Summary report of the workshop D2: Conceptual Issues, Foundational Questions and Quantum Cosmology

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1 Introduction

In this workshop rather diverging problems related to quantum gravity and quantum cosmology were discussed. Since it is almost impossible to summarize all these discussions in coherent and integrated expressions, in this report, I give a brief description of the content of each paper in order, with some additional comments on their backgrounds.

In order to give a readable presentation, I classified the oral and poster papers presented in the workshop into six groups by their contents. The next section treats discussion on spacetime topology and includes a single paper by S. Carlip. Section 3 includes three papers on quantum cosmology by L.O. Pimentel and C. Mora, by R. Mansouri and F. Naseri, and by the author (H.K.). Section 4 treats papers on the interpretation problem in quantum gravity and cosmology by D.P. Datta, by M. Kenmoku and others, by M. Morikawa and others, by J. Larsson, and by A. Corichi and M.P. Ryan. Section 5 summarizes discussions related to supersymmetry by R. Ferraro and D.M. Sforza, by A.Yu. Kamenshchik and S.L. Lyakhovich, by H. Luckock, and by H. Luckock and H. Farajollahi. Section 6 includes papers on various non-standard approaches to quantum gravity and other arguments on foundational problems presented by O. Richter, by P. Kuusk and others, by J. Geddes, by R.S. Tung, by D.R. Finkelstein, by R.R. Karnik, and by E.G. Mychelkin. The final section is devoted to the single paper by S. Brave and T.P. Singh on the relation between the cosmic censorship and the second law of thermodynamics.

2 Spacetime Topology

S. Carlip reported on his very interesting observations on topological fluctuations of spacetime in Euclidean quantum gravity[1].

He applied the saddle point approximation to the Euclidean path-integral expression for the partition function of the pure gravity system with cosmological constant $\Lambda (\neq 0)$. After rescaling the metric so that each saddle point is represented by an Einstein space whose Ricci scalar curvature is equal to $\pm 12$, he wrote the corresponding expression for $Z$ as a sum over the volume $\tilde{\nu}$ of the rescaled space: $Z = \sum_{\tilde{\nu}} \rho(\tilde{\nu}) \exp(9\tilde{\nu}/8\pi \Lambda L_{pl})$, where $\rho(\tilde{\nu})$ is the number of normalized Einstein spaces with volume $\tilde{\nu}$. One important point here is that $\tilde{\nu}$ is a good index to classify topology because each manifold seems to admit only a

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[1]To be published in the Proceedings of the 15th General Relativity and Gravity Meeting.
limited number of Einstein metrics. Thus the sum over $\tilde{v}$ can be regarded as the sum over topologies.

On the basis of this expression and estimations of $\rho(\tilde{v})$, he argued that quantum fluctuations of topology have quite different nature for $\Lambda > 0$ and $\Lambda < 0$. In particular, he showed that for $\Lambda < 0$, $\ln \rho$ grows faster than $\tilde{v} \ln \tilde{v}$. This implies that complicated topologies dominate quantum fluctuations in this case. He further argued that the Hartle-Hawking wavefunction may be sharply peaked at some special 3-topologies.

Though Carlip’s argument is still at the level of speculation, it is quite fascinating and seems to have far-reaching implications.

3 Quantum Cosmology

Two papers discussed special quantum cosmological models in the Wheeler-DeWitt approach.

First, L.O. Pimentel and César Mora considered a spatially homogeneous scalar-tensor theory on the FRW universe[2]. They exactly solved the Wheeler-DeWitt equation for this model and found that some special subset of the solutions satisfy the boundary condition proposed by the Hawking and Page, which requires that wave functions are regular at degenerate geometries and fall off as the volume of 3-space increases[3]. They interpreted that these solutions represent wormholes, although I could not understand the reason.

Second, R. Mansouri and F. Naseri applied the Wheeler-DeWitt quantization to a peculiar model proposed by R. Mansouri and his collaborators[4], in which the universe has a fractal structure and its dimension is dynamical, and discussed the dependence of operator-ordering parameters on the spatial dimension.

