On spherically symmetrical accretion in fractal media

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ABSTRACT

We use fractional integrals to generalize the description of hydrodynamic accretion in fractal media. The fractional continuous medium model allows the generalization of the equations of balance of mass density and momentum density. These make it possible to consider the general case of spherical hydrodynamic accretion onto a gravitating mass embedded in a fractal medium. The general nature of the solution is similar to the “Bondi solution”, but the accretion rate may vary substantially and the dependence on central mass may change significantly depending on dimensionality of the fractal medium. The theory shows consistency with the observational data and numerical simulation results for the particular case of accretion onto pre-main-sequence stars.

Key words: Accretion, accretion discs – hydrodynamics – turbulence – stars: pre-main-sequence – ISM: clouds – ISM: structure.

1 INTRODUCTION

The interstellar medium (ISM) is believed to have a self-similar hierarchical structure over several orders of magnitude in scale (Larson 1981; Falgarone, Puget & Perault 1992; Heithausen et al. 1998). Direct H i absorption observations and interstellar scintillation measurements suggest that the structure extends down to a scale of 10 AU (Crovisier, Dickey & Kazés 1985; Langer et al. 1995; Faison et al. 1998) and possibly even to a scale of sub-AU (Hill et al. 2005). However, the latter is limited by the spatial resolution of the observations. Hence the issue is far from being definite even after observational detection of lower limit of self-similarity scale in some ISM components (Goodman, Barranco, Wilner & Heyer 1998). Numerous theories have attempted to explain the origin, evolution and mass distribution of these clouds (began with the hierarchical fragmentation picture, Hoyle 1953) and it has been established, from both observations (Elmegreen & Falgarone 1996; Burkert, Bate & Bodenheimer 1998; Klessen, Burkert & Bate 1998; Semelin & Combes 2000), that the interstellar medium has a clumpy hierarchical self-similar structure with a fractal dimension 2.5 ≤ D ≤ 2.7 (Sánchez-Alfaro & Pérez 2005) in 3 dimensional space. The main reason for this is still not properly understood but can result from the underlying fractal geometry that may arise due to turbulent processes in the medium.

Here we have investigated physical processes, in particular hydrodynamics, in such a medium with fractal dimension. We have considered the simplest situation of hydrodynamic spherical steady accretion of the medium onto a large gravitating mass embedded in this medium. We have assumed the dimensionality of the medium to be isotropic and homogeneous. It implies that the mass density (or, equivalently, dimensionality) is same for any surface independent of the orientation. The famous solution of the continuous medium case is the “Bondi Solution” (Bondi 1952). Here we have extended that analysis for a medium with fractal dimension D = 3d ≤ 3.

To describe physical processes in fractal medium, we have used fractional integration and differentiation (Zaslavsky 2002 and references therein). We replace the fractal medium by some continuous medium and the integrals on the network of fractal medium is approximated by fractional integrals (Ren et al. 2003). The interpretation of fractional integration is connected with fractional mass dimension (Mandelbrot 1983). Fractional integrals can be considered as integrals over fractional dimension space (up to a numerical factor; Tarasov 2004). We have chosen the numerical factor properly to get the right dimension of any physical parameter and to derive the standard expression in the limit d → 1. The isotropic and homogeneous nature of dimensionality is also incorporated properly. This model allows us to describe the hydrodynamics in a self-consistent way in a fractal medium.

In this paper we first derive, in §2, the steady state mass accretion rate for fractal spherical hydrodynamic accretion using simple dimensional analysis. In §3 we derive the steady state hydrodynamic equations to describe the spherical accretion in a fractal medium and the steady state accretion rate in terms of boundary conditions at infinity is derived in §4. We discuss the actual astrophysical implication of our analysis and compare our central result with observational data and numerical simulation results in §5. Possible limitations of our analysis is discussed in §6. Finally, we summarize and present our conclusions in §7.
2 DIMENSIONAL ANALYSIS OF FRACTAL ACCRETION

We assume a medium that, on a range of length scale $R$, has a fractal structure of dimensionality $D = 3d < 3$ embedded in 3 dimensional space. Here $D$ refers to the mass dimension and it implies that in such a medium of constant density $\rho$, the mass enclosed in a sphere of radius $r$ will be

$$M_D = kr^D \sim \frac{\rho}{l_c^{3(d-1)}R^{3d}}$$

where $l_c$ is a characteristic inner length of the medium and can take arbitrary value in the limit $d \to 1$. It is the scale below which the medium will be continuous. We can define the modified “density” for this fractal medium as $\overline{\rho} \equiv \rho/l_c^{3(d-1)}$ so that $M_D \sim \overline{\rho}R^{3d}$.

