On $p_t(x_L)$ dependence in h-A interactions and lateral features of most energetic particles in EAS cores

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Abstract. Lateral divergence of most energetic particles in EAS cores ($\gamma$-ray families) are shown to be independent of secondary particles’ $\langle p_t \rangle$ in h-A interactions, but strongly determined by $p_t(x_L)$ dependence at $0.05 \lesssim x_L \lesssim 0.30$. Average $x_L$- and $x_L^2$-weighted $p_t$-dependent parameters calculated in “truncated” $x_L$ intervals are proposed to characterize interaction model features related to the lateral dimension of $\gamma$-ray families.

1. Introduction

All the ground-based astrophysical experiments exploit simulations of air cascade development at energies $E_0 \gtrsim 10^{11}$ eV of particles of primary cosmic radiation (PCR). The best hadron-air nucleus (h-A) interaction model is not chosen yet. A number of models used by the CORSIKA package are tuned by using sea-level EAS data. High-mountain X-ray emulsion chamber (XREC) results are not taken into account although the XREC techniques makes it possible to detect groups of EAS-core high-energy ($E \gtrsim 4$ TeV) $e^\pm$ and $\gamma$-rays, i.e., so-called $\gamma$-ray families, which seem to be very sensitive to model variations. Besides, the majority of $\gamma$-ray families is produced by PCR protons. This decreases uncertainties related to the PCR mass composition.

The transverse momentum, $p_t$, is most important factor influencing on lateral features of $\gamma$-ray families. On the other hand, the contribution of high-$x_L$ particles (here $x_L = p_Z/p_{tot}$ in Lab system) into the cascade transverse evolution at the initial stage is obviously higher than that of low-$x_L$ particles. Let us search for correlations between lateral features of proton-initiated $\gamma - h$ families and $p_t$- and $x_L$-dependent parameters at $E_0^{p-air} = 10^{16}$ eV, as this is the average energy of the first interaction of PCR-spectrum protons producing families considered below.

Simulations of $\gamma$-ray families were made using PCR particles at $E_0 \geq 10^{15}$ eV. The following families at a depth of 600 g·cm$^{-2}$ were selected: $\gamma$-ray multiplicity $N_\gamma \geq 4$, distance of particles with energy $E_\gamma \geq 4$ TeV from the event center $R_\gamma \leq 15$ cm; energy $\sum E_\gamma = 100 - 400$ TeV.

2. $p_t$-dependent parameters vs $\langle \bar{R}_\gamma \rangle^p$

MC0 [1], FANSY/QGSJ 1.0 and 1.01 models [2, 3] as well as CORSIKA’s QGSJET 01, QGSJET II, SYBILL 2.1, EPOS 1.99 models are used below. Figure 1 shows a significant difference between charged-particle $p_t(x_L)$ dependencies realized by models in p-air interactions.
at $E_{p-air} = 10^{16}$ eV. The FANSY/QGSJ 1.0 and 1.01 models differ only in behaviour of $p_t(x_L)$ dependence at $0.01 \leq x_L \leq 0.5$ and are identical in all other characteristics.

Let us define three types of (1) $x_L$-independent, (2) $x_L$- and (3) $x_L^2$-weighted $p_t$-dependent parameters for $N$ particles generated in “truncated” kinematic ranges, $x_{min} \leq x_L \leq x_{max}$.

\[
\langle p_t \rangle_{x_{min}-x_{max}} = \frac{1}{N} \sum_{i=1}^{N} p_{ti}/N
\]

\[
\langle p_t \rangle_{x x_{min}-x_{max}} = \frac{1}{\sum_{i=1}^{N} x_{Li}} \sum_{i=1}^{N} p_{ti} x_{Li}/\sum_{i=1}^{N} x_{Li},
\]

\[
\langle p_t \rangle_{x^2 x_{min}-x_{max}} = \frac{1}{\sum_{i=1}^{N} x_{Li}^2} \sum_{i=1}^{N} p_{ti} x_{Li}^2/\sum_{i=1}^{N} x_{Li}^2.
\]

Parameters (1) – (3) are calculated at $x_{min} = 0.0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07$ and $x_{max} = 0.20, 0.25, 0.30, 0.40, 0.50, 1.0$. Non-zero $x_{min}$ values are used to exclude lowest-energy particles, which give the main contribution into calculated parameters but do not influence on radical cascade development. Non-unity $x_{max}$ values are used to exclude the fluctuating contribution of most energetic particles.

