Analysis of noise characteristics of spur face gear under different installation errors

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Abstract. Face gear drives have been widely used in helicopter drive system, but there are few research literatures on meshing noise. In order to study the relationship between installation errors and gear meshing noise, the tooth surface equation of spur face gear is derived based on the principle of gear shaping; Based on the virtual meshing relative position of gear shaper, pinion and spur face gear, the control equation of installation distance between pinion and spur face gear is derived; Based on the method of orthogonal test, the noise test scheme with installation errors is set up, and the meshing noise characteristics of spur face gear are obtained. The research provides a theoretical basis for the development of noise reduction technology of spur face gears.

1. Introduction

Face gear drive is a new kind of drive by a cylindrical gear and a face gear. Compared with the traditional bevel gear drive, it has many advantages, for example small volume, light weight, high load-carrying capacity, no axial force and excellent split power et al. [1]. Based on the advantages, the face gear drives have been applied widely. NASA has been successfully applied in Apache power split system, which obtains better performance in the consequence [2].

At the present, the research of domestic scholars on face gears mainly concentrate on tooth surface characteristic and method of face gear machining. For example, Litvin [3] studied a shaping method of face gears, analyzed the phenomenon of root undercutting and tip tipping. At the same time, he proposed the method of a localized bearing contact to achieve the point contact to guarantee the gear ratio. Wang et al. [4, 5] proposed the milling method of face gears, deduced the tooth equation of the cutter, and studied the influence of the installation error on the machining accuracy and proposed the compensation method of face gear machining parameters at last. Wang [6] studied the grinding mechanism by disc wheel and analyzed the influence of the wheel trajectory and the installation error on the tooth surface deviation of spur face gears. Litvin [7] established the theory of grinding spur face gear, and designed a disc grinding wheel. The patent of Apparatus and method for precision grinding face gear applied, which made the transformation of spur face gear manufacture technology from rough finishing to fine finishing. Litvin et al [8] proposed a new generation method of spur face-gears by applying for a grinding worm and the tooth contact analysis and stress analysis were performed. The results confirmed existence of a longitudinal bearing contact, avoidance of edge contact, and reduction of contact stresses. Shen et al. [9] presented a method of using two-parameter worm grinding to form the face gear. A helical face gear was formed via a two-parameter worm envelope, and he analyzed the two-parameter enveloping process and the envelope formation.
visualization results that a ideal face gear tooth surface was obtained. Gleason [10] proposed a CONIFACE grinding method for face gears, and it used an involute disc wheel with a tool inclination to grind face gears. The grinding wheel did not need to be fed along the radial direction of face gears, and the efficiency was high. The method has been applied to the Phoenix gear grinding machine. Tang et al. [11] developed a progressive grinding method, and established the mathematical model of parabolic rack cutter, spur gear cutter, disk wheel and modified spur face-gear, and built the calculation model of swing angle range of the complete grinding of the profile. At last, the simulation processing test based on VERICUT software verifies the feasibility and correctness of the method. Ming et al. [12] built the model of grinding surface roughness of face gear based on the mathematical formula of the residual height of motion trajectory of abrasive grains. The experiment’s results suggested that the mathematical model was reasonable, and the effect of the disk wheel spindle speed and feed velocity of the disk wheel were more obvious than abrasive code.

We can find that the research of face gears is focused on the field of machining by the above literature. The paper derived the tooth surface equation of spur face gear based on the principle of gear shaping, and the control equation of installation distance between pinion and spur face gear. The noise test experiment with installation errors was set up at last, and the meshing noise characteristics of spur face gear were obtained.

2. Tooth equation of spur face gears

2.1. Tool equation

The tooth surface of spur face gear is formed by the enveloping of gear shaper and face gear, as shown in Figure 1. $S_s$ is the coordinate system of the spur face gear and $S_2$ is the coordinate system of the shaper cutter. The axis of the cutter and the face gear are $Z_s$ and $Z_2$, respectively. Their shaft intersection angle is $\gamma_m$. The teeth number of shaper and face gear are $N_s$ and $N_2$, respectively. They rotate around their own axes, and the angular velocity satisfy the following relation:

$$\frac{\omega_s}{\omega_2} = \frac{N_2}{N_s}$$  \hspace{1cm} (1)

Where, $S_s$ and $S_2$ are the angular velocity of shaper cutter and spur face gear, respectively.

