Design of Fractional Swarm Intelligent Computing With Entropy Evolution for Optimal Power Flow Problems

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ABSTRACT Optimal reactive power dispatch (ORPD) problems in power system have been solved by using several variants of traditional nature inspired particle swarm optimization (PSO) with aim to achieve a promising solution for a given objective such as line loss, voltage deviation and overall cost minimization. Several schemes have been designed to improve the performance of the optimization technique in tuning the operational variables and analyzed by evaluating the final results. In this article, a different method is designed to solve ORPD problems, by introducing Shannon entropy based diversity in the fractional order PSO dynamics, i.e., FOPSO-EE. The results show that synergy of both, the Shannon entropy and the fractional calculus can be used as the useful tools for enhancing the optimization strength of algorithm while solving the ORPD problems in standard IEEE 30 and 57 bus power systems. The performance of the design FOPSO-EE is further validated through results of statistical interpretations in terms of histogram analysis, box plot illustration, quantile-quantile probability plot and empirical probability distribution function.

INDEX TERMS Computational intelligence, optimal power flow, fractional calculus, Shannon entropy, particle swarm optimization.

I. INTRODUCTION

A. MOTIVATION AND INCITEMENT
Optimal reactive power dispatch (ORPD) aims to improve the performance of a power system by reducing the voltage deviation, transmission line losses, operational cost, improving system security, stability index, and line capacity. Most of these objectives are achieve by tuning the operational variables of the network such as the transformers tap positions, generator output voltages, capacitor banks and solid state flexible AC transmission systems (FACTS) devices [1]. However, the optimal tuning of these variables is a complex problem due to the multi-modal, non linear and discrete nature of the power systems.

B. LITERATURE REVIEW
In the literature, several arithmetic, stochastic, evolutionary, social-based and meta-heuristic optimization techniques are developed since last few decades including the differential evolution [2], genetic algorithm [3]–[8], particle swarm optimization (PSO) etc. A summary of the proposed optimization techniques for ORPD is documented in Table 1. Since the development of the canonical PSO, a considerable number of its variants based on different numerical tools have been proposed in order to enhance the algorithm performance, and significantly applied in a plethora of applications [9]–[14].
### TABLE 1. Summary of proposed algorithms for ORPD.

| Ref   | Algorithm                      | Objectives | Year |
|-------|--------------------------------|------------|------|
| [15]  | Evolutionary programming        | $f_1$      | 1995 |
| [3]   | Adaptive genetic algorithm      | $f_1$      | 1998 |
| [16]  | PSO                             | $f_1$      | 2000 |
| [10]  | Multi agent PSO                 | $f_1$      | 2005 |
| [4]   | Improved GA                     | $f_1$, $f_2$ | 2005 |
| [17]  | GA-interior point method        | $f_1$      | 2006 |
| [18]  | Modified PSO                    | Stability   | 2007 |
| [19]  | Self adaptive real coded GA     | $f_1$      | 2009 |
| [20]  | Turbulent crazy PSO             | $f_1$, $f_2$ | 2009 |
| [9]   | Comprehensive learning PSO      | $f_1$      | 2010 |
| [21]  | Ant colony optimization         | $f_1$      | 2011 |
| [22]  | MNSGA-II                        | $f_1$, stability | 2011 |
| [23]  | Biogeography-based optimization | $f_1$, $f_2$ | 2011 |
| [24]  | Harmony search algorithm        | $f_1$, $f_2$, $f_3$ | 2011 |
| [25]  | Adaptive approaches             | $f_1$, $f_2$ | 2012 |
| [26]  | HFMOEA                          | $f_1$, stability | 2013 |
| [27]  | Opposition based GSA            | $f_1$, $f_2$, stability | 2013 |
| [28]  | MICA-IWO                        | $f_1$      | 2014 |
| [29]  | Differential evolution (DE)     | $f_1$      | 2015 |
| [30]  | Enhanced firefly algorithm      | $f_1$, $f_2$ | 2015 |
| [31]  | Gray wolf optimizer (GWO)       | $f_1$, $f_2$ | 2015 |
| [32]  | Teaching learning optimization  | $f_1$      | 2015 |
| [33]  | Hybrid firefly algorithm        | $f_1$, $f_2$ | 2015 |
| [34]  | Quasi-oppositional DE           | $f_1$, $f_2$, stability | 2016 |
| [35]  | Two-point estimate method       | $f_1$, $f_2$ | 2016 |
| [36]  | Chaotic krill herd              | $f_1$, $f_2$ | 2016 |
| [37]  | Exchange market algorithm       | $f_1$, $f_2$, stability | 2016 |
| [38]  | Backtracking search algorithm   | $f_1$, $f_2$ | 2016 |
| [39]  | Oppositional krill herd         | $f_1$, $f_2$ | 2016 |
| [40]  | Moth-flame optimization         | $f_1$      | 2017 |
| [41]  | GBWC                            | $f_1$, $f_2$ | 2017 |
| [42]  | Whale optimization              | $f_1$      | 2018 |
| [43]  | Chemical reaction optimization  | $f_1$, $f_2$, stability | 2018 |
| [44]  | ABC-FF                          | $f_1$, $f_2$, stability | 2018 |
| [45]  | Sine cosine algorithm           | $f_1$, $f_2$ | 2019 |
| [46]  | ALC-PSO algorithm               | $f_1$, $f_2$ | 2019 |
| [47]  | Moth Swarm Algorithm            | $f_1$, $f_2$ | 2019 |
| [48]  | Lightning Attachment Procedure  | $f_1$      | 2019 |
| [49]  | Enhanced GWO                    | $f_1$, $f_2$ | 2019 |
| [50]  | Artificial bee colony           | $f_1$, stability | 2020 |
| [51]  | Chaotic Bat Algorithm           | $f_1$, $f_2$, stability | 2020 |

* $f_1$ and $f_2$ are loss and deviation minimization, respectively.

