We consider the measuring procedure that in principle allows to avoid the homodyne detection for the simultaneous selection of both quadrature components in the far-field. The scheme is based on the use of the coherent sources of the non-classical light. The possibilities of the procedure are illustrated on the basis of the use of pixellised sources, where the phase-locked sub-Poissonian lasers or the degenerate optical parametric oscillator generating above threshold are chosen as the pixels.

The theory of the pixellised source of the spatio-temporal squeezed light is elaborated as a part of this investigation.

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I. INTRODUCTION

The subject of quantum optics always implies the use of some kind of nonclassical light. Therefore under the conditions of the real experiment one of the basic goals is to find a comprehensible source that gives nonclassical light with acceptable spatio-temporal properties. For now there are several kinds of sources such as optical parametric amplifiers and oscillators generating below threshold (see, e.g., [1]) that are non-alternatively used in the practice. These devices are preferable mostly because of the nature of parametric interaction itself, which automatically ensures a generation of the squeezed light states [2, 3, 4, 5].

Together with the obvious advantages of the parametrical sources their applications meet the well-known difficulties. One of them, for example, is a necessity of the balanced homodyne detection use. In the experimental practice a preparation of the local oscillator with the acceptable spatio-temporal configuration can be not a simple operation especially for quantum imaging. So it is reasonable to consider the other alternatives. In this article, the light sources are considered to be with quantum fluctuations occurring on high coherent level. We believe that the similar sources could provide us with new possibilities in comparison with traditional ones. This can be realized by devices such as lasers [6, 8, 9] or by any parametric device generating above threshold [10, 11]. These sources we will call coherent sources. As we shall demonstrate further they allow us to avoid an obligatory use of the homodyne detection procedure, since the coherent component of the radiation itself constitutes for the local oscillator amplitude.

The development of quantum optics during the last decade has clearly demonstrated that for reasons of any applications the greater number of degrees of freedom in the non-classical radiation is a positive and very important factor allowing to essentially improve an efficiency of optical measuring procedures. From this standpoint the use of the single-mode coherent sources does not seem to be an attractive perspective. However as it was demonstrated in Ref. [6] such conclusion is too hasty because even in this case the radiation outside the high-Q cavity in free space turns out to be broadband.

Having the good temporal statistics the single-mode lasers or the optical parametric oscillation are often ill acceptable for the analysis of spatial structures, for example, for the aims of quantum imaging. To adapt them to the spatially multi-mode schemes it is suggested to use the so-called pixellised sources that were discussed in Refs. [8, 10]. These sources are formed by putting periodically the pixels (lasers or optical parametric oscillators) on a plane surface. That provides us with a possibility to effectively use both the spatial and temporal degrees of freedom in distinguish from the optical parametric oscillator generating below threshold that was in particular used in the dense-coding protocol [12], where the temporal degrees freedom could not be enable.

In this article we use the following abbreviations: coherent source (CS); homodyne detection (HD); pixellised source (PS); degenerate optical parametric oscillator (DOPO); sub-Poissonian laser (SPL).
This article is organized as follows. In Secs. II and III, two ways (with one and with two sources and correspondingly with one and two detectors) of a direct detection of the quadrature components are considered. In Secs. IV, V, and VI, the measurement procedure is applied for the specific sources, namely for the isolated pointlike phase-locked SPL and DOPO, for the PSs on the basis of the SPL and DOPO.

II. DIRECT DETECTION OF THE QUADRATURE COMPONENTS IN THE FAR-FIELD (SCHEME WITH SINGLE COHERENT SOURCE)

In this article, our main aims are coupled with the problems of quantum optics and quantum information that is the squeezed and entangled light states are specially important for us. However, we would like to emphasize that the results obtained in Secs. II and III and devoted to the selection of the quadrature components without the use of the homodyne detection technique can have more wide application and are not directly coupled with any quantum aspects in the field.

We shall consider two ways to select the quadrature components presented correspondingly in Fig. 1 and Fig. 2. One of them is coupled with following the even and odd parts of a photocurrent in the single beam configuration and the other with two-beam one. In order to characterize our approach it is important in advance to stress that in distinguish from the other approaches we are going to use the so-called coherent sources, which could be formally defined by the inequality

$$\langle \hat{E}_N(\vec{r}) \rangle \gg \delta \hat{E}_N(\vec{r}, t),$$

where on the left we have an averaged Heisenberg amplitude and on the right its fluctuation. The Heisenberg amplitude is written in the form

$$\hat{E}_N(\vec{r}, t) = \langle \hat{E}_N(\vec{r}) \rangle + \delta \hat{E}_N(\vec{r}, t).$$

We have a right to say it for both the near and far-fields. Hereinafter the lower index N (F) means the near- (far-) field.

