Optomechanical reference accelerometer

O Gerberding¹,², F Guzmán Cervantes¹,², J Melcher¹, J R Pratt¹ and J M Taylor¹,²,³

¹ National Institute of Standards and Technology, Gaithersburg, MD 20899, USA
² Joint Quantum Institute, University of Maryland, College Park, MD 20742, USA
³ Joint Center for Quantum Information and Computer Science, University of Maryland, College Park, MD 20742, USA

E-mail: felipe.guzman@nist.gov and jmtaylor@umd.edu

Received 3 April 2015, revised 3 July 2015
Accepted for publication 13 July 2015
Published 8 September 2015

Abstract
We present an optomechanical accelerometer with high dynamic range, high bandwidth and readout noise levels below $7.8 \times 10^{-5} \text{ m s}^{-2}/\sqrt{\text{Hz}}$. The straightforward assembly and low cost of our device make it a prime candidate for on-site reference calibrations and autonomous navigation. We present experimental data taken with a vacuum sealed, portable prototype and deduce the achieved bias stability and the accuracy of the sensitivity. Additionally, we present a comprehensive model of the device physics that we use to analyze the fundamental noise sources and accuracy limitations of such devices.

Keywords: accelerometer, optomechanics, interferometry, inertial sensing

(Some figures may appear in colour only in the online journal)

1. Introduction

Accelerometers and gyroscopes form the fundamental building blocks of inertial sensing [1, 2]. Devices with a wide range of bandwidth, precision, accuracy and dynamic range are available and deployed in various applications, including commercial products, medical devices [3], construction engineering [4], natural resource exploration [5], inertial sensing for autonomous navigation [6] and fundamental research [7, 8]. The critical parameters of an accelerometer with a given bandwidth are its precision, often denoted as acceleration noise $\overline{\text{a}}_{\text{acc}}$; its bias stability, the long-term drift of the dc acceleration $a_{\text{dc}}$; and the uncertainty of the magnitude of sensitivity $S$, which describes errors in the value relating physical observable quantities, such as the output voltage $v_{\text{acc}}$ to acceleration ($v_{\text{acc}} = S \cdot a_{\text{ext}}$, $(S) = 1 \text{ V (m s}^{-2})^{-1}$).

Commercially available accelerometers require laborious calibrations to ensure a minimal deviation of the above described parameters. Such primary accelerometer calibrations are often performed at National Metrology Institutes (NMIs) [11], where the test unit is mounted onto an interferometrically-interrogated reference shaker system [9, 12–14].

Such calibrations reach relative uncertainties of the order of $10^{-2}–10^{-3}$. Already calibrated devices, known as reference accelerometers, can, in turn, be used to calibrate further devices using more simple back-to-back measurement setups. However, such secondary calibrations are accompanied by an inevitable degradation in uncertainty.

Optomechanical accelerometers interrogated using fiber interferometric methods have recently demonstrated high levels of readout precision [15–17], using optical instead of electro-static readout. The devices investigated by Guzman et al [17] use monolithic fused silica in-plane oscillators interrogated by fiber optic micro-cavities [18, 19]. These devices have achieved levels of acceleration measurement sensitivities below $9.81 \times 10^{-7} \text{ m s}^{-2}/\sqrt{\text{Hz}}$ and, more importantly in this context, they provide direct traceability to SI units, a property we denote as ‘self-calibrating’. This is achieved in a two-step process: first, external accelerations of the oscillator are converted to displacement via its transfer function, characterized by its resonance frequency $\omega_0$ and mechanical quality factor $Q$. Second, displacement is turned into a readout voltage via laser interferometry. The crucial features that enable self-calibration in these devices are the parallelogram flexure design, creating a linear, one-dimensional motion of the mechanical oscillator, and the cavity characterization via laser tuning, allowing us to calibrate the displacement readout.
Such devices have also recently been used as optomechanical force sensors for atomic force microscopy (AFM) [20].

In this article, we present a similar device to [17] that is interrogated by a low-finesse cavity formed by a gap of approx. 50 μm between two flat-cleaved fibers [18]. Its simple and cost-effective construction, combined with self-calibration, high dynamic range, and high bandwidth (exceeding 10kHz) make this device a promising candidate to perform autonomous navigation and on-site acceleration calibrations of other accelerometers. We demonstrate the portability and accuracy of such a device by vacuum packaging it and by characterizing its readout noise and bias stability. Without additional feedback, the low-finesse readout allows our device to have a magnitude of sensitivity deviation, due to the non-linear feedback, lower than $10^{-3}$ for accelerations below $1.962 \times 10^2$ m s$^{-2}$. In comparison with the dynamic range of the high-finesse device (about $1.962 \times 10^{-2}$ m s$^{-2}$ for the same magnitude of sensitivity deviation) this is an improvement by three orders of magnitude. Section 2 describes our prototype, the experimental set-up and investigations, as well as the methodology behind the self-calibration. Section 3 presents a summary of our current understanding of the device physics, optics, mechanics and readout. Section 4 discusses the expected limits for acceleration readout noise, accuracy and self-calibration and, in addition, we extrapolate this analysis to a high-finesse cavity readout, paving the path for substantial improvements.

2. Accelerometer prototype

2.1. Accelerometer prototype design

Our accelerometer device (shown in figure 1) uses a similar in-plane, monolithic fused-silica oscillator design described by Guzman et al [17]. It is frit-bonded onto an additional micromachined fused-silica part that contains a relief for enabling free test mass motion. This part is, in turn, frit-bonded onto a 4 cm radius quartz plate with a thickness of 3 mm. The accelerometer is enclosed by a glass bell, which is glued with Torr Seal onto the base plate. Two flat-cleaved fibers are fed into the bell at a cut-out and their ends are glued into v-grooves next to the test mass. Flat-cleaved fibers glued onto opposite facing v-grooves on the test mass act as cavity end mirrors. The acceleration-driven test mass motion translates to optical phase changes of the fiber micro-cavity, which is read out in reflection by monitoring the reflected power. The fiber ends acting as cavity mirrors, have a low reflectivity of approximately 4%. The oscillator was designed to have a mass of 25 mg and a resonance frequency of about 10 kHz.

After assembling the fiber cavities, we vacuum sealed our device by pumping on a glass pipe originally connected to the top of the bell and performing a vacuum pulling by flame heating the pipe. This is evident by the residual glass structure on top of the bell shown in figure 1(b). The device is now available as a portable accelerometer.

