Aerodynamic Performance Prediction of XH-59A Helicopter in Pop-up Flight

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Abstract. The pop-up progress of the coaxial rigid rotor helicopter was decomposed into 5 stages. Quasi-steady flow assumption was adopted to analyze the helicopter performance. XH-59A helicopter was regarded as a solid body. Flow interactions between fuselage and rotor, rotor and tail were ignored. The aerodynamic performance of coaxial rotor, fuselage and horizontal tail were calculated based on Blade element moment theory, wind tunnel test data and theoretical calculation correspondingly. The Pitt/Peters inflow model was used to represent the inflow velocity on the rotor and flow interaction between upper and lower rotor were considered using interference factor. The net force and moment on the helicopter was derived and formulated into flight mechanics equations. To validate the calculation, a wind tunnel test of a coaxial rigid rotor helicopter was employed. The rotor lift coefficient, helicopter pitching and rolling moment coefficients were close to the experimental data. The controlled variables of the helicopter was composed of helicopter pitch angle (shaft angle), collective pitch, cyclic pitch of both rotors, deflection angle of the horizontal tail during the pop-up progress. The mathematical model presented in this paper provides the basic system structure to analyze the pop-up progress. The required power of the helicopter were formulated with altitude, rotor thrust, flight velocity, which help to obtain a rapid and primary evaluation of the integral performance of the progress.

Nomenclature

| Symbol | Description |
|--------|-------------|
| A₁     | Longitudinal cyclic blade pitch |
| aₒ, a₁ | Taper angle, Back chamfer angle of the rotor |
| B₁     | Lateral cyclic blade pitch |
| b₁     | Roll angle of rotor |
| CLR    | Rotor lift coefficient |
| CXR    | Net force coefficient on wind direction of the rotor |
| CYR    | Rotor side force coefficient |
| CMZ    | Helicopter yawing moment coefficient |
| CMY    | Helicopter pitching moment coefficient |
| CMX    | Helicopter rolling moment coefficient |
| H      | Rotor backward force |
| k      | Number of blades per rotor |
| θ, γ, φ| Pitch angle, Roll angle, Yaw angle of helicopter |
| θ*     | Rotor blade collective pitch |

| Symbol | Description |
|--------|-------------|
| R      | Rotor radius |
| S      | Rotor side force |
| T      | Rotor thrust |
| Vᵢ     | Induced velocity at rotor disk |
| V₀     | Dimensionless mean induced velocity of the rotor |
| Vₓ, Vᵧ, V₂ | Flight speed |
| α      | Attack angle of helicopter |
| αₑ     | Shaft angle of attack |
| α₀     | Attack angle of rotor |
| β      | Sideslip angle of helicopter |
| δ      | Interference factor |
| τ      | Swing angle |
| φ*     | Blade azimuth |
1. Introduction

Rigid coaxial rotor helicopter is an important development direction of armed helicopter due to high speed and high mobility. However, there are limited data regarding its performance in experimental test or flight test. In order to capture the helicopter aerodynamic performance with controlled variables, the theoretical method based on Blade Element Moment (BEM) theory of the rotor and rigid-body kinematics were carried out.

The XH-59A helicopter created by Sikorsky Company applies the Advancing Blade Concept (ABC) [1] rotor and achieved symmetrical lift force from the contra rotating rotors. Two auxiliary jet engine are located at both side of the fuselage to supply extra thrust in flight direction. In 1970s, the demonstration helicopters reached speeds of 296 km/h and 356 km/h in level flight and dive without auxiliary thrust [2]. Although it was replaced by X-2 and S-97 for better performance later on. But it has the most adequate parameters and performance published in wind tunnel test and flight test. Figure 1 shows the picture of XH-59A helicopter and table 1 shows the detailed parameters, the parameters are directly or calculated from literature [3].

To analyze pop-up flight, the process is assumed to be decomposed into 5 stages: (1) A steady level flight at original altitude, (2) a circular movement in vertical direction, (3) vertical climb with constant pitch angle, (4) a circular movement in vertical direction, and (5) a steady level flight at increased altitude, shown in figure 2. The quasi-steady forward flight model based on BEM [4-5] will be applied for the maneuver process.

2. Motion equations of the helicopter

To carry out the helicopter performance prediction, assumptions are proposed. (1) Helicopter is a rigid body; (2) The flow interaction among rotor, fuselage and tail is ignored; (3) Forces and moments on the helicopter are composed of that on rotor, fuselage and tail. (4) The inflow model is suitable for the coaxial rigid rotor with flow interaction factor derived from other coaxial rotor helicopter experiment.

| Table 1. XH-59A helicopter specifications |
|------------------------------------------|
| Gross weight, lb | 13000 lb |
| Rotor radius, cutout radius | 5.486 m, 1.1 m |
| Number of rotors, blades per rotor | 2, 3 |
| Rotor separation | 2.5 ft |
| Blade tip Chord | 0.938 ft |
| Blade taper Ratio | 2:1 |
| Blade twist (nonlinear) | -10° |
| Total rotor solidity | 0.1267 |
| Blade precone angle, prelag angle | 3°, 1.4° |
| Shaft Tilt | 0° |
| Design Rotor Tip Speed | 650 ft/sec |
| Rotor drive system power | 1500 HP |
| Airfoil section | NACA series |
| Horizontal/vertical surface area of tail | 60 ft², 30 ft² |
| Horizontal Tail Incidence | -5° |
| Single blade mass | 133.60 kg |
| Single blade mass moment to flapping hinge | 187.26 kg·m |
| Single blade polar moment of inertia | 615.51 kg·m² |
Then the forces and moments on each component of the helicopter were calculated and transformed from different coordinate systems.

