A method for selecting a proper modulation technique for the parametric acoustic array

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Abstract. The implementation of the parametric acoustic array has invariably being based on Berktay’s far-field solution, despite its limitations. In addition to the problems due to the approximations of the model, recent studies suggest that the frequency response of the transducers add substantial errors in the demodulation process, which are not considered by Berktay’s equation. This paper introduces a method to overcome the frequency response problem of the ultrasonic transducers, in order to reduce the harmonic distortion produced by the parametric acoustic array, by using a suitable modulation technique. The frequency response of a parametric loudspeaker has been acoustically measured. The characteristics of this response have been applied as a filter to the signal to be demodulated, using Berktay’s far-field solution. A modulation technique has been proposed, taking into consideration the frequency response of the parametric loudspeaker. The results show an improvement in the estimation of the Total Harmonic Distortion (THD) when the parametric loudspeaker frequency response is considered in the modelling. Moreover, the experimentally measured Total Harmonic Distortion of the proposed modulation technique is consistent with the predictions of the model.

1. Introduction
Westervelt described the theory of the parametric acoustic array in 1963 [1]. Additionally, Berktay derived a quasi-linear approach in 1965 [2], which introduced a far-field solution describing a self-demodulation process in the propagation field.

From early experimentations of the parametric acoustic array in liquids until the latest development of parametric loudspeakers [3, 4], Berktay’s solution has remained to be the main model in the implementation of the parametric acoustic array. However, minor amendments have been done to this model [5], despite the limitations of the equation describing the parametric acoustic array. In addition to inaccuracy due to the approximations in the origin of the model, the bandwidth limitations of the transducers affect the demodulation process, by increasing the harmonic distortion [6, 7].

This paper introduces a method to consider the frequency response of the ultrasonic transducer to obtain a suitable modulation technique, in order to lower the harmonic distortion. The rest of the paper is structured as follows: Section 2 describes Westervelt and Berktay’s equations, Section 3 describes the method proposed and the experimental setup, Section 4 shows the results, and Section 5 contains the conclusions and future work.
2. Westervelt equation

The parametric acoustic array was first described by Westervelt as an approximation of the full second order wave equation, for the cases where local nonlinear effects are insignificant compared to cumulative nonlinear effects:

\[ \Box^2 p + \delta \frac{\partial^4 p}{c_0^4 \partial t^4} = -\frac{\beta}{\rho_0 c_0^4} \frac{\partial^2 p^2}{\partial t^2} \]  

(1)

Where \( p \) is pressure, \( \rho_0 \) is the mass density, \( \Box^2 = \nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \) is the d’Alambertian operator, \( \beta = 1 + \frac{B}{2A} \) is the coefficient of nonlinearity, and \( \delta \) is the diffusivity of sound. A quasi-linear solution has been considered to solve Westervelt equation:

\[ p = p_1 + p_2 \]

(2)

This solution includes a linear part \( p_1 \) and a nonlinear correction \( p_2 \).

2.1. Berktay’s far-field solution

In 1965, a quasi-linear solution was implemented by Berktay [2], in the form of equation 2. Berktay described the parametric acoustic array as a self-demodulation process in the propagation medium. If the source is considered as a modulated signal from a circular piston with radius \( a \), the source condition is [8]:

\[ p(r, 0, t) = p_0 f(t) H(a - r) \]

(3)

Where \( p_0 \) is the characteristic source pressure, \( H \) is the unit step function and \( f(t) \) is the time-varying signal, defined by:

\[ f(t) = E(t) \sin[\omega_0 t + \phi(t)] \]

(4)

Where \( E(t) \) is the envelope and \( \phi(t) \) is the phase, both are slowly varying functions compared to \( \sin(\omega_0 t) \).

Westervelt’s analysis was based on strong absorption criterion [9], so nonlinear effects are limited to the near-field. To abide with this assumption, the following condition should be considered:

\[ \alpha_0 z_0 \geq 1 \]

(5)

Where \( \alpha_0 \) is the absorption coefficient at frequency \( \omega_0 \), and \( z_0 = \frac{1}{2} k_0 a^2 \) is the Rayleigh distance at this frequency, which establishes the limit between near-field and far-field.

For solving Westervelt equation, the quasi-linear approach of equation 2 is considered, so the following form of \( p_1 \) is proposed, which includes exponential attenuation [8]:

\[ p_1(r, z, t) \approx p_0 e^{\alpha(t) z} E(t) \sin[\omega_0 t + \phi(t)] H(a - r) \]

(6)

In a fluid, \( \alpha(t) = [\Omega(t)/\omega_0]^2 \alpha_0 \).

After solving Westervelt equation, and considering strong absorption, the nonlinear part of equation 2 is obtained:

\[ p_2(0, z, t) = \frac{\beta p_0^2 a^2}{16 \rho_0 c_0^4} \frac{\partial^2}{\partial t^2} E^2(t) \]

(7)

Which is Berktay’s far-field solution [2]. This equation is valid in the far-field, as several assumptions were made in this regard. The audible sound wave is proportional to the second derivative of the square of the envelope.

