Proposal for Compositeness of String out of Objects – Fake Scattering, Finite String Field Theory Formulation

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Bled , July , 2020

Abstract

We review a bit our earlier novel string field theory[1, 4] stressing the interesting property, that it becomes expressed in terms of particle like objects called by us “objects” which in our formalism do not at all develop in time. So in this way there is in our picture, in spite of it being supposed to reproduce string theory with an arbitrary number strings present - in this sense a string field theory -, in fact no time! This strange missing of time in the formalism gives rise to slight speculations about the philosophy of the concept of time. There is course then also no need for a Hamiltonian, but we construct or rather attempt to do so, a fake Hamiltonian or phantasy Hamiltonian,

Novel String Field Theory and Unitarity, although in non-relativistic case (Fake Scattering, Hope for finiteness).

1 Introduction

We have long worked on new/novel formulation [1] of the bosonic string theory as a second quantized theory in the sense that we describe states with an arbitrary number of strings, an achievement made earlier by String Field Theory by
Kaku and Kikkawa and by Witten and others (for Bosonic string theory [2]).

Our approach has some similarity to the work by C. Thorn [3] in as far as we also use a splitting of the string into small bits. We, however, did it after the description of the single string has been resolved into right and left moving fields $X^R_\mu(\tau - \sigma)$ and $X^L_\mu(\tau + \sigma)$ (using the often used notation in string theory in so called conformal gauge) and most remarkably we achieve that we do not need any time development for what is the analogue of Charles Thorns small bits, we call ours as said somewhat different small parts for “objects”. The “objects” stand, we could say, still[4].

It is this property of our string field theory which seems so interesting, that we indeed shall concentrate on precisely this no time development property in the present article.

We thus seek in the present article to go backwards in the sense that we begin by considering such objects, that do not develop, and then afterwards shall seek by what we call “phantasy” to return to the string theory able to describe several strings. We believe it would be not so terribly hard to make the same “object” description of the various superstrings, presumably by having “objects” being fermions with odd spin, but of course they should still not develop in time. Both because it would complicate the presentation and because we have not developed yet the string field theory of ours with such spin-objects yet - we did not even quite finish the bosonic version - we shall keep to the boson string theory in the present article.

The real motivation is to seek generalizations of the string theory. Indeed the main reason that superstring theory is so popularly speculated to be the “theory of everything” is that it avoids the ultraviolet divergence problems plaguing the point particle quantum field theories. This avoidance of ultraviolet divergences is connected with or a consequence of that the string scattering amplitudes - the Veneziano models - are strongly indeed exponentially cut off for large momentum transfer. In our “object” picture this cut off at large momentum transfer can be traced back to that the distribution of the momenta of the objects is cut off in a similar way.

The scattering in our picture of objects has the character of two composite particles (\sim strings) composed from the “objects” exchange some collections of objects with each other. Thus after the scattering the momenta of the final composite particles (\sim strings) can only deviate from those in the initial state by the momenta of the exchanged collections of objects. We shall namely remember the for this argument to be valid extremely important point that the single object never can change its momentum, because it does not develop at all, especially it does not scatter by itself. The whole seeming scattering of the composite particles composed from objects is purely “fake” in the sense that they are a result of some objects being transfered from one compositum to another one.

(1 - 1 )Fake Scattering Concept

We are so fascinated by this idea of making a quantum field theory like theory, so that in a fundamental sense there is No Timedevolution, but when looking at it appropriately, then you can “see” it as e.g. string field theory (a theory of second quantized strings).

This fake-scattering concept is implemented in the “Novel string field theory”
which we have put forward long ago.

(1 - 2) Hamiltonian = 0 gives no time-development

So quantum mechanically the no time-development theory is just a Hilbert space of the states, and they never develop - there is basically no time needed -.

In the “Novel string field theory” of ours the states in this Hilbert space are described formally by a second quantized theory of particles that can occur in different numbers just like in usual second quantized theory. We call these particles “objects” and they are crudely to be considered small pieces of strings like in the Charles Thorn’s string bit theory. But very importantly we first split the string into bits, after we have gone to the light cone variables on the string: \( \tau - \sigma \) and \( \tau + \sigma \).

(1 - 3) Introduction of Fake Degrees of Freedom

In the philosophy that the true fundamental theory has no time (or say no time development) means that all development with time has to be fake. That is to say it has to be in some degrees of freedom, that do not really exist in nature, but which we the physicists introduce formally so as to make a theory more in agreement with our usual picture of how physics is.

Basically the idea is that we introduce some extra degrees of freedom that only are there in phantasy, so that we construc formally a system/a world with some extra variables or some extra information on its states. These extra informations shall however only in some way be adjusted to help describing the original degrees of freedom, which we call the true degrees of freedom. In the case we here hope to realize; the original or true degrees of freedom are the ones for the object. But by the addition of the extra degrees of freedom we have in mind giving an information about how the chains of the objects are glued together in possibly different ways. These different ways we hope to describe by means of the extra phantasy degrees of freedom. It is mainly how the cyclically ordered chains of objects are, one can say the extra degrees of freedom should tell which objects belong to which cyclically ordered chain. Thereby the extra phantasy degrees of freedom also come to tell which objects belong to which string. Thus some strings exchanging objects can be a pure phantasy happening. This is what is called than scattering is a fake.

Abstractly we replace each basis vector in a basis for the second quantized Hilbert space by a series of basis-vectors. So to each “fundamental basis vector” we have a lot of basis-vectors in the extended theory only deviating from each other by invented or fake degrees of freedom.

Then we allow the “fake-development” - the fake Hamiltonian - to only move around the basis-vectors into each other which belong to the same fundamental basisvector.

(1 - 4) A String Field Theory Inspired Example

To a good enough approximation the readers can imagine that our “objects” (after some technical details of only using the “even” ones among them) are (scalar) particles with position and momenta in a 25+1 dimensional world (or if we choose an infinite momentum frame in 24 transverse dimensions), and that there in any single particle state for such a particle can be a number of particles \( n = 0, 1, 2, ... \), just as in second quantization.

