Nonequilibrium dynamics: preheating in the SU(2) Higgs model

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Abstract

The term ‘preheating’ has been introduced recently to denote the process in which energy is transferred from a classical inflaton field into fluctuating field (particle) degrees of freedom without generating yet a real thermal ensemble. The models considered up to now include, besides the inflaton field, scalar or fermionic fluctuations. On the other hand the typical ingredient of an inflationary scenario is a nonabelian spontaneously broken gauge theory. So the formalism should also be developed to include gauge field fluctuations excited by the inflaton or Higgs field. We have chosen here, as the simplest nonabelian example, the SU(2) Higgs model. We consider the model at temperature zero. From the technical point of view we generalize an analytical and numerical renormalized formalism developed by us recently to coupled channel systems. We use the ’t Hooft-Feynman gauge and dimensional regularization. We present some numerical results but reserve a more exhaustive discussion of solutions within the parameter space of two couplings and the initial value of the Higgs field to a future publication.

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1 Introduction

Nonequilibrium dynamics in quantum field theory within the closed time path (CTP) formalism has become recently a fastly developing area of research. Pioneering work by [1] has been followed by applications to inflationary cosmology [2, 3, 4] and to the hadronic phase transition, especially the possibility of formation of chiral condensates [5, 6]. With the increase of the experimental lower bound of the Higgs mass the electroweak phase transition may be second order and could then become a realistic field of application as well.

The typical numerical computations - or ‘experiments’ - in this new field have included up to now, in addition to the ‘Higgs’ of ‘inflaton’ field, scalar and fermionic fluctuations. The cosmological application has been prepared [7, 8, 9] by considering the nonequilibrium time development in a constantly curved space. Besides Friedmann - Robertson - Walker cosmology another typical ingredient of inflationary scenarios or of the cosmological electroweak phase transition are nonabelian spontaneously broken gauge theories. So the formalism should also be developed to include gauge field fluctuations. It is the aim of our present work to describe the analytical and computational tools for such applications.

The analytical part includes the formulation of the theory and renormalization. As a convenient gauge, used extensively in perturbative and nonperturbative calculations in the electroweak theory, we have chosen the ‘t Hooft-Feynman background gauge. We will not be able to discuss gauge invariance, especially since already the Ansatz for the classical Higgs field implies a choice of gauge. In the - closely related - formulation of the effective potential it has been proposed recently [10] to use the absolute value squared of the Higgs field as a gauge invariant order parameter, a choice that merits consideration also in the present context.

The renormalization conditions were chosen such the tree effective potential remain unchanged around the minimum corresponding to the broken Higgs phase. The renormalization has been based on dimensional regularization.

The numerical computation and the analytical one are both contained in a common scheme that we have proposed recently [11] for such nonequilibrium processes. The main characteristics of the method are: (i) a clean separation of the divergent and finite parts of the fluctuation integrals in close relation to CTP perturbation theory; (ii) analytic computation of the leading order
contributions using standard covariant regularization schemes; (iii) numerical computation of the finite parts avoiding small differences of large numbers - the leading orders are not subtracted from the integrand but omitted from the outset. A fourth property has been mentioned in but not yet used: the fact that the method can be extended easily to coupled channel systems. This application of the method will be demonstrated in this paper within the context of the SU(2) Higgs model.

The plan of this work is as follows: in section we recall the basic definitions and relations; in section we present the one-loop nonlinear relaxation equations; we prepare the regularization in section by expanding the fluctuation modes in orders of the vertex function governed by the classical field and by deriving the large momentum behaviour of the first terms; regularization is then straightforward, the renormalization requires some algebra, both are presented in section; the numerical computation is discussed in section; we conclude in section with a discussion of the numerical results and an outlook to more realistic and more general applications of the method.

2 Basic relations

The SU(2) Higgs model is defined by the Lagrangean density

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi) . \]  

(1)

Here \( \Phi \) denotes a complex Higgs doublet. The covariant derivative is defined as

\[ D_\mu = \partial_\mu - i \frac{g}{2} A_\mu^a \tau^a \]  

(2)

and the corresponding field strength tensor is given by

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c . \]  

(3)

We write the Higgs potential in the form

\[ V(\Phi^\dagger \Phi) = \frac{1}{2} \mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 . \]  

(4)

In the SU(2) Higgs model \( \mu^2 = -2m_h^2 \) where \( m_h \) is the mass of the Higgs field. In the case of unbroken symmetry - not considered here - the mass term would be defined by \( \mu^2 = m_s^2 \), where \( m_s \) is the mass of the complex scalar
doublet. We denote the "classical" or "inflaton" field as \( H_0 \); it is supposed to be constant in space and depends only on the time \( t \). We will treat here the fluctuations in one-loop order, generalizations to the \( 1/N_c \) expansion and to the Hartree approximation are straightforward; they are discussed e.g. in [12, 13]. We therefore decompose the Higgs field into the inflaton and the fluctuation parts as

\[
\Phi(x) = (H_0(t) + h(x) + i\tau^a \varphi^a(x)) \begin{pmatrix} 0 \\ 1 \end{pmatrix} .
\]

Here \( h(x) \) is the isoscalar Higgs field fluctuation, the isovector fluctuations \( \varphi^a(x) \), the "would-be Goldstone fields" will couple to the gauge fields. Since there is no classical gauge field, the gauge field reduces to

\[
F_{\mu\nu}^a = \partial_\mu a_\nu^a - \partial_\nu a_\mu^a + g\epsilon^{abc} a_\mu^b a_\nu^c
\]

in terms of the fluctuations \( a_\mu^a(x) \). Furthermore we have to introduce a gauge fixing and Faddeev-Popov Lagrangean. It is convenient to use the ’t Hooft-Feynman gauge with the gauge condition

\[
F^a(a_\mu, \varphi) = \partial_\mu a^{aa} + \frac{1}{2} g(H_0 + h) \varphi^a ,
\]

and the gauge fixing term

\[
\mathcal{L}_{gf} = -\frac{1}{2} F^a F^a .
\]

The corresponding Faddeev-Popov Lagrangean reads

\[
\mathcal{L}_{FP} = \eta^{1a} \left( -\Box - \frac{g^2}{4}(H_0 + h)^2 \right) \eta^a + \eta^{1a} \left( g \epsilon^{abc} a^{\mu c} \partial_\mu + g \epsilon^{abc} \partial_\mu a^{\mu c} + \frac{g^2}{4} \varphi^b \varphi^a + \frac{g^2}{4} \epsilon^{abc} (H_0 + h) \varphi^c \right) \eta^a.
\]

