Constraining the initial conditions of globular clusters using their radius distribution

Poul E. R. Alexander1,* and Mark Gieles1,2
1Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, UK
2Department of Physics, University of Surrey, Guildford GU2 7XH, UK

Accepted 2013 February 9. Received 2013 February 8; in original form 2013 January 21

ABSTRACT
Studies of extragalactic globular clusters (GCs) have shown that the peak size of the GC radius distribution (RD) depends only weakly on galactic environment. We model RDs of GC populations using a simple prescription for a Hubble time of relaxation-driven evolution of cluster mass and radius. We consider a power-law cluster initial mass function (CIMF) with and without an exponential truncation, and focus in particular on a flat and a steep CIMF (power-law indices of 0 and −2, respectively). For the initial half-mass radii at birth, we adopt either Roche volume (RV) filling conditions (‘filling’, meaning that the ratio of half-mass to Jacobi radius is approximately $r_h/r_J \approx 0.15$) or strongly RV under-filling conditions (‘under-filling’, implying that initially $r_h/r_J \ll 0.15$). Assuming a constant orbital velocity about the galaxy centre, we find for a steep CIMF that the typical half-light radius scales with the galactocentric radius $R_G$ as $R_R \approx R_G^{1/3}$. This weak scaling is consistent with observations, but this scenario has the (well-known) problem that too many low-mass clusters survive. A flat CIMF with ‘filling’ initial conditions results in the correct MF at old ages, but with too many large (massive) clusters at large $R_G$. An ‘under-filling’ GC population with a flat CIMF also results in the correct MF, and can also successfully reproduce the shape of the RD, with a peak size that is (almost) independent of $R_G$. In this case, the peak size depends (almost) only on the peak mass of the GC MF. The (near) universality of the GC RD is therefore because of the (near) universality of the CIMF. There are some extended GCs in the outer halo of the Milky Way that cannot be explained by this model.

Key words: globular clusters: general – galaxies: star clusters: general.

1 INTRODUCTION
It has long been known that the shape of the globular cluster (GC) luminosity function is relatively insensitive to galactic environment. Accordingly, the peak luminosity of $M_V \approx -7$ is often used as a standard candle (Shapley & Sawyer 1927; Rejkuba 2012). It is, however, not known whether this typical luminosity is the result of dynamical evolution (Fall & Zhang 2001) or the outcome of the cluster formation process (see Brodie & Strader 2006; Portegies Zwart, McMillan & Gieles 2010, for reviews).

Jordán et al. (2005) showed that the half-light radii ($r_{hl}$) of GCs are also sufficiently independent of galactic environment to be used as a standard ruler. Typical radii of $r_{hl} \approx 3$ pc are found for clusters in the Milky Way (MW; Harris 1996, 2010 version, hereafter H96) and other galaxies (Kundu & Whitmore 2001; Jordán et al. 2005; Peng et al. 2011). Within galaxies the typical radii depend only weakly on the galactocentric radius $R_G$. For the MW GCs, van den Bergh, Morbey & Pazder (1991) and McLaughlin (2000) find $R_{hl}^{0.3}$ and $R_{hl}^{0.4}$, respectively. Similar and shallower correlations are found for clusters in external galaxies: $r_h \propto R_G^{0.38}$ (in the Sombrero galaxy; Spitler et al. 2006), $r_h \propto R_G^{0.17}$ (for the inner regions of NGC 5128; Gómez & Woodley 2007) and $r_h \propto R_G^{0.11}$ (in a sample of six ellipticals; Harris 2009). If clusters fill their Jacobi radius $r_J$, their radii scale with their mass $M$ and their orbital frequency $\Omega$ as $M^{1/3} \Omega^{-2/3}$. Hence, for a constant $M$ and orbital velocity $V_G$, we expect $r_{hl} \propto R_G^{1/3}$, i.e. a stronger dependence on $R_G$ than observed.

The objective of this work is to understand how the shape of the cluster radius distribution (RD) depends on the cluster initial mass function (CIMF), the distribution of clusters within the galaxy and the initial cluster sizes. This will allow us to use the RD to put additional constraints on the initial conditions of GCs.

The structure of this Letter is as follows. In Section 2, we discuss the RD arising for a synthetic sample of GCs at one $R_G$. In Section 3, we consider an entire population of synthetic clusters and compare our results with the MW GC system. We finally compare our results to previous studies and provide a summary in Section 4.

