Analysis of engineering mechanics problems solved by descriptive geometry methods

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Abstract. The article shows the possibility of using descriptive geometry methods for solving problems of theoretical mechanics. The solution of problems of mechanics associated with the analysis of plane and spatial systems of forces is considered. The graphic solution of problems was carried out using the image of force vectors using orthogonal projections. A comparison of the results of graphical constructions with an analytical solution is presented.

1. Introduction

Descriptive geometry is a universal instrument for solving flat and spatial graphic problems, as well as a wide range of problems from various fields of science and technology [1]. Its methods are used to solve problems of theoretical optics, problems from the field of mechanical engineering and metal processing technology, mining and underground work, geometric crystallography, physico-chemical analysis and metallography, and civil engineering.

2. Solution Methods

The methods for graphically solving spatial problems of mechanics can be divided into two groups: methods based on the image of a vector using orthogonal projections and methods based on various methods of representing a spatial vector on the same plane. All things being equal, preference may be given to methods based on orthogonal projections. This can be explained by the fact that almost all technical drawings are performed in orthogonal projections. When applying descriptive geometry methods based on orthogonal projections, the initial data are taken directly from the working drawing and the result of the construction is also transferred to it [2].

When solving problems of theoretical mechanics using the graphical method, projection transformation methods can be used (a method of replacing projection planes, a method of rotation around an axis perpendicular to the projection plane, a method of rotation around a level line and alignment), and in some cases it is convenient to use the rectangular triangle method, which serves to determining the natural value of the distance between two points and does not apply to methods of transforming projections [3].
3. Research Results and Discussion
The article considers the possibility of using descriptive geometry methods for solving spatial problems of theoretical mechanics, and analyzes the graphic and analytical solution of problems to justify obtaining reliable results.

3.1. Problem 1
Two tractors A and B, traveling along the banks of the straight channel at a constant speed, pull the barge using two ropes (Figure 1, a). The tension forces of the ropes are 800 and 960 N, respectively. Determine the resistance force of water $P$ experienced by the barge during its movement, and the true values of the angles $\alpha$ and $\beta$, which should be the ropes with the banks of the channel, if the barge moves parallel to the banks [4].

For graphical solution of the problem, we present the tensile forces of the ropes pulling the barge with straight lines $a$ and $b$, arrange the directions of the forces (Figure 1, b).

To determine the length of the segments (Figure 2) that define the force modules, use the right triangle method. Thus, the force modules will correspond to the segments: $|P_A| = CA$, $|P_B| = CB$. Choose the scale: 1kN = 50 mm. Thus, projections $C_2A_2$ and $C_1A_1$ are the frontal and horizontal projections of the tension force module of the rope pulling the barge by tractor A, projections $C_2B_2$ and $C_1B_1$ are the frontal and horizontal projections of the tension force module of the rope pulling the barge by tractor B.

Since the value of the water resistance force will be equal to the modulus of the resultant tension forces of the ropes pulling the barge, it is necessary to determine the true values of the segments $CA$ and $CB$, corresponding to the modules of the forces $P_A$ and $P_B$, respectively. Since the segments $CA$ and $CB$ form a plane, we transform this plane into a level plane by replacing the projection planes.

$$ f \in \alpha (CA \cap CB); $$

$$ x \frac{\pi_2}{\pi_1} \to x \frac{\pi_2}{\pi_4}; \pi_4 \perp f; $$

$$ \pi_5 \mid \alpha (CA \cap CB); $$
We construct parallelogram on the resulting projection of the plane \( \alpha \) on the plane \( \pi_5 \) and we determine the full value of the \( CD \) segment, which defines the modulus of the resultant force, which determines the modulus of the water resistance force. Consider the scale 1kN = 50 mm, determining the modulus of the water resistance force \( |P| = 1,53 \text{ kN} \).

The true values of the angles \( \alpha \) and \( \beta \) are also determined from the drawing, as the angles of the direction of forces relative to the water resistance force. We get, \( \alpha = 27^\circ \), \( \beta = 33^\circ \).

For the analytical solution of the problem, we use the condition of the balance of forces in space (Figure 3).

From the law of cosines we have:
\[
p^2 = p_A^2 + p_B^2 + 2p_A \cdot p_B \cdot \cos \alpha = 2,3296
\]
\[
P = 1,53 \text{ kN}
\]

Further, from the law of sines follows
\[
\sin \varphi = \frac{p_A}{p} \sin 60^\circ = 0,5434
\]
\[ \phi = 32.91^\circ; \beta = \alpha - \phi = 60^\circ - 32.91^\circ = 27.08^\circ \approx 27^\circ \]

As you can see, the result of analytical and graphical solutions to this problem is the same.