The complete Lagrangean is then given by

\[
\mathcal{L}_{tot} = \mathcal{L} + \mathcal{L}_{gf} + \mathcal{L}_{FP} = \mathcal{L}_0 + \mathcal{L}_1 .
\]

The propagators and vertices can be read off from the free Lagrangean

\[
\mathcal{L}_0 = -\frac{1}{2} \partial_\mu a_\nu^a \partial^\mu a^{\nu a} + \frac{g^2}{8} v^2 a_\mu^a a^{\mu a}
\]
\[
\begin{align*}
+ \frac{1}{2} \partial_0 H_0 \partial^0 H_0 - \frac{\lambda}{4} (H_0^2 - v^2)^2 \\
+ \frac{1}{2} \partial_\mu h \partial^\mu h - \lambda v^2 h^2 \\
+ \frac{1}{2} \partial_\mu \varphi^a \partial^\mu \varphi^a - \frac{g^2}{8} v^2 \varphi^a \varphi^a \\
+ \partial_\mu \eta^{ta} \partial^\mu \eta^a - \frac{g^2}{4} v^2 \eta^{ta} \eta^a 
\end{align*}
\] (11)

and the interaction Lagrangean

\[
\mathcal{L}_1 = -g \epsilon^{abc} \partial_\mu a^b \partial^\mu a^c - \frac{g^2}{4} \epsilon^{abc} \partial_\mu a^b \partial^\mu a^c a^d \partial^\mu a^e \\
+ \partial_0 H_0 \partial^0 h + g(\partial_0 H_0) a^0 \varphi^a + g(\partial_\mu h) a^\mu \varphi^a + \frac{g^2}{2} \epsilon^{abc}(\partial_\mu \varphi^a) a^\mu \varphi^c \\
+ \frac{g^2}{8} (H_0^2 - v^2) a^\mu a^\mu + \frac{g^2}{4} H_0 h a^\mu a^\mu + \frac{g^2}{8} h^2 a^\mu a^\mu + \frac{g^2}{8} a^\mu a^\mu \varphi^a \varphi^b \\
- \lambda h H_0^3 - \frac{3}{2} \lambda h^2 (H_0^2 - v^2) - \lambda v^2 H_0 \\
- \lambda h H_0 \varphi^a \varphi^a - \frac{\lambda}{4} h^4 - \frac{\lambda}{2} h^2 \varphi^a \varphi^a - \frac{\lambda}{4} v^2 \varphi^a \varphi^b \varphi^b + \lambda v^2 h H_0 \\
- \frac{g^2}{8} (H_0^2 - v^2) \varphi^a \varphi^a - \frac{g^2}{8} h^2 \varphi^a \varphi^a - \frac{g^2}{4} H_0 h \varphi^a \varphi^a \\
- \frac{g^2}{2} H_0 h \eta^{ta} \eta^a - \frac{g^2}{4} h^2 \eta^{ta} \eta^a - \frac{g^2}{4} \eta^{ta}(H_0^2 - v^2) \eta^a \\
+ \epsilon^{abc} \left( g \eta^{ta} \eta^b \partial_\mu a^{\mu c} + g \eta^{ta}(\partial_\mu \eta^b) a^{\mu c} + \frac{g^2}{4} (H_0 + h) \eta^{ta} \eta^b \varphi^c \right) \\
+ \frac{g^2}{4} \varphi^a \varphi^b \eta^{ta} \eta^b .
\] (13)

3 Equations of motion

The formalism of nonequilibrium dynamics in quantum field theory and the use of the tadpole method [14] have been presented or reviewed recently by various authors [15]. We give here just the one-loop equations of motion which are obtained from this formalism.
The basic graph from which the equation of motion of the inflaton field is derived is depicted in Fig. 1. The propagators are here the exact propagators in the inflaton background field. In contrast to the free propagators some of them involve coupled channels. We adapt the notation to this general case by using for the Green functions the notation $G_{jl}^{++}$. They will be defined below. The lower latin subscripts correspond to the different fluctuating fields $h, \varphi, a^0, a^i$ and $\eta$ introduced in the previous section. For all one loop integrals the contributions of the space components of the gauge fields and the Faddeev-Popov fields will combine since they involve the same propagators (Green functions). We will therefore introduce the set of subscripts $h, \perp, a$ and $\varphi$ for the isoscalar component of the Higgs field, the ‘transverse’ components of the gauge fields (i.e.the combination of their space components and the Faddev-Popov fields), the time components of the gauge fields and the isoscalar components of the Higgs field, respectively.

The vertices are realized by a matrix of vertex operators $Q_{jl}(t)$ which has the following nonvanishing components

\begin{align*}
Q_{hh}(t) &= 3\lambda H_0(t) \\
Q_{\perp\perp}(t) &= \frac{3}{4}g^2 H_0(t) \\
Q_{aa}(t) &= -\frac{3}{4}g^2 H_0(t) \\
Q_{\varphi\varphi}(t) &= 3(\lambda + \frac{g^2}{4}) H_0(t) \\
Q_{\varphi a}(t) &= Q_{a\varphi}(t) = -\frac{3g}{2} \frac{d}{dt}.
\end{align*}

With these notations the differential equation for the inflaton field reads

$$\ddot{H}_0(t) + \lambda(H_0^2(t) - v^2)H_0(t) - i \sum_{jl} Q_{jl}(t) G_{jl}^{++}(t, t) = 0.$$  \hspace{1cm} (15)

The propagators $G_{jl}^{++}$ are the usual time ordered Green functions. We have omitted the spatial variables $\vec{x}$ and $\vec{x}'$ since the Green functions are taken at $\vec{x} = \vec{x}'$ and, due to translation invariance, are then independent of $\vec{x}$. These Green functions at $\vec{x} = \vec{x}'$ can be written in terms of Fourier components as

$$G_{jl}^{++}(t, t') = \int \frac{d^3k}{(2\pi)^3} G_{jl}^{++}(\vec{k}, t, t').$$  \hspace{1cm} (16)
The Green functions $G_{hh}^{++}(\vec{k}, t, t')$ for momentum $\vec{k}$ are obtained in the usual way from the mode functions for the various fluctuations in the time dependent background field $H_0(t)$. We discuss briefly the different channels.

