Subleading corrections to the $|V_{ub}|$ determination from exclusive $B$ decays

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(Dated: October 29, 2018)

Abstract

It has been proposed to determine the CKM matrix element $|V_{ub}|$ in a model-independent way from a combination of rare and semileptonic $B$ and $D$ decays near the zero recoil point. An essential ingredient in such a determination is a heavy quark symmetry relation connecting the form-factors appearing in $B \rightarrow K^* e^+ e^-$ to semileptonic form factors relevant for $B \rightarrow \rho e \nu$. We estimate the leading corrections to this symmetry relation, of order $\alpha_s(m_b)$ and $\Lambda/m_b$, pointing out that they can be as large as 20%, depending on the value of the matrix element of a dimension-4 operator. Dimensional analysis estimates of this matrix element give a corresponding uncertainty in $|V_{ub}|$ of the order of a few percent.
The Cabibbo-Kobayashi-Maskawa matrix element $V_{ub}$ is an important ingredient of the flavor structure of the Standard Model. Its magnitude determines one of the sides of the unitarity triangle. Although at present it is one of most poorly known CKM parameters, several promising methods have been proposed to extract it from experimental data (see [1] for a recent review).

We focus in this paper on a particular model-independent method for determining $|V_{ub}|$ from exclusive rare and semileptonic $B$ decays proposed in [2, 3, 4]. There are two basic ingredients going into such a determination:

1. Heavy quark symmetry relations between heavy-light form factors [4, 5] connect the rate for $\bar{B}\to K^*\ell^+\ell^-$ at the zero recoil point $q^2_{\text{max}}$ to the rate for $\bar{B}\to\rho\ell\nu$, up to $\Lambda/m_b$ and $SU(3)$ breaking corrections.

$$\frac{\text{d} \Gamma(\bar{B}\to\rho\ell\nu)/\text{d}q^2}{\text{d} \Gamma(\bar{B}\to K^*\ell^+\ell^-)/\text{d}q^2} = \frac{|V_{ub}|^2}{|V_{tb}V_{ts}|^2} \frac{8\pi^2}{\alpha^2} \frac{1}{|C_9|^2 + |C_{10}|^2} \frac{|f^{B\to\rho}(y)|^2}{|f^{B\to K^*}(y)|^2} \frac{1}{1 + \Delta(y)}$$

(1)

In this relation the ratio of decay rates is taken at a common value of the parameter $y$ defined by $v \cdot p_Y = m_Y y$. The correction $\Delta(y)$ parametrizes the contribution of the magnetic penguin operator $Q_7$ near $y = 1$.

2. The $SU(3)$ breaking corrections introduced in step 1 can be eliminated with the help of semileptonic decay $D\to(\rho,K^*)\ell\nu$ data, by using an approximate equality to unity of the double ratio [7]

$$R(y) \equiv \left| \frac{|f^{B\to\rho}(y)|/|f^{B\to K^*}(y)|}{|f^{D\to\rho}(y)|/|f^{D\to K^*}(y)|} \right| = 1 + \mathcal{O} \left( \frac{m_s}{m_c} - \frac{\Lambda}{m_b} \right).$$

(2)

First evidence for the mode $B\to K^*\ell^+\ell^-$ has been recently reported by the BaBar collaboration, and an upper limit was given by BELLE [8, 9]

$$B(\bar{B}\to K^*\ell^+\ell^-) = (1.68^{+0.68}_{-0.58} \pm 0.28) \times 10^{-6} \quad \text{(BaBar)}$$

$$< 14 \times 10^{-7} \quad \text{(90\% CL) \quad (BELLE)}.$$  

(3)

This suggests that a determination of $|V_{ub}|$ using these decays might become feasible in a not too distant future.

