Disorder Effects in Superconductors with Anisotropic Pairing:
From Cooper Pairs to Compact Bosons

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Abstract

In the weak coupling BCS-approximation normal impurities do not influence superconducting $T_c$ in significant manner in case of isotropic $s$-wave pairing. However, in case of $d$-wave pairing these are strongly pair-breaking. This fact is in rather strong contradiction with many experiments on disordered high-$T_c$ superconductors assuming the $d$-wave nature of pairing in these systems.

With the growth of electron attraction within the Cooper pair the system smoothly crosses over from BCS-pairs to compact Boson picture of superconductivity. As pairing strength grows and pairs become compact significant deviations from universal Abrikosov-Gorkov dependence of $T_c$ on disorder appear in case of $d$-wave pairing with superconducting state becoming more stable than in the weak coupling case. As high-$T_c$ superconductors are actually in the intermediate region with Cooper pairs size of the order of few interatomic lengths, these results can explain the relative stability of $d$-wave pairing under rather strong disordering.

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It is well known that in the usual weak-coupling BCS-approximation normal impurities do not influence superconducting $T_c$ in case of isotropic $s$-wave pairing (Anderson theorem) \cite{1}. In case of the so called anisotropic $s$-wave pairing $T_c$ reduction due to disorder is also relatively weak \cite{2,3}. However in case of $d$-wave pairing normal impurities are strongly pair-breaking \cite{2–4} and the universal dependence of $T_c$ on disorder is expressed by the famous Abrikosov-Gorkov equation:

$$\ln\left(\frac{T_c}{T_c^0}\right) = \left[\psi\left(\frac{1}{2} + \frac{\gamma}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right)\right]$$

(1)

where $\psi(x)$ is digamma function, $\gamma = \pi n_{imp} v^2 N(E_F)$ is the usual scattering rate of electrons, due to impurities with point-like potential $v$, which are chaotically distributed in space with some density $n_{imp}$, $N(E_F)$ - density of states at the Fermi level $E_F$. From Eq.(1) it follows directly that $T_c$ is completely suppressed at some critical scattering rate $\gamma = 0.88T_{c0}$, which determines the appropriate critical impurity concentration or residual resistivity of the normal state

$$\rho_{AG} = \frac{2m\gamma_c}{ne^2} = \frac{8\pi\gamma_c}{\omega_p^2}$$

(2)

where $n$ and $m$ are electron concentration and mass, $\omega_p$ is plasma frequency of electrons \cite{4}.

At present there is an emerging consensus on the $d$-wave nature of the pairing state in high-temperature superconducting copper oxides \cite{5}. However the scale of the critical scattering rate of $\gamma_c \sim T_{c0}$ is in rather strong contradiction with the large amount of data on disorder suppression of $T_c$ in these systems \cite{6}, which apparently demonstrate superconducting state being conserved up to disorder induced metal-insulator transition, i.e $\gamma \sim E_F \gg T_{c0}$. The aim of the present report is to propose some possible explanation of this discrepancy.

Consider the (opposite to the usual BSC-picture) limit of extremely strong pairing interaction, leading to compact Boson formation \cite{7}. In this case $T_c$ is determined by the temperature of Bose condensation of free Bosons. In case of impure system condensation point can be determined by the following equation \cite{8}:

$$\mu_p - \Sigma(0) = 0$$

(3)
where \( \mu_p \) is the chemical potential of pairs and \( \Sigma(0) \) is the zero-frequency limit of Boson self-energy due to impurity scattering, which in the weak scattering approximation reduces to the one-loop expression, corresponding to diagram shown in Fig.1:

\[
\Sigma(\varepsilon_n) = n_{\text{imp}}v^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{i\varepsilon_n - \frac{p^2}{2m^*} + \mu_p}
\]

where \( \varepsilon_n = 2\pi nT \) is the even Matsubara frequency, \( m^* = 2m \) is the mass of the pair, and we assume temperatures \( T > T_c \). In the following we consider only three-dimensional systems. Direct calculations give:

\[
\Sigma(0) = Re \tilde{\Sigma}(0) + E_{0c}
\]

where \( E_{0c} = -\frac{m^*}{\pi^2}n_{\text{imp}}v^2p_0 \) is the band-edge shift due to impurity scattering \( p_0 \) is some cut-off in momentum space of the order of inverse lattice spacing \( a^{-1} \) and

\[
Re \tilde{\Sigma}(0) = \frac{1}{\sqrt{2\pi}} n_{\text{imp}}v^2 m^{*3/2} \sqrt{|\mu_p|}
\]

Actually, \( E_{0c} \) leads just to renormalization of the chemical potential: \( \tilde{\mu} = \mu_p - E_{0c} \), so that in renormalized form Eq.(3) reduces to:

\[
\tilde{\mu} \left( 1 - \frac{1}{\sqrt{2|\tilde{\mu}|\pi}} n_{\text{imp}}v^2 m^{*3/2} \text{sign} \tilde{\mu} \right) = 0
\]

with the only relevant (\( \tilde{\mu} < 0 \) for Bosons at \( T > T_c \)) solution of \( \tilde{\mu} = 0 \), i.e. \( \mu_p - E_{0c} = 0 \), determining the Bose condensation temperature of the impure system by the standard equation:

\[
\frac{n}{2} = g \int_{-\infty}^{\infty} d\varepsilon N(\varepsilon) \frac{1}{e^{\frac{\varepsilon}{T_c}} - 1}
\]

where \( g = 2s+1 \) (for Bosons of spin \( s \)), \( N(\varepsilon) \) is the impurity averaged density of states, which in case of the simplest approximation of Eq.(4) just reduces to \( N(E - E_{0c}) \) - the usual free particle expression with energy \( \varepsilon \) calculated with respect to the shifted band-edge. Obviously we obtain the standard expression for \( T_c \) [10]:

\[
T_c = \frac{3.31 \left( \frac{n}{2} \right)^{2/3}}{g^{2/3} m^*}
\]
which is \textit{independent of disorder}. The only possible disorder effect may be connected with exponentially small “Lifshits tail” in the density of states in Eq.(8) due to localization \cite{11}, which is neglected in our simplest approximation of Eq.(4). Thus, our conclusion is that in case of extremely strong pairing interaction (compact Boson picture of superconductivity) \( T_c \) is practically disorder independent for \textit{any} value of the spin of Cooper pair, e.g. \( s \)-wave, \( d \)-wave etc.

