Phi Mesons from a Hadronic Fireball

Peter Filip\(^1\) and Evgeni E. Kolomeitsev\(^2\)

\(^1\)Max-Planck-Institut für Physik, D-80805 Munich, Germany
\(^2\)European Centre for Theoretical Studies in Nuclear Physics and Related Areas Villa Tambosi, I-38050 Villazzano (TN) and INFN, G.C. Trento, Italy

Production of \(\phi\) mesons is considered in the course of heavy-ion collisions at SPS energies. We investigate the possible difference in momentum distributions of \(\phi\) mesons measured via their leptonic \((\mu^+\mu^-)\) and hadronic \((K^+K^-)\) decays. Rescattering of secondary kaons in the dense hadron gas together with the influence of in-medium kaon potential can lead to a relative decrease of a \(\phi\) yield observed in the hadronic channel. We analyze how the in-medium modifications of meson properties affect apparent - reconstructed momentum distributions of \(\phi\) mesons. Quantitative results are presented for central Pb+Pb collisions at \(E_{\text{beam}} = 158\) GeV/A.

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I. INTRODUCTION

Growing body of experimental information on fixed-target nucleus-nucleus collisions from AGS and CERN-SPS accelerators provides a reliable basis for the systematic investigation of strongly interacting hadronic matter under extreme conditions. Particularly, production of particles containing strange quarks is expected to reflect the reaction dynamics at the early stage of collisions. In this context, \(\phi\) mesons, particles consisting mainly of \(s\bar{s}\) pairs, are of a special interest. Since the interaction of \(\phi\) mesons with non-strange hadronic matter is suppressed according to OZI rule, \(\phi\) mesons are expected to decouple easily from the hadronic fireball.

Several calculations have been done for \(\phi\) meson properties in dense hadronic environment (mainly at higher baryon densities) \([1, 2, 3, 4, 5, 6, 7]\). It has been found that modifications of the \(\phi\) width and mass are sensitive to the strangeness content of the surrounding medium.

Recently, new experimental data from CERN-SPS on \(\phi\) meson production in central Pb+Pb collisions at 158GeV/A beam energy became available. Advantage of the rich experimental program at CERN is that it allows to study the \(\phi\) production via different \(\phi\) decay channels. Results of NA49 collaboration \([8]\) are based on the \(\phi\) meson identification via its hadronic decay \(\phi \rightarrow K^+K^-\), while NA50 collaboration has recently reported \([9]\) about preliminary analysis of \(\phi\) mesons identified via the dileptonic decay channel \(\phi \rightarrow \mu^+\mu^-\).

The purpose of this paper is to show, that the \(\phi\) meson production spectra reconstructed via \(K^+K^-\) and \(\mu^+\mu^-\) decay channels can be significantly different up to the level of present experimental observations.

After introductory considerations in Section II we derive in Section III expressions for momentum distributions of \(\phi\) mesons detected via \(KK\) and \(\mu\mu\) channels. We take into account rescattering of decay kaons and in-medium modification of \(\phi\) meson properties discussed in Section IV. Numerical results are presented in Sections V and VI and conclusions are drawn in Section VII.

II. PHI MESONS IN HADRON GAS

For our estimates of the observed \(\phi\) meson spectra we assume a simple two-stage picture of central heavy ion collisions, which is close to those considered within cascade-transport \([10, 11]\), hydrodynamical \([12, 13, 14]\) and thermodynamical approaches \([15, 16, 17]\). Initial stage of a collision is characterized by temperature close to the QCD phase transition \(T \gtrsim T_c \approx 170 \pm 10\) MeV. Then the system expands up to the point, when numbers of different kinds of particles freeze in - a chemical freeze-out. Thermodynamical parameters of this stage could be obtained by fitting the final total hadron multiplicities. Typical temperature is found to be \(T_{\text{chem}} \sim 160 \pm 10\) MeV. During the second stage of the expansion, elastic scatterings change momentum distributions of hadrons until they cease and distributions freeze in. The freeze-out temperature can be extracted from a simultaneous fit to the single-particle \(mT\)-spectra of different particles supplemented by the analysis of particle correlation data. According to Refs. \([11, 14, 20, 21]\) one has \(T = T_{\text{therm}} \sim 110 \pm 30\) MeV.

In our considerations we assume that the fireball created in heavy-ion collision consists, mainly, of pions, kaons and excited mesonic resonances in central rapidity region.

Mean free path \(\lambda_{\phi}\) of \(\phi\) mesons in hadron gas is estimated in Ref. \([11]\). Comparison with mean free paths of pions and kaons, \(\lambda_{\pi,K}\), from Ref. \([22, 23]\) gives \(\lambda_{\pi,K} \lesssim \lambda_{\phi}\) for temperatures \(T_{\text{therm}} < T < T_{\text{chem}}\). Hence, one can expect that \(\phi\) mesons decouple from the pion-kaon subsystem at some earlier stage between the chemical and thermal freeze-out. Upon this stage \(\phi\) mesons stream freely out from the fireball. Pions and kaons, on the other hand, may still participate in mutual secondary interactions up to the stage of total thermal freeze-out. Therefore, if a \(\phi\) meson decays inside a fireball via hadronic channel we have to take into account possible interaction of its decay products with surrounding hadronic environment.
As a result of secondary interactions, the reconstructed invariant mass of a given pair of $\phi$ daughter kaons falls out from the original $\phi$ meson peak into the region identified as a combinatorial background. Therefore, the $\phi$ mesons decaying in medium, can be partially unrecognized in experimental analysis of $K^+K^-$ pairs. Together with negligible final state interaction of secondary dimuons originating from $\phi \rightarrow \mu^+\mu^-$ decays this behaviour results in the relative suppression of $\phi$ meson yield observed via hadronic $K^+K^-$ channel. Such mechanism has been quantitatively studied in Ref. [27] where suppression at the level 40–60% has been obtained from simulation using RQMD code.

