The $(g - 2)_\mu$ anomaly, Higgs bosons and heavy neutrinos

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Abstract

Within the model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group and having the bidoublet and two triplets of the Higgs fields (left-right model) the Higgs sector impact on the value of the muon anomalous magnetic moment (AMM) is considered. The contributions coming from the doubly charged Higgs bosons, the singly charged Higgs bosons and the lightest neutral Higgs boson are taken into account. The obtained value of the muon is the function of the Higgs boson masses and the Higgs boson couplings constants (CC’s). We express the most of part of the CC’s as the functions of the heavy neutrino sector parameters. We show that at the particular parameters values the model under study could explain the BNL’00 result.
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1 Introduction

Measurements of the spin magnetic dipole moment of particles have a rich history as harbingers of impressive progress in the quantum theory. Thus, registration of the anomalous values of the nucleons magnetic moments was powerful argument for the benefit of the $\pi$-mesonic theory of the nuclear forces formulated by Yukawa. Determination of the anomalous magnetic moment (AMM) of the electron has played an important role in development of modern quantum electrodynamics and renormalization theory. It appeared to be reasonable that the ongoing muon $(g - 2)_\mu$ measurement E821 at Brookhaven National Laboratory (BNL) on Alternating Gradient Synchrotron would be a sensitive test for the results of the standard model (SM) electroweak corrections. Because this experiment is the culmination of a series of measurements of ever-increasing accuracy over the
past several years then its results has triggered the interest in theoretical calculations of
the muon AMM. The first BNL result based on the data taken through 1997 was [1]

\[ a_{\mu}^{\text{exp}} = (116 \, 592 \, 500 \pm 1500) \times 10^{-11} \mu_0 \quad (\text{BNL’97}), \]

where \( \mu_0 \) is the muon magnetic moment predicted by the Dirac’s theory. The 1998 and
1999 runs had the much higher statistics and gave the results with the increased precession

\[ a_{\mu}^{\text{exp}} = (116 \, 591 \, 910 \pm 590) \times 10^{-11} \mu_0 \quad (\text{BNL’98 [2]}). \]
\[ a_{\mu}^{\text{exp}} = (116 \, 592 \, 020 \pm 160) \times 10^{-11} \mu_0 \quad (\text{BNL’99 [3]}). \]

The BNL’98 and BNL’99 results averaged with older measurements made at CERN [4]
brought to the following value of the muon AMM

\[ a_{\mu}^{\text{exp}} = (116 \, 592 \, 023 \pm 151) \times 10^{-11} \mu_0, \]

In the SM the expression for the muon AMM can be presented as a sum

\[ a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{had}}, \]

in which \( a_{\mu}^{\text{QED}} = 11 \, 658 \, 470.57(0.29) \times 10^{-10} \mu_0 \) (see [5] and references therein) and \( a_{\mu}^{\text{EW}} = 15.2(0.4) \times 10^{-10} \) (see [6] and references therein).

The term \( a_{\mu}^{\text{had}} \) arises from virtual hadronic contributions to the photon propagator
in 4\(^{\text{th}}\) \( a_{\mu}^{\text{had}}(VP1) \) and 6\(^{\text{th}}\) order, where the latter includes hadronic vacuum polarization \( a_{\mu}^{\text{had}}(VP2) \) and light-by-light scattering \( a_{\mu}^{\text{had}}(LbyL) \). The dominant contribution to \( a_{\mu}^{\text{had}} \) as well as one of the largest ambiguities in its value come from the \( a_{\mu}^{\text{had}}(VP1) \). The \( a_{\mu}^{\text{had}}(VP1) \) has been derived in Ref. [7] from the \( e^+e^- \) hadronic cross section and the hadronic \( \tau \) decay data

\[ a_{\mu}^{\text{had}}(VP1) = 6 \, 924(62) \times 10^{-11}. \]

Evolution of 3-loop hadron vacuum polarization contribution \( a_{\mu}^{\text{had}}(VP2) \) has given the result [8]

\[ a_{\mu}^{\text{had}}(VP2) = -100(6) \times 10^{-11} \mu_0. \]

It is important to keep in mind that all the estimations of the \( LbyL \) scattering contribution \( a_{\mu}^{\text{had}}(LbyL) \) made so far are the model dependent. The calculations are based on the chiral perturbation or extended Nambu-Jona-Lasinio model. Also the vector meson dominance is assumed and the phenomenological parametrization of the pion form factor \( \pi\gamma^*\gamma^* \) is introduced in order to regularize the divergence. The previous average value for \( a_{\mu}^{\text{had}}(LbL) \) is given by [9,10]

\[ a_{\mu}^{\text{had}}(LbyL) = -85(25) \times 10^{-11} \mu_0. \]

With this value of the \( LbyL \) hadronic correction the total SM prediction of \( a_{\mu}^{\text{SM}} \) was

\[ a_{\mu}^{\text{SM}} = 116 \, 591 \, 597(67) \times 10^{-11} \mu_0. \]
Comparing Eq. (9) with the experimental average in Eq. (4) one could find
\[ \delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = 426(165) \times 10^{-11} \mu_0. \]  
Eq. (10) means that there is the 2.6 \( \sigma \) deviation between experiment and the SM prediction.

Recently the theoretical prediction for the \( a_\mu^{had}(LbyL) \) has undergone a significant revision because of the change in sign. The re-evaluations have given the following values for \( a_\mu^{had}(LbyL) \)
\[
a_\mu^{had}(LbyL) = \begin{cases} 
83(12) \times 10^{-11} \mu_0 \quad [11], \\
89(15) \times 10^{-11} \mu_0 \quad [12], \\
83(32) \times 10^{-11} \mu_0 \quad [13].
\end{cases}
\]
Taking the average of these new results one finds
\[ a_\mu^{SM} = 116,591,770(70) \times 10^{-11} \mu_0. \]  
Using Eqs. (4) and (12) one could obtain
\[ \delta a_\mu = 260(160) \times 10^{-11} \mu_0. \]
Thus the deviation value has dropped from 2.6 \( \sigma \) up to 1.6 \( \sigma \).

On July 30, 2002 Muon g-2 Collaboration announced the new result based on the \( \mu^+ \) data collected in the year 2000 [14]
\[ a_\mu^{exp} = 116,592,040(70)(50) \times 10^{-11} \mu_0. \quad \text{(BNL’00)} \]
An uncertainty of BNL’00 is almost two times smaller than in BNL’99 and only two times larger than the final aim of the E821 experiment. With this new result the present world average experimental value is
\[ a_\mu^{exp} = 116,592,030(80) \times 10^{-11} \mu_0. \]  
The improved calculations of the \( a_\mu^{had}(VP1) \) have been presented recently [15,16]. These are data-driven analysis using the most recent data from the \( e^+e^- \) hadronic cross section observed at CMD-2, BES, SND [17]. Their precision \( \sim 58 \times 10^{-11} \) are now even smaller than those in Eq. (6). Further on for the \( a_\mu^{had}(VP1) \) we shall use the result of Ref. [15] where the experimental input is based only on the \( e^+e^- \) data
\[ a_\mu^{had}(VP1) = 6,889(58) \times 10^{-11} \mu_0. \]
For the estimation of the \( a_\mu^{had}(LbyL) \) we invoke the new result obtained in [18]
\[ a_\mu^{had}(LbyL) = 80(40) \times 10^{-11} \mu_0. \]  
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Then with help of Eqs. (7), (16) and (17) the full hadronic contributions is given by

\[ a_{\mu}^{\text{had}} = 6.869(71) \times 10^{-11} \mu_0. \]  

(18)

This leads us to the SM prediction

\[ a_{\mu}^{\text{SM}} = 116.591 \ 726.7(70.9) \times 10^{-11} \mu_0. \]  

(19)

So, at present the deviation between experimental data and the SM prediction reached the value

\[ \delta a_{\mu} = 303.3(106.9) \times 10^{-11} \mu_0, \]  

(20)

that is, the deviation is roughly about 3\sigma.

