Time-Free and Timer-Based Assumptions Can Be Combined to Solve Authenticated Byzantine Consensus

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Abstract

To circumvent the FLP impossibility result in a deterministic way several protocols have been proposed on top of an asynchronous distributed system enriched with additional assumptions. In the context of Byzantine failures for systems where at most $t$ processes may exhibit a Byzantine behavior, two approaches have been investigated to solve the consensus problem. The first, relies on the addition of synchrony, called Timer-Based, but the second is based on the pattern of the messages that are exchanged, called Time-Free. This paper shows that both types of assumptions are not antagonist and can be combined to solve authenticated Byzantine consensus. This combined assumption considers a correct process $p_i$, called $\diamondsuit 2t$-BW, and a set $X$ of $2t$ processes such that, eventually, for each query broadcasted by a correct process $p_j$ of $X$, $p_j$ receives a response from $p_i \in X$ among the $(n - t)$ first responses to that query or both links connecting $p_i$ and $p_j$ are timely. Based on this combination, a simple hybrid authenticated Byzantine consensus protocol, benefiting from the best of both worlds, is proposed. Whereas many hybrid protocols have been designed for the consensus problem in the crash model, this is, to our knowledge, the first hybrid deterministic solution to the Byzantine consensus problem.

Keywords: Asynchronous distributed system, Byzantine process, Consensus, Distributed algorithm, hybrid protocol, time-free assumption, timer-based assumption, Fault tolerance.

1 Introduction

1.1 Context of the Study and Motivation

The Consensus problem is one of the most attractive problems in the field of asynchronous distributed systems. It may be used as building block to design or to implement several applications on on top of fault prone asynchronous distributed systems, since it abstracts several basic agreement problems. Solving the Consensus problem in an asynchronous distributed system where processes (even only one) may crash is impossible\cite{12}. This impossibility result comes from the fact that it is impossible to distinguish a crashed process from a process that is slow or with which communication is slow. To overcome this impossibility, asynchronous distributed systems have to be enriched with additional power such as Synchrony assumptions \cite{11}, Common coins \cite{30}, randomization \cite{5, 21}, unreliable failure detectors \cite{7}, and input vector restrictions \cite{27}. When considering the Consensus problem in a setting where some processes can behave arbitrarily (Byzantine behavior), solving this problem becomes more beneficial for designers or for developers of applications on top of Byzantine fault prone asynchronous distributed system, but the capacity of a such
behavior, make this task more complex and more difficult comparatively with crash failures. This difficulty comes from the fact that a Byzantine process propose a wrong value and it tries to impose it on correct processes.

1.2 Related Works

To solve the consensus with deterministic way, in the context of crash failures, synchrony assumptions must be added [11] or information about failures must be provided by a failures detectors associated with the processes of the system [7]. A failure detector can be seen as a black box that gives (possibly incorrect) information about process failures. Three approaches have been investigated to implement failures detectors. The first, called Timer-Based, considers the partially synchronous system model [2], which generalizes the model of [?] where there are bounds on the relative speed of processes and message transfer delays, but these bounds are not known and hold only after some finite but unknown time, called Global Stabilization Time (GST). The second approach, introduced in [22], does not assume timing assumptions about process speeds and communication delays. This approach, called “Time-Free”, is based on the pattern of messages that are exchanged. It considers the query-response-based winning messages proposed in [22, 28] and the teta-model proposed in [34]. In the third approach, called hybrid, assumptions of both approaches cited above are combined to implement failure detectors [25, 23].

In the context of Byzantine failure, the most of solutions for the consensus problem consider the partially synchronous system model where all links are eventually timely [3, 6, 9, 10, 17, 16, 18, 19]. In such a context, the notion of failure detector, originally designed for crash failures, is extended to mute failures [9, 17]. A muteness failure detector provides information about processes that are silent (did not send some consensus protocol messages). This category is used directly in [13, 14] to solve Byzantine consensus.

For the classical partially synchronous models [7, 11] composed of $n$ partially synchronous processes where at most $t$ may crash, many models, that require only some links which have to be timely, have been proposed [2, 15, 25] in contrast of the related works cited above which assume that the whole system is eventually synchronous. The system model considered in [2] assumes at least an eventual $t$-source. An eventual $t$-source is a correct process with $t$ outgoing eventually timely links (processes communicate using point-to-point communication primitives). On the other hand, the system model considered in [25] assumes a broadcast communication primitive and at least one correct process with $t$ bidirectional but moving eventually timely links. These two models are not comparable [15]. In such a context, [2] proved that an $t$-source (eventual $t$-source) is necessary and sufficient to solve consensus which means that it is not possible to solve consensus if the number of eventually timely links is smaller than $t$ or if they are not outgoing links of a same correct process.