The PSO consists on a machine learning procedure which is basically inspired by the behavior of social species such as fish schooling or birds flocking in search of food. Each fish or bird is represented by a particle that move on the search space for finding an optimal solution. The particle is characterized by two vectors, namely by its current velocity and position. A set of particles is known as swarm which evolves during several iterations constituting a powerful computational technique. Since 1995, many mathematical tools are presented to complement and/or refine the conventional PSO technique, namely by adopting integration with other evolutionary strategies and by analyzing the tuning parameters. In this regard, Machado and team [52], [53] presented a new method for improving the convergence rate of a canonical PSO based on fractional calculus (FC) theory and developed fractional order optimization technique, known as FO-PSO.

Afterwards, FO-PSO has been successfully designed for a number of fields such as computational biology, color image quantization, pattern recognition simulation and animation of natural flocks or swarms, computer graphics, and social modeling [54]–[68]. However, the strength of FC based optimization algorithms have not yet been explored in solving problems of energy and power sector specifically in the domain of ORPD. Besides FC, the Shannon entropy, a mathematical tool, has been tested in myriad of fields, such as biology, sociology, communications and economics among others in the recent years, however, its use has been overlooked in evolutionary computation.

### C. CONTRIBUTION AND PAPER ORGANIZATION

Considering the discussed concepts, a new algorithm i.e. fractional order particle swarm optimization with entropy evolution, known as FOPSO-EE, is designed and tested on primary problems of ORPD such as the minimization of the transmission line losses, overall cost of operation and voltage deviation in the IEEE standard power system and results are compared with those of other counterpart algorithms.

Therefore, this work uses the fractional calculus as a tool to improve the convergence rate of the canonical PSO while entropy metric to avoid the sub optimal solution. The salient contributions of this research work are:

- New diversity indices stimulated by biologic systems and physics based on entropy evolution are exploited to enhance the optimization strength of FOPSO.
- The better performance of designed FOPSO-EE is verified over its integer counterpart while solving the ORPD problems in standard power system considering the FACTS devices, namely, the TCSC and SVC.
- The results of statistical interpretations including the histogram analysis, box plot illustration, quantile-quantile probability and empirical probability distribution function are used to further endorse the accuracy, reliability and stability of proposed FOPSO-EE.
- Simplicity in the concept, smooth implementation and wider applicability in energy and power sector are other valuable perks of designed FOPSO-EE.

Bearing these ideas in mind, the organization of this paper is set as follows. Section 2 formulates the objective functions to be optimized. Section 3 develops the methodology followed in the work including a brief description of the fractional calculus, fractional PSO and entropy. Section 4 presents the experimental results for the
FOPSO-EE. Section 5 presents the statistical analysis. Finally, section 6 summarizes the main conclusions and discusses future recommendations.

II. MATHEMATICAL MODELS OF ORPD PROBLEMS

This section presents the objective functions that are considered during the tests of FOPSO-EE. The optimization functions consist in minimizing transmission line losses, voltage deviation index and overall cost of operation in standard power system, namely, IEEE-30 bus (Fig. 1), which consists of 4 tap changing transformers, 6 generators, 3 capacitor banks and 41 transmission lines [69].

A. TRANSMISSION LINE LOSS MINIMIZATION, \( P_{\text{loss}} \)

This function is mathematically expressed as:

\[
\text{Minimize } f_1(z) : P_{\text{loss}}(z) + \lambda(z) \quad (1)
\]

here,

\[
P_{\text{loss}} = \sum_{r=1}^{nl} G_r \left[ U_m^2 + U_n^2 - 2 \times U_m \times U_n \cos(\delta_m - \delta_n) \right] 
\]

\[z \in [U, T, Q] \quad (2)\]

hereafter, \( G_r \) is the conductance of \( r \)th line between \( m \) and \( n \) bus, \( U_m \) and \( U_n \) are the voltage magnitudes at \( m \)th and \( n \)th bus respectively, \( \delta_m \) and \( \delta_n \) are the voltage angles at \( m \)th and \( n \)th bus respectively, \( nl \) represents no. of transmission lines. \( \lambda \) is the penalty factor for control variables such as reactive power compensators \( Q \) (capacitor bank), transformer tap positions \( T \), and bus voltages \( U \) in the IEEE standard power system.

The general form of objective function based on penalty factor is expressed as:

\[
F = P_{\text{loss}} + \sum_{m=1}^{nload} Q_m(Q_m - Q_m^{\text{lim}})^2 + \sum_{m=1}^{nload} R_m(U_m - U_m^{\text{lim}})^2 + \sum_{m=1}^{nload} T_m(T_m - T_m^{\text{lim}}) 
\]

\[
Q_m^{\text{lim}} = \begin{cases} Q_m^{\text{max}} & \text{if } Q_m > Q_m^{\text{max}}, \quad m = 1, 2, \ldots, K_Q \\ Q_m^{\text{min}} & \text{if } Q_m < Q_m^{\text{min}}, \quad m = 1, 2, \ldots, K_Q 
\end{cases} 
\]

\[
U_m^{\text{lim}} = \begin{cases} U_m^{\text{max}} & \text{if } U_m > U_m^{\text{max}}, \quad m = 1, 2, \ldots, K_U \\ U_m^{\text{min}} & \text{if } U_m < U_m^{\text{min}}, \quad m = 1, 2, \ldots, K_U 
\end{cases} 
\]

\[
T_m^{\text{lim}} = \begin{cases} T_m^{\text{max}} & \text{if } T_m > T_m^{\text{max}}, \quad m = 1, 2, \ldots, K_T \\ T_m^{\text{min}} & \text{if } T_m > T_m^{\text{min}}, \quad m = 1, 2, \ldots, K_T 
\end{cases} 
\]

Here, \( Q \) is reactive power rating of capacitor bank, \( V \) is the voltage at generator bus, and \( T \) represents tap setting of transformer, \( K \) denotes total no. of buses, \( K_T \) is the number of transformer with tap changer and \( K_G \) is the number of generators in the power system. The minimum and maximum values of \( Q_m^{\text{lim}}, U_m^{\text{lim}}, \) and \( T_m^{\text{lim}} \) in above expressions represents the control variable’s permissible limits.

B. VOLTAGE DEVIATION, \( V_D \)

\[
\text{Minimize } f_2 = V_D = \sum_{m=1}^{nload} |V_m - 1.0| 
\]

here, \( nload \) represents the total number of load-buses [70].

