WZ Sagittae as a DQ Herculis star

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ABSTRACT

We argue that quiescent WZ Sge is a rapidly spinning magnetic rotator in which most of the matter transferred from the secondary is ejected from the system. Assuming that the observed 27.87-s oscillation period results from the spinning white dwarf, we propose that the other observed principal period of 28.96 s is a beat caused by reprocessing of the rotating white dwarf beam on plasma blobs in Keplerian rotation at the outer disc rim. The weaker, transient, 29.69-s period is identified as a beat with the Keplerian period of the magnetosphere. WZ Sge evolves through a cycle of spin-up and spin-down phases. During the spin-down phase it is a DQ Her star; during the spin-up phase it should be an ER UMa star.

Key words: stars: individual: WZ Sge – stars: magnetic fields – novae, cataclysmic variables – stars: oscillations – stars: rotation.

1 INTRODUCTION

WZ Sagittae is a remarkable dwarf nova binary system. Dwarf novae are a subclass of cataclysmic variables (CVs) which show more or less regular outbursts. In CVs, a late-type, Roche-lobe filling secondary star loses mass which, in general, is accreted by a white dwarf primary. Usually, when the white dwarf is not too strongly magnetized, the accreting matter forms an accretion disc. It is well established that such discs are the sites of dwarf nova outbursts. WZ Sge outbursts are very rare (the recurrence time is around 30 yr) and they are only of the ‘superoutburst’ type, i.e., they are long (~ 30 d), high-amplitude (~ 7 mag) outbursts during which a ‘superhump’ in the optical light curve is observed at a period slightly longer than the orbital one. WZ Sge is the prototype of a class of systems with similar outburst properties (sometimes also called ‘TOADs’). The orbital period of WZ Sge is 81.63 min, close to the minimum period (~80 min), below which no (non-degenerate) CV has been observed.

It is generally believed that dwarf nova outbursts result from a thermal–viscous instability present in accretion discs at temperatures corresponding to hydrogen partial ionization. In the standard version of the disc instability model (DIM) (see Hameury et al. 1998 for the most recent version of this model, and Cannizzo 1993 and Lasota & Hameury 1998 for reviews) one assumes that mass transfer from the secondary is constant prior to, and during, the outburst. Properties of outbursts depend on the viscosity mechanism which transports angular momentum and heats the disc. In the DIM the kinematic viscosity coefficient is taken as \( \nu = \alpha c_s H \), where \( c_s \) is the (adiabatic) speed of sound, \( H \) the disc semithickness, and the ‘viscosity coefficient’ \( \alpha < 1 \) (Shakura & Sunyaev 1973). The DIM requires different values of \( \alpha \) in outburst and in quiescence (Smak 1984b). The quiescent value for the great majority of systems should be ~0.01 (Livio & Spruit 1991). Smak (1993) showed, however, that the outburst cycle of WZ Sge cannot be described by such a version of the DIM (see also Osaki 1996). The main reason is that for \( \alpha \sim 0.01 \) the disc is able to accumulate enough to be stable with respect to the thermal instability believed to be responsible for dwarf-nova outbursts. They noticed that at low mass transfer rates outer regions of accretion discs in CVs are cold enough to be stable with respect to the thermal instability believed to be responsible for dwarf-nova outbursts. They proposed that the WZ Sge accretion disc does not extend down to the surface of the white dwarf, but is truncated at a radius corresponding to a stable outer disc. An enhancement of mass transfer would bring such a disc into an unstable state and thus trigger an outburst. One should
stress that such an outburst would still be due to the thermal
viscous instability, the enhanced mass transfer serves only as a
trigger. The recurrence time is then related to the characteristic
time-scale of mass transfer fluctuations. One can also obtain long
recurrence times if the truncated disc is marginally unstable
(Warner, Livio & Tout 1996), but in practice such a disc is
indistinguishable from a marginally stable one proposed by Lasota
et al. (1995). There is still, however, a problem to be solved: since
\( \alpha \approx 0.01 \), there would be only \( \approx 10^{15} \) g in the disc available. In this
case, therefore, mass must be added to the disc during outburst.
There is observational evidence (Smak 1997; see also Hessman et
al. 1984) that irradiation of the secondary during outburst increases
mass transfer from the secondary (see Hameury et al. 1997 for a
simple model).

