Aspects of thermal leptogenesis in braneworld cosmology

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The mechanism of thermal leptogenesis is investigated in the high-energy regime of braneworld cosmology. Within the simplest seesaw framework with hierarchical heavy Majorana neutrinos, we study the implications of the modified Friedmann equation on the realization of this mechanism. In contrast with the usual leptogenesis scenario of standard cosmology, where low-energy neutrino data favors a mildly strong washout regime, we find that leptogenesis in the braneworld regime is successfully realized in a weak washout regime. Furthermore, a quasi-degenerate light neutrino mass spectrum is found to be compatible with this scenario. For an initially vanishing heavy Majorana neutrino abundance, thermal leptogenesis in the brane requires the decaying heavy Majorana neutrino mass to be $M_1 \gtrsim 10^{10}$ GeV and the fundamental five-dimensional gravity scale $10^{12} \lesssim M_5 \lesssim 10^{16}$ GeV, which corresponds to a transition from brane to standard cosmology at temperatures $10^9 \lesssim T_1 \lesssim 10^{14}$ GeV.

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I. INTRODUCTION

The most recent Wilkinson Microwave Anisotropy Probe (WMAP) results and big bang nucleosynthesis analysis of the primordial deuterium abundance imply that the baryon-to-photon ratio of number densities.

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_{\gamma}} = (6.1 \pm 0.3) \times 10^{-10},$$

for the baryon-to-photon ratio of number densities. Among the viable mechanisms to explain such a matter-antimatter asymmetry in the universe, leptogenesis has become one of the most compelling scenarios due to its simplicity and close connection with low-energy neutrino physics. In the simplest framework, consisting of the out-of-equilibrium decays of the heavy Majorana neutrinos $N_1$ at temperatures below their mass scale $M_1$. The lepton asymmetry generated in the presence of $CP$-violating processes is then partially converted into a baryon asymmetry by the sphalerons.

Right-handed neutrinos can also provide a natural explanation for the smallness of the neutrino masses through the well-known seesaw mechanism. In this respect, the heavy Majorana neutrino mass spectrum turns out to be crucial for a successful implementation of both leptogenesis and the seesaw mechanism. In particular, the standard thermal leptogenesis scenario with hierarchical heavy Majorana neutrino masses ($M_1 \ll M_2 \ll M_3$) constrains the lightest heavy Majorana mass to be $M_1 \gtrsim 4 \times 10^8$ GeV, if $N_1$ is in thermal equilibrium before it decays, or $M_1 \gtrsim 2 \times 10^9$ GeV for a zero initial $N_1$ population. Moreover, an upper bound on the light neutrino masses, $m_i < 0.12$ eV, is implied by successful leptogenesis.

In determining the departure from thermal equilibrium, the interplay between the expansion rate of the universe $H$ and the particle reaction rates involved at the relevant epoch is crucial. Of course, such an interplay depends on the specific properties of early cosmology. Braneworld cosmology (BC) has opened up the possibility for a new phenomenology of the early universe. In particular, the Randall-Sundrum type II braneworld model has recently received much attention. In this model, the expansion dynamics at early epochs is changed by the presence in the Friedmann equation of a term quadratic in the energy density $\rho$.

$$H^2 = \frac{8\pi}{3M_P^2} \rho \left(1 + \frac{\rho}{2\lambda}\right), \quad \lambda = \frac{3}{4\pi} \frac{M_5^2}{M_P^2},$$

where $M_P \approx 1.22 \times 10^{19}$ GeV is the four-dimensional Planck mass, $M_5$ is the five-dimensional Planck mass and we have set the four-dimensional cosmological constant to zero and assumed that inflation rapidly makes any dark radiation term negligible. Notice that Eq. (2) reduces to the usual Friedmann equation at sufficiently low energies, $\rho \ll \lambda$, while at very high energies we have $H \propto \rho$. This behavior has important consequences on early universe phenomena such as inflation and the generation of the baryon asymmetry.

In this paper, we study how the standard mechanism of thermal leptogenesis is affected by modifications in the expansion rate of the universe due to early braneworld cosmology. We perform a thorough analysis of this scenario by solving the relevant Boltzmann equations following the approach of Refs. and the standard cosmology (SC) case.

