On the Decay Modes $B_{d,s}^0 \rightarrow \gamma D^{*0}$

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The various decay modes of the type $B \rightarrow \gamma D^*$ are dynamically different. In general there are factorizable contributions, and there are pole diagrams and pseudoscalar exchange contributions at meson level. The purpose of this paper is to point out that the decay modes $B_{d,s}^0 \rightarrow \gamma D^{*0}$ have negligible contributions from such mechanisms, in contrast to the decay modes $B_{d,s}^0 \rightarrow \gamma D^{*0}$ and $B^- \rightarrow \gamma D_{s,d}^{*0}$. However, for the decay modes $B_{d,s}^0 \rightarrow \gamma D^{*0}$ there are non-factorizable contributions due to emission of soft gluons, and such non-factorizable contributions are found to dominate the amplitudes for these latter decay modes.

We estimate the branching ratio for these modes in the heavy quark limits, both for the $b$- and the $c$- quarks, and obtain a value $\simeq 1.6 \times 10^{-6}$ for $B_d^0 \rightarrow \gamma D^{*0}$, and $\simeq 8 \times 10^{-7}$ for $B_s^0 \rightarrow \gamma D^{*0}$. We expect large corrections to this limit because the energy gap between the $b$- and $c$- quark masses are significantly bigger than 1 GeV. However, we expect that our estimate for $B_{d,s}^0 \rightarrow \gamma D^{*0}$ gives the right order of magnitude for the amplitudes.

Keywords: $B$-decays, factorization, gluon condensate.
PACS: 13.20.Hw , 12.39.St , 12.39.Fe , 12.39.Hg.

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I. INTRODUCTION

There is presently great interest in decays of $B$-mesons, due to numerous experimental results coming from BaBar and Belle. Later LHC will provide data for such processes. $B$-decays of the type $B \to \pi \pi$ and $B \to K \pi$, where the energy release is big compared to the light meson masses, has been treated within QCD factorization and soft collinear effective theory (SCET) \[1\]. In these cases the amplitudes factorize into products of two matrix elements of weak currents in the high energy limit, and non-factorizable corrections of order $\alpha_s$ can be calculated perturbatively.

The decays $B \to \pi \pi, K \pi$ are typical heavy to light decays. It was pointed out in previous papers \[2\] that for various decays of the type $\bar{B} \to D \bar{D}$, which are of heavy to heavy type, the methods of \[1\] are not expected to hold because the energy release is of order 1 GeV. (Here $\bar{B}, D$, and $\bar{D}$ contain a heavy $b, c$, and anti-$c$ quark respectively). In this paper we consider decay modes of the type $B \to \gamma D^*$. Such modes have been studied in the literature \[3, 4\] for some time. We restrict ourselves to processes where the $b$-quark decays. This means the quark level processes $b\bar{q} \to \gamma c\bar{u}$, $b\bar{q} \to \gamma u\bar{c}$, and $b\bar{u} \to \gamma c\bar{q}$, where $q = d$ or $q = s$. Processes where the anti-$b$-quark decays proceed analogously.

Formally, decays of the type $\bar{B} \to \gamma D$ is a heavy to heavy transition in the heavy quark limits $(1/m_b) \to 0$ and $(1/m_c) \to 0$, and in ref. \[4\] the decay of a charged $B$-meson was studied within heavy quark effective theory (HQEFT) \[5\] and heavy light chiral perturbation theory (HL\(\chi\)PT) \[6\]. This framework was also used to study the Isgur-Wise function for the $B \to D$ transition currents, which is also a heavy to heavy transition where chiral loops (in terms of HL\(\chi\)PT) and $1/m_{b,c}$ corrections (in terms of HQEFT) have been added \[7\]. In the present paper we will also stick to this framework, although it is not expected to hold for precise numerical estimates because the energy gap between the $b$- and the $c$-scale is substantial, namely about three times the chiral symmetry breaking scale.

First, decay modes of the type $B \to \gamma D^*$ might have substantial factorizable contributions, of pole or non-pole type. These pole diagrams are present only for radiation from charged $B$- or $D$-mesons. Second, there are also meson exchange contributions. These will be chiral loop contributions in the HQEFT limits (for the $b$ and $c$-quarks). Such meson exchange diagrams, which are non-factorizable and $1/N_c$ suppressed, are present for the decay modes $\bar{B}^0_{d,s} \to \gamma \bar{D}^{*0}$ and $B^- \to \gamma D^*_{s,d}$. 
The purpose of this paper is to point out that the decay modes $\overline{B}_{d,s}^0 \rightarrow \gamma D^{*0}$ have almost zero contribution from the factorizable and the meson exchange mechanisms. However, these decay modes have significant contributions from soft gluon emission. Such non-factorizable (colour suppressed $\sim 1/N_c$) contributions to $B - \bar{B}$ mixing [8], $B \rightarrow D\bar{D}$ [2] and $B \rightarrow D\eta'$ [9] decays are calculated in terms of the (lowest dimension) gluon condensate within a recently developed heavy light chiral quark model (HL$\chi$QM) [10], which is based on the HQEFT [5]. They have also been studied in the light sector for $K - \bar{K}$ mixing and $K \rightarrow 2\pi$ decays [11]. We estimate the branching ratios for $\overline{B}_{d,s}^0 \rightarrow \gamma D^{*0}$ in the heavy $b$- and $c$-quark limits. Note that the decay modes $\overline{B}_{d,s}^0 \rightarrow \gamma D^{*0}$ and $\overline{B}_{d,s}^0 \rightarrow \gamma D^{*0}$ proceed differently. In the last case there are substantial meson exchange contributions.

