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ABSTRACT
The nonlinear propagation of magnetosonic waves in a magnetized strongly coupled dusty plasma consisting of inertialess electrons and ions as well as strongly coupled inertial charged dust particles is presented. A generalized viscoelastic hydrodynamic model for the strongly coupled dust particles and a quantum hydrodynamic model for electrons and ions are considered. In the kinetic regime, we derive a modified Kadomstev-Petviashvili (KP) equation for nonlinear magnetosonic waves of which the amplitude changes slowly with time due to the effect of a small amount of dust viscosity. The approximate analytical solutions of the modified KP equations are obtained with the help of a steady state line-soliton solution of the second type KP equation in a frame with a constant velocity. The dispersion relationship in the kinetic regime shows that the viscosity is no longer a dissipative effect.

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I. INTRODUCTION

Magnetosonic waves propagating perpendicular to the applied magnetic field in a dusty plasma differ from those in an electron-ion plasma because of not only the unusual time and space scale but also the correlations of the dust grains. Self-gravitation due to heavy dust grains could also modify the traditional Jeans instability of magnetosonic modes, which is generalized to multiple dust species with the effects of mass distribution. For magnetosonic waves propagating parallel to the magnetic field, the presence of dust grains can greatly cause nonresonant firehose instability. When the drift velocity of the dust beam parallel to the applied magnetic field is larger than that of the phase velocity of magnetosonic waves, the instability of the sheared flow grows, and the amplitude of magnetosonic waves also grows. Accordingly, the linear magnetosonic waves and nonlinear magnetosonic waves in a dusty magnetoplasma were well studied by linear dispersion relations or nonlinear equations in a different regime.

Quantum effects on magnetosonic waves or dusty acoustic waves have attracted much attention recently. As a plasma is cooled to a relatively low-temperature or if its number density is relatively high, the de Broglie wavelength of the particles can be comparable to the distance between the charged particles. Accordingly, the quantum effects should be considered. Quantum effects can greatly affect the collective nonlinear excitation in dusty quantum plasmas. The electrostatic or electromagnetic waves in a dusty degenerate or nondegenerate plasma have been well studied in recent times.

As the mass of dust particles is much larger than the mass of electrons and ions, the quantum effects on dust particles are usually less important than those on electrons and ions. Accordingly, many authors have only considered quantum effects on electrons or ions by Bohm potential. The dusty grains was described by a fluid equation or taken as a static background.

In magnetized quantum dusty plasmas, the propagating properties of magnetosonic waves were already well studied for both slow and fast modes, which shows that all kinds of quantum effects, such as diffraction effects, statistic effects, and spin-1/2 effects, can have great effects on magnetosonic solitary waves.

In this paper, we will use the Coulomb coupling parameter \( \Gamma_d = q_d^2 / k_B T_d r_{d0} \) in a classical regime for dusty species, where \( q_d \) is the equilibrium charge of the dust grains, \( T_d \) is the temperature, \( k_B \) is the Boltzmann constant, and \( r_{d0} \) is the Wigner - Seitz radius. We will use the coupling parameter \( \Gamma_j = q_j^2 / k_B T_j r_{j0} \) for electrons (\( j = e \)) and ions (\( j = i \)), respectively. For a dusty plasma with a relatively...
low-temperature and high number density, we will consider \( \Gamma_d < 1 \) and \( \Gamma_d > 1 \) for typical parameters in an astrophysical object. In fact, the linear or nonlinear waves in strongly coupled dusty plasmas are quite different from those in weakly coupled systems.\(^{3,14,15}\) In astrophysical objects, such as the interior of heavy planets, white dwarfs, and neutron stars, the dusty plasma is in a strongly coupled state.\(^7\) Thus, in this paper, we consider the dusty grains to be strongly coupled.

