Electromagnetic form factors for pions couplings to constituent quarks and weak magnetic field

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Abstract

Electromagnetic form factors of pion couplings to constituent quarks are investigated by means of a one loop background field method for a nonperturbative one gluon exchange between quarks. Form factors are obtained from a large quark effective mass expansion and numerical estimations are presented for two different nonperturbative gluon propagators and also two pion field definitions. The corresponding Electromagnetic axial and pseudoscalar averaged quadratic radii are calculated as functions of the quark effective mass. The case in which the Electromagnetic field reduces to a weak magnetic field is also addressed and it yields weak magnetic field induced corrections to the usual Strong form factors: pseudoscalar, scalar, vector and axial ones. The momentum dependent Goldberger Treiman relation and quadratic radii also receive anisotropic weak magnetic field induced corrections.

1 Introduction

Although lattice QCD is expected to provide the final quantitative answers for the description of hadrons in terms of the more fundamental degrees of freedom it is very interesting to develop analytical tools to describe the gaps between the fundamental level and the measured hadron/nuclear properties. Among these, there are electromagnetic hadrons properties and interactions. Electromagnetic and strong form factors make possible a suitable comparison of many important observables calculated theoretically from different approaches, for example [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], with experiments, for example [12, 13, 14, 15, 16]. Among these observables, hadron charge distribution, spin structure and electroweak interaction properties can be understood in terms of electromagnetic and axial form factors. Due to the enormous difficulties in solving QCD in the non perturbative regime, low energy effective models have been considered based on general QCD symmetries and properties and phenomenology, in particular Chiral Symmetry and the Dynamical Chiral Symmetry Breaking (DChSB). The constituent quark model (CQM) describes many aspects of phenomenology and it is usually extended to incorporate the dynamics of the ubiquitous pion as a Goldstone boson [17, 18]. One example is the Weinberg effective field theory (EFT) that copes constituent quark picture with the large Nc expansion [19]. This EFT has
been derived in [20, 21] without and with electromagnetic interaction by starting from a quark-quark interaction due to a dressed one gluon exchange. This EFT is built in the constituent quark level and one might expect that a comparison between the electromagnetic and strong constituent quark form factors and baryons form factors might shed light on diverse aspects of baryons structure and interactions as well as it might provide further criteria to understand or improve the reliability of the CQM to describe low energy QCD.

In the last decade a high interest on the effect of magnetic fields on hadron properties and dynamics increased due to estimates for intense magnetic fields expected to appear in peripheral heavy ions collisions and in magnetars [22, 23]. Large magnetic fields of the order of \((eB_0) \approx 10^{17} - 10^{19} \text{G} \simeq (0.1 - 15) m_\pi^2\) would not be large as compared to the hadron mass scale, such as the nucleon mass, although it would only appear for a short time interval in non central heavy ions collisions [24]. One cannot expect large magnetic fields in low/intermediary energies hadron collisions in which the usual pion interactions as well as it might provide further criteria to understand or improve the reliability of the CQM to describe low energy QCD.

The non perturbative one gluon exchange quark-quark interaction is one of the leading terms of QCD effective action. With the minimal coupling to a background electromagnetic field, it is given by the following generating functional [31, 32, 33]:

\[
Z = N \int \mathcal{D}[\bar{\psi}, \psi] \exp i \int_x \left[ \bar{\psi} \left( i \slashed{D} - m \right) \psi - \frac{g^2}{2} \int_y j^b_\mu(x) \tilde{R}^{\mu\nu}_{bc}(x-y) j^c_\nu(y) + \bar{\psi} J + J^* \psi \right]
\]

(1)

Where \(\int_x\) stands for \(\int d^4x\), \(i, j, k = 0, \ldots (N_f^2 - 1)\) will be used for SU(2) flavor indices, \(a, b = 1, \ldots (N_c^2 - 1)\) stands for color in the adjoint representation and the quark sources are written in the last terms. The color quark current is given by \(j^a_\mu = \bar{\psi} \lambda_a \gamma^\mu \psi\), and the sums in color, flavor and Dirac indices are implicit. \(D_\mu = \partial_\mu \delta_{ij} - ieQ_{ij} A_\mu\) is the covariant quark derivative with the minimal coupling to photons, with the diagonal matrix \(Q = \text{diag}(2/3, -1/3)\). The gluon propagator is an external input and it is written as \(\tilde{R}^{\mu\nu}_{ab}(x-y)\). It must be a non perturbative one by incorporating to some extent the gluonic non Abelian character and, in particular, it will be required that, with a corrected quark-gluon coupling, it has enough strength to yield dynamical chiral symmetry breaking (DChSB), as it has been found in several approaches eventually by considering corrections to the interaction above, few examples in [34, 2, 35, 36, 37, 38, 39]. In several gauges this kernel can be written in terms of transversal and longitudinal components, as: \(\tilde{R}^{\mu\nu}_{ab}(x-y) \equiv \tilde{R}^{\mu\nu}_{ab} = \delta_{ab} \left[ (g^{\mu\nu} - \frac{\partial^{\mu}\partial^{\nu}}{m^2}) R_T(x-y) + \frac{\partial^{\mu}\partial^{\nu}}{m^2} R_L(x-y) \right]\). The method employed in the present paper was considered in previous works and it provided light vector mesons interactions with constituent quarks and the corresponding interactions with the electromagnetic field [20, 30]. It was also able to produce the whole Weinberg’s large \(N_c\) EFT for constituent quarks and pions and the corresponding interactions with electromagnetic field [20, 21]. The corresponding pion-constituent quarks form factors in the vacuum are investigated in a related article [40] and in the present work the corresponding momentum dependent interactions with the electromagnetic field will be addressed.
The method employed below was described with details in \cite{29,30,20,21} and it will be very succintly reminded in the next section. Two pion field definitions will be considered, the Weinberg pion field definition and the more usual one in terms of the functions $U, U^\dagger$. The resulting Strong vector and axial form factors are found to be equal to each other for a given pion field definition. The work is organized as follows. In the next section the determinant of sea quarks is presented for structureless pion field and then it is expanded for large quark effective mass. The complete momentum dependence of the leading electromagnetic effective couplings of pion interactions with constituent quarks are presented as momentum integrals of components of the quark and gluon propagators. In the following section two non perturbative gluon propagators are considered to provide numerical results, the Tandy-Maris propagator \cite{41} and an effective confining one \cite{36}, both of them produce DChSB. Next, the case in which photons give rise to a magnetic field, weak with respect to constituent quark mass, is considered. In this case the form factors for the electromagnetic couplings reduce to anisotropic corrections to pion-constituent quarks Strong form factors. Also the electromagnetic quadratic radii for the pion-constituent quark couplings are calculated as functions of the quark effective mass. The corresponding anisotropic corrections to the Strong quadratic radii due to a weak magnetic field are also presented. In the final section there is a Summary.

