A Continuous Time Scheduling Model for Printing and Dyeing Plants

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Abstract—Domestically, virtually very little has been done in this area in the printing and dyeing production process scheduling, while some work using dispatch rule for modeling and solution is reported abroad. In this paper, a mathematical model is presented to describe the printing & dyeing production process scheduling. According to features of printing and dyeing production process. Based on the authentic data coming from a printing & dyeing enterprise and mathematical programming, a case study is provided. The programming solution shows the simulation result is correct.

Keywords—printing & dyeing; production process; scheduling; mathematical programming

I. INTRODUCTION

Main purpose of production scheduling is to assign a set of tasks to machines of many stages over scheduling time [1]. So, how to efficiently allocate one or more resources to activities over time is a mathematic problem[2], and it’s application is widely used to many fields such as airlines, transportation, and manufacturing industry including printing & dyeing.

Since production techniques vary in different enterprises and industries, so do the elements considered for modeling of different production processes, it is getting more difficult to model and optimize the resource allocation in a production process, not to mention the validation of the modeling and optimization solutions, especially when the problem size and complexity increase [3]. Production scheduling in printing & dyeing is to optimize the allocation of manufacturing resources of production process for orders that competes for these resources to improve production efficiency and conserve resources.

Domestically, virtually very little has been done in this area in the printing and dyeing industry, while some work using dispatch rule for modeling and solution is reported abroad [4]. Based on our study of the techniques used, and management styles in its production process, it is concluded that the printing and dyeing production is a continuous, batch-mixed, sequence-dependent and orders-driven production process that takes into consideration of dyeing-machine cleaning, setup time between stages and no resource balance.

In the past two decades, many mathematical formulations about production scheduling have been presented [9]. Pinto and Grossmann presented a continuous time mathematical model for short term scheduling of multistage batch plants [5]. In this model, units and orders are matched by tetra-index variables (stage, order, and time). But the model had difficulties solving order-sequence-dependent constraints. Pinto and Grossmann purposed an alternate mathematical formulation which replaced the tetra-index variable by a tri index variable (order, stage, and unit) and imposing pre-ordering of the orders. Thus the number of binary variables are reduced and the solution time is shortened. These models mentioned above were only suitable for pre-ordered orders. Cerda, Henning and Grossmann [6] presented an mathematic formulation for the short-term batch process scheduling of a multi-stage multi-product. A set of tri-index variables (order, order, and unit) are applied to handle order sequence-dependencies. Ierapetritou and Floudas [7][8] purposed a continuous time mathematical model for the short-term batch process scheduling.

This paper gives a continuous-time mathematic formulation for the short-term scheduling of mult-istage, multi-product printing and dyeing plants with considering setup, changover and sequence-dependent constraints by the application of tri-index variables (order, stage, and time). Based on the authentic data coming from a printing & dyeing enterprise and ILOG CPLEX programming, a case study is provided, and the scheduling results demonstrate that the scheduling efficiency much more than the one man-made.

II. MANUFACTURING FEATURES OF PRINTING AND DYEING PRODUCTION PROCESS

Printing and dyeing production is a continuous, batch-mixed, sequence-dependent and customer orders-driven process. Basically, gray cloth to the final products should be passed these stages, singeing, desizing, scouring, bleaching, mercerizing, dyeing, printing, softening, stretching and preshrunk, not every product should pass all stages, and different products have different stages and different machines processing speeds. Two consecutive stages can be continuous or batch. If it is batch, there is a setup time between these two stages and setup time is fixed. While a dyeing machine firstly dyes deep color gray cloth then dyes light color one, it need not to be cleaned, conversely, it must be cleaned.
III. MATHEMATICAL MODEL

Some parameters from mathematical formulation are defined as following [9].

\( i, i', j, j' \): order number, \( I \): set of orders
\( I_u \): orders that can be processed in unit \( u \)
\( u, u', v \): unit number, \( U \): set of units
\( U_l \): units on stage \( l \), \( U_{il} \): units that can process order \( i \)
\( l, l', m, m' \): stage, \( l_l \): the last stage of order \( i \)
\( L \): set of stages, \( L_i \): stages of order \( i \) should pass
\( D \): due date of order \( i \)
\( T \): process time of order \( i \) in unit \( u \)
\( H \): weight of order \( i \) on stage \( l \)

Some variables defined as following.

