The Immirzi parameter in quantum general relativity

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Barbero has generalized the Ashtekar canonical transformation to a one-parameter scale transformation $U(\iota)$ on the phase space of general relativity. Immirzi has noticed that in loop quantum gravity this transformation alters the spectra of geometrical quantities. We show that $U(\iota)$ is a canonical transformation that cannot be implement unitarily in the quantum theory. This implies that there exists a one-parameter quantization ambiguity in quantum gravity, namely a free parameter that enters the construction of the quantum theory. The purpose of this letter is to elucidate the origin and the role of this free parameter.

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I. INTRODUCTION

One of the most interesting recent results in quantum gravity is the computation of the quantum spectra of certain geometrical quantities [1]. These spectra turn out to be discrete, suggesting a discrete structure of geometry at the Planck scale which might be, in principle, physically measurable [2]. Recently, Immirzi has pointed out that the overall scale of these spectra is not determined by the theory [3]. More precisely, Immirzi has considered a certain scale transformation introduced by Barbero [4], and noticed that if we quantize the theory starting from scaled elementary variables, we end up with different spectra for the same geometrical quantities.

Here, we analyze the issue in detail. We find that the quantum theory is in fact undetermined by one parameter. This is due to the fact that the holonomy algebra on which this approach is based depends on a free parameter. This fact gives rise to a one-parameter family of inequivalent quantum theories, which are all, up to additional physical inputs, physically viable. In a sense, there is a one-parameter family of “vacua” in quantum general relativity, parameterized by a free (real) parameter, which we call “Immirzi parameter”, and denote as $\iota$ (“iota”). Equivalently, there is a symmetry in the classical theory which is realized as a canonical transformation but cannot be realized as a unitary transformation in the quantum theory.

The existence of this quantization ambiguity is due to the peculiar kind of representation on which non-perturbative quantum gravity is based [5]. This representation is characterized by the fact that the holonomy is a well-defined operator in the quantum theory. Conventional perturbative Maxwell and Yang-Mills theories are not defined using this kind of representation and the $\iota$ parameter does not appear in that context. But physical and mathematical arguments indicate that this representation is relevant at the diffeomorphism-invariant and background-independent level [6]. Thus, the Immirzi parameter appears in the general covariant context.

In this letter, we describe in some detail how this ambiguity is originated and its consequences. In particular, we address a certain number of questions that have been recently posed concerning the $\iota$ parameter, and we try to rectify a number of proposed incorrect interpretations of the appearance of this free parameter.

For a similar discussion, but centered on the loop quantization of Maxwell theory, see [8], where an interesting speculation tying the Immirzi parameter with charge quantization is presented.

II. THE IMMIRZI AMBIGUITY

Consider a three-dimensional compact smooth manifold $\Sigma$, and two fields defined over it: an $su(2)$ valued vector density $E$, with components $E_i^a$, where $a, b, c, \ldots$ denote tensor indices, and $i, j, k, \ldots$ are $su(2)$ indices; and an $SU(2)$ connection $A$ with components $A_i^a$. In terms of these fields, one can describe Euclidean or Lorentzian general relativity, as well as $SU(2)$ Yang-Mills theory. In both cases $A_i^a$ and $E_i^a$ are canonically conjugate and subject to the Gauss constraint.

$$G_i := \partial_a E_i^a + \epsilon_{ijk} A_i^a E_i^{ak}. \quad (1)$$

In Yang-Mills theory, $E$ is the electric field; in general relativity, $E_i^a = \det(e_i^a) e_i^a$, were $e_i^a$ and $e_i^a$ are the triad field (related to the 3-metric by $g_{ab} = e_i^a e_i^b$) and its inverse.

In general relativity it is natural to take dimensionless coordinates. $E$ has dimension length$^2$ and $A = A_a dx^a$ is dimensionless. The fundamental Poisson brackets are \{ $A_i^a(x), E_j^b(y)$ \} = $G \delta_i^b \delta_j^a \delta(x, y)$, where $G$ is $16 \pi c^{-3}$ times the Newton constant.

