Indentation Size Effect of the Vickers Indentation to Improve the Accuracy of Inverse Materials Properties Modelling Based on Hardness Value

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Abstract. In this work the Indentation Size effect (ISE) in the Vickers hardness tests and the scale and consistency/reliability of ISE has been systematically investigated by fitting data following the power law and proportional resistance model. In both cases, the results show that the ISE can be linked to the hardness-to-modulus ratio. A new concept of using ISE data for estimating the n values of steel has been explored and shown reasonable results for narrowing the range of predicted material properties based on hardness values.

A new concept to use the indentation size data of the Vickers indentation has been explored to improve the accuracy/robustness of inverse properties modelling based on hardness. Systematic experimental work has been performed on steel samples of different carbon contents and heat-treatments. The ISE data was analysed by fitting data following the power law and proportional resistance model. In both cases, the results show that the ISE can be linked to the hardness-to-modulus ratio (H/E). A new concept of using ISE data for estimating the work hardening coefficients (n) values of steel has been explored and shown reasonable results for narrowing the range of predicted material properties based on hardness values, thus improve the robustness of the inverse program.

Keywords : Vickers hardness, ISE, H/E, P-h curve, work hardening coefficients

1. Introduction

In a hardness measurement the impression size was measured after the applied load has been released. Theoretically, the hardness of the materials should be comparable when using different applied load, however in some cases, the hardness has been found to be dependent on the load applied. This was defined as indentation size effect (ISE)[1] the most common indentation size effect, the hardness increases with decreasing applied load (i.e. decrease in indentation size). There is another type of ISE called reverse ISE[2] in which the hardness decreased with lower loads. Study the indentation size effect (ISE) it is known that an increase in hardness with increasing indentation size, especially in the in the micro hardness depth regime[3], [4], [5]. The source of ISE is still a subject of study, which has been attributed to a number of
phenomena and mechanisms, including: elastic recovery, work hardening during indentation and strain gradients associated with dislocations [6], [7]. The scale of ISE has been correlated to different material properties such as elastic modules, dislocation density, etc.[8] . It is of interest to extend the work to steels with different carbon contents and treat-treatments due to their widespread applications. Another purpose is to explore if this can potentially provide additional measurable data to improve the robustness of the inverse material parameter evaluation process.

2. Materials and Experiment
Sample steel used is solid rod-shaped elliptical of 5 mm in diameter and 90 mm long. The tensile tests were performed using a Lloyd LR 30K Universal material testing machine with extensometer. The machine has a maximum loading capacity of 30 kN, with the readings being accurate to 0.5% the material used were steel. The chemical compositions of materials as stated in Table 1.

| Material       | Condition          | Element Composition (%) |
|----------------|--------------------|-------------------------|
| Carbon Steel (CS) | 0.10% C |       | C  | Mn | P  | S  | Si | Ni |
| Normalized at 900° | 0.1  | 0.5  | <0.04 | <0.05 | 0.1 | 0.01 |
| Mild Steel         | N/A    | 0.3  | 0.03  | 0.05  | 0.05 | 0.122 | 490 ppm |

The two main materials were used in this experiment include a carbon steel (0.1% C Steel) and mild steel specimen. The stress strain curves of the two main materials tested are shown in Figure 1 (a), (b).

Figure 1. Typical Plastic stress strain data of the specimen (a) 0.1% Carbon Steel and (b) Mild Steel
The yield stress ($\sigma_y$) identified for the 0.1% C carbon steel is 308.03 MPa, work hardening coefficients ($n$) is 0.07; the yield stress ($\sigma_y$) for the mild steel is 601.66 MPa, the work hardening coefficient ($n$) is 0.025. The stress–strain curves and the material properties data are to be used as input to the Finite Element (FE) model and to assess the accuracy of the P-h curves based hardness evaluation and inverse material properties. Specimens were sectioned, in order to hardness testing performed using hardness test equipment. Each data point represents the mean value of six measurements. The error bar used is 5%, which represents the upper limit of the potential error of the material. The Vickers hardness tests were carried out using a Duramin-1 Struers Vickers hardness tester. The Duramin-1 Struers Vickers hardness tester uses a direct load method with a load range of 490.3 mN to 19.61 N. The indenter has the form of right pyramid with a square base and an angle of 136° between opposite face. In order to determine the effect of the indentation load, a range of loads was tested. Figure 2 shows the hardness values with different loads (50, 100, 200, 300, 500, 1000 and 2000 gram) of the two main materials presented in this section include a carbon steel 0.10% carbon and mild steel specimen. Each data point represents the mean value of six measurements. The error bar used is 5%, which represents the upper limit of the potential error of the material. As shown in the figure, the hardness of the 0.1% C steel is much lower than that of the mild steel samples. The hardness values (Hv) is within 98.368 % of the measured value; In the case of Mild steel, the hardness value (Hv) is within 98.611 % of the measured value. In both cases, the fundamental reason for ISE is not conclusive, however, a commonly accepted concept (also proved by the link of ISE with H/E).