Let us search for correlations between the doubly averaged radius\(^1\) of family $\gamma$-rays, $\langle R_{\gamma} \rangle^P$, and parameters (1–3) as $Y = m \cdot X + const$. Here $m$ is found by the least-squares method for relative variables $X$ and $Y$ defined for any model as $X = P_{model}/P_{FANSY 1.01}$ ($P$ is one of Eqs. 1–3)), $Y = \langle R_{\gamma} \rangle^P_{model}/\langle R_{\gamma} \rangle^P_{FANSY 1.01}$. The determinancy coefficient, $r^2$, is additionally calculated. Obviously, the ideal case would be at $m = r^2 = 1$.

While using the whole kinematic range of particles, $x_{min} = 0.0$ and $x_{max} = 1.0$, Eq. (1) gives the usual average transverse momentum, $\langle p_t \rangle$, and Eqs. (2) and (3) give $x_L$- and $x_L^2$-weighted parameters $\langle p_t \rangle_{x_{0.0}-1}$ and $\langle p_t \rangle_{x^2_{0.0}-1}$, for $N$ particles.

Figure 2 shows correlation of $\langle R_{\gamma} \rangle^P$ and traditional charged-particle $\langle p_t \rangle$ values at $E_0 = 10^{16}$ eV for all the models under consideration. The scatter in $\langle R_{\gamma} \rangle$ values is much more than that in $\langle p_t \rangle$ values. In this case $m = 1.99 \pm 1.26$ at $r^2 = 0.33$, i.e., no understandable correlation between $\langle p_t \rangle$ and $\langle R_{\gamma} \rangle$ is found.

\(^1\) Double averaging of a parameter means that at the first stage averaging is made over particles of each separate event. At the second stage, the found values are averaged over all events.

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**Figure 1.** Model charged-particle $p_t(x_L)$ dependencies at $E_{p-air} = 10^{16}$ eV.

**Figure 2.** Dependence of $\langle R_{\gamma} \rangle^P$ on charged-hadron $\langle p_t \rangle$ at $E_{p-air} = 10^{16}$ eV.
Figure 3. Dependence of $\langle R_\gamma \rangle^p$ on $\langle p_t \rangle_x 0.07-0.20$.

Consideration of correlations of $\langle R_\gamma \rangle^p$ with $x_L$- and $x_T^2$-weighted $p_t$-dependent parameters (Eqs. (2) and (3)) in the same kinematic interval ($0 \leq x_L \leq 1$) give close results: $m = 2.03 \pm 1.05$, $r^2 = 0.43$ and $m = 0.05 \pm 0.97$, $r^2 = 0.001$, respectively. Analysis of the similar correlations in a “truncated” kinematic range $0 \leq x_L \leq 0.5$ range give close results as well, namely: $m = 0.62 \pm 0.29$, $r^2 = 0.47$ and $m = 0.72 \pm 0.96$, $r^2 = 0.10$, respectively.

Thus, $r^2$ values are very low. So no understandable correlation is found for these cases as well.

Analysis of “truncated” parameters for different $x_{\text{min}}$ and $x_{\text{max}}$ show that the general trend is a growth of correlation with approaching of $x_{\text{min}}$ to $\sim 0.05$ and $x_{\text{max}}$ to $\sim 0.20$. This relates to all the models excluding QGSJET II which deviates from this trend. So, $m$ is calculated below for all the models, excluding QGSJET II, and denoted $m_{tr}$. An example of the real correlation ($m_{tr} \leq 1.20$ and $r^2 \geq 0.97$) is given in figure 3, which shows correlation of $\langle R_\gamma \rangle^p$ with $\langle p_t \rangle_x 0.07-0.20$ ($m_{tr} = 1.14 \pm 0.09$, $r^2 = 0.97$). Dotted lines are drawn by hand to illustrate evident trends.

Let us emphasize the difference in FANSY 1.0 and 1.01’s data as these two models differ only in $p_t(x_L)$ dependence at 0.01 $\lesssim x_L \lesssim 0.5$.

