Mixed fluid cosmological model in $f(R,T)$ gravity

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We construct Locally Rotationally Symmetric (LRS) Bianchi type-I cosmological model in $f(R,T)$ gravity by employing a time varying deceleration parameter (DP). We observe through the behavior of the state finder parameters $(r,s)$ that our model begins from the Einstein static era and goes to ΛCDM era. The EoS parameter $(\omega)$ for DE varies from phantom $(\omega < -1)$ phase to quintessence $(\omega > -1)$ phase which is consistent with the observational results. It is found that the discussed model can reproduce the current accelerating phase of expansion of the universe.

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I. INTRODUCTION

The latest cosmological observation detects the expansion of the universe as an accelerating rate [1,2]. This has led us to consider the exotic matter, dark energy, as a clarification and proof of this late time acceleration. DE existence has been supported by various observational data which includes Cosmic Microwave Background (CMB) anisotropy [3,4], Large Scale Structure (LSS) [5,6], Sloan Digital Sky Survey (SDSS) [7,8], Wilkinson Microwave Anisotropy Probe (WMAP) and Chandra X-ray observatory [9]. A further investigation has established our universe composition as 73% dark energy, 23% dark matter and only 4% as baryonic matter. DE is a scalar field of cosmological nuclear energy [31] and cosmological constant [32–35]. Out of these, the approach of cosmological constant is the simplest and most general to explain the acceleration but it incorporates the problems related to cosmological coincidence and fine-tuning [33,36,37]. However, these choices are insufficient to explain the mystery of dark energy completely. DE model is outlined using the Equation of State (EoS) parameter.

DE can be explained in two ways. The first one is by choosing any of the exotic matter options viz. quintessence [15–17], phantom [18], k- essence [19–21], tachyons [22], quintom [23], chaplygin gas [24–29], chameleon [30], cosmological nuclear energy [31] and cosmological constant [32–35]. Out of these, the approach of cosmological constant is the simplest and most general to explain the acceleration but it incorporates the problems related to cosmological coincidence and fine-tuning [33,36,37]. However, these choices are insufficient to explain the mystery of dark energy completely. DE model is outlined using the Equation of State (EoS) parameter $\omega$ which defined as in terms of pressure and energy density such that: $\omega(t) = \frac{p}{\rho}$. This parameter need not be a constant [38]. It can be parametrized as in terms of time or scale factor $(a)$ or redshift $(z)$.

The second way to explain DE is by modifying the theory of gravity. These theories serve natural gravitational alternatives to DE and attempt to justify current acceleration. Various modified theories are $f(R)$, $f(G)$, $f(T)$ and $f(R,T)$. The generalization of the Lagrangian in Einstein-Hilbert action where, function $f(R)$ is used instead of $R$, Ricci scalar gives $f(R)$ theory of gravity [39]. This theory serves as a consolidation of early time inflation with late time acceleration. The model handles higher order curvature invariants as a function of $R$. A further generalization of $f(R)$ gravity theory yields $f(R,T)$ theory which was originally introduced by Harko et al. [40]. The authors considered Lagrangian density as a function $f(R,T)$, where $T$ denotes the trace of energy momentum tensor. This model, contrast to other theories, discuss matter and geometry coupling. This results in source term independence, where source term is the matter stress energy tensor variant. They claim that cosmic acceleration is also a result of matter content besides geometrical input. Thereafter, many researchers are interested to do more investigations of this theory through various aspects [41–48]. In [49] FLRW cosmological model has been studied in the framework of $f(R,T)$ gravity through phase space analysis. We can see in refs. [50–52] it has been studied for different matter components. Recently, V. Fayaz et al. [53] studied Bianchi-I space-time in this theory where they regenerated $f(R,T)$ function using holographic dark energy. They reproved that the rate of evolution of the anisotropic universe is greater than that of FRW and ΛCDM model.