In this paper we discuss effects which may enhance the suppression of observed $\phi$ mesons identified via kaon channel. Possible increase of a $\phi$ meson width in medium will enlarge the probability of $\phi$ decay inside a fireball enhancing thus consequences of mechanism studied in Ref. [27]. Alternatively, we consider a possibility that the kaon decay channel of a $\phi$ meson becomes kinematically quenched in the medium. Additionally we argue that substantial relative difference in properties of $K^+$ and $K^-$ meson in hadronic environment caused by the isospin asymmetry and/or by the large baryonic admixture, would also prevent the reconstruction of $\phi$ mesons decaying into kaon pairs inside the medium.

III. DISTRIBUTION OF $\phi$ DECAY PRODUCTS

Let us denote the phase-space distribution of $\phi$ mesons in the center-of-mass system of two colliding nuclei at freeze-out as $f_\phi(\vec{x}, \vec{p})$. Then the primary momentum distribution of $\phi$ mesons is given by

$$\eta_\phi(p) = \int_{\Sigma} d^3\sigma^\mu p_\mu f_\phi(\vec{x}_\phi, \vec{p}_\phi),$$

where integration goes over the fireball volume within a freeze-out hyper-surface $\Sigma$ (surface normal vector $d^3\sigma^\mu$ contracted with a $\phi$ meson momentum $p^\mu$) [23]. Here we assume the case of a time-like freeze-out hyper-surface, $d^3\sigma^\mu p_\mu > 0$, which is relevant for applications below (for discussions of alternative cases cf. [24]). In the absence of any in-medium modifications of decay products and final state rescattering the shape of observed momentum distributions $\eta_\phi(p) \Gamma^\mu_\mu/\Gamma_{tot}$ of muon pairs ($\mu^+\mu^-$) and $\eta_\phi(p) \Gamma^\ast_{KK}/\Gamma_{tot}$ of kaon pairs ($K^+K^-$) would be the same. Here $\Gamma_{tot}$ is $\phi$ meson total width and $\Gamma^\ast_{KK}$ are the $\phi$ partial decay widths in kaon and muon decay channels. Accordingly, in the experimental analysis, $\phi$ meson distribution is reconstructed from momentum distribution of decay products multiplied by the corresponding inverse branching ratio.

In this section we derive expressions for the apparent momentum distributions of $\phi$ mesons reconstructed via kaon and dimuon decay channels taking into account possible modifications of meson properties in medium and consequences of $\phi$ meson rescattering.

Let us consider $\phi$ meson suffered the last interaction at position $\vec{x}_\phi$ inside the hadronic fireball. The probability, that $\phi$ meson lives for a time $t$ is

$$D_\phi(t) = \exp \left[ -\int_0^t \bar{\Gamma}_{tot}(t') dt' \right], \quad (2)$$

where $\bar{\Gamma}_{tot} = \Gamma_{tot} m_\phi/E_\phi$ is total width of a moving meson with energy $E_\phi = (m^2_\phi + p^2)_{1/2}$. Asterisk * denotes in-medium values of the quantities.

After traveling for a given time $t$ with velocity $\vec{v}_\phi = \vec{p}/E_\phi$ the $\phi$ meson decays at the position $\vec{x}_\phi + \vec{v}_\phi t$ with a probability $\Gamma^\ast_{KK}(t)/\Gamma_{tot}(t)$ into two kaons. In-medium values of widths $\Gamma^*$ are determined by the current local temperature and density of the system. Daughter kaons from $\phi$ meson decay have momenta $\pm p_{K\bar{K}} n_{K\bar{K}}$ in the rest frame of the $\phi$ meson. Value of $p_{K\bar{K}}$ follows from equation $m_\phi = \omega^\ast_{K\bar{K}}(p_{K\bar{K}}) + \omega^\ast_{\bar{K}K}(p_{K\bar{K}})$ where $\omega^\ast_{K\bar{K}}(p)$ and $\omega^\ast_{\bar{K}K}(p)$ are in-medium spectra of kaons and anti-kaons. In the center-of-mass system of two colliding nuclei (CMS) the kaon momenta are equal to

$$\vec{p}^\pm_K = \frac{\vec{p}_\phi}{2} \pm \delta\vec{p},$$

$$\delta\vec{p} = p_{KK} \{ n_{K\bar{K}} + \vec{n}_\phi (\gamma_\phi - 1) (\vec{n}_K \cdot \vec{n}_\phi) \},$$

where $\gamma_\phi = (1 - v^2_\phi)^{-1/2}$. The unit vector $n_{K\bar{K}}$ is uniformly distributed in the $\phi$ rest frame, direction of $n_{\phi}$ is determined by $\phi$ meson momentum $\vec{p}_\phi$ in CMS.

For a successful identification of $\phi$ mesons in invariant mass spectrum of observed $K^+K^-$ pairs it is essential that momenta of daughter kaons do not change while leaving the hadronic fireball. Here we investigate two mechanisms which may change momenta of daughter kaons: rescattering in surrounding hadronic environment and change of momentum due to in-medium $K$ meson potential.

