Ultrahigh-precision Rydberg atomic localization using optical vortices

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We propose a robust localization of the highly-excited Rydberg atoms, interacting with doughnut-shaped optical vortices. Compared with the earlier standing-wave (SW)-based localization methods, a vortex beam can provide an ultrahigh-precision two-dimensional localization solely in the zero-intensity center, within a confined excitation region down to the nanometer scale. We show that the presence of the Rydberg-Rydberg interaction permits counter-intuitively much stronger confinement towards a high spatial resolution when it is partially compensated by a suitable detuning. In addition, applying an auxiliary SW modulation to the two-photon detuning allows a three-dimensional confinement of Rydberg atoms. In this case, the vortex field provides a transverse confinement while the SW modulation of the two-photon detuning localizes the Rydberg atoms longitudinally. To develop a new subwavelength localization technique, our results pave one-step closer to reduce excitation volumes to the level of a few nanometers, representing a feasible implementation for the future experimental applications.

\section{INTRODUCTION}

Recent years have seen a vast progress in the precise localization of atoms, with potential applications in the fundamental and applied science. Some important examples are precise addressing of ultracold atoms in optical lattices \cite{1,2}, patterning of Bose-Einstein condensates (BECs) \cite{3,4}, optical lithography \cite{5} or fluorescence microscopy \cite{6}. The diffraction limit, however, is a barrier to the possible resolution. For example, in an optical microscope, the highest achievable point-to-point resolution is limited by diffraction. The diffraction restricts the ability of optical instruments to distinguish between two objects separated by a distance smaller than a half of the wavelength of light employed to image the sample.

Coherent-adiabatic light-matter interactions provide some ways to overcome the diffraction limit by reducing the excitation volume in atom-light coupling schemes. The concepts are based on spatially modulated dark-states created by the electromagnetically induced transparency (EIT) \cite{7,8,9,10} or the coherent population trapping (CPT) \cite{11}. The space-dependent interaction between the light field and the atomic internal states is produced by a standing-wave (SW) field. New schemes have been proposed for the SW localization by applying different measuring ways, \textit{e.g.} absorption spectrum \cite{12,13}, level population \cite{14,15,16,17,18,19,20}, or adopting complex energy-level structures \cite{21,22}. Beyond the theory proposals, there are only a few experiments on EIT-based localization \cite{23,24,25}, demonstrating an atomic localization to regions of 60 nm, \textit{i.e.} 13 times smaller than the wavelength of incident light \cite{25}. The precision level better than 30 nm has been achieved in imaging of molecules and biological dynamics \cite{26,27}. Yet improving the resolution of subwavelength atomic localization down to the range of a single nanometer remains an important challenge.

A localization protocol with SW produces a periodic pattern of tightly localized regions. This was fine for the first experimental demonstrations \cite{23,24,25}, but it is not appropriate for applications, which usually require single excitation regions. In order to find atoms in a single excitation region we utilize a three-level ladder atom-light coupling scheme with a special space-dependent beam of doughnut-shaped optical vortex beams which carry an orbital angular momentum \cite{28,29}. This beam can ensure the atoms to be confined solely within a single site at the core of optical vortex where the intensity goes to zero. On the other hand, it should be noted that in the search for systems suitable for quantum information and precision measurement \cite{30,31,32,33}, Rydberg atom has emerged as one of the favorites mainly due to its strong long-range interaction that blocks the possibility of multiple excitations \cite{34,35,36,37,38}. However this superiority also arises a poor quality of localizing Rydberg atoms because it is difficult to confine them in a small region with a high density. Hence it is still under way to precisely localize highly-excited Rydberg atoms using currently available experimental techniques.

