A Rule of Thumb Derivation of Born-Infeld Action for D-branes

RIKA ENDO, RIE KURIKI, SHIN’ICHI NOJIRI, and AKIO SUGAMOTO

Department of Physics, Ochanomizu University
2-1-1, Otsuka, Bunkyo-ku, Tokyo 112, Japan
\# Department of Mathematics and Physics, National Defence Academy
Hashirimizu, Yokosuka, 239, Japan

Abstract

A rule of thumb derivation of the Dirac-Born-Infeld action for D-branes is studied à la Fradkin and Tseytlin, by simply integrating out of the superstring coordinates in a narrow strip attached to the D-branes. In case of superstrings, the coupling of Ramond-Ramond fields as well as the Dirac-Born-Infeld type coupling of the Neveu Schwarz-Neveu Schwarz fields come out in this way.

1 Introduction

Recently, solitonic extended objects have been found in string theories and they are called D-branes. (“D” comes from the Dirichlet boundary condition.) Investigation of the dynamics of D-branes is very important in order to learn the nonperturbative behavior of string theories in the strong coupling region. It is also known that the interaction of D-brane with massless (bosonic) modes of superstrings is described by a kind of the Born-Infeld action.

Let us recapitulate the various derivations of the Born-Infeld action in string theories. In Ref.\cite{2} the authors have studied the effective action for an Abelian vector field coupled with the open bosonic string in the space-time dimensions $D = 26$, and derived the Born-Infeld type action in a constant background field strength. For open superstrings with Dirichlet (fixed) boundary conditions, boundary states have been introduced, and the infinities and anomalies as well as their relationship to the moduli space boundaries have been analyzed in detail\cite{3}. Through the investigation of the boundary states of open superstrings, the authors in Ref.\cite{4} have found the Born-Infeld type effective action, from which loop corrected field equations in the $\sigma$-model approach have resulted.

On the other hand, in Ref.\cite{5}, a new class of $\sigma$-models is introduced, which is characterized by the choice of boundary conditions for strings. In particular, Leigh has shown that the effective action, which is consistent with the vanishing condition of open string's
\(\beta\) functions, is the Dirac-Born-Infeld action. In Ref. [6] a set of universal couplings between Ramond-Ramond (R-R) gauge fields of superstrings and the internal gauge field of D-branes have been determined as a geometrical consequence. In Ref. [7] the massless sector of boundary states of D-branes is constructed, and the picture changing between various vertex operators is studied. After sensibly summarizing the Born-Infeld actions for D-branes, the self-duality for type IIB superstring as well as the duality relationship between type IIA superstring and the 11-D supermembrane are investigated using the Born-Infeld actions [8].

Here we will study a rule of thumb derivation of the Born-Infeld action. It is a way to understand the derivations of the Born-Infeld action \(\text{à la} \) Fradkin and Tseytlin, by simply integrating out of the superstring coordinates in a narrow strip attached to the D-branes. The way of thinking is easy to follow and as a result it may be useful for many researchers.

### 2 Born-Infeld Action for Bosonic String

We first start with the bosonic string case, where the open string described by the coordinate \(X^\mu(\tau, \sigma)\) is coupled with the dilaton \(\phi(X)\), the graviton \(G^{\mu\nu}(X)\), and the antisymmetric tensor field (the two-form field of the Neveu Schwarz-Neveu Schwarz (NS-NS) sector) \(B^{\mu\nu}(X)\). The open string has an end point which is stuck to the D-brane, but can move freely inside the \(p\) dimensionally extended D-brane (D-p brane). The position of the D-brane may be given by the coordinates \((x^0, \ldots, x^p, x^{p+1}, \ldots, x^9)\). Here, \(x^m (m = 0, \ldots, p)\) are the inner coordinates, whereas \(x^i = Y^i(x^0, \ldots, x^p)(i = p+1, \ldots, 9)\) are the outer coordinates of the D-brane.

