Abstract Precision measurements of $Z^0$ boson properties could enable a determination of the mass of the b quark at the scale of the $Z^0$ boson mass $m_b(m_{Z^0})$. The dependence of Standard Model predictions on the b quark mass using the program Gfitter is studied. The precision of the currently available measurements by the LEP experiments and SLD, together with measurements from the LHC experiments of the mass of the top quark and the Higgs boson, is not sufficient for a relevant determination. The predicted precision of $Z^0$ boson resonance measurements at future e$^+$e$^-$ colliders will allow a competitive direct determination of $m_b(m_{Z^0})$.

1 Introduction

The theory of strong interactions, Quantum Chromo Dynamics (QCD) [1–4] is a part of the Standard Model (SM) of particle physics via QCD corrections to all electroweak interactions [5–7] involving quarks. QCD has as the only free parameters the strong coupling constant, usually given as its value at the $Z^0$ mass scale $\alpha_s(m_{Z^0})$, and the masses of the six quarks of the SM. The values of the strong coupling as well as of the quark masses depend on and decrease with the energy scale of the interaction, which is known as asymptotic freedom.

The mass of the b quark in the $\overline{\text{MS}}$ renormalisation scheme [8] was determined from analysis of B-hadron mass spectra and thus at energy scales corresponding to the B-hadron masses with the current world average $m_b(m_b) = (4.183 \pm 0.004)$ GeV [9]. Measurements of the mass of the b quark at the energy scale of the $Z^0$ boson mass $m_b(m_{Z^0})$ were performed by the LEP experiments using jet production with events tagged by B-hadron decays and next-to-leading order QCD calculations. A review [10] quotes $m_b(m_{Z^0}) = (2.90 \pm 0.31)$ GeV where the error is dominant by experimental and hadronisation systematic uncertainties. These results for the b quark mass in the $\overline{\text{MS}}$ scheme are different by more than four standard deviations and thus already provide strong evidence for the presence of a running b quark mass. This finding was recently reproduced and improved [11].

The new analysis [11] studied the dependence of the branching ratio of Higgs boson decays to a b$\bar{b}$ pair $\Gamma(H \rightarrow b\bar{b})$ normalised to the branching ratio for Higgs boson decays to a pair of $Z^0$ bosons. The branching ratio $\Gamma(H \rightarrow b\bar{b}) \sim m_b(m_H)^2$ and therefore a measurement is highly sensitive to $m_b(m_H)$. The analysis obtains $m_b(m_H) = (2.60^{+0.36}_{-0.31})$ GeV from combined recent ATLAS and CMS measurements of $\Gamma(H \rightarrow b\bar{b})/\Gamma(H \rightarrow Z^0Z^0)$, which has the same precision as the combined LEP measurements of $m_b(m_{Z^0})$.

We propose to study the dependence of $Z^0$ boson properties connected with b quarks on the value of the b quark mass $m_b(m_{Z^0})$ used in a SM prediction. The observables are the partial width $\Gamma(Z \rightarrow b\bar{b})$, the branching ratio $BR(Z \rightarrow b\bar{b}) \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma_{Z,tot}$, where $\Gamma_{Z,tot}$ is the total width of the $Z^0$ boson, and $R_{0,b} = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$. A comparison of these predictions with precision measurements of these quantities can be used to extract $m_b(m_{Z^0})$.

A determination of $m_b(m_H)$ from $\Gamma(H \rightarrow b\bar{b})$ assumes the Yukawa coupling of the Higgs boson to b quarks to have the expected SM value [11,12]. A complementary determination of the b quark mass at high scales, i.e. $m_b(m_{Z^0})$, but without direct dependence on the b quark Yukawa coupling, will thus help to resolve possible ambiguities.

We will briefly review the dependence of SM predictions for $Z^0$ boson properties on $m_b(m_{Z^0})$, then explain the programs used to obtain predictions, compare the predictions to measurements with uncertainties as valid now and with uncertainties expected for the proposed future facility FCC-ee [13].

