Oscillating Quintessence

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An oscillating scalar field as a quintessence model for dark energy is proposed. The case of a power-law potential is particularly intriguing and is the focus of the present article. In this model the equation of state $w_{Oq}$ of dark energy is a constant determined simply by the power $n$ in the potential through $w_{Oq} = (n-2)/(n+2)$. Accordingly, when $0 < n < 1$, the oscillating quintessence can provide repulsive gravity and drive the cosmic acceleration. The condition for oscillation and the constraints from observations are investigated. For this new scenario a specific natural model with much less fine tuning is presented.

PACS numbers: 98.80.-k,98.80.Cq

Introduction. The existence of the accelerated expansion of the present universe was discovered in 1998 via type Ia supernova (SN Ia) distance measurement [1,2] and further reinforced by recent observations [3,4,5,6]. This mysterious phenomenon is one of the most important unsolved problems in this century. Since this discovery a variety of models have been proposed and many of them so far survive under continually updated observational results. These models involve diverse approaches and strategies, including (1) introducing new energy contents which provide negative pressure and repulsive gravity, e.g. a positive cosmological constant [7,8,9] and a dynamical scalar field such as quintessence [10,11,12], phantom [13], etc., as generally called dark energy, (2) introducing new ingredients in the geometry/gravity part, such as modifying the gravity action [14,15,16] and invoking extra dimensions [17,18,19,20,21,22,23], and (3) taking into consideration the effects of inhomogeneities of the universe (e.g. see [24,25] and references therein). Among these models, quintessence will be the focus in the present article.

For a field (or a physical object) and its possible evolution patterns, the oscillation-like behavior (including orbiting around) is the most familiar because it has been seen frequently in nature from particle physics of subatomic scales, daily lives of human scales, to our universe of astronomical scales. Oscillation is therefore the most natural mode for a field to consider. Nevertheless, with regard to the quintessence as a realization of dark energy with negative pressure and repulsive gravity, it is straightforward to think that oscillating modes correspond to excitations or particles that provide nonnegative pressure and attractive gravity and therefore can hardly do the job. For example [26], for $V = \frac{1}{2}m^2\phi^2$, the oscillating scalar field is a linear combination of sin- and cosine-wave modes, $\cos(k \cdot x + \theta_0)$, while each mode provides nonnegative pressure with the equation of state ranging from 0 to 1, corresponding to the wave number $|k|$ from the small ($\ll m$) to the large ($\gg m$). If insisting to utilize this potential to construct the quintessence model, one requires extremely small mass $m \lesssim H_0 \sim 10^{-35}$eV and extremely large amplitude $|\phi| \gtrsim M_{pl} \sim 10^{26}$eV to accommodate a slowly (enough) evolving mode (instead of oscillation) for dark energy. Accordingly, most of the efforts at the quintessence model construction are devoted to slowly rolling evolution patterns under a rather smooth and flat potential that dominates over the kinetic energy of the quintessence at all times in the recent epoch. For example, in the tracker quintessence model [27,28] the potential goes to zero asymptotically when the scalar field goes to infinity along with time, as a realization of the running away evolution pattern.

On the contrary, in the present article, a new quintessential dark energy scenario involving an oscillating scalar field is proposed. In particular, the present article is focused on the power-law potential $V(\phi) = M^{4-n}|\phi|^n$ $(n > 0, M > 0)$. In this model, as going to be presented, the equation of state of dark energy is a constant determined simply by the power in the potential:

$$w_{Oq} = \frac{n-2}{n+2},$$

where the subscript “OQ” stands for oscillating quintessence. As a result, when $0 < n < 1$, we have $-1 < w_{Oq} < -1/3$ and accordingly the oscillating quintessence can drive the cosmic acceleration.

