Prediction of mass of $\eta_c(2S)$ using variational method

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The suitability of using non-relativistic quantum mechanics to investigate heavy quark mesons is illustrated through a study of the charmonium meson. We consider a limiting form of the QCD potential which is a simple combination of the linear and Coulomb potential. The experimentally determined masses of $J/\psi(1S)$ and $\chi_{c1}(1P)$ are reproduced for $m_c \approx 1.1\text{GeV}$. For $\psi(2S)$ we have three different sets of variational parameters and to choose the appropriate one we use the leptonic decay width of $\psi(2S)$ and $J/\psi(1S)$. Finally we use a spin-spin interaction to investigate the hyperfine splitting of Charmonium and use it to calculate the mass of $\eta_c(2S)$. Our theoretical results agree with the experimentally measured values of $\eta_c(2S)$ and thereby verifies the usefulness of non-relativistic quantum mechanics in the study of heavy quark meson.

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I. INTRODUCTION

With the experimental discovery of $J/\psi$ particle [1,2] and the correct identification of it with a bound state of charm quark and its anti-particle [3], the study of bound states using non-relativistic quantum mechanics acquired a renewed interest. The basic idea is to solve the non-relativistic Schrödinger equation using an interaction potential and fit the bound state energy to the mass of $J/\psi$ system. However, unlike the electromagnetic interaction for which the interaction potential is known to be the Coulomb potential, the interaction potential for the $c\bar{c}$ system is not known in its completeness. A form for the $c\bar{c}$ potential is assumed taking into account that the quarks are permanently confined inside $J/\psi$. A simple confining potential is a linear potential, $V(r) = kr$, where $k$ is a constant.

Lucha, Schöberl and Gromes [4] have suggested a Coulomb plus linear potential for the $c\bar{c}$ system as

$$\begin{align*}
V(r) &= -\frac{a}{r} + kr + V_0 
\end{align*}$$

The parameters $a$, $k$ and $V_0$ have been fitted with $c\bar{c}$ system with the mass of the charmed quark taken as $\approx 1.64$ GeV and their values are:

$$\begin{align*}
a &= 0.27\text{(dimensionless)}, \\
k &= 0.25 GeV^2, \\
V_0 &= -0.76 GeV.
\end{align*}$$

Recently Sumino [5] has analysed QCD potential taken as an expectation value of the Wilson loop and showed that in the large $N$ limit, the QCD potential behaves like a Coulomb and linear potential. In view of this we take the $c\bar{c}$ potential to be

$$\begin{align*}
V(r) &= -\frac{a}{r} + kr
\end{align*}$$

with $a$ and $k$ as given (2). We do not include $V_0$ and so consistently treat the mass of the charmed quark as a parameter to be determined.

An exact analytic solution of the Schrödinger equation with the potential (3) is not possible and so, we use “Variational method”. Since the potential (3) is spherically symmetric, the radial wavefunction is guided by known solutions of the $1/r$ potential for bound states. The motivations of this study are:

1. To determine the mass of the charmed quark by fitting the $J/\psi$ mass spectrum [6], which is

$$\begin{align*}
J/\psi(1S) &= 3.09687 \pm 0.00004 \text{ GeV}, \\
\chi_c(1P) &= 3.51051 \pm 0.00012 \text{ GeV}, \\
\psi(2S) &= 3.68596 \pm 0.00009 \text{ GeV}.
\end{align*}$$

2. The $J/\psi(1S)$ and $\psi(2S)$ states are spin triplet states. The recent Belle collaboration [7] observed a new charmonium state $\eta_c(2S)$, in which the spins of $c\bar{c}$ form a singlet. The value of the mass for $\eta_c(1S) = 2.9797 \pm 0.0015 \text{ GeV}$ [6]. These results show a “hyperfine splitting”, as $J/\psi(1S)$ and $\eta_c(1S)$ have different masses, although both are $c\bar{c}$ bound states. This cannot be explained by the potential model in Eqn. (3). In this work, we introduce a spin-spin interaction and use that to explain the hyperfine splitting. The second motivation is to determine the mass of $\eta_c(2S)$ from the results of hyperfine splitting analysis, consistent with the leptonic decay widths of $J/\psi$ system.