To avoid the problems with relativity, Dirac sea etc., we like to for pedagogical reasons effectively consider a non-relativistic theory, or almost equivalent
an infinite momentum frame formulation.

\textbf{1- 5) The Pedagogical Non-relativistic model with Zero Hamiltonian}

We consider a model with say non-relativistic bosons - so that they can occur in any number in any single particle state -. To make the theory not develop in time we want to simplify to make the Hamiltonian zero

\[ H = 0, \]  

(1)

which in addition to having no interactions mean that we let the non-relativistic mass

\[ m \to \infty, \]  

(2)

so that even the kinetic term \( \frac{\vec{p}^2}{2m} \) goes to zero.

We can choose a basis for the single particle states to be e.g. either the momentum eigenstates or the position eigenstates (a priori as we wish).

\textbf{1- 5 - 1) Second Quantizing our \( H = 0 \) Particle Model:}

As basis to use in single particle Hilbert space we shall here choose the position eigenstates because we like to investigate about a “nearness” concept (we want to say if two particles described by such basis vectors chosen are close or far apart.)

Then the corresponding basis in the second quantization state space is enumerated by a function, that to every position \( \vec{x} \) assigns a number \( n(\vec{x}) \) giving the number of particles with exactly the position \( \vec{x} \).

In other words a second quantized basis-vector can be described by the number \( n(\vec{x}) \) of “objects” (=particles) in each position \( \vec{x} \):

\[ n : \mathbb{R}^{24} \to \{0, 1, 2, ...\} \]  

(3)

and we cannot require it continuous unless we take it to be only constant, because a continuous function mapping the real number type of space \( \mathbb{R}^{24} \) into a discrete space, the positive integers and 0, can only be constant if it is continuous.

However, we shall at this stage not describe about continuity.

\textbf{1- 5 - 2) Second Quantized Basis}

A basis - and this is the one we now have chosen to use - in the second quantized state space consists of vectors like

\[ |n> = \prod_{\vec{x}} \frac{a^{\dagger}(\vec{x})^{n(\vec{x})}}{\sqrt{n(\vec{x})}} |n = 0 > \]  

(4)

where \( a^{\dagger}(\vec{x}) \) is the creation operator for a particle at the position \( \vec{x} \). The symbol \( \mathbb{R} \) stands for the set of real numbers.

Remember

\[ n : \mathbb{R}^{24} \to \{0, 1, 2, ...\}. \]  

(5)

\textbf{1- 5 - 3) Introduction of the Fake Degree of Freedom “The Successor Function” \( f \)}

Our extremely simple \( H = 0 \) theory just introduced has a priori nothing to do with strings (nor much other sensible physics for that matter), but now we want by just explaining to make it into a string field theory!
The dots are point’s with an \( n(x) = 1 \). The arrows denote the action of \( f \).

Figure 1: The points denote the objects and the arrows denote the action of the permutation function \( f \). Since a permutation can be resolved into cyclic permutations, the objects will on this drawing get into cyclically ordered chains.

For each single one \(|n>\) of our basis states in the second quantized space we want to introduce a “successor function” \( f \), which is a permutation of the particles present in that state.

In the state \(|n>\) there are \( N(n) = \sum_{\vec{x}} n(\vec{x}) \) particles present. Here we assumed that there were not infinitely many particles present.

The “Successor function” \( f \) is a Permutation of the Particles present in the state \(|n>\).

Assuming that there are only finitely many particles in a second quantized state vector \(|n>\) we can think of these \( N(n) \) particles as true particles, and you could define \( N(n)! \) permutations \( f \) of the \( N(n) \) particles present.

We Think of a Phantasy-space with \(|n>\) replaced by \( N(n)! \) new phantasy basis vectors representing the same true physics.

So the new basis-vectors in the second quantized space should be denoted

\[
|n, f > = (|n>, f) \quad \text{where} \quad f \in P_{N(n)}
\]

where again

\[
N(n) = \sum_{\vec{x}} n(\vec{x})
\]
is the number of particles in the state $|n>$.  

**Working with Phantasy space Makes Life Easier**

Of course it is $f$ which is the phantasy degree of freedom. It was just introduced by us.

"Thus we can decide in the following rule:

We throw away all the choices of permutations $f$ unless it fulfills the following rule (we do so for some reason to be explained later): The position $x_{first}$ of a particle $first$ being mapped by $f$ to $f(first)$ must have a position $x_f(first)$ close to $x_{first}$. That is to say, we require only to include in our phantasy space such combinations that $f$ should satisfy $f(first)$ is close to first, i.e. $|x_{first} - x_{f(first)}|$ should small.

If $f$ does not obey this restriction, we simply take it out and let there be fewer state vectors in the phantasy Hilbert space."

**We can phantasize that $f$ describes successors in long almost connected chains**

We can choose the $f$ permutations, we allow, to be such that they describe connected closed loop chains of the particles in the state, so well it is possible.

From our purpose of making theory to be part of a speculated theory for everything we could be allowed to postulate something - if beautiful enough - also about the state of the universe, such as that the most likely type of state is one in which the particles sit in long circular chains with rather small distance between the neighbors and even further assumptions involving the momenta.

(1 - 6) About Fundamental Physics We can further only make assumptions about the Initial and/or Final states

After we settled on no time-development ( $\sim$ Hamiltonian being zero) we can not as physicists looking for the right theory of nature anymore speculate about the Hamiltonian, because that we already took to zero (as operator).

But we may want to have a bit of chance to assume a little bit to adjust to fit our hoped for model to experimental information etc.

Then we have the chance of speculating about the initial state (which is also the final state though, when no development).  

**Assumptions about Initial and final state**

For our purpose with the string theory towards which we are driving in mind we like to make assumptions about initial condition like this:

- (a) An approximate constraint on the relative state of a couple of particles $A$ and $f(A)$, namely

$$k(x_{f(A)} - x_A) \approx \vec{p}_{f(A)} + \vec{p}_A,$$

(9)

where $k$ is a constant, actually related (as to be seen) to the Regge slope $\alpha'$ so important in string theory.

- (b) The particles shall approximately form cyclic chains.

