Abstract  Pulsar "standard model", that considers a pulsar as a rotating magnetized conducting sphere surrounded by plasma, is generalized to the case of oscillating star. We developed an algorithm for calculation of the Goldreich-Julian charge density for this case. We consider distortion of the accelerating zone in the polar cap of pulsar by neutron star oscillations. It is shown that for oscillation modes with high harmonic numbers \((l, m)\) changes in the Goldreich-Julian charge density caused by pulsations of neutron star could lead to significant altering of an accelerating electric field in the polar cap of pulsar. In the moderately optimistic scenario, that assumes excitation of the neutron star oscillations by glitches, it could be possible to detect altering of the pulsar radio-emission due to modulation of the accelerating field.

Keywords stars:neutron, oscillations, magnetic fields · pulsars:general

1 Introduction

Neutron stars (NS) are probably the most dense objects in the Universe. There are extreme physical conditions inside NS, i.e. the magnetic field is close to the quantum limit, the pressure is of the order of the nuclear one and the typical radius of a NS is only about 2-3 times larger than its gravitational radius. Knowledge of properties of matter under such extreme conditions if very important for fundamental physics. It is impossible to reconstruct such physical circumstances in terrestrial laboratories, therefore, study of the NS’s internal structure would give an unique opportunity for experimental verification of several fundamental physical theories.

To study interiors of a celestial body one have to perform some kind of seismological study, by comparing observed frequencies of eigenmodes with frequencies inferred from theoretical considerations. Eigenfrequencies of oscillations in the crust of NS as well as in its interiors were calculated in several papers (see e.g. McDermott et al. 1988; Chugunov 2006, and references there). However, for seismological study there should exist both i) a mechanism for excitation of oscillations, ii) a mechanism modulating radiation of the celestial object.

There are two types of known NSs: member of binary systems and isolated ones. The former radiate due to accretion of the matter from the companion. For these stars there are many possibilities to excite oscillations, for example by instabilities in the accretion flow. However, in this case it would be difficult to distinguish whether a particular feature in the power spectrum of the object is due to oscillations of the NS or it is caused by some processes in the accretion disc/column. Because of this ambiguity we think that the study of isolated NS should be more promising in regard of the seismology.

The vast majority of known isolated neutron stars are radiopulsars. The glitch (sudden change of the rotational period) is probably the only possible mechanism for excitation of oscillations for isolated pulsars. Radiation of radiopulsars is produced mostly in the magnetosphere. In order to judge whether oscillations of the NS could produce detectable changes in pulsar radiation, the impact of the oscillations on the magnetosphere must be considered. There is a widely accepted model of radiopulsar as a highly magnetized NS surrounded by non-neutral plasma (Goldreich & Julian 1969). Although, there is still no self-consistent theory of radiopulsars, there is a general agreement regarding basic picture for the processes in the magnetosphere. Oscillations of the star can generate electric field as it happens in the case of rotation. Generalization of the formalism developed for rotating NS to the case of oscillating star should help to obtain the desired information.

The first attempt to generalize Goldreich-Julian (GJ) formalism to the case of oscillating NS was made in Timokhin et al. (2000). It was developed a general algorithm for calculation of the GJ charge density in the near zone of an oscillating NS. Using this algorithm GJ
charge density and electromagnetic energy losses were calculated for the case of toroidal oscillations of the NS. Here we apply this formalism to the case of spheroidal oscillation modes, representing wide class of stellar modes (r-,g-,p- modes). We consider also impact of stellar oscillations on the acceleration mechanism in the polar cap of pulsar and discuss the possibility of observation on the NS oscillations.

