Abstract

We investigate the semi-classical instability of vacuum domain walls to processes where the domain walls decay by the formation of closed string loop boundaries on their worldvolumes. Intuitively, a wall which is initially spherical may ‘pop’, so that a hole corresponding to a string boundary component on the wall, may form. We find instantons, and calculate the rates, for such processes. We show that after puncture, the hole grows exponentially at the same rate that the wall expands. It follows that the wall is never completely thermalized by a single expanding hole; at arbitrarily late times there is still a large, thin shell of matter which may drive an exponential expansion of the universe. We also study the situation where the wall is subjected to multiple punctures. We find that in order to completely annihilate the wall by this process, at least four string loops must be nucleated. We argue that this process may be relevant in certain brane-world scenarios, where the universe itself is a domain wall.

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I. INTRODUCTION

Hybrid topological defects can occur in a variety of physical scenarios ([1], [2]) where multiple phase transitions are allowed to occur simultaneously. Typically, one may envision lower dimensional defects ending on higher dimensional defects (so-called ‘Dirichlet’ topological defects [2]), or conversely higher dimensional defects may end on lower dimensional defects [3]. In this paper, we are chiefly concerned with the latter situation, where domain walls may have string boundary components.

In the first papers to study walls ending on strings [3], the authors considered an explicit symmetry breaking pattern whereby the unifying gauge symmetry Spin(10) is broken to $SU(3) \times SU(2) \times U(1)$ by way of the group $SU(4) \times SU(2) \times SU(2)$. The first symmetry breaking pattern gives rise to the existence of $\mathbb{Z}_2$ strings, which may form boundaries for the domain walls which are produced by the second symmetry breaking.

In fact, this basic picture will hold in any scenario where you have a series of phase transitions of the form

$$G \xrightarrow{X_1} H \xrightarrow{X_2} I$$

(1.1)

where $\pi_0(G) = \pi_1(G) = I$ and $\pi_0(H) \neq I$ (here $I$ denotes the identity group). Since the sequence (1.1) is exact it follows that

$$\pi_1(G/H) = \pi_0(H)$$

That is to say, the first symmetry breaking give rise to strings. The second phase transition then gives rise to domain walls. More explicitly, if we consider the behaviour of the field $X_2$ as we move around a string, we see that $X_2$ must have a discontinuity since the field is not invariant under the action of $H$. In other words, we have to ‘add in’ domain walls to account for this discontinuity.

It follows that domain walls in these multiple phase transition scenarios are not topologically stable defects. Instead, such walls are unstable to a quantum mechanical decay process, whereby a closed string loop boundary component appears on the worldvolume of the domain wall, and then expands. To calculate the probability, $P$, that this process occurs, one typically invokes the semiclassical approximation

$$P \propto A e^{-\frac{1}{2}(S_E - S_B)}$$

where $S_E$ is the Euclidean action of the instanton of the post-tunneling configuration (a domain wall with a hole in it), $S_B$ is the Euclidean action of the background instanton (a domain wall without a hole in it), and $A$ is a prefactor which is calculated by considering fluctuations about the instanton. Typically, one ignores the prefactor and focusses on the action terms (since these terms lead to exponential suppression, they typically dominate).

The trajectory of a virtual string loop moving in Euclidean space is a two-sphere of radius $R$, where $R$ is the radius at which the string nucleates. The Euclidean section of a planar domain wall is (topologically) still just $\mathbb{R}^3$. Thus, the instanton for the final state is $\mathbb{R}^3$ with a ball of radius $R$ removed, and the instanton for the initial state is $\mathbb{R}^3$. It follows that the difference between the two actions must contain a boundary term proportional to the
area of the two-sphere, and a bulk term proportional to the volume of the removed region; indeed, one obtains

$$S_E - S_B = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \sigma$$

(1.2)

where $\mu$ is the string tension and $\sigma$ is the energy density of the domain wall. For typical symmetry breaking scales associated with strings and walls, one finds that $S_E - S_B >> 1$, and so this decay process is usually suppressed [1].

