Charged Vector Particles Tunneling From A Pair of Accelerating and Rotating and $5D$ Gauged Super-Gravity Black Holes

Wajiha Javed$^1$, G. Abbas$^2$ and Riasat Ali$^{1,2}$

$^1$Division of Science and Technology University of Education Township Campus, Lahore-54590, Pakistan
$^2$Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur, Pakistan.

Abstract

The aim of this paper is to study the quantum tunneling process for charged vector particles through the horizons of more generalized black holes by using Proca equation. For this purpose, we consider a pair of charged accelerating and rotating black holes with NUT parameter and a black hole in $5D$ gauged super-gravity theory, respectively. Further, we study the tunneling probability and corresponding Hawking temperature for both black holes by using WKB approximation. We find that our analysis is independent of the particles species either background black hole geometries are more generalized.

Keywords: Charged vector particles; Quantum tunneling; Proca equation; Electromagnetic background; Hawking radiation.

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*wajiha.javed@ue.edu.pk; wajihajaved84@yahoo.com
†abbasg91@yahoo.com; ghulamabbas@iub.edu.pk
‡riasatyasin@gmail.com
1 Introduction

A black hole (BH) is considered as an object which absorbs all the matter/energy from the environing area into it due to its intense gravitational field. General relativity (GR) depicts that a BH swallows all particles that collide the horizon of the BH. In 1974, Hawking predicted that a BH behaves like a black body having a specific temperature, known as Hawking temperature, which allows a BH to emit radiation (called Hawking radiation) from its horizon by assuming quantum field hypothesis in the background of the curved spacetime.

A particle’s action of quantum mechanical nature is used in order to calculate Hawking radiation spectrum from different BHs [1, 2]. The analysis of Hawking radiation as a quantum tunneling phenomenon and accretion onto some particular BHs has attracted the attention of many researchers [3]–[8]. Various efforts have been carried out to examine this radiation spectrum from BHs by considering quantum mechanics of scalar, Dirac, fermion and photon particles etc. Many researchers [9]–[12] have been studied vector particles tunneling to obtain more information about the Hawking temperature and radiation spectrum from different BHs. The charged vector particles tunneling from Kerr-Newman BH [13] and charged black string [14] are important contributions towards the BH physics.

The charged fermions tunneling from Reissner-Nordström de-Sitter BH with a global monopole [15] is studied by using WKB approximation and Dirac equation to evaluate the tunneling process for charged particles as well as Hawking temperature. In this paper the authors have evaluated the tunneling probability and Hawking temperature for charged fermion tunneling from event horizon. The tunneling process for Plebanski-Demianski BHs is determined by the graphical behavior of Hawking temperature of ingoing and outgoing charged fermion from event horizon [16]. The Hawking temperature for charged NUT (Newman-Unti-Tamburino) BH solutions to the field equations, is considered with rotation and acceleration. A BH can be studied on the small measurement through quantum field theory on a curved background [17]. The tunneling probability for outgoing particle is ruled by the imaginary part of particle’s action. A large number of attempts [18]–[26] have been made to calculate tunneling of charged and uncharged scalar and Dirac particles with different BHs configurations. The tunneling of spin-$\frac{1}{2}$ particles by event horizon of the Rindler spacetime was explained and Unruh temperature has been calculated [27]. Kraus and Wilczek [28, 29] projected
a semi-classical process to analyze Hawking radiation as a tunneling event. This process contains the calculation for the phenomenon of s-wave emission across event horizon. In [30], it has been shown that the Hawking radiation from rotating wormhole may emit all types of particles.

This paper deals with the study of the Hawking radiation process of charged vector particles from the horizons of a pair of accelerating and rotating BHs and a BH in 5D gauged super-gravity.

Vector particles (spin-1 bosons) such as $Z$ (uncharged) and $W^{\pm}$ (charged) bosons are of very importance in Standard Model. In the background of BHs geometries, the behavior of the bosons can be determined by using Proca equation. First, we formulate the field equations of charged $W^{\pm}$-bosons by using Lagrangian of the Glashow-Weinberg-Salam model [31]. Then we shall investigate particle emission process by using the Hamilton-Jacobi definition and WKB approximation to the derived equation for charged case in the considered BHs geometries. By putting the determinant (of coefficient matrix) equals to zero, we can solve for radial function. Consequently, we compute the tunneling rate of the charged vector particles from the horizons of BHs and find the corresponding Hawking temperature values in both cases.

The paper is planned follows: We discuss in the section 2, the tunneling rate and Hawking temperature for charged accelerating and rotating BH solutions with NUT parameter. Section 3 is devoted to investigate the charged vector particles tunneling and Hawking temperature for BH in 5D gauged super-gravity spacetime, by investigating the $W^{\pm}$ bosons observation. Section 4 provides the summary of the results for both cases.

2 Accelerating and Rotating Black Holes with NUT Parameter

In universal, the NUT parameter is affiliated with the gravitomagnetic monopole, related to the bending properties of the environing spacetime due to the fundamental mass, its accurate physical significance could not be determined. The generalization for multi dimensional Kerr-NUT de-Sitter spacetime [32, 33] and its physical implication [34] is also investigated. As a BH, the dominance on the NUT parameter the revolution parameter departs the spacetime free on bending singularities and the agreeing result is appointed as NUT alike result. If the revolution parameter commands the NUT pa-
rameter, the result is Kerr-like and a closed chain bending singularity forms. The behavior of this form of the singularity structure is independent of the existence on the cosmology constant.

There are lots of BHs which comprise of the NUT parameter and lots of investigation have been made to examine their physical effects in the space of colliding waves. Accurate significance of the NUT parameter exists, when a motionless Schwarzschild mass is absorbed in a stationary source and allows electromagnetic universe [35]. The NUT parameter is referred to the bend of the electromagnetic universe leaving out the fundamental Schwarzschild mass. In the absence of electromagnetic field, it reduces to the bend of the vacuum spacetime [36]. The bend of the surrounding space pair with the mass of reference yields NUT parameter.