For the second approach [22], used to implement the failure detectors defined in [7], where there are no eventual bounds on process speeds and communication delays (Message Pattern), [23] proposed a leader protocol with very weak assumption on the pattern of messages that are exchanged. This protocol assumes a correct process $p_i$ and a set $Q$, possibly contains crashed processes, of $t$ processes (with $p \ni Q$) such that, each time a process $p_j \in Q$ broadcasts a query, it receives a response from $p_i$ among the first $(n - t)$ corresponding responses (such a response is called a winning response). The two previous approaches ($t$-source and Message Pattern) are combined, in [28], to obtain an eventual leader protocol. This combined assumption considers a star communication structure involving $(t + 1)$ processes (these $t + 1$ processes can differ from a run of the system to another run) and is such that each of its $t$ links can satisfy a property independently of the property satisfied by the $t - 1$ other links.

In the context of Byzantine consensus where $t$ processes can exhibit an arbitrary behavior, Aguilera et al. [2] propose a system model with weak synchrony properties that allows solving the consensus problem.
The model assumes at least an \( \diamond \)bisource (eventual bisource). An \( \diamond \)bisource is a correct process with all its outgoing and incoming links eventually timely. This means that the number of eventually timely links could be as low as \( 2(n - 1) \) links. Their protocol does not need authentication and consists of a series of rounds each made up of 12 communication steps and \( \Omega(n^3) \) messages. In [24] Moumen et al. proposed a system model that considers an eventual bisource with a scope of \( 2t \). The eventual bisource assumed by [2] has the maximal scope \( (x = n - 1) \). An eventual \( 2t \)-bisource (\( \diamond 2t \)-bisource) is a correct process where the number of privileged neighbors is \( 2t \) where \( t \) is the maximum number of faulty processes. Their protocol needs authentication and consists of a series of rounds each made up of 5 communication steps and \( \Omega(n^2) \) messages. In [20], Moumen and Mostefaoui propose a weak system model that does not rely on physical time but on the pattern of messages that are exchanged. This model is based on the query-response mechanism and assumes at least an \( \diamond 2t \)-winning process (eventual \( 2t \)-winning process). An \( \diamond 2t \)-winning is a correct process where the number of privileged neighbors is \( 2t \), such that eventually, for each query broadcasted by any of its privileged neighbors, any of its privileged neighbors receives a response from the \( \diamond 2t \)-winning process among the \( (n - t) \) first responses to that query. Their protocol needs authentication and consists of a series of rounds each made up of 5 communication steps and \( \Omega(n^2) \) messages. Note that this assumption does not prevent message delays from always increasing without bound. Hence, it is incomparable with the timer-based \( \diamond 2t \)-bisource assumption.

### 1.3 Contribution of the Paper

The two previous approaches (Timer-Based and Time-free) have been considered both in the case of crash failures and Byzantine failures, but they have never been combined in the case of Byzantine faults. This paper shows that timer-based and Time-Free assumptions can be combined and proposes a system model where processes are partially synchronous and the communication model satisfies the requirements of the combined assumption. This combined assumption consider a correct processes \( p_i \), called \( \diamond 2t \)-BW (B for Bisource and W for Winning), and a set \( X \) of \( 2t \) processes (some processes may be Byzantine), such that \( \diamond 2t \)-winning. For the assumed model, a simple hybrid authenticated Byzantine consensus protocol, benefiting from the best of both worlds, is proposed. To our knowledge, this is the first protocol that combines between Timer-Based and Time-Free Assumptions to solve authenticated Byzantine consensus.