- (c) And they shall even especially locally along the chains have a certain wave function like they would have in string theory if they were identified with the “objects” of ours (which we have not yet described in detail.)
If the points of the state $|n\rangle$ happens to be in chains one can easily find a successor function $f$ so that successors are close.

Figure 2: We think of the objects sitting (due to some assumed principle about the likely configuration of the objects) in cyclically ordered chains, that can be described by giving the permutation function $f$ as here illustrated by the arrows. The reader can imagine continuing marking the arrows to tell how $f$ acts.
Assumptions about (Initial) State Formulated by Density Matrix $\rho$

Whatever assumption about a quantum system one might want to make it can in principle be written by means of a **density matrix** $\rho$.

$\rho$ is a positive operator on the Hilbert space of state vectors for the system normalized to $Tr(\rho) = 1$.

We have one $\rho_{\text{fundamental}}$ for the “fundamental degrees of freedom, and we can partly choose one $\rho_{\text{full}}$ for the combined system of the fundamental and the phantasy degrees of freedom system. Then you can act

$$\rho_{\text{fundamental}} |n>$$

or

$$\rho_{\text{full}} (|n>, f)$$

**Density Matrix Relation**

We shall naturally require for consistency

$$<p | \rho_{\text{fundamental}} | n> = \sum_f <p | f \rho_{\text{full}} (|n>, f) \quad \text{(10)}$$

or formulated differently:

$$\rho_{\text{fundamental}} = Tr_{\text{w.r.t. phantasy}} \rho_{\text{full}} \quad \text{(11)}$$

So far we talk about timeless density matrices.

But could we make a purely phantasy time development of only the phantasy or f-degrees of freedom without disturbing the fundamental ( |$n>$, |$p>$,... ) degrees of freedom?

**Stringy Initial State Assumptions, and Phantasy Notation give String Field Theory**

The point is we put a fairly large amount of string theory into assumptions about the initial state, partly because we cannot do it in the proper Hamiltonian.

The assumption,

“An approximate constraint on the relative state of a couple of particles $A$ and $f(A)$, namely

$$k(\vec{x}_{f(A)} - \vec{x}_A) \approx \vec{p}_{f(A)} + \vec{p}_A, \quad \text{(12)}$$

where $k$ is a constant, actually related (as to be seen) to the Regge slope $\alpha'$ so important in string theory.”

This assumption would if the particles did not have infinite masses mean that the cyclic chain would move along itself.

**Yet a complication in relating the trivial static theory to string theory**

The cyclic chains of particles are **not** simply the strings when we identify with string theory - as it would be in Charles Thorn’s theory - No.

We have to choose a starting point and go along the cyclical chain from that with two marks in opposite directions along the chain, and then construct for each step an average of the two “people” that started at the start. It is the series of average under this trip of the two “people” that makes up the string.

In this way we get an open string from making this two “people” walk on a cyclical ordered chain.
Figure 3: How to construct an open string in terms of objects: You need only one cyclically ordered chain for giving an open string with apart from a factor 2 having the points of the string being the averages of the two object points in the pair. Of course one can pair the objects in different ways even keeping to the continuous type of way illustrated, and that then give the string at a different moment of time. To ensure that the reader identifies the right small spots with the objects one may count that there are 26 objects on this drawing. Corresponding to that there must be 14 points on the open string. The long curved arrows just point to two objects forming a pair, but here are 14 “pairs”, corresponding to 14 points on the string, the objects at the ends of the string being paired with themselves.
Figure 4: How a closed string is constructed from objects: You need for that two cyclically ordered chains of objects (as ordered by $f$, but $f$ is not drawn on this figure) and except for normalization by a factor 2 you construct the averages for pairs of objects with one object in the one cyclically ordered chain and the other member of the pair in the other cyclically ordered chain.
Main Point: Brought although a bit complicated a correspondence to String Theory

To a set of strings in a known state - e.g. the ground state of their oscillations - one can calculate the state of the corresponding particles (which we usually call “objects”) sitting - ordered by the faked $f$ description - in a cyclic chain for each open string (we postpone the closed strings for the moment).

I.e. **We can pretend to see string theory in our game with infinitely heavy particles.**

Most remarkably: When we calculated the overlap between two different sets of strings represented as second quantized states of objects, we got - apart from a wrong sign (a missing $i$) - the form of the **Veneziano model**.

As the typical example we considered an initial state with two cyclically ordered chains of objects representing two open string in the states of the ground state of bosonic strings. Then as final state we took a similar state of objects corresponding to two ground state open string. We allowed, however, these open strings to have arbitrary momenta. Then the overlap indeed run out to become the four point Veneziano model for the two incoming and two outgoing particles identified with the strings. We did though get two “small” problems: 1) We had expected to get a Veneziano scattering amplitude being a sum of three Beta-function terms, but we got only one term. 2) If we should expect the overlap we calculated to be identified with the S-matrix element - as the S-matrix in theory without time development would be expected to be 1 - we should have gotten the Veneziano model with an extra factor $i = \sqrt{-1}$ because there is in the expression for the S-matrix in terms of the amplitude which is real in lowest order Veneziano model with an extra $i$.

But that was what we got at first.

**Correspondence with Veneziano Model rather short via thinking on surfaces of string development**

(1) - 9) **Important step in Showing Veneziano Model from Our Novel String field theory**

You think of external ground state strings. They can be produced as in general ground states - by a long imaginary time development with the appropriate
Hamiltonian. This development is then written as in complex time development of the string, very reminiscent of what it always used in string theory to compute say Veneziano model.

Very crudely we just give a motivation for this kind of functional integral description of the strings.

Really we do it with a doubled string; i.e. we have a closed string diagram describe the open string. So there are some complications but we did manage to one of the three terms.

Changing Phantasy Degrees of Freedom can Change Number of Cyclic Chains and thus of (Open) Strings

For different “successor functions” $f_1$ and $f_2$ you can find different numbers of cyclic chains even for completely the same configuration of the infinitely heavy particles (="objects") and thus in fundamental physics-wise the same situation $|n>$. 

