Problems of the Sensitivity Parameter and Its Relation to the Time-varying Fundamental Couplings Problems

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The sensitivity parameter is widely used for quantifying fine tuning. However, examples show it fails to give correct results under certain circumstances. We argue that these problems only occur when calculating the sensitivity of a dimensionful mass parameter at one energy scale to the variation of a dimensionless coupling constant at another energy scale. Thus, by mechanisms such as dynamical symmetry breaking etc, the high sensitivity of the energy scale parameter Λ to the dimensionless coupling constant can affect the reliability of the sensitivity parameter through the renormalization invariant factor of the dimensionful parameter. Theoretically, these phenomena are similar to the problems associated with the time-varying coupling constant discovered recently.

We argue that, the reliability of the sensitivity parameter can be improved if it is used properly.

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I. INTRODUCTION

In quantum field theories, as a consequence of quadratic divergent quantum corrections to the fundamental scalar masses, in order to obtain light weak scale observables, delicate fine-tuning mechanisms are usually required. Due to the absence of reasonable explanations to these fine-tuning mechanisms, quadratic divergent quantum corrections are usually problematic, which indicate the theories are incorrect. In light of this, Wilson and ’t Hooft[1] introduced the principle of naturalness, which requires the radiative corrections to a measurable parameter should not be much greater than the measurable parameter itself, thus a magical fine-tune mechanism is not required to have the theory agree with the current observations.

The simplest example of such problems is the fundamental scalar of φ⁴ model:

\[ L = \frac{1}{2} \left( \partial_{\mu} \phi \right)^{2} - m_{0}^{2} \phi^{2} - \frac{g}{4!} \phi^{4} \]

At one-loop the renormalization of the scalar mass is of the form:

\[ m^{2} = m_{0}^{2} - g^{2} \Lambda^{2} \] (2)

where \( m_{0} \) is the bare mass, \( \Lambda \) is the cut-off energy scale. Because of the tremendous scale difference between the light observable scalar mass and the bare mass, a fine-tuning mechanism that can adjust \( m_{0} \) and \( \Lambda \) very precisely is required to stabilize the observable mass. Otherwise any minute variations of \( m_{0} \) or \( \Lambda \) will completely change the value of the observable mass.

Motivated by the importance of such problems, find a way to quantitatively describe the severity of fine-tuning is important. Think of a weak scale measurable parameter \( y \) which is affected by the fine-tuning problem, it exhibits a strong dependence on a fundamental Lagrangian parameter \( x \) at the Planck scale. To calculate the severity of fine-tuning for such instance, R. Barbieri and G.F.Giudice et al.[2] proposed the following sensitivity parameter:

\[ c(x) = \left| \frac{x \frac{\partial y}{\partial x}}{y} \right| \] (3)

Under this definition, larger sensitivity means higher severity of fine-tuning. Traditionally, a particular value \( c = 10 \) is chosen as the upper limit for a measurable parameter to be categorized as “natural” (or not fine-tuned), although the choice of \( c = 10 \) itself is quite arbitrary. The measure \( c(x) \) along with its cut-off value constitute the sensitivity criterion.

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The sensitivity criterion has been subsequently adopted by many researchers. Although the largeness of the sensitivity parameter is usually in good correspondence with fine-tuning, many researchers soon found it can not accurately represent the severity of fine-tuning under certain circumstances. The most famous examples among them are given by G. Anderson et al[3] and P. Ciafaloni[6].

The example given by G. Anderson et al[3] is regarding the high sensitivity of proton mass $m_p$ to the strong coupling constant $g$. Because the relation between the weak scale proton mass $m_p$, the Planck scale($M_P$) strong coupling constant $g$ is:

$$m_p \approx M_P e^{-\frac{(4\pi)^2}{bg^2(M_P)}}$$

which yields the sensitivity:

$$c(g) = \frac{4\pi}{b} \frac{1}{\alpha_s(M_P)} \gtrsim 100$$

The large value of $c$ shows the proton mass is extremely sensitive. According to the sensitivity criterion, the proton mass should be highly fine-tuned. But it is well known that the lightness of the proton mass is the result of the gauge symmetry, not the result of any delicate fine-tuning mechanisms. Obviously here the sensitivity parameter failed to reflect the severity of fine-tuning correctly.

