A New Algorithm for Size Optimization of the Truss Structures with Buckling Constraint using Finite Element Method

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Abstract. This paper proposed a way to solve the problem of the optimum size of the truss, taking into account the local buckling constraint of compression elements of the truss. The consideration of dynamic constraint for buckling increases the complexity of the iterative algorithm to solve the truss optimization problem, because the dynamic constraint expresses condition involving the cross-sectional variable. The author has established an iterative algorithm to optimize trusses with stresses constraints (under strength conditions for tensile elements, buckling conditions for compression elements) and displacements. The iterative algorithm is established based on the correlation coefficients of internal forces between elements. The constraints of the problem are established on the basis of the results of internal forces, displacement and governing equation by finite element method. Based on the established algorithm, the authors had written the program to solve the optimization problem of plane trusses with 10 and 17 elements, spreading the space trusses of 25 elements. Comparing the results of weight optimization of the trusses by the proposed method of this paper with the results based on genetic algorithms, there is no significant difference, especially when increasing the number of elements, the weight of the truss calculated by the author's method is significantly smaller.

1. Introduction

Trusses are common structures in construction due to their outstanding advantages in material saving and maximum utilization of structure load capacity. Optimization of trusses structure is a problem of seeking best solution in Preliminary Design stage. Size optimization of truss has been the subject of numerous studies over the years using a wide range of optimization methods. Most research works published on the size optimization of trusses consider only the use of static constraints [1,3-8]. The studies on size optimization of the trusses, which has included buckling constraints, were not widely conducted. Since the buckling constraint is dynamic one that varies in each iteration step corresponding to the design variable, the consideration of the buckling constraint increases the complexity of the problem. In recent years, it has been announced that the research team [9,10] used the optimal genetic algorithms to consider the buckling constraint.
In [13], the author has proposed an optimal search algorithm and set up the program of size optimization of truss in terms of static constraints on stress and displacement. The following article will develop and set up algorithms to solve the truss optimization problem including dynamic constraint on local buckling of the bar. On that basis, examples of weight optimization for a number of plane and spatial trusses were conducted.

2. Problem formulation

2.1. Determination of internal forces and displacement using finite element method

The procedure of solving truss’s problem using finite element method [2,11] include following main steps:

Discrete truss into elements, establish the displacement matrix, force matrix, stiffness matrix of all system.

Establish solving equations in global coordinates, taking into account boundary conditions. Solve system of equations to determinate nodal displacements in global coordinates:

\[ y = K^{-1}P \]  

Determining the stresses and internal forces

\[ \sigma = \frac{E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ \delta \end{bmatrix} \]  

\[ f = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ \delta \end{bmatrix} \]  

In which, \( K \) – stiffness matrix of truss; \( y \) – vector of nodal displacement; \( P \) – vector of nodal forces of truss.

2.2. Euler Buckling Constraints

Many optimal solutions for size optimization problems of trusses have small cross sections of elements subjected to large compression forces. It is hypothesised that these are weak points in the structure, therefore consequent effects of buckling should be tested during the optimization process to avoid unusable results. Since the Euler critical buckling load equation (3) considers cross sectional characteristics and the size optimization creates a new set of cross sections for all iterations for all elements, buckling needs to be checked for all iterations. The proposed Euler buckling constraint defined by Euler’s critical load is given in the following expressions:

\[ f_{icr}^\text{com} \leq f_i^\text{cr} = \frac{\pi^2 E I_i}{L_i^2} \quad \text{or} \quad \sigma_i \leq \sigma_i^\text{cr} = \frac{\pi^2 E}{\lambda_i^2} \quad \text{for } i=1,\ldots,n \]  

Where \( E \) is elastic modulus, \( f_i \) is axial compression force, \( f_i^\text{cr} \) is Euler’s critical load, \( I_i \) is the minimum cross-sectional moment of inertia, \( L_i \) is the length of the truss member, \( \sigma_i \) is compression stress, \( \sigma_i^\text{cr} \) is Euler’s critical compression stress, \( \lambda_i \) is slenderness ratio of cross section of the \( i^{th} \) element.

