Effect of relaxation time on the squeezed correlations of bosons for evolving sources in relativistic heavy-ion collisions

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Abstract The squeezed back-to-back correlation (SBBC) of a boson–antiboson pair is sensitive to the time distribution of the particle-emitting source, and the SBBC function for an evolving source is expected to be affected by the relaxation time of the system. In this study, we investigated the effect of relaxation time on the SBBC function. We propose a method for calculating the SBBC function with relaxation-time approximation for evolving sources. SBBC functions of $D^0\bar{D}^0$ in relativistic heavy-ion collisions were investigated using a hydrodynamic model. We found that the relaxation time reduces the amplitudes of the SBBC functions. This becomes apparent for long relaxation times and large initial relative deviations of the chaotic and squeezed amplitudes from their equilibrium values in the temporal steps.

Keywords Relaxation time \, Squeezed back-to-back correlation \, Evolving source \, Relativistic heavy-ion collisions

1 Introduction

In relativistic heavy-ion collisions, the interactions between the particles in the sources lead to a modification of the boson mass in the sources and thus give rise to a squeezed boson–antiboson correlation [1, 2]. This squeezed correlation is caused by the Bogoliubov transformation between the creation and annihilation operators of the quasiparticles in the source and the free observable particles and forces the bosons and antibosons to move in opposite directions. Therefore, it is also known as a squeezed back-to-back correlation (SBBC) [1–3]. Measuring the SBBC of bosons can be used to get information about the interaction between the meson and the source medium and will be useful for understanding the properties of the particle-emitting sources [1–6].

Hydrodynamics has been widely used in relativistic heavy-ion collisions to describe the evolution of a particle-emitting source. In the hydrodynamic description, it is assumed that the source system is under the local equilibrium, that is, it evolves in the so-called quasi-static process. However, the quasi-static process is a rough approximation. Because the SBBC is sensitive to the time distribution of the source, appropriately representing the temporal factors is of interest in the calculation of the SBBC function for an evolving source.

$D$ mesons contain a heavy quark (charm quark) produced during the early stage of relativistic heavy-ion collisions. The SBBC of the $D$ mesons is stronger than that of light mesons and useful for probing the source properties in the early stage [6–10]. This study proposes a method for calculating the SBBC function with the relaxation-time approximation for evolving sources. The effects of the

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relaxation time on the SBBC functions of $D^0\bar{D}^0$ in relativistic heavy-ion collisions are investigated using the hydrodynamic model VISH2+1 [11, 12]. We found that the SBBC functions decrease when the relaxation time is considered. The change in the SBBC functions increases with increasing relaxation time and becomes considerable for large relative deviations of chaotic and squeezed amplitudes at the beginning of their deviation from their equilibrium values.

The remainder of this paper is organized as follows: In Sect. 2, we present the formulas of the SBBC functions of evolving sources with relaxation-time approximation. In Sect. 3, we investigate the influence of the relaxation time on the SBBC functions of $D^0\bar{D}^0$ in relativistic heavy-ion collisions. Finally, a summary and discussion are given in Sect. 4.

2 Formulas

The SBBC function of a boson–antiboson pair with momenta of $p_1$ and $p_2$, respectively, is defined as [2, 3]

$$G_{SBB}(p_1, p_2) = 1 + \frac{\|G_s(p_1, p_2)\|^2}{G_c(p_1, p_1)G_c(p_2, p_2)}, \quad (1)$$

where

$$G_c(p_1, p_2) = \sqrt{\omega_p \omega_{p'}} \langle a^{\dagger}(p_1) a(p_2) \rangle$$
$$\equiv \sqrt{\omega_p \omega_{p'}} \langle g_c(p_1, p_2) \rangle, \quad (2)$$

$$G_s(p_1, p_2) = \sqrt{\omega_p \omega_{p'}} \langle a(p_1) a(p_2) \rangle$$
$$\equiv \sqrt{\omega_p \omega_{p'}} \langle g_s(p_1, p_2) \rangle, \quad (3)$$

are the so-called chaotic and squeezed amplitudes, respectively [2, 3], where $\omega_p = \sqrt{p^2 + m^2}$ is the energy of a free bosons with a mass of $m$, $a$ and $a^{\dagger}$ are the annihilation and creation operators of the free boson, respectively, and $\langle \cdots \rangle$ represents the ensemble average.

