Studying the physics potential of long-baseline experiments in terms of new sensitivity parameters

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Received June 15, 2016; Revised August 21, 2016; Accepted August 21, 2016; Published October 26, 2016

We investigate physics opportunities to constrain the leptonic CP-violation phase $\delta_{CP}$ through numerical analysis of working neutrino oscillation probability parameters, in the context of long-baseline experiments. Numerical analysis of two parameters, the “transition probability $\delta_{CP}$ phase sensitivity parameter ($A_{M}$)” and the “CP-violation probability $\delta_{CP}$ phase sensitivity parameter ($A_{CP}$),” as functions of beam energy and/or baseline have been carried out. It is an elegant technique to broadly analyze different experiments to constrain the $\delta_{CP}$ phase and also to investigate the mass hierarchy in the leptonic sector. Positive and negative values of the parameter $A_{CP}$, corresponding to either hierarchy in the specific beam energy ranges, could be a very promising way to explore the mass hierarchy and $\delta_{CP}$ phase. The keys to more robust bounds on the $\delta_{CP}$ phase are improvements of the involved detection techniques to explore lower energies and relatively long baseline regions with better experimental accuracy.

1. Introduction

The phenomenon of neutrino oscillations in vacuum and matter can be described by six fundamental parameters: three lepton flavor mixing angles, viz. $\theta_{12}$, $\theta_{13}$, and $\theta_{23}$; two neutrino mass-squared differences, $\Delta m^2_{21}$ and $\Delta m^2_{31}$; and one Dirac-type CP-violating phase $\delta_{CP}$, collectively known as neutrino oscillation parameters. Owing to a number of dedicated neutrino oscillation experiments in past decades, both ($\theta_{12}$, $\Delta m^2_{21}$) and ($\theta_{23}$, $|\Delta m^2_{31}|$) have been measured with reasonably good accuracy [1]. The investigation of the moderately large value of the smallest leptonic mixing angle $\theta_{13}$ in the investigation of the lepton mixing matrix [2–4] by the Daya Bay [5] and RENO [6] reactor neutrino experiments has rejuvenated the opportunities to investigate the unknowns in neutrino physics. This great discovery enhances the possible capability of the next-generation experiments to pin down the neutrino mass hierarchy (i.e., the sign of $\Delta m^2_{31}$) and eventually to determine the leptonic Dirac CP-violating phase $\delta_{CP}$. A global fit of neutrino oscillations with data from world-class experiments [7,8] has put stringent bounds on the neutrino oscillation parameters.

In the present work we shall discuss the possible measurement of the CP-violating phase $\delta_{CP}$ in the context of the recently proposed LAGUNA-LBNO [9] and LBNE [10] experiments.

Long baseline (LBL) neutrino experiments like LBNL, LBNNE, etc., due to their long baselines, have advantages over short baseline experiments; the latter can be approximated to vacuum oscillation neutrino experiments. In vacuum, CP violation depends only on the $\delta_{CP}$ phase, and hence vacuum
oscillation CP violation amplitudes give a pure or intrinsic measurement of $\delta_{\text{CP}}$. Due to very small values of CP-violating effects at these short baselines, it is very difficult to carry out the experimental analysis. Over long distances, contamination by terrestrial matter effects becomes large, which in turn increases the oscillation amplitude and fakes the $\delta_{\text{CP}}$ phase effects. In LBL experiments, pure CP violation effects arising due to the $\delta_{\text{CP}}$ phase only get mixed with CP violation matter effects arising due to asymmetric forward scattering of neutrinos and anti-neutrinos with matter constituents, also known as fake or extrinsic CP violation effects. In the case of matter oscillation phenomenology, the CP conjugate of the particle oscillation probability can be obtained by merely changing the sign of the $\delta_{\text{CP}}$ phase and the matter potential $A$, as can be seen in Eqs. (1) and (2) below. Due to these changes, matter effects in the case of a normal mass hierarchy produce an overall enhancement in the vacuum effects, which makes the transition probability amplitude so large at moderate baseline lengths that we expect to be able to measure them experimentally. But now if we shift from the normal mass hierarchy (NH, i.e. $\Delta m^2_{13} > 0$) to the inverted mass hierarchy (IH, i.e. $\Delta m^2_{13} < 0$), the mass hierarchy parameter $\alpha$ in Eq. (1) also changes sign, due to which part of the matter effects get reduced, which in turn lowers the value of the probability amplitude. This addition in the NH case and subtraction in the IH case at given base line length $L$ and beam energy $E$ separates the NH and IH probability amplitudes to an amount such that we can differentiate between them experimentally.