Yadav et al. [54] discussed the DE model in Bianchi type-III universe with constant DP $(q)$. The EoS parameter $\omega$ is established as a time dependent factor in the respective case. Naidu et al. examined spatially homogeneous and anisotropic Bianchi type II [55] and III [56] models based on Saez and Ballester theory. Based on the same theory, the authors also investigated Bianchi type V [57] model with variable $\omega$ and constant $q$. Kotambkar et al. [58]...
constructed anisotropic Bianchi type I model with bulk viscosity and quintessence and discussed various physical properties of the model. Singh et al. \[59\] examined Bianchi type II model for a perfect fluid source in $f(R, T)$ gravity. The solutions were obtained using the power law relation between mean Hubble parameter ($H(t)$) and average scale factor ($a(t)$). The same conditions were worked upon by Reddy et al. \[60\] using the special law of variation for Hubble’s parameter given by Berman \[61\]. The special law generates constant DP which implies exclusion of open universes. Samanta \[62\] constructed a model of the universe filled with dark energy from a wet dark fluid in $f(R, T)$ gravity. Samanta and Dhal \[63\] studied Bianchi type- V universe with a binary mixture of perfect fluid and dark energy in $f(R, T)$ gravity. Sahoo and Mishra \[64\] investigated Kaluza-Klein dark energy model in the form of wet dark fluid in $f(R, T)$ gravity. Singh and Sharma \[59\] constructed Bianchi type-II dark energy cosmological model with variable EoS parameter in $f(R, T)$ gravity. By considering constant DP they obtained two models of the universe, namely, power law model and exponential model. Yadav et al \[65\] obtained dark energy dominated universe with variable EoS parameter in $f(R, T)$ gravity. Sahoo \[69\] considered Kaluza-Klein universe filled with wet dark fluid in $f(R, T)$ gravity with variable DP. Chaubey et al. \[68\] considered general class of Bianchi cosmological models in $f(R, T)$ gravity with the dark energy in the form of standard and modified Chaplygin gas. Sahoo \[69\] considered Kaluza-Klein universe filled with wet dark fluid in $f(R, T)$ gravity and obtained the exact solutions from a time varying DP.

In this work, we use both the approach concurrently. That is we considered the source of gravitational matter as a mixture of perfect fluid and dark fluid in a modified theory called $f(R, T)$ theory. This type of simultaneous use of both the approach have already been used by the authors \[70\] and \[71\].

The work being organized in the following manner: In Section-I, Introduction and motivations from the literature are briefly elaborated. Section-II contains the basic formalism of $f(R, T)$ gravity general field equations. The solution of the field equation for LRS Bianchi type-I metric by employing time varying DP are presented in Section-III. At last the Physical behavior of the model and conclusions are outlined in Section-IV and Section-V respectively.

**II. THE $f(R, T) = R + 2f(T)$ GRAVITY**

Field equation for $f(R, T)$ gravity can be formulated from the Hilbert-Einstein in the following manner.

$$S = \frac{1}{16\pi G} \int \sqrt{-g} f(R, T) d^4x + \int \sqrt{-g} L_m d^4x,$$

where $L_m$ is the matter Lagrangian density, $g$ is the determinant of the metric tensor $g_{ij}$, $R$ is Ricci scalar and $T$ is the trace of energy-momentum tensor $T_{ij}$. The energy-momentum tensor $T_{ij}$ is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}.$$  

Here, Instead of considering the derivative of matter Lagrangian, we have assumed that the matter Lagrangian $L_m$ depends only on the metric components. Such as

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}}.$$  

The $f(R, T)$ gravity field equations are obtained from the eqn. \[1\] by varying the action $S$ with respect to metric component. It is given as

$$f_R(R, T)R_{ij} - \frac{1}{2} f(R, T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R, T) = 8\pi T_{ij} = f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij},$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}},$$

and $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$, $\Box \equiv \nabla^i\nabla_i$, where $\nabla_i$ is the covariant derivative.

To construct different kind of cosmological models according to the choice of matter source, Harko et al. \[40\] constructed three types of $f(R, T)$ gravity as follows

$$f(R, T) = \begin{cases} 
R + 2f(T) 
\quad f_1(R) + f_2(T) 
\quad f_1(R) + f_2(R)f_3(T) 
\end{cases}$$
The general field equation for first frame of \( f(R, T) = R + 2f(T) \) gravity is given as

\[
R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} + f(T)g_{ij}, \tag{7}
\]

### III. FIELD EQUATIONS AND SOLUTIONS

We consider the spatially homogeneous LRS Bianchi type-I metric as

\[
ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2,
\]

where \( A, B \) are functions of cosmic time \( t \) only. The stress-energy momentum tensor is in the form

\[
T^{ij} = \text{diag}[\rho_m, -p_m, -p_m, -p_m],
\]

and

\[
T^{(de)}_{ij} = \text{diag}[\rho_d, -p_d, -p_d, -p_d],
\]

where \( p_m, \rho_m \) are pressure and energy density for perfect fluid and \( p_d, \rho_d \) are pressure and the energy density for dark energy components respectively.