For the isoscalar part of the Higgs field $G_{hh}^{++}(\vec{k}, t, t')$ is expressed as

$$G_{hh}^{++}(\vec{k}, t, t') = \frac{i}{2\omega_{h0}(\vec{k})} U_h(\vec{k}, t) U^*_h(\vec{k}, t') \theta(t - t')$$

$$+ \frac{i}{2\omega_{h0}(\vec{k})} U_h(\vec{k}, t') U^*_h(\vec{k}, t) \theta(t' - t),$$

(17)

where the mode functions $U_h(\vec{k}, t)$ satisfy the differential equation

$$\left[ \frac{d^2}{dt^2} + \omega^2_h(\vec{k}, t) \right] U_h(\vec{k}, t) = 0.$$  

(18)

The mode frequency $\omega_h(\vec{k}, t)$ is defined by

$$\omega^2_h(\vec{k}, t) = \vec{k}^2 + m^2_h + 3\lambda(H_0(t) - v^2)$$

(19)

and the ‘frequency at $t=0$’ is given by

$$\omega_{h0}(\vec{k}) = \omega_h(\vec{k}, 0).$$

(20)

The initial conditions are

$$U_h(\vec{k}, 0) = 1 \quad \dot{U}_h(\vec{k}, 0) = -i\omega_{h0}(\vec{k})$$

$$U^*_h(\vec{k}, 0) = 1 \quad \dot{U}^*_h(\vec{k}, 0) = i\omega_{h0}(\vec{k}).$$

(21)

For $t = t'$ we get

$$G_{hh}^{++}(\vec{k}, t, t') = \frac{i}{2\omega_{h0}(\vec{k})} |U_h(\vec{k}, t)|^2.$$

(22)

For the transversal components of the gauge fields the mode functions satisfy

$$\left[ \frac{d^2}{dt^2} + \omega^2_\perp(\vec{k}, t) \right] U_\perp(\vec{k}, t) = 0$$

(23)

where

$$\omega^2_\perp(\vec{k}, t) = \vec{k}^2 + m^2_W + \frac{g^2}{4}(H_0^2(t) - v^2).$$

(24)
The Green function is then given by
\[ G_{++}(\vec{k}, t, t) = \frac{i}{2\omega_{\perp}(\vec{k})} |U_{\perp}(\vec{k}, t)|^2 \] (25)
where again \( \omega_{\perp} \) is defined as
\[ \omega_{\perp}(\vec{k}) = \omega_{\perp}(\vec{k}, 0). \] (26)

The Green functions for the time component \( a_0 \) of the gauge field and of the isovector part of the Higgs field \( \phi \) satisfy the coupled system of differential equations
\[ \left[ \left( \frac{d^2}{dt^2} + \vec{k}^2 + m^2_W \right) + W(t) \right] G_{jn}^{++}(\vec{k}, t, t') = \delta(t - t')\delta_{mn}. \] (27)
Here we have introduced the metric \( g = \text{diag}\{-1, 1\} \) taking into account the minus sign of the kinetic term and of the propagator of \( a_0 \) in the Feynman gauge and where the matrix \( W(t) \) is defined as
\[ W(t) = \begin{pmatrix} -\frac{g^2}{4}(H_0^2(t) - v^2) & \frac{g\partial_0 H_0(t)}{g\partial_0 H_0(t)} \\ \frac{g\partial_0 H_0(t)}{g\partial_0 H_0(t)} & (\lambda + \frac{g^2}{4})(H_0^2(t) - v^2) \end{pmatrix}. \] (28)

For the Green functions of this system we make the Ansatz [16, 17]
\[ G_{jn}^{++}(\vec{k}, t, t') = U_{\alpha j}^{\alpha}(\vec{k}, t)c_{\alpha \beta}(\vec{k})U_{\beta n}^{\beta}(\vec{k}, t')\Theta(t - t') + U_{\alpha j}^{\alpha*}(\vec{k}, t)c_{\beta \alpha}(\vec{k})U_{\beta n}^{\beta}(\vec{k}, t')\Theta(t' - t). \] (29)

Here the mode equations
\[ \left[ \left( \frac{d^2}{dt^2} + \vec{k}^2 + m^2_W \right) + W(t) \right] U_{\alpha j}^{\alpha}(\vec{k}, t) = 0 \] (30)
have a fundamental system of two independent solutions labelled by the superscript \( \alpha \) which takes again the values \( a \) and \( \varphi \). We choose as a basis the two independent solutions characterized by the initial conditions
\[ U_{\alpha j}^{\alpha}(\vec{k}, 0) = \delta_{\alpha j}^{\alpha}, \]
\[ U_{\alpha j}^{\alpha*}(\vec{k}, 0) = -i\delta_{\alpha j}^{\alpha} \omega_{\alpha j}(\vec{k}). \] (31)
where
\[ \omega_{j0}(\vec{k}) = \left[ \vec{k}^2 + m_{j0}^2 \right]^{\frac{1}{2}} \] (32)
with
\[ m_{a0}^2 = m_{W0}^2 = m_{W}^2 + \frac{g^2}{4}(H_0^2(0) - v^2) \] (33)
\[ m_{\varphi0}^2 = m_{W}^2 + (\lambda + \frac{g^2}{4})(H_0^2(0) - v^2). \] (34)

By a simple extension of the derivation given in [17] one can show that the coefficients \( c_{\alpha\beta}(\vec{k}) \) are then related to the Wronskians
\[ W(U^\alpha, U^{\beta*}, \vec{k}) = g_{mn} \left( U_m^\alpha(\vec{k}, t)U_n^{\beta*}(\vec{k}, t) - U_m^\beta(\vec{k}, t)U_n^{\alpha*}(\vec{k}, t) \right) \] (35)
via
\[ c_{\alpha\beta}(\vec{k})W(U^\alpha, U^{\gamma*}, \vec{k}) = -\delta_\beta^\gamma. \] (36)

The Wronskians can be computed from the initial conditions for the \( U^\alpha \); we obtain for the coefficients \( c_{\alpha\beta}(\vec{k}) \)
\[ c_{\alpha\beta}(\vec{k}) = g_{\alpha\beta}/(2\omega_{\alpha0}(\vec{k})). \] (37)