There are several sources of theoretical uncertainties connected with such an approach. The dominant theoretical uncertainty is connected with long-distance contributions coming from four-quark operators in the weak Hamiltonian $Q_1 = (\bar{s}_\alpha \gamma_\mu P_L b_\beta)(\bar{c}_\beta \gamma_\rho P_L c_\alpha)$, $Q_2 = (\bar{s}_\gamma \mu P_L b)(\bar{c}_\gamma \rho P_L c)$ [10]. Their effect can be absorbed into a redefinition of the Wilson coefficient $C_9$ [10] but this modification is generally process-dependent. It can be computed reliably in perturbation theory as long as the $e^- e^+$ invariant mass is sufficiently low. Such computations have been performed for inclusive $B\to X_s e^+ e^-$ [11, 12] and exclusive $B\to K^* e^+ e^-$ decays [14, 15]. At small recoil, as considered in this Letter, this method is not applicable and one has to resort instead to a phenomenological parameterization of these effects in terms of sums over $J/\psi$ resonances [16]. While this is not a controlled approximation, the validity of such an expansion can be tested by measuring other observables such as $q^2$ spectra and/or angular asymmetries [4, 15].
A better understood source of uncertainty is introduced in step 2 as corrections to the double ratio \( R(y) \). From power counting, such corrections are of order \( \frac{m_s}{m_c} \left( \frac{\Lambda}{m_b} - \frac{\Lambda}{m_c} \right) \approx 7\% \). A more precise estimate has been performed in [3] using the chiral perturbation theory for vector mesons [17]. The corresponding deviation of the ratio \( R(1) \) from unity was found to be small, under 1%.

Finally, another source of corrections is introduced in the first step of the method. As shown in [2], the contribution of the electromagnetic penguin operator \( Q_7 \) to the \( B \to K^* e^+e^- \) decay rate (parameterized by \( \Delta(y) \)) can be computed in a model-independent way at the zero recoil point \( y = 1 \) using heavy quark symmetry. The corrections to this prediction are of order \( O(\Lambda_{QCD}/m_b) \) and were thus thought to be negligible.

Recently, the structure of the leading corrections to the heavy quark symmetry relations for heavy-light decays [4, 19] has been studied in [19]. They come from dimension 4 operators and from hard gluon effects appearing as Wilson coefficients in the HQET. We are therefore now in a position to study the leading corrections in step 1 of the method. As mentioned, the leading corrections are of order of 3%.

The amplitude for \( B \to K^* e^+e^- \) receives in general contributions from all operators in the matching of the weak current onto heavy quark effective theory (HQET) operators, and from hard gluon effects appearing as Wilson coefficients in the HQET. We are therefore now in a position to study the leading corrections in step 1 of the method. As mentioned, the leading corrections are of order of 3%.

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The two helicity amplitudes $H^{(V,A)}_{\pm 1}(q^2)$ correspond to the vector and axial coupling to the leptons, respectively. Expressed in terms of the form factors defined in (7)–(9), they are given explicitly by

$$H^{(V)}_{\pm 1}(q^2) = C_7 \frac{2m_b}{q^2} (g_+(q^2)[\pm 2m_B|\vec{q}| + m_B^2 - m_V^2] + g_-(q^2)q^2)$$
$$+ C_9 [\pm 2m_B g(q^2)|\vec{q}| + f(q^2)]$$
$$H^{(V)}_0(q^2) = C_7 \frac{2m_b \sqrt{q^2}}{2m_V} (g_+(q^2)(m_B^2 + 3m_V^2 - q^2) + g_-(q^2)(m_B^2 - m_V^2 - q^2) + 4h(q^2)m_B^2q^2)$$
$$+ C_9 \frac{1}{m_V \sqrt{q^2}} \left( (f(q^2) \frac{1}{2}(m_B^2 - m_V^2 - q^2) + 2m_B^2q^2a_+(q^2) \right)$$

and

$$H^{(A)}_{\pm 1}(q^2) = C_{10} (\pm 2m_B g(q^2)|\vec{q}| + f(q^2))$$
$$H^{(A)}_0(q^2) = C_{10} \frac{1}{m_V \sqrt{q^2}} \left( (f(q^2) \frac{1}{2}(m_B^2 - m_V^2 - q^2) + 2m_B^2q^2a_+(q^2) \right)$$

At the zero recoil point $y = 1$ (corresponding to maximal $q^2$, $q^2_{\text{max}} = (m_B - m_V)^2$), the form of the helicity amplitudes simplifies drastically and can be written only in terms of one axial form factor $f(y) = 1$ as

$$H^{(V)}_{\pm 1}(y = 1) = H^{(V)}_0(y = 1) = C_9 f(1)(1 + \delta(1))$$
$$H^{(A)}_{\pm 1}(y = 1) = H^{(A)}_0(y = 1) = C_{10} f(1)$$

where we defined

$$\delta(y) \equiv \frac{2m_b}{m_B - m_V} C_7 \cdot \frac{\mathcal{F}(y)}{f(y)}$$

