Double Dalitz Plot Analysis of the Decay $B^0 \rightarrow DK^+\pi^-$, $D \rightarrow K_{S}^{0}\pi^+\pi^-$

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It is shown that it is possible to perform a model-independent extraction of the CKM Unitarity Triangle angle $\gamma$ using only the decays $B^0 \rightarrow DK^+\pi^-$ with $D \rightarrow K_S^0\pi^+\pi^-$ and $B^0 \rightarrow DK^+\pi^-$ with flavour-specific $D$ decays. The proposed method can also utilise the $B^0 \rightarrow DK^+\pi^-$ data with $CP$-eigenstate decays of the $D$ meson in a model-independent way.

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I. INTRODUCTION

Among the fundamental parameters of the Standard Model of particle physics, the angle $\gamma = \arg (-V_{ud}V_{cb}^{*}/V_{cb}V_{ub}^{*})$ of the Unitarity Triangle formed from elements of the Cabibbo-Kobayashi-Maskawa quark mixing matrix [1, 2] has a particular importance. Not only is it one of the least well determined fundamental parameters of the Standard Model, it is also the only $CP$-violating parameter that can be measured using only tree-level decays [3]. The precise determination of $\gamma$ is thus a critical element of the programme towards understanding the baryon asymmetry of the Universe, and is one of the main objectives of planned future $B$ physics experiments (see, for example, [4–6]).

A method to measure the CKM phase $\gamma$ in $B^0 \rightarrow DK^+\pi^-$ decays by comparing the Dalitz plot distribution obtained when the neutral $D$ meson is reconstructed in $CP$ eigenstates to that when flavour-specific states are used has been proposed recently [7]. The method builds on the original proposal of Gronau, London and Wyler (GLW) [8, 9] for a similar analysis using $B^+ \rightarrow DK^+$ decays, and exploits the fact that $CP$ violation effects are expected to be enhanced in $B^0 \rightarrow DK^{*0}$ decays since the interfering amplitudes are of comparable magnitude [10]. Since the charge of the kaon in $B^0 \rightarrow DK^+\pi^-$ decays unambiguously tags the flavour of the decaying $D$ meson, there is no need for time-dependent analysis [11].

Compared to previous proposals for similar analyses [12–14] this method has several attractive features: 1) the amplitudes can be measured directly in the Dalitz plot analysis, relative to the flavour-specific $B^0 \rightarrow D_S^{*0}K^+$ amplitude; 2) there is no need to normalise to decays with another final state; 3) there is sufficient information to extract $\gamma$ using only $D$ decays to $CP$-even states; 4) the sensitivity to $\gamma$ does not depend strongly on the values of $\gamma$ and strong phases involved; 5) $\gamma$ is determined with only a single unresolvable ambiguity ($\gamma \rightarrow \gamma + \pi$). These advantages have much in common with those of the $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0\pi^+\pi^-$ method, which is currently providing the most constraining measurements of $\gamma$ [15, 16]. Moreover, this technique is especially promising for the LHCb experiment since it does not require the reconstruction of neutral particles in the final state.

There is, however, a potential drawback of the method, in that the fit to the Dalitz plot distribution is inherently model-dependent. The magnitude of the model-dependence has been estimated to be small [17], but in view of the aim to achieve an $O(1\%)$ measurement of $\gamma$ at future experiments, it is certainly desirable to explore possibilities to remove such systematic uncertainties.

A model-independent analysis procedure has been proposed for the extraction of $\gamma$ from $B^+ \rightarrow DK^+$, $D \rightarrow K_S^0\pi^+\pi^-$ decays [18, 19]. The method uses data from charm factory experiments – specifically, $CP$-tagged charm mesons – to obtain information about the strong phase variation across the $D$ decay Dalitz plot. A similar approach to probe the strong phases present in the $B^0 \rightarrow DK^+\pi^-$ Dalitz plot is not immediately available since there is no realistic way to obtain sufficient samples of $CP$-tagged $B$ decays.

In this paper we show that the study of Dalitz plot distributions in the decay chain $B^0 \rightarrow DK^+\pi^-$, $D \rightarrow K_S^0\pi^+\pi^-$ allows to translate the model-independent measurement of the $D \rightarrow K_S^0\pi^+\pi^-$ amplitude to the $B^0 \rightarrow DK^+\pi^-$ mode. This approach allows not only to measure $\gamma$ model-independently in this decay chain, but also to use the obtained constraints on the $B^0 \rightarrow DK^+\pi^-$ amplitude in the $B^0 \rightarrow D_{CP}K^+\pi^-$ Dalitz plot analysis. For clarity, our method is independent of model assumptions in both $B^0 \rightarrow DK^+\pi^-$ and $D \rightarrow K_S^0\pi^+\pi^-$ decays. The more straightforward cases which are either model-dependent in $B^0 \rightarrow DK^+\pi^-$ but model-independent in $D \rightarrow K_S^0\pi^+\pi^-$, or vice-versa, are also possible but are not discussed in detail. A fully model-dependent analysis is, of course, also possible.