In Section II, the energies of $J/\psi$ system are calculated for the Coulomb plus Linear potential in variational method. For $\psi(2S)$ state, we get three set of values for the variational parameters. In Section III, the ratio of the leptonic decay width of $J/\psi(1S)$ and $\psi(2S)$ are calculated for the three sets. Comparison with the data, allows us to choose one of the three sets. In Section IV, this set is used to explain the hyperfine splitting after introducing a spin-spin contact interaction and the mass of $\eta_c(2S)$ is predicted. This prediction agrees with the recent experimental values. The results are summarized in Section V.
II. COULOMB PLUS LINEAR POTENTIAL PREDICTION OF CHARMONIUM SPECTRUM

The non-relativistic Schrödinger equation for the $c\bar{c}$ system is given by

$$\left( -\frac{\hbar^2}{2\mu} \nabla^2 + V(r) \right) \psi = E\psi$$  \hspace{1cm} (4)

where $\mu$ is the reduced mass of the $c\bar{c}$ and is equal to $m_c/2$, with $m_c$ as the mass of the Charmed quark. Using Spherical polar co-ordinates the Eqns. (3) and (4) are reduced to radial equations. The radial function $U_{nl}(r)$ satisfies

$$\frac{d^2U_{nl}}{dr^2} + \frac{2\mu}{\hbar^2} \left( E_{nl} - Kr + \frac{a}{r} - \frac{\hbar^2(l+1)}{2\mu r^2} \right) U_{nl} = 0$$  \hspace{1cm} (5)

The choice of the trial wave function is dictated by symmetry requirements. The wave function must vanish at infinity. The wave function must have the angular part of it as $Y_{ml}(\theta, \phi)$.

A. 1S Bound State

The trial wave function is taken to be, $\psi_{1s} = R_{1s}(r)Y_{00}(\theta, \phi)$ with

$$R_{1s} = e^{-\lambda r}$$  \hspace{1cm} (6)

where $\lambda$ is the variational parameter. Then we compute the expectation value of the Hamiltonian as follows:

$$\langle H \rangle = \frac{\langle \psi_{1s} | H | \psi_{1s} \rangle}{\langle \psi_{1s} \psi_{1s} \rangle} = \frac{\hbar^2 \lambda^2}{2\mu} - a\lambda + \frac{3k}{2\lambda}$$  \hspace{1cm} (7)

On extremising with respect to $\lambda$ we obtain a cubic equation for $\lambda$ as

$$\frac{\hbar^2 \lambda^3}{\mu} - a\lambda^2 + \frac{3k}{2\lambda} = 0.$$  \hspace{1cm} (8)

Using (5) in (7), we obtain the minimum value of variational energy as

$$\langle H \rangle_{\text{min}} = \frac{9k}{4\lambda} - \frac{a\lambda}{2}$$  \hspace{1cm} (9)

On account of the choice of (6), we must use only the real root of the cubic equation (8), which is

$$\lambda = \sqrt[3]{\frac{3k\mu}{4\hbar^2}} \left[ \left( 1 + \frac{4\mu^2a^3}{81\hbar^4} \right)^{\frac{1}{3}} + \left( 1 + \frac{8\mu^2a^3}{81\hbar^4} \right)^{\frac{1}{3}} \right] + \frac{\mu a}{3\hbar^2}$$  \hspace{1cm} (10)

which expresses $\lambda$ in terms of the potential parameters $a$ and $k$. We shall then use $\hbar = c = \mu = 1$ system of units.

Since the mass of charm quark lies between 1.0 GeV and 1.4 GeV [6], the values of $\lambda$ and the variational energy given by (10) is calculated for $m_c$ from 1.0 GeV and 1.4 GeV. The variational energy is added to the rest mass energy $2m_c\hbar^2$ after making conversions to CGS units. The values of the variational parameter $\lambda$ and the Energy $J/\psi(1S)$ state for values of mass $m_c$ are given in Table I.
TABLE I. Mass of \( J/\psi \) (1S) for \( m_c \) from 1 GeV to 1.4 GeV

| Mass of Charm quark \( m_c \) GeV/c\(^2\) | Variational Parameter \( \lambda \) (dimensionless) | \( J/\psi \) (1S) GeV |
|---------------------------------|---------------------------------|------------------|
| 1.0                            | 1.2420                          | 2.8218           |
| 1.1                            | 1.1720                          | 2.9855           |
| 1.2                            | 1.1119                          | 3.1530           |
| 1.3                            | 1.0599                          | 3.3254           |
| 1.4                            | 1.0142                          | 3.4964           |

From Table I and using experimental value of \( J/\psi \) (1S) = 3.09687 \( \pm 0.00004 \) GeV [6], the Charm quark mass is found to be \( m_c = 1.165 \text{ GeV/c}^2 \).

