On the Age of Information in Wireless Networks Using Rateless Codes

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This work was supported in part by the National Natural Science Foundation of China under Grant 61701071, in part by the China Postdoctoral Science Foundation under Grant 2017M621129 and Grant 2019T120204, in part by the Open Research Fund of National Mobile Communications Research Laboratory, Southeast University, under Grant 2019D03, and in part by the Fundamental Research Funds for the Central Universities under Grant DUT19RC(4)014 and Grant 3132019348.

ABSTRACT The information freshness, quantified by the emerging idea of Age of Information (AoI), has recently received widespread attentions from both academia and industry. However, most prior works on AoI considered a single source-destination pair and provided the performance analysis merely through queueing-theoretic models, which do not lend themselves to the analysis of large-scale wireless networks. This paper focuses on the AoI analysis in a large-scale setting using random spatial models in stochastic geometry. Specifically, we first adopt rateless coding for packet delivery to improve the transmission reliability while reducing the transmission delay. Then, to quantify the enhancement of the information freshness by rateless coding, we derive the average peak age of information (PAoI) in Poisson bipolar and cellular networks, which characterizes the maximum value of AoI immediately before an update packet is received accounting for the spatial variation of network nodes and temporal dynamics of service packets. With these results, we further give the conditions that make the average PAoI to be finite in each type of the network. Numerical results show the significant benefits of rateless codes relative to the commonly used fixed-rate codes in terms of the PAoI and the feasible region of system parameters such as the traffic arrival rate, transmit probability, etc., in Poisson bipolar and cellular networks.

INDEX TERMS Peak age of information, rateless codes, Poisson bipolar and cellular networks, stochastic geometry, queueing theory.

I. INTRODUCTION

A. MOTIVATION

One of the typical applications in Internet of Things (IoT) is to enable smart devices (information sources) to sense the real-time physical measurements and update them to the fusion center (information destination) for further processing such as tracking, positioning, monitoring, etc. The effectiveness of such processing highly depends on the freshness or timeliness of the sensed data received at the destination, which, however, cannot be comprehensively characterized by the conventional performance metrics such as transmission rate, reliability and delay. This tension has led to the necessity to find a new performance metric that quantifies the freshness of information, which is of fundamental importance in networked monitoring and cyber-physical systems [2].

Age of information (AoI) is recently proposed to measure the information freshness [3], defined as the time that has elapsed since the generation of the last sensed data successfully received at the destination. It is easy to identify the intuitive negative correlation between AoI and information freshness, namely, the smaller AoI, the more timely information update. From the definition, the AoI involves the end-to-end delay from the generation of a message at information sources (e.g. smart devices) to the arrival at the information destinations (e.g. data server) and the waiting time for the next successfully transmitted message at the information destinations. The analysis of the AoI needs to consider the traffic arrival pattern, service discipline, queueing management, packet delivery effectiveness and delay, etc. The dynamic and randomness in wireless networks further complicate the analysis. Hence, it is important and necessary to develop appropriate mathematical tools and models for typical scenarios to investigate the information freshness in terms of AoI and
evaluate how the enabling technologies affect the information freshness.

B. RELATED WORK

Using the queueing-theoretic models, the average AoI and its variants (e.g., peak AoI) has been well studied under different arrival/departure processes, numbers of sources and service disciplines (please refer to [3]–[5] and references therein). However, all these works ideally assume that packet delivery occurs in an error-free channel, which, obviously, is unrealistic in wireless networks due to the practical factors like the stochastic fluctuation of channel conditions, thermal noise and external interference. More importantly, the unsuccessful delivery would undoubtedly worsen the freshness of information, which cannot be neglected.

To capture the unreliable channel, a deliver failure occurred with a fixed probability in [6], [7], where different scheduling policies were employed to cope with the unreliability and hence reduce the AoI. In [8], a graph-based interference model was used to capture the interference characteristic when evaluating the AoI performance in a multi-hop wireless network. Although research efforts have been made in modeling real-world situations with transmission errors, the potential coupling between unreliability and certain wireless propagation environment is ignored. The authors of [9], [10] further considered the propagation factors such as the path loss, channel fading and thermal noise, and focused on the information freshness issues enabled by the emerging technologies such as unmanned aerial vehicle [9] and wireless energy transfer [10]. In these works, different scheduling policies and resource allocations were also used to reduce the AoI, however, the concerned scenarios merely have finite network nodes, which is far from the practical network settings. As a consequence, the proposed approaches do not lend themselves to the analysis of large-scale wireless networks. In particular, the mutual interference of concurrent transmissions is the major bottleneck that affects the success probability of packet delivery and thus the information freshness. This motivates a comprehensive investigation on the AoI performance in large-scale wireless networks using random spatial models and tools from stochastic geometry.