2.3. Structural optimization problem

In size optimization problems, optimization objective function is chosen to find minimum of truss weight \( G \).

Optimization objective function: \( G = \sum_{i=1}^{N} y_i L_i A_i \rightarrow \min \)  

In the case of homogeneous structures \( V = \sum_{i=1}^{N} L_i A_i \)
Where $N$ is number of truss element, $A_i$ is the area of the $i^{th}$ cross section, $y_i$ is specific weight of material of the $i^{th}$ truss element, $\sigma_i$ is stress of the $i^{th}$ truss element, $\sigma_{\text{ten}}^i$ is allowable stress in tension, $y_j$ is the displacement of the $j^{th}$ node, $[y]$ is allowable displacement of the node.

### 2.4. Algorithm for optimization

Results of determination of displacements, stresses, internal forces of truss elements can be expressed as functions of design variables $A$ as follows:

$$y_i = a_i A^{-1}; \quad \sigma_i = b_i A^{-1}; \quad f_i = c_i$$

(7)

Where $a$, $b$, $c$ are constants determined from equations using finite element method

According to strength of material constraints (elements with the same strength of material) it is received:

$$\frac{f_i}{A_i} = \frac{f_j}{A_j} = \ldots = \frac{f_k}{A_k}$$

(8)

From (7) and (8) can derive the relationship:

$$n_i = \frac{f_i}{A_i} c_i$$

(9)

According to (9) the relationship between design variables $A_i$ can be calculated from the correlation of element internal forces. Thereof, the authors proposed a new method to solve problems of weight minimization of trusses with constraints established using finite element method. The basis content of the method can be expressed in algorithm of seeking of optimum values of design variables by correlation between element internal forces. The authors call this method as "the method of correlation coefficient between element internal forces". The order of solving problems of weight optimization of trusses by method of correlation between element internal forces includes following steps:

Step 1: Preliminary selection of cross-section sizes for design variable groups. Displacements $y$, stresses $\sigma$, element internal forces $f$ is determined using finite element method in the case of that the elements have the same design variables $A$. Determine $A_{\text{min}}^{(j)}$ for design variable groups according to the constraints of strength material and displacement of all elements in groups as follows:

$$\begin{cases} y_i \leq [y] \\ \sigma_i \leq [\sigma] \end{cases} \Rightarrow A_{\text{min}}^{(j)} = \max \left( \frac{a_i}{y}; \frac{b_i}{\sigma} \right)$$

(10)

Preliminary determination of truss volume: $V_i = \sum_{n=1}^{N} L_i A_{(i)}^{(j)}$

Step 2: Repeat selection of cross-section sizes $A_i = n_i A_j$ of design variable group “$j$” according to correlation coefficient between element internal forces of design variable groups.

$$\bar{n}_j = \frac{\sum f_i}{N f_i}$$

(11)

Where $\sum f_i$ is total element internal force in the same group; $N$ is number of elements in the same group.

Checking the buckling constraints and choosing $A_i$ are carried out as follows:

If $A_i \leq \max(A_{\text{Euler}}^i, A_{\text{min}})$ then $A_i = \max(A_{\text{Euler}}^i, A_{\text{min}})$

And if $A_i = n_i A_j$

Where $A_{\text{Euler}}^i$ is determined by Euler’s buckling constraint, $A_i$ is minimum value depending on design requirement.
Determine truss volume: \[ V_2 = \sum_{j=1}^{N} L_j A_j^{(0)} \]

Determine the percentage of decrease of cross-section area.

\[ \frac{V_i - V_2}{V_i} \times 100\% \quad (12) \]

These steps are implemented iteratively to \( n \) circle until achieving required percentage of decrease of cross-section area. The iterative method according to correlation between element internal forces can be represented in the form of a diagram in figure 1.

