Gauge independence and chiral symmetry breaking in a strong magnetic field

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Abstract

The gauge independence of the dynamical fermion mass generated through chiral symmetry breaking in QED in a strong, constant external magnetic field is critically examined. We present a (first, to the best of our knowledge) consistent truncation of the Schwinger-Dyson equations in the lowest Landau level approximation. We demonstrate that the dynamical fermion mass, obtained as the solution of the truncated Schwinger-Dyson equations evaluated on the fermion mass shell, is manifestly gauge independent.

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Chiral symmetry breaking in an external magnetic field has attracted a lot of attention in the past decade. Being inherently a nonperturbative phenomenon, the generation of a dynamical fermion mass is usually studied with the help of the Schwinger-Dyson (SD) equations truncated in certain schemes. The dynamical fermion mass has been calculated in the literature in several truncation schemes, such as the rainbow [1,2,3] and the improved rainbow [3,4,5] approximations with a momentum independent fermion self-energy, as well as their extensions to a momentum dependent fermion self-energy [4,5,6]. However, to the best of our knowledge, issues regarding the consistency of truncation schemes as well as the gauge independence of the dynamical fermion mass have not been properly addressed in the previous literature in this field.

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The demonstration of gauge independence of physical quantities is of fundamental importance in gauge theories [7,8]. In particular, the gauge independence of physical quantities obtained in a nonperturbative calculation (e.g., the SD equations) is a highly nontrivial problem. This is because an infinite subset of diagrams arising from every order in the loop expansion has to be resummed consistently, thus leading to potential gauge dependence of physical quantities whenever not all relevant diagrams are accounted for. Therefore, we emphasize that in gauge theories no truncation schemes of the SD equations should be considered consistent unless the gauge independence of physical quantities calculated therein is unequivocally demonstrated.

In this article, we critically study dynamical chiral symmetry breaking in weak coupled QED in a strong, constant external magnetic field. We demonstrate that there exists a consistent truncation of the SD equations in the lowest Landau level approximation. We show that within this consistent truncation scheme the dynamical fermion mass, obtained as the solution of the truncated SD equations evaluated on the fermion mass shell, is manifestly gauge independent.

We take the constant external magnetic field of strength \( H \) in the \( x_3 \)-direction. The corresponding vector potential is given by \( A^\text{ext}_\mu = (0, 0, H x_1, 0) \), where \( \mu = 0, 1, 2, 3 \). In our convention, the metric has the signature \( g_{\mu \nu} = \text{diag}(-1, 1, 1, 1) \). The SD equations in QED in an external magnetic field are well-known in the literature. The equations for the full fermion propagator \( G(x, y) \) are given by

\[
G^{-1}(x, y) = S^{-1}(x, y) + \Sigma(x, y),
\]

(1)

\[
\Sigma(x, y) = ie^2 \int d^4 x' d^4 y' \gamma^\mu G(x, x') \Gamma^\nu(x', y, y') D_{\mu \nu}(x, y'),
\]

(2)

where \( S(x, y) \) is the bare fermion propagator in the external field \( A^\text{ext}_\mu \), \( \Sigma(x, y) \) is the fermion self-energy and \( \Gamma^\nu(x, y, z) \) is the full vertex. The full photon propagator \( D_{\mu \nu}(x, y) \) satisfies the equations

\[
D_{\mu \nu}^{-1}(x, y) = D_{\mu \nu}^{-1}(x, y) + \Pi_{\mu \nu}(x, y),
\]

(3)

\[
\Pi_{\mu \nu}(x, y) = -ie^2 \text{tr} \int d^4 x' d^4 y' \gamma^\mu G(x, x') \Gamma^\nu(x', y', y) G(y', x),
\]

(4)

where \( D_{\mu \nu}(x, y) \) is the free photon propagator (defined in covariant gauges) and \( \Pi_{\mu \nu}(x, y) \) is the vacuum polarization.

