A High-Accuracy and Conservative Unstructured Overset Grid data transfer Method for Numerical Simulation of Turbomachinery

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Abstract. The overset grid method is getting more and more popular in the numerical simulation of turbomachinery, especially in the simulation of wind turbines. Reasonable interpolation between overset grid interfaces is one of the bases to ensure correct calculation of flow fields, non-conservative interpolation is insufficient to guarantee the mass fluxes in the numerical simulation of turbomachinery. This paper develops a conservative interpolation method for hybrid overset mesh based on supermesh technology and centre-based finite volume method. A method to build the local supermesh during overset grid interface by the cell-cut algorithm is presented, and flow field variables could be interpolated conservatively during overset meshes with second-order accuracy via the local supermesh. Numerical results show that the proposed method is strictly conservative for second-order distributed flow variables. Compared with trilinear interpolation method and reverse distance-weighted interpolation method, the method in this paper decreases the numerical errors and the variables are interpolated more accurately. Engineering cases show that the method in this paper is robust and practical for engineering applications.

1. Introduction

For the numerical simulation of turbomachinery, two main approaches exist to transfer the flow through successive bladed rows in relative motion[1]. These are namely the sliding interface method[2-6] and the overset grid method[7-8]. The sliding interface method is widely used in the simulation of turbomachinery[8-11] because it’s easy to operate than overset grid method. However, the sliding interface method is a simplified computational model, the grid cells move together within an independent domain[12]. The overset grid method separates the complicated flowfield into several simple sections, the grids of each section is generated independently and the flowfield of each section is computed independently, the data of flowfield is transferred between the interfaces of overset grids by interpolation. With respect to the sliding interface method, the overset grid method is known to be more flexible since the size of the overlapping zone can vary and the mesh generation of complex geometries is eased[1], and the overset grid method can achieve more accurate simulation[12].

The overset grid method is getting more and more popular in the simulation of turbomachinery, especially in the simulation of wind turbines and marine current turbines. Li[13] and Zahle[14] simulated
the aerodynamic characteristics of NREL Phase VI wind turbine based on overset grid method, the numerical results agree well with experiment results. Chow\cite{15} studied the shape optimization of blades of wind turbine, and researched the aerodynamic characteristics with overset grid method. Tran\cite{16} researched the rotational motion of OC3-Hywind turbine with overset grid method in STAR-CCM+ software.

However, the interpolation between overset mesh interfaces is still a challenge to Computational Fluid Dynamics (CFD). There are usually 2 interpolation methods: conservative interpolation and non-conservative interpolation. The non-conservative interpolation methods are widely used in many non-discontinuity flow field simulations because it is easy to operate. However, it strongly depends on the grids scale in the overset mesh area. When the grid scale matches badly, the non-conservative method is insufficient to guarantee the accuracy, and may cause numerical oscillation in numerical simulation.

In general, conservative interpolation is an important basis to ensure the correct flow field. Farrell\cite{17,18} proposed the concept of “supermesh” and developed an integral conservation interpolation method of two-dimensional mesh, and then he did further research in the conservative interpolation method of three-dimensional hybrid mesh in 2011. Wang and Xu\cite{19,20,21} proposed the concept of “efficient search” of donor cells in overset grid, which improved the parallel efficiency of interpolation in overset grid, and increased the interpolation accuracy of overlapping grid areas by increasing the number of interpolation cells. Usually in the flow field of strong shock or large separation, the traditional second-order gradient interpolation, distance interpolation and trilinear interpolation could not guarantee the conservation of physical between overset mesh interfaces, which would lead to a wrong shock wave position and a rough flow field\cite{22,23,24,25}.

Based on supermesh and three dimensional unstructured grids, this paper develops an implicit conservative interpolation method for overset grid, the results show that this method decreases the numerical errors and is strictly conservative for second-order flow variables, and engineering cases show that the method in this paper is robust and practical for engineering applications.

