Modelling Bank Performance: A Novel Fuzzy Two-Stage DEA Approach

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ABSTRACT
Evaluating the banks’ performance has always been of interest due to their crucial role in the economic development of each country. Data envelopment analysis (DEA) has been widely used for measuring the performance of bank branches. In the conventional DEA approach, decision-making units (DMUs) are regarded as black boxes that transform sets of inputs into sets of outputs without considering the internal interactions taking place within each DMU. Two-stage DEA models are designed to overcome this shortfall. Thus this paper presented a new two-stage DEA model based on a modification on Enhanced Russell Model. On the other hand, in many situations, such as in a manufacturing system, a production process or a service system, inputs, intermediates and outputs can be given as a fuzzy variable. The main aim of this paper is to build and present a new fuzzy two-stage DEA model for measuring the efficiency of 15 branches of Melli bank in Hamedan province.

1. Introduction
Banks and financial and credit institutions play a very important role in the economic development of each country. Currently, due to the significant growth of the number of banks and financial and credit institutions in Iran and due to the privatisation process of state banks and the conversion of credit cooperatives and credit institutions to the bank, their performance evaluation has become very important. One of the well-known methods in assessing the performance of firms is data envelopment analysis that was developed by Charnes et al. [1] as CCR model, see [2–7] for more details. CCR model is a radial model. Radial models have some disadvantages like failure to recognise weak efficient DMUs [8, 9]. Another type of DEA models is non-radial DEA models [10]. One of the important non-radial DEA model is Enhanced Russell Model (ERM model) that is proposed by [11]. This model has some useful properties. One of them is ability to recognise the weak efficient DMUs. On the other hand, this model has some disadvantages like failure to rank efficient DMUs. Izadikhah et al. [12] proposed a modified version of ERM model that enables ERM model to rank efficient DMUs. Our proposed DEA methodology is an extension of their model that considers internal structures of DMUs. DEA has been widely used for assessing the performance of

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banks. Chortareas et al. [13] presented a good survey in this topic. Also, the readers are referred to [14] for a complete review for applications of DEA in industries.

Typically, a single stage production process is assumed to transform inputs to final outputs and is treated as a black box. In contrast to the black-box approach, real-world production systems often have network structure [15]. There is an increasing literature body that is devoted to efficiency assessment in multistage production processes. Castelli et al. [16] provided a comprehensive review of models and methods developed for different multi-stage production structures. In recent years, many researchers studied various DEA models for evaluating efficiencies of two-stage systems especially for evaluating efficiencies of banking systems. First, Seiford and Zhu [17] presented the first two-stage DEA model to evaluate the marketability and profitability of the U.S. commercial banks. For an in-depth review in multi-stage DEA model, see [9, 18–21]. In this study, we propose a new two-stage DEA model based on the modified ERM model. However, in many situations, such as in a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex so that it is difficult to measure them in an accurate way. Instead, the data can be given as a fuzzy variable. The concept of fuzzy theory was initialised in Zadeh [22]. After that many fuzzy approaches have been introduced in the DEA literature. Sengupta [23] applied principle of fuzzy set theory to introduce fuzziness in the objective function and the right-hand side vector of the conventional DEA model and developed the tolerance approach that was one of the first fuzzy DEA models. Many authors have combined DEA and Fuzzy modelling to study the efficiency of banking systems, see [24] and for a review on fuzzy DEA modelling, see [25–34]. Conventional DEA needs accurate measurement of inputs and outputs. However, the values of the input and output data in banking systems are sometimes imprecise or uncertain and since the quantity of some of our data in this paper was not known exactly the data was stated as fuzzy data. Thus we extend our proposed two-stage DEA model in fuzzy environment. Then we present a method for solving the proposed two-stage fuzzy DEA model based on the concept of alpha cut and possibility approach. Also for the purpose of final ranking and calculating the overall and stage efficiencies, this paper proposes a stochastic closeness coefficient. This coefficient is very useful and integrates all results of various values of \( \alpha \). The objective of this paper is to present a new two-stage fuzzy DEA model based on a modified version of enhanced Russell measure model to measure the performance of 15 branches of Melli bank in Hamedan province in the time period 2015–2016. In the proposed case study, the data are known as triangular fuzzy numbers.

The main contributions of this paper are as follows. This paper presents a new two-stage DEA model based on a recent modified version of ERM model. Also, this paper presents a new two-stage fuzzy DEA model based on the modified ERM. Our model uses the concept of \( \alpha \)-cut and possibility approach to defuzzification. Also for the purpose of calculating the overall and stage efficiencies, this paper proposes a stochastic closeness coefficient. This coefficient removes the difficulty of various ranking by various values of \( \alpha \). The proposed methodology applies to evaluate the efficiencies of 15 branches of Melli bank in Hamedan province.

This paper unfolds as follows: Section 2 briefly reviews the possibility approach. Section 3 proposes our new DEA methodology. In Section 4, a case study is presented and final conclusion is appeared in Section 5.
2. Preliminaries

Zadeh [22] presented the concept of possibility approach in terms of fuzzy set theory. Now, we review the definition of possibility space [22, 35].

**Definition 1:** (Possibility space) Let $\Theta$ be a nonempty set, and $P$ the power set of $\Theta$. Each element in $P$ is called an event. To present an axiomatic definition of possibility, it is necessary to assign to each event $A$, a number $\pi(A)$ which indicates the possibility that $A$ will occur. Then the triplet $(\Theta, p, \pi)$ is called a possibility space.

**Definition 2:** Let $A$ be a fuzzy variable defined on a possibility space $(\Theta, p, \pi)$. The membership of this variable introduced by Zadeh is as follows:

$$\mu_A(s) = \pi(\theta_i \in \Theta_i | A(\theta_i) = s) = \sup_{\theta_i \in \Theta_i} \{ \pi(\theta_i) | A(\theta_i) = s \}, \forall s \in R$$

**Definition 3:** Let $(\Theta, p, \pi)$ be a possibility space such that $\Theta = \Theta_1 \times \cdots \times \Theta_n$, therefore, for any set $A$ we have

$$\pi(A) = \sup_{\theta_i \in \Theta_i} \{ \pi_i(A_i) | A = A_1 \times \cdots \times A_n, A_i \in p \}$$

**Definition 4:** We denote an $\alpha -$ cut of fuzzy number $A$ by $A_\alpha$ which is defined as follows:

$$A_\alpha = \{ x | A(x) \geq \alpha \}$$

An $\alpha -$ cut of $A$ can be stated as $A_\alpha = [A_\alpha^L(\alpha), A_\alpha^U(\alpha)] = [[A]_\alpha^L, [A]_\alpha^U]$ for all $\alpha \in [0, 1]$.

Considering the fuzzy theory, there is a lemma that can be very useful to interpret the possibility function. Now, this lemma is represented.

**Lemma 1:** Let $A_1, \ldots , A_n$ be normal and convex fuzzy variables. Then, for any given possibility levels $\varepsilon_1, \varepsilon_2$ and $\varepsilon_3$ ($0 \leq \varepsilon_i \leq 1$) we have

(i): $\pi(A_1 + \ldots + A_n \leq a) \geq \varepsilon_1$ if and only if $[A_1]_{\varepsilon_1}^L + \ldots + [A_n]_{\varepsilon_1}^L \leq a$

(ii): $\pi(A_1 + \ldots + A_n \geq a) \geq \varepsilon_2$ if and only if $[A_1]_{\varepsilon_2}^U + \ldots + [A_n]_{\varepsilon_2}^U \geq a$

(iii): $\pi(A_1 + \ldots + A_n = a) \geq \varepsilon_3$ if and only if $[A_1]_{\varepsilon_3}^L + \ldots + [A_n]_{\varepsilon_3}^L \leq a$ and $[A_1]_{\varepsilon_3}^U + \ldots + [A_n]_{\varepsilon_3}^U \geq a$

where $[A_j]_{\varepsilon_j}^L$ and $[A_j]_{\varepsilon_j}^U$ are the lower and upper bounds of the $\varepsilon_j$-level set of $A_j$ ($j = 1, \ldots , n$). The above lemma is very useful to defuzzification of the fuzzy DEA model’s constraints.