In the context of large-scale wireless networks, stochastic geometry is a powerful mathematical tool to characterize the conventional performance metrics when accounting for mutual interference and gains a widespread attraction in different research fields [11]–[14]. However, fairly few literature focuses on the novel metrics for the information freshness in large-scale wireless networks. Recently, the articles of [15]–[17] develop the spatiotemporal models to capture the temporal traffic dynamics and spatial node variation in large-scale wireless networks and the information freshness is analyzed in terms of average AoI or its variants based on the stochastic geometry and queueing theories. In [15], [16], Poisson bipolar model is considered to analyze the average AoI or peak AoI with infinite retransmission mechanism, which guarantees the reliability of packet transmission but worsens the information freshness. To cope with this issue, [15] adopts the queuing management with deadline constraint, and [16] designs a decentralized scheduling policy with local observations to minimize the peak AoI. The authors in [17] analyze the peak AoI for large-scale uplink cellular IoT network under time-triggered and event-triggered traffic with the infinite retransmission mechanism, however this work mainly provides a unified and effective analytical framework rather than proposing countermeasure to improve information freshness. Different from these works, this paper adopts rateless codes to improve the transmission reliability due to its capability to adapt the transmission of parity symbols to time-varying channel conditions and thus retransmission mechanism is circumvented. In this regard, this paper investigates the impact of rateless codes on the information freshness in large-scale wireless networks.

C. CONTRIBUTIONS

In this paper, we establish a mathematical framework to analyze the peak AoI (PAoI) in large-scale wireless networks using rateless codes, where the spatial distribution of the information sources is modeled as a Poisson point process and the temporal arrival of traffic for each source is modeled as an independent Bernoulli process. The specific contributions are summarized as follows.

- We focus on two typical scenarios, i.e., the Poisson bipolar networks with ALOHA and the Poisson cellular networks, according to the communication mode between the information source and its destination. By dividing the PAoI into the packet end-to-end delay and the inter-arrival time of two successful packet deliveries, we provide analytical expressions for the average PAoI with rateless codes for the two types of networks based on the stochastic geometry and queueing theory.

- We give the conditions of the traffic arrival rate that should be meet to realize a finite average PAoI. Specifically, in Poisson bipolar networks, this condition merely depends on the transmit probability while in Poisson cellular networks, things would be more complicated since it is also related to other system parameters, such as the frame length, path loss exponent, etc.

- To highlight the significant enhancement on the information freshness by rateless codes, we also analyze the average PAoI with fixed-rate codes in Poisson bipolar and cellular networks. Besides, we also provide some useful design insights by studying the effect of few network parameters, such as the frame length, arrival rate and node density on the PAoI in each type of the network.

II. SYSTEM MODEL

A. NETWORK MODEL

The information sources are assumed to be uniformly distributed in the two-dimensional Euclidean space $\mathbb{R}^2$ according to a homogeneous Poisson point process (PPP) $\Phi_s$. 

$\Phi_s$
of density $\lambda$, with unit transmit power. Each source has its dedicated destination at distance $r_0$ in a random orientation, i.e., the sources and destinations form a Poisson bipolar network [18, Def. 5.8]. The time line is divided into discrete frames with equal duration $T_f$. We mainly consider two factors affecting the strength of the signal: one is the path loss modeled by the path loss function $\ell(x) = |x|^{-\alpha}$ with exponent $\alpha$, the other is the fading assumed to be an exponential random variable with unit mean (Rayleigh fading). In addition, the fading coefficients keep unchanged over the whole frame and are spatially and temporally independent.

We further consider an interference-limited wireless network and ignore the thermal noise.

We assume that packets arrive at different sources according to independent Bernoulli processes with arrival rate $\zeta$ per frame, i.e., $\zeta$ is the probability that a packet with $K$ information bits is generated in each frame. We further assume the arrival of packets occurs at the moment immediately before the frame boundaries. A buffer with infinite capacity is assumed to store the service packets for each source and the first-come-first-service (FCFS) discipline is applied to transmit the head-of-line (HOL) packet. If the source finds its buffer nonempty, it attempts to deliver the HOL packet at the beginning of the frame with probability $p$ (i.e., the ALOHA strategy). To fully utilize the channel condition, rateless code is employed to deliver a packet with the maximum duration of one frame, and no retransmission mechanism is adopted to decrease the message delivery latency, because the inherent property of the rateless code makes it more sensible to continue the rateless transmission rather than retransmitting (from scratch). During each delivery, the source uses the rateless coder to encode the information bits and continuously transmits the output symbols from the rateless coder, and the destination uses the rateless decoder to process the symbols collected from the physical channel and feedbacks an acknowledgment (ACK) in an error-free way if the packet is successfully decoded. When an ACK is received or the frame runs out, the source stops the symbol transmission, deletes the current HOL packet and does not cause the interference to ongoing packet delivery of other source-destination pairs.

**B. AGE OF INFORMATION**

Assuming that each packet has the time stamp when it arrives at the source, the age of information (AoI) is adopted to measure the freshness of the information, defined as [3]

$$\Delta(t) \triangleq t - U(t),$$

where $U(t)$ is the time stamp of the most recently successfully received packet at the destination. In this paper, the peak AoI is adopted to measure information freshness in the worst case [9], defined as the value of AoI achieved immediately before the next packet with the successful delivery.

