Fine-tuning Pruned Networks with Linear Over-parameterization

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Abstract

Structured pruning compresses neural networks by reducing channels (filters) for fast inference and low footprint at run-time. To restore accuracy after pruning, fine-tuning is usually applied to pruned networks. However, too few remaining parameters in pruned networks inevitably bring a great challenge to fine-tuning to restore accuracy. To address this challenge, we propose a novel method that first linearly over-parameterizes the compact layers in pruned networks to enlarge the number of fine-tuning parameters and then re-parameterizes them to the original layers after fine-tuning. Specifically, we equivalently expand the convolution/linear layer with several consecutive convolution/linear layers that do not alter the current output feature maps. Furthermore, we utilize similarity-preserving knowledge distillation that encourages the over-parameterized block to learn the immediate data-to-data similarities of the corresponding dense layer to maintain its feature learning ability. The proposed method is comprehensively evaluated on CIFAR-10 and ImageNet which significantly outperforms the vanilla fine-tuning strategy, especially for large pruning ratio.

1 Introduction

Deep Neural Networks (DNNs) have achieved remarkable performance in recent years. Despite the great success, their large model sizes and huge computational requirements add a significant burden to practical computing systems, especially for embedded and Internet of things systems. One promising solution is compressing the network with minimum accuracy damage using pruning [Han \textit{et al.}, 2015b], quantization [Han \textit{et al.}, 2015a], knowledge distillation [Tung and Mori, 2019], matrix low-rank approximation [Zhang \textit{et al.}, 2015], etc.

Network pruning aims to remove “unimportant” weights from a pretrained model. After that, the fine-tuning process is indispensable since the accuracy dramatically drops after hard-pruning. However, with the increase of pruning ratio, it becomes increasingly tough to restore the accuracy of the trimmed network for two reasons. Firstly, with a large pruning ratio, the model gradually prunes the “unimportant” weights and turns them into “useful” weights. Removing such weights leads to mass information loss. Secondly, the more compact the network is, the harder it is to gather useful features during fine-tuning.

In this paper, we focus on the second problem by using linear over-parameterization [Guo \textit{et al.}, 2018; Ding \textit{et al.}, 2021]. Linear over-parameterization was an approach to train compact neural networks from scratch. It expands each linear layer of the compact network into multiple consecutive linear layers without adding any nonlinear operations. This over-parameterized structure is well trained and then contracts back to the original structure algebraically at inference. Normally, it achieves better results than training directly from scratch. In the pruning process, over-parameterization can be considered as a booster during fine-tuning to improve the accuracy recovery ability. However, over-parameterization for fine-tuning after model pruning faces two challenges:

- When we expand the layers in the trimmed network, we need to keep their behavior unchanged, so as to inherit the pruning result which is beneficial for fine-tuning. In short, the over-parameterized network output needs to be the same as the trimmed network for the same input.
- The over-parameterized network needs to not only keep the output unchanged but also preserve the feature extraction ability. An irrational layer expanding scheme will lead to sudden accuracy collapse at the beginning of fine-tuning. Thus, constraints need to be imposed to guide the fine-tuning of the over-parameterized network.

To address the first challenge, we leverage matrix factorization to equally expand the original layer into consecutive linear layers based on the fact that a layer in networks can be expressed as a matrix. To be specific, we first randomly initialize all but one matrix and then calculate the remaining one by matrix manipulation including matrix inverse and matrix multiplication. In Sec. 3.1, we introduce the decomposition schemes of three common layers (convolutional layer, depth-wise separable convolutional layer and linear layer) of neural networks in detail.

To address the second challenge, we use similarity-preserving knowledge distillation [Tung and Mori, 2019] to guide the fine-tuning of the over-parameterized network. This method is inspired by the observation that semantically similar inputs tend to elicit similar activation patterns in a trained...
neural network. Applying similarity-preserving knowledge distillation loss on each layer can maintain the representation ability in fine-tuning after the expansion. The detailed process of our algorithm is shown in Fig. 1.