with

$$\mathcal{F}(y) \equiv g_+(y)(m_B + m_V) + g_-(y)(m_B - m_V).$$

Inserting the expressions for the helicity amplitudes at the end-point (13), (16) into the rate for $B \to K^* \ell^+ \ell^-$, gives the following result for the correction $\Delta(y)$ appearing in (1) [2, 3]

$$\Delta(y) = \frac{1}{|C_9|^2 + |C_{10}|^2} \left\{ \frac{4m_b}{m_B - m_V} \text{Re} \left( C_7 C_5 \right) \frac{\mathcal{F}(y)}{f(y)} + \frac{4m_b^2}{(m_B - m_V)^2} |C_7|^2 \left( \frac{\mathcal{F}(y)}{f(y)} \right)^2 \right\}. \quad (19)$$

The form-factor ratio $\mathcal{F}(1)/f(1)$ is predicted from heavy quark symmetry to be 1 at leading order in the heavy quark limit [2, 3]. The purpose of this Letter is to estimate the leading corrections to this result, and study their effect on the $|V_{ub}|$ determination.

Using the results of [19] one finds the following prediction from heavy quark symmetry

$$\mathcal{F}(y) = \left( \kappa_5 + \frac{\bar{\Lambda} - m_V}{m_B} \right) f(y) + 2m_B m_V \left( 1 - \frac{1}{\kappa_1} \right) g(y) + \frac{2}{m_B} \mathcal{D}_1(y)$$
$$+ 2m_B m_V \left( 1 - \frac{\bar{\Lambda} + m_V}{m_B} \right) (y - 1) g(y) + 2m_V (y - 1) \mathcal{D}(y) + \cdots,$$
where the ellipses denote contributions suppressed by $\Lambda^2/m_b^2$ relative to the leading term.

The subleading form factors $\mathcal{D}(y)$ and $\mathcal{D}_1(y)$ appearing in (20) are defined by matrix elements of the dimension-4 currents

$$\langle V(p',\varepsilon)|\bar{q}iD_\mu h_v|\bar{B}(v)\rangle = \mathcal{D}(y)i\varepsilon_{\mu\nu\lambda\sigma}\varepsilon^{\nu}_vp_\lambda p'_\sigma \tag{21}$$

$$\langle V(p',\varepsilon)|\bar{q}iD_\mu\gamma_5 h_v|\bar{B}(v)\rangle = \mathcal{D}_1(y)(\varepsilon^*\cdot p)(p_\mu + p'_\mu) + \mathcal{D}_2(y)(\varepsilon^*\cdot p)(p_\mu - p'_\mu) \tag{22}.$$  

The equation of motion for the heavy quark field $iv\cdot D_hv = 0$ implies a relation among the form factors of the $\bar{q}D_\mu\gamma_5 h_v$ current, such that only two of them are independent.

The coefficients $\kappa_1$ and $\kappa_5$ contain hard gluon corrections. The first coefficient $\kappa_1 = -c_0(m_b)/c_0'(m_b)$ is defined as the ratio of two Wilson coefficients appearing in the matching for $J_\mu = \bar{q}\gamma_\mu b$ and $J'_\mu = \bar{q}i\sigma_{\mu\nu}v^\nu b$

$$J_{\mu}^{(l)} = c_0^{(l)}(\mu)\bar{q}\gamma_\mu h_v + c_0^{(l)}(\mu)\bar{q}v_\mu h_v + O(1/m_b). \tag{23}$$

The values of the Wilson coefficients can be extracted from the next-to-leading computation of (20) and their explicit values can be found in (19). For our estimate we only require their expressions to one-loop order, which give $\kappa_1 = 1 + O(\alpha_s^2(m_b))$.

The coefficient $\kappa_5$ is defined analogously in terms of the Wilson coefficients appearing in the matching of the currents $J_{5\mu} = \bar{q}i\sigma_{\mu\nu}\gamma_5 \bar{b}$ and $J'_{5\mu} = (g_{\mu\nu} - v_\mu v_\nu)\bar{q}\gamma_\nu\gamma_5 \bar{b}$

$$J_{5\mu}^{(l)} = c_0^{(l)}(\mu)\bar{q}\gamma_\mu\gamma_5 h_v + c_1^{(l)}(\mu)\bar{q}v_\mu\gamma_5 h_v + O(1/m_b). \tag{24}$$

