Lightlike Membranes in Black Hole and Wormhole Physics, and Cosmology

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Abstract. We shortly outline the principal results concerning the reparametrization-invariant world-volume Lagrangian formulation of lightlike brane dynamics and its impact as a source for gravity and (nonlinear) electromagnetism in black hole and wormhole physics.

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1 Introduction

Extended objects (strings and p-branes) are of primary importance for the construction of self-consistent unified modern theory of fundamental forces in Nature. In a series of recent papers of ours we have proposed for the first time in the literature a systematic world-volume Lagrangian description and studied in detail the physical properties of a new class of brane theories called lightlike branes (LL-branes), which are qualitatively distinct from the standard Nambu-Goto type brane models which describe intrinsically massive world-volume modes.

As it is well known, LL-branes (also called null-branes) are of substantial interest in general relativity as they describe impulsive lightlike signals arising in various violent astrophysical events, e.g., final explosion in cataclysmic processes such as supernovae and collision of neutron stars. LL-branes also play important role in the description of various other physically important cosmological and astrophysical phenomena such as the “membrane paradigm” of black hole physics and the thin-wall approach to domain walls coupled to gravity. For a detailed account, see \cite{5}. More recently they became significant also in the context of modern non-perturbative string theory \cite{6}.

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Here we will shortly describe some of our principal results concerning the physics of LL-branes and their implications in black hole and wormhole physics, and cosmology:

(a) Horizon “straddling” effect: the dynamics of LL-branes requires the bulk space-time geometry to possess one or more horizons, for instance, to be of black hole type, and it dictates that LL-branes automatically occupy (one of these) horizon(s).

(b) LL-branes are natural candidates for matter and charged sources of “thin-shell” traversable wormholes of various types (one- or multi-“throat” “tube-like”, rotating etc.) [2].

(c) LL-branes naturally produce regular black holes, i.e., black holes free of “inside” (below the inner horizon) physical space-time singularities [3].

(d) LL-branes trigger spontaneous compactification of space-time, as well as compactification/decompactification transitions [4].

(e) LL-branes are consistent matter sources for lightlike braneworlds [7].

(f) LL-branes produce new wormhole “universes” exhibiting charge-hiding and charge-confining effects [8], physically analogous to the quark confinement mechanism in quantum chromodynamics.

2 Gravity and Nonlinear Gauge Fields Coupled to LL-Brane Sources

In Refs.[2, 3, 4, 7, 8] we proposed and extensively studied a manifestly reparametrization invariant world-volume Lagrangian action of LL-branes:

\[ S_{LL}[q] = -\frac{1}{2} \int d^{p+1}\sigma \sqrt{-\gamma} \left[ \gamma^{ab} \bar{g}_{ab} - b_0 (p - 1) \right] , \quad (1) \]

\[ \bar{g}_{ab} \equiv g_{ab} - \frac{1}{T^2} (\partial_\sigma u + q A_\sigma) (\partial_\sigma u + q A_\sigma) , \quad A_\sigma \equiv \partial_\sigma X^{\mu} A_\mu . \quad (2) \]

Here and below the following notations are used:

- \( \gamma_{ab} \) is the intrinsic world-volume Riemannian metric; \( g_{ab} = \partial_\sigma X^{\mu} G^{\mu\nu}(X) \partial_\nu X^a \) is the induced metric on the world-volume, which becomes singular on-shell (manifestation of the lightlike nature); \( b_0 \) is world-volume “cosmological constant”;

- \( X^a(\sigma) \) are the p-brane embedding coordinates in the \( D \)-dimensional bulk space-time with Riemannian metric \( G^{\mu\nu}(x) \) \( (\mu, \nu = 0, 1, \ldots, D - 1) \); \( (\sigma) \equiv (\sigma^0 = \tau, \sigma^i) \) with \( i = 1, \ldots, p \); \( \partial_\sigma \equiv \partial/\partial \sigma^i \).

- \( u \) is auxiliary world-volume scalar field defining the lightlike direction of the induced metric;
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- $T$ is dynamical (variable) brane tension;
- $q$ – the coupling to bulk spacetime gauge field $A_{\mu}$ is LL-brane surface charge density.