$m_b(m_{Z^0})$ revisited with Zedometry

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2 SM predictions as a function of $m_b$

The SM prediction for $\Gamma(Z \to f\bar{f})$, where $f\bar{f}$ is a pair of SM fermions, can be written as

$$\Gamma(Z \to f\bar{f}) = \frac{G_F m_Z^3}{24\sqrt{2}\pi} N_{C,f}(v_f^2 R^F_f + a_f^3 R^A_f)$$

where $G_F$ is the Fermi constant, $N_{C,f}$ is the number of colours, and $v_f$ and $a_f$ are the vector and axial-vector couplings of the $Z^0$ to the fermions. The radiator functions $R^F_f$ and $R^A_f$ contain in the case of a $q\bar{q}$ pair the QCD corrections, see e.g. [14]. The leading QCD corrections to the radiator functions for $Z^0 \to b\bar{b}$ decays\footnote{The fermion index $f$ is dropped now for clarity.} are

$$R^{V/A}(s) = 1 + R_1^{V/A} \frac{a_S(s)}{Q} + \left( \frac{m_b(s)}{Q} \right)^2 \left( R_2^{V/A}(m) + R_1^{V/A}(m) \frac{a_S(m)}{Q} \right) + \cdots$$

where $Q = \sqrt{s}$. The mass corrections up to $a_S^3(m_b/Q)^4$ and $a_S^2(m_b/Q)^2$ are known [15].

The observables depend on the $b$ quark mass $m_b$, and one expects that predictions for the three observables will decrease quadratically with increasing $m_b$. In comparison to $\Gamma(H \to b\bar{b})$ the sensitivity of $\Gamma(Z \to b\bar{b})$ to $m_b(s)^2$ is suppressed by about a factor $s/m_b^2$.

Predictions of the SM for $Z^0$ boson properties are available in the programs zfitter [16,17] and Gfitter [18]. The last available version 6.42 of the zfitter program dates from 2006 and does not contain updates to SM predictions which appeared since this date. The Gfitter program was used for the last update of the electroweak precision fit in 2018 [19]. We use Gfitter version 2.2\footnote{http://project-gfitter.web.cern.ch/project-gfitter/Software/Gfitter2_2.tar.bz2.} and for comparison and cross checks zfitter version 6.42.\footnote{https://elsevier.digitalcommonsdata.com/895c9aea-260c-4c90-91ee-261c32e0f19.}

2.1 Gfitter predictions

The Gfitter program uses $m_b(m_b)$ in the $\overline{\text{MS}}$ scheme as input with a default value of $m_b(m_b) = 4.2$ GeV. The Gfitter parameter $m_b(\overline{\text{MS}})$ is defined in the Gfitter datacard with a scan range from 1.0 to 6.0 GeV. We set in section GEWFlags of the Gfitter datacard the flag FullTwoLoop = "F", which turns this option off, because the full two-loop prediction in Gfitter is a parametrisation without variation of $m_b$. Using the Gfitter action PctOfFreePara\footnote{In order to avoid double counting we disabled separate variation of the results for $R_{b,b}$, $\Gamma_{Z,\text{tot}}$ and $\Gamma_{Z,\text{had}}$ with the same parameters in Gfitter.} = "T:mb_MSb:Nbins=50" the parameter $m_b(\overline{\text{MS}})$ is varied within its range with 50 points and the predictions for the active theory parameters are calculated. As active theory parameters we define GEW::Rob, GEW::GammaZhad and GEW::GammaZtot, where we implemented the class GammaZhad in the GEW package of Gfitter to provide the prediction for $\Gamma(Z^0 \to \text{hadrons})$. Using these three predicted parameters we can derive the observables $\Gamma(Z \to b\bar{b})$ and $BR(Z \to b\bar{b})$. The scan of $m_b(\overline{\text{MS}})$ is repeated for three values of $\alpha_S(m_Z^2)$ given by $\alpha_S(m_Z^2) = 0.1179 \pm 0.0010$. For the evaluation of theory uncertainties the N3LO terms $C_4 \alpha_S^4$ and $I_4 \alpha_S^4$ in the radiator functions are multiplied simultaneously by factors of zero or two with the Gfitter parameters DeltaAlphasTheoC05_Scale and DeltaAlphasTheoCMT4_Scale\footnote{https://github.com/DavidMStraub/rundec-python.} and the scans of $m_b(\overline{\text{MS}})$ are repeated.

