Probing Seesaw in an Adjoint SUSY SU(5) Model at LHC

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Abstract

The SU(5) GUT model extended with fermions in the adjoint 24_{F} representation predicts triplet fermions in the 100 GeV mass range, opening up the possibility of testing seesaw at LHC. However, once the model is supersymmerized, the triplet fermion mass is constrained to be close to the GUT scale for the gauge couplings to unify. We propose an extension of the SUSY SU(5) model where type II seesaw can be tested at LHC. In this model we add a matter chiral field in the adjoint 24_{F} representation and Higgs chiral superfields in the symmetric 15_{H} and 15_{H} representations. We call this the symmetric adjoint SUSY SU(5) model. The triplet scalar and triplet fermion masses in this model are predicted to be in the 100 GeV and 10^{13} GeV range respectively, while the mass of the singlet fermion remains unconstrained. This gives a type I plus type II plus type III seesaw mass term for the neutrinos. The triplet scalars with masses \sim 100 GeV range can be produced at the LHC. We briefly discuss the collider phenomenology and predictions for proton decay in this model.
1 Introduction

Despite its tremendous success, the Standard Model (SM) of particle physics with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, is generally regarded as the low energy limit of a more fundamental and complete theory. Among the major stumbling blocks of the SM is the observation of neutrino masses and mixing. The other major problem faced by the SM is the hierarchy problem. Since the electroweak scale is $10^{16}$ times smaller than the Planck scale, it is expected that the quantum corrections to the Higgs mass should be $\sim 10^{18}$, unless one plots to cancel quadratic radiative corrections by fine-tuning.

Grand Unified Theories (GUTs), based on higher gauge groups, allow for the unification of the quarks and leptons, and more importantly, the unification of the gauge couplings. Since the rank of $SU(3)_C \times SU(2)_L \times U(1)_Y$ is 4, the smallest simple Lie Group which contains the SM gauge group as its subgroup is $SU(5)$. The $SU(5)$ GUT model was proposed long ago \cite{1} with three copies of $\bar{5}_F$ and $10_F$ representations containing the three generations of quarks and leptons, a $24_C$ containing the gauge bosons, and two Higgs representations $5_H$ and $24_H$. The $24_H$ acquires a Vacuum Expectation Value (VEV) at a high scale, known as the GUT scale. This brings about spontaneous breaking of the $SU(5)$ gauge group, thereby giving mass to 12 of the 24 gauge bosons contained in the $24_C$. These massive gauge bosons are known in the literature as the $X$ and $Y$ gauge bosons. The remaining 12 massless gauge bosons belong to the SM. The $W$ and $Z$ subsequently get massive when SM is spontaneously broken to $SU(3)_C \times U(1)_{em}$ as a result of the VEV of the SM doublet Higgs contained in the $5_H$ of $SU(5)$.

While the minimal $SU(5)$ GUT is simple and elegant, it fails on a few counts. Firstly, the SM gauge couplings $g_i \ (i = 1, 2, 3)$ do not unify at one scale. While $g_2$ and $g_3$ unify at around $10^{17}$ GeV, the coupling $g_1$ unifies with $g_2$ much earlier. This problem can solved by introducing supersymmetry, which anyway is required in order to stabilize the Higgs mass and hence solve the gauge hierarchy problem. Another lacuna in the minimal $SU(5)$ GUT concerns the generation of neutrino mass. The minimal $SU(5)$ predicts the neutrinos to be massless. Neutrino masses can be generated by extending either the fermion sector or the Higgs sector. Presence of $SU(5)$ singlet fermions can give rise to the type I seesaw mechanism \cite{2}, while introduction of $15_H$ can produce neutrino mass via the type II seesaw \cite{3}. A lot of recent interest has been generated from the possibility of a third kind of seesaw in $SU(5)$. This so-called type III seesaw \cite{4} can be realized by extending $SU(5)$ with fermions in the adjoint representation, $24_F$ \cite{5–9}. This model has been popularly called the adjoint $SU(5)$. The $24_F$ contains the $(1,3,0)$ fermion representation of the SM, which can mediate type III seesaw. The additional advantage one gets by introducing the $24_F$ is that the contribution of the representations of $24_F$ to the individual SM gauge coupling running is such that the gauge couplings unify at around $10^{15.5}$ GeV, even without invoking supersymmetry. This happens when the $(1,3,0)$ fermion representation has mass in the 100s of GeV range, making it accessible at LHC \cite{10,11}. This opens up the possibility of testing seesaw at LHC. A further intrinsic problem in minimal $SU(5)$ concerns the masses of the d-type quarks and charged leptons, which become degenerate at the GUT scale. This is a well known problem and can be easily solved either by allowing higher dimensional
operators or an additional $45_H$ to break this unwanted degeneracy.

While requirement of supersymmetry for achieving gauge coupling unification in $SU(5)$ is alleviated by adding a $24_F$, supersymmetry is still required for addressing the issue of the hierarchy problem. Therefore, one should consider a supersymmetric GUT as the complete theory. The supersymmetric version of the adjoint $SU(5)$ has been proposed in the literature and has been called adjoint SUSY $SU(5)$ \cite{12}. In this paper we study the issue of gauge coupling unification and the condition it imposes on the particle spectra. We find that once supersymmetry is imposed, the mass of the $(1,3,0)$ fermion representation of $24_F$ turns out to be very close to the GUT scale, making it impossible to produce it at the LHC. Therefore, type III seesaw predicted by the adjoint SUSY $SU(5)$ model cannot be tested at the current and even future collider experiments.

We next embark upon constructing a model based on the supersymmetric $SU(5)$ which allows for TeV-scale seesaw mechanism that can be probed at the LHC. As mentioned above, addition of symmetric $15_H$ representation to minimal $SU(5)$ allows for the type II seesaw mechanism. The ramifications of the $15_H$ representation for the gauge coupling unification in $SU(5)$ without supersymmetry has been studied \cite{13,15}. In the absence of supersymmetry, gauge coupling unification can be achieved if the $(3,2,1/6)$ representation of $15_H$ has mass in the $10^2$-$10^3$ GeV range, making them accessible at the LHC. However, the $(1,3,1)$ scalars which mediate type II seesaw have masses in the intermediate range and are therefore inaccessible at the LHC. In addition, once supersymmetry is imposed, even the masses of the leptoquarks should be around the GUT scale in order to get gauge coupling unification.