The field eqn. (7) with \( f(T) = \alpha T \), where \( \alpha \) is an arbitrary constant, becomes

\[
R_{ij} - \frac{1}{2} R g_{ij} = (8\pi + 2\alpha) T_{ij} + (2\alpha p + \alpha T) g_{ij}. \tag{12}
\]

In the framework of \( f(R, T) \) gravity, in the term \( 2\alpha p + \alpha T \), \( p \) is the isotropic pressure and \( T \) is the trace of energy-momentum tensor. According to [72] and [73] the trace of energy momentum tensor is of isotropic pressure and energy density i.e. \( T = \rho - 3p \).

The field eqn. (12) for the line element (8) is given as

\[
-H_1 - H_2^2 - H_2 - H_1 H_2 = (8\pi + 2\alpha)(p_m + p_d) - \alpha(p_m - p_m), \tag{13}
\]

\[
-2H_1 - 3H_2^2 = (8\pi + 2\alpha)(p_m + p_d) - \alpha(p_m - p_m), \tag{14}
\]

\[
2H_1 H_2 + H_2^2 = (8\pi + 2\alpha)(p_m + p_d) + \alpha(p_m - p_m). \tag{15}
\]

Here \( H_1 = \dot{A}/A \), \( H_2 = \dot{B}/B \) and the over dot represent derivatives with respect to cosmic time \( t \). We have six unknowns \( H_1, H_2, \rho_m, \rho_d, p_m, \) & \( p_d \) and three equations. In order to obtain the exact solution, we have assumed in first step the Bianchi identity \( G_{ij}^{DE} = 0 \) as it is followed from the definition of the Einstein tensor \( G_{ij} \) and \( R_{ij} \) [74]. From which we have obtained the following relation.

\[
\dot{\rho}_m + 3(1 + \omega_m)\rho_m H = 0, \tag{16}
\]

where \( H = \frac{\dot{a}}{a} = \frac{1}{3}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}) \) is mean Hubble parameter, \( a = (A^2 B^2)^{\frac{1}{4}} \) is the average scale factor and \( \omega_m = \frac{p_m}{\rho_m} \) is EoS parameter of perfect fluid considered as a constant [75]. From eqn. (16) we obtain the value of \( \rho_m \) as

\[
\rho_m = c_1 a^{-3(1+\omega_m)}, \tag{17}
\]

where \( c_1 \) is an integration constant. Following [76] we have considered the time varying DP of the form

\[
q = -\frac{a\dot{a}}{a^2} = b(t), \tag{18}
\]

This geometric parameter has vital role in the description of the evolution of the universe, which defines the phase transition of the universe from past decelerating expansion to the recent accelerating one. Thus, it is well motivated to consider a time-dependent DP \( q \) is due to the fact that the universe exhibits phase transitions, as revealed by the cosmic observations of SNe Ia. Also, the transitional phase of the universe can be determined by the signature flipping nature of DP, i.e. positive DP defines decelerating phase and negative sign of DP represents the accelerating phase of
late universe. Thus, the choice of time-dependent DP is physically reliable for cosmological models. The expression of \( q \) given in eqn. (18) can be written as

\[
\frac{\ddot{a}}{a} + b \frac{\dot{a}^2}{a^2} = 0. \tag{19}
\]

By assuming \( b = b(a) \) or \( b = b(a(t)) \), the general solution of (19) is given as

\[
\int e^{f t} \, dt = t + c,
\]

where \( c \) is a constant of integration. In order to derive the solution (20), without any loss of generality, one can choose \( \int \frac{1}{2} \, dt = \ln f(a) \), and eqn. (20) yields

\[
\int f(a) \, da = t + c. \tag{21}
\]

In this eqn. (21), the arbitrary function \( f(a) \) can be chosen in such a way that, it will provide a physically viable and observationally consistent cosmological model. Thus, \( f(a) \) is considered as

\[
f(a) = \frac{n a^{n-1}}{\beta \sqrt{1 + a^{2n}}}, \tag{22}
\]

where \( \beta \) is an arbitrary constant and \( n \) is a positive constant. Using eqn. (22) in eqn. (21) and taking \( c = 0 \), we obtain the following exact solution is

\[
a(t) = \sinh(\beta t)^{\frac{1}{n}}. \tag{23}
\]