The structure of the paper is as follows. In Section II we briefly present the basic formulae for the mechanism of thermal leptogenesis in braneworld cosmology. In section III we present a simple fit for the efficiency factor...
as a function of the decay parameter, obtained by solving numerically the Boltzmann equations. In Section IV we find lower bounds on the decaying heavy Majorana neutrino mass, the fundamental five-dimensional gravity scale and the transition temperature from brane to standard cosmology. In Section V we derive the upper bounds on the light neutrino masses implied by leptogenesis. Finally, in section VI we present our conclusions.

II. THERMAL LEPTOGENESIS IN THE BRANE

In its simplest standard model version, leptogenesis is dominated by the CP-violating interactions of the lightest of the heavy Majorana neutrinos. Assuming that right-handed neutrinos are hierarchical, $M_1 \ll M_{2,3}$, and that the decays of the heavier neutrinos do not influence the final value of the $B - L$ asymmetry, the baryon asymmetry generated by $N_1$ can be conveniently parameterized as

$$\eta_B = \frac{\xi}{f} N_{B-L}, \quad (3)$$

where $\xi = 28/79$ is the fraction of the $B - L$ asymmetry converted into a baryon asymmetry by sphaleron processes $[3]$ and $f = 2387/86$ is the dilution factor calculated assuming standard photon production from the onset of leptogenesis till recombination. The amount of $B - L$ asymmetry per comoving volume, $N_{B-L}^f = N_{B-L}(T \ll M_1)$, is obtained from the solution of the relevant Boltzmann equations, which in the above minimal framework can be written in the compact form $[2]$,

$$\frac{dN_{N_1}}{dz} = -(D + S) \left( N_{N_1} - N_{N_1}^{eq} \right), \quad (4)$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 D \left( N_{N_1} - N_{N_1}^{eq} \right) - W N_{B-L}, \quad (5)$$

where $z = M_1/T$, $N_{N_1}^{eq}$ is the equilibrium number density, normalized so that $N_{N_1}^{eq}(z \ll 1) = 3/4$, and $\epsilon_1$ is the CP-asymmetry parameter in $N_1$ decays. Different physical processes contribute to these equations. The terms $D$, $S$ and $W$, defined through the corresponding rates $\Gamma_i$ as

$$D = \frac{\Gamma_D}{H}, \quad S = \frac{\Gamma_S}{H}, \quad W = \frac{\Gamma_W}{H}, \quad (6)$$

account for decays and inverse decays ($D$), $\Delta L = 1$ scatterings ($S$) and washout processes ($W$), respectively. While decays yield the source for the generation of the $B - L$ asymmetry, all other processes, including $\Delta L = 2$ processes mediated by heavy neutrinos, contribute to the total washout term.

During radiation domination $\rho = \pi^2 g_* T^4/30$ and the expansion rate $[2]$ is given by

$$H(z) = \frac{\bar{H}}{z^2} \sqrt{1 + \left(\frac{z}{z_I}\right)^4 \frac{1}{1 + z_t^4}}, \quad (7)$$

where

$$\bar{H} \equiv H(z = 1) = \sqrt{\frac{4\pi^3 g_* M_P^2}{45 M_f^2}} \sqrt{1 + z_t^4}, \quad (8)$$

and $g_* = 106.75$ is the effective number of relativistic degrees of freedom. The parameter $z_t$ is defined as

$$z_t = \frac{M_4}{T_t} = \left(\frac{\pi^3 g_* M_P^2 M_f^4}{45 M_P^2}\right)^{1/4}, \quad (9)$$

where $T_t$ is the temperature at which the transition from brane to standard cosmology takes place ($\rho(T_t) \simeq 2\lambda$).

In establishing a connection between leptogenesis and neutrino physics, the dependence of the different contributions given in Eq. (6) on the neutrino parameters turns out to be crucial. In particular, one can show that terms $D$ and $S$ are proportional to the so-called effective neutrino mass parameter

$$\bar{m}_1 = \frac{(m^+_{1\nu} m_{1\nu})_{11}}{M_1}, \quad (11)$$

where $m_{1\nu}$ is the Dirac neutrino mass matrix. On the other hand, the washout term $W$ contains in general two contributions, one which depends only on $\bar{m}_1$ and another which is proportional to the absolute neutrino mass scale $m^2 = m^2_1 + m^2_2 + m^2_3$.

