Gauge propagator of the nonbirefringent CPT-even sector of the Standard Model Extension

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The CPT-even gauge sector of the Standard Model Extension is composed of nineteen components comprised in the tensor \((K_F)_{\mu\nu\rho\sigma}\), of which nine do not yield birefringence. In this work, we examine the Maxwell electrodynamics supplemented by these nine nonbirefringent CPT-even components in aspects related to the Feynman propagator and full consistency (stability, causality, unitarity). We adopt a prescription that parametrizes the nonbirefringent components in terms of a symmetric and traceless tensor, \(K_{\mu\nu}\), and second parametrization that writes \(K_{\mu\nu}\) in terms of two arbitrary four-vectors, \(U_{\mu}\) and \(V_{\nu}\). We then explicitly evaluate the gauge propagator of this electrodynamics in a tensor closed way. In the sequel, we show that this propagator and involved dispersion relations can be specialized for the parity-odd and parity-even sectors of the tensor \((K_F)_{\mu\nu\rho\sigma}\). In this way, we reassess some results of the literature and derive some new outcomes showing that the parity-even anisotropic sector engenders a stable, noncausal and unitary electrodynamics.

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I. INTRODUCTION

Lorentz symmetry violation has been an issue of permanent interest in the latest years, with many investigations in the context of the Standard Model Extension (SME)\textsuperscript{1, 2}. The SME incorporates terms of Lorentz invariance violation (LIV) in all sectors of interaction and has been studied in many respects\textsuperscript{3, 4}. The investigations in the context of the SME concern mainly the fermion sector\textsuperscript{5, 6}, the gauge sector\textsuperscript{7-24}, and extensions involving gravity\textsuperscript{25}. The violation of Lorentz symmetry has also been addressed in other theoretical frameworks\textsuperscript{26}, with many interesting developments\textsuperscript{27, 28}.

The CPT-odd gauge sector of the SME, represented by the Carroll-Field-Jackiw (CFJ) electrodynamics\textsuperscript{7}, has its properties largely examined in literature, addressing consistency aspects\textsuperscript{8} and modifications induced in QED\textsuperscript{9, 10}, dimensional reduction\textsuperscript{11}, supersymmetry\textsuperscript{12}, controversies discussing the radiative generation of the CFJ term\textsuperscript{13}, the vacuum emission of Cherenkov radiation\textsuperscript{14}, the electromagnetic propagation in waveguides\textsuperscript{15}, modifications on the Casimir effect\textsuperscript{16}, effects on the Planck distribution and finite-temperature contributions\textsuperscript{17, 18}, possible effects on the anisotropies of the Cosmic Microwave Background Radiation\textsuperscript{19}. Since 2002 the CPT-even sector of SME has been also much investigated, mainly in connection with issues able to provide good bounds on its 19 LIV coefficients. The studies about the properties of the CPT-even electrodynamics, represented by the tensor \((K_F)_{\alpha\nu\rho\sigma}\), were initiated by Kostelecky \& Mewes in Refs.\textsuperscript{20, 21}, where it was stipulated the existence of ten linearly independent combinations of the components of \((K_F)_{\alpha\nu\rho\sigma}\) sensitive to birefringence. A broader and interesting study in this respect was performed recently in Ref.\textsuperscript{22}. These elements are contained in two \(3 \times 3\) matrices, \(\tilde{\kappa}_{e+}\) and \(\tilde{\kappa}_{e-}\). Using high-quality spectropolarimetry data of cosmological sources\textsuperscript{23} and microwave cavities experiments, stringent upper bounds \((10^{-32} \text{ and } 10^{-37})\) were imposed on these birefringent LIV parameters. The study of Cherenkov radiation\textsuperscript{24} and the absence of emission of Cherenkov radiation by UHECR (ultrahigh energy cosmic rays)\textsuperscript{29, 30} has been a point of great interest in latest years, as well as the photon-fermion vertex interactions yielding new bounds on the LIV coefficients\textsuperscript{31, 33, 34}. Investigations on finite temperature properties and the implied modifications on the Planck law were developed as well for the CPT-even sector\textsuperscript{35, 36}.

In a recent work, the gauge propagator of the CPT-even electrodynamics of the SME has been explicitly carried out in the form of a \(4 \times 4\) matrix\textsuperscript{37}. The dispersion relations were determined from the poles of the propagator, and used to analyze the stability, causality and unitarity of this theory for the nonbirefringent parity-odd components and for the isotropic parity-even one. The pole analysis showed that the parity-odd sector is stable, non-causal and...
unitary, whereas the parity-even isotropic sector, represented uniquely by the trace component, provides a stable, causal and unitary theory for the range $0 \leq \kappa_{tr} < 1$.

In the present work, we evaluate the gauge propagator for the nine nonbirefringent coefficients of the CPT-even sector of SME in an exact tensor form, using the parametrization of the tensor $(\tilde{K}_F)_{\alpha\nu\rho\phi}$ in terms of a symmetric and traceless rank-2 tensor, $k_{\alpha\beta}$, introduced in Ref. [33]. In order to evaluate the propagator, the tensor $k_{\alpha\beta}$, in turn, is parametrized in terms of two four-vectors, $U_\mu, V_\nu$, that contain the Lorentz-violating components. Once the propagator is explicitly written, it is specialized for the parity-odd and parity-even sectors of this theory for some choices of $U_\mu, V_\nu$, yielding the dispersion relations attained in Ref. [37]. New investigations concerning the anisotropic parity-even components are performed, revealing that this sector is stable, noncausal and unitary.

### II. THEORETICAL MODEL AND THE GAUGE PROPAGATOR

The Lagrangian of the CPT-even electrodynamics of SME is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(K_F)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2,$$

(1)

where $\xi$ is the gauge fixing parameter, $F_{\alpha\nu}$ is the electromagnetic field tensor, $(K_F)_{\alpha\nu\rho\phi}$ is a renormalizable and dimensionless coupling, responsible for Lorentz violation. The tensor $(\tilde{K}_F)_{\alpha\nu\rho\phi}$ has the same symmetries as the Riemann tensor

$$(K_F)_{\alpha\nu\rho\phi} = -(K_F)_{\nu\alpha\rho\phi}, \quad (K_F)_{\alpha\nu\rho\phi} = -(K_F)_{\alpha\nu\rho\phi}, \quad (K_F)_{\alpha\nu\rho\phi} = (K_F)_{\rho\phi\alpha\nu},$$

(2)

$$(k_F)_{\alpha\beta\rho\phi} + (k_F)_{\alpha\nu\beta\phi} + (k_F)_{\alpha\nu\rho\phi} = 0,$$

(3)

and a double null trace, $(K_F)^{\rho\phi}_{\rho\phi} = 0$. The tensor $(\tilde{K}_F)_{\alpha\nu\rho\phi}$ has 19 independent components, from which nine do not yield birefringence. A very useful parametrization for addressing this theory is the one presented in Refs. [20, 21], in which these 19 components are contained in four $3 \times 3$ matrices:

$$(\bar{\kappa}_e^+)_{jk} = \frac{1}{2}(\kappa_{DE} + \kappa_{HB})_{jk}, \quad (\bar{\kappa}_e^-)_{jk} = \frac{1}{2}(\kappa_{DE} - \kappa_{HB})_{jk} - \frac{1}{3}\delta_{jk}(\kappa_{DE})^{ii}, \quad \kappa_{tr} = \frac{1}{3}\text{tr}(\kappa_{DE}),$$

(4)

$$(\bar{\kappa}_o^+)_{jk} = \frac{1}{2}(\kappa_{DB} + \kappa_{HE})_{jk}, \quad (\bar{\kappa}_o^-)_{jk} = \frac{1}{2}(\kappa_{DB} - \kappa_{HE})_{jk},$$

(5)

where $\bar{\kappa}_e$ and $\bar{\kappa}_o$ designate parity-even and parity-odd matrices, respectively. The $3 \times 3$ matrices $\kappa_{DE}, \kappa_{HB}, \kappa_{DB}, \kappa_{HE}$ are defined in terms of the $(K_F)$ –tensor components:

$$(\kappa_{DE})_{jk} = -2(\kappa_{FB})_{0j0k}, \quad (\kappa_{HB})_{jk} = \frac{1}{2}\epsilon^{ijk}\epsilon^{k\ell m}(F_{F})_{\ell\ell m},$$

(6)

$$(\kappa_{DB})_{jk} = -\kappa_{HE})_{kj} = \epsilon^{k\ell m}(F_{F})_{\ell\ell m}.$$

(7)

The matrices $\kappa_{DE}, \kappa_{HB}$ contain together 11 independent components while $\kappa_{DB}, \kappa_{HE}$ possess together 8 components, which sums the 19 independent elements of the tensor $(K_F)_{\alpha\nu\rho\phi}$. From these 19 coefficients, 10 are sensitive to birefringence and 9 are nonbirefringent. These latter ones are contained in the matrices $\bar{\kappa}_o^+$ and $\bar{\kappa}_o^-$. The analysis of birefringence data reveals the coefficients of the matrices $\bar{\kappa}_e^+$ and $\bar{\kappa}_o^-$ are bounded to the level of 1 part in $10^{32}$ [20, 21] and 1 part in $10^{35}$ [28]. This leads to the following constraints: $\kappa_{DE} = -\kappa_{HB}, \kappa_{DB} = \kappa_{HE}$, which implies

$$(\bar{\kappa}_o^+)_{jk} = (K_{DB})_{jk}, \quad (\bar{\kappa}_e^-)_{jk} = (\kappa_{DE})_{jk} - \delta_{jk}\kappa_{tr},$$

(8)

where the matrix $\bar{\kappa}_e^-$ is symmetric and traceless (has 5 components) and the matrix $\kappa_{DB}$ has now become antisymmetric, possessing only three components written in terms of a 3-vector [38],

$$\kappa^i = \frac{1}{2}\epsilon^{ijk}(\kappa_{DB})_{jk}.$$  

(9)
An interesting way to parametrize the 9 nonbirefringent components of the tensor \((K_f)\) is the one introduced in Ref. [38], in which it is written in terms of a symmetric traceless tensor \(k_{\nu\rho}\):

\[
(K_f)^{\lambda\sigma\rho} = \frac{1}{2} \left[ g^{\lambda\delta} k^{\nu\rho} - g^{\rho\delta} k^{\lambda\nu} + g^{\nu\rho} k^{\lambda\delta} - g^{\lambda\rho} k^{\nu\delta} \right].
\]  