Then, we have the following action,

\[
S = \frac{1}{4\pi} \int d^2 \xi \sqrt{g(\xi)} R^{(2)}(\xi) \phi(X) + \frac{1}{\kappa} \int d^2 \xi \left( \eta^{ab} G_{\mu\nu}(X) + \epsilon^{ab} B_{\mu\nu}(X) \right) \partial_a X^\mu \partial_b X^\nu - \frac{1}{\kappa} \int d\tau \left( A_m(X) \partial_\tau X^m(\tau, \sigma = 0) - N_i(X) \partial_\sigma X^i(\tau, \sigma = 0) \right).
\]

Here the end point trajectory of the open string is specified simply by \(\sigma = 0\), but the other end point \(\sigma = \pi\) is also possible to stick to the same D-brane. The trajectory may couple with the two kinds of vector fields, \(A_\mu(X)\) and \(N_\mu(X)\), and \(\kappa^{-1}\) represents the energy stored inside the unit length of the string, being related to the Regge slope \(\alpha'\) as \(\kappa = 2\pi \alpha'\).

Since the worldsheet of the open string (in the relevant strip region) is a disc, the Euler number appearing in front of the dilaton field is unity, \(\chi = \frac{1}{4\pi} \int d^2 \xi \sqrt{g(\xi)} R^{(2)}(\xi) = 1\). In the low-energy effective theory, all the external fields vary slowly compared with the rapid oscillation of the string modes, so that the \(X^\mu\) can be replaced by its center-of-mass value \(x^\mu\).

By using the Stoke’s and Gauss’ theorems, the action \(S\) becomes

\[
S = \phi(x) + \frac{1}{\kappa} \int d^2 \xi \ M_{\mu\nu}(x) \left( \eta^{ab} + \epsilon^{ab} \right) \partial_a X^\mu \partial_b X^\nu,
\]
where

\[ M_{\mu\nu}(x) = G_{\mu\nu}(x) + N_{\mu\nu}(x) + B_{\mu\nu}(x) + F_{\mu\nu}(x), \]

(3)

and

\[ N_{\mu\nu}(x) = \partial_\mu N_\nu + \partial_\nu N_\mu, \]

\[ F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

(4)

In the derivation of Eq. (3), one needs the classical equation of motion for the matter field.

The string variable \( X^\mu \), having free (Neumann) boundary condition for \( \mu = m = 0, \ldots, p \), and fixed (Dirichlet) boundary condition for \( \mu = i = p + 1, \ldots, 9 \), satisfies

\[ \partial_\sigma X^m|_{\sigma=0, \pi=0}, \quad \text{and} \quad \partial_\tau X^i|_{\sigma=0, \pi=0}, \]

(5)

from which we read the well-known fact in the oscillation problem of chords, i.e., the phase of oscillation does not change at the free boundary, whereas it changes by \( \pi \) at the fixed boundary. This fact is expressed as \( \alpha_n^\mu = \tilde{\alpha}_n^\mu \) for the Neumann boundary condition, whereas \( \alpha_n^\mu = -\tilde{\alpha}_n^\mu \) for the Dirichlet boundary condition, since \( \alpha_n^\mu \) and \( \tilde{\alpha}_n^\mu \) are right and left movers, which correspond to the incoming (outgoing) and the outgoing (incoming) oscillation modes, respectively.

More explicitly we have the following mode expansion:

\[ X^m(\tau, \sigma) = x^m + \frac{K}{\pi} p^m \tau + i \sqrt{\frac{K}{\pi}} \sum_{n \neq 0} \frac{\alpha_n^m}{n} e^{-in\tau} 2 \cos n\sigma, \]

(6)

\[ X^i(\tau, \sigma) = x^i + \frac{K}{\pi} \omega^i \sigma + i \sqrt{\frac{K}{\pi}} \sum_{n \neq 0} \frac{\alpha_n^i}{n} e^{-in\tau} 2i \sin n\sigma, \]

(7)

where \( p^m \) is center-of-mass coordinate, and \( \omega^i \) is the winding vector. We take \( \omega^i = 0 \) when both ends of the open string attach in the neighborhood of the same D-brane, otherwise we need the nonvanishing winding vector.

