Research article

All-optical dynamic analysis of the photothermal and photoacoustic response of a microcantilever by laser Doppler vibrometry

Yang Liu\textsuperscript{a,b,c}, Tommaso Seresini\textsuperscript{a}, Jun-Yan Liu\textsuperscript{b,c,++}, Liwang Liu\textsuperscript{a}, Fei Wang\textsuperscript{b,c}, Yang Wang\textsuperscript{b,c}, Christ Glorieux\textsuperscript{a,**}

\textsuperscript{a} Laboratory for Soft Matter and Biophysics, Department of Physics and Astronomy, KU Leuven, Leuven, B-3001, Belgium
\textsuperscript{b} State Key Laboratory of Robotics and System, Harbin Institute of Technology, Harbin, 150001, China
\textsuperscript{c} School of Mechatronics Engineering, Harbin Institute of Technology, Harbin, 150001, China

1. Introduction

By virtue of their high sensitivity in response to pressure variations at a particular frequency, cantilever beams (CBs) are utilized as vibration or force sensors in various narrowband applications of pressure or motion detection \cite{1,2}. In addition to well-established applications in atomic force microscopy (AFM) \cite{3,4}, scanning force microscopy (SFM) \cite{5}, photothermal spectroscopy (PTS) \cite{6}, and biomolecular sensors \cite{7}, CBs have also been explored for enhanced design of weak acoustic waves \cite{8}, such as photothermal signals \cite{9–11} in cantilever enhanced photothermal spectroscopy (CE-PAS) \cite{12–16} which is a performant spectroscopy method \cite{17} for trace detection of explosives, pollutants \cite{18,19}, illicit drug testing \cite{20} and industrial monitoring \cite{21,22}. It has been reported that CB enhanced microphones can offer a sensitivity that is 100 times higher than a good quality regular microphone \cite{23}.

In applications of CE-PAS \cite{24–26}, in which pressure variations are detected via optical or piezoelectric detection by the accompanying cantilever motion, appropriate choice of the working frequency and optimization of the quality factor of the vibrational resonance of cantilevers is key to improving their sensitivity \cite{27}. Moreover, knowledge of the modal shape at resonance can be exploited for better performance in applications by optimizing the vibration detection position on traditional cantilever \cite{28} or special-shaped resonators \cite{29–33}. Adequate characterization of vibrational behavior is thus of great importance.

Till now, a wide spectrum of actuation approaches has been developed to characterize the dynamic response of CBs. Piezoelectric actuation \cite{34,35} is the most used method, in which a cantilever is attached to a piezoelectric transducer or film. The involved mechanical contact has been reported \cite{36} to affect the dynamic behavior of CBs, deteriorating the representativeness of the measured response. This type of shortcoming exists also in other contact actuation methods such as electrostatic actuation \cite{37,38} and magnetic actuation \cite{39,40} methods, where the cantilever is attached to an electrode or magnetic film.

Also optical actuating methods have been exploited, in which the...
surface of the cantilever \([41,42]\) or the adjacent light-absorbing accessory \([43-45]\) was illuminated by an intensity-modulated light beam. In this approach, the driving forces originate from the photothermally induced thermal expansion of the cantilever \([41,42]\) or the photoacoustic effect of the adjacent air heated by the illuminated accessory \([44,45]\). Due to the unrestricted choice of the location of the excitation optical spot or the geometry of the light pattern, non-contact optical actuation offers great versatility. The majority of current research mainly focuses on the actuation parameters and frequency response of the cantilever. To the best of our knowledge, no spatial mapping of optically generated cantilever modes has been reported and no quantitative estimation has been made in which the contributions to the displacement from thermal expansion in and in the air near the cantilever have been discriminated.

In this work, we present results on the optical mapping of the mechanical response of a photo-thermo-elastically and photo-acoustically excited vibrating cantilever. The measurement results are interpreted in the framework of numerically and analytically obtained simulations. In Section II, airborne driven vibration modes of a rectangular cross-section cantilever are modeled based on a Bernoulli-Euler approximation and simulated through a finite element method (FEM). In Sections III & IV, we present the optical excitation assay, in which intensity-modulated light emitting diode (LED) light was used to excite bending modes of a commercial-available cantilever, and a laser Doppler vibrometer (LDV) was used to quantitatively map the vibrational response along the long axis of the cantilever beam. Position scans were carried out for white noise modulation of the exciting LED light, yielding the combined positional and frequency dependence of the cantilever response. The experimental results are compared with the theoretical predictions. In Section V, the origin of the different vibrations is analyzed by quantitative analytical modeling and numerical FEM simulation. The article ends with a concluding section.

