Generalized parton distributions and strong forces inside nucleons and nuclei

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Abstract

We argue that generalized parton distributions (GPDs), accessible in hard exclusive processes, carry information about the spatial distribution of forces experienced by quarks and gluons inside hadrons. This way the measurements of hard exclusive processes open a possibility for direct "measurements" of strong forces in different parts of nucleons and nuclei. Also such studies open a venue for addressing questions of the properties of the quark (gluon) matter inside hadrons and nuclei. We give a simple example of relations between GPDs and properties of "nuclear matter" in finite nuclei.

1. The generalized parton distributions (GPDs), accessible in hard exclusive reactions (see the original works \cite{1,2,3,4} and reviews \cite{5,6,7,8}), describe the response of the target hadron to the well-defined QCD operators on the light-cone. Generically the GPDs contain information about the matrix elements of the following type:

\[ \langle B|\bar{\psi}_{\alpha}(0) \ P_{\mu}g_f \int_0^z dx_{\mu} A^\mu \ \psi_{\beta}(z)|A \rangle, \]

\[ \langle B|G_{\alpha\beta}^a(0) \ \left[ P_{\mu}g_f \int_0^z dx_{\mu} A^\mu \right]^{ab} G_{\mu\nu}^b(z)|A \rangle, \]

where the operators are on the light-cone, i.e. \( z^2 = 0 \), and \( A,B \) are various hadronic states. In this way the hard exclusive processes provide us with a set of new fundamental probes of the hadronic structure. An important question is a physical interpretation of these probes. Many ideas have already been put forward, for example, viewed in the infinite momentum frame GPDs allow us to probe the distribution of partons in the transverse plane this way we can obtain detailed spatial partonic images of hadrons, see e.g. \cite{9,10,11}. In this note we shall discuss why the lowest Mellin moments of GPDs provide us with information about the spatial distribution of energy, momentum and forces experienced by quarks and gluons inside hadrons.

To be specific we consider a spin-1/2 hadronic target, e.g. a nucleon. For the GPDs we shall use the notation of X. Ji, see Ref. \cite{5}. We note that all spin independent equations in this paper apply to the spin-0 targets as well.

2. The \( x \) -moments of the GPDs play a special role as they are related to the form factors of the symmetric energy momentum tensor. The nucleon matrix element of the symmetric
energy momentum tensor is characterized by three scalar form factors \[12, 2\]. The nucleon matrix elements of the quark and gluon parts of the QCD energy-momentum tensor (EMT) can be parametrized the following way \[2\]:

\[
\langle p'|\hat{T}^{Q,G}_{\mu\nu}(0)|p\rangle = \bar{N}(p') \left[ M_{2}^{Q,G}(t) \frac{\bar{P}_{\mu}P_{\nu}}{m_{N}} + J^{Q,G}(t) \frac{i\bar{P}_{(\mu}\sigma_{\nu)\rho}\Delta^{\rho}}{m_{N}} \right] N(p) + d^{Q,G}(t) \frac{1}{5 m_{N}} \left( \Delta_{\mu}\Delta_{\nu} - g_{\mu\nu}\Delta^{2} \right) \pm \bar{c}(t) g_{\mu\nu} N(p). \tag{2}
\]

Here \(\hat{T}^{Q}_{\mu\nu} = \frac{i}{2} \bar{\psi}\gamma_{\mu}^{(e)} \nabla_{\nu} \psi\) is the quark part of the QCD energy-momentum tensor (massless case) and \(\hat{T}^{G}_{\mu\nu} = G_{\mu\alpha}^{a} G_{\alpha\nu}^{a} + \frac{1}{4} g_{\mu\nu} G^{2}\) is the gluon part of the QCD EMT. Dirac spinors \(\bar{N}\) and \(N\) are normalized by \(\bar{N}N = 2 m_{N}\) and the kinematical variables are defined as \(\bar{P} = (p + p')/2\), \(\Delta = (p' - p)\), \(t = \Delta^{2}\). The form factor \(\bar{c}(t)\) accounts for nonconservation of the separate quark and gluon parts of the EMT. This form factor enters the quark and gluon parts with opposite signs in order to account for conservation of the total (quark+gluon) EMT. The form factors in eq. (2) are related to the Mellin moments of GPDs through\[2\]:

\[
\begin{align*}
\int_{-1}^{1} dx \ x \ (H(x, \xi, t) + E(x, \xi, t)) &= 2 J^{Q}(t), \\
\int_{-1}^{1} dx \ x \ H(x, \xi, t) &= M_{2}^{Q}(t) + \frac{4}{5} d^{Q}(t) \xi^{2}.
\end{align*}
\tag{3}
\]