2.2. Experimental set-up

We connect the device to our measurement set-up shown in figure 2. A laser beam from a widely tunable laser [21] is sent to one of the fiber cavities via a 50/50 coupler and the reflected power is detected on a photo receiver ($v_R$). The light preparation is protected from these reflections with an isolator (ISO). To increase optical power for the performance test an erbium-doped fiber amplifier [22] (not shown) was integrated after the laser source. Part of the laser light is split off to stabilize the laser power by actuating on a fiber-based electro-optic amplitude modulator (EOAM). A fiber-based unequal arm-length Mach–Zehnder interferometer, with 10 m fiber delay between the arms, is used as frequency reference. It is read out via a balanced detection scheme and the error signal is used to stabilize the laser frequency by actuating on the laser pump current and on the laser cavity piezo. The accelerometer is placed on an isolation platform to reduce coupling of unwanted accelerations. A piezo crystal (PZT) mounted on the platform is used to excite the accelerometer. The second, unused output port of the 50/50 beam splitter was additionally terminated by wrapping the fiber with a very small bending radius and thereby introducing excess losses of the light traveling through the fiber core.
2.3. Analysis of the optical cavity response

We use a widely tunable laser [21] to measure the dependency of the reflected light power \( P_R \), and the corresponding photo detector output voltage \( v_R \) at the laser wavelength \( \lambda \). Figure 3 shows the response for one of the two cavities, which was used throughout this study. Each measurement consists of one scan with decreasing and a subsequent one with increasing wavelength. Using this method we can suppress effects due to hysteresis or delays that are not accounted for in our data acquisition.

We first fit our data using the following sinusoidal model approximation of the fiber cavity response.

\[
v_{R,s} = v_{dc} - v_{dc} \cdot \kappa \cos \left(4\pi \frac{z_m}{\lambda} \right),
\]

where \( v_{dc} \) is the dc output voltage measured in reflection, \( \kappa \) is the optical contrast, and \( z_m \) is the macroscopic cavity length. By fitting our measurement data with this response we determine \( v_{dc} = 4.34 \text{ V}, \kappa = 0.90 \) and \( z_m \approx 42.3 \mu \text{m} \). By tuning the laser wavelength to a quadrature point of the response (\( \lambda_0, z_m \mod \lambda_0 = 0 \)), we maximize the ratio between voltage and length change to a now calibrated value [23].

\[
\Delta v_R = v_{dc} \cdot \kappa \frac{4\pi}{\lambda_0} \Delta z_m.
\]

We also analyze our measurement data with the Airy function, which takes multiple reflections into account [19]:

\[
v_{R,a} = v_{offset} + v_r \left(1 + R^2\right) \frac{1 - \cos \left(4\pi \frac{z_m}{\lambda} \right)}{1 + R^2 - 2R \cos \left(4\pi \frac{z_m}{\lambda} \right)}.
\]

Here we define an offset voltage \( v_{offset} \), an amplitude scaling voltage \( v_r \) that includes the optical contrast and the reflectivity

2.4. Analysis of mechanical oscillator response

To estimate the oscillator mechanical characteristic parameters we perform a ring-down experiment with our device. We excite it close to the resonance frequency, turn off the excitation, and then monitor the oscillation decay. A measurement for our device is shown in figure 4. We fit our data using the formula

\[
v_{id} = v_0 \cdot e^{\frac{\omega_d}{2\Omega} \sin(\omega_d t + \phi)}.
\]

From this fit we estimate the natural frequency of the oscillator \( \omega_0 \approx 2\pi \times 106.4656 \text{ Hz} \) with a relative fit standard error on the order of \( 10^{-8} \). We then perform a software demodulation.
of the signal at $\omega_0$ to extract only the exponential decay of the oscillation amplitude $\tilde{n}_d$, which is governed by the following equation.

$$\tilde{n}_d = \tilde{n}_0 \cdot e^{-\frac{\omega_0 f}{Q}}.$$  \hfill (7)

By fitting this formula to the decay we determine the quality factor $Q \approx 123 \; 79.07$ with a relative standard uncertainty of about $1.5 \times 10^{-3}$. The resulting fit curve is shown in figure 4, as the envelope of the ring down. A correction of $\omega_0$ due to a damping-induced frequency shift is not necessary, since the correction factor is fully negligible at these levels of $Q$. To further refine and test the long term stability of $\omega_0$ and $Q$ we perform the above described analysis for multiple, consecutive ring down measurements. For 250 ring down, spaced over 25 min (1500 s) of measurement time, we determine a relative standard deviation for $\omega_0$ on the order of $1 \times 10^{-7}$ and $2 \times 10^{-4}$ for $Q$, each an order of magnitude larger than the single-shot statistical variation. Using the transfer function of a damped harmonic oscillator, we have the relation between external accelerations and displacement:

$$T_{\text{HO}}(\omega) = \frac{\Delta z(\omega)}{a_{\text{ext}}(\omega)} = -\frac{1}{\omega_0^2 - \omega^2 + \frac{i}{Q} \omega}.$$  \hfill (8)

### 2.5. Traceability by self-calibration

We now summarize the self-calibration method and introduce terminology that we use to discuss its limitations. A simplified picture of how external acceleration $a_{\text{ext}}$ is converted into measured acceleration $a_m$ is shown in figure 5. Any external acceleration experienced by our device is contaminated by some measurement noise $\tilde{a}$, which we discuss in more detail in section 4.1, and is converted into a voltage via a physical sensitivity operator $S_p$ (We use an operator to account for the frequency dependent sensitivity of the mechanical oscillator).

A constant bias $v_b$ is added to form the total measured output voltage $v_m$. During post processing we subtract an estimated voltage bias $v_{b,c}$ and we convert into measured acceleration $a_m$ by applying the inverse of the estimated sensitivity operator $S^{-1}$. We neglect any influence of magnitude of sensitivity inaccuracies on the determined noise/error levels in the following. We can now write down the measured acceleration,

$$a_m = S^{-1}_e (v_b - v_{b,c} + S_p (a_{\text{ext}} + \tilde{a})).$$  \hfill (9)

The estimated sensitivity operator is constructed from the earlier described characterizations of the cavity (equation (5)) and the mechanical oscillator (equation (14), the time-domain equivalent to equation (8)).

By fitting this formula to the decay we determine the quality factor $Q \approx 123 \; 79.07$ with a relative standard uncertainty of about $1.5 \times 10^{-3}$. The resulting fit curve is shown in figure 4, as the envelope of the ring down. A correction of $\omega_0$ due to a damping-induced frequency shift is not necessary, since the correction factor is fully negligible at these levels of $Q$. To further refine and test the long term stability of $\omega_0$ and $Q$ we perform the above described analysis for multiple, consecutive ring down measurements. For 250 ring down, spaced over 25 min (1500 s) of measurement time, we determine a relative standard deviation for $\omega_0$ on the order of $1 \times 10^{-7}$ and $2 \times 10^{-4}$ for $Q$, each an order of magnitude larger than the single-shot statistical variation. Using the transfer function of a damped harmonic oscillator, we have the relation between external accelerations and displacement:

$$T_{\text{HO}}(\omega) = \frac{\Delta z(\omega)}{a_{\text{ext}}(\omega)} = -\frac{1}{\omega_0^2 - \omega^2 + \frac{i}{Q} \omega}.$$  \hfill (8)

### 2.5. Traceability by self-calibration

We now summarize the self-calibration method and introduce terminology that we use to discuss its limitations. A simplified picture of how external acceleration $a_{\text{ext}}$ is converted into measured acceleration $a_m$ is shown in figure 5. Any external acceleration experienced by our device is contaminated by some measurement noise $\tilde{a}$, which we discuss in more detail in section 4.1, and is converted into a voltage via a physical sensitivity operator $S_p$ (We use an operator to account for the frequency dependent sensitivity of the mechanical oscillator).