### 1.4 Organization of the Paper

The paper is made up of six sections. Section 2 presents the basic computation model and the Consensus problem. Then, Section 3 presents the consensus protocol, with a \( \diamond 2t \)-BW, we propose and Section 4 proves its correctness. Finally, Section 5 concludes the paper.
2 Basic Computation Model and Consensus Problem

2.1 Asynchronous Distributed System with Byzantine Process

We consider a message-passing system consisting of a finite set \( \Pi \) of \( n \) (\( n > 1 \)) processes, namely, \( \Pi = \{p_1, \ldots, p_n\} \). A process executes steps (send a message, receive a message or execute local computation). Value \( t \) denotes the maximum number of processes that can exhibit a Byzantine behavior. A Byzantine process may behave in an arbitrary manner. It can crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, send different values to different processes, perform arbitrary state transitions, etc. A correct process is one that does not Byzantine. A faulty process is the one that is not correct.

Processes communicate and synchronize with each other by sending and receiving messages over a network. The link from process \( p \) to process \( q \) is denoted \( p \rightarrow q \). Every pair of processes is connected by two links \( p \rightarrow q \) and \( q \rightarrow p \). Links are assumed to be reliable: they do not create, alter, duplicate or lose messages. There is no assumption about the relative speed of processes or message transfer delays.

2.2 An authentication mechanism

In order to deal with the power of Byzantine processes, We assume that an authentication mechanism is available. A public key cryptography such as RSA signatures \([31]\) is used by a process to verify the original sender of the message and to force a process to relay the original message received. In our authenticated Byzantine model, we assume that Byzantine processes are not able to subvert the cryptographic primitives. To prevent a Byzantine process to send different values to different processes, each message has to carry a value and the set of \( (n - t) \) values received by a process during the previous step. The included signed values can be used by a receiving process to check whether the value sent by any process complies with the values received at the previous step. This set of signed values is called certificate and its role is to prove to the receiver that the value is legal.

To ensure the message validity, each process has an underlying daemon that filters the messages it receives. For example, the daemon will discard all duplicate messages (necessarily sent by Byzantine processes as we assume reliable send and receive operations between correct processes). The daemon, will also discard all messages that are not syntactically correct, or that do not comply with the text of the protocol.

2.3 A Time-Free Assumption

Query-Response Mechanism In this paper, we consider that each process is provided with a query-response mechanism. More specifically, any process \( p \) can broadcast a QUERY () message and then wait for corresponding RESPONSE () messages from \( (n - t) \) processes. Each of this RESPONSE () messages is a winning response for that query, and the corresponding sender processes are the winning processes for that query. The others responses received after the \( (n - t) \) RESPONSE () messages are the losing responses for that query, and automatically discarded. A process issues a new query only when the previous one has terminated (the first \( (n - t) \) responses received). Finally, the response from a process to its own queries is assumed to always arrive among the first \( (n - t) \) responses that is waiting for.

Henceforth, we reuse the definition of \([22, 26, 23, 28]\) to define formally a winning link, an \( x \)-winning.

**Definition 1** Let \( p \) and \( q \) be two processes. The link \( p \rightarrow q \) is eventually winning (denoted \( \diamond WL \)) if there is a time \( \tau \) such that the response from \( p \) to each query issued by \( q \) after \( \tau \) is a winning response (\( \tau \) is finite but unknown).
Definition 2 A process \( p \) is an \( x \)-winning at time \( \tau \) if \( p \) is correct and there exists a set \( X \) of processes of size \( x \), such that: for any process \( q \) in \( X \), the link \( p \rightarrow q \) is winning. The processes of \( X \) are said to be privileged neighbors of \( p \).

Definition 3 A process \( p \) is an \( \diamond \times \)-winning if there is a time \( \tau \) such that, for all \( \tau' \geq \tau \), \( p \) is an \( x \)-winning at \( \tau' \).

2.4 A Timer-Based Assumption

Hereafter, we rephrase the definition of [15] to define formally a timely link and an \( x \)-bisource.

Definition 4 A link from a process \( p_i \) to any process \( p_j \) is timely at time \( \tau \) if (1) no message sent by \( p_i \) at time \( \tau \) is received at \( p_j \) after time \( (\tau + \delta) \) or (2) process \( p_j \) is not correct.

Definition 5 A process \( p_i \) is an \( x \)-bisource at time \( \tau \) if:
- (1) \( p_i \) is correct
- (2) There exists a set \( X \) of processes of size \( x \), such that: for any process \( p_j \) in \( X \), both links from \( p_i \) to \( p_j \) and from \( p_j \) to \( p_i \) are timely at time \( \tau \). The processes of \( X \) are said to be privileged neighbors of \( p_i \).