2. Theoretical analysis of cantilever modes and coupling with sound waves

Analysis of modal frequencies and mode shapes is crucial for applications involving cantilever resonance-enhanced detection \([46,47]\). The lowest order bending mode is the mode that is most exploited. Several approximate models for cantilever bending have been developed, among which are the Bernoulli-Euler, Rayleigh, shear, and Timoshenko models \([48]\). Taking into account the quite small thickness \(h\) to length \(L\) ratio of the considered cantilever \((h=2.4\ \mu\m, L=2400\ \mu\m\), \(h/L=1\times10^{-3}\)), the shear deformation and rotary inertia can be neglected, making it plausible to utilize Bernoulli-Euler (B-E) bending theory to predict the modal frequencies and mode shapes. Assumptions that the cross-section of a cantilever is a plane perpendicular to the axis of the cantilever beam before and after bending were adopted according to B-E bending theory. The expressions for different modal shapes and angular modal frequencies are respectively given by: \([49]\)

\[
\phi(x) = K [\cos ax - \cosh ax - \frac{(\cos al + \cosh al)}{(\sin al + \sinh al)}(\sin x - \sinh x)] \\
\omega_a = (al)^2 \left(\frac{EI}{\rho al^4}\right)^{1/2} \\
\text{where } K \text{ is an amplitude factor, } a \text{ [m}^{-1}\text{] has the characteristics of a wavenumber, } L \text{ [m] is the length, } EI \text{ [Nm}^2\text{] is the flexural stiffness (E [Pa] is the Young’s modulus and I [m}^4\text{] is the moment of inertia), } \rho \text{ [kg/}	ext{m}^3\text{] is the density of the cantilever, } A \text{ [m}^2\text{] is the area of the cross-section.}
\]

Modal values for the quantity \(al\) can be determined by solving the transcendental equation: \([49]\)

\[
\cos al \cosh al + 1 = 0
\]

By numerical iteration, possible values can be approximately obtained:

\[\begin{align*}
(al)_1 &= 1.875, \\
(al)_2 &= 4.694, \\
(al)_3 &= 7.855, \\
(al)_4 &= \pi \left(\frac{2n - 1}{2}\right), n = 4, 5, 6 \ldots
\end{align*}\]

Fig. 1a presents the geometrical features of the considered cantilever and snapshots of the three simplest modes of the cantilever based on Eq. (1). In view of developing and optimizing applications of CE-PAS detection and dynamic analysis by photoacoustic actuation, the coupling between the cantilever and the acoustic field is important to be investigated. The Helmholtz equation for a sound wave in the frequency domain can be expressed as:

\[
\nabla^2p(\omega) + k^2p(\omega) = 0
\]

with \(p\ [\text{Pa}]\) the pressure and \(k\ [\text{rad. m}^{-1}]\) the wavenumber. The interaction of an acoustic wave with the cantilever is described by Newton’s law and by the continuity of displacement, velocity, and acceleration at the air-cantilever interface. The acoustic pressure gradient loaded on the cantilever can be approximated by:

\[
\nabla p_n = \frac{1}{h}[p_r(\omega) - p_f(\omega)] \cdot \vec{n}
\]

where \(p_r(\omega)\) and \(p_f(\omega)\) represent the sound pressure on the front and rear boundaries of the cantilever, respectively. \(\vec{n}\) is the normal unit vector of the cantilever. The effect of the cantilever on the sound field can be written as:

\[
a_r \cdot \vec{n} = a_f \cdot \vec{n} = a_c \cdot \vec{n}
\]
the cantilever in a contactless way aimed at exploiting the photo-thermo-elastic effect, which involves the dynamic temperature response of a material due to the light-induced heating, and the subsequent photo-thermoelastic/photo-acoustic effect, i.e. thermal expansion induced thermoelastic stress in or nearby the illuminated material. A schematic diagram of the setup is shown in Fig. 2a. Intensity-modulated light emitted by a LED was used to photothermally excite the cantilever and its vibration was monitored by a commercial laser Doppler vibrometer (LDV).

Noise measurements were first carried out to characterize the measurement system by recording cantilever displacement signals with a zero signal supplied to the sound card that was driving the LED, at 150 positions along the cantilever axis with a step of 12 μm. The noise spectrum is shown in Fig. 2b, from which one can identify the classical sources of electronic noise at the net frequency 50 Hz and its harmonics, and classical 1/f vibrational noise (pink noise). The peak around 920 Hz turned out to be caused by a spurious pick-up signal between the sound card and LED driver circuit, and thus is not related to the cantilever mechanics and irrelevant for further discussion. The local and spatially averaged noise spectra also show distinct peaks around (3.64 ± 0.04) kHz, and (10.60 ± 0.05) kHz, revealing resonant behavior, driven by spontaneous vibrations in the environment.