The on-shell singularity, i.e., the lightlike property of the induced metric $g_{ab}$, directly follows from the equations of motion resulting from (1):

$$g_{ab}(\bar{g}^{bc}(\partial_c u + qA_c)) = 0.$$  \hspace{1cm} (3)

Now, the full action of gravity and (nonlinear) gauge fields interacting self-consistently with LL-branes reads (we specialize to $D = 4$ space-time dimensions and use units with the Newton constant $G_N = 1$):

$$S = \int d^4x\sqrt{-G}\left[\frac{R(G) - 2\Lambda_0}{16\pi} + L(F^2)\right] + \sum_{k=1}^{N} S_{\text{LL}}[q^{(k)}],$$  \hspace{1cm} (4)

where the superscript $(k)$ indicates the $k$-th LL-brane. Here $R(G) = G^{\mu\nu}R_{\mu\nu}$ and $R_{\mu\nu}$ denote the Riemannian scalar curvature and the Ricci tensor of the bulk space-time geometry. $L(F^2)$ is the Lagrangian of a remarkable non-standard nonlinear electrodynamics containing square root of ordinary Maxwell Lagrangian $[10]$:

$$L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2}, \quad F^2 \equiv F_{\mu\nu}F_{\nu\lambda}G^{\mu\lambda}.$$  \hspace{1cm} (5)

This is an explicit realization of ‘t Hooft’s proposal (in flat space-time) for infrared charge confinement $[11]$ (see also next talk $[12]$ at this congress).

3 Charge-Confinement via “Tube-Like” Wormhole

The general scheme to construct “lightlike thin-shell” wormholes of static “spherically-symmetric” type (in Eddington-Finkelstein coordinates $dt = dv - \frac{d\eta}{A(\eta)}$ and “radial”-like coordinate $\eta \in (-\infty, +\infty)$):

$$ds^2 = -A(\eta)d\eta^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^id\theta^j, \quad F_{vn} = F_{vn}(\eta),$$

$$-\infty < \eta < \infty, \quad A(\eta^{(k)}) = 0 \text{ for } \eta^{(1)} < \ldots < \eta^{(N)}.$$  \hspace{1cm} (6)

(7)

is as follows (cf. Section 5 in Ref.[3]):

1. Take “vacuum” solutions of Einstein and (nonlinear) Maxwell equations resulting from (4) (i.e., without the delta-function LL-brane contributions) in each space-time region (separate “universe”) given by $(-\infty < \eta < \eta^{(1)}), \ldots, (\eta^{(N)} < \eta < \infty)$ with common horizon(s) at $\eta = \eta^{(k)}$ ($k = 1, \ldots, N$).
(2) Each $k$-th LL-brane automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ – intrinsic property of LL-brane dynamics defined by the action (1).

(3) Match discontinuities of the derivatives of the metric and the gauge field strength across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the LL-brane stress-energy tensor and charge current density systematically derived from the action (4) with (1).

Let us now consider the gravity/nonlinear-gauge-field system coupled to two oppositely charged LL-branes, i.e., $N = 2$ and $q_1 = -q_2 \equiv q$ in (4). We obtain a particularly interesting “two-throat” wormhole-type solution exhibiting a QCD-like charge confinement effect. The total space-time manifold consists of three “universes” with different geometry glued together at their common horizons occupied by the two oppositely charged LL-branes:

(i) “Left-most” non-compact “universe” comprising the exterior region of a new kind of non-standard Schwarzschild-de Sitter-type black hole, with additional constant vacuum radial electric field $E_{\text{vac}}$, beyond the Schwarzschild-type horizon $r_0$ for the “radial-like” $\eta$-coordinate interval $-\infty < \eta < -\eta_0 \equiv -\left[4\pi \left(\sqrt{2}f_0|E| - \bar{E}^2\right) + \Lambda_0\right]^{-\frac{1}{2}}$, where (using notations as in (6)):

\begin{align}
A(\eta) &= 1 - \frac{2m}{r_0 - \eta_0 - \eta} - \frac{\Lambda_{\text{eff}}}{3}(r_0 - \eta_0 - \eta)^2, \quad (8) \\
C(\eta) &= (r_0 - \eta_0 - \eta)^2, \quad |F_{\eta\eta}(\eta)| \equiv |E_{\text{vac}}| = \frac{f_0}{\sqrt{2}} < |\bar{E}|. \quad (9)
\end{align}