C. OVERALL COST MINIMIZATION, \( C_{\text{total}} \)

\[
\text{Minimize } f_3 = C_{\text{total}} = C_{\text{energy}} + C_{\text{cpx}} \quad (8)
\]

where

\[
C_{\text{energy}} = 0.06 \cdot 365 \cdot 24 \cdot P_{\text{loss}}
\]

Here, days/year are 365, hours/day are 24, cost associated with power loss is 0.06 $/kWh. The cost \( C_{\text{cpx}} \), represents the capital cost of the reactive power compensators i.e., FACTS and taken from the Siemens AG database [7] as

\[
C_{\text{cpx}} = q \cdot \beta(MVA_{\text{rating}}) + \alpha(MVA_{\text{rating}}^2) 
\]

here, \( q, \beta \) and \( \alpha \) represents the cost coefficients while \( MVA_{\text{rating}} \) is the operating range in mega volt ampere (MVA) of FACTS devices. The cost functions for SVC, TCSC and UPFC can be expressed respectively as:

\[
C_{\text{SVC}} = 127.38 - 0.3051(MVA_{\text{rating}}) + 0.0003(MVA_{\text{rating}}^2) 
\]

\[
C_{\text{TCSC}} = 153.75 - 0.7139(MVA_{\text{rating}}) + 0.0015(MVA_{\text{rating}}^2) 
\]
\[ C_{UFC} = 188.22 - 0.2691(MVA_{rating}) + 0.0003(MVA_{rating})^2 \]  

(12)

1) INEQUALITY CONSTRAINTS

The expressions for inequality constraints are given below:

- **Transformer boundaries**
  \[ T_{\text{max}} n m \leq T_i \leq T_{\text{min}} n m , \quad m = 1, 2, \ldots, K_T \]  
  (13)

- **Generator boundaries**
  \[ U_{\text{max}} G m \leq U_{G m} \leq U_{\text{min}} G m , \quad m = 1, 2, \ldots, K_G \]  
  (14)

\[ Q_{\text{max}} G m \leq Q_{G m} \leq Q_{\text{min}} G m , \quad m = 1, 2, \ldots, K_G \]  
  (15)

- **Shunt-VAR boundaries**
  \[ Q_{\text{max}} c m \leq Q_{c m} \leq Q_{\text{min}} c m , \quad m = 1, 2, \ldots, N_c \]  
  (16)
FIGURE 3. Best, average and worst learning curves for (a) \( \alpha = 0.1 \) (b) \( \alpha = 0.2 \) (c) \( \alpha = 0.3 \) (d) \( \alpha = 0.4 \) (e) \( \alpha = 0.5 \) (f) \( \alpha = 0.6 \) (g) \( \alpha = 0.7 \) (h) \( \alpha = 0.8 \) (i) \( \alpha = 0.9 \) during power loss minimization.

- \textbf{FACTS devices constraints}

\[
-X_{max} \leq X_{TCSC} \leq X_{min} \quad (17)
\]

\[
-Q_{max} \leq Q_{SVC} \leq Q_{min} \quad (18)
\]

Here, \( max \) and \( min \) defines the upper and lower bounds, respectively, \( N_C \) is the no. of buses where reactive power compensators are coupled, \( X \) is the per unit (p.u) reactance offered by the TCSC, \( Q \) is the reactive power provided by the SVC.

\textbf{D. EQUALITY CONSTRAINTS}

The equality constraints includes the power balance equations [70]. Mathematically, for any bus \( m \), the real and reactive power balance equations are stated as:

\[
-U_m \sum_{n=1}^{K} U_n [B_{mn} \sin (\delta_m - \delta_n) + G_{mn} \cos (\delta_m - \delta_n)] - P_{Dm} + P_{Gm} = 0
\]

\[
-U_m \sum_{n=1}^{K} U_m [B_{mn} \cos (\delta_m - \delta_n) + G_{mn} \sin (\delta_m - \delta_n)] - Q_{Dm} + Q_{Gm} = 0
\]

Here, \( P_{Dm} \) and \( P_{Gm} \), are the demanded and injected real powers at \( m^{th} \) bus, \( Q_{Dm} \) and \( Q_{Gm} \) are the demanded and injected reactive powers at \( m^{th} \) bus.
\[ P_{mn} = U_m^2 G_{mn} - U_m U_n G_{mn} \cos(\delta_m - \delta_n) - U_m U_n B_{mn} \sin(\delta_m - \delta_n) \]
\[ Q_{mn} = -U_m^2 B_{mn} - U_m U_n G_{mn} \sin(\delta_m - \delta_n) + U_m U_n B_{mn} \sin(\delta_m - \delta_n) \]

Here, the conductance and susceptance of the transmission line are given by

\[ G_{mn} = \frac{R}{R^2 + (X - X_{TCSC})^2} \]
\[ B_{mn} = \frac{X - X_{TCSC}}{R^2 + (X - X_{TCSC})^2} \]

III. METHODOLOGY

This section presents the fundamental concepts about the computational tools, integrated to develop FOPSO-EE, such as the fractional calculus, particle swarm optimization and entropy.

A. FRACTIONAL CALCULUS

Fractional calculus (FC) is a generalization of the ordinary integer integration and differentiation to a non-integer order. FC was an important topic in the last few centuries and many mathematicians, such as Weyl, Riemann and Liouville contributed to its development. Fractional calculus has attained the focus of research community, being applied in various scientific fields such as irreversibility, modeling,
wave propagation, viscoelasticity, electronics, fractals, chaos, signal processing, control, biology, percolation, diffusion, and physics. There are number of alternate interpretations of fractional derivatives, one of the most important being, the Grunwald-Letnikov(GL), is based on the concept of fractional differential with order $\alpha \in C$ of a general signal $f(z)$, given by the equation [52], [71]:

$$D^\alpha f(z) = \lim_{h \to 0} \left[ \frac{1}{h^\alpha} \sum_{w=0}^{\infty} (-1)^w \Gamma(\alpha+1) f(z - wh) \right].$$

Here, $h$ represents the step time augmentation while $\Gamma(\cdot)$ is the gamma function. This, Grunwald-Letnikov, interpretation reveal an important property that the integer derivative just infers a finite series, while the fractional derivative involves an infinite numeral of terms, that is, they hold implicitly, a memory of previous events.

