We prove the uniqueness theorem for static higher dimensional charged black holes spacetime containing an asymptotically flat spacelike hypersurface with compact interior and with both degenerate and non-degenerate components of the event horizon.

I. INTRODUCTION

The pioneering investigations of mathematical topics related to the the black hole equilibrium states were attributed to Israel [1], Müller zum Hagen et al. [2] and Robinson [3]. Bunting and Masood-ul-Alam [16] proposed an alternative proof of the uniqueness of black hole solutions. Then, the method was strengthened to the Einstein-Maxwell (EM) black holes [5,6]. Heusler [7] comprised the magnetically charged Reissner-Nordström (RN) solution and static Einstein-σ-model case [8]. The classification of static of vacuum black holes was finished in [9], where the condition of non-degeneracy of the event horizon was removed and it was shown that Schwarzschild black hole exhausted the family of all appropriately regular black hole spacetimes. In Ref. [10] it was revealed that RN solution comprised the family of regular black hole spacetimes under the restrictive condition that all degenerate components of black hole horizon carried a charge of the same sign.

The problem of stationary and axisymmetric black hole spacetimes being the solution of vacuum Einstein equations was considered in Refs. [11–14], while the systematic way of obtaining the desire results in electromagnetic case was provided by Mazur [15] and Bunting [16]. For a review of the uniqueness of black hole solutions story see [17] and references therein.

In the recent years there was a considerable resurgence of mathematical works concerning black hole equilibrium state in the low-energy string theory. The uniqueness of the black hole solutions in dilaton gravity was proved in works [18–20], while the uniqueness of the static dilaton $U(1)^2$ black holes being the solution of $N = 4, d = 4$ supergravity was provided in [21]. The extension of the proof to the theory to allow for the inclusion of $U(1)^N$ static dilaton black holes was established in Ref. [22].

The latest development of string theory as well as the possibility that the weak scale is the fundamental scale of nature and the Planck scale is to be derived from it [23] trigger the interests in higher dimensional black hole solutions. The so-called TeV gravity attracts attention to higher dimensional black hole which may be produced in high energy experiments [24]. It was revealed that the five-dimensional stationary vacuum black hole are not unique. Myers-Perry [25] solution generalized the Kerr solution to arbitrary dimension, while Emparan et al. [26] revealed a counterexample showing that a five-dimensional rotating black hole ring solution with the same angular momentum and mass but the
horizon of which was homeomorphic to $S^2 \times S^1$. The uniqueness theorem for $n$-dimensional Schwarzschild-Tangherlini black hole was provided by Gibbons et al. [27] and for asymptotically flat static $n$-dimensional charged (dilaton) black holes was presented in Ref. [28]. The uniqueness theorem for self-gravitating nonlinear $\sigma$-models in higher dimensional spacetime was obtained in [29]. As far as the stationary $n$-dimensional black hole uniqueness theorem is concerned, the proof for $N = 1$, $d = 5$ supersymmetric black holes was given in [30].

In this issue our strategy will be to modify the considerations of Ruback [32] and Chruściel [10] in order to comprise the problem of the uniqueness of the charged $n$-dimensional static black hole solutions containing an asymptotically flat spacelike hypersurface with compact interior and with both degenerate and non-degenerate components of the event horizons. In the attitude proposed by Ruback [32] the metric induced on the hypersurface orthogonal to the asymptotically timelike Killing vector field was considered. In our case we shall consider the orbit space metric [9] (defined below) on the spacelike hypersurface $\Sigma$. The key considerations will be held near both degenerate and non-degenerate components of $\partial \Sigma$. Our key result will be the proof that a static higher dimensional charged black hole spacetime containing an asymptotically flat spacelike hypersurface with compact interior and with both degenerate and non-degenerate components of the event horizon exists, subject to the additional assumption that all degenerate components of the horizon should have charges of the same signs.

