In this work we propose a hybrid scheme to implement a photonic controlled-z (CZ) gate using photon storage in highly-excited Rydberg states, which controls the effective photon-photon interaction using resonant microwave fields. Our scheme decouples the light propagation from the interaction and exploits the spatial properties of the dipole blockade phenomenon to realize a CZ gate with minimal loss and mode distortion. Excluding the coupling efficiency, fidelities exceeding 95% are achievable and are found to be mainly limited by motional dephasing and the finite lifetime of the Rydberg levels.

Although optical photons are ideal for quantum communication, their utility for computation is limited by the lack of strong photon-photon interactions [1,2]. However, recently there has been a substantial progress in this area using Rydberg ensembles [3-5], where the strong interactions between highly-excited Rydberg atoms are mapped into strong interactions between individual optical photons [6-10]. In addition, quantum gate protocols based on Rydberg atoms have been proposed [11] and realised [12,13] where the information was encoded in the ground state of the atoms instead of photons. The idea of exploiting the large dipole-dipole interactions between Rydberg atoms for photon processing has been analysed theoretically for a variety of scenarios [14-17]. In general the interaction is dissipative, however dissipation can be reduced at the cost of interaction strength by detuning off-resonance [18,19]. An additional problem is the implicit link between the interaction and propagation, which inevitably leads to a distortion of the photon wave packet and thereby precludes the realisation of high fidelity gates, which is one of the requirements for quantum information processing. In fact it has been argued on fundamental grounds [20,21] that this problem occurs whenever conventional optical non-linearities (such as cross-phase modulation) are used, and cannot be circumvented.

In this paper, we present a photon gate scheme that decouples light propagation and interaction, allowing the realization of high-fidelity photon-photon gates with negligible loss or distortion. We use the dark-state polariton protocol [22,23] to convert two photonic qubits (control and target) in the dual rail encoding into collective excitations with Rydberg character in different positions or sites in an ensemble of cold atoms. We subsequently perform a $2\pi$-rotation on the target qubit using a microwave field coupled to an auxiliary state, which by default gives an overall phase shift of $\pi$-radians to the qubit pair (a Z phase gate). However, the microwave field also induces resonant dipole-dipole interactions [8] between the target and the control sites that are closest together, preventing the rotation for one of the four qubit-pair states, and thereby implementing a CZ phase gate. The excitations are then converted back to photons and emitted by the ensemble in the phase-matched direction.

Our scheme relies on the ability to modify the range of the dipole-dipole interactions between highly-excited Rydberg atoms using a resonant microwave field [8,24,25]. By using this field to couple to an auxiliary Rydberg state, we exploit the spatial independence of the dipole blockade mechanism [18] to induce a homogeneous phase shift on the stored photon, and thereby circumvent the local-field limitation of the optical Kerr effect [20,21].

This Letter is organised as follows: first we outline the storage procedure, and then we show how, for two photons stored in adjacent sites in an atomic cloud, resonant dipole-dipole interactions can be used to obtain a $\pi$ phase shift to the desired qubit state. Afterwards, we show that off-resonant, van der Waals interaction between Rydberg levels in adjacent sites disrupts the ideal process of the gate, but that its short-range effect can be overcome thanks to the long range scaling of the resonant interactions. Finally, we give an estimate of the gate fidelity for the example of ultra-cold $^{87}\text{Rb}$ atoms, where we consider the effects of finite coupling strengths, extended spatial samples and finite temperature.