2.2 Zfitter predictions

The zfitter program implements heavy quark masses in the on-shell (OS) scheme using a value of $m_b^{OS} = 4.7$ GeV. The on-shell mass is converted in zfitter to the running mass in the $\overline{\text{MS}}$ scheme and then evolved to the $Z^0$ mass scale. The zfitter program calculates and prints the predictions of $Z^0$ properties such as $\Gamma(Z^0 \to \text{hadrons})$ via the routine ZVWEAK. The value of the input OS quark mass $m_b^{OS}$ is varied over a range from 1.0 to 7.0 GeV in steps of 0.2 GeV. The scan of $m_b^{OS}$ is repeated for three values of $\alpha_S(m_Z^2)$ given by $\alpha_S(m_Z^2) = 0.1179 \pm 0.0010$.

2.3 Mass scheme conversion and mass evolution

In Gfitter the $\overline{\text{MS}}$ mass with a nominal value of $m_b(\overline{\text{MS}}) = 4.2$ GeV is used as input. We use 4-loop evolution of $\overline{\text{MS}}$ quark masses to present Gfitter results at the $Z^0$ mass scale. For all computations the same value of $\alpha_S(m_Z^2) = 0.1179 \pm 0.0010$ as above is applied. The calculations available in CRunDec [20–22] are used for all mass transformations.\footnote{If in order to avoid double counting we disabled separate variation of the results for $R_{b,b}$, $\Gamma_{Z,\text{tot}}$ and $\Gamma_{Z,\text{had}}$ with the same parameters in Gfitter.} For estimating systematic uncertainties we change the value of $\alpha_S(m_Z^2)$ inside its errors, or change the number of loops, i.e. the perturbative accuracy, from the standard value of four to three.

In zfitter the OS mass $m_b^{OS}$ is used as an input parameter. In order to present results at the $Z^0$ mass scale we convert from the OS to the $\overline{\text{MS}}$ mass definition at the scale of $m_b^{OS}$ using the 4-loop calculations implemented in CRunDec. The resulting $m_b^{OS}(m_b)$ value is evolved at 4-loop accuracy to $m_b(\overline{\text{MS}})$.\footnote{In order to avoid double counting we disabled separate variation of the results for $R_{b,b}$, $\Gamma_{Z,\text{tot}}$ and $\Gamma_{Z,\text{had}}$ with the same parameters in Gfitter.}
3 Results

The LEP and SLD results for the three observables are \( \Gamma(Z^0 \to b \bar{b}) = 377.3 \pm 1.2 \text{ MeV} \), \( BR(Z \to b \bar{b}) = 15.121 \pm 0.048\% \) and \( R_{0,b} = 0.21628 \pm 0.00066 \) [23]. The values and errors shown are from the determinations assuming lepton universality. The relative total uncertainties are 0.3\% for all three observables.

The predictions by Gfitter are presented below. All plots on Fig. 1 show as solid black lines the Gfitter predictions for the observables as indicated as a function of \( m_b(m_{Z^0}) \). The expected decrease of the observables \( \sim (m_b(m_{Z^0}))^2 \) is clearly visible.

On all plots of Fig. 1 the horizontal dash-dotted lines indicate the LEP and SLD results [23]. On Fig. 1a–c the horizontal and vertical solid lines indicate the value of the observable leading to a result \( m_b(m_{Z^0}) = 2.88 \text{ GeV} \) (\( m_b(m_b) = 4.2 \text{ GeV} \)), i.e. the nominal Gfitter value. The horizontal dashed lines indicate the actual uncertainties from the LEP and SLD results applied to the nominal value calculated by Gfitter and the vertical dashed lines thus indicate the corresponding uncertainty region for \( m_b(m_{Z^0}) \). The uncertainties of a hypothetical determination of \( m_b(m_{Z^0}) = 2.88 \text{ GeV} \) have values between approximately one and two GeV.

The actual LEP and SLD results with their uncertainties cannot be used to derive a determination of \( m_b(m_{Z^0}) \), since already their one s.d. upper uncertainty intervals cover regions not accessible by the predictions. The incompatibility is less pronounced for \( R_{0,b} \), where possible biases to the hadronic and b\( \bar{b} \) widths of the \( Z^0 \) boson can cancel more effectively in the ratio.

