Quantum Theory in Design

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Abstract

In this paper, we use braiding diagrams to present rules of shapes and designs. That is, we design colour, design size, design brightness, design codes by means of braiding.

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0 Introduction

The Yang-Baxter equation first came up in a paper by Yang as factorization condition of the scattering S-matrix in the many-body problem in one dimension and in the work of Baxter on exactly solvable models in statistical mechanics. It has been playing an important role in mathematics and physics (see [BD82], [YG89]). Attempts to find solutions of the Yang-Baxter equation in a systematic way have led to the theory of quantum groups [Ka95].

Shape grammars have been used as a computational design tool for over two decades. Shape grammars are a production system created by taking a sample of the whole for which one is trying to write a language [St06]. From this sample a vocabulary of shapes can be written that represent all the basic forms of that sample. By defining the spatial relationships between those forms and how the forms are related to each other, shape rules can be written. A shape rule consists of a left and right side. If the shape in the left side matches a shape in a drawing then the rule can be applied, and the matching shape changes to match the right side of the rule. The shape rules allow the addition and subtraction of shapes, which in the end are perceived as shape modifications. These shape rules, combined with an initial shape, produce a shape grammar that represents the language of the design [St06]. Shapes themselves can exist as points, lines, planes, volumes, or any combination thereof [St06]. All shape generation must start with an initial shape: a point, a coordinate axis, or some foundation from which to start the shape grammar. If
the grammar is going to end, it can end with a terminal rule, which prevents any other rules from being applied after it. This forces there to be closure in the rule sequence. Alternatively, a design sequence can continue indefinitely and designs could be chosen at any point in the design process. The method discussed here fundamentally changes the method of developing the shape grammar. Mackenzie [Ma69] demonstrates in a simplified case that if the fundamental shapes in the language are defined, the relationship between the shapes can be inferred through examples. These inferred relationships can then be mapped to trees which in turn can be used to automatically create shape grammar rules for that language. While this was shown effective in a particular use, it is more common in practice that the vocabulary of the design to be used as the foundation of a shape grammar is determined by the creator of the grammar. The creator of the grammar looks at the sample and subjectively derive the vocabulary. From that vocabulary, the rules are formed based upon the creators experience and intention. It is quite possible that two different persons looking at the same sample of shapes would create two very different shape grammars.

In [OCB08], the results from the principal component analysis are used to create a new coupe shape grammar based upon these discovered shape relationships. The shape grammar is then used to create new coupe vehicles. Although the focus of this paper is on vehicle design, the methods developed here are applicable to any class of physical products based on a consistent form language. In [Ho10], visual 3D spatial grammars are studied.

In this paper we use braiding diagrams to represent rule of shapes and designs. This simplifies the expression of designs. The shape rules allow braiding of two shapes, which in the end are perceived as shape modifications. We represent new design by means of restricted PBW of Nichols algebras. That is, we designs in braided tensor categories. We design colour, design size, design brightness, design codes by means of braiding. This completely is a new method and will be applied more and more in various designs.

1 Preliminaries

We begin with the tensor category (see [Ma98], [Ka95] and [Zh99]). We define the product \( \mathcal{C} \times \mathcal{D} \) of two category \( \mathcal{C} \) and \( \mathcal{D} \) whose objects are pairs of objects \((U, V) \in (ob\mathcal{C}, ob\mathcal{D})\) and whose morphisms are given by

\[
\text{Hom}_{\mathcal{C} \times \mathcal{D}}((V, W)(V', W')) = \text{Hom}_{\mathcal{C}}(V, V') \times \text{Hom}_{\mathcal{D}}(W, W').
\]

Let \( \mathcal{C} \) be a category and \( \otimes \) be a functor from \( \mathcal{C} \times \mathcal{C} \) to \( \mathcal{C} \). This means

(i) we have object \( V \otimes W \) for any \( V, W \in ob\mathcal{C} \); 

(ii) we have morphism \( f \otimes g \) from \( U \otimes V \) to \( X \otimes Y \) for any morphisms \( f \) and \( g \) from \( U \) to \( X \) and from \( V \) to \( Y \);
(iii) we have
\[(f \otimes g)(f' \otimes g') = (ff') \otimes (gg')\]
for any morphisms \(f : U \to X, g : V \to Y, f' : U' \to V\) and \(g' : V' \to V\);
(iv) \(id_{U \otimes V} = id_U \otimes id_V\).

