Azimuthal flow of decay photons in relativistic nuclear collisions

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An overwhelming fraction of photons from relativistic heavy ion collisions has its origin in the decay of \( \pi^0 \) and \( \eta \) mesons. We calculate the azimuthal asymmetry of the decay photons for several azimuthally asymmetric pion distributions. We find that the \( k_T \) dependence of the elliptic flow parameter \( \nu_2 \) for the decay photons closely follows the elliptic flow parameter \( \nu_2^{\pi^0} \) evaluated at \( p_T \approx k_T + \delta \), where \( \delta \approx 0.1 - 0.2 \) GeV, for typical pion distributions measured in nucleus-nucleus collisions at relativistic energies. Similar results are obtained for photons from the 2-\( \gamma \) decay of \( \eta \) mesons. Assuming that the flow of \( \pi^0 \) is similar to those for \( \pi^+ \) and \( \pi^- \) for which independent measurements would be generally available, this ansatz can help in identifying additional sources for photons. Taken along with quark number scaling suggested by the recombination model, it may help to estimate \( \nu_2 \) of the parton distributions in terms of azimuthal asymmetry of the decay photons at large \( k_T \).

I. INTRODUCTION

The azimuthal flow of particles produced in relativistic heavy ion collisions has provided a strong evidence for the creation of a hot and dense system very early in non-central collisions \([1]\). The importance of this observation stems from the fact that pressure gradients generated in the system at very early times, transform the eccentricity in-coordinate space for such collisions to the momentum space for distribution of the produced particles \([2]\). The \( p_T \) dependence of the elliptic flow parameter \( \nu_2 \) at lower transverse momenta has been quantitatively explained using hydrodynamics \([3]\). The observed saturation of \( \nu_2 \) at higher \( p_T \) has been attributed to effects of viscosity \([4]\) or incomplete thermalization \([5]\). The observed scaling of \( \nu_2 \) with the number of valence quarks in the hadrons is understood in terms of the recombination model for hadronization \([6,7]\).

It has recently been suggested that the study of the elliptic flow of thermal photons \([8]\) may provide valuable insight into the build-up of azimuthal flow with time. This will require a subtraction of the contribution of photons from the decay of \( \pi^0 \) and \( \eta \) mesons produced in the collisions \([8]\).

The deviation of elliptic flow of inclusive photons from that of decay photons, will thus confirm the presence of additional sources of photons. We shall see later that the elliptic flow of photons from the decay of \( \pi^0 \) and \( \eta \) mesons may also provide useful estimates for \( \nu_2 \) of the partons in the frame-work of the recombination model.

In the present work, we suggest an ansatz for the evaluation of the elliptic flow of photons from the decay of pions, which could be useful for such studies. In the next section we study the transverse momentum dependence of the elliptic flow parameter for decay photons for several momentum distribution functions for the \( \pi^0 \) and \( \eta \) mesons. In section III; we evaluate the \( \nu_2 \) for photons arising from 2-\( \gamma \) decay of \( \pi^0 \) and \( \eta \) mesons, which are formed from recombination of partons to get a direct relation with \( \nu_2 \) for partons. Finally, we give our conclusions.

II. PHOTONS FROM DECAY OF \( \pi^0 \) AND \( \eta \) MESONS

Consider a \( \pi^0 \) having four-momentum \( p \) and mass \( m \) decaying into two photons. The momentum distribution of photons in an invariant form is given by \([10]\):

\[
k_0 \frac{dN}{d^3k} \propto \frac{1}{\pi} \delta(p \cdot k - \frac{1}{2}m^2),
\]

where \( k \) is the four-momentum of the photon.

Thus, the Lorentz invariant cross-section for photon production is

\[
k_0 \frac{d\sigma}{d^3k} = \int \frac{d^3p}{E} \frac{d\sigma}{d^3p} = \frac{1}{\pi} \delta(p \cdot k - \frac{1}{2}m^2).
\]