Experiments show that our method outperforms training compact models from scratch, normal fine-tuning, and over-parameterization from scratch. Meanwhile, with the increase of the pruning ratios, its effect on accuracy restoration becomes significantly obvious. Moreover, our method is orthogonal to other pruning related technologies that involve fine-tuning to restore accuracy, such as importance weight selection [Zhang et al., 2018; He et al., 2017], adaptive pruning ratio [Wang et al., 2013; Liu et al., 2019; Yu and Huang, 2019; Luo et al., 2017] and so on. As a result, it can be simultaneously applied to these approaches and improves the results.

2 Related Work

2.1 Structured Pruning

Structured and non-structured pruning are two mainstream weight pruning methods. Non-structured pruning is not friendly to CPUs or GPUs and requires dedicated hardware [Pal et al., 2018; Zhang et al., 2020] to support sparse matrix multiplication, which is highly demanding to design [Wang et al., 2018]. In this paper, we focus mainly on structured pruning. Early work [Han et al., 2015b] proposed an iterative and static magnitude-based weight pruning to explore the redundancy. Later, [Zhang et al., 2018] presented a systematic weight pruning framework of deep networks using the Alternating Direction Method of Multipliers (ADMM). It can reduce some weights and push them towards zero during training, to reduce the decline in accuracy caused by pruning. However, the above methods cannot fully maintain the accuracy, so it is usually necessary to fine-tune the model after pruning. [He et al., 2017] reconstructed the outputs with remaining channels with linear least squares without fine-tuning. [Zhu and Gupta, 2017] proposed an automated gradual pruning method, in which pruning and fine-tuning are carried out alternately to gradually prune the network. However, these methods either bypass the fine-tuning process or try to alleviate the decline inaccuracy, and do not focus on the accuracy recovery during fine-tuning.

2.2 Model Re-parameterization

Re-parameterization means training a heavyweight network and inference a slim network by contracting the layers without nonlinearity. Existing re-parameterization methods can be divided into two types: series and parallel. Asym Conv Block (ACB) [Ding et al., 2019], DO-Conv [Cao et al., 2020] and ExpandNet [Guo et al., 2018] can be viewed as series re-parameterization. They are designed for component-level improvements and used as a drop-in replacement for convolutional layers and fully-connected layers. Compared to these methods, parallel re-parameterization expands a layer to several branches instead of to a simple plain layer, such as DiracNet [Zagoruyko and Komodakis, 2017] and RepVGG [Ding et al., 2021]. In this paper, we adopt a series re-parameterization method to expand the pruned layer before fine-tuning.

2.3 Similarity-preserving

Similarity-preserving knowledge distillation [Tung and Mori, 2019] guides the training of a student network such that input pairs that produce similar (dissimilar) activations in the teacher network produce similar (dissimilar) activations in the student network. The student is not required to mimic the representation space of the teacher, but rather to preserve the pairwise similarities in its own representation space. In this paper, although we expand the pruned network by matrix factorization to ensure consistency of the output feature maps, it still destroys the feature extraction ability of the expanded layers. By using similarity-preserving, pairwise activation similarities within each input mini-batch are used to supervise the training of the over-parameterized network with the pretrained network.
3 Methodology

3.1 Expand Layers

In this section, we introduce our proposed approach that linearly expands the layers in the structurally pruned compact neural networks for fine-tuning. We focus on normal convolutions, depthwise convolutions, and fully-connected layers as they are commonly used in lightweight neural networks.

Expanding normal convolutional layers. We denote $F_{n \times m \times k \times k}$ as the filters of a convolutional layer with input channels of $m$, output channels of $n$ and kernel size of $k \times k$. Our goal is to expand it to multiple consecutive linear layers like ExpandNets without adding any nonlinear operations. However, we cannot simply add a convolutional layer with kernel size $k \times k$ as a series of convolutional layers with arbitrary kernel sizes. Because the resulting receptive field size would differ from the original one generally. To solve this problem, [Guo et al., 2018] proposes to expand a $k \times k$ convolutional layer into 3 consecutive convolutional layers: a $1 \times 1$ convolution; a $k \times k$ one; and another $1 \times 1$ one because the $1 \times 1$ convolutions retain the computational benefits of convolutional layers while not modifying the receptive field size (see Fig. 1(c)). Mathematically, for a $k \times k$ convolutional layer with $m$ input channels and $n$ output channels, we define the number of output channels of the first $1 \times 1$ layer as $p = rm$ and the number of output channels of the intermediate $k \times k$ layer as $q = rn$ with an expansion rate $r > 0$, the relationship of these layers can be expressed as