In this paper we propose a supersymmetric $SU(5)$ GUT model where we add a matter chiral field in the adjoint $\hat{24}_F$ representation and Higgs chiral superfields in the symmetric $\hat{15}_H$ and $\hat{\bar{15}}_H$ representations. We call this “symmetric adjoint SUSY $SU(5)$” model. Since this model has $(1,1,0)$ and $(1,3,0)$ fermionic representations as well as $(1,3,1)$ scalar multiplet, neutrino masses could get contributions from type I, type II, as well as type III seesaw. We show that gauge coupling unification constrains the particle masses such that the triplet scalar and the triplet fermion masses in this model are predicted to be in the $100$ GeV and $10^{13}$ GeV range, respectively. The triplet scalars with masses $\sim 100$ GeV range can be produced at the LHC making seesaw testable at the LHC. We briefly discuss the phenomenological aspects of this model.

The paper is organized as follows. In section 2 we briefly outline the particle content of the SUSY $SU(5)$ GUT model and some of its extensions that are relevant to this paper. In section 3 we study the RG evolution of the SM gauge couplings and the constraints it imposes on the particle masses in the adjoint $SU(5)$ and the adjoint SUSY $SU(5)$. In section 4 we propose the symmetric adjoint SUSY $SU(5)$. We give the particle mass spectra expected in this model and show that these particle masses consistently give gauge coupling unification at the GUT scale. In section 5 we study the phenomenological consequences of this model. Finally, in section 6 we end with our conclusions.
2 SUSY SU(5) and its Extensions

We begin with a very brief overview of the particle content of the supersymmetric SU(5) model. The minimal version of the model [16] is comprised of three families of \( \hat{5} \equiv (3,1,1/3) \oplus (1,2,-1/2) \equiv (\hat{d}^C, \hat{L}) \) and \( \hat{10} \equiv (3,1,-2/3) \oplus (3,2,1/6) \oplus (1,1,1) \equiv (\hat{u}^C, \hat{Q}, \hat{e}^C) \) matter chiral multiplets, while the Higgs sector comprises of a \( \hat{5}_H \equiv (3,1,-1/3) \oplus (1,2,1/2) \equiv (\hat{T}, \hat{H}_u) \), a \( \hat{5}_H \equiv (3,1,1/3) \oplus (1,2,-1/2) \equiv (\hat{T}, \hat{H}_d) \) and a \( 24_H \equiv (8,1,0) \oplus (1,3,0) \oplus (3,2,-5/6) \oplus (3,2,5/6) \oplus (1,1,0) \equiv (\hat{\Sigma}_8, \hat{\Sigma}_3, \hat{\Sigma}_{(3,2)}, \hat{\Sigma}_{(3,2)}, \hat{\Sigma}_0) \). The gauge sector is obviously contained in the \( 24_G \), which has 12 gauge bosons belonging to the SM and another 12 which are new (called the X and Y gauge bosons) and get mass at the SU(5) breaking scale. In this minimal model, SU(5) is broken when the \( \Sigma \) Higgs picks up a VEV at around \( 10^{15.5} \) GeV.

As discussed in the introduction, the minimal SUSY SU(5) is still incomplete, as it does not give neutrino mass. In order to generate neutrino masses one needs to extend the model with additional multiplets. In the adjoint SUSY SU(5) model, one introduces additional matter chiral fields in the adjoint representation \( 24_F \equiv (8,1,0) \oplus (1,3,0) \oplus (3,2,-5/6) \oplus (3,2,5/6) \oplus (1,1,0) \equiv (\hat{\rho}_8, \hat{\rho}_3, \hat{\rho}_{(3,2)}, \hat{\rho}_{(3,2)}, \hat{\rho}_0) \). Neutrino masses are generated via the type I (mediated by \( \rho_0 \)) and type III (mediated by \( \rho_3 \)) seesaw. Alternatively, one could also introduce the \( 15_H \equiv (1,3,1) \oplus (3,2,1/6) \oplus (6,1,-2/3) \equiv (\hat{\Delta}_3, \hat{\Delta}_{(3,2)}, \hat{\Delta}_0) \) Higgs chiral multiplet. The \( \Delta_3 \) could give rise to neutrino masses by the type II seesaw mechanism.

3 RG running of gauge couplings in Adjoint SUSY SU(5)

The RG evolution of the three SM gauge couplings between the electroweak scale and the GUT scale in the \( \overline{MS} \) scheme up to one loop corrections and including threshold effects is given by

\[
2\pi (\alpha_i^{-1}(M_Z) - \alpha_G^{-1}(M_G)) = b_i \ln \left( \frac{M_G}{M_Z} \right) + \sum_j b_i(j) \ln \left( \frac{M_G}{M_j} \right)
\]

where the second term in the RHS of Eq. (1) give threshold corrections at one loop coming from particles whose mass \( M_j > M_Z \). The above equation relates the gauge couplings at \( M_Z \) to the gauge coupling at the GUT scale \( M_G \). The \( b_i \)'s are the co-efficients of the \( \beta \)-functions for gauge couplings \( g_i \) at the one loop level. They can be calculated for a given gauge group \( \mathcal{G}_i \otimes \mathcal{G}_j \) as

\[
b_i = -\frac{11}{3} T_G(R_i) d(R_j) + \frac{2}{3} T_F(R_i) d(R_j) + \frac{1}{3} T_S(R_i) d(R_j),
\]

where \( T(R_i) \) are the Casimir of the representation \( R_i \) under the group \( \mathcal{G}_i \) and \( d(R_j) \) is the dimension of the representation \( R_j \) under group \( \mathcal{G}_j \). The first, second and third terms
in Eq. (2) gives the contribution coming from gauge bosons, fermions and scalars in the model.

In Table I we give the individual contributions to $b_i$ for the different possible SM representations. We give these values separately for gauge bosons (G), fermions (F) and scalars (S) for the three gauge couplings. One can note from the Table I that the contribution $b_i$ for gauge bosons is always negative, while that for fermions and scalars is always positive. Since any extension of a given gauge theory entails addition of matter and Higgs fields without tampering with the gauge sector, and since every fermion and Higgs field brings a further negative contribution on the RHS of Eq. (1), extension of any GUT model results in reducing the slope of $\alpha_i^{-1}$ as a function of the energy scale. The extent of this reduction for a given gauge coupling $g_i$ depends on the representation of the relevant multiplet under the gauge group $G_i$. Therefore, even though every additional multiplet has the effect of reducing the slope of all the three $\alpha_i^{-1}$, the relative reduction between them is different for different multiplets.