Fig. 1 shows an example of the AoI for a source-destination pair, where $a_n$ denotes the time stamp of the $n$-th packet arriving at the source and $d_n$ denotes the time instant of completing its delivery at the destination. For the packet with delivery failure, $d_n$ is the time instant of the ending moment of the frame, and for the packet with successful delivery, $d_n$ is the time instant when the destination successfully decodes the packet. As shown in Fig. 1, when the $n$-th packet is successfully received at the destination, the AoI is reset to $\Delta(d_n) = d_n - a_n$ and then increases linearly to a peak value until the time instant of the next successfully received packet. We have the expression of the AoI during two successive successful deliveries, given by

$$\Delta(t) = \begin{cases} d_n - a_n, & t = d_n, \\ \Delta(d_n) + t - d_n, & d_n < t < d_m. \end{cases}$$

where $n$ and $m$ are the indexes of the two successive successfully received packets. We denote by $A_n$ the peak AoI corresponding to the $n$-th packet, given by

$$A_n = \sup \{ \Delta(t) : t \in [d_n, d_m) \} = d_m - a_n = (d_m - a_m) + (a_m - a_n).$$

This shows that the peak AoI is decomposed into two parts: one is the end-to-end delay of a packet $D_m = d_m - a_m$; the other is the inter-arrival time of two successful deliveries $X_{\text{inter}} = a_m - a_n$. Both terms are related to whether a packet is successfully delivered, and thus the delivery time model with rateless codes in interference-limited networks is established in the following.

**C. DELIVERY TIME MODEL WITH RATELESS CODES**

We consider a destination at the origin that attempts to receive from an additional information source $x_0$ located at $(r_0, 0)$ and it becomes the typical destination under expectation over the PPP due to Slivnyak’s theorem [18, Thm. 8.10]. For the typical source $x_0$, assuming that a packet is granted to be sent in frame $k$, and denoting by $T_{x_0,k}$ the time needed to decode a packet with $K$ information bits, this packet is successfully delivered in frame $k$ if $T_{x_0,k} \leq T_f$. Letting $t_k = (k - 1)T_f$ be the starting time of the $k$-th frame, the instantaneous

![Age of Information](image)

**FIGURE 1.** The age of information versus time.
interference of the typical destination in frame $k$ is

$$I_k(t) = \sum_{x \in \Phi_k} \ell(x) h_{x,k} B_{x,k} 1_{Q_{x,k} > 0} e_{x,k}(t), \quad t_k \leq t \leq t_{k+1},$$

(4)

where $h_{x,k}$ be the fading coefficient of source $x$ in frame $k$, $B_{x,k} = 1$ with probability $p$ (and $B_{x,k} = 0$ otherwise), $Q_{x,k}$ is the number of packets in the queue of source $x$, $e_{x,k}(t) = 1$ if $t \leq t_k + T_{x,k}$ denotes the active state of source $x$ at time $t$, and $T_{x,k}$ is the packet delivery time associated with $x$. Thus the achievable rate $C_k(t)$ with the nearest-neighbor decoder [19] is

$$C_k(t) = W \log_2 \left( 1 + \ell(x) h_{x,k} / \hat{I}(t) \right),$$

(5)

where $W$ is the spectrum bandwidth and $\hat{I}(t)$ is the time-average interference from $t_k$ to $t$, expressed as

$$\hat{I}(t) = \frac{1}{t - t_k} \int_{t_k}^{t} I_k(\tau) d\tau = \sum_{x \in \Phi_k} \ell(x) h_{x,k} B_{x,k} 1_{Q_{x,k} > 0} \eta_{x,k}(t),$$

(6)

where

$$\eta_{x,k}(t) = \frac{1}{t - t_k} \int_{t_k}^{t} e_{x,k}(\tau) d\tau = \min \left\{ 1, \frac{T_{x,k}}{t - t_k} \right\}. \quad \text{(7)}$$

Since each interfering source ceases interference to other ongoing deliveries after receiving its ACK, $\hat{I}(t)$ monotonically decreases with $t$, and thus $C_k(t)$ is a monotonically increasing function of $t$. Hence we have

$$T_{x_0,k} = \min \{ t - t_k : (t - t_k) \cdot C_k(t) \geq K \}. \quad \text{(8)}$$

Since each source attempts to transmit independently at the beginning of each frame, $C_k(t)$ and $T_k$ are statistically identical in each frame. Thus, the frame index is omitted, and the achievable rate is rewritten as

$$C(t) = W \log_2 \left( 1 + \ell(x_0) h_{x_0} / \hat{I}(t) \right), \quad 0 \leq t \leq T_f, \quad \text{(9)}$$

where $\eta_{x}(t) = \min \left\{ 1, \frac{T_x}{t} \right\}$ and

$$\hat{I}(t) = \sum_{x \in \Phi_k} \ell(x) h_{x,k} B_{x,k} 1_{Q_{x,k} > 0} \eta_{x}(t). \quad \text{(10)}$$

The time needed to decode a packet with $K$ information bits is also expressed by

$$T_{x_0} = \min \{ t : t \cdot C(t) \geq K \}. \quad \text{(11)}$$

Letting $C$ be the event that a packet is successfully delivered to the typical destination, we have

$$C = \{ T_{x_0} \leq T_f \} = \{ T_f \cdot C(T_f) \geq K \}. \quad \text{(12)}$$

III. PEAK AöI ANALYSIS

The average PAöI, denoted by $\hat{A}$, is adopted to characterize the peak AöI, and from (3), we have

$$\hat{A} = E[A_n] = E[D_m] + E[X_{\text{inter}}]. \quad \text{(13)}$$

Hence the average PAöI is composed of the expected end-to-end delay of a packet, plus the expected inter-arrival time of two successful deliveries.