In LBL experiments, the experimental configurations of LBNE ($L = 1280 \text{ km}$, $E = 3.55 \pm 1.38 \text{ GeV}$) and LBNO ($L = 2300 \text{ km}$, $E = 5.05 \pm 1.65 \text{ GeV}$) are chosen so that the asymmetry between the $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probabilities is larger than the CP violation effects produced by the $\delta_{\text{CP}}$ phase, which makes these suitable for determining the mass hierarchy as well as the $\delta_{\text{CP}}$ phase (see [12] and references therein). The recently proposed neutrino oscillation experiment DUNE [13–15], with baseline nearly equal to LBNE, produces similar discussion and conclusions to those of LBNE. Thus, while studying LBNE we are also simultaneously studying the oscillation phenomenology of the DUNE experiment.

2. Oscillation phenomenology of platinum channel

The sub-dominant platinum channel ($\nu_\mu \rightarrow \nu_e$), because of its sensitivity to the still unknown neutrino oscillation parameters (e.g. mass ordering, $\delta_{\text{CP}}$ phase, octant of $\theta_{23}$, etc.) and the ability to analyze the experimental data logically, has advantages over other appearance and disappearance oscillation channels. The analytic expressions for the neutrino flavor transition probabilities up to first and/or second order in small oscillation parameters, viz. the mass ordering parameter ($\alpha = \Delta m^2_{21}/\Delta m^2_{31}$) and third mixing angle $\theta_{13}$ (also known as the reactor mixing angle), have already been calculated in the literature very elegantly [17–20]. All these analytic formalisms make use of the method of perturbation theory expansion of the neutrino evolution $S$ matrix. In the present work, we have made use of the platinum channel oscillation probability from analytic results by [20]; these can be written as

$$P_{\mu \rightarrow e} \equiv \left| \mathcal{S}_{e,\mu}^\prime \right|^2 = \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \sin^2 \left[ \frac{A \Delta L}{2} \right] \sin \frac{[A - 1] \Delta L}{2} + 4s_{13}^2 s_{23}^2 \frac{\sin^2 \left[ (A - 1) \Delta L \right]}{(A - 1)^2}$$

$$+ 2 \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \left( \frac{\Delta L}{2} + \delta_{\text{CP}} \right) \frac{\sin \left( [A - 1] \Delta L \right)}{A} \frac{\sin \left( (A - 1) \Delta L \right)}{(A - 1)}. \quad (1)$$

Another reason for preferring the platinum channel lies in the fact that nowadays charged mu mesons can easily be stored in world class accelerator facility beam dump sources [21–26], which can be controlled to accelerate these charged entities to the desired energy values.
The transition probability for anti-neutrinos can be obtained by merely changing $\delta_{CP} \rightarrow -\delta_{CP}$ and $V \rightarrow -V$ (or $A \rightarrow -A$) in Eq. (1) above; hence we can write

$$P_{\mu \rightarrow e} = a^2 \sin^2 2\theta_{12} \frac{\sin^2 [A\Delta_{L}]}{A^2} + 4s_{13}^2 \frac{\sin^2 [(A+1)\Delta_{L}]}{(A+1)^2}$$

$$+ 2a_{s13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \left( \Delta_{L} - \delta_{CP} \right) \frac{\sin [A\Delta_{L}] \sin [(A+1)\Delta_{L}]}{A (A+1)},$$

with $A \equiv 2EV/\Delta m_{31}^2$, where $V = \sqrt{2}G_FN_e$, $N_e$ the number density of electrons in the medium, $G_F$ the Fermi weak coupling constant = 11.6639 $\times$ 10$^{-24}$ eV$^{-2}$, $\Delta \equiv \Delta m_{31}^2/2E \approx \Delta m_{32}^2/2E$, $\alpha = \Delta m_{21}^2/\Delta m_{32}^2$, $L$ the baseline length, and $E$ the beam energy.