A discrete time implementation of the GL expression can be approximated as:

$$D^\alpha f(z) = \frac{1}{T^\alpha} \sum_{r=0}^{\infty} (-1)^r \Gamma(\alpha+1) f(z - rT) \Gamma(w+1) \Gamma(\alpha - w + 1),$$

(24)

Here $r$ is the truncation order and $T$ denotes sampling period.

The inherent memory property of fractional order systems make them much suitable to define phenomena such as chaos and irreversibility. Therefore, the random behavior of particle’s movement during search evolution constitute a scenario where FC tool fit appropriately.
B. PSO ALGORITHM

The traditional PSO algorithm, proposed originally by Kennedy and Eberhart in 1995, is a metaheuristic computational technique inspired by the movement of the particles in a swarm both for finding global solutions and as a defensive approach. This movement is represented by two vectors, specifically by its position \( x \) and velocity \( v \).

Algorithm 1 illustrates a canonical PSO mechanism. The cognitive learning of each particle is incorporated by considering the distance between its best position found up to now \( LB^n_t \) and the current position \( x^n_t \), while, the social learning of each particle, is obtained by taking the distance between the swarm global best position obtained up to now \( GB^n_t \) and its current position \( x^n_t \). Both learning factors are assigned a randomly generated weight \( \rho_1 \) and \( \rho_2 \), respectively.

\[
v^n_{t+1} = \omega v^n_t + \rho_1 r_1 (LB^n_t - x^n_t) + \rho_2 r_2 (GB^n_t - x^n_t),
\]

Hereafter, \( x \) denotes the particle’s position, \( t \) represents the flight index, \( n \) is the particle index with corresponding velocity \( v \), \( r_1 \) and \( r_2 \) are the random numbers between 0-1, \( \rho_1 \) is the local while \( \rho_2 \) is the global acceleration coefficients, \( LB \) represents the local and \( GB \) denotes the global best particle while \( \omega \) is inertial weight.

C. PSO WITH FRACTIONAL VELOCITY

In 2010, Machado and team [52], introduced a new technique to improve the convergence of conventional PSO by integrating the concept of fractional derivative in velocity update equation of conventional PSO. At first, the original velocity expression is reshuffled to alter the coefficient of the velocity derivative, specifically

\[
v^n_{t+1} = \omega v^n_t + \rho_1 r_1 (LB^n_t - x^n_t) + \rho_2 r_2 (GB^n_t - x^n_t),
\]
FIGURE 7. FOPSO-EE comparison with PSO-EE during $f_2$ minimization for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 2. Function parameters.

| Type | iter | $P$ | Parameters $[U, T, Q]$ | Range $(\epsilon)$ $\min - \max$ | $\rho_1, \rho_2$ | $\omega$ |
|------|------|-----|------------------------|-------------------------------|------------------|---------|
| $f_1$ | 50 | 30 | [6, 4, 3] | [0.9-1.1, 0.9-1.05, 0-5] | 0.9 - 0.1 | 0.9 - 0.4 |
| $f_2$ | 20 | 20 | [6, 4, 3] | [0.9-1.1, 0.9-1.05, 0-5] | 0.9 - 0.1 | 0.9 - 0.4 |

Considering the inertial influence, $\omega = 1$, equation (25) can be rearranged as:

$$v_{i+1}^n - v_i^n = \rho_1 r_1 (LB_i^n - x_i^n) + \rho_2 r_2 (GB_i^n - x_i^n) = \nabla(v_{i+1}^n)$$

while assuming $T = 1$, the expression $v_{i+1}^n - v_i^n$ is the discrete form of the derivative with coefficient $\alpha = 1$, implies to the following relation:

$$D^\alpha(v_{i+1}^n) = \rho_1 r_1 (LB_i^n - x_i^n) + \rho_2 r_2 (GB_i^n - x_i^n)$$

Considering $(24)$ with first four terms i.e. $r = 4$, (27) can be written as:

$$v_{i+1}^n - \alpha v_i^n - \frac{1}{2} \alpha(1-\alpha) v_{i-1}^n - \frac{1}{6} \alpha(1-\alpha)(2-\alpha) v_{i-2}^n$$

\[- \frac{1}{24} \alpha(1-\alpha)(2-\alpha)(3-\alpha) v_{i-3}^n\]

$$= \phi_1 r_1 (LB_i^n - s_i^n) + \phi_2 r_2 (GB_i^n - s_i^n)$$

or

$$v_{i+1}^n = \alpha v_i^n + \frac{1}{2} \alpha(1-\alpha) v_{i-1}^n + \frac{1}{6} \alpha(1-\alpha)(2-\alpha) v_{i-2}^n$$

$$+ \frac{1}{24} \alpha(1-\alpha)(2-\alpha)(3-\alpha) v_{i-3}^n + \phi_1 r_1 (LB_i^n - s_i^n) + \phi_2 r_2 (GB_i^n - s_i^n)$$

The coefficient $\alpha$ can be generalized to a real number $0 \leq \alpha \leq 1$, if the fractional calculus viewpoint is adopted, leading to a longer memory effect and smoother variation. It can
FIGURE 8. FOPSO-EE comparison with PSO-EE during minimization of $f_1$ in power system with FACTS for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 3. Comparison of results for case 1.

