The effect of transverse rectification of electromagnetic waves in a two-dimensional superlattice

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Abstract. Using the constant-collision frequency approximation, an analytical expression was obtained for the direct current density occurring in a two-dimensional superlattice when two waves with mutually perpendicular polarization planes are exposed to the sample, the frequency ratio of which is 2.

1. Problem statement

Recently, the attention of researchers has been focused on the study of various nonlinear phenomena in low-dimensional semiconductor structures that have a prospect of application in the field of generation and detection of terahertz waves [1]. Thus, nonlinear optical properties of two-dimensional structures were studied in [2-7]. In connection with the development of graphene manufacturing technology and the appearance of first samples of graphene superlattices (GSL) (see, for example [8]), interest in two-dimensional superlattices (SL) [9] has been resumed. A distinctive feature of the energy spectrum of graphene and, accordingly, the GSL, is its non-additivity, which leads to the mutual dependence of the movements of current carriers in directions perpendicular to each other. In this paper, we consider the effect of transverse current rectification in a two-dimensional semiconductor superlattice with a non-additive energy spectrum [9]:

\[ \varepsilon(p) = \Delta \left[ 1 - \cos \left( \frac{p_x d}{\hbar} \right) \cos \left( \frac{p_y d}{\hbar} \right) \right], \]

where \( \Delta \) is the width of the miniband, \( d \) is the period of the SL.

We assume that periodic potential is two-dimensional, and its period as well as miniband width in direction along X and Y axis are equal. Expression (1) is a model spectrum, which the most closely correspond to 2D quantum dot superlattice (see, for example [10]) and at certain condition can be used for description of GSL [11]. Problem geometry is presented on figure 1. The intensity of the applied electric fields \( E_1 = E_{10} \cos(\omega t) \), \( E_2 = E_{20} \cos(\omega t + \varphi) \), the field \( E_1 \) is applied along the X axis, the field \( E_2 \) along the Y axis. We are interesting in determine constant component of the current density \( j_x \) along X axis. A similar problem was solved in [12] in the case of graphene, which has a band gap in its spectrum, in the first non-vanishing approximation on the wave field strength.
Figure 1. Geometry of the problem

2. Investigation of the direct current density

The current density is determined as follows:

\[ j_x = e \sum_p \nu_x(p) f(p,t). \] (2)

where

\[ \nu = \frac{\partial \varepsilon(p)}{\partial p} = \Delta d \frac{h}{\varepsilon} \left\{ \sin \frac{p_x d}{h} \cos \frac{p_y d}{h}; \cos \frac{p_y d}{h} \sin \frac{p_x d}{h} \right\}. \] (3)

The non-equilibrium distribution function \( f(p,t) \) is a solution of the Boltzmann kinetic equation, the collision term of which we choose in the approximation of the collision frequency \( \nu \), which we will further consider to be constant:

\[ f(p,t) = \nu \int_{-\infty}^{t} \exp[-\nu(t-t')] f_0(p'(t';p,t)) dt', \] (4)

where \( p'(t';p,t) \) is the solution of the classical equation of motion \( dp'/dt = eE(t') \) with an initial condition \( t' = t, p' = p \) that has the following form:

\[ p_x' = \frac{h}{d} \left( p_x \frac{d}{h} + \frac{eE_{10}}{\hbar \omega_1} \left[ \sin(\omega_1 t') - \sin(\omega_1 t) \right] \right), \]
\[ p_y' = \frac{h}{d} \left( p_y \frac{d}{h} + \frac{eE_{20}}{\hbar \omega_2} \left[ \sin(\omega_2 t' + \varphi) - \sin(\omega_2 t + \varphi) \right] \right). \] (5)

The equilibrium distribution function \( f_0 = A \exp(-\varepsilon(p)/T) \) is normalized by the condition:
\[
\frac{A}{(2\pi\hbar)^5} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{-\frac{\Delta}{T} \left[ 1 - \cos \left( \frac{p_x d}{\hbar} \right) \cos \left( \frac{p_y d}{\hbar} \right) \right] \right\} dp_x dp_y = n,
\]

where \( T \) is the temperature expressed in energy units, \( n \) is a surface concentration of charge carriers.

The final expression for normalization constant \( A \) is:

\[
A = n\hbar^4 e^{\frac{\Delta}{T}} \left( d^2 J_0^2 \left( \frac{\Delta}{2T} \right) \right),
\]

Thus, the nonequilibrium function is represented as:

\[
f(p,t) = \frac{Av}{\exp\left(\frac{\Delta}{T}\right)} \int_{-\infty}^{0} \exp(-\nu a) \exp \left\{ \frac{\Delta}{T} \cos \left[ \frac{p_x d}{\hbar} + \frac{edE}{\hbar \omega} \sin[\omega t] - \sin(\omega t) \right] \right\} \cdot \cos \left[ \frac{p_x d}{\hbar} + \frac{edE}{\hbar \omega} \sin[\omega t + \varphi] - \sin(\omega t + \varphi) \right] \right\} da,
\]

where \( a = t - t' \).