In Lasota et al. (1995) and Hameury et al. (1997) the truncation
was assumed to be due either to evaporation (Meyer & Meyer-
Hofmeister 1994) or to the presence of a magnetic field strong
enough to disrupt the quiescent disc (<10^7 G would be sufficient for
a low-mass white dwarf). The second hypothesis seemed to be
favoured by the presence, before the last, 1978 outburst, of a 27.87-s
coherent oscillation which could be attributed to the rotation of an
accreting white dwarf (Patterson 1980, hereafter P80). There
were two problems with this interpretation. First, in addition to
the 27.87-s period, several other (some of them transient) ‘satellite
period’ oscillations have been observed (Robinson, Nather &
Patterson 1978; P80). The interpretation of these periods in terms
of beat frequencies resulting from reprocessing of the white dwarf’s
pulsed light on various features in the accretion flow encountered
several difficulties (P80). Second, all the oscillations disappeared
during the outburst. After the outburst the principal, 27.87-s,
oscillation has been absent for 16 yr (although the 28.96-s oscillation
was seen when WZ Sge was at twice its pre-outburst brightness).
Although it was easy to understand that during the outburst the increased accretion rate could suppress the magnetosphere, the
lack of the principal pulse afterwards was difficult to understand,
and it cast a shadow of doubt on the presence of a rapidly rotating
white dwarf in WZ Sge.

In 1995 the 27.87-s oscillation reappeared in the company of the
previously present 28.96-s pulsation (Patterson et al. 1998, here-
after P98). Weak, transient ‘satellite period’ pulses are also seen at
28.2 and 29.69 s. The most important news, however, was the
detection of a 27.86 \pm 0.01-s period in the 2–6 keV ASCA energy
band (P98), which confirmed the presence of a rotating, magnetized
white dwarf in WZ Sge. The presence of a rapidly rotating white
dwarf was confirmed by UV spectral observations (Cheng et al.
1997). WZ Sge is therefore a DQ Her star, a CV containing a
rapidly rotating magnetized white dwarf.

In a recent article Meyer-Hofmeister, Meyer & Liu (1999) (see
also Mineshige et al. 1998) argue that the inner disc ‘hole’ in WZ
Sge is due to evaporation and not to the presence of a magnetic field.
These authors, however, underestimate by a significant factor the
coherece of the observed pulsations, and their model seems to be
contradicted by observations. For example, their model predicts the
presence of superhumps well before the beginning of the super-
outburst, whereas in WZ Sge this feature appeared only 10–12 d
after the maximum (Patterson et al. 1981). Also, the remarkable
constancy of the X-ray luminosity after the 1978 outburst is in
contradiction with their model.

In this article we argue that Patterson’s (P80, also P98)
hypothesis that WZ Sge contains a rapid, oblique magnetic rotator
is consistent with most of the observed properties of this system. We
show that sideband oscillations observed in WZ Sge can be
interpreted as resulting from reprocessing of the white dwarf’s
pulsed light on various features of the transferred flow. In Section 2
we present the observed properties of WZ Sge, and identify various
oscillations with orbital sidebands. In Section 3 we discuss possible
configurations of WZ Sge as a system containing a magnetic
rotator, and show that only one is consistent with the system
properties at outburst. In Section 4 we discuss the accretion history
of WZ Sge and its relation to other CVs. We summarize our model
in Section 5.

2 WZ SGE AS A MAGNETIC ROTATOR

If WZ Sge is an oblique magnetic rotator, one should, in principle,
observe in its optical light curve the fundamental spin frequency
and some sidebands resulting from the beat between this and the
orbital frequencies (Warner 1986). In what follows we will assume
that all characteristic periods observed in WZ Sge have a rotational
origin. The possibility that these oscillations (or some of them;
Warner 1995) are due to white dwarf pulsations cannot be excluded (see Wood 1999 for recent arguments in favour of this
model). Such a model is, however, hard to test, because it does not
make any predictions (P98). In what follows we will try to show that
all observed periodicities observed in WZ Sge can be explained in
the framework of a magnetic rotator model.