The example given by P. Ciafaloni et al[6] is about the high sensitivity of the Z-boson mass. When the Z-boson mass $M_Z$ is dynamically determined through gaugino condensation in a “hidden” sector, the mass $M_Z$ can be written as:

$$M_Z \approx M_P e^{-l/g_H^2}$$

where $g_H$ is the hidden sector gauge coupling constant renormalized at $M_P$. $l$ is a constant. Similarly, the sensitivity $c$ of this example is also much greater than the fine-tuning cut-off $c = 10$. If we follow the sensitivity criterion, Z-boson mass $M_Z$ will be always fine-tuned. This is definitely not true. All these examples show that the sensitivity parameter is not a reliable measure of fine-tuning.

In order to avoid such misleading results, many authors have attempted to explain these problems, proposed alternative prescriptions that supposed to be able to give correct results under these circumstances. G. Anderson et al[3, 4, 5], first introduced the idea of probability distribution. They argued that, some physical parameters do have intrinsic large sensitivity. We should use $\bar{e}$, the probability average of the sensitivity $c$, to rescale the sensitivity parameter. The result will reflect the fine-tuning correctly:

$$\gamma = \frac{c}{\bar{e}}$$

Under this prescription, only those with $\gamma \gg 1$ can be categorized as fine-tuned. This criterion gives correct judgments for the examples discussed above, although they didn’t explain why some parameters may have intrinsic large sensitivity and why the rescaling can remove the problem.

Other authors proposed a modified version of the sensitivity parameter to solve the problem [6, 7, 8, 9, 10]:

$$c(x_0) = \left| \frac{\Delta x}{y} \frac{\partial y}{\partial x_0} \right|$$

where $\Delta y$ is the experimentally allowed range of the parameter $x$. They argued that properly choose the experimentally allowed range $\Delta x$ can solve these problems. But as we know, the fine-tuning problem is an intrinsic property. It should not depend on any experimental technologies we used to measure a physical quantity.

Although many authors have attempted to give correct numerical descriptions of fine tuning in various ways, yet none of them can claim quantitative rigor. No explanation has ever been proposed to explain why sometimes we have such large sensitivities for the parameters which are apparently not fine-tuned. It is still unclear how to quantitatively describe the fine-tuning problem in a correctly way. The calculated fine-tuning level usually depends on what criterion we use, and how we use it. Their judgments may reflect the naturalness properties correctly or incorrectly. Because the sensitivity criterion plays such an important role, it is worth to investigate the relationship between the severity of fine-tuning and the sensitivity, find the reason why the sensitivity is so large for those examples we just discussed.
II. WHY THE SENSITIVITY CRITERION FAILS

It is meaningless to directly compare two physical quantities with completely different mass dimensions. Convert them to a nondimensionalized format is the most reasonable approach. For physical quantities $x_i (i = 1, 2, \cdots)$ with different mass dimensions, usually they are first nondimensionalized to $\delta x_i / x_i$. By doing so it is believed that the problem of comparing physical quantities with different mass dimensions has been solved.

The examples we discussed in the introduction have many similarities. For example, Eq. 4 is almost identical to Eq. 6. Both of them are just the simplest renormalization relations between a dimensionless coupling and a dimensionful mass parameter. The fact that the sensitivity parameter fails in such simple equations reminds us that the problem maybe related with the mass dimension. The nondimensionalization method we used may be problematic. Some effects related with dimensionality haven’t been fully removed. To further investigate the origin of the problem, it is crucial to revisit the relation between the mass dimension and the renormalization related fine-tuning problems.