Since the dynamics of fermion pairing in a strong magnetic field is dominated by the lowest Landau level (LLL) [1,2,3,4], we will consider the propagation of, as well as radiative corrections originating only from, fermions occupying the LLL. This is referred to as the lowest Landau level approximation (LLLA) in the literature. Consequently, for the purpose of this article, the fermion
propagator and the fermion self-energy in the SD equations (1)-(4) will be taken to be those for the LLL fermion. For notational simplicity, no separate notation will be introduced.

It is well-known that the SD equations (1)-(4) do not form a closed system of integral equations unless a truncation scheme is employed by specifying the full vertex $\Gamma^\mu$ in terms of other entities already appeared in the SD equations. To this end we will work in the bare vertex approximation (BVA), in which the vertex corrections are completely ignored. This is achieved by replacing the full vertex in the SD equations (1)-(4) by the bare one, viz,

$$\Gamma^\mu(x, y, z) = \gamma^\mu \delta^4(x - z) \delta^4(y - z). \quad (5)$$

This truncation is also known as the (improved) rainbow approximation and has been employed extensively in the literature [1,2,3,4,5,6]. However, we emphasize that unlike what has usually been done in the literature, here we will not confine ourselves to a particular gauge (usually the Feynman gauge) [1,2,3], nor will we make the assumption that the BVA (5) is valid only in a certain gauge [4,5,6]. Instead, it is our aim to prove that the BVA (5) is a consistent truncation of the SD equations (1)-(4) in the LLLA. The dynamical fermion mass, obtained as the solution of the truncated SD equations evaluated on the fermion mass shell, is manifestly gauge independent. In the weak coupling regime that we consider, such a gauge independent approach allows one to resum consistently an infinite subset of diagrams that arises from every order in the loop expansion and whose contributions are of leading order in the gauge coupling, thus leading to a consistent and reliable calculation of the dynamical fermion mass.

The main ingredient in the proof of the gauge independence of physical quantities is the Ward-Takahashi (WT) identity satisfied by the vertex and the inverse fermion propagator. The WT identity for the bare vertex (5) takes the form

$$\delta^4(x - y) e^{-iq \cdot x} \gamma \cdot q = (e^{-iq \cdot x} - e^{-iq \cdot y}) G^{-1}(x, y), \quad (6)$$

where $q^\mu$ is the momentum carried by the photon. It was shown in Ref. [9] that in order to satisfy the WT identity in the BVA (6), the LLL fermion self-energy in momentum space has to be a momentum independent constant. We note that due to an oversight in Refs. [2,9] regarding the matrix structure in the orthonormal condition of the Ritus $E_p$ functions [10] for the LLL fermions, the calculations therein require further investigations. It can be shown [11] that with the correct orthonormal condition the conclusion obtained in Ref. [9] on the WT identity in the BVA (6) remains valid within the LLLA. The reliability of such a momentum independent approximation and, consequently, of the WT identity in the BVA (6) has been verified in certain gauges in the momentum region relevant to the dynamics of fermion pairing in a strong magnetic field [4,5,6].
As per the WT identity in the BVA (6), we can write the self-energy for the LLL fermion as \( \Sigma(p_{\parallel}) = m(\xi) \), where \( p_{\parallel} \) is the momentum of the LLL fermion and \( m(\xi) \) is a momentum independent but gauge dependent constant, with \( \xi \) being the gauge parameter in covariant gauges. Here and henceforth, the subscript \( \parallel (\perp) \) refers to the longitudinal: \( \mu = 0, 3 \) (transverse: \( \mu = 1, 2 \)) components. It is noted that \( m(\xi) \) depends implicitly on \( \xi \) through the full photon propagator \( D_{\mu\nu} \) in (2). We emphasize that because of its \( \xi \)-dependence, \( m(\xi) \) should not be taken for granted to be the dynamical fermion mass, which is a gauge independent physical quantity.

