Consolidation of ultrafast optics in electron spectroscopies based on free electron energy exchange with matter has matured significantly over the past two decades, offering an attractive toolbox for the exploration of elementary events with unprecedented spatial and temporal resolution. Here, we propose a technique for monitoring electronic and nuclear molecular dynamics that is based on self-heterodyne coherent beating of a broadband pulse rather than incoherent population transport by a narrowband pulse. This exploits the strong exchange of coherence between the free electron and the sample. An optical pulse initiates matter dynamics, which is followed by inelastic scattering of a delayed high-energy broadband single-electron beam. The interacting and noninteracting beams then interfere to produce a heterodyne-detected signal, which reveals snapshots of the sample charge density by scanning a variable delay $T$. The spectral interference of the electron probe introduces high-contrast phase information, which makes it possible to record the electronic coherence in the sample. Quantum dynamical simulations of the ultrafast nonradiative conical intersection passage in uracil reveal a strong electronic beating signal imprinted onto the zero-loss peak of the electronic probe in a background-free manner.

Strongly coupled electronic and nuclear motions in molecules give rise to ultrafast relaxation pathways, some of which are optically dark. State-of-the-art single-electron microscopes equipped with ultrafast optical elements introduce hybrid probes involving both electrons and photons. In this theoretical study, we show that when a broadband fast electron ($\sim 0.3 \text{ MeV}$) passes near an optically pumped molecule, an interference between the interacting and noninteracting free electron trajectories stores information regarding inelastic scattering in the transmitted electron spectrum. This single-pulse inelastic coherent electron scattering (SPICES) technique is demonstrated for the conical intersection (CoIn) passage with joint nuclear–electronic contributions in the RNA base uracil.

Several experimental techniques (1–5) have been proposed for monitoring ultrafast charge-density dynamics by combining electron microscopy with advanced ultrafast optics, pioneered by Zewail and coworkers (1, 3, 4). Photon-induced near-field electron microscopy (1, 3–5) acquires temporal resolution via the embedded ultrafast optics, which makes it possible to record the electronic coherence in the sample. Quantum dynamical simulations of the ultrafast nonradiative conical intersection passage in uracil reveal a strong electronic beating signal imprinted onto the zero-loss peak of the electronic probe in a background-free manner.

### Significance

In a theoretical study, we present an ultrafast technique for probing time-dependent molecular charge densities. An ultrafast optical pump first brings the molecule into an electronic nonstationary state. This is followed by coherent inelastic scattering of a broadband single-electron probe pulse with a variable delay $T$, which is detected spectrally. The technique is applied to reveal phase-sensitive background-free coherent electron beating in the conical intersection passage in uracil and reveals the otherwise elusive coherent beating of strongly coupled electrons and nuclei.
direction by the long-range Coulomb interaction with the sample. The scattered electron pulse initially centered at a variable delay $t = T$ is energy dispersed. The process is described by the loop diagram in Fig. 2. The spectrally resolved electron is derived in SI Appendix, section I, which yields the SPICES signal

$$S_k(T) = \frac{2e}{h\gamma} \Re \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega T} \int_{\mathbb{R}^3} \frac{d\mathbf{q}}{(2\pi)^3} \sigma_{\mu}(q,\omega) \frac{1}{v\mathbf{k} \cdot \mathbf{q} - \omega - i\gamma} \phi_{\mathbf{k}} - \phi_{\mathbf{k}}^{\dagger},$$

where $\sigma_{\mu}$ is the total molecular charge density, $\phi_{\mathbf{k}}$ is the free electron amplitude, $\mathbf{k}$ is the detected electron momentum related to the energy–momentum exchange via $\omega = \mathbf{q} \cdot \mathbf{v}_k = \frac{1}{2} \mathbf{k} \cdot \mathbf{q}$, and $v_k$ is the corresponding free electron velocity. The SPICES signal scales $\propto q^{-2}$ rather than the $q^{-4}$ for conventional incoherent (narrowband) EELS (9).

Here, our primary goal is spectroscopy rather than microscopy. To that end, we utilize a collimated, unfocused electron rather than a tightly focused beam as used when spatial information is desired. This offers insights into the molecular dynamics thanks to the high-contrast information delivered by the hybrid probe regarding the strong nuclei–electron coupling.

