Tunneling and transmission resonances of a Dirac particle by a double barrier

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Abstract

We calculate the tunneling process of a Dirac particle across two square barriers separated by a distance $d$, as well as the scattering by a double-cusp barrier, where the centers of the cusps are separated by a distance larger than their screening lengths. Using the scattering matrix formalism, we obtain the transmission and reflection amplitudes for the scattering processes of both configurations. We show that the presence of transmission resonances modifies the Lorentzian shape of the energy resonances and induces the appearance of additional maxima in the transmission coefficient in the range of energies where transmission resonances occur. We calculate the Wigner time delay and show how their maxima depend on the position of the transmission resonance.

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1. Introduction

Barrier penetration of relativistic electrons is a very important problem in scattering theory and provides a theoretical framework for different physical phenomena that are not present in the nonrelativistic regime such as the Klein paradox and supercriticality [1–3].

The discussion of tunneling of relativistic particles by one-dimensional potentials has been restricted to some simple configurations such as delta potentials and square barriers, mainly in the study of the possible relativistic corrections to mesoscopic conduction [4] and the analysis of resonant tunneling through multi-barrier systems [5]. Recently [6], electron transport through electrostatic barriers in single- and bi-layer graphene has been studied using the Dirac equation and barrier penetration effects analogous to the Klein paradox.

The study of transmission resonances in relativistic wave equations in external potentials has been discussed extensively in the literature [2, 7–9]. In this case, for given values of the energy and shape of the barrier, the probability of transmission reaches unity even if the potential strength is larger than the energy of the particle, a phenomenon that is not present in the nonrelativistic case. The relation between low momentum resonances and supercriticality has been established by Dombey et al [2] and Kennedy [10]. Recently [9–11], some results on the scattering of Dirac particles by a one-dimensional potential exhibiting resonant behavior have been reported.

The study of the tunneling effects of Dirac particles by potential barriers has been almost restricted to those cases where the wave equations are solvable in terms of special functions and a straightforward identification of the asymptotic states is possible. The composition of barrier potentials [12] using the scattering matrix method permits us to consider physical configurations that cannot be solved in closed form. We discuss resonant tunneling of Dirac particles by a double square barrier and double cusp potential when transmission resonances are also present. We show that the presence of transmission resonances close to the position of the poles of the scattering matrix results in the appearance of additional peaks in the transmission coefficient and in a modification of the Breit–Wigner Lorentzian profile. We calculate the Wigner time delay and show that some of the peaks cannot be associated with energy resonances.

The paper is structured as follows. In section 2, we calculate the transmission of a Dirac particle by a double barrier of equal strength in a range of energies where the barriers support transmission resonances. We calculate the Wigner time delay of the resonances and show that the transmission coefficient as a function of energy, exhibits peaks that cannot be identified as energy resonances even though they correspond to peaks of the Wigner time delay.
In section 3, applying the composition of scattering matrices, we discuss the transmission of a Dirac particle by a double cusp potential. We show that when the system exhibits energy resonances in the range of energy where the cusps support transmission resonances, the transmission coefficient and the Wigner-time delay also present maxima that cannot be identified as Breit–Wigner resonances. In section 4 we briefly summarize our results.

2. Double square barrier

Resonant scattering of a Dirac particle by a square potential barrier has been discussed by different authors in the literature [4]. Here we are interested in studying the resonant transmission of relativistic particles by two square barriers separated by a distance \( d \) when barriers support transmission resonances [1].

Since we are working in \((1 + 1)\) dimensions, we choose the following representation for \( \gamma \) matrices:

\[
y^0 = i\sigma^3, \quad y^1 = \sigma^1, \tag{1}
\]

where \( \sigma_1 \) and \( \sigma_3 \) are Pauli matrices. Using representation (1), the Dirac equation in the presence of a potential \( V(x) \) takes the form [11]

\[
[e^2(E - eV(x)) + \sigma^1 \partial_x + m] \Psi = 0, \tag{2}
\]

where we have adopted the natural units \( h = 1 \) and \( c = 1 \).

