Segmentation of colour image using fuzzy sets concept

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Abstract. The article deals with the problem of segmentation of digital images, which is one of the main tasks in the field of digital image processing (IP) and computer vision. To solve this problem, an algorithm was proposed based on the use of a concept based on the theory of fuzzy sets. The main idea of the proposed algorithm is the formation of subsets of interconnected pixels based on the fuzzy-to-mean method. A distinctive feature of the proposed algorithm is the definition of a set of features that define areas with similar characteristics in the space of the characteristic features of the analyzed image. The proposed segmentation algorithm (SA) consists of two stages: 1) the formation of characteristic features for all channels of the base color; 2) clustering of image elements. The practical significance of the obtained results lies in the fact that the developed models of algorithms can be used in various applied problems, where the classification of objects represented as images is provided. To test the efficiency of the developed algorithm, experimental studies were carried out in solving a number of applied problems related to color image segmentation, in particular, license plate recognition problems.

1. Introduction

To date, a large number of methods and algorithms have been developed and investigated for solving the image segmentation (IS) problem [1-8]. Despite this, the problems of IS are one of the most critical problems in the field of processing and recognition of digital images [9, 10]. In recent years, many researchers are increasingly focusing on the development of new approaches and methods in IP to find effective methods for solving various problems of IS.

One of the such ways is the use of "soft computing" technologies in digital IP [11]. As it is well known, these technologies are based on the theory of fuzzy sets, fuzzy logic [12], and they are now widely used in solving problems of IP and recognition of objects on them, in particular, IS [9,10,13-15]. The possibility of successful application of methods of fuzzy logic is largely determined by a flexible mathematical apparatus used in the analysis and processing of images.

The main goal of this work is the development of algorithms for the segmentation of color images based on the apparatus of the theory of fuzzy sets.

To achieve this goal, it is necessary to solve the following problems:

1) analyze the state of existing methods of IS and determine the range of problems that need to be solved;
2) to develop an algorithm for color IS within the concept of the theory of fuzzy sets;
3) assess the effectiveness of the developed algorithm for solving the problem of color IS based on experimental studies.

2. Review of existing methods of IS
The analysis of literary sources shows that the methods of IS based on the theory of fuzzy sets are divided into the following groups.

2.1. Thresholding
The methods of this group are among the simplest methods of IS and the image is divided into object and background areas. Thus, these methods, based on a comparison with a threshold, form a binary image in which the value of the pixels belonging to the objects is equal to 1, and the value of the pixels belonging to the background is 0. There are several traditional methods for determining thresholds adequate to the original image [15, 16]. To determine the optimal threshold, it is possible to calculate measures such as a linear / quadratic fuzzy index [17], fuzzy compactness [15] or index of the coverage zone [17]. Fuzzy deviations and probabilistic measures can also be used to segment images onto an object and background [16].

2.2. Segmentation using clustering methods
The first method of fuzzy clustering is the Fuzzy C-means (FCM) method [17-19], which currently has many modifications [20]. The FCM method is based on the use of ideas and the mathematical apparatus of fuzzy logic. In the course of this algorithm each pixel of the image is associated with a vector from the membership functions to each class, on the basis of which it is possible to draw conclusions about the nature of this object.

The result of segmentation using the FCM algorithm is highly dependent on the chosen measure. Euclidean distance is effective only when the clusters are well separated and roughly equal in size. Otherwise, other algorithms, such as the algorithm proposed in [15], or the decomposition algorithm for a Gaussian mixture may be used.

Despite the successes achieved in the segmentation of images using the apparatus of fuzzy sets, there are a number of unsolved problems [11]. These include the problem of adequate mapping of the subject area to a fuzzy system, the choice of models of fuzzy logic inference and their integration into a single intellectual system. Moreover, many of the methods developed based on the theory of fuzzy sets require large computational resources, which makes their application in practical systems more difficult. Thus, the problems of IS using the apparatus of fuzzy sets have not been adequately studied. Therefore, the development and improvement of IS methods based on the theory of fuzzy sets are very urgent problem.

