Light hadron spectroscopy

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I review the progress that has been made in light hadron spectroscopy from lattice QCD, since the LATTICE’94 conference in Bielefeld.

1. INTRODUCTION

Calculation of the hadron spectrum has been a central goal of Lattice QCD for 20 years. A precision calculation of the hadron spectrum would be a validation of QCD, and a validation of Lattice QCD as a calculational technique. It would give us confidence when we apply lattice QCD to calculating other quantities. In addition we would be able to clarify the nature of observed hadrons and predict where others might be found. True precision hadron mass calculations have been performed for heavy-quarkonium systems [1–3], and are beginning to appear for heavy-light systems [4]. Despite some monumental efforts such as that by the GF-11 group [5], the light hadron spectrum remains considerably less well determined than one might hope for. This is unfortunate, since this part of the hadron spectrum is the most complex and least well understood.

The reasons for this situation are well known. The light hadrons (with possible exception of the glueballs) are large, their size being determined by the pion Compton wavelength, so that large lattices are needed. Small lattice spacings are needed to make contact with the continuum limit. The u and d quark masses are small, making the Dirac operator ill-conditioned and expensive to invert. There are a large number of excited states, which makes it difficult to isolate the ground state contribution to (almost) any hadron propagator. At small quark masses, hadron propagators become more noisy – especially for large (time) separations. Simulations with dynamical quarks suffer from critical slowing down as the quark mass approaches zero.

For these reasons, much of the work on light hadron spectroscopy has been performed in the quenched approximation. Unfortunately, it is in just this regime of light quarks that this approximation becomes the most suspect. Even here, few measurements have used lattices of much more than 3.5fm in spatial extent. Many calculations are restricted to u and d quark masses so large that \( m_\pi/m_\rho > 0.5 \), requiring considerable extrapolation to obtain physical results (\( m_\pi/m_\rho \approx 0.18 \)).

Contributions since LATTICE’94 are listed in Tables 1–8. In each table, the columns give \( \beta \), lattice size, number of configurations and number of source positions/configuration.

In addition, three groups, COLUMBIA, S. Kim – DKS and Gottlieb have contributions on the chiral limit. Kilcup has presented calculations of the connected and disconnected contributions to the \( \eta' \) propagators using staggered quarks.

Before proceeding to summarize the above contributions, let me first make a few general observations on directions pursued since LATTICE’94. For work prior to this one should consult recent review articles such as those by Michael [6] and by Weingarten [7], and the collections of parallel talks (and poster sessions) in the proceedings of LATTICE’94 and earlier conferences in the series.

In the past year the trend has been towards high statistics, rather than larger lattices and/or smaller quark masses. Most of the work has been in the quenched approximation. Many of these...
Table 1
QUENCHED STAGGERED

| Group          | Volume | Lattice | # of Points | # of Sites |
|----------------|--------|---------|-------------|------------|
| Gottlieb       | 5.7    | $3^3 \times 48$ | 600         | 6          |
|                |        | $12^3 \times 48$ | 400         | 6          |
|                |        | $16^3 \times 48$ | 400         | 6          |
|                |        | $20^3 \times 48$ | 200         | 6          |
|                |        | $24^3 \times 48$ | 200         | 6          |
|                | 5.85   | $12^3 \times 48$ | 200         | 6          |
|                |        | $20^3 \times 48$ | 200         | 6          |
|                |        | $24^3 \times 48$ | 200         | 6          |
|                | 6.15   | $32^3 \times 64$ | 115         | 8          |
| COLUMBIA       | 5.7    | $16^3 \times 40$ |             |            |
| JLQCD          | 5.85   | $16^3 \times 64$ | 60          |            |
|                | 5.93   | $20^3 \times 40$ |             |            |
|                | 6.0    | $24^3 \times 64$ | 50          |            |
|                | 6.2    | $32^3 \times 64$ | 40          |            |
| Kim – DKS      | 6.0    | $16^3 \times 64$ | 410         | 2          |
|                |        | $24^3 \times 64$ | 339         | 1          |
|                |        | $32^3 \times 64$ | 200         | 1          |
| Kim – Ohta     | 6.5    | $48^3 \times 64$ | 50          | 1          |

Table 2
QUENCHED WILSON

| Group | Volume | Lattice | # of Points |
|-------|--------|---------|-------------|
| GERMAN| 6.0    | $16^3 \times 32$ | 1000        |
| LANL  | 6.0    | $32^3 \times 64$ | 150         |
| JLQCD | 6.0    | $24^3 \times 64$ | 200         |
|       | 6.1    | $24^3 \times 64$ | 50          |
|       | 6.3    | $32^3 \times 80$ | 50          |
| JLQCD | 6.0    | $24^3 \times 64$ | 1000        |
| SUZUKI| 5.7    | $16^3 \times 32$ | 20          |

