Calculation of the critical power density of natural light in a polymer optical fiber

A I Sterhov¹, K A Petrov¹ and I Yu Loshkarev²

¹LLC SOLARGY Group, office 323, 71a, Dzerzhinsky, Izhevsk, 426939, Russia
²Saratov State Agricultural University n. a. N I Vavilov, 1, Theatralnaya Square, Saratov, 410012, Russia

E-mail: igyulo@mail.ru

Abstract. This article discusses the advantages of polymer fiber over quartz one. The comparison is made between the absorption spectra of polymethylmethacrylate, perfluoropolymer and quartz fibers. The scheme of thermophysical processes occurring during the interaction of radiation with matter is considered. The formulas for calculating the threshold (critical) power density necessary for heating the surface to a given temperature for pulsed and continuous heating are derived. Further, the calculations of the critical power density and the allowable size of the area are made, with which the concentration of natural light in the POF could be determined.

1. Introduction
For a long time, a polymer optical fiber (polymer optical fiber, POF) was in the shadow of quartz fiber and was used only for low-speed connections and for short distances, since the transmission of light over a multimode fiber was limited by intermode dispersion [1]. The first developments in POF were reported by the Boston company Pilot Chemical in the early 1960s, and at the end of the decade Du Pont began to improve it. Despite numerous studies of POF based on various materials, by 1978 the minimum level of losses was achieved only by 500–1000 dB / km, with polymethyl methacrylate (PMMA) being the best material. Such a high level of losses limited the scope of the application of light transmission to the transmission of light over very short distances — mainly to illuminate the scales of measuring instruments. In 1978, the company Du Pont decided to sell its units engaged in the development of the POF to the Japanese company Mitsubishi Rayon. Over the next few years, Mitsubishi Rayon managed to reduce losses to almost a theoretical minimum, which is about 150 dB / km at a wavelength of 650 nm (the so-called “red minimum”). As a result, a fiber with a stepped refractive index profile and a 50 Mbit / s bandwidth at a distance of more than one hundred meters was created. The next major breakthrough was achieved by Professor Koike group (Y. Koike) at Keio University (Japan) in 1990. She developed a process for the production of gradient polymer fibers (GI-POF) based on the same PMMA. The Koike group announced it was receiving a 3 GHz bandwidth in a fiber 100 m long, but the minimum loss remained at 150 dB / km (at a wavelength of 650 nm). The losses in the fiber based on PMMA are associated with the intrinsic absorption of the material and are due to the interaction with the harmonics of the vibrations of carbon – hydrogen (C – H) groups. It was possible to reduce losses by replacing hydrogen with a heavier isotope, deuterium. However, the use of deuterium is very expensive and this method has not found practical application. Another possibility - replacing hydrogen with fluorine - was successfully implemented by the Koike group in 1995. As a result, perfluoropolymer...
fiber was developed, the loss of which was reduced to 50 dB / km for the wavelength range of 650–1300 nm. The theoretical minimum loss in a perfluoropolymer is about 10 dB / km. The next achievement was the simultaneous development in 2001 by groups from Australia and Korea of a microstructured hollow optical polymer fiber.

2. Materials

The most important parameter of the optical fiber used to transmit light is the attenuation coefficient associated with the loss of signal power. The main sources of losses are scattering and absorption of light in the fiber. The light signal can be scattered on material inhomogeneities, on bulk defects, as well as at the boundary with the cladding or at the fiber ends. The losses associated with these mechanisms depend on the production technology and can be significantly reduced by optimizing it. The contribution to total losses is also made by absorption by impurities and fundamental absorption. Among hydrocarbon POF the smallest losses are in polymethyl methacrylate (PMMA). Two working windows of transparency of this fiber fall on the wavelengths of 570 and 650 nm (visible range). The practical use of these two transparency windows is due to the presence of inexpensive LEDs and red semiconductor lasers emitting at given wavelengths. Quite a large attenuation (the minimum attenuation currently achieved in PMMA is approximately 70 dB/km at a wavelength of 560 nm) limits the range of information transmission in such a bandwidth with a distance of about 100 m.

