Resonant Bar Constraints on Macro Dark Matter

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The current standard model of cosmology, \( \Lambda \)CDM, requires dark matter to make up around 25% of the total energy budget of the universe. Yet, quite puzzlingly, there appears to be no candidate particle in the current Standard Model of particle physics. Assuming the validity of the CDM paradigm, dark matter has evaded detection thus far either because it is intrinsically a weakly-interacting substance or because its interactions are suppressed by its high constituent mass and low number density. Most approaches to explain dark matter to date assume the former and therefore require beyond-the-Standard-Model particles that have yet to be observed directly or indirectly. Given the dearth of evidence for this class of candidates it is timely to consider the latter possibility, which allows for candidates that may or may not arise from the Standard Model. In this work we extend a recent study of this general class of so-called macro dark matter – candidates with characteristic masses of grams and geometric cross sections of cm\(^2\). We consider new bounds that can be set using existing data from the resonant bar gravitational wave detectors NAUTILUS and EXPLORER.

I. INTRODUCTION

The last two decades have ushered in a new era of precision cosmology with a plethora of modern experiments and observations all leading to the so called \( \Lambda \)CDM concordance model of the universe. A crucial component of this model is the cold dark matter (CDM) making up around 25% of the total energy density of our universe, as inferred from observations on large scales (see e.g. \([1–4]\)). On intermediate scales its existence is observed indirectly due to the discrepancies between the gravitational and luminous masses of large astrophysical systems. The dark matter cannot be made of ordinary baryonic matter (see e.g. \([5,6]\), \([7]\), and references therein) nor can it consist of any fundamental particle of the Standard Model of particle physics.

Although a tremendous effort has been devoted to the study of particle candidates that are intrinsically weakly-interacting, such as WIMPs and axions, alternative dark matter candidates deserve increased attention given the lack of direct detection of those canonical possibilities to date (see e.g. \([8,9]\)). Other candidates can couple to Standard Model particles with high probability but are nevertheless effectively weakly-interacting or “dark” because they are very massive. Several examples exist in the literature, most notably nuclear-dense candidates with a Standard Model basis (e.g. \([10,12]\)). Other similarly massive candidates include compact objects with some connection to the Standard Model (e.g. \([13,15]\)), primordial black holes \([16]\), and other candidates that may be found in the literature (e.g. \([17,21]\)).

A wide range of Earth-based constraints on nuclear-dense candidates (nuclearites) were considered in \([22]\) with more recent bounds presented in \([23]\). However, only recently has a model-independent study of the general class of “macroscopic” candidates been considered in \([24]\). These macro dark matter candidates, referred to as Macros, have characteristic mass and geometric cross sections of grams and cm\(^2\) and may or may not have a Standard Model origin. A number of Earth-based, astrophysical, and cosmological observations were used in \([24]\) to place constraints on Macros. In this work we explore the use of resonant bar gravitational wave detectors for constraining a portion of the Macro parameter space and improve on the constraints found in \([24]\) on the coupling of Macros to baryons.

The use of large aluminum bars as detectors of gravitational waves has been studied for more than five decades \([25]\), and their sensitivity to cosmic rays has been appreciated for some time \([29]\). Because of the thermoacoustic effect, such bars (when cryogenically cooled) are also sensitive to other exotic cosmic impactors, such as monopoles \([27]\) and nuclearites \([28]\). The resonant bar experiments NAUTILUS \([29]\), which ran from 2003-2012
in Frascati, Italy, and EXPLORER [30], which ran from 2003-2009 at CERN, were recently used in a study of nuclearite dark matter [31]. The data from the two experiments was used to rule out nuclearites with masses up to $\sim 10^{-4} \text{ g}$. Here, we generalize that analysis to constrain the elastic scattering cross section of macro dark matter with baryons.

