This paper reports a study of the cement-concrete coating on bridges using FRP reinforcement. That has made it possible to design optimal structures by selecting the height for reinforcement arrangement in the layers of a roadbed in order to ensure strength characteristics.

An engineering method for calculating a hard roadbed with composite reinforcement has been devised, which makes it possible to take into consideration its work both in a joint package of the structure with a slab and separately – when it exfoliates from the slab of the bridge's span structure. Underlying this research are effort-determining methods, estimation dependences from the theory of bending layered structures, as well as dependences from elasticity theory to assess the strength of materials for a roadbed. The consideration of shear strains when designing slabs has helped establish that the deflections according to the devised method were 1.4 times larger than those in the classical approach.

The method was tested by a numerical experiment, which confirmed the need to use composite reinforcement in the upper layers of a road surface on bridges, which improves its durability by 1.2 times. The results of the numerical experiment indicate that the equivalent stresses in the lower layers of a free-moving roadbed were 2.91 MPa, and, when operating in a joint assembly with a slab, they took a negative value (~0.2 MPa).

Practical application of the devised calculation method makes it possible to determine the refined normal stresses in the layers of a roadbed, taking into consideration the characteristics of structure operation. Owing to this, additional opportunities open up for calculating the roadbeds of bridges whose design utilizes the most common types of span structures in the bridge industry.

Keywords: cement-concrete coating, roadbed, layered structures, composite materials, stressed-strained state.
concrete coating of bridges with composite materials on the stressed-strained state of the structure.

2. Literature review and problem statement

The work of slabs in bridges’ roadways with the use of fiberglass reinforcement (GFRP) under the static and fatigue load was investigated in [2]. The behavior of the bridge roadway slabs, reinforced with glass and carbon-bar grids, on steel run structures was also studied. As a result of study [2], it was concluded that the main structural effect in the slabs, which are opposed to the concentrated load of the wheel, is not bending, as is traditionally believed, but a complex internal stressed state associated with shear strains. The assessment of fatigue of concrete slabs of bridges reinforced with composite GFRP reinforcement was performed in [3–5] whose authors analyzed a change in the rigidity of bridge slabs with a given type of reinforcement. In particular, paper [4] determined that the rigidity of the slabs depends on the effect of temperature conditions; the authors of [5] found the fatigue properties of the concrete slab in a carbon-plastic reinforced concrete bridge (CFRP) do not deteriorate under repeated transport loads. A positive result of the cited study was the establishment of a relatively low degradation rate of slabs with GFRP reinforcement compared to metal. As a result of study [6], it was proven that FRP-reinforcement is the best alternative for reinforcing concrete structural elements of bridges. Summarizing the above studies [2–6], it should be noted that they do not sufficiently substantiate the dependence of the distribution of stresses in the slabs of a bridge roadway using a given type of reinforcement.

The authors of [7] report a theoretical study of slabs reinforced with composite materials by the method of finite elements and propose using an FRP-reinforcement coefficient of 0.3 % per m² in the upper and lower layers of a bridge roadway slab. It was also established that the bearing capacity at the maximum load on bridge roadway slabs is seven times greater than a working load. However, the results of the cited study do not contain a clear algorithm for determining the effect of the reinforcement coefficient on the bearing capacity of the structure.

An applied study of FRP-reinforcement performance [8] showed that the deflections of bridge slabs do not exceed the maximum permissible ones according to the Canadian standard (CSA 2000).

When studying the rigidity of a cement-concrete coating, the author of [9] found that when using additional reinforcement with GFRP bars, it is possible to reduce the thickness of the cement-concrete coating while maintaining the predefined strength. However, papers [8, 9] did not determine how the reduction in the thickness of the cement-concrete coating affects the strength of the structure under a cyclic load.

The bending characteristics of bridge slabs reinforced with FRP mesh made of glass, carbon, and hybrid fibers were studied in [10]. In the cited study, it was found that the deflections and width of crack opening were greater with a given reinforcement since FRP meshes have a lower elasticity modulus than steel ones. However, the authors did not consider options for the combined use of various reinforcement materials in the lower and upper layers of the structure.

Work [11] investigated the operation of basalt reinforcement (BFRP) in the stressed and relaxed states under the action of simulated aggressive media at 25, 40, and 55 °C. It was found that the mechanism of degradation of structures reinforced with BFRP-rebar is accelerated at stress exceeding 20 % of the limit. Study [12] examines the stressed-strained state of beams reinforced with BFRP-reinforcement.