If one imposes unification and eliminates the unified gauge coupling $\alpha_G^{-1}(M_G)$ from the Eq. (1) for the three SM gauge couplings, one gets two equations

$$\ln \frac{M_G}{M_Z} = \frac{\Delta_{12}}{B_{12}},$$

$$B_{23} = \frac{\Delta_{23}}{\Delta_{12}} B_{12},$$

with,

$$B_{ij} = B_{ij} + \sum_k B_{ij}(k) \frac{\ln(M_G/M_k)}{\ln(M_G/M_Z)},$$

where, $B_{ij} = b_i - b_j$ and $\Delta_{ij} = 2\pi(\alpha_i^{-1} - \alpha_j^{-1})$. Inserting the experimental measured values of $\alpha_i^{-1}$ at $M_Z$ ($\alpha_1^{-1}(M_Z) = 58.85$, $\alpha_2^{-1}(M_Z) = 29.46$, $\alpha_3^{-1}(M_Z) = 8.50$), we get

$$B_{23} = 0.713 B_{12}.$$  

One can use Table I to calculate the $b_i$'s for the SM and the MSSM. These values come out to be

$$b_{i}^{SM} \equiv \left(\frac{41}{10}, -\frac{19}{6}, -7\right),$$

$$b_{i}^{MSSM} = \left(\frac{33}{5}, 1, -3\right).$$

Using Eq. (7) one gets $B_{23}/B_{12} = 0.53$ for the SM, which is inconsistent with Eq. (6). Therefore, unification fails in the SM. For the MSSM on the other hand one gets using Eq.
| SM multiplet                | Particle type | $b_{U(1)y}$ | $b_{SU(2)_{L}}$ | $b_{SU(3)_{C}}$ |
|-----------------------------|---------------|-------------|-----------------|-----------------|
| (1,1,0)                     | All           | 0           | 0               | 0               |
|                            | G             | -11/5       | 0               | 0               |
|                            | F             | 2/5         | 0               | 0               |
|                            | S             | 1/5         | 0               | 0               |
| (1,1,1)                     |               |             |                 |                 |
| (1,2,1/2) & (1,2,-1/2)      | G             | -11/10      | -11/6           | 0               |
|                            | F             | 1/5         | 1/3             | 0               |
|                            | S             | 1/10        | 1/6             | 0               |
| (1,3,0)                     | G             | 0           | -22/3           | 0               |
|                            | F             | 0           | 4/3             | 0               |
|                            | S             | 0           | 1/3             | 0               |
| (8,1,0)                     | G             | 0           | 0               | -11             |
|                            | F             | 0           | 0               | 2               |
|                            | S             | 0           | 0               | 1/2             |
| (3,1,-1/3) & (3,1,1/3)      | G             | -11/15      | 0               | -11/6           |
|                            | F             | 2/15        | 0               | 1/3             |
|                            | S             | 1/15        | 0               | 1/6             |
| (3,2,1/6) & (3,2,-1/6)      | G             | -11/30      | -11/2           | -11/3           |
|                            | F             | 1/15        | 1               | 2/3             |
|                            | S             | 1/30        | 1/2             | 1/3             |
| (3,1,-2/3) & (3,1,2/3)      | G             | -44/15      | 0               | -11/6           |
|                            | F             | 8/15        | 0               | 1/3             |
|                            | S             | 4/15        | 0               | 1/6             |
| (3,2,-5/6) & (3,2,5/6)      | G             | -55/6       | -11/2           | -11/3           |
|                            | F             | 5/3         | 1               | 2/3             |
|                            | S             | 5/6         | 1/2             | 1/3             |
| (1,3,1) & (1,3,-1)          | G             | -33/5       | -22/3           | 0               |
|                            | F             | 6/5         | 4/3             | 0               |
|                            | S             | 3/5         | 1/3             | 0               |
| (6,1,-2/3) & (6,1,2/3)      | G             | -88/15      | 0               | -11/2           |
|                            | F             | 16/15       | 0               | 1               |
|                            | S             | 8/15        | 0               | 1/2             |

Table 1: The $\beta$-function co-efficients $b_{i}$ for the different SM representations. The SM representation is given in first column. The ‘G’, ‘F’ and ‘S’ in the second column stand for gauge bosons, fermions and scalars, respectively.
\( B_{23}/B_{12} = 0.714 \) nearly consistent with Eq. (6) and hence the experimental values of the coupling constants at the electroweak scale.

For the SM all particles are massless above the electroweak scale and hence \( B_{ij} = b_i - b_j \). As discussed above this fails to give unification of the gauge couplings. However, if one extends the model to include other massive particles which contribute to \( B_{ij} \) such that the increase \( \Delta(B_{23}) \) is greater than the increase \( \Delta(B_{12}) \), then one could get unification even without supersymmetry. In the minimal SU(5) all the massive particles above the electroweak scale are at the GUT scale and hence do not contribute to the gauge coupling running. In order to achieve unification one must therefore add additional matter multiplets with masses in the intermediate scale such that \( B_{23}/B_{12} \) can be raised. One could add either a \( 15_H \) to the minimal SU(5) or a \( 24_F \) in order to achieve unification. While the former leads to type II seesaw, the latter gives rise to type I+III seesaw. In the rest of this section we will focus on the SU(5) extensions with \( 24_F \) as it allows for the testing of the seesaw framework at the LHC.

The non-supersymmetric adjoint SU(5) with one family of \( 24_F \) receives threshold corrections from \( \rho_8 \equiv (8, 1, 0), \rho_3 \equiv (1, 3, 0) \) and \( \rho_{(3,2)} \equiv (3, 2, -5/6) \). The \( \rho_8 \) contributes to the running of \( \alpha_3^{-1} \) only while \( \rho_3 \) impacts the running of \( \alpha_2^{-1} \) only. The effect of \( \rho_{(3,2)} \) on the other hand is felt by all the three. If one restricts \( \rho_{(3,2)} \) to have mass close to the GUT scale then its impact on the RG running of gauge couplings can be reduced. Therefore, only \( \rho_3 \) gives threshold corrections to the running of \( \alpha_2^{-1} \) and since it is a fermion its \( b_2 \) contribution helps to reduce the value of \( B_{12} \). Likewise, the effect of threshold corrections due to \( \rho_8 \) impacts the running of \( \alpha_3^{-1} \) and together \( \rho_3 \) and \( \rho_8 \) can be constrained to have masses below the GUT scale such that \( B_{23}/B_{12} = 0.713 \) and unification constraint can be satisfied. In this model \( B_{12} \) gets threshold contribution from \( \rho_3 \) and \( \rho_{(3,2)} \) and one can easily relate the GUT scale \( M_G \) to \( M_{\rho_3} \). Using Eq. (3) and Table I one gets

\[
\log M_G = 16.4 - 0.3 \log M_{\rho_3}. \tag{9}
\]

For \( M_G = 10^{15.8} \) GeV the above equation gives \( M_{\rho_3} = 10^2 \) GeV. The Eq. (4) can next be used to find the mass of \( \rho_8 \) as

\[
\ln M_{\rho_8} = 5.7 + \log M_{\rho_3}, \tag{10}
\]

leading to the relation \( M_{\rho_8}/M_{\rho_3} \sim 10^{5.7} \) between the masses of the two particles. Therefore, while production of \( \rho_8 \) is impossible at LHC, the production of \( \rho_3 \) is possible via gauge interactions, making it possible to probe seesaw.

