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Empirical findings on infrastructure efficiency at a bicycle T-junction

A. Gavriilidou a,∗, W. Daamen a, Y. Yuan a, N. van Nes b, S.P. Hoogendoorn a

a Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands
b SWOV Institute for Road Safety Research, Bezuidenhoutseweg 62, 2594 AW The Hague, The Netherlands

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A B S T R A C T
In the pursuit of promoting cycling and providing better cycling infrastructure, its design is of great importance. One of the critical locations in a network are intersections, and specifically T-junctions, due to the inverted perception of priority. In this paper, we investigate the behaviour at T-junctions dedicated to cyclists and the effect on bicycle flow efficiency resulting from the introduction of lane markings that advise through cyclists to shift to the left so that merging cyclists can occupy the space on the right hand side of the cycle path. A comprehensive framework is proposed for the assessment of the T-junction efficiency. Empirical trajectory data from a large-scale cycling experiment are used for the analyses. The findings suggest that cyclist heterogeneity can be even more influential on the efficiency than the infrastructure design. Moreover, a form of self-organisation is observed, as through cyclists are willing to move to the left and allow merging cyclists to fit in the cycle path without the provision of any instructions. This means that the lane marking is both redundant and not improving the efficiency of the T-junction.
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1. Introduction

The increased interest and promotion of cycling by governments and municipalities throughout the world goes hand-in-hand with the provision of better cycling infrastructure. The design of this infrastructure is of great importance, because it influences cyclist behaviour, as well as the attractiveness to use it. Moreover, the design should offer a safe and efficient transition through the network. Critical locations in the network in terms of safety and efficiency are discontinuities of the infrastructure, obstacles placed within the infrastructure and locations where different transport modes or traffic streams interact and negotiate the use of space.

At intersections there is a higher risk for cyclists to be involved in a collision [1,2]. At signalised intersections, this is mostly attributed to rule violations by the cyclists or the motorised vehicles. Especially in Asian countries, the cause of bicycle crashes at signalised intersections has been found to be the red-light running and retrograde behaviours of electric bicycles [3,4]. However, the label of electric bicycles is also used for scooters [5], which explains why red-light running behaviour of cyclists is not found so dangerous in Western world countries [3]. At unsignalised intersections, on the other hand, this higher risk may relate, among other things, to the presence of sharp turns and the lack of road marking or lighting [6]. Another cause might be the high number of interactions. A study in Amsterdam showed that

* Corresponding author.
E-mail address: a.gavriilidou@tudelft.nl (A. Gavriilidou).

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when the number of cyclists at an intersection is high, the level of stress and discomfort of the cyclists rise, and they adhere less to traffic rules [7].

One type of intersections are T-junctions, which consist of three road segments (also known as arms) two of which belong to a straight road. In car traffic, it has been found that uncontrolled T-junctions can pose a problem when priority is assigned to the intersecting (right) arm, because drivers on the straight road have a high perception of priority and fail to yield [8]. Priority perception is very important for the type of interactions that occur. When priority perception is high, drivers as well as cyclists tend to have higher speeds and minimal head movements to observe their surroundings, which can result in unsafe interactions [9]. These studies, however, investigated either only car traffic or mixed car–bicycle flows, where the priority was indicated by road signs. Given the findings of [10] where it is suggested to keep the different transport modes in separated infrastructure, the transferability of the findings to dedicated bicycle T-junctions becomes questionable. Moreover, the effect of design possibilities other than road signs to indicate priority has not been researched.

We focus on T-junctions dedicated to cyclists and investigate cyclist behaviour using empirical trajectory data collected during a large-scale cycling experiment [11]. The reason for using observations from an experimental setting rather than real-world, is that there is control over the infrastructure design, the characteristics of the participants and their bicycles [12]. During the experiment, lane marking was implemented, aiming to guide through cyclists to the left and leave the right side of the path for merging cyclists. The hypothesis was that this lane marking could inflict separation of conflicting streams in space, leading to safer interactions and more efficient flow. The safety effects are presented in [13]. In the present paper, we study the effect on infrastructure efficiency. For the assessment of efficiency we develop a comprehensive framework, which is the first contribution of this paper. Another contribution are the empirical findings we derive on the infrastructure efficiency of a bicycle T-junction. Based on these findings, our third contributions is that we make recommendations for the design of bicycle T-junctions.

The paper is structured as follows. In Section 2 the framework to assess the infrastructure efficiency is described and in Section 3 the data from the controlled experiment are presented. The method to apply the framework is described in Section 4. The results are provided and discussed in Section 5, followed by conclusions and recommendations for the infrastructure design in Section 6.

2. Infrastructure efficiency framework

This study aims to assess the efficiency of bicycle infrastructure. Our focus is on T-junctions dedicated for bicycle use. In order to evaluate efficiency, a comprehensive framework is proposed that turns the most relevant influence factors into specific key performance indicators. This framework is depicted in Fig. 1.

Infrastructure efficiency depends on the infrastructure design, as well as on the behaviour of its users. The design captures elements like the marking, the merging angle, the width and surface of the cycle path. Marking may be used to indicate which direction has priority. This is usually indicated by “shark’s teeth” (i.e. white isosceles triangles) drawn on the surface of the path, at the end of the minor (i.e. non-priority) stream or by road signs. When such markings and signs are missing, general traffic rules apply. Marking can also be used to exemplify the expected space utilisation, by drawing lines on the surface. Solid lines generally indicate a compulsory movement, while dashed lines are suggestive, for example to guide minor stream cyclists into the through lane. The merging angle may affect the perception of priority as well as the speed of the merging cyclists and the ease of their merging manoeuvre. The width of the cycle path affects the ability to form lanes as well as the overtaking behaviour. Last but not least, the type of surface may affect the cycling speeds and the perception of priority.