Since the E821 data have been thoroughly collected and studied over many years, it is most unlikely that this discrepancy could be also explained as a mere statistical fluctuation, as several earlier deviations from the SM turned out to be. Attention is drawn to the fact of the extremely small variation of the muon AMM central value in all the BNL results presented up to now. This circumstance could be the weighty argument in favour of a trustworthiness of the E821 experiment. While it is often argued that the SM should be augmented by New Physics at higher energy scales because of some unanswered fundamental questions, the \((g − 2)_\mu\) anomaly with such phenomena as the neutrino oscillations [19], 3\sigma departure of \(\sin^2 \theta_W\) from the SM predictions measured in the deep inelastic neutrino-nucleon scattering [20], and the observation of the neutrinoless double beta decay [21] may serve as the New Physics signal already at the weak scale. If the deviation of Eq. (20) can be attributed to effects of the physics beyond the SM, then at 95\% \(CL\), \(\delta a_{\mu}/\mu_0\) must lie in the range

\[ 93.8 \times 10^{-11} \leq \frac{\delta a_{\mu}}{\mu_0} \leq 512.8 \times 10^{-11}. \]  

(21)

This contribution is positive, and has the same order as the electroweak corrections to \(a_{\mu}\), namely \(\sim G_F m_{\mu}^2/(4\pi^2\sqrt{2})\).

Suggestions already made in literature for explaining \(\delta a_{\mu}/\mu_0\) include supersymmetry [22], additional gauge bosons [23], anomalous gauge boson couplings [24], leptoquarks [25], extra dimensions [26], muon substructure [27], exotic flavour-changing interactions [28], exotic vectorlike fermions [29], possible nonpertubative effects at the 1 TeV order [30] and so on.

Amongst explanations of E821-experiment at Brookhaven AGL the approach based on the possibility of the violation of \(CPT\) and Lorentz invariance (see, for example [31]) should be particularly noted. In E821-experiment \(a_{\mu}\) is determined by measuring the difference \(\omega_a\) between the spin precession angular frequency \(\omega_s\) and the cyclotron angular frequency \(\omega_c\) of highly polarized muons in a storage ring with a uniform magnetic field. The field strength is determined from the nuclear magnetic resonance (NMR) frequency of
protons in water, calibrated relative to the free proton NMR frequency $\omega_p$. The quantity $a_\mu$ is determined via

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p},$$  \number{22}

where the ratio $\mu_\mu/\mu_p$ is taken from the measurement by W. Liu et al. \[32\]. According to Ref. \[31\] CPT/Lorentz violating terms in the Lagrangian induce a shift $\delta\omega_a$ to frequency $\omega_a$. This shift is predicted to be different for positive and negative muons and to oscillate with the Earth’s sidereal frequency. The level of CPT/Lorentz violating effects is characterized by the dimensionless quantity $r = \omega_a/m_\mu$ which interpret $\delta\omega_a$ as an muon energy shift in respect to the rest energy $m_\mu$. Already with the 1999 data set $r$ could be probed down to the level of $0.19 \times 10^{-22}$. Nowadays the common believe is that CPT and Lorentz invariance are the immovable laws of Nature and, therefore, detecting of their violation must lead to the radical alterations of the contemporary quantum field theory.

Some of explanations of E821-experiment turn out to be excluded by the current experimental data. To cite some examples. The possibility of muon substructure can be immediately ruled out since the necessary compositeness scale of muon should already have been seen in processes involving high energetic muons at LEP, HERA, and the Tevatron.

For anomalous $W$-boson dipole magnetic moment

$$\mu_W = \frac{e}{2m_W} (1 + \kappa_\gamma)$$

the additional one loop contribution to $a_\mu$ is given by the expression

$$a_\mu(\kappa_\gamma) \approx \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \ln \left( \frac{\Lambda^2}{m_W^2} \right) (\kappa_\gamma - 1),$$

where $\Lambda$ is the high momentum cutoff required to give a finite result. For $\Lambda \approx 1$ TeV, in order to obtain the accord between theory and observation one should demand

$$\delta\kappa_\gamma \equiv \kappa_\gamma - 1 \approx 0.4.$$  \number{4.}

However such a big values of $\delta\kappa_\gamma$ is already eliminated by $e^+e^- \rightarrow W^+W^-$ data at LEP II which gives \[33\]

$$\delta\kappa_\gamma = 0.08 \pm 0.17.$$  \number{5.}

In this manner, at the moment the $(g - 2)_\mu$ anomaly plays the role of an Occam’s razor for the existing SM extensions.

The purpose of this work is to investigate the $(g - 2)_\mu$ anomaly within the left right model (LRM) based on the gauge group $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$. One-loop contributions to $a_\mu$ from extra gauge bosons have been calculated in \[34\]. However, the contribution coming from the $Z_2$ gauge boson is negative while in order to accommodate the discrepancy in Eq. (20) the mass value of the $W_2$ gauge boson should lies around 100
GeV, which is clearly ruled out by direct searches and precision measurements [33]. In the LRM the Higgs bosons may apply to the role of the following candidates which may give significant contributions to the muon AMM.

In the SM, the Higgs boson contribution to $a_\mu$ is negligible because $\mu \mu h$ coupling is extremely small, namely $\sim m_\mu/v$, where $v$ is a vacuum expectation value being equal to 246 GeV. In the LRM the Higgs sector is much richer than in the SM. It includes four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $h^{(\pm)}$, and $\tilde{\delta}^{(\pm)}$, four neutral scalars $S_i$ ($i = 1, 2, 3, 4$) and two neutral pseudoscalars $P_{1,2}$. The current experimental data allow some of these Higgs bosons to have masses around the electroweak scale and couplings of at least electroweak strength. It is well to bear in mind that amongst the extensions of the SM the LRM is of special interest because its Higgs sector contain the elements belonging to other most popular nowadays models. The presence of the bidoublet in the LRM causes the existence of the same physical Higgs bosons as in the two Higgs doublet modification of the SM (THDM) [35] and in the MSSM [36]. Owing to the availability of the triplets the LRM has the Higgs bosons which are present in the model based on the $SU(3)_L \times U(1)_N$ gauge group [37].

One more a fascinating property of the LRM resides in the fact that the LRM belongs among the models in which the Higgs bosons coupling constants (CC’s) determining the interaction of the Higgs bosons both with leptons and with gauge bosons are connected to the neutrino oscillation parameters (NOP’s). Therefore, in such models the obtained bounds on the Higgs sector parameters could be extended to the bounds on the NOP’s.

The paper is organized as follows. In the next Sect. the one-loop electroweak corrections to the muon AMM caused by the LRM Higgs bosons are calculated. There we establish the connection between the CC’s and the NOP’s. Then comparing the theoretical and the experimental values of $a_\mu$ we find the bounds on the Higgs sector parameters which provide in its turn information on the heavy neutrino masses and the mixing angles. Sec.4 is devoted to analysis of the results obtained.

2 Higgs bosons corrections to $a_\mu$

In the LRM the choice of the Yukawa potential has the influence upon the form of the Lagrangian describing the Higgs boson interactions both with fermions and gauge bosons. The most general Yukawa potential $\mathcal{L}_Y^n$ has been proposed in [38]. In spite of the fact that $\mathcal{L}_Y^n$ has the very complicated form the diagonalization of the charged Higgs bosons mass matrix presents no special problems. However, for the neutral Higgs bosons mass matrix $M_n$ this procedure could be only realized when some simplifications in $\mathcal{L}_Y^n$ have been done [39]. For example, the matrix $M_n$ could be diagonalized at the following conditions (we use the same notation as in Ref.[38])

$$\alpha_1 = \frac{2\alpha_2 k_2}{k_1}, \quad \alpha_3 = \frac{2\alpha_2 k_2}{k_1 k_2}, \quad \beta_1 = \frac{2\beta_2 k_2}{k_1},$$

(23)
where $\alpha_{1,2,3}$ and $\beta_{1,2}$ are the constants entering the Yukawa potential, $k_1$ and $k_2$ are the vacuum expectation values (VEV’s) of the neutral components of the Higgs bidoublet and $k_+ = \sqrt{k_1^2 + k_2^2}$ ($k_+ = 174$ GeV).