Probability that secondary kaon and anti-kaon leave a fireball without rescattering is determined by their mean free paths $\lambda_{K\bar{K}}$, the time which kaons need to reach the fireball border $\tau^\pm_{K\bar{K}}$, and their velocities $\vec{v}^\pm_{K\bar{K}} = \vec{p}^\pm_K/\omega^\ast_{K\bar{K}}$ (in CMS) as follows:

$$P_\lambda(t) = \exp \left[ -\frac{\tau^\pm_{K\bar{K}}}{\lambda_{K\bar{K}}(t')} - \frac{\tau^\ast_{K\bar{K}}}{\lambda^\ast_{K\bar{K}}(t')} \right]. \quad (4)$$

Thus, the probability to register $\phi$ meson in the kaon channel when it decays in medium can be expressed as:

$$P_1 \cdot (1 - P_\lambda(t)) + P_{\Pi} \cdot P_\lambda(t), \quad (5)$$

where $P_1$ and $P_{\Pi}$ are probabilities to identify a $\phi$ meson from rescattered ($P_1$) and non-rescattered ($P_{\Pi}$) kaons.

Single rescattering of one of the kaons changes momentum of the kaon pair by vector $\Delta \vec{p}_{\text{scat}}$, with $|\Delta \vec{p}_{\text{scat}}| \sim p_T$
being a average thermal momentum of pions. Correspondingly the invariant mass changes by $\Delta M^2_{\text{rescat}} \sim p_T^2$. At high temperatures ($T \sim m_\pi$) average thermal momentum of pions is $p_T \sim 400$ MeV and hence a single rescattering may shift $M_{KK-}$ far from the $\phi$ meson mass. In this case we put $P_{\Pi} = 1$.

Neglecting in-medium effects, one takes $P_{\Pi} = 1$, assuming that without a hard rescattering all kaon pairs from $\phi$ decays can be identified. However the mechanism of particle properties can provide another mechanism preventing the invariant mass changes the invariant mass and momentum of the pair.

Going out of medium, kaon stays on the same energy level, and its momentum outside the fireball becomes

$$\vec{q}_{\alpha}^2 = \vec{p}_{K}^2 + \omega^2_{K,K}(p) t^2 - m^2_K p^2_{KK}/p^2_{K} \quad (\alpha = K, K),$$

where the kaon energy $\omega_{K}(p)$ is evaluated at the moment of $\phi$ decay. As a result, momentum of the kaon pair is changed by $\Delta \vec{p}_{\text{pot}} \approx \Delta \omega^2_{K} \vec{p}_{K}^2/p^2_{K} + \Delta \omega^2_{K} \vec{p}_{K}^2/p^2_{K} ^2$, where we put $\Delta \omega^2_{K} \ll p^2_{K}$ and $\Delta \omega^2_{K} = \omega^2_{K}(0) - m^2_K$. The invariant mass shift is then given by $\Delta M^2_{\text{pot}} = (\Delta \vec{p}_{\text{pot}})^2$.

In the nucleon free, isospin symmetric meson gas, in-medium spectra of kaons and anti-kaons are identical $\omega_{K}(p) = \omega_{K}(p)$ and the invariant mass shift reduces to $\Delta M^2_{\text{pot}} = -\frac{2 \Delta \omega^2_{K}}{2 p_{K}} \left( \frac{1}{2} \frac{p_K^4 + 2 \Delta p^2 \omega^2_{K} }{p_K^4 - 2 \Delta p^2} \right).$ [6]

This expression can be interpolated between the limit cases $p_K \gg p_{KK}$ and $p_K \ll p_{KK}$ as $\Delta M^2_{\text{pot}} \approx -\Delta \omega^2_{K} \frac{4 p^2_{K} \Delta p^2}{p^2_{K} + 6 p^2_{KK}}$.

Therefore in the symmetrical case $\Delta \vec{p}_{\text{pot}}$ and $\Delta M^2_{\text{pot}}$ vanish for small $p_K$. Moreover in the isospin symmetric case kaons suffer only a small mass modification ($\lesssim 30$ MeV for $T \sim m_\pi$). Hence, the release of kaons from a small potential well formed inside a fireball does not affect the momentum and invariant mass of a pair strongly enough to prevent a $\phi$ meson reconstruction. For this case we take $P_{\Pi} \approx 1$.

In the case of rather strong isospin asymmetry and/or significant baryonic admixture $K^+$ and $K^-$ mesons can have quite different spectra in medium [23]. This may lead to a wide spread of the kaon pair invariant masses, making the reconstruction of $\phi$ mesons decaying inside fireball impossible ($P_{\Pi} \rightarrow 0$). Note, that in this case our results are insensitive to the details of kaon propagation in medium. They depend only on the total $\phi$ meson width via function $D_\phi(t)$.

Finally the probability that a $\phi$ meson created at position $x_\phi$ will be detected via the kaon channel is:

$$\int_0^\infty dt \Gamma_{KK-}^*(t) D_\phi(t) P_\lambda(t) P_{\Pi},$$

where $\tau_\phi$ is the time of flight of a $\phi$ meson through the fireball, for $t > \tau_\phi$ we have $P_\lambda = 1 = P_{\Pi}$ and $D_\phi(t) = D_\phi(\tau_\phi) \exp(-\int_{-\infty}^{\tau_\phi} dt \Gamma_{KK-}^*(t) D_\phi(t) P_\lambda(t) P_{\Pi})$.

$$D_\phi(t) = \int_0^\infty dt \Gamma_{KK-}^*(t) D_\phi(t) P_\lambda(t) P_{\Pi}. \quad (6)$$

Note that according to this definition $\eta_\phi(p) = 1$.