Equation (1) can be rewritten in the form

$$P_{\mu e} = a + b_1 \cos \delta_{CP} + c_2 \sin \delta_{CP},$$

(3a)

where $a$ and $b$ are the first and second terms in Eq. (1) above, which are independent of the $\delta_{CP}$ phase, and the remaining coefficients $c_1$ and $c_2$ of the $\delta_{CP}$-dependent terms have the following expressions:

$$c_1 = 2a_{s13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin(A\Delta_{L}/2) \sin[(A-1)\Delta_{L}/2]}{(A-1)} \cos(\Delta_{L}/2)$$

$$c_2 = -2a_{s13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin(A\Delta_{L}/2) \sin[(A-1)\Delta_{L}/2]}{(A-1)} \sin(\Delta_{L}/2).$$

(3b)

Equation (3a) can be further compacted to the following form:

$$P_{\mu e}(\delta_{CP}) = a + b + \sqrt{c_1^2 + c_2^2} \sin(\beta + \delta_{CP}),$$

(3c)

where $\beta = \tan^{-1}(c_1/c_2)$.

We can analogously compact the anti-particle probability given in Eq. (2) to a form similar to the above equation.

### 3. Transition probability; $\delta_{CP}$ phase sensitivity parameter $A^{M}$

This parameter enables us to predict the sensitivity of the transition probability towards the $\delta_{CP}$ phase variations for a given experimental configuration. We can find the maximum possible transition probability amplitude bandwidth ($A^{M}$) for full variation in the CP violation phase $\delta_{CP}$ from 0 to $2\pi$ radians, at any chosen value of beam energy $E$ and baseline $L$, with the help of Eq. (3c) in the following form:

$$\Delta P_{\mu e}^{m}(\delta_{CP}) \equiv A^{M} \quad \text{(say)}$$

$$= P_{\mu e}^{\max}(\delta_{CP}) - P_{\mu e}^{\min}(\delta_{CP}) = 2\sqrt{c_1^2 + c_2^2}.$$

(4)

A similar type of parameter has been studied in Refs. [31–33]. This parameter is plotted as the green and the yellow curves for the NH and IH cases respectively in Fig. 1. In the NH case (i.e. the green curve) for LBNE, the first oscillation maximum of the parameter $A^{M}$ lies at 1.6 GeV with value $\approx 5\%$, and the second maximum at $\sim 0.8$ GeV with value $\approx 10\%$. Similarly, for LBNO the first maximum is at 2.8 GeV with value $\approx 6\%$ and the second is at $\sim 1.3$ GeV with a value of $\approx 10\%$. 
Fig. 1. The variation in amplitude of the transition probability for the platinum channel as a function of beam energy $E$. The left panel represents the LBNE ($L = 1280$ km, $E = 3.55 \pm 1.38$ GeV) and that on the right the LAGUNA-LBNO ($L = 2300$ km, $E = 5.05 \pm 1.65$ GeV) experimental setups as tabulated in Table 2. The green curve corresponds to the bandwidth (i.e. parameter $A^M$) in the NH case and the yellow curve the IH case, for full variation (i.e. $0–2\pi$) in the $\delta_{CP}$ phase, as calculated in Eq. (4). The remaining oscillation parameters have the best-fit values shown in Table 1.

Table 1. The best-fit and 3 $\sigma$ values of mixing angles and mass square differences from a global fit of neutrino oscillation data, adapted from [16].

| Parameter | Best fit $\pm 1\,\sigma$ | $3\,\sigma$ |
|-----------|--------------------------|-------------|
| $\theta_{12}$ | $34.6 \pm 1.0$ | $31.8–37.8$ |
| $\theta_{23}^{[NH]}$ | $48.9^{+1.9}_{-1.4}$ | $38.80–53.30$ |
| $\theta_{23}^{[IH]}$ | $49.2^{+2.6}_{-2.5}$ | $39.40–53.10$ |
| $\theta_{13}^{[NH]}$ | $8.8 \pm 0.4$ | $7.70–9.90$ |
| $\theta_{13}^{[IH]}$ | $8.9 \pm 0.4$ | $7.80–9.90$ |
| $\Delta m_{21}^2$ | $7.60^{+0.19}_{-0.18}$ | $7.11–8.18$ |
| $|\Delta m_{31}^2|_{NH}$ | $2.48^{+0.05}_{-0.04}$ | $2.30–2.65$ |
| $|\Delta m_{31}^2|_{IH}$ | $2.38^{+0.05}_{-0.06}$ | $2.20–2.54$ |

Hence, we can conclude that both experiments are equally sensitive to the variations in $\delta_{CP}$ phase in the NH case.

