The role of chiral symmetry in two-pion exchange nuclear potential

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We evaluate the two pion exchange contribution to the nucleon-nucleon potential in configuration space using a minimal chiral model containing only pions and nucleons. We argue that this model has nowadays a rather firm conceptual basis, which entitles it to become a standard ingredient of any modern potential. The main features of this model is that the scalar-isoscalar component of the interaction is relatively small, due to cancellations between large terms, and fails to reproduce the intermediate range attraction in the central channel. We show that chiral symmetry is the responsible for these large cancellations in the two-pion exchange nucleon-nucleon interaction, which are similar to those occurring in free pion-nucleon scattering. Another feature of the model is that these results do not depend on how chiral symmetry is implemented.

I. INTRODUCTION

The long range part of the nucleon-nucleon (NN) interaction is ascribed to the exchange of one pion (OPEP) and is very well known. The medium range region, on the other hand, is much more controversial, since in the literature one finds various competing theoretical approaches, as dispersion relations, field theory or just phenomenology. In all cases, the medium range interaction is associated with the exchange of two pions (TPEP), whereas there is a wide variation in the way short distance effects are treated. In order to reproduce the data, parameters are used which either reflect knowledge about other physical processes or are adjusted ad hoc. The differences in all these approaches are enhanced in the isospin-symmetric nuclear matter, where the OPEP vanishes and one must know the asymptotic behavior of the TPEP.

The role of the TPEP in the framework of chiral symmetry has recently attracted considerable attention, especially as far as the restricted pion-nucleon sector is concerned. It is well known that this process is closely related to the pion-nucleon scattering amplitude.

Chiral symmetry at hadron level may be implemented by means of either linear or non-linear Lagrangians. The fact that no serious candidate has been found for the σ meson favours the latter type of approach. There are two forms of non-linear Lagrangians which are especially suited for the πN system. One of them is based on a pseudoscalar (PS) πN coupling supplemented by a scalar (S) interaction, equivalent to the exchange of an infinitely massive σ meson, and denoted as PS+S scheme (Eq. 1). The other one employs a pseudovector (PV) πN interaction and a vector (V) term, which could represent the exchange of an infinitely massive ρ meson, constituting the PV+V scheme (Eq. 2).

Both approaches yield the very same amplitude for the σN scattering for the isospin-symmetric amplitude. The fact that physical results should be independent of the representation used to implement chiral symmetry was discussed in very general terms by Coleman, Wess, and Zumino. As far as nucleon-nucleon scattering is concerned, the two-pion exchange amplitude up to order $1/f^4_{\pi}$ is given by five diagrams, usually named box (□), crossed box
We consider the adimensional variable $\mu$, respectively. The first two diagrams contain only nucleon propagators and are independent of chiral symmetry, whereas the triangles and the bubble involve the scalar interaction and hence are due to the symmetry. When one considers the potential instead of the amplitude, the iterated OPEP has to be subtracted from the box diagram.

II. PARAMETRIZATION OF THE TPEP

Our calculation of the TPEP is based on the Blankenbecler-Sugar reduction of the Bethe-Salpeter equation. Its dynamical content is associated with the five diagrams displayed in Fig. 1. Therefore we label the corresponding individual contributions by $\cd$, $\triangle$, $\square$ and $\Box$, where the last one also includes the subtraction of the iterated OPEP. It has the general form

$$V(r) = \left[ \left( V^C_0 + V^C_\Delta \right) + \hat{O}_{LS} \left( V^{LS}_0 + V^{LS}_\Delta \right) \right] + \left[ 3 + 2 \tau(1) \cdot \tau(2) \right] \left[ V^C_\Pi + \hat{O}_{SS} V^{SS}_\Pi + \hat{O}_{LS} V^{LS}_\Pi + \hat{O}_T V^T_\Pi \right]$$

where the spin operators are given by $\hat{O}_{SS} = \sigma(1) \cdot \sigma(2)$, $\hat{O}_{LS} = L \cdot \{ \sigma(1) + \sigma(2) \}$, and $\hat{O}_T = 3 \sigma(1) \cdot \hat{\tau}(2) \cdot \hat{\tau} - \sigma(1) \cdot \sigma(2)$, whereas $\sigma(i)$ and $\tau(i)$ represent spin and isospin matrices for nucleon $(i)$.

