Target mass corrections for spin-dependent structure functions in collinear factorization

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We derive target mass corrections (TMC) for the spin-dependent nucleon structure function \( g_1 \) and polarization asymmetry \( A_1 \) in collinear factorization at leading twist. The TMCs are found to be significant for \( g_1 \) at large \( x_B \), even at relatively high \( Q^2 \) values, but largely cancel in \( A_1 \). A comparison of TMCs obtained from collinear factorization and from the operator product expansion shows that at low \( Q^2 \) the corrections drive the proton \( A_1 \) in opposite directions.

I. INTRODUCTION

Understanding the transition from the perturbative to the nonperturbative regimes of Quantum Chromodynamics (QCD) remains one of the most challenging problems in nuclear and hadron physics. Recent progress in describing this transition has focused on quark-hadron duality, which relates observables computed from quark and gluon degrees of freedom to those parametrized in terms of hadronic variables. A classic example of this is the phenomenon of Bloom-Gilman duality [1], in which low-lying nucleon resonances follow deep inelastic structure functions describing high energy data, to which the resonance structure functions average [2].

In QCD the observation of this duality can be formulated within the operator product expansion (OPE), in which moments of structure functions are expanded in inverse powers of \( Q^2 \), the four-momentum squared of the exchanged photon. The leading term is given by matrix elements of twist-two local operators, and is associated with single parton scattering, while the \( \mathcal{O}(1/Q^2) \) and higher terms are related to higher twist nonperturbative multi-parton correlations. (The twist of a local operator in the OPE is defined as its mass dimension minus its spin.) The magnitude of the higher twist contributions then determines the degree to which duality holds [3].

In order to reliably extract information on the duality-violating higher twist contributions to structure functions, it is vital to remove from the data kinematical corrections associated with nonzero values of \( Q^2/\nu^2 = 4x_B^2 M^2/Q^2 \), where \( \nu \) is the energy transfer, \( M \) is the nucleon mass, and \( x_B = Q^2/2M\nu \) is the Bjorken scaling variable. While formally related to twist-two operators [4], these “target mass corrections” (TMCs) are suppressed by powers of \( M^2/Q^2 \), hence TMCs are sometimes inaccurately referred to as “kinematical higher twists”, and can obscure information on genuine higher twist terms.

The importance of TMCs has been highlighted recently by high-precision data from Jefferson Lab on both spin-averaged and spin-dependent structure functions [5] taken at moderate \( Q^2 \) values, \( Q^2 \sim 1 - 5 \) GeV\(^2\), and at large \( x_B \), where TMCs are most significant. Furthermore, with high-intensity neutrino-nucleus scattering experiments planned in similar kinematics [6], TMCs for weak interactions also need to be understood.

Target mass corrections for spin-averaged nucleon structure functions were first considered by Georgi & Politzer within the OPE [3, 7], and later extended to the full set of electroweak structure functions [8, 9]. For spin-dependent scattering, these were evaluated within the same OPE formalism in Refs. [10, 11], extended to the full set by Blümlein & Tkabladze [8], and computed by Detmold [12] for the deuteron.

One of the limitations of the OPE formulation of TMCs is the so-called “threshold problem”, in which the target mass corrected structure functions remain nonzero at \( x_B \geq 1 \). This arises from the failure to consistently incorporate the elastic threshold in moments of structure functions at finite \( Q^2 \), resulting in nonuniformity of the \( Q^2 \to \infty \) and \( n \to \infty \) limits, where \( n \) is the rank of the moment. After performing an inverse Mellin transform on the moments, the extracted structure functions consequently acquire incorrect support at large \( x_B \) [13]. A number of attempts have been made to redress the threshold problem by considering various prescriptions to tame the unphysical behavior as \( x_B \to 1 \) [13, 14, 15]. These approaches are not unique, however, and sometimes introduce additional complications (see Ref. [16] for a review).

