Local Variation of Collective Attention in Hashtag Spike Trains

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Abstract
In this paper, we propose a methodology quantifying temporal patterns of nonlinear hashtag time series. Our approach is based on an analogy between neuron spikes and hashtag diffusion. We adopt the local variation, originally developed to analyze local time delays in neuron spike trains. We show that the local variation successfully characterizes nonlinear features of hashtag spike trains such as burstiness and regularity. We apply this understanding in an extreme social event and are able to observe temporal evaluation of online collective attention of Twitter users to that event.

Introduction
Hashtag diffusion in Twitter social network is nonlinear in time. Pairwise or higher order temporal correlations, bursts, and regular patterns are observed in data analysis. The distribution of time delays between two successive hashtag activities gives a power-law scaling with fat tails (Domenico et al. 2013), on the contrary to an exponential distribution suggested for an independent Poisson process. A potential reason addressed is that earlier hashtags influence coming hashtags such that past hashtags can both cooperate and compete with present hashtags (Myers and Leskovec 2012; Coscia 2013). Heterogeneity of individual online user behavior in micro scale and self-organized cascades (Cheng et al. 2014) due to unequal selection (Ratkiewicz et al. 2010; Weng et al. 2012; Gleeson et al. 2014; Coscia 2013; Cetin and Bingol 2014; Gleeson et al. 2015) in the hashtag pool in macro scale, and the underlying cyclic rhythm of tweeting habit (Myers and Leskovec 2014; Franca et al. 2014; Mollgaard and Mathiesen 2015; Sanli and Lambiotte 2015) are further factors driving time-dependent hashtag propagation. Although preserving highly nonlinear nature, building tools to characterize hashtag time-series, except obtaining the distribution functions, has not been considered in detail, yet.

Extreme social events such as elections and protests (Borge-Holthoefer et al. 2011; Gonzalez-Bailon et al. 2011), announcement of scientific innovations (Domenico et al. 2013), and panic events such as crisis (Kenett et al. 2014) and earthquakes (Sasahara et al. 2013) artificially deform Twitter network and encourage massive amount of hashtag activity in a short time window, as shown in Figure 1. The resultant emergent online behavior is both empirically (Yang and Leskovec 2011; Mollgaard and Mathiesen 2015) and theoretically (Mollgaard and Mathiesen 2015) studied and distinct temporal properties of collective attention are quantified. These
properties are significantly important to be able to predict these extreme, but rare social events (Kenett et al. 2014; Miotto and Altmann 2014).

Our main motivation is to establish a systematic methodology to distinguish real noisy hashtag signals to independent random signals and to extract temporal patterns from the real signals. We apply an approach called the local variation $L_V$, originally introduced to analyze noisy neuron spike trains and to detrend for salient dynamics of neurons (Shinomoto, Shima, and Tanji 2003; Miura, Okada, and ichi Amari 2006; Omi and Shinomoto 2011). After convincing the usage of $L_V$ in semantic analysis, which has been performed extensively in our recent work (Sanli and Lambiotte 2015), we present a promising study on evaluation of collective attention by performing $L_V$ on a political election. Remarkable difference in $L_V$ in rush period suggests that local nonlinear features could predict extreme social events.

Data Set

Data Collection

The data is collected via publicly open Twitter API. A fine time window, between April 30, 2012 and May 10, 2012, is determined on purpose to be able to cover two social events such as the political debate on the French presidential election-2012 held on May 2 and the election day held on May 6. Having 10 days data helps us to visualize activity in regular days, both between and after these extreme events, and compare the difference in hashtag dynamics. During this period, all twitting activity, but only the users addressed in France is considered not to deal with time differences between countries and regions and other potential social events held on in the same period. The time resolution is 1 second and no language selection is applied.

