Unit commitment direct current optimal power flow using mixed-integer linear programming

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Abstract. This paper presents unit commitment direct current optimal power flow (UCDCOPF) using mixed-integer linear programming (MILP) for daily power system generation planning. This method is used to solve optimum operation in an electric power system by scheduling generator status and calculating the optimal power output of each generator in a power system. The calculation of this method considers several constraints in the power system. A modified IEEE 24 bus system is used as a test system to show the ability of the proposed approach.

1. Introduction

The main objective of power system operation is to minimize the operating cost. In achieving this, the optimization must not violate all constraints involved in the system. There are some operation level methods which are applied in the power system. Those operations are unit commitment (UC), economic dispatch (ED), and optimal power flow (OPF) [1].

Unit commitment (UC) is one of the power system operations to schedule all generators optimally in the system to meet the electricity demand. There are several operational constraints of the power system that must be accounted for UC. Those are minimum uptime, minimum downtime, startup cost, and shutdown cost. Minimum uptime is the minimum operation time of the unit to operate, whereas minimum downtime is the minimum operation time to not operate during the operation. The startup cost is an additional cost which is added to the unit when it is operated for the first time, whereas shutdown cost is an additional cost which added to the unit when it is committed from online to the offline condition [2].

Another power system operation is an economic dispatch (ED). This optimization is used to adjust the output of each online generator optimally. The constraints which are considered in ED are minimum stable generation, maximum generation, and power system balance. Minimum stable and maximum generation is a condition in which the online unit must inject the power with the value between both conditions. Power system balance is a condition in which the total supplied power to the system is equal to the system demand. Also, optimal power flow (OPF) is a development of ED by adding a transmission constraint [3].

Many studies related to power system operation has been conducted. R.S. Wibowo et al. [4–6] developed a dynamic DC optimal power flow using quadratic programming. This method solved OPF with multi-load levels by adding ramp rate constraint. V.S. Borra and K. Debnath [7] analyzed the comparison between dynamic programming and particle swarm optimization for solving unit commitment problems. Their simulation results demonstrated that the PSO technique is more accurate.
than DP technique in solving UC problems. T. Kim, G. Won and Y. Chung [8] designed an optimal power dispatch integrated with the unit using dynamic programming.

According to the existing result, in this paper, an integrated unit commitment direct current optimal power flow (UCDCOPF) will be calculated using a high-performance solver, mixed-integer linear programming (MILP), to achieve a better optimal result. To show the ability of the proposed approach, a modified IEEE 24 bus system is used as a test system.

2. Unit Commitment Direct Current Optimal Power Flow (UCDCOPF) Formulation

2.1. Objective Function

The objective of UCDCOPF is formulated as the minimization of \( f(x) \) as presented in the equation below [9,10].

\[
f(x) = f_p(p) + f_{uc}(v,w)
\]

Where,

- **power dispatch cost:**

\[
f(p) = \sum_{t \in T} \sum_{i \in I} [C_{pi}^t(p^t_i)]
\]

\( t \): Index of periods

\( T \): Set of indices of periods

\( i \): Index of units

\( I \): Set of indices of units

\( C_{pi}^t \): Cost function of real power injection

\( p^t_i \): Real power injection

- **startup and shutdown cost:**

\[
f_{uc}(v,w) = \sum_{t \in T} \sum_{i \in I} (C_{vi}^t v^t_i + C_{wi}^t w^t_i)
\]

\( C_{vi}^t \): Startup cost

\( C_{wi}^t \): Shutdown cost

\( v^t_i \): Binary startup state

\( w^t_i \): Binary shutdown state

2.2. DCOPF Constraints

The constraint of DCOPF consists of power balance and transmission limit. The power balance defines that the total load and the total generation must be equal to zero, while the transmission limit defines that the power flow in the branch must be below the line capacity limit. The formulation of these constraints can be seen as follows [1,2].

- **power balance**

\[
g^t(p^t, \theta^t) = P_{Gn}^t + P_{Ln}^t + P_{Inn}^t = 0
\]

\[
P_{Inn}^t = \frac{1}{x_{nm}} (\theta_n^t - \theta_m^t)
\]

- **transmission limit**

\[
h^t(p^t, \theta^t) \leq 0
\]

\[
F_{nm}^t \leq F_{nm}^{max}
\]

Where,
\( g^t \): DC power flow equations
\( h^t \): Transmission limit
\( p^t, \theta^t \): Active power injection and voltage angle
\( p^t_{Gn} \): Power generation at bus \( n \)
\( p^t_{Ln} \): Power demand at bus \( n \)
\( p^t_{INn} \): Sum of power flows at transmission lines connected to bus \( n \)
\( F^t_{nm} \): Power flow at branch \( nm \)
\( F^t_{nm \text{ max}} \): Line capacity of branch \( nm \)
\( x^t_{nm} \): Reactance of branch \( nm \)
\( \theta^t_n \): Voltage angle at bus \( n \)
\( \theta^t_m \): Voltage angle at bus \( m \)

2.3. Unit Commitment Constraints
The purpose of unit commitment is to schedule all generations optimally in the power system for a given period. Several constraints of unit commitment, including maximum and minimum power generation, minimum uptime, minimum downtime, startup and shutdown condition, are considered as presented in the equation below [1,2].