2 Form factors for the electromagnetic couplings of pion interactions with constituent quarks

The method will be succintly reminded below. To make possible a more complete investigation of all the flavor channels for diquark systems, a Fierz transformation for the model \cite{1} is performed and the non color singlet terms are neglected, being of non leading order due to a factor $1/N_c$. Chiral structures with combinations of bilocal currents are obtained. The quark field must be responsible for the formation of mesons and baryons and these different possibilities are envisaged by considering the Background Field Method (BFM), being that the background component will be a constituent quark ($\psi_1$) and the sea quark can be integrated out ($\psi_2$). For the one loop BFM it is enough to perform a shift of each of the quark current obtained from the Fierz transformation. For each of the isospin and Dirac channels the following shift is performed: $\bar{\psi}\Gamma^m\psi \rightarrow (\bar{\psi}\Gamma^m\psi)_2 + (\bar{\psi}\Gamma^m\psi)_1$. The integration of the sea quark is improved with respect to the usual one loop BFM by introducing light quark-antiquark mesons and excitations by means of the auxiliary field method. The following auxiliary fields, representing quark-antiquark states, are obtained: $S, P_i, S_i, P, V_\mu, V_\mu, \bar{A}_\mu, A_\mu$. However only the (chiral) scalar and isotriplet pseudoscalar sector will be kept.

The structureless mesons limit will be considered and this is reached by expanding bilocal fields in an infinite orthogonal basis for all the excitations of a given channel, for example for the pseudoscalar isotriplet fields one has: $P_i(x, y) = P_i\left(\frac{x+y}{2}, x-y\right) = P_i(u, z) = \sum_k F_k(z) P_{i,k}(u)$, where $F_k$ are vacuum functions invariant under translation for each of the local field $P_{i,k}(u)$. Only the lowest energy modes, highest $k = 0$ can be expected to contribute in the low energy regime. In the pseudoscalar channel it corresponds to the pions: $P_{i,k=0} = \pi_i$, and their form factors to reduce to constants $F_k(z) = F_k(0)$ which yield the canonical normalization of the fields. The heavier vector and axial mesons with their couplings to the constituent quarks were considered in \cite{29,30} and they can be neglected in the lower energy regime indeed. With that, by integrating out the sea quarks, their coupling to background
photons make possible to consider the electromagnetic coupling of the light mesons and constituent quarks. The auxiliary fields are undetermined and the saddle point equations can be used for this. In the mean field they can be written from the conditions: \( \frac{\partial S_{\alpha}^{\text{eff}}}{\partial \phi_\alpha} = 0 \), where \( S_{\alpha}^{\text{eff}} \) is the effective action obtained with the integration of the sea quark with the auxiliary fields and \( \phi_\alpha \) stands for each of the (constant) auxiliary fields. These equations for the NJL model and for the model \([1]\) have been analyzed in many works in the vacuum or under finite energy densities. In the vacuum, the scalar auxiliary field is the only one whose gap equation has a non trivial solution corresponding to a scalar quark-antiquark condensate as the order parameter of DChSB. At non zero constant magnetic fields a contribution to the quark effective mass arises associated to the so called magnetic catalysis that is well established from NJL-type and other models and also lattice QCD. The scalar quark-antiquark field does not necessarily correspond to a light meson and a chiral rotation can be performed. Two resulting pion field definitions will be considered below. The first one is the Weinberg pion field in which the chiral invariant terms correspond to a light meson and a chiral rotation can be performed. Two resulting pion field definitions chosen as:

\[
S_{\text{det}} = Tr \ln \{-iS_{c,q}^{-1}(x - y)\},
\]

\[
S_{c,q}^{-1}(x - y) \equiv S_{0,c}^{-1}(x - y) + \Xi_{q}^{f}(x - y) + \sum_q a_q \Gamma_q j_q(x, y),
\]

where \( Tr \) stands for traces of all discrete internal indices and integration of spacetime coordinates and \( \Xi_{q}^{f}(x - y) \) stands for the coupling of sea quark to the pions for a particular definition (p.f.). After the each of the chiral rotations considered it can be written for the two pion field definitions chosen as:

\[
\Xi_{q}^{W}(x - y) = \left[ \gamma^\mu \tilde{\sigma} \cdot D_\mu \bar{\pi} i\gamma_5 + i\gamma^\mu \tilde{\sigma} \cdot \frac{\bar{\pi} \times \partial_\mu \bar{\pi}}{1 + \bar{\pi}^2} + 4m_\pi \left( \frac{\bar{\pi}^2}{1 + \bar{\pi}^2} - \frac{\epsilon_{ijk} \sigma_k \pi_i \pi_j}{1 + \bar{\pi}^2} \right) \right] \delta(x - y),
\]

\[
\Xi_{q}^{U}(x - y) = F(P_R U + P_L U^\dagger) \delta(x - y),
\]

where \( P_{R/L} \) are the chiral right/left hand projectors and \( F \) the pion field normalization constant. The quark kernel can be written in terms of the effective quark mass generated by the scalar field gap equation as

\[
S_{0,c}^{-1}(x - y) = (i \not{\Phi} - M^*) \delta(x - y).
\]

In expression \((3)\) the following quantity with the usual leading chiral constituent quark currents has been defined:

\[
\frac{\sum_q a_q \Gamma_q j_q(x, y)}{\alpha g^2} = 2R(x - y) \left[ \bar{\psi}(y) \psi(x) + i\gamma_5 \sigma_i \bar{\psi}(y) i\gamma_5 \sigma_i \psi(x) \right] - \bar{R}^{\mu\nu}(x - y) \gamma_\mu \sigma_i \left[ \bar{\psi}(y) \gamma_5 \sigma_i \psi(x) + \gamma_5 \bar{\psi}(y) \gamma_5 \gamma_\mu \sigma_i \psi(x) \right]
\]

In this expression \( \alpha = 4/9 \), for flavor SU(2) and color SU(3), and combinations of the longitudinal and transversal parts of the gluon propagator were defined as:

\[
R(x - y) = 3R_T(x - y) + R_L(x - y),
\]

\[
\bar{R}^{\mu\nu}(x - y) = g^{\mu\nu}(R_T(x - y) + R_L(x - y)) + 2 \frac{\partial^\mu \partial^\nu}{\partial^2}(R_T(x - y) - R_L(x - y)).
\]
3 Leading form factors

The large effective quark mass expansion of the determinant within the zero order derivative expansion is performed in the following. The momentum dependent form factors for pion couplings to constituent quarks are investigated in [10]. The corresponding electromagnetic couplings for the Weinberg field definition are given by:

$$\mathcal{L}_{W,B}^{q-\pi} = T_{jki}G^W_{\pi V}(K, Q, Q_1) F^{\mu\nu}(Q_1) \pi_j(q_a) \partial_{\mu} \pi_k(q_b) \bar{\psi}(K)\gamma_\mu\sigma^i \psi(K + Q + Q_1)$$
+ $$\epsilon_{ijk}G^W_{\pi A}(K, Q, Q_1) F^{\mu\nu}(Q_1) \partial_{\mu} \pi_i(Q) \bar{\psi}(K)i\gamma_5\gamma_\nu\sigma^j \psi(K + Q + Q_1)$$
+ $$G^W_{\pi b F}(K, Q, Q_1, Q_3) F^{\mu\nu}(Q_1) F^{\mu\nu}(Q_3) \bar{\pi}(q_a) \cdot \bar{\pi}(q_b) \bar{\psi}(K)\psi(Q_T),$$

where in the couplings with two pions $$Q = q_a + q_b$$ and in the last one $$Q_T = K + Q + Q_1 + Q_3$$. The following tensor was defined in the first coupling: $$T_{jki} = \delta_{ij}\delta_{3k} - \delta_{j3}\delta_{ik}$$. The form factors are written in terms of general functions $$F_i(K, Q, Q_1)$$ written below, after resolving traces in internal indices, by:

$$G^W_{\pi V}(K, Q, Q_1) = 8d_1eN_c(\alpha g^2)F_3^I(K, Q, Q_1),$$

$$G^W_{\pi A}(K, Q, Q_1) = \frac{8}{3}d_1eN_c(\alpha g^2)F_3^I(K, Q, Q_1),$$

$$G^W_{\pi b F}(K, Q, Q_1, Q_3) = \frac{80}{9}d_1e^2N_c(\alpha g^2)F_4^I(K, Q, Q_1, Q_3),$$

