Direct $CP$ violation in $B$ decays in $R$-parity violating models

Gautam Bhattacharyya

Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Calcutta 700064, India

Abstract. In the standard model, $CP$ asymmetries in the $B^\pm \rightarrow \pi^\pm K$ channels are $\sim 2\%$ based on perturbative calculation. Rescattering effects might enhance it to at most $\sim (20 - 25)\%$. We show that lepton-number-violating $\lambda'$ couplings in supersymmetric models are capable of enhancing it to as large as $O(100\%)$. Upcoming $B$ factories will test this scenario.

Based on a work done in collaboration with A. Datta.

1. Introduction to direct $CP$ violation

Measurements of $CP$ violation in the upcoming $B$ factories could reveal new physics with new phases. The best places to look for those are some $CP$ asymmetries which in the standard model (SM) are predicted to be too small, but nonetheless are going to be measured with high precision. Measurements significantly larger than the SM predictions will definitely point towards new physics with new phases. The decay $B^+ \rightarrow \pi^+ K^0$ (at the quark level $\bar{b} \rightarrow \bar{s} d d$) constitutes one such mode. To develop the formalism, let us consider a generic decay process $B^+ \rightarrow f$. The amplitude can be written as $A(B^+ \rightarrow f) = \sum_i |A_i| e^{i\phi_i^W} e^{i\phi_i^S}$. The summation implies that in general there could be more than one diagram, labelled by the index $i$, contributing to this process, and $\phi_i^W$ and $\phi_i^S$ are the weak and strong

1 E-mail: gb@tnp.saha.ernet.in [Invited Talk presented at “Beyond the Desert 1999”, Castle Ringberg, Tegernsee, Germany, 6-12 June 1999.]
phases, respectively, for the $i$th diagram. The CP asymmetry is defined as $a_{CP} \equiv [B(B^+ \rightarrow f) - B(B^- \rightarrow f)]/[B(B^+ \rightarrow f) + B(B^- \rightarrow f)]$. Requiring CPT invariance and assuming, for the sake of simplicity, that only two terms dominate in a given decay amplitude, the above asymmetry can be expressed as

$$a_{CP} = \frac{2|A_1||A_2| \sin(\phi_1^W - \phi_2^W) \sin(\phi_1^S - \phi_2^S)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2| \cos(\phi_1^W - \phi_2^W) \cos(\phi_1^S - \phi_2^S)}.$$  \hspace{1cm} (1)

It is clear that in order to produce large $a_{CP}$ the following conditions will have to be satisfied: (i) $|A_1| \approx |A_2|$, (ii) $\sin(\phi_1^W - \phi_2^W) \approx 1$ and (iii) $\sin(\phi_1^S - \phi_2^S) \approx 1$.

2. The standard model prediction

In the SM, the decay $\bar{b} \rightarrow s \bar{d}d$ receives contributions only from penguin operators. Using the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one can write the decay amplitude as $[2]

$$A^\text{SM}(B^+ \rightarrow \pi^+ K^0) = -A\lambda^2(1 - \lambda^2/2) \left[1 + \rho e^{i\theta} e^{i\gamma}\right] |P_{tc}| e^{i\delta_{tc}},$$  \hspace{1cm} (2)

where $\lambda = 0.22$ is the Wolfenstein parameter; $A \equiv |V_{cb}|/\lambda^2 = 0.81 \pm 0.06$; $\gamma \equiv -\text{Arg}(V_{ub}V_{td}/V_{cb}V_{cd})$ is the CKM weak phase; $\theta$ and $\delta_{tc}$ are CP-conserving strong phases; $P_{tc} \equiv P^S_{t} - P^S_{c} + P^W_{t} - P^W_{c}$ (the difference between top- and charm-mediated strong and electroweak penguins); and finally, $\rho$ depends on the dynamics of the up- and charm-penguins. For calculating $P_{tc}$ we employ the factorization technique, which has been suggested to be quite reliable $[3]$. One can express $|P_{tc}| \approx G_F f(C_t) F/\sqrt{2}$, where $f(C_t) \approx 0.09$ is an analytic function of the Wilson coefficients, and $F = (m_d^2 - m_s^2) f_K F_{B\pi}$ with $F_{B\pi} = 0.3$. The NLO estimate of $B(B^\pm \rightarrow \pi^\pm K^0) \equiv 0.5[B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- \bar{K}^0)]$ varies in the range $(1.0 - 1.8) \times 10^{-5}$ for $\rho = 0$ $[4]$.

