Theoretical determination and analysis of the length of the non-working move and the width of the headland when performing fishtail turn with a rectilinear reverse move, which is not parallel to the boundary of a field with an irregular shape

Krasimir Trendafilov
Trakia University, Faculty of Technics and Technologies, 38 Graf Ignatiev Street, Yambol, Bulgaria
e-mail: krasimir.trendafilov@trakiauni.bg

Abstract. The way of movement of the machine-tractor units in the field influences the efficiency of the performed operations. A way of movement must be chosen in which the length of the non-working moves and the width of the headland are minimal. When moving in an irregularly shaped field, the length of the non-working moves and the width of the headland depend on the angle between the direction of movement of the unit and the field boundary. In the article dependences for determining the length of the non-working move when performing open and closed fishtail turns with a rectilinear reverse move, which is not parallel to the field boundary, as well as for determining the width of the headland required for their execution are derived. It was found that the non-working move is the shortest and the width of the headland is the smallest when performing a closed fishtail turn and the direction of alternation of the working moves in the field from right to left.

1. Introduction
The efficiency of the performed operations in the field largely depends on the way of movement of the machine-tractor units. The efficiency function is the ratio between the length of the working moves and the sum of the time for performing working and non-working moves [1]. Therefore, the shorter length of non-working moves, and hence the shorter time to perform them leads to increased productivity of the units. Basically, the non-working moves of the units are the turns carried out in the headland when changing the direction of movement. In a study by Bochtis D.D. and Sorensen C.G turns represent 5.27% and 6.48% of the total distance traveled by the unit [2]. In cases where the unit requires loading, for example a fertilizing unit, the distance traveled by the unit for turning is the second non-productive distance after the distance traveled to the place of loading [3].

There are three basic types of turns - pear-shaped turn (Ω-turn), semicircular turn (U-turn) and fishtail turn (T-turn) [4]. The first two are carried out of machine-tractor units with trailed machines and the last of units with mounted machines. The fishtail turn is carried out in reverse, reduce the width of the headland and is in line with European practices [5, 6].

For analytical determination of the length of the turns, the different authors use mainly two methods - without the use of clothoids and with clothoids. In the first method, the turn is represented as a set of straight lines and arcs of a circle of equal radius. In addition to these geometric shapes, clothoids are used in the second method. The clothoid is a curve with a variable radius of curvature proportional to its length [7]. The length of the clothoid depends on the speed of movement of the unit and the speed of
deviation of its steering wheels. The actual turn length is longer than calculated. It is also influenced by factors such as driver skills, soil conditions, interaction between tires and soil, etc., which lead to lengthening of turns relative to their theoretical length. This additional path traveled by the unit cannot be determined analytically and is a stochastic variable [8].

The headland must be of minimum width and the distance traveled by the machine-traktor units in it must be minimal. Repeated passage of units through the headland leads to soil compaction and lower yield than this area [8].

The length and shape of the turn as well as the width of the headland depend on the angle between the direction of movement of the unit and the field boundary [9]. The direction of the turns is also important, i.e. the sequence of arrangement of the working moves in the field.

The purpose of the present work is to derive analytical dependences for determining the length of fishtail turns with a rectilinear reverse move that is not parallel to the field boundary, as well as for determining the width of the headland required to perform them in an irregularly shaped field and to analyze the influence of the angle between the direction of movement and the field boundary on the length of the non-working move and the width of the headland at different direction of movement of the unit in the field.

2. Material and method

A fishtail open turn and a fishtail closed turn with a rectilinear reverse movement in different directions of their performance in a field with an irregular shape are considered. To determine the length of the turn, a geometric method is used, in which the turn is represented by straight lines and arcs of a circle of equal radius. The different types of turns are presented in Figure 1, Figure 2, Figure 3 and Figure 4. The length of the turn is defined as the sum of the lengths of its geometric elements. The width of the headland required to perform the turn is defined as the sum of the segments perpendicular to the field boundary and depending on the elements of the turn.

