Polarization Effects in $B \rightarrow K_1(1270) + J/\psi$ Decays

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Abstract

The joint angular distribution of the decay $B \rightarrow J/\psi V$, where $V$ is an axial vector or vector resonance, followed by the subsequent decay processes of the $J/\psi$ and $V$ is calculated using the covariant density method. In particular, the case where $V$ is the axial vector meson $K_1(1270)$ which decays into $K\rho$ is considered as well as the case that $V$ is the vector meson $K^*(890)$ which decays into $K\pi$. 

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I. INTRODUCTION

Recently the decays $B^0 \to K_1^0(1270)J/\psi$ and $B^+ \to K_1^+(1270)J/\psi$ have been observed in the Belle detector at the KEKB asymmetric $e^+e^-$ collider with significant branching ratios of the order of $10^{-3}$ \[1\]. So these decays might be useful for CP violation studies similar to the decay $B \to K^*(892)J/\psi$. The final state in the decay $B \to K_1J/\psi$ is a mixture of $CP = \pm 1$ eigenstates depending on the angular momentum of the $K_1$ and $J/\psi$. For CP violation studies, the relative strengths of the two eigenstates must be known. These can be determined from an analysis of the joint angular correlation of the decay products of $K_1 \to K\rho$ and $J/\psi \to l^+l^-$, where $l$ may be an electron or muon, respectively \[2\]. Since the branching fraction of the decay $K_1 \to K\rho$ is quite large about 42\% \[3\] and the decay $J/\psi \to l^+l^-$ has a rather distinct signature, which easily can be measured, it is to be expected that the full angular correlation will be measured as soon as a large data sample has been collected at Belle and at BABAR. In fact, this has been achieved recently for the decay $B \to K^*J/\psi$ with $K^* \to K\pi$ and $J/\psi \to l^+l^-$ with high precision at BABAR \[4\] and Belle \[5\] improving earlier measurements by the CLEO \[6\] and CDF \[7\] collaborations.

The joint angular distribution of a $B$ meson decaying into the vector (or axial vector) particles $V_1$ and $V_2$ with subsequent decays of $V_1$ and $V_2$, respectively, is described by three angles. In the original derivation of the angular correlation for the decay $B \to K^*J/\psi$ with $K^* \to K\pi$ and $J/\psi \to l^+l^-$ \[8\] the usual polar and azimuthal angles for the decays of $K^*$ and $J/\psi$ were used, describing the decay matrix elements of the decay products for $B$, $K^*$ and $J/\psi$ in the familiar helicity basis \[9\]. The CP parity information, however, is more easily extracted by introducing the $B$ decay amplitudes $A_0$, $A_{\parallel}$ and $A_{\perp}$ in the transversity basis \[2\]. In this basis, using in the transversity system angles $\theta_{tr}$ and $\phi_{tr}$, defined as polar and azimuthal angles of the $l^+$ in the $J/\psi$ rest frame, the angular correlation was presented in \[10\]. Following the earlier work the angular correlation for the decay chain $B \to K_1J/\psi$, $K_1 \to K\rho$ and $J/\psi \to l^+l^-$ can be derived as well either in the helicity or the transversity
basis. Since the matrix elements of the two decay processes $B \rightarrow K^* J/\psi$ and $B \rightarrow K_1 J/\psi$ have the same Lorentz covariant form, although the invariant form factors can be different in size and CP properties, it is more convenient to apply the method of covariant density matrices [11] instead. Then it is quite simple to incorporate the fact that the two subsequent decays $K^* \rightarrow K \pi$ and $K_1 \rightarrow K \rho$ produce different contributions in the decay density matrix of the $J/\psi$.

In this short note we derive the angular distribution of the decay $J/\psi \rightarrow l^+ l^-$ in terms of the density matrix $\rho_{\lambda \lambda'}$. The explicit form of $\rho_{\lambda \lambda'}$ for the two cases, $B \rightarrow K_1(1270) J/\psi$ and $B \rightarrow K^*(892) J/\psi$, is calculated covariantly. With this result the full angular distribution for both decays are derived in the transversity basis. It is clear that this method is quite general and can be applied to any $B$ decay into two spin-1 particles (either vector or axial vector) and their subsequent decays.