![Iterative diagram using correlation coefficient between element internal forces](image)

**Figure 1.** Iterative diagram using correlation coefficient between element internal forces

3. Establishment of calculation programme for weight optimization of trusses using Matlab software

Based on the proposed above method of correlation coefficient between element internal forces the block diagram of algorithm is established (figure 2) for weight optimization of plane and space trusses with any design variable group using Matlab software. Truss consists from solid circular cross section elements with

\[ A_{\text{Euler}} = L \sqrt{\frac{2f}{\pi E}} \]
Figure 2. Block diagram of algorithm for weight optimization using method of correlation coefficient between element internal forces
4. Test examples and analyses
The authors used the above established programs for optimization to solve several examples of determination of optimum weight of plane and space trusses. For the purposes of this research, the 10-bar plane truss, 17-bar plane truss, and 25-bar space truss models, known as the typical test case studied by many researchers, were considered. Calculated results were compared with results implemented by other method in published studies [1,3,5,6,9].

The cantilever 10-bar plane truss with 10 independent design variables is shown in figure 3. The material characteristics are: modulus of elasticity $E=206842$MPa, density $\rho=2.7$g/cm$^3$. Members are constrained in stress $[\sigma]=\pm172$MPa and displacement $d=\pm50.8$mm. Two load cases were considered as follows:

- Load case 1 (LC1): $P_1=444$kN, $P_2=0$kN
- Load case 2 (LC2): $P_1=667$kN, $P_2=222$kN

**Figure 3.** 10-bar plane truss model

The 17-bar plane truss with 17 independent design variables is shown in figure 4. A single loading $P=444$kN is applied on the node 9. The material characteristics are: elastic modulus $E=206842$MPa, density $\rho=7.4$g/cm$^3$. The stress constraints were not considered in this example. A displacement limitation $d=\pm50.8$mm, minimal radius of cross section $d=4.5225$mm is the lower boundary of variables.

**Figure 4.** 17-bar plane truss model

The 25-bar space truss is shown in figure 5. The material of the truss elements is the same with the 10-bar truss. This space truss is subjected to the two loading conditions shown in table 4. The compressive and tensile stress limitations are shown in table 5. A displacement limitation $d=\pm88.9$mm. Minimal radius of cross section $d=1.433$mm is the lower boundary of variables. The problem was solved with two options of dividing design variables groups as below:

- Option 1 - 25 independent design variables
- Option 2 - 8 groups 1(A1), 2(A1-A5), 3(A6-A9), 4(A10-A11), 5(A12-A13), 6(A14-A17), 7(A18-A21), 8(A22-A25)

**Table 1.** Load conditions for the 25-bar truss problem

| Node | 1 | 2 | 3 | 6 |
|------|---|---|---|---|
| Load Components $P_x$, $P_y$, $P_z$ | 0, 20, -5 | 0, -20, -5 | 0, 0, 0 | 0, 0, 0 |
Table 2. Member stress limitations for the 25-bar space truss

| Grouping of variables | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Compressive stress limitations (MPa) | 241.91 | 79.91 | 119.31 | 241.95 | 241.95 | 46.60 | 47.98 | 76.40 |
| Tensile stress limitations (MPa) | 40    | 40    | 40    | 40    | 40    | 40    | 40    | 40    |

Figure 5. 25-bar space truss model

5. Results
The results of weight optimization of 10-bar plane truss for load condition 1 (LC1) and LC2 are shown in tables 1&2. The results of weight optimization of 17-bar plane truss are shown in tables 3. The results of weight optimization of 25-bar plane truss are shown in tables 4. Also provide the comparison between the optimization results published in the literature in two cases (with and without buckling constraint) and the present work. For bars that do not meet buckling conditions the values of their cross sections are given in bold. Comparison the efficiency of the optimization algorithm established by author based on the