In the case of the bubble diagram, the leading contribution to the asymptotic potential can be calculated analytically $\Box$; its result sets the pattern for the parametrization of the other components of the force.

Our numerical expressions represent the various components of the potential in MeV, and are given in terms of the adimensional variable $x \equiv \mu r$, where $\mu$ is the pion mass. We keep the axial $\pi N$ coupling constant $g_A$ as a free parameter and adopt the values $\mu = 137.29$ MeV and $f_\pi = 93$ MeV for the pion mass and the pion decay constant respectively.

In general, the parametrized expressions reproduce quite well the numerical results for the complete theoretical calculations, given in Ref. $\Box$, except for a few cases and regions where the discrepancies become of the order of 0.25%. Our results are listed below.

A. Central Potential

The profile function for the central potential has the following common multiplicative expression

$$F_C(x) = \left( \frac{g_A \mu}{2 f_\pi} \right)^4 \frac{e^{-2x}}{x^2 \sqrt{x}}.$$  

The parametrization of each diagram gives:

$$V^C_0(x) = F_C(x) \left\{ -275.364 - \frac{51.0923}{x} + \frac{6.54068}{x^2} - \frac{1.26190}{x^3} + \frac{0.130706}{x^4} \right\}$$  

$$V^C_\Delta(x) = F_C(x) \left\{ 343.558 - \frac{14.0446}{x} + \left( 135.249 + 14.6514 \cdot x + 6.43825 \cdot x^2 \right) \cdot e^{-0.397835 \cdot x} \right\}$$  

$$V^C_\Pi(x) = \frac{V^C_\Pi(x)}{12} + F_C(x) \left\{ - \frac{265.304}{\sqrt{x}} + \frac{518.531}{x} - \frac{577.210}{x \sqrt{x}} + \frac{378.004}{x^2} - \frac{133.374}{x^2 \sqrt{x}} + \frac{19.5061}{x^3} \right\}$$  

FIG. 1. Loop diagrams for the two pion exchange NN potential calculated in the minimal chiral model.
\[ V_C^C(x) = F_c(x) \left\{ -25.9987 + \frac{8.33777}{x} - \frac{0.870724}{x^2} \right\} \cdot e^{-[0.101214 \cdot x + 0.00123687 \cdot x^2]} \]  

(8)

**B. Spin-spin potential**

The multiplicative factor for the spin-spin potential is the same as that of the central potential, and receives contributions from the box and crossed diagrams only, which are given by

\[ V_{\Omega}^{SS}(x) = F_c(x) \left\{ 0.40804 + \frac{1.05042}{x} + \frac{0.421043}{x^2} - \frac{0.0284309}{x^3} - 0.215829 \cdot e^{-1.2344 \cdot x} \right\} \]  

(9)

\[ V_{\Delta}^{SS}(x) = F_c(x) \left\{ 0.399845 + \frac{1.07191}{x} + \frac{0.216302}{x^2} - \frac{0.0306271}{x^3} + 0.0371333 \cdot x \cdot e^{-0.16808 \cdot x} \right\} \]  

(10)

**C. Spin-orbit Potential**

The spin-orbit multiplicative function is

\[ F_{LS}(x) = \left( \frac{g_A \mu}{2 f_\pi} \right)^4 \left( 1 + \frac{1}{2x} \right) \frac{e^{-2x}}{x^3 \sqrt{x}}, \]  

and individual contributions are:

\[ V_0^{LS}(x) = F_{LS}(x) \left\{ -5.88744 - \frac{5.51078}{x} + \frac{0.994157}{x^2} - \frac{0.217562}{x^3} + \frac{0.0336067}{x^4} - \frac{0.00244620}{x^5} \right\} \]  

(12)

\[ V_{\Delta}^{LS}(x) = F_{LS}(x) \left\{ 7.34548 + \frac{2.15233}{x} - \frac{0.381025}{x^2} + \left( 7.48798 - 0.448484 \cdot x + 0.391431 \cdot x^2 \right) \cdot e^{-0.419984 \cdot x} \right\} \]  