3. Proposed Methodology

In this section, we first present our proposed two-stage DEA model, and then we bring our new fuzzy two-stage DEA model alongside the procedure for solving its program.
3.1. Proposed Two-Stage DEA Model

Let’s consider DMUs have an internal production structure. So, in this section we extend black-box production structure and performance measures to a two-stage production process. Here, assume that there are \( n \) DMUs \((j = 1, \ldots, n)\) consisting of two divisions and \( m_1 \) and \( s_1 \) are numbers of inputs and outputs of the first Division and \( m_2 \) and \( s_2 \) are numbers of inputs and outputs of the second Division, respectively. Also, \( x_{ij}^1 \) \((i = 1, \ldots, m_1)\) denotes input that is consumed by the first stage of DMU\(_j\) entirely, and \( y_{ir}^1 \) \((r = 1, \ldots, s_1)\) denotes output that is produced by first stage of DMU\(_j\) directly. And \( x_{ij}^2 \) \((i = 1, \ldots, m_2)\) denotes input that is consumed by the second stage of DMU\(_j\), entirely; \( y_{ir}^2 \) \((r = 1, \ldots, s_2)\) denotes output that is produced by second stage of DMU\(_j\), directly. \( z_{ij}^f \) \((f = 1, \ldots, F)\) shows intermediate products from the first division to the second division. We propose a new non-radial two-stage DEA model. Tone and Tsutsui [36] proposed a non-radial network DEA model. Their model evaluated the stage (divisional) efficiencies along with overall efficiency of decision making units. Izadikhah et al. [12] presented a modified version of Enhanced Rusek Measure of efficiency (ERM) model that could rank all DMUs. So, in this paper inspired from the idea of [36] and [12] we propose a new two-stage DEA model that carries the properties of two above models. Therefore, our proposed two-stage model under variable returns-to-scale (VRS) case is presented as follows:

\[
R_p^* = \min \frac{w_1 \left\{ \left( 1/m_1 \right) \sum_{j=1}^{m_1} \theta_j^1 \right\} + w_2 \left\{ \left( 1/m_2 \right) \sum_{j=1}^{m_2} \theta_j^2 \right\}}{w_1 \left\{ \left( 1/s_1 \right) \sum_{r=1}^{s_1} \varphi_r^1 \right\} + w_2 \left\{ \left( 1/s_2 \right) \sum_{r=1}^{s_2} \varphi_r^2 \right\}}
\]

s.t.
\[
\begin{align*}
\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 & \leq \theta_j^1 x_{ip}^1, \quad i = 1, \ldots, m_1; \\
\sum_{j=1}^{n} \lambda_j^2 x_{ij}^2 & \leq \theta_j^2 x_{ip}^2, \quad i = 1, \ldots, m_2; \\
\sum_{j=1}^{n} \lambda_j^1 y_{ir}^1 & \geq \varphi_r^1 y_{ip}^1, \quad r = 1, \ldots, s_1; \\
\sum_{j=1}^{n} \lambda_j^2 y_{ir}^2 & \geq \varphi_r^2 y_{ip}^2, \quad r = 1, \ldots, s_2; \\
\sum_{j=1}^{n} \lambda_j^1 z_{ij}^f & \leq \sum_{j=1}^{n} \lambda_j^2 z_{ij}^f, \quad f = 1, \ldots, F; \\
\sum_{j=1}^{n} \lambda_j^1 = 1; & \sum_{j=1}^{n} \lambda_j^2 = 1;
\end{align*}
\]

\[
\begin{align*}
\theta_j^1 - 1 & \leq M \delta_j^1; \quad i = 1, \ldots, m_1; \quad \theta_j^2 - 1 \leq M \delta_j^2; \quad i = 1, \ldots, m_2; \\
- \theta_j^1 + 1 & \leq M (1 - \delta_j^1); \quad i = 1, \ldots, m_1; \quad - \theta_j^2 + 1 \leq M (1 - \delta_j^2); \quad i = 1, \ldots, m_2; \\
\varphi_r^1 - 1 & \leq M \delta_r^1; \quad r = 1, \ldots, s_1; \quad \varphi_r^2 - 1 \leq M \delta_r^2; \quad r = 1, \ldots, s_2; \\
\varphi_r^1 - 1 & \leq M (1 - \delta_r^1); \quad r = 1, \ldots, s_1; \quad \varphi_r^2 - 1 \leq M (1 - \delta_r^2); \quad r = 1, \ldots, s_2; \\
\delta_j^1, \delta_j^2 & \in \{0, 1\}; \quad \theta_j^1, \theta_j^2, \lambda_j^1, \lambda_j^2 \geq 0; \quad \forall i; \quad \forall j
\end{align*}
\]
where $\lambda_1^j, \lambda_2^j (\forall j)$ are intensity variables for the first and second stages of production process. Moreover, $w_1$ and $w_2$ are weights addressing total preference over the two stages. When stages 1 and 2 have similar importance, $w_1$ and $w_2$ will be equal and they add up to 1.

In model (1), binary variables $\delta^1$ and $\delta^2$ guarantee that only one group of two groups of constrains is held:

\[
\begin{align*}
\text{(I)} : & \quad \theta^k \leq 1; \forall i; \forall k = 1, 2 \\
& \quad \psi^k \geq 1; \forall r; \forall k = 1, 2 \\
\text{or} \quad \text{(II)} : & \quad \theta^k \geq 1; \forall i; \forall k = 1, 2 \\
& \quad \psi^k \leq 1; \forall r; \forall k = 1, 2
\end{align*}
\]

If DMU is located inside production possibility set (PPS), thus constraints of group (I) will be active. If DMU is located outside PPS, thus constraints of group (II) will be active. This point enables our model to rank DMUs (see [12]). $R^*_p$ shows overall efficiency score for DMU$_p$. Based on this model, overall efficiency can be defined as follows.

**Definition 5**: (Overall efficiency): DMU$_p$ is said to be an overall efficient DMU if $P^*_p \geq 1$, otherwise it is inefficient.

As mentioned before, our model calculates overall efficiency scores and it is able to present a complete ranking that is difficulty of existing models. $P^*_p \geq 1$ implies that DMU under evaluation is overall efficient and shows rank of DMU. In order to check the performance of each stage, we need to define stage efficiency. However, we define stage efficiency as follows:

\[
\begin{align*}
R^1_p &= \frac{(1/m_1) \sum_{i=1}^{m_1} \theta^{1*}_i}{(1/s_1) \sum_{r=1}^{s_1} \psi^{1*}_r} \\
R^2_p &= \frac{(1/m_2) \sum_{i=1}^{m_2} \theta^{2*}_i}{(1/s_2) \sum_{r=1}^{s_2} \psi^{2*}_r}
\end{align*}
\]

where $\theta^{k*}_i$ and $\psi^{k*}_r$ are appeared in optimum solution of model (1). $R^k_p (k = 1, 2)$ shows $k$th-stage efficiency score for DMU$_p$ and based on this model stage efficiency can be defined as follows:

**Definition 6**: (Stage efficiency): DMU$_p$ is the $k$th-stage efficient DMU if $P^k_p \geq 1$, otherwise it is inefficient.

### 3.2. A New Fuzzy Two-Stage Modified ERM Model

The classic DEA models can only be used for cases where the data are precisely measured while in real-world situations, the observed values of the input and output data are sometimes inexact, incomplete, vague or ambiguous. These types of uncertainty data can be
represented as linguistic variables characterised by fuzzy numbers for reflecting a kind of general sense or experience of experts. The concept of fuzzy set theory was first developed by [22] to deal with the issue of uncertainty in systems modelling. Fuzzy DEA is a powerful tool for evaluating the performance of DMUs in uncertainty environments. In this section, we propose a new fuzzy DEA model for evaluating a set of DMUs with fuzzy inputs, intermediates and outputs. Hence, we extend model (1) to a fuzzy model.

3.2.1. Justification of the Fuzzy Model
Let the evaluation of efficiency of a homogeneous set of \( n \) DMUs (DMU\(_j\); \( j = 1, \ldots, n \)) is to be assessed where each DMU consists of two divisions. Also, \( \tilde{x}^1_{ij}, (i = 1, \ldots, m_1) \) denotes \( m_1 \) fuzzy inputs that are consumed by the first stage of DMU\(_j\) entirely, and \( y^1_{ir}, (r = 1, \ldots, s_1) \) denotes \( s_1 \) fuzzy outputs that are produced by first stage of DMU\(_j\) directly.

And, \( \tilde{x}^2_{ij}, (i = 1, \ldots, m_2) \) denotes \( m_2 \) fuzzy inputs that are consumed by the second stage of DMU\(_j\), entirely; \( y^2_{ir}, (r = 1, \ldots, s_2) \) denotes \( s_2 \) fuzzy outputs that are produced by second stage of DMU\(_j\), directly. \( \tilde{z}_{jf}, (f = 1, \ldots, F) \) shows \( F \) fuzzy intermediate products from the first division to the second division. The proposed fuzzy two-stage DEA model for calculating the overall efficiency of DMU\(_p\) is as follows:

\[
\begin{align*}
R^*_{p} = & \min \frac{w_1 \left\{ (1/m_1) \sum_{i=1}^{m_1} \theta^1_i \right\} + w_2 \left\{ (1/m_2) \sum_{i=1}^{m_2} \theta^2_i \right\}}{w_1 \left\{ (1/s_1) \sum_{r=1}^{s_1} \phi^1_r \right\} + w_2 \left\{ (1/s_2) \sum_{r=1}^{s_2} \phi^2_r \right\}} \\
\text{s.t.} & \\
& \sum_{j=1}^{n} \lambda^1_{ij} \tilde{x}^1_{ij} \leq \theta^1_i \tilde{x}^1_{ip}; i = 1, \ldots, m_1 \\
& j \neq p \\
& \sum_{j=1}^{n} \lambda^2_{ij} \tilde{x}^2_{ij} \leq \theta^2_i \tilde{x}^2_{ip}; i = 1, \ldots, m_2 \\
& j \neq p \\
& \sum_{j=1}^{n} \lambda^1_{ij} \tilde{y}^1_{ir} \geq \psi^1_r \tilde{y}^1_{ip}; r = 1, \ldots, s_1 \\
& j \neq p \\
& \sum_{j=1}^{n} \lambda^2_{ij} \tilde{y}^2_{ir} \geq \psi^2_r \tilde{y}^2_{ip}; r = 1, \ldots, s_2 \\
& j \neq p \\
& \sum_{j=1}^{n} \lambda^1_{ij} \tilde{z}^1_{jf} \leq \sum_{j=1}^{n} \lambda^2_{ij} \tilde{z}^1_{jf}; f = 1, \ldots, F \\
& j \neq p \\
& j \neq p
\end{align*}
\]
\[
\sum_{j=1, j \neq p}^{n} \lambda_j^1 = 1;
\]

\[
\sum_{j=1, j \neq p}^{n} \lambda_j^2 = 1;
\]

\[
\theta_1^i - 1 \leq M \delta_1^i; i = 1, \ldots, m_1;
\]

\[
- \theta_1^i + 1 \leq M(1 - \delta_1^i); i = 1, \ldots, m_1;
\]

\[
\theta_2^i - 1 \leq M \delta_2^i; i = 1, \ldots, m_2;
\]

\[
- \theta_2^i + 1 \leq M(1 - \delta_2^i); i = 1, \ldots, m_2;
\]

\[
\varphi_1^r + 1 \leq M \delta_1^r; r = 1, \ldots, s_1;
\]

\[
\varphi_2^r - 1 \leq M(1 - \delta_1^r); r = 1, \ldots, s_1;
\]

\[
\varphi_1^r + 1 \leq M \delta_2^r; r = 1, \ldots, s_2;
\]

\[
\varphi_2^r - 1 \leq M(1 - \delta_2^r); r = 1, \ldots, s_2;
\]

\[
\delta_1, \delta_2 \in \{0, 1\}
\]

\[
\theta_1^i, \varphi_r^i, \lambda_j^1, \theta_1^i, \varphi_r^i, \lambda_j^2 \geq 0; \forall i, r, j
\]