3.2. Problem 2

This problem shows the use of descriptive geometry techniques to simplify the solution.

The gasholder has the shape of a ball with a diameter of 10 m, its weight together with the gas is \( m = 200 \) t. The gasholder is mounted on three supports A, B and C, located in the same horizontal plane and on the surface of the ball. The supports form an equilateral triangle with a side of 10 m. Support A has a fixed ball joint, support B and C are movable ball rollers, whose smooth support planes are perpendicular to radiuses \( OB \) and \( OC \). Determine the reaction of supports if the force \( F = 370 \) kN acts perpendicular to the frontal plane parallel to the line \( BC \) in the direction from the \( B \) to \( A \) [5].

First, let’s consider a graphical approach to solving this problem. In order to construct an orthogonal drawing of the proposed structure, it is necessary to know the modules of forces acting on the structure.

Since the weight of the gasholder together with the gas is 200 t, we translate this value into a unit of force.

\[ G = m \cdot g = 20 \cdot 10^3 \cdot 9.8 = 1960 \text{ kN} \]

Force acting on the base of the gasholder \( F = 370 \) kN.

Support A has three support reactions directed along the coordinate axes – \( R_{xA}, R_{yA}, R_{zA} \). Supports B and C have one reaction each, directed perpendicular to the reference plane – \( R_{zB} \) and \( R_{zC} \).

Since the reaction of the support A on the y-axis \( R_{yA} \) is directed opposite to the force \( F \), therefore, in order for the system to be in balance, it is necessary that the sum of them is equal to zero. Therefore, \( R_{yA} = F = 370 \) kN.

The reaction component of the support A on the x-axis \( R_{xA} \) = 0, since other forces are not directed on the x-axis.

Force of gravity \( G \) must balance the reaction components of all three supports along the z-axis. Therefore, they will be equalled:

\[ R_{zA} = R_{zB} = R_{zC} = \frac{1}{3} G = \frac{1}{3} 1960 = 653 \text{ kN} \]

We construct an orthogonal drawing of the structure in three projections (Figure 4) at the scale: 100 kN = 2 mm.

At the selected scale, we apply the force vectors to the drawing.

The only unknown quantity is the support reaction \( R_A \), which is defined as the resultant of the \( R_{zA} \) and \( R_{yA} \) reactions. It is defined graphically. We measure the vector \( R \) and translate the value from mm to kN according to the accepted scale.

The length of the vector \( R_A = 15 \) mm, therefore, the modulus of the reaction vector of the support A is equal to \( R_A = 750 \) kN.

Let’s check the correctness of the problem solution analytically.

The modules of support reactions directed along the coordinate axes are determined from the equations of force projections on the coordinate axes:

\[ \sum F_x = R_{xA} = 0 \]
\[ \sum F_y = F - R_{yA} = 0 \]
\[ \sum F_z = R_{zA} + R_{zB} + R_{zC} - G = 0 \]

We also compose equations of moments of forces relative to coordinate axes (origin of coordinates is point A).
\[ \sum M_x = -R_{x_B} \cdot 8.7 - R_{x_C} \cdot 8.7 + G \cdot 5.8 = 0 \]
\[ \sum M_y = -R_{x_B} \cdot 5 + R_{x_C} \cdot 5 = 0 \]
\[ \sum M_z = 0 \]

Therefore, \( R_{z_A} = 0 \). The reaction component of the support A on the y-axis is equal in modulus to the force \( F \), therefore \( R_{z_A} = F = 370 \text{ kN} \), sum of modules of component reactions of supports A, B and C on the z-axis is equal to the modulus of the force \( G \), therefore

\[ G = R_{z_A} + R_{z_B} + R_{z_C} \]
\[ R_{z_A} = R_{z_B} = R_{z_C} = \frac{1}{3} G = \frac{1}{3} 1960 = 653 \text{ kN} \]
Defining the resultant force

\[ R_A = \sqrt{R_{yA}^2 + R_{zA}^2} = \sqrt{370^2 + 653^2} = 750 \text{ kN}. \]

As you can see, in this problem, the result of the graphical and analytical solutions is also the same.

4. Inference

Conducted researches have shown the possibility, and in some cases the feasibility, of using descriptive geometry methods to solve problems of theoretical mechanics.

References

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