Accordingly, our aim in this work is to investigate magnetoionic waves in a strongly coupled magnetized dusty plasma. Here, we focused on a kinetic regime in which magnetoionic solitary waves can be excited. In fact, the nonlinear waves in a strongly coupled nonmagnetized dusty plasma\(^{3,34,35}\) were already studied in a kinetic regime.\(^{35}\) In this paper, we will study the quantum effects and strongly coupled effects on nonlinear magnetized waves in a magnetized dusty plasma. Then, one may expect that dust viscosity together with the quantum effects on electrons and ions has a strong effect on the properties of magnetovacancies, which is important for astrophysical objects, such as the interior of heavy planets, white dwarfs, and neutron stars.\(^1\)

### II. BASIC PLASMA THEORY

We consider the nonlinear propagation of magnetovacancies in a magnetized dusty plasma consisting of inertial strongly coupled classical dusts and inertialless quantum electrons and ions with weak interparticle interactions. We also consider the applied magnetic field along the \( Z \) axis. The nondegenerated electrons and ions obey the pressure law \( P_e = P_n (n_e/n_n)\gamma, \) where \( P_e \) is the pressure of electrons/ions, and \( P_n = n_n k_B T_n \). The number density \( n_j \) is normalized by its equilibrium density \( n_0 \) of \( j \) species particles, where \( j = e, \) and \( i \) stands for electrons and ions, respectively.

Then, the momentum equation for electrons and ions can be given by the quantum hydrodynamic equation,\(^1\)

\[
0 = E + \Omega_d v_e \times B + \sigma_e \frac{1}{n_e} \nabla n_e^2 - H^2 \nabla \left( \frac{1}{n_e} \nabla^2 \sqrt{n_e} \right),
\]

(1)

\[
0 = E + \Omega_d v_i \times B - \sigma_i \frac{1}{n_i} \nabla n_i^2 - H^2 \nabla \left( \frac{1}{n_i} \nabla^2 \sqrt{n_i} \right).
\]

(2)

On the other hand, the dynamics of strongly coupled dusts can be described by a generalized hydrodynamic model as

\[
\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d v_d) = 0,
\]

(3)

\[
\left( 1 + \tau_m \frac{d}{dt} \right) \left[ n_d \frac{dv_d}{dt} + n_d E + n_d \Omega_d v_d \times B + \nabla n_d \right]
\]

\[
= \eta^s \nabla^2 v_d + \left( \xi^s \eta^s \right) \nabla \left( \nabla \cdot v_d \right),
\]

(4)

where \( \tau_m \), which is normalized by the inverse dusty plasma frequency \( \omega_{pd}\), denotes the viscoelastic relaxation time.\(^{46}\)\(^{46}\)

\[
\tau_m = \frac{\omega_{pd} \eta}{n_d k_B T_d} \left[ 1 - \sqrt{1 + 4 \frac{u_0 (\Gamma_d)}{15}} \right]^{-1},
\]

(5)

where \( \eta = \xi^s + 4 \eta^s / 3 \) is the coefficient of effective dust viscosity, in which \( \eta^s \) and \( \xi^s \) are, respectively, the shear and bulk viscosities. The viscosity \( \eta \) stands for the correlation effects, which gives the additional corrections of the dispersion relation and changes the phase velocity of the waves through \( \eta \). If one considers a weak coupling limit, the viscosity \( \eta \) reduces to the Navier-Stokes viscosity, which comes from the collision of the dusty particles.\(^{29}\) For a strongly coupling limit, a transverse shearlike mode can appear in a dusty plasma.\(^{26}\) Again, it has been found that the viscosity coefficient \( \eta \) in Eq. (5) has a wide minima (\(-1\)) in \( 1 < \Gamma_d < 10 \), and it tends to increase with \( \Gamma_d \). Also, it becomes high for \( \Gamma_d < 1 \). Typical values of \( \eta \) for a dusty plasma are \(-45 \) for \( \Gamma_d = 0.1 \), \(-46.4 \) for \( \Gamma_d = 100 \), and \(-118.5 \) for \( \Gamma_d = 202.\(^{3,29,30}\) Thus, \( \tau_m \) becomes high in weak coupling (\( \Gamma_d < 1 \)) as well as in strong coupling (\( \Gamma_d > 1 \)) regimes. So, the kinetic modes exist only for \( (\Gamma_d < 1) \) or \( (\Gamma_d > 1) \) where the condition \( w_{\tau_m} \gg 1 \) is satisfied.