A constant bias $v_b$ is added to form the total measured output voltage $v_m$. During post processing we subtract an estimated voltage bias $v_{b,c}$ and we convert into measured acceleration $a_m$ by applying the inverse of the estimated sensitivity operator $S^{-1}$. We neglect any influence of magnitude of sensitivity inaccuracies on the determined noise/error levels in the following. We can now write down the measured acceleration,

$$a_m = S^{-1}_e (v_b - v_{b,c} + S_p (a_{\text{ext}} + \tilde{a})).$$  \hfill (9)

The estimated sensitivity operator is constructed from the earlier described characterizations of the cavity (equation (5)) and the mechanical oscillator (equation (14), the time-domain equivalent to equation (8)).

Assuming we can subtract dc biases well, we simplify equation (9) to

$$a_m = S^{-1}_e S_p a_{\text{ext}} + \tilde{a}.$$  \hfill (10)

If we now compute the effective error of our acceleration measurement ($a_{\text{err}} = a_m - a_{\text{ext}}$) we get

$$a_{\text{err}} = a_{\text{ext}} \cdot (S^{-1}_e S_p - 1) + \tilde{a}.$$  \hfill (11)

We call $S^{-1}_e S_p - 1$ the sensitivity accuracy $S_s$ operator, and, together with the noise (this includes bias drifts) and the dynamics of the external acceleration, they determine the total readout error

$$a_{\text{ext}} = a_{\text{ext}} = S_s + \tilde{a}.$$  \hfill (12)

One should note that this analysis and the self-calibration directly depend on the accuracy of the two applied response models (Airy function and harmonic oscillator). Potential deviations and limits are discussed in section 3.

### 2.6. Noise performance

With our device resting on a vibration isolation platform we measure the spectra of the output voltage to determine the readout noise floor. Using equation (2) we convert the voltage spectra into the corresponding displacement. The results of this are shown in figure 6 for different laser stabilization schemes (see figure 2). With both laser amplitude and frequency stabilization the spectrum shows the thermally excited peak of the mechanical oscillator is visible at $f_0$. The thermally excited resonance peak of the mechanical oscillator is visible at $f_0$. 

$$S^{-1}_e = \frac{\sigma_m}{\lambda_0} \left( \frac{d}{dt} + \frac{\omega_0}{Q} \frac{d}{dt} + \omega_0^2 \right).$$  \hfill (10)

Assuming we can subtract dc biases well, we simplify equation (9) to

$$a_m = S^{-1}_e S_p a_{\text{ext}} + \tilde{a}.$$  \hfill (11)

If we now compute the effective error of our acceleration measurement ($a_{\text{err}} = a_m - a_{\text{ext}}$) we get

$$a_{\text{err}} = a_{\text{ext}} \cdot (S^{-1}_e S_p - 1) + \tilde{a}.$$  \hfill (12)

We call $S^{-1}_e S_p = 1$ the sensitivity accuracy $S_s$ operator, and, together with the noise (this includes bias drifts) and the dynamics of the external acceleration, they determine the total readout error

$$a_{\text{ext}} = a_{\text{ext}} = S_s + \tilde{a}.$$  \hfill (13)

One should note that this analysis and the self-calibration directly depend on the accuracy of the two applied response models (Airy function and harmonic oscillator). Potential deviations and limits are discussed in section 3.

With our device resting on a vibration isolation platform we measure the spectra of the output voltage to determine the readout noise floor. Using equation (2) we convert the voltage spectra into the corresponding displacement. The results of this are shown in figure 6 for different laser stabilization schemes (see figure 2). With both laser amplitude and frequency stabilization the spectrum shows the thermally excited peak of the mechanical oscillator, which sticks out of an almost flat noise floor of about 15 fm/$\sqrt{\text{Hz}}$. The noise increases at lower frequencies, though the magnitude is strongly reduced by the stabilizations. We estimate the shot noise as explained in section 3.6 with a measured dc power on the photodiode of 60 $\mu$W. The dark noise measurement includes contributions from the photo receiver and the measurement devices. This was measured by blocking all light going onto the measurement
photodiode. The laser frequency stabilization allowed us to strongly reduce the excess frequency noise, likely induced by coupling of acoustics and vibrations into the tunable laser head [25, 26]. The unequal arm length Mach–Zehnder configuration is able to operate at any given wavelength and can, therefore, be used with the tunable laser, necessary to perform the self-calibration.

Using the harmonic oscillator transfer function model of the mechanical oscillator (see equation (8)) we can determine the corresponding acceleration noise spectra for a given displacement noise. Using a data acquisition system we performed long-term measurements. We converted the measured voltage into acceleration by filtering and scaling it accordingly. The resulting acceleration noise spectra are shown in figure 7. Our device achieves an acceleration noise floor better than $7.848 \times 10^{-5} \text{ m s}^{-2} \text{Hz}^{-1/2}$ above 1 kHz. Its low frequency performance is limited by a 1/f noise, leading to levels of $7.8 \times 10^{-3} \text{ m s}^{-2} \text{Hz}^{-1/2}$ at 1 Hz and better than $4 \times 10^{-4} \text{ m s}^{-2} \text{Hz}^{-1/2}$ at 10 mHz (corresponding to a cavity displacement noise of 2 pm/√Hz and better than 90 pm/√Hz respectively). The noise floor is about two orders of magnitude higher than the one achieved with a high-finesse cavity by Guzman et al (200 am/√Hz) [17]). This difference is easily understood: the high-finesse cavity reflects about half of the incident light and has a response slope that is strongly increased by multiple round trips, while the low-finesse cavity only reflects about 4% and senses almost only a single round trip. Hence, the high-finesse readout is less susceptible to shot and amplitude noise, as described in more detail in section 3.