II. MACRO DETECTION VIA RESONANT BAR DETECTORS

A. Macro velocity evolution

Given an incoming Macro galactic velocity, $v_0$, its evolution as it impacts the Earth as a function of the reduced cross section, $\sigma_X/M_X$, is approximately given by

$$v_X(r) = v_0 e^{-\frac{rr}{\rho_0 r}} ,$$  \hspace{1cm} (1)

where $\langle pr \rangle$ is the column density encountered by the Macro. This column density can have three distinct contributions, the first being atmospheric. Simple atmospheric models give an atmospheric density that depends exponentially on altitude:

$$\rho(z) = \rho_0 e^{-z/H} ,$$  \hspace{1cm} (2)

where $z$ is the altitude above sea level, $\rho_0 \simeq 10^{-3} \text{ g/cm}^3$, and the scale height, $H \simeq 10 \text{ km}$. Writing $z = r \cos \theta$, where $\theta$ is the impact angle, the column density encountered through the atmosphere is

$$\langle pr \rangle_{\text{atm}} = \int_{r}^{\infty} dr' \rho \simeq \frac{\rho_0 H}{\cos \theta} e^{-z/H} .$$  \hspace{1cm} (3)

In addition, we shall conservatively assume that at angles $\theta > \pi/3$ the Macro would encounter an additional column density of 100 meters water equivalent (MWE), or

$$\langle pr \rangle_{\text{MWE}} = 10^3 \text{ g/cm}^2 ,$$  \hspace{1cm} (4)

due to surrounding structures, mountains, or other obstacles. Lastly, at angles $\theta > \pi/2$ the Macro would pass though the Earth and encounter a column density,

$$\langle pr \rangle_{\oplus} = 2 R_\oplus \rho_\oplus |\cos \theta| .$$  \hspace{1cm} (5)

B. Thermo-acoustic detection of cosmic particles

Upon its passage through the (identical) NAUTILUS or EXPLORER detectors, a very dense object would deposit energy in a line along its track in the detector. Because the Macro velocity is super-sonic in the aluminum, the energy deposition would be nearly instantaneous. The resulting near-instantaneous thermal expansion would source a pressure wave that would excite the bar’s longitudinal vibrational modes. As described in e.g. [27, 28, 31], for a bar of radius $R$ and length $L$, the energy of the lowest mode is

$$\Delta E = \frac{4}{9\pi} \frac{\gamma^2}{\rho L v_s^2} \left( \frac{dE_X}{dx} \right)^2 G(z_0, l_0, \theta_0) ,$$  \hspace{1cm} (6)

where $\gamma$ is the Gruneisen parameter, $\rho$ is the bar density, $v_s$ is the longitudinal sound speed, and the energy lost by a nuclearite or, more generally, a Macro is

$$|\frac{dE_X}{dx}| = \rho \sigma_X v_s^2 .$$  \hspace{1cm} (7)

The geometric function that depends on the track through the bar is

$$G(z_0, l_0, \theta_0) = \left[ \frac{L}{\pi R} \sin \left( \frac{\pi z_0}{L} \right) \sin \left( \frac{\pi l_0 \cos \theta_0}{L} \right) \right]^{-2} ,$$  \hspace{1cm} (8)

where $z_0$ is the distance of the track midpoint to one end of the bar, $\theta_0$ is the angle between the track and the bar axis and $l_0$ is the length of the track. For the bars described in [31], $R = 0.3 \text{ m}$ and $L = 3 \text{ m}$; the full geometric acceptance is therefore 19.54 m$^2$ sr.

C. Application of nuclearite analysis to Macros

In Ref. [31] it is reported that, for the parameters specific to the NAUTILUS and EXPLORER aluminum bars, a vertical impact at the bar center results in an energy deposition (in units of temperature) into the fundamental mode of

$$\Delta E = 10.7 \text{ K} \left( \frac{v_X \xi(M)}{10^{-3}c} \right)^4 ,$$  \hspace{1cm} (9)

where, in our notation, $\xi(M) = (M/1.5 \text{ ng})^{1/3}$ is a function that characterizes the radius of a passing nuclearite as a function of its mass$^1$.

It is straightforward to apply the analysis in [31] to macro dark matter, considering a nuclearite of mass 1.5 ng would have a cross section, $\sigma_0 \simeq \pi \times 10^{-16} \text{ cm}^2$ at the reference density of $3.5 \times 10^{14} \text{ g/cm}^3$. The energy deposition by a Macro can then be readily translated by making the replacement

$$\xi(M)^2 \rightarrow \frac{\sigma_X}{\sigma_0}$$  \hspace{1cm} (10)

in [9]. We will furthermore use 250 km/s as a reference velocity and reinsert the geometric function, $G$, writing the excitation energy as

$$\Delta E = 5.2 \text{ K} \left( \frac{v_X}{250 \text{ km/s}} \right)^4 \left( \frac{\sigma_X}{\sigma_0} \right)^2 G(z_0, l_0, \theta_0) .$$  \hspace{1cm} (11)

$^1$ For an object of constant density $\sigma_X \propto M^{2/3}$, hence from [9] and [7] the $\xi(M)^4$ dependence in [9] can be understood.
Given the detection threshold of 2 K used in [31] and the Macro velocity upon impact given in [1], the fraction of impacts for a given pair of Macro parameters ($\sigma_X, M_X$) that exceed this threshold is determined by performing a Monte Carlo simulation over the possible bar impact points and Macro trajectories.