The solution to the Dirac equation in the presence of a potential barrier of height \( V \) can be obtained after decomposing the spinor in three regions. For \( x < 0 \) (region I), the solution to the Dirac equation (2) has the form

\[
\Psi_I = A_I \left( \frac{i}{\sqrt{E - m}} \right) e^{ikx} + B_I \left( \frac{i}{\sqrt{E - m}} \right) e^{-ikx}. \tag{3}
\]

The spinor solution in sector II \((0 < x < a)\) is

\[
\Psi_{II} = \alpha \left( \frac{1}{E-eV} \right) e^{ipx} + \beta \left( \frac{1}{E-eV} \right) e^{-ipx}, \tag{4}
\]

where \( p = ((E-eV)^2 - m^2)^{1/2} \). In region III \((x > a)\), we have

\[
\Psi_{III} = C_I \left( \frac{i}{\sqrt{E - m}} \right) e^{ikx} + D_I \left( \frac{i}{\sqrt{E - m}} \right) e^{-ikx}. \tag{5}
\]

Imposing the continuity of the spinor solution at \( x = 0 \) and \( a \), we obtain that the relation between the amplitudes of the incoming and outgoing waves is

\[
B_I = \frac{-2i(1-\gamma^2)\sin pa}{(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}} A_I - \frac{4\gamma e^{-ika}}{(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}} D_I, \tag{6}
\]

\[
C_I = -\frac{4\gamma e^{-ika}}{(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}} A_I + \frac{-2i(1-\gamma^2)\sin pa e^{-2ika}}{(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}} D_I. \tag{7}
\]

Using equations (6) and (7) we readily obtain that the scattering matrix \( S_I \) associated with a square barrier of strength \( V \) in \([0, a]\) is

\[
S_I = \frac{1}{(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}} \times \begin{pmatrix} -2i(1-\gamma^2)\sin pa & -4\gamma e^{-ika} \\ -4\gamma e^{ika} & 2i(1-\gamma^2)\sin pa e^{-2ika} \end{pmatrix}, \tag{8}
\]

where \( \gamma = k(E-eV-m)/p(E-m) \). Applying the property of transformation of the scattering matrix under a translation [12], we obtain that the matrix \( S_2 \) associated with a potential barrier of strength \( V_1 \) in \([d, d+b]\) is

\[
S_2 = \frac{1}{(1-\delta)^2e^{i\eta b} - (1+\delta)^2e^{-i\eta b}} \times \begin{pmatrix} -2ie^{2ikd}(1-\delta^2)\sin qb & -4\delta e^{-ika} \\ -4\delta e^{ika} & 2ie^{-2ikd}(1-\delta^2)\sin qbe^{-2ikd} \end{pmatrix}, \tag{9}
\]

where \( \delta = k(E-eV_1-m)/q(E-m) \). The composition of two scattering matrices \( S_1 \) and \( S_2 \) can be written as [13]

\[
S = \begin{pmatrix} r & r' \\ t & t' \end{pmatrix} = \begin{pmatrix} r_1 + \frac{r't_2t_1}{1 - r_1' r_2} & \frac{t_1't_2}{1 - r_1' r_2} \\ \frac{t_2}{1 - r_1' r_2} & r_2' + \frac{r't_1}{1 - r_1' r_2} \end{pmatrix}, \tag{10}
\]

where indices 1 and 2 correspond to potentials \( V \) and \( V_1 \), respectively.