We next turn our attention at the SUSY adjoint SU(5) and impose the unification constraints given by Eqs. (3) and (4). In the SUSY adjoint SU(5) we have an extra \( \bar{5}_H \) multiplet. In addition, for every particle contribution we have to include the contributions coming from the corresponding superparticle. We have already seen that in the MSSM itself these additional superparticle contributions are enough to get gauge coupling unification at the right scale. In SUSY adjoint SU(5) we get threshold corrections from the additional particles as well. Assuming all superpartners of the standard model particles to be at the
Figure 1: Gauge coupling running for the adjoint SU(5) (dashed lines) and adjoint SUSY SU(5) (solid lines) models. For the adjoint SU(5) we have taken $M_{\rho_3} = 100$ GeV, $M_{\rho_8} = 10^{7.7}$ GeV and $M_{\rho_{(3,2)}} = 5 \times 10^{14}$, while all other parameters are at the GUT scale. For the adjoint SUSY SU(5) model $M_{SUSY} = 1$ TeV while all other particles have masses close to the GUT scale.

As a result, for $M_G \sim 10^{16}$ GeV we get $M_{\rho_3} \sim 10^{16}$ GeV. Therefore, testing seesaw at LHC will be impossible once SUSY is imposed in the adjoint SU(5) model.

We show in Fig. 1 the running of the gauge couplings for the adjoint SU(5) (thick lines) and the adjoint SUSY SU(5) (thin lines) models. The kinks in the running show the position of the masses of the SU(2) triplet and the color octet fermions. Note that the unified gauge coupling $\alpha_{G}^{-1}$ is lower for the adjoint SUSY SU(5). This is in fact a generic feature of any GUT model which is extended by adding more multiplets. Extending the multiplet content of the model always results in additional fermion and scalar contributions, both of which lower the slope of $\alpha_{G}^{-1}$ as well as the value of $\alpha_{G}^{-1}$.

4 The Symmetric SUSY Adjoint SU(5)

In this section we propose a supersymmetric SU(5) GUT model which generates neutrino masses by the seesaw mechanism, such that the mass of the seesaw mediating particle(s)
is in the TeV range, making it possible to produce them at the LHC, and hence test the seesaw mechanism. We propose to extend the SUSY SU(5) by adding three additional multiplets, a matter chiral supermultiplet $\hat{24}_F$ and the Higgs chiral supermultiplets $\hat{15}_H$ and $\bar{15}_H$. Since $15$ is symmetric and $24$ the adjoint representation under $SU(5)$, we name this model the symmetric adjoint SUSY SU(5). The renormalizable superpotential for this model is given by

$$W_{ren} = Y_{\hat{5}_F \hat{10}_F \bar{5}_H} + Y'_{\hat{10}_F \bar{10}_F \hat{5}_H} + Y_{\Delta \hat{5}_F \hat{15}_H \bar{5}_F} + Y_{\rho \hat{5}_F \hat{24}_F \hat{5}_H} + m_H \hat{\delta}_H + \mu \hat{\delta}_H \bar{15}_H \hat{\delta}_H$$

$$+ \mu \hat{\delta}_H \bar{15}_H \hat{\delta}_H + m_\rho Tr(24^2_F) + \lambda_\rho Tr(24^2_F \hat{24}_H) + m_\Sigma Tr(24^2_H) + \lambda_\Sigma Tr(24^3_H)$$

$$+ Y_{\Sigma \hat{5}_H \hat{24}_H \hat{5}_H} + m_\Delta Tr(\Sigma \hat{15}_H \Sigma \hat{24}_H) + \lambda_\Delta Tr(\hat{15}_H \hat{24}_H \hat{15}_H) + \lambda''_\Delta Tr(\hat{15}_H \hat{24}_H \hat{15}_H) + Y''_{\hat{24}_F \hat{10}_F \bar{15}_H}.$$ (12)

The $SU(5)$ gauge symmetry breaks to the SM when the $24_H$ Higgs gets a VEV. Subsequently, the VEV of $H_u$ and $H_d$ in $5_H$ and $\bar{5}_H$ breaks SM and generates the seesaw type I and type III masses for the neutrinos. In addition, the VEV of $\Delta_3$ in $15_H$ generates the type II seesaw masses for the neutrinos. Therefore, in our model neutrinos could get masses from all the three types of seesaw mechanisms. In what follows, we will first extract the mass spectrum of the particles in this model. We next look at the RG running of the SM gauge couplings.

### 4.1 Particle Mass Spectra

The relevant super-potential to generate mass of $\hat{24}_F$ particles is

$$W^M_\rho = m_\rho Tr(24^2_F) + \lambda_\rho Tr(24^2_F \hat{24}_H) + \frac{\gamma F}{\Lambda} Tr(24^2_F \hat{24}_H) + \frac{\delta F}{\Lambda} Tr(24^2_F) Tr(24^2_H) + \frac{\lambda_F}{\Lambda} [Tr(24_F \hat{24}_H)]^2,$$ (13)

where we have also included the dimension five effective operators that contribute to the mass of the particles belonging to $\hat{24}_F$. The scale $\Lambda$ could be associated with the Planck scale. The higher dimension effective operators have to be added anyway in the $SU(5)$ GUT in order to break the degeneracy between the masses of the charged lepton and the d-type quark masses. The above leads to the following expressions for the masses of $\hat{\rho}_0$, $\hat{\rho}_3$, $\hat{\rho}_8$ and $\hat{\rho}_{(3,2)}$

$$M_{\rho_0} = m_\rho - \frac{\lambda_\rho \nu_\Sigma}{\sqrt{30}} + \frac{\nu^2_\Sigma}{\Lambda} \left( \delta_F + \frac{7}{30} (\gamma_F + \lambda_F) \right),$$

$$M_{\rho_3} = m_\rho - \frac{3 \lambda_\rho \nu_\Sigma}{\sqrt{30}} + \frac{\nu^2_\Sigma}{\Lambda} \left( \delta_F + \frac{3}{10} (\gamma_F + \lambda_F) \right),$$

$$M_{\rho_8} = m_\rho + \frac{2 \lambda_\rho \nu_\Sigma}{\sqrt{30}} + \frac{\nu^2_\Sigma}{\Lambda} \left( \delta_F + \frac{2}{15} (\gamma_F + \lambda_F) \right),$$

$$M_{\rho_{(3,2)}} = m_\rho - \frac{\lambda_\rho \nu_\Sigma}{2\sqrt{30}} + \frac{\nu^2_\Sigma}{\Lambda} \left( \delta_F + \frac{13 \gamma_F - 12 \lambda_F}{60} \right).$$ (14)
respectively. The higher dimensional terms bring additional parameters such that we can tune all the above particles to be at any arbitrary scale. We will see below that to be able to test seesaw at LHC one should allow for $\hat{\rho}_3$ to have masses $\sim 10^{13}$ GeV and $\hat{\rho}_8$ at some intermediate scale, while the mass of $\hat{\rho}_{(3,2)}$ is constrained to be at the GUT scale.

