Characteristics of Prospective Student Teacher's Representation in Solving Ill-Well Algebraic Problems

Ika Santia¹,²*, Purwanto², Subanji², Sudirman², and Akbar Sutawidjadja²

¹Universitas Nusantara PGRI Kediri, Kediri, Indonesia.
²Universitas Negeri Malang, Malang, Indonesia.

*ika.santia.1703119@students.um.ac.id

Abstract. Mathematical representation is an important aspect of mathematical problem-solving. But prospective student teachers' ability of a mathematical representation in ill-structured problem solving is still very limited compared to that in well-structured problem-solving. However, ill-structured problems supported mathematical abstraction used in mathematical concept understanding. This study described the characteristics of prospective student teachers' representations and translation among representation in ill-structured and well-structured social algebraic problem-solving. Thirty Indonesian prospective students teachers have to solve the ill-well algebraic problems by using think-aloud while recorded using a video recorder and the researchers observed by writing on a field note to record the important events. Data were analyzed using a comparative method so that it was obtained the different characteristics of representations between solving ill-structured and well-structured problems. The result showed that verbal and symbolic representations used by subjects to compute, detect and correct an error, and justify their answers in ill-structured problem solving but visual representation only used by the first subject to detect and correct error. The subjects lack to uncover necessary information to solve the ill-Structured problem compared to that of a well-structured problem. It means that representation is important to be studied further because it affected to ill-structured problem ability.

Keywords: characteristics, prospective student teacher's, algebraic problems

1. Introduction

Representation is the disclosure of mathematical ideas by using various means such as spoken language, written language, symbols, pictures, diagrams, models, charts, or using physical members [3,5,11]. NCTM explains that the use of a mathematical representation forms, such as charts, graphs, tables and symbols as well as a transition among the representation of an important capital in expressing mathematical thinking [8]. The use of representation reinforces
students' understanding to construct mathematics concepts and mathematics problem solving [1]. So in the mathematics learning process, mathematical representation is one aspect that must be prioritized. This is because of the ability of an accurate representation of mathematics students then they can expand their capabilities in solving mathematical problems. Research related to the importance of representation in mathematics, among others, performed by Caglayan & Olive, Bal, Villegas, et al., Fazio, et al., and Birgin, these studies found that the ability of the student representation is the key to success in understanding mathematical concepts and problem-solving [2,3,4,12].

Based on the importance of mathematical representation in problem-solving, one of the problems that support multiple representations is an ill-structured problem. Jonassen stated that Ill-structured problems are the kinds of problems that are conflicting goals, multiple solution methods, unanticipated problems, distributed knowledge, collaborative activity systems, and multiple forms of problem representation. So, by using the ill structure problem will support students' mathematical representation ability. Ill-structured problems contents authenticity, complexity, and openness [6,10]. The complexities made multiple problem representations.

Hammard introduced an ill-structured problem as an ill-structured problem. The aspects of conflicting goals and multiple solution methods in ill-structured problems were seen on an ill-structured problem. The Ill-Structured problem is a problem that either has no solutions in the desired class, or has many solutions, or the solution procedure is unstable. Most difficulties in solving ill-Structured problems are caused by solution instability. Therefore, the term "ill-structured problems" is often used for unstable problems [7]. The inverse of ill-structured problems is a well-structured problem. Well-Structured problems are the problem that has exists a unique solution to this problem that continuously depends on its data [6,7].

This study using ill-structured problems and well-structured problems describes of mathematical representation of mathematics education students. To describe the mathematical representation, we took into account the following types of external representations: 1) verbal representation of the well-Structured problems and ill-structured problems: consisting fundamentally of the well-Structured problems and ill-Structured problems as stated, whether in writing or spoken (Vb); 2) visual representation: consisting of drawings, diagrams or graphs, as well as any kind of related action (Vs); 3) symbolic representation, consisting of numbers, operation and relation signs; algebraic symbols, and any kind of action referring to these (Sb); and 4) their translation among multiple representations in solving ill-Structured problem and well-Structured problem (Vb-Vs, Vb-Sb, Vs-Sb), we used the analytical framework from Villegas’ framework modified that shown in Table 1.