Since the end points of the open string can be located on the D-brane, various worldsheets of open strings can appear as the quantum fluctuations. Then, the external fields such as graviton, antisymmetric tensor field, etc. may couple to these fluctuated worldsheets of the open string, so that these external fields can be considered to interact with the original D-brane indirectly. If we integrate out the string variables, the messenger of the interaction between D-brane and the external fields, we will obtain the effective action of D-brane in the external fields. Therefore, open string worldsheets located apart from the world-volume of the D-brane, do not contribute to the effective action of the D-brane, even though they interact with the external fields. It is then sufficient to take into account, as the messenger of the interaction, the string coordinates inside the narrow strip sticking directly to the D-brane, and to integrate them out. Now, we can understand that apart from a constant factor, the effective action so obtained does not depend on the shape of the worldsheet, since the essential configuration is the narrow strip sticking to the D-brane, but is irrelevant to the topology of the worldsheet of string as a whole. Probably the coincidence of the effective Born-Infeld actions up to a constant factor between tree and one-loop labels found in Ref. [2] comes from this property.

The narrow strip may be specified by \(-T/2 < \tau < +T/2 \) \((T \sim \infty)\), \( 0 < \sigma < \varepsilon \ll 1 \), the string coordinates in which we are going to integrate out. In performing the path integration over the string coordinates on the narrow strip, we will restrict the integration
variables to the particular modes, satisfying equation of motions like Eq.(5)-(7). This restriction corresponds to the primary order of WKB approximation in the expansion of $\kappa$, or the expansion in terms of the momentum $p^m$. Therefore, the approximation is valid in deriving low energy effective action for D-branes. Then, we have

$$\int_{-\infty}^{+\infty} d\tau \int_{0}^{\infty} (\eta^{ab} + \epsilon^{ab}) \partial_a X^\mu \partial_b X^\nu = \int_{-\infty}^{+\infty} d\tau \int_{0}^{\infty} (\partial_\tau - \partial_\sigma)X^\mu(\partial_\tau + \partial_\sigma)X^\nu$$

(8)

$$= T\varepsilon \kappa \pi \sum_{-\infty}^{+\infty} \pm \alpha^\mu_0\alpha^\nu_n.$$  

(9)

where the signature $+$ and $-$ are associated with Neumann and Dirichlet boundary conditions for the direction $\nu$, respectively.

In Eq.(9), we can see that the zero modes $\alpha^\mu_0$ do not contribute. The zero modes are of course the center of mass momentum and the winding (or length) vector,

$$\alpha^\mu_0 = \begin{cases} \frac{1}{2} \sqrt{\frac{2}{\pi}} p^m & \text{for } \mu = m = 0, \cdots, p, \\ -\frac{1}{2} \sqrt{\frac{2}{\pi}} w^i & \text{for } \mu = i = p; 1, \cdots, 9. \end{cases}$$

(10)

For the massless excitation of the open string, we may choose $w^i = 0$, namely the length of the string is vanishing. The meaning of the field $N^\mu(X)$ can be seen from the variation of the ordinary string action with respect to $X^\mu$,

$$\delta S = \frac{1}{\kappa} \int d\tau \delta X_\mu \partial_\sigma X^\mu(\tau, \sigma = 0).$$

(11)

Therefore, the variation of the end point $\delta X_\mu$, or the variation of the D-brane’s position is identified with the field $N_\mu(X)$, namely,

$$N^m(x^0, \cdots, x^p) = 0, \text{ and } N^i(x^0, \cdots, x^p) = Y^i(x^0, \cdots, x^p),$$

(12)

where the first equation means that the variation in the tangential direction is nonphysical and its mode can be absorbed into the reparametrization of the D-brane. Now, nonvanishing components of $N^{\mu\nu}$ are for $(\mu, \nu) = (m, i)$ or $(i, m)$. Then, we understand that the contribution brane of the zero modes in Eq.(9) vanishes due to the massless condition,

$$p^m G_{mn}(x)p^n = 0.$$  

(13)