B. 1P Bound State

In close analogy with the Coulomb system, the trial wavefunction for the 1P state is taken to be, 
\[
\psi_{1P} = r e^{-\lambda r}
\]
where the variational parameter \( \lambda \) is now for the 1P-state. The 1P energy \( \langle H \rangle \) is found to be,
\[
\langle H \rangle = \frac{\langle \psi_{1S} | H | \psi_{1S} \rangle}{\langle \psi_{1S} | \psi_{1S} \rangle} = \frac{\hbar^2 \lambda^2}{2\mu} - \frac{a \lambda^2}{2} + \frac{5 k}{2 \lambda}
\]
This is extremized with respect to \( \lambda \) and we obtain a cubic equation for \( \lambda \),
\[
\frac{\hbar^2 \lambda^3}{\mu} - \frac{a \lambda^2}{2} + \frac{5 k}{2 \lambda} = 0.
\]
From (12) and (13) we find,
\[
\langle H \rangle_{\text{min}} = \frac{15 k}{4 \lambda} - \frac{a \lambda}{4}.
\]
The real root of Eqn. (14) of \( \lambda \) expressed in terms of \( a \) and \( k \) is found to be
\[
\lambda = \left[ \frac{5 k \mu}{4 \hbar^2} \right]^\frac{1}{3} \left[ 1 + \frac{\mu^2 a^3}{270 k \hbar^4} \right]^{\frac{1}{6}} + \left( 1 + \frac{\mu^2 a^3}{270 k \hbar^4} \right)^{\frac{1}{3}} \left[ 1 + \frac{\mu^2 a^3}{135 k \hbar^4} \right]^{\frac{1}{2}} + \frac{\mu a}{6 \hbar^2}.
\]

TABLE II. Mass of \( \chi_{c1} \) (1P) for \( m_c \) from 1 GeV to 1.4 GeV

| Mass of Charm quark \( m_c \) GeV/c\(^2\) | Variational Parameter \( \lambda \) (dimensionless) | \( J/\psi \) (1S) GeV |
|---------------------------------|---------------------------------|------------------|
| 1.0                            | 1.4022                          | 3.2899           |
| 1.1                            | 1.3186                          | 3.4437           |
| 1.2                            | 1.2499                          | 3.6026           |
| 1.3                            | 1.1844                          | 3.7657           |
| 1.4                            | 1.1295                          | 3.9323           |

From Table II and the experimental value of \( \chi_{c1} = 3.51051 \pm 0.00012 \text{ GeV} \) [6], the mass of the charmed quark is found to be \( m_c = 1.1425 \text{ GeV/c}^2 \) which is in close agreement with the result obtained for 1S state in II.1.
C. 2S Bound State