For a homogeneous thermal-equilibrium source with a fixed volume of $V$ and in the temporal interval of $[0-\Delta t]$ with the time distribution $F(t)$, the amplitudes $G_c(p, p)$ and $G_s(p, -p)$ can be expressed as [2, 3]

$$G_c(p, p) = \frac{V}{(2\pi)^3} \omega_p \left[ c_{p}^2 n_p + |s_p|^2 (n_p + 1) \right]$$
$$\equiv \omega_p \langle g_c(p, p) \rangle, \quad (4)$$

$$G_s(p, -p)$$
$$= \frac{V}{(2\pi)^3} \omega_p \left[ c_{p}^2 n_p + c_{-p} s_{-p} (n_{-p} + 1) \right] \tilde{F}(\omega_p, \Delta t)$$
$$\equiv \omega_p \langle g_s(p, -p) \rangle, \quad (5)$$

where

$$c_{\pm p} = c_{\pm p} = \cosh f_p,$$
$$s_{\pm p} = s_{\pm p} = \sinh f_p,$$
$$f_p = \frac{1}{2} \ln(\omega_p/\Omega_p),$$

where $\Omega_p = \sqrt{p^2 + m^2}$ is the energy of the quasi-particle, $m$ is the effective mass in the source medium, $n_p$ is the boson distribution of the quasiparticle, and $\tilde{F}(\omega_p, \Delta t)$ is the Fourier transform of time distribution.

For an evolving source, $g_c(p, p)$ and $g_s(p, -p)$ can be expressed as

$$g_c(p, p) = g_c^0(p, p) - \tau_c \frac{\partial g_c(p, p)}{\partial t}, \quad (7)$$

$$g_s(p, -p) = g_s^0(p, -p) - \tau_s \frac{\partial g_s(p, -p)}{\partial t}, \quad (8)$$

where $g_c^0(p, p)$ and $g_s^0(p, -p)$ are their quantities in the equilibrium state given by Eqs. (4) and (5), and $\tau_c$ and $\tau_s$ are the relaxation-time parameters related to the system’s capacity to return to its equilibrium state. Relaxation-time approximation is an usual approach for determining the quantities of evolving systems. In this approximation, $\tau_{c,s}$ must be smaller than the width of the temporal interval $\tau_{c,s} < \Delta t$, and they tend to zero under the quasi-static condition.

Assuming $\tau_c = \tau_s = \tilde{\tau}$, Eqs. (7) and (8) give

$$g_c(p, p) = g_c^0(p, p) + \Delta_c^0 e^{-\tilde{\tau} / \tau_c},$$
$$\approx g_c^0(p, p) (1 + \delta_c^0 e^{-\tilde{\tau} / \tau_c}), \quad (9)$$

$$g_s(p, -p) = g_s^0(p, -p) + \Delta_s^0 e^{-\tilde{\tau} / \tau_s},$$
$$\approx g_s^0(p, -p) (1 + \delta_s^0 e^{-\tilde{\tau} / \tau_s}), \quad (10)$$

where $\Delta_c$ and $\Delta_s$ are the differences in $g_c$ and $g_s$ between the beginning of the evolution and equilibrium, and it is assumed that the differences are approximately proportional to $g_c$ and $g_s$ with the proportionality parameters of $\delta_c$ and $\delta_s$, respectively. $\delta_c$ and $\delta_s$ denote the relative deviation of the chaotic (squeezed) amplitude at the beginning of its deviation from its equilibrium value. Then, the SBBC function $C_{SBB}(\mathbf{p}, -\mathbf{p})$ for an evolving source is given by
Fig. 1 (Color online) The SBBC functions of the D$^0$F$^0$ pair in the hydrodynamic model of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with 0–80% and 40–80% centralities and two momentum ranges.

\[ C_{\text{SBBC}}(p, -p) = 1 + \frac{\left| \langle \phi_{0}^{p}(p, -p)(1 + \delta_{0}^{c} e^{-\delta_{0}^{c}/t}) \rangle \right|^2}{\langle \phi_{0}^{p}(p, p)(1 + \delta_{0}^{c} e^{-\delta_{0}^{c}/t}) \rangle \langle \phi_{0}^{p}(-p, -p)(1 + \delta_{0}^{c} e^{-\delta_{0}^{c}/t}) \rangle}. \]