A. THE PACKET END-TO-END DELAY

The end-to-end delay of a packet includes the waiting time from the arrival time to the time that a packet is granted to be delivered and the delivery time from the granted time to the time that the destination successfully decodes the packet. For notational convenience, we define $\delta \triangleq 2/\alpha$ and $\theta_0 = 2 \pi \delta - 1$. The average end-to-end delay is given in the following theorem.

Theorem 1: The average packet end-to-end delay with rateless codes is approximated as

$$E[D_m] \approx \frac{1 - \xi}{p - \xi} T_f - \int_0^{T_f} \exp \left( -\lambda_s \xi \left[ \min \{ 1, \frac{\bar{T}}{b} \} \frac{\pi^2 \delta^2 \theta_0^2}{\sin(\pi \delta)} \right] \right) db, \quad \text{(14)}$$

where

$$\bar{T} = T_f - \int_0^{T_f} \exp \left( -\lambda_s \xi \frac{\pi^2 \delta^2}{\sin(\pi \delta)} \theta_0^2 \right) db. \quad \text{(15)}$$

Proof: See Appendix A.

Remark 1: The approximation is obtained by adopting the approach in [20] to decouple the interaction between the interference and the actual delivery time of rateless codes, and the accuracy of the approximation has been demonstrated in [20]. Specifically, the approximation assumes that the packet delivery time $T_k$ is i.i.d. for different interfering sources with the complementary cumulative distribution function (CCDF) $P(T > b)$. As suggested in [20], such CCDF can be chosen as the distribution of the packet delivery time of the typical source where the interfering sources always transmit dummy signal until the frame ending, and $\bar{T}$ is the expected value under the CCDF $P(T > b)$.

Remark 2: It should be noted that the above result holds conditioned on $\xi < p$ to guarantee the finite packet waiting time (finite queueing length). Otherwise, the packets will be accumulated in the buffer and the packet waiting time tends to be infinite, and thus the information freshness tends to be completely outdated. Furthermore, the average packet end-to-end delay a monotonically decreasing function of $p$.

B. INTER-ARRIVAL TIME OF TWO SUCCESSFUL DELIVERIES

The inter-arrival time of two successful deliveries depends on the success probability of delivering the packet to the destination. Since different packet deliveries of the typical source have the same deployment of interfering sources, the success events for different packets are independent given the realization of $\Phi_k$. Hence, we first focus on the success event conditioned on the point process $\Phi_k$, termed $C_{\Phi_k}$, then gives the conditional probability distribution of the inter-arrival time of two successful deliveries, and finally the expectation w.r.t. the point process yields unconditional average inter-arrival time of two successful deliveries. The following theorem gives the average inter-arrival time of two successful deliveries.
The average inter-arrival time of two successful deliveries with rateless codes is approximated as
\[
E[X_{\text{inter}}] \approx \frac{T_f}{\zeta} \exp \left( \frac{\lambda_s \pi^2 \delta}{\sin(\pi \delta)} (1 - \zeta) (\frac{\theta_f \bar{T}}{T_f} \delta^2 r_0^2) \right),
\]
(16)

Proof: See Appendix B.

Remark 3: It is observed that the average inter-arrival time of two successful deliveries is independent with the probability \(p\) and is finite for \(\zeta = 1 - \epsilon\) with \(\epsilon > 0\). But if \(\zeta = 1\), it is infinite. Let \(\zeta_c\) be the critical arrival rate denote the phase transition from finite to infinite average PAoI, and we obtain that average PAoI is finite in Poisson bipolar networks as long as \(\zeta < \zeta_c = p\) by combining the finite conditions on the average end-to-end delay and inter-arrival time of two successful deliveries.

C. COMPARISON WITH FIXED-RATE CODE

To highlight the enhancement of information freshness brought by rateless codes, we also derive the corresponding results for fixed-rate codes. When fixed-rate coding is adopted, each source is always active during the entire frame. Thus, the packet delivery time is constant, i.e., \(T_s = T_f\), and the average PAoI is obtained similar to the case with rateless codes, given by
\[
\bar{A} = \frac{1 - \zeta}{p - \zeta} T_f + \frac{T_f}{\zeta} \exp \left( \frac{\lambda_s \pi^2 \delta^2 T_f \delta^2 r_0^2}{\sin(\pi \delta)} (1 - \zeta)^{\delta - 1} \right),
\]
(17)
and the critical arrival rate with fixed-rate codes is \(\zeta_c = p\).

In the following, we give some numerical results to demonstrate the performance of rateless codes discussed above in wireless ad hoc networks, where \(\lambda_s = 0.1 m^{-2}\), \(r_0 = 2 m\), \(\alpha = 4\), \(T_f = 0.01 s\), \(\zeta = 0.9\), \(W = 1 MHz\), \(\delta = 0.4\), and \(K = 1000\) bits are the default values.

Fig. 2 shows the relationship between the average PAoI and the frame length with different densities of information sources. We observe that the AoI with rateless codes is always lower than the one with fixed-rate codes, and the gap between two coding schemes grows with the increasing of frame length. It can also be seen that the average PAoI of both coding schemes decreases first until a minimum is reached and then increases with the increasing frame length. This is because that the frame length has a competing impact on the two parts of the AoI: on one hand, a short frame length decreases the waiting time in the buffer, but it causes low success delivery probability and thus increases the time of the destination to wait for the next successful packet delivery; on the other hand, a long frame length leads to high successful delivery probability, but it increases the packet waiting time in the buffer. Hence, there exists a suitable value to balance these two parts and the minimum of the AoI is reached.