Here, the parameter \( y_1 \) is the dust adiabatic index. \( u(\Gamma_d) \) is a measure of the excess internal dust energy and can be given as\(^{3,17,38}\)

\[
u(\Gamma_d) = 1.5 - 0.90 \Gamma_d + 2980 \Gamma_d^2 \text{ for } (160 < \Gamma_d < 300)
\]

\[
u(\Gamma_d) = -0.90 \Gamma_d + 0.951 \Gamma_d^{1/4} + 0.181 \Gamma_d^{-1/4} - 0.80 \text{ for } (1 < \Gamma_d < 160).
\]

Furthermore, the compressibility parameter \( \mu_d \) appearing in Eq. (5) is given by

\[
\mu_d = 1 + \frac{1}{3} u(\Gamma_d) \Gamma_d \frac{\partial T_d}{\partial \Gamma_d}.
\]

(6)

Since \( u(\Gamma_d) \) is negative for increasing values of \( \Gamma_d, \mu_d \) can change its sign. It has been shown that for values of \( \Gamma_d \) in \( 1 < \Gamma_d < 10 \), this change of sign can cause the dispersion curve to turn over with the group velocity going to zero and then to negative values.\(^{39}\) We consider dust pressure to include the thermodynamic contribution as well as the pressure due to mutual electrostatic repulsion of like charged dust particles as \( P_d = y_d k_B T_d n_d \), where \( T_d = T_s + \mu_d T_d \) is the effective dust temperature in which \( T_s \) appears due to electrostatic interactions between strongly coupled dusts and is given by \( T_s = N_{nn} \ast T_{d*} + T_d \).\(^{46,49,50}\) Here, \( N_{nn} \) is determined by the dust structure and corresponds to the number of nearest neighbors (e.g., in the crystalline state, \( N_{nn} = 12 \) for the fcc and hcp lattices, and \( N_{nn} = 8 \) for the bcc lattice). Although the parameter \( \mu_d \) can be negative for increasing values of \( \Gamma_d, T_s \), may be comparable or even dominate \( \mu_d T_d \) for \( \Gamma_d \gg 1 \), so the effective temperature \( T_d \) in the limit of \( \kappa \to 0 \) is most likely due to the strong coupling of dust grains. Then, the system is enclosed by the following Maxwell’s equations:

\[
\nabla \times E = -\partial B / \partial t
\]

(7)

\[
\nabla \times B = \Omega_d \partial E / \partial t + \Omega_d \beta[-\alpha n_e v_e + (1 + \alpha) n_i v_i - n_d v_d].
\]

(8)

Equations (1)–(4), (7), and (8) have been dimensionless. The number density \( n_j \) is normalized by the equilibrium number density \( n_0 \) for electrons, ions, and dusty particles. The equilibrium number densities of different species satisfy the electrical neutrality condition as \( n_e + Z_d n_d = n_0 \), where \( Z_d \) is the number of charges of the dust grains, which is determined in units of electron charge. The parameter \( \alpha = n_e / Z_d n_d \) is used to satisfy the electrical neutrality condition. The fluid velocity \( v_i \) is also normalized by the effective thermal speed \( v_T = \sqrt{\gamma_e k_B T_d/m_i} \), where \( m_i \) is the mass of the dust grains. The space and time coordinates are also normalized by the effective Debye length \( \lambda_D = \sqrt{\gamma_e k_B T_d/4\pi n_e Z_d e^2} \) and the dust plasma oscillation frequency \( \omega_{pd} = v_T / \lambda_D \), respectively.
Here, the amplitude of the electric field vector $E$ was normalized by $m_d\omega_{pd}V_j/I_dZ_{de}$. The amplitude of the magnetic field $B$ was normalized by the applied magnetic field vector $B_0$. The dust gyrofrequency $\Omega_d = Z_{de}B_0/m_d\omega_{pd}$ is normalized by the dusty plasma frequency $\omega_{pd}$. In Eqs. (1) and (2), the coefficient of pressure term is given by $\sigma_j = (3Z_{de}T_j)/(2\gamma_dT_{de})$ for electrons and ions. The coefficient $\beta = V_{thd}^2/V_{thd}$ with $V_{thd} = \sqrt{8kT_{de}/m_d\omega_{pd}}$. The coefficients $H_c = \sqrt{Z_{de}m_d/2m_dh\omega_{pd}/y_dk_BT_{de}}$ and $H_i = \sqrt{Z_{de}m_d/2m_dh\omega_{pd}/y_dk_BT_{de}}$ are quantum parameters. The normalized shear and bulk viscosities are $\eta^* = \eta\omega_{pd}/y_dk_BT_{de}$ and $\kappa^* = \kappa\omega_{pd}/y_dk_BT_{de}$.