Departure from thermal equilibrium is controlled by the decay parameter $K$,

$$K = \frac{\bar{\Gamma}_D}{\bar{H}} = \frac{\bar{m}_1}{m_*}, \quad (12)$$

where $\bar{\Gamma}_D = \Gamma_D(z = \infty)$ is the $N_1$ decay width at zero temperature,

$$\bar{\Gamma}_D = \frac{1}{8\pi} \frac{\bar{m}_1 M_f^2}{v^2}, \quad (13)$$

$v \approx 174$ GeV and $m_*$ is the equilibrium neutrino mass

$$m_* = \frac{8\pi \bar{H} v^2}{M_1^2} \simeq 1.08 \times 10^{-3} \text{ eV} \sqrt{1 + z_t^4}. \quad (14)$$

In the high-energy regime of brane cosmology, i.e. for $z_t \gg 1$, we find

$$m_* \simeq 1.1 \times 10^{-3} \text{ eV} \left(\frac{M_1}{10^8 \text{ GeV}}\right)^2 \left(\frac{10^{12} \text{ GeV}}{M_5}\right)^3. \quad (15)$$

The solution of Eq. (8) for $N_{B-L}$ is given by

$$N_{B-L}(z) = N_{B-L}^i e^{-\int_{z_i}^{z} dz' W(z')} - \frac{3}{4} \epsilon_1 \kappa(z), \quad (16)$$
where $N^i_{B-L}$ accounts for any possible initial $B-L$ asymmetry (e.g. due to the decays of the heavier neutrinos $N_{2,3}$) and $\kappa(z)$ is the efficiency factor, which measures $B-L$ production from $N_1$ decays,

$$\kappa(z) = \frac{4}{3} \int_{z_1}^{z} dz' \frac{D}{D + S} \frac{dN_1}{dz'} e^{-\int_{z'}^{z} dz'' W(z'')} \frac{dz''}{dz'} , \quad (17)$$

 Assuming no pre-existing asymmetry, i.e. $N^i_{B-L} = 0$, from Eqs. (9) and (16) the final baryon asymmetry can be conveniently written as

$$\eta_B = d \epsilon_1 \kappa_f , \quad (18)$$

where $d \approx 0.96 \times 10^{-2}$ and $\kappa_f = \kappa(\infty)$ is the final value of the efficiency factor, normalized in such a way that $\kappa_f \rightarrow 1$ in the limit of thermal initial $N_1$ abundance and no washout. The computation of $\kappa_f$ is a rather difficult task, since it involves solving the Boltzmann equations. Nevertheless, simple analytical formulae can be found in the limiting cases of strong ($K > 1$) and weak ($K \ll 1$) washout regimes [6], and numerical fits valid for all values of $\tilde{m}_1$ can also be derived in the standard cosmology case [3]. Clearly, one expects these results to be modified in the braneworld scenario. In the next section we will derive a simple fit for the efficiency factor, assuming that the baryon asymmetry is created during the high-energy regime of brane cosmology.

### III. THE EFFICIENCY FACTOR

Neutrino oscillation experiments presently constrain two mass squared differences for the light neutrinos [13],

$$m_{sol} \equiv \sqrt{\Delta m^2_{sol}} \approx 8.9 \times 10^{-3} \text{ eV} ,$$

$$m_{atm} \equiv \sqrt{\Delta m^2_{atm}} \approx 5.0 \times 10^{-2} \text{ eV} , \quad (19)$$

from solar and atmospheric measurements, respectively. The most stringent bounds on the absolute neutrino mass scale come from 0νββ-decay experiments [10], which yield an upper bound on the light Majorana neutrino masses of about 1 eV, and from the 2df galaxy redshift survey and WMAP results [1], which indicate that $\sum m_i < 0.7$ eV.

The effective neutrino mass parameter $\tilde{m}_1$ is an essential quantity for thermal leptogenesis. Remarkably, it is possible to find a natural range for this quantity in terms of the light neutrino masses. Indeed, the lower bound $\tilde{m}_1 \geq m_1$ is always verified [17]. Moreover, in the absence of strong cancelations or specific fine-tunings, one has the upper bound $\tilde{m}_1 \lesssim m_3$ [6]. Thus, one finds $m_1 \lesssim \tilde{m}_1 \lesssim m_3$.