The authors of [13] focus on the impact of structural factors on the nature of changes in the patterns of bearing capacity, crack resistance, and deformations of concrete structures using BFRP. It should be noted that the cited papers [2, 11–13] do not give algorithms for determining the components of the stressed-strained state of structures reinforced with composite materials.

Our review of the above studies [2–13] indicates a high scientific interest in this issue. However, it should be noted that scientists have not sufficiently investigated the impact of FRP-reinforcement of cement-concrete coating of bridges on the stressed-strained state of the structure in general.

3. The aim and objectives of the study

The purpose of this study is to determine the impact of reinforcing a cement-concrete coating of bridges on the stressed-strained state of structures by devising an engineering method for calculating a hard roadbed with composite reinforcement, taking into consideration the running structure of the bridge.

To achieve the set aim, the following tasks have been solved:

- to define problems in ensuring the durability of road bridge coatings;
- to investigate the practical use of composite materials as reinforcement of road surface on bridges;
- to devise a method for determining the components of the stressed-strained state of the system “roadbed – slab”, taking into consideration the peculiarities of work of composite materials;
- to determine the components of the stressed-strained state of the roadbed structure of bridges according to the proposed method.

4. The study materials and methods

The object of this study is the processes of distribution of stresses in the layers of a roadbed with composite reinforcement, taking into consideration the span structure of a bridge.

The calculation of layered composite structures is carried out according to the refined models, which take into consideration the effect of transverse shear strain. The main hypothesis of this study assumes that in order to devise an effective approach to determining the stressed state of structural elements in a roadbed as a layered system, it is necessary to use a rational combination of methods for calculating multilayer and single-layer structures. This approach makes it possible to solve practical problems to study the stressed-strained state of heterogeneous structures with different rigidity and bearing capacity. We determined the refined stresses in the layers of a bridge roadway slab using the theory of bending layered structures taking into consideration transverse normal stresses due to compression. Thus, the main assumption in this study into the stressed-strained state of the slab is the need to examine it as a structure consisting of isotropic layers of different rigidity in terms of thickness. This makes it possible to consider the work of all layers with different thicknesses and physical-mechanical
characteristics. The method for calculating the structure of a hard roadbed is based on the estimation dependences from the theory of bending layered structures (beams, slabs). Strength assessment of structural materials is carried out according to the dependences from elasticity theory [14], which make it possible to take into consideration the peculiarities of the characteristics of materials. Considering the three-dimensional nature of the stressed-strained state of the structure, the proposed method makes it possible to simplify the calculation task to a two-dimensional model since the desired functions are those of the coordinate surface. The reliability of solutions based on these methods has been confirmed by comparison with the simulated bridge structure in the ANSYS software package.

5. Results of studying the impact of reinforcement on the stressed-strained state of the bridge roadbed structure

5.1. Defining the problem of ensuring the durability of road bridge coatings

The issue of durability of bridge road surfaces is a rather complex, multiparametric problem, which is influenced by various factors, in particular, the choice of technology, layer material, operational, natural and climatic factors, etc. When using reinforced cement concrete as a coating, which in its properties is more durable than asphalt concrete, there is a specific issue related to corrosion processes in metal reinforcement. For example, a large-scale study of bridges in the United States revealed that approximately 15 % of the 583,000 bridges examined had undergone destruction caused by corrosion of metal reinforcement (Fig. 1) [15].

They also noted the negative consequences of destruction, which are associated with the loss of the volume by the working part of the reinforcement, the unsatisfactory condition of compensatory seams on the supports, the low resistance of concrete to bending loads. Structural cracks in the cement-concrete coating, which, as a rule, arise due to excessive load, lead to overloading the structure in general. Non-structural cracks are usually the result of internal overstress in concrete structures under the influence of the linear expansion of materials and heat loads. As a result, moisture, chlorides, salt penetrate through the formed cracks; an unfavorable alkaline environment is created, which leads to corrosion of the reinforcement.

According to study [16], it was determined that about 10 % of the volume of reinforcement may be formed by corrosion products. That creates internal pressure in the structure and leads to various kinds of coating destruction (cracking, exfoliation, crack mesh, etc.). Such damage, in turn, could have a serious impact on the performance of structural elements of bridges and reduce the durability of structures.

Based on the results from the visual, instrument-based, and thermal-imaging survey of bridges carried out in Ukraine [17], a number of issues related to cement-concrete coatings affecting the durability of structures were identified. In particular, these include:

- the destruction of bridge slabs as a result of the detachment of the protective layer of metal reinforcement (Fig. 2);
- the corrosion of metal reinforcement due to opening cracks;
- uneven settling of earthen bed on approaches to bridges;
- unsatisfactory state of deformation joints (Fig. 3).