(10)

Here, the nine nonbirefringent components are all contained in the symmetric traceless tensor \(k_{\nu\rho}\), defined as the contraction

\[
k^{\mu\nu} = (K_f)^{\alpha\mu\nu}. \]

(11)

The components of matrices \(\kappa_{DE}, \kappa_{HB}, \kappa_{DB}, \kappa_{HE}\) are linked with the components of the tensor \(k^{\mu\nu}\) by means of the following relations:

\[
\begin{align*}
(k_{DE})^{jk} &= \delta^{jk} k^{00} - k^{jk}, \\
(k_{DB})^{jk} &= -\epsilon^{jkq} k^{0q}, \\
(k_{HE})^{jk} &= -\delta^{jk} k^{ll} + k^{kj}, \\
(k_{ve})^{jk} &= -\epsilon^{jkq} k^{qq}. \quad (12)
\end{align*}
\]

(13)

Considering that the \(k^{\mu\nu}\) is traceless \((k^{ll}_{\mu} = 0)\), it holds \(k^{00} = k^{ii}\), which leads to \(\kappa_{DE} = -\kappa_{HB}\) in accordance with relation (12). Furthermore, the matrix \(\kappa_{DB}\) is written in terms of the three components \(k^{0q}\). All this is consistent with the nonbirefringent character of parametrization (10). Regarding the relations (9) and (13), it is possible to show that

\[
k^{0q} = -k^q. \quad (14)
\]

The gauge propagator for Lagrangian density (11) was evaluated in a matrix form in Ref. [37] for the nonbirefringent parity-odd and the isotropic parity-even component. To compute this gauge propagator in an exact closed tensor form, we will use the prescription (10). We begin writing Lagrangian (1) in a squared form

\[
\mathcal{L} = \frac{1}{2} A_{\mu} D^{\mu\nu} A_\nu, \quad (15)
\]

with \(D^{\mu\nu}\) being a second order tensor operator defined as

\[
D^{\mu\nu} = \square g^{\mu\nu} + \left( \frac{1}{\xi} - 1 \right) \partial^\mu \partial^\nu - S^{\mu\nu}, \quad (16)
\]

where \(g^{\mu\nu} = (+, -, - -)\) is the metric tensor and \(S^{\mu\nu}\) is the symmetric Lorentz-violating operator

\[
S^{\mu\nu} = 2 (K_f)^{\lambda\mu\alpha\nu} \partial_\lambda \partial_\beta = S^{\nu\mu}. \quad (17)
\]

The gauge propagator is the defined as

\[
\langle 0 | T(A_\mu (x) A_\nu (y)) | 0 \rangle = i \Delta_{\mu\nu} (x - y), \quad (18)
\]

where \(\Delta_{\mu\nu}\) is the operator that fulfills the relation:

\[
D^{\mu\beta} \Delta_{\beta\nu} (x - y) = \delta^{\mu\nu} \delta (x - y). \quad (19)
\]

We should compute the gauge propagator in the Feynman gauge, \(\xi = 1\), which implies

\[
D^{\mu\nu} = \square g^{\mu\nu} - S^{\mu\nu}. \quad (20)
\]

Regarding the prescription (10), the \(S^{\lambda\rho}\) operator becomes

\[
S^{\lambda\rho} = \left[ g^{\lambda\delta} k^{\nu\rho} - g^{\rho\delta} k^{\lambda\nu} + g^{\nu\rho} k^{\lambda\delta} - g^{\lambda\rho} k^{\nu\delta} \right] \partial_\beta \partial_\delta. \quad (21)
\]

In the Fourier representation, we have

\[
\delta (x - y) = \int \frac{dp}{(2\pi)^4} e^{-ip (x - y)}, \quad \Delta_{\beta\nu} (x - y) = \int \frac{dp}{(2\pi)^4} \tilde{\Delta}_{\beta\nu} (p) e^{-ip (x - y)}. \quad (22)
\]
with
\[
\tilde{D}^\lambda{}^\rho = -(p^2 g^{\lambda\rho} + \tilde{S}^{\lambda\rho}),
\]
\[
\tilde{S}^{\lambda\rho} = -2 (K_F)_{\lambda\rho} p_\nu p_\delta,
\]
\[
\tilde{S}^{\lambda\rho} = - [p^\lambda p_\nu k^{\nu\rho} - p^2 k^{\lambda\rho} + p^\rho p_\delta k^{\lambda\delta} - g^{\lambda\rho} p_\delta p_\nu k^{\nu\delta}],
\]
so that
\[
\tilde{D}^\lambda{}^\rho = -p^2 g^{\lambda\rho} + p^\lambda p_\nu k^{\nu\rho} - p^2 k^{\lambda\rho} + p^\rho p_\delta k^{\lambda\delta} - g^{\lambda\rho} p_\delta p_\nu k^{\nu\delta}.
\]
For inverting this tensor operator, we must solve the relation
\[
\tilde{D}^\lambda{}^\rho \Delta_{\rho\beta} = \delta^\lambda{}_{\beta}.
\]
For it, we use the general parametrization for a symmetric traceless tensor,
\[
k^{\lambda\rho} = \frac{1}{2} (U^\lambda V^\rho + U^\rho V^\lambda) - \frac{1}{4} g^{\lambda\rho} (U \cdot V),
\]
in terms of two arbitrary four-vectors, \( U^\lambda, V^\rho \), which comprise the Lorentz-violating coefficients. This prescription obviously assures the traceless feature \( k^{\lambda}_{\chi} = 0 \), as expected. Moreover, it holds:
\[
k^{00} = k^{ii} = \frac{3}{4} (U^0 V^0) + \frac{1}{4} (U \cdot V),
\]
\[
k^{ij} = \frac{1}{2} (U^i V^j + U^j V^i),
\]
\[
(\kappa_{DE})^{ik} = \frac{1}{2} (U^i V^j + U^j V^i),
\]
Comparing Eq. \((21)\) with Eq. \((5)\), we note that
\[
(\tilde{\kappa}_{\epsilon -})^{ik} = \frac{1}{2} (U^i V^j + U^j V^i), \quad \kappa_{tr} = \frac{1}{2} (U^0 V^0 + U \cdot V).
\]
Remembering that the matrix \( \tilde{\kappa}_{\epsilon -} \) is traceless, we should impose \( U \cdot V = 0 \), which simply implies
\[
\kappa_{tr} = U^0 V^0 / 2.
\]
After these preliminary definitions, we come back to the propagator evaluation. Replacing the parametrization \((28)\) in Eq. \((26)\), we have:
\[
\tilde{D}^\lambda{}^\rho = \left[ p^2 \left( 1 - \frac{1}{2} U \cdot V \right) + (p \cdot U) (p \cdot V) \right] g^{\lambda\rho} - \frac{1}{2} (U \cdot V) p^\lambda p^\rho + \frac{1}{2} (p \cdot U) (p^\rho V^\lambda + p^\lambda V^\rho)
\]
+ \( \frac{1}{2} (p \cdot V) (p^\rho U^\lambda + p^\lambda U^\rho) - \frac{1}{2} p^2 (U^\lambda V^\rho + U^\rho V^\lambda). \]
In order to solve Eq. \((27)\), we must first find a closed operator algebra, composed by the following projectors:
\[
\Theta_{\rho\beta}, \omega_{\rho\beta}, U_\rho V_\beta, U_\rho V_\beta, p_\rho U_\beta, p_\beta U_\rho, p_\rho V_\beta, p_\beta V_\rho, V_\beta V_\rho, U_\beta U_\rho,
\]
where
\[
\Theta_{\mu\nu} = g_{\mu\nu} - \omega_{\mu\nu}, \quad \omega_{\mu\nu} = p_{\mu} p_{\nu} / p^2,
\]
are the transverse and longitudinal projectors. In this way, it is proposed for the gauge propagator the general form:
\[
\tilde{\Delta}_{\rho\beta} (p) = (a_1 \Theta_{\rho\beta} + a_2 \omega_{\rho\beta} + a_3 U_\rho V_\beta + a_4 U_\beta V_\rho + a_5 p_\rho U_\beta + a_6 p_\beta U_\rho + a_7 p_\beta V_\beta + a_8 p_\beta V_\rho + a_9 U_\beta U_\rho + a_{10} V_\beta V_\rho),
\]
with the coefficients \(a_i\) being functions (of the momentum and of the four-vectors \(U_\mu, V_\nu\)) to be determined. The closed algebra of the projectors is explicitly shown in Table I and Table II.

Performing all the tensor contractions encompassed in the expression (40), we obtain a system of ten equations for the ten coefficients \(a_i\), whose solutions are:

\[
a_1 = -\frac{1}{[p^2 - \frac{1}{2}p^2(U \cdot V) + (p \cdot U)(p \cdot V)]}, \quad a_2 = a_1 \left[\frac{N}{\mathcal{B}(p)}\right],
\]

\[
a_3 = a_4 = \frac{a_1}{2} \left[\frac{p^2[p^2 + \frac{1}{2}(p \cdot V)(p \cdot U)]}{\mathcal{B}(p)}\right],
\]

\[
a_5 = a_6 = \frac{a_1}{2} \left[\frac{p^2(p \cdot V) + (p \cdot U)(p \cdot V)^2 - \frac{1}{4}(p \cdot U)V^2p^2}{\mathcal{B}(p)}\right],
\]

\[
a_7 = a_8 = \frac{a_1}{2} \left[\frac{(p \cdot U)p^2 + (p \cdot U)^2(p \cdot V) - \frac{1}{4}U^2(p \cdot V)p^2}{\mathcal{B}(p)}\right],
\]

\[
a_9 = a_{10} = -\frac{a_1}{4} \left[\frac{p^2[(p \cdot V)^2 - p^2V^2]}{\mathcal{B}(p)}\right],
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
& \Theta^{\lambda}_\rho & \omega^{\lambda}_\rho & U_\rho V_\lambda & U_\lambda V_\rho & p_\lambda p_\rho \\
\hline
\Theta^{\lambda}_\rho & \omega^{\lambda}_\rho & 0 & -p_\lambda p_\rho & U_\lambda p_\rho & 0 \\
\hline
p^2V_\rho & p^2V_\lambda & -p_\lambda p_\rho & -p_\lambda V_\rho & p_\rho V_\lambda & 0 \\
\hline
U_\rho V_\lambda & U_\lambda V_\rho & p_\lambda p_\rho & p_\lambda V_\rho & p_\rho p_\lambda & 0 \\
\hline
U_\lambda p_\rho & U_\rho p_\lambda & 0 & 0 & 0 & 0 \\
\hline
U_\lambda p_\rho & U_\rho p_\lambda & 0 & 0 & 0 & 0 \\
\hline
U_\rho p_\lambda & U_\lambda p_\rho & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
p_\lambda U_\rho & p_\rho V_\lambda & p_\lambda V_\rho & U_\lambda U_\rho & V_\lambda V_\rho \\
\hline
p_\lambda U_\rho & p_\rho V_\lambda & p_\lambda V_\rho & U_\lambda U_\rho & V_\lambda V_\rho \\
\hline
p_\lambda U_\rho & p_\rho V_\lambda & p_\lambda V_\rho & U_\lambda U_\rho & V_\lambda V_\rho \\
\hline
p_\lambda U_\rho & p_\rho V_\lambda & p_\lambda V_\rho & U_\lambda U_\rho & V_\lambda V_\rho \\
\hline
p_\lambda U_\rho & p_\rho V_\lambda & p_\lambda V_\rho & U_\lambda U_\rho & V_\lambda V_\rho \\
\hline
\end{array}
\]

TABLE I: Algebra of tensor projectors.