2.1 General properties and parameters

Following Warner (1995a), we classify WZ Sge as a DQ Her star.
DQ Her stars are rapidly rotating intermediate polars (IPs), i.e.,
magnetic CVs in which the white dwarf rotation is not syn-
chronous with the orbital motion (synchronous systems are called
‘polars’ or ‘AM Her stars’). Warner (1995a) adds also that DQ Her
stars are characterized by the absence of hard X-rays in the sense
that, contrary to ‘usual’ IPs, their X-ray temperature is much lower
than 10 keV. In WZ Sge the X-ray temperature is \( \approx 4.5 \) keV (P98),
so it is a hard X-ray emitter, even if its temperature is lower than that
in ‘usual’ IPs (see below).

WZ Sge has been one of the best observed CVs, but the values of
its fundamental parameters are still uncertain. Smak’s (1993)
photometric solution gives \( M_1 = 0.45 \) (where \( M_1 \) is the white
dwarf mass in solar units) and \( q = M_2/M_1 = 0.13 \), where \( M_2 \) is
the secondary’s mass, whereas Spruit & Rutten (1998), who
model the ‘hotspot’ at which the mass transfer stream interacts with the
outer disc regions, get \( M_1 = 1.2 \) and \( q = 0.075 \). As we shall see, Smak’s value is too small if the white dwarf in WZ Sge
is spinning at 27.87 s, but the Spruit & Rutten (1998) value might
be too high.

In Fig. 1 we show various mass–radius relations relevant for WZ
Sge. The white dwarf mass–radius relation is that of Nauenberg
(1972), which is suitable for helium white dwarfs (using other
mass–radius relations gives very similar results; e.g. P98). Also
plotted is the corotation radius

\[
R_\text{co} = \left( \frac{GM_2 M_1 P_1^2}{4\pi^2} \right)^{1/3} = 1.5 \times 10^9 P_1^{2/3} M_1^{1/3} \text{ cm},
\]

(1)

where \( P_1 \) is the white dwarf spin period. The corotation radius
corresponds to a distance at which a free particle in circular
Keplerian orbit corotates with the white dwarf. In particular, the
white dwarf radius must satisfy \( R_\text{i} < R_\text{co} \). If one assumes that
\( P_1 = 27.87 \text{ s} \), Fig. 1 shows that for Smak’s \( M_1 = 0.45 \) the white
dwarf would be rotating just below the break-up speed, so \( M_1 \) must
be larger than 0.45 (see also P98).
The 28.19-s oscillation might be composed of signals at 28.14 and 28.24 s, oscillations are present, sometimes with additional periods up to 2.2 s. Orbital sidebands Figure 1. dashed line), the corotation radius (\(R_c\); equation 1), the critical radius (\(R_{\text{crit}}\)) above which a disc is marginally stable (equation 6) for three values of \(\alpha\) (see text; dot-dashed lines), and the Kepler radii at which the beat between the white dwarf spin frequency and the Kepler frequency gives rise to the observed oscillations at 28.96 and 29.69 s. The inferred solution from Smak (1993) is also indicated. With dotted lines we indicate the white dwarf masses as derived by Smak (1993) and Spruit & Rutten (1998), i.e., 0.45 and 1.2 \(M_\odot\) respectively.

### 2.2 Orbital sidebands

In addition to the 27.87-s oscillation, several other optical oscillations between 27.8 and 30 s have been observed in WZ Sge (P98, and references therein). In general, either the 27.87- or 28.96-s oscillations are present, sometimes with additional periods up to ~30 s (i.e., 28.19, 28.52, 29.69 s). A 28.2-s oscillation is also seen in the UV (Welsh et al. 1997) [strictly speaking, the UV oscillation (at \(q = 3\)] does not exist in WZ Sge, truncating the inner parts of an accretion disc and making it similar to an IP. It also confirms that the 27.87 s period is the white dwarf spin period; checks made by assuming the white dwarf spin period is at the other observed frequencies show that although the expected sideband frequencies match the observed frequencies, there is the no explanation for the 27.87 s period. For example, assuming 28.19-s as the white dwarf spin period gives sidebands at 28.03, 28.35 and 28.52 s. These are all observed in the power spectra of P80 and P98. In addition, \(\omega - 2\Omega\) gives 28.96 s. \(P_k = 28.09-s\) gives sidebands at 28.25, 27.93, 28.42 s which in principle could match the observed peaks in the power spectrum, as well as \(\omega - 3\Omega\) which gives 28.58 s. In both cases, however, the 27.87-s oscillation remains unexplained.