An alternative approach, which avoids the threshold ambiguities from the outset, involves formulating TMCs directly in momentum space [17] using the collinear factorization (CF) formalism [18, 19]. This method was heuristically applied by Aivazis, Olness & Tung [20] and by Kretzer & Reno [21] to spin-averaged structure functions. More recently Accardi & Qiu [17] applied this formalism to deep inelastic structure functions at large \( x_B \), carefully taking into account the elastic threshold and thereby solving the threshold problem. However, in the handbag approximation, without introducing a suitable jet function accounting for the invariant mass of the final hadronic state [17], leading order structure functions can still be nonzero at \( x_B = 1 \).
In this Letter we use the CF framework to derive target mass corrections to the leading twist $g_1$ and $g_2$ structure functions and the $A_1$ polarization asymmetry. In Sec. III we outline the main steps in the derivation; a more detailed account will be presented elsewhere.\cite{22} In Sec. III we compare and contrast the predictions for the TMCs in CF with those using the standard OPE formulation. We find that the TMCs using the CF method are generally larger for the $g_1$ structure function than in the OPE. However, since the TMCs are qualitatively similar for $g_1$ and $F_1$, the effects largely cancel in the $A_1$ asymmetry, although the residual effects can still be up to 20% at large $x_B$, and for $A_1$ even differ in sign for the CF and OPE approaches. Finally, in Sec. IV we summarize our findings and preview future work.

II. TMC IN COLLINEAR FACTORIZATION

The computation of TMCs in collinear factorization makes use of the factorization theorem relating the hadronic tensor $W^{\mu\nu}$ for $g^*N$ scattering to the partonic tensor $w^{\mu\nu}$ for the scattering of a virtual photon from a parton of flavor $f$. The target mass corrected structure functions are then obtained by suitable projections of the hadronic tensor without neglecting the target mass $M$ relative to $Q^2$ at any stage.

The hadronic tensor for spin-dependent inclusive scattering of leptons from nucleons is given by

$$W^{\mu\nu}(p, q) = \frac{1}{p \cdot q} e^{\mu\nu\rho\sigma} q_\rho \times \left[ S_\sigma g_1(x_B, Q^2) + \left( S_\sigma - \frac{S \cdot q}{p \cdot q} p_\sigma \right) g_2(x_B, Q^2) \right] ,$$

where $p$ and $q$ are the target nucleon and virtual photon four-momenta, respectively, and $S$ is the nucleon spin vector, with $S^2 = -M^2$ and $S \cdot p = 0$. We work in collinear frames, defined such that $p$ and $q$ do not have transverse momentum. This allows us to decompose $p, q$ and the parton four-momentum $k$ in terms of light-cone vectors $n^\mu$ and $\bar{n}^\mu$ as\cite{18}

$$p^\mu = p^+ \bar{n}^\mu + \frac{M^2}{2p^+} n^\mu ,$$

$$q^\mu = -\xi p^+ \bar{n}^\mu + \frac{Q^2}{2\xi p^+} n^\mu ,$$

$$k^\mu = xp^+ \bar{n}^\mu + \frac{k^2 + k_T^2}{2xp^+} n^\mu + k_T^\mu ,$$

where $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 1$. The transverse parton momentum $k_T^\mu$ satisfies $k_T \cdot n = k_T \cdot \bar{n} = 0$.

The nucleon plus-momentum $p^+ = (p_0 + p_3)/\sqrt{2}$ can be interpreted as a parameter for boosts along the $z$-axis, connecting the target rest frame to the hadron infinite-momentum frame. In terms of the plus-components of the momenta, the parton fractional light-cone momentum is defined as $x = k^+/p^+$, while the virtual photon fractional momentum

$$\xi = -\frac{q^+}{p^+} = \frac{2x_B}{1 + \sqrt{1 + \gamma^2}}$$

coincides with the Nachtmann scaling variable\cite{3}, with $\gamma^2 = 4x_B^2 M^2 / Q^2$. In the Bjorken limit ($Q^2 \to \infty$ at fixed $x_B$), $\xi \to x_B$ and we recover the standard kinematics in the $M \to 0$ approximation.

