Features of the Multi-Criteria Optimization Mathematical Model of the Thermal and Electrical Loads Distribution at a Combined Heat and Power Plant with a Mixed Equipment Composition

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Abstract. The features of the mathematical model of multi-criteria optimization of the distribution of current thermal and electrical loads at a combined heat and power plant with a mixed composition of equipment based on traditional heating units and a heating CCGT are considered. The previously proposed mathematical apparatus for solving the problem of multi-criteria optimization at a thermal power plant is analyzed. It is shown that with a mixed composition of equipment, along with the criteria of efficiency and environmental friendliness, it is also necessary to take into account the factors of reliability and mobility (maneuverability). The substantiation of the choice of reliability and mobility criteria for optimizing the operation modes of a thermal power plant is given. Approaches to solving the multi-criteria task are considered. The description of the features of the algorithm for solving the optimization problem is given in relation to thermal power plants with a mixed composition of equipment, including heating turbines of the T type and PGU.

1. Introduction

Currently, the main share of electric and thermal energy in Russia is generated by thermal power stations. Most of the equipment of power plants has been in operation for many years and is obsolete, while, due to the above, many stations do not provide the economic, environmental and especially reliability indicators of the operation of stations required by the power system.

In market conditions, planning and management of operating modes of power plant equipment is carried out according to the following algorithm [1-6]:

- based on the initial data available at the CHPP and based on the experience of operating the CHPP equipment in similar conditions, the station specialists (planning and technical department or other services) calculate the operating modes of the CHPP equipment for each group point of electricity supply, taking into account the predicted values of the thermal load, as a rule, according to methods that rely on the maximum and minimum loads of the CHPP, while a small number of modes with intermediate load values are considered. This scenario-expert approach allows us to take into account the current economic and technical condition of the power plant equipment to the maximum extent;

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• the station transmits to the system operator (SO) all the necessary data on the prediction of the composition of generating equipment for each delivery point (for 3-4 days in advance). The SO on the conditions of power plant operation as a whole considers the proposals of the station for each unit of equipment at each point of delivery. In turn, the SO transmits the boundary conditions (boundary capacities) - the power reserve for each group delivery point (GDP), which the station must provide to regulate the total load of the system and transmits the agreed data back to the station. Thus, for 3...4 days ahead, the station should predict the composition of the equipment for the operating day \( X \). But with the current state of our equipment, such forecasting is associated with a very high risk and high requirements for ensuring the reliability of equipment.

• based on the agreed composition of the equipment, the station predicts the operation of the generating equipment, its work schedule for the operating day \( X \). At this stage, the internal optimization task is especially relevant, it allows you to get the main share of resource savings (fuel) due to the optimal distribution according to the criteria of efficiency and reliability of the units.

• on 2 days ahead, the station prepares its proposals for participation in the electricity and capacity market "for the day ahead" for each point of supply in terms of electricity volume and the cost of specific fuel consumption per 1 kW*h of electricity.

• on 1 day (trading day) SO and the administrator of the trading network organize the purchase and sale process for each node point. At the end of the day, the station receives a dispatch schedule. Thus, as a result, each CHPP receives an hourly load “for a day ahead” with an indication of electrical loads, the values of selections for heating turbines, the steam capacity of steam and hot water boilers, and other parameters for the modes calculated above (minimum, maximum and 2-3 intermediate modes). Recalculation, optimization or adjustment of the loading modes of the CHPP equipment, as a rule, is not performed.

Under these conditions, the intra-station optimization task is especially relevant at the stage of operational planning and control of the station mode, since it allows you to save resources (fuel) and ensure the reliability of the units due to the choice of their composition, as well as reactive and active capacities. When solving this problem, it is assumed that the station operates with loads that are specified by the united generating company (or regional generating company) and all the specified system conditions and restrictions are met.

2. Formulation of the equipment operation intra-station modes optimization problem in modern conditions

In general, the task of optimizing the intra-station operating modes of equipment in modern conditions can be formulated as follows: according to the specified total thermal and electrical loads at the station, it is necessary to minimize certain optimality criteria, taking into account the restrictions imposed on both the optimized parameters (electric and thermal power of each unit) and additional parameters (inherent in a specific CHPP).

