COMMENT ON ”Zc-like spectra from QCD Laplace sum rules at NLO”

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Abstract

In Phys. Rev. D103 (2021) 074015 (arXiv:2101.07281), huge (or largest) mass gaps between the ground states and first radial excited states for the tetraquark molecular states are obtained, $\Delta = 1.70 \sim 2.35$ GeV. We explore the problems in the operator product expansion, Borel windows, etc, in details.

In the QCD, we deal with the non-perturbative vacuum, and have to resort to non-zero vacuum expectation values of the normal-ordered quark-gluon operators to describe the hadron properties in a satisfactory way. We usually parameterize the vacuum matrix elements in terms of

$$\langle 0 | : \bar{q}^i \gamma^j q^j : | 0 \rangle = \frac{1}{4N_c} \langle 0 | : \bar{q} q : | 0 \rangle \delta_{ij} \delta_{\alpha \beta} ,$$

(1)

$$\langle 0 | : \bar{q}_{\alpha i} \gamma_\beta G_{\mu \nu} : | 0 \rangle = \frac{1}{24(N_c^2 - 1)} \langle 0 | : \bar{q} g_{\sigma} \cdot G q : | 0 \rangle \left( \sigma_{\mu \nu} \right) \frac{\lambda_\alpha^\beta}{2} ,$$

(2)

$$\langle 0 | : \bar{q}_{\alpha i} \gamma_\beta \sigma_{\mu \nu} q^m : | 0 \rangle = \frac{\kappa}{16 N_c} \left\{ \bar{q} : \bar{q} q | 0 \rangle \right\} 2 \left( \delta_{ij} \delta_{mn} \delta_{\alpha \beta} \delta_{\lambda \tau} - \delta_{in} \delta_{jm} \delta_{\alpha \tau} \delta_{\beta \lambda} \right) ,$$

$$= \frac{1}{16 N_c} \left\{ \bar{q} \bar{q} q : \bar{q} | 0 \rangle \right\} \left( \delta_{ij} \delta_{mn} \delta_{\alpha \beta} \delta_{\lambda \tau} - \delta_{in} \delta_{jm} \delta_{\alpha \tau} \delta_{\beta \lambda} \right) ,$$

(3)

etc, where the $i$, $j$, $m$ and $n$ are color indexes, the $\alpha$, $\beta$, $\lambda$ and $\tau$ are Dirac spinor indexes, the $\lambda^a$ are the Gell-mann matrixes. Except for the quark condensates, which indicate spontaneous breaking of the Chiral symmetry through the Gell-Mann-Oakes-Renner relation $f_\pi^2 m_\pi^2 = - 2(m_u + m_d) | \langle 0 | : \bar{q} q : | 0 \rangle |^2$, other vacuum condensates, such as $\langle 0 | : \bar{q} q g \cdot G q : | 0 \rangle$, $\kappa (\langle 0 | : \bar{q} q : | 0 \rangle)^2$, $\cdots$ are just parameters introduced by hand to describe the non-perturbative vacuum, we can parameterize the non-perturbative properties in one way or the other.

According to the arguments of Shifman, Vainshtein and Zakharov and Ioffe [2, 3], the accuracy of factorization hypothesis is of order $\frac{1}{N_c}$, the vacuum saturation (factorization) works well in the large $N_c$ limit $\frac{1}{N_c} \sim 0$, in reality, $N_c = 3$, $\frac{1}{N_c} \sim 10\%$, it is obvious that the value $\kappa = 3 \sim 4$ chosen in Phys. Rev. D103 (2021) 074015 is too large.

In the QCD sum rules for the heavy-light mesons, we often choose the currents $J_\Gamma (x)$ or $J_{\Gamma'}(x)$,

$$J_\Gamma (x) = \bar{Q}(x) \Gamma q(x) ,$$

$$J_{\Gamma'}(x) = \bar{q}(x) \Gamma' Q(x) .$$

(4)

to interpolate the heavy-light mesons, where the $\Gamma$ and $\Gamma'$ are some Dirac matrixes, and resort to the two-point correlation functions $\Pi_{\Gamma/\Gamma'}(p)$,

$$\Pi_{\Gamma/\Gamma'}(p) = i \int d^4 x e^{i p \cdot x} \langle 0 | T \left\{ J_{\Gamma/\Gamma'}(x) J_{\Gamma/\Gamma'}^\dagger(0) \right\} | 0 \rangle ,$$

(5)

to study the masses and decay constants.

