Modelling a Decentralized Constraint Satisfaction Solver for Collision-Free Channel Access

Jaume Barcelo, Nuria Garcia, Azadeh Faridi, Member, IEEE, Simon Oechsner, and Boris Bellalta, Senior Member, IEEE

Abstract—In this paper, the problem of assigning channel slots to a number of contending stations is modeled as a Constraint Satisfaction Problem (CSP). A learning MAC protocol that uses deterministic backoffs after successful transmissions is used as a decentralized solver for the CSP. The convergence process of the solver is modeled by an absorbing Markov chain (MC), and analytical, closed-form expressions for its transition probabilities are derived. Using these, the expected number of steps required to reach a solution is found. The analysis is validated by means of simulations and the model is extended to account for the presence of channel errors. The results are applicable in various resource allocation scenarios in wireless networks.

Index Terms—Medium Access Control, decentralized constraint satisfaction solver, learning MAC protocol

I. INTRODUCTION

Since the inception of wireless local area networks (WLANs), random medium access mechanisms have played a key role in arbitrating access to shared channels. The core principles of the medium access control (MAC) that were introduced in the first release of the IEEE 802.11 standard are still valid today [1]. The contenders for the channels use carrier sense to avoid interrupting ongoing transmissions. Until recently, slotted time combined with a random backoff has been used to reduce the chances that two stations simultaneously start a transmission. However, it was pointed out recently that the random choice of the backoff value is not necessary after successful transmissions [2]. In fact, if all the nodes that have successfully transmitted choose a common deterministic backoff value for their next transmission, the chances of collisions are reduced, since in their next transmission they may only collide with the remaining unsuccessful nodes. Furthermore, under certain conditions, a collision-free operation can be reached and maintained.

The idea of using a deterministic backoff after successful transmissions has been explored in more detail in, e.g., [3]–[6]. The goal of this class of protocols is to distributively build a collision-free schedule which can then repeat periodically without further collisions, as long as the network does not change. This is equivalent to the decentralized assignment of stations to slots within one period of the schedule in such a way that no slot is assigned to more than one station. Such an assignment is obtainable if the number of contending stations does not exceed the number of slots in one period of the schedule. We are interested in forecasting the expected number of rounds required to reach a collision-free assignment.

Similar problems can be found in other areas of networking where limited resources need to be distributed among a group of stations. Examples of such resources include channel time slots [2], frequency channels [7], and code division multiple access scramble codes [8].

In [7], a general framework is presented that encompasses problems such as graph coloring, channel assignment to WLANs cells, the search for feasible inter-flow codes in network coding, and the construction of collision-free schedules in CSMA networks. This framework consists in modeling the resource-assignment problem as a Constraint Satisfaction Problem (CSP) [9]. When the nodes that participate in the CSP cannot communicate with one another to solve the problem, a decentralized approach is required. Decentralized solvers are different from distributed solvers [10], which require message exchange among the participating nodes. The concept of decentralized CSP solvers is very recent and is yet to be explored in depth. In [7], a decentralized solver for this CSP is presented and analyzed, and a bound on the convergence time of the solver is found.

In this work, we focus on analyzing a MAC protocol that uses deterministic backoffs after successful transmissions, however, our results are applicable for many other distributed resource-allocation problems. Similarly to [7], we model the channel access problem as a CSP for which the aforementioned protocol serves as a decentralized solver. To calculate the expected number of rounds the solver requires to reach a solution, we model the convergence process using an absorbing Markov chain (MC). The first contribution of this paper is the derivation of closed-form expressions for transition probabilities of the absorbing MC, which are then used to calculate the expected convergence time of the solver. The second contribution is the adaptation of the model to an environment in which errors can occur.

II. SYSTEM MODEL AND THE CORRESPONDING CSP

We consider a wireless network in which channel time is slotted. The slots are grouped in rounds, and each round contains $B$ consecutive slots. Our focus is on the distributed assignment of $N$ contending wireless stations to the $B$ channel time slots per round. In each round, each stations randomly selects one of the $B$ slots in the round. In each slot a single transmission can be completed and acknowledged. A station succeeds if the slot it has chosen is not selected by any other
The system is in state $S_d$ if exactly $d$ stations ($0 \leq d \leq N$) were successful in the previous round and, therefore, will deterministically choose their transmission slot in the current round. From the CSP perspective, this is equivalent to saying that there are exactly $d$ variables that were not involved in any constraint that was not satisfied in the previous round.

We are interested in calculating the expected number of rounds required to reach a solution. To this end, we construct a Markov chain to model the behavior of the protocol (or equivalently, the CSP solver).

By the pigeonhole principle, a solution exists only when $N \leq B$, i.e., when there are at least as many slots in a round as the total number of stations. Considering $N \leq B$ contending stations, the associated MC model has $N + 1$ different states, $S_0, \ldots, S_N$. The system is in state $S_d$ if exactly $d$ stations ($0 \leq d \leq N$) were successful in the previous round and, therefore, will deterministically choose their transmission slot in the current round. From the CSP perspective, this is equivalent to saying that there are exactly $d$ variables that were not involved in any constraint that was not satisfied in the previous round.

We are interested in the computation of the transition probability $P_{d,N}^{B,N}$, from one state $S_d$ to another state $S_{\delta}$, $0 \leq \delta \leq N$. In other words, $P_{d,\delta}^{B,N}$ is the probability of obtaining $\delta$ successful transmissions given $N$ stations and $B$ slots when $d$ of the stations use a deterministically chosen slot while the remaining $N - d$ stations transmit in a randomly chosen slot.