Analysis. Consider a Friedmann-Lemaître-Robertson-Walker (FLRW) universe which is described by the Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2d\Omega^2 \right),$$

and at the present epoch is dominated by pressureless matter and quintessence. The quintessence is represented

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2 The behaviors of the small- and the large-wave-number modes are consistent with those of the non-relativistic and the relativistic particles, respectively [26].
by a scalar field with the Lagrangian density:

\[ \mathcal{L} = \sqrt{|g|} \left[ \frac{1}{2} g^{\mu\nu} (\partial_{\mu} \phi)(\partial_{\nu} \phi) - V(\phi) \right]. \]  

(3)

For the evolution of this universe, the governing equations are as follows.

\[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8 \pi G}{3} \rho = \frac{8 \pi G}{3} (\rho_m + \rho_\phi), \]  

(4)

\[ \frac{\ddot{a}}{a} = -\frac{4 \pi G}{3} (\rho + 3p) = -\frac{4 \pi G}{3} (\rho_m + \rho_\phi + 3p_\phi), \]  

(5)

\[ \frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - \frac{1}{a^2} \nabla^2 \phi + V'(\phi) = 0, \]  

(6)

where the Hubble expansion rate \( H \equiv \dot{a}/a \) and the energy density and pressure of the quintessence are given by

\[ \rho_\phi = \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2a^2} (\nabla \phi)^2 + V(\phi), \]  

(7)

\[ p_\phi = \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{6a^2} (\nabla \phi)^2 - V(\phi). \]  

(8)

For simplicity, in the rest of the present article, the spatial curvature and the spatial derivative terms will be ignored respectively by the assumption that our universe is flat spatially \((k = 0)\) and the spatial dependence of the quintessence is very weak.

In the present article the following power-law potential is considered.

\[ V(\phi) = M^{4-n} |\phi|^n, \quad n > 0, \ M > 0. \]  

(9)

For \( 0 < n \leq 1 \), in order to regularize the potential around \( \phi = 0 \), the potential is modified as follows.

\[ V(\phi) = \frac{M^{4-n}|\phi|^{n+\epsilon}}{|\phi| + \epsilon}, \quad \epsilon > 0. \]  

(10)

This modification can be ignored when \( |\phi| \gg \epsilon \).

Here we consider an oscillating mode of the quintessence whose period \( T \) is much smaller than the Hubble time, i.e., \( T \ll H^{-1} \), and study the quantities averaged over a period, \( \langle \mathcal{O} \rangle \). From the field equation [4] where the damping term \( 3H \dot{\phi} \) can be ignored for \( T \ll H^{-1} \), it is straightforward to obtain the relation between the kinetic and the potential contributions to the averaged quintessence energy density:

\[ \langle K \rangle = \frac{n}{2} \langle V \rangle, \]  

(11)

where \( K \equiv \dot{\phi}^2/2 \) and the over-head dot denotes the time derivative. Thus, for an oscillating quintessence under a power-law potential, the equation of state (averaged over a period \( T \) or a time scale much larger than the period) is a constant determined simply by the power \( n \), as given in Eq. (1):

\[ w_{\text{QO}} = \frac{n - 2}{n + 2}. \]

Regarding different \( w_{\text{QO}} \) corresponding to different power \( n \), several cases are listed in the following table.

| \( n \) | 0.01 | 0.05 | 0.1 | 0.5 | 1.0 | 2.0 | 4.0 |
|-------|------|------|-----|-----|-----|-----|-----|
| \( w_{\text{QO}} \) | -1.0 | -0.99 | -0.95 | -0.9 | -0.6 | -1/3 | 0.1/3 |

In particular, when \( 0 < n < 1 \), the oscillating quintessence provides negative pressure and repulsive gravity with the equation of state between \(-1 \) and \(-1/3 \), and therefore has the ability to drive the cosmic expansion to accelerate.