In the next section (II) we present the weak four quark Lagrangian and its factorizable and non-factorizable matrix elements. In section III we present the framework of HQEFT and HL$\chi$PT, and in section IV we calculate the non-factorizable matrix elements due to soft gluons expressed through the (model dependent) quark condensate. In section V we give the results and conclusion.

II. THE WEAK QUARK LAGRANGIAN AND ITS MATRIX ELEMENTS

Based on the electroweak and quantum chromodynamical interactions, one constructs an effective non-leptonic Lagrangian at quark level in the standard way:

$$\mathcal{L}_W = \sum_i C_i(\mu) Q_i(\mu),$$

where all information of the short distance (SD) loop effects above a renormalization scale $\mu$ is contained in the Wilson coefficients $C_i$. In our case there are four relevant operators

$$Q_1 = 4(\bar{\tau}_L\gamma^\alpha b_L)(\bar{\tau}_L\gamma_\alpha u_L), \quad Q_2 = 4(\bar{\tau}_L\gamma^\alpha b_L)(\bar{\tau}_L\gamma_\alpha u_L),$$

$$Q_3 = 4(\bar{\tau}_L\gamma^\alpha b_L)(\bar{\tau}_L\gamma_\alpha c_L), \quad Q_4 = 4(\bar{\tau}_L\gamma^\alpha b_L)(\bar{\tau}_L\gamma_\alpha c_L),$$

for $q = d, s$. This effective Lagrangian is based on the interactions in Fig. [1] and hard gluon corrections to these diagrams. Operators from penguin diagrams may also contribute, but have small Wilson coefficients. The coefficients $C_{1,2}$ and $C_{3,4}$ have different KM structures. We may write

$$C_i = -\frac{G_F}{\sqrt{2}}(V_{cb}V_{uj}^*) a_i; \quad C_j = -\frac{G_F}{\sqrt{2}}(V_{ub}V_{cj}^*) a_j,$$
FIG. 1: Tree level W-exchange leading to the effective Lagrangian in eq. (1). The left diagram 1a) gives rise to $Q_{1,2}$, and the right diagram 1b) gives rise to $Q_{3,4}$.

for $i = 1, 2$ and $j = 3, 4$ respectively. Here the “reduced” Wilson coefficients $a_i$ (for $i = 1, 2, 3, 4$) are dimensionless numbers. Furthermore, in terms of the Wolfenstein parameter $\lambda$, we have $V_{cb} V_{ud}^* \sim \mathcal{O}(\lambda^2)$. For $q = s$ the KM factors $V_{cb} V_{us}^*$ and $V_{ub} V_{cd}^*$ are both $\sim \mathcal{O}(\lambda^3)$, while $V_{ub} V_{cd}^* \sim \mathcal{O}(\lambda^4)$. At the scale $\mu = M_W$, when perturbative QCD is switched off, one has $a_{1,3} = 0$ and $a_{2,4} = 1$. At the scale $\mu = m_b$, $a_{1,3} \sim 10^{-1}$ and negative, and $a_{2,4}$ are still $\sim 1 \ [12]$. (In practice, $a_1 = a_3$ and $a_2 = a_4$.) Extrapolating the Wilson coefficients (naively) down to $\mu \sim \Lambda \sim 1$ GeV, which is the matching scale between short and long distance effects within our framework [2, 8, 9], we obtain $a_{2,4} \simeq 1.17$ and $a_{1,3} \simeq -0.37$ [9]. Alternatively, one might perform perturbative QCD within HQEFT as done in [13] and used in [2] for $B \to D \bar{D}$, but numerical differences will be small.

One may also think of operators like

$$e F_{\mu\nu} (\bar{q}_L \gamma^\mu b_L) (\bar{c}_L \gamma^\nu u_L) , \quad e F_{\mu\nu} (\bar{q}_L \sigma^{\alpha\beta} F_{\alpha\beta} \gamma^\mu b_L) (\bar{c}_L \gamma^\nu u_L) .$$

However, such operators are of dimension eight, and dominated at low momenta which make a short distance treatment dubious.