As we know, the scalar mass $m$ of Eq. 2 roughly observes the following scaling law between the mass and the energy scale parameter $\Lambda$:

$$m \sim \Lambda^1 \tag{9}$$

If the observable mass $m$ is light, due to the tremendous energy scale difference between the weak and the Planck scale, precise matching of the initial condition is required. As a consequence, this type of scaling relation will end up in a fine-tuning problem. Unlike the scalar mass, the fermion masses are protected by the gauge symmetry, therefore they do not have the similar scaling relation. Now suppose we have a gauge theory with a dimensionless parameter $g$ and dimensionful mass parameters $m_i$. Without considering any specific interactions, the lowest order renormalization group equations can be written as [11, 15]:

$$\frac{dm_i}{dt} = \gamma_{ij}(g)m_j + \cdots \tag{10}$$

$$\frac{dg}{dt} = \beta(g) + \cdots \tag{11}$$

where $t = \ln \Lambda / \Lambda_0$. $\beta$ and $\gamma_{ij}$ are dimensionless functions of the coupling constants. For example, in QCD they are [12, 13, 14, 15]:

$$\beta(g) = \frac{-b}{16\pi^2} g^3 + \cdots \tag{12}$$

$$\gamma_{ij} = \gamma_{ij}^0 \frac{g^2}{16\pi^2} + \cdots \tag{13}$$

Solving Eq. 10 and Eq. 11, we have:

$$m = m_0 e^{\int_{t_0}^{t} \Gamma dt} + \cdots \tag{14}$$

where matrices $\mathbf{m} = (m_i)$, $\Gamma = (\gamma_{ij})$. $m_0$ is the initial value of $m$ at $\Lambda_0$. Its value does not depend on the renormalization (renormalization invariant). If this model only has one mass parameter $m$, then Eq. 14 can be further simplified as [11]:

$$m(t) = m(t_0) \left( \frac{g(t)}{g(t_0)} \right)^{-\gamma^0 / b} \tag{15}$$

If ignore the dependency of the renormalization invariant quantity $m(t_0)$ to the variation of $g(t_0)$, the sensitivity of the mass parameter to the coupling constant is:

$$c \approx \gamma^0 / b \tag{16}$$
Because the anomalous dimensions are usually small[23], based on the sensitivity criterion, the result of Eq. 16 is certainly not fine-tuned.

The above descriptions are well-known but they are not complete. In any models, besides these explicit mass parameters, there always exists an implicit dimensionful parameter: the energy scale parameter \( \Lambda \). Its effects on the sensitivity parameter are widely ignored. Unlike the other explicit mass parameters, because in any renormalization group equation, each term should be dimensionally consistent. This requires that the right hand side of the renormalization group equation of a dimensionless coupling constant (for example, Eq. 11) can not have the first order term. The consequence of this requirement is that the mathematical relation between a dimensionless parameter and the energy scale parameter \( \Lambda \) must be an exponential function. This means the energy scale parameter \( \Lambda \) is always sensitive with respect to minute variations of the related dimensionless coupling constants. The large sensitivity of \( \Lambda \) here is originated from different mass dimensions between a dimensionless coupling and \( \Lambda \), it should not be understood as a fine-tuning problem. Because the relation between \( \Lambda \) and \( g \) is nonlinear, If for some reason a not-fine-tuned dimensionful parameter is linearly proportional to \( \Lambda \), then the nondimensionalized method used in the sensitivity parameter will not be able to remove the effect of different mass dimensions. The large sensitivity of \( \Lambda \) to \( g \) then will affect the reliability of the sensitivity criterion.

Generally, from the renormalization point of view, a physical mass parameter can be divided into two factors: the factor that depends on the running of the energy scale (for example, \((g(t)/g(t_0))^{-\gamma t/b}\) in Eq. 15), and the factor that is renormalization invariant (for example, \(m(t_0)\) in Eq. 15). Usually because of the protection of the gauge symmetry, the one loop correction to the mass parameter will diverge logarithmically rather than quadratically:

\[
\delta m \approx g^2 \ln \frac{\Lambda}{\Lambda_0} \tag{17}
\]

The logarithm function relieves the high sensitivity of \( \Lambda \) to \( g \). Therefore the renormalization dependent factor of a not-fine-tuned physical mass won’t contribute to the sensitivity problem discussed in the previous part.

But it is still possible that a physical mass could be linearly proportional to the energy scale parameter in the renormalization invariant factor. If so the sensitivity of the physical mass will be greatly affected by the high sensitivity of \( \Lambda \) to \( g \). The severity of fine-tuning will be overestimated, even though the mass parameter itself is protected by gauge symmetry and is not fine-tuned.