2. Numerical method

The numerical simulations in this paper were conducted by the China Aerodynamics Research & Development Center (CARDC) MFlow-code, which is an unstructured finite volume cell-center CFD solver and has participated in the 5th and 6th CFD drag prediction workshop. Second-order accuracy in space is achieved by linear reconstruction in cells. The vertex-based Gauss method is adopted for gradient computations\cite{26}. Roe scheme is used for inviscid flux computations.

2.1. Governing equations

The unsteady Navier-Stokes equations are discretized by a cell centered finite-volume method. The integral form of unsteady Navier-Stokes equations for a bounded domain \( V \) with a boundary \( \partial \Omega \) can be expressed as:

\[
\frac{\partial}{\partial t} \iiint_V Q dV + \iiint_{\partial V} (H(Q) \cdot \hat{n} - Q v_{r} \cdot \hat{n}) dS = \iiint_{\partial V} H_{v}(Q) \cdot \hat{n} dS
\]  

(1)

Where \( Q = [\rho \ u \ v \ w \ E] \) is conservative vector, \( v_{r} \) is wall velocity \( H(Q) \) is inviscid flux vector, and \( H_{v}(Q) \) is viscous flux vector. Here, \([\rho \ u \ v \ w \ E] \) denotes the density, velocity of three directions, and specific total energy of the fluid, \( \hat{n} \) is outward pointing normal unit vector of boundary. The inviscid flux is replaced by a numerical Riemann flux function Roe schemes, and the viscous flux is discretized with central difference scheme. The Venkatakrishnan limiter is used to restrict oscillation.

2.2. Grid generation

The computational grid is unstructured hybrid grid, including triangular prism, pyramid, and tetrahedral grids. The triangular prisms and tetrahedrons are used to simulate the boundary layer and the isotropic region of the spatial flowfield, respectively, and the pyramid grid is used to transition the triangular prism and the tetrahedron.
3. Conservative interpolation algorithm

3.1. Conservative conditions
Conservative interpolation requires the integration of the physical variables could be kept constant in the overlapping areas of the overlapping grids. Suppose there are two sub-meshes $\Gamma_A$, $\Gamma_B$, and the overset area is $\Gamma_C$. The distribution of physical variable $\phi$ satisfies the function $\phi^A(x)$ in mesh $\Gamma_A$, and the distribution of physical variable $\phi$ satisfies the function $\phi^B(x)$ in mesh $\Gamma_B$. If the physical variable $\phi$ satisfies the following condition in any area $\Gamma_D$ in mesh $\Gamma_C$:

$$\int_{\Gamma_D} \phi^A(x)dV = \int_{\Gamma_D} \phi^B(x)dV$$

(2)

where the physical variable $\phi$ is strictly conservative in the overset area $\Gamma_C$ of meshes $\Gamma_A$ and $\Gamma_B$.

3.2. Supermesh
Farrell proposed the concept of "supermesh"\[^{17-18}\] for any meshes $\Gamma_A$ and $\Gamma_B$, $N_A$ and $N_B$ is the grid nodes, $E_A$ and $E_B$ is the grid edges, supermesh $\Gamma_C$ is composed with their common nodes and edges, $N_C$ and $E_C$ are the node and edge of grid $\Gamma_C$, which satisfy following conditions:

$$N_C \supseteq N_A \cup N_B \quad \Gamma_C \in \{\Gamma_A, \Gamma_B\}$$

(3)

Figure 1. Schematic of supermesh.

In short, all the nodes in the supermesh $\Gamma_C$ are contained in the parents meshes $\Gamma_A$ and $\Gamma_B$, and all the cells in the supermesh $\Gamma_C$ are contained in the parents meshes $\Gamma_A$ and $\Gamma_B$. Each grid cell in the supermesh $\Gamma_C$ is intersected by the grid cells of $\Gamma_A$ and $\Gamma_B$. Cells of supermesh do not intersect with any cells of $\Gamma_A$ and $\Gamma_B$. Obviously, the construction of a supermesh is not unique, but if we treat one of the parents meshes as “target mesh”, and treat the other mesh as “donor mesh”, let each cell of donor mesh intersect with cells of target mesh, the construction of a supermesh is unique. Fig. 1 shows...
the super grid diagram, Fig.1 (a) and (b) show two parents meshes $\Gamma_A$ and $\Gamma_B$, if $\Gamma_A$ is treated as “target mesh”, we can get a supermesh in Fig.1 (c), and the color part of the mesh belong to $\Gamma_A$. If $\Gamma_B$ is treated as “target mesh”, we can get a supermesh in Fig.1 (d), and the color part of the grid belong to $\Gamma_B$.