| Variables     | MFO [28] | GWO [31] | M-IWO [28] | ISA [70] | IWO [75] | CA [28] | PSO [70] | GA [9] | DE [69] | FOPSO-EE | FPSO-EE |
|---------------|----------|----------|------------|----------|----------|---------|----------|--------|--------|----------|---------|
| V1            | 1.1      | 1.1      | 1.07372    | 1.0726   | 1.06965  | 1.0785  | 1.0313   | 1.0721 | 1.095319 | 1.01     | 1.1     |
| V2            | 1.0946   | 1.096149 | 1.07055    | 1.0625   | 1.06038  | 1.06943 | 1.0113   | 1.063   | 1.085946 | 1.04231  | 1.1     |
| V5            | 1.0756   | 1.080036 | 1.04836    | 1.0399   | 1.03692  | 1.06943 | 1.0221   | 1.0377 | 1.062628 | 1.0401   | 1.0844  | 1.0833  |
| V8            | 1.772    | 1.080444 | 1.04865    | 1.0422   | 1.03864  | 1.04714 | 1.0031   | 1.0445 | 1.065076 | 1.0956   | 1.0820  | 1.08533 |
| V11           | 1.0868   | 1.093452 | 1.07518    | 1.0318   | 1.02973  | 1.03485 | 0.9744   | 1.0132 | 1.0266   | 1.011    | 1.0834  | 1.0931  |
| V13           | 1.1      | 1.1      | 1.07072    | 1.0681   | 1.05574  | 1.07106 | 0.9987   | 1.0898 | 1.014253 | 1.0491   | 1.0936  | 1.1     |
| T6-9          | 1.0411   | 1.04     | 1.03       | 1.01     | 1.05     | 1.08    | 0.97     | 1.0221 | 1.017796 | 1.061    | 1.0278  | 1.0434  |
| T6-10         | 0.95007  | 0.95     | 0.99       | 1        | 0.96     | 0.95    | 1.02     | 0.9917 | 0.979277 | 0.9295   | 1.0602  | 1.0294  |
| T4-12         | 0.95541  | 0.95     | 1.09       | 0.97     | 1        | 1.01    | 0.9964   | 0.97783 | 0.9665  | 0.90183  | 1.0752  |
| T27-28        | 0.95754  | 0.95     | 0.98       | 0.97     | 0.97     | 0.97    | 0.99     | 0.971  | 1.008938 | 0.9555   | 1.0362  | 1.0210  |
| Qc3           | 7.1032   | 12       | -7         | 34       | 8        | 6       | 17       | 5.3502 | 20.22359 | 8.4272   | 7.5420  | 4.2822  |
| Qc10          | 30.796   | 30       | 23         | 12       | 35       | 36      | 13       | 36.0   | 9.584327 | 25.1542  | 2.8600  | 2.6762  |
| Qc24          | 9.8981   | 8        | 12         | 10       | 11       | 11      | 23       | 12.4175| 13.02992 | 9.2331   | 9.4543  | 6.6747  |
| $P_{loss}$    | 4.608    | 4.613    | 4.846      | 5.109    | 4.92     | 4.849   | 5.8815   | 4.8775 | 4.888081 | 4.606    | 4.6448  | 4.5971  |
be seen from expression (29) that the traditional PSO is a special scenario of the fractional PSO with $\alpha = 1$. Because, the FOPSO integrates the fractional calculus tool to control the particle convergence, the fractional order $\alpha$ must needs to be identified to guarantee a high level of exploration during the evolution of search. The additional literature of basic FOPSO can be seen in [53], [71]–[74].

## D. ENTROPY

Several entropy definitions have been presented over the years, such as information, freedom spreading, mixing, chaos and disorder. The foremost interpretation of entropy was presented by Boltzmann as transformation of the system from ordered to disordered states. Lewis interpreted that, during the impulsive expansion of gas in an isolated system, the uncertainty or, missing information increases while knowledge about particles location decreases. Guggenheim used spreading to show the evolution in volume of an energy system from a small scale to a large scale. Shannon introduced the information theory for the quantization of the information loss during any message transmission in a communication network and concentrated on statistical and physical constraints that restrict the signal processing. Shannon described $H$ as a degree of uncertainty, information

![Figure 9: FOPSO-EE comparison with PSO-EE during minimization of $f_3$ in power system with FACTS for 100 independent runs](image)

**TABLE 4.** Percentage line loss reduction in test system.

| Parameter     | initial value | DE      | GA      | HSA     | M-IWO   | GWO-MFO | FO-DPSO  | PSO-EE   | FOPSO-EE |
|---------------|---------------|---------|---------|---------|---------|---------|----------|----------|----------|
| $P_{loss}$ (MW)| 5.663         | 4.88808 | 4.8775  | 5.109   | 4.846   | 4.613   | 4.608    | 4.606    | 4.6448   | 4.5971   |
| Loss reduction (%) | -             | 13.68   | 13.87   | 9.78    | 14.44   | 18.54   | 18.64    | 18.66    | 17.97    | 18.82    |
FIGURE 10. FOPSO-EE comparison with PSO-EE during $f_1$ minimization in IEEE 57 bus system for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 5. Comparative results of $f_2$ optimization.

| Methods    | $V_D$ (p.u) | % improvement | Methods    | $V_D$ | % improvement |
|------------|-------------|---------------|------------|-------|---------------|
| Base case  | 1.1606      | -             | -          | -     | -             |
| OGSMA      | 0.8540      | 26.41         | KHA        | 0.2963| 74.4          |
| ALC-PSO    | 0.3001      | 74.14         | CKHA       | 0.3524| 69.6          |
| CLPSO      | 0.245       | 78.89         | NGBWCA     | 0.3003| 74.1          |
| PSO        | 0.2424      | 79.11         | SGA        | 0.7369| 36.5          |
| PSO-EE     | 0.1177      | 89.8          | -          | -     | -             |
| FOPSO-EE   | 0.1057      | 90.8          | -          | -     | -             |

and choice:

$$H(X) = -K \sum_{x \in X} p_i(x) \log p_i(x)$$ (30)

For random variables $(x, y) \in (X, Y)$

$$H(X, Y) = -K \sum_{x \in X} \sum_{y \in Y} p_i(x, y) \log p_i(x, y)$$ (31)

where, $K$ is the constant parameter, normally equal to 1, $x \in X$ is a discrete variable and $p(x)$ is the probability distribution.