Take a fundamental physics situation $|n>$ like this:

In choice $f_1$ of the phantasy you have one open string

In another choice $f_2$ of the Phantasy gives Two chains, thus Two open strings

Unification of strings can be change of $f$, thus phantasy, because changing actually actually only a couple of values of the successor function $f$ can cause that e.g. a previously closed cyclically ordered chain of objects get split into two such chains. Since this would correspond to one open string being split into two, we see that splitting can be easily described by changing $f$. Oppositely of course the opposite change in $f$ would mean a unification of two to become one open string.
Successor function $f_1$ chosen so as to make only one cyclic chain.

Figure 7:

Successor function $f_2$ chosen so that there are two cyclic chains. But fundamental physics - the particles - is the same.

Figure 8: On this figure the reader sees the same points illustrating the "objects" as on foregoing figure, and we hope the reader can see that one can have two separate cyclically closed chains that though come very close to each other at some point. It is such cases that the successor function $f$ can be changed a few places and still be in agreement with the approximate requirement that the successor function maps one object to the next in a chain to which it belongs.
Two strings can become one just in the phantasy of $f_2 \rightarrow f_1$.

Figure 9:

2 Hamiltonian

How to make a purely Phantasy Hamiltonian

The exercise we want to do now is to see what Hamiltonian is allowed working on the extended Hilbert space containing also the phantasy degrees of freedom, so that the basis states are

$$(\text{Extended}) \quad \text{Basis states of the form} \quad |n>,f$$

while

$$\quad |n>$$

Thinking of matrices the extended operator (matrix) consists of a lot of blocks, one block for each matrix element in the (original) fundamental Hamiltonian (which is actually zero).

Attempting to find a phantasy Hamiltonian only moving the Phantasy Degrees of Freedom

For any operator depending only on the physical degrees of freedom $O$ we want the “only phantasy” Hamiltonian $H_{\text{phantasy}}$ candidate to commute with it:

$$[O,H_{\text{phantasy}}] = 0.$$  \hspace{1cm} (15)

This condition is, however, too strong, since it would not allow the Hamiltonian to depend at all on the “fundamental” degrees of freedom, because if so, the conjugate variable to the one it depended on would be made to vary (and that we wanted to exclude).

We must be satisfied with only having this requirement **approximately**.

**Not so good Argument that we can have a wanted** $H_{\text{phantasy}}$ **approximately**.

We want the development in the $f$ or phantasy degrees of freedom only to depend on that some cyclic chains come very close / touch; and that is dependent on only very few particles/“objects”, so at least it does only involve at first few among an extremely big number of “objects” in the interesting situation.
2.1 An Attempt to an $H_{phantasy}$

If one thinks of using position eigen-state say, one could enumerate the particles by the position $\vec{x}$ or we could say that the number $I$ were a function of the position $\vec{x}$ defined for the values of the latter a particle, i.e. defined whenever $n(\vec{x}) = 1$ or bigger. In the case when there are positions with more than one particle the particle number $I$ would in addition have to depend on a small $i$ number enumerating the particles at one special position $\vec{x}$ say. So we would in this more general case write

$$I = I(\vec{x}, i) = \text{The number } I \text{ of the } i\text{'th particle at the position } \vec{x}. \quad (16)$$

where the $I$ is then a name or a number assigned to the $i$th object at the position $\vec{x})$. In order that at least Hamiltonian $H_{phantasy}$ should commute with those operators $O$ that could be constructed out of the position operators of the “objects” alone, we would have to make the $H_{phantasy}$ operator at least not change the second quantized state in the position basis formulation, when acting on it. As soon as we would make an image of $f$ say $f(I)$ to be change to be an object sitting at a different position than before the action, this could give rise to the change by the action of some only on the positions depending operator, and thus this would not be allowed.

This consideration would leave us with only the possibility that the action with $H_{phantasy}$ to change the image $f(I)$ from what it starts being to the number for an object with the same position.. That is to say that if say

$$f(J) = I(\vec{x}, i) \text{ before the action with } H_{phantasy},$$

then

$$f(J) = I(\vec{x}, k) \text{ (where } \vec{x} \text{ is the same, but } k \text{ can be different from } i). \quad (18)$$

This means that under the operation of the phantasy Hamiltonian we can as far as these $\vec{x}$-representation considerations go change the $f$ into another $f$ let us say $f'$ obtained by multiplying it - in the permutation composition way - from the right by a permutation of a subset of objects sitting on the same position. If $P_\vec{x}$ denotes the subgroup of the permutations of the objects with position $\vec{x}$, we can say it would be allowed that the successor function $f$ changes into $f'$ with $f' = f \circ p$ for some $p \in P_\vec{x}$ for some position $\vec{x}$.

That is to say that worrying only about the (fundamental) position dependent operators w.r.t. whether they commute with the phantasy hamiltonian we can allow matrix elements like:

$$< n, f \circ p | H_{phantasy} | n, f > = g \text{ (some nonzero value) } \quad (19)$$

where $p$ is a permutation of objects with the same position. (The value $g$, which we must here introduce, will turn out to be proportional to the coupling constant, also usually called $g$ in the formulation of veneziano models.).

This proposal is, however, although it looks at first o.k. not good: The point is that if we concentrate on some successor function $f$ having resulted by multiplication with such a permutation $p$ then you will in that state find that there is an infinite uncertainty in the relative momentum for the two objects that had the same $\vec{x}$ position. It would act much like they had scattered with a pointlike interaction. This would mean indeed that the $H_{phantasy}$ had changed...
the state of the fundamental degrees of freedom and we wanted to avoid that. Because if our playing or phantasy Hamiltonian truly change the state of the true fundamental degrees of freedom it is not truly only phantasy.