2 Main Results

2.1 General formalism

Let us start by considering the case of a non-rotating oscillating NS. Motion of the conducting NS surface in the strong magnetic field of the star generates electric field as in the case of oscillations. Only oscillation modes with non-vanishing velocity $V_{osc}$ at the surface will disturb the magnetosphere. For the same reason as in the pulsar “standard model”, the electric field in the magnetosphere of an oscillating star should be perpendicular to the magnetic field. Otherwise charged particles will be accelerated by a longitudinal (parallel to $\mathbf{B}$) electric field and their radiation will give rise to electron-positron cascades producing enough particles to screen the accelerating electric field [Sturrock 1971]. As in the case of rotating stars we will define the Goldreich-Julian electric field $\mathbf{E}_{GJ}$ as the field which is everywhere perpendicular to the magnetic field of the star, $\mathbf{E}_{GJ} \perp \mathbf{B}$, and the GJ charge density as a charge density, which supports this field

$$\rho_{GJ} \equiv \frac{1}{4\pi} \nabla \cdot \mathbf{E}_{GJ} \quad (1)$$

For simplicity we consider only a zone near the NS, at the distances $r \ll 2\pi c/\omega$, where $\omega$ is the frequency of NS oscillations. For many global oscillation modes (see e.g. McDermott et al. 1988) the polar cap accelerating zone is well within this distance, therefore, we can study the changes of the accelerating electric field in the polar cap caused by oscillation. In the near zone all physical quantities change harmonically with time, i.e. the time dependence enters only through the term $e^{-i\omega t}$.

We make an additional assumption, that changes of the magnetic field induced by currents in the NS crust are much larger than the distortion caused by currents flowing in the magnetosphere. This assumption is considered as a first order approximation according to the small parameter $(\xi/R_{NS})$, where $\xi$ is the amplitude of oscillation and $R_{NS}$—the NS radius. In other words, outside of the NS

$$\nabla \times \mathbf{B} = 0 \quad (2)$$

in the first order in $(\xi/R_{NS})$. This assumption can be rewritten in terms of a condition on the current density in the magnetosphere as

$$j \ll \rho_{GJ} c \left( \frac{c}{\omega r} \right) \equiv \tilde{j}_{GJ} \left( \frac{c}{\omega r} \right) \quad (3)$$

Condition (3) implies that the current density in the near zone of the magnetosphere is less than the GJ current density connected with oscillations multiplied by a large factor $c/(\omega r)$. So, if the current density in the magnetosphere is of the order of the GJ current density, assumption (3) is valid. Under these assumption it is possible to solve the problem analytically in general case, i.e. to develop an algorithm for finding an analytical solution for the GJ charge density for arbitrary configuration of the magnetic field and arbitrary velocity field on the NS surface.

Under assumption (2) the magnetic field can be expressed through a scalar function $P$ as

$$\mathbf{B} = \nabla \times \nabla \times (P \mathbf{e}_r) \quad (4)$$

The GJ electric field depends also on a scalar potential $\Psi_{GJ}$ through the relation

$$\mathbf{E}_{GJ} = -\frac{1}{c} \nabla \times (P \partial_t \mathbf{e}_r) - \nabla \Psi_{GJ} \quad (5)$$

The GJ charge density is then expressed as

$$\rho_{GJ} = -\frac{1}{4\pi} \Delta \Psi_{GJ} \quad (6)$$

An equation for $\psi_{GJ}$ is

$$\Delta_{GJ} \partial_t \psi_{GJ} - \partial_t \partial_P \partial_P \psi_{GJ} - \frac{1}{\sin^2 \theta} \partial_r \partial_P \partial_P \psi_{GJ}$$

$$+ \frac{1}{c \sin \theta} (\partial_r \partial_P \partial_r \partial_P - \partial_t \partial_P \partial_t \partial_P) = 0 \quad (7)$$

where $\Delta_{GJ}$ is an angular part of the Laplace operator. This is the first order linear partial differential equation for the GJ electric potential $\psi_{GJ}$. As the equation for $\psi_{GJ}$ is linear, each oscillation mode can be treated separately. This equation is valid for arbitrary configuration of the magnetic field and for any amplitude of the surface oscillation, provided that condition (2) is satisfied. Dependence on oscillation mode appears in the boundary conditions and also through the time derivative $\partial_t P$. For different oscillation modes both the equation and boundary conditions for $\psi_{GJ}$ are different. Derivation of eq. (4) boundary condition for functions $\psi_{GJ}$ as well as more detailed discussion of used approximations can be found in [Timokhin et al. (2000)].

