Exploring inconsistencies with the Cabibbo-Kobayashi-Maskawa model

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A small upper-bound is obtained for the CP phase $\delta$ in a CKM matrix, $|\sin \delta| < 0.44$ based on CP independent data on $|V_{ub}/V_{cb}|$ alone. Potential inconsistencies with the CKM model in existing data which need a theory beyond the standard model to overcome emerge in the analysis using this CKM matrix. In addition, all CP asymmetry measurables for the $B_d - \bar{B}_d$ system are expressed solely by the CP phases in the decay amplitudes. Remarkably the CP asymmetry of the benchmark process $B_d \to \psi K_S$ equals $\sin 2\delta$. This will soon be measured by BaBar and Belle to confront the above upper-bound.

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The Cabibbo-Kobayashi-Maskawa (CKM) model is the best available framework for the study of weak interactions involving quarks and for the study of CP violation. The CKM model is also an indispensable part of the standard model (SM). An approach in the effort for finding new physics beyond CKM is to find data which are inconsistent with the CKM model. In doing so, better and more accurate data are of course crucial. However the success is also very much dependent on the specific CKM matrix one is using in the analysis. For example, the matrix elements $V_{ud}$ and $V_{us}$ are already measured to high accuracies. The two exact values seem not enough to provide a test of their consistency with the CKM model if one uses the original form of the Kobayashi-Maskawa matrix because functions of other angles are involved, in addition to the Cabibbo angle. With the Wolfenstein matrix, the two matrix elements are essentially different functions of one and the same parameter $\lambda$. Consequently, the two values provide a crucial consistency check with the CKM model.

Several forms of the CKM matrix have appeared in the literature. (For simplicity, different forms will be called different CKM matrices in this note.) However, popular CKM matrices are only convenient for $K^0 - \bar{K}^0$ studies, not for B-physics studies (see later). Indeed the phase convention dependent mixing parameter $\epsilon_K$ is very small in the popular CKM matrices; however $\epsilon_{B_d}$ in these matrices is not small enough for any convenience. In this note we will recommend two new CKM matrices, based on a previous work of the author with Chen. One is convenient for $K^0, B_d$ and $D^0$ and the other is convenient for $B_d, B_s$ and $D^0$. The values of the parameters of such matrices will be briefly discussed. A $5.2 \sigma$ difference is there in the $\lambda$ value measured by $d$-quark decays and by $K$ decays, which, if persists, will be a signal of inconsistency of experimental data with the CKM matrix. A small upper-bound for $\sin \delta$ is obtained from the CP independent data on $\frac{|V_{ub}/V_{cb}|}{\sqrt{\epsilon^2}}$, where $\delta$ is the only CP phase in these matrices. This bound may also jeopardize the standard model, if the already small value becomes even smaller along with more accurate data. This bound or its equivalent bound with similarly clear physical meaning does not appear if one works with other CKM matrices. A big advantage of using these matrices is that the CP phases in the mixing mass and mixing width are extremely small for the neutral

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2 With the Wolfenstein matrix, $V_{ud}$ and $V_{us}$ will not be functions of parameters other than $\lambda$ up to the order of $\lambda^5$, and the coefficient of the $\lambda^6$ term is of order 1, see later.

3 For a recent study on CKM parameters, see Ref.
particles involving the b quark. Consequently, all CP measurables are exclusively expressed by the CP phases in the decay amplitudes. This will greatly simplify data analysis in the near future. One of the remarkable results of this study is that the asymmetry of the benchmark process $B_d \to \psi K_S$ will be exactly $\sin 2\delta$ in Eq (17) (the master equation). This asymmetry is expected to be measured in the first phase of BaBar and Belle experiments. One will soon be able to confront the measured $\delta$ with the above upper-bound.

The note is organized as the following. In the beginning, academic considerations are presented to evaluate the characters of different CKM matrices, in terms of whether they provide a small mixing parameter $\epsilon$ for a specific mixing system. A reader may skip this part if he does not want to go into academic details. The advantage of having extremely small $\epsilon$ for the B-physics is clearly expressed by Eq (8). The CKM matrix which is convenient for $K$, $B_d$ and $D$ studies is presented in (9) and its simplification, in (10). In discussion of the values of the parameters in the CKM matrix (10), an upper-bound for the CP phase $|\sin \delta|$ is obtained in Eq (15). The implications of Eq (8) are further explored, with Eq (17) presented. In Eq (20) we present a CKM matrix which is convenient for heavy flavor studies.