$$ F_{n \times m \times k \times k} = F_{n \times m \times k \times k}^3 \circ F_{p \times q \times k \times k}^2 \circ F_{p \times m \times 1 \times 1}^1. \quad (1) $$

We noticed that the convolutional filters can be expressed in matrix form, hence can be expanded by matrix decomposition, which is able to preserve and leverage the original information (weights) in pruned pretrained networks after expanding. Specifically, let $X_{m \times h \times w}$ be the input tensor with input channels of $m$, height of $h$, and width of $w$. Ignoring the bias for simplicity, which can be taken into account by incorporating an additional channel to both $X$ and $F$, the convolution can be expressed as

$$ Y_{n \times h' \times w'} = F_{n \times m \times k \times k} \ast X_{m \times h \times w} = \text{reshape} \left( W_{n \times m \times k \times k}^F \times X_{m \times h \times w}^M \right), \quad (2) $$

where $Y_{n \times h' \times w'}$ is the corresponding output tensor with shape of $h' \times w'$, $X_{m \times h \times w}^M$ is the unrolled matrix representation of the input tensor $X_{m \times h \times w}$, and $W_{n \times m \times k \times k}^F$ is the matrix representation of the convolutional filters $F_{n \times m \times k \times k}$.

Fig. 2 shows an example of this computational process. With this matrix representation, we can therefore expand a layer linearly by replacing the matrix with the other three matrices. To be more specific, we can express Eq. 1 as the form of matrix multiplication

$$ W_{n \times m \times k \times k}^F = W_{n \times q}^F \times \text{reshape} \left( W_{q \times pkk}^F \right)_{q \times p} \times W_{p \times m}^F, \quad (3) $$

where the operator $\text{reshape}(W)_{p \times q}$ represents reshaping the matrix $W$ into shape of $p \times q$. Then we can expand the layers by solving and assigning these three matrices. We first initializing $W_{n \times q}$ and $W_{p \times m}^F$ randomly and then calculate the weight of the rest layer by using matrix decomposition. This process can be formulated as

$$ W_{q \times pkk}^F = (W_{n \times q}^F)^{-1} \times \text{reshape} \left( W_{n \times mkk}^F \right)_{n \times m} \times (W_{p \times m}^F_L)^{-1}_{n \times pk}, \quad (4) $$

where $(W_{n \times q}^F)^{-1}$ indicates the right inverse of $W_{n \times q}^F$ and with the actual shape of $q \times n$. $(W_{p \times m}^F_L)^{-1}$ means the left inverse of $W_{p \times m}^F$ with the actual shape of $m \times p$.

In this way, we can expand the convolution structure without changing the receptive field, and compress them back through matrix multiplication and reshape operation to the original compact network algebraically after fine-tuning (see Fig. 1(d)).

In practice, padding and strides are used to control the size of the output feature map in convolutional layers. To expand a convolutional layer with padding $p$, we apply padding $p$ in the first layer of the expanded unit while not padding the remaining layers. To handle a stride $s$, we set the stride of the middle layer to $s$ and that of the others to 1.

Expanding depthwise convolutional layers. Depthwise convolutions are often used to design compact networks, such as MobileNetV2 [Sandler et al., 2018] and ShuffleNetV2 [Ma et al., 2018]. Pretraining models based on them are often used for pruning. To expand the depthwise convolutional layers, we set the group parameter of the expanded layers same as the original layer, then the normal convolutional expansion strategy is applied on these three consecutive layers within each group. This makes the expanded layers equivalent to the original ones for each group.