Now, we can perform the path integral over the open string modes within the narrow strip,

$$S_{eff} = \prod_{\mu = 0}^{9} \int dx^\mu e^{-\phi(x)} \prod_{n=1}^{\infty} \int d\alpha^\mu_n d\alpha^\nu_n e^{-T\varepsilon \kappa \pi \sum_{n=1}^{\infty} \pm (\alpha^\mu_n M_{\mu\nu}(x) \alpha^\nu_n + \alpha^\nu_n M_{\nu\mu}(x) \alpha^\mu_n)}$$

$$= \prod_{\mu = 0}^{9} \int dx^\mu e^{-\phi(x)} \left( \frac{\det M_{\mu\nu}(x)}{m_{\mu\nu}(x)^P} \right)^P,$$

(14)

(15)

where the power $P$ can be estimated by

$$P = \sum_{n=1}^{\infty} 1 = \left( \sum_{n=1}^{\infty} n^{-s} \right)_{s \to 0} = \zeta(0) = -\frac{1}{2}.$$  

(16)
Then we obtain
\[ S_{\text{eff}} = \int d^{p+1}x \, e^{-\phi(x)} \sqrt{\det_{\mu,\nu=0,\ldots,9} (G_{\mu\nu}(x) + N_{\mu\nu}(x) + B_{\mu\nu}(x) + F_{\mu\nu}(x))}. \] (17)

Here, the center-of-mass coordinates are restricted to those inside the D-brane.

In the effective action of the D-brane, the external fields coupling to it must have their indices along its tangential direction as much as possible. Here, we choose the flat background metric \( G_{\mu\nu} = \delta_{\mu\nu} \), the nonvanishing components of the two-form field and the two vector fields as \( B_{mn}(x^0, \ldots, x^p) \), \( A_m(x^0, \ldots, x^p) \), and \( N_i(x^0, \ldots, x^p) \) given in Eq.(12).

Then, we have
\[ \det_{\mu,\nu=0,\ldots,9} M_{\mu\nu} = \det_{\mu,\nu=0,\ldots,9} \left( \frac{\eta_{mn} + B_{mn} + F_{mn}}{\partial_m Y^i \partial_n Y^i} \right), \] (18)
\[ \det_{m,n=0,\ldots,p} (\hat{G}_{mn} + F_{mn}), \] (19)

where \( \hat{G}_{mn} \) is the induced metric on the D-brane,
\[ \hat{G}_{mn} = \eta_{mn} - \sum_{i=p+1}^9 \partial_m Y^i \partial_n Y^i = \partial_m Y^\mu \eta_{\mu\nu} \partial_n Y^\nu, \] (20)
\[ F_{mn} = B_{mn} - F_{mn}. \] (21)

We have finally obtained the bosonic part \( S_{\text{eff}}^{(1)} \) of the effective Born-Infeld action for the D-brane, which is valid in the neighborhood of the D-brane,
\[ S_{\text{eff}}^{(1)} = \int d^{p+1}x \, e^{-\phi(x)} \sqrt{\det_{m,n=0,\ldots,p} (\hat{G}_{mn}(x) + F_{mn}(x))}. \] (22)

### 3 Born-Infeld Action for Superstrings

In order to generalize the rule of thumb derivation of Born-Infeld action to superstrings, we have to add two more interactions: (a) interaction of the fermionic string coordinate, \( \psi^\mu(\tau, \sigma) \) with the background fields of NS-NS sectors which appeared in the previous section. (b) In addition, we have to introduce the interaction of the fermionic string coordinate, \( \psi^\mu(\tau, \sigma) \), with the Ramond-Ramond (R-R) background fields. Those are the totally antisymmetric tensor fields with \( n \) indices (\( n \)-form fields), \( A_{\mu_1,\mu_2,\ldots,\mu_n} \), and \( n \) is known to be odd and even for the type IIA and type IIB superstring, respectively.

