Flavor Symmetries in Extra Dimensions

Alfredo Aranda\textsuperscript{a} and J. Lorenzo Diaz-Cruz\textsuperscript{b}

\textsuperscript{(a)}Facultad de Ciencias, Universidad de Colima
Colima, Col., México
\textsuperscript{(b)}Center for Particles and Fields, FCFM-BUAP
Puebla, Pue., México

Abstract

We present a class of models of flavor that rely on the use of flavor symmetries and the Froggart-Nielsen mechanism in extra dimensions. The particle content is that of the standard model plus an additional flavon field; all the fields propagate in universal extra dimensions and the flavor scale is associated with the cutoff of the theory, which in 5D is $\sim 10$ TeV. The Yukawa matrices are generated by higher dimension operators involving flavon fields, whose vacuum expectation values break the flavor symmetry. We apply this framework and present a specific 5D model based on a discrete local symmetry that reproduces all fermion masses and mixing angles both in the quark and charged lepton sectors.
1 Introduction

It is possible that there exist extra dimensions and that they might even play an important role in electroweak physics. This possibility has led to an impressive amount of work over the past few years resulting in new and exciting ways of tackling (or interpreting) some of the main problems in particle physics. Models with one or more extra dimensions, flat or warped, with and without SUSY have been discussed in the literature. For a partial list of references see [1].

The problem of flavor is one of the pressing problems in particle physics. In the context of extra dimensions some interesting solutions have been presented. For example it is possible to generate hierarchies among the fermion masses and mixing parameters by restricting them to a brane while imposing some flavor symmetry broken by the mechanism of shinning [4]. Another way is to localize the fermion fields in different positions along the extra dimensions, and in this way generate hierarchies among masses and mixing parameters through wave-function overlaps [5]. Yet another alternative, similar in spirit to the previous “cartographic” solution, is to localize the fermion fields along different points inside a fat brane [6] or by placing different matter fields in different branes [7]. In the fat-brane case, the hierarchies and mixing properties are obtained, as before, through the wave-function overlaps. In the case of several branes this is accomplished by having the branes intersect in some specific way. Other new mechanisms employ warped extra dimensions [2] to, for example, produce small Yukawa couplings and hierarchies, as well as small neutrino masses without the need of a seesaw [3].

However, despite the new ideas brought by extra dimensions to the flavor problem, it does not seem to be possible to generate realistic masses within the minimal frameworks proposed thus far, and one may need to invoke a certain amount of flavor symmetries to solve the problem. In this letter we discuss a class of extra-dimensional models of flavor, with the following properties:

- Spacetime consists of $4 + \delta$-dimensions in which all fields propagate, i.e. they are universal. The extra dimensions are compactified on generalized orbifolds $T^n/Z_2$; which in 5D have a radius of compactification $R$.

- A flavor symmetry is added to the SM gauge group. This symmetry is broken by the vacuum expectation values (vevs) of a flavon field, leading to the generation of Yukawa matrix textures for all matter fields through the Froggart-Nielsen (FN) mechanism.

- The particle content of the model is that of the SM plus one additional scalar (flavon) field. This is the minimal set we require in order to reproduce the observed hierarchies.

We show that given the above conditions, it is possible to generate viable models of flavor that reproduces all observed masses and mixing angles, both in the quark and
charged lepton sectors. Furthermore, this is accomplished with a flavor scale determined by the current perturbativity bound on the scale of the universal extra dimension, which for 5D is of order $\sim 10$ TeV [8]. The properties of the model are chosen so as to minimize the amount of additions beyond the SM. It should be noted that the flavor physics presented in this paper is based on the traditional Froggart-Nielsen mechanism with flavor symmetries, and the addition of extra dimensions is done in order to obtain a low flavor scale.

The framework of our extra-dimensional flavor models is presented in section 2. There we show how to generate the Yukawa matrices from operators in the extra-dimensional Lagrangian using the FN mechanism. In Section 3, we present an specific model in 5 dimensions, and show the results from a fit to the experimental data. Additional comments on the model and its extension to 6 dimensions are presented in Sect. 4, which also contains our conclusions.

2 The FN mechanism in extra-dimensions

As mentioned in the introduction, the model consists of the SM fields plus one flavon field, all propagating in a $(4 + \delta)$-dimensional spacetime. We assume the extra-dimensional coordinates $(y_i, i = 1, \ldots, \delta)$ are compactified on a $\delta$-torus/\textit{Z}$_2$; in 5d it corresponds to an orbifold $S^1/\text{Z}^2$ with a radius of compactification $R = 1/M_c$, where $M_c$ is the compactification scale.