For steady state hydrodynamic spherical accretion onto a central mass $M$ from its surroundings fractal medium with a mass dimension of $D$, the relevant physical parameters will be (1) sound speed at a large distance away from the accretor $(d_{\infty})$, (2) modified “density” of the fractal medium at a large distance away from the accretor $(\overline{\rho}_{\infty} \equiv \rho_{\infty}/l_c^{3(d-1)})$ and (3) mass of the accretor scaled by the gravitational constant $(GM)$. The dimensions of these three parameters are

$$[a_{\infty}] = [M]^1[L]^1[T]^{-1}$$
$$[\overline{\rho}_{\infty}] = [M]^1[L]^0[T]^0$$
$$[GM] = [M]^0[L]^3[T]^{-2}$$

It is possible to uniquely construct, from these parameters, a quantity $\overline{\rho}$ with dimension $[M]^1[L]^0[T]^{-1}$. So, from simple dimensional analysis we get the mass accretion rate

$$\dot{M} \sim \overline{\rho}_{\infty}(GM)^{D-1}(a_{\infty})^{3-2D} = \frac{4\pi\overline{\rho}_{\infty}}{l_c^{3(d-1)}}(\overline{\rho}_{\infty})^{D-1}.$$ (5)

Dimensional analysis cannot be used to fix the dimensionless constant $C$ in the above equation and does not give a detailed physical picture. But we have used the fractional integrals to derived, in a more detailed analysis in the following sections, the mass accretion rate and found that to be consistent with the accretion rate derived from the dimensional analysis.

3 FRACTIONAL INTEGRALS AND HYDRODYNAMIC EQUATIONS

The integrals on network of fractals can be approximated by fractional integrals [Ren et al 2003] and the interpretation of the fractional integration is connected with fractional mass dimension. We consider the fractional integrals as integrals over fractional dimension space (up to a numerical factor) and the fractional infinitesimal length for a medium with isotropic mass dimension will be given by

$$d\overline{\rho} = \frac{1}{l_c^{3(d-1)}}r^{d-1}dr, \quad d = D/3 < 1$$

where the constant is chosen to derive the standard expression in the limit $d \to 1$. Note that the infinitesimal area and volume elements in this “fractional continuous” medium of mass dimension $D = 3d$ will be

$$dA_x = \frac{1}{l_c^{3(d-1)}} \frac{r^{2(d-1)}}{d^2} dr^2 sin \theta d\phi$$

$$dA_{\theta} = \frac{1}{l_c^{3(d-1)}} \frac{r^{2(d-1)}}{d} r dr sin \theta d\phi$$

$$dA_{\phi} = \frac{1}{l_c^{3(d-1)}} \frac{r^{2(d-1)}}{d} r dr d\theta$$

$$dV = \frac{1}{l_c^{3(d-1)}} \frac{r^{3(d-1)}}{d^2} r^2 dr sin \theta d\theta d\phi$$

and hence the mass enclosed in a sphere of radius $R$ for constant density $\rho$ will be

$$M_D = \int_V \rho dV = \frac{4\pi\rho}{l_c^{3(d-1)}} \frac{R^{3d}}{3d^3} \sim R^D$$

which will give the standard expression in the limit $d \to 1$.

Below we consider the mass density and momentum density balance in such a fractal medium of mass dimension $3d$ in the case of accretion onto a central gravitating mass $M$ for the sake of our interest (i.e. the steady state hydrodynamic spherical accretion).