![Absorption spectra](image)

**Figure 1.** Absorption spectra.

The main fiber-optic element is a single-core light guide. It is a homogeneous light guide vein (also called a core) made of a material that is transparent to transmitted radiation with a high refractive index \(n_1\), surrounded by a light-insulating sheath with a low refractive index \(n_2\). Two materials can be used for a light guide if the refractive index of the cladding differs from the refractive index of the fiber core.
by at least 3%. The diameter of the light guide core (2-100 microns) is several times greater than the wavelength of the transmitted light [4]. The radiation incident on the input end propagates along the light guide (from an optically more dense medium to a medium with a lower optical density) along a broken (zigzag) curve, due to the effect of multiple total internal reflection from the interface between the core and the shell, and leaves the opposite end (Figure 2).

**Figure 2.** Propagation of the light beam in transparent optical fibers.

The input parameters of the optical signal are mainly stored at the output of the light guide. Such a fiber-optic element is called a stepped one because in it, at the core-cladding interface, the change in the refractive index occurs in steps.

The light guide of the fiber determines the numerical aperture (NA) - the maximum angle of the conical beam of rays that can pass through the fiber [5].

\[ \text{NA} = \left( n_1^2 - n_2^2 \right)^{1/2} \]

The numerical aperture of polymer fibers is usually 0.5-0.6. For the manufacture of fibers with a larger aperture polymers with a higher refractive index are required. They can be obtained, for example, by introducing heavy metal ions (barium, tin, lanthanum) into the polymer [6].

| Fiber Characteristic | PMMA | Polystyrene | Polystyrene | Quartz |
|---------------------|------|-------------|-------------|--------|
| Core: - material     | 1.49 | 1.59        | 1.59        | 1.46   |
| - refractive index   |      |             |             |        |
| Shell: - material    | Fluorocarbon | PMMA | Silicon rubber | Silicon rubber |
| - refractive index   | 1.39 | 1.49        | 1.40        | 1.40   |
| Numerical aperture   | 0.54 | 0.55        | 0.75        | 0.41   |
| Theoretical          |      |             |             |        |
| dispersion, ns km\(^{-1}\) | 340 | 340         | 640         | 200    |
| Minimum absorption, dB km\(^{-1}\) | 100 | -          | 150         | 8      |

### 3. Method

#### 3.1. Theoretical justification of the method

Thermophysical processes occurring during the interaction of radiation with matter are schematically presented in Figure 3. The nature of these processes, and, consequently, the result of this interaction, strongly depends on the material absorption coefficient at the radiation wavelength, its power and duration of exposure to the material. The radiation incident on the treated surface is absorbed in accordance with the exponential Bouguer – Lambert law, which is presented below in its general form:

\[ I(x) = I_0 \cdot e^{-\alpha \cdot x}, \]

where \( I(x) \) — the intensity of radiation penetrating the material to a depth \( x \); \( I_0 \) - the intensity of the radiation incident on the object. The released heat spreads deep into the material due to thermal conductivity (Figure 3a).
Figure 3. Basic physical processes arising from the interaction of laser radiation with matter.

Then the material is heated, and after the surface temperature it reaches the melting point, the liquid-phase boundary begins to spread into the material (Figure 3b). Upon further irradiation of the material, the heating process continues, this time up to the evaporation (boiling) temperature. When this temperature is reached, the process of evaporation of the substance from the treated surface is initiated (Figure 3c), accompanied by ionization of surface impurities and contaminants, which are almost always present in one form or another. Further, the radiation is absorbed by the base material, and if the intensity is not high enough, the material melts, evaporates, and the vapors ionize. In this case, the vapor pressure contributes to splashing out the melt, and a hole is gradually formed in the material. If the radiation intensity is too high, then a high-temperature opaque plasma is formed as a result of evaporation. The plasma cloud absorbs the incident radiation and shields the sample surface, thereby preventing further effects on the substance (Figure 3d) [7].

The surface temperature \( T \) depends on the radiation power \( P \) absorbed by a unit of area \( S \):

\[
q = \frac{P}{S},
\]

where \( q \) – radiation power density, \( P \) – radiation power.

The ratio between \( T \) and power density \( q \) can be determined from the heat equation:

\[
\frac{\partial T}{\partial t} - a \cdot \Delta T = \frac{Q(x, y, z, t)}{\rho \cdot c},
\]

where \( \Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \) – Laplace operator; \( Q(x, y, z, t) \) – the bulk density of the absorbed light flux; \( a \) – thermal diffusivity; \( \rho \) – is the density; \( c \) – is the heat capacity.