In all of this analysis, we have not said anything about the gravitational fields of these domain walls. This is because we have been assuming that the walls are so ‘light’ that they have effectively decoupled from gravity. Of course, there is no reason why heavy domain walls should not suffer the same instabilities as light walls. We are therefore led to generalize the known work on light domain walls, to include the effects of gravity. However, the gravitational effects of heavy domain walls are highly non-trivial; indeed, a gravitating domain wall generically closes the universe! Of course, this overwhelming property of domain walls is just another reason why we should be interested in finding new decay modes to get rid of them. We now turn to a short discussion of the gravitational effects of domain walls, before outlining the new work on how they may decay by the formation of closed string loop boundary components.

Throughout this paper we use units in which $\hbar = c = G = 1$.

II. VIS DOMAIN WALLS: A BRIEF INTRODUCTION

Solutions for the gravitational field of a domain wall were found by Vilenkin [4] (for an open wall) and Ipser and Sikivie [5] (for closed walls). The global structure of these Vilenkin-Ipser-Sikivie (or ‘VIS’) domain walls has been extensively discussed recently ([6], [7]) so we will only present a brief sketch here.

To begin, we look for a solution of the Einstein equations where the source term is an energy momentum tensor describing a distributional source located at $z = 0$:

$$T_{\mu\nu} = \sigma \delta(x) \text{diag}(1,1,1,0)$$

(2.1)

It is impossible to find a static solution of the Einstein equations with this source term; indeed, the VIS solution is a time-dependent solution describing a uniformly accelerating domain wall. In order to understand the global causal structure of the VIS domain wall, it is most useful to use coordinates $(t, x, y, z)$ so that the metric takes the form

$$ds^2 = e^{-2k|z|} \left( dt^2 - dz^2 \right) - e^{2k(t-|z|)}(dy^2 + dx^2),$$

(2.2)

Here, $k = 2\pi\sigma$, and the wall is located at $z = 0$. The gravitational field of this solution has amusing properties. For example, if you take the Newtonian limit of the Einstein equations for (2.2) you obtain the equation

$$\nabla^2 \phi = -2\pi \sigma$$

where $\phi$ is the Newtonian gravitational potential and $\sigma$ is the energy density of the wall. From this equation it is clear that a wall with positive surface energy density will have...
a repulsive gravitational field, whereas a wall with negative energy density will have an attractive gravitational field. An even simpler way to see that the (positive $\sigma$) VIS wall is repulsive is to notice that the $t - z$ part of the metric is just the Rindler metric.

Further information is recovered by noticing that the $z=$ constant hypersurfaces are all isometric to 2 + 1 dimensional de Sitter space:

$$ds^2 = dt^2 - e^{2kt}(dy^2 + dx^2).$$ \hspace{1cm} (2.3)

Given that 2 + 1 de Sitter has the topology $S^2 \times \mathbb{R}$ it follows that the domain wall world sheet has this topology. In other words, at each instant of time the domain wall is topologically a two-dimensional sphere. Indeed, in the original Ipser-Sikivie paper a coordinate transformation was found which takes the $(t, x, y, z)$ coordinates to new coordinates $(T, X, Y, Z)$ such that in the new coordinates the metric becomes (on each side of the domain wall):

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2.$$ \hspace{1cm} (2.4)

Furthermore the domain wall, which in the old coordinates is a plane located at $z = 0$, is in the new coordinates the hyperboloid

$$X^2 + Y^2 + Z^2 = \frac{1}{k^2} + T^2.$$ \hspace{1cm} (2.5)

Of course, the metric induced on a hyperboloid embedded in Minkowski spacetime is just the de Sitter metric, and so this is consistent with what we have already noted. This metric provides us with a useful way of constructing the maximal extension of the domain wall spacetime:

