The Influence Analysis of Time Synchronization Error on Direct Source Position Algorithm

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ABSTRACT. It is very difficult to eliminate the time synchronization error completely. The influence of time synchronization error on direct source position algorithm was researched. The signal model with time synchronization error was given firstly. Then the CRLB of position estimation under the condition that the time synchronization error is exist was derived. In the last, the influence was analyzed by Monte Carlo simulations. The simulation results show that the Cramer-Rao lower bound, under the condition the time synchronization error is exist, is deteriorated obviously compared with the condition the time is synchronized perfectly; when the time synchronization error is shorter than the signal sample period, the CRLBs are same with different time synchronization errors; however the position accuracy of direct source position algorithm drops sharply, if the time synchronization error is not being considered.

1. INTRODUCTION

The direct radiation source localization algorithm does not need to estimate the time difference and the frequency difference then directly estimates the position of the radiation source by using the signal received by the receiving station. Literature [1] has proved that the direct positioning algorithm is an optimal positioning method, especially when the signal-to-noise ratio of the received signal is low, the positioning accuracy of the direct positioning algorithm is significantly better than that based on time difference, frequency difference and other observations algorithm. Therefore, scholars have carried out research on the direct localization algorithm of radiation source. Weiss A J. team extended the method in array signal processing to the problem of radiation source localization, and gave the direct localization algorithm of radiation source [1]; On this basis, the team successively gave the direct localization algorithm in the multi-radiation source scene [2]; The direct source position algorithm of the scenario that the signal model has errors-known waveforms[3] and the direct localization algorithm in the receiving station motion scene [4-6], but these algorithms did not consider the coherence among the signals. Scholar Li Jinzhou [7] studied the influence of signal coherence on the accuracy of direct localization algorithm, and gave the corresponding direct localization algorithm.

The direct localization algorithm proposed by the above scholars assumes that the time of all receiving stations is completely synchronized, which is difficult to implement in the real world. In this paper, the influence of the time synchronization error of the receiving station on the direct localization algorithm is studied and analyzed. Firstly, the signal model with time synchronization error under the condition of narrowband signal is given. Then CRLB is derived. Then the direct localization based on maximum likelihood estimation is given. Algorithm; Finally, Monte Carlo simulation is used to simulate the influence of time synchronization error on direct localization algorithm.
2. Signal model with time synchronization error

When the signal source radiates the bandwidth of the signal and satisfies the narrowband condition [1] within the signal duration, then in the case of time synchronization error, the baseband signal received by the first receiving station can be expressed as:

\[ r_1(t) = \alpha_s(t - \delta t - \tau) e^{j\omega t f_s(t)}(e^{j\omega t \delta t} - e^{j\omega t \tau}) + \phi(t) \]  

(1)

Where \( \alpha_s \) is the signal amplitude loss factor, which \( s(t) \) is the signal envelope, which \( \phi \) is the unknown initial phase, which \( \omega t \) is the time synchronization error of the receiving station. It is assumed that the error value remains unchanged within the signal duration, which is a zero-mean statistical independent complex-cycle Gaussian noise \( \tau \) is the delay from the propagation of the signal from the source to the receiving station, which \( f_s \) is the Doppler shift between the source and the receiving station. The expressions \( \tau \) and \( f_s \) for delay and Doppler shift are:

\[ \tau = -\frac{c}{c_{up}} T \]  

(2)

\[ f_s = \frac{c}{c_{up}} \left( \frac{\delta t}{T} - \frac{\delta t - \tau}{T} \right) \]  

(3)

Where in the position of the radiation source is the speed of light, the position of the receiving station, and the speed of the receiving station, which is the carrier frequency of the signal. It is assumed here that the delay and Doppler are constant over the signal duration; this assumption is reasonable when the signal duration is small (for example).

Assuming the sampling frequency is \( f_s \), the resulting discrete signal is:

\[ r[nT] = \alpha_s[nT - \delta t - \tau] e^{j\omega t nT}(e^{j\omega T \delta t} - e^{j\omega t \tau}) + \phi[n], n=0, \ldots, N_t-1 \]  

(4)

The number of samples is the sampling period. Use the symbol definition similar to [7]:

\[ n = [0, 1, \ldots, N_t-1]^T \]

\[ \hat{n} = \left[ \frac{N_t}{2}, \ldots, N_t - 1, 0, \ldots, \frac{N_t}{2} \right]^T \]

\[ D_n = \text{diag} \left\{ \exp \left( -j \frac{2\pi}{N_t} n \right) \right\} \]

\[ F = \frac{1}{\sqrt{N_t}} \exp \left( -j \frac{2\pi}{N_t} \hat{n} \hat{n}^T \right) \]

\[ D_{\delta t} = \text{diag} \left\{ \exp \left( -j \frac{2\pi}{N_t} \delta t \right) \right\} \]

\[ D_{\tau} = \text{diag} \left\{ \exp \left( j f_s \left( nT - (\delta t + \tau) 1_{N_t} \right) \right) \right\} \]

\[ E_n = \alpha_s e^{j\omega t} 1_{N_t} \]