We will derive a general linear dispersion relation for strongly coupled plasmas to identify different collective modes. In this paper, we will focus on the modes propagating perpendicular to the applied magnetic field. Here, the transverse perturbations are very weak and can be regarded as higher order perturbations. Accordingly, we have $\nu_d \ll \nu_d$, $E_y$, and $k_y \ll k_x$. By linearizing Eqs. (1)–(4), (7), and (8), we obtain the following dispersion relation:

$$\frac{\omega^2}{k^2} (1 + \Omega_d) + \frac{i\omega \eta}{1 - i\omega \tau_m} - \frac{1}{2} \left[ aH_c^2 + (1 + \alpha)H_i^2 \right] k^2 - \frac{5}{3} \sigma_e \eta + \frac{5}{3} \left( 1 + \alpha \right) \sigma_i + 1 + \frac{1}{\beta} = 0.$$  

(9)

Dispersion relation (9) only describes the fast magnetosonic waves propagating perpendicular to the applied magnetic field. We find that dispersion relation (9) for fast magnetosonic waves is greatly modified by the quantum diffusion effects, nondegenerate pressure, and dust fluid viscosity ($\propto \tau_m$ or $\eta$) associated with the strong coupling of the dust particles. As mentioned before, the memory function $\tau_m$ (proportional to the dust fluid viscosity) defines two characteristic time scales to distinguish between two classes of wave modes, namely, "hydrodynamic modes" ($\omega \tau_m \ll 1$) and "kinetic modes" ($\omega \tau_m \gg 1$).

In the hydrodynamic regime ($\omega \tau_m \ll 1$), dispersion relation (9) reduces to

$$\frac{\omega^2}{k^2} (1 + \Omega_d) = -i\omega \eta + \frac{1}{2} \left[ aH_c^2 + (1 + \alpha)H_i^2 \right] k^2 + \frac{5}{3} \sigma_e \eta + \frac{5}{3} \left( 1 + \alpha \right) \sigma_i + 1 + \frac{1}{\beta} = 0.$$  

(10)

Assuming that the wave frequency has real and imaginary parts, i.e., $\omega = \omega_r + i\omega_i$, and the wave number $k$ is real, separating the real and imaginary parts, we obtain the following from Eq. (10):

$$\omega_r = \frac{k}{\sqrt{1 + \Omega_d^2}} \left[ \frac{1}{2} \left( aH_c^2 + (1 + \alpha)H_i^2 \right) k^2 + \left( \frac{5}{3} \sigma_e \eta + \frac{5}{3} \left( 1 + \alpha \right) \sigma_i + 1 + \frac{1}{\beta} \right) \eta \right]^{1/2},$$  

(11)

$$\omega_i = -\frac{1}{2} \left( 1 + \Omega_d^2 \right) k^2 \eta.$$  

(12)

Consequently, the magnetosonic waves suffer viscous effects in the hydrodynamic regime, which were also observed in strongly coupled dusty plasmas with Boltzmann distributed electrons and ions. In the long-wavelength limit, the phase velocity of magnetosonic waves (obtained from the real part of $\omega$) assumes a constant value, i.e., the wave becomes dispersionless. From Eq. (11), we also find that the magnetosonic wave frequency increases with increasing values of $k$.

In the upper panel of Fig. 1, we give a plot of the real part of $\omega$ with respect to $k$. It is found that as the pressure parameter $\sigma_e$ increases, the wave frequency increases with a cut-off at a larger wave number. From Eq. (11), one can also find that the wave has cut-off frequencies at $k = k_c$ as follows:

$$k_c = \left[ \frac{5}{3} \sigma_e \eta + \frac{5}{3} \left( 1 + \alpha \right) \sigma_i + 1 + \frac{1}{\beta} \right]^{1/2} \left( \frac{\eta^2}{4(1 + \Omega_d^2)^2} - \frac{1}{2} \left( aH_c^2 + (1 + \alpha)H_i^2 \right) \right).$$  

(13)