We denote by $h_{\iota}(A) \in SU(2)$ the transport propagator of $A$ along a path $e$. If $e$ is a closed path then $\text{tr}(h_{\iota}(A))$ is the holonomy of $A$ around $e$, or the Wilson loop functional. Wilson loops are natural $SU(2)$ gauge...
invariant quantities, and any gauge invariant function of \( A \) can be approximated by sum of products of Wilson loops. In conventional perturbative Yang-Mills theory, there is no well-defined operator corresponding to the Wilson loop, because field operators need to be smeared in more than one dimension to be well defined. In a general covariant theory, on the other hand, one expects the Wilson loop operators to be well-defined in the quantum theory. Thus, one can define the quantum theory as a unitary representation of the (non-canonical) Poisson algebra generated by the classical observables \( h_\epsilon(A) \) and \( E_i^a(x) \) (with a suitable smearing of the latter).

By doing so, one obtains a quantum representation that can be described as follows. The Hilbert space of the theory is taken as \( \mathcal{H} = L^2(\mathcal{A}, d\mu_0) \), where \( \mathcal{A} \) is (the closure in a suitable norm of) the space of the smooth connection fields, and \( d\mu_0 \) is a faithful, diffeomorphism invariant \( \sigma \)-additive probability measure on \( \mathcal{A} \). One can show that the measure \( \mu_0 \) is the unique probability measure on \( \mathcal{A} \) which implements the classical reality conditions \( (A_i^a, E_i^a) \) are real and the given Poisson algebra. The holonomy observable \( h_\epsilon(A) \) becomes the quantum operator \( \hat{h}_\epsilon \) defined by

\[
\hat{h}_\epsilon \Psi = h_\epsilon \Psi, \quad \Psi \in \mathcal{H}.
\]

A characteristic example of a geometric operator defined in the theory is the area of a two-dimensional surface. Suppose we define the theory requiring that the parallel transport \( \hat{h}_\epsilon \Psi = h_\epsilon(A) \) be a well defined operator in \( \mathcal{H} \).

Now, the triad \( e_i^a \) defines a three dimensional spin connection \( \Gamma_i^a \) by

\[
\partial_{[a}e_{b]}^i = \epsilon^{ijk}\Gamma_j^a e_{b]^k},
\]

which transforms under \( SU(2) \) as the connection \( A \) does. Pick an arbitrary positive real number \( \iota \) and define the scaled fields

\[
A_i^a = \iota A_i^a + (1 - \iota) \Gamma_i^a,
\]

\[
E_i^a = \frac{1}{\iota} E_i^a.
\]

It is then easy to verify the following. i) \( A_i \) and \( E_i \) are canonically conjugate, namely the map \( U_\iota : (A, E) \mapsto (A_\iota, E_\iota) \) is a canonical transformation. ii) \( A_i \) and \( E_i \) transform under gauge transformations as \( A \) and \( E \) do. In particular, \( A_i \) is a connection. This follows from the fact that \( A_i \) is a convex linear combination of two \( SU(2) \) connections. Thus \( U_\iota \) preserves the affine structure of the space of connections. iii) Clearly \( U_\iota \) preserves the reality conditions as well. Thus \( U_\iota \) leaves the kinematical structure of general relativity invariant.

The canonical transformation \( U(\iota) \) was studied by Barbero. The case \( \iota = \sqrt{-1} \) corresponds to Ashtekar’s canonical transformation \( [12] \). Thus, the fact that \( U(\iota) \) is a canonical transformation is a simple extension of the important discovery due to Ashtekar which gave rise to the connection formulation of general relativity. To see the relation with these works, notice that by introducing the extrinsic curvature \( K_i^a = A_i^a - \Gamma_i^a \), we can write as

\[
A_i^a = \Gamma_i^a + K_i^a,
\]

\[
A_i^a = \Gamma_i^a + iK_i^a.
\]

The generating function of the (infinitesimal) canonical transformation is \( C = \int K_i^a E_i^a \). Finally, notice that the area of a surface \( S \) is given in terms of the scaled variables by

\[
A(S) = \iota f(S, E_i).
\]
from $\mathcal{H}$ to $\mathcal{H}_s$ which sends physically corresponding states and physically corresponding operators into each other. Namely, we expect that the canonical transformation $U(\iota)$ can be realized as a unitary transformation of $L_2[\mathcal{A}, d\mu]$ into itself.

If this transformation exists, an immediate consequence is that the spectra of observables which are unitary images of each other are identical. In particular, the spectrum of the area operator defined in one theory should agree with the spectrum of the area operator defined in the other theory.