*E-mail: pera@ast.cam.ac.uk

© 2013 The Authors
Published by Oxford University Press on behalf of the Royal Astronomical Society

doi:10.1093/mnrasl/slt022
2 THE RADIUS DISTRIBUTION OF CLUSTERS 
IN A SINGLE TIDAL ENVIRONMENT

We start by considering the RD, i.e. the number of clusters with a half-mass radius between \( r_h \) and \( r_h + dr_h \), of clusters evolving in a fixed tidal environment (for example, evolving at a constant \( R_G \) within a galaxy halo). For this situation, the RD can be related to the CIMF (i.e. the number of clusters with the initial mass in the interval \( M_0 \) and \( M_0 + dM_0 \)) as

\[
\frac{dN}{dr_h}(R_h) = \frac{dN}{dM_0} \left| \frac{\partial M}{\partial M} \right| \left| \frac{\partial M}{\partial r_h}(R_h) \right|.
\]

Here \( |\partial M_0/\partial M| \) and \( |\partial M/\partial r_h(R_h)| \) depend on the mass and radius evolution, respectively. These will be discussed in the following sections.

2.1 Initial mass function and mass evolution

In this section, we assume that the CIMF is a power-law distribution: \( dN/dM_0 \propto M_0^{-\alpha} \) in the range \( M_{\text{low}} \leq M_0 \leq M_{\text{up}} \). We adopt a constant \( V_0 = 220 \text{ km s}^{-1} \) such that \( \Omega \propto R_G^{-2} \). Throughout this work, we assume that clusters evolve with a constant mass-loss rate that depends on the galactocentric radius as \( M_{\text{RG}} = -20 M_\odot \text{ Myr}^{-1} \text{ kpc}^{-1} \) (Gieles, Heggie & Zhao 2011, hereafter G11). The mass then evolves as

\[
M = M_0 - \Delta, \quad \Delta \equiv -M t \propto r_h^{-3}.
\]

Here, \( t \) is the age of the cluster, for which we adopt 13 Gyr. We then have \( |\partial M/\partial M| = 1 \). The total dissolution time \( t_{\text{dis}} = -M_0/M \). We finally need to know how the radii of clusters depend on \( M \), for which we compare two scenarios. In Section 2.2, we consider RV filling clusters, whereby the ratio of half-mass to Jacobi radius \( (R \equiv r_h/r_1) \) is constant and approximately the Hénon (1961) value, \( R = 0.145 \). In Section 2.3, we consider initially RV under-filling clusters, where initially \( R = 0 \) and \( R \) increases during the evolution.

2.2 RV filling clusters

We first consider the scenario whereby all clusters fill their RV from birth. Such clusters will evolve at a constant density whilst losing mass (Hénon 1961) and so

\[
M = Ar_h^3, \quad A \equiv \frac{2\Omega^2}{G R_h^3} \propto R_h^{-2},
\]

where \( G \) is the gravitational constant. From this we find \( |\partial M/\partial r_h(R_h)| \propto r_h^{-2} R_h^{-2} \), which we combine with equation (1) to derive the RD

\[
\frac{dN}{dr_h}(R_h) \propto \left( A r_h + \Delta \right)^\alpha r_h^{-2} R_h^{-2} \propto \left\{ \begin{array}{ll} r_h^{3\alpha+2} R_h^{2(\alpha+1)}, & \Delta \leq Ar_h^3 \leq M_{\text{up}} \\ r_h^{3\alpha+2} R_h^{2(\alpha+1)}, & \Delta \leq Ar_h^3 \leq M_{\text{up}}. \end{array} \right.
\]

For \( \alpha = 0 \) the RD is a single power law with slope +2, while for \( \alpha \neq 0 \) it is a double power-law distribution: at small radii the shape of the RD is independent of the shape of the CIMF and only depends on how clusters lose mass. This is because the globular cluster mass function (GCMF) evolves to \( dN/dM \propto \text{constant} \) at low masses, regardless of the CIMF (Hénon 1961; Fall & Zhang 2001). At large radii, the shape of the RD depends on the index of the CIMF, for which we find two regimes: for \( \alpha < -2/3 \) the RD is a decreasing function of \( r_h \) and the peak of the distribution occurs where the two power laws join, i.e. at \( R_h \approx (\Delta/A)^{1/3} \propto R_G^{1/3} \), where we introduce the symbol \( \bar{r}_h \) to denote the mode of the RD (most probable radius). For \( \alpha > -2/3 \), the RD rises until the largest radius present, which is the radius of the most massive cluster \( (M_{\text{up}}) \) and thus \( \bar{r}_h \propto R_G^{2/3} \).