To compare model and experimental data, it is necessary to account for PCR composition as well as the XREC response and measurement procedures, which result in some increase of $\langle R_\gamma \rangle$ by a factor of $\approx 1.14 \pm 0.04$ [5, 6]. Figure 4 shows correlation between corrected model $\langle R_\gamma \rangle^0$ values for all-nuclei-initiated $\gamma$-ray families with $\sum E_\gamma = 100 - 400$ TeV and $\langle p_t \rangle_x 0.05-0.50$. The Pamir’s $\langle R_\gamma \rangle_{\text{exp}}$ value [7] is shown by shadowed area including statistical errors. FANSY 1.01, MC0, EPOS 1.99 and QGSJET 01 models give more or less reasonable $\langle R_\gamma \rangle^0$ values. QGSJET II, SYBILL 2.1 and especially FANSY 1.0 models give too high $\langle R_\gamma \rangle$ values. A clear trend of points shown by dotted line is demonstrated by MC0, FANSY 1.0, FANSY 1.01, QGSJET 01, SYBILL 2.1, EPOS 1.99 models while the QGSJET II point deviates from the general trend.

$\langle R_\gamma \rangle$ value could be also influenced the following factors. Growth of inelastic cross section $\sigma_{\text{inel}}^{p-\text{air}}$ and average inelasticity coefficient $\langle K_{\text{inel}}^{p-\text{air}} \rangle$ (or decrease of relative energy of the most energetic baryon, $\langle x_{m.e.b.}^{p-\text{air}} \rangle \approx 1 - \langle K_{\text{inel}}^{p-\text{air}} \rangle$) as well as softening of spectra of secondary particles accelerate the cascade development and energy dissipation. As a result, energy of particles decreases on an average, and, consequently, $\langle R_\gamma \rangle$ increases at the same $p_t$.

Figure 5 shows model correlation of $\langle R_\gamma \rangle^p$ and $\sigma_{\text{inel}}^{p-\text{air}} (E_0 = 10^{16}\text{eV})$. As is seen, the difference in $\sigma_{\text{inel}}^{p-\text{air}}$ values is small ($\lesssim 5\%$) while the scatter of $\langle R_\gamma \rangle$ values is much larger.

Figure 6 shows model neutral-pion spectra in p-air interactions at $E_0 = 10^{16}$ eV. Plainly QGSJET 01 and QGSJET II realize identical spectra and give different $\langle R_\gamma \rangle$. Similarly FANSY
1.0 and FANSY 1.01 being identical in pion spectra show the maximum difference in $\langle R_\gamma \rangle_p$ values. So, these two factors are not related to the difference in size of families.

Figure 7 shows dependence of model $\langle R_\gamma \rangle_p$ values on $\langle x_{m.e.b.} \rangle$. Obviously, the scatter of $\langle x_{m.e.b.} \rangle$ values is rather large (~ 40%), but chaotic. As a result, in this case one cannot find some definite correlation between $\langle R_\gamma \rangle_p$ and $\langle x_{m.e.b.} \rangle$ as well.

**Conclusion**

High-mountain XRECs data on high-energy $\gamma$-ray families present a good chance to test hadron-air nucleus interaction models at $E_0 \gtrsim 10^{15}$ eV by simulating $\gamma$-ray families.

The average transverse momentum, $\langle p_t \rangle$, is not related to lateral features of EAS core high-energy particles. $p_t$-dependent parameters (1) – (3) calculated in some “truncated” range (0.05 $\lesssim x_L \lesssim 0.3$) seem to be more preferable. In this range the $p_t(x_L)$ dependence is very important and determines lateral features of EAS initial-stage cores.

Results on $\langle R_\gamma \rangle$ by FANSY 1.01, MC0, EPOS 1.99 and QGSJET 01 models are more close to the experimental value. QGSJET II, SYBILL 2.1 and FANSY 1.0 give too high $\langle R_\gamma \rangle$ values.

All the models (excluding QGSJET II) show an understandable linear dependence of $\langle R_\gamma \rangle_p$ on parameters (1 – 3) at 0.05 $\lesssim x_L \lesssim 0.3$.

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