Composing matrices \( S_1 \) and \( S_2 \), we obtain that the relativistic double barrier exhibits energy resonances for values of \( E \) satisfying the equation

\[
((1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa})(1-\gamma^2)e^{ipa} - (1+\gamma^2)e^{-ipa}) + 4(1-\gamma^2)\sin pa(1-\delta^2)\sin qbe^{2ik(d-a)} = 0. \tag{11}
\]

Transmission resonances for the double barrier occur when the transmission coefficient \( T \) of the whole system is equal to unity; this condition takes place, for \( a = b \), when \( p = q \) and \( \sin(p/a) = 0 \), i.e. when the two barriers have equal strength \( V_1 \) and the energy \( E \) satisfies the relation

\[
E = V - \sqrt{n^2\pi^2/a^2 + m^2}. \tag{12}
\]

Transmission resonances of a single barrier are maxima of the transmission coefficient. They are not poles of the scattering matrix and therefore cannot be associated with quasibound states of the system, nonetheless, their presence modifies the profile and peak distribution of transmission amplitude against energy.

The relativistic double barrier exhibits a discrete number of resonances, whose position and shape reduce to those obtained in the double delta configuration as \( a \to 0 \) and \( b \to 0 \) with \( aV \to \mu \) and \( bV_1 \to \delta \), where \( \mu \) and \( \delta \) are the strengths of the delta barriers. Since transmission resonances for a delta barrier do not depend on the energy, all maxima of the transmission coefficient for the double delta are associated with energy resonances [14, 15].
Figure 1. Double-barrier configuration with $V = V_1 = 5$, with $a = b = 3$ and separated a distance $d = 5$. (a) The dot line corresponds to $\Re E_1 = 1.505$, (b) the dashed line corresponds to the transmission resonance $E_a = 1.7033$, (c) the dash–dot line corresponds to $\Re E_2 = 2.0414$, (d) the long dashed line corresponds to the transmission resonance $E_b = 2.6791$ and (e) the space–dot line corresponds to $\Re E_3 = 2.7595$. Notice that $E_b$ is close to $\Re E_3$.

Figure 2. Coefficient of transmission versus energy for the double-barrier relativistic system, with $a = b = 3$ and $d = 5$. The solid line corresponds to $V = 5$ and $V_1 = 5$. The dashed line corresponds to $V = 5$ and $V_1 = 4$.

Figure 3. Time delay for the relativistic double barrier with $a = b = 3$ and $d = 5$. The solid line corresponds to $V = 5$ and $V_1 = 5$. The dashed line corresponds to $V = 5$ and $V_1 = 4$.

the scattering matrix $S$ for $V = V_1 = 5$ for $1 < \Re(E) < 3$ are $E_1 = 1.5058 - 0.0688i$, $E_2 = 2.0414 - 0.0810i$ and $E_3 = 2.7595 - 0.1141i$ and the transmission resonances satisfying equation (12) are located at $E_a = 1.7030$ and $E_b = 2.6791$. Figure 1 depicts the double-barrier configuration with $V = V_1 = 5$, $d = 5$ and $a = b = 3$. The position of the resonances for $\Re E < 3$ is also shown. From figure 2, it can be observed that there is a peak at the transmission resonance value $E_a$. The transmission resonance at $E_b$ is located close to $E_3$ and the peak does not exhibit a Lorentzian Breit–Wigner shape (see figure 2). The dashed line corresponds to the case $V = 5$ and $V_1 = 4$. It is worth mentioning the difference we can observe between the two plots around $E_b$ (figure 3).

The phase shift $\psi$ of the transmitted amplitude $r = |r| \exp(i\psi)$ changes rapidly near the energy resonances. The Wigner time delay [16, 17]

$$\tau = \frac{d\psi}{dE}$$

(13)

has maxima at positions very close to the resonance energies. The particle is trapped for a long time in the region between the barriers before being transmitted.

Figure 3 shows the Wigner time delay corresponding to the resonances depicted in figure 2. The maxima of the Wigner time delay are very close to the values of $E_1$, $E_2$ and $E_3$. There are also two additional peaks at $E_a$ and $E_b$ that cannot be associated with energy quasibound states but exhibiting a behavior analogous to that observed for energy resonances. Figure 3 shows that the heights of peaks at $E_a$ and $E_b$ are smaller than those of the energy resonances between $1 < \Re(E) < 3$.