3. Statement of the problem
Let an image $J$ consist of $m$ ($m = m_h \times m_v$) elements:

$$J = \{ p_{uv} \mid u \in [0,\ldots,(m_h-1)], v \in [0,\ldots,(m_v-1)] \}.$$

It is assumed that each element $J$ can be represented by a set of characteristics (or features) $X : X = \{ x_1, \ldots, x_j, \ldots, x_n \}$, which is called the vector of characteristics (features) of segmented elements. It should be noted that in many sources of image recognition, the processed data elements are called objects. Further, relying on this terminology, the elements of the image under study are called objects, and the image itself is called the set of objects. Then the images under study can be considered as a set $I$ consisting of $m$ elements:

$$I = \{ J_1, \ldots, J_i, \ldots, J_m \},$$

where $i = u + m_v \cdot v$ ($u \in [0,\ldots,(m_h-1)], v \in [0,\ldots,(m_v-1)]$). In this case, each object $J_i$ corresponds to a vector of features $\overline{x}_i$. 


\[ \bar{x}_j = (x_{j1}, \ldots, x_{jm}) \]

It is known [21] that the set of objects \( \{ J_1, \ldots, J_i, \ldots, J_m \} \) can be represented as a data table \( T_{mn} = \{ \tau_{uv} \mid u \in [1, \ldots, m], v \in [1, \ldots, n] \} \) of dimension \( m \times n \). Each line of \( T_{mn} \) is equal to the value of the features’ vector of the corresponding object.

Let the initial information \( T_{mn} \) be given. The task of fuzzy segmentation is to define an unclear partition of the set of objects under consideration into a given number of subsets that reach the extremum of the objective function (3) among all fuzzy partitions.

In order to clarify the problem of fuzzy segmentation, we introduce some concepts. Let the desired fuzzy segments represent a subset \( S_j \) consisting of the initial set of segmentation objects for which the following condition is met:

\[ \sum_{j=1}^{\ell} \mu_{S_j}(J_u) = 1 \quad (\forall J_u \in I) \]

where \( \ell \) – number of fuzzy segments \( S_j \ (j \in \{2, \ldots, \ell\}) \).

It is assumed that each fuzzy segment is preliminarily defined and is characterized by the so-called typical element (or center \( z_j, z_j = (z_{j1}, \ldots, z_{jm}) \)) of the desired fuzzy segment \( S_j \ (j \in \{2, \ldots, \ell\}) \), which is calculated for each feature:

\[ z_{j} = \left(\sum_{u=1}^{m} (\mu_{S_j}(J_u))^k \cdot \tau_{uv}\right) \left/ \sum_{u=1}^{m} (\mu_{S_j}(J_u))^k \right. \]

where \( \tau_{uv} \) is the value of the \( i \)-th feature in the \( u \)-th object (elements); \( k \) is the parameter of the algorithm, called the exponential weight, and is equal to some real number \( (k < 1) \).

As the objective function, we consider the sum of the squares of errors:

\[ R = \sum_{j=1}^{\ell} \sum_{u=1}^{m} (\mu_{S_j}(J_u))^k \| J_u - z_j \| \]

where \( \| x \| = \text{norm of the vector } x (\| x \| = \sqrt{x^T x}) \); \( k \) – parameter of the algorithm, whose value is given depending on the number \( m \) of elements \( I \). The larger \( m \), the smaller the value of \( k \).

4. Solution method

At this stage, preliminary processing of images is carried out, and although this processing does not give us the characteristic features that can be used in IS, it creates favorable conditions for determining the values of these features more accurately than without preliminary processing [2, 22]. This circumstance is due to the fact that when creating images and performing various operations on them, various distortions and noise often arise [23]. In such conditions, the application of SAs to distorted and noisy images becomes more complicated and their efficiency is reduced. In addition, the source images are specified in different formats, which form different color spaces. In such cases, for the application of SAs, the transformation of the original color space into RGB space is required. The procedures used in this step include the following preprocessing options: 1) smoothing the image to be segmented; 2) transformation of the color coordinate system. It should be noted that the algorithms considered at this stage are used to improve the quality of segmentation and are the initial step in solving the problem of IS.

4.1. Formation of a set of fuzzy features

At this stage, a number of fuzzy characteristics are defined to describe each valid element \( \beta \) of the original image. To convert the elements of the original image into a set of feature vectors, the following fuzzy characteristics for each pixel of the analysed original image can be distinguished:
1. The fuzzy brightness of the pixel \( p_i \) on the \((x, y)\) coordinates for each of the three base colours \((i = 1, 2, 3)\):

\[
p_i = \{ z \mid \mu_{z}(z) \geq 0, z \in \mathbb{N} \}, \quad z = J_i(x, y), \mathbb{N} = \{0, 1, \ldots, 255\},
\]

where \( J_i(x, y) \) – brightness of the \(i\)-color of the pixel with coordinates \((x, y)\).