The occurrence of the need for selecting a permutation of the objects sitting on the position \( \vec{x} \) say a priori is a little freedom to be specified, but in what we think should be the most important situation:

- (2.1.a) That there are in most positions \( \vec{x} \) no objects at all, i.e. most \( n(\vec{x}) = 0 \).
- (2.1.b) and the dominant part of the rest of the positions have just one object, i.e. next to \( n = 0 \) it is the value \( n = 1 \) that is most common.
- (2.1.c) Continuing this way with falling numbers of positions the higher \( n \), the first and dominant value of \( n \) for which a non-trivial permutations of the objects at the position is \( n = 2 \) for the position in question. And in this case there is only one nontrivial (i.e. not unity) permutation of the objects at the position. So in this most copious non-trivial case the permutation \( p \) is not ambiguous.
- (2.1.d) Finally we expect higher \( n \)-values than 2 to be extremely seldom, and we may ignore approximately this possibility.

Of course it is so to speak the choice of the density matrix \( \rho \) which shall give the a priori probability for how to find the system of objects that should be made so that we have this probability for \( n \) taking a given value to fall rapidly with the size of this value. It is actually very natural with such a property in the limit of the \( \vec{x} \)-space going to a continuum.

The reader may check that we also without causing any problem for the conservation under the phantasy Hamiltonian development of the operators depending on the positions for the objects by also allowing

\[
< n, f | H_{\text{phantasy}} | n, p \circ f > = g^* \quad (\text{the complex conjugate of } g), \quad (20)
\]

so that we can achieve that the phantasy Hamiltonian is a Hermitian one in the phantasy Hilbert space constructed as tensor product of the space with the \( f \)'s as basis vector marks and the fundamental Hilbert space. So we can claim we arrange:

\[
H_{\text{phantasy}} = H_{\text{phantasy}}^\dagger. \quad (21)
\]

2.2 The problem of keeping fundamental degrees of freedom fixed

But this proposed \( H_{\text{phantasy}} \) will not commute with the relative momentum of the typically two objects being permuted by \( p \), because the expression we proposed depends on the relative position and thus will not commute with the conjugate momentum.

Actually as already said it is impossible to solve this problem except at best approximately somehow. If we truly arranged that the phantasy Hamiltonian should commute with all operators from the fundamental degrees of freedom, we would be forced to have a phantasy Hamiltonian only depending on the pure phantasy degrees of freedom, and that would not be so fun.
But we anyhow want to speculate that such a phantasy Hamiltonian can act approximately without disturbing the fundamental degrees of freedom significantly. E.g. one could speculate that as described in the now following subsection, it would approximately commute enough under assumption of the density matrix distribution.

2.3 A rather bad example for idea of concrete $H_{\text{phantasy}}$

Since we actually have just seen that a fully satisfactory phantasy Hamiltonian is impossible (see the argument below formula (14)), just propose one that has difficulties in the sense of not commuting fully with the fundamental degrees of freedom - meaning operators acting only on the original basis $|n>$ space - would still be of interest. To give the possibility to work on with the idea of constructing a phantasy Hamiltonian that functions approximately let us indeed build the proposal from an operator $N(I,J)$ supposed to act on the space of fundamental states and being effectively zero in all cases when the objects $I$ and $J$ are not close to each other and only significant when these two objects are close to each other. You may take it that it is so to speak a “nearness operator”, and that is why we called in by the first letter $N$ in the word “nearness”. Such an operator $N(I,J)$ is to be considered an operator of the same kind as an interaction between the two objects $I$, and $J$, and thus could be written as a convolutions by some function possibly involving smeared delta functions. The operator $N(I,J)$ of course only act on the wave functions for just those two objects $I$ and $J$, so it could be written acting on the space of all the objects represented by wave function like $\psi(x_1, \ldots, x_N)$ as

$$N(I,J)\psi(\vec{x}_1, \ldots, \vec{x}_N) =$$

$$\int \int d\vec{x}_I^J d\vec{x}_J dK(\vec{x}_I, \vec{x}_J; \vec{x}_I^J, \vec{x}_J)\psi(\vec{x}_1, \ldots, \vec{x}_{I-1}, \vec{x}_I, \vec{x}_{I+1}, \ldots, \vec{x}_{J-1}, \vec{x}_J, \vec{x}_{J+1}, \ldots, \vec{x}_N)$$

Of course in order that $N(I,J)$ be a nearness operator the to be chosen function of four spatial vector $K(\vec{x}_I, \vec{x}_J; \vec{x}_I^J, \vec{x}_J)$ should vanish for any of the four arguments being far away from the other ones.

We should also think of it as being in spite of its locality rather smooth so that it does not change the momenta of the objects $i$ and $J$ too much. Actually the reader should understand that we are hoping for - the impossible - that we have an operator just testing if the two objects numbered $I$ and $J$ are near each other, but preferably without disturbing them. But we know from the discussions of Niels Bohr etc. that in quantum mechanics you cannot measure without disturbing.

Anyway let us go on for pedagogical reasons as if we had arranged an operator $N(I,J)$ that could just observe without disturbing. This would be an only classical intuition that could have that.

If we anyway fall back on classical intuition, we could as well really take it as if the operator also asked for nearness in momentum space, i.e. it should be arranged to only be significant in size for the two objects having approximately the same momenta also.

Supposing we now had a for practical purposes such nice operator checking if two objects are in the approximately same point in the phase space $N(I,J)$. The idea then is that we shall by means of it construct a term in the phantasy Hamiltonian that if $N(I,J)$ is non-zero will permute the actions of the successor
function \( f \) on the two objects involved. That is to say that with a weight \( N(I, J) \)
the successor function \( f \) shall be changed so that the images of \( I \) and \( J \) are no
longer as at first \( f(I) \) and \( f(J) \) respectively, but oppositely \( f(J) \) and \( f(I) \).

This means that we define a term to be put into the phantasy Hamiltonian

\[
H_{IJ} = N(I, J)P_f \rightarrow f_{opIJ},
\]

where \( P_f \rightarrow f_{opIJ} \) is an operator only acting on the phantasy-degrees of freedom,
\( i.e. \) on the \( f \)-part, by permuting the two object(numbers) \( I \) and \( J \) before the
action of \( f \). Here the permutation \( p_{IJ} \) means the permutation permuting the
two objects \( i \) and \( J \).

The full proposal for the phantasy Hamiltonian should then be the sum over
all pairs of different objects \( I, J \), with \( I \neq J \).