In the present work, we propose and analyze a theoretical scheme of combining Rydberg atoms with a special vortex beam, for the realization of Rydberg three-dimensional (3D) localization with ultrahigh precision and spatial-resolution. In contrast to the earlier SW-based localization protocols, a doughnut beam geometry makes it possible to detect atoms in a single spatial region with a 100\% probability, where any fluctuations from the laser noise can be largely suppressed. It is shown that the strong Rydberg-Rydberg interaction can be partially compensated by a suitable detuning, and a two-
FIG. 1. Schematic diagram for a collection of Rydberg superatoms interacting with a TW field \( \Omega_p \), as well as a LG field \( \Omega_e(r, \phi) \). Both beams are propagating along the same direction \( \hat{z} \). The concept of superatom is that, within the blockade radius \( R_b \), only one of the atoms can obtain one excitation to the uppermost Rydberg state. Inset: Level structure of each atom with states \(|g\rangle, |e\rangle, |r\rangle\) denoting the ground, intermediate and Rydberg states, which enables the transitions of \(|g\rangle \leftrightarrow |e\rangle \) and \(|e\rangle \leftrightarrow |r\rangle\). \( V_{vdW} \) stands for the intrinsic \( ns - ns \) type \( vdW \)'s Rydberg interaction between the unique excited atoms of adjacent superatoms. Other parameters are described in the text.

II. THEORETICAL FORMULATION

Let us consider an ensemble of atoms characterized by a typical three-level ladder configuration of energy levels as shown in Fig. 1. For each atom, states \( \{|g\rangle, |e\rangle, |r\rangle\} \) represent the ground, the excited and the highly excited Rydberg states, respectively. The transition between \(|g\rangle\) and \(|e\rangle\) is induced by a traveling-wave (TW) field characterised by the Rabi-frequency \( \Omega_p \) and the frequency detuned by \( \Delta_p \) from \(|e\rangle\). The upper transition between \(|e\rangle\) and \(|r\rangle\) is driven by a vortex control field \( \Omega_e(r, \phi) \), which is detuned by \( \Delta_e \) with respect to \(|r\rangle\). The control field detuning \( \Delta_e(z) \) can be adjusted to be \( z \)-dependent enabling an auxiliary spatial confinement along the \( z \)-axis.

We assume a frozen-atom limit where the atomic center-of-mass motion is negligible due to the fast operation of experiments \( \sim \mu s \) [39–41]. Therefore, applying the rotating-wave approximation, the Hamiltonian is given by (\( \hbar = 1 \))

\[
\mathcal{H} = \mathcal{H}_a + \mathcal{V}_{af} + \mathcal{V}_{vdW},
\]

where

\[
\mathcal{H}_a = - \sum_j \left[ \Delta_p \sigma^e_{pj} + (\Delta_e + \Delta_p) \sigma^r_{ej} \right],
\]

\[
\mathcal{V}_{af} = - \sum_j \left[ \Omega_p \sigma^e_{pj} + \Omega_e \sigma^r_{ej} + H.c. \right],
\]

\[
\mathcal{V}_{vdW} = \frac{C_6}{\sum_{m \neq j} |r_j - r_m|^6} \sigma^m_{rr} \otimes \sigma^m_{rr}
\]

are the unperturbed atomic Hamiltonian \( \mathcal{H}_a \), the atom-field interaction \( \mathcal{V}_{af} \) and the intermolecular van der Waals(\( vdW \))-type interaction \( \mathcal{V}_{vdW} \), respectively. For the \( j \)-th atom, \( \sigma^j_{\alpha\beta} = \langle \alpha | \langle \beta \rangle \rangle \) is the transition (\( \alpha \neq \beta \)) or projection (\( \alpha = \beta \)) operator, while \( C_6 \) denotes the \( vdW \)s coefficient which depends on \( |r_j| \). Under the mean-field treatment [42], one can safely replace \( \mathcal{V}_{vdW} \) with

\[
\hat{\sigma}^{gg}_{j} = \Gamma_e \sigma^e_{gg} - 2Im(\Omega_p^* \sigma^e_{ge}),
\]

