The mathematical description of the part as the source data in the computer-aided design process

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Abstract. The paper reviews the issues of source data generation for automated design of the technological process of machining and geometric information transfer from one design stage to another. When dealing with design issues, the input of source data related to the work-piece and manufacturing conditions plays an important role. The formalized description of these objects is carried out using mathematical models. A drawing of a machine-building part or work-piece is a geometric model of the manufacturing item. However, this model is inconvenient when establishing links in the design of its manufacturing processes. Drawing models are the basis of graphic modules of computer-aided design systems. However, they are inconvenient for achieving process design goals, since the object is known to be surface in machining or assembly process, but not zero-dimensional objects on the plane. It represents a mathematical model of a machine-building part consisting of the basic elements of the form, which can be used in the automated development of the technological process of machining.

1. Introduction
Improving labour efficiency in the production of work-pieces is associated with the introduction of integrated computer-aided design systems. When designing in CAD/CAM/CAE systems, it is very important to automate the process of transferring geometric information from one design stage to another, as well as to establish feedbacks from later design stages to earlier ones.

In the work-piece production process, starting from the structural design and ending with the manufacturing stage, a special role is played by the drawing as the main model. When dealing with design issues, the input of source data related to the work-piece and manufacturing conditions plays an important role. A computer-aided design system efficiency depends on the degree of its automation and software. A subsection

2. Problem statement
Characterization of the manufacturing object (part, assembly unit, or design process) is a challenge and it has not been fully solved yet. The formalized description of these objects is carried out using mathematical models. The challenge is that when building models, their elements should be chosen based on the laws of designing technological processes. A drawing of a machine-building part or work-piece is a geometric model of the manufacturing item. However, this model is inconvenient when establishing links in the design of its manufacturing processes, since the drawing contains the initial elements (segments, arcs, points, etc.), i.e. one-dimensional manifolds on the plane that make up the projection of the part, cross-sections, and sections. Using these manifolds, recognition of the
geometric structure of a part is a difficult task to formulate. Also, the drawing implicitly reflects the information required to build the technological process. For example, surface machinability for a given basing structure, the length of the surface, etc. Thus, one and the same part can have an infinite number of mathematical models, depending on the selection of the multiple source elements.

3. Theory
A machine-building part is defined as a material bounded by a number of surfaces or a combination of them, one located against the other, based on the intended service and the most economical manufacturing process.

A solid mathematical model of a part can be represented as:

\[ M=(M, R), \]

where \( M \) is the manifold surfaces bounding the part;
\( R \) – manifold relations between them (surface parameters, quality indicators, shape errors, etc.) [1,2,3].

The geometric model of the part specified in the drawing can be represented as follows:

\[ M_{\text{draw}}=(M, R), \]

where \( M \) is the manifold of initial primitive elements (segments, arc lines, points, etc.);
\( R \) – a manifold of relations determining the properties of primitives (type and thickness of lines, segment point coordinates, etc.). For example, you can set a circle with three parameters: coordinates of the midpoint and radius. These relationships define the drawing model of the part. The manifold of relations, in this case, is massive and difficult to formalize.

Solid and drawing models are different models of the same part. Drawing models are the basis of graphic modules of computer-aided design systems. However, they are inconvenient for achieving process design goals, since the object is known to be surface in machining or assembly process, but not zero-dimensional objects on the plane.

Therefore, when designing the assembly and machining process, such models are not used, since the objects of such processes are parts and surfaces, but not line segments.

Geometric models also differ depending on the goals and stages of design. The solution involves the development of geometric information converters for the transition from one design stage to another. These converters are developed based on relational database management systems, where the source data related to the part is entered in the conversational mode for each surface. In this case, the automated transfer of geometric information from the graphic module to the process design module consists of automatic recognition of surfaces and their geometric properties in the source file made in the CAD module. Currently, the issue of automated information transfer is partially solved[4].

