Precision neutron interferometric measurements of the n-p, n-d, and n-\(^{3}\)He zero-energy coherent neutron scattering amplitudes

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Abstract

We have performed high precision measurements of the zero-energy neutron scattering amplitudes of gas phase molecular hydrogen, deuterium, and \(^{3}\)He using neutron interferometry. We find \(b_{np} = (-3.7384 \pm 0.0020)\) fm\(^{1}\), \(b_{nd} = (6.6649 \pm 0.0040)\) fm\(^{2,1}\), and \(b_{n^{3}He} = (5.8572 \pm 0.0072)\) fm\(^{3}\). When combined with the previous world data, properly corrected for small multiple scattering, radiative corrections, and local field effects from the theory of neutron optics and combined by the prescriptions of the Particle Data Group, the zero-energy scattering amplitudes are: \(b_{np} = (-3.7389 \pm 0.0010)\) fm, \(b_{nd} = (6.6683 \pm 0.0030)\) fm, and \(b_{n^{3}He} = (5.853 \pm .007)\) fm. The precision of these measurements is now high enough to severely constrain NN few-body models. The n-d and n-\(^{3}\)He coherent neutron scattering amplitudes are both now in disagreement with the best current theories. The new values can be used as input for precision calculations of few body processes. This precision data is sensitive to small effects such as nuclear three-body forces, charge-symmetry breaking in the strong interaction, and residual electromagnetic effects not yet fully included in current models.

Key words: neutron interferometry, scattering amplitude, neutron optics, NN potentials, three-nucleon force, effective field theory

1. Introduction

The last decade has seen a revolution in the accuracy with which low energy phenomena in nuclear few body systems can be calculated. Insight into certain features of few-nucleon systems has come both from greatly-improved calculations using potential models based on the measured nucleon-nucleon (NN) interaction\(^{4}\) and also from the development of effective field theory (EFT) approaches based on the chiral symmetry of QCD\(^{5,6,7}\). Such theories have been used to develop a physical understanding rooted ultimately in QCD for the relative sizes of many quantities in nuclear physics, such as nuclear N-body forces \(^{8}\) and in particular the nuclear 3-body force (3N), which is now investigated intensively. Although it is well understood that 3N forces must exist with a weaker strength and shorter range than the NN force, little else is known.

EFT has been used to solve the two and three nucleon problems with short-range interactions\(^{7,9}\). For
the two-body system, EFT is equivalent to effective range theory and reproduces its well-known results for NN forces [10,11,12]. The chiral EFT expansion does not require the introduction of an operator corresponding to a 3N force until next-to-next-to leading order in the expansion, and at this order it requires only two low energy constants [13,14], which are taken to be the triton binding energy and the zero energy doublet n-d scattering amplitude. There have also been significant advances in other approaches to the computation of the properties of few-body nuclei with modern potentials such as the AV18 potential [15,16], which includes electromagnetic effects and terms to account for charge-independence breaking and charge symmetry breaking of the strong interaction. These calculations accurately reproduce the well-measured energy levels of few body bound states only with the phenomenological inclusion of a nuclear 3-body force, so it is clear that more information on this force is needed for further progress.

Precision measurements of low-energy strong interaction properties, such as the zero-energy scattering amplitudes and electromagnetic properties of small A nuclei, are therefore becoming more important for low energy, strong interaction physics both as precise data that can be used to fix parameters in the EFT expansion and also as new targets for theoretical prediction. In this article, we briefly summarize a series of precision measurements of the zero-energy coherent scattering amplitudes in the two, three, and four-body systems using neutron interferometric techniques [1,2,3]. We summarize these zero-energy coherent scattering amplitude results and discuss possibilities for future measurements.

2. Neutron Optics Theory

The zero-energy coherent scattering amplitude is the linear combination of scattering amplitudes that gives rise to the optical potential of a neutron in a medium [17]. The zero-energy bound scattering amplitude, $b$, is related to the free scattering amplitude $a$ by

$$b = \frac{m + M}{M} a.$$  

(1)

Here, $m$ is the mass of the neutron and $M$ is the mass of the atom. For hydrogen, $a$ is the linear combination of the singlet and the triplet scattering amplitudes given by,

$$a_{np} = \frac{1}{4} a_{np} + \frac{3}{4} a_{np}.$$  

(2)

for deuterium it is the linear combination of the doublet and quartet scattering amplitudes,

$$a_{nd} = \frac{1}{3} a_{nd} + \frac{2}{3} a_{nd}.$$  

(3)

and for $^3$He, it is the linear combination of the singlet and triplet scattering amplitudes,

$$a_{n^3He} = \frac{1}{4} a_{n^3He} + \frac{3}{4} a_{n^3He}.$$  

(4)

The bound zero-energy scattering amplitude is one particular linear combination of the triplet and singlet (or doublet and quartet) scattering amplitudes. Knowledge of some other combination allows one to independently extract the individual bound (or free) scattering amplitudes for each state.