The trial wave function for the 2S state is taken to be, \( \psi(2S) = R_{2S} = R_{2S} Y_0^0(\theta, \varphi) \), with
\[
\psi_{2S} = (b - \lambda r)e^{-\lambda r}
\]
(16)
where \( b \) and \( \lambda \) are the variational parameters. The 2S energy \( \langle H \rangle \) is calculated to be
\[
\langle H \rangle = \frac{\hbar^2 \lambda^2}{8\mu}(b^2 - b + 1) + \frac{3k}{2\lambda^2}(b^2 - 4b + 5) - a\lambda(b^2 - 2b + 3/2)
\]
(17)
Extremization of Eqn. (17) with respect to \( \lambda \) and \( b \) leads to
\[
\frac{\hbar^2 \lambda}{\mu}(b^2 - b + 1) - \frac{3k}{2\lambda^2}(b^2 - 4b + 5) - a(b^2 - 2b + 3/2) = 0
\]
(18)
\[
\frac{\hbar^2 \lambda}{\mu}(-b^2 + 2b) + \frac{3k}{2\lambda^2} - a(-b^2 + 3b - 3/2) = 0
\]
(19)
On simplifying (18) using (18) and (19), the minimum value of the variational energy \( \langle H \rangle_{\text{min}} \) is found to be
\[
\langle H \rangle_{\text{min}} = \frac{9k}{4\lambda}(b^2 - 4b + 5) - \frac{a\lambda}{2}(b^2 - 2b + 3/2)
\]
(20)
From (18) and (19), we have
\[
b = \frac{3k - \lambda^3}{\lambda^3 - a\lambda^2}
\]
(21)
We have to first determine \( \lambda \) and \( b \) consistently satisfying (18) and (19). We used two independent methods for this analysis. First, \( \lambda \) is assumed to be in the range 0.1 and 2.0 through a trial and error method. For each value of \( \lambda \), Eqn. (21) determines \( b \). Then (18) and (19) are required to be satisfied. This happens for three sets of values of \( (\lambda, b) \). In the second method, a ‘random number generator’ is used to choose \( \lambda \). Determining \( b \) from Eqn. (21), the random number generator finds \( \lambda \) such that equations (18) and (19) are required to be satisfied. This gave three sets of values for \( (\lambda, b) \) which agrees with the first method. Among the three sets of values, two of them have both \( \lambda \) and \( b \) positive, meanwhile in the third set while \( \lambda \) is still positive, \( b \) was \(-ve\). Using all the three sets of results, the \( \psi(2S) \) energy is calculated.

Set 1: \( (\lambda > 0 ; b > 0) \)

| Mass of Charm quark \( m_c \) GeV/c² | Variational Parameter | Variational Parameter | \( J/\psi \) (1S) GeV |
|--------------------------------------|----------------------|----------------------|---------------------|
| 1.0                                 | 1.1322               | 1.4012               | 3.7100              |
| 1.1                                 | 1.0663               | 1.3992               | 3.8485              |
| 1.2                                 | 1.0995               | 1.3974               | 3.9927              |
| 1.3                                 | 0.9605               | 1.3956               | 4.1439              |
| 1.4                                 | 0.9171               | 1.3938               | 4.2986              |

Using the experimental value of \( \psi(2S) = 3.68596 \pm 0.00009 \text{GeV} \) [6] and Table III the values of \( b \) and \( \lambda \) are
\[
b = 1.4012 \text{dimensionless}
\]
\[
\lambda = 1.1322 \text{dimensionless}
\]
\[
m_c = 1.0 \text{GeV/c²}
\]
Set 2: \((\lambda > 0 ; b > 0)\)

### TABLE IV. Mass of \(\psi(2S)\) for \(m_c\) from 1 GeV to 1.6 GeV

| Mass of Charm quark \(m_c\) GeV/\(c^2\) | Variational Parameter \(\lambda\) (dimensionless) | Variational Parameter \(b\) (dimensionless) | \(J/\psi\) (1S) GeV |
|-----------------------------------------|----------------------------------|----------------------------------|-----------------|
| 1.0                                    | 0.9147                          | 4.1423                          | 2.8063          |
| 1.1                                    | 0.8636                          | 4.1466                          | 2.9704          |
| 1.2                                    | 0.8196                          | 4.1509                          | 3.1384          |
| 1.3                                    | 0.7815                          | 4.1551                          | 3.3094          |
| 1.4                                    | 0.7480                          | 4.1594                          | 3.4831          |
| 1.5                                    | 0.7183                          | 4.1634                          | 3.6587          |
| 1.6                                    | 0.6917                          | 4.1678                          | 3.8363          |

From the above table and using the experimental value of \(\psi(2S) = 3.68596 \pm 0.00009\) GeV \[6\] the mass of the charm quark was found to be 1.515 GeV/\(c^2\) and the corresponding values of the variational parameters are \(\lambda = 0.714078\) dimensionless and \(b = 4.164253\) dimensionless.

