The Anapole Form Factor of the Nucleon

C.M. Maekawa and U. van Kolck

Kellogg Radiation Laboratory, 106-38
California Institute of Technology
Pasadena, CA 91125

Abstract

The anapole form factor of the nucleon is calculated in chiral perturbation theory in leading order. To this order, the form factor originates from the pion cloud, and is proportional to the non-derivative parity-violating pion-nucleon coupling. The momentum dependence of the form factor —and in particular, its radius— is completely determined by the pion mass.

1maekawa@krl.caltech.edu
2vankolck@krl.caltech.edu
There has been much recent interest in parity-violating electron scattering on the nucleon and light nuclei as a source of information about nucleon structure, in particular strangeness content. The SAMPLE collaboration has measured the longitudinal electron asymmetry for scattering on the proton \([1]\) and the deuteron at \(Q^2 = -q^2 = 0.1 \text{ GeV}^2\), where \(q\) is the momentum transferred to the target. Experiments at JLab (HAPPEX \([2]\) and G0 \([3]\)) extend these measurements to higher \(Q^2\).

A class of contributions to the measured electron asymmetry comes from the anapole form factor of the nucleon. The anapole is a parity-violating electromagnetic moment of a charged particle with spin, related at classical level to the magnetic field induced within a torus by a winding wire \([4]\). It vanishes for on-shell photons and thus cannot be separated from contact electron-particle operators. Nevertheless, in the case of hadrons at low energies, which can be formulated as an effective field theory (EFT), the anapole does constitute a class of contributions that is independent of the choice of photon (gauge), nucleon and pion fields, and therefore can be examined in itself. The anapole contribution to the asymmetry needs to be understood before one can draw definitive conclusions about the strangeness current.

Here we focus on the anapole form factor of the nucleon at \(Q \ll M_{QCD}\), where \(M_{QCD} \sim 1 \text{ GeV}\) is the typical mass scale in QCD. In this kinematic regime we can perform a systematic expansion of the form factor in powers of \(Q/M_{QCD}\) (times functions of \(Q/m_\pi\)). This expansion receives analytic contributions from short-range physics (rho mesons, etc.) and non-analytic contributions from the long-range pion cloud. The latter are calculable in a model-independent way in chiral perturbation theory (\(\chi\)PT). We discuss the relative order these contributions appear, and show that to lowest order only the pion cloud contributes. The anapole form factor at this order is then a calculable function of \((Q/m_\pi)^2\), completely determined by pion properties and pion-nucleon couplings in principle known from other processes.

The anapole form factor at \(Q = 0\) (the anapole moment) has been calculated before, together with subsets of sub-leading contributions \([5, 6]\), and depends on the leading, yet poorly-determined, parity-violating pion-nucleon coupling. However, it is not the anapole moment \(per\ se\) that is relevant for the above scattering experiments. Because it is generated by the pion cloud, the scale that governs the momentum dependence of the form factor is set by \(m_\pi\), and not \(M_{QCD}\). In the chiral limit, the form factor would vary wildly between \(Q = 0\) and \(Q = M_{QCD}\), but even in the real world it could be quite different (say, by a factor of a few) at \(Q \sim 300\ \text{ MeV}\) (SAMPLE experiment) than at \(Q = 0\). For the proper interpretation of the new experimental data one thus needs to investigate the momentum dependence of the anapole contribution.

We present here the leading-order results for the nucleon anapole form factor, which turns out to be purely isoscalar. We recover the known result for the anapole moment \([5, 6]\) and further predict its momentum dependence, for which we give an analytic expression. This momentum dependence is given by the known pion mass and is an example of a low-energy theorem. In particular, the radius of the anapole form factor is, as expected, \(\propto 1/m_\pi\) and blows up in the chiral limit. These predictions could in the future be tested. We will also show that dimensionless factors are such that the scale of
the $Q^2$ variation is $\sim 6m_\pi$, numerically (but not parametrically) close to the rho mass, $m_\rho$. As a consequence, the form factor at this order does not display much variation with momentum.

Here we denote by $iJ^\mu_{an}$ the parity-violating nucleon current that interacts with the electron current $-ie\bar{\sigma}\gamma^\mu e$ via the photon propagator $i\gamma^\mu = -i(\eta^\mu/q^2 + \ldots)$ to produce a contribution

$$i T = -ie\bar{\sigma}(k')\gamma^\mu e(k)\gamma^\mu (q)\bar{N}(p')J^\mu_{an}(q)N(p),$$

(1)
to the electron-nucleon $S$ matrix. We have $q^2 = (p - p')^2 \equiv -Q^2 < 0$.