In $(4+\delta)$-dimensions, the Lagrangian has mass-dimensions $4+\delta$; therefore the fermion fields have dimensions $(3 + \delta)/2$, while for the scalar fields it is: $(2 + \delta)/2$ and the XD Yukawa coupling have dimensions $-\delta/2$.

Thus, the fermion fields (quarks and leptons) can be decomposed in $(4+\delta)$-dimensions as follows [8]

$$Q(x^\mu, y) = \frac{1}{(\pi R)^{\delta/2}} \left( Q_L^{(0)}(x^\mu) + \sqrt{2} \sum_{[n_i]} \left[ P_L Q_L^{(n_i)}(x^\mu) f_{L}^{n_i}(y_i) + P_R Q_R^{(n_i)}(x^\mu) f_{R}^{n_i}(y_i) \right] \right),$$

$$E(x^\mu, y) = \frac{1}{(\pi R)^{\delta/2}} \left( E_R^{(0)}(x^\mu) + \sqrt{2} \sum_{[n_i]} \left[ P_R E_R^{(n_i)}(x^\mu) f_{R}^{n_i}(y_i) + P_L E_L^{(n_i)}(x^\mu) f_{L}^{n_i}(y_i) \right] \right),$$

where $Q$ and $E$ denote an SU(2)$_W$ doublet and singlet field respectively. $P_L$ and $P_R$ are the 4D chiral projection operators $P_{R,L} = (1 \pm \gamma_5)/2$. The sum is performed over the complete set of indexes $[n_i]$ needed to specify the KK expansion. This decomposition should ensure that the zero modes correspond to SM fields. For instance, in 5-dimensions the expansion functions correspond to: $f_{L}^{n_i} = \cos \left( \frac{n_i y}{R} \right)$, $f_{R}^{n_i} = \sin \left( \frac{n_i y}{R} \right)$, while their generalizations to six-dimensions can be found in Ref. [8] too.

Since the Higgs and the flavon fields must couple to these zero-modes, they must be
even under the $Z_2$. Their decomposition is then given by

$$S(x^\mu, y) = \frac{1}{(\pi R)^{\delta/2}} \left( S^{(0)}(x^\mu) + \sqrt{2} \sum_{[n_i]} S^{(n_i)}(x^\mu) f^{n_i}(y_i) \right). \quad (3)$$

Now we can consider the generation of Yukawa matrices. Once a flavor symmetry is included in the model, the lowest dimensional Yukawa terms are absent, though they can appear as higher-order operators. All the terms in the Lagrangian responsible for the fermion (modulo neutrinos) masses will in general have the form:

$$\mathcal{L}_{XD} \sim \hat{\lambda}_{ab} \overline{Q}_a i\sigma_2 H^* U_b \left[ \Phi \frac{\Lambda}{\Lambda^{(2+\delta)/2}} \right]^n + \text{h.c.}, \quad (4)$$

where $a$ and $b$ are flavor indexes. Dimensionless couplings are introduced by defining: $\hat{\lambda}_{ab} = \lambda_{ab} [\pi R]^{\delta/2}$, $\lambda_{ab}$ denotes the 4D Yukawa coupling (which we assume to be a number of order one), $\Lambda$ is the cutoff of the theory, and $\Phi$ represents a flavon field. To see how the 4D Yukawa matrices are generated, let's consider Eq. (4) after compactification. We are assuming that the flavor symmetry is broken by the vevs of the flavon field at or very close to the scale $\Lambda$. Thus, after compactification, we obtain

$$\mathcal{L}^4 \sim \lambda_{ab} \overline{Q}_a i\sigma_2 H^* U_b \left[ \left( \frac{M_c}{\pi \Lambda} \right)^{n\delta/2} \right] + \text{h.c.}, \quad (5)$$

where all fields now correspond to the zero-modes of those in Eq. (4), and where the vev of $\Phi$ has been set equal to $\Lambda$. We now define a new 4D Yukawa coupling given by

$$\lambda'_{ab} = \lambda_{ab} \left( \frac{M_c}{\pi \Lambda} \right)^{n\delta/2} = O(1) \left( \frac{M_c}{\pi \Lambda} \right)^{n\delta/2} = O(1) \epsilon^{n\delta}, \quad (6)$$

where $\lambda_{ab} = O(1)$ and $\epsilon = \sqrt{M_c/\pi \Lambda}$ should be of order 0.1. Thus, the hierarchies in masses and mixing angles are obtained by assigning different charges to different generations in such a way as to obtain realistic textures for the Yukawa matrices of quarks and leptons. These matrices appear as an expansion in the small parameter $\epsilon$.