The line element for accelerating and rotating BHs with NUT parameter is defined as [37]

$$ds^2 = -\frac{1}{\Omega^2} \left[ \frac{Q}{\rho^2} \left( dt - (a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2}) d\phi \right)^2 - \frac{\rho^2}{Q} dr^2 \right]$$

$$+ \frac{\tilde{P}}{\rho^2} \left( adt - (r^2 + (a + l)^2) d\phi \right)^2 - \frac{\rho^2}{P} \sin^2 \theta d\theta^2$$, \hspace{1cm} (2.1)

where

$$\Omega = 1 - \frac{\alpha}{\omega}(l + a \cos \theta)r, \quad \rho^2 = r^2 + (l + a \cos \theta)^2,$$

$$Q = \left[ (\omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2)(1 + 2al \frac{r}{\omega}) - 2Mr + \frac{\omega^2 \tilde{k}r^2}{a^2 - l^2} \right]$$

$$\times \left[ 1 + \frac{a - l}{\omega} r \right] \left[ 1 - \frac{a + l}{\omega} r \right],$$

$$\tilde{P} = \sin^2 \theta (1 - a_3 \cos \theta - a_4 \cos^2 \theta) = P \sin^2 \theta,$$

$$a_3 = 2M \frac{\alpha a}{\omega} - 4 \frac{\alpha l \alpha^2}{\omega^2} (\omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2),$$

$$a_4 = -\frac{\alpha^2 a^2}{\omega^2} (\omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2).$$

Here, $M$ denotes the mass of pairs of BHs, $e$ and $g$ indicate the electric and magnetic charges, respectively, while $l$ is a NUT parameter of BH, $\alpha$ and $\omega$ indicate acceleration and rotation of sources, respectively. Also, $a$ is the
Kerr-like rotation parameter and \( \tilde{k} \) is given by

\[
\left( \frac{\omega^2}{a^2 - l^2} + 3\alpha^2 l^2 \right) \tilde{k} = 1 + 2\frac{\omega}{\omega} M - 3\frac{\alpha^2 l^2}{\omega^2} (e^2 + \tilde{g}^2).
\]

Here, \( \alpha, \omega, M, \tilde{e}, \tilde{g} \) and \( \tilde{k} \) are arbitrary real parameters. We would like to mention that \( \omega \) depends on NUT parameter \( l \) and Kerr-like rotation parameter \( a \). The \( \alpha \) twisting property of BHs is proportional to the rotation \( \omega \). Also, \( \omega \) depends on rotation parameters \( l \) and \( a \). The parameters \( \alpha, \omega, M, \tilde{e}, \tilde{g} \) and \( \tilde{k} \) vary independently. If \( \alpha \) is equal to zero, then metric in Eq. (2.1) leads to the Kerr-Newman-NUT solution. If \( l = 0 \), then metric in Eq. (2.1) gives the couple of charged and rotating BHs. In this case, if \( \tilde{e} \) and \( \tilde{g} \) are equal to zero, we have a Schwarzschild BH and if \( l \) and \( a \) are equal to zero it leads to C-metric.

The metric (2.1) can be rewritten as

\[
ds^2 = -f(r, \theta)dt^2 + \frac{dr^2}{g(r, \theta)} + \Sigma(r, \theta) d\theta^2 + k(r, \theta) d\phi^2 - 2H(r, \theta) dt d\phi, \tag{2.2}
\]

where \( f(r, \theta), g(r, \theta), \Sigma(r, \theta), K(r, \theta) \) and \( H(r, \theta) \) are given by the following equations:

\[
f(r, \theta) = \frac{Q - Pa^2 \sin^2 \theta}{\rho^2 \Omega^2}, \quad g(r, \theta) = \frac{Q \Omega^2}{\rho^2}, \quad \Sigma = (r, \theta) = \frac{\rho^2}{\Omega^2 P},
\]

\[
k(r, \theta) = \frac{1}{\Omega^2 \rho^2} \left( \sin^2 \theta P \left( r^2 + (a + l)^2 \right)^2 - Q(a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2})^2 \right),
\]

\[
H(r, \theta) = \frac{1}{\Omega^2 \rho^2} \left( \sin^2 \theta Pa \left( r^2 + (a + l)^2 \right) - Q(a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2}) \right).
\]

The electromagnetic potential for these BHs is given

\[
A = \frac{1}{a (r^2 + (l + a \cos \theta)^2)} \left[ -\tilde{e} r \left( adt - d\phi \left( l + a \right) - (l^2 + a^2 \cos^2 \theta + 2al \cos \theta) \right) \right.
\]

\[
-\tilde{g} \left( l + a \cos \theta \left( adt - d\phi \left( r^2 + (l + a)^2 \right) \right) \right]. \tag{2.3}
\]

The event horizons are obtained for \( g(r, \theta) = \frac{Q \Omega^2}{\rho^2} = 0 \), which implies that \( \Omega \neq 0 \), so \( Q = 0 \), which yields the following real roots of \( r \), i.e.,

\[
r_{a1} = \frac{\omega}{\alpha (a - l)}, \quad r_{a2} = \frac{-\omega}{\alpha (a - l)}, \quad r_{\pm} = \frac{a^2 - l^2}{\omega^2 \tilde{k}} \left[ -\left( \omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2 \right) \frac{\alpha l}{\omega} - M \right]
\]

\[
\pm \sqrt{\left( \omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2 \right) \frac{\alpha l}{\omega} - M} \left( M - \frac{\omega^2 \tilde{k} + \tilde{e}^2 + \tilde{g}^2}{\alpha^2 - l^2} \right), \tag{2.4}
\]

5
where $r_{\alpha 1}$ and $r_{\alpha 2}$ are acceleration horizons and $r_{\pm}$ represent the outer and inner horizons, respectively such that

$$\left((\omega^2 k + \bar{e}^2 + \bar{g}^2)\frac{\alpha l}{\omega} - M\right)^2 - (\omega^2 k + \bar{e}^2 + \bar{g}^2)\frac{\omega^2 k}{\alpha^2 - l^2} > 0.$$ 

The angular velocity at BH outer (event) horizon is defined by

$$\Omega = \frac{a}{r_+^2 + (a + l)^2}. $$ (2.5)

In order to investigate the tunneling spectrum for charged vector particles through the BH horizon, we will consider Proca equation with electromagnetic effects. In a curved spacetime with electromagnetic field, the motion of massive spin-1 charged vector fields is depicted by the given Proca equation by using the Lagrangian of the W-bosons of Glashow-Weinberg-Salam model [10]