This model is a fuzzy version of model (1) that the fuzzy numbers are incorporated into the model (1). This fuzzy integrated DEA model cannot be solved like a crisp model. It is needed to design a procedure to solve that model.

### 3.2.2. Solving Procedure for the Proposed Fuzzy Model

As it is mentioned before the proposed fuzzy DEA model cannot be solved like a crisp model. So, in order to solve it one can apply a possibility approach formulated in terms of fuzzy set theory proposed by [22]. This procedure converts the fuzzy integrated DEA model to the standard linear programming (LP) by \( \alpha \)-cut technique. In this case, each fuzzy coefficient can be viewed as a fuzzy variable and each constraint can be considered as a fuzzy event, see [35]. Using possibility theory, possibilities of fuzzy events (i.e. fuzzy constraints) can be determined. Regarding the proposed model and the concept of possibility space of fuzzy event, some constrains are defined as a crisp value and other constrains are considered as an uncertain. For this reason by introducing the predetermined acceptable levels of possibility for constraints as \( \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \) and \( \varepsilon_5 \). Therefore the proposed model converted as follows:

\[
R_p^* = \min \left\{ \frac{w_1 \left\{ (1/m_1) \sum_{i=1}^{m_1} \varphi_1^i \right\} + w_2 \left\{ (1/m_2) \sum_{i=1}^{m_2} \Theta_1^i \right\}} {w_1 \left\{ (1/s_1) \sum_{r=1}^{s_1} \Phi_1^r \right\} + w_2 \left\{ (1/s_2) \sum_{r=1}^{s_2} \Phi_2^r \right\}} \right\}
\]

s.t.
\[
\pi \left( \sum_{j=1 \atop j \neq p}^{n} \lambda_j^1 x_j^1 - \theta_i^1 x_i^1 \leq 0 \right) \geq \varepsilon_1; i = 1, \ldots, m_1
\]
\[
\pi \left( \sum_{j=1 \atop j \neq p}^{n} \lambda_j^2 x_j^2 - \theta_i^2 x_i^2 \leq 0 \right) \geq \varepsilon_2; i = 1, \ldots, m_2
\]
\[
\pi \left( \sum_{j=1 \atop j \neq p}^{n} \lambda_j^1 y_j^1 - \varphi_i^1 y_i^1 \geq 0 \right) \geq \varepsilon_3; r = 1, \ldots, s_1
\]
\[
\pi \left( \sum_{j=1 \atop j \neq p}^{n} \lambda_j^2 y_j^2 - \varphi_i^2 y_i^2 \geq 0 \right) \geq \varepsilon_4; r = 1, \ldots, s_2
\]
\[
\pi \left( \sum_{j=1 \atop j \neq p}^{n} \lambda_j^1 z_j^1 - \sum_{j=1 \atop j \neq p}^{n} \lambda_j^2 z_j^2 \leq 0 \right) \geq \varepsilon_5; f = 1, \ldots, F
\]
\[
\sum_{j=1 \atop j \neq p}^{n} \lambda_j^1 = 1;
\]
\[
\sum_{j=1 \atop j \neq p}^{n} \lambda_j^2 = 1;
\]
\[
\theta_i^1 - 1 \leq M \delta^1; i = 1, \ldots, m_1;
\]
\[
- \theta_i^1 + 1 \leq M(1 - \delta^1); i = 1, \ldots, m_1;
\]
\[
\theta_i^2 - 1 \leq M \delta^2; i = 1, \ldots, m_2;
\]
\[
- \theta_i^2 + 1 \leq M(1 - \delta^2); i = 1, \ldots, m_2;
\]
\[
- \varphi_i^1 + 1 \leq M \delta^1; r = 1, \ldots, s_1;
\]
\[
\varphi_i^1 - 1 \leq M(1 - \delta^1); r = 1, \ldots, s_1;
\]
\[
- \varphi_i^2 + 1 \leq M \delta^2; r = 1, \ldots, s_2;
\]
\[ \phi_r^2 - 1 \leq M(1 - \delta^2); r = 1, \ldots, s_2; \]
\[ \delta^1, \delta^2 \in \{0, 1\} \]
\[ \theta_i^1, \phi_i^1, \lambda_j^1, \theta_i^2, \phi_i^2, \lambda_j^2 \geq 0; \forall i, r, j \quad (3) \]

In model (3), parameters \( \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \) and \( \varepsilon_5 \) are the predefined levels that the related constraints should attain the possibility level. According to Lemma 1, model (3) can be stated as follows:

\[ \rho_p^* = \min \frac{w_1 \{ (1/m_1) \sum_{i=1}^{m_1} \theta_i^1 \} + w_2 \{ (1/m_2) \sum_{i=1}^{m_2} \theta_i^2 \}}{w_1 \{ (1/s_1) \sum_{r=1}^{s_1} \phi_r^1 \} + w_2 \{ (1/s_2) \sum_{r=1}^{s_2} \phi_r^2 \}} \]

s.t.
\[ \left( \begin{array}{c}
\sum_{j=1}^{n} \lambda_j^1 x_{ij} - \theta_j^1 x_{ip} \\
(\delta_1^1) \\
\sum_{j=1}^{n} \lambda_j^2 x_{ij} - \theta_j^2 x_{ip} \\
(\delta_1^2) \\
\sum_{j=1}^{n} \lambda_j^1 y_{ij} - \phi_j^1 y_{ip} \\
(\delta_1^3) \\
\sum_{j=1}^{n} \lambda_j^2 y_{ij} - \phi_j^2 y_{ip} \\
(\delta_1^4) \\
\sum_{j=1}^{n} \lambda_j^1 z_{ij} - \sum_{j=1}^{n} \lambda_j^2 z_{ij} \\
(\delta_1^5)
\end{array} \right) \leq 0; i = 1, \ldots, m_1 \]
\[ \left( \begin{array}{c}
\sum_{j=1}^{n} \lambda_j^1 x_{ij} - \theta_j^1 x_{ip} \\
(\delta_1^1) \\
\sum_{j=1}^{n} \lambda_j^2 x_{ij} - \theta_j^2 x_{ip} \\
(\delta_1^2) \\
\sum_{j=1}^{n} \lambda_j^1 y_{ij} - \phi_j^1 y_{ip} \\
(\delta_1^3) \\
\sum_{j=1}^{n} \lambda_j^2 y_{ij} - \phi_j^2 y_{ip} \\
(\delta_1^4) \\
\sum_{j=1}^{n} \lambda_j^1 z_{ij} - \sum_{j=1}^{n} \lambda_j^2 z_{ij} \\
(\delta_1^5)
\end{array} \right) \leq 0; i = 1, \ldots, m_1 \]
\[ \left( \begin{array}{c}
\sum_{j=1}^{n} \lambda_j^1 x_{ij} - \theta_j^1 x_{ip} \\
(\delta_1^1) \\
\sum_{j=1}^{n} \lambda_j^2 x_{ij} - \theta_j^2 x_{ip} \\
(\delta_1^2) \\
\sum_{j=1}^{n} \lambda_j^1 y_{ij} - \phi_j^1 y_{ip} \\
(\delta_1^3) \\
\sum_{j=1}^{n} \lambda_j^2 y_{ij} - \phi_j^2 y_{ip} \\
(\delta_1^4) \\
\sum_{j=1}^{n} \lambda_j^1 z_{ij} - \sum_{j=1}^{n} \lambda_j^2 z_{ij} \\
(\delta_1^5)
\end{array} \right) \leq 0; i = 1, \ldots, m_1 \]
\[
\begin{align*}
\sum_{j=1}^{n} & \quad \lambda_j^1 = 1; \\
& \quad j \neq p \\
\sum_{j=1}^{n} & \quad \lambda_j^2 = 1; \\
& \quad j \neq p \\
\theta_i^1 & \quad -1 \leq M\delta^1; i = 1, \ldots, m_1; \\
& \quad -\theta_i^1 + 1 \leq M(1 - \delta^1); i = 1, \ldots, m_1; \\
\theta_i^2 & \quad -1 \leq M\delta^2; i = 1, \ldots, m_2; \\
& \quad -\theta_i^2 + 1 \leq M(1 - \delta^2); i = 1, \ldots, m_2; \\
\varphi_r^1 & \quad -1 \leq M\delta^1; r = 1, \ldots, s_1; \\
& \quad -\varphi_r^1 + 1 \leq M(1 - \delta^1); r = 1, \ldots, s_1; \\
\varphi_r^2 & \quad -1 \leq M\delta^2; r = 1, \ldots, s_2; \\
& \quad -\varphi_r^2 + 1 \leq M(1 - \delta^2); r = 1, \ldots, s_2; \\
\delta^1, \delta^2 & \quad \in \{0, 1\} \\
\theta_i^1, \varphi_r^1, \lambda_j^1, \theta_i^2, \varphi_r^2, \lambda_j^2 & \quad \geq 0; \forall i, r, j
\end{align*}
\] (4)

