Heavy Fermion Production and the Symmetry Breaking Sector of the Electroweak Interactions*

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Abstract

We point out that heavy fermion production through the fusion of the longitudinal gauge bosons might be relevant in probing the strongly interacting symmetry breaking sector of the electroweak interactions, by showing the dependence of the one loop amplitude for \((w^+w^- \rightarrow \bar{t}t)\) on the symmetry breaking mechanism. The one loop amplitude for \((w^+w^- \rightarrow \bar{t}t)\) is calculated for the standard model and extended technicolour theory. Techni-rho meson exchange is also briefly discussed. We find at \(m_t = 150 \text{ GeV}\) the cross section of top pair production in \(e^+e^-\) collisions is comparable in order of magnitude to that of the longitudinal gauge boson scattering.

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1. It is well known that the symmetry breaking sector (SBS) of the electroweak interaction is far from being fully understood. We simply do not know what underlying dynamics causes the spontaneous symmetry breaking that gives masses to the weak gauge bosons. Although there are many scenarios proposed for the SBS, they may be broadly divided into two classes depending on their strength of interactions, namely, the weak scenario and the strong scenario.

Typically, in theories that belong to the weak scenario, there are one or a few scalar particles with light masses (roughly $\leq 200 GeV$) and small coupling constants so that amplitudes may be calculated perturbatively. The minimal Higgs model with one light elementary Higgs boson and the supersymmetric models are typical examples of this scenario. Since the searches for the Higgs boson have already started, we expect this scenario to be throughly tested in the near future.

The theories in the category of the strong scenario are characterized by the absence of particles with small masses. The minimal Higgs model with a heavy Higgs boson and technicolour theories belong to this group. In technicolour theories, a new QCD-like gauge interactions appear at the TeV scale and break chiral symmetry, generating the Goldstone bosons necessary for the weak gauge boson masses. Observation of a heavy Higgs bosons or techni-rho mesons would be direct evidences for this scenario. In the case that searches for direct evidence for the strong scenarios fail, it may still be possible to study the strongly interacting SBS’s using the method first proposed by Dobado and Herrero and independently by Donoghue and Ramirez [1, 2]. They noticed that the scattering of the longitudinal weak gauge bosons (LWB), which are essentially the Goldstone bosons from the symmetry breaking, can be described, in analogy to the $\pi\pi$ scattering, by a low energy effective Lagrangian (LEL) with a few unknown coefficients which, in turn, depend on the symmetry breaking mechanism. Then, the scattering amplitudes at low energies (typically $0.4 TeV \leq \sqrt{s} \leq 1.2 TeV$) are calculated through chiral perturbation theory.

This note is concerned with the strong scenario. The motivation of this paper is the simple observation that heavy fermion production may be relevant in probing the underlying dynamics of the SBS’s because the interactions between fermions and the LWB’s are proportional to the fermion masses. The heavy fermion we are considering
here is the top quark which has quite a heavy expected mass of 150 GeV. Thus there is a possibility that the interaction between top quark and the SBS is large enough that low energy scattering processes involving top quark may reveal some information on the SBS. As a first step to explore this possibility we calculate the cross section for the top pair production by LWB fusion to the one loop in chiral perturbation theory and compare it with that of the LWB scattering to see its magnitude and sensitivity to the SBS. Also computed is the effective vertex of the ordinary fermions and the LWB’s through techi-rho meson exchanges. The background analysis which is absolutely necessary for applicability of our work to actual experiment is not discussed here. This note is organized as follows. In sec.2, the LWB scattering is briefly reviewed and in sec.3, the amplitude for \((W_LW_L \to t \bar{t})\) is calculated in the minimal standard model and extended technicolour theory. In sec.4, the effective interaction of the ordinary quarks and the LWB’s induced by techi-rho meson exchanges is derived and in sec.5, the cross section for \((e^+e^- \to \bar{\nu} \nu (W_LW_L) \to \bar{\nu} \nu \bar{t}t)\) is calculated and compared with that of the LWB fusion.