The remainder of the paper is organised as follows. In Section II we outline the basic idea and the formalism of the analysis. In Section III we discuss how possible complications arising from the use of $D \rightarrow K \pi$ decays as quasi-flavour-specific states can be resolved. In Sec-

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1 $CP$-odd $D$ decays with large branching fractions such as $D \rightarrow K_S \pi^0$ are difficult to reconstruct, particularly in experiments at hadron colliders.
tion \cite{11} we describe the Dalitz plot models that we use in our feasibility study. We finally present the results of the study in Section \cite{11} before concluding in Section \cite{11}.

**II. FORMALISM**

We begin by recalling the essentials of the $B^+ \rightarrow D K^+$, $D \rightarrow K_S^0 \pi^+ \pi^-$ model-independent method \cite{11}. The amplitude of the $B^+ \rightarrow D K^+$, $D \rightarrow K_S^0 \pi^+ \pi^-$ decay can be written

$$A_{D \text{ Daliz}} = A_D + r_B e^{i(\delta_B + \gamma)} A_D,$$

where $A_D = \overline{A}_D(m^2_{K_S^0}, m^2_{\pi^+\pi^-}) \equiv \overline{A}_D(m^2_{\pi^+\pi^-})$ is the amplitude of the $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ decay, $A_D = A_D(m^2_{\pi^+\pi^-})$ is the amplitude of the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decay, $r_B$ is the ratio of the absolute values of the interfering $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow D^0 K^+$ amplitudes, and $\delta_B$ is the strong phase difference between these amplitudes. Assuming no CP violation in $D$ decay, $A_D(m^2_{\pi^+\pi^-}) = \overline{A}_D(m^2_{\pi^+\pi^-})$. The density of the Dalitz decay Dalitz plot from $B^+ \rightarrow D K^+$ decay is given by the absolute value squared of the amplitude

$$|A_{D \text{ Daliz}}|^2 = |\overline{A}_D + r_B e^{i(\delta_B + \gamma)} A_D|^2 = |\overline{A}_D|^2 r_B^2 |A_D|^2 + 2 |A_D| |\overline{A}_D| (x c - y s),$$

where

$$x = r_B \cos(\delta_B + \gamma); \quad y = r_B \sin(\delta_B + \gamma).$$

The functions $c = c(m^2_{\pi^+\pi^-})$ and $s = s(m^2_{\pi^+\pi^-})$ are the cosine and sine of the strong phase difference $\delta_D = \arg A_D - \arg \overline{A}_D$ between the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $\overline{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$ amplitudes\footnote{This paper follows the convention for strong phases in $D$ decay amplitudes used by CLEO \cite{22}. Note that alternative conventions are used in the literature, e.g. Ref. \cite{11} uses a convention where the sign of $\delta_D$ is opposite.}:

$$c = \cos \delta_D(m^2_{\pi^+\pi^-}); \quad s = \sin \delta_D(m^2_{\pi^+\pi^-}).$$

The equations for the charge-conjugate mode $B^- \rightarrow D K^-$ are obtained with the substitution $\gamma \rightarrow -\gamma$. Considering both $B$ charges, one can obtain $\gamma$ and $\delta_B$ separately.

Once the Dalitz plot is divided into $2N$ bins symmetrically to the exchange $m^2_1 \leftrightarrow m^2_2$, the expected number of events in the $i$th bin of the Dalitz plot of $D$ decay from $B^+ \rightarrow D K^+$ is

$$\langle N_i \rangle = h_{D \text{ Daliz}} \left[ K_i + r_B^2 K_{-i} + 2 \sqrt{K_i K_{-i}} (x c_i - y s_i) \right],$$

where $K_i$ is the number of events in the corresponding bin of the Dalitz plot where the $D$ meson is in a flavour eigenstate (conveniently obtained using $D^{\pm} \rightarrow D \pi^{\pm}$ samples) and $h_{D \text{ Daliz}}$ is a normalisation constant. The bin index $i$ ranges from $-N$ to $N$ (excluding 0); the exchange $m^2_1 \leftrightarrow m^2_2$ corresponds to the exchange $i \leftrightarrow -i$. The coefficients $c_i$ and $s_i$, which include information about the cosine and sine of the phase difference, are given by

$$c_i = \frac{\int |A_D| |\overline{A}_D| \cos(\delta_D dD)}{\int |A_D|^2 dD \int |\overline{A}_D|^2 dD},$$

where the dependences on Dalitz plot position of $|A_D|$, $|\overline{A}_D|$ and $\delta_D$ have been suppressed. The parameters $s_i$ are defined similarly with cosine substituted by sine. Here $D$ represents the Dalitz plot phase space and $D_i$ is the bin region over which the integration is performed.