We examine 295,697 unique hashtags out of 2,942,239 tweets include at least one hashtag, which is 30% of all tweets. 228,525 online users, almost half of the total online users, are associated with hashtag diffusion. The network in the period contains hashtags directly related to the debate, election, and two candidates Francois Hollande and Nicolas Sarkozy for the presidency of France. Ranking them by the number of appearance (frequency) or equivalently popularity $p$, from the most popular to the least, we have #ledebat (180946), #hollande (143636), #sarkozy (116906), #votehollande (99908), #avescsarkozy (67549), #ledebat (66668) [in French], #france2012 (20635), #presidentielle (13799), and many others with lesser $p$. The numbers inside the parenthesis present the corresponding $p$. These popular hashtags are at the top of the others in the pool, e.g. 0.0001% of all hashtags.

Real Hashtag Spike Trains

Single hashtag diffusion in time can be represented as a spike train, as shown in Figure 1. Each spike represents that the corresponding hashtag used at that time without specifying ways and users. Having the resolution 1 second, the spike time of multiple events occurring in a second cannot be distinguished and therefore in this situation only one appearance is counted. We construct spike trains for all hashtags observed in the data ordering from the earliest appearance time to the latest time, e.g. $\tau_0$, $\tau_1$, $\tau_2$, ..., $\tau_k$. Each hashtag has a unique number of (exact) appearance, popularity $p$.

Randomized Hashtag Spike Trains

To be able to compare real dynamics with an artificial and independent one, the randomized version of real hashtag spike trains is established serving as a null model. First, all spikes coming from any hashtags are combined, giving a single merged hashtag spike train. Uniforming spike appearance, one spike at a spike time, is still valid. Children randomized hashtag spike trains are obtained by uniformly permuting the matrix $T$ of the spike times of the merged train by $p$ times, the number of spikes of the desired real train we compare. We apply randperm($T$, $p$) in Matlab and have $p$ times uniformly distributed unique independent random spike times, e.g. $\tau_0$, $\tau_1$, $\tau_2$, ..., $\tau_k$.

Local Variation

The local variation $L_V$, specifically defined to quantify nonlinear neural time-series and to uncover temporal patterns in neuron spike trains, is defined at spike time $\tau_i$ (Omi and Shinomoto 2011)

$$L_V = \frac{3}{N-2} \sum_{i=2}^{N-1} \left( \frac{\Delta \tau_i + \Delta \tau_{i+1}}{\Delta \tau_i + 1} \right)^2,$$

where \(\Delta \tau_i = \tau_{i+1} - \tau_i\) and \(\Delta \tau_{i+1} = \tau_i - \tau_{i-1}\). \(\Delta \tau_{i+1}\) quantifies forward delay and \(\Delta \tau_i\) represents backward waiting time. Importantly, the denominator normalizes the quantity such as to account for local variations of the rate at which events take place.

By definition, $L_V$ takes values in the interval [0,3]. Furthermore, it is derived that $L_V$ is on average equal to 1, $\langle L_V \rangle = 1$, if the underlying process described by an independent Poisson distribution, which the distribution of the inter-spike intervals gives an exponential function (Shinomoto, Shima, and Tanji 2003). Here, the brackets describe the average taken over the given distribution. All other situations can be generalized by Gamma processes (Shinomoto, Shima, and Tanji 2003; Miura, Okada, and ichi Amari 2006) and $\langle L_V \rangle$ should be significantly different than 1. For instance, $\langle L_V \rangle \approx 3$ if the hashtag spike trains are extremely bursty (irregular), on the other hand $\langle L_V \rangle \approx 0$ while the trains present regular (homogeneous) temporal patterns (Sanli and Lambiotte 2015).

Figure 2 shows the results of our $L_V$ analysis, for both real and randomized hashtag spike trains. The probability distribution of $P(L_V)$ of the calculated values of $L_V$ on the two data sets, with classifying hashtag groups in popularity $p$, presents distinct behavior. Whereas $\langle L_V \rangle = 1$ for any groups of $p$ for the randomized trains, suggesting Poisson processes, $\langle L_V \rangle$ never indicates 1 for the real trains. The randomization dampens nonlinearity of the real trains, temporal correlations, burstiness, and regularity in series and construct statistically stationary and independent processes,
yet time-dependent events. Therefore, we characterize time-dependent Poissons in Figure 2(b), \( P(L_V) \), of hashtag spike trains (Sanli and Lambiotte 2015). (a) Real hashtag spike trains. We observe a clear shift, to the higher values of \( L_V \), in the peak positions while decreasing hashtag popularity \( p \), which indicates that the process becomes bursty (irregular). In any \( p \), the mean values never gives 1, none of the real signal is Poisson process. (b) Randomized hashtag spike trains. Independent of \( p \), all curves suggest fluctuations around 1, as expected for temporarily independent signals. To satisfy a better visualization, the results are grouped based on ranking \( p \) from the most popular to the least popular ones: High \( p \), red and orange symbols, moderate \( p \), yellow and green symbols, and low \( p \), blue and purple symbols.

