Density waves in layered systems with fermionic polar molecules

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Abstract. A layered system of two-dimensional planes containing fermionic polar molecules can potentially realize a number of exotic quantum many-body states. Among the predictions, are density-wave instabilities driven by the anisotropic part of the dipole-dipole interaction in a single layer. However, in typical multilayer set-ups it is reasonable to expect that the onset and properties of a density-wave are modified by adjacent layers. Here we show that this is indeed the case. For multiple layers the critical strength for the density-wave instability decreases with the number of layers. The effect depends on density and is more pronounced in the low density regime. The lowest solution of the instability corresponds to the density waves in the different layers being in-phase, whereas higher solutions have one or several adjacent layers that are out of phase. The parameter regime needed to explore this instability is within reach of current experiments.

1 Introduction

After the great successes of cold atomic gas physics using neutral atoms with short-range interaction [1,2], many groups have now set their goals on obtaining ultracold samples of polar molecules that have an anisotropic long-range interaction [3–10]. These can, however, lead to strong losses and the design of experimental geometries that reduce these effects are now becoming a reality. In particular, the use of two-dimensional geometries can reduce losses and at the same time very interesting many-body phases in both single- and multilayer configurations have been proposed [11–27]. One such proposal concerns the potential instability of a single two-dimensional layer with polar fermions toward the formation of density-waves as the polarization of the molecules with respect to the layer plane is varied [28,29]. However, the systems of current experimental interest are not single-layer [10], and the effect of adjacent layers is therefore of concern.

Using linear response within the the random-phase approximation, we consider how interlayer interactions influence the density-waves instability and how the critical strength is modified by interlayer terms. In order to estimate the effects of exchange terms, we use many-body local field factors. This approach has been successfully applied to electron systems. We find that the instability is enhanced by the presence of in-phase density-waves in neighboring layers. The effect depends on the density of fermions in each layer and is most pronounced in the low density limit where the critical value is inversely proportional to the number of layers. The latter effect is largely insensitive to the inclusion of exchange terms, Fermi sur-

face deformation, or changes in the effective mass. The density-wave instability will therefore occupy a larger region of the zero-temperature phase diagram for a multilayered system as compared to a single layer system.

2 Linear response and effective interaction

We consider a multilayer system of fermions with dipole moment $D$ and mass $m$ confined in planes parallel to the $xy$-plane and separated by the distance $d$. In the direction normal to the planes, all dipoles reside in the lowest quantum level which we take to be a Gaussian of width $w$, i.e. $\phi(z) \propto \exp(-z^2/2w^2)$. The dipole moments $D$ are aligned by an external field forming an angle $\theta$ with respect to the normal of the planes and with a projection onto the planes which is parallel to the $x$-axis. The experimental setup is illustrated in Figure 1. Two dipoles separated by $r$ interact with the potential $V(r) = D^2(1 - 3 \cos^2 \theta_{rd})/r^3$ where $\theta_{rd}$ is the angle between $D$ and $r$. We assume that the layers all have the same density $n$ of fermions.

To obtain the instabilities of the multilayered system we use linear response theory and the random-phase approximation (RPA) as was done for the case of a single layer in [28,29]. Within the RPA framework, the density-wave instability occurs at the poles of the density-density response function. To treat several layers we extend the RPA to a multilayer (or multicomponent) system. We can write a general density fluctuation in response to an external potential, $\phi_{ext}$, in momentum $(q)$ and frequency $(\omega)$ space as

$$\delta \rho(q, \omega) = \chi(q, \omega) \phi_{ext}(q, \omega),$$

where $\delta \rho$ is a vector quantity containing the disturbances in each layer as entries. Likewise, $\chi(q, \omega)$ is now in general...
where the matrix $V$ produced potential which we write as

$$V_{ij}(q) = V_{ji}(q)$$

where $\chi^0(q,\omega)_{ij} = \delta_{ij} \chi^0(q,\omega)$ is the matrix of response functions of the non-interacting system which is of course diagonal. Combining equations (1) and (3) we arrive at the following matrix equation the response function

$$\chi(q,\omega) = [I - \chi^0(q,\omega)V(q)]^{-1} \chi^0(q,\omega).$$

In the case of a single layer this equation reduces to the standard RPA expression for the density-density response function. Here we are interested in density-wave instabilities in the static limit $\omega = 0$ and we have to determine the singularities of $\chi(q)$. By inversion, we see that these occur when

$$\det[I - \chi^0(q)V(q)] = 0,$$

and this is the equation that we will solve below.

We assume here that the density in each layer is the same, so that the non-interacting response functions are all the same, i.e. $\chi^0_i(q) = \chi^0(q)$, and are given by

$$\chi^0(q) = \int \frac{d^2k}{(2\pi)^2} \frac{f(k+q) - f(k)}{\epsilon_{k+q} - \epsilon_k}.$$  

where $\epsilon_k = \hbar^2 k^2 / 2m$ and $f$ is the Fermi distribution. In the two-dimensional case of interest here we have the explicit expression [30]

$$\chi^0(q) = \frac{m}{2\pi\hbar^2} \left[ \sqrt{1 - \left( \frac{2k_F}{q} \right)^2} \theta(q - 2k_F) - 1 \right],$$

where $k_F$ is the Fermi momentum and $\theta(x)$ is the Heaviside step-function. For simplicity, we ignore any Fermi surface deformation due to the dipolar interaction [28]. We will briefly comment on the influence of such effects in Section 3.5.

\subsection{2.1 Exchange corrections}

The RPA analysis above neglects the role of exchange interactions. In the single-component Fermi system we consider here, the exchange effect can be significant. As an example, we note that for a momentum-independent potential, the exchange correction would completely cancel the direct term in a Hartree-Fock calculation. The effective dipolar interaction that we discuss in the next section depends, however, linearly on momentum. The effects of exchange can be included via the Hartree-Fock RPA approximation. This unfortunately involves a non-local interaction making the resulting numerics somewhat involved. We will not pursue such calculations here, but rather follow the simpler local field factor approach that has been very successful for the electron liquid [31]. It attempts to include the intrinsically non-local effects of the exchange term through the introduction of an effective local “exchange” potential, in similar spirit to the highly successful density-functional method. This approach has