Consideration of the di-muon pair momentum distribution is more straightforward. With a change of total $\phi$ width in medium the branching ratio of the $\phi \rightarrow \mu\mu$ decay $\Gamma_{\mu\mu}/\Gamma_{\phi}$ changes too. Therefore, the apparent $\phi$ distribution reconstructed in the muonic channel is:

$$\eta_\mu(p) = \langle \eta_\phi(p) + \int_0^{\tau_\phi} \frac{\Gamma_{\phi}}{\Gamma_{\phi}} dt D_\phi(t) \rangle P_\mu(t) P_{\Pi}.$$

$$\eta_\mu(t) = \int_0^{\frac{\int dt D_\phi(t) P_\mu(t)}{4 \pi}} d\Omega \tilde{\sigma}_\phi(t) P_\mu(t)(\cdots) \rangle.$$ (7)

For a direct comparison with experimental results on $m_T$ distributions of identified $\phi$ mesons, one has to integrate over the rapidity interval accessible to the experiments, utilizing $p = \sqrt{m_\phi^2 \cosh^2 y - m_\phi^2}$. Dependence of the ratio (8) on $m_T$ reads

$$\mathcal{R}(m_T) = < \eta_\phi(p) >_y / < \eta_\mu(p) >_y.$$ (10)

We also define ratio of the apparent and primary $\phi$ momentum distributions for $K^+K^-$ and $\mu^+\mu^-$ decay channels as a function of $m_T$:

$$\mathcal{R}_{K,\mu}(m_T) = < \eta_{K,\mu}(p) >_y / < \eta_\mu(p) >_y.$$ (11)

### IV. φ DECAYS IN MEDIUM

Main hadronic decay channels of $\phi$ meson are $\phi \rightarrow K\bar{K}$ and $\phi \rightarrow \rho\pi$. Let us now consider the change of a $\phi$ decay
width in a hot meson gas due to the modification of kaon, pion and $\rho$-meson properties. In-medium properties of pions can be effectively incorporated by a small mass shift $m_\pi^* = m_\pi + \delta m_\pi$ with $\delta m_\pi << m_\pi$. Similar behaviour is expected for kaons.

Spectral function of $\rho$ meson in medium was extensively investigated in the context of di-lepton production in heavy-ion collisions at SPS energies [24, 25, 26]. In baryonic matter, coupling of $\rho$ mesons to resonance-nucleon-hole modes [23] together with the modification of pions, play dominant role. The $\rho$ meson becomes very broad in medium and its spectral density strength is driven effectively to lower energies in analogy to Brown-Rho-scaling picture [31]. In purely mesonic systems the $\rho$ mass is found to be almost independent of the temperature due to cancellation of $\pi-\pi$ and $\pi-\pi$-a1-loop contributions [23]. The $\rho$ width, on the other hand, is expected to increase in meson gas considerably e.g. by 80 MeV at $T=150$ MeV and by 160 MeV at $T=180$ MeV [32].

The partial width of the $\phi \rightarrow K\bar{K}$ decay in medium depends on the kaon mass $m_K$ as:

$$\Gamma_{KK}^\star = \frac{2}{3} \rho_{KK}^3 \frac{m_\rho^2}{p_{cm}(m_\rho^*, m_k^*, m_k^*, m_k)} \frac{m_\pi^2}{p_{cm}(m_\pi^*, m_k, m_k)}, \quad \text{(11)}$$

where $p_{cm}(s, m_1, m_2)$ is a kaon momentum in the rest frame of $\phi$ meson decay, obeying the equation: $\sqrt{s} = \sqrt{m_1^2 + p_{cm}^2 + m_2^2 + \rho_{cm}^3}$. Assuming isospin symmetry we have $m_K = (m_K^+ + m_K^-)/2 = 495.6$ MeV. Then vacuum width is equal to $\Gamma_{KK}^\star = (\Gamma_{KK}^\star + \Gamma_{KK}^{\star,2}/2 = 1.84$ MeV. Note that for $\delta m_K > 0$ the in-medium width $\Gamma_{KK}^\star$ decreases fast and vanishes for $\delta m_K \approx 14$ MeV. The second hadronic decay channel $\phi \rightarrow \pi^+\pi^-$ is effected by the decrease of the $\rho$ meson mass, increase of the $\rho$ meson width and the Bose-Einstein enhancement factor for pions. For the $\phi$ meson at rest we write

$$\Gamma_{\rho\pi}^\star = \Gamma_{\rho\pi}^0 \kappa_{\rho\pi}(m_\rho^*, \Gamma_{\rho\pi}^\star, T)/\kappa_{\rho\pi}(m_\rho^0, \Gamma_{\rho\pi}^\star, 0), \quad \text{(12)}$$

and

$$\kappa_{\rho\pi}(m_\rho^0, \Gamma_{\rho\pi}^\star, T) = \int_{2m_\pi}^{m_\rho^0 - m_\pi} \frac{d\omega}{\pi} \left[ (\omega - m_\rho^0)^2 - 2m_\pi^2 \right]^{1/2} \times$$

$$(1 + n_\pi(\omega)) \frac{\omega}{(\omega - E_\rho(m_\rho^0))^2 + \omega^2 \Gamma_{\rho\pi}^\star^2 / (4m_\rho^0)^2}, \quad \text{(13)}$$

