First-order spatial coherence of excitons in planar nanostructures: 
a k-filtering effect

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Abstract

We propose and analyze a $k_{\parallel}$-filtering effect which gives rise to the drastic difference between the actual spatial coherence length of quasi-two-dimensional (quasi-2D) excitons or microcavity (MC) polaritons in planar nanostructures and that inferred from far-field optical measurements. The effect originates from the conservation of in-plane wavevector $k_{\parallel}$ in the optical decay of the particles in outgoing bulk photons. The $k_{\parallel}$-filtering effect explains the large coherence lengths recently observed for indirect excitons in coupled quantum wells (QWs), but is less pronounced for MC polaritons at low temperatures, $T \lesssim 10\, \text{K}$.

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Long-range spatial coherence is a fingerprint of well-developed Bose-Einstein (BE) statistics. Measurements of the first-order spatial coherence function $g^{(1)}$ and the coherence length $\xi$ have allowed to visualize the BE condensation transition in a trapped Bose gas of Rb atoms. There are several recent reports on the observation of long-range spatial optical coherence in a low-temperature quasi-2D system of microcavity polaritons and indirect excitons. In this case, the resonant optical decay of MC polaritons or QW excitons in bulk photon modes allows to map the in-plane coherence function $g^{(1)}$ of the particles, by measuring the optical coherence function $\tilde{g}^{(1)}$ of the emitted photons. It is commonly assumed that the coherence length of QW excitons (MC polaritons), $\xi_x$, is identical to that, $\xi_{\gamma}$, of the optical coherence function $\tilde{g}^{(1)}$.

In this Letter, we report a $k_\parallel$-filtering effect, which can strongly influence the optical coherence function $\tilde{g}^{(1)}$ measured from a planar nanostructure, and calculate $g^{(1)}$ and $\tilde{g}^{(1)}$ for QW excitons and MC polaritons. For QW excitons, the $k_\parallel$-filtering effect tremendously increases the optical coherence length $\xi_{\gamma}$, leading to $\xi_{\gamma} \gg \xi_x$, and can naturally explain the $\mu$m coherence lengths observed for indirect excitons and attributed to spontaneously developed coherence. The effect is less pronounced for MC polaritons, still with $\xi_{\gamma} \gtrsim \xi_p$.

The $k_\parallel$-filtering effect stems from the energy and in-plane momentum $\hbar k_\parallel$ conservation in the resonant conversion “quasi-2D QW exciton (MC polariton) $\rightarrow$ outgoing bulk photon”. For a (coupled) quantum well surrounded by thick co-planar barrier layers, the case illustrated in Fig. 1, only low energy optically-active excitons from the radiative zone $k_\parallel \leq k_0 = (\sqrt{\varepsilon_b}/c)\omega_0$, with $\varepsilon_b$ the dielectric constant of (AlGaAs) barrier layers and $\hbar\omega_0$ the exciton energy at $k_\parallel = 0$, are bright, i.e., can emit far-field light. In a far-field optical experiment with the detection angle $2\alpha$ [see Fig. 1 (b)], the fraction of QW excitons which contribute to the optical signal is drastically reduced further to the wavevector band $\Delta k_\parallel$ given by $0 \leq k_\parallel \leq k_\parallel^{(\alpha)} = (k_0/\sqrt{\varepsilon_b})\sin \alpha \ll k_0$. The $\alpha$-dependent narrowing of the detected states results in an effective broadening of the first-order spatial coherence function $\tilde{g}^{(1)}$. In addition, the sharp cutoff of the detected states at $k_\parallel = k_\parallel^{(\alpha)}$ yields an unusual oscillatory behavior of $\tilde{g}^{(1)}$. The $k_\parallel$-filtering effect has no analogy in optics of bulk excitons or polaritons.