Let \(\otimes \tau\) denote the functor from \(C \times C\) to \(C\) such that \((\otimes \tau)(U,V) = (V \otimes U)\) and \((\otimes \tau)(f,g) = g \otimes f\), for any objects \(U,V,X,Y\) in \(C\), and for any morphisms \(f : U \to X\) and \(g : V \to Y\).

An associativity constraint \(a\) for tensor \(\otimes\) is a natural isomorphism
\[a : \otimes(\otimes \times id) \to \otimes(id \times \otimes)\].

This means that, for any triple \((U,V,W)\) of objects of \(C\), there is a morphism \(a_{U,V,W} : (U \otimes V) \otimes W \to U \otimes (V \otimes W)\) such that
\[a_{U',V',W'}((f \otimes g) \otimes h) = (f \otimes (g \otimes h))a_{U,V,W}\]
for any morphisms \(f, g\) and \(h\) from \(U\) to \(U'\), from \(V\) to \(V'\) and from \(W\) to \(W'\) respectively.

Let \(I\) be an object of \(C\). If there exist natural isomorphisms
\[l : \otimes(I \times id) \to id\quad\text{and}\quad r : \otimes(id \times I) \to id\ ,\]
then \(I\) is called the unit object of \(C\) with left unit constraint \(l\) and right unit constraint \(r\).

**Definition 1.1.** \((C, \otimes, I, a, l, r)\) is called a tensor category if \(C\) is equipped with a tensor product \(\otimes\), with a unit object \(I\), an associativity constraint \(a\), a left unit constraint \(l\) and a right unit constraint \(r\) such that the Pentagon Axiom and the Triangle Axiom are satisfied, i.e.
\[(id_U \otimes a_{V,W,X})a_{U,V \otimes W,X}(a_{U,V,W} \otimes id_X) = a_{U,V,W \otimes X}a_{U \otimes V,W,X}\]
and
\[(id_V \otimes l_W)a_{V,I,W} = r_V \otimes id_W\]
for any \(U,V,W,X \in \text{ob}C\).

Furthermore, if there exists a natural isomorphism
\[C : \otimes \to \otimes \tau\]
such that the Hexagon Axiom holds, i.e.
\[a_{V,W,U}C_{U,V \otimes W}a_{U,V,W} = (id_V \otimes C_{U,W})a_{V,U,W}(C_{U,V} \otimes id_W)\]
and
\[a_{W,U,V}^{-1}C_{U \otimes W,V}a_{U,V,W}^{-1} = (C_{U,W} \otimes id_V)a_{U,W,V}^{-1}(id_U \otimes C_{V,W}),\]
for any \( U,V,W \in \text{ob}\, C \), then \((C, \otimes, I, a, l, r, C)\) is called a braided tensor category. In this case, \( C \) is called a braiding of \( C \). If \( C_{U,V} = C_{V,U}^{-1} \) for any \( U,V \in \text{ob}\, C \), then \((C, C)\) is called a symmetric braided tensor category or a symmetric tensor category. Here functor \( \tau : C \times C \to C \times C \) is the flip functor defined by \( \tau(U \times V) = V \times U \) and \( \tau(f \times g) = g \times f \), for any \( U,V \in C \), and morphisms \( f \) and \( g \). Note that we denote the braiding \( C \) in braided tensor category \((C, \otimes, I, a, l, r, C)\) by \( ^C C \) sometimes.