It was shown rather long ago by Nozieres and Schmitt-Rink \cite{7} for non impure superconductor that as the strength of the pairing interaction grows, there is a smooth crossover of \( T_c \) from the weak-coupling BCS-picture to that of compact Bosons. In the impure case similar analysis for \( T_c \) can be performed solving the following coupled system of equations generalizing similar equations of Ref. \cite{7} — the usual equation for BCS instability:

\begin{equation}
1 - \chi(0, 0) = 0
\end{equation}

and the equation for Fermion density (chemical potential of electrons \( \mu \)):

\begin{equation}
\frac{1}{2}(n - n_f) = \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{1}{\exp(\frac{\omega}{T_c}) - 1} \frac{\partial}{\partial \mu} \delta(q\omega)
\end{equation}

where \( n_f(\mu, T_c) \) is the free Fermion part of density,

\begin{equation}
\delta(q\omega) = \arctg \frac{Im\chi(q\omega)}{1 - Re\chi(q\omega)}
\end{equation}

and Cooper susceptibility \( \chi(q\omega) \) is determined by diagrams shown in Fig.2. In this figure the vertices contain the symmetry factors for different types of pairing, e.g. in case of cubic lattice \cite{12}:

\begin{align*}
\psi_s(p) &= 1 \quad \text{(isotropic } s\text{-wave)} \\
\psi_s'(p) &= \cos px a + \cos py a + \cos pz a \quad \text{(anisotropic } s\text{-wave)} \\
\psi_{d_{x^2-y^2}}(p) &= \cos px a - \cos py a \quad \text{(} d\text{-wave)} \\
\psi_{d_{x^2+y^2}}(p) &= 2 \cos px a - \cos px a - \cos py a \quad \text{etc.}
\end{align*}

Pairing interaction is assumed to have the following form:
\[ V_i(p, p') = V_{pp'} \psi_i(p) \psi_i(p') \]  \hfill (14)

with \( \psi_i(p) \) defined as above and pairing potential

\[ V_{pp'} = -\frac{V_0}{\sqrt{(1 + \frac{a^2}{p_0^2})(1 + \frac{a'^2}{p_0'^2})}} \]  \hfill (15)

similar to that used in Ref. [7] with \( p_0 \sim a^{-1} \).

Numerical work required to solve Eqs. (10), (11) is very heavy even for non impure case [7]. However, it is clear that these equations will produce also the smooth crossover in \( T_c \) dependence on disorder, interpolating between the BCS and compact Boson limits discussed above. In isotropic s-wave case \( T_c \) will remain practically independent from disorder, i.e. the Anderson theorem remains valid also for compact Boson limit. In case of d-wave pairing the universal dependence of \( T_c \) on disorder defined by Eq.(1) ceases to be valid in the crossover region from large Cooper pairs to compact Bosons. The physical reason for this is quite clear — depairing mechanism of \( T_c \) suppression by disorder ceases to operate with the growth of attractive interaction within pairs, and in the strong coupling region \( T_c \) is determined by Bose condensation of pairs in impure system. Qualitative behavior of \( T_c \) dependence on disorder is shown in Fig.3. It illustrates the smooth crossover in \( T_c \) dependence on normal state resistivity from universal Abrikosov-Gorkov dependence (curve d) to \( T_c \) independent on disorder (curve s). Dashed lines correspond to transition region and the values of coupling constant \( V_0 \) growing from curve 1 to curve 2. It is clear that for d-wave system belonging to this transitional region we can easily obtain superconducting state persisting for rather large disorder with \( \rho > \rho_{AG} \).

Crossover region is qualitatively defined by the simple inequality introduced in Ref. [13]:

\[ \pi^{-1} < p_F \xi < 2\pi, \] where \( p_F \) is Fermi momentum and \( \xi \) is superconducting coherence length. It appears that high-temperature superconductors lie on the the so-called Uemura plot [14] near the “instability” line \( p_F \xi = 2\pi \) [13]. This can explain deviations of \( T_c \) dependence on disorder in these systems from universal Abrikosov-Gorkov curve and their relative stability to disordering [3], despite the possible d-wave symmetry of the pairing state.
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Figure Captions:

Fig.1. One-loop Boson self-energy due to random impurity scattering.

Fig.2. (a) Diagram representation of Cooper susceptibility $\chi(q,\omega)$. $V$ — pairing potential. $\Gamma$ — impurity scattering vertex-part in Cooper channel, defined by the “ladder” approximation (b).

Fig.3. Qualitative dependence of transition temperature $T_c$ on disorder (normal state residual resistivity $\rho$). Curve $d$ — universal Abrikosov-Gorkov dependence of Eq. (1). Curve $s$ — the case of isotropic $s$-wave pairing. Dashed curves — $d$-wave pairing in crossover region from BCS pairs to compact Bosons.
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