The nucleon polarization vector $S$ can be decomposed into longitudinal ($S_L$) and transverse ($S_T$) components,

$$S^\mu = \sigma \lambda S_L^\mu + S_T^\mu ,$$

where $S_L^2 = -M^2$, $S_L \cdot p = S_T \cdot p = 0$, and the nucleon helicity $\lambda = \pm 1$ indicates polarization parallel or antiparallel to the nucleon direction of motion. The degree of longitudinal polarization is given by $\sigma = \sqrt{1 + S_T^2 / M^2}$, with the limits $\sigma_{\min} = 0$ and $\sigma_{\max} = 1$ describing nucleons with purely transverse ($S_T^2 = -M^2$) or longitudinal ($S_T^2 = 0$) polarization, respectively.

Collinear factorization for the hadronic tensor can be obtained by expanding the parton momentum $k$ around its on-shell ($k^2 = m_f^2 = 0$) and collinear ($k_T \to 0$) component,

$$k^\mu \to \tilde{k}^\mu = xp^+ \bar{n}^\mu .$$

In terms of the on-shell parton momentum $\tilde{k}$ we can then define the collinear invariant

$$x_f = \frac{-q^2}{2k \cdot q} = \frac{\xi}{x} ,$$

where the second equality hold for massless quarks, to which we restrict this analysis.

The spin vector of a collinear parton, $s^\mu$, is defined analogously such that $s^2 = 0$ and $s \cdot \tilde{k} = 0$. For a massless spin-1/2 quark, as well as for a massless spin-1 gluon, the spin vector can be written as

$$s^\mu = \lambda_f \lambda \bar{k}^\mu ,$$

where the parton helicity $\lambda_f = \pm 1$ corresponds to a parton with spin parallel or antiparallel to the proton longitudinal spin.

According to the QCD factorization theorem\cite{19} the hadronic tensor can be factorized as

$$W^{\mu\nu}(p, q, S) = \sum_{f, \lambda_f} \int_{\xi}^{\xi_{\max}} \frac{dx}{x} w_f^{\mu\nu}(\tilde{k}, q, s) \varphi^{\lambda_f}(x, Q^2) ,$$

where $w_f^{\mu\nu}$ is the partonic tensor for scattering from a parton of flavor $f$. (Note that the vector $s$ in the argument of $w_f^{\mu\nu}$ depends on $\lambda_f$.) The upper limit of integration in Eq. (8) is\cite{13}, viz., $x_{\max} = \xi / x_B$, guarantees that structure functions vanish for $x_B > 1$\cite{17}, in contrast to Refs.\cite{20,21} where $x_{\max} = 1$. The neglected terms
of order $O(k^\mu - \bar{k}^\mu)$ in the collinear expansion are suppressed by powers of $\Delta^2/Q^2$, with $\Delta$ some hadronic scale, and contribute to the restoration of gauge invariance in higher-twist diagrams [23]. The factorized expression [8] is obtained with the additional approximation of neglecting the intrinsic parton $k_T$ and parton off-shellness in the kinematics of the handbag diagram. The $(x_B, Q^2)$ region where this approximation is valid has been estimated in Ref. [17]. A detailed account of non-zero distributions to the hadronic tensor at leading order in $\alpha$ or inclusion of higher order corrections in 1 [24, 25, 26]. Extension to transversity distributions, where this approximation is valid has been estimated in Ref. [18]. Even [24, 25], require a generalization of Eq. (8). Even [24, 25], require a generalization of Eq. (8).

Polarized scattering is described by the antisymmetric part of the tensor, which can be decomposed in terms of the partonic $g_{1,f}$ and $g_{2,f}$ structure functions,

$$\psi_{f}(k, q) = \frac{1}{k \cdot q} e^{\mu\nu\rho\sigma} q_\rho$$

$$\times \left[ s_\sigma g_{1,f}(x_f, Q^2) + \left( s_\sigma \frac{k \cdot q}{k \cdot q} \right) g_{2,f}(x_f, Q^2) \right].$$

