Manifestation of Confinement in the Gluon propagator

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The gluon propagator in Landau gauge is calculated on the lattice for SU(2) gauge theory. In particular the manifestation of confinement in the gluon propagator is studied. Removing the confining center vortices from the Yang-Mills ensemble leads to a drastic reduction of the gluonic form factor in the intermediate momentum regime.

1. INTRODUCTION

Understanding the physics of hadrons in particular the mechanism of confinement and spontaneous breaking of chiral symmetry requires a non-perturbative treatment of QCD, which is the theory of strong interaction. To date the only rigorous non-perturbative approach to QCD are the Monte Carlo lattice calculations. Despite recent progress by improved lattice algorithm and the increase in computer time, lattice calculations including dynamical quarks are exceedingly cumbersome and finite baryon densities are hardly accessible in realistic SU(3) lattice simulations.

Dynamical fermions and finite baryon densities can be relatively easily treated in the Schwinger-Dyson-equation approach to continuum QCD [3], which is also non-perturbative, but less rigorous in the sense that truncations of the tower of coupled equations are necessary in practical calculations, which however are not well under control. Despite this shortcoming the Schwinger-Dyson-approach has been successfully applied to the description of hadron phenomenology [3]. The central quantity of interest in this approach is the gluon propagator. It is therefore of primary interest to obtain rigorous results on the gluon propagator, which to date can be obtained only by lattice Monte Carlo calculations. Of particular interest is how confinement manifests itself in the gluon propagator. This is the main subject of the present study.

Since the gluon propagator is gauge dependent, its properties can be specified only after fixing the gauge. In the Schwinger-Dyson-approach it is common to use the Landau gauge

$$\partial_\mu A_\mu(x) = 0,$$

and the gluon propagator has been studied in this gauge on the lattice in [3]. Here we studied the signature of quark confinement which might be encoded in the gluon propagator in this gauge.

Recent lattice calculations give strong support for the vortex picture of confinement according to which the “confiner” are center vortices [4]. We will therefore investigate the effect of center vortices on the gluon propagator.

2. LATTICE DEFINITION OF THE GLUON PROPAGATOR

Lattice gauge theory is defined in terms of link variables $U_\mu(x)$ from which the gauge field $A_\mu(x)$ of the continuum theory is usually defined by $U_\mu(x) = \exp\{-a A_\mu^a(x)T^a\}$ where $a$ is the lattice spacing and $T^a$ denotes the generators of the gauge group in the fundamental representation. Note that the so defined gauge field $A_\mu^a(x)$ becomes singular when $U_\mu(x) \to -1$. Furthermore under gauge transformations it does not properly transform as a connection of the continuum theory. In particular, it is not invariant under center
gauge transformations. Therefore we prefer to perform a coset decomposition of the links

\[ U_\mu(x) = Z_\mu(x)\tilde{U}_\mu(x), \quad Z_\mu(x) = \text{sign} \text{tr} U_\mu(x), \]

where \( Z_\mu(x) \in \{1, -1\} \) denotes the center element closest to \( U_\mu(x) \), and define the gauge field \( A_\mu(x) \) from the coset part \( \tilde{U}_\mu(x) \) or equivalently from the links in the adjoint representation

\[ \tilde{U}_\mu(x)^{ab} = -2\text{tr} \{ U_\mu(x)T^aU_\mu(x)^\dagger T^b \} = \exp\{-aA_\mu(x)T^c\}, \]

where \((\tilde{T}_c)^{ab} = f^{acb}\) are the structure constants.

With the continuum gauge field \( A^a_\mu(x) \) at hand the gluon propagator is given by

\[ D^{ab}_{\mu\nu}(x - y) = \langle A^a_\mu(x)A^b_\nu(y) \rangle \]

where \( < ... > \) denotes the Monte Carlo average over properly thermalized lattice gauge configurations. The Fourier transform of \( D^{ab}_{\mu\nu}(x - y) \) is expressed in terms of the lattice momentum \( p_\mu = \frac{2}{a(\beta)} \sin \frac{n_\mu \pi}{N_\mu} \) which reduces to the Matsubara frequency \( \bar{p}_\mu = \frac{2\pi}{N_\mu a(\beta)}n_\mu \) for \( n_\mu \ll N_\mu \). In this variable the free lattice gluon propagator has the form \( D_0(p) = \frac{1}{p^2} \). In the following we will be interested in the non-perturbative information contained in the gluon propagator \( D(p) \), which is measured by the form factor \( F(p) = p^2D(p) \). High precision Monte Carlo measurements of the form factor are obtained by extending the method of \( [3] \) by choosing a purely temporal momentum transfer \( \bar{p} = (0, 0, 0, \bar{p}_4) \) and expressing the gluonic form factor as

\[ F(\bar{p}_4^2) = \sum_{\alpha, \mu}\frac{1}{N^2} \left\langle \left[ \sum_x \Delta_t A^a_\mu(x) \cos \bar{p} x \right]^2 \right. \]

\[ + \left. \left[ \sum_y \Delta_t A^a_\mu(y) \sin \bar{p} y \right]^2 \right\rangle \]

where \( \Delta_t A(x) = A(x + a\vec{e}_4) - A(x) \).