From Eq. (13), one can clearly see that this zero-frequency mode occurs due to the existence of dust viscosity, which is related to the first term of the denominator on the right side of Eq. (13). In the lower panel of Fig. 1, we also find that as the pressure parameter of ions $\sigma_i$ increases, the wave frequency increases with a cut-off at a larger wave number. In Fig. 1, we used the physical parameters $m_e = 9.1 \times 10^{-28}$ g, $m_i = 1836m_e$, $m_d = 10^6m_i$, $Z_{de} = 300$, $n_{e0} = 2.0 \times 10^{19}$ cm$^{-3}$, $n_{i0} = 5.6 \times 10^{18}$ cm$^{-3}$, and $n_{d0} = n_{e0} + Z_{de}n_{i0}$. $T_e \sim 10^5$ K and $T_d \sim 10^5$ K are ion and dust grain temperatures, respectively. The weakly coupled pressure parameters of electrons are given as $\sigma_e = 0.044$ (solid line), $\sigma_e = 0.448$ (dashed line), $\sigma_e = 1.795$ (dotted line), and $\sigma_e = 3.590$ (dashed-dotted line), which corresponds to the electron temperatures $T_e = 10^4$ K, $10^5$ K, $4 \times 10^5$ K, and $8 \times 10^5$ K, respectively. The lower panel of Fig. 1 is plotted by the parameters $T_i = 10^5$ K, $10^6$ K, $4 \times 10^5$ K, and $8 \times 10^5$ K, and the electron temperature is $T_e = 8 \times 10^5$ K, from which we can obtain the weakly coupled pressure parameter of ions $\sigma_i = 0.044$ (solid line), $\sigma_i = 0.448$ (dashed line), $\sigma_i = 1.795$ (dotted line), and $\sigma_i = 3.590$ (dashed-dotted line).
In Fig. 2, we also give the plot of the real part of \( \omega \) with respect to \( k \) to consider the quantum diffraction effects on the dispersion relation. One can find that as the quantum diffraction parameter of electrons \( H_e \) increases, the wave frequency decreases with a cut-off at a lower wave number. The cut-off frequencies at \( k = k_c \) can also be obtained from Eq. (11). The quantum diffraction parameters are given as \( H_e = 0.0019 \) (solid line), \( H_e = 0.0021 \) (dashed line), \( H_e = 0.0023 \) (dotted line), and \( H_e = 0.0024 \) (dashed-dotted line), which correspond to the number density \( n_{d0} = 5.6 \times 10^{19} \text{ cm}^{-3} \), \( n_{d0} = 9.6 \times 10^{19} \text{ cm}^{-3} \), \( n_{d0} = 15.6 \times 10^{19} \text{ cm}^{-3} \), and \( n_{d0} = 20.6 \times 10^{19} \text{ cm}^{-3} \), respectively. These dispersion curves in Figs. 1 and 2 show that the quantum diffraction and the weakly coupled pressure lead to dispersive correction and the changes in the phase velocity. We can also find that the wave modes have a smooth evolution trend from kinetic modes to the hydrodynamic mode with the increasing of the quantum effects although the physical parameters is quite different for the two wave modes.

On the other hand, for relatively high-frequency magnetosonic waves, we obtain the following dispersion relation for the kinetic wave modes (\( \omega \tau_m \gg 1 \)):

\[
\frac{\omega^2}{k^2} (1 + \Omega_d) = \frac{\bar{\eta}}{\tau_m} + \frac{1}{2} (\alpha H_e^2 + (1 + \alpha) H_i^2) k^2 + \left[ \frac{5}{3} \alpha \sigma_i + \frac{5}{3} (1 + \alpha) \sigma_j + 1 + \frac{1}{\beta} \right].
\]

Clearly, viscosity is no longer a dissipative effect; however, it modifies the dispersion curve. Furthermore, Eq. (14) shows that the phase speed of the wave remains greater than the effective dust thermal speed and assumes a constant value in the long-wavelength limit \( k \to 0 \). The kinetic wave modes have no cut-off frequencies. The kinetic magnetosonic wave frequency increases with increasing values of \( k \). As the nonlinear hydrodynamic modes are well known already, we focus on the kinetic modes in a nonlinear regime.

III. NONLINEAR MAGNETOSONIC WAVES

We note that the normalized relaxation time \( \tau_m \) in Eq. (5) represents two characteristic time scales, which describe two classes of wave modes, namely, the hydrodynamic modes and kinetic modes. In this section, we focused on the kinetic modes. We will use the reductive perturbation method, which was already used to investigate the nonlinear waves in a nonmagnetized dusty plasma, to investigate the nonlinear magnetosonic waves in a kinetic regime. As we mainly consider the effects of transverse perturbation on nonlinear magnetosonic solitary waves, we introduce stretched coordinates \( \tau = \varepsilon(x - v_p t), \) \( Y = \varepsilon^2 y, \) and \( T = \varepsilon^4 t, \) where \( \varepsilon \) is an arbitrary small parameter, and \( v_p \) is the phase velocity of solitary waves. The dependent variables, which are expanded in powers of \( \varepsilon \), have the form