Fig. 3 shows the impact of the arrival rate on the average PAoI with different communication distances of the source-destination pair. We observe that rateless codes have a better performance on the information freshness than fixed-rate codes for various arrival rates. A shorter communication distance leads to higher delivery transmission probability and thus decreases the average PAoI. As the arrival rate increases, the average PAoI of both coding schemes decreases first and then increases after reaching a minimum value. The reason is that the arrival rate also has a competing impact on the two parts of the AoI, where lower arrival rate causes longer inter-arrival time of packets and higher arrival rate causes longer packet wait time in the buffer.

Fig. 4 shows the relationship between the average PAoI and the path loss exponent with different arrival rates. We also observe that rateless codes have a better performance on the information freshness than fixed-rate codes for various path loss exponents, and the gap between two coding schemes keeps nearly unchanged. As the path loss exponents increases, the average PAoI of both coding schemes have the minimum value. The reason is that the path loss exponent has the same impact of the desired and interference signal strengths and thus the impact on achievable rate depends on which one is the dominant factor. From the figure, when \(\alpha < 3\), we can see the increasing path loss exponent has a bigger impact on the interference signal strength than the desired one, and the success probability is improved, and the peak AoI is lowered. The influence of the increasing path loss exponent is contrary when \(\alpha > 3\), and thus the peak AoI becomes large.

Fig. 5 shows how the density affect the average PAoI with different ALOHA scheduling probabilities. It can be seen that a larger scheduling probability leads to the lower peak AoI for...
both coding schemes, due to the fact that faster service rate causes the smaller packet end-to-end delay. As the density increases, the average PAoI slightly becomes large at first and then quickly increases when \( \lambda_s > 0.1 \). The reason is that the average inter-arrival time of two successful deliveries increases with the density in the exponential law from the results in Theorem 2.

IV. POISSON CELLULAR NETWORKS

A. SYSTEM MODEL

In this section, we consider another typical wireless network, i.e., cellular network, where the information sources are the base stations forming a PPP of density \( \lambda_s \). The cellular users are associated with their nearest BSs and thus each BS adopts the rateless code to transmit messages to the users distributed in its Voronoi cell. This leads to the key difference from Poisson bipolar networks that the communication distance between information sources and destinations is random, and all interfering BSs are farther from the serving BSs to users. Hence, it is critical to study how these differences affect the Peak AoI. We focus on the typical user is located at the origin and assume that the packets for the typical user arrive at the serving BS following a Bernoulli process of arrival rate \( \xi \) (packets per frame). We further consider the random muting mechanism of BSs to mitigate the interference (similar to the ALOHA scheme in bipolar case), where each BS keeps silent with probability \( 1 - p \) and active with probability \( p \).

In practical cellular networks, in the muted frame, only control channels and cell-specific reference signals are transmitted without user data transmission. The achievable rate for the typical user is given as

\[
C(t) = W \log_2 \left( 1 + \frac{\ell(x_0) h_{x_0}}{I(t)} \right), \quad 0 \leq t \leq T_f, \tag{18}
\]

where \( x_0 \) is the tagged BS serving the typical user, and

\[
I(t) = \sum_{x \in \Phi \setminus \{x_0\}} \ell(x) h_{x} B_{x} 1_{Q_x > 0} \eta_x(t), \tag{19}
\]

and the time needed to decode a packet with \( K \) information bits in cellular networks is \( T_{x_0} = \min\{t : t \cdot C(t) \geq K\} \).

B. PEAK AOI ANALYSIS

The following theorems give the average packet end-to-end delay and the average inter-arrival time of two successful deliveries with rateless codes, respectively.

**Theorem 3:** Let \( F(\alpha, \theta) \triangleq _2F_1(1, -\delta, 1 - \delta, -\theta) \), where \( _2F_1(\cdot) \) is the Gaussian hypergeometric function. The average packet end-to-end delay with rateless codes is

\[
\mathbb{E}[D_m] \approx \frac{1 - \xi}{p - \xi} T_f \int_0^{T_f} \frac{1}{1 + \xi F(\alpha, \theta_b \min\{1, \bar{T}/b\})} \, db. \tag{20}
\]

where

\[
\bar{T} = T_f - \int_0^{T_f} \frac{1}{1 + \xi F(\alpha, \theta_b)} \, db. \tag{21}
\]

**Proof:** See Appendix C.

**Theorem 4:** The average inter-arrival time of two successful deliveries with rateless codes is approximated as

\[
\mathbb{E}[X_{\text{inter}}] \approx \frac{T_f}{\xi} \frac{1 - \xi}{1 - \xi F(\alpha, \theta_b (1 - \xi) \bar{T})}. \tag{22}
\]

**Proof:** See Appendix D.

**Remark 4:** The average packet end-to-end delay and inter-arrival time of two successful deliveries are independent with the BS density in Poisson cellular networks, and thus it also holds for the average PAoI. Furthermore, the increasing transmit probability \( p \) decreases the average packet end-to-end delay but does not affect the average inter-arrival time of two successful deliveries. It is also very important and interesting to explore the critical arrival rate (denoting the phase transition from finite to infinite average PAoI) in Poisson cellular networks, given in the following corollary.