In the limit of a quasi-monochromatic and quasi-plane travelling wave it is convenient to present it in the form

$$\hat{E}_N(z, \vec{ρ}, t) = i \sqrt{\frac{\hbar \omega_0}{2 \varepsilon_0 c}} e^{i k_0 z - i \omega_0 t} \hat{S}_N(\vec{ρ}, t).$$

The operator $\hat{S}_N(\vec{ρ}, t)$ is normalized amplitude such that the value $\langle \hat{S}_N^\dagger \hat{S}_N \rangle$ takes sense of the mean photon number per sec and per cm$^2$. Under the propagation in free space the normalized amplitude obeys the canonical commutation relations

$$\left[\hat{S}_N(\vec{ρ}, t), \hat{S}_N^\dagger(\vec{ρ}', t')\right] = \delta^2(\vec{ρ} - \vec{ρ}') \delta(t - t'),$$

$$\left[\hat{S}_N(\vec{ρ}, t), \hat{S}_N(\vec{ρ}', t')\right] = 0.$$  

Furthermore, we shall assume that the emission from our sources possess a cylindrical symmetry in the near- and far-field:

$$\langle \hat{S}(\vec{ρ}) \rangle = \langle \hat{S}(-\vec{ρ}) \rangle.$$  

By choosing such experimental situation, where a lens is placed at the focal distances $f$ between the source and the detection plane the field in front of the detection plane may be considered as a far-field. We shall denote the corresponding normalized amplitude $\hat{S}_F(\vec{ρ}, t)$, hence the far- and near-fields turn out to be coupled by the integral

$$\hat{S}_F(\vec{ρ}, t) = -\frac{i}{\lambda f} \int d^2 \rho' \hat{S}_N(\vec{ρ}', t) e^{-i \vec{Q} \vec{ρ}} = \frac{2\pi i}{\lambda f} \hat{S}_N(\vec{Q}, t), \quad \vec{Q} = \frac{2\pi}{\lambda f} \vec{ρ}.$$
Hereinafter we shall label $G(q)$ as the Fourier image of a function $G(\rho)$, i.e.,

$$G(q) = \frac{1}{2\pi} \int d^2 \rho \, G(\rho) \, e^{-i\vec{q}\vec{\rho}} \quad \text{and} \quad G(\rho) = \frac{1}{2\pi} \int d^2 q \, G(q) \, e^{i\vec{q}\vec{\rho}}.$$  

(7)

In particular in Eq. (6) we have expressed the Heisenberg far-field amplitude in physical space via the Heisenberg near-field amplitude in the Fourier domain with $\vec{q} \to \vec{Q}$.

Let us remind how the wanted quadrature component is selected in the HD technique in the near-field. The photocurrent fluctuation operator $\delta \hat{i} = \hat{i} - \langle \hat{i} \rangle$ reads

$$\delta \hat{i}_N(\vec{\rho}, t) = \beta(\vec{\rho}) \, \delta \hat{S}_N(\vec{\rho}, t) + \beta^* \delta \hat{S}_N^\dagger(\vec{\rho}, t),$$  

(8)

where $\beta$ is the local oscillator complex amplitude. By choosing the acceptable $\beta$ we can select any quadrature component at the detector. For the coherent source it is possible partially to avoid the use of the HD technique because, in this case, the mean amplitude $\langle \hat{S}_N(\vec{\rho}, t) \rangle$ takes a role of the local oscillator amplitude. The amplitude quadrature turns out to be automatically selected but the phase one remains inaccessible.

If our measurement procedure is replaced to the far-field, then we shall illustrate that there it is possible to find an acceptable approach when the phase quadrature is selected automatically as well as the amplitude one.

In the far-field the photocurrent fluctuations for the coherent sources are given by

$$\delta \hat{i}_F(\vec{\rho}, t) = \langle \hat{S}_F(\vec{\rho}) \rangle^* \, \delta \hat{S}_F(\vec{\rho}, t) + \langle \hat{S}_F(\vec{\rho}) \rangle \, \delta \hat{S}_F^\dagger(\vec{\rho}, t).$$  

(9)

Taking into account (6) one can obtain

$$\delta \hat{i}_F(\vec{\rho}, t) = \left( \frac{2\pi}{\lambda f} \right)^2 \left[ \langle \hat{S}_N(\vec{Q}) \rangle^* \, \delta \hat{S}_N(\vec{Q}, t) + \langle \hat{S}_N(\vec{Q}) \rangle \, \delta \hat{S}_N^\dagger(\vec{Q}, t) \right].$$  

(10)

Furthermore we shall use the simpler physical situation, where the specific phases are chosen for our sources. In this section we require the mean field amplitude in the near field is real,

$$\langle \hat{S}_N(\vec{\rho}, t) \rangle = \langle \hat{S}_N(\vec{\rho}, t) \rangle^*.$$  

(11)