Comparing the three observables we find that the uncertainties of the hypothetical determinations of \( m_b(m_{Z^0}) \) increase from \( \Gamma(Z^0 \to b \bar{b}) \) to \( R_{0,b} \). This can be explained by the fact that \( BR(Z \to b \bar{b}) \) and \( R_{0,b} \) are normalised by the total or hadronic width of the \( Z^0 \), which also depend on \( m_b \) leading to a reduction of the experimental sensitivity.

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**Fig. 1** Figures a–c show Gfitter predictions for the three observables as a function of \( m_b(m_{Z^0}) \). The horizontal dash-dotted lines indicate the measurements from LEP and SLD. The horizontal and vertical solid and dashed lines display the correspondence between a hypothetical determination based on the Gfitter default value \( m_b(m_{Z^0}) = 2.88 \text{ GeV} \) (\( m_b(m_b) = 4.2 \text{ GeV} \)), and results for \( m_b(m_{Z^0}) \). The dotted lines show the same correspondence for experimental uncertainties reduced by a factor 1/10 (see text for details).

**Table 1** Results for \( m_b(m_{Z^0}) \) uncertainties [GeV] with Gfitter for the three observables with either the actual uncertainties from LEP and SLD or uncertainties reduced by a factor 1/10. \( m_b(m_{Z^0}) = 2.88 \text{ GeV} \) is used in all cases

| Observable | Expected value | \( m_b(m_{Z^0}) \) exp. unc. | \( m_b(m_{Z^0}) \) aS(m_{Z^0}) unc. | \( m_b(m_{Z^0}) \) N3LO unc. |
|------------|----------------|-----------------------------|---------------------------------|-----------------------------|
| \( \Gamma(Z^0 \to b \bar{b}) \) [MeV] | 375.8 ± 1.2 | +0.92 | ±0.09 | ±0.01 |
| | 375.79 ± 0.12 | −1.41 | ±0.09 | ±0.01 |
| \( BR(Z \to b \bar{b}) \) [%] | 15.064 ± 0.048 | +1.06 | ±0.03 | ±0.02 |
| | 15.0641 ± 0.0048 | −1.84 | ±0.03 | ±0.02 |
| \( R_{0,b} \) | 0.21582 ± 0.00066 | +0.09 | ±0.01 | ±0.04 |
| | 0.215818 ± 0.000066 | −1.98 | ±0.01 | ±0.04 |
The horizontal dotted lines in Fig. 1a–c present the same hypothetical determination as above with total uncertainties taken as 1/10th of the actual uncertainties. The studies in [13] for possible precision measurements of $Z^0$ boson properties at FCC-ee show that such a precision could be reached. The resulting uncertainties for a possible determination of $m_b(m_{Z^0})$ are shown by the dotted vertical lines. We conclude that with measurements of the observables with such small uncertainties precise determinations of $m_b(m_{Z^0})$ can be obtained.

With a numerical evaluation of the hypothetical determination of $m_b(m_{Z^0})$ we study theoretical uncertainties. The value of $\alpha_S(m_{Z^0}) = 0.1179 \pm 0.0010$ is varied within its uncertainties for the Glitter predictions. The results are shown in Table 1. The theory uncertainty (see Sect. 2.1) is presented in the last column labeled “N3LO unc.”.

We find that an assumed reduction of the experimental uncertainties from LEP and SLD by a factor 1/10 leads to a corresponding reduction of the experimental uncertainty of $m_b(m_{Z^0})$ by about the same factor. Furthermore, as observed above, the experimental uncertainties increase slightly from $\Gamma(Z^0 \rightarrow b\bar{b})$ to $R_{0,b}$ also with the reduced measurement errors. However, the uncertainty of the predictions due to variation of $\alpha_S(m_{Z^0}) = 0.1179 \pm 0.0010$ is smaller for $BR(Z \rightarrow b\bar{b})$ w.r.t. $\Gamma(Z^0 \rightarrow b\bar{b})$ and is negligible for the determination of $m_b(m_{Z^0})$ derived from a measurement of $R_{0,b}$. This effect was already discussed for $R_{0,b}$ in [24]. The relative total uncertainty on $m_b(m_{Z^0})$ with reduced uncertainties is about 5%.