### Results and Discussion

**The Setup.** The system is described by the Hamiltonian $H = H_e + H_f + H_{\mu} + H_I$, composed of the electromagnetic field ($\varphi$), free electron ($f$), matter ($\mu$), and interaction ($I$) contributions. The free electron’s Hilbert space is assumed to be separable from the matter, excluding exchange pathways from the physical discussion. The interaction Hamiltonian of the sample with the electron field is given by

$$H_I = -\frac{e^2}{2} \int d^3r \frac{\sigma_f(r) \cdot A(r)}{|R|} \int d^3rd^3R \frac{\sigma_{\mu}(r + R) \sigma_{\mu}(r)}{|R|},$$

where we have used the Coulomb gauge for the electromagnetic vector potential $A(r)$, $\sigma_{f}$ and $\sigma_{\mu}$ represent the free electron and sample charge-density operators, respectively. The single free electron is described by the wave packet $|1_e\rangle = \sum_p \phi_p f_{pR}^\dagger |\bar{p}\rangle$, where $\phi_p$ is the single-electron amplitude, $|\bar{p}\rangle$ is the free electron vacuum, and $f_{pR}^\dagger$ ($f_{pR}$) is a free (spinless) electron creation (annihilation) operator obeying Fermion anticommutators $\{f_{pR}, f_{p'R}^\dagger\} = \delta_{pp'}$. Gauge fixing is crucially important for the physical interpretation of the detected quantities. Note that the interactions of the bound electrons and nuclei are included in $H_{\mu}$. The free electron and matter are initially uncorrelated, and their wave functions are thus factorized.

The free electron measurement is represented by instantaneous annihilation of a momentum state using the electron operator $f_k$ within an energy window $m_{k,k'}$, given by $\mathcal{M}_k(t) = \int d^3k' m_{k,k'} f_{k'}(t)$. These operators are defined by the measurement setup and the detector properties. The signal is given by the integrated current with momentum $k$:

$$S_k(T) = \int dt \delta I_k(t),$$

where $\delta I_k = \langle \Phi(t) | I_k^{(\text{int})} - I_0 | \Phi(t) \rangle$ is the change in the free electron spectral density due to the interaction of the interfering and noninteracting contributions. $I_0$ is the initial spectral density of the probe, $I_k^{(\text{int})}$ represents the interaction with the sample, and $|\Phi(t)\rangle$ is the joint electron plus sample wave function. We expand the joint electron–sample wave function in powers of the interaction Hamiltonian (2). In the Coulomb gauge, the vector field $A(r)$ does not contribute to first order since the electromagnetic field for $T > 0$ is in its vacuum state. The coupling is thus solely given by the Coulomb term $H_I = -\frac{e^2}{2} \int d^3r d^3R \frac{\sigma_f(r + R) \sigma_{\mu}(r)}{|R|}$.

Eq. 1 is obtained by using 1) high spectral resolution $m_{k,k'}$ such that $\nu \Delta k \ll \gamma$ (SI Appendix, section I), and 2) the energy exchange between the sample and electron is much smaller than the free electron central momentum $k_i \gg q$ such that $\tilde{\epsilon}_k + v_k \cdot q \approx \epsilon_k + v_k \cdot q$, where $v_k = \frac{\hbar}{m} k$ and $\tilde{\epsilon}_k \parallel \zeta$.

Eq. 1 includes the free electron amplitude $\phi_k$, reflecting the self-heterodyning interference between noninteracting and interacting electron states. Only transitions within the electron’s bandwidth are recorded. In the single-electron regime considered here, the effective electron bandwidth characterized by the FWHM $\Delta \epsilon$ is determined by several factors. One important factor is the match
between the energy of the ionizing photon $h\omega_p$ and the cathode work function $\Phi$ (10). By tuning this parameter $\alpha = h\omega_p - \Phi$, a reduction of the electron energy distribution was reported in both the longitudinal and transverse momentum components (11). Minimal broadening values of $\Delta p/e\{10^{-4},10^{-6}\}$ have been realized (including acceleration-induced broadening) in the range of 30 to 300 keV (11). Our proposed measurement benefits from a broad energy spread, for which $\alpha$ can be tuned.

**Monitoring the Nonadiabatic Dynamics.** The time-dependent wave function of the optically pumped sample can be expanded in a superposition in the adiabatic basis:

$$\Psi(r, R, t) = \sum_i c_i(t) \varphi_i(r, R) \chi_i(r, R). \quad [4]$$

Here, $\varphi_i(r, R)$ is the $i$th electronic state; $\chi_i(r, R)$ is the corresponding nuclear wave packet; $r$ and $R$ are electronic and nuclear positions, respectively; and $c_i(t)$ is its time-dependent amplitude due to the optical pumping. Eq. 1 can be expressed explicitly:

$$S(k, T) \propto \Im \left\{ \int dt d\epsilon e^{i\epsilon k q(T-\omega)} \frac{\hat{\rho} \rho^*}{q^2} \right\} \sum_{ij} \rho_{ij}^* (t) \langle \chi_i (t) | \hat{\sigma} (q) | \chi_j (t) \rangle,$$

where $\rho_{ij} (t) = c_i (t) c_j^* (t)$ is the electronic coherence and $\hat{\sigma} (q)$ is the Fourier transform of the total charge density. The charge-density operator $\sigma = \sigma_e + \sigma_n$ (electron plus nuclei) is assumed to be diagonal in the nuclear space (no coherence), and the momenta $\{k, q\}$ are parallel to the electron trajectory (here, the $z$ axis by convention).

