Field and Cost Optimization of a 5 T/m Normal Conducting quadrupole for the 10-MeV Beam Line of the Electron Linac of the Mexican Particle Accelerator Community

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Abstract. The Mexican Particle Accelerator Community is currently designing the first Mexican RF eLINAC composed of three beam lines at 10, 60 and 100 MeV. In this work, we present an optimized design in terms of field quality and production cost for the 5 T/m normal conducting quadrupoles of the 10 MeV beamline. Several candidate materials for the yoke were studied in terms of the availability and machinability, with the aim to optimize in-house production cost (Mexico) while restricting a low multipole content.

1. Introduction
The Mexican Particle Accelerator Community (CMAP) is composed of a group of young scientists and students that have the common goal of developing science and technology of particle accelerators in Mexico. Since foundation, in 2015, CMAP has been boosting the development of the area by promoting fundamentals schools and workshops for students all around from Mexico and, recently, with a project to develop of a 100 MeV electron linear accelerator (linac), currently under design. The lattice linac design will be using 2 m focusing-defocusing (FODO) cells, and each of it is formed of two quadrupoles [1]. This work presents an optimized design for the normal-conducting quadrupoles that will make part of the Mexican linac FODO Cells. The design is based on a cost optimization model, regarding production cost and field uniformity, developed to promote in-house manufacture.

2. Parametrization Model
The development of accelerator technology represents a challenge, on the one hand the technological and on the other hand economic perspectives [2, 3, 4]. To merge both concepts, a comprehensive design of a normal-conducting quadrupole must consider a large number of parameters to fulfill beam requirements, and it should provide feedback of how this affects the
total production and operation cost. A schematic model of one-quarter of the “standard type” quadrupole geometry is shown in Fig. 1, that relates the field requirement with the geometry parameters, providing a direct path for field and cost optimization.

Figure 1. Geometry parameters for a quarter section of a standard type geometry of normal-conducting quadrupole.

The main parameters that define the geometry of a quadrupole are: \( R \) the aperture radius, \( W \) the width of the hyperbola pole-profile, and \( A \), the latter defined as in terms of the minimum pole distance \([5]\). This last parameter has a strong influence of the field quality at the region of the aperture. By geometric construction one can express the width of the pole as:

\[
W = \sqrt{2A} \left| 1 - \frac{R^2}{2A^2} \right|
\]  

(1)

On the other hand, the length of the pole can be expressed in terms of the number of wires \( N_p \), laying on the pole face of a block-coil distribution, which depends on the conductor radius \( R_w \). The thickness of the flux return \( B_t \), depends on the field saturation and can be defined on the magnetic relaxation model. This model provides a correlation between the main geometrical parameters and the beam requirements, and it could be easily extended to Collins and Panofsky geometries \([6]\), or to higher order normal-conducting electromagnets.

3. Cost Optimization
To produce an optimized quadrupole, regarding minimum production and operation costs, along with optimum field performance, we have merged the latter parametrization model, with the cost optimization procedure given by Brianti and Gabriel \([7]\), and extended to normal-conducting quadrupoles.
The total cost is divided into two main components: the equipment cost $M_e$, and the running cost, $M_r$. The first takes into account the cost of the power supply and associated equipment $M_1$, cost of finish coil mounted on the yoke $M_2$, cost of the finished yoke $M_3$, cost of AC and DC distribution $M_4$ and cost of cooling $M_5$, shown in Eq. (2). The latter considers the operation cost for a given running time and involves the cost of electricity for the power supply.

$$M_{tot} = M_r + \sum_{i=1}^{5} M_i$$  \hspace{1cm} (2)

We have chosen to normalize the cost components $M_i$ with respect to power supply cost, to avoid fluctuations in price and economy. This normalization relates the cost between different elements and leaves undetermined the actual money. The values and the functional dependence of the $M_i$ coefficients with the main parameters are given on Table 1.

| Parameter                              | Symbol | Value      |
|----------------------------------------|--------|------------|
| Field Gradient                         | $K$    | 3.2 T/m    |
| Aperture radius                        | $R$    | 0.025 m    |
| Field at pole tip                      | $B_{tip}$ | 0.080 T  |
| Minimum pole distance$^1$              | $A$    | 0.0074 m   |
| Flux return thickness                  | $B_f$  | 0.013 m    |
| Wires along pole                       | $N_p$  | 18         |
| Wires along base                       | $N_b$  | 6          |
| Total Wires/pole                       | $N = N_b N_p$ | 108       |
| Conductor radius                       | $R_w$  | 0.002 m    |
| $M_1$ Power Supply & Equip.            | $M_{01}$ | 1.5161    |
| Power cost/kW                          | $m_1$  | 1.22x10$^{-5}$ |
| Power factor                           | $\alpha$ | 0.99      |
| $M_2$ Conductor cost/m$^3$             | $m_2$  | 114.744    |
| Volume of finished coil                | $V_c$  |            |
| $M_3$ Cost of finished yoke/m$^3$      | $m_3$  | 5.458      |
| Volume of finished yoke                | $V_y$  |            |
| $M_4$ a.c. distribution/kW             | $m_4$  | 0.3058     |
| $M_5$ Cost of cooling/kW              | $m_5$  | 0.002      |
| $M_r$ Running time                     | $T$    | 36500h     |
| Cost of electricity/kWh                | $m_6$  | 1.22x10$^{-5}$ |
| Power Correction factor                | $\beta$ | 0.5      |