| Table 1. Framework for Mathematical Representation Analysis |
|-------------------------------------------------------------|
| **Part 1: Verbal representation of well-structured and ill-structured problems solving (Vb)** |
| Description: Prospective student teacher finds the problem. |
| Indicators: 1) Prospective student teacher reads the well-ill Structured problem aloud or silently, and 2) Prospective student teacher verbalizes the well-ill Structured problem used the usual style of talk. |
| **Part2: Visual representation of well-structured and ill-structured problems solving (Vs)** |
| Description: Mathematics students use or modify visual representations. |
| Indicators: 1) Mathematics student draws problems with visual representation, or modifies such representations made earlier, 2) Prospective student teacher operates formulas with visual representations, 3) Prospective student teacher observes a visual representation or verbalizes terms associated with visual representations, and 4) Prospective student teacher uses their gesture to show visual representations. |
Part 3: Symbolic representation of well-structured and ill-structured problems solving (Sb)

Description: Prospective student teacher produces, operates with or modifies symbolic representations.

Indicators: 1) Prospective student teacher solves a symbolic expression, 2) Prospective student teacher verbalizes how he can solve an equation, or checks how it was solved, and 3) Prospective student teacher modifies, eliminates a symbolic expression.

Part 4: Translation between a verbal representation and visual representation of well-Structured and ill-Structured problems solving (Vb-Vs)

Description: Prospective student teacher relates a visual representation to a verbal representation.

Indicators: 1) Prospective student teacher makes a visual representation directly from the well-structured and ill-structured problems, either without modifying it, 2) Prospective student teacher transforms or modifies a visual representation according to a new interpretation of the well-Structured and ill-structured problems, and 3) Prospective student teacher establishes relationships between the well-Structured and ill-structured problems and a visual representation, using verbalizations or gestures.

Part 5: Translation between a visual representation and symbolic representation of well-Structured and ill-Structured problems solving (Vs-Sb)

Description: Prospective student teacher somehow relates a pictorial representation to a symbolic representation.

Indicators: 1) Prospective student teacher formulates on paper a symbolic expression or part of one based on a visual representations or makes a visual representation based on a symbolic expression, 2) Prospective student teacher establishes relationships between a symbolic expression and a visual expression using verbalizations or gestures, 3) Prospective student teacher makes changes or eliminates a visual representation made earlier, based on symbolic results obtained, and 4) Prospective student teacher modifies or eliminates symbolic representations due to results obtained in visual representations or due to a new visual representation.

Part 6: Translation between a symbolic representation and verbal representation of well-Structured and ill-Structured problems solving (Sb-Vb)

Description: Prospective student teacher somehow relates a symbolic representation to a verbal representation.

Indicators: 1) Prospective student teacher formulates a symbolic expression based on their interpretation, without modifying it, 2) Prospective student teacher modifies a symbolic expression due to a new interpretation of the well-structured and ill-structured problems, and 3) Prospective student teacher reformulates the well-structured and ill-structured problems in a new way due to some result obtained from a symbolic expression.

2. Method

In qualitative research, the instrument is the researchers itself. Researchers take apart as a planner, a data collector, and an analyzer last but not least is as a person concludes. In this case, researchers as instruments were supported by other instruments, namely ill-well structured problems, think-aloud video recording, and its transcripts, and field notes. This research approach is a qualitative study. The steps to obtain the data were: (1) the researchers gave ill-well structured a problem to 30 students who took the “Magang II” courses in Universitas Nusantara PGRI Kediri, and then they called prospective student teacher. The problems in the form of a
design problem for ill-structured problems and algebra problems for a well-structured problem. The problem is an ill-structured test and well-structured test) as shown in Figure 1 and Figure 2, respectively; (2) subjects solve mathematical problems by think-aloud. During the processed the researcher recorded the subject activity using a video recorder and recorded interesting things that happened on the field note sheet; (3) the researcher reviewed the subject's answer sheet, video recordings, and field notes to know the activity of representation in ill-well structured problem-solving process; (4) researchers conducted transcription data after whole data were obtained. The data transcribed were think-aloud recording data; (6) the researchers performed data reduction process by making abstraction in the form of summary of core data, process, and statements that need to be maintained to be used as data; (7) researchers analyzed the mathematical representation in ill-well structured problem-solving process of subjects, (8) the researcher performed the triangulation process. This research used a technique triangulation process in the form of several uses of data collection techniques in the form of answer sheets of the subject in solving ill-well structured problems, think-aloud transcriptions, and field notes, (9) researchers drew conclusions through the results of the split between the subject sheet in solving ill-well structured problems, transcription of think aloud, as well as field notes.