Using Eq. (2), the CP asymmetry in the $B^+ \rightarrow \pi^+ K^0$ channel is given by (neglecting tiny phase space effects)

$$a_{CP}^{\text{SM}} = -2\rho \sin \theta \sin \gamma/(1 + \rho^2 + 2\rho \cos \theta \cos \gamma).$$  \hspace{1cm} (3)

In the perturbative limit, $\rho = \mathcal{O}(\lambda^3 R_b) \sim 1.7\%$, where $R_b = |V_{ub}|/\lambda|V_{cb}| = 0.36 \pm 0.08$. On the other hand, rescattering effects $[2, 3]$ such as, $B^+ \rightarrow \pi^0 K^+ \rightarrow \pi^+ K^0$, i.e., long-distance contributions to the up and charm penguins, can enhance $\rho$ to as large as $\mathcal{O}(10\%)$ (this order-of-magnitude estimate is based on Regge phenomenology). As a result, $a_{CP}$ can be as large as $\mathcal{O}(20\%)$. Therefore, if the upcoming experiments measure a much larger $a_{CP}$, an undisputed evidence of new physics with new phase(s) will be established.
3. Effects of $R$-parity violation

3.1. New diagram at tree level

In the minimal supersymmetric standard model, there are additional penguins mediated by superparticles and they contain new phases. As a result, $a_{CP}$ can go up to $\sim 30\%$ \cite{6}. But switching on $R$-parity-violating ($\not R$) $\lambda'_{ijk} L_i Q_j D_k$ superpotential \cite{7,8} triggers new diagrams which contribute to $B^+ \rightarrow \pi^+ K^0$ (or other non-leptonic $B$ decays) at tree level. The interference between $\not R$ tree and the SM penguins may generate large $a_{CP}$. Considering the current upper bounds on the relevant $\lambda'$ couplings \cite{8}, it is very much possible that the $\not R$ tree contributions are of the same order of magnitude as the SM penguins.

3.2. New weak phase

A non-leptonic $B$ decay amplitude involves the product of the type $\lambda'_{ij3} \lambda'^*_{ilm}$. The $\lambda'_{ijk}$ couplings are in general complex. Even if a given $\lambda'$ is predicted to be real in a given model, the phase rotations of the left- and right-handed quark fields required to keep the mass terms real and to bring the CKM matrix to its standard form automatically introduce a new weak phase in this coupling, barring accidental cancellation. Thus the tree level $R$-parity-violating $B^+ \rightarrow \pi^+ K^0$ amplitude in general carries a new weak phase.

3.3. New strong phase

The isospin of a $|B^+\rangle$ state is $1/2$, while that of a $|\pi^+ K^0\rangle$ state is either $1/2$ or $3/2$. The SM penguin operator does not carry any isospin, while the $\not R$ tree operator carries an isospin ($0$ or $1$). As a consequence, the SM penguins produce $\pi^+ K^0$ final states always in the isospin $1/2$ state, while the $\not R$ tree operator can produce the same final states in the isospin $3/2$ state. The final state interaction between states with different isospins may generate a relative strong phase between the SM penguins and the $\not R$ tree diagrams.

3.4. Computation of the new diagram

To generate $B^+ \rightarrow \pi^+ K^0$ at tree level, consider that $\lambda'_{i13}$ and $\lambda'_{i12}$ are the only non-vanishing couplings. This constitutes a sneutrino ($\tilde{\nu}_i$) mediated decay. The new amplitude can be written as (the negative sign in front is just our convention)

$$A^{\not R}(B^+ \rightarrow \pi^+ K^0) = -(|\lambda'_{i13} \lambda'^*_{i12}|/8\tilde{m}^2)F e^{i\gamma_R} \equiv -|A_R|e^{i\gamma_R}, \quad (4)$$
where $\gamma_R$ denotes the weak phase associated with the product of $\lambda$'s. The total amplitude then becomes

$$A(B^+ \rightarrow \pi^+ K^0) = -A\lambda^2(1 - \lambda^2/2)|P_{tc}|e^{i\delta_{tc}}(1 + \rho e^{i\gamma}e^{i0} + \rho e^{i\gamma}e^{i\theta})$$,

(5)

where

$$\rho_R \equiv |\mathcal{A}_R|/A\lambda^2(1 - \lambda^2/2)|P_{tc}|.$$

(6)

It is important to note that $\rho_R$ is free from uncertainties due to factorization. Numerically, $\rho_R$ could easily be order one.