Mathematical models and their components can be presented in various forms: linear models, in the form of differential and finite-difference equations, elements of mathematical statistics and logic, in the form of graphs, simulation, etc. And they can also be divided into analytical and simulation.

Numerous mathematical methods have been developed to solve problems using models. The variety of these methods is caused by requirements applied to the method of solving a specific task. Therefore, the selection of the method comes to an optimization goal: to find a given task solution method that would meet the requirements of the ECM and have the highest accuracy [5].

Many of the methods for finding optimal options are iterative and based on continual improvement of options. Finite methods leading to an extremum in a finite number of steps are used. Deterministic methods include methods with a rigid algorithm. Random search methods or static methods are those in which trial steps or working steps are performed with random elements. In addition to these examples, there are many others.

The logical-algebraic language for describing machine-building parts is universal and allows you to identify all the geometric properties of the part elements that are necessary for the design of the
Let's present the geometric model of a part using the logical-algebraic language in the following form:

\[ M = (M, R), \quad (3) \]

where \( M \) is the manifold surfaces bounding the part and which can be used to "design" the part.;
\( R \) is the manifold of relations that we divide into two groups:
1) relations that define the properties of the surface (geometrical parameters and the location parameters);
2) relationships that are formed using logical operations & (and), V (or), - (not).

Any quite simple part can be represented as a combination of elementary elements that are easy to describe mathematically. In the formation of machine-building parts, the following surfaces are most common: plane, cylindrical, conical, spherical, toroidal, composite surface.

A plane in three-dimensional space is defined by three points that do not lie on a single line. Based on the information from the drawing, there are various ways to encode the plane.

The cylindrical surface is defined by seven parameters: coordinates of two mismatched points lying on the axis of rotation; the radius of the circle of the plane intersection that is perpendicular to the axis of rotation, and the cylindrical surface.

A spherical surface is defined by four parameters: coordinates of the sphere midpoint and the radius value.

The toroidal surface is defined by eight parameters: coordinates of the great circle center passing through the small circle centers; the radius of a great circle; the radius of the small circle, the normal vector to the plane of the centers.

A surface of standard form can be obtained by approximating it with splines.

For the analytical equation of a surface based on the parameters of its external representation, the matrix \( M \) is used for converting coordinates from a relative system to an absolute one.

\[
\begin{bmatrix}
m_{11} & m_{12} & m_{13} & X_1 \\
m_{21} & m_{22} & m_{23} & X_2 \\
m_{31} & m_{32} & m_{33} & X_3 \\
0 & 0 & 0 & 1
\end{bmatrix}
\quad (4)
\]

where \( m_{11} = \cos\beta \cos\gamma; \quad m_{12} = -\cos\beta \sin\gamma; \quad m_{13} = 0; \)
\( m_{21} = -\sin\alpha \sin\beta \cos\gamma + \cos\alpha \sin\beta \sin\gamma; \quad m_{22} = \sin\alpha \sin\beta \sin\gamma - \cos\alpha \cos\beta; \quad m_{23} = \sin\alpha \cos\beta; \)
\( m_{31} = -\cos\alpha \sin\beta \cos\gamma - \sin\alpha \sin\gamma; \quad m_{32} = \cos\alpha \sin\beta \sin\gamma - \sin\alpha \cos\gamma; \quad m_{33} = \cos\alpha \cos\beta. \)

Then the inverse matrix \( M^{-1} \) is calculated to transform coordinates from the external representation to the internal one:

\[
M^{-1} = M_y^{-1} \cdot M_\beta^{-1} \cdot M_\alpha^{-1} \cdot M_0^{-1}
\quad (5)
\]

Let's take a cylindrical surface (Figure 1) as an example of such a transformation.

**External representation:**

- \( A_1 = [x_1^1, x_2^1, x_3^1] \) — the first point on the surface axis;
- \( A_2 = [x_1^2, x_2^2, x_3^2] \) — the second point on the surface axis;
- \( R \) is the radius of the perpendicular section circle.