The symbols used in the figures are as follows:

- \( \alpha \) is the angle between the direction of working move of the unit and the boundary of the field;
- \( p.A \) – the beginning of the turn;
- \( p.B \) – the end of turn;
- \( p.O_1, p.O_2 \) – the centers of the first and second curvilinear movement within the turn;
- \( \beta_1, \beta_2 \) – the central angles of the arcs described in the first and second curvilinear movements;
- \( R \) – the radius of curvilinear movement (turning radius of the machine-tractor unit);
- \( M \) – the width of the tractor measured from the outside of the wheels. The track gauge (track) measured between the axis of symmetry of the wheels may also be used;
- \( B \) – the working width of the machine-tractor unit;
- \( la \) – the kinematic length of the unit - the distance between the center of the unit and the last row of working tools of the machine. The center of the machine-tractor unit with wheeled tractor is the center of the rear axle of the tractor;
- \( E \) – the minimum width of the headland;
- \( E' \) – the width of the headland, limited by the wheels of the tractor when making the turn;
- \( H \) – the longitudinal base (distance between axles of the front and rear wheels) of the tractor;
- \( l_g \) – the length of the rectilinear move in the headland before and after the turn.

For the analysis of the length of the non-working move and the width of the headland an example of movement of a machine-tractor unit with the following parameters is considered: \( B = 3.5 \text{ m}; la = 3.4 \text{ m}; R = 4 \text{ m}; H = 2.4 \text{ m}; M = 1.8 \text{ m} \). The unit can be, for example, with inter-row cultivator and must not enter the field when maneuvering so as not to damage the plants. In this case, the headland is of width \( E' \). If the unit does not work in rows, when performing turn it can enter the inside of the field it is processing. Then the headland will be with smaller width \( E \), respectively the length of non-working moves will be smaller. Non-working move performed by the unit in the headland include turn with length \( l_T \) (including curvilinear and rectilinear moves) and rectilinear moves with length \( l_g \) before and after performing the turn. Such rectilinear moves will exist in the cases when the headland is of width...
$E'$. If the machine-tractor unit is not in the transport position, the area $S$ will be cultivated during these moves. This area will be cultivated a second time when cultivating the headland. This can be avoided by using modern navigation systems. To perform the study, it is assumed that the angle between the direction of working move and the field boundary varies from $10^\circ$ to $90^\circ$.

3. Results and discussion

3.1. Open fishtail turn

3.1.1. Movement from left to right

When machine-tractor unit crossed the boundary of the headland with the entire working width of the machine, the unit is brought to the transport position, a curvilinear movement is carried out around point $O_1$, followed by reverse movement of the tractor to reaching the field boundary and again curvilinear forward movement around point $O_2$ (Figure 1, (a)). Immediately afterwards, the machine is brought into working position at the boundary of the headland and the next working move begins. The length of the turn is measured from point $A$ to point $B$ and is determined by the dependence

$$l_T = \pi R + \bar{O}_1 \bar{O}_2 = \pi R + \frac{2R + B}{\sin \beta_1}$$

The central angle $\beta_1$ is determined by the dependence

$$\beta_1 = \tan^{-1} \left( \frac{2R + B}{\tan \alpha + 2l_a} \right)$$

After substitution in equation (1) for $l$, is obtained

$$l_T = \pi R + \left[ (2R + B)^2 + \left( \frac{B}{\tan \alpha} + 2l_a \right)^2 \right]^{1/2}$$

To determine the width of the headland, segments $a$, $b$ and $c$ are used, the lengths of which are respectively

![Figure 1](image.png)

**Figure 1.** Fishtail open turn with rectilinear reverse movement, which is not parallel to the field boundary - movement from left to right.
\[ a = l_a \cdot \sin \alpha - 0,5B \cdot \cos \alpha \] (4)

\[ b = R \cdot \cos \alpha \] (5)

\[ c = (R + 0,5M) \cos(\beta_1 - \alpha) \] (6)

The width of the headland is

\[ E = b + c - a = (R + 0,5B) \cos \alpha + (R + 0,5M) \cos(\beta_1 - \alpha) - l_a \cdot \sin \alpha \] (7)

At a larger angle \( \alpha \), the headland is bounded by the front right wheel of the tractor, which reaches the field boundary at the end of the first curvilinear movement (Figure 1(b)). The following dependencies are used to determine the width of the headland:

\[ a = 0,5M \cdot \cos(\alpha - \beta_1) \] (8)

\[ h = H \cdot \sin(\alpha - \beta_1) \] (9)

\[ c = R \cdot \cos(\alpha - \beta_1) \] (10)

\[ b = R \cdot \cos \alpha \] (5)

\[ p = 0,5B \cdot \cos \alpha + l_a \cdot \sin \alpha \] (11)

\[ E = a + h + c - b + p = \]

\[ = (0,5M + R) \cos(\alpha - \beta_1) + H \cdot \sin(\alpha - \beta_1) + (0,5B - R) \cos \alpha + l_a \cdot \sin \alpha \] (12)

The transition between the modes of movement shown in Figure 1(a) and Figure 1(b) is performed at an angle

\[ \alpha = \tan^{-1}\left(\frac{2R + H \sin \beta_1}{H \cos \beta_1 + 2l_a}\right), \] (13)

in which the width of the headland in both variants is the same.