The symmetry under $\pi^+ \leftrightarrow \pi^-$ requires $c_i = c_{-i}$ and $s_i = -s_{-i}$, thus for $2N$ bins there are $2N + 3$ unknowns. For each bin there are two observables in $B \rightarrow D K$ decays, being the numbers of events in $B^+$ and $B^-$ samples, and thus there are a total of $4N$ observables. The system can consequently be solved for $N \geq 2$. However, due to the small value of $r_B$ in $B^+ \rightarrow D K^+$ decays, there is very little sensitivity to the $c_i$ and $s_i$ parameters, leading to a reduction in the precision on $\gamma$ that can be obtained. Use of external constraints on $c_i$ and $s_i$, that can be provided by charm-factory experiments \cite{22}, leads to much improved sensitivity to $\gamma$.

Turning now to $B^0 \rightarrow D K^+ \pi^-$ decays, we find that the simplest approach towards performing a model-independent analysis is not viable. Using the convention $D_{CP} = (D^0 \pm \bar{D}^0) / \sqrt{2}$, where $+ \rightarrow -$ and $- \rightarrow +$ correspond to CP-even and CP-odd final states, respectively, the amplitude for $B^0 \rightarrow D_{CP} K^+ \pi^-$ decay is given by

$$A_{B \text{ Daliz}} = \frac{1}{\sqrt{2}} (\pm \overline{A}_B + e^{i\gamma} A_B),$$

where $\overline{A}_B = \overline{A}_B(m^2_{D_{CP}}, m^2_{K^+ \pi^-})$ is the amplitude of the $B^0 \rightarrow \overline{D}^0 K^+ \pi^-$ decay and $A_B = A_B(m^2_{D_{CP}}, m^2_{K^+ \pi^-})$ is the amplitude of the $B^0 \rightarrow D^0 K^+ \pi^-$ decay. In these expressions we have factorised out the $CP$-violating phase $\gamma$ so that the amplitudes $\overline{A}_B$ and $A_B$ are $CP$-conserving. Both the strong phase difference between $B^0 \rightarrow \overline{D}^0 K^+ \pi^-$ and $B^0 \rightarrow D^0 K^+ \pi^-$ decays and the ratio of the suppressed and allowed amplitudes $r_B$ are now functions of $B$ decay Dalitz plot variables and enter the expressions for amplitudes.

Let us now consider dividing the $B$ decay Dalitz plot into bins (denoted by the index $\alpha$, $1 \leq \alpha \leq M$). The number of expected events in each $B^0 \rightarrow \overline{D}_{CP} K^+ \pi^-$ Dalitz plot bin is

$$\langle M_\alpha \rangle = h_{B \text{ Daliz}} \left[ N_\alpha + N_\alpha \pm 2 \sqrt{N_\alpha N_{-\alpha} (\kappa_\alpha \cos \gamma - \sigma_\alpha \sin \gamma)} \right].$$

In this expression the factors $N_\alpha$ and $N_{-\alpha}$ give the numbers of events in the corresponding bin in $B^0 \rightarrow$
\( D^0 K^+ \pi^- \) and \( B^0 \to D^0 K^+ \pi^- \) decays, respectively (complications arising due to the difficulty to reconstruct \( D \) mesons in pure flavour eigenstates are discussed in Section III below). The factor \( h_{D^0Dlz} \) is a global normalisation constant. In Eq. (8) we have introduced the parameters \( \alpha_\alpha \) and \( \sigma_\alpha \) that describe the average strong phase difference and amplitude difference between \( B^0 \to D^0 K^+ \pi^- \) and \( B^0 \to D^0 K^+ \pi^- \) decays in each bin, and that are defined by:

\[
\alpha_\alpha = \int \frac{|A_B| |\overline{A}_B|}{D_\alpha} \cos \delta_B \, dD \sqrt{\frac{1}{D_\alpha} \int \frac{|A_B|^2}{dD} \int \frac{|\overline{A}_B|^2}{dD}},
\]

(9)

where the dependence on the \( B \) decay Dalitz plot position of \( |A_B|, |\overline{A}_B| \) and \( \delta_B = \arg A_B - \arg \overline{A}_B \) have been suppressed. \( D \) again represents the phase space (now of the \( B \) decay Dalitz plot) and \( D_\alpha \) is the bin region over which the integration is performed. The parameters \( \alpha_\alpha \) are given by similar expressions with cosine substituted by sine.