\( T_{si} \): starting time order \( i \) on stage \( l \)
\( T_{di} \): tardiness of order \( i \)
\( W_{iu} \in \{0,1\} \): whether order \( i \) can be processed in unit \( u \) or not
\( X_{ijl} \in \{0,1\} \): whether order \( j \) precedes order \( i \) on stage \( l \) or not
\( S_{iu} \in \{0,1\} \): whether order \( i \) is the first processed order of unit \( u \) or not

The mathematic formulation for the scheduling of printing and dyeing plants is gived as following:

1. The allocation of consecutive orders to the same unit

\[
W_{iu} + \sum_{v \in (U_l \cap U_{il}) \setminus u} W_{iv} + X_{ijl} \leq 2
\]

\( \forall i \in I, j \in I, i \neq j, u \in (U_i \cap U_j \cap U_{ij}), l \in L \) (1)

This means that if task ‘\( i \)’ and task ‘\( j \)’ are consecutive orders and task ‘\( i \)’ is allocated to unit ‘\( u \)’, then task ‘\( j \)’ is not processed in any other unit.

2. In each stage, each order has only one unique successor

\[
\sum_{j \neq i} X_{ijl} \leq 1 \quad \forall i \in I, l \in L, j \in I_l
\] (2)

Each order can be processed by only one unit in each stage.

3. Each order has a only one predecessor

\[
\sum_{j \in I_l \cup U_{il}} X_{ijl} + \sum_{u \in (U_l \cap U_{il})} S_{iu} = 1 \quad \forall i \in I, l \in L
\] (3)

Order ‘\( i \)’ is the first order to be processed at a unit or is preceded by a unique order ‘\( j \)’.

4. Relation between variables \( W_{iu} \) and \( S_{iu} \)

\[
W_{iu} \geq S_{iu} \quad \forall i \in I, u \in U
\] (4)

If order ‘\( i \)’ is allocated as a first order to unit ‘\( u \)’, the equality sign holds, this formulation keep only one first order to be handled in a unit.

5. Each unit is assigned to only one starting order

\[
\sum_{i \in I} S_{iu} \leq 1 \quad \forall u \in U
\] (5)

This formulation keep the allocation of only one first-order ‘\( i \)’ to unit ‘\( u \)’.

6. In each stage every order handled by a unique unit

\[
\sum_{u \in U_{il}} W_{iu} = 1 \quad \forall i \in I, l \in L
\] (6)

7. Constraint for the starting time and completion time of an order processed in each stages

\[
T_{si} - (T_{sj} + \sum_{u \in (U_l \cap U_{il})} W_{iu} \times T_{iu}) \geq 0
\]

\( \forall l, m \in L_i, m > l, i \in I \) (7)

This formulation is to ensure the sequential processing dependency of the every orders in the every stages.

8. Relation between the starting time of two consecutive orders processed in the same stage

\[
(1 - X_{ijl}) \times M + T_{si} \geq T_{sj} + \sum_{u \in (U_l \cap U_{il})} W_{iu} \times T_{iu}
\]

\( \forall i, j \in I, i \neq j, l \in L \) (8)

The changeover time between two consecutive stages is independent of the handling unit. This formulation is to keep order ‘\( j \)’ is the successor of order ‘\( i \)’ if that happened.
(9) Tardiness

\[ Td_i \geq (T_s_{il} + \sum_{w \in (U, r, d_{il})} W_{iw} \times T_{iw}) - D_i \quad \forall i \in I, l \in L \] (9)

The completion time of an order \( i \) must be before the due date of the order \( i \).

(10) Objective function

Due to the order-driven production, especially, export order must be delivery on time, so, the objective function has to ensure that the completion time of the orders is as closer as due date of the orders, if not, it will impose a high penalty on the tardy orders. The objective function is as following:

\[ \sum_{i \in I} \sum_{l \in L} H_{il} \times (T_s_{il} + \sum_{w \in (U, r, d_{il})} W_{iw} \times T_{iw}) - \sum_{i \in I} Td_i \times N \] (10)

Here \( N \) is a large number to keep job tardiness is always minimized. Parameter \( H_{il} \) is taken from Pinto and Grossmann [5]:

\[ H_{il} = 0.2 \times \left[ \text{Max}(d) \right] / d_i \times \text{Ord}(l) \] (11)

Where Max(d) is the latest due date of all scheduled orders and Ord(l) is the order of processing stage \( l \).

IV. CASE STUDY

One example with data from a printing and dyeing plant is presented here to demonstrate the capabilities of the scheduling model.

Five orders should be scheduled in this case, these orders have process flow in figure I. Gray cloth experiences the process flow in figure I to become the final products. In figure I, rectangle indicates the stage and use number in cycle represents this stage for programming, for example number 1 represents the stage singeing, whereas not each product passes the same process flow. Stages a unit may be used by these fives orders is listed in table I.