3.1.2. Movement from right to left

In this way of movement after reaching the boundary of the headland, a curvilinear movement is performed until reaching the boundary of the field, after which a rectilinear reverse movement is performed and again a curvilinear movement forward to the boundary of the headland (Figure 2). At the end of the reverse movement, the tractor wheels may be outside the headland with width \( E \), i.e. in the field. The length of the turn is

\[ l_T = \pi R + \overline{O_1 O_2} = \pi R + \frac{2R + B}{\sin \beta_2} \] (14)

The central angle \( \beta_2 \) is determined by the dependence

\[ \beta_2 = \tan^{-1}\left(\frac{2R + B}{\frac{B}{\tan \alpha} - 2l_a}\right) \] (15)

After substitution in equation (14) for the length of turn is obtained

\[ l_T = \pi R + \left[(2R + B)^2 + \left(\frac{B}{\tan \alpha} - 2l_a\right)^2\right]^{1/2} \] (16)

4
Figure 2. Fishtail open turn with rectilinear reverse movement, which is not parallel to the field boundary - movement from right to left.

From equation (16) it can be seen that the length of the turn has a minimum at \( \alpha = \tan^{-1}\left(\frac{B}{2l_a}\right) \). Then \( \beta_2 = 90^\circ \). For \( \alpha > \tan^{-1}\left(\frac{B}{2l_a}\right) \) the central angle is determined by the dependence

\[
\beta_2 = 180 + \tan^{-1}\left(\frac{2R + B}{R + 2l_a}\right)
\] (17)

The lengths of the following segments are used to determine the width of the headland:

\[
a = 0.5M \cos(\beta_2 - \alpha)
\] (18)

\[
h = H \sin(\beta_2 - \alpha)
\] (19)

\[
c = R \cos(\beta_2 - \alpha)
\] (20)

\[
b = R \cos \alpha
\] (5)

\[
p = 0.5B \cos \alpha + l_a \sin \alpha
\] (11)

The width of the headland is determined by the dependence

\[
E = a + h + c + b + p = (R + 0.5B) \cos \alpha + (R + 0.5M) \cos(\beta_2 - \alpha) + H \sin(\beta_2 - \alpha) + l_a \sin \alpha
\] (21)
When the working conditions do not allow the unit to move beyond the headland when maneuvering, the width of the headland is limited from the side of the field from the rear right wheel of the tractor at the end of the reverse move. Then the width of the headland is determined by the dependences (18), (19) and

$$s = \frac{(2R+B) \sin(\beta_2 - \alpha)}{\sin\beta_2} = 2R \cos\alpha + 2l_a \sin\alpha$$  \hspace{1cm} (22)

$$E' = 2\alpha + h + s = M \cos(\beta_2 - \alpha) + H \sin(\beta_2 - \alpha) + 2R \cos\alpha + 2l_a \sin\alpha$$  \hspace{1cm} (23)

After entering the headland with width \(E'\) and before leaving it, the unit makes a rectilinear move with length

$$l_g = \left(\frac{0.5M-R}{\sin\alpha} \cos(\beta_2 - \alpha) + \frac{R-0.5B}{\tan\alpha} + l_a = \frac{(0.5M-R) \cos\beta_2 + R-0.5B}{\tan\alpha} + (0.5M-R) \sin\beta_2 + l_a \right)$$  \hspace{1cm} (24)

The rectilinear move becomes \(l_g = 0\) at

$$\alpha = \tan^{-1} \left( \frac{(0.5M-R) \cos\beta_2 + R-0.5B}{(R-0.5M) \sin\beta_2 - l_a} \right)$$  \hspace{1cm} (25)