2.3 RV under-filling clusters

We next consider clusters that are born RV under-filling. We use the model of G11 for the relaxation-driven evolution of \( r_h \) as a function of \( M_0 \) and \( R_G \), in which clusters are assumed to form with a half-mass density much greater than the tidal density. In (roughly) the first half of their life, clusters expand and become increasingly RV filling, while in the second half they contract at a constant density, and near final dissolution the radius evolves as in equation (3).

The functional form for the evolution of \( r_h \) is (equation 26 in G11)

\[
r_h = \left( \frac{M}{A} \right)^{1/3} \left[ 1 - \left( \frac{M}{M_0} \right)^{5/2} \right]^{2/3}.
\]

Note that \( A = A(R_G) \), \( M = M_0(R_G, \Delta) \) and \( \Delta = \Delta(R_G, t) \). For \( M \approx M_0 \), we find \( r_h \propto M_0^{-1/2} t^{-2/3} \), which follows from the fact that in the expansion phase clusters evolve towards a constant ratio of \( t_{\text{dis}}/t \) with \( r_h \propto (M_{\text{rg}})^{1/2} \) (assuming a constant mean stellar mass and Coulomb logarithm, Spitzer & Hart 1971). If there are clusters in both the expansion and contraction phases, the size of the largest cluster \( r_h^{\text{max}} \) is not that of the most massive cluster, as was the case for RV filling clusters (Section 2.2). To find \( r_h^{\text{max}} \) in a population of clusters with different \( M_0 \) at a given \( R_G \) and \( t \), we numerically solve \( dr_h/dM_0 = 0 \) (equation 5), and find \( M_0(r_h = r_h^{\text{max}}) \approx 3.175 \Delta \). We note that this is not the mass of the cluster that has just reached its maximum size, but instead a slightly more massive cluster that is still expanding (see Fig. 1). This is because the maximum \( r_h \) is reached roughly halfway the total lifetime (G11), so the cluster that has reached its maximum size has an initial mass of \( M_0 \approx 2 \Delta \). From Fig. 1 we also see that clusters can be placed in three categories:

(i) \( M_0 \geq 2 \Delta \). Clusters have past their individual maximum \( r_h \) and are now approximately RV filling (dashed green lines).

(ii) \( 2 \Delta < M_0 \leq 3.175 \Delta \). Clusters that are expanding towards their individual maximum \( r_h \), and show a positive correlation between mass and \( r_h \) (solid magenta lines).

Figure 1. Radius evolution of clusters of different masses \((10^4 < M < 10^7 M_\odot)\) at \( R_G = 8.5 \text{ kpc} \), \( V_0 = 220 \text{ km s}^{-1} \), as a function of fractional dissolution time (left-hand panel) and physical time (right-hand panel). The thick orange line represents the \( r_h \) of clusters after a Hubble time, and is curved in the left-hand panel on account of the longer \( t_{\text{dis}} \) of more massive clusters. The green dashed, magenta solid and cyan dot–dashed lines represent the evolutionary tracks of individual (sample) clusters in states (i), (ii) and (iii), respectively. These tracks are thicker for the evolution that has occurred during a Hubble time (on the left-hand side of the orange line).
(iii) \( M_0 > 3.175\Delta \). Clusters that are expanding, and have a negative correlation between \( M \) and \( r_h \) (dot–dashed cyan lines).

