The self-interacting dark matter (SIDM) model with flavor mixing (astro-ph/0010613) was proposed to resolve problems of the CDM model on small scales by keeping attractive features of both SIDM and annihilating dark matter, and simultaneously avoid their drawbacks. A dark particle produced in a flavor eigenstate will separate into two mass eigenstates because they propagate with different velocities and, in a gravitational filed, along different geodesics, see Fig. 1. Thus, in the flavor-mixed SIDM, dark halos are made of heavy eigenstates, whereas light eigenstates may leave the halo. Collisions (elastic scattering) of mass states results in eigenstate conversion, see Fig. 2, which leads to the gradual decrease of the halo mass in high-density cusps. On the other hand, in the early Universe, one may expect a problem of over-production of light (hot) particles over heavy (cold) when the temperature of dark matter falls below the mass of the heavy component. We show how this problem is avoided.

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**Fig. 1**

**Fig. 2**
Constraints on the Model

- There are constraints imposed on the model by processes in the early Universe. In particular, when temperature drops below the mass difference, the number of heavy particles is suppressed exponentially. Dark matter then will consist predominantly of light particles which ruins the model.

I. Two ways to avoid over-production of light mass states

1. If spreading of a wave packet (because of quantum uncertainty in the momentum of a particle at the moments of creation and detection) is faster than the separation of mass states due to the velocity difference, then the two mass states, essentially, never separate. In this case, DM particles created in an interaction state always remain in this interaction state. Such DM particles in the early Universe behave as a single-species collisional dark matter, i.e., the standard SIDM. The mass states will be separated later in the gravitational potential of assembling dark halos, because these states propagate along different geodesics. At some point, light states will escape from the halo and heavy states remain bound. At even later times, the density in cores becomes so high that collisions and conversions become significant. Conversions will decrease the central density.

2. In the opposite case, when particles do separate, it seems the only way to avoid over-production is what you’ve suggested: the degenerate case: $m_h \approx m_l$. Assuming that the DM-matter cross-section $\sigma_m$ is much smaller than the DM self-interaction cross-section $\sigma_{si} \gg \sigma_m$, we have that after the moment when DM particles (both flavors) freeze-out (when $n_f\langle\sigma_m v_f\rangle \sim H_f$), these particles remain highly collisional and conversions go both ways, i.e., $m_h \rightarrow m_l$ and $m_l \rightarrow m_h$. We then need to require that $m_h - m_l < T_c$ at the moment when conversions halt, i.e., $n_c\langle\sigma_{si} v_c\rangle \sim H_c$.

Let’s consider these two cases separately.
II. Wave packet spreading

Assuming a Gaussian wave packet at $t = 0$ having the velocity $v_0 < c$ and width $a_0$,

$$\Psi(0) = A \exp \left( -\frac{x^2}{2a_0^2} + \frac{imv_0 x}{\hbar} \right),$$

one can show that, at some time $t$,

$$|\Psi(t)|^2 = \frac{|A|^2}{\sqrt{1 + \frac{\hbar^2 t^2}{m^2 a_0^4}}} \exp \left[ -\frac{(x - v_0 t)^2}{a_0^2 \left(1 + \frac{\hbar^2 t^2}{m^2 a_0^4}\right)} \right].$$

That is, the wave packet moves with a constant velocity $v_0$ and spreads in space as

$$\Delta = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} = \frac{a_0}{\sqrt{2}} \left(1 + \frac{\hbar^2 t^2}{m^2 a_0^4}\right)^{1/2} \approx \frac{\hbar t}{\sqrt{2} m a_0} = \frac{\hbar t}{2m \Delta_0}$$

as $t \to \infty$. 

(1)

This is the main equation of this section.

The wave-packet width, $\Delta_0$, is, in general, determined by the processes of production and detection, which are simply scattering events in our case. For low-energy elastic scattering, the scattering cross-section is isotropic and does not depend on energy, $\sigma = 4\pi l^2$. Here $l$ is the scattering length determining the wave packet width, $\Delta_0 \sim l$.

For the cross section we use the SIDM parameterization

$$\sigma_{si} = 10^{-24} \sigma_{si,-24} m_{\text{GeV}} \text{ cm}^2,$$

where $m_{\text{GeV}}$ is the mass of DM particle in units of GeV.

(2)
III. When do mass states not separate in the early Universe?

We now consider the first scenario: the mass eigenstates do not separate from each other in the early Universe (i.e., in the absence of gravitational forces) but do separate in dark halos. There are two cases.