Until this stage the discussion for \( B^0 \to DK^+ \pi^- \) decays has run in parallel to that for \( B^+ \to DK^+ \), \( D \to K^0_S \pi^+ \pi^- \). However, unlike the \( B^+ \to DK^+ \), \( D \to K^0_S \pi^+ \pi^- \) case, the two amplitudes \( \overline{A}_B \) and \( A_B \) are inherently different and thus there is no symmetry in the Dalitz plot to be exploited. In the case of \( CP \)-eigenstate \( D \) decays, we therefore have 2\( M \)+1 unknowns (even if the normalisation is fixed externally), that cannot be solved for with 2\( M \) observables. Hence external constraints on \( \alpha_\alpha \) and \( \sigma_\alpha \) are necessary [12].

Let us now consider both Dalitz plots simultaneously in the \( B^0 \to DK^+ \pi^- \), \( D \to K^0_S \pi^+ \pi^- \) decay chain. The amplitude can be written as

\[
A_{D^0Dlz} = \overline{A}_B A_D + e^{\gamma} A_B A_D.
\]

(10)

The decay probability is proportional to the absolute value squared of the amplitude \( A \):

\[
|A_{D^0Dlz}|^2 = |\overline{A}_B|^2 |A_D|^2 + |A_B|^2 |A_D|^2 + 2 |A_B| |A_D| |\overline{A}_B| |A_D| \times \Re \left(e^{i\gamma} e^{i\delta_B (m^2_{D^0}, m^2_D)} e^{i\delta_D (m^2_{K^0_S}, m^2_{\pi^-})}\right) = |\overline{A}_B|^2 |A_D|^2 + |A_B|^2 |A_D|^2 + 2 |A_B| |A_D| |\overline{A}_B| |A_D| \times \left[(\kappa_\alpha - \sigma_\alpha) \cos \gamma - (\kappa_\alpha + \sigma_\alpha) \sin \gamma\right].
\]

(11)

To implement the model-independent analysis both \( B \) and \( D \) plots have to be binned. Corresponding bin indices are denoted here, as before, by Greek and Latin characters, respectively. The expected number of events in the bins of \( B^0 \to DK^+ \pi^- \), \( D \to K^0_S \pi^+ \pi^- \) double Dalitz plot is then

\[
\langle M_{\alpha\alpha} \rangle = h_{D^0Dlz} \left[N_\alpha K_i + N_\alpha K_{-i} + 2 \sqrt{N_\alpha K_i N_\alpha K_{-i}} \times \left[(\kappa_\alpha c_i - \sigma_\alpha s_i) \cos \gamma - (\kappa_\alpha s_i + \sigma_\alpha c_i) \sin \gamma\right]\right],
\]

(12)

where the definition of the event numbers \( N_\alpha, \overline{N}_\alpha, K_i \) and phase terms \( c_i, s_i, \kappa_\alpha, \sigma_\alpha \) is the same as for the processes mentioned above, and \( h_{D^0Dlz} \) is a normalisation constant.

If the number of bins in the \( B^0 \to DK^+ \pi^- \) Dalitz plot is \( M \) and the number of bins in the \( D \to K^0_S \pi^+ \pi^- \) Dalitz plot is \( 2N \), then after exploiting the symmetry of the \( D \to K^0_S \pi^+ \pi^- \) decay as before, the number of equations represented by Eq. (12) is \( MN \). The number of unknowns (including the normalisation factor \( h_{D^0Dlz} \)) is \( 2M + 2N + 2 \). Naturally, information on \( \gamma \) cannot be extracted from the decays of only one \( B \) flavour, as is apparent since Eq. (12) is invariant under the rotation of \( \gamma \) with the simultaneous rotation of all \((\kappa_\alpha, \sigma_\alpha)\) pairs. To determine the \( CP \) violation parameter it is, of course, necessary to add the opposite \( B \) flavour (with the substitution \( \gamma \to -\gamma \)). The system can now be fully resolved: in total, there are \( 2MN \) observables and \( 2M + 2N + 2 \) unknowns, i.e. it is solvable for \((M - 1)(N - 1) \geq 2\).

Since the coefficients \( c_i \) and \( s_i \) are the same as in the model-independent \( B \to DK \) analysis, and can be obtained at a charm factory, the number of unknowns can be reduced even further.

As a by-product of the \( B^0 \to DK^+ \pi^- \), \( D \to K^0_S \pi^+ \pi^- \) analysis, the coefficients \( \kappa_\alpha \) and \( \sigma_\alpha \) will be determined. The values of these parameters can then be used to enable a model-independent analysis of both \( B^0 \to DC_{CP}K^+ \pi^- \) decays. Technically the combination of \( B^0 \to DK^+ \pi^- \), \( D \to K^0_S \pi^+ \pi^- \) and \( B^0 \to DC_{CP}K^+ \pi^- \) modes is conveniently done by using a combined likelihood fit that includes both the expressions Eq. (12) and Eq. (8).