2.2 Goldreich Julian charge density

In [Timokhin et al. (2000)] equation (7) was solved for the case of small-amplitude toroidal oscillations and dipolar configuration of unperturbed magnetic field. Solutions had been obtained with a code written in computer algebra language MATHEMATICA. Now we have developed a new version of this code, which allows to obtain analytical solutions of equation (7) for a more complicated case of spheroidal modes. Any vector field on a sphere can be represented as a composition of toroidal ($\nabla \times \mathbf{V}_{osc} = 0$) and spheroidal ($\nabla \times \mathbf{V}_{osc} = 0$) vector fields [Unno et al.
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Fig. 1 The shape of the star surface during oscillations left for spheroidal mode (7, 3), right for spheroidal mode (7, 2). The shape changes harmonically with time.

Fig. 2 Electric potential $\Psi_{GJ}$ along a dipolar magnetic field line as a function of the polar angle $\theta$ is shown for 5 field lines with azimuthal angle $\phi = 0$. left for spheroidal mode (7, 3), right for spheroidal mode (7, 2). The quantity changes harmonically with time.

So, now we are able to calculate GJ electric field and charge density for arbitrary oscillations of a NS with dipole magnetic field.

Similar to the case of toroidal oscillations the small current approximation turned to be valid for a half of all oscillation modes. For oscillation modes with velocity field, which is symmetric relative to the equatorial plane (see an example of such mode in Fig. 1 (left)), solution of eq. (7) is smooth everywhere (see Fig. 2 (left)). For modes with antisymmetrical velocity field (an example of such mode is shown in Fig. 1 (right)), $\Psi_{GJ}$ is discontinuous at the equatorial plane (see Fig. 2 (right)). There is the following reason for such behavior. Dipolar magnetic field is antisymmetric relative to the equatorial plane. Antisymmetric motion of the field line footpoints give rise to a twisted configuration of the magnetic field, which cannot be curl-free. So, for such modes a strong current will flow along closed magnetic field lines, $j \gg j_{GJ}$. However, there is no physical reason why a smooth solution for GJ electric field can not exists also for such modes. An argument supporting this hypothesis is a solution for twisted force-free magnetic field found
Fig. 3 Charge density $\rho_{GJ}$ near the NS for azimuthal angle $\phi = 0$. Positive values of the charge density are shown in red and negative ones in blue. left: $\rho_{GJ}$ for the spheroidal mode $(7,3)$; right: $\rho_{GJ}$ for the spheroidal mode $(7,2)$. The quantity changes harmonically with time. NB: at the equatorial plane ($\theta = \pi/2$) $\rho^2_{GJ}$ is infinite.

Fig. 4 Charge density $\rho_{GJ}(r = R_{NS}, \theta, \phi = 0)$ on the NS surface is shown in a polar coordinate system $\left( |\rho^{|lm}_{GJ}(R_{NS}, \theta, 0)|, \theta \right)$. Positive values of charge density are shown by the solid line and negative ones by the dashed line. left: $\rho_{GJ}$ for the spheroidal mode $(7,3)$; right: $\rho_{GJ}$ for the spheroidal mode $(7,2)$. The quantity changes harmonically with time. Circles correspond to the values of $\rho_{GJ} = (0.1, 0.2, 0.3, 0.4, 0.5)$ in normalized units. NB: on the equatorial plane ($\theta = \pi/2$) $\rho^2_{GJ}$ is infinite.

by Wolfson (1995). In his solution, which corresponds to the toroidal mode $(2,0)$, the configuration of force-free twisted magnetic field is supported by a strong current flowing along magnetic field lines.

For the modes with smooth solutions our approximation should be valid. On the other hand, a strong electric current will flow only along closed magnetic field lines. The current density along open magnetic field lines should be close to $j_{GJ}$, and condition for the small current approximation (eq. (3)) will be satisfied in the open field line domain. Then, $\Psi_{GJ}$ in the polar cap could be obtained by solving eq. (7). $\Psi_{GJ}$ in the polar cap will differ from the solutions obtained here, because the boundary conditions should be set at the polar cap boundaries and not on the whole surface on the NS. The main properties of $\Psi_{GJ}$ regarding its qualitative dependence on the coordinates, used in our discussion, would be, however, similar to the properties obtained from our solutions.