![Figure 1. Spur face gear shaping diagram.](image)

The involute profile equation of shaper cutter is shown as below, and its radial vector function can be expressed:

$$r_s(u_s, \theta_{ks}) = \begin{bmatrix} \pm r_{ks} [\sin(\theta_{ks} + \theta_{cs}) - \theta_{cs} \cos(\theta_{ks} + \theta_{cs})] \\ -r_{ks} [\cos(\theta_{ks} + \theta_{cs}) + \theta_{cs} \sin(\theta_{ks} + \theta_{cs})] \\ u_s \\ 1 \end{bmatrix} \hspace{1cm} k = (\gamma, \beta)$$  \hspace{1cm} (2)
Where, $r_{bs}$ is the base circle radius of involute profile; $\mu_s$ is the axial parameter of the shaper cutter; $\theta_{os}$ is the angle parameter of involute profile; $\theta_{ks}$ is the starting angle of involute profile.

2.2. Spatial coordinate system

The coordinate system of shaping the spur face gear is as shown in Figure 2. $S_{s0}$ and $S_{20}$ are the initial position coordinate systems of the shaper cutter and the spur face gear, respectively. $S_s$ and $S_2$ are their moving coordinate systems, which rotates with them around their own axis. In order to calculate the transmit matrix from $S_s$ to $S_2$ in an easy way, we assume that the original points of coordinate system of spur face gear and shaper cutter are the same point.

![Figure 2. Shaping coordinate system of spur face gear.](image)

The coordinate system from $S_s$ to $S_2$ is as follows according to the relative position of the shaping coordinate systems.

$$
M_{2s} = \begin{bmatrix}
\cos \phi_2 \cos \phi_s & -\cos \phi_2 \sin \phi_s & -\sin \phi_2 \\
-\sin \phi_2 \cos \phi_s & \sin \phi_2 \sin \phi_s & -\cos \phi_2 \\
\sin \phi_s & \cos \phi_s & 0
\end{bmatrix} (3)
$$

The rotational angular satisfy the following relation:

$$
\phi_2 = m_{2s}\phi_s = \phi_s \frac{N_s}{N_2} (4)
$$

Where, $m_{2s}$ is the drive ratio of the shaper cutter and the spur face gear.

2.3. Tooth equation of spur face gear

According to the space meshing theory, the expression of the tooth surface vector function of the spur face gear is established.

$$
r_s(u_s, \theta_{ks}, \phi_s) = M_{2s}(\phi_s)r_s(u_s, \theta_{ks}) (5)
$$

The normal vector of the tooth surface is obtained.

$$
n_s = L_{2s}n_s (6)
$$

Where, $L_{2s}$ is the $3 \times 3$ order submatrix of Matrix $M_{2s}$.

As is known that the shaper cutter and face gear wheel pair rolling motion. According to the meshing principle, the meshing equation can be obtained.

$$
n_s \cdot v_s^{(12)} = 0 (7)
$$

Where, $n_s$ is the normal vector of tooth surface of the cutter, $V_s$ is the relative velocity vector between the shaper cutter and spur face gear in $S_s$. 


Where, $V^{(s)}_0$ and $V^{(s)}_2$ are the velocity vector of the shaper cutter and face gear in $S_s$.

After finishing, the below equation can be obtained.

$$
V^{(s)}_2 = \omega_s \begin{bmatrix}
-x' - z'm_{2s} \cos \phi_s \\
-x' + z'm_{2s} \sin \phi_s \\
m_{2s}(x' \cos \phi_s - y' \sin \phi_s)
\end{bmatrix}
$$

Thus, the meshing equation of spur face gear can be expressed:

$$
f(u', \theta_{ks}, \phi_s) = r_{hs} - u' m_{2s} \cos(\phi_s \pm (\theta_{as} + \theta_{ks})) = 0
$$

The tooth equation of spur face gear can be expressed:

$$
\mathbf{r}_2(\phi_s, \theta_{ks}) = \begin{bmatrix}
r_{hs} \left[ \cos \phi_s \left( \sin \xi_{kr} \mp \theta_{ks} \cos \xi_{kr} \right) \right] \\
-r_{hs} \left[ \sin \phi_s \left( \sin \xi_{kr} \pm \theta_{ks} \cos \xi_{kr} \right) \right] + \\
-\frac{\sin \phi_2}{m_{2s}} \cos \xi_{kr} - \\
-\frac{\cos \phi_2}{m_{2s}} \sin \xi_{kr}
\end{bmatrix}
$$

3. Control equation of installation distance

In order to avoid the edge contact, the number of teeth of the shaving cutter is one more than that of the pinion. Their relative position are shown as Figure 3.