Of all the Higgs bosons, $\Delta^{(\pm)}$, $h^{(\pm)}$ and $S^{(\pm)}$- and $S_1$-bosons have been of our main interest here because the current data allow their masses to lie on the electroweak scale (recall that the $S_1$-boson is the analog of the SM Higgs boson). Assuming the conditions (23) to be fulfilled one obtain the squared masses of these particles

\[
m^2_{R} = \alpha(v_R^2 + k_0^2) + \frac{\beta_1^2 k_+^4 k_0^2}{k_+^4 (\alpha + \rho_1 - \rho_3/2)}, \tag{24}
\]

\[
m^2_{\delta} = (\rho_3/2 - \rho_1)v_R^2 - \frac{\beta_1^2 k_+^4 k_0^2}{k_+^4 (\alpha + \rho_1 - \rho_3/2)}. \tag{25}
\]

\[
m^2_{\Delta_1} = \frac{\alpha_3 k_+^2 + 4\rho_2 v_R^2}{2} + \frac{\beta_1^2 k_+^4 (\beta_3 k_+^2 + \beta_1 k_1 k_2)^2}{2k_1^4 (4\rho_2 + \rho_3 - 2\rho_1)v_R^2}, \tag{26}
\]

\[
m^2_{\Delta_2} = \alpha_3 k_+^2 - (2\rho_1 - \rho_3)v_R^2 - \frac{\beta_1^2 k_+^4 (\beta_3 k_+^2 + \beta_1 k_1 k_2)^2}{2k_1^4 (4\rho_2 + \rho_3 - 2\rho_1)v_R^2}, \tag{27}
\]

\[
m^2_{S_1} = 2\lambda_1 k_+^2 + 8k_1^2 k_2^2(2\lambda_2 + \lambda_3)/k_+^2 - 8\lambda_4 k_1 k_2 + \frac{4k_1 k_2 k_4^2[2(2\lambda_2 + \lambda_3)k_1 k_2/k_+^2 - \lambda_4]}{\alpha_2 v_R^2 k_+^2} \tag{28},
\]

where

\[
\alpha = \frac{\alpha_3 k_+^2}{2k_+^2} = \frac{\alpha_3(1 + \tan^2 \beta)}{2(\tan^2 \beta - 1)}, \quad \beta_0 = \frac{\beta_1 k_+^2}{k_+^2} = \frac{\beta_1(1 + \tan^2 \beta)}{(\tan^2 \beta - 1)},
\]

\[
k_0 = \frac{k_+}{\sqrt{2k_+}} = \frac{k_+(\tan^2 \beta - 1)}{\sqrt{2(\tan^2 \beta)}},
\]

$\rho_{1,3}$ are the constants entering the Yukawa potential, $\tan \beta = k_1/k_2$ and $v_R$ is the VEV of the neutral component of the right-handed Higgs triplet, $v_R \gg \max(k_1, k_2)$. From the relations (25) and (27) it follows that the masses $\tilde{\delta}^{(\pm)}$ and $\Delta^{(\pm)}$-bosons are very close to each other. For the $h^{(\pm)}$, $\tilde{\delta}^{(\pm)}$ and $\Delta_{1,2}^{(\pm)}$-boson masses to be around the electroweak scale the constants $\alpha_3$, $\rho_2$ and $(\rho_3/2 - \rho_1)$ should have the order of $\sim 10^{-2}$ which follows from the expressions (24) — (27).

The Lagrangians which are required for our purposes are given by the expressions

\[
\mathcal{L}_{\gamma\Delta} = 2ie[(\partial_\mu \Delta_1^{(-)})^*(x))\Delta_1^{(-)}(x) - \Delta_1^{(-)}(x)\partial_\mu \Delta_1^{(-)}(x))] + (1 \to 2), \tag{29}
\]

\[
\mathcal{L}_{dc}^l = -\sum_{a,b} \frac{f_{ab}}{2}[\bar{T}_a(x)(1 + \gamma_5)l_b(x)c_\theta_d - \bar{T}_a(x)(1 - \gamma_5)l_b(x)s_\theta_d] \Delta_1^{(-)}(x) + (1 \to 2, \theta_d \to \theta_d - \frac{\pi}{2}) + \text{conj.}, \tag{30}
\]
\[ \mathcal{L}_{W_{1\gamma}} = \frac{g_R e m_W (1 - \tan^2 \beta)(\alpha - \rho_3/2 + \rho_1 + 1)s_\xi}{g_L (1 + \tan^2 \beta)} h^{(-)*}(x) W_{1\mu}(x) A_\mu(x) + \text{conj.,} \quad (31) \]

\[ \mathcal{L}_{W_{2\gamma}} = \mathcal{L}_{W_{1\gamma}}(s_\xi \to c_\xi), \quad \mathcal{L}_{W_{1\delta}} = g_R e g_L^2 \beta_1 m_W s_\xi \delta^{(-)*}(x) W_{1\mu}(x) A_\mu(x) + \text{conj.,} \quad (33) \]

\[ \mathcal{L}_{W_{2\delta}} = \mathcal{L}_{W_{1\delta}}(s_\xi \to c_\xi), \quad (34) \]

\[ \mathcal{L}_i^{se} = \sum_{a,b} \left[ \frac{h_{ab}' k_2 - h_{ab} k_1}{2k_+} \mathcal{T}_a(x)(1 - \gamma_5) l_b(x) - \frac{h_{ab} k_2 - h_{ab}' k_1}{2k_+} N_a(x)(1 + \gamma_5) l_b(x) h^{(-)*}(x) + \frac{f_{ab}}{\sqrt{2}} \mathcal{T}_a(x)(1 + \gamma_5) \nu_{e\mu}(x) \left( \frac{\beta_0 k_0}{(\alpha + \rho_1 - \rho_3/2)v_R} h^{(-)*}(x) - \delta^{(-)*}(x) \right) + \mathcal{T}_a(x)(1 - \gamma_5) N_b(x) \left( \frac{k_0}{v_R} h^{(-)*}(x) + \frac{\beta_0 k_0}{(\alpha + \rho_1 - \rho_3/2)v_R} \delta^{(-)*}(x) \right) \right] + \text{conj.}, \quad (35) \]

\[ \mathcal{L}^n = -\frac{1}{\sqrt{2}k_+} \left\{ \sum_{a,b} \mathcal{T}_a(x) l_b(x) [(h_{ab} k_1 + h_{ab}' k_2) s_{\theta_d} + (h_{ab}' k_1 - h_{ab} k_2) c_{\theta_d}] S_1(x) \right\}, \quad (36) \]

where the superscript c denotes the charge conjugation operation, \( c_{\theta_d} = \cos \theta_d, s_{\theta_d} = \sin \theta_d \), \( \theta_d \) is the mixing angle of the doubly charged Higgs bosons (\( \tan \theta_d \sim k_2^2/v_R^2 \)), \( f_{ab} \) is the Yukawa triplet coupling constant, \( g_R \) is the gauge coupling of the SU(2)_R subgroup (further we shall speculate that \( g_L = g_R \)). \( N_a(x) \) describes the heavy neutrino with the flavor \( a \), \( \xi \) is the mixing angle of the charged gauge bosons, and the angle \( \theta_0 \) is determined by the Yukawa potential parameters and the VEV’s

\[ \tan 2\theta_0 = \frac{4 k_1 k_2 k^2_2 [-2(2\lambda_2 + \lambda_3) k_1 k_2 + \lambda_4 k_2^2]}{k_1 k_2 [(4\lambda_2 + 2\lambda_3)(k_1^2 - 4k_1^2 k_2^2) - k_2^2 (2\lambda_1 k_2^2 - 8\lambda_4 k_1 k_2)] + \alpha_2 v_R^2 k_2^4}. \quad (37) \]

The influence of the Yukawa potential choice on the physical results could be easily seen by the example of the Lagrangian (36). Really, when in the condition (23) the change \( k_2 \to -k_2 \) is carried out, then instead of (36) one obtains

\[ \mathcal{L}^n = -\frac{1}{\sqrt{2}k_+} \left\{ \sum_a m_a \mathcal{T}_a(x) l_a(x) c_{\theta_d} + \sum_{a,b} \mathcal{T}_a(x) l_b(x) (h_{ab} k_1 - h_{ab}' k_2) s_{\theta_d} \right\} S_1(x). \quad (38) \]

Since \( \tan \beta_0 \sim k_2^2/v_R^2 \) and for the muon \( m_\mu/k_+ \sim 6 \times 10^{-3} \), then, as the example, the cross section of the electron-muon recharge

\[ e^- \mu^+ \rightarrow e^+ \mu^-, \]

never could have the resonance peak connected with the \( S_1 \)-boson when the Lagrangian (38) is used [39], while the existence of such a peak could be quite possible when one works with the Lagrangian (36) [40].