2.7. Allan variance

To investigate the bias stability we determined the Allan deviation of our readout. The results are shown in figure 8 with and without stabilization. At short integration times we are limited only by shot noise. The exact nature of the increase at longer integration times is currently under investigation, and it is presumed to be caused by residual frequency noise or parasitic stray beams in our fiber set-up. Each of these contributions is susceptible to acoustic and thermal fluctuations, causing the noise in our set-up to not be constant. This is evident from the second, shorter measurement with stabilizations shown in figure 8, which presents the best levels of long-term stability achieved during our measurement campaign, even though the readout noise floor was significantly higher. Our frequency reference Mach–Zehnder interferometer was placed in the same thermal environment as our laser source and was isolated passively against thermal fluctuations. The use of active thermal stabilizations for the fiber interferometer may help to improve the performance in future implementations. We reach the minimum of our Allan deviation at about 1 ms of integration time with a value of $2.9 \times 10^{-3} \text{ m s}^{-2}$.

During our investigation we found that the use of fiber circulators, to feed light to the accelerometer and to detect the reflection, caused an increased coupling of laser frequency noise into the measurement. We attribute this to parasitic beams, introduced by the excess leakage in the return path of the circulator, that are phase modulated relative to our signal of interest by the frequency noise. These beams, or stray light, can reach significant amplitudes in comparison to the reflected signal of the low-finesse cavity, which is only about 4% of the incident power, and contaminate the interfered signal, leading to enhanced coupling of laser frequency noise and other phase noise in our fiber set-up. Using a 50/50 fiber beam splitter we were able to actively tune and ultimately minimize this coupling by terminating the open, unused output of the splitter. This improvement in stability comes at the cost of reducing the detected power to about one quarter of the levels achieved with a circulator. Future implementations have to take these parasitic signals into account to reduce the coupling of laser frequency noise to the signal from the actual cavity.

As an example of what may be possible with such a device having a higher-finesse optical readout, figure 8 shows Allan deviations for the high-finesse cavity readout implemented by Guzman et al [17], which were measured with a finesse of
3. Metrology chain model

Any external acceleration measured with our devices is converted into a readout voltage. In the following, we describe and model this conversion and its self-calibration as a chain of individual transducer steps. This allows us to (i) determine readout noise of the devices, sensitivity uncertainty, bias stability and dynamic range for a given set of parameters, and (ii) to determine the parameters that are necessary to reach a desired measurement uncertainty. For our analysis we assume to operate in the linear regime of both the optical and mechanical readout. For the low-finesse case we assume to have a maximum displacement of 4.5 pm, which corresponds to a maximum displacement of 4.5 nm and a total voltage change of about 150 mV (assuming the values shown in section 2). For this range the relative magnitude of sensitivity deviation due to the nonlinearity of the cavity response is lower than 10^{-3}. This was derived by calculating the difference between the linearized slope and the derivative of the Airy function for the maximum displacement around the point of highest slope \( \lambda_0 \). For the high-finesse device described by Guzman et al [17] a range of about 1.96 \times 10^{-2} m s^{-2}, corresponding to a maximum displacement of 4.5 pm, is assumed.

3.1. Mechanical oscillator

As mentioned earlier, we describe the behavior of the mechanical oscillator as a damped harmonic oscillator (HO), that converts external accelerations \( a_{ext} \) into measured test mass displacement \( \Delta z \) (see figure 9). This model is appropriate due to the high flexure stiffness and the correspondingly small test mass displacements. A measured displacement in the time domain is converted back into an acceleration by

\[
a_{acc}(t) = -\frac{a_0}{Q} \Delta \ddot{z}(t) + \frac{\omega_0^2}{Q} \Delta \dot{z}(t) + \frac{\omega_0^4}{Q} \Delta z(t).
\]  
(14)

3.2. Thermal noise

The acceleration readout is fundamentally limited by thermal acceleration noise \( \tilde{a}_th \) induced by the finite temperature of the oscillator and mechanical losses [27].

\[
\tilde{a}_th = \sqrt{\frac{4k_B T_0}{mQ}}.
\]  
(15)

This is approximately a white noise that is simply added to \( a_{ext} \), and therefore, only contributes to the readout noise floor, but it does not influence the sensitivity, nor the bias stability.

3.3. Conversion into displacement

At low frequencies the scaling of acceleration into displacement depends only on \( \omega_0 (\omega \ll \omega_0) = -1/\omega_0^2 \) and at resonance the influence of \( Q \) is maximal (\( T_HO(\omega = \omega_0) = -Q/\omega_0^2 \)).

The measurement of \( \omega_0 \) is fundamentally limited by the uncertainty of the used frequency standard. The measurement uncertainty of \( Q \) is only limited by the integration time and the readout noise floor. Larger values of \( Q \) require in general a longer measurement time, due to the increase in relaxation time \( \tau = \frac{20}{mQ} \).

3.4. Deviations from the harmonic oscillator

As briefly mentioned above, the harmonic oscillator model is only an approximation of the real behavior of the sensor. A full model that allows us to estimate the error of our approximation is beyond the scope of this article. However, we present the relevant parasitic effects, included as additional transfer function \( T_{par} \) in our model, by separating them into three categories and we discuss their expected behavior.

Higher order modes

The parallelogram design of our oscillator ensures that higher order modes are well separated in frequency and that their main axis of motion is perpendicular to the fundamental mode.
Figure 10 shows the results of a finite element model analysis of our sensors, which verifies the mode spacing, as well as the reduced coupling of higher order modes into the critical z-axis. As we can see from the plot, the detection of the oscillator motion close to the top surface is not ideal for reducing coupling of the higher modes, especially \( \omega_z \), into \( z \). Hence, future devices might use larger v-grooves or otherwise optimized geometries to detect the displacement at the optimal point, if these modes prove to be limiting in the future.

Non-viscous damping

Our oscillator devices are designed to be operated in vacuum and under small effective displacements, significantly less than 1 nm. Hence, residual gas damping and mechanical losses inside the oscillator are the dominant contributions, which are well described by a linear viscous damping model. Variations in these parameters are not at all critical, since they are incorporated into the measured \( Q \) (providing that these parameters do not change significantly between the measurement time and the self-calibration). Non-viscous damping effects, like structural damping, can result in a different transfer function from acceleration into displacement. One direct way to investigate such effects is to excite the accelerometer with known amplitudes at various frequencies to verify the transfer function, an experiment that will be conducted at a later point.

Nonlinearity

Simple beam deflection theory, which is part of the basis for the linear harmonic oscillator model, breaks down for large displacements, at which point additional higher order dependencies on the input frequency become relevant and the fundamental mode loses energy, effectively decreasing \( Q \). These effects limit the dynamic range of the oscillator, which relates, for a given readout uncertainty, to a maximum acceleration. One can characterize these behaviors experimentally, by operating the device under the desired maximum acceleration. For low frequencies such a measurement has to determine whether higher harmonics of the excitation frequency are present in the readout. For frequencies close to the resonance this can be combined with a measurement of \( Q \) under varying levels of excitation. The total deflection for accelerations of up to \( 1.962 \times 10^4 \text{ m s}^{-2} \) is on the order of 5 nm. This is significantly smaller than the thickness of the flexure in \( z \)-direction (255 \( \mu \text{m} \)), which leads us to expect this contribution to be very small.