1. Non-relativistic degenerate case

Here we consider the case when \( m_h \approx m_l \approx m \), \( \delta m = m_h - m_l \ll m \). First, the widths of the wave packets are nearly the same,

\[
\Delta_h \approx \Delta_l \approx \frac{\hbar t}{m} \sqrt{\frac{4\pi}{\sigma_{si}}}.
\]

Second, from the energy conservation in conversions and assuming \( v_h \ll v_l \), we have

\[
v_l \approx c \sqrt{\frac{2(\delta m)}{m}}.
\]

Non-separation of wave packets means that they remain overlapped for all times, i.e., the distance between them is smaller than or comparable to the packets’ width

\[
\delta x = |v_l - v_h| t \approx v_l t \approx c t \sqrt{\frac{2\delta m}{m}} \leq \Delta \approx \frac{\hbar t}{2m} \sqrt{\frac{4\pi}{\sigma_{si}}}.
\]

This yields the constraint on masses

\[
\frac{\delta m}{m} \leq \frac{\pi \hbar^2}{2m^2 c^2 \sigma_{si}} \quad \text{or} \quad \frac{\delta m}{m} \leq 6.1 \times 10^{-4} m_{\text{GeV}}^{-3} \sigma_{si,-24}^{-1}.
\]  \hspace{1cm} (3)

If this constraint is satisfied, the mass eigenstates do not separate from each other (in the absence of gravitational field) and their wave packets remain overlapped all the time.

For the state conversions to occur in dark halos, the mass states must separate (segregate) in the gravitational potential, i.e., they must move along significantly different geodesics. This implies that the light state must be nearly unbound whereas the heavy state be bound. Thus the velocity \( v_l \) must be comparable to or greater than the typical escape velocity. We assume it to be \( v_l \geq 10^{-3} c v_{-3} \). This gives the lower
bound on the mass difference

\[ \frac{\delta m}{m} \geq 5 \times 10^{-7} v^2_{-3}. \]  (4)

This is a complementary condition to (3). These two constraints can be combined together to yield the upper bound on the mass of a DM particle in this scenario:

\[ m \leq 11 \left( \sigma_{si,-24} v^2_{-3} \right)^{1/3} \text{ GeV}. \]  (5)

2. Non-degenerate case

In this case \( m_h \ll m_l \), i.e., \( m_l \) is essentially zero and \( m_h = m \). The heavy particle is non-relativistic, therefore \( v_h \ll v_l = c \). Thus the states separate with the velocity \( \sim c \). The non-relativistic wave packet also spreads according to equation (1), while the relativistic packet quickly spreads in the directions perpendicular to the direction of motion but the spreading in the direction of motion is much smaller due to Lorentz contraction effects. To avoid separation of states we again require that

\[ \delta x = ct \leq \Delta \approx \frac{\hbar t}{2m} \sqrt{\frac{4\pi}{\sigma_{si}}}. \]

This puts the upper bound on mass

\[ m \leq 0.10 \sigma_{si,-24}^{-1/3} \text{ GeV}. \]  (6)

(This limit corresponds to (3) with \( \delta m/m \sim 1 \).) If the mass of the heavy eigenstate satisfies this constraint, a relativistic light state and a non-relativistic heavy state do not separate without gravitational potential. In halos, the states separate easily because \( v_l \) is much greater than the escape velocity.
IV. What if the mass states separate?

The states will separate when
a) $m_h \gg m_l$ and $m_h \geq 0.16 \text{ GeV}$,
b) $m_h \approx m_l$ and $\delta m/m \geq 2.4 \times 10^{-3} m_{\text{GeV}}^{-3}$.

In both cases one has to worry about the exponentially large number of lighter particles over the heavier ones once the temperature drops below their mass difference.

DM particles freeze out when $H_f > \Gamma_f = \langle \sigma_m v_f \rangle n_f$, where $H_f$ is the Hubble constant at freeze-out and $\sigma_m \sim 10^{-36}$ is the cross-section of DM interactions with ordinary matter and I estimated it from the requirement: $\Omega_m \sim 0.3$. The DM particles remain collisional until a much later time $H_c \sim \Gamma_c = \langle \sigma_{si} v_c \rangle n_c$, where $\sigma_{si} \sim 10^{-24} \gg \sigma_m$. Collisions are accompanied by conversions from heavy states to light and back. However, if the energy of the light component drops below the heavy particle’s rest mass, the conversions of light to heavy are suppressed by the Boltzmann factor.

One possibility to avoid this is to have the mixing angle vanishing and, hence, no conversions will occur. In fact, the effective mixing angle may be much smaller than that in vacuum due to interactions with matter (similar to neutrinos). But after the DM freeze-out, such interactions are nearly absent and this idea does not work.

Thus the only way out is to assume that $\delta m \leq T_c \ll T_f$, which is possible only in case (b). (Here and below $c = \hbar = k_B = 1$.) We explore this possibility below.