**Corollary 1:** If \( \theta_b(1 - \frac{1}{T_f} \int_0^{T_f} \frac{1}{1 + \xi F(\alpha, \theta_b)} \, db) > \alpha/2 - 1 \), \( \xi_o \in [0, 1] \) denotes the root of

\[
\int_1^\infty \frac{\zeta \, d\zeta}{\left( \theta_b(1 - \frac{1}{T_f} \int_0^{T_f} \frac{1}{1 + \xi F(\alpha, \theta_b)} \, db) \right)^{1 - \frac{\alpha}{2}}} + 1 - \zeta = 1, \tag{23}
\]

otherwise, \( \xi_o = 1 \). The critical arrival rate in Poisson cellular networks is given by \( \xi_c = \min(p, \xi_o) \).

**Proof:** See Appendix E.

**Remark 5:** The critical arrival rate in Poisson cellular networks is related to \( p, T_f, \alpha, \) and \( K/W \), and it can be obtained by numerical methods (e.g. bisection search).
To highlight the enhancement of information freshness brought by rateless codes, we also derive the corresponding results for fixed-rate codes. When fixed-rate coding is adopted, each source is always active during the entire frame. Thus, the packet delivery time is constant, i.e., \( T_s = T_f \), and the average PAoI is obtained similar to the case with rateless codes, given by

\[
\bar{A} = \frac{1 - \zeta}{p - \zeta} T_f + \frac{T_f}{\zeta} \left( 1 - \zeta \right) F(\alpha, \theta T_f (1 - \zeta)) \tag{24}
\]

and the critical rate is \( \zeta_c = \min(p, \zeta_0) \), where \( \zeta_0 \) is the root of \( \int_0^\infty \frac{c}{(1 + c)^{\alpha / 2} + 1 - \zeta} \, dc \) if \( \theta T_f > \alpha / 2 - 1 \), otherwise \( \zeta_0 = 1 \).

In the following, we give some numerical results to demonstrate the performance of rateless codes in cellular networks, where \( \alpha = 4, T_f = 0.01s, p = 0.9, W = 1MHz, \zeta = 0.4, \) and \( K = 1000 \) bits are the default values. Fig. 6 shows how the critical arrival rate changes with the frame length with different path loss exponents. We observe that the critical arrival rate with rateless codes is always larger than the one with fixed-rate codes, which means the rateless codes expand the feasible region of the packet arrival rates and thus support a wider range of service types. It can be seen that smaller path loss exponent yields smaller critical arrival rate. The reason is that a smaller path loss exponent yields smaller delivery transmission probability and thus more likely causes the infinite average inter-arrival time of two successful delivery. As the frame length increases, the critical arrival rate becomes larger for both coding schemes until reaching a peak value constrained by the muting probability. This is because the condition for finite average PAoI is dominated by the average inter-arrival time of two successful delivery with small frame length. And then a large frame length can have relatively high success probability of information transmission, thereby the average inter-arrival time of two successful delivery is more likely to be finite and can tolerate larger arrival rate. In this moment, the condition for finite average packet end-to-end delay is the dominant factor for the finite average PAoI and the critical arrival rate is limited by the muting probability.
The average peak AoI decreases when $\zeta > 0.4$ and tends to be infinite, because the arrival rate tends to the critical value.

Fig. 9 shows how the transmission bandwidth affects the average PAoI with different ALOHA scheduling probabilities. It can be seen that a smaller muting probability $1 - \rho$ leads to the lower peak AoI for both coding schemes, due to the fact that faster service rate of the BSs causes the smaller packet end-to-end delay. As the bandwidth increases, the average PAoI quickly decreases at first and tends to be almost unchanged when $W > 0.2$ MHz. The reason is that almost all the delivery transmissions occur successfully with a larger bandwidth and the average inter-arrival time of two successful deliveries becomes almost unchanged.

V. CONCLUSION

In this paper, we proposed a general framework to analyze the PAoI of rateless codes in the large-scale wireless networks incorporating both the spatial variations of network nodes and temporal characteristics of service dynamics. The average peak AoI is adopted to quantify the information freshness and divided into the end-to-end delay of an incoming packet and the inter-arrival time of two successful packet delivers, where the corresponding analytical expressions are derived using the stochastic geometry and queueing theories. Accordingly, the average peak AoI of fixed-rate codes is also provided and compared with rateless codes to show the freshness enhancement in Poisson bipolar and cellular networks. In these two typical networks, we give the conditions on the traffic arrival rate for a finite average PAoI and the adoption of rateless codes supports a wider range of traffic arrival rate than fixed-rate codes.

The results show that the transmit probability merely affects the average packet end-to-end delay in the PAoI as long as it satisfies the conditions for finite average PAoI and the PAoI is monotonically decreasing with the increasing of the transmit probability in both networks. The density of information sources does not affect the average PAoI in the Poisson cellular networks, but in Poisson bipolar networks, the average PAoI is an increasing function of the density. Numerical results show the minimum of the average peak AoI can be achieved by judiciously choosing the frame length, arrival rate and path loss exponent. It should be noted that rateless codes can be combined with the scheduling policies to further enhance the information freshness.

ACKNOWLEDGMENT

This article was presented in part at the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC’2020) [1] in 2020.