**Application to the Coln Passage in Uracil.**

**CoIns.** CoIns are degeneracy regions between electronic potential energy surfaces of molecules. Around CoIns, the electronic and nuclear frequencies are comparable and strongly coupled. Thus, they play an essential role in photochemistry, enabling ultrafast nonradiative relaxation pathways ($<100$ fs). This renders them strong candidates for energy conversion and switching applications that rely on CoIns as their operational principle. Moreover, CoIns offer control knobs over product yields and rates of a large class of photochemical processes (reviewed in ref. 12). Numerous photoinduced reactions enabled by CoIns have been studied, such as cyclohexadiene ring opening (13), photosynthesis of vitamin D (14), retinal photosomization in the primary event of vision (15), photodamage of DNA (16), and DNA repair (17). Due to their high quantum yield and ultrafast switching, optical molecular switches that rely on CoIns have been proposed for numerous applications ranging from optical memories to chemical energy storage (18, 19). From the theoretical perspective, CoIns are exceptionally interesting since the electronic and nuclear motions are strongly coupled and thus, inseparable [beyond the Born–Oppenheimer approximation (20)]. Spectroscopically, direct detection of CoIns is a daunting challenge since the associated signatures—based on coherences rather than populations—are intrinsically weak. Here, we show how the direct detection of CoIns is possible by using pulsed electrons.

**Application to uracil.** Our model is based on ab initio multireference quantum chemical data on the photorelaxation of the RNA nucleobase uracil (21) that is crucially relevant for RNA photostability. An effective Hamiltonian that includes two nuclear degrees of freedom is constructed. The latter captures the relaxation through a Coln after the optical excitation with kinetic rates that match corresponding experiments (22). Exact nonadiabatic quantum dynamical propagation of the photoexcited wave packet is performed, fully capturing the quantum character of the nuclei and therefore, the Coln passage. The Hamiltonian, along with analysis of the wave packet motion, has been described in detail elsewhere (21, 23). Briefly, a 20-fs FWHM optical pump pulse, resonantly tuned to the electronic $S_0 \rightarrow S_2$ transition, launches a nuclear wave packet in the $S_2$ excited state. After a short free evolution period, it reaches the $S_2/S_1$ Coln around 100 fs and relaxes to $S_1$. This generates a vibronic coherence between $S_2$ and $S_1$ due to the nonvanishing overlap of the nuclear wave packet in the two states. Small parts of the wave packet continue to reach the Coln, and it takes several hundred femtoseconds for the relaxation to be completed. The population and coherences dynamics are depicted in Fig. 3 A–C and in Fig. 3 D–F, respectively.

The average beam momentum $\langle k_0 \rangle$ introduces fast oscillations in Eq. 5, from which the relevant integration time can be inferred. When the electron’s velocity is approximately half the speed of light in vacuum $v_0 \approx 0.5 c$, these oscillations determine the temporal resolution $\delta T \equiv t - T$, which depends on the observed momenta exchange with the sample. The longitudinal momenta exchange, parallel to the electron propagation direction ($q \parallel k$), determines the significant integration interval—here, collimated and thus, quasione dimensional. For momentum exchange $q$ in the range of several inverse angstroms ($10^{-10}$ m), the effective temporal width of the probe is $\delta T > 10^{-18}$ s. It is, therefore, reasonable to estimate the temporal resolution within the order of 0.1 fs, which is longer than the time step used for the numerical calculations $\Delta t_{\text{num}} = 0.48$ fs. Assuming that within these time steps, the charge density does not vary significantly, the temporal summation can be replaced by sampling the charge density at $t = T$. This assumption is validated by our numerical calculation of the temporal dynamics. In this parameter regime, the signal is given by

$$S_k (T) \propto \Im \left\{ \int dq \frac{\hat{\phi}_k - \phi_k^* q^2}{q^2} \sum_{ij} \rho_{ij}^* (T) \langle \chi_i (T) | \hat{\sigma} (q) | \chi_j (T) \rangle \right\} \equiv \sum_{ij} S_{ij} (k, T). \quad [6]$$

Eq. 6 is derived from Eq. 1 by performing the Fourier transform of the charge density to time domain and integrating the radial frequency $\omega$ for small $\gamma$ followed by eliminating the fast oscillating contributions with respect to the resolution as discussed above (Eq. 5).