As expected, the GJ charge density distribution follows the distribution of the velocity field (see e.g. Figs. 3, 4, 5). With increasing of the harmonic numbers, $\rho_{GJ}$ falls more rapidly with the distance, what is also expected for multipolar solutions. A remarkable property of GJ charge density distribution near oscillating star is that the local maxima of $\rho_{GJ}$ increases with increasing of both $l$ and $m$ (see Figs. 6, 7). The reasons for this are as follows. The electric field induced by oscillations is of the order

$$E_{GJ} \sim \frac{V_{osc}}{c}B.$$  

The charge density supporting this electric field is of the order of $E/\Delta x$, where $\Delta x$ is a characteristic distance of electric field variation. For a mode with harmonic num-
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Fig. 5 View of the polar cap of pulsar from the top. Goldreich-Julian charge density for oscillation mode (44, 4) is shown by the color map (positive values in red, negative – in blue). The quantity changes harmonically with time. The polar cap boundary for a pulsar with period 3 ms is shown by the dashed line.

The discrepancy between these charge densities gives rise to a longitudinal electric field (see Scharlemann et al. 1978; Muslimov & Tsygan 1992).

The GJ charge density for oscillation modes with large \( l \), \( m \) falls very rapidly with the distance. Hence, the charge density of a charge-separated flow for oscillating NS will exceed the local GJ charge density at some distance from the star. This produces a decelerating electric field. Therefore, in the case of rotating and oscillating NS, if \( \rho_{\text{osc}}^\text{GJ} < \rho_{\text{rot}}^\text{GJ} \), the effective accelerating electric field will be reduced periodically due to superposition of accelerating and decelerating electric fields.

The oscillational GJ charge density \( \rho_{\text{osc}}^\text{GJ} \) for modes with large \( l \), \( m \) decreases practically to zero already at 2–3 NS radii and the whole oscillational GJ charge density contributes to the decelerating electric field. While in the case of rotation only \( \sim 15% \) of the GJ charge density contributes to the accelerating electric field for SCLF in the polar cap (Muslimov & Tsygan 1992). The most important factor increasing modification of the accelerating electric field is that the amplitude of \( \rho_{\text{osc}}^\text{GJ} \) increases with increasing of the harmonic numbers of the mode. In Fig. 8 we show the difference between the charge density of SCLF and the local GJ charge density for dipolar magnetic field as a function of the distance for rotating and oscillating stars. In order to demonstrate the importance of the discussed effects we consider a rather grotesque case when the linear velocity of rotation is equal to the maximum velocity of oscillations. It is evident from this plot, that for large enough \( l \) and \( m \) the decelerating electric field caused by stellar oscillations could be of comparable strength with the accelerating electric field even if \( V_{\text{osc}} \ll V_{\text{rot}} \).

Let us estimate the harmonic number of the oscillation mode where decelerating electric field would have a given impact on the accelerating electric field induced by NS rotation. The decelerating electric will be \( \kappa \) times less that the rotational accelerating electric field,

\[
\kappa \equiv \frac{E_{\text{dec}}}{E_{\text{acc}}} \sim \frac{\rho_{\text{osc}}^\text{GJ}}{0.15 \rho_{\text{rot}}^\text{GJ}},
\]

if

\[
l \sim 0.15 \frac{V_{\text{rot}}}{V_{\text{osc}}},
\]


2.3 Particle acceleration in the polar cap of pulsar

The GJ charge density induced by NS oscillations influences particle acceleration mechanism in the polar cap of pulsar. As we will show, oscillations will have the strongest impact on the accelerating electric field in the polar cap of pulsar for models with free particle escape from the NS surface (Scharlemann et al. 1978; Muslimov & Tsygan 1992). The accelerating electric field in pulsars arises due to deviation of the charge density of the plasma from the local GJ charge density. For pulsar models with Space Charge Limited Flow (SCLF) the charge density of the flow \( \rho \) at the NS surface is equal to the local value of the GJ charge density, \( \rho(R_{\text{NS}}) = \rho_{\text{GJ}}(R_{\text{NS}}) \). Magnetic field lines diverge and the charge density of the flow decreases with increasing of the distance from the star. However, the local GJ charge density decreases in a different way and at some distance \( r \) from the NS \( \rho_{\text{GJ}}(r) \neq \rho(r) \). The discrepancy between these charge densities gives rise to
1. About observable parameters is necessary to study the oscillation modes (from left to right) (64, 14), (54, 14), (64, 2), (54, 2). The same relation for an aligned rotator is shown by the dashed line. Negative values of $\Delta \rho$ give rise to an accelerating electric field, negative ones — to a decelerating field.