(13)

\[ V_{\Omega}^{LS}(x) = -\frac{V_{\Delta}^{LS}(x)}{4} + F_{LS}(x) \left\{ \frac{5.67235}{\sqrt{x}} - \frac{10.6417}{x} + \frac{14.7389}{x \sqrt{x}} - \frac{12.6506}{x^2} + \frac{6.27374}{x^2 \sqrt{x}} - \frac{1.66637}{x^3} + \frac{0.184264}{x^3 \sqrt{x}} \right\} \]  

(14)

\[ V_{\square}^{LS}(x) = F_{LS}(x) \left\{ -1.19527 - \frac{1.58089}{x} + \frac{0.319790}{x^2} - \frac{0.0196461}{x^3} \right\} \cdot [1 - 0.0160259 \cdot x]^{-1} \]  

(15)

**D. Tensor Potential**

The common factor for the tensor potential is

\[ F_T(x) = \left( \frac{g_A \mu}{2 f_\pi} \right)^4 \left( 1 + \frac{3}{2x} + \frac{3}{4x^2} \right) \frac{e^{-2x}}{x^2 \sqrt{x}}. \]  

(16)

It receives contributions from the box and crossed diagrams only, which have the form

\[ V_0^{T}(x) = F_T(x) \left\{ -0.204041 - \frac{0.510720}{x} + \frac{0.0597556}{x^2} + \left( 1.32932 - 0.939553 \cdot x + 0.706050 \cdot x^2 \right) \cdot e^{-2.29686 \cdot x} \right\} \]  

(17)

\[ V_{\square}^{T}(x) = F_T(x) \left\{ -0.246349 - \frac{0.521123}{x} + \frac{0.352463}{x^2} - \frac{0.135028}{x^3} + \frac{0.0303012}{x^4} - \frac{0.00297840}{x^5} \right\} \]  

(18)
FIG. 2. Profile functions for the bubble (○) and triangle (▽) scalar-isoscalar potentials and for their sum (S), showing a strong cancellation between these two contributions. The graph at right is an amplification.

III. RESULTS

Using the minimal chiral model potential described in last section we evaluated the scalar-isoscalar component of the TPEP and the phase shifts for some singlet waves. We adopted $g_A = 1.33$ [6]. It is possible to notice two important cancellations within the scalar-isoscalar sector of the $\pi\pi$E-NNP. The first of them happens between the triangle and bubble contributions, as shown in Fig. 2. The other one occurs when the remainder from the previous cancellation (S) is added to the sum of the box and crossed box diagrams (PS). In this last case, the direct inspection of the profile functions for the potential, given in Fig. 3, provides just a rough estimate of the importance of the cancellation, since the iterated OPEP is not included there.

The second cancellation can be better studied in the $NN$ scattering problem, since the amplitudes obtained by solving a dynamical equation include automatically the iterated OPEP and depend very little on the way the potential is defined. In our derivation of the $NN$ potential we used a Lagrangian which did not contain these contact terms and hence it is suited for medium and long distances. Thus, in order to avoid these undetermined short range effects, we consider only the $^1D_2$, $^1G_4$, $^1F_3$, and $^1H_5$ waves. For each channel, we decompose the full $NN$ potential $V$ as

$$V = U_\pi + U_{PS} + U_S + U_C,$$

where $U_\pi$ is the OPEP, $U_C$ represents the short ranged core contributions, $U_{PS}$ is due to the box and crossed box diagrams whereas $U_S$ is associated with the chiral triangle and bubble interactions. Using the variable phase method, it is possible to write the phase shift for angular momentum $\ell$ as [11,12]