Electrochemical corrosion of steel is the main reason for the deterioration of engineering infrastructure. This is becoming a major issue for the construction industry around the world. Climatic conditions and the use of a large amount of salts as an anti-ice agent in the winter months can contribute to the acceleration of the corrosion process. This usually requires significant financial costs to recover, as it can lead to the catastrophic destruction of concrete structures. An effective solution to this problem is the use of corrosion-resistant materials, such as high-performance fiber-reinforced plastic (FRP) composites. The use of non-metal composite reinforcement over the past 10 years has defined this technology as an advanced one as a mechanism for overcoming problems in cement-concrete structures caused by corrosion of steel reinforcement.

The scope of FRP reinforcement in construction in developing countries is quite limited by regulatory and technical documentation (which is practically lacking) and low funding for bridge restoration programs [2–5].

The comparison of the physical and mechanical characteristics of FRP reinforcement with the characteristics of the metal reinforcement used in transport construction is given in Table 1.

Fig. 1. Typical damage to the hard road surface of a bridge resulting from: a — the corrosion of reinforcement; b — structural cracks; c — the exfoliation of the top layer [15]

Table 1

Comparison of non-metallic composite reinforcement and metal reinforcement

| Characteristics                        | Fiberglass reinforcement (GFRP) | Basalt-plastic reinforcement (BFRP) | Metal reinforcement (А400С) |
|----------------------------------------|---------------------------------|-------------------------------------|----------------------------|
| Raw material                           | Glass roving, epoxy resin       | Basalt roving, epoxy resin          | Metal                      |
| Stretching strength, MPa               | 600–1,200                       | 700–1,300                           | 395                        |
| Elongation, %                          | 2.2                             | 2.2                                 | 25                         |
| Elasticity, MPa                        | 45,000                          | 60,000                              | 200,000                    |
| Linear extension coefficient, α×10^6 °C^-1 | 9–12                           | 9–12                               | 13–15                      |
| Density, t/m^3                         | 1.9                             | 1.9                                 | 7.85                       |
| Diameter, mm                           | 4–20                            | 4–20                                | 6–80                       |
| Corrosion resistance to aggressive environments | Stainless material of the first group of chemical resistance | Stainless material of the first group of chemical resistance | Collapses with the release of corrosion products |
| Heat conductivity                      | Thermally conductive            | Thermally conductive                | Not-thermally conductive   |
| Electric conductivity                  | Dialectic                       | Dialectic                           | Electrically conductive    |
| Durability                             | At least 80 years               | At least 80 years                   | In line with construction norms |
Along with the main characteristics important for transport infrastructure such as absolute corrosion resistance, FRP reinforcement has a number of other advantages. However, the disadvantage of basalt plastic reinforcement (BFRP) is the presence of characteristics that make it impossible to use it for bending bearing elements of transport structures. These include the FRP-reinforcement elasticity module, which is almost three times lower than the steel elasticity module, and unacceptably low fire resistance of the material (Table 1). This leads not only to excessive deflections but also to a sharp decrease in stresses in the calculations for crack resistance.

Figure 4 shows the ratio of stresses and deformations of different types of FRP composites made from solid glass fibers (GFRP), aramid (AFRP), carbon (CFRP), basalt (VFRP), or combinations of these materials (CFRP HS, CFRP HM), compared to steel. The “stress $\sigma$ – strain $\varepsilon$” diagram (Fig. 4) [18] reveals the fragile destruction of the structure reinforced by BFRP with a significant load on it. The facility does not signal its entry into the first boundary state and collapses sharply, which has been repeatedly confirmed experimentally in various studies. This is the main obstacle to the use of FRP reinforcement in bending bearing elements.

The above comparative analysis gives reason to assert that, due to the low physical and mechanical characteristics of basalt plastic reinforcement – a module of elasticity and fire resistance, it cannot be used in structures of the consequence class SS2. The reason for the deviation of this reinforcement is that such structures work under the conditions of dynamic (cyclic) loads and the impact of a possible fire.
However, there is an obvious fundamental possibility of using basalt plastic reinforcement for a wide class of bearing structures of the consequence class SS1, such as slabs on an elastic base.

That is, the properties of FRP reinforcement are the better, the higher the fiber content that binds to the polymer. At the same time, excessive use of fiber in bars leads to the opposite effect due to the fact that the polymer binder cannot spread to all fibers and they do not begin to work together. The impact of this type of reinforcement on the stressed-strained state of bridge structures requires additional research.