We use the notation \( U^\alpha_j \) for the single channel fields \( h \) and \( \perp \) as well, implying that \( U^h_j(\vec{k}, t) \equiv U^h_j(\vec{k}, t) \) and \( U^\perp_j(\vec{k}, t) \equiv U^\perp_j(\vec{k}, t) \) and extend the metric to these components via \( g = \text{diag}\{1, 1, -1, 1\} \) corresponding to the ordering \( h, \perp, a, \varphi \). Collecting the various expressions for the Green functions we define the (yet unrenormalized) fluctuation integral
\[ \mathcal{F}(t) = \sum_{j\alpha} Q_{j\alpha}(t) \int \frac{d^3k}{(2\pi)^3} g_{\alpha\beta} \frac{U^\alpha_j(\vec{k}, t)U^{\beta*}_j(\vec{k}, t) - U^\beta_j(\vec{k}, t)U^{\alpha*}_j(\vec{k}, t)}{2\omega_{\alpha0}(\vec{k})}. \] (38)

The equation of motion for \( H_0(t) \) is then
\[ \ddot{H}_0(t) + \lambda(H_0^2(t) - v^2)H_0(t) + \mathcal{F}(t) = 0. \] (39)

In order to obtain the energy we first obtain the Hamiltonian from the Lagrangean (31), insert the field expansion in terms of the mode functions and
the corresponding annihilation and creation operators and take the expectation value in the initial state. For the \( a, \varphi \) subsystem we expand the fields as

\[
\{ a(t, \vec{x}) \varphi(t, \vec{x}) \} = \int \frac{d^3k}{(2\pi)^3} \sum_\alpha \frac{1}{2\omega_\alpha(\vec{k})} \left[ c_\alpha(\vec{k}) \left\{ \begin{array}{c} U^\alpha_a(t) \\ U^\alpha_\varphi(t) \end{array} \right\} e^{i\vec{k}\vec{x}} + c_\alpha^\dagger(\vec{k}) \left\{ \begin{array}{c} U^{\alpha^*}_a(t) \\ U^{\alpha^*}_\varphi(t) \end{array} \right\} e^{-i\vec{k}\vec{x}} \right].
\]

(40)

With our initial conditions (31) for the mode functions \( c_a, c_\varphi^\dagger \) are at \( t = 0 \) the annihilation and creation operators of the field \( a^0 \) and \( c_\varphi, c_\varphi^\dagger \) those of the field \( \varphi \). So averaging in the initial vacuum state of the system we have in the Feynman gauge

\[
\langle c_\alpha(\vec{k}) c_\alpha^\dagger(\vec{k}') \rangle_0 = g^{\alpha\beta}(2\pi)^3 2\omega_\alpha(\vec{k}) \delta^3(\vec{k} - \vec{k}').
\]

Therefore, the metric enters twice: once in order to take into account the sign of the kinetic terms of the field components (latin subscripts) in the Hamiltonian and a second time due to the averaging of the field operators (greek indices) in the initial state. The unrenormalized total energy of the system consisting of the inflaton field and the fluctuations is therefore given by

\[
E = \frac{1}{2} \dot{H}_0^2(t) - \frac{1}{4} m^2 H_0^2(t) + \frac{\lambda}{4} H_0^4(t)
\]

(41)

\[
+ \int \frac{d^3k}{(2\pi)^3} \sum_{j,\alpha} g_{jj} g_{\alpha\alpha} \frac{d_j}{2\omega_\alpha(\vec{k})} \left( \frac{1}{2} |U^\alpha_j(\vec{k}, t)|^2 + \frac{\omega_j^2(\vec{k}, t) |U^\alpha_j(\vec{k}, t)|^2}{2} \right)
\]

where the summation is over all fields \( h, \perp, a, \varphi \). \( d_j \) is the isospin degeneracy, i.e. \( d_j = 3 \) for \( j = \perp, a, \varphi \) and \( d_j = 1 \) for \( j = h \). The frequency \( \omega_\alpha(t) \) is defined by

\[
\omega_\alpha^2(t) = \vec{k}^2 + m_\alpha^2(t).
\]

(42)

Using the equations of motion for \( H_0 \) and the mode functions it can be checked easily that the energy is conserved. In both the fluctuation integral and the energy we have already taken into account the cancellation between the transversal gauge modes and the Faddeev-Popov ghosts so that the latter ones do not appear any more.

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4The vertex operators are defined such to include such degeneracy factors. The vertex operators \( Q_{jl} \) and the potentials \( V_{jl} \) are obviously related by a functional derivative via

\[
d_j \delta [\int dt' V_{jl}(t')] / \delta H_0(t) = Q_{jl}(t).
\]
4 Perturbative expansion

In order to prepare the renormalized version of the equations given in the previous section we introduce a suitable expansion of the mode functions $U_\alpha^j(t)$. For a single channel this expansion has been presented in [11]. We extend here the discussion to coupled channel systems. Adding the term $(W_{mj}(t) - W_{mj}(0))U_\alpha^j(t)$ on both sides of the mode equations (30) they take the form

$$g_{jn} \left[ \frac{d^2}{dt^2} + \omega_{n0}^2(\vec{k}) \right] U_\alpha^j(\vec{k}, t) = -V_{jl}(t)U_\alpha^l(\vec{k}, t)$$

(43)

with

$$V(t) = \mathbf{W}(t) - \mathbf{W}(0)$$

(44)

for the coupled system and

$$V_{hh}(t) = 3\lambda(H_0^2(t) - H_0^2(0))$$

(45)

$$V_{\perp\perp}(t) = \frac{g^2}{4}(H_0^2(t) - H_0^2(0))$$

(46)

for the single component systems. We assume that $dH_0(t)/dt$ to vanish at $t = 0$. Including the initial conditions [31] the mode functions satisfy the equivalent integral equation

$$U_\alpha^j(\vec{k}, t) = \delta_\alpha^j e^{-i\omega_{j0}t} + \int_0^\infty dt' \Delta_{j,\text{ret}}(\vec{k}, t - t')g_{jn}V_{nl}(t')U_\alpha^l(\vec{k}, t')$$

(47)

with

$$\Delta_{j,\text{ret}}(\vec{k}, t - t') = -\frac{1}{\omega_{j0}}\Theta(t - t') \sin(\omega_{j0}(t - t')).$$

(48)

We separate $U_\alpha^j(\vec{k}, t)$ into the trivial part corresponding to the case $V_{jl}(t) \equiv 0$ and a function $f_\alpha^j(\vec{k}, t)$ which represents the reaction to the potential by making the Ansatz

$$U_\alpha^j(\vec{k}, t) = e^{-i\omega_{j0}t}(\delta_\alpha^j + f_\alpha^j(\vec{k}, t)).$$

(49)

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The function \( f_j^\alpha(\vec{k}, t) \) satisfies then the integral equation

\[
f_j^\alpha(\vec{k}, t) = \int_0^t dt' \Delta_j,\text{ret}(\vec{k}, t-t') e^{i\omega_j t'} g_{jn} V_{nl}(t') e^{-i\omega_l t'} (\delta_l^\alpha + f_l^\alpha(\vec{k}, t'))
\]

and an equivalent differential equation

\[
\left( \frac{d^2}{dt^2} - 2i\omega_{j0}(\vec{k}) \frac{d}{dt} \right) f_j^\alpha(\vec{k}, t) = -g_{jn} V_{nl}(t) e^{i(\omega_j - \omega_l) t} (\delta_l^\alpha + f_l^\alpha(\vec{k}, t))
\]

with the initial conditions \( f_j^\alpha(\vec{k}, 0) = \dot{f}_j^\alpha(\vec{k}, 0) = 0 \).