3.5. Fiber cavity readout

Our model for the propagation of the test mass displacement \( \Delta z \) into reflected optical power \( P_R \) is shown in figure 11.

Cavity length

The total cavity length is a combination of a constant term \( z_0 \), the test mass displacement \( \Delta z \), and any unwanted, parasitic influences \( z_{\text{par}} \) caused, for example, by thermal expansion of the sensor. Time dependent parasitic changes will cause an unwanted proportional output signal, inducing noise and decreasing the bias stability, and, for significant changes, a sensitivity deviation.

Cavity response

The effective cavity reflectivity for a low-finesse external fiber micro-cavity has been studied in great detail [24, 28, 29]. Models are available that include the influence of angular misalignment and multiple reflections. In the following, we will present a few equations for the sinusoidal model, which are simple and useful for calculating errors due to shot and laser intensity noise for the low-finesse cavity, and we will present the more general formulas for the Airy function model, which is applied to determine the sensitivity and for cavities with higher finesse. For the linearized sinusoidal model the reflected intensity around the quadrature point \( \lambda_q \) can be estimated

\[
R = R_{\text{dc}} + \Delta R = R_{\text{dc}} + \frac{R_{\text{dc}} \cdot k \cdot 4 \pi \cdot z_m}{\lambda_q}.
\]

(16)

From this approximation it becomes clear that any changes in \( \lambda \) will move the operating point that was used during the characterization, which changes the effective magnitude of sensitivity and the bias.

Using the Airy function model (see equation (3)), which includes multiple reflections, we generalize the cavity reflected intensity around a given operating wavelength \( \lambda_0 \) as

\[
R = R(z_m, \lambda_0) + \frac{\lambda_0}{z_m} \left( \frac{dR}{d\lambda} \right) \Delta z_m
\]

(17)

In case of a hypothetical perfect cavity model, the laser wavelength uncertainty represents the fundamental limit for the displacement readout accuracy. Any differences between the model and the real response of the cavity will also limit the sensitivity accuracy.

Nonlinearity

For large displacements at a given uncertainty the linear approximations of equations (16) and (17) introduce deviations in the magnitude of sensitivity and generate signals at higher harmonics of the input signal. The dynamic range of our device is relatively large compared to systems using higher values of finesse, and rough error estimates can be
easily calculated from the deviations of the linear approximations relative to the expected response functions.

Laser frequency noise

Changes in the laser frequency (which are inverse to wavelength fluctuations) couple into the readout as phase noise, causing an effective displacement noise. This is caused by the interference between the reflections, which are delayed relative to each other by twice the cavity length. The effective coupling for small delays in comparison to the readout frequencies is given as

$$\tilde{f} = \frac{\lambda_0}{c} f.$$  

(18)

Here \(c\) is the speed of light and \(\tilde{f}\) is the laser frequency noise.

3.5.1. Input power. The reflectivity of the cavity is sensed with the power sent into the fiber.

$$P_R = PR_{dc} + P\Delta R.$$  

(19)

We determine the change in reflected power for the Airy function model as

$$\Delta P_R = P \frac{\lambda_0}{z_m} \left( \frac{dR}{d\lambda} \right) \Delta z_m = \frac{\lambda_0}{z_m} \left( \frac{dP}{d\lambda} \right) \Delta z_m.$$  

(20)

Intensity fluctuations

Relative amplitude/intensity noise (RIN) has to be taken into account as well. Its influence can be described as two separate effects. The first effect is that RIN directly influences the scaling factor, as evident from equation (20). The second effect is a coupling into the readout as additive amplitude noise. To quantify this effect we write the optical power \(P\) in terms of a constant component and an additive power noise \(\tilde{P}\),

$$P = P_0 + \tilde{P}.$$  

(21)

The dc component of the optical signal can now be rewritten as a constant term and a fluctuation noise term.

$$P \cdot R_{dc} = P_0 \cdot R_{dc} + \tilde{P} \cdot R_{dc} = P_{0,dc} + \tilde{P}_{RIN}.$$  

(22)

The effective length noise \(\tilde{z}_p\) due to RIN can be estimated, by computing the ratio of \(\tilde{P}_{MIN}\) and \(\Delta P_R/z_m\). For the sinusoidal model this corresponds to

$$\tilde{z}_p = \frac{\tilde{P}_{q}}{P} = \frac{\lambda_q}{4\pi k} = \text{RIN} \cdot \frac{\lambda_q}{4\pi k}.$$  

(23)

For the more general cavity response we can write this as

$$\tilde{z}_p = \frac{\tilde{P}}{P} \frac{R(z_m, \lambda_0)}{\frac{dR}{d\lambda}} \frac{z_m}{\lambda_0} = \text{RIN} \cdot \frac{R(z_m, \lambda_0)}{\frac{dR}{d\lambda}} \frac{z_m}{\lambda_0}.$$  

(24)

Here we omit any power losses in our fiber set-up, which simply scale the effective power \(P\). Experimentally, one can simply measure the reflected power at quadrature \(P_{dc}\) to determine the correct scaling.

3.6. Light to voltage conversion

Photodiode

A photo detector is used to convert the reflected optical signal into a photo current, via the photodiode responsivity \(r_{PD}\) ((\(r_{PD}\) = ampere/watt = A W^{-1}). The current response for the Airy function model gives

$$\Delta i_R = r_{PD} \frac{\lambda_0}{z_m} \left( \frac{dR}{d\lambda} \right) \Delta z_m.$$  

(25)

Shot noise

The dc power on the photodiode generates a shot noise, which can be modeled as an effective white photodiode current noise \(\tilde{i}_n\). This noise depends only on the dc input power and the photodiode responsivity.

$$\tilde{i}_n = \sqrt{2qP_{dc}r_{PD}}.$$  

(26)

The shot noise induced displacement noise depends on the ratio of this current noise to \(\Delta i_R/z_m\). For the sinusoidal model this gives

$$\tilde{z}_n = \frac{\tilde{i}_n \cdot z_m}{\Delta i_R} = \frac{\sqrt{2qR(z_m, \lambda_0)}}{r_{PD} P \left( \frac{dR}{d\lambda} \right) \lambda_0}.$$  

(27)

For the general cavity response this corresponds to

$$\tilde{z}_n = \sqrt{\frac{2qR(z_m, \lambda_0)}{r_{PD} P \left( \frac{dR}{d\lambda} \right) \lambda_0}}.$$  

(28)

Trans-impedance amplifier

The conversion into readout voltage is done using a trans-impedance amplifier (TIA), which is often combined with the photodiode in a photo receiver. At this point, we assume that the TIA has either sufficient bandwidth to minimize any frequency dependent scaling effects of amplitude and phase, or that the back-end corrects for this. The TIA is then simply characterized by a gain \(R_{TIA}\) that determines the current-to-voltage ratio.