TABLE II: Algebra of tensor projectors.
where the denominator element is
\[
\Xi(p) = p^4(1 - \frac{V^2U^2}{4}) + \frac{p^2}{4}[4(p \cdot U)(p \cdot V) + (p \cdot V)^2U^2 + (p \cdot U)^2V^2].
\] (43)

With these results, the gauge propagator is properly written as
\[
\langle 0 | T(A_\mu(x)A_\nu(y)) | 0 \rangle = \frac{i}{[p^2 - \frac{1}{2}p^2(U \cdot V) + (p \cdot V)(p \cdot U)]} \Xi(p) \Theta_{\rho \beta} + N(p)\omega_{\rho \beta} + F(p)(U_\rho V_\beta + U_\beta V_\rho) + G(p)(p_\rho U_\beta + p_\beta U_\rho) + H(p)(p_\rho V_\beta + p_\beta V_\rho) + I(p)U_\rho U_\beta + L(p)V_\beta V_\rho \right) \right] \Xi(p)
\] (44)

with the following coefficients:
\[
G(p) = \frac{1}{2}(p \cdot V)p^2 + (p \cdot U)(p \cdot V)^2 - \frac{1}{2}(p \cdot U)p^2V^2,
\] (45)
\[
H(p) = \frac{1}{2}(p \cdot U)p^2 + (p \cdot V)(p \cdot U)^2 - \frac{1}{2}(p \cdot V)p^2U^2,
\] (46)
\[
F(p) = -\frac{p^2}{2}[p^2 + \frac{1}{2}(p \cdot V)(p \cdot U)], \quad I(p) = \frac{1}{4}p^2[p^2V^2 - (p \cdot V)^2], \quad J(p) = \frac{1}{4}p^2[p^2U^2 - (p \cdot U)^2],
\] (47)
\[
N(p) = \frac{a_1}{4a_2} \left[ 4 \Xi[1 - \frac{1}{2}(U \cdot V)] - (p \cdot U)(p \cdot V)p^2U^2 V^2 + (p \cdot V)^2p^2V^2
\] + \[p \cdot V \right)^2p^2U^2 V^2 + (p \cdot U)(p \cdot V)^3U^2 + (p \cdot V)(p \cdot U)^3V^2 \right],
\] (48)
with \( U^2 = U \cdot U = U_\mu U^\mu, V^2 = V \cdot V = V_\mu V^\mu. \)

Taking into account the expression \[28\], an important comment is that the products \( U_\beta U_\rho, V_\beta V_\rho, U_\rho V_\beta \) are first order terms in the Lorentz-violating coefficients of the matrix \( k^{\lambda \rho} \). We thus notice that our exact results involve terms until third order in the coefficients of the matrix \( k^{\lambda \rho} \), although only second order terms contribute to any observable associated with the matrix \( S \). It is still important to mention that this gauge propagator is symmetric before an indices permutation \( (\tilde{\Delta}_{\rho \beta} = \tilde{\Delta}_{\beta \rho}) \) and before the \( U \leftrightarrow V \) permutation, as it really must be.

### III. DISPERSION RELATIONS

The dispersion relations are read off from the poles of the propagator, that is:
\[
p^2[1 - \frac{1}{2}(U \cdot V)] + (p \cdot U)(p \cdot V) = 0,
\] (49)
\[
p^2(1 - \frac{V^2U^2}{4}) + \frac{1}{4}[4(p \cdot U)(p \cdot V) + (p \cdot V)^2U^2 + (p \cdot U)^2V^2] = 0.
\] (50)

From these relations we can analyze the energy stability, causality and unitarity of this theory. First, however, it is interesting to regard the choices of \( U^\mu, V^\mu \), that represent the parity-odd and parity-even components. From now on, we adopt the general notation: \( U^\mu = (U_0, \mathbf{u}), V^\mu = (V_0, \mathbf{v}). \)

We initiate discussing the isotropic parity-even coefficient, \( \kappa_{\text{tr}} \), that can be related only with the temporal components of \( U^\mu, V^\mu \). Taking \( U^\mu = (U_0, 0), V^\mu = (V_0, 0) \) as a first choice, the tensor \( k^{\lambda \rho} \) presents a single nonvanishing component: \( k^{00} = 3U_0V_0/4 = 3\kappa_{\text{tr}}/2 \). The dispersion relation (49) yields
\[
p_0 = |p|\sqrt{\frac{1 - U_0V_0/2}{1 + U_0V_0/2}},
\] (51)
which has to be compared with the dispersion relation of Ref. [37] involving this isotropic component,
\[
p_0 = |p|\sqrt{\frac{1 - \kappa_{\text{tr}}}{1 + \kappa_{\text{tr}}}}.
\] (52)
From this, we state the equality
\[ U_0 V_0 = 2 \kappa_{4\tau}, \]
which coincides with Eq. (33). The second dispersion (50) yields
\[ p_0^2 = p^2 - \frac{(4 - U_0^2 V_0^2)}{4(1 + U_0 V_0) + U_0^2 V_0^2} = p^2 \left[ 2 - \frac{U_0 V_0}{2 + U_0 V_0} \right], \]
which is exactly reduced to Eq. (52) when the replacement (53) is performed. This confirms the result of Ref. [37]: Eq. (52) is the unique dispersion relation for the parity-even isotropic coefficient.

Taking now \( U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0) \), we specify the parity-odd components, having \( k^{0i} = \frac{1}{2} V_0 u^i \). In order to verify it, we write the dispersion relation (49) for this choice:
\[ p_0^2 = p^2 + p_0 (\mathbf{p} \cdot \kappa). \]
This relation becomes equal to the dispersion relation of Ref. [37] for the parity-odd components represented in terms of the 3-vector \( \kappa \),
\[ p_0^2 = p^2 - 2p_0 (\mathbf{p} \cdot \kappa), \]
whenever the following identification is done:
\[ \kappa = -\frac{1}{2} V_0 \mathbf{u}. \]

It is easy to note that the relation (57) is consistent with Eq. (14).

Into the choice \( U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0) \), the dispersion relation (50) is read as
\[ 4p_0^2 - 4p_0 V_0 (\mathbf{p} \cdot \mathbf{u}) = (4 + V_0^2 \mathbf{u}^2) p^2 - (\mathbf{p} \cdot \mathbf{u})^2 V_0^2. \]
Replacing the condition (57) in Eq. (50), it turns out:
\[ p_0^2 + 2p_0 (\mathbf{p} \cdot \kappa) = p^2 + \kappa^2 p^2 - (\mathbf{p} \cdot \kappa)^2, \]
which is exactly the second dispersion relation for the parity-odd sector attained in Ref. [37]. Obviously, the parity-odd components can be also particularized by \( U^\mu = (U_0, 0), V^\mu = (0, \mathbf{v}) \), for which the dispersion relations (49) and (50) become
\[ p_0^2 = p^2 + p_0 (\mathbf{p} \cdot (U_0 \mathbf{v})), \]
\[ 4p_0^2 - 4p_0 U_0 (\mathbf{p} \cdot \mathbf{v}) = (4 + U_0^2 v^2) p^2 - (\mathbf{p} \cdot \mathbf{v})^2 U_0^2. \]

By replacing the condition \( \kappa = -\frac{1}{2} (U_0 \mathbf{v}) \) in Eqs. (60) (61), one recovers the relations (50) (59) of Ref. [37]. We thus notice that both choices, \([U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0)] \) or \([U^\mu = (U_0, 0), V^\mu = (0, \mathbf{v})] \), specify the parity-odd components of the theory.

The third choice is the one that particularizes the anisotropic parity-even components, \( U^\mu = (0, \mathbf{u}), V^\mu = (0, \mathbf{v}) \). With it, the dispersion relations (49) and (50) take the form:
\[ p_0^2 = \left[ p^2 - (\mathbf{p} \cdot \mathbf{u})(\mathbf{p} \cdot \mathbf{v}) \right], \]
\[ p_0^2 = p^2 + \frac{1}{(4 - \mathbf{u}^2 \mathbf{v}^2)} [(\mathbf{p} \cdot \mathbf{v})^2 \mathbf{u}^2 + (\mathbf{p} \cdot \mathbf{u})^2 \mathbf{v}^2 - 4(\mathbf{p} \cdot \mathbf{u})(\mathbf{p} \cdot \mathbf{v})], \]
where the \( \kappa_{\tau \tau} \) traceless condition, \((\mathbf{u} \cdot \mathbf{v}) = 0\), was taken into account. Such dispersion relations were not evaluated in Ref. [37], once the anisotropic parity-even sector was not analyzed there. However, these relations coincide with the ones of the Appendix of Ref. [32] for \((\mathbf{u} \cdot \mathbf{v}) = 0\), except for a negative signal. In this reference, it was analyzed the finite-temperature properties of this parity-even anisotropic electrodynamics using the prescription \((\kappa_{\tau \tau})_{\tau \tau} = (a^1 b^k + a^k b^1)/2\), with \( \mathbf{a} \cdot \mathbf{b} = 0 \). The relative signal difference is compatible with Eq. (32). It may be recovered by a suitable choice in which one of the vectors is taken as opposite, that is, \( \mathbf{u} \rightarrow - \mathbf{u}, \) or \( \mathbf{v} \rightarrow - \mathbf{v} \).