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \psi^{\mu}) + \frac{m^2}{\hbar^2} \psi^{\nu} + \frac{i}{\hbar} e A_\mu \psi^{\nu} + \frac{i}{\hbar} e F^{\mu\nu} \psi_\mu = 0, \quad (2.6)$$

where $g$ is determinant of coefficients matrix, $m$ is particles mass and $\psi^{\mu\nu}$ is anti-symmetric tensor, i.e.,

$$\psi^{\mu\nu} = \partial_\nu \psi_\mu - \partial_\mu \psi_\nu + \frac{i}{\hbar} e A_\nu \psi_\mu - \frac{i}{\hbar} e A_\mu \psi_\nu \quad \text{and} \quad F^{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu.$$

Here, $A_\mu$ is considered as the electromagnetic potential of the BH, $e$ denotes the charge of the W-bosons and $\nabla_\mu$ is geometrically covariant derivative. Since the equation of motion for the $W^+$ and $W^-$ bosons is similar, the tunneling processes should be similar too. For simplification, here we will consider the $W^+$ boson case, the results of this case can be extended to $W^-$ bosons due to the digitalization of the line element. For $W^+$ field, the values of the components of $\psi^{\mu}$ and $\psi^{\nu\mu}$ are obtained as follows

$$\psi^0 = \frac{-k \psi_0 - H \psi_3}{fk + H^2}, \quad \psi^1 = g \psi_1, \quad \psi^2 = \Sigma^{-1} \psi_2, \quad \psi^3 = \frac{-H \psi_0 + f \psi_3}{fk + H^2},$$

$$\psi^{01} = \frac{-kg \psi_{01} - Hg \psi_{13}}{fk + H^2}, \quad \psi^{02} = \frac{-kg \psi_{02} - H \psi_{23}}{\Sigma(fk + H^2)}, \quad \psi^{03} = \frac{-\psi_{03}}{fk + H^2},$$

$$\psi^{12} = g \Sigma^{-1} \psi_{12}, \quad \psi^{13} = \frac{g(f \psi_{13} - H \psi_{01})}{fk + H^2}, \quad \psi^{23} = \frac{g \psi_{23} - H \psi_{02}}{\Sigma(fk + H^2)}.$$
The electromagnetic vector potential for this BH is given by \[38\]

\[
A = \frac{1}{a[r^2 + (l + a \cos \theta)^2]}[-\tilde{e} r [adt - d\phi(l + a)^2 - (l^2 + a^2 \cos^2 \theta \\
+ 2la \cos \theta)] - \tilde{g}(l + a \cos \theta)[adt - d\phi r^2 + (l + a)^2]].
\] (2.7)

Using, the WKB approximation \[39\], i.e.,

\[
\psi_\nu = c_\nu \exp\left[\frac{i}{\hbar} S_0(t, r, \theta, \phi) + \sum \hbar^n S_n(t, r, \theta, \phi)\right],
\] (2.8)

to the Proca Eq.(2.6) and neglecting the terms for \(n = 1, 2, 3, 4, \ldots\), we obtain
the following set of equations

\begin{align*}
  k g & \left[ c_1(\partial_1 S_0)(\partial_1 S_0) + eA_0 \right] - c_0(\partial_1 S_0)^2 - H g[c_3(\partial_1 S_0)^2] \\
  - & c_1(\partial_1 S_0)((\partial_3 S_0) + eA_3] + k \sum c_2(\partial_2 S_0)((\partial_0 S_0) + eA_3) - c_0(\partial_2 S_0)^2] \\
  - & H \sum \left[ c_3(\partial_2 S_0)^2 - c_2(\partial_2 S_0)((\partial_3 S_0) + eA_3) \right] + [c_3(\partial_3 S_0)((\partial_0 S_0) + eA_0) \\
  - & c_0(\partial_3 S_0)((\partial_3 S_0) + eA_3)] + eA_3 k g[c_1((\partial_0 S_0) + eA_0) - c_0(\partial_1 S_0)] \\
  - & m^2 k c_0 - m^2 H c_3 - eA_3 H g[c_3(\partial_1 S_0) - c_1((\partial_3 S_0) + eA_3)] = 0, \quad (2.9) \\
  k g & \left[ c_1(\partial_0 S_0)((\partial_0 S_0) + eA_0) - c_0(\partial_1 S_0)(\partial_0 S_0) \right] - H g[c_3(\partial_1 S_0)(\partial_0 S_0) \\
  - & c_1(\partial_0 S_0)((\partial_3 S_0) + eA_3)] + \frac{g(fk + H^2)}{\sum} \left[ c_2(\partial_1 S_0)(\partial_2 S_0) - c_1(\partial_2 S_0)^2 \right] \\
  + & g f[c_3(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_3 S_0)((\partial_3 S_0) + eA_3)] + g H[c_1(\partial_3 S_0)((\partial_0 S_0) \\
  - & eA_1 - c_0(\partial_1 S_0)((\partial_3 S_0)] + eA_0 k g[c_1((\partial_0 S_0) - eA_0) - c_0(\partial_1 S_0)] \\
  - & m^2 g c_1(fk - H) - eA_0 H g[c_3(\partial_1 S_0) - c_1((\partial_3 S_0) - eA_3)] \\
  + & eA_1 g f[c_3(\partial_1 S_0) - c_1((\partial_3 S_0) + eA_3)] + eA_3 H c_1((\partial_0 S_0) + eA_0) \\
  - & c_0(\partial_1 S_0)] = 0 \quad (2.10) \\
  k \sum & \left[ c_2(\partial_2 S_0)^2 - c_0(\partial_2 S_0)(\partial_0 S_0) + eA_0(\partial_0 S_0)c_2 \right] - H \sum[c_3(\partial_1 S_0)(\partial_2 S_0) \\
  - & c_2(\partial_1 S_0)((\partial_3 S_0)] - \frac{g}{\sum} \left[ c_2(\partial_1 S_0)^2 - c_1(\partial_1 S_0)(\partial_2 S_0) \right] (fk - H) \\
  + & \frac{f}{\sum} \left[ c_3(\partial_2 S_0)(\partial_3 S_0) - c_2(\partial_3 S_0)^2 - eA_3 c_2(\partial_3 S_0) \right] + \frac{H}{\sum} \left[ (\partial_3 S_0)(\partial_0 S_0)c_2 \\
  - & c_0(\partial_2 S_0)(\partial_3 S_0) + c_2 eA_0(\partial_2 S_0)(\partial_3 S_0)] - m^2 c_2 \Sigma^{-1}(fk - H) \\
  + & eA_0 \frac{k}{\sum} [c_2(\partial_0 S_0) - c_0(\partial_2 S_0) + eA_0 c_2] - eA_0 \frac{H}{\sum} [c_3(\partial_2 S_0) - c_2(\partial_3 S_0) \\
  - & c_2 eA_3 + eA_3 \frac{f}{\sum} [c_3(\partial_2 S_0) - c_2(\partial_3 S_0) - eA_3 c_2] + eA_3 \frac{H}{\sum} [c_2(\partial_0 S_0) \\
  - & c_0(\partial_2 S_0) + eA_0 c_2] = 0 \quad (2.11)
\end{align*}
Using separation of variables technique, we can choose