The phase shift measured in neutron interferometry is proportional to the real part of the S-wave coherent scattering amplitude in the medium,

$$\phi = k(1 - n)D = -\lambda ND b.$$  

(5)

where $k$ is the incident wave vector, $n$ is the index of refraction, $N$ is the number density, $D$ is the thickness of the sample, and $\lambda$ is the neutron wavelength. Thus to experimentally measure $b$, the neutron optical phase shift $\phi$, the atom density, the sample thickness, and the neutron wavelength must each be measured to high precision.

3. Experimental Procedure

Scattering amplitude measurements were performed at the National Institute of Standards and Technology (NIST) Center for Neutron Research (NCNR) Interferometer and Optics Facility [18]. A cold monochromatic neutron beam ($E = 11.1$ meV) enters the perfect silicon crystal neutron interferometer and is coherently divided via Bragg diffraction into two beams that travel along paths $I$ and $II$ as shown schematically in Fig. 1. These beams are again diffracted and then coherently recombined to form the interference pattern. A detailed description of the facility, experimental arrangement, and procedures for the determination of zero-energy neutron scattering amplitudes can be found in Ref. [1].

A secondary sampling method is used to measure the phase shift $\phi$ due to the gas sample. This is accomplished by positioning a rotatable quartz phase shifter across the two beams as shown in Fig. 1. The intensities of the beams that arrive at the two $^3$He detectors are a function of the phase shifter angle $\delta$ and are given by

$$I_O(\delta) = A_O + B \cos(C f(\delta) + \phi_{gas} + \phi_{cell})$$  

(6)

$$I_H(\delta) = A_H + B \cos(C f(\delta) + \phi_{gas} + \phi_{cell} + \pi).$$

The values of $A_O$, $A_H$, $B$, and $C$ are extracted from fits to the data. The function $f(\delta)$ depends on the Bragg angle $\theta_B$ and is a measure of the neutron optical path length difference between the beams induced by the phase shifter and is given by

$$f(\delta) = \frac{\sin(2\theta_B)}{2\sin(\theta_B)} \delta.$$
Fig. 1. A schematic view of the experimental setup as the neutron beam passes through the perfect crystal silicon interferometer. Parameters associated with the neutron optics are discussed in the text.

\[ f(\delta) = \frac{\sin(\theta_B) \sin(\delta - \delta_0)}{\cos^2(\theta_B) - \sin^2(\delta - \delta_0)}. \]  

The hydrogen, deuterium, or \(^3\)He gases are housed in a cell specifically designed to minimize the phase shift \(\phi_{\text{cell}}\) due to the aluminum walls of the cell (see Fig. 1). The phase shifts arising from the presence of the gas and cell (\(\phi_{\text{gas}}\) and \(\phi_{\text{cell}}\)) were determined by collecting \(\approx 10^3\) interferogram pairs with the cell positioned both within the interferometer and removed from the beam paths. The phase difference between cell-in/cell-out sets of interferograms is extracted for each pair, with a typical set shown in Fig. 2. The phase shift from the cell, \(\phi_{\text{cell}}\), was determined using an evacuated cell.

The atom density was determined using the measured purity of the gas and the ideal gas law with virial coefficient corrections up to the third pressure coefficient. The absolute temperature was continuously monitored using two calibrated 100 \(\Omega\) platinum thermometers that have an absolute accuracy of 0.023 \% at 300 K. The pressure was continuously monitored using a calibrated silicon pressure transducer capable of measuring the absolute pressure to better than 0.01 \%. The wavelength of neutrons traversing the interferometer was measured using a pyrolytic graphite (PG 002) crystal. This analyzer crystal, calibrated separately against a Si crystal with a precisely-known lattice constant, was placed in the H-beam of the interferometer and rotated so that both the symmetric and anti-symmetric Bragg reflections were determined. In a separate test, the stability of the wavelength over the measurement time was shown to be 0.001 \%. More details on the measurement techniques and systematic uncertainties involved in the determination of the neutron wavelength, atom density, temperature, and cell thickness can be found in refs. [1] and [3].