The gluon propagator is gauge dependent and is properly defined only after fixing the gauge. In fact in the unfixxed gauge lattice theory the gluon propagator vanishes by center symmetry. In order to be able to compare with the Schwinger-Dyson-approach to continuum Yang-Mills-theory we use the (lattice) Landau gauge \( \sum_x \text{tr} U_\mu(x) \rightarrow \max \) which reduces in the continuum limit \( a \rightarrow 0 \) to the familiar Landau gauge, eq. \( [1] \). The lattice Landau gauge exploits the gauge freedom to bring each link as close as possible to the unit matrix. Consequently in this gauge most of the center elements in the coset decomposition, eq. \( [2] \), are trivial (\( Z_\mu(x) = 1 \)) and most of the physics is transferred to the adjoint links. Fig. 1 shows the gluon form factor in the Landau gauge evaluated from eq. \( [3] \). The gluon form factor vanishes at zero momentum transfer \( q = 0 \), has a rather pronounced peak at intermediate momenta and asymptotically approaches the perturbative tail. Also shown in Figure 1 is the gluon propagator obtained by solving the coupled gluon-ghost Schwinger-Dyson-equations of continuum Yang-Mills-theory in the Landau gauge, eq. \( [4] \), \( [5] \). The Schwinger-Dyson-approach qualitatively reproduces the lattice gluon propagator, but misses a substantial part of the peak at intermediate momenta. This missing strength of the Schwinger-Dyson gluon propagator could originate from: truncation of the Schwinger-Dyson equations or approximations in the numerical solutions of the
coupled Schwinger-Dyson equations (e.g. angle approximation).

3. MANIFESTATION OF CONFINEMENT IN THE GLUON PROPAGATOR

Let us now investigate how confinement manifests itself in the gluon propagator. Lattice calculations have given strong support for the center vortex picture of confinement. The confining center vortices can be easily detected (and removed) by the method of center projection on top of the maximum center gauge fixing. The maximum center gauge is equivalent to the adjoint Landau gauge, which brings the adjoint links as close as possible to the unit matrix

$$\sum_{x,y} \text{tr} A^\mu U_{\mu}(x) \to \text{max}.$$  

Since in this gauge the center elements $Z_\mu = \pm 1$ are treated on equal footing, a substantial portion of the center elements are non trivial ($Z_\mu = -1$). It is observed that the confinement physics is concentrated on the center degrees of freedom, or equivalently, on the center vortices. Indeed performing in this gauge a so-called center projection replacing each (gauged) link $U_{\mu}(x)$ by its “nearest” center element $Z_\mu(x) = \text{sign} \text{tr} U_{\mu}(x)$ results in a center vortex ensemble, which reproduces the full string tension. On the other hand, removing the center vortices by replacing

$$U_{\mu}(x) \to Z_\mu(x) U_{\mu}(x)$$

eliminates the confining properties, resulting in a vanishing string tension. In the continuum limit, the maximal center gauge reduces to the background gauge $[\partial_\mu + V_\mu, A_\mu] = 0$ where $V_\mu(x)$ is an “optimally chosen” center vortex field. Hence after the center vortices have been removed by the replacement eq. the maximum center gauge reduces in the continuum theory to the Landau gauge eq. The gluon propagator in the Landau gauge obtained from the non-confining vortex-free ensemble is also shown in Fig. 1. Removal of the confining center vortices has basically removed the peak at intermediate momenta. In view of this result we can conclude, that parts of the confining properties escape the Schwinger-Dyson-approach, at least within the approximations used in present state of the art calculations.

It would be interesting to solve the Schwinger-Dyson equation for the quark propagator using the gluon propagator obtained above in the lattice theory as input and calculate the quark condensate. Using the gluon propagator obtained in the non-confining theory, where the center vortices have been removed we expect that the quark condensate $<\bar{q}q>$ vanishes as lattice calculations indicate.

Further details of the work outlined in this talk can be found in [8].

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3Center vortices are the only non-trivial field configurations of a $Z(2)$ theory.