(11)

For a hydrodynamic source, the chaotic and squeezed amplitudes $G_{c}(p_{1}, p_{2})$ and $G_{s}(p_{1}, p_{2})$ can be expressed in the relaxation time approximation as [2–6],

\[ G_{c}(p_{1}, p_{2}) = \int \frac{d^{4}\sigma_{\mu}(r)}{(2\pi)^{3}} K_{1,2}^{\mu} e^{iq_{1,2} \cdot r} \left[ \left| c'_{p_{1}', p_{2}'} n'_{p_{1}', p_{2}'} + |s'_{p_{1}', p_{2}'}|^{2}(n'_{p_{1}', p_{2}'} + 1) \right] \times \left[ 1 + \delta_{c}^{0} e^{-\delta_{c}^{0}/t} \right], \]

\[ G_{s}(p_{1}, p_{2}) = \int \frac{d^{4}\sigma_{\mu}(r)}{(2\pi)^{3}} K_{1,2}^{\mu} e^{2iK_{1,2} \cdot r} \left[ s_{p_{1}', p_{2}'}^{c} c'_{p_{1}', p_{2}'} n'_{p_{1}', p_{2}'} + c'_{p_{1}', p_{2}'} s_{p_{1}', p_{2}'}^{c} n'_{p_{1}', p_{2}'} + |s'_{p_{1}', p_{2}'}|^{2}(n'_{p_{1}', p_{2}'} + 1) \right] \times \left[ 1 + \delta_{s}^{0} e^{-\delta_{s}^{0}/t} \right]. \]

(12)

(13)

where $d^{4}\sigma_{\mu}(r) = f_{\mu}(r)d^{4}r dr$ is the four-dimension element of the freeze-out hypersurface, $q_{1,2}^{\mu} = p_{1}^{\mu} - p_{2}^{\mu}$, $K_{1,2}^{\mu} = (p_{1}^{\mu} + p_{2}^{\mu})/2$, $c'_{p_{1}', p_{2}'}$ and $s'_{p_{1}', p_{2}'}$ are the coefficients of the Bogoliubov transformation between the creation and annihilation operators of the quasiparticles and the free particles, $n'_{p_{1}', p_{2}'}$ is the boson distribution associated with the particle pair in the local frame, and $p_{i}^{\prime}(i = 1, 2)$ is local-frame momentum [2–6]. In Eqs. (12) and (13), $[1 + \delta_{c,s}^{0} e^{-\delta_{c,s}^{0}/t}]$ is the factor for relaxation-time influence, where $t$ is the local frame time. Eqs. (12) and (13) reduce to their usual forms as in [2–6] when $\tilde{t} = 0$.

Dividing the entire time evolution into a series of time steps ($j = 1, 2, \cdots$) with the same step width, we have

\[ G_{c}(p_{1}, p_{2}) = \sum_{j} \int \frac{f_{\mu}(j)d^{4}r}{(2\pi)^{3}} K_{1,2}^{\mu} e^{iq_{1,2} \cdot r} e^{i\delta_{1,2}^{0}/t} \left[ c'_{p_{1}', p_{2}'} n'_{p_{1}', p_{2}'} + |s'_{p_{1}', p_{2}'}|^{2}(n'_{p_{1}', p_{2}'} + 1) \right] \times \left[ 1 + \delta_{c}^{0} \int_{0}^{\Delta t} d't' D(t') e^{\delta_{1,2}^{0}/t} e^{-t'}/\tilde{t}', \right] \]

(14)
\[ G_S(p_1, p_2) = \sum_j f_i(r) \frac{d^3 r}{(2\pi)^3} K_{1,2}^\mu e^{-2ik_{1,2} x} e^{2ik_{1,2} y} \]

\[ = \left[ s_{\gamma_1,\gamma_2}^\mu p_{\gamma_1,\gamma_2} + c_{\gamma_1,\gamma_2}^\mu p_{\gamma_1,\gamma_2} \times (n_{\gamma_1,\gamma_2} + 1) \right] \left[ 1 + \gamma^0 e^{\Delta t} - i \tau \right]. \]