In the factorized limit (-no strong interactions between the two quark currents) we obtain the amplitude for $B^0_q \to \gamma D^{*0}$ obtained from (1) and (3):

$$\langle \gamma D^{*0} | \mathcal{L}_W | B^0_q \rangle_F = 4 \left( C_1 + \frac{C_2}{N_c} \right) \left( \langle \gamma | c_L \gamma_\mu u_L | 0 \rangle \langle \gamma | q_L \gamma_\mu b_L | B^0_q \rangle \right) + \langle \gamma D^{*0} | \bar{c}_L \gamma_\mu u_L | 0 \rangle \langle 0 | \bar{q}_L \gamma_\mu b_L | B^0_q \rangle ,$$

where the subscript $F$ means “factorized”. For $B^0_q \to \gamma D^{*0}$ we obtain the same expression with $C_{1,2}$ replaced by $C_{3,4}$ and with $c$ and $u$ interchanged. Thus, the neutral decays have
negligible factorized contributions proportional to $a_{nf} = (a_{1,3} + a_{2,4}/N_c)$, which is of order $10^{-2}$.

For the charged case $B^− \rightarrow \gamma D^*_q$ we obtain:

$$
\langle \gamma D^*_q | \mathcal{L}_W | B^− \rangle_F = 4 \left( C_4 + \frac{C_3}{N_c} \right) \left( \langle D^*_q | \bar{q}_L \gamma_\mu c_L | 0 \rangle \langle \gamma | \bar{u}_L \gamma_\mu b_L | B^− \rangle + \langle D^*_q | \bar{q}_L \gamma_\mu c_L | 0 \rangle \langle 0 | \bar{u}_L \gamma_\mu b_L | B^− \rangle \right).
$$

Thus the charged decays have substantial factorizable contributions proportional to $a_f = (a_4 + a_3/N_c) \sim 1$.

In order to study non-factorizable contributions at quark level, we use the following relation between the generators of $SU(3)_c$ ($i, j, l, n$ are colour indices running from 1 to 3):

$$
\delta_{ij} \delta_{lm} = \frac{1}{N_c} \delta_{in} \delta_{lj} + 2 t^a_{in} t^a_{lj},
$$

where $a$ is the color octet index. Then the operators $Q_{1,2}$ may, by means of a Fierz transformation, be written in the following way:

$$
Q_{1,3} = \frac{1}{N_c} Q_{2,4} + 2 \widetilde{Q}_{2,4}, \quad Q_{2,4} = \frac{1}{N_c} Q_{1,3} + 2 \widetilde{Q}_{1,3},
$$

where the operators with the “tilde” contain colour matrices:

$$
\widetilde{Q}_1 = 4 (\bar{q}_L \gamma^\alpha t^a b_L) (\bar{c}_L \gamma_\alpha u_L), \quad \widetilde{Q}_2 = 4 (\bar{q}_L \gamma^\alpha t^a b_L) (\bar{u}_L \gamma_\alpha t^a u_L),
$$

$$
\widetilde{Q}_3 = 4 (\bar{q}_L \gamma^\alpha t^a b_L) (\bar{c}_L \gamma_\alpha t^a c_L), \quad \widetilde{Q}_4 = 4 (\bar{u}_L \gamma_\alpha t^a u_L) (\bar{q}_L \gamma_\alpha t^a c_L).
$$

To obtain physical amplitudes, one has to calculate the hadronic matrix elements of the quark operators $Q_i$ and $\widetilde{Q}_i$, within some framework describing long distance (LD) effects.
The non-factorizable amplitude for $B^0_q \rightarrow \gamma D^{*0}$ with one gluon emission obtained from the coloured operators is the same as above with $C_2 \rightarrow C_4$ and with $u$ and $c$-quarks interchanged. The non-factorizable amplitude for $B^- \rightarrow \gamma D_q^{*-}$ with gluon emission from the coloured operators is proportional to $C_3$ and therefore relatively small.

We observe the following generic pattern: Some decay modes have substantial factorizable contributions proportional to the favorable Wilson coefficient linear combination $a_f \equiv (a_{2,4} + a_{1,3}/N_c)$, which is close to one. In this case there are contributions from the coloured operators $\tilde{Q}_{1,3}$ proportional to $2a_{1,3}$ of moderate importance. For other modes there might be factorizable matrix elements proportional to the non-favorable coefficient $a_{nf} \equiv (a_{1,3} + a_{2,4}/N_c)$ which is close to zero (of order $10^{-2}$ or smaller) at our matching scale $\mu = \Lambda_{\chi}$. In these cases there are substantial contributions proportional to $2a_{2,4}$ from the coloured operators $\tilde{Q}_{1,3}$.

In terms of the $B$-meson field $\Phi$, the $D^*$-meson field $V^\mu$, and the electromagnetic field tensor $F_{\mu\nu}$, we can write down the effective Lagrangian to first order in the photon momentum,
consistent with the heavy quark limits:

$$\mathcal{L}_{\text{eff}} = A^{(+)} i\epsilon_{\mu\nu\alpha\beta} \Phi F^{\mu\nu} V^{\alpha} v_\beta^b + A^{(-)} \Phi F^{\mu\nu} V^{\mu} v_\nu^b,$$

where the positive and negative parity amplitudes $A^{(\pm)}$ depend on hadronic parameters, and the meson masses $M_{B,D}$.