Thus, we can assert that the present prescription recovers all the exact dispersion relations known for CPT-even electrodynamics, and states the new relations (62) (63).
A. Causality and stability analysis

In ref. [37], the dispersion relations [62] were used to investigate the energy stability, causality and unitarity of the CPT-even electrodynamics. It was verified that the parity-odd sector represented by relations [60, 59] is stable, noncausal and unitary, whereas the parity-even sector, described by relation [62], is stable, causal and unitary for some limited values of $\kappa_{tr}$. Once the dispersion relations here derived are shown to recover the ones of Ref. [37] for the parity-odd and parity-even isotropic components, the consistency analysis performed for these two sectors will not be retraced here. However, we now use the tensor propagator [44] for analyzing the dispersion relations and the consistency of the parity-even anisotropic sector.

As it is known, the causality analysis is related to the sign of the propagator poles [40], given in terms of $p^2$, in such a way one must have $p^2 \geq 0$ in order to preserve the causality (preventing the existence of tachyons). We should now adopt a more detailed and confident analysis on causality: the group velocity ($u_g = dp_0/d|p|$) and the front velocity ($u_{front} = \lim_{|p| \to \infty} u_{phase}$). The causality is assured if $u_g \leq 1$ and $u_{front} \leq 1$.

In Ref. [37], the causality of the parity-odd sector was examined, revealing a noncausal theory. The same kind of analysis showed that the isotropic parity-even coefficient provides a causal theory for $0 \leq \kappa_{tr} < 1$.

The causality of the anisotropic parity-even sector, however, was not investigated, remaining to be verified. We take as starting point the dispersion relations [62] and [63], which are now analyzed in the following coordinate system: $x-$axis parallel to $u$, $y-$axis along $v$, and the $z-$axis parallel to $u \times v$. The 3-momentum expressed in spherical coordinates, $p = |p|(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, allows to rewrite the dispersion relation [62] as

$$p_0 = |p|\sqrt{1 - \frac{1}{2}|u||v|\sin^2 \theta \sin 2\phi}, \quad (64)$$

which shows that the energy is always positive since the product $|u||v|$ is small, so the stability is assured. The group and front velocities are

$$u_g = \frac{dp_0}{d|p|} = \sqrt{1 - \frac{1}{2}|u||v|\sin^2 \theta \sin 2\phi}, \quad (65)$$

$$u_{front} = \sqrt{1 - \frac{1}{2}|u||v|\sin^2 \theta \sin 2\phi}, \quad (66)$$

Even for a small background $(|u||v| << 1)$, for $\phi \in \{\pi/2, \pi\} \cup \{3\pi/2, 2\pi\}$ it may occur that $|u_g| > 1$ and $u_{front} > 1$. So, this model is in general noncausal.

For the relation [63], we have

$$p_0 = |p|\sqrt{1 - \frac{\gamma}{2} \left[ \sin 2\phi - \frac{1}{2}|u||v| \right] |u||v|\sin^2 \theta}, \quad (67)$$

where $\gamma = (1 - \frac{1}{4}u^2v^2)^{-1}$. This relation clearly indicates a positive energy for a small product $|u||v|$, implying stability. The group and front velocities are given by

$$u_g = u_{front} = \sqrt{1 - \frac{\gamma}{2} \left[ \sin 2\phi - \frac{1}{2}|u||v| \right] |u||v|\sin^2 \theta}. \quad (68)$$

At the same way, this expression provides $|u_g| > 1, u_{front} > 1$ for $\phi \in \{\pi/2, \pi\} \cup \{3\pi/2, 2\pi\}$. Thus, we conclude that the anisotropic parity-even sector is stable but noncausal.

B. Unitarity analysis

The unitarity analysis of this model at tree-level is here carried out through the saturation of the propagators with external currents [41], which must be implemented by means of the saturated propagator (SP), a scalar quantity given as follows:

$$SP = J^\mu \text{Res}(i\Delta_{\mu\nu}) J^\nu, \quad (69)$$
where \( \text{Res}(i\Delta_{\mu\nu}) \) is the matrix residue evaluated at the pole of the propagator. The gauge current \( (J^\mu) \) satisfies the conservation law \( (\partial_\mu J^\mu = 0) \), which in momentum space is read as \( p_\mu J^\mu = 0 \). In accordance with this method, the unitarity analysis is assured whenever the imaginary part of the saturation \( SP \) (at the poles of the propagator) is positive. A way to carry out the saturation consists in determining the residue of the propagator matrix, evaluated at its own poles.

We begin writing the saturated gauge propagator (taking into account the current conservation):

\[
SP = [-iR] \left[ \text{Res}(p^2) J^2 + 2F(p)(U \cdot J)(V \cdot J) + I(p)(U \cdot J)^2 + L(p)(V \cdot J)^2 \right],
\]

where the terms \( \Xi, F, I, L \), given by Eqs. (43-47), are to be evaluated in the one of the poles of the propagator, and

\[
R = \text{Res} \left[ \frac{1}{p^2(1 - \frac{1}{2}uv\cdot V + (p \cdot u)(p \cdot V))} \right],
\]

is the residue evaluated in the pole.

To examine the unitarity of the anisotropic parity-even sector, we use the parametrization \( U^\mu = (0, u), V^\mu = (0, v) \), where \( u \) and \( v \) are orthogonal 3D-vectors, \( u \cdot v = 0 \), due to the traceless property of matrix \( \kappa_{\alpha\beta} \) [see Eq. (32)].

We examine the pole stemming from the dispersion relation (49),

\[
p^2 = -\eta(p \cdot U)(p \cdot V),
\]

with \( \eta = (1 - \frac{1}{2}(U \cdot V))^{-1} \). In the anisotropic sector, \( \eta = 1 \). In this pole, the residue is

\[
R = \frac{1}{4p^2([p \cdot v]^2u^2 + (p \cdot u)^2v^2 - u^2v^2(p \cdot u)(p \cdot v))}.
\]

The saturation (70) is read as

\[
SP = [-iR] \left[ R^{-1} J^2 + 2F(U \cdot J)(V \cdot J) + I(U \cdot J)^2 + L(V \cdot J)^2 \right],
\]

with

\[
F = \frac{1}{4} p^2(p \cdot v)(p \cdot u),
\]

\[
I = \frac{1}{4} p^2(p \cdot v)((p \cdot u)v^2 - (p \cdot v)),
\]

\[
L = \frac{1}{4} p^2((p \cdot u)v^2 - (p \cdot u)).
\]

Replacing all these expressions in the saturation, we achieve:

\[
SP = -\frac{iR}{4} \left\{ 4R^{-1} J^2 + (p \cdot v)(p \cdot u)[u^2(J \cdot v)^2 + v^2(J \cdot u)^2] - [(p \cdot v)(J \cdot u) - (p \cdot u)(J \cdot v)]^2 \right\}.
\]

In order to verify the positivity of the expression above, it is suitable to define a 3-dimensional basis, generated by the vectors \( \hat{v}, \hat{u} \) and \( \hat{c} \):

\[
\hat{v} = v/|v|, \quad \hat{u} = u/|u|, \quad \hat{c} = (v \times u)/|u||v|.
\]

In this basis, it holds

\[
J \cdot v = J_v |v|; \quad J \cdot u = J_u |u|; \quad p \cdot v = p_v |v|; \quad p \cdot u = p_u |u|,
\]

\[
J^2 = J_0^2 - J^2 = J_0^2 - J^2_c - J^2_u,
\]

\[
p^2 = p^2_\gamma + p^2_\alpha + p^2_\nu - |u|v|p_\alpha p_\nu|.
\]
\[ R = - \frac{1}{\frac{1}{4} |\mathbf{u}|^2 |\mathbf{v}|^2 [p_u^2 + p_v^2 - |\mathbf{u}|v|p_u p_v|]}. \]  

(83)

From Eq. (83), and using the relations (80, 82), we obtain:

\[ SP = -i \frac{1}{4} R \left\{ 4R^{-1}J^2 + p_0p_v |\mathbf{u}|^3 |\mathbf{v}|^3 [J_u^2 + J_v^2] - |\mathbf{u}|^2 |\mathbf{v}|^2 [p_v J_u - p_u J_v]^2 \right\}, \]

(84)

which is equivalent to:

\[ SP = i \left\{ -J_0^2 + J_c^2 - \frac{|\mathbf{u}|^2 |\mathbf{v}|^2}{4} R [p_v J_u + p_u J_v]^2 \right\}. \]

(85)

Making more algebraic manipulations, and using the current conservation, \( p_0 J_0 = p_v J_v + p_u J_u + p_c J_c \), the saturation takes the form

\[ SP = i \left[ \frac{J_c}{[p_v^2 + p_u^2 - |\mathbf{u}|v|p_u p_v|] [p_v^2 + p_u^2 + p_c^2 - |\mathbf{u}|v|p_u p_v|]} > 0, \right. \]

(86)

which is compatible with the unitarity validity. In the result above the denominator term \( p_c^2 + p_u^2 - |\mathbf{u}|v|p_u p_v| \) was taken as positive. It occurs whenever it holds the condition: \(|\mathbf{u}|v| < 2\). As the magnitude of the Lorentz-violating parameters is always much smaller than 1, this condition is fulfilled. A similar development can be accomplished for the pole stemming from the dispersion relation (50). We thus assert that the anisotropic parity-even sector is noncausal and unitary.

**IV. CONCLUSIONS**

In this work, we have exactly evaluated the gauge propagator for the nonbirefringent CPT-even electrodynamics of SME using a prescription proposed in Ref. [38] and a parametrization for the symmetric \( k^{\lambda \delta} \) in terms of two arbitrary four-vectors. These parametrization allowed to obtain an exact tensor form for the propagator of the nonbirefringent components which recovers the gauge propagator expressions is the suitable parametrization choices are adopted. The involved dispersion relations coincide with the ones obtained in Ref. [37] for the isotropic parity-even component and for the three parity-odd nonbirefringent components. Furthermore, the dispersion relations for the anisotropic parity-even components were achieved as well. The analysis of stability, causality and unitarity for the anisotropic parity-even components was performed, revealing that this sector is stable, noncausal and unitary. This study completes the analysis initiated in Ref. [37]. The achievement of a tensor form propagator assures the facilities of the tensor calculus for some interesting applications, as scattering amplitude evaluation in a Quantum Electrodynamics context. Some investigations in this direction are now under development.