We can, however, give a semi-analytic estimate of the constrained region in the $\sigma_X - M_X$ plane by recognizing that at large $\sigma_X/M_X$ the only Macros that can make it to the detector with sufficiently high velocity (quantified below) will come from nearly directly overhead. For those impacts $G(z_0, l_0, \theta_0)$ is maximized at unity. The requirement for resonant bar detection is then

$$ \left( \frac{v_0}{250 \text{ km/s}} \right)^2 e^{-2 \pi X (\rho r)} \left( \frac{\sigma_X}{\sigma_0} \right) \gtrsim 0.62. \quad (12) $$

This inequality is saturated at the critical value $\sigma_{X,c}$, which may be solved implicitly as a function of $M_X$:

$$ \sigma_{X,c} = \frac{M_X}{2 (\rho r)} W(x). \quad (13) $$

Here

$$ x \simeq -1.24 \times \langle \rho r \rangle \frac{\sigma_0}{M_X} \left( \frac{v_0}{250 \text{ km/s}} \right)^2, \quad (14) $$

and $W(x)$ is known as the Lambert-W function, defined implicitly by the relation

$$ x = W(x) \exp W(x). \quad (15) $$

$W(x)$ has an infinite number of branches, however, the branch $W_{-1}(x)$ delineates the top of the constrained region in the $\sigma_X - M_X$ plane according to [13].

We must also ensure that the Macro velocity exceeds the sound velocity of aluminum, $v_s \approx 2 \times 10^{-5} c$, otherwise the energy loss rate formula (7) breaks down [31]. From [1] this requires that

$$ \sigma_X \lesssim \frac{1}{\langle \rho r \rangle} \log \frac{v_0}{v_s} M_X \lesssim 3.7 \times 10^{-3} \text{ cm}^2 \left( \frac{M_X}{1 \text{ g}} \right) \quad (16) $$

where we have used an approximate atmospheric depth\(^2\) of $\langle \rho r \rangle = 10^3 \text{ g/cm}^2$ and $v_0 = 250 \text{ km/s}$. For masses larger than roughly $10^{-19} \text{ g}$ this turns out to be more restrictive than using (13) so it sets the upper edge of the region of constraint in that range.

Lastly, we consider the Macro flux limitation. At large $\sigma_X/M_X$, we cannot use the full geometric acceptance of $\approx 19.54 \text{ m}^2 \text{sr}$ because of significant additional drag that would be experienced by the Macros during their passage through (i) the environment surrounding the detector (e.g. other buildings, mountains, etc.) and (ii) the integrated depth through the Earth. Likewise, Macros with small $\sigma_X/M_X$ are more likely to make it to the detector with high enough velocity and energy deposition. Therefore, for the largest $\sigma_X/M_X$ we use only 1/4 of the geometric acceptance since we can only trust their ability to make it through the atmosphere. For $\sigma_X/M_X \lesssim 10^{-4} \text{ cm}^2/\text{g}$ a Macro could easily pass through surrounding obstacles so that using 1/2 of the full acceptance is justified. Finally, for $\sigma_X/M_X \lesssim (\rho_\odot R_\odot)^{-1} \simeq 3 \times 10^{-10} \text{ cm}^2/\text{g}$ the Macro could pass freely through the Earth and in that case we may use the full acceptance. These approximations ignore the detector efficiency as a function of impact trajectory, however this is taken into account in the Monte Carlo simulation.