where $m_\rho^*$ and $\Gamma_{\rho}^*$ are in-medium $\rho$ meson mass and width, respectively. Vacuum values are $m_\rho^0 = 770$ MeV, $\Gamma_{\rho}^\star = 150$ MeV, and $\Gamma_{\rho\pi}^\star = 0.75$ MeV. We use the constant width approximation for the $\rho$ meson spectral density and denote $E_\rho(m_\rho) = (m_\rho^2 + m_\pi^2 - m_\rho^2)/2m_\rho$. The Bose-Einstein distribution of pions with temperature $T$ is $n_\pi(\omega)$. In the zero-width limit we obviously have $\kappa_{\rho\pi}(\Gamma \rightarrow 0) = p_{cm}(m_\rho^*, m_\rho^*, m_\rho^*, m_\rho^*). \kappa_{\rho\pi}(m_\rho^0, \Gamma_{\rho\pi}^\star, 0)$. Numerical evaluation of Eq. (13) leads to the approximated relation $\Gamma_{\rho\pi}^\star \approx \Gamma_{\rho\pi}^0 \left( 1 - 0.91 \frac{\delta m_\rho}{100 \text{ MeV}} + 0.25 \frac{\delta m_\rho}{100 \text{ MeV}} + 0.07 \left( \frac{T}{100 \text{ MeV}} - 1 \right) \right)$ valid for $\delta m_\rho = m_\rho^* - m_\rho \lesssim 200$ MeV.

$\delta \Gamma_\rho = \Gamma_\rho^\star - \Gamma_\rho^0 \lesssim 200$ MeV, and $100$ MeV $< T \lesssim 200$ MeV. The shift of a $\rho$-ion mass produces a minor effect and it is, therefore, neglected here. Finally, the total width of $\phi$ is given by $\Gamma_{\rho\pi} = 2\Gamma_{KK}^\star + \Gamma_{\rho\pi}^\star$. Here and below we do not consider the $\phi$ meson mass shift, which is small and cancels as soon as we consider the ratio of the momentum spectra, cf. Eq. (11).

For completeness we remark that the di-lepton decay channel $\phi \rightarrow l^+l^-$ also suffers a modification in medium, because the vector-meson–photon coupling is suppressed by meson fluctuations [33, 34]. For the $\phi - \gamma$ coupling the suppression factor is determined only by kaon fluctuations $\chi_{\phi\gamma} = (1 - 2 \langle |K|^2 \rangle / f_\pi^2)$. Here $\langle |K|^2 \rangle_T = \int d^3k \exp(-\omega_k(k)/T) [(2\pi^3)^2 \omega_k(k)]^{-1}$, $\omega_k(k) = \sqrt{m_k^2 + k^2}$ and $f_\pi = 93$ MeV is the pion decay constant. Because of the large kaon mass this correction is negligibly small: $\langle |K|^2 \rangle / f_\pi^2 \sim 1\%$.

Finally we, specify, how the in-medium properties of kaons and $\rho$ mesons relax to their vacuum values during the fireball expansion. Assuming that $\delta m_{K,\rho}$ and $\delta \Gamma_{\rho}$ are proportional to the density of the system, we have

$$\delta m_{K,\rho}(t) = \delta m_{K,\rho}^0 \frac{R_0^3}{R(t)^3}, \quad \delta \Gamma_{\rho}(t) = \delta \Gamma_{\rho}^0 \frac{R_0^3}{R(t)^3},$$

where $\delta m_{K,\rho}^0$ and $\delta \Gamma_{\rho}^0$ are input parameters.

Before finishing this section we would like to point out that the estimations done here are valid only for an almost baryon free fireball. In the presence of baryons the calculation of a phi self-energy becomes more elaborated, cf. Ref. [24]. However, in this case the spectra of kaons and anti-kaons suffer a modification in the fireball expansion. Assuming that $\delta m_{K,\rho}$ and $\delta \Gamma_{\rho}$ are valid for a total width only. To simulate effectively the in-medium modification of $\Gamma_{K,\rho}^*$ we will use eq. (11) and vary the kaon mass.

V. SPACE-TIME EVOLUTION OF THE FIREBALL

After the considerations above let us now specify the model of the fireball expansion, which we will use in our numerical calculations. Assume a simple homogeneous spherical fireball with the constant density and temperature profiles. The $\phi$ meson momentum distribution

$$f_\phi(x, \vec{p}) = \exp \left[ - \frac{E_\rho - \vec{p} \cdot \vec{u}(x)}{T_0 \sqrt{1 - u^2(x)}} \right],$$

is determined by the temperature $T_0$, flow velocity profile $\vec{u}(x) = v_{\perp}(x)/R_0$ and radius $R_0$. Time of flight of a $\phi$ meson and its daughter kaons through the medium contained in Eqs. (11) and (12) can be expressed as $\tau_\phi = \tau_R(\vec{u}_\phi, \vec{x})$ and $\tau_{KK} = \tau_R(\vec{u}_\phi, \vec{x}) + \tau_{KK}^\star(\vec{u}_\phi, \vec{x})$, where $\tau_R(\vec{v}, \vec{x})$ stands for a time, during which a particle with velocity $\vec{v}$ passes a distance from position $\vec{x}$ to the border of a sphere with the radius $R$. Since the fireball is...
expanding with radial velocity $v_f$, this time satisfies the equation $(\vec{x} + \vec{v} \tau)^2 = (R + v_f \tau)^2$. This implies:

$$
\tau_\text{R}(\vec{v}, \vec{x}) = \left( \sqrt{(\vec{v} \cdot \vec{x} - v_f R)^2 + (R^2 - \vec{x}^2)} \right) \left( \vec{v}^2 - v_f^2 \right)^{-1}.
$$

Solution (15) is valid for $|\vec{x}| < R$ and $|\vec{v}| > v_f$. In the case $|\vec{v}| < v_f$ we put $\tau = \infty$.