Of particular interest is the favored neutrino mass range $m_{sol} \lesssim \tilde{m}_1 \lesssim m_{atm}$. In this situation, leptogenesis in the standard cosmology case would occur in the mildly strong washout regime $8 \lesssim K \lesssim 46$ (see Eqs. (12) and (14)). On the other hand, in the high energy regime of brane cosmology ($z_t \gg 1$) one has $8/z_t^2 \lesssim K \lesssim 46/z_t^2$, which for $z_t > 7$ already implies that leptogenesis in the brane lies in the weak washout regime. As an illustrative example, in Fig. we compare the evolution of the abundances with temperature in the SC strong and BC weak washout regimes. Assuming zero initial $N_1$ population at $T \gg M_1$, we have numerically solved the system of Boltzmann equations (6) and (7) following the approach presented in Ref. [8], which properly includes the leading finite-temperature corrections and takes into account all the relevant processes. The evolution of the $N_1$ abundance (red curves) and the ratio $|\eta_B/\epsilon_1|$ (black curves) are shown as functions of $z$ for $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = m_{atm}$. The dashed (dashed) lines correspond to the brane (standard) cosmology result.

![FIG. 1: (Color online) The evolution of $N_{N_1}$ (red curves), $N_{N_2}$ (green curve) and $|\eta_B/\epsilon_1|$ (black curves) as functions of the temperature for $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = m_{atm}$. The solid (dashed) lines correspond to the brane (standard) cosmology result.](image)

It is possible to find a simple numerical fit for the efficiency factor $\kappa_f$ in terms of the decay parameter $K$. In both SC and BC cases, the behavior of $\kappa_f$ is well described by the power law fit

$$\kappa_f^{-1} = \frac{a}{K} + \left( \frac{K}{b} \right)^c , \quad (20)$$

valid for a heavy neutrino mass $M_1 \lesssim 10^{14}$ GeV. For SC we find

$$a = 3.5 , b = 0.6 , c = 1.2 , \quad (21)$$

(see also Ref. [3]), while in the high-energy regime of BC we obtain

$$a = 0.8 , b = 0.025 , c = 1.65 . \quad (22)$$
These fits are plotted in Fig. 2 and compared with the efficiency factor obtained by solving numerically the full set of Boltzmann equations for different values of \( \tilde{m}_1 \). As expected, the curves in the BC case are displaced to the weak washout region where thermal leptogenesis is more efficiently realized.

Using the numerical fit (20), one can see that maximal efficiency is achieved at

\[
K_{\text{peak}} = (a b e^{-c})^{1/(c+1)},
\]

which in BC corresponds to

\[
K_{\text{peak}} \simeq 0.08, \quad \kappa_f^{\text{peak}} \simeq 0.06,
\]

while in SC one finds

\[
K_{\text{peak}} \simeq 1.23, \quad \kappa_f^{\text{peak}} \simeq 0.19.
\]

IV. LOWER BOUNDS ON \( M_1, M_5 \) AND \( T_\ell \)

In order to discuss the bounds implied by leptogenesis, it is convenient to write the CP asymmetry \( \epsilon_1 \) as the product

\[
\epsilon_1 = \epsilon_1^{\text{max}} \sin \delta_L,
\]

where \( \epsilon_1^{\text{max}} \) is the maximal CP asymmetry and \( \delta_L \) is an effective leptogenesis phase. In general, \( \epsilon_1^{\text{max}} \) depends on \( M_1, \tilde{m}_1 \) and the light neutrino masses. In the case of hierarchical and quasi-degenerate light neutrinos,

\[
\epsilon_1^{\text{max}} = \frac{3}{16 \pi} \frac{M_1 m_3}{\nu^2} \left[ 1 - \frac{m_1}{m_3} \sqrt{1 + \frac{m_3 - m_1}{m_1}} \right],
\]

where for a normal hierarchy

\[
m_3^2 = m_1^2 + m_{\text{atm}}^2 + m_{\text{sol}}^2,
\]

\[
m_2^2 = m_1^2 + m_{\text{sol}}^2,
\]

\[m^2 = 3 m_1^2 + m_{\text{atm}}^2 + 2 m_{\text{sol}}^2.\]

