Multiparty controlled quantum secure direct communication using Greenberger-Horne-Zeilinger state

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Base on the idea of dense coding of three-photon entangled state and qubit transmission in blocks, we present a multiparty controlled quantum secret direct communication scheme using Greenberger-Horne-Zeilinger state. In the present scheme, the sender transmits her three bits of secret message to the receiver directly and the secret message can only be recovered by the receiver under the permission of all the controllers. All three-photon entangled states are used to transmit the secret messages except those chosen for eavesdropping check and the present scheme has a high source capacity because Greenberger-Horne-Zeilinger state forms a large Hilbert space.

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Quantum communication has been one of the most promising applications of quantum information science. Quantum key distribution (QKD) which provides unconditionally secure key exchange has progressed quickly since the first QKD protocol was proposed by Bennett and Brassard in 1984 [1]. A good many of other quantum communication schemes have also been proposed and pursued, such as Quantum secret sharing (QSS) [2, 3, 4, 5, 6, 7], quantum secure direct communication (QSDC) [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. QSS is the generalization of classical secret sharing to quantum scenario and can share both classical and quantum messages among sharers. Many researches have been carried out in both theoretical and experimental aspects after the pioneering QSS scheme proposed by Hillery, Buzêk and Berthiaume in 1999 [2]. Different from QKD, QSDC’s object is to transmit the secret messages directly without first establishing a key to encrypt them. QSDC can be used in some special environments which has been shown by Boström and Deng et al. [9, 10]. The works on QSDC attracted a great deal of attentions and can be divided into two kinds, one utilizes single photon [11, 12], the other utilizes entangled state [8, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Deng et al. proposed a QSDC scheme using batches of single photons which serves as one-time pad [11]. Cai et al. presented a deterministic secure direct communication scheme using single qubit in a mixed state [12]. The QSDC scheme using entanglement state is certainly the mainstream. Boström and Felbinger proposed a “Ping-Pong” QSDC protocol which is quasi-secure for secure direct communication if perfect quantum channel is used [18]. Cai et al. pointed out that the “Ping-Pong” Protocol is vulnerable to denial of service attack or joint horse attack with invisible photon [10, 20]. They also presented an improved protocol which doubled the capacity of the “Ping-Pong” protocol [13]. Deng et al. put forward a two-step QSDC protocol using Einstein-Podolsky-Rosen (EPR) pairs [10]. We presented a QSDC scheme using EPR pairs and teleportation [14]. Chuan Wang et al. proposed a QSDC scheme with quantum superdense coding [17] and a multi-step QSDC scheme using GHZ state [10]. Ting Gao et al. and Zhan-jun Zhang et al. each presented a QSDC scheme using entanglement swapping [17, 18].

In this paper, we present a multiparty controlled QSDC (MCQSDC) scheme using GHZ state and its transformation. In the present scheme, the sender’s secret message is transmitted directly to the receiver and can only be reconstructed by the receiver with the help of all the controllers. Different to QSS, the sender transmits his or her secret message to the receiver directly and the information of the receiver is asymmetric to those of the controllers. Our scheme employs dense coding of three-photon entangled state proposed by H. J. Lee et al. [21] and qubit transmission in blocks [22]. Eight possible states of GHZ state which form a complete orthonormal basis, carry three bits of information and all GHZ states are used to transmit the secret messages except those chosen for eavesdropping check. We also discuss the security of the scheme, which is unconditionally secure.

We first present a single party controlled QSDC (CQSDC) and then generalize it to a MCQSDC scheme. In the scheme, we utilize dense coding of three-photon GHZ state. There are eight independent three-photon GHZ states which form a complete orthonormal basis, namely

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) = \frac{1}{\sqrt{2}}(|\psi^+\rangle|+\rangle + |\phi^-\rangle|\rangle),$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) = \frac{1}{\sqrt{2}}(|\psi^-\rangle|+\rangle + |\phi^+\rangle|\rangle),$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |011\rangle) = \frac{1}{\sqrt{2}}(|\psi^+\rangle|+\rangle - |\psi^-\rangle|\rangle),$$