The leading momentum dependent couplings with the electromagnetic field for the second pion field definition are the following:

$$\mathcal{L}_{U,A}^{q-\pi} = F^U_{\pi s}(K, Q, Q_1, Q_3) F^{\mu\nu}(Q_1) F^{\mu\nu}(Q_3) \pi_i(q_a) \pi_i(q_b) \bar{\psi}(K)\psi(K + Q + Q_1 + Q_3),$$
+ $$F_{psph}(K, Q, Q_1, Q_3) F^{\mu\nu}(Q_1) F^{\mu\nu}(Q_3) \epsilon_{ijk} \pi_i(Q_1) \bar{\psi}(K)\sigma_j i\gamma_5\psi(K + Q + Q_1 + Q_3)$$
+ $$T_{jki}F^U_{\pi V}(K, Q, Q_1) F^{\mu\nu}(Q_1) \pi_j(q_a) (\partial_{\mu} \pi_k(q_b)) \bar{\psi}(K)\gamma_\mu\sigma^i \psi(K + Q + Q_1),$$
+ $$\epsilon_{ijk}F^U_{\pi A}(K, Q, Q_1) F^{\mu\nu}(Q_1) \partial_{\mu} \pi_i(Q) \bar{\psi}(K)i\gamma_5\gamma_\nu\sigma^j \psi(K + Q + Q_1),$$

where in the couplings with two pions $$Q = q_a + q_b$$. The form factors in this expression are given by:

$$F^U_{\pi s}(K, Q, Q_1, Q_3) = \frac{5}{9}16d_1N_c(\alpha g^2)e^2FF_4^I(K, Q, Q_1, Q_3),$$

$$F_{psph}(K, Q, Q_1, Q_3) = \frac{4}{3}16d_1N_c(\alpha g^2)e^2FF_4^I(K, Q, Q_1, Q_3),$$

$$F^U_{\pi V}(K, Q, Q_1) = \frac{4}{3}16d_1N_cF(\alpha g^2)e^2F_4^I(K, Q, Q_1),$$

$$F^U_{\pi A}(K, Q, Q_1) = \frac{4}{3}16d_1N_cF(\alpha g^2)e^2F_4^I(K, Q, Q_1),$$

In this leading order of the determinant expansion, there are also unusual or anomalous couplings to the electromagnetic field that also break chiral and isospin symmetries explicitly and that correspond to sort of mixing couplings induced by the photon. The leading ones can be written as:

$$\mathcal{L}_{A_j} = -i\epsilon_{ijk}F_{6P}(K, Q, Q_1) A_{\mu}(Q_1) (\partial_\mu \pi_i(q_a)) \pi_j(q_b) \bar{\psi}(K)\psi(Q_T),$$
+ $$2i\epsilon_{ijk}F_{6M}(K, Q, Q_1) A_{\mu}(Q_1) \partial_\mu \pi_i(Q) \bar{\psi}(K)i\gamma_5\sigma^i \psi(Q_T),$$
+ $$iJ_{jik}F_{7P}(K, Q, Q_1) F_{A_{\mu}(Q_1)} \pi_i(q_a) \pi_j(q_b) \bar{\psi}(K)i\gamma_\mu\sigma^k \psi(Q_T),$$
+ $$2i\epsilon_{ijk}F_{7M}(K, Q, Q_1) F_{A_{\mu}(Q_1)} \pi_j(Q) \bar{\psi}(K)i\gamma_\mu\gamma_5\sigma^i \psi(Q_T),$$

5
where

\[
J_{113} = J_{311} = -J_{131} = 1, \quad J_{223} = J_{322} = -J_{232} = \frac{1}{3}, \quad J_{333} = 1,
\]

if \( i \neq j \neq k \) : \( J_{ijk} = \frac{i}{3} \epsilon_{ijk} \) \hspace{1cm} (19)

The form factors were defined in terms of functions \( H_{6P,M} \) and \( H_{7P,M} \):

\[
F_{6P}(K, Q, Q_1) = 4d_1 e N_c K_0 H_{6P}^t(K, Q, Q_1),
\]

(20)

\[
F_{6M}(K, Q, Q_1) = 4d_1 e N_c K_0 H_{6M}^t(K, Q, Q_1),
\]

(21)

\[
F_{7P}(K, Q, Q_1) = 4d_1 e N_c K_0 H_{7P}^t(K, Q, Q_1),
\]

(22)

\[
F_{7M}(K, Q, Q_1) = 4d_1 e N_c K_0 H_{7M}^t(K, Q, Q_1),
\]

(23)

![Figure 1](image)

Figure 1: Diagrams (1a,1b,1c) stand for the couplings of expressions (9) and diagrams (1a,1b,1c,1d) stand for the couplings of expressions (13,18). The wavy line with a full dot is a (dressed) non perturbative gluon propagator, the solid lines stand for quarks, dashed lines for pions and the dotted line stands for the photon strength tensor. A full square in a vertex represent momentum dependent pion coupling. The momenta of each of the particles are indicated by \( K \) (quarks), \( Q \) (pion(s)) and \( Q_1 \) (photons).

The diagrams for expressions (9,13,18) are presented in Fig. (1). The incoming constituent quark has momentum \( K \) and \( K + Q_T \) is the outgoing constituent quark momentum, where \( Q_T \) is the total momentum transferred by the pion(s) and photon(s) to the constituent quark. \( Q \) denotes the total transferred momentum from the pion(s) and \( Q_1, Q_3 \) are each of the photon momenta.
The functions $F_i^t(K, Q, Q_1), H_i^t(K, Q, Q_3)$ used above were defined as:

\[
F_3^t(K, Q, Q_1) = \frac{1}{2} (F_3(K, Q, Q_1) + F_3(k, Q_1, Q)),
\]

\[
F_4^t(K, Q, Q_1, Q_3) = \frac{1}{2} \left( F_4(K, Q, Q_1, Q_3) + F_4(k, Q_3, Q) \right),
\]

\[
F_5^t(K, Q, Q_1) = \frac{1}{2} (F_5(K, Q, Q_1) + F_5(k, Q_1, Q)),
\]

\[
H_{6p}^t(K, Q, Q_1) = \frac{1}{2} (F_6(K, Q, Q_1) + F_6(k, Q_1, Q)),
\]

\[
H_{6m}^t(K, Q, Q_1) = \frac{1}{2} (F_6(K, Q, Q_1) - F_6(k, Q_1, Q)),
\]

\[
H_{7p}^t(K, Q, Q_1) = \frac{1}{2} (F_7(K, Q, Q_1) + F_7(k, Q_1, Q)),
\]

\[
H_{7m}^t(K, Q, Q_1) = \frac{1}{2} (F_7(K, Q, Q_1) - F_7(k, Q_1, Q)),
\]