Let us write the pion distribution as:

\[
\frac{d\sigma}{d^2p_T dy} = \frac{d\sigma}{d^2p_T dy} \left[ 1 + 2\nu_2(p_T) \cos(2\phi) + \ldots \right]
\]

for a non-central collision of identical nuclei, where \( \nu_2(p_T) \) is the momentum dependent elliptic flow parameter. We consider two typical parameterizations for the pion distribution: an exponential distribution and a power-law distribution. For the first case, we assume

\[
\frac{d\sigma}{p_T dp_T dy} \sim \exp(-\sqrt{p_T^2 + m^2/T_0}) \times \exp(-y^2/2\alpha)
\]

where the slope parameter \( T_0 \) is of the order of 290 MeV \([11]\) and the width parameter of the rapidity distribution \( \alpha \) is taken as \( \sim 4 \). For the power-law distribution, we take \([12]\)

\[
\frac{d\sigma}{p_T dp_T dy} \sim \left( \frac{p_0}{p_0 + p_T} \right)^n \times \exp(-y^2/2\alpha)
\]

where \( p_0 \sim 5 \) GeV and \( n \) is about 29. The elliptic flow parameter \( \nu_2 \) is approximated as

\[
\nu_2(p_T) = [1 - \exp(-p_t/b)]
\]
which approximately reflects the increase of $v_2$ with $p_T$ for lower transverse momenta and saturation of its value at larger $p_T$. We have taken $a = 0.2$ and $b = 1$ GeV for the first set of calculations. All the results given in the present work are for photon rapidity equal to zero.

Eq. 2 is then evaluated numerically and $v_2(k_T)$ for the resulting photon distribution obtained. In Fig. 1 we give our results for the spectrum of photons from decay of $\pi^0$ along with the variation of transverse momentum dependence of their elliptic flow parameter. We find that for both distributions, the resulting $v_2(k_T)$ for the decay photons can be approximated as:

$$v_2(k_T) \approx v_2^0(p_T)$$

where

$$p_T \approx k_T + \delta$$

and $\delta \approx 0.1 - 0.2$ GeV, for $k_T > 0.2$ GeV, to an accuracy of better than 1–3%.

This result can be understood as follows. The $\delta$-function in Eq. 1 provides that

$$m_T k_T \cosh(y_p - y_k) - p_T k_T \cos(\phi_p - \phi_k) - \frac{1}{2}m^2 = 0,$$

in an obvious notation. Next, we note that once the momentum of the pions is large, the opening angle for the decay-photons will be very small [10]. Thus, for example, in the extreme case, we have, $y_p - y_k \approx 0$ and $\phi_p - \phi_k \approx 0$ or $\pi$, for the photons which are co-linear with the pion. This leads to the solutions: $k_T \approx p_T$ and $k_T \approx (m^2/4p_T^2)(1 - m^2/4p_T^2)$ for large $p_T$.

However, in general, the two photons between them will cover the entire range of the allowed transverse momentum. Both the photons, which will be almost co-linear with the pion, will “inherit” the $v_2$ of the pion, which has a larger transverse momentum. The photon with the lower $k_T$ will, however, be submerged in a much larger yield of photons coming from decay of $\pi^0$ having lower $p_T$, as the transverse momentum distribution of pions falls steeply with increase in $p_T$. Thus, in general the $v_2$ of a photon with a given transverse momentum will

FIG. 1: (Color on-line) Upper Panel: Spectrum of photons from the decay of $\pi^0$ for an exponential (Eq. 4) and a power-law (Eq. 5) distribution for pions. Lower Panel: Elliptic flow of photons from decay of $\pi^0$ having elliptic flow given by Eq. 6 at $y = 0$. $f(p_T)$ stands for the momentum distribution of the $\pi^0$. The symbols give the result of numerical calculation, while the curves give the fits.

FIG. 2: (Color on-line) Same as Fig. 1 for $\eta$ mesons.
be larger than the \( v_2 \) of a pion with the same transverse momentum. This accounts for the shift, Eq. 8.

This argument will hold even if the \( v_2(p_T) \) for the pions decreases with increase in \( p_T \), which is likely for larger \( p_T \). The only difference would be that \( v_2 \) for the decay photons will now be smaller than the \( v_2 \) for the pions at the same transverse momentum (see later; Fig. 5).

The spectra and the transverse momentum dependence of the elliptic flow parameter for the \( \eta \) mesons are given in Fig. 2. The rich structure seen for the \( \eta \) mesons at smaller transverse momenta has its origin in the large mass of the \( \eta \) mesons. Thus, we find that the \( v_2 \) for decay photons coming from the 2-\( \gamma \) decay of \( \eta \) mesons, approaches the \( v_2 \) for the mesons only at larger transverse momenta (\( > 0.8 - 1.0 \) GeV). For the simple parameterizations of the momenta considered here, \( v_2 \) is negative for very low \( k_T \) as the decay photons are distributed away from the major axis in the momentum space when the \( p_T \) of the \( \eta \)-mesons is low and the opening angles are large.