For ease of notation, in the following we will omit the dependence on $Q^2$ of the structure functions and parton distributions functions. The function $\varphi_{f}^{\lambda^i}(x)$ in Eq. [8] is the parton distribution function for a parton of flavor $f$ and helicity $\lambda_f$ inside a nucleon. In the light-cone gauge, and at leading order in $\alpha$, this is defined as

$$\varphi_{f}^{\lambda^i}(x) = \int \frac{dz^+}{2\pi} e^{-ixp^+z^-}$$

$$\times \langle p, S|\psi_f(z^- n) \frac{1}{2}(1 + \gamma_5) \frac{1}{2} \psi_f(0)|p, S \rangle,$$

where $\psi_f$ is the quark Dirac field. For polarized scattering the spin-dependent quark distribution function $\Delta\varphi_f$ is then given by

$$\Delta\varphi_f(x) = \frac{1}{\sigma} [\varphi^+_f(x) - \varphi^-_f(x)].$$

Note that the factorized expression in Eq. [8] is suitable for discussing the contribution of helicity parton distributions to the hadronic tensor at leading order in the expansion of parton correlators in powers of $1/p^+$ [24, 25, 26]. Extension to transversity distributions, or inclusion of higher order corrections in $1/p^+$ (corresponding to “dynamical twist” $\geq 3$ in the language of Refs. [14, 22]), require a generalization of Eq. [8]. Even at $O(1/p^+)$ the TMCs can become nontrivial [18], and we will discuss these higher order corrections elsewhere.

Using suitable projection operators, structure functions can be projected from the hadronic and partonic tensors in Eqs. [1] and [9], and using Eq. [8] one finds

$$g_1(x_B) = \frac{1}{1 + \gamma^2} \sum_f \int \frac{x_B^2}{x_B} dx_f \left( \frac{\xi}{x} \right) \Delta\varphi_f(x),$$

$$g_2(x_B) = -g_1(x_B)$$

for the target mass corrected structure functions at $O(1)$ in $1/p^+$. At leading order in $\alpha$, the partonic structure function $g_{1,f}$ is proportional to $\delta(x - \xi)$, in which case the target mass corrected nucleon $g_1$ structure function is given by

$$g_1(x_B) = \frac{1}{1 + \gamma^2} g_1^{(0)}(\xi),$$

where $g_1^{(0)}$ is the structure function in the massless target limit, $M^2/Q^2 \rightarrow 0$. Note that Eq. [14] is strictly valid only at leading order. At higher orders the massless limit $g_1$ structure function generalizes to

$$g_1^{(0)}(x_B) = \sum_f \int_0^1 \frac{dx}{x} g_{1,f} \left( \frac{x_B}{x} \right) \Delta\varphi_f \left( \frac{x_B}{x} \right),$$

with $g_2^{(0)}(x_B) = -g_1^{(0)}(x_B)$. Clearly, in general one has $g_{1,f}(x) \neq g_1^{(0)}(\xi)$ because of the $1/(1 + \gamma^2)$ factor in Eq. [12], and the upper limits of integration (i.e., $\xi/x_B$ versus 1).

In actual polarized deep-inelastic scattering experiments one typically measures not the structure functions directly, but the virtual photon polarization asymmetries $A_1$ and $A_2$, defined as ratios of spin-dependent to spin-averaged structure functions,

$$A_1(x_B) = \frac{g_1(x_B) - \gamma^2 g_2(x_B)}{F_1(x_B)},$$

$$A_2(x_B) = \gamma g_1(x_B) + g_2(x_B)$$

Using the results in Eqs. [12]–[13] one can write the asymmetries in collinear factorization as

$$A_1(x_B) = \left( 1 + \gamma^2 \right) \frac{g_1(x_B)}{F_1(x_B)},$$

$$A_2(x_B) = 0,$$

where $F_1$ is the spin-averaged structure function, which in collinear factorization is given by [17]

$$F_1(x_B) = \sum_f (F_{1,f} \otimes \varphi_f)(\xi),$$

using a shorthand notation $\otimes$ for the integral over $x$ as in Eq. [12]. The function $\varphi_f$ is defined as the sum of the helicity distributions in Eq. [10], $\varphi_f(x) = \varphi^+_f(x) + \varphi^-_f(x)$.