Definition 6 A process \( p_i \) is an \( \diamond \times \)-bisource if there is a time \( \tau \) such that, for all \( \tau' \geq \tau \), \( p_i \) is an \( x \)-bisource at \( \tau' \).

2.5 Combining Time-Free and Timer-Based Assumptions

Definition 7 A process \( p_i \) is an \( \diamond \times \)-BW at time \( \tau \) if:
- (1) There exists a set \( Y \) of processes of size \( y \) and a set \( Z \) of processes of size \( z \) such that, \( Y \cap Z = \emptyset \) and \( y + z = x \)
- (2) There is a time \( \tau \) such that, for all \( \tau' \geq \tau \), \( p_i \) is an \( y \)-bisource and an \( z \)-winning at the same time \( \tau' \). If \( y = 0 \) then \( p_i \) is an \( x \)-winning and if \( z = 0 \) then \( p_i \) is an \( x \)-bisource.

For the rest of the paper, we consider an asynchronous distributed system where the only additional assumptions are those needed by the \( \diamond \times \)-BW.

2.6 The Consensus Problem

We consider the multivalued consensus problem, where there is no bound on the cardinality of the set of proposable values. In the multivalued consensus problem, every process \( p_i \) proposes a value \( v \) and all correct processes have to eventually decide on a single value among the values proposed by the processes.

Formally, the consensus problem is defined by the following three properties:

- **Termination**: Every correct process eventually decides.
• Agreement: No two correct processes decide different values.
• Validity: If all the correct processes propose the same value \( v \), then only the value \( v \) can be decided.

3 An Authenticated Byzantine Consensus Protocol With \( \diamond 2t \)-BW

Figure 1 presents an authenticated Byzantine consensus protocol in asynchronous distributed system where the only additional assumptions are those needed by the \( \diamond x \)-BW. The principle of the proposed protocol is similar to those that have been proposed in [20, 24] except the coordination phase at the beginning of each round. Each process \( p_i \) executes the code of the protocol given by Figure 1. This protocol is composed of three tasks: a main task (T1), a coordination task (T2), and a decision task (T3).

Before executing the first round \( (r = 1) \), each process \( p_i \) keeps its estimate of the decision value in a local variable \( c_est_i \) and starts by the init phase in order to guarantee the validity property. In this phase, each process \( p_i \) sends the \( \text{INIT}(v_i) \) message, that containing its estimate, to all processes. If \( p_i \) receives at least \( (n - 2t) \) \( \text{INIT} \) messages for \( v \) then it changes its estimate to \( v \), else it keeps its own estimate. After this phase, the protocol proceeds in consecutive asynchronous rounds. Each round \( r \) is composed of four communication phases and is coordinated by a predetermined process \( p_c \) (line 4).

First phase of a round \( r \) (lines 5-8). Each process that starts a round (including the coordinator of the round) first sends its own estimate (with the associated certificate) to the coordinator \( (p_c) \) of the current round and sets a timer to \( (\Delta_i[c]) \).

In a separate task \( T2[r] \) (line 21), each process receives a valid \( \text{QUERY} \) message (perhaps from itself) containing an estimate \( \text{est}_i \), it sends a \( \text{RESPONSE} \) message to the sender. If the process that responds to a query message is the coordinator of the round to which is associated the query message, the value it sends in the \( \text{RESPONSE} \) message is the coordination value. If the process that responds is not the coordinator, it responds with any value as the role of such a message is only to define winning links. As the reader can find it in lines 22-23, the value sent by the coordinator is the value contained in the first valid query message of the round it coordinates.

In the main task at line 6, a process \( p_i \) waits for the response from \( p_c \) (the coordinator of the round) or for expiration of the timer of \( p_c \) \( (\Delta_i[c]) \) and for \( (n - t) \) responses from other processes. In the latter case, process \( p_i \) is sure that \( p_c \) is not the \( \diamond 2t \)-BW as its response is not winning and its link with \( p_i \) is not timely.

If a process \( p_i \) receives a response from the coordinator then it keeps the value in a variable \( \text{aux}_v \); otherwise, it sets \( \text{aux}_v \) to a default value \( \perp \) (this value cannot be proposed). If the timer times out while waiting for the response from \( p_c \), \( \Delta_i[c] \) is incremented and \( p_i \) considers that its link with \( p_c \) is not eventually timely or \( p_c \) is Byzantine or the value \( \Delta_i[j] \) is not set to the right value. As \( \Delta_i[c] \) is incremented each time \( p_c \)'s responses misses the deadline, it will eventually reach the bound on the round trip between \( p_i \) and \( p_c \) if the link between them is timely.