The loop momentum integrals of each of the form factors can be written in Euclidean momentum space, always for incoming quark momentum $K = 0$, as:

\[
F_3^t(0, Q, Q_1) = \int \frac{\dd^4 k}{(2\pi)^4} [k \cdot (k + Q_1) - M^*^2] \tilde{S}_0(k) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1) \tilde{R}(-k),
\]

\[
F_4^t(0, Q_1, Q_3) = -\int \frac{\dd^4 k}{(2\pi)^4} [k \cdot (k + Q_1 + Q) - M^*^2] \tilde{S}_0(k) \tilde{S}_0(k + Q_1 + Q) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1 + Q_3) \tilde{R}(-k),
\]

\[
\tilde{F}_4^t(0, Q, Q_1, Q_3) = \int \frac{\dd^4 k}{(2\pi)^4} [k^2 + k \cdot Q - M^*^2] \tilde{S}_0(k) \tilde{S}_0(k + Q) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1 + Q_3) \tilde{R}(-k),
\]

\[
F_5^t(0, Q, Q_1) = \int \frac{\dd^4 k}{(2\pi)^4} M^* \tilde{S}_0(k) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1) \tilde{R}(-k),
\]

\[
F_6^t(0, Q, Q_1) = \int \frac{\dd^4 k}{(2\pi)^4} [k \cdot (k + Q + Q_1) - M^*^2] \tilde{S}_0(k) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1 + Q_3) \tilde{R}(-k),
\]

\[
F_7^t(0, Q, Q_1) = \int \frac{\dd^4 k}{(2\pi)^4} [3k^2 + k \cdot (4Q_1 + 2Q) + Q_1^2 + Q \cdot Q_1 - M^*^2] \tilde{S}_0(k) \tilde{S}_0(k + Q_1) \tilde{S}_0(k + Q + Q_1 + Q_3) \tilde{R}(-k),
\]

where $\int_k = \int \frac{\dd^4 k}{(2\pi)^4}$ and the following function was used: $\tilde{S}_0(k) = \frac{1}{k^2 + M^*^2}$. It is also possible to truncate the resulting momentum dependence of the form factors by the following approximation:

\[
S_0^tr(k) \sim M^* \tilde{S}_0(k).
\]

It yields truncated form factors $F_i^{tr}(K, Q, Q_1)$ that make possible to obtain always positive quadratic mean radii at this level. Several of them will be compared to the results from the complete expressions above. Their overall normalizations however are different from the complete expressions although they are of the same order of magnitude.
4 Numerical results

Two very different gluon propagators $D_I(k)$ and $D_{II}(k)$ will be considered for the numerical calculations, such that:

$$g^2 R_{\mu\nu}(k) = h_a D_{a\mu\nu}(k)$$

(35)

where $h_a$ is a constant factor used in previous works [21, 30] to fix the quark gluon (running) coupling constant such as to reproduce a well established value for one particular effective coupling constant, for example $g_A = 1$ at the constituent quark level. In the present work it will be considered that $h_a = 1$. The first of these propagators is a transversal one proposed by Tandy and Maris [41] and it is considered where for the first expression

$$D_I(k) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2\gamma_m E(k^2)}{\ln[\tau + (1 + k^2/\Lambda_{QCD}^2)]},$$

(36)

$$D_{II}(k) = \frac{K_F}{(k^2 + M_F^2)^2},$$

(37)

where for the first expression $\gamma_m = 12/(33 - 2N_f)$, $N_f = 4$, $\Lambda_{QCD} = 0.234\text{GeV}$, $\tau = e^2 - 1$, $E(k^2) = [1 - \exp(-k^2/[4m_t^2])]/k^2$, $m_t = 0.5\text{GeV}$, $\omega = 0.5\text{GeV}$, $D = 0.55^2/\omega$ (GeV$^2$). For the second expression $K_F = (2\pi M_k/(3k_e))^2$ where $k_e = 0.15$ and $M_k = 0.220\text{GeV}$.

The photon momentum $Q_1$ was chosen in six different regimes, along the direction of the pion momentum $Q$ and in an anti-parallel direction, i.e. respectively for $Q \cdot Q_1 > 0$ and $Q \cdot Q_1 < 0$. The following ranges of photon momentum were chosen:

(i) $Q \cdot Q_1 = +Q^2$, (a) $Q \cdot Q_1 = +1.5Q^2$, (b) $Q \cdot Q_1 = 0.5Q^2$,
(c) $Q \cdot Q_1 = -Q^2$, (d) $Q \cdot Q_1 = -1.5Q^2$, (e) $Q \cdot Q_1 = -0.5Q^2$.

(38)

The cases (c,d,e) can be seen as pion photoproduction [3].

In figures (2,3) the electromagnetic form factor for the axial pion coupling to constituent quarks for the Weinberg pion field definition $C_{\pi A}^W(0, Q, Q_1)$ is presented as a function of the pion momentum $Q = |Q|$ for the gluon propagators $D_{II}(k)$ and $D_I(k)$ respectively and $M^* = 0.31\text{GeV}$. For most of the cases shown there is a crossing to (small) negative values in the range of momenta exhibited. The same overall behavior is obtained for each of the gluon propagators although there is no crossing to negative values for $D_I(k)$ in fig. (3). Besides that, there are some points with unusual behavior for figure (3) (non smooth behavior) for the case of $Q_1 \cdot Q = -1.5Q^2$ due to numerical instabilities.

In figures (4,5) the truncated version of the form factor $G_{A,\tau}^W(0, Q, Q_1)$ is presented respectively for the gluon propagators $D_{II}(k)$ and $D_I(k)$ and $M^* = 0.31\text{GeV}$. There is a monotonic decreasing in the momentum dependence for the same range of photon momentum $Q_1$ used in previous figures. At zero pion and photon momentum, the corresponding effective coupling constant have different values from (larger than) the non truncated expression. The form factors remain positive and they tend to zero faster or slower depending on larger or smaller values of the photon momenta.

In figures (6,7) the electromagnetic coupling for the axial pion coupling to constituent quarks for the second pion definition $F_{\pi A}^U(0, Q, Q_1)$ is shown as a function of the pion momentum for different ranges
Figure 2: In this figure the photon coupling to the axial form factor $G_{\pi A}^W(0, Q, Q_1)$ as a function of the pion momentum with the Weinberg pion field is presented for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$, in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$. Of the photon momentum $Q_1$, respectively for gluon propagators $D_{II}(k)$ and $D_I(k)$ and $M^* = 0.31\text{GeV}$. For most of the cases there is a monotonic decreasing. However for the case of large photon momentum transfer antiparallel to the pion $Q_1 \cdot Q = -1.5Q^2$, the form factor crosses to negative values. This behavior is independent of the gluon propagators. There are no truncated versions of the expressions for $F_{\pi A}^U(0, Q, Q_1)$.

The electromagnetic form factor of the pseudoscalar pion coupling to constituent quarks is presented as a function of the pion momentum in figures (8,9), for the ranges of the photon momentum $Q_1$ considered in the previous figures, and again $M^* = 0.31\text{GeV}$. The overall normalization at $Q = 0$ is larger than the axial coupling as it is expected from the phenomenology. For the cases of antiparallel photon momenta $Q_1 \cdot Q \propto -Q^2$, there is a broad range of $Q$ for which the form factors $F_{psph}^U$ change sign and then it becomes very negative. Larger the photon momentum transfer $Q_1$ more negative the form factor becomes. Both gluon propagators provide the same general behavior.

The truncated expression for the electromagnetic form factor of the pseudoscalar pion coupling to constituent quarks is presented in figures (10,11) for the two gluon propagators and $M^* = 0.310\text{GeV}$. A monotonic decreasing behavior is found.