We usually carry out the operator product expansion up to the vacuum condensates of dimension 6,

$$\Pi_{\Gamma/\Gamma'}(p) = C^{\Gamma/\Gamma'}_0 + C^{\Gamma/\Gamma'}_{\bar{q} q} \langle \bar{q} q \rangle + C^{\Gamma/\Gamma'}_{G G} \left( \frac{\alpha_s}{\pi} GG \right) + C^{\Gamma/\Gamma'}_{\bar{q} G q} \langle \bar{q} g_{\sigma} G q \rangle + C^{\Gamma/\Gamma'}_{\bar{q} q} \kappa \alpha_s \langle \bar{q} q \rangle^2 ,$$

(6)

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The Feynman diagram contributes to the $\kappa\langle\bar{q}q\rangle^2$, where the solid lines and dashed lines denote the light quarks and heavy quarks, respectively.

where the $C_{0}^{\Gamma'/\Gamma}$, $C_{\bar{q}q}^{\Gamma/\Gamma'}$, $C_{\bar{q}Gq}^{\Gamma/\Gamma'}$, and $C_{\bar{q}q}^{\Gamma/\Gamma'}$ are Wilson’s coefficients, the parameter $\kappa$ parameterizes deviation from the vacuum saturation hypothesis [5, 6], while in Ref.[7], we assume vacuum saturation and also take account of the three-gluon condensate $\langle g^3 s \rangle$. The accuracy of factorization hypothesis is of order $1/N_c^2 \sim 10\%$, where $N_c = 3$ is the number of colors [3]. In the QCD sum rules for the traditional mesons, the $\langle \bar{q}q \rangle^2$ is always companied with the fine-structure constant $\alpha_s$, see Eq.(6) for example, and plays a minor important role, the deviation from $\kappa = 1$ cannot make much difference, although the value $\kappa > 1$ can lead to better QCD sum rules in some cases [8].

In the QCD sum rules for the hidden-charm or hidden-bottom tetra quark molecular states, we often choose the color-singlet-color-singlet type local four-quark currents, $J(x) = J_\Gamma(x)J_{\Gamma'}(x)$, (7) to interpolate the tetraquark molecular states, and resort to the two-point correlation functions $\Pi(p)$,

$$\Pi(p) = i \int d^4xe^{ip\cdot x}\langle 0|T\{J(x)J(0)\}|0\rangle,$$

(8)
to study the masses and decay constants (or pole residues).

At the QCD side of the correlation functions, we can make a rough estimation about how to truncate the vacuum condensates in the operator product expansion,

$$\Pi(p) \propto \Pi_\Gamma(p) \otimes \Pi_{\Gamma'}(p) + \cdots ,$$

$$\propto \cdots + C_{\bar{q}q^2}\langle \bar{q}q\rangle^2 + C_{\bar{q}qGq}\langle \bar{q}q\rangle\langle \bar{q}g_s\sigma Gq\rangle + C_{\bar{q}Gq^2}\langle \bar{q}g_s\sigma Gq\rangle^2 + \cdots ,$$

(9)
where the $C_{\bar{q}q^2}$, $C_{\bar{q}qGq}$ and $C_{\bar{q}Gq^2}$ are the Wilson’s coefficients. In the QCD sum rules for the heavy-light mesons, the quark condensate and mixed condensate play an important role [5, 6, 7], so in the QCD sum rules for the tetraquark molecular states, we should take account of the vacuum condensates $\langle \bar{q}q\rangle^2$, $\langle \bar{q}q\rangle\langle \bar{q}g_s\sigma Gq\rangle$ and $\langle \bar{q}g_s\sigma Gq\rangle^2$ at least.

In Phys. Rev. D103 (2021) 074015, the operator product expansion is accomplished up to the vacuum condensates of dimension 6 [9]. The contributions of the vacuum condensates $\kappa\langle \bar{q}q\rangle^2$ can be shown diagrammatically in Fig.1. If the deviation from the vacuum saturation is large, for example, $\kappa = 3 \sim 4$ in Phys. Rev. D103 (2021) 074015 [9], which means that the contributions of the four-quark condensates are amplified about $3 \sim 4$ times, it is no reason to discard the contributions of the vacuum condensates $\langle \bar{q}q\rangle\langle \bar{q}g_s\sigma Gq\rangle$ and $\langle \bar{q}g_s\sigma Gq\rangle^2$, which are shown diagrammatically in Fig.2. The Feynman diagrams shown in Fig.1 and Fig.2 are analogous, they should be taken into