2. The scattering amplitudes of the LWB’s at energies higher than the mass of the weak gauge bosons \(m_W\) can be easily calculated with the help of the equivalence theorem \cite{3}. The theorem states that the S-matrix element of a process with the LWL’s as the external particles is equivalent, up to \(O\left(\frac{m_W}{\sqrt{s}}\right)\), to that of the process with the external LWB’s being replaced by the corresponding Goldstone bosons. It should be noted that the LWB’s need not be the sole external particles. The general proof of the theorem is given in Ref.4. According to the theorem, the LWB’s scattering can be described by the following Lagrangian,

\[
\mathcal{L} = \frac{v^2}{2} \mathcal{L}_2 + \mathcal{L}_4 + O(\partial^6),
\]

where

\[
\mathcal{L}_2 = \frac{1}{2} Tr \partial \mu U^+ \partial \mu U,
\]

\[
\mathcal{L}_4 = \alpha_1 \left( Tr \left( \partial \mu U^+ \partial \mu U \right) \right)^2 + \alpha_2 \left( Tr \left( \partial \mu U^+ \partial \nu U \right) \right) \left( Tr \left( \partial \mu U^+ \partial \nu U \right) \right),
\]

with \(v = (\sqrt{2} G_F)^{\frac{1}{2}} = 246\) GeV, and

\[
U(x) = \exp \left( \frac{w^a}{v} \sigma^a \right).
\]
Here, $w^a(x), a = 1, 2, 3$, are the Goldstone bosons corresponding to the LWB’s and $\sigma^a$ are the Pauli matrices. While the leading term $L_2$ is model independent, the $\alpha_i$ in $L_4$ are quite sensitive to the underlying dynamics. As is well known, the coefficients $\alpha_i$ are renormalized beyond the tree level, and the one loop renormalized coefficients were calculated for the minimal standard model and scaled-up QCD [1, 4, 5]. They are given by [5],

$$\alpha_{1,SM}(\mu) = \frac{1}{4} \left[ \frac{v^2}{2m_H^2} + \frac{1}{16\pi^2} \left( \frac{9\pi}{4\sqrt{3}} - \frac{37}{9} \right) - \frac{1}{48\pi^2} \ln(\frac{\mu}{m_H}) \right]$$

$$\alpha_{2,SM}(\mu) = \frac{1}{4} \left[ \frac{1}{16\pi^2} \left( \frac{2}{9} \right) - \frac{2}{48\pi^2} \ln(\frac{\mu}{m_H}) \right]$$

(4)

for the minimal standard model, with $m_H$ being the Higgs mass, and

$$\alpha_{1,TC}(\mu) = \frac{1}{4} \left[ -0.011 - \frac{1}{48\pi^2} \ln \left( \frac{\mu (Gev)f_\pi}{v} \right) \right]$$

$$\alpha_{2,TC}(\mu) = \frac{1}{4} \left[ 0.0046 - \frac{1}{24\pi^2} \ln \left( \frac{\mu (Gev)f_\pi}{v} \right) \right]$$

(5)

for scaled-up QCD where $\mu$ is the renormalization scale coming from the one loop diagrams of $L_2$. We note that the coefficients $\alpha_{i,TC}$ were determined experimentally.