In addition to the three main optimality criteria [4-8] (economy, environmental friendliness and reliability), as mentioned above, in the presence of CCGT in the equipment of the CHPP, it is also possible to distinguish the manoeuvrability criterion, since CCGTs have significant differences in loading and unloading speeds compared to traditional steam turbine units [8].

Thus, we have the following optimality criteria:

\[
F_1(x) = \sum_{j=1}^{n} f_{1j} \rightarrow \min_x, \quad F_2(x) = \sum_{j=1}^{n} f_{2j} \rightarrow \min_x, \\
F_3(x) = \sum_{j=1}^{n} f_{3j} \rightarrow \min_x, \quad F_4(x) = \sum_{j=1}^{n} f_{4j} \rightarrow \min_x
\]  

(1)
where $F_1(x)$, $F_2(x)$, $F_3(x)$, $F_4(x)$ are optimal criteria for fuel consumption (efficiency), reliability, ecology and manoeuvrability; $f_{1j}$, $f_{2j}$, $f_{3j}$, $f_{4j}$ - energy characteristics of the $j$-th power unit, reflecting its fuel consumption, average damage due to unreliability of operation (due to emergency downtime and failures) and the amount of emissions of harmful substances into the environment; $f_{4j}$ - manoeuvrability characteristics of the equipment.

The characteristics of the equipment $f_{1j}$, $f_{2j}$, $f_{3j}$, $f_{4j}$ are determined by the operating mode of the unit, the electrical ($N$) and thermal load ($Q$); as well as additional parameters that characterize the operation of the unit, affecting the deviation of the characteristics from the nominal values.

The conditions imposed on the optimization problem can be represented in the following form:

$$
\sum_{j=1}^{n} N_j = N_{\Sigma}, \quad \sum_{j=1}^{n} Q_j = Q_{\Sigma},
\tag{2}
$$

$$
N_j^{\min} \leq N_j \leq N_j^{\max}, \quad Q_j^{\min} \leq Q_j \leq Q_j^{\max}, j = 1, 2, ..., n,
\tag{3}
$$

where $N_j$, $Q_j$ - electric and thermal power of the $j$-th unit; $N$, $Q$ - total required electric and thermal power of the CHP; $N_j^{\min}$, $N_j^{\max}$, $Q_j^{\min}$, $Q_j^{\max}$ - permissible load limits of the $j$ power unit, which are determined by the operating mode of the unit.

It should be noted that the ranges of changes in the electrical load also depend on the magnitude of the thermal load of the turbine, forming at the same time the area of determining the loads for a specific mode of the unit.

3. Three stages of the intra-station optimization problem solution

The solution of the intra-station optimization problem can be represented by three stages.

3.1. When choosing the composition of generating equipment (the task of the CCGE).

To solve the problem of choosing the composition and operating modes of the CHPP equipment, as a rule, a scenario approach is used, for which a matrix of possible acceptable variants of the composition and operating modes of the station units is compiled (Table.1) taking into account the adjustment ranges of the loads of each unit, the technological and operational features of a particular station, the nature of its participation in covering the loads of the power system, the time of year, the actual technical condition of the equipment and a number of other factors.

| Table 1. Matrix of options for the composition of the CHP equipment |
|---------------------------------------------------------------|
| variant | 1st unit | 2nd unit | ... | n unit |
|---------|----------|----------|-----|--------|
| 1 variant | $S_{11}$ | $S_{12}$ | ... | ... |
| 2 variant | $S_{21}$ | $S_{22}$ | ... | ... |
| ... | ... | ... | $S_{ij}$ | ... |
| m variant | $S_{m1}$ | $S_{m2}$ | ... | $S_{mn}$ |

In the matrix: $S_{ij}$ – operating mode (state) of $j$-th unit of the station with the $i$-th variant of the equipment composition, $n$ – the number of units of the station, including steam turbine power units, CCGT power units and hot water boilers; $m$ – the number of permissible variants of the equipment composition for generating the required amount of electric and thermal energy.

The engineers of the station where the developed methodology is used are involved as experts. Experts should have a clear understanding of the current state of the equipment and the features of their operation at this CHPP. Also, operators of CCGT units who have experience in control a power unit, taking into account the peculiarities of CCGT operation in various modes, can be involved as experts.