Consider in the proposed model, each fuzzy number is considered as a triangular fuzzy number. So, let \( \bar{x}_i^k = (x_{ij}^{kl}, x_{ij}^{km}, x_{ij}^{ku}) \) \((k = 1, 2)\) is a triangular fuzzy number of the \( i \)th input of DMU\(_j\) at the \( k \)th stage, \( \bar{y}_i^k = (y_{ij}^{kl}, y_{ij}^{km}, y_{ij}^{ku}) \) \((k = 1, 2)\) are the triangular fuzzy numbers of the \( r \)th output of DMU\(_j\) at the \( k \)th stage. Also, \( \bar{z}_i^f = (z_{ij}^f, z_{ij}^m, z_{ij}^u) \) is a triangular fuzzy number of the \( f \)th intermediate product of DMU\(_j\). Also, without loss of generality, let us assume that \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \alpha \). By these transformations, our model for evaluating DMU\(_p\) and measuring its overall efficiency becomes as follows:

\[
R_p^* = \min \left\{ \frac{w_1 \left\{ (1/m_1) \sum_{i=1}^{m_1} \theta_i^1 \right\} + w_2 \left\{ (1/m_2) \sum_{i=1}^{m_2} \theta_i^2 \right\}}{w_1 \left\{ (1/s_1) \sum_{r=1}^{s_1} \varphi_r^1 \right\} + w_2 \left\{ (1/s_2) \sum_{r=1}^{s_2} \varphi_r^2 \right\}} \right\}
\]

s.t.
\[
\sum_{j=1}^{n} \lambda_j^1 (x_{ij}^{1L} + \alpha(x_{ij}^{1M} - x_{ij}^{1L})) - \theta_i^1 (x_{ip}^{1L} + \alpha(x_{ip}^{1M} - x_{ip}^{1L})) \leq 0; i = 1, \ldots, m_1;
\]
\[
\sum_{j=1}^{n} \lambda_j^2 (x_{ij}^{2L} + \alpha(x_{ij}^{2M} - x_{ij}^{2L})) - \theta_i^2 (x_{ip}^{2L} + \alpha(x_{ip}^{2M} - x_{ip}^{2L})) \leq 0; i = 1, \ldots, m_2;
\]
\[
\sum_{j=1, j\neq p}^{n} \lambda_r^1 (y_{ij}^{1U} - \alpha (y_{ij}^{1U} - y_{ij}^{1M})) - \phi_r^1 (y_{ip}^{1U} - \alpha (y_{ip}^{1U} - y_{ip}^{1M})) \geq 0; r = 1, \ldots, s_1;
\]
\[
\sum_{j=1, j\neq p}^{n} \lambda_r^2 (y_{ij}^{2U} - \alpha (y_{ij}^{2U} - y_{ij}^{2M})) - \phi_r^2 (y_{ip}^{2U} - \alpha (y_{ip}^{2U} - y_{ip}^{2M})) \geq 0; r = 1, \ldots, s_2;
\]
\[
\sum_{j=1, j\neq p}^{n} \lambda_j^1 (z_{ij}^{f} + \alpha (z_{ij}^{M} - z_{ij}^{f})) - \sum_{j=1, j\neq p}^{n} \lambda_j^2 (z_{ij}^{f} + \alpha (z_{ij}^{M} - z_{ij}^{f})) \leq 0; f = 1, \ldots, F;
\]
\[
\sum_{j=1, j\neq p}^{n} \lambda_j^1 = 1;
\]
\[
\sum_{j=1, j\neq p}^{n} \lambda_j^2 = 1;
\]
\[
\theta_i^1 - 1 \leq M\delta^1; i = 1, \ldots, m_1;
\]
\[
\theta_i^2 - 1 \leq M\delta^2; i = 1, \ldots, m_2;
\]
\[
- \theta_i^1 + 1 \leq M(1 - \delta^1); i = 1, \ldots, m_1;
\]
\[
- \theta_i^2 + 1 \leq M(1 - \delta^2); i = 1, \ldots, m_2;
\]
\[
- \phi_r^1 + 1 \leq M\delta^1; r = 1, \ldots, s_1;
\]
\[
- \phi_r^2 + 1 \leq M\delta^2; r = 1, \ldots, s_2;
\]
\[
\phi_r^1 - 1 \leq M(1 - \delta^1); r = 1, \ldots, s_1;
\]
\[
\phi_r^2 - 1 \leq M(1 - \delta^2); r = 1, \ldots, s_2;
\]
\[
\delta^1, \delta^2 \in \{0, 1\},
\]
\[
\theta^1, \theta^2, \lambda^1_j, \lambda^2_j \geq 0; \forall i; \forall j
\] (5)

In addition, \(w_1\) and \(w_2\) are weights assigned to the efficiency scores in the first and second stages, respectively. Several approaches such as point allocation, paired comparisons, trade-off analysis and regression estimates can be used to specify the weights [37, 38]. Alternatively, pairwise comparisons and eigenvalue theory proposed by [39] can be used to determine suitable weights for efficiency scores of the two stages [9].

For each value of \(\alpha \in [0, 1]\) model (5) calculates the overall efficiency score for DMUP. This value is called \(\alpha\)-overall efficiency. Also, by using the optimal values of model (5), the \(\alpha\)-first stage efficiency and \(\alpha\)-second stage efficiency can be determined. For the purpose of integrating the obtained scores and ranking DMUs, we use the following criterion \(\psi_p\) for each DMUP and is called stochastic closeness coefficient. This criterion is inspired by the closeness coefficient of TOPSIS method. Assume that \(n+1\) different value for \(\alpha \in [0, 1]\) as \(\{\alpha_0, \alpha_1, \ldots, \alpha_n\}\) are applied to obtain the \(\alpha\)-efficiencies. We denoted the selected values for
\( \alpha \) by \( \Delta \), i.e. \( \Delta = \{\alpha_0, \alpha_1, \ldots, \alpha_n\} \). This criterion is measured as follows:

\[
\psi_p = \frac{\left( \frac{\sum_{\alpha \in \Delta} R^\alpha_p}{(n + 1)} - \min_{\alpha_d} \{R^\alpha_j\} \right)}{\left( \frac{\sum_{\alpha \in \Delta} R^\alpha_p}{(n + 1)} - \min_{\alpha_d} \{R^\alpha_j\} \right) + \left( \max_{\alpha_d} \{R^\alpha_j\} - \min_{\alpha_d} \{R^\alpha_j\} \right)}
\]

In fact, is the worst result of \( \alpha \)-efficiencies among all DMUs and under all considered values for \( \alpha \), so it is a kind of negative ideal value. On the other hand, the best result of \( \alpha \)-efficiencies among all DMUs is \( \max_{\alpha_d} \{R^\alpha_j\} \), so it is a kind of positive ideal value. The idea behind the criterion \( \psi_p \) is if the average obtained values for DMUp has the shortest distance from the positive ideal value and the farthest distance from the negative ideal value then DMUp should have the best ranking situation. The stochastic closeness coefficient is simply can be converted to the following relation:

\[
\psi_p = \frac{\left( \frac{\sum_{\alpha \in \Delta} R^\alpha_p}{(n + 1)} - \min_{\alpha_d} \{R^\alpha_j\} \right)}{\max_{\alpha_d} \{R^\alpha_j\} - \min_{\alpha_d} \{R^\alpha_j\}}
\]

Clearly, for each \( p \) we have \( 0 \leq \psi_p \leq 1 \). And thus we can rank DMUs according to decreasing order of the stochastic closeness coefficient of overall efficiency. By a same manner, we can obtain the stochastic closeness coefficient of stage efficiencies.

4. Case Study

Bank Melli Iran (BMI) is the first national Iranian bank. The bank was established in 1927 by the order of the Majlis (the Iranian Parliament) and since then has consistently been one of the most influential Iranian banks. Since 1933, BMI has grown to become a large retail bank with several domestic and international branches. BMI opened its first foreign branch in Hamburg, Germany, in 1965. BMI is now the largest commercial retail bank in Iran and in the Middle East with over 3300 branches and 43,000 employees. The aim of this paper is to evaluate 15 branches of Melli bank which are located in Hamedan province. The data are belonging to the period 2015–2016 and are obtained from a direct survey of the banks. As shown in Figure 1, the two-stage performance measurement system of branches of Melli bank is comprised of two stages. Stage 1 represents the profitability and Stage 2 represents marketability banking.