The amplitude for $(w^+w^- \rightarrow w^+w^-)$ to one loop in chiral perturbation is given by,

$$T_{++--} = -\frac{u}{v^2} + \frac{4}{v^4} \left[ 2\alpha_1(\mu) \left( s^2 + t^2 \right) + \alpha_2(\mu) \left( s^2 + t^2 + 2u^2 \right) \right]$$

$$+ \frac{1}{(4\pi v^2)^2} \left[ -\frac{1}{12} \left( 9s^2 + u^2 - t^2 \right) \ln \left( \frac{s}{\mu^2} \right) \right]$$

$$- \frac{1}{12} \left( 9t^2 + u^2 - s^2 \right) \ln \left( \frac{t}{\mu^2} \right) - \frac{u^2}{2} \ln \left( \frac{u}{\mu^2} \right)$$

(6)

where $s, t$ and $u$ are the Mandelstam variables and $\alpha_i(\mu)$ are given in (4), (5). The cross section for $(e^+e^- \rightarrow \bar{\nu} \nu(W_LW_L) \rightarrow \bar{\nu} \nu W_LW_L)$ using (3) and the effective W-approximation [6] is presented in Fig.6.

3. In this section, the low energy one loop amplitude for $(w^+w^- \rightarrow \bar{t} t)$ is calculated in the minimal standard model and the extended technicolour theory. At this level, the amplitude is already sensitive to the symmetry breaking mechanism.

Let us begin with the minimal standard model. The Higgs sector and the Yukawa
coupling of top quark in the standard model is,
\[
L_{(\text{Higgs+top})} = \partial_\mu \Phi^* \partial^\mu \Phi - \frac{g^2 (m_H^0)^2}{8 (m_W^0)^2} \left( \Phi^* \Phi - 2 \frac{(m_W^0)^2}{g^2} \right)^2 + \left( - \frac{g m_t^0}{2 m_W^0} \bar{t}_L \tau_R + \text{h.c.} \right)
\]
(7)
where \( g \) is the \( SU(2) \) gauge coupling and \( m_t^0, m_W^0 \) and \( m_H^0 \) is the bare mass of the top quark, gauge boson and Higgs respectively. Here,
\[
\Phi = \begin{pmatrix} w^+ & \frac{1}{\sqrt{2}}(\rho + w^0) \end{pmatrix}
\]
(8)
is the \( SU(2)_L \) doublet scalar field, with \( w^\pm, w^0 \) being the Goldstone bosons giving mass to the weak gauge bosons, and \( \rho \) representing the massive Higgs field. To simplify the calculation we neglect the diagrams whose loop is composed of fermion and scalar propagators, which are suppressed at least by a factor of \( m_t/\sqrt{s} \) relative to the other loop diagrams. Some diagrams of this kind are shown in Fig.1. Then, the only nontrivial diagrams we need to calculate are such that a single massive Higgs line is connected to the external top quarks, so the calculation essentially reduces to the evaluation of the off-shell Higgs decay into two LWB’s. For this calculation, we closely follow Marciano and Willenbrock [7], who studied the on-shell Higgs decay into \( W_L^+W_L^- \) to one loop. We note that in our approximation there is no mass renormalization of the top quark because the tadpole diagram, which is the only remaining source of the renormalization, is absorbed into the scalar self energy in the Marciano and Willenbrock scheme. From the diagrams in Fig.2, we can easily see that the one loop amplitude can be written as,
\[
\tilde{T}_{(w^+w^+\rightarrow t)} = \frac{-i g m_t^0}{2 m_W} \bar{u}(p_1) v(p_2) \cdot A(q^2),
\]
(9)
where \( u(p_1), v(p_2) \) are the spinors for the top and \( A(q^2) \) is given as the multiplication of the wave function renormalization factor \( z_w \), of \( w^\pm \), the Higgs propagator \( G_H \) and the vertex function \( \Gamma \), with \( q^2 = (q_1 + q_2)^2 \) and \( q_i \) the momentum of \( w^\pm \),
\[
A(q^2) = \frac{m_W}{m_W^0} \cdot G_H(q^2) \cdot (\Gamma(q^2)) \cdot z_w.
\]
(10)
From Marciano and Willenbrock, we have
\[
\frac{m_W}{m_W^0} = \frac{1}{\sqrt{z_w}}, \quad z_w = 1 + \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \left( -\frac{1}{8} \right).
\]
(11)
A detailed calculation shows that
\[
G_H(q^2) = \frac{-i}{m_H^2} \left[ 1 + \frac{q^2}{m_H^2} - \frac{g^2 m_H^2}{16\pi^2 m_W^2} \left( \frac{3}{16} \right) \left( 1 + 2 \ln \left( \frac{-q^2}{m_H^2} \right) \right) + \frac{g^2 q^2}{16\pi^2 m_W^2} \left( \frac{3}{16} \right) \left( 1 + 2 \ln \left( \frac{-q^2}{m_H^2} \right) \right) \right],
\]
(12)
and
\[
-\frac{i}{m_W} \left[ 1 - \frac{g^2 m_H^2}{16\pi^2 m_W^2} \left( \frac{37}{16} - \frac{9\pi}{8\sqrt{3}} - \frac{3}{8} \ln \left( \frac{-q^2}{m_H^2} \right) \right) - \frac{g^2 q^2}{16\pi^2 m_W^2} \left( \frac{1}{16} \right) \left( 1 - 2 \ln \left( \frac{-q^2}{m_H^2} \right) \right) \right],
\]
(13)
to \(O(q^2)\). Substituting these results into (14), we get,
\[
A(q^2) = -\frac{g}{2m_W} \left[ 1 + \frac{g^2 m_H^2}{16\pi^2 m_W^2} \left( \frac{33}{16} - \frac{9\pi}{8\sqrt{3}} \right) + \frac{q^2}{m_H^2} + \frac{g^2 q^2}{16\pi^2 m_W^2} \left( 2 - \frac{9\pi}{8\sqrt{3}} - \frac{1}{4} \ln \left( \frac{-q^2}{m_H^2} \right) \right) + O(q^4) \right].
\]
(14)
Here \(m_H\) and \(m_W\) are the physical masses of the Higgs and the gauge bosons defined as the poles of their respective propagators.