In the IH case (i.e. the yellow curve) for LBNE, the first and second oscillation maxima exist respectively at $2.2$ GeV (2%) and $0.9$ GeV (7%), where the values in parentheses are the corresponding values for the parameter $A^M$. Similarly, for LBNO the first, second, and third oscillation maxima are respectively located at $4.2$ GeV ($\approx 1\%$), $1.7$ GeV ($\approx 6\%$), and $1.0$ GeV ($11\%$).

Thus we can say that in the NH and IH cases, both baselines have almost equal $\delta_{CP}$ phase sensitivity at given oscillation maxima. Even though for both NH and IH cases the two baselines have almost equal $\delta_{CP}$ phase sensitivity, the locations of given oscillation maxima lie at higher values of beam energy in the case of a longer baseline, i.e. LBNO. It is also evident from Fig. 1 that the gradient of parameter $A^M$ with respect to the beam energy around the peak value of the oscillation maxima changes very rapidly (suggesting very fast oscillations), and this rapidity further increases as we move from first- to higher-order maxima. For this reason, we do not investigate higher oscillation maxima, yet these have a large sensitivity toward $\delta_{CP}$ variations. Therefore, we can’t investigate higher-order
maxima with sharp peaks to the desired precision, in the context of the currently available energy resolutions of neutrino detectors.

If we look at the shapes of the curves in the shaded region drawn for the spread in beam energy for a given experiment, the curves are almost straight lines. Because of this, we can predict results in terms of average values over the possible beam energy spreads. We can find from Fig. 1 that \( \langle A^M \rangle \approx 2\% \) (NH case); \( \langle A^M \rangle \approx 1\% \) (IH case) at \( \langle E \rangle \approx 3.6 \text{ GeV} \) for LBNE and \( \langle A^M \rangle \approx 4\% \) (NH case); \( \langle A^M \rangle \approx 2\% \) (IH case) at \( \langle E \rangle \approx 5.0 \text{ GeV} \) for LBNO. We can conclude that there is observable sensitivity toward \( \delta_{CP} \) phase variations for both experimental configurations, but to achieve more sensitivity toward the variation of the \( \delta_{CP} \) phase and high precision in constraining the \( \delta_{CP} \) phase we need to explore observables around higher maxima, which can be realized only with a nearly mono-energetic beam.

Since in accelerator beam sources both \( \bar{\nu}_\mu \) and \( \nu_\mu \) beams are equally available, it is possible to study the \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) channel experimentally. In the case of anti-neutrinos, Fig. 1 can be replotted by replacing the NH curves with IH ones and vice versa. It is evident from Eq. (1) that when we transform from the NH case to the IH case the parameters \( \alpha, \Delta, \) and \( A \) change sign, and in the case when we transform from particle to anti-particle only the parameters \( A \) and \( \delta_{CP} \) change sign. If we compare the final results of the above two transformations, we find that it is the third term that changes sign, while the first two terms appear with the same sign in the expressions for the two transformations. We also find that in the chosen beam energy ranges, shown by shaded regions for the two experiments, the contribution of the third term is negligible in comparison to the sum of the first two terms.

In Fig. 2, an oscillogram for the parameter \( A^M \) in the \( E–L \) plane is plotted. It is evident from this figure that for LBNE in the NH case the average value (i.e. at the central red dot corresponding to the average beam energy) of \( A^M \) is \( \approx 2\% \), while for the IH case it is \( \approx 1.5\% \). In the LBNO experiment in the NH case, the parameter \( A^M \) assumes an average value of \( \approx 3\% \), while in IH case it has a value \( \approx 1.5\% \). Hence, in both experiments the sensitivity toward variations in the \( \delta_{CP} \) phase for the NH case is greater than in the IH case. Also, this sensitivity further increases at the lower end of the energy spectrum in the case of NH and remains almost the same over the whole range of the energy spread for the IH case. It is recommended to investigate the \( \delta_{CP} \) phase at lower values of the energy spectrum, especially for the confirmed NH case.

![Fig. 2. An oscillogram of transition probability CP violation phase sensitivity parameter \( A^M \) in the \( E–L \) plane. The central red dot corresponds to \( \langle E \rangle \) and the error bar to \( \Delta E \) as tabulated in Table 2. The remaining oscillation parameters have the best-fit values shown in Table 1.](https://academic.oup.com/ptep/article-abstract/2016/10/103B03/2468883)
Table 2. The long baseline (LBL) experimental configurations [11] considered in the present work.

| Experiment       | Baseline $L$ (km) | Beam energy $\langle E \rangle \pm \Delta E$ (GeV) |
|------------------|-------------------|-----------------------------------------------|
| LBNE (DUNE)      | 1280              | 3.55 ± 1.38                                   |
| LBNO             | 2300              | 5.05 ± 1.65                                   |

4. CP violation probability; $\delta_{CP}$ phase sensitivity parameter $A^{CP}$

We can write the expected event rate at a detector site in the following way [27–30]:

$$N \sim \langle \phi P(\nu_\alpha \rightarrow \nu_\beta)\sigma(\nu_\beta \rightarrow \beta) \rangle,$$

where angular brackets denote the average over neutrino beam energy ($E_\nu$), $\phi$ is the neutrino flux at the detector site, and $\sigma$ is the neutrino–nucleon interaction cross section.

