Immediate causality network of stock markets

Li Zhou¹, Lu Qiu², Changgui Gu¹ and Huijie Yang¹(α)

¹ Business School, University of Shanghai for Science and Technology - Shanghai 200093, China
² School of Finance and Business, Shanghai Normal University - Shanghai, 200234, China

received 8 November 2017; accepted in final form 22 March 2018
published online 13 April 2018

PACS 87.23.Kg – Dynamics of evolution
PACS 05.45.-a – Nonlinear dynamics and chaos

Abstract – Extensive works show that a network of stocks within a single stock market stores rich information on evolutionary behaviors of the system, such as collapses and/or crises. But a financial event covers usually several markets or even the global financial system. This mismatch of scale leads to lack of concise information to coordinate the event. In this work by using the transfer entropy we reconstruct the influential network between ten typical stock markets distributed in the world. Interesting findings include, before a financial crisis the connection strength reaches a maximum, which can act as an early warning signal of financial crises. The markets in America are monodirectionally and strongly influenced by that in Europe and act as the center. Some strongly linked pairs have also close correlations. The findings are helpful in understanding the evolution and modelling the dynamical process of the global financial system. This method can be extended straightly to find early warning signals for physiological and ecological systems, etc.

Copyright © EPLA, 2018

Introduction. – Globalization networks the financial systems distributed all over the world into a complex system. Stock markets are barometers of their corresponding local financial systems, respectively, relationship network between which displays subsequently the state of the global finance [1–3]. In this paper, we are interested in the non-trivial influential patterns among stock markets in the world and their evolutionary behaviors, from which we try to find early warning signals of financial collapses and/or crises. The preliminary step is to reconstruct concisely the financial network with a proper method.

Complex network theory has been adopted in some works to monitor evolutionary behaviors of financial systems [4–19]. For instance, Song et al. [6] convert cross-correlations between a total of 57 industry portfolios in the American stock market into a series of planar maximally filtered graphs (a network embedded in a high-dimensional surface) to represent the states of the corresponding successive durations, and find that mutual entropy between successive states reaches a peak at a financial crisis. Qiu et al. [19,20] construct a series of stock networks from successive segments of return series covering two months each to describe the evolutionary process of the Dow Jones stock market in the period from 1994 to 2013. Linking the stock networks between which the distances are less than a threshold, results in a state network. Distribution of the stock networks corresponding to famous crises in history on the state network illustrates specific characteristics of the crises.

Zheng et al. [9] find that the signs of the components in the eigenvectors of stock cross-correlation matrix corresponding to the few largest eigenvalues can divide stocks into two groups. The major group, defined to be the one that contains more industries in the US stock market, covers however the blue chips and ST stocks in the Chinese stock market. Mantegna et al. [12] calculate the cross-correlation matrix of a total of 49 industry indices of the American stock market portfolio. They find that the largest components in the eigenvectors corresponding to the largest two eigenvalues are closely related with the financial crises in history. Ren et al. [14] report that the cross-correlation coefficient and the largest eigenvalue of the cross-correlation matrix of stock prices in the Shanghai Stock market increase significantly in the periods of the dot-com bubble in 2001 and the global financial crisis of 2007–2009.

The works mentioned above try to construct with cross-correlations a network between stocks within a specific stock market, while a collapse and/or crisis occurs generally in a scale covering several markets or even the global financial system. This mismatch of scale implies several limitations. First, the price and/or volume for every stock

(α)E-mail: hjyang@ustc.edu.cn (corresponding author)
is determined by the special and flexible conditions of the corresponding company and strongly polluted by noise, which makes the extracted relationships between it and the other stocks have unacceptable low confidence. The non-trivial patterns in the stock network are generally contaminated by statistical fluctuations. Second, the state of a specific stock market is an integrated result of interplays between social/economical/financial systems all over the world. A global crisis may induce significant changes or even strong shocks to the stock market, but the change of the stock market solely is not enough to coordinate the crisis. Third, the non-trivial information stored in the relationships, especially the causalities and thus information flow, between markets is lost.