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |011\rangle) = \frac{1}{\sqrt{2}}(|\psi^+\rangle|\rangle - |\psi^-\rangle|+\rangle),$$

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\[ |\Psi_1 \rangle = \frac{1}{\sqrt{2}} (|010 \rangle + |101 \rangle), \]
\[ |\Psi_2 \rangle = \frac{1}{\sqrt{2}} (|010 \rangle - |101 \rangle), \]
\[ |\Psi_3 \rangle = \frac{1}{\sqrt{2}} (|+1 \rangle + |y1 \rangle), \]
\[ |\Psi_4 \rangle = \frac{1}{\sqrt{2}} (|+1 \rangle - |y1 \rangle), \]
\[ |\Psi_5 \rangle = \frac{1}{\sqrt{2}} (|+y1 \rangle + |y1 \rangle), \]
\[ |\Psi_6 \rangle = \frac{1}{\sqrt{2}} (|+y1 \rangle - |y1 \rangle), \]
\[ |\Psi_7 \rangle = \frac{1}{\sqrt{2}} (|+y1 \rangle + |y1 \rangle), \]
\[ |\Psi_8 \rangle = \frac{1}{\sqrt{2}} (|+y1 \rangle - |y1 \rangle), \]

where
\[ |+ \rangle = \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle), |\rangle = \frac{1}{\sqrt{2}} (|0 \rangle - |1 \rangle), \]
\[ |\phi^+ \rangle = \frac{1}{\sqrt{2}} (|00 \rangle + |11 \rangle), |\phi^- \rangle = \frac{1}{\sqrt{2}} (|00 \rangle - |11 \rangle), \]
\[ |\psi^+ \rangle = \frac{1}{\sqrt{2}} (|01 \rangle + |10 \rangle), |\psi^- \rangle = \frac{1}{\sqrt{2}} (|01 \rangle - |10 \rangle). \]

Table I: The transformation of GHZ states by performing operations on two photons

| \text{unitary operations on the first and the second photon} | \text{on the first and the second photon} |
|----------------------------------------------------------|------------------------------------------|
| \textbf{| } | \textbf{| } |
| $|\Psi_1 \rangle$ | $\sigma_z \otimes \sigma_z$ or $I \otimes I$ |
| $|\Psi_2 \rangle$ | $I \otimes \sigma_z$ or $\sigma_z \otimes I$ |
| $|\Psi_3 \rangle$ | $i\sigma_y \otimes \sigma_z$ or $\sigma_z \otimes I$ |
| $|\Psi_4 \rangle$ | $\sigma_z \otimes \sigma_z$ or $i\sigma_y \otimes I$ |
| $|\Psi_5 \rangle$ | $I \otimes \sigma_z$ or $\sigma_z \otimes i\sigma_y$ |
| $|\Psi_6 \rangle$ | $\sigma_z \otimes \sigma_z$ or $I \otimes i\sigma_y$ |
| $|\Psi_7 \rangle$ | $\sigma_z \otimes \sigma_z$ or $i\sigma_y \otimes i\sigma_y$ |
| $|\Psi_8 \rangle$ | $i\sigma_y \otimes \sigma_z$ or $\sigma_z \otimes i\sigma_y$ |

In order to distinguish the sender's operations correctly in our scheme, we select eight two-photon operations, $U_1, U_2, \ldots, U_8$ from sixteen operations, where
\[
U_1 = \sigma_z \otimes \sigma_z, \quad U_2 = I \otimes \sigma_z, \\
U_3 = i\sigma_y \otimes \sigma_z, \quad U_4 = \sigma_z \otimes \sigma_z, \\
U_5 = I \otimes \sigma_z, \quad U_6 = \sigma_z \otimes \sigma_z, \\
U_7 = \sigma_z \otimes \sigma_z, \quad U_8 = i\sigma_y \otimes \sigma_z.
\]

Thus we obtain $|\Psi_k \rangle$ ($k = 1, \ldots, 8$) if $U_k$ is performed on the $|\Psi_1 \rangle$. We now describe the CQSDC scheme in detail. Suppose the sender Alice want to transmit her secret message directly to the receiver Charlie under the control of the controller Bob.