2.1. Force and moment equations in body axis system
In order to describe conveniently, the external forces and moments are unified in the body axis system. Terrestrial coordinate system can coincide with body axis system after rolling three times ($\phi, \theta, \gamma$). Suppose the velocity of the incoming flow is $V$, then the component of velocity along the body axis should be $V_x = V\cos \alpha \cos \beta$, $V_y = V\sin \beta$, $V_z = V\sin \alpha \cos \beta$. Thus the force and moment equations in body axis system can be written as (1) and (2).

$$m\frac{dv_x}{dt} + V_x \omega_y - V_y \omega_x + mgsin \theta = F_x$$

$$m\frac{dv_y}{dt} + V_y \omega_z - V_z \omega_y + mgsin \theta \cos \gamma = F_y$$

$$I_x d\omega_x/dt + \omega_y \omega_z (I_z - I_x) + (\omega_x \omega_z - d\omega_z/dt)I_{xy} = \sum M_x$$

$$I_y d\omega_y/dt + \omega_z \omega_x (I_x - I_y) + (\omega_x \omega_z - d\omega_z/dt)I_{yx} = \sum M_y$$

$$I_z d\omega_z/dt + \omega_x \omega_y (I_y - I_z) + (\omega_x \omega_y - d\omega_y/dt)I_{zy} = \sum M_z$$

2.2. Aerodynamic performance of fuselage
The aerodynamic characteristic matrix of the XH-59A fuselage can be achieved which is based on the wind tunnel experiments [3].

2.3. Aerodynamic performance of horizontal tail
The velocity of air flowing through the horizontal tail includes the relative flow of helicopter forward flight, the relative flow velocity caused by fuselage angular motion and the velocity caused by rotor downwash, side wash and fuselage downwash. Therefore, the velocity of the horizontal tail can write as (3). Then the force and moment equations can also be given in equation (4) and (5). Three variables $x_H, y_H, z_H$ mean the longitudinal, vertical, and lateral positions of the horizontal tail relative to the center of gravity of the helicopter.

$$V_{xH} = V_x(K_{OH})^{0.5} + \omega_z z_H - \omega_y y_H$$

$$V_{yH} = V_y(K_{OH})^{0.5} + \omega_z x_H - \omega_x z_H + K_{MH}V_1 + \epsilon V_y(K_{OH})^{0.5}$$

$$V_{zH} = V_z(K_{OH})^{0.5} + \omega_x y_H - \omega_y x_H$$

$$F_{x,H} = -\cos(\alpha_H - \phi_H) \sin(\alpha_H - \phi_H) 0 \begin{bmatrix} D_{H} \\ L_{H} \end{bmatrix}$$

$$F_{y,H} = -\sin(\alpha_H - \phi_H) -\cos(\alpha_H - \phi_H) 0 \begin{bmatrix} D_{H} \\ L_{H} \end{bmatrix}$$

$$F_{z,H} = 0 0 1 0 \begin{bmatrix} D_{H} \\ L_{H} \end{bmatrix}$$

$$M_{x,H} = F_{y,H} Z_H$$

$$M_{y,H} = -F_{x,H} Z_H$$

$$M_{z,H} = F_{x,H} x_H - F_{y,H} y_H$$

2.4. Aerodynamic performance of rotors
Firstly, the airfoils performance on each section of the blades were calculated using CFD software under various working conditions. The predicted lift coefficient, drag coefficient varying with angle of attack, Mach number, Reynolds number were formulated into a database for performance prediction of the rotating blades. According to the BEM theory, the helicopter is controlled by the forces and moments. And seven manipulated variables ($A_{11}, A_{1u}, B_{11}, B_{1u}, \theta_{pl}, \theta_{pu}, \alpha_q$) can determine the forces and moments on the rotors.
The above equations require the variable \( \psi \) to get the final result, however it is not a constant value. So equations (10-11) were added based on Pitt/Peters inflow model [6]. The equation (10) shows the induced velocity of upper rotor, while (11) shows that of lower one, where \( \delta_a \) is the flow interaction factor of upper rotor to lower rotor, and \( \delta_l \) is the verse versa.