II. HIGHER DIMENSIONAL EINSTEIN-MAXWELL SYSTEM

We shall consider the $n$-dimensional Einstein-Maxwell system described by the following action:

$$ I = \int d^n x \sqrt{-\hat{g}} \left[ (n) R - F_{\mu\nu} F^{\mu\nu} \right], $$

(1)

where $\hat{g}_{ij}$ is $n$-dimensional metric tensor, $F_{\mu\nu}$ is the Maxwell field. The metric of $n$-dimensional static spacetime with asymptotically timelike Killing vector field $k_\alpha = \left( \frac{\partial}{\partial t} \right)_\alpha$ and $\tilde{V}^2 = -k_\mu k^\mu$ has the following form:

$$ ds^2 = -\tilde{V}^2 dt^2 + h_{ij} dx^i dx^j, $$

(2)

where $\tilde{V}$ and $h_{ij}$ are independent of the $t$-coordinate as the quantities of the hypersurface $\Sigma$ of constant $t$. The hypersurface $\Sigma$ is a connected and simply connected spacelike hypersurface, $\bar{\Sigma}$ denotes the closure of it. The topological boundary of $\Sigma$, $\partial \Sigma = \Sigma \setminus \Sigma$ is a nonempty topological manifold with $h_{ij} k^i k^j = 0$ on $\partial \Sigma$. For the static metric the electromagnetic potential will be of the form $A_0 = \phi dt$.

We treat first the problem of the orbit space. Namely, as in Ref. [9] we shall consider a point $p$ of the manifold such that $h_{ij} k^i k^j \neq 0$ and perform a decomposition

$$ T_p M = K(k_\mu) \oplus K^\perp, $$

(3)

where $K(k_\mu)$ is the vector space spanned by the Killing vector field $k_\mu(p)$ and $K^\perp$ is the space orthogonal to the vector space spanned by $k_\mu$. We define the orbit space metric $g_{ij}(p)$ in the form

$$ g(X, Z) = h(X_\perp, Z_\perp). $$

(4)
The metric $g_{ij}$ should not be identified with the metric on the space of orbits because one is not assuming any regularity properties of that space (in particular we do not even assume that the space of orbits is a differentiable manifold). One has then

$$Y_{\perp} = Y - \frac{h(X,Y)}{h(X,X)}X,$$

so we arrive at the following expression for the metric tensor $g_{ij}$

$$g(Y,Z) = h(Y,Z) - \frac{h(X,Y)h(X,Z)}{h(X,X)}.$$

As was proved in Ref. [9] (lemma 5.1) if one had to do with the static spacetime and if we supposed that $(h_{ij}, \tilde{V})$ satisfied the same coordinate independent system of equations, then the orbit space metric $g_{ij}$ with the function $V$ (where $V^2$ was the square of the norm of $k_\mu$ on $\Sigma$) satisfied the same system of equations. This theorem enables us to write the equations of motion in the following form:

$$(g)\nabla_i (g)\nabla^i V = \frac{C^2}{V} (g)\nabla_i \phi (g)\nabla^i \phi,$$

$$(g)\nabla_i (g)\nabla^i \phi = \frac{1}{V} (g)\nabla_i \phi (g)\nabla^i V,$$

$$(n-1)R_{ij}(g) = -\frac{1}{V} (g)\nabla_i (g)\nabla_j V = -\frac{2}{V^2} (g)\nabla_i \phi (g)\nabla_j \phi + \frac{2}{(n-2)V^2} (g)\nabla_i \phi (g)\nabla_j \phi ,$$

where we have denoted by $C^2 = 2(n-3)/(n-2)$, while $\phi$ is the electrostatic potential. The covariant derivative with respect to $g_{ij}$ is denoted by $(g)\nabla$, while $(n-1)R_{ij}(g)$ is the Ricci tensor defined on the hypersurface $\Sigma$.

Let us assume further that we take into account the asymptotically flat spacetime, i.e, the spacetime contains a data set $(\Sigma_{end}, g_{ij}, K_{ij})$ with gauge fields such that $\Sigma_{end}$ is diffeomorphic to $\mathbb{R}^3$ minus a ball and the following asymptotic conditions are fulfilled:

$$|g_{ij} - \delta_{ij}| + r|\partial_a g_{ij}| + \ldots + r^k|\partial_{a_1 \ldots a_k} g_{ij}| + r|K_{ij}| + \ldots + r^k|\partial_{a_1 \ldots a_k} K_{ij}| \leq O\left(\frac{1}{r}\right),$$

$$|F_{\alpha \beta}| + r|\partial_a F_{\alpha \beta}| + \ldots + r^k|\partial_{a_1 \ldots a_k} F_{\alpha \beta}| \leq O\left(\frac{1}{r^2}\right).$$

Consequently, under the above assumptions, there is a standard coordinates system in which we have the usual asymptotic expansion

$$V = 1 - \frac{\mu}{r^{n-3}} + O\left(\frac{1}{r^{n-2}}\right),$$

and accordingly for the metric tensor

$$g_{ij} = \left(1 + \frac{2}{(n-3) r^{n-3}} \frac{\mu}{r^{n-3}}\right) \delta_{ij} + O\left(\frac{1}{r^{n-2}}\right),$$

and for the electrostatic potential

$$\phi = \frac{Q/C}{r^{n-3}} + O\left(\frac{1}{r^{n-2}}\right),$$

where $\mu$ is the ADM mass seen by the observer from the infinity, $Q$ is the electric charge while $r^2 = x_i x^i$. 