We consider the infinitesimal volume $dV$ in $E^3$ and the balance of mass density or the conservation of mass that volume in the case of steady state spherical accretion will imply

$$\frac{d}{dr}[\rho_0(r/l_c)^{d-1}dA_x] = 0 \implies \frac{d}{dr}[\rho_0r^{3(d-1)}] = 0.$$ (12)

This is the generalized mass density balance equation or the generalized continuity equation that can be integrated over a surface of constant $r$ to give a constant radial accretion rate

$$\dot{M} = \frac{4\pi r^3}{d^2} \rho_0[l_c^{3(d-1)}r^{3d-1}] = \text{Constant (independent of } r).$$ (13)

In the infinitesimal volume $dV$ in $E^3$, the balance of momentum density will imply

$$\frac{d}{dr}[\rho \overline{V} d\overline{V}] = F^M + F^S.$$ (14)

where $\overline{V}$ is the velocity vector, $F^M$ is the total gravitational force acting on the mass contained in the infinitesimal volume $dV$ and $F^S$ is the total surface force due to pressure acting on the surfaces bounding the volume $dV$ and are given by

$$F^M_x = -\frac{GM\rho}{r^2} d\overline{V}$$

$$F^S_a = \overline{e}_a \cdot (r/l_c)^{d-1}[(pdA_x\overline{e}_x) - (pdA_x\overline{e}_x)_{r+d}r + (pdA_x\overline{e}_x)_{\theta+d}r + (pdA_x\overline{e}_x)_{\phi+d}r]$$

and, for spherical accretion where $p = p(r)$, it will be reduced to

$$F^S_x = -\frac{dp}{dr} d\overline{A}_x (r/l_c)^{d-1}.$$ (17)

The total change of radial momentum is given by

$$\frac{d}{dr}(\rho u d\overline{V}) = \frac{\partial}{\partial t}(\rho u d\overline{V}) + \frac{\partial}{\partial x}[(\rho u d\overline{V})]_{x}.$$ (18)

Combining this and equation (12), in case of steady state we get

$$\rho u \frac{d}{dr}(r/l_c)^{d-1}d\overline{A}_x dr = -\frac{dp}{dr} d\overline{A}_x (r/l_c)^{d-1} + \frac{GM\rho}{r^2} d\overline{V}.$$ (19)

and this can be simplified to the generalized momentum density balance equation

$$\frac{d}{dr} + \frac{1}{\rho} \frac{dp}{dr} + \frac{GM}{r^2} = 0.$$ (20)

This equation, using the equation of state $p = K \rho^{\gamma}$ ($1 \leq \gamma \leq \frac{2}{3}$) and the boundary condition at infinity, can also be integrated to give
Figure 1. Velocity profile \( u/v_r = \rho_r G M / a_\infty^2 \) for \( D = 2.55 \) and \( \gamma = \frac{5}{3} \) at \( \lambda = 0.25 \lambda_c, 1.00 \lambda_c, 4.00 \lambda_c \).

\[
\frac{1}{2} u^2 + \frac{1}{\gamma - 1} a^2 - \frac{GM}{r} = \frac{1}{\gamma - 1} a_\infty^2
\]  
\[\text{(21)}\]

where \( a \) is the sound speed given by \( a \equiv (dp/\rho)^{1/2} \) and \( a_\infty \) is the sound speed at infinity. This is a local conservation law and, as expected, has exactly the same form as that of the continuous medium.

4 MASS ACCRETION RATE

We have solved equations (12) and (20) for a smooth, monotonic solution without any singularities in the flow. Following the standard derivation of the “Bondi solution”, we found that there exist unique eigen value solution of the problem for a given \( d \) and \( \gamma \) and that the solution must pass through a critical point where

\[
u_c^2 = a_c^2 = S(d, \gamma)a_\infty^2, \quad S(d, \gamma) = \frac{2}{(6d - 1) - 3\gamma(2d - 1)}.
\]
\[\text{(22)}\]