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Feynman propagator for the nonbirefringent CPT-even electrodynamics of the Standard Model Extension

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The CPT-even gauge sector of the standard model extension is composed of nineteen components comprised in the tensor \((K_F)_{\mu\nu\rho\sigma}\), of which nine do not yield birefringence. In this work, we examine the Maxwell electrodynamics supplemented by these nine nonbirefringent CPT-even components in aspects related to the Feynman propagator and full consistency (stability, causality, unitarity). We adopt a prescription that parametrizes the nonbirefringent components in terms of a symmetric and traceless tensor, \(K_{\mu\nu}\), and second parametrization that writes \(K_{\mu\nu}\) in terms of two arbitrary four-vectors, \(U_\mu\) and \(V_\nu\). We then explicitly evaluate the gauge propagator of this electrodynamics in a tensor closed way. In the sequel, we show that this propagator and involved dispersion relations can be specialized for the parity-odd and parity-even sectors of the tensor \((K_F)_{\mu\nu\rho\sigma}\). In this way, we reassess some results of the literature and derive some new outcomes showing that the parity-even anisotropic sector engenders a stable, noncausal and unitary electrodynamics.

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I. INTRODUCTION

Lorentz symmetry violation has been an issue of permanent interest in the past few years, with many investigations in the context of the standard model extension (SME) \cite{1, 2}. The SME incorporates terms of Lorentz invariance violation (LIV) in all sectors of interaction and has been studied in many respects \cite{3, 4}. The investigations in the context of the SME concern mainly the fermion sector \cite{5, 6}, the gauge sector \cite{7}-\cite{26}, and extensions involving gravity \cite{27}. The violation of Lorentz symmetry has also been addressed in other theoretical frameworks \cite{28}, with many interesting developments \cite{29, 30}.

The CPT-odd gauge sector of the SME, represented by the Carroll-Field-Jackiw (CFJ) electrodynamics \cite{7}, has its properties largely examined in literature, addressing consistency aspects \cite{8} and modifications induced in QED \cite{8, 10}, dimensional reduction \cite{11}, supersymmetry \cite{12}, controversies discussing the radiative generation of the CFJ term \cite{13}, the vacuum emission of Cherenkov radiation \cite{14}, the electromagnetic propagation in waveguides \cite{15}, modifications on the Casimir effect \cite{16}, effects on the Planck distribution and finite-temperature contributions \cite{17}, \cite{18}, possible effects on the anisotropies of the Cosmic Microwave Background Radiation \cite{19}. Since 2002 the CPT-even sector of SME has been also much investigated, mainly in connection with issues able to provide good bounds on its 19 LIV coefficients. The studies about the properties of the CPT-even electrodynamics, represented by the tensor \((K_F)_{\alpha\nu\rho\sigma}\), were initiated by Kostelecky and Mewes in Refs. \cite{20, 21}, where it was stipulated the existence of ten linearly independent combinations of the components of \((K_F)_{\alpha\nu\rho\sigma}\) sensitive to birefringence. A broader and interesting study in this respect was performed recently in Ref. \cite{22}. These elements are contained in two \(3 \times 3\) matrices, \(\kappa_{\alpha+}\) and \(\kappa_{\alpha-}\). Using high-quality spectropolarimetry data of cosmological sources \cite{23}, stringent upper bounds \((10^{-32}\) and \(10^{-37}\)) were imposed on these birefringent LIV parameters. Since 2003, precise experiments involving rotating optical and microwave resonators have been performed \cite{24}, yielding bounds at the level of until one part in \(10^{17}\) on the CPT-even parameters. The study of Cherenkov radiation \cite{25} and the absence of emission of Cherenkov radiation by ultrahigh energy cosmic rays \cite{31, 32} has been a point of great interest in latest years, as well as the photon-fermion vertex interactions yielding new bounds on the LIV coefficients \cite{33, 35, 36}. Investigations on finite-temperature properties and the implied modifications on the Planck law were developed as well for the CPT-even sector \cite{37, 38}. A full evaluation of the dispersion relations of the CPT-even electrodynamics in connection with the birefringent role played by the LIV coefficients is also presented in Refs. \cite{37, 38, 39}. More recently, the birefringence of the CPT-even coefficients at leading and higher orders is discussed in Ref. \cite{40}.
In a recent work, the gauge propagator of the CPT-even electrodynamics of the SME has been explicitly carried out in the form of a $4 \times 4$ matrix \[39\]. The dispersion relations were determined from the poles of the propagator, and used to analyze the stability, causality and unitarity of this theory for the nonbirefringent parity-odd components and for the isotropic parity-even one. The pole analysis showed that the parity-odd sector is stable, noncausal, and unitary, whereas the parity-even isotropic sector, represented uniquely by the trace component, provides a stable, causal, and unitary theory for the range $0 \leq \kappa_{\text{tr}} < 1$.

In the present work, we evaluate the Feynman gauge propagator for the nine nonbirefringent coefficients of the CTP-even sector of SME in an exact tensor form, using the parametrization of the tensor $(K_F)_{\alpha\nu\rho\phi}$ in terms of a symmetric and traceless rank-2 tensor, $k_{\alpha\beta}$, introduced in Ref.\[11\]. In order to evaluate the propagator, the tensor $k_{\alpha\beta}$, in turn, is parametrized in terms of two four-vectors, $U_\mu, V_\nu$, that contain the Lorentz-violating components. Once the propagator is explicitly written, it is specialized for the parity-odd and parity-even sectors of this theory for some choices of $U_\mu, V_\nu$, yielding the dispersion relations attained in Ref.\[39\]. New investigations concerning the anisotropic parity-even components are performed, revealing that this sector is stable, noncausal, and unitary.

II. THEORETICAL MODEL AND THE GAUGE PROPAGATOR

The Lagrangian of the CPT-even electrodynamics of SME is
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (K_F)_{\mu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} - \frac{1}{2\xi} (\partial_{\mu} A^{\mu})^2, \tag{1}
\]
where $\xi$ is the gauge fixing parameter, $F_{\alpha\nu}$ is the electromagnetic field tensor, and $(K_F)_{\alpha\nu\rho\phi}$ is a renormalizable and dimensionless coupling, responsible for Lorentz violation. The tensor $(K_F)_{\alpha\nu\rho\phi}$ has the same symmetries as the Riemann tensor $(K_F)_{\alpha\nu\rho\phi} = - (K_F)_{\nu\alpha\rho\phi}$, $(K_F)_{\alpha\nu\rho\phi} = (K_F)_{\rho\nu\alpha\phi}$, $(K_F)_{\alpha\nu\rho\phi} = (K_F)_{\rho\nu\alpha\phi}$, $(k_F)_{\alpha\beta\rho\phi} + (k_F)_{\alpha\beta\phi\rho} + (k_F)_{\alpha\rho\beta\phi} = 0$, and a double null trace, $(K_F)^{\rho\phi}_{\rho\phi} = 0$. The tensor $(K_F)_{\alpha\nu\rho\phi}$ has 19 independent components, from which nine do not yield birefringence. A very useful parametrization for addressing this theory is the one presented in Refs. \[20, 21\], in which these 19 components are contained in four $3 \times 3$ matrices:
\[
(\kappa_{\text{e}+})^{jk} = \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk}, \quad (\kappa_{\text{e}-})^{jk} = \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \frac{1}{3} \delta^{jk}(\kappa_{DE})^{ii}, \quad \kappa_{tr} = \frac{1}{3}\text{tr}(\kappa_{DE}), \tag{2}
\]
\[
(\kappa_{o+})^{jk} = \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk}, \quad (\kappa_{o-})^{jk} = \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk}, \tag{3}
\]
where $\kappa_{\text{e}}$ and $\kappa_{o}$ designate parity-even and parity-odd matrices, respectively. The $3 \times 3$ matrices $\kappa_{DE}, \kappa_{HB}, \kappa_{DB}, \kappa_{HE}$ are defined in terms of the $(K_F)$ tensor components:
\[
(\kappa_{DE})^{jk} = -2(K_F)^{ij0k}, \quad (\kappa_{HB})^{jk} = \frac{1}{2} \epsilon^{ipq} \epsilon^{klm} (K_F)^{pqlm}, \tag{4}
\]
\[
(\kappa_{DB})^{jk} = - (\kappa_{HE})^{kj} = \epsilon^{pq} (K_F)^{ipq}. \tag{5}
\]
The matrices $\kappa_{DE}, \kappa_{HB}$ contain together 11 independent components while $\kappa_{DB}, \kappa_{HE}$ possess together eight components, which sums the 19 independent elements of the tensor $(K_F)_{\alpha\nu\rho\phi}$. From these 19 coefficients, ten are sensitive to birefringence and nine are nonbirefringent. These latter ones are contained in the matrices $\kappa_{o+}$ and $\kappa_{e-}$. The analysis of birefringence data reveals the coefficients of the matrices $\kappa_{e+}$ and $\kappa_{e-}$ are bounded to the level of one part in $10^{32}$ \[20, 21\] and one part in $10^{37}$ \[22\]. This leads to the following constraints: $\kappa_{DE} = - \kappa_{HB}, \kappa_{DB} = \kappa_{HE},$ which implies
\[
(\kappa_{o+})^{jk} = (\kappa_{DB})^{jk}, \quad (\kappa_{e-})^{jk} = (\kappa_{DE})^{jk} - \delta^{jk}\kappa_{tr}, \tag{6}
\]
where the matrix $\kappa_{e-}$ is symmetric and traceless (has five components) and the matrix $\kappa_{DB}$ has now become antisymmetric, possessing only three components written in terms of a 3-vector \[42\].
\[
\kappa^{i} = \frac{1}{2} \epsilon^{ipq} (K_F)_{pq}. \tag{7}
\]
An interesting way to parametrize the nine nonbirefringent components of the tensor $K_F$ is the one introduced in Ref. [41], in which it is written in terms of a symmetric traceless tensor $k_{\nu\rho}$:

$$\langle K_F \rangle^{\lambda\delta\rho} = \frac{1}{2} \left[ g^{\lambda\delta} k^{\nu\rho} - g^{\nu\delta} k^{\lambda\rho} + g^{\nu\rho} k^{\lambda\delta} - g^{\lambda\rho} k^{\nu\delta} \right].$$