For extended technicolour theories [3], we consider the simplest case with a single \(SU(2)_L \times U(1)_Y\) family of techniquarks \(Q = (U, D)\) and ordinary quarks \(q = (t, b)\). We assume chiral symmetry is a good symmetry of TC interactions and it breaks down to isospin symmetry. The interaction between ordinary fermions and techniquarks at the symmetry breaking scale is point-like and given as
\[
\sum_A c_A j_A^A j_A^\mu,
\]
(15)
with \(c_A = g_{ETC}^2/m_A^2\) and
\[
j_A^\mu = f_1^A \bar{t}_L \gamma^\mu U_L + f_2^A \bar{t}_R \gamma^\mu U_R + \cdots + h.c,
\]
(16)
where \(f_1^A, f_2^A\) are constants and the terms neglected are not important in our calculation. Expanding (15) with (16) and using the Fierz transformation, we can write (15) as,
\[
\sum_A c_A \left( 2 \left( f_1^A f_2^{A*} \bar{t}_L \gamma^\mu t_R \overline{U}_L + h.c \right) - \frac{1}{2} \left( |f_1^A|^2 \bar{t}_L \gamma^\mu t_L + |f_2^A|^2 \bar{t}_R \gamma^\mu t_R \right) \overline{U}_\mu U + \cdots \right).
\]
(17)
Now the first term in (17) is the interaction which is subject to chiral perturbation. The second term is relevant for techni-rho meson exchanges and it is discussed in next section. To derive the effective interaction between top quark and the Goldstone bosons we are considering, we simply substitute $U_R U_L$ in (17) with $<U_R U_L>$, with $<U_R U_L>$ being the techni-quark condensate, noting that $Q_R Q_L$ and $U$ defined in (3) transform identically under the chiral symmetry $SU(2)_L \times SU(2)_R$. Then the LEL, at the tree level, is given by

$$L_{eff,TC} = \frac{v^2}{2} L_2 + \mathcal{L}_t + \alpha_{TC} \mathcal{L}_t \mathcal{L}_2,$$

$$\mathcal{L}_t = -m_t \bar{t}_L t_R U_{11} + h.c,$$  \hspace{1cm} (18)

with $\alpha_{TC} = 0$ and,

$$m_t = - \left( \sum_A c_A \text{Re} \left( f_A^1 f_A^{A*} \right) \right) \cdot <UU>.$$  \hspace{1cm} (19)