In the next section we shall consider the scheme with two sources and for one source we survive the requirement of the reality but for the second one we shall foresee the phase shift equal to $\pi/2$, i.e., our requirement for the second source will be $\langle \hat{S}_N(\vec{\rho}, t) \rangle = -\langle \hat{S}_N(\vec{\rho}, t) \rangle^*$. Then Eq. (9) can be rewritten in the form

$$\delta \hat{i}_F(\vec{\rho}, t) = \left( \frac{2\pi}{\lambda f} \right)^2 \langle \hat{S}_N(\vec{Q}) \rangle \left[ \delta \hat{S}_N(\vec{Q}, t) + h.c. \right].$$  

(12)

Let us introduce the quadrature components $\hat{X}_N(\vec{\rho}, t)$ and $\hat{Y}_N(\vec{\rho}, t)$ for the near-field

$$\hat{S}_N(\vec{\rho}, t) = \hat{X}_N(\vec{\rho}, t) + i \hat{Y}_N(\vec{\rho}, t),$$  

(13)

and correspondingly in the Fourier domain

$$\hat{S}_N(\vec{Q}, t) = \hat{X}_N(\vec{Q}, t) + i \hat{Y}_N(\vec{Q}, t).$$  

(14)

Under the assumption

$$\langle \hat{S}_N(\vec{Q}) \rangle = \langle \hat{S}_N(\vec{Q}) \rangle^* = \langle \hat{X}_N(\vec{Q}, t) \rangle,$$  

(15)

one can see both quadratures are presented equally in current (12).

Let us vary the direct measurement procedure by investigating not the photocurrent itself but its even and odd parts independently. The even and odd parts are proportional to summarized and differential photocurrent respectively:

$$\delta \hat{i}_\pm(\vec{\rho}, t) = \delta \hat{i}_F(\vec{\rho}, t) \pm \delta \hat{i}_F(\vec{\rho}, t).$$  

(16)

Now one can get

$$\delta \hat{i}_+(\vec{\rho}, t) = 2 \left( \frac{2\pi}{\lambda f} \right)^2 \langle \hat{X}_N(\vec{Q}) \rangle \left[ \delta \hat{X}_N(\vec{Q}, t) + h.c. \right],$$  

(17)

$$\delta \hat{i}_-(\vec{\rho}, t) = 2 \left( \frac{2\pi}{\lambda f} \right)^2 \langle \hat{X}_N(\vec{Q}) \rangle \, i \left[ \delta \hat{Y}_N(\vec{Q}, t) - h.c. \right].$$  

(18)
One can conclude that the summarized photocurrent is determined by real part of the X-quadrature and the differential photocurrent - by imaginary part of the Y-quadrature. On the one hand, it is not the same result that could be obtained in the HD technique, where it is possible to select the quadratures as whole. On the other hand, in principle, the mathematics provides us with a possibility to renew the full analytical function by knowing only its real or imaginary part (the Cauchy-Riemann theorem).

Although this development seems as quite encouraging, nevertheless in the next section we want to discuss another measurement procedure, where the quadratures are selected as whole. It is more convenient because does not require any additional mathematical treatment.

III. DIRECT DETECTION OF THE QUADRATURE COMPONENTS IN THE FAR-FIELD (SCHEME WITH TWO COHERENT SOURCES)

Let two independent coherent sources emit two beams (see fig. 2) presented correspondingly by the slow Heisenberg amplitudes $\hat{S}_{1N}(\hat{\rho}, t)$ and $\hat{S}_{2N}(\hat{\rho}, t)$ in the near-field. As usual the corresponding quadratures are introduced by relations

$$\delta \hat{S}_{mN}(\hat{Q}, t) = \delta \hat{X}_{mN}(\hat{\rho}, t) + i \delta \hat{Y}_{mN}(\hat{\rho}, t), \quad m = 1, 2.$$  \hspace{1cm} (19)

As for our coherent sources we assume that they are perfectly identical but there is a phase shift between them equal to $\pi/2$. In particular this means that the following equalities can take place

$$\langle \hat{S}_{1N}(\hat{\rho}) \rangle = \langle \hat{X}_{1N}(\hat{\rho}) \rangle, \quad \langle \hat{S}_{2N}(\hat{\rho}) \rangle = i \langle \hat{Y}_{2N}(\hat{\rho}) \rangle, \quad \langle \hat{Y}_{2N}(\hat{\rho}) \rangle = \langle \hat{X}_{1N}(\hat{\rho}) \rangle,$$  \hspace{1cm} (20)

and

$$\langle \delta \hat{X}_{1N}(\hat{\rho}, t) \delta \hat{X}_{1N}(\hat{\rho}', t') \rangle = \langle \delta \hat{Y}_{2N}(\hat{\rho}, t) \delta \hat{Y}_{2N}(\hat{\rho}', t') \rangle,$$

$$\langle \delta \hat{Y}_{1N}(\hat{\rho}, t) \delta \hat{Y}_{1N}(\hat{\rho}', t') \rangle = \langle \delta \hat{X}_{2N}(\hat{\rho}, t) \delta \hat{X}_{2N}(\hat{\rho}', t') \rangle.$$  \hspace{1cm} (21)