### 3 A 5D Model

We are now in a position to present a flavor model in 5 dimensions. Using the values for $M_c = 0.3$ TeV, and $\Lambda = 10$ TeV from Ref. [8], we obtain that the expansion parameter is $\epsilon \approx 0.1$. The 5D model is based on a $Z_6$ local flavor symmetry whose anomalies are assumed to be canceled by a Green-Schwarz mechanism [9] (For a discussion on discrete gauge symmetries see [10]).
The charge assignments for the matter fields are given by

\[ Q \sim (0, 5, 4), \overline{Q} \sim (0, 1, 2) \]
\[ \mathcal{L} \sim (0, 5, 4), \overline{\mathcal{L}} \sim (0, 1, 2) \]
\[ \mathcal{U} \sim (2, 3, 4), \mathcal{D} \sim (1, 2, 2), \mathcal{E} \sim (1, 2, 2) \]

where the numbers in parenthesis correspond to the charges of each generation.

The charges for the Higgs (\( \mathcal{H} \)) and flavon field (\( \Phi \)) are

\[ \mathcal{H} \sim 0, \quad \Phi \sim 1 \quad (7) \]

Using these assignments together with Eqs. (5) and (6) to compute the Yukawa matrices we obtain

\[ \lambda_U \sim \begin{pmatrix} \phi^4 & \phi^3 & \phi^2 \\ \phi^3 & \phi^2 & \phi \\ \phi^2 & \phi & 1 \end{pmatrix} \rightarrow \begin{pmatrix} e^4 & e^3 & e^2 \\ e^3 & e^2 & e \\ e^2 & e & 1 \end{pmatrix}, \quad (8) \]
\[ \lambda_D \sim \begin{pmatrix} \phi^5 & \phi^4 & \phi^4 \\ \phi^4 & \phi^3 & \phi^3 \\ \phi^3 & \phi^2 & \phi^2 \end{pmatrix} \rightarrow \begin{pmatrix} e^5 & e^4 & e^4 \\ e^4 & e^3 & e^3 \\ e^3 & e^2 & e^2 \end{pmatrix}, \quad (9) \]
\[ \lambda_E \sim \begin{pmatrix} \phi^5 & \phi^4 & \phi^4 \\ \phi^4 & \phi^3 & \phi^3 \\ \phi^3 & \phi^2 & \phi^2 \end{pmatrix} \rightarrow \begin{pmatrix} e^5 & e^4 & e^4 \\ e^4 & e^3 & e^3 \\ e^3 & e^2 & e^2 \end{pmatrix}, \quad (10) \]

where O(1) coefficients have been omitted, and only the powers of \( \phi \) are shown here for clarity; \( \phi \) represents the zero-mode of \( \Phi \).

On the other hand, a possible way to compute the neutrino mass matrix, is through the following operator

\[ \mathcal{L}^5 \sim l_{ab} \overline{\mathcal{L}}_a L_b H^2 \frac{\Phi^n}{\Lambda^{3n/2+2}} \quad (11) \]

where \( l_{ab} \) is an O(1) parameter. After compactification this operator becomes indeed a mass matrix operator, namely

\[ \mathcal{L}^4 \sim l'_{ab} \overline{\mathcal{L}}_a L_b H^2, \quad (12) \]

where \( l'_{ab} = O(1) \frac{\Lambda^{(n+2)}}{\Lambda} \), is a Yukawa parameter. Using these operators one can generate textures that reproduce the mixings and mass hierarchies in this sector, but in order to get the right neutrino scale, one might need, for example, to introduce other flavons propagating in the bulk of more than one extra dimension. Alternatively one could introduce right-handed neutrinos \( \nu_R \) propagating in more extra dimensions. In this work, we will not pursue further the problem of neutrino masses. We just mention that it is
indeed possible to construct a model that does the job [15] and concentrate in presenting a detailed discussion on the mass matrices for the quark and charged lepton sectors.