Although $\alpha_{TC} = 0$ at the tree level, we will see that it can not be zero at the one loop level because of renormalization. The amplitude to one loop in $L_2, \mathcal{L}_t$ and to the tree level in $L_2 \mathcal{L}_t$ in the $\overline{MS}$ scheme is given by (see Fig.3)

$$\frac{im_t}{v^2} \pi(p_1) v(p_2) \left[ 1 + \alpha_{TC}(\mu) q^2 + \frac{2q^2}{16\pi^2 v^2} \left( 1 - \frac{1}{2} \ln \left( -\frac{q^2}{\mu^2} \right) \right) \right].$$  \hspace{1cm} (20)

To find $\alpha_{TC}$ to one loop, we follow the Weinberg prescription [9] that requires the amplitude to be indepedent of the renormalization scale $\mu$. This gives,

$$\alpha_{TC}(\mu) = \frac{1}{16\pi^2 v^2} \ln \left( \frac{\mu_0^2}{\mu^2} \right),$$  \hspace{1cm} (21)

where $\mu_0$ is a constant. Though the exact value of the cut-off $\mu_0$ should be determined by exprement, it is quite obvious that the scale of $\mu_0$ should be the symmetry breaking scale $4\pi v$, at which a new physics appears. Since no experimental data for $\mu_0$ is available, we simply take $\mu_0 = 4\pi v$ in sec.5. Substituting (21) into (20), we have

$$\mathcal{T}_{(w^- w^+ \rightarrow t \bar{t})_{TC}} = \frac{ig^2 m_t}{4m_W^2} \pi(p_1) v(p_2) \left[ 1 + \frac{2q^2}{16\pi^2 v^2} \left( 1 - \frac{1}{2} \ln \left( -\frac{q^2}{\mu_0^2} \right) \right) \right].$$  \hspace{1cm} (22)

4. The techni-rho meson exchanges occur through the second term in (17),

$$- \frac{1}{2} \sum_A c_A \left( |f_1^A|^2 \bar{t}_L \gamma^\mu t_L + |f_2^A|^2 \bar{t}_R \gamma^\mu t_R \right) U \gamma_\mu U.$$  \hspace{1cm} (23)
Using the isospin symmetry in the TC sector and the “current-field identity” in the vector meson dominance hypothesis \[10\],

\[\mathcal{J}_\mu = -\frac{m_\rho}{f_\rho} \rho_{\mu}, \quad (24)\]

where \(j_\mu^a\) is the isospin current and \(f_\rho\) is the \(\rho\)-meson decay constant, we get the vertex of top quark and the techni-rho meson,

\[\frac{1}{2} \frac{m_\rho^2}{f_\rho} \sum_A c_A \left( |f_1^A|^2 \overline{t}_L \gamma_\mu t_L + |f_2^A|^2 \overline{t}_R \gamma_\mu t_R \right) \rho_\mu^0, \quad (25)\]

with the obvious notation that \(m_\rho, f_\rho\) now representing the mass and the decay constant of the techni-rho meson. Noting that

\[\sum_A c_A \left( |f_1^A|^2 + |f_2^A|^2 \right) \geq 2 \sum_A c_A |\text{Re} \left( f_1^A f_2^{A*} \right) | \geq 2 \frac{m_t}{<UU>}, \quad (26)\]

we may write the general form of the interactions between ordinary quarks and the techni-rho meson as