Of course, a more complete analysis would include the photons coming from the 3-\( \pi^0 \) decay of the \( \eta \) mesons, which will mainly populate the low \( k_T \) window. While a simulation of the contribution of this process to the \( v_2 \) can be straightforward using standard event generators, a direct evaluation will not be easy as it would involve a 10-dimensional numerical integration.

However, we can easily make an estimate of the range of the transverse momenta of photons, where the photons arising from decay \( \eta \rightarrow 3\pi^0 \rightarrow 6\gamma \) would contribute. The maximum kinetic energy of a \( \pi^0 \) in the rest frame of the \( \eta \) meson is given by 13:

\[
T = \frac{(m_\eta - m_\pi)^2 - 4m_\pi^2}{2m_\eta} \approx 80 \text{ MeV}. \tag{10}
\]

Thus, for example, the maximum energy of a pion emerging from an \( \eta \) meson having a momentum of 2 GeV would be of the order of 0.62 GeV and would contribute to photons having momenta less than that. Next, we recall that \( \eta/\pi^0 \) is of the order of 0.44 for \( p_T > 2 \) GeV and is expected to drop to zero at \( p_T = 0 \), at least according to calculations based on PYTHIA 12. This coupled with the branching ratio of about 0.3 for this mode of decay should ensure that the contribution of photons from the 3-\( \pi^0 \) decay of \( \eta \) mesons would be limited to smaller momenta. A more detailed study of this effect would definitely be useful.

Let us return to the discussion of the \( v_2 \) of photons from the 2-\( \gamma \) - decay of \( \pi^0 \) and \( \eta \) mesons. We now plot the ratio of the elliptic flow parameters of the calculated decay photons and the mesons as a function of their transverse momenta in Fig. 3. We see once again that \( v_2 \) for the two are quite similar when transverse momenta exceed 1 GeV. Similar results for the transverse momentum dependence of \( v_2^\gamma/v_2^{\pi^0} \) had earlier been noted by the WA98 group 14. We now understand this in terms of the momentum shift \( \delta \) (Eq. 5).

Considering the likely usefulness of the relation given by Eq. 5 we have examined it to determine the dependence of \( \delta \) on the slope-parameter \( T_0 \) or the power-law parameter \( n \) for the photons coming from the decay of \( \pi^0 \).

\[
\frac{v_2^\gamma}{v_2^{\pi^0}} = 0.2\text{[1-exp}(-p_T/1.0)]
\]

The only difference would be that \( v_2 \) for the decay photons is obtained from the \( v_2 \) for the pions by a shift of about 0.27 GeV. We again see that the transverse momentum dependence of the \( v_2 \) for pions, which rises with \( p_T \), reaches a maximum, and then starts decreasing. We illustrate our discussion (see Fig. 5) by taking,

\[
v_2(p_T) = 0.4p_T^2\exp(-p_T). \tag{11}
\]

We again see that the transverse momentum dependence of \( v_2 \) for the decay photons is obtained from the \( v_2 \) for the pions by a shift of about 0.27 GeV.

It has been recently reported 15 that the so-called quark-number scaling of the elliptic flow parameter suggested by the recombination model works much more accurately when plotted in terms of “transverse kinetic en-
FIG. 5: Elliptic flow parameter for photons from the 2-γ decay of π⁰. The symbols denote the calculated \( v₂ \) while the dashed curve denotes the fit. The \( v₂(p_T) \) for pions is taken to rise first, reach a maximum and drop for larger transverse momenta.

FIG. 6: Elliptic flow parameter for photons from the 2-γ decay of π⁰ and η mesons. The hadron \( v₂ \) is taken to scale with "transverse kinetic energy" (Ref. [17]). Symbols, as before, denote the calculated values.

III. RECOMBINATION MODEL AND DECAY PHOTONS

As remarked earlier, one of the more interesting results of elliptic flow studies has been the approximate quark-number scaling of the hadronic \( v₂ \). This finds a natural explanation in terms of the recombination model for hadronization of the quark gluon plasma [6, 7], which suggests that in the region of \( p_T \) where the recombination of the partons dominates the process of hadronization, that is \( p_T < 4 \) (6) GeV for mesons (baryons) and the effect of mass is minimal, \( v₂ \) obeys a simple scaling law:

\[
v₂(p_T) \approx n v₂^q(p_T/n)
\]

where \( n \) is the number of valence quarks in the hadron and \( v₂^q \) is the elliptic flow parameter for them.