3 Discussion

We have shown, that oscillations of the NS can induce changes in the accelerating electric field, which are more stronger than a naive estimation ($V_{\text{osc}}/c$) $B$. Indeed, for high harmonics the induced electric field will be $\sim l$ times stronger. In order to make definitive predictions about observable parameters is necessary to study the polar cap acceleration zone more detailed. An accelerating electric potential and the height of the pair formation front should be calculated.

However, we can do simple estimations using equation (14). Let us estimate the harmonic number of the mode which can cancel the accelerating electric field, assuming the mode is excited by a glitch. As we pointed in Sec. 2.1 only modes with non-zero amplitude on the NS surface can produce changes in the magnetosphere. The distribution of oscillational motion plays crucial role. If oscillations are trapped in the NS crust a rather small energy will be required to pump the oscillation amplitude to the level high enough for strong disturbance of the accelerating electric field. If a fraction $\epsilon$ of the NS mass $M_{\text{NS}}$ is involved in the oscillations the amplitude of the oscillational velocity is of the order

$$V_{\text{osc}} \sim \sqrt{\frac{2W_{\text{osc}}}{\epsilon M_{\text{NS}}}},$$

where $W_{\text{osc}}$ is the total energy of the mode. The energy transferred during the glitch of the amplitude $\Delta \Omega$ is

$$W_{\text{glitch}} = i I_{\text{NS}} \Omega \Delta \Omega = i I_{\text{NS}} \left(\frac{2\pi}{P}\right)^{2} \frac{\Delta \Omega}{\Omega},$$

(13)

where $i$ is the fraction of the total momentum of inertia of the NS $I_{\text{NS}}$ coupled to the crust. Let us assume that some fraction $\eta$ of this energy goes into excitation of oscillations. Using eqs. (13), (12), (11) we get conditions for harmonic number of modes which would periodically cancel the accelerating electric field, as

$$l > 300 \eta_{\%}^{-1/2} \epsilon^{1/2} \eta_{\%}^{-1/2} \left(\frac{\Delta \Omega}{\Omega}\right)^{-1/2},$$

(14)

for SCLF model, and for Ruderman & Sutherland (1975) model:

$$l > 2000 \eta_{\%}^{-1/2} \epsilon^{1/2} \eta_{\%}^{-1/2} \left(\frac{\Delta \Omega}{\Omega}\right)^{-1/2},$$

(15)
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Here $\eta_\%$ is measured in per cents and the relative magnitude of the glitch $(\Delta \Omega/\Omega)_0$ is normalized to $10^{-6}$. It is widely accepted, that the origin of pulsar glitches is angular momentum transfer from the NS core to the crust. In the frame of this model the fraction of the energy which can go into excitation of NS oscillation $\eta$ is of the order of $\Delta \Omega/\Omega$, i.e. it is very small, of the order of $\sim 10^{-6}$. We may speculate however, that excited oscillations are trapped in the NS crust, i.e. $\epsilon$ is also very small. Such global oscillation modes ($l \sim$ several hundreds) could induce substantial changes in the accelerating electric field.

As we mentioned in Sec. 2.1 all physical quantities in the solutions obtained here oscillate with the frequency of the star oscillations. The accelerating electric field close to the local geometrical maxima of the oscillational GJ charge density will be weakened periodically by the decelerating effect due to stellar pulsations. The field oscillation will influence the particle distribution in the open field line zone of the pulsar magnetosphere and it should produce some observable effects. Depending on oscillation mode and position of the line of sight a complicated pattern will appear periodically in individual pulse profiles. Although individual pulses are highly variable, the presence of periodical features should be possible to discover in the power spectra of pulsars, provided the oscillations are excited to a high enough level and observations have been made with hight temporal resolution. If one observes some feature, which appears just after the glitch, then decreases and disappears after some time, and it never appears in the normal pulsar emission, then one can undoubtedly attribute this feature to the NS oscillations.

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