The unusual anomalous (mixing) electromagnetic form factor for the pion coupling to scalar quark current $F_{6P}(0, Q, Q_1)$ is exhibited in figures (12) and (13) for the same range of momenta of pions and photons of the previous figures respectively for $D_{II}(k)$ and $D_I(k)$ and $M^* = 0.310\text{GeV}$. A similar behavior to the form factor $F_{psph}(0, Q, Q_1)$ in figures (8,9), with change of sign, is found. It is interesting
In this figure the photon coupling to the axial form factor $G_{\pi A}^W(0, Q, Q_1)$ as a function of the pion momentum for the Weinberg pion field is presented for the gluon propagator $D_I(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$, in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

Figure 3: In this figure the photon coupling to the axial form factor $G_{\pi A}^W(0, Q, Q_1)$ as a function of the pion momentum for the Weinberg pion field is presented for the gluon propagator $D_I(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$, in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

to note the largest values of these chiral and isospin symmetry breaking couplings are one or two order of magnitude smaller at the zero momentum transfer limit $Q = Q_1 = 0$ than the previous electromagnetic couplings of usual axial and pseudoscalar pion couplings.

The truncated anomalous electromagnetic form factor $F_{6P}^{tr}$ for the pion coupling to scalar quark current for the two gluon propagators are exhibited in figures (14,15) again for $M^* = 0.310\text{GeV}$. The resulting monotonic behavior with momentum is characteristic from the truncated form factors. Another similar trend to the previous truncated form factor is that the absolute values are slightly larger than the ones obtained from the untruncated expressions but considerably smaller than the usual form factors.

The second unusual anomalous electromagnetic coupling of pions to vector quark current, $F_{7P}(0, Q, Q_1)$ is presented in figures (16,17) for the same range of pion and photon momenta as the previous figures respectively for $D_{II}(k)$ and $D_I(k)$ and $M^* = 0.310\text{GeV}$. The corresponding truncated versions are exhibited in figures (18,19). In this case the monotonic decreasing is similar to both cases and to the truncated form factor $F_{6P}^{tr}(0, Q, Q_1)$. 
Figure 4: The truncated form factor for photon coupling to the axial interaction $F_{\pi A}^{\text{Tr}}(0,Q,Q_1)$ as a function of the pion momentum, with the Weinberg pion field, is presented for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0,1.5,0.5)Q^2$ in dashed lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0,1.5,0.5)Q^2$.

5 Weak magnetic field

The leading form factors for pion-constituent quark interactions in the vacuum were presented in [40] by means of the same method employed in the present work. It is shown now that corrections to those usual form factors induced by weak background magnetic field are obtained by considering the electromagnetic couplings of the previous section. For that, the photon field will be associated to $A_\mu = -B_0 (0,x,0,0)$.

The leading contribution for the weak magnetic field effect from the complete sum over the Landau orbits for the quark propagator have been found in different works [42, 43] and it can be written as:

$$G(k) = S_0(k) + S_1(k)(eB_0) = \frac{k + M^*}{k^2 - M^{*2}} + i\gamma_1\gamma_2\frac{(\gamma_0k^0 - \gamma_3k^3 + M^*)}{(k^2 - M^{*2})} (eB_0).$$ (39)

By substituting the vacuum quark propagator by a $G(k)$ several weak magnetic field-dependent corrections to pion constituent quark couplings appear. For several of the leading pion-constituent quark couplings found in [40] however the first order correction to the quark propagator $S_1(k)(eB_0)$ does not contribute in the leading terms, i.e. linear in $(eB_0)$. The few non zero leading corrections from the leading quark propagator component $S_1(k)$ are at most of the order of magnitude of the corrections found below. There are mostly corrections of the order of $(eB_0)^2$ and higher appear will not be presented below. The background photon field considered in the previous section, however, induces corrections of leading order in the weak magnetic field as shown below.
Figure 5: The truncated form factor for photon coupling to the axial interaction $F_{\pi A}^{W, tr}(0, Q, Q_1)$ as a function of the pion momentum, with the Weinberg pion field $W$, is presented for the gluon propagator $D_I(k)$, $M^* = 0.310 \text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = + (1.0, 1.5, 0.5)Q^2$ in dashed lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

The leading contributions in this section arise from the zeroth order quark propagator of expression \textbf{(39)}, $S_0(k)$, and a background photon $A_\mu = -B_0 (0, x, 0, 0)$. By considering the pion momentum $Q$, or $Q = q_a + q_b$ for the two-pion couplings, the resulting anysotropic corrections to pion-constituent quark form factors are obtained from expressions \textbf{(9,13)}, for each of the pion field definitions ($W$ and $U$). They are given by:

$$
\mathcal{L}^{q-\pi}_{W,B} = T_{jki} F_{\pi V-B}(K, Q) \pi_j(q_a) \partial_x \pi_k(q_b) \bar{\psi}(K) \gamma_\sigma i \psi(K + Q) \\
+ \epsilon_{3ij} F_{\pi A-B}^W(K, Q) \partial_x i \pi_i(Q) \bar{\psi}(K) i \gamma_5 \gamma_\sigma i \psi(K + Q), \\
+ m F_{\pi I-\pi B}(K, Q) \bar{\pi}(q_a) \cdot \bar{\pi}(q_b) \bar{\psi}(K) \psi(K + Q),
$$

\textbf{(40)}

$$
\mathcal{L}^{q-\pi}_{U,B} = F_{\pi V-B}(K, Q) F_{\pi I}(q_a) \pi_i(q_b) \bar{\psi}(K) \psi(K + Q) \\
+ \epsilon_{ij3} F_{\pi A-B}(K, Q) \pi_j(q_a) \partial_\mu \pi_k(q_b) \bar{\psi}(K) \gamma_\mu \sigma^i \psi(K, Q), \\
+ T_{jki} F_{\pi A-B}(K, Q) \pi_j(q_a) \partial_\mu \pi_k(q_b) \bar{\psi}(K) \gamma_\mu \sigma^i \psi(K, Q), \\
+ \epsilon_{ij3} F_{\pi A-B}(K, Q) \partial_\mu \pi_i(Q) \bar{\psi}(K) i \gamma_5 \gamma_\nu \sigma^j \psi(K, Q),
$$

\textbf{(41)}

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Figure 6: In this figure the electromagnetic form factor for the vector form factor $F_{\pi A}^U(0, Q)$ as a function of the pion momentum, with the usual pion field, in terms of $UU^\dagger$, is presented for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310 \text{GeV}$, for the same ranges of photon momenta of the previous figures. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = + (1.0, 1.5, 0.5) Q^2$, in solid lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = - (1.0, 1.5, 0.5) Q^2$.