Expanding fully-connected layers. Fully-connected layers are ubiquitous in neural networks. Compared with convolutional layers, a fully-connected layer is easier to expand because it can be naturally represented in matrix form. We can directly expand a linear layer with $m$ input and $n$ output dimensions into $l$ linear layers as

$$ W_{n \times m} = W_{n \times p_1-1} \times W_{p_1-1 \times p_2-2} \times \cdots \times W_{p_l \times m}. \quad (5) $$

Similar to the convolutional layers, we allocate the weights of the expanding layers by randomly initializing $l - 1$ of them.
and calculating the weight of the remaining \( \theta \)-th layer by matrix decomposition

\[
W_{p_{\theta} \times p_{\theta-1}} = \prod_{i=\theta-1}^{l} (W_{p_i \times p_{i-1}})^{-1}_R \\
\times W_{n \times m} \prod_{i=1}^{\theta-1} (W_{p_i \times p_{i-1}})^{-1}_L.
\]

For convenience, we denote \( p_t = n \) and \( p_0 = m \) in Eq. 5 to get Eq. 6. In practice, considering the computational complexity of fully-connected layers, we expand each layer into only two or three layers with a smaller expansion rate.

**Existence of left and right inverses.** The left and right inverses of matrices need to be computed in Eq. 4 and Eq. 6 so they must exist. If not, the only alternative is to use the pseudo-inverse of the matrices, which inevitably induce differences between the expanded layers and the original layer. In our work, all matrices that need determining the inverses are initialized randomly. Considering matrix \( W_{n \times m} \), the necessary and sufficient condition for the existence of its left inverse is \( n \leq m \) and \( \text{rank}(W_{n \times m}) = n \), i.e., matrix \( W_{n \times m} \) is row full rank. As \( W_{n \times m} \) is randomly initialized with normal distribution, the inverse \( W'_{n \times m} \) can be determined after we remove the last \( m - n \) columns. Under these circumstances, we can compare their full rank probability as

\[
P[\text{rank}(W_{n \times m}) = n] \geq P[\text{rank}(W'_{n \times m}) = n].
\]

Meanwhile, [Feng and Zhang, 2007] has proved that a rational random matrix has full rank with probability 1, that is, \( P[\text{rank}(W_{n \times m}) = n] = 1 \). Thus with Eq. 7 we get \( P[\text{rank}(W_{n \times m}) = n] = 1 \) and its right inverse exist certainly. In this way, in order to ensure the existence of a matrix’s left and right inverse, what we need to do is to control the expansion rate. E.g., when we expand a fully-connected layer \( W_{n \times m} \) to \( W_{n \times p} \times W_{p \times m} \), if we randomly initialize \( W_{n \times p} \) and calculate \( W_{n \times m} \), we should ensure \( p \geq n \). Conversely, we should set the value \( p \geq m \).

### 3.2 Feature Extraction Preserving

After expanding the layers, the over-parameterized network generates exactly the same output as the pruned compact network. However, the expanded weights are constructed based on random initialization and matrix decomposition, therefore they do not hold a stable feature extraction ability as the original network. Specifically, we observed that the accuracy of an over-parameterized network experienced a sudden decline at the beginning of the fine-tuning. In this case, the fine-tuning result is not much different from training the compact network from scratch and the model converges much lower. Accordingly, we assume that over-parameterization will destroy the stable feature extraction ability of the pretrained and pruned model. To handle this, similarity-preserving knowledge distillation is applied. It guides the fine-tuning of over-parameterized networks such that produce similar (dissimilar) activations in the un-pruned pretrained network produce similar (dissimilar) activations in the over-parameterized network.