**Empirical Application: Collective Attention**

We now utilize \( L_V \) for more practical purposes and ask: Can \( L_V \) predict extreme social events? Our investigation will be presented below is far from a complete understanding. However, we will be able to capture temporal evaluation of online emergent behavior as a result of collective attention of twitting on the French presidential election-2012, in the first week of May 2012.

We specifically compare hashtag diffusion in extreme days, the debate day (May 2) and the election day (May 6) with the dynamics in a regular day between these events, e.g. May 4. Instead of considering all hashtags in the pool, as done in the previous Section, we concentrate on topic related hashtags such as #ledebat (180946), #hollande (143636), #sarkozy (116906), #votehollande (99908), #avecsgarndo (67549), and #ledebat (66668) [in French]. The numbers in the parenthesis indicate \( p \) of the corresponding hashtag.

Local variation \( L_V \) is obtained for these topic-oriented hashtag spike trains. The trains are constructed separately for the three days. \( L_V \) for each train and for each day is calculated considering time window with duration 1 hour. Figure 3 presents the results in the debate (left), regular (middle), and election (right) days. The top row [Figure 3(a)] shows \( L_V(t) \) in the days in hour resolution. The below row [Figure 3(b)] summarizes the twitting activity as the tweets including listed hashtags in the legend versus time, again in hour resolution.

Rush hours in online communications during the debate and the announcement of the election result are highlighted in the shaded yellow rectangle and with the yellow vertical line, respectively. Significant decays in \( L_V(t) \) for both the debate and election days, synchronizing perfectly with the peak of the counts, indicate regular activation of the online users on the discussion of the election and so describe no burstiness, \( L_V(t) \approx 0 \). This trend is not observed at all for the regular day and mainly the cyclic rhythm of Twitter network (Sanli and Lambiotte 2015) characterize the values of \( L_V(t) \). While large amount of fluctuations present in inactve hours [0 am:6 am], the rest of the day \( \langle L_V(t) \rangle \approx 1 \) suggesting time-dependent Poisson processes. These results are preliminary, but promising since the stages of collective attention are clearly visible on \( L_V(t) \).

**Discussion and Future Work**

The main purpose of this paper is to establish a tool for noisy social time-series and uncover nonstationary features and temporal patterns, specifically in an online emergent limit. Our comparative test on the real and randomized data sets shows that the local variation \( L_V \), a metric introduced to quantify the fluctuations of neuron spike trains as compared to a local characteristic time, works successfully in hashtag spike trains, as well. This encourages us to develop further tools, for instance to predict extreme online events by evaluating the early noisy signal prior to an extreme event. As an example, we consider the week of the French presidential election-2012. This fine time window is well suitable for our
aim and we find that $L_V$ is sensitive enough to distinguish collective attention period, users are active homogeneously in time, from the preceding period where temporal heterogeneity is present and therefore a prediction would be satisfied by performing better statistics in the decay of $L_V(t)$.

We obtain $L_V(t)$ is almost 0 in rush periods. Such artificial regularity originates from our assumption due to lack of time resolution below 1 second. Although we observe heterogeneity in hashtag spike trains in rush hours in the empirical data, uniforming spike appearance setting to 1 in any spike time creates unnatural homogeneity in emergent limit. To resolve this, the trains should be constructed preserving the heterogeneity in the data and so $L_V$ must be re-introduced for nonuniform number of spikes at different spike times in a train.