A phase convention dependent quantity $\epsilon$ has played an important role in $P - P$ (Here $P$ stands for a neutral pseudo-scalar particle) mixing and CP violation studies. $\epsilon$ is defined as a parameter which appears in the definition of mass eigenstates[9], assuming CPT conservation.

$$|P_\pm\rangle = \frac{(1 + \epsilon)|P\rangle \pm (1 - \epsilon)|\bar{P}\rangle}{(1 + |\epsilon|^2)^{1/2}}. \quad (1)$$

$\epsilon$ is commonly called the mixing parameter, although it does not bear an immediate physical meaning: it is not measurable, for example. What is physically important for $\epsilon$ is the value of $|(1 + \epsilon)/(1 - \epsilon)|$. Obviously, this value does not change when, for instance, $\epsilon \to \epsilon^*$, and $\epsilon \to \frac{1}{\epsilon}$.

An essential and related quantity, $\sigma$, called the overlap, is defined as

$$\sigma = \frac{1}{2}\langle P_+ | P_- \rangle. \quad (2)$$

$\sigma$ is a phase convention independent quantity[10]

$$\sigma = \frac{\text{Im} M_{12} \Gamma_{12}}{4 |M_{12}|^2 + |\Gamma_{12}|^2} \quad (3)$$
which is a real number for all mixing systems. For the $B_d$ and $B_s$ systems, \( \sigma = \frac{1}{4} \text{Im}(\frac{M_{12}}{\Gamma_{12}}) \). It is also directly measurable through the measurement of the asymmetry in the leptonic decay channels\(^{[11]}\) $P_+ \rightarrow l^\pm + x(x)$. For the $B_d$ system $\sigma$ is approximately (to calculate $M_{12}$ and $\Gamma_{12}$, see Ref\(^{[12]}\)\(^{[13]}\))

\[
\sigma_{B_d} \simeq \pi \frac{m_c^2}{m_t} \sin \delta \leq 2.2 \times 10^{-4}.
\] (4)

Values of $\sigma$ for other mixing systems are given in Table 1.

It has been shown\(^{[14]}\) that in a complex plane, the possible values of $\epsilon$ makes a loop whose diameter is exactly $|\sigma - \frac{1}{\sigma}|$. This loop is big, especially when $\sigma$ is as small as that in Eq (4). The extremes of $|\epsilon|$ appears when $\epsilon$ is real and is exactly $\epsilon = \sigma$ (the minimal magnitude) and $\epsilon = \frac{1}{\sigma}$ (the maximal magnitude). This can also be seen from the complex expression for $\epsilon$,

\[
\epsilon = \frac{i \text{Im} M_{12} + \text{Im} \Gamma_{12}/2}{\text{Re} M_{12} - \Delta m/2 - (i/2)\text{Re} \Gamma_{12} + i\Delta \gamma/4},
\] (5)

where

\[
\Delta m = m_- - m_+ \quad \Delta \gamma = \gamma_- - \gamma_+
\] (6)

and $m_{\pm}$ are the masses of $P_\pm$, and $\gamma_{\pm}$ are the widths of $P_\pm$ respectively. These definitions seem to be complete in defining a definite $\epsilon$. However, in practice there are problems. First, $M_{12}$ is phase dependent, so both $\text{Im} M_{12}$ and $\text{Re} M_{12}$ can change their values and signs when the CKM convention is changed; similarly for $\Gamma_{12}$. Second, one really does not know whether $\Delta m$ is positive or negative; in other words, one does not know whether $P_+$ or $P_-$ is heavier in a specific situation. One does not know the sign of $\Delta \gamma$ either.