We expand now \( f_j^\alpha(\vec{k}, t) \) with respect to orders in \( V_{jl}(t) \) by writing

\[
f_j^\alpha(\vec{k}, t) = f^{(1)\alpha}_j(\vec{k}, t) + f^{(2)\alpha}_j(\vec{k}, t) + f^{(3)\alpha}_j(\vec{k}, t) + \ldots
\]

where \( f^{(n)\alpha}_j(\vec{k}, t) \) is of n’th order in \( V_{jl}(t) \) and \( f^{(n)\alpha}_j(\vec{k}, t) \) is the sum over all orders beginning with the n’th one. The \( f^{(n)\alpha}_j(\vec{k}, t) \) are obtained by iterating the integral equation (50) or the differential equation (51). The function \( f^{(1)\alpha}_j(\vec{k}, t) \) is identical to the function \( f_j^\alpha(\vec{k}, t) \) itself which is obtained by solving (51), the function \( f^{(2)\alpha}_j(\vec{k}, t) \) can be obtained as

\[
f^{(2)\alpha}_j(\vec{k}, t) = \int_0^t dt' \Delta_j,\text{ret}(\vec{k}, t-t') e^{i\omega_j t'} g_{jn} V_{nl}(t') e^{-i\omega_l t'} f^{(1)\alpha}_l(\vec{k}, t')
\]

or by solving the inhomogeneous differential equation

\[
\left( \frac{d^2}{dt^2} - 2i\omega_{j0}(\vec{k}) \frac{d}{dt} \right) f^{(2)\alpha}_j(\vec{k}, t) = -g_{jn} V_{nl}(t) e^{i(\omega_j - \omega_l) t} f^{(1)\alpha}_j(\vec{k}, t).
\]

Note that in this way one avoids the computation of \( f^{(2)\alpha}_j(\vec{k}, t) \) via the small difference \( f^{(1)\alpha}_j(\vec{k}, t) - f^{(1)\alpha}_j(\vec{k}, t) \). This feature is especially important if deeper subtractions are required as in the case of fermion fields.

The order on the potentials \( V_{jl}(t) \) will determine the behaviour of the functions \( f^{(n)\alpha}_j(\vec{k}, t) \) at large momentum. We will give here the relevant
leading terms for \( f_j^{(1)\alpha}(\vec{k}, t) \) and \( f_j^{(2)\alpha}(\vec{k}, t) \). We have

\[
f_j^{(1)\alpha}(\vec{k}, t) = \frac{i}{2\omega j 0} \int_0^t dt' (e^{2i\omega j 0(t-t')} - 1) e^{i(\omega j 0 - \omega_\alpha) t'} g_{jn} V_{n\alpha}(t') .
\] (56)

Integrating by parts we obtain

\[
f_j^{(1)\alpha}(\vec{k}, t) = -\frac{i}{2\omega j 0} \int_0^t dt' g_{jn} V_{n\alpha}(t') e^{i(\omega j 0 - \omega_\alpha) t'} - \frac{1}{2\omega j 0(\omega j 0 + \omega_\alpha)} g_{jn} V_{n\alpha}(t) e^{i(\omega j 0 - \omega_\alpha) t} + \frac{1}{2\omega j 0(\omega j 0 + \omega_\alpha)} \int_0^t dt'' e^{2i\omega j 0(t-t'')} g_{jn} V_{n\alpha}(t') e^{i(\omega j 0 - \omega_\alpha) t'} .
\] (57)

For \( f_j^{(2)\alpha}(\vec{k}, t) \) we need to know only that the leading behaviour is

\[
f_j^{(2)\alpha}(\vec{k}, t) = -\sum_m \frac{1}{4\omega j 0\omega m 0} \int_0^t dt' \int_0^{t'} dt'' g_{jl} V_{lm}(t') e^{i(\omega j 0 - \omega m 0) t'} g_{mn} V_{n\alpha}(t'') e^{i(\omega m 0 - \omega_\alpha) t''} + O((\omega_\alpha)^{-3}) .
\] (58)

The leading terms of \( f_j^{(1)\alpha}(\vec{k}, t) \) and \( f_j^{(2)\alpha}(\vec{k}, t) \) in this expansion in powers of \((\omega_\alpha)^{-1}\) are the same as for \( f_j^{(1)\alpha}(\vec{k}, t) \) and \( f_j^{(2)\alpha}(\vec{k}, t) \) respectively.

## 5 Renormalization

Using the expansion in orders of the potential \( V_{jl} \) the fluctuation term (38) occuring in the equation of motion (39) can be written as

\[
\mathcal{F}(t) = \sum_{j\alpha} Q_{jl}(t) g_{\alpha\alpha} \int \frac{d^3 k}{(2\pi)^3} \left( \delta_j^\alpha + f_j^\alpha(\vec{k}, t) \right) \left( \delta_j^\alpha + f_j^\alpha(\vec{k}, t) \right) e^{i(\omega j 0 - \omega_\alpha) t} .
\] (59)

To zeroth order in \( V_{jl} \) the functions \( f_j^\alpha \) vanish and we have

\[
\mathcal{F}^{(0)}(t) = \sum_l Q_{ll}(t) g_{ll} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2\omega l 0} .
\] (60)

These correspond to the tadpole graphs depicted in Fig. 2; they are removed by including into the Lagrangean a mass counterterm for the Higgs field.
To first order in the potentials $V_{jl}$ we find

$$F^{(1)}(t) = \sum_l Q_{ll}(t) g_{ll} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_0^2} 2\text{Re} f^{(1)}_l(\vec{k}, t)$$

$$+ Q_{a\phi}(t) \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{g_{a\phi}}{2\omega_{a0}} 2\text{Re} \left[ f^{(1)a}_a(\vec{k}, t)e^{i(\omega_{a0}-\omega_{\phi0})t} \right] - \frac{g_{aa}}{2\omega_{a0}} 2\text{Re} \left[ f^{(1)a}_a(\vec{k}, t)e^{i(\omega_{a0}-\omega_{\phi0})t} \right] \right\}. \quad (61)$$

The first part, the sum over the diagonal terms proportional to $Q_{ll}(t)$, corresponds to the graphs of Fig. 3a and their divergent parts are removed by the coupling constant renormalization; the second term corresponds to the graph of Fig. 3b and its divergent part is removed by a wave function renormalization counter term.