where \( \Delta t \) is the step width of time in the local frame, \( t(t') = \gamma t', \gamma = 1/\sqrt{1 - v^2}, v \) is the velocity of the fluid element, and \( D(t') \) is the local time distribution at each time step. Taking \( D(t') \) to be a uniform distribution, the relaxation-time factors of \( G_c(p, p) (i = 1, 2) \) and \( G_S(p_1, p_2) \) are

\[ 1 + \delta_0^c \frac{\tilde{\tau}}{\Delta t} \int d t' e^{-\tau / \tilde{\tau}} \approx \left[ 1 + \delta_0^c \frac{\tilde{\tau}}{\Delta t} \right], \quad (\tilde{\tau} = < \Delta \tau), \]

\[ 1 + \delta_0^S \frac{\tilde{\tau}}{\Delta t} \int d t' e^{2iK_{j,2} t(\tilde{\tau})} - i \frac{\tilde{\tau}}{\Delta t} \]

\[ \approx \left[ 1 + \delta_0^S \frac{\tilde{\tau}}{\Delta t} \frac{1 + i 2K_{j,2} \tau}{1 + (2K_{j,2} \tau)^2} \right]. \]

3 Results

Figure 1a and 1b shows the SBBC functions \( C(\Delta \phi) \) of the \( D^0D^0 \) pair in the hydrodynamic model for Au+Au collisions at RHIC energy \( \sqrt{s_{NN}} = 200 \text{ GeV} \) with 0–80% centrality and in momentum intervals of 0.55–0.65 GeV/c and 1.15–1.25 GeV/c, respectively. Here, \( \Delta \phi \) is the azimuthal angle difference between the transverse momenta \( p_{1T} \) and \( p_{2T} \) of the two \( D \) mesons, and the dashed, solid, dot-dashed, and two-dot-dashed lines represent \( (\delta_0^c = \delta_0^S = \delta_0 = 0.5, \tilde{\tau} = 0 \text{ fm/c}, \) without relaxation-time modification), \( (\delta_0^c = \delta_0^S = \delta_0 = 0.5, \tilde{\tau} = 0.2 \text{ fm/c}), \) \( (\delta_0^c = \delta_0^S = \delta_0 = 0.25, \tilde{\tau} = 0.4 \text{ fm/c}), \) and \( (\delta_0^c = \delta_0^S = \delta_0 = 0.5, \tilde{\tau} = 0.4 \text{ fm/c}), \) respectively. In our model calculations, the event-by-event initial conditions of MC-Gib [13] are employed, the ratio of the shear viscosity to the entropy density of the QGP is taken to be 0.08 [14, 15], and the freeze-out temperature is taken to be \( T_f = 150 \text{ MeV} \) according to the comparisons of the model transverse momentum spectrum of \( D^0 \) [8] with the RHIC-STAR experimental data [16] (see the Fig. 3 in Ref. [8]). In the calculations, the time step width was assumed to be 1 fm/c, the free \( D^0 \) meson mass was taken to be 1.865 GeV/c², and the in-medium average mass and width were obtained from the results of the FMFK calculations [8, 17, 18].

Figure 1a and 1b shows that the relaxation time decreases the SBBC functions. This change increases with an increase in \( \tilde{\tau} \) and \( \delta_0 \). The relaxation-time parameter indicates the capacity of the system to return to the equilibrium. In thermodynamics, high temperature and violent collisions may increase this capacity. \( \delta_0 \) is related to the expanding velocity of the system. The SBBC functions in the lower momentum interval are higher than those in the higher momentum interval because of a greater in-medium mass modification at low momenta than that at high momenta [8], and greater single-event SBBC function oscillations at high momenta [5] may also lead to a lower average SBBC function [6, 8].

Figure 1c and 1d shows the SBBC functions \( C(\Delta \phi) \) of the \( D^0D^0 \) pair in the hydrodynamic model for Au+Au collisions at RHIC energy \( \sqrt{s_{NN}} = 200 \text{ GeV} \) with 0–80% centrality, and momentum intervals of 0.55–0.65 GeV/c and 1.15–1.25 GeV/c, respectively. From these figures, it can be observed that changes in the relaxation-time decrease the SBBC functions, and the influence of this change increases with increasing \( \tilde{\tau} \) and \( \delta_0 \). Compared to the results for collisions with 0–80% centrality, the SBBC functions for collisions with 40–80% centrality are higher. This is mainly because the temporal distribution of the source is narrower for peripheral collisions [6]. Contributions to SBBC functions at lower \( \Delta \phi \) are mainly from the more peripheral collisions, which have small spatial and temporal sizes [8]. The differences between the SBBC functions of the collisions with 0–80% and 40–80% centralities become small in the higher momentum interval.