We cannot derive an easy analytical expression for the RD, because we cannot invert the relation \( r_h(M_0) \) (equation 5). We are, however, able to give the behaviour of the RD at the extremes of \( r_h \). For expanding clusters, we use \( r_h \propto M_0^{1/3} \) and \( M \simeq M_0 \) to find \( \left[ \frac{dM}{dr_h} \right] \propto r_h^{-3} \), while contracting clusters approximately follow the same relationships as in Section 2.2. We substitute these expressions into equation (1) and find

\[
\frac{dN}{dr_h}(R_G) \propto \begin{cases} \frac{r_h^2}{6} R_G^{(2+\alpha)} & 0 \leq r_h \lesssim r_h^{\text{max}}, \\ \frac{r_h^{3(2+\alpha)}}{6} & r_h(M_\text{up}) \leq r_h \lesssim r_h^{\text{max}}. \end{cases}
\]

(6)

Here we assumed that clusters with \( M = M_\text{max} \) have dissolved already. These two parts of the RD both occupy the region \( 0 \lesssim r_h \lesssim r_h^{\text{max}} \). The first approximation applies to the clusters whose \( r_h \) has approximately reached the RV filling condition (\( R \simeq 0.145 \)), i.e. \( M_0 \lesssim 3.175\Delta \), while the second approximation holds for the expanding clusters (\( M_0 \gtrsim 3.175\Delta \)). For \( \alpha = -2 \) we find that both extremes of the RD behave as \( r_h \). For very high \( M_\text{up} \) (where \( r_h(M_\text{up}) \simeq 0 \)), this RD has the same shape as the RD for RV filling clusters with \( \alpha = 0 \) (Section 2.2), but has a smaller \( r_h^{\text{max}} \).

As for the RV filling case, the mode of the RD is either at the radius of the most massive cluster or at the radius of the cluster with \( M_0 \approx \text{few} \times \Delta \), depending on whether the RD of expanding clusters is rising or falling. The difference is that \( r_h(M_\text{up}) \) is smaller than \( r_h^{\text{max}} \). For \( \alpha > -4/3 \), the RD of the RV filling clusters is rising and \( \tilde{r}_h = r_h^{\text{max}} \propto R_G^{1/3} \). For \( \alpha < -4/3 \), this part of the RD is a declining function, and if the most massive cluster is still expanding the RD peaks at the radius of the most massive cluster: \( \tilde{r}_h = r_h(M_\text{up}) < r_h^{\text{max}} \approx \text{constant} \) (Fig. 2). This is the most straightforward way to explain a near universal radius scale for GCs. The condition that the most massive clusters have not yet expanded until their \( r_h \approx R_G \) is not satisfied in strong tidal fields, and will be discussed further in Section 3.4.

The RD is computed for RV under-filling clusters and for three values of \( \alpha \) using a Monte Carlo approach (Fig. 2). The power-law behaviours at the expanding and contracting extremes are shown as the dashed black and dotted red lines, respectively. The power laws do not follow the Monte Carlo data where \( r_h \approx r_h^{\text{max}} \), as here \( M_0 \approx 3.175\Delta \) and mass-loss is neither dominant nor negligible.

We will consider GC populations in a MW-type galaxy in the next section.

### 3 Radius Distribution of a Cluster System

In this section, we consider the RD of clusters in a Galactic halo, i.e. with a range of \( R_G \). The RD can be found by multiplying the RD at a single \( R_G \) with the number of clusters at each \( R_G \) \( \frac{dN}{dR_G} = 4\pi R_G^2 n(R_G) \), where \( n(R_G) \) is the number density of clusters in the halo) and integrating over all \( R_G \):

\[
\frac{dN}{dr_h} = \int_{r_h^{\text{min}}}^{r_h^{\text{max}}} \frac{dN}{dr_h}(R_G) \frac{dN}{dR_G} dR_G.
\]

(7)

The analytic evaluation of this integration gives results only for a limited number of cases. We therefore perform a number of Monte Carlo simulations, for which we adopt a Hernquist (1990) model (i.e. a double-power-law density profile with an \( r^{-1} \) central cusp and an \( r^{-4} \) outer halo) with a scale radius \( \alpha = 5 \) kpc and a truncation at \( R_G = 100 \text{kpc} \) for the distribution of clusters in the Galaxy

![Figure 2. Monte Carlo simulations of clusters with different CIMF slopes (\( \alpha \)) at \( R_G = 8.5 \text{kpc} \). The colour coding is the same as in Fig. 1 (green = stage (i) clusters, magenta = stage (ii) clusters, cyan = stage (iii) clusters). In the left-hand panels, clusters in the three phases of evolution are shown separately, while they are added in the right-hand panels. The dashed black line represents the expected slope of expanding clusters, while the dotted red line corresponds to the expected slopes of approximately RV filling (contracting) clusters (equation 6). The sample size is \( 10^5 \) clusters and clusters are evolved for 13 Gyr.](https://academic.oup.com/mnrasl/article-abstract/432/1/L1/1135437/1135437)
slope of $3\alpha + 3.5 = -2.5$. The typical radius scales as $r_h \propto R_G^{1/3}$. We recall from Section 2.2 that this is due to the scaling of the break in the double-power-law RD with $R_G$. This scaling is close to what is found for MW GCs, but in this scenario there are too many large clusters (see Fig. 3, Panel A). In addition, too many low-mass clusters survive at large $R_G$ in this scenario (Vesperini 2001; Gieles & Baumgardt 2008).