![Figure 3. Relative position of shaper, pinion and face gear.](image-url)

The distance of $O_2O_1$ can be obtained as follow.

$$
\Delta B = \frac{1}{2} m_{2s} - \frac{1}{2} m_{2s}
$$

The distance from the central of the shaper cutter to the root of the spur face gear can be obtained as follow.

$$
L_1 = \frac{1}{2} m_{2s} + 1.25m
$$

Thus, the installation distance of the pinion and the spur face gear can be obtained as follow.

$$
L_1 = L_s - \Delta B
$$
4. Noise detection of specimen

4.1. Detection equipment
The face gear sample is shown as Figure 4. The meshing noise of gear pair under different central distance deviation $\Delta H$ and offset distance $\Delta E$ were detected. The detection equipment is shown in Figure 5. On the experiment system, the input shaft speed is measured by DT-2234B photoelectric digital tachometer and the noise is measured by GM1351 digital noise meter. While noise is measured, the distance between the noise meter and the top of the spur face gear is 200mm. The velocity of spur face gear is 50r/min, and the standard installation distance is 100.944mm.

![Figure 4. Detection specimen](image)

![Figure 5. Detection equipment](image)

4.2. Detection scheme and result
The orthogonal test scheme was adopted. The number of variables of $\Delta E$ is 5, and they are 0mm, +0.05mm, +0.1mm, -0.05mm and -0.1mm, respectively. The number of variables of $\Delta H$ is 5, and they are 0mm, +0.02mm, +0.04mm, -0.02mm and -0.04mm, respectively. Thus, the total number is 25. The specific test scheme and results are shown as Figure 6.

When the offset distance is constant (0mm, +0.02mm, +0.04mm, -0.02mm, -0.04mm), the relationship of noise and central distance deviation is shown as Table 1.

![Figure 6. Noise graph of with the variation of central distance deviation](image)
Table 1. Test scheme and results.

| No. | ΔE(mm) | ΔH(mm) | Noise(dB) |
|-----|--------|--------|-----------|
| 1   | 0      | 0      | 83.3      |
| 2   | 0      | +0.02  | 83.9      |
| 3   | 0      | +0.04  | 85.1      |
| 4   | 0      | -0.02  | 83.7      |
| 5   | 0      | -0.04  | 84.3      |
| 6   | +0.05  | 0      | 83.6      |
| 7   | +0.05  | +0.02  | 84.4      |
| 8   | +0.05  | +0.04  | 84.8      |
| 9   | +0.05  | -0.02  | 83.9      |
| 10  | +0.05  | -0.04  | 84.4      |
| 11  | +0.1   | 0      | 82.3      |
| 12  | +0.1   | +0.02  | 83.5      |
| 13  | +0.1   | +0.04  | 83.3      |
| 14  | +0.1   | -0.02  | 83.4      |
| 15  | +0.1   | -0.04  | 83.2      |
| 16  | -0.05  | 0      | 82.6      |
| 17  | -0.05  | +0.02  | 83.4      |
| 18  | -0.05  | +0.04  | 82.5      |
| 19  | -0.05  | -0.02  | 83.3      |
| 20  | -0.05  | -0.04  | 83.1      |
| 21  | -0.1   | 0      | 85.0      |
| 22  | -0.1   | +0.02  | 84.2      |
| 23  | -0.1   | +0.04  | 84.6      |
| 24  | -0.1   | -0.02  | 85.1      |
| 25  | -0.1   | -0.04  | 85.6      |

When the central distance deviation is constant (0mm, +0.05mm, +0.1mm, -0.05mm, -0.1mm), the relationship of noise and offset distance deviation is shown as Figure 7.

Figure 7. Noise graph of with the variation of central distance deviation.
5. Conclusions
When the offset distance is constant, the decibel of noise increases with the increase of installation distance deviation. When the central distance deviation is constant, the decibel of noise increases with the increase of offset distance deviation. Compared with the offset distance, the noise sensitivity of the central distance deviation is higher.

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