First, take two copies of Minkowski space, and in each copy consider the interior of the hyperboloid determined by equation (2.3), match these solid hyperboloids to each other across their respective boundaries; there will be a ridge of curvature (much like the edge of a lens) along the matching surface, where the domain wall is located. Thus, an inertial observer on one side of the wall will see the domain wall as a sphere which accelerates towards the observer for $T < 0$, stops at $T = 0$ at a radius $k^{-1}$, then accelerates away for $T > 0$. We illustrate this construction below, where we include the acceleration horizons to emphasize the causal structure.
Now, the repulsive effect of this vacuum domain wall is very similar to the inflationary effect of a positive cosmological constant seen in de Sitter space. Indeed, we often find it is useful to think of a VIS spacetime as an inflating universe where all of the vacuum energy has been ‘concentrated’ on the sheet of the domain wall.

III. ‘POPPING’ A VIS DOMAIN WALL: INSTANTON AND ACTION

A. Euclidean section of the ingoing state

Before we construct instantons for popping domain walls, it is useful if we first recall a few basic facts about the Euclidean section and action of the ordinary VIS domain wall.

In the simplest scenarios, a VIS domain wall will form when there is a breaking of some discrete symmetry. Usually, one thinks of the symmetry breaking in terms of some Higgs field $\Phi$. If $\mathcal{M}_0$ denotes the ‘vacuum manifold’ of $\Phi$ (i.e., the submanifold of the Higgs field configuration space on which the Higgs acquires a vacuum expectation value because it will minimize the potential energy $V(\Phi)$), then a necessary condition for a domain wall to exist is that $\pi_0(\mathcal{M}_0) \neq 0$. In other words, vacuum domain walls arise whenever the vacuum manifold is not connected. Given these assumptions, one usually writes the Lagrangian density for the matter field $\Phi$ as
\[ \mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - V(\Phi). \]  

(3.1)

The exact form of \( V(\Phi) \) is not important. All that we require in order for domain walls to be present is that \( V(\Phi) \) has a discrete set of degenerate minima, where the potential vanishes. Given this matter content, the full (Lorentzian) Einstein-matter action then reads:

\[ S = \int_M d^4x \sqrt{-g} \left[ \frac{R}{16\pi} + \mathcal{L}_m \right] + \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K. \]  

(3.2)

Here, \( M \) denotes the four-volume of the system, and \( \partial M \) denotes the boundary of this region. One obtains the Euclidean action, \( I \), for the Euclidean section of this configuration by analytically continuing the metric and fields and reversing the overall sign. The ‘simplified’ form of this Euclidean action in the thin wall limit has been derived in a number of recent papers (\([6], [7]\)) and so we will not reproduce the full argument here. Basically, one first assumes that the cosmological constant vanishes (\( R = 0 \)) and then one uses the fact that the fields appearing in the matter field Lagrangian depend only on the coordinate ‘\( z \)’ normal to the wall, and one integrates out this \( z \)-dependence to obtain the expression

\[ I = -\sum_{i=1}^{n} \frac{\sigma_i}{2} \int_{D_i} d^3x \sqrt{h_i}. \]  

(3.3)

Here, \( D_i \) denotes the \( i \)-th domain wall, \( \sigma_i \) is the energy density of the domain wall \( D_i \), \( h_i \) is the determinant of the three-dimensional metric \( h_{ab}^{(i)} \) induced on the domain wall \( D_i \) and \( n \) is the total number of domain walls. It is not hard to prove that variation relative to \( h_{ab}^{(i)} \) on each domain wall will yield the Israel matching conditions.

Now, as we have seen the Lorentzian section of a single VIS domain wall is just two portions of flat Minkowski space glued together. It is therefore natural to propose that the Euclidean section is obtained by gluing two flat Euclidean four-balls together along a common \( S^3 \) boundary component. In this way one obtains the ‘lens instanton’, which describes (in the context of the no-boundary proposal) the creation of a single VIS domain wall from ‘nothing’. This lens instanton is the Euclidean section of the incoming state - the VIS domain wall before the decay process (puncture) has taken place. In order to calculate the rate for the process, we now need to construct the instanton for the perforated domain wall.