III. EFFECTS OF DOUBLY-CABIBBO-SUPPRESSED CONTRIBUTIONS TO \( D \to K\pi \) DECAYS

In practice it is hard to reconstruct secondary \( D \) mesons from \( B \) decays in pure flavour eigenstates. (The possibility to exploit the charge of the associated pion in \( D^{*\pm} \to D_{\pi\pm} \) is not available.) Semileptonic \( D^0 \) decays in principle provide a source of pure flavour-specific states, but owing to small branching fractions and large backgrounds these are experimentally difficult to deal with, especially at hadron experiments such as LHCb. It is more convenient to use hadronic final states such as \( K\pi \), which are not pure flavour tags due to the presence of a small admixture of doubly-Cabibbo-suppressed amplitude. This contribution can significantly affect a precision measurement.

In the proposed analysis, hadronic final states can be used after some correction of the procedure. The amplitude of \( B^0 \to DK^+ \pi^- \), \( D \to K^+ \pi^- \) decay is

\[
A_{fav} = \overline{A}_B + r_{K\pi} e^{-i\delta_{K\pi}} e^{i\gamma} A_B,
\]

(13)

where \( r_{K\pi} \) and \( \delta_{K\pi} \) are the ratio of the magnitudes of the suppressed and favoured \( D \) decay amplitudes and the
strong phase between them, respectively. The number of events in the $B^0 \to DK^+\pi^-$ Dalitz plot bins with $D$ detected in a $K^+\pi^-$ state is:

$$\langle N_{K^+}^{\text{fav}} \rangle = N_{K^+} + 2r_{K^+}N_{\pi^+} + 2r_{K^+}\sqrt{N_{K^+}N_{\pi^+}} \times \left[ \sigma_\alpha \cos(-\delta_{K^+} + \gamma) - \sigma_\alpha \sin(-\delta_{K^+} + \gamma) \right].$$

(14)

Similarly for the suppressed decay $B^0 \to DK^+\pi^-$, $D \to K^-\pi^+$, the amplitude is

$$A_{\text{sup}} = r_{K^+}e^{i\delta_{K^+}}A_B + e^{i\gamma}A_B,$$

and the number of events is

$$\langle N_{K^+}^{\text{sup}} \rangle = r_{K^+}^2N_{\pi^+} + N_{\pi^+} + 2r_{K^+}\sqrt{N_{K^+}N_{\pi^+}} \times \left[ \sigma_\alpha \cos(\delta_{K^+} + \gamma) - \sigma_\alpha \sin(\delta_{K^+} + \gamma) \right].$$

(16)

The expressions for $B^0 \to DK^-\pi^+$ decays are obtained by the substitution $\gamma \to -\gamma$.

The relations of Eq. (14) and Eq. (16) add another 4$M$ equations to the system of equations to be used in the analysis (2$M$ equations for each $B$ flavour), but since the constants $r_{K^+}$ and $\delta_{K^+}$ are known from charm analyses [21, 22], only 2$M$ new unknowns are added (the numbers $N_{\pi^+}$ and $N_{\pi^+}$). Hence, the inclusion of the suppressed final states into the analysis can help to improve the overall sensitivity to $\gamma$ [23, 24]. Recent studies suggest further improvement could be gained from including also the $D \to K\pi\pi^0$ decays [25].

IV. $B$ DECAY DALITZ PLOT MODELS

In order to perform a feasibility study of the proposed method it is necessary to define a $B$ decay Dalitz plot model. The sensitivity depends crucially on the structure of the $B^0 \to D^0K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ amplitudes, specifically on the interference between them. If the strong phase difference $\delta_{K^+}$ is nearly constant (e.g. in the case when both amplitudes are dominated by $B \to DK^*$ decay), the double Dalitz analysis reduces to a conventional Dalitz plot analysis technique similar that used for $B^{\pm} \to DK^{\pm}$ decays. Note that in this type of analysis the possible contribution from non-$K^*$ states can be taken into account model-independently by introducing the coherence factor $0 \leq \kappa \leq 1$ to the interference term [12] (analyses using this approach have been performed, albeit with low statistics [26, 27]). However, if the resonance structure in $B^0 \to D^0K^+\pi^-$ decays is rich with multiple overlapping states in different channels ($K^*$, $D^*$ and $D_s^*$), the proposed technique should provide a significant benefit compared to analysis of the $D \to K^{\ast}_0\pi^+\pi^-$ Dalitz plot with the coherence factor.