The first-order spatial coherence function $g^{(1)}$ of quantum well excitons, at a fixed time, is given by $g^{(1)}(r'_\parallel, r''_\parallel) = G^{(1)}(r'_\parallel, r''_\parallel)/[G^{(1)}(r'_\parallel, r'_\parallel)G^{(1)}(r''_\parallel, r''_\parallel)]^{1/2}$ with $G^{(1)}(r'_\parallel, r''_\parallel) = \langle \hat{\Psi}^1(r'_\parallel) \hat{\Psi}^1(r''_\parallel) \rangle$, where $\hat{\Psi}^1(r'_\parallel) = (1/\sqrt{S}) \sum_{k_\parallel} e^{ik_\parallel r'_\parallel} B_{k_\parallel}$, $r_\parallel$ is the in-plane coordinate, $S$ is the
FIG. 1: (color online) Schematic of the $k_{\parallel}$-filtering effect. (a) The exciton and photon dispersions. Only low-energy QW excitons from the radiative zone $k_{\parallel} \leq k_0$ can emit outgoing bulk photons. (b) A far-field optical experiment with the detection angle $2\alpha$: A small fraction of QW excitons with $|k_{\parallel}| \leq k_{\parallel}^{(\alpha)} = (k_0/\sqrt{\varepsilon_b}) \sin \alpha$ contributes to the optical signal.

area, and $B_{k_{\parallel}}$ is the exciton operator. Thus for isotropically distributed QW excitons one receives:

$$g^{(1)}(r_{\parallel}) = \frac{1}{2\pi n_{2d}} \frac{1}{\pi} \int_0^\infty J_0(k_{\parallel} r_{\parallel}) n_{k_{\parallel}} k_{\parallel} dk_{\parallel},$$

where $r_{\parallel} = |r_{\parallel}'' - r_{\parallel}'|$, $n_{2d}$ is the concentration of particles, $n_{k_{\parallel}} = \langle B_{k_{\parallel}}^\dagger B_{k_{\parallel}} \rangle$ is the occupation number, and $J_0$ is the zeroth-order Bessel function of the first kind. For a classical gas of QW excitons at thermal equilibrium, Eq. (1), with $n_{k_{\parallel}}$ given by the Maxwell-Boltzmann (MB) distribution function $n_{k_{\parallel}}^{MB}$, yields the well-known result:

$$g^{(1)} = g^{(1)}_{cl}(r_{\parallel}) = e^{-\pi r_{\parallel}^2/\lambda_{dB}^2},$$

where the thermal de Broglie wavelength is given by $\lambda_{dB} = [(2\pi\hbar^2)/(M x k_B T)]^{1/2}$ with $T$ the temperature and $M x$ the exciton in-plane translational mass. For helium temperatures, one estimates from Eq. (2) the coherence length of MB-distributed indirect excitons in coupled QWs as $\xi_x \sim \lambda_{dB} \sim 0.1 \mu m$.

Comparing with Eq. (1), the spatial coherence function $\tilde{g}^{(1)}$ of photons emitted by QW excitons is given by

$$\tilde{g}^{(1)}(r_{\parallel}) = \frac{\int_0^\infty G_t(k_{\parallel}) J_0(k_{\parallel} r_{\parallel}) n_{k_{\parallel}} k_{\parallel} dk_{\parallel}}{\int_0^\infty G_t(k_{\parallel}) n_{k_{\parallel}} k_{\parallel} dk_{\parallel}}.$$

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where $G_f = \Theta(k_{\parallel}^{(\alpha)} - k_{\parallel})\Gamma_{x-\gamma}(k_{\parallel})$ is the \(k_{\parallel}\)-filtering function with $\Theta(x)$ the step function and $\Gamma_{x-\gamma}(k_{\parallel})$ the efficiency of the resonant conversion of a QW exciton in an outgoing bulk photon. The function $G_f$ reduces the integration limits on the right-hand side (r.h.s.) of Eq. (3) to the narrow band $\Delta k_{\parallel} = [0, k_{\parallel}^{(\alpha)}]$ and describes the \(k_{\parallel}\)-filtering effect in high-quality planar nanostructures. If both the function $\Gamma_{x-\gamma}(k_{\parallel})$ and the occupation number $n_{k_{\parallel}}$ do not change significantly in the narrow band $\Delta k_{\parallel}$, Eq. (3) yields:

$$\tilde{g}^{(1)} = \tilde{g}^{(1)}_f(r_{\parallel}) = 2J_1(k_{\parallel}^{(\alpha)} r_{\parallel})/(k_{\parallel}^{(\alpha)} r_{\parallel}),$$

where $J_1$ is the first-order Bessel function of the first kind. From Eq. (4) one concludes that the optical coherence length $\xi_{\gamma}$, evaluated as the half width at half maximum of $\tilde{g}^{(1)} = \tilde{g}^{(1)}_f(r_{\parallel})$, is given by

$$4J_1(k_{\parallel}^{(\alpha)} \xi_{\gamma}) = k_{\parallel}^{(\alpha)} \xi_{\gamma} \rightarrow k_{\parallel}^{(\alpha)} \xi_{\gamma} \simeq 2.215.$$  