Here, the nine nonbirefringent components are all contained in the symmetric traceless tensor $k_{\nu\rho}$, defined as the contraction

$$k^{\mu\nu} = \langle K_F \rangle^{\mu\nu}_{\alpha}. \quad \text{(9)}$$

The components of matrices $\kappa_{DE}, \kappa_{HB}, \kappa_{DB}, \kappa_{HE}$ are linked with the components of the tensor $k^{\mu\nu}$ by means of the following relations:

$$(\kappa_{DE})^{jk} = \delta^{jk} k^{00} - k^{jk}, \quad (\kappa_{DB})^{jk} = -\epsilon^{jkq} k^{0q}, \quad (\kappa_{HB})^{jk} = -\delta^{jk} k^{11} + k^{kj}, \quad \text{(10)}$$

$$(\kappa_{DE})^{jk} = -\epsilon^{jkq} k^{0q}, \quad (\kappa_{DB})^{jk} = \delta^{jk} k^{00} - k^{jk} - \delta^{jk} k^{i\nu}, \quad \text{(11)}$$

Considering that the $k^{\mu\nu}$ is traceless ($k^{\mu\mu} = 0$), it holds $k^{00} = k^{ii}$, which leads $\kappa_{DE} = -\kappa_{HB}$ in accordance with relation (10). Furthermore, the matrix $\kappa_{DB}$ is written in terms of the three components $k^{0q}$. All this is consistent with the nonbirefringent character of parametrization (5). Regarding the relations (7) and (11), it is possible to show that

$$k^{0q} = -\kappa^3. \quad \text{(12)}$$

The gauge propagator for Lagrangian density (11) was evaluated in a matrix form in Ref. [39] for the nonbirefringent parity-odd and the isotropic parity-even component. To compute this gauge propagator in an exact closed tensor form, we will use the prescription (8). We begin writing Lagrangian (11) in a squared form

$$\mathcal{L} = \frac{1}{2} A_{\mu} D^{\mu\nu} A_{\nu}, \quad \text{(13)}$$

where $D^{\mu\nu} = \Box g^{\mu\nu} + (1/\xi - 1) \partial^\mu \partial^\nu - S^{\mu\nu}$ is a second order tensor operator, $S^{\mu\nu}$ is the symmetric Lorentz-violating operator, $S^{\mu\nu} = 2\langle K_F \rangle^{\mu\alpha\beta\nu} \partial_\alpha \partial_\beta$, and $g^{\mu\nu} = (+, -, -)$ is the metric tensor adopted here. The gauge propagator is defined as $\langle 0 | T[A_\mu(x) A_\nu(y)] | 0 \rangle = i \Delta_{\mu\nu}(x - y)$, where $\Delta_{\mu\nu}$ is the operator that fulfills the relation: $D^{\mu\beta} \Delta_{\beta\nu}(x - y) = \delta^{\mu\nu} \delta(x - y)$. We should compute the gauge propagator in the Feynman gauge, $\xi = 1$, which implies $D^{\mu\nu} = \Box g^{\mu\nu} - S^{\mu\nu}$. Regarding the prescription (5), the $S^{\lambda\rho}$ operator becomes

$$S^{\lambda\rho} = \left[ g^{\lambda\delta} k^{\nu\rho} - g^{\nu\delta} k^{\lambda\rho} + g^{\nu\rho} k^{\lambda\delta} - g^{\lambda\rho} k^{\nu\delta} \right] \partial_\nu \partial_\delta. \quad \text{(14)}$$

In the Fourier representation, we have $\hat{D}^{\lambda\rho} = -(p^2 g^{\lambda\rho} + \hat{S}^{\lambda\rho})$, $\hat{S}^{\lambda\rho} = -2\langle K_F \rangle^{\lambda\rho\delta} p_\nu p_\delta$, leading to

$$\hat{S}^{\lambda\rho} = - \left[ p^{\lambda} p_\nu k^{\nu\rho} - p^2 k^{\lambda\rho} + p^\rho p_\delta k^{\lambda\delta} - g^{\lambda\rho} p_\delta p_\nu k^{\nu\delta} \right], \quad \hat{D}^{\lambda\rho} = -p^2 g^{\lambda\rho} + p^{\lambda} p_\nu k^{\nu\rho} - p^2 k^{\lambda\rho} + p^\rho p_\delta k^{\lambda\delta} - g^{\lambda\rho} p_\delta p_\nu k^{\nu\delta}. \quad \text{(15)}$$

For inverting this tensor operator, we must solve the relation $\hat{D}^{\lambda\rho} \Delta_{\rho\beta} = \delta^{\lambda\beta}$. For it, we use the general parametrization for a symmetric traceless tensor,

$$k^{\lambda\rho} = \frac{1}{2} (U^\lambda V^\rho + U^\rho V^\lambda) - \frac{1}{4} g^{\lambda\rho} (U \cdot V), \quad \text{(16)}$$

in terms of two arbitrary four-vectors, $U^\lambda, V^\rho$, which comprise the Lorentz-violating coefficients. This prescription obviously assures the traceless feature ($k^{\lambda\lambda} = 0$), as expected. Moreover, it holds:

$$k^{00} = k^{ii} = \frac{3}{4} U^0 V^0 + \frac{1}{4} (U \cdot V), \quad \text{(17)}$$

$$k^{ij} = \frac{1}{2} (U^i V^j + U^j V^i) + \frac{1}{4} \delta^{ij} (U^0 V^0 - U \cdot V), \quad \text{(18)}$$

$$(\kappa_{DE})^{ij} = -\frac{1}{2} (U^i V^j + U^j V^i) + \frac{1}{2} \delta^{ij} (U^0 V^0 + U \cdot V). \quad \text{(19)}$$
Comparing Eq. (19) with Eq. (20), we note that
\[
(\tilde{\kappa}_e^-)^{jk} = -\frac{1}{2} (U^i V_j + U^j V_i), \quad \kappa_{tr} = \frac{1}{2} (U^0 V^0 + U \cdot V).
\] (20)

Remembering that the matrix \(\tilde{\kappa}_e^-\) is traceless, we should impose \(U \cdot V = 0\), which simply implies
\[
\kappa_{tr} = U^0 V^0 / 2.
\] (21)

After these preliminary definitions, we come back to the propagator evaluation. Replacing the parametrization (10) in Eq. (15), we have:
\[
\tilde{D}_a = -\left[ p^2 \left( 1 - \frac{1}{2} U \cdot V \right) + (p \cdot U)(p \cdot V) \right] \delta^{\lambda \rho} - \frac{1}{2} (U \cdot V) p^\lambda p^\rho + \frac{1}{2} (p \cdot U) \left( p^6 V^\lambda + p^\lambda V^6 \right)
+ \frac{1}{2} (p \cdot V) \left( p^6 U^\lambda + p^\lambda U^6 \right) - \frac{1}{2} p^2 \left( U^6 V^\rho + U^\rho V^6 \right).
\] (22)

In order to solve the relation \(\tilde{D}_a \tilde{\Delta}_{\rho \beta} = \delta^{\lambda \beta}\), we must first find a closed operator algebra, composed by the following projectors:
\[
\Theta_{\rho \beta}, \omega_{\rho \beta}, \ U_\rho V_\beta, \ U_\beta V_\rho, \ p_\rho U_\beta, \ p_\beta U_\rho, \ p_\rho V_\beta, \ p_\beta V_\rho, \ V_\beta V_\rho, \ U_\beta U_\rho,
\] (23)
where \(\Theta_{\mu \nu} = g_{\mu \nu} - \omega_{\mu \nu}\), \(\omega_{\mu \nu} = p_\mu p_\nu / p^2\) are the transverse and longitudinal projectors. In this way, it is proposed for the gauge propagator the general form:
\[
\tilde{\Delta}_{\rho \beta} (p) = (a_1 \Theta_{\rho \beta} + a_2 \omega_{\rho \beta} + a_3 U_\rho V_\beta + a_4 U_\beta V_\rho + a_5 p_\rho U_\beta + a_6 p_\beta U_\rho + a_7 p_\rho V_\beta + a_8 p_\beta V_\rho + a_9 U_\beta U_\rho + a_{10} V_\beta V_\rho),
\] (24)
with the coefficients \(a_i\) being functions (of the momentum and of the four-vectors \(U_\mu, V_\nu\)) to be determined. The closed algebra of the projectors is explicitly shown in Table I and Table II.

Performing all the tensor contractions, we obtain a system of ten equations for the ten coefficients \(a_i\), whose solutions is
\[
a_1 = -\frac{1}{[p^2 - \frac{1}{2} p^2 (U \cdot V) + (p \cdot U)(p \cdot V)]}, \quad a_2 = a_1 \left[ \frac{N}{\Xi(p)} \right],
\] (25)
\[
a_3 = a_4 = -\frac{a_1}{2} \left[ \frac{p^2 [p^2 + \frac{1}{2} (p \cdot V)(p \cdot U)]}{\Xi(p)} \right],
\] (26)
with the following coefficients:

\[
\begin{align*}
\Theta^\lambda_{\rho} & \quad p_{\rho}U^\lambda - (p \cdot U)p_{\rho}p^\lambda/ho^2 \\
\omega^\lambda_{\rho} & \quad (p \cdot U)p_{\rho}p^\lambda/ho^2 \quad p_{\rho}V^\lambda - (p \cdot V)p_{\rho}p^\lambda/ho^2 \\
p^\lambda V^\rho & \quad (p \cdot U)V^\lambda p_{\rho} - (p \cdot V)V^\lambda p_{\rho} \\
p^\lambda U^\rho & \quad (p \cdot U)V^\lambda p_{\rho} - (p \cdot V)V^\lambda p_{\rho} \\
p^\rho p^\rho & \quad (p \cdot U)V^\lambda p_{\rho} - (p \cdot V)V^\lambda p_{\rho} \\
U^\rho V^\rho & \quad (p \cdot U)V^\lambda p_{\rho} - (p \cdot V)V^\lambda p_{\rho} \\
U^\rho V^\rho & \quad (p \cdot U)V^\lambda p_{\rho} - (p \cdot V)V^\lambda p_{\rho}
\end{align*}
\]