Accordingly, a very recent effort is to construct causality networks of international stock market indices and/or government bonds [21–23], rather than cross-correlation ones (see [24,25] and references therein). For instance, Výrost et al. [22] network, a total of 20 stock markets from four continents by Granger causalities extracted from their indices. The network structures depend on a temporal proximity of closing times, the role of which for spillovers, i.e., the time distance between the markets matters, is confirmed further by spatial probit models. Using the indices for a total of 83 stock markets in a diversity of counties, Junior et al. [21] find that transfer entropy is an effective measure to quantify the flow of information. Stavrogoul et al. [23] conduct a systematic analysis on the causalities networks inferred by eight different causality methods. Detailed calculations ascertain the existence of significant causal behavior among assets, which implies possible arbitrage opportunities in financial markets. What is more, the average causality measured by nonlinear intertemporal cross correlation (or nonlinear cointegration) exhibits a marked change a half year (or three months) before the famous financial crash of 2007–2009 that can subsequently serve as an early warning signal of the crisis. As to whether the other methods can act as early warning signals or not, further scrutiny is required.

Transfer entropy is firstly introduced by Schreiber in 2000 [26]. As a model-free measure of causality, it can detect correctly the nonlinearities and distinguish positive and negative causality from (non-) stationary records, which are the advantages over the other members of causality family such as the (non-) linear Granger causality, cointegration, and intertemporal cross-correlation [23,27]. In the mutual entropy-based shadow causality a time lag is required to account for directional- ity and thus causality. The transfer entropy, however, does not need the time lag, which makes it a natural method for causality inference [23].

Summarily, the nonlinear causality among stock markets stores much more and reliable information on behaviors of the global financial system, which is consistent with the wide existence of nonlinearity in finance. Transfer entropy is the first candidate in the causality family for its advantages over the others, such as its being nonparametric, sensitive to nonlinearities and valid for nonstationary series. Hence, by means of transfer entropy in the present work we construct a causality network between ten typical stock markets in the world. The stock market network is used to represent the state of the global financial system. An event in the system is then displayed by its damage to the network. Technically, we collect the indices of a total of ten stock markets distributed in Asia, America, and Europe. Let a window covering one year slide along the multivariate series with a step of one month. From every segment covered by the window, the influences of every stock market on the others are measured quantitatively by transfer entropies. The stock market networks corresponding to the successive segments form a series, which contains the evolutionary information of the global financial system.

The contribution of our work is multifold. First, we show that the transfer entropy can serve as an early warning signal of financial crises, i.e., before each famous financial crisis the influential strength between the markets reaches a maximum. The key procedure is to filter out the short-term noises in the average of influences. Second, the influential network has a heterogeneous structure. Though averagely the influences between the markets are symmetrical, the markets in USA are strongly and monodirectionally influenced by that in Europe and act as the center. Third, some strongly influential relationships have strong cross-correlations. These findings shed new lights on the coupling structure of the global financial system and are helpful in modelling the dynamical processes on it.

Data. – Ten stock markets distributed in different continents are considered, i.e., the Dow Jones (DJI) and NASDAQ (NASD) in USA, the Tokyo’s Nikkei (NIKK), Hongkong’s HangSeng (HSI), Shanghai’s Stock Market (SHI), Shenzhen’s Stock Market (SZI), and Taiwan’s TWII in Asia, and German’s DAX, London’s FTSE, and Paris’ CAC40 (CAC) in Europe. The original data is daily indices in the duration from January 1992 to March 2017 (a total of 5040 simultaneous records) [28].