If the current coordinator is a \( \diamond 2t \)-BW then at least \( (t + 1) \) correct processes will get the value \( v \) of the coordinator and thus set their variable \( \text{aux} \) to \( v \) (\( \neq \perp \)). The next phases will serve to propagate this value from the \( (t + 1) \) correct processes to all correct processes. Indeed, among the \( 2t \) privileged neighbors of the current coordinator at least \( t \) are correct processes and all of them will receive the value of the coordinator. If the current coordinator is Byzantine, it can send nothing to some processes and/or perhaps send different certified values to different processes. If the current coordinator is not a \( \diamond 2t \)-BW or if it is Byzantine, the three next phases allow correct processes to behave in a consistent way. Either none of them decides or if some of them decides a value \( v \) despite the Byzantine behavior of the coordinator, then the only certified value for the next round will be \( v \) preventing Byzantine processes from introducing other values.
Second phase of a round $r$ (lines 9–11). During the second phase, all correct processes relay, at line 9, either the value they received from the coordinator (with its certificate) or the default value $\perp$ if they timed out and they received $(n - t)$ RESPONSE messages from others processes. Each process collects $(n - t)$ valid messages and stores the values in a set $V_i$ (line 9). At line 9 if the coordinator is correct only one value is valid and can be relayed.

Moreover, if the current coordinator is a $\diamondsuit 2t$-BW then any correct process $p_i$ will get in its set $V_i$ at least one copy of the value of the coordinator as among the $(t + 1)$ copies sent by the $(t + 1)$ correct processes that got the value of the coordinator a correct process cannot miss more than $t$ copies (recall that a correct process collect $(n - t)$ valid messages). If the coordinator is not a $\diamondsuit 2t$-BW or if it is Byzantine, some processes can receive only $\perp$ values, others may receive more than one value (the coordinator is necessarily Byzantine in this case) and some others can receive a unique value. This phase has no particular effect in such a case. The condition $(V_i - \{\perp\} = \{v\})$ of line 11 means that if there is only one non-$\perp$ value $v$ in $V_i$ then this value is kept in $aux_i$, otherwise, $aux_i$ is set to $\perp$. The aim of this second phase is that if the coordinator is an $\diamondsuit 2t$-BW then all the correct processes will get its value.

Third phase of a round $r$ (lines 12–14). This phase is a filter; it ensures that at the end of this phase, at most one non-$\perp$ value can be kept in the $aux$ variables in the situations where the coordinator is Byzantine. If the coordinator is correct, this is already the case. When the coordinator is Byzantine two different correct processes may have set their $aux_i$ variables to different values. This phase consists of an all-to-all message exchange. Each process collects $(n - t)$ valid messages the values of which are stored in a set $V_i$. If all received messages contain the same value $v (V_i = \{v\})$ then $v$ is kept in $aux_i$ otherwise, $aux_i$ is set to $\perp$. At the end of this phase, there is at most one (or none) certified value $v (\neq \perp)$.

Fourth phase of a round $r$ (lines 15–19).

This phase ensures that the Agreement property will never be violated. This prevention is done in the following way. If a correct process $p_i$ decides $v$ during this round then if some processes progress to the next round, then $v$ is the only certified value. In this decision phase, a process $p_i$ collects $(n - t)$ valid messages and store the values in $V_i$. If the set $V_i$ of a process $p_i$ contains a unique non-$\perp$ value $v$, $p_i$ decides $v$. Indeed among the $(n - t)$ same values $v$ received by $p_i$, at least $n - 2t$ have been sent by correct processes. As $(n - t) + (n - 2t) > n$ any set of $(n - t)$ valid signed messages of this phase includes at least one value $v$. Hence, all processes receive at least one value $v$ (the other values could be $v$ or $\perp$) and the only certified value for the next rounds is $v$. This means that during the next round (if any) no coordinator (whether correct or Byzantine) can send a valid value different from $v$.

If during the fourth phase, a process $p_i$ receives only $\perp$ values, it is sure that no process can decide during this round and thus it can keep the value it has already stored in $est_i$ (the certificate composed of the $(n - t)$ valid signed messages received during phase four containing $\perp$ values, allow $p_i$ to keep its previous values $est_i$).