3.2 Scenario B: RV filling clusters and $\alpha = 0$

For RV filling clusters and $\alpha = 0$, the RD at a single $R_G$ is simply a $+2$ power law. In this case, the integration of equation (7) can be performed, with the result that at large radii we find $dN/dr_h \propto r_h^{-1}$. Because of this slow decline, this scenario produces even larger clusters than scenario A. However, owing to the invariance of this CIMF under the influence of mass-loss, this scenario does predict the correct GCMF (meaning that the peak mass, $\bar{M}$, is independent of $R_G$, Vesperini 2000). Because $r_h$ scales with the Jacobi radius of the most massive cluster (Section 2.2) at any given $R_G$, we find $r_h \propto R_G^{2/3}$ (see Fig. 3, panel B), which is a stronger correlation than found for Galactic clusters. This is the only scenario of the four we considered for which the $R_G^{2/3}$ scaling is found.

3.3 Scenario C: RV under-filling and $\alpha = -2$

For under-filling clusters with a steep CIMF ($\alpha = -2$), both the expanding and contracting clusters have a RD that behaves as a $+2$ power law (just as in scenario B). However, the total RD is different because the typical radius scales with $r_h \propto R_G^{\alpha}$ and therefore $r_h \propto R_G^{1/3}$ (Section 2.3). Accordingly, there are fewer large clusters, because high-mass clusters are still expanding to their tidal boundary and are smaller than clusters of equivalent mass in scenario B.

3.4 Scenario D: RV under-filling and $\alpha = 0$

In the final scenario (under-filling clusters with $\alpha > -4/3$), the sample population contains a fraction of expanding clusters, which is larger for higher values of $\alpha$ (Fig. 1). The radii of expanding clusters are independent of their local tidal field and also independent of their initial radii (Hénon 1965, hereafter H65; Gieles et al. 2010). This is the case for $R_G \gtrsim 5$ kpc (Fig. 3, panel D). The typical $r_h$ is then simply the radius of the cluster with the typical mass. For $R_G < 5$ kpc, the majority of clusters have expanded to become RV filling and are therefore described by scenario B with a typical radius scaling as $r_h \propto R_G^{1/3}$. The number of clusters in this regime depends on the slope of the CIMF and the Hernquist scale radius. This scenario is the only scenario that produces a clear peak in the RD below 5 pc, and the only scenario to produce a rapid decline in the RD at high $r_h$ consistent with MW GCs.

3.5 Comparison to MW GCs

In the right-hand panels of Fig. 3, we show the RD and the individual $r_h$ against $R_G$ values of the MW GCs. The width of the MW GC RD is narrow, with radii rarely in excess of 20 pc (90 per cent of the GCs have radii <10 pc, H96). Comparing this with the models, we see that in scenarios A and B there are too many clusters with $r_h \gtrsim 20$ pc (see Fig. 3), especially in scenario B, where we find clusters with size up to $r_h \sim 100$ pc. Scenarios C and D both show a narrower range of $r_h$, in better agreement with that found for MW GCs, with scenario D providing a very comparable shape overall.

Several MW GCs at high $R_G$ are larger than those predicted. These outlying clusters cannot be explained by our simple model for cluster evolution, and so may result from effects neglected by this study. The scaling of $r_h$ with $R_G$ of the MW GCs shows considerable scatter, but is approximately $r_h \propto R_G^{0.4}$, similar to the scalings of scenarios A and C and shallower than that of scenario B. However, the majority of MW GCs are found within the innermost $\sim 10$ kpc.
In this regime, the radii in scenario D are similar to those in scenario C. It is therefore difficult to satisfactorily distinguish which scenario optimally represents the $r_\nu$ scaling of MW GCs, but combined with the superior match of the RD shape, we conclude that scenario D is preferred. The $r_\nu(R_G)$ relationship of scenario D is also similar to that described for M87 in Webb, Sills & Harris (2012). We therefore conclude that the good match of the shape of the RD in scenario D suggests that most clusters form RV under-filling, and with a relatively flat CIMF, similar to the present-day GCMF.