**Example 1.2. (The tensor category of vector spaces )** The most fundamental example of a tensor category is given by the category \( C = \text{Vect}(k) \) of vector spaces over field \( k \). \( \text{Vect}(k) \) is equipped with tensor structure for which \( \otimes \) is the tensor product of the vector spaces over \( k \), the unit object \( I \) is the ground field \( k \) itself, and the associativity constraint and unit constraint are the natural isomorphisms

\[
a_{U,V,W}((u \otimes v) \otimes w) := u \otimes (v \otimes w) \quad \text{and} \quad l_V(1 \otimes v) := v := r_V(v \otimes 1)
\]

for any vector space \( U,V,W \) and \( u \in U, v \in V, w \in W \).

Furthermore, the most fundamental example of a braided tensor category is given by the tensor category \( \text{Vect}(k) \), whose braiding is usual twist map from \( U \otimes V \) to \( V \otimes U \) defined by sending \( a \otimes b \) to \( b \otimes a \) for any \( a \in U, b \in V \).

Now we define some notations. If \( f \) is a morphism from \( U \) to \( V \) and \( g \) is a morphism from \( V \) to \( W \), we denote the composition \( gf \) by

\[
\begin{array}{ccc}
U & U & U \\
\downarrow g & \downarrow f & \downarrow g \\
V & V & V \\
\end{array}
\]

We usually omit \( I \) and the morphism \( id_I \) in any diagrams. In particular, If \( f \) is a morphism from \( I \) to \( V \), \( g \) is a morphism from \( V \) to \( I \),we denote \( f \) and \( g \) by

\[
\begin{array}{ccc}
\uparrow f & \uparrow f & \uparrow f \\
V & V & V \\
\end{array}
\]

If \( f \) is a morphism from \( U \otimes V \) to \( P \), \( g \) is a morphism from \( U \otimes V \) to \( I \) and \( \zeta \) is a morphism from \( U \otimes V \) to \( V \otimes U \), we denote \( f \), \( g \) and \( \zeta \) by

\[
\begin{array}{ccc}
\uparrow f & \uparrow f & \uparrow f \\
P & U & V \\
\end{array}
\]

In particular, we denote the braiding \( C_{U,V} \) and its inverse \( C_{U,V}^{-1} \) by

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

In particular, we denote the braiding \( C_{U,V} \) and its inverse \( C_{U,V}^{-1} \) by

\[
\begin{array}{ccc}
U & V & U \\
\downarrow & \downarrow & \downarrow \\
V & U & V \\
\end{array}
\]

4
\(\xi\) is called an \(R\)-matrix of \(\mathcal{C}\) if \(\xi\) is a natural isomorphism from \(\otimes\) to \(\otimes \tau\) and for any \(U, V, W \in \text{ob}\mathcal{C}\), the Yang-Baxter equation of \(\mathcal{C}\):

\[
(YBE):
\]

\[
\begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array}
= \begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array}
\]

holds. In particular, the above equation is called the Yang-Baxter equation on \(V\) when \(U = V = W\).

**Lemma 1.3.** (i) The braiding \(C\) of braided tensor category \((\mathcal{C}, \tau)\) is an \(R\)-matrix of \(\mathcal{C}\), i.e.

\[
\begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array} = \begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array}
\]

holds.

(ii)

\[
\begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array} = \begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array} = \begin{array}{cccc}
U & V & W & U \\
\xi & \xi & \xi & \xi \\
W & V & U & W \\
\end{array}
\]

Proof. (i)
(ii) We have

\[
\begin{array}{ccc}
U \otimes V W & U \otimes W W & U \otimes V W \\
\text{by naturality} & \text{by Hexagon Axiom} & \\
C_{U \otimes V W} & C_{U,V} \otimes W & C_{U,V}
\end{array}
\]

Similarly, we can show the second equation. □

Now we give some concepts as follows: Assume that \( H, A \in \text{ob } C \), and

\[
\begin{align*}
\alpha : H \otimes A &\rightarrow A, & \beta : H \otimes A &\rightarrow H, \\
\phi : A &\rightarrow H \otimes A, & \psi : H &\rightarrow H \otimes A, \\
m_H : H \otimes H &\rightarrow H, & m_A : A \otimes A &\rightarrow A, \\
\Delta_H : H &\rightarrow H \otimes H, & \Delta_A : A &\rightarrow A \otimes A, \\
\eta_H : I &\rightarrow H, & \eta_A : I &\rightarrow A, \\
\epsilon_H : H &\rightarrow I, & \epsilon_A : A &\rightarrow I.
\end{align*}
\]

are morphisms in \( C \).