\[
f = f^{(0)} + \sum_{n=1}^{\infty} \varepsilon^n f^{(n)},
\]

for \( n \), \( n_{d0}, B_{z0}, E_y, \) and \( \psi_{jk} \), where \( f^{(0)} = 1 \) for \( n, n_{d0}, B_{z0}, \) and \( E_y, \) and \( f^{(0)} = 0 \) for \( \psi_{jk} \) and \( E_x \). The other dependent variables \( E_x \) and \( \psi_{yp} \) are expanded in a different way in powers of \( \varepsilon \)

\[
g = \sum_{n=1}^{\infty} \varepsilon^{2n+1} g^{(n)},
\]

In what follows, we substitute the stretched coordinates and the expansions given in (15) and (16) into Eqs. (1)–(4), (7), and (8). In the lowest order of \( \varepsilon \), we obtain from Eqs. (1)–(4), (7), and (8) the following first-order quantities: \( n_{d1} = B_{z1}, \) \( \psi_{jk1} = v_p B_{z1}, \) and \( E_{y1} = \frac{\Omega_d}{\varepsilon} \psi_{jk1} B_{z1}. \) One can find that quasineutrality is satisfied from the first order quantities. The other relationships for the first order physical quantities are given as

\[
- \alpha v_{yj1} + (1 + \alpha) v_{ij1} - v_{dij1} = \frac{\beta v^2_{ij1} \Omega_d^2}{\beta \Omega_d} - 1 \partial B_{z1} \partial X,
\]

\[
\frac{\partial E_{y1}}{\partial X} = \left( \frac{v^2_{ij1} - \frac{\beta}{\varepsilon^2}}{v^2_{ij1} + v^2_{ij}} - \frac{1}{\varepsilon^2} \right) \tau_{ij1} - v_{dij1} \partial\Omega_d \partial X.
\]

where we have introduced the parameter \( \bar{\eta} = \tilde{\xi}^* + 4 \eta / 3 \). By using the first order results, one can obtain the dispersion relation as

\[
v_{ij}^2 = \frac{1}{\beta} \left[ \alpha \sigma_i (1 + \alpha) + 1 + \frac{\tilde{\xi}^*}{\varepsilon^2} \right].
\]

One can combine the second order expansion equations and use the first order physical quantities to derive a modified Kadomtsev-Petviashvili (mKP) equation as

\[
\frac{\partial}{\partial X} \left[ \partial \Phi + A \partial \partial \Phi + B \partial^3 \partial \Phi + 3 \partial^\prime \partial \Phi \right] + \delta \partial^\prime \partial \Phi = \Phi \left( \frac{\partial \Phi}{\partial X} \right)^2,
\]

where \( \Phi \) stands for \( B_{z1}. \) Here, the coefficients of nonlinearity, dispersion, transverse perturbation, and viscosity effects, i.e., \( A, B, \delta, \) and \( y \), are given as

\[
A = \frac{3 v^2_{ij} - 1 + 2 \frac{\beta}{\varepsilon^2} - \alpha \sigma_i (1 + \alpha) \frac{\beta}{\varepsilon^2}}{2 v_p (\Omega_d^2 + 1)},
\]
Using the above relationship, the KP equation without the strongly coupled effects and quantum effects. The KP equation can be used to investigate a soliton in a weak two dimensional nonlinear excitation, which stands for transverse perturbations. If we neglect the dust viscosity effect ($\eta = 0$), the coefficient $\gamma$ vanishes, and Eq. (20) reduces to the usual KP equation as

$$
\frac{\partial}{\partial X} \left[ \frac{\partial \phi}{\partial T} + A \frac{\partial \phi}{\partial X} + B \frac{\partial^3 \phi}{\partial X^3} + \frac{\delta}{2} \frac{\partial^2 \phi}{\partial Y^2} \right] = 0.
$$

(25)

The KP equation can have an analytical line soliton solution and lump solution, which depend on the sign of the transverse perturbation coefficient $\delta$. For our work, the KP equation is the KPII equation as the transverse perturbation $\delta$ is positive, which has a steady line-soliton solution.