Let us now discuss how the textures obtained above reproduce the observed mass patterns and mixing angles in both the quark and charged lepton sectors. To do this, we need to include O(1) coefficients in the entries of the Yukawa matrices. These coefficients are determined by performing a fit to the observables. We emphasize that the hierarchies among the masses and mixing angles are determined by the textures and not by the coefficients. In order to determine that this is the case, we perform several fits starting from randomly selected initial sets of parameters. An important property of the fits is that the O(1) parameters are not treated freely and they are allowed to vary only within a range, say between 1/3 and 3. They are then included into a $\chi^2$ function and thus treated as additional pieces of data. The particular range is of course arbitrary and is meant only to determine what we explicitly mean by O(1). For details about the fit and how to treat the O(1) coefficients see Ref. [11].

In the quark sector we fit to the six quark masses and three CKM-angles (CP violation is neglected) whereas in the charged lepton sector we use the $e$, $\mu$, and $\tau$ masses. The experimental uncertainties on the observables (or estimates in the case of the quark masses) used in the fits are either those of Ref [12] or 1% of the central value, whichever is larger. We find very good fits for a large number of initial points and conclude that the textures presented in the previous section do reproduce the observed patterns. In Tables 1 and 2 we present the results of one of the fits where we used the following parametrizations:

$$
\lambda_U = \begin{pmatrix}
    u_1 \epsilon^1 & u_2 \epsilon^3 & u_3 \epsilon^2 \\
    u_4 \epsilon^3 & u_5 \epsilon^2 & u_6 \epsilon \\
    u_7 \epsilon^2 & u_8 \epsilon & u_9
\end{pmatrix}, \quad 
\lambda_D = \begin{pmatrix}
    d_1 \epsilon^5 & d_2 \epsilon^4 & d_3 \epsilon^4 \\
    d_4 \epsilon^4 & d_5 \epsilon^3 & d_6 \epsilon^3 \\
    d_7 \epsilon^3 & d_8 \epsilon^2 & d_9 \epsilon^2
\end{pmatrix}, \quad 
\lambda_E = \begin{pmatrix}
    l_1 \epsilon^5 & l_2 \epsilon^4 & l_3 \epsilon^4 \\
    l_4 \epsilon^4 & l_5 \epsilon^3 & l_6 \epsilon^3 \\
    l_7 \epsilon^3 & l_8 \epsilon^2 & l_9 \epsilon^2
\end{pmatrix}.
$$

We will present a complete analysis involving a large number of fits, including the neutrino sector as well, in a longer version of this letter [15].

4 Comments and Conclusions

Here we shall comment on some additional features of the model. First, the inclusion of large discrete symmetries might have an accidental continuous symmetry giving pseudo-Goldstone bosons. In the present model such flavor invariant operators that might lead to an accidental symmetry, will be determined by physics above the cut-off. However, as opposed to the 4D case, where the masses of the pseudo-Goldstone bosons are suppressed by ratios of flavon VEV’s to the fundamental scale, in our case, the ratios may not be suppressed since the fundamental scale may not be far above the cut-off.

Another issue regarding flavor models with a low scale, is their possible contribution to flavor changing processes (FCNC) such as the ones described in [4]. For instance, consider an operator of the type $\sum_a c_a (Q_a D_a) (\overline{Q_a D_a}) / \Lambda^3$. Though this appears to be diagonal in the weak basis, when one goes to the mass-eigenstate basis, it will induce
dangerous FCNC, unless the scale $\lambda$ is of order $\simeq 100$ TeV, or there is some reason that forces the coefficients $c_a$ to cancel at the one percent level. One possibility to achieve such cancellations would be to embed the abelian flavor symmetry into a non-abelian one. However, for the present model we shall simply assume that physics above the cut-off is responsible for the cancellation between the coefficients.

Furthermore, a renormalization group analysis should also be incorporated into the fit. However, since in our model the flavon fields have a mass of $O(\Lambda)$, they do not participate in the running and the hierarchies are affected only by scale-independent factors of $O(1)$ [19]. The running might change the overall scales, and thus it might be necessary to modify the overall scale of the Yukawa matrices. This can be easily done by changing either the charge assignments or by enlarging the flavor group. Also, we don’t view this model as unique, in fact, it might be possible to create a model with a discrete Non-Abelian gauge symmetry that can be broken sequentially. In this case, depending on the scales, the flavon fields can participate in non-trivial ways through the RGE analysis.