5. 2. Investigating the practical use of composite materials as reinforcement for road surface on bridges

FRP reinforcement was used on hundreds of bridges in Canada and the United States. Those bridges were designed using the Canadian Road Design Code (CAN), or the AASHTO LRFD Bridge Design Guide [19].

Below are the results from research based on the project reported in [20]. The Sierrita de la Cruz Creek bridge (Fig. 5) is located 25 miles northwest of Amarillo, Texas. It was built in 2000 to replace the original bridge, which was structurally faulty, and had significant damage caused by corrosion of steel reinforcement. It is the first bridge in the state of Texas to introduce FRP reinforcement as a structural solution to increase the durability of the structure. The bridge operates under the following environmental conditions: the thermal range is from 22 to 95 °F (−6 to +35 °C); the cycles are “wet-dry”, as well as the cycles “freezing-thawing”; exposure to salt against icing. FRP reinforcement was applied in the upper reinforcement grid in two spans of concrete fabric. The removal of concrete cylindrical samples took place in May 2015 in order to analyze the physical and mechanical properties of reinforcement (Table 2).

Concrete cylindrical samples were used to perform pH and carbonation depth measurements to characterize the chemical environment in concrete. In addition, microscopic studies of FRP samples and mechanical tests were carried out to monitor possible changes in the microstructure and mechanical properties. In particular, scanning electron microscopy (SEM), energy-dispersion X-ray spectroscopy (EPC), interlayer shear, vitreous transition temperature (Tg), and fiber content. The results of the interlayer shear test and fiber content measurement were compared with the results of similar tests performed in 2000 at the time of construction. Since no historical data were available for EPC and Tg analyses, these tests were carried out on FRP-bars manufactured in 2015 by the same manufacturer to serve as a benchmark for comparison (Table 3).

5. 3. Devising a method for determining the components of the stressed-strained state of the system “roadbed – slab”

The road surface of bridges is a layered structure on an elastic base, which can be attributed to the class of piecewise-heterogeneous systems. A distinctive feature of such systems is the joint operation of the road surface with the slab and the span structure of the bridge. However, it is not always possible to determine which part of the structural element of the bridge can be attributed to a particular layer.

Thus, there is a need to solve the problem of calculating the system “roadbed – slab” of a bridge roadway, which consists of layers with different physical and mechanical characteristics. This problem can be solved with the help of the method of analysis of components of the stressed-strained state, which is based on the estimation dependences from the theory of bending layered structures and elasticity theory to assess the strength of structural materials [14].

The method of determining the stressed-strained state of a bridge roadbed structure should be represented in the form of a model that provides for four variants of estimation schemes of the structure (Fig. 6).

---

**Table 2**

| Diameter (mm) | Year of production | Fiber Type | Resin Formulation | Additive and filler | Catalyst |
|---------------|--------------------|------------|-------------------|---------------------|----------|
| 16            | 2000               | E-glass    | Ashland Hetron 922 | Styrene, ASP 400, BYK A555 and colloidal silicon dioxide | P16 and TBPB |
| 18            | 2000               | E-glass    | Ashland Hetron 922 | Styrene, ASP 400, BYK A555 and colloidal silicon dioxide | P16 and TBPB |
| 16            | 2015               | E-CR glass| VEX 10-962 CoRezyn | Styrene and ASP 400 | BPO |

---

**Table 3**

| Simple type | Span length (m) | Rated diameter (mm) | Sample No. | Breaking load (N) | Mean factor (%) | Interlayer shear strength Mean (MPa) | Variance factor (%) |
|-------------|-----------------|--------------------|------------|-------------------|----------------|--------------------------------------|---------------------|
| FRP No. 4  | 1.5             | 14                 | 5          | 8838              | 2.4            | 46.6                                 | 2.4                 |
| FRP No. 5  | 1.875           | 16                 | 5          | 12,390            | 2.5            | 41.8                                 | 2.5                 |
| FRP No. 6  | 2.25            | 18                 | 5          | 20,493            | 3.6            | 47.98                                | 3.6                 |

The results of the horizontal shear test of the taken reinforcement samples confirm that FRP materials retained their microstructural integrity and mechanical properties after 15 years of operation in the field. The microscopic study did not detect the destruction of FRP reinforcement. The fibers did not lose any area of the cross-section, the matrix was intact, and there was no damage on the fiber-matrix boundary surface. In addition, the boundary surface between the concrete and FRP reinforcement was in good condition; there was no loss of intersurface bonds.
The work of a roadbed in a joint assembly with the structure of the bridge’s span structure in the elastic stage and detached from the slab on the surface of its contact is analyzed.