During the expansion $R(t) = R_0 + v_f t$, fireball density drops as $\rho(t) = \rho_0 R_0^3 / R^3(t)$ and the temperature decreases as $T(t) = T_0 R_0 / R(t)$ as expected for relativistic pion gas, cf. Ref. [2]. The kaon mean free path is $\lambda_K \propto 1/\rho$ and therefore

$$
\lambda_K(t) = \frac{\lambda_K^0 R_0^3(t)}{R_0^3}.
$$

To incorporate the freeze-out effect we will assume that as soon as $T(t) \leq T_\text{therm}$ kaons become free and $\lambda_K \to \infty$ as well as $\rho \to 1$. The freeze-out time is then given by

$$
\tau_{\text{f.o.}} = \frac{R_0}{v_f} \left( \frac{T_0}{T_\text{therm}} - 1 \right). \tag{16}
$$

Parameters $R_0^0$, $T_0^0$, $v_f$, and $\lambda_K^0$ serve as input for numerical evaluations below. The parameter $T_\text{therm}$ is used for an effective parameterization of the freeze-out time $\tau_{\text{f.o.}}$. It should not be considered as a true freeze-out temperature, since our estimation is based on the simplified hydrodynamical description of a fireball.

The model set up here is a rather crude approximation. However, the final results are found to be rather insensitive to the details of hydrodynamical evolution of a fireball, being determined mainly by the values of $\Gamma_\text{tot}^\alpha R_0$, $\Gamma_\text{tot}^\tau \tau_{\text{f.o.}}$, and $v_f$. We shall vary the input parameters within a broad range to illustrate different possibilities.

## VI. NUMERICAL ESTIMATES

In this section we perform numerical evaluation of our expressions for $\phi$ mesons yields reconstructed via $K^+K^-$ and $\mu^+\mu^-$ channels in central Pb+Pb collisions at 158GeV/n SPS energy.

First we investigate to what extent rescattering of secondary kaons enhanced by the in-medium modification of a $\phi$ meson width can suppress experimentally observed yield of $\phi$ mesons identified via $K^+K^-$ channel.

We remind that results of Ref. [27] give maximal suppression factor 40% for the $\phi$ meson observation in the kaon decay channel.

In our evaluation we use several combinations of input parameters. Freeze-out temperatures $T_0$ of $\phi$ mesons distributed according to Eq. (14) vary between $T_\text{chem}$ and $T_\text{therm}$: (i) $T_0 = 150$ MeV, (ii) $T_0 = 160$ MeV, (iii) $T_0 = 170$ MeV. Size of the fireball $R_0$ at the stage of the $\phi$ freeze-out has to be comparable with $\phi$ meson mean free path $\lambda_\phi$ at given temperature $R_0 \approx \alpha_R \lambda_\phi$ with $\alpha_R \sim 1$.

![FIG. 1: Ratio $R_0$ as a function of $m_T$ calculated for three sets of parameters ($T_0, v_f, R_0$) without inclusion of in-medium modifications of kaons and $\rho$ mesons. The upper grey area corresponds to variations of kaon mean free paths $\lambda_K$ and freeze-out temperature $T_\text{therm}$ as described in the text.](image)

For different temperature parameters above we take according to Ref. [3]: $\lambda_\phi^{(i)} = 13$ fm, $\lambda_\phi^{(ii)} = 10$ fm, and $\lambda_\phi^{(iii)} = 7$ fm.

First we evaluate suppression factor (14) without any modifications of particle properties in medium, i.e., $\delta m_\phi^0 = \delta m_\phi = 0$. In this case we have $R_\text{eff}(m_T) \equiv 1$ and $R(m_T) = \mathcal{R}(m_T)$. Flow velocities corresponding to selected temperatures $T_0$ are adjusted to reproduce the slope of $\phi$ meson $m_T$ distribution measured by the NA50 collaboration: $T_\text{eff} = 218$ MeV; $v_f^{(i)} = 0.50$, $v_f^{(ii)} = 0.46$, $v_f^{(iii)} = 0.41$. We take $\alpha_R = 1$ and vary the mean free path of kaons $\lambda_K$ within the interval $0 < \lambda_K^0 < \lambda_K(T_0)$, where $\lambda_K(T_0)$ follows from estimations of Ref. [2]: $\lambda_K^{(i)} = 2$ fm, $\lambda_K^{(ii)} = 1$ fm, and $\lambda_K^{(iii)} = 0.5$ fm. We vary also $T_\text{therm}$ between 100 MeV and 80 MeV in agreement with analysis [27]. This translates into the interval of freeze-out time values $10 \text{ fm} < \tau_{\text{f.o.}} < 20$ fm. All these variations produce the upper grey areas shown in Fig. 1.