Equations (4)-(5) illustrate the net \(k_{\parallel}\)-filtering effect: $\xi_{\gamma} \propto 1/k_{\parallel}^{(\alpha)} \propto 1/\sin \alpha$ strongly increases with decreasing aperture angle $2\alpha$. Below we analyze in more detail the polarization function $g^{(1)}$ against the optical $\tilde{g}^{(1)}$, assuming no phase transition to a collective (superfluid) quasi-2D state$^{13}$.

**First-order spatial coherence of non-interacting quasi-2D bosons (excitons) in equilibrium.**

In this case, the chemical potential $\mu_{2d}$ is given by $\mu_{2d}^{(0)} = k_B T \ln(1 - e^{-T_0/T})$ with $k_B T_0 = (2\pi/g)(\hbar^2/M_x) n_{2d}$ the quantum degeneracy temperature and $g$ the spin degeneracy factor of bosons ($g = 4$ for indirect excitons). By substituting $n_{k_{\parallel}} = n_{k_{\parallel}}^{BE}$ into Eq. (1), where $n_{k_{\parallel}}^{BE}$ is the Bose-Einstein occupation number, one receives:

$$g^{(1)} = g^{(1)}_{\text{hint}}(r_{\parallel}) = \frac{T}{T_0} g_1(1 - e^{-T_0/T}, e^{-\pi r_{\parallel}^2/\lambda^2_{\text{dB}}})$$

$$= \frac{T}{T_0} \sum_{n=1}^{\infty} \frac{(1 - e^{-T_0/T})^n}{n} e^{-\pi r_{\parallel}^2/n\lambda^2_{\text{dB}}}.$$  

(6)

Here, the generalized Bose function$^{14}$ $g_\alpha(x, y)$ with $\alpha = 1$ is defined as $g_\alpha(x, y) = \sum_{k=1}^{\infty} (x^k y^{1/k})/k^\alpha$.

For small distances, $r_{\parallel} \ll \lambda_{\text{dB}}$, Eq. (6) yields:

$$g^{(1)}(r_{\parallel} \ll \lambda_{\text{dB}}) \simeq 1 - \frac{T}{T_0} \frac{\pi r_{\parallel}^2}{\lambda^2_{\text{dB}}} \text{Li}_2(1 - e^{-T_0/T}),$$

(7)

where $\text{Li}_\alpha(x) = \sum_{k=1}^{\infty} x^k/k^\alpha$ with $\alpha = 2$ is the polylogarithm. For $T \gg T_0$, Eq. (7) recovers the classical limit, $g^{(1)}_{cl}(r_{\parallel} \to 0) \simeq 1 - (\pi r_{\parallel}^2)/\lambda^2_{\text{dB}}$, which is consistent with Eq. (2). For large
distances, \( r_\parallel \gtrsim r_\parallel^{(q)} = \lambda_{db} \left[ -\frac{(2/\pi) \ln(1 - e^{-T_0/T})}{2} \right]^{1/2} \), Eq. (6) reduces to
\[
g^{(1)}(r_\parallel \gtrsim r_\parallel^{(q)}) \simeq 2 \frac{T}{T_0} K_0 \left( \frac{r_\parallel}{r_0} \right),
\] (8)
where \( K_0 \) is the modified Bessel function of the second kind and \( r_0 = \lambda_{db} \left[ -\frac{4\pi \ln(1 - e^{-T_0/T})}{2} \right]^{1/2} \). Equation (8) explicitly includes quantum corrections to the first-order correlation function \( g^{(1)} \), through \( T_0 \propto \hbar^2 \). For \( r_\parallel \gg r_0 \), Eq. (8) reduces further to the quantum limit:
\[
g^{(1)} = g^{(1)}_q(r_\parallel \gg r_0) = \sqrt{2\pi} \frac{T}{T_0} \sqrt{\frac{r_0}{r_\parallel}} e^{-r_\parallel/r_0}.
\] (9)
For temperatures \( T \gg T_0 \), the spatial coherence function is well approximated by Eq. (2), and the quantum corrections given by Eq. (9) refer to large \( r_\parallel \gtrsim \lambda_{db} \sqrt{(2/\pi) \ln(T/T_0)} \gg \lambda_{db} \), and, therefore, to very small values of \( g^{(1)} \). The latter conclusion is consistent with the \( e^{-\pi r_\parallel^2/n\lambda_{db}^2} \) – series on the r.h.s. of Eq. (6). For \( T \lesssim T_0 \), when Bose-Einstein statistics is well-developed, Eq. (9) is valid for distances larger than \( \lambda_{db} \sqrt{(2/\pi) e^{-T_0/2T}} \ll \lambda_{db} \), so that \( g^{(1)} \) is well-approximated by \( g^{(1)}_q \) for any \( r_\parallel \).

