BOUND STATES IN THE BETHE-SALPETER FORMALISM

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ABSTRACT

We solve the Bethe-Salpeter equation in order to determine the spectrum of pseudoscalar and vector meson bound states for light as well as heavy quarks. The fermion propagators are obtained by solving the Schwinger-Dyson equation consistently with the Bethe-Salpeter equation, a procedure necessary for demonstrating the Goldstone nature of the pion in the chiral limit. Our results agree qualitatively and quantitatively with expectations both from current algebra for light quarks and from composite models for heavy quarks.

Presented at the DPF92 meeting, Fermilab, Chicago, Nov. 10-14, 1992
1. Introduction

There has been considerable theoretical study of the strongly interacting bound states. However most of the work is based on non-relativistic approximations to the bound state equations. This approach should be accurate if we consider the bound states which only involve heavy quarks, but is expected to fail for light mesons. In particular it does not take into account the fact that the light pseudoscalar mesons are almost Goldstone particles and that the light quark masses are mostly dynamical. Furthermore in most of the studies there is no clear connection between the constituent quark mass which is fed into the bound state equation and the current algebra mass. In order to correctly describe the light mesons it is necessary to go to a field theoretic framework. In the present paper we study the quark-antiquark bound states by solving the Bethe-Salpeter (BS) equation consistently with the Schwinger-Dyson (SD) equation for the quark propagator, in such a way that we get zero mass pseudoscalar states in the limit of zero bare quark masses. To simplify the problem we work in the ladder approximation. We employ for our calculations theoretically and phenomenologically motivated models for the gluon propagator.

The BS equation in the ladder approximation can be written as,

$$S_a^{-1}(q + \xi p)\chi(p, q)S_b^{-1}(q - (1 - \xi)p) = -i \int \gamma_\mu \chi(p, k)\gamma_\nu G_{\mu\nu}(k - q) \frac{d^4k}{(2\pi)^4},$$

(1)

The quark propagators are determined by solving the SD equation, also in the ladder approximation,

$$S_a^{-1}(q) = \frac{q}{\tilde{m}_a(\Lambda_c)} + i \int \gamma_\mu S_a(k)\gamma_\nu G_{\mu\nu}(k - q) \frac{d^4k}{(2\pi)^4},$$

(2)

where $\Lambda_c$ is the ultraviolet cutoff. The gluon propagator, $G_{\mu\nu}$, is modelled in terms of several parameters which can be fitted to experimental data. It is given in the Landau gauge by $G_{\mu\nu}(k) = -(g_{\mu\nu} - k_\mu k_\nu/k^2)G(k^2)$ where,

$$G(k^2) = \frac{16\pi^2}{3} \left[ \frac{d}{k^2 \ln(x_0 + x)} \left( 1 + b \frac{\ln[\ln(x_0 + x)]}{\ln(x_0 + x)} \right) + (2\pi\eta)^2 \delta^{(4)}(k) + a(1 - \omega k^2/k_0^2)e^{-k^2/k_0^2} \right],$$

and $x = k^2/\Lambda_{QCD}^2$. This model goes to the two loop form of the running coupling at large momentum and leads to an approximately harmonic oscillator potential in three dimensional configuration space, a form which is necessary to get the realistic spectrum of heavy mesons. The BS wave function for the pseudoscalar bound state can be expressed as,

$$\chi(p, q) = \gamma_5[\chi_0 + \not{p}\chi_1 + \not{q}\chi_2 + [\not{p}, \not{q}]\chi_3]$$

(3)

with similar decomposition for other spin and parity states. The resulting BS equation is simplified by expanding the wave functions, $\chi_i$ in terms of Tschebyshev polynomials,

$$\chi_i(q^2, M_B^2, \cos \theta) = \sum_n \chi_n^{(i)}(q^2, M_B^2) T_n^{(i)}(\cos \theta),$$

(4)
where we have set $p^2 = -M_B^2$, the bound state mass squared. We keep only the leading order polynomial for our calculation. The contribution due to the dropped terms was estimated to be small. Details of the calculation for pseudoscalars are given in Ref. [1].