4.1. Data

The inputs to the first stage are: \((x_1^1)\) Branch costs, which consists of personnel costs, administrative costs and operating costs; \((x_1^2)\) Employee, consists of the number of employees by considering their education levels. The inputs to the second stage are: \((x_2^1)\) Staff, consists of the number of employees by considering their education levels; \((x_2^2)\) facilities, that consists of features such as queuing system, seating for customers, the new computer equipment, etc. The intermediate products are: \((z_1)\) Deposit, which consists of short-term investment, long-term investment, etc.; \((z_2)\) Loans, which consists of Loans presented, partnership loan, loans to buy housing, etc. The outputs of the first stage are: \((y_1^1)\) Documents, which consists
Figure 1. Comparison among all stochastic closeness coefficients.

Figure 2. Proposed two-stage structure of each Bank branch.

of Cash and transfer documents centralised and decentralised systems, and two-fifths of
the service bills in proportion to the size of their turnover. The outputs of the second stage
are: \( y^2 \) Net profit. The two-stage structure of the banking system is shown in Figure 2.

To deal with the uncertainty, in this study the inputs, intermediate and outputs are con-
sidered as fuzzy numbers. Historical data of 15 branches of Melli bank (DMUs) are reported
in Tables 1–3.

4.2. Results and Analysis

The results of solving model (5) can be seen in Tables 4–6. These tables show the overall
efficiency score and efficiency scores of the first and second stages, respectively. We ran
model (5) for some values of \( \alpha \), i.e. \( \alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\} \). In this
section, the efficiencies for every DMU are obtained by executing a GAMS program of model
(5) at different levels of \( \alpha \) for the time period 2015–2016. The model (5) is solved given
\( w_1 = 0.5 \) and \( w_2 = 0.5 \). According to Table 4, we can see DMU #12 is the only DMU that is
overall efficient in all values of \( \alpha \), i.e. only 7% of DMUs. Also, DMU #3 is overall efficient in
some values of \( \alpha \). Table 6 shows better performances in the first stage. Based on Table 5,
eight DMUs, i.e. 4, 5, 7, 8, 9, 12, 13, 14 are efficient in all values of \( \alpha \), i.e. 53% of DMUs. Table 6
| Bank branches (DMUs) | \( x_1^L \) | \( x_1^M \) | \( x_1^U \) | \( x_2^L \) | \( x_2^M \) | \( x_2^U \) | \( y_1^L \) | \( y_1^M \) | \( y_1^U \) |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1  | Emamzadeh_Abdollah | 587,233,292.05 | 605,395,146.00 | 708,312,321.05 | 2.88 | 3.35 | 3.62 | 196,329.94 | 202,402.00 | 242,882.40 |
| 2  | Shahrdari | 251,664,000.00 | 256,800,000.00 | 290,184,000.00 | 3.78 | 4.20 | 4.70 | 123,627.30 | 130,134.00 | 143,147.40 |
| 3  | Bolvar_Keshavarz | 449,309,747.87 | 493,746,975.70 | 592,496,370.84 | 2.93 | 3.12 | 3.68 | 133,654.00 | 157,240.00 | 183,970.80 |
| 4  | Aramgah | 130,819,001.64 | 137,704,212.26 | 154,228,717.32 | 3.82 | 3.86 | 4.28 | 310,575.72 | 316,914.00 | 367,620.24 |
| 5  | Bou_Ali | 154,216,400.00 | 164,060,000.00 | 176,341,200.00 | 3.09 | 3.47 | 3.57 | 324,154.60 | 334,180.00 | 374,281.60 |
| 6  | Dadgostari | 493,048,793.52 | 560,282,720.04 | 633,119,474.52 | 4.33 | 4.42 | 4.46 | 152,807.34 | 162,561.00 | 186,945.15 |
| 7  | Ghadir | 290,204,960.70 | 296,127,511.00 | 340,546,637.60 | 4.31 | 4.68 | 5.43 | 237,390.30 | 263,767.00 | 316,520.40 |
| 8  | Takhti | 241,238,613.56 | 256,636,822.48 | 279,734,136.16 | 3.07 | 3.37 | 3.77 | 205,789.80 | 236,540.00 | 255,463.20 |
| 9  | Aref | 191,883,200.00 | 223,120,000.00 | 227,582,400.00 | 4.09 | 4.35 | 5.05 | 216,897.12 | 246,474.00 | 276,050.88 |
| 10  | BabaTaher | 524,838,179.94 | 610,276,952.00 | 689,612,956.77 | 3.11 | 3.46 | 3.91 | 226,214.76 | 240,654.00 | 257,499.78 |
| 11  | Shariati | 117,693,775.26 | 118,882,601.28 | 124,826,713.40 | 3.69 | 4.15 | 4.86 | 190,521.82 | 221,537.00 | 234,829.22 |
| 12  | Pasdaran | 471,250,073.60 | 512,228,340.00 | 537,839,757.25 | 2.74 | 3.15 | 3.24 | 158,040.16 | 162,928.00 | 177,591.52 |
| 13  | Bazar | 335,177,234.10 | 394,326,158.00 | 469,248,128.34 | 3.33 | 3.47 | 3.68 | 271,714.25 | 286,015.00 | 311,756.35 |
| 14  | Meidan_Sepah | 427,304,456.48 | 445,108,808.80 | 476,266,425.16 | 3.09 | 3.12 | 3.43 | 191,279.70 | 212,533.00 | 235,911.63 |
| 15  | Meidan_Bar | 354,525,040.90 | 407,500,047.00 | 448,250,051.00 | 3.58 | 4.02 | 4.46 | 121,305.68 | 131,854.00 | 141,083.78 |
Table 2. Related fuzzy dataset for intermediate variables.

| Bank branches (DMUs) | $z_1$: Deposit | $z_1'$: Deposit | $z_2$: Loans | $z_2'$: Loans |
|---------------------|----------------|-----------------|--------------|---------------|
| 1 Emamzadeh_Abdollah| 763,730,55854.86 | 888,058,78901.00 | 923,581,14057.04 | 863,916,7893.93 |
| 2 Shahrdari         | 739,045,69755.88 | 803,310,44539.00 | 811,343,54984.39 | 103,469,14057.62 |
| 3 Bolvar_Keshavarz | 513,918,97277.20 | 540,967,33976.00 | 638,341,46091.68 | 236,794,05346.96 |
| 4 Aramgah          | 149,979,820000.00 | 159,553,000000.00 | 189,868,070000.00 | 228,456,50066.11 |
| 5 Bou_Ali          | 166,223,960000.00 | 176,834,000000.00 | 178,602,340000.00 | 433,988,01629.10 |
| 6 Dadgostari       | 664,619,13743.45 | 781,904,86757.00 | 899,190,59770.55 | 198,782,15904.00 |
| 7 Ghadir           | 572,891,98855.95 | 643,698,86355.00 | 733,816,70444.70 | 258,297,81923.52 |
| 8 Takhti           | 542,698,50241.88 | 596,371,98068.00 | 661,972,89855.48 | 177,380,96057.02 |
| 9 Aref             | 104,066,810000.00 | 116,929,000000.00 | 122,775,450000.00 | 118,236,55263.46 |
| 10 BabaTaher       | 541,970,35389.00 | 602,189,28210.00 | 620,254,96056.30 | 159,684,30884.25 |
| 11 Shariati        | 117,898,000000.00 | 124,104,000000.00 | 132,791,280000.00 | 285,411,50072.01 |
| 12 Pasdaran        | 610,039,24555.24 | 663,086,13647.00 | 789,072,50239.93 | 167,186,47678.82 |
| 13 Bazar           | 650,177,89072.29 | 699,116,01153.00 | 734,071,81210.65 | 376,290,00836.96 |
| 14 Meidan_Sepah    | 406,772,86305.87 | 467,555,01501.00 | 542,363,81741.16 | 177,016,13585.63 |
| 15 Meidan_Bar      | 259,089,02079.06 | 284,713,20966.00 | 298,948,87014.30 | 309,722,44542.68 |
Table 3. Related fuzzy dataset for bank’s Marketability (Stage 2).

| Bank branches (DMUs) | \( x_1^2 \): Staff | \( x_2^2 \): Facilities | \( y_1^2 \): Net profit |
|----------------------|-------------------|----------------------|-------------------|
|                      | \( x_1^2 \) | \( x_2^M \) | \( x_2^U \) | \( x_1^2 \) | \( x_1^{2M} \) | \( x_1^{2U} \) | \( x_2^2 \) | \( x_2^{2M} \) | \( x_2^{2U} \) |
| Emamzadeh Abdollah   | 5.90  | 6.56  | 7.22  | 2.85  | 3.10  | 3.63  | 77,143,114.89 | 88,670,247.00 | 104,630,891.46 |
| Shahrdari            | 6.83  | 6.90  | 8.28  | 4.02  | 4.10  | 5.82  | 122,400,000.00 | 136,000,000.00 | 159,120,000.00 |
| Bolvar Keshavarz     | 5.83  | 6.14  | 7.37  | 1.76  | 2.00  | 2.28  | 341,786,012.48 | 388,393,196.00 | 411,696,787.76 |
| Aragham              | 7.51  | 7.88  | 8.37  | 4.47  | 5.20  | 5.82  | 45,311,735.56 | 48,203,974.00 | 57,362,729.06 |

From Tables 4–6, we can see there are some DMUs which are efficient in the first stage but they are inefficient in the second stage. One reason for this issue is that since these DMUs are efficient in the first stage, they produce a large amount of output and these outputs are inputs for the second stage. Thus they consume a large amount of inputs for the second stage to produce outputs in the second stage. This point leads to decrease in their efficiency score in the second stage. Similar reason holds for inefficient DMUs in the first stage [9].