Fig. 4 depicts the various contributions $S_{ij} (k, T)$ defined in Eq. 6. Fig. 4 A–C captures the population dynamics, and the electronic coherences are given in Fig. 4 D–F. From Eq. 6, we can appreciate that low momenta values contribute significantly to the signal due to the $q^{-2}$ weight. This corresponds to long-wavelength dominance, similar to the general form of transient redistribution of ultrafast electronic coherences in attosecond Raman signals (TRUECARS) introduced in ref. 24. TRUECARS is dominated by the polarizability and generated by the longitudinal component of the off-resonant X-ray scattering that follows optical excitation. Here, the long wavelength of the electronic charge density plays a similar role to the one of the polarizability in the TRUECARS signal. From Fig. 4, it is clear that electronic population contribution to the signal is antisymmetric and vanishes exactly at $k = k_0$, which stems from the fact that this is a
Fig. 3. Population and coherence dynamics. The molecule is initially in its ground state, depicted by the $|0\rangle\langle0|$ population in A. Around $t=100$ fs, the second excited state $|2\rangle$ is populated (B) followed by probability flow to state $|1\rangle$ (C). D–F present the real parts of the respective electronic coherence rescaled by the green highlighted prefactor for visibility.

First-order contribution (the imaginary part of the integration is taken). The coherences, on the other hand, are maximal at $k = k_0$ and thus, contribute. This results in a background-free electronic coherence signal on the zero-loss peak.

The overall signal obtained by the summation in Eq. 6 is depicted in Fig. 5. By comparing Fig. 4 F with Fig. 5, it is evident that the $|1\rangle\langle2|$ coherence dominates the signal. This coherence is developed during the dynamics around $\approx 180$ fs when the molecule reaches the CoIn between these electronic states. It is evident from the numerical integration that the $|0\rangle\langle2|$ and $|0\rangle\langle1|$ coherences are not negligible, yet they contribute significantly less than the $|1\rangle\langle2|$ coherence. The main reason for this is that the temporal resolution required to sample this oscillator is <0.4 fs and thus, seems to be averaged/filtered out.

Fig. 4. Signal contributions. A–C depict the contributions to the signal in Eq. 6 from the diagonal contributions $S_{ii}$ ($\sigma_{00}, \sigma_{11}, \sigma_{22}$, respectively), corresponding to the electron aligned along the x direction of the charge density, shown in Right. D–F depict the signals arising from the transition charge densities $\sigma_{01}, \sigma_{02}, \sigma_{12}$, respectively, corresponding to $S_{ij}$ ($i \neq j$) under similar conditions.
Fig. 5. Total signal. The recorded signal is obtained from superposition of all the contributing processes given by Eq. 6.

The signal contributions in Fig. 4 exhibit temporal oscillations with different frequencies. These are especially pronounced for the coherences in Fig. 4 D–F, where the $|0⟩⟨2|$ and $|0⟩⟨1|$ contributions oscillate with higher frequency due to the energy difference between the adiabatic states, while the frequency of the $|1⟩⟨2|$ contribution that stems from the CoIn is lower. To characterize them further, we performed a temporal gating analysis inspired by a cross-correlation frequency-resolved optical gating (XFROG) measurement (25–35) according to

$$I_{\text{FROG}}(\omega, T) = \left| \int_{-\infty}^{\infty} dt S_k(t) E_{\text{gate}}(t - T) e^{-i\omega t} \right|^2, \tag{7}$$

where $E_{\text{gate}}$ is a Gaussian function with a 3-fs FWHM and $S(t)$ is a temporal signal trace taken at $k = k_0$ where the coherence dominates. $I_{\text{FROG}}$ is numerically evaluated only at the postprocessing stage, requiring no additional measurements. It gives information about the transient frequency of the oscillations and is depicted in Fig. 6 for all contributions and the total signal.