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We now proceed to the calculations of the contribution to the muon AMM caused by the Higgs bosons. The diagrams corresponding the exchange of the doubly charged Higgs bosons are shown in Fig. 1. They give the following corrections to the AMM value

$$\frac{\delta a_{\mu}^{(D)}}{\mu_0} = \frac{1}{8\pi^2} \left( 4f_{\mu e}^2 \sum_{i=1}^{2} I_{e i}^{D_i} + f_{\mu \mu}^2 \sum_{i=1}^{2} I_{\mu i}^{D_i} + 4f_{\mu \tau}^2 \sum_{i=1}^{2} I_{\tau i}^{D_i} \right),$$  \hspace{1cm} (39)

where

$$I_{i a}^{D_i} = \int_0^1 \left( \frac{2m_{\mu}^2(z^2 - z^3)}{m_{\mu}^2(z^2 - z) + m_{\Delta_i}^2 z + m_{\Delta_i}^2 (1 - z)} + \frac{m_{\mu}^2(z^2 - z^3)}{m_{\mu}^2(z^2 - z) + m_{\Delta_i}^2 (1 - z) + m_{\Delta_i}^2 z} \right) dz,$$

and $I_{i a}^{D_i} > 0$.

The singly charged Higgs bosons also influence the value of the AMM. The relevant diagrams are depicted in Fig. 2. For the diagrams which contain the loops with the $W_{1}^{\pm}$- and $h^{(\pm)}$-bosons the following relation takes place

$$\frac{M_{W_{1}^{\nu_{\mu} h}}}{M_{W_{2}^{\nu_{\mu} h}}} = s_{\xi},$$

where $M_{W_{1}^{\nu_{\mu} h}}$ ($M_{W_{2}^{\nu_{\mu} h}}$) are the matrix elements appropriate the diagrams with the exchange of the light (heavy) neutrino. As the mixing angle of the charged gauge bosons is very small $|\xi| \approx 10^{-2} - 10^{-5}$ [33], then one could neglect the contributions coming from the diagrams with the virtual heavy neutrino. Taking into account the analogous relations

$$\frac{M_{W_{2}^{\nu_{\mu} h}}}{M_{W_{2}^{\nu_{\mu} h}}} = s_{\xi}, \quad \frac{M_{W_{1}^{\nu_{\mu} h} h}}{M_{W_{2}^{\nu_{\mu} h} h}} = s_{\xi}, \quad \frac{M_{W_{2}^{\nu_{\mu} h} h}}{M_{W_{2}^{\nu_{\mu} h} h}} = s_{\xi},$$

we give the dominant contribution from the diagrams shown in Fig. 2 to the muon AMM

$$\frac{\delta a_{\mu}^{(hh)}}{\mu_0} = \frac{1}{8\pi^2} \sum_{a=e,\mu,\tau} \left( \alpha_{\mu N_{\alpha}}^2 h_{N_{\alpha}}^h h_{N_{\alpha}}^h I_{h h}^h + \alpha_{\mu \nu_{\alpha} h}^2 \right),$$  \hspace{1cm} (40)

$$\frac{\delta a_{\mu}^{(\delta \delta)}}{\mu_0} = \frac{1}{8\pi^2} \sum_{a=e,\mu,\tau} \left( \alpha_{\mu N_{\alpha}}^2 I_{N_{\alpha}}^{\delta \delta} I_{N_{\alpha}}^{\delta \delta} + \alpha_{\mu \nu_{\alpha} h}^2 \right),$$  \hspace{1cm} (41)

$$\frac{\delta a_{\mu}^{(W_{1}^{h})}}{\mu_0} = \frac{(\alpha - \rho_3/2 + \rho_1 + 1)(1 - \tan^2 \beta) s_{\xi} m_{W_{1}} \alpha_{\mu \nu_{\alpha} h} I_{W_{1}^{h}}}{16\sqrt{2}\pi^2(1 + \tan^2 \beta)},$$  \hspace{1cm} (42)

$$\frac{\delta a_{\mu}^{(W_{2}^{h})}}{\mu_0} = \frac{(\alpha - \rho_3/2 + \rho_1 + 1)(1 - \tan^2 \beta) c_{\xi} m_{W_{1}} \alpha_{\mu N_{\alpha} h} I_{W_{2}^{h}}}{16\sqrt{2}\pi^2(1 + \tan^2 \beta)},$$  \hspace{1cm} (43)

$$\frac{\delta a_{\mu}^{(W_{1}^{\delta})}}{\mu_0} = \frac{\beta_1 (1 - \tan^2 \beta) s_{\xi} m_{W_{1}} \alpha_{\mu \nu_{\alpha} h} I_{W_{1}^{\delta}}}{16\sqrt{2}\pi^2(1 + \tan^2 \beta)},$$  \hspace{1cm} (44)
\[ \frac{\delta a^{(W_2 \delta)}}{\mu_0} = \beta_1 (1 - \tan^2 \beta) \xi m_{W_1} \alpha_{\mu N_a} \delta I_{W_2 \delta}, \] 

(45)

where

\[ \alpha_{\mu \nu h} = \frac{h'_{ab} k_2 - h_{ab} k_1}{2k_+}, \quad \alpha_{\mu \xi} = \frac{h'_{ab} k_1 - h_{ab} k_2}{2k_+}, \quad \alpha_{\mu \nu h} = \frac{f_{ab}}{\sqrt{2}}, \]

\[ \alpha_{\mu \nu h} = \frac{f_{ab} \beta_0 k_0}{2(\alpha + \rho_1 - \rho_3/2)v_R}, \]

\[ I^{ hh}_i = \int_0^1 \frac{m^2 \rho (z^3 - z^2)dz}{m^2 + (m^2 - m^2)z + m^2}, \quad \alpha_{\mu \nu h} = \frac{f_{ab} \beta_1 k_+}{2(\alpha + \rho_1 - \rho_3/2)v_R}, \]

\[ I^{ S_h}_i = I^{ hh}_i (m_h \rightarrow m_{\delta}), \quad I^{ hh}_i < 0 \]

\[ I^{ W_1 h} = \frac{m^2}{m_1^2 - m_\mu^2} \left[ \ln \left( \frac{m_1^2}{m^2} \right) - \int_0^1 \frac{z^2 [m^2 (2z - 1) + m_1^2]dz}{m^2 + (m^2 - m_\mu^2)z + m_\mu^2} \right] + (m_{W_1} - m_{W_2}), \]

\[ I^{ S_1} = \int_0^1 \frac{m^2 (z^2 - 1)}{m^2 + m_{S_1}^2 (1 - z) + m_{S_1}^2 z}, \quad I^{ S_1} > 0. \]

The total correction value motivated by the Higgs bosons to the muon AMM \( \delta a_{\mu} / \mu_0 \) is defined by the sum of the expressions (39) — (46). To perform an exhaustive analysis of the obtained result one should have information both about the coupling constants \( \alpha_{L_a L_b H_i} (L_a = \nu_a, N_a) \) and the Higgs boson masses \( m_{H_i} \). At present such information follows from looking for the deviations from the SM predictions. It is usually reported in terms of the upper limits for quantities of the type \( \alpha_{L_a L_b H_i} / m_{H_i} \), or that is more frequent, for quantities of the type

\[ \sum_i C_i \epsilon_i^{ab} \beta_i^{ab'} = \sum_i C_i \left( \frac{\alpha_{L_a L_b H_i} \alpha_{L_b' L_b H_i}}{m^4_{H_i}} \right)^2, \]

where \( C_i \) are the constants (see, for review [41]). As a rule the determination of the upper bound only for one quantity \( \alpha_{L_a L_b H_i} / m_{H_i} \) is a very involved task. As a case in point we consider the decay

\[ \mu^- \rightarrow \nu e^- \gamma. \] 

(47)
In the third order of the perturbation theory the appropriate diagrams follow from those in Figs.1, 2 and 3 when in the final state the muon is replaced by the electron. The width of the decay (47) includes the whole complex quantities $\epsilon_i^{ab\ell\nu}$. Besides, the presence of the diagrams with the singly charged Higgs bosons leads to the destructive interference of the reaction amplitudes that complicates a lot the task of extracting information about the diagrams with the singly charged Higgs bosons.