The term “cycling behaviour”, in this case, encompasses different behaviours, like the choice of speed and lane, distance keeping, lane changing, yielding and gap acceptance. This behaviour is not only influenced by the infrastructure design, but also by the traffic rules and the person’s attitude to adhere to them. Other personal (age, experience) and bicycle characteristics (type, condition) also have an effect.

The interaction between infrastructure design and cyclist behaviour determines the infrastructure efficiency. We hypothesise that efficiency cannot be captured by a single metric, but is rather assessed by a compilation of indicators. The proposed framework aims to comprehensively cover different aspects of efficiency. These aspects are grouped in three categories with increasing order of complexity. The first refers to single cyclists, the second to the use of the infrastructure, and the third to the interactions between cyclists.

Within the first group we consider the speed upstream and downstream of the T-junction, and the travel time. The use of the infrastructure in the second category refers to the capacity of the T-junction and the utilisation of the two-dimensional space. The capacity can be determined from the longitudinal time-headways that occur downstream the junction, while the space utilisation is reflected by the lanes that are formed in the lateral direction. The last group of indicators focuses on the cyclist encounters in conflicting directions. The group sizes that arrive upstream of the T-junction negotiating priority influence the delay that is inflicted. This delay can also be captured by the number and duration of stops that cyclists make to give way to the priority stream.

3. Cycling data at T-junction

The dataset used to assess the T-junction efficiency consists of cyclist trajectories collected during a controlled experiment conducted in Rotterdam, The Netherlands in April 2018 [11]. In this section, details about the experiment are provided (Section 3.1), followed by the description of the trajectory extraction (Section 3.2) and a summary of the characteristics of the final dataset for which the key performance indicators are calculated (Section 3.3).
3.1. Controlled experiment

The experiment took place inside the Ahoy Convention Centre, on a track marked by white tape covering a surface of 100 m × 40 m. The cycle path was 2 m wide at all locations. Several scenario runs were performed to investigate different types of interactions, cyclist and bicycle populations and infrastructure designs. In total 178 cyclists participated and video data were collected by eight overhead cameras for a duration of six hours. This paper focuses on one hour of video data from one of the cameras, namely the one placed over the T-junction. The selection of the one hour is such that it covers the scenario runs during which the route on the track that made use of the T-junction was open. These runs will be referred to as the “merging runs”. During the whole experiment, there were six merging runs of about ten minutes each (hence the one hour of video), which were spread throughout the day to keep the schedule of the participants interesting, alternating between different routes and interactions. For details regarding the experiment, the reader is directed to [11].

An overview of the track layout that was used during the merging runs is shown in Fig. 2a. In each run, 40%–60% of the participants were asked to follow the main track route and the others followed the merging track route. The track was always marked with a continuous tape, so it is only for readability that the merging track is indicated in the figure with a dashed line. The camera view of interest for this study covers the T-junction and is indicated by the blue box in Fig. 2a. No other marking was placed on the track except for an infrastructural nudge that was implemented prior to the last merging run in order to study the effect of lane marking on the cycling behaviour at the T-junction. The lane marking consisted of a dashed line, starting at the edge of the track upstream of the T-junction and leading toward the centre of the track, continuing till downstream the T-junction as a centre line. A snapshot of the lane marking is shown in Figs. 2b and 2c, which correspond to the camera views of the camera placed on top of the junction (blue box in Fig. 2a) and the camera upstream of it (orange box in Fig. 2a).

As shown in Fig. 2a, the two camera views overlap. This is to facilitate the coupling of trajectories extracted from different cameras. The boxes in Figs. 2b and 2c were marked with yellow tape on the ground, while the yellow letters A and B are added in the figure to demonstrate which boxes are the same in the two snapshots. The marked cross-sections in Fig. 2c are not present on the ground, but are added to the figure to assist in the explanation of the calculation of the infrastructure efficiency indicators in Section 4.

The aim of the lane marking is to direct through cyclists to the left lane and achieve separation of the conflicting flows at the junction. Ideally, this design would alleviate the need for through cyclists to stop, as they can freely continue into the left lane, while merging cyclists enter at the right lane. The through cyclist seen in Fig. 2b indeed steers away from the dashed line. Another observation concerns the design of the T-junction in Fig. 2c. The width of both streams was kept constant at all time at 2 m, which is enough for two cyclists to comfortably ride side-by-side and execute overtaking manoeuvres. The turning angle was also kept constant at 90 degrees. The motivation behind the choice of perpendicular cycle paths was to create a clear sense of priority, to be assigned to cyclists coming from the right. No priority marking was added to further influence the behaviour. Moreover, the taped edges of the track were chosen to be perpendicular rather than curved in order to observe to what extent cyclists cut-off the edge and identify the curve they find most comfortable for turning. This choice was made also for practical reasons, as a straight angle is easier to tape. The feasibility of the turn
was tested and approved, so it was possible to execute the merging manoeuvre without straying off the track edges. Fig. 2c shows a snapshot of a merging cyclist cutting the corner.

Regarding the cycling behaviour, participants were instructed to cycle as they would normally do, and to try to stay within the edges of the cycle path as much as possible. The given instruction left the choice of speed, priority allocation and overtaking to the participants. This means that they were neither forced to obey the traffic rules nor prohibited from overtaking other cyclists. The tape was thick enough to let passing cyclists know that they are crossing it, but not raised as a curb that could cause safety incidents.