Set 3: \((\lambda > 0 ; b < 0)\)

### TABLE V. Mass of \(\psi(2S)\) for \(m_c\) from 1 GeV to 1.6 GeV

| Mass of Charm quark \(m_c\) GeV/\(c^2\) | Variational Parameter \(\lambda\) (dimensionless) | Variational Parameter \(b\) (dimensionless) | \(J/\psi\) (1S) GeV |
|-----------------------------------------|----------------------------------|----------------------------------|-----------------|
| 1.0                                    | 1.9392                          | -0.6839                          | 2.7831          |
| 1.1                                    | 1.8267                          | -0.6962                          | 2.9049          |
| 1.2                                    | 1.7298                          | -0.7083                          | 3.1172          |
| 1.3                                    | 1.6461                          | -0.7202                          | 3.2895          |
| 1.4                                    | 1.5720                          | -0.7319                          | 3.4638          |
| 1.5                                    | 1.5062                          | -0.7435                          | 3.6400          |
| 1.6                                    | 1.4481                          | -0.7548                          | 3.8189          |

Using the experimental value of \(\psi(2S) = 3.68596 \pm 0.00009\) GeV \[6\] and Table V, we find the mass of charm quark as 1.7425 GeV/\(c^2\) and the corresponding values of the variational parameters are, \(\lambda = 1.48996\) dimensionless, \(b = -0.74657\) dimensionless. \(\) \(\) \(\) \(\)

Thus, we have three set of results for \(\psi(2S)\) state. In order to choose one among the three sets we calculate the “Leptonic Decay Width” of \(J/\psi\) (1S) and \(\psi(2S)\) states and compare with their experimental values.

### III. LEPTONIC DECAY WIDTH

The leptonic decay width of a typical \((nS)\) state is given by \[8\]

\[
\Gamma(nS \rightarrow e^+ e^-) = \frac{4\alpha^2 e_q^2}{M_n^2} |R_n(0)|^2 \gamma_n
\]  

where \(\alpha\) is the QCD coupling strength, \(e_q\) is the charge of the quark, \(M_n\) of the \(n^{th}\) state meson, \(R_n(0)\) is the value of the \(n^{th}\) state radial wavefunction at the origin and \(\gamma_n\) is the QCD correction factor.

For the \(2S\) and \(1S\) states of \(c\bar{c}\) meson, we have

\[
R = \frac{\Gamma(\psi(2S \rightarrow e^+ e^-))}{\Gamma(\psi(1S \rightarrow e^+ e^-))} = \frac{|R_{2S}(0)|^2}{|R_{1S}(0)|^2} \times \frac{M_{2S}^2}{M_{1S}^2} \times \frac{\gamma_2}{\gamma_1}
\]

where we have assumed the fact that \(\alpha(1S)\) and \(\alpha(2S)\) are nearly equal to one another and \(\gamma_1 = 0.581\) and \(\gamma_2 = 0.606\) \[8\]. From particle physics data group \[6\], the experimental values of the leptonic decay width for \(1S\) and \(2S\) states
give the ratio

\[ R = \frac{\Gamma_\psi(2S \rightarrow e^+e^-)}{\Gamma_\psi(1S \rightarrow e^+e^-)} = 0.4163498. \]  

(26)

For the 2S state we have three set of results and so consequently three set of values for \( \lambda \) and \( b \).

We use Eqn. (25) to calculate the ratio between the 2S leptonic decay width and 1S leptonic decay width of \( c\bar{c} \), using the wave function for \( J/\psi(1S) \) and \( \psi(2S) \) after normalization with \( \lambda \) and \( b \) from the three sets as shown in the table below: From the Leptonic decay width ratios for the three sets, we find that the result for the second set agrees with the experimental value. So we choose the mass value corresponding to second set of result. The mass of charm quark is \( m_c = 1.515 \text{GeV/c}^2 \).