One does know the relative sign of $\Delta m$ and $\Delta \gamma$ which is defined by\(^{[10]}\)

\[
4 \text{Re} M_{12}^\ast \Gamma_{12} = \Delta m \Delta \gamma.
\] (7)

Because of the smallness of $\sigma$ for all $P - \bar{P}$ systems, one has

\[
|\Delta m \Delta \gamma| = 4 |M_{12} \Gamma_{12}|
\]
to a very good accuracy. Assuming $m_-$ is larger than $m_+$ (which of course needs justification\(^{[4]}\)), then it is desired to find a CKM matrix with which one obtains a small $\text{Im} M_{12}$ as well as a

\(^{4}\text{For the interested reader, see Ref}\(^{[13]}\).
negative $ReM_{12}$. Such a matrix will make the parameter $\epsilon$ small because $2ReM_{12}$ and $-\Delta m$ in the denominator will add together. They could cancel each other if only $ImM_{12}$ is small, but $ReM_{12}$ is positive. In that case, $2ReM_{12}-\Delta m$ would be zero (so would be $2Re\Gamma_{12}-\Delta \gamma$), which causes $\epsilon$ to explode.

In all popular CKM matrices[4], $\epsilon_K$ is close to its minimum, which brings some convenience in the $K_S - K_L$ studies. However, with the same CKM matrices, such advantage is completely eluded in the $B$ physics studies where promising and copious CP violation phenomena are going to be the center of intensive studies. Such convenience includes avoiding talking about a big unknown mixing parameter $\epsilon$ and a correspondingly elusive “big CP violation in the mixing in the SM”[5]. In addition, the expression for CP asymmetry quantities is simplified[14]

$$\lambda_f = \frac{(1 - \epsilon) A_f}{(1 + \epsilon) A_f} = \frac{A_f}{A_f}$$

where $A_f(\bar{A}_f)$ is the decay amplitude of $B_d(\bar{B}_d)$ to a neutral final state $f$, which can be either self conjugated or not. In other words, in such CKM matrices, the CP asymmetries are all caused by CP phases in the decay amplitudes. This formula is applicable to the $B_d - \bar{B}_d$ system, but not to the Kaon or D system, because $\sigma$ for these two latter systems are compatible with their CP violation event rates $R$, (see Table I).

Such a CKM matrix can be easily reached by reparametrization of the CKM matrix[4]. Indeed, a convenient CKM matrix for the $K$ and $B_d$ studies is recommended as the following based on the Wolfenstein matrix[2] and a work by Chen and Wu[5]:

$$V_{BK} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 & \lambda & A\lambda^3(e^{-i\delta} - \zeta + \frac{1}{2} \lambda^2 e^{-i\delta}) \\
-\lambda + A^2 \lambda^5(\zeta e^{-i\delta} - \frac{1}{2}) & 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} A^2 \lambda^4 & A\lambda^2(e^{-i\delta} + \zeta \lambda^2 + \frac{1}{2} \lambda^2 e^{-i\delta}) \\
A\zeta \lambda^3(1 + \frac{1}{2} \lambda^2) & -A\lambda^2 e^{i\delta}(1 + \frac{1}{2} \lambda^2) & 1 - A^2 \lambda^4
\end{pmatrix}$$

where all parameters $\lambda, A, \zeta$ and $\delta$ are real. The unitarity of this CKM matrix is accurate to $\lambda^5$. One finds $J = A^2 \zeta \lambda^6 \sin \delta$ for every quartet of the matrix. A simplified version of this CKM matrix, which is accurate enough for most practical purposes, is

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5 A phase convention independent definition of “CP violation in the mixing” is recently emphasized by Nir. [7]
\[ V_{BK} = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (e^{-i \delta} - \zeta) \\ -\lambda + A^2 \zeta \lambda^5 e^{-i \delta} & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 e^{-i \delta} \\ A \zeta \lambda^3 & -A \lambda^2 e^{i \delta} & 1 \end{pmatrix} \]  

(10)

The approximation from Eq (9) to (10), requires at least one of \((\cos \delta - \zeta)\) and \(\sin \delta\) to be of order 1. The signs of \(A\), \(\lambda\), and \(\zeta\) are adjustable by using \((-1)\) to change the phase of a line or/and a column together with a redefinition of \(\delta\). It is therefore allowed to make

\[ A, \quad \lambda, \quad \text{and} \quad \zeta > 0. \]

In the following, we will give a brief discussion of the values of the parameters in the matrix. Most experimental data are extracted from a recent fit by Ali and London[16].

1) From the width of some \(d\) decays, one has 

\[ \lambda_d = 0.2298 \pm 0.0010 \]

and from the Kaon decays,

\[ \lambda_s = 0.2205 \pm 0.0018 \]

There is a 5.2 standard deviation discrepancy between the two values of \(\lambda\). The significance of the discrepancy must be refined. A scenario which needs non-standard physics to explain may emerge (or is already there). Before the discrepancy is clarified, the weighted average of the two

\[ \lambda = 0.2270 \pm 0.0011, \]

will be used in accurate calculations. \(\lambda = 0.23\) will be used as a round off number.

