The Metal-Insulator Transition in High-$T_c$ Cuprates – An unusual Quantum Transition

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Abstract. The Hilbert spaces representing the quantum states in the CuO$_2$ planes are not spanned by crystal symmetry adapted basis states, but by self-consistently renormalized states which form compound systems of Hilbert subspaces. In addition, these self-consistent basis representations are not necessarily stationary in time but behave dynamic in many respects. In particular, the coordinate systems of the Hilbert subspaces form a definite dynamic relative state in space and time, thus the coordinate system itself becomes a variable. This implicates a deterministic space-time relation of quantum states and the quantization of time by an internal time constant, the *eigentime* $\tau_{eigCBF}$. In undoped and hole doped CuO$_2$ planes $\tau_{eigCBF}$ occurs as a conservation quantity, whereas under electron doping a partial fluid exists in which $\tau_{eigCBF}$ does not occur as quantized quantity. The deterministic space-time behaviour of the Hilbert subspaces represents the emergence of a classical space-time structure in quantum systems. The metal-insulator transition in high-$T_c$ cuprates, usually attributed to an antiferromagnetic Mott transition, results here from an additional splitting into Hilbert subspaces. The transition from the insulator to the conductor is causally related to coordinate transformations from the copper to the oxygen sites. The antiferromagnetism in the CuO$_2$ planes is not caused by the half-filled valence bands, as usually assumed, but is created by *off-diagonal spin compensations* within filled bands.

1. Introduction

Ever since the experimental investigations of superconducting cuprates, these materials were proven to be unconventional in many respects, in the normal as well as superconducting state. This is primarily concerned within hole-doped cuprates which show, for example, an anomalous low temperature resistivity behaviour $\rho \sim T$, a temperature dependent Hall effect, an optical conductivity $\sigma(\omega) \sim 1/\omega$, non-Korringa copper relaxation rates found in NMR, non-pronounced or unconventional isotopic effects, coexisting spin and superconducting gaps found in neutron scattering, a temperature independent superconducting gap up to $T_c$ distinctly visible in IR measurements, and several other unconventional findings. A comprehensive review can be found in several monographs and review articles [1]–[6] and some particular results have already been discussed in our previous articles [7, 8].

An important phenomenological difference between high-$T_c$ superconducting cuprates and conventional superconductors is the obviously non-s-wave symmetry of the order parameter $\Delta(\alpha)$ in the high-$T_c$ superconductors in dependence on the wave vector direction $\alpha$. The found $\pi$ shift, responsible for a half-integer flux quantization of the
magnetic flux already existing without an outer magnetic field, is attributed to non-s-wave symmetries [9]. An anisotropic gap $\Delta(\alpha)$ found by angle-resolved photoemission spectroscopy (ARPES), which is reflected by the anisotropy of the momentum dependence of the gap being largest along the $\Gamma - M$ (parallel to the $a$ or $b$ direction) and smallest along $\Gamma - Y$ (diagonal line between them) [10]–[12], can also be attributed to a non-s-wave pairing. However, despite these indications that apparently favour a d-wave symmetry of $\Delta(\alpha)$, the symmetry problem of the superconducting pair state is not yet unambiguously answered. This and other particular problems will appear in a completely different light, if the Topological Resonance (TR) conception [8] is taken as a basis which assumes arrays of parallel one-dimensional electronic states. Under these conditions the symmetry of the gap $\Delta(\alpha)$ has to be related in the first place to one-dimensional electronic states. Hence, all theoretical attempts which are stringently founded on a stationary two-dimensional $d_{x^2−y^2}$ symmetry of the pair-wave function [13]–[16] can not reflect the superconducting state within the CuO$_2$ planes, if the electronic state is causally founded on one-dimensional electronic states.

The above experimental results were related to hole-doped ($p$-type) cuprate superconductors. The experimental findings of electron-doped ($n$-type) superconductors, Nd$_{2−x}$Ce$_x$CuO$_{4−\delta}$ (NCCO) and Pt$_{2−x}$Ce$_x$CuO$_{4−\delta}$ (PCCO), are different to their hole-doped counterparts in many ways. For example, the highest $T_c$ values in the hole-doped cuprates are more than five times as high as in the highest $T_c$ electron-doped cuprates [17, 18]. The in-plane resistivity is approximately quadratic in temperature, contrary to the pronounced linear behaviour within the hole-doped cuprates. The flat band just below the Fermi-surface ($E_F$) at $(\pi, 0)$ and $(0, \pi)$ always found in hole-doped cuprates is located at 300 meV below $E_F$ in NCCO [19]. However, the symmetry dependence of the electronic states near the Fermi surface does not seem to essentially be different to the hole-doped case. These different characteristics of the $p$- and $n$-type superconductors are causally related to the different influence of the time-quantized phases as it will be discussed in this article.

From the beginnings of theoretical considerations about the electronic states in high-$T_c$ cuprates, the unusual properties in the normal state ($T > T_c$) have questioned the Fermi liquid description as being the right theoretical basis [1]. The occurrence of an antiferromagnetic state in the CuO$_2$ planes in the hole undoped case has motivated most scientists to attribute the antiferromagnetic couplings to the unusual normal state properties as well as to the existence of a superconducting state. It is assumed that the undoped electronic state of a CuO$_2$ plane corresponds to an antiferromagnetic Mott insulator and hole doping creates a variety of possible spin frustrations including local spin pair states which may resonate [1]–[6], [13]–[16]. A separation between spin and charge states, which are highly correlated, is explicitly included. This is reflected by holon and spinon states in the RVB conception [1, 2] and the formation of linear spin and charge states, termed as stripes (smectic phases), as well as square structures (nematic phases) [3]. The theoretical postulation of stripes as well as square structures were motivated by extraordinary experimental results which were summarized by the general term 1/8 problem and the formation of checkerboard structures, respectively (see e.g. Refs. [3, 7]). In the theoretical descriptions based on antiferromagnetic couplings, this obvious great variety of electronic states in the CuO$_2$ planes is comprehended by postulating quantum critical points which lead to quantum critical fluctuations/phases [3], [13]–[16]. In order to differentiate between theories which assume an antiferromagnetic Mott insulator as the fundamental basis and the TR conception, which represents the theoretical basis in this article, some common characteristics of various theories are firstly summarized which are based on the Mott-Hubbard philosophy [1]–[6], [13]–[16].
(i) The undoped state of the CuO$_2$ plane represents an antiferromagnetic Mott insulator.

(ii) A hole doped Mott insulator may be conducting. The charge carriers carry also the spin (one-band representation), i.e. the remnant antiferromagnetic couplings are created by the charge carriers themselves.

(iii) Under hole doping, the remnant antiferromagnetic spin interactions are primarily causing the formation of particular quantum phases and the creation of the superconducting phase.

(iv) Various spin-charge separations are possible but they are basically of non-crystalline nature (smectic phases, nematic phases) [3], [13]−[16].

Contrary to these assumptions, which are most popular in describing the electronic states in high-$T_c$ cuprates, we have shown [7, 8] that an antiferromagnetic Mott insulator is not the right frame in order to characterize the electronic states in undoped CuO$_2$ planes, but a subtle renormalized electronic state exists which was termed as charge and bonding fluctuation state (CBF) [7]. The CBF state includes the antiferromagnetism, however, the antiferromagnetic couplings are not causal for the electronic renormalizations but only a consequence of that. In comparison to the above characteristics of the Mott-Hubbard basis some topics resulting from this new electronic state are summarized:

(a) The basis state in the hole undoped case represents a charge and bonding fluctuation state (CBF) caused by complex electronic renormalizations [7]. The complete quantum state represents a dynamic charge and bonding fluctuation state (DCBF) [8].

(b) A hole doped CBF state may be conducting. The basic principles underlying an insulator-conductor transition are described in this article. The charge carriers do not create the antiferromagnetic spin couplings itself, the antiferromagnetic spin interactions are continuously created by lower energetic bands, but the quantum states of the charge carriers and the antiferromagnetic spin states are strongly correlated.

(c) Under hole doping, the antiferromagnetic spin interactions are not causal for the formation of the valence quantum states. Instead quite the reverse applies, the electronic correlations dominate the spin structure. The superconducting state is causally related to electron and phonon states [8].

(d) Spin-charge separations are generally given by (b). The generally existence of topological charge separations which lead to self-consistent crystalline charge orders is deciding. Primarily, the spin structure follows the charge order but not vice versa.

A simple comparison of the topics (i) to (iv) with (a) to (d) shows the different basic assumptions between the Mott-Hubbard model and the theoretical basis underlying the considerations in this article.

In addition to the points selected above, there are further fundamental differences to existing theoretical approaches. The CBF state subdivides the total electronic state into particular electronic substates, corresponding to a subdivision of the unified Hilbert space into Hilbert subspaces. Quantum states which are founded on (i)-(iv) are continuously described within an unified stationary Hilbert space. Contrary to that, an electronic system $S$ which is associated to the Hilbert space $\mathcal{H}$ being decomposed into two subsystems $S_1$, $S_2$, and being associated to the Hilbert spaces $\mathcal{H}_1$, $\mathcal{H}_2$, is represented by the compound system $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, i.e. the tensor product of the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$. This separation of the Hilbert space $\mathcal{H}$ into Hilbert subspaces $\mathcal{H}_1$ and $\mathcal{H}_2$ implies two important quantum effects, which are essential within the TR state. Firstly, interferences between states in $\mathcal{H}_1$ and $\mathcal{H}_2$ are prevented, but in return entanglements between quantum states in $\mathcal{H}_1$ and $\mathcal{H}_2$ may be enhanced.

It is of high significance that the Hilbert subspaces themselves can not be considered
as being stationary in time but their basis states have to be considered as dynamically variable. Firstly, basis states of particular Hilbert subspaces are continuously exchanged, which particularly may occur in a probabilistic manner but above that there is also a strongly deterministic exchange in space and time. For example, the dynamics of the DCBF state (see(a)) is strongly deterministic in space and time, but the exchange within the \( k \) and \( -k \) electron pair states may occur probabilistically in the non-superconducting state (IS quantum hole states). However, these \( k, -k \) pair exchanges are strongly quantized in time in the superconducting state (DS quantum hole states) [8]. The exchange of quantum states is causally related to the exchange of the coordinate systems of Hilbert subspaces. In consequence, the particular coordinate systems of the Hilbert subspaces develop into dynamic variables, either strongly periodic and deterministic in space and time or of probabilistic nature. If the coordinate system occurs as a deterministic space-time variable, internally quantized time scales of the dynamic Hilbert subspaces are created which are termed as eigentimes [8].

The goal of this article is to explain the static and dynamic separation of the Hilbert space into Hilbert subspaces and the resulting consequences for the quantum states following from that. It is particularly important to emphasize the conditions for the insulator-conductor transition under the special condition of crystalline topological hole structures and topologically ordered electron structures.

2. The electronic states of the CuO\(_2\) planes

The electronic states of the CuO\(_2\) planes were derived in preceding articles [7, 8]. The crystal symmetry adapted electronic states, usually existing in quantum systems, are renormalized in a way that a charge and bonding fluctuation state (CBF) with the period \( 2a, 2b \) is created within the CuO\(_2\) plane, depicted by the +,− signs in Fig. 1. This state implies a general renormalization of the electronic states with the result that the initially equivalent copper sites are separated into two different copper sites, as demonstrated by +,− signs in Fig. 1, which separates the crystal lattice into two sublattices. Only local electronic states associated to one sublattice are involved within every particular quantum state. In the hole undoped case representing half-filled valence states, only quantum valence states that are associated to one sublattice, the lattice corresponding to the − copper sites, are occupied. Therefore, the undisturbed CBF state is characterized by general renormalizations of the electronic states leading to two electronic subspaces with the initial unit cell vectors being divided by half as well as by a charge fluctuation state with the periodicity \( 2a, 2b \) along the crystallographic axis \( a \) and \( b \), respectively, as depicted in Fig. 1. For all low energetic bands where the Cu-O bonds possess bonding character, the two subspaces associated to the + and − lattice sites still represent two-dimensional electronic states. Especially, band states with nearly exclusively \( d_{x^2−y^2} \) orbital symmetry of the Cu orbital parts belong to these two-dimensional low energetic band states. This two-dimensionality of the electronic states is additionally broken for the energetically highest half-filled valence bands where the Cu-O bonds are antibonding. As a result, the two-dimensional quantum states of the energetically highest valence bands are transformed into arrays of one-dimensional quantum states (see Ref. [7]). The one-dimensional electronic states are created along the lines \( I_x, I_x' \) and \( I_y, I_y' \) as well as \( I_y, I_y' \) and \( I_y, I_y' \) in Fig. 1, leading to the formation of one-dimensional bands. The particular one-dimensional bands are half-filled for the undoped CuO\(_2\) plane, leading to the higher populated − copper sites. According to Fig. 1, two arrays of correlated one-dimensional subspaces exist correspondingly to the polarization of the one-dimensional states in \( x(a) \) and \( y(b) \) direction, respectively.

1 Although \( k, -k \) pair states generally exist an outer impulse can be transferred to the paired electrons, if the pair state possesses IS quantum hole symmetry (non-superconducting state), as discussed in Ref. [8].
Figure 1. Topological hole configuration of a CuO$_2$ plane for a nominal hole concentration $n_h = 0.25$ holes/copper, representing an electronic ground state configuration of a static CBF state [7]. The two energetically highest valence bands of the CuO$_2$ plane are decomposed in two perpendicular arrays of one-dimensional electronic subspaces, along the $x$ coordinate ($I_x, I'_x, I_x^{'}, I'_x$), and the $y$ coordinate ($I_y, I'_y, I_y^{'}, I'_y$). The $+,-$ signs indicate the two different copper sites, corresponding to the renormalized electronic CBF state. The pictogram representation along the bold lines reflects attractive hole - hole interactions with $E_C < 0$ (see Ref. [7]). These holes are termed as b-holes (bonded holes), with $n_h(b$-holes$) = n_h/2 = 0.125 = 1/8$ holes/copper (lines $I_y, I'_y$). The remaining holes are also correlated holes but with $E_C = 0$, and termed as f-holes (free holes). The b-holes and f$_1$-holes are aligned along the y(b) coordinate, within the one-dimensional electronic states $I_y, I'_y$ and $I_y^{'}, I'_y$, respectively. The f$_1$-holes belong to equivalent states within $I'_x$ along the x(b) coordinate. The phases/coordinates $\frac{\Delta \phi S_i}{\Delta r S_i}$ and $\frac{-\Delta \phi S_i}{-\Delta r S_i}$ are topological relative phases/coordinates which differentiate between the + an − copper sublattices. The phase/coordinate $\phi S_i / r S_i$ corresponds to the center point phase/coordinate of the entire b-hole structure, $-\Delta \phi S_i / -\Delta r S_i$ and $\Delta \phi S_i / \Delta r S_i$ are the relative center point phases/coordinates of the b-hole substates along $I_y$ and $I'_y$, respectively. Equivalent center point phases/coordinates $\phi S_i / r S_i$, $-\Delta \phi S_i / -\Delta r S_i$ and $\Delta \phi S_i / \Delta r S_i$ are defined perpendicular to the b-hole polarisation direction. The relative phases $-\Delta \phi S_i$ and $\Delta \phi S_i$ define the positions of the two b-hole sublattices/substates within every b-hole state along $I_y$ or $I'_y$ having a periodicity of $8b$ and relative shift of $3b$ to each other. This creates the different topological hole states $h_1$ and $h_3$ as well as $h_2$ and $h_4$, respectively.
Under hole doping, which occurs if electrons are removed from the CuO$_2$ plane, the hole density is not widely distributed over the CuO$_2$ plane but rather localized on particular oxygen atoms. This leads to an additional hole localization energy $E_S < 0$ which is square dependent on the anisotropic local hole density $q_h$, located predominantly on particular oxygen atoms (Fig. 1). It has been argued [7] that a nominal hole density of $q_h = 1$ is localized on a particular oxygen atom. This means, a one-to-one correspondence exists between the number of doped holes and the number of localized topological hole states. However, these topological hole states are generally not identical with localized quantum states, because the electronic quantum states usually remain delocalized caused by the strong periodicity of the topological hole states. If linear hole - hole arrangements occur, as it is given for the hole pairs $(h_1,h_2)$ or $(h_3,h_4)$ in Fig. 1, an additional hole - hole coupling energy $E_C < 0$ occurs [7]. For non-linear hole - hole arrangements like $(h_5,h_6)$, i.e. a cyclic hole - hole topology [7], the holes are also strongly correlated, but with $E_C = 0$. The localization energy $E_S$ (hole self-energy) and the correlation energy $E_C$ are predominantly based on quadrupolar-polarizations on copper atoms [7]. The different correlations for linear and cyclic hole - hole arrangements create two types of holes, b-holes and f-holes. The linearly arranged b-holes with $E_C < 0$ exist undisturbed within the hole concentration range of $0.125 \leq n_h \leq 0.25$ holes/copper with a definite periodic topological hole order, related to a unite cell of $(4a,8b)$ in Fig. 1, and a constant mean b-hole density of $n_h$(b-holes) = 0.125($-1/8$) holes/copper. Ordered b-hole structures which include b-hole vacancies occur for $n_h < 0.125$ holes/copper. The f-holes with $E_C = 0$ can only exist for $n_h > 0.125$ holes/copper forming strong periodic structures when the hole density of $n_h = 0.25$ holes/copper is reached as given in Fig. 1. The f-holes can exist with different topological hole configurations within every particular b-hole configuration [7, 8].

