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Errata

Magnetic Properties of a Superconductor with no Inversion Symmetry, by S. K. Yip, J. Low Temp. Phys. 140, 67 (2005)

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The claim that $B_y^{(1)} = 0$ in Sec 4A of the above paper (hereafter as I) is incorrect. The calculation over there only establishes a particular solution Eqs. I(33), I(34) for $\tilde{A}_z^{(1)}$ to Eqs. I(31), I(32) with $B_y^{(1)} = 0$. It is necessary to add a solution to the homogeneous equations corresponding to Eqs. I(31) and I(32), i.e.,

$$
\frac{\partial B_y^{(1),h}}{\partial \tilde{z}} - \tilde{A}_x^{(1),h} = 0 \quad (1)
$$

$$
\frac{\partial B_y^{(1),h}}{\partial \tilde{x}} + \tilde{A}_z^{(1),h} = 0 \quad (2)
$$

correspondingly

$$
\frac{\partial^2 B_y^{(1),h}}{\partial \tilde{z}^2} + \frac{\partial^2 B_y^{(1),h}}{\partial \tilde{x}^2} - B_y^{(1),h} = 0 \quad (3)
$$

This homogeneous solution must be included so that $B_y^{(1)} - 4\pi M_y^{(1)}$ contains no discontinuity at the vortex. Using $M_y^{(1)} = \frac{\tilde{q}_x^{(0)}}{2\pi}$ hence

$$
M_y^{(1)} = -\tilde{\kappa} \frac{\tilde{\Phi}_0}{8\pi^2} K_1(r) \cos \theta , \quad (4)
$$

the homogeneous solution required is

$$
B_y^{(1),h} = -\tilde{\kappa} \frac{\tilde{\Phi}_0}{2\pi} K_1(r) \cos \theta \quad (5)
$$

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correspondingly \( B_y^{(1)} = B_y^{(1),h} \). We see now that \( B_y^{(1)} - 4\pi M_y^{(1)} \) becomes identically zero. To check the consistency with the Maxwell equation I(8), we note that \( \vec{A}^{(1),h} \) can be obtained from Eqs I(1) and (2). Together with Eqs. I(33) and I(34), we obtained the correct first order vector potentials as

\[
\vec{A}_x^{(1)} = \frac{1}{2\pi} \Phi_0 K_0(r) \tag{6}
\]

and

\[
\vec{A}_z^{(1)} = 0 \tag{7}
\]

Eq. (6) gives \( J_z^{(1)} = 0 \). Eq. (7), together with I(1) and \( B_y^{(0)} = \frac{1}{2\pi} \Phi_0 K_0(r) \) gives \( j_x^{(1)} = 0 \).

The total field up to first order is therefore

\[
B_y^{(0)} + B_y^{(1)} = \frac{1}{2\pi} \left[ K_0(r) - \kappa K_1(r) \cos \theta \right] \tag{8}
\]

\[
\approx \frac{1}{2\pi} \left[ K_0(|r + \kappa \hat{z}|) \right] \tag{9}
\]

Therefore, to first order in \( \kappa \), the field pattern is the same as a centrosymmetric superconductor except that it is shifted from the vortex center towards \( -z \) by \( \kappa \) in dimensionless units (\( \kappa \lambda_x \) in ordinary units). This shift has been found numerically by Oka et al [1] in their numerical solution of Ginzburg-Landau equations, instead of the London equations here.

In contrast, the total gauge invariant superfluid velocities along \( x \) and \( z \) are, up to first order in \( \kappa \), proportional to \(-K_1(r) \cos \theta + \kappa K_0(r)\) and \( K_1(r) \sin \theta \), respectively. The singularity is dominated by \( K_1(r) \) terms and is still at the vortex center.

The correct \( \vec{A}_x^{(1),z} \) in Eq (6) and (7) has to be used to obtain the next higher correction \( B_y^{(2)} \), which can be shown to be finite instead of zero as claimed in I. The expression is not particularly informative and we shall not obtain it here.