Table 3
QUENCHED CLOVERLEAF

| Group | Volume | Lattice | # of Points |
|-------|--------|---------|-------------|
| UKQCD | 5.7    | $12^3 \times 24$ | 480         |
|       | 6.0    | $16^3 \times 48$ | 125         |
|       | 6.2    | $24^3 \times 48$ | 184         |

Table 4
QUENCHED VALENCE

| Group     | Volume | Lattice |
|-----------|--------|---------|
| Liu – Dong| 6.0    | $16^3 \times 24$ |

Table 5
IMPROVED-QUENCHED CLOVERLEAF

| Group | Volume | Lattice |
|-------|--------|---------|
| SCRI  | 6.80   | $16^3 \times 32$ |
|       | 7.40   | $8^3 \times 16$  |
|       | 7.60   | $16^3 \times 32$ |
|       | 7.75   | $16^3 \times 32$ |
|       | 7.90   | $8^3 \times 32$  |

Table 6
IMPROVED-QUENCHED IMPROVED

| Group | Volume | Lattice |
|-------|--------|---------|
| CORNELL| 6.8    | $5^3 \times 14$  |
|       | 7.1    | $6^3 \times 16$  |

Table 7
FULL QCD STAGGERED

| Group    | Volume | Lattice |
|----------|--------|---------|
| COLUMBIA | 5.7    | $16^3 \times 40$ |

Table 8
FULL-QCD-STAGGERED CLOVERLEAF

| Group    | Volume | Lattice |
|----------|--------|---------|
| SCRI     | 5.6    | $16^3 \times 32$ |
|          |        | 100     |
light hadron spectrum calculations were performed as an integral part of other projects, such as matrix-element calculations and/or heavy-light physics calculations.

This high statistics, sometimes combined with use of multiple sources/sinks to isolate the ground state, has enabled the spectra to be measured with sufficient precision to permit extrapolations in quark mass and lattice spacing. Tests have been made as to how well one reproduces the spectrum of hadrons containing strange valence quarks.

Using Wilson quarks for spectrum calculations has the advantage that flavour symmetry is exact, although this is at the expense of losing all chiral symmetry. A perhaps more serious limitation is that Wilson quarks introduce errors $\mathcal{O}(a^2)$. An alternative solution is to use an improved Wilson-quark action, the Sheikholeslami-Wohlert or cloverleaf action, which has the flavour symmetry of the Wilson action but errors $\mathcal{O}(a^2)$.

Preliminary studies aimed at implementing the Lepage programme of using improved actions to allow use of larger lattice spacings and smaller (in number of sites) lattices have been undertaken.

Section 2 presents highlights of the above mentioned spectrum calculations. In section 3 we discuss the chiral limit of the pion spectrum of quenched lattice QCD. Section 4 gives a brief discussion of the work by Kilcup et al. on the $\eta'$ mass, while section 5 is devoted to discussions and conclusions.

2. HIGHLIGHTS OF SPECTRUM CALCULATIONS

What follows are brief descriptions of recent contributions on light-hadron spectroscopy, which have been summarised in Tables 1–8. Most of the figures in this section were provided by the people mentioned, or prepared by me from data provided by said persons. All work presented in this section should be considered as preliminary. The light-hadron spectroscopy contributions of two of the JLQCD projects was only a sideline to other calculations, and was not available to me at the time of preparing this paper. I therefore refer you to the parallel talks by Hashimoto and Aoki for details (I thank M. Okawa for bringing these works to my attention).

2.1. QUENCHED STAGGERED

Gottlieb has calculated the light hadron spectrum with staggered quarks in the quenched approximation to lattice QCD. He used “corner” wall sources and point sinks. His lattice sizes ranged from $8^3 \times 48$ to $32^3 \times 64$. At $\beta = 5.7$ and $\beta = 5.85$ his quark masses ranged from 0.01 to 0.16, while at $\beta = 6.15$ they ranged from 0.005 to 0.08.

High statistics on multiple lattice sizes has enabled him to analyse finite size effects. For $m_\pi/m_\rho \gtrsim 0.5$, finite size effects appear to be fairly small for spatial boxes of extent $> 1.5$–2fm. Use of several masses at each beta has enabled him to extrapolate $m_N/m_\rho$ to $m_q = 0$ (a linear extrapolation appears adequate). Finally, use of several $\beta$'s and hence $a$'s has permitted extrapolation to $a = 0$. Both a linear and the accepted quadratic extrapolations in $a$ give acceptable fits and mass ratios which are too high by 5–8%, which is normally considered acceptable considering the systematic uncertainties of the ratios, fits and error estimates. (see Figure 4).