The explicit results for the Wilson coefficients depend on the $\gamma_5$ definition and are given by (20)

$$c_0(m_b) = c_1(m_b) = 1 - \frac{4\alpha_s(m_b)}{3\pi} + O(\alpha_s^2) \quad \text{(NDR,HV)} \tag{25}$$

$$c_0'(m_b) = c_1'(m_b) = \left\{ \begin{array}{l} 1 + O(\alpha_s^2) \quad \text{(NDR)} \vspace{0.2cm} \\
1 - \frac{4\alpha_s(m_b)}{3\pi} + O(\alpha_s^2) \quad \text{(NDR)} \end{array} \right. \tag{26}$$

This gives for the coefficient $\kappa_5$

$$\kappa_5 = \frac{c_0(m_b)}{c_0'(m_b)} = \left\{ \begin{array}{l} 1 + O(\alpha_s^2) \quad \text{(NDR)} \\
1 - \frac{4\alpha_s(m_b)}{3\pi} + O(\alpha_s^2) \quad \text{(NDR)} \end{array} \right. \tag{27}$$

For completeness we give also the corrected HQET symmetry relations used in deriving (20) (19)

$$\kappa_1(g_+ - g_-) + 2mBg = -2(mV y - \bar{\Lambda})g(y) - \frac{1}{m_B}f(y) - 2\mathcal{D}(y) + O(m_b^{-3/2}) \tag{28}$$

$$g_+ + g_- - 2mV y \mu - \kappa_5 \frac{1}{m_B}f =$$

$$-2\frac{mV}{m_B} (\bar{\Lambda}y - mV)g(y) + \frac{\bar{\Lambda}}{m_B^2}f + \frac{2}{m_B}(mV m_B y \mathcal{D}(y) + \mathcal{D}_1(y)) + O(m_b^{-5/2}) \tag{29}.$$  

From (20) one finds for the ratio of form factors appearing in $\delta(1)$ at the zero recoil point $y = 1$  

$$\frac{\mathcal{F}(1)}{f(1)} = \kappa_5 + \frac{\bar{\Lambda} - mV}{m_B} + 2m_B mV \left(1 - \frac{1}{\kappa_1}\right)\frac{g(1)}{f(1)} + \frac{2}{m_B} \cdot \mathcal{D}_1(1) \tag{30}$$

$$= \left\{ \begin{array}{l} 1 - 0.1 + \frac{2}{m_B} \cdot \frac{\mathcal{D}_1(1)}{f(1)} \quad \text{(NDR)} \\
1 - 0.09 - 0.1 + \frac{2}{m_B} \cdot \frac{\mathcal{D}_1(1)}{f(1)} \quad \text{(HV)} \end{array} \right.$$

5
term depends on the subleading form factor $D_1$ for the form factor ratio (30), using two sets of Wilson coefficients $\kappa$ are negligibly small (the numerical value for $m^-$ scheme could be as large as

if this term turns out to be negative, the overall correction to the ratio (30) in the NDR dimensional analysis its contribution is expected to be of order $\Lambda/m$. Therefore, if this term turns out to be negative, the overall correction to the ratio (30) in the NDR scheme could be as large as $-20\%$.

We give in Table I the values of the coefficient $\Delta(1)$ corresponding to a generic range for the form factor ratio (30), using two sets of Wilson coefficients $C_{7,9,10}(m_b)$. The first set corresponds to the next-to-leading log (NLL) approximation $C^{NLL}_{7} = -0.314$, $C^{NLL}_{9} = 4.154$, $C^{NLL}_{10} = -4.261$. These values correspond to the NDR scheme, and were obtained at $\mu = 4.6$ GeV, with $m_b(m_b) = 4.4$ GeV, $\Lambda_{QCD}^{(m=5)} = 220$ MeV following [14]. The perturbative correction from $\kappa_5$ multiplies $C_7$, thus this order consistency requires that $\kappa_5$ not be included. The one-loop matrix elements of $Q_{1-6}$ are included by absorbing them into a scheme-independent effective Wilson coefficient $C^{eff}_9$ defined as in [14,11]. At the zero recoil point this is given by $C^{eff}_9(1) = C_9(m_b)\tilde{\eta}(1) + 0.208 + i0.363$, with $\tilde{\eta}(1) = 1 - 0.61\alpha_s(m_b)/\pi = 0.95$, where we used $m_c = 1.4$ GeV. The corresponding numerical results for $\Delta(1)$ are shown in the first line of Table I.