The results of the other sections in I are unaffected.

References

1. M. Oka, M. Ichioka and K. Machida, Phys. Rev. B 73, 214509 (2006)
Magnetic Properties of a Superconductor with no Inversion Symmetry

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Abstract

We study the magnetic properties of a superconductor in a crystal without $z \rightarrow -z$ symmetry, in particular how the lack of this symmetry exhibits itself. We show that, though the penetration depth itself shows no such effect, for suitable orientation of magnetic field, there is a magnetic field discontinuity at the interface which shows this absence of symmetry. The magnetic field profile of a vortex in the $x-y$ plane is shown to be identical to that of an ordinary anisotropic superconductor to second order in a small parameter $\tilde{\kappa}$. For a vortex along $z$, there is an induced magnetization along the radial direction.

Keywords: Superconductivity, Magnetic Screening, Vortices

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I. INTRODUCTION

Lately, there is much attention on the theory of superconductivity in systems without inversion symmetry in the normal state, e.g., $^1,^2,^3,^4,^5,^6,^7$. This surge of interest is due in no small part to the discovery of new superconductors in crystal where this symmetry is absent, for example, the heavy fermion superconductor CePt$_3$Si$_8$ with $T_c \sim 0.75K$. The normal state of CePt$_3$Si (ignoring the possible antiferromagnetic ordering at $T_N \sim 2.2K$) has tetragonal $P4mm$ symmetry. Due to the displacement of Si along $\hat{c}$ direction, the reflection symmetry $z \rightarrow -z$ has already been already lost in the normal state before the superconducting transition.

Indeed on very general grounds, the properties of a superconductor in crystals without inversion symmetry is expected to be very different from those where such a symmetry is respected. In the latter case, which is applicable to most known superconductors, inversion symmetry and Kramers degeneracy allow the classifications of superconducting states into singlet, even parity on the one hand and triplet, odd parity on the other $^9$. The physical properties of such superconductors can then be drawn from the generalization of theories of conventional superconductors or superfluid $^3$He. This is no longer the case if inversion symmetry is already broken in the normal state $^1$. Some peculiar aspects of these superconductors have already been discussed theoretically in the literature $^2,^3,^4,^5,^6,^7,^10$.

In particular, in ref $^2,^10$, it is shown that for systems where $z \rightarrow -z$ is broken, a superfluid flow in the plane, say along $x$, is expected to produce a spin polarization along $y$ when a Rashba $^{11}$ type spin-orbit coupling is present. Conversely, a Zeeman magnetic field along $y$ is expected to generate a superfluid flow or phase gradient along $x$. Though the calculation in $^2$ was specifically for two dimensions, the general argument there is obviously generalizable to a three dimensional superconductor. (see also Section V below) Here we elaborate further on the consequence these effects discussed in Ref $^2$. We shall (Section III) first study the screening of an external magnetic field by the superconductor, i.e., the Meissner effect. More specifically, consider a magnetic field along $\hat{y}$ with the superconductor occupying either $z > 0$ or $z < 0$. These two situations are not equivalent if the crystal lacks the $z \rightarrow -z$ symmetry. We shall however see that, (at least to the surprise to the present author), the penetration depth itself shows no direct effect of the absence of this $z \rightarrow -z$ symmetry. The latter only manifests itself as discontinuities in the magnetic field near the crystal surface with opposite
signs for the two mentioned geometries.

Next (Section IV) we discuss the structure of a vortex in London theory. We shall show that, at least to second order in a small parameter $\tilde{\kappa}$ to be defined below, a vortex for field along $\hat{y}$ has a magnetic field distribution again symmetric with respect to $z \to -z$. Hence the constant magnetic field contour is elliptical similar to that of an ordinary crystal with different effective masses along $\hat{x}$ and $\hat{z}$. For magnetic field along $\hat{z}$, there is a spin magnetization pointing along the radial direction, the sign of which reflects the absence of $z \to -z$ symmetry. We finally estimate the order of magnitude of these broken symmetry effects (Section V).