We now begin the proof that the BVA is a consistent truncation of the SD equations (1)-(4), in which \( m(\xi) \) is \( \xi \)-independent and hence can be identified unambiguously as the dynamical fermion mass, if and only if the truncated SD equation for the fermion self-energy is evaluated on the fermion mass shell.

We first recall that, as proved in Ref. [8], in gauge theories the singularity structures (i.e., the positions of poles and branch singularities) of gauge boson and fermion propagators are gauge independent when all contributions of a given order of a systematic expansion scheme are accounted for. Consequently, this means the dynamical fermion mass has to be determined by the pole of the full fermion propagator obtained in a consistent truncation scheme.

The full propagator for the LLL fermion is given by

\[
G(p_{\parallel}) = \frac{1}{\gamma_{\parallel} \cdot p_{\parallel} + \Sigma(p_{\parallel})} \Delta[\text{sgn}(eH)],
\]

(7)

where \( \Delta[\text{sgn}(eH)] = [1 + i\gamma^1 \gamma^2 \text{sgn}(eH)]/2 \) is the projection operator on the fermion states with the spin parallel to the external magnetic field. Assume for the moment that the BVA is a consistent truncation of the SD equations in the LLLA, such that the position of the pole of \( G(p_{\parallel}) \) in (7) is gauge independent. In accordance with the WT identity in the BVA (6), we have

\[
\Sigma(p_{\parallel}) = \Sigma(p_{\parallel}^2 = -m^2) = m,
\]

(8)

where \( m \) is the gauge independent, physical dynamical fermion mass, yet to be determined by solving the truncated SD equations self-consistently. What remains to be verified in our proof is the following statements: (i) the truncated vacuum polarization is transverse; (ii) the truncated fermion self-energy is gauge independent when evaluated on the fermion mass shell, \( p_{\parallel}^2 = -m^2 \).

We highlight that the fermion mass shell condition is one of the most important points that has gone unnoticed in the literature, where the truncated fermion self-energy used to be evaluated off the fermion mass shell at, say, \( p_{\parallel}^2 = 0 \) [1,2,3,4,5,6].

In terms of (7) and (8), the vacuum polarization \( \Pi_{\mu\nu}(q) \) in the BVA is found
to be given by [11]

\[
\Pi^{\mu\nu}(q) = -\frac{ie^2}{2\pi} |eH| \exp \left( -\frac{q^2}{2|eH|} \right) \text{tr} \int \frac{d^2p_\parallel}{(2\pi)^2} \gamma^\mu_\parallel \frac{1}{\gamma^\mu_\parallel \cdot p_\parallel + m} \gamma^\nu_\parallel \\
\times \frac{1}{\gamma^\mu_\parallel \cdot (p - q)_\parallel + m} \Delta[\text{sgn}(eH)].
\]

(9)

In obtaining (9), we have made use of the following properties

\[
\Delta[\text{sgn}(eH)] \gamma^\mu_\parallel \Delta[\text{sgn}(eH)] = \gamma^\mu_\parallel \Delta[\text{sgn}(eH)],
\]

(10)

\[
\Delta[\text{sgn}(eH)] \gamma^\mu_\perp \Delta[\text{sgn}(eH)] = 0.
\]

(11)

The presence of \(\Delta[\text{sgn}(eH)]\) in (9) is a consequence of the LLLA, which, as explicitly displayed in (9), leads to an effective dimensional reduction [1,3].

With the LLL fermion self-energy given by (8), the WT identity in the BVA (6) reduces in momentum space to [11]

\[
\gamma^\mu_\parallel \cdot q^\mu = (\gamma^\mu_\parallel \cdot p_\parallel + m) - [\gamma^\mu_\parallel \cdot (p - q)_\parallel + m],
\]

(12)

where, due to (11), the transverse components \(\gamma^\mu_\perp \cdot q^\mu\) on the left-hand side decouple in the LLLA.

