Research on Simulation Modeling for Carrying Spare Parts Optimization Considering Random Common Cause Failures

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Abstract. It has been proved to be of great significance to determine the carrying spare parts reasonably for promoting the probability of the system successfully completing a mission. Taking into account the effects of the in-situ repair capacity and random common cause failures occurred in the mission, a novel carrying spare parts optimization model is developed. The proposed model is to maximize the probability of the system successfully completing the next mission considering storage space constraints. Additionally, a solution algorithm integrating the famous Monte Carlo simulation and marginal algorithm is presented to solve the proposed model. Finally, a real example is given to verify the validity of the model, as well as the solution algorithm, and a detailed discussion is carry out to demonstrate the effects of the storage space on carrying spare part quantity and the associated probability of the system successfully completing the mission. The results show that: (1) the proposed model can be used for carrying spare part optimization considering in-situ maintenance capacity and random common cause failure; (3) the reasonable spare part storage space and carrying spare part quantity contribute to a great probability of the system successfully completing a mission.

1. Introduction

In many practical military applications, equipment is always requested to execute sequential missions. To guarantee the equipment successfully completing the mission, some maintenance activities needed to be carried out during the mission or mission break. When replacement is selected, spare parts will be needed. The spare parts carried during the missions can be interpreted as carrying spare parts. Obviously, carrying spare parts, as an important maintenance resource, play an important role in sustaining or recovering the operation state of the equipment. However, it is not an easy work to determine how many and what kind of spare parts to be carried for a specific mission. And managers are always confronted with a contradiction of limited storage space and a high requirement for the equipment successfully completing a mission. For such case, the carrying spare parts optimization is proposed to determine the type and quantity of carrying spare parts quantity.

The carrying spare parts optimization problems have been extensively investigated by many researchers so far [1, 2]. Zhang et al. [3] discussed the joint optimization model of maintenance time and carrying spare parts for k-out-of-N system, in which operational availability was taken as a constraint condition and minimal maintenance costs as objective function. And the marginal algorithm was applied to solve the model. They[4] also developed a joint optimization model of maintenance...
interval and carrying quantity of spare parts under opportunistic maintenance policy for shipboard system that subject to silent failures. And the model optimized the support project of spare parts under periodic supply interval. Besides, Zhang et al. [5] investigated an improved model under the background of naval capacity, which is based on multi-constraints including carrying capacity, spare parts cost, availability of equipment, and the serviceability rate of the same type equipment. Guo et al. [6] proposed a optimization model for the quantity of carrying spare parts of combat unit with constraints on mission availability, and solved it using marginal algorithm. Ruan et al. [7] developed a carrying spare parts optimization model of warship on the basis of the multi-target constraints, such as mass, volume and cost of spare parts and equipment availability. Guo et al. [8] developed a mathematical algorithm for mission reliability given limited spares, which could help us make a decision on the quantity of carrying spare parts. Liu et al. [9] proposed an approximation method of carrying project for repairable spares with scrap under multi-constraints, and Lagrange factors and marginal algorithm are introduced during the model solving. Zhang et al. [10] investigated the relationship between the quantity of carrying spare parts and mission availability when replacing strategy was adopted. Hu et al. [11] developed a model of calculating spare parts demand based on age replacement considering preventive maintenance, and a discrete algorithm for the model was presented. They [12] also proposed a three-phase multicriteria classification framework for spare parts management using the dominance-based rough set approach.

It can be concluded from the aforementioned literatures that most works focus on carrying spare parts optimization problem considering the effect of element’s inherent degradation. While, it has been proved by many works that the external working conditions can significantly affect the degradation process and failure mode of the systems and elements. Intuitively, the quantity and type of spare parts needed during mission are also depended on the external working conditions. Additionally, when system breaks down, the in-situ repair may be carried out on the failed elements firstly. If the in-situ repair cannot recover failed elements to operational states, then replacement is performed. Hence, the probability of the failed element can be recovered by in-situ repair, i.e. in-situ repair capacity, may significantly affect the spare parts quantity needed during the mission. However, to the best of our knowledge, the maintenance capacity has not yet been considered during carrying spare parts optimization.

Motivated by the real life applications, a carrying spare parts optimization problem considering the effect of random common cause failures and in-situ repair capacity is addressed in this paper. This is real because a general case encountered in military applications is that, equipment, such as artillery, tank etc., may suffer some random common cause failures (RCCFSs) [13] during the mission. The random common cause events occurred randomly may cause simultaneously failures of different elements with different probabilities. The in-situ repair is performed on the failed element firstly if the system fails. This is because the in-situ repair can maintain the elements as soon as possible. If the in-situ repair fails to recover the failed element, then replacement is selected, and spare parts are needed. Additionally, the store space is limited, and only sets of spare parts can be carried in the mission. Hence, considering the limited storage space and mission requirement, managers need to optimally determine the carrying spare parts quantity for each kind of element.