As for the interaction (a), using the worldsheet supersymmetry, the action should be invariant under the following replacement,
\[ (\partial_\tau - \partial_\sigma)X^\mu(\tau, \sigma) \rightarrow \psi^\mu(\tau - \sigma), \quad \text{and} \quad (\partial_\tau + \partial_\sigma)X^\mu(\tau, \sigma) \rightarrow \tilde{\psi}^\mu(\tau + \sigma). \] (23)

Therefore, we have the following effective action from Eqs. (2)
\[ S_{\psi}^{(1)} = \frac{1}{\kappa} \int d\tau d\sigma \, M_{\mu\nu} (X(\tau, \sigma)) \psi^\mu(\tau - \sigma) \tilde{\psi}^\nu(\tau + \sigma), \] (24)

---

1 This part follows the lecture given by N. Ishibashi [8] with a modification.
representing the interaction of the fermionic coordinates with the background NS-NS fields.

According to the supersymmetry in Eq.(23), the Neumann and Dirichlet boundary conditions are transferred to the fermionic coordinates, and we have \( \psi_k^\mu = \pm \tilde{\psi}_k^\mu \), for the \( k \)th oscillation modes. (\( k \) is integer and half-integer for R and NS sectors, respectively.) Here, \(+(-)\) is chosen for the Neumann (Dirichlet) boundary condition, giving the well-known fact of the phase shift \( 0 (\pi) \) at the free (fixed) boundary of the chord as was discussed in the previous section. This naive discussion is valid for the R sector, where the bosonic and fermionic fields have both integer modes and the worldsheet supersymmetry is manifest. In the NS-sector, however, the fermionic coordinate has half-integer modes, different from the integer modes of the bosonic one, so that we cannot apply our naive argument, but then the local supersymmetric generator \( G_{\pm 1/2} \) with mode number \( \pm 1/2 \) plays the role of the supersymmetry.

Here, in both NS and R sectors, we will adopt the following mode expansion of the fermionic coordinates for \( m = 0, \cdots, p \) and \( i = p + 1, \cdots, 9 \):

\[
\left( \psi^m(\tau - \sigma), \tilde{\psi}^m(\tau + \sigma) \right) = \sqrt{\frac{\kappa}{2\pi}} \sum_k \psi_k^m \left( e^{-ik(\tau - \sigma)}, e^{-ik(\tau + \sigma)} \right), \tag{25}
\]

\[
\left( \psi^i(\tau - \sigma), \tilde{\psi}^i(\tau + \sigma) \right) = \sqrt{\frac{\kappa}{2\pi}} \sum_k \psi_k^i \left( e^{-ik(\tau - \sigma)}, -e^{-ik(\tau + \sigma)} \right). \tag{26}
\]

Next we will discuss the second interaction (2) with the R-R fields. As was found by Polchinski [1], these fields may couple to the D-brane’s volume element,

\[
\int_{\text{D-brane}} dy^{m_1} \cdots dy^{m_{p+1}} A_{m_1 \cdots m_{p+1}}(y_1, \cdots, y_p). \tag{27}
\]

Since there exist smaller D-branes within the D-brane [2], the interaction (2) becomes the sum of their contributions,

\[
S_{\psi}^{(2)} = \sum_{n \leq p+1} \int_{n\text{-brane} \subseteq \text{D-brane}} dy^{m_1} \cdots dy^{m_n} A_{m_1 \cdots m_n}(y_1, \cdots, y_p), \tag{28}
\]

where the D-brane’s position is specified by the proper coordinate system of \( y = (y^1, \cdots, y^p, y^i = 0 (i = p + 1, \cdots, 9)) \). We can redundantly multiply the extra volume element (infinite but constant) \( dy^{p+1} \cdots dy^9 \) to the above equation and obtain

\[
S_{\psi}^{(2)} = \sum_n \int dy^{\mu_1} \cdots dy^{\mu_n} dy^{p+1} \cdots dy^9 A_{\mu_1, \cdots, \mu_n}(y_1, \cdots, y_p). \tag{29}
\]

Then, the position of the D-brane is automatically specified and \( n \)-branes within the D-brane are naturally chosen, without the restriction of \( n\text{-brane} \subseteq \text{D-brane} \).