4 SUMMARY AND DISCUSSION

We have investigated the RD of GC populations in MW-type haloes for a variety of initial conditions for the clusters’ masses and radii. We find that RV filling initial conditions result in too many large clusters ($\gtrsim$20pc) at large $R_G$, regardless of CIMF. Models in which clusters are initially RV under-filling and form with a steep CIMF show a scaling of $r_\nu \propto R_G^{1.3}$, roughly consistent with MW GCs, but combined with the RD distribution. A fast cluster evolution code is being developed (Alexander & Gieles 2012) with the goal of including all these effects in order to be able to model the RD, GCMF and $R_G$ distribution.

ACKNOWLEDGEMENTS

We thank Holger Baumgardt and Henny Lamers for insightful discussions and helpful comments. We also thank our anonymous reviewer for helpful suggestions. The authors are grateful to the University of Queensland, Brisbane, where part of this study was performed. PERA acknowledges financial support from STFC, and MG thanks the Royal Society for financial support.

REFERENCES

Alexander P. E. R., Gieles M., 2012, MNRAS, 422, 3415
Baumgardt H., Parmentier G., Gieles M., Vesperini E., 2010, MNRAS, 401, 1832
Brodie J. P., Strader J., 2006, ARA&A, 44, 193
Da Costa G. S., Grebel E. K., Jerjen H., Rejkuba M., Shara M. E., 2009, AJ, 137, 4361
Elmegreen B. G., 2008, ApJ, 672, 1006
Ernst A., Just A., 2013, MNRAS, 429, 2953
Fall S. M., Zhang Q., 2001, ApJ, 561, 751
Fukushige T., Heggie D. C., 2000, MNRAS, 318, 753
Gieles M., Baumgardt H., 2008, MNRAS, 389, L28
Gieles M., Baumgardt H., Heggie D. C., Lamers H. J. G. L. M., 2010, MNRAS, 408, L16
Gieles M., Heggie D. C., Zhao H., 2011, MNRAS, 413, 2509 (G11)
Gómez M., Woodley K. A., 2007, ApJ, 670, L105
Harris W. E., 1996, AJ, 112, 1487 (H96)
Harris W. E., 2009, ApJ, 699, 254
Harris W. E., Spitler L. R., Forbes D. A., Bailin J., 2010, MNRAS, 401, 1965
Hénon M., 1961, Ann. Astrophys., 24, 369
Hénon M., 1965, Ann. Astrophys., 28, 62 (H65)
Hernquist L., 1990, ApJ, 356, 359
Jordán A. et al., 2005, ApJ, 634, 1002
Jordán A. et al., 2007, ApJS, 171, 101
Kundu A., Whitmore B. C., 2001, AJ, 121, 2950
Mackey A. D. et al., 2010, ApJ, 717, L11
McLaughlin D. E., 2000, ApJ, 539, 618
Madrid J. P., Hurley J. R., Sippel A. C., 2012, ApJ, 756, 167
Peng E. W. et al., 2011, ApJ, 730, 23
Portegies Zwart S. F., McMillan S. L. W., Gieles M., 2010, ARA&A, 48, 431
Rejkuba M., 2012, Ap&SS, 341, 195
Schechter P., 1976, ApJ, 203, 297
Shapley H., Sawyer H. B., 1927, Harv. Coll. Obs. Bull., 852, 22
Shin J., Kim S. S., Yoon S-J., Kim J., 2013, ApJ, 762, 135
Spitler L. R., Larsen S. S., Strader J., Brodie J. P., Forbes D. A., Beasley M. A., 2006, AJ, 132, 1593
Spitzer L. J., Hart M. H., 1971, ApJ, 164, 399
van den Bergh S., Morby C., Puzder J., 1991, ApJ, 375, 594
Vesperini E., 2000, MNRAS, 318, 841
Vesperini E., 2001, MNRAS, 322, 247
Webb J. J., Sills A., Harris W. E., 2012, ApJ, 746, 93
Zhang Q., Fall S. M., 1999, ApJ, 527, L81

This paper has been typeset from a TeX/LaTeX file prepared by the author.