However, the Higgs bosons coupling constants $\alpha_{L\alpha LbH}$ could be expressed in terms of the lepton sector parameters. It is easily done at least in the two-flavor approximation. For this purpose we need the neutrino mass matrix $\mathcal{M}$. Once one chooses the basis $\Psi^T = (\nu^T_{aL}, N^T_{aR}, \nu^T_{bL}, N^T_{bR})$, the $\mathcal{M}$ takes the form

$$\mathcal{M} = \begin{pmatrix}
    f_{aa}v_L & m_D^a & f_{ab}v_L & M_D \\
    m_D^a & f_{aa}v_R & M_D & f_{ab}v_R \\
    f_{ab}v_L & M_D & f_{bb}v_L & m_D^b \\
    M_D & f_{ab}v_R & m_D^b & f_{bb}v_R
\end{pmatrix},$$

(48)

where $v_L$ is the VEV of the neutral component of the left-handed Higgs triplet ($v_L \ll \max(k_1, k_2)$) and

$$m_D^a = h_{aa}k_1 + h'_{aa}k_2,$$

(49)

$$M_D = h_{ab}k_1 + h'_{ab}k_2.$$  

(50)

In its turn the elements of the matrix $\mathcal{M}$ are connected with the neutrino oscillations parameters [39,42]

$$m_D^a = c_{\phi a}s_{\phi a}(-m_1c_{\theta_\nu}^2 - m_3s_{\theta_\nu}^2 + m_2c_{\theta_\nu}^2 + m_4s_{\theta_\nu}^2),$$

(51a)

$$m_D^b = m_D^a(\phi_a \rightarrow \phi_b, \theta_{\nu N} \rightarrow \theta_{\nu N} + \pi),$$

(51b)

$$M_D = c_{\phi a}s_{\phi a}c_{\theta_\nu}s_{\theta_\nu}(m_1 - m_3) + s_{\phi a}c_{\phi a}c_{\theta_\nu}s_{\theta_\nu}(m_4 - m_2),$$

(52)

$$f_{ab}v_R = s_{\phi a}c_{\phi a}s_{\theta_\nu}c_{\theta_\nu}(m_3 - m_1) + c_{\phi a}c_{\phi a}s_{\theta_\nu}s_{\theta_\nu}(m_4 - m_2),$$

(53)

$$f_{aa}v_R = (s_{\phi a}c_{\theta_\nu})^2 m_1 + (c_{\phi a}c_{\theta_\nu})^2 m_2 + (s_{\phi a}s_{\theta_\nu})^2 m_3 + (c_{\phi a}s_{\theta_\nu})^2 m_4,$$

(54)

$$f_{bb}v_R = f_{aa}v_R(\phi_a \rightarrow \phi_b + \frac{\pi}{2}, \theta_N \rightarrow \theta_N + \frac{\pi}{2}),$$

$$f_{ll}\nu_L = f_{ll}\nu_R(\phi_{l,l'} \rightarrow \phi_{l,l'} + \frac{\pi}{2}), \quad l, l' = a, b,$$

(55)

where $\phi_a$ is the mixing angle in the $a$ generation between the light and the heavy neutrino entering into the left-handed and the right-handed lepton doublet

$$\begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L, \quad \begin{pmatrix} N_a \\ l_a \end{pmatrix}_R,$$

respectively, $\theta_{\nu}(\theta_N)$ is the mixing angle between the $\nu_{aL}$ and the $\nu_{bL}$ neutrino ($N_{aR}$ and $N_{bR}$), $c_{\phi a} = \cos \phi_a$, $s_{\phi a} = \sin \phi_a$ and so on. As $m_\nu \ll m_N$, then with the help of Eqs.(53)
and (55) it is possible to find the relationship for an estimation of the mixing angles between light and heavy neutrinos. Further on we shall assume that the mixing takes place between $\mu$ and $\tau$ generations ($a = \mu, b = \tau$) only. Then for the mixing angles we obtain

\[
\sin 2\varphi_\mu \approx \frac{f_{\mu\mu}\sqrt{v_R v_L}}{c_{\vartheta_N}^2 m_{N_\mu} + s_{\vartheta_N}^2 m_{N_\tau}},
\]

\[
\sin 2\varphi_\tau \approx \frac{f_{\tau\tau}\sqrt{v_R v_L}}{s_{\vartheta_N}^2 m_{N_\mu} + c_{\vartheta_N}^2 m_{N_\tau}}.
\]

The estimation of $v_L$ could be done with the help of quantity

\[
\rho = \frac{m_Z^2 c_W^2}{m_W^2}.
\]

In LRM the quantity $\rho$ is defined by the relation [43]

\[
\rho = \frac{1 + 4x}{1 + 2x},
\]

where

\[
x = \left(\frac{v_L}{k_+}\right)^2.
\]

As the experiment for today yields

\[
\rho = 1.0107 \pm 0.0006,
\]

that the value $v_L$ can reach 13 GeV.

Taking into consideration both the definition of $m_D^a$ (Eqs. (49), (51)) and the formulae for the charged lepton masses

\[
m_{l_a} = h_{aa}k_2 + h'_{aa}k_1
\]

it is not difficult to obtain

\[
\alpha_{\tau_{a\nu_a h}} = \frac{h_{aa}k_2 - h'_{aa}k_1}{2k_+} = \frac{1 + \tan^2 \beta}{2k_+(1 - \tan^2 \beta)} \left(\frac{2m_{l_a} \tan \beta}{1 + \tan^2 \beta} + m_D^a\right) \approx \frac{1 + \tan^2 \beta}{2k_+(1 - \tan^2 \beta)} \left[2m_{l_a} \tan \beta c_{\varphi_a} s_{\varphi_a} (m_2 c_{\vartheta_N}^2 + m_4 s_{\vartheta_N}^2)\right].
\]

The analogous mathematics for $\alpha_{\tau_{aN_a h}}$ lead to the expression

\[
\alpha_{\tau_{aN_a h}} = \frac{h'_{aa}k_1 - h_{aa}k_2}{2k_+} = \frac{1 + \tan^2 \beta}{2k_+(1 - \tan^2 \beta)} \left(\frac{2m_D^a \tan \beta}{1 + \tan^2 \beta} - m_{l_a}\right) \approx \frac{1 + \tan^2 \beta}{2k_+(1 - \tan^2 \beta)} \left[2c_{\varphi_a} s_{\varphi_a} (m_2 c_{\vartheta_N}^2 + m_4 s_{\vartheta_N}^2) \tan \beta\right].
\]
It is pertinent to note that there is the connection between the coupling constants \( \alpha_{laNbh} \) and \( \alpha_{laNbh}S_1 \)

\[
\alpha_{laNbh} \approx \frac{\alpha_{laNbh}S_1}{\sqrt{2}}. \tag{62}
\]

The next step is the determination of the non-diagonal Higgs bosons coupling constants. To suppress the mixing in the charged lepton sector (between \( l_a \) and \( l_b \)) it is necessary to demand

\[
h_{ab}k_2 + h_{ab}k_1 = 0. \tag{63}
\]

Then, with the regard to the definitions of the quantity \( M_D \) (Eqs. (45) and (47)) one obtains

\[
\alpha_{laNbh} = -\frac{M_D}{2k_+} \approx -\frac{s_{\phi_a}c_{\phi_b}\theta_N s_{\theta_N}(m_4 - m_2)}{2k_+}, \tag{64}
\]

\[
\alpha_{laNbh} = -\frac{M_D \tan \beta}{k_+(1 + \tan^2 \beta)} \approx -\frac{s_{\phi_a}c_{\phi_b}\theta_N s_{\theta_N}(m_4 - m_2) \tan \beta}{k_+(1 + \tan^2 \beta)}. \tag{65}
\]

From the expressions (60) — (62), (64) and (65) it is obvious that the values of the coupling constants \( \alpha_{laNbh} \) are basically defined by the oscillation parameters of the heavy neutrinos. Nowadays the information concerning the heavy neutrino sector is very poor. All we have is the upper bound for the heavy electron neutrino mass resulting from the experiments aimed at finding the neutrinoless double \( \beta \) decay

\[
m_{N_e} > 63 \text{ GeV} \left( \frac{1.6 \text{ TeV}}{m_{W_2}} \right)^4. \tag{66}
\]

Hence, our sole way out in an existing situation is to set any minimal number of parameters of heavy neutrinos sector, and other parameters to express through them with the help of the equations (51) — (55). As those we shall take \( m_{N_\mu}, m_{N_\tau} \) and \( \theta_N \).

Now we shall pass to the discussion of the constraints on the Higgs bosons masses. The current limit on the singly charged Higgs boson mass has been obtained within the THDM’s under investigation of the reaction

\[
e^+e^- \rightarrow H^+H^-. \tag{67}
\]

The lowest value for the mass of the charged Higgs boson, independent of the its branching ratio, is currently 78.6 GeV [33]. It is evident that this limit may be broken down to the singly charged Higgs bosons of the LRM \( \tilde{\delta}(\pm) \) and \( h(\pm) \). Really, in the THDM the charged Higgs boson interacts with the quarks at the tree level while in the LRM such interaction exists for the \( h^+(\pm) \)-boson only. Furthermore, in the both models the coupling constants of the charged Higgs bosons with the \( Z \)-boson are not equal to each other

\[
\frac{(g_{HHZ})_{2HDM}}{(g_{\delta\delta Z})_{LRM}} = \frac{\cot 2\theta_W}{g' \cos^{-1} \theta_W(\alpha + \rho_1 - \rho_3/2)(g'^{-1} \sin \theta_W \cos \Phi + g_R^{-1} \sin \Phi).} \tag{68}
\]
\[
\frac{(g_{HHZ})_{2HDM}}{(g_{hAZ})_{LRM}} = -\frac{\cot 2\theta_W}{\cos^{-1} \theta_W (\sin^{-1} \theta_W \cos 2\theta_W \cos \Phi + g_R g'^{-1} \sin \Phi) / 2}.
\]

However, since the analysis of the process (67) from the LRM point of view is absent up to now, we shall assume that the lower bound on the masses of the singly charged Higgs bosons of the LRM is 78.6 GeV too.