### 3.1. Main parameters

Both components of the capital cost, $M_e$ and $M_r$, depends on the active power, the volume of the conductor and the volume of the yoke, as described in [7]. The power $P$, defined by the Joule-Lenz law [8], can be express in terms of the main beam requirements: field gradient $K$, current density $S_f$, magnet length $L$, aperture radius $R$ and the field quality ($A$), setting the
magnet length and the current density as the two scaling parameters. To determine the power $P$, we define the number of wires per pole in terms of the field gradient and aperture radius. The former is determined by the electrical excitation in the coils according to Ampere’s law [9]. If the integration path connecting the segments: $P_0P_1$, $P_1P_2$, $P_2P_3$ and $P_3P_0$ on Fig. 1 is used, one can estimate the field gradient in terms of the operational current $I_{op}$, as:

$$I_{op} = \frac{KR^2}{2\mu_0}$$  \hspace{1cm} (3)

The number of wires per pole is obtained as the ratio between the operational current, and the total current per wire, in terms of the current density passing through a conductor of radius $R_w$:

$$N = \frac{KR^2}{2\mu_0(S_f\pi R_w^2)}$$  \hspace{1cm} (4)

If a conductor of radius $R_w$ and resistivity $\rho_c$ is used, the resistance is:

$$R = \frac{\rho_c L}{A} = \frac{\rho_c}{\pi R_w^2} [L + W] 8N$$  \hspace{1cm} (5)

The power $P$ defined by the length of the magnet, the resistance of the conductor and the current density, can be expressed as:

$$P = 4K\rho_c S_f R^2 \left( L + \sqrt{2}A \left| 1 - \frac{R^2}{2A^2} \right| \right)$$  \hspace{1cm} (6)

One can estimate the volume of conductor as:

$$V_c = \frac{4KR^2}{\mu_0 S_f} \left( L + \sqrt{2}A \left| 1 - \frac{R^2}{2A^2} \right| \right)$$  \hspace{1cm} (7)

From Fig. 1, the volume of the iron yoke is calculated as a function of the quadrupole length $L$. Assuming that the cross-sectional area of pole is approximately $2N_p R_W W$, one can write the volume of the yoke as given by:

$$V_y = L \left[ 8B_t N_p R_W + 4B_t W + 16B_t N_b R_w + \sqrt{2} B_t^2 + 8B_t \left( A + \sqrt{2} R_W (N_p - N_b) \right) \right]$$  \hspace{1cm} (8)

From $N = N_p N_b$ one can express $N_b$ as:

$$N_b = \frac{KR^2}{2\mu_0 (S_f\pi R_w^2)} \frac{1}{N_p}$$  \hspace{1cm} (9)

3.2. Case study: CMAP quadrupoles

The latter procedure is applied to the case of the normal-conducting quadrupoles using the values described on Table 1. For the optimization procedure, several candidate materials for the yoke were considered regarding the availability and machinability. The steels: A-1010, A-1008, A-1006 and A-1018 are available and at low cost. A preliminary study of the multipole content, considering the latter materials [10], revealed a negligible difference between them in terms of performance. Steel A-1010 was selected for the iron yoke. As far as machinability, laser-cut milling offers a 10x cheaper production, while maintaining tolerances, in comparison with Computer Numerical Control (CNC) milling, for a laminated quadrupole design.

As it can be seen on Fig. 2, the total cost has a minimum for the current density at 1.8 A/mm², and a minimum concerning the length at $L= 0.1$ m. A 1% fluctuation of the cost around the
minimum allows setting a current density in the range from 1.2 to 2.4 A/mm$^2$. If the length of quadrupole increases to next value, 0.15 m, the total cost increases by 2.8%. A dissection of the total cost regarding the power supply, the magnet, and the cost of electricity, reveals that for the low-field gradient and small aperture requirements, the power supply represents the significant expense. The magnet cost and the electricity cost merge at current densities of higher than 2 A/mm$^2$.