The N3LO theory uncertainties are smaller than the experimental uncertainties with the expected measurements at a future FCC-ee. The N3LO theory uncertainties are smaller than the $\alpha_S(m_{Z^0})$ uncertainty for $\Gamma(Z^0 \rightarrow b\bar{b})$, they are about the same size for $BR(Z \rightarrow b\bar{b})$, and for $R_{0,b}$ they are larger than the $\alpha_S(m_{Z^0})$ uncertainty.

The $b$ quark mass is set in Glitter as $m_b(m_b)$ which is a technical limitation in the analysis, since this value must be evolved to $m_b(m_{Z^0})$, even though the SM prediction could be used directly with $m_b(m_{Z^0})$ as a free parameter. As a test of the additional uncertainty introduced by the evolution from $m_b(m_b)$ to $m_b(m_{Z^0})$ the value of $\alpha_S(m_{Z^0}) = 0.1179 \pm 0.0010$ is changed within its errors and the perturbative order of the calculation is changed from 4-loop to 3-loop precision. The corresponding uncertainties for $m_b(m_{Z^0})$ are $^{+0.03}_{-0.04}$ ($\alpha_S(m_{Z^0})$) and $\pm 0.10$ (pert. order). Since these uncertainties would not appear if $m_b(m_{Z^0})$ could be varied directly we do not show them in Table 1.

It was checked that the results for the uncertainties are consistent with the results from zfitter.

4 Conclusions

We have studied the possibility to determine $m_b(m_{Z^0})$ from precision measurements of $Z^0$ boson properties from LEP and SLD or the proposed future $e^+e^-$ facility FCC-ee. The theory predictions for the three observables $\Gamma(Z^0 \rightarrow b\bar{b})$, $BR(Z \rightarrow b\bar{b})$, and $R_{0,b}$ are obtained from the program Glitter as functions of the $b$ quark mass $m_b(m_{Z^0})$, evolved from the Glitter input parameter $m_b(m_b)$. A comparison of the predictions with actual measurements from LEP and SLD does not provide a meaningful extraction of $m_b(m_{Z^0})$. Using expected central values with current uncertainties corresponding to the input $b$ quark mass of the predictions we find that with the data from LEP and SLD the uncertainties for $m_b(m_{Z^0})$ from the analysis of $Z^0$ boson properties would be between one and two GeV. These uncertainties are much larger than other existing determinations of $m_b(m_{Z^0})$ from jet production in $b$-tagged events at LEP or $m_b(m_H)$ from Higgs boson decays to $b$ quark pairs.

Using the expected uncertainties for measurements of $Z^0$ boson properties at the future $e^+e^-$ collider FCC-ee, which are taken to be smaller by a factor 1/10 w.r.t. the current uncertainties, a determination of $m_b(m_{Z^0})$ with a relative error of 5% is possible. The total uncertainty would be dominated by the experimental errors in all cases. Depending on the observable either the uncertainty from the current world average value of $\alpha_S(m_{Z^0})$ or from the N3LO theory error would be the second largest uncertainty.

At future $e^+e^-$ colliders determinations of $m_b(m_H)$ with O(10) MeV precision could be obtained [11]. The uncertainty for a determination of $m_b(m_{Z^0})$ from jet production in $b$-tagged hadronic final states in $e^+e^-$ annihilation at the proposed ILC based on a large sample of $Z^0$ boson decays was estimated as $\Delta m_b(m_{Z^0}) = 0.12$ GeV [25]. This uncertainty is dominated by theory uncertainties. Our proposed determination of $m_b(m_{Z^0})$ is thus expected to have a comparable uncertainty with complementary sources of uncertainties.

A precision determination of $m_b(m_{Z^0})$ from $Z^0$ boson decays combined with improved determinations of $m_b(m_H)$ from Higgs boson decays would constitute a new and stringent test of the SM. The $b$ quark couples to the $Z^0$ via the electroweak interaction and the observed mass is related to the strong interaction while the Higgs boson couples to the $b$ quark directly via its Yukawa coupling. It will be interesting to see such a test become reality with data from future experimental facilities.
Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data used in the analysis are published in [23]. The numerical data of the predictions generated with Gfitter can be regenerated using the instructions in Sect. 2.1.]

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