Before deciding (line 17), a process first sends to all other processes a signed message DEC that contains the decision value (and the associated certificate). This will prevent the processes that progress to the next round from blocking because some correct processes have already decided and stopped sending messages. When a process $p_i$ receives a valid DEC message at line 24 it first relays it to all other processes and then decides. Indeed, task $T_3$ is used to implement a reliable broadcast to disseminate the eventual decision value.
preventing some correct processes from blocking while others decide.

4 Correctness of the protocol

Lemma 1 If tow corrects processes $p_i$ and $p_j$ decide $v$ and $v'$, respectively, then $v = v'$.

Proof The proof is by contradiction. Suppose that $p_i$ and $p_j$ have decided $v$ and $v'$, respectively, such that $v \neq v'$. This means that $v$ appears at least $(n - t)$ times in $V_i$ and $v'$ also appears at least $(n - t)$ times in $V_j$ at line 16. This means that $|V_i| + |V_j| \geq 2(n - t)$. Since, the $(n - t)$ correct processes send (according to the protocol) the same message to both processes and the $t$ Byzantine processes can send different messages to them, we have $|V_i| + |V_j| \leq (n - t) + 2t = (n + t)$. This leads to $(n + t) \geq 2(n - t)$ i.e. $n \leq 3t$ a contradiction as we assume $n > 3t$.

Lemma 2 If a correct process $p_i$ decides $v \neq \perp$ during a round $r$, then all correct processes start the next round with the same estimate $v$ if they have not deciding.

Proof Let us first note that if any correct process decides on the value $v \neq \perp$ at the round $r$ then all correct processes, that have not decided, set their estimates to $v$ because each of these processes receives at line 22 at least one $\text{FILT}(r, aux)$ message carrying the value $v$. Moreover, all correct processes start a round $r + 1$ with the same estimate $v$.

The proof is by contradiction. Suppose that a correct process $p_i$ decides $v$ at a round $r$ (line 17) and a correct process $p_j$ has not decided at this round and sets own estimate to $v' \neq v$. This means that the set $V_j$ of $p_j$ contains only values different to $v$. By assumption, the value $v$, appears in $V_i$ at least $(n - t)$ times because it has decided. As there are $t$ Byzantine processes, $v$ is received by $p_i$ at least $(n - 2t)$ times from correct processes. From these $(n - 2t)$ messages for $p_i$ at most $t$ are loosed by $p_j$, because it wait for $(n - t)$ messages at line 16. From this, we can conclude that $V_j$ contains at least $(n - 3t) \geq$ times the value $v$ $(n > 3t)$. Moreover, $p_j$ sets its estimate to $v$. A contradiction.

Corollary 1 If a correct process $p_i$ decides a certified value $v$ during a round $r$, then only $v$ can be decided in the same or in subsequent rounds.

Proof Let us consider that a process $p_i$ decides a value $v$ in a round $r$. If a correct process $p_j$ decides at the same round $r$ then, by lemma 1 it decides the same value $v$ decided by $p_i$. If a correct process $p_j$ does not decide at the same round $r$ then, by lemma 2 all correct processes start the next round $r + 1$ by the same estimate value $v$ decided by $p_i$ at a round $r$.Indeed, in the latter case, $v$ will be the only certified value as even $\perp$ is not certified as a certificate for the value that will be used during the next round is composed by a set of $(n - t)$ messages as we said above that any such set includes at least one value $v$. From now on, the only value that can be exchanged is $v$ and only $v$ can be decided.

Theorem 1 (agreement) No two correct processes decide differently.

Proof If a correct process decides at line 24 it decides a certified value decided by another process. Let us consider the first round where a process decides at line 17 By Corollary 2, if a process decides a certified
Function Consensus$(v_i)$

\begin{align*}
\text{Init: } r_i &\leftarrow 0; \Delta_i[1..n] \leftarrow 1; \\
\text{Task } T1: & \quad \% \text{ basic task } \\
\end{align*}

\begin{enumerate}
\item send $\text{INIT}(v_i)$ to all;
\item wait until (INIT messages received from at least $(n - t)$ distinct processes);
\item if $(\exists v: \text{received at least } (n - 2t) \text{ times})$ then $\text{est}_i \leftarrow v$ else $\text{est}_i \leftarrow v_i$ endif;
\end{enumerate}