### III. THEORETICAL FRAMEWORK FOR THE HEAVY QUARK LIMITS

Our calculations will be based on HQEFT [5], which is a systematic $1/m_Q$ expansion in the heavy quark mass $m_Q$. The heavy quark fields $Q (= b, c \bar{c})$ are replaced with a “reduced” field $Q_v^{(\pm)}$ for a heavy quark, and $Q_{v}^{(-)}$ for a heavy antiquark (in the case $c \bar{c}$). The Lagrangian for heavy quarks is:

$$L_{\text{HQEFT}} = \pm Q_v^{(\pm)} i v \cdot D Q_v^{(\pm)} + O(m_Q^{-1}).$$

where $v$ is the velocity of the heavy quark, and $D_\mu$ is the covariant derivative containing the gluon and the photon fields. In [8] the $1/m_Q$ corrections were calculated for $B - \bar{B}$-mixing. In this paper these will not be considered.

Integrating out the heavy and light quarks, the effective Lagrangian up to $O(m_Q^{-1})$ can be written as [6, 10]

$$L = \mp Tr \left[ H_a^{(\pm)} i v \cdot D_{ba} H_b^{(\pm)} \right] - g_A Tr \left[ H_a^{(\pm)} H_b^{(\pm)} \gamma_\mu \gamma_5 A_\mu^{ba} \right] + \ldots,$$

where the ellipses denote terms not relevant in this paper. The indices $a, b = 1, 2, 3$ are indices corresponding to the quark flavours $u, d, s$ and $H_a^{(\pm)}$ is the heavy meson field containing a spin zero and spin one boson and $A^\mu$ is an axial field:

$$H_a^{(\pm)} \equiv P_\pm (P_{a\mu}^{(\pm)} \gamma^\mu - i P_{a5}^{(\pm)} \gamma_5 ); \ A_\mu \equiv - \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger),$$

where $P_\pm$ are projecting operators $P_\pm = (1 \pm \gamma \cdot v)/2$, and $v$ is the velocity of the heavy quark. Here $\xi \equiv exp(i \Pi f)$, where $f$ is the bare pion coupling, and $\Pi$ is a 3 by 3 matrix which contains the Goldstone bosons $\pi, K, \eta$ in the standard way. The axial chiral coupling is $g_A \simeq 0.6$. Eqs. (15) and (16) are used for the chiral loop contributions within HL$\chi$PT. The covariant derivative is given by $i D_\mu^{ba} = i \delta_{ba} \partial_\mu - e \tilde{Q}_\xi A^\mu$, where $\tilde{Q}_\xi = \xi Q \xi^\dagger R + \xi^\dagger Q \xi L$ and $A^\mu$ is the photon field.
The simplest way to calculate the matrix element of four quark operators like $Q_{1-4}$ in eq. (11) is by inserting vacuum states between the two currents, as indicated in section II. This vacuum insertion approach (VSA) corresponds to bosonizing the two currents in $Q_{1-4}$ separately and multiply them, i.e. the factorized case. Based on the symmetry of HQEFT, the bosonized current for decay of the $b\bar{q}$ system is [6, 10]:

$$\overline{q}_L \gamma^\mu Q_{bv}^{(+)} \rightarrow \frac{\alpha_H}{2} Tr \left[ \xi^\dagger \gamma^\alpha L H_{bq}^{(+)} \right],$$

where $Q_{bv}^{(+)}$ is a heavy $b$-quark field, $v = v_b$ is its velocity, and $H_{bq}^{(+)}$ is the corresponding heavy meson field for $B_q$. This bosonization has to be compared with the matrix elements defining the meson decay constants $f_H$ ($H = B, D$). Before QCD for scales $\mu < m_Q$ and chiral loop corrections, one has $\alpha_H = f_H \sqrt{M_H}$ (see [5, 10]). For the $W$-boson materializing to a $D$ or $\bar{D}$ mesons, we obtain the bosonized current

$$\overline{q}_L \gamma^\alpha Q_{c\bar{c}}^{(\pm)} \rightarrow \frac{\alpha_H}{2} Tr \left[ \xi^\dagger \gamma^\alpha L H_{c\bar{c}}^{(\pm)} \right],$$

where $v$ is the velocity of the heavy $c$ or $\bar{c}$ quarks ($v = v_c$ or $v = v_{\bar{c}}$), and $H_{c\bar{c}}^{(\pm)}$ is the corresponding field for the $D_q$ or $\bar{D}_q$ meson.