\((A, m_A, \eta_A)\) is called an algebra living in \( C \), if the following conditions are satisfied:

\[
\begin{align*}
\begin{array}{ccc}
AA & A & A \\
\text{by naturality} & \text{by naturality} & \\
\eta_A & \psi_A & \eta_A \\
\hline
A & A & A
\end{array}
\end{align*}
\]

In this case, \( \eta_A \) and \( m_A \) are called unit and multiplication of \( A \) respectively.
\((H, \Delta_H, \epsilon_H)\) is called a coalgebra living in \(C\), if the following conditions are satisfied:

\[
\begin{align*}
\begin{array}{c}
H \\
\downarrow \Delta_H
\end{array} & = 
\begin{array}{c}
H \\
\downarrow \Delta_H
\end{array}, & \begin{array}{c}
H \\
\downarrow \Delta_H
\end{array} & = 
\begin{array}{c}
H \\
\downarrow \Delta_H
\end{array}, & \begin{array}{c}
H \\
\downarrow \Delta_H
\end{array} & = 
\begin{array}{c}
H \\
\downarrow \Delta_H
\end{array}.
\end{align*}
\]

In this case, \(\epsilon_H\) and \(\Delta_H\) are called counit and comultiplication of \(H\) respectively.

If \(A\) is an algebra and \(H\) is a coalgebra, then \(\text{Hom}_C(H, A)\) becomes an algebra under

\[
f * g = \begin{array}{c} H \\ \downarrow \Delta \\ \downarrow \epsilon \\
\downarrow \eta \\
\downarrow \epsilon \\
A
\end{array}.
\]

and its unit element \(\eta = \eta \epsilon\). If \(S\) is the inverse of \(\text{id}_H\) in \(\text{Hom}_C(H, H)\), then \(S\) is called antipode of \(H\).

If \((H, m_H, \eta_H)\) is an algebra, and \((H, \Delta_H, \epsilon_H)\) is a coalgebra living in \(C\), and the following condition is satisfied:

\[
\begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array} = 
\begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array}, & \begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array} & = 
\begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array}, & \begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array} & = 
\begin{array}{c}
H \\
\downarrow m_H \\
\downarrow \Delta_H \\
\downarrow m_H
\end{array}.
\]

then \(H\) is called a bialgebra living in \(C\). If \(H\) is a bialgebra and there is an inverse \(S\) of \(\text{id}_H\) under convolution product in \(\text{Hom}_C(H, H)\), then \(H\) is called a Hopf algebra living in \(C\), or a braided Hopf algebra.

When \(H\) is a Hopf algebra algebra in braided tensor category \((C, C)\), then the condition above is equivalent to

(YD):

\[
\phi \alpha = 
\begin{array}{c} H \\
\downarrow S \\
\downarrow \alpha \\
\downarrow \eta \\
\downarrow \epsilon \\
M
\end{array}.
\]
Let $YD(C)$ denote the category of all Yetter-Drinfeld $H$-modules in $C$. If $(C, C) = \mathcal{V}ect(k)$, we write $YD(C) = YD$, called Yetter-Drinfeld category. It follows from [RT93] and [BD98, Theorem 4.1.1] that $YD(C)$ is a braided tensor category with $YD_C(U, V) = (\alpha_U \otimes \text{id}_V)(\text{id}_H \otimes C(U, V)(\phi_U \otimes \text{id}_V)$ for any two Yetter-Drinfeld modules $(U, \phi_U, \alpha_U)$ and $(V, \phi_V, \alpha_V)$ when $H$ has an invertible antipode. In this case, $YD_C^{-1}(U, V) = (\text{id}_V \otimes \alpha_U)(C_H^{-1} \otimes \text{id}_V)(S^{-1} \otimes C_U^{-1})(C_U^{-1} \otimes \text{id}_V)(\text{id}_U \otimes \phi_V).$ Algebras, coalgebras and Hopf algebras and so on in $YD(C)$ are called Yetter-Drinfeld ones or YD ones in short.