That is to say we propose the phantasy Hamiltonian to be of the form:

\[
H_{\text{phantasy}} = \sum_{\{I, J\} \text{ with } I \neq J} H_{IJ}
\]

\[
= \sum_{\{I, J\} \text{ with } I \neq J} N(I, J)P_f \rightarrow f_{opIJ}.
\]

This phantasy Hamiltonian is made so as give some topology change - change
in the way the objects are thought to hang together in chains (the cyclically
ordered chains) as the phantasy-time goes on, but only provided the chains
almost coincide where the change takes place. This will correspond also when
translated into strings shifting the topology of how they hang together to only
 glue the strings in a new way at places where they touch. This is what you
expect for physical strings also: they only interfere when they touch.

As already stressed the \( H_{\text{phantasy}} \) here is at best approximately o.k.. We can
argue that it is not so bad again by remarking that if one thinks on strings with
infinitely many objects in them and that we can arrange the interaction between
the strings to be sufficiently weak - by putting the coupling \( g \) above absorbed
in \( N(I, J) \) sufficiently small - so that one only has about one interaction at a
time, meaning that only one out or two of infinitely many objects get disturbed
by the operators \( N(I, J) \).

2.4 Mathematically Formulated Approximate \( H_{\text{phantasy}} \) Restriction

One idea to make a concrete statement of the sought for purely phantasy hamiltonian,
that should preferably only move the phantasy degrees of freedom \( f \) but
not the fundamental degrees of freedom \( n \), would be to replace the hoped for
\([O, H_{\text{phantasy}}] = 0\) requirement by the milder

\[
Tr(p[O, H_{\text{phantasy}}]) = 0,
\]

for all genuinely fundamental operators \( O \) and the assumed density matrix \( \rho \)
expressing our assumption about the state of the system of “objects”. We should
presumably most wisely only take this relation in the limit of infinitely many
objects and then we can hope as just mentioned that a single object being a bit
pushed would not count very much, if it stands inside the quantum fluctuations.
It is easy to see by a bit of trivial algebra that if we choose $H_{\text{phantasy}}$ to commute with the density matrix $\rho$ we get fulfilled \((25)\).

This means that we should look for arranging that our $N(I,J)'s$ in the phantasy Hamiltonian commute with the density (matrix) operator $\rho$.

2.5 Unitarity

Once we have settled on a formalism with a constructed phantasy Hamiltonian, we can of course construct corresponding time development operators, say the time-development operator from time $t_1$ to time $t_2$ would be

$$U(t_2,t_1) = \exp(-iH_{\text{phantasy}}),$$

\((26)\)

(of course a phantasy development). This time-development - which is also an approximate S-matrix - would of course be a unitary operator acting in the space extended with the phantasy degree of freedoms. Thinking of the development with lowest order perturbation in the parameter $g$ leading to the Veneziano model as we have previously argued, it is essentially obvious that the higher orders will give unitarity corrections to this Veneziano model. So the scheme with the phantasy Hamiltonian should automatically lead to include these Veneziano model unitarity corrections.

( Let us though at this point remind of the problem we had in deriving the Veneziano model: when we did it in the infinite momentum frame - which is very close to the non-relativistic game used in this article - we did not get but one of the three terms we ought to have got. Of course then we shall also miss some of the unitarity correction terms if we just use the here a bit simplified form.)

3 On the Concept of Time.

As a little parenthesis at this point let us point out that our picture with the stressing of no time development, really means that in our object-description there is at first no time. One can say that the time first comes in when we introduce the phantasy degrees of freedom, and the phantasy Hamiltonian. In this sense the concept of time comes into our scheme as a “phantasy” a fake. The fundamental world has no time. Only by looking at situations in which the various pieces of cyclic chains are screwed together in different combinations as existing at different moments of time a time-concept pops up. That is to say that if one wants to make some ontological model for how a concept of time comes into physics, then we here have the roots for some idea about that:

The time concept could be a phantasy degree of freedom which for some reason could be a reasonable way of describing an a priori timeless physics.

Interestingly enough this attitude of time being a phantasy or fake concept is not actually quite new in as far as we can claim that it is already present in general relativity:

In general relativity all the coordinates and not only (but also) the time coordinate $t = x^0$ are arbitrary and phantasy or fake, in the sense even that the physicist that chooses the coordinates, can decide what these coordinate shall be.
Crudely imposing quantum mechanics and reaching the Wheeler-DeWitt equation one has by this Wheeler-DeWitt equation a restriction on the state, which seemingly tell that the state of the gravity theory is the same at all times. The most close to a Hamiltonian in the gravity theory is namely an integral over the Wheeler-DeWitt equation quantity. This then means that one has got a constraint that the Hamiltonian shall be zero as a constraint. So taking this at face-value one has in gravity a very similar situation as to the one in our scheme: There is no time development, except in some gauge-chosen or fake way.

4 Motivations

Purpose of this Faked Scattering String Theory Formulation

Hope you got the idea of considering a completely trivial $H = 0$ quantum field theory and built up a story of e.g. strings just by defining some extra “phantasy degrees of freedom”.

What is the purpose ?:

- (4-a) It is a method to make a second quantized string theory (competing with works by Kaku and Kikkawa and by Witten, ...[2]). You can describe states with several strings.
- (4-b) You may use the idea to look for further models sharing the great property of string theory of not having the usual divergencies. Likely this is the only hope for making theories, that make sense, in high dimensions.

Problem of Ultraviolet Divergencies Worse the Higher Dimension of Space-time

Each momentum-formulated loop integral in a Feynman diagram bring a $\int \ldots d^q q$ integration and unless there are very many propagators in the loop we cannot avoid divergence for large loop momenta $q$.

The higher dimension the more different loop integrals lead to divergencies.

To absorb the divergencies into bare coupling constants you need in high dimensions so many that the theory ends up with infinitely many parameters, and is in principle useless.

Direction of Hope for High Dimensional Theories: Formfactors

One needs some factor that can make converge the loops in the high dimensional theory, otherwise you have ultraviolet divergencies and in high dimensions it gets too many different divergencies.