3. Indentation Size Effect (ISE) On Vickers Hardness Prediction And Results
On the inverse material properties prediction explored the feasibility of using the hardness values to predict constitutive material parameters (Yield stress and work hardening coefficients) with a particular focus on uniqueness issue making use of the methodologies and equations established in this work. The program consists of three main parts ¼ experimental data, FE modelling data/simulation space and an inverse program based on objective function comparing the numerical and experimental input data. The simulation space included a group of hardness data covering a wide range of material properties. In this work, the best
correlation between Vickers hardness (Hv) and representative stress (σr) is found as a linear line with an equation of:

\[ Hv = 0.3115 \sigma_r + 11.186 \]  

(1)

If \( \varepsilon_r \) is a particular plastic strain point in elastic-plastic of power law material, the stress at the point representative stress \( \sigma_r \), at this plastic strain point, the representative stress (\( \sigma_r \)) be expressed as \([9],[10]\)

\[ \sigma_r = \sigma_y \left( 1 + \frac{E}{\sigma_y} \varepsilon_r \right)^n \]  

(2)

Where \( E \) is the Young’s modulus, \( n \) the strain hardening exponent and \( \sigma_y \) the initial yield stress. The correlation coefficients between Eq. (1) and the data is over 99%. Using this approach is much easier than repeatedly running the FE models to generate the data and can be applied to much smaller property increments. In other words, it can effectively map out all the possible materials. In this work, the yield stress was varied from 100 to 900 MPa with an increment of 10 MPa. The strain hardening coefficients used were from 0.01 to 0.3 with an increment of 0.01. This covers over 2400 hardness (Hv) data. The results were then recorded and stored into a database to form a simulation space. The simulation space was structured in an excel program, which allows easy interpretation. In the searching process, the program calculates the difference between the input data and calculated hardness values different sets of material properties within the simulation space. In each case, the optimum material parameters, which produces the P-h curves match or close to the experimental results (in this case, hardness values), were determined by mapping the objective function (Eq. 3) \([11]\)

\[ G = \frac{Hv(\text{experimental}) - Hv(\text{predicted})}{Hv(\text{experimental})} \]  

(3)

G is the objective function that needs to be minimized. This simple format allows easier interpretation of the relative error of the results. The materials with lower objective functions are potentially the target material parameter sets. The advantage of this approach is in its ability to map out any potential material property sets over the possible range of the materials properties. Thus could be a useful feature in feasibility studies in particular to establish the uniqueness of the inverse method with confidence. This is very important for practical applications, where uniqueness is crucial; otherwise, the results may converge to a wrong property sets. In this work, the Vickers hardness value (Hv) has been used as single indenter approach.

4. Potential use of indentation size effect (ISE) in inverse FE Modeling

From the work presented in the last section, the extent of ISE can be consistently measured and it can potentially be linked to the H/E ratio. The scale of ISE of the samples tested is fairly repeatable and the link with material properties (e.g H/E) is significant and reliable. This potentially could provide additional set of experimental data reflecting the materials properties. The fundamental reason for ISE is not conclusive, however, a commonly accepted concept (also proved by the link of ISE with H/E) is that it is associated with the gradient strain and dislocation density during the indentation process. Based on the geometric necessary dislocations (GNDs) model, the influence of an increase dislocation density due to additional hardening will affect the length and volume of dislocation stored \([12],[13]\). Recently, Kim et al., has evaluated the feasibility of estimating plastic flow properties by characterizing indentation size effect using
a sharp (Berkovich) indenter based on nano-indentation data and resulted showed some success based on the P-h curves [14]. This can potentially be used with the aid of the P-h curve prediction method developed in this work to estimate the range of constitutive material properties, in particular the work hardening coefficients. This can further improve the accuracy of the inverse FE modeling process. The process and key concepts is outlined below. Based on the strain gradient (MSG) plasticity model, in theory, the non ISE hardness and the ISE hardness can be expressed by following equation:

\[
\frac{H}{H_0} = \sqrt{1 + \frac{h^*}{h}}
\]  \hspace{1cm} (4)