**II. DECAY OF $J/\psi \rightarrow L^+ L^-$**

Neglecting weak effects the process $J/\psi(q) \rightarrow l^+ (q_1) l^- (q_2)$ can be described by the amplitude

$$M = \frac{f_1}{2} e^\mu (q_2) \bar{u}(q_2) \gamma_\mu v(q_1),$$

(1)

where $f_1$ is a coupling constant. Then the decay distribution of the $J/\psi$ with polarization vector $e^\mu (q)$ after it is produced in any process is obtained from

$$|M|^2 = |f_1|^2 \langle e^\mu (q) e^\nu (q) \rangle (q_1^\mu q_2^\nu + q_1^\nu q_2^\mu - \frac{q_2^2}{2} q^{\mu \nu}),$$

(2)

where the lepton polarizations have been summed. Here, the ensemble averaged value $\langle e^\mu (q) e^\nu (q) \rangle$ can be replaced by the covariant density matrix $\rho_{\mu \nu}$ of the $J/\psi$ which is obtained explicitly from the $B$ decay process into the $J/\psi$. The most general form of $\rho^{\mu \nu}$ can be written as [11].
\[ \rho_{\mu\nu} = \frac{1}{3}(-g^{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_{1}^{2}}) - \frac{i}{2m_{1}}\epsilon_{\mu\nu\sigma\tau}q^{\sigma}P^{\tau} - \frac{1}{2}Q_{\mu\nu}, \]  

where the polarization vector \( P^{\tau} \) and the polarization tensor \( Q^{\mu\nu} \) can be obtained explicitly. The covariant density matrix \( \rho_{\mu\nu} \) is related to the density matrix \( \rho_{\lambda\lambda'} \) in the spin momentum basis as follows

\[ \rho_{\mu\nu} = \epsilon^{\mu}(q, \lambda)\rho_{\lambda\lambda'}\epsilon^{\nu}(q, \lambda'). \]

Here the projection operation \( \epsilon^{\mu}(q, \lambda)\epsilon^{\nu}(q, \lambda') \) is given by [11]

\[ \epsilon^{\mu}(q, \lambda)\epsilon^{\nu}(q, \lambda') = \frac{1}{3} \left( -g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{m_{1}^{2}} \right) \delta_{\lambda\lambda'} - \frac{i}{2m_{1}}\epsilon^{\mu\nu\sigma\tau}q_{\sigma}n_{i}^{i}(S^{i})_{\lambda\lambda'} - \frac{1}{2}n_{i}^{\mu}n_{j}^{\nu}(S^{ij})_{\lambda\lambda'}. \]

where the \( (S^{i}) \) are the standard matrix representations of spin-1 angular momentum operators and \( (S^{ij}) \) are traceless and symmetric matrices defined as

\[ S^{ij} = S^{i}S^{j} + S^{j}S^{i} - \frac{4}{4}\delta^{ij}I. \]

In Eq. (5), \( (n_{0}^{\mu}, n_{1}^{\mu}, n_{2}^{\mu}, n_{3}^{\mu}) \) form a tetrad with \( n_{0}^{\mu} = (E/m, \vec{q}/m) \) and \( n_{3}^{\mu} \) is the Lorentz boost from \( \hat{q} \) in the \( J/\psi \) rest frame [11]. Then \( P^{\mu} \) and \( Q^{\mu\nu} \) in Eq. (3) can be written as

\[ P^{\mu} = n_{i}^{\mu}\text{Tr}(S^{i}\rho) \]
\[ Q^{\mu\nu} = n_{i}^{\mu}n_{j}^{\nu}\text{Tr}(S^{ij}\rho). \]