3.5. The CP asymmetry

Assuming that $\rho_R \gg \rho$, one can write

$$a_{CP} \approx -\frac{2\rho_R \sin \theta_R \sin \gamma_R}{1 + \rho_R^2 + 2\rho_R \cos \theta_R \cos \gamma_R}.$$  

(7)

To have a feeling of the size of $\rho_R$, we first choose $\tilde{m} = 100$ GeV throughout our analysis. Employing the current upper limits on $\lambda'_{113}\lambda'_{112}$, we obtain, for $i = 1$, $2$, and $3$, $\rho_R \approx 0.17$, $3.45$, and $4.13$, respectively. Therefore, it is possible to have a situation when $\rho_R = 1$ (for $i = 2, 3$). This implies that a $100\%$ CP asymmetry is very much attainable, once we set $\gamma_R = \theta_R = \pi/2$. We assert that such a drastic hike of CP asymmetry constitutes a characteristic feature of R-parity violation and it is hard to find such large effects in other places.

The minimum $\rho_R$ required to generate a given $a_{CP}$ is given by (for $\rho = 0$)

$$\rho_R \approx (1 - \sqrt{1 - a_{CP}^2})/|a_{CP}|.$$  

(8)

Eq. (8) has been obtained by minimizing $\rho_R$ with respect to $\gamma_R$ and $\theta_R$ for a given $a_{CP}$. Numerically,

$$\rho_R \approx 1.0 \ (1.0), 0.50 \ (0.8), 0.33 \ (0.6), 0.21 \ (0.4), 0.10 \ (0.2);$$  

(9)

where the numbers within brackets refer to the corresponding $a_{CP}$'s.

3.6. New bounds on $\lambda'$ product couplings

We should also be alert that $\rho_R$ does not become so large that the prediction for $\mathcal{B}(B^+ \rightarrow \pi^+ K)$ exceeds the experimental upper limit. According to the latest CLEO measurement, $\mathcal{B}^{\exp}(B^+ \rightarrow \pi^+ K) = (1.4 \pm 0.5 \pm 0.2) \times 10^{-5}$, which means $\mathcal{B}^{\exp} \leq 1.9 \times 10^{-5} \ (1\sigma)$. On the other hand, $\mathcal{B}^{SM} \approx (1.0 - 1.8) \times 10^{-5}$ in view of the present uncertainty of the SM (for $\rho = 0$; varying $\rho$ within $0 - 0.1$ cannot change $\mathcal{B}^{SM}$ significantly). Hence one can accommodate a multiplicative new physics effect by at most a factor 1.9.
One can check from the denominator of Eq. (7) that $R$ effects modify the SM prediction of the branching ratio by a multiplicative factor $(1 + \rho_H^2 + 2\rho_H \cos \theta_H \cos \gamma_H)$. To evaluate the maximum allowed value of $\rho_H$, we set one of the two angles appearing in this factor to zero and the other to $\pi$ (i.e., we arrange for a maximum possible destructive interference in the branching ratio). This leads to the conservative upper bound

$$\rho_H \lesssim 2.4; \quad \text{which implies} \quad |\lambda'_{113}\lambda_{112}^*| \lesssim 5.7 \times 10^{-3} \ (1\sigma). \quad (10)$$

For $i = 2, 3$, these limits are already stronger than the existing ones. Moreover, the existing bounds from semi-leptonic processes necessarily depend on the exchanged squark masses and have been extracted by assuming a common mass of 100 GeV for them. Present Tevatron data disfavour such a low mass squark. On the contrary, our bounds from hadronic $B$ decays rely on a more realistic assumption that the exchanged sneutrinos have a common mass of 100 GeV. We also note that the choice of phases that leads to Eq. (10) predicts vanishing $CP$ asymmetry. If, on the other hand, we are interested in finding the upper limit on $\rho_H$, keeping $a_{CP}$ maximized with respect to $\gamma_H$ and $\theta_H$, each of the two angles should be $\pi/2$. This way the interference term vanishes, and we obtain a stronger limit $\rho_H \lesssim 1.0$. This, in conjunction with Eqs. (9) and (10), defines a range of the $R$ couplings to be investigated in the upcoming $B$ factories.

### 4. Extraction of the CKM parameter $\gamma$

$B \to \pi K$ decays are expected to provide useful information on $\gamma$, the least known angle of the unitarity triangle. In the SM, the three angles ($\alpha$, $\beta$ and $\gamma$) measured independently should sum up to $\pi$. However, if a given channel is contaminated by new physics, it might lead to a wrong determination. The $R$ couplings $\lambda'_{113}$ and $\lambda'_{112}$, the only non-vanishing ones under consideration, do not affect the $B_d \to \pi^+\pi^-$ and $B_d \to \Psi K_s$ channels, which are used for the direct measurements of $\alpha$ and $\beta$, respectively. Once $\alpha$ and $\beta$ are measured this way, $\gamma$ can be indirectly determined via the relation $\gamma = \pi - \alpha - \beta$.