4. Dynamic characterization of the cantilever

4.1. Spectrum of the optically excited vibrations of the cantilever

In order to determine the broadband response of the cantilever, white noise was used as a broadband modulation signal for the LED source. The excitation lasted 10 s and the sampling frequency was 44.1 kHz. Fig. 3 shows an example of the analysis of the vibrational response of the cantilever to modulated (white-noise) illumination, with the LDV probe spot at 96 μm from the free end of the cantilever. The excitation signal and response signal were cross-correlated to obtain the impulse response (Fig. 3a). A Fourier transform was then applied to convert each impulse response to a corresponding spectrum (Fig. 3b). The peak at (0.51 ± 0.01) kHz is consistent with the nominal value of the natural frequency as specified by the manufacturer. The peaks at (3.64 ± 0.04) kHz and (10.60 ± 0.05) kHz comply with the prediction by Bernouilli-Euler approximation in the sense that the experimental ratios with the frequency of the lowest mode, 3.64/0.51 = (7.1 ± 0.4) and 10.6/0.51 = (21 ± 1) are within 15 % from the respective ones of the simulation, 3566/569 = 6.3 and 9997/569 = 17.6. The discrepancy may be due to small differences in geometrical aspect ratios of the cantilever beam between the simulated and real cantilever, or due to the fact that we have ignored the elastic anisotropy of the silicon cantilever material in the simulation.

In order to get a deeper insight into the cantilever motion of the resonant modes, the vibrational response of the cantilever to the optical excitation was scanned along 150 positions, in steps of 12 μm, along the 1.8 mm cantilever axis, covering approximately the free-end and fixed-end of the cantilever beam. Fig. 4b and c present the amplitude and phase distribution on the position and frequency, respectively. Fig. 4a reveals the same 3 resonant peaks as the ones discussed above in Fig. 3b, with quality factors of 40, 101, and 201, suggesting a substantial resonant sensitivity of the 3rd mode for incoming sound around 10.60 kHz. The spatial pattern of the mode at 0.51 kHz corresponds with a simple transverse displacement mode, with maximum amplitude at the free end of the cantilever, and with a quarter of the axial wavelength matching the length of the cantilever. The half wavelength of the transverse displacement of the mode at 3.64 kHz is about 1.6 mm, corresponding with the quarter wavelength fitting 3 times in the 2.4 mm length of the cantilever, with maximum displacement at the free end. Given its high Q-factor, the most efficient
way to use the cantilever as a sound sensor is thus to have an airborne sound wave pressure gradient antinode around one of the two antinodes (at \(x = 0.45\) mm and \(x = 1.35\) mm) of the 10.60 kHz mode and a pressure gradient node at its node (at \(x = 0.90\) mm). Note that the wavelength of airborne waves at 10.60 kHz is 32 mm, substantially longer than the cantilever. The axial gradient in air pressure oscillations required to optimize the cantilever response at 10.60 kHz thus needs to be achieved by arranging geometrical aspects of the neighborhood of the cantilever, so that they locally shadow the pressure oscillation in the nodal locations of the mode.

4.2. Detailed modal vibration patterns

Fig. 5 shows the frequency dependence of the modal vibration patterns in the neighborhood of the resonant modes. These patterns were obtained by combining the amplitude pattern (Fig. 4b) with the sign of the phase (Fig. 4c).

Fig. 6 shows the root mean square averages of the vibration displacements around the 3 resonance frequencies of the vibration patterns. The modal shapes were fitted with the theoretical model Eq. (1), yielding a very good correspondence. The coefficient \(a_L\) and \(K\) were treated as the fitting variables and the \(L\) was fixed 2.4 mm. As illustrated in Table 2, the fitted \(a_L\) values are 2.6, 4.59, and 7.90, with respective standard deviations \(\sigma\) of 0.1, 0.01, and 0.01. The discrepancy between
the determined \((aL)_1\) and the theoretical value, \((aL)_1 = 1.875\), is due to
the fitting length being substantially smaller than the wavelength, the
other two values are quite close to the theoretical values of
\((aL)_2 = 4.694\), \((aL)_3 = 7.855\).

In cantilever enhanced applications, reasonable selection of the
excitation frequency and the detection position are critical to achieving
a higher detection amplitude and experimental quality factor for the
improvement of the detection sensitivity, SNR, and control of energy
dissipation. The above frequency response analysis provides insight into
the optimal excitation frequency and detection position to achieve a
higher vibration amplitude and quality factor.

5. Analysis of the photothermally/photoacoustically induced
vibration mechanism

It is well known that the mechanism to generate mechanical waves
by intensity-modulated light is based on dynamical thermal expansion,
caused by the conversion of light into heat due to optical absorption. In
the investigated cantilever configuration, thermal expansion could be
converted into mechanical energy in three ways, depicted in Fig. 7a: (i)
by temperature and thus thermal expansion gradients in the cantilever,
causings photo-thermo-elasticly induced cantilever bending, (ii) by a
pressure difference between the frontside and backside of the cantilever,
due to unequal temperature oscillations in the air layers adjacent to the
front and backside of the cantilever, caused by unequal photothermal
heating of the illuminated and dark side, (iii) by photoacoustically
induced pressure changes caused by the air layer adjacent to the part of