In its coordinate system, the equation of a cylindrical surface has the form:
\[
\begin{align*}
    x_1 &= t \\
    x_2 &= R \cos \alpha \\
    x_3 &= R \sin \alpha 
\end{align*}
\]  
\hspace{1cm} (6)

where \(0 < \alpha < 2\pi\).

The coordinate transformation matrix \(M\) is obtained using sequential movements: parallel translation of the \(00_1= [c_1, c_2, c_3]\) system, consecutive rotations of the absolute coordinate system around the \(O_x, O_y\) axes to the \(\gamma\) and \(\beta\) angles.

\[\text{Figure 1.}\] The coordinate transformation when moving from the external representation of a cylindrical surface to the internal one.

As for geometrically complicated parts, having more than ten bounding surfaces, their description becomes massive. Therefore, intermediate spatial form elements are introduced, which are formed from the original set of surfaces. In this case, the part is designed not from the original surfaces, but larger spatial elements: parallelepipeds, cylinders, truncated cones, spheres, tori, pyramids, prisms, etc. Each of the basic elements is characterized by geometric parameters that determine the length of the elements in space and the parameters of their location.

To get the part using these elements, you need to perform the following operations: stretching or compressing along the coordinate axes; parallel translation along coordinate axes; rotation about the coordinate axes.

4. Experimental results
Let's describe a part of the flange type (Figure 2), for example.

\[\text{Figure 2.}\] The flange type part drawing detail
This part consists of several form elements: C01, C02, C03, C04, C05 cylinders (Figure 3).

**Figure 3.** Model of the flange type part

The first letter is the name of the element, and the digits are the number of the element. You can record the relative location of the part’s basic elements:

\[ D = C01VC02VC03 - (C04&05) \]

Let’s present the basic elements that form the part and their geometric parameters and location parameters in Table 1.

### Table 1. The basic elements of the part and their geometrical parameters.

| Part number | Geometrical parameters | Location parameters |
|-------------|------------------------|---------------------|
|             | P1 | P2 | P3 | x | y | z | α | β | Y' |
| C01         | 35 | 60 | -  | 0 | 0 | 0 | 0 | 0 | 0 |
| C02         | 15 | 120| -  | 0 | 0 | 0 | 0 | 0 | 0 |
| C03         | 50 | 80 | -  | 0 | 0 | 0 | 0 | 0 | 0 |
| C04         | 100| 40 | -  | 0 | 0 | 0 | 0 | 0 | 0 |
| C05         | 15 | 10 | -  | 0 | 50| 0 | 0 | 0 | 0 |

You can build a coordinate transformation matrix \( M \) for a cylindrical surface (spatial case). We assume that the absolute coordinate system is obtained as a result of sequential movements:

- parallel translation along the \( OO'=(C_1; C_2; C_3) \) vector - \( M_0 \) matrix;
- rotation of the \( OX_i^0; OX_j^0 \) coordinate axes, respectively on the \( M_\gamma \) and \( M_\beta \) angles of the matrix. \( M_0; M_0 \) \( M_\gamma \) and \( M_\beta \),

where

\[
M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} ; \quad M_\gamma = \begin{pmatrix} \cos \gamma & \sin \gamma & 0 & 0 \\ -\sin \gamma & \cos \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} ; \quad M_\beta = \begin{pmatrix} \cos \beta & 0 & -\sin \beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]

Let us find the equation of a cylindrical surface for a special case:

\[
C01 \quad A_1(2; 0; 1), A_2(37; 0; 36), R=30 \\
C02 \quad A_3 = A_2(37; 0; 36), A_4(52; 0; 51), R=60 \\
C03 \quad A_5 = A_3(52; 0; 51), A_6(102; 0; 101), R=40 \\
C04 \quad A_6 = A_1(2; 0; 1), A_7(102; 0; 101), R=20
\]
C05 $A_1(2; 0; 1), A_2(37; 0; 36), R = 30$