Table II: Algebra of tensor projectors.

\[
\begin{align*}
a_5 &= a_6 = \frac{a_1}{2} \left[ \frac{p^2(p \cdot V) + (p \cdot U)(p \cdot V)^2 - \frac{1}{2}(p \cdot U)V^2p^2}{\Xi(p)} \right], \\
a_7 &= a_8 = \frac{a_1}{2} \left[ \frac{(p \cdot U)p^2 + (p \cdot U)(p \cdot V) - \frac{1}{2}U^2(p \cdot V)p^2}{\Xi(p)} \right], \\
a_9 &= a_{10} = -\frac{a_1}{4} \left[ \frac{p^2[(p \cdot V)^2 - p^2V^2]}{\Xi(p)} \right],
\end{align*}
\]

where the denominator element is

\[
\Xi(p) = p^4(1 - \frac{V^2U^2}{4}) + \frac{p^2}{4}[4(p \cdot U)(p \cdot V) + (p \cdot V)^2U^2 + (p \cdot U)^2V^2].
\]

With these results, the gauge propagator is properly written as

\[
\langle 0 | T(A_\rho(x)A_\beta(y)) | 0 \rangle = -\frac{i}{[p^2 - \frac{1}{2}(p \cdot U)(p \cdot V) + (p \cdot U)(p \cdot V)] \Xi(p)} \left\{ \Xi(p)\Theta_{\rho\beta} + N(p)\omega_{\rho\beta} + F(p)(U_\rho V_\beta + U_\beta V_\rho) \\
+ G(p)(p_\rho U_\beta + p_\beta U_\rho) + H(p)(p_\rho V_\beta + p_\beta V_\rho) + I(p)U_\beta U_\rho + L(p)V_\beta V_\rho \right\},
\]

with the following coefficients:

\[
\begin{align*}
G(p) &= \frac{1}{2}(p \cdot V)p^2 + (p \cdot U)(p \cdot V)^2 - \frac{1}{2}(p \cdot U)p^2V^2, \\
H(p) &= \frac{1}{2}(p \cdot V)p^2 + (p \cdot V)(p \cdot U)^2 - \frac{1}{2}(p \cdot V)p^2U^2, \\
F(p) &= -\frac{p^2}{2}[p^2 + \frac{1}{2}(p \cdot V)(p \cdot U)], \\
I(p) &= \frac{1}{4}p^2[p^2V^2 - (p \cdot V)^2], \\
J(p) &= \frac{1}{4}p^2[p^2U^2 - (p \cdot U)^2], \\
N(p) &= \frac{a_1}{4a_2} \left[ 4\Xi[1 - \frac{1}{2}(U \cdot V)] - (p \cdot U)(p \cdot V)p^2U^2V^2 + (p \cdot V)^2p^2U^2 \\
&\quad + (p \cdot U)^2p^2V^2 + (p \cdot U)(p \cdot V)^3U^2 + (p \cdot V)(p \cdot U)^3V^2 \right],
\end{align*}
\]
with \( U^2 = U \cdot U = U_\mu U^\mu, V^2 = V \cdot V = V_\mu V^\mu \).

Taking into account the expression (16), an important comment is that the products \( U_\beta U_\rho, V_\beta V_\rho, U_\rho V_\beta \) are first order terms in the Lorentz-violating coefficients of the matrix \( k^{\lambda \rho} \). We thus notice that our exact results involve terms until third order in the coefficients of the matrix \( k^{\lambda \rho} \), although only second order terms contribute to any observable associated with the matrix \( S \). It is still important to mention that this gauge propagator is symmetric before an indices permutation (\( \tilde{\Delta}_{\rho \beta} = \tilde{\Delta}_{\beta \rho} \)) and before the \( U \leftrightarrow V \) permutation, as it really must be.

### III. DISPERSION RELATIONS

The dispersion relations are read off from the poles of the propagator, that is

\[
p^2 \left[ 1 - \frac{1}{2} (U \cdot V) \right] + (p \cdot U)(p \cdot V) = 0, \tag{36}
\]

\[
p^2 \left( 1 - \frac{V^2 U^2}{4} \right) + \frac{1}{4} [4(p \cdot U)(p \cdot V) + (p \cdot V)^2 U^2 + (p \cdot U)^2 V^2] = 0. \tag{37}
\]

From these relations we can analyze the energy stability, causality, and unitarity of this theory. First, however, it is interesting to regard the choices of \( U_\mu, V_\mu \), that represent the parity-odd and parity-even components. From now on, we adopt the general notation:

\[
U_\mu = (U_0, \mathbf{u}), \quad V_\mu = (V_0, \mathbf{v}).
\]

We initiate discussing the isotropic parity-even coefficient, \( \kappa_{tr} \), that can be related only with the temporal components of \( U_\mu, V_\mu \). Taking \( U_\mu = (0, \mathbf{u}), V_\mu = (V_0, 0) \) as a first choice, the tensor \( k^{\lambda \rho} \) presents a single nonvanishing component:

\[
k^{00} = \frac{3}{4} U_0 V_0 = \frac{3}{2} \kappa_{tr}/2.
\]

The dispersion relation (36) yields

\[
p_0 = |p| \sqrt{\frac{1 - U_0 V_0/2}{1 + U_0 V_0/2}} , \tag{38}
\]

which has to be compared with the dispersion relation of Ref. [39] involving this isotropic component,

\[
p_0 = |p| \sqrt{(1 - \kappa_{tr})/(1 + \kappa_{tr})}. \tag{39}
\]

From this, we state the equality

\[
U_0 V_0 = 2 \kappa_{tr} , \tag{40}
\]

which coincides with Eq. (21). The second dispersion (37) yields

\[
p_0^2 = p^2 \frac{4 - U_0^2 V_0^2}{4(1 + U_0 V_0) + U_0^2 V_0^2} = p^2 \left[ \frac{2 - U_0 V_0}{2 + U_0 V_0} \right] , \tag{41}
\]

which is exactly reduced to Eq. (39) when the replacement (40) is performed. This confirms the result of Ref. [39]: Eq. (39) is the unique dispersion relation for the parity-even isotropic coefficient.

Taking now \( U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0) \), we specify the parity-odd components, having \( k^{0i} = \frac{1}{4} V_0 u^i \). In order to verify it, we write the dispersion relation (36) for this choice:

\[
p_0^2 = p^2 + p_0 p \cdot (V_0 \mathbf{u}) . \tag{42}
\]

This relation becomes equal to the dispersion relation of Ref. [39] for the parity-odd components represented in terms of the 3-vector \( \kappa \),

\[
p_0^2 = p^2 - 2p_0 (p \cdot \kappa) , \tag{43}
\]

whenever the following identification is done:

\[
\kappa = -\frac{1}{2} V_0 \mathbf{u} . \tag{44}
\]
It is easy to note that the relation (44) is consistent with Eq. (12). Into the choice \( U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0) \), the dispersion relation is read as

\[
4p_0^2 - 4p_0 V_0 (\mathbf{p} \cdot \mathbf{u}) = (4 + V_0^2 \mathbf{u}^2) p^2 - (\mathbf{p} \cdot \mathbf{u})^2 V_0^2.
\]

Replacing the condition (44) in Eq. (37), it turns out

\[
p_0^2 + 2p_0 (\mathbf{p} \cdot \kappa) = p^2 + \kappa^2 p^2 - (\mathbf{p} \cdot \kappa)^2,
\]

which is exactly the second dispersion relation for the parity-odd sector attained in Ref. [39]. Obviously, the parity-odd components can be also particularized by \( U^\mu = (U_0, 0), V^\mu = (0, \mathbf{v}) \), for which the dispersion relations (36) and (37) become

\[
p_0^2 = p^2 + p_0 \mathbf{p} \cdot (U_0 \mathbf{v}),
\]

\[
4p_0^2 - 4p_0 U_0 (\mathbf{p} \cdot \mathbf{v}) = (4 + U_0^2 \mathbf{v}^2) p^2 - (\mathbf{p} \cdot \mathbf{v})^2 U_0^2.
\]

By replacing the condition \( \kappa = -\frac{1}{2} (U_0 \mathbf{v}) \) in Eqs. (37), one recovers the relations (43, 46) of Ref. [39]. We thus notice that both choices, \( [U^\mu = (0, \mathbf{u}), V^\mu = (V_0, 0)] \) or \( [U^\mu = (U_0, 0), \ V^\mu = (0, \mathbf{v})] \), specify the parity-odd components of the theory.

The third choice is the one that particularizes the anisotropic parity-even components, \( U^\mu = (0, \mathbf{u}), V^\mu = (0, \mathbf{v}) \). With it, the dispersion relations (36) and (37) take the form

\[
p_0^2 = \left[ p^2 - (\mathbf{p} \cdot \mathbf{u})(\mathbf{p} \cdot \mathbf{v}) \right],
\]

\[
p_0^2 = p^2 + \frac{1}{4 (1 - \mathbf{u}^2 \mathbf{v}^2)} [(\mathbf{p} \cdot \mathbf{v})^2 \mathbf{u}^2 + (\mathbf{p} \cdot \mathbf{u})^2 \mathbf{v}^2 - 4(\mathbf{p} \cdot \mathbf{u})(\mathbf{p} \cdot \mathbf{v})],
\]

where the \( \tilde{\kappa}_{e^-} \) traceless condition, \( (\mathbf{u} \cdot \mathbf{v} = 0) \), was taken into account. Such dispersion relations were not evaluated in Ref. [39], once the anisotropic parity-even sector was not analyzed there. However, these relations coincide with the ones of the Appendix of Ref. [37] for \( (\mathbf{u} \cdot \mathbf{v} = 0) \), except for a negative signal. In this reference, it was analyzed the finite-temperature properties of this parity-even anisotropic electrodynamics using the prescription \( (\tilde{\kappa}_{e^-})^k = (a^i b^k + a^k b^i)/2 \), with \( \mathbf{a} \cdot \mathbf{b} = 0 \). The relative signal difference is compatible with Eq. (20). It may be recovered by a suitable choice in which one of the vectors is taken as opposite, that is, \( \mathbf{u} \to - \mathbf{u}, \text{ or } \mathbf{v} \to - \mathbf{v} \).