![Fig. 4. Spatially resolved (b) and averaged (a) amplitude spectrum of the vibrational response of the cantilever to modulated illumination, and spatially resolved phase (c).](image)

![Fig. 5. Color maps of modal vibration patterns in frequency bands around the 3 resonant modes.](image)
assuming full absorption of sinusoidally modulated laser light with intensity $0.22 \text{ W/mm}^2$, depicted in Fig. 7 c, d, and e respectively. The calculation was performed taking into account the finite length and the mechanical boundary conditions of the cantilever, a 3D finite element model (FEM) was built, based on the configuration and dimensions of the presented experimental cantilever, holder and frame in a spherical area with low acoustic reflection outer boundary. In order to make the results more generally referential, the cantilever and holder/frame were assumed to be silicon and glass respectively, with the parameters as shown in Table 4 (we assumed absence of acoustic damping). Using FEM simulations, we explored two of the above mentioned vibration generation mechanisms: photo-thermo-elastically cantilever bending, and photoacoustically induced pressure changes caused by the air layer adjacent to the part of the cantilever holder and frame.

The mesh of the domain is shown in Fig. 8a. For investigating the photo-thermo-elastic contribution to the cantilever bending, a periodic heat flux of $0.22 \text{ W/mm}^2$, cfr the light beam intensity from the experimental LED described in Section III, was loaded on the cantilever upper surface (which is highlighted red in Fig. 8a). Heat transfer, linear expansion, and dynamic equations in the solid were utilized in the cantilever domain. For the simulation of photoacoustic actuation from the holder and frame by the adjacent air, the same uniform thermal flux was loaded on the air interfaces on the upper surface of the frame and the holder, which are colored respectively with blue and green in Fig. 8a. The thermal expansion of the air and accompanying pressure fluctuation were then calculated by the equation of state and the acoustic equation in the sphere air domain with a low reflection spherical boundary layer. The coupling between the air and the cantilever was defined by the continuity of the pressure/normal stress and acceleration at the air-cantilever interfaces. In order to calculate the heat flux into the gas adjacent to the holder and frame, which was driving the photoacoustic effect, the acoustic simulation was preceded by a thermal diffusion calculation, in a locally axisymmetric frequency domain heat conduction model at the glass holder/frame – air interface for a total heat flux of $0.22 \text{ W/mm}^2$ on the interface as shown in Fig. 8b. In Fig. 8c, the blue curve shows that the heat flux into the air is only a very small fraction from the optical absorption generated heat flux.

Assuming a sinusoidally varying heat flux, the frequency response of the displacement at the free side midpoint of the cantilever in different directions was calculated for both the photoacoustic and thermoelastic generation mechanism. The peaks in Fig. 8d correspond with axial resonances of mechanical bending waves. As mentioned above, the photoacoustic signal results from photothermal temperature changes, which decrease as $f^{-1/2}$, within an air piston with the thermal diffusion length as thickness, which decreases as $f^{-1/2}$, resulting in an overall decrease as $1/f$, not taking into account mechanical resonance effects and effects of 3D propagation and diffraction of acoustic waves around the cantilever. The thermoelastic contribution is proportional with the differences in mechanical stiffnesses and wave velocities. In the silicon cantilever, not only normal but also axial gradients in thermoelastic stress grating are quenched by stress equalizing mechanical waves, with longitudinal wavelengths ranging from 10 mm (in the MHz range) till 10 m (in the kHz range), much longer than the grating wavelength (of the order of $2 L = 4.8 \text{ mm}$ for the considered wavevector $k = 838 \text{ rad/m}$). The wavelength of the photoacoustically induced pressure waves near the cantilever are much shorter. Interestingly, the spectrum of the photoacoustically induced displacements generated in the air near the cantilever shows a peak around $f = 45 \text{ kHz}$. It turns out that at this frequency the wavelength of sound in air, $\lambda_{\text{air}} = c_{\text{air}}/f = 340/(45 \times 10^3) = 7.5 \text{ mm}$, equals $\lambda_{\text{bending}} = 2\pi/k$, the wavelength of the spatially periodic pattern in this $k = 838 \text{ rad/m}$ calculation. This peak thus reflects so-called coincidence between the acoustic waves in air and the guided waves at the air-material interface, which is well known in structural acoustics. This implies that photoacoustic generation of cantilever bending waves could thus be maximized by choosing the cantilever length (and/or thickness) such that the structural axial resonance frequency would match the coincidence frequency.

In order to get further insight in the bending generation mechanism, we calculated amplitude and phase of the temperature, stress, and displacement fields, respectively causing and accompanying the cantilever thermoelastic bending and photoacoustic actuation contributions were estimated individually or combined.

The calculated amplitude and phase of the temperature, stress, and displacement fields, respectively causing and accompanying the cantilever thermoelastic bending and air heated by the cantilever, are depicted in Fig. 7c, d, and e respectively. In order to make the results comparable with the experimental data, a spatially periodic light pattern was assumed, with a wavenumber value $k = 838 \text{ rad/m}$ that matched the lowest cantilever resonance.