We identify the 28.2-s optical and UV oscillations with \(\omega - 2\Omega\), which would result from reprocessing on the hotspot. Welsh et al. (1997) identify this period with the white dwarf spin period, but the presence of the 27.87 s period in X-rays does not support this identification. If the 28.2-s oscillation results from reprocessing on the hotspot, one must then explain why the fundamental spin period (27.87 s) is not seen in the UV. In order to understand this absence, one should, presumably, understand the structure of the accretion flow. As we shall see below, the spin period of WZ Sge implies that this system is in an ‘ejector’ phase, ejecting most of the transferred matter. Such a case would be totally different from the usual IPs (see, e.g., Hellier 1998). No model of such a flow seems to exist at present.

It is also interesting to note that a reprocessing projected area might also vary with the \(2\Omega\) component (Warner 1995a). This will introduce components at frequencies \(\omega + 3\Omega\) and \(\omega - 2\Omega\) in the power spectrum. The latter produces a peak near 28.35 s, which is visible (at ~28.34 s) as a shoulder of the peak near 28.19 s in the ‘grand average’ power spectrum of the observations prior to the outburst end of 1978 (P80; see also P98). We note that the observed oscillation at 28.52 s is consistent with \(\omega - 4\Omega\) (P80), but we have no interpretation for this fact.

Having explained to our satisfaction various sidebands, we are still left with the 28.96- and 29.69-s periods which are not commensurable with the orbital period. The 28.96-s signal appears as one of the two main peaks in the power spectrum and is generally present. The 29.69-s signal is weak and transitory.

### 3 WZ SGE AS A MAGNETIC EJECTOR

#### 3.1 A discless system?

P80 suggested that the 28.96-s period could result from the beat between the white dwarf spin frequency and the Keplerian frequency at the magnetosphere. The required Keplerian period is \(P_k = 733.47\) s and could correspond to the rotation period of plasma blobs at the disc inner edge. The problem with such an interpretation is that, if true, not much of a disc would be left. The corresponding magnetospheric radius would be

\[
R_{\text{M}}(28.96) = 1.22 \times 10^{10} M_1^{\frac{3}{4}} \text{cm}.
\]

According to Smak (1993), for \(M_1 = 0.45\), the outer disc radius is \(R_0 = 1.07 \pm 0.19 \times 10^{10} \text{cm}\), whereas Spruit & Rutten (1998) obtain, for \(M_1 = 1.2\), \(R_0 = 1.75 \pm 0.14 \times 10^{10} \text{cm}\) so that in both cases we would rather have a ring, then a disc, or no disc at all, depending on the details of plasma interaction with the magnetic field (see, e.g., King 1993). In addition, if the Keplerian radius \(R_k\) corresponding to the 28.96-s period were the magnetospheric radius, the so-called ‘fastness parameter’ (see, e.g., Frank, King...
& Raine 1992) would be
\[ \omega_s = \frac{P_K}{P_s} = 26.3. \] (3)

Since \( \omega_s > 1 \), matter transferred from the secondary could not be accreted on to the white dwarf and would rather be ejected (as noticed by P98). WZ Sge could then be similar to another DQ Her star, AE Aqr, which is a discless ‘jector’ (Wynn, King & Horne 1997). Such a solution would be rather attractive: two, of three known, DQ Her stars would be discless magnetic ejectors; apparent similarities between WZ Sge and AE Aqr have already been already pointed out in P80.

We think, however, that such a model cannot describe WZ Sge. First, Wynn et al. (1997) show that in AE Aqr the Hα Doppler maps of the system are consistent with their simulations of a discless flow. Similar maps for WZ Sge (Spruit & Rutten 1998) look completely different, and clearly show the presence of a disc and not just a ring, although the authors point out that the brightness distribution is less ‘disc-like’ than in other CVs. One should bear in mind, however, that, as pointed out by Spruit & Rutten, the Hα brightness may not be a good tracer of matter distribution. On the other hand, Mennickent & Arenas (1998) find that the accretion flow in WZ Sge forms a ‘ring’ with a ratio of the inner to the outer radii \( \sim 0.3 \).