The sum of all contributions of order higher than 1 in the potential $V_{jl}(t)$ is finite. It is given by

$$F^{(2)}(t) = \sum_{a\ell} Q_{ll}(t) g_{ll} \int \frac{d^3k}{(2\pi)^3} \frac{g_{a\alpha}}{2\omega_{a0}} \left\{ \delta_\alpha^2 2\text{Re} f^{(2)\ell}_l(\vec{k}, t) + f^{(2)\ell}_l(\vec{k}, t) f^{(1)\ell}_l(\vec{k}, t) \right\}$$

$$- \frac{3}{2} \frac{d}{dt} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{g_{a\phi}}{2\omega_{a0}} 2\text{Re} \left[ e^{i(\omega_{a0}-\omega_{\phi0})t} (f^{(2)\phi}_a(\vec{k}, t) + f^{(1)\phi}_a(\vec{k}, t) f^{(1)\phi}_a(\vec{k}, t)) \right] \right.$$

$$\left. + \frac{g_{\phi\phi}}{2\omega_{\phi0}} 2\text{Re} \left[ e^{i(\omega_{a0}-\omega_{\phi0})t} (f^{(2)\phi}_a(\vec{k}, t) + f^{(1)\phi}_a(\vec{k}, t) f^{(1)\phi}_a(\vec{k}, t)) \right] \right\}. \quad (62)$$

The divergent zeroth and first order terms have now to be defined by a regularization and the divergent parts to be absorbed into appropriate counter terms. In [11] we found dimensional regularization to be suitable both for Lorentz covariance and for technical elegance. The zeroth order term can be handled as in [11] via

$$F^{(0)}_{\text{reg}}(t) = \sum_l g_l Q_{ll}(t) \mu^{\epsilon} \int \frac{d^3\epsilon k}{(2\pi)^3-\epsilon} \frac{1}{2\omega_0^2}$$

$$= - \sum_l \frac{g_l Q_{ll}(t) m_{l0}^2}{16\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi \mu^2}{m_{l0}^2} - \gamma + 1 \right\}. \quad (63)$$

The divergent part of this expression depends on the ‘initial masses’ $m_{l0}$, the discussion of the counterterm is postponed, therefore, until we have discussed the first order term as well. The first order part consists of a sum over diagonal terms proportional to $Q_{ll}$ and a nondiagonal part containing $Q_{a\phi}$. 
The diagonal part can be handled as in \[11\]; using (57) we obtain
\[
\sum_l g_{ll} Q_{ll}(t) \int \frac{d^3k}{(2\pi)^3} \frac{2}{2\omega_{l0}} 2 \text{Re} f_l^{(1)}(\vec{k}, t) = \sum_l \frac{Q_{ll}(t) \mu^e}{16\pi^2} V_{ll}(t) \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_{l0}^2} - \gamma \right\}
\]
\[+ \sum_l Q_{ll}(t) \int \frac{d^3k}{(2\pi)^3} \frac{1}{4\omega_{l0}^3} \int_0^t dt' \cos(2\omega_{l0}(t-t')) \tilde{V}_{ll}(t'). \tag{64}\]

Again using the expansion (57) we obtain for the nondiagonal part
\[
Q_{a\varphi}(t) \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{g_{a\varphi}}{2\omega_{a0}} 2 \text{Re} \left[ f_a^{(1)\varphi}(\vec{k}, t)e^{i(\omega_{a0} - \omega_{\varphi0})t} \right] + \frac{g_{a\varphi}}{2\omega_{a0}} 2 \text{Re} \left[ f_a^{(1)\varphi}(\vec{k}, t)e^{i(\omega_{a0} - \omega_{\varphi0})t} \right] \right\}
\[= -\frac{3}{2} g^2 \tilde{H}_0(t) \int \frac{d^3k}{(2\pi)^3} \frac{g_{a\varphi}g_{a\varphi}}{\omega_{a0}\omega_{\varphi0}(\omega_{a0} + \omega_{\varphi0})} \int_0^t dt' \tilde{H}_0(t') \cos((\omega_{a0} + \omega_{\varphi0})(t-t')). \tag{65}\]

The first integral on the right hand side is divergent; the connection to the wave function renormalization in usual time-ordered perturbation theory becomes transparent if we rewrite it as
\[
\int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_{a0}\omega_{\varphi0}(\omega_{a0} + \omega_{\varphi0})} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{a0}^2 + i\epsilon)(k^2 - m_{\varphi0}^2 + i\epsilon)}. \tag{66}\]

Using dimensional regularization we find
\[
\int \frac{d^{3-\epsilon}k}{(2\pi)^{3-\epsilon}} \frac{1}{\omega_{a0}\omega_{\varphi0}(\omega_{a0} + \omega_{\varphi0})} \tag{67}\]
\[= \frac{1}{16\pi^2} \left\{ \frac{2}{\epsilon} - \gamma + 1 + \ln \frac{4\pi\mu^2}{m_{a0}^2} + \frac{m_{\varphi0}^2}{m_{a0}^2 - m_{\varphi0}^2} \ln \frac{m_{\varphi0}^2}{m_{a0}^2} \right\}. \]

As in \[11\] there are cancellations between the zeroth order and first order divergencies such that the counter terms can be chosen independent of the initial conditions, i.e. they can be written in terms of the masses \(m_l\) rather than the masses ‘at time zero’, \(m_{l0}\). We renormailze at \(q^2 = 0\) and chose the counter terms in such a way that the corrections to the tree effective potential vanish at its minimum \(|\Phi| = H_0 = v = m_h^2/2\lambda\). In particular we find the wave function renormalization counter term
\[
\delta Z = \frac{3g^2}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_W^2} - \gamma + 1 \right\}. \tag{68}\]
The mass renormalization counter terms takes the form
\[
\delta m_h^2 = \frac{3}{16\pi^2} \lambda m_h^2 \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_h^2} - \gamma + 1 \right\}
+ \frac{3}{16\pi^2} \left( \lambda + \frac{g^2}{4} \right) m_h^2 \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_h^2} - \gamma + 1 \right\}.
\]
(69)