Such relations between GPDs and the form factors of EMT open a possibility to study these form factors in hard exclusive processes. For example, the form factor \(d^{Q}(t)\) contributes to the \(x_{Bj}\) independent part of the real part of the DVCS amplitude, which is accessible through the beam charge asymmetry \[14\]. Simultaneously this form factor corresponds to the first coefficient in the Gegenbauer expansion of the so-called D-term \[14\] in the parametrizations of the GPDs, see for details \[15, 16\]. The real part of the DVCS amplitude (spin-0 target for simplicity) at small \(x_{Bj}\) and \(t\) to the leading order in \(\alpha_{s}(Q)\) can be written, under certain simplifying assumptions, as:

\[
\text{Re} A \propto \pi H(\xi, \xi, t) \tan \left( \frac{\pi \omega}{2} \right) + 2 d^{Q}(t) + \ldots , \tag{4}
\]

where \(\omega\) corresponds to the exponent in the small \(x_{Bj}\) behaviour of the singlet quark distribution \(q(x) + \bar{q}(x) \sim 1/x^{1+\omega}\). The “slice” \(H(\xi, \xi, t)\) of quark GPD is directly measurable in the DVCS beam spin asymmetry. Ellipsis in eq. (4) stands for the higher Gegenbauer coefficients of the D-term expansion, which die out logarithmically with increasing of the photon virtuality and seems to be small even at a low normalization point, see estimates in ref. \[16\]. Note that the constant contribution \(\sim d^{Q}(t) + \ldots\) is similar to the contribution of fixed poles in the angular momentum plane to the virtual Compton scattering discussed, e.g. in \[13, 14\].

*We write explicitly results only for quarks, corresponding expressions for the gluon GPDs are similar*
Let us now analyze the physics content of the form factors $M_2^Q(t)$, $J^Q(t)$ and $d^Q(t)$. To reveal the physics content of these form factors, in the same way as for electromagnetic form factors \[17\], it would be useful to consider the nucleon matrix elements of the energy momentum tensor in Breit frame. In this frame the energy transfer $\Delta^0 = 0$, therefore one can introduce the static energy momentum tensor defined as:

$$T^Q_{\mu \nu}(\vec{r}, \vec{s}) = \frac{1}{2E} \int \frac{d^3 \Delta}{(2\pi)^3} e^{i \vec{r} \cdot \vec{\Delta}} \langle p', S'| \hat{T}^Q_{\mu \nu}(0)| p, S \rangle ,$$  

where $\hat{T}^Q_{\mu \nu}(0)$ is the QCD operator of the symmetric energy momentum tensor of quarks.

In Breit frame $E = E' = \sqrt{m_N^2 + \vec{\Delta}^2/4}$. The polarization vectors of the initial and final nucleons, $S^\mu$ and $S'^\mu$, we choose in such a way that both of them correspond to the same polarization vector $(0, \vec{s})$ in the rest frame of the corresponding nucleon. We also note that in the Breit frame the four-momentum transfer squared $t = -\vec{\Delta}^2$.