A plot of $m_N/m_\rho$ versus $m_\pi/m_\rho$ for these simulations is shown in Figure 4.

Gottlieb has also attempted to calculate the quantity $J$, introduced by Lacock and Michael as a consistency check on the meson spectrum. ($J = m_K \frac{dm_N}{dm_P}$ evaluated at $m_V/m_P = m_K/m_K$.) His very preliminary results are: 0.56(1), for $\beta = 5.7$ ($24^3 \times 48$ lattice), 0.55(2) for $\beta = 5.85$ ($20^3 \times 48$ lattice) and 0.49(1) ($24^3 \times 48$ lattice), and 0.45(1) for $\beta = 6.15$ ($32^3 \times 64$ lattice). A quadratic extrapolation to $a = 0$ gives 0.43(1) compared with an experimental value of 0.48–0.50. He has also studied the PCAC relationship for the pion mass (see Section 3).

S. Kim and D. K. Sinclair have studied the
light hadron spectrum with quenched staggered quarks at $\beta = 6.0$ (their previous simulations had been at $\beta = 6.5$ [10]). They used wall sources and point sinks. Lattice sizes of $16^3 \times 64$, $24^3 \times 64$, and $32^3 \times 64$ were used to study finite size effects. Quark masses of 0.0025, 0.005, 0.01 were used.

High statistics enabled us to study finite size effects. The $\pi$ showed no significant finite size effects for box sizes $\gtrsim 24^3$; similarly for the $\rho$ (at least for 2 heavier quark masses). Because of the difficulty in finding a plateau in the nucleon effective mass on the $32^3 \times 64$ lattice, and because of the persistent difference between the two different estimates (sources) for the nucleon mass, it remains unclear how significant finite size effects are for the nucleon for these box sizes and quark masses.

As one can see from the effective mass plots for the nucleon (Figure 3), at least for the largest lattice, it is unclear whether a plateau is attained before the signal is swamped by noise, despite the high statistics. We thus concluded that a wall source was a poor choice for a $32^3 \times N_t$ lattice at $\beta = 6.0$.

From the extrapolated $\rho$ mass, $a^{-1} = 1.87(2)$GeV at $\beta = 6.0$. This yields $a_{6.0}/a_{6.5} = 2.02(3)$ compared with scaling prediction ($\overline{MS}$) of 1.8976. Finally we include an Edinburgh plot (Figure 4). The 2 points for each mass/lattice size come from the 2 different nucleon sources. A rough estimate of the $J$ parameter, is $J \approx 5.6$, compared with the experimental value of 4.8–5.0. Our results for the chiral behaviour of the theory are presented in section 3.

S. Kim and S. Ohta [17] are performing...
quenched simulations at $\beta = 6.5$ on a $48^3 \times 64$ lattice. On their current sample of 50 lattices they have calculated the light hadron spectrum with staggered quark masses 0.01, 0.005, 0.0025 and 0.00125 (in lattice units) enabling them to probe down to $m_\pi/m_\rho < 0.3$. They use wall sources and point sinks.

When combined with the earlier work of Kim and Sinclair [16], this work should enable them to study finite size effects at $\beta = 6.5$, where scaling is apparent and violations of flavour symmetry (indicated by the difference in masses between Goldstone and non-Goldstone pions) are minimal. Their preliminary results, summarized in the Edinburgh plot of Figure 5 would seem to indicate considerable finite size effects in the $m_N/m_\rho$ mass ratio. A word of caution is appropriate, since these are relatively low statistics, preliminary results. $m_N/m_\rho$ decreases both because $m_N$ decreases and because $m_\rho$ increases. Since the low statistics forces them to fit propagators at relatively small time separations, at least some of the difference could be due to not having yet reached the plateau in their propagators. Only higher statistics can resolve this issue.

2.2. QUENCHED WILSON

The German collaboration (G. Schierholz et al.) have performed quenched light hadron spectroscopy with Wilson fermions as part of a nucleon structure function calculation [18]. They used a $16^3 \times 32$ lattice at $\beta = 6.0$, and used a Jacobi smeared source and sink.

Using high statistics with Jacobi smearing (which appears to be a good choice), they obtained well behaved hadron propagators with good evidence for plateaux (see Figure 6 which shows the nucleon effective mass). They also use 2-component “non-relativistic” sources (projected with $\frac{1}{2}(1 + \gamma_4)$), which halves the number

Figure 3. Nucleon ($N_1$) effective mass plot – quenched staggered.
of required inversions. The hadron mass values they obtain are given in Table 9.