In Figure 1 shows the fuzzy brightness function used to form the brightness features of an image.

The brightness \( J_i(x, y) \): \(C = \{C_1, C_2, C_3\}\) corresponds to each \(i\)-colour on the coordinate \((x, y)\).

Here \(C\) - fuzzy sets of brightness: \(C_1\) - "weak" fuzzy subset; \(C_2\) - "average" fuzzy subset; \(C_3\) - "strong" fuzzy subset.

2. The fuzzy brightness \(S_i\) of an arbitrary image fragment consisting of \(3 \times 3\) elements \((i = 1, 2, 3)\):

\[
S_i = \{ s_i \mid \mu_{s_i}(s_i) \geq 0, s_i \in S \}, S \subseteq \mathbb{N},
\]

where \(s_i\) – the average brightness of the \(i\)-th color in the vicinity of the pixel with the coordinates \((x, y)\):

\[
s_i = \left( \sum_{u=-1}^{1} \sum_{v=-1}^{1} J_i(x+u, y+v) / 9 \right).
\]

To determine the fuzzy brightness (when forming the averaged brightness features from the image fragment), the membership function shown in Fig. 1 can be used in the vicinity of the pixel with the coordinates \((x, y)\).

![Membership functions used to generate a fuzzy pixel brightness set](image)

**Figure 1.** Membership functions used to generate a fuzzy pixel brightness set \(J_i(x, y)\) on the coordinate \((x, y)\) for each \(i\)-th colour \(C = \{C_1, C_2, C_3\}\). Here - \(C\) are fuzzy sets of brightness; \(C_i\) is fuzzy "weak" subset; \(C_2\) is fuzzy "average" subset; \(C_3\) is fuzzy "strong" subset.

3. A fuzzy estimate of the average quadratic brightness value on an image fragment consisting of \(3 \times 3\) elements \((i = 1, 2, 3)\):

\[
E_i = \{ e_i \mid \mu_{e_i}(e_i) > 0, e_i \in D \}, D \subseteq \mathbb{N},
\]

where \(e_i\) – average brightness of \(i\)-th color of the pixel with coordinates \((x, y)\):

\[
\overline{e_i} = \left( \sum_{u=-1}^{1} \sum_{v=-1}^{1} (J_i(x+u, y+v))^2 \right).
\]

A fuzzy estimate of the scattering of brightness on an image fragment consisting of \(3 \times 3\) elements \((i = 1, 2, 3)\):

\[
D_i = \{ \overline{e_i} \mid \mu_{\overline{e_i}}(\overline{e_i}) > 0, D_i \in D \}, D \subseteq \mathbb{N},
\]

where \(\overline{e_i}\) – average brightness of \(i\)-th color of the pixel with coordinates \((x, y)\):

\[
\overline{e_i} = \left( \sum_{u=-1}^{1} \sum_{v=-1}^{1} (f_i(u,v))^2 / 9 \right), \quad f(u,v) = J_i(x+u, y+v) - s_i(x, y),
\]

where \(s_i(x, y)\) – the average brightness of the \(i\)-th color in the vicinity of the pixel with the coordinates \((x, y)\).
4.2. Determining image segments

The dividing of the image into \( l \) segments is performed on the basis of the formation of subsets of the connected pixels. The basic idea of forming subsets of strongly correlated elements is that the elements of each segment will be closer to its "center" than to the "centers" of the other segments. The problem of forming subsets of strongly correlated elements is considered solved if it is possible to determine the "centers" of segments and the boundaries of the corresponding subsets of elements in the set \( \mathbb{R} \). The proximity to the central element of the segments is determined on the basis of the concept of a fuzzy set [12, 20].

We consider the set of admissible elements (i.e., allowable pixels) of the image \( \mathbb{R} \). We assume that each admissible element \( p (p \in \mathbb{R}) \) corresponds to the features’ vector \( \vec{a} = (a_1, \ldots, a_i, \ldots, a_n) \), computed in the second step.