The corresponding terms in the superpotential which contribute to the mass of the $\hat{24}_H$ are given by

$$W^M_\Sigma = \alpha Tr(\hat{24}_H^2) + \beta Tr(\hat{24}_H^3) + \frac{\gamma}{\Lambda} Tr(\hat{24}_H^4) + \frac{\delta}{\Lambda} (Tr(\hat{24}_H^2))^2, \quad (15)$$

where in the third and the fourth terms we have introduced higher dimensional effective operators suppressed by some heavy scale $\Lambda$. The masses of the $\hat{\Sigma}_8$, $\hat{\Sigma}_3$ and $\hat{\Sigma}_0$ then turn out to be

$$M_{\Sigma_8} = \alpha + \frac{6\beta v_\Sigma}{\sqrt{30}} + \frac{4\gamma v_\Sigma^2}{5\Lambda} + \frac{2\delta v_\Sigma^2}{\Lambda},$$
$$M_{\Sigma_3} = \alpha - \frac{9\beta v_\Sigma}{\sqrt{30}} + \frac{9\gamma v_\Sigma^2}{5\Lambda} + \frac{2\delta v_\Sigma^2}{\Lambda},$$
$$M_{\Sigma_0} = \alpha - \frac{3\beta v_\Sigma}{\sqrt{30}} + \frac{7\gamma v_\Sigma^2}{5\Lambda} + \frac{6\delta v_\Sigma^2}{\Lambda}. \quad (16)$$

With four free parameters, $\alpha$, $\beta$, $\gamma$ and $\delta$, we can tune the masses of the particles such that $\hat{\Sigma}_3$ and $\hat{\Sigma}_0$ are at the GUT scale while $\hat{\Sigma}_8$ is light, and at the same time satisfy the minimization condition of the scalar potential required for the spontaneous breaking of the SU(5) gauge symmetry.

Finally, we give the mass spectrum of $\hat{15}_H$. Following terms from the superpotential Eq. (12) give contribution to the masses of the $\hat{15}_H$ (and $\hat{\bar{15}}_H$) multiplets:

$$W^M_\Delta = m_\Delta Tr(\hat{15}_H \hat{\bar{15}}_H) + \lambda_\Delta Tr(\hat{15}_H \hat{\bar{15}}_H \hat{24}_H) + \lambda'_\Delta Tr(\hat{15}_H \hat{\bar{24}}_H \hat{\bar{\bar{24}}}_H). \quad (17)$$

We have not shown the higher dimensional operators in the expression above. While they are indeed present, they do not make any difference to our discussion here and hence we do not explicitly show them. The masses of the $\hat{\Delta}_3$, $\hat{\Delta}_6$ and $\hat{\Delta}_{(3,2)}$ multiplets are obtained as follows:

$$M_{\Delta_3} = m_\Delta - \frac{6\lambda_\Delta v_\Sigma}{\sqrt{30}},$$
$$M_{\Delta_6} = m_\Delta + \frac{4\lambda_\Delta v_\Sigma}{\sqrt{30}},$$
$$M_{\Delta_{(3,2)}} = m_\Delta - \frac{\lambda_\Delta v_\Sigma}{\sqrt{30}}. \quad (18)$$

where $\lambda_\Delta = \lambda'_\Delta + \lambda''_\Delta$. Here again we see that we can tune one of the multiplets to be at TeV scale. The masses of other multiplets present in $\hat{15}_H$ are then of GUT scale.
4.2 Constraints from Gauge Coupling Unification

In the following we demand that the SM gauge couplings unify at the GUT scale. This constrains the particle masses of the model. We try to find solutions of the particle mass spectra of our model where seesaw mediating particle can be below the TeV scale. We find that in this supersymmetric version of extended SU(5) GUT model, the only seesaw mediating particle that can be made light turns out to be $\hat{\Delta}_3 \equiv \overline{(1,3,1)}$.

We discuss this possibility in a little more detail. The role of the different SM representations belonging to $\hat{24}_F$ in the running of the gauge couplings have been discussed in the previous section. The representations coming from $15_H$ (and $\bar{15}_H$), are $(1,3,1) \oplus (3,2,1/6) \oplus (6,1,-2/3) \equiv (\Delta_3, \Delta_{(3,2)}, \Delta_6)$. Therefore, while $\Delta_3$ and $\Delta_{(3,2)}$ affect the running of $\alpha_{1/2}^{-1}$, $\Delta_{(3,2)}$ and $\Delta_6$ lower the slope of $\alpha_3^{-1}$. All the three supermultiplets are non-trivial under $U(1)_Y$ and hence affect the running of $\alpha_{1/1}^{-1}$. For $M_{\Delta_3} \sim 100$ GeV, the gauge couplings are found to unify if $\hat{\rho}_3$, $\hat{\rho}_8$, and $\hat{\Sigma}_8$ are allowed to be at an intermediate scale, while all other non-SM particles are at the GUT scale. The GUT scale is related to $M_{\Delta_3}$ and $M_{\rho_3}$ as

$$\log M_G = 21.36 + 0.06 \log M_{\Delta_3} - 0.396 \log M_{\rho_3}.$$ (19)

The Eq. (19) shows that the GUT scale actually has a very mild dependence on the mass of the triplet scalar. On the other hand a small change to the GUT scale makes a big change to the mass of $\hat{\Delta}_3$. If one chooses $M_{\Delta_3} \sim 100$ GeV, then a GUT scale of $M_G \sim 10^{16}$ GeV can be obtained if the $\hat{\rho}_3$ mass is constrained to be $M_{\rho_3} \sim 10^{13}$ GeV. We will discuss the implications for this in the next section when we discuss neutrino masses and related phenomenology. Note also from Eq. (19) that smaller values of $M_{\Delta_3}$ are associated with smaller values of $M_G$.