3. The formation of electron subspaces

The electronic renormalizations lead to crystal symmetry breaking topological charge orders [7]. This becomes obvious by the formation of the CBF state and the occurrence of ordered topological hole structures which are decisively determined by the b-holes. In addition, the symmetry breaking leads to transformations of the coordinate systems of the particular topological charge orders with respect to the fundamental crystal structure. The CBF state which differentiates the copper atoms into two different sites, signed by + and − in Fig. 1, creates two sublattices with the lattice periodicity $2a, 2b$ along the crystal axes, accompanied by shifts of the particular coordinate systems of the two sublattices by $\pm \Delta r_{||}$ with respect to the center point $r_{||}$ along the $x(b)$ or $y(b)$ direction (Fig. 1). Similarly, the b-hole sublattice creates topological/spectral phases $-\Delta \phi_{S_{||}}, +\Delta \phi_{||}$ and $\Delta \phi_{S_{||}}, \pm \Delta \phi_{||}$ within each particular one-dimensional direction $I_y$ and $I_y'$ in Fig. 1, respectively, and a common center phase $\phi_{S||}$ can be defined for the whole b-hole lattice. All phases are related to the hole periodicity of 8b in $y(b)$ direction of Fig. 1. The one-dimensional b-hole states along $I_y$ and $I_y'$ can be differentiated by the perpendicular phases $\phi_{S_{\perp}}, \pm \Delta \phi_{S_{\perp}}$, which are related to the b-hole periodicity of 4e in $x(a)$ direction. Similar topological phases can be defined for periodic f-holes states $(f_{||}$-holes and $f_{\perp}$-holes), but they are uniquely defined exclusively for $n_h = 0.25$ holes/copper.

The renormalizations of the electronic states which lead to two different copper sites, − and + in Fig. 1, were deduced from MO cluster calculations where every renormalized MO orbital involves predominantly next neighbour copper sites but one, but not consecutive copper sites. In consequence, the periodic lattice of local electronic states within an infinitely extended CuO$_2$ plane falls into two sublattices with the lattice periodicity of 2a, 2b along the crystal axes a and b, respectively, correspondingly to the − and + lattice sites in Fig. 1. The two sublattices result from self-consistent electronic renormalizations of the initial crystal symmetry adapted quantum states, which implies a
strong association of the quantum states of the two sublattices. Usually, Bloch states are constructed from orthonormalized local electronic states possessing the stationary lattice periodicity. In contrast to that, the above renormalizations leading to symmetry breaking of the electronic states of the CuO$_2$ plane require the general inclusion of two neighboured lattice sites during the formation of one local state. This means, the local states within a Bloch representation comprehend generally two lattice sites. On the other hand, the local states associated to the $-$ and $+$ copper sites are orthogonal [7]. In consequence, every particular two-dimensional Bloch state can be constructed with respect to the local states of one periodic sublattice $-$ or $+$. Therefore, two different two-dimensional Bloch states can be defined for a given wave vector $\mathbf{k}$:

\begin{align}
\varphi_k(r_1, -\Delta r/2) &= e^{i(kr_1 - \Delta \phi/2)} u_k(r_1 - \Delta r/2) \quad \in \mathcal{H}_{-\Delta r/2} \\
\text{and} \\
\varphi_k(r_2, +\Delta r/2) &= e^{i(kr_2 + \Delta \phi/2)} u_k(r_2 + \Delta r/2) \quad \in \mathcal{H}_{+\Delta r/2}
\end{align}

with $u_k(r_1 - \Delta r/2)$ and $u_k(r_2 + \Delta r/2)$ being periodically local states corresponding to the sublattices $-$ and $+$, respectively, within a coordinate system that is rotated by 45° with respect to the coordinate system as shown in Fig. 1. The sublattices are assumed to be shifted by $\pm \Delta r/2 = \pm (\pi/4, 0)$ in $x(a)$ direction or $\pm \Delta r/2 = \pm (0, \pi/4)$ in $x(b)$ direction, within a coordinate system as shown in Fig. 1. Correspondingly, the exponential functions in Eqs. (1) and (2) are phase shifted in relation to $\Delta \theta = \frac{\pi}{\Delta r} \cdot \Delta r = \pi$, leading to $\Delta \phi = \frac{\pi}{2}$ and $-\Delta \phi = -\frac{\pi}{2}$. All in all, the original Hilbert space $\mathcal{H}$ being characterized by Bloch states that are not U1 symmetry broken is transformed into a Hilbert space $\mathcal{H}_{ren}$, with a renormalized basis of U1 symmetry broken states which is separated into two basis representations creating the two subspaces $\mathcal{H}_{-\Delta r/2}$ and $\mathcal{H}_{+\Delta r/2}$, correspondingly to the two sublattices $-$ and $+$. The two subspaces $\mathcal{H}_{-\Delta r/2}$ and $\mathcal{H}_{+\Delta r/2}$ form the orthogonal complement to each other, but in addition, the coordinate systems of the two Hilbert subspaces are different caused by the shift vectors $\pm \Delta r/2$.

Two electrons can principally be placed within each of the two states described by Eqs. (1) and (2), resulting in a spin-up and a spin-down electron, because the total number of quantum states has to be conserved during the renormalizations. Thus, the dimension of the basis in $\mathcal{H}_{ren}$ is identical to the the dimension of the initial Hilbert space $\mathcal{H}$.

The number of Hilbert spaces is doubled, $\mathcal{H}_{-\Delta r/2}$ and $\mathcal{H}_{+\Delta r/2}$, but the number of $\mathbf{k}$ states is divided by half caused by the doubling of the lattice constants given by 2$a1$ and 2$b2$. An electron occupies either the Bloch state given by Eq. (1) or Eq. (2) making different electronic populations possible. A spin-up and a spin-down electron can simultaneously be placed within the state $\varphi_k(r, -\Delta r/2) \in \mathcal{H}_{-\Delta r/2}$ while leaving the state $\varphi_k(r, +\Delta r/2) \in \mathcal{H}_{+\Delta r/2}$ empty, or vice versa. In this case spin paired electron states occur at the lattice sites $-$ or $+$, respectively, i.e. a diagonal spin compensation/pairing occurs as it is found in ordinary closed shell states. The "or", however, proves to be as an exclusive "OR", because, either of the two Hilbert subspaces, $\mathcal{H}_{-\Delta r/2}$ OR $\mathcal{H}_{+\Delta r/2}$, is exclusively populated until one
subband is completely filled. This is caused by the energetically favoured imbalanced population of a particular lattice site, without loss of generality the – lattice site in Fig. 1 (see Section 5). Contrary to these local spin paired states, one electron, for example, a spin-down electron, can be placed within the \( | \Psi_k \rangle (r, -\Delta r) \) state and the spin-up electron in the \( | \Psi_k \rangle (r, +\Delta r) \) state, or vice versa. If Cu – O bonding states exist, ideally, the "or" is again exclusive. The exclusive OR follows from the fact that under these conditions local unpaired spin populations are favored for electronic populations of \( \mathcal{H}_{\text{ren.}} \) higher than half filling (see Section 4). Overall, a spin-down and a spin-up electron belong separately to the lattice sites – and + for every given k state, i.e. the spins are locally separated so that an off-diagonal spin compensation/pairing occurs.

The energetically highest valence states reveal an additional symmetry breaking, the two-dimensional electron states are transformed into arrays of one-dimensional electron states as discussed in Section 2. Each of the two perpendicularly polarized arrays along the \( x(a) \) and \( y(b) \) coordinate consists of one-dimensional states along \( I_i (I_i') \) and \( I_j (I_j') \). This means, the electronic states along a particular one-dimensional state \( I_i \) or \( I_j \) belong to separated subspaces \( \mathcal{H}_{I_i} \) or \( \mathcal{H}_{I_j} \), respectively. Again, every one-dimensional Bloch state can be related to the – or + copper site. Therefore, four different one-dimensional Bloch states can be distinguished for every array corresponding to the shifts of the particular coordinate systems by \( \pm \Delta r_k \) within \( I_i \) and \( I_j \), according to

\[
\begin{align*}
| \varphi_{k|I_i} \rangle (r_{1||}) & = e^{i(k||r_{1||} - \Delta \theta || / 2)} | \varphi_{k|I_i} || r_{1||} - \Delta r || / 2 \rangle \\
\in \mathcal{H}_{I_i, -\Delta r || / 2} & \\
| \varphi_{k|I_i} \rangle (r_{2||}) & = e^{i(k||r_{2||} + \Delta \theta || / 2)} | \varphi_{k|I_i} || r_{2||} + \Delta r || / 2 \rangle \\
\in \mathcal{H}_{I_i, +\Delta r || / 2} & \\
| \varphi_{k|I_j} \rangle (r_{3||}) & = e^{i(k||r_{3||} + \Delta \theta || / 2)} | \varphi_{k|I_j} || r_{3||} + \Delta r || / 2 \rangle \\
\in \mathcal{H}_{I_j, +\Delta r || / 2} & \\
| \varphi_{k|I_j} \rangle (r_{4||}) & = e^{i(k||r_{4||} - \Delta \theta || / 2)} | \varphi_{k|I_j} || r_{4||} - \Delta r || / 2 \rangle \\
\in \mathcal{H}_{I_j, -\Delta r || / 2} &
\end{align*}
\]  

with \( r_{i||} = (x, 0) \), \( \Delta r || / 2 = (\frac{a}{2}, 0) \) or \( r_{i||} = (0, y) \), \( \Delta r || / 2 = (0, \frac{b}{2}) \) for the \( x(a) \) or \( y(b) \) polarized one-dimensional states in Fig. 1, respectively, and \( \Delta \theta || / 2 = \frac{\pi}{4} \). This means, for every pair of one-dimensional electronic states along \( I_i \) and \( I_j \), with \( l = 1 \) and \( l = -1 \) for the neighboired subspaces, it can be distinguished between four separated Hilbert subspaces \( \mathcal{H}_{I_i, -\Delta r || / 2} \), \( \mathcal{H}_{I_i, +\Delta r || / 2} \), \( \mathcal{H}_{I_j, -\Delta r || / 2} \) and \( \mathcal{H}_{I_j, +\Delta r || / 2} \). Here, \( \mathcal{H}_{I_i, -\Delta r || / 2} \) and \( \mathcal{H}_{I_j, +\Delta r || / 2} \) as well as \( \mathcal{H}_{I_i, +\Delta r || / 2} \) and \( \mathcal{H}_{I_j, -\Delta r || / 2} \) form the orthogonal complement to each other, respectively.

Again, the coordinate systems are different and given by definite relative shifts of \( \pm \Delta r_k \).
Equivalently to the discussion above, different electronic populations of these particular states are possible. A spin-up and a spin-down electron within $I_i$ or $I_j$ can be simultaneously assigned, either to the $-\Delta r$ or to the $+\Delta r$ lattice site, i.e. the two electrons belong either to $\mathcal{H}_{I_i,-\Delta r}$ OR to $\mathcal{H}_{I_i,+\Delta r}$ and correspondingly either to $\mathcal{H}_{I_j,-\Delta r}$ OR to $\mathcal{H}_{I_j,+\Delta r}$. In this case a diagonal spin compensation occurs again. On the other hand, the spin-up and the spin-down electrons may be locally separated so that they belong to the different lattice sites $-\Delta r$ and $+\Delta r$, respectively, i.e. the spin-up electron belongs, for example, to $\mathcal{H}_{I_i,-\Delta r}$ and the spin-down electron to $\mathcal{H}_{I_j,+\Delta r}$, or vice versa. Correspondingly, the same applies to the $\mathcal{H}_{I_j}$ subspaces. This leads again to an off-diagonal spin compensation/pairing.

4. The electronic state of filled bands

If the electron density of a CuO$_2$ plane is not reduced, the energetically upper valence bands are half-filled, but the energetic deeper lying bands are filled [7]. In the case of filled bands every $-$ and $+$ copper site is occupied by two electrons, associated to the particular band states of Eqs. (1) and (2), respectively. Therefore, no charge fluctuations occur within these bands; the influence of charge fluctuations is merely given by the charge fluctuations within the upper valence bands (see Section 5). This raises the question, in which way the spin-up and spin-down states are compensated within these filled band states. If the spin-up and spin-down states were compensated within every particular lattice site (diagonal spin compensation), i.e. within the $-$ and $+$ site, respectively, the $-\Delta r$ and $+\Delta r$ lattice sites would be completely decoupled/uncorrelated with respect to the spin interactions. This corresponds to the case where a particular $k$ state belongs to same Hilbert subspace $\mathcal{H}^[-]$ or $\mathcal{H}^+$ with its associated spin state functions $\chi^-$ and $\chi^+$, respectively, diagonal spin compensation. As already noticed, this is the ordinary case of locally spin paired closed shell states. However, this behaviour can not be expected for states with Cu-O bonding character, because it was deduced [7] that the self-consistent electronic renormalizations, which create the $-\Delta r$ and $+\Delta r$ copper sites should lead to a net bonding interaction between these two lattice sites. In particular, the tendency to maintain the total bonding energy for the Cu-O bonding states with $d_{x^2-y^2}$ orbital character as far as possible after renormalization preserves a net bonding interaction between these renormalized states, which belong to low energetic bands. Possibly, quantum state transformations similar as described in the Appendix may occur leading to remnant continuous covalent couplings, which is presently, however, still unproved. All in all, this favours paired spin states where the spins are separately associated to the $-\Delta r$ and $+\Delta r$ copper sites, i.e. an antiparallel orientation of the spin states at the $-\Delta r$ and $+\Delta r$ copper sites (Fig.3,(3)). Thus, for every $k$ state one kind of spins, for example the spin-down states, are favoured to be associated to the local states $\mathcal{H}^[-]$ and the spin-up states correspondingly to the $\mathcal{H}^+$, or (exclusive OR) vice versa, meaning the $-\Delta r$ and $+\Delta r$ lattice sites ideally carry only one spin state. Consequently, the spin compensation will occur between the $-\Delta r$ and $+\Delta r$ copper sites, corresponding to an off-diagonal spin compensation, but not on the same copper site. This requires the occupation of higher $k$ states which belong to the second Brillouin zone within every subband. The latter, however, does not lead to particular problems, because diminutive overlap densities between the local states, the distances of the local states along the bond directions are $2a$ and $2b$, within each of the two particular bands result in a very high effective band mass ($m_k \rightarrow \infty$), i.e. the bands representing the electronic states of Fig. 1 will be nearly dispersionless. The two dimensional band
states with primarily $d_{x^2-y^2}$ orbital character of the copper parts belong to these low-
energetic filled bands. The spin orbit coupling in conjunction with the planar Cu-O bonds of the $d_{x^2-y^2}$ orbital states can cause planar off-diagonal spin pairings resulting in an anti-
ferromagnetically ordered spin state aligned within the CuO$_2$ plane, as experimentally
found (see Section 5). This is a crucial difference to the ad hoc assumptions within other
theoretical approaches where the antiferromagnetic spin couplings are generally attributed
to half-filled valence bands [1, 3],[5, 6], [13]–[16]. The renormalized half-filled valence bands
$I_1$, $I_2$ in Fig. 1, however, prove to have no antiferromagnetic couplings (see Section 5). The
same argumentation as given for the $d_{x^2-y^2}$ orbital states is also applicable for the low
energetic band states with $d_{x^2}, s$ orbital character, but the stronger localization of the $d_{x^2}$ orbital within the $x, y$ plane leads only to small renormalization influences of the Cu-O
bonds and in combination with the additionally existing $d_{x^2}, s$ hybridizations a pronounced
two-dimensionally antiferromagnetic ordering of the spins can not be expected.