Table 3. Optimization results of the 10-bar truss for LC1

| Radius of bar [mm] | [1] | [6] | [3] | [5] | [9] | [9] Present work |
|-------------------|-----|-----|-----|-----|-----|-----------------|
| 1                 | 78.991 | 79.169 | 83.067 | 79.050 | 77.561 | 48.724 | 91.335 |
| 2                 | 4.522 | 4.532 | 18.240 | 4.532 | 4.532 | 4.522 |
| 3                 | 68.883 | 69.030 | 68.577 | 69.089 | 71.614 | 116.328 | 91.335 |
| 4                 | 55.741 | 55.913 | 54.001 | 56.178 | 60.994 | 70.763 | 76.803 |
| 5                 | 4.522 | 4.532 | 18.240 | 4.532 | 4.532 | 4.522 |
| 6                 | 10.663 | 10.642 | 18.240 | 10.868 | 4.677 | 44.093 | 4.522 |
| 7                 | 39.063 | 39.159 | 40.456 | 39.089 | 36.948 | 73.445 | 99.601 |
| 8                 | 65.646 | 65.724 | 68.577 | 65.618 | 69.157 | 92.320 | 99.601 |
| 9                 | 66.269 | 66.493 | 67.216 | 66.498 | 66.732 | 29.761 | 99.601 |
| 10                | 1.430 | 4.532 | 18.240 | 4.532 | 4.532 | 105.932 | 4.522 |
| Weight [kg]       | 2294.56 | 2295.64 | 2490.55 | 2295.61 | 2327.91 | 4759.458 | 5023.115 |
### Table 4. Optimization results of the 10-bar truss for LC2

| Radius of bar [mm] | Without buckling constraint | With buckling constraint |
|-------------------|----------------------------|-------------------------|
|                   | [1] | [6] | [3] | [5] | [9] | [9] | Present work |
| 1                 | 69.058 | 69.509 | 79.142 | 69.505 | 69.781 | 45.217 | 84.997 |
| 2                 | 4.522 | 4.532 | 4.532 | 4.532 | 4.532 | 50.072 | 4.522 |
| 3                 | 72.328 | 72.072 | 70.204 | 70.846 | 71.064 | 114.176 | 96.575 |
| 4                 | 54.763 | 52.411 | 53.620 | 54.529 | 57.515 | 81.026 | 76.803 |
| 5                 | 4.522 | 4.532 | 4.532 | 4.532 | 4.532 | 50.072 | 4.522 |
| 6                 | 20.070 | 20.113 | 10.133 | 20.241 | 20.110 | 68.914 | 64.582 |
| 7                 | 50.084 | 50.471 | 39.245 | 50.328 | 52.322 | 61.104 | 110.227 |
| 8                 | 50.858 | 51.319 | 66.447 | 51.047 | 56.538 | 102.370 | 83.753 |
| 9                 | 64.500 | 64.615 | 66.447 | 64.652 | 64.193 | 48.840 | 99.602 |
| 10                | 4.522 | 4.532 | 4.532 | 4.532 | 4.532 | 50.072 | 4.522 |
| Weight [kg]       | 2120.73 | 2121.81 | 2298.50 | 2122.04 | 2165.65 | 5104.39 | 5260.504 |

### Table 5. Optimization results of the 17-bar truss

| Radius of bar [mm] | Without buckling constraint | With buckling constraint |
|-------------------|----------------------------|-------------------------|
|                   | [1] | [9] | [9] | Present work |
| 1                 | 56.996 | 53.786 | 50.398 | 57.321 |
| 2                 | 4.646 | 21.095 | 14.887 | 2.374 |
| 3                 | 49.693 | 53.020 | 54.496 | 49.642 |
| 4                 | 4.532 | 4.532 | 22.309 | 1.433 |
| 5                 | 40.818 | 45.165 | 42.272 | 40.533 |
| 6                 | 33.705 | 26.918 | 4.962 | 39.847 |
| 7                 | 49.316 | 45.359 | 43.563 | 49.642 |
| 8                 | 4.532 | 4.532 | 12.225 | 1.433 |
| 9                 | 42.421 | 36.208 | 31.510 | 40.532 |
| 10                | 4.532 | 27.110 | 33.587 | 2.374 |
| 11                | 28.914 | 35.305 | 42.005 | 30.757 |
| 12                | 4.532 | 17.666 | 33.649 | 1.831 |
| 13                | 34.166 | 34.309 | 31.083 | 39.887 |
| 14                | 28.693 | 31.054 | 36.328 | 30.757 |
| 15                | 33.866 | 28.286 | 44.067 | 39.887 |
| 16                | 4.532 | 21.496 | 46.529 | 2.374 |
| 17                | 33.853 | 18.808 | 34.937 | 39.887 |
| Weight [kg]       | 1169.711 | 1183.071 | 1507.665 | 1323.831 |