The LANL collaboration (communicated by R. Gupta) have performed hadron spectroscopy with quenched Wilson quarks on a $32^3 \times 64$ lattice at $\beta = 6$ [19]. They use four $\kappa$’s — 0.153, 0.155, 0.1558, 0.1563 — for u, d, s physics (they also do some charm physics). Hadron propagators are obtained for wall source/point sink (WL), Wuppertal source/point sink (SL) and Wuppertal source/Wuppertal sink (SS). Since SL and SS effective masses approach the infinite separation limit from above, WL from below, a linear combination was used to reduce the contributions from excited states.

No significant finite size effects were observed between $24^3 \times N_t$ and $32^3 \times 64$ lattices. Linear extrapolation to the correct pion mass gave $m_N/m_\rho = 1.42(4)$, $m_\Delta/m_\rho = 1.80(5)$, (both 10–15% too high) and $a^{-1} = 2.319(44)$GeV (from $m_\rho$). Figure 4 shows the linear dependence of meson and baryon masses on the quark mass.

Figure 4. Edinburgh plot – quenched staggered.

Figure 5. Edinburgh plot – quenched staggered.

Figure 8 is the Edinburgh plot from these simulations.

The values of $m_s$ required to give the correct values for $m_K/m_\rho$, $m_K^*/m_\rho$, and $m_\phi/m_\rho$ differ by $\sim 20\%$. Even with the largest of these estimates for $m_s$, the baryon octet and decuplet splittings are still as much as $\sim 30\%$ smaller than experiment.

Several forms for the lattice dispersion relation were tested for mesons. The best was found to be $\sinh^2(E/2) = \sin^2(p/2) + \sinh^2(m/2)$, which worked well up to charmonium masses.

The JLQCD collaboration (presented by A. Ukawa) have simulated quenched lattice QCD on a $24^3 \times 64$ lattice at $\beta = 6.0$ [20]. They have calculated hadron propagators and hence masses for 3 different $\kappa$ values of Wilson quarks. Most of their preliminary results were for $\kappa = 0.1550$ where $m_\pi/m_\rho \approx 0.70$.

They have used their high statistics to examine the “wiggles” in the plateaux of effective mass plots. Such behaviour is common. It is worrisome
because these fluctuations are often considerably larger than 1 standard deviation, and thus unexpected.

What Ukawa and his colleagues do is to calculate the complete covariance matrix from the data. They then model the data with a propagator which is a 2-exponential fit to the data, with gaussian fluctuations in the directions of the eigenvectors of the true covariance matrix. The width of these gaussian fluctuations is determined by the eigenvalues of this true covariance matrix. Thus this model has the same average propagator and the same covariance matrix as the actual data. Its effective mass plot for any finite sample shows deviations from the plateau by more than 1 standard deviation, as does the real data. They thus conclude that the existence of such large fluctuations is a property of the covariance matrix of the theory, rather than an indication that no plateau exists or that their datasets are not independent.

Suzuki and collaborators [21] have studied quenched QCD with Wilson quarks on a $16^3 \times 32$ lattice at $\beta = 5.7$. Their study is aimed at determining whether abelian monopoles, which have been suggested by ’t Hooft [22] to be responsible for confinement, are responsible for generation of hadron masses in the chiral limit.

Their procedure is as follows. After gauge fixing to the maximal abelian gauge, they factorize the gauge fields into a diagonal abelian gauge field, and a residual field. They then calculate field strengths corresponding to these diagonal gauge fields. These abelian field strengths are separated into “smooth”, “photonic” contributions and Dirac string, monopole contributions. Abelian gauge fields are reconstructed from each of these field strengths.

They have calculated Wilson quark and hence hadron propagators for each of the full gauge field, the abelian gauge field, the monopole (Dirac string) gauge field and the photonic gauge field. What they have found is that the $\pi$ and $\rho$ masses for the full and abelian gauge configurations approach the same values in the chiral limit. The results for the proton are unclear. More work needs to be done before it will be known whether the masses for the monopole gauge field yield the same results. The “photon” field case, as expected, does not reproduce the correct spectrum in the chiral limit.