Then Eq. (2) becomes

\[ |\mathcal{M}|^{2} \sim |f|^{2}m_{1}^{2}\rho_{\lambda\lambda'}\left[ \frac{1}{3}\delta_{\lambda\lambda'} + \frac{1}{m_{1}^{2}}q_{1}^{\mu}q_{1}^{\nu}(S^{ij})_{\chi\lambda'} \right], \]

In the transversity basis, defined as in Fig. 1 in the \( J/\psi \) rest frame, it is

\[ d\Gamma \sim \left[ 1 + \rho_{00} + (1 - 3\rho_{00})\sin^{2}\theta_{tr}\cos^{2}\phi_{tr} \right. \]
\[ + 2\text{Re}(\rho_{1-1})(\sin^{2}\theta_{tr}\sin^{2}\phi_{tr} - \cos^{2}\theta_{tr}) \]
\[ - 2\text{Im}(\rho_{1-1})\sin^{2}\theta_{tr}\sin2\phi_{tr} \]
\[ \left. + \sqrt{2}\text{Re}(\rho_{10} - \rho_{-11})\sin2\theta_{tr}\cos\phi_{tr} \right] \]
\[ - \sqrt{2}\text{Im}(\rho_{10} + \rho_{-11})\sin2\theta_{tr}\sin\phi_{tr} \],

where \( (\theta_{tr}, \phi_{tr}) \) are the polar and azimuthal angles of the outgoing lepton \( l^{+} \) in the transversity basis as defined in [11] where \( (\hat{n}_{3}, \hat{n}_{1}, \hat{n}_{2}) \) is along the \( (\hat{x}, \hat{y}, \hat{z}) \) axis.
III. DENSITY MATRIX OF $J/\psi$

The explicit values of the density matrix elements $\rho_{\lambda\lambda'}$ are calculated from the amplitude of the $J/\psi$ production process. The matrix element of the $B \rightarrow J/\psi V$ ($V$ is either $K_1$ or $K^*$) decay is given in terms of three independent Lorentz scalars $A$, $B$ and $C$ as

$$\mathcal{M} = A\epsilon_1^* \cdot \epsilon_2^* + \frac{B}{m_1 m_2} \epsilon_1^* \cdot q \epsilon_2^* \cdot k + \frac{iC}{m_1 m_2} \epsilon_{\mu\nu\sigma\tau}^\ast \epsilon_1^* \epsilon_2^* \cdot q \cdot k,$$

where $\epsilon_i (i = 1, 2)$ are the polarization vectors of $J/\psi$ and $V$, respectively, and $m_2$ and $k^\mu$ are the mass and momentum of the $V$ particle. Instead of the scalar amplitudes $a$, $b$ and $c$ it is more convenient to introduce the three transversity amplitudes $A_0, A_\parallel$ and $A_\perp$ if one wants to specify the $CP$ property of the decay process. They are related to $A, B, C$ by

$$A_0 = -xA - (x^2 - 1)B,$$
$$A_\parallel = \sqrt{2} A,$$
$$A_\perp = \sqrt{2(x^2 - 1)} C,$$

where $x$ is defined as

$$x = \frac{k \cdot q}{m_1 m_2} = \frac{m_2^2 - m_1^2 - m_2^2}{2m_1 m_2}.$$  

When the decay of the $B$ into $J/\psi$ and $V$ is specified by the polarization vectors $\epsilon_1 (q, \lambda_1)$ and $\epsilon_2 (k, \lambda_2)$, respectively, the density matrix elements of the $J/\psi$ and $V$ final states can be obtained from the square of Eq. (10) and by using Eq. (5) for each $\epsilon_i (i = 1, 2)$ as follows,