We want to evaluate $J^\mu_{an}$ at $Q \ll M_{QCD}$. Because the nucleon mass is heavy, $m_N \sim M_{QCD}$, in this regime the nucleon is essentially non-relativistic, and is parametrized by a given velocity $v^\mu$ and spin $S^\mu$ ($S^\mu = (0, \hat{\sigma}/2)$ in the nucleon rest frame where $v^\mu = (1, 0)$).

As we are going to see, we can write

$$J^\mu_{an}(q) = \frac{2}{m_N^2} \left( a_0 F^{(0)}_A(-q^2) + a_1 F^{(1)}_A(-q^2) \tau_3 \right) (S^\mu q^2 - S \cdot q q^\mu),$$

(2)

where $F^{(i)}_A(0) = 1$ and $\tau_i$ is the $i$th Pauli matrix in isospin space. $a_0$ ($a_1$) is the isoscalar (isovector) anapole moment of the nucleon and $a_0 F^{(0)}_A(Q^2) (a_1 F^{(1)}_A(Q^2))$ the corresponding form factor.

At $Q \sim m_\pi$, the resolution of the virtual photon is enough to see pions, and pionic contributions have to be taken into account explicitly. Because the delta-nucleon mass difference is comparable to the pion mass, the delta isobar is also a relevant low-energy degree of freedom. All these contributions can be calculated in a model-independent way starting from the most general Lagrangian involving nucleons $N$, pions $\pi$ and deltas that transforms under the symmetries of QCD in the same way as QCD itself. This EFT has been described countless times in the literature; here we only highlight the features strictly relevant to our calculation, and refer the reader to a good review —such as Ref. [7]— for details.

Since the symmetries allow an infinite number of interactions, it is imperative to have an ordering scheme for the various contributions. Chiral symmetry plays a fundamental role here. Such a power counting argument is possible because all strong interactions bring in a small scale. Interactions that preserve chiral symmetry involve derivatives of the pion field, so they bring to amplitudes powers of the small momentum, or powers of the delta-nucleon mass difference; and interactions that break chiral symmetry involve the quark masses, so they bring powers of the pion mass. One can then order the strong interactions in the Lagrangian, $\mathcal{L} = \sum_\Delta \mathcal{L}^{(\Delta)}$, according to the chiral index $\Delta \equiv d + n/2 - 2$, where $d$ is the sum of the number of derivatives and powers of the pion mass and of the delta-nucleon mass difference, and $n$ is the number of fermion fields [8]. Electromagnetic interactions also break chiral symmetry; they do not necessarily contain quark masses, but they are proportional to the small charge $e$. It is convenient to account for factors of $e$ by enlarging the definition of $d$ accordingly. It can then be shown that in leading order in the strong and electromagnetic interactions, contributions come from

$$\mathcal{L}^{(0)}_{\text{str/em}} = \frac{1}{2} (D_\mu \pi)^2 - \frac{1}{2} m_\pi^2 \pi^2 + N i v \cdot D N - \frac{g_A}{f_\pi} N (\tau \cdot S \cdot D \pi) N + \ldots$$

(3)
Here $f_\pi = 93$ MeV is the pion decay constant, $D_\mu = (\partial_\mu - ieQA_\mu)$ with $Q^{(\pi)}_{ab} = -i\varepsilon_{3ab}$ and $Q^{(N)} = (1 + \tau_3)/2$, and "..." stand for interactions with more pion, nucleon and/or delta fields, that are not needed explicitly in the following. Note that at this order the nucleon is static and couples only to longitudinal photons. Kinetic corrections and magnetic couplings have relative size $O(Q/M_{QCD})$ and appear in $L^{(1)}_{\text{str/em}}$. The same is true for the delta isobar, including the nucleon-delta transition through coupling to a transverse photon. The parity-conserving pion-nucleon coupling is given by the axial-vector coupling of the nucleon; according to naive dimensional analysis $g_A = O(1)$, which is indeed observed, $g_A = 1.26$ (see, e.g., Ref. [4]). A term in $L^{(2)}_{\text{str/em}}$ provides an $O((m_\pi/M_{QCD})^2)$ correction that removes the so-called Goldberger-Treiman discrepancy.