Our next step will be the analysis of the behaviour of $g_{ij}$ and $\phi$ near $\partial \Sigma$. By the theorems derived by Vishweshwara [33] and Carter [34] one obtains that $\partial \Sigma$ has to be a subset of a Killing horizon. On the other hand, the Killing horizon is a smooth manifold. As it is well known $\phi$ is constant on any connected components of the Killing horizon and it implies its constancy on $\partial \Sigma$. In what follows, we shall call a connected $S$ component of $\partial \Sigma$ degenerate and non-degenerate when $S$ intersects a degenerate or non-degenerate Killing horizon, respectively. If necessary we can deform slightly the hypersurface $\Sigma$ in spacetime and we ensure that $\partial \Sigma$ is a smooth submanifold both of $\Sigma$ and of $M$ near degenerate horizons. As far as the non-degenerate horizon is concerned, $\partial \Sigma$ will not be a smooth submanifold of $M$. It is caused by the existence of points on $\partial \Sigma$ at which the Killing vector field $k_\mu$ vanishes. As was proved in Ref. [9] one could equip $\Sigma$ with a differentiable structure so that $\partial \Sigma$ was a smooth submanifold of $\Sigma$. The boundary $\partial \Sigma$ with the differentiable structure is a totally geodesic boundary of $(\Sigma, g_{ij})$ across which $g_{ij}$ and the electric potential $\phi$ can be smoothly extended across $S$ when a doubling of $\Sigma$ across $S$ is done.

Based on the observation presented by Ruback [32] let us consider now the function

$$ F_\pm = V^2 - \left(1 \pm C\phi\right)^2. \quad (15) $$

Having in mind equation of motion, by the direct calculations one can show that $F_\pm$ is harmonic in the metric $V^{-2}g_{ij}$. $F_\pm$ tend to zero as one approaches the asymptotically flat regions. On the other hand, on every components of $\partial \Sigma$ one obtains $F_\pm \leq 0$.

First, let us suppose that $F_- = 0$ on all components of $\partial \Sigma$. Next by means of the maximum principle one has that $F_\pm \equiv 0$ on $\Sigma$. In the case for which $F_+$ and $F_-$ are negative somewhere on $\partial \Sigma$, from maximum principle we get that $F_\pm < 0$, provided that

$$ V^2 < \left(1 - C\phi\right)^2, \quad V^2 < \left(1 + C\phi\right)^2, \quad (16) $$

on $\Sigma$. By hypothesis $V$ has no zeros on $\Sigma$ and having in mind (16) it shows that $\left(1 - C\phi\right)$ and $\left(1 + C\phi\right)$ have no zeros there. Both $\left(1 - C\phi\right)$ and $\left(1 + C\phi\right)$ tends to 1 at infinity which in turn implies that $-1 < C\phi < 1$ on $\Sigma$. The above equations implies that $0 < V < \min\left(1 + C\phi; 1 - C\phi\right) = 1 - |\phi|$ on $\Sigma$ and we reach to the conclusion that

$$ 0 \leq V + C|\phi| \leq 1. \quad (17) $$

The inequalities are strict except when the metric is locally Majumdar-Papapetrou (MP) [37]. The right inequality is strict on non-degenerate horizons.

If $\mu = Q$ we have that $F_+ = \mathcal{O}\left(1/\phi^2\right)$ and $F_+ \equiv 0$ follows from the harmonicity of $F_+$ in the metric $V^{-2}g_{ij}$ and the asymptotic strong maximum principle. Then, using the above arguments (as in proving Eq.(17)) one reaches to the conclusion that the metric is locally MP. The case $\mu = -Q$ follows by providing the similar arguments as in the case of $F_+$.

Now we shall consider the case for which $\mu \geq |Q|$. In this stage of the proof we shall produce a data set to which one can apply considerations proposed by Rubback [32] and Chruściel [10]. Namely, we take into account two copies of the hypersurface $\Sigma_+$ and $\Sigma_-$ and define metric and the electric field $\hat{E}_{\alpha \pm}$ on them.