Best hope:

**some exponentially falling off factor**

\[ \text{Factor extra in loop } \propto \exp(-k * q_{E}^2) \]  \hspace{1cm} (27)

much like what one gets from formfactors when one has effective theories for hadrons.

Suggestion:
Replace the particles in the high dimensional theory by composite (bound) states, like the hadrons are composite in QCD.

**Just Bound States Not Good Enough: Partons**

If as we now believe hadrons are bound states but of quarks and gluons called in this connection partons the effective vertices will NOT go down exponentially for very big momentum transfers but will be dominated by the coupling to a single parton and behave at the end more like in the theory of just particles. Thus it will only help a part of the way, but finally at high momenta the divergencies reappear.

**Only if there are infinitely many constituents (=partons) in the bound states and they have Bjorken variable $x = 0$, you can postpone parton dominance from popping up, and thus only then we can use the replacement of the original particles in high dimensional theory by bound states.**

**Hadrons Scatter Crudely by Exchange of Bunches of Constituents**

Hadron scattering at energies below where partons collisions become important was described by exchange of other hadrons, pions, vector bosons like $\omega$, again hadrons which again consists of many partons. So it was mainly exchange of lots of partons between one hadron and another one, while the single partons hardly were seen.

**Moderate energy Hadron scattering in terms of partons is much like the fake-scattering of just exchanging bunches from one bound state to another one.**

(we here ignored the relativity and effects of vacuum)

### 4.1 Bound States Not Perfectly True for Our Fake Model

Let as a side remark call attention to that our model of the string in string theory as “composed” of constituents which we called “objects” should presumably not really be called composite in as far as when we stress that there is no interaction it is not truly a bound state. You could of course think that one could the limit of letting the interaction be weaker and weaker and thus at the end have the non-interacting constituents. One could think of some weakly bound states such as some molecules or atoms and then consider a process in which - for some reason a very fast - exchange of say an electron or some other combination of electrons and nuclei, e.g. some whole atom takes place between a couple of different molecules. If this exchange goes fast compared to the internal quantum mechanical motions of the electrons around the nuclei in two scattering molecules one would make the approximation of taking the scattering or exchange amplitude to be given simply by the overlap of the wave function for the two incomming molecules with that of the two molecules after the scattering. Such an overlap approximation for the scattering or exchange process when it goes fast, would be completely analogous to the type of approximation which we use in our calculations of the Veneziano model amplitude in our fake scattering theory.

In a high energy hadron collision the meeting of the two hadrons goes rather fast compared to the moving around of the constituents / partons in the hadrons. So the usual low transverse momentum type of hadronic collisions are not so
far from the described case of a rather fast exchange between the molecules
compared to the moving around of the constituents. In this sense one might
speculate whether the rather fast passage of the hadrons could be described as
being close to being fake in the sense that the genuine interaction between the
constituents first shows up before or after the main hitting passage.

When a couple of partons really hit each other there is a fast interaction
taking place between the constituents it would not be analogous to the fake
process even in the short passage time.

5 Unitarity

A Major Achievement of Phantasy Hamiltonian Formulation is Uni-
tarity of Time-development Operator.

If the theory has a formal / phantasy time development given by a Hamilto-
nian $H_{\text{phantasy}}$ then we have automatically that developing during some time
interval will result in a unitary operator development.

Essentially unitary S-matrix.

Perturbation Expansion in Coefficient on the “Phantasy Hamiltonian” $H_{\text{phantasy}}$

Really the overall scale of the $H_{\text{phantasy}}$ is a matter of the time unit. In
fact there is no time in the theory before we introduce the phantasy degrees of
freedom and make them move.

Natural to make perturbation theory in the coefficient on $H_{\text{phantasy}}$.

Then we get one shift in the topology or way of connection of the cyclic chains
for each order in the perturbation. That corresponds to different topologies of
string surface diagrams as describing unitarity corrections to the Veneziano
model.

6 Conclusion

This article was truly inspired by our novel string field theory on which we by
now have worked for long, and believe to have formulated a theory in which
many strings can be present, so that it is really a string field theory, in terms of
what we called objects, which is really pieces of strings taken for right and left
movers separately.

The remarkable fact turning out of this our old formalism was that the
objects, meaning the bits making up the right and left mover degrees of freedom
turned out having zero Hamiltonian, zero timedevelopment.

As a pedagogical exercise to study such a system like our objects with zero
Hamiltonian we started by considering a second quantized system of infinitely
heavy non-relativistic particles they namely have vanishing Hamiltonian if they
do not have any interaction:

• We have put forward a very trivial second quantized theory (of infinitely
  heavy non-relativistic particles identified as our earlier “objects”) and as-
  sumed for it a Hamiltonian that is zero as operator. So no timedevelop-
ment in this “fundamental” theory. (It is this one which is the analogue of the theory of the objects from our Novel String Field Theory.)

• We can only make it more interesting or adjustable by assuming something about the state of it. Say by a density matrix $\rho_{\text{fundamental}}$. We use this option to assume that the particles (= “objects”) sit in (long) closed chains (cyclic chains).

• We interpret each cyclic chain to describe an open string in a string theory.

• We introduced a phantasy system of degrees of freedom by introducing a “successor function” $f$, which puts all the “objects” (∼ particles) into a series of closed chains, thereby making explicit that such chains are assumed to be present by the assumption about the likely state of the trivial second quantized system.

• Mostly we imagine the cyclic ordering is given by the “fundamental” state of the trivial theory, but in some cases it will be ambiguous which chains there are. Then it is we introduce the fake/phantasy/$f$-variable to distinguish possibilities.

• Then the idea was to make a Hamiltonian supposed to mainly make this fake degree of freedom move, but approximately to avoid varying the “fundamental” degrees of freedom.

With this we then get a quite phantasy time. We only get time development due to the phantasy degrees of freedom.

This could be used to realize the philosophy that the very concept of time is indeed a fake concept, so that at the fundamental level there is no time, but only a static state of the universe. Then only by introduction of a fake overbuilding (analogous to our phantasy successor function $f$ we obtain a world seemingly having a time-concept.