After mixing in the near-field on the symmetrical beamsplitter we get two other beams with amplitudes

$$\hat{E}_{1N}(\hat{\rho}, t) = \frac{1}{\sqrt{2}}(\hat{S}_{1N}(\hat{\rho}, t) + \hat{S}_{2N}(\hat{\rho}, t)), \quad \hat{E}_{2N}(\hat{\rho}, t) = \frac{1}{\sqrt{2}}(\hat{S}_{1N}(\hat{\rho}, t) - \hat{S}_{2N}(\hat{\rho}, t)).$$  \hspace{1cm} (22)

These beams can turn out to be entangled if the initial beams were squeezed in the mutually orthogonal quadratures.

Now we again follow the summarized and differential currents in the far-field that are defined as

$$\delta \hat{i}_{\pm}(\hat{\rho}, t) = \delta \hat{i}_{1F}(\hat{\rho}, t) \pm \delta \hat{i}_{2F}(\hat{\rho}, t),$$  \hspace{1cm} (23)

where $i_{1F}$ and $i_{2F}$ are correspondingly the currents on the first and the second pointlike detectors. After non-difficult algebraic operations one can obtain

$$\delta \hat{i}_{\pm}(\hat{\rho}, t) = \left(\frac{2\pi}{\lambda f}\right)^2 \langle X_{1N}(\hat{Q}) \rangle \left[(1 - i) \left[\delta \hat{X}_{1N}(\hat{Q}, t) + i \delta \hat{Y}_{2N}(\hat{Q}, t)\right] + h.c\right].$$  \hspace{1cm} (24)
Let us calculate the correlation functions

\[ \langle \delta \hat{i}_\pm(\vec{\rho}, t) \delta \hat{i}_\pm(\vec{\rho}', t') \rangle = 4 \left( \frac{2\pi}{\lambda_0^2} \right)^4 \frac{4}{\langle X_1(\vec{Q}) \rangle \langle X_1(\vec{Q}') \rangle} \left[ \delta(t - t') \sqrt{2} \delta(\vec{Q} - \vec{Q}') + 4 \left( \langle \delta \hat{X}_1(\vec{Q}, t) \delta \hat{X}_1(\vec{Q}', t') : \rangle \right) \right]. \tag{25} \]

The notation \( \langle \cdots : \rangle \) means the normally ordered averaging. Under deriving it we have used Eqs. \( \text{(21)} \) staying identification of the CS.

From Eqs. \( \text{(26)} \) one can conclude that in the measuring scheme presented in fig.2 it is possible to select any quadrature component by only choosing the summarized or differential current. We shall call this direct measuring procedure the plus-minus detection.

IV. THE PLUS-MINUS DETECTION SCHEME ON THE BASIS OF TWO DOPO OR TWO SPL

In the previous sections our requirements relative to sources were as follows: they are coherent and cylindrically symmetrical. Below we consider the specific situations, namely in this section we discuss the plus-minus detection on the basis of two phase-locked SPL or two DOPO.

The quasi-monochromatic radiation with bounded aperture inside the cavity at the output mirror can be formally described by the Heisenberg amplitude

\[ \hat{E}_s(z = L, \vec{\rho}, t) = i \sqrt{\frac{\hbar \omega_0}{2\epsilon_0 L}} e^{-i\omega_0 t} f(\vec{\rho}) \hat{a}(t). \tag{26} \]

Here \( \vec{\rho} \) is the transverse spatial vector, \( z \) is a longitudinal co-ordinate, \( L \) is the cavity perimeter, \( \omega_0 \) is the mode frequency. The normalized amplitudes \( \hat{a}(t) \) and \( \hat{a}^\dagger(t) \) obey the canonical commutation relation

\[ [\hat{a}(t), \hat{a}^\dagger(t)] = 1 \tag{27} \]

and \( \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle \) gives the mean photon number inside the cavity.

The transverse size of the mode is formally bounded by the function \( f(\vec{\rho}) \) that is normalized such that

\[ \int d^2 \rho \, |f(\vec{\rho})|^2 = 1, \quad d^2 \rho = dx \, dy. \tag{28} \]

For the sake of simplicity we shall discuss here the Gaussian mode, then

\[ f(\vec{\rho}) = \frac{1}{\sqrt{\pi/2 \omega_0^2}} \exp \left( -\frac{\rho^2}{\omega_0^2} \right), \quad \rho^2 = x^2 + y^2. \tag{29} \]

The value \( \omega_0 \) takes a sense of the light spot size.