By introducing a number of the denotation, presented below, and making the replacement, we get:

\[
f(p,t) = C\nu \int_{-\infty}^{0} \exp(-\nu a) \exp \left[ \gamma \cos(q_x + F_1 g_1) \cos(q_y + F_2 g_2) \right] da,
\]

where \( C = A / \exp(\gamma) = n\hbar^4 / d^2 J_0^2 (\gamma / 2), \gamma = \Delta / T, q_{x,y} = p_{x,y} d / \hbar, F_{1,2} = eE_{10,20} d / \hbar \omega_{1,2}, g_1 = \sin(b(t - a)) - \sin(bt), g_2 = \sin(t - a + \varphi) - \sin(t + \varphi), a \rightarrow \omega t', t \rightarrow \omega t, b = \omega_1 / \omega_2.\)

We substitute the expression for the nonequilibrium distribution function in the current density formula (2), where we go from summative to integration in the standard way. Taking into account the introduced notation, we obtain:

\[
j_i = j_{i0} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp(-\beta a) \exp \left[ \gamma \cos(q_x + F_1 g_1) \cos(q_y + F_2 g_2) \right] \sin(q_y) \cos(q_x) dq_x dq_y da,
\]

where \( j_{i0} = e \frac{\Delta d}{\hbar} C \frac{1}{(2\pi\hbar)^5} \frac{1}{\omega_2}, \beta = \frac{\nu}{\omega_2}.\)

Thus, taking the integrals entering into the expression and averaging over the period of the electromagnetic wave, we bring the expression to the following form:

\[
j_i = j_0 \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \frac{(-1)^{k+m}}{i} J_n(F_1) J_k(F_2) J_m(F_i) \exp[i\varphi(k + n)] \delta(Q, 0) \cdot
\left\{ \frac{1}{in + ibl + \beta} - \frac{1}{ik + imb + \beta} - \frac{1}{ik + imb + \beta} + \frac{1}{ik + imb + \beta} \right\},
\]

where \( j_0 = eC \beta \hbar \Delta / 4d, Q = i(tn + k + lb + mb).\)

In [13], it was shown that in the case of coincidence of the polarization planes of two waves incident on a one-dimensional SL, a constant current should arise, directed along the axis of the SL, when the ratio of the frequencies of the incident waves is 2 (generally speaking, the occurrence of current is also
possible at another frequency, but the case \( b = 2 \) is most distinctive). In the 2D superlattice under consideration, when the ratio of the frequencies of the incident waves is 2, due to the non-additivity of the energy spectrum, a non-zero direct current also arises when the planes of polarization are mutually perpendicular. From the expression for the current density \( j_x \), it can be seen that the dependence of the rectified current on the amplitudes of the incident waves is substantially non-linear and, under certain conditions, manifests oscillating behavior (figure 2).

![Figure 2. Current density dependence in units of \( j_0 \) from the field \( F_1 \)](image)

**3. Special cases analysis**

We shall analyze some limiting cases for the expression of the density of the longitudinal current.

The case of weak fields. Initially, we assume weak high-frequency fields \( F_1 \) and \( F_2 \) (\( F_1, F_2 \ll 1 \)) \( b = 2 \) and \( \varphi = 0 \). Since the expansion of the Bessel functions \( J_n(F_2), J_k(F_2), J_l(F_1), J_m(F_1) \) begins with the terms of the order \( F_1^n, F_2^m, F_2^n, F_2^l \), in order to provide the first non-vanishing approximation in the fields \( F_1, F_2 \), it suffices to leave only the terms with \( n, k, l, m = -1, 0, 1 \) in the sum. Considering cases that satisfy the condition, the expression for current density takes the following form:

\[
j_x = j_0 \left( \sum_{k=-1}^1 \sum_{l=-1}^1 \sum_{m=-1}^1 \frac{(-1)^{l+m}}{i} J_n(F_2) J_k(F_2) J_l(F_1) J_m(F_1) \right) \cdot \left( \frac{1}{in + ibl + \beta} - \frac{1}{in + ibm + \beta} + \frac{1}{ik + ibl + \beta} - \frac{1}{ik + ibm + \beta} \right). \tag{12} \]

In this case, we obtain the expression for the density of the longitudinal current \( j_x \):

\[
j_x = j_0 \frac{2F_1F_2^2}{\beta^2 + 1}. \tag{13} \]
The calculation shows that for small values of $F_1$ and $F_2$ the dependence of the current density $j_x$ on the strengths of the applied fields $j_x \sim F_1^2 F_2^2$, that is, linear in the field strength of the wave polarized along the X axis, having a double frequency, and quadratic in the wave field strength with a lower frequency and polarized along the Y axis. This is a characteristic property of the considered effect of wave rectification in materials with a nonparabolic energy spectrum. In particular, for a one-dimensional superlattice in the case of coinciding polarization planes of the incident waves, it was established in [13]. The case of a high collision frequency ($\beta >> 1$). In this case, the greatest contribution to the value of the current density will be given by elements with small quantities $n, k, l, m$. To show this, consider the following factor:

$$\left(\frac{1}{in + ibl + \beta} - \frac{1}{in + ibm + \beta} + \frac{1}{ik + ibl + \beta} - \frac{1}{ik + ibm + \beta}\right).$$

Due to the fact that the expression $(\beta)^{-1} << 1$, the terms $in + ibl$, $in + ibm$, $ik + ibl$, and $ik + ibm$ must be taken the smallest.

Considering condition (12), we obtain the expression for the density of the longitudinal current:

$$j_x = J_0 \frac{16J_0(F_1)J_1(F_1)J_2^2(F_2)}{\beta^2 + 1}. \quad (15)$$

Expanding in this expression the Bessel functions into series on the fields $F_1, F_2$ limiting by to the first non-vanishing approximation, we obtain the expression for the current density for the case of weak fields:

$$j_x = J_0 \frac{2F_1 F_2^2}{\beta^2 + 1}. \quad (16)$$

4. Conclusion

Thus, using the constant-collision frequency approximation, an analytical expression was obtained for the direct current density occurring in a two-dimensional superlattice when two waves with mutually perpendicular polarization planes are exposed to the sample, the frequency ratio of which is 2. Based on the general analytical solution, special cases of small amplitudes of the incident waves and high frequencies are considered. The results obtained for a two-dimensional superlattice in particular cases coincide with the results obtained previously for a one-dimensional superlattice and graphene.

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