Where \(H\) is the hardness, \(H_0\) is the macroscopic hardness and ‘\(h\)’ is the indentation depth, while \(h^*\) is a length which characterizes the depth dependence of the hardness.

\[
h^* = h^* \left( \frac{H}{H_0} \right)^2 - 1
\]  \hspace{1cm} (5)

By defined the representative stress and strain deformation state of the material underneath the sharp indenter, the representative stress \(\sigma_r\) can be described by Hollomon equation as:

\[
\sigma_r = K \varepsilon_r^n
\]  \hspace{1cm} (6)

Where \(\varepsilon_r\) is the representative strain, \(n\) is work hardening coefficient. Kim’s work showed the work hardening coefficients (\(n\)) was found to following this equation:

\[
n = \frac{1}{2\ln \varepsilon_r} \ln \left( \frac{h^*}{b} \right) - \frac{1}{\ln \varepsilon_r} \ln \left( \frac{K}{\mu} \right) + \frac{1}{\ln \varepsilon_r} \ln \left( \frac{3}{\sqrt{2}} \alpha \cot \Theta \right)
\]  \hspace{1cm} (7)

Where \(n\) is the work hardening coefficient, \(b\) is the Berger’s vector (0.248), while \(\mu\) is the shear modulus (for steel \(\mu = 79.3\)GPa), \(\varepsilon_r\) is representative strain. In this work use \(\varepsilon_r = 0.029\), length \(h^*\) can be determine from the hardness values at different loads. where \(H\) is hardness and \(H_0\) hardness on the non-ISE. The indentation depth (\(h\)) can be estimated from the impression size. \(\alpha\) is a constant with value from 0.3-0.6. and \(\Theta\) is indenter geometry (\(\Theta\) for Vickers indenter is 68°). When the material parameters, \(b\), \(K\), \(\mu\), and \(\alpha\), and the indenter geometry \(\Theta\) are known work hardening coefficient (\(n\)) by using Eq. (7) can be determined. By using this range, the work hardening of the two materials (Figure 3 (a)) has been calculated/estimated. Figure 3 shows the estimated range of the work hardening coefficients based on the ISE of the two materials samples (0.1% Carbon steel and mild steel). In the work, the ‘\(\alpha\)’ constant ranged between 0.3 to 0.4, any value higher that 0.4 will result in a negative ‘\(n\)’ value, which is physically not possible.
Figure 3 (a) Estimated range of the work hardening coefficient based on the ISE of the two materials. (the $\alpha$ constant in Eq. 7 is ranged between 0.3 to 0.4, Figure 3 (b) shows the estimated Potential properties based on the ISE of the two materials samples (0.1% Carbon steel and mild steel).

With this range of ‘n’ values, the predicted materials can be narrowed to a much sensible range, as illustrated in Figure 3 (b). The yield stress for mild steel is found to be within 500-600 MPa with n within 0.08. This is much more close to the real measured values. Further work is required to evaluate this concept and determine the exact value for $\alpha$ constant more robustly, which can then further reduce range of the n values and the yield stress.

5. Conclusion

A new approaches to predict the indentation P-h from constitutive material properties has been developed and evaluated for Vickers indentation by combining representative stress analysis and FE modelling using steel as a typical model material group which has proven to be a useful tool for predicting the Vickers hardness value (Hv) of steels. Results show for single indenter (Vickers indentation) the result is not unique. A new concept to use the indentation size data of the Vickers indentation has been explored to improve the accuracy/ robustness of inverse properties modelling based on hardness. The hardness data with different applied loads of the steels with different conditions has been evaluated. The data shows that hardness increased with decreasing carbon content in the steels. The sample with different heat-treatments showed that hardness decreased with annealing and tempering temperature. In all cases, there is a clear indentation size effects and the extent of hardness decrease with applied load is different among the material. A new concept of using ISE data for estimating the n values of steel has been explored and shown reasonable results for narrowing the range of predicted material properties based on hardness values.

6. References

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