5. The dynamic quantum states of the undoped CuO$_2$ plane
In contrast to filled bands, disproportionate populations will occur within the half-
filled one-dimensional valence bands $I_1$ and $I_2$. By closed shell Hartree-Fock calculations
it could be shown that for the case of half filled bands the electrons are associated to one
of the two different lattice sites in Fig. 1, leading to a charge and bonding fluctuation state
(CBF) [7]. The $−$ copper site in Fig. 1 is assumed to be populated which corresponds
simultaneously to a higher electronic charge of this site. In a previous paper it was
deduced [7] that a negative electronic correlation energy per copper atom $U_{\text{eff}}$ in the
order of $−1\text{eV}$ per copper atom or even higher is generated, if the electronic system is
transformed into a state of disproportionate electron densities, favouring the occupation of the $−$ copper site. Consequently, the electrons are driven to populate exclusively one
of the two Hilbert subspaces in $I_1$ and $I_2$, i.e. per definition the $[−]I_1−\frac{\Delta \rho_{\perp}}{2}$ and $[−]I_2+\frac{\Delta \rho_{\parallel}}{2}$
subspace. Under this condition, the subspaces $[−]I_1−\frac{\Delta \rho_{\perp}}{2}$ and $[−]I_2+\frac{\Delta \rho_{\parallel}}{2}$ comprehend the
two spin functions $\chi_+$ and $\chi_−$ in their basis representation, realizing a diagonal spin
compensation. In this case, the correlation energy $U_{\text{eff}}$ has to be compared with half of the
antiferromagnetic correlation energy per copper atom $U_{\text{mag}}=4\cdot U_{S(-)S(+)}$ for the case of a
filled band, with $U_{S(-)S(+)}$ representing half of the antiferromagnetic spin correlation energy
between the spin $S(−)$ at the $+$ copper sites and the opposite spin $S(−)$ at the $−$ copper
site, i.e $U_{\text{mag}}$ represents the energy lowering under off-diagonal populations/off-diagonal
spin compensations in comparison to diagonal populations/diagonal spin compensations of a filled band. Although the energy $U_{S(-)S(+)}$ is not well defined at the moment the highest possible value at all can be estimated, which corresponds to states with Cu $−0$
bonding character. Néel temperatures were measured in the order of 300 K (25.85 meV)
for systems with single CuO$_2$ planes and experimental values for an antiferromagnetic
coupling constant $J$ were deduced to be in the range of 120-165 meV [20] (see also
Table I in Ref. [3]), for different high-$T_c$ materials. These values lead to the general
condition $|U_{\text{eff}}|> |U_{\text{mag}}|$. Therefore, the tendency to populate the $−$ copper site
suppresses the equal population of the $−$ and $+$ copper sites for half-filled valence bands.

The coupling constants $J$ in Ref. [20] are related to a Heisenberg antiferromagnet with $S(+)=$1/2,
which proves to be too large in relation to the measured Néel temperatures. A Heisenberg antiferromagnet of the here deduced filled band states with off-diagonal spin compensation requires $S(+)=$1. However, the experimentally deduced magnetic moments per copper atom are only $\mu \approx 0.2$–0.5 $\mu_B$ [21]. The lowered magnetic moments can be attributed to the annihilation of the spin-up and spin-down spin densities within the oxygen parts of the two spin functions $\chi_+$ and $\chi_−$ which are separately associated to the neighboured $−$ $\text{Cu}$ and $+\text{Cu}$ site.
In conclusion, a diagonal spin compensation occurs within the half-filled one-dimensional valence band states in $I_i, I_j$, which prevents an antiferromagnetic coupling within these bands (Fig. 3,(1)). This remains also valid for all populations of the bands $I_i, I_j$ below half-filling.

The CBF state as given in Fig. 1 is, however, not stable in time, but is driven to turn into its mirror symmetric state CBF$^*$, caused by the induced lattice deformations that create an additionally positive time dependent potential $V^{deff}(t)$ [8]. The influence of this time dependent positive potential has to be minimized which can be achieved by the solution of the equation of motion

$$i\hbar \frac{dC_i(t)}{dt} = \sum_{j=1,1} H_{ij}(t)C_j(t)$$

with the Hamilton matrix elements $H_{ij}(t)$ and the coefficients $C_j(t)$, which are time dependent, where $1 = \text{CBF}$ and $1 = \text{CBF}^*$. The complete time behaviour can be assigned to time dependent off-diagonal matrix elements $H_{ij}(i \neq j)$ and assuming time stationary diagonal Hamiltonians $H_{ij}(i = j)$ [8]. During the CBF $\leftrightarrow$ CBF transitions $U_{eff}$ is rising but in opposite direction the deformation potential $V^{deff}(t)$ (phonon excitations) will be reduced. Under these conditions an energetic minimum is created which can be given by the minimum conditions

$$\frac{dH_{11}}{dt'} = 0 \quad \text{and} \quad \frac{dH_{11}}{dt''} = 0.$$

with respect to the time coordinates $t', t''$. This leads to $t_{min}' = t_o + \tau_{\text{CBF}}$ and $t_{min}'' = t_o + \tau_{\text{CBF}} + \tau_{\text{CBF}^*} = t_o + 2\tau_{\text{CBF}}$, with $t_o$ the time zero point and $\tau_{\text{CBF}}$ as well as $\tau_{\text{CBF}^*} = \tau_{\text{CBF}}$ the lifetimes of the CBF and CBF state, respectively. Identical quantum states and time independent amplitudes will occur for

$$t = n \cdot 2\tau_{\text{CBF}} \quad (n: \text{integer}).$$

In conclusion, the time independent quantum states $\Psi_{\text{CBF}}$ and $\Psi_{\text{CBF}^*}$, which reflect the total wave function of all the occupied one-dimensional subspaces $\{I_i, I_j\}$, form the basis states for a stationary but dynamic quantum state $\Psi_{\text{DCBF}}$ which is characterized by an internal time scale, the eigentime $\tau_{\text{eigDCBF}} = 2\tau_{\text{CBF}}$. Hence, the quantum state $\Psi_{\text{DCBF}}$ is a dynamic time-quantized but stationary quantum state with an internal time periodicity determined by $\tau_{\text{eigDCBF}} = 2\tau_{\text{CBF}}$, i.e. $\Psi_{\text{DCBF}}$ is strongly deterministic in time within the time period $\tau_{\text{eigDCBF}}$.

The transformation of the $\Psi_{\text{CBF}}$ state to the $\Psi_{\text{CBF}^*}$ state, and vice versa, means that the particular one-dimensional states within $I_i$ and $I_j$ of each of the two arrays in Fig. 1 has to be transformed in the same way. This can be reached by the transformation of the Hilbert subspaces according to

$$\mathcal{H}_{I_i, -\alpha_{ij}/2} \leftrightarrow \mathcal{H}_{I_i, +\alpha_{ij}/2},$$

$$\mathcal{H}_{I_j, +\alpha_{ij}/2} \leftrightarrow \mathcal{H}_{I_j, -\alpha_{ij}/2}.$$
and
\[
\mathcal{H}_{I_i, + \Delta r_i/2} \leftrightarrow \mathcal{H}_{I_i, - \Delta r_i/2} \quad (13)
\]
\[
\mathcal{H}_{I_j, - \Delta r_j/2} \leftrightarrow \mathcal{H}_{I_j, + \Delta r_j/2} \quad (14)
\]
Eqs. (10) and (13) as well as Eqs. (11) and (14) describe the transformation within the states \( I_i \) and \( I_j \), respectively. Effectively, the coordinate systems which are related to the periodically local states \( [\pm u_k] \) have been exchanged, represented by the shifts \( \pm \Delta r_i/2 \). During the transformation of the Hilbert spaces, the electrons which were initially placed at the \( - \) lattice site are transferred to the initial \( + \) lattice site, i.e. the CBF state is inverted. The electron transformation between the \( - \) and \( + \) lattice sites cannot be described by a simple electron promotion but an exchange of the quantum states is necessary, because the local functions \( [\pm u_k] \) depend on the population of the states themselves in a self-consistent manner. Therefore, the ordinary conception of static Hilbert spaces commonly underlying quantum theoretical descriptions has to be replaced by the assumption of Hilbert spaces being periodically transformed in time.

The CBF \( \leftrightarrow \text{CBF} \) transitions have to occur as fast as possible, because the effective electronic correlation energy \( U_{\text{eff}} \) possesses its minimum value within the two particular CBF configurations and it is rising proportional to the time period of the existence of an intermediary state between the CBF and CBF state. If it was assumed that the particular one-dimensional states \( I_i, I_j \) are uncorrelated in time among themselves, the CBF \( \leftrightarrow \text{CBF} \) transitions would be based on individual transitions within the particular one-dimensional states \( I_i, I_j \). If at all possible, such uncorrelated transitions would be time consuming leading to a large increase of \( U_{\text{eff}} \). Over and above that, the CBF \( \leftrightarrow \text{CBF} \) transitions would be merely the result of a mean field behaviour where the electrons within the existing CBF state, adopting the lowest \( U_{\text{eff}} \) value, would tend to prevent the transition to the other CBF state. All in all, the transformations of Eqs. (10)–(14) will be only conceivable, if there is a time correlation between the particular one-dimensional states \( I_i, I_j \). This can be realized by a time-correlated exchange of paired Hilbert spaces related to \( I_i \) and \( I_j \) according to the vertical arrows in Eqs. (10)–(14). Physically this corresponds to an exchange of the coordinate systems, i.e. an exchange of the \( \pm \Delta r_i/2 \) coordinates, between the corresponding Hilbert spaces. The dynamically paired Hilbert spaces \( \mathcal{H}_{I_i, - \Delta r_i/2} \) and \( \mathcal{H}_{I_j, + \Delta r_j/2} \) as well as \( \mathcal{H}_{I_i, + \Delta r_i/2} \) and \( \mathcal{H}_{I_j, - \Delta r_j/2} \) are definitely entangled in space and time in this case. The time-periodic exchange coupling between these entangled Hilbert spaces leads to new dynamic but stationary quantum states with an inherent time scale \( \tau_{\text{DCBF}} \).

For strong correlated identical systems one can define an antisymmetric tensor product \( \mathcal{H}_1 \otimes_a \mathcal{H}_1 = \mathcal{H}_1^{(2)} \) for two coupled identical Hilbert spaces \( \mathcal{H}_1 \) according to the two particle state space \( \mathcal{H}_1^{(2)} = \{ (\varphi \otimes_a \psi) : \varphi \in \mathcal{H}_1, \psi \in \mathcal{H}_1 \} \) with \( (\varphi \otimes_a \psi) = \frac{1}{2}(\varphi \otimes \psi - \psi \otimes \varphi) \) [22]. Correspondingly, it results for the one-dimensional states in \( I_i, I_j \)
\[
\mathcal{H}_{I_i} \otimes_a \mathcal{H}_{I_i} \quad \text{and} \quad \mathcal{H}_{I_j} \otimes_a \mathcal{H}_{I_j}, \quad (15)
\]
where the two Hilbert spaces have the same basis representation, but different coordinate systems. The tensor products of Eq. (15) can exist for arbitrary \( i, j \) values, i.e. a two-dimensional entanglement of all the one-dimensional states in space and time with respect...
to their coordinate systems is reached for a given polarization direction. If a Hilbert space \( H_{DCBF} \) is defined as a compound system consisting of subspaces of Eq. (15) and if the particular subspaces of Eqs. (10)–(14) are considered to be basis states, normalized antisymmetric products can be defined according to

\[
\begin{align*}
\mathcal{H}_{I_i,-\frac{\Delta r_i}{2}} \otimes_{a} \mathcal{H}_{I_j,\frac{\Delta r_j}{2}} &= \frac{1}{\sqrt{2}} \left( \mathcal{H}_{I_i,-\frac{\Delta r_i}{2}} \otimes \mathcal{H}_{I_j,\frac{\Delta r_j}{2}} - \mathcal{H}_{I_i,\frac{\Delta r_i}{2}} \otimes \mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \right) \\
\mathcal{H}_{I_i,\frac{\Delta r_j}{2}} \otimes_{a} \mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} &= \frac{1}{\sqrt{2}} \left( \mathcal{H}_{I_i,\frac{\Delta r_i}{2}} \otimes \mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} - \mathcal{H}_{I_i,-\frac{\Delta r_i}{2}} \otimes \mathcal{H}_{I_j,\frac{\Delta r_j}{2}} \right)
\end{align*}
\]

(16)

(17)

Here, the Hilbert subspaces appear as state vectors, because the antisymmetric product is the result of the exchange of the Hilbert subspaces as a whole, which corresponds to an exchange of the coordinate systems. In the state representation of Eqs. (16) and (17) the time, i.e. the eigenstate \( \tau_{\epsilon_{DCBF}} \), does not occur, i.e. these antisymmetric products represent time integrated states over the time period \( \tau_{\epsilon_{DCBF}} \). This is in opposite to the usually applied principle in quantum mechanics which postulates an objective simultaneity of the states. The eigenstate \( \tau_{\epsilon_{DCBF}} \) proves to be a real existing and quantized quantity within a stationary DCBF state, but within the pair states given by Eqs. (16) and (17) the eigentime \( \tau_{\epsilon_{DCBF}} \) occurs merely as a hidden variable. However, the occurrence of the pair states of Eqs. (16) and (17) themselves is strongly coupled to the existence of the eigentime \( \tau_{\epsilon_{DCBF}} = 2 \cdot \tau_{CBF} \). The two product wave functions given by Eqs. (16) and (17) must have the same probability, i.e. they exist for the same lifetime \( \tau_{CBF} \), including a complementary deterministic time behaviour within the period \( \tau_{\epsilon_{DCBF}} \), and Eqs. (16) and (17) are produced exclusively by exchange processes of quantum states. All pair states represented by Eq. (15) have the same probability to form all possible \( I_i, I_j \) pairings, where possible pairings at a given time \( t \) can perpetually be exchanged without any mass inertia effect, meaning these pair exchanges can occur infinitely fast. So, the complete compound state is given by the Tensor products

\[
\begin{align*}
\mathcal{H}_{I_1} \otimes_{a} \mathcal{H}_{I_1} \otimes_{a} \cdots \otimes_{a} \mathcal{H}_{I_n} \otimes_{a} \mathcal{H}_{I_n} \otimes_{a} \mathcal{H}_{I_n} = \mathcal{H}_{I_1} \otimes_{a} \mathcal{H}_{I_1} \otimes_{a} \cdots \otimes_{a} \mathcal{H}_{I_n} \otimes_{a} \mathcal{H}_{I_n} \otimes_{a} \mathcal{H}_{I_n}
\end{align*}
\]

(18)

for the two polarization directions \( x \) and \( y \), respectively. The number \( n \) corresponds to the half of the one-dimensional states of a given polarization direction. The state vectors within the antisymmetric tensor product of Eq. (18) result from the simultaneous exchange of all particular Hilbert subspaces \( \mathcal{H}_{I_i}, \mathcal{H}_{I_j} \). There are \( n! \) independent possibilities of simultaneous exchanges of paired Hilbert subspaces which have to be included in Eq. (18), i.e. Eq. (18) is restricted to these \( n! \) possibilities of simultaneous pair exchanges.

Horizontal transformations of the Hilbert subspaces of Eqs. (10)-(14), depicted by horizontal arrows, correspond to transformations of the individual coordinate systems of the Hilbert subspaces along the particular one-dimensional states where the electrons follow the movements of the coordinate systems. The transformation/exchange of the quantum states can be assumed to be infinitely fast, however, the electron movement between two lattice sites is finite in time, which is given by the mass inertia of the transferred electrons. Hence, the exchange of the Hilbert subspaces occurring with a finite transition time \( t'_{tr} \) has to be taken into consideration, leading to the time dependence of the definitely correlated Hilbert subspaces given by:
If a linear time dependence of the expectation values of the states during the exchange process is assumed the coefficients follow a periodic time behaviour

\[
c_1(t') = \begin{cases} 
  1 & \text{for } t'_r/2 \leq t' < \tau_{CBF} - t'_r/2 \\
  \frac{\tau_{CBF} - t'}{t'_r} + \frac{1}{2} & \text{for } \tau_{CBF} - t'_r/2 \leq t' < \tau_{CBF} + t'_r/2 \\
  0 & \text{for } \tau_{CBF} + t'_r/2 \leq t' < 2\tau_{CBF} - t'_r/2 \\
  \frac{t'_r - (2\tau_{CBF} - t'_r)}{t'_r} - \frac{1}{2} & \text{for } 2\tau_{CBF} - t'_r/2 \leq t' < 2\tau_{CBF} + t'_r/2 
\end{cases}
\]  

(24)

with \( t' = t - n \cdot \tau_{e_iCBF} \in [0, \tau_{e_iCBF}] \) and \( n \) an integer.

The representation of Eqs. (19)-(22) by a direct sum describing a superposition of quantum states without interferences within a normalized state according to Eq. (23) results from the following arguments: A superposition of quantum states which includes interference terms requires a free accessibility of the quantum states at any time \( t \), reflecting the simultaneity of superimposing quantum states, but additionally, a common Hilbert space for the superimposing states with a unified coordinate system has stringently to be established. Assuming Eqs. (16) and (17) to be basically fulfilled within every time period \( \tau_{e_iCBF} \), the time dependence of the Hilbert spaces \( \mathcal{H}_I_i(t') \), \( \mathcal{H}_{I_j}(t') \) is exclusively caused by exchanges of the coordinate systems between the one-dimensional states in \( I_i \) and \( I_j \) without any excitation, according to the vertical arrows in Eqs. (10)-(14), i.e. the Hilbert spaces are exchanged. At any time \( t' \), an electron with the amplitudes \( c_1(t') \) and \( c_2(t') \) simultaneously belongs to the two superimposing Hilbert spaces, respectively, but the requirement of identical coordinate systems is never given, which is caused by the time-deterministic and time-quantized exchange of Hilbert subspaces under maintaining of a definite coordinate relative state. As a result, interference terms of the type \( c_1(t')c_2(t')\varphi_{\Delta \varphi/2} \) disappear at any time \( t' \), which requires a direct sum of the superimposing quantum states.