The reminder of this paper is organized as follows: The problems description and some assumptions are presented in Section 1. Then, the simulation model for carrying spare parts optimization is shown in Section 2. Next, an example and its relevant analysis are given in Section 3. Finally, we draw the conclusions about this optimization model in Section 4.

2. Problem Description and Assumptions

Assume that a binary state equipment composed of $n$ elements is requested to execute a mission with random duration. During the mission, the equipment may suffer from some fire attacks coming from the enemy, which generated randomly may lead to the simultaneously failure of different elements with different probabilities. When the equipment fails, the failed elements will be maintained by the in-situ repair firstly. If the in-situ repair is unable to recover the failed element to the working
condition, then replacement will be adopted. To guarantee the equipment successfully completing the mission, some spare parts are carried with the equipment. However, the storage space is limited, and only sets of spare parts can be carried. Taking into account the next mission requirement and the storage space constraint, managers need to determine the carrying spare parts quantity for each element.

In addition to above problem descriptions, all basic assumptions are summarized as follows:

1). During the mission, the equipment may break down due to two independent reasons, that’s the inherent degradation and random common cause failures;

2). The mission duration $t_d$, as well as the occurrence time between two adjacent random common cause failure events, is assumed to be a random variable characterized by an exponential distribution.

3. Simulation Modeling for Carrying Spare Parts Optimization

3.1. Carrying Spare Parts Optimization Model and its Optimization Algorithm

3.1.1. Carrying Spare Parts Optimization Model. Based on the problems description in Section 1, the optimization model is to maximize the probability of the system successfully completing the mission considering storage space. From the optimization model, managers can obtain: (1) the optimal carrying spare parts quantity for each element; (2) the achievable probability of the system successfully completing the mission; and (3) the space needed for storing the carrying spare parts. Hence, the carrying spares parts optimization model can be given as:

$$\max \ SU$$

$$s.t. \ \sum_{j=1}^{n} Sp_j V_j \leq V_m$$

Where, Eq. (1) is the probability of the system successfully completing a mission, Eq. (2) is the limited storage space. In Eq. (2), $Sp_j$, denoting the carrying spare parts quantity for element $j (j=1,2,\ldots,n)$, is the variable to be optimized, and $V_m$ is the maximum acceptable store space.

3.1.2 Spare Parts Optimization Algorithm Marginal algorithm, as a popular optimization algorithm, has been applied in many well-known foreign spare parts optimization software, such as OPUS, VMetric, etc., which can also be used to calculate the carrying spare parts quantity. Therefore, marginal algorithm is adopted in this paper to solve the spare parts optimization model. [13]

The core of Marginal algorithm is to determine the diminishing law of marginal benefit. In present paper, the marginal benefit value can be interpreted as the ratio of increment of mission success probability to the increment of storage space caused by the increase of carrying spare parts. It can be given as:

$$\Delta s_j = \frac{\Delta SU_j}{\Delta V_j}$$

Where, $\Delta s_j$ denotes the marginal benefit. $\Delta SU_j$ is the increment of mission success probability $\Delta V_j$ represents the increment of storage space when added a carrying spare part.

After each type of spare parts is added by one, the spare parts type $j$ which maximize the $\Delta s_j$ is found to add a spare part, at the same time, the quantity of other carrying spare parts remains unchanged. And then start the next iteration computation, until the total volume of spare parts is exceeded by $V_m$. Finally, we can gain the spare parts support project optimized. The flow-process diagram of carrying spare parts quantity optimization based on marginal algorithm is shown in Fig 1.
3.2. The Simulation Flow-process of Mission Success Probability Assessment

Mission success probability is the key to optimize carrying spare parts quantity which is analyzed by marginal algorithm. Because the mission duration, lifetime of each element and the occurrence time of each common cause failure event are random variables. It is not an easy task to estimate the carrying spare parts quantity by analytic method. Hence, in this paper, the simulation method is adopted to assess mission success probability. The procedure of the proposed simulation model is presented in Fig. 2, and the detailed simulation process is:

1). Initialization. The total simulation number is set to $N_{tot}$. The number of mission success is set to 0, and the times of the current simulation is set to 1. Then input the initial age $A_{o}(j)$ and the current age $A(j) = 1, 2, \cdots n$. The failure rate $pc(j)$ of element $j$ under random common cause failure and the probability $pr(j)$ of successfully repair of element $j$ when adopting in-situ repair mode also need to be input;

2). If $i \leq N_{tot}$, go to Step 3), otherwise go to Step 13);

3). According to life distribution parameters (the scale parameter of the elements subject to Weibull distribution are represent by $\eta_j$, and the shape parameter of the elements subject to Weibull distribution are represent by $m_j$) of various elements, mission parameters and their distribution
replacement, the element will restore in good condition, marked as \( X_j(1) \), if repair is successful, then the element will be in good condition, still \( X_j(0) \), when common cause failure occurs and system structure function \( \varphi(X) \) of equipment. If both the conditions \( \min(T) \geq t_d \) and \( t_e \geq t_d \) are satisfied, then \( s = \varphi(X) = 1 \), that is to say, the mission is completed successfully, \( N_{\text{suc}} = N_{\text{suc}} + 1 \), next, step 12) is executed, otherwise, perform step 6);

6). Judge the current state of system based on the current state of each element in the system, system structure function \( \varphi(X) \) and the value of current mission time \( t_s \). If both the conditions \( s = 1 \) and \( t_e \leq t_d \) are satisfied at the same time, then step 7) is executed, otherwise, perform step 11);

7). Determine the value of current mission time \( t_s \), according to the smallest remaining life \( \min(T) \) of parts generated randomly and the recent moment \( t_e \) when common cause failure event occurred. If \( t_e \leq \min(T) \) is satisfied, then step 8) is executed, otherwise, perform step 10);

8). \( t_s = t_e \), Check whether common cause failure has been caused under common cause failure events through the elements one by one. If there is no failure for the element \( j \), marked as \( X(j) = 1 \), that is to say, the elements is in good condition. When the element \( j \) fails, it needs to be repaired in in-situ repair mode firstly. If repair is successful, then the element will be in good condition, still marked as \( X(j) = 1 \), then the remaining life \( T(j) \) of the element needs to be update. If in-situ repair fails, replacement repair is performed. When there are spare parts for replacement, the element will restore in good condition, marked as \( X(j) = 1 \), then the remaining life \( T(j) \) of the element needs to be update. When there is no spare parts for replacement, the element will keep failure, marked as \( X(j) = 0 \);

9). To check the current state of the system, if \( s = \varphi(X) = 1 \), the next event of common cause failure will occur, and then step 8) is executed. If \( s = \varphi(X) = 0 \), perform step 12);

10). \( t_s = \min(T) \), the element whose remaining life is the minimum will be found, then it will be repaired in in-situ repair mode firstly. If repair is successful, the element will be in good condition, marked as \( X(j) = 1 \), then the remaining life \( T(j) \) of the element will be update, and step 6) will be executed. If in-situ repair fails, replacement repair is performed. When there are spare parts for replacement, the element will restore in good condition, marked as \( X(j) = 1 \), then the remaining life \( T(j) \) of the element will be update, and step 6) will be executed. When there is no spare parts for replacement, the element will keep failure, marked as \( X(j) = 0 \), then step 11) will be executed;

11). check whether the condition \( s = 1 \) is satisfied. If do, mission is successful, \( N_{\text{suc}} = N_{\text{suc}} + 1 \);

12). \( i = i + 1 \), skip to step 2);

13). Mission success probability is calculated by \( SU = N_{\text{suc}} / N_{\text{tot}} \).
Figure 2. The procedure of the proposed simulation model
4. An Illustrative Example and Discussions

4.1. An Illustrative Example

In this section, we will take the electric system of an engine as an example to validate the proposed model. The electric system is composed of 5 elements connected in series, and the reliability block diagram is shown in Fig. 3. The lifetime of all elements are characterized by Weibull distributions. The age, lifetime distributions parameters, volume, failure probability due to RCCFS, and the probability of in-situ repair of each element are shown in Table 1. The parameter related to the mission duration and the random common cause failures are $\lambda_d = 0.18 \text{day}^{-1}$ and $\lambda_c = 2.5 \text{day}^{-1}$, respectively. The available storage space for carrying spare parts is $V_m = 40 \text{m}^3$. The optimization algorithm is achieved by Matlab programming, and the total simulation times is $N_{tot} = 100000$.