Weak interactions in this EFT have been discussed in Ref. [6]. The four-fermion interactions between quarks generated by $W$ and $Z$ exchange break chiral symmetry with a strength given by the Fermi constant $G_F = 1.17 \cdot 10^{-5}$ GeV$^{-2}$. The effective operators it entails at low energies are proportional to the Fermi constant times the square of a mass scale. A natural scale is the pion decay constant, so we assume that these operators have coefficients of order of $G_F f_\pi \sim 10^{-7}$ times other natural scales. There is one pion-nucleon interaction with negative index,

$$L^{(-1)}_{\text{weak}} = -\frac{h_{\pi NN}}{\sqrt{2}} N (\tau \times \pi)_3 N + \ldots$$

(4)

Here $h_{\pi NN}$ is the parity-violating pion-nucleon coupling. Again, according to naive dimensional analysis we expect $h_{\pi NN} f_\pi = G_F f_\pi^2 M_{QCD}$, or $h_{\pi NN} = O(G_F f_\pi M_{QCD}) \sim 10^{-6}$. The value of $h_{\pi NN}$ is not well determined (for a review of constraints on parity-violating parameters, see, for example, Ref. [8] [9].

There are various weak interactions with indices 0, 1, and 2. The first pion-nucleon-delta coupling, for example, has to contain in the fermion rest frame the $4 \times 2$ transition matrix $\vec{S}_{tr}$; in other words, we cannot combine nucleon and delta in a spin zero object. Rotational invariance then demands the presence of a gradient to form a scalar, $\vec{S}_{tr} \cdot \vec{\nabla}$. Such interaction can appear only in $L^{(0)}_{\text{weak}}$, and its effects are suppressed by $O(Q/M_{QCD})$ relative to those stemming from the Lagrangian (4). The same reasoning applies to other operators. Particularly relevant among higher-order interactions is

$$L^{(2)}_{\text{weak}} = \frac{2}{m_N^2} \overline{N} (\tilde{a}_0 + \tilde{a}_1 \tau_3) S \mu N \partial_\nu F^{\mu\nu} + \ldots,$$

(5)

because $\tilde{a}_0$ ($\tilde{a}_1$) is a short-range contribution to the isoscalar (isovector) anapole moment. From dimensional analysis, $\tilde{a}_i/m_N^4 = O(eG_F f_\pi^2/m_N^4) = O(eG_F/(4\pi)^2)$. Direct short-range contributions to the momentum dependence of the anapole form factor first appear in $L^{(4)}_{\text{weak}}$, being further suppressed by $O((Q/M_{QCD})^2)$.

\footnote{There are clearly factors of $4\pi$ that might not be accounted for properly this way, but they will not affect the relative order of weak interactions.}

\footnote{At lowest order, our parameters relate to those of Ref. [10] (DDH) by $h_{\pi NN} g_A/f_\pi = f_{DDH}^2 g_{DDH}^{(\pi)}/m_N$.}
Figure 1: Diagrams contributing to the nucleon anapole form factor in leading order. Solid, dashed and wavy lines represent nucleon, pion, and (virtual) photon, respectively; circles and squares stand for interactions from $\mathcal{L}_{\text{str/em}}^{(0)}$ and $\mathcal{L}_{\text{weak}}^{(-1)}$, respectively. For simplicity only one of two possible orderings are shown here.

Let us now consider the sizes of specific contributions to $J_{\text{an}}^{\mu}(q)$. The first tree level contribution is given by the vertex generated by the weak Lagrangian (5). It has a size $O\left(\frac{eG_F Q^2}{(4\pi)^2}\right)$, and thus contributes $O\left(\frac{eG_F m_p^2}{(4\pi)^2}\right)$ to the anapole form factor (specifically, the anapole moment). The lowest-order one-loop graphs are built out of one vertex from the weak Lagrangian (4) and all other vertices from the strong Lagrangian (3). They are depicted in Fig. 1. Consider for example the graph in Fig. 1(a). The coupling of the photon to the pion contributes $O(eQ)$, the parity-violating pion-nucleon coupling $O(e G_F m_{\pi} Q_{\text{CD}})$, each pion propagator $O(1/Q^2)$, the nucleon propagator $O(1/Q)$, and the loop integration $O(Q^4/(4\pi)^2)$. We expect the whole graph then to be $O(e G_F M_{\text{QCD}} Q/(4\pi)^2)$, and contribute $O(e G_F M_{\text{QCD}} m_{\pi}^2 Q_{\text{CD}}/(4\pi)^2)$ to the anapole form factor. One can easily verify that Figs. 1(b),(c) give contributions of the same size. These long-range contributions are $O(M_{\text{QCD}}/Q)$ larger than the most important short-range contribution. The factor $1/Q$ represents an infrared enhancement. The anapole moment, which is a constant, is $O(e G_F Q_{\pi} M_{\text{QCD}}/m_{\pi}) \sim 5 \times 10^{-7} e$, and diverges in the chiral limit. The anapole form factor is $O(e G_F f_{\pi}^2 M_{\text{QCD}}/m_{\pi})$ times a function $F(Q^2/m_{\pi}^2) = O(1)$.