The polarization asymmetry in collinear factorization can then be written at $O(1)$ in the $1/p^+$ expansion as

$$A_1(x_B) = \frac{\sum_f g_{1,f} \otimes \Delta\varphi_f(\xi)}{\sum_f (F_{1,f} \otimes \varphi_f)(\xi)}.$$
The massless $A_1^{(0)}$ asymmetry is also directly related to the lepton asymmetry $A_1$ for scattering leptons with longitudinal polarization aligned and anti aligned with the nucleon polarization,

$$A_1^{(0)}(x_B) = \frac{A_1(x_B)}{D},$$

where $D$ is a depolarization factor of the virtual photon. A commonly used approximation in experimental data analysis relates the longitudinal lepton asymmetry with the ratio of the $g_1$ and $F_1$ structure functions,

$$(1 + \gamma^2)\frac{g_1}{F_1} \approx \frac{A_1}{D}. \tag{24}$$

From Eqs. (18) and (23) this is equivalent to assuming that

$$A_1 \approx A_1^{(0)}. \tag{25}$$

In the next section we shall test the validity of this approximation numerically, and compare the results of the collinear factorization with the target mass corrections obtained from the OPE.

### III. COMPARISON WITH THE OPE

A common prescription for evaluating target mass corrections uses the operator product expansion to compute moments of structure functions at leading twist, including the trace terms which introduce the kinematical $M^2/Q^2$ corrections, and extracts the TMC structure functions through an inverse Mellin transform [3, 7, 8, 11, 12, 16]. The resulting target mass corrected $g_1$ and $g_2$ structure functions can be written as [8]:

$$g_1^{\text{OPE}}(x_B) = \frac{1}{(1 + \gamma^2)^{3/2}} \frac{x_B}{\xi} g_1^{(0)}(\xi) + \frac{\gamma^2}{(1 + \gamma^2)^2} \int_{1/\xi}^{1} \frac{dv}{v} \left[ \frac{x_B + \xi}{\xi} + \frac{\gamma^2 - 2}{2\sqrt{1 + \gamma^2}} \log\left(\frac{v}{\xi}\right) \right] g_1^{(0)}(v), \tag{26}$$

$$g_2^{\text{OPE}}(x_B) = -g_1^{\text{OPE}}(x_B) + \int_{x_B}^{1} \frac{dy}{y} g_1^{\text{OPE}}(y). \tag{27}$$

The expression for $g_2^{\text{OPE}}$ in Eq. (27) is known as the Wandzura-Wilczek relation [27], and was shown in Ref. [8] to survive target mass corrections. This expression differs from Eq. (13), obtained at $\mathcal{O}(1)$ in the $1/p^+$ expansion in collinear factorization, by the presence of the integral term. In collinear factorization such a term emerges at $\mathcal{O}(1/p^+)$, and Eq. (27) holds if one neglects quark-gluon-quark correlators [29, 30] and matrix elements related to the Wilson line in the expansion of the quark-quark correlators [31].

The prefactors for $g_1^{(0)}$ in the first term of Eq. (26) differ from those in the corresponding collinear factorization expression, Eqs. (12) and (13). The factor $(1 + \gamma^2)^{-1}$ can be traced back to the tensor decomposition of $W^\mu$ and has the same origin as the factor appearing in Eq. (12), while the remaining $(1 + \gamma^2)^{-1/2}x_B/\xi$ factor arises from the OPE treatment of TMCs. Substituting Eq. (27) in Eq. (10) one obtains for the $A_1$ asymmetry in the OPE:

$$A_1^{\text{OPE}}(x_B) = \frac{1}{F_1^{\text{OPE}}(x_B)} \left[ g_1^{\text{OPE}}(x_B) - \gamma^2 \int_{x_B}^{1} \frac{dy}{y} g_2^{\text{OPE}}(y) \right], \tag{28}$$

which again differs from Eq. (15) in the integral term. One should also note that the Wandzura-Wilczek relation (27) is not a direct consequence of the OPE [25, 32], which leaves open the possibility of $\delta$-function contributions at $x_B = 0$ to the right-hand-side of Eq. (27). To explore the phenomenological consequences of contributions to $A_1$ from subleading powers of $1/p^+$, we consider both definitions in the numerical evaluation of $A_1$.