4 Summary and discussion

In this study, we investigated the effects of relaxation time on the SBBC functions of boson–antiboson pairs in relativistic heavy-ion collisions. A method for calculating the SBBC functions of bosons with relaxation-time approximation for evolving sources is proposed. Using the method in a hydrodynamic model, we investigated the SBBC functions of \( D^0D^0 \) in Au+Au collisions at the RHIC energy \( \sqrt{s_{NN}} = 200 \text{ GeV} \) and centralities of 0–80% and 40–80%. We found that the relaxation time reduces the amplitudes of the SBBC functions. This change increases with increasing relaxation time and becomes considerable for relatively large deviations of the chaotic and squeezed amplitudes at the beginning of their deviation from their equilibrium values.

Relaxation-time approximation is an usual approach for calculating quantities in a near-equilibrium evolving
system. In viscous hydrodynamic models, relaxation times of shear and bulk viscosities are introduced, which may change the system’s space-time structure. However, the relaxation times associated with the quantities must be considered during calculations using the hydrodynamic model because they transition from a nonequilibrium to an equilibrium state in each temporal step. In addition, the relaxation time must be appropriately considered when calculating a sensitive time-depend observable.

Using a hydrodynamic model, one can obtain the source temperature as a space-time function. The final observed particles are assumed to be emitted thermally from a four-dimensional hypersurface at the fixed freeze-out temperature, which can be determined by comparing the calculated observables, for example, particle transverse-momentum spectra, with experimental data. In this study, we used the viscous hydrodynamic model VISH2+1 [11, 12] to determine the freeze-out hypersurface and calculate SBBC functions with and without the relaxation-time term \( \delta^0 e^{-t^2/\tau} \). The relaxation time reduces the SBBC functions. This effect may be retained in an ideal hydrodynamic model although the viscosity changes of the freeze-out hypersurface.

Because the SBBC is caused by changes in the particle mass in the source medium, analyzing SBBC may become a new technique for getting information about particles in medium interactions; however, there are no experimental data for comparison with the findings of the model when it is used in this manner. However, it is difficult to account for particle scattering in detail in a bulk evolution model. In our model calculations, we assume that \( D \) mesons have a mass shift and width, which are obtained from the FMFK calculations [8, 17, 18], in the sources because of the in-medium interactions. More detailed studies of the influence of particle scattering on the SBBC based on a cascade model (for instance, a multi-phase transport model [19, 20]) or a hybrid model (for instance, the hydro-+ UrQMD model [21, 22]) will be pursued in the future.

Authors’ contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Wei-Ning Zhang and Peng-Zhi Xu. The first draft of the manuscript was written by Wei-Ning Zhang, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