The problem is: what is the \( n \)-form \( dy^{\mu_1} \wedge \cdots dy^{\mu_n} \) on the D-brane? In order to discuss the interaction of the D-brane with the R-R fields, the \( n \)-form should be rewritten in terms of the fermionic coordinates \( \psi^\mu(\tau, \sigma) \). The one-form on the D-brane \( dy^{m} \) \( (m = 0, \cdots, p) \) can be represented as the motion of the end point of the open string from \( y^m \) to \( y^m + dy^m \), which is generated by the zero mode momentum \( p^m \) of the string. Similarly, the form \( dy^i \) \( (i = p; 1, \cdots, 9) \) pointing outwardly from the D-brane is generated by the winding
(or the length) vector $w^i$ of the string. Since the zero mode momentum and the winding vector is written as $\alpha_0^m + \tilde{\alpha}_0^m$ and $\alpha_0^i - \tilde{\alpha}_0^i$, respectively, they are transferred to fermionic coordinates by the worldsheet supersymmetry:

\[
d y^m \leftrightarrow p^m \rightarrow \theta_0^m \equiv \psi_0^m + \tilde{\psi}_0^m, \\
d y^i \leftrightarrow w^i \rightarrow \theta_0^i \equiv \psi_0^i - \tilde{\psi}_0^i.
\]

Therefore, we can use $\theta_0^\mu$ as the one-form on the D-brane, and $\theta_0^{\mu_1} \ldots \theta_0^{\mu_n}$ as the $n$-form on the D-brane.

Now the interaction (2) of the R-R fields with the D-brane, given in Eq.(22), is rewritten as

\[
S^{(2)}_\psi = \int d\tau d\sigma \sum_n \theta_0^{\mu_1} \ldots \theta_0^{\mu_n} \theta_0^{\nu_1+1} \ldots \theta_0^{\nu_l} \frac{1}{n!} A^{(n)}_{\mu_1, \ldots, \mu_p} (X(\tau, \sigma)).
\]

Now, from Eqs.(24) and (29), we have the fermionic string action coupled to the external fields of both types NS-NS and R-R as

\[
S_\psi = S^{(1)}_\psi + S^{(2)}_\psi.
\]

If we integrate out the fermionic coordinates $\psi^{\mu}(\tau, \sigma)$ in the narrow strip $(-T/2 < \tau < T/2, 0 < \tau \ll \varepsilon)$, we will obtain the effective action for the D-brane.

First, using the identity,

\[
\int \prod_{\mu=0}^9 d\theta_0^\mu \theta_0^{\mu_0} \ldots \theta_0^{\mu_l} = \epsilon^{\mu_0, \ldots, \mu_l},
\]

and noting the zero mode of $\psi^{\mu}(\tau - \sigma) \tilde{\psi}^{\mu}(\tau + \sigma)$ to be $\frac{1}{2} \theta_0^\mu \theta_0^\nu$, integration over zero modes $\theta_0^\mu$ gives the following effective action,

\[
S^{(2)}_{\text{eff}} = \int \prod_{\mu=0}^9 d\theta_0^\mu e^{-S_\psi}
= \int d^{p+1} x \epsilon^{\mu_0 \ldots \mu_l} \left( \frac{1}{(p+1)!} A^{(p+1)}_{\mu_0, \ldots, \mu_p} (x) - \frac{1}{(p-1)!} A^{(p-1)}_{\mu_0, \ldots, \mu_{p-2}} (x) \mathcal{F}^{(2)}_{\mu_{p-1}, \mu_p} (x) \right)
+ \frac{1}{(p-3)!} \frac{1}{8} A^{(p-3)}_{\mu_0, \ldots, \mu_{p-4}} (x) \mathcal{F}^{(2)}_{\mu_{p-3}, \mu_{p-1}} (x) \mathcal{F}^{(2)}_{\mu_{p-1}, \mu_p} (x) + \ldots
\]

\[
= \int d^{p+1} x \left( \sum_{n \leq p+1} A^{(n, \text{-form})} e^{-\frac{1}{2} \mathcal{F}^{(\text{two-form})}_{(p+1, \text{-form})}} \right).
\]