In Fig. 1 we compare the results of a leading order evaluation of target mass corrected versus uncorrected proton $g_1$ (left panel) and $F_1$ (center panel) structure functions, and polarization asymmetries $A_1$ (right panel) for several $Q^2$ values, using the leading order GRSV2000 (standard scenario) polarized parton distributions for $g_1$ [33] and the GRV98LO unpolarized distributions for $F_1$ [34]. For both the CF and OPE corrections, the $g_1$ ratio dips below unity at intermediate $x_B$, 0.2 $\lesssim x_B \lesssim$ 0.5, before rising dramatically at larger $x_B$. The magnitude of the dip and the steepness of the rise for are naturally greater at lower $Q^2$. However, while the size of the TMCs at $x_B \lesssim$ 0.5 is $\lesssim$ 2–3% for $Q^2 >$ 10 GeV$^2$, at larger $x_B$ the corrections remain significant even at much larger $Q^2$. For these reasons, the commonly adopted cut $Q^2 >$ 1 GeV$^2$ for polarized parton distribution function analysis requires inclusion of TMCs for extracting precise PDFs.

For the $A_1$ polarization asymmetry the TMC effects largely cancel in the ratio because the TMCs in the $F_1$ structure function are similar to those in $(1 + \gamma^2)g_1$ [7, 9, 17, 21]. Nevertheless, the residual effects can still be up to 20% at $Q^2 = 1$ GeV$^2$, decreasing to $\sim$ 2–3% at $Q^2 = 10$ GeV$^2$. This provides a quantitative test of the validity of the commonly used approximation in Fig. 1 for the longitudinal asymmetry $A_1$ in terms of $A_1$. Interestingly, the TMC effects drive $A_1$ in opposite directions for $x_B \lesssim 0.7$, with $A_1$ increasing relative to $A_1^{(0)}$ in the OPE approach but decreasing in the CF formulation. This is due to the fact that for most $x_B$ values $g_1^{\text{OPE}} > g_1^{\text{CF}}$, while $F_1^{\text{OPE}} < F_1^{\text{CF}}$. Such ordering arises mainly from the different prefactors for $g_1$ in Eq. (12) and in the first term of Eq. (26) (for the analogous formulas for $F_1$ see Ref. [17]).

The effect of using the Wandzura-Wilczek relation in the computation of $A_1$ instead of the CF result $g_1 = -g_2$, indicated by the shaded band in Fig. 1 has $\lesssim 5\%$ effect in general, and is negligible for $Q^2 \gtrsim 3$ GeV$^2$. The contribution of the Wandzura-Wilczek term is small compared...
FIG. 1: Ratio of target mass corrected to massless proton $g_1$ (left panel) and $F_1$ (center panel) structure functions and $A_1$ polarization asymmetry (right panel) in collinear factorization (solid) and in the OPE (dashed), for $Q^2 = 1, 3$ and $10$ GeV$^2$. For $A_1$ the shaded band for the OPE result indicates the effect of using the Wandzura-Wilczek relation, Eq. (27) (lower bound), or the identity $g_1 + g_2 = 0$, Eq. (13) (upper bound).

with the differences between the two TMC schemes considered.

The differences between the two TMC implementations can be seen more dramatically in Fig. 2 where the ratios of OPE and CF target mass corrected $g_1$ (left panel) and $A_1$ (right panel) are presented. At $Q^2 = 1$ GeV$^2$ the $g_1$ structure function corrected using the OPE prescription can be up to $\sim 20\%$ larger than that using the CF approach at $x_B \sim 0.4$, with the difference decreasing at larger $x_B$. The differences diminish with increasing $Q^2$, so that by $Q^2 = 10$ GeV$^2$ the methods give essentially the same results at the $2\%$ level for all $x_B \lesssim 0.8$.