II. CONSTITUTIVE EQUATIONS

First we recall the constitutive equations relating the (number) current $J$, (local) magnetization $M$, gauge invariant phase gradient $q \equiv \hbar \nabla \phi + \frac{2e}{\hbar c} A$ (electron charge $= -e$) and the magnetic field $B$ in a superconductor with broken $z \to -z$ symmetry in its normal state. For currents and magnetic field in the $x - y$ plane, the relations are expected to have the general form

$$J_x = n_s \frac{q_x}{2m_x} - \kappa B_y$$

$$J_y = n_s \frac{q_y}{2m_y} + \kappa B_x$$

$$M_x = -\frac{\kappa}{2} q_y + \chi_x B_x$$

$$M_y = \frac{\kappa}{2} q_x + \chi_y B_y$$

Here $n_s$ is the superfluid density and $m_x$ etc are the effective masses, and $\chi_x$ etc. the spin susceptibilities ($m_x = m_y$ and $\chi_x = \chi_y$). The terms proportional to $\kappa$ in eq (1) and (2) represent the supercurrent induced by Zeeman field and the corresponding ones in eq (3) and (4) represent the magnetization induced by superflow. Note the difference in sign for the terms proportional to $\kappa$ between eq (1) and (2) and between eq (3) and (4). These terms are specific to the absence of $z \to -z$ symmetry. The purpose of the present paper is to study effects due to these terms. As already mentioned in ref, these equations are
expected from the general form of the free energy

\[ F = n_s \left( \frac{q_x^2}{8m_x} + \frac{q_y^2}{8m_y} + \frac{q_z^2}{8m_z} \right) \]

\[- \left( \frac{1}{2} \chi_x B_x^2 + \frac{1}{2} \chi_y B_y^2 + \frac{1}{2} \chi_z B_z^2 \right) + \frac{B^2}{8\pi} \]

\[- \frac{\kappa}{2} (q_x B_y - q_y B_x) \]  

appropriate to the present symmetry. The term proportional to \( \kappa \) here is symmetry allowed in the present case (see also\(^3,6\)).

Currents and magnetization along the z axis are given by the usual relations

\[ J_z = n_s \frac{q_z}{2m_z} \]  \hspace{1cm} (6)

\[ M_z = \chi_z B_z \]  

The equation governing the magneto-statics is given by

\[ \nabla \times B = 4\pi \nabla \times M + \frac{4\pi}{c} (-e)J \]  

where \( J \) and \( M \) are given by eqs (11)–(4), (6) and (7). Eq (8) also follows from the free energy (5) by variation with respect to \( A \) when one takes into account the basic relation \( B = \nabla \times A \).

It is worth mentioning that the screening of a magnetic field in a superconductor without inversion symmetry has also been considered in ref\(^12\). However, they assume the presence of a term with \( J \parallel B \). This term is absent for our case of \( P4mm \) symmetry because the presence of vertical reflection planes such as \( x \rightarrow -x \) and \( y \rightarrow -y \) and the fact that \( J \) and \( B \) transform differently under reflections.

III. MEISSNER SCREENING

(A) Let us now consider screening of a magnetic field in the basal plane, taken to be \( B = B_y \hat{y} \) without loss of generality, with the sample occupying either \( z > 0 \) or \( z < 0 \). These two (not a priori equivalent) geometries are particular interesting since the broken \( z \rightarrow -z \) symmetry may manifest. With translational invariance along \( x \) and \( y \), one can verify that the \( z \) component of eq (8) is trivially satisfied. The \( y \) component is satisfied by \( B_x = 0, M_x = 0, J_y = 0 \). The \( x \) component reduces to

\[ \frac{\partial B_y}{\partial z} = 4\pi \frac{\partial M_y}{\partial z} + \frac{4\pi e}{c} J_x \]  \hspace{1cm} (9)
In our case we can take the gauge \( \phi = 0 \) and \( A = A_x(z) \hat{x} \). With eq (11) and (14) and taking into account \( B_y = \frac{\partial A_x}{\partial z} \), it can be seen that the terms proportional to \( \kappa \) cancel out in eq (9). Further taking the \( z \) derivative gives

\[
(1 - 4\pi \chi_y) \frac{\partial^2 B_y}{\partial z^2} = \frac{4\pi n_s e^2}{m_x c^2} B_y.
\]