![Process Flow of Orders](image)

**TABLE I. STAGES AND UNITS FOR ORDERS**

| Stage No. | Stage Name | Unit No. |
|-----------|------------|---------|
| 1         | Singeing   | 1       |
| 2         | Desizing, Scouring, Bleaching | 2       |
| 3         | Stack cooling | 3       |
| 4         | Unwinding  | 4       |
| 5         | Mercerizing | 5       |
| 6         | Preshaping | 6       |
| 7         | Dyeing     | 7       |
| 8         | Softening, Stretching | 8       |
| 9         | Preshrunk  | 9       |

Basic data on five orders is showed in table II, each order with quantity and due date is processed in about nine consecutive stages before being turned into the final product, but not all orders pass the same stages. And processing speed of each order in a unit are demonstrated in table III, for example, order I1 processing speed in unit U1 is 6000 meters/per hour.

**TABLE II. DATA RELATED TO ORDERS**

| Order | Products name | Color     | Quantity (meter) | Due date | Stages |
|-------|---------------|-----------|------------------|----------|--------|
| I1    | All cotton canvas | Black     | 4000             | 20       | 1, 3, 4, 5, 6, 7, 8, 9 |
| I2    | Cotton poplinette | Stone color | 7000             | 23       | 1, 2, 5, 7, 8, 9 |
| I3    | Cotton satin   | Yellowish | 5000             | 25       | 1, 2, 5, 7, 8, 9 |
| I4    | Cotton twill   | Camel yellow | 6000             | 30       | 1, 2, 5, 7, 8, 9 |
| I5    | Cotton double elastic twill | Orange | 8000             | 36       | 1, 3, 4, 5, 6, 7, 8, 9 |
TABLE III. PROCESSING SPEED (m/h) OF ORDERS IN UNITS

| Order | I1 | I2 | I3 | I4 | I5 |
|-------|----|----|----|----|----|
| U1    | 6000 | 6000 | 5000 | 6000 | 6000 |
| U2    | 0   | 3000 | 2700 | 2700 | 0   |
| U3    | 2700 | 0   | 0   | 0   | 2700 |
| U4    | 3000 | 0   | 0   | 0   | 3000 |
| U5    | 4500 | 4200 | 4200 | 3300 | 3600 |
| U6    | 3000 | 0   | 0   | 0   | 2700 |
| U7    | 2400 | 2400 | 2400 | 2400 | 2400 |
| U8    | 3000 | 3600 | 3600 | 2400 | 1800 |
| U9    | 3600 | 3600 | 3600 | 3600 | 3600 |

According to the dyeing machine cleaning rule, the dyeing sequence relation and cleaning time is expressed in Table IV, for example, if order I1 is dyed firstly, then order I2, dyeing machine must be cleaned.

TABLE IV. CLEANING TIME OF A DYEING MACHINE WITH SEQUENCE-DEPENDENT ORDERS

| Order | I1 | I2 | I3 | I4 | I5 |
|-------|----|----|----|----|----|
| I1    | 0  | 0  | 0  | 0  | 0  |
| I2    | 0.8| 0  | 0.5| 0.7| 0.6|
| I3    | 0  | 0  | 0  | 0.4| 0.5|
| I4    | 0.3| 0  | 0  | 0  | 0  |
| I5    | 0.2| 0  | 0  | 0.2| 0  |

After using ILOG CPLEX to program [10], the scheduling results of these five orders are shown in figure V with 627 constraints, 366 variables and consuming 0.89 second to compute.

The solutions of decision variable $S_{iu}$ are given in figure II, this table indicates which one of the five orders is the first order to be processed in one of nine units, for example, order I2 is the first order to be processed in unit U2.

The solutions of decision variable $W_{iu}$ are given in figure III. This table shows that one of the five orders will be processed by a set of units, for example, processed by U1, U3, U4, U5, U6, U7, U8 and U9, order I1 become the final product.

And solution in figure IV demonstrates that starting processing time of nine stages are given to each of the five order, for example, the starting time of nine stages for order I1 are provide on line 1.

Based on solutions mentioned above, a Gantt chart is presented in figure V.

In this figure, order I1 experiences 9 stages and processed by units U1, U2, U3, U4, U5, U6, U7, U8 and U9. Time between dotted line and full line indicates the setup time between consecutive stages. Setup time is fixed according to printing and dyeing production feature. From figure II, we can also know that dyeing machine U7 firstly dyes order I1 deep color then immediately order I2 light color, therefore the dyeing machine must be cleaned with time of 0.8 hour. Moreover, the final products colors of order I2, I3, I4, I5 become lighter and lighter, the scheduling result shows that dyeing sequence in the dyeing machine U7 is I2, I3, I4, I5 and the cleaning is omitted.
V. CONCLUSION

In this paper, a mathematic model with continuous-time, sequence-dependent, setup, changover, multistage and multi-product features is given to printing and dyeing plants scheduling. The model is based on the production process features of the printing and dyeing enterprises. The scheduling result demonstrates the effectiveness and efficiency that much more than the scheduling man-made.

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