Thus, with temperature \( T \) decreasing from \( T \gg T_0 \) to \( T \lesssim T_0 \), the coherence function \( g^{(1)} \) changes from the \( n_{2d} \)-independent Gaussian \( g^{(1)}_{cl}(r_\parallel) \), given by Eq. (2), to the \( n_{2d} \)-dependent exponentially decaying \( g^{(1)}_q(r_\parallel) \), given by Eq. (9). The quantum statistical effects considerably increase the correlation length \( \xi_x \), as shown in Fig. 2. For \( T \lesssim T_0 \) one has \( \xi_x \sim r_0 \simeq \left[ \lambda_{db}/(2\sqrt{\pi}) \right] e^{T_0/2T} \), i.e., \( \xi_x \) increases exponentially with increasing density \( n_{2d} \). This is due to large population of the low-energy states, in particular the ground-state mode \( k_\parallel = 0 \): \( n_{k_\parallel = 0}^{BE} = e^{T_0/T} - 1 \).

The coherence function \( g^{(1)} \) of weakly-interacting thermal QW excitons. For circularly polarized excitons in a single quantum well, the case relevant to MC polaritons, the repulsive interaction between the particles is well approximated by a contact potential \( U_{sqw} = (u_0/2) \delta(r_\parallel) \) with \( u_0 = u_{0sqw} > 0 \). In this case, the mean-field (Hartree-Fock) interaction only shifts the chemical potential, \( \mu_{2d} = \mu_{2d}^{(0)} + u_{0sqw} n_{2d} \), leaving unchanged Eqs. (6)-(9) for the coherence function \( g^{(1)} \).

For indirect excitons in coupled QWs, the mid-range dipole-dipole repulsive interaction \( U_{cqw} \) of the particles cannot be generally approximated by a contact potential. Following [15], we use the two-parametric model potential \( U_{cqw}(r_\parallel) = [(\sqrt{\pi} u_0 w)/r_\parallel^2] \left( 1 - e^{-r_\parallel^2/w^2} \right) \) with parameters \( u_0 = u_{0cqw} \simeq 4\pi(e^2/\varepsilon_b) d_z \) [16,17] and \( w \simeq a_{cqw}^{(2d)} \), where \( \varepsilon_b \) is the static dielectric con-
FIG. 2: (color online) (a) The first-order spatial coherence function \( g^{(1)} = g_{\text{ind}}^{(1)}(r_\parallel) \) of indirect excitons in a GaAs coupled QW structure with \( d_z = 11.5 \text{ nm} \) and \( w = 15 \text{ nm} \): \( n_{2d} = 10^{10} \text{ cm}^{-2} \) and \( T = 1 \text{ K} \) (dotted line), 0.4 K (dash-dotted line), 0.2 K (dashed line), and 0.1 K (solid line). Inset: The renormalized mass \( M_x^* \) against temperature \( T \), calculated with Eq. \((11)\) for \( n_{2d} = 10^{10} \text{ cm}^{-2} \) (solid line) and \( 2 \times 10^{10} \text{ cm}^{-2} \) (dashed line). (b) \( g^{(1)} = g_{\text{cl}}^{(1)}(r_\parallel) \) calculated with Eq. \((2)\) (solid line), \( g^{(1)} = g_{\text{int}}^{(1)}(r_\parallel) \) evaluated with Eq. \((6)\), and \( g^{(1)} = g_{\text{ind}}^{(1)}(r_\parallel) \) calculated with Eqs. \((6)\), \((10)\) and \((11)\) (dotted line): \( n_{2d} = 10^{10} \text{ cm}^{-2} \) and \( T = 0.1 \text{ K} \). Inset: the same functions evaluated for \( n_{2d} = 10^{10} \text{ cm}^{-2} \) and \( T = 1 \text{ K} \).