Let us denote the price series with

\[
P = \begin{bmatrix}
P_{1,1} & P_{1,2} & \cdots & P_{1,T+1} \\
P_{2,1} & \ddots & \cdots & P_{2,T+1} \\
\vdots & \vdots & \ddots & \vdots \\
P_{M,1} & P_{M,2} & \cdots & P_{M,T+1}
\end{bmatrix},
\]

where \( P_{i,t} \) is the index of the \( i \)th stock market on the \( t \)-th day, \( M = 10 \) represents the number of the stock markets, \( T + 1 = 5040 \) is the total length of the series. The daily return series reads,

\[
R = \{ r_{i,t} \equiv \ln P_{i,t+1} - \ln P_{i,t}, \\
i = 1, 2, \cdots, M; t = 1, 2, \cdots, T \}.
\]

Based upon the top ten financial events each year reported in the Financial Times (in Chinese) [29] and the
Table 1: Famous crises in the duration from 1992 to 2017.

| ID  | Starting time | Event                                      |
|-----|---------------|--------------------------------------------|
| C1  | 1994/12/30    | Mexico’s financial crisis                   |
| C2  | 1997/07/02    | Asian financial crisis                      |
| C3  | 2002/09/23    | Internet bubble burst                       |
| C4  | 2005/05/29    | The European referendum                     |
| C5  | 2008/09/14    | The Global financial crisis                 |
| C6  | 2009/12/08    | The world’s three largest Rating firms downgraded |
|     |               | the Greece’s sovereign rating               |
| C7  | 2012/05/01    | Japanese financial crisis                   |
| C8  | 2014/12/16    | Russian financial crisis                    |

Discussions in references [6,7,10–12] of the present paper, eight crises are identified (see table 1).

**Series of stock market network.** — Let a window with predefined length and step slide along the return series. Successive segments covered by the window form a series of segments. From each segment we calculate the transfer entropies from every stock market index to the others to measure quantitatively the stock market’s influences on the others. Transfer entropies between all the stock markets form a transfer entropy matrix, called herein stock market network. By this procedure, the multivariate series is converted to a series of stock market network, which stores the evolutionary behavior of the global financial system.

Denoting the length and step of the window with \( L \) and \( \Delta \), the segment series reads

\[
R_s = \begin{cases} 
  r_{1,\Delta}(s-1) + 1 & r_{1,\Delta}(s-1) + 2 & \cdots & r_{1,\Delta}(s-1) + L \\
  r_{2,\Delta}(s-1) + 1 & \cdots & r_{2,\Delta}(s-1) + L \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{M,\Delta}(s-1) + 1 & r_{M,\Delta}(s-1) + 2 & \cdots & r_{M,\Delta}(s-1) + L 
\end{cases}
\]

\( s = 1, 2, \cdots, W \),

where \( W \) is the total number of segments, \( [\cdot] \) the rounding of a real number.

The segment series is then mapped to a series of stock market network, \( S_{TE}(s), s = 1, 2, \ldots, W \), whose element \([S_{TE}(s)]_{mn}\) is the influence of the \( m \)-th market on the \( n \)-th market in the duration corresponding to the \( s \)-th segment, measured quantitatively herein with transfer entropy. The specific form of transfer entropy proposed in the paper [30] is employed. Let us denote the return series of the \( m \)-th and \( n \)-th stock markets in the \( s \)-th segment with \( X \) and \( Y \), namely,

\[
\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} r_{m,\Delta}(s-1) + 1 & r_{m,\Delta}(s-1) + 2 & \cdots & r_{m,\Delta}(s-1) + L \\ r_{n,\Delta}(s-1) + 1 & r_{n,\Delta}(s-1) + 2 & \cdots & r_{n,\Delta}(s-1) + L \end{bmatrix}
\]

and define two corresponding series with a delay of \( \tau \),

\[
\begin{bmatrix} X^{\tau}(l) \\ Y^{\tau}(l) \end{bmatrix} = \begin{bmatrix} R_s(m, l - \tau) \\ R_s(n, l - \tau) \end{bmatrix}, l = \tau + 1, \tau + 2, \cdots, L.
\]