On the other hand, once we include the flavon field in the spectrum, it may be possible to generate Higgs-flavon mixing, which would produce a “more flavored” Higgs profile, which could produce interesting Higgs signals such as $h \to \tau\mu$ [20].

Finally, one way to avoid the large flavor charges that appeared in the 5D model would be to extend the model to six dimensions (or higher). Namely, since the entries of the Yukawa matrices scale as $\epsilon^{n\delta}$, while the value of $\epsilon$ can be maintained of order 0.1, one could lower values for $n$ as $\delta$ increases; the 5D charges would be reduced by one half by just letting the model live in six-dimensions.

In summary, we have presented in this paper a new framework in extra dimensions to address the flavor problem, which is based in the traditional FN mechanism. We have presented a specific model in 5D based on a $Z_6$ local symmetry, whose particle content is that of the SM plus an additional flavon field. In 5D there is one universal extra dimension compactified on an $S^1/Z_2$ orbifold with radius of compactification $R = 1/Mc = 3.33$ TeV$^{-1}$, and with a high energy cutoff of $\Lambda = 10$ TeV which is also the flavor scale. When the flavon fields acquire vacuum expectation values they generate Yukawa matrices, which are then used to perform fits to the observables in both the quark and charged lepton sectors. The model successfully accommodates all the the data for the quarks and charged leptons, while for the neutrino masses our scheme seems adequate to generate the right textures, however a detailed model, as well as additional fits, and possibly an improved version of the model in six-dimensions, will be left for a future publication [15].

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\[ \epsilon = 0.1 \quad \chi^2 = 2.29 \]

| Parameter | Expt. value | Fit value |
|-----------|-------------|-----------|
| \( u_1 \) | \( +0.96 \) | \( +0.94 \) | \( l_1 = -1.07 \) |
| \( u_2 \) | \( -1.19 \) | \( -0.89 \) | \( l_2 = -0.89 \) |
| \( u_3 \) | \( +0.73 \) | \( +1.72 \) | \( l_3 = +0.81 \) |
| \( u_4 \) | \( -1.07 \) | \( +1.12 \) | \( l_4 = +1.21 \) |
| \( u_5 \) | \( +1.23 \) | \( +2.28 \) | \( l_5 = +0.69 \) |
| \( u_6 \) | \( -0.64 \) | \( -0.95 \) | \( l_6 = +1.22 \) |
| \( u_7 \) | \( -0.98 \) | \( +1.19 \) | \( l_7 = +1.15 \) |
| \( u_8 \) | \( -0.82 \) | \( -1.97 \) | \( l_8 = +0.83 \) |
| \( u_9 \) | \( +0.99 \) | \( +1.41 \) | \( l_9 = +0.58 \) |

Table 1: Parameters obtained in one fit for the \( Z_6 \) model.

| Observable | Expt. value | Fit value |
|-----------|-------------|-----------|
| \( m_e \) | \( (5.11 \pm 1\%) \times 10^{-4} \) | \( 5.11 \times 10^{-4} \) |
| \( m_\mu \) | \( 0.106 \pm 1\% \) | \( 0.106 \) |
| \( m_\tau \) | \( 1.78 \pm 1\% \) | \( 1.78 \) |
| \( m_u \) | \( (3.25 \pm 1.8) \times 10^{-3} \) | \( 3.28 \times 10^{-3} \) |
| \( m_d \) | \( (6.0 \pm 3.0) \times 10^{-3} \) | \( 5.69 \times 10^{-3} \) |
| \( m_c \) | \( 1.25 \pm 0.15 \) | \( 1.25 \) |
| \( m_s \) | \( 0.115 \pm 0.055 \) | \( 1.04 \times 10^{-1} \) |
| \( m_t \) | \( 173.8 \pm 5.2 \) | \( 173.9 \) |
| \( m_b \) | \( 4.25 \pm 0.15 \) | \( 4.24 \) |
| \( |V_{us}| \) | \( 0.22 \pm 0.0035 \) | \( 0.221 \) |
| \( |V_{ub}| \) | \( (3.1 \pm 1.35) \times 10^{-3} \) | \( 3.1 \times 10^{-3} \) |
| \( |V_{cb}| \) | \( 0.04 \pm 0.03 \) | \( 0.03 \) |

Table 2: Experimental values versus fit central values for observables using the inputs of Table 1. Masses are in GeV and all other quantities are dimensionless. Error bars are taken as indicated in the text.