Temporal behavior of synchronization between chaotic fiber lasers

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A new configuration for synchronizing two chaotic fiber lasers, which includes both coupling and losses, is presented. Experimental and calculated results reveal that the synchronization time can be significantly shorter than with configuration that have only coupling.

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In general, chaotic lasers have received considerable attention in the past few years, because they are potentially useful for applications such as encryption and secure communication [1, 2, 3]. More recent, attention has been shifted to chaotic fiber lasers that have a rich and very broad bandwidth [2, 3, 4, 5, 6, 7, 8, 9]. In these applications synchronization plays a dominant role where the most crucial factor is the time that is necessary for synchronizing two or more chaotic lasers [8, 9, 10, 11].

In this letter, we investigate the temporal dynamics of synchronization between two chaotic fiber lasers. We consider two different configurations. One is a modified version of the configuration which is normally used for synchronizing two lasers, in which the coupling between the lasers is performed outside their cavity - namely outer-cavity configuration. The other configuration is one in which the coupling between the lasers is performed inside their combined cavity - namely intra-cavity configuration. We develop relatively simple models for both configurations that predict that synchronization will occur much faster with the intra-cavity configuration, in agreement with the experimentally obtained results.

The basic outer-cavity configuration is presented schematically in Fig. 1. It includes two fiber lasers with an intermediate output coupler, a 50/50 beam splitter and an additional combined output coupler for coupling and synchronizing the two lasers. Also included is a two lens 4f optical arrangement in which a chopper which spins at 1000Rpm is placed at the focal plane between the lasers for rapidly switching on and off the coupling between the lasers. With such an arrangement it was possible to experimentally obtain either no coupling or strong coupling with coupling strength rise time of 120ns, as shown in the inset. Each fiber laser consists of Erbium doped fiber of about ten meters in length, where one end is attached to a high reflection fiber Bragg grating (FBG) with central wavelength of 1550nm and bandwidth of about 1nm that serves as a back reflector mirror and the other end is spliced to a collimating graded index (GRIN) lens with anti-reflection layer to suppress any reflections back into the fiber cores, and an intermediate output coupler of 20% reflectivity. Each fiber laser is pumped with a rapidly oscillating diode laser which imposes chaotic behavior, to obtain two chaotic fiber lasers. The beam splitter combines the beams from the two chaotic lasers into one beam and the additional output coupler reflects back part of the light of the combined beam back into the fiber lasers, thereby obtaining coupling between them [12, 13].

The intra-cavity configuration is similar to that shown in Fig. 1 but without the intermediate output couplers. Accordingly, the beam splitter is now inside the cavities of the lasers and the combined output coupler is common for both fiber lasers. In this configuration, the beam splitter introduces high losses to each of the lasers when they are not synchronized, i.e. the losses depend on the synchronization. In general, the lasers prefer to operate at minimum losses. The minimization of losses occurs very rapidly, so the two lasers will synchronize even at a much faster rate than at the outer-cavity configuration, were there are no such losses at all.

We began our experiments by simultaneous detecting the output power as a function of time for each laser. This was done by reflecting a small portion of the light that is propagating in each lasers towards fast photodetectors. Representative results are shown in Figs. 2 and 3. Figure 2 shows the results when the lasers are not coupled. As expected, the two signals are not synchronized. Figure 3 shows the results when the lasers are strongly coupled. As evident, the lasers are now synchronized.

Figure 4 shows the results when the lasers are strongly coupled. As evident, the lasers are now synchronized. This is confirmed by plotting the output power of one laser as a function of the output power of the other, at

FIG. 1: (color online) Basic outer-cavity configuration for synchronizing between two chaotic fiber lasers and measuring the temporal behavior.
FIG. 2: Experimental results of the output power of the two lasers as a function of time when the lasers are not synchronized. Inset shows the output power of one fiber laser as a function of the output power of the other.

The inset in Fig. 3 which yield a narrow linear distribution. The corresponding calculated correlation coefficient for this distribution was over 0.98. It should be noted that the intensity fluctuation of the lasers in Figs. 2 and 3 are mainly due to beating between different longitudinal modes. Thus, the synchronization between the fiber lasers in our configurations is not only due to synchronization of intensity fluctuation but also due to phase locking of the lasers longitudinal modes.

We now determined the temporal behavior of synchronization between the two lasers around the transition from no coupling to full coupling. This was done by measuring simultaneously the output powers as a function of time for the two lasers around the transition region and calculating their correlation coefficient within a moving window of 100 ns width. This correlation coefficient is essentially the instantaneous synchronization between the lasers.

The results of synchronization as a function of time starting at the onset of coupling for both the outer-cavity and intra-cavity configuration, are shown in Fig. 4. The dots denote the experimentally obtained results for the outer-cavity configuration, indicating that it takes 350 ns to reach 80% synchronization. The circles denote the experimentally obtained results for the intra-cavity configuration, indicating that it takes 100 ns to reach 80% synchronization, probably limited by the finite rise time of the coupling as dictated by the chopper optical arrangement. These results clearly show that it is at least three times faster to reach reasonable synchronization with the intra-cavity configuration. To elucidate these results we also present experimental results for configuration which has intermediate output coupler with much lower reflection than in the outer-cavity configuration but still non-zero as it was in the intra-cavity configuration. These results are presented in Fig. 4 as the stars. In this intermediate configuration, the time in takes for the synchronization to reach 80% is 200 ns, which is indeed in-between the two extreme cases.