\[\frac{m_t}{<UU>} \frac{m_\rho^2}{f_\rho} \left( \left( C_+^V \overline{t}_\gamma \gamma^\mu b + C_-^A \overline{t}_\gamma \gamma^\mu \gamma_5 b \right) \rho_\mu^+ + \left( C_0^V \overline{t}_\gamma \gamma^\mu t + C_0^A \overline{t}_\gamma \gamma^\mu \gamma_5 t \right) \rho_\mu^0 + h.c. \right), \quad (27)\]

where \(C_+^V, C_-^A\) are model-dependent, dimensionless constants of order one. Given a model, it should be straightforward to calculate these constants. It should also be noted that there is no simple relations between these constants since the extended techniour interactions usually breaks the isospin symmetry. Now using the vertex between the rho mesons and pions in QCD,

\[f_\rho \varepsilon_{abc} \rho_0^a \partial_\mu \pi^b \pi^c, \quad (28)\]

the effective vertex between ordinary quarks and the LWB’s can be easily derived. For example, the effective vertex between \(\overline{t}, t\) and \(w^+, w^-\) is,

\[- \frac{im_t}{<UU>} \left( C_0^V \overline{t}_\gamma \gamma^\mu t + C_0^A \overline{t}_\gamma \gamma^\mu \gamma_5 t \right) \left( \partial_\mu w^+ w^- - \partial_\mu w^- w^+ \right). \quad (29)\]

The vacuum condensate can be evaluated by scaling up the QCD result, which gives

\[<UU> \approx 17 v^3, \quad (30)\]

with \(N_{TC} = 3\).
5. Considering the heavy background and the smallness of the cross section for the LWB scattering in $e^+e^-$ collisions, it seems to be very difficult to probe the SBS using LWB scattering in electron-positron collider. However, just to see the sensitivity of the process $(w^+ w^- \rightarrow \bar{t} t)$ to the symmetry breaking mechanism, we calculate the cross section of top pair creation through LWB fusion in $e^- e^+$ collision shown in Fig.5 and compare it with that of $w^+ w^-$ scattering. Adding the trivial term in Fig.4 to the amplitude we have calculated in section 3, we get the scattering amplitude,

$$ T_{(w^- w^+ \rightarrow \bar{t} t)} = \tilde{T}_{(w^- w^+ \rightarrow \bar{t} t)} + \frac{1}{2} |U_{tb}|^2 \left( \frac{ig_{\text{em}}}{2m_W} \right)^2 \hat{\pi}(p_1) (1 - \gamma_5) \frac{i}{\not{p}_1 - \not{q}_2} (1 + \gamma_5) \nu(p_2), \quad (31) $$

with $U_{tb}$ the Kobayashi-Maskawa matrix element and $\tilde{T}_{(w^- w^+ \rightarrow \bar{t} t)}$ given in (9) and (22) for the minimal standard model and the technicolour theory respectively. Here, for simplicity, the amplitude from the techni-rho meson exchanges is not included. In the second term in (31), the mass of bottom quark has been neglected. The cross section for $(e^- e^+ \rightarrow \nu \bar{\nu} (W_L W_L) \rightarrow \nu \bar{\nu} \bar{t} \tilde{t})$ can be easily found using the effective W-approximation. In this approximation, the cross section is given by

$$ \frac{d\sigma}{d\hat{s}} = \int_0^1 dx_1 \int_0^1 dx_2 \hat{\sigma} (w_L w_L \rightarrow \bar{t} t) \delta(\hat{s} - x_1x_2s) F_{w_L}(x_1)F_{w_L}(x_2), \quad (32) $$

where $\sqrt{s}$ is the C.M energy of the $e^+, e^-$ and

$$ F_{w_L}(x) = \frac{\alpha}{4\pi \sin^2 \theta_W} \frac{1 - x}{x} \quad (33) $$

is the distribution function of the gauge boson $W_L^\pm$ and $\hat{\sigma}$ is the cross section for the subprocess $W_L^- W_L^+ \rightarrow t \bar{t}$ and $\sqrt{s} = M_{t\bar{t}}$. Integration over $x_1$ and $x_2$ reduces (32) to

$$ \frac{d\sigma}{dM_{t\bar{t}}} = -\frac{4}{\sqrt{s}} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \left[ \left( 1 + \frac{\hat{s}}{s} \right) \ln \left( \frac{\hat{s}}{s} \right) + 2 \left( 1 - \frac{\hat{s}}{s} \right) \right] \hat{\sigma}(\hat{s}). \quad (34) $$