It is also possible to involve only one expert - a CHPP engineer who is responsible for choosing the operating modes of the CHPP units. In the case of involving several experts to coordinate their
opinions, it is necessary to use expert assessment methods with a survey of all experts with the formation of one resulting position on the selection of matrix rows.

The essence of the expert evaluation method is to conduct an intuitive and logical analysis of the equipment composition options by experts with a quantitative assessment of their effectiveness on a point scale and formal processing of the results. The generalized expert opinion obtained as a result of processing is accepted as a solution to the problem (the final form of the matrix).

Thus, a set of matrices is obtained, compiled with the involvement of experts immediately before the load distribution optimization procedure is carried out. The number of rows of matrices should not be too large, since this significantly increases the computational costs required to solve the task.

Taking into account the predictive, uncertain nature of the information, at this stage of optimal load distribution for each scenario, it is recommended to solve the problem with consideration of two criteria – economic for the minimum fuel consumption at the power plant ($B_{pp}$) and reliability for the maximum availability factor ($K_a$):

$$F_1 (x) = \sum_{j=1}^{n} f_{1j} \rightarrow \min_x (B_{pp})$$

$$F_2 (x) = \sum_{j=1}^{n} f_{2j} \rightarrow \max_x (K_a)$$

3.2. At the stage of short-term planning based on forecast estimates.

The solution of the problem at this stage is based on forecast and statistical operational information and allows the operational personnel to plan measures for the rational management of the station for a period, most often for the next operational day. As a result of solving the problem of optimal load distribution for the composition of generating equipment agreed with the dispatching service, at this stage the station prepares and transmits to the dispatching service of the power system its proposals for participation in various sectors of the electricity and capacity market for the "day ahead" (MDA).

The optimal distribution of thermal and electrical loads at this stage is proposed to be based on three criteria – economic for the maximum marginal profit of the power plant ($P_{pp}$), reliability for the maximum availability coefficient and environmental for the minimum total emissions ($V_{pp}$):

$$F_1 (x) = \sum_{j=1}^{n} f_{1j} \rightarrow \max_x (P_{pp})$$

$$F_2 (x) = \sum_{j=1}^{n} f_{2j} \rightarrow \max_x (K_a)$$

$$F_3 (x) = \sum_{j=1}^{n} f_{3j} \rightarrow \min_x (V_{pp})$$

3.3. At the stage of execution of the dispatching schedule of loads set as a result of the auction.

Based on the results obtained at the previous stage of optimal distribution with a known composition of generating equipment, the problem of pre-optimization of the operating parameters (pressure in the condenser, fresh steam pressure, boiler efficiency, etc.), at which the operating power units will have to work, is solved. At the same time, predictive and, mainly, current information about the real conditions of each type of equipment and aggregates as a whole is used, i.e. the task of optimizing the parameters is solved not only based on the task set (performing the planned load release at the first stage), but also on the basis of the actual technical conditions of the equipment. If the real state cannot provide the specified load or can perform, but not in an optimal way, then the loads or parameters are corrected without changing the composition of the generating equipment. Since optimization at this stage is carried out without changing the composition of generating equipment to simplify calculations, the reliability criterion can be ignored or considered in the form of a restriction.
The maneuverability criterion at this stage is taken into account during the hours of loading and unloading, while there are two approaches:
- \( F_A(x) \) is presented as a restriction:

\[
\sum_{j=1}^{m} W_j \geq W_i, \tag{6}
\]

where \( W_j \) is the loading (unloading) speed of the CHPP equipment involved in the load set and discharge, \( W_i \) is the loading (unloading) speed specified by the daily schedule for the considered CHPP, \( m \) is the number of units involved in the load set and discharge;
- \( F_A(x) \) is represented as a criterion - the minimum difference between the total rate of change of the load and the required rate of change according to the load schedule.

The use of the second approach, as shown by further calculations, in the presence of restrictions on the loading/unloading speed of generating units leads to a significant complication of the optimal distribution algorithm, and therefore the first approach was used in further calculations.

In addition to the above limitations, in-station optimization requires a detailed analysis and consideration of many station limitations and features of generating equipment. One feature of this task is that most of the control processes of the station modes are automated and therefore the solution of the problem should be made taking into account the possibilities of its implementation by means of automation.