3.2. Closed fishtail turn

3.2.1. Movement from left to right

After reaching the boundary of the headland, the unit turns right, moving along a curve reaches the boundary of the field, makes a rectilinear reverse move and again through a curvilinear forward move to the right reaches the boundary of the headland and begins the next working move (Figure 3). When the unit is reversing, the tractor wheels goes outside the headland with width \(E\). Since this part of the field is cultivated and the wheels enter the cultivated field, it is better to determine the width of the headland not according to the capabilities of the unit to perform a turn without a straight move forward in the headland with width \(E\), but by take into account the innermost point in the field reached by the tractor wheels. In this case, the width of the headland is larger and is marked with \(E'\). It can be seen from the figure that before and after the turn in the headland, the unit makes a rectilinear moves of equal length \(l_g\). Since the sum of the central angles is \(\beta_1 + \beta_1 = \pi\), the length of the turn is determined by the dependence

$$l_T = \pi R + \frac{2R-B}{\sin\beta_1}$$  \hspace{1cm} (26)

The central angle \(\beta_1\) is determined by the triangle \(O_1C_2O\) by the dependence

$$\beta_1 = \tan^{-1} \left( \frac{2R-B}{B + 2l_a} \right)$$  \hspace{1cm} (27)

Finally for the length of the turn is obtained

$$l_T = \pi R + \left[ (2R-B)^2 + \left( \frac{B}{\tan\alpha} + 2l_a \right)^2 \right]^{1/2}$$  \hspace{1cm} (28)

The width of the headland for a small angle between the direction of movement and the field boundary is the sum of segments of the following lengths (Figure 3, (a)):

$$a = 0.5M \cos(\alpha + \beta_1)$$  \hspace{1cm} (29)

$$b = H \sin(\alpha + \beta_1)$$  \hspace{1cm} (30)

$$e = l_a \sin\alpha + (0.5B + R) \cos\alpha - R \cos(\alpha + \beta_1)$$  \hspace{1cm} (31)
Figure 3. Fishtail closed turn with rectilinear reverse movement, which is not parallel to the field boundary - movement from left to right.

For the width of the headland, where there is no rectilinear move \((l_\text{g} = 0)\) is obtained

\[
E = a + b + e = (0.5M - R).\cos(\alpha + \beta_1) + H.\sin(\alpha + \beta_1) + l_a \sin \alpha + (0.5B + R) \cos \alpha
\]  

(32)

The width of the headland, taking into account the entry of the tractor into the field during its reverse movement, is determined by the dependences (30), (31) and

\[
c = \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \beta_1}
\]  

(33)

\[
E' = 2a + b + c = M.\cos(\alpha+\beta_1) + H.\sin(\alpha+\beta_1) + \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \beta_1}
\]  

(34)
The length of the rectilinear moves of the unit before and after the turn in the headland with width $E'$ is

$$l_g = \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \alpha \sin \beta_1} + \left(\frac{0.5M+R}{\sin \alpha}\right) \cos(\alpha+\beta_1) - l_a - (0.5B + R) \cot \alpha \quad (35)$$

From Figure 3(a) it can be seen that when performing a rectilinear reverse, the left rear wheel of the tractor reaches the boundary of the headland. In increasing the angle $\alpha$ between the direction of movement and the field boundary, the position of the tractor changes and at some point its rear axle becomes parallel to the boundary of the headland. Then the length of the segment $a$ becomes equal to zero. From dependence (29) it can be seen that this happens at $\cos(\alpha+\beta_1) = 0$. Then $\beta_1 = 90 - \alpha$. After substitution in equation (27) determines the angle at which the rear axle of the tractor is parallel to the field boundary

$$\alpha = \tan^{-1}\left(\frac{l_a + \sqrt{l_a^2 + (2R-B)^2}}{2R-B}\right) \quad (36)$$

When the angle $\alpha$ increases above this value, the headland is limited by the rear right wheel of the tractor, as it is located more inward in the field (Figure 3, (b)).

The width of the headland in this case is defined as the sum of segments $b$ (equation (30)), $e$ (equation (31)), as well as the segments:

$$a = -0.5M \cos(\alpha+\beta_1) \quad (37)$$

$$d = \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \beta_1} - 0.5M \cos(\alpha+\beta_1) \quad (38)$$

Hence, for the two variants of determining the width of the headland for Figure 3, (b) is obtained:

$$E = a + b + e =$$

$$= H. \sin(\alpha+\beta_1) + l_a \sin \alpha + (0.5B + R) \cos \alpha - (0.5M + R) \cos(\alpha+\beta_1) \quad (39)$$

$$E' = a + b + d = H. \sin(\alpha+\beta_1) + \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \beta_1} - M \cos(\alpha+\beta_1) \quad (40)$$