The product of the masses of $\hat{\rho}_8$ and $\hat{\Sigma}_8$ are related to $\hat{\Delta}_3$ and $\hat{\rho}_3$ masses as

$$\log(M_{\rho_8} M_{\Sigma_8}) = -8.03 + 1.23 \log M_{\Delta_3} + 1.3 \log M_{\rho_3}^2.$$ (20)

For $M_{\Delta_3} \sim 100$ GeV and $\hat{\rho}_3 \sim 10^{13.3}$ GeV, the product of the $\hat{\rho}_8$ and $\hat{\Sigma}_8$ masses turns out to be $10^{11.7}$ GeV. Therefore in this framework, it is possible to have two particles, the seesaw mediating $(1,3,1)$, and a SU(3) octet $(8,1,0)$ at the LHC scale. In fact, Eq. (20) leads to the following situations:

- $\hat{\rho}_8$ is within the reach of the LHC and $\hat{\Sigma}_8$ has mass $\sim 10^{10}$ GeV,
- $\hat{\Sigma}_8$ is within the reach of the LHC and $\hat{\rho}_8$ has mass $\sim 10^{10}$ GeV,
- both $\hat{\rho}_8$ and $\hat{\Sigma}_8$ have masses in the intermediate regime.

In all the three possible cases mentioned above, the masses of $\hat{\Sigma}_8$ and $\hat{\rho}_8$ must below the GUT scale for unification of the gauge couplings. This requires that the $\hat{\Sigma}_8$ be split from the masses of $\hat{\Sigma}_0$ and $\hat{\Sigma}_3$, which are close to the GUT scale. Similarly, we require $M_{\rho_3}$ and $M_{\rho_8}$ masses to be split from the mass of $\hat{\rho}_{(3,2)}$ which is constrained to be close to the GUT scale. The $\hat{\rho}_0$ mass is unconstrained from gauge coupling contraints. We have discussed
the mass spectra of $\hat{2}_4 H$ and $\hat{2}_4 F$ in the previous subsection and have shown that such a splitting can be consistently obtained from the superpotential.

We show in Fig. 2 the running of the gauge couplings for the symmetric adjoint SUSY SU(5) model where we choose $M_{SUSY} = 1$ TeV, $M_{\Delta_3} = 100$ GeV, $M_{\rho_3} = 10^{13.3}$ GeV, and $M_{\rho_8} = M_{\Sigma_8} = 7.7 \times 10^5$ GeV.

Figure 2: Gauge coupling running for the symmetric adjoint SUSY SU(5) with $M_{SUSY} = 1$ TeV, $M_{\Delta_3} = 100$ GeV, $M_{\rho_3} = 10^{13.3}$ GeV, and $M_{\rho_8} = M_{\Sigma_8} = 7.7 \times 10^5$ GeV.

In the left panel of Fig. 3 we show the contours for the value of $M_{\rho_3}$ for which one gets unification in the log $M_G$ - log $M_{\Delta_3}$ plane. The leftmost dashed line corresponds to $M_{\rho_3} = 10^{14.4}$ GeV, while the rightmost dashed line is for $M_{\rho_3} = 10^{12.6}$ GeV. The value of log $M_{\rho_3}$ increases by unit of 0.2 for every dashed line, as we move upward from 14.4 which borders the lightest color of the figure to 12.6 which borders the darkest one (almost at the lower-right edge of the plot). This covers the range of $M_G$ from $10^{15.8}$ to $10^{16.5}$ GeV. The plot shows that for $M_{\rho_3} \lesssim 10^{13.2}$ GeV, $M_G \gtrsim 10^{16.2}$ GeV, while all values of $M_{\Delta_3}$ in the LHC range are possible. The right panel of Fig. 3 shows the contours of $(M_{\rho_8} M_{\Sigma_8})$ in the log $M_G$ - log $M_{\Delta_3}$ plane. The right-most dashed line bordering the darkest part of the plot corresponds to $(M_{\rho_8} M_{\Sigma_8}) = 10^{11}$ GeV and for each subsequent dashed line log$(M_{\rho_8} M_{\Sigma_8})$ increases by 0.5. Note that Eq. (20) shows that fixing $(M_{\rho_8} M_{\Sigma_8})$ fixes the value of $M_{\rho_3}$ for a given value of $M_{\Delta_3}$, and therefore both panels in this figure are equivalent way of showing the parameter region.
Figure 3: Left panel shows contours of $M_{\rho_3}$ for which one gets unification in the log $M_G$ -log $M_{\Delta_3}$ plane. The leftmost dashed line corresponds to $M_{\rho_3} = 10^{14.4}$ GeV, while the rightmost dashed line is for $M_{\rho_3} = 10^{12.6}$ GeV. The value of log $M_{\rho_3}$ increases by unit of 0.2 for every dashed line. The right panel of the figure shows the contours of $(M_{\rho_8} M_{\Sigma_8})$ in the log $M_G$-log $M_{\Delta_3}$ plane. The right-most dashed line bordering the darkest part of the plot corresponds to $(M_{\rho_8} M_{\Sigma_8}) = 10^{11}$ GeV and each for each subsequent dashed line log$(M_{\rho_8} M_{\Sigma_8})$ increases by 0.5.

5 Phenomenological Consequences of the Model

5.1 Neutrino Masses and Mixing

In this subsection we discuss neutrino mass generation in our symmetric adjoint SUSY SU(5) model. The part of the superpotential which is involved in the generation of neutrino mass is

$$W_\nu = W_\nu^Y + W_\rho^M + W_\Delta,$$

(21)

where $W_\rho^M$ is given in Eq. (13) and $W_\nu^Y$ is the Yukawa part of the superpotential given by

$$W_\nu^Y = Y_{\Delta_3} \delta_{\hat{F}_8, \hat{F}_7} 1\hat{5}_H + Y_\alpha \delta_{\hat{F}_8, \hat{2}\hat{4}_F} \hat{2}\hat{4}_F \hat{\tilde{5}}_H + \frac{1}{\Lambda} \hat{\delta}_F \left( Y_{\alpha} \hat{2}\hat{4}_F \hat{2}\hat{4}_H + Y_{2}\hat{2}\hat{4}_H \hat{2}\hat{4}_F + Y_{3} \text{Tr}(\hat{2}\hat{4}_F \hat{2}\hat{4}_H) \right) \hat{\tilde{5}}_H,$$