Upon using the WT identity in the BVA (12), one can verify that \(\Pi^{\mu\nu}(q)\) is transverse, i.e., \(q^\mu \Pi_{\mu\nu}(q) = 0\). Explicit calculation in dimensional regularization shows that the \(1/\epsilon\) pole corresponding to an ultraviolet logarithmic divergence cancels, leading to \(\Pi^{\mu\nu}(q) = \Pi(q^2_\parallel, q^2_\perp)(g^{\mu\nu} - q^\mu_\parallel q^\nu_\parallel / q^2_\parallel)\). This in turn implies that the full photon propagator takes the following form in covariant gauges (\(\xi = 1\) is the Feynman gauge):

\[
\mathcal{D}^{\mu\nu}(q) = \frac{1}{q^2 + \Pi(q^2_\parallel, q^2_\perp)} \left( g^{\mu\nu} - \frac{q^\mu_\parallel q^\nu_\parallel}{q^2_\parallel} \right) + \frac{q^\mu_\perp q^\nu_\perp}{q^2} + \frac{q^\mu_\parallel q^\nu_\perp}{q^2 q^2_\parallel} \\
+ (\xi - 1) \frac{1}{q^2} \frac{q^\mu q^\nu}{q^2}.
\]

(13)

The polarization function \(\Pi(q^2_\parallel, q^2_\perp)\) is given by

\[
\Pi(q^2_\parallel, q^2_\perp) = \frac{2\alpha}{\pi} |eH| \exp \left( -\frac{q^2_\perp}{2|eH|} \right) F \left( \frac{q^2_\parallel}{4m^2} \right),
\]

(14)

where \(\alpha = e^2/4\pi\) is the fine-structure constant and

\[
F(u) = 1 - \frac{1}{2u\sqrt{1 + 1/u}} \log \frac{\sqrt{1 + 1/u} + 1}{\sqrt{1 + 1/u} - 1}.
\]

(15)
The above result for \( \Pi(q^2, q^2) \) agrees with those obtained in Refs. [12,3,4,5,13]. The function \( F(u) \) has the following asymptotic behavior: \( F(u) \simeq 0 \) for \( |u| \ll 1 \) and \( F(u) \simeq 1 \) for \( |u| \gg 1 \). The polarization effects modify the propagation of virtual photons in an external magnetic field. Whereas photons of momenta \( |q^2| \ll m^2 \) remain unscreened, photons of momenta \( m^2 \ll |q^2| \ll |eH| \) and \( q^2 \ll |eH| \) are screened with a characteristic length \( L = (2\alpha|eH|/\pi)^{-1/2} \).

In terms of (7) and (8), the fermion self-energy in the BVA, when evaluated on the fermion mass shell, is found to be given by [11]

\[
m \Delta[\text{sgn}(eH)] = ie^2 \int \frac{d^4q}{(2\pi)^4} \exp \left( -\frac{q^2}{2|eH|} \right) \gamma^\mu \frac{1}{\gamma^{\parallel} \cdot (p - q)^{\parallel} + m} \gamma^\nu \times D_{\mu\nu}(q) \Delta[\text{sgn}(eH)] \bigg|_{p^2 = -m^2},
\]

(16)

where \( D_{\mu\nu}(q) \) is given by (13) and use has been made of (10) and (11). The presence of \( \Delta[\text{sgn}(eH)] \) in (16) is again a consequence of the LLLA. Using the WT identity in the BVA (12), one can rewrite the would-be gauge dependent contribution (denoted symbolically as \( \Sigma_\xi \)) on the right-hand side of (16) as

\[
\Sigma_\xi = ie^2(\xi - 1)(\gamma^{\parallel} \cdot p^{\parallel} + m) \int \frac{d^4q}{(2\pi)^4} \exp \left( -\frac{q^2}{2|eH|} \right) \frac{1}{(q^2)^2} \\
\times \frac{1}{\gamma^{\parallel} \cdot (p - q)^{\parallel} + m} \gamma^{\parallel} \cdot q^{\parallel} \Delta[\text{sgn}(eH)].
\]

(17)

Since \( \Sigma_\xi \) is proportional to \( (\gamma^{\parallel} \cdot p^{\parallel} + m) \), it vanishes identically on the fermion mass shell \( p^2 = -m^2 \) or, equivalently, \( \gamma^{\parallel} \cdot p^{\parallel} + m = 0 \). This, together with the transversality of the vacuum polarization, completes our proof that the BVA is a consistent truncation of the SD equations. Consequently, the dynamical fermion mass, obtained as the solution of the truncated SD equations evaluated on the fermion mass shell, is manifestly gauge independent.

