Identification of the anisotropic plasticity using the virtual fields method for 2024 aluminium alloy

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Abstract. In this work, the virtual fields method was applied to identify the anisotropic plastic constitutive parameters for the wrought 2024 aluminium alloy. First, low cyclic tension-compression tests were conducted for the specifically designed aluminium alloy specimens on a hydraulic universal testing machine and the full-field deformations measured by digital image correlation. Then, the measured experimental data were used to simultaneously identify the constitutive parameters of the Hill 1948 yield criterion and the nonlinear kinematic hardening model based on the virtual fields method. Reasonable identified parameters were obtained. The fitting curves of the virtual work during the minimization process verify the reliability of the identification results.

1. Introduction

2024 aluminium alloys have been widely used as important structural materials in industries due to their excellent mechanical properties and lightweight. It is frequently used in the sheet form such as automotive shell parts, aircraft skin etc. Aluminium alloy sheets usually present high plasticity anisotropy such as the different yield strengths along different material orientations as well as the early yielding (Bauschinger effect) during a load reversal [1-2]. To improve the forming precision, an effective way is to perform finite element simulations of the forming process, which, however require proper material constitutive models and the corresponding constitutive parameters. Using conventional methods to characterize material anisotropic plasticity usually involve multiple experiments and sophisticated machines, which are time consuming but also expensive.

As an effective inverse identification strategy, the Virtual Fields Method (VFM) is increasingly applied to characterize the mechanical properties for various materials and constitutive models from experimentally measured strain fields [3], such as the calibration of the anisotropic yield criteria from notched specimen uniaxial tension tests and the anisotropic hardening models from cyclic forward-reverse simple shear tests [4-6]. High computational efficiency and less sensitivity to loading boundary conditions make it more and more popular in the material science and engineering community along with other inverse identification strategies such as the Finite Element Model Updating and the Constitutive Equation Gap Method, etc. [7].

In this paper, the VFM is utilized to characterize the anisotropic plasticity of the 2024 aluminium alloy. A specific tension-compression test configuration is adopted to simultaneously identify the constitutive parameters of the Hill 1948 yield criterion and the nonlinear kinematic hardening model for
the rolled 2024 aluminium alloy sheet specimen, which effectively simplifies the material constitutive characterization process.

2. Methodology

2.1. The virtual fields method
The basic idea of the VFM is to establish an equilibrium equation between the internal virtual work (IVW) and the external virtual work (EVW). In quasi-static state, the principle of the VFM can be expressed as [3]:

\[-\int_V \sigma : \varepsilon^* \, dV + \int_{\partial V} T \cdot u^* \, dS = 0\]  

(1)

For the nonlinear case of plasticity, a cost function between the IVW and EVW can be defined as:

\[C(X) = \sum_{i=1}^{N} \left( -t \int_A \sigma : \varepsilon^* \, dS + \int_{\partial V} T \cdot u^* \, dS \right)^2\]  

(2)

where \(N\) represents the number of loading steps, \(t\) the specimen thickness, \(A\) the area of interest, \(X\) the unknown parameters, \(T\) the distributed load on the loading boundary \(\partial V\), \(u^*\) the defined virtual displacement vector, \(\varepsilon^*\) the corresponding virtual strain tensor, \(\sigma\) the stress tensor calculated from real strain tensor \(\varepsilon\) through the constitutive relation. The unknown parameters \(X\) can be obtained by minimizing \(C(X)\) using appropriate optimization algorithms.

2.2. Constitutive models
Hill1948 yield criterion was chosen to describe the anisotropic yield behaviours of the 2024 aluminium alloy specimens, which writes:

\[f(\sigma_y, \varepsilon_p) = \sqrt{(G + H)\sigma_{xx}^2 + (F + H)\sigma_{yy}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\sigma_{xy}^2 - \bar{\sigma}_s(\varepsilon_p)} = 0\]  

(3)

where the Swift hardening law is adopted for the current flow stress, namely

\[\bar{\sigma}_s(\varepsilon_p) = K_0(\varepsilon_p + \varepsilon_p)^n\]  

(4)

The nonlinear kinematic hardening law was adopted to capture the anisotropic hardening behaviour of the material. Thus,

\[f(\sigma, a, \varepsilon_p) = \sqrt{\frac{3}{2}(\sigma_{dev} - a_{dev}) : (\sigma_{dev} - a_{dev}) - \bar{\sigma}_v(\varepsilon_p)} = 0\]  

(5)

and the Voce hardening law is used,

\[\bar{\sigma}_v(\varepsilon_p) = Y_v + R(1 - e^{-m\varepsilon_p})\]  

(6)

In equation (5), the evolution of the backstress tensor \(a\) can be expressed as:

\[da = \frac{C}{\bar{\sigma}_v}(\sigma - a) \, d\varepsilon_p - \gamma a \, d\varepsilon_p\]  

(7)

More details regarding the above constitutive models can be found in [8, 9].