The time dependence of the coordinate systems is given by the periodic time dependence of the expectation value of the point of origin \( \Delta \varphi/2 \) according to:

\[
\frac{\Delta \varphi}{2} (t') |\mathcal{H}_{I_i}\rangle = \frac{\Delta \varphi}{2} (c_2(t') - c_1(t'))
\]  

(25)

\[
\frac{\Delta \varphi}{2} (t') |\mathcal{H}_{I_j}\rangle = \frac{\Delta \varphi}{2} (c_1(t') - c_2(t'))
\]  

(26)

\[
\frac{\Delta \varphi}{2} (t') |\mathcal{H}_{I_i}\rangle = \frac{\Delta \varphi}{2} (c_1(t') - c_2(t'))
\]  

(27)

\[
\frac{\Delta \varphi}{2} (t') |\mathcal{H}_{I_j}\rangle = \frac{\Delta \varphi}{2} (c_2(t') - c_1(t'))
\]  

(28)
The individual coordinate shifts of the Hilbert subspaces correlate with phase shifts according to

\[
\Delta \theta_\parallel = \frac{2 \pi}{2a} \cdot \Delta r_\parallel = k_{i\parallel DCBF} \cdot \Delta r_\parallel = \frac{2 \pi}{\tau e_{iDCBF}} \cdot \tau_{CBF} = \omega \cdot \Delta r_\parallel
\]

or

\[
\Delta \theta_\parallel = \frac{2 \pi}{2b} \cdot \Delta r_\parallel = k_{i\parallel DCBF} \cdot \Delta r_\parallel = \frac{2 \pi}{\tau e_{iDCBF}} \cdot \tau_{CBF} = \omega \cdot \Delta r_\parallel
\]

which leads to:

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\Delta \theta_\parallel}{2}(c_2(t') - c_1(t'))
\]

(31)

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\Delta \theta_\parallel}{2}(c_1(t') - c_2(t'))
\]

(32)

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\Delta \theta_\parallel}{2}(c_1(t') - c_2(t'))
\]

(33)

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\Delta \theta_\parallel}{2}(c_2(t') - c_1(t'))
\]

(34)

for the corresponding coordinate center point phase \( \frac{\delta \theta_\parallel}{2}(t') \). The Bloch states within the time dependent Hilbert spaces given by Eqs. (19)–(22) are defined now with respect to the expectation values of the local periodic function for a given time \( t' \) according to

\[
\frac{\delta r_\parallel}{2}(t')[H_{I_1}]/\frac{\delta r_\parallel}{2}(t') = \frac{\Delta r_\parallel}{2}(c_1(t') - c_2(t'))
\]

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\Delta \theta_\parallel}{2}(c_2(t') - c_1(t'))
\]

(35)

with respect to the time dependent Hilbert space of Eq. (19) and with the expectation value of the coordinate center point \( \frac{\delta r_\parallel}{2}(t') \) as given in Eq. (25). Equivalent representations result for the Eqs. (20)–(22). Exchanged are only the local states \( \frac{\delta r_\parallel}{2}(t') \) and \( \frac{\delta \theta_\parallel}{2}(t') \), but not the exponential function of the Bloch states. Nevertheless, the exponents within the exponential part of the Bloch states are continually symmetry broken in relation to the center coordinate \( r_{c1} \) in Fig. 1. The actual phase will be given by the expectation value \( \frac{\delta \theta_\parallel}{2}(t') \) corresponding to the effective local state function \( \frac{\delta r_\parallel}{2}(t') \). Therefore, the common Bloch state will adopt the general form:

\[
\frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = \frac{\delta \theta_\parallel}{2}(t')[H_{I_1}] = e^{i(k_{i\parallel DCBF} \cdot \delta r_\parallel(t)} = e^{i(k_{i\parallel DCBF} \cdot \delta r_\parallel(t)} = e^{i(k_{i\parallel DCBF} \cdot \delta r_\parallel(t)}
\]

(36)

where \( \frac{\delta r_\parallel(t)}{2} \) is now transformed to a continuous coordinate which is related to the absolute time coordinate \( t \), taking into account that \( \frac{\delta r_\parallel(t)}{2} \) is equivalent to \( \frac{\delta r_\parallel(t)}{2} \). Conventionally, the requirement for the existence of stationary quantum states is determined by the time independence of the expectation values where the expectation
value is assumed to be instantaneously given at any time. The basis representation of the corresponding Hilbert space is then required to be time independent, which is the traditional foundation of quantum mechanics. The dynamic Hilbert space $\mathcal{H}_{DCBF}$ is formed by the paired one-dimensional subspaces according to Eqs. (15) and (18). The particular basis states, i.e. the basis of the particular subspaces $|\bar{\tau}_I (t')\rangle, |\bar{\tau}_I (t')\rangle$, $|\bar{\tau}_I (t')\rangle$, $|\bar{\tau}_I (t')\rangle$ and $|\bar{\tau}_I (t')\rangle$, are not stationary in time, as discussed, but behave strongly periodic in time according to Eqs. (19)-(24). The condition for stationarity of the states is now defined by the requirement that the expectation values must be identical for periodically equivalent times. This means, after the time interval $\tau_{edCBF} = 2\tau_{CBF}$ the basis representation of all the subspaces $\{|\bar{\tau}_I (t')\rangle, |\bar{\tau}_I (t')\rangle, |\bar{\tau}_I (t')\rangle, |\bar{\tau}_I (t')\rangle\}$ must be identically recreated. Hence, for every electron belonging to the dynamic DCBF state the eigenvalue $\tau_{edCBF}$ occurs as a conservation quantity. If the conservation of $\tau_{edCBF}$ is violated, i.e. if the strong time-periodicity of the Hilbert subspaces according to Eqs. (19)-(24) is not given, the two summands in Eq. (16) and Eq. (17) are differently weighted, i.e. pair states according to Eq. (16) and Eq. (17) do not exist. The two dimensional coupling in time of the particular one-dimensional subspaces is then broken up and, in consequence, the DCBF state can not exist.

A time dependence of the Hilbert subspaces means that the periodically local states $|\bar{-}\rangle_{k\parallel}$ and $|\bar{+}\rangle_{k\parallel}$ are time dependent corresponding to the time dependent amplitudes $c_1(t')$, $c_2(t')$ according to Eq. (35). Hence, the two one-dimensional subspaces in $I_{\parallel}$ and $\bar{I}_{\parallel}$ are strongly time-correlated multi-electronic pair states with respect to their time dependent phases $\delta\theta_{\parallel}(t')$ of Eqs. (31) - (34), but not yet in relation to the their particular $k_{\parallel}$ states.

The strongly time-periodic phase $\delta\theta_{\parallel}(t')$ prevents that this phase can appear as a free variable. The phase $\delta\theta_{\parallel}(t')$ is the only existing phase under half-filling, i.e. every accidental electronic transition from an occupied local site $|\bar{-}\rangle_{k_{\parallel}}$ to free states of the local site $|\bar{+}\rangle_{k_{\parallel}}$ is connected with changes of the phase $\Delta\theta_{\parallel}$ by $\pm \pi$ (Eqs. (3)- (6)). In consequence, every local electronic transition which is not time dependent according to the amplitudes $c_1(t')$, $c_2(t')$ is forbidden. In addition, any single electronic excitation of $k_{\parallel}$ states within the filled bands of the subspaces $\mathcal{H}_{I_{\parallel}}$ or $\mathcal{H}_{\bar{I}_{\parallel}}$ also requires electronic transitions to local states $|\bar{+}\rangle_{k_{\parallel}}$, because, the closed shell electronic state representing a diagonal spin pairing within $|\bar{-}\rangle_{I_{\parallel}}$ or $|\bar{-}\rangle_{\bar{I}_{\parallel}}$ is removed in favour to open shell states, which include off-diagonal spin pairings. Moreover, the electronic excitations are strongly local due to the large effective band mass $m_{\parallel} \rightarrow \infty$. Even if delocalized $k_{\parallel}$ excitations were assumed, the accompanied modulation of the electron density would include the population of local states $|\bar{+}\rangle_{k_{\parallel}}$, because the local states $|\bar{-}\rangle_{(R_{\parallel})}$ at the lattice sites $R_{\parallel}$ are completely filled. In either case, this requires local transitions from a filled local state $|\bar{-}\rangle_{k_{\parallel}}$ to an empty local state $|\bar{+}\rangle_{k_{\parallel}}$. In this case, the time quantization of the states given by $\tau_{edCBF}$ would be violated again.

In consequence, a hole undoped CuO$_2$ plane, representing half-filled valence bands, is an insulator as it has always experimentally been found.

6. Separated worlds – unified worlds

Symmetry broken Bloch states according to Eq. (36) implicate a commutator based on the space and impulse operator $\partial_{\parallel}$ which describes the dynamics of the
coordinate system given by

\[ [\delta \mathbf{r}_\parallel(t), \frac{\hbar}{i} \frac{\partial}{\partial (\delta \mathbf{r}_\parallel)}] = i\hbar, \]

implying the uncertainty relation

\[ \Delta(\delta \mathbf{r}_\parallel(t)) k_{|||DCBF} \cdot \Delta p_{|||DCBF} \geq \frac{\hbar}{2}, \]

with \( \Delta p_{|||DCBF} \) representing the impulse uncertainty of a dynamic coordinate system, with \( \Delta p_{|||DCBF} = 0 \) for an infinitely extended CuO\(_2\) plane. This noncommutative behaviour seems to be very surprising at first glance, because \( \delta \mathbf{r}_\parallel(t') \) is deterministic in time which should lead to a classical relation between space coordinate and impulse, i.e. one would expect \( \delta \mathbf{r}_\parallel(t) \) and \( \frac{\hbar}{i} \frac{\partial}{\partial (\delta \mathbf{r}_\parallel)} \) to commute. The phenomenon can be explained by the cyclic behaviour of the absolute time \( t, t = t' + n \cdot \tau_{|||DCBF} \) \( (\tau_{|||DCBF} \equiv \tau_{|||DCBF}) \), which leads to identical quantum states. The wave function in Eq. (36) is also cyclic with respect to the space coordinate, \( \delta \mathbf{r}(t) = \delta \mathbf{r}(t') + m \cdot (2\Delta \mathbf{r}_\parallel) \). The numbers \( m \) and \( n \) are not necessarily equivalent, but can adopt completely different values. Therefore, at a given time \( t \) the space coordinate \( \delta \mathbf{r}_\parallel(t) \) can have all values \( \delta \mathbf{r}_\parallel(t') \) modulo \( 2\Delta \mathbf{r}_\parallel \). The relative coordinate systems of the particular Hilbert spaces \( \mathcal{H}_{I_1}, \mathcal{H}_{I_2}, \mathcal{H}_{I_3}, \mathcal{H}_{I_4} \) are definitely entangled in space and time, but the absolute reference coordinate system given by \( \mathbf{r}_{|||} \) is uncertain for multiples of the time quantization \( \tau_{|||DCBF} \) and the space quantization \( 2\Delta \mathbf{r}_\parallel \), respectively. In brief, the total common quantum state encompasses a classical space-time behaviour. In view of this extraordinary characteristic, it is only justified to compare the quantum properties for times \( t \) modulo \( \tau_{|||DCBF} \) and for space coordinates \( \delta \mathbf{r}_\parallel \) modulo \( 2\Delta \mathbf{r}_\parallel \) or one considers quantum states that are integrated states over the time period \( \tau_{|||DCBF} \) as it is implicitly done in Eqs. (16) and (17). The latter requirement reflects the fact that the complete symmetry of the stationary quantum state is only defined after the time period \( \tau_{|||DCBF} \). A very important result is that at any time \( t \) the total quantum state is self-consistently separated in particular branches which are assigned to four entangled worlds, i.e. entangled by the unified time function Eq. (24), represented by four distinguishable Hilbert spaces \( \mathcal{H}_{I_1}, \mathcal{H}_{I_2}, \mathcal{H}_{I_3}, \mathcal{H}_{I_4} \). The state vectors of the complete compound system representing entangled states in Eq. (18) do not explicitly include the eigentime \( \tau_{|||DCBF} \), i.e. these state vectors describe the whole world time independent. The internal time \( \tau_{|||DCBF} \) of these state vectors exists merely as a hidden variable.

The eigentimes introduced in this paper describe periodic movements of the coordinate systems of the quantum states. This may also occur in ordinary Bloch states which adapt the crystal symmetry, i.e. states without broken U1 symmetry. A given eigentime \( \tau_{ei} \) would be related to the coordinate transformation from a given lattice site to the next one. This, however, coincides with the general uncertainty of the space coordinate which is given by \( r \) modulo \( \Delta \mathbf{r} \), where \( \Delta \mathbf{r} \) corresponds to one lattice spacing. If the coordinate system is moved from lattice site \( i \) to lattice site \( i + 1 \) the eigentime \( \tau_{ei} \) can be simultaneously defined with respect to the lattice site \( i \) or to the lattice site \( i + 1 \), or to any other lattice site \( i + k \). In consequence, the eigentime \( \tau_{ei} \) is completely indefinite. Hence, it is not appropriate to define the eigentime as a variable due to its arbitrariness. If eigentimes possibly exist they will merely occur as hidden indefinite variables which can not be detected. In consequence, the definition of non-symmetry broken Bloch states within a stationary Hilbert space when assuming an unified coordinate system is justified. The energy of the Bloch states is decisively determined by time independent transition matrix
elements $H_{i,i\pm 1}$ between the local site $i$ and the neighbouring local sites $i \pm 1$. This means, an electron belongs simultaneously to the lattice site $i$ and to the lattice sites $i \pm 1$. This simultaneity, or better the simultaneous accessibility of the states $i$ and $i \pm 1$ is not queried, if one assumes indefinite eigentimes in ordinary Bloch states, i.e. the existence of dynamic but probabilistically varying coordinate systems. The complete uncertainty of the coordinate systems guarantees a simultaneous accessibility of different lattice sites at any time $t$. Although the description of all Bloch states can be done within a common and time stationary Hilbert space every particular Bloch state could thus also possess its own coordinate system, i.e. in reality the whole quantum system could represent a hidden dynamic many-worlds quantum state. Therefore, the high-$T_c$ materials may offer the chance to get more clarity about the many-worlds problematic [23]-[26].

7. Dynamic quantum states of the CuO$_2$ plane under hole doping

The individual orthonormalized local states within the periodic functions $|u_{k||}^\pm\rangle$ and $|u_{k\bar{||}}^\pm\rangle$ of the valence states comprehend three atomic wave functions, a copper orbital state being $s, d_{x^2-r^2}, d_{z^2}$ hybridized and two oxygen orbitals $p_{\|}$ that correspond to the oxygen atoms on the left and right hand side of the included copper atom along a one-dimensional state in $I_i$ or $\bar{I}_j$. This is visualized by an assumed probability distribution $u_o^{2\|}$ in Fig. 2(a). The shifts $\pm \Delta r_{\|}$ of the particular coordinate systems within the Bloch states of the Eqs. (3)-(6) are related to the positions of the copper atoms. This reflects the fact that the left and right positioned oxygen atoms contribute to the same amount to a given local state, and the periodicity of the local states is determined by the copper positions of the corresponding copper sublattices – and +, respectively. In more common terms, the probability to find an electron at a given oxygen site is the same for all oxygen atoms at any time $t$, i.e. the left and right oxygen atoms are indistinguishable with respect to their populations. As a result, the coordinate center point is identical with the center of mass of the local state, i.e. given by the copper site within a O$_{\|}$–Cu–O$_{\bar{\|}}$ bonding-unit. This remains valid as long as the electronic states are independent/uncorrelated Bloch states, which is given for the hole undoped uncorrelated half-filled one-dimensional valence bands within the subspaces $\mathcal{H}_{I_i}, \mathcal{H}_{I_{\bar{j}}}$.