The transfer entropy from \( X \) to \( Y \) reads,

\[
S_{TE}^{\tau}(m, n) = H(Y|Y^{\tau}) - H(Y|Y^{\tau}, X^{\tau}),
\]

where \( H(Y|Y^{\tau}) \) and \( H(Y|Y^{\tau}, X^{\tau}) \) are the Shannon entropies of \( Y \) under conditions of \( Y^{\tau} \) and \( Y^{\tau}, X^{\tau} \),

\[
H(Y|Y^{\tau}) = - \sum_{y \in Y, y^{\tau} \in Y^{\tau}} p(y, y^{\tau}) \ln p(y^{\tau}|y),
\]

\[
H(Y|Y^{\tau}, X^{\tau}) = - \sum_{y \in Y, y^{\tau} \in Y^{\tau}, x^{\tau} \in X^{\tau}} p(y, y^{\tau}, x^{\tau}) \times \ln p(y^{\tau}|y),
\]

respectively.

Here two remarks are in order. First, we must consider the time zone effect [22]. The ten stock markets distribute in different time zones. The opening and closing times for the stock markets are different. A stock market opening later may receive information on the other markets opening earlier, and be affected subsequently by the markets. From the viewpoint of the time zone, the stock markets cluster into three groups distributed in Asia, Europe, and America, respectively. Within each group the time zones for the stock markets are identical or have a slight difference, e.g., the Shanghai Stock Market in China Mainland opens an hour earlier than the NIKK in Japan. However, the difference of opening times of stock markets in different groups can reach six or even twelve hours. Hence, a proper method to consider the time zone effect is: within each group the markets can not communicate effectively in the same day; a market in Europe can receive the information of the markets in Asia group in the same day; and a market in America can receive information from all the markets in Asia and Europe.

Second, we are interested specially in the immediate influence, namely, how the present record influences the state of the next record. Because of the high-efficiency of information spreading, if we consider a large value of \( \tau \), the influence will be an integration of mutual-influences within a total of \( \tau \) days. The influences between a pair of stock markets will be entangled together. Hence, the immediate influence is a reasonable selection. The value of \( \tau \) depends subsequently on the time zones of stock markets. If the market \( A \) opens several hours earlier than the market \( B \), the influence of \( A \) on \( B \) is calculated with \( \tau = 0 \), otherwise \( \tau = 1 \). By this way all the markets are networked together, called immediate causality network of stock markets.
Li Zhou et al.

Fig. 1: (Colour online) Average of influence as a prior indicator of financial crises. (a) Distribution of transfer entropy estimated from all the transfer entropies. (b) Evolution of average influence. The time duration for each stock market network is estimated from all the transfer entropies. (a) Distribution of transfer entropy estimated from all the transfer entropies. (b) Evolution of average influence between the stock markets. We are specially interested in the behavior before or in an event of financial collapse.

From fig. 1(a) one can find that the transfer entropy distributes normally, centers at 0.214 with a standard deviation of 0.043. Figure 1(b) presents the evolution of average influence between the stock markets, which implies a high efficiency and a high risk at the same time. When a risk occurs and spreads in the financial system, people working in the system and being aware of the risk try to eliminate and/or block it by weakening the relations between different markets (a decrease of AVI). A failure of the efforts may induce a crisis.

Extensive works in the literature have found that a crisis occurs generally around a peak of cross-correlation. But there exist much more peaks around which one can find no crises. Hence, flittering noise (high-frequency-components) from the original curve of influence is the key procedure to find an early warning signal of financial crisis, which is ignored by almost all the researchers.

A failure of the efforts may induce a crisis. Asymmetry of influence (ASI) is used to show if the influence between a pair of markets is monodirectional or bidirectional. Herein it is measured by the ratio of the average difference between over average summation of transfer entropies of a pair of stock markets, and expose subsequently the trend, as shown with the red curve. Interestingly, the trend has several peaks, several months later from each of which one can find a famous financial crisis in the world. The peaks and the crises are one-to-one correspondence, except the last famous financial crisis occurred in Russia in 2014. The summits of peaks corresponding to the crises C1, C2, C3, C4, C5, C6 and C7 listed in Table 1 occurred at 1994/04, 1997/04, 1991/05, 2003/09, 2007/04, 2009/09 and 2012/05, namely, 8, 3, 16, 20, 17, 3 and 0 months before the occurrences of the crises. As for the crisis C8 in Russia, it is well-known that it is a localized collapse with ignorable influence on the global financial system. Hence, the AVI seems to be a good indicator prior to a financial collapse.