The CP-asymmetry reaches its maximum for \( m_1 = 0 \) and \( m_3 \simeq m_{\text{atm}} \), i.e. for fully hierarchical neutrinos. In this case,

\[
\epsilon_1^{\text{max}} = \frac{3}{16 \pi} \frac{M_1 m_{\text{atm}}}{\nu^2} \simeq 10^{-6} \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \left( \frac{m_{\text{atm}}}{0.05 \text{ eV}} \right).
\]

A lower bound on \( M_1 \) can be obtained from the condition

\[
\eta_B^{\text{max}} = \eta_B^{\text{obs}} \left( \frac{\kappa_f^{\text{peak}}}{\eta_B^{\text{obs}}} \right) \geq \eta_B^{\text{obs}},
\]

where \( \eta_B^{\text{obs}} \) is given in Eq. (1) and \( \eta_B^{\text{max}} \) is the maximal baryon asymmetry with the efficiency factor \( \kappa_f^{\text{peak}} \) given in Eqs. (24) and (25) for the BC and SC regimes, respectively.

The condition (30) then leads to the following lower bound on \( M_1 \)

\[
M_1 \gtrsim 6.3 \times 10^8 \text{ GeV} \left( \frac{\eta_B^{\text{obs}}}{6 \times 10^{-10}} \right) \left( \frac{0.05 \text{ eV}}{m_{\text{atm}}} \right) (\kappa_f^{\text{peak}})^{-1}.
\]

Taking into account the minimum value for \( \eta_B \) allowed by observations \( \eta_B^{\text{obs}} \geq 5.8 \times 10^{-10} \) (cf. Eq. (1)) and the atmospheric neutrino mass difference result given in Eq. (19), one obtains

\[
M_1 \gtrsim 6.1 \times 10^8 \text{ GeV} (\kappa_f^{\text{peak}})^{-1},
\]

In the SC leptogenesis regime, \( \kappa_f^{\text{peak}} \simeq 0.19 \) and one gets \( M_1 \gtrsim 3.2 \times 10^9 \text{ GeV} \). On the other hand, in the high-energy regime of brane cosmology, \( \kappa_f^{\text{peak}} \simeq 0.06 \) and one obtains the more restrictive bound

\[
M_1 \gtrsim 1.0 \times 10^{10} \text{ GeV}.
\]
This bound implies, in turn, a lower bound on the gravity scale $M_5$. Indeed, imposing $K = K_{\text{peak}}$ with $K$ given in Eq. (12), and using Eq. (9) and the bound (33), we get

$$M_5 \gtrsim 9.7 \times 10^{12} \text{ GeV} \left( \frac{\bar{m}_1}{10^{-3} \text{ eV}} \right)^{-1/3}.$$ \hspace{1cm} (34)

Finally, from this result and using Eq. (10), we can derive the following lower bound on the transition temperature

$$T_t \gtrsim 3.0 \times 10^9 \text{ GeV} \left( \frac{\bar{m}_1}{10^{-3} \text{ eV}} \right)^{-1/2}.$$ \hspace{1cm} (35)

In the favored neutrino mass range $m_{\text{sol}} \lesssim \bar{m}_1 \lesssim m_{\text{atm}}$, the bounds (34) and (35) imply

$$M_5 \gtrsim (2.6 - 4.7) \times 10^{12} \text{ GeV},$$ \hspace{1cm} (36)

$$T_t \gtrsim (4.2 - 9.9) \times 10^8 \text{ GeV}.$$ \hspace{1cm} (37)

For $\bar{m}_1 \lesssim 1 \text{ eV}$, these bounds yield $M_5 \gtrsim 1.0 \times 10^{12} \text{ GeV}$ and $T_t \gtrsim 1.0 \times 10^8 \text{ GeV}$.