As regards the doubly charged Higgs boson mass the situation is somewhat more simple. The \(\Delta^{(\pm \pm)}\)-bosons are typical representatives of the LRM. For this reason the experiments aimed at their appearence are analyzed from the LRM point of view only. The current lower bounds on their masses obtained by OPAL Collaboration at 95\% CL [44] are 98.5 GeV.

We also should discuss the implementation of the lower bound 115 GeV on the mass of the lightest neutral Higgs (LNH) in the SM extensions. This bound has been obtained on LEP II under investigation of the reaction

\[
{e^+e^- \rightarrow Z^+ \rightarrow Zh,}
\]

from the SM point of view [45]. The reaction (70) is analyzed for the four \(Zh\) decay channels

\[
Zh \rightarrow q\bar{q}q'^\prime, q\bar{q}\nu\bar{\nu}, q\bar{q}_a\bar{\nu}_a (l_a = e, \mu), \tau^+\tau^- q\bar{q},
\]

where the final state \(h \rightarrow q\bar{q}\) includes both the quark-antiquark and the gluon-gluon pairs. For the THDM’s, as an example, the substantial deviations from the SM are present both in the cross section \(\sigma_{e^+e^- \rightarrow Zh}\) and in the decay widths \(\Gamma_h\). Since in these models the coupling constant of the neutral CP-even \(h\) boson (analog of the SM Higgs boson) with the \(Z\) boson has the form

\[
g_{ZZh} = \frac{g_L m_Z \sin(\beta - \alpha)}{\cos \theta_W}
\]

where \(\sin(\beta - \alpha) \sim 0\), then \((\sigma_{e^+e^- \rightarrow Zh})_{THDM}\) is much less relatively to the SM value. On the other hand, as the relation takes place

\[
\frac{(g_{f\bar{f}h})_{THDM}}{(g_{f\bar{f}h})_{SM}} \approx \tan \beta, \quad f = b, \tau.
\]

the \(h\) boson decay widths through quarks and leptons have greater values than in the SM. It is evident, that the analysis of the reaction (70) leads to the different mass values for the SM Higgs and the LNH of the THDM. Actually, when \(|\sin(\beta - \alpha)| \leq 0.06\) the LEP data result in \(m_h \sim 10\) GeV at 98\% CL [46].

The only reason for the LRM cross section \(\sigma_{e^+e^- \rightarrow ZS_1}\) not to coincide with that of the SM may be the coupling constant of the \(S_1\) boson with the \(Z_1\) boson

\[
g_{Z_1Z_1S_1} = \frac{g_L s_W^2 m_{Z_1} c_{\theta_0} (1 - t_g^2 \beta) - 2 s_{\theta_0} \tan \beta)}{2 c_W (\tan^2 \beta + 1)} (2 g_R g'^{-1} c_\Phi s_\Phi s_W^{-1} -
\]
\[ -c_\Phi^2 s_W^2 - g_{R}^2 g'^{-2} s_\Phi^2 \],

(72)

where \( \Phi \) is the mixing angle of \( Z_1 \) and \( Z_2 \) bosons \( (\Phi \approx 10^{-2} - 10^{-3}) \) and \( g' \) is the gauge constant of subgroup \( U(1)_{B-L} \). When

\[ \Phi = k_2 = 0 \]

then \( g_{Z_1 Z_1 S_1} \) converts to the constant describing the interaction between the Higgs bosons and the \( Z \) boson in the SM. Since the symmetric LRM reproduces the SM under the following values of \( g_R \) and \( g' \)

\[ g_L = g_R = e s_W^{-1}, \quad g' = e \sqrt{c_W^2 - s_W^2}, \]

(73)

then the quantity \( g_R g'^{-1} \) may moderately differ from unity. Therefore, the major contributor to the deviation \( g_{Z_1 Z_1 S_1} \) from their SM values is the factor

\[ \Delta g = \frac{[c_\theta_0 (1 - \tan^2 \beta) - 2 s_\theta_0 \tan \beta]}{(\tan^2 \beta + 1)}. \]

(74)

From the expression (37) follows that the angle value \( \theta_0 \) is basically determined by the parameter \( \alpha_2 \) which enters the Higgs potential. When \( \alpha_2 \approx 10^{-2} \) then the angle \( \theta_0 \) may reach the value \( \pi/4 \). At this condition the \( S_1 \) boson could remain light as usual but the \( S_2 \) boson ceases to be superheavy

\[ m_{S_2}^2 = \frac{\alpha_2 v_R^2 k_+^2}{k_1 k_2} - \frac{4 k_1 k_2 k_4^2 [2 (2 \lambda_2 + \lambda_3) k_1 k_2 / k_+^2 - \lambda_4]}{\alpha_2 v_R^2 k_+^2} \]

(75)

Recall, that the demand

\[ m_{S_2} \geq 10 \text{ TeV}, \]

(76)

is caused by the necessity to suppress at the tree level the flavor changing neutral currents (FCNC) in the Lagrangian

\[
\mathcal{L}_q^n = - \frac{1}{\sqrt{2} k_+} \sum_{a,b} \bar{u}_a \left\{ m_{u_a} \left[ c_\theta_0 + \frac{2 k_1 k_2}{k_+^2} s_\theta_0 \right] S_1 - m_{u_a} (s_\theta_0 - \frac{2 k_1 k_2}{k_+^2} c_\theta_0) S_2 - 
- \frac{k_+^2}{k_2} (\mathcal{K} \mathcal{M}_d \mathcal{K}^*)_{ab} (S_1 s_\theta_0 + S_2 c_\theta_0) \right\} u_b + 
+ (u_a \to d_a, m_{u_a} \leftrightarrow m_{d_a}, \gamma_5 \to -\gamma_5),
\]

(77)

where \( \mathcal{K} \) is the Cabibbo-Kobayashi-Maskawa matrix and \( \mathcal{M}_d \) is the diagonal mass matrix for the down quarks. The absence of the FCNC in its turn allows to describe properly the \( K^0 \leftrightarrow K^0 \) transitions. However, as it is shown in [39] the successfull LRM building
demands the redefinition of the traditional Yukawa Lagrangian for quarks. The expression (76) must be changed for

$$L_q^n = -\frac{1}{\sqrt{2}k_+} \sum_a \eta_a \left\{ m_{u_a} \left[ \left( c_{\theta_0} - \frac{k_1}{k_2} s_{\theta_0} \right) S_1 - \left( s_{\theta_0} + \frac{k_1}{k_2} c_{\theta_0} \right) S_2 \right] + \right. $$

$$\left. + \frac{im_{u_a} k_1}{k_2} \gamma_5 P_1 \right\} u_a + (u_a \to d_a, \theta_0 \to -\theta_0).$$

Since the Lagrangian (78) does not induce any FCNC, the inequality (76) breaks down. Then from Eq.(74) follows that with the increasing of the angle $\theta_0$ the deviation $\Delta g$ from unity could be large enough.

From the form of the Lagrangian (78) it is evident that the decay widths of the $S_1$ boson into quarks and gluons may significantly differ from those of the SM. Since the coupling constant of the $S_1$ boson with the $\tau$ lepton is determined by Eq.(62) then the value $\Gamma_{S_1 \to \tau^+\tau^-}$ also could not coincide with the corresponding value in the SM. Thus, it is apparent that the LNH mass lower bound in the LRM may not agree with that in the SM. However, since up to now any works containing the analysis of the process (70) from the point of view of the LRM are absent, we shall also take the value 115 GeV as the low bound on the $S_1$-boson mass.