One can now show that all other contributions (including all delta effects) are of higher order. First, other purely short-range contributions start at $\mathcal{L}_{\text{weak}}^{(4)}$, and contribute at most $O(e G_F Q^2/(4\pi M_{\text{QCD}})^2)$ to the form factor. Second, other long-range contributions contribute at most $O(e G_F Q/(4\pi)^2 M_{\text{QCD}}))$. Consider other one-loop graphs: they all will have at least (i) one vertex from $\mathcal{L}_{\text{str/em}}^{(1)}$; or (ii) one vertex from $\mathcal{L}_{\text{weak}}^{(0)}$. Examples are, respectively, graphs where (i) the photon couples to a virtual nucleon or delta magnetically, or causes a nucleon-delta transition; and (ii) the parity-violating pion coupling is a nucleon-delta transition. Diagrams containing one insertion of either of these couplings are formally suppressed by $Q/M_{\text{QCD}}$ compared to the leading-order contributions obtained from the charge coupling present in the Lagrangian (3) and the parity-violating...
pion-nucleon vertex in (4): they first contribute in sub-leading order. A consequence is that, although the delta is treated on the same footing as the nucleon, its (virtual) contributions to the nucleon anapole are all subleading in power counting. Finally, even the most important two-loop graphs, made out of $L^{(0)}_{str/em}$ and $L^{(-1)}_{weak}$, should be suppressed by the usual $(Q/4\pi f_\pi)^2$ associated with loops in $\chi$PT.

The leading contribution to the anapole form factor comes thus from the graphs in Fig. 1. They are evaluated with $v \cdot q = 0$, as a consequence of the static nature of the nucleon at this order. In particular, Fig. 1(c) vanishes. It is straightforward to calculate the other diagrams as well.

The other two sets of graphs combine to give a result in the form of Eq. (2). We find a purely isoscalar result,

$$a_0 = \frac{egAh_{\pi NN}m_N^2}{48\sqrt{2\pi}m_\pi f_\pi} \quad (6)$$

$$a_1 = 0. \quad (7)$$

This result for the anapole moment agrees with Ref. [6]. Ref. [3] contains a partial set of sub-leading contributions because it pre-dates the heavy-fermion expansion employed here and in Ref. [3]. Note that Eq. (3) is $\sim \sqrt{2}/3$ larger than our naive estimate. Eqs. (6, 7) are predictions of $\chi$PT, but unfortunately the value of $h_{\pi NN}$ is currently not well determined.

Because the form factor is given by lowest-order loop graphs, it depends on the combination $Q^2/m_\pi^2$ only. Because all $Q$ dependence comes from Fig. 1(a), we find that it actually is a function of $Q^2/(2m_\pi)^2$:

$$F_A^{(0)}(Q^2) = \frac{3}{2} \left\{ -\left(\frac{2m_\pi}{\sqrt{Q^2}}\right)^2 + \left(\frac{2m_\pi}{\sqrt{Q^2}}\right)^2 + 1 \right\} \frac{2m_\pi}{\sqrt{Q^2}} \arctan \frac{\sqrt{Q^2}}{2m_\pi} \quad (8)$$

This is also a testable prediction of $\chi$PT: it is plotted as function of $Q$ in Fig. 2.

The variation of the form factor with $Q$ is somewhat smaller than expected. This can be seen by looking at the anapole square radius (defined in analogy to the charge square radius):

$$\langle r_{an}^2 \rangle = -6\left(\frac{dF_A^{(0)}}{dQ^2}\right)_{Q^2=0} = \frac{3}{10m_\pi^2}. \quad (9)$$

We see the square radius is smaller than expected by a factor 5. At $Q \ll m_\pi$ we can write

$$F_A^{(0)}(Q^2) = 1 - \frac{1}{5}\left(\frac{Q^2}{(2m_\pi)^2}\right) + O\left(\frac{Q^4}{(2m_\pi)^4}\right). \quad (10)$$

This radius approximation to the form factor is also displayed in Fig. 2.