Next, we will perform the nonzero mode integration over $\alpha_{\pm n}^\mu$ and $\psi_\pm^\mu$. As was found in Ref.[4], the bosonic and fermionic mode integrations completely cancel in the R-sector, giving no contribution to the effective action. Whereas in the NS-sector, bosonic mode integration gives the effective action $S^{(1)}_{\text{eff}}$ given in Eq.(22), but the fermionic mode integration gives nothing since

\[
\sum_{k=\text{positive half-integer}} 1 = 2^s \sum_{n=1}^{\infty} \left( n^{-s} - (2n)^{-s} \right) |_{s \to 0}
= (2^s - 1) \zeta(s) |_{s \to 0} = 0.
\]
Usefulness of the zeta function regularization in the dynamics of extended objects may be found in Ref. [10].

From the above discussions, we have finally obtain the Born-Infeld like effective action for D-brane in the superstrings:

\[
S_{\text{eff}} = S_{\text{eff}}^{(1)} + S_{\text{eff}}^{(2)}
\]

\[
= \int d^{p+1}x \left( e^{-\phi(x)} \sqrt{\det_{m,n=0,\ldots,p} \left( G_{mn}(x) + F_{mn}(x) \right)} \right)
\]

\[
+ \left( \sum_{n \leq p+1} A^{(n)} e^{-\frac{1}{2}F^{(2)}} \right)_{(p+1)\text{-form}}.
\]

\[
(41)
\]

4 Conclusion

Here we have discussed the rule of thumb derivation of the Born-Infeld action for D-branes in the supersymmetric models. The “derivation” is simply performed by integrating out of the superstring coordinates in the narrow strip attached to the D-brane, so that it can be viewed as a way to understand how the Born-Infeld action is derived. The way of thinking is, however, easy to follow. Thus, this may be helpful for us to study various problems in D-branes.

Note added

In the paper by A.Tseytlin [11], a similar idea of deriving explicitly the Born-Infeld action for D-brane has been given. In the superstring case, however, our treatment may be more direct than his. We are most grateful to Prof. A.A.Tseytlin for pointing out his interesting paper.

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References

[1] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.

[2] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B163 (1985) 123; B158 (1985) 316; Nucl. Phys. B261 (1985) 1.

[3] J. Polchinski and Y. Cai, Nucl. Phys. B296 (1988) 91.

[4] C. G. Callan, C. Lovelace, C. R. Nappi, and S. A. Yost, Nucl. Phys. B308 (1988) 221.
[5] J. Dai, R. G. Leigh, and J. Polchinski, *Mod. Phys. Lett.* **A4** (1989) 2073; 
R. G. Leigh, *ibid.* **A4** (1989) 2767.

[6] M. R. Douglas, [hep-th/9512077](https://arxiv.org/abs/hep-th/9512077).

[7] M. Li, *Nucl. Phys.* **B460** (1996) 351.

[8] C. Schmidhuber, *Nucl. Phys.* **B467** (1996) 146.

[9] N. Ishibashi, Lecture given at Chubu Summer School at Shiga Highland, Japan, Au-
gust (1996).

[10] See for example, E. Elizalde, S. D. Odintsov, A. Romeo, A. A. Bytsenko, and S. 
Zerbini, *Zeta Regularization Techniques with Applications* World Scientific (1994).

[11] A.A.Tseytlin, *Nucl.Phys.* **B469**(1996)51.