In order to calculate the matrix elements of the quark operators in (11) beyond the factorizable limit, we will use a model which incorporates emission of soft gluons modeled by a gluon condensate. This will be performed within the HL$\chi$QM recently developed in [10]. See also [14, 15]. The Lagrangian for the HL$\chi$QM is

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQEFT} + \mathcal{L}_{\chi QM} + \mathcal{L}_{\text{int}}.$$  

The first term is given in equation (14). The light quark sector is described by the Chiral Quark Model ($\chi$QM), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons:

$$\mathcal{L}_{\chi QM} = \overline{\chi} [\gamma^\mu (i D_\mu + \gamma_5 A_\mu) - m] \chi + ... ,$$

where the ellipses denote terms which are irrelevant here. Here $m$ is the SU(3) invariant constituent light quark mass, and $\chi$ is the flavour rotated quark fields given by $\chi_L = \xi^\dagger q_L$, $\chi_R = \xi q_R$, where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections $q_L$ and $q_R$ are transforming after $SU(3)_L$ and $SU(3)_R$ respectively. In [20] we have discarded terms involving the light current quark mass which is irrelevant.
in the present paper. The covariant derivative $D_\mu$ in (20) is given as in (15) and contains in addition the soft gluon field forming the gluon condensates. The gluon condensate contributions are calculated by Feynman diagram techniques as in [8, 9, 10, 11, 16, 17].

The interaction between heavy meson fields and heavy quarks are described by the following Lagrangian [10]:

$$\mathcal{L}_{\text{Int}} = -G_H \left[ \overline{\chi}^a H_a^{(\pm)} \bar{Q}^{(\pm)}_v + \overline{Q}^{(\pm)}_v H_a^{(\pm)} \chi_a \right],$$

where $G_F$ is a coupling constant satisfying $G_F^2 = 2m_\rho/\pi f_\pi^2$, $\rho$ being a hadronic parameter of order one. In [10] it was shown how (15) could be obtained from the HL$\chi$QM. Performing this bosonization of the HL$\chi$QM, one encounters divergent loop integrals which will in general be quadratic-, linear- and logarithmic divergent [10]. Also, as in the light sector [11] the quadratic and logarithmic integrals are related to the quark condensate and the gluon condensate respectively.

To calculate the factorizable contributions in (6) and (7) corresponding to Fig. 2 within our framework, we need the bosonized currents in (17) and (18), and in addition the bosonized currents involving an emission of a photon from the $B$- or the $D$-meson. For photon emission from the $B$-meson we have (for $v = v_b$)

$$\left( \overline{q}_L \gamma^\alpha Q^{(\pm)}_{bc} \right)_\gamma \rightarrow -\frac{G_H e}{32\pi} F_{\mu\nu}T \left[ \xi^\dagger \gamma^\alpha L H_a^{(\pm)} \bar{Q}^{(\pm)}_v \left( \sigma^{\mu\nu} - \frac{2\pi f_\pi^2}{m^2 N_c} \{\sigma^{\mu\nu}, \gamma \cdot v\} \right) \right] ,$$

where $F$ is the electromagnetic tensor and $Q_\xi = (\xi Q \xi^\dagger + \xi^\dagger Q \xi)/2$. For emission from the $D$-meson there is a similar expression.

Bosonizing currents with one gluon emission from a coloured current operator, for instance to be used in the left part in Fig. 3 b) one obtains:

$$\left( \overline{q}_L t^a \gamma^\alpha Q^{(\pm)}_{bu} \right)_G \rightarrow -\frac{G_H g_s}{64\pi} C_{\mu\nu}^a T \left[ \xi^\dagger \gamma^\alpha L H_a^{(\pm)} \bar{Q}^{(\pm)}_{bv} \left( \sigma^{\mu\nu} - \frac{2\pi f_\pi^2}{m^2 N_c} \{\sigma^{\mu\nu}, \gamma \cdot v\} \right) \right] ,$$

where $C_{\mu\nu}^a$ is the octet gluon tensor, and $H_a^{(\pm)}$ represents the heavy $B_q$-meson fields. Similarly the (heavy) $D$- and $\bar{D}$-mesons are represented by $H_a^{(+)\bar{c}}$ and $H_a^{(-)\bar{c}}$ corresponding to a heavy quark field $Q^{(+)\bar{c}}$ and heavy anti-quark field $Q^{(-)\bar{c}}$ respectively. $v_c$ and $\bar{v} = v_{\bar{c}}$ are the velocities of the $c$ and $\bar{c}$ quarks, respectively. The symbol $\{ , \}$ denotes the anti-commutator.
FIG. 4: Pole contributions to $B \to \gamma D$ present for charged mesons. The combined full and dashed lines are the heavy mesons, and the wavy lines represent photons.