For all three sets of parameters ($T_0, v_f, R_0$) we observe that $\mathcal{R}(m_T)$ does not fall below 0.6 significantly. This is related to the large expansion velocity of the fireball. In this case $0.2 < \Gamma_\text{tot}^\alpha \tau_{\text{f.o.}} < 0.4$ and $\phi$ mesons decay after the thermal freeze-out. To illustrate this effect we recalculate ratio $\mathcal{R}(m_T)$ for the same three cases fixing $v_f = 0.1$ what corresponds to $\tau_{\text{f.o.}} \sim 1$. Results obtained ($\mathcal{R} \sim 0.6$) are shown as lower gray areas in Fig. 1.

To reproduce results of RQMD calculations described in Ref. [27] we take somewhat larger size of a fireball with $\alpha_R = 1.5$, freeze-out temperature $T_\text{chem} = 80$ MeV and $\lambda_K^0 = 0.5$ fm. This corresponds to the lowest limit allowed by the analysis [27]. The results are shown in Fig. 1 by solid lines. The limiting scenario considered in [27] when the freeze-out volume is determined
by the last kaon interactions, can be reproduced with $T_{\text{therm}} = 40$ MeV. This case is shown by dash lines in Fig. 2. We take solid lines in Fig. 2 as a reference point for our further investigation of in-medium effects.

First we consider modifications of $\rho$ meson properties. We choose $\rho$ mass shift to be $\delta m_\rho = -200$ MeV for our three parameter sets. The $\rho$ meson width depends on the temperature. Relying on Ref. 13 we take: $\delta \Gamma_\rho = 80$ MeV, $\delta \Gamma_\rho^{(i)} = 180$ MeV, and $\delta \Gamma_\rho^{(iii)} = 200$ MeV. Comparison with Fig. 2 (solid lines) shows slight decrease of ratio $R(m_T)$ which corresponds to small increase of the total $\phi$ meson width by 40% due to $\phi \to \rho \pi$ channel.

Fig. 3 shows results obtained taking into account modification of $K$ meson properties in-medium.

First we investigate the case when $\phi \to K \bar{K}$ channel is closed initially ($\delta m_K^0 = 15$ MeV) and it opens only during the fireball expansion. Results are shown in the left part of Fig. 3, where the upper plot is calculated for $P_{\Pi} = 1$ and the lower one corresponds to a strong suppression of the kaon channel in medium with $P_{\Pi} = 0$. Comparing ratios $R(m_T)$ in Fig. 2 and in Fig. 3, shown by thick solid line (A) calculated for parameter set (i) we observe that kinematical quenching of $\phi \to K^+K^-$ channel by increasing kaon mass decreases ratio $R(m_T)$ very slightly for both values of $P_{\Pi}$. This happens because $\phi$ meson width becomes very small in this case and therefore probability of $\phi$ meson decay inside the expanding fireball and consequently also probability for rescattering of daughter kaons are small. Compare lines (C), calculated for parameter set (i), with a corresponding line in Fig. 2. Lines (B) in Fig. 3 (left side) show that $R_\mu$ becomes larger than one for increasing kaon mass. Since increase of $R_\mu$ by 10%–20% does not change considerably the effective $m_T$ slope of $\langle \eta_\mu \rangle_\mu$ distribution, we do not need to readjust the flow velocity parameter. This increase of $R_\mu$ leads at the end to a small decrease of $R$.

Let us now consider the case when the $\phi$ meson width increases strongly in hadronic medium due to the increase of $\Gamma_{KK}$. We simulate this effect by decrease of the kaon mass in medium, which can result e.g. from rescattering of kaons on pions through $K^*$ and heavier kaonic resonances. Here we restrict ourselves to rather conservative modification of kaon masses $-30$ MeV < $\delta m_K^0 < 0$ which corresponds to the $\phi$ width 4 MeV $\leq \Gamma_{\phi} \leq 20$ MeV. In this case $R_\mu < 1$ and at small $m_T - m_\phi$ region $R_\mu$ can be suppressed up to 40–60%. Thus, for a given freeze-out temperature $T_0$ and total width $\Gamma_{\phi}$ we just adjust flow velocity $v_f^0$ to reproduce the slope of the $m_T$ distribution measured in $e^+e^-$ channel by NA50.

For our three sets of parameters ($T_0, R_0$) we obtain new flow velocities: (i) $v_f^0 = 0.38$, (ii) $v_f^0 = 0.35$, (iii) $v_f^0 = 0.28$.

Right part of Fig. 3 shows our results obtained for $\delta m_K = -30$ MeV. Corresponding partial width is $\Gamma_{KK} \approx 10$ MeV, and the total width is $\Gamma_{\phi} \approx 21$ MeV. This leads to $\Gamma_{\phi} \approx R_\phi \sim 1\,\Gamma_{\phi_{\mu}} \sim 2$, which provides a strong suppression of $R_K$ as it is shown for the parameter set (i) in Fig. 3 (right side, lines C). We find $R_K(m_T \to m_\phi) \sim 0.2$ for $P_{\Pi} = 1$ and $\sim 0.15$ for $P_{\Pi} = 0$. However, since the ratio $R_\mu(m_T)$ also shown in Fig. 3 (lines B) is also suppressed the resulting ratio $R(m_T)$ remains on the level $\sim 0.3$ for small $m_T - m_\phi$, provided we put $P_{\Pi} = 0$ and $\sim 0.5$ for $P_{\Pi} = 1$. Taking even larger values of the total decay width in the most preferable case (i) we obtain $R(m_T \to m_\phi) \approx 0.28$ for $\Gamma_{\phi} \approx 27$ MeV, $v_f^0 = 0.35$, and $R(m_T \to m_\phi) \approx 0.23$ for $\Gamma_{\phi} \approx 34$ MeV, $v_f^0 = 0.32$. 