Consider now the operator $f(S, E_s)$ defined in the scaled theory. Since it is has the same form as the operator $f(S, E)$ in the unscaled theory, it will certainly have the same spectrum. But then equations (8) and (9) show that the spectrum of the area in the scaled theory is obtained from the one in the original theory by multiplying it by $\iota$. Since the spectrum of the area is discrete, it cannot be invariant under a scaling by an arbitrary real parameter. Therefore the two quantizations yield unitarily inequivalent theories.

The two theories give different physics. For instance, the predicted spectrum of the area differ. The other geometrical quantities having discrete spectrum, such as volume and length, have different spectra in the scaled theory as well. Since $\iota$ enters linearly in $E_s$, which has dimension $L^2$, the discrete spectrum of a geometrical quantity homogeneous in $E$ with dimension $L^n$ scales as $\iota^{n/2}$.

Thus, there is a free parameter $\iota$ in the quantization of the theory. It could be measured, in principle, simply by measuring, for instance, the size of the “quanta” of area. So far, there does not seem to be any compelling reason for choosing a particular value for $\iota$. Better knowledge of the theory may indicate such a reason.

It has been suggested (for instance, see [8]) that one could fix $\iota$ using black hole entropy. It was shown in [13] that one can derive the Bekenstein-Hawking entropy formula Entropy = $\text{const} \times A$ from loop quantum gravity – with a constant $\text{const}$ that turns out to be finite, but with an incorrect value. This constant is affected by a rescaling of $\iota$, and therefore one might choose a value for $\iota$ yielding the correct value $\text{const} = \frac{1}{4\pi} \hbar G$. This would fix a value of $\iota$ approximately given by

$$\iota \sim \frac{1}{4\pi}. \quad (11)$$

III. INCORRECT INTERPRETATIONS

Several interpretations of the appearance of the free parameter $\iota$ in quantum gravity have been recently proposed. Some of these are incorrect. Here we discuss some of these interpretations.

A. $\iota$ is a free constant that multiplies the connection in the definition of the holonomy

Let $\epsilon : [0, 2\pi] \mapsto \Sigma$ be a closed path. The holonomy $h_\epsilon$ is formally written as

$$h_\epsilon = \mathcal{P} e^{\oint_{\epsilon} \mathcal{A}}, \quad (12)$$

and it is more precisely defined as $h_\epsilon = h_\epsilon(2\pi)$ where the $SU(2)$-valued function $h_\epsilon(s)$ is the solution of the differential equation

$$\frac{d}{ds} h_\epsilon(s) - i\dot{\epsilon}(s) A_\alpha(e(s)) h_\epsilon(s) = 0, \quad h_\epsilon(0) = 1. \quad (13)$$

Here $\dot{\epsilon}(s)$ is the tangent to $\epsilon$ at $s$ and 1 the identity in $SU(2)$. It has been repeatedly suggested that in defining the holonomy one if free to add a dimensionless parameter $\iota$ in the exponent of (12) – keeping any other thing fixed in the theory.

However, this is not true. The transformation properties of $A$ under an $SU(2)$ gauge transformation are uniquely fixed by the classical action, because from the action one derives the gauge constraint (1). If (12) is gauge covariant, then

$$\tilde{h}_\epsilon = \mathcal{P} e^{\iota \oint_{\epsilon} \mathcal{A}}, \quad (14)$$

or, more precisely, the solution of

$$\frac{d}{ds} \tilde{h}_\epsilon(s) - i\dot{\epsilon}(s) A_\alpha(e(s)) \tilde{h}_\epsilon(s) = 0, \quad \tilde{h}_\epsilon(0) = 1, \quad (15)$$

is not gauge covariant, as a direct computation shows. In other words, if we multiply a connection by a real number, we do not obtain a quantity that transforms as a connection: connections form an affine space, not a linear space.