(22)

where we have kept the higher dimensional terms. The first term gives rise to the type II seesaw Majorana mass term when $\Delta_3$ in $\hat{1}\hat{5}_H$ acquires a VEV $v_\Delta$. The other terms in Eq. (22) give contribution to the type I and type III seesaw due to electroweak symmetry breaking. The part $W_\Delta$ in Eq. (21) contains the Higgs part relevant for type-II seesaw

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and is given by

$$W_\Delta = \mathcal{W}_\Delta^M + \mu_\Delta \tilde{\delta}_H \bar{15}_H \bar{\delta}_H^T + \mu_\Delta \tilde{\delta}_H^T \bar{15}_H \hat{\delta}_H,$$

(23)

where we have kept only the renormalizable terms. The first term in Eq. (22) gives a contribution $m_{II} = Y_\Delta v_\Delta$ to the low energy neutrino mass matrix. One can use the Eq. (17) and Eq. (23) to get the triplet Higgs VEV $v_\Delta$ as

$$v_\Delta = \frac{Y_\Delta \mu v_u^2}{M_{\Delta_3}^2},$$

(24)

where

$$\mu = \mu_\Sigma M_{\Delta_3}.$$

(25)

Hence, in this model the type II seesaw contribution to neutrino masses comes out to be

$$m_{II} = \frac{Y_\Delta \mu v_u^2}{M_{\Delta_3}^2}$$

(26)

where $v_u$ is the VEV of the standard Higgs $H_u$. Note that the coupling $\mu_\Delta$ is dimensionless.

There are of course several higher dimensional terms such as

$$\mathcal{W}_{\Delta NR} = \frac{\mu_\Delta}{\Lambda} \bar{\delta}_H \bar{15}_H \bar{\delta}_H^T 24_H + \frac{Y_\Delta}{\Lambda} \tilde{\delta}_F \tilde{15}_F \tilde{\delta}_F^T 24_F + \frac{1}{\Lambda} \tilde{\delta}_H \bar{15}_H \bar{\delta}_H^T Tr(24_H^2) + \frac{1}{\Lambda} \tilde{\delta}_H \tilde{\delta}_H \bar{15}_H \bar{\delta}_H^T Tr(24_F^2)$$

$$+ \frac{1}{\Lambda} \tilde{\delta}_H \tilde{\delta}_H Tr(24_H^2) + \frac{1}{\Lambda} \tilde{\delta}_F \tilde{\delta}_F Tr(24_F^2) + \frac{1}{\Lambda} \tilde{\delta}_F \tilde{\delta}_F \bar{15}_F \bar{\delta}_F^T Tr(24_F^2)$$

(27)

which should be included in Eqs. (22) and (23) for consistency. All these terms are suppressed by either $v_\Sigma/\Lambda$ (or $v_3^2/\Lambda$) or $v_\Delta/\Lambda$ (or $v_2^2/\Lambda$). Since $v_\Delta$ is constrained to be extremely small by the smallness of neutrino mass (cf. Eq. (26)), terms proportional to $v_\Delta/\Lambda$ (or $v_2^2/\Lambda$) can be safely neglected. We have explicitly checked that the effect of all other terms can also be neglected since $v_\Sigma/\Lambda \sim 10^{-3}$ and hence they all give very small correction compared to the leading term, which anyway are present in the renormalizable part of the superpotential.

As discussed before, in this model the neutrino mass matrix can get contribution from type I and type III seesaw as well. Thus the complete neutrino mass matrix is given by

$$M_\nu = \frac{1}{2} m_{D_0} M_{\rho_0}^{-1} m_{T_{D_0}} + \frac{2 Y_\Delta \mu v_u^2}{M_{\Delta_3}^2} + \frac{1}{2} m_{D_3} M_{\rho_3}^{-1} m_{T_{D_3}}$$

(28)

where $m_{D_0}$ and $m_{D_3}$ are as follows,

$$m_{D_0} = -\frac{3 v_u Y}{\sqrt{30}} + \frac{v_\Sigma v_u}{\Lambda} \left( Y^3 + \frac{9}{30} (Y^1 + Y^2) \right)$$

(29)

$$m_{D_3} = -\frac{v_u Y}{\sqrt{2}} + \frac{3 v_3 v_u}{\sqrt{60} \Lambda} \left( Y^1 + Y^2 \right),$$

(30)
and $M_{\rho_0}$ and $M_{\rho_3}$ given in Eq. (14) are the masses of the singlet and triplet fermions of $24_F$, respectively. The first, second and third terms in Eq. (28) are the contributions from type I, II and III seesaw, respectively. Since $M_{\Delta_3} \sim 100$ GeV in this model, $Y_{\Delta} \mu \sim M_{\nu}$. Therefore, for $Y_{\Delta} \sim 1$ and $\mu \sim 0.1 \text{ eV}$, we get significant contribution to $M_{\nu}$ from type II seesaw. We have seen that gauge coupling unification gives $M_{\rho_3} \sim 10^{13}$ GeV. Therefore, the contribution to the neutrino mass matrix from type III seesaw is also expected to be significant under the natural assumption of $Y \sim 1$. Gauge coupling unification puts absolutely no constraint on the mass of $\rho_0$. We have seen in Eq. (14) that the mass of $\rho_0$ can be tuned to any desired value. Therefore, we have type I plus type II plus type III seesaw in this model.

5.2 Collider Signatures and Lepton Flavor Violation

In the previous section we have seen that the $\Delta_3$ belonging to the $\bar{5}_H$ representation is predicted to be of 100 GeV mass range in our model. This opens up the possibility of testing the neutrino mass generation mechanism at LHC by directly observing the type II seesaw mediating $\Delta_3$. The potential of testing type II seesaw at LHC has been extensively studied in the literature [11, 17, 18]. The best way to probe type II seesaw is by observing the doubly charged Higgs scalar through its decay modes

$$\begin{align*}
\Delta^{++} & \rightarrow l^+l^+ \\
\Delta^{++} & \rightarrow W^+W^+ \\
\Delta^{++} & \rightarrow \Delta^+W^+ \\
\Delta^{++} & \rightarrow \Delta^+\Delta^+ \\
\Delta^{++} & \rightarrow \bar{\Delta}^+\bar{\Delta}^+ ,
\end{align*}$$