The slightly more symmetric way of defining the magnetic field $A^\mu = -B_0(0, y, x, 0)/2$ guarantees the anisotropies to be kept in the plane perpendicular to the magnetic field. The quark effective mass $M^*$ receives corrections from the weak magnetic field in the scalar gap equation and it will not addressed with details in the present work. These expressions provide numerical values one order of magnitude

where $T_{jki} = \delta_{ij}\delta_{3k} - \delta_{ij}\delta_{3k}$ and

\begin{align}
F_{\pi V-B}^W(K, Q) &= (eB_0)8d_1N_c(\alpha g^2)F_3(K, Q, Q_1 = 0), \\
F_{\pi A-B}^W(K, Q) &= (eB_0)\frac{8}{3}d_1N_c(\alpha g^2)F_3(K, Q, Q_1 = 0), \\
F_{\beta s\rho F-B}^W(K, Q) &= -(eB_0)^2\frac{80}{9}d_1N_c(\alpha g^2)F_4(K, Q, Q_1 = Q_3 = 0), \\
F_{F-\pi s-\pi-B}^U(K, Q, 0) &= (eB_0)^2\frac{80}{9}d_1N_c(\alpha g^2)F_4(K, Q, Q_1 = Q_3 = 0), \\
F_{\rho s ph-B}^U(K, Q) &= (eB_0)^2\frac{64}{3}d_1N_c(\alpha g^2)F_4(K, Q, Q_1 = Q_3 = 0), \\
F_{\pi V-B}^U(K, Q) &= (eB_0)\frac{64}{3}d_1N_cF(\alpha g^2)F_5(K, Q, Q_1 = 0), \\
F_{\pi A-B}^U(K, Q) &= (eB_0)\frac{64}{3}d_1N_cF(\alpha g^2)F_5(K, Q, Q_1 = 0).
\end{align}
smaller than of the original pion - constituent quark couplings because they have small multiplicative extra factors $B_0$ or $B_0^2$ that can be factorized in constants such as as $eB_0/M^2$ or $(eB_0)^2/M^4$ within the current large quark effective mass regime. These make explicit that the $B_0$ induced corrections are considerably smaller than the original coupling constants and form factors.

The anomalous electromagnetic pion-quark couplings from expression (18) generate magnetic field induced mixing pion couplings breaking explicitly chiral and isospin symmetries. For pion momentum $Q$, or $Q = q_a + q_b$ in the two pion couplings, it yields:

$$L_{A_j} = -i\epsilon_{j3}F_6PB(K,Q)(\partial^\mu \pi_i(q_a))\pi_j(q_b) \bar{\psi}(K)\psi(K + Q)$$

$$- 2i\epsilon_{j3}F_6MB(K,Q)\pi_i(q_a)\bar{\psi}(K)i\gamma_5\sigma^j\psi(K + Q)$$

$$+ iJ_{ijk}F_7PB(K,Q)\pi_i(q_a)\pi_j(q_b) \bar{\psi}(K)i\gamma^\mu\sigma^k\psi(K + Q)$$

$$+ 2i\epsilon_{j3}F_7MB(K,Q)\pi_j(Q) \bar{\psi}(K)i\gamma^\mu\gamma_5\sigma^j\psi(K + Q).$$

The following functions were used in expressions (49) written in terms of the momentum derivative
Figure 8: The electromagnetic form factor for the pseudoscalar coupling $F_{psph}(0, Q, Q_1, Q_1)$ as a function of the pion momentum, for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310$GeV, for different ranges of photon momenta. Three situations in which each of the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

$\partial Q_x = \partial Q_1^1$:

\[ F_{6BP}(0, Q) = 4d_1(eB_0)FN_cK_0 \left( \partial Q_2 H_{6P}^I(0, Q, Q_1) \right)_{Q_1=0}, \quad (50) \]

\[ F_{6BM}(0, Q) = 4d_1(eB_0)FN_cK_0 \left( \partial Q_2 H_{6M}^I(0, Q, Q_1) \right)_{Q_1=0}, \quad (51) \]

\[ F_{7BP}(0, Q) = 4d_1(eB_0)FN_cK_0 \left( \partial Q_2 H_{7P}^I(0, Q, Q_1) \right)_{Q_1=0}, \quad (52) \]

\[ F_{7MP}(0, Q) = 4d_1(eB_0)FN_cK_0 \left( \partial Q_2 H_{7M}^I(0, Q, Q_1) \right)_{Q_1=0}. \quad (53) \]

5.1 Numerical results for weak $B_0$

In the following, numerical estimations for the form factors above will be shown. With a weak magnetic field dependence on the quark effective mass $M^* = M^*(B)$, the complete expression to the axial form factor for the Weinberg pion field definition can be written as

\[ G_A(Q, B) = G_A^{M^*}(Q) + \frac{eB_0 F_{\pi A-B}^W(0, Q)}{M^*^2 (eB_0/M^*^2)}. \quad (54) \]

Where the first term of the right hand side is the form factor presented and investigated in [40] for the Weinberg pion field definition and the correction $F_{\pi A-B}^W(0, Q)$ is shown in figure [20], for the gluon propagators $D_{II}(k), D_I(k)$ and different quark effective masses. This magnetic field correction in the
Figure 9: The electromagnetic form factor for the pseudoscalar coupling $F_{psph}(0, Q, Q_1, Q_2)$ as a function of the pion momentum, for the gluon propagator $D_I(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which each of the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

The figure is divided by a factor $(eB_0)/M^{*2}$ to make easier the interpretation and the comparison of each of the contributions. The effective mass $M^*$ is however kept constant for both contributions in expression (54).

In figure (21) the equivalent correction to the axial form factor for the second pion definition, $F_{\pi A-B}^{U}(K, Q)/(eB_0/M^{*2})$, being that the total axial form factor can be written in the same form as expression (54). There is however a different monotonic decrease of the form factors with momenta.

In figure (22) the weak magnetic field correction to the pseudoscalar form factor is presented for the two gluon propagators as a function of the pion momentum, divided by $(eB_0/M^{*2})^2$ and for different values of $M^*$. A more complete estimation for the total pseudoscalar form factor is obtained with the following expression:

$$G_{ps}(Q, B) = G_{ps}^{M^*}(Q) + \left(\frac{eB_0}{M^{*2}}\right)^2 \frac{F_{psphB}^{U}(0, Q)}{(eB_0/M^{*2})^2}.$$  

(55)

An expression for $G_{ps}^{M^*}(Q)$ is found in [40] and the isolated factor $(eB_0)^2/M^{*4}$ helps to identify the order of magnitude of the magnetic field correction for large quark mass regime. Note the $B_0$ correction to the pseudoscalar form factor has a factor $(eB_0/M^{*2})^2$ that is considerably smaller than $(eB_0/M^{*2})$ in the axial form factor.

The anysotropic anomalous form factors at finite (weak) magnetic field $F_{6PB}(0, Q)$ and $F_{7PB}(0, Q)$ are exhibited in figure (23) as function of the pion momentum $Q = |Q_2|$, for the two gluon propagators.
Figure 10: The truncated electromagnetic form factor for the pseudoscalar coupling $F_{\text{psph}}(0, Q, Q_1, Q_1)$ as a function of the pion momentum, for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in dashed lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

with $M^* = 310\text{MeV}$. They disappear in the zero momentum limit. The dependence of the form factor $F_{\text{psph}}(0, Q)$ on the gluon propagator is seemingly larger than for the previous form factors analysed in the present work. Although the order of magnitude might be larger than the corresponding electromagnetic form factors (18) these values must be multiplied by $(eB_0)/M^*$.2.

In figure (24) the axial and pseudoscalar form factors in the vacuum - from [40] - and with a weak magnetic field $eB_0/M^* = 0.2$ from expressions (54,55) are presented for the two gluon propagators for the second pion definition with an unique value of the quark effective mass $M^* = 0.31\text{GeV}$. The correction for the axial form factor is larger mainly because the pseudoscalar form factor has a correction overal correction of the order of $(eB_0/M^*)^2$ instead of $eB_0/M^*$ from the axial form factor.