A second set of Wilson coefficients corresponds to the NNLL approximation and takes $C^{NNLL}_{7} = -0.308$, $C^{NNLL}_{9} = 4.214$, $C^{NNLL}_{10} = -4.312$ [2,3,4]. These partial results corresponding to the NDR scheme do not contain the (as yet unknown) three-loop mixing into $Q_9$. The associated uncertainty in $C^{NNLL}_9$ was estimated in [14] and found to be $\sim \pm 0.1$. Also, the complete results for the $O(\alpha_s)$ matrix elements of the $Q_{1-6}$ operators are not available, although they were recently computed in [13] in an expansion in powers of $q^2/m_b^2, m_c^2/m_b^2$. These approximate results are not applicable in the zero recoil region considered here.

At NNL order the radiative correction from $\kappa_5$ has to be included as well, which raises the issue of $\gamma_5$ scheme independence of the result. This has been demonstrated explicitly at NLL order in [14]. In analogy with this result, one expects that the scheme dependence in $\kappa_5$ will be cancelled by that in the matrix elements of the four-quark operators $\sum_{i=1}^6 C^{NNLL}_i(Q_i)^{NLL}$ and by that in $C_7$. In the numerical evaluation one should use for consistency the NDR scheme for all quantities involved.

The uncertainty in the value of $|V_{ub}|$ extracted from [14] coming from $\Delta(1)$ is dominated by that in $D_1(1)$. For $|2D_1(1)/(m_b f(1))| \leq 0.1$ one finds from Table I a 3% effect in $|V_{ub}|^2$. A precise computation of $D_1(1)$ could help eliminate this source of uncertainty.

In the quark model the ratio of form factors $D_1(1)/f(1)$ appearing in (30) is always positive. Keeping only a $S$-wave component for the vector meson wave function, one finds in the static limit for the $b$ quark

$$D_1(1)/f(1) = \frac{1}{6m_q} \cdot \frac{\langle \phi_V^+, \bar{p}^2 \phi_B \rangle}{\langle \phi_V^+, \phi_B \rangle},$$  \hspace{1cm} (31)

with $m_q$ the mass of the light quark produced in the weak decay $b \rightarrow q$. The expectation values can be computed explicitly in the ISGW model [23] with the result

$$D_1(1)/f(1) = \frac{1}{2m_q} \cdot \frac{\beta_B^2 \beta_X^2}{\beta_B^2 + \beta_X^2} = 0.093 \text{ GeV} \quad (B \rightarrow \rho)$$  \hspace{1cm} (32)
\[ = 0.062 \text{ GeV} \ (B \rightarrow K^*) \].

We used here the parameters of the ISGW model \( \beta_B = 0.41 \text{ GeV}, \ m_u = m_d = 0.33 \text{ GeV}, \ m_s = 0.55 \text{ GeV}, \ \beta_\rho = 0.31 \text{ GeV}, \ \beta_{K^*} = 0.34 \text{ GeV} \). This amounts to a small positive contribution of 2-3\% from the last term in (30).

The correction to the ratio of form factors (30) can be also extracted from the QCD sum rule calculation of Ref. [22]. This gives \( \mathcal{F}(1)/f(1) = 1.17 \) for \( B \rightarrow \rho \) and 1.18 for \( B \rightarrow K^* \). A similar QCD sum rule calculation in [23] quotes the range \( \mathcal{F}(1)/f(1) = 1.15^{+0.13}_{-0.07} \) for \( B \rightarrow K^* \). This amounts to a large positive subleading contribution of \( \sim 30\% \) from the last term in (30). The large discrepancy in \( D_1 \) with the dimensional analysis estimate and the quark model result is rather puzzling. It is not clear if this anomalously large subleading correction is an artifact of the sum rule computation or of the interpolation formulas in [22, 23]. A precise determination of this formfactor is clearly important.

In conclusion, we studied in this Letter the effect of subleading corrections to a heavy quark symmetry relation relevant for the determination of \( |V_{ub}| \) from exclusive rare and semileptonic \( B \) decays. The structure of the corrections to this symmetry relation is analyzed using the heavy quark expansion. We point out a possible large effect, depending on the value of an unknown matrix element of a dimension-4 operator. Lattice computations could eventually help to eliminate this source of uncertainty in exclusive determinations of \( |V_{ub}| \).

We thank Iain Stewart for comments on the manuscript. D. P. is grateful to Andrey Grozin for discussions about the results of Ref. [20]. This work has been supported by the DOE under Grant No. DOE-FG03-97ER40546.

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