To analyze the stressed-strained state of the composite slab of a bridge roadway on metal span structures, it is advisable to use a finite-element method underlying most computational software packages. The system “roadbed – slab” of a bridge roadway is discrete, which creates preconditions for its simulation by the finite elements of the shell of zero curvature [22].

The system “roadbed – slab” of a bridge roadway is determined by the finite-element scheme of nodes in increments $a$ (Fig. 7).

According to [23], it is necessary to determine the refined deflections at the points of the system “roadbed – slab” of a bridge roadway:

$$
q = \frac{D_{gs} \Delta}{D_{g1}} - D_{g1} \left( \frac{M_x + M_y}{D_{g1} (1 + \nu_n)} \right)
$$

Fig. 6. Model to calculate a roadbed reinforced with composite materials

$$
D_{g1} = \int_{h_n} E_{in} \cdot \dot{\varphi}_{gn}(z) \cdot \dot{\varphi}_{n}(z) dz,
$$

where $h_n$ is the thickness of layer $n$ in the “roadbed – slab” system; $E_{in}$ is the reduced elasticity module for layer $n$; $\dot{\varphi}_{gn}(z), \dot{\varphi}_{n}(z)$ are the specified functions of the distribution of displacements by the direction of vectors.

The reduced elasticity module for layer $n$ is determined from the formula:

$$
E_{in} = \frac{E_0}{1 - \nu_n}.
$$

where $\nu_n$ is the Poisson coefficient of layer $n$.

The cylindrical rigidity of the layer assembly is determined from the formula:

$$
D_{g1} = \int_{h_n} E_{in} \cdot \dot{\varphi}_{gn}(z) \cdot \dot{\varphi}_{n}(z) dz = \frac{B_n^2}{B},
$$

where $B_n, B$ are the coefficients that are the generalized physical and geometric characteristics of an assembly of layers.
The function \( \vartheta_{mn}(z) \) of the distribution of displacements by vector directions is as follows:

\[
\vartheta_{mn}(z) = \frac{1}{E_{mn}} \left[ \int_{k_{m,n}}^{z} E_{mn} \, dz + \sum_{r=1}^{n-1} E_{mr} \, dz \right] + c_p - c_n, \tag{5}
\]

where \( G_p \) is the shear modulus for the transversal direction; \( E_{mn} \) is the reduced elasticity modulus for layers \( r \in \{1; n-1\} \);
\( c_p, c_n \) are the integration constants.

Based on the results of several iterations, one can obtain integration constants from the conditions of contact of layers in the assembly:

\[
c_p = \frac{1}{2G_p} \left[ \frac{2E_{mn}}{3} \sum_{r=1}^{n-1} E_{mr} (h_{r+1}^2 - h_{r}^2) \right] + \sum_{r=1}^{n-1} E_{mr} \left( \frac{h_{r+1}^3 - h_{r+1}^2}{3} \right) + \sum_{r=1}^{n-1} \sum_{r'=1}^{n-1} E_{mn} \left( \frac{h_{r}^2 - h_{r+1}^2}{2} \right) \left( h_{r+1} - h_{r} \right), \tag{6}
\]

where \( G_p \) is the shear modulus for the transverse direction of layer \( p \); \( E_{mn} \) is the reduced elasticity modulus for layer \( n \);
\( c_p = \frac{1}{2G_p} \left[ \frac{2E_{mn}}{3} \sum_{r=1}^{n-1} E_{mr} (h_{r+1}^2 - h_{r}^2) \right] + \sum_{r=1}^{n-1} E_{mr} \left( \frac{h_{r+1}^3 - h_{r+1}^2}{3} \right) + \sum_{r=1}^{n-1} \sum_{r'=1}^{n-1} E_{mn} \left( \frac{h_{r}^2 - h_{r+1}^2}{2} \right) \left( h_{r+1} - h_{r} \right), \tag{7}
\]

where \( G_p \) is the shear modulus for the transverse direction for layers \( r \in \{1, n-1\} \).