### 3. Result and Discussion

#### 3.1 Result

In this article, we describe two subjects’ solutions, namely S1 and S2. For the first description, we show S1’s solution and think-aloud transcription. Figure 3 and Figure 5 demonstrate the differences in mathematical representation in ill-structured and well-structured problems of S1’s solutions.

![Figure 3. Subject S1’s solution in ill-Structured problem](image)

To explore further information, researchers analyze think aloud data on the subject of answers to the problem given. The data snippet as presented in Table 2.

### Table 2. Subject S1’s Transcription Think Aloud Data for ill-Structured Problem Solving

| Mathematical Representation |
|-----------------------------|
| Determine the quadratic function that it's graph through point K (2, 3) |

![Figure 1. TPM-1 (ill-Structured problem)](image)

If $x \in R$ and satisfying $\frac{6}{1+x} = 2$. Find the value of $x$

![Figure 2. TPM-1 (well-Structured problem)](image)
Time | Description | Type of Representations
00:01 | Silent and read the problem. | -
00:05 | “These question only have one point known, namely point K(2,3).” | Vb-Sb, Sb-Vb
00:11 | “Emm...it means a lot of solutions to this problem because the equation must go through a minimum of two points.” | Vb
00:23 | “My first step is to write a general form of quadratic equations \( y - q = a(x - p)^2 \).” | Vb-Sb
00:32 | “Earlier q was known to be 3 and p was 2, and a is unknown, so I am writing \( y - 3 = a(x - 2)^2 \)” | Vb-Sb, Sb-Vb
01:05 | “...then everything changed and moved to one segment, which resulted in \( y = a\left(x - \frac{3}{4}\right)^2 - 4ax + 4a + 3 \)” | Vb-Sb
01:11 | “Hemm...but a=...He... I’m confused” | Vb

Figure 3 and Table 2 indicates that S1 used verbal representation to expressed that the problem known was point K (2,3), and the quadratic function found passed through that point K. S1 detected and corrected error by verbal representation with said that must be a minimum of two known points to form a quadratic function. S1 used algebraic equation \( y - q = a(x - p)^2 \) to represent symbolically the quadratic function in the problem. S1 made translation between a verbal representation to simbolic representation. S1 also made translation from verbal representation to symbolic representation. S1 lacked to uncover necessary information about scale to compute and justify her answers.

![Figure 3](image)

**Figure 4.** Subject S1’s Solution in Well-Structured Problem Solving

To explore further information, researchers analyze think aloud data on the subject of answers to the problem given. The data snippet as presented in Table 3.

**Table 3.** Subject S1’s Transcription Think Aloud Data for Well-Structured Problem Solving

| Time  | Description | Type of Representations |
|-------|-------------|-------------------------|
| 00:01 | Silent and see the problem | - |
| 00:35 | In the right side, I replace 3 by 1plus 2, so \( \frac{4}{1+x} \) equals to 2. | Vs-Sb |
| 01:06 | Then, in the right side I replace again 2 with 4 divided by 2, so \( x \) equals to 1. | Vs-Sb |

Figure 4 and Table 3 indicates that S1 used verbal representation to formulates a symbolic expression. S1 detected and corrected error by symbolic representation with substitution \( x = 1 \).
to first equation. S1 transforms or modifies a symbolic expression due to a new interpretation. S1 make translation between a visual representation to symbolic representation to compute and justify her answers.

And then we give a described of Subject S2 solutions. Figure 7 and Figure 8 demonstrate the differences mathematical representation in ill-Structured and well-Structured problems of S2 solutions.

Figure 5. Subject S2’s Solution in Ill-Structured Problem Solving

To explore further information, researchers analyze think aloud data on the subject of answers to the problem given. The data snippet as presented in Table 4.