Finally the coupling is renormalized via
\[
\delta \lambda = \frac{9}{16\pi^2} \lambda \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_h^2} - \gamma \right\}
+ \frac{3}{128\pi^2} g^4 \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_a^2} - \gamma \right\}
+ \frac{3}{16\pi^2} \left( \lambda + \frac{g^2}{4} \right)^2 \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m_h^2} - \gamma \right\}.
\]
(70)

This choice of the counter terms corresponds to a renormalization of the Green functions at \( q^2 = 0 \) as usual in the renormalization of the effective potential. If these counter terms are introduced into the equation of motion, all divergencies of \( F \) are cancelled, but we are still left with the respective finite parts
\[
\Delta Z = -\frac{3g^2}{32\pi^2} \left\{ \frac{m_{\phi_0}^2}{m_{\phi_0}^2 - m_{\phi_0}^2} \ln \frac{m_{\phi_0}^2}{m_{\phi_0}^2} + \ln \frac{m_a^2}{m_{\phi_0}^2} \right\}
\]
(71)
\[
\Delta m_h^2 = -\frac{3}{16\pi^2} \lambda m_h^2 \ln \frac{m_h^2}{m_{\phi_0}^2} + \frac{9}{8\pi^2} \lambda^2 H_0^2(0)
+ \frac{3}{16\pi^2} g^2 m_a^2 H_0^2(0)
- \frac{3}{16\pi^2} (\lambda + \frac{g^2}{4}) m_h^2 \ln \frac{m_h^2}{m_{\phi_0}^2} + \frac{3}{8\pi^2} (\lambda + \frac{g^2}{4})^2 H_0^2(0)
\]
(72)
\[
\Delta \lambda = -\frac{9}{16\pi^2} \lambda^2 \ln \frac{m_h^2}{m_{\phi_0}^2} - \frac{3}{128\pi^2} g^4 \ln \frac{m_a^2}{m_{\phi_0}^2}
- \frac{3}{16\pi^2} (\lambda + \frac{g^2}{4})^2 \ln \frac{m_h^2}{m_{\phi_0}^2}.
\]
(73)

With these definitions the equation of motion reads
\[
(1 + \Delta Z_{H_0}) \ddot{H}_0(t) - \frac{1}{2} (m_h^2 + \Delta m_h^2) H_0(t) + (\lambda + \Delta \lambda) H_0^3(t) + F_m(t) = 0,
\]
(74)
where the finite part of the fluctuation integral is given by

\[
\mathcal{F}_{\text{fin}}(t) = \int \frac{d^3k}{(2\pi)^3} \left\{ \sum l Q_{lt}(t) \frac{g_{ll}}{4\omega_{l0}^3} \int_0^t dt' \cos(2\omega_{l0}(t - t')) \dot{V}_{lt}(t') \right. \\
+ \sum_{l\alpha} Q_{lt}(t) g_{ll} \left\{ 2\delta_l^{\alpha} \text{Re} f_l^{(2)l}(\vec{k}, t) + f_l^{(1)l}(\vec{k}, t) f_l^{(1)l*}(\vec{k}, t) \right\} \right\} \\
+ \frac{3}{2} g^2 \partial_t \frac{1}{\omega_{a0} \omega_{\phi 0}} \int_0^t dt' \dot{H}_0(t') \cos((\omega_{a0} + \omega_{\phi 0})(t - t')) \\
- \frac{3}{2} \frac{g^2}{\omega_{a0} \omega_{\phi 0}} \text{Re} \left( e^{i(\omega_{a0} - \omega_{\phi 0})t} \left\{ f_{\phi a}^{(2)l}(\vec{k}, t) + f_{\phi a}^{(1)l}(\vec{k}, t) f_{\phi a}^{(1)l*}(\vec{k}, t) \right\} \right) \\
- \frac{3}{2} \frac{g^2}{\omega_{\phi 0}} \text{Re} \left( e^{i(\omega_{a0} - \omega_{\phi 0})t} \left\{ f_{\phi a}^{(2)l}(\vec{k}, t) + f_{\phi a}^{(1)l}(\vec{k}, t) f_{\phi a}^{(1)l*}(\vec{k}, t) \right\} \right) \right\} .
\]

The infinite and finite counter terms introduced so far have to be included into the expression for the energy as well. In addition a new counter term is required to compensate the quartically divergent zero point energy. It has again an infinite part determined by the renormalization condition at \( q^2 = 0 \) and \( |\Phi| = \nu \)

\[
\delta \Lambda = \frac{m_h^4}{64\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi \mu^2}{m_h^2} - \gamma + \frac{3}{2} \right\} 
\]

(76)

for the divergent part and

\[
\Delta \Lambda = \frac{m_h^4}{256\pi^2} \left\{ \ln \frac{m_h^2}{m_{\phi 0}^2} + 3 \ln \frac{m_h^2}{m_{\phi 0}^2} \right\} 
\]

(77)

for the finite part. In addition the quadratically and logarithmically terms contain some finite parts which depend on the initial value of \( H_0(0) \); they are constant and do not appear in the equation of motion, therefore. They lead to another finite correction to the energy

\[
\Delta \Lambda' = \frac{1}{128\pi^2} \left\{ \left( 9\lambda^2 + \frac{3g^4}{8} + 3 \left( \lambda + \frac{g^2}{4} \right)^2 \right) H_0^4(0) + \left( 3\lambda + 3 \left( \lambda + \frac{g^2}{4} \right) \right) H_0^2(0) \right\} .
\]

(78)

Altogether we obtain for the renormalized energy

\[
\mathcal{E} = \frac{1}{2} (1 + \Delta Z_{H_0}) \dot{H}_0^2(t) - \frac{1}{4} (m_h^2 + \Delta m_h^2) H_0^2(t)
\]
\[ + \lambda + \Delta \lambda \frac{4}{H_0^4(t)} + \Delta \Lambda + \Delta \Lambda' \]

\[ + \int \frac{d^3k}{(2\pi)^3} \sum_{j\alpha} \frac{d_j g_{jj} g_{aa}}{2\omega_0} \left\{ \omega_{j0}(2Re\delta_j^\alpha f_j^{(2)}(\vec{k}, t) + \bar{f}_j^{(2)}(\vec{k}, t))^2 \right\} \]

\[ + \frac{1}{2} |f_j^{(1)}(\vec{k}, t)|^2 - \omega_{j0}Re(i\delta_j^\alpha f_j^{(2)}(\vec{k}, t) + i f_j^{(1)}(\vec{k}, t)f_j^{(2)}(\vec{k}, t)) \]

\[ + \frac{1}{2} \left( \delta_t^\alpha + 2\delta_j^\alpha Re f_j^{(1)}(\vec{k}, t) + |f_j^{(1)}(\vec{k}, t)|^2 \right) + \frac{V_{ja}(t)}{4\omega_{j0}(\omega_{j0} + \omega_0)} \}