Second, contrary to AE Aqr, WZ Sge is a dwarf nova so that we expect an accretion disc to be present prior to, and during, the outburst (this is independent of the validity of the DIM – there is ample observational evidence that dwarf nova outbursts require the presence of accretion discs around white dwarfs). One could imagine that enhanced mass transfer prior to outburst could squeeze the magnetosphere and allow the reappearance of an accretion disc. However, even if this happened, the superoutbursts observed in WZ Sge would not occur. This is shown in Figs 2–4, where we have plotted the magnetospheric radius as a function of the accretion rate for different primary masses. For the magnetospheric radius we use the approximate formula (see, e.g., Frank et al. 1992)
\[ R_m = 9.8 \times 10^8 M_{15}^{-2/7} M_1^{1/7} \mu_{30}^{-4/7} \text{cm}, \] (4)

where \( M_{15} = \) the accretion rate in \( 10^{15} \text{g s}^{-1} \), and \( \mu = \mu_{30}(10^3 \text{G cm}^3) = B R_1^3 \) is the magnetic moment of a white dwarf with a surface field \( B \). Magnetic moments corresponding to the 29.69-s oscillation are: \( \mu_{30} = 34 \) for \( M_1 = 0.45 \), \( \mu_{30} = 55 \) for \( M_1 = 0.80 \), and \( \mu_{30} = 77 \) for \( M_1 = 1.2 \) (see also Section 3.4).

Near maximum of the 1978/1979 outburst most of the disc material was accreted on to the white dwarf. According to Smak (1993) the maximum accretion rate during the 1978 outburst was \( 3.2 \times 10^{18} \text{g s}^{-1} \). For primary masses larger than Smak’s \( M_1 = 0.45 \), this maximum would be lower (Smak, private communication). As the mass accretion increases during the rise to outburst, the magnetospheric radius moves inward. Accretion occurs only when this radius moves inside the corotation radius. Since no clear oscillations are seen during most of the outburst (Patterson et al. 1981), this suggests that in WZ Sge the disc was able to reach the white dwarf surface (P98). It can be seen, however, from Figs 2–4 that at outburst maximum the magnetospheric radius given by equation (2), i.e., the inner disc radius, would just reach the corotation radius. For a given maximum accretion rate this is the case for all white dwarf masses (because the relevant radii are Keplerian), but in reality, for higher masses, the magnetospheric radius would not even get there, since in this case the maximum accretion rate would be smaller than the value plotted in Figs 2–4. It is obviously impossible to have the maximum of accretion luminosity at the very moment at which accretion on to the white dwarf just begins.

We conclude therefore that WZ Sge is not in a discless ejector phase. As we will show below, WZ Sge is most probably in a state intermediate between those of the other two DQ Her stars: AE Aqr, a pure discless ejector (Wynn et al. 1997), and DQ Her itself which seems to have a steady accretion disc (Warner 1995a).

### 3.2 Blobs at the outer disc edge

As can be seen in Fig. 2, the Keplerian radius corresponding to the frequency giving the 28.96-s beat with the white dwarf spin frequency is very close to the outer disc radius as determined from modelling observational data. For \( M_1 = 0.45 \) it coincides, within the error bars, with \( R_D \), and one can expect that this will be true also for white dwarf masses smaller than \( \sim 1 M_\odot \). We propose, therefore, that the 28.96-s beat frequency results from reprocessing of the spinning, magnetized, white dwarf beam on plasma blobs orbiting at, or close to, the outer disc radius. According to Spruit & Rutten (1998) the ‘tail’ of the observed hotspot’s Hα emission is probably due to material orbiting at the outer edge of the disc at the Keplerian velocity. This material had crossed the stream–disc interaction region undergoing a sequence of heating and cooling events that are likely to form a non-uniform flow. In a stationary accretion disc the outer radius is well defined and constant in time, so that one can expect the motion of plasma blobs there to give coherent oscillations. Warner (1995b) suggested that QPOs observed during some dwarf nova outbursts could be due to blobs orbiting at the outer disc.
radius. During outbursts, however, the outer disc radius moves (in and out; see, e.g., Smak 1984a), and so coherent oscillations should not be expected in such a case.