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\[ \dot{E}_\alpha \pm = \frac{1}{\sqrt{2}(1 + C\phi \pm V)} \left[ \sqrt{2} (^{(q)} \nabla_\alpha \phi (1 + C\phi) - \sqrt{\frac{n-2}{n-3}}^{(g)} \nabla_\alpha V \right], \] (18)

\[ \dot{g}_{ij} \pm = \left( \frac{1 + C\phi \pm V}{2} \right) \frac{1}{g_{ij}}. \] (19)

By \( \partial_{\text{nodeg}} \Sigma \) we shall denote all these components of the boundary of \( \Sigma \) which correspond to non-degenerate components of the event horizons of the black hole.

If \( \partial_{\text{nodeg}} \Sigma \neq 0 \) we can paste \( \bar{\Sigma}^+ + \partial_{\text{nodeg}} \Sigma \) and \( \bar{\Sigma}^- = \Sigma_- + \partial_{\text{nodeg}} \Sigma \) by indentifying \( \partial_{\text{nodeg}} \Sigma \) from \( \bar{\Sigma}^- \) with \( \partial_{\text{nodeg}} \Sigma \) from \( \bar{\Sigma}^+ \) using the identity map. Then, we obtain \( \hat{\Sigma} = \Sigma_+ \cup \Sigma^- \cup \partial_{\text{nodeg}} \Sigma \). The metric defined on \( \Sigma_+ \cup \Sigma^- \) can be extended by continuity to smooth metric on \( \hat{\Sigma} \), similarly this can be done with the electric field. Thus, we set

\[ \hat{g}_{ij} \big|_{\Sigma_+} = \tilde{g}_{ij} \quad \hat{g}_{ij} \big|_{\Sigma^-} = \tilde{g}_{ij}. \] (20)

and consequently

\[ \hat{E} \big|_{\Sigma_+} = \tilde{E}_+, \quad \hat{E} \big|_{\Sigma^-} = \tilde{E}_-. \] (21)

In the case of \( \partial_{\text{nodeg}} \Sigma = 0 \), one has that \( \hat{\Sigma} = \Sigma \), \( \hat{g}_{ij} = \tilde{g}_{ij} \) and \( \hat{E} = \tilde{E}_+ \).

A tedious but simple calculations can envisage the following fact:

\[ ^{(n-1)} \dot{R}_{ij}(\tilde{g}) = 2 \dot{E}_\alpha \dot{E}^\alpha, \] (22)

\[ ^{(\beta)} \nabla_\alpha \dot{E}^\alpha = 0, \] (23)

where \( ^{(\beta)} \nabla_\alpha \) is the covariant derivative with respect to the metric \( \tilde{g}_{ij} \). Eqs.(22) and (23) consist (as was observed for the first time in Ref. [32]) the constraint relations for a time symmetric problem.

Using the asymptotical behaviour of \( V \) and \( \phi \) it could be observed that as if we approach \( \Sigma_+ \) the line element \( ds^2_+ \) could be expressed as

\[ ds^2_+ = \left( 1 + \frac{\mu + Q}{(n-3)r^{n-1}} \right) dx_idx_i + \mathcal{O} \left( \frac{1}{r^{n-2}} \right). \] (24)

From Eq.(24) it follows in particular that, the ADM mass is equal to \( M_{ADM} = \frac{\mu + Q}{2} \). It can be verified that, the electric field is given by

\[ \dot{E}_\alpha = \frac{1}{2\sqrt{2}} \left[ - \sqrt{(n-3)(n-2)} \left( \frac{\mu + Q}{r^{n-2}} \right) \right] + \mathcal{O} \left( \frac{1}{r^{n-2}} \right). \] (25)

On \( \Sigma_- \) the metric is as follows:

\[ ds^2_- = \frac{1}{2^{2\beta}} \left[ \frac{\mu + Q}{r^{n-3}} \right]^{2\beta} dx_idx_i + \mathcal{O} \left( \frac{1}{r^{n-2}} \right). \] (26)

where \( \beta = \frac{1}{n-3} \).