3. Scattering by a double cusp barrier

In this section, we proceed to study resonant tunneling by a double-cusp potential barrier:

$$V(x) = V_0 \exp(-|x|/a) + V_1 \exp(-|x-b|/d).$$

(14)
The Dirac equation (2) in the presence of potential (14) cannot be solved exactly, therefore in order to discuss tunneling effects, we proceed to compose the scattering matrices associated with the cusp at \( x = 0 \) and \( b \). This composition gives a very good approximation to the exact result when the separation \( b \) between the centers of the cusp is large in comparison with the shape parameters \( a \) and \( d \).

We proceed to calculate the scattering matrix \( S_1 \) associated with the cusp potential

\[
A^\mu = V_0 \exp \left( -\frac{|x|}{a} \right) \delta^\mu_0.
\]

We use the following representation for the Dirac matrices [11],

\[
\gamma^0 = i \sigma^2, \quad \gamma^1 = \sigma^1,
\]

related to matrix representation (1) via the unitary transformation \( T \):

\[
T = \frac{1}{\sqrt{2}} \left( 1 - i \sigma^1 \right).
\]

Components of the spinor \( \Psi \) solution to the Dirac equation (2) in the presence of the cusp potential are

\[
\left( \frac{d}{dx} - i \sigma V_0 \exp \left( -\frac{|x|}{a} - E \right) \right) \Psi_1 + m \Psi_2 = 0,
\]

\[
\left( \frac{d}{dx} + i \sigma V_0 \exp \left( -\frac{|x|}{a} - E \right) \right) \Psi_2 + m \Psi_1 = 0,
\]

where \( \Psi_1 \) and \( \Psi_2 \) represent the upper and lower components of \( \Psi \). The solutions of the system of equations (18) and (19) can be expressed in terms of Whittaker functions \( M_{k,\mu}(z) \) [18]. We solve the Dirac equation in potential (15) considering an incoming plane wave as \( x \rightarrow \infty \). Potential (15) induces boundary conditions on the solution to the Dirac equation at \( x = 0 \). We split the solutions to equations (18) and (19) into two sectors: I (\( x < 0 \)) and II (\( x > 0 \)). Looking at the asymptotic behavior of the Whittaker function \( M_{k,\mu}(z) \) as \( z \rightarrow 0 \) [18],

\[
M_{k,\mu}(z) \rightarrow z^{1/2 + i\mu} \exp \left( -\frac{z}{2} \right),
\]

the solution behaving asymptotically as an incoming wave from the left can be written as

\[
\Psi_{\text{inc}} = \theta \left( \frac{\tilde{y}^{-1/2} M_{k,\mu}(\tilde{y})}{ma} \right) \exp \left( \frac{\tilde{y}^{-1/2} M_{k+1,\mu}(\tilde{y})}{ma} \right),
\]

where \( \tilde{y} = 2iaeV_0 \exp(x/a) \). The normalization constant \( \theta \) is

\[
\theta = \frac{m}{(2iaeV_0)^{\mu} \sqrt{2E(E + \sqrt{E^2 - m^2})}}.
\]

The spinor \( \Psi_{\text{inc}} \) exhibits the following asymptotic behavior as \( x \rightarrow -\infty \):

\[
\Psi_{\text{inc}} \rightarrow \left( \frac{\tilde{y}^{-1/2} M_{k,\mu}(\tilde{y})}{ma} \right) (2iaeV_0)^{\mu} \exp \left( -\frac{i\sqrt{E^2 - m^2}x}{ma} \right).
\]