### 2.3. QUENCHED CLOVERLEAF

I will start by discussing briefly the work of Lacock and Michael [14], who introduced the quantity $J = m_K \frac{d m_P S}{d m_P S}$ evaluated at $m_V/m_P S = m_K^*/m_K \approx 1.8$. Experimentally $J=0.49(2)$, if they use the assumption that $m_P S^2 \propto (m_1+m_2)/2$.

| Table 9 |
|------------------|
| The hadron masses in lattice units at $\beta = 6.0$. |
| $\kappa$          | 0.1515 | 0.153 | 0.155 |
| $m_\pi$          | 0.504(1) | 0.422(2) | 0.297(2) |
| $m_\rho$          | 0.570(2) | 0.507(2) | 0.422(2) |
| $m_N$            | 0.900(4) | 0.798(4) | 0.658(4) |
Figure 7. Dependence of hadron masses on the quark mass

with $m_1$ and $m_2$ the quark and antiquark masses in the pseudoscalar meson, $PS$ and $m_V = c + d(m_1 + m_2)$ with $c$ and $d$ constants. Ignoring for the moment any isospin breaking, $m_x/m_\rho$ fixes the degenerate u and d quark masses, while $m_K/m_\pi$ fixes the strange quark mass. Hence, on the lattice, $J$ is determined. From the UKQCD data for the unimproved cloverleaf action they found $J = 0.37(3)(4)$, and similar values for Wilson data from other groups.

The UKQCD collaboration (presented by H. Shanahan [23]) have studied the light-light hadron spectrum with tadpole-improved cloverleaf quarks. They ran at $\beta = 5.7$, 6.0 and 6.2 on lattices sizes up to $24^3 \times 48$. Two $\kappa$ values were used at each $\beta$ ($\kappa = 0.14144$ and 0.14226 at $\beta = 6.2$). Point and Jacobi smeared and/or “fuzzed” sources and sinks were used.

Using multiple sources and sinks allowed stable 2-particle fits to propagators and better determination of the ground state mass. They used smeared-smeared (SS), smeared-local (SL) and local-local (LL) propagators in their fits. Effective mass plots for each of these propagators for the vector states (from their earlier runs with an unimproved cloverleaf action) are shown in Figure 8 [14]. Note that these various propagators clearly have different couplings to the excited states, which allows stable 2-mass fits (shown in the figure).

$J$ (defined above) was measured. The results were $0.368(5)$ ($\beta = 5.7$), $0.330(19)(17)$ ($\beta = 6.0$) and $0.363(60)$ ($\beta = 6.2$), still below the experimental value.

$m_V^2 - m_{PS}^2 \approx 0.55\text{GeV}^2$ experimentally, from light mesons through light-charm mesons. UKQCD’s measured values of this quantity are in good agreement with this observation. Wilson quarks and unimproved cloverleaf quarks are in considerably worse agreement with this observation. This is of particular interest, since this splitting is due to the spin-orbit coupling.

Figure 10 shows the $a$ dependence of $m_\rho$ (ob-
Figure 9. Effective mass v’s time separation for vector mesons, at $\kappa = 0.14266$. The symbols are LL(+), SL(×) and SS(○). The curves are the 2-state fits.

2.4. QUENCHED VALENCE

K.-F. Liu and S.-J. Dong have studied light hadron spectroscopy in the valence quark approximation [24]. They work on a $16^3 \times 24$ lattice at $\beta = 6$, with $\kappa = 0.148$, 0.152 and 0.154.

A word of explanation is needed since the quenched approximation itself is often referred to as the valence quark approximation (even in this talk). Liu and Dong’s valence quarks are quenched Wilson quarks with all backward moving contributions removed. Hence for meson propagators, exactly 1 quark and 1 antiquark pass through any timeslice between source and sink. For baryons exactly 3 quark lines pass through any timeslice.

Light hadron spectroscopy with valence quarks shows even better SU(6) symmetry than one might expect. The $\Delta$ and nucleon masses are almost degenerate, as are the $\rho$ and pion masses (in SU(6) $\Delta$ and N are in the same multiplet, as are $\rho$ and $\pi$). This is shown in Figure 11.

They have also applied this approximation to the calculation of low energy matrix elements.

2.5. IMPROVED-QUENCHED CLOVERLEAF

The SCRI collaboration (presented by R. Edwards [25]) are calculating the light hadron spectrum in the quenched approximation. Gauge configurations are being generated using the tadpole improved action of Lepage et al. [9,26], with $\beta = 6.8–7.9$ (note that these $\beta$ values correspond to much smaller $\beta$’s for the Wilson action). Quark propagators are being calculated using use tadpole improved cloverleaf action. Lattice sizes are $8^3 \times 16$, $8^3 \times 32$ and $16^3 \times 32$. They are using two gaussian smeared sources (and sinks?). Four
Valence approximation masses

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{valence_approximation_masses.png}
\caption{Δ, N, ρ and π masses as a function of quark mass in the valence quark approximation.}
\end{figure}

\kappa’s corresponding to fixed pion masses are being used. Fits to multiple correlation functions allow 2-exponential fits and better isolation of the ground state.