As a note, according to the widespread view obtained by Google search, the birth of the global financial crisis of 2007–2009 (herein numbered C5) is identified to be the 9th of August in 2007 [23], about one year before the starting time given in Table 1. The sum of the peak corresponding to this crisis in fig. 1(b) occurs 5 months before its birth.

Result. – In our calculations, the window size is selected to be $L = 12$-calendar-month, the sliding step $\Delta = 1$-calendar-month. The return series is then separated into a total of 292 segments. From the resulting series of $S_{TE}(s), s = 1, 2, \ldots, W$, several properties of the networks are calculated.

Average of influences (AVI) between the stock markets is defined simply from summation of all the elements in the stock market network,

$$ I_{ave}(s) \equiv \frac{1}{M(M-1)} \cdot \sum_{m,n=1}^{M} |S_{TE}(s)|_{m,n}. \quad (8) $$

A small (large) value of $I_{ave}$ implies a weak (strong) average influence between the stock markets. We are specially interested in the behavior before or in an event of financial collapse.

From fig. 1(a) one can find that the transfer entropy distributes normally, centers at 0.214 with a standard deviation of 0.043. Figure 1(b) presents the evolution of AVI in the years from 1992 to 2017. The last day of the $[12 + (s - 1) \cdot 1]$-th month in the return series is used to represent the duration corresponding to the $s$-th segment, namely, the value of $I_{ave}(t)$ is obtained from the segment ended at $t$. $I_{ave}(t)$ fluctuates frequently with $s$ (the gray curve).

A reasonable assumption is that an intrinsic behavior of the financial system should evolve slowly and is perturbed strongly by the daily changes of environments. A Fast-Fourier Transformation procedure is conducted to filter out the noises (high-frequency components) and expose subsequently the trend, as shown with the red curve. Interestingly, the trend has several peaks, several months later from each of which one can find a famous financial collapse in the world. The peaks and the crises are one-to-one correspondence, except the last famous financial crisis occurred in Russia in 2014. The summits of peaks corresponding to the crises C1, C2, C3, C4, C5, C6 and C7 listed in Table 1 occurred at 1994/04, 1997/04, 1991/05, 2003/09, 2007/04, 2009/09 and 2012/05, namely, 8, 3, 16, 20, 17, 3 and 0 months before the occurrences of the crises. As for the crisis C8 in Russia, it is well-known that it is a localized collapse with ignorable influence on the global financial system. Hence, the AVI seems to be a good indicator prior to a financial collapse.

As a note, according to the widespread view obtained by Google search, the birth of the global financial crisis of 2007–2009 (herein numbered C5) is identified to be the 9th of August in 2007 [23], about one year before the starting time given in Table 1. The sum of the peak corresponding to this crisis in fig. 1(b) occurs 5 months before its birth.

Extensive works in the literature have found that a crisis occurs generally around a peak of cross-correlation. But there exist much more peaks around which one can find no crises. Hence, flittering noise (high-frequency-components) from the original curve of influence is the key procedure to find an early warning signal of financial crisis, which is ignored by almost all the researchers.

A tentative understanding of the behavior is that: When the financial system works in a formal way, people in the world tend to cooperate with each other (increase of AVI), which implies a high efficiency and a high risk at the same time. When a risk occurs and spreads in the financial system, people working in the system and being aware of the risk try to eliminate and/or block it by weakening the relations between different markets (a decrease of AVI). A failure of the efforts may induce a crisis.