The reflected wave is

\[
\Psi_{\text{ref}} = \phi \left( \frac{\tilde{y}^{-1/2} M_{k,\mu}(\tilde{y})}{ma} \right) \exp \left( -\frac{i\sqrt{E^2 - m^2}x}{ma} \right),
\]

where the normalization constant \( \phi \) is

\[
\phi = \frac{m}{(2iaeV_0)^{-\mu} \sqrt{2E(E - \sqrt{E^2 - m^2})}}.
\]

The asymptotic behavior of \( \Psi_{\text{ref}} \) (24) as \( x \rightarrow -\infty \) is

\[
\Psi_{\text{ref}} \rightarrow \left( \frac{1}{(i\mu + k)} \right) (2iaeV_0)^{\mu} \exp \left( -i\sqrt{E^2 - m^2}x \right).
\]

The transmitted wave is

\[
\Psi_{\text{trans}} = \left( \frac{1}{ma} \right) (2iaeV_0)^{\mu} \exp \left( \frac{i\sqrt{E^2 - m^2}x}{ma} \right),
\]

which corresponds to an asymptotic plane wave traveling to the right.

Using spinors \( \Psi_{\text{inc}} \) (21), \( \Psi_{\text{ref}} \) (24) and \( \Psi_{\text{r}} \) (27), the solutions to the Dirac equation in the presence of the cusp potential in regions I (\( x < 0 \)) and II (\( x > 0 \)) are

\[
\Psi_I(x) = \frac{a_1 \theta \left( \frac{\tilde{y}^{(1/2)M_{k,\mu}(\tilde{y})}}{ma} \right) + b_1 \phi \left( \frac{\tilde{y}^{(1/2)M_{k,\mu}(\tilde{y})}}{ma} \right)}{\sqrt{2E(E + \sqrt{E^2 - m^2})}}
\]

\[
\Psi_{II}(x) = \frac{c_1 \phi \left( \frac{\tilde{y}^{(1/2)M_{k,\mu}(\tilde{y})}}{ma} \right) + d_1 \theta \left( \frac{\tilde{y}^{(1/2)M_{k,\mu}(\tilde{y})}}{ma} \right)}{\sqrt{2E(E + \sqrt{E^2 - m^2})}},
\]

where \( a_1, b_1, c_1 \) and \( d_1 \) are constants. The continuity of the spinor at \( x = 0 \) results in the following system:

\[
a_1 \theta M_{k,\mu}(v) + b_1 \phi M_{k,\mu}(v) = c_1 \phi \left( \frac{\tilde{y} + k + \mu}{ma} \right) M_{k+1,\mu}(v)
\]

\[
+ d_1 \theta \left( \frac{\tilde{y} + k + \mu}{ma} \right) M_{k,\mu}(v),
\]

\[
- \frac{1}{ma} a_1 \theta M_{k+1,\mu}(v) - \frac{1}{ma} b_1 \phi M_{k+1,\mu}(v) = c_1 \phi M_{k,\mu}(v) + d_1 \theta M_{k,\mu}(v).
\]

Expressing the outgoing amplitudes \( b_1 \) and \( d_1 \) in terms of the incoming amplitudes \( a_1 \) and \( c_1 \), we have

\[
b_1 = r_1 a_1 + t_1 d_1,
\]

\[
c_1 = t_1 a_1 + r_1 d_1,
\]
where the components \( r_1, r'_1, t_1 \) and \( t'_1 \) of matrix \( S_1 \) for the cusp barrier are

\[
\begin{align*}
    r_1 &= r'_1 \\
    &= -\theta \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0} M_{k+1,\mu}(v) M_{k+1,-\mu}(v) + M_{k,\mu}(v) M_{k,-\mu}(v) \\
    &= -\frac{\phi}{\phi} \\
    &= \left[ \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \right] \\
    &= -\frac{\phi}{\phi} \\
    &= \left[ \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \right]
\end{align*}
\]

(35)

\[
\begin{align*}
    t_1 &= t'_1 = \frac{\phi}{\phi} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k,-\mu} M_{k+1,\mu} M_{k+1,-\mu} + M_{k,\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu}
\end{align*}
\]