$$\rho_{\lambda_1 \lambda_1', \lambda_2 \lambda_2'} \sim \frac{1}{9} (|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2) \delta_{\lambda_1 \lambda_1', \lambda_2 \lambda_2'} \times$$
$$\times \left[ -\frac{2}{3\sqrt{\Delta}} \text{Re}(A_0^* A_\perp) \{ m_1 n_1 \cdot k (S^i)_{\lambda_1 \lambda_1'} \delta_{\lambda_2 \lambda_2'} + m_2 n_2 \cdot q \delta_{\lambda_1 \lambda_1'} (S^k)_{\lambda_2 \lambda_2'} \} \right.$$
$$+ \frac{1}{\Delta} \left\{ (|A_\parallel|^2 + |A_\perp|^2) m_1 m_2 n_1 \cdot k n_2 \cdot q \right.$$  
$$+ \sqrt{\frac{\Delta}{2}} \text{Im}(A_0^* A_\perp) \langle q n_1 \cdot n_2 \rangle$$  
$$- \frac{\sqrt{\Delta}}{4} \Delta \text{Re}(A_0^* A_\parallel) (n_1 \cdot n_2 - \frac{4q \cdot k}{\Delta} n_1 \cdot k n_2 \cdot q) \right\} (S^i)_{\lambda_1 \lambda_1'} (S^k)_{\lambda_2 \lambda_2'}.$$
Here the density matrix is multiplied by \( \langle V \rangle \). Now the density matrix element of \( K \) decays, can be obtained by specifying the amplitude of the \( V \) particle decay process. If the explicit form of the \( V \) decay process is given, one can explicitly obtain the density matrix of the \( V \), \( \rho_{\lambda_1 \lambda_2}^V (V) \), for its decay process. Then the density matrix of the \( J/\psi \) can be obtained from the product

\[
\rho_{\lambda_1 \lambda_1', \lambda_2 \lambda_2'} \rho_{\lambda_1' \lambda_2'}^V (V).
\]

The amplitude of the decay process \( K_1 (k, \lambda_2) \rightarrow K (\bar{k}) \rho (\bar{k}') \) is described by

\[
-\frac{1}{3\Delta} \left[ 2|A_0|^2 - |A_\parallel|^2 - |A_\perp|^2 \right] \left\{ m_1^2 n_1^i k n_1^j k (S^{ij})_{\lambda_1 \lambda_1'} \delta_{\lambda_2 \lambda_2'} + m_2^2 n_2^k q n_2^l q \delta_{\lambda_1 \lambda_1'} (S^{kl})_{\lambda_2 \lambda_2'} \right\}
\]

\[
-\frac{1}{\Delta} \left\{ \sqrt{2} \text{Im}(A_\parallel^* A_\parallel) \langle g k n_1^i n_2^k \rangle 
+ \frac{2}{\sqrt{\Delta}} \text{Re}(A_\parallel^* A_\perp) m_1 m_2 n_1^i k n_2^k q 
- \frac{\sqrt{2}}{\Delta} \text{Re}(A_\perp^* A_\parallel) \left( n_1^i n_2^k - \frac{4q \cdot k}{\Delta} n_1^i k n_2^k q \right) \right\}
\times \{ m_2 n_2^l q (S^i)_{\lambda_1 \lambda_1'} (S^{kl})_{\lambda_2 \lambda_2'} + m_1 m_2 n_2^i q (S^i)_{\lambda_1 \lambda_1'} (S^{kl})_{\lambda_2 \lambda_2'} \}
\]

\[
\left\{ \frac{4}{\Delta^2} |A_0|^2 m_1^2 m_2^2 n_1^i k n_1^i k n_2^k q n_2^l q 
+ \frac{1}{8} |A_\parallel|^2 (n_1^i n_2^k - \frac{4q \cdot k}{\Delta} n_1^i k n_2^k q)(n_1^i n_2^l - \frac{4q \cdot k}{\Delta} n_1^i k n_2^l q) 
\right. 
\]

\[
\left. + \frac{1}{2\Delta} |A_\perp|^2 \langle g k n_1^i n_2^k \rangle \langle g k n_1^i n_2^l \rangle 
- \frac{2\sqrt{2}}{\Delta} \text{Re}(A_\parallel^* A_\parallel) m_1 m_2 n_1^i k n_2^k q(n_1^i n_2^l - \frac{4q \cdot k}{\Delta} n_1^i k n_2^l q) 
\right. 
\]