Table 4. Subject S2’s Transcription Think Aloud Data for ill-Structured Problem Solving

| Time  | Description                                                                 | Type of Representations |
|-------|------------------------------------------------------------------------------|-------------------------|
| 00:01 | Silent and read the problem.                                                 |                         |
| 00:15 | “These question known, hemm...y = ax^2 + bx + c so 3 = ....”                 | Vs-Sb, Sb-Vb            |
| 00:21 | “The first alternative solution is y = \( \frac{1}{2} x^2 \), x substituted with 2, then the result is 4; 4 divided by 2 is 2; because these result must be 3, 1 must be added, so this means y = \( \frac{1}{2} x^2 + 1 \).” | Vb-Sb, Sb-Vs            |
| 00:23 | “...and then the second alternative solution is y = x^2 – 3x. It means -3(2) + 22 = -6 + 4, which is -2; so for the answer to be 3, this means 5 must be added y = x^2 – 3x + 5” | Vb-Sb, Sb-Vs, Vb-Vs     |

Figure 5 and Table 4 indicates that S2 represented symbolically the quadratic function in question as \( y = ax^2 + bx + c \), and then used another symbolical representation by writing ‘3 = ?’. S2 used verbal representation to calculate the equation. S2 used symbolic representation to verbalizes the final answer. S2 make translation between a simbolic representation to verbal representation. S2 also make translation from verbal representation to symbolic representation. S2 lacks to uncover necessary information about scale to justify her answers.

Figure 6. Subject S2’s Solution in Well-Structured Problem Solving

To explore further information, researchers analyze think aloud data on the subject of answers to the problem given. The data snippet as presented in Table 5.
Table 5. Subject S2’s Transcription Think Aloud Data for Well-Structured Problem Solving

| Time   | Description                                                                 | Type of Representations |
|--------|-----------------------------------------------------------------------------|-------------------------|
| 00:27  | I multiplied the left and right segments by \(1 + \frac{4}{1+x}\)            |                         |
| 00:45  | Then, I reduced the left and right segments by 2. So I get \(4 = \frac{8}{1+x}\) | Vs-Sb                   |
| 01:03  | And then I written \(1 + x = \frac{8}{4}\)                                  | Vs-Sb                   |
| 01:05  | Okey ma’am, the answer is 3                                                  |                         |

Figure 6 and Table 5 indicates that S2 transforms or modifies a symbolic expression due to a new interpretation. S2 used verbal representation to formulate a symbolic expression. S2 can’t detect and correct error the final answer. S1 make translation between a verbal representation to symbolic representation to compute his answers.

3.2 Discussion

After examining the participants’ worksheet and think aloud result, the researcher found that only two participants were able to answer completely solving well-structured and ill-structured problems. From that fact, it showed that participants had difficulty in solving the problem of algebraic function and algebraic equations in ill-well structured.

Based on Table 2 and Table 4, the subject solution stages to solve an ill-structured problem can be described as follows:
1. The subject used verbal representation to identify the problem.
2. Subject used translation between verbal representation and visual representation to describe the data.
3. Subject used translation between visual representation and symbolic representation to compute the answer.
4. The subject lacks to uncover necessary information about scale to justify her answers.

Figure 7. Stages of Subject’s Mathematical Representation in Ill-Structured Problem Solving.

Diagram details:
- Identify the problem
- Make a plan to solve the problem
- Carry out the plan to solve the problem (to compute and justify the answer)
- Detect and correct error of procedures and the answer
Based on Table 3 and Table 5, the subject solution stages to solve a well-structured problem can be described as follows:

1. The subject used verbal representation to identify the problem.
2. Subject used translation between verbal representation and symbolic representation to describe the data.
3. The subject used a translation between symbolic representation and verbal representation to compute the answer.
4. The subject used verbal representation to justify her answers.