We denote the sum of the first five terms as the "inflaton energy" and the last term as "fluctuation energy". Of course the inflaton energy includes besides the tree level energy the finite terms left over after renormalization. In terms of potentials we can distinguish between:

the tree level potential (4), i.e.

\[ V_{\text{tree}}(H_0) = \frac{1}{2} m_h^2 H_0^2 + \frac{\lambda}{4} H_0^4, \]  

the effective potential appearing in the inflaton energy

\[ \tilde{V}_{\text{eff}} = -\frac{1}{4} (m_h^2 + \Delta m_h^2) H_0^2(t) + \lambda + \Delta \lambda \frac{4}{H_0^4(t)} + \Delta \Lambda + \Delta \Lambda' \]  

and the renormalized one-loop potential

\[ V_{1-1}(H_0) = \frac{m_h(H_0)^4}{64\pi^2} \ln \frac{m_h^2}{m_h^2(H_0)} + \frac{3m_a(H_0)^4}{32\pi^2} \ln \frac{m_a^2}{m_a^2(H_0)} \]

\[ + \frac{3m_p(H_0)^4}{64\pi^2} \ln \frac{m_p^2}{m_p^2(H_0)} + \frac{3H_0^4}{128\pi^2} \left( 9\lambda^2 + \frac{3}{8} g^4 + 3 \left( \lambda + \frac{g^2}{4} \right)^2 \right) \]

\[ - \frac{3}{128\pi^2} m_h^2 H_0^2 \left( 2\lambda + \frac{g^2}{4} \right). \]

Here \( m_h^2(H_0) = m_h^2 + 3\lambda(H_0^2 - v^2) \) while \( m_a(H_0) \) and \( m_p(H_0) \) are given by Eqns. (33,34) with \( H_0(0) \) replaced by \( H_0 \).

### 6 Results and Conclusions

The computation scheme presented in this paper can be implemented numerically along the lines described in our previous work [11] for the case of a
self-coupled scalar field. We have to deal here with a coupled-channel sys-
tem which, however, can be treated using perfectly analogous methods; there
is of course a noticeable increase in CPU time. If the Higgs mass is chosen
as the mass scale the $SU(2)$ Higgs model has two parameters, $g$ and $\lambda$, the
initial value of the Higgs field is an additional variable. We will not attempt
here to exhaust this parameter space by presenting results for a systematic
choice of samples. Rather we restrict ourselves here to two examples; a sys-
tematic study should include the development of new concepts like the ‘true
effective potential’ \[12\] or at least the static one loop effective potential (see
below). Especially for gauge theories with their numerous degrees of freedom
one-loop corrections tend to be large and the use of the tree potential and
the bare parameters may lead to wrong interpretations of the results.

We have chosen two parameters sets. The first one has a large initia-
value for the Higgs field leading, in spite of the mexican-hat potential, to
symmetric oscillations; the parameters are $g = 0.1$, $\lambda = 1$ and $H_0(0) = 10.$
The results are presented in Figs. 4 - 6. The classical amplitude decreases
slowly with time, the energy is transferred almost entirely to the fluc-
tuations.

The second example is chosen in such a way that the inflaton field oscil-
lates around its vacuum expectation value. Here the parameters are: $g = 1,$
$\lambda = 1$ and $H_0(0) = 0.65.$ We observe again a strong decrease of the classical
amplitude and a corresponding transfer of energy.

For both parameter sets we present also the inflaton energy as a function
of $H_0.$ For time-independent $H_0$ this energy would be given by the sum of
tree and the renormalized one-loop effective potential $V_{\text{tree}} + V_{1-1}.$ The actual
motion is governed, however, by the potential $\tilde{V}_{\text{eff}}.$ It is surprising that this
potential which is determined entirely by the initial conditions governs the
motion even after the quantum fluctuations are fully developed.

In conclusion we have extended here our method for the numerical study
of nonequilibrium processes to the $SU(2)$ Higgs model and thereby to a sys-
tem which is typical for the inflationary scenario. It was again possible to
separate the numerical computation of finite part of the fluctuation integrals
and the analytic evaluation of their renormalization parts. We have not stud-
ied here initial conditions where the inflaton field starts out at the metastable
maximum $H_0 = 0$ or anywhere in the region where the effective potential has
negative curvature or is even complex. As we have mentioned above such a
study would have to be accompanied by a study of effective potentials. Such
investigations could help to develop our understanding of negative curvature
or complex effective potentials, as analyzed theoretically by Weinberg and
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Figure captions

Fig. 1 Diagrammatic representation of the one-loop equation (51).

Fig. 2a Renormalization parts: Tadpole diagram, Eq. (60).

Fig. 3a Renormalization parts: Fish diagram, Eq. (61).

Fig. 3b Renormalization parts: Fish diagram, Eq. (61).

Fig. 4 The inflaton amplitude $H_0(t)$ for $\lambda = 1$, $g = 0.1$ and $H_0(0) = 10$.

Fig. 5 The fluctuation integral $\Delta F(t)$ for the same set of parameters.

Fig. 6 The inflaton and fluctuation energies as a function of time for the same parameters: inflaton energy (short dashed line), the fluctuation energy (long dashed line) and total energy (solid line).

Fig. 7 The inflaton amplitude $H_0(t)$ for $\lambda = 1$, $g = 1$ and $H_0(0) = 0.65$.

Fig. 8 The inflaton and fluctuation energies as a function of time for the same parameters: inflaton energy (short dashed line), the fluctuation energy (long dashed line) and total energy (solid line).

Fig. 9 The inflaton energy as a function of $H_0(t)$ for the first parameter set; we also display (a) the tree level potential $V_{\text{tree}}$, (b) the sum of tree level and one-loop effective potentials and (c) the potential $\tilde{V}_{\text{eff}}$.

Fig. 10 The same as Fig. 9 for the second parameter set.
Fig. 1

Fig. 2
Fig. 3a

Fig. 3b
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 10