Now we are ready to embark on the investigation of the Higgs boson contributions to the muon AMM. Let us determine some key moments in our strategy at calculation of the couplings constants of the Higgs bosons. To evaluate the VEV of the right-handed Higgs triplet $v_R$ we invoke the relation [39]

$$v_R = \frac{m_{W_2}^2 - m_{W_1}^2}{\sqrt{g^2 f_{\mu \mu} (1 + \tan^2 \xi)}},$$

The current bounds on the $W_2$ gauge boson and the mixing angle $\xi$ are varied within a broad range in relation to what kind of reactions and what assumptions have been used at analysis [33]. For example, the lower bound on $m_{W_2}$ being equal to 484 GeV is obtained from the investigation of the polarized muon decay under assumption $\xi = 0$. The analysis of the process $b \to s\gamma$ leads to the constraints

$$-0.01 \leq \xi \leq 0.003.$$  

Having specified $m_{W_2} = 0.8$ TeV and $\xi = 10^{-2}$ we evaluate $v_R$. Then setting values $m_{N_\mu}$, $m_{N_\tau}$, $\theta_N$ and $v_L$ we can present the quantities $\varphi_\mu$, $\varphi_\tau$, $f_{\mu\tau}$, $m_D^\mu$ and $M_D$ as the functions on $f_{\mu\mu}$.

First we assume that the dominant contribution comes from the $\Delta_2^{(-)}$-boson. A negligible value of the corrections from the $S_1$, $\Delta_1^{(-)}$, $\tilde{\delta}^{(-)}$ and $h^{(-)}$-bosons could be caused both by the large values of their masses and by the small values of their couplings constants. It is natural to require that $f_{\mu\mu}$ should be less than 1. Then analysis shows
that the interval of the $\Delta_2^{(-)}$-boson mass at which the satisfaction to BNL’00-results is possible, critically depends on the value of $f_{\mu\tau}$. Let us note, that the value of $f_{\mu\tau}$, as follows from Eq. (53), very weakly depends on the angles $\varphi_\mu$ and $\varphi_\tau$ and is basically determined by the difference of the heavy neutrino masses. When one sets $v_L$ equal to 1.7 GeV then the $\Delta_2^{(-)}$-boson mass would reach the greatest value $(m_{\Delta_2})_{\text{max}}$ at $f_{\mu\tau} \approx 0.15$. For this case in the $m_{\Delta_2}$ vs. $f_{\mu\mu}$ parameter space two contour lines marked 93.8 and 512.8 corresponding to 95% CL limits for the contribution of New Physics to $\delta a_\mu/\mu_0$ are exhibited in Fig.4. The range of the Higgs sector parameters allowed by the BNL’00 result lies between contours 93.8 and 512.8.

When $f_{\mu\tau} > 0.15$ and $m_{\Delta_2} > 140$ GeV the value of $f_{\mu\mu}$ becomes more than 1 for the upper bound of $\delta a_\mu/\mu_0$. Decreasing of $f_{\mu\tau}$ results in the reduction of $(m_{\Delta_2})_{\text{max}}$. At fixed $f_{\mu\tau}$ the reduction of $v_L$ practically has no effect on the final result. However, increasing of $v_L$ up to 10 GeV gives rise to the growth of $f_{\mu\mu}$. For example, at $m_{\Delta_2} = 100$ GeV the value of $f_{\mu\mu}$ lies in the interval $(0.318, 0.743)$.

Inasmuch as the masses of the $\Delta_1^{(-)}$- and $\tilde{\delta}^{(-)}$- bosons are close to each other then the following possibility should be considered: the observable value of the muon AMM stems from $\Delta_1^{(-)}$, $\Delta_2^{(-)}$, and $\tilde{\delta}^{(-)}$-bosons. Notice, that the quantities $\delta a_{W_1,2}^{(W_1,2)}$ change the sign as one passes from the region $\tan \beta < 1$ to the region $\tan \beta > 1$. We shall restrict our consideration to the specific case, namely, when the quantities $\beta_1 \alpha_{\mu\nu_{\mu\delta}}$ and $\beta_1 \alpha_{\mu N_\mu \tilde{\delta}}$ are positive (recall, that $I^{(W_1,2;\tilde{\delta})} > 0$).

In numerical calculations we shall use the following parameters values

$$
\begin{align*}
m_{N_{\mu}} &= 110 \text{ GeV}, \\
m_{N_{\tau}} &= 125 \text{ GeV}, \\
v_L &= 0.17 \text{ GeV}, \\
\tan \beta &= 0.8, \\
\alpha - \rho_3/2 + \rho_1 &= 1, \\
\beta_1 &= 1, \\
\theta_N &= 0.78
\end{align*}
$$

and shall assume such hierarchy of the Higgs boson masses

$$
m_{\Delta_1} = 1.1 m_{\Delta_2}, \quad m_{\tilde{\delta}} = 1.05 m_{\Delta_2}.
$$

In the $m_{\Delta_2}$ vs. $f_{\mu\mu}$ parameter space two contour lines marked 93.8 and 512.8 are shown in Fig.5. At the chosen values of the heavy neutrino masses the quantity $f_{\mu\tau}$ is approximately equal to 0.01.

With the increase of the heavy neutrino masses, but provided that

$$
m_{N_{\tau}} - m_{N_{\mu}} = \text{const}
$$

the function $f_{\mu\mu}(m_{\Delta_2})$ grows faster. However, this rise becomes essential only when the heavy neutrino masses are approximately changed on the order of magnitude. So, for example, choosing $m_{N_{\mu}} = 900$ and $m_{N_{\tau}} = 915$ GeV, we obtain that at $m_{\Delta_2} = 200$ GeV the value of $f_{\mu\mu}$ lies in the interval $(0.092, 0.444)$. In this case $f_{\mu\tau}$ is approximately equal to 0.006. On the other hand, with the increase of $m_{N_{\tau}} - m_{N_{\mu}}$ the rate of growth of the function $f_{\mu\mu}(m_{\Delta_2})$ goes down. For example, when we set

$$
m_{N_{\mu}} = 900 \text{ GeV}, \quad m_{N_{\tau}} = 1100 \text{ GeV}
$$
and leave all the remaining parameters without change, the value of \( f_{\mu\mu} \) will lie in the interval \((0.029, 0.235)\) at \( m_{\Delta_2} \) being equal to 100 GeV. The reduction of \( \tan \beta \) results in decreasing of \( f_{\mu\mu} \) as a function of \( m_{\Delta_2} \). For example, in the case \( m_{N_\mu} = 110 \) and \( m_{N_\tau} = 125 \) GeV (all the remaining parameter values are unchanged) when \( \tan \beta \) has been set to 0.3 we have

\[
f_{\mu\mu} \in (0.009, 0.049) \quad \text{when} \quad m_{\Delta_2} = 100 \text{ GeV}
\]

and

\[
f_{\mu\mu} \in (0.014, 0.079) \quad \text{when} \quad m_{\Delta_2} = 200 \text{ GeV}.
\]

The obtained results are practically not changed at increasing of \( v_L \) up to its maximum value.

At present we assume that the muon AMM value could be explained by the \( S_1 \)-boson contribution only. To suppress the contributions coming from the remaining Higgs bosons it is enough to assume that

\[
\alpha \sim 1, \quad \rho_2 \sim 1, \quad \rho_3/2 - \rho_1 \sim 1
\]

(this will make \( h^{(-)}, \Delta_{1,2}^{(-)} \) and \( \tilde{\delta}^{(-)} \)-bosons to be superheavy). The contours 93.8 and 512.8 in \( m_{S_1} \) vs. \( \tan \beta \) parameter space are represented in Fig.6. In numerical calculations the following parameters values have been used

\[
f_{\mu\mu} = 0.04, \quad m_{N_\mu} = 110 \text{ GeV}, \quad m_{N_\tau} = 125 \text{ GeV}, \quad v_L = 1.7 \text{ GeV}.
\]

Increasing (decreasing) of \( v_L \) results in the reduction (the enhancement) of the allowed values of \( \tan \beta \). For example, when \( v_L \) is equal to 10 GeV we obtain

\[
\tan \beta \in (0.804, 0.913) \quad \text{at} \quad m_{S_1} = 115 \text{ GeV}
\]

and

\[
\tan \beta \in (0.88, 0.947) \quad \text{at} \quad m_{S_1} = 200 \text{ GeV}.
\]

Increasing of the heavy neutrino masses and of the values of \( f_{\mu\mu} \) does not cause the appreciable change of the obtained results.