Thus the penetration depth \( \lambda_x \) is given by

\[
1/\lambda_x^2 = \frac{4\pi n_s e^2}{m_x c^2} / (1 - 4\pi \chi_y)
\]

Here the subscript of \( \lambda_x \) denotes that the current is along \( x \). Typically \( \chi_x \ll 1 \) and thus \( \lambda_x \) reduces to the usual expression \( 1/\lambda_x^2 = \frac{4\pi n_s e^2}{m_x c^2} \).

Since the term linear in \( \kappa \) drops out, the penetration depth shows no direct effect of the lack of \( z \to -z \) symmetry. It is the same for samples occupying \( z > 0 \) or \( z < 0 \).

This, however, does not mean that there is no broken symmetry effects at all. Since \( B_y - 4\pi M_y \) has to be continuous across the vacuum-sample interface, we then have, for sample occupying \( z > 0 \), \( B_{\text{ext}} = B_{\text{in}} - 4\pi M_y(0^+) \). Here \( B_{\text{in}} = B(z = 0^+) \) is the value of magnetic field just inside the sample. \( M_y \) is given by eq (4), hence (recall that \( \phi = 0 \))

\[
M_y(0^+) = \frac{\kappa e}{c} A_x(0^+) + \chi_y B_{\text{in}}
\]

Inside the sample, \( B_y(z) = B_{\text{in}} e^{-z/\lambda_x} \) with \( \lambda_x \) already determined in eq (11) above. Thus

\[
A_x(z) = -\lambda_x B_{\text{in}} e^{-z/\lambda_x}.
\]

We finally have

\[
B_{\text{in}} = B_{\text{ext}} / (1 - 4\pi \chi_y + \tilde{\kappa})
\]

where \( \tilde{\kappa} = \frac{4\pi n_s c}{e} \lambda_x \) is a dimensionless parameter.

Similar calculation for the case where the sample occupies \( z < 0 \) shows

\[
B_{\text{in}} = B_{\text{ext}} / (1 - 4\pi \chi_y - \tilde{\kappa})
\]

Hence the \( \kappa \) term results in a discontinuity of the magnetic field with a contribution of opposite signs in the two geometries. This is a manifestation of the broken symmetry. Of course the magnetic field is discontinuous only under our (London) approximation: the variation would spread out probably over a length scale of order of the coherence length in a more microscopic treatment.
(B) For ease of latter reference we also consider the screening of magnetic field $\mathbf{B} = B_y \mathbf{\hat{y}}$ with the sample occupying either $x > 0$ or $x < 0$. Since there cannot be any current along $\mathbf{\hat{x}}$, $J_x = 0$ and so from eq (11)

$$\frac{n_s e}{m_z c} \left( A_x + \frac{\hbar c \partial \phi}{2e \partial x} \right) = \kappa B_y \tag{15}$$

Note it follows that we cannot set both $A_x$ and $\frac{\partial \phi}{\partial x}$ zero, in contrast to the more usual case where the $\kappa$ term is absent. In our case, it is convenient to make use of the translational invariance along $z$ and choose $A_x$ and $\phi$ both dependent only on $x$, so that, e.g., $B_y = -\frac{\partial A_z}{\partial x}$.