This gauge action has errors \( O(a^4) \) (and \( O(\alpha_s^2 a^2) \)), compared with \( O(a^2) \) for the standard (Wilson) gauge action. The cloverleaf quark action has errors \( O(a^2) \) (and \( O(\alpha_s^2 a^2) \)). They are measuring the \( a \) dependence of the spectrum. The meson spectrum appears consistent with being linear in \( a^2 \) (the nucleon mass has yet to be measured). The \( a \) dependence of the \( \rho \) mass is given in Figure 12.

The lattice spacings are relatively large (0.16 – 0.41fm). (The chief advantage of using the improved action is to allow this use of coarser lattices.) Of course, one really needs an \( O(a^4) \) quark action to make full use of this fact.

\[ J = m_K \frac{d m_K}{d m_{\rho}} \text{, defined above, is measured and is } \lesssim 0.39 \text{ for the parameters of these simulations (experiment } = 0.48(2). \text{ The potential and string tension are calculated as a scale for the hadron masses.}

\section{2.6. IMPROVED-QUENCHED IMPROVED}

The Cornell group (presented by M. Alford \cite{27}) has studied quenched QCD with the tadpole improved action of Lepage et al.\cite{9,26}. The quark and hence hadron propagators are generated on these configurations using the D234 improved quark action. They work with \( 5^3 \times 14 \) and \( 6^3 \times 16 \) lattices at \( \beta = 6.8 \text{ and } 7.1 \) respectively.

The improved gauge action produces results which differ from the continuum theory by terms \( O(a^4) \) (and \( O(\alpha_s^2 a^2) \)). The D234 quark action is given by

\begin{equation}
S_q = \bar{\psi} [-\slashed{D} + m + \frac{a^2}{6} D^{(3)} - \frac{ra}{2} (D^{(2)} + g F \sigma) + sa^2 D^{(4)}] \psi
\end{equation}

whose physics differs from the continuum by terms \( O(a^3) \) (and \( O(\alpha_s a^2) \)). This allows the use of much larger lattice spacings than was possible with the Wilson gauge action, and Wilson, cloverleaf or staggered quark action. In fact, with the chosen \( \beta \)'s the sizes of the spatial boxes are \( \sim 2 \text{fm} \), which is larger than a \( 16^3 \) spatial box at \( \beta = 6.0 \), with the Wilson gauge action.

The spectrum appears similar to that obtained from conventional simulations – \( m_N/m_{\rho} \sim 1.5 - 1.6 \) for \( m_{\pi}/m_{\rho} \sim 0.6 - 0.65 \). The value of the quantity \( J \), defined above, is 0.38(2) at \( \beta = 6.8 \) and 0.40(2) at \( \beta = 7.1 \).

Figure 12. \( a \) dependence of the (\( \rho \)) mass. \( \sigma \) is the string tension.
2.7. FULL QCD STAGGERED

The COLUMBIA group (presented by D. Chen [28]) have continued their studies of the light hadron spectrum with 2 flavours of dynamical staggered quarks. They ran on a $16^3 \times 40$ lattice, at $\beta = 5.7$, with staggered quark mass $m_q = 0.01$. Two sources were employed for spectrum calculations, a $16^3$ (wall) source and an $8^3$ (octant) source, with the gauge field fixed to coulomb gauge.

Using 2 sources, with high statistics, enables 2-particle fits for the pion propagator and 4-particle fits for other hadron propagators, enabling better isolation of the ground state in each channel, and giving a preliminary estimate for the mass of the first(?) excited state.

For the $\Delta$ they tried Landau as well as Coulomb gauge. The mass from the Landau gauge propagators was smaller than that from the Coulomb gauge propagators. What is unclear is whether this is a true effect due to the unphysical excitations in the Landau gauge, or just that the onset of the plateau is delayed in the Landau gauge. Figure 13 shows the $\Delta$ effective mass in coulomb gauge.

![Figure 13. $\Delta$ effective mass in Coulomb gauge](image)

The chiral behaviour of the pion propagator as a function of valence quark mass has been calculated and will be discussed further in section 3.

2.8. FULL-QCD-STAGGERED CLOVERLEAF

The SCRI group (presented by J. Sloan [29]) are calculating the light hadron spectrum on the $16^3 \times 32$ HEMCGC configurations which were generated with dynamical staggered quarks. The spectrum is being calculated with tadpole improved cloverleaf quarks. $\beta = 5.6$, and $m_q = 0.01$ for the dynamical quarks. Their spectrum calculations use wall and shell-smeared sources.