The photonic qubit is defined using the dual-rail encoding [1], where the two states of the computational basis in each qubit (|0⟩ and |1⟩) travel through two spatially separated regions of an atomic cloud. For a two-qubit gate we consider four separate spatial paths (see FIG.1). Similar geometries with only two sites have already been implemented [12,13]. We label the four channels as the elements of the set £B = \{ |1C⟩, |0C⟩, |1T⟩, |0T⟩\}, where the subscript represent the (C)ontrol and (T)arget qubits. We arrange the paths for the |1⟩ (interacting) components to be adjacent while the |0⟩ paths are farther apart. We store the different photonic components in the medium as collective excitations (also called dark-state polaritons) with Rydberg character using electromagnetically induced transparency (EIT) in a ladder configuration [4,7,10]. To this end, the signal light is resonant with the closed atomic transition $|g⟩ \leftrightarrow |e⟩$, and classical coupling lasers resonant with the transitions $|e⟩ \leftrightarrow |r⟩$ or $|e⟩ \leftrightarrow |r'⟩$ are employed to store the control and target

All-optical quantum information processing using Rydberg gates

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Microwaves \( \Omega_\mu \)

Addressable regions

\( |0_T\rangle \)
\( |1_T\rangle \)
\( |1_C\rangle \)
\( |0_C\rangle \)

\( R_b^{(6)}(r'r) \)
\( R_b^{(3)}(r'p) \)

FIG. 1. Optical layout: Control (|C\rangle) and Target (|T\rangle) photonic qubits, in dual-rail encoding, are stored as Rydberg polaritons (dark red) in a cold atomic ensemble (yellow). The spatial modes corresponding to the qubit states |1_C\rangle and |1_T\rangle are stored in adjacent sites at a distance \( d_{11} \), and the others arbitrarily further apart. After storage we attempt a 2\( \pi \) rotation on the target qubit using the microwave field with Rabi frequency \( \Omega_\mu \) and an intermediate state. This succeeds except for |1_C-1_T\rangle, in which the intermediate state is shifted by resonant dipole-dipole interactions, which have a characteristic lengthscale \( R_b^{(3)} \). We need to ensure that the van der Waals interactions between stored states (characterized by the blockade radius \( R_b^{(3)} \)) are small.

Photons in two different Rydberg states |\text{r}\rangle \) and |\text{r}'\rangle \) (see FIG. 2). We assume that these states are of the form |\text{r}\rangle = |nS\rangle \) and |\text{r}'\rangle = |n'S\rangle \), where \( n, n' \) are the principal quantum numbers and \( S \) denotes the \( L = 0 \) angular momentum state. Using different Rydberg states for target and control qubits allows us to perform operations on the individual qubits using a global microwave field.

The gate works with two photonic qubits, so there is at most one excitation in each of the sites. The excitation is shared amongst all the atoms in that site, which maps the state into the superposition |\text{S}_j\rangle = \frac{1}{\sqrt{N_j}} \sum_k |\text{r}'_k\rangle e^{i\phi_k} \), where at each site \( j \in \mathbb{B} \) with \( N_j \) atoms, the sum spans all possible singly-excited states |\text{r}'_k\rangle \) to the Rydberg level |\text{r}'\rangle \): |\text{r}\rangle \) in the target qubit, |\text{r}'\rangle \) in the control. The phase \( \phi_k \) depends on the probe and coupling fields at the position of atom \( k \). This process maps the photonic state |\text{CT}\rangle = |C\rangle \otimes |T\rangle \) into a spin-wave state involving all of the four spatial channels |\text{S}_{CT}\rangle = |\text{S}_C\rangle \otimes |\text{S}_T\rangle \), and can be achieved with efficiencies per site exceeding 90% given a sufficiently high optical depth [17].

Once we have a mapping of the two-qubit state into the cloud, we make use of an auxiliary state |\text{p}\rangle \) in the target qubit to perform the gate operation. A microwave pulse is applied to the system to attempt a \( \int_0^t \Omega_\mu dt = 2\pi \) rotation on the transition |\text{r}\rangle \leftrightarrow |\text{p}\rangle \) transition. Since there is only one excitation at each site, each ensemble behaves like an effective spin system, coupling the target states |\text{S}_T\rangle \) and the superposition of singly-excited |\text{p}_k\rangle \) states, |\text{P}_T\rangle = \frac{1}{\sqrt{N_T}} \sum_k |\text{p}_k\rangle e^{i\phi_k} \), with the single-atom Rabi frequency \( \Omega_\mu \). Since the wavelength of the microwave field is much greater than the separation between sites, the coupling to both target sites is the same.