Eq (8) gives

$$\frac{\partial B_y}{\partial x} = 4\pi \frac{\partial M_y}{\partial x} - \frac{4\pi e}{c} J_z \tag{16}$$

Eq (11), together with eq (15), gives

$$M_y = (\chi_y + \tilde{\kappa}^2) B_y \tag{17}$$

Therefore eq (16) reduces to

$$(1 - 4\pi \chi_y - \tilde{\kappa}^2) \frac{\partial B_y}{\partial x} = -\frac{4\pi n_s e^2}{m_z c^2} A_z \tag{18}$$

Taking the $z$ derivative shows that the penetration depth $\lambda_z$ is given by

$$1/\lambda_z^2 = \frac{4\pi n_s e^2}{m_z c^2} / (1 - 4\pi \chi_z - \tilde{\kappa}^2) \tag{19}$$

Thus the $\kappa$ term only gives a correction to the penetration depth proportional to $\tilde{\kappa}^2$. Hence again there is no asymmetry between the geometries where the samples occupy $x > 0$ or $x < 0$. Similar argument as in the last subsection shows that there is a discontinuity in magnetic field near the sample surface $\propto \tilde{\kappa}^2$.

(C) We finally consider a field along $\mathbf{\hat{z}}$. Without loss in generality, we take the sample to occupy $y > 0$. Translational invariance along $x$ and $z$ are respected and all quantities depend only on $y$. It can be shown easily that the magnetic field obeys the usual screening equations and thus $B_z(y) = B_z(0) e^{-y/\lambda_z}$ where $1/\lambda_z^2 = \frac{4\pi n_s e^2}{m_z c^2} / (1 - 4\pi \chi_z)$ is the same as that in section (A) [except that the small correction due to spin susceptibility is here now $(1 - 4\pi \chi_z)$ instead of $(1 - 4\pi \chi_y)$ in eq (11)]. The peculiar feature here, however, is that from eq (14) that $M_y \neq 0$ since $q_x \neq 0$. One easily finds $M_y(y) = \frac{\tilde{\kappa}}{4\pi} B_z(0) e^{-y/\lambda_z}$. Thus there is a magnetization towards (if $\kappa > 0$) or away from (if $\kappa < 0$) the inside of the sample if $B_{z\text{ext}} > 0$. 

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IV. FIELD DISTRIBUTION OF A SINGLE VORTEX

(A) Now we study the magnetic field profile around a vortex for magnetic field in the basal plane, chosen to be again along $\hat{y}$. For simplicity, we shall ignore the small spin susceptibilities $\chi_y$ in the present section. The basic equations are again eq (9) and (16), with $M_y$, $J_x$, $J_z$ given by eq (4), (11), (6). In constrast to section IIIA, the vector potential must depend both on $x$ and $z$, and thus

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$  \hfill (20)

Due to the presence of these two contributions to $B_y$, the terms linear in $\kappa$ does not drop out in eq (9) (or equivalently eq (26) below). Therefore it is not a priori obvious that the vortex field distribution will obey $z \rightarrow -z$ symmetry. We shall however show below that this symmetry is respected at least to order $\tilde{\kappa}^2$.

We begin by performing a rescaling of coordinates by the penetration depths, thus we write

$$z = \lambda_x \tilde{z}$$  \hfill (21)
$$x = \lambda_z \tilde{x}$$  \hfill (22)

It is convenient also to rescale the components of $\mathbf{A}$:

$$A_x = \lambda_x \tilde{A}_x$$  \hfill (23)
$$A_z = \lambda_z \tilde{A}_z$$  \hfill (24)

so that

$$B_y = \frac{\partial \tilde{A}_x}{\partial \tilde{z}} - \frac{\partial \tilde{A}_z}{\partial \tilde{x}}$$ \hfill (25)