The profit of the station from the sale of electricity and heat is accepted as a criterion of efficiency. Since all calculations are carried out for each hour of the day on the electricity market, the station's profit for an hour of the operating day will be (excluding penalties for deviations):

\[
P_i = \sum_{t=1}^{p} \Delta C^T_T \cdot Q_t + \sum_{k=1}^{k} \left( \Delta C^E_R \cdot N_{RA} + \Delta C^E_B \cdot N_{BA} + \Delta C^E_{MDA} \cdot N_{MDA} + \Delta C^E_{BM} \cdot N_{BM} \right), \tag{7}
\]

where \( \Delta C^T_T = C_{TT} - C^T_i \), \( C_{TT} \) - the tariff for thermal energy; \( C^T_i \) - the thermal component for the production of 1 Gcal of thermal energy; \( \Delta C^E_i = C^E_i - C_{IT} \), \( C^E_i \) - the selling price of 1 MW*h of electricity in the relevant market: electricity under regulatory agreements (RA), bilateral agreements (BA), for the "day ahead" (MDA) and balancing (BM); \( C_{IT} \) - the fuel component in the cost of electricity generation at the CHPP; \( k = 1, 2, ..., p \) - the number of GDP; \( l = 1, 2, ..., r \) - the number of thermal branches; \( N_{RA}, N_{BA}, N_{MDA}, N_{BM} \) - the hourly power sold in the relevant market sectors (regulatory agreements, bilateral agreements, MDA, balancing).

With a constant tariff for thermal energy \( C_{TT} \), set values \( \Delta C^E_{RA} \) and \( \Delta C^E_{BA} \) variables for the hours of the day, but constant values \( \Delta C^E_{MDA} \), \( \Delta C^E_{BM} \) for a given hour, the position of the maximum of the criterion \( P_i \) coincides with the position of the minimum fuel costs at the CHPP.

The reliability of each element of the station's equipment can be represented by stationary values of indicators: the availability coefficient \( K_a \), the operating time for failure of the \( T_o \), h (or the failure rate \( \omega \), 1/year), the recovery time of the \( T_R \), h, etc. Taking into account that the choice of the composition of generating equipment is ahead of the actual operating time in time, it is proposed to take into account the reliability factor by the operational readiness coefficient, which shows the probability that the system will be operational at an arbitrarily selected time in the steady-state operation mode and that, starting from this moment, the system will work flawlessly for a given time interval \( t_i \) [9]:

\[
K_a = \frac{\theta}{\theta + \tau_R} \cdot P(t_i,t) \tag{8}
\]

where \( P(t_i,t) \) is the conditional probability of failure-free operation of the system at the interval \( (t_i, t_i + \tau) \), provided that the system was operational at the time of \( t_i \); \( \tau_R \) is the duration of recovery; \( \theta \) is the
average operating time before failure (the operating time of the power unit (object) from the start of operation to failure (or between two neighboring failures)).

As a criterion for reliability with optimal load distribution of the station, we take the maximum values of the station readiness coefficient, i.e.

\[ F_3 (x) = V_{pp} = \sum_{i=1}^{m} M_{Si} (N_{pp}, Q_{pp}) \rightarrow \min \]  

when restricted

\[ M_{pp} \leq M_{pp,S} \]  

where \( M_{pp,S} \) is the permissible (standard) value of the power plant's emissions at the considered hour of the day.

In practice, the solution of a multi-criteria problem is associated with some fundamental problems, including the problem of the optimality principle and the problem of algorithms for solving the problem. It should be noted that the indicators-criteria of optimality are usually contradictory, so it is only possible to choose a reasonable compromise, i.e. the definition of such an acceptable vector of controlled parameters \( x \), in which all the criteria will take acceptable values.

Analysis of the existing basic approaches to finding such a solution by forming a single criterion of optimality - methods of "weighting coefficients" and the "main criterion", the minimax approach, the "sequential ledge" algorithm [10-14], etc. showed that, in relation to the multi-criteria task, it is most expedient to use the last of the above approaches. This is due to the fact that with a large number of criteria, the process of solving the problem becomes complicated due to the multiple solution of the problem for individual criteria with a large number of additional restrictions, which significantly increases the calculation time. In addition, when solving a problem, it is often difficult to determine the surrounding area of the optimal values of the criterion for the steps of solving the problem. The advantage of the "sequential ledge" method is that the number of criteria is not limited, and each criterion acts in its own specific dimension, whereas for the first three methods given, it is necessary to bring all the criteria to a single dimension, which is a rather complex and uncertain process, especially for the reliability criterion.