To solve this equation, it is necessary to define one initial condition, 6 boundary conditions and determine \( Q(x, y, z, t) \):

\[
T_{x, y, z, 0} = T_H; \quad T_{x=0, y, z} = T_{y=0, z, t} = T_{x=0, y, t} = T_H; \quad \frac{\partial T}{\partial x_{0,t}} = \frac{\partial T}{\partial y_{0,t}} = \frac{\partial T}{\partial z_{0,t}} = 0;
\]

\[
Q = q_0 (1 - R)e^{-\alpha x}.
\]

After solving these equations, the relationship between \( T \) and \( q \) is expressed in the form:

\[
T = \left[ \begin{array}{c} q(1 - R) \\ \rho, c, a, R, \alpha \\ x, y, z, t \end{array} \right].
\]

where \( q(1 - R) \) – absorbed power; \( \rho, c, a, R, \alpha \) – thermophysical and optical parameters; \( x, y, z, t \) – arguments.

The solution of such an equation under the previously specified conditions and for a round heat
source with radius \( r_0 \) will be:

\[
T = \frac{2 q_0 (1-R) \sqrt{a\tau}}{k} \left( \frac{1}{\sqrt{\pi}} - \text{erfc} \left( \frac{r_0}{2 \sqrt{a\tau}} \right) \right) + T_H. 
\]

When \( r_0 > \sqrt{a\tau} \) (pulse heating):

\[
T = \frac{2 q_0 (1-R) \sqrt{a\tau}}{k} + T_H. 
\]

When \( r_0 < \sqrt{a\tau} \) (continuous heating):

\[
T = \frac{2 q_0 (1-R) \cdot r_0}{k} + T_H. 
\]

Based on the obtained relations, it is possible to calculate the threshold (critical) power density \( q \) necessary for heating the surface to a given temperature \( T \) for pulsed \( q_{\text{crit.puls}} \) and continuous heating \( q_{\text{crit.cont}} \):

\[
q_{\text{crit.puls}} = \frac{(T - T_H) \cdot k \cdot \sqrt{a\tau}}{2(1-R) \cdot \sqrt{a\tau}}; \\
q_{\text{crit.cont}} = \frac{(T - T_H) \cdot k}{(1-R) \cdot r_0},
\]

where \( k \) - the coefficient of thermal conductivity; \( T \) – the required temperature; \( T_H \) – the initial temperature; \( R \) - the reflection coefficient; \( r_0 \) – the spot radius [7].

### 3.2. Calculation of critical power density

As it is mentioned above, absorption occurs in the polymer fiber in the infrared and ultraviolet parts of the spectrum. Indeed, on the attenuation spectrum (Figure 4) provided by the “Technology Center of Polymeric Optical Fiber”, it can be seen that the main radiation losses occur in the blue and red parts of the spectrum.

![Figure 4. PMMA-based attenuation spectrum of POF.](image)

We assume that the radiation of the blue and red regions absorbed by the POF is completely expended on heating the fiber. We will not take into account the infrared part of the spectrum, since it is assumed that filters are silicating this part of the stream. The calculation will be made for a fiber layer 1 cm thick.
Table 2. Data for calculation in Mathcad.

| No. | Wavelength, λ, nm | Attenuation of each wavelength, T_{DB}, dB/km | The proportion of energy of each wavelength in the visible spectrum, χ |
|-----|------------------|---------------------------------------------|--------------------------------------------------|
| 1   | 400              | 890                                         | 0.025                                            |
| 2   | 410              | 730                                         | 0.026                                            |
| 3   | 420              | 620                                         | 0.027                                            |
| 4   | 430              | 550                                         | 0.027                                            |
| 5   | 440              | 500                                         | 0.028                                            |
| 6   | 450              | 465                                         | 0.028                                            |
| 7   | 460              | 435                                         | 0.028                                            |
| 8   | 470              | 413                                         | 0.029                                            |
| 9   | 480              | 415                                         | 0.029                                            |
| 10  | 490              | 418                                         | 0.029                                            |
| 11  | 500              | 420                                         | 0.029                                            |
| 12  | 510              | 415                                         | 0.029                                            |
| 13  | 520              | 410                                         | 0.029                                            |
| 14  | 530              | 400                                         | 0.029                                            |
| 15  | 540              | 380                                         | 0.029                                            |
| 16  | 550              | 370                                         | 0.028                                            |
| 17  | 560              | 290                                         | 0.028                                            |
| 18  | …                | …                                           | …                                                |

The calculation for the blue part of the spectrum will be made within: \( \lambda = 400; 410; 420; 430; 440; 450 \). And for the red part of the spectrum it will be made within: \( \lambda = 680; 690; 700; 710; 720; 730; 740; 750; 760; 770; 780 \).