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**Fig. 2:** Ratio $\Gamma^{(i)}_\tau$ calculated for cases (i)-(iii) with $\alpha_R = 1.5$, $\lambda_\phi = 13$ fm and $\lambda_K = 0.5$ fm. Solid lines are calculated with $T_{\text{therm}} = 80$ MeV and dash lines correspond to $T_{\text{therm}} = 40$ MeV. No medium effects are included.

**Fig. 3:** Thick lines (label A) show the ratio $R(m_T)$ calculated for in-medium modification of $\phi$ meson properties. Results for different parameter sets (i)-(iii) are depicted by solid, dash and dotted lines, respectively. The left plane corresponds to the case when the kaon mass increases in medium $\delta m_K^0 = 15$ MeV, whereas the right plane shows results for a decreasing kaon mass $\delta m_K^0 = -30$ MeV. Thin lines show suppression factors in the muon $R_\mu$ (B) and kaon $R_K$ (C) channels calculated for parameter set (i). Upper plots are calculated for the case $P_{\Pi} = 1$, lower plots correspond to $P_{\Pi} = 0$. In all cases modification of $\rho$ meson properties in medium is taken into account.
For decreasing kaon mass the reduction is even stronger. For increasing kaon mass and correspondingly vanishing \( \Gamma \) we get \( \eta_\gamma(y = 0) = 1 \). We observe a considerable broadening of the rapidity distribution measured in the kaon channel.

### VII. CONCLUSIONS

We have studied distributions of \( \phi \) mesons in heavy-ion collisions at SPS energies reconstructed via hadronic \( K^+K^- \) and dilepton \( \ell^+\ell^- \) decay channels. The analysis of \( \phi \) meson mean free path allows to suppose that \( \phi \) mesons decouple from the hadronic system at somewhat earlier stage before the common breakup of the hadronic fireball. Therefore, kaon pairs originated from the \( \phi \) decays inside a fireball can be rescattered or absorbed. Such kaon pairs will not contribute to a \( \phi \) meson reconstruction, whereas the leptonic probes can leave a fireball freely. We derive the expressions (7) and (8) for the apparent momentum distribution of \( \phi \) mesons in kaonic and muonic channels respectively.

Within a simple model of spherically expanding fireball we investigate dependence of a relative suppression factor of the hadronic channel with respect to the dileptonic one on parameters of the system and on the \( \phi \) meson in-medium properties. For a vacuum \( \phi \)-meson width \( \sim 4 \text{ MeV} \) the maximal suppression 0.6–0.8 is obtained for the fireball size and expansion time \( T_0 \sim \tau_{\text{fo}}, \sim 20 \text{ fm} \). These values are in agreement with results of RQMD simulations [27]. The crucial parameter is the fast expansion of a fireball with \( v_f \sim 0.4–0.5 \) corresponding to the \( \phi \) freeze-out temperature range \( T_0 \sim 150–170 \text{ MeV} \).

Width of hadronic \( \phi \) meson decay channels \( \phi \to K\bar{K} \) and \( \phi \to \pi\rho \) can be modified in medium due to changes of the meson properties. We have found that increase of the \( \pi\rho \) channel width due to the broadening of \( \rho \) meson and decrease of \( \rho \) meson mass leads alone to a tiny increase of the suppression.

Other possibility is kinematical quenching of the kaon decay channel, which we simulate by simultaneous increase of \( K^+ \) and \( K^- \) masses. Since total width \( \Gamma_{\text{tot}} \) of \( \phi \) meson in medium becomes small (increase of the \( \pi\rho \) channel width is not strong enough) \( \phi \) mesons decay mainly outside the fireball, where vacuum properties of \( \phi \) mesons are restored and rescattering of daughter kaons is negligible. Together with relative amplification of the muon decay channel by 20% the resulting suppression factor found for a quenched kaon decay channel was \( \sim 0.5 \).

The increase of the \( \phi \) meson width in medium provides, on the other hand, a mechanism for strong suppression \( \sim 0.15 \) of the kaonic detection channel due to the enhancement of \( \phi \) decay probability inside a fireball increasing thus rescattering of daughter kaons. However, increase of the \( \phi \) total width reduces simultaneously the branching ratio of \( \phi \to \mu^+\mu^- \) decay and suppresses the spectrum of \( \phi \) mesons reconstructed via \( \mu^+\mu^- \) de-
cay channel. Obtained suppression of the muonic decay channel at the level ~ 40–60% requires readjustment of the flow velocity to be compatible with experimental slope of NA50. Adjusted flow velocity $v_f \sim 0.3–0.4$ for $\Gamma_{\pi^*} \sim 20$ MeV and $T_0 \sim 150–170$ MeV (compare to $v_f \sim 0.4–0.5$ obtained for a vacuum $\phi$ width) gives final net relative suppression factor of kaon channel to muon channel ~ 0.3. This value is close to experimental observations at CERN SPS for $\pi^*$. Strong increase of $\phi$ meson in-medium width can take place if the kaon mass decreases in medium by 30 MeV. The mechanism for such kaon mass modification can be similar to that studied in Ref. [27]. We have found that reconstructed rapidity distributions of $\phi$ mesons become effectively wider, if in-medium properties of mesons and rescattering of kaons are taken into account.

Finally we suppose that to improve understanding of experimental results on $\phi$ meson production in heavy ion collisions at CERN SPS further detailed investigations taking into account in-medium effects within transport or hydrodynamical models are necessary.

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