The Heisenberg amplitude \( \hat{a}(t) \) describes the intracavity field. Certainly, its time evolution depends on the specific physical processes inside the cavity. For the coherent field

\[ \hat{a}(t) = \langle \hat{a} \rangle + \delta \hat{a}(t), \quad \langle \hat{a} \rangle \gg \delta \hat{a}. \tag{30} \]

The near-field at the output of the cavity with amplitude \( \hat{E}_N(\vec{\rho}, t) \) is formed as a linear combination of the field leaving the cavity for a propagation in free space and the multi-mode vacuum field reflected from the output mirror outside the cavity

\[ \hat{E}_N(\vec{\rho}, t) = \sqrt{T} \hat{E}_s(z = L, \vec{\rho}, t) - \sqrt{R} \hat{E}_\text{vac}(z = -0, \vec{\rho}, t). \tag{31} \]

Here \( R \) and \( T \) are correspondingly the reflection and transmission coefficients. We put there are no losses of the field under passing the mirror, i.e., \( R + T = 1 \).

Taking into account \( \text{(9)} \) and \( \text{(25)} \), we can rewrite \( \text{(31)} \) via the normalized amplitudes in the form

\[ \hat{S}_N(\vec{\rho}, t) = \sqrt{R} f(\vec{\rho}) \hat{a}(t) - \hat{S}_\text{vac}(\vec{\rho}, t), \tag{32} \]
Here the normalized amplitude $\hat{S}_{vac}(\vec{p}, t)$ as well as $\hat{S}_N(\vec{p}, t)$ obeys free-space commutation relations \(^\text{[1]}\). In order to ensure this we should put

$$\sqrt{r} \left[ \hat{a}(t), \hat{a}^\dagger(t') \right] = \left[ \hat{a}(t), \hat{S}_{vac}^\dagger(t') \right] + \left[ \hat{S}_{vac}(t), \hat{a}^\dagger(t') \right]. \quad (34)$$

Because we want to consider the ± detection we need to foresee two sources and correspondingly we have to discuss the amplitudes $\hat{S}_{r,vac}(\vec{p}, t), \hat{S}_{r,N}(\vec{p}, t)$, and $\hat{a}_r(t)$, where index $r = 1, 2$ indicates the source. We shall assume that both sources are perfectly identical with each other, and the parameters $\kappa$ and $f(\vec{p})$ are the same for both. However it is suggested that there is the phase shift between them such that

$$\langle \hat{a}_1 \rangle = \sqrt{n}, \quad \langle \hat{a}_2 \rangle = i\sqrt{n}, \quad (35)$$

and equalities \(^\text{[21]}\) take place.

Applying these considerations to our sources, it is not difficult to get the quadrature components

$$\delta \hat{X}_{r,N}(\vec{p}, t) = \sqrt{r} f(\vec{p}) \delta \hat{x}_r(t) - \hat{X}_{r,vac}(\vec{p}, t), \quad (36)$$
$$\delta \hat{Y}_{r,N}(\vec{p}, t) = \sqrt{r} f(\vec{p}) \delta \hat{y}_r(t) - \hat{Y}_{r,vac}(\vec{p}, t), \quad (37)$$
$$\langle \hat{X}_{1,N}(\vec{p}) \rangle = \langle \hat{Y}_{2,N}(\vec{p}) \rangle = \sqrt{r\kappa} f(\vec{p}), \quad (38)$$
$$\langle \hat{X}_{2,N}(\vec{p}) \rangle = \langle \hat{Y}_{1,N}(\vec{p}) \rangle = 0, \quad (39)$$

where $\delta \hat{x}_r(t)$ and $\delta \hat{y}_r(t)$ are the quadrature components of the intracavity field for both sources

$$\delta \hat{a}_r(t) = \delta \hat{x}_r(t) + i\delta \hat{y}_r(t). \quad r = 1, 2. \quad (40)$$

Passing on to the Fourier domain, one can obtain

$$\delta \hat{x}_{i,N}(\vec{Q}, t) = \sqrt{r} f(\vec{p}) \delta \hat{x}_i(t) - \hat{x}_{i,vac}(\vec{Q}, t), \quad (41)$$
$$\delta \hat{y}_{i,N}(\vec{Q}, t) = \sqrt{r} f(\vec{p}) \delta \hat{y}_i(t) - \hat{y}_{i,vac}(\vec{Q}, t), \quad (42)$$
$$\langle \hat{x}_{1,N}(\vec{Q}) \rangle = \langle \hat{y}_{2,N}(\vec{Q}) \rangle = \sqrt{r\kappa} f_{\vec{Q}}, \quad (43)$$