The mass accretion rate may not be steady in a small timescale because of the “clumpy” structure of the medium but a time average mass accretion rate can also be calculated following the standard derivation and is given by

\[
\dot{M} = 4\pi \lambda_c(d, \gamma) \rho_\infty \frac{GM}{a_\infty^2} \left( \frac{GM}{a_\infty^2} \right)^{3d-1}
\]
\[
= 4\pi \lambda_c(d, \gamma) \rho_\infty a_\infty \left( \frac{GM}{a_\infty^2} \right)^2 F^{3(1-d)}
\]
\[\text{(23)}\]

where \( F(M, a_\infty, l_c) \) and \( \lambda_c(d, \gamma) \) are dimensionless parameters

\[
F(M, a_\infty, l_c) = \frac{GM}{a_\infty^2 l_c}
\]
\[\text{(24)}\]

\[
\lambda_c(d, \gamma) = \frac{(3d - 1)^{1-3d}}{d^2} S(d, \gamma) \left[ \frac{GM}{a_\infty^2} \right]^{-3(1-d)}
\]
\[\text{(25)}\]

This is consistent with the accretion rate derived from the dimensional analysis (see Eqn. 5 in Sec. 2). The solution also uniquely corresponds to the physically likely situations properly matching the boundary conditions (small velocity at large distance and high velocity at small \( r \) in accretion flow and the opposite in “wind flow”). The velocity profile is shown in figure 1 for a particular case where \( D = 3d = 2.55 \) and \( \gamma = \frac{5}{3} \). The general nature of the solution is similar to the “Bondi solution” (Bondi 1952).

Note that the smooth, physically selected solution exists only for the transonic case with \( \lambda > \lambda_c \). It also corresponds to the maximum accretion rate as there exists no meaningful solution either for \( \lambda > \lambda_c \) or for \( \lambda < \lambda_c \).

Note that value of \( \lambda_c \) is of order unity. In particular, for \( d = 1 \) and \( \gamma = \frac{5}{3} \), \( \lambda_c = \frac{4}{\text{Bondi 1952}} \) and for \( \gamma = 1 \), \( \lambda_c \) is given by

\[
\lambda_c(d, \gamma = 1) = \frac{(3d - 1)^{1-3d}}{d^2} c^{3d-\frac{2}{3}}
\]
\[\text{(26)}\]

which is the limit of equation (25) as \( \gamma \to 1 \), so that \( \lambda_c \) is continuous at \( \gamma = 1 \).

The most interesting consequence is the dependence of accretion rate on the factor \( F(M, a_\infty, l_c) \). In the interstellar medium typical ambient sound speed (which goes as \( v_c \)) is \( \sim 10 \text{ km s}^{-1} \) for a temperature of \( \sim 10^4 \text{ K} \). In this case the value of \( F \) is given by

\[
F(M, a_\infty, l_c) = 8.907 \left( \frac{M}{M_\odot} \right) \left( \frac{10 \text{ km s}^{-1}}{a_\infty} \right)^2 \left( \frac{1 \text{ AU}}{l_c} \right)
\]
\[\text{(27)}\]

This is simply the ratio of the length-scale of “sphere of influence” of the accreting object to the scale length of the fractal structure. As we are mainly interested in the medium inside the sphere of influence, our approach to the problem of accretion in fractal medium will be valid only if \( F \gtrsim 1 \).

In such situations, this factor can change the mass accretion rate substantially and it will crucially depend on the fractional dimensionality \( d \). More interestingly, for the cases where this factor is important, the accretion rate \( \dot{M} \) will not be proportional to \( M^2 \) but will be significantly different from that. For example, for \( 2.5 \lesssim D \lesssim 2.7 \) (Sanchez, Alfaro & Perez 2005) or equivalently \( 0.83 \lesssim d \lesssim 0.90 \), the accretion rate \( M \sim M^0 \) where \( 1.5 \lesssim \alpha \lesssim 1.7 \).