We now consider a direct measurement of $\gamma$, using the observable $R \equiv \frac{\mathcal{B}(B_d \to \pi^- K^+) + \mathcal{B}(B_d \to \pi^+ K^-)}{\mathcal{B}(B^+ \to \pi^+ K^0) + \mathcal{B}(B^- \to \pi^- K^0)}$. The present experimental range is $R = 1.0 \pm 0.46$ [11]. Neglecting rescattering effects, as a first approximation, $\sin^2 \gamma \lesssim R \frac{1}{4}$ in the SM. Within errors, it may still be possible that $R$ settles to a value significantly smaller than one, disfavouring values of $\gamma$ around 90°. This will certainly be in conflict if, for example, $\gamma \approx 90^\circ$ is preferred by indirect determination.

Now we see the effects of $R$-parity violation. The SM bound is modified to ($\rho = 0$)

$$\sin^2 \gamma \lesssim R \left(1 + \rho_H^2 + 2\rho_H \cos \theta_H \cos \gamma_H\right). \quad (11)$$
Here we assumed that the left-handed selectron is somewhat heavier than the sneutrino (due to the D-term contribution), so that the neutral $B$ decay (which appears in the numerator of $R$) is not significantly affected. From Eq. (11), we see that the bound on $\gamma$ either gets relaxed or further constrained depending on the magnitude of $\rho_R$ and the signs of $\gamma_R$ and $\theta_R$. For $\rho_R \approx 1$ and $\gamma_R = \theta_R \approx \pi/2$, it turns out that $\sin^2 \gamma \lesssim 2R$, and hence there is no constraint on $\gamma$ once $R > 0.5$. Thus the lesson is that if one observes large CP asymmetry in the $B^+ \to \pi^+ K^0$ channel, extraction of $\gamma$ becomes an even more nontrivial exercise.

5. Conclusions

We identified one distinctive feature of $R$-parity violation that it can enhance both the CP asymmetry and the branching ratio simultaneously. As a result it is easier to capture these effects experimentally. Our study is a prototype analysis in a particular channel, which can be easily extended to a multichannel analysis combining all kinds of $B \to \pi\pi, \pi K, DK$ modes. In case no large CP asymmetry is observed, or no disparity between $B^{\text{SM}}$ and $B^{\text{exp}}$ is established, one can place improved constraints on many $R$-parity-violating couplings. Upcoming $B$ factories may very well turn out to be a storehouse of many surprises!

Acknowledgements

First, I thank Amitava Datta for a very fruitful collaboration. I also thank Prof. H.V. Klapdor-Kleingrothaus and his team for organizing yet another successful meeting at Ringberg Castle. I take this opportunity to thank all my friends and colleagues at MPI/Heidelberg for their warm hospitality during my stay this summer. Finally I acknowledge an additional financial support of the Max-Planck Society, Germany, which enabled me to attend this meeting.

Note added: After this manuscript had been written up, an interesting article studying the impact of various new physics models, including the $R$-parity-violating ones, on observables related to $B \to \pi K$ decays appeared in the archive [11].

References

[1] Bhattacharyya G and Datta A 1999 Phys. Rev. Lett. 83 2300.

[2] Buras A, Fleischer R and Mannel T 1998 Nucl. Phys. B 533 3; Fleischer R 1998 Eur. Phys. J. C6 451; Fleischer R 1998 Phys. Lett. B435 221.
[3] Ali A, Kramer G and Lü C.-D 1998 Phys. Rev. D58 094009.
[4] Fleischer R and Mannel T 1998 Phys. Rev. D57 2752.
[5] Falk A, Kagan A, Nir Y and Petrov A 1998 Phys. Rev. D57 4290; Gronau M and Rosner J 1998 Phys. Rev. D 57 6843; Atwood D and Soni A 1998 Phys. Rev. D58 036005.
[6] Barbieri R and Strumia A 1997 Nucl. Phys. B508 3.
[7] Farrar G and Fayet P 1978 Phys. Lett. B76 575; Weinberg S 1982 Phys. Rev. D26 287; Sakai N and Yanagida T 1982 Nucl. Phys. B197 533; Aulakh C and Mohapatra R 1982 Phys. Lett. 119B 136.
[8] For recent reviews, see Bhattacharyya G hep-ph/9709395 and 1997 Nucl. Phys. Proc. Suppl. 52A 83; Dreiner H hep-ph/9707434; Barbier R et al. hep-ph/9810232.
[9] Chemtob M and Moreau G 1999 Phys. Rev. D59 116012.
[10] CLEO Collaboration, Artuso M et al., in Proceedings of the International Conference on High Energy Physics: ICHEP’98, Vancouver, 1998 (CLEO CONF 98-20).
[11] Grossman Y, Neubert M and Kagan A hep-ph/9909297.