(S1) Bob prepares an ordered $N$ three-photon states. Each of the three-photon states is in the state
\[ |\Psi \rangle = \frac{1}{\sqrt{2}} (|000 \rangle + |111 \rangle)_{ABC}. \]

We denotes the ordered $N$ three-photon qubits with \{$|P_1(A)\rangle, |P_2(B)\rangle, |P_3(C)\rangle$, \ldots, $|P_N(A)\rangle, |P_N(B)\rangle, |P_N(C)\rangle$\}, where the subscripts indicate the order of each three-photon in the sequence, and $A, B, C$ represents the three photons of each state, respectively. Bob takes one particle from each state to form an ordered partner photon sequence $|P_1(A)\rangle, |P_2(A)\rangle, \ldots, |P_N(A)\rangle$, called $A$ sequence. The remaining partner photons compose $B$ sequence, $|P_1(B)\rangle, |P_2(B)\rangle, \ldots, |P_N(B)\rangle$ and $C$ sequence, $|P_1(C)\rangle, |P_2(C)\rangle, \ldots, |P_N(C)\rangle$. Bob selects randomly one of four unitary operations, $I, \sigma_z, \sigma_x, i\sigma_y$ and performs it on each of the photons in the $B$ sequence. He then sends the $A$ sequence and the $B$ sequence to Alice and keeps the $C$ sequence.

(S2) After receiving the $A, B$ sequence, Alice selects a sufficiently large subset from $A, B$ sequence for eavesdropping check.

The procedure of the eavesdropping check is as follows: (a) Alice announces publicly the positions of the selected photons. (b) Bob publishes his operations which performed on the sampling photons in the $B$ sequence. (c) Alice then chooses randomly a measuring basis $Z$-basis ($|0 \rangle, |1 \rangle$) or Bell-basis ($|\phi^\pm \rangle$, $|\psi^\pm \rangle$) to measure the selected photons in the $A, B$ sequence and announces publicly the measuring basis for each of the sampling photons. (d) If Alice performs a $Z$-basis measurement, Bob also performs $Z$-basis measurement on the corresponding photons in the $C$ sequence: If Alice performs a Bell-basis measurement, Bob performs a $X$-basis ($|+ \rangle$, $|\rangle$) measurement. After measurements, Bob publishes his measurement results. Because of Bob’s unitary operations, $|\Psi \rangle$ is changed to one of four GHZ states $|\Psi_1 \rangle$, $|\Psi_2 \rangle$, $|\Psi_5 \rangle$, $|\Psi_6 \rangle$. According to Eq(A) Alice and Bob can check the existence of eavesdropper by comparing their measurement results. If the error rate exceeds the threshold, they have to abort the communication. Otherwise they continue to execute the next step.

(S3) Bob chooses randomly one of four unitary operations, $I, \sigma_z, \sigma_x, i\sigma_y$ and performs it on each of the photons in the $C$ sequence. He then sends the $C$ sequence photons to Charlie.

(S4) Alice and Charlie analyze the error rate of the transmission of $C$ sequence. The method of eavesdropping check is similar to that of the step 2, but Bob should announce his operations on the selected photons in the $C$ sequence after Charlie publish his measurement results. If the error rate is below the threshold, they proceed to execute the next step. Otherwise they abort the communication.

