Study on Mathematical Model for Condensation Heat Transfer in Two-phase Closed Thermosyphon

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Abstract. The two-phase closed thermosyphon is a special heat pipe utilizing gravity instead of capillary force to return the liquid to evaporator. Some assumptions for Nusselt model are not true in the condenser section, resulting in the complexity of condensation. In this paper, a numerical model based on Nusselt’s model is developed considering vapour-liquid interfacial shear stress. The distributions of local parameters such as shear stress, liquid film thickness are analysed. The variation of average heat transfer coefficient with temperature difference is displayed. And the average heat transfer coefficients predicted by numerical model are compared with values calculated from other references. The results show that both the shear stress and liquid film thickness increase as condensation proceeds. But the growth rate decreases along the axial direction. The average heat transfer coefficients predicted by this model is lower than that calculated by Nusselt’s correlation especially at higher heat load, indicating that shear stress really reduces the heat transfer performance.

1. Introduction

The heat pipe is a hollow evacuated device, with working fluid filled. The working fluid is vaporized in evaporator section. The produced vapour flows to the condenser section owing to the pressure gradient. Then the vapour condenses into fluid in condenser. The produced liquid returns to the evaporator by the capillary force of powder wick [1, 2]. The two-phase closed thermosyphon is a special heat pipe, which utilizes gravity instead of capillary force to return the liquid to evaporator [3, 4]. Due to its simple structure and low fabrication cost, the thermosyphon has been widely used in many industries, including waste heat recovery, solar heating, electronic component cooling and so on.

There are three heat transfer phenomena inside the thermosyphon, including convection heat transfer, evaporation and condensation. The correlation for cross flow tube bundle arrays [5] is adopted for the calculation of convection heat transfer. According to the filling ratio of working fluid and heat flux, two types of heat transfer model with five flow patterns happen in evaporator section. Natural convection boiling happens at the lower heat fluxes while nucleate boiling at the higher heat fluxes. Empirical correlations are used to determine the heat transfer coefficient in evaporator section.

The Nusselt model for condensation on a vertical wall [6] is usually used for the condensation determination inside the thermosyphon. However, some assumptions for Nusselt model are not true here, resulting in the complexity of characteristic of condensation inside the thermosyphon. In particular, the shear force owing to the countercurrently flowing between liquid and vapour increases...
the liquid film thickness and weakens the condensation performance. In this paper, a numerical model based on Nusselt model considering the vapour-liquid interfacial shear stress is developed. The distributions of local parameters such as shear stress, liquid film thickness and heat transfer coefficient are analysed. The average heat transfer coefficients predicted by this model are compared with that calculated from correlations in other references.

2. Mathematical model and calculation procedure

2.1. Mathematical model

The curvature effect is negligible because the liquid film thickness is much smaller than the inner diameter of tube. And the following assumptions are made: (1) steady state; (2) the liquid film flow is laminar without surface fluctuation; (3) the inertia term in momentum equation is neglected; (4) the temperature at the interface is equal to the vapour saturation temperature; (5) the temperature distribution in liquid film is linear; (6) the thermophysical properties of vapour and fluid are constant.

The momentum equation of liquid film is described as:

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \left( \rho_l - \rho_v \right) g = 0
$$

The energy equation of liquid film is written as:

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0
$$

The associated boundary conditions are as following:

$$
\begin{align*}
    r &= R: u = 0; T = T_w \quad (3a-3b) \\
    r &= R - \delta: \frac{\partial u}{\partial r} = \mu \frac{\partial u}{\partial r}; T = T_v \quad (4a-4b)
\end{align*}
$$

The energy balance in the control volume taken from the liquid film is illustrated in Fig.1. The energy conservation equation of the control volume is described as:

$$
dm \times h_{fg} = \frac{\delta}{\delta} \left( T_v - T_w \right) Rdx + \frac{d}{dx} \left\{ \int_0^\delta \rho \alpha_c (T - T_v) rdr \right\} dx
$$

$$
\text{Figure 1. Energy balance in control volume}
$$

The interfacial shear stress is consists of frictional shear stress and phase-change shear stress and can be written as:

$$
\tau_\delta = \tau_f + \tau_p = \frac{\dot{\lambda}_v (T_v - T_w)}{h_{fg} \delta} (U_v + u_i) + \frac{c_i}{2} \rho_v (U_v + u_i)^2
$$

Here, $U_v$ is the average velocity of vapour, $c_i$ is the correlation index, they can be calculated from:
Using the boundary conditions 3(a) and 4(a), Eq. (1) is solved as:

\[ u = \frac{\rho_i - \rho_v}{4 \mu_l} g(R^2 - r^2) + \left[ (R - \delta) \frac{\tau_\delta}{\mu_l} + \frac{\rho_l - \rho_v}{2 \mu_l} g(R - \delta)^2 \right] \ln \frac{r}{R} \]  

(9)