The event rate for the neutrino and anti-neutrino cases from Eq. (5) can be written as

$$N_\nu \sim \langle \phi_\nu P(\nu_\alpha \rightarrow \nu_\beta)\sigma_{\nu} \rangle \quad \text{and} \quad N_\tau \sim \langle \phi_\tau P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)\sigma_{\tau} \rangle.$$  \hspace{1cm} (6)

If we consider the case of a nearly mono-energetic neutrino beam, which is true for certain off-axis beams, and assume that both the neutrino and anti-neutrino beam fluxes are nearly equal (i.e. $\phi_\nu \sim \phi_\tau = \phi$), then we can write

$$\Delta N^{CP} = N_\nu - N_\tau = \phi \sigma (2P_{\alpha\beta} - P_{\alpha\bar{\beta}}) \propto 2P_{\alpha\beta} - P_{\alpha\bar{\beta}} = A^{CP} \text{ (say)},$$

where the fact that $\sigma_\tau \simeq \sigma_\nu/2 = \sigma$ [27,28,34,35] has been used.

We can estimate the parameter $A^{CP}$ in the case of the platinum channel ($\nu_\mu \rightarrow \nu_e$) with the help of Eqs. (1) and (2):

$$A^{CP} = 2P_{\mu e}(\delta_{CP}) - P_{\bar{\mu}\bar{e}}(\delta_{CP})$$

$$= \alpha^2 \sin^2 2\theta_{12} \frac{\sin^2 [A\Delta \frac{L}{2}]}{A^2} + 4s_{13}^2 s_{23}^2 \left[ 2 \sin^2 \left( \frac{(A - 1)\Delta \frac{L}{2}}{A - 1} \right) - \sin^2 \left( \frac{(A + 1)\Delta \frac{L}{2}}{A + 1} \right) \right]$$

$$+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin [A\Delta \frac{L}{2}]}{A} \left[ 2 \cos \left( \frac{L}{2} + \delta_{CP} \right) \frac{\sin \left( (A - 1)\Delta \frac{L}{2} \right)}{(A - 1)} \right.$$\n
$$- \cos \left( \Delta \frac{L}{2} - \delta_{CP} \right) \frac{\sin \left( (A + 1)\Delta \frac{L}{2} \right)}{(A + 1)} \right]$$

$$= g + r \cos \delta_{CP} + r 2 \sin \delta_{CP},$$  \hspace{1cm} (8a)
where \( g \) comprises the first two terms independent of the CP violation phase \( \delta_{CP} \) and the coefficients of the other \( \delta_{CP} \)-dependent terms have the expressions:

\[
\begin{align*}
    r_1 &= 2 \alpha_{s13} \sin \theta_{12} \sin \theta_{23} \frac{\sin[A \Delta L]}{A} \left( \frac{2 \sin[(A - 1) \Delta L/2]}{A - 1} - \frac{\sin[(A + 1) \Delta L/2]}{A + 1} \right) \cos(\Delta L/2), \\
    r_2 &= -2 \alpha_{s13} \sin \theta_{12} \sin \theta_{23} \frac{\sin[A \Delta L]}{A} \left( \frac{2 \sin[(A - 1) \Delta L/2]}{A - 1} + \frac{\sin[(A + 1) \Delta L/2]}{A + 1} \right) \sin(\Delta L/2).
\end{align*}
\]

This parameter enables the measurement of the CP violation phase as long as the constant matter density approximation holds. Matter effects along with increasing the oscillation amplitude also increase the sensitivity toward the \( \delta_{CP} \) phase variations. Equation (8a) can be further compacted to the new form in the following way:

\[
A_{CP} = g + \sqrt{r_1^2 + r_2^2} \sin(\gamma + \delta_{CP}),
\]

where \( \gamma = \tan^{-1}(r_1/r_2) \).