The renormalization invariant factor of a physical mass usually is its initial value at the a given energy scale. Therefore principally, any mechanisms that can linearly relate the initial value to the energy scale parameter will cause the problem we discussed in the previous part. For example, for theories that are asymptotical free, the running of the coupling constant \( g \) can produce a mass via dynamical symmetry breaking[16]:

\[
m(g, \Lambda_0) = \Lambda_0 e^{-\int dg/\beta g} \tag{18}
\]

The mass defined by Eq. 18 is certainly highly sensitive to \( g \). If the initial value of a physical mass is given by Eq. 18, then apparently the physical mass will also be highly sensitive to \( g \). Even if the gauge symmetry stabilizes the mass, prevents quadratic divergent in the renormalization, it still can not prevent the physical mass linearly proportional to \( \Lambda \) in the renormalization invariant factor. The sensitivity of the physical mass to the coupling constant will be extremely large.

The mechanism of the dynamical symmetry breaking is widely used in many models. For example, in supersymmetric standard models, dynamical symmetry breaking is used to specify the soft masses[17]. As a consequence, any masses related with this mechanism will be highly sensitive to the variations of the dimensionless coupling \( g \). the severity of fine-tuning will be greatly over-estimated. This is the reason why the sensitivity parameter fails in Eq. 4 and Eq. 6.

Besides the dynamical symmetry breaking, those mechanisms that end up with a linear relation between a physical mass and \( \Lambda \) also can cause the same sensitivity problem. For example, chiral symmetry breaking[18], which has the relation:

\[
\langle 0 | \bar{q}q | 0 \rangle^{1/3} \sim \Lambda \tag{19}
\]

If the initial value of a nucleon mass is defined by Eq. 19, then the nucleon mass will highly sensitive to the variation of \( g \). Other mechanism like gaugino condensation also has the same effect.

The problems of the sensitivity parameter have a very close theoretical connection to the problems related with the time variations of the fine structure constant. Recent astrophysical observations have shown many evidences of small time variation of the gauge coupling constant[19]. Many researchers have pointed out that the small time variations of the fundamental couplings will produce large time variations of various physical parameters, such as proton mass
and magnetic moment[20, 21, 22]. Therefore, we need to explain why these physical quantities can have such large time variations. Technically, this phenomenon is identical to the problems we just discussed. The time variations of the fine structure constant \( \dot{\alpha} \) corresponds to the variations of the initial value \( \delta x \) here. Similar to our analysis, the large time variations of the physical quantities are caused by the large time variations of the energy scale parameter \( \Lambda \), and the large time variations of the energy scale parameter \( \dot{\Lambda} \) are caused by the small time variations of the fine structure constant \( \dot{\alpha} \). The last step is the consequence of the different mass dimensions between the fine structure constant \( \alpha \) and the energy scale parameter \( \Lambda \). Based on our analysis, we can conclude that the large relative changes of nucleon mass and many other dimensionful quantities in the time-varying coupling constant problem are originated from the the native scale difference between the energy scale parameter and the dimensionless coupling constants.

Obviously, these problems only happen when two physical quantities have different mass dimensions. If two physical quantities have the identical mass dimension, then theoretically they won’t have the problem. To verify this, suppose we have two dimensionful mass parameters \( m_i \) and \( m_j \), the solution of Eq. 10 now becomes[11]:

\[
m_i(t) = \sum_{j,k} U_{ij} e^{\int \frac{dt}{\Gamma_0/b} U_{jk}^{-1} m_k(t_0)}
\]

where \( \Gamma_0 \) is the matrix formation of \( \gamma_{ij}^0 \), \( U_{ij} \) is the element of the matrix that diagonalizes \( \Gamma_0 \).

Because in Eq. 20, the exponent \( \Gamma_0/b \) does not explicitly contain any mass parameter, thus the value of \( \partial m_i(t)/\partial m_j(t_0) \) will not be affected by the factor \( \partial \Lambda/\partial g \). So it is quite straightforward to conclude that the sensitivity of a dimensionful mass parameter \( m_i(t) \) to another dimensionful mass parameter \( m_j(t_0) \) won’t have the same problem we discussed in the previous part. Similarly, we can also conclude that the problems caused by the time-varying coupling constant only exist in the dimensionful parameters. The dimensionless parameters won’t have such problems.