Asymmetry of influence (ASI) is used to show if the influence between a pair of markets is monodirectional or bidirectional. Herein it is measured by the ratio of the average difference between over average summation of transfer entropies of a pair of stock markets,

$$ I_{asy}(s) \equiv \frac{\sum_{m,n=1}^{M} |S_{TE}(s)|_{m,n} - |S_{TE}(s)|_{n,m}}{\sum_{m,n=1}^{M} \left\{ |S_{TE}(s)|_{m,n} + |S_{TE}(s)|_{n,m} \right\} \cdot \frac{1}{M(M-1)}}. \quad (9) $$

In the scatter plot of $(I_{ave}, I_{asy})$, the points will cluster around the original point (0,0) if there is almost no influence between the stock markets, i.e., the global financial system is in a loosely linked state. If the stock markets
influence bidirectionally with a strong strength on each other, the points will distribute in the bottom-right corner. A point at the top-right corner implies that for each specified pair of stock markets, the influence tends to be monodirectional.

As shown in fig. 2, the values of $I_{\text{ave}}(s), s = 1, s, \ldots, W$ distribute in a wide range of $[0.190, 0.255]$, while that for the values of $I_{\text{asy}}(s), s = 1, s, \ldots, W$ distribute in a sharp interval of $[0.01, 0.04]$. And the cross-correlation between $I_{\text{ave}}$ and $I_{\text{asy}}$ is very weak (if there exists any). Hence, the influence between the stock markets is averagely symmetrical.

Activity of influence of the $m$-th stock market on the $n$-th one is evaluated by the average and the variance of transfer entropy, which are estimated, respectively, with

$$A_{\text{str}}(m, n) = \frac{1}{W} \sum_{s=1}^{W} [S_{TE}(s)]_{m,n},$$

$$A_{\text{flu}}(m, n) = \sqrt{\frac{1}{W} \sum_{s=1}^{W} \left( [S_{TE}(s)]_{m,n} - [S_{TE}(s)]_{m,n} \right)^2},$$

(10)

where $\bar{\cdot}$ is statistical average. A large (small) value of $A_{\text{str}}$ implies a strong (weak) average influence of the $m$-th market on the $n$-th one. A large (small) value of $A_{\text{flu}}$ shows that the influence of the $m$-th market on the $n$-th one is weak (stable). For a strong and persistent influence, the point of $(A_{\text{str}}, A_{\text{flu}})$ will appear in the bottom-right corner in the scatter plot.

Figure 3 presents the points of $(A_{\text{str}}, A_{\text{flu}})$ for all the pairs of stock markets. There exists a strong cross-correlation between the two measures; When $A_{\text{str}}$ is less than the average value of 0.214 (indicated with the vertical dotted line), $A_{\text{flu}}$ is inversely proportional to it. After then $A_{\text{flu}}$ is proportional to it.

Influential pair network (IPN) is then constructed to show the relations of the influential pairs. From the series of $S_{TE}(s), s = 1, 2, \ldots, W$, one can extract two series $[S_{TE}(s)]_{m,n}, s = 1, 2, \ldots, W$ and $[S_{TE}(s)]_{i,j}, s = 1, 2, \ldots, W$, describing the influences of the $m$-th on the $n$-th and the $i$-th on the $j$-th market, respectively. The cross-correlation coefficient of the two series, $C(m, n; i, j)$ is then calculated to show how the relation from the $i$-th to $j$-th stock markets on that from the $m$-th to $n$-th markets.
Fig. 5: (Colour online) Influential pair network. (a) Each influential pair is assigned an identification number. The element at the row \( m \) and the column \( n \) is the identification number of the influential pair from the market \( m \) to the market \( n \). (b) The cross-correlation coefficients between all the pairs distribute normally. (c) Discarding all the links whose absolute values of cross-correlation coefficients are in \([0.074 - 0.32249, 0.074 + 0.32249]\), namely, two standard deviations around the average value of cross-correlation coefficients in (b), results into a influential pair network. The width of a link is proportional to the corresponding absolute value of the cross-correlation coefficient.