\[ [c_3(\partial_3 S_0)^2 - c_0(\partial_3 S_0)(\partial_0 S_0) + eA_0 c_3(\partial_0 S_0) - eA_3 c_0(\partial_0 S_0)] + g - H[c_1(\partial_0 S_0)(\partial_1 S_0) - c_0(\partial_1 S_0)^2 + eA_0 c_1(\partial_0 S_0)] - f g[c_3(\partial_0 S_0)^2] - c_1(\partial_1 S_0)(\partial_3 S_0) - eA_3 c_1(\partial_0 S_0)] - \frac{H}{\Sigma} [c_2(\partial_0 S_0)(\partial_2 S_0) - c_0(\partial_2 S_0)^2] + eA_0 c_2(\partial_2 S_0)] - f g[c_3(\partial_1 S_0)^2 - c_1(\partial_1 S_0)(\partial_3 S_0) - eA_3 c_1(\partial_0 S_0)] - \frac{H}{\Sigma} [c_2(\partial_0 S_0)(\partial_2 S_0) - c_0(\partial_2 S_0)^2 + eA_0 c_2(\partial_2 S_0)] - \frac{f}{\Sigma} [c_3(\partial_0 S_0)^2] - c_2(\partial_2 S_0)(\partial_3 S_0) - eA_3 c_2(\partial_2 S_0)] + eA_0 [c_3(\partial_0 S_0) - c_0(\partial_3 S_0) + eA_0 c_3 - eA_3 c_0 + m^2[Hc_0 + c_3 f] = 0. \tag{2.12} \]

Using separation of variables technique, we can choose

\[ S_0 = -(E - j \bar{\Omega})t + W(r) + N \dot{\phi} + \Theta(\theta), \tag{2.13} \]

where \( E \) and \( j \) represent particle’s energy and angular momentum, respectively. From the Eqs. (2.9)-(2.12), we can obtain a matrix equation

\[ G(c_0, c_1, c_2, c_3)^T = 0, \]

which implies a 4 \times 4 matrix labeled as “\( G \)”, whose components are given as follows:

\[
\begin{align*}
G_{11} &= -\dot{W}^2 k g - \frac{k N^2}{\Sigma} - \dot{\Theta}^2 - \dot{\Theta} eA_3 - m^2 k - eA_3 k g \dot{W}, \\
G_{12} &= -\dot{W} k g (E - j \bar{\Omega}) + k g \dot{W} eA_0 + H g \dot{W} \dot{\Theta} + H g W eA_3 \\
&- eA_3 k g (E - j \bar{\Omega}) + k g eA_3 eA_0 + eA_3 H g \dot{\Theta} + H g eA_3, \\
G_{13} &= -\frac{k}{\Sigma} (E - j \bar{\Omega}) N + \frac{k}{\Sigma} N eA_3 + \frac{H}{\Sigma} \dot{\Theta} N + \frac{H}{\Sigma} N eA_3, \\
G_{14} &= -\dot{W}^2 H g - \frac{H}{\Sigma} N^2 - \dot{\Theta}^2 (E - j \bar{\Omega}) + \dot{\Theta} eA_0 - m^2 H - eA_3 g \dot{W}, \\
G_{21} &= k g (E - j \bar{\Omega}) \dot{W} - g H \dot{W} \dot{\Theta} - eA_0 k g \dot{W} - eA_3 H \dot{W}, \\
G_{22} &= -k g (E - j \bar{\Omega}) (E - j \bar{\Omega}) + eA_0 - g H (E - j \bar{\Omega}) (\dot{\Theta} + eA_3) \\
&+ \frac{g}{\Sigma} N^2 (f k - H) - g f \dot{\Theta} (\dot{\Theta} + eA_3) + g H \dot{\Theta} (-E - j \bar{\Omega}) \\
&- eA_0 + eA_0 k g (E - j \bar{\Omega}) + eA_0 g H (\dot{\Theta} - eA_3) - eA_3 g f (\dot{\Theta} + eA_3)
\end{align*}
\]
\[ eA_3H(-(E - j\tilde{\Omega}) - m^2g(fk - H)), \quad G_{23} = \frac{g}{\Sigma} \dot{W}N(fk - H^2), \]

\[ G_{24} = Hg(E - j\tilde{\Omega})\dot{W} + g\dot{W}\dot{\Theta} - eA_0gH\dot{W} + eA_3gf\dot{W}, \]

\[ G_{31} = \frac{k}{\Sigma}(E - j\tilde{\Omega})N - \frac{H}{\Sigma}N\dot{\Theta} - eA_3HN, \quad G_{32} = \frac{g}{\Sigma} \dot{W}N(E - j\tilde{\Omega}), \]

\[ G_{33} = -\frac{k}{\Sigma}[(E - j\tilde{\Omega})^2 - eA_0(E - j\tilde{\Omega})] + \frac{H}{\Sigma} \dot{W}\dot{\Theta} \]

\[ - \frac{g}{\Sigma}[\dot{W}^2(fk + H^2)] - \frac{f}{\Sigma}[\dot{\Theta}^2 + eA_3(\dot{\Theta})] + \frac{H}{\Sigma}[\dot{\Theta}((E - j\tilde{\Omega}) + eA_0N)] \]

\[ - m^2\Sigma^{-1}(fk + H^2) - eA_0 \frac{k}{\Sigma}[(E - j\tilde{\Omega}) - eA_0] + eA_0 \frac{H}{\Sigma}[(\dot{\Theta} + eA_3] \]

\[ - eA_3 \frac{f}{\Sigma}[(\dot{\Theta} + eA_3] \]