In Fig. 7c, due to the thermal diffusion length being longer than the cantilever thickness, the temperature amplitudes on the illuminated front and dark backside of the cantilever (decreasing from around 10 K at 200 Hz till mK at 1 MHz) are quasi equal, with differences of less than $10^{-4} \text{ K}$, with almost equal phases below $10^5 \text{ Hz}$. While due to the smallness of these differences, the resulting thermoelastically and photoacoustically induced cantilever displacements (Fig. 7e) are also small – ranging from the nanometers to picometers in the frequency range between kHz and MHz – they are beyond the noise level of interferometric vibrometers. Fig. 7d shows that the amplitudes of the normal stress fluctuations were calculated for both the photoacoustic and thermoelastic generation mechanism. The peaks in Fig. 8d correspond with axial resonances of mechanical bending waves. As mentioned above, the photoacoustic signal results from photothermal temperature changes, which decrease as $f^{-1/2}$, within an air piston with the thermal diffusion length as thickness, which decreases as $f^{-1/2}$, resulting in an overall decrease as $1/f$, not taking into account mechanical resonance effects and effects of 3D propagation and diffraction of acoustic waves around the cantilever. The thermoelastic contribution is proportional with the

![Graph](image-url)

**Fig. 6.** Experimental mode shapes (amplitude of the displacement) and theoretical curves fitted by Eq. (1).

**Table 2**

| mode | fit $R^2$ | $K$ | $\sigma (K)$ | $aL$ | $\sigma (aL)$ |
|------|----------|-----|--------------|-----|--------------|
| 1    | 0.8149   | -0.38 | 0.01         | 2.6 | 0.1          |
| 2    | 0.9939   | 5.71 | 0.02         | 4.59 | 0.01         |
| 3    | 0.9959   | 2.58 | 0.01         | 7.90 | 0.01         |
temperature oscillation amplitude, which varies like $f^{-1/2}$, but the conversion of a thermal expansion gradient to bending is enhanced when the thermal diffusion length (proportional with $f^{-1/2}$) is of the order or smaller than the cantilever thickness. Along the directions of x and y, the thermoelastic amplitudes are negligible with respect to the photoacoustic excited displacements. The stretching vibration along the y-direction is over 2 orders of magnitude larger than the bending on z-direction, except around the peaks. In Fig. 8 d, the frequency peaks of the first three modes by photoacoustic excitation are 553, 3531, and 9969 Hz, by photothermal excitation are 582, 3642, and 10,204 Hz. They are reasonably consistent (maximum difference 12 %) with the experimentally determined values, (0.51 ± 0.01) kHz, (3.64 ± 0.04) kHz, and (10.60 ± 0.05) kHz, respectively, shown in Fig. 3 d. The magnitudes of the FEM calculated spectrum agree within one order of magnitude with the experimentally measured spectrum. As mentioned above, the latter was measured by laser Doppler vibrometry (taking into account the non-flat spectrum of the Polytec LDV system in displacement mode).

In order to verify the FEM results, the actual displacement of the cantilever was experimentally estimated by using chirp modulated LED as the optical excitation. As shown in Fig. 8 e, the displacement excited by 3.7 kHz modulated light is 130 nm, showing good agreement with the displacement along the z-direction (Fig. 8 d) around the cantilever 2nd modal frequency. The peak around 1.85 kHz is most probably caused by nonlinearity of the LED, causing it to emit at the double frequency (3.7 kHz). In spite of the amplitude of this harmonic in the LED intensity spectrum being small, due to the strong cantilever response at that frequency, it results in a substantial peak.

The analytical calculation and the numerical FEM simulation results

Table 3
Assignment of thermal expansion coefficient ($\gamma$).

| origin of the vibration          | layer 0 | layer 1 | layer 2 |
|----------------------------------|---------|---------|---------|
| cantilever thermoelastic bending  | $0$     | $\gamma_{\text{silicon}}$ | $0$     |
| air heated by cantilever         | $\gamma_{\text{air}}$ | $0$     | $\gamma_{\text{air}}$ |

Table 4
Thermal and mechanical properties used in FEM model. In the FEM simulation the damping is all from the air, no other mechanical damping is concerned.

| parts            | material | thermal conductivity $K$ (W/(m K)) | density $\rho$ (kg/m$^3$) | specific heat capacity $C$ (J/(kg K)) | Young’s modulus $E$ (10$^9$ Pa) | Poisson’s ratio $\nu$ | coefficient of thermal expansion $\alpha$ (10$^{-6}$ K$^{-1}$) |
|------------------|----------|-----------------------------------|---------------------------|-------------------------------------|---------------------------------|-----------------------|-----------------------------------------------|
| holder/ frame    | glass    | 1.4                               | 2210                      | 730                                 | \                                 | \                     | \                                             |
| cantilever       | silicon  | 130                               | 2329                      | 700                                 | 170                             | 0.28                  | 2.6                                           |
indicate that the photoacoustically induced pressure changes caused by the illumination of the cantilever contribute most to the vibration, and the photoacoustic pressure changes caused by the illumination of the holder and frame may significantly contribute to the vibration at a level similar to the one of the cantilever thermoelastic bending, if the pressure variations are generated on a holder and frame with high optical absorption.