Next, a normalized ratio of pseudoscalar and axial form factors can be defined by:

$$GT(Q) = \frac{G_{\text{ps}}(Q)}{G_{\text{A}}(Q)} \frac{F}{M^*}$$ (56)

such that if quark-level Goldberger Treiman relation (GTR) is satisfied at $Q = 0$ then $GT(0) = 1$. This normalized GTR as a function of momentum, presented in [40], is shown together with the weak magnetic field correction to GTR in figure (25) as a function of the pion momentum for the two gluon propagators. Also $M^* = 0.310\text{GeV}$ for $B_0 = 0$ and for $(eB_0/M^*) = 0.2$ with $M^* = 0.350\text{GeV}$. The weak magnetic field correction for the axial coupling is larger than the corresponding correction for the pseudoscalar form factor, and therefore it provides the leading contribution for the $B_0$-correction to the
Figure 11: The truncated electromagnetic form factor for the pseudoscalar coupling $F_{\text{tr}}^{\pi\phi}(0, Q, Q_1, Q_1)$ as a function of the pion momentum, for the gluon propagator $D_I(k)$, $M^* = 0.310 \text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = + (1.0, 1.5, 0.5)Q^2$ in dashed lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$. GTR. Within this formalism for $Q = 0$ the GTR is satisfied in the large quark effective mass limit at zero magnetic field.

5.2 Quadratic averaged radii and weak magnetic field anisotropic correction

From the electromagnetic form factors for the the axial and pseudoscalar couplings it is possible to define electromagnetic quadratic averaged radii. It can be expected to provide an estimate for an average distance in which the charge is distributed in each of the interactions. Since the form factors are not dimensionless the resulting quadratic radii can be extracted from the following expressions for the two pion definitions ($W, U$):

\[
< r^2_{\gamma} >_{A}^{W,U} = - \frac{6}{F_{\pi A}^{W,U}(0, 0, 0)} \frac{dF_{\pi A}^{W,U}(0, Q_\pi, Q_\gamma)}{dQ_\gamma^2} \bigg|_{Q_\pi=Q_\gamma=0}, \tag{57}
\]

\[
< r^2_{\gamma} >_{U\pi}^{U} = - \frac{6}{F_{\pi U}^{U}(0, 0, 0)} \frac{dF_{\pi U}^{U}(0, Q_\pi, Q_\gamma)}{dQ_\gamma^2} \bigg|_{Q_\pi=Q_\gamma=0}. \tag{58}
\]

These quadratic radii, as functions of the quark effective mass $M^*$, are presented in figures [26, 27] respectively for the two gluon propagators $D_I(k)$ and $D_{II}(k)$, being that for the first pion definition,
Figure 12: In this figure the anomalous electromagnetic mixing form factor for the vector coupling of pion to the scalar current $F_{6P}(0,Q,Q_1)$ as a function of the pion momentum is presented for the effective gluon propagator $D_{II}(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in solid lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

$W$, results from both the untruncated and the truncated expressions are displayed. It is interesting to compare these resulting values with the numerical results for the axial and pseudoscalar quadratic radii obtained with the same method uncoupled with the electromagnetic field as presented in [40]. The present electromagnetic quadratic radii are systematically three or four times larger in average. For the sake of comparison, the pion quadratic charge radius in lattice was found to be $<r^2> = 0.37\text{fm}^2$ [44] being it has an experimental value given by $<r^2> \simeq 0.45\text{fm}^2$ [45, 3]. The experimental nucleon axial radius is $<r^2_A>^{1/2} \simeq 0.68\text{fm}$ [12, 7].

From the electromagnetic form factors above, induced weak-$B_0$ corrections to the axial and pseudoscalar form factors can be obtained for expressions (40,41) and it is also possible to calculate anysotropic corrections to the square axial and pseudoscalar radii due to the weak magnetic field. The weak magnetic field due to the electromagnetic coupling provides dimensionless form factors they make possible to write them with a dimensionless factor $eB_0/M^*2$ or $(eB_0/M^*2)^2$. The magnetic field along the $\hat{z}$ direction can be chosen to be $B_0 = -B_0(0,y,x,0)/2$ for which it can be obtained a more general result. The resulting anysotropic corrections to the form factors are dimensionless and the
Figure 13: In this figure the anomalous electromagnetic mixing form factor for the vector coupling of pion to the scalar current $F_{6P}(0, Q, Q_1)$ as a function of the pion momentum is presented for the gluon propagator $D_I(k)$, $M^* = 0.310 \text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in solid lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$. Corresponding corrections to the quadratic radii can be defined simply as:

$$\Delta \langle r^2 \rangle^{W,U}_{A,x-y} = -6 \frac{dF^{W,U}_{\pi A-B}(0, Q_\pi)}{dQ^2_\pi} \bigg|_{Q_\pi=0},$$

where it was emphasized the anisotropy in the plane $x-y$ perpendicular to the constant weak magnetic field. The same effect has been shown for light vector and axial mesons in [30]. The resulting value for the anisotropic weak magnetic field dependent axial and pseudoscalar square radii for a particular pion field definition, are given by:

$$\langle r^2 \rangle_{A}^{W,U}(B) = \langle r^2 \rangle_{A}^{W,U}(M^*) + \left(\frac{eB_0}{M^*} \right) \frac{\Delta \langle r^2 \rangle_{A,x-y}^{W,U}(M^*)}{(eB_0/M^*)^2} ,$$

$$\langle r^2 \rangle_{ps}^{W,U}(B) = \langle r^2 \rangle_{ps}^{W,U}(M^*) + \left(\frac{eB_0}{M^*} \right)^2 \frac{\Delta \langle r^2 \rangle_{ps,x-y}^{W,U}(M^*)}{(eB_0/M^*)^2} ,$$

where $A(x-y)$ and $ps(x-y)$ correspond to axial and pseudoscalar radii, respectively.
Figure 14: The truncated electromagnetic anomalous mixing form factor for the scalar coupling of pion to the scalar current \( F_{6P}^{tr}(0,Q,Q_1) \) as a function of the pion momentum is presented for the effective gluon propagator \( D_{II}(k) \), \( M^* = 0.310 \text{GeV} \), for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum \( Q_1 \cdot Q = +\left(1.0, 1.5, 0.5\right)Q^2 \) in dashed lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum \( Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2 \).
Figure 15: The truncated electromagnetic anomalous mixing form factor for the scalar coupling of pion to the scalar current $F_{6P}^S(0,Q,Q_1)$ as a function of the pion momentum is presented for the gluon propagator $D_I(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q\cdot Q = + (1.0, 1.5, 0.5)Q^2$ in dashed lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q\cdot Q = -(1.0, 1.5, 0.5)Q^2$.