Among them is the unit matrix, which is an all-one vector. Therefore, the formula (4) is expressed as a vector form as follows:

\[ r = Qs + w \]

(5)

among them

\[ Q = ED_{\delta t} F^T D_{\delta t} D_{\tau} F \]

\[ s = \left[ s[0], s[T], \ldots, s[(N_t - 1)T] \right]^T \]

\[ r = \left[ r[0], r[T], \ldots, r[(N_t - 1)T] \right]^T \]

\[ w = \left[ \phi[0], \phi[T], \ldots, \phi[(N_t - 1)T] \right]^T \]

\( r \) are all points of adoption for the first receiving station to receive signals. The definition of equation (1) shows that it is an independent cyclic complex Gaussian random vector whose
distribution function is, and is the noise power. Make
\[ r = \left[ r_1^H, r_2^H, \ldots, r_L^H \right]^H \]
\[ Q = \left[ Q_1^H, Q_2^H, \ldots, Q_u^H \right]^H \]
\[ w = \left[ w_1^H, w_2^H, \ldots, w_w^H \right]^H \]
\[ E\{ww^H\} = \Lambda = \text{diag}\{\Lambda_1, \Lambda_2, \ldots, \Lambda_L\} \]
Which is the number of all receiving stations. The signals received by a receiving station are represented as a vector form:
\[ r = Qs + w \]  
(6)

3. CRLB derivation
Define the unknown vector as follows:
\[ \theta = \left[ \hat{\theta}^T, \varphi^T, u^T \right]^T \]  
(7)
among them
\[ \varphi = \left[ \phi_1, \phi_2, \ldots, \phi_L \right]^T \]
\[ \delta t = \left[ \delta t_1, \delta t_2, \ldots, \delta t_L \right]^T \]
The unknown vector contains the radiation source position vector, the constant term of the initial phase in the receiving station signals, and the time synchronization error of the receiving stations. Therefore, the probability density function measured by the equation (6) is:
\[ p(\theta, \theta) = \frac{1}{\pi \Lambda} \exp\left\{ -\frac{1}{\pi \Lambda} (r - Qs)^H (r - Qs) \right\} \]
(8)
The Fisher information matrix is known from the literature [7]:
\[ J = 2 \sum_{d} \text{Re} \left\{ \frac{1}{\sigma_d^2} \left( \frac{\partial Q_s}{\partial \theta} \right)^H \left( \frac{\partial Q_s}{\partial \theta} \right) \right\} \]
(9)
According to the definition in equation (7), there are:
\[ \frac{\partial Q_s}{\partial \theta} = \begin{bmatrix} Q_s & Q_s & Q_s \\ \frac{\partial Q_s}{\partial \theta} & \frac{\partial Q_s}{\partial \varphi} & \frac{\partial Q_s}{\partial u} \end{bmatrix} \]
(10)
\[ A = \text{diag} \left\{ 2 \sum_{d} \text{Re} \left\{ \frac{1}{\sigma_d^2} \left( \frac{\partial Q_s}{\partial \theta} \right)^H \left( \frac{\partial Q_s}{\partial \theta} \right) \right\} \right\} \]
\[ B = 2 \sum_{d} \text{Re} \left\{ \frac{1}{\sigma_d^2} \left( \frac{\partial Q_s}{\partial \theta} \right)^H \left( \frac{\partial Q_s}{\partial \varphi} \right) \right\} \]
\[ C = 2 \sum_{d} \text{Re} \left\{ \frac{1}{\sigma_d^2} \left( \frac{\partial Q_s}{\partial \theta} \right)^H \left( \frac{\partial Q_s}{\partial u} \right) \right\} \]
among them
\[ \frac{\partial Q_s}{\partial \theta} = \left[ 0, \ldots, -j(2\pi f_d, r_i + \hat{r}_i), \ldots, 0 \right] \]
(11)
\[ \hat{r}_i \triangleq \frac{2\pi}{N_i} \text{diag} \{ n \} r_i \]
(12)
\[ \frac{\partial Q_s}{\partial \varphi} = \left[ 0, \ldots, j r_i, \ldots, 0 \right] \]
(13)
\[
\frac{\partial Q_s}{\partial u} = \begin{bmatrix} \frac{\partial Q_s}{\partial u} \\ \frac{\partial Q_s}{\partial f_{j,i}} \end{bmatrix}
\]
(14)

\[
\frac{\partial Q_s}{\partial \tau_i} = -j \left( 2\pi f_{j,i} \mathbf{r}_i + \mathbf{\dot{r}}_i \right)
\]
(15)

\[
\frac{\partial Q_s}{\partial f_{j,i}} = j2\pi T_{di} \text{diag} \{ \mathbf{n} \} \mathbf{r}_i - j2\pi (\tau_i + \delta_i) \mathbf{r}_i
\]
(16)

According to the definitions of delay and Doppler in equations (2) and (3), it is available:

\[
\frac{\partial \tau_i}{\partial u} = \frac{\mathbf{u} - \mathbf{p}}{c} \quad (17)
\]

\[
\frac{\partial f_{j,i}}{\partial u} = \frac{\mathbf{u} - \mathbf{p}}{c} \quad (18)
\]