And the liquid velocity at the interface can be written as:

\[ u_l = \frac{\rho_l - \rho_v}{4 \mu_l} g(2R - \delta)\delta + \left[ (R - \delta) \frac{\tau_\delta}{\mu_l} + \frac{\rho_l - \rho_v}{2 \mu_l} g(R - \delta)^2 \right] \ln \frac{R - \delta}{R} \]  

(10)

The mass flow rate of liquid film of unit radian is:

\[ \dot{m} = \frac{1}{3} \rho_l^2 g R \delta^3 + \frac{T_\delta}{\mu_l} \left( \frac{\delta^2}{2} + \frac{\delta^4}{6R} - \frac{\delta^4}{4R^2} \right) \]  

(11)

Using the boundary conditions 3(b) and 4(b), Eq. (2) is solved as:

\[ T = \frac{T_w - T_v}{\ln R - \ln(R - \delta)} + \frac{T_v \ln R - T_w \ln(R - \delta)}{\ln R - \ln(R - \delta)} \]  

(12)

Substitute Eq. (11) and Eq. (12) into Eq. (5), an implicit equation is obtained:

\[ f_1(\delta, \tau, x) = 0 \]  

(13)

Substitute Eq. (7) and Eq. (10) into Eq. (6), another implicit equation is acquired:

\[ f_2(\delta, \tau, x) = 0 \]  

(14)

Using the above two equations and iterative solution, the function of film thickness \( \delta \) and shear stress \( \tau \) can be obtained along the axial direction.

Finally, the local condensation heat transfer coefficient is determined as:

\[ h_c(x) = \frac{\lambda_i}{\delta(x)} \]  

(15)

2.2. Calculation procedure

The calculation procedure consists of the following steps:

Step 1: Input the structure parameters of condenser section, such as length and inner diameter.

Obtain the temperatures of vapour and wall. Acquire the thermophysical properties of vapour and fluid;

Step 2: Divide the condenser section into \( n \) control volumes along the axial direction;

Step 3: From \( i=0 \) to \( i=n \), using iterative method, obtain the film thickness \( \delta \) and shear stress \( \tau \) at each control volume;

Step 4: Calculate the local heat transfer coefficient \( h_c \) at each control volume;
Step 5: Acquire the average heat transfer coefficient by taking arithmetic mean of local heat transfer coefficient at each control volume.

3. Results and discussion

3.1. Distribution of shear stress

The distribution of interfacial shear stress along the axial direction at different working temperatures is showed in Fig. 2. It is clear that the interfacial shear stress increases as axial distance increases. This is because the relative velocity of vapor and liquid film increases, which leads to the increase of the degree of friction. The shear stress increases rapidly at the initial stage, but then the growth rate slows down. In addition, the shear stresses at lower working temperature are higher than that at higher working temperature. The reason is that the dynamic viscosity of liquid film at lower working temperature is relatively higher.

![Figure 2. Distribution of shear stress](image)

3.2. Distribution of liquid film thickness

![Figure 3. Distribution of liquid film thickness](image)
Figure 3 displays the distribution of liquid film thickness along the axial direction at different working temperatures. It is found that the variation trend of liquid film thickness is similar with that of shear stress. At the top of the condenser section, the thickness of the liquid film is zero. The thickness increases rapidly at the initial stage, but then the growth rate slows down. This is because when condensation proceeds, the liquid film between vapor and wall serves as the heat transfer resistance and reduces the condensation performance. Similarly, the thicknesses at lower working temperature are higher than that at higher working temperature due to the larger shear stress.

3.3. Condensation heat transfer coefficient

The variation trend of average condensation heat transfer coefficient with temperature difference is depicted in Fig. 4. It can be seen that the heat transfer coefficient decreases as temperature difference increases. This is because the larger temperature difference means the larger heat transfer rate and thicker liquid film thickness. Meanwhile, the heat transfer coefficient at lower working temperature is smaller than that at higher working temperature with the same temperature difference, which is due to that the liquid film thickness at lower working temperature is relatively thicker.
Figure 5 illustrates the comparison between heat transfer coefficient predicted by this model and that calculated from correlations in other references. It is found that the Nusselt’s correlation overestimates the heat transfer coefficient, inferring that shear stress could reduce the heat transfer performance. The gap between calculated values and predicted values is greater at the smaller heat transfer coefficient, indicating that the shear stress cannot be ignored especially at higher heat transfer rate. And the predicted values are larger than that calculated from Chen’s correlation. But the gap is greater at the higher heat transfer coefficient.

4. Conclusions
In this paper, a numerical model based on Nusselt model considering vapour-liquid interfacial shear stress is established. The distributions of local parameters such as shear stress, liquid film thickness are analysed. The variation of average heat transfer coefficient with temperature difference is displayed. And the average heat transfer coefficients predicted by this model are compared with values calculated from other references. The results show that both the shear stress and liquid film thickness increase as axial distance increases. But the growth rate decreases as condensation proceeds. The average heat transfer coefficients predicted by this model is lower than that by Nusselt’s correlation especially at higher heat load, indicating that shear stress could reduce the heat transfer performance.

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