\[
\hat{\sigma}^{ee}_{j} = \Gamma_r \sigma^e_{rr} - \Gamma_e \sigma^e_{er} - 2Im(\Omega_e^* \sigma^e_{er}) + 2Im(\Omega_p^* \sigma^e_{ge}),
\]

\[
\hat{\sigma}^{ge}_{j} = (\Delta_p - \gamma_{ge} \sigma^e_{ge} + i [\Omega_p^* \sigma^e_{ge} - \Omega_e^* \sigma^e_{ge} - \gamma_{ge} \sigma^e_{ge}]),
\]

\[
\hat{\sigma}^{er}_{j} = [i (\Delta_e - \gamma_{er}) \sigma^e_{er} - \gamma_{er} \sigma^e_{er} - i [\Omega_p^* \sigma^e_{gr} + \Omega_e^* \sigma^e_{er} - \gamma_{ee} \sigma^e_{ee}]),
\]

\[
\hat{\sigma}^{gr}_{j} = [i (\Delta_2 - \gamma_{gr}) \sigma^e_{gr} + \gamma_{gr} \sigma^e_{gr} + i (\Omega_e \sigma^e_{ge} - \Omega_p \sigma^e_{ge})],
\]

where

\[
s = \sum_{m \neq j} \frac{C_6}{|r_j - r_m|^6} \sigma^m_{rr}
\]

is the accumulated \( vdW \)-induced energy shift for the atom \( j \) induced by the adjacent Rydberg-state atoms \( m \), and \( \gamma_{\alpha\beta} = (\Gamma_\alpha + \Gamma_\beta)/2 \) is the dephasing rate, with \( \alpha, \beta \in \{g, e, r\} \). If the spontaneous decay rates obey the condition \( \Gamma_e \gg \Gamma_r, \Gamma_g \approx 0 \), one has approximately \( \gamma = \gamma_{er} = \gamma_{ge}, \Gamma_e = 2\gamma \) and \( \gamma_{gr} = \Gamma_r \approx 0 \). In addition the two-photon detuning is \( \Delta_2 = \Delta_p + \Delta_e \). For simplicity, in what follows the superscript \( j \) is ignored. Solving the system of equations (5) under the steady limit \( (\hat{\sigma}_{\alpha\beta}(t) \equiv 0) \), one arrives at a steady-state solution \( \sigma_{rr} \) indicating the stable population for the state \(|r\rangle\):
with the laser intensity \( I_{p(c)} = |\Omega_{p(c)}|^2 \). For \( s = 0 \), the solution \( \sigma_{rr} \) has a Lorentzian dependence on \( \Delta_2 \), with its half-peak width given by \( w = (I_p + I_c)/\sqrt{\gamma^2 + \Delta_p^2 + 2I_p} \) [43].

For estimating the Rydberg-Rydberg interaction between the neighboring excited atoms belonging to different superatoms, the blockade radius is defined by \( R_b = (C_0/w)^{1/6} \) if assuming \( \hbar w = C_0/R_b^6 \) \((\hbar = 1)\) [38]. Then only one atom can be excited within a single superatom volume \( V_b \). It is apparent that the blockade radius \( R_b \) is also position-dependent due to the spatial dependence of the control field and hence \( w \). The Rydberg interaction \( s \) felt by the \( j \)th excited atom within a single \( V_b \) is calculated by integrating over all excitation probabilities from the volume \( V \neq V_b \) [44]

\[
s = \int_{\n\neq V_b} \frac{C_0}{r^6} \sigma_{rr} \rho d^3r. \tag{8}
\]

Here \( \rho \) is the ground atom density and \( r = |r_j - r_m| \) denotes the relative distance. The average Rydberg excitation fraction \( \bar{\sigma}_{rr} \) is described by [45, 46]

\[
\bar{\sigma}_{rr} = \frac{f_0}{1 + (N_{sa} - 1)f_0}, \tag{9}
\]

where the Rydberg population fraction is \( f_0 = \sigma_{rr} \) at \( s = 0 \), and \( N_{sa} = V_b \rho \) represents the number of atoms in a single superatom. Note that if \( N_{sa} = 1 \) we get \( \bar{\sigma}_{rr} = f_0 \) meaning that only one atom inside can obtain a determined excitation; otherwise, assuming \( N_{sa} \gg 1/f_0 \), one gets \( \bar{\sigma}_{rr} = 1/N_{sa} \) indicating that the blocked volume definitely contains one Rydberg excitation and the Rydberg excitation fraction for each atom is \( 1/N_{sa} \).