Further on, one can introduce the coordinates \( x_I \), where \( I = 1, ..., N-2 \), so that the metric tensor \( g_{ij} \) can be rewritten in the form
\[ g_{ij} = \frac{1}{W^2}dV^2 + \gamma_{ij}dx_1dx^j. \]  
(27)

From the exact form of \( \hat{E}_{\alpha\pm} \) Eq.(18) and having in mind Eqs.(22) and (23) satisfied by them one can deduce that the gauge potential is a function of \( V \), namely \( \phi(V) \). This fact in turn enables us to rewrite equations of motion as follows:

\[
\frac{\partial \phi}{\partial V} = \frac{V}{W\sqrt{\gamma}} \frac{\partial}{\partial V} \left( W\sqrt{\gamma} \frac{\partial \phi}{\partial V} \right),
\]
(28)

\[
C^2 \left( \frac{\partial \phi}{\partial V} \right)^2 = \frac{V}{W\sqrt{\gamma}} \frac{\partial}{\partial V} \left( W\sqrt{\gamma} \right),
\]
(29)

where \( \gamma = \text{det} \gamma_{ij} \).

Accordingly to Eqs.(28) and (29) we obtain

\[
\frac{\partial \phi}{\partial V} - C^2 \left( \frac{\partial \phi}{\partial V} \right)^3 = V \frac{\partial^2 \phi}{\partial V^2}.
\]
(30)

From the above relation it follows in particular that

\[
\frac{\partial \phi}{\partial V} = \frac{aV}{\sqrt{1 + a^2 C^2 V^2}},
\]
(31)

where \( a \) is an integration constant. Using the asymptotic behaviour of \( V \) we get the following expression:

\[
\lim_{V \to 1} \frac{\partial \phi}{\partial V} = -\frac{(n - 3)Q}{\mu}.
\]
(32)

Integrating Eq.(31) and taking the limit \( \lim_{V \to 1} \phi = 0 \) we arrive at the following expression for \( \phi(V) \):

\[
\phi(V) = \frac{\mu - \sqrt{\mu^2 + Q^2(n - 3)^2(V^2 - 1)}}{(n - 3)CQ}.
\]
(33)

By virtue of Eq. (22) one can see that the Ricci tensor \( (n - 1) \hat{R}_{ij}(\hat{g}) \) on the hypersurface is manifestly non-negative. Furthermore the asymptotic behaviour of the metric \( \hat{g}_{ij} \) on \( \Sigma_+ \) becomes

\[
\hat{g}_{ij} = \delta_{ij} + O \left( \frac{1}{V^{n-2}} \right),
\]
(34)

While on \( \Sigma_- \) it reduces to

\[
\hat{g}_{ij} = \left[ \left( \frac{\mu + Q}{2} \right) \frac{1}{V^r} \right] \delta_{ij} + O \left( \frac{1}{V^2} \right).
\]
(35)

It follows directly from Eq.(34) that the total mass on \( \hat{\Sigma} \) vanishes. Thus, as a consequence of the positive mass theorem [36], the manifold \( \hat{\Sigma} \) is isometric to flat manifold. One can rewrite \( g_{ij} \) in a conformally flat form [27]

\[
g_{ij} = \mathcal{U} \frac{\pi^2}{r^2} \delta_{ij},
\]
(36)

where we have defined a smooth function on \( (\hat{\Sigma}, \delta_{ij}) \), namely \( \mathcal{U} = \frac{2}{1 + \sqrt{1 + \gamma_{ij}}} \). One can show that the Einstein-Maxwell equations of motion reduces to the Laplace equation on the \( (n - 1) \) Euclidean manifold \( \nabla_i \nabla^i \mathcal{U} = 0 \), where \( \nabla \) is the connection on a flat manifold. Having in mind the above we can adopt for the metric \( \delta_{ij} \) in the flat base space the following metric:
\[ \delta_{ij}dx^i dx^j = \tilde{\rho}^2 dt^2 + \tilde{h}_{AB} dx^A dx^B. \] 

(37)

First we shall consider the case of the single horizon. The event horizon is located at \( U = 2 \) and one can show that the embedding of \( \mathcal{H} \) into the Euclidean \((n - 1)\) space is totally umbilical [31]. This embedding must be hyperspherical, i.e., each of the connected components of the horizon \( \mathcal{H} \) is a geometric sphere with a certain radius determined by the value of \( \rho \mid_\mathcal{H} \), where \( \rho \) is the coordinate which can be introduced on \( \Sigma \) as follows:

\[ g_{ij} dx^i dx^j = \rho^2 dV^2 + h_{AB} dx^A dx^B. \]

(38)