\[
\left. + \frac{2\sqrt{2}}{\Delta \sqrt{\Delta}} \text{Im}(A_\parallel^* A_\perp) m_1 m_2 n_1^i k n_2^k q \langle g k n_1^i n_2^l \rangle 
\right. 
\]

\[
\left. - \frac{1}{2\Delta} \text{Im}(A_\parallel^* A_\perp)(n_1^i n_2^k - \frac{4q \cdot k}{\Delta} n_1^i k n_2^k q) \langle g k n_1^i n_2^l \rangle \right\} (S^{ij})_{\lambda_1 \lambda_1'} (S^{kl})_{\lambda_2 \lambda_2'}.
\]

Here the density matrix is multiplied by \( (|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2) \) for convenience and \( \Delta \) and \( \langle g k n_1^i n_2^l \rangle \) are defined as

\[
\Delta = (m_B^4 + m_1^4 + m_2^4 - 2m_B^2 m_1^2 - 2m_B^2 m_2^2 - 2m_1^2 m_2^2) 
= 4[(q \cdot k)^2 - m_1^2 m_2^2] = 4m_1^2 m_2^2 (x^2 - 1),
\]

\[
\langle g k n_1^i n_2^l \rangle = e^{i\omega \sigma \cdot q} k_{\mu} (n_1^i)_{\mu} (n_2^l)_{\nu}. \]

Now the density matrix element of \( J/\psi \), after \( V \) decays, can be obtained by specifying the amplitude of the \( V \) particle decay process.
\[ M_2 = a \epsilon_2^* \epsilon_3^* + \frac{b}{m_2 m_\rho} \epsilon_2^* \tilde{k}' \epsilon_3^* k, \]

where \( \epsilon_3^* \) and \( \tilde{k}' \) are the polarization vector and momentum of \( \rho \). Equation (13) can be used for the decay process \( K_1 \to K \rho \) if some notations are changed, and after the polarization of \( \rho \) is summed over, we obtain

\[ \rho_{x,z}^D (K_1) = \frac{1}{3} \delta_{\chi_\rho^*} - \frac{2m_2^2}{\tilde{\Delta}} (1 - \xi) \tilde{k} \cdot n_2^k \tilde{n}_2^l (S_2^k)^\chi_\rho^* \chi_\rho^*, \]

where \( \tilde{\Delta} \) and \( \xi \) are defined as

\[ \tilde{\Delta} = (m_2^4 + m_K^4 + m_\rho^4 - 2m_2^2m_K^2 - 2m_2^2m_\rho^2 - 2m_K^2m_\rho^2), \]

\[ \xi = \frac{3|a_||^2}{2(|a_0|^2 + |a_||^2)}. \]

The linear polarization amplitudes in the \( K_1 \) decay are related to the form factors in Eq. (17) and helicity amplitudes \( H_0 \) and \( H_\pm \) as

\[ a_0 = -\tilde{x} a - (\tilde{x}^2 - 1)b = H_0, \]

\[ a_\| = \sqrt{2} a = (H_{+1} + H_{-1})/\sqrt{2} = \sqrt{2} H_+, \]

where \( \tilde{x} = (m_2^2 - m_K^2 + m_\rho^2)/2m_2m_\rho \).

Likewise, the amplitude of the decay process \( K^* (k, \lambda_2) \to K(\bar{k})\pi(\bar{k}') \) is given by

\[ M_3 = f_{3\epsilon^\mu} (k, \lambda_2) (\bar{k} - \bar{k}')_\mu = 2f_{3\epsilon^\mu} (k, \lambda_2) \bar{k}_\mu, \]

and the density matrix of the decay process becomes

\[ \rho_{x,z}^D (K^*) = \frac{1}{3} \delta_{\chi_\rho^*} - \frac{2m_2^2}{\tilde{\Delta}} \bar{k} \cdot n_2^k \bar{n}_2^l (S_2^k)^\chi_\rho^* \chi_\rho^*, \]

where \( \tilde{\Delta} \) is defined in the same way as in Eq. (19) after replacing the subscripts \( K, \rho \) by \( K, \pi \). We see that the density matrix for \( K^* \) to \( K \pi \) is obtained from that of \( K_1 \) to \( K \rho \) in Eq. (18) if we replace \( \xi = 0 \).

