Comment on “Triviality of the Ground State Structure in Ising Spin Glasses”

In a recent, very interesting paper, Palassini and Young [1] have shown that it is possible to get useful information about the nature of the low T phase of 3D Ising spin glasses by studying the behavior of the ground state (GS) after changing the boundary conditions (BC) of an $L^3$ lattice system from periodic ($P$) to anti-periodic ($AP$). They analyze GS obtained with the same realization of the quenched disorder and different BC.

Let $P(M, L)$ be the probability that the spins in an $M^3$ cube remain in the same configuration (apart from a full reversal, due to the global $Z_2$ symmetry at zero magnetic field) when we change BC from $P$ to $AP$ [2]. The behavior of $P(M, L)$ when $L$ goes to infinity is very important from the theoretical point of view. In the droplet model (DM) [3] $P(M, L) \propto L^{-\lambda}$, where $\lambda \equiv D - D_s$, $D$ is the space dimension and $D_s$ is the fractal dimension of the interface, while in the usual form of the Replica Symmetry Breaking (RSB) approach [4] $P(M, L) \rightarrow A(M)$, where $A(M)$ is a non zero function (i.e. the interface is space filling). In [1] it is shown that the data for $P(2, L)$ can be well fitted by a power law with a non-zero $\lambda$ (as suggested by the DM), although they can also be fitted as $a + bL^{-1} + cL^{-2}$, with a non zero $a$. In this comment we point out that one can better discriminate among the DM and the RSB approach if one extends their analysis by considering the value of additional quantities. At this end we have computed the GS in systems with side up to $L = 12$ (with Gaussian disorder) and we have compared the GS obtained with $P$ and $AP$ BC.

If in the large volume limit the interface is a homogeneous fractal that can be characterized by a single fractal dimension (i.e. if it has not a multi-fractal behavior), and if the relation $\lambda = D - D_s$ is correct, the probability that the interface does not intersect a region $\mathcal{R}$, whose size is proportional to the system size, goes to a limit which is a non-trivial function of the shape of the region $\mathcal{R}$. If the interface is space filling, such a probability always goes to zero. This argument implies that under the previous assumptions in the DM (for large $L$) $P(M, L) \propto g(ML^{-1})$.

We plot in figure (1) our results for boxes of size $M = 2, 3, 4$ versus $ML^{-1}$. The data are very far from collapsing on a single universal curve (they are consistent with a smooth behavior in $L^{-1}$, and are well fitted by a second order polynomial in $L^{-1}$). Stronger hints are obtained if we consider the probability $P_L$ of finding that a full $y-z$ plane of $L^2$ spins does not hit the interface when we go from $P$ to $AP$ in the $x$ direction. This corresponds in the previous argument to consider a region $\mathcal{R}$ of size $L \times L \times 1$. In figure (2) we plot $P_L$ versus $L$. $P_L$ can be roughly fitted as $L^{-\gamma}$, with a relative large value of $\gamma$ (i.e. $\gamma \approx 1.5 - 2.0$). In other words the probability that the interface hits $\mathcal{R}$ goes to one (or to a value very close to one) when the volume goes to infinity.

We have shown that extending the innovative analysis of Palassini and Young to a larger set of observables, one finds serious problems with the usual DM interpretation: the most natural scenario is based on the fact that the interface is space filling as predicted by the RSB approach. Other possibilities like the presence of very strong corrections to the scaling, or that the relation $\lambda = D - D_s$ is not valid and/or the interface is multi-fractal, are less plausible. We are grateful to M. Palassini and P. Young for pointing out an error in the interpretation of the data in a first version of this comment and for a very useful correspondence.

E. Marinari and G. Parisi, Università di Roma La Sapienza

[1] M. Palassini and A. P. Young, Phys. Rev. Lett. 83, 5126 (1999).
[2] E. Marinari, G. Parisi, F. Ricci-Tersenghi and J. J. Ruiz-Lorenzo, J. Phys. A 31, L481 (1998).
[3] W. L. McMillan, J. Phys. C 17, 3179 (1984); A. J. Bray and M. A. Moore, in Heidelberg Colloquium on Glassy Dynamics and Optimization, L. Van Hemmen and I. Morgenstern eds. (Springer-Verlag, Heidelberg 1986); D. S. Fisher and D. A. Huse, Phys. Rev. B 36, 386 (1988).
[4] M. Mézard, G. Parisi and M. A. Virasoro, Spin Glass Theory and Beyond (World Scientific, Singapore 1987).