The strong couplings can be catalogued into three kinds. The first kind occurs between mutual pairs, i.e., the pair from \( m \) to \( n \) and the pair from \( n \) to \( m \), including the links between the nodes numbered 1 and 10, 41 and 50, and 72 and 89, namely between DJI→NASDAQ and NASDAQ→DJI, SHI→SZI and SZI→SHI, and DAX→CAC and CAC→DAX, respectively. The second kind occurs between the pairs from stocks in Europe to that in America, including the links between 74 and 82, 73 and 83, 64 and 83 and 65 and 82. The pairs from the markets in Europe to that in America are mediated into a cluster by the markets in America. The third kind occurs when two pairs share one stock market, including all the other strong links (13 out of the total of 20 strong links).

Conclusion and discussion. – A financial system contains many elements that are linked by complicated relationships between them, which can be modelled with a network. The nodes and edges are the elements and their relationships, respectively. Extensive works show that a crisis may induce significant changes to the topological structure, which in turn can be used as clue of financial collapse and/or crisis. Most of the existing works pay attention on intra-network structure of stocks within a single financial market, while a financial collapse/crisis reaches generally several stock markets or even the whole global financial system. This mismatch of scale implies several limitations, such as large fluctuations in the network structure and the lack of important information stored in the relations between markets.

In the present work, by using transfer entropy a total of ten typical stock markets distributed in Asia, Europe, and America are linked to a series of market network, to represent the evolution of the state of the global financial system. The findings include the linking strength between the markets which provides us with a significant indicator prior to financial crises, averagely the influences between the markets are almost symmetrical, but the markets in America are strongly and monodirectionally influenced by that in Europe, and occupy the center of the system; some influential relations closely cross-correlated with each other; and the markets in Mainland China have limited influence on the global financial system, if there is any. These findings provide some detailed information on the coupling structure between stock markets in the world, which can be used further in modelling the dynamical processes of the system.
A system contains generally many elements that are coupled together by their relationships, which can be modelled by a complex network. Monitoring dynamical process of the network produces a multivariate time series. Reconstructing from the multivariate records the network to represent the system's state has attracted special attention in recent years [31–33]. From the series of network one can obtain information on the detailed structure of the system and its evolution, which is the preliminary step to develop a mathematical model of the system and has potential applications in diverse fields.

For instance, from physiological signals recorded from the brain, eyes, heart, leg, chin and lung, Ivanov et al. [35] construct a series of networks between the six organs. The link number of the network can identify different sleep stages and the transitions between the stages. From high-throughput output co-expressions of genes for volunteers that are initially healthy and infected by influenza, Yu et al. [36] construct a series of relation network between genes for every person. The network structure can be used to distinguish healthy individuals from the others. Using networks constructed with the fMRI/EEG/DTI records for different regions of brain, Zhang et al. [37] monitor successfully mental disorders. Actually, our solution can be extended straightforwardly to these problems.

***

The work is supported by the National Science Foundation of China under Grant No. 10975099 (HY) and No. 11505114 (CG), the Program for Professor of Special Appointment (Oriental Scholar) at Shanghai Institutions of Higher Learning under Grant Nos. D-USST02 (HY) and QD2015016 (CG). One of the authors (HY) thanks Prof. C. P. Zhu from Nanjing University of Aeronautics and Astronautics and Prof. C.-K. Hu from Academia Sinica for helpful discussions.

REFERENCES

[1] Haldane A. G. and May R. M., Nature, 469 (2011) 351.
[2] Catanzaro M. and Buchanan M., Nat. Phys., 9 (2013) 121.
[3] Battiston S., Farmer J. D., Flache A., Garlaschelli D., Haldane A. G., Heesterbeek H. et al., Science, 351 (2016) 818.
[4] Eryigit M. and Eryigit R., Physica A, 388 (2009) 3551.
[5] Tumminello M., Lillo F. and Mantegna R. N., J. Econ. Behav. Organ., 75 (2010) 40.
[6] Song D. M., Tumminello M., Zhou W. X. and Mantegna R. N., Phys. Rev. E, 84 (2011) 026108.
[7] Kumar S. and Deo N., Phys. Rev. E, 86 (2012) 026101.