Numerical Simulation of Hydrofoil Cavitation

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Abstract—A contrastive analysis was conducted on the lift and drag performance, pressure and gas-phase volume fraction of NACA4412 hydrofoil under different conditions of the cavitation number and the angle of attack based on CFD. The results are obtained as follows: after hydrofoil cavitation, the lift coefficient decreased and the drag coefficient increased with the decrease of the cavitation number, and supercavitation can reduce the drag coefficient locally; the pressure difference between the upper and lower surfaces of the hydrofoil decreased and the cavitation zone on the upper surface became larger with the decrease of the cavitation number; the cavitation zone of the hydrofoil originated from the leading edge; the length of the cavitation zone under large cavitation numbers first increased and then decreased with the increase of the angle of attack, and the zone moved towards the leading edge; the cavitation zone under small cavitation numbers covered almost the entire upper surface of the hydrofoil.

1. Introduction
Cavitation will occur when liquids move at high speed on the surface of an object, which will produce a series of adverse effects[1] like the cavitation erosion of turbine blades. Therefore, it is very important to study hydrofoil cavitation.

Wang et al. [2] studied the adaptability of jet control of hydrofoil cavitation flow under multiple working conditions. Xie et al. [3] analyzed the evolution of the unsteady cavitation flow field around hydrofoils based on the dynamic mode decomposition method. Zheng et al. [4] conducted a numerical study on the 3D cavitation characteristics of NACA66 hydrofoil based on different cavitation models. Most of the studies above were aimed at 3D hydrofoils. In this paper, the cavitation of 2D hydrofoils was numerically simulated.

2. Numerical Model and Algorithm
In this paper, Reynolds-averaged N-S equations were solved with SIMPLEC algorithm[5] through the combination of Singhal full cavitation model[6] based on pressure and RNG \(k-\varepsilon\) turbulence model[7].

2.1. Singhal Full Cavitation Model
The full cavitation model proposed by Singhal et al. [5] was used to obtain the following governing equations:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (1) \]
Momentum equation:
\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\mu + \mu_t)(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})\right] + \rho g_i
\]  
(2)

Mass fraction of cavities:
\[
\frac{\partial}{\partial t}\left[(\mu + \mu_t)\frac{\partial f}{\partial x_j}\right] + R_e + R_c, \quad \frac{1}{\rho} = \frac{f_u}{\rho_u} + \frac{f_g}{\rho_g} + \frac{1 - f_g - f_u}{\rho_i}
\]  
(3)

2.2. RNG k-ε Turbulence Model
In RNG k-ε turbulence model, the governing equation of turbulence energy and dissipation rate is as follows:[6]
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_i}\left[(\mu + \mu_t)\frac{\partial k}{\partial x_i}\right] + G_k - \rho \varepsilon, \quad \frac{\partial}{\partial x_i}\left[(\mu + \mu_t)\frac{\partial \varepsilon}{\partial x_i}\right] = c_1 G_k \frac{\varepsilon}{k} - c_2 \rho \frac{\varepsilon^2}{k}
\]  
(4)

2.3. Calculation Method and Parameters
SIMPLEC algorithm was used to solve the pressure-velocity coupling problem. The steady incompressible RANS equation was directly solved with the enhanced wall function method by the commercial software Fluent14.0.

NACA4412 hydrofoil was used, with chord length c = 100mm, inflow velocity V = 15m/s, Reynolds number of about 1.5×10^6 and cavitation number σ of 1.5, 1.0, 0.8, 0.5 and 0.2 respectively.

2.4. Computational Domain and Grids
O-shaped fully structured grids were adopted. The computational domain was a circle composed of left and right semicircles, the center of which was the trailing edge point of the hydrofoil and the radius of which was about 20c. Boundary conditions of the left semicircle, the right semicircle and the hydrofoil surface were the velocity inlet, the pressure outlet and the wall respectively. According to the requirement of y+ < 1, the grid height of the first layer was about 1.62×10^4m. The number of grids was finally determined as 29,512 by comparison, as shown in Fig.1.
3. Analysis of Calculation Results

3.1. Lift and Drag Performance

Fig.2 shows the variation of lift coefficient $C_l$ and drag coefficient $C_d$ of the hydrofoil with the angle of attack $\alpha$ under different cavitation numbers.

According to Fig.2(a), the lift coefficient was high when $\sigma = 1.5$. When the angle of attack was less than $6^\circ$, it increased rapidly and approximately linearly. The lift coefficient increased with the increase of the angle of attack. It reached the maximum value of 0.104 when the angle of attack was $9^\circ$. Then, when the hydrofoil began stalling which fluctuated to some extent, the lift coefficient decreased gradually. When the angle of attack was less than $12^\circ$, the fluctuation was small and then became greater, but the lift coefficient decreased slightly. With the decrease of the cavitation number, the lift coefficient decreased as a whole. When $\sigma = 1.0$ and the angle of attack was less than $3^\circ$, it increased approximately linearly, which was basically equal to that when $\sigma = 1.5$. It increased with the increase of the angle of attack and reached the maximum value of 0.089 when the angle of attack was $13^\circ$. Then, the hydrofoil began stalling which was gentle. The lift coefficient decreased gradually but modestly. When $\sigma = 0.8$ and the angle of attack was less than $1^\circ$, it was basically equal to that when $\sigma = 1.5$. It increased with the increase of the angle of attack. The hydrofoil did not have stalled until the angle of attack increased to $20^\circ$. When $\sigma = 0.5$, the lift coefficient decreased more greatly compared to that when $\sigma = 1.5$, and increased with the increase of the angle of attack. The hydrofoil did not have stalled until the angle of attack increased to $20^\circ$, but the lift coefficient was small. When $\sigma = 0.2$, the lift coefficient decreased more greatly compared with that when $\sigma = 1.5$, and increased with the increase of the angle of attack. It increased slowly when the angle of attack was less than $4^\circ$ and then rapidly. The hydrofoil did not have stalled until the angle of attack increased to $20^\circ$, but the lift coefficient was small.