B. $\iota$ is the constant that multiplies the classical action

The quantity $\tilde{h}_\epsilon$ defined in (14) is gauge covariant under the gauge transformation generated by a scaled Gauss constraint

$$G_i := \partial_\alpha F^\alpha_i + \iota \epsilon_{ijk} A^j_\alpha F^{\alpha k}. \quad (16)$$

This constraint, in turn, can be obtained by scaling the action of the theory. Accordingly, it has been suggested

\textit{*The generator of the classical canonical transformation can be promoted to a self-adjoint operator. It coincides with the generator of the Wick rotation transform considered in [3]. The corresponding operator $\mathcal{C}$ is defined on $\mathcal{H}$ in [4].}
that the $\ell$ ambiguity is a consequence of the freedom in choosing the constant in front of the action of general relativity. Equivalently: scaling $A$, but not $E$, without absorbing this into a redefinition of the coupling constant.

Again, this interpretation is wrong. Physically, the constant in front of the general relativity action determines the strength of the macroscopic Newtonian interaction. The freedom in the choice of the Immirzi parameter in the quantum theory consists in the fact that the overall scale of the spectra is not determined by low energy physics. In other words, we can measure the Newton constant by means of classical gravitational experiments, and measure the Planck constant by means of non-gravitational quantum experiments. From these two quantities we obtain a length, the Planck length $l_P = \sqrt{\hbar G}$. The point of the Immirzi ambiguity is that the ratio of, say, a given eigenvalue of the area to $l_P$ is not determined by the quantization procedure.

C. Any Yang-Mills theory has a free $\ell$ parameter when quantized in the loop representation

As the previous comments indicate, the Immirzi parameter is not related to a multiplicative scaling of the connection. Rather, it is generated by the affine transformation. In order to be able to write such an affine transformation, we need to have a second, independent connection. Rather, it is generated by the affine transformation. The freedom in the choice of the Immirzi parameter in the quantum theory consists in the fact that the overall scale of the spectra is not determined by low energy physics. In other words, we can measure the Newton constant by means of classical gravitational experiments, and measure the Planck constant by means of non-gravitational quantum experiments. From these two quantities we obtain a length, the Planck length $l_P = \sqrt{\hbar G}$. The point of the Immirzi ambiguity is that the ratio of, say, a given eigenvalue of the area to $l_P$ is not determined by the quantization procedure.

D. The algebra has inequivalent representations

A characteristic phenomenon in field theory—and in finite dimensional quantum mechanics, when the phase space is non-linear—is that the main observable algebra may have inequivalent representations. It has been suggested that the Immirzi parameter distinguishes inequivalent representations of the same algebra. However, we do not think that this is the case. We have defined the $\ell$-scaled theory as a representation of a physically distinct algebra (the algebra of the scaled variables) obtaining the quantum theory on $\mathcal{H}_c$. If this theory could be obtained also as a representation of the original algebra of unscaled variables, then it would carry a representation of the scaled as well as the unscaled, $h_c$ and $h_r$, operators. But we do not see any reason for the operators $h_c$ to be well-defined in the Hilbert space of the usual loop quantum theory.

IV. MODELS

We now discuss a few simple models in which the Immirzi ambiguity does or does not appear, in order to illustrate some of the statements made above.

A. Harmonic oscillator: no $\ell$ ambiguity

Consider the phase space $\Gamma = \mathbb{R}^2$ of an harmonic oscillator with canonical coordinate and momentum $(x, p)$ and Hamiltonian

$$H = \frac{1}{2} p^2 + \frac{\omega^2}{2} x^2.$$  \hfill (17)

The quantum theory is defined by a representation of the canonical algebra $[\hat{q}, \hat{p}] = i\hbar$. Let the Hilbert space be $\mathcal{H} = L^2(\mathbb{R}, dq)$ and $(\hat{q} = x, \hat{p} = -i\hbar \partial_q)$. The spectrum of the Hamiltonian operator is

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega.$$  \hfill (18)

Suppose we decide to quantize the theory starting from the scaled variables

$$x_i = \ell x, \quad p_i = \frac{1}{\ell} p.$$  \hfill (19)

Is the spectrum of the Hamiltonian altered? The transformation $(x, p) \rightarrow (x, p)$ is clearly a canonical transformation. The algebra of the scaled variables is the same as the algebra of the unscaled ones. Therefore we can write $\mathcal{H}_c = L^2(\mathbb{R}, dq)$ and $(\hat{q}_c = x, \hat{p}_c = -i\hbar \partial_q)$. In the scaled variables, the Hamiltonian reads

$$H = \ell^2 \left[\frac{1}{2} \hat{p}_c^2 + \frac{\omega^2}{2\ell^2} \hat{x}_c^2\right].$$  \hfill (20)