The two variants of the headland are equalized when $e = d$, i.e. when there is no rectilinear move before and after the turn in the headland. The length of this move is determined by the dependence

$$l_g = \frac{(2R-B)\sin(\alpha+\beta_1)}{\sin \alpha \sin \beta_1} + \left(\frac{0.5M+R}{\sin \alpha}\right) \cos(\alpha+\beta_1) - l_a - (0.5B + R) \cot \alpha \quad (41)$$

This dependence can also be represented in the form

$$l_g = \frac{(2R-B)\sin(\alpha+\beta_1)}{\tan \alpha} + \frac{(R-0.5M) \cos \beta_1}{\tan \alpha} - (R - 0.5B) \sin \beta_1 + l_a \quad (42)$$

where

$$\sin \beta_1 = \frac{2R-B}{\left(\frac{B}{\tan \alpha} + 2l_a\right)^2 + (2R-B)^2}^{1/2} \quad (43)$$

$$\cos \beta_1 = \frac{B}{\left(\frac{B}{\tan \alpha} + 2l_a\right)^2 + (2R-B)^2}^{1/2} \quad (44)$$

### 3.2.2. Movement from right to left

The way of movement is shown in Figure 4. In the headline, the unit performs a left turn, followed by a rectilinear reverse movement and again a left turn to the beginning of the next working move.
Figure 4. Fishtail closed turn with rectilinear reverse movement, which is not parallel to the field boundary - movement from right to left.

The length of the turn is

\[ l_T = \pi R + \frac{O_1O_2}{\sin \beta_2} = \pi R + \left(2R - B\right)^2 + \left(\frac{B}{\tan \alpha} - 2l_a\right)^2 \right)^{1/2} \],

(45)

because the central angle \( \beta_2 \) determined by the triangle \( O_1CO_2 \) is

\[ \beta_2 = \tan^{-1} \left( \frac{2R - B}{\frac{B}{\tan \alpha} - 2l_a} \right) \]

(46)

The length of the turn has a minimum at \( \sin \beta_2 = 1 \), i.e. at \( \beta_2 = 90^\circ \). The turn has a minimum length at

\[ \alpha = \tan^{-1} \left( \frac{B}{2l_a} \right) \],

(47)

as well as in the open turn performed in the same direction (Figure 2).

For \( \alpha > \tan^{-1} \left( \frac{B}{2l_a} \right) \) the angle \( \beta_2 \) is determined by the dependence

\[ \beta_2 = 180 + \tan^{-1} \left( \frac{2R - B}{\frac{B}{\tan \alpha} - 2l_a} \right) \]

(48)

The width \( E \) of the headland at a small angle \( \alpha \) (Figure 4, (a)) is determined by the dependence
\[ E = 0.5(M + B) \cos \alpha + (H + l_a) \sin \alpha \] (49)

When performing the fishtail turn, the tractor goes deeper into the field. The total width of the headland, taking into account the innermost point in the field reached by the tractor, can be defined as the sum of the lengths of the following sections:

\[ a = 0.5M \cos(\alpha + \beta_2) \] (50)
\[ b = H \sin(\alpha + \beta_2) \] (51)
\[ c = R \cos(\alpha + \beta_2) \] (52)
\[ d = (R - 0.5B) \cos \alpha - l_a \sin \alpha \] (53)

For the total width of the headland is obtained
\[ E' = a + b + c + d + E = \]
\[ = (R + 0.5M) \cos \alpha + H \sin \alpha + (R + 0.5M) \cos(\alpha + \beta_2) + H \sin(\alpha + \beta_2) \] (54)

When the rectilinear moves before and after the turn, located in the headland with width \( E' \) are working moves, they do not overlap during the final cultivating of the headland. Then only the headland of width \( E' \) is cultivated. In this case their length must be determined, which is
\[ l_g = \frac{E' - E}{\sin \alpha} = \frac{a + b + c + d}{\sin \alpha} = \frac{(R + 0.5M) \cos(\alpha + \beta_2) + H \sin(\alpha + \beta_2)}{\sin \alpha} + (R - 0.5B) \cot \alpha - l_a \] (55)

When the angle \( \alpha \) between the direction of movement and the field boundary increases the way of determining the width of the headland is changed. The boundary of the headland with width \( E' \) is determined by the front left wheel of the tractor, and the rear right wheel reaches the field boundary when reversing (Figure 4, (b)).