**B. Euclidean section of the outgoing state**

As we saw above, a VIS domain wall moving in imaginary time sweeps out an \( S^3 \) ‘ridge’ of curvature along the equator of the lens instanton. In a similar way, we expect a virtual loop of string moving in imaginary time to sweep out a two-sphere, \( S^2 \). For our purposes, we want this string loop to correspond to a puncture which appears on the Euclidean portion of the domain wall, expands to some maximum size, then collapses again. In other words, the Euclidean section of the punctured domain wall is obtained by taking the lens instanton, and
‘truncating’ each flat four-ball so that the ‘hole’ swept out by the string is made manifest. This is illustrated below:

More explicitly, if we write the metric on flat space in the usual form

\[ ds^2 = dT^2 + dx^2 + dy^2 + dZ^2 \]

where \( T = iT \) denotes imaginary time, then the domain wall (on the lens instanton) is located at the boundary of each ball of radius \( R = 1/k \). In order to obtain the instanton for the punctured wall, we truncate the range of the variable \( Z \):

\[ Z \leq C < R \]

where \( C \) is some constant. In other words, we shave off a portion of the lens instanton so that the virtual string worldsheet is located at the surface \( Z = C \).

C. Calculation of the action and amplitude

We are now in position to calculate the action and decay rate for this process. Since there is no topology change in this situation, it is easy to construct an interpolating instanton by finding slices of the initial instanton which match smoothly to slices of the final instanton. We may therefore use the no-boundary ansatz without relying on more sophisticated arguments,
such as the ‘patching proposal’ put forward in [10]. As described in the introduction, we want to calculate the action difference, \( \Delta S \), between the Euclidean action of the final configuration \((S_F)\) and the Euclidean action of the initial configuration \((S_I)\).

Since the final instanton is obtained from the initial instanton simply by removing a ‘cap’ from the boundary of each four-ball before performing identifications, it follows that the only action difference can come from this contribution. As in the non-gravitating case described in the introduction, the removed region will contribute a bulk term (corresponding to the hole in the domain wall) and a surface term (the boundary of the hole).

The volume of the removed cap is calculated to be

\[
2\pi^2 R^3 \left(1 - \frac{2}{\pi} \left(\frac{-C}{2R} \sqrt{1 - C^2/R^2 + \cos^{-1}(C/R)/2} \right)\right)
\]

Likewise, the surface swept out by the string is just a two-sphere of radius \((R^2 - C^2)^{1/2}\).

Weighting the bulk term with domain wall energy density \(\sigma\), and the surface term with string tension \(\mu\) we therefore obtain

\[
\Delta S = S_F - S_I = 4\pi(R^2 - C^2)\mu - \pi^2 R^3 \left(1 - \frac{2}{\pi} \left(\frac{-C}{2R} \sqrt{1 - C^2/R^2 + \cos^{-1}(C/R)/2} \right)\right)\sigma \tag{3.4}
\]

Ignoring the prefactor term, the decay probability is then given as

\[
P = e^{-\Delta S} \tag{3.5}
\]

(Actually, the prefactor term may contain interesting information in this situation, for the simple reason that when the domain wall ‘pops’, the overall symmetry of the spacetime is broken from spherical symmetry to cylindrical symmetry; presumably, fluctuations about the instanton would respect this symmetry breaking. We will have more to say about the evolution of quantum fields in a punctured domain wall spacetime later in this paper).

Now, just as for popping light domain walls [1], our use of the thin wall approximation to derive (3.4) is only justified if the scale of symmetry breaking for the strings is much larger than the scale of symmetry breaking for domain walls. It follows that, generically, we will have \(\Delta S >> 1\), and hence \(P << 1\).