3.2. Trajectory extraction

During the experiment, all participants wore a red cap which could be tracked in each frame of the videos, thereby allowing us to reconstruct the trajectory of each cyclist. The extraction of the trajectories followed six automated steps:
Table 1
Basic characteristics of runs.

| Run | Session | Bicycles | Lane marking | Number of trajectories | % Merging | Duration of run [min] | Flow [cycl/s] |
|-----|---------|----------|--------------|------------------------|-----------|----------------------|--------------|
| 1   | Morning | Mix 1    | No           | 480                    | 50.8      | 8.8                  | 0.91         |
| 2   | Morning | Mix 2    | No           | 417                    | 46.8      | 6.6                  | 1.05         |
| 3   | Morning | Mix 1    | No           | 629                    | 56.1      | 10.0                 | 1.04         |
| 4   | Afternoon | Pure 1 | No           | 598                    | 55.9      | 10.1                 | 0.99         |
| 5   | Afternoon | Pure 2 | No           | 610                    | 45.3      | 10.0                 | 1.02         |
| 6   | Afternoon | Pure 1 | Yes          | 803                    | 51.8      | 13.1                 | 1.02         |

1. Identification of red caps in each video frame based on detection of red coloured pixels and clustering them into objects as described in [14].
2. Connection of centre points of the identified red caps in the consecutive video frames to construct cyclist trajectories using an adjusted version of the MODT-2 software developed in [15].
3. Transformation of head trajectories to ground trajectories by correcting for the height difference between the cap and the ground. This process made use of height at which the camera was placed (i.e. 12 m), and of the average height of persons on a bicycle which was measured to be 179 cm.
4. Orthorectification of the ground trajectories to correct for the camera distortion and convert from pixel units to metres. The correction is based on parameters determined by ImageTracker [16].
5. Removal of short, misidentified trajectories by ensuring that each cyclist is observed at two cross-sections (3 m and 18 m lines in Fig. 2c for through cyclists and 10.5 m and 18 m for merging cyclists).
6. Replacement of blobs (i.e. clouds of points) observed during a stop by a single stopping location, thereby ignoring the movement of the head when the bicycle stands still.

3.3. Run characteristics

Table 1 shows an overview of basic characteristics of the merging runs. The first three runs took place in the morning session of the experiment, where participants could bring regular bicycles as well as racing and e-bikes. The 88 participants who joined this session were divided into groups, referred to as “Mix 1” and “Mix 2” because of the mixture of bicycle types. In the afternoon sessions, the 90 participants, who were asked to bring regular bicycles only, were split into the groups “Pure 1” and “Pure 2”. The last run in the afternoon is the run with the lane marking.

Since the identified trajectories do not correspond to specific person IDs, the number of trajectories shown in the 5th column of Table 1 does not reflect the number of participants, but rather the number of passages through the camera view. The share of identified merging trajectories ranges from 45% to 56%, which does not coincide with the share of cyclists taking the merging route, because the shorter distance of this route and the cycling speed affect the number of laps they make and, hence, the number of cyclists seen by the camera per run. The share of merging cyclists is not known, as it varied during the run, because participants took the liberty of alternating between the two routes (i.e. the main and merging tracks). These identified shares of merging trajectories can be seen as an outcome of the efficiency of the T-junction, rather than a cause. For this reason they are not further discussed or used to assess the efficiency. Instead, the assessment of the infrastructure efficiency is based on the framework presented in Section 2 and corresponding indicators (to be introduced in Section 4).

It can be seen that the number of trajectories differs per run, but so does the duration of the run. Therefore, the comparison should be based on the ratio of the two, which is the average flow of cyclists and is shown in the last column of the table. The flow values in all runs are around 1 cyclist per second and are, thus, comparable. Therefore, it is considered that the different number of trajectories has no effect on the findings of this paper.

Having six runs allows investigating not only the effect of the lane marking, but also of the different bicycle types (“Mix” versus “Pure”) and persons (morning and afternoon, and group 1 versus group 2). Moreover, the results of the 3rd run compared to those of the 1st reveal whether any learning effect takes place as participants get familiar to the experimental conditions. The conclusion on whether there was learning effect is then used in the comparison between the 6th and 4th run, to separate the impact of the lane marking, from that of the learning effect. The impact of different bicycle types is found by comparing runs 1 and 2 with runs 4 and 5, while the impact of different personal characteristics is investigated through the comparison of run 1 with run 2, and run 4 with run 5.

4. Method to calculate efficiency indicators

In Section 2 the framework was presented to assess the efficiency of a T-junction. In this section, the method to calculate each of the efficiency indicators is described and the results are discussed in Section 5. The extracted trajectories have been the basis for the calculation of all metrics.
4.1. Single cyclist performance indicators

Speed distributions are constructed from the speeds of the individual cyclists in a specific segment. A speed measurement is made based on the passing time, \( t \), between two cross-sections in Fig. 2c. For example, for the through cyclists upstream of the T-junction, the cross-sections at 3 m and 9 m are used and the time instances at which each cyclist passed them (\( t_3 \) and \( t_9 \), respectively) are known from the trajectory data. The ratio of the distance covered and the corresponding time interval define the average cycling speed, \( v \), for that stretch:

\[
\begin{align*}
    v_{\text{upstream, through}} &= \frac{9 - 3}{t_9 - t_3} \\
    v_{\text{upstream, merging}} &= \frac{10.5 - 7.5}{t_{2.5} - t_{7.5}} \\
    v_{\text{downstream, } i} &= \frac{18 - 12}{t_{18} - t_{12}}, \text{ for } i = \text{through, merging}
\end{align*}
\]  

(1)

If the infrastructure is efficient, the cycling speeds should be similar for the up- and downstream stretches of the same cycling direction, such that the cyclists in each stream can pass the T-junction uninterrupted (i.e. keeping the same speed). However, as the streams are intersecting, cyclists should also ensure safe interactions which require anticipation and deceleration. The infrastructure design that requires the least amount of deceleration is, thus, the most efficient. The expected effect is further elaborated for each stream. Through cyclists are expected to yield to the merging cyclists who have priority. The yielding through cyclists who come to a complete stop (i.e. zero speed) are not taken into account, as their speeds are very different in the two stretches and the properties of their stops are considered in other indicators. The cyclists who decelerate are included in the analysis as they show the effect of the interactions with merging cyclists on their speed. This effect should ideally be minimal, thus similar speeds upstream and downstream. Regarding the merging cyclists, it is expected that they slow down to make the turn. Any differences observed between the two stretches are, then, depicting the efficiency deterioration of the infrastructure design (i.e. the 90 degree turning angle).