In addition to the equation of state, the other two essential quantities are the energy density \( \rho_{\text{QO}} \) and the period \( T \) of the oscillating quintessence:

\[ \rho_{\text{QO}} = \langle \rho_\phi \rangle = \langle K \rangle + \langle V \rangle = \left( \frac{1}{2} + \frac{n}{2} \right) \langle V \rangle = V_{\text{max}} = M^{4-n} |\phi|_{\text{max}}^n, \]  

(12)

where the subscript “max” denotes the maximal value within a period \( T \) and \( |\phi|_{\text{max}} \) stands for the amplitude of the oscillation:

\[ T = \frac{\sqrt{8 \pi}}{\rho \overline{\gamma}} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n} + \frac{1}{n})} \cdot M^{1-4/n} \rho_{\text{QO}}^{-1/n-1/2}, \]  

(13)

or

\[ HT = \frac{8 \pi}{\sqrt{3n}} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n} + \frac{1}{n})} \cdot M_{\text{pl}}^{-1} \rho^{1/2} \rho_{\text{QO}}^{1/2}, \]  

\[ = \frac{8 \pi}{\sqrt{3n}} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n} + \frac{1}{n})} \cdot \left( \frac{\rho_{\text{QO}}}{\rho} \right)^{-1/2} \left( \frac{\rho_{\text{QO}}}{M_{\text{pl}}} \right)^{1/4} \left( \frac{\rho_{\text{QO}}}{M} \right)^{-1/4}, \]  

\[ = \frac{8 \pi}{\sqrt{3n}} \frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n} + \frac{1}{n})} \cdot \left( \rho \rho_{\text{QO}} \right)^{-1/2} \left( \frac{|\phi|_{\text{max}}}{M_{\text{pl}}} \right)^{1/4}, \]  

(14)

where the Planck scale \( M_{\text{pl}} \equiv G^{-1/2} 1.2 \times 10^{19} \text{GeV} \) and the total energy density \( \rho \) in unit of the current value \( [\rho(0) = \rho_c] \) as a function of the redshift \( z \) is as follows.

\[ \frac{\rho(z)}{\rho(0)} = \frac{\rho(z)}{\rho_c} = \Omega_{\text{QO}}(1 + z)^{6n/(2+n)} + \Omega_m(1 + z)^3 + \Omega_{\gamma}(1 + z)^4, \]  

(15)

if only the contributions from quintessence, matter and
photons are considered. For the present time we have
\[ H_0 T_0 = \frac{\sqrt{\pi}}{n} \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2} + \frac{1}{n}\right)} \left(\frac{3}{8\pi}\right)^{\frac{1}{2}} \cdot \frac{\Omega_{\text{Q}0}}{M_{\text{pl}}} \left(\frac{M}{H_0}\right)^{-\frac{2}{3}} \cdot \left(\frac{1}{M_{\text{pl}}^{1/2}}\right) \left(\frac{M_{\text{pl}}}{M}\right)^{-\frac{1}{2}} \right], \]
where \(z\) denotes the present time.

Requirements and Constraints. According to the observational results, we have the following information as our input [29]: \(H_0 \approx 73 \text{km s}^{-1} \text{Mpc}^{-1}, \rho_c \approx 4.3 \times 10^{-11} \text{eV}^4, \Omega_{\text{m}} \approx 0.24, \Omega_{\text{z}} \approx 4.6 \times 10^{-5}\), the dark energy ("c") density fraction \(\Omega_c \approx 0.75\) and the equation of state \(w_c < -0.9 (1\sigma)\) for the present time. For the oscillating quintessence to play the role of dark energy, from Eqs. (11) and (12) the constraints \(w_c < -0.9 (1\sigma)\) and \(\Omega_c \approx 0.75\) respectively requires
\[ 0 < n < 0.1, \]
\[ V_{\text{max}}(0) = M^{4-n} |\phi_{\text{max}}^n(0)| \approx 0.75 \rho_c \approx 3.2 \times 10^{-11} \text{eV}^4, \]
as two conditions for the three parameters, \(n, M\) and \(|\phi|_{\text{max}}(0)\), in the power-law potential.