The following segments are used to determine the widths \( E \) and \( E' \).
\[ a = -0.5M \cos(\alpha + \beta_2) \] (56)
\[ b = \left(\frac{2R - B}{\sin \beta_2} + H\right) \sin(\alpha + \beta_2) \] (57)
\[ c = -(R + 0.5M) \cos(\alpha + \beta_2) \] (58)
\[ d = (R + 0.5B) \cos \alpha - l_a \sin \alpha \] (59)

For the widths of the headlands and the length of the rectilinear move, it is obtained, respectively
\[ E = c + d = (R + 0.5B) \cos \alpha - l_a \sin \alpha - (R + 0.5M) \cos(\alpha + \beta_2) \] (60)
\[ E' = 2a + b = -M \cos(\alpha + \beta_2) + \left(\frac{2R - B}{\sin \beta_2} + H\right) \sin(\alpha + \beta_2) = \]
\[ = 2R \cos \alpha - 2l_a \sin \alpha - M \cos(\alpha + \beta_2) + H \sin(\alpha + \beta_2) \] (61)
\[ l_g = \frac{E' - E}{\sin \alpha} = \frac{(R - 0.5M) \cos(\alpha + \beta_2) + H \sin(\alpha + \beta_2)}{\sin \alpha} + (R - 0.5B) \cot \alpha - l_a \] (62)

The length of the rectilinear move is \( l_g = 0 \), i.e. \( E' = E \) at an angle
\[ \alpha = \tan^{-1} \left( \frac{(R-0,5M) \cos \beta_2 + H \sin \beta_2 - R-0,5R}{(R-0,5M) \sin \beta_2 + 2l_a - H \cos \beta_2} \right) \] (63)

At a higher value of the angle \( \alpha \) the tractor moves within the headland with width \( E \), which is determined by dependence (60) (Figure 4, (c)). The reverse move of the tractor is parallel to the field boundary when \( \alpha + \beta_2 = 180^\circ \). When the angle \( \alpha \) increases above the value

\[ \alpha = \tan^{-1} \left( \frac{R}{l_a} \right) \] (64)

the headland is limited by the front right wheel of the tractor (Figure 4, (d)).

The width of the headland is determined by the dependencies:

\[ a = -0,5M \cdot \cos(\alpha + \beta_2) \] (56)

\[ b = -H \cdot \sin(\alpha + \beta_2) \] (65)

\[ c = -R \cdot \cos(\alpha + \beta_2) \] (66)

\[ d = (0,5B - R) \cdot \cos \alpha + l_a \cdot \sin l_a \] (67)

\[ E = a + b + c + d = (0,5B - R) \cos \alpha + l_a \cdot \sin \alpha - (0,5M + R) \cos(\alpha + \beta_2) - H \sin(\alpha + \beta_2) \] (68)

3.3. Length of the non-working move

When the rectilinear reverse move is not parallel to the field boundary, there is no rectilinear non-working move when entering and exiting the headland with width \( E \), i.e. when the unit may goes outside from the headland when turning. In this case, the length of the non-working move is the length of the turn itself, i.e.

\[ l_n = l_T \] (69)

In cases when the machine-tractor unit does not have to leave the headland (with width \( E' \)), rectilinear non-working moves of the same length \( l_g \) are performed when entering and leaving the headland. The length of the non-working move in this case is determined by the dependence

\[ l_n = l_T + 2l_g \] (70)

![Graph](image_url)

**Figure 5.** Length of the non-working move in a fishtail open turn depending on the angle between the direction of movement of the unit and the field boundary.
When performing an open fishtail turn with a rectilinear reverse move which is not parallel to the field boundary and moving from left to right, the unit does not leave the headland and the non-working moves are determined by dependence (69). When moving in the opposite direction (from right to left), the unit leaves the headland with width $E$ when reversing. If the operation allows entry into the field, the length of the non-working move is also determined by (69). From Figure 5 it can be seen that the length of the non-working move is significantly smaller when moving from right to left, especially at a smaller angle between the direction of movement and the field boundary. In cases where the unit does not have to leave the headland and moves in a headland with width $E'$, the length of the non-working move is determined by dependence (70). In these cases, left-to-right movement should be preferred, as the length of non-working moves is shorter and the unit moves within a headland with width $E$.