The maximum possible \( \delta_{CP} \) phase sensitivity of the above CP violation probability parameter at given beam energy \( E \) and baseline \( L \) can be written as:

\[
A_{CP m} \quad \text{(say)} = A_{CP max} - A_{CP min} = 2 \sqrt{r_1^2 + r_2^2}.
\]

This parameter helps in finding the optimal beam energy for a given baseline and the optimal experimental baseline for a given beam energy for which the \( \delta_{CP} \) phase sensitivity is maximum.

This CP violation probability, the \( \delta_{CP} \) phase sensitivity parameter \( A_{CP} \) (for \( \delta_{CP} \rightarrow 0, 2\pi \)) for both the NH and IH cases, is illustrated as a function of beam energy \( E \) in Fig. 3 for the chosen LBL experimental setups, LBNE and LBNO.

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**Fig. 3.** Variation of the working parameters \( A_{CP} \) defined in Eq. (8c) [shown by the magenta band for 0–2\( \pi \) variation in the \( \delta_{CP} \) phase] and the maximum possible \( \delta_{CP} \) phase sensitivity parameter, i.e. \( A_{CP m} \) (bandwidth) calculated in Eq. (9) [shown by the black curve], as a function of the beam energy \( E \). Thus, \( A_{CP m} \) is a measure of bandwidth in the broad curve. The remaining oscillation parameters have the best-fit values shown in Table 1.
We will restrict our discussion mainly to the region of the first oscillation maximum, i.e. for $E > 1$ GeV in the case of LBNE and $E > 2$ GeV in the LBNO case. In these figures we observe that, in both the NH and IH cases, for LBNE we expect an average sensitivity of $\sim 3\%$ at $\langle E \rangle \approx 3.55$ GeV and for LBNO there is a sensitivity of $\sim 6\%$ at $\langle E \rangle \approx 5.05$ GeV.

There are other oscillation maxima, for example at $E \approx 0.8$ GeV with a sensitivity of $\sim 25\%$ for LBNE in both the NH and IH cases, while for the LBNO experiment at $E = 1.3$ GeV we expect a sensitivity of $\sim 20\%$ for the NH case and a sensitivity of $\sim 12\%$ for the IH case. Also, there are other oscillation maxima with sensitivities of $\sim 32\%, 42\%$ for the NH case and $\sim 20\%, 30\%$ for the IH case at 0.8 and 0.6 GeV respectively, but due to the fast oscillations around these maxima, only almost mono-energetic beams could make experimental realization possible. Energy spreads in the currently available beam sources are relatively broad; hence, we do not discuss these oscillation maxima in detail.

The other thing we notice in these figures is that, in the specific beam energy range, parameter $A_{CP}^m$ attains positive values for one hierarchy and negative for the other hierarchy. For example, in the case of the LBNO experiment, in the beam energy range of 2–8 GeV, parameter $A_{CP}^m$ assumes positive values in the NH case, while it takes negative values in the IH case over the whole $\delta_{CP}$ (0–$2\pi$) range. These positive and negative values of the parameter $A_{CP}^m$ can be confirmed experimentally. Thus these energy ranges provide the opportunity to investigate mass ordering along with the $\delta_{CP}$ phase investigation.

In Fig. 4, an oscillogram for the parameter $A_{CP}^m$ in the $E–L$ plane has been shown. It is evident from the oscillogram that, at the central red dot for LBNE (DUNE), $A_{CP}^m$ is $\approx 5\%$ for the NH case and $\approx 4\%$ in the IH case. At the lower end of the energy spectrum we expect a sensitivity of $\sim 9\%$, while at the higher end $\sim 2\%$ for both hierarchies. In the LBNO experiment, in the NH case, at the central red dot $A_{CP}^m \approx 8\%$ and, further, has values of $\sim 11\%$ and $\approx 4\%$ respectively at the lower and higher ends of energy spectrum. In the IH case, parameter $A_{CP}^m$ has value $\approx 5\%$ at the central red dot and has values of $\sim 7\%$ and $\sim 3\%$ at the lower and higher ends of the energy spectrum respectively. Thus, for LBNO, normal hierarchy has more sensitivity as compared to the inverted hierarchy and the sensitivity further increases at the lower end of the energy spectrum. This suggests that the lower energy spectrum ends are more suitable than higher ones to investigate the $\delta_{CP}$ phase.