stant, \( d_z \) is the distance between coupled quantum wells, and \( a_x^{(2d)} \) is the radius of an indirect exciton. The model potential reproduces \( 1/r_\parallel^3 \) behavior at \( r_\parallel \gtrsim a_x^{(2d)} \) and \( 1/r_\parallel \) Coulomb repulsive potential at \( r_\parallel \lesssim a_x^{(2d)} \). The self-consistent Hartree-Fock (HF) analysis\(^1\) of the Hamiltonian \( H_x = \sum_{p_\parallel} \left( (\hbar^2 p_\parallel^2)/(2M_x) \right) B_{p_\parallel}^\dagger B_{p_\parallel} + 1/(2S) \sum_{p_\parallel, l_\parallel, q_\parallel} U_{\text{cqw}}(q_\parallel) B_{p_\parallel}^\dagger B_{l_\parallel}^\dagger B_{l_\parallel+q_\parallel} B_{p_\parallel-q_\parallel} \) yields the \( n_{2d}^{-}\) and \( T^{-}\) dependent change of the in-plane translational mass \( M_x \). In this case, \( \mu_{2d} \) is
given by
\[ \mu_{2d} = \mu_{2d}^{(0)} + u_0 n_{2d} + \frac{u_0}{2(\lambda_{DB})^2} \left[ \frac{T_0^*}{T} + \sqrt{\pi} \frac{w}{\lambda_{DB}} \right. \]
\[ \times \left. \left[ \frac{w}{2 \lambda_{DB}} \text{Li}_2 \left( 1 - e^{-T_0^*/T} \right) - \text{Li}_3/2 \left( 1 - e^{-T_0^*/T} \right) \right] \right], \tag{10} \]

where, alongside Eq. (6), both the de Broglie wavelength \( \lambda_{DB}^* \) and the degeneracy temperature \( T_0^* \) are changed according to \( M_x \rightarrow M_x^* \). The particle mass \( M_x^* \) renormalized by the dipole-dipole interaction is given as a single solution of the transcendental equation:
\[ \frac{1}{M_x^*} = \frac{1}{M_x} + \frac{u_0 w}{8 \sqrt{\pi} \hbar^2 \lambda_{DB}^*} \left[ \sqrt{\pi} \frac{w}{\lambda_{DB}^*} \frac{T_0^*}{T} - \text{Li}_{1/2} \left( 1 - e^{-T_0^*/T} \right) \right]. \tag{11} \]

In Fig. 2 (a) we plot \( g^{(1)} = g^{(1)}_{\text{ind}}(r_{\parallel}) \) evaluated numerically by using Eqs. (6), (10) and (11) for indirect excitons in a GaAs coupled QW structure. In Fig. 2 (b), the coherence function \( g^{(1)}_{\text{ind}} \) is compared with \( g^{(1)}_{\text{cl}} \) evaluated with Eq. (2) and \( g^{(1)}_{\text{nint}} \) calculated with Eq. (6) for non-interacting excitons. The main result is that the dipole-dipole repulsive interaction induces an increase of the translational mass \( \Delta M_x = M_x^* - M_x \geq M_x \) and, therefore, decreases the coherence length \( \xi_x \) comparing to that of non-interacting particles [see also Fig. 3 (a)]. The effect, however, becomes visible only at temperatures well below 1 K. For \( T = 1 \text{K} \) all three correlation functions, \( g^{(1)}_{\text{ind}}, g^{(1)}_{\text{cl}}, \) and \( g^{(1)}_{\text{nint}} \), nearly coincide, as is clearly seen in the inset of Fig. 2 (b). In other words, for \( n_{2d} \sim 10^{10} \text{cm}^{-2} \) and \( T \gtrsim 1.5 \text{K} \), which are relevant to the experiments,\(^4\,^5,^6,^7\), the quantum limit, i.e., \( g^{(1)} = g_q^{(1)} \) given by Eq. (9), cannot build up. For example, for \( n_{2d} = 10^{10} \text{cm}^{-2} \) and \( T = 1.5 \text{K} \) one estimates \( T_0 \simeq T_0^* \simeq 0.65 \text{K} \) and \( n_{k_{\parallel}=0}^{\text{BE}} \simeq 0.54 < 1 \), so that BE statistics is rather weakly developed to influence the coherence length \( \xi_x \).