- Maximum generating capacity, minimum stable generation and commitment
  \( u^t p^t_{\text{min}} \leq p^t_i \leq u^t p^t_{\text{max}} \) (8)
- Minimum up and downtime
  \( u^t i - u^{(t-1)} i = v^t i - w^t i \) (9)
- Startup and shutdown condition
  \( \sum_{y=t-i=+} v^y i \leq u^t i, \sum_{y=t-i=-} w^y i \leq 1 - u^t i \) (10)

Where,
\( u^t i \): Binary commitment state
\( v^t i \): Binary startup state
\( w^t i \): Binary shutdown state
\( p^t_{\text{min}} \): Minimum real power injection
\( p^t_{\text{max}} \): Maximum real power injection
\( \tau^+_i \): Minimum uptime
\( \tau^-_i \): Minimum downtime

2.4. Intertemporal Constraints
The intertemporal constraints in this system define the upward and downward ramp rate of the unit in each period as presented in the equation as follows [1,2].

- Ramp rate limit
  \( 0 \leq \delta^t_i \leq \delta^t_{\text{max}+} \) (11)
  \( 0 \leq \delta^t_{i+} \leq \delta^t_{\text{max}^-} \) (12)

Where,
\( \delta^t_i \): Upward load-following ramping reserve
\( \delta^t_{i+} \): Downward load-following ramping reserve
\( \delta^t_{\text{max}+} \): Maximum upward load-following ramping reserve
\( \delta^t_{\text{max}^-} \): Maximum downward load-following ramping reserve
3. Study Simulation

The UCDCOPF problem is a mixed-integer linear programming (MILP) problem. The definition of MILP is a problem with conditions as follows [11].

\[
\begin{align*}
\min_x & \quad f^T x \\
\text{subject to} & \quad \begin{cases}
(\text{x are integers}) \\
A \cdot x \leq \text{beq} \\
A_{eq} \cdot x = \text{beq} \\
\text{lb} \leq x \leq \text{ub}
\end{cases}
\end{align*}
\]

1. The linear objective function, \( f^T x \), where \( f \) is a column vector of constants, and \( x \) is the column vector of unknowns.
2. Bounds and linear constraints, but no nonlinear constraints.
3. Restrictions on some components of \( x \) to have integer values.

The problem formulation of UCDCOPF is implemented using MATLAB. Several toolboxes are used, including MATPOWER Optimal Scheduling Toolbox (MOST) and CPLEX to perform the MILP problem. The use of MOST in this simulation is to program the UCDCOPF. On the other hand, CPLEX is one of the high-performance solvers added in MATLAB to perform MILP calculation.

In this simulation, a modified IEEE 24 bus system is used [12]. This system consists of 12 thermal generation units and six wind farms. To show the effectiveness of UCDCOPF method, its result will be compared with dynamic DCOPF which uses linear programming for optimization and ignores the unit commitment constraints.

4. Result and Analysis

The main results of UCDCOPF are generation scheduling, power generation units and operation cost which can be seen in Table 1, Figure 1 and Figure 3. On the other hand, the main result of dynamic DCOPF are only power generation unit and operation cost which can be seen in Figure 2 and Figure 3. This is because the dynamic DCOPF method ignores the unit commitment constraints so that all the generation units are online over the 24 hours.

| Table 1. Generation scheduling result of UCDCOPF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Unit i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Gen1 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen2 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen3 | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF |
| Gen4 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen5 | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF |
| Gen6 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen7 | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF | OFF |
| Gen8 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen9 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen10 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen11 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
| Gen12 | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON | ON |
Figure 1. Power generation units of UCDCOPF

Figure 2. Power generation units of dynamic DCOPF

Figure 3. Operation cost of UCDCOPF and dynamic DCOPF
Based on the result, it can be seen that the UCDCOPF and dynamic DCOPF have different simulation results on generation scheduling, power generation units, and operation cost. This is because the UCDCOPF considers more constraints than the dynamic UCDCOPF, specifically the unit commitment constraints such as startup cost, shutdown cost, minimum uptime and minimum downtime. As a result, this will affect the value of power generation units and operation costs.

The most important result in this simulation is the operating cost, respectively $475392.30 for the UCDCOPF and $495465.90 for dynamic DCOPF. Based on Figure 3, it can be seen that the UCDCOPF has a lower cost than dynamic DCOPF with a difference of $20073.60.

5. Conclusions
In this paper, a unit commitment direct current optimal power flow (UCDCOPF) has been simulated and calculated by using the mixed-integer linear programming. Based on the simulation result, it can be concluded that the UCDCOPF has a more optimal operation cost than the dynamic DCOPF.

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