\[
v_{iu} = v_{iu} + \delta_a v_{il} + (r / R) \left[ (v_{cu} + \delta_a v_{cl}) \cos \phi_a + (v_{su} - \delta_a v_{sl}) \sin \phi_a \right]
\]

\[
v_{il} = v_{il} + \delta_l v_{iu} + (r / R) \left[ (v_{cl} + \delta_l v_{cu}) \cos \phi_l + (v_{ul} - \delta_l v_{sl}) \sin \phi_l \right]
\]

Rotor blade pitches can also be expressed as the function of blade azimuth, lateral cyclic blade pitch, longitudinal cyclic blade pitch.

\[
\theta_c^* = (\theta_c^* + \theta_y) - (A + A_y) \cos \phi - B + B_y \sin \phi
\]

\[
\theta_l^* = (\theta_l^* - \theta_y) - (A - A_y) \cos \phi - B - B_y \sin \phi
\]

3. Performance validation by wind tunnel test at advancing ratio 0.7

Falarski [7] investigated the full-scale advancing blade concept rotor system at high advance ratios in 1971. The experiment covers an advance ratio range of 0.2 to 0.9, with all the force and moment trimmed. The experimental data of Run 21 was used to validate the BEM theory. The parameters of the experiment is shown in Table 2. Figure 3 shows the ABC helicopter during the test. The rotor performance predicted by Blade Element Moment (BEM) theory are compared with the experimental data in Figure 4. The x coordinate PT (point) represents the serial number.

Figure 4 shows that the prediction of C_L, C_MX, and C_MZ are close to experimental result. However, C_N, C_Y and C_MY have a certain disparity with experiment. The reasons are supposed to be: (1) flow interaction of the rotor with faired body in the test, while the body is ignored in the BEM method; (2) The moment coefficients on the helicopter in the experiment are compared with
moment coefficients on the rotor in BEM theory; (3) The trimmed result refers to variables with quite small value, which exaggerates the error of BEM results.

Table 2. Rotor parameters in Run 21 of ABC helicopter in test

| Parameter                      | value          |
|-------------------------------|----------------|
| Advancing ratio               | 0.7            |
| Free steam velocity           | 138m/s         |
| Rotor radius, $M_{tip}$       | 6.098m, 0.58   |
| Rotating speed, Tip speed     | 309RPM, 197.2m/s |
| Shaft angle                   | $-4^\circ$-$4^\circ$ |
| Precone angle                 | $5^\circ$(upper), $0^\circ$(lower) |
| Collective angle              | $3^\circ$-$10^\circ$ |
| longitudinal cyclic pitch $A_1$ | $-7.5^\circ$-$1^\circ$ |
| lateral cyclic pitch $B_1$    | $-5.6^\circ$-$0.1^\circ$ |
| Airfoil section               | NACA 0030- NACA 0006 |

Figure 3. ABC helicopter in test

Figure 4. Result comparison of theoretical prediction and experiment

4. Simplified verification formula of forces and moments
The control variable of the helicopter is assumed to be eight, despite the previous variable mentioned in section 2.4, deflection angle of the horizontal tail were added. The forces and moments of the helicopter can be received from solving the integral equation (6)-(9). However the process is difficult and because of the lack of important data, those equations can't be solved for now. So simplified verifications formula of forces and moments are necessary. Here the simplified equation for proximate calculation were given in equation (14).
\begin{align*}
T_i (u_s - a_s) + T_i (u_s - a_s) &= mg \theta \\
T_i + T_i &= \left(1 + \frac{1}{1000}\right) mg \\
T_b a_s + T_b a_s &= -mg \gamma \\
T_b a_s y_{sl} + T_b a_s y_{sl} &= 0 \\
T_b a_s x_{sl} + T_b a_s x_{sl} &= 0 \\
T_i (x_{sl} - y_{sl} a_s) + T_i (x_{sl} - y_{sl} a_s) + T_i a_s y_{sl} + T_i a_s y_{sl} &= 0
\end{align*}

(14)

Simplified verification formula of power and torque of the helicopter were given in equation (15), where lift, backward force, side force of rotor can be obtained.

\begin{align*}
P &= m_k \frac{1}{2} \rho \pi R^2 (\pi R)^3 \\
M_k &= m_k \frac{1}{2} \rho \pi R^2 (\pi R)^3 R \\
m_k &= \frac{1}{4} \rho C_k \rho_0 \left(1 + 5 \mu^2\right) + C_T \frac{1}{3} \left(1 + 3 \mu^2\right) + C_f \left(-\lambda_0\right) - C_{\mu \mu}
\end{align*}

The parameters $K_{p0}, J_0$ mean type resistance power correction factor and induced power correction factor, and $C_{x7}$ means lift coefficient of characteristic section.

5. Conclusions
The pop-up progress of the coaxial rigid rotor helicopter was decomposed into 5 stages. The mathematical equations for helicopter performance prediction was created based on assumptions. The BEM theory was employed for rotor performance prediction. The evaluation of the model was completed by the wind tunnel test of a rigid coaxial rotor helicopter. Results shows that method proposed in this paper has a satisfying accuracy for rotor lift coefficient, helicopter pitching and rolling moment coefficients prediction. It can be developed for future investigation of the pop-up process of the helicopter.

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