In the absence of other interactions, performing the 2\( \pi \)-rotation adds a \( \pi \)-phase shift to the wavefunction, |\text{CT}\rangle \rightarrow -|\text{CT}\rangle \). However, if the target state |\text{p}\rangle \) is coupled to the control Rydberg state |\text{r}'\rangle \) via an electric-dipole interaction at a distance \( d \), dipole-dipole interactions shift the energy of the coupled state |\text{r}'p\rangle \) by \( H_{dd} = \hbar \Delta_{\text{r}'p} |\text{r}'\rangle \langle \text{r}'| + |\text{r}'\rangle \langle \text{r}'| \), which can prevent the rotation, and thus the phase shift, conditional on the presence of a control excitation in a nearby channel. This operation, which implements a CZ gate, occurs with arbitrarily high fidelity if the distance between the adjacent control and target channels, \( d_{11} \), is smaller than the characteristic length,

\[
d_{11} < R_b^{(3)}(r'p) = \sqrt{C_3(r'p)/\hbar \Omega_\mu},
\]

where \( \Omega_\mu \) is the microwave Rabi frequency.

However the discussion outlined above is only valid if there are no other interactions between sites. If we have two excitations at a distance \( d \) in the medium (one for each qubit), off-resonant, van der Waals (vdW) int-
teractions between the Rydberg levels $|r\rangle$ and $|r'\rangle$ are important, and can hinder the process of the gate by introducing spatially-dependent detunings to the interacting modes. These interactions detune the doubly-excited state $|r'r\rangle$ by an amount $H_{vdW} = \Delta r'_{r'} |r'r\rangle = (C_b(r'r)/d^3)|r'r\rangle$ by coupling the states $|r'r\rangle \leftrightarrow |p'p\rangle$, where $|p\rangle$ and $|p'\rangle$ are dipole-coupled to both $|r\rangle$ and $|r'\rangle$. Here, $C_b(r'r) \propto 1/\delta_f$ is the vdW coefficient of an $|n'S, nS\rangle$ pair state, where $\delta_f$ is the Förster energy defect $[20,27]$. If the energy shift $\Delta r'_{r'}$ is comparable to the energy defect $\delta_f$, dipole-dipole interactions populate neighbouring states. Also, if this energy defect is zero (a situation called Förster resonance), we expect excitation hopping between $|r'r\rangle$ and $|p'p\rangle$ to occur spontaneously. Therefore, we need to avoid these situations choosing an appropriate level system.

Even if we have a system without Förster resonances, vdW interactions affect the proper functioning of the gate: during the storage and retrieval processes and during the rotation in the microwave domain.

If we have an excitation $|r'\rangle$ in one site, the interaction shift between sites prevents an excitation to $|r\rangle$ within a certain region characterized by the blockade lengthscale, $R_b^{(6)}(r'r') = (C_b(r'r)/\hbar\Omega)^{1/6}$, where $\Omega$ is the (power broadened) linewidth of the EIT transparency window. We minimize these inter-site interactions by ensuring that the distance $d$ between any two spatial channels satisfies the inequality

$$d > R_b^{(6)}(r'r').$$  \hspace{1cm} (2)

This condition ensures that the interaction during the storage and retrieval stages of the gate is negligible, thus avoiding distortion of the spatial modes of the qubits.

Also, the vdW interactions between $|r\rangle$ and $|r'\rangle$ are present even during the microwave rotation, when the coupling laser is off. This space-dependent energy shift would cause a dephasing to the interacting component $|r'\rangle$. Taking $\Gamma$ to perform the rotation in the microwave domain.

If we condition the interaction between $|1_{C}\rangle$ and $|1_{T}\rangle$ by $\Gamma$, and make sure that the effect of vdW interactions are negligible during the storage/retrieval and the microwave rotation, the photonic component $|11\rangle$ picks up a homogeneous $\pi$-phase with respect to $|00\rangle$, $|10\rangle$ and $|01\rangle$. Then, the overall change in the system corresponds to that of a CZ-gate $\Gamma$.