Figure 1: The Feynman diagram contributes to the $\kappa\langle\bar{q}q\rangle^2$, where the solid lines and dashed lines denote the light quarks and heavy quarks, respectively.
Figure 2: The Feynman diagrams contribute to the $\langle \bar{q}q \rangle \langle \bar{q}g_s Gq \rangle$ and $\langle \bar{q}g_s Gq \rangle^2$. Other diagrams obtained by interchanging of the light and heavy quark lines are implied.

account or discarded simultaneously. The Feynman diagrams shown in Fig.2 cannot be discarded just because "our poor knowledge of their size", "the validity of the vacuum saturation used for its estimate is questionable [9]". In fact, no direct theoretical calculations can improve that: the vacuum condensates $\langle \bar{q}q \rangle^2$ are associated with a factor $\kappa = 3 \sim 4$, and the vacuum condensates $\langle \bar{q}q \rangle \langle \bar{q}g_s Gq \rangle$ and $\langle qg_s Gq \rangle^2$ are not associated with the same factor $\kappa = 3 \sim 4$. The truncation of the operator product expansion in Ref.[9] is too crude to make reliable predictions. In Ref.[10], H. J. Lee observes that the vacuum condensates $\langle \bar{q}q \rangle \langle \bar{q}g_s Gq \rangle$ play a crucial role in the QCD sum rules for the light tetraquark states. In the QCD sum rules for the hidden-charm or hidden-bottom tetraquark (molecular) states, we should carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way [11, 12], and take account of the vacuum condensates $\langle \bar{q}q \rangle$, $\langle \frac{a_{GG}}{\pi} \rangle$, $\langle \bar{q}g_s Gq \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}q \rangle \langle \frac{a_{GG}}{\pi} \rangle$, $\langle \bar{q}q \rangle \langle \frac{a_{GG}}{\pi} \rangle$, $\langle \bar{q}g_s Gq \rangle$, $\langle qg_s Gq \rangle^2$ and $\langle \bar{q}q \rangle^3 \langle \frac{a_{GG}}{\pi} \rangle$.

In Phys. Rev. D103 (2021) 074015, whether or not the operator product expansion is convergent is not checked, in such an extreme condition that the contributions of the four-quark condensates, the vacuum condensates of the highest dimension, are amplified about $3 \sim 4$ times. In addition, the continuum threshold parameters and energy scales of the QCD spectral densities are postponed to very large values to obtain energy scale independent QCD sum rules. However, from Fig.11-b, Fig.13-b, Fig.14-b, Fig.16-b, Fig.19-b in Ref.[9], we cannot see any Borel platforms to extract the molecule masses.

In Table[11] we present the continuum threshold parameters, energy scales of the QCD spectral densities and (ground state) tetraquark molecule masses obtained in Ref.[9]. From the Table, we
can see clearly that $\sqrt{t_c} - M \gg 0.5 \sim 0.6$ GeV, such large continuum threshold parameters are beyond permission of the QCD sum rules.

In Table 2, we present the energy gaps between the ground states and first radial excited states of the conventional mesons from the Particle Data Group and LHCb collaboration [13, 14]. From the Table, we can see that the mass gaps are less than 0.7 GeV if there exist one or two heavy quarks, while the largest mass gaps are about 1 GeV for the $\pi$ and $K$ mesons with the $J^{PC} = 0^{-+}$ due to the fact that the light pseudoscalar mesons play a double role, as both Goldstone bosons and $q\bar{q}$ bound states [15]. In Phys. Rev. D103 (2021) 074015, huge mass gaps between the ground states and first radial excited states of the hidden-charm molecular states are obtained,

$$\Delta = 1.70 \sim 2.35 \text{GeV},$$

the largest mass gaps known up to now, and they are beyond the accommodation of the quark model.

The $Z_c(3900)$ and $Z_c(4430)$ are usually assigned to be the ground state and first radial excited state of the tetraquark (molecular) states respectively according to the analogous decays,

$$Z_c(3900) \to J/\psi \pi,$$

$$Z_c(4430) \to \psi' \pi,$$

analogous mass gaps $M_{Z_c(4430)} - M_{Z_c(3900)} = m_{\psi'} - m_{J/\psi}$ [16, 17, 18]. While $m_{\psi'} - m_{J/\psi} \ll \Delta$.

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