In its final form, the function \( \vartheta_{mn}(z) \) function takes the form:

\[
\vartheta_{mn}(z) = \frac{1}{E_{mn}} \left[ \frac{z^3}{3} - h_{r+1}^2 - h_{r+1}^2 \right] + \frac{1}{2G_p} \sum_{r=1}^{n-1} E_{mr} \left( h_{r+1}^2 - h_{r}^2 \right) \left( h_{r+1} - h_{r} \right) + c_p - c_n. \tag{8}
\]

The components of the stress tensor are determined taking into consideration the derivatives of the found deflection functions \( w_{mn} \) in the nodes of the estimation scheme of the system "roadbed - slab" of a bridge roadway:

\[
\begin{align*}
\sigma_{mn}^{(1)}(z) &= E_{mn} \left[ \varepsilon_{11}(z) + \varepsilon_{22}(z) \right] + \left( \eta_{11}(z) + \eta_{22}(z) \right) \vartheta_{mn}(z) \text{,} \\
\sigma_{mn}^{(2)}(z) &= E_{mn} \left[ \varepsilon_{11}(z) - \varepsilon_{12}(z) \right] + \left( \eta_{11}(z) - \eta_{12}(z) \right) \vartheta_{mn}(z) \text{,} \\
\sigma_{mn}^{(3)}(z) &= E_{mn} \left[ \varepsilon_{11}(z) + \varepsilon_{12}(z) \right] + \left( \eta_{11}(z) + \eta_{12}(z) \right) \vartheta_{mn}(z) \text{.}
\end{align*} \tag{9}
\]

where \( \varepsilon, \eta \) are the deformations of the coordinate surface.

In a general case, the expression for transverse tangent stresses at \( g=1.2 \) is defined as follows:

\[
\sigma_{mn}^{(t)}(z) = \xi_{mn} f_{mn}(z), \tag{10}
\]

where \( \xi_{mn} \) is the shear function, \( t \in \{1, 2, 3\} \);
\( f_{mn}(z) \) is the function of the distribution of tangential stresses along the normal.

The function of the distribution of tangential stresses along the normal \( f_{mn}(z) \) is determined from the following formula:

\[
f_{mn}(z) = \int_{k_{m,n}}^{z} E_{mn} \, dz + \sum_{r=1}^{n-1} E_{mr} \, dz = \frac{E_{mn}}{2} \left( z^2 - h_{r+1}^2 \right) + \sum_{r=1}^{n-1} \frac{E_{mr}}{2} \left( h_{r+1}^2 - h_{r}^2 \right). \tag{11}
\]

Deformations of the coordinate surface \( (\varepsilon, \eta) \) are determined by the finite difference method:

\[
\varepsilon_{11} = -w_{11} = -\frac{w_{11} - 2w_{12} + w_{13}}{\alpha^2}, \tag{12}
\]

where \( w \) is the normal movement of the coordinate surface.

\[
\varepsilon_{12} = -w_{12} = \frac{(w_{11} + w_{12}) - (w_{11} + w_{12})}{4\alpha^2}, \tag{13}
\]

\[
\eta_{11} = -\xi_{11} = -\frac{\xi_{11} + 2\xi_{12} + \xi_{13}}{\alpha^2}, \tag{14}
\]

\[
\eta_{12} = -\xi_{12} = -\frac{\xi_{11} + \xi_{12} + \xi_{13}}{4\alpha^2}, \tag{15}
\]

\[
\eta_{13} = -\xi_{13} = -\frac{\xi_{11} - 2\xi_{12} + \xi_{13}}{\alpha^2}. \tag{16}
\]

The displacement and shear functions’ derivatives are indicated by a bar.

The next step in the calculation is to determine the main stresses \( \sigma_{11}, \sigma_{22}, \sigma_{33} \), which are the roots of the cubic equation [24]:

\[
\sigma_1^3 - I_1 \sigma_1^2 + I_2 \sigma_1 - I_3 = 0, \tag{18}
\]

where \( I_1, I_2, I_3 \) are the coefficients of the cubic equation, the so-called stress invariants.

The coefficients in formula (18) are determined from the system of equations:

\[
\begin{pmatrix}
I_1 \\
I_2 \\
I_3
\end{pmatrix} = \begin{pmatrix}
\sigma_{11} + \sigma_{33} \\
\sigma_{21} + \sigma_{33} \\
\sigma_{21} + \sigma_{32} + 2\sigma_{33}
\end{pmatrix}, \quad \begin{pmatrix}
\sigma_{11} \\
\sigma_{21} \\
\sigma_{21}
\end{pmatrix} = \begin{pmatrix}
\sigma_{11} \\
\sigma_{21} \\
\sigma_{21}
\end{pmatrix}, \quad \begin{pmatrix}
\sigma_{11} \\
\sigma_{21} \\
\sigma_{21}
\end{pmatrix} = \begin{pmatrix}
\sigma_{11} \\
\sigma_{21} \\
\sigma_{21}
\end{pmatrix}. \tag{19}
\]

The strength of the reinforcement is estimated according to theory IV:

\[
\sigma_{mn}^w = \sqrt{\frac{1}{2} \left( \left( \sigma_{11} - \sigma_{22} \right)^2 + \left( \sigma_{22} - \sigma_{33} \right)^2 + \left( \sigma_{33} - \sigma_{11} \right)^2 \right)} \leq R_c. \tag{20}
\]
where \( R_p \) is the calculated resistance of the material, which is determined from the expression:

\[
R'_p = R_p \cdot m \cdot \beta.
\]  

(21)

where \( R_p \) is the material’s resistance; \( \beta \) is a coefficient that takes into consideration the reliability of the structure; \( m \) is the coefficient that takes into consideration working conditions.