Then the directional scale factors \( A \) and \( B \) are derived from the relation \( V = a^3 \) as follows

\[
A(t) = \sinh(\beta t)^{\frac{1}{n}}, \tag{24}
\]
\[
B(t) = \sinh(\beta t)^{\frac{2}{n}}. \tag{25}
\]

Now the line element (8) can be rewritten as

\[
ds^2 = dt^2 - [\sinh(\beta t)]^{\frac{1}{n}} (dx^2 + dy^2) - [\sinh(\beta t)]^{\frac{2}{n}} \, dz^2. \tag{26}
\]

From equations (14) and (15), the values of \( p_d, \rho_d \) and \( \omega_d \) are obtained as

\[
\rho_d = \frac{c_1(\alpha(\omega_m - 3) - 8\pi)(\frac{1}{z+1})^n - 3(\omega_m + 1)}{2(\alpha + 4\pi)} + \frac{9\beta^2((\frac{1}{z+1})^{-2n} + 1)}{4n}, \tag{27}
\]

\[
p_d = \frac{c_1(-3\alpha\omega_m + \alpha - 8\pi\omega_m)(\frac{1}{z+1})^n - 3(\omega_m + 1)}{2(\alpha + 4\pi)} - \frac{\beta^2((\frac{1}{z+1})^{-2n} - 3(\frac{1}{z+1})^{2n} + 1)}{4n}, \tag{28}
\]

\[
\omega_d = -\frac{4c_1 n^2(\alpha(3\omega_m - 1) + 8\pi\omega_m)(\frac{1}{z+1})^{2n} + \beta^2(3(\frac{1}{z+1})^{2n} - 4n + 3) ((\frac{1}{z+1})^n - 3(\omega_m + 1))}{9\beta^2((\frac{1}{z+1})^{2n} + 1) - 4c_1 n^2(\alpha(\omega_m - 3)) ((\frac{1}{z+1})^{2n} + 1)}. \tag{29}
\]

The variation of energy density, pressure and equation of state (EoS) parameter with cosmic time \( t \) are shown in the following figures. In Fig. 1 the energy density \( \rho_d \) is a positive decreasing function of time and tends to zero at \( t \) tends to \( \infty \). As per the observation, the negative pressure is due to DE in the context of accelerated expansion of the universe. Hence, the behavior of pressure in our model agrees with this observation. In Fig. 2 the EoS parameter lies in the accelerated phase dominated by DE era. From Fig. 3 one can observe that the EoS parameter shows a transitional behavior. In Figs. 2, 4 and 5 the variation of energy density, pressure and equation of state (EoS) parameter with redshift parameter \( z \) are depicted respectively, and it provides the reliability of the model.
IV. PHYSICAL PROPERTIES OF THE MODELS

The values of various physical parameters viz. DP, energy density of perfect fluid ($\rho_m$), mean Hubble parameter ($H$), expansion scalar ($\theta$), shear scalar ($\sigma^2$) and mean anisotropic parameter ($A_m$) for the model are obtained as

$$q = n \text{sech}^2(\beta t) - 1 = \frac{n}{\left(\frac{1}{z+1}\right)^{2n} + 1} - 1,$$

(30)

$$\rho_m = c_1 \sqrt{\sinh(\beta t) - 3\omega - 3} = c_1 \left(\beta \left(\frac{1}{z+1}\right)^n \right)^{-\frac{3\omega - 3}{n}},$$

(31)

FIG. 1: Variation of $\rho$ against time with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$.

FIG. 2: Variation of $\rho$ against $z$ with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$.

FIG. 3: Variation of $p$ against time with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$.

FIG. 4: Variation of $p$ against $z$ with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$.

FIG. 5: Variation of Eos Parameter against time with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$.