In the actual astrophysical situation, however, the Bondi-Hoyle accretion rate will probably be more relevant (Bondi & Hoyle 1952; Bondi 1952; Edele 2004 and references therein). Bondi (1955) proposed the interpolation formula (corrected up to a numerical factor by Shima, Matsuda, Takeda & Sawada 1985) replacing \( a_\infty \) by \( (a_\infty^2 + v_c^2)^{1/2} \) and Bondi radius \( r_B = GM/a_\infty^2 \) by Bondi-Hoyle radius \( r_{BH} = GM/(a_\infty^2 + v_c^2) \) where \( v_c \) is the gas velocity relative to the star at a large distance. For a medium with density and velocity structure, it is not possible to define a unique \( v_c \) and the present analysis cannot be extended to such a situation. But it may be extended to a more idealized case where the medium has a fractal structure and the accretor is moving in a velocity \( v_c \) relative to the medium at a large distance. Following a similar interpolation method, the accretion rate in this case will be

\[
\dot{M}_{BH} = 4\pi \lambda_c G^2 \rho_\infty M^2 \left( \frac{v_c}{a_\infty^2 + v_c^2} \right)^{3/2} T^{-(3-1-d)}
\]
\[\text{(28)}\]
Comparison of data and theory for accretion rates of PMS stars and brown dwarfs. The stellar mass is in $M_\odot$ and the accretion rate is in $M_\odot$ yr$^{-1}$.

5 ASTROPHYSICAL IMPLICATIONS

Our results have important implications for a number of astrophysical problems. One of these, for example, is the problem of pre-main-sequence (PMS) accretion. Numerical simulation shows that approximating the PMS accretion process as Bondi-Hoyle accretion leads to agreement between simulation and observation (Padoan et al. 2005). For a solar mass star and for typical sound speed $a_\infty \sim 0.2$ km s$^{-1}$ in ambient molecular cloud filaments (Padoan et al. 2005), a fractal scale length of $\sim 10$ AU will be smaller than the size of the sphere of influence and hence our approach should be valid in this particular case. Here we compare our central result with observational data and numerical simulation results. For that we have taken the accretion rates of PMS stars and brown dwarfs compiled by Padoan et al. (2003), (from Natta et al. 2002, White & Hillenbrand 2004, Muzerolle et al. 2004, and references therein) includes all detections but no upper limit. Though the data do not provide any strong support for one theory as opposed to another one due to scatter in the accretion rate attributed to (i) an age dependence of the accretion rate, (ii) variation of $\rho_\infty$, $a_\infty$ and $v_\infty$ and (iii) interaction of accretion flow with jets and outflows on smaller scale (Padoan et al. 2005), we find that our theory is consistent with these data. For example, the best fit value of $\alpha$ is $1.49 \pm 0.13$ for $\log (\dot{M}_{accr}/M_\odot$ yr$^{-1}) \geq -10$. Even when we include all the compiled data of Padoan et al. (2005), the best fit value of $\alpha$ is consistent with our theory within $3\sigma$ error for $2.5 \lesssim D \lesssim 2.7$. In Figure 2 we have shown the data and the best fit for $\log (\dot{M}_{accr}/M_\odot$ yr$^{-1}) \geq -10$. Data from Muzerolle et al. (2004) and White & Hillenbrand (2004) are shown as filled squares and empty squares respectively and the rest of the data are shown as empty circles. We also found that for higher accretion rate and higher central mass, the exponent is significantly different from that of the Bondi-Hoyle accretion. For a smaller central mass, the self-gravity that we have neglected in our analysis may change the accretion rate significantly.

It is possible to use a high resolution simulation (e.g. Padoan et al. 2005) to find out the fractional dimensionality and the scale length $l_c$ by using equation (11) and counting the number of particle $n(R)$ on boxes of different scale length ($R$). On the other hand, comparing the result with existing numerical simulation results is not very straightforward as in most of the cases it is assumed that $\dot{M}_{accr} = AM^2$ and the coefficient $A$ is computed for the assumed $M^2$ dependence on accretion rate. But with the existing published simulation results, it is still possible to check if the variation of this coefficient $A$ within computational uncertainty can consistently accommodate our result and we found that the best fit of the observational data and our theoretical estimate are within $1.5\sigma$ scatter around the computed accretion rate of Padoan et al. (2005). On the other hand, to get a normalized mean accretion rate (see Krumholz et al. 2006, for details) of 0.1 for a Mach number of 5 and $D = 2.55$, we get, from Eqn. (28), $r_B/l_c \sim 17740$. For a solar mass star and for a typical sound speed $a_\infty \sim 0.2$ km s$^{-1}$, this translates to $l_c \sim 1.3$ AU. Fractal structure at this scale is not observationally ruled out. But one should take this as an order of magnitude estimate of the scale length as factors like velocity structure or magnetic field in real complicated astrophysical situations may alter the accretion rate significantly.