The author(H.K.) discussed the quantum dynamics of a spatially compact Bianchi I model from a different viewpoint. As is well know, the canonical approach to quantum gravity based on the Wheeler-DeWitt equation has various difficulties in mathematical formulation and interpretation. In order to eliminate these difficulties of the conventional Dirac quantization, the author recently proposed a new mathematically rigorous formalism, called the Web formalism, for quantum dynamics of totally constrained systems[5]. In this workshop, he talked about the relation of quantum dynamics in this formalism to that in the Wheeler-DeWitt approach for the special compact Bianchi model.

He first pointed out that in locally homogeneous pure gravity systems obtained by compactifying Bianchi I models, their moduli freedom is essentially frozen and their dynamics is described by a hamiltonian system corresponding to the diagonal Bianchi I model[6]. Then by restricting consideration to the diagonal model on the basis of this result, he showed that quantum dynamics in the Web formalism can be described by solutions to the Wheeler-DeWitt equation only for special choices of the lapse function. Further he discussed a general criterion for good time variables or instant operators for this model.
4 Interpretation Problem

Finding natural correspondences between quantum theory and classical theory plays an important role in constructing and interpreting quantum theory. In particular, the extraction of time concept is the major problem in the interpretation of dynamics of closed systems including quantum gravity systems. In this workshop, three different approaches to this problem were discussed.

The first one is the famous WKB approach, which corresponds to the Born-Oppenheimer approximation used in the analysis of quantum behavior of composite systems. In this approach, one decomposes the whole system $U$ into a semi-classical part $C$ and a quantum part $Q$ and writes the wave function $\Psi$ for the whole system as the product of those for each part as $\psi \chi$. If one applies the WKB approximation to the semi-classical part $\psi = e^{iS}$, one obtains classical Hamiltonian-Jacobi trajectories, on each of which a natural classical time can be defined, and the evolution equation for $\chi$ is written as the Schrödinger equation with respect to this time variable.

In this prescription, however, there is an ambiguity or a gauge freedom in the choice of phase in the decomposition, which affects the decomposed dynamics[7]. D.P. Data investigated the relation of this gauge freedom and quantum fluctuations of the quantum subsystem and argued that under an appropriate gauge fixing, one can pick up a natural time variable which is related to quantum fluctuations of the system.

The second approach to the classical-quantum correspondence problem is the de Broglie-Bohm interpretation for wave functions. In this approach, one does not decompose the whole system $U$ unlike in the WKB approach. Instead, one writes its wave function $\Psi$ simply as $\Psi = A \exp(iS)$ and considers classical trajectories (dBB trajectories) corresponding to the Hamilton-Jacobi function $S$. This leads to equations of motion which are different from the classical ones by a correction to the potential, called the quantum potential. This quantum potential represents the effect of quantum fluctuations, and if it is small for some variables describing a subsystem, this subsystem behaves classically. The amplitude $A$ of $\Psi$ is interpreted as giving the probability of each classical trajectory.

Mathematically speaking, this approach is exactly equivalent to the standard formalism for quantum mechanics, because the wave function $\Psi$ is assumed to satisfy the exact Schrödinger equation. Physically speaking, however, it introduces a new interpretation which is different from the conventional Copenhagen interpretation: the dBB trajectories are regarded as objective physical reality.

M. Kenmoku, H. Kubotani, E. Takasugi and Y. Yamazaki reported their work on the application of this de Broglie-Bohm interpretation to a spherically symmetric quantum black hole[8]. They first solved the Wheeler-DeWitt equation for this system and found generic exact solutions which are eigenfunctions of the mass operator. Then they examined the behavior of the dBB trajectories for these solutions and showed that the global event horizon and the apparent horizon exactly coincide for a special set of solutions whose dBB trajectories correspond to the classical solution. They further calculated the quantum potential for these special solutions and its influence on light rays propagating on the spacetimes corresponding to the dBB trajectories. They also discussed the dependence of the results on the operator ordering in the Hamiltonian and the mass operators.