Travel time is also calculated as a measure of delays. Its variability across the runs can reveal any deviations from the free-flow travel time. Since the latter is not known, the average of the six runs is taken as a proxy. The travel time, \( t_t \), is defined as the time interval during which a cyclist is present between two cross-sections:

\[
\begin{align*}
    t_{t, \text{merging}} &= t_{18} - t_{10.5} \\
    t_{t, \text{through}} &= t_{18} - t_3
\end{align*}
\]

(2)

4.2. Infrastructure use indicators

Lane formation and time headways are the two metrics for the use of the infrastructure. The former can be observed through a heatmap of the space utilisation, and the latter through the time interval between consecutive cyclists at a selected cross-section.

According to the standard dimensions of the width of a bicycle handlebar [17], space is discretised in squares of 67 cm. A maximum of three sublanes can then be formed and observed in the 2 m width of the cycle path. Using squares reduces the complexity of having to rotate the shape with the changing direction of the merging cyclists. The heatmap is constructed by counting the number of cyclists (trajectories) that ride over each square. A more efficient infrastructure would ensure that the entirety of the cycle path width is used.

The (minimum) time headways are a proxy for the capacity of the T-junction (capacity equals one over the average minimum time headway) and should, therefore, be measured downstream of it. The cross-section at 15 m (Fig. 2c) is chosen and the time interval between consecutive cyclists passing it is recorded. The shorter the time headways, the higher the capacity and, thereby, the higher the efficiency of the T-junction.

4.3. Indicators for interaction with other cyclists

Under the assumption that the priority stream remains unaffected by the presence of through cyclists, it is the latter that need to react, i.e. either make a stop or reduce their speed or make space or accept a gap. The indicators reflecting the interactions with other cyclists have as starting point the presence of a through cyclist upstream of the T-junction. In order to consistently compare the interactions that each through cyclist faces, a cross-section needs to be selected as the critical moment for the decision of the through cyclist to react, if necessary, to the anticipated conflict. To the authors’ knowledge there is no literature investigating how far upstream cyclists anticipate on conflicts and when they make their decisions. In this study, we take the cross-section at 6 m (Fig. 2c) as the critical decision moment. One argument for this choice is that it is four metres upstream of the cross line for through cyclists (thus upstream of the intersection), which is the same distance as that available in the camera view for merging cyclists. Another argument is that it is in the middle of the range for which the speed distributions have been investigated. The start of this range at 3 m is considered too far upstream for proper anticipation, while the 9 m at the end of the range are too close to the cross line which is at 10 m,
and there is not sufficient reaction time if cyclists would make their decision there. The choice for the 6 m cross-section is further evaluated in the discussion of the results in Section 5.

When a through cyclist passes the 6 m line, it is checked whether any merging cyclist is present upstream of the junction. If so, the number of cyclists present in each stream is counted and this represents an instance of two group sizes, one for the through stream and one for the merging. This process is repeated every time a through cyclist passes the 6 m line, leading to a frequency of different group size combinations to occur. Given the length of the visible stretch for through cyclists upstream the T-junction, which is 10 m, the 2 m width of the cycle path and the average bicycle length of 2 m, a maximum of 10 through cyclists can be observed. Similarly, given the length of 4 m visible in the upstream merging stretch, a maximum of 4 merging cyclists is expected. In an efficient design the group sizes should be larger for the merging than for the through cyclists. The reasoning behind this is that if many through cyclists yield for only few merging cyclists, more delay is experienced than time savings.

As already stated, one of the possible reactions of through cyclists is to stop, yielding to merging cyclist(s). These stops are a form of delay and two metrics are defined to assess their effect on efficiency. The first one is the stop duration, corresponding to the time a through cyclist remains stationary. The second metric is the stopping frequency. This is a normalisation of the number of stops in the run to remove the effect of the run duration. The applied normalisation is the inverse of the time difference between two consecutive stops, so that the variation of stops within each run is captured as well. More specifically, the stopping frequency, $S_k$, in stops per minute is calculated for each stop, $k$, using the following equation:

$$S_k = \frac{60}{t_k - t_{k-1}}$$  \hspace{1cm} (3)

The efficiency is higher when stops are shorter, which is linked to smaller group sizes and when there are less stops per minute. High numbers indicate that the interval between consecutive stops is high, so many cyclists need to yield at the same time or the outflow from the queue goes in waves and the same cyclist needs to make several stops.

5. Results and discussion of efficiency

The results for each set of indicators of the assessment framework in Fig. 1 are presented in the following subsections, along with their discussion.

5.1. Efficiency based on single cyclist performance

Fig. 3 shows the cumulative distribution functions of the average speed per run, direction and section of the infrastructure. The top row corresponds to the section upstream of the merging location, and the bottom to the downstream section. The left column is for through cyclists and the right for merging cyclists. The $y$-axis shows the cumulative probability and the $x$-axis the cycling speed through the corresponding segment in km/h. Runs with the same group of cyclists have been assigned the same colour and different line styles. Full lines correspond to the first run of that group of participants and dashed lines to the second run, when applicable.
It can be seen that the through cyclists in the upstream section have the lowest speed, which can be attributed to the fact that they sometimes need to decelerate and give way to merging cyclists coming from the right. Downstream of the junction, the through cyclists reach twice as high speeds. The opposite phenomenon occurs for the merging cyclists. Upstream of the junction they have higher speeds than downstream. This is due to the sharp right turn, when merging cyclists try to enter the right lane, which requires a speed reduction.