Our identification of the 28.96-s period origin is therefore qualitatively consistent with the Spruit & Rutten (1998) model of WZ Sge. A quantitative agreement will be difficult to achieve, because the interpretation of the Hα ‘tail’ as due to matter in Keplerian motion leads to their high white dwarf mass (1.2 M⊙ (1999 RAS, MNRAS a WZ Sge. A quantitative agreement will be difficult to achieve, qualitatively consistent with the Spruit & Rutten (1998) model of

As Fig. 2, but now for Mwd = 0.8 M⊙, Rwd is determined from the mass–radius relation appropriate for helium white dwarfs (Nauenberg 1972). No solution of Rdisc exists in the literature for Mwd = 0.8 M⊙.

With such a mass the Keplerian period of the outer disc rim cannot be ~733 s. The value of the white dwarf mass in WZ Sge is, however, subject to controversy (see, e.g., the discussion in Spruit & Rutten 1998), but for our purpose (see below) the value of 1.2 M⊙ is too high. A lower value of ~0.8 M⊙ would be consistent with our hypothesis concerning the origin of the 28.96-s period and the estimates of P98.

3.3 A (marginally) stable accretion disc?

As shown by Lasota et al. (1995), the very long outburst recurrence time of WZ Sge can be reconciled with the ‘standard’ values of the viscosity parameter α if the disc is truncated so that between outbursts it is (marginally) stable. As we will see below, at present, the flow of transferred matter in WZ Sge does not, probably, form a standard accretion disc. We know, however, that in the past there was an accretion disc in WZ Sge (and we expect one to form in the future), so it is interesting to see what is the inner radius required by the stability criterion.

The critical value of accretion rate Mcrit below which a disc is in cold, stable equilibrium can be expressed as (Hameury et al. 1998; for a similar expression see, e.g., Smak 1984b and Ludwig, Meyer-Hofmeister & Ritter 1994):

\[ M_{\text{crit},15} = 4.0 \alpha^{-0.90} M_1^{0.89} R_1^{-0.27} \]

where \( M_{\text{crit},15} \) is the critical \( M \) in units of \( 10^{15} \) g s\(^{-1} \), and \( R_1 \) the accretion disc radius in units of 10\(^7\) cm. Note that \( M_{\text{crit}} \) depends only weakly on the value of \( \alpha \) (in similar formulae by other authors the critical \( M \) is independent of \( \alpha \) (e.g. Ludwig et al.

The corresponding Kepler frequency at \( R_{\text{crit}} \) is given by

\[ \omega_{\text{crit}} = 0.025 \dot{M}_{15}^{0.56} \alpha^{-0.022} \]

This equation shows that \( \omega_{\text{crit}} \) is independent of the mass of the white dwarf and only very weakly dependent on \( \alpha \). Since the quiescent \( \dot{M} \) in WZ Sge is \( \sim 10^{15} \) g s\(^{-1} \) (Smak 1993), this leads to a Kepler period at \( R_{\text{crit}} \), i.e., \( P(R_{\text{crit}}) \sim 330 - 370 \) s for values of \( \alpha \sim 0.01 - 1 \). Therefore, in the framework of the truncated disc model, any beat period between the white dwarf spin period and a Kepler period of matter in the accretion disc in the quiescent state of WZ Sge should have periods shorter than \( \sim 30.4 \) s, as observed.

We notice that the recently reappeared oscillation at 29.69 s (P98) is very close to this value. We therefore suggest that the 29.69 s period is the beat between the white dwarf spin period and the Kepler period of matter near \( R_{\text{crit}} \). Since the inner disc radius, \( R_{\text{in}} \), is larger than \( R_{\text{crit}} \) in quiescence, the 29.69-s period would mark the inner part of the disc, where matter is dominated by the magnetic field.

We propose that the magnetospheric radius is located at

\[ R_{\text{in}}(29.69) = 8.87 \times 10^3 M_1^{1/3} \text{ cm}, \]

corresponding to the Keplerian period \( P_M = 454.65 \) s. The fastness parameter is now

\[ \omega_s = \frac{P_K}{P_s} = 16.3, \]

still much larger than 1, so that most of the mass transferred from the secondary cannot be accreted by the white dwarf, but is ejected from the system (see below).