The shifted-scaling of the \( v₂ \) of the decay photons with the \( v₂ \) for the π⁰ and η seen earlier for a variety of momentum distributions and momentum-dependences of \( v₂ \), taken along with the valence-quark scaling of the \( v₂ \) for hadrons, holds out the promise of the "lowly" decay photons providing a direct measure of the \( v₂^q \) of the partons. In order to examine this possibility, we now explicitly calculate the \( v₂ \) for photons from the decay of π⁰ and η, which in turn, are formed by the recombination of partons.

The transverse momentum distribution of mesons formed by the recombination from thermal parton dis-
distribution can be written as \[7\]:

\[
\frac{dN}{d^2p_T dy} (y = 0) \sim K_1 \left[ \frac{2(m_q^2 + p_T^2/4)^{1/2} \cosh \eta_T}{T} \right] \\
\times m_T I_0 \left[ \frac{p_T \sinh \eta_T}{T} \right].
\]

In the above \(m_T\) is the transverse mass of the hadron, \(m_q = 260\text{ MeV}\) is the quark mass, \(T = 175\text{ MeV}\) is the transition temperature, and \(\eta_T\) is the transverse rapidity such that \(\tanh \eta_T = 0.55\), appropriate for a system created in collision of gold nuclei at \(\sqrt{s_{NN}} = 200\text{ GeV}\). While writing the above we have utilized the \(\delta\)-function for the asymptotic form of the perturbative meson distribution amplitude.

We now return to the present calculation. In these illustrative studies, we account for the elliptic flow of the mesons by multiplying the above distribution with \([1 + 4v_2^2(p_T/2) \cos(2\phi)]\). We have calculated \(v_2^\gamma(p_T)\), the elliptic flow for the partons, using Eq. 81 of Fries et al. \[7\].

In Fig. 7 we give our results for the elliptic flow parameter for pions obtained, using the recombination model along with the \(v_2\) for the photons coming from the decay. We should not take the results for the \(v_2\) for pions for lower \(p_T\) too literally, as the pion mass is much smaller than \(2m_q\) which goes into making it. However, for larger \(p_T\) this difference is not very relevant and we can trust the model. We see that \(v_2^\gamma\) closely follows the \(v_2^\pi\) as before.

The mass of \(\eta\) mesons on the other hands is large enough to be free from this complication. We see that once again, the decay photons closely follow the azimuthal asymmetry of the \(\eta\) mesons at larger \(p_T\) (see Fig. 8).

**IV. SUMMARY**

We have studied the azimuthal asymmetry of distribution of photons originating from the \(2-\gamma\) decay of \(\pi^0\) and \(\eta\) mesons produced in relativistic heavy ion collisions. Several distribution functions and parameterizations of the transverse momentum dependence of the elliptic flow parameter, including those coming from the recombination model have been considered.

We have empirically found that the elliptic flow parameter for the photons from the decay of pions is quite close to the same for the pions evaluated at a transverse momentum shifted by about \(0.1 - 0.2\text{ GeV}\). This has its origin in the small opening angle for the photons for pions having large momenta. Similar results are found for photons from the \(2-\gamma\) decay of \(\eta\) mesons, at \(p_T > 1\text{ GeV}\).

These empirical findings could be useful in identifying additional sources of photons, as the elliptical flow of \(\pi^0\) should be similar to that for \(\pi^+\) and \(\pi^-\). The flow of \(\eta\) mesons could perhaps be approximated with those for kaons based on considerations of their masses and the number of valence quarks. In any case, in general, the yield of eta mesons is much less compared to that for pions. Thus the presence of an additional source of photons will be indicated by the deviation of the \(v_2\) for inclusive photons from the \(v_2\) of decay photons from the \(2-\gamma\) decay of \(\pi^0\) and \(\eta\) mesons, which could be obtained using the \(v_2\) for charged pions and kaons.

We add that a relation \[13\] between the \(p_T\) integrated anisotropy for \(\pi^0\) and photons from its decay, obtained using simulations, has been successfully used to analyze the azimuthal anisotropy of decay photons in the WA98 experiment \[14\].

Thus, an experimental verification of the empirical findings for the transverse momentum dependence of the elliptic flow reported in the present work, may be of considerable interest.

In general, such a study would also account for the acceptance and efficiency of the photon detectors, which is beyond the scope of this work.

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