2) The life-time of the \(b\) quark is simply proportional to \(A^2 \lambda^4\), from which one obtains

\[ A = 0.81 \pm 0.058. \]  

(12)

3) The \(B_d - \bar{B}_d\) mixing mass \(\Delta m = 0.464 \pm 0.018 \text{ (ps)}^{-1}\) is simply proportional to \(A^2 \zeta^2 \lambda^6\), which simply provides a value of \(\zeta\), according to the formula

\[ 2 \text{Re} M_{12} = -\text{Re} \left( \frac{G_F^2 f_{B_d}^2}{6 \pi^2} |B_{B_d} M_{B_d}|^2 \right) (V_{tb} V_{td}^*)^2. \]  

(13)

\[ ^6\text{With a previous value of} \lambda_s[4], \text{the discrepancy was 4.5 standard deviations.} \]
This equation involves a product of unknown parameters $f_{B_d}^2 B_{B_d}$. According to lattice calculations\[4\][16], $\sqrt{B_B f_B} = 140 - 240$ MeV, one obtains

$$\zeta = 0.73 \pm 0.29.$$  \hspace{1cm} (14)

This number has a relatively large theoretical uncertainty.

4) The value of $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$, provides simply a constraint to $\sin\delta$

$$\sqrt{(\cos \delta - \zeta)^2 + \sin^2 \delta} = 0.363 \pm 0.073.$$  

Substituting the $\zeta$ value in (14) into this equation does not shed any light on the values of the CP phase $\delta$. One simply has

$$|\sin \delta| \leq 0.363 \pm 0.073.$$  \hspace{1cm} (15)

Note that this upper-bound of the CP phase is obtained from a CP independent measurement. One notices that a small $b$ to $u$ decay strongly excludes the scenario of a maximal CP violation of $\sin \delta = \pm 1$. Furthermore, there is a potential danger that a small product of $A^2 \zeta \sin \delta$ ($J < 1.1 \times 10^{-5}$ to $4.9 \times 10^{-5}$) may not be able to explain the observed CP violation in the $K^0$ system. A direct measurement of the CP phase $\sin \delta$ is badly needed. Such a measurement will give an accurate value of $\sin \delta$ and will immediately test whether the CKM matrix is solely responsible for all CP violation phenomena, by checking with the relatively tight upper-bound Eq (15). In addition, by obtaining a $\sin \delta$ value directly, one can use the $|V_{ub}/V_{cb}|$ data as the information source for the parameter $\zeta$, if the $\sin \delta$ value from another source agrees with the bound in (15).

A quantitative comparison of information from CP violation in K decays with Eq (15) needs an extensive effort, because of the controversies in $B^K$ as well as in the “long distance” contribution to $M_{12}(K)$. A recent effort is made in Ref\[16\] and references therein. The results from Eqs (11, 12, 14, 15) are summarized in Table II.

It is easy to find that the expressions for both $\text{Re} \epsilon_K$ and $\text{Re} \epsilon_{B_d}$ are simple with this matrix ($\text{Re} \epsilon_K$ is simple already in popular CKM matrices). Indeed, one has

$$\text{Re} \epsilon_{B_d} = \frac{\text{Im}(\Gamma_{12}(B_d))/2\Delta m_{B_d}}{\Delta m_K \left( \frac{\text{Im} M^*_{12}(K)}{|\Gamma_{12}|} + \frac{|\Gamma_{12}|}{2|\text{Im} M_{12}|} \right)} \approx \frac{\text{Im} M^*_{12}(K)/2\Delta m_K}{\Delta m_K \left( \frac{\text{Im} M^*_{12}(K)}{|\Gamma_{12}|} + \frac{|\Gamma_{12}|}{2|\text{Im} M_{12}|} \right)}.$$  \hspace{1cm} (16)
the formula for $\text{Re}\,\epsilon_K$ is accurate to 0.3%. It is also found that $\epsilon_D$ is small with this matrix, because the dominant CKM factor in the mixing mass is $(V_{cs} V_{us}^*)^2$, which is real and positive. Noting that $\text{Re}\,\epsilon_K \simeq +0.16\%$, the formula for $\text{Re}\,\epsilon_K$ requires $\sin\delta > 0$, if $B^K > 0$ (as indicated by lattice calculations).