In these variables, eq (9) and eq (16) become

$$\frac{\partial B_y}{\partial \tilde{z}} - \left( \tilde{A}_x + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{x}} \right) = \tilde{\kappa} \frac{\partial}{\partial \tilde{z}} \left( \tilde{A}_x + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{x}} \right) - \tilde{\kappa} B_y$$  \hfill (26)
$$\frac{\partial B_y}{\partial \tilde{x}} + \left( \tilde{A}_z + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{z}} \right) = \tilde{\kappa} \frac{\partial}{\partial \tilde{x}} \left( \tilde{A}_z + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{z}} \right) + \tilde{\kappa}^2 \left( \tilde{A}_z + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{z}} \right)$$  \hfill (27)

where $\tilde{\Phi}_0 \equiv \frac{\pi \hbar c}{e \lambda_x \lambda_z}$ is a scaled flux quanta (magnetic field). Note that as usual, $\mathbf{B}$ and $\mathbf{J}$ must vanish at large distances and the total flux is therefore given by the flux quanta $\frac{\pi \hbar c}{e}$. In our scaled variables this condition becomes $\int B \, d\tilde{x} d\tilde{z} = \tilde{\Phi}_0$. 

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It seems difficult to solve eq (26) and (27) for general \( \tilde{\kappa} \). We shall thus make use of the smallness of \( \tilde{\kappa} \) to solve these equations order by order in this parameter. (Strictly speaking the penetration depth \( \lambda_z \) already contains a \( \tilde{\kappa}^2 \) correction, but the present rescaling using this corrected \( \lambda_z \) simplifies the calculations below substantially).

In the lowest (zeroth) order, we can drop all terms on the right hand sides of eq (26) and (27). The resulting equations are the standard equations for the vortex. For a vortex with positive flux along \( y \), we need to choose \( \phi = -\theta \) and the solutions are

\[
B_{y}^{(0)} = \frac{\tilde{\Phi}_0}{2\pi} K_0(r) \tag{28}
\]

\[
A^{(0)} = A^{(0)}_\theta \hat{\theta} \tag{29}
\]

where

\[
A^{(0)}_\theta = \frac{\tilde{\Phi}_0}{2\pi} \left( \frac{1}{r} - K_1(r) \right) \tag{30}
\]

and we have introduced the cylindrical coordinates for the scaled variables: \( r \equiv (\tilde{z}^2 + \tilde{x}^2)^{1/2} \), \( \cos\theta = \tilde{z}/r \), \( \sin\theta = \tilde{x}/r \), \( \hat{\theta} = -\sin\theta \hat{z} + \cos\theta \hat{x} \). \( K_0, K_1 \) are the modified Bessel functions.

To first order, we have, from eq (26) and (27),

\[
\frac{\partial B_{y}^{(1)}}{\partial \tilde{z}} - \tilde{A}_{\tilde{x}}^{(1)} = \tilde{\kappa} \frac{\partial}{\partial \tilde{z}} \left( \tilde{A}_{\tilde{x}}^{(0)} + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{z}} \right) - \tilde{\kappa} B_{y}^{(0)} \tag{31}
\]

\[
\frac{\partial B_{y}^{(1)}}{\partial \tilde{x}} + \tilde{A}_{\tilde{z}}^{(1)} = \tilde{\kappa} \frac{\partial}{\partial \tilde{x}} \left( \tilde{A}_{\tilde{z}}^{(0)} + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{x}} \right) \tag{32}
\]

We claim that \( B_{y}^{(1)} = 0 \). Assuming this, using eq (28), (29) and therefore \( \tilde{A}_{\tilde{x}}^{(0)} + \frac{\tilde{\Phi}_0}{2\pi} \frac{\partial \phi}{\partial \tilde{x}} = -\frac{\tilde{\Phi}_0}{2\pi} K_1(r)\cos\theta \), eq (31) and (32) become

\[
\tilde{A}_{\tilde{x}}^{(1)} = \tilde{\kappa} \frac{\tilde{\Phi}_0}{2\pi} \left[ K_0(r) + \frac{\partial}{\partial \tilde{z}} (K_1(r)\cos\theta) \right] \tag{33}
\]

\[
\tilde{A}_{\tilde{z}}^{(1)} = -\tilde{\kappa} \frac{\tilde{\Phi}_0}{2\pi} \left[ \frac{\partial}{\partial \tilde{x}} (K_1(r)\cos\theta) \right] \tag{34}
\]