Various components of the static energy momentum tensor $T^Q_{\mu \nu}(\vec{r}, \vec{s})$ can be interpreted as spatial distributions (averaged over time) of the quark contribution to various mechanical characteristics of the nucleon. However, doing this we have to be careful because of nonconservation of the separate quark and gluon parts of the EMT encoded in the form factor $\bar{c}(t)$ in eq. \[2\]. The point is that the “charges” obtained from the tensor densities through the relation like

$$Q_\mu \left( x^0 \right) = \int d^3 r \ D_\mu \left( x^0, \vec{r} \right) .$$  

are time dependent for the non-conserved tensor density $D_{\nu \mu}$ and therefore the Lorentz covariance generically is broken. Nevertheless for the case of the quark or gluon part of the EMT one is free from such a problem for the $T^Q_{0k}(\vec{r}, \vec{s})$ and $\left( T^Q_{ik}(\vec{r}, \vec{s}) - \frac{1}{3} \delta_{ik} T^Q_{ll}(\vec{r}, \vec{s}) \right)$ components of the static tensor densities, because the “problematic” term $\bar{c}(t)$ drops out in these combinations.

The components $T^Q_{0k}(\vec{r}, \vec{s})$ correspond to the distribution of the quark momentum in the nucleon. The components $\left( T^Q_{ik}(\vec{r}, \vec{s}) - \frac{1}{3} \delta_{ik} T^Q_{ll}(\vec{r}, \vec{s}) \right)$ characterize the spatial distribution (averaged over time) of “shear forces” experienced by quarks inside the nucleon. For a spin-1/2 hadron, only the component $T^Q_{0k}(\vec{r}, \vec{s})$ is sensitive to the polarization state. For the higher spin hadrons (e.g. higher spin nuclei) all components of $T^Q_{\mu \nu}(\vec{r}, \vec{s}, \ldots)$ are polarization dependent.

Now we can easily relate the form factors $J^Q(t)$ and $d^Q(t)$ to the spatial distribution of the energy-momentum and forces encoded in $T^Q_{\mu \nu}(\vec{r}, \vec{s})$. The relation for the form factor $J^Q(t)$ is the following:

$$J^Q(t) + \frac{2}{3} t \frac{d J^Q(t)}{dt} = \int d^3 r \ e^{-ir \cdot \vec{\Delta}} \varepsilon^{ijk} s_i r_j T^Q_{0k}(\vec{r}, \vec{s}) .$$  

We see that the form factor $J^Q(t)$ gives us information about the spatial distribution of the quark angular momentum inside the nucleon. Now if we take the limit $t \to 0$ in eq. \[7\] we obtain:

\[\varepsilon^{ijk} r_j T^Q_{0k}(\vec{r}, \vec{s}) \text{ corresponds to angular momentum density.}\]
\[ J^Q(0) = \int d^3r \, \varepsilon^{ijk} s_i r_j T^Q_{0k}(\vec{r}, \vec{s}). \] (8)

This relation illustrates the interpretation of \( J^Q(0) \) as a fraction of the angular momentum of the nucleon carried by quarks and antiquarks \[3\].

Concerning the form factor \( M_2^Q(t) \), one can easily see, going to the infinite momentum frame in eq. (2), that at \( t = 0 \) it is related to the momentum fraction carried by quarks which is measured in inclusive deep inelastic scattering. The constant \( M_2^Q(0) \) is related to the parton distributions via
\[ M_2^Q(0) = \sum_q \int_0^1 dx \, x \, (q(x) + \bar{q}(x)). \]

Obviously the form factors \( M_2^Q(t), J^Q(t) \) and \( d^Q(t) \) are renormalization scale dependent. This corresponds to the fact that the individual distributions of quarks (gluons) depend on the resolution scale. The scale independent quantities are obtained adding the contributions of quarks and gluons. These are \( M_2(t) = M_2^Q(t) + M_2^G(t) \), \( J(t) = J^Q(t) + J^G(t) \) and \( d(t) = d^Q(t) + d^G(t) \). The scale independent form factors \( M_2(t) \) and \( J(t) \) are expressible in terms of the total static energy momentum tensor \( T^\mu_\nu(\vec{r}, \vec{s}) = T^{Q^\mu}_{\nu}(\vec{r}, \vec{s}) + T^{G^\mu}_{\nu}(\vec{r}, \vec{s}) \) \[\dagger\]. The corresponding expressions have the form
\[ M_2(t) - \frac{t}{4m_N^2} \left[ M_2(t) - 2J(t) + \frac{4}{5} d(t) \right] = \frac{1}{m_N} \int d^3r \, e^{-i\vec{r} \cdot \vec{\Delta}} \, T_{00}(\vec{r}, \vec{s}), \]
\[ J(t) + \frac{2}{3} t \frac{dJ(t)}{dt} = \int d^3r \, e^{-i\vec{r} \cdot \vec{\Delta}} \, \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}, \vec{s}), \] (9)