Kolmogorov–Smirnov tests are performed to compare the empirical distributions against each other and draw conclusions regarding the presence of a statistically significant difference [18]. Four types of comparisons are performed between the distributions of:

- **Runs of through cyclists**: When comparing the different runs to each other, it is found that there is no statistically significant difference at a 5% significance level between runs 1 and 3 which have the same population. This means that there is no evidence for a learning effect due to the experimental conditions, so through cyclists do not change their speed upstream of the T-junction as they get more familiar with the infrastructure and the occurrence of encounters. Contrary to this, the speed distributions in runs 4 and 6 are found to differ. Therefore, the lane marking appears to have an effect on the speed of the through cyclists upstream of the junction. The effect is positive, as more observations have higher speeds in the case of run 6. Another finding is that the upstream speed distributions of runs 5 and 6 are not statistically significantly different, which shows that the characteristics of the cyclist population (i.e. cyclist heterogeneity) are at least as important as the presence of lane marking. This heterogeneity is further attributed to personal characteristics, rather than the different bicycle types, as the speed distributions of runs 1 and 4 have no statistically significant difference. Similar results are found when comparing the speed distributions of the through cyclists downstream of the T-junction. The only difference is in the comparison of runs 5 and 6, which are statistically significant different downstream the T-junction. In this stretch, the cyclist heterogeneity leads to higher speeds than the presence of the lane marking.

- **Runs of merging cyclists**: Most speed distributions of merging cyclists are statistically significant different at a 5% significance level. Heterogeneity is observed in terms of personal characteristics as well as in the different bicycle types. Personal characteristics seem to be more prominent, as the presence of faster bicycles in the mix (i.e. racing and electric bicycles) does not necessarily result in higher speeds (run 4 with only regular bicycles is the fastest run, but run 2 with a mix of bicycle types is faster than run 5 with only regular bicycles). In this case, there seems to be a learning effect, as the “Mix 1” cyclists have higher speeds both upstream and downstream in the case of run 3 compared to run 1. This means that merging cyclists become more comfortable with the right turn and develop a strategy to take it with the least speed reduction possible. At the same time, and despite the positive contribution of the learning effect, merging cyclists are worse-off by the introduction of the lane marking, as they cycle slower in run 6 compared to run 4. The explanation might be the introduction of the lane separation downstream: they feel obliged to merge into the right lane without using the left lane that has been assigned to through traffic by the lane marking, so they are forced to decelerate already upstream. In other words, the lane marking makes the 90 degree turn even sharper.

- **Merging and through streams**: An interesting observation from Fig. 3 is the complementary effect between the speeds of the two directions; the stream that is fastest in a run, is the slowest in the other direction of the same run. For example, run 5 was the fastest for through cyclists and the slowest for merging cyclists. This is more evident in the upstream section and has its roots in the way that cyclists handle their encounters. When merging cyclists are very fast, through cyclists experience very small gaps, which they cannot accept. So they have to yield until a sufficient gap becomes available, resulting in reduced average speed in the upstream segment.

- **Upstream and downstream stretches**: The comparison of speed distributions of the same run between the two stretches shows that they are all different at a 5% significance level. The speed in one stretch is almost half of that in the other for both cycling directions. This large difference in speed shows that the efficiency is low at the T-junction. Regarding the merging cyclists, it shows that the 90 degree is not an efficient infrastructure design. The same holds for the lane marking in the through direction, as it does not achieve that the speeds upstream and downstream the T-junction remain similar. The through cyclists have quite some interactions where they need to decelerate and give priority to merging cyclists.

The different cycling speeds affect travel time, whose average values per run and direction are shown in Table 2, along with the corresponding standard deviation. It can be seen that the variance is higher for through cyclists, which can be explained by the dependence of travel time on the number and duration of the stops to give way to the merging cyclists. Moreover, through cyclists need on average twice the time to pass, which is attributed partially to the longer distance in the camera view and partially to the delay from the stops.

Levene’s test is performed to test for equality of variances in travel time [19]. The outcome is that there is homogeneity of the travel time variances of merging cyclists, and heterogeneity of variances for through cyclists. This means that an ANOVA needs to be applied to compare the average travel times of merging cyclists, and the Kruskal–Wallis test needs to be performed on the travel times of through cyclists [19]. Both tests result in a p-value < 0.01%, which indicates that the difference is statistically significant between the runs for both directions.

For further insights, KS-tests are performed to compare the underlying travel time distributions. According to the results of the tests, the travel time distributions of through cyclists are the same for runs 1, 3 and 4. This means that there
is no learning effect, and that the presence of different bicycle types does not appear to affect the travel time. Moreover, the lane marking in run 6 leads to travel time savings compared to run 4 given the same cyclist population and the aforementioned lack of learning effect. However, the travel time distribution of run 6 is not statistically different from those of runs 2 and 5. This proves that the heterogeneity of the cyclist population is at least as important in decreasing travel time as the presence of lane marking. Regarding merging cyclists, runs 2, 3 and 4 are the fastest, and their travel time distributions are not statistically different. Runs 1 and 6 follow the same travel time distribution and are slower than 2, 3 and 4. The fact that run 3 is faster than run 1 confirms that there is a learning effect for merging cyclists which makes them faster. The lane marking, despite this learning effect, slows them down (run 6 is slower than 4). As runs 2 and 4 follow the same distribution, there is no evidence for an effect of bicycle type on travel time. The heterogeneity stemming from different personal characteristics is, on the other hand, shown to have a positive effect on travel time. This is because of the difference found in the distributions of runs 1 and 2, but also of runs 4 and 5. To conclude, the lane marking is overall less effective in decreasing the travel time compared to familiarity with a situation (i.e. learning effect) and cyclist heterogeneity.