Considering the two processes simultaneously, we write the decay density matrix elements of the two \( V \) particles (\( K_1 \) and \( K^* \)), Eqs. (18) and (24), as
\[
\rho^D_{\lambda_2 \lambda_2} = \frac{1}{3} \delta_{\lambda_2 \lambda_2} + D \tilde{k} \cdot n_2^k \tilde{k} \cdot n_2^l (S^{kl})_{\lambda_2 \lambda_2},
\] 

and then we have

\[
D = -\frac{2m_2^2}{\Delta}, \quad \text{for } K^* \rightarrow K\pi,
\]
\[
= -\frac{2m_2^2}{\Delta} (1 - \xi), \quad \text{for } K_1 \rightarrow K\rho.
\] 

The explicit form of the density matrix elements of \( J/\psi \) neglecting the subscript 1 in \( \lambda_1 \) and \( n_1 \) is now obtained as follows,

\[
\rho^{\lambda \nu} \sim \rho_{\lambda_1 \lambda_2 \lambda_2} \rho_{\lambda_2 \lambda_2}^D
\]
\[
= \frac{1}{3} \delta_{\lambda \nu} \left[ \frac{1}{3} (|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2) + \frac{D\Delta}{12m_2^2} \left\{ 1 - \frac{48m_2^4}{\Delta\Delta} (q \cdot J)^2 \right\} (2|A_0|^2 - |A_\parallel|^2 - |A_\perp|^2) \right]
\]
\[
+ (S^i)_{\lambda \lambda} \left[ -\frac{2}{3\sqrt{\Delta}} \text{Re}(A_\parallel^* A_\perp) m_1 k \cdot n^i \right.
\]
\[
\left. + \frac{2\sqrt{2}m_2^2 D}{\Delta} \left\{ 2 \text{Im}(A_\parallel^* A_\parallel) q \cdot J \langle q k \tilde{k} n^i \rangle \right. \right.
\]
\[
\left. + \frac{m_1}{24} \sqrt{2\Delta\Delta} \text{Re}(A_\parallel^* A_\perp) \left\{ 1 - \frac{48m_2^4}{\Delta\Delta} (q \cdot J)^2 \right\} k \cdot n^i \right.
\]
\[
\left. + \sqrt{\Delta} \text{Re}(A_\parallel^* A_\perp) q \cdot J (J \cdot n^i - \frac{4q \cdot k \cdot J}{\Delta} k \cdot n^i) \right\} \right]
\]
\[
+ (S^{ij})_{\lambda \lambda} \left[ -\frac{m_1^2}{3\Delta} \left( 1 + \frac{D\Delta}{2m_2^2} \right) (2|A_0|^2 - |A_\parallel|^2 - |A_\perp|^2) k \cdot n^i k \cdot n^j \right.
\]
\[
\left. + D \left\{ \frac{16m_1^2 m_2^2}{\Delta^2} (q \cdot J)^2 k \cdot n^i k \cdot n^j |A_0|^2 \right. \right.
\]
\[
\left. + \frac{1}{2} |A_\parallel|^2 (J \cdot n^i - \frac{4q \cdot k \cdot J}{\Delta} k \cdot n^i) (J \cdot n^j - \frac{4q \cdot k \cdot J}{\Delta} k \cdot n^j) \right.
\]
\[
\left. + \frac{2}{\Delta} |A_\perp|^2 \langle q k \tilde{k} n^i \rangle \langle q k \tilde{k} n^j \rangle \right.
\]
\[
\left. - \frac{4\sqrt{2}}{\Delta} m_1 m_2 (q \cdot J) \text{Re}(A_\parallel^* A_\parallel) k \cdot n^i (J \cdot n^j - \frac{4q \cdot k \cdot J}{\Delta} k \cdot n^j) \right.
\]
\[
\left. - \frac{8\sqrt{2}}{\Delta\sqrt{\Delta}} m_1 m_2 (q \cdot J) \text{Im}(A_\parallel^* A_\perp) k \cdot n^i \langle q k \tilde{k} n^j \rangle \right.
\]
\[
\left. + \frac{2}{\sqrt{\Delta}} \text{Im}(A_\parallel^* A_\parallel) (J \cdot n^i - \frac{4q \cdot k \cdot J}{\Delta} k \cdot n^i) \langle q k \tilde{k} n^j \rangle \right\} \right],
\]