E. FOPSO WITH ENTROPY EVOLUTION

The method followed in this study is inspired by the need to explore and exploit the entropy during the fractional particle swarm optimizer time evolution and to adopt this
TABLE 6. Comparative results of case 3 and 4.

| Variable | SPSO [13] | WOA [76] | QOGWO [76] | QODE [76] | PSO-EE | FOPSO-EE |
|----------|-----------|-----------|------------|-----------|---------|----------|
| \(Q_C\) (2) | 0.6 | 0.6 | 0.0339 | 0.3535 | 7.2287 | 1.6343 |
| \(Q_C\) (5) | 0.0 | 0.6250 | -0.0027 | 0.2365 | 2.4845 | 3.3546 |
| \(Q_C\) (8) | 0.0 | 0.5 | 0.2907 | 0.4462 | 5.3979 | 0.3094 |
| \(Q_C\) (11) | 0.4 | 0.0029 | 0.0515 | 0.3497 | 4.0767 | 0.4432 |
| \(Q_C\) (13) | 0.0 | 0.0177 | 0.2342 | 0.2460 | 5.4464 | 7.1187 |
| T (11) | 0.9 | 0.9 | 0.9 | 0.9012 | 0.9803 | 0.9829 |
| T (12) | 0.9 | 0.9448 | 0.9452 | 0.9514 | 0.9791 | 0.9628 |
| T (15) | 0.9 | 0.9 | 0.9 | 0.9004 | 0.9683 | 0.9727 |
| T (36) | 0.9223 | 0.9285 | 0.9271 | 0.9278 | 0.9643 | 0.9660 |

TABLE 7. Comparative results of loss reduction and overall cost reduction.

| Methods | \(P_{loss}\) (p.u) | \(C_{overall}\) (USD) | Loss reduction (%) | Cost reduction (%) |
|---------|---------------------|------------------------|--------------------|--------------------|
| Base case | 0.0711 | 3737016 | - | - |
| WOA [76] | 0.06333 | 3.33E+06 | 10.928 | 10.723 |
| QOGWO [76] | 0.06331 | 3.33E+06 | 10.956 | 10.723 |
| SPSO [13] | 0.05198 | 2.73E+06 | 26.891 | 26.809 |
| QODE [76] | 0.0528 | 2.78E+06 | 25.788 | 25.469 |
| PSO-EE | 0.051675 | 2.69E+06 | 27.320 | 27.832 |
| FOPSO-EE | 0.051673 | 2.68E+06 | 27.323 | 28.260 |

synergy to enhance the algorithm characteristics, namely, the convergence. For this reason, the Shannon entropy is evolved in the internal structure of the optimizer while adopting the ORPD problems. Since the FOPSO is a non-deterministic solver, therefore, a set of 100 independent runs is performed to develop a representative statistical data set. Mean while, the influence of entropy signal is observed in the behavior of algorithm, namely by the swarm reini-
tialization, along the FOPSO execution for enhancing its convergence.

Indeed, the entropy measure the changing tendency of a system energy i.e., the spreading of particles within the search space, during the present case. Bearing this idea in mind, a distance \(d_i\) is considered between \(i^{th}\) particle position and the best global particle. Then, probability \(p_i\) for each particle is given by the distance \(d_i\) to maximum possible distance, that is:

\[ p_i = \frac{d_i}{d_{\text{max}}} \]  \hspace{1cm} (32)

For a \(n\) swarm size, and \(k = 1\), the diversity index (30) of particle is quantified as:

\[ H(X) = -\sum_{i=1}^{n} p_i \log p_i \]  \hspace{1cm} (33)

The overall workflow diagram is depicted in Fig. 2 while the procedural steps are illustrated in algorithm 2.

IV. RESULTS AND DISCUSSION

This section demonstrates the efficacy of FOPSO-EE over the other counter part algorithms for 4 different cases, each considering one objective function based on optimal reactive power dispatch scenarios defined previously using the parameters listed in Table 2.

Since particle swarm optimization is a stochastic method, every time it is applied it shows a different learning behavior. Therefore, a set of 100 independent trials was conducted, for each fractional order \(\alpha = 0.1, \ldots, 0.9\) and the median, arithmetic mean, maximum and minimum values were taken as the final output.

A. CASE 1: MINIMIZATION OF \(P_{\text{loss}}\)

The first objective function to be adapted belongs to the minimization of transmission line loss, as described by
TABLE 8. Comparison of results for case 5.

| Variables | Initial | NGBWCA | DE | CKHA | GSA | PSO | PSO-EE | FOPSO-EE |
|-----------|---------|--------|----|------|-----|-----|-------|---------|
| V1        | 1.04    | 1.06   | 1.0397 | 1.06 | 1.06 | 1.0834 | 1.0877 | 1.0915 |
| V2        | 1.01    | 1.0591 | 1.0463 | 1.059 | 1.06 | 1.0849 | 1.0853 | 1.0908 |
| V3        | 0.985   | 1.0492 | 1.0511 | 1.0487 | 1.06 | 1.0823 | 1.0800 | 1.0863 |
| V6        | 0.98    | 1.0399 | 1.0236 | 1.0431 | 1.0081 | 1.0818 | 1.0806 | 1.0829 |
| V8        | 1.05    | 1.0586 | 1.0538 | 1.06 | 1.0549 | 1.0887 | 1.0889 | 1.0875 |
| V9        | 0.98    | 1.0461 | 0.9451 | 1.0447 | 1.0098 | 1.0835 | 1.0798 | 1.0849 |
| V12       | 1.015   | 1.0413 | 0.9907 | 1.041 | 1.0185 | 1.0796 | 1.0833 | 1.0898 |
| T4-18     | 0.97    | 0.9712 | 1.02 | 0.9179 | 1.1 | 1.0429 | 1.0418 | 1.0327 |
| T4-18     | 0.978   | 0.9243 | 0.91 | 1.0256 | 1.0826 | 1.0107 | 1.0273 | 1.0238 |
| T21-20    | 1.043   | 0.9123 | 0.97 | 0.9 | 0.9219 | 1.0265 | 1.0260 | 1.0217 |
| T24-26    | 1.043   | 0.9001 | 0.91 | 0.902 | 1.0167 | 1.0380 | 1.0273 | 1.0249 |
| T7-29     | 0.967   | 0.9112 | 0.96 | 0.9104 | 0.9962 | 1.0120 | 1.0253 | 1.0345 |
| T34-32    | 0.965   | 0.9004 | 0.99 | 0.9005 | 1.1 | 1.0290 | 1.0236 | 1.0196 |
| T11-41    | 0.955   | 0.9128 | 0.98 | 0.9 | 1.0746 | 1.0394 | 1.0159 | 1.0203 |
| T15-45    | 0.955   | 0.96 | 0.96 | 0.9 | 0.9543 | 1.0117 | 1.0342 | 1.0233 |
| T14-46    | 0.9    | 1.0218 | 1.05 | 1.0797 | 0.9377 | 1.0436 | 1.0299 | 1.0333 |
| T10-51    | 0.9    | 0.9902 | 1.07 | 0.9887 | 1.0167 | 1.0165 | 1.0093 | 1.0195 |
| T13-49    | 0.895   | 0.9368 | 0.99 | 0.9914 | 1.0525 | 1.0269 | 1.0260 | 1.0360 |
| T11-43    | 0.958   | 0.9    | 0.96 | 0.9    | 1.1 | 1.0345 | 1.0429 | 1.0165 |
| T40-56    | 0.958   | 0.9    | 0.99 | 0.9002 | 0.9799 | 1.0282 | 1.0355 | 1.0375 |
| T39-57    | 0.98    | 1.0118 | 0.97 | 1.0173 | 1.0246 | 1.0286 | 1.0222 | 1.0421 |
| T9-55     | 0.94    | 0.9    | 1.07 | 1.0023 | 1.0373 | 1.0375 | 1.0402 | 1.0400 |
| Qc18      | 0      | 0.0914 | 0   | 0.0994 | 0.0782 | 4.6205 | 3.9490 | 3.0492 |
| Qc25      | 0      | 0.0587 | 0   | 0.059 | 0.0058 | 1.0404 | 3.1515 | 5.0487 |
| Qc53      | 0      | 0.0634 | 0   | 0.063 | 0.0468 | 4.0080 | 6.8901 | 7.6275 |
| Ploss,    | 27.86  | 26.74 | 35.94 | 27.48 | 29.4 | 26.896 | 26.4616 | 26.4390 |