To better comparison, these results are provided in Figure 3. This figure shows the efficiency scores of bank branches at the different levels of \( \alpha \) in an integrated form. According to these tables and figure, it seems that the performance of DMUs in the first stage is relatively better than second stage. From Figure 3, it is clear that DMU #8, i.e. ‘Takhti’ has been recognised for its best performance in the first stage in different levels of \( \alpha \). (It has maximum
efficiency in different values of $\alpha$.) Also, we can see DMU #12, i.e. 'Pasdaran' has been recognised for its best performance in the second stage in different levels of $\alpha$ and because of big difference between its efficiency and others' efficiencies, this DMU has been recognised as the best overall performance (Figure 3(a)).

The values of the stochastic closeness coefficient for overall, first stage and second stage efficiencies alongside rankings are shown in Table 7. From Table 7, we can see DMU #12 (Pasdaran) and DMU #10 (BabaTaher) have the best and worst overall performances among all DMUs, respectively. A slight note to the last row of Table 7 shows that in average the performances of DMUs in the first stage are better than their performances in the second stages. This result had been stated from Figure 3, too.

From Table 7, we can conclude that 47% of branches have efficiency more than average in Stage 1 and that 40% of branches have efficiency more than average in Stage 2. The other notable result of Table 8 is to see there are eight bank branches that are recognised as the efficient branches in the first stage in all values of $\alpha$, which means there is no efficiency loss during operating. Therefore, the branches in profitability stage have compromise performances. Also, there are three bank branches that are recognised as the efficient branches in the second stage in all values of $\alpha$. Therefore, the branches in marketability stage do not
Figure 3. Various efficiency scores at different levels of $\alpha$. 

(a) Overall efficiency

(b) First stage efficiency

(c) Second stage efficiency
Table 7. Stochastic closeness coefficients (SCCs) and related rankings.

| Bank Branches (DMUs)          | Overall SCC | Rank | First Stage SCC | Rank | Second Stage SCC | Rank |
|-------------------------------|-------------|------|-----------------|------|------------------|------|
| Emamzadeh_Abdollah           | 0.049594108 | 8    | 0.279853        | 9    | 0.019074         | 8    |
| Shahrdari                    | 0.343314876 | 6    | 0.04775         | 14   | 0.232701         | 6    |
| Bolvar_Keshavarz             | 0.691044836 | 2    | 0.238543        | 10   | 0.559783         | 3    |
| Aramgah                      | 0.014743831 | 10   | 0.528295        | 4    | 0.003727         | 10   |
| Bou_Ali                      | 0.572543143 | 4    | 0.561103        | 3    | 0.248345         | 5    |
| Dadgostari                   | 0.037981829 | 9    | 0.202832        | 11   | 0.015715         | 9    |
| Ghadir                       | 0.186777593 | 7    | 0.429845        | 7    | 0.06973          | 7    |
| Takhti                       | 0.00100129  | 14   | 0.990262        | 1    | 2.83E-05         | 15   |
| Aref                         | 0.577925918 | 3    | 0.432654        | 6    | 0.254941         | 4    |
| Bazar                        | 3.36003E-05 | 15   | 0.188804        | 12   | 0.000519         | 14   |
| Shariati                     | 0.005510456 | 12   | 0.052741        | 13   | 0.002864         | 11   |
| Pasdaran                     | 0.942886135 | 1    | 0.320322        | 8    | 0.940106         | 1    |
| Bazar                        | 0.008406806 | 11   | 0.444189        | 5    | 0.00284          | 12   |
| Meidan_Sepah                 | 0.003138272 | 13   | 0.602678        | 2    | 0.00117          | 13   |
| Meidan_Bar                   | 0.506632708 | 5    | 0.004794        | 15   | 0.62765          | 2    |
| Average                      | 0.263622923 |      | 0.356047103     |      | 0.198613         |      |

Table 8. Comparison among fuzzy two-stage DEA models.

| Novel Network Model | Parametric/Non-Parametric | Efficiency Decomposition | integrated index | Applied in Banking Industry |
|---------------------|---------------------------|--------------------------|------------------|-----------------------------|
| Tavana et al. [40]  | ✓                         | Parametric               | ✓                | x                           | x                           |
| Shermeh et al. [41] | ✓                         | Parametric               | ✓                | x                           | x                           |
| Liu [42]            | ×                         | Parametric               | ✓                | x                           | x                           |
| Soltanzadeh and Omrani [43] | ✓               | Parametric               | x                | x                           | x                           |
| Simsek and Tüysüz [44] | ✓                     | Parametric               | x                | x                           | x                           |
| Hatami-Marbini and Saati [45] | ✓               | Parametric               | x                | x                           | x                           |
| Zhou et al. [46]    | ✓                         | Parametric               | x                | x                           | x                           |
| Tavana and Khalili-Damghani [24] | ✓               | Parametric               | ✓                | x                           | ✓                           |
| Tavana et al. [47]  | ✓                         | Non-Parametric           | ✓                | x                           | x                           |
| Proposed Model      | ✓                         | Non-Parametric           | ✓                | ✓                           | ✓                           |

have compromise performances. But, only there is one bank branch, which is ‘Pasdaran’ branch, that worked efficiently in both stage one and two in all values of $\alpha$, and therefore, is recognised as the only overall efficient branch among these 15 branches.

Refer to Figure 1 for an illustrative comparison among the results. Figure 1 illustrates the graphical representations of the stochastic closeness coefficient results using fuzzy input–intermediate-output data; and it is clear that the first stage efficiency score of DMU ‘Takhti’ is the highest value among all efficiency scores and the next place is for DMU ‘Pasdaran’ for its second stage efficiency score. Finally, the poor performance of DMU ‘BabaTaher’ in all situations is quite clear.

4.3. Comparison with Other Methods

By examining the literature, it can be seen that there are a number of articles on the evaluation of decision-making units with fuzzy two-stage DEA models. Table 8 provides a comparison among the proposed model and some of the existing fuzzy two-stage DEA models. For this comparison, some important criteria such as novelty in two-stage model,
According to Table 8, the method proposed in this paper has several advantages over the existing models. On the other hand, the method proposed in this study is more general compared with the existing models. The non-parametric nature of the proposed model is a very important feature that makes the evaluation and ranking not dependent on $\alpha$ values. Additionally, the proposed model provides an integrated index that helps decision maker to make robust decisions.

4.4. Sensitivity Analysis on Stages’ Weights

This study is done by considering the equal values for weights of stages. In this section, we check the importance of weight change and its effect on the final ranking of DMUs. For this purpose, we consider four extra cases for weights as Table 9. These four cases have been selected due to consider the large difference as well as the small difference between the weights.

The proposed model is run for all cases and the obtained rankings are illustrated in Table 10. The case of equal weights is called the main case in these tables. In order to check the relation among the results, we employ the Spearman’s correlation coefficient.

The Spearman’s correlation coefficient among the results of obtained rankings of the current case and the new extra four cases are calculated as the last row of Table 10. The results indicate that, there is a high and meaningful correlation between obtained results. This fact indicates the high similarity among the obtained rankings. This fact shows that

| Table 9. Extra cases for stages’ weights. |
|----------------------------------------|
| Case   | $w_1$ | $w_2$ |
|--------|-------|-------|
| Case 1 | 0.15  | 0.85  |
| Case 2 | 0.45  | 0.55  |
| Case 3 | 0.55  | 0.45  |
| Case 4 | 0.85  | 0.15  |
| Main Case | 0.5  | 0.5   |

| Table 10. The results of rankings for all cases. |
|-----------------------------------------------|
| No. | DMU             | Main Case | CASE 1 | CASE 2 | CASE 3 | CASE 4 | Sensitivity |
|-----|-----------------|-----------|-------|-------|-------|-------|-------------|
| 1   | Emamzadeh_Abdollah | 8         | 8     | 8     | 8     | 8     | 0.000       |
| 2   | Shahrdari       | 6         | 6     | 6     | 6     | 6     | 0.000       |
| 3   | Bolvar_Keshavarz | 2         | 3     | 5     | 5     | 4     | 1.304       |
| 4   | Arangh          | 10        | 10    | 10    | 10    | 10    | 0.000       |
| 5   | Bou_Ali         | 4         | 5     | 4     | 2     | 1     | 1.643       |
| 6   | Dadgostari      | 9         | 9     | 9     | 9     | 9     | 0.000       |
| 7   | Ghadir          | 7         | 7     | 7     | 7     | 5     | 0.894       |
| 8   | Takhiti         | 14        | 15    | 15    | 15    | 15    | 0.447       |
| 9   | Aref            | 3         | 4     | 2     | 3     | 2     | 0.837       |
| 10  | Baba_Taher      | 15        | 14    | 15    | 15    | 15    | 0.447       |
| 11  | Shariati        | 12        | 12    | 12    | 12    | 13    | 0.447       |
| 12  | Pasdaran        | 1         | 1     | 1     | 1     | 3     | 0.894       |
| 13  | Bazar           | 11        | 11    | 11    | 11    | 11    | 0.000       |
| 14  | Meidan_Sepah    | 13        | 13    | 13    | 13    | 12    | 0.447       |
| 15  | Meidan_Bar      | 5         | 2     | 3     | 4     | 7     | 1.924       |
| Spearman’s correlation coefficient | 0.975 | 0.975 | 0.975 | 0.95 |
the weight change has a low effect on the final ranking of DMUs. However, the rankings are not completely the same and there are some differences among the results. Last column of Table 10 shows the sensitivity of DMUs with respect to weight changes. We can see that five DMUs, i.e. \{1, 2, 4, 6 and 13\} are not sensitive to weight change and the other DMUs are very little sensitive to weight changes. DMU 15 is the most sensitive DMU among all.