The situation will distinctly be changed if ordered topological hole states are formed, caused by reduced electronic populations, i.e. far from half-filling. An undisturbed topological hole ordering as created by the b-holes in Fig. 1 occurs for hole concentrations of $0.125 \leq n_h \leq 0.25$ holes/copper. The f-holes, occurring exclusively for $n_h > 0.125$ holes/copper, also tend to ordered states, as given in Fig. 1, but a clustering or an increased periodicity has to be expected which is inverse proportional to $n_h (< 0.25)$ holes/copper. In the subsequent considerations, infinitely extended ordered topological hole structures are considered, i.e. hole structures with periodic boundary conditions. Such extended strongly ordered topological hole states, clearly given for the b-holes within the above concentration range, are created by coherent interferences of $k_{||}^\pm, -k_{||}^\pm$ paired Bloch states according to [8]:

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\[
\varphi_{k_{||}-\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{1||} + \Delta r_{\Delta \phi_{||}}) = \sum_i c_i \left[ \varphi_{k_{||}+i[q_{||}],-\Delta \phi_{S_{||}},\Delta \phi_{||},-\Delta \phi_{S_{\perp}}} (r_{1||} + \Delta r_{\Delta \phi_{||}}) e^{i \Delta \theta_{\Delta \phi_{||}}} \right] \\
= \sum_i c_i \left[ \varphi_{k_{||},-\Delta \phi_{S_{||}},\Delta \phi_{||},-\Delta \phi_{S_{\perp}}} (r_{1||} + \Delta r_{\Delta \phi_{||}}) \right] \in \mathcal{H}_{I_{\perp},\Delta r_{\Delta \phi_{||}}} 
\] (39)

\[
\varphi_{k_{||},-\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{2||} + \Delta r_{-\Delta \phi_{||}}) = \sum_i c_i \left[ \varphi_{k_{||}+i[q_{||}],-\Delta \phi_{S_{||}},\Delta \phi_{||},-\Delta \phi_{S_{\perp}}} (r_{2||} + \Delta r_{-\Delta \phi_{||}}) e^{i \Delta \theta_{-\Delta \phi_{||}}} \right] \\
= \sum_i c_i \left[ \varphi_{k_{||},-\Delta \phi_{S_{||}},\Delta \phi_{||},-\Delta \phi_{S_{\perp}}} (r_{2||} + \Delta r_{-\Delta \phi_{||}}) \right] \in \mathcal{H}_{I_{\perp},\Delta r_{-\Delta \phi_{||}}} 
\] (40)

\[
\varphi_{k_{||},\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{3||} + \Delta r_{\Delta \phi_{||}}) = \sum_i c_i \left[ \varphi_{k_{||}+i[q_{||}],\Delta \phi_{S_{||}},\Delta \phi_{||},\Delta \phi_{S_{\perp}}} (r_{3||} + \Delta r_{\Delta \phi_{||}}) e^{i \Delta \theta_{\Delta \phi_{||}}} \right] \\
= \sum_i c_i \left[ \varphi_{k_{||},\Delta \phi_{S_{||}},\Delta \phi_{||},\Delta \phi_{S_{\perp}}} (r_{3||} + \Delta r_{\Delta \phi_{||}}) \right] \in \mathcal{H}_{I_{\perp},\Delta r_{\Delta \phi_{||}}} 
\] (41)

\[
\varphi_{k_{||},-\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{4||} + \Delta r_{-\Delta \phi_{||}}) = \sum_i c_i \left[ \varphi_{k_{||}+i[q_{||}],-\Delta \phi_{S_{||}},-\Delta \phi_{||},\Delta \phi_{S_{\perp}}} (r_{4||} + \Delta r_{-\Delta \phi_{||}}) e^{i \Delta \theta_{-\Delta \phi_{||}}} \right] \\
= \sum_i c_i \left[ \varphi_{k_{||},-\Delta \phi_{S_{||}},-\Delta \phi_{||},\Delta \phi_{S_{\perp}}} (r_{4||} + \Delta r_{-\Delta \phi_{||}}) \right] \in \mathcal{H}_{I_{\perp},\Delta r_{-\Delta \phi_{||}}} 
\] (42)

For the + lattice sites equivalent states result correspondingly to:

\[
\varphi_{k_{||},-\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{5||} + \Delta r_{\Delta \phi_{||}}) \in \mathcal{H}_{I_{\perp},\Delta r_{\Delta \phi_{||}}} 
\] (43)

\[
\varphi_{-k_{||},-\Delta \phi_{S_{||}} \Delta \phi_{||}-\Delta \phi_{S_{\perp}}} (r_{6||} + \Delta r_{-\Delta \phi_{||}}) \in \mathcal{H}_{I_{\perp},\Delta r_{-\Delta \phi_{||}}} 
\] (44)
\[ \psi_{\pm, \Delta \Phi_{\parallel}} (r^\perp_{\parallel} + \Delta r_{\pm, \Delta \Phi_{\perp}}) \in \mathcal{H}_{\parallel, \pm, \Delta \Phi_{\perp}} \]  
\[ \psi_{-k, \Delta \Phi_{\parallel}} (r^\perp_{\parallel} + \Delta r_{-k, \Delta \Phi_{\perp}}) \in \mathcal{H}_{\parallel, -k, \Delta \Phi_{\perp}} \]

All these states are related to the b-holes in Fig. 1. Here \( \parallel \) means parallel to the polarization direction of the b-hole structure, i.e. along the one-dimensional electronic states, and \( \perp \) perpendicular to that. Every periodic b-hole structure along a particular line \( I_y \) or \( I'_y \) in Fig. 1 can be considered as a superposition of two phase shifted periodic b-hole substructures, each having a periodicity of \( 8b \) but a relative shift to each other of \( 3b \). This leads to the four coherent states given by Eqs. (39)-(42), as well as Eqs. (43)-(46) [8]. The phases \( \pm \Delta \Phi_{\parallel} \) are spectral phases of U1 symmetry broken one-dimensional Bloch states, parallel to the b-hole polarization direction along \( I_y, I'_y \) and \( \pm \Delta \Phi_{\perp} \) are the relative center phases of the one-dimensional Bloch states perpendicular to the b-hole polarization direction. The absolute assignment between the \( k_{\parallel} \) wave vectors and the sign of the phases \( \pm \Delta \Phi_{\parallel} \) is arbitrarily chosen. For the b-hole state in Fig. 1 one obtains: \( k_{\parallel} = (0, k_y), q_{\parallel} = (0, q_y) \) and \( r_{\parallel} = (0, y) \). The coherence conditions are defined both by the spectral phases \( \pm \Delta \Phi_{\parallel} \), the relative center point phases \( \pm \Delta \Phi_{\perp} \) as well as the absolute center point phase \( \Phi_{\perp} \), defining the relative and absolute local positions of the particular b-hole states along the \( y \) direction in Fig. 1, and particularly by the fundamental excitation wave vector \( q_{\parallel} \). If the b-holes are polarized in \( y(b) \) direction \( q_{\parallel} = (0, 2\pi \frac{8}{77}) \) is given and for \( x(a) \) directed b-hole states \( q_{\parallel} = ( \frac{2\pi}{55}, 0) \) results.

A coherent superposition of Bloch states according to Eqs. (39)-(46) leads to electronic density modulations along the periodically local states \( u_{k_{\parallel}} \) and \( u_{k_{\parallel}} \). If the local states were continuously given by the local state function \( u_{o_{\parallel}} \) in Fig. 2(a), the hole density would symmetrically be distributed at the given copper site, i.e. the two neighboured oxygen atoms would equivalently be involved. This would represent quantum states corresponding to the hole undoped case. However, the topological holes are exclusively assigned to a particular oxygen site as depicted in Fig. 1 [7], i.e. a topological hole state is assigned to the left or right oxygen atom of a given bonding-unit \( O_{\parallel} - Cu - O_{\parallel} \). Therefore, a coherent superposition of Bloch states with the fundamental excitation vector of \( q_{\parallel} = (0, 2\pi \frac{8}{77}) \) is a necessary but not a sufficient condition for representation of the topological b-hole structure. A splitting of the local state \( u_{o_{\parallel}} \) into two parts which are separately associated to the left and right oxygen atom has additionally to occur. Then, the local state \( u_{o_{\parallel}} \) is represented by \( u_{o_{\parallel}} = (u_{R_{\parallel}, \Delta r_{\parallel}} + u_{L_{\parallel}, -\Delta r_{\parallel}}) \) with the local wave functions \( u_{R_{\parallel}, \Delta r_{\parallel}} \) and \( u_{L_{\parallel}, -\Delta r_{\parallel}} \) which are associated to the left and right oxygen site, respectively. Once again for the sake of clarity, the local function \( u_{o_{\parallel}} \) is not changed but it is merely split into two parts \( u_{R_{\parallel}, \Delta r_{\parallel}} \) and \( u_{L_{\parallel}, -\Delta r_{\parallel}} \) which are associated to different coordinate systems given by the shifts \( \Delta r_{\parallel} \) and \( -\Delta r_{\parallel} \), respectively. There is not a real renormalization of the quantum state \( u_{o_{\parallel}} \). Hence, one has currently to start from \( u_{o_{\parallel}} \) if the electron density distribution is discussed, as it is done below. The separation of the local states \( u_{o_{\parallel}} \) into the particular states \( u_{R_{\parallel}, \Delta r_{\parallel}} \) and \( u_{L_{\parallel}, -\Delta r_{\parallel}} \) without a real renormalization of \( u_{o_{\parallel}} \) means that the transformation of the local state function requires no additional energy, i.e. the transformation reflects merely a symmetrically splitting of the existing coordinate system into two subsystems which are relatively shifted to each other by \( \Delta r_{\parallel} \). Such coordinate transformations reflect merely a topological polarization of the local state functions. This fact will be very significant for
Figure 2. (a) Visualization of local wave functions (density distributions) at a given copper site. In the hole undoped case, the local function $u_o$ is centered at the copper site including two neighboured oxygen orbitals which are not differentiated (full line). If periodic topological hole states exist, the local function $u_o$ is split up into two oxygen centered particular local states $u_{L,-\Delta r}$ and $u_{R,\Delta r}$ (triangles, circles). (b) The local state function $u_o$ is split into the local states $u_{L,-\Delta r}$ and $u_{R,\Delta r}$ which implicates a coordinate transformation of $-\Delta r$ and $\Delta r$, respectively. This leads to a transformation of the coordinate systems of the particular Hilbert subspaces which were initially centered at the copper sites to new coordinate systems, now, being centered at the oxygen sites (vertical dashed lines).

Various unusual properties of high-$T_c$ materials. Principally the same considerations as given for the b-hole system can be applied to a periodically ordered f-hole system. However, broken periodicity of the topological hole states have to be included for $n_h < 0.25$ holes/copper.

The splitting of the periodically local functions $u_{k||}$ and $u_{k\perp}$ into two sets of periodic wave functions which are locally separated by $\Delta r_{||} = a$ or $\Delta r_{\perp} = b$ makes it possible that only one oxygen atom left or right to a copper atom in a $O_{||} - Cu - O_{\perp}$ bonding-unit is predominantly involved in the spectral superposition of the Bloch states according to Eqs. (39)-(46). Only under these conditions the periodic hole structures along the particular one-dimensional states $I_i, I_j$ in Fig. 1 can be realized by a constructive interference of Bloch states.

The periodic lattices, defining the periodically local states $u_{k||}$ and $u_{k\perp}$, which were initially defined by the copper sites, are now associated to the neighbouring oxygen sites that belong either to the $-$ or $+$ copper sites, respectively, as depicted in Fig. 2(b). These transformations define the periodically local states of Eqs. (39)-(46). The periodically local states of Eqs. (39)-(46) are causally related to the spectral phases $\Delta \phi_{||}$ and $-\Delta \phi_{\perp}$ which result from the formation of coherent Bloch states of $k, -k$ paired electron states, which create the topological hole orders of Fig. 1 [8]. Hence, the redefined coordinate systems in Eqs. (39)-(46) being related to the phases $\Delta \phi_{||}$ or $-\Delta \phi_{\perp}$ correspond to new relative states of the particular coordinates and phases according to:

$$\Delta r_{-\Delta \phi_{||}} = 0 \quad (\text{modulo } 2\Delta r_{||})$$  \hspace{1cm} (47)

$$\Delta r_{\Delta \phi_{||}} = -\Delta r_{||} \quad (\text{modulo } 2\Delta r_{||})$$  \hspace{1cm} (48)
and

\[ \Delta \theta - \Delta \phi = 0 \quad \text{(modulo } 2\pi) \]  
\[ \Delta \theta \Delta \phi = -\pi \quad \text{(modulo } 2\pi) \]  

with respect to the original coordinate system given by \( r_o \), visualized by the vertical dashed lines in Fig. 2(b). Equations (47)-(50), originating from the shifts of the particular coordinate systems, represent new coordinate systems for the transformed states within the DCBF state. Under these conditions, the initial Hilbert spaces are transformed as follows:

\[
\begin{align*}
\mathcal{H}_{I_i, -\Delta r ||} &\Rightarrow \mathcal{H}_{I_i, -\Delta r ||}, \\
\mathcal{H}_{I_i, 0} &\Rightarrow \mathcal{H}_{I_i, 0}, \\
\mathcal{H}_{I_i, +\Delta r ||} &\Rightarrow \mathcal{H}_{I_i, +\Delta r ||} = -\Delta r || \\
\mathcal{H}_{I_i, +\Delta r ||} &\Rightarrow \mathcal{H}_{I_i, -\Delta r ||} \equiv -\Delta r || \\
\end{align*}
\]  

and equivalently for \( I'_i \). If the b-hole state is assigned to the one-dimensional states \( \bar{I}_i \), it results

\[
\begin{align*}
\mathcal{H}_{\bar{I}_i, -\Delta r ||} &\Rightarrow \mathcal{H}_{\bar{I}_i, -\Delta r ||}, \\
\mathcal{H}_{\bar{I}_i, 0} &\Rightarrow \mathcal{H}_{\bar{I}_i, 0}, \\
\mathcal{H}_{\bar{I}_i, +\Delta r ||} &\Rightarrow \mathcal{H}_{\bar{I}_i, +\Delta r ||} = -\Delta r || \\
\mathcal{H}_{\bar{I}_i, +\Delta r ||} &\Rightarrow \mathcal{H}_{\bar{I}_i, -\Delta r ||} \equiv -\Delta r || \\
\end{align*}
\]  

However, the dynamics of the DCBF state is further on related to center coordinates with respect to the copper sites, because the time dependence of the split Hilbert spaces is still related to \( u_{o||} \) (see below). Therefore, the time dependence of the coefficients \( c_1(t') \), \( c_2(t') \) of the DCBF state is conserved and only the relative coordinate systems of the split Hilbert subspaces are different, i.e \( \tau_{\text{c,DCBF}} \) is unchanged and still represents a conservation quantity for all transformed Hilbert subspaces in Eqs. (51)-(56).

The dynamics within the DCBF state is based on the exchange of the relative coordinates \( \Delta r || \) and \( \Delta r = -\Delta \phi || \) as well as the relative phases \( \Delta \theta \Delta \phi || \) and \( \Delta \theta - \Delta \phi || \) which must be strongly symmetric in time. Every exchange has to obey to a strong symmetric time function as it is given in Eq. (24), in order to realize the pair symmetry within Eqs. (16) and (17). In addition, only a symmetric splitting of the initial coordinates on the left side of Eqs. (51),(53),(54) and (56) by \( \pm \frac{\Delta r ||}{2} \), leading to the Hilbert subspaces on the right side, may conserve the relative state quantization in space of the particular Hilbert subspaces. In this case, effectively a horizontal exchange of the local state functions.
\[ |u\rangle_{R, \Delta r_1} \text{ and } |u\rangle_{L, -\Delta r_1}, \text{ as well as } |u\rangle_{R, -\Delta r_1} \text{ and } |u\rangle_{L, \Delta r_1} \] of neighboured oxygen sites in \( I_y \) and \( \bar{I}_y \) in Fig. 1 occurs, which corresponds to a global shift of the center point coordinate \( \mathbf{r}_{o_1} \) by \( \Delta r_1 = b \). Altogether, the transformed coordinate systems of the particular Hilbert subspaces are exclusively located at the oxygen sites (Fig. 2b). Therefore, the center points of the coordinate systems of the local function \( u_{o_1} \) and the particular local functions \( u_{R, \Delta r_1} \), \( u_{L, -\Delta r_1} \) are localized at the copper sites and oxygen sites, respectively, but nowhere else.