V. UPPER BOUND ON THE LIGHT NEUTRINO MASSES

The washout term $W$ receives contributions from inverse decays, $\Delta L = 1$ processes and $\Delta L = 2$ processes. At low temperatures ($z \gg 1$) the contribution $\Delta W$, which depends on the absolute neutrino mass scale $\bar{m}$, dominates over the contribution proportional to $\bar{m}_1$. This contribution is approximately given by

$$\Delta W(z \gg 1) \approx \frac{c_W}{z^2 \sqrt{1 + (z_t/z)^2}},$$ \hspace{1cm} (38)

where

$$c_W = \frac{9\sqrt{3}}{8} \frac{M_1}{M_5} \frac{m^2}{\sqrt{g_*} v^4} \approx 0.19 \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \left( \frac{\bar{m}}{1 \text{ eV}} \right)^2.$$ \hspace{1cm} (39)

The presence of $\Delta W$ modifies the total efficiency factor,

$$\eta_f = \kappa_f e^{-\int_{z_B}^{\infty} dz \Delta W},$$ \hspace{1cm} (40)

where $z_B$ is the value at which the lepton asymmetry is no longer produced. The washout term in Eq. (38) leads then to

$$\eta_f = \kappa_f e^{-c_W f(z_B, z_t)},$$ \hspace{1cm} (41)

with

$$f(z_B, z_t) = \int_{z_B}^{\infty} \frac{dz}{z^2 \sqrt{1 + (z_t/z)^2}}$$ \hspace{1cm} (42)

$$= \frac{1}{4 \sqrt{1} z_t} B \left[ - \left( \frac{z_t}{z_B} \right)^4 ; \frac{1}{4}, \frac{1}{2} \right],$$ \hspace{1cm} (43)

where $B(z; a, b)$ is the incomplete beta function. For $z_t \ll z_B$ one recovers the standard cosmology result \cite{10}.

while for $z_t \gg z_B$ (BC regime) one finds

$$f(z_t) \approx \frac{\Gamma(1/4) \Gamma(5/4)}{\sqrt{\pi} z_t} \approx \frac{1.85}{z_t},$$ \hspace{1cm} (45)

leading to

$$\eta_f \approx \kappa_f \exp \left[ -3.4 \times 10^{-3} \left( \frac{M_5}{10^{12} \text{ GeV}} \right)^{3/2} \left( \frac{\bar{m}}{1 \text{ eV}} \right)^2 \right].$$ \hspace{1cm} (46)

We notice that the dependence of the washout term on $M_1$ is canceled out in the BC regime. In the latter case, the $\Delta W$ contribution to the total washout depends only on the fundamental scale $M_5$ and the absolute neutrino mass scale $\bar{m}$. Clearly, the exponential suppression in the efficiency factor becomes relevant only for large values of $M_5$ or $\bar{m}$. In Fig. 3 we present the contour lines of constant $\eta_f^{\text{max}} = \eta_f^{\text{obs}} = 5.8 \times 10^{-10}$ in the $(m_1, M_5)$-plane for different values of $\bar{m}_1$. We assume that thermal leptogenesis occurs in the high-energy regime of brane cosmology.
VI. CONCLUSIONS

The explanation of the matter-antimatter asymmetry observed in the universe remains an open question. Leptogenesis is a simple and elegant mechanism for explaining the cosmological baryon asymmetry. Since this asymmetry must have been created at early times, a successful realization of this mechanism crucially depends on the properties of early cosmology.

In this paper, we have studied the implications of Randall-Sundrum type II braneworld cosmology on the thermal leptogenesis scenario. In contrast with the usual leptogenesis scenario of standard cosmology, where low-energy neutrino data favors a strong washout regime and quasi-degenerate light neutrinos are not compatible with leptogenesis, we have found that in the high-energy regime of braneworld cosmology leptogenesis can be successfully realized in a weak washout regime, and a quasi-degenerate neutrino mass spectrum is allowed.