According to Fig.2(b), the drag coefficient was small and increased with the increase of the angle of attack when $\sigma = 1.5$. It increased slowly when the angle of attack was less than $7^\circ$ and then increased rapidly. When the angle of attack was greater than $15^\circ$, the drag coefficient fluctuated. It increased as a whole with the decrease of the cavitation number. When $\sigma = 1.0$ and the angle of attack was less than $4^\circ$, the drag coefficient was basically equal to that when $\sigma = 1.5$, and then increased rapidly. When $\sigma = 0.8$ and the angle of attack was less than $2^\circ$, the drag coefficient was basically equal to that when $\sigma = 1.5$. Then, it increased at a rate basically equal to that when $\sigma = 1.0$. When $\sigma = 0.5$, the drag coefficient increased more greatly than that when $\sigma = 1.5$ and approximately linearly. When $\sigma = 0.2$, it increased greatly compared to that when $\sigma = 1.5$. When the angle of attack was less than $5^\circ$, the drag coefficient decreased with the increase of the angle of attack, which was higher than that when $\sigma$
= 1.5. Then, it increased with the increase of the angle of attack, which was smaller than that when $\sigma = 1.5$.

### 3.2. Pressure

Fig.3 shows the distribution of the surface pressure $p$ of the hydrofoil relative to the position $x/c$ along the chord length direction under different conditions of the cavitation number and the angle of attack. According to Fig.3, the smaller the cavitation number is, the lower the pressure on the upper and lower surfaces of the hydrofoil will be, and the smaller the pressure difference between the upper and lower surfaces at the same angle of attack will be. Therefore, the lift coefficient in Fig.2(a) decreased with the decrease of the cavitation number. Under the same cavitation number, the larger the angle of attack is, the smaller the pressure on the upper surface will be; the higher the pressure on the lower surface is, the greater the pressure difference between the upper and lower surfaces will be. However, when $\sigma = 1.5$ and the angle of attack was 18°, the pressure on the upper surface of the hydrofoil increased greatly, resulting in a decrease in pressure difference. Therefore, except when the cavitation number was 1.5, the lift coefficient increased monotonically with the angle of attack in Fig.2(a).

At the same angle of attack, the smaller the cavitation number is, the larger the cavitation zone on the upper surface of the hydrofoil will be. Therefore, the drag coefficient in Fig.2(b) increased with the decrease of the cavitation number. Under the same cavitation number, the length of the cavitation zone on the upper surface of the hydrofoil first increased and then decreased slightly with the increase of the angle of attack. Therefore, all the drag coefficients in Fig.2(b) increased monotonically with the variation of the angle of attack.
Most cavitation zones were located at the leading edge. When $\sigma = 0.2$ and 0.5 and the angle of attack was small, cavitation zones occurred on the upper and lower surfaces of the hydrofoil. Under other cavitation numbers, the cavitation zone was located on the upper surface of the hydrofoil.

3.3. Gas-phase Volume

Fig. 4 shows the gas phase distribution on the hydrofoil surface under different conditions of the cavitation number and the angle of attack. According to Fig. 4, cavitation only occurred on the upper surface of the hydrofoil and was located at the leading edge under large cavitation numbers. Cavitation occurred only at a large angle of attack. The length of the cavitation zone decreased when the angle of attack was too large, and the cavitation zone moved towards the leading edge with the increase of the angle of attack. Under small cavitation numbers, cavitation may occur on the upper and lower surfaces of the hydrofoil when the angle of attack was small, but the cavitation zone on the lower surface was small and concentrated on the leading edge. When the angle of attack was large, cavitation only occurred on the upper surface of the hydrofoil, covering almost the entire upper surface. At the same angle of attack, the smaller the cavitation number is, the larger the cavitation zone will be. The variation of the gas-phase volume fraction on the hydrofoil surface was consistent with the distribution of the surface pressure of the hydrofoil in Fig. 3.

When $\sigma = 0.2$ and the angle of attack was 0°, the whole hydrofoil was surrounded by bubbles. Such a phenomenon is called supercavitation, which reduces the drag to some extent. Therefore, the drag coefficient decreased locally when the angle of attack was small in Fig. 2(b).

4. Conclusion

The following conclusions were drawn through the numerical simulation analysis on cavitation of NACA4412 2D hydrofoil:

1. After hydrofoil cavitation, the lift and drag coefficients increased with the increase of the angle of attack. The lift coefficient decreased and the drag coefficient increased with the decrease of the cavitation number. Supercavitation can reduce the drag coefficient locally.

2. After hydrofoil cavitation, the pressure difference between upper and lower surfaces increased and the length of the cavitation zone on the upper surface first increased and then decreased slightly with the increase of the angle of attack. The pressure difference between upper and lower surfaces
decreased and the cavitation zone on the upper surface of the hydrofoil increased with the decrease of the cavitation number.

(3) The cavitation zone of the hydrofoil originated from the leading edge. The cavitation zone under large cavitation numbers only occurred on the upper surface of the hydrofoil, while that under small cavitation numbers can occur on the upper and lower surfaces of the hydrofoil.

(4) The length of the cavitation zone under large cavitation numbers first increased and then decreased with the increase of the angle of attack, and moved towards the leading edge. The cavitation zone under small cavitation numbers covered almost the entire upper surface of the hydrofoil.

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