In conclusion, the total \( \mathbf{k}_{||} \) space within the one-dimensional bands \( I_y, \bar{I}_y \), already differentiated with respect to the \(-\) and \(+\) copper sites, is additionally split up into two subspaces having coordinate systems which are relatively shifted to each other by \( \Delta r_1 \) according to Eqs. (51), (53), (54) and (56). This results in new basis representations of the particular Hilbert subspaces which possess local states that are associated to one of the two oxygen sites within a \( O|| - Cu - O|| \) bonding-unit. Uncorrelated one-electron states, usually given for half-filled bands, continuously belong to the non-differentiated local states \( u_{o_1} \), which are associated to the copper sites. Periodic topological hole orders as given in Fig. 1 are connected with \( \mathbf{k}_{||}, -\mathbf{k}_{||} \) pair states \([8]\). The coherent pair excitations lead to the localized hole densities at particular oxygen atoms and in consequence to the separated states \( u_{R, \Delta r_1} \) and \( u_{L, -\Delta r_1} \). Nevertheless, one has further on to start from \( u_{o_1} = u_{R, \Delta r_1} + u_{L, -\Delta r_1} \) if the electron density distribution is considered, because \( u_{o_1} \) is not renormalized as mentioned above. The electron density \( u_{o_1}^2 \) is separated into three parts \( u_{R, \Delta r_1}^2 + 2u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} + u_{L, -\Delta r_1}^2 \). The diagonal parts \( u_{R, \Delta r_1}^2 \) and \( u_{L, -\Delta r_1}^2 \) are definitively associated to the coordinate systems which result from the splitting and the shifts \( \Delta r_1 \) and \(-\Delta r_1 \) of the initial coordinate system, respectively. The off-diagonal term \( 2u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} \) can not exist, because there is not an unambiguously defined coordinate system. Only off-diagonal states \( u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} \) or \( u_{R, -\Delta r_1} \cdot u_{L, -\Delta r_1} \) can really exist. If the local state \( u_{R, \Delta r_1} \) is populated the local state \( u_{L, -\Delta r_1} \) is also particularly involved, and if the local state \( u_{L, -\Delta r_1} \) is populated the state \( u_{R, -\Delta r_1} \) is particularly involved, too. In a dynamic picture this means, the off-diagonal terms \( 2u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} \) are dynamically associated/transformed to the product of the local state functions \( u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} \) or \( u_{R, -\Delta r_1} \cdot u_{L, -\Delta r_1} \). This implicates that these off-diagonal density terms may be exchanged between the two local state functions \( u_{R, \Delta r_1} \) and \( u_{L, -\Delta r_1} \) in a probabilistic manner. The phases \( \Delta \theta_{\Delta \phi_1} \) and \( \Delta \theta_{\Delta \phi_1 - \Delta \phi_1} \) in Eqs. (39)-(46) which are related to the \( \mathbf{k}_{||} \) states within the Bloch functions, however, are not probabilistic but strongly associated to the coordinate system of \( u_{R, \Delta r_1} \) or \( u_{L, -\Delta r_1} \). The additional probabilistic exchange coupling between the local state functions \( u_{R, \Delta r_1} \) and \( u_{L, -\Delta r_1} \) guarantees the conservation of an electronic coupling between these two local functions, which is a precondition for conductivity. These exchanges of the coordinate systems may be infinitely fast, because no mass transfer occurs, i.e. there is only an exchange of information. Each of the local functions \( u_{R, \Delta r_1} \) and \( u_{L, -\Delta r_1} \), which obey to a strongly deterministic time function with respect to the dynamics of their coordinate systems according to Eq. (24), additionally include a probabilistic off-diagonal part, i.e. \( u_{R, \Delta r_1} \) creates \( u_{L, -\Delta r_1} \) parts and \( u_{L, -\Delta r_1} \) creates \( u_{R, \Delta r_1} \) parts with the coordinate systems being conserved, respectively. The probability to find \( u_{R, \Delta r_1} \cdot u_{L, -\Delta r_1} \)
or \( u_{L, \Delta_r} \) will depend on the relative strength of polarization caused by the local states \( u_{R, \Delta_r} \) and \( u_{L, -\Delta_r} \), i.e. it will be proportional to the relative amplitudes of \( u^2(r, \Delta_r) \) and \( u^2(r, \Delta_r) \) at a given position \( r \). Hence, the total effective local density associated to the right and left oxygen atoms can be represented by

\[
\tilde{u}^2_{R, \Delta_r}(r) = \frac{u^2}{R, \Delta_r}(r) + 2 \cdot \frac{u}{R, \Delta_r}(r) \cdot \frac{u}{L, \Delta_r}(r) \\left( \frac{u^2}{R, \Delta_r}(r) + \frac{u^2}{L, \Delta_r}(r) \right), \tag{57}
\]

\[
\tilde{u}^2_{L, -\Delta_r}(r) = \frac{u^2}{L, -\Delta_r}(r) + 2 \cdot \frac{u}{R, -\Delta_r}(r) \cdot \frac{u}{L, -\Delta_r}(r) \\left( \frac{u^2}{R, -\Delta_r}(r) + \frac{u^2}{L, -\Delta_r}(r) \right), \tag{58}
\]

with

\[
u^2_{o||}(r) = \tilde{u}^2_{o||}(r) = \tilde{u}^2_{R, \Delta_r}(r) + \tilde{u}^2_{L, -\Delta_r}(r), \tag{59}
\]

described within a common coordinate system, which is graphically depicted in Fig. 2(a).

The physical background of Eqs. (57) and (58) implicate two properties which seem to be contradictory:

(a) The formation of particular Hilbert subspaces which belong to different coordinate systems.

(b) Simultaneously, the unification of the two Hilbert subspaces by common/exchanged off-diagonal densities.

This dualism of the electronic states is only comprehensible if we consider the coordinate systems as dynamically variable. A more detailed examination is given in the Appendix with the result that the local state function \( \tilde{u}_{o||}(r) \) is given by the direct sum

\[
\tilde{u}_{o||}(r) = \tilde{u}_{R, \Delta_r}(r) \oplus \tilde{u}_{L, -\Delta_r}(r), \tag{60}
\]

of particular local functions \( \tilde{u}_{R, \Delta_r}(r) \) and \( \tilde{u}_{L, -\Delta_r}(r) \) reflecting local states that are associated to the two coordinate systems, respectively, and which are on their part a direct sum of local states again being associated to the two coordinate systems (Eqs. (A.12) and (A.13)). The Hilbert space axiomatic is furthermore defined within the initial Hilbert spaces represented by the left side of Eqs. (51), (53), (54) and (56), but on the condition of varying coordinate systems. All in all the quantum states are dynamically separated in terms of relative states of the coordinate systems. This behaviour can be termed as an incomplete separation of one Hilbert space into several Hilbert subspaces, depicted by the tilde on the Hilbert space symbol on the left side of Eqs. (51), (53), (54) and (56).

The symmetric splitting of the Hilbert spaces according to Eqs. (51), (53), (54) and (56) does not influence the space-time quantization of the DCBF state, as discussed above. This means, at every time \( t' \) the two particular basis representations defined by \( u_{o||} \) and \( \tilde{u}_{R, \Delta_r} \),
\( \tilde{u}_{L, -\frac{\Delta r_1}{2}} \), which belong to the different coordinate systems given by \( r_{\|} \) and \( -\frac{\Delta r_1}{2}, -\frac{\Delta r_2}{2} \), may exist simultaneously with a certain probability. Hence, these two representations may superimpose according to

\[
u_{\|}(t') = \alpha_1(t') u_{\|} + \alpha_2(t') (\tilde{u}_{R, \frac{\Delta r_1}{2}} + \tilde{u}_{L, -\frac{\Delta r_2}{2}})
\]

with real coefficients \( \alpha_1(t'), \alpha_2(t') \) and \( \alpha_1(t') + \alpha_2(t') = 1 \). Again the direct sum occurs, because, any interference term must disappear based on the different coordinate systems which form a definite coordinate relative state. The quantum state of Eq. (61) is the most common quantum state including the completely uncorrelated local state for \( \alpha_1 = 1 \) and \( \alpha_2 = 0 \) and the strongly correlated/entangled partial local states for \( \alpha_1 = 0 \) and \( \alpha_2 = 1 \). Equation (61) with \( \alpha_1 \neq 0 \) and \( \alpha_2 \neq 1 \) may be important, for example, if correlated electron states are induced even under half-filling of the valence bands.

The fact that the transformed Hilbert subspaces according to the right sides of Eqs. (51), (53), (54) and (56) create common coordinate systems is of decisive importance. According to Fig. 2(b), the transformed local states \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) as well as \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_2}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_2}{2}}\) belong to common coordinate systems. The local states \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) as well as \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_2}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_2}{2}}\) are quasi-stationary related to each other within the absolute coordinate systems \( r_{\|} - \Delta r_{\|} \) (modulo 2\( \Delta r_{\|} \)) or \( r_{\|} \) (modulo 2\( \Delta r_{\|} \)), respectively. These local functions are synchronously changed in dependence on \( c(t') \), with respect to a common coordinate system. Therefore, electronic transitions between the local states \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) as well as \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_2}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_2}{2}}\) can occur independently on \( c(t') \), \( c_2(t') \) and the time-quantization \( \tau_{\text{BCB}} \). A probabilistic electronic transition from \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) to \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) as well as \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) to \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) or vice versa is not connected with a jump of \( \pm \Delta \theta_{\|} \). As a result, the states \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) as well as \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_2}{2}}\) and \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_2}{2}}\) can freely interfere at any time \( t \). In this case, the Hilbert subspaces \( \mathcal{H}_{I,0} \) and \( \mathcal{H}_{I,0} \) as well as \( \mathcal{H}_{I, -\Delta r_{\|}} \) and \( \mathcal{H}_{I, -\Delta r_{\|}} \) in Eqs. (51) and (53) form a common Hilbert space, respectively, which is the linear span of the corresponding subspaces, \( \text{lin}(\mathcal{H}_{I,0} \cup \mathcal{H}_{I,0}) \) and \( \text{lin}(\mathcal{H}_{I, -\Delta r_{\|}} \cup \mathcal{H}_{I, -\Delta r_{\|}}) \) (Eq. 52). Equivalent linear spans, \( \text{lin}(\mathcal{H}_{I,0} \cup \mathcal{H}_{I,0}) \) and \( \text{lin}(\mathcal{H}_{I, -\Delta r_{\|}} \cup \mathcal{H}_{I, -\Delta r_{\|}}) \) (Eq. 55), result for the one-dimensional states along \( i \) according to Eqs. (54) and (56). As a result, at any time \( t \) an electron can be transferred from the occupied local site \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\) to the empty local site \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) or from the occupied local site \([\frac{\cdot}{\cdot}]_{R, \frac{\Delta r_1}{2}}\) to the empty local site \([\frac{\cdot}{\cdot}]_{L, -\frac{\Delta r_1}{2}}\), under conservation of the phase \( \delta \theta_{\|}(t') \). In terms of the copper-oxygen bonding that means, an electron initially giving a contribution to the Cu–O bonding is transferred to the Cu–O bonding. In this way the charge center point of the electronic system is changed. In consequence, an electron transfer along the \( r_{\|} \) coordinate occurs, i.e. the electronic system becomes conducting. The formation
of common coordinate systems of particular Hilbert subspaces and the existence of the off-diagonal densities in Eqs. (57) and (58) form the basis for dynamic quantum states throughout all atomic sites along the coordinate $r_i$. The deterministic quantized time-behaviour of the quantum states given by $\delta \theta_{||}(t')$, being initially a barrier for conductivity in the hole undoped case, is bypassed, even under the continuous existence of strongly ordered topological hole states.

Under hole doping the electronic states are basically $k_{\parallel}, -k_{\parallel}$ pair states, if ordered topological hole structures as given in Fig. 1 exist, as it was previously elaborated [8]. Only pair excitations are possible within the $k_{\parallel}$ space which leave the center of mass point and charge center point unchanged. Hence, a directed electron movement can not be created within the $k_{\parallel}$ space, but will only result from unrestricted superpositions of the local states

$$ u_{L, -\frac{\Delta \tau_{||}}{2}} \text{ and } u_{R, \frac{\Delta \tau_{||}}{2}} \text{ as well as } u_{L, -\frac{\Delta \tau_{||}}{2}} \text{ and } u_{R, \frac{\Delta \tau_{||}}{2}}. $$

An impulse that is transferred to the electron pair states will only lead to a finite effective mass $m_{\parallel eff} < \infty$ of the paired charge carriers, if an electronic transition between the local states $u_{L, \frac{\Delta \tau_{||}}{2}}$ and $u_{R, \frac{\Delta \tau_{||}}{2}}$ as well as $u_{L, -\frac{\Delta \tau_{||}}{2}}$ and $u_{R, \frac{\Delta \tau_{||}}{2}}$ takes place. Principally the same considerations can be applied to a periodically ordered f-hole system. Singular topological f-holes or strongly clustered topological f-hole states, which can exist for low hole doping $0.125 < n_h < 0.25$ holes/copper [7], however, can form non-conducting regions within the CuO$_2$ plane. The same will occur for the b-holes under the condition $n_h < 0.125$ holes/copper.

In conventional superconductors, the conductivity is attributed to the phase of the superconducting condensate which appears as a free variable. The center phase $\phi_{S_{||}}$ of the topological b-hole state in Fig. 1 is strongly coupled to the center coordinate $r_{o||}$ of the CBF state. Therefore, $\phi_{S_{||}}$ is strongly determined by the center coordinate $r_{o||}$ with an uncertainty given by modulo $2\Delta r_{||}$, reflecting the periodicity of the CBF state. If the electronic system is transferred from the CBF to the CBF state the phase $\phi_{S_{||}}$ is also transferred, by $\delta \phi_{S_{||}} = \frac{2\pi}{R}$ (OR (exclusive) by $\delta \phi_{S_{||}} = -\frac{2\pi}{R}$) with respect to the b-hole state in order to occupy an equivalent next neighbour topological hole state, if the b-hole state is moved in $y(b)$ direction of Fig. 1 [8]. In addition, a phase transformation according to $\phi_{S_{||}} = \phi_{S_{||}} + \delta \phi_{S_{||}}$ during the lifetime $\tau_{CBF}$ of a CBF configuration is generally not possible, since it would be connected with a jump of $\pm \Delta \theta_{||}$ which is forbidden due to the deterministic time behaviour of $\delta \theta_{||}(t')$. In other words, such probabilistic transitions would violate the strongly deterministic space-time separation of the Hilbert spaces. Hence, the variation $\delta \phi_{S_{||}}(t')$ will be time-dependent obeying the same functionality as given for $\delta r_{||}(t')$ and $\delta \theta_{||}(t')$, i.e. $\delta \phi_{S_{||}}(t')$ is strongly deterministic in space and time, too. Thus, the phase $\phi_{S_{||}}(t')$ can not be a free variable of the b-hole condensate, and neither the phases $\pm \delta \phi_{S_{||}}(t')$ or $\pm \delta \phi_{S_{||}}(t')$ which are directly related to $\phi_{S_{||}}(t')$.

The phases $\phi_{S_{||}}, \pm \Delta \phi_{S_{||}}$ and $\pm \Delta \phi_{S_{||}}$ in Fig. 1 which characterize the b-hole state are strongly periodic in space with a periodicity of $8b$ with respect to the $I_g$ or $I'_g$ direction. In the full TR state, introduced in Ref. [8], the topological b-hole state is transferred in $y(b)$ direction (see Fig. 1) by 8 subsequent steps of one lattice spacing, if we consider the non-superconducting state, i.e. a state with IS quantum hole symmetry (see Ref. [8]). After 8 phase steps $\delta \phi_{S_{||}} = \frac{2\pi}{8}$ (OR $\delta \phi_{S_{||}} = -\frac{2\pi}{8}$) the initial b-hole state is recreated. The strong coupling of $\delta \phi_{S_{||}}(t')$ to the time function Eq. (24) of the DCBF state requires that a particular topological b-hole state as given in Fig. 1 has exclusively a lifetime of $\tau_{CBF}$ in the non-superconducting state, although the uncertainty $2n \cdot \Delta r_{||}$ (n integer) of the DCBF state would principally permit jumps of $\pm 2n \cdot \delta \phi_{S_{||}} = \pm 2n \cdot \frac{2\pi}{8}$ during the time $\tau_{CBF}$. This conclusion, i.e. the stationarity of the b-hole state during the lifetime
$\tau_{CBF}$, was justified by the correlation energy $E_C < 0$ for the b-holes [8]. However, there are even more fundamental reasons. Every transformation to a different b-hole state along $y$ coordinate changes the hole topology. Among others, this leads to variations of the quadrupolar polarizations on the copper atoms (see Ref.[7]) along the perpendicular one-dimensional states $I_x$, $I_y$ of Fig. 1. This is connected with bonding transformations $\{\text{Cu} - \text{O} \leftrightarrow \text{Cu} - \text{O}\}$ (see Fig.5(c) in Ref. [7]). As deduced here, such transformations are only allowed, if a basis representation given by $[\{-\}], \{[\{+\}]_L, \{\{+\}]_R$, exists, because the deterministic time behaviour according to Eq. (24) is violated otherwise. However, most one-dimensional subspaces $I_x$, $I_y$ carry no holes, so the basis states of these hole free one-dimensional subspaces have only $u_{\sigma|}$ character. Therefore, additional polarizations which would inevitably occur, if the b-hole state is topologically transformed in $y(b)$ direction of Fig. 1 are forbidden, i.e. the b-hole state must be stationary during the time $\tau_{CBF}$. Hence, the topological b-hole state can only be periodically changed in steps of $\tau_{CBF}$ correspondingly to the $\text{CBF} \leftrightarrow \text{CBF}$ transitions with a characteristic inherent time function similar as given by Eq. (24). This defines a stationary periodicity of the dynamic b-hole state corresponding to the eigentime $\tau_{el} = 8 \cdot \tau_{CBF}$. The dynamic b-hole state is absolutely stationary with respect to this eigentime, if a strong periodic b-hole structure is assumed and if all eight equivalent b-hole states along the $b$ direction in Fig. 1 have the same probability. However, even if the b-hole state were not stationary with respect to the eigentime, i.e. if different eigentimes would exist, a promotion into states with other eigentimes $\tau_{el}'$ can only happen if the existing periodic cycle is finished. Only under these requirements the stationarity of the Bloch states in Eqs. (39)-(46) is realized, if the phase $\phi_{S_{\parallel}}(t)$ is time dependent. Therefore, $\phi_{S_{\parallel}}(t)$ is strongly periodic in time during a given cycle defined by $\tau_{el}$. This prevents that $\phi_{S_{\parallel}}(t)$ and the stationary related phases $\pm \Delta \phi_{\parallel}$ and $\pm \Delta \phi_{\parallel}$ can exist as free variables in the hole doped ($p$-type) cuprates.