Here $\bar{E}$ is the constant electric field in the “middle” “tube-like” “universe” (ii) (Eq.(12) below); $\Lambda_{\text{eff}} \equiv \Lambda_0 + 2\pi f_0^2$ in (8) is dynamically generated/shifted cosmological constant, which is non-vanishing even in the absence of the “bare” cosmological constant $\Lambda_0$. Let us stress that constant vacuum radial electric fields such as in (9) do not exist as solutions of ordinary Maxwell electrodynamics on generic non-compact space-times – the former are due exclusively to the nonlinear “square-root” term in (5).

(ii) “Middle” “tube-like” “universe” of Levi-Civita-Bertotti-Robinson type [9] with geometry $dS_2 \times S^2$ ($dS_2$ denotes two-dimensional de Sitter space; $S^2$ – sphere with constant radius $r_0$), comprising the finite extent (w.r.t. $\eta$-coordinate) region between the two horizons of $dS_2$ at $\eta = \pm \eta_0$ occupied by the two LL-branes with charges $\pm q$:

\begin{align}
-\eta_0 < \eta < \eta_0 \equiv \left[4\pi \left(\sqrt{2}f_0|E| - \bar{E}^2\right) + \Lambda_0\right]^{-\frac{1}{2}}, \quad (10)
\end{align}

where the metric coefficients and electric field are:

\begin{align}
A(\eta) &= 1 - \left[4\pi \left(\sqrt{2}f_0|E| - \bar{E}^2\right) + \Lambda_0\right] \eta^2, \quad A(\pm \eta_0) = 0, \quad (11) \\
C(\eta) &= r_0^{-2} = \frac{1}{4\pi \bar{E}^2 + \Lambda_0}, \quad |\bar{E}| = |q| + \frac{f_0}{\sqrt{2}} = \text{const}. \quad (12)
\end{align}
Figure 1. Shape of $t = \text{const}$ and $\theta = \frac{\pi}{2}$ slice of charge-confining wormhole geometry. The whole electric flux is confined within the middle cylindric tube.

(iii) “Right-most” non-compact “universe” of the same type as (i) above for the “radial-like” $\eta$-coordinate interval $\eta_0 < \eta < \infty$ ($\eta_0$ as in (10)). Its metric is given by Eq.(8) upon changing $-\eta \rightarrow \eta$ and the electric field is the same as in (9).

The equations for the electric field (second relations in (9) and (12)) have profound consequences:

- The “left-most” and “right-most” non-compact “universes” are two identical copies of the electrically neutral exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. They both carry a constant vacuum radial electric field with magnitude $|\vec{E}| = \frac{f_0}{\sqrt{2}}$ pointing inbound/outbound w.r.t. pertinent horizon. The corresponding electric displacement field $\vec{D} = \left(1 - \frac{\vec{E}}{|\vec{E}|}\right) \vec{E} = 0$, so there is no electric flux there.

- The whole electric flux produced by the two charged $LL$-branes with opposite charges $\pm q$ at the boundaries of the above non-compact “universes” is confined within the finite-extent “tube-like” middle “universe” of Levi-Civitta-Robinson-Bertotti type with geometry $dS_2 \times S^2$, where the constant electric field is $|\vec{E}| = \frac{f_0}{\sqrt{2}} + |q|$ with associated non-zero electric displacement field $|\vec{D}| = |q|$. This is QCD-like confinement.

The charge-confining wormhole geometry is visualized on Fig.1.
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To conclude let us emphasize that the existence of charge-confining “thin-shell” wormholes is entirely due to the combined effect of the exceptional properties of LL-brane dynamics and the “square-root” nonlinear electrodynamics.

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