The method of determining the stressed-strained state of the system “roadbed – slab” of a bridge roadway is two-dimensional in terms of its mathematical base since all the desired functions are determined by the functions of the coordinate surface. Thus, the applied method makes it possible to reduce the three-dimensional problem from elasticity theory to two-dimensional.

Next, the strength of materials in each layer of the roadbed of a bridge is tested.

5.4. Determining the components of the stressed-strained state of a bridge roadbed structure

The vast majority of road surfaces on bridges are built using an asphalt-concrete coating, the bearing capacity of which is no more than 115 kN (11.5 tons) per axle, and the total elasticity module is up to 300 MPa. Load from 13 tons per axle requires an increase in the total elasticity module to 390–400 MPa. Stress in the layers of a road surface during overload can reach 7.0–7.5 MPa, which is 2.5–3.0 times higher than the limits of strength and shear resistance of standard asphalt concrete. To address this issue, it is proposed to use specialized technical solutions [25] such as the use of a cement-concrete coating, including composite reinforcement.

To determine stress in the layers of a roadbed structure, two typical structures were considered (Fig. 8). The road surface is arranged on a reinforced concrete slab of a bridge roadway. In this case, the joint work of these two elements is considered.

We implemented the proposed method for determining the stressed-strained state of a bridge roadbed structure using the Mathcad software (USA); the calculation was performed for a cement-concrete coating with composite reinforcement. The span structure of the beam type in the transverse direction consists of 4 freely supported beams. The slabs are designed using the concrete of class B35.

As a result of the calculation, we derived the dependences of the distribution of normal and equivalent stresses for the height of a roadbed (Fig. 9–11).

The results of the derived stresses according to the proposed method on the upper and lower surfaces of a bridge’s roadbed layers are summarized in Table 4.

Our results indicate that the strength at the bottom of a roadbed with free movement is close to the limit when strengthening it at the top. This demonstrates that for the effective operation of the structure, a roadbed must be arranged taking into consideration its work in a joint assembly with a slab.

In order to test the proposed method, a bridge structure was simulated in the ANSYS software package. For modeling, the integrated element Solid65 was selected to construct the concrete beams of the span structure and a slab. The internal reinforcement of the bridge’s span slab was simulated using the Link180 integrated element (Fig. 12) [26].

![Fig. 8. Structure of a roadbed on the reinforced concrete slab: 1 – cement-concrete coating; 2 – waterproofing; 3 – aligning layer; 4 – bridge roadway slab: a – with waterproofing; b – without waterproofing](image)

![Fig. 9. The nature of the distribution of normal stresses in the structure of a roadbed in the joint assembly with a slab](image)
Table 4
Results of calculating the cement-concrete roadbed with BFRP reinforcement

| Roadbed structure, layer characteristics | Estimation resistance to stretching $R_p$, MPa | Equivalent stresses in layers, MPa | Roadbed in a joint assembly with a slab | Free-moving roadbed |
|----------------------------------------|-----------------------------------------------|----------------------------------|--------------------------------------|-------------------|
| Roadbed in a joint assembly with a slab | Layer top | Layer bottom | Layer top | Layer bottom |
| Cement-concrete, $h_1=3$ cm, $E_1=30,000$ MPa | 3.0 | -0.544 | -0.377 | -0.544 | -0.504 |
| Reinforcement $d=0.4$ cm, $h_2=0.01508$ cm | Steel | 265 | 13.950 | 13.926 | 8.364 | 8.311 |
| BFRP | 600 | 2.697 | 2.693 | 1.432 | 1.421 |
| Ratio, times | – | 5.17 | 5.17 | 5.84 | 5.85 |
| Cement-concrete, $h_1=7.98492$ cm, $E_1=30,000$ MPa | 3.0 | -0.377 | -0.201 | -0.504 | 2.906 |
| Waterproofing, $h_1=0.5$ cm, $E_1=2,000$ MPa | 1.3 | – | – | 0.479 | 0.488 |

Comparing the results of calculations according to the proposed method and when using the ANSYS software package reveals that the stresses that occur in a roadbed have a ratio of 1.13, which proves the reliability of the mathematical values obtained.