4.5. Analysis and Further Discussions

Now, by neglecting the intermediate products, we consider the DMUs as black boxes and measure the sustainability of suppliers. It is shown that in such cases, using the conventional DEA models may lead to the biased results. For instance, the conventional DEA models cannot specify the inefficiency reasons in network structured DMUs [24, 37, 48, 49]. Also, there are several studies that show the deficiency of traditional DEA models (e.g. [50–54]).

Unlike the traditional DEA models, the two-stage DEA model can examine the structure and processes within DMUs. This helps managers to identify the inefficiency sources within DMUs [55–58]. To investigate the effects of intermediate products on DMUs’ performance, we assess the performance of branches of BMI assuming there is no intermediate measure. To this end, the Branch costs, Employee and Staff, Facilities are considered as inputs. The Documents and Net profit are considered as outputs. To compare the overall sustainability scores of the two-stage DEA model and the ‘black box’, the following fuzzy single-stage DEA model is formulated:

\[
\min \left( \frac{1}{m_1} \sum_{i=1}^{m_1} \theta_i^1 \right) \left( \frac{1}{s_1} \sum_{r=1}^{s_1} \varphi_r^1 \right)
\]

s.t.

\[
\sum_{j=1}^{n} \lambda_j^1 (x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L})) - \theta_i^1 (x_{ip}^{L} + \alpha(x_{ip}^{M} - x_{ip}^{L})) \leq 0; \quad i = 1, \ldots, m_1
\]

\[
\sum_{j=1}^{n} \lambda_j^1 (y_{ij}^{U} - \alpha(y_{ij}^{U} - y_{ij}^{M})) - \phi_r^1 (y_{rp}^{U} - \alpha(y_{rp}^{U} - y_{rp}^{M})) \geq 0; \quad r = 1, \ldots, s_1
\]

\[
\sum_{j=1}^{n} \lambda_j^1 = 1;
\]

\[
\theta_i^1 - 1 \leq M\delta_i^1; \quad i = 1, \ldots, m_1;
\]

\[
-\theta_i^1 + 1 \leq M(1 - \delta_i^1); \quad i = 1, \ldots, m_1;
\]

\[
-\phi_r^1 + 1 \leq M\delta_r^1; \quad r = 1, \ldots, s_1;
\]

\[
\varphi_r^1 - 1 \leq M(1 - \delta_r^1); \quad r = 1, \ldots, s_1;
\]
Table 11. Black-box results.

|   | Main   | Black Box |
|---|--------|-----------|
| 1 | 0.04959| 0.55989   |
| 2 | 0.34331| 0.10704   |
| 3 | 0.70385| 0.43743   |
| 4 | 0.01474| 0.71114   |
| 5 | 0.57254| 0.93810   |
| 6 | 0.03798| 0.02485   |
| 7 | 0.18678| 0.77245   |
| 8 | 0.00100| 0.84774   |
| 9 | 0.57793| 0.95568   |
| 10| 0.00003| 0.39495   |
| 11| 0.00551| 0.05241   |
| 12| 0.94289| 0.57438   |
| 13| 0.00841| 0.96168   |
| 14| 0.00314| 0.59329   |
| 15| 0.50663| 0.00959   |
|Average| 0.26362| 0.52938|

\[ \delta_1^1 \in \{0, 1\} \]

\[ \theta_i^1, \phi_r^1, \lambda_j^1 \geq 0; \forall i, r, j \]  

The results of stochastic closeness coefficient (6) are shown in Table 11. Based on Table 11, the average of obtained scores by model (7) is more than the fuzzy two-stage DEA model. This fact indicates that there are some inefficiencies that the single-stage model cannot recognise them.

The results of Table 11 show that the overall performance of DMUs, in their single stage form is very far from the results obtained by the proposed two-stage DEA model. The differences imply that the black-box DEA model cannot properly represent the overall performance of bank branches. The results show that if the intermediate factors are not involved in the evaluation process, there might be biased results.

4.6. Managerial Insights

Not only performance measurement in bank branches is essential but it also plays a critical role as an element of productive banking industry operations on strategic and operational levels. Mathematical models and analytic approaches are powerful tools in the performance measurement of bank branches and they enable managers to obtain helpful information to inform strategic and operational decisions. One of the popular and rigorous approaches used by managers to evaluate the efficiency in banking industry is DEA model. The findings of this study can also increase operations managers’ confidence in the right decision-making for performance measurement of bank branches.

Classical DEA models consider each DMU as a black box and do not care about internal structure of DMUs. Also, primary DEA models assume that data are known exactly. However, in real world, there might be stochastic data. Our proposed two-stage DEA model can evaluate the bank branches in presence of fuzzy data. Moreover, the feature of using two-stage structure can help managers to identify any inefficient resources in each stage of a banking operation and address these by making the right decisions. Another issue is that since
the initial investment in banking industries under uncertain conditions can be costly, time-consuming and risky, powerful performance measurement techniques, including the fuzzy two-stage DEA model presented in this study, can serve as appropriate decision support system tools.

In the case study section, the proposed model has been applied to evaluate the efficiency of 15 bank branches in Hamedan. According to the derived results, only one bank branch was recognised as efficient during the examined period 2014–2015 at all significant levels. This fact provides managers with information about which branches need to be actively developed so as to trigger innovation and growth. It also allows managers to identify productive investment and appropriate management activities. In the first stage, the sub-process of profitability measurement, eight branches were identified as efficient (at all significant levels). In the second stage, the sub-process of marketability measurement, three branches were recognised to perform efficiently (at all significant levels). From a statistical viewpoint, the efficiency of the first stage must be higher than that of the second stage. This indicates that the low efficiency scores obtained for the two-stage processes were mainly due to the low efficiency scores of the corresponding second stages, that is, the marketability efficiency values.

5. Conclusion

The assessment of the banks’ performance has always been of interest due to their crucial role in most economic activities and the maintenance of the health of monetary markets and economic conditions. Data envelopment analysis has been found to be a well-known methodology for measuring performance of bank branches. However, conventional DEA model makes difficult, if not impossible, to understand what are the sub-processes and the interactions causing the inefficiency of a DMU. In addition, in the banking environment, there is always a need for tools that allow one to uncover the inefficiencies that can affect different components of an operation. Moreover, many banking operations take the form of two-stage processes. In this paper, we considered a novel two-stage DEA model based on the modified version of ERM method. The aim of this study was evaluating the 15 branches of Melli bank in Hamedan province. After determining the input, intermediate and output variables, it is realised that generally they weren’t precisely known and as a result they couldn’t be considered in exact form. For this reason, the data was stated as fuzzy data. We used triangular fuzzy data to state the complexity of data. Therefore, we further extended our proposed two-stage DEA model to deal with fuzzy data. After that we presented a method for solving the proposed fuzzy DEA model based on the concept of alpha cut and possibility approach.

In the case study section, the proposed model was applied to evaluate the efficiency of 15 bank branches in Hamedan. The proposed fuzzy two-stage DEA model was coded using GAMS 23.6 software and was conducted by considering some different values of $\alpha$. The obtained values were integrated by using the proposed stochastic closeness coefficient. Based on the results of the proposed stochastic closeness coefficient the best and worst performances were determined. According to the derived results, 14 out of the 15 bank branches analysed turned out to be overall inefficient. Only one bank branch was recognised as efficient during the examined period 2014–2015 at all significant levels.
We conclude with a few possible research directions towards which to extend the results of this study. Our approach could be extended to consider other kinds of data such as dual-role data, stochastic data and so on. In this paper, we applied our proposed model for evaluating the performance of bank branches. It seems that our proposed model can be used in other problems such as evaluating the sustainability of suppliers, regional R&D processing, evaluating non-life insurance companies, efficiency evaluation of production lines, efficiency evaluation of hospitals which have many wards interacting with each other and have network structure, and so on. Developing a fuzzy dynamic two-stage DEA model will be another interesting research topic. The proposed model may be extended to cases where the intermediate products could be lost or added from external sources.

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No potential conflict of interest was reported by the author(s).

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References
[1] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. Eur J Oper Res. 1978;2(6):429–444.
[2] Aggelopoulos E, Georgopoulos A. Bank branch efficiency under environmental change: A bootstrap DEA on monthly profit and loss accounting statements of Greek retail branches. Eur J Oper Res. 2017;261(3):1170–1188.
[3] Degl’Innocenti M, Kourtzidis SA, Sevic Z, et al. Bank productivity growth and convergence in the European union during the financial crisis. J Bank Financ. 2017;75:184–199.
[4] San-Jose L, Retolaza JL, Torres Pruñonosa J. Efficiency in Spanish banking: a multistakeholder approach analysis. J Int Financial Markets Institutions Money. 2014;32:240–255.
[5] Stewart C, Matousek R, Nguyen TN. Efficiency in the Vietnamese banking system: A DEA double bootstrap approach. Res Int Business Finance. 2016;36:96–111.
[6] Zhou Z, Lin L, Xiao H, et al. Stochastic network DEA models for two-stage systems under the centralized control organization mechanism. Comput Ind Eng. 2017;110:404–412.
[7] Atwood J, Shaik S. Theory and statistical properties of quantile data envelopment analysis. Eur J Oper Res. 2020;286(2):649–661.
[8] Izadikhah M, Farzipoor Saen R. A new preference voting method for sustainable location planning using geographic information system and data envelopment analysis. J Clean Prod. 2016;137:1347–1367.
[9] Izadikhah M, Farzipoor Saen R. Evaluating sustainability of supply chains by two-stage range directional measure in the presence of negative data. Transport Res D Transport Environ. 2016;49:110–126.
[10] Tone K, Toloo M, Izadikhah M. A modified slacks-based measure of efficiency in data envelopment analysis. Eur J Oper Res. 2020;287:560–571.
[11] Pastor JT, Ruiz JL, Sirvent I. An enhanced DEA Russell graph efficiency measure. Eur J Oper Res. 1999;115(1):596–607.