\begin{enumerate}[resume]
\item repeat forever
\item $c \leftarrow (r_i \mod n) + 1; r_i \leftarrow r_i + 1;$
\item send $\text{QUERY}(r_i, \text{est}_i)$ to all; set $\text{timer}(\Delta_i[c])$;
\item wait until (RESPONSE$(r_i, \text{est})$ received from $p_i$ )
\quad or (time-out and RESPONSE$(r_i, \text{est})$ received from $(n - t)$ distinct processes);
\item if RESPONSE$(r_i, \text{est})$ received from $p_i$ then $\text{aux}_i \leftarrow \text{est}$ else $\text{aux}_i \leftarrow \bot$;
\item if (timer times out) then $\Delta_i[c] \leftarrow \Delta_i[c] + 1$ else disable $\text{timer}$ endif;
\item send $\text{RELAY}(r_i, \text{aux}_i)$ to all;
\item wait until (RELAY$(r_i, \ast)$ received from at least $(n - t)$ distinct processes) store values in $V_i$;
\item if $(V_i = \{\bot\} = \{v\})$ then $\text{aux}_i \leftarrow v$ else $\text{aux}_i \leftarrow \bot$ endif;
\item send $\text{FILT1}(r_i, \text{aux}_i)$ to all;
\item wait until (FILT1$(r_i, \ast)$ received from at least $(n - t)$ distinct processes) store values in $V_i$;
\item if $(V_i = \{v\})$ then $\text{aux}_i \leftarrow v$ else $\text{aux}_i \leftarrow \bot$ endif;
\item send $\text{FILT2}(r_i, \text{aux}_i)$ to all;
\item wait until (FILT2$(r_i, \ast)$ received from at least $(n - t)$ distinct processes) store values in $V_i$;
\item case $(V_i = \{v\})$ then send $\text{DEC}(v)$ to all; return$(v)$;
\item $(V_i = \{v, \bot\})$ then $\text{est}_i \leftarrow v$;
\item endcase;
\end{enumerate}

end repeat

\begin{enumerate}[resume]
\item Task $T2[c]: \quad \% \text{ Query-response coordination task - There is one such task per round } r \%$
\item upon receipt of $\text{QUERY}(r, \text{est})$ from $p_j$;
\item if $p_i$ is the coordinator of the round $r$ and $c_{\text{est}_i} \leftarrow \bot$ then $c_{\text{est}_i} \leftarrow \text{est}$;
\item send RESPONSE$(r_i, c_{\text{est}_i})$ to $p_j$
\end{enumerate}

Task $T3$:

\begin{enumerate}[resume]
\item upon receipt of $\text{DEC}(\text{est})$: send $\text{DEC}(\text{est})$ to all; return$(\text{est})$;
\end{enumerate}

Figure 1: An Authenticated Byzantine Consensus Protocol With $\diamond 2t$-BW
value during the same round, it decides the same value. If a process decides after receiving a DEC message at line 24 it decides the same value. Any process that starts the next round with its local variable $est_i \neq v$ will see its messages rejected (no value different from $v$ can be certified).

\[ \square \text{Theorem 1} \]

**Lemma 3** If no process decides during a round $r' \leq r$, then all correct processes start round $r + 1$.

**Proof** Let us first note that a correct process cannot be blocked forever in the init phase. Moreover, it cannot be blocked at line 6 because of the time-out and at least $(n - t)$ processes respond to QUERY messages.

Suppose that no process has decided a value $v$ during a round $r' \leq r$, where $r$ is the smallest round number in which a correct process $p_i$ blocks forever. The proof is by contradiction.

By assumption, $p_i$ is blocked at lines 10, 13 or 16.

Let us first examine the case where $p_i$ blocks at line 10 which is the first statement of round $r$ where a process can block forever. This means that at least $(n - t)$ correct processes eventually execute line 9 because processes are partially synchronous. Consequently as communication is reliable between correct processes the messages sent by correct processes will eventually arrive at $p_i$ that blocks forever at line 10. The cases where $p_i$ blocks at line 13 or 16 are similar to this first case. It follows that if $p_i$ does not decide, it will proceed to the next round. A contradiction.

\[ \square \text{Lemma 3} \]

**Theorem 2 (termination)** If there is a $\circ 2t$-BW in the system, then all correct processes eventually decide.

**Proof** As the protocol uses authentication, if some process receives a valid DEC message, it can decide even if the message has been sent by a faulty process. Recall that a Byzantine process cannot forge a signature. If a correct process decides at line 17 or at line 24 then, due to the sending of DEC messages at line 17 or line 24 respectively, prior to the decision, any correct process will receive such a message and decide accordingly (line 24).