Let $\mathbb{Z}, \mathbb{N}$ and $\mathbb{C}$ denote integer set, natural number set and complex field, respectively. Throughout basic field is a complex field $\mathbb{C}$, which is denoted by $k$ sometimes. If $V$ is a vector space with a basis $v_1, v_2, \cdots, v_n$ and $q_{ij} \in \mathbb{C}^*$ for $1 \leq i, j \leq n$ such that map $C : V \otimes V \rightarrow V \otimes V, \quad v_i \otimes v_j \mapsto q_{ij} v_j \otimes v_i,$ then $(V, C)$ is called a braided vector space of diagonal type. Denote by $(q_{ij})_{n \times n}$ the braiding matrix of $(V, C)$ under the basis $v_1, v_2, \cdots, v_n$. Then $(V, C)$ is also written as $(V, (q_{ij})_{n \times n}).$ We can get Nichols algebra $B(V)$ (see [He06b] and [ZZC04]). Let $B$ denote the set of all generators of restricted PBW basis and $P$ denote restricted PBW basis of $B(V)$.

Let $v_1, v_2, \cdots, v_n$ be a basis of $V \in kG \cdot YD$ with comodule operation and module operation $\delta(v_i) = g_i \otimes v_i, \ h \cdot v_i = \chi_i(h)v_i,$ where $\chi_i$ is a multiplication character of $G$, i.e. a homomorphism from $G$ to $\mathbb{C}^*$, for $1 \leq i \leq n.$ It is clear that $(V, C)$ is a braided vector space of diagonal type. Otherwise, for any $q_{ij} \in \mathbb{C}, 1 \leq i, j \leq n$ and a basis $v_1, v_2, \cdots, v_n$ of $V$, $V$ can become a Yetter-Drinfeld module over group $\mathbb{Z}^n$ by defining $\delta(v_i) = e_i \otimes v_i$ and $e_j \cdot v_i = q_{ji}v_i, 1 \leq i, j \leq n,$ where $e_1 = (1, 0, \cdots, 0), \cdots, e_n = (0, 0, \cdots, 1) \in \mathbb{Z}^n.$

## 2 Quantum Design of Industry

In this section we give some examples to design by means of braiding. That is, we design by means of quantum theory.

**Example 2.1.** (Category of quantum design grammars) Let $\{D_i \mid 1 \leq i \leq m\}$ be the set of all shapes in some design $D$. Let $v_1, v_2, \cdots, v_n$ be a basis of $V \in kG \cdot YD$ with $\delta(v_i) = g_i \otimes v_i, \ h \cdot v_i = \chi_i(h)v_i.$ Assume that $g_i = g_j$ and $\chi_i = \chi_j$ when there exists rule $R_{ij} : D_i \rightarrow D_j$. Define a morphism $f_{ij}$ from $kv_i$ to $kv_j$ in $kG \cdot YD$ such that $f_{ij}(v_i) = R_{ij}(D_i)$. It is clear that $f_{ij}$ is a morphism in $kG \cdot YD$ and $kv_i$ and $V$, are in $kG \cdot YD$. Define a map $\psi : P \rightarrow \{D_i \mid 1 \leq i \leq m\}$ such that $\psi(v_i) = D_i. \ \psi(u)$ denote the combination of shapes of $D_{i_1}, D_{i_2}, \cdots, D_{i_r}$ when $u = v_{i_1}v_{i_2}\cdots v_{i_r} \in P.$