For any mesh $\Gamma_B$, there exists a supermesh $\Gamma_C$, whose grid cell can be found on the specific location and the corresponding physical variables in mesh $\Gamma_A$, the variable $\phi$ has the same distribution in mesh $\Gamma_A$ and $\Gamma_B$, so the interpolation function can be write as:

$$
\int_{\Gamma_C} \phi^A(x) dV = \sum_{i=1}^{n} \phi^A_i V_i
$$

Thus the variable $\phi$ could be interpolated conservatively from $\Gamma_A$ to $\Gamma_B$ through the medium supermesh $\Gamma_C$.

Cells of the supermesh $\Gamma_C$ are constructed by intersection of parents meshes $\Gamma_A$ and $\Gamma_B$. Common three-dimensional polyhedral grid units include tetrahedrons, triangular prisms, pyramids, hexahedrons and so on, of which triangular prisms, pyramids and hexahedrons have quadrilaterals, and in the grid intersection process we can not guarantee the four vertices are in the same plane. Therefore, the polyhedron mesh units are decomposed into multiple tetrahedral units uniformly, and the intersection of polyhedrons is obtained by calculating the intersection of tetrahedral units. The intersection method between tetrahedrons has been mature, and Sutherland-hodgman method is employed in this paper, for example in two dimensions the main idea is to cut the “target polygon” by the edge of “cut polygon” and to get a new polygon. Fig.2 shows a two-dimension case of Sutherland-hodgman method, $A_1A_2A_3$ is “target polygon” and $B_1B_2B_3$ is “cut polygon”, $A_1A_2A_3$ is cut by each edge of $B_1B_2B_3$ and we can get a new polygon $B_1CDEF$.

Intersection method between three-dimensional tetrahedrons is same as two dimensional polygons, Fig 3 shows the tetrahedral mesh intersection process, in Fig.3 (a) the black tetrahedron is target tetrahedron, the red tetrahedrons are the cutting tetrahedrons, the target tetrahedron is cut by each surface of cutting tetrahedrons to get each unit of the supermesh. Fig.3 (b) shows a supermesh which is constructed by the intersection of target tetrahedron and cutting tetrahedrons.
An important consideration for the construction of supermesh is run-time efficiency, which depends on the intersections calculations. Menon tested the numerical cost of intersections calculations\cite{27}, and it just costs 25.1 seconds to deal with 3.03 million grids using an Intel Core 2 Quad computer running at 2.83GHz with 4Gb of RAM, which is a tiny computational cost compared with normal numerical simulation.

3.3. Interpolation method
Interpolation of variable $\phi$ from $\Gamma_A$ to $\Gamma_B$ is achieved by the medium supermesh $\Gamma_C$. $K$ is a grid cell of mesh $\Gamma$, the physical variable is firstly interpolated from $\Gamma_A$ to supermesh $\Gamma_C$ with a second-order Taylor expansion:

$$\phi(X_{K_c}) = \phi_{K_A} + (\nabla \phi)_{K_A} \cdot (X_{K_c} - X_{K_A}) \cdot \psi_{K_A}$$

(5)

where $(\nabla \phi)_{K_A}$ is the gradient of the physical variable and $\psi_{K_A}$ is the limiter function, and Vencat Limiter is employed in this paper. The physical variables are then interpolated from the supermesh $\Gamma_C$ to the target mesh $\Gamma_B$. In this paper, the second-order Taylor method is adopted in the interpolation method. And the interpolation from source mesh to target mesh is strictly conservative for second-order distributed physical variables.