Here, we neglect the strongly coupled effects to find the analytical line-soliton solution of (25). Then, we consider the physical quantity to be $\phi(\zeta)$, where $\zeta$ is defined as $\zeta = X + Y - V T$ with $V$ being the normalized velocity of steady magnetosonic solitary waves. Using the above relationship, the KP equation without the strongly coupled effects can be written as

$$
-(V - \delta) \frac{d \phi}{d \zeta} + A \frac{d \phi}{d \zeta} + B \frac{d^3 \phi}{d \zeta^3} = 0.
$$

(26)

With the help of boundary conditions $\phi \to 0$ and $d\phi/d\zeta \to 0$ at $\zeta \to \infty$, the solution of the steady state KP equation is calculated as

$$
\phi = \frac{3(V - \delta)}{A} \text{sech}^2 \left( \frac{\sqrt{V - \delta}}{4B} \zeta \right).
$$

(27)

The steady solution (27) is the well known line-soliton profile.

We mention that an exact analytic solution of Eq. (20) is much complicated. However, we can seek for an approximate time-dependent soliton solution of Eq. (20) with a small effect of $\eta$ and high-order transverse effects of $\delta$, which is confirmed from the stretched transverse coordinate $Y = e^\gamma$. To this end, we rewrite Eq. (20) after integrating with respect to $X$ as

$$
\frac{\partial \phi}{\partial T} + A \frac{d \phi}{d \zeta} + B \frac{d^3 \phi}{d \zeta^3} = \int_{-\infty}^{\infty} \left[ \gamma \left( \frac{\partial \phi}{\partial X'} \right)^2 - \frac{\delta}{2} \frac{\partial^2 \phi}{\partial Y'^2} \right] dX',
$$

(28)

where we have used the following boundary conditions: $\phi \to 0$, $d\phi/d\zeta \to 0$ as $X \to \pm \infty$. In order to study the effects of viscosity on the character of magnetosonic solitary waves, we consider the normalized solitary speed $V$ to be dependent on time $T$,

$$
\phi = \frac{3[V(T) - \delta]}{A} \text{sech}^2 \left( \frac{\sqrt{V(T) - \delta}}{4B} \zeta \right).
$$

(29)

To determine the time dependent speed $V(T)$, we use the momentum conservation law in the presence of viscosity as

$$
\frac{d l}{d T} = \gamma \int_{-\infty}^{\infty} \phi \left[ \int_{-\infty}^{\infty} \left( \frac{\partial \phi}{\partial X'} \right)^2 dX' \right] dX - \delta \int_{-\infty}^{\infty} \phi \left[ \int_{-\infty}^{\infty} \frac{\partial^2 \phi}{\partial Y'^2} dX' \right] dX,
$$

(30)

where the momentum of this system $I = (1/2) \int_{-\infty}^{\infty} \phi^2 dX$. Substituting Eq. (29) into Eq. (30), we obtain the analytical expression for time dependent speed as

$$
V(T) = \delta + \left( V_0 - \delta \right) \left( 1 - \frac{T}{T_0} \right)^{-\frac{1}{2}},
$$

(31)

where the parameters $T_0 = 5A\sqrt{B}/(V_0 - \delta)^{3/2}$ and $V_0$ is the velocity of the magnetosonic waves when $T = 0$. From the expression of the speed $V(T)$, one can see that the amplitude of magnetosonic solitary waves will change slowly with time, which is due to the small amount of dust viscosity. From Eq. (31), it can also be seen that as the wave amplitude increases, the propagation speed also increases in widening the pulse width.

To obtain the typical parameter for the strongly coupled parameter, we use the dense plasmas in the interiors of heavy planets, white dwarfs and neutron stars. In such dense astrophysical scenarios, the plasma density lies in the range $10^{22} - 10^{29}$ cm$^{-3}$, the temperature lies in the range $10^5$-$10^7$ K, and the magnetic field is estimated to lie in the range $10^8$-$10^{10}$ G. Here, we use the following parameters: $n_e = 9.1 \times 10^{-28}$ g, $m_i = 1836 + m_e$, $m_d = 10^9 m_e$, $Z_{eb} = 200$, $n_d = 2.0 \times 10^{28}$ cm$^{-3}$, $n_0 = 5.6 \times 10^{19}$ cm$^{-3}$, and $n_0 = n_{d0} + Z \delta d_{\Omega} d_{\Omega}$; $T_e \approx T_i \sim 10^8$ K and $T_d \sim 10^6$ K, where $T_e$, $T_i$, and $T_d$ are

![FIG. 3](image-url)
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