For one gluon and one photon emission from the $B_q$-meson appearing in left part in Fig. 3a) one obtains:

$$(q_L t^a \gamma^\alpha Q_{\psi b}^{(+)})_{G\gamma} \rightarrow G_H g_s e F_{\mu\nu} G_{\alpha\beta}^a Tr \left[ \xi^\dagger \gamma^\alpha L H_{bq}^{(+)} R^{\mu\nu\alpha\beta} \right],$$

where the tensor $R$ contains products of Dirac matrices and propagators (with momentum integrated out). Multiplying the currents for each vertex, for instance those in eqs. (23) and (24), and using the prescription:

$$g_s^2 G_{\mu\nu} G_{\alpha\beta}^a \rightarrow 4\pi^2 \left( \frac{\alpha_s}{\pi} G^2 \right)^{12} \left( g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right),$$

we obtain the bosonized version for the operator $\tilde{Q}_1$ in eqs. (10) and (11) as the product of two traces. (The expression may be simplified by using the Dirac algebra, but we do not enter these details here).

**IV. AMPLITUDES FOR $B \to \gamma D^*$**

We restrict ourselves to processes where the $b$-quark decays, namely the charged modes $B^- \to \gamma D_{q^-}^*$ and the neutral modes $B_0^q \to \gamma D_{q^0}^*$, and $\bar{B}_q^0 \to \gamma D_{q^0}^*$, for $q = d, s$. Processes where the anti-$b$ quark decays proceed analogously.

Considering simple quark diagrams only, we observe that in terms of the Wolfenstein parameter $\lambda$, the amplitudes for $B^- \to \gamma D_{q^-}^*$ and $\bar{B}_q^0 \to \gamma D_{q^0}^*$ are $O(\lambda^4)$ and small. In contrast, the amplitude for $\bar{B}_d^0 \to \gamma D_{q^0}^*$ is $O(\lambda^2)$, and is KM non-suppressed compared to other $b \to \gamma D^*$ modes. For $q = s$, all the amplitudes are $O(\lambda^3)$. We will however see that strong interactions might make this simple picture more complicated.

For the charged decay(s) there are *pole diagrams*, which are absent for the neutral decays. These are (within HQEFT) obtained by the bosonized currents in (17) and (18) and the
photon emission is obtained from (15). Here the $B^− \to D_{s,d}^{*-}$ transitions are proportional to the favorable coefficient $a_f = (a_{2,4} + a_{1,3}/N_c) \sim 1$, while the non-factorizable contributions proportional to $2a_{1,3}$ due to coloured operators are relatively small.

There are some meson exchange decay mechanism contributions. In the heavy quark limit these are identical with chiral loop contributions. These are shown in Fig. 5. For the process $B^0 \to B^0_{d,s} \to \gamma D^{*-}$ there is an intermediate $B^0_{d,s} \to D^{*-}$ transition accompanied with emission and re-absorption of $\pi^-$, or an intermediate $B^0_{d,s} \to D^{*-}$ transition accompanied with emission and re-absorption of $K^-$. For the process $B^0 \to B^0_{d,s} \to \gamma D^{*-}$ there is an intermediate $B^0_{d,s} \to D^{*-}$ transition accompanied with emission and re-absorption of $\pi^+$. For $B^0 \to B^0_{d,s} \to \gamma D^{*-}$ there is an intermediate $B^0_{d,s} \to D^{*-}$ transition accompanied with emission and re-absorption of $K^+$. Because the transitions $B^0 \to D^{*-}$ are non-suppressed in the factorized limit, the decays $B^0_{d,s} \to \gamma D^{*-}$ is semi-suppressed, having meson exchange amplitudes reducing to chiral loops in the HQEFT limits. In this limit the meson exchange amplitudes are proportional to

$$\chi(M) = \left( \frac{g_{AM_M}}{4\pi f_\pi} \right)^2 \ln\left( \frac{\Lambda^2}{m_M} \right),$$  

for exchange of $M = K, \pi$ respectively. Here $g_A$ is the light $(M = K, \pi)$ meson axial coupling to heavy mesons and $\Lambda \chi \simeq 1$ GeV. Numerically, $\chi(K) \simeq 0.09$ and $\chi(\pi) \simeq 0.02$, respectively. For the processes $B^0_{d,s} \to \gamma D^{*-}$ there are only Zweig-forbidden and $SU(3)_F$ violating neutral meson exchange which give small contributions.

For the processes $B^0_{d,s} \to \gamma D^{*-}$ the amplitudes $A^{(\pm)}$ are of the form

$$A^{(\pm)}_G = \frac{-eC_G}{28\pi} G^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \left( Q^B_B Z^{(\pm)}_B + Q^D_D Z^{(\pm)}_D \right),$$

where $Q^B_B$ and $Q^D_D$ are the charges of the light quarks within the $B$ and $D$ mesons, respec-
tively. For the processes $\overline{B}^0_{d,s} \to \gamma D^{*0}$ we have $Q^B_{q} = -1/3$ and $Q^D_{q} = 2/3$. The quantities $Z^{(\pm)}$ are of order one and given by

$$Z_B^{(+)} = \left(\frac{89\pi}{288} - \frac{13}{18}\right)k\omega y + \left(\frac{7\pi}{144} + \frac{5}{18}\right)k\omega^2 + \left(\frac{11}{18} - \frac{13\pi}{96}\right)k + \left(\frac{2}{3} - \frac{\pi}{18}\right),$$