Here

$$f_{\vec{Q}} = \frac{1}{2\pi} \int d^2 \rho f(\vec{p}) e^{-i\vec{Q}\vec{p}} \quad (44)$$

and for the Gaussian mode \(^\text{[29]}\)

$$f_{\vec{Q}} = \sqrt{\frac{w_0^2}{2\pi}} e^{-\frac{1}{4} w_0^2 Q^2} \left( \frac{1}{4} \frac{\rho^2}{w_0^2} \right), \quad \vec{w}_0 = \frac{\lambda f}{2\pi} \frac{1}{w_0}. \quad (45)$$

Now we have a complete information about our sources to calculate a final correlation function. Substituting \(^\text{[11]}\)–\(^\text{[43]}\) to Eq. \(^\text{[25]}\), one can obtain

$$\langle \delta \hat{X}_{\pm}(\vec{p}, t) \delta \hat{X}_{\pm}(\vec{p}', t') \rangle = 4\kappa n \left( \frac{2\pi}{\lambda f} \right)^4 f_{\vec{Q}} f_{\vec{Q}'} \left[ \delta(t - t') \delta^2(\vec{Q} - \vec{Q}') + 4\kappa \left( \langle \delta \hat{x}_1(t) \delta \hat{x}_1(t') \rangle \right) f_{\vec{Q}} f_{\vec{Q}'} \right]. \quad (46)$$

Let us consider the spectrum of the spatio-temporal correlation function determined as

$$(\delta \hat{X}_{\pm})^2_{\Omega, \Omega} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \int d^2 \rho d^2 \rho' \langle \delta \hat{X}_{\pm}(\vec{p}, t) \delta \hat{X}_{\pm}(\vec{p}', t') \rangle e^{i\Omega(t - t')} e^{i\vec{Q}(\vec{p} - \vec{p}')}. \quad (47)$$
Substituting (46) to (47) after all integrations one can obtain the spectra in the explicit form. For DOPO it can be read via the physical parameters:

\[
\langle \delta i_{\pm}^2 Q_\Omega \rangle = 1 \pm e^{-\tilde{u}_0^2 q^2 \left( \frac{\kappa^2}{\kappa^2 (\mu_{th} - 1)^2 + \Omega^2} \right)}
\]

Under calculations of these formulas we have used the expressions for the correlation functions \( \langle \delta \hat{x}_1(t) \delta \hat{x}_1(t') \rangle \) and \( \langle \delta \hat{y}_1(t) \delta \hat{y}_1(t') \rangle \) obtained in Ref. [11]. Here \( \mu_{th} > 1 \) is a factor determining a relationship between the pump power and the threshold one.

By the same way we could find the spectra for phase-locked SPLs in the scheme of plus-minus detection. Using the results obtained in Ref. [6, 7] one can read

\[
\langle \delta i_{\pm}^2 Q_\Omega \rangle = 1 \pm e^{-\tilde{u}_0^2 q^2 \left( \frac{\kappa^2}{\kappa^2 + \Omega^2} \right)}
\]

It is not difficult to check that the measurements in the HD technique in the near-field will give exactly the same results. The last formulas exhibit the possibility of the plus-minus detection in the cases of the simplest single-mode sources. One can see that it is possible to follow both the squeezed and stretched quadratures simultaneously without the use of the HD technique.

V. PIXELISED SOURCE: ARRAY OF THE COHERENT POINTLIKE SOURCES

Let us consider more complicated sources that we call the PS. Let \( N^2 \) pointlike identical sources be placed periodically on the plane such that the linear distance between the adjoining pixels is much more than the beam spot each of the individual pixels

\[
l \gg w_0.
\]

This requirement allows us to say that all sources are perfectly independent from each other. Simultaneously we assume that all the lasers or DOPO used as the pixels are perfectly synchronized by means of the pump field for the DOPO or by means of the special synchronizing external field for the lasers [6, 7].

In order to construct theoretically the full beam from the PS we can use the linear superposition principle and derive the full Heisenberg amplitude as a sum of the individual amplitudes from each of the pixels,

\[
\hat{E}(\vec{r}, t) = \sum_m \hat{E}_m(\vec{r}, t).
\]

The positions of the pixels on the surface is determined by the vectors

\[
\vec{\rho}_m = l \vec{m}, \quad m_x, m_y = 0, \pm 1, \pm 2, \ldots, \pm (N - 1)/2.
\]

In Eq. (32) we have written the explicit form for the slow Heisenberg amplitude in the case of the single pixel. It is not difficult to generalize it on the case of the PS

\[
\hat{S}(\vec{\rho}, t) = \sqrt{\kappa} \sum_{\vec{\rho}_m} f(\vec{\rho} - \vec{\rho}_m) \hat{a}_m(t) - \hat{S}_{vac}(\vec{\rho}, t).
\]

The function \( f(\vec{\rho} - \vec{\rho}_m) \) describes the \( m \)-beam aperture. Since we choose \( l \gg w_0 \), then all the pixels are independent from each other and we have a right to require that

\[
\int |f(\vec{\rho} - \vec{\rho}_m)|^2 d^2 \rho = 1
\]
The operators $\hat{a}_m$ are the normalized amplitudes describing a single mode field inside each of the individual cavities. As before we consider the coherent fields that is

$$\hat{a}_m(t) = \langle \hat{a}_m \rangle + \delta \hat{a}_m(t), \quad \langle \delta \hat{a}_m \rangle \gg \delta \hat{a}_m(t).$$

(56)

Because under plus-minus detection we have to foresee two independent PSs we are under obligation to specify belonging the $m$-th pixel to one of these PSs that is to add additional index: $\hat{a}_m \to \hat{a}_{r,m}$, where $r = 1, 2$.