[12] Izadikhah M, Farzipoor Saen R, Ahmadi K. How to assess sustainability of suppliers in the presence of dual-role factor and volume discounts? A data envelopment analysis approach. Asia-Pac J Oper Res. 2017;34(3):1740016.

[13] Chortareas G, Kapetanios G, Ventouri A. Credit market freedom and cost efficiency in US state banking. J Empirical Finance. 2016;37:173–185.

[14] Emrouznejad A, Yang G-L. A survey and analysis of the first 40 years of scholarly literature in DEA: 1978-2016. Socioecon Plann Sci. 2018;61(1):1–5.

[15] Henriques IC, Sobreiro VA, Kimura H, et al. Two-stage DEA in banks: terminological controversies and future directions. Expert Syst Appl. 2020;161:113632.

[16] Castelli L, Pesenti R, Ukovich W. A classification of DEA models when the internal structure of the decision making units is considered. Ann Oper Res. 2010;173(1):207–235.

[17] Seiford LM, Zhu J. Profitability and marketability of the top 55 U. S. Commercial Banks. Manage Sci. 1999;45(9):1270–1288.

[18] Castelli L, Pesenti R. Network, shared flow and multi-level DEA models: a critical review. In: Cook W, Zhu J, editors. Data envelopment analysis. Vol. 208. International Series in Operations Research & Management Science. Boston (MA): Springer; 2014. p. 329–376. doi:10.1007/978-1-4899-8068-7_15

[19] Li Y, Cui Q. Carbon neutral growth from 2020 strategy and airline environmental inefficiency: A network range adjusted environmental data envelopment analysis. Appl Energy. 2017;199:13–24.

[20] Mahdiloo M, Jafarzadeh AH, Saen RF, et al. A multiple criteria approach to two-stage data envelopment analysis. Transport Res D Transport Environ. 2016;46:317–327.

[21] Toloo M, Emrouznejad A, Moreno P. A linear relational DEA model to evaluate two-stage processes with shared inputs. Comp Appl Math. 2015;36:1–17.

[22] Zadeh LA. Fuzzy sets. Inf Control. 1965;8(3):338–353.

[23] Sengupta JK. A fuzzy systems approach in data envelopment analysis. Comput Math Appl. 1992;24(8–9):259–266.

[24] Tavana M, Khalili-Damghani K. A new two-stage Stackelberg fuzzy data envelopment analysis model. Measurement (Mahwah N J). 2014;53(0):277–296.

[25] Hatami-Marbini A, Agrell P, Tavana M, et al. A flexible cross-efficiency fuzzy data envelopment analysis model for sustainable sourcing. J Clean Prod. 2016.

[26] Hatami-Marbini A, Ebrahimnejad A, Lozano S. Fuzzy efficiency measures in data envelopment analysis using lexicographic multiobjective approach. Comput Ind Eng. 2017;105:362–376.

[27] Izadikhah M, Farzipoor Saen R, Ahmadi K. How to assess sustainability of suppliers in volume discount context? A new data envelopment analysis approach. Transport Res D Transport Environ. 2017;51:102–121.

[28] Khalili-Damghani K, Tavana M, Santos-Arteaga FJ. A comprehensive fuzzy DEA model for emerging market assessment and selection decisions. Appl Soft Comput. 2016;38:676–702.

[29] Mashayekhi Z, Omrani H. An integrated multi-objective Markowitz–DEA cross-efficiency model with fuzzy returns for portfolio selection problem. Appl Soft Comput. 2016;38:1–9.

[30] Wanke P, Barros CP, Nwaogbe OR. Assessing productive efficiency in Nigerian airports using fuzzy-DEA. Transp Policy (Oxf). 2016;49:9–19.

[31] Zerafat Angiz LM, Mustafa A, Ghadiri M, et al. Relationship between efficiency in the traditional data envelopment analysis and possibility sets. Comput Ind Eng. 2015;81:140–146.

[32] Izadikhah M, Farzipoor Saen R, Ahmadi K, et al. How to use fuzzy screening system and data envelopment analysis for clustering sustainable suppliers? A case study in Iran. J Enterp Inf Manag. 2020.

[33] Mohtashami A, Ghiasvand BM. Z-ERM DEA integrated approach for evaluation of banks & financial institutes in stock exchange. Expert Syst Appl. 2020;147:113218.
[34] Marins FAS, da Silva AF, Miranda R, et al. A new approach using fuzzy DEA models to reduce search space and eliminate replications in simulation optimization problems. Expert Syst Appl. 2020;144:113137.

[35] Azadi M, Jafarian M, Farzipoor Saen R, et al. A new fuzzy DEA model for evaluation of efficiency and effectiveness of suppliers in sustainable supply chain management context. Comput Oper Res. 2015;54:274–285.

[36] Tone K, Tsutsui M. Network DEA: A slacks-based measure approach. Eur J Oper Res. 2009;197:243–252.

[37] Ebrahimnejad A, Tavana M, Lotfi FH, et al. A three-stage data envelopment analysis model with application to banking industry. Measurement (Mahwah NJ). 2014;49(0):308–319.

[38] Kleindorfer PR, Kuneuther HC, Schoemaker PJH. Decision sciences: an integrative perspective. New York: Cambridge University Press; 1993.

[39] Saaty TL. Rank from comparisons and from ratings in the analytic hierarchy/network processes. Eur J Oper Res. 2006;168(2):557–570.

[40] Tavana M, Khalili-Damghani K, Santos Arteaga FJ, et al. Efficiency decomposition and measurement in two-stage fuzzy DEA models using a bargaining game approach. Comput Ind Eng. 2018;118:394–408.

[41] Shermeh HE, Najafi SE, Alavidoost MH. A novel fuzzy network SBM model for data envelopment analysis: A case study in Iran regional power companies. Energy. 2016;112:686–697.

[42] Liu S-T. Restricting weight flexibility in fuzzy two-stage DEA. Comput Ind Eng. 2014;74:149–160.

[43] Soltanzadeh E, Omrani H. Dynamic network data envelopment analysis model with fuzzy inputs and outputs: An application for Iranian Airlines. Appl Soft Comput. 2018;63:268–288.

[44] Simsek B, Tüysüz F. An application of network data envelopment analysis with fuzzy data for the performance evaluation in cargo sector. J Enterp Inf Manag. 2018;31(4):492–509.

[45] Hatami-Marbini A, Saati S. Efficiency evaluation in two-stage data envelopment analysis under a fuzzy environment: A common-weights approach. Appl Soft Comput. 2018;72:156–165.

[46] Zhou X, Wang Y, Chai J, et al. Sustainable supply chain evaluation: A dynamic double frontier network DEA model with interval type-2 fuzzy data. Inf Sci (Ny). 2019;504:394–421.

[47] Tavana M, Khalili-Damghani K, Santos Arteaga FJ, et al. A fuzzy multi-objective multi-period network DEA model for efficiency measurement in oil refineries. Comput Ind Eng. 2019;135:143–155.

[48] Sexton T, Lewis H. Two-Stage DEA: An application to Major League Baseball. J Product Anal. 2003;19(2-3):227–249.

[49] Avkiran NK. An illustration of dynamic network DEA in commercial banking including robustness tests. Omega (Westport). 2015;55:141–150.

[50] Chen Y, Liang I, Yong F. A DEA game model approach to supply chain efficiency. Ann Oper Res. 2006;145:5–13.

[51] Kao C, Hwang S-N. Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. Eur J Oper Res. 2008;185(1):418–429.

[52] Kao C, Hwang S. Efficiency measurement for network systems: IT impact on firm performance. Decis Support Syst. 2010;48:437–446.

[53] Izadikhah M, Tavana M, Di Caprio D, et al. A novel two-stage DEA production model with freely distributed initial inputs and shared intermediate outputs. Expert Syst Appl. 2018;99:213–230.

[54] Tavana M, Izadikhah M, Di Caprio D, et al. A new dynamic range directional measure for two-stage data envelopment analysis models with negative data. Comput Ind Eng. 2018;115:427–448.

[55] Färe R, Grosskopf S. Network DEA. Socioecon Plann Sci. 2000;34(1):35–49.

[56] Li Y, Chen Y, Liang L, et al. DEA models for extended two-stage network structures. Omega (Westport). 2012;40(5):611–618.

[57] Souza GS, Staub RB. Two-stage inference using data envelopment analysis efficiency measurements in univariate production models. Int Trans Oper Res. 2007;14(3):245–258.

[58] Liu S-T. Fuzzy efficiency ranking in fuzzy two-stage data envelopment analysis. Optim Lett. 2014;8(2):633–652.