(S5) Alice first selects randomly two sufficiently large subsets from $A, B$ sequence for eavesdropping checks.
She then performs randomly one of eight operations, \( U_1, U_2, \ldots, U_8 \) on the sample photons of \( A, B \) sequence. The operation \( U_k \) \((k = 1, \ldots, 8)\) executed on the \( A, B \) sequence makes the state \( |\Psi\rangle \) become one of the eight independent GHZ states, \( |\Psi_1\rangle, |\Psi_2\rangle, \ldots, |\Psi_8\rangle \). Alice, Bob and Charlie agree that the eight GHZ states \( |\Psi_1\rangle, |\Psi_2\rangle, \ldots, |\Psi_8\rangle \), represent a 3-bit of secret message, that is \(|\Psi_1\rangle \rightarrow 000, |\Psi_2\rangle \rightarrow 001, |\Psi_3\rangle \rightarrow 010, |\Psi_4\rangle \rightarrow 011, |\Psi_5\rangle \rightarrow 100, |\Psi_6\rangle \rightarrow 101, |\Psi_7\rangle \rightarrow 110, |\Psi_8\rangle \rightarrow 111 \). Alice encodes her secret messages on the remaining photons of \( A, B \) sequence by performing the unitary operations \( U_k \) \( (k = 1, \ldots, 8) \). She then sends the \( B \) sequence to Charlie. After hearing from Charlie, Alice measures the sampling photons of one subset of \( A \) sequence in \( Z \)-basis or \( X \)-basis randomly. She then publishes the positions of the sampling photons and the measuring basis to each of the sampling photons. If Alice performs a \( Z \)-basis (\( X \)-basis) measurement, Charlie performs a \( Z \)-basis (\( B \)-basis) measurement on the corresponding sampling photon in the \( B, C \) sequence. After measurements, he publishes his measurement results. Alice then lets Bob announce his operations on the sampling photons of \( B, C \) sequence. Thus Alice and Charlie can check eavesdropping by comparing their measurement results. If they make certain that there is no eavesdropper, Alice sends the \( A \) sequence to Charlie. After hearing from Charlie, Alice publishes the positions of the sampling photons of the other subset and lets Charlie make GHZ basis measurements on the sampling photons of \( A, B, C \) sequence. After Charlie announces his measurements results, Alice lets Bob publish his operations on the sampling photons in the \( B, C \) sequence. They then estimate the error rate of the transmission of \( A \) sequence. If there is no eavesdropping, they continue to the next step, otherwise they abandon the communication.

(S6) Thus Charlie owns the \( A, B, C \) sequence. Without Bob’s permission, Charlie can not obtain Alice’s secret messages. Only after Bob published his operations on the photons of \( B, C \) sequence, could Charlie acquire Alice’s secret messages by performing a GHZ basis measurement on each of the GHZ states.

So far we have presented the CQSDC scheme. The security of the present scheme is similar to those in Refs. \[10, 22\]. The participators check the existence of eavesdropper during each transmission of photon sequence. The security for the transmission of photons can be reduced to the security of the BBM92 protocol \[23\]. The process for the transmission of \( A, B \) sequence from Bob to Alice is similar to that in Ref. \[2\]. Alice and Bob measure the sampling photons in a randomly selected measuring basis, which ensures the security of the transmission of photon sequence. Bob first performs a random unitary operation on the \( B \) sequence and then sends it to Alice, which prevent Charlie from acquiring partial secret message without the permission of Bob. Without Bob’s operations, Charlie performs GHZ basis measurement on photons in the \( A, B, C \) sequence and she can obtain partial information of Alice at the step 6 of the scheme. The security for the transmission of \( C \) sequence in our scheme is similar to that of the \( C \) sequence in Deng et al.’s two-step QSDC protocol \[10\]. The transmission of \( A, B \) sequence from Alice to Charlie is the same as the transmission of \( M \) sequence in the two step protocol. Because of the qubit correlation of each GHZ state, eavesdropper’s eavesdropping will be detected during the eavesdropping check. Charlie cannot obtain Alice’s secret without Bob’s permission because Bob performs random unitary operations on the photons in the \( B, C \) sequence. Thus the present scheme is unconditional security.

We then generalize the CQSDC scheme to a MCQSDC one. Suppose Alice is the sender; Bob, Charlie, Dick, \( \ldots \), York are the controllers; Zach is the receiver. The first two step of the MCQSDC scheme is the same as those of CQSDC scheme. We describe the MCQSDC scheme from the step 3.

(S3’)

Bob chooses randomly the Hadamard operation,

\[
H = \frac{1}{\sqrt{2}} \left( |0\rangle \langle 0| - |1\rangle \langle 1| \right)
\]

or the identity operation. He also chooses randomly one of four unitary operations, \( I, \sigma_z, \sigma_x, i\sigma_y \). Bob then performs these two operations on each of the photons in the \( C \) sequence. He then sends the \( C \) sequence photons to Charlie.