The lengths of the non-working move when making a fishtail closed turn with a rectilinear reverse move, which is not parallel to the field boundary, are shown in Figure 6. It can be seen that the non-working move is shorter when the unit moving from right to left in both variants of the headland ($E$ and $E'$). In this way of movement after reaching a certain value of the angle $\alpha$ (in this case about 30 °) the unit moves in a headland with width $E$.

![Figure 6. Length of the non-working move in a fishtail closed turn depending on the angle between the direction of movement of the unit and the field boundary.](image)

3.4. Width of the headland

The actual width of the headland must be a multiple of the working width of the machine-tractor unit. The number of moves for which the headland is cultivated, depending on whether the movement of the unit should be restricted in it or not, is determined by the dependencies

$k = \frac{E}{B}$

or

$k' = \frac{E'}{B}$

The result obtained is rounded to the larger integer, which represents the actual number of moves of the unit for cultivating the headland - $k_r$ and $k'_r$, respectively. The actual width of the headland is determined by the dependencies

$E_r = k_r \cdot B \ , \ m$

or

$E'_r = k'_r \cdot B \ , \ m$
When $E_r = E$, respectively $E_r' = E'$, the number of moves is an integer. This equation is satisfied for a certain angle $\alpha$, which can be determined at $k_r = 1, 2, 3 \ldots$ using the dependences to determine $E, E', E_r$, and $E_r'$. At this angle, a transition is obtained between the different actual widths of the headland, shown stepwise in the figures below. A curved line shows the theoretical width of the headland.

When performing an open fishtail turn with a rectilinear reverse move, which is not parallel to the boundary of the field with direction of movement in the field from left to right, the unit also does not go out of the headland with width $E$. From Figure 7 it can be seen that when performing this turn, the width of the headland is smaller by one working width of the unit at average values of angle $\alpha$ than at very large or very small values of $\alpha$. When the open turn is performed in the direction from right to left, the unit exits the headland with width $E$. If there is a requirement during cultivating with the respective machine-tractor unit not to leave the headland, then a headland with greater width $E'$ is used. The actual

![Figure 7](image-url)

**Figure 7.** Width of the headland in fishtail open turn depending on the angle between the direction of movement of the unit and the field boundary - movement from left to right.

![Figure 8](image-url)

**Figure 8.** Width of the headland in fishtail open turn depending on the angle between the direction of movement of the unit and the field boundary - movement from right to left.
width of the headland in both cases is the same at a small angle $\alpha$ (Figure 8). At a larger angle between the direction of movement and the field boundary, the actual width of the headland $E'$ is greater than the actual width of the headland $E$. When an open turn is used and the unit must move within the boundaries of the headland with a width $E$, it is advisable to left-to-right movement is used due to the smaller width of the headland.

![Figure 9](image1)

**Figure 9.** Width of the headland in fishtail closed turn depending on the angle between the direction of movement of the unit and the field boundary - movement from left to right.

![Figure 10](image2)

**Figure 10.** Width of the headland in fishtail closed turn depending on the angle between the direction of movement of the unit and the field boundary - movement from right to left.

When unit moves from left to right in the field a closed fishtail turn with a rectilinear reverse move, which is not parallel to the field boundary, can be performed in a headland with width $E$ or $E'$, depending on the operation performed. From Figure 9 it is seen that only at a very large angle $\alpha$ the actual widths of the two headlands are the same. At a small angle, the difference between them is two working widths of the unit. When the closed turn is performed in the direction from right to left, with increasing angle $\alpha$, the difference between the headlands with width $E$ and $E'$ decreases and at a certain angle they are equalized (Figure 10).
It can be seen from the figures that the actual width of the headland is the smallest (in both ways its determination) when performing a closed turn with direction of movement from right to left. At large values of angle $\alpha$, the actual width $E$ of the headland is the same for open and closed turns in both directions of movement in the field.

4. Conclusions
When use of fishtail turns with a rectilinear reverse movement which is not parallel to the field boundary there is not rectilinear non-working move when entering and exiting the headland with width $E$. When maneuvering, the machine-tractor unit exits this headland. If the processing requirements do not allow the unit to exit the headland with width $E$, wider headland $E'$, must be use. The length of non-working move is shorter when moving on the field from right to left in both variants of the headland.

When performing a closed fishtail turn with direction of movement on the field from right to left the headland has the smallest width in both variants of the headland - $E$ and $E'$.

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