![Fig. 4. CP violation probability, $\delta_{CP}$ phase sensitivity parameter $A_{CP}^m$, oscillogram in the $E–L$ plane. The central red dot corresponds to $\langle E \rangle$ and the error bar to $\Delta E$ as tabulated in Table 2. The remaining oscillation parameters have the best-fit values shown in Table 1.](https://academic.oup.com/ptep/article-abstract/2016/10/103B03/2468883)
5. Sensitivity of \( A^M \) and \( A^{\text{CP}} \) toward mixing angles and mass square difference variations

In Figs. 5, 6, and 7 the sensitivity of working parameters \( A^{\text{CP}}_m \) and \( A^M \) toward the 3σ variations in three mixing angles and two mass square differences is illustrated. It is clear from Fig. 5 that parameter \( A^{\text{CP}}_m \) is weakly sensitive toward variations in the mixing parameters in the chosen range, for both LBNE and LBNO experiments. From the numerical analysis, it can also be confirmed that parameter \( A^M \) also has small sensitivity to the variations in mixing parameters for both the experimental configurations.

In Fig. 6, the sensitivity of the parameters \( A^{\text{CP}}_m \) and \( A^M \) toward variations in the mixing parameters for the experimental configuration (\( E = 0.5 \text{ GeV}, L = 1280 \text{ km} \)) has been illustrated. We can analyze from the figure that both parameters have small sensitivity toward mixing angle \( \theta_{12} \) and \( \theta_{23} \) variations. Parameter \( A^{\text{CP}}_m \) has noticeable sensitivity toward the \( \theta_{13} \) variations, while \( A^M \) has small sensitivity. Also, the parameter \( A^{\text{CP}}_m \) has noticeable sensitivity to the \( \Delta m_{12}^2 \) variations, while parameter \( A^M \) has small sensitivity. We can easily notice that \( A^{\text{CP}}_m \) has large sensitivity towards the \( \Delta m_{13}^2 \) variations, while \( A^M \) has moderate sensitivity to that parameter. We can conclude that, if we know \( \theta_{13} \) very precisely, then parameter \( A^{\text{CP}}_m \) is left sensitive to the mass square differences.

It is evident from the analysis of Fig. 7 that parameter \( A^{\text{CP}}_m \) attains moderate sensitivity toward the \( \theta_{12} \) and \( \theta_{23} \) variations, while \( A^M \) has small sensitivity toward these variations. Also, parameter \( A^{\text{CP}}_m \) has large sensitivity to the \( \theta_{13} \) variations, while parameter \( A^M \) has moderate sensitivity. Similarly,
Fig. 6. The sensitivity of parameters $A_{\text{CP}}^m$ and $A^M$ within 3 $\sigma$ variations of mixing angles and mass square differences for the experimental configuration ($E = 0.5 \text{ GeV}, L = 1280 \text{ km}, \rho = 3 \text{ g cm}^{-2}$). The parameter $A_{\text{CP}}^m \equiv A^m_{\mu \rightarrow e}$ along the $y$-axis is in units of $10^{-2}$. The first column (NH case) and second column (IH case) correspond to parameter $A_{\text{CP}}^m$, while the third (NH case) and fourth (IH case) columns correspond to the parameter $A^M$. All other oscillation parameters, except the one considered along the $x$-axis, assume the best-fit values tabulated in Table 1.

$A_{\text{CP}}^m$ has noticeable sensitivity toward the $\Delta m_{12}^2$ variations, while $A^M$ has moderate sensitivity. It is evident from the figure that both parameters $A_{\text{CP}}^m$ and $A^M$ attain large sensitivity toward variations in the atmospheric mass square difference ($\Delta m_{13}^2$). Thus, if we know the precise value of the reactor mixing angle $\theta_{13}$ then we are left only with large sensitivities of the parameters toward the variations in solar and atmospheric mass square differences. Parameter $A_{\text{CP}}^m$ has large sensitivity toward the solar mass square difference (i.e. $\Delta m_{12}^2$) variations in comparison to parameter $A^M$, while both $A_{\text{CP}}^m$ and $A^M$ have large sensitivity toward variations in atmospheric mass square difference (i.e. $\Delta m_{13}^2$) in the 3 $\sigma$ range.

We can also conclude that for a given parameter, sensitivity in the NH case is always almost equal to sensitivity in IH case. It is also clear from the analysis of Figs. 5, 6, and 7) that the sensitivity of the parameters increases with an increase in baseline length $L$ and lowering of the beam energy $E$.