The resulting \( s \) becomes \( s = \int_{\n\neq V_b} \frac{C_0}{r^6} \sigma_{rr} \rho d^3r \). In appendix A, we discuss the calculation of the shifted energy \( s(r_j) \) with respect to the \( j \)th atom in details.

III. ULTRA-PRECISION 2D TRANSVERSE LOCALIZATION

A. Atomic spatial resolution

Let us first consider a perfect antiblockade condition \( \Delta_2 - s = 0 \) where the detuning \( \Delta_2 \) compensates the Rydberg shift \( s (\Delta_2 - s = 0) \). In this case, the population of the Rydberg state given by Eq. (7) takes the form

\[
\sigma_{rr} = \frac{1}{1 + \eta}. \tag{10}
\]

Here \( \eta = I_c/I_p \) represents a relative ratio between the two laser intensities. Equation (10) acquires its maximum value in the core of the vortex beam where \( \Omega_c = 0 \) and hence \( \eta = 0 \). Thus, ensuring the perfect antiblockade condition and so long as the steady state is reached, monitoring the population of Rydberg state is a sufficient tool to measure the position of atoms. Inspired by this, we take the control laser \( \Omega_c(r, \phi) \) to be a doughnut-shaped Laguerre-Gaussian (LG) beam of the form [47]

\[
\Omega_c(r, \phi) = \Omega_{c0} \left( \frac{r}{w_c} \right)^{|l|} e^{-r^2/w_c^2} e^{i\phi} \tag{11}
\]

with a zero-intensity at the beam core \( r = 0 \). Here \( \Omega_{c0} \), \( w_c \), and \( l \) are, respectively, the peak amplitude, the beam waist and the winding number of the vortex beams, while \( (r, \phi) \) are, respectively, the cylindrical radius and the azimuthal angle. The probe field denoted by \( \Omega_p = \Omega_{p0} \) describes a TW propagating along the same direction \( z \) as the LG laser beam. Note that the spatial modulation of \( \Delta_c \) has been ignored by setting \( \Delta_c = \Delta_{c0} \) for accomplishing an ideal 2D localization.

Using (11) for \( \Omega_c(r, \phi) \), the intensity ratio \( \eta \) takes the form

\[
\eta = \kappa^2 \left( \frac{r}{w_c} \right)^{2|l|} e^{-2(r/w_c)^2} \tag{12}
\]

with \( \kappa = \Omega_{c0}/\Omega_{p0} \). One gets \( \eta = 0 \) at the beam core corresponding to \( r = 0 \). This yields a perfect confinement with a 100%-probability of finding atoms at the vortex core where \( r = 0 \).

The localization quality depends on a high spatial resolution characterized by a narrow linewidth of excitation spectra \( \sigma_{rr}(r) \). A very narrow linewidth indicates that the position of atoms can be determined well within a very small excitation volume. For the lowest-order mode of the LG beam with \( l = 1 \), \( \eta \) can be expanded around \( r = 0 \) in a Taylor series up to the fourth order, giving

\[
\eta \approx \kappa^2[(\frac{r}{w_c})^2 - 2(\frac{r}{w_c})^4]. \tag{13}
\]

Then the full width at half maxima (FWHM) of the excitation spectra \( a_r \) can be approximated as

\[
a_r \approx \frac{w_c}{\sqrt{1 - \frac{\sqrt{\kappa^2 - 8}}{\kappa}}}. \tag{14}
\]