5.2. Efficiency based on infrastructure use

The space utilisation is visualised per run and direction in the heatmaps in Fig. 4. The colour indicates the share of cyclists that rode over each square of the infrastructure. Note that the maximum share value is different for the two directions; it reaches 60% for through cyclists and 80% for merging cyclists. This means that through cyclists make better use of the full width of the cycle path, while merging cyclists are more concentrated in an optimal trajectory.

Fig. 4a shows the heatmap of the infrastructure usage by through cyclists. It is remarkable that the sublane closest to the left edge of the infrastructure is used by more than half of the cyclists, especially just upstream of the merging location and downstream till the end of the camera view. An exception to this observation is run 4, where the entire width of the cycle path is equally used by through cyclists. In general, the utilisation of the right and middle sublanes seems greatly dependent on the run and group of cyclists. In some runs they are equally likely to be used by through cyclists, while in others the middle sublane is more prominent. The section upstream has the highest variability in use, as some cyclists might not mind to yield and others might shift to the left sublane hoping to avoid a stop. In the case of run 6, the lane marking causes a very small spread throughout the path, as cyclists are concentrated in the left lane both upstream and downstream. It is particularly interesting to notice the difference from run 4, which has the same population and where through cyclists seem to claim the whole width for their own use. This means that with the addition of the lane marking, most cyclists are already in the left lane upstream of the T-junction as intended by the marking. The lane marking, thus, counters the learning effect observed when comparing runs 1 and 3. These runs show that as cyclists get more familiar with the infrastructure (run 3), they optimise its usage by spreading more equally over the three sublanes to improve efficiency. The presence of the lane marking prevents them from doing so and pushes them even more into the left sublane.

The concentration of through cyclists on the left lane in run 6, gives space to the merging cyclists to use the right sublane, as proves to be the case in Fig. 4b. This figure displays the patterns of infrastructure use by the merging cyclists. It can be seen that in all runs, most merging cyclists use the middle and right sublanes downstream of the T-junction. In most cases the share of cyclists in these two sublanes is equal, with a slight preference for the right sublane. The exception is, again, the cyclist population of run 4, where the middle sublane is used by half of the merging cyclists. For these cyclists, the lane marking pushes them to the right of the cycle path and leads to a utilisation of the infrastructure that other cyclist populations could achieve without the lane marking. Regarding the use of the upstream section by merging cyclists, more than half of them use the middle sublane and start shifting to the right sublane at about 2 m upstream of the T-junction.

As already mentioned, merging cyclists seem to follow an “optimal” trajectory. They start in the middle sublane and move to the right to take the turn. As expected, the right angle is too sharp and most cyclists violate the track boundary at the junction. A curved edge that would permit passage into the 67 × 67 cm² would be optimal for most merging cyclists. In order to accommodate all cyclists, the turn should have a radius of 2 m. This value could actually be smaller, since the cyclists lean inwards when taking the turn and their heads are being tracked through the red caps for the construction.
5.3. Efficiency based on interaction with other cyclists

Table 3 shows the number of through cyclists that are observed per run in relation to the interacting merging group size. The merging group size is the number of merging cyclists upstream of the T-junction when a through cyclist passes the trajectory. The observed trajectory is, therefore, shifted slightly to the right compared to where the bicycle is. However, the effect is considered negligible due to the low cycling speeds when taking the turn, and is ignored.

Fig. 5 displays the cumulative probability headway distribution functions per run. Kolmogorov–Smirnov tests indicate that only the distribution of the first run has a statistically significant difference from the rest. The explanation for this might be the learning effect, as this is the first run during which the participants could make use of the T-junction and approached more cautiously than later in the experiment. Another explanation might be related to the demand (i.e. the offered gaps), as one over the mean headway is equal to the average flow of one cyclist per second. In either case, this result shows that the lane marking does not have an influence on the observed headways and, therefore, it is unlikely that it influences the capacity of the junction. Estimation of composite headway models will be performed in the future to provide further evidence.
Fig. 5. Headway distribution per run downstream of the T-junction.

Table 3
Number of through cyclists per run and interacting merging group size.

| Run | Total through cyclists | Average through cyclists per minute |
|-----|------------------------|-------------------------------------|
|     | Merging cyclist group size |                                      |
|     | 0  | 1  | 2  | 3  | 4  | 0  | 1  | 2  | 3  | 4  |
| 1   | 106 | 104 | 23 | 2  | 0  | 12 | 12 | 3  | 0  | 0  |
| 2   | 112 | 80  | 30 | 0  | 0  | 17 | 12 | 5  | 0  | 0  |
| 3   | 107 | 124 | 40 | 5  | 0  | 11 | 12 | 4  | 1  | 0  |
| 4   | 126 | 97  | 34 | 7  | 0  | 12 | 10 | 3  | 1  | 0  |
| 5   | 165 | 113 | 43 | 12 | 1  | 17 | 11 | 4  | 1  | 0  |
| 6   | 149 | 160 | 67 | 10 | 0  | 11 | 12 | 5  | 1  | 0  |

6 m cross section. Therefore, a group size of 0 means that there are no merging cyclists upstream of the T-junction when the through cyclist is at the 6 m cross-section, and thus that the considered through cyclist does not have an interaction. The total of each row in the table is the total number of through cyclists in the corresponding run. As the duration of the runs differs (see Table 1), the absolute number of through cyclists is a function of the corresponding run duration. For this reason, the totals are normalised per minute of run duration. The absolute numbers are shown on the left part of the table and the normalised ones on the right. As an example, in run 3, it occurred 40 times that a through cyclist was interacting with 2 merging cyclists when crossing the 6 m cross section. The duration of run 3 was 10 min, so on average there were 4 through cyclists per minute interacting with 2 merging cyclists when crossing the 6 m cross section.