Now it is not difficult to generalize formulas (41)-(43)

$$\delta \tilde{X}_{r,N}(\vec{q}, t) = \sqrt{\kappa} \int \frac{d\vec{q}}{2\pi} e^{-i\vec{q}\vec{\rho}_m} \delta \tilde{x}_{r,m}(t) - \tilde{X}_{r,vac}(\vec{q}, t),$$

(57)

$$\delta \tilde{Y}_{r,N}(\vec{q}, t) = \sqrt{\kappa} \int \frac{d\vec{q}}{2\pi} e^{-i\vec{q}\vec{\rho}_m} \delta \tilde{y}_{r,m}(t) - \tilde{Y}_{r,vac}(\vec{q}, t),$$

(58)

$$\langle \tilde{X}_{1,N}(\vec{q}) \rangle = \langle \tilde{Y}_{2,N}(\vec{q}) \rangle = \sqrt{\kappa n} f_{\vec{q}} \Lambda_{\vec{q}}.$$  

(59)

Here $\delta \tilde{x}_{r,m}(t)$ and $\delta \tilde{y}_{r,m}(t)$ are the quadrature components for the intracavity single-mode field

$$\delta \hat{a}_{r,m} = \delta \tilde{x}_{r,m}(t) + i \delta \tilde{y}_{r,m}(t)$$

(60)

and

$$\Lambda_{\vec{q}} = \sum_{\vec{r}_m} e^{-i\vec{q}\vec{\rho}_m}.$$  

(61)

Evaluating the series in Eqs. (61) one can get it in the explicit form,

$$\Lambda_{\vec{q}} = \frac{\sin Q_x lN/2}{\sin Q_x l/2} \frac{\sin Q_y lN/2}{\sin Q_y l/2},$$

(62)

where $Q_x$ and $Q_y$ are the components of vector $\vec{q}$. On the basis of Eqs. (57)-(59) the correlation functions can be read

$$\langle \delta \hat{i}_\pm(\vec{p}, t) \delta \hat{i}_\pm(\vec{p}', t') \rangle =$$

$$= 4\kappa n \left(\frac{2\pi}{\lambda f}\right)^4 f_{\vec{q}} \Lambda_{\vec{q}} f_{\vec{q}} \Lambda_{\vec{q}} \left[ \langle \delta (t - t') \delta^2(\vec{q} - \vec{q}') \rangle + 4\kappa \left( \langle \delta \tilde{x}_r(t) \delta \tilde{x}_r(t') \rangle \right) f_{\vec{q}} f_{\vec{q}} \Lambda_{\vec{q}} f_{\vec{q}} \Lambda_{\vec{q}} \right].$$

(63)

Passing to the Fourier domain, one can obtain again for the DOPO and SPL Eqs. (48)-(49), where we should do the following exchange

$$e^{-\vec{u}_0^2\vec{q}^2} \rightarrow \frac{1}{N^2} \sum_{\vec{Q}_m, \vec{Q}_n} e^{-\vec{u}_0^2(\vec{q} + \vec{Q}_m - \vec{Q}_n)^2}, \quad \vec{Q}_m = \frac{2\pi}{\lambda f} \vec{\rho}_m.$$  

(64)

In Fig. 3 one can see that the photocurrent spectrum exhibits the periodical structure in dependence on spatial frequency $q$ that reflects directly the periodical geometry of the PS. The reduction of the shot noise is more effective for bigger $N$, where the bigger number of holes with nearly perfect noise reduction appears.

**VI. DIRECT DETECTION IN THE SINGLE BEAM**

Let us return to the measuring procedure presented in Fig. 1, where we follow the even and odd parts of current under the detection in the beam from the single PS.
Taking into account the formulas from the previous section it is possible to get the explicit expression for the pair correlation function of the summarized and differential currents in the form

$$\langle \delta i_{\pm}(\vec{\rho}, t) \delta i_{\pm}(\vec{\rho}', t') \rangle = 2\kappa n \left( \frac{2\pi}{\lambda f} \right)^4 f_{\vec{Q}} \Lambda_{\vec{Q}'} f_{\vec{Q}'} \Lambda_{\vec{Q}} \left[ \delta(t - t') \left( \delta^2(\vec{Q} - \vec{Q}') + \delta^2(\vec{Q} + \vec{Q}') \right) + \right. \\
\left. + 4\kappa \left( \langle : \delta \hat{X}_r(t) \delta \hat{X}_r(t') : \rangle - \langle : \delta \hat{Y}_r(t) \delta \hat{Y}_r(t') : \rangle \right) \right] f_{\vec{Q}} f_{\vec{Q}'} \Lambda_{\vec{Q} + \vec{Q}'} \pm \Lambda_{\vec{Q} - \vec{Q}'} \right]. \tag{65}$$