Our study of the state of coatings of motor road bridges (Fig. 1–3) has made it possible to define the problems of ensuring the durability of structures, which are mostly
related to the corrosion processes in metal reinforcement. To address this issue, it is proposed to use composite FRP-reinforcement, which has the best strength characteristics and is resistant to aggressive media (Table 1, Fig. 4).

Analysis of the practical application under actual conditions of FRP-reinforcement confirms that FRP-materials retain their microstructural integrity and mechanical properties after the long-term operation (Table 3). This proves that FRP reinforcement can be used as a reinforcement material in the roadbed of bridges.

An engineering method has been devised for determining the components of the stressed-strained state of the system "roadbed – slab" reinforced with BFRP (Fig. 6). Unlike traditional methods [14, 22, 24] in which the task of estimating the stressed-strained state is resolved on the basis of universal theoretical models, the devised calculation method makes it possible to take into consideration its work in a joint assembly of the structure with a slab. In addition, according to a given method, it is possible to determine the operation of a separately exfoliated roadbed from the slab of a bridge’s span structure. When developing the calculation method, the estimation dependences from the theory of bending layered structures (1) were taken into consideration, as well as the refined transverse normal stresses due to layer compression were used. The transverse tangent stresses (10) were determined taking into consideration the deformations of shear. To assess the strength of structural materials, the dependences from elasticity theory (20), (21) and the features of composite materials (Table 1) included in the layered structures of span structures and roadbeds were used.

Our mathematical experiment, using an example of the bridge's span structure reinforced with both steel and BFRP, demonstrated a ≈1.4-time discrepancy between the classical approach to the calculation and the method devised. The results of analyzing the established deflections confirm the expediency of taking into consideration the deformations of the shear of the slabs of bridges’ span structures.

The equivalent stresses at the top of a layer of a cement-concrete roadbed without reinforcement when working together with a slab reach the maximum permissible values of about 3.0 MPa, which indicates its operation on the verge of its strength. To ensure the reliability of our results, the stresses in the layers of the “roadbed – slab” system were within the permissible limits. In addition, according to the devised method, it is possible to take into consideration the effects of temperature changes on the stressed-strained state of the structure.

Further studies into the stressed-strained state of the roadbed of bridges should be carried out towards refining the stresses, taking into consideration the effects of temperature changes on the stressed-strained state of the structure.

7. Conclusions

1. The main disadvantage of functioning structures is corrosion processes in metal rears. Our visual, thermal-imaging, and instrument-based studies revealed issues related to the destruction of a cement-concrete coating and the exfoliation of a protective layer of metal reinforcement in the roadway slabs due to the overload on the bridges’ span structures. It was established that an effective solution to the specified problems is the use of composite FRP reinforcement. The main advantage of non-metal reinforcement is that its strength at stretching is 2–3 times higher than that of steel while the coefficient of linear expansion is 1.25 times lower. The specific weight of FRP reinforcement is 4 times less, which increases the bearing capacity of the structure in general.

2. It was found that the FRP materials used as reinforcement retained their microstructural integrity and mechanical properties after a long-term operation of the bridge. Additionally, it was established that the intersurface bonds between the concrete and FRP reinforcement were in satisfactory condition. These conclusions are a prerequisite for ensuring the strength characteristics of a structure during its long-term operation. This is justified by the fact that due to the resistance to aggressive environments, the main disadvantages of using metal reinforcement are excluded, which leads to an increase in the operational life of bridges by 1.2 times.

3. The devised method for determining the components of the stressed-strained state of the system “roadbed – slab” makes it possible to take into consideration the work of a roadbed both in a joint assembly of the structure with a slab and separately – when it is exfoliated from the slab of the bridge’s span structure. A given method was tested in a numerical experiment, which confirmed the need to use composite reinforcement in the upper layers of the road surface on bridges to ensure the strength of the structure. Based on the results of our study, the discrepancy between the values of deflections of 1.4 times was established between the classical approach to the calculation and the developed method.

4. The determined equivalent stresses in the lower layer of a roadbed with free movement were 2.91 MPa; when working in a joint assembly with a slab, they took a negative value (~0.2 MPa). It was established that the values of equivalent stresses were within the permissible limits. In addition, from a practical point of view, it has been proven that the devised method makes it possible to obtain refined stresses in the layers of the “roadbed – slab” system. The adequacy of the calculation results was proven by comparing the obtained data with the structure simulated in the ANSYS software package. It was established that the discrepancy in the stresses of the structural layers was 1.04, which confirms the reliability of our results.
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