y S_y + \gamma \sigma_z S_z) \) where \( \gamma \) represents anisotropy of the XXZ-type interaction and \( S_i \) are the impurity spins. According to Ref. [5], vertex corrections for a Rashba model with magnetization is explicitly calculated. The vertex corrected velocity operator is given by [Eqs. (25) and (26) in Ref. [5]]

\[ \tilde{v}_x = k_x/m_e - \tilde{\alpha}_R \sigma_y \]

where

\[ \frac{\tilde{\alpha}_R}{\alpha_R} = 1 - \frac{B}{2 - C}, \]  \hspace{1cm} (S52)

\[ B = -\frac{2\gamma^2}{\gamma^2 + 2} \left[ 1 - \frac{1}{1 + (2J\tau/\hbar)^2} \right], \]  \hspace{1cm} (S53)

\[ C = -\frac{2\gamma^2}{\gamma^2 + 2} \frac{1}{1 + (2J\tau/\hbar)^2}. \]  \hspace{1cm} (S54)

up to \( O(\alpha_R) \). Here \( \tau \) is the momentum scattering time due to the magnetic impurity. Now, vertex corrected \( \tilde{\beta}_{int} \) is given by \( \tilde{\beta}_{int}/\beta_{int} = \tilde{\alpha}_R/\alpha_R \). Recalling that \( 2J\tau/\hbar = \beta_{int}^{-1} \) and assuming \( \beta_{int}^2 \ll 1 \), one obtains

\[ \tilde{\beta}_{int} = \left( 1 + \frac{2\gamma^2}{2 + \gamma^2} \right) \beta_{int}, \]  \hspace{1cm} (S55)

which corresponds to Eq. (29) in Ref. [5] and ranges from \( \beta_{int} \) to \( 3\beta_{int} \). This clearly shows that our effect survives against or may be enhanced by vertex corrections from magnetic impurities. One remark is in order. According to Ref. [6], the effects of magnetic impurities competes with those of nonmagnetic impurities so that the result depends on the relative ratio between nonmagnetic and magnetic impurities. Nevertheless, the intrinsic non-adiabatic spin torque survives unless all impurities are perfectly nonmagnetic.

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