The f-hole states are strongly coupled to the b-hole state with their own intrinsic eigentimes, if the f-hole structures are strongly periodic in space. Under these conditions the same argumentation holds true for the f-hole states as discussed for the b-holes. However, different f-hole configurations are possible during the lifetime $\tau_{CBF}$ [8]. The one-dimensional states along $I'_x$ in Fig. 1 where $f_{\perp}$-holes are placed implicate a basis representation of $[\{-\}]_L, \{[\{+\}]_R$, with the possibility of $\text{Cu} - \text{O} \leftrightarrow \text{Cu} - \text{O}$ transitions. In consequence, transformations between different topological f-hole states are possible during the life time $\tau_{CBF}$, if the transformations occur within the one-dimensional bands $I_y$, $I_y'$, $I'_x$ in Fig. 1 where f-holes already exist. Altogether, the phases within a strong periodic b-hole condensate can generally not occur as free variable. The f-hole states possess a certain variability of their phases which is, however, distinctly restricted.

In consequence, the conductivity in undisturbed largely extended periodic hole states as given in Fig. 1 is exclusively realized by the phase conserving local transitions $[\{-\}]_L, \{[\{+\}]_R$, with respect to the b-holes, and also for the f-holes the restricted variability of the phases will largely constrain the conductivity on these local transitions.\footnote{This is only valid in the non-superconducting state having IS quantum hole symmetry and if polarization influences do not exist which may possibly induce $u_{\parallel} \leftrightarrow u_{L}, \{[\{+\}]_R, \{\{+\}]_R$ transitions.}

\footnote{If the topological hole states in Fig. 1 are clustered, as has to be expected for $n_a \ll 0.25$ holes/copper with respect to f-holes and for $n_a \ll 0.125$ holes/copper with regard to b-holes, electrons may be promoted or may tunnel between the clusters including not well-defined periodic phase shifts. The eigentime $\tau_{el}$}
8. The quantum state dynamics of the CuO$_2$ plane under electron doping

The dynamics of the quantum states under electron doping into the initially undoped CuO$_2^\text{-}$ plane proves to be very different to the quantum dynamic behaviour in hole doped CuO$_2^\text{2-}$ planes. This can be found in the n-type Ce based compounds NCCO and PCCO. The states which are associated to the − copper sites within the undoped CuO$_2^\text{-}$ plane belonging to $|\mathcal{H}_{I_i,-\Delta r_{ij}}\rangle$ or $|\mathcal{H}_{I_j,+\Delta r_{ij}}\rangle$ are filled, so that every additional electron has to be injected into states at the + copper site, which belong to $|\mathcal{H}_{I_i,+\Delta r_{ij}}\rangle$ or $|\mathcal{H}_{I_j,-\Delta r_{ij}}\rangle$, as depicted in Figs. 3,(2) and 3,(2'). This is connected with a reduced strength of the charge fluctuation potential within the existing CBF state. Every additional electron being injected at the + copper site compensates the charge fluctuation influence of one electron within the DCBF state which was already localized at the − copper site. If the number of additionally injected electrons at the + copper site is $n_e$, 2 · $n_e$ electrons will not any longer contribute to the exchange energy of the DCBF state. Or more precisely, the 2 · $n_e$ electrons will either contribute to off-diagonal populations as depicted in Fig. 3,(2) or to pairwise diagonal populations given in Fig. 3,(2'), both representing time independent states. Therefore, these electrons are free, i.e. they are not quantum coupled to the fluctuations of the DCBF state. The charge fluctuations of the DCBF state occur merely as a mean field influence for these electrons, but they are not quantum entangled to the dynamics of the quantum states within the DCBF state. The quantum states populated by these electrons can be considered as static with the result that the eigentime $\tau_{\text{eicDCBF}}$ is not a conservation quantity for these states, because it does not exist for those.

For the moment we assume that these 2 · $n_e$ electrons represent one-electron states, i.e. not paired electron states. At any time $t$ an electron which belongs, for example, to the − copper site can be promoted to the + copper site or vice versa, i.e. these transitions are not coupled to $\tau_{\text{eicDCBF}}$, as it is given for half-filled or hole doped valence bands. In consequence, these states which are independent on $\tau_{\text{eicDCBF}}$ may freely interfere, i.e. they may belong simultaneously in a quantum probabilistic sense to the Hilbert spaces $|\mathcal{H}_{I_i,-\Delta r_{ij}}\rangle$ and $|\mathcal{H}_{I_i,+\Delta r_{ij}}\rangle$ as well as $|\mathcal{H}_{I_j,+\Delta r_{ij}}\rangle$ and $|\mathcal{H}_{I_j,-\Delta r_{ij}}\rangle$ or to $|\mathcal{H}_{I_i,+\Delta r_{ij}}\rangle$ and $|\mathcal{H}_{I_j,+\Delta r_{ij}}\rangle$ as well as the local phase $\phi_{S_{ij}}$ as well as their definite interrelation become uncertain. In the same way this uncertainty will be important within tunnel junctions at crystal boundaries, for example, with respect to Josephson pair tunnelling.

\textbf{Figure 3.} Population of the two different copper sites (+) and (−) in Fig.1. (1) For half filled bands, only the (−) copper sites are populated forming diagonal spin pairings, within the antibonding valence bands. (2),(2') For populations above half filling, the (+) and (−) lattice sites are simultaneously populated where off-diagonal (2) or diagonal (2') spin pairings may occur. (3) For filled bands, the (+) and (−) lattice sites are simultaneously populated where ideally off-diagonal spin pairings occur in the case of bonding states, which is the case for lower energetically filled bands. Filled areas indicate populated states.
electron pair states and transformations of the coordinate systems of the Hilbert spaces according to the right sites of Eqs. (51) to (56). At present, the topological electron structure for \( n \) electrons may probabilistically occupy states within Hilbert spaces which are the linear spans \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\), \([\mathcal{H}_{I_j,-\Delta r_j} \cup \mathcal{H}_{I_j,\Delta r_j}]\), \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\), \([\mathcal{H}_{I_j,-\Delta r_j} \cup \mathcal{H}_{I_j,\Delta r_j}]\) or \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\). The uniform band states, given for half-filled and hole doped bands, are split up into energetic lower band states which belong further to the Hilbert spaces on the left side of Eqs. (51),(53),(54) and (56) and energetic upper band states that belong to the corresponding static linear spans of these Hilbert spaces. The energetically lower band states are filled with \( n' = n - n_e \) electrons and the energetically upper band states with \( n'' = 2n_e \) electrons, with \( n_e \) being the number of electrons for half-filled bands. The \( n' \) electrons are lowered in energy based on their additional negative correlation energy within the DCBF state. The \( n'' \) electrons have lost the DCBF correlation energy and are probably additionally shifted to higher energies resulting from the continually existing dynamic charge fluctuations which create a time dependent potential that generates a non-stationary electron dynamics.

If the degree of electron doping is increased, one has to expect that ordered topological electron structures may also be formed, similar as it was given under hole doping. The driving force for such topologically structured electron states should again be quadrupolar-polarization induced correlations, similar as described for hole doped states [7]. Again, this leads to \( \mathbf{k}_j \), \(-\mathbf{k}_j\) electron pair states and transformations of the coordinate systems of the Hilbert spaces according to the right sites of Eqs. (51) to (56). At present, the topological electron structure for \( n \)-type cuprates is not yet known which prevents a deeper elaboration of the quantum states. Nevertheless, an ordered topological electron structure may again lead to a periodic transformation of a topologically ordered electron state in phase steps of \( \delta \phi_{S_{ij}} \). This again will lead to an eigentime \( \tau_{ei_{||}} = n \cdot \tau_{CBF} \) of this dynamic state, with \( n \cdot \Delta r_{||} \) being the periodicity of the topological electron structure along its polarization direction. The lifetime \( \tau_{CBF} \) remains the determining time constant, because an ordered topological electron structure will also be strongly coupled to the CBF superstructure \((-\),\(+\)) caused by the same electronic polarizations (quadrupolar-polarizations), as it is given for hole doped states. One can say, an ordered topological electron structure will topologically triggered in time by \( \tau_{CBF} \). The main difference to the hole doped case is again given by the fact that the electron pair states \( \mathbf{k}_j \), \(-\mathbf{k}_j\) creating a topological electron structure are not quantum-coupled to the DCBF state. These electron pair states belong at all times probabilistically to the linear spans \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\), \([\mathcal{H}_{I_j,-\Delta r_j} \cup \mathcal{H}_{I_j,\Delta r_j}]\), \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\), \([\mathcal{H}_{I_j,-\Delta r_j} \cup \mathcal{H}_{I_j,\Delta r_j}]\) or \([\mathcal{H}_{I_j,-\frac{\Delta r_j}{2}} \cup \mathcal{H}_{I_j,\frac{\Delta r_j}{2}}]\), as discussed for single electron states. As a result, \( \tau_{e_{DCBF}} \) does not exist for these states and, in consequence, the phase \( \theta_{ij} \) is uncertain for these quantum states. Hence, the electron doped quantum pair states can be described as follows: There will be a strong deterministic space-time relation of the ordered topological electron state, which is given by the eigentime \( \tau_{ei_{||}} = n \cdot \tau_{CBF} \) and the fixed phase steps \( \delta \phi_{S_{ij}} \) including the periodicity of \( n \cdot \delta \phi_{S_{ij}} \). The deterministic space-time relation reflects the coherent transformation of all Hilbert subspaces in space and time. However, the expectation values of the phases \( \langle \theta_{ij} \rangle >, \langle \phi_{S_{ij}} \rangle > \) are free variables at any time \( t \), based on the possibility of probabilistic superpositions of states which belong initially to different Hilbert subspaces which are associated to the \((-\) and \((+\) lattice sites, respectively. Therefore, at any time \( t \) the strongly time-periodic oscillations of the topological electron structure, given by \( \tau_{ei_{||}} \) and \( n \cdot \delta \phi_{S_{ij}} \), can be superimposed by shifts of the expectation values of the particular phases which imply charge density variations.
In conclusion, the phase of the condensate can occur as free variable, with respect to the energetically higher \( n' \) electrons. The corresponding phases \( \theta_{||} \) and \( \phi_{||} \) of the energetically deeper \( n' \) electrons are further on time-quantized, at least for the b-holes, and can not behave as a free variable. Altogether, the electron dynamics and especially the conductivity in hole doped and electron doped high-\( T_c \) materials will be distinctly different, as pointed out in the introduction.

9. Experimental proofs

Direct experimental proofs of the here deduced extraordinary characteristics of the quantum states in space and time require well-directed experiments which have yet to be done. Nevertheless, there is a broad experimental support for the here deduced quantum states as already discussed in our previous articles \([7, 8]\). More detailed experimental comparisons usually require a broader discussion, which will be done in separated articles.

A rather definite proof of the here deduced quantum behaviour can be given by experimental methods which are sensitive to the effective mass of the charge carriers, because, the effective mass \( m_{||} \) along the particular one-dimensional states \( I_x, \bar{I}_x \) will be infinite, \( m_{||} = \infty \), if the local state function \( u_{o||} \) exclusively exists but \( m_{||} \) will be finite, if \( u_{o||} \) is split up into the coordinate shifted relative states states \( u_{L,-\Delta r_{||}} \) and \( u_{R,\Delta r_{||}} \), according to Fig. 2. If at no time holes exist within the particular one-dimensional states \( I_x \) or \( \bar{I}_x \), the local states within these one-dimensional states will have exclusively \( u_{o||} \) character, but the local states \( u_{o||} \) will be split up into the states \( u_{L,-\Delta r_{||}} \) and \( u_{R,\Delta r_{||}} \), if an ordered topological hole structure exists. Even if topological holes are only temporarily existent within the particular states \( I_x \) or \( \bar{I}_x \), contributions of the split local states \( u_{L,-\Delta r_{||}}, u_{R,\Delta r_{||}} \) have to be assumed.\(^6\)

A decisive proof can be given by Hall effect measurements, because the b-holes cannot create a Hall current, if the b-hole state is permanently polarized along one polarization direction, e.g. \( y(b) \) direction in Fig. 1, and if no f-holes exist which is given for \( n_h \leq 0.125 \) holes/copper. In this case, the states within \( I_x, \bar{I}_x \) in Fig. 1 never carry holes, i.e. \( m_{||} = \infty \) for these states. This means, the effective mass \( m_{\perp} \) for perpendicular electronic state transformations with respect to the initial electronic states which belong to \( I_x, \bar{I}_x \) is infinite, \( m_{\perp} = \infty \). Therefore, a Hall current can not exist. If additionally f-holes exist (\( f_{||} \)-holes + \( f_{\perp} \)-holes), the effective mass \( m_{||} \) along the states \( I_x, \bar{I}_x \) which carry \( f_{\perp} \)-holes will be permanently or temporarily finite, i.e. a Hall current can be formed. Even this is found by Hall effect measurements of T. Noda et al. on \( La_{2-x-y}Nd_ySr_xCuO_4 \) with \( x = 0.10, 0.12, 0.13, 0.14 \) and \( y = 0.4, 0.6 \) \([27]\). Below a characteristic temperature \( T_0 \) the Hall coefficient \( R_H \) changes significantly in dependence on \( x \). For \( x = 0.1, 0.12 \) \( R_H \) drops to zero when lowering the temperature, whereas for \( x = 0.13, 0.15 \) \( R_H \) remains finite for \( T \to 0 \). This reflects the different behaviour of the b-hole and f-hole system. Below a temperature \( T_0 \) the hole states are assumed to be "pinned" \([27]\). In the theory derived here this means that there is only one polarization direction for the topological b-hole state (\( k_BT_0 \) represents an activation barrier for switching the polarization direction). For \( x = 0.1, 0.12 \) (\( n_h = 0.1, 0.12 \) holes/copper) only b-holes exist, i.e. \( m_{\perp} = \infty \), whereas for \( x = 0.13, 0.15 \) (\( n_h = 0.13, 0.15 \) holes/copper) b-holes and f-holes exist meaning \( m_{\perp} \) will become finite, even if the b-hole polarization direction is stationary.

If the b-hole structure fluctuates between the two polarization directions \( x(a) \) and \( y(b) \)

\(^5\) The effective mass \( m_{||} \) is related to transitions between local states \( u_{||} \) and \( u_{||} \), but not identical with the effective band mass \( m_{\perp} \) which must be set infinite for \( -k_{||}, k_{||} \) pair states.