For an initially vanishing heavy Majorana neutrino abundance, we have obtained bounds on the decaying heavy Majorana neutrino mass, the fundamental five-dimensional gravity scale and the transition temperature from brane to standard cosmology. As far as the upper bound on the light neutrino masses is concerned, we have seen that thermal leptogenesis in the brane imposes a limit less restrictive than \( m_\nu < 0.23 \text{ eV} \), which is the one presently implied by WMAP results. Finally, it is worth emphasizing that all the bounds have been derived under the simple assumption of hierarchical heavy Majorana neutrinos. For a partially degenerate spectrum these bounds can be relaxed. Indeed, if at least two of the heavy Majorana neutrinos are quasi-degenerate in mass, i.e. \( M_1 \approx M_2 \), then the leptonic CP asymmetry relevant for leptogenesis exhibits the resonant behavior \( \epsilon_1 \sim (M_2 - M_1)^{-1} \), and values of \( \epsilon_1^{\text{max}} \sim O(1) \) can be reached. In this case, the lower bound on \( M_1 \) coming from leptogenesis can be relaxed.

Let us also briefly comment on the consistency of inflation and reheating with the present framework of thermal leptogenesis. Assuming that inflation is driven by a simple quadratic chaotic inflation potential, \( V = \frac{1}{2} m_\nu^2 \varphi^2 \), the scale of inflation in the \( \rho^2 \)-dominated period is roughly given by \( V \sim (0.1 M_3)^4 \), thus ensuring that sufficient inflation can take place while fulfilling the requirement \( V < M_4^4 \). Another important constraint comes from the reheating process, often associated with the decay of the inflaton into elementary particles, and which eventually results in the creation of a thermal bath. If the inflaton decays through Yukawa interactions with a coupling to matter fields of order \( \sim O(1) \), then the decay rate is fast, \( \Gamma_\varphi \approx m_\nu \), and reheating is practically instantaneous. Assuming that inflation ends in the \( \rho^2 \)-regime, the reheating temperature is estimated as \( T_{rh} \sim O(10^{-2}) \times M_5 \), so that the energy density of radiation \( \rho_{rh} = \pi^2 g_* T_{rh}^4 / 30 < M_5^4 \).

Since consistency with braneworld thermal leptogenesis requires \( T_\nu \lesssim M_1 \lesssim T_{rh} \), from Eq. (10) one obtains the upper bound \( M_5 \lesssim 10^{16} \text{ GeV} \), which coincides with the leptogenesis bound implied by the observed baryon asymmetry (cf. Fig. 3). Notice however that, for values of \( M_5 \) close to the upper limit, i.e. \( M_5 \sim 10^{16} \text{ GeV} \), one would have \( T_\nu \approx M_1 \approx T_{rh} \approx 10^{14} \text{ GeV} \) and the mechanism of leptogenesis would occur during the transition from brane to standard cosmology.

During the reheating process the distribution of particles is far from thermal equilibrium. Thus, number-conserving as well as number-violating interactions are necessary to achieve kinetic and chemical equilibrium among the different species. In an expanding universe, full thermal equilibrium requires a sufficiently low temperature. Indeed, \( 2 \leftrightarrow 2 \) GUT interactions mediated by massless gauge bosons would have occurred in the very early universe at a rate \( \Gamma \sim \alpha_{\text{GUT}}^2 T, \) with \( \alpha_{\text{GUT}} \sim O(10^{-2}) \). Since during the radiation-dominated era of standard cosmology \( H \sim T^2 / M_P, \) such interactions are expected to be in equilibrium for \( T \lesssim \alpha_{\text{GUT}}^2 M_P \sim 10^{15} \text{ GeV} \). On the other hand, in the high-energy regime of braneworld cosmology the expansion rate scales as \( H \sim T^2 / M_3^3 \) and, consequently, thermal equilibrium is expected at \( T \lesssim \alpha_{\text{GUT}}^{2/3} M_5 \lesssim 5 \times 10^{14} \text{ GeV} \), for \( M_5 \lesssim 10^{16} \text{ GeV} \). Clearly, the above analysis could be subject to modifications, if there exist new interactions which could thermalize the universe at the earliest epochs. For instance, it is conceivable that quantum gravity effects near the fundamental Planck scale could have resulted in a state of maximal entropy. Nevertheless, in the absence of a complete theory of quantum gravity, this remains an open issue.

Note added: While this work was in preparation, a related preprint has appeared \cite{20}. However, the conclusions drawn by the authors about mass scales and parameters somewhat differ from ours because in Ref. \cite{20} the corresponding bounds were derived by using the right-handed neutrino thermalization condition instead of solving the full set of Boltzmann equations.

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