They employ fits to multiple propagators which enables better isolation of the ground state.

Extrapolations of hadron masses to the chiral limit are being studied. Vector meson masses are fit to forms which are polynomial in the pseudoscalar meson mass $M_{PS}$. They have tried fits to polynomials up to fourth order, but which do not include a linear term. Preferred fits were to the forms

$$M_V = C_0 + C_2 M_{PS}^2 + C_3 M_{PS}^3$$  \hspace{1cm} (2)$$

and

$$M_V = C_0 + C_2 M_{PS}^2 + C_4 M_{PS}^4$$  \hspace{1cm} (3)$$

The cubic fit above yields coefficients $C_0 = 0.393(4)$ and $C_2 = 1.10(3)$ for cloverleaf quarks. The data and these fits are given in Figure 14. The fits for Wilson quarks are given for comparison. From these chiral extrapolations of the $\rho$ mass they obtain $a^{-1} = 2.25(3)$GeV for Wilson quarks, = 1.92(2)GeV for cloverleaf. This is to be compared with 1.80(2)GeV obtained for staggered quarks.

A linear approximation to $J$ defined above, yields $0.425(6)(10)$ for Wilson quarks and $0.444(5)(26)$ for cloverleaf compared with the experimental value 0.499. Cloverleaf light vector meson masses are in reasonable agreement with experimental values and closer than Wilson masses. Higher mass mesons are predicted to be lighter than experimentally measured.
3. THE CHIRAL LIMIT — CHIRAL LOGARITHMS v’s FINITE SIZE v’s FINITE LATTICE SPACING

In quenched QCD, chiral perturbation theory predicts that

\[ m_\pi^2 = 2\mu m_q \left( \frac{m_\pi^2}{A^2} \right)^{-\delta} \]  

rather than the usual

\[ m_\pi^2 = 2\mu m_q \]  

[30,31] where \( \delta \) is estimated to be \( \approx 0.2 \).

On the other hand finite lattice size effects would be expected to modify the above relation (Equation 5) to

\[ m_\pi^2 = \text{constant} + 2\mu m_q \]  

where constant \( \to 0 \) as the spatial volume \( V \to \infty \). The COLUMBIA group point out that these two departures from the canonical behaviour can be very difficult to distinguish [32].

In our \( 32^3 \times 64 \) pion mass data we observed departures from the normal relation above, which were well fit by the quenched chiral log formula [33]. Since there was good agreement between our \( 32^3 \times 64 \) and \( 24^3 \times 64 \) data, we rejected the interpretation of this as being a finite size effect. We show this data in Figure 15.

Figure 14. Cubic fit of \( m_V v’s m_{PS}^2 \)

Figure 15. \( m_\pi^2/m_q \) v’s \( m_\pi^2 \) at \( \beta = 6.0 \) (quenched).

The COLUMBIA group first observed similar departures from the canonical relationship in full QCD at \( \beta = 5.48 \) on a \( 16^3 \times 32 \) lattice and at \( \beta = 5.7 \) on a \( 16^3 \times 40 \) lattice when the dynamical quark mass was held fixed (at \( m_q = 0.004 \) and 0.01 respectively) [32]. Although one would expect anomalous chiral behaviour in this case, it should not be identical to the quenched case. They also performed quenched simulations at \( \beta = 5.7 \) on a \( 16^3 \times 40 \) lattice and obtained similar results. For all their pion spectra they found good fits to the finite-size-modified formula. They also managed to fit our results to the formula with a finite intercept, although the fit had a much worse \( \chi^2 \). For their own simulations, they were able to correlate these finite size effects in \( m_\pi \) with those
in $\langle \bar{\psi} \psi \rangle$. Figure 16 shows the variation of $m^2_{\pi}/m_q$ with quark mass.

Figure 16. $m^2_{\pi}/m_q$ v’s $m_q$. Columbia results (32\textsuperscript{3} x 64 results are from S. Kim and DKS).

Gottlieb’s quenched data at $\beta = 5.7$, 5.85 and 6.15, combined with a collage of data at $\beta = 6.0$, indicates rather different variation of $m^2_{\pi}/m_q$ with $m_q$ at different $\beta$’s suggesting that at least some of the variation is a finite lattice spacing effect \cite{12}. His data at $\beta = 5.7$ shows mass dependence out to masses for which finite size effects must be small (see Figure 17).

4. THE $\eta'$ MASS

Kilcup et al. \cite{34} have calculated the $\eta'$ mass using a method similar to that used by Kuramashi et al. \cite{35}, but using staggered, rather than Wilson quarks. They use $16^3 \times 32$ lattices at $\beta = 6.0$ (quenched) and $\beta = 5.7$ (full QCD – Columbia group). The full QCD configurations had $m_q = 0.025$ and 0.01.