D. Decay of domain walls with multiple punctures

It is also possible for several holes to spontaneously form on the surface of a domain wall. In the simplest situation, all of the string loop boundary components will nucleate at the same initial radius \(((R^2 - C^2)^{1/2})\), with the same string tension \(\mu\) so that the total action will just be

\[
\Delta S_{TOT} = N\Delta S
\]

where \(\Delta S\) is given by (3.4), and \(N\) is the total number of holes. Of course, one might also imagine more exotic scenarios where the holes nucleate at different initial radii \(R_i = (R^2 - C_i^2)^{1/2}\), and string tension \(\mu_i\), so that the total action for the decay would be given as
\[ \Delta S_{TOT} = \sum_{i=1}^{N} \Delta S_i \]

where

\[ \Delta S_i = 4\pi(R^2 - C_i^2)\mu_i - \pi^2 R^3 \left( 1 - \frac{2}{\pi} \left( \frac{-C_i}{2R} \sqrt{1 - \frac{C_i^2}{R^2}} + \frac{\cos^{-1}(C_i/R)}{2} \right) \right) \sigma \]

In either situation, the creation of multiple holes is heavily suppressed relative to the nucleation of a single hole.

Of course, it is of some interest to know whether or not a domain wall can ever be completely annihilated by the processes which we are discussing here. In fact, it is not hard to prove that a domain wall will be completely destroyed by this decay process whenever at least four string loop boundary components are nucleated.

In order to understand this, recall that the worldvolume of the domain wall is isometric to 2+1-dimensional de Sitter spacetime (embedded in 3+1-dimensional Minkowski), and that the trajectory of a given string loop boundary component may be obtained by taking the intersection of the hyperboloid with a surface of constant \( Z \) (relative to the coordinates (2.4) on Minkowski space), which is simply a copy of 2+1-dimensional Minkowski spacetime. If we view all of this from ‘above’ the hyperboloid, then we see that the domain wall is a two-sphere which expands uniformly outwards, and that the string loop boundary components are intersections of this two-sphere with flat timelike hypersurfaces. If there are at least four (non-parallel) such hypersurfaces, then on each spacelike surface the hypersurfaces will bound a tetrahedron of fixed size. In three dimensions, a tetrahedron may bound a two-sphere, and so initially at least the domain wall may still lie (partially) within the tetrahedron. However, if the sphere continues to expand it will always eventually envelop the tetrahedron, and hence the domain wall will have been annihilated (i.e., the string boundary components ‘collide’ precisely at the corners of the tetrahedron). If you only had three (or fewer) timelike hypersurfaces, you could never completely bound the two-sphere in this way. The intersection of the sphere with a given timelike hypersurface would continue unbounded in some direction (because you would never encounter another timelike hypersurface). Thus, at least a portion of the domain wall would survive eternally. In other words, you need to nucleate at least four puncture wounds in a domain wall in order to completely annihilate the domain wall.

**IV. LORENTZIAN EVOLUTION OF PUNCTURED DOMAIN WALLS**

In the last section, we showed that a VIS domain wall may decay via the formation of closed string loop boundary components on the worldvolume of the wall. We found instantons, and calculated the corresponding actions and rates, for such processes. We now turn our attention to the Lorentzian, or ‘real time’, picture of this process.

First, recall the representation (Fig. 1) of the VIS domain wall as two solid hyperboloids in Minkowski space, identified along their respective boundaries. The constraint on the coordinated \( Z, Z \leq C < R \), which we imposed on the Euclidean section, extends to the Lorentzian section as well. Thus, in the Lorentzian coordinates \( (T,X,Y,Z) \), the equation of motion for the loop of string may be written as
\[ X^2 + Y^2 = (R^2 - C^2) + T^2 \]  \hspace{2cm} (4.1)