It is interesting to estimate $\sin\delta$ from the measured value of $\text{Re}\,\epsilon_K$, neglecting the long distance contribution to $\text{Re}M_{12}(K)$. The value of $B^K$ becomes unimportant, once $B^K$ is positive. One then obtains

$$\text{Re}\,\epsilon_K = \frac{1}{2} A^2 \zeta \lambda^4 (1 + \frac{\eta_3}{\eta_1} \frac{m_t^2}{m_c^2}) \sin\delta$$

where $\eta_3$ and $\eta_1$ are QCD correction factors. Additional uncertainties in this formula include the error in QCD corrections and the ambiguity in the $c$-quark mass. Taking $m_c = 1.4$ MeV/$c^2$, one obtains $\zeta \sin\delta = 0.27 \pm 0.02$. Combining this with that from $|V_{ub}/V_{cb}|$, one obtains two solutions: $\sin\delta = 0.20 \pm 0.04$, $\zeta = 1.35 \pm 0.31$ and $\sin\delta = 0.42 \pm 0.08$, $\zeta = 0.64 \pm 0.13$. Both solutions seem reasonable, compared with the bounds obtained before, although some values are close to the margins and may subject to a challenge, if the margin becomes more stringent with better data.

Using the parameters defined in (10), the CP violating event rate $R$ and the overlap $\sigma$ for different systems are listed in the following table.

| System          | R                        | $|\sigma(overlap)|$      |
|-----------------|--------------------------|---------------------------|
| $K^0 - \bar{K}^0$ | $A^2 \zeta \lambda^4 \sin\delta$ | $1.7 \times 10^{-3}$ $\sim A^2 \zeta \lambda^4 \sin\delta$ |
| $B_d^0 - \bar{B}_d^0$ | $\zeta \lambda^2 \sin\delta$ | $\pi \sin\delta \frac{m_t^2}{m_t^2}$ $< 2.2 \times 10^{-4}$ |
| $B_s^0 - \bar{B}_s^0$ | $\zeta \lambda^2 \sin\delta$ | $\pi \zeta \lambda^2 \sin\delta \frac{m_t^2}{m_t^2}$ $< 1 \times 10^{-5}$ |
| $D^0 - \bar{D}^0$ | $A^2 \zeta \lambda^6 \sin\delta$ | $A^2 \zeta \lambda^4 \sin\delta$ $< 1 \times 10^{-3}$ |

In the last column of table I, the number in front of the quantity is measured and the number after the quantity is estimated. Here $R$ is the upper-bound branching ratio of any
CP violating processes in terms of CKM parameters. For example, for the Kaon, $R = \Gamma(K_L \rightarrow 2\pi)/\Gamma_{K_S}$, and for $B_d$, $R(B_+ \rightarrow \psi K_S) = \Gamma(B_+ \rightarrow \psi K_S)/\Gamma_{B_d}$. One notices from Table I that none of the neutral mixing systems has a large overlap $\sigma$ parameter.

Table II. Values of CKM Parameters

| $\lambda$   | $A$     | $\zeta$    | $\sin\delta$ |
|-------------|---------|------------|---------------|
| 0.2270±0.0011 | 0.81±0.058 | 0.73*±0.29 | 0 to 0.44     |

* values that are strongly dependent on theoretical prejudices.

It is worth exploring further the implications of Eq (8), in which the measurable CP asymmetry quantities are exclusively expressed in terms of CP phases in the decay amplitudes. As explained before, this formula is valid when the matrix (10) is used for the $B_d$ system. $\epsilon_{B_d}$ in this case is much smaller than the phases in $A_f$. One therefore has, for measurable CP violating quantities

$$\text{Im}\lambda_{B_d\rightarrow \psi K_S} = \text{Im}(-V_{cb}V_{cd}^*/V_{cb}V_{cd}) = \sin 2\delta,$$

and

$$\text{Im}\lambda_{B_d\rightarrow \pi\pi} = \frac{2\sin \delta(\cos \delta - \zeta)}{1 + \zeta^2 - 2\zeta \cos \delta}. \quad (18)$$