(S4’)

After receiving the \( C \) sequence, Charlie performs the similar operations as Bob and sends it to the next controller, Dick. Dick and the remaining controllers repeat the similar operations as Charlie until the receiver, Zach receives the \( C \) sequence. The \( H \) operation is very important for the security of the scheme. Suppose the participators only performs \( I, \sigma_z, \sigma_x \) or \( i\sigma_y \) operations on the \( C \) sequence randomly and an eavesdropper, Eve intercepts the \( C \) sequence. She then prepares a fake \( C \) sequence which belongs to one part of EPR pairs and sends it to the next participator, say Charlie. Eve can also intercepts the sequence on which Charlie performed his operations. She then performs Bell basis measurement on the intercepted sequence and her other part of EPR pairs. Thus Eve can acquire the operation information of the controller Charlie.

(S5’)

After hearing from Zach, Alice selects randomly a sufficiently large subset from \( A, B \) sequence to check eavesdropping. She then publishes the position of sampling photons. She lets Zach measure the sampling photons in the \( C \) sequence by using either \( Z \)-basis or \( X \)-basis and publishes his measurement results. For each of the sampling photon, Alice randomly selects a controller to announce his operation information firstly and then the others publish their operation information on the sampling photons in turn. Suppose the number of \( H \) operations performed on each sampling photon in the \( C \) sequence by the controllers is odd. If Alice lets Zach measure the sampling photon in the \( Z \)-basis (\( X \)-basis), she performs Bell basis (\( Z \)-basis) measurement on her corresponding two photons. Suppose the number of \( H \) operations is even. If Zach performs \( Z \)-basis (\( X \)-basis) measurement, Alice measures the corresponding photons.
in Z-basis (Bell basis). After doing these, Alice can determine the error rate of the transmission of C sequence. If she confirms there is no eavesdropping, the process is continued. Otherwise, the process is stopped.

(S6') Alice chooses the subsets of the sampling photons, encodes her secret messages on the AB sequence and transmits the A, B sequence to Zach step by step in the same way as the step 5 of the CQSDC scheme. During the eavesdropping checks for the transmission of A sequence, Alice lets the controllers Bob, Charlie, · · ·, York announce their operations on the sampling photons. If the number of H operations is odd, Zach first performs H operation on each of the selected photon in the C sequence and then measures each of the corresponding three-photon in GHZ basis. If the number of H operations is even, Zach performs GHZ basis measurement directly. Alice then lets Zach publish his measurement results. Thus Alice and Charlie can estimate the error rate of the transmission of A sequence. Actually, Eve can only interrupt the transmission of A, B sequence and cannot steal any information even if she attacks the photons.

(S7') If the controllers permit Zach to reconstruct Alice's secret messages, they tell Zach their operation information. If the number of H operations is odd, Zach first performs H operation on the corresponding photon and then measure the three-photon in GHZ basis. If the number of H operations is even, Zach performs GHZ basis measurement directly. Thus Zach can obtain Alice's secret messages under the permission of the controller Bob, Charlie, · · ·, York.

The H operations performed by controllers can prevent Eve or a dishonest controller from obtaining the control information, which we have described it in the step 4'.

During the eavesdropping check for the transmission of C sequence, Alice first lets Zach publish his measurement results and then the controllers announce their operation information, which ensure each controller can really act as a controller. If the controllers first publish their operation information, Zach can obtain Alice’s secret as long as he acquires Bob’s permission. In the present scheme, only with the permissions of all the controllers could Zach acquire the secret messages. The security of MCQSDC scheme is the same as that of CQSDC.

So far we have presented a MCQSDC scheme based on dense coding of three-photon entangled state and qubit transmission in blocks. We first present a CQSDC scheme controlled by a single controller, and then generalize it to a MCQSDC scheme. The sender encodes three bits of secret message on two photons of GHZ state and the receiver can reconstruct the sender’s secret with the permissions of all the controllers. It seems like a QSS scheme, but the sender’s secret can send to the receiver directly and the information of the receiver is asymmetric to the controllers. We also analyzed that the security of this scheme is the same as that of the BBM92 protocol, which is unconditionally secure. In the present scheme, all of the GHZ states are used to transmit the secret except those used to check eavesdropping. The scheme has a high source capacity in that GHZ state forms a large Hilbert space.

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