FIG. 3: (color online) Experimental results of the output power of the two lasers as a function of time when the lasers are synchronized. Inset shows the output power of one fiber laser as a function of the output power of the other.

FIG. 4: (color online) Experimental and calculated results of the synchronization between two chaotic fiber lasers as a function of time after the onset of coupling. Dots denote experimental results for the outer-cavity configuration. Circles denote experimental results for the intra-cavity configuration. Stars denote experimental results for configuration between the outer-cavity and the intra-cavity configurations. Solid curve denotes calculated results for the outer-cavity configuration. Dashed curve denotes calculated results for the intra-cavity configuration.

In order to support our results, we developed a model...
to help elucidate the temporal behavior of synchronization between two fiber lasers. To analyze the complex behavior of fiber lasers one should consider a detailed model [14], introducing the parameters of our system this model is reduced to the well known equations of coupled single longitudinal mode lasers [15], as

\[ \frac{dE_{1,2}}{dt} = \frac{1}{\tau_c} [(G_{1,2} - \alpha) E_{1,2} + \kappa E_{2,1}] + i\omega_{1,2}E_{1,2}, \]  

(1)

\[ \frac{dG_{1,2}}{dt} = \frac{1}{\tau_f} (P - G_{1,2} - G_{1,2}E_{1,2}^2), \]  

(2)

where \( E \) is the electric field in each of the lasers, \( G \) the gain, \( \tau_c \) the life time of the cavity, \( \tau_f \) the life time of the excited state in the gain, \( \alpha \) the losses in the cavities, \( P \) the pumping rate, \( \kappa \) the coupling strength between the lasers and \( \omega \) the frequency in each laser. Then we substitute \( E_{1,2} = A_{1,2} \exp(i\varphi_{1,2}) \) with \( A_{1,2} \) the amplitude of each laser, and define \( \beta \) as the ratio between these amplitudes, as

\[ \beta = \frac{A_1}{A_2}, \]  

(3)

where \( d\beta/dt \) is a measure for the synchronization, i.e. \( d\beta/dt = 0 \) indicates that the two lasers are synchronized.

In order to analytically evaluate the temporal behavior of synchronization between the lasers in the outer-cavity configuration, we use Eqs. (1) and (3) to derive an equation of motion for \( \beta \), as

\[ \frac{d\beta}{dt} = \frac{\kappa}{\tau_c} (1 - \beta^2) \cos (\varphi_1 - \varphi_2). \]  

(4)

Thus, when the two lasers are synchronized \( \beta = 1 \). Since phase locking occurs very rapidly [13], so \( \varphi_1 - \varphi_2 < 1 \), we can neglect the cos term. Thus, the solution of Eq. (4) is

\[ \beta(t) = \frac{\exp \left( \frac{2\kappa t}{\tau_c} \right) + C}{\exp \left( \frac{2\kappa t}{\tau_c} \right) - C}, \]  

(5)

where \( C \) is a constant of integration that is set by the initial conditions. Using Eq. (5) we calculated \( \beta \) as a function of time. The results are presented as the solid curve (blue) in Fig. 4. As evident, there is a good agreement between the experimental results and those predicted by the model, indicating that the assumption of \( \varphi_1 - \varphi_2 < 1 \) was justified.

For the intra-cavity configuration, the cavity losses are reduced when there is destructive interference between the two beams in the loss channel. The efficiency of the destructive interference is a function of the relative phase and the relative intensity of the two lasers. Assuming that the phase locking occurs much faster than the synchronization, the losses are a function of only \( \beta \). Accordingly, in order to analytically evaluated the temporal behavior of the synchronization between the lasers in the intra-cavity configuration we replaced the constant losses \( \alpha \) in Eq. (1) with a losses which are a function of \( \beta \), as

\[ \alpha(\beta) = \frac{(\beta - 1)^2}{2}. \]  

(6)

Now we use Eqs. (1), (3) and (6) to derive the equation of motion of \( \beta \), as

\[ \frac{d\beta}{dt} = \frac{1}{\tau_c} \left( \frac{(\beta - 1)^2 \beta}{2} + (1 - \beta^2) \kappa \cos (\varphi_1 - \varphi_2) \right). \]  

(7)

Again assuming \( \varphi_1 - \varphi_2 < 1 \) and also assuming a small coupling of \( \kappa < 0.1 \), then an approximate analytic solution of Eq. (7) is

\[ \beta(t) = \frac{\exp \left( \frac{6\kappa t}{\tau_c} \right) + C}{\exp \left( \frac{6\kappa t}{\tau_c} \right) - C}. \]  

(8)

where \( C \) is a constant of integration that is set by initial conditions. Using Eq. (8), where exponentials are three times larger than these of Eq. (5), we calculated the evolution of \( \beta \) as a function of time. The results are presented as the dashed curve (red) shown in Fig. 4. As evident, the experimentally obtained rise-time is somewhat longer than the predicted, probably because it is limited by the finite rise-time of the coupling which is about 120ns.

To conclude, we investigated the temporal behavior of synchronization between two chaotic fiber lasers when the coupling between them is suddenly switched on. Two configurations were considered, one where the synchronization is achieved with just coupling and the other by coupling as well as additional losses. The results reveal that with the additional losses the rise-time was about three times shorter than with just coupling. We performed our experiment for a wide range of pump modulation parameters, some of them chaotic and some not, and obtained very similar results for both regimes. Such reduction of rise time should be useful in number of applications such as higher possible bit rate in secure communication.

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