\[ G_{34} = -\frac{H}{\Sigma} N\dot{W} + \frac{f}{\Sigma} N\dot{\Theta} - eA_0N\frac{H}{\Sigma} + eA_3 + \frac{f}{\Sigma}, \]

\[ G_{41} = (E - j\tilde{\Omega})\dot{\Theta} + (E - j\tilde{\Omega})eA_3 + gH\dot{W}^2 + N^2\frac{H}{\Sigma} + m^2H + (E - j\tilde{\Omega})eA_0 \]

\[ - e^2A_0A_3, \quad G_{42} = gH\dot{W}(E - j\tilde{\Omega}) - gHeA_0\dot{W} + fg\dot{W}\dot{\Theta} \]

\[ - fgeA_3(E - j\tilde{\Omega}), \]

\[ G_{43} = \frac{H}{\Sigma}(E - j\tilde{\Omega})N - \frac{H}{\Sigma}NeA_0 + \frac{f}{\Sigma} N\dot{\Theta} + NeA_3, \]

\[ G_{44} = (E - j\tilde{\Omega})^2 - (E - j\tilde{\Omega})eA_0 - fg\dot{W}^2 - \frac{f}{\Sigma} N - m^2f \]

\[ - eA_0[(E - j\tilde{\Omega}) - eA_0], \]

where \( \dot{W} = \partial_r S_0, \dot{\Theta} = \partial_\theta S_0 \) and \( N = \partial_\phi S_0 \). For the non-trivial solution, the absolute value \( G \) is equals to zero, and solving the resultant equation for the radial part so that one can get the following integral

\[ ImW^\pm = \pm \int \sqrt{(E - eA_0 - j\tilde{\Omega})^2 + X} \frac{dr}{f(r)g(r)} \]  \hspace{1cm} (2.14)

where + and − represent the radial function of outgoing and incoming particles, respectively, while the function \( X \) can be defined as \( X = -\Sigma^{-1}fN - m^2f - Hg(E - j\tilde{\Omega}) - gf\dot{\Theta} + eA_0gH - eA_3gf \), \( \tilde{\Omega} \) is the angular velocity on the event horizon.

Expanding the functions \( f(r) \) and \( g(r) \) in Taylor’s series near horizon, we get

\[ f(r_+) \approx f'(r_+)(r - r_+), \quad g(r_+) \approx g'(r_+)(r - r_+). \]  \hspace{1cm} (2.15)
Using above expressions in Eq. (2.14), one can see that resulting equation has no poles at \( r = r_+ \). For the calculation of Hawking temperature by using tunneling method, it is required to regularize the singularity by a specific complex contour to bypass the pole. For our standard co-ordinates of BH metric, the tunneling of outgoing particles can be obtained by taking an infinitesimal halfcircle below the pole \( r = r_+ \), while for the ingoing particle such contour is taken below above the pole. Further, in order to calculate the semiclassical tunneling probability, it is required that resulting wave equation must be multiplied by its complex conjugate. In this way, the part of trajectory that starts from outside of the BH and continues to the observer, will not contribute to the calculation of the final tunneling probability and can be ignored because it will be completely real. Therefore, the only part trajectory that contributes to the tunneling probability is the contour around the BH horizon.

Hence using Eq. (2.14) and Eq. (2.15), and integrating the resulting equation around the pole, we get

\[
\text{Im}W^\pm = \pm i\pi \frac{E - eA_0 - j\bar{\Omega}}{2\kappa(r_+)}, \tag{2.16}
\]

and the surface gravity is [36]

\[
\kappa(r_+) = \left[ \frac{[\alpha l \omega (\omega^2 \bar{k} + \bar{e}^2 + \bar{g}^2) - M + \omega^2 \bar{k} \bar{r}_+]}{r_+^2 + (a + l)^2} \times \left[ 1 + \frac{\alpha(a - l)}{\omega} r_+ \right] \times \left[ 1 - \frac{\alpha(a + l)}{\omega} r_+ \right] \right].
\]

The tunneling probability for charged vector particles is given by

\[
\Gamma = \frac{\text{Prob}[\text{emission}]}{\text{Prob}[\text{absorption}]} = \frac{\exp[-2(\text{Im}W^+ + \text{Im}\theta)]}{\exp[-2(\text{Im}W^- - \text{Im}\theta)]} = \exp[-4\text{Im}W^+]
\]

\[
\exp \left[ -2\pi \frac{E - eA_0 - j\bar{\Omega}}{\left[ \frac{[\alpha l \omega (\omega^2 \bar{k} + \bar{e}^2 + \bar{g}^2) - M + \omega^2 \bar{k} \bar{r}_+]}{r_+^2 + (a + l)^2} \times \left[ 1 + \frac{\alpha(a - l)}{\omega} r_+ \right] \times \left[ 1 - \frac{\alpha(a + l)}{\omega} r_+ \right] \right]} \right].
\]

Now, finally we can calculate the Hawking temperature by comparing the above result with the Boltzmann formula \( \Gamma_B = e^{-(E - eA_0 - j\bar{\Omega})/T_H} \), to get

\[
T_H = \left[ \frac{\left[ \frac{[\alpha l \omega (\omega^2 \bar{k} + \bar{e}^2 + \bar{g}^2) - M + \omega^2 \bar{k} \bar{r}_+]}{2\pi r_+^2 + (a + l)^2} \times \left[ 1 + \frac{\alpha(a - l)}{\omega} r_+ \right] \times \left[ 1 - \frac{\alpha(a + l)}{\omega} r_+ \right] \right]}{\left[ \frac{[\alpha l \omega (\omega^2 \bar{k} + \bar{e}^2 + \bar{g}^2) - M + \omega^2 \bar{k} \bar{r}_+]}{r_+^2 + (a + l)^2} \times \left[ 1 + \frac{\alpha(a - l)}{\omega} r_+ \right] \times \left[ 1 - \frac{\alpha(a + l)}{\omega} r_+ \right] \right]} \right].
\tag{2.17}
\]
The Hawking temperature depend on $A_0$ vector potential, $E$ energy, $\Omega$ angular momentum, $M$ is a mass a pair of BHs, $e$ and $g$ are electric and magnetic charges respectively, $a$ is the rotation of a BH, $l$ is a NUT parameter, $\alpha$ represents acceleration of the sources and $\omega$ rotation of the sources.