The equation of the C01 cylindrical surface in its own coordinate system looks like:

$$X_1 = t; \quad X_2 = R \cos \alpha; \quad X_3 = R \sin \alpha, \quad R = 30$$

The equation for the first cylinder:

$$\frac{1}{1800} (X_3 - X_1 + 1)^2 + \frac{1}{900} X_2^2 = 1 \quad (10)$$

We find the equation for the C02 element:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -\sqrt{2} \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 30 \cos \alpha \\ 30 \sin \alpha \end{pmatrix} \times \begin{pmatrix} t \\ 60 \cos \alpha \\ 60 \sin \alpha \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{\sqrt{2}}{2} t - 30\sqrt{2} \cos \alpha + 37 \\ x_2 = 60 \cos \alpha \\ x_3 = \frac{\sqrt{2}}{2} t + 30\sqrt{2} \sin \alpha + 36 \end{cases} \quad (11)$$

Then we get the equation for the second cylinder:

$$\frac{1}{7200} (X_3 - X_1 + 1)^2 + \frac{1}{3600} X_2^2 = 1 \quad (12)$$

In a similar manner we find the equation for the third element C03:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -\sqrt{2} \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 40 \cos \alpha \\ 40 \sin \alpha \end{pmatrix} \times \begin{pmatrix} t \\ 40 \cos \alpha \\ 40 \sin \alpha \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{\sqrt{2}}{2} t - 20\sqrt{2} \sin \alpha + 52 \\ x_2 = 40 \cos \alpha \\ x_3 = \frac{\sqrt{2}}{2} t + 20\sqrt{2} \sin \alpha + 51 \end{cases} \quad (13)$$

So we obtain the equation for the cylinder C03:

$$\frac{1}{3200} (X_3 - X_1 + 1)^2 + \frac{1}{1600} X_2^2 = 1 \quad (14)$$

We find the equation for the element C04:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -\sqrt{2} \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 20 \cos \alpha \\ 20 \sin \alpha \end{pmatrix} \times \begin{pmatrix} t \\ 20 \cos \alpha \\ 20 \sin \alpha \end{pmatrix} \Rightarrow \begin{cases} x_1 = \frac{\sqrt{2}}{2} t - 10\sqrt{2} \sin \alpha + 2 \\ x_2 = 20 \cos \alpha \\ x_3 = \frac{\sqrt{2}}{2} t + 10\sqrt{2} \sin \alpha + 1 \end{cases} \quad (15)$$

We obtain the equation for the fourth cylinder:

$$\frac{1}{800} (X_3 - X_1 + 1)^2 + \frac{1}{400} X_2^2 = 1 \quad (16)$$
\[
\begin{align*}
\frac{1}{1800} (x_3 - x_1 + 1)^2 + \frac{1}{900} x_2^2 &= 1 \\
\frac{1}{7200} (x_3 - x_1 + 1)^2 + \frac{1}{3600} x_2^2 &= 1 \\
\frac{1}{3200} (x_3 - x_1 + 1)^2 + \frac{1}{1600} x_2^2 &= 1 \\
\frac{1}{800} (x_3 - x_1 + 1)^2 + \frac{1}{400} x_2^2 &= 1
\end{align*}
\]

5. Discussion of results
The logical-algebraic model of the part (17) allows you to determine the necessary properties of surfaces required for the design of the technological process of machining. Based on the obtained universal model, it is possible to change the parameters of the studied part in a wide range quickly, without significant costs and to automate the processes of transferring geometric information at different design stages using computer technologies. This allows working out the design of the part for producibility at the early stages of design.

6. Summary and conclusions
Achieved part model is required for solving numerous issues of computer-aided design of the machining process. For example, when preparing initial information for design, generating a database for technological purposes, keeping it up-to-date and searching for information, selecting the structure and calculating process parameters, as well as optimizing technological processes.

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