The accidentally small square radius makes the mass scale that governs the $Q$ dependence near $Q = 0$ larger than $2m_\pi$. Data on the proton Sachs form factors are well fitted

\[5\] The Lagrangian of Ref. [6] can be obtained from ours by a redefinition of the sign of the pion field.
Anapole Form Factor

Figure 2: The isoscalar anapole form factor $F_A^{(0)}$ as function of $Q$: $\chi$PT in leading order, Eq. (8) (solid line); quadratic approximation, Eq. (10) (dotted line); and dipole approximation, Eq. (11) (dashed line).

by a dipole profile with a mass scale close to $m_\rho$. A dipole form factor with the correct anapole square radius is

$$F_A^{(0)}(Q^2) = \frac{1}{(1 + (Q/M)^2)^2} + O\left(\frac{Q^4}{(2m_\pi)^4}\right), \quad (11)$$

with the mass scale $M = 2\sqrt{10}m_\pi = 880$ MeV, instead of $2m_\pi$. $M$ is numerically close to $m_\rho$, but this is clearly a coincidence that would be destroyed if the quark masses where much smaller than what they are. In fact, short-range contributions such as that from the rho appear only in the counterterms at higher orders, so $M$ has no obvious connection to $m_\rho$. The dipole approximation is also shown in Fig. 2. It improves over the radius approximation but clearly Eq. (8) is softer than a dipole.

In conclusion, we have shown results for the anapole form factor of the nucleon in leading order in $\chi$PT. We have not yet examined extensively the form factor in next
order. There undetermined short-range isoscalar and isovector parameters appear, and
the anapole moment can no longer be predicted in a model-independent way. However,
the momentum dependence continues to be determined by the pion cloud— with nucleons
and deltas in internal lines—and an improved low-energy theorem can be derived. Under
the assumption that higher-order results are afflicted by the same dimensionless factors
seen above, the error in Fig. 2 at momentum $Q$ would be $\sim Q/m_\rho$. Within such an
error, the approximation of using the anapole moment instead of the full form factor
would be good to about 15% in the SAMPLE experiment, but larger changes appear
at momenta relevant to the JLab experiments. On the other hand, if in next order the
dimensionless factors that affect the $Q^2$ variation turn out closer to 1 than to 1/5, the
momentum dependence from sub-leading order could be (accidentally) comparable to the
leading order considered above. Only an explicit calculation can verify if the contributions
that are formally subleading are indeed numerically small. We are currently investigating
these issues [11].

Acknowledgements
We thank Bob McKeown and the group at the Kellogg Lab for getting us interested in
this problem and for comments on this work, Paulo Bedaque for a useful suggestion, and
Wick Haxton and Barry Holstein for pointing out the unusual convention for the sign of
$\gamma_5$ in DDH. CMM acknowledges a fellowship from FAPESP (Brazil), grant 99/00080-5.
This research was supported in part by the NSF grant PHY 94-20470.

Note added
After this work was completed, it was pointed out to us that the nucleon form factor in
leading order had previously been calculated in [12].

References
[1] D.T. Spayde et al (SAMPLE Collaboration), nucl-ex/9909010; B. Mueller et al
(SAMPLE Collaboration), Phys. Rev. Lett. 78 (1997) 3824.
[2] K.A. Aniol et al (HAPPEX Collaboration), Phys. Rev. Lett. 82 (1999) 1096.
[3] G0 Collaboration, www.npl.uiuc.edu/exp/G0/G0Main.html
[4] Ya.B. Zel’ dovich, Sov. Phys. JETP 6 (1958) 1184; Sov. Phys. JETP 12 (1961) 777.
[5] M.J. Musolf and B.R. Holstein, Phys. Rev. D43 (1991) 2956; W.C. Haxton, E.M.
Henley, and M.J. Musolf, Phys. Rev. Lett. 63 (1989) 949.
[6] D.B. Kaplan and M.J. Savage, Nucl. Phys. A556 (1993) 653.
[7] V. Bernard, N. Kaiser, U.-G Meißner, Int. J. Mod. Phys. E4 (1995) 193.
[8] S. Weinberg, *Phys. Lett.* **B251** (1990) 288; *Nucl. Phys.* **B363** (1991) 3.

[9] W. Haeberli and B.R. Holstein, in *Symmetries and Fundamental Interactions in Nuclei*, W.C. Haxton and E.M. Henley (eds.), World Scientific, Singapore (1995); W.T.H. van Oers, hep-ph/9910328.

[10] B. Desplanques, J.F. Donoghue, and B.R. Holstein, *Ann. Phys.* **124** (1980) 449.

[11] C.M. Maekawa, J.S. da Veiga, and U. van Kolck, in progress.

[12] M. J. Savage and R. P. Springer, nucl-th/9907069.