Unlike the $D \to K^{\ast}_0\pi^+\pi^-$ amplitude, the amplitudes for $B^0 \to DK^+\pi^-$ decays are not yet well established. For our feasibility study, based on pseudo-experiments generated using Monte Carlo (MC) simulation, we use several models based on two-body amplitudes with $K^*$ and $D^*$ resonances. We define first model A, which includes only $K^*(892)$ and $D_S^*(2460)$ states in the favoured amplitude and $K^*(892)$ in the suppressed amplitude with relative magnitudes consistent with measured values for the branching fractions [28] and phases chosen arbitrarily. Model B is similar to that used in a recent study [17], where the contributions of further resonances are based on results from $B \to D^+K^{0}\pi^-$ [29] using the assumption of isotopic symmetry. Model C includes a contribution from the $D_sJ(2700)$ resonance [30] in addition to all states of model B. Its contribution is estimated from the ratio

$$B(B \to D_{sJ}\pi) \approx B(B \to D_{sJ}^0\pi)$$

with a phase space correction. Models D and E use the same states as models B and C, respectively, excluding the nonresonant term. For the $D \to K^{\ast}_0\pi^+\pi^-$ amplitude, we use the model from the most recent Belle analysis [31].

The fit fractions and phases for model C are shown in Table I. Other models contain subsets of the states from model C with the same amplitudes and phases. The Dalitz plots of $B^0 \to D^0K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ decays generated according to model C are shown in Figure 1. Other models contain subsets of the states from model C with the same amplitudes and phases. The Dalitz plots of $B^0 \to D^0K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ decays generated according to model C are shown in Figure 1.

In the case of the $B^0 \to D^0K^+\pi^-$ Dalitz plot, the number of events in each bin is the product of the number of events in the $DK^+\pi^-$ Dalitz plot with the phase space correction. For the $B^0 \to D^0K^+\pi^-$ Dalitz plot with 7 bins for model C is shown in Figure 2. Other models contain subsets of the states from model C with the same amplitudes and phases. The Dalitz plots of $B^0 \to D^0K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ decays generated according to model C are shown in Figure 1.

The sensitivity of the technique will depend on the choice of binning for both $D \to K^{\ast}_0\pi^+\pi^-$ and $B^0 \to DK^+\pi^-$ Dalitz plots. The choice of optimal binning for $D \to K^{\ast}_0\pi^+\pi^-$ Dalitz plot has been previously considered in application to $B^+ \to DK^+$ modes [10]. Here the requirements for the optimal binning are the same: the strong phase difference $\delta_{K^+}$ in case of $B^0 \to DK^+\pi^-$ and $\delta_D$ for $D \to K^{\ast}_0\pi^+\pi^-$ and the ratio of suppressed to allowed amplitudes should be as constant as possible over each bin to maximise values of $\sigma_\alpha$ and $c_2^2 + s_1^2$. In the case of the $D \to K^{\ast}_0\pi^+\pi^-$ Dalitz plot, a binning based on uniform division of strong phase difference has been found to give a good approximation to the optimal [19]. However, in the case of $B^0 \to DK^+\pi^-$ amplitudes, such an approach gives poor results because of very large variations of the absolute value of amplitude ratio across the bin. A binning based on maximising the “binning quality factor" $Q$ [19], gives significantly better results. The optimal binning of the $B^0 \to DK^+\pi^-$ Dalitz plot with 7 bins for model C is shown in Figure 2 (a), while Figure 2 (b) shows the values of $(\sigma_\alpha, \sigma_a)$ obtained in each bin. For the $D \to K^{\ast}_0\pi^+\pi^-$ Dalitz plot, we also use optimised binning obtained with the same technique (see Figure 3) with binning quality factor $Q$=0.89.

We use the values $r_B = 0.4$ and $\gamma = 60^\circ$ in the simulation. Note that the value of $r_B$ can differ for different $K^*$ states (e.g. $K^*(892)$, $K^{*0}_0(1430)$, $K^*_2(1430)$ etc.); we use the same value for all these states. We generate

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3 We use a sign convention for $\delta_{K^+}$ that is consistent with that used in the majority of the literature (see, e.g. Ref. [4]), but is opposite in sign to that used for $\delta_D$. Another different convention is used by CLEO [23, 24].
FIG. 1: Generated Dalitz plots for (a) $B^0 \rightarrow \overline{D}^0 K^+ \pi^-$ and (b) $B^0 \rightarrow D^0 K^+ \pi^-$ decays using amplitude model C.

FIG. 2: (a) Binning of $B^0 \rightarrow DK^+ \pi^-$ Dalitz plot and (b) values of $\kappa_i, \sigma_i$ parameters for optimal binning (crosses are calculated values, scattered points are the values obtained from the fit to toy MC samples).

FIG. 3: (a) Binning of $D \rightarrow K^0_S \pi^+ \pi^-$ Dalitz plot and (b) values of $c_i, s_i$ parameters for optimal binning (crosses are calculated values, scattered points are the values obtained from the fit to toy MC samples).
pseudo-experiments with samples of $10^4$ events for each $B$ flavour for both $B^0 \to DK^+\pi^-$ with $D \to K_S^0\pi^+\pi^-$ and $B^0 \to D_{CP}K^+\pi^-$ modes. Note that once the Dalitz plot models are fixed, the ratios of the numbers of events between the different samples is also fixed. The numbers of $B^0 \to D_{CP}K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ decays are $10^5$ and $\sim 8000$, respectively. These numbers of events roughly correspond to 20 times the expected annual yield of the LHCb experiment.\textsuperscript{4} Hence the precision obtained from this study is indicative of that which could be achieved in an upgraded phase of LHCb operation. It is, however, trivial to scale the sensitivities to lower luminosities.