**Figure 2.** On top, cost optimization model applied to CMAP quadrupoles. On the bottom, the dissection of the total cost in terms of its main components.

4. Field Optimization
The field quality of a quadrupole depends on a number of variables such as the positioning of the coils, the orientation of the poles, the tolerances on the machine parts, and the quality of the magnetic model. To evaluate the magnetic model alone, one can express the magnetic field inside the aperture, in terms of its components $B_x$ and $B_y$, which can be specified by the multipole expansion, Eq. 10, as described by [1]. Where $B_{ro}$ is the fundamental harmonic at the reference radius $R_0$ (1.8 cm for this calculations), $b_n$ and $a_n$ are the normal and skew multipoles,
respectively.

\[ B_x + iB_y = 10^{-4}B_{ro} \sum_{n=1}^{\infty} (b_n + ia_n) \times \left[ \cos(n\theta) + i\sin(n\theta) \right] \left( \frac{r}{R_r} \right)^n \] (10)

Applying the parametric model to the geometry shown in Fig. 1 and using Comsol Multiphysics [11], we performed a parametric sweep on the minimum pole distance \( A \). For a 3.2 T/m gradient, the allowed higher-order multipoles \( b_5, b_9 \) and \( b_{13} \) [12] are kept within 1 unit when the minimum pole distance \( A \) is 7.4 mm, if the pole width slightly increases beyond 7.6 mm, the multipoles rapidly grow, as shown at Fig. 3. The latter imposes a tolerance limit for machining parts. Tolerances of 76 \( \mu \)m could readily be achieved by laser-cut milling [13], offering safe variation within 7.4 and 7.6 mm.

![Figure 3. Multipole content as a function of the minimum pole distance \( A \).](image)

5. Conclusion
An optimization procedure, regarding field quality, production, and operation cost, was applied to encourage the in-house development of the normal-conducting quadrupoles in Mexico. The process merged a parametrization model with well known-cost optimization procedure and extended it to normal-conducting quadrupoles. A 5 T/m quadrupole, was initially proposed. Nevertheless, adjustments on the field requirements shifted the field gradient to a new value of 3.2 T/m. The cost optimization expressed in normalized units, considered several steels for iron yoke, being A-1010 the final candidate in terms of availability and low cost. For a 3.2 T/m field gradient, the total estimated cost is 1.66 the cost of the power supply at a current density of 2.2 A/mm².

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References

[1] C.A. Valerio-Lizarraga et al, Study of the First Mexican RF linear accelerator. in *Rev. Mex. Fis.* 64 (2018) 116-121.

[2] Caroline M. Sallee, Scott D. Watkins and Alex L. Rossen. The Economic Impact of Fermi National Accelerator Laboratory. in *Anders Economic Group, LLC.* 2011.

[3] L.R. Evans. LHC Accelerator physics and technology challenges. in *Proceedings of the 1999 Particle Accelerator Conference, New York,* 1999.

[4] Massimo Florio, Andrea Bastianini and Paolo Castelnuovo. The socio-economic impact of a breakthrough in the particle accelerator technology: a research agenda. in *arXiv:1802.00352v2*

[5] Helmut Wiedemann. Particle Accelerator Physics. in *Springer-Verlag Berlin Heidelberg. 3rd Edition 2007.*

[6] Th. Zinkeider. Basic design and engineering of normal-conducting, iron-dominated electromagnets. in *CERN, Geneva, Switzerland.*

[7] G. Brianti and M. Gabriel. Basic expressions for evaluating iron core magnets a possible procedure to minimize their cost. in *CERN/IS/Int. DL/70-10. CERN.*

[8] S K Thangaraju and K M Munisamy. Electrical and Joule heating relationship investigation using Finite Element Method. in *2015 IOP Conf. Ser.: Mater. Sci. Eng.* 88012036

[9] Thomas Weiland. On the numerical solution of Maxwell’s equations and applications in the field of accelerator physics. in *Particle Accelerators.* Vol.15 pp. 245-292.

[10] D. Chavez et al. Status report on the R&D of a 5 T/m normal conducting quadrupole magnet for the 10 – MeV electron linac of the Mexican particle accelerator community. in *XXVI International Material Research Congress. Symposium E.6 Particle Accelerators for Science & Engineering of Materials.*

[11] Comsol Multiphysics Software. [https://www.comsol.com/](https://www.comsol.com/)

[12] J. Tanabe. Iron dominated Electromagnets Design, Fabrication, Assembly and Measurements. in *World Scientific Pub. Co.* (2005) 48-50.

[13] Syrma. [http://www.syrma.com.mx/#](http://www.syrma.com.mx/#)