Given an isotropic dark matter flux of

$$ \frac{1}{4\pi} n_x v_0 \approx 1.2 \times 10^{-9} \text{ m}^{-2} \text{sr}^{-1} \text{day}^{-1} \times \left( \frac{1 \text{ g}}{M_X} \right) \left( \frac{\rho_\odot}{0.4 \text{ GeV/cm}^2} \right) \left( \frac{v_0}{250 \text{ km/s}} \right) $$

(17)

and the combined 3,921 live-time days of NAUTILUS and EXPLORER, the maximum number of events expected is

$$ N \approx 10^{-4} \left( \frac{1 \text{ g}}{M_X} \right), \quad (18) $$

for a local dark matter density, $\rho_\odot = 0.4 \text{ GeV/cm}^3$, and $v_0 = 250 \text{ km/s}$. A Macro impact would be a random (Poisson) process, therefore a null detection of Macro passages for which $N > 3$ events are expected will rule them out at greater than 95% confidence. It follows that, accounting for the useable fraction of the geometric acceptance mentioned above, the following macro dark matter candidates are ruled out by this analysis:

$$ M_X < \begin{cases} 3 \times 10^{-5} \text{ g}, & \text{if } \frac{\sigma_X}{M_X} \lesssim 10^{-9} \text{ cm}^2/\text{g}; \\ 2 \times 10^{-5} \text{ g}, & \text{if } 10^{-9} \text{ cm}^2/\text{g} \lesssim \frac{\sigma_X}{M_X} \lesssim 10^{-4} \text{ cm}^2/\text{g}; \\ 10^{-5} \text{ g}, & \text{if } 10^{-4} \text{ cm}^2/\text{g} \lesssim \frac{\sigma_X}{M_X} \lesssim 10^{-3} \text{ cm}^2/\text{g}. \end{cases} \quad \text{(19)} $$

Note that at the smallest $\sigma_X/M_X$ our bound differs slightly from the one found in [31] because of both our choice of $v_0 = 250 \text{ km/s}$ and 95% confidence requirement. Deviations from these estimates are expected at low enough $\sigma_X$ because of the decreasing excitation energy as a function of $\sigma_X$, as seen in [11]. The results of our more accurate Monte Carlo analysis are illustrated in

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2 We have neglected the additional integrated depth the Macro must pass through due to concrete in the roofing of the building in which EXPLORER was housed, as well as the experimental components mounted to the top of the bars [22]. On the other hand, EXPLORER was itself run at 430 m above sea level at CERN, so we have overestimated the atmospheric depth to it. These two complications (at least) partially offset each other and, in any case, they can be neglected considering the level of accuracy required here.
Figure [1] where, however, we have used the fixed value of $v_0 = 250 \text{ km/s}$.

Here the velocity distribution of Macros was taken to be isotropic; in reality the Earth (with the solar system) moves through the galaxy at approximately 200-250 km/s and this should result in an anisotropic distribution observed on the Earth. Also, the Macro velocity distribution presumably has a tail from which it is common to draw an initial Macro velocity, $v_0$ significantly larger than 250 km/s. This means that the constraints inferred here are conservative and the region of constraint should extend upwards to larger $\sigma_X$, i.e. the constraint is stronger than $\sigma_X/M_X \lesssim 3.7 \times 10^{-3} \text{ cm}^2/\text{g}$. Therefore, greater overlap is expected with those determined from large-scale structure, which are $\sigma_X/M_X \lesssim 3.3 \times 10^{-3} \text{ cm}^2/\text{g}$ [34]. On the other hand, the range of $\sigma_X/M_X$ constrained here is only log-sensitive to $v_0$, so we do not expect this to significantly enhance our results.

Here we have also made modest improvements to the constraints inferred from both Skylab [35] and ancient mica [36] compared to those presented in [24]. For both detectors we have more carefully calculated their acceptance, taking into account the column density encountered by a Macro as a function of its impact angle – this resulted in more rounded corners of the constrained regions. For the mica samples studied in [36] the orientation during their $\sim 500 \text{Myr}$ exposure time is unknown; we therefore computed the minimum region that is ruled out for a flat mica sample oriented either parallel or perpendicular to the Earth’s surface and have presented those constraints in Figure [1].

The new constraints presented here on macro dark matter from resonant bar detectors provide overlap with both the Skylab and mica constraints, and also fills in part of the gap between the large-scale structure and mica bounds on the coupling of Macros to baryons. In future studies, however, a dedicated experiment of this type is far from ideal due to its relatively limited exposure, which is proportional to the detector surface area and experiment lifetime.

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