APPENDIX A

PROOF OF THEOREM 1

Proof: Denoting by $T_w$ and $T_d$ the waiting time and the delivery time of packet $m$ for the typical source, respectively, we have

$$E[D_m] = E[T_w] + E[T_d].$$  \hfill (25)

Since the packet arrival process follows a Bernoulli process and the ALOHA scheme is adopted to grant the channel access for the HOL packet, the queueing process at each source follows a Geo/Geo/1 queueing model with arrival rate $\zeta$ and service rate $\rho$. Letting $T_s$ be the sojourn time, we have $T_w = T_s - T_f$. This is because that each packet delivery occurs at the beginning of a frame and should be finished no later than the frame ending. According to [21, Corollary 2], we have

$$E[T_w] = E[T_s] - T_f - \int_0^{T_f} \mathbb{P}(T_x > b) \, db \right) 
\tag{27}$$

Then the average delivery time is given by

$$E[T_d] = \int_0^{T_f} \mathbb{P}(T_x > b) \, db 
\tag{28}$$

where step (a) is obtained according to (11), step (b) follows since $h_t$ is an exponential distributed variable, $\theta_1 = 2 - \alpha$, and $\Lambda_X(s) = \mathbb{E}_X(\exp(-sX))$ is the Laplace transform (LT) of the random variable $X$. The interaction between the interference and the actual delivery time of rateless codes causes technical difficulty to derive the exact expression of the LT of $I(b)$. Similar to the approach used in [20], an approximation to the LT is given via assuming that the packet delivery time

![FIGURE 9. The average PAoI versus the spectrum bandwidth.](image-url)
Letting $\xi = \theta T_r \hat{r}_T / T_f$, we have

$$
E[X_{\text{inter}}] = \mathbb{E}_{\Phi_s}\left[ \frac{T_f}{\xi} \mathbb{P}(C_{\Phi_s}) \right] 
\approx \frac{T_f}{\xi} \mathbb{E}_{\Phi_s}\left[ \prod_{x \in \Phi_s} \frac{1}{1 - \xi + \frac{\xi}{1 + \xi \ell(x)}} \right] 
= \frac{T_f}{\xi} \exp\left( -2\pi \lambda_s \int_{0}^{\infty} \left[ 1 - 1 - \xi + \frac{1}{1 + \xi \ell(x)} \right] r \, dr \right) 
= \frac{T_f}{\xi} \exp\left( -2\pi \lambda_s \int_{0}^{\infty} \left[ 1 - \xi + \frac{\xi}{1 + \xi \ell(x)} \right] r \, dr \right),
$$

(35)

where the final step follows the result of [22, Theorem 1].

**APPENDIX C**

**PROOF OF THEOREM 3**

*Proof:* Due to the random muting mechanism, the typical user in the tagged BS is scheduled to receive the message with probability $p$ in a frame, and thus the queueing process for each user at their serving BSs follows a Geo/Geo/1 queueing model with arrival rate $\xi$ and service rate $p$. Similar to Theorem 1, the end-to-end delay is also divided into packet waiting time and packet delivery time, and the average packet waiting time is

$$
E[T_w] = \frac{1 - p}{p - \xi} T_f, \quad \xi < p,
$$

(36)

and the average delivery time is given by

$$
E[T_d] = \int_{0}^{\infty} \mathbb{P}(T_{x_0} > \hat{b}) \, db 
\overset{(a)}{=} \int_{0}^{\hat{T}} \mathbb{P}\left( K > bW \log_{2} \left( 1 + \ell(x_0)h_{x_0} / \hat{I}(t) \right) \right) \, db 
\overset{(b)}{=} \int_{0}^{\hat{T}} 1 - \mathbb{E}\left[ \exp\left( - \theta_b \|x_0\|^2 \hat{I}(b) \right) \right] \, db 
= T_f - \int_{0}^{\hat{T}} \int_{0}^{\infty} \mathcal{L}_{l_{b}(\hat{b})}(\theta br^a) f_{||x_0||}(r) \, dr \, db,
$$

(37)

where $f_{||x_0||}(r) = 2\pi \lambda_s r e^{-\pi \lambda_s r^2}$ is the probability density function of the serving distance in Poisson cellular networks with nearest association [23], and $\mathcal{L}_{l_{b}(\hat{b})}(\cdot)$ is the Laplace transform of the interference outside the circle centered at the origin with radius $r$. The Laplace transform evaluated at $s = \theta br^a$ is approximated via the same decoupling technique in bipolar case, namely, assuming that the packet delivery time $T_s$ of different interfering BSs is i.i.d. with the CCDF $\mathbb{P}(T > b)$, we have

$$
\mathbb{P}(C_{\Phi_s}) = \mathbb{P}(T_s \cdot C(T_s) \geq K \mid \Phi_s) 
= \mathbb{E}\left[ \exp\left( - \theta T_r \hat{r}_T \hat{I}(T_s) \right) \mid \Phi_s \right] 
\approx \prod_{r \in \Phi_s} \left[ 1 - \xi + \mathbb{E}_{\Phi_s}\left( \frac{\xi}{1 + \theta r \hat{r}_T \ell(x) / \hat{T}_f} \right) \right],
$$

(33)

where step (a) follows the approximation used in [20] via assuming that $T_s$ is i.i.d. for different interfering sources. Letting $g(T_s)$ be $g(T_s)$ for $T_s \in [0, T_f]$, we have $E[g(T_s)] \geq g(\hat{T})$, where $\hat{T}$ is the expected value with the CCDF $\mathbb{P}(T > b)$. We further obtain the approximation of $\mathbb{P}(C_{\Phi_s})$, given by

$$
\mathbb{P}(C_{\Phi_s}) \approx \prod_{r \in \Phi_s} \left[ 1 - \xi + \frac{\xi}{1 + \theta T_r \hat{r}_T \ell(x) / \hat{T}_f} \right].
$$