### III. LIMITATIONS OF THE SENSITIVITY CRITERION

Although the sensitivity criterion has these problems, it doesn’t mean this criterion will fail under all circumstances. The problems of the sensitivity criterion occurs only when two parameters have different mass dimensions, one is a dimensionful quantity, and the other is a dimensionless coupling constant. They are at the different energy scales and need to be mathematically linked by renormalization. Besides, there should be a mechanism which makes the renormalization invariant factor of the dimensionful mass quantity linearly proportional to the energy scale parameter \( \Lambda \).

However, the fine-tuning phenomena exist not only in renormalization related problems, but also in problems such as mass mixing[24, 25], where all parameters involved in a fine-tuning problem are at the same energy scale and a renormalization evolution is not required. Therefore the effects introduced by different mass dimensions have no place to play such a role. The severity of fine-tuning won’t be overestimated if judged by the sensitivity parameter. So estimate the severity of fine-tuning by comparing two parameters with different mass dimensions at different energy scales is the only situation that we need to pay special attention to the validity of the sensitivity criterion.

To solve these problems, many researchers have proposed many alternatives methods[3, 4, 5, 6, 7, 8, 9, 10]. These methods restrict the value of the sensitivity parameter either by presetting an upper limit to the value of a language parameter or by rescaling the sensitivity parameter by a background sensitivity. Technically these prescriptions do relieve the problem, but they all based on incorrect theories. Without the knowledge of why the sensitivity parameter fails to reflect the severity of fine-tuning, the results based on these prescriptions could also be misleading. Certainly, due to the complexity of real models, different mechanisms could have different types of fine-tuning problems. It is difficult to invent a simple yet universal prescription that can be easily applied to any problems. The advantages of the sensitivity criterion is obvious, though it has problems under these special situations. If we want to find a way to measure the fine-tuning correctly without losing the beauty of simplicity and straightforwardness, it is better to keep the sensitivity criterion but avoid using it when two parameters with different mass dimensions at different energy scales are compared. Because this is the simplest and the most efficient solution to measure the naturalness correctly while avoiding all these problems. Certainly this solution won’t restrict our ability to describe the severity of the fine-tuning, because the fine-tuning can also be measured by comparing parameters with the identical mass dimension.

One may argue that, these problems could be explained in other way. For instance, The sensitivity parameter is correct, while those mechanisms such as dynamical symmetry breaking are fine-tuned and should be avoided. Because the methods like dynamical symmetry breaking are used so widely, if this argument were true, the consequences would be disastrous. The astrophysical observations of the time-varying gauge coupling constant give us a clue to examine our theory experimentally. Maybe in the future we can find an astrophysical way to measure the time variations.
of the proton mass or similar dimensionful physical quantities. By comparing the time variations of a dimensionful parameter with the time variations of the gauge coupling, we will not only have the experimental evidence to verify our conclusion, understand the actual role of the mechanisms like dynamical symmetry breaking, but also have the knowledge of the largest sensitivity allowed in our universe, not just the arbitrary value \( c_{\text{max}} = 10 \) chosen by R. Barbieri and G.F. Giudice.

IV. CONCLUSION

The problems of the sensitivity parameter has been discovered for more than ten years. Many analyses, explanations and alternative prescriptions have been proposed to solve these problems. But none of them has been accepted widely. The reason why the sensitivity parameter fails under specific circumstances still remains unclear.

we investigated the problems exist in the sensitivity parameter, demonstrated that the reason why the sensitivity parameter fails to represent the true level of fine-tuning is because the energy scale parameter directly appears in the renormalization invariant factor of a dimensionful parameter via mechanisms other than the renormalization. Thus the large sensitivity of the energy scale parameter to the dimensionless coupling will greatly influence the sensitivity of the dimensionful parameter, severity of fine-tuning will be over-estimated. Theoretically, the problems of the sensitivity parameter are similar to the problems related with time-varying coupling constant. Results in that field can be applied in the sensitivity research.

Finally, we want to point out that this effect only exists when parameters with different mass dimensions at different energy scales are compared. Based on these analyses, we argue that the best way to avoid these problems is always compare parameters with same mass dimensions if they are at the different energy scales.

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