The Possible Shapes of Numerical Ranges

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Recommended Citation
Helton, J. W., & Spitkovsky, I. M. (2011). The possible shapes of numerical ranges. arXiv preprint arXiv:1104.4587.

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THE POSSIBLE SHAPES OF NUMERICAL RANGES

J. WILLIAM HELTON AND I. M. SPITKOVSKY

Abstract. Which convex subsets of \( \mathbb{C} \) are the numerical range \( W(A) \) of some matrix \( A \)? This paper gives a precise characterization of these sets. In addition to this we show that for any \( A \) there exists a symmetric \( B \) of the same size such that \( W(A) = W(B) \) thereby settling an open question from [2].

Mathematics subject classification (2010): Primary 47A12.

Keywords and phrases: Numerical range, linear matrix inequalities.

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