In obtaining Eq (17), the fact of $K_S$ being the mixed state of $K^0$ and $\bar{K}^0$ is considered. Eq (17) shows that the bench-mark mode is a token of the angle $\delta$ in the CKM matrix with a two-fold ambiguity, as the sign of $\sin\delta$ is known. There is a small uncertainty with penguin contributions[18], however basically this formula is neat, compared with Eq (13) for example. We expect this formula to be frequently used in the BaBar and Belle analysis in the near future. A measured value of Eq (18) will provide an independent information on two values of $\zeta$ if $\sin 2\delta$ in (17) is known\[7]. According to Ref[16],

$$0.28 < \sin 2\delta < 0.88.$$  

\[7\] An interesting point is made by Dr. Xing that their matrix in Ref[6] can express $\text{Im}B_d \rightarrow \pi\pi$ as $\sin 2\delta$ where $\delta$ is the CP phase in their matrix.
which implies a \(\sin \delta\) of

\[
\sin \delta = 0.14 - 0.51. \tag{19}
\]

(Another solution which gives a bigger \(\sin \delta\) is excluded by a small \(b\) to \(u\) decay rate.) This value is consistent with the bound in (15). Combining Eq (19) with the ratio \(|V_{ub}/V_{cb}|\), a bound for \(\zeta\) is obtained which is parallel to (15)

\[0.48 < \zeta < 1.32.\]

This bound for \(\zeta\) is less restrictive than (14).

If one wants to concentrate on heavy flavor physics to acquire all CKM parameters, it is suggestive to introduce a CKM matrix which is convenient for heavy flavor studies only, leaving the Kaon unattended. It has been proved that among \(\epsilon_K, \epsilon_{B_d},\) and \(\epsilon_{B_s}\) only two of them can be made small at the same time\[^{[5]}\], based on the unitarity of the CKM matrix. The following matrix is convenient for \(B_d, B_s\) as well as \(D^0\) studies:

\[
V_{HF} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda e^{i\delta} & A\lambda^3(e^{i\delta} - \zeta) \\
-\lambda + A^2 \zeta \lambda^5 e^{-i\delta} & (1 - \frac{1}{2} \lambda^2) e^{i\delta} & A\lambda^2 e^{i\delta} \\
A\zeta \lambda^3 & -A\lambda^2 & 1
\end{pmatrix}. \tag{20}
\]

It should be noted that the beloved formula \(\text{Re}\epsilon_K \propto A^2 \zeta \lambda^4 \sin \delta\) is not valid with this matrix. This is because \(\text{Re}\epsilon_K\) can be very large with this matrix. One must now distinguish the \(\epsilon_K\) defined by Eq(1) and the physically measurable \(\epsilon^0_K\) (the superscript ‘\(^0\)’ is for I-spin zero final state) which equals to

\[
\epsilon^0_K = (2\eta^{+-} + \eta^{00})/3.
\]

The difference between this matrix and that in Eq(10) is that the phases of the elements in the second column are all shifted by the same amount. With this matrix, in addition to Eqs (19, 20), one also finds

\[
\text{Im}\lambda_{B_s \rightarrow \psi\phi} = 2\zeta \lambda^2 \sin \delta. \tag{21}
\]

This formula provides another neat means to access the \(\zeta\) value, if \(\sin \delta\) is measured. It is worth mentioning that a CKM matrix which surrenders small \(\epsilon_{B_d}\) and \(\epsilon_{B_s}\) is recommended by Fritzsch and Xing\[^{[6]}\] through a different approach.
From the above study, we conclude that the CKM model has already been challenged by the over constrained measurements available to date. The confrontations may be sharpened if a suitable choice of the CKM matrix is taken for a specific $P - \bar{P}$ system. The choices considered in this note are based on the observation that the overlaps $\sigma$ for all the $P - \bar{P}$ mixing systems are small. One therefore can always choose phase conventions to make the CP phases in the mixing mass and width simultaneously small. The measurable CP asymmetries are therefore completely expressed by the CP phases in the decay amplitudes. The results Eq (8) and Eq (17) are thus obtained, which will be the most useful formulas in the BaBar and Belle data analysis, and will be convenient for further confrontations of the new data with the old ones. The question of whether these CKM matrices facilitate the discussion of nonstandard models will be studied elsewhere.

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