The cross sections for the minimal standard model and the technicolour theory at $\sqrt{s} = 2 \text{ TeV}$ using (31) with $U_{tb} = 1$, are plotted in Fig.6. The plots show that the cross section and the sensitivity to the symmetry breaking mechanism of the top pair production are smaller but comparable in order of magnitude at $m_t = 150 \text{ GeV}$ to those of the LWB scattering. We also notice that there is not much difference between Fig.6(a) and 6(b),
which means that the top production is quite insensitive to the SBS at this energy. This can be easily understood when we compare (3) and (22). The difference between (3) and (22) is approximately proportional to $q^2(M^2_{tt})$ and so it is small at low $q^2$. Now the luminosity of $w^+, w^-$ with large $q^2$ is much suppressed at low $\sqrt{s}$ to make the top production relatively insensitive to the SBS. However, the luminosity of $w^+, w^-$ with large $q^2$ increases as $\sqrt{s}$ becomes larger. Therefore, the top production would be more sensitive to the SBS at higher energies.

Whether the top production is relevant in probing the SBS can be only said after the background has been studied. As is well known, there is a strong background in the LWB scattering and there would probably be as strong or stronger background in the case of the top production too. In this note we do not attempt to study the background and leave it as an open problem.

Finally, we would like to comment on $t, b$ production in proton collider through the techni-rho meson exchanges. Any possible observation of $t, t$ production in proton collider must be greatly obstructed by the strong background of the gluon-gluon fusion. However, $t, b$ production does not have such background, though it does have other strong, but weaker than the g-g fusion, background such as W-g fusion. One additional advantage of $t, b$ production over $t, t$ production is that there is no $t, b$ channel, in leading order, in the standard model except the trivial tree diagram, while $t, t$ can be produced by the Higgs exchange. Whether these facts will be of any use need to be seen.

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Figure Captions

Fig.1: Examples of loop diagrams that are suppressed.

Fig.2: The leading Feynman diagrams for \((w^+w^- \rightarrow \bar{t}t)\). Thick solid lines and dotted lines represent the massive Higgs boson and the Goldstone bosons respectively.

Fig.3: The leading Feynman diagrams for \((w^+w^- \rightarrow \bar{t}t)\) in extended technicolour theory.

Fig.4: The blobs in dotted lines represent the one loop wave function renormalization of \(w^\pm\).

Fig.5: LWB scattering and \(\bar{t}t\) production through LWB fusion in \(e^+e^-\) collision.

Fig.6: \(\frac{d\sigma}{dM_{\bar{t}t}}\) and \(\frac{d\sigma}{dM_{w^+w^-}}\) for \(\bar{t}t\) and \(w^+w^-\) production from vector boson fusion in \(e^+e^-\) collisions at \(\sqrt{s} = 2\ TeV\). Dotted lines in (a),(b),(c) and (d) represent the tree level cross sections. (a),(b):\(\bar{t}t\) production in the extended technicolour theory and the minimal standard model respectively. The solid and dot-dashed lines in (b) are at \(m_H = 2\ TeV, 1.2\ TeV\) respectively. (c),(d):\(w^+w^-\) scattering in scaled-up QCD and the minimal standard model respectively. The solid and dot-dashed lines in (d) are at \(m_H = 2\ TeV, 1.5\ TeV\) respectively.