Indeed, substituting these equations into (26), we get

\[
B_{y}^{(1)} = \tilde{\kappa} \frac{\tilde{\Phi}_0}{2\pi} \left[ \left( \frac{\partial}{\partial \tilde{z}} K_0(r) \right) + \left( \frac{\partial^2}{\partial \tilde{z}^2} + \frac{\partial^2}{\partial \tilde{x}^2} \right) (K_1(r)\cos\theta) \right] \tag{35}
\]

However, by properties of the Bessel function, \( \left( \frac{\partial^2}{\partial \tilde{z}^2} + \frac{\partial^2}{\partial \tilde{x}^2} \right) (K_1(r)\cos\theta) = K_1(r)\cos\theta \). Further using \( K'_0 = -K_1 \) shows that indeed \( B_{y}^{(1)} = 0 \).
Now we proceed to the second order. Eq (26) and (27) read
\[
\begin{align*}
\frac{∂B_y^{(2)}}{∂z} - \tilde{A}_x^{(2)} &= \tilde{κ} \frac{∂}{∂z} \tilde{A}_x^{(1)} \\
\frac{∂B_y^{(2)}}{∂x} + \tilde{A}_z^{(2)} &= \tilde{κ} \frac{∂}{∂x} \tilde{A}_z^{(1)} + \tilde{κ}^2 \left( \tilde{A}_z^{(0)} + \frac{Φ_0}{2\pi} \frac{∂φ}{∂z} \right)
\end{align*}
\]  
(36)
(37)

We again claim that \( B_y^{(2)} = 0 \). If so, we get, using \( \tilde{A}_z^{(0)} + \frac{Φ_0}{2\pi} \frac{∂φ}{∂z} = \frac{Φ_0}{2\pi} K_1(r) \sinθ \), and eq (33) and (34),

\[
\begin{align*}
\tilde{A}_z^{(2)} &= -\tilde{κ}^2 \frac{Φ_0}{2\pi} \left[ \frac{∂K_0(r)}{∂z} + \frac{∂^2}{∂z^2} (K_1(r) \cosθ) \right] \\
\tilde{A}_x^{(2)} &= -\tilde{κ}^2 \frac{Φ_0}{2\pi} \left[ \frac{∂K_0(r)}{∂x} + \frac{∂^2}{∂x∂z} (K_1(r) \cosθ) + K_1(r) \sinθ \right]
\end{align*}
\]  
(38)
(39)

From eq (20) for \( B_y^{(2)} \), we get
\[
B_y^{(2)} = -\tilde{κ}^2 \frac{Φ_0}{2π} \left[ \tilde{κ}^2 K_0(r) + \frac{∂^2}{∂z^2} (K_1(r) \cosθ) + \frac{∂}{∂x} (K_1(r) \sinθ) \right]
\]  
(40)

Using again the properties of the modified Bessel functions, this reduces to
\[
B_y^{(2)} = -\tilde{κ}^2 \frac{Φ_0}{2π} \left[ K_0(r) + \frac{∂}{∂z} (K_1(r) \cosθ) + \frac{∂}{∂x} (K_1(r) \sinθ) \right]
\]  
(41)

The last two term combines to \( \frac{1}{r} \frac{d}{dr} (r K_1(r)) = -K_0 \) by recursion relation of modified Bessel function. Hence \( B_y^{(2)} = 0 \) as claimed.

Hence to second order in \( \tilde{κ} \), \( B_y \) has the same form as an ordinary anisotropic material:
\[
B_y = \frac{Φ_0}{2πλ_x λ_z} K_0 \left( \left[ (z/λ_z)^2 + (x/λ_z)^2 \right]^{1/2} \right)
\]  

The constant magnetic field contours are ellipses with center at the point where the order parameter has a singularity.