Having proved the gauge independence of the dynamical fermion mass in the BVA, we are now ready to find \( m \) by solving (16) self-consistently. Note that the transverse components in \( D_{\mu\nu}(q) \) decouple in the LLLA. Following the same argument given above in the proof of the on-shell gauge independence, one can verify that contributions from the longitudinal components in \( D_{\mu\nu}(q) \) proportional to \( q^{\mu} q^{\nu} / q^2 \) vanish identically on the fermion mass shell. Therefore, only the first term in \( D_{\mu\nu}(q) \) proportional to \( q_{\parallel}^{\mu} q_{\parallel}^{\nu} / q_{\parallel}^2 \) contributes to the on-shell SD equation (16). Consequently, the matrix structures on both sides of (16) are consistent. With this, we find from (16) the gap equation that determines the dynamical fermion mass [11]

\[ m \Delta[\text{sgn}(eH)] = i e^2 \int \frac{d^4q}{(2\pi)^4} \exp \left( -\frac{q_{\parallel}^2}{2|eH|} \right) \gamma^{\mu} \frac{1}{\gamma^{\parallel} \cdot (p - q)^{\parallel} + m} \gamma^{\nu} \times D_{\mu\nu}(q) \Delta[\text{sgn}(eH)] \bigg|_{p^2 = -m^2}, \]

(16)
\[ m = \frac{\alpha}{2\pi^2} \int d^2q_{\parallel} \frac{m}{q_{\parallel}^2 + (q_4 - m)^2 + m^2} \int_0^{\infty} dq_{\perp}^2 \frac{\exp(-q_{\perp}^2/2|eH|)}{q_{\perp}^2 + q_{\perp}^2 + \Pi(q_{\parallel}^2, q_{\perp}^2)}, \]  

where \( q_{\parallel}^2 = q_{\parallel}^2 + q_{\parallel}^2 \). Here we have made a Wick rotation to Euclidean space and used the mass shell condition \( p_\parallel^2 = (m, 0) \).

The generalization of our result to the case of QED with \( N_f \) fermion flavors can be done straightforwardly by the replacement \( \Pi(q_{\parallel}^2, q_{\perp}^2) \to N_f \Pi(q_{\parallel}^2, q_{\perp}^2) \) in (13) and (18). We have numerically solved (18) to obtain \( m \) as a function of \( \alpha \) for several values of \( N_f \) (see Fig. 1). Numerical analysis shows that the solution of (18) can be fit by the following analytic expression:

\[ m = a \sqrt{2|eH|} \beta(\alpha) \exp \left[ -\frac{\pi}{\alpha \log(b/N_f \alpha)} \right], \]  

where \( a \) is a constant of order one, \( b \simeq 2.3 \), and \( \beta(\alpha) \simeq N_f \alpha \).

Our result differs from those obtained in Refs. [4,5], in which the so-called improved rainbow approximation is used. We note that the improved rainbow approximation used in Refs. [4,5] is exactly the same as the BVA used in this article. It can be verified fairly easily that the truncated SD equations in both approximations resum identically the same infinite subset of diagrams. Before concluding this article, we argue that to a large extent those earlier results can be attributed to gauge dependent artifacts. A detailed comparison to previous works and further discussions will be presented elsewhere [11].