M. Morikawa, F. Shibata, T. Shiromizu and M. Yamaguchi reported their recent
work on another application of this de Broglie-Bhom interpretation, which they called the
potential formalism. Quantum tunneling of the universe occurs both in the quantum
creation of universe and in GUT phase transitions. This quantum tunneling process is
usually analyzed in the imaginary-time formalism using the instanton method. However,
this imaginary-time formalism in general leads to a non-unitary evolution of the wave
function. M. Morikawa and his collaborators analyzed this quantum tunneling process
by the potential formalism and showed that this formalism leads to a completely unitary
description of the tunneling process.

The third approach is the geometrical formulation proposed by T.W.B. Kibble[9] and
extended by A. Ashtekar and T.A. Schilling[10]. In this formulation, utilizing the fact that
the inner product of a Hilbert space $\mathcal{H}$ induces a Kähler metric on the projective ray space
of $\mathcal{H}$, quantum theory is translated into a classical theory on this Kähler space. Hence
the classical phase is infinite-dimensional and does not correspond to the phase space of
the classical limit of quantum theory. Apart from this peculiar feature, this formulation is
quite fascinating in that all concepts and physical quantities, such as observables, spectrum,
quantum fluctuations and probability, are expressed in terms of a geometric language, and
that the correspondence between the quantum theory and the classical theory is natural
and exact.

A. Corichi and M.P. Ryan reported their preliminary work on the application of this
formulation to a minisuperspace quantum gravity corresponding to the diagonal Bianchi I
model with axial symmetry. In order to avoid technical difficulties arising from the infinite-
dimensionality of the classical phase space, they investigated the reduction of the classical
phase space to a finite-dimensional subset consisting of coherent states. They argued that
the investigation along this line would provide new insights on dynamics and interpretation
of quantum cosmology and gravity.

Finally, J.A. Larsson discussed Bell’s inequality in the hidden variable theory in his
poster paper, which is closely related to the above interpretation problems. The main point
of his work is to generalize Bell’s inequality by taking account of detector inefficiency. In
order to incorporate the detector inefficiency into the formulation, he introduced the con-
cept of ‘change of ensemble’, which provides both qualitative and quantitative information
on the nature of the ‘loophole’ in the proof of the original Bell inequality. He showed that
only models which contain change of ensemble lowers the violation, and derived a bound
on the violation, which does not depend upon any symmetry assumptions such as constant
efficiency, or the assumption of independent errors.

5 Supersymmetric Quantization

In this section I summarize the papers which treated problems related to supersymmetry.

Two papers discussed the BFV canonical quantization of the Einstein gravity and
related systems. First, R. Ferraro and D.M. Sforza considered a constrained system
with finite degrees of freedom, which has one quadratic constraint $H_\perp$ in addition to linear
constraints, and discussed the operator-ordering problem in the definition of the BRST
charge operator $\Omega$. Utilizing the results of their previous work[11], they showed that
one can find a consistent ordering which gives $\Omega^2 = 0$, if the potential term in $H_\perp$ is
monotonically increasing along a Killing vector of the supermetric.

A.Yu. Kamenshchik and S.L. Lyakhovich discussed a similar problem for a specific but genuine Einstein gravity system[12]. They considered a linearized Einstein gravity system with $N$ independent scalar fields on locally Euclidean $d$-dimensional torus $T^d$. They first showed that the constraint algebra is decomposed into a subalgebra corresponding to area-preserving diffeomorphisms and a Virasoro-like subalgebra, if the constraint operators are expanded by the Harmonic tensors on $T^d$. Then, from the requirement $\Omega^2 = 0$, they obtained the consistency condition, $N = 30 + 5(d + 1)(d - 2)/2$. This condition implies that the pure gravity system does not give a consistent quantum theory in this framework. They also argued that corrections introduced by going beyond the linear perturbation are of order $L_{pl}/V^{1/d}$, where $L_{pl}$ is the Planck length and $V$ is the space volume.