We would however like to add two cautionary remarks. Firstly, it is not true that other physical quantities such as \( J \) or \( M \) also have elliptic distributions around the vortex. For example, using eq (14) and the solution to \( A^{(0)} \), we find \( M_y^{(1)} = -\tilde{κ} \frac{Φ_0}{2π} K_1(r) \cosθ \). Therefore \( M_y(x, z) \) is odd under \( z \rightarrow -z \). Secondly, it is not true that the corrections to magnetic field vanish to higher orders. Proceeding to the third order, one can verify that \( B_y^{(3)} = 0 \) is inconsistent with \( A^{(2)} \) of eq (39) and (38). We, however would not proceed to calculate these very small corrections.

(B) We now consider a vortex with field along \( \hat{z} \). Using translational invariance along \( z \), one finds that there is no magnetic field induced along the \( x - y \) plane, and \( B_z \) and \( A \) are the same as those of an ordinary superconductor with in-plane penetration depth \( λ_x \) given in Sec
III before. However, due to the presence of $q$, there is an in-plane magnetization induced by the presence of the $\kappa$ term (see eq (3) and (4)). We find that this magnetization is along the radial direction: $M = M_r \hat{r}$ (thus its curl vanishes) where $M_r = \tilde{\kappa} \frac{2\Phi_0}{8\pi^2} K_1 \left( \frac{(x^2+y^2)^{1/2}}{\lambda_x} \right)$. The magnitude of this magnetization is therefore $\tilde{\kappa}/4\pi$ times the local magnetic field $B_z$ along the $z$ direction. It points radially outwards if $\kappa > 0$. For field along $-\hat{z}$, this magnetization changes sign and points radially $inwards$ if $\kappa > 0$.

V. ORDER OF MAGNITUDE

Finally we estimate $\tilde{\kappa}$, assuming the clean limit. The calculations in Ref\textsuperscript{2} can be easily generalized to the present 3D case once the Fermi surface and the spin-orbit splittings are given. We however would not do this calculation here but just satisfy ourselves with some estimates. It should be noted that, for the crystal symmetry $P4mm$ in question, the allowed spin-orbit interaction, besides one in the Rashba form $-\alpha \hat{z} \cdot (\vec{p} \times \sigma) = -\alpha(p_x\sigma_y - p_y\sigma_x)$, (here $\vec{p}$ is the momentum, $\sigma$ the Pauli matrix, and $\alpha$ a coefficient) can also have terms of the form $-\beta p_x p_y (p_x\sigma_x - p_y\sigma_y)$ and $-\gamma p_x p_y p_z (p_x^2 - p_y^2)\sigma_z$. Here $\alpha, \beta, \gamma$ can be functions of $\vec{p}$ but must be invariant under the symmetries of the crystal. (c.f.,\textsuperscript{13}). Both the first two terms can generate the terms proportional to $\kappa$ in eq (1)–(4). However, we expect that (c.f., eq (12) of ref\textsuperscript{2}) that $\kappa$ (at $T \rightarrow 0$) has an order of magnitude given by $\sim \mu_B(p_{F\pm}^2 - p_{F-}^2)/\hbar^3$ where $\mu_B$ is the Bohr magneton, $p_{F\pm}$ are the typical Fermi momenta for the spin-orbit splitted Fermi surfaces. Using the expression for $\lambda_x$ and $n_s \sim p_{Fz}^3$, we find that $\tilde{\kappa}$ is of order

\[ \tilde{\kappa} \sim \left( \frac{e^2}{\hbar c} \right) (p_{F\pm} R_B)^{1/2} \left( \frac{\delta}{\mu} \right) \]  

(42)

where $\delta$ is the typical splitting in energy by the spin-orbit interaction, $\mu$ the chemical potential, and $r_B$ the Bohr radius. For $\delta/\mu \sim 0.1$, this ratio is then of order $\sim 10^{-3}$ assuming typical electron densities.

VI. CONCLUSION

In conclusion, we have studied some magnetic properties for a superconductor with no inversion symmetry in its normal state. In particular we investigated how the broken symmetry and magneto-electric effects discussed in ref\textsuperscript{2,10} exhibit themselves in Meissner screening
and vortices. An unusual magnetization spatial pattern are found in some geometries. This magnetization can in principle be detected by Knight shift measurements.

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