This method, proposed by [Tung and Mori, 2019], is a new form of knowledge distillation loss based on the idea that semantically similar inputs tend to elicit similar activation patterns in a trained network. Given an input mini-batch, it denotes the activation map produced by the un-pruned network \( T \) (teacher network) at a particular layer \( l \) as \( (A^{(l)}_T)_{b \times m \times h \times w} \) and the activation map produced by the over-parameterized network \( S \) (student network) at the last corresponding expand layer \( l' \) as \( (A^{(l')}_S)_{b \times m' \times h' \times w'} \). Here \( m \) is not equal to \( m' \) because we have already removed some “unimportant” filters from \( T \) through structured pruning. To guide \( S \) with the activation correlations in \( T \), the distillation loss, an L2 regularization penalty, is imposed on the outer products of \( (A^{(l)}_T)_{b \times m \times h \times w} \) and \( (A^{(l')}_S)_{b \times m' \times h' \times w'} \). First, let \( (Q^{(l)}_T)_{b \times m \times h \times w} = \text{reshape}((A^{(l)}_T)_{b \times m \times h \times w})_{b \times m \times h \times w} \). Then we get

\[
(G^{(l)}_T)_{b \times b} = (Q^{(l)}_T)_{b \times m \times h \times w} \cdot (Q^{(l)\top}_T)_{m \times h \times w \times b},
\]

\[
G^{(l)}_T[i,:]=\frac{(\hat{G}^{(l)}_T[i,:])^\top \hat{G}^{(l)}_T[i,:]}{\|G^{(l)}_T[i,:]\|_2^2}.
\]

Intuitively, entry \((i,j)\) in \( (G^{(l)}_T)_{b \times b} \) encodes the similarity of the activation layer \( l \) elicited by the \( i\)th and \( j\)th images in the mini-batch. We apply a row-wise L2 normalization to obtain \((G^{(l)}_T)_{b \times b} \), where the notation \( [i,:) \) denotes the \( i\)th row in a matrix. Similarly, let \( (Q^{(l')}_S)_{b \times m' \times h' \times w'} = \text{reshape}((A^{(l')}_S)_{b \times m' \times h' \times w'})_{b \times m' \times h' \times w'} \). Then we get

\[
(G^{(l')}_S)_{b \times b} = (Q^{(l')}_S)_{b \times m' \times h' \times w'} \cdot (Q^{(l')}_{S\top})_{m' \times h' \times w' \times b},
\]

\[
G^{(l')}_S[i,:]=\frac{(\hat{G}^{(l')}_S[i,:])^\top \hat{G}^{(l')}_S[i,:]}{\|G^{(l')}_S[i,:]\|_2^2}.
\]

The similarity-preserving knowledge distillation loss is designed as

\[
\mathcal{L}_{SP}(G_T,G_S)=\frac{1}{b^2} \sum_{(l,l') \in \mathcal{I}} \left\| G^{(l)}_T - G^{(l')}_{S} \right\|_F^2,
\]

where \( \mathcal{I} \) collects the \((l,l')\) layer pairs (i.e. layers in the un-pruned network and the last layers in the corresponding expanded layers in the over-parameterized network.) and \( \left\| \cdot \right\|_F \) is the Frobenius norm. Eq. 12 is a summation, over all \((l,l')\) pairs, of the mean element-wise squared difference between the \( G^{(l)}_T \) and \( G^{(l')}_{S} \) matrices. Finally, the total loss for student network training is defined as

\[
\mathcal{L} = \mathcal{L}_{\text{task}}(Y, \sigma_S(X)) + \gamma \mathcal{L}_{SP}(G_T,G_S),
\]

where \( \mathcal{L}_{\text{task}}(\cdot,\cdot) \) denotes the loss function of vision task, \( Y \) is the ground truth, and \( X \) is the input mini-batch. \( \sigma_S(\cdot) \) denotes the output from over-parameterized network \( S \). \( \gamma \) is a balancing hyper-parameter and is set to 3000 following [Tung and Mori, 2019].
8 GPUs with the input image size 224.

MobileNetV2 for CIFAR-10. All the models are trained on a large dataset. We use ResNet-50 on ImageNet and MobileNetV2 for CIFAR-10. All the models are trained on 8 GPUs with the input image size 224×224 and the minibatch size 256. Our work is focused on the fine-tuning stage which can be applied to any pruned networks. However, the models obtained by different pruning strategies will seriously affect the final performance. So in all our experiments, we use ADMM [Zhang et al., 2018] as the only pruning strategy. ADMM process gradually reduces the weights of some filters to zero by a preset amount during training and then removes them finally. In our experiments, there are two cases to deal with the pruned network: 1) utilize its weights during fine-tuning, and the method we proposed belongs to this one. 2) keep only the structure of the pruned network and reinitialize its parameters during retraining. For the first case, we fine-tune the network for 150 epochs, starting with a learning rate of 0.1 decayed by a factor 10 at epochs 60, 100 and 120. For the second case, we retrain the network for 280 epochs, starting with a learning rate of 0.1 decayed by a factor 10 at epochs 100, 160, 210, 250. All the results we report below have chosen the best performance in this phase.