6. Conclusions

The results show that non-contact optical generation of vibrations in combination with non-contact optical probing is an adequate manner to perform a detailed modal analysis of a cantilever response. Using this approach, both the frequency dependence of the response and the modal shapes were found to be consistent between experiment data and Bernoulli-Euler theory. Knowledge of the resonance frequencies and the corresponding locations of the antinodes and nodes is of importance for designing the geometry of photoacoustic cells and other narrowband sound wave detectors in which a cantilever is to be implemented, in order to achieve a good match between the exciting airborne pressure pattern and the pattern of a high Q factor cantilever mode.

Analytical calculations of the bending response of the cantilever to optical excitation show that there are two main mechanisms to the generation of bending vibrations: (i) thermoelastic bending, resulting from dynamic temperature and thermal expansion gradients in the cantilever, and (ii) bending due to gradients in photoacoustic pressure between the front and rear side of the cantilever, generated by the dynamical heat flux from the optically heated cantilever to the adjacent air. The latter mechanism turns out to be the dominant one. The calculations also confirm experimental observations of the generation of cantilever bending vibrations by pressure waves caused by the oscillating photoacoustic volume velocity piston that results from dynamically illuminating the nearby cantilever holder and frame. Both the shape and the absolute value of the amplitude spectrum are adequately predicted by finite element modeling, taking into account all aspects of the phenomenon: light to heat conversion, photothermal modeling predicting temperature oscillations, photoacoustic modeling linking temperature oscillations to displacements, and mechanical resonances in the finite length cantilever.

Interestingly, the calculations also predict that further enhancement of CE-PAS detectivity is possible by choosing a configuration in which the frequency of one of the modes matches the coincidence frequency for bending waves of the cantilever in air.

Declaration of Competing Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

CG acknowledges KU Leuven C1 project OPTIPROBE (C14/16/063). LL acknowledges the financial support from FWO (Research Foundation-Flanders) postdoctoral research fellowship (12V4419N). This work was also supported by the Foundation for Innovative Research Groups of theNational Nature Science Foundation of China under Grant No. 51521003, the Chinese National Natural Science Foundation under Contract No. 61571153, No. 51173034, Self-planned Task of State Key Laboratory of Robotics and System (HIT) and the Programme of Introducing Talents of Discipline of Universities (grant No. B07108) for the research support. The authors are also grateful to Aaron Beyen and Gérard de Maere d’Aertrycke for their enthusiastic help with the semi-analytical modeling.

Appendix

The differential equations for heat transport in air-cantilever-air configuration (shown in Fig. 7b in material layers \( m = 0,1,2 \) are given by:
Here, for the sake of conciseness, Einstein’s convention for implicit summation with repeated indices was used, and material indices for the 3 layers

with \( m \) referring to material layers 0.1.2 for temperature \( T_m \) and thermal diffusivity \( \alpha_m \).

The boundary conditions for spatially and temporally periodic incoming photothermal induced heat flux at air-Si-interface \( z = 0 \) are:

\[
\begin{aligned}
T_0(z = 0) &= T_1(z = 0) \\
\frac{\partial T_0}{\partial z}(z = 0) &= k_0 \frac{\partial T_0}{\partial z}(z = 0) + I_0 \exp(i\omega t + ikx) \\
T_1(z = -d) &= T_2(z = -d) \\
\frac{\partial T_1}{\partial z}(z = -d) &= k_2 \frac{\partial T_1}{\partial z}(z = -d)
\end{aligned}
\]

(A2)

where \( k_{0,1,2} \) are the thermal conductivities, \( k \) is the wavenumber and \( I_0 \) is the intensity of the incident light.

The general solutions of Eq. (A2) (with 4 undetermined coefficients) are given by:

\[
\begin{aligned}
T_0(z, x, t) &= (T_0^* \exp(-\sigma c z)) \exp(i\omega t + ikx) \\
T_1(z, x, t) &= (T_1^* \exp(\sigma c z) + T_2^* \exp(-\sigma c z)) \exp(i\omega t + ikx) \\
T_2(z, x, t) &= (T_2^* \exp(\sigma c (z + d))) \exp(i\omega t + ikx)
\end{aligned}
\]

(A3)

Applying the 4 boundary conditions yields a 4 \times 4 set of equations in the unknown coefficients,

\[
\begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 1 & -1 + b_0 & 0 \\
0 & 0 & \exp(\sigma c d) - 1 & 1 + b_0 \\
0 & 0 & -2\exp(\sigma c d) & -b_2 + 1
\end{bmatrix}
\begin{bmatrix}
T_0 \\
T_1 \\
T_1 \\
T_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
\frac{I_0}{\kappa_0 c (1 + b_0)} \\
\frac{I_0 \exp(-\sigma c d)}{\kappa_2 c (1 + b_0)} \\
0
\end{bmatrix}
\]

(A4)

with

\[
\begin{aligned}
\sigma_n &= \sqrt{k^2 + i\omega / \alpha_n} \\
b_n &= \frac{\kappa_n \sigma_n}{\kappa_i \sigma_i}
\end{aligned}
\]

(A5)

with \( m \) referring to material layers 0.1.2 for \( \sigma_m \) and 0.2 for \( b_m \).