6 Summary

The leading electromagnetic (e.m.) form factors of pion-constituent quark effective interactions were investigated for timelike momenta within a dynamical approach. First of all, the background field method applied to the global color model was able to provide constituent quark interactions with sea quarks that form quark-antiquark states including the light mesons described by auxiliary fields, in particular pions. Two gluon non-perturbative propagators and two different pion field definitions, the Weinberg pion field $W$ and the more usual one $U, U^\dagger$, were considered. The e.m. couplings of the pion axial and pseudoscalar effective interaction with constituent quark were presented as function of the pion timelike momentum for different ranges of the photon momentum. The axial form factor for the $W$-pion field and the pseudoscalar form factor were presented both with the complete expression and with a truncated expression. For all form factors, the photon momentum was chosen in different ranges to be parallel or anti-parallel to the pion momentum. In several situations for antiparallel pion and photon, there are zeros in the (non truncated) form factors for low-intermediary momenta, such as for example $C_{\pi A}^W(0,Q,Q_1)$ for $Q = |Q| \sim 0.3 - 0.45\text{GeV}$ or $F_{psph}(0,Q,Q_1)$ for $Q \sim 0.3 - 0.35\text{GeV}$, being that all form factors go to zero for larger (pion, photon) momenta. The form factors with truncated expressions, however, might go to zero faster than the untruncated for not very large photon (antiparallel) momenta and they do not cross to negative values. In all these estimations the quark effective mass from the gap equation was kept constant, i.e. momentum independent, with the large contribution from the
Figure 16: The anomalous mixing form factor for scalar coupling of pion to the vector quark current 
\( F_{7P}(0,Q,Q_1) \) as a function of the pion momentum is presented for the effective gluon propagator \( D_{II}(k) \), \( M^* = 0.310 \text{GeV} \), for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum \( Q_1 \cdot Q = + (1, 1.5, 0.5) Q^2 \) in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum \( Q_1 \cdot Q = -(1, 1.5, 0.5) Q^2 \). 

Scalar condensate constant. The difficulties of establishing precise values and behavior for the quark gluon running coupling constant and an exact non perturbative gluon propagator manifest mainly in the ambiguity of finding an unambiguous value for the zero momentum value of the form factors. Specific (smaller) differences in the behavior of form factors with pion momenta due to the momentum behavior of gluon propagator were also found. The complete calculation for the nucleon form factors starting from the present dynamical approach was left outside the scope of the present work.

The case in which the coupled photon represents a weak magnetic field along the \( \hat{z} \)–direction was also addressed. This limit provides magnetic field induced corrections to usual pion-constituent quark couplings. The limit of very weak magnetic field with the leading Landau orbit(s) was shown to be reproduced by the leading photon couplings and eventually the resummation of higher order photon couplings would provide the strong magnetic field regime \(^{[12]}\). Few anomalous unusual e.m. couplings to pions-constituent quarks effective interactions were also obtained and they yield sort of anisotropic mixing pion couplings to constituent quarks induced by a weak magnetic field. These anomalous form factors were shown both in truncated and non truncated forms that provide very different behaviors as discussed above.

Average electromagnetic quadratic axial and pseudoscalar radii were also calculated as functions of the quark effective mass. The e.m. form factor for pseudoscalar coupling was found to be smaller than the axial one. It must be noted that pions were considered in the structureless limit in the quark determinant \(^{[2]}\). These quadratic radii decrease considerably with the values of \( M^* \) although
The anomalous mixing form factor for the scalar coupling of pion to the vector quark current $F_7P(0,Q,Q_1)$ as a function of the pion momentum is presented for the gluon propagator $D_I(k)$, $M^* = 0.310\text{GeV}$, for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum $Q_1 \cdot Q = +(1.0, 1.5, 0.5)Q^2$ in solid lines, and, in dot-dashed lines, three situations in which the photon momentum is anti-parallel to the pion momentum $Q_1 \cdot Q = -(1.0, 1.5, 0.5)Q^2$.

the different pion field and gluon propagators yield different slopes and normalizations. In particular the truncated expressions produce weaker decrease of the quadratic radii with increasing $M^*$. The corresponding weak magnetic field induced correction to pion and constituent quark form factors and to the quadratic axial and pseudoscalar radii were also obtained and their behaviors with increasing quark effective mass were also presented. The more general calculation for strong magnetic fields is intended to be investigated elsewhere.

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Figure 18: The truncated anomalous mixing form factor for the scalar coupling of pion to the vector quark current \( F_{7P}^{tr}(0, Q, Q_1) \) as a function of the pion momentum is presented for the effective gluon propagator \( D_{II}(k) \), \( M^* = 0.310 \text{GeV} \), for different ranges of photon momenta. Three situations in which the photon momentum is parallel to the pion momentum \( Q_1 \cdot Q = +(1, 1.5, 0.5)Q^2 \) in dashed lines, and, in dotted lines, three situations in which the photon momentum is anti-parallel to the pion momentum \( Q_1 \cdot Q = -(1, 1.5, 0.5)Q^2 \).

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Figure 20: The leading magnetic field correction to the axial form factor $F_{WA-B}^W(0, Q)$ as a function of the pion momentum for the Weinberg pion field is presented for the two gluon propagators $D_I(k)$ (thin lines) and $D_{II}(k)$ (thick lines). Different values of the quark effective mass are considered $M^* = 350\text{MeV}$ in dot-dashed lines, $M^* = 310\text{MeV}$ in dashed lines, $M^* = 280\text{MeV}$ in solid lines $M^* = 70\text{MeV}$ in dotted lines.

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Figure 22: In this figure the leading magnetic field correction to the pseudoscalar form factor $F_{pseud-B}^U(0,Q)$ as a function of the pion momentum for the usual pion field is presented for the two gluon propagators $D_I(k)$ (thin lines) and $D_{II}(k)$ (thick lines). Different values of the sea quark effective mass are considered $M^*=350\text{MeV}$ in dot-dashed lines, $M^*=310\text{MeV}$ in dashed lines, $M^*=280\text{ MeV}$ in solid lines $M^*=70\text{ MeV}$ in dotted lines.

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Figure 24: Pseudoscalar and axial form factors for $B_0 = 0$, from [40], and for $(e B_0/M^*)^2 = 0.2$ with different contributions: from the quark mass dependence on the weak magnetic field and from the correction to the form factor from expressions (54,55) for the two gluon propagators $D_I(k)$ and $D_{II}(k)$ and for the second pion field definition with $M^* = 310\text{MeV}$.
Figure 25: Normalized Goldberger-Treiman relation as function of momentum for $M^* = 0.31\text{GeV}$ at $B = 0$, from [40], and by considering expressions (54,55) for $eB_0/M^*^2 = 0.2$ and for the two gluon propagators. The quark mass dependence on the weak magnetic field was used with $M^*(B_0) = 350\text{MeV}$.
Figure 26: The electromagnetic-axial and electromagnetic-pseudoscalar square radii for the two pion definitions and gluon propagator $D_I(k)$ from expressions (57, 58) as functions of the effective quark mass from the gap equation. For the Weinberg field the complete and the truncated expressions are shown.
Figure 27: The electromagnetic-axial and electromagnetic-pseudoscalar square radii for the two pion definitions and gluon propagator $D_{II}(k)$ from expression (57,58) as functions of the effective quark mass from the gap equation. For the Weinberg field the complete and the truncated expressions are shown.
Figure 28: The axial squared radius and anisotropic weak magnetic field induced corrections for the two pion definitions and gluon propagator $D_{II}(k)$ as functions of the effective quark mass from the gap equation. The weak magnetic field corrections from expressions [59] are shown divided by the factor $eB/M^2$. 
Figure 29: The axial squared radius and anisotropic weak magnetic field induced corrections for the two pion definitions and gluon propagator $D_I(k)$ as functions of the effective quark mass from the gap equation. The weak magnetic field corrections from expressions (59) are shown divided by the factor $eB/M^2$. 
Figure 30: The pseudoscalar squared radius (untruncated and truncated expressions) and the anisotropic magnetic field induced correction for the two gluon propagators as functions of the effective quark mass from the gap equation. Some results are divided by a factor 10. The weak magnetic field corrections are shown divided by the factor \((eB/M^2)^2\).