equation (3). Equation (33) is considered to monitor the fractional PSO evolution. The learning curves are plotted in Fig. 3, showing the best, average and worst iterative updates of transmission line losses during 100 independent trials for \( \alpha = 0.1, \ldots, 0.9 \). In Table 3, the results can be seen for the \( P_{\text{loss}} \) function \( f_1 \) minimization, yielded by FOPSO-EE, along a comparison with other algorithms such as MFO, GWO, MICA-IWA, HSA, IWO, GA, DE, FO-DPSO, and PSO-EE. Observing Table 3, percentage line loss reduction in Table 4 and Fig. 3, one can conclude that, FOPSO-EE revealed a better behavior by computing minimum losses as compared to counter part algorithms, hence, the synergy of fractional calculus and entropy evolution contributes to an enhanced convergence dynamics.

Algorithm 1 Traditional PSO

1: procedure In steps with input and outputs
2: Inputs: Inertia weights, particle number, acceleration coefficients
3: Output: Global best solution
4: Start of PSO
5: Initialization: Swarm, random position \( x \) and velocity \( v \)
6: Fitness evaluation: Repeat until stopping criteria
   • for all particles calculate fitness
7: Updating mechanism: for all particles update,
   • pbest, gbest and lbest
   • velocity based on
   \[
   v^n_{t+1} = v^n_t + \rho_1 r_1 (LB^n_t - x^n_t) + \rho_2 r_2 (GB^n_t - x^n_t),
   \]
   • position based on
   \[
   x^n_{t+1} = x^n_t + v^n_{t+1}
   \]
8: Termination criterion: Convergence, stagnation, iteration
9: End PSO

B. CASE 2: MINIMIZATION OF \( V_D \)

The second optimization function to be adopted is the minimization of voltage deviation index, as described by expression (7), to monitor the FOPSO-EE evolution. The results are demonstrated in Fig. 4 for archive size of 20 particles and order \( \alpha = 0.1, \ldots, 0.9 \), where best global solution is found at \( \alpha = 0.9 \). The comparison of results revealed by FOPSO-EE and that by other algorithms is documented in Table 5. It can be verified that FOPSO-EE yielded better results with respect to the counter part algorithms.

C. CASE 3: MINIMIZATION OF \( P_{\text{loss}} \) WITH FACTS

In third case, the transmission line losses are minimized considering FACTS devices, namely, the TCSC and SVC, as the
Algorithm 2 Pseudocode of FOPSO-EE for ORPD

1: procedure In steps with inputs and outputs
2: Inputs: Bus, branch and generator data for IEEE standard power system i.e., IEEE 30 bus.
3: Output: Minimum power loss as expressed by equation (1), Minimum voltage deviation as expressed by equation (7) and Minimum overall cost as expressed by equation (8).
4: Start of FOPSO-EE
5: Initialization: Initialize
   • Position x and velocity v matrices with real values
   • Swarm with set of possible solutions P, known as particle in n-dimensional search space as:
     \[ S_n^{\text{min}} = \begin{bmatrix} V_1^{\text{min}}, & V_2^{\text{min}}, & \ldots, & V_n^{\text{min}} \end{bmatrix}, \quad S_n^{\text{max}} = \begin{bmatrix} V_1^{\text{max}}, & V_2^{\text{max}}, & \ldots, & V_n^{\text{max}} \end{bmatrix} \]
6: Fitness evaluation: Add exterior penalty function with fitness function to restrict the constraint violation as
   \[ \text{Minimize} : f(f_1/f_2/f_3) + \sum \lambda_V(V_i - V_i^{\text{lim}})^2 + \sum \lambda_T(T_i - T_i^{\text{lim}})^2 + \sum \lambda_Q(Q_i - Q_i^{\text{lim}})^2 \] (34)
   with, \( T_i^{\text{lim}} = \begin{cases} T_i^{\text{max}}, & T_i > T_i^{\text{max}} \\ T_i^{\text{min}}, & T_i < T_i^{\text{min}} \end{cases} \), \( Q_i^{\text{lim}} = \begin{cases} Q_i^{\text{max}}, & Q_i > Q_i^{\text{max}} \\ Q_i^{\text{min}}, & Q_i < Q_i^{\text{min}} \end{cases} \) and \( V_i^{\text{lim}} = \begin{cases} V_i^{\text{max}}, & V_i > V_i^{\text{max}} \\ V_i^{\text{min}}, & V_i < V_i^{\text{min}} \end{cases} \).
7: Updating mechanism: FOPSO-EE is updated based on two mechanisms
   • Velocity using equation (29) as:
     \[ \nu(p, k + 1) = \alpha v(p, k) + \frac{1}{2} \alpha(1 - \alpha)(p, k - 1) + \frac{1}{6} \alpha(1 - \alpha)(2 - \alpha)v(p, k - 2) + \frac{1}{24} \alpha(1 - \alpha)(2 - \alpha)(3 - \alpha)v(k - 3) + \phi_1 r_1(LB(p, k) - x(p, k)) + \phi_2 r_2(GB(p, k) - x(p, k)) \]
     here, \( p \) denotes the particle, \( k \) is the flight index, \( LB \) is used for \( p_{\text{best}} \) and \( GB \) for \( g_{\text{best}} \).
   • Particle position using the expression:
     \[ x(p, k + 1) = x(p, k) + \nu(p, k + 1) \]