We would like to mention that Hawking temperature of charged vector particles given in Eq.(2.17) is same as the Hawking temperature of fermion particles in Eq.(4.20) of [36]. Thus the Hawking temperature is independent of the particle species.

### 3 Black Holes in 5D Gauged Super-gravity

The gauged theory is stated as a super-gravity theory in which the gravitino, the superpartner of the graviton is charged under some internal gauge group. However, the gauged super-gravity is more significant as compared to the ungauged case, because this theory has a negative cosmological constant, so it is defined on an anti-de Sitter space. Here, for the discussion of charged vector particles tunneling spectrum form a BH in 5D gauged super-gravity, we evaluate the tunneling probability of particles and the corresponding Hawking temperature at BH horizon. Such BH solutions occur in $D = 5\, N = 8$ gauged super-gravity (symmetry) [40]. Firstly, this solution was formulated in [41] as a particular case (STU-model) of solutions of $D = 5\, N = 2$ gauged super-gravity equations of motion. The metric for BH in 5D gauged super-gravity is [40]

$$ds^2 = -(H_1H_2H_3)^{-\frac{2}{3}} f dt^2 + (H_1H_2H_3)^{\frac{1}{3}} \left(f^{-1} dr^2 + r^2 d\Omega_{3, k}^2\right),$$

(3.1)

where

$$f = k - \frac{\mu}{r^2} + g^2 r^2 H_1 H_2 H_3, \quad H_i = 1 + \frac{q_i}{r^2}, \quad \text{for } i = 1, 2, 3$$

and $d\Omega_{3, k}^2$ is the metric on $S^3$ with unit radius if $k = 1$, or the metric on $R^3$ if $k = 0$, here $\mu$ is the non-extremality parameter [41], which is related to ADM mass, $g = 1/L$ is the inverse radius of $AdS_5$ related to the cosmological constant $\Lambda = -6g^2 = -6/L^2$, and $q_i$ are charges entering the metric. The three gauge field potentials $A_i^\mu$ from the solution of equation of motion are of the form

$$A_i^\mu = -\frac{q_i}{r^2 + q_i} \quad \text{for } i = 1, 2, 3$$
where $\tilde{q}_i$ are physical charges which are conserved and Gauss law is applicable to such charges.

The line element can be rewritten as

$$ds^2 = -\tilde{A}(r)dt^2 + \tilde{B}^{-1}(r)dr^2 + \tilde{C}(r)d\theta^2 + \tilde{D}(r)d\phi^2 + \tilde{E}(r)d\zeta^2. \quad (3.2)$$

where

\begin{align*}
\tilde{A}(r) &= f(H_1H_2H_3)^{-\frac{2}{3}} \\
\tilde{B}^{-1}(r) &= f^{-1}(H_1H_2H_3)^{\frac{1}{3}} \\
\tilde{C}(r) &= r^2(H_1H_2H_3)^{\frac{1}{3}} \\
\tilde{D}(r) &= r^2 \sin^2 \theta (H_1H_2H_3)^{\frac{1}{3}} \\
\tilde{E}(r) &= r^2 \sin^2 \theta \sin^2 \phi (H_1H_2H_3)^{\frac{1}{3}}
\end{align*}

The horizons of metric (3.2) can be determined when $f(r) = 0$. For this purpose we follow [40] and assume that $g^2 = 1$ (by the choice of units as in [40]). Hence, in this case the outer horizon is located at

$$r_+ = \sqrt{\frac{(1+q_i)^2 + 4\mu - (1+q_i)}{2}},$$

for $(1+q_i)^2 + 4\mu > (1+q_i)$ and $i = 1, 2, 3$.

In Proca Eq.(2.6) the components of $\psi^\nu$ and $\psi^{\mu\nu}$ are given by

\begin{align*}
\psi^0 &= -\tilde{A}^{-1}\psi_0, \quad \psi^1 = \tilde{B}\psi_1, \quad \psi^2 = \tilde{C}^{-1}\psi_2, \quad \psi^3 = \tilde{D}^{-1}\psi_3, \quad \psi^4 = \tilde{E}^{-1}\psi_4, \\
\psi^{01} &= -\tilde{B}\tilde{A}^{-1}\psi_{01}, \quad \psi^{02} = -(\tilde{A}\tilde{C})^{-1}\psi_{02}, \quad \psi^{03} = -(\tilde{A}\tilde{D})^{-1}\psi_{03}, \\
\psi^{04} &= -(\tilde{A}\tilde{E})^{-1}\psi_{04}, \quad \psi^{12} = \tilde{B}\tilde{C}^{-1}\psi_{12}, \quad \psi^{13} = \tilde{B}\tilde{D}^{-1}\psi_{13}, \quad \psi^{14} = \tilde{B}\tilde{E}^{-1}\psi_{14}, \\
\psi^{23} &= (\tilde{C}\tilde{D})^{-1}\psi_{23}, \quad \psi^{24} = (\tilde{C}\tilde{E})^{-1}\psi_{24}, \quad \psi^{34} = (\tilde{D}\tilde{E})^{-1}\psi_{34}.
\end{align*}

By using Eq.(2.6), we obtain the following set of equations (for simplicity, we assume $A_0 \equiv A_0^i$ for all $i$)

\begin{align*}
&\tilde{B}[c_0(\partial_1S_0)^2 - c_1(\partial_0S_0)(\partial_1S_0) - c_2A_0c_1(\partial_2S_0)] + \tilde{C}^{-1}[c_0(\partial_2S_0)^2] \\
&- c_2(\partial_0S_0)c_0(\partial_2S_0) - c_2A_0c_2(\partial_2S_0) + \tilde{D}^{-1}[c_0(\partial_3S_0)^2 - c_3(\partial_3S_0)(\partial_0S_0)] \\
&- c_3A_0c_3(\partial_3S_0) + \tilde{E}^{-1}[c_0(\partial_4S_0)^2 - c_4(\partial_4S_0)(\partial_0S_0) - c_4A_0c_4(\partial_4S_0)] \\
&+ m^2c_0 = 0, \quad (3.3)
\end{align*}
\[
\tilde{A}^{-1}[c_0(\partial_4 S_0) - c_1(\partial_0 S_0)^2 - eA_0c_1(\partial_0 S_0)] + \tilde{C}^{-1}[c_1(\partial_2 S_0)^2 \\
-c_2(\partial_1 S_0)(\partial_2 S_0)] + \tilde{D}^{-1}[c_3(\partial_0 S_0)(\partial_3 S_0) - c_2(\partial_3 S_0)^2] + \tilde{E}^{-1}[c_4(\partial_2 S_0)(\partial_4 S_0) \\
-c_2(\partial_4 S_0)^2] + eA_0\tilde{A}^{-1}[c_2(\partial_0 S_0) - c_0(\partial_2 S_0) + eA_0c_2] - m^2c_2 = 0, \\
(3.4)
\]