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**References**

Borge-Holthoefer, J.; Rivero, A.; Garcia, I.; Cauhe, E.; Ferrer, A.; Ferrer, D.; Francos, D.; Iniguez, D.; Perez, M. P.; Ruiz, G.; Sanz, F.; Serrano, F.; Vinas, C.; Tarancon, A.; and Moreno, Y. 2011. Structural and dynamical patterns on online social networks: The Spanish May 15th Movement as a case study. *PLoS ONE* 6(8):e23883–1–8.

Cetin, U., and Bingol, H. O. 2014. Attention competition with advertisement. *Phys. Rev. E* 90:032801–1–7.

Cheng, J.; Adamic, L.; Dow, P. A.; Kleinberg, J. M.; and Leskovec, J. 2014. Can cascades be predicted? In *Proceedings of the 23rd International Conference on World Wide Web, WWW ’14*, 925–936. New York, NY, USA: ACM.

Coscia, M. 2013. Competition and success in the meme pool: A case study on quickmeme.com. In *International AAAI Conference on Weblogs and Social Media (ICWSM)*.

Domenico, M. D.; Lima, A.; Mougel, P.; and Musolesi, M. 2013. The anatomy of a scientific rumor. *Sci. Rep.* 3:2980–1–9.

Franca, U.; Sayama, H.; McSwiggen, C.; Daneshvar, R.; and Bar-Yam, Y. 2014. Visualizing the “Heartbeat” of a City with Tweets. *ArXiv e-prints*.

Gleeson, J. P.; Ward, J. A.; O’Sullivan, K. P.; and Lee, W. T.
Gleeson, J. P.; O’Sullivan, K. P.; Banos, R. A.; and Moreno, Y. 2015. Determinants of Meme Popularity. ArXiv e-prints.

Gonzalez-Bailon, S.; Borge-Holthoefer, J.; Rivero, A.; and Moreno, Y. 2011. The dynamics of protest recruitment through an online network. Sci. Rep. 1:197–1–7.

Kenett, D. Y.; Morstatter, F.; Stanley, H. E.; and Liu, H. 2014. Discovering social events through online attention. PLoS ONE 9(7):e102001–1–7.

Miotto, J. M., and Altmann, E. G. 2014. Predictability of extreme events in social media. PLoS ONE 9(11):e111506–1–7.

Miura, K.; Okada, M.; and ichi Amari, S. 2006. Estimating spiking irregularities under changing environments. Neural Comput. 18:2359–2386.

Mollgaard, A., and Mathiesen, J. 2015. Emergent user behavior on Twitter modelled by a stochastic differential equation. ArXiv e-prints.

Myers, S., and Leskovec, J. 2012. Clash of the contagions: Cooperation and competition in information diffus. In Data Mining (ICDM), 2012 IEEE 12th International Conference on, 539–548.

Myers, S. A., and Leskovec, J. 2014. The bursty dynamics of the twitter information network. In Proceedings of the 23rd International Conference on World Wide Web, WWW ’14, 913–924. New York, NY, USA: ACM.

Omi, T., and Shinomoto, S. 2011. Optimizing time histograms for non-poissonian spike trains. Neural Comput. 23:3125–3144.

Ratkiewicz, J.; Fortunato, S.; Flammini, A.; Menczer, F.; and Vespignani, A. 2010. Characterizing and modeling the dynamics of online popularity. Phys. Rev. Lett. 105:158701–1–4.

Sanli, C., and Lambiotte, R. 2015. Local variation of hashtag spike trains and popularity in Twitter. ArXiv e-prints.

Sasahara, K.; Hirata, Y.; Toyoda, M.; Kitsuregawa, M.; and Aihara, K. 2013. Quantifying collective attention from tweet stream. PLoS ONE 8(4):e61823–1–10.

Shinomoto, S.; Shima, K.; and Tanji, J. 2003. Differences in spiking patterns among cortical neurons. Neural Comput. 15:2823–2842.

Weng, L.; Flammini, A.; Vespignani, A.; and Menczer, F. 2012. Competition among memes in a world with limited attention. Sci. Rep. 2:335–1–8.

Yang, J., and Leskovec, J. 2011. ACM International Conference on Web Search and Data Mining (WSDM).