The optical spatial coherence function \( \tilde{g}^{(1)} \) of indirect excitons. In order to explain the experiments\(^4\,^5,^6,^7\), which demonstrate a coherence length \( \xi_{\gamma} \) much larger than \( \xi_x \sim 0.1 \mu \text{m} \), we implement the concept of \( k_{\parallel} \)-filtering. In this case, \( \tilde{g}^{(1)} = \tilde{g}^{(1)}_{\text{ind}}(r_{\parallel}) \) is given by Eq. (3) with the efficiency of the “indirect exciton \( \rightarrow \) bulk photon” conversion \( \Gamma_{x-\gamma} = (2k_0^2 - k_{\parallel}^2)/(k_0(k_0^2 - k_{\parallel}^2)^{1/2}) \) \[^8\,^9,\,10,\,11\]. In Fig. 3 (b), we plot \( \tilde{g}^{(1)}_{\text{ind}} \) calculated for various aperture angles, \( 2^\circ \lesssim 2\alpha \lesssim 40^\circ \). The dependence \( \tilde{g}^{(1)} = \tilde{g}^{(1)}_{\text{ind}}(r_{\parallel}) \) is well-approximated by Eq. (4). The above approximation of \( \tilde{g}^{(1)} \) by the “device function” \( \tilde{g}_t^{(1)} \) is valid when \( n_{k_{\parallel}} = n_{k_{\parallel}=0}^{\text{BE}} \ell^2/2M_x \) is
FIG. 3: (color online) (a) The dependence of the correlation length $\xi_x$ against temperature $T$, calculated for noninteracting (dashed line) and dipole-dipole interacting (solid line) indirect excitons. (b) The $k_\parallel$-filtering effect: $\tilde{g}^{(1)}(r_\parallel) = \tilde{g}^{(1)}(r_\parallel)$ evaluated for $\alpha = 18.9^\circ$ (solid line), $8.3^\circ$ (dashed line), $2.1^\circ$ (dotted line), $1.4^\circ$ (dashed-dotted line), and $0.8^\circ$ (dashed-double-dotted line). Inset: The real-space 2D image of $\tilde{g}^{(1)}$. (c) The coherence length $\xi_\gamma$ against the aperture angle $2\alpha$. Nearly constant in the rather narrow energy interval $0 \leq E \leq E^{(\alpha)}$, i.e., when

$$E^{(\alpha)} = \frac{(\hbar k_\parallel^{(\alpha)})^2}{2M} \ll k_B T_0 e^{-T_0/T}. \quad (12)$$

For indirect excitons, the inequality (12) with $T_0$ replaced by $T_0^*$ is definitely held for $n_{2d} \sim 10^{10}$ cm$^{-2}$ and $T \sim 1$ K (e.g., for $\alpha = 20^\circ$ the cutoff energy $E^{(\alpha)}$ is only 1.2 $\mu$eV). Thus the $k_\parallel$-filtering effect yields the correlation length $\xi_\gamma \simeq 2.215/\sqrt{\varepsilon_b/(k_0 \sin \alpha)}$ with $k_0 \simeq 2.8 \times 10^5$ cm$^{-1}$, according to Eq. (5). As a result, $\xi_\gamma$ is intrinsically scaled by the photon wavelength, i.e., is in the $\mu$m length scale [see Fig. 3 (c), where $\xi_\gamma$ is plotted against the angle $\alpha$].