(31)

where $\Delta^{++}$ and $\Delta^+$ are respectively the doubly and singly charged Higgs scalar in the mass basis. This model has two Higgs doublets and two Higgs triplets at the electroweak scale. Therefore, we have 2 doubly charged Higgs, 3 singly charged Higgs and 7 chargeless Higgs - 4 of which are CP even and 3 which are CP odd. The branching ratios of the various competing channels in Eq. (31) depends crucially on the VEV $v_{\Delta}$, mass $M_{\Delta_3}$ and Yukawa coupling $Y_{\Delta}$. For $M_{\Delta_3} \approx 300$ GeV, the decay of $\Delta^{++}$ into the dilepton channel dominates over $W$ production channels if $v_{\Delta} \lesssim 10^{-4}$ GeV [17,18]. This limiting value of $v_{\Delta}$ increases as $M_{\Delta_3}$ increases. The decay width of the dilepton channel depends on the strength of the Yukawa coupling matrix $Y_{\Delta}$. More importantly, the lepton flavor in the final state of the dilepton channel depends on the individual matrix elements of $Y_{\Delta}$. This provides a one-to-one correspondence between the neutrino mass matrix and LHC signatures in pure type II seesaw models. However, in our model neutrino masses could have contributions from all three types of seesaw. Therefore, it is possible that we do not have a one-to-one correspondence between the neutrino experiments and the signal at LHC. One can turn this argument around and say that any discrepancy between the neutrino and LHC experiments would point towards a hybrid seesaw model with scalar triplets, such as ours.
The particles $\tilde{\Delta}^+$ in Eq. (31) are charginos in the mass basis. Including the contribution coming from the Higgs triplet field and the MSSM charginos, in our model we have 3 singly charged charginos ($\tilde{\Delta}^+$). In addition we also have one doubly charged chargino ($\tilde{\Delta}^{++}$) and 6 neutralinos ($\tilde{\Delta}^0$) in our model.

The singly charged Higgs scalar decays via
\[
\Delta^+ \to \ell^+\nu,
\Delta^+ \to \Delta^0W^+.
\] (32)

Since we have imposed supersymmetry we have 100 GeV mass range fermionic partners of the charged Higgs particles. Some of their decay modes are
\[
\tilde{\Delta}^{++} \to \tilde{\ell}\ell,
\tilde{\Delta}^+ \to \tilde{\nu}/\tilde{\ell}\tilde{\nu},
\tilde{\Delta}^{++} \to W^+\Delta^+,
\tilde{\Delta}^{++} \to \Delta^+\Delta^+,
\tilde{\Delta}^0 \to \tilde{\Delta}^0\Delta^0,
\tilde{\Delta}^+ \to \tilde{\Delta}^+\Delta^0,
\tilde{\Delta}^+ \to \tilde{\Delta}^0\Delta^+.
\] (33)

From Eq. (12) one can see that the octet fermions $\rho_8$ and $\tilde{\Sigma}_8$ decay via channels such as
\[
\rho_8 \to d^C T,
\rho_8 \to q^C \tilde{\Delta}_6,
\tilde{\Sigma}_8 \to \tilde{\Delta}_{(3,2)}\tilde{\Delta}_{(3,2)}
\] (34)

while the octet scalars decay through
\[
\tilde{\rho}_8 \to q^C \tilde{\Sigma}_6,
\tilde{\rho}_8 \to d^C \tilde{T}/d^C T,
\Sigma_8 \to \tilde{\Delta}_{(3,2)}\tilde{\Delta}_{(3,2)}
\] (35)

Since all these modes involve a GUT scale particle, which have to be produced off-shell, the decay of the octet fermions and scalar happen via effective operators which are suppressed by the GUT scale and hence these particles have very long lifetimes.

5.3 Proton Decay

There are no additional proton decay mediating diagrams in our model. If we assume that the contribution of dimension five operators in SUSY SU(5) is negligible [20], then proton decay is mediated only by dimension six operators involving particles with masses of the GUT scale, viz., the superheavy gauge bosons $X$ and $Y$, the SU(3) triplets $T$ and
Therefore, the decay width for proton decay in our model is same as that in minimal SUSY SU(5) where contribution from dimension five operators are negligible. This has been discussed widely in the literature (for a review see [21]). The predicted decay width for the following channels are [22]

\[
\tau(p \to \pi^+\bar{\nu}) = 1.44 \times 10^{-31} \left( \frac{M_G/\text{GeV}}{\alpha_G^2} \right)^4 \text{years},
\]

\[
\tau(p \to K^+\bar{\nu}) = 4.31 \times 10^{-30} \left( \frac{M_G/\text{GeV}}{\alpha_G^2} \right)^4 \text{years}.
\]

The relation between \(M_{\rho_3}, M_G\) and \(M_{\Delta_3}\) has been discussed in the previous section. In particular, one can see this in Eq. (19) and Fig. 3. Since \(\Delta_3\) is testable at LHC and both \(\Delta_3\) and \(\rho_3\) are related to neutrino masses, it could be possible to related the predictions at proton decay signatures at large scale future detectors [23] with the results from the LHC and neutrino oscillation experiments.

6 Conclusions

The minimal SU(5) cannot explain the presence of neutrino masses and must be necessarily extended. Addition of SU(5) singlets gives rise to type I seesaw, addition of 15\(_H\) of SU(5) gives type II seesaw masses, while the extension with 24\(_F\) makes it possible to generate type III seesaw mass term. With the LHC running, it is pertinent to expect that one could probe seesaw at this collider experiment. An obvious pre-requisite for this is that the mass of the seesaw mediating particle should be within the reach of the LHC, and hence should have masses within 1 TeV. In this context it was rather exciting to note that in the SU(5) model extended with the 24\(_F\) multiplet, known as adjoint SU(5) in the literature, gauge unification imposed that the mass of the seesaw mediating \(\rho_3\) should be close to 100 GeV range, making it accessible at LHC. However, once this model is supersymmetrized, we found that mass of \(\rho_3\) should be close to the GUT scale in order to get gauge coupling unification. Therefore, seesaw in the SUSY adjoint SU(5) cannot be probed at the LHC. The SU(5) GUT model extended by a 15\(_H\) also does not predict 100 GeV scale type II seesaw mediating triplet scalar masses.

We proposed a SUSY SU(5) GUT model extended with a 24\(_F\) matter chiral field, and 15\(_H\) and \(\bar{15}_H\) Higgs chiral fields. We call this model the symmetric adjoint SUSY SU(5). In principle, this model can get contribution from type I, II as well as III seesaw mechanisms. We showed that it is possible to consistently predict masses for triplet scalar and triplet fermion in the 100 GeV and 10\(^{13}\) GeV range respectively, while the mass of \(\rho_0\) is unconstrained. This gives type I plus type II plus type III seesaw mass term for the neutrinos. The triplet fermion masses in the 10\(^{13}\) GeV range allows for type III seesaw with Yukawa couplings of the order of 1. The triplet scalars with masses \(\sim 100 \text{ GeV}\) range can be produced at the LHC. We briefly discussed the collider phenomenology of this model and prediction for proton decay.
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