\(^6\) This, however, includes the problem of quantum relaxations, which is generally very complex.
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10. Conclusions

The comprehensive analysis of the electronic states within the CuO$_2$ planes [7] has shown that the quantum states in the CuO$_2$ planes can not be assumed any longer to be adapted to the crystal symmetry. Self-consistent renormalizations of the electronic states lead to an electronic superstructure, the CBF state, which subdivides the initial crystal lattice into two sublattices. Inevitably, the initial Hilbert space $\mathcal{H}$ is renormalized ($\mathcal{H}_{\text{ren.}}$) and $\mathcal{H}_{\text{ren.}}$ is transformed into the Hilbert subspaces $\mathcal{H}_{-\Delta r}$ and $\mathcal{H}_{+\Delta r}$. The two subspaces form the orthogonal complement to each other, but beyond that they represent a definite relative state with respect to their particular coordinate systems given by the relative coordinate shifts $\pm \Delta r$. Regarding the half-filled valence bands, the two dimensional Hilbert subspaces are additionally transformed into arrays of one-dimensional Hilbert subspaces $\left\{ \mathcal{H}_{I_i,-\Delta r_j}, \mathcal{H}_{I_i,+\Delta r_j}, \mathcal{H}_{I_j,-\Delta r_i}, \mathcal{H}_{I_j,+\Delta r_i} \right\}$. Again, the subspaces $\mathcal{H}_{I_i,-\Delta r_j}$ and $\mathcal{H}_{I_i,+\Delta r_j}$ as well as $\mathcal{H}_{I_j,-\Delta r_i}$ and $\mathcal{H}_{I_j,+\Delta r_i}$ form the orthogonal complement to each other, and their individual coordinate systems are definitely related to each other by the relative shifts $\pm \Delta r$ of the coordinate systems of the two Hilbert subspaces along $I_i$ and $I_j$ in Fig. 1, respectively. Altogether, the set of one-dimensional subspaces $\left\{ \mathcal{H}_{I_i,-\Delta r_j}, \mathcal{H}_{I_i,+\Delta r_j}, \mathcal{H}_{I_j,-\Delta r_i}, \mathcal{H}_{I_j,+\Delta r_i} \right\}$ define a definite relative state with respect to their coordinate systems. For half-filled valence bands and under hole doping, the exceptional property is that these Hilbert subspaces are deterministically time dependent, i.e. their coordinate systems are periodic functions of time along the space coordinate $r$ within a space interval of $2\Delta r$, with $\Delta r$ being the lattice constant in $a$ or $b$ direction. In this case, the stationarity of a basis state is not defined as a time independent state which is instantaneously given, as usually supposed in quantum mechanics, but is defined for periodically equivalent internal times $t_{\text{int}}$ according to

$$t_{\text{int}} = t' + n \cdot \tau_{\text{DCBF}} \quad (n \text{ integer}).$$

Here, $\tau_{\text{DCBF}}$ defines the oscillation time (eigentime) for periodic oscillations of the coordinate systems of the Hilbert subspaces. The time $t' \in [0, \tau_{\text{DCBF}}]$ is a definite internal time variable which describes the periodic time dependence of the coordinate systems of the Hilbert subspaces (Eq. (24)). The integer number $n$ occurs as a probabilistic variable describing the uncertainty of the absolute position of the common center point coordinate of the Hilbert subspaces $\left\{ \mathcal{H}_{I_i,-\Delta r_j}, \mathcal{H}_{I_i,+\Delta r_j}, \mathcal{H}_{I_j,-\Delta r_i}, \mathcal{H}_{I_j,+\Delta r_i} \right\}$ in time, which is
quantized by $\tau_{elDCBF}$. Two possible quantum state interpretations are valid:

(a) Time integrated states over $t'$ between 0 and $\tau_{elDCBF}$.
(b) Time instantaneous states with $t' = const.$ according to Eq. (62).

In both cases the time scale $t_{int}$ has to be considered as quantized in time by $\tau_{elDCBF}$. In case (a) the eigentime $\tau_{elDCBF}$ appears to be a hidden variable, because the integrated quantum state is equivalent for all times $n \cdot \tau_{elDCBF}$, i.e. the quantum state is stationary. However, one has to bear in mind that the time scale is not continuous but quantized by $\tau_{elDCBF}$. In (b) the time $t'$ occurs explicitly, but equivalent quantum states are only given for multiples of $\tau_{elDCBF}$, i.e. the stationarity is again defined for multiples of $\tau_{elDCBF}$. The full symmetry of the whole quantum state is, however, only defined for case (a), because the antisymmetric pair states according to Eqs. (16)-(18) exist only as integrated states over the full time period $\tau_{elDCBF}$. If crystal symmetry adapted quantum states are given, individual dynamic coordinate systems of particular quantum states may exist but the corresponding eigentimes are completely uncertain. This is equivalent with a disappearing of the definite time variable $t'$. This corresponds to $\tau_{el} = 0$, i.e. the internal time variable $t_{int}$ disappears, as usually supposed in quantum mechanics, leading to time stationary quantum states which are instantaneously defined and unchanged at all times.

If additional electrons $n_e$ are injected into the two times negatively charged CuO$_2^-$ plane, i.e. electron-doped (n-type) cuprates, the initially unpopulated Hilbert subspaces $\mathcal{H}_{I_i, \pm \Delta r ||}$ and/or $\mathcal{H}_{I_j, \pm \Delta r ||}$ will be populated. As a result, $2n_e$ electronic states in $\mathcal{H}_{I_i, \pm \Delta r ||}$ and $\mathcal{H}_{I_j, \pm \Delta r ||}$ remain static, i.e. they are not quantum entangled to the DCBF state. The same occurs for the one-dimensional states along $\bar{I}_j$, if electrons are additionally injected into $\mathcal{H}_{I_j}$. Therefore, the eigentime $\tau_{elDCBF}$ does not exist for these quantum states, so that $t_{int}$ becomes zero for these quantum states. At any time, states within the linear spans $\text{lin}(\mathcal{H}_{I_i, \pm \Delta r ||})$ and/or $\text{lin}(\mathcal{H}_{I_j, \pm \Delta r ||})$ may be occupied. In this respect more customary single or paired quantum states occur for these $2n_e$ electrons. However, the dynamic DCBF state is existent further on which generates some exceptional dynamic conditions, for these $2n_e$ electrons as well.

If the intrinsic eigentime $\tau_{elDCBF}$ exists for the quantum states, free electronic transitions between different Hilbert subspaces are forbidden at any time $t$, as long as the Hilbert subspaces belong to different coordinate systems. Any electron transition between Hilbert subspaces that does not obey the strongly deterministic time behaviour according to Eq. (24) breaks/destroys the quantum state itself, because $\tau_{elDCBF}$ is no longer a conservation quantity. This is the reason way the undoped twice negatively charged CuO$_2^-$ plane behaves like an insulator. These conditions would also be valid under hole doping, if the Hilbert subspaces $\{\mathcal{H}_{I_i, \pm \Delta r ||}, \mathcal{H}_{I_i, \pm \Delta r_j}, \mathcal{H}_{I_j, \pm \Delta r ||}, \mathcal{H}_{I_j, \pm \Delta r_j}\}$ remained unchanged. However, the highly correlated $-k ||, k ||$ electron pair states under hole doping lead to an additional splitting of every particular Hilbert subspace into two further subspaces according to Eqs. (51),(53),(54) and (56). This creates Hilbert subspaces according to $\mathcal{H}_{I_i,0}$ and $\mathcal{H}_{I_i,0}$ as well as $\mathcal{H}_{I_i,\pm \Delta r ||}$ and $\mathcal{H}_{I_i,\pm \Delta r_j}$ (modulo $2\Delta r ||$), and along $\bar{I}_j$ equivalent transformations occur. Now, the common coordinate systems in $\mathcal{H}_{I_i,0}$ and
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The Hilbert spaces $\mathcal{H}_{I_i,0}$ as well as $\mathcal{H}_{I_i,-\Delta r_i}$ and $\mathcal{H}_{I_i,-\Delta r_i}$ enables accidental electronic transitions between the Hilbert spaces $\mathcal{H}_{I_i,0}$ and $\mathcal{H}_{I_i,-\Delta r_i}$ as well as $\mathcal{H}_{I_i,-\Delta r_i}$ and $\mathcal{H}_{I_i,-\Delta r_i}$ at any time $t$ without breaking $\tau_{\text{DCBF}}$. As a result, electronic transformations along the one-dimensional states $I_i$, $\bar{I}_j$ will be possible and, in consequence, conductivity can occur. In conclusion, the insulator-conductor transition in high-$T_c$ cuprates is the result of an additional splitting of the Hilbert subspace being accompanied with coordinate transformations where the copper site centered Hilbert subspaces have been transformed into oxygen site centered Hilbert subspaces. In electron doped ($n$-type) cuprates, $\tau_{\text{DCBF}}$ is not a conservation quantity for the $2n_e$ conduction electrons and, therefore, electronic transitions between particular Hilbert subspaces may always occur. The dynamic influence of the DCBF state on the $2n_e$ conduction electrons is merely a mean field effect, the electron doped quantum states are not time-quantization by $\tau_{\text{DCBF}}$. However, isolating states can also be formed if the doped holes or electrons localize, which may be possible for small hole or electron doping, if clustered topological electron structures occur.

In summary, the TR conception of the quantum states in high-$T_c$ cuprates, recently derived [8], enables a complete new interpretation of the electron dynamics in the CuO$_2$ planes of these materials. Separated Hilbert subspaces form a highly correlated compound system including a definite/deterministic relative state of their coordinate systems in space and time. The coordinate systems themselves occur as variables. A particular splitting of the Hilbert subspaces accompanied with coordinate transformations are responsible for the insulator-conductor transition under hole doping. Mostly unusual is the quantization of time by an internal time constant $\tau_{\text{DCBF}}$ termed as eigentime, which occurs for undoped (half-filled) and hole doped cuprates as a conservation quantity but for electron doping a separated electron fluid appears in which $\tau_{\text{DCBF}}$ is not a conservation quantity. The time quantization of the quantum states implicates a deterministic space-time behaviour of quantum states, meaning that the quantum realm encompasses classical behaviour. This result motivates the reflection about the general interpretation of quantum mechanics.

**Appendix: Probabilistic superposition of Hilbert spaces**

The separation of Hilbert spaces in Hilbert subspaces has been elaborated as a striking characteristic of the quantum states within the CuO$_2$ planes. This is causally connected with the formation of definite relative coordinates of the separated Hilbert spaces, i.e. a definite relative state of the individual coordinate systems, which may be dynamical. Most unusually, a time-deterministic dynamic relative state of the coordinate systems was concluded as described in Sec. 5. Over and above that time independent, i.e. probabilistic dynamic relative states of the coordinate systems, have to be taken into consideration as well, as concluded in Sec. 7.

Under half filling, separated Hilbert spaces occur which are related to the copper sites. Subsequently, these particular Hilbert spaces are defined by the Hilbert space $\mathcal{H}_o$.

\[
\mathcal{H}_o \equiv \frac{\mathcal{H}_{I_i,0}}{\mathcal{H}_{I_i,-\Delta r_i}} + \frac{\mathcal{H}_{I_i,-\Delta r_i}}{\mathcal{H}_{I_i,0}} \quad \text{or} \quad \frac{\mathcal{H}_{I_i,0}}{\mathcal{H}_{I_i,-\Delta r_i}} + \frac{\mathcal{H}_{I_i,-\Delta r_i}}{\mathcal{H}_{I_i,0}}
\]

and equivalently for $I'_i$ and $\bar{I}_i$. If the electron density is reduced, the formation of topological hole states is accompanied by a splitting of the local wave function $u_{\theta_{||}}$ into left and right local wave functions $u_{\theta_{||}}$ as discussed in Sec. 7, where the latter are associated with the oxygen sites contrary to the $u_{\theta_{||}}$ function which is assigned to the
copper sites. This leads to the splitting of the Hilbert space $\mathcal{H}_o$ into separated subspaces $\mathcal{H}_{o,\frac{\Delta r_i}{2}}$ and $\mathcal{H}_{o,-\frac{\Delta r_i}{2}}$ which are related to the Hilbert spaces in Eqs. (39) - (46) according to

$$\mathcal{H}_o \rightarrow \mathcal{H}_{o,\frac{\Delta r_i}{2}}, \mathcal{H}_{o,-\frac{\Delta r_i}{2}} = \left[ \begin{array}{c} - \mathcal{H}_{I_i,\Delta r_{-\phi ||}}, \mathcal{H}_{I_i,\Delta r_{\phi ||}} \\ + \mathcal{H}_{I_i,\Delta r_{-\phi ||}}, \mathcal{H}_{I_i,\Delta r_{\phi ||}} \end{array} \right],$$

and equivalently for $I'_i$ and $\bar{I}'_i$, where the assignment between the Hilbert spaces on the left and right side of Eq. (A.3) depends on the considered states in Eqs. (A.1) and (A.2). The subsequent local states are defined in dependence on the given coordinate system according to

$$u_{R,\frac{\Delta r_i}{2}}(r), u_{L,\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,\frac{\Delta r_i}{2}} \quad (A.4)$$

$$u_{R,-\frac{\Delta r_i}{2}}(r), u_{L,-\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,-\frac{\Delta r_i}{2}} \quad (A.5)$$

There is no time ordering for a dynamic association of the states $u_L$ and $u_R$ in Eqs. (A.4) and (A.5) to the two Hilbert subspaces which allows a probabilistic superposition of these two quantum states within a common or separated Hilbert subspace, i.e. if the coordinate system is preserved or changed into the complementary coordinate system. Hence, the subsequent states, respective Hilbert spaces, may occur simultaneously in a quantum probabilistic sense.

$$u'_{R,\frac{\Delta r_i}{2}}(r) = u_{R,\frac{\Delta r_i}{2}}(r) + f_{R,\frac{\Delta r_i}{2}}(r)u_{L,\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,\frac{\Delta r_i}{2}} \quad (A.6)$$

$$u''_{L,-\frac{\Delta r_i}{2}}(r) = f_{R,\frac{\Delta r_i}{2}}(r)u_{L,-\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,-\frac{\Delta r_i}{2}} \quad (A.7)$$

with

$$f_{R,\frac{\Delta r_i}{2}}(r) = \left( \frac{u^2_{R,\frac{\Delta r_i}{2}}(r)}{u^2_{R,\frac{\Delta r_i}{2}}(r) + u^2_{L,-\frac{\Delta r_i}{2}}(r)} \right) \quad (A.8)$$

and

$$u'_{L,-\frac{\Delta r_i}{2}}(r) = u_{L,-\frac{\Delta r_i}{2}}(r) + f_{L,-\frac{\Delta r_i}{2}}(r)u_{R,-\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,-\frac{\Delta r_i}{2}} \quad (A.9)$$

$$u''_{R,\frac{\Delta r_i}{2}}(r) = f_{L,-\frac{\Delta r_i}{2}}(r)u_{R,\frac{\Delta r_i}{2}}(r) \in \mathcal{H}_{o,\frac{\Delta r_i}{2}} \quad (A.10)$$

with

$$f_{L,-\frac{\Delta r_i}{2}}(r) = \left( \frac{u^2_{L,-\frac{\Delta r_i}{2}}(r)}{u^2_{R,\frac{\Delta r_i}{2}}(r) + u^2_{L,-\frac{\Delta r_i}{2}}(r)} \right). \quad (A.11)$$
From this, two particular quantum states can be defined according to
\[
\tilde{u}_{R,\frac{\Delta r}{2}}(\mathbf{r}) = \beta\frac{\Delta r}{2} \cdot u'_{R,\frac{\Delta r}{2}}(\mathbf{r}) \oplus \beta\frac{\Delta r}{2} \cdot u''_{L,-\frac{\Delta r}{2}}(\mathbf{r}) \tag{A.12}
\]
with \(\beta\frac{\Delta r}{2} = 1\) and \(\beta\frac{\Delta r}{2} = -1\) and
\[
\tilde{u}_{L,-\frac{\Delta r}{2}}(\mathbf{r}) = \beta\frac{\Delta r}{2} \cdot u'_{L,-\frac{\Delta r}{2}}(\mathbf{r}) \oplus \beta\frac{\Delta r}{2} \cdot u''_{L,-\frac{\Delta r}{2}}(\mathbf{r}) \tag{A.13}
\]
with \(\beta\frac{\Delta r}{2} = -1\) and \(\beta\frac{\Delta r}{2} = 1\), respectively. Then, the electron density distribution is given by
\[
\tilde{\rho}_{R,\frac{\Delta r}{2}}(\mathbf{r}) = \frac{\beta\frac{\Delta r}{2}}{u^2_{R,\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{R,\frac{\Delta r}{2}}(\mathbf{r})} + \frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})}\right)\right) \tag{A.14}
\]
and
\[
\tilde{\rho}_{L,-\frac{\Delta r}{2}}(\mathbf{r}) = \frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{R,\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} + \frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})}\right)\right) \tag{A.15}
\]
respectively, which leads to the common state function
\[
\tilde{\rho}_{\frac{\Delta r}{2}}(\mathbf{r}) = \tilde{\rho}_{R,\frac{\Delta r}{2}}(\mathbf{r}) \oplus \tilde{\rho}_{L,-\frac{\Delta r}{2}}(\mathbf{r})
\]
\[
= \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{R,\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{R,\frac{\Delta r}{2}}(\mathbf{r})} + \frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})}\right)\right)\right) \oplus \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} \cdot \left(\frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})} + \frac{\beta\frac{\Delta r}{2}}{u^2_{L,-\frac{\Delta r}{2}}(\mathbf{r})}\right)\right) \tag{A.16}
\]
For the electron density distribution results
\[
\tilde{\rho}_{\frac{\Delta r}{2}}(\mathbf{r}) = \tilde{\rho}_{\frac{\Delta r}{2}}(\mathbf{r}) = \tilde{\rho}_{\frac{\Delta r}{2}}(\mathbf{r}) \oplus \tilde{\rho}_{\frac{\Delta r}{2}}(\mathbf{r}) \quad \text{is} 1/2 \text{ which means that only one electron is nominally associated with particular oxygen site, with the consequence that maximally one topological hole may be placed at a given oxygen site, as it has previously already been deduced [7].}
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