### Table 6. Optimization results of the 25-bar truss

| Radius of bar [mm] | Without buckling constraint | With buckling constraint |
|-------------------|----------------------------|-------------------------|
|                   | [1] | [9] | [9] | Present work | Present work |
| Members           | Cross section | Grouping | Cross section | Grouping | Non-Grouping |
| 1                 | 1.430 | 1.433 | 11.741 | 6.381 | 6.044 |
| 2                 | 20.155 | 16.014 | 34.151 | 35.375 | 38.733 |
| 3                 | 20.155 | 16.014 | 34.151 | 31.969 | 38.733 |
| 4                 | 20.155 | 16.014 | 34.151 | 29.468 | 38.733 |
| 5                 | 20.155 | 16.014 | 34.151 | 49.004 | 38.733 |
| 6                 | 24.759 | 28.185 | 32.821 | 37.634 | 40.393 |
| 7                 | 24.759 | 28.185 | 32.821 | 33.520 | 40.393 |
| 8                 | 24.759 | 28.185 | 32.821 | 41.167 | 40.393 |
Table 6. Optimization results of the 25-bar truss – continued

| Radius of bar [mm] | Without buckling constraint | With buckling constraint |
|-------------------|----------------------------|--------------------------|
|                   | [1] | [9] | [9] | Present work | Present work |
| Members | Cross section | Grouping | Cross section | Grouping | Non-Grouping |
| 9 | 24.759 | 28.185 | 32.821 | 42.807 | 40.393 |
| 10 | 1.430 | 1.433 | 3.851 | 9.107 | 8.326 |
| 11 | 1.430 | 1.433 | 3.851 | 19.558 | 8.326 |
| 12 | 1.430 | 1.433 | 6.559 | 9.450 | 18.252 |
| 13 | 1.430 | 1.433 | 6.559 | 25.703 | 18.252 |
| 14 | 11.798 | 12.985 | 23.082 | 16.034 | 14.328 |
| 15 | 11.798 | 12.985 | 23.082 | 10.162 | 14.328 |
| 16 | 11.798 | 12.985 | 23.082 | 16.398 | 14.328 |
| 17 | 11.798 | 12.985 | 23.082 | 10.659 | 14.328 |
| 18 | 18.499 | 21.652 | 37.515 | 25.449 | 27.478 |
| 19 | 18.499 | 21.652 | 37.515 | 30.100 | 27.478 |
| 20 | 18.499 | 21.652 | 37.515 | 14.015 | 27.478 |
| 21 | 18.499 | 21.652 | 37.515 | 24.109 | 27.478 |
| 22 | 23.340 | 20.925 | 34.442 | 22.761 | 18.687 |
| 23 | 23.340 | 20.925 | 34.442 | 14.684 | 18.687 |
| 24 | 23.340 | 20.925 | 34.442 | 13.607 | 18.687 |
| 25 | 23.340 | 20.925 | 34.442 | 20.753 | 18.687 |
| Weight [kg] | 247.153 | 261.269 | 690.649 | 530.175 | 595.596 |

Figure 6. Comparison chart of results of truss optimization with buckling constraint

6. Conclusions

The optimization results showed that more than one bar do not meet the buckling condition in tested solutions without buckling constraints. In tested solutions with buckling constraint all bar cross sections were increased, but they meet buckling conditions. Therefore, it can be concluded that a buckling condition is necessary in size optimization of truss structures.

The results of weight optimization of plane and space trusses obtained from optimization algorithm established by the authors are negligibly different in comparison with the results of optimization obtained from other methods. The weight of trusses obtained from optimization results of this method...
is smaller in comparison with results obtained from other methods in the case of increasing the number of truss members. The division of design variables groups also affects the results of problem.

The method of correlation coefficient between element internal forces established by the authors is a simple and effective method for weight optimization of plane and space trusses with dynamic constraint for buckling. In the next studies, the authors will continue to develop the established algorithms to solve problems of shape and structure optimization of trusses.

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