$$\delta_\ell = -\frac{m}{k} \int_0^\infty dr V P^2_\ell.$$

In this expression, the structure function $P_\ell$ is given by

$$P_\ell = j_\ell \cos D_\ell - \hat{n}_\ell \sin D_\ell,$$

where $j_\ell$ and $\hat{n}_\ell$ are the usual Bessel and Neumann functions multiplied by their arguments and $D_\ell$ is the variable phase. Using the decomposition of the potential given in Eq. 19, one writes the perturbative result
\[
\delta_\ell = -\frac{m}{k} \int_0^\infty dr \left\{ U_{\pi} \frac{\partial^2}{\partial r^2} \right. \\
+ \left[ U_{\pi} \left( P_\ell^2 - j_\ell^2 \right) + U_{PS} P_\ell^2 + U_{S} P_\ell^2 \right] + U_{C} P_\ell^2 \bigg\} \\
\equiv \delta_\ell^{\pi L} + [\delta_\ell^{\pi I} + \delta_\ell^{PS} + \delta_\ell^{S}] + \delta_\ell^{C}. \tag{22}
\]

In this expression, the first term represents the perturbative long range OPEP \((\pi L)\), the second the iterated OPEP \((\pi I)\), the third the part due to the box and crossed-box diagrams (PS), the fourth the contribution from chiral symmetry (S). The last one is due to the core and vanishes for waves with \(\ell \neq 0\).

In Fig. 4 we show the partial contributions of the singlet even waves \(1D_2\) and \(1G_4\) to the phase shifts as functions of energy. Observing Fig. 4 one sees that the minimal chiral model with a core added fails to reproduce the energy dependence of \(1D_2\) wave and is reasonable for the \(1G_4\) wave. This not-so-well agreement was already expected since the minimal chiral model does not include the complete dynamics of the \(\pi N\) scattering. Mesons and baryons resonances like \(\rho\) and \(\Delta\) are very important for higher energies and lower orbital angular momenta \(\ell\). As one goes to higher values of \(\ell\), the role of the one and two pion exchange becomes more important. This can be seen in Fig. 5, where we show the partial contributions of the singlet odd waves \(1F_3\) and \(1H_5\) to the phase shifts as functions of energy.

A remarkable feature of these two waves is that the net result is given just by the long-OPEP contribution, meaning that the medium range contributions cancel entirely.

In these channels, the iterated OPEP is noticeable and the relationship \(\delta_\ell^{\pi I} + \delta_\ell^{PS} = -\delta_\ell^{S}\) holds. This is an important feature of the chiral symmetry and explains why the first theoretical models for the TPEP in the 50’s, which do not have the S term, spoiled the good agreement achieved by the OPEP alone for phase shifts with large \(\ell\). Moreover, it is possible to see another important features of the TPEP, namely the iterated OPEP contribution is comparatively small, indicating that ambiguities in the definition of the potential do not have numerical significance.

Our results show that chiral symmetry, in the restricted pion-nucleon sector, is responsible for large cancellations in the two-pion exchange interaction. This process is therefore similar to threshold pion-nucleon or pion-deuteron scattering amplitudes, where the main role of the symmetry is to set the scale to the problem to be small.

**IV. PERSPECTIVES**

A shortcoming of the minimal chiral model is that it fails to reproduce experimental information in the case of the intermediate \(\pi N\) amplitude that enters the TPEP. In order to overcome this difficulty, one may extend the approach so as to encompass other degrees of freedom. This possibility was recently considered by the introduction of the \(\Delta\) resonance \(\text{[6]}\), which improved considerably the predictive power of the method. Another way to achieve this goal is to introduce the empirical information that
FIG. 6. Diagrams associated with the $\pi N$ amplitude. (a), (b), and (c) represent the minimal chiral model, whereas (d) corresponds to the net effect of the HJS coefficients and is denoted by $R$, for “rest”.

is missing in the intermediate $\pi N$ amplitude in a model independent way, with the help of the Höhler, Jacob and Strauss [14,15,16] subthreshold coefficient, as proposed by Robilotta [9]. The detailed knowledge of the $\pi N$ amplitude provided by these coefficients allows a precise determination of the missing part of the TPEP, as illustrated in Fig. 6. This approach is very similar to in spirit to that followed long ago by Tarrach and Ericson [17], who explored in detail the analogy between the TPEP and Van der Waals force.

A detailed calculation for the leading term in this approach was already done [9], and a complete calculation will be presented soon [18].

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