### IV. HYPERFINE SPLITTING

In order to provide further check on the second set for the \( \psi(2S) \) state, we consider the hyperfine splitting of charmonium levels. In the \( c\bar{c} \) system there are two types of states; the \( J/\psi(1S) \) and the \( \psi(2S) \) are spin triplet states and the \( \eta_c \) states are the spin singlet states. Their masses are not the same. This cannot be explained by the potential given in Eqn. (3). We therefore introduce a spin dependent interaction of the form \( \alpha \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r}) \) in the Hamiltonian where \( \alpha \) is taken to be a constant for simplicity [8]. So the Hamiltonian is of the form,

\[ H = \frac{p^2}{2\mu} + kr - \frac{a}{r} + \alpha \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r}) \]  

(27)

The expectation of the Hamiltonian in Eqn. (27) is

\[ E = E_0 + \alpha \langle \vec{S}_1 \cdot \vec{S}_2 \rangle \delta^3(\vec{r}). \]  

(28)

where \( E_0 \) represents the spin independent part of the energy. For the spin triplet state,

\[ \langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \frac{1}{4} \]

For the spin singlet state

\[ \langle \vec{S}_1 \cdot \vec{S}_2 \rangle = -\frac{3}{4}. \]

Then

\[ E_{1S}(J/\psi) = E_0 + \frac{\alpha}{4} [R_{1S}(J/\psi)_{r=0}]^2 \]  

(29)

\[ E_{1S}(\eta_c) = E_0 - \frac{3\alpha}{4} [R_{1S}(\eta_c)_{r=0}]^2. \]  

(30)

Similarly for 2S state,

\[ E_{2S}(\psi) = E_0 + \frac{\alpha}{4} [R_{2S}(J/\psi)_{r=0}]^2 \]  

(31)

\[ E_{2S}(\psi) = E_0 - \frac{3\alpha}{4} [R_{2S}(\eta_c)_{r=0}]^2. \]  

(32)

From the above set of relations we get

\[ E_{1S}(J/\psi) - E_{1S}(\eta_c) = 2\alpha [R_{1S}(r = 0)]^2 = \Delta H(1S) \]

\[ E_{2S}(J/\psi) - E_{2S}(\eta_c) = 2\alpha [R_{2S}(r = 0)]^2 = \Delta H(2S) \]  

(33)
\[
\frac{\Delta H(1S)}{\Delta H(2S)} \approx \frac{|R_{2S}(r = 0)|^2}{|R_{1S}(r = 0)|^2}.
\] (34)

In we assumed that \(\alpha\) does not change much for 1\(S\) and 2\(S\) states. The ratio of \(\frac{\Delta H(1S)}{\Delta H(2S)}\) for our set of values i.e., the second set is

\[
\frac{\Delta H(1S)\Delta H(2S)}{\Delta H(2S)} = 0.5578 \text{dimensionless}
\] (35)

From the particle data group [6] we know that \(\Delta(1S) = 117.2\,\text{MeV}\). Then Eqn. (35) gives

\[
\Delta H(2S) = 65.37\,\text{MeV}
\] (36)

Since, \(\Delta H(2S) = E_{2S}(\psi) - E_{2S}(\eta_c)\) our calculation gives

\[
E_{2S}(\eta_c) = 3.62059\,\text{GeV}.
\] (37)

The recent observation of Charmonium \(\eta_c(2S)\) by the Belle Collaboration yielded the following values for its mass

\[
M(\eta_c(2S)) = 3.654(14)\,\text{GeV}[9] \quad M(\eta_c(2S)) = 3.622(12)\,\text{GeV}[7].
\] (38)

The value of \(E_{2S}(\eta_c)\) obtained from our calculation agrees with the value of \(M(\eta_c(2S))\) experimentally obtained.

V. CONCLUSION

We have used a Coulomb plus linear potential to explain the mass of \(c\bar{c}\) spectrum, employing the variational method. \(J/\psi(1S)\) and \(\chi_{c1}(1P)\) masses are correctly reproduced for \(m_c \approx 1.1\,\text{GeV}\). \(\psi(2S)\) state is explained with three set of values for the variational parameters. In order to find the correct set, we calculate leptonic decay width of \(\psi(2S)\) and \(J/\psi(1S)\). By comparing with experiment, the second set is found to explain both the mass and leptonic decay width. This set is used in conjunction with spin-spin contact interaction to explain the hyperfine splitting of Charmonium. As a consequence, we are able to calculate the mass of \(\eta_c(2S)\) as 3.6026\,\text{GeV} which agrees reasonably well with the experiment. This study indicates the ability of non-relativisitic approach in explaining the \(c\bar{c}\) system.

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