V. RESULTS OF THE FEASIBILITY STUDY

Different strategies for using $B^0 \to DK^+\pi^-$ decays to measure $\gamma$ are studied using MC simulation. The results are shown in Table I. The uncertainties are obtained from the spread of results in the samples of 200 pseudo-experiments.

An unbinned fit of $B^0 \to D_{CP}K^+\pi^-$ decays with free parameters $r_B$, $\gamma$ and $\delta_B$ (option 1) gives an estimate of the statistical error for a model-dependent Dalitz plot analysis of this decay.\textsuperscript{7} A binned fit of the same mode (option 2) with fixed values of $\alpha$ and $\sigma$ shows a drop of sensitivity due to binning of the $B^0 \to D^+K^-\pi^-$ Dalitz plot. For most amplitude models, the loss in sensitivity is about 20%.

We next consider selecting only events in the $K^*$ region of $B^0 \to DK^+\pi^-$ phase space, introducing a coherence factor\textsuperscript{12}, and using $D \to K_S^0\pi^+\pi^-$ decays. A conventional single Dalitz plot analysis (option 3) gives comparable sensitivity to the unbinned $B^0 \to D_{CP}K^+\pi^-$ analysis (option 1). However, only a fraction of all $B^0 \to DK^+\pi^-$ events are used indicating that further gains are potentially possible. Changing from unbinned to a binned model-independent approach (option 4) reduces the sensitivity by about 15%. Relaxing the $c_i$ and $s_i$ coefficients in the binned approach is also possible (option 5)\textsuperscript{18}. Although the sensitivity in this case reduces significantly, the reduction is far less than in the $B^0 \to D_{CP}K^+\pi^-$ mode due to the larger interference term in neutral $B$ decays and also due to the optimal binning used in this study.

The double Dalitz plot analysis (option 6) uses all available events in the $B^0 \to DK^+\pi^-$ phase space, providing better $\gamma$ accuracy compared to option 4, and also giving as a by-product the values of $B^0 \to DK^+\pi^-$ amplitude coefficients $\kappa_\alpha$ and $\sigma_\alpha$. The values of $\kappa_\alpha$ and $\sigma_\alpha$ returned from the fit and a comparison with their calculated values is shown in Figure 2 (b). Relaxing the $D \to K_S^0\pi^+\pi^-$ amplitude coefficients $c_i$, $s_i$ (option 7) results in a much smaller reduction of the sensitivity than that with single Dalitz plot analysis selecting only the $K^*$ region. The fitted values of the $c_i$, $s_i$ coefficients for this option are shown in Figure 3 (b).

Combined with $B^0 \to D_{CP}K^+\pi^-$ decays (option 8), the double Dalitz analysis allows to further improve the statistical precision. However the gain in sensitivity for most models is about 15% worse than the weighted error for options 6 and 2, which means that the precision of $\kappa_\alpha$ and $\sigma_\alpha$ terms obtained from the Dalitz analysis affects the $B^0 \to D_{CP}K^+\pi^-$ mode. With very large statistics it may be possible to mitigate this effect by using a finer binning of the $B^0 \to DK^+\pi^-$ Dalitz plot.

An example of the residual distribution for $\gamma$ using option 6 (double Dalitz plot fit) with amplitude model C is shown in Figure 4 (a). No systematic bias is seen in any of the fits.