The photo receiver elements can introduce additional electronic noise that can spoil the measurement performance. Care should be taken to reduce any such influence to negligible levels. A simple method to do this is to calculate the equivalent input current noise of all contributions in the photo receiver [30] and to compare them to the expected shot noise level (see equation (26)).

Nonlinearities in the back-end

The conversion into a readout voltage, as well as further digitization and processing should provide sufficient dynamic
range to ensure that the signal is not distorted. The applied dc readout can be challenging in this regard, due to the presence of the large dc bias. However, techniques like a direct subtraction of photo diode current can be applied to reduce the signal amplitude before the current-to-voltage conversion, dramatically reducing the required dynamic range of subsequent readout elements.

Table 2. Relevant noise sources and boundary conditions necessary to achieve a sensitivity of better than 9.81 × 10⁻⁵ m s⁻²√Hz.

| Error       | Source       | Symbol | Parameter | Value for reaching a of $a_0$ |
|-------------|--------------|--------|-----------|-------------------------------|
| Thermal     | $\tilde{a}_t$ | $Q$    |           | <9.81 × 10⁻⁵ m s⁻²√Hz         |
| Displacement| $\tilde{a}_d$ | $z_d$  |           | <22 fm/√Hz                    |
| Frequency   | $\tilde{a}_f$ | $f$    |           | <101 kHz/√Hz                  |
| Amplitude   | $\tilde{a}_p$ | RIN    |           | <1.7 × 10⁻⁷/√Hz               |
| Shot        | $\tilde{a}_s$ | $P_{dc}$|           | >11.3 µW                      |

4.2. Magnitude of sensitivity accuracy

To discuss the estimated magnitude of sensitivity accuracy achievable with our device we determine the combined standard uncertainty $u_c$ for the sensitivity operator $S_c$. We summarize the relevant effects, models and fundamental parameters in table 3, split by the two characterization steps that we use to estimate $S_c$⁻¹ (see section 2.5). For a given model of the behavior of our system we can calculate the magnitude of sensitivity uncertainty based on measured, or estimated uncertainties of the relevant parameters. For a number of $N$ parameters $x$ with relative uncertainties $u_x$ and estimated values $x_e$, given as $x_e = x(1 + e_x)$, applied with $S_c$⁻¹, we estimate the combined standard uncertainty as

$$u_c = \sqrt{\sum_{x=1}^{N} \left( \frac{\partial S_c}{\partial x} u_x \right)^2}.$$  \hspace{1cm} (29)

To give an example, we evaluate the coupling of $\omega_0$ into the standard uncertainty for low frequency signals. For this case $S_c$ simplifies to

$$S_c + 1 = \frac{\omega_{0,e}^2}{\omega_0^2} = (1 + e_{\omega_0})^2 = 1 + 2e_{\omega_0} + e_{\omega_0}^2.$$  \hspace{1cm} (30)

Computing the derivative and omitting terms of the order $O^2$ we determine a scaling factor of

$$\frac{\partial S_c}{\partial \omega_0} \approx 2.$$  \hspace{1cm} (31)

We have summarized the uncertainties of the relevant parameters and their coupling factors in table 4. For the mechanical oscillator part we include two cases, at low frequencies and at resonance. For the interferometer we include the uncertainties for the here-presented low-finesse readout and, for comparison, the values for the high-finesse readout from the earlier study [17]. In the following we discuss the individual contributions.

4. Noise, accuracy & bias stability

4.1. Noise sources

The noise contributions for the acceleration readout performance are compiled for the parameters listed in table 1. The parameters are either chosen by design or they represent experimentally reproducible values. For the major error contributions we can now derive the necessary values of the other readout parameters to achieve a sensitivity of 9.81 × 10⁻⁵ m s⁻²√Hz. The target parameter values to achieve this sensitivity are summarized in table 2. Thermal noise is completely negligible. Shot noise and RIN are broadband and can both dominate the high frequency behavior. The influence of RIN can, however, be easily reduced by implementing an amplitude stabilization. Laser frequency noise is expected to dominate at low frequencies for free running lasers, as observed. For a well-stabilized laser source the low frequency performance could at some point also be dominated by parasitic displacement noise, induced, for example, by thermal fluctuations that drive the non-zero coefficient of thermal-expansion of the device.
The fundamental limitation of this approach is given by the wavelength uncertainty. The currently used laser system provides a resolution of 0.01 nm during a wavelength scan. From this we can estimate the wave length uncertainty limit to \( u_\lambda = 7 \times 10^{-6} \). Changes of the cavity length due to thermal expansion or of the reflected power, due to intensity noise, influence the magnitude of sensitivity over time. Power stabilizations that reach levels of better than \( 10^{-6} \) over long time scales are feasible and can be used, if higher magnitude of sensitivity precisions are aimed for.

We have characterized the current experimental limits for our cavity readout characterization using the Airy function model described in equation (3) and the resulting \( \frac{df}{dx} \) (see equation (5)). Within this model the uncertainties of four parameters are relevant: \( u_\lambda, u_{\lambda m}, u_R \) and \( u_{\omega} \). The standard errors of our Airy function fit point to an uncertainty for \( z_m \) of better than \( 1.0 \times 10^{-5} \). Taking the fundamental limit of \( u_\lambda \) for this determination into account, we estimate \( u_{\lambda m} \approx 1.2 \times 10^{-5} \). The reflectivity of our fiber ends is only poorly estimated by the fit to \( u_R \approx 10 \% \), but we can determine it better using additional reflection and transmission measurements to about \( u_R \approx 0.5 \% \). The coupling of reflection variations into the magnitude of sensitivity is complex and is given by the derivative of the Airy function \([19]\), we estimate the uncertainty scaling to \( 0.003 \). However, our dominating noise term arises from uncertainties in \( \nu_\nu \), which, based on our fits, can be determined with an uncertainty of \( u_{\nu} \approx 5 \times 10^{-4} \). To achieve this value we reduced the influence of a wavelength dependent amplitude response of our experimental set-up by correcting the reflected amplitude from the cavity with the response measured with only a single flat-cleaved fiber end and no second mirror. This method allowed us to account for the wavelength dependent behavior of the photodiodes, as well as the optical isolator and the couplers that are in between the point of amplitude detection and the cavity. The wavelength dependent discrepancy between the interference amplitudes estimated by our fits (see figure 3) indicates that we are limited either by diffraction related effects or by a non-ideal amplitude correction, potentially caused by parasitic cavities. Models that include wavelength dependent mode propagation, as well as corrections due to cavity misalignments \([24]\) can be applied in future studies to improve the understanding and accuracy of the reflection response.