4 Experiments

4.1 Implemented Details

We conduct the experiments and fully analyze our method on CIFAR-10 [Krizhevsky et al., 2009] and ImageNet [Deng et al., 2009] dataset. We use ResNet-50 on ImageNet and MobileNetV2 for CIFAR-10. All the models are trained on 8 GPUs with the input image size 224×224 and the minibatch size 256. Our work is focused on the fine-tuning stage which can be applied to any pruned networks. However, the models obtained by different pruning strategies will seriously affect the final performance. So in all our experiments, we use ADMM [Zhang et al., 2018] as the only pruning strategy. ADMM process gradually reduces the weights of some filters to zero by a preset amount during training and then removes them finally. In our experiments, there are two cases to deal with the pruned network: 1) utilize its weights during fine-tuning, and the method we proposed belongs to this one. 2) keep only the structure of the pruned network and reinitialize its parameters during retraining. For the first case, we fine-tune the network for 150 epochs, starting with a learning rate of 0.01 decay by a factor 10 at epochs 60, 100 and 120. For the second case, we retrain the network for 280 epochs, starting with a learning rate of 0.1 decayed by a factor 10 at epochs 100, 160, 210, 250. All the results we report below have chosen the best performance in this phase.

4.2 Results

In this section, we compare the proposed with four other competing methods. They are 1) training from scratch: reinitialize the pruned model and train it from scratch, 2) ExpandNets: expand the pruned model, reinitialize it and contract it back after fine-tuning, 3) vanilla distillation: reinitialize the pruned model and retrain it guided by the similarity-preserving knowledge distillation loss and 4) vanilla fine-tuning. It should be noted that all models are pretrained and pruned with a preset pruning ratio in advance, and have the same structure, float operations per second (FLOPs) and latency when they are compared in our experiments. We carried out thorough experiments on the different pruning ratios. It’s worth mentioning that after removing some filters in one layer, the input channel of the next layer will be removed passively, so the ultimate proportion of removed weights is always larger than the preset pruning ratio. In Sec. 4.3, we perform an ablation study on the expansion rate and find 3 to be an appropriate value. So it is set to 3 of all models throughout the experiments.

MobileNetV2 for CIFAR-10. The results of top-1 accuracy of all five methods are shown in Table. 1. The first column represents the original models before pruning. At this time, all models have not been retrained or fine-tuned except ExpandNets. This table shows that of all the pruning ratios tested, our method can best restore accuracy. Moreover, its effectiveness becomes more obvious with the increase of the pruning ratio. When 89.8% weights are removed, the accuracy of our method gets only 0.4% higher than that of vanilla fine-tuning. But when the pruning ratio increases to an extreme 98.66%, our method achieves 2.49% higher accuracy restoration than the vanilla fine-tuning and more than 5% compared with vanilla distillation and ExpandNets. We believe that this result benefits from the increased amount of parameters induced by over-parameterization and the guidance provided by the similarity-preserving penalty. ExpandNets and vanilla distillation produce close results, which is also mentioned in [Guo et al., 2018]. Both of them perform much worse than vanilla fine-tuning, which indicates that applying over-parameterization and knowledge distillation alone is not enough. Our experiment shows that for pruning, utilizing them together can achieve the best results.