The primary aim of this table is to demonstrate that most through cyclists have either no interaction or interact with one merging cyclist. Moreover, it is noted that the maximum merging group size is 4 cyclists (though it occurs only once) and that the number of observations decreases as the merging group size increases. A Chi-square test is performed on the normalised results, which shows that there is no statistically significant difference between them.

In order to get a clearer picture of the groups that interact, the distribution of these totals over different sizes of through cyclists is needed. The group size for through cyclists is calculated when each through cyclist crosses the 6 m cross section. The group size is equal to the number of cyclists upstream of the T-junction at that moment. Table 4 summarises this distribution for each run. The values in each cell correspond to the share of through cyclists that interacted with a particular merging group size. So 0% means that the specific combination of through and merging group sizes was not observed during that run, while for example in run 3 (table (c)), there are in total 40 through cyclists that interacted with a group size of 2 merging cyclists and for 5% of these cyclists (i.e. 2 times) there were 7 through cyclists upstream of the T-junction, including the through cyclist that is crossing at that moment the 6 m cross section.

The overall maximum group size of through cyclists observed is 10 cyclists, which means that the maximum expected size of through cyclists is observed but not that of merging cyclists. This can be explained by the fact that through cyclists stop to give way, and therefore, form a queue upstream of the junction. The most common situation in all runs is to have one merging cyclist and up to three through cyclists. The effect of the lane marking in run 6 is that longer queues of through cyclists are observed.

Table 5 summarises the properties of the stops made in each run. The first column shows the run number. Columns two and three display the average and standard deviation of the stopping position in the longitudinal (i.e. x) direction which is the direction of movement. The next two columns refer to the average stopping position in the lateral (i.e. y) direction, averaged separately over each lane. Fig. 2c has already introduced the basic cross-sections of the cycle path.
Table 4
Distribution of through cyclists interactions per run (subtable), group size of merging cyclists (column) and group size of through cyclists (rows).

|       | (a) Run 1 |       | (b) Run 2 |       | (c) Run 3 |       |
|-------|-----------|-------|-----------|-------|-----------|-------|
|       | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  |
| 1     | 19%| 22%| 50% | 24%| 16.6%| 0% | 16%| 12.5%| 20%|
| 2     | 19%| 13%| 0%  | 31%| 20%  | 0% | 34%| 17.5%| 0% |
| 3     | 35%| 43%| 50% | 27.5%| 36.7%| 0% | 24%| 35%  | 80%|
| 4     | 13%| 22%| 0%  | 12.5%| 26.6%| 0% | 10.5%| 10%  | 0% |
| 5     | 11%| 0% | 0%  | 4% | 0%  | 0% | 10.5%| 7.5%  | 0% |
| 6     | 3% | 0% | 0%  | 1% | 0%  | 0% | 2%  | 5%   | 0% |
| 7     | 0% | 0% | 0%  | 0% | 0%  | 0% | 1%  | 0%   | 0% |
| 8     | 0% | 0% | 0%  | 0% | 0%  | 0% | 1%  | 0%   | 0% |
| 9     | 0% | 0% | 0%  | 0% | 0%  | 0% | 0%  | 0%   | 0% |
| 10    | 0% | 0% | 0%  | 0% | 0%  | 0% | 0%  | 0%   | 0% |
| Total | 104| 23 | 2   | Total| 80 | 30 | 0  | Total| 124| 40 | 5 |

Table 5
Properties of yielding stops.

| Run | Position in x [m] | Position in y [m] | % stops in left lane | Duration [s] | Frequency $S$ [stops/min] |
|-----|------------------|------------------|----------------------|--------------|--------------------------|
|     | Average | Std. | Right lane | Average | Std. | Left lane | 45 | 6.0 | 3.8 | 46 |
| 1   | 8.4    | 1.4  | 6.9       | 6.0    | 45   | 30       | 4.6 | 4.5 | 2.5 | 77 |
| 2   | 8.5    | 0.7  | 7.0       | 6.2    | 30   | 48       | 3.5 | 4.7 | 1.4 | 54 |
| 3   | 8.7    | 1.4  | 6.9       | 5.9    | 44   | 26       | 4.6 | 4.7 | 1.4 | 50 |
| 4   | 8.6    | 0.7  | 6.8       | 5.9    | 48   | 21       | 4.4 | 1.2 | 2.2 | 50 |
| 5   | 9.0    | 0.8  | 6.9       | 6.0    | 37   | 51       | 6.5 | 1.2 | 2.2 | 50 |
| 6   | 8.8    | 0.7  | 7.0       | 5.9    | 51   | 51       | 6.5 | 1.2 | 2.2 | 50 |

which are crucial in interpreting the values of Table 5. More specifically, it is known that the longitudinal distance at which the through cyclists enter the conflict area of the T-junction is at 10 m, while in the lateral direction the right lane is between 6.5 m and 7.5 m, and the left lane between 5.5 m and 6.5 m. The sixth column shows the share of the stops that take place in the left lane. Column seven and eight refer to the average and standard deviation of the stop duration, respectively. The last column displays the stopping frequency $S$ expressed in stops per minute.