With the current models, at low frequencies and for the low-finesse cavity we can calculate

\[
u_\nu = \sqrt{\left(2u_{\omega 0}\right)^2 + \left(u_\nu \right)^2 + \left(u_{\nu R}\right)^2 + \left(0.003u_R\right)^2 + \left(u_{\nu\nu}\right)^2}.
\] (32)

Table 4 shows the resulting combined standard uncertainties for signals at low frequencies and at oscillator resonance.

### Table 3. Overview of the components that limit the fundamental magnitude of sensitivity accuracy.

| Parameter | Transducer | Mech. oscillator | Interferometer |
|-----------|------------|-----------------|----------------|
| \( \omega_0, Q \) | \( \frac{df}{dx} \) \( \lambda_0, z_m \) |
| SI reference | \( f_{rel} \) | \( \lambda_0 \) |
| Simple model | (Viscously) damped harmonic oscillator | Two-beam interference (sinusoidal model) |
| Extensions | Higher order modes | Multiple reflections |
| Non-viscous damping | Nonlinearity | Fiber coupling efficiency |
| Nonlinearity | \( \lambda_0 \) |
| Stability influence | Thermal expansion | Thermal expansion |
| Clamping | Intensity noise | |

### Table 4. List of the individual relative uncertainties and their scaling, together with the derived combined standard uncertainties for signals at low frequencies and at oscillator resonance.

| Harmonic oscillator | \( \omega \ll \omega_0 \) | \( \omega = \omega_0 \) |
|---------------------|-----------------|-----------------|
| Uncertainty | Value | Value |
| \( u_{\omega 0} \) | \( 0.1 \times 10^{-6} \) | \( 0.1 \times 10^{-6} \) |
| \( u_\nu \) | \( <0.2 \times 10^{-3} \) | \( <0.2 \times 10^{-3} \) |

| Fabry–Pérot interferometer | | |
|-----------------------------|-----------------------------|
| Uncertainty | Value | Value |
| \( R = 4 \% \) | \( 7 \times 10^{-6} \) | \( 7 \times 10^{-6} \) |
| \( \omega \ll \omega_0 \) | \( 5.06 \times 10^{-4} \) | \( 5.41 \times 10^{-4} \) |
| \( \omega = \omega_0 \) | \( 5.16 \times 10^{-4} \) | \( 5.16 \times 10^{-4} \) |
that the high-finesse cavity in [17] was also mode matched, using a curved mirror to form a hemispherical resonator, potentially reducing the influence of beam propagation effects neglected in our Airy function analysis.

4.3. Bias stability

For a given model of the dominating noise sources one can compute a corresponding Allan deviation behavior [10, 31–34]. By comparing the two most dominant noise types, white noise and 1/f noise, one can compute the Allan deviation minimum and derive from that the expected bias stability.

A white noise of \( \tilde{a}_w = a_w \cdot 1/\sqrt{\text{Hz}} \) induces an Allan deviation slope of

\[
\sigma_a(\tau) = \frac{a_w}{\sqrt{2\tau}}. \tag{33}
\]

A 1/f noise of \( \tilde{a}_{1/f} = a_{1/f} f \cdot 1/\sqrt{\text{Hz}} \) induces an Allan deviation slope of

\[
\sigma_{1/f}(\tau) = 2\pi \frac{a_{1/f}}{\sqrt{6}}. \tag{34}
\]

For the minimum value, which is often denoted as the bias stability, we can now calculate the optimal integration time,

\[
\tau_{bs} = \frac{\sqrt{3}}{2\pi} a_w/a_{1/f}. \tag{35}
\]

This leads to an estimate of the bias stability of

\[
\sigma_{bs} \approx 2\sigma_w(\tau_{bs}) = \frac{4\pi}{\sqrt{3}} a_w a_{1/f}. \tag{36}
\]

Assuming a white acceleration noise floor of \( \tilde{a}_w = 7.8 \times 10^{-5} \text{ m s}^{-2}/\sqrt{\text{Hz}} \) and a 1/f noise floor of \( a_w = 8.8 \times 10^{-3} \text{ m s}^{-2}/f/\sqrt{\text{Hz}} \), the integration minimum can be found at \( \tau_{bs} \approx 2.5 \text{ ms} \) and the corresponding bias stability is on the order of \( 1.962 \times 10^{-3} \text{ m s}^{-2} \), which is consistent with our data presented in figure 8. Using the presented formulas we can now extrapolate the 1/f laser frequency stability \( f_{1/f} \) that is required to achieve a certain bias stability with our model. This is calculated as

\[
f_{1/f} = \frac{c}{\lambda_{0} \sigma_{b} \sigma_{a}^{2}} \frac{\sqrt{3}}{4\pi a_w}. \tag{37}
\]

With our cavity parameters and the earlier assumed white noise floor we require a 1/f laser frequency noise of less than \( f_{1/f} \approx 1700 \text{ Hz}/\sqrt{\text{Hz}} \) to achieve a bias stability of \( 9.81 \times 10^{-8} \text{ m s}^{-2} \) at integration times of 1.3 s.

5. Summary and conclusion

We have presented an accelerometer device that is a promising candidate for future applications requiring high bandwidth and in situ self-calibration. We have conducted a detailed analysis of the device physics and derived the effects that influence noise, accuracy and stability. We achieve a bias stability of better than \( 2.9 \times 10^{-3} \text{ m s}^{-2} \) and an estimated magnitude of sensitivity uncertainty of \( 5 \times 10^{-4} \). For an input signal of \( 1.962 \times 10^3 \text{ m s}^{-2} \) we can estimate a total acceleration measurement error on the order of \( 3.2 \times 10^{-2} \text{ m s}^{-2} \), dominated by nonlinearities in the optical readout and systematic effects in the characterization of the cavity response.

The bias stability of our device is currently limited by laser frequency noise and non-stationary thermal and acoustic noise coupling into the measurement through parasitic beams in the fiber set-up. At frequencies above 1 kHz our readout noise is dominated by shot noise. The accuracy of our device is limited by the characterization of the reflectivity response of the Fabry–Pérot cavity.

The limits of the achievable accuracies for magnitude of sensitivity and bias stability have been discussed, both for low-finesse and for high-finesse readout. While the ultimate limits of self-calibration to SI standards are rather well understood based on the applied simple behavior models, targeted studies will be conducted in the future to measure the experimental variations of the magnitude of sensitivity over long time scales and its dependency on temperature. Applying a cavity model that takes diffraction effects and misalignments into account and further study of the wavelength dependent behavior of the fiber set-up may further improve the self-calibration of the cavity response. Further experiments are also required to test the limits of the simple behavior models by, for example, performing detailed measurements of the harmonic oscillator response function.