The polarization asymmetry is similarly found to be up to $\sim 20 - 30\%$ larger within the OPE approach at $x_B \gtrsim 0.7$ for $Q^2 = 1$ GeV$^2$, depending on the prescription used for $g_2$, but again decreasing to $\lesssim 2\%$ for $Q^2 = 10$ GeV$^2$. In all cases the TMCs are larger for $A_1$ in the OPE than in the CF approach. These results clearly highlight the need for a careful treatment of TMCs in the low-$Q^2$ and large-$x_B$ kinematics.

IV. CONCLUSION

In this study we have derived for the first time the target mass corrections to the spin-dependent nucleon $g_1$ and $g_2$ structure functions, as well as to the polarization asymmetry $A_1$, in the framework of collinear factorization. In the CF framework the threshold problem affecting the OPE framework is naturally avoided by directly implementing four-momentum conservation in the handbag diagram, rendering the structure functions zero for $x_B > 1$. A further advantage of this formalism is that it can be readily extended to processes such as semi-inclusive DIS, where the OPE is not available, and indeed
FIG. 2: Ratio of the $g_1$ structure functions (left panel) and $A_1$ polarization asymmetries (right panel) computed with target mass corrections in the OPE and CF formalisms at $Q^2 = 1$ (largest ratios), 5 and 10 GeV$^2$ (smallest ratios). For $A_1$ in the OPE is as in Fig. 1.

to any other hard scattering process. Additional corrections to structure functions at large $x_B$, such as from jet mass corrections or threshold resummation, can also be naturally incorporated together with TMCs.

The numerical results for the target mass corrections to the leading-order $g_1$ structure function in CF are found to be qualitatively similar to those obtained from the OPE, but up to 20% larger at $Q^2 = 1$ GeV$^2$. The corrections become smaller at larger $Q^2$, with differences between the CF and OPE results $\lesssim 2$–3% for $Q^2 = 10$ GeV$^2$. Nevertheless, the TMCs remain significant at $x_B > 0.7$ even for $Q^2 > 10$ GeV$^2$, and need to be taken into account when analyzing large-$x_B$ data. The numerical difference between the two schemes is likely to increase in a next-to-leading order computation, where the convolution over $x$ in Eq. (12) is performed only up to $\xi/x_B$ instead of 1.

Since the TMCs are qualitatively similar for the $g_1$ and $F_1$ structure functions, they largely cancel in the $A_1$ asymmetry, although the sign of the correction is opposite in the CF and OPE approaches over most of the range of $x_B$. The CF target mass effects in $A_1$ can be as large as 20% at $Q^2 = 1$ GeV$^2$ for $x_B \sim 0.8$–0.9, again decreasing to less than a few percent by $Q^2 = 10$ GeV$^2$.

The commonly used approximation relating $A_1$ directly to the longitudinal asymmetry $A_{1||}$ will therefore break down at low $Q^2$, so that accurate determination of polarized structure functions will require measurement of both $A_{1||}$ and the transverse asymmetry $A_{1\perp}$.

In the future, this analysis can be extended in several directions. Firstly, while the CF formalism avoids unphysical regions in dealing with the threshold problem, the corrected structure functions remain nonzero at $x_B = 1$. To tame this behavior one can follow the approach of Ref. [17] by introducing jet mass corrections, which render the TMC structure functions zero in the limit $x_B \to 1$. Furthermore, while we have restricted ourselves to massless quarks, the generalization to heavy flavors can be accommodated within the collinear factorization framework. In addition, future quantitative analysis of large-$x_B$ and low-$Q^2$ data will require TMCs to be computed for structure functions at subleading powers in $1/p^+$, which will be necessary for a more complete treatment of $g_2$, for instance. Finally, work is currently in progress[33] to extend the collinear factorization formalism to semi-inclusive deep inelastic scattering, where the target and hadron mass effects are yet to be evaluated, and it will be interesting to address the case of transverse momentum dependent parton distributions[24], which will enable the role of the parton intrinsic transverse momentum to be quantified.

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