Not all matter is ejected – a few per cent finds its way to the white
dwarf and accounts for the pulsed emission. The configuration we have in mind is different from disc accretors, or disc accretors with overflowing accretion stream (see, e.g., Frank, King & Lasota 1987 and Hellier 1999). It is also different from the pure, discless ejector models of King (1993) and Wynn & King (1995). It could be a truncated ‘excretion disc’. It might be a structure one obtains in the models of King (1993) and Wynn & King (1995). It could be a and Hellier 1999). It is also different from the pure, discless ejector throw accretion stream (see, e.g., Frank, King & Lasota 1987

4 THE ACCRETION HISTORY OF WZ SGE

If WZ Sge is in an ejector phase, its white dwarf should be spinning down. Indeed, Patterson (1980) reports \( P_s = 8 \times 10^{-12} \) s. This value is very large compared with the \( P_s = 5.64 \times 10^{-11} \) observed in AE Aqr (de Jager et al. 1994). The implied spin-down power of WZ Sge would be \( \approx 5 \times 10^{35} \) erg s\(^{-1} \), two orders of magnitude higher than in AE Aqr. Wynn et al. (1997) find that (assuming that plasma interacts with magnetic field in the form of blobs) \( L_{\text{spin}} \) scales with \( M \). Since in WZ Sge the mass transfer rate is two orders of magnitude lower than in AE Aqr, the two orders of magnitude higher spin-down power is hard to understand. One should note, however, an interesting point: in WZ Sge the light cylinder radius \( R_c = c / \omega = 1.3 \times 10^{11} \) cm is larger than the size of the system (\( a \sim 3.4 \times 10^{10} \) cm), which allows, in principle, acceleration of particles outside the system. It may well be that results for systems in which \( R_c \approx a \) do not apply to WZ Sge. For particles accelerated to speeds close to the speed of light

The inner disc radius is given by \( R_{\text{in}} = R_M \) with \( R_M \) set by the observed 29.69 s oscillation, we derive magnetic field strengths of \( \sim 3.6 \times 10^4 \), \( \sim 1.6 \times 10^6 \) and \( \sim 1.3 \times 10^8 \) G respectively for \( M_1 = 0.45, 0.8 \) and 1.2. In the last case, the white dwarf in WZ Sge would have a magnetic field strength comparable to that of most IPs. The magnetic moment of the white dwarf in WZ Sge is therefore \( \mu = (3 - 8) \times 10^{31} \) G cm\(^3 \).

The magnetospheric radius is given by equation (8), then, for all white dwarf masses of interest,

\[
R_M > 4R_1
\]

which, according to Wickramasinghe, Wu & Ferrario (1991) (see also Warner 1995b) means that WZ Sge should be a hard X-ray emitter, as is indeed observed. In this respect, as mentioned before, WZ Sge is different from the other two DQ Her stars, but for a reason that is obvious: a much lower mass transfer rate.

On the other hand, in outburst, WZ Sge is not a DQ Her star but enters the dwarf nova oscillation (DNO) regime in which the ‘slippage of the surface’ i.e., differential rotation of the white dwarf’s outer layers is expected (Warner 1995a, b). For WZ Sge the minimum field required to maintain rigid rotation is \( B \sim 2 \times 10^8 M_1^{2/3} \) G, so in WZ Sge \( B < B \) (Fig. 5). No confirmed DNOs have been seen, however, during the last outburst (Patterson et al. 1981).
starts expanding to longer orbital periods. This ‘bounce’ is helped by the secondary becoming degenerate (see, e.g., King 1997). It would seem, therefore, that high accretion rates are impossible at short orbital periods.

The situation, however, is not as hopeless as it would seem. We are interested here in time-scales of the order of the white dwarf spin-up time, i.e., $10^7 - 10^5$ yr. Fluctuations on such short time-scale do not modify the secular evolution of the binary.