which at \( t = 0 \) read
\[ M_2(0) = \frac{1}{m_N} \int d^3r \, T_{00}(\vec{r}, \vec{s}) = 1 \]
\[ J(0) = \int d^3r \, \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}, \vec{s}) = \frac{1}{2}. \]

Written in this form the above equations have obvious interpretation. The first one tells us that the total energy of the nucleon in the rest frame is equal to its mass. The second equation states that the total spin of the nucleon is 1/2. Also it shows that the anomalous gravimagnetic moment of the nucleon is zero \[23\].

3. Now let us turn to the physics content of the form factor \( d^Q(t) \). It is easy to see that this form factor is related to the traceless part of the static stress tensor \( T^Q_{ik}(\vec{r}, \vec{s}) \) which characterizes the spatial distribution (averaged over time) of shear forces experienced by quarks in the nucleon \[22\]. In detail this relation is the following:
\[ d^Q(t) + \frac{4}{3} t \frac{dd^Q(t)}{dt} + \frac{4}{15} t^2 \frac{d^2d^Q(t)}{dt^2} = -\frac{m_N}{2} \int d^3r \, e^{-i\vec{r} \cdot \vec{\Delta}} \, T^Q_{ij}(\vec{r}) \left( r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right). \] (10)

\[\dagger\] Note that for the total conserved EMT all components of this tensor have a meaning of “good” tensor densities

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If one considered the nucleon as a continuous medium then $T_{ij}^Q(\vec{r})$ would characterize the force experienced by quarks in an infinitesimal volume at distance $\vec{r}$ from the centre of the nucleon. At $t = 0$ eq. (10) gives:

$$dQ(0) = -\frac{m_N}{2} \int d^3r \, T_{ij}^Q(\vec{r}) \left( r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right).$$

The expressions for the gluon and total energy momentum tensors are analogous.

As our understanding of the forces inside hadrons in QCD is still rather limited we can not make first principles prediction for the value of $d(t)$. The estimate which is based on the calculation of GPDs in the chiral quark soliton model [19] at a low normalization point $\mu \approx 0.6$ GeV, gives [10, 21] a rather large and negative value of $d_Q(0) \approx -4.0$. The negative value of this constant has a deep relation to the spontaneous breaking of chiral symmetry in QCD, see [18, 16, 7].

4. To illustrate physics behind the form factor $d(t) = d_Q(t) + d^G(t)$ let us consider an idealized model of a very large nucleus. Generically the static stress tensor for spin-0 and spin-1/2 targets can be decomposed as:

$$T_{ij}(\vec{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}.$$  

(12)

The functions $s(r)$ and $p(r)$ are related to each other by conservation of the total energy-momentum tensor. The function $p(r)$ can be interpreted as the radial distribution of the “pressure” inside the hadron. The function $s(r)$ is related to the distribution of the shear forces and, in the simple model of a large nucleus considered below, is related to the surface tension.

For a very large nucleus we can assume that the pressure $p(r)$ is constant, $p_0$, in the bulk of the nucleus, and it changes only in the thin “skin” around radius $R$ of the nucleus. It is known from the famous electron scattering experiments [20] that the distribution of electric charge has such a shape in large nuclei. Surely this does not imply that, say, the distribution of pressure follows the shape of the electric charge distribution, although such an assumption is rather sensible. The measurements of coherent hard exclusive processes (like DVCS) on nuclei will provide us with detailed information about deviations of the energy, pressure, and shear forces distributions from that of electric charge, see eqs. (9,10). Since here our aim is merely illustrative, we consider an (over)idealized case of a nucleus with sharp edges, like a liquid drop. In this case, the pressure can be written as:

$$p(r) = p_0 \theta(R - r) - \frac{p_0 R}{3} \delta(R - r).$$

(13)

From the condition $\partial_k T_{kl}(\vec{r}) = 0$ we obtain that

$$s(r) = \frac{p_0 R}{2} \delta(R - r) = \gamma \delta(R - r).$$

(14)

Note that for non-homogeneous media the pressure, defined as the force per unit area and directed orthogonally to the surface element, gets a contribution from shear forces as well.