Conditions $\Gamma$, $\Pi$, and $\Delta$ suggest using

$$R_b^{(3)}(r'r) / max(R_b^{(6)}, R_\mu) = \left( C_b \min(\Omega^2, \Omega_\mu^2) / C^2_3 \Omega_\mu \right)^{1/6} R_b^{(3,6)}$$

as the figure of merit, but in reality, both $R_b^{(3,6)}$ and $R_\mu$ are bounded by the shortest lifetime $\tau = 1/\Gamma$ involved in the system. Therefore, we can optimize the CZ gate operation by choosing a system that maximizes this dimensionless figure of merit,

$$O = \frac{C_b(r'r)^2}{C_b(r'r)\hbar\Gamma^2}.$$  \hspace{1cm} (4)

Note that this figure of merit does not depend on any experimental parameters, and just depends on physical properties of the atomic species used and the level system chosen. Both $R_b^{(3,6)}$ and $O$ for $^{87}$Rb are shown as a function of principal quantum number in FIG. 3.

To better understand the possible implementation of the phase gate including real-world sources of decoherence, we estimate the fidelity using a simplified optical Bloch-equation approach. Our aim is not to provide a full many-body simulation of the gate protocol, but rather to estimate the errors in the case of a physical realisation using a cloud of cold $^{87}$Rb atoms. We shall note that we do not fully simulate storage and retrieval processes; instead, we use a one-photon transition to the Rydberg states to this effect (more details can be found in the Supplemental Information).

We choose $|r'\rangle = |nS_{1/2}\rangle$, $|r\rangle = |(n+1)S_{1/2}\rangle$ and $|p\rangle = |nP_{1/2}\rangle$ to maximize the ratio $R_b^{(3)}(r'r)/R_b^{(6)}(r'r)$ and avoid Förster resonances in the region of interest. For example, for $n = 70$, we obtain $R_b^{(6)} \sim 7 \mu m$ and $R_b^{(3)} \sim 20 \mu m$ by coupling to the $M = 0$ state, i.e., the characteristic length of the resonant microwave transition is around 3 times larger than the optical blockade.

In FIG. 4 the results of this exploration are shown,

FIG. 3. (Left) The characteristic lengthscales $R_b$ as a function of principal quantum number, $n$, for different pair states in $^{87}$Rb. In solid red, the blockade radii for vdW interactions $R_b^{(6)}(nS_{1/2}, (n+1)S_{1/2})$. In yellow, standard vdW blockade radii for same-level pair state with coefficient $C_6(nS_{1/2}, nS_{1/2})$ are shown for reference. In dashed blue, the long-range resonant interactions lengthscale $R_b^{(3)}(nS_{1/2}, nP_{1/2})$ for the $M = 0$ pair state. All radii are calculated for a coupling of 1 MHz. Note the Förster resonance for the coupling $38S_{3/2} \leftrightarrow 38P_{3/2}$. (Right) The figure of merit $O$ [see (4)] for $|nS\rangle$, $|(n+1)S\rangle$ and $|nP\rangle$ as a function of the principal quantum number $n$. 