\footnote{SU(2) Yang-Mills theory has essentially the same phase space as general relativity. However, notice that in order to define $\Gamma^i_a$ we need to invert the $3 \times 3$ matrix $E_i^a$; in general relativity this matrix is invertible, because of the non-degeneracy condition on the metric, but in SU(2) Yang-Mills it is not so in general.}
Thus its spectrum is $i^2$ times the spectrum of an Hamiltonian of angular frequency $\omega/\ell^2$. That is

$$E_n = i^2 \left( n + \frac{1}{2} \right) \hbar \frac{\omega}{\ell^2}.$$ \hfill (21)

precisely as before. Therefore, in the quantization based on the scaled variables the wave functions $\psi(q)$ have a different interpretation, but physical predictions, such as the spectrum of the Hamiltonian, are not altered. This example shows that in general a one-parameter scaling of the quantization variables that does not alter the observable algebra will not alter the theory. This was to be expected from Von-Neumann’s uniqueness theorem.

**B. Particle on a circle: no \( \ell \) ambiguity**

As a second example, we consider another simple system in which a scaling of the basic variables does not alter the quantum theory: the theory of a particle on a circle. We include this case because, unlike the harmonic oscillator case, here the observable algebra has a one-parameter family of inequivalent irreducible representations. However, this fact turns out to irrelevant as far as the \( \ell \) ambiguity is concerned.

Consider a particle constrained to move on a circle of length \( L \). Let \( q \in [0, L] \) its position and \( p \) its momentum. The Hamiltonian is \( H = \frac{p^2}{2} \), and has eigenvalues \( E_n = n^2 \frac{\omega^2}{2L^2} \). Now, suppose we change basic variable to \( (s, p) = (q, \frac{1}{L} p) \). Then the Hamiltonian reads

$$H = \frac{i^2}{2} p^2.$$  

At first sight one is tempted to say that the spectrum of \( H \) has now changed, because \( p_2 \) and \( q_2 \) have the same algebra as \( p \) and \( q \) and therefore \( p_1 \) must have the same spectrum as \( q \). But of course this is wrong, because \( q \) and \( q_2 \) are defined on a circle, and the spectrum of their conjugate momentum depends on the size of the circle. Clearly this exactly compensates the \( i^2 \) in \( H(p_1) \).

More precisely, we cannot quantize the \((q, p)\) algebra in this case because \( q \) is not a global coordinate on the phase space \( \Omega \). We must, for instance, replace \( q \) with \( g = \frac{L}{2\pi} \exp(i2\pi q/L) \in U(1) \), which is a global coordinate. If we introduce also the angular momentum \( \hat{\ell} = \frac{L}{2\pi} p \), we obtain the algebra

$$\{g, \ell\} = g.$$ \hfill (22)

This algebra has a one-parameter family of distinct irreducible representations, which can all be defined on \( L_2[\tilde{S}_1, d\phi] \) by

$$\hat{\ell} = -i\partial_\phi, \quad \hat{g} = re^{i\phi},$$ \hfill (23)

where \( r \) labels the representation. The representation appropriate for a given \( L \) is the one obtained by choosing \( r = L/2\pi \), because this realizes the non linear condition on the basic variables \( |g| = L/2\pi \). In terms of these variables, the Hamiltonian is

$$H = \frac{1}{2} \frac{\omega}{L^2} \ell^2.$$ \hfill (24)

The spectrum of \( \hat{\ell} \) does not depend on the representation chosen, and we have again the correct spectrum for the Hamiltonian.

Notice that we could have written the Hamiltonian (perhaps “more correctly”) as

$$H = \frac{1}{2} \frac{1}{|g|^2} \ell^2.$$ \hfill (25)

Then, the corresponding operator is different in each representation, but since the representation is tied to \( L \), the spectrum is again correct. In any case, no free parameter \( \ell \) appears.

**C. A simple model with the free \( \ell \) parameter**

Consider a phase space \( \mathbb{R}^6 \) with coordinates \((K, E)\) and Hamiltonian \( H = E_1 E_2 / 2 \), and let a theory be defined by the constraint \( G_1 = \epsilon_{ijk} K^j E^k \). The theory has one physical degree of freedom.