The authors of Ref. [4] claimed that (i) in covariant gauges there are one-loop vertex corrections arising from the term \( q_\parallel^\mu q_\parallel^\nu / q_\parallel^2 q_{\parallel}^2 \) in the full photon propagator that are not suppressed by powers of \( \alpha \) (up to logarithms) and hence need to be accounted for; (ii) there exists a noncovariant and nonlocal
gauge in which, and only in which, the BVA is a reliable truncation of the SD equations that consistently resums these one-loop vertex corrections. The gauge independent analysis in the BVA, as presented in this article, shows clearly that such contributions vanish identically on the fermion mass shell. To put it another way, had the authors of Ref. [4] calculated properly the physical, on-shell dynamical fermion mass (as we have done in our study), they would not have found the “large vertex corrections” they obtained, and therefore their claim that the BVA (or improved rainbow approximation) is a good approximation only in the special noncovariant and nonlocal gauge they invoke is not valid. Together with the fact that in the BVA there are no diagrams with vertex corrections being resummed by the SD equations, our analysis calls into question about the validity of their conclusions.

We emphasize that the WT identity is a necessary condition for the gauge independence of the dynamical fermion mass, but it is far from sufficient. While the WT identity guarantees the truncated vacuum polarization is transverse, it guarantees only the truncated on-shell fermion self-energy is gauge independent. Hence, the dynamical fermion mass is gauge independent only when determined by the position of the fermion pole obtained in a consistent truncation. This is tantamount to evaluating the truncated fermion self-energy on the fermion mass shell. Even though the WT identity in the BVA is verified in Ref. [4] in a particular noncovariant and nonlocal gauge, this does not guarantee that the dynamical fermion mass obtained therein from the truncated fermion self-energy evaluated off the fermion mass shell will be gauge independent. In fact, the particular noncovariant and nonlocal gauge is invoked by hand such that the gauge dependent contribution cancels contributions from terms proportional to $q_\mu^\nu q_\nu^\parallel /q^2_\parallel$ in the full photon propagator. Our gauge independent analysis in the BVA reveals clearly that such a gauge fixing is not only ad hoc and unnecessary, but also leaves the issue of gauge independence unaddressed.

In a recent article [5], the authors claimed that (i) in QED with $N_f$ fermion flavors a critical number $N_{cr}$ exists for any value of $\alpha$, such that chiral symmetry remains unbroken for $N_f > N_{cr}$; (ii) the dynamical fermion mass is generated with a double splitting for $N_f < N_{cr}$. As can be gleaned clearly from Fig. 1, both these conclusions are incorrect. They are gauge dependent artifacts of an inconsistent truncation. On the one hand, the SD equation for the fermion self-energy was obtained (in an unspecified “appropriate” gauge) in the BVA within the LLLA. On the other hand, the vacuum polarization was calculated in the BVA but beyond the LLLA. This, however, is not a consistent truncation of the SD equations because the WT identity in the BVA can be satisfied only within the LLLA [11].

The result of Ref. [5] suggests that in the inconsistent truncation as well as in the unspecified gauge used there, the unphysical, gauge dependent contribu-
tions from higher Landau levels are so large that they become dominant over the physical, gauge independent contribution from the LLL and lead to the authors’ incorrect conclusions. Hence, we emphasize that the LLL dominance in a strong magnetic field should be understood in the context of consistent truncation schemes as follows. Contributions to the dynamical fermion mass from higher Landau levels that are obtained in a (yet to be determined) consistent truncation of the SD equations are subleading when compared to that from the LLL obtained in the consistent BVA truncation. To the best of our knowledge, such a consistent truncation has not appeared in the literature.

In conclusion, we have presented a consistent truncation of the SD equations in the LLLA that allows us to study in a gauge independent manner the physics of chiral symmetry breaking in a strong external magnetic field. The gauge independent approach to the Schwinger-Dyson equations discussed in this article is general in nature, and hence not specific to the problem at hand. We believe that this approach will be useful in other areas of physics that also require a nonperturbative understanding of gauge theories.

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