Thus, we can assert that the present prescription recovers all the exact dispersion relations known for CPT-even electrodynamics, and states the new relations (49, 50).

### A. Causality and stability analysis

In ref. [39], the dispersion relations (36, 43, 46) were used to investigate the energy stability, causality and unitarity of the CPT-even electrodynamics. It was verified that the parity-odd sector represented by relations (43, 46) is stable, noncausal and unitary, whereas the parity-even sector, described by relation (39), is stable, causal and unitary for some limited values of \( \kappa_{tr} \). Once the dispersion relations here derived are shown to recover the ones of Ref. [39] for the parity-odd and parity-even isotropic components, the consistency analysis performed for these two sectors will not be retaken here. However, we now use the tensor propagator (51) for analyzing the dispersion relations and the consistency of the parity-even anisotropic sector.

As it is known, the causality analysis is related to the sign of the propagator poles (43), given in terms of \( p^2 \), in such a way one must have \( p^2 > 0 \) in order to preserve the causality (preventing the existence of tachyons). We should now adopt a more detailed and confident analysis on causality: the group velocity \( (u_g = dp_0/dp) \) and the front velocity \( (u_{front} = \lim_{p \to \infty} u_{phase}) \). The causality is assured if \( u_g \leq 1 \) and \( u_{front} \leq 1 \).

In Ref. [39], the causality of the sector parity-odd was examined, revealing a noncausal theory. The same kind of analysis showed that the isotropic parity-even coefficient provides a causal theory for \( 0 \leq \kappa_{tr} < 1 \).

The causality of the anisotropic parity-even sector, however, was not investigated, remaining to be verified. We take as starting point the dispersion relations (39, 50), which are now analyzed in the following coordinate system:
$x$–axis parallel to $u$, $y$–axis along $v$, and the $z$–axis parallel to $u \times v$. The 3-momentum expressed in spherical coordinates, $p = |p| (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, allows one to rewrite the dispersion relation (49) as

$$p_0 = |p| \sqrt{1 - \frac{1}{2} |u||v| \sin^2 \theta \sin 2\phi},$$

(51)

which shows that the energy is always positive since the product $|u||v|$ is small, so the stability is assured. The group and front velocities are

$$u_g = \frac{d p_0}{d |p|} = \sqrt{1 - \frac{1}{2} |u||v| \sin^2 \theta \sin 2\phi},$$

(52)

$$u_{\text{front}} = \sqrt{1 - \frac{1}{2} |u||v| \sin^2 \theta \sin 2\phi},$$

(53)

Even for a small background ($|u||v| \ll 1$), for $\phi \in \langle \pi/2, \pi \rangle \cup \langle 3\pi/2, 2\pi \rangle$ it may occur that $|u_g| > 1$ and $u_{\text{front}} > 1$. So, this model is in general noncausal.

For the relation (50), we have

$$p_0 = |p| \sqrt{1 - \gamma^2 \left[ \sin 2\phi - \frac{1}{2} |u||v| \right] |u||v| \sin^2 \theta},$$

(54)

where $\gamma = (1 - \frac{1}{4} u^2 v^2)^{-1}$. This relation clearly indicates a positive energy for a small product $|u||v|$, implying stability. The group and front velocities are given by

$$u_g = u_{\text{front}} = \sqrt{1 - \gamma^2 \left[ \sin 2\phi - \frac{1}{2} |u||v| \right] |u||v| \sin^2 \theta}.$$

(55)

At the same way, this expression provides $|u_g| > 1, u_{\text{front}} > 1$ for $\phi \in \langle \pi/2, \pi \rangle \cup \langle 3\pi/2, 2\pi \rangle$. Thus, we conclude that the anisotropic parity-even sector is stable but noncausal.

B. Unitarity analysis

The unitarity analysis of this model at tree-level is here carried out through the saturation of the propagators with external currents (44), which must be implemented by means of the saturated propagator $(SP)$, a scalar quantity given as follows:

$$SP = J^\mu \text{Res}(i\Delta_{\mu\nu}) J^\nu,$$

(56)

where $\text{Res}(i\Delta_{\mu\nu})$ is the matrix residue evaluated at the pole of the propagator. The gauge current ($J^\mu$) satisfies the conservation law ($\partial_\mu J^\mu = 0$), which in momentum space is read as $p_\mu J^\mu = 0$. In accordance with this method, the unitarity analysis is assured whenever the imaginary part of the saturation $SP$ (at the poles of the propagator) is positive. A way to carry out the saturation consists in determining the residue of the propagator matrix, evaluated at its own poles.

We begin writing the saturated gauge propagator (taking into account the current conservation):

$$SP = [-iR] \left[ \Xi(p) J^2 + 2F(p) (U \cdot J) (V \cdot J) + I(p) (U \cdot J)^2 + L(p) (V \cdot J)^2 \right],$$

(57)

where the terms $\Xi, F, I, L$, given by Eqs. (30-34), are to be evaluated in one of the poles of the propagator, and

$$R = \text{Res} \left[ \frac{1}{\delta^2 (1 - \frac{1}{2} U \cdot V) + (p \cdot U) (p \cdot V)} \right],$$

(58)

is the residue evaluated in the pole.
To examine the unitarity of the anisotropic parity-even sector, we use the parametrization $U_\mu = (0, \mathbf{u}), V_\mu = (0, \mathbf{v})$, where $\mathbf{u}$ and $\mathbf{v}$ are orthogonal 3D-vectors, $(\mathbf{u} \cdot \mathbf{v} = 0)$, due to the traceless property of matrix $\tilde{\kappa}_{\mu \nu}$ [see Eq. (20)].

We examine the pole stemming from the dispersion relation (36),

$$p^2 = -\eta(p \cdot U)(p \cdot V),$$

with $\eta = (1 - \frac{1}{2}(U \cdot V))^{-1}$. In the anisotropic sector, $\eta = 1$. In this pole, the residue is

$$R = \frac{1}{4p^2((p \cdot v)^2u^2 + (p \cdot u)^2v^2 - u^2v^2(p \cdot u)(p \cdot v))},$$

The saturation (57) is read as

$$SP = [-iR \left\{ R^{-1}J^2 + 2F(U \cdot J)(V \cdot J) + I(U \cdot J)^2 + L(V \cdot J)^2 \right\}],$$

with

$$F = \frac{1}{4}p^2(p \cdot v)(p \cdot u),$$

$$I = \frac{1}{4}p^2(p \cdot v)[(p \cdot u)v^2 - (p \cdot v)],$$

$$L = \frac{1}{4}p^2(p \cdot u)[(p \cdot v)u^2 - (p \cdot u)].$$

Replacing all these expressions in the saturation, we achieve

$$SP = -\frac{iR}{4} \left\{ 4R^{-1}J^2 + (p \cdot v)(p \cdot u)[u^2(J \cdot v)^2 + v^2(J \cdot u)^2] - [(p \cdot v)(J \cdot u) - (p \cdot u)(J \cdot v)]^2 \right\}.$$  (65)

In order to verify the positivity of the expression above, it is suitable to define a three-dimensional basis, generated by the vectors $\hat{v}, \hat{u}$ and $\hat{c}$:

$$\hat{v} = v/|v|, \quad \hat{u} = u/|u|, \quad \hat{c} = (v \times u)/|u||v|. $$

In this basis, it holds

$$J \cdot v = J_v |v|; \quad J \cdot u = J_u |u|; \quad p \cdot v = p_v |v|; \quad p \cdot u = p_u |u|,$$

$$J_0^2 = J_0^2 = J^2 - J_2^2 - J_1^2 - J_0^2,$$

$$p_0^2 = p_c^2 + p_a^2 + p_v^2 - |u||v|p_a p_v.$$  (69)

$$R = \frac{1}{4} |u|^2 |v|^2 \left\{ p_a^2 + p_v^2 - |u||v|p_a p_v \right\}$$

From Eq. (65), and using the relations (67-69), we obtain:

$$SP = -\frac{iR}{4} \left\{ 4R^{-1}J^2 + p_a p_v |u|^3 |v|^3 [J_2^2 + J_a^2] - |u|^2 |v|^2 [p_v J_u - p_u J_v]^2 \right\},$$

which is equivalent to:

$$SP = i \left\{ -J_0^2 + J_2^2 - \frac{|u|^2 |v|^2}{4} R[J_v J_u + p_u J_a]^2 \right\}.$$  (72)

Making more algebraic manipulations, and using the current conservation, $p_0 J_0 = p_v J_v + p_a J_a + p_c J_c$, the saturation takes the form

$$SP = i \left\{ J_0 \left\{ p_c^2 + p_a^2 - |u||v|p_a p_v \right\} + p_v (p_v J_v + p_u J_a) \right\} \frac{1}{2} \left\{ p_a^2 + p_v^2 - |u||v|p_a p_v \right\} \left\{ p_c^2 + p_a^2 + p_v^2 - |u||v|p_a p_v \right\} > 0,$$

which is compatible with the unitarity validity. In the result above, the denominator term $p_c^2 + p_a^2 - |u||v|p_a p_v$ was taken as positive. It occurs whenever it holds the condition: $|u||v| < 2$. As the magnitude of the Lorentz-violating parameters is always much smaller than 1, this condition is fulfilled. A similar development can be accomplished for the pole stemming from the dispersion relation (37). We thus assert that the anisotropic parity-even sector is noncausal and unitary.
IV. CONCLUSIONS

In this work, we have exactly evaluated the gauge propagator for the nonbirefringent CPT-even electrodynamics of SME using a prescription proposed in Ref. [41] and a parametrization for the symmetric $k^{ab}$ in terms of two arbitrary four-vectors. These parametrizations allowed to obtain an exact tensor form for the propagator of the nonbirefringent components which recovers the gauge propagator expressions is the suitable parametrization choices are adopted. The involved dispersion relations coincide with the ones obtained in Ref. [39] for the isotropic parity-even component and for the three parity-odd nonbirefringent components. Furthermore, the dispersion relations for the anisotropic parity-even components were achieved as well. The analysis of stability, causality, and unitarity for the anisotropic parity-even components was performed, revealing that this sector is stable, noncausal, and unitary. This study completes the analysis initiated in Ref. [39]. The achievement of a tensor form propagator assures the facilities of the tensor calculus for some interesting applications, as scattering amplitude evaluation in a Quantum Electrodynamics context. Some investigations in this direction are now under development.

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