Consider the canonical transformation

$$(K, E) \rightarrow (A := K + \Gamma, E)$$ \hfill (26)

where

$$\Gamma_i := \frac{\partial \sqrt{E^i E^i}}{\partial E_i}.$$ \hfill (27)

Here we are mimicking the canonical transformation to the “connection” variable \( A \), where \( \Gamma \) is the “spin-connection” of \( E \). In the new variables, the constraint becomes

$$G_1 = \epsilon_{ijk} A^j E^k,$$ \hfill (28)

which can be viewed as the Gauss law in \( d = 0 \) space dimensions.

Now consider a representation of the canonical commutation relations \([A_j, E^k] = i\delta^k_j \) on the Hilbert space \( \mathcal{H} := L_2(G, d\mu_H(g)) \) where \( g = \exp(A, \tau_i) \) is the “holonomy” of \( A \) and \( \tau_i \) are the generators of \( su(2) \).

Consider the map \((K, E) \rightarrow (K', E') = (iK, E/\ell)\). Under this canonical transformation the Hamiltonian scales by \( H = E' E'/2 = i^2 E_1 E_2 / 2 \). The spectrum of the Hamiltonian (Casimir of \( SU(2) \)) is discrete; it is given by \( \lambda_n = 1/2(n/2)[(n/2) + 1] \), where \( n \) is a positive integer. But in the scaled variables the spectrum of the Hamiltonian changes to \( \lambda_n = i^2/2(n/2)[(n/2)+1] \). Thus, we have a one parameter quantization ambiguity in the theory.

Of course, the choice we have made of the observable algebra for the quantization is a bit strange: it is only justified by the analogy with gravity. A more conventional quantization would not lead to any such quantization ambiguity. This shows how the \( \ell \) parameter is intimately linked with the affine structure of the configuration space and with the choice of the holonomies as basic operators in quantum gravity.
In the loop representation approach to quantum gauge theories, one does not gauge fix the theory prior to quantization, but rather maintains the geometrical structure of the gauge theory explicit in the quantum theory. In particular, one works on the group rather than on the algebra. The quantization is based on the physical assumption that the Wilson loops, or the “Faraday lines” of the theory are the physical elementary quantum excitations, and thus correspond to finite norm states. Equivalently, the holonomy operator is well-defined. If there is more than one connection ($A$ and $\Gamma$) that can be defined on the phase space, and which transform in the same way, then one can construct a $\iota$ scaled connection ($A_\iota$) by interpolating between distinct connections. Then, the assumption that the elementary physical excitations of the theory are the Wilson loops of $A_\iota$ turns out to be physically distinct for different values of $\iota$.

This is manifested in the dependence of some physical spectra upon $\iota$. More precisely, what happens in gravity is that the metric information is in the conjugate variable $E$. Being conjugate to the connection, $E$ is given in the quantum theory by derivative operators acting on functions over the group. Geometrical quantities which are functions of $E$ turn out to be elliptic operators over the group manifold. Hence their discrete spectrum. But these elliptic operators have non-vanishing scaling dimension with respect to the affine scaling of the connection. Therefore, the $\iota$ quantization ambiguity shows up in the spectrum of the elliptic geometric operators.

The resulting quantization ambiguity, which we have denoted here as Immirzi ambiguity, affects the operators with a discrete spectrum that scale with $\iota$. A free parameter $\iota$ appears in the loop quantization of general relativity, where it affects the scale of the discreteness of space, and in the loop quantization of Maxwell theory, where it has been suggested that it might be related to charge quantization. Its origin is not tied to the infinite dimensionality of the phase space, nor (at least to a first analysis) to the existence of inequivalent representations of the observables algebra.

Finally, the indeterminacy is given by a single parameter, and therefore it does not reduce the predictive power of the theory more than, say, $\lambda_{\text{QCD}}$ in quantum chromodynamics, or the string constant in string theory. Notice in this regard, that also in perturbative string theory there are two independent length scales: string tension and Planck constant. Similarly, there are two length scales in quantum gravity: the Planck constant $l_p = \sqrt{\hbar G/c^3}$ and the quantum of area $A_0 = 8\sqrt{3\pi}l_p^2$. Unless some non yet understood requirement fixes the value of the Immirzi parameter, these two length scales are independent.

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