Using again the definition (47) it is possible to get the spectra for the DOPO and SPL in the form Eqs. (48) and (49), where we should do the following exchange

$$e^{-\tilde{w}^2_0 q^2} \rightarrow \frac{1}{2N^2} \sum_{\vec{Q}_m, \vec{Q}_n} \left[ -\tilde{w}^2_0 \left( q + \frac{1}{2}(\vec{Q}_m - \vec{Q}_n) \right)^2 \pm e^{-\tilde{w}^2_0 \left( q + \vec{Q}_m - \vec{Q}_n \right)^2} \right]. \tag{66}$$

The formula exhibits a set of resonant Gaussian picks or holes on the level of the shot noise (see figs. 4 and 5). Each of the picks has a width

$$\Delta q = \frac{1}{\tilde{w}_0} = \frac{2\pi}{\lambda f} w_0, \tag{67}$$

and is centered on the frequency \(\vec{Q}_m\) or \(\vec{Q}_m/2\) that matches the pixel positions:

$$\vec{Q}_m = \frac{2\pi}{\lambda f} \vec{\rho}_m. \tag{68}$$

The distance between the adjoining picks along the x- or y-axes \(d\) and the full actual frequency range \(D\) are given by

$$d = \frac{2\pi}{\lambda f} l \quad \text{and} \quad D = \frac{2\pi}{\lambda f} lN. \tag{69}$$

Let us remind that values \(w_0, l (w_0 \ll l)\), and \(N\), respectively, corresponds to the linear size of the pixel, a distance between the adjoining pixels, and the pixel number along one of the transverse directions.

We want to follow the reduction of the shot noise, i.e., the summarized current in the case of the SPL pixels and the differential one in the case of the DOPO pixels. In (fig. 4), an interchange for the holes on the main frequencies (in case \(l/w_0 = 10\) they are frequencies with numbers divisible by 10 and zero frequency) and half less deep holes on the additional frequencies (here with the numbers divisible by 5) occurs.

Both sequences of the holes decline to the shot noise level on the frequencies far from zero. The effective hole number is determined by the parameter \(D\) (Eq. (69)). One can see that for small \(N\) (see fig. 4), complete reduction of the shot noise takes place only around zero frequency \(q_x\). For the greater number of \(N\) (see fig. 5), the interval which damping of the holes in the shot noise level occurs on gets larger. Therefore the well reduced fluctuations can be observed at the wider range of \(q_x\).

The situation when the DOPO is chosen as the pixels (fig. 5) is unfavorable since deeps at the shot noise level dependency are alternating with picks. Moreover, the depth of the holes is limited by half of shot noise level. Therefore the perfect suppression of the shot noise at this case is simply impossible.
Figure 4: Spatial summarized photocurrent variances spectrum in the far field, in case when SPL is chosen as the pixel, \( q_y = 0, \Omega = 0, l/w_0 = 10 \) in arbitrary units, \( N = 7 \) (a), \( N = 99 \) (b).

Figure 5: Spatial differential photocurrent variances spectrum in the far field, in case when DOPO is chosen as the pixel, \( q_y = 0, \Omega = 0, l/w_0 = 10 \) in arbitrary units, \( N = 7 \).

VII. CONCLUSION

In this article we purpose two main aims. First of all we wanted to find the measuring procedure that could be in some cases an alternative to the balanced homodyne detection. The latter is an universal approach, which allows to select any quadrature component of the field by mixing on the detector with the field from the local oscillator. For that we have to choose the acceptable phase and the acceptable spatio-temporal configuration of the local oscillator amplitude. Often especially in quantum imaging, where the spatial configuration can be extremely complicated, this task turns out to be not very simple. Then it would be nice to find an approach, where these obstacles could be graded to some way.

To our mind we have discussed the quite adequate scheme that allows to follow both squeezed and stretched quadrature components simultaneously without the use of the HD technique. For that we need first to use the so-called coherent sources and second to follow not current itself but the summarized and differential combinations currents under the detection of two entangled beams in the far-field.

The second goal was as follows. We constructed the theory of so-called pixellised source of the spatio-temporal squeezed light. We assumed to use the phase-locked sub-Poissonian lasers and the optical parametric oscillators generating above threshold as pixels. On this basis we have examined our measuring scheme.

VIII. ACKNOWLEDGMENT

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