There is at least one piece of evidence which shows that the accretion history of WZ Sge might have had episodes of high mass transfer. There exist, at orbital periods between 79 and 92 min, systems which are supposed to have high (~$10^{17}$ g s$^{-1}$) mass transfer rates. These are ER UMa systems, which show extremely short intervals between superoutbursts (19–44 d) and very short (3–4 d) ‘normal’ outburst interval. They seem to be the high accretion rate equivalent of the low accretion rate systems showing superoutbursts: the SU UMa systems (WZ Sge is a SU UMa system showing superoutbursts only). The idea that DI UMa (one of the four known ER UMa systems with $P_{orb} = 79$ min), which is more luminous than WZ Sge by a factor ~50, spends most of its life as ‘an ordinary WZ Sge star’, but was caught during an ‘upward surge in accretion’, has been proposed by Patterson (1998). He speculates that the short period CV mass transfer cycles in which $M$ surges to high values (~1–5 per cent of the cycle time) are somehow associated with classical nova eruptions because, in addition to the four ER UMa’s, four other systems are ‘too bright’ for their orbital periods, and three of them are classical nova remnants.

Whatever the reason, what is important for our argument is the presence of systems which accrete at high rates but have orbital periods close to that of WZ Sge. With such accretion rates the white dwarf in WZ Sge can be spun up to $P_{eq} = 27.87$ s.

When the white dwarf gets to this equilibrium spin rate the mass transfer is stable (Wynn & King 1995). After some time the mass transfer rate will return to its secular value. This value, however, is two orders of magnitude lower than that required for $P_{eq} \sim$ few x $10^5$ s. In a very short time (viscous time of the disc ~days) the magnetosphere will start expanding and becoming larger than the corotation radius. The system enters into the ejector phase. As shown by Wynn & King (1995), this phase is dynamically stable (for 0.05 $\leq q \leq 1$). It will stay in this WZ Sge phase (for about $10^5$ yr) until it gets to a new spin equilibrium corresponding to the low accretion rate (~ few minutes). Such a system has been recently observed: IP RX J0757.0+6306 (Tovmassian et al. 1998) has an orbital period very close to that of WZ Sge (81 ± 5 min), but its spin period, 8.52 ± 0.15 min, corresponds to the equilibrium value as given by equation (12) for the expected secular mass transfer rate. The magnetic field of the white dwarf in this system could therefore have a strength close to that of WZ Sge. The binary RX J0757.0+6306 could therefore be a WZ Sge-type system at a particular phase of its spin-up/spin-down history. [Note that this system is different from the other IP with a very short orbital period (84 min), RX J1238 – 38. There the spin period is 36 min (Buckley et al. 1998), so it would be a ‘classical’ IP with $\mu_{32} \sim 10$.]

A jump in the mass transfer rate could bring such an equilibrium system back to a spin-up phase. If RX J0757.0+6306 has indeed a magnetic field close to the one we obtained for WZ Sge, and if its mass transfer rate is close the secular one, its accretion disc should be truncated at a radius close to the value given by equation (8), and therefore stable with respect to the thermal-viscous instability. It is not yet clear if this system shows any kind of dwarf-nova activity.

5 SUMMARY OF THE MODEL

We can summarize our model (or ‘scenario’) as follows: WZ Sge contains a $\lesssim 0.8 \cdot M_\odot$ magnetized white dwarf spinning with a period of 27.87 s. The magnetic moment is $\lesssim 5 \times 10^{31}$ G cm$^3$, i.e., a magnetic field at the surface $\lesssim 2 \times 10^9$ G.

In quiescence the mass transferred from the secondary forms a disc-like structure with outer radius and inner radii at 1.22 and $8.57 \times 10^{10}$ cm respectively. The accretion flow is disrupted by the rapidly rotating magnetosphere and, since the fastness parameter is $\gg 1$ (the corotation radius is much smaller than the magnetospheric radius), most of the matter cannot be accreted by the white dwarf. A small fraction ($\lesssim 10$ per cent) finds its way to the white dwarf surface, where it is responsible for the observed X-ray emission.

The white dwarf is in a spin-down phase, and most of the transferred matter is ejected from the system. The disc-like structure formed by the flow might be composed of plasma ‘blobs’. At outburst, which is triggered by an enhanced mass transfer event, an accretion disc is (re)created. At outburst maximum the disc reaches down to the surface of the white dwarf.

The white dwarf in WZ Sge will spin down to an equilibrium rotation period of a few minutes. In this phase an increase of accretion rate will transform it into an ER UMa star. After a spin-up phase the system will become a DQ Her star once more.

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