H. Luckock discussed supersymmetry of quantum constraints in locally supersymmetric theories. In general, local supersymmetry cannot be represented on the configuration space of a theory, because the transformation of configuration variables depends on momentum variables. From this, they argued that a supergravity wave function cannot be regarded as a superfield unlike that in globally supersymmetric theories.

Finally H. Luckock and H. Farajollahi discussed the stochastic quantization of locally supersymmetric theories and argued that the Wick rotation of the physical time variable in such theories plays the role of an extra fictitious time in the standard formulation of stochastic quantization.

6 Other Approaches to Quantum Gravity

In addition to the investigations along major approaches to quantum gravity and quantum cosmology presented so far, some unconventional approaches were also discussed.

First, O. Richter reported his preliminary work on quantum gravity within the framework of non-commutative geometry of A. Connes[13]. He considered non-commutative geometry on $M \times \Gamma_4$ with a discrete symmetry group $G$ acting on $\Gamma_4$, where $M$ is an ordinary 4-manifold, and $\Gamma_4$ is a set of four points. By formulating an analogue of the Einstein-Hilbert action for this non-commutative geometry, he obtained discretized versions of the $U(1)$ Kaluza-Klein model for $G = \mathbb{Z}_4$ and the non-linear $\sigma$ model for $G = T_4$(the tetrahedral group).

Second, P. Kuusk, J. Örd and E. Paal discussed an extension of the concept of translation to a generic curved spacetime[14]. The basic idea is to consider a non-associative binary operation $x \cdot y$ for two points $x$ and $y$ in a Gaussian normal neighborhood at a point $e$ defined as $R_y x \equiv x \cdot y = (\exp_y \circ \tau^e_y \circ \exp_e^{-1})(x)$, where exp is the exponential mapping, and $\tau^e_y$ is the parallel transport of vectors along the geodesic from $e$ to $y$. For a flat spacetime, this binary operation coincides with the sum of the position vectors for $x$ and $y$ with respect to the origin $e$. By defining a momentum operator as the generator of the right translation $R_a$, they constructed a non-standard Poisson algebra for the position and the momentum of a particle in a curved spacetime and discussed its quantization in the background spacetime with a weak plane gravitational wave.

Other foundational problems related to quantum gravity were discussed in poster papers. First, starting from an action which is expressed in terms of a Dirac spinor 1-
form (spin 3/2-field) and a $SL(2,C)$-connection and is equivalent to the Einstein-Hilbert action\cite{15}. R.S. Tung defined a gravitational energy-momentum 3-form and discussed its relevance to the issue of microscopic spacetime in quantum gravity. Second, J. Geddes proposed a new definition of the functional integral for a special class of functionals of the form $F[\phi] = \exp[\int dx f(\phi(x))]$. Finally, D.R. Finkelstein discussed a generalization of the general relativity principle, R.R. Karnik gave a speculation on spacetime properties, and E.G. Mychelkin discussed some geometrodynamical aspects of spacetime.

7 Cosmic Censorship

The validity of the cosmic censorship hypothesis has a great importance both in classical cosmology and astrophysics and in quantum gravity. Since lots of counter examples are known at least in the spherically symmetric case, there is a possibility that the cosmic censorship hypothesis does not hold in a pure mathematical sense even for generic cases.

From this observation, S. Barve and T.P. Singh argued that the cosmic censorship may not be a consequence of the local laws of physics and might be guaranteed by some additional general principle. To be explicit, they proposed that the Weyl curvature hypothesis proposed by R. Penrose\cite{17} should be imposed as a general principle to pick up a subset of initial conditions which are actually realized in the universe. Here the term ‘initial condition’ is used in such a wider sense that the Weyl curvature should vanish in the past limit along any past inextendible causal curve. The motivation of their proposal is a close connection between the Weyl curvature hypothesis and the second law of thermodynamics. In order to see whether this proposal really works, they calculated the behavior of the Weyl curvature near naked singularities for known solutions and found that the Weyl curvature diverges when one approaches the naked singularities along outgoing null rays\cite{16}. This result implies that these naked singularities do not appear in nature according to their proposal.

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