One can always locate one connected component of the horizon at \( r = r_0 \) surface without loss of generality. Thus, we have to do with a boundary value problem for the Laplace equation on the base space \( \Omega = E^{n-1}/B^{n-1} \) with the Dirichlet boundary condition \( U \mid_\mathcal{H} = 2 \) and the asymptotic decay condition \( U = 1 + \mathcal{O}\left(\frac{1}{r^{n-3}}\right) \). Suppose further that \( U_1 \) and \( U_2 \) be two solutions of the boundary value problem. By means of the Green identity and integration over the volume element we reach to the following expression:

\[ \left( \int_{r \to \infty} - \int_\mathcal{H} \right) \left( U_1 - U_2 \right) \frac{\partial}{\partial r} \left( U_1 - U_2 \right) dS = \int_\Omega | \nabla \left( U_1 - U_2 \right) |^2 d\Omega. \]

(38)

Because of the boundary condition the left-hand side vanishes, and we draw a conclusion that two solutions must be identical.

The case of not single horizon can be treated as in Ref. [27,28]. Namely, one should consider the evolution level surface in Euclidean space. From the Gauss equation in Euclidean space we shall obtain the evolution equation for shear \( \sigma_{AB} \). Making use of the harmonicity of \( U \), i.e., \( \nabla^2 U = 0 \), we can draw a conclusion that

\[ \sigma_{AB} = 0, \quad \tilde{D}_A \rho = 0, \quad \tilde{D}_A k = 0, \] 

(39)

where \( \tilde{D}_A \) denotes the covariant derivative on each level set of \( V \), \( k_{AB} \) is the second fundamental form of the level set. This implies in turn that each level surface of \( U \) is totally umbilic and hence spherically symmetric.

As was mentioned in Ref. [27,28] this is the local result, since one consider only the region without saddle points of the harmonic function \( U \). In order to achieve the global result one should take into account the assumption about analyticity.

Now we turn to the problem of a charge of the connected components of black hole event horizon and its generalization to the non-connected case. In four-dimensions Heusler [35] showed that if all horizons were degenerate and \( Q_i Q_j \geq 0 \), where \( Q_i \) was the charge of the adequate connected component of black hole, then the black hole was a standard MP black hole. One should has in mind that a standard connected MP black hole is an extreme RN one [37].

First, if we suppose that \( S_a \), where \( a = 1, 2 \), is a connected components of \( \partial \Sigma \) such that the electrical potentials of the horizons \( \phi_a = \phi \mid_{S_a} \) imply

\[ \phi_1 = \inf_{\Sigma} < 0, \quad \phi_2 = \sup_{\Sigma} > 0. \] 

(40)

Then as was shown in [10] \( Q_a \) of the \( S_a \) are non-vanishing and have the opposite signs. It can be verified by writing out the expression for the charge \( Q_a \).
\[
Q_a = -\lim_{i \to \infty} \int_{S_{a,i}} \frac{C}{V} \mathbf{\nabla} \phi dS_i.
\]  

(41)

and using the divergence theorem and maximum principle (see [10] for the details).

By exactly similar arguments as presented in work [10] we can treat the case of a non-connected \( \partial \Sigma \). Finally, the above considerations enable us to formulate the main conclusion of our work.

**Theorem:**

Let us consider a static solution to the \( n \)-dimensional Einstein-Maxwell equations of motion with an asymptotically timelike Killing vector field \( k_\mu \). Suppose further that the manifold under consideration consists of a connected and simply connected spacelike hypersurface \( \Sigma \) to which \( k_\mu \) is orthogonal. The topological boundary \( \partial \Sigma \) of \( \Sigma \) is a nonempty topological manifold with \( h_{ij}k^i k^j = 0 \) on \( \partial \Sigma \). Then, one arrives at the following conclusions:

1. If \( \partial \Sigma \) is connected, then there exist a neighbourhood of \( \Sigma \) which is diffeomorphic to an open subset of \( n \)-dimensional Reissner-Nordström spacetime (extreme or non-extreme).

2. If \( \partial \Sigma \) is not connected and moreover we have fulfilled the following inequality:

\[
\forall_{i,j} Q_i Q_j \geq 0,
\]

(42)

where \( Q_i \) is the charge of the adequate component of \( \partial \Sigma \), that intersects the degenerate horizon, then there is an open neighbourhood of \( \Sigma \) which is diffeomorphic to \( n \)-dimensional Majumdar-Papapetrou spacetime.

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