(36)

Applying the translation property of the scattering matrix, we obtain that the scattering matrix \( S_2 \) associated with the cusp barrier \( V_1 \exp(-|x-b|/d) \) has the following components:

\[
\begin{align*}
    r_2 &= -e^{i\theta} \frac{\phi^*}{\phi} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,\mu} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu}
\end{align*}
\]

(37)

\[
\begin{align*}
    r'_2 &= -e^{i\theta} \frac{\phi^*}{\phi} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k,\mu} M_{k+1,\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu}
\end{align*}
\]

(38)

\[
\begin{align*}
    t_2 &= t'_2 = \frac{\phi^*}{\phi} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k,-\mu} M_{k+1,\mu} M_{k+1,-\mu} + M_{k,\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu} \\
    &= \frac{(\frac{1}{2}k^2 - \mu^2)}{m_0^2} M_{k+1,-\mu} M_{k+1,-\mu} + M_{k,-\mu} M_{k,-\mu}
\end{align*}
\]

(39)

where

\[
\kappa = iEd - \frac{1}{2}, \quad \eta = id\sqrt{E^2 - m^2}, \quad \epsilon = i2dV_1
\]

(40)

and

\[
\begin{align*}
    \theta^* &= \frac{m}{(2iecV_1)^* \sqrt{2E(E + \sqrt{E^2 - m^2})}} \\
    \phi^* &= \frac{m}{(2iedV_1)^* \sqrt{2E(E - \sqrt{E^2 - m^2})}}
\end{align*}
\]

(41)

Using expressions (35)–(38), we calculate the scattering matrix (10) for the double cusp (14). Using the components \( r \) and \( t \) of the \( S \) matrix, we calculate the energy resonances.

The solid line in figure 5 shows the behavior of the transmission coefficient \( T \) versus energy for potential (14) with \( a = d = 0.4 \) and \( b = 4 \). The solid line depicts the case when both cusps have the same height \( V_0 = V_1 = 6.4271 \), which is a transmission resonance value for an energy of \( E_{\text{res}} = 1.3 \). The solid line has four peaks corresponding to the scattering matrix poles \( E_1 = 1.2290 - 0.0451i \), \( E_2 = 1.3404 - 0.07391i \), \( E_3 = 1.79129 - 0.04321i \), and \( E_4 = 2.6031 - 0.03531i \). Figure 4 depicts the double-cusp configuration with \( V_0 = V_1 = 6.4271 \), \( a = d = 0.4 \) and
Figure 6. Time delay for the relativistic double-cusp system with $a = d = 0.4$ and $b = 4$. The solid line corresponds to $V_0 = V_1 = 6.4271$. The dashed line corresponds to $V_0 = 6.4271$ and $V_1 = 6$.

Figure 6 shows the Wigner time delay for the two cusp barriers described by figure 5. The solid line shows that the transmission resonance located at $E_{\text{res}} = 1.3$ modifies the shape of the peak corresponding to the energy resonance $E_2$. The dashed line shows the Wigner time delay for $V_0 = 6.4271$ and $V_1 = 6$.

4. Concluding remarks

In this paper, we studied the resonant tunneling of Dirac particles by a double square barrier and a double cusp potential. We have shown that the presence of transmission resonances associated with the square barrier or the cusp potential modifies the shape and distribution of the energy resonances. We have shown that the Wigner time delay of the energy resonances is modified when they are located close to a transmission resonance. We have also shown that there are no transmission resonances when the cusps or barriers have different strengths. The transmission with no reflection across a barrier whose amplitude is stronger than the energy of the tunneling particle is a relativistic phenomenon with no analogue in the Schrödinger framework.

The application of semianalytical methods such as phase-integral approximation [19] or uniform approximation [20] can be applied in the study of transmission resonances of composite systems where no analytic solutions for the potentials are available.

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