Assume that DM has an $s$-wave annihilation cross-section $\langle \sigma_m v \rangle \propto T^n$ with $n = 0$. Freeze-out of cold relics occurs when $\Gamma \sim H \approx T^2/m_{\text{Pl}}$, i.e.,

$$\sigma_m v_f n_f \sim T_f^2/m_{\text{Pl}}$$  \hspace{1cm} (7a)

After freeze-out, the total number of dark matter particles is conserved; hence their number density scales as $n \propto T^3$. The collisional cross-section $\sigma_{si} \sim \text{constant}$ and $v \propto T^{-1/2}$, i.e., $n = 1/2$. Collisions and conversions become inefficient (we assume large mixing $\vartheta \sim 1$) when
Γ_c \sim H_c \text{ which now reads as }
\sigma_{si} v_f \left( \frac{T_c}{T_f} \right)^{1/2} n_f \left( \frac{T_c}{T_f} \right)^3 \sim \frac{T_c^2}{m_{Pl}}. \quad (7b)

Dividing (7a) by (7b), we obtain
\frac{T_c}{T_f} = (\frac{\sigma_m}{\sigma_{si}})^{2/3}. \quad (8)

Now, using Kolb & Turner:
\begin{align*}
x_f &\approx \ln \left[ 0.038(n + 1)(g/g_s^{1/2}) m_{Pl} m \sigma_0 \right], \\
\Omega h^2 &\approx 10^9 \frac{(n + 1)x_f^{n+1} \text{GeV}^{-1}}{(g_sS/g_s^{1/2}) m_{Pl} \sigma_0}
\end{align*}

for \( g \approx 2, g_s \approx 60, v_f \sim 1 \), we have for the freeze-out
\begin{align*}
x_f &\approx 20, \quad T_f \approx 5 \times 10^{-2} m_i \quad \sigma_m \approx \sigma_0 \approx 5 \times 10^{-36} \text{ cm}^2,
\end{align*}
where \( m_i \) is the effective mass of the interacting flavor state. From these expressions and equation (8), and noting that \( \sigma_{si} \sim 2 \times 10^{11} \sigma_m \), we obtain
\begin{align*}
T_c &\approx 2.4 m_{GeV}^{1/3} (\sigma_{m,-35}/\sigma_{si,-24})^{2/3} \text{ eV \ and \ } x_c \sim 46.
\end{align*}

Finally, form the condition \( \delta m/T_c \sim x_c \), we have the upper limit on the mass difference:
\begin{align*}
\frac{\delta m}{m} &\leq 1.1 \times 10^{-7} m_{GeV}^{-2/3} (\sigma_{m,-35}/\sigma_{si,-24})^{2/3}. \quad (9)
\end{align*}

If this constraint is satisfied, there is no over-production of light states with respect to heavy. We have to compare this condition with (b) in the beginning of this section, that is
\begin{align*}
0.6 \times 10^{-3} m_{GeV}^{-3} \sigma_{si,-24}^{-1} \leq 1.1 \times 10^{-7} m_{GeV}^{-2/3} (\sigma_{m,-35}/\sigma_{si,-24})^{2/3}.
\end{align*}

This is possible for
\begin{align*}
m &> 40 \text{ GeV} \left( \sigma_{m,-35}^2 \sigma_{si,-24} \right)^{1/7}, \quad \text{which yields } \frac{\delta m}{m} \leq 4 \times 10^{-9}.
\end{align*}

Previously, we have estimated (from \( v \sim v_{\text{escape}} \)) the mass difference, which is required to significantly alter the structure of a dark halo, see
equation (4). Both conditions for $\delta m/m$ are satisfied simultaneously for $v \sim 10^{-4}$ which corresponds to about 40 km/s. Clearly this is smaller than the escape or even circular velocity, but, it seems, that it is not too small to rule out this case with confidence. Moreover, some changes in the cross-sections may affect this constraint. We conclude that this last scenario is somewhat less probable than the scenarios in section III, and numerical N-body simulations are required to confirm this result.

V. Conclusions

It was shown that the light particle over-production may be avoided in two cases.

(1) Flavor states do not separate into mass states in time. This system behaves similar to the conventional SIDM. This scenario requires the DM mass to be $\leq 17$ GeV in the extreme degenerate case [constraints (3) and (4) must be satisfied as well] and to be $\leq 0.16$ GeV in the non-degenerate case.

(2) In the opposite case, mass states separate from each other and the only plausible way out is the highly degenerate case. It requires DM mass $\geq 90$ GeV and results in rather small particle velocities (less than the escape velocity). Therefore the halo will not loose mass in this scenario, however, particle conversions may “puff up” the halo and hence change its inner structure.

N-body simulations are needed to study the structure formation and evolution of halos in both cases, but especially for (2) to, possibly, rule it out.