![Figure 3. The equipment reliability block diagram](image)

Table 1. Parameters of each element in the equipment

| part | 1    | 2    | 3    | 4    | 5    |
|------|------|------|------|------|------|
| age/day| 90   | 245  | 115  | 140  | 230  |
| distribution parameter | $\eta_1 = 320$, $\eta_2 = 265$, $\eta_3 = 270$, $\eta_4 = 235$, $\eta_5 = 340$, $m_1 = 9$, $m_2 = 13$, $m_3 = 8$, $m_4 = 11$, $m_5 = 8$ |
| volume/ m$^3$ | 2.0  | 1.6  | 3.2  | 4.0  | 2.7  |
| Failure rate under RCCFS | 0.03 | 0.05 | 0.04 | 0.02 | 0.04 |
| Success probability of situ repair | 0.45 | 0.36 | 0.40 | 0.35 | 0.48 |

Based on the simulation program of mission success probability assessment and marginal algorithm, the optimization model of carrying spare parts is solved. Through calculation, the best carrying spare parts support project is provided, that is $Sp = [3,5,3,2,3]$. What it means is that the quantity of spare parts for type 1 is 3, the quantity of spare parts for type 2 is 5, the quantity of spare parts for type 3 is 3, the quantity of spare parts for type 4 is 2 and the quantity of spare parts for type 5 is 3. In this situation, the mission success probability is 0.9897 and the space occupied by spare parts is 39.7m$^3$. The relationship among carrying spare parts quantity, storage space and mission success probability is shown in table 2. And the iterative optimization process of spare parts based on marginal algorithm is shown in Fig. 4.

Table 2. The carrying spare parts quantity, storage space and mission success probability

| spare parts quantity | (0,0,0,0) | (0,1,0,0) | (1,1,0,0) | (1,1,0,1) | (1,1,1,0) | (1,2,1,0) |
|----------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| storage space        | 0         | 1.6       | 3.6       | 6.3       | 9.5       | 11.1      |
| success probability  | 0.4359    | 0.5174    | 0.5635    | 0.6332    | 0.7380    | 0.7693    |
| spare parts quantity | (1,2,1,1) | (1,2,2,1) | (2,2,2,1) | (2,2,2,2) | (2,3,2,1) | (2,3,2,2) |
| storage space        | 15.1      | 18.3      | 20.3      | 23.0      | 24.6      | 28.6      |
| success probability  | 0.8588    | 0.8866    | 0.9053    | 0.9301    | 0.9476    | 0.9628    |
| spare parts quantity | (2,4,2,2) | (2,4,3,2) | (2,4,3,2) | (3,4,3,2) | (3,5,3,2) | (3,5,3,3) |
| storage space        | 30.2      | 33.4      | 36.1      | 38.1      | 39.7      |           |
| success probability  | 0.9682    | 0.9770    | 0.9840    | 0.9881    | 0.9897    |           |

According to the principle of marginal algorithm, each dot represents a carrying spare parts support project in Fig. 4. In the project, the total space occupied is minimal when the mission success
probability is given. It can be seen from table 2, when the mission success probability is 0.9840, the total space occupied by spare parts is 36.1 m$^3$. And when the mission success probability is 0.9682, the total space occupied by spare parts is 30.2 m$^3$. Comparing these two sets of data, we can find the space occupied has increased by 5.9 m$^3$, however, the mission success probability increased by only 0.0158. In addition, it can be concluded from the Fig. 4 that when storage space is more than 30 m$^3$, there is little difference in the mission success probability when the spare parts storage space is increased. Thus, on the basis of satisfying the terminative mission success probability (e.g., more than 96%), the carrying spare parts quantity can be reduced to save more storage space for other equipment system.

![Figure 4. Carrying spare parts quantity vs. storage space](image)

It can be observed from the results that the proposed model can be used to optimize the carrying spare parts quantity when the in-situ repair capacity and RCCFs are considered. And the effectiveness of the proposed method has been verified. Secondly, comparing the mission success probability of different carrying spare parts support project, we know mission success probability can be improved by increasing the number of spare parts, which accords with practice. Thirdly, aiming to save storage space, it is very vital to choose appropriate mission success probability as the target according to the importance of mission. Fourthly, planning carrying spare parts quantity reasonably is of great significance to improve the mission success probability.

5. Conclusion
Considering the mission success probability and the spare parts storage space, a simulation modeling for carrying spare parts optimization is established under the condition of RCCFs. Monte-Carlo simulation and marginal algorithm are introduced during the model solving. Taking the series system of the equipment as an example, the feasibility of this method is verified, which could provide model support for optimizing carrying spare parts quantity. Phased-mission optimization for carrying spare parts under RCCFs could be a study emphasis in further research.
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