**Example 2.2.** (Quantum design grammars of Coca-Cola) Let $D_1, D_2, D_3$ and $D_4$ be initial shapes; $D_5, D_6, D_7$ and $D_8$ be cap shape, above shape, middle shape and below shape of battle of Coca-Cola; Rule $R_{15} : D_1 \rightarrow D_5$, Rule $R_{26} : D_2 \rightarrow D_6$, Rule $R_{37} : D_3 \rightarrow D_7$ and $R_{48} : D_4 \rightarrow D_8.$
(i) Let $G = \mathbb{Z}_7$ be cycle group and $C(v_s \otimes v_t) = e^{\frac{2\pi i s t}{7}} = \omega^{g_s g_t}$, where $\omega := e^{\frac{2\pi i}{7}}$ and $i := \sqrt{-1}$. Assume that $g_s = s$, $\chi_s(g_t) = \omega^{st}$ when $1 \leq s, t \leq 4$; $g_{4+p} = g_p$, $\chi_{4+p} = \chi_p$ for $1 \leq p \leq 4$.

Here $u = v_6 v_7 v_8$ and $\psi(u)$ is the battle of Coca-Cola without cap, $u' = v_5 v_6 v_7 v_8$ and $\psi(u')$ is the battle of Coca-Cola with cap. Figure 1 is an ordinary design and Figure 2 is a quantum design. Define that $\omega^1 u', \omega^2 u', \ldots, \omega^7 u'$ means that the color of the battle of Coca-Cola is Red, orange, yellow, green, cyan, blue, purple, respectively. Therefore, $\omega^{18} u' = \omega^4 u'$ and the color of the battle of Coca-Cola is green.

(ii) Let $G = \mathbb{Z}$ and $\chi_1(g_1) = q; 1 \neq q$ be a positive real number; $g_i = g_1$ and $\chi_i = \chi_1$ for $1 \leq i \leq 8$. Then $C(v_s \otimes v_t) = q(v_t \otimes v_s)$. Similarly, we can get figure 3 by replacing $\omega^{18}$ by $q^6$ in figure 2. Define that $q^6 u'$ means that the size of the battle of Coca-Cola is $q^6$ times of original battle $\psi(u')$ of Coca-Cola.

Example 2.3. Let $D_0, D_1, D_3, D_5, D_7$ and $D_9$ be frame, left steering wheel, left front gate, left back gate, left front lamp and left back lamp of car. We shall obtain a design of car by means of these left components of car as follows. Let $G = \mathbb{Z}_{10}$ and $C(v_0 \otimes v_t) = q v_t \otimes v_0$, $C(v_s \otimes v_t) = v_t \otimes v_s$, where $q \neq 1$ is positive real number for $s \neq 0, s, t = 1, 3, 5, 7, 9$. 
where \( u = v_0v_1v_3v_5v_7v_9 \) and \( q^4u = v_0v_1qv_3qv_5qv_7qv_9v_9 \). Define that \((qv_1), (qv_3), (qv_5), (qv_7), (qv_9)\) denote right steering wheel, right front gate, right back gate, right front lamp and right back lamp of car, respectively. Figure 4 is a braiding diagram which represents a combination with left components of car and whole car consists of them. Notice that the steering wheel is in left side. If we require that the steering wheel is in right side of car, we must place the \( v_1 \) in the left hand side of \( v_0 \) in braiding diagram and have braiding \( C(v_1 \otimes v_0) \).

**Example 2.4.** Let \( D_4, D_1, D_2 \) and \( D_3 \) be the first floor, the second floor and the third floor of ship; \( D_5 \) the brightness of ship. We shall obtain a design about the brightness of ship by means of braiding diagram as follows. Let \( G = \mathbb{Z}^5 \) and \( C(v_s \otimes v_t) = q_{st}(v_t \otimes v_s) \), where \( q_{st} \) is positive real number for \( 1 \leq s,t \leq 5 \).

where \( u_3 := q_{35}v_3, \ u_2 := q_{25}v_2, \ u_1 := q_{15}v_1, \ u_4 := q_{45}v_4. \) Define that the brightness in the
first floor, the second floor, the third floor and negative first floor are $q_{15}$ unit, $q_{25}$ unit, $q_{35}$ unit and $q_{45}$ unit according to $q_{35}v_1$, $q_{25}v_2$, $q_{35}v_2$ and $q_{45}v_4$ in braiding diagram. We can choose the value of $q_{st}$ according to the physical truth and requirement of customer.