The value of the bound zero-energy scattering amplitude was calculated for each data set on a run-by-run basis. These values were combined using a weighted average to obtain \(b\) for each gas species. Our reported results are \(b_{np} = (−3.7384 \pm 0.0020)\) fm[1], \(b_{nd} = (6.6649 \pm 0.0040)\) fm[2,1], and \(b_{n^3\text{He}} = (5.8572 \pm 0.0072)\) fm[3].

4. Results and Discussion

Since these measurements of the n-p, n-d, and n-\(^3\)He zero-energy scattering amplitudes were performed in an identical manner using the same apparatus (cell, neutron wavelength analyzer, and pressure and temperature monitors), one can take the ratio of the measurement values to obtain results which are even less sensitive to any potential remaining systematic errors. Using \(b_{np}\) as the reference, we obtain the ratios

\[ \frac{b_{n^3\text{He}}}{b_{np}} = (−1.5668 \pm 0.0021) \]
\[ \frac{b_{nd}}{b_{np}} = (−1.7828 \pm 0.0014). \]

Although these ratios possess slightly larger statistical uncertainties, it will be independent of any unknown systematic uncertainty to first order and can provide a more robust target for comparison to theories which attempt to calculate all of the scattering lengths within a common theoretical framework.

Recently, theoretical predictions of the n-d zero-energy scattering amplitude were performed using the high precision NN forces CD Bonn 2000, AV18, Nijm I, II and 93 in combination with 3N force models[19]. The results of these calculations are summarized in
Fig. 3. Theoretically determined values for the n-d scattering amplitude $b_{nd}$ and the triton binding energy using different NN and 3N forces[19]. The experimental value consists of our measurement of $b_{nd}$ and the very precisely known value of the triton binding energy, $(8.481855 \pm 0.000013)$ MeV[20].

Fig. 3 alongside the experimental data. For NN forces alone with and without electromagnetic interactions, they recovered the approximate correlation between the triton binding energy and $2a_{nd}$ known as the Phillips line. The approximate correlation between these observables is now understood to be a generic feature for systems such as the deuteron and other low A nuclei whose size is significantly larger than the 1 fm scale of the NN interaction range. However it is understood that this correlation is only an approximation and should not be obeyed exactly: for example 3N forces will introduce corrections. Although the addition of 3N forces of a wide variety of types does shift the values closer to the observed $2a_{nd}$, none of these calculations agrees with the new high precision measurements. The authors note that the $2a_{nd}$ zero-energy scattering amplitude must be considered as an independent low energy observable for future calculations in few body systems.

A second group has recently published new calculations of the spin-dependent $n-^3$He scattering amplitudes using the Resonating Group Method and a variety of modern NN and 3N potentials[21]. Comparisons of the experimental free nuclear singlet and triplet scattering amplitudes as determined from our measurement and the $n-^3$He incoherent scattering amplitude from Zimmer et al.[22] with theoretical calculations of these parameters were performed using an R-matrix formalism and the AV18 NN potential with two types of 3N forces[21].

The precision of these measurements is now high enough to severely constrain these few-body models. Both the n-d and n-$^3$He coherent neutron scattering amplitudes are in disagreement with the best current theories. Only models which correctly take into account nuclear three-body forces, charge-symmetry breaking, and residual electromagnetic effects have the possibility to successfully confront the data.

We can combine these measurements with the previous world’s data to obtain new values for the coherent scattering amplitudes. We note that some of the past measurements of high precision using a gravity refractometer performed in Garching a few decades ago[23] must be corrected for local field effects[24], which are significant for the coherent neutron scattering amplitude in hydrogen $b_{np}$ as measured in the gravity refractometer but are negligible in neutron interferometry. This correction shifts the value from $b_{np} = (-3.7409 \pm 0.0011)$ fm[23] to $b_{np} = (-3.7390 \pm 0.0011)$ fm[24], an approximately two sigma effect. In addition there is another two sigma correction to the neutron interferometry measurement in hydrogen gas due to multiple scattering and correlation corrections to the neutron index of refraction which are negligible for the gravity refractometer measurements. These corrections have already been applied to obtain the value of $b_{np} = (-3.7384 \pm 0.0020)$ fm[1] quoted above. After these corrections, both independent measurements are in excellent agreement. We can therefore present an improved value of the coherent neutron scattering length of hydrogen $b_{np} = (-3.7389 \pm 0.0010)$ fm. These corrections turn out to be negligible for deuterium.
5. Future Measurements