While the population signatures are around zero frequency, the $|0⟩⟨1|$ and $|0⟩⟨2|$ coherences appear between 2 and 3 eV in Fig. 6 D and E. The energy difference between the adiabatic states is around 5 eV, where the discrepancy can be explained by evaluating the signal "only" every 1 fs and thereby, undersampling with respect to these fast oscillations. Nevertheless, they are well separated from the $|1⟩⟨2|$ coherence in Fig. 6 F. This is the most interesting dynamical signature. It starts at 0.1 eV around 80 fs and then, splits into a constant signature staying at 0.1 eV and another one that decays to 0.01 eV at 120 fs. As discussed in ref. 23, the 20-fs pump pulse excites a local nuclear wave packet in $S_2$, which then travels to the CoIn. This arrival is directly mapped by the FROG spectogram in Fig. 6 F. The $|1⟩⟨2|$ coherence emerges at 0.1 eV, where the nonadiabatic coupling between $S_2$ and $S_1$ already is nonvanishing. Part of it then travels to the actual crossing region where the energy difference between the adiabatic states is vanishing. The signal thus provides direct access to nuclear wave packet pathways in ultrafast molecular dynamics. The FROG of the total signal in Fig. 6 G is dominated

![Fig. 6. XFROG analysis. Using Eq. 7 with $S(t)$ taken at $k = k_0$ from the individual contributions, we depict the XFROG decomposition of the total signal. A–C correspond to XFROG spectrograms of the populations and D–F to the ones of the coherence contributions to the signal in Fig. 4. G presents the XFROG of the total signal in Fig. 5.](https://doi.org/10.1073/pnas.2205510119)
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Fig. 7. XFrog analysis according to Eq. 7 with $S(t)$ taken at $k = 2.8$ Å (A) and 5.6 Å (B) from the total signal in Fig. 5.

by this $|1⟩|2⟩$ contribution since we take the signal trace at $k = k_0$ where the populations are weak, allowing for direct access to the coherences. By gradually shifting $S_k(t)$ to $k$ values away from $k_0$, populations get more visible in the total XFROG and eventually, dominate. This is demonstrated in SI Appendix and constitutes a convenient and readily accessible analysis tool for both populations and coherences (Fig. 6).

Conclusions

SPICES is a technique that monitors electronic coherences of optically pumped aligned molecules in a background-free manner using frequency-resolved broadband electron and manifest heterodyne electron detection. The signal is maximal when the optical pulse propagates parallel to the electron beam. The temporal resolution of the nonadiabatic evolution is initiated by the optical pulse by scanning the delay time ($\tau$) between the optical pulse and the center of the free electron pulse. In the unfocused, collimated, broad-beam regime considered here (weak electron–sample coupling), the long-range longitudinal component (Coulomb term) is dominant, and due to the dispersive broadband nature of the coupling, the free electron’s phase is significantly shaped by the coherence of the charge density.

The hybrid combination of electron and photon probes is particularly useful to monitor ultrafast coherence phenomena. The photonic degrees of freedom offer the well-developed quantum optical toolbox with unparalleled temporal control and weak nondestructive coupling as well as advanced generation and detection schemes (26–31). The electronic degrees of freedom present a highly versatile broadband source with unparalleled spatial resolution. The dispersive coupling—responsible for the electron’s decoherence properties even in the weak coupling regime—is useful for sensing electronic coherences.

The weak coupling regime enabled the spatial extension of the electron pulse in the transverse plane (low effective cross-section).

Therefore, the interaction can be interpreted as the interference of free electron trajectories recoiled by the long-ranged Coulomb term. This picture is related to the recently proposed “aloof spectroscopy” (32), whereby an electron beam is focused outside the sample, generating optical excitations that depend on the distance between the focusing point and the sample. Here, since the temporal resolution is important to sample the ultrashort electronic coherence, the entire free electron bandwidth is used.

The present signal corresponds to heterodyne detection of the generated electron field with the incoming beam. Ordinary EELS signals require two interactions with the incoming beam. Here, we only need one interaction, and the signal is given by its interference with the incoming beam. For photons, this is known as heterodyne detection. This signal is analogous to TRUECARS introduced in ref. 24, where the electron field is replaced by a photon field. In SPICES, the electronic coherence is more pronounced due to the long-ranged Coulomb potential.

For nonradiative decay pathways, such as Coln, higher-order processes are not required. Generally, the relaxation process from the initial optical pump may be radiative, which will result in a resonant term $\alpha (j \cdot \mathbf{A})^2$ in the interaction Hamiltonian for the photon generation (Wigner–Weisskopf like). Interference between these two processes may also lead to interesting effects, although not studied in this work. Recently, some of these advantages were experimentally demonstrated, showing significant background reduction (pathway filtering) in energy-dispersive X-ray spectrometry (33). Moreover, photon–electron and electron–electron entanglement is expected to enhance pathway separation, introducing noise reduction mechanisms and joint time–frequency resolution that exceed the classical boundaries (34).

Data Availability. All study data are included in the article and/or SI Appendix.

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