Indeed different moments would correspond to just different ways of looking at the very same state, whatever the moment in question.

Conclusion on Hopes and Applications

• Really the formulation of ours is a solution of second quantized string theory, in the sense that we could say we solved the time development by identifying string theory with several strings with a theory without time development.

• Hope to generalize our “object” picture to different models which have the same great property as string theory of not having usual divergencies! This would be absolutely needed in high dimensions, because with point particles high dimensions cause rather hopeless divergencies.

• As a special case we may generalize to p-adic [5, 6] Veneziano model [10].
Acknowledgement

One of us (H.B.N.) acknowledges the Niels Bohr Institute for allowing him to work as emeritus and for partial economic support. Also thanks food etc. support from the Corfu conference and to Norma Mankoc Borstnik for asking for a way to get meaningful quantum field theories in higher than 4 dimensions. The thinking on hadronic like bound states could namely be looked upon as an attempt to find such a scheme using bound states as the theory behind the particles for which to make the convergent theory.

M. Ninomiya acknowledges Yukawa Institute of Theoretical Physics, Kyoto University, and also the Niels Bohr Institute and Niels Bohr International Academy for giving him very good hospitality during his stay. M.N. also acknowledges at Yuji Sugawara Lab. Science and Engineering, Department of physics sciences Ritsumeikan University, Kusatsu Campus for allowing him as a visiting Researcher.

References

[1] H. B. Nielsen and M. Ninomiya, A New Type of String Field Theory, in Proceedings of the 10th Tohwa International Symposium in String Theory, July 3-7, 20011 Fukuoka Japan AIP conference Proc vol. 607 p. 185-201; arXiv: [hep-th 0111240v1], Nov.2001

H. B. Nielsen and M. Ninomiya, An idea of New String Field Theory - Liberating Right and Left movers, in Proceedings of the 14th Workshop, “What Comes Beyond the Standard Models”Bled July 11-21, 2011, eds. N. M. Borstnik, H. B. Nielsen and D. Luckman arXiv 1112. 542 [hep-th]

H. B. Nielsen and M. Ninomiya, “A Novel String Field Theory Solving String Theory by Liberating Left and Right Movers’, JHEP 20131.05131.v2

[2] As for bosonc string field theory in the light-cone gauge:
M. Kaku and K. Kikkawa, Phys. Rev.D10(1974)110;
M. Kaku and K. Kikkawa, Phys. Rev. D10(1974)1110;
M. Kaku and K. Kikkawa, Phys- Rev. D10(1974) 1823;
S. Mandelstam, Nucl. Phys. B64 (1973)205;
E. Cremmer and Gervais, Nucl. Phys., B90 (1975) 410
Witten type mid-point interaction of covariant string field theory for open string:
E. Witten, Nucl. Phys. B268 (1986) 253.

[3] R. Giles and C. B. Thorn, A lattice Approach to String Theory, Phys. Rev. D16 (1977) 366
C.B. Thorn, On the Derivation of Dual Model from Field Theory, Phys. Lett. 70B (1977) 85
C. B. Thorn, On the Derivation of Dual Models from Field Theory 2, Phys. Rev. D17 (1978) 1073
C. B. Thorn, Reformulating string theory with 1/N expansion, In Moscow 1991. Proceedings, Sakharov Memorial Lecture in Physics, Vol 1* 447 hep-th/9405069

O. Bergman, C. B. Thorn, String Bit Model for Superstring, Phys. Rev. D52 (1995) 5980

C. B. Thorn, Space from String Bits, JHEP11 (2014) 110

[4] H. B. Nielsen (Bohr Inst.) and M. Ninomiya (Ritsumeikan U.) (2019) Contribution paper to 22nd Workshop on What comes Beyond the Standard Models p. 232-236

H. B. Nielsen (Bohr Inst.) and M. Ninomiya (Osami, Osaka City U.), “Novel String Field theory with also Negative Energy Constituents/Objects gives Veneziano Amplitude”, JHEP 02(2018) 097, e-print: 1705.01739[hep-th]

H. B. Nielsen and M. Ninomiya, “An Object Model of String Field Theory and Derivation of Veneziano Amplitude, published in Proc. of Corfu 2016 (2017) 134, arXiv 1705.01739.

H. B. Nielsen and M. Ninomiya, “Instructive Review of Novel SFT with non-interacting constituents objects and the generalization to p-adic theory”, Corfu Summer Institute 2019 “School and Workshops on Elementary Particle Physics and Gravity” (Corf 2019) 31. August - 25. September Corfu Greece, arXiv 2006.09546.

[5] Peter Freund and M Olson, “Non-archimedian strings”, Physics Letters B 199 (1987)

I. Volovich, “p-adic space-time and string theory”, Math. Phys. 71 574–576 (1987)

P. Freund and E. Witten, “Adelic string Amplitudes”, Phys. Letter B 199 191 (1987)

[6] “p-Adic, Adelic and Zeta Strings” Branko Dragovich, Institute of Physics, P.O. Box 57, 11001 Belgrade, SERBIA

[7] Holger B. Nielsen, Masao Ninomiya, “Dirac Sea for Bosons. I: - Formulation of Negative Energy Sea for Bosons” Progress of Theoretical Physics, Volume 113, Issue 3, March 2005, Pages 603-624, https://doi.org/10.1143/PTP.113.603

Holger B. Nielsen, Masao Ninomiya, “Dirac Sea for Bosons. II: - Study of the Naive Vacuum Theory for the Toy Model World Prior to Filling the Negative Energy Sea” — Progress of Theoretical Physics, Volume 113, Issue 3, March 2005, Pages 625-643, https://doi.org/10.1143/PTP.113.625

[8] Yichiïro Nambu for earli work Prog. Theoretical Phys. 5[4] 614 (1950) July-August. H. Bethe and E. Salpeter, “A relativistic Equation for bound state problem”, Phys. Rev. 84, 1232 (1951)

[9] J. Bjorken “Inelastic Electron-Proton and γ-Proton Scattering and the Structure of Nucleon, Phys.Rev.185 (1969)

[10] H.B. Nielsen and M. Ninomiya, paper to appear.