Disclaimer

Certain commercial equipment, instruments, or materials are identified in this paper in order to specify the test and measurement procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

Acknowledgments

The authors would like to thank A Kirchhoff for his help with the device assembly and vacuum sealing. We would also like to thank J Gorman, T LeBrun, and R Lutwak for useful discussion. This work was supported by DARPA and ARO under grant W911NF-14-1-0681.

References

[1] Barbour N and Schmidt G 2001 Inertial sensor technology trends IEEE Sensors J. 1 332–9
[2] Barbour N M 2010 Inertial navigation sensors Low-Cost Navigation Sensors and Integration Technology (NATO Science and Technology Organization)
[3] Cooper G, Sheret I, McMillian L, Silverdivs K, Sha N, Hodgins D, Kenney L and Howard D 2009 Inertial sensor-based knee flexion/extension angle estimation J. Biomech. 42 2678–85
[4] Strasberg M and Feit D 1996 Vibration damping of large structures induced by attached small resonant structures J. Acoust. Soc. Am. 99 335–44
[5] Nakstad H and Kringeleben J T 2008 Oil and gas applications: probing oil fields Nat. Photonics 2 147–9
[6] Dong Y, Zwahlen P, Nguyenv, Rudolf F and Stauffer J M 2010 High performance inertial navigation grade sigma-delta mems accelerometer IEEE/ION Position Location and Navigation Symp. pp 32–6
[7] Lenoir B, Christophe B and Reynaud S 2011 Measuring the absolute non-gravitational acceleration of a spacecraft: goals, devices, methods, performances SF2A-2011: Proc. of the Annual Meeting of the French Society of Astronomy and Astrophysics vol 1 pp 663–7
[8] Ignatiev A Yu 2015 Testing MOND on earth 1 Can. J. Phys. 93 1–3
[9] ISO/TC 108/SC 3 1999 ISO 16063-11 Methods for the calibration of vibration and shock transducers—part 11: primary vibration calibration by laser interferometry
[10] IEEE standard specification format guide and test procedure for linear, single-axis, non-gyroscopic accelerometers IEEE Std 1293-1998 (R2008) pp 1–249
[11] Robinson D C, Serbyn M R and Payne B F 1987 A Description of NBS Calibration Services in Mechanical Vibration and Shock (Gaithersburg, MD: US Department of Commerce and National Bureau of Standards)
[12] Ripper G P, Dias R S and Garcia G A 2009 Primary accelerometer calibration problems due to vibration exciters Measurement 42 1363–9 (Concerning foundational concepts of measurement special issue section)
[13] von Martens H-J, Link A, Schlaak H-J, Taeubner A, Wabinski W and Goebel U 2004 Recent advances in vibration and shock measurements and calibrations using laser interferometry Proc. SPIE 5503 1–19
[14] von Martens H-J 2014 Evaluation of measurement uncertainty in calibrations of laser vibrometers AIP Conf. Proc. 1600 123–42
[15] Krause A G, Winger M, Blasius T D, Lin Q and Painter O 2012 A high-resolution microchip optomechanical accelerometer Nat. Photonics 6 768–72
[16] Zhang Q, Zhu T, Hou Y and Chiang K S 2013 All-fiber vibration sensor based on a fabry-perot interferometer and a microstructure beam J. Opt. Soc. Am. B 30 1211–5
[17] Guzman Cervantes F, Kumanchik L, Pratt J and Taylor J M 2014 High sensitivity optomechanical reference accelerometer over 10 kHz Appl. Phys. Lett. 104 221111
[18] Rugar D, Mamin H J, Erlandsson R, Stern J E and Terris B D 1988 Force microscope using a fiber-optic displacement sensor Rev. Sci. Instrum. 59 2337–40
[19] Smith D T, Pratt J R and Howard L P 2009 A fiber-optic interferometer with subpicometer resolution for dc and low-frequency device measurements Rev. Sci. Instrum. 80 035105
[20] Melcher J, Stirling J, Cervantes F G, Pratt J R and Shaw G A 2014 A self-calibrating optomechanical force sensor with femtowton resolution Appl. Phys. Lett. 105 233109
[21] Newport 2014 TLB-6700 velocity widely tunable lasers datasheet assets.newport.com/webDocuments-EN/images/DS_041104_Velocity_Datasheet.pdf
[22] Optilab EDFA-I-R inline erbium-doped fiber amplifier, rackmount datasheet 2010 (www.oquest.com/getDatasheet/id/5868-32550acc0e0cf57.pdf)
[23] Rugar D, Mamin H J and Guethner P 1989 Improved fiber-optic interferometer for atomic force microscopy Appl. Phys. Lett. 55 2588–90
[24] Wilkinson P R and Pratt J R 2011 Analytical model for low finesse, external cavity, fiber Fabry–Perot interferometers including multiple reflections and angular misalignment Appl. Opt. 50 4671–80
[25] Harvey K C and Myatt C J 1991 External-cavity diode laser using a grazing-incidence diffraction grating Opt. Lett. 16 910–2
[26] Hawthorn C J, Weber K P and Scholten R E 2001 Littrow configuration tunable external cavity diode laser with fixed direction output beam Rev. Sci. Instrum. 72 4477–9
[27] Yasumura K Y, Stowe T D, Chow E M, Pfafman T, Kenny T W, Stipe B C and Rugar D 2000 Quality factors in micron- and submicron-thick cantilevers J. Microelectromech. Syst. 9 117–25
[28] Di Donato A, Morini A and Farina M 2013 Optical fiber extrinsic micro-cavity scanning microscopy Prog. Electromagn. Res. 133 347–66
[29] Wilkinson P R, Shaw G A and Pratt J R 2013 Determination of a cantilevers mechanical impedance using photon momentum Appl. Phys. Lett. 102 184103
[30] Guzman Cervantes F, Livas J, Silverberg R, Buchanan E and Stebbins R 2011 Characterization of photoreceivers for LISA Class. Quantum Grav. 28 094010
[31] Woodman O J 2007 An introduction to inertial navigation Technical Report UCAMCL-TR-696 University of Cambridge pp 14–5
[32] El-Sheimy N, Hou H and Niu X 2008 Analysis and modeling of inertial sensors using allan variance IEEE Trans. Instrum. Meas. 57 140–9
[33] Ferre-Pikal E S and Walls F L 1999 Frequency standards, characterization Wiley Encyclopedia of Electrical and Electronics Engineering (Hoboken, NJ: Wiley)
[34] Quinchia A G, Falco G, Falletti E, Dovis F and Ferrer C 2013 A comparison between different error modeling of mems applied to gps/ins integrated systems Sensors 13 9549–88