If current best particle i.e. \( f(x(p, k + 1)) > \) previous best particle i.e. \( f(LB(p, k)) \), then \( LB(p, k + 1) = x(p, k + 1) \) else \( LB(p, k + 1) = x(p, k) \) End if \( f(LB(p, k + 1)) > f(GB(p, k)) \) then \( GB(p, k + 1) = LB(p, k + 1) \) else \( GB(p, k + 1) = LB(p, k) \) and repeat the update for each particle in a swarm
8: Termination criterion: The algorithm will stop the searching process if saturation point is reached. Print the particle with latest \( g_{\text{best}} \). Until termination criteria, repeat from step 6 with updated swarm.
9: Storage: The variables of global best particle are stored on the basis of minimum power losses, voltage deviation and overall cost.
10: Analysis: Repeat steps 5 to 9 for the given variations to produce a large dataset for comprehensive analysis of the FOPSO-EE performance:
   • Different fractional orders \( \alpha \) of the FOPSO-EE
   • Perform 100 independent trials for each variant of the FOPSO-EE
11: End of FOPSO-EE

Ayillary reactive power sources. The incorporation of FACTS devices alters the equality constraints and adds additional constraints, as defined in section II. The best global particle and corresponding fitness value evaluated by FOPSO-EE is listed in Table 6, while the percentage line loss reduction is listed in Table 7. It can be seen that, the proposed scheme has outperformed other algorithms, namely, the SPSO, WOA, QOGWO, QODE, and PSO-EE.

D. CASE 4: MINIMIZATION OF OVERALL COST
The results for the overall cost minimization function \( f_3 \) are illustrated in Table 6. The percentage cost reduction, fourth column in Table 7, show that FOPSO-EE has computed a lower overall cost of operation in comparison with those obtained with the SPSO, WOA, QOGWO, QODE, and PSO-EE. We verify that the FOPSO-EE lead to a significantly better solution for ORPD problems.

E. CASE 5: VALIDATION OF FOPSO-EE IN LARGE SCALE TEST SYSTEM
The effectiveness of FOPSO-EE is further ascertained by testing it on large scale power system i.e., IEEE 57 bus by adopting line loss minimization as a fitness function. The evolution of proposed algorithm execution is monitored by using the Equation (33). The obtained learning curves for all the fractional orders i.e., \( \alpha = [0.1, 0.2, \ldots, 0.9] \) are plotted.
in Fig. 5, depicting the best, average and worst iterative updates of transmission line losses during 100 independent trials. The optimum value of the operational variable with corresponding losses are documented in Table 8, along with the other well-known algorithms such as NGWCA, DE, CKHA, GSA, PSO, and PSO-EE. Observing the Table 8 and graphical illustrations in Fig. 5, one can conclude that, FOPSO-EE revealed a better behavior by computing minimum losses as compared to counterpart algorithms, hence, the synergy of fractional calculus and entropy evolution contributes to an enhanced convergence dynamics while endorsing a better optimization strength of FOPSO-EE for large scale power system.

V. STATISTICAL ANALYSIS
To analyze the consistency and reliability of the FOPSO-EE, a detailed statistical analysis has been carried out for all four cases, considering the best fractional order $\alpha$ in the set. In this line of thought, for all test cases, 100 independent trials were carried out and the median of the fitness evolution is taken as reference, for nominating the fractional order. The statistical assessment is based on the minimum fitness evolution in each independent simulation, convergence curves, quantile-quantile plots, box plot illustrations, and empirical cumulative distribution function, as depicted in Figs 6-10.

The sub figures 6(a)-10(a) reveal that FOPSO-EE compute minimum fitness in most of the independent run when compared with PSO-EE. A considerable difference in learning behaviors can be verified through sub figures 6(b)-10(b) where in all charts the FOPSO-EE performed better than PSO-EE. It can be verified, through sub figures 6(c)-10(c), that the minimum fitness evolution versus the quantiles of a standard normal distribution is more ideal in case of FOPSO-EE. Sub figures 6(d)-10(d) indicates that median gauges were always occurred at lower side for FOPSO-EE. The sub figures 6(e)-10(e) depicts that the probability of finding a best fitness through FOPSO-EE is on higher side than PSO-EE.

Bearing these results in mind, it is verified that both, the entropy and fractional calculus characterizes a natural tool which allows to design new variants of traditional algorithms, and leads to future promising advancements based on a new vantage point.

VI. CONCLUSION
A novel optimization approach FOPSO-EE is presented for solving the ORPD problems in the power system by exploitation of entropy diversity in fractional swarm intelligence. In the designed method FOPSO-EE, two important mathematical tools, namely the Shanon entropy and fractional calculus are integrated with traditional PSO algorithm. The proposed FOPSO-EE is viability implemented in standard IEEE 30 bus power system for minimizing the transmission line losses, voltage deviation and overall operational cost by tuning the operational variables such as transformer tap positions, bus voltages and reactive power compensators to near optimum-value. The results demonstrated that, the synergies of applying both, the Shanon entropy and fractional calculus concept, improved the performance of the optimizer in terms of fitness evolution and convergence rate during the proposed FOPSO-EE executions.

In this line of thought, both, the entropy evolution and fractional order dynamics will be considered in designing new integrated fractional swarming/evolutionary algorithms to solve significant optimization problems related to engineering sector in future research works such as the distributed generation [77], coordination of directional over current relay [78] and parameter extraction [79] etc. Moreover, the proposed algorithm can be further investigated for slandered benchmark functions with performance evaluation in terms of Wilcoxon sign rank tests.
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Y. Muhammad et al.: Design of Fractional Swarm Intelligent Computing With Entropy Evolution
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