On the strange quark mass with improved staggered quarks

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We present results on the sum of the masses of light and strange quark using improved staggered quarks. Our calculation uses 2+1 flavours of dynamical quarks. The effects of the dynamical quarks are clearly visible.

1. INTRODUCTION

1.1. Motivation

Simulations of hardronic matter with realistic sea quark effects form a long standing problem in lattice gauge theory. To achieve this goal on the upcoming generation of computers, improved staggered quarks are most promising. Compared to fermion formulations based on the Wilson action, staggered quarks have fewer variables and are faster to simulate for small quark masses.

Naïve staggered quarks suffer from large flavour changing strong interactions, which can be suppressed by the use of fat links \cite{1}. This action can be further improved to $O(\alpha_s a^2, a^4)$ \cite{2}, the most precise to date, see \cite{3} for recent developments. These actions have proven to be successful to reduce the splittings in the pion spectrum \cite{4} and the large renormalisations \cite{5,6}, as observed for naïve staggered quarks. Due to the advantages of improved staggered quarks it has been possible to generate data sets with 2+1 flavours of dynamical quarks with a low $m_{\pi}/m_{\rho}$ ratio \cite{7,8}. The simulation parameters of the configurations have been tuned to keep the lattice spacing fixed in units of the gluonic observable $r_1$.

2. LATTICE SPACING

2.1. Upsilon spectrum

$\Upsilon$-spectroscopy on the configurations of \cite{7} is discussed in \cite{10}. For the data set with $n_f = 2+1$ and the smallest $m_{\pi}/m_{\rho}$ ratio one gets $a^{-1} = 1.6$ GeV. In the quenched approximation the inverse lattice spacing is substantially larger.

2.2. Kaon spectrum

In real world, the difference $m_V^2 - m_{PS}^2$ between the square of the mass of the vector and the pseudo-scalar meson is almost independent from the masses of the valence quarks. In principle this should yield a good method for determining the lattice spacing. Vector mesons are unstable however. Their decays will eventually be seen in dynamical lattice results, but currently it is unclear what effect this has on the measured masses.

In the quenched approximation, $m_V^2 - m_{PS}^2$ is much more sensitive to the valence quark masses.
than the real world. For data sets with dynamical fermions on the other hand the splittings $m_{\rho}^2 - m_{\pi}^2$, $m_{K}^2 - m_{\eta}^2$ and $m_{\phi}^2 - m_{\eta_s}^2$ differ by only $\approx 2\%$. On the $n_f = 2+1$ sets one gets $a^{-1} = 1.4$ GeV from $m_{K}^2 - m_{\bar{K}}^2$. For the quenched data this procedure gives an inverse lattice spacing which is about 100 MeV smaller.

2.3. Gluonic scale
The gluonic observable $r_1$ is determined in a similar way to $r_0$, but gives the point where $r^2 F(r) = 1$. Since the configurations have been generated for fixed $r_1$, it is natural to use $r_1$ as an alternative to set the scale. MILC estimated $r_1 = 0.35$ fm, using $r_0 = 0.5$ fm. Neither of these quantities is easily related to a physical observable, however, and so there must be considerable uncertainty in their physical value. Taking $r_1 = 0.35$ fm gives $a^{-1} = 1.5$ GeV for all configurations.

3. QUARK MASS

3.1. Determination of the bare quark mass
We determine the sum of the bare masses of the light and strange quark by interpolation of the pseudo-scalar meson mass to the physical $K$ with the ansatz
\[(a m_{PS})^2 = a + b a m_{av} + c (a m_{av})^2.\] (1)
With $m_{av} := \frac{1}{2}(m_1 + m_2)$ we denote the average of the valence masses. This is shown in figure 1 for the most chiral dynamical data set. The data points represent the $\pi$, $K$ and $\eta_s$. The difference between a quadratic and linear interpolation is just visible in the upper right corner of the figure. At the physical $K$ this difference is completely irrelevant.

At this point the advantage of not having additive mass renormalisations for staggered quarks pays back. No simulation is needed to determine the point of vanishing quark mass.

3.2. Renormalisation and matching
The renormalisation and matching to the $\overline{\text{MS}}$-scheme is done in a three step procedure. We use the quark self-energy to convert from the lattice mass $m_0$ to the pole mass $m_0$. In a second step we convert to the $\overline{\text{MS}}$-mass at the scale of the pole mass and finally run to the scale $\mu = 2$ GeV. At 1-loop level we get
\[
m_{\overline{\text{MS}}} = \left[1 + \alpha_s \left(b - \frac{4}{3\pi} - \frac{2}{\pi} \log(a \mu)\right)\right] m_0
=:\left[1 + \alpha_s Z_0^{(2)}\right] m_0.
\] (2)
Here $b := \delta m_0^2/m_0 - 2/\pi \log(a m_0)$. This is the 1-loop contribution to the mass shift between the bare lattice mass and the pole mass with the logarithmic divergence taken out. For small values of $a m_0$ this becomes a constant $\tilde{3}$. For $n_f = 2+1$ we obtain
\[-0.12 < Z_0^{(2)} < -0.04,\] (3)
depending on the quantity used to fix the lattice spacing. It is interesting to note, for naive staggered quarks, that is without fat links to suppress the flavour changes, $Z_0^{(2)} \approx 3$.

For $\alpha_s$ we use the average of $\alpha P$ for the scales $aq = 1$ and $2$, determined on these configs $\tilde{3}$. The difference between the two is included into the error. Preliminary calculations of the modified BLM scale $aq^* \tilde{2}$ give values lying between 1 and 2. Because of the small $Z_0^{(2)}$, variations in $q$ have only very small effects.

![Figure 1. Interpolation of the pseudoscalar mass to the Kaon](image-url)
Our final result for the sum of the masses of the light and the strange quark in the $\overline{\text{MS}}$-scheme is presented in figure 2. Different shapes of the symbols refer to the numbers of sea quarks. The different shadings of the symbols indicate the determination of the lattice spacing. The dashed line corresponds to the real world $m_\pi/m_\rho$-ratio. The error bars encompass statistical uncertainties, the uncertainties of the determination of $a$ and the uncertainty of the scale setting of $\alpha_s$. One needs to add an uncertainty for the 2-loop effects. At the present time we estimate this as $O(\alpha_s^2) \approx 20\%$ for $n_f = 2 + 1$ and smaller otherwise.

In the quenched case, the results show a wide spread, depending on how $a$ is determined. This spread is clearly reduced for $n_f = 2$ and even smaller for $n_f = 2 + 1$. There might even be a trend for a decrease with decreasing $m_\pi/m_\rho$. In table 1 we give preliminary results from the most chiral point of the corresponding data set.

### 4. SUMMARY

We presented the first ever calculation of the light quark masses using two flavours of dynamical light quarks and one flavour of dynamical strange. This has been possible due to the use of improved staggered quarks. Due to this improvement, the renormalisations are small and well controlled. The dynamical quarks clearly reduce the dependency on the quantity used to determine the lattice spacing. Compared to other lattice calculations and sum rule calculations, see [13] resp. [14] for review, our results favour a low value of the strange quark mass.

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**Table 1**

Preliminary results on $m_l + m_s$ in $\overline{\text{MS}}$-scheme.

| $n_f$ | $a(K^*-K^2)$ | $a(\Upsilon(1P)-\Upsilon(1S))$ |
|-------|---------------|-----------------|
| 0     | 94(9) MeV     | 71(7) MeV       |
| 2     | 83(11) MeV    | 71(10) MeV      |
| 2+1   | 78(14) MeV    | 71(13) MeV      |