One can easily include higher terms which take into account the width of the “skin” and other characteristics of the spatial distribution of forces, see below.
This equation immediately shows that the function \( s(r) \) in eq. (12) has a meaning of the surface tension \( \gamma \) of the nucleus. Substituting the solution (14) into eq. (11) we obtain for the constant \( d(0) \) the following value:

\[
d(0) = -\frac{4\pi}{3}m_A \gamma R^4. \tag{15}
\]

First we see that the \( d \)-constant is negative. The effect of the finite width of the nuclear “skin” also has a negative sign. The corresponding formula can be easily derived:

\[
d(0) = -\frac{4\pi}{3}m_A \gamma R^4 \left( 1 + \frac{5\pi^2}{3} \frac{a^2}{R^2} \right), \tag{16}
\]

where \( a \) is a “skin” width introduced by replacing the step function by Fermi-like function \( \theta(R - r) \to 1/(1 + \exp((r - R)/a)) \). If we assume that the surface tension depends slowly on the atomic number (as it is suggested by the Weizsäcker formula), we come to the conclusion that \( d(0) \sim A^{7/3} \), i.e. it rapidly grows with the atomic number. This fact implies that the contribution of the D-term to the real part of the DVCS amplitude grows with an increase of the atomic number as \( A^{4/3} \). This should be compared to the behaviour of the amplitude \( \sim A \) in the impulse approximation. If true, rather interesting phenomenon! In principle, it can be checked by measuring the charge beam asymmetry in coherent DVCS on nuclear targets. Taking the value of the nuclear surface tension of \( \gamma \approx 1\text{MeV}/\text{fm}^2 \) (as it follows from the Weizsäcker formula) and Hofstadter’s \( R = 1.12 A^{1/3} \text{fm}, \ a \approx 0.54 \text{fm} \) [20], we get an estimate \( d(0) \approx -0.2 A^{7/3} \left( 1 + 3.8/A^{2/3} \right) \).

Although being very rough and naive, estimate (16) shows a big potential of hard exclusive processes for studies of properties of quark and gluon “matter” inside nuclei.

Another possible application of our eqs. (7,9,10) is the estimations of the EMT form factors in various effective models of the nucleon structure, like chiral soliton models. In the latter case the static EMT can be obtained from the expression for EMT of the effective chiral Lagrangian (EChL) computed on the static soliton field.

5. Hard exclusive processes allow us to extend a set of fundamental probes of hadronic structure. As an example we considered physics encoded in the lowest Mellin moments of generalized parton distributions. In particular, we showed that one has an access to the spatial distributions of energy, angular momentum and forces inside hadrons, see eqs. (7,9,10). These equations give us a tool for systematic studies of the properties (distributions of energy, angular momentum, pressure, shear forces, etc.) of the quark-gluon “matter” inside hadrons.

As an illustration, we considered a rough idealized picture of a large nucleus. We showed that even in this picture the parameters of GPDs carry detailed information about nuclear matter in the nucleus—knowledge which is still incomplete. We note that the picture used here can be considerably refined, for instance one can, instead of macroscopic approach used here, apply microscopic description.

\[\parallel\]Recall the well known Kelvin relation between the pressure in a liquid spherical drop, its surface tension and the radius of the drop \( P = 2\gamma/R \) [24].
Hopefully experimental studies of the hard exclusive processes will fill the gap in our understanding of the strong forces creating our world as we see it. See the first experimental data on deeply virtual Compton scattering (DVCS) [25, 26, 27, 28, 29]. We hope that this studies will be extended for the nuclear targets.

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