The nonlinear interaction is expected to be limited by the near-field, although it can extend over the far-field in some cases. The propagation of the primary ultrasonic waves is limited by the absorption of the medium.
3. Method proposed

Since Berktay’s far-field solution establishes the propagated wave demodulates in air, the implementation of the parametric acoustic array considers a modulation stage. This stage is based on the form given by equation 4, which corresponds to amplitude modulation.

Westervelt and Berktay’s analyses are based on assumptions and criteria which are not always met. One example is the case of the strong absorption criterion given by equation 5, so the far-field solution becomes inaccurate. Moreover, the characteristics of the parametric loudspeaker are not considered in the theoretical model, so the bandwidth limitations of the transducers have not been taken into consideration for the selection of the proper modulation technique. The bandwidth effect has already been described in the demodulation process [6, 7].

Different studies have proposed several modulation techniques [6, 10], all of them based on Berktay’s solution, but the frequency response of the transducers has not been considered. Since these techniques are based on the generation of additional frequencies to the primary waves, the frequency limitations of the transducers have a deep impact in the modulation stage.

To overcome the inaccuracies due to the physical characteristics of the ultrasonic transducers, a new method for selecting a proper modulation technique is proposed. It consists in measuring the frequency response of the parametric array loudspeaker before selecting a suitable modulation technique. Additionally, the second-order nonlinear generation is calculated using Berktay’s far-field solution. Finally, a filter describing the frequency response of the parametric loudspeaker is applied to the signal to be demodulated, in order to include the frequency limitations of the transducer.

3.1. Implementation

The parametric loudspeaker used for this experiment was a NICERA AS195. This is an array of 195 ultrasonic transducers of 10 mm of diameter, with a resonance frequency of 40 kHz, as expressed by the manufacturer. The frequency response of the array of transducers was acoustically measured in an isolated silent room, at 1.5 meters from the source, using a Brüel & Kjær 2670 microphone. A sweep from 20 kHz to 70 kHz with a total duration of 20 seconds was used for obtaining the frequency response. The result of this test is shown in figure 1.

![Figure 1. Frequency Response of NICERA AS195 array, measured at 1.5 m.](image)

The resonance frequency of the array of transducers is 39440 Hz, as observed from figure 1. The cut-off frequencies are 38440 and 40060 Hz, as a -3 dB attenuation is observed at those points. Subsequently, a band-pass digital filter is designed in Matlab, based on a Van Dyke’s model describing ultrasonic transducers [11].

Secondly, seven different modulation techniques were implemented using Matlab. Five of these techniques have been previously presented [6], which are: Quadrature Amplitude Modulation (QAM), Double Sideband Amplitude Modulation (DSBAM), Square-root Amplitude Modulation (SRAM) and Modified Amplitude Modulation (MAM2 and MAM3). Additionally, two more modulation techniques have been introduced, in order to compare the
effects of considering the frequency response in the process of developing a specific technique. These techniques are based on a QAM scheme, which also serves to produce MAM techniques [6]. The scheme used is shown in figure 2:

Figure 2. Quadrature Amplitude Modulation Scheme.

In all cases, the first signal is $g_1(t) = 1 + mg(t)$ and the second signal $g_2(t)$ is specific to each technique. For the development of MAM from QAM, $g_2(t) = 1 - m^2g^2(t)$ was used, as it would eliminate the harmonics generated by $g_1(t)$, according to Berktay’s solution. The final forms of MAM2 an MAM3 are obtained using a Taylor series approximation [6]. Our proposal for the second signal has considered the addition of distortion in a smaller amount, in order to eliminate the distortion produced by $g_1(t)$, but considering the frequency response of the transducer.

For this purpose, $g_2(t) = 1 - \frac{m^2g^2(t)}{2}$ was selected as the second envelope for the first proposal, and $g_2(t) = 1 - \frac{m^3g^3(t)}{3}$ for the second one. The first proposal adds up to the fourth harmonic but in a small amount, and the second proposal adds up to the third harmonic, but in a higher level, compared to the previous technique. Table 1 summarizes the envelopes of all used modulation techniques.

| Modulation Technique | Envelope |
|----------------------|----------|
| QAM                  | $\sqrt{(1 + mg(t))^2 + 1 - m^2g^2(t)}$ |
| DSBAM                | $1 + mg(t)$ |
| SRAM                 | $\sqrt{1 + mg(t)}$ |
| MAM2                 | $\sqrt{2}\sqrt{1 + mg(t) + \frac{1}{16}m^6g^6(t) + \frac{1}{128}m^8g^8(t)}$ |
| MAM3                 | $\sqrt{2}\sqrt{1 + mg(t) + \frac{5}{128}m^8g^8(t) + \frac{1}{128}m^{10}g^{10}(t) + \frac{1}{512}m^{12}g^{12}(t)}$ |
| Proposal 1           | $\sqrt{2 + 2mg(t) + \frac{1}{4}m^4g^4(t)}$ |
| Proposal 2           | $\sqrt{2 + 2mg(t) + m^2g^2(t) - \frac{2}{3}m^3g^3(t) + \frac{1}{9}m^6g^6(t)}$ |