As mentioned above, the measured value of $b_{np}$ was corrected for multiple scattering and correlation corrections to the neutron index of refraction. A more precise interferometric measurement in comparison with the gravity refractometer value would be able to isolate this term experimentally. Such a measurement would constitute the first experimental observation of corrections to the neutron index of refraction due to virtual excitations and multiple internal scattering within a molecule. Such effects were predicted by Nowak 20 years ago[25]. The calculation of Nowak, which was done in the long wavelength limit $kR_0 \rightarrow 0$ ($R_0 =$ bond length of $\text{H}_2 = 0.74611$ nm, for D$_2$ $R_0 = 0.74164$ nm [26]), must be extended to the conditions of the experiment (which correspond to $kR_0=1.73$) to make a precise prediction. It would be possible to measure $b_{np}$ using $\text{H}_2$ gas at least a factor of two more precisely, which may be accurate enough to isolate this neutron optics effect experimentally for the first time.

Our measurement of $b_{nd}$ will hopefully soon be complemented by a measurement in progress at PSI of the incoherent n-d scattering amplitude using pseudomagnetic precession in a polarized deuterium target[27], which is also an interferometric technique that operates in neutron spin space as opposed to real space. This measurement will allow one to separate the two spin channels and determine the interesting doublet amplitude, $a_{nd}$, with high precision. We have argued that the quartet amplitude should be independent of 3N forces and the theoretical value should be reasonably robust[28] and were thus able to extract a value of $a_{nd} = (0.645 \pm 0.003(\text{expt}) \pm 0.007(\text{theory})) \text{fm}[3]$, however an experimental determination is badly needed. We expect that these new measurements will improve $a_{nd}$ by an order of magnitude to $\approx 10^{-3}$. The doublet amplitude is very important from a theoretical point of view: in the EFT approach it fixes the doublet amplitude, $a_{d}$, and consequently there is still room for improvement. In the case of the incoherent scattering amplitude ($b_i$) determination from pseudomagnetic precession performed by Zimmer et al.[22], the accuracy is unfortunately limited by the poor experimental knowledge of the relative contributions of singlet and triplet channels to the n-3He absorption cross section. A better measurement of this ratio, currently known to $\approx 1 \%[29,30]$, could be immediately combined with the Zimmer et al. measurement to improve the accuracy of $b_i$ by as much as a factor of three. In addition, a new measurement currently being planned at NIST to directly measure the spin-dependent n-3He scattering amplitudes is independent of this ratio [31]. Dramatically reduced uncertainties for $a_{s3He}$ and $a_{t3He}$ in n-3He are therefore possible.

Measurements in other light nuclei are also quite feasible. Two targets that we are presently exploring are 4He and tritium. From a theoretical point of view at present, a 4He measurement is not that interesting because the framework for solving five-body problems is not yet in place. The tritium measurement on the other hand is quite interesting theoretically because the small inelastic effects make it much easier to calculate than n-3He, but it is experimentally difficult because of the radioactive nature of the target. We are investigating the possibility of performing a measurement of $b_{nt}$ using the same techniques used in the current measurements. We expect that the limiting factor in these measurements will be how well we can determine the isotopic composition of the radioactive tritium gas.

Perhaps the single most interesting low energy NN scattering amplitude to measure is the neutron-neutron scattering amplitude $b_{nn}$. A value for $b_{nn}$ is the last remaining obstacle to theoretically calculating the n-A scattering amplitudes (A > 3) from first principles. At present, no direct measurements of $b_{nn}$ exist. An experiment to determine $b_{nn}$ by viewing a high-density neutron gas near the core of a reactor and measuring a quadratic dependence of the neutron fluence on source power is currently being designed[32]. A second experiment to let the neutrons in an extracted beam scatter from each other has also been considered [33]. An EFT analysis to extract $b_{nn}$ from the $\pi^- d \rightarrow nn\gamma$ reaction has also recently been performed [34]. Any direct measurement would need to be performed to an accuracy of a few percent to be physically interesting.

One additional measurement using neutron interferometry is presently underway at NIST, a determination of the neutron’s mean square charge radius by measuring the neutron-electron scattering amplitude, $b_{ne}$[35]. This experiment is expected to lead to a five-fold improvement in the precision of $b_{ne}$. This precision may be sensitive to radiative corrections calculable in a recent EFT approach[36].

Theoretical advances over the last decade have made few-nucleon systems into a quantitative testing ground for low energy QCD. Low energy neutrons can be used to perform high-precision measurements of scattering amplitudes. We look forward to new high-precision measurements in this field in the next few years.
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