In the case when the muon is stipulated by the contributions from \( S_1 \)- and \( h \)-bosons the contours 93.8 and 512.8 in \( m_{S_1} \) vs. \( \tan \beta \) parameter space are represented in Fig.7. The choice of the model parameters is as follows

\[
m_{N_\mu} = 110 \text{ GeV}, \quad m_{N_\tau} = 125 \text{ GeV}, \quad v_L = 1.7 \text{ GeV}, \quad f_{\mu\mu} = 0.04
\]

With the decrease of \( v_L \) the value \( \tan \beta \) comes closer and closer to 1, while the increase of \( v_L \) results in the removal of the values of \( \tan \beta \) from 1 to 0. The increase of the heavy neutrino masses does not practically influence the behaviour of the contours shown in Fig.7. At the increase of \( f_{\mu\mu} \) there is the reduction of the allowed values of \( \tan \beta \) with the growth of the \( m_{S_1} \) values. For example, when \( f_{\mu\mu} = 0.1 \) (all the remaining parameters are unchanged), then at \( m_{S_1} = 165 \) GeV the values of \( \tan \beta \) lie within the interval \((0.0015, 0.4542)\) and at \( m_{S_1} = 200 \) GeV those lie within the interval \((0.0211, 0.5562)\).
3. Conclusions

We have considered the Higgs sector of the LRM as a source of the muon value observed at BNL. The contributions from the interactions of the doubly charged Higgs bosons ($\Delta_{1,2}^{(-)}$), the singly charged ($h^{(-)}$ and $\tilde{\delta}^{(-)}$) and the neutral ($S_1$) Higgs bosons both with leptons and gauge bosons were taken into the account. The found value of the muon represents the function of the Higgs boson masses and the Higgs boson couplings constants (CC’s). For the majority of the SM extensions the information about the Higgs boson masses is at the level of knowledge of the lower borders only. The situation with the CC’s is even more pessimistic. The experimental data derived up to now do not allow to obtain the constraints on all the CC’s. We managed to show that the most of part of the CC’s is the functions of the neutrino oscillation parameters. By this it turned out that the values of these CC’s are practically not sensitive to the masses and the mixing angles in the light neutrino sector and are mainly defined by the values of the heavy neutrino masses and by the mixing angles between the light and the heavy neutrinos. It should be particularly emphasized that this property is common for all the models with the "see-saw" mechanism, i.e. for the models with the heavy neutrino.

To explain the observed value of the muon AMM by the contributions either from $S_1$ and $h^{(-)}$-bosons or from both of them, it is necessary to assume that $\tan \beta$ is close to 1. To put this another way, the coincidence with the BNL’00 result will take place at quasi-degeneracy of the bidoublet VEV’s ($k_1 \approx k_2$), i.e. at the fine tuning of the bidoublet VEV’s. As in this case obtained borders on the Higgs boson parameters weakly depends on the neutrino sector parameters then the recovery of some information concerning the masses and the mixing angles of the neutrinos will be rather difficult. However the reverse side of this history is the fine capability for detecting of $S_1$ and $h^{(-)}$-bosons. It is appeared that when $\tan \beta$ is close to 1 then the values of the CC’s for $h^{(-)}$- and $S_1$-bosons are those that these bosons can be observed as the resonance peaks in the whole series of the processes. For example, the $S_1$-boson could be observed as the resonance splashes in the cross sections of the reactions

$$\mu^+\mu^- \rightarrow \mu^+\mu^-, \tau^+\tau^-; \quad (84)$$

$$\mu^+e^- \rightarrow \mu^-e^+; \quad (85)$$

which practically have now background. The reactions (84) and (85) may be investigated right now, because the energy of the muon beams used in the current experiments is rather high. The Spin Muon Collaboration at CERN has been working with the muon beams having energy 190 GeV [47] and the FNAL experiments investigating the muon-proton interaction has been using the muons with energies of 470 GeV [48]. The reactions (84) and (85) can be also investigated at the muon colliders (MC’s) which are now under design. For the detection of the $h^{(-)}$-bosons one could employ the reactions

$$e^-\nu_e \rightarrow W^{-}_1Z_1; \quad (86)$$
\[ e^- \nu_e \rightarrow \mu^- \nu_\mu, \]  

(87)

which have the s-channel diagrams with the exchange of the \( h^{(-)} \)-boson [39]. The ultra high energy cosmic neutrinos could be used for studying these two reactions at such neutrino telescopes as BAIKAL NT-200, NESTOR and AMANDA.

However, one important point to remember that the fine tuning of the parameters always belongs to the extremely rare expedient which is used by Nature. For this reason the variant with wider range of the parameters ensuring the agreement between theory and experiment is preferable. The situation with the dominating contribution to the muon AMM from \( \Delta_{1,2}^{(-)} \)- and \( \tilde{\delta}^{(-)} \)-bosons is just such a case. However and in this case it is still far to the final definition of the heavy neutrino parameters. Having established the values of \( f_{\mu\mu}, v_L \) and \( v_R \) we shall obtain only two equations for the definition of the quantities \( \varphi_\mu, \theta_N, m_{N_\mu}, m_{N_\tau} \), that is obviously not enough. Certainly it is possible to select the conventional way too. Out of the five parameters of the heavy neutrino sector (\( \varphi_\mu, \varphi_\tau, \theta_N, m_{N_\mu}, m_{N_\tau} \)) we may fix the four parameters and vary the one, say, \( m_N \). At such an approach instead of the contours shown in Fig.5 we shall have the contours constructed in \( m_{N_\mu} \) vs. \( m_{\Delta_2} \) parameter space, that will not introduce anything essentially new to our analysis. The most important here is something else, namely, when the contribution to the muon is truly caused by the \( \Delta_{1}^{(-)}, \tilde{\delta}^{(-)} \)- and/or \( \Delta_{2}^{(-)} \)-bosons then the further way of defining the parameters of the heavy neutrino without their direct observation is evident. For example, we could investigate the reactions

\[ \mu^- \mu^- \rightarrow \mu^- \mu^-, \]  

(88)

\[ \mu^- \mu^- \rightarrow \mu^- \tau^-, \]  

(89)

\[ \mu^- \mu^- \rightarrow \tau^- \tau^-, \]  

(90)

which may be observed at the MC’s. All these reactions are going through the s-channels with the exchanges of the \( \Delta_{1,2}^{(-)} \)-bosons. Therefore, their cross sections have two resonance peaks related to the Higgs bosons. Detecting of the reaction (88) will allow to determine \( f_{\mu\mu} \), while the investigation of reactions (89) and (90) will yield the information about \( f_{\mu\tau} \) and \( f_{\tau\tau} \) respectively. Then the use of Eqs. (51) — (55) will allow to define the regions in which the values of the heavy neutrino masses and the mixing angles are constrained.

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Fig.1. One-loop diagrams contribute to the muon AMM due to the doubly charged Higgs bosons $\Delta_{1,2}^{(-,-)}$. The wavy line represent the electromagnetic field.

Fig.2. One-loop diagrams contribute to the muon AMM due to the singly charged Higgs bosons $\tilde{\delta}^{(-)}$ and $h^{(-)}$.

Fig.3. One-loop diagrams contribute to the muon AMM due to the lightest neutral Higgs boson $S_1$.

Fig.4. Contours of the one-loop contribution from the $\Delta_2^{(-,-)}$-boson to the muon AMM.

Fig.5. Contours of the one-loop contribution from the $\Delta_{1,2}^{(-,-)}$ and the $\tilde{\delta}^{(-)}$-boson to the muon AMM.

Fig.6. Contours of the one-loop contribution from the $S_1$-boson to the muon AMM.

Fig.7. Contours of the one-loop contribution from the $S_1$- and $h^{(-)}$-boson to the muon AMM.
Fig. 1
\[ \gamma \]

\[ h^{(-)}, \bar{\delta}^{(-)} \]

\[ \mu^- \]

\[ \nu_a \]

\[ \mu^- \]

\[ h^{(-)}, \bar{\delta}^{(-)} \]

\[ h^{(-)}, \bar{\delta}^{(-)} \]

\[ \mu^- \]

\[ N_a \]

\[ \mu^- \]

\[ \gamma \]

\[ h^{(-)}, \bar{\delta}^{(-)} \]

\[ W_{1,2}^- \]

\[ \mu^- \]

\[ \nu_{\mu, N\mu} \]

\[ \mu^- \]

\[ W_{1,2}^- \]

\[ h^{(-)}, \bar{\delta}^{(-)} \]

\[ \mu^- \]

\[ \nu_{\mu, N\mu} \]

\[ \mu^- \]

Fig. 2
Fig. 3
Fig. 4
Fig. 5
\[ \tan \beta \]

Fig. 6