If a sinusoidal input is considered, the second derivative of the squared envelope of the first modulation technique proposed is:

$$\frac{\partial^2 E_1(t)}{\partial t^2} = -2msin(t) + \frac{1}{2}m^4cos(2t) - \frac{1}{2}m^4cos(4t)$$ (8)
Which shows the generation of the second and fourth harmonic. The harmonic generation of the second proposal is given by:

$$\frac{\partial^2 E_2(t)}{\partial t^2} \approx \frac{m^3 + 4m}{2} \sin(t) + 2m^2 \cos(2t) + \frac{3}{2} m^3 \cos(3t)$$ (9)

An approximation was necessary in this case, since higher harmonics were present but with a very low level. Subsequently, all seven modulation techniques were tested using the same setup as for the frequency response measurement. In this case the acoustic measurement was done at 3.6 meters from the source, in order to obtain data comparable to Berktay’s far-field solution results. Only one point was measured, due to limitations of the room.

Finally, total harmonic distortion (THD) was calculated for the data measured, along with the Berktay’s far-field solution predicted by equation 7, for each modulation technique. Additionally, the filter describing the frequency response of the transducers array was applied to all seven modulation techniques, and their Berktay’s far-field solution was computed. The THD was calculated according to equation 10:

$$THD = \sqrt{\frac{P_2^2 + P_3^2 + P_4^2 + \cdots + P_{n-1}^2 + P_n^2}{P_1^2 + P_2^2 + P_3^2 + P_4^2 + \cdots + P_{n-1}^2 + P_n^2}} \times 100$$ (10)

Where $P_1$ is the fundamental frequency and $P_2$ to $P_n$ the harmonics.

4. Results
The Total Harmonic Distortion calculated for the data measured and for Berktay’s solution computed with and without the filter is presented in table 2, for all modulation techniques used.

| Modulation Technique | Berktay (No filter) | Berktay (Filter) | Measurement |
|----------------------|---------------------|------------------|-------------|
| QAM                  | 0.44                | 17.53            | 27.78       |
| DSBAM                | 44.59               | 33.31            | 43.48       |
| SRAM                 | 0.74                | 9.38             | 8.29        |
| MAM2                 | 1.05                | 9.44             | 6.23        |
| MAM3                 | 0.75                | 9.39             | 6.21        |
| Proposal 1           | 4.43                | 17.89            | 13.83       |
| Proposal 2           | 12.58               | 16.44            | 18.10       |

In most cases the harmonic distortion predicted by Berktay’s solution is very low compared to the measurements, except for DSBAM. When the filter is applied, higher THD is obtained, as more harmonics appear due to the frequency response of the transducers. In general, a better approximation to harmonics generation is found when a filter is applied to the modulated signal.

Only DSBAM shows a more accurate Berktay’s far-field solution without the filter applied. This is because the second harmonic of this modulation technique is very high and it is almost the sole contribution towards Total Harmonic Distortion. The filter applied attenuates more of the second harmonic than anything else, so the results for DSBAM become less accurate. A similar case occurs with QAM, where the second harmonic is also very high, but not as important as for DSBAM, and also the third harmonic contributes towards THD calculation.
Finally, the proposed techniques have a close THD value, 13.83% for the first one, and 18.10% for the second one, according to the measurements. Although Berkty’s far-field solution initially shows a bigger difference between the two, 4.43% for the first one, and 12.58% for the second one, the frequency response affects especially to the first proposal. In all experiments, the parametric loudspeaker is capable of producing up to the third harmonic of the fundamental demodulated wave (1 kHz), and the first proposed technique includes up to the fourth harmonic. The inability to produce the expected higher harmonics causes the generation of lower harmonics, which ultimately increases the THD. Although proposal one seems to be much less distorted than proposal two, the difference is much closer at the end.

5. Conclusions and future work
A new method for selecting an appropriate modulation technique has been proposed. This method increases the accuracy of Berkty’s solution, by including the characteristics of the transducers used to implement the parametric acoustic array. With a more accurate far-field solution it is possible to select an appropriate modulation technique for the specific transducer. In six of the seven cases studied, the harmonic generation predicted was more accurate when the filter was applied, with an increase of up to 17% of THD, in the case of QAM. Despite the good results, there are some challenges to improve accuracy. The generation of the second harmonic in many cases was weak when the method was applied, so it needs to be improved. Additionally, more measurement points should be considered in future work, so the propagation effect is taken into consideration. The use of a different model, like the Khokhlov-Zabolotskaya-Kuznetsov equation would be suitable, as it includes absorption, diffraction and nonlinearity effects of sound beams. The proposed method could be used as a tool to describe THD and anticipate the undesired harmonic generation, so other mitigation actions can be taken.

Acknowledgments
This work is funded by the National Commission for Scientific and Technological Research of Chile (CONICYT), Becas Chile program.

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