where \( J = -\tilde{k} + \frac{k \cdot \tilde{k}}{m_2^2} k \). This density matrix is manifestly covariant and from the explicit form of matrices \((S^i), (S^{ij})\), one can obtain the angular distribution of \( B \rightarrow K_1 J/\psi \) as well as that.
of $B \to K^*J/\psi$. The angular distribution for the decay process $B \to K_1(1270)J/\psi$ ($K_1 \to K\rho, J/\psi \to l^+l^-$) in the transversity basis is derived from Eqs. (3) and (27):

$$
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_1 d\cos \theta_{tr} d\phi_{tr}} = \frac{9}{32\pi} \frac{1}{1+3\alpha} \left[ 2|A_0(t)|^2 \left\{ (\cos^2 \theta_1 + \alpha)(1 - \sin^2 \theta_{tr} \cos^2 \phi_{tr}) \right\} 
+ |A_\parallel(t)|^2 \left\{ \sin^2 \theta_1 (1 - \sin^2 \theta_{tr} \sin^2 \phi_{tr}) 
+ \alpha(1 + \sin^2 \theta_{tr} \cos^2 \phi_{tr}) \right\} 
+ |A_\perp(t)|^2 \left\{ \sin^2 \theta_1 \sin^2 \theta_{tr} + \alpha(1 + \sin^2 \theta_{tr} \cos^2 \phi_{tr}) \right\} 
+ \text{Im}(A_\parallel^*(t)A_\perp(t)) \sin 2\theta_{tr} \sin \phi_{tr} 
- \frac{1}{\sqrt{2}} \text{Re}(A_0^*(t)A_\parallel(t)) \sin 2\theta_1 \sin^2 \theta_{tr} \sin 2\phi_{tr} 
+ \frac{1}{\sqrt{2}} \text{Im}(A_0^*(t)A_\perp(t)) \sin 2\theta_1 \sin 2\theta_{tr} \cos \phi_{tr} \right],
$$

(28)

where $\theta_1$ is the angle between the $K_1$ direction and the $K$ direction in the $K_1$ rest frame and the three amplitudes are normalized as $|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1$. Here, $\alpha$ is defined as

$$
\alpha = \frac{1}{3} \frac{\xi}{1 - \xi} = \frac{|a_\parallel|^2}{2|a_0|^2 - |a_\parallel|^2}.
$$

(29)

The angular distribution in the process $B \to K^*J/\psi(K^* \to K\pi, J/\psi \to l^+l^-)$ in the transversity basis can be obtained by putting $\xi = 0$ or $\alpha = 0$ in Eq. (28). The angular correlation for $B \to K_1J/\psi$ depends also on details of the decay $K_1 \to K\rho$ in contrast to $K^* \to K\pi$ and this is related to the fact that the decay $K_1 \to K\rho$ depends on two independent matrix elements. In order to analyze the transversity amplitudes for the $B$ decay, one must know the ratio of three decay matrix elements for $K_1 \to K\rho$ or otherwise one must determine them from the angular correlation together with the $B$ decay transversity amplitudes.

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Figure Captions

Fig. 1: The definitions of the angles in the transversity basis. $\theta_{tr}$ and $\phi_{tr}$ are polar and azimuthal angles of $l^+$ in the $J/\psi$ rest frame. $\theta_1$ is the angle between the $K_1$ direction and the $K$ direction in the $K_1$ rest frame.
in the $K_1$-rest frame

FIG. 1.