ResNet-50 on ImageNet. The top-1 accuracy results on this dataset are shown in Table. 2. The results are basically consistent with the above conclusions. The effectiveness of our method can not be highlighted with a small pruning ratio. It performs even worse than vanilla fine-tuning when the percent of removed weights is 28.37%. Another thing that should be noted is that with the ResNet-50 structure, the performance of ExpandNets is worse than training from scratch before pruning. One possible reason is that when the parameters are sufficient, over-parameterization may complicate the network training and slow down the convergence.

| Method                  | Percent of weights removed (%) |
|------------------------|-------------------------------|
|                        | 0  | 68.37 | 72.74 | 78.23 | 86.2 | 89.8 | 92.18 | 94.8 | 98.66 |
| Training from scratch  | 94.17 | 86.22 | 84.93 | 83.47 | 80.91 | 78.75 | 76.35 | 72.08 | 49.08  |
| ExpandNets             | 94.36 | 87.61 | 86.02 | 84.99 | 82.74 | 80.85 | 78.71 | 75.14 | 51.25  |
| Vanilla distillation   | 94.17 | 87.80 | 86.21 | 85.03 | 83.01 | 80.73 | 79.12 | 75.03 | 51.31  |
| Vanilla fine-tuning    | 94.17 | 93.27 | 93.01 | 92.74 | 92.3  | 91.67 | 88.91 | 86.03 | 54.39  |
| **Our method**         | 94.17 | 93.31 | 93.4  | 93.26 | 92.59 | 92.07 | 90.12 | 87.97 | 56.88  |

Table 1: Top-1 accuracy(%) on the test set of CIFAR-10 based on MobileNetV2.

| Method                  | Percent of weights removed (%) |
|------------------------|-------------------------------|
|                        | 0  | 28.37 | 54.25 | 71.23 | 93.18 |
| Training from scratch  | 79.18 | 74.14 | 71.42 | 67.39 | 56.79 |
| ExpandNets             | 78.87 | 74.28 | 73.82 | 71.06 | 58.07 |
| Vanilla distillation   | 79.18 | 75.32 | 73.69 | 72.61 | 58.65 |
| Vanilla fine-tuning    | 79.18 | 77.63 | 75.03 | 72.88 | 62.47 |
| **Our method**         | 79.18 | 77.47 | 75.76 | 73.36 | 64.08 |

Table 2: Top-1 accuracy(%) on the test set of ImageNet based on ResNet-50.
and pruning ratio in practice. The results of the third/fourth row of Table.4 are inconsistent with the conclusion in [Guo et al., 2018]. There is no obvious difference between the networks that expand the convolutional layer with and without the fully-connected layer. However, as it induces almost no increase in FLOPs, we apply the expansion together with the fully-connected layer throughout all other experiments.

4.4 Discussion and Limitation

Here we discuss the limitations mainly on the aspect of weight distribution and architecture design during the layer expansion. Firstly, we expand a layer to \( n \) layers by randomly generating \( n - 1 \) layers with normal distribution, and compute the rest layer to ensure that they produce the same output feature map. This decomposition scheme destroys the original layer’s feature extraction ability. A better solution can be initializing the expanded layers to conform to the weight distribution of the well trained layers. Secondly, we adopt a single-path over-parameterization scheme for layer expansion which replaces a layer with consecutive linear layers. There are other expansion architectures such as multi-branch over-parameterization [Ding et al., 2021], which outperforms the single-path ones. We assume that expanding the pruned networks into a combined structure of single-path and multi-branch forms may perform better in this task, but the decomposition is more complex at the same time. It will be one direction of our future work. Moreover, this work can be simultaneously applied to other pruning approaches and improves the results, which can also be further explored in future research.

5 Conclusion

In this paper, we propose a linear over-parameterization method that expands the layers of structurally pruned neural networks to solve the problem that there are too few fine-tuning parameters to restore the accuracy. Specifically, the proposed method equivalently expands the convolutional/linear layers with consecutive layers by matrix decomposition and does not alter the output feature maps. Furthermore, we utilize the similarity-preserving knowledge distillation to maintain the representation ability during fine-tuning. Experimental results show that our fine-tuning method significantly outperforms the vanilla fine-tuning counterpart, especially for large pruning ratios. Our work focuses on the fine-tuning stage of pruning and is orthogonal to the research of pruning optimization, weight importance evaluation, adaptive pruning ratios, and so on. Thus, it can be simultaneously applied to these methods and improve the results.
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