4. Direct localization algorithm when time synchronization error exists

The position of the radiation source is obtained by estimating the unknown vector. According to the probability density function of the received signal in equation (8), the maximum likelihood estimate obtainable is:

\[
\hat{\theta}_{\text{ML}} = \arg \max_{\mathbf{u}} \left\{ \sum_{j=1}^{L} \frac{1}{\sigma^2_j} \mathbf{r}_j^T Q_s \right\}
\]
(19)

Since the unknown phase vector is a redundant variable, the influence of the unknown phase term can be eliminated by using the absolute value function, so the above equation (19) can be corrected to:

\[
\hat{\mathbf{u}} = \arg \max_{\mathbf{a}} \left\{ \sum_{j=1}^{L} \frac{1}{\sigma^2_j} \mathbf{r}_j^T Q_s \right\}
\]
(20)

In the process of locating the radiation source, although the time synchronization error of each receiving station is a redundant variable that we do not care about, since the time synchronization error and the signal propagation delay have the same influence on the received signal, it is difficult to distinguish the two. Therefore, when estimating the position of the radiation source using equation (20), it is necessary to simultaneously estimate the time synchronization error, which will increase the amount of calculation. When the grid search algorithm is used to estimate the position of the radiation source, the time synchronization error will result in a multiplication of the computational amount, which represents the number of grids searched for each unknown parameter.

5. Simulation analysis

This section analyzes the influence of time synchronization error on the direct positioning performance of the radiation source through Monte Carlo simulation. For the sake of analysis, we only performed 2D simulation, and the 3D case also applies. Suppose there are three receiving stations and one radiation source. The positions of the three receiving stations are [-8000, 1000]m, [7500, 4500]m, [-1000, -6000]m; three receiving stations do the uniform speed. For linear motion, the speeds are: [50, -10] m/s, [-40, 20] m/s, [30, 40] m/s. The source of radiation is stationary at the origin of the coordinates. The time synchronization error of each receiving station is randomly generated, the standard deviation of the synchronization error is, and the synchronization error remains unchanged for the duration of the received signal. The number of Monte Carlo simulations is 1000. The received signal duration is T=10ms.

The envelope of the radiation source signal is generated by the ARMA (Auto-regressive moving-average) model, and its discrete transfer function is:

\[
H(z) = \frac{1+0.9z^{-1}+0.8z^{-2}}{1-0.7z^{-1}}
\]

The carrier frequency of the source signal, the sampling frequency of the receiving station.
Figure 1 shows the CRLB comparison for different time synchronization errors. It can be seen from the figure that the CRLB when all receiving stations are fully synchronized (i.e., the time synchronization error is 0) is 13 dB lower than the CRLB when there is a time synchronization error, and the existence of the time synchronization error causes the precision threshold of the direct positioning algorithm to drop greatly. In addition, the magnitude of the time synchronization error does not affect the positioning accuracy threshold. This is because the sampling period in the simulation scenario is far more than the time synchronization error in the simulation. Therefore, the right side of equation (16) is approximated, so the time synchronization error size does not affect the positioning accuracy threshold.
The simulation results of (a)~(d) in Fig. 2 are time synchronization errors of 10 ns, 50 ns, 100 ns, and 200 ns, respectively, and the simulation results of the time synchronization error localization algorithm are not considered in the localization algorithm (20). It can be seen from the figure that
when the signal-to-noise ratio is greater than 0dB, the positioning accuracy of the proposed algorithm can reach the CRLB threshold; however, the positioning accuracy of the time synchronization error is not considered. When the time synchronization error is 10ns, 50ns, 100ns, 200ns, respectively, it is higher than CRLB. The threshold is 2dB, 6dB, 9dB, and 12dB. It can be seen that although the time synchronization error does not affect the positioning accuracy threshold, if the time synchronization error is not considered, the positioning accuracy of the algorithm will be drastically reduced.

6. Conclusion
In this paper, the influence of time synchronization error on the direct localization algorithm is analyzed. The theoretical analysis and simulation verification show that:

(1) The existence of time synchronization error causes the direct positioning accuracy threshold CRLB to deteriorate significantly. In the simulation scenario of Section 5, the precision threshold deteriorates by 13 dB;
(2) When the time synchronization error is less than the signal sampling period, the magnitude of the time synchronization error does not affect the positioning accuracy threshold;
(3) If the time synchronization error is not considered, the accuracy of the direct positioning algorithm will drop sharply. In the simulation scenario of Section 5, when the time synchronization error is 200 ns, the positioning accuracy is reduced by 12 dB;
(4) Although the positioning accuracy of the positioning algorithm estimated by the time synchronization error and the radiation source position can reach the theoretical positioning threshold, the estimation of the time synchronization error will cause the calculation amount to increase sharply, which is about doubled. When the number of receiving stations is large, or the number of search grids is large, the real-time performance of the algorithm will deteriorate. It can be seen from the simulation results in Fig. 2(a) that when the time synchronization error is small, the time synchronization error has little effect on the positioning accuracy of the algorithm. Therefore, the accuracy of the algorithm can be improved by improving the time synchronization accuracy between the receiving stations.

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