**Figure 8. Stages of Subject’s Mathematical Representation in Well-Structured Problem Solving**

Diagram details:

| Diagram Details                                      |
|------------------------------------------------------|
| Identify the problem                                 |
| Make a plan to solve the problem                    |
| Carry out the plan to solve the problem (to compute and justify the answer) |
| Detect and correct error of procedures and the answer |

Based on the theoretical framework and results, we found that the two subjects had different methods of using mathematical representations to solve an ill-structured problem. It showed that the success of problem-solvers is based on their ability to build problem representations in a problem-solving situation [9,13]. The process of selecting problem representations causes differently characteristics when solving ill-structured problems, which are visio-verbal and verbal problem solvers. These characteristics show that there is a strong relationship between success in solving ill-structured problems and translation of representation skills. The findings are by following Villegas who states that there was a strong relationship between success in solving problems and skills in construction, use, and articulation of representation [12].
4. Conclusion
The purpose of this study was to describe how prospective student-teacher used mathematical representations in solving well-Structured and ill-Structured problems. For this purpose, we first developed a theoretical framework for mathematical representation analysis and then we take representations and the translation between representations as six parts or segments in solving ill-structured problems. In the process of segmentation and coding, we noted that certain segments do not correspond to the section identified initially, therefore it was necessary to add in these episodes as "events not classifiable as representations", comprising verbalizations about planning and execution, affectivity or verification.

Based on these theoretical frameworks for mathematical representation analysis, we found that the two prospective student teachers have different typologies in how they use representations in solving well-structured and ill-structured problems. This leads us to think that there are well-defined typologies of solvers.

Characterization of the solvers makes evident that there is a strong relationship between success in solving well-structured and ill-structured problems and skill in the translation of representations. There is a strong relationship between success in problem solving and skill in construction, use, and accurate representations. Furthermore, this problem needs to be studied by other researchers in the next studies, because the mathematical representation is the key to success in mathematical problem-solving.

Furthermore, the studies will be carried out by increasing the number of subjects and extending the study period so that explored the translation of mathematical representations in solving ill-structured problems is obtained.

Acknowledgments
The researchers would like to express their gratitude to the Graduate Program of Mathematics Education at The State University of Malang and the Department of Mathematics Education at The Faculty of Teacher Training and Education of University of Nusantara PGRI Kediri.

References
[1] Bal, A. P. (2015). The Examination of Representations used by Classroom Teacher Candidates in Solving Mathematical Problems. Educational Sciences: Theory & Practice. https://doi.org/10.12738/estp.2014.6.2189
[2] Birgin, O. (2012). Investigation of Eighth-Grade Students’ Understanding of the Slope of the Linear Function Investigando a Compreensão de Alunos do Oitavo Ano sobre a Inclinação de Funções Lineares, 139–162.
[3] Caglayan, G., & Olive, J. (2010). Eighth grade students’ representations of linear equations based on a cups and tiles model. Educational Studies in Mathematics, 74(2), 143–162. https://doi.org/10.1007/s10649-010-9231-z
[4] Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. Journal of Experimental Child Psychology, 123(1), 53–72. https://doi.org/10.1016/j.jecp.2014.01.013
[5] Goldin, M. S., & Beilock, S. L. (2010). Action's influence on thought: The case of gesture. Perspectives on Psychological Science, 5(6), 664–674. https://doi.org/10.1177/1745691610388764
[6] Jonassen, D. (2011). Learning to Solve Problems A Handbook for Designing Problem-Solving Learning Environments. London: Pfeiffer Publisher
[7] Kabanikhin, S. I. (2008). Definitions and examples of inverse and ill-Structured problems, 16, 317–357.
[8] NCTM (2000). Principles and standards for school mathematics. Reston, VA: The author.
[9] Nizarrudin. (2014). Role of Multiple Representations in Mathematical Problem Solving. Conference on Mathematics and Natural Sciences Education Faculty, PGRI, Semarang University Indonesia, pp 163-168

[10] Plano, V. & Creswell, J. (2011). Designing and conducting mixed methods research, 2nd edn. London: Sage.

[11] Rahmad, B. A., Ipung, Y., Abdur, R. A., Siswoyo, & Dwi, R. (2016). Mathematical representation by students in building relational understanding on concepts of area and perimeter of rectangle. Educational Research and Reviews, 11(21), 2002–2008. https://doi.org/10.5897/ERR2016.2813

[12] Villegas, J. L., Castro, E., & Gutiérrez, J. (2009). Representations in problem solving: a case study with optimization problems. Electronic Journal of Research in Educational Psychology, 7(1), 279–308.

[13] Zhang, J. (1997). The Nature Problem of External in Solving Representations. Cognitive Science, 21(2), 179–217. https://doi.org/10.1207/s15516709cog2102_3