The other potential implication is the black hole accretion. The model growth of galactic center black holes assume that the black holes accrete at Bondi rate (e.g., Springel, Di Matteo & Hernquist 2005). If the gas around the black hole is fractal in nature, then one should rather use the modified accretion rate for fractal medium. Here we would like to mention the caveat that there is no definitive observational evidence against or for the fractal nature of the medium in this case. Even if the medium is fractal, the validity of our approach, as mentioned in §3, crucially depends on the mass of the accretor, the scale length of the fractal and the ambient sound speed.

6 DISCUSSION

We have derived the steady state hydrodynamic equations to describe the spherical accretion in a fractal medium by replacing the fractal medium by a “fractional continuous” model. We have derived the steady state accretion rate in terms of boundary conditions at infinity in this simplified situation without considering the self-gravity of the material for a medium where the dimensionality is independent of position and orientation. Magnetic fields, which we have not included in these models, will certainly play a major role in determining the dynamics. But even without the inclusion of magnetic field we have got the following results.

We have found that there exists a unique solution with maximum mass accretion rate and the general nature of the solution is similar to the “Bondi solution” (Bondi 1952) even for a fractal medium with $D = 3d < 3$. The mass accretion rate, in cases, may differ substantially depending on the fractional dimensionality. In particular, the accretion rate is likely to be significantly different from the “Bondi accretion rate” and will be proportional to $M^{D-1}$ in case of accretion in fractional medium with scale length $l_c$ very different from $GM/a_{\infty}^2$.

One limitation of our analysis is that we have not considered the self-gravity of the medium. This is justified provided the central mass $M$ is very large. In cases where self-gravitation is not negligible, it can change the accretion rate significantly. A more important limitation is that the present analysis does not account for turbulent velocity structure. It will certainly play an important role to determine the accretion rate but is probably unlikely to change the...
mass dependence. In a real astrophysical situation with both density and velocity structure in the medium, the analytical mass accretion rate will hence not be applicable. Neglecting the magnetic fields, as mentioned earlier, is the other major limitation of the analysis. The presence of magnetic field is more likely to suppress the accretion rate and magnetohydrodynamic simulation in fractal medium will be required to make a more definitive statement. But the central result, that the accretion rate will be proportional to $M^{D-1}$ will not be affected by the addition of magnetic fields.

Our assumption of the equation of state of the form $p = K\rho^\gamma$ may seems to be a considerable limitation but that is not likely to be the case. In ordinary Bondi-Hoyle accretion in continuous medium, changing the equation of state changes the accretion rate by a numerical factor of the order unity (Ruffert, Arnett 1994) and hence will not change the result much. A more detailed justification of the assumption can be found in

7 CONCLUSION

We have shown that the accretion rate onto a mass embedded in a fractal medium may differ, in some cases, significantly from the Bondi accretion rate even in the simplest situation. We have used the simple model of accretion onto a mass from a fractal medium of mass dimensionality $D \leq 3$ and derived a self-consistent solution that matches the “Bondi solution” as $D \to 3$. The primary result of our investigation is that theoretically accretion rate will be proportional to $M^{D-1}$. The observational accretion rate data and the numerical simulation for particular astrophysical problem of accretion onto PMS stars is consistent with our result. Our findings suggest that the fractal structure of the medium around the accreting mass is playing a major role to determine the accretion rate and its dependence on the central mass. The agreement of the theoretical prediction with existing numerical simulation implies the consistency of the approach. A number of previously published numerical and analytical results have not considered this effect and may need to be reconsidered. We leave a more detailed treatment of the problem, including the effects discussed in this work, the effect of self-gravity and stability of fractal accretion, to a future work.

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