If we compare parameters $A_{\text{CP}}^m$ and $A^M$, then the former shows large sensitivity in comparison to the latter.

6. Conclusions and perspectives

In this work we have studied two parameters, viz. the “transition probability, $\delta_{\text{CP}}$ phase sensitivity parameter $A^M$,” and the “CP violation probability, $\delta_{\text{CP}}$ phase sensitivity parameter $A_{\text{CP}}^m$,” especially to investigate Dirac’s $\delta_{\text{CP}}$ phase. We can conclude from the analysis of Figs. 1, 2, 3, and 4 that LBNO provides better sensitivity as compared to LBNE toward the $\delta_{\text{CP}}$ phase variations. We also notice that
Fig. 7. The sensitivity of parameters $A^M$ and $A^\text{CP}_m$ within 3 $\sigma$ variations of mixing angles and mass square differences, for the experimental configuration ($E = 0.5$ GeV, $L = 2300$ km, $\rho = 3.3$ g cm$^{-3}$). The parameter $A^\text{CP}_m \equiv A^\text{CP}_m(\mu \to e)$ along the $y$-axis is in units of $10^{-2}$. The first column (NH case) and the second column (IH case) correspond to parameter $A^\text{CP}_m$, while the third (NH case) and fourth (IH case) columns correspond to parameter $A^M$. All the other oscillation parameters, except the one considered along the $x$-axis, assume the best-fit values tabulated in Table 1.

for a given baseline, this sensitivity is more in the NH case than in the IH case for the parameter $A^M$. This sensitivity further increases at the lower end of the energy spectrum. Parameter $A^\text{CP}_m$ enables us to clearly differentiate between the two mass hierarchies from the positive and negative values of this parameter for either hierarchy, in the specific beam energy range, as is apparent from Fig. 3. This latter distinction of the two hierarchies by the sign of $A^\text{CP}_m$ is more pronounced in the case of the LBNO experiment. We also notice from the comparison of Figs. 2 and 4 that for given baseline length $L$ and beam energy $E$, sensitivity toward the $\delta^\text{CP}$ phase variations is larger for the parameter $A^\text{CP}_m$ in comparison to parameter $A^M$.

It is not convenient to investigate higher oscillation maxima (i.e., second and third order) due to large spreads in the energy spectra of beam sources and comparatively lower resolutions of current detectors, but sufficiently large values of parameters $A^M$ and $A^\text{CP}_m$ at these maxima encourage the investigation of these experimentally, to put more stringent bounds on the $\delta^\text{CP}$ phase. Investigation of these higher oscillation maxima needs more accurate detection techniques and narrower energy spectra.

At a relatively small value of beam energy, $E \simeq 0.5$ GeV, the sensitivity of the parameter $A^\text{CP}_m$ is large toward the atmospheric mass square difference ($\Delta m^2_{13}$) variations in both the LBNE and LBNO experiments, while this parameter has moderate sensitivity toward the solar mass square difference ($\Delta m^2_{12}$) variations in the LBNE experiment case and has large sensitivity in the case of LBNO. Parameter $A^M$ has moderate sensitivity toward the $\Delta m^2_{13}$ variations and has small sensitivity to the
$\Delta m_{12}^2$ variations in the case of LBNE, while this parameter has large sensitivity to $\Delta m_{13}^2$ variations and moderate sensitivity to the $\Delta m_{12}^2$ variations in the case of the LBNO experiment. Also, parameter $A^\text{CP}_m$ has a large sensitivity toward the $\theta_{13}$ variations in both LBNE and LBNO experiments, while the parameter $A^M$ has moderate sensitivity. For both LBNE and LBNO experiments, the sensitivity of both parameters is small toward the $\theta_{12}$ and $\theta_{23}$ variations. Thus, if we know the precise value of the reactor mixing angle $\theta_{13}$, we are left with large sensitivities of the parameters toward the variations in solar and atmospheric mass square differences only. If we need large/small sensitivities, we can choose low/high beam energy regions.

As parameters $A^\text{CP}$ and $A^\text{CP}_m$ are the differences of two CP conjugate channels and parameter $A^M$ is that of a single oscillation channel, no doubt errors/uncertainties get cancelled to an extent for both parameters, but being a difference involving the same channel such cancellation is large in the case of parameter $A^M$.

Acknowledgements

I would like to thank Prof. Brajesh Chandra Choudhary (Department of Physics and Astrophysics, Delhi University) for useful discussions.

Funding

Open Access funding: SCOAP3.

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