The peculiarity of thermal power plants is that the variety of modes of heating equipment leads to the presence of a set of local optima for all the criteria under consideration, which significantly complicates the process of searching for a global optimum. Another feature of the CHPP modes is the seasonality of the schedules of heat supply to thermal consumers up to its complete absence in summer modes, i.e., in fact, the CHPP turns into a condensing power plant.

4. The solution algorithm applied to the CHP

Based on the above, the following algorithm is proposed for the thermal power plant, based on the improved method of "sequential ledge":

- Step 1. In accordance with the algorithm of the method, the "main" criterion is selected, based on the optimization problem being solved and the specific operating conditions of the CHPP; let's assume this is an economic criterion for solving the 2nd stage or a reliability criterion for the first stage. For the remaining criteria, either one-sided restrictions are imposed, or, if there is an acceptable area, two-sided restrictions are imposed.
- Step 2. Taking into account the constraints, i.e. with the transition of the optimization problem to an unconditional form, the objective function and constraints are specified.
- Step 3. Drawing up an algorithm and solving a single-criteria static problem. As mentioned above, the solution of this problem is complicated by the presence of a large number of local optima, and as the calculations carried out with respect to several thermal power plants with a mixed composition of equipment have shown, the use of generally accepted methods of static
optimization, especially if there are steam-gas installations at the thermal power plant in addition to traditional steam turbine equipment (dynamic programming, gradient methods, Lagrange method, etc.) does not provide a stable solution. As a result of the conducted research, [15] recommends a combined method using a genetic algorithm to obtain an intermediate solution and a deformable polyhedron method for further searching for a global optimum.

- Step 4. Selecting the value of the "ledge" according to the main criterion (for example, ±10-15% of the value of the obtained optimum) and compiling a matrix of solutions that fall within the specified interval.
- Step 5. Analysis and refinement of the values of the 2nd most important criterion and ranking of the solutions obtained by this criterion, taking into account the restrictions imposed on the remaining third criterion;
- Step 6. Selecting the value of the "ledge" according to the 2nd reliability criterion and compiling a matrix of solutions that fall within the selected interval.
- Step 7. Ranking of the solutions selected in step 5 and selecting either the optimal solution or several solutions that are close to each other in terms of the value of the third criterion, leaving the final decision to the person making it.

The advantage of the proposed algorithm is its simplicity, the universality of the algorithm in relation to the number of criteria and the type of power plant. Its use is recommended for solving an optimization problem in cases where the main goal of optimization is to find an acceptable solution in relation to the selected main criterion with acceptable values of other criteria.

5. Some results and conclusions of researches on multi-criteria optimization
The conducted researches on multi-criteria optimization of load distribution and selection of the composition of the CHPP equipment based on the developed algorithm have shown that:

- the most effective distribution of thermal and electrical loads is the distribution of loads with the maximum load of the CCGT on the electrical and thermal load, while the optimization effect increases with the increase in the number of operating equipment. A comparison of the optimization results with uniform loading of the units showed that the developed method is quite effective – up to 4.5-8.9% of the hourly fuel economy with the full composition of the operating equipment (for example, a CHPP with a PGU-450T and thermal turbines of the PT-80 type), while the greatest effect from the optimal distribution is obtained when solving this problem at the stage of choosing the composition of generating equipment;
- taking into account the reliability factor at the stage of selecting the composition of generating equipment leads to the maximum use of the advantages of PGU-450 power units during night load failures due to the possibility of its deep unloading using the GTU-CHPP mode-maximum unloading of gas turbines and waste heat boilers within the permissible adjustment range of gas turbines with the transfer of the steam turbine to the motor mode with the discharge of high and low pressure steam into network heaters in addition to the steam turbine [16];
- taking into account the manoeuvrability factor when distributing loads in the morning hours of increasing the electric load leads to loading, first of all, the PGU-450;
- the use of CCGT in the condensation mode in the presence of a thermal load at the CHPP is economically impractical;
- the selection of the composition of generating equipment and load distribution in market conditions should be carried out based on those criteria that are most important for the current operation of equipment at a particular station;
- when solving the optimization problem of choosing the composition of generating equipment, special attention should be paid to the reliability criterion.
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