2. Results and discussion

Here we describe some qualitative and quantitative features of the results for pseudoscalar and vector mesons. The detailed numerical results for pseudoscalar mesons and the parameter choices are given in Ref. [1]. The mass functions for different quarks are displayed in Fig. 1. The asymptotic behavior of the mass functions and the wave functions agrees with the one found on the basis of operator-product-expansion considerations. For light mesons our results satisfy the Gell-Mann, Oakes and Renner relation,

$$M_{ab} = \tilde{m}_a(\Lambda_c) + \tilde{m}_b(\Lambda_c) < q\bar{q}>_{ch}/f_{ch}^2,$$

where $\tilde{m}$ is the bare mass of the quark defined in Eqn. 2 and the subscript $ch$ means that the quantity is computed by using $\tilde{m}(\Lambda_c) = 0$. Both $\tilde{m}$ and the condensate $< q\bar{q}>_{ch}$ depends on the ultraviolet cutoff $\Lambda_c$ but their product was found to be insensitive to $\Lambda_c$. For heavy quarks, the relationship between $M_{ab}$ and $[m_a(0) + m_b(0)]$ is roughly linear, in agreement with nonrelativistic limit expectations.

We have also obtained preliminary results for vector mesons ground states. The overall fit was found to be better with parameters choices, $a=-32.9 \text{ MeV}^2$, $\eta=270 \text{ MeV}$ with the remaining parameters same as in Ref. [1]. The results for pseudoscalar mesons were found to be within a few % of the ones given in [1]. The results for some of the vector mesons are as follows: $m_\rho = 787 \text{ MeV} (770 \text{ MeV}), m_{K^*} = 978 \text{ MeV} (892 \text{ MeV}), m_\phi = 1250 \text{ MeV} (1019 \text{ MeV}), m_{D^*} = 2075 \text{ MeV} (2010 \text{ MeV}), m_{B^*} = 5325 \text{ MeV} (5324 \text{ MeV}), m_{J/\psi} = 3245 \text{ MeV} (3097 \text{ MeV}), m_{\Upsilon} = 9500 \text{ MeV} (9460 \text{ MeV}).$ The numbers in parenthesis are the experimental results. These results were obtained by keeping only the dominant term in the invariant decomposition of the vector meson wave function. More complete results will be given in a forthcoming publication.

Finally we display our preliminary results for the electromagnetic and isospin splittings for pseudoscalar and vector mesons. Our results for the pseudoscalar mass splittings decrease significantly as we go from the Kaon to the B meson, in reasonable agreement with experiments. Quantitatively we find, $K^0 - K^+ = 4.02 \text{ MeV}$ (input), $D^+ - D^0 = 3.95 \text{ MeV} (4.77 \text{ MeV}) B^0 - B^+ = 1.0 \text{ MeV} (0.1 \text{ MeV}).$ For the vector meson we do not get as strong a decrease as is displayed by the experiments. The results are as follows: $K^{*0} - K^{*+} = 11 \text{ MeV} (6.7 \text{ MeV}), D^{*+} - D^{*0} = 10.8 \text{ MeV} (2.9 \text{ MeV}) B^{*0} - B^{*+} = 7.4 \text{ MeV} (?)$ We are currently examining different models and calculating corrections to our result to determine if the agreement can be improved.

In conclusion, we have presented a covariant treatment of $q\bar{q}$ bound states which is applicable for both light and heavy mesons. Qualitatively our results are in good agreement with current algebra results for light quarks and nonrelativistic limit expectations for heavy quarks. Further tests of the approach described here,
as well as parametrization of $\alpha_s(q^2)$, will be done by obtaining a more complete spectrum and by computing the electromagnetic and weak form factors of these mesons.

5. Acknowledgements

We thank Douglas W. McKay and John P. Ralston for useful discussions. This work was supported in part by the Department of Energy under grant No. DE-FG02-85-ER40214.

6. References

1. H.J. Munczek and P. Jain, Phys. Rev. D46 (1992) 438, and references therein.