The exponential law of change in the power of a light signal becomes linear if the power is also expressed in logarithmic units.

The logarithmic level of the energy value \( P_2 \) (power or intensity of light) relative to the initial level \( P_1 \) of the same value is determined by the expression:

\[
dB = 10 \cdot \log\left(\frac{P_2}{P_1}\right).
\]

Where \( P_2 \) can be expressed as:

\[
P_2 = 10^{(0.1(-dB))}.
\]

The attenuation for each wavelength in the visible spectrum with an intensity of 1 W is determined as follows:

\[
Q_i = 1 \cdot 10^{-\left[0.1\left(\frac{-dB-0.01}{1000}\right)\right]}.
\]

where \( Q_i \) - intensity of each wavelength \( \lambda \), which has passed a POF layer of 1 sm. Then the intensity absorbed in the POF at a layer of 1 cm is \( r = 1 - Q_i \). The results of the calculation are in Table 3.
Table 3. Results of the calculation.

| No. | $Q_1$     | $r$     |
|-----|-----------|---------|
| 1   | 0.99795   | 0.00205 |
| 2   | 0.99832   | 0.00168 |
| 3   | 0.99857   | 0.00143 |
| 4   | 0.99873   | 0.00127 |
| 5   | 0.99885   | 0.00115 |
| 6   | 0.99893   | 0.00107 |
| 7   | 0.99900   | 0.00100 |
| 8   | 0.99905   | 0.00095 |
| 9   | 0.99904   | 0.00096 |
| 10  | 0.99904   | 0.00096 |
| 11  | 0.99903   | 0.00097 |
| 12  | 0.99904   | 0.00096 |
| 13  | 0.99906   | 0.00094 |
| 14  | 0.99908   | 0.00092 |
| 15  | 0.99913   | 0.00087 |
| 16  | ...       | ...     |

Then the total fraction of the intensity of natural light spent on heating the fiber is $P_{pogl} = 0.76 \text{ mW}$.

According to the “Technology Center of Polymeric Optical Fiber”, the POF retains its physical and optical characteristics up to 80°. Taking the ambient temperature of + 40°, and the critical temperature to which the POOF + 70° can be heated, it is possible to estimate the critical power density:

$$q = \frac{(T_k - T_n) \cdot k}{R} = 0.57 \text{ W/sm}^2,$$

where $T_k$=273+70K; $T_n$=273+40K; $R$=0.1 sm - radius POF; $k$- coefficient of thermal conductivity = 0.0019 W/(sm·K).

The area of the fiber end with a diameter of 2 mm:

$$S_{fiber} = \pi \cdot R^2 = 0.03142 \text{ sm}^2.$$

Then the value of permissible power is equal to

$$p = S_{fiber} \cdot q = 0.01791 \text{ W}.$$

Then the value of the permissible power of natural light in the fiber is equal to:

$$P_{crit} = \frac{P}{P_{pogl}} = 23.648 \text{ W}.$$

Knowing the allowable power of natural light in the POF and the value of solar radiation on the Earth's surface, one can estimate the allowable size of the area from which the concentration of natural light in the POF can be:

$$S_{max} = \frac{P_{crit}}{P \cdot 10^{-4}},$$

where $P_{crit}$ – the allowable amount of natural light in the POF; $P$ – the average solar radiation per year in W/m², equal to $P = \eta \cdot W/H$, where $\eta$ – the fraction of natural light; $W$- the value of solar radiation in Wh/m² year; $H$ – he amount of sun shine per year in hours.
4. Conclusion

The critical power density of natural light is calculated: 0.57W/m$^2$. The area of the fiber end with a diameter of 2 mm is determined. It is equal to 0.03142sm$^2$. The size of permissible power is calculated: 0.01791W. The power of natural light in the fiber is 23.648W. Based on the permissible power of natural light in the POF and the value of solar radiation on the surface of the Earth, the conclusion has been made on the assessment of the allowable size of the area from which the concentration of natural light in the end of the POF is possible.

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