They calculate both the connected (1-loop) and disconnected (2-loop) contributions to the $\eta'$ propagator. The connected piece is calculated in the standard fashion, but using a noisy source. The disconnected piece requires the inverse of the propagator at “zero” separation averaged over all sites of the source and sink time-slices (actually pairs of time slices with staggered quarks). Since this is impractical, it is replaced by a stochastic estimator, using 96 noise vectors per lattice.

The ratio of the 2-loop to 1-loop contributions to the propagator rises linearly in time separation $t$ for quenched QCD. For full QCD it approaches a constant as $t \to \infty$, exponentially. Such behaviour is evident in their results. They estimate the $\eta'$ mass from the slope of the linear rise in the quenched theory and from the mass scale in the exponential approach to the asymptotic value for full QCD. Using the approximate value $a^{-1} = 2$GeV, they get $m_0 = 1050(170)$MeV for the quenched theory, and $m_0 = 730(250)$MeV for the full theory, where

$$m^2_{\eta'} = m_0^2 + m^2_8$$  \hspace{1cm} (7)
and $m_8$ is the degenerate flavour octet ($\pi/K/\eta$) mass.

In addition they have studied the mass spectrum on cooled gauge field configurations and have confirmed that the $\eta'$ gets its mass from large instantons. This agrees with the earlier work with Wilson fermions.

5. SUMMARY AND CONCLUSIONS

Meson spectroscopy is becoming precise enough to ask such questions as whether the quenched approximation adequately describes mesons made of $u,d,s$ quarks and their antiquarks. Baryon spectroscopy is approaching this stage. However, understanding of the systematic errors associated with choices of sources, sinks, and choices of fits, is not yet good enough to preclude significant disagreements between groups. (From Section 2 one would conclude that the quenched approximation is less capable of reproducing the correct light hadron spectrum than was claimed by the GF-11 group.)

With Wilson quarks there does not appear to be any strange quark mass which gives the correct light hadron spectrum in the quenched approximation, at the lattice spacings currently used. Going to cloverleaf quarks does not appear to improve the situation, and even using the Lepage improved gauge and quark actions does not lead to a significant improvement. All these cases yield values of $J$ in the 0.36–0.40 range, whereas the experimental value is in the range 0.48–0.50. Preliminary results for staggered quarks extrapolated to $a = 0$ give $J = 0.43(1)$. The splittings in the baryon multiplets (Wilson quarks) appear to be too small, for any reasonable value of the strange quark mass.

With good statistics it appears possible to accurately extrapolate to the “physical” $u$ and $d$ quark masses. Such mass extrapolations, and the interpolations used to include the effects of the $s$ quark, are helped by the fact that, at least for light quark physics, the assumption that hadron masses depend on only the sum of valence quark masses appears valid.

For full QCD, cloverleaf quarks are an improvement over Wilson quarks. The value of $J$ is better for cloverleaf than Wilson quarks, and for both Wilson and cloverleaf quarks it is better than the quenched values.

We have seen the first attempts at implementing the Lepage scheme for improved actions. For more serious implementations this will require going to an $O(a^2)$ fermion action (to match the gauge action). This scheme shows great promise as a calculational tool.

For the chiral limit of quenched lattice QCD with staggered quarks, it is difficult to sort out chiral logarithms from finite size effects and finite lattice spacing effects. If we have not seen chiral logarithms, the obvious question is where are they, and what would it take to see them. Part of the difficulty in observing chiral logarithms is the flavour symmetry breaking, which gives the non-Goldstone pions and the disconnected part of the $\eta'$, larger masses than the Goldstone pion. It would therefore be helpful to study quenched QCD with Wilson/cloverleaf quarks, which have no flavour symmetry breaking, down to such light pion masses. In conclusion, although the difficulty in observing quenched chiral logarithms is bad news for theory, it means that their effects are small, which is good news for phenomenology.

Although we are still quite a way from doing precision calculations of the light hadron spectrum with dynamical quarks, progress has been made, particularly within the quenched approximation. Improved actions probably are our best chance for full QCD. More work appears necessary to understand systematics.

Note added in proof: Since the preparation of this manuscript we have received a revised draft of the LANL preprint [19]. In this, a more extensive analysis of the baryon octet mass splittings reveals that these splittings are not linear in the quark mass splittings, as had been previously assumed. When this is properly taken into account, the baryon octet splittings are in reasonable agreement with experiment. However, for the baryon decuplet, the linear formula works well, and the disagreement with experiment remains.
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