\[
\tilde{A}^{-1}[c_2(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_2 S_0)] + \tilde{B}[c_2(\partial_1 S_0)^2 \\
-c_1(\partial_1 S_0)(\partial_2 S_0)] + \tilde{D}^{-1}[c_3(\partial_2 S_0)(\partial_3 S_0) - c_2(\partial_3 S_0)^2] + \tilde{E}^{-1}[c_4(\partial_2 S_0)(\partial_4 S_0) \\
-c_2(\partial_4 S_0)^2] + eA_0\tilde{A}^{-1}[c_2(\partial_0 S_0) - c_0(\partial_2 S_0) + eA_0c_2] - m^2c_2 = 0, \\
(3.5)
\]

\[
\tilde{A}^{-1}[c_3(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_3 S_0)] + eA_0c_3(\partial_0 S_0)] - \tilde{B}[c_3(\partial_1 S_0)^2 \\
-c_1(\partial_1 S_0)(\partial_3 S_0)] - \tilde{C}^{-1}[c_3(\partial_2 S_0)^2 - c_2(\partial_2 S_0)(\partial_3 S_0)] \\
+ \tilde{E}^{-1}[c_4(\partial_3 S_0) - c_3(\partial_4 S_0)^2] + eA_0\tilde{A}^{-1}[c_3(\partial_0 S_0) \\
-c_0(\partial_3 S_0) + eA_0c_3] - m^2c_3 = 0, \\
(3.6)
\]

\[
\tilde{A}^{-1}[c_4(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_4 S_0) + eA_0c_4(\partial_0 S_0)] - \tilde{B}[c_4(\partial_1 S_0)^2 \\
-c_1(\partial_1 S_0)(\partial_4 S_0)] - \tilde{C}^{-1}[c_4(\partial_2 S_0)^2 - c_2(\partial_2 S_0)(\partial_4 S_0)] \\
- \tilde{D}^{-1}[c_4(\partial_3 S_0) - c_3(\partial_4 S_0)] + eA_0\tilde{A}^{-1}[c_4(\partial_0 S_0) \\
-c_0(\partial_4 S_0) + eA_0c_4] - m^2c_4 = 0. \\
(3.7)
\]

We carry out the separation of variables as
\[
S_0 = -(E - j\tilde{\Omega}_1)t + W(r) + \Theta(\zeta, \vartheta) + N\phi, \\
(3.8)
\]

where $\tilde{\Omega}_1$ is the angular velocity for BH given by Eq. (3.2).

For the above $S_0$ the preceding set of Eqs. (3.3)-(3.7) can be written in terms of matrix equation $\Lambda(c_0, c_1, c_2, c_3, c_4)^T = 0$, the elements of the required matrix have the following form

\[
\Lambda_{00} = \tilde{B}W^2 + \tilde{C}^{-1}(\partial_2 \Theta)^2 + \tilde{D}^{-1}(\partial_3 \Theta)^2 + \tilde{E}^{-1}N + m^2 \\
\Lambda_{01} = \tilde{B}[E - j\tilde{\Omega}_1]W - eA_0\tilde{W}, \\
\Lambda_{02} = \tilde{C}^{-1}(E - j\tilde{\Omega}_1)(\partial_2 \Theta) \\
\Lambda_{03} = \tilde{D}^{-1}(E - j\tilde{\Omega}_1)(\partial_3 \Theta) - \tilde{D}^{-1}eA_0(\partial_3 \Theta), \\
\Lambda_{04} = \tilde{E}^{-1}(E - j\tilde{\Omega}_1)N - \tilde{E}^{-1}jeA_0, \\
\Lambda_{10} = -\tilde{A}^{-1}(E - j\tilde{\Omega}_1)\tilde{W} + eA_0\tilde{A}^{-1}\tilde{W}, \\
\Lambda_{11} = -\tilde{A}^{-1}(E - j\tilde{\Omega}_1)^2 + eA_0(E - j\tilde{\Omega}_1)\tilde{A}^{-1} + \tilde{C}^{-1}(\partial_2 \Theta)^2 \\
+ \tilde{D}^{-1}(\partial_3 \Theta)^2 + \tilde{E}^{-1}N^2 + eA_0\tilde{A}^{-1}(E - j\tilde{\Omega}_1) + m^2, \\
\Lambda_{12} = -\tilde{C}^{-1}\tilde{W}(\partial_2 \Theta), \\
\Lambda_{13} = -\tilde{D}^{-1}\tilde{W}(\partial_3 \Theta), \\
\Lambda_{14} = -\tilde{E}^{-1}WN, \\
\Lambda_{20} = \tilde{A}^{-1}(E - j\tilde{\Omega}_1)(\partial_2 \Theta) - \tilde{A}^{-1}eA_0(\partial_2 \Theta), \\
\Lambda_{21} = \tilde{B}\tilde{W}(\partial_2 \Theta),
\]
\lambda_{22} &= \tilde{\Lambda}^{-1}(E - j\tilde{\Omega}_1)^2 - eA_0(E - j\tilde{\Omega}_1)\tilde{\Lambda}^{-1} - \tilde{B}W^2 \\
&\quad - \tilde{D}^{-1}(\partial_3\Theta)^2 - \tilde{E}^{-1}N^2 + eA_0\tilde{A}^{-1}[eA_0 - (E - j\tilde{\Omega}_1)] - m^2, \\
\lambda_{23} &= \tilde{D}^{-1}(\partial_2\Theta)(\partial_3\Theta), \quad \lambda_{24} = \tilde{E}^{-1}(\partial_2\Theta)N, \\
\lambda_{30} &= \tilde{A}^{-1}(E - j\tilde{\Omega}_1)(\partial_3\Theta) - eA_0\tilde{A}^{-1}(\partial_3\Theta), \quad \lambda_{31} = \tilde{B}W(\partial_3\Theta), \\
\lambda_{32} &= \tilde{C}^{-1}(\partial_2\Theta)(\partial_3\Theta), \\
\lambda_{33} &= \tilde{A}^{-1}(E - j\tilde{\Omega}_1)^2 - eA_0\tilde{A}^{-1}(E - j\tilde{\Omega}_1) - \tilde{B}W^2 - \tilde{E}^{-1}N^2 \\
&\quad - \tilde{C}^{-1}(\partial_2\Theta)^2 - m^2 - eA_0\tilde{A}^{-1}[(E - j\tilde{\Omega}_1) - eA_0], \\
\lambda_{34} &= \tilde{E}^{-1}j(\partial_3\Theta), \quad \lambda_{40} = \tilde{A}^{-1}((E - j\tilde{\Omega}_1)N - eA_0\tilde{A}^{-1}j), \quad \lambda_{41} = \tilde{B}W N, \\
\lambda_{42} &= \tilde{C}^{-1}(\partial_2\Theta)N, \quad \lambda_{43} = \tilde{D}^{-1}(\partial_3\Theta)N, \\
\lambda_{44} &= \tilde{A}^{-1}(E - j\tilde{\Omega}_1)^2 - eA_0\tilde{A}^{-1}(E - j\tilde{\Omega}_1) - \tilde{B}W^2 - \tilde{C}^{-1}(\partial_2\Theta)^2 \\
&\quad - \tilde{D}^{-1}(\partial_3\Theta)^2 - m^2 - eA_0\tilde{A}^{-1}[(E - j\tilde{\Omega}_1) - eA_0].