According to whether the “donate cells” also act as “target cells”, the interpolation between overset grids can be divided into implicit interpolation and explicit interpolation. An implicit interpolation method is employed in this paper, the donate cells and target cells are not strictly distinguished, each single overset grid sends data to other overset grids and also receives data from other overset grids during parallel computing. The implicit parallel interpolation is more efficient than explicit interpolation, and can be appropriate for massive grids of complex configurations.

4. Numerical results

4.1. Linear equation
Assume that the physical variables are linearly distributed in the grid area and satisfy the equation:

$$\phi(x, y, z) = 2x + 3y + z$$

(6)

$\Gamma_A$ is a structured mesh and $\Gamma_B$ is an unstructured tetrahedron mesh, and they share the same grid area. Let the distribution of the physical variables on the mesh $\Gamma_B$ satisfy equation (6), then interpolate
100 times between $\Gamma_A$ and $\Gamma_B$, and calculate the absolute value of the mean value of the conservation and each cells.

$$\phi(x, y, z) = 1 + \sin(2px) \sin(2py) \sin(2pz)$$

(7)

Fig.4 shows a comparison of the proposed method in this paper and traditional interpolation methods on accuracy and conservation. Fig.4 (a) and Fig.4 (b) show the integration and average error during the interpolation process. Integration denotes the sum of variable of all cells, and average error denotes the average error of each cell. As is shown in Fig.4 (a), the integral value of the physical variable on the grid shows that the interpolation developed in this paper satisfies the conservation of the integral value of the physical variable in the overlapping area. Fig.4 (b) shows the absolute value of the interpolation error of the physical quantity on the grid. Compared with the distance-weight method and trilinear method, the method in this paper decreases the errors of interpolation.

4.2. Trigonometric function

Assume that the physical variables are trigonometric distributed in the grid area and satisfy the equation:

Fig.5 shows a comparison of the proposed method in this paper and traditional interpolation methods on accuracy and conservation. Fig.5 (a) and Fig.5 (b) show the integration and average error during the interpolation process. Integration denotes the sum of variable of all cells, and average error denotes the average error of each cell. As is shown in Fig.5 (a), the integral value of the physical variable on the grid shows that the interpolation developed in this paper satisfies the conservation of the integral value of the physical variable in the overlapping area. Fig.5 (b) shows the absolute value of the interpolation error of the physical quantity on the grid. Compared with the distance-weight method and trilinear method, the method in this paper decreases the errors of interpolation.
Fig. 5 also shows a comparison of the proposed method in this paper and traditional unconservative interpolation methods on accuracy and conservation. It can be seen that the average error of proposed interpolation method is much less than other methods, and is almost conservative for the integral of variables.

4.3. Numerical simulation of engineering applications

The overset grid method is getting more and more popular in the simulation of turbomachinery, especially in the simulation of wind turbines. An isolated six-bladed propeller at different speeds is numerically simulated to investigate the robustness and engineering practicability of the developed method. Wind tunnel experiments are conducted in 8m×6m Low Speed Wind Tunnel of CARDC. Fig. 6 (a) shows the overset grids of propeller, the grid in red color is the sub-grid of blades, and the grid in blue color is the sub-grid of the hub, which is also the background grid and is refined in the zone of slipstream in order to accurately simulate the propeller slipstream. The number of total grids is 18.2 million.

Fig. 6 (b) and Fig. 6 (c) show the Q-criterion of propeller slipstream at advance ratio 1.58, the vortex is observed clearly because of the mesh refinement behind the propeller. A comparison of numerical simulation and experiment result of torque coefficient is presented in Fig. 6 (d), and the abscissa is the propeller advance ratio. The numerical result is well consistent with the experiment result, which verifies that the proposed method is robust and practical for engineering applications.

5. Conclusion

A new conservative data transfer method for overset grid is developed based on unstructured grid and supermesh theory in this paper, which improves the accuracy of data transfer between the overset interfaces, the method is strictly conservative for second-order distributed flow variables, and engineering cases show that the method in this paper is robust and practical for engineering applications.
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