(27)

$$Z_B^{(-)} = -\frac{5\pi}{9}k\omega y + \left(\frac{\pi + 2}{9}\right)k\omega^2 + \left(\frac{\pi}{9} - \frac{4}{3}\right)k - \left(\frac{\pi + 8}{9}\right),$$

(28)

$$Z_D^{(+)} = -\left(\frac{11\pi}{288} + \frac{17}{36}\right)k\omega y + \left(\frac{1}{36} - \frac{53\pi}{288}\right)k - \left(\frac{\pi}{64} + \frac{7}{72}\right)\omega y + \left(\frac{41\pi}{576} + \frac{23}{72}\right),$$

(29)

$$Z_D^{(-)} = -\left(\frac{\pi}{3} + \frac{4}{9}\right)k + \left(\frac{2}{9} - \frac{\pi}{18}\right)\omega y + \frac{4}{3}k\omega y + \left(\frac{2}{9} - \frac{\pi}{18}\right),$$

(30)

where the dimensionless parameters $k, \omega,$ and $y$ are given by

$$k = \frac{2\pi f_p^2}{N_c m^2}, \quad \omega = v_b \cdot v_c = \frac{M_B^2 + M_D^2}{2M_B M_D}, \quad y = \frac{M_B}{M_D},$$

(31)

where we for $M_D$ have used the mass of $D^*$. Using $m = 230$ MeV, $\rho = 1.1$, and $\langle \alpha_s \pi G^2 \rangle^{1/4} = 310$ MeV, we obtain

$$BR(\overline{B}^0_d \to \gamma D^{*0}) \simeq 1.6 \times 10^{-6}, \text{ and } BR(\overline{B}^0_s \to \gamma D^{*0}) \simeq 8 \times 10^{-8}. \quad (32)$$

For the decays $\overline{B}^0_{d,s} \to \gamma D^{*0}$, there is a delicate balance between different amplitudes, and it is hard to conclude anything within our framework. The decays $B^- \to \gamma D_{d,s}^-$, the factorizable contributions dominate, and the amplitudes obtained from the diagrams in FIG. 2 alone are

$$A_F^{(\pm)} = -(C_4 + C_3/N_c) \frac{e N_c G_H \alpha_H}{16\pi} Y^{(\pm)},$$

(33)

where

$$Y^{(\pm)} = Q^B_q + Q^D_q (1 + k - k\omega y)$$

(34)

which within our framework is roughly four times the pole contribution. For the parity violating case, one has

$$Y^{(-)} = -Q^B_q (1 + 2k) + Q^D_q (1 + k\omega y).$$

(35)

To obtain a parity violating pole term, the intermediate heavy meson(s) must have positive parity. We find that $B^- \to \gamma D_d^-$ has a branching ratio of order $2 \times 10^{-7}$ within our framework.
V. CONCLUSION

We have considered six decay modes of the type \( B \rightarrow \gamma D \) generated by three (main) mechanisms:

- a) Factorized contributions of pole and non-pole type. These might be proportional to the favorable Wilson coefficient combination \( a_f \sim 1 \), or the non-favorable coefficient combination \( a_{nf} \) of order \( 10^{-2} \).

- b) Meson exchanges, that is, some intermediate \( B \rightarrow D \) transition accompanied with an emission and re-absorption of a pseudoscalar boson (\( \pi \) or \( K \)).

- c) Emission of soft gluons, modeled by a gluon condensate. In the HQEFT limits, the mechanisms b) and c) are (formally) \( 1/N_c \) suppressed. If the corresponding factorizable amplitude is proportional to the favorable coefficient \( a_f \), this mechanism gives a non-important contribution \( \sim a_{1,3} \). On the other hand, if the factorizable contribution is proportional to the non-favorable coefficient \( a_{nf} \), the non-factorizable soft emission amplitude contribution is proportional to the favorable coefficient \( 2a_{2,4} \), and gives a significant contribution. Contributions to the various \( B \rightarrow \gamma D^* \) modes are qualitatively summarized in Table I.

The present analysis is performed within HQEFT, both for the \( b \)- and the \( c \)-quark. Formally, the modes \( B \rightarrow \gamma D^* \) are “heavy to heavy”, but for precise estimates our framework is unrealistic because the energy gap between the \( b \)- and the \( c \)-quark are significantly bigger.
than 1 Gev, which is the scale of HLχPT and the HLχQM. Therefore large 1/m_Q corrections (especially 1/m_c corrections) must be expected. Phrased in another way, damping form factors are expected to be present, and most probably, our estimates are overestimates. Alternative estimates, based on other frameworks, for instance considering the charm quark as “light”, should be performed. Still, we expect that we have obtained amplitudes of the right order of magnitude, while the branching ratios might be an order of magnitude off. Still, our conclusion about the importance of the non-factorizable gluon emission for $B_{d,s}^0 \rightarrow \gamma D^{*0}$ should hold.