References

1. M. Asakawa, T. Csörgő, Strangeness correlation: A clue to hadron modification in dense matter? Heavy. Ion. Phys. 4, 233 (1996). arXiv: hep-ph/9612331
2. M. Asakawa, T. Csörgő, M. Gyulassy, Squeezed correlations and spectra for mass-shifted bosons. Phys. Rev. Lett. 83, 4013 (1999). https://doi.org/10.1103/PhysRevLett.83.4013
3. S.S. Padula, G. Krein, T. Csörgő et al., Back-to-back correlations for finite expanding fireballs. Phys. Rev. C 73, 044906 (2006). https://doi.org/10.1103/PhysRevC.73.044906
4. D.M. Dudek, S.S. Padula, Squeezed \( K^+K^- \) correlations in high energy heavy ion collisions. Phys. Rev. C 82, 034905 (2010). https://doi.org/10.1103/PhysRevC.82.034905
5. Y. Zhang, J. Yang, W.N. Zhang, Squeezed correlations of \( \phi \) meson pairs for hydrodynamic sources in high-energy heavy-ion collisions. Phys. Rev. C 92, 024906 (2015). https://doi.org/10.1103/PhysRevC.92.024906
6. Y. Zhang, W.N. Zhang, Predictions for squeezed back-to-back correlations of \( \phi \phi \) and \( K^+K^- \) in high-energy heavy-ion collisions by event-by-event hydrodynamics. Eur. Phys. J. C 76, 419 (2016). https://doi.org/10.1140/epjc/s10052-016-4270-y
7. S.S. Padula, D.M. Dudek Jr., O. Socolowski, Squeezed correlations of strange particles-antiparticles. J. Phys. G 37, 094056 (2010). https://doi.org/10.1088/0954-3899/37/9/094056
8. A.G. Yang, Y. Zhang, L. Cheng et al., Squeezed back-to-back correlation of \( D \bar{D} \) in relativistic heavy-ion collisions. Chin. Phys. Lett. 35, 052501 (2018). https://doi.org/10.1088/0256-307X/35/5/052501
9. P.Z. Xu, W.N. Zhang, Y. Zhang, Squeezed back-to-back correlation between bosons and antibosons with different in-medium masses in high-energy heavy-ion collisions. Phys. Rev. C 99, 011902(R) (2019). https://doi.org/10.1103/PhysRevC.99.011902
10. P.Z. Xu, W.N. Zhang, Squeezed back-to-back correlations of bosons with nonzero widths in relativistic heavy-ion collisions. Phys. Rev. C 100, 014907 (2019). https://doi.org/10.1103/PhysRevC.100.014907
11. H. Song, U. Heinz, Suppression of elliptic flow in a minimally viscous quark-gluon plasma. Phys. Lett. B 658, 279 (2008). https://doi.org/10.1016/j.physletb.2007.11.019
12. H. Song, U. Heinz, Causal viscous hydrodynamics in 2 + 1 dimensions for relativistic heavy-ion collisions. Phys. Rev. C 77, 064901 (2008). https://doi.org/10.1103/PhysRevC.77.064901
13. C. Shen, Z. Qiu, H. Song et al., The iEBE-VISHNU code package for relativistic heavy-ion collisions. arXiv:1409.8164; https://u.osu.edu/vishnu/
14. C. Shen, U. Heinz, P. Huovinen et al., Radial and elliptic flow in Pb + Pb collisions at energies available at the CERN Large Hadron Collider from viscous hydrodynamics. Phys. Rev. C 84, 044903 (2011). https://doi.org/10.1103/PhysRevC.84.044903
15. J. Qian, U. Heinz, J. Liu, Mode-coupling effects in anisotropic flow in heavy-ion collisions. Phys. Rev. C 93, 064901 (2016). https://doi.org/10.1103/PhysRevC.93.064901
16. L. Adamczyk, J.K. Adkins, G. Agakishiev et al., (STAR Collaboration), Observation of \( D \) Meson Nuclear Modifications in Au + Au Collisions at \( \sqrt{s_{NN}} = 200 GeV \). Phys. Rev. Lett. 113, 142301 (2014). https://doi.org/10.1103/PhysRevLett.113.142301
17. C. Fuchs, B.V. Martemyanov, A. Faessler et al., \( D \)-mesons and charmonium states in hot pion matter. Phys. Rev. C 73, 034204 (2006). https://doi.org/10.1103/PhysRevC.73.034204
18. B.V. Martemyanov, A. Faessler, C. Fuchs et al., Medium modifications of kaons in pion matter. Phys. Rev. Lett. 93, 052301 (2004). https://doi.org/10.1103/PhysRevLett.93.052301
19. Z.W. Lin, C.M. Ko, B.A. Li et al., Multiphase transport model for relativistic heavy ion collisions. Phys. Rev. C 72, 064901 (2005). https://doi.org/10.1103/PhysRevC.72.064901
20. H. Wang, J.H. Chen, Study on open charm hadron production and angular correlation in high-energy nuclear collisions. Nucl. Sci. Tech. 32, 2 (2021). https://doi.org/10.1007/s41365-020-00839-x
21. H.C. Song, S.A. Bass, U. Heinz, Viscous QCD matter in a hybrid hydrodynamic+Boltzmann approach. Phys. Rev. C 83, 024912 (2011). https://doi.org/10.1103/PhysRevC.83.024912

22. I.A. Karpenko, P. Huovinen, H. Petersen et al., Estimation of the shear viscosity at finite net-baryon density from $A + A$ collision data at $\sqrt{s_{NN}} = 7.7 - 200$ GeV. Phys. Rev. C 91, 064901 (2015). https://doi.org/10.1103/PhysRevC.91.064901