Comparing to standard interference patterns in Young’s double-slit experiment, with contrast determined by $\tilde{g}^{(1)}$, the oscillatory behavior of the optical coherence function $\tilde{g}^{(1)}$ =
FIG. 4: (color online) The MC polariton coherence function $g^{(1)} = g_{MC}^{(1)}(r_\parallel)$ (dashed lines) against that of emitted photons, $\tilde{g}^{(1)} = \tilde{g}_{MC}^{(1)}(r_\parallel)$ (solid lines). Inset: The coherence lengths $\xi_p$ ans $\xi_\gamma$ versus temperature $T$. The calculations, which model the experiments, refer to a GaAs microcavity with positive detuning $\delta = 7$ meV and Rabi splitting $\Omega_{MC} = 4$ meV. The density of MC polaritons $n_{2d} = 10^8$ cm$^{-2}$ and the aperture half-angle $\alpha = 16.7^\circ$, so that $T_0 = 27.6$ K and $E^{(\alpha)} = 0.96$ meV.

$\tilde{g}^{(1)}(r_\parallel)$ is rather unusual [see Eq. (4) and Fig. 3 (b)]. This is a signature of the $k_\parallel$-filtering effect: The $k_\parallel$-filtering function $G_f \propto \Theta(k_\parallel(\alpha) - k_\parallel)$ gives a sharp cutoff at $k_\parallel = k_\parallel^{(\alpha)}$ in the integrals of Eq. (3) that results in oscillations of $\tilde{g}^{(1)}(r_\parallel)$. In some aspects, the effect is similar to Friedel oscillations in a Fermi liquid, with $\hbar k_\parallel^{(\alpha)}$ akin to the Fermi momentum.

*The coherence function $\tilde{g}^{(1)}$ of MC polaritons.* In this case, the “MC polariton → bulk photon” conversion function in Eq. (3) is $\Gamma_{x-\gamma} = \Psi(k_{\parallel})/\tau_{\gamma}(k_{\parallel})$ with $\Psi (0 \leq \Psi \leq 1)$ the photon component along a MC polariton branch and $\tau_{\gamma}$ the radiative (escape) lifetime of a MC photon. In Fig. 4, $g^{(1)} = g_{MC}^{(1)}(r_\parallel)$ calculated with Eq. (6) for circularly polarized MC polaritons is compared with $\tilde{g}^{(1)} = \tilde{g}_{MC}^{(1)}(r_\parallel)$ evaluated with Eq. (3). According to the experiments, we assume the BE distribution of MC polaritons along the lower polariton branch which is taken in the parabolic approximation with an effective in-plane mass $M_{MC}^{lb}$. Comparing to the case of QW excitons, the difference between $g_{MC}^{(1)}$ and $\tilde{g}_{MC}^{(1)}$ is much smaller, still giving $\xi_\gamma > \xi_p$. This is because the cutoff energy $E^{(\alpha)}$ in the $k_\parallel$-filtering effect is much larger than that relevant to QW excitons [in Eq. (12) $M_x$ should be replaced by $M_{MC}^{lb} \ll M_x$]. The functions $g_{MC}^{(1)}$ and $\tilde{g}_{MC}^{(1)}$ nearly coincide, if $k_B T \ll E^{(\alpha)}$ (see Fig. 4).
We qualitatively explain a sharp increase of the coherence length with decreasing temperature, found in the experiments with GaAs coupled quantum wells, by combining the $k_\parallel$-filtering effect with screening of disorder by dipole-dipole interacting indirect excitons. The screening of the random in-plane potential $U_{\text{rand}}(r_\parallel)$ can be quantified by replacing $U_{\text{rand}}$ with $U_{\text{eff}} = U_{\text{rand}}(r_\parallel)/(1 + (2/\pi)(u_0 M^* \chi^2/\hbar^2)(e^{T_0^*/T} - 1)) \simeq U_{\text{rand}}(r_\parallel) [(k_B T)/(k_B T + u_0 n_2 d)] \quad [17]$.

In high-quality GaAs coupled QWs the screening process effectively develops at $T \lesssim 5$ K, giving rise to a well-defined single-particle momentum $\hbar k_\parallel$, as has been observed, e.g., in the experiments. Thus the large correlation length $\xi = \xi_\gamma \sim 1 \mu$m of indirect excitons, which strongly depends on $\alpha$, can naturally be explained by the $k_\parallel$-filtering effect and can occur even for the Maxwell-Boltzmann distributed particles. In order to see an increase of $\xi$ due to quantum statistics, the bath temperature should be decreased to tens of mK.

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