According to the symmetry, we have \( a_r = a_x = a_y \) where \( a_x \) and \( a_y \) are the FWHM of excitation along \( \hat{x} \) and \( \hat{y} \) directions, respectively. From Eq. (14), it is intuitive that \( a_r \rightarrow 0 \) only when \( \kappa \gg 2\sqrt{2} \), indicating a high resolution peak. A very large \( \kappa \) is obtained for sufficiently weak probe pulses (\( \Omega_{p0} \ll \Omega_{c0} \)). As confirmed by Fig.2(a), starting from its peak value 1.0, \( \sigma_{rr} \) is found to decrease rapidly as \( r \) grows, allowing an ultra-precise
very weak. Oppositely, a smaller value of $\kappa$ or $\sigma_s$ becomes $500$, as in the case of perfect blockade. However, for a larger $\omega_c$ (e.g., $\omega_c = 5\mu m$) it is insufficient to compensate $s(r_j)$ within the whole localization regime, as the distortion and shrink of the blockade sphere caused by a broader beam waist $\omega_c$ can bring a significant shift to $s(r_j)$. Hence, we apply the partial blockade approach at $r_j = 0$ realized by $\Delta_c = s(r_j) = 0$. Fortunately, the

The dependence of $a_r$ on OAM number $l$ is demonstrated in Fig.2(c). One can see that larger topological charge numbers destroy the spatial resolution, i.e., the larger the OAM number $l$ is, the bigger the value of $a_r$ and the wider the localization width are. Because when the OAM number $l$ increases to larger numbers, the dark hollow center is increased in size as indicated by Eq.(11). For example when $l = 5$ and for $\kappa = 10$, $a_r$ becomes $1.39\mu m$, in contrast to the best case where we achieved $a_r = 4.0nm$ only for $l = 1$. Hence, in what follows we take $l = 1$ to get the best results.

FIG. 2. (a) Representation of the steady Rydberg probability $\sigma_{rr}(r_j)$ versus $r_j/\lambda_c$ for $l = 1$. The subscript $j$ means for the $j$th Rydberg atom. The full-width at half maximum of excitation where $\sigma_{rr} = 0.5$ is defined by $a_r$. Here a half-width $a_r/2$ is labeled. (b1-b3) The 2D plot of atom transverse localization with different $\kappa$ values. (c) The full-width $a_r$ (in unit of $\lambda_c$) with the increase of the winding number $l$. Detailed values $a_r$ are shown in the inset. Here the beam waist and the wavelength are $\omega_c = 1\mu m$, $\lambda_c = 480nm$. Cases of $\kappa = 10$, 100, 500 are given by red-dotted, green-dashed and blue-solid curves, respectively.

range of localization $a_r \sim 0.0083\lambda_c \approx 4.0nm$, as long as a sufficiently large $\kappa (=500)$ is adopted. Yet for such large values of $\kappa$, the time required for the steady state to be formed is increased to tens of microseconds, as $\Omega_{p0}$ is very weak. Oppositely, a smaller value of $\kappa$ e.g. $\kappa = 100$ or $\kappa = 10$, would cause a poor resolution although it may speed up the time to reach the steady state. The full numerical results by solving equations (5) confirm the analytical predictions. Figure 2(b1-b3) shows the 2D imaging of the atomic localization with different $\kappa$ values. It is clear that the localization quality becomes worse as $\kappa$ decreases indicating the importance of a sufficiently weak probe field for a tight 2D confinement of atoms.

B. Influence of the Rydberg shift $s$

Because of the position-dependent nature of the control field, the Rydberg shift $s$ is difficult to be completely compensated by the detuning $\Delta_2$ (a perfect antiblockade condition). When $\Delta_2 - s(r) \neq 0$ and considering $I_c \ll I_p$ (at $r \approx 0$), $\sigma_{rr}$ can be approximated as

$$\sigma_{rr}(r) = \frac{1}{1 + (\Delta_2 - s(r))^2/\omega^2}.$$  

The resolution $a_r$ is then given by $|