3. Experimental results
In the present study, a low cycle tension-compression test configuration was used to characterize the anisotropic plasticity of the 2024 aluminium alloy, based on which the constitutive parameters of the Hill1948 and the nonlinear kinematic hardening models were identified simultaneously from the same tests. To realize this, a specimen configuration was designed that can generate a balanced heterogeneous tension/shear stress state under uniaxial loading and is not liable to buckle when compressed, as shown in Figure 1. More details regarding the design of the test configuration have been provided elsewhere in a submitted work.
Several groups (for checking the repeatability of the test results) of rolled 2024 aluminium alloy specimens were machined by wire cutting. Each group includes one specimen in the rolling direction and one in the transverse. Cyclic tension-compression tests were carried out for each group on the MTS universal test machine. Full-field deformations were measured using digital image correlation (VIC-3D, Correlated Solution). The load-displacement curves of one tension-compression test is shown in Figure 2. The measured full-field strain and the load data were then input into the VFM program to extract the anisotropic yield and hardening constitutive parameters.

Table 1 lists the identified parameters of the Hill 1948 yield criterion and the Swift hardening law, which were obtained by combining the experimental data of the first tensile stage of the two specimens in the rolling and the transverse directions. The virtual fields used here were manually defined, which writes:

$$u_x^* = \frac{y \ast (y - l) \ast x}{200}, \quad u_y^* = \frac{(l \ast y^2 / 2 - y^3 / 3)}{200}$$

where $l$ represents the length of the AOI along the loading direction.

To check the robustness of the results, three different initial estimates were randomly chosen. In Table 1, one can see that the identified results are stable. The fitting curves of the IVW and the EVW...
when the minimization process reached its convergence is illustrated in Figure 3. One can see that the IVW and the EVW fit well with each other, indicating the reliability of the identified results.

Table 1. Identified parameters of the Hill 1948 yield criterion

| direction | K  | $\varepsilon_0$ | n  | H  | F  | N  |
|-----------|----|----------------|----|----|----|----|
| I.E.1     | 600| 0.002          | 0.25| 0.5| 0.5| 2.5|
| I.R.1     | 639| 0.006          | 0.15| 0.15| 0.87| 0.62|
| I.E.2     | 800| 0.001          | 0.3 | 0.8 | 0.8 | 5  |
| I.R.2     | 638| 0.006          | 0.15| 0.14| 0.88| 0.61|
| I.E.3     | 1000| 0.001         | 0.4 | 0.3 | 0.3 | 7.5|
| I.R.3     | 653| 0.008          | 0.16| 0.12| 0.90| 0.60|

(Notes: I.E. — Initial Estimates, I.R. — Identified Results)

Figure 3. Fitting curves of the IVW and EVW for the Hill 1948 yield criterion when the minimization converges

Table 2 lists the identification results of the nonlinear kinematic hardening model. These were obtained using the whole experimental data of the cyclic tension-compression in the rolling direction. As can be seen, very stable results were obtained from different initial estimates. The good match between the IVW and EVW curves in Figure 4 also verifies the reliability of the results. It should be mentioned that the current identification algorithm is highly efficient. Using the trust-region-reflective algorithm and a regular desktop, the minimization can reach its convergence within approximately 3 minutes.

Table 2. Identified results of the nonlinear kinematic hardening model

| direction | C  | $\gamma$ | $Y_0$ | R   | m   |
|-----------|----|----------|-------|-----|-----|
| I.E.1     | 10000| 150      | 180   | 50  | 30  |
| I.R.1     | 8117 | 87.8     | 304   | $10^{-5}$ | $10^3$ |
| I.E.2     | 15000| 200      | 120   | 20  | 20  |
| I.R.2     | 8111 | 87.7     | 304   | $10^{-5}$ | $10^5$ |
| I.E.3     | 5000 | 100      | 100   | 100 | 100 |
| I.R.3     | 8117 | 87.8     | 342   | $10^{-5}$ | $10^5$ |

(Notes: I.E. — Initial Estimates, I.R. — Identified Results)
4. Conclusions
In this paper, a method that combines the low cycle tension-compression test and the VFM inverse identification strategy was applied to characterize the anisotropic plasticity of the 2024 aluminium alloy sheet specimens. A specific test configuration was adopted so that the constitutive parameters of the Hill 1948 yield criterion and the nonlinear kinematic hardening law can be identified simultaneously. It was found that using the current method, reliable and robust identification results were obtained. This can simplify the conventional complex identification procedures of the material anisotropic plasticity.

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