Acknowledgments

JOE is supported in part by the Norwegian research council and by the European Union RTN network, Contract No. HPRN-CT-2002-00311 (EURIDICE).

[1] M. Beneke, G. Buchalla, M. Neubert, C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); C. W. Bauer, D. Pirjol, I. Z. Rothstein, I. W. Stewart et al., Phys. Rev. D 70, 054015 (2004); Y.Y. Keum, H-n. Li, A.I. Sanda, Phys.Lett. B504 6-14 (2001).
[2] J.O. Eeg, A. Hiorth, and A.D. Polosa: Phys.Rev. D65 (2002) 054030 [hep-ph/0109201]; J.O. Eeg, S. Fajfer, and A. Hiorth: Phys.Lett. B570: (2003) 46-52 [hep-ph/0304112]. Jan O. Eeg, Svjetlana Fajfer, and Anita Prapotnik: Eur. Phys. J. C42 (2005) 29-36 [hep-ph/0501031]. J.O. Eeg, S. Fajfer, J. Zupan, Phys. Rev. D 64 (2001) 034010.
[3] R.R. Mendel and P. Sitarski, Phys.Rev. D36 (1987) 953, H.-Y. Cheng et al. Phys.Rev. D51 (1995) 1199. H. Routh and V.P. Gautam, Phys. Scripta. 60 (1999) 401. R.F. Lebed, Phys.Rev. D61 (2000) 033004.
[4] B.Grinstein and R.F. Lebed, Phys.Rev. D60 (1999) 031302.
[5] For a review, see M. Neubert, Phys. Rep. 245 (1994) 259.
[6] R. Casalbuoni, A. Deandrea, N. Di Bartelomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rep. 281 (1997) 145.
[7] C.G. Boyd and B. Grinstein, Nucl. Phys. B451 (1995) 177.
[8] A. Hiorth and J. O. Eeg, *Eur. Phys. J. Direct C* **30** (2003) 006.
[9] J. O. Eeg, A. Hiorth, A. D. Polosa, *Phys. Rev. D* **65** (2002) 054030.
[10] A. Hiorth and J. O. Eeg, *Phys. Rev. D* **66** (2002) 074001.
[11] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *Nucl. Phys. B* **449** (1995) 197,
V. Antonelli, S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin,
*Nucl. Phys. B* **469** (1996) 143, S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, *Nucl. Phys. B* **514** (1998) 63, *ibid B* **514** (1998) 93.
[12] G. Buchalla, A. J. Buras, M. E. Lautenbacher *Rev. Mod. Phys.* **68** (1996) 1125.
[13] B. Grinstein, W. Kilian, T. Mannel, and M.B. Wise, *Nucl. Phys. B* **363** (1991) 19. R. Fleischer,
*Nucl. Phys. B* **412** (1994) 201.
[14] J. A. Cronin, *Phys. Rev.** 161** (1967) 1483,
S. Weinberg, *Physica** 96A** (1979) 327,
D. Ebert and M.K. Volkov, *Z. Phys. C* **16**(1983) 205,
A. Manohar and H. Georgi, *Nucl. Phys. B** 234**(1984) 189,
J. Bijnens, H. Sonoda and M.B. Wise, *Can. J. Phys.* **64** (1986) 1,
D. Ebert and H. Reinhardt, *Nucl. Phys. B** 71**(1986) 188,
D. I. Diakonov, V. Yu. Petrov and P. V. Pobylitsa, *Nucl. Phys. B** 306**(1988) 809, D. Espriu,
E. de Rafael and J. Taron, *Nucl. Phys. B** 345**(1990) 22.
[15] W. A. Bardeen and C. T. Hill, *Phys. Rev. D* **49** (1994) 409 .
A. Deandrea, N. Di Bartelomeo, R. Gatto, G. Nardulli, and A. D. Polosa,
*Phys. Rev. D** 58** (1998) 034004. A. Polosa, *Riv. Nuovo Cim.* **23** N11 (2000) 1.
D. Ebert, T. Feldmann R. Friedrich and H. Reinhardt, *Nucl. Phys. B** 434**(1995) 619, D. Ebert
and M.K. Volkov, *Phys.Lett. B** 272**(1991) 86.
[16] J. O. Eeg and I. Picek, *Phys. Lett. B** 301**(1993) 423, *ibid. B** 323**(1994) 193,
A.E. Bergan and J.O. Eeg, *Phys. Lett. 390** (1997) 420.
[17] A. Pich and E. de Rafael, *Nucl. Phys. B** 358**(1991) 311.
[18] J.A.M. Vermaseren, “Symbloic Manipulation with FORM”,
CAN 1991, Amsterdam. (ISBN 90-74116-01-9).
[19] John Atle Macdonald Sørensen, Master Thesis, Phys. Dept, Univ of Oslo, 2005. Unpublished.