We emphasise that, although the fit procedure uses a binning that depends on the decay amplitude, the method does not generate systematic bias even if the amplitude used to define the binning is wrong or the model-blind binning is used. To demonstrate this, we perform the fit of the data generated using model C, but using model-blind rectangular Dalitz plot bins (the bin bound-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & $B^0 \to D_{CP}K^+\pi^-$ & & $B^0 \to D^0K^+\pi^-$ & & \\
 & Amplitude & Phase & Fit frac. & Amplitude & Phase & Fit frac. \\
\hline
$K^*$ (892) & 1 & 0° & 47% & $r_B \times 1$ & 120° & 56% \\
$K_0^*(1430)$ & 0.2 & 284° & 0.015% & $r_B \times 0.2$ & 284° & 0.018% \\
$K_0^*(1430)$ & 0.16 & 221° & 10% & $r_B \times 0.16$ & 221° & 12% \\
$K^*$ (1680) & 0.61 & 128° & 2.4% & $r_B \times 0.61$ & 128° & 2.9% \\
$D_{CP}^0(2400)$ & 4.8 & 267° & 2.3% & - & - & - \\
$D_{CP}^0(2400)$ & 1.1 & 325° & 30% & - & - & - \\
$D_{CP}^0(2700)$ & - & - & - & $r_B \times 4.0$ & 200° & 25% \\
Non-res & 1.25 & 140° & 5.3% & $r_B \times 1.25$ & 140° & 6.3% \\
\hline
\end{tabular}
\caption{Parameters of $B^0 \to DK^+\pi^-$ amplitudes. The suppressed $B^0 \to D^0K^+\pi^-$ amplitude uses $r_B = 0.4$.
\label{tab:table1}}
\end{table}
of bins is 7 as in the other fits). The quality factor for the dependence on the strong phases in $B^m_m$ is also not large. Decays, which were taken arbitrarily in our study, should significantly on the value of $m^2_{K^+\pi^-}$. We have proposed a technique to obtain the angle $\gamma$ in the sensitivity. The precision on $\gamma$ is shown in Figure 4(c) for model C, options 6, the residual distribution for $B^0 \rightarrow D_{CP}^{-}\pi^+\pi^-$ (floated $c_{i}, s_{i}$). The fit is unbiased, and the statistical accuracy compared to the fit with binning based on the correct underlying model is roughly proportional to the ratio of the $Q$ values: the $\gamma$ resolution is $\sigma(\gamma) = 1.93 \pm 0.10^\circ$. The statistical accuracy of the method does not depend significantly on the value of $\gamma$. The precision on $\gamma$ as a function of $\gamma$ is shown in Figure 4(b) for model C, options 6 (double Dalitz plot fit only) and 8 (double Dalitz plot fit combined with $B^0 \rightarrow D_{CP}^{-}\pi^+\pi^-$). We expect that the dependence on the strong phases in $B^0 \rightarrow D K^+\pi^-$ decays, which were taken arbitrarily in our study, should also not be large.

| Composition | $B^0 \rightarrow D K^+\pi^-$ binning quality $Q$ | Model A | Model B | Model C | Model D | Model E |
|-------------|-----------------------------------------------|---------|---------|---------|---------|---------|
| $K^+$, $D_S$ only | 0.90 | 0.91 | 0.81 | 0.86 | 0.77 |
| $D_S$ only | 0.90 | 0.91 | 0.81 | 0.86 | 0.77 |
| All states included | 0.90 | 0.91 | 0.81 | 0.86 | 0.77 |
| No $D_s$ and no nonres. | 0.90 | 0.91 | 0.81 | 0.86 | 0.77 |
| No nonres. | 0.90 | 0.91 | 0.81 | 0.86 | 0.77 |

VI. CONCLUSION

We have proposed a technique to obtain the angle $\gamma$ of the Unitarity Triangle in a model-independent way by using the decay chain $B^0 \rightarrow D K^+\pi^-$, $D \rightarrow K^0_S\pi^+\pi^-$. Our method is independent of model assumptions in both $B^0 \rightarrow D K^+\pi^-$ and $D \rightarrow K^0_S\pi^+\pi^-$ amplitudes. The proposed approach allows not only to measure $\gamma$ in this decay chain, but also to use the constraints on the $B^0 \rightarrow D K^+\pi^-$ amplitude obtained in this measurement for a model-independent $B^0 \rightarrow D_{CP}^{-}\pi^+\pi^-$ Dalitz plot analysis.

The precision of the $\gamma$ measurement using our technique will depend strongly on the structure of $B^0 \rightarrow D K^+\pi^-$ amplitude, which is as-yet unknown. However, MC simulation results show that if interference effects between $B^0 \rightarrow D^0K^+\pi^-$ and $B^0 \rightarrow \overline{D}^0K^+\pi^-$ amplitudes are strong enough to allow a meaningful model-independent measurement of $\gamma$ using $B^0 \rightarrow D_{CP}^{-}\pi^+\pi^-$ amplitudes, the number of reconstructed $B^0 \rightarrow D K^+\pi^-$, $D \rightarrow K^0_S\pi^+\pi^-$ decays is comparable to the number of $B^0 \rightarrow D_{CP}^{-}\pi^+\pi^-$ decays, the model independent technique proposed here should enable a significant improvement in the sensitivity.

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FIG. 4: Accuracy of $\gamma$ measurement for the binned double Dalitz fit of $B^0 \rightarrow DK^+\pi^-$, $D \rightarrow K^0_S\pi^+\pi^-$ (option 6) using model C. (a) Residual distribution for the optimal binning, (b) residual distribution for the rectangular binning with $Q = 0.60$, (c) dependence of the accuracy on the value of $\gamma$ and comparison with the combined fit with $B^0 \rightarrow D_{CP}K^+\pi^-$ decays (option 8).

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