The explicit solutions for the undetermined coefficients are:

\[
\begin{aligned}
T_0 &= \frac{I_0}{\kappa_0 c \sigma_0} \frac{b_0 + 1}{b_0 + 1} \exp(\sigma c d) + \frac{b_2 - 1}{b_0 - 1} \exp(-\sigma c d) \\
T_1^* &= \frac{I_0}{\kappa_1 c \sigma_1} (b_2 + 1) \exp(\sigma c d) - \frac{b_2 - 1}{b_0 - 1} \exp(-\sigma c d) \\
T_2^* &= \frac{I_0}{\kappa_2 c \sigma_2} (b_0 + 1) \exp(\sigma c d) - \frac{b_2 - 1}{b_0 - 1} \exp(-\sigma c d) \\
T_2^* &= \frac{2}{\kappa_2 c \sigma_2} (b_0 + 1) \exp(\sigma c d) - \frac{b_2 - 1}{b_0 - 1} \exp(-\sigma c d)
\end{aligned}
\]

(A6)

The dynamic temperature field acts as a driving source for strains \( \epsilon_{ij} \), stresses \( \sigma_{ij} \) and displacements \( u_i \) in the silicon layer (thermoelastic effect) and the adjacent air (photoacoustic effect). This phenomenon is described by Newton’s and Hooke’s laws:

\[
\begin{aligned}
\rho \dddot{u}_i &= \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma_{ij}}{\partial x_j} = \epsilon_{ij} \\
\sigma_{ij} &= \epsilon_{ij} E_i - \rho y T \delta_{ij} \\
\epsilon_{ij} &= \rho(C_1 \dot{u}_i^2 + 2C_2 \dot{u}_i + C_2^2 u_i^2) + y T \delta_{ij} \\
\sigma_{ij} &= \rho(C_1 \dot{u}_i^2 + 2C_2 \dot{u}_i + C_2^2 u_i^2 + u_i^2 + u_j^2) - y T \delta_{ij}
\end{aligned}
\]

(A7)

Here, for the sake of conciseness, Einstein’s convention for implicit summation with repeated indices was used, and material indices for the 3 layers...
were omitted. The involved material properties are $\gamma = (3c_L^2 - 4c_T^2)\gamma_0$ [m$^2$/s$^{-2}$·K$^1$] and $\gamma_0$ [K$^{-1}$] the linear thermal expansion coefficient, the density $\rho$ [kg/m$^3$], the longitudinal and transverse speed of sound, $c_L$ [m/s] and $c_T$ [m/s]. For the sake of simplicity, and without strong impact on the conclusions, isotropy is assumed also for the silicon layer.

The above equations can be simplified by invoking so-called Helmholtz decomposition [50] of the vectoral fields, defining a longitudinal potential $\psi(x,z,t)$ and transverse potential $\psi(x,z,t)$, as:

$$\begin{pmatrix}
\psi(x,z,t) \\
\psi(x,z,t)
\end{pmatrix} = \nabla \phi + \nabla \times \psi$$

(A8)

Where $\nabla \cdot \psi = 0$ (A9) and of which it can be shown that:

$$\begin{aligned}
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} &= \frac{1}{c_L^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{\gamma}{c_L^2}
\\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} &= \frac{1}{c_T^2} \frac{\partial^2 \psi}{\partial t^2}
\end{aligned}$$

(A10)

so that the general solutions can be written as:

$$\begin{aligned}
\phi_0(z,x,t) &= \phi_0 \exp(-p_{10} z) + F_0 \exp(-\sigma_0 z) \\
\phi_1(z,x,t) &= \phi_1 \exp(p_{11} z) + \phi_2 \exp(-p_{11} z) + F_1 \exp(\sigma_1 z) + F_1^* \exp(-\sigma_1 z) \\
\phi_2(z,x,t) &= \phi_3 \exp(p_{12}(z + d)) + F_2 \exp(\sigma_2(z + d))
\end{aligned}$$

(A11)

and the shear velocities and shear potentials in Newtonian fluid air layers 0 and 2, $c_{T0}=c_{T2}$, $\psi_0 \psi_2 = 0$.