Note that the considered MC is an absorbing MC, as $p_{N,N}^{B,N} = 1$. This is because, once a collision-free schedule is found, the same collision-free schedule is repeated endlessly without collisions.

We are interested in the expected number of steps before convergence can be computed if the values of $P_{d,\delta}^{B,N}$ are known [13].

### A. Calculating the Transition Probabilities

To calculate the transition probabilities, we number the stations from 1 to $N$ and define $A_i$ to be the event that station $i$ succeeds, and the set $A = \{A_i\}_{i=1}^N$ to be the collection of all such events. These events are partially overlapping since more than one station may successfully transmit in the same round. For a given $d$, the transition probability $P_{d,\delta}^{B,N}$ is the probability that exactly $\delta$ out of the $N$ events in $A$ happen. As
mentioned before, when \( d = N \), the system is in the absorbing state \( S_N \), and \( P_{N,N,\delta}^{B,N} = 1 \), when \( \delta = N \), and is zero otherwise. When \( d < N \), this probability can be calculated applying a generalized version of the inclusion-exclusion principle (see, e.g., the theorem in Sec. IV.3 of [14]) as follows:

\[
P_{d,\delta}^{B,N} = \sum_{j=\delta}^{N} (-1)^{j-\delta} \binom{j}{\delta} S(j),
\]

where \( S(j) \) is given by

\[
S(j) = \sum_{\forall A^j \subseteq A} \Pr \left\{ \bigcap_{A_i \in A^j} A_i \right\},
\]

Using (2), (3), and (4), \( S(j) \) can be calculated as

\[
S(j) = \sum_{k=\max(0,j+d-N)}^{\min(d,j)} \binom{d}{k} \binom{N-d}{j-k} \times \frac{(B-d)!(B-j)^{N-d-(j-k)}}{(B-d-(j-k))! B^{N-d}}, \quad j < N
\]

and for \( j = N \),

\[
S(N) = \frac{(B-d)!}{(B-N)! B^{N-d}}.
\]

Finally, the transition probabilities for \( d < N \) can be calculated by replacing \( S(j) \) in (1). When \( d = 0 \), i.e., when all the \( N \) stations randomly select a slot, this result exactly matches the one obtained in [15].

### B. Calculating the Number of Steps until Absorption

To compute the expected number of rounds needed for the solver to reach a solution, we calculate the expected number of transitions that the MC takes to reach the absorbing state \( S_N \) (see, e.g., [13] for the theory behind this calculation).

Let \( P_{B,N}^{B,N} \) be the transition probability matrix of the MC. This matrix is a square matrix of size \( N+1 \). If we number the rows and columns of \( P_{B,N}^{B,N} \) starting with zero, the element in row \( d \) and column \( \delta \) is simply \( P_{d,\delta}^{B,N} \) as in (1).

In this matrix, rows \( 0 \) to \( N-1 \) represent transitions from the transient states and row \( N \) the transitions from the absorbing state. Therefore, \( P_{B,N}^{B,N} \) has the following general form:

\[
P_{B,N}^{B,N} = \begin{bmatrix}
TR & ABS \\
Q_{(N \times N)} & e_{(N \times 1)}
\end{bmatrix}
\]

where \( Q \) is a matrix containing the first \( N \) rows and columns of \( P_{B,N}^{B,N} \), from which we calculate the fundamental matrix of the absorbing MC as \( N^{-1} \), where \( I_{N \times N} \) is the \( N \times N \) identity matrix. The expected number of steps to absorption, if the system starts in state \( S_0 \), is the sum of all the elements in the first row of \( N \).

### C. The Markov Chain in the Presence of Channel Errors

So far we have not considered the possibility that the channel introduces errors. In presence of channel errors, a transmission may be unsuccessful even if it has not suffered a collision. In fact, after an unsuccessful transmission, a wireless station cannot know whether it has suffered a collision or a channel error, and the response of the protocol will be exactly the same, i.e., moving the station back to the random behavior.

In this case, the probability of moving from the state \( S_d \) to the state \( S_i \) is the probability that \( i \in [\delta, N] \) stations do not collide, but exactly \( i - \delta \) of those stations suffer a channel error, i.e.,

\[
P_{d,\delta}^{B,N,\epsilon} = \sum_{i=\delta}^{N} \binom{i}{\delta} \epsilon^{i-\delta} (1 - \epsilon)^{\delta} P_{d,i}^{B,N}.
\]

where \( \epsilon \) is the channel error probability. Note that the resulting MC is no longer absorbing.
To validate the expression in (8) we compute the average number of successful transmissions in each step from the MC and compare it with averages obtained from a simulation of 10,000 rounds. The results for a channel error probability $\epsilon = 0.1$, different numbers of slots ($B$) and different numbers of contenders ($N$) are presented in Fig. 3.

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V. CONCLUSION

We have studied a decentralized CSP solver to assign channel slots to contending stations. With this solver, the system eventually converges to collision-free operation under ideal channel conditions. We have modeled the convergence process as an absorbing MC and have derived closed expressions for the transition probabilities, which are used to compute the expected number of steps required for the system to converge to a solution. We have also considered the presence of channel errors and constructed an MC that accounts for channel errors, and have calculated its transition probabilities. The presented results have been validated by means of simulation. The results can be adapted to various scenarios in wireless networks where a finite number of resources need to be distributively assigned to a number of contending stations.

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