The algorithm of fuzzy segmentation of image elements can be described as follows. Let the parameters \( l, C_j (j = 2, l) \), which characterize the number of segment and segment centers respectively, be given. Then, to determine the initial partition \( R_0 (\mathbb{R}) = \{ S_j, S_j \subset \mathbb{R} \} \), we can calculate the segment centers by the formula (1) and the value of the objective function by the formula (2). If for some \( j \ (j \in \{2, \ldots, l\}) \) and some \( \chi_{xy} (\chi_{xy} \in \mathbb{R}) \), the value:

\[
\sum_{i=1}^{N} (\chi_{xy} - v_{ij}(x, y)) = 0
\]

is met, then for the corresponding fuzzy segment \( S_j \) it is assumed that \( \mu_j (\chi_{xy}) = 1 \), and for the segments \( S_q \) \( (q = 1, l, j \neq q) \), it is assumed that \( \mu_j (\chi_{xy}) = 0 \).

Further, for the fuzzy segments obtained, the refinement of the "centers" of the segments and the value of the objective function, respectively, by formula (1) and (2). The calculation of the "centers" of segments is carried out on the basis of the iterative method, which is based on the sequential refinement of the value \( z_{ij} \) (see formula 1) in each iteration. In this case, the value of \( \mu_j (J_a) \) is calculated as follows:

\[
\mu_j (J_a) = \delta_{aw} / \left( \sum_{i=1}^{l} \delta_{aw} \right) , \quad \delta_{aw} = 1 / \left( \sum_{i=1}^{N} (z_{ij} - \tau_{uw}) \right), \quad j \in \{1, 2, \ldots, l\}, u \in \{1, 2, \ldots, m\}.
\]

During the implementation of the fuzzy SA, the refinement process of the "centers" of the segments is performed. This process is terminated if:

the following condition is satisfied: \( |D(R_{q-1}) - D(R_q)| \leq \varepsilon \);

the number of completed iterations \( q \) exceeds a predetermined number \( q_0 \).

5. Experiments and Results

To evaluate the efficiency of the proposed algorithm, experimental studies were carried out to solve the problem of recognition of wagon number plates [24, 25]. It is known that the concept of "the digit colors on railway wagons" does not have a clear analytical description. In this regard, questions about the accuracy of the results of the algorithm were determined on the basis of visual comparisons. The essence of the application of the developed algorithm is to determine the areas corresponding to the digit colors on the color images of the wagons. The algorithm considered in [26] was used to solve the recognition problem. In this case, the problem was solved both with the application of the proposed algorithm and without its application.

As a source data, a set of 200 images of wagon license plates is given. These images were divided into 2 equal parts: training and test sets. In the training set, there were 10 images for each class (digits). The remaining 100 images are considered as a test set. Naturally, the number of classes (digits) is 10.
Examples of preprocessing and selection of image areas corresponding to the area of the car license plates are shown in Figures 2 and 3.

![Image](image_url)

**Figure 2.** Image (a) and obtained result after extracting the image fragment corresponding to the area of license plate (b).

**Figure 3.** Segmentation and recognition results.

The conducted experimental investigations showed the efficiency of the developed SA when solving problems of extracting of digits on color images of wagons.

6. **Conclusion**

The main results of the study on this work are as follows:

1. Analysis of existing literary sources showed that one of the urgent tasks in the field of digital IP and computer vision is IS. Therefore, many researchers are increasingly focusing on the development of new and improvement of existing algorithms focused on IS. One of these ways is to use the concept of “soft computing” in solving various problems of IS.

2. An algorithm for segmentation of color images using the fuzzy clustering method has been developed. A distinctive feature of the proposed algorithm is to build an attribute description of each pixel of the image under consideration. The proposed algorithm consists of two stages: 1) formation of a feature space based on fragmentary processing of data on all channels of the base color; 2) fuzzy clustering of image elements using the FCM method. The assessment of the efficiency of the developed algorithm was tested when solving a number of applied problems related to the segmentation of color images, in particular, the problems of recognizing license plates of cars.

3. The developed algorithm is an addition to the existing methods of segmentation of color images and can be used in the preparation of various software systems oriented to solve a class of tasks related to
the analysis of product quality by its image, diagnosing plant diseases from leaf images, recognizing forest fires and determining the area floods on aerial photography.

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