By inserting the proposed general solutions in the wave equations, the wavenumbers in the z-direction and the source terms can be derived to be:

$$p_{Lm} = \sqrt{k^2 - \omega^2/c_L^2}$$

(A13)

For sign $\neq 0$ or $\omega = 0$ and for material layer $m = 0,1,2$.

Combining Eqs. (A11), (A12), (A13) with Eq. (A8), general solutions can be obtained for the displacements, and further inserting these in Hooke’s law in Eq. (A7), also general solutions can be derived for the stresses. These general solutions can now be made explicit at the solid-fluid-interfaces $z = 0$ and $z = d$, where continuity of normal displacement and stress must hold:

$$\begin{aligned}
&u_{10}(z = 0) = u_{00}(z = 0) \\
&u_{10}(z = -d) = u_{00}(z = -d) \\
&\sigma_{00}(z = 0) = \sigma_{00}(z = 0) = 0 \\
&\sigma_{00}(z = -d) = \sigma_{00}(z = -d) = 0 \\
&\sigma_{11}(z = 0) = \sigma_{11}(z = 0) \\
&\sigma_{11}(z = -d) = \sigma_{11}(z = -d)
\end{aligned}$$

(A14)

From this $6 \times 6$ set of equations, the coefficients $\psi_0$, $\psi_1$, $\psi_2$, $\psi_3$, $\psi_4^+$ can be solved, so that all potentials, displacements in stresses in the 3 layers can be calculated for any given set of material properties, frequency $f = \omega/(2\pi)$ [Hz] and wavenumber $k$.

References

[1] B.N. Johnson, R. Mutharasu, Biosensing using dynamic-mode cantilever sensors: a review, Biosens. Bioelectron. 32 (1) (2012) 1–18.
[2] H. Nazemz, A. Joseph, J. Park, et al., Advanced micro-and nano-gas sensor technology: a review, Sensors 19 (6) (2019) 1285.
[3] J. Shen, D. Zhang, F.H. Zhang, et al., AFM tip-sample convolution effects for cylinder protrusions, Appl. Surf. Sci. 422 (2017) 482–491.
[4] D. Chavan, D. Andres, D. Immachi, Note: Ferrule-top atomic force microscope. II. Imaging in tapping mode and at low temperature, Rev. Sci. Instrum. 82 (4) (2011), 046107.
[5] J. Tamayo, R. Garcia, Deformation, contact time, and phase contrast in tapping mode scanning force microscopy, Langmuir 66 (18) (2012) 309–312.
[6] D.S. Volkov, O.B. Rogova, M.A. Praskurin, Photoacoustic and photothermal methods in spectroscopy and characterization of soils and soil organic matter, Photoacoustics 17 (2020), 100151.
[7] C. Ziegler, Cantilever-based biosensors, Anal. Bioanal. Chem. 379 (7) (2004) 946–959.
[8] L. Basategar, A. Vafanejad, S.J. Chen, et al., Resonance-enhanced piezoelectric microphone array for broadband or prefiltered acoustic sensing, J. Microelectromechanical Syst. 22 (1) (2012) 107–114.
Fei Wang was born in Yucheng, Shandong Province, China in 1990. He received the B.Sc. degree in Mechanical Design and Manufacturing from Ludong University, Yantai, China in 2012, and M.Sc. degree in Manufacturing Engineering of Aerospace Vehicle from Harbin Institute of Technology, Harbin, China in 2015. His main research areas were infrared thermal imaging and non-destructive testing & evaluation (NDT&E). From 2018–2019, he was a joint-Ph.D. student in Centre de Recherche en Acquisition et Traitement de l’Image pour la Santé (CREATIS), INSA de Lyon, Lyon, France funded by CSC. His research interests include non-destructive evaluation, photo-thermal imaging, and infrared thermography.

Yang Wang is a professor in the School of Mechatronics Engineering, Harbin Institute of Technology (since 1982, head of the Dept.a 2008–2018). He has trained over 60 graduate students (M.Sc. and Ph.D.) and more than 200 publications. He received the B.S., M.S., and Ph.D. degrees in Mechanical Manufacturing Engineering from Harbin Institute of Technology, Harbin, China in 1982, 1988 and 1999. His research interests include non-destructive testing and evaluation (NDT&E) technique, laser manufacturing technology, laser staining additive manufacture, and water guided laser processing.

Christ Glorieux obtained his Ph.D. degree in 1994 at the Physics and Astronomy department of KU Leuven, Belgium on the topic "Depth profiling of inhomogeneous materials and study of the critical behavior of gadolinium by photoacoustic and related techniques". He is active in research and teaching physics and sounds and waves, electromagnetism, optics and experimental physics to undergraduate and graduate students. He is leading a research team in the field of photothermal and photoacoustic applications for the fundamental study of the thermophysical properties of complex soft, biological and heterogeneous matter, the development of measurement techniques in acoustics and optics and for the characterization and depth profiling of thin (sub-micron) layered structures, with applications in the field of non-destructive testing.