In fact, we also provide the brightness of every components of car be means of braiding diagram.

3 Quantum Design of Codes

In this section we obtain a code by means of braiding.

Example 3.1. We need send a word $w$ from $X$ to $Y$ by internet and require to keep secret, where $X$ and $Y$ are two persons or two companies. Let $\{D_i \mid 1 \leq i \leq n\}$ be a vocabulary and $Q = (q_{ij})_{n \times n}$ its quantum matrix.

Let $Q$ and Figure 6 are the private key cryptography. i.e. $X$ and $Y$ have $Q$ and Figure 6 without using internet. Assume $w = D_s$. We choose $p$ such that $s \in \{p, p+1, \ldots, p+9\}$ and positive real numbers $a_1, \ldots, a_{10}$ randomly with $a_1 \neq 1, \ldots, a_{10} \neq 1$. Let $b_i = a_i$ when $i \neq s$ and $b_s = 1$. Let $v_i := D_{p+i-1}$ and $\tilde{q}_{ij} := q_{p+i-1,p+j-1}$ for $1 \leq i, j \leq 10$. It is clear $C^{-1}(v_i \otimes v_j) = \tilde{q}_{ji}^{-1}(v_j \otimes v_i)$ and $C^{-1}(v_i \otimes u) = \prod_{j=1}^{10} \tilde{q}_{ji}^{-1}(u \otimes v_i)$. Compute the value $c_i$ of $b_i \prod_{j=1}^{10} \tilde{q}_{ji}^{-1}$ and send $c_i$ to $Y$ for $i = 1, 2, \ldots, 10$ and send $p$ to $Y$. In $Y$ compute the
value $d_i$ of $c_i \prod_{j=1}^{10} \tilde{q}_{ij}$ and $d_i = 1$ if and only if $s = i + p - 1$ since coefficient of $C^{-1}(v_i \otimes u)$ is $\prod_{j=1}^{10} \tilde{q}_{ji}^{-1}$ and coefficient of $c(u \otimes v_i)$ is $\prod_{j=1}^{10} \tilde{q}_{ji}$. Consequently, we can get the word $w$ in $Y$. Furthermore, we can send a article from $X$ to $Y$ by means of internet and keeping secret because they consist of some single words. We also can send file from $X$ to $Y$ by means of internet and keeping secret because the password consist of some single words.

Otherwise, we can modify the method above as follows.

Remark. (i) In case above, we choose the last layer 14-th layer and left line $u$. In fact, we also can choose $t$-th layer with $1 \leq t \leq 14$ and left line $v$ which need be sent to $Y$. That is, private key cryptography contains $Q$, figure 6, $p$, $t$-th layer and left line $v$. For example, set $t = 7$, i.e. $v = v_5v_6$. It is clear $C^{-1}(v_i \otimes v) = \prod_{j=5}^{6} \tilde{q}_{ji}^{-1}(u \otimes v_i)$ and $c(v \otimes v_i) = \prod_{j=5}^{6} \tilde{q}_{ji}(v_i \otimes v)$. Compute the value $c_i$ of $b_i \prod_{j=5}^{6} \tilde{q}_{ji}^{-1}$ and send $c_i$ to $Y$ for $i = 1, 2, \cdots, 10$. In $Y$ compute the value $d_i$ of $c_i \prod_{j=5}^{6} \tilde{q}_{ji}$ and $d_i = 1$ if and only if $s = i$.

(ii) figure 6 and vocabulary $\{D_i \mid 1 \leq i \leq n\}$ can become public key cryptography. The order of vocabulary $\{D_i \mid 1 \leq i \leq n\}$ can be the same as Xinhua dictionary.

(iii) Vocabulary $\{D_{r+i} \mid 1 \leq i \leq n\}$ can become public key cryptography and $r$ can become the private key cryptography.

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