![Graph showing the characteristic lengthscales R_b as a function of principal quantum number, n, for different pair states in 87Rb. In solid red, the blockade radii for vdW interactions R_b^{(6)}(nS_{1/2}, (n+1)S_{1/2}). In yellow, standard vdW blockade radii for same-level pair state with coefficient C_6(nS_{1/2}, nS_{1/2}) are shown for reference. In dashed blue, the long-range resonant interactions lengthscale R_b^{(3)}(nS_{1/2}, nP_{1/2}) for the M = 0 pair state. All radii are calculated for a coupling of 1 MHz. Note the Förster resonance for the coupling 38S_{3/2} \leftrightarrow 38P_{3/2}. (Right) The figure of merit O [see (4)] for |nS\rangle, |(n+1)S\rangle and |nP\rangle as a function of the principal quantum number n.](image-url)
where we have calculated the fidelity $F_0$ for the initial state $|\psi\rangle = (|0\rangle + |1\rangle + |10\rangle + |11\rangle)/2$ in the double-qubit basis to become $|\psi'\rangle = (|0\rangle + |01\rangle + |10\rangle - |11\rangle)/2$ after the action of the gate. We initially obtain $F_0$ as a function of distance, keeping the rest of the parameters constant. To account for the finite width of the sites, we convolve $F_0$ with a Gaussian of width $w = \sqrt{2q}/R_{\text{h}}^{(6)}(r')$, where $q = w_0/R_{\text{h}}^{(6)}(r')$ is the ratio between the probe waist $w_0$ at each site and $R_{\text{h}}^{(6)}(r')$. The $\sqrt{2}$ factor states that the interaction takes place between two sites. Finally, since the stored excitations are spin-waves, and these can suffer from motional dephasing, we multiply the fidelity by a motional dephasing coefficient $\eta_m \propto \exp[-(t^2/\tau^2)/(1 + t^2/\xi^2)]$ (taken from [29]), where atoms at a temperature $T$ and average speed $v = \sqrt{k_B T/m}$ ($m$ is the atomic mass) can exit the site with mode diameter $w_0$ in a time $\tau = w_0/v$, or can move across the stored spin-wave with wavelength $\Lambda$ in a time $\tau = \Lambda/(2\pi v)$. With these factors taken into account, we obtain processing fidelities over 95% over a broad range of experimental parameters (see FIG. 4).

Inspecting FIG. 4 we note the following general remarks: ensuring a strong coupling $\Omega_\mu$ is key, as it allows the two interacting sites to be stored close together, and profit from a higher resonant dipole shift. Increasing the principal quantum number increases the fidelity, although we expect a weak scaling with $n$, as seen inspecting $O$ in FIG. 4. This happens because we can drive transitions in the microwave domain with a weaker $\Omega_\mu$ due to the favourable scaling of the lifetimes. However, this moves the peak of the fidelity towards lower driving frequencies making the gate operation slower, which puts this parameter in competition with motional dephasing. Finally, the smaller the waist of the sites, the higher the fidelity, but this is limited by the diffraction limit; also, when the sites are very small, achieving a high OD is challenging, and motional dephasing becomes a problem.

In addition to the limitations outlined above, a significant source of inefficiency is likely to arise from the mapping between the light field and the stored polaritons [17]. However, by making the cloud sufficiently dense ($N \sim 10^{14}$ cm$^{-3}$), it is possible to obtain optical depths OD $\sim 1000$ that would give an efficiency per-channel of $\eta_c \approx 0.9$ and an overall $\eta^2_c \approx 81\%$, although denser samples might show more dephasing. This coupling efficiency can be further increased by using photonic waveguides or by optimizing the temporal shape of the probe pulse [17]. These numbers compares favourably with previous implementations using linear optics [31–33], which have a 1/9 efficiency before post-selection, and experimentally achieves $\eta^2 \sim 85\%$ after post-selection. The process of storage and retrieval of polaritons with Rydberg content is still not fully understood, and further optimisations may be possible.

One can imagine using this scheme in combination with integrated chip atom trapping and waveguides [34] to join several quantum gates, both sequentially and in parallel. Using existing waveguide technology, one could also implement single qubit operations [35, 36] in the same chip, which brings us closer towards a fully integrated quantum processor. Also, the proposed geometry and processing method could be extended to implement a photon switch and other operations.

In conclusion, we have shown that it is feasible to realize a quasi deterministic, high-fidelity universal quantum gate for photons. We circumvent the restrictions of conventional optical non-linearities by using the non-local dipole blockade effect and by separating the propagation and interaction phases of the gate. We exploit microwave fields to switch between short range van der Waals interactions and longer range resonant dipole-dipole interactions, which allows us to achieve a conditional phase shift on the stored target.
photon. Fidelities in excess of 95% are predicted for currently achievable experimental conditions, and overall efficiencies exceeding 75% are theoretically possible. Deterministic photon processing will facilitate a wide range of efficient quantum information protocols.

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