(34)
where $\Phi_1 = \Phi_\lambda \setminus \{x_0\}$, and the expectation operator in the last step is average over $T$. Through analyzing the 2nd derivative of $z(T) = \frac{\theta_b(r/T)^\alpha \min\{1, \frac{T}{r}\}}{1 + \theta_b(r/T)^\alpha \min\{1, \frac{T}{r}\}}$ with respect to $T$, $z(T)$ is a concave function for $T \in [0, T_f]$. Then, we have $\mathbb{E}[z(T)] \leq z\mathbb{E}(T)$, and $\mathcal{L}_{I(b)}(\theta_br^\alpha)$ is lower bounded as

$$\mathcal{L}_{I(b)}(\theta_br^\alpha) = \exp\left(-2\pi \lambda \int_0^\infty \frac{\theta_b(r/T)^\alpha \min\{1, \frac{T}{r}\}}{1 + \theta_b(r/T)^\alpha \min\{1, \frac{T}{r}\}} dt\right)$$

where $\tilde{T}$ is $\mathbb{E}(T)$ is the expected value with the CCDF $\mathbb{P}(T > b)$ and step (a) uses the identity [24]

$$1 + \theta^\delta \int_{0}^{\infty} \frac{1}{1 + r^\delta} dr \equiv 2F(1, -\delta, 1 - \delta, -\theta).$$

Finally, the final results are obtained by substituting (39) and (41) into (37).

**APPENDIX D PROOF OF THEOREM 4**

**Proof:** Given a realization of $\Phi_\lambda$, a packet arrives at the tagged BS $x_0$ with probability $\xi$ per frame for the typical user and it is successfully delivered with probability $\mathbb{P}(C_{x_0})$. Hence the inter-arrival time of two successful deliveries also follows a geometric distribution, and the average inter-arrival time of two successful deliveries is

$$\mathbb{E}[X_{\text{inter}}] = \mathbb{E}\left[\frac{T_f}{\xi \mathbb{P}(C_{x_0})}\right].$$

(42)

Letting $\Phi_1 = \Phi_\lambda \setminus \{x_0\}$ and $r = \|x_0\|$, the conditional success probability is

$$\mathbb{P}(C_{x_0}) = \mathbb{P}\left(T_1 \cdot C(T_1) \geq K \mid \Phi_1\right)$$

$$= \mathbb{E}\left[\exp\left(-\theta_{T_1}r^\alpha I(T_1)\right) \mid \Phi_1\right]$$

$$= \mathbb{E}\left[\prod_{x \in \Phi_1} \left(1 - \xi + \frac{\xi}{1 + \theta_{T_1}r^\alpha \ell(x)T_f/T_1}\right) \mid \Phi_1\right]$$

(a) $\approx \prod_{x \in \Phi_1} \left(1 - \xi + \mathbb{E}_{\mathcal{F}_1} \left(\frac{\xi}{1 + \theta_{T_1}r^\alpha \ell(x)T_f/T_1}\right)\right)$

(b) $\approx \prod_{x \in \Phi_1} \left(1 - \frac{\xi \theta_{T_1}r^\alpha \ell(x)T_f/T_1}{1 + \theta_{T_1}r^\alpha \ell(x)T_f/T_1}\right).$  

(43)

where step (a) follows the independent approximation for different $T_x$ used in [20], and step (b) uses the convexity of $g(T_x) = \frac{\xi}{1 + \theta_{T_1}r^\alpha \ell(x)T_f/T_1}$ for $T_x \in [0, T_1]$ and $\tilde{T}$ is the expected value with a suitable CCDF $\mathbb{P}(T > b)$. Letting $\tilde{\theta} = \theta_{T_1} \tilde{T}$, we further obtain

$$\mathbb{E}[X_{\text{inter}}] \approx \frac{T_f}{\xi} \prod_{x \in \Phi_1} \left[1 - \frac{\xi \tilde{\theta}r^\alpha \ell(x)}{1 + \theta_{T_1}r^\alpha \ell(x)}\right]^{-1}$$

$$= \frac{T_f}{\xi} \int_0^\infty 2\pi \lambda xe^{-\pi \lambda x^2}$$

$$\times \exp\left(-2\pi \lambda e^{\int_0^{\infty} \left(1 - \left(1 - \frac{\xi \tilde{\theta}r^\alpha \ell(x)}{1 + \theta_{T_1}r^\alpha \ell(x)}\right)\right) dr}\right) dr$$

$$= \frac{T_f}{\xi} \int_0^\infty \frac{1 - \xi}{1 - \xi F(\alpha, \theta_{T_1}(1 - \frac{\xi}{\tilde{T}}))}.$$

(44)

where the final step follows the result of [22, Theorem 3] if the following condition is satisfied that

$$\int_1^\infty \frac{\xi}{\theta_{T_1}T/T_1}^{-1}r^{\alpha/2} + (1 - \xi) \, dr < 1.$$  

(45)

Otherwise, $\mathbb{E}[X_{\text{inter}}]$ becomes infinite.

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