Thus, the initial radius of the loop of string (at \( T = 0 \)) is seen to be \( R^2 - C^2 > 0 \), which is what we expect given that the Lorentzian section must match smoothly to the Euclidean section. At late times, the hole is expanding at the speed of light. Of course, the hole never completely devours the wall for the simple reason that the spherical wall also expands exponentially. The global structure of this spacetime is illustrated below:

Figure 3

It is interesting to consider the gross properties of particles which are propagating in the background of a wall which spontaneously decays in this way. For example, suppose we consider some scalar field \( \psi \) which couples to the Higgs field (of the domain wall) \( \phi \) through the interaction

\[ \mathcal{L}_{int} = -\frac{\lambda}{2} \phi^2 \psi^2 \]

(Here, we are thinking of \( \phi \) as a Higgs field with a standard \( \phi^4 \) ‘double-well’ potential). In [1], it is shown that the reflection coefficient for scattering of \( \psi \) particles off of a \( \phi \) domain wall is zero in the limit that the domain wall has zero thickness. In other words, it makes sense to think of the (infinitely thin) VIS domain walls which we have been considering in this paper as spherical, accelerating, ‘moving mirrors’. Put another way, each side of the domain wall is a spherical cavity, with Dirichlet boundary conditions for the fields at the boundary of the cavity.
In [11] it is shown that a mirror moving with uniform acceleration $a$ will generate (relative to an inertial particle detector) a thermal bath of radiation at temperature

$$T = \frac{a}{2\pi k_b}$$

where $k_b$ is Boltzmann’s constant. Of course, we could have predicted that the VIS spacetime would have an entropy and temperature of this form, simply because the repulsive energy of the wall generates a cosmological horizon which leads to loss of information. Inertial observers in a VIS spacetime can never recover information about what is happening on the other side of the domain wall, and they will represent this ignorance by tracing over states associated with the horizon.

If a domain wall decays by the formation of a closed string loop boundary component on the worldvolume of the wall, the isotropic thermal nature of the initial (VIS) spacetime should be lost. This is because the initial spherical symmetry will be broken to cylindrical symmetry when the hole forms. The hole is in some sense then a ‘window’ between the two spherical cavities, through which information can propagate. It would be interesting to have some explicit calculation for the scattering of $\psi$-particles in the background of a VIS domain wall with a single puncture.

As we have shown, if four (or more) punctures spontaneously nucleate on a wall, the string boundary components must eventually collide and annihilate the wall, so that all of the energy density initially stored in the wall is thermalized. The final ‘annihilation’ of a domain wall by this process is rather analogous to the endpoint of inflation, in the sense that the source driving the exponential expansion is suddenly ‘switched off’. However, a domain wall with several punctures will presumably generate a highly non-isotropic radiation background, in contrast with the inflationary scenario.

V. CONCLUSIONS: A POTENTIAL BRANE-WORLD INSTABILITY?

We have shown how to describe the decay of vacuum domain walls by the formation of closed string boundary components, when the effects of gravity are included. We found new instantons, as well as the corresponding Lorentzian solutions, which describe the formation and evolution of multiple closed string loop boundary components on a given VIS domain wall background. It would be nice to have a complete picture of how quantum fields will evolve in the background of one of these punctured walls.

In general domain walls arise as (D-2)-dimensional defects (or extended objects) in D-dimensional spacetimes. In fact, domain walls are a common feature in the menagerie of objects which appear in the low-energy limit of string theory, as has been discussed in detail in [8] and [9].

Of course, there has recently been an enormous amount of interest in the possibility that the universe itself might be a domain wall moving in some five-dimensional bulk spacetime. In particular, Randall and Sundrum [12] have recently put forward a model where the universe is a $\mathbb{Z}_2$-symmetric positive tension domain wall bounding two bulk regions of anti-de Sitter (adS) spacetime. In their original scenario, the bulk cosmological constant is fine-tuned relative to the domain wall energy density so that the effective cosmological constant on the brane is precisely zero. However, during a period of inflation on the brane-world
the effective cosmological constant would be positive, and the domain wall would simply be a four-dimensional de Sitter hyperboloid embedded in five-dimensional \( \text{adS} \) (for a recent interesting discussion of various semiclassical instabilities associated with these de Sitter brane-worlds see [13]). Thus, the causal structure of these de Sitter brane-worlds is identical to that of the VIS domain wall \(^1\).