So, suppose that no correct process decides. The proof is by contradiction. By hypothesis, there is a time $\tau$ after which there is a process $p_x$ that is a $\circ 2t$-BW. Let $p_j$ be a correct process and one of the $2t$ privileged neighbors of $p_x$. Let $r$ be the first round that starts after $\tau$ and that is coordinated by $p_x$. As by assumption no process decides, due to Lemma 3.

All correct processes $p_i$ (and possibly some Byzantine processes) start round $r$ and send a valid QUERY message to $p_x$ (line 5). This QUERY message contains a value $est$ which is the estimate of process $p_i$. When the coordinator $p_x$ of round $r$ receives the first QUERY message (line 21) possibly from itself, it sets a local variable $c_{est_x}$ to the valid value contained in the message. Then each time process $p_x$ receives a QUERY message related to this round $(r)$, it sends a RESPONSE message to the sending process. If we consider any correct process $p_i$ privileged neighbor of $p_x$, the RESPONSE message from $p_x$ the coordinator to the QUERY message of $p_i$ will be received by $p_x$ among the first $n - t$ responses because the link between $p_i$ and $p_x$ is winning or the RESPONSE message from $p_x$ will be received by $p_i$ before expiration of $\Delta_i[x]$ , because the link between $p_i$ and $p_x$ is timely (line 21).

In the worst case, there are $t$ Byzantine processes among the $2t + 1$ privileged neighbors of $p_x$. A Byzantine process can either relay the value of $p_x$ ($t$ Byzantine processes, $t$ correct processes and itself). During the next phase, a Byzantine process can either relay the value of $p_x$ or relay $\bot$ arguing that $\Delta_i[x]$ has expired and it did not receive the response of $p_x$ among the first $(n - t)$ RESPONSE messages (the value of $p_x$ and $\bot$ are the only two valid values for this round). This allows to conclude that the value $v$ sent by $p_x$
the coordinator of the present round is relayed at line 9) by, at least, the $t + 1$ correct processes with which $p_x$ has timely or winning links (the only other possible value is $\bot$).

During the third phase (lines [12][14], as the value $v$ of $p_x$ is the only certified value, all the processes that send a certified message at line 12. This allows to conclude that all processes will have to set their variables $aux$ variable to $v$ (line 14). By the same way, all processes that send certified messages at line 15 will send $v$. From there we can conclude that correct processes will all decide at line 17 which proves the theorem.

\[\square_{\text{Theorem} 2}\]

**Theorem 3 (Validity)** If all correct processes propose $v$, then only $v$ can be decided.

**Proof** Let $v$ be the only proposed value by correct processes at line 1. Since all correct processes propose $v$, $v$ is sent at least $(n - t)$ times at line 1. Since processes receive at least $(n - t)$ values from distinct processes, we can conclude that at line 3 the values $v$ is received at least $(n - 2t)$ times by any correct processes. Moreover, any value proposed by Byzantine processes will be received at most $t$ times. As $n > 3t$, we have $t < n - 2t$. Consequently, the only certified value is $v$. This means that all correct processes set their variable $est$ to $v$.

\[\square_{\text{Theorem} 3}\]

5 Conclusion

This paper has shown that timer-based assumption and time-free assumption can be combined to solve authenticated Byzantine consensus in asynchronous distributed systems. It has presented the first deterministic authenticated Byzantine protocol that benefiting from the best of both worlds. This combined assumption considers a correct process $p_i$, called $\diamond 2t$-BW, and a set $X$ of $2t$ processes such that, eventually, for each query broadcasted by a correct process $p_j$ of $X$, $p_j$ receives a response from $p_i \in X$ among the $(n - t)$ first responses to that query or both links connecting $p_i$ and $p_j$ are timely. The proposed protocol has very simple design principle and it provides an assumption coverage better than the one offered by any protocol based on a single of these assumptions. In favorable setting, the proposed protocol can reach decision in only 6 communication steps and needs only $\Omega(n^2)$ messages in each step. The major contribution of this paper is to show that Byzantine Consensus is possible with a very weak hybrid additional that satisfying the properties required by a $\diamond 2t$-BW.

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