FIG. 6: Variation of Eos Parameter against $z$ with $\beta = 0.09804$, $n = 1.15$, $c_1 = 0.001$, $\omega_m = 0.5$. 
One of the important quantities for the dynamical description of the universe is known as state finder pair or $r-s$ parameter. It helps to study the coincidence between obtained model with ΛCDM model. For flat ΛCDM model, the value of state finder pair yields as $\{r, s\} = \{1, 0\}$ [77]. The values of the $r-s$ parameter of our model becomes

$$r = \frac{\ddot{a}}{aH^3} = n(2n - 3) \text{sech}^2(\beta t) + 1 = \frac{n(2n - 3)}{(\frac{1}{z+1})^{2n} + 1},$$

$$s = \frac{r - 1}{3(q - \frac{1}{2})} = \frac{n(2n - 3) \text{sech}^2(\beta t)}{3(n \text{sech}^2(\beta t) - 1.5)} = \frac{2(3 - 2n)n}{(\frac{1}{z+1})^{2n} - 6n + 1}.$$  

The matter energy density ($\Omega_m$) and dark energy density ($\Omega_d$) are obtained as

$$\Omega_m = \frac{c_1n^2 \left(\frac{1}{z+1}\right)^{2n} \left[\left(\frac{1}{z+1}\right)^n\right]^{1/n} - 3\omega_m - 3}{3\beta^2 \left(\frac{1}{z+1}\right)^{2n} + 1},$$

$$\Omega_d = \frac{\left(\frac{1}{z+1}\right)^{3(\omega_m+1)}}{n} \left[9\beta^2 \left(\frac{1}{z+1}\right)^{2n} + 1 \left(\frac{1}{z+1}\right)^n \frac{3(\omega_m+1)}{n} - 4c_1n^2(8\pi - \alpha(\omega_m - 3)) \left(\frac{1}{z+1}\right)^{2n}\right].$$

Adding eqns. (38) and (39) we get the total energy ($\Omega$) as
\[ \Omega = \frac{n^2 \left( \frac{1}{\alpha + 2} \right)^{2n}}{3\beta^2 \left( \frac{1}{\alpha + 2} \right)^{2n} + 1} \left[ c_1 (\alpha (\omega_m - 3) - 8\pi) \left( \frac{1}{\alpha + 2} \right)^n \right]^{3(\omega_m + 1)} - \frac{9\beta^2 \left( \frac{1}{\alpha + 2} \right)^{2n} + 1}{4n^2} + c_1 \left( \frac{1}{\alpha + 2} \right)^{3/n} - 3n \right]^{-3\omega_m} \]

From eqn. (30) we find that \( q < 0 \), hence the model represents an accelerating universe. Since \( \lim_{t \to \infty} \frac{\sigma^2}{\Omega^4} \neq 0 \) the universe is anisotropic throughout the evolution. Fig. 7 shows the variation of \( s \) with respect to \( r \). It is clear from this figure that \( s \) is negative when \( r \) is greater than one. As \( r \to \infty, s \to -\infty \) and when \( r = 1 \) we have \( s = 0 \). Hence the universe starts from the Einstein static era and goes to \( \Lambda CD M \) era.

V. CONCLUSION

The LRS Bianchi type-I cosmological model in \( f(R, T) \) gravity theory is constructed in this paper with the exact solutions of the field equations. In \( f(R, T) \) gravity, cosmic acceleration depends on geometric contribution as well on matter content of the universe. Cosmological models with a source of dark energy yield a very good approximation to the accelerated expansion of the universe. As a result, in this obtained model we can see the behavior of the energy density and pressure with respect to time in Fig. 1 and Fig. 3 and with respect to redshift parameter \( z \) in Fig. 2 and Fig. 4. From Fig. 1 one can observe that at initial epoch the energy density of the universe is very high. As time increases it decreases and approaches to zero when \( t \to \infty \). Energy density remains positive throughout the evolution of the universe. From Fig. 3 we see that the universe starts with a very large negative pressure, decreases with increase in cosmic time \( t \) and approaches to zero for large \( t \). This reveals the characteristic behavior of the dark energy. The nature of EoS parameter \( (\omega) \) for DE with the evolution of cosmic time \( t \) is shown in Fig. 5 and with respect to redshift parameter \( z \) is shown in Fig. 6. The parameter \( \omega > -1 \) and \( \omega < -1 \) correspond quintessence and phantom energy respectively attributes present accelerated expansion of the universe. Also, The evolution of energy density, pressure and EoS parameter correspond to redshift parameter \( z \) are depicted in details in the Figs. 2, 4, and 6 respectively. It can be observed that the universe is dominated by dark energy which may be the strongest evidence for present cosmic expansion. All of the solutions obtained are consistent with the observational results. Hence we feel that these results will be helpful for the researchers to realize the characteristics of the universe in the framework of \( f(R, T) \) theory.

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