The stopping positions in the longitudinal direction are tested with Levene’s test and found to meet the homoscedasticity assumption. Only cyclists stopping at the front of the queue are considered in the analysis of the longitudinal stopping positions. ANOVA shows that there is no statistically significant difference at a 5% significance level. The average stopping position is, thus, at around 8.7 m, which justifies the choice of the 6 m cross-section as the decision point for the reaction to the encounter.

In the lateral direction, the two lanes that are formed by the lane marking are separately considered. All stopping cyclists are taken into account in the analysis. The stops in the right lane are at 7 m and at 6 m in the left lane, which are the middle points of the width of the corresponding lane. Though the difference in these values is not statistically significant between the runs, the share of cyclists that stop in each lane changes. This share ranges from 30% to 51%. The highest value is observed in run 6, but it is very close to the 48% of run 4, which has the same cyclist population. The lane marking does, thus, not affect the stopping position. The cyclist population plays a more important role in this decision.

It is remarkable to note that only through cyclists stopped to give priority and no merging cyclist. Based on this, it could be concluded that the priority perception is not inverted at bicycle T-junctions, though one should be aware of the experimental conditions.

The differences in the *stop duration* are statistically tested. Levene’s test rejects the assumption of homoscedasticity. The applied Kruskal–Wallis test leads to the conclusion that at a 5% significance level the medians of the stop duration of the six runs have a statistically significant difference. In this case, the introduction of the lane marking makes the stops
of the through cyclists last longer, which means that it reduces the efficiency of the T-junction. A possible explanation for
the longer stops is that the group sizes that interact are larger and the queues that are formed are longer, so the cyclists
that are at the back of the queue need to wait a long time.

The stopping frequency (number of stops per minute) varies a lot among the runs. The averages, however, should not
be used for the comparison as they are sensitive to outliers and Levene’s test rejects the hypothesis of equal variances.
Therefore, the Kruskal–Wallis test is performed. The outcome is that there is no statistically significant difference in the
medians of the stopping frequency of the six runs at a 5% significance level. Apparently, the lane marking does not have
a major influence on the number of stops. This means that despite the lane marking, cyclists felt obliged to stop just
as frequently and give priority to merging cyclists who would not be able to merge into the main track comfortably
otherwise.

6. Conclusions and recommendations

We investigated the cyclist behaviour at T-junctions dedicated to bicycles and assessed the efficiency of the T-junction
design on the bicycle flow. The analyses were performed using empirical trajectory data collected during a large-scale
cycling experiment. The effect was studied of adding lane marking that advised through cyclists to shift to the left so that
merging cyclists could occupy the space to the right of the cycle path. The expected outcome was that the two conflicting
streams would be separated in space, thereby increasing flow efficiency. In order to assess the efficiency, a framework
was proposed with eight indicators that cover different aspects of the infrastructure efficiency.

The findings overall suggest minimal to no effect on the efficiency once the lane marking is introduced, but great
effect resulting from the heterogeneity of the cycling population, especially stemming from personal characteristics and
less so from the different bicycle types. The speeds of through cyclists increased by adding the lane marking, and not by
the repetitive nature of the experiment. However, cyclists in other runs could reach even higher speeds, indicating that
the personal characteristics are at least equally important. With respect to merging cyclists, it was shown that there is
a learning effect towards an "optimal" trajectory, which had a positive effect on speed. Despite the increased confidence
due to familiarity, the introduction of the lane marking slowed merging cyclists down, as it made the perception of the
90 degree turn even sharper, forcing them to use only the space of the right lane. This way, the desired outcome of clear
lane separation was achieved, while at the same time it was observed that cyclists in some runs could self-organise,
without guidance, into such separated flow. Regarding the capacity, no change was observed in the time headways. The
same can be stated about the stopping frequency of through cyclists, even though the duration of the stops increased and
longer queues were formed. The combination of these findings means that there is no obvious advantage, as originally
hypothesised, in the introduction of the lane marking, but possibly a negative one, as the duration of the stops, and hence
the delay of the through cyclists, increased.

Based on these findings, the design recommendation would be against using such a lane marking at bicycle T-junctions.

Through cyclists are capable of making space to allow merging cyclists to fit in the cycle path without instructions. This
self-organisation might even prove more efficient as they can use the full width of the cycle path when no encounter is
about to take place. Moreover, the trajectories of merging cyclists showed that a turn with a radius of 2 m would
accommodate all cyclists. In [17], a minimum radius of 5 m is advised, such that the speed of cyclists is not interrupted.
It is indeed the case that the speed is reduced, but this more cautious approach might lead to safer interactions at the
T-junction.

An important outcome of our analysis is that no merging cyclist stopped to give priority to through cyclists. This
could indicate that cyclists respect the priority traffic rules and there is no inverted perception of priority at bicycle T-
junctions. An alternative explanation for this might be the experimental setting under which participants felt obliged to
abide by the traffic rules and yield to traffic coming from the right. In order to conclude on perceived priority, real-world
observations should be collected and analysed. Moreover, the effect of a wider range of densities on the cycle path should
be investigated, along with the effect of different cycle path widths. Additionally, a topic for future research is to compare
the findings of this study with the efficiency at a T-junction with priority given to the through traffic, either by means of
signage or in countries where traffic from the left has priority and based on that conclude which rule leads to the highest
efficiency. Apart from this, it is of interest to investigate when a decision to yield is made instead of a decision to make
space. Modelling this decision making process will be the focus of future research.

CRediT authorship contribution statement

A. Gavriilidou: Conceptualization, Methodology, Formal analysis, Investigation, Data curation, Writing - original
draft, Visualization. W. Daamen: Conceptualization, Methodology, Investigation, Writing - review & editing, Supervision.
Y. Yuan: Methodology, Writing - review & editing, Supervision. N. van Nes: Investigation, Funding acquisition. S.P.
Hoogendoorn: Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have
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