Now, one can certainly imagine that the domain wall of the Randall-Sundrum model arises because of a symmetry breaking pattern which allows for the universe itself to end on a two-brane (all that is required is an associated exact sequence of the form (1.1)). In such a scenario, the brane-world would be unstable to the formation of ‘holes’ - two-dimensional surfaces where the universe would end. These holes would accelerate out, devouring the universe and converting brane-world fields into bulk degrees of freedom. Since a fundamental polytope in four dimensions has five faces, it follows that you would have to nucleate at least five of these holes to completely annihilate a de Sitter universe in this fashion.

One could even go further and imagine scenarios where the brane-world is itself the boundary of a puncture wound in some four-brane (in this way you could have more than one large extra dimension).

In general, we would expect many of the (self-gravitating) \( p \)-branes of supergravity models to be unstable to the sort of decay processes which we have described in this paper. From this point of view, the process which we have studied is just another example of the sort of ‘brane damage’ which we expect to be a generic feature of the \( p \)-branes of M-theory.

Research on these and related questions is currently underway.

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\(^1\)The causal structure of these walls, in the four-dimensional case, was first discussed in [14]; it was shown in [15] that this causal structure is universal in any dimension.
REFERENCES

[1] A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and Other Topological Defects*, Cambridge University Press, Cambridge (1994).

[2] Sean M. Carroll and Mark Trodden, *Dirichlet topological defects*, Phys. Rev. D57, 5189-5194, (1998); [hep-th/9711099](#) (1997).

[3] T.W.B. Kibble, G. Lazarides and Q. Shafi, *Walls bounded by strings*, Phys. Rev. D26, 435 (1982).

[4] A. Vilenkin, Phys. Lett. B133, 177-179, (1983).

[5] J. Ipser and P. Sikivie, Phys. Rev. D30, 712-719, (1984).

[6] R.R. Caldwell, A. Chamblin and G.W. Gibbons, *Pair creation of black holes by domain walls*, Phys. Rev. D53: 7103-7114, (1996); [hep-th/9602126](#).

[7] Shawn J. Kolitch and Douglas M. Eardley, *Quantum decay of domain walls in cosmology: 1. Instanton approach*, Phys. Rev. D56, 4651-4662, (1997); [gr-qc/9706011](#).

[8] P.M. Cowdall, H. Lu, C.N. Pope, K.S. Stelle and P.K. Townsend, *Domain walls in massive supergravities*, Nucl. Phys. B486: 49-76, (1997); [hep-th/9608173](#).

[9] Mirjam Cvetič and Harald H. Soleng, *Supergravity domain walls*, Phys. Rept. 282: 159-223, (1997); [hep-th/9604090](#).

[10] R. Bousso and A. Chamblin, *Patching up the No-Boundary proposal with virtual Euclidean wormholes*, [gr-qc/9803047](#).

[11] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge (1982).

[12] L. Randall and R. Sundrum, *An alternative to compactification*, [hep-th/9906064](#).

[13] J. Garriga and M. Sasaki, *Brane-world creation and black holes*, [hep-th/9912118](#).

[14] Mirjam Cvetič, Stephen Griffies, and Harald H. Soleng, *Local and global gravitational aspects of domain wall space-times*, Phys. Rev. D48, 2613-2634, (1993); [gr-qc/9306003](#).

[15] Mirjam Cvetič and Jing Wang, *Vacuum Domain Walls in D-dimensions: Local and Global Space-Time Structure*, [hep-th/9912187](#).