For the non-trivial solution, the determinant \( \Lambda \) is equal to zero and using the same technique as discussed in the previous section, we get

\[ ImW^\pm = \pm \int \frac{(E - eA_0 - j\tilde{\Omega}_1)^2 + \tilde{X}}{AB} = \pm \frac{\epsilon \pi (E - eA_0 - j\tilde{\Omega}_1)}{2\kappa(r_+)} \] (3.9)

where

\[ \tilde{X} = -\tilde{A}\tilde{C}^{-1}(\partial_2\Theta)^2 - \tilde{A}\tilde{D}^{-1}(\partial_3\Theta)^2 - \tilde{A}m^2 - \tilde{E}^{-1}(\partial_3\Theta)N. \] (3.10)

Since BH given by Eq. (3.2) is non-rotating, so \( \tilde{\Omega}_1 = 0 \). The surface gravity for this BH is given by \[40\]

\[ \kappa(r_+) = \frac{2r_+^6 + r_+^4(1 + \sum_{i=1}^3 q_i) - \prod_{i=1}^3 q_i}{r_+^2\sqrt{\prod_{i=1}^3(r_+^2 + q_i)}}. \] (3.11)

The required tunneling probability as discussed in the previous section is

\[ \tilde{\Gamma} = \frac{\tilde{\Gamma}_{\text{emission}}}{\tilde{\Gamma}_{\text{absorption}}} = e^{-4ImW^+} = e^{-\frac{2\pi (E - eA_0)\sqrt{\prod_{i=1}^3(r_+^2 + q_i)}}{2r_+^6 + r_+^4(1 + \sum_{i=1}^3 q_i) - \prod_{i=1}^3 q_i}}. \]

The Hawking temperature in this case is given by

\[ \tilde{T}_H = \frac{[2r_+^6 + r_+^4(1 + \sum_{i=1}^3 q_i) - \prod_{i=1}^3 q_i]}{2\pi r_+^2\sqrt{\prod_{i=1}^3(r_+^2 + q_i)}}. \] (3.12)
The Hawking temperature is related to energy $E$, potential $A_0$, angular momentum $j$, the radial coordinate at the outer horizon $r_+$ and charge $q_i$. We would like to mention that the Hawking temperature of charged vector particles given by Eq.(3.12) is same as the Hawking temperature of 5D gauged super-gravity BH in Eq.(9) in Ref.[40].

4 Outlook

During the tunneling process when a particle with electropositive energy crosses the horizon, it seems as Hawking radiation. Likewise, a particle with electronegative energy burrows inweave, it is assimilated by the BH, so its mass falls and at last disappears. Thus, the movement of the particles may be in the configuration of outgoing and incoming, the carry out particle’s action turns out complex and real, respectively. The emission rate of a tunneling particles from the BH is associated with the imaginary component of the particles action, which in fact is related to the Boltzmann factor based on the Hawking temperature.

In this paper, we have extended the work of vector particles tunneling for more generalized BHs in $4D$ and $5D$ spaces and recovered their corresponding Hawking temperatures at which particles tunnel through horizons. For this purpose, we have used Proca equation with the background of electromagnetism to investigate the tunneling of charged vector particles from accelerating and rotating BHs $4D$ and $5D$ BHs having electric and magnetic charges with a NUT parameter. We have implemented the WKB approximation to Proca equation, which leads to the set of field equations, then use separation of variables to solve these equations. Solving for the radial part by using the determinant of coefficient matrix equal to zero. Using surface gravity, we have formulated the tunneling probability and Hawking temperature for both BHs at the outer horizon. All these quantities depend on the defining parameters of the BHs. It is worth while to mention here that the back-reaction effects of the emitted particle on the BH geometry and self-gravitating effects have been neglected, the derived Hawking temperature only is a leading term. So that one does not need to calculate the appropriate solution of the semi-classical Einstein field equations for the geometry of background BH in equilibrium with its Hawking radiation [42].

From our analysis we have concluded that, Hawking temperature at which particles tunnel through the horizon is independent of the species of parti-
cles. In particular nature of background BHs geometries, for the particles having different spins (either spin-up or down) or zero spin, the tunneling probabilities will be same by considering semi-classical effects. Thus, their corresponding Hawking temperatures must be same for all kinds of particles. For both cases, we have carried out the calculations for more general BHs, i.e., a pair of charged accelerating and rotating BHs with NUT parameter (which is more general BHs as compared to BH taken in [43]) and a BH in 5D gauged super-gravity. Our findings are similar with the statement i.e., temperature of tunneling particles is independent of species of the particles, this result is also valid for different coordinate frames by using specific coordinate transformations. The authors of Ref.[43] have been proved for Kerr BH (only rotating), while we have proved for more generalized BHs. Hence, the conclusion still holds if background BHs geometries are more generalized.

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