The condition for the oscillation is
\[ HT \ll 1 \quad \text{from the present back to some early time.} \]
The condition at the present time \((H_0 T_0 \ll 1)\) requires
\[ M \gg \left[ \frac{8\pi}{\sqrt{3n}} \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2} + \frac{1}{n}\right)} \cdot \Omega_{\text{Q}0}^{-1/2} \rho_c^{-1/2} M_{\text{pl}}^{-1} \right]^{1/2}, \]
or, equivalently,
\[ |\phi|_{\text{max}}(0) \ll \frac{8\pi}{\sqrt{3n}} \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2} + \frac{1}{n}\right)} \cdot \sqrt{\Omega_{\text{Q}0} M_{\text{pl}}}, \]
i.e., the current amplitude of the oscillation \(|\phi|_{\text{max}}(0)\) should be smaller than or roughly around the same scale of the Planck scale \(M_{\text{pl}}\).

In addition, the condition \(HT \ll 1\) gives a lower limit to the time, i.e. an upper limit to the redshift \(z\), with respect to the beginning of the oscillation. From Eqs. (14) – (16), we have
\[ \frac{HT}{H_0 T_0} = (1 + z)^{2(2-n)/(2+n)} \sqrt{\rho(z)/\rho(0)} \]
\[ \equiv F_n(z), \]
where the ratio of the total energy density, \(\rho(z)/\rho(0)\), is given in Eq. (15). Note that, for \(0 < n < 1\), \(F_n(z)\) is a monotonically increasing function of \(z\). Accordingly, \(HT \ll 1\) requires
\[ z \ll z_{\text{max}} \equiv F_n^{-1} \left(\frac{1}{H_0 T_0}\right). \]

Several examples with various values of \(n\) and \(M\) fulfilling the above constraints and requirements are listed in Table 1. Among these examples the case with \(n \approx 0.1\) and \(M = \rho_c^{1/4}\) is particularly interesting. In this case both the two free dimensionless parameters, \(M\) and \(|\phi|_{\text{max}}(0)\), are on the same scale of \(\rho_c^{1/4}\), i.e., the energy scale of the present universe regarding the energy density. In addition, the period of the oscillation at the present time \(T_0\) is also roughly around the same scale, \(\rho_c^{-1/4}\), and the oscillation started around \(z = 7 \times 10^6\) when the temperature of the universe was around \(10^7\)K (i.e. 1keV).

| \(n\) | \(w_{\text{Q}0}\) | \(M > 1.6 \times 10^{-7} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 19\) |
|---|---|---|
| 0.5 | -0.6 | 1.6 \times 10^{-7} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 19 |
| 2.56 \times 10^{-3} \text{eV} | eV | MeV | TeV |
| (M_{\text{pl}}) | 10^{-5} \text{eV} |
| log \(\frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}}\) | -31 | -49 | -91 | -133 | -245 |
| log \(H_0 T_0\) | -30 | -48 | -90 | -132 | -244 |
| log \(z_{\text{max}}\) | 8.3 | 13 | 24 | 35 | 65 |

| \(n\) | \(w_{\text{Q}0}\) | \(M > 4.3 \times 10^{-4} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 40\) |
|---|---|---|
| 0.1 | -0.9 | 4.3 \times 10^{-4} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 40 |
| 2.56 \times 10^{-3} \text{eV} | eV | MeV | TeV |
| (M_{\text{pl}}) | 10^{-5} \text{eV} |
| log \(\frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}}\) | -32 | -133 | -367 | -601 | -1225 |
| log \(H_0 T_0\) | -30 | -131 | -365 | -599 | -1223 |
| log \(z_{\text{max}}\) | 6.9 | 28 | 78 | 128 | 260 |

| \(n\) | \(w_{\text{Q}0}\) | \(M > 10^{-7} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 57\) |
|---|---|---|
| 0.05 | -0.95 | 10^{-7} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 57 |
| 2.56 \times 10^{-3} \text{eV} | eV | MeV | TeV |
| (M_{\text{pl}}) | 10^{-5} \text{eV} |
| log \(\frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}}\) | -33 | -238 | -712 | -1186 | -2450 |
| log \(H_0 T_0\) | -31 | -236 | -710 | -1184 | -2448 |
| log \(z_{\text{max}}\) | 6.9 | 49 | 147 | 244 | 505 |

| \(n\) | \(w_{\text{Q}0}\) | \(M > 2 \times 10^{-3} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 126\) |
|---|---|---|
| 0.01 | -0.99 | 2 \times 10^{-3} \text{eV}, \frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}} \ll 126 |
| 2.56 \times 10^{-3} \text{eV} | eV | MeV | TeV |
| (M_{\text{pl}}) | 10^{-5} \text{eV} |
| log \(\frac{|\phi_{\text{max}}(0)|}{M_{\text{pl}}}\) | -43 | -1077 | -3471 | -5863 | -120 |
| log \(H_0 T_0\) | -41 | -1075 | -3469 | -5863 | -120 |
| log \(z_{\text{max}}\) | 8.7 | 217 | 698 | 1180 | 2465 |

Summary and Discussion. It has been shown in the present article that it is possible for an oscillating scalar field to generate repulsive gravity and play the role of dark energy. This is contrary to the usual thinking that oscillating modes correspond to excitations or particles that provide nonnegative pressure and attractive gravity.

The case of the power-law potential is particularly intriguing. In this case the equation of state of the oscillating quintessence \(w_{\text{Q}0}\) is a constant determined simply by the power \(n\) in the potential. In particular, when \(0 < n < 1, -1 < w_{\text{Q}0} < -1/3\) and accordingly the oscillating quintessence has the ability to provide repulsive gravity and drive the cosmic acceleration. In addition,
the cases \((n, w_{\text{eq}}) = (1, -1/3), (2, 0), (4, 1/3)\) are also essential and interesting.

Several constraints on the parameters in the potential — the power \(n\), the energy scale \(M\) and the current amplitude of the oscillation \(|\phi|_{\text{max}}(0)\) — are given by observations and the condition for oscillation. The observational results about the equation of state and the energy density of dark energy, \(w_x < -0.9\) (1σ) and \(\Omega_x \simeq 0.75\), respectively, requires \(0 < n < 0.1\) and \(M^{4-n}|\phi|_{\text{max}}(0) \sim 10^{-13}\, eV^3\). The condition for oscillation, \(HT \ll 1\) from the present back to some early time, further requires \(|\phi|_{\text{max}}(0) \lesssim M_{\text{pl}}\) and gives an upper limit to the redshift, \(z_{\text{max}}\), with respect to the beginning of the oscillation.

Among the examples satisfying the observational constraints and the oscillation condition, the case where \(n \simeq 0.1\) (\(w_{\text{eq}} \simeq -0.9\)) and \(M \simeq |\phi|_{\text{max}}(0) \simeq \rho_c^{1/4} \sim 10^{-3}\, eV\) is particularly interesting. In this case the only two free dimensionful parameters in this model, the energy scale \(M\) in the power-law potential and the current amplitude of the oscillation \(|\phi|_{\text{max}}(0)\), are both on the scale of the energy density of the present universe. In addition, the period of the oscillation at the present time is also roughly around the same scale, \(\rho_c^{1/4}\). This is much less fine tuned and therefore much more natural, especially when compared with the case of \(V = \frac{1}{4} m^2 \phi^2\) where it is required that \(m \lesssim H_0 \sim 10^{-33}\, eV\) (extremely small) and \(|\phi| \gtrsim M_{\text{pl}} \sim 10^{28}\, eV\) (extremely large) for realizing dark energy and driving the cosmic acceleration.

The oscillating quintessence opens a new scenario that is very different from the running away quintessence and other slowly rolling evolution patterns involved in most of the current quintessence models. In this scenario the quintessence in the recent epoch is oscillating in time, but not slowly rolling and not always potential-dominated. This oscillating scenario may be extended to phantom and other dark energy models involving one or more scalar fields (or even other kinds of fields). This new scenario largely extends the scope of the model construction for dark energy played by quintessence and other fields, in particular, while the oscillation-like behavior (including orbiting around) is much more familiar and much more frequent to see in nature of all scales from the very small to the very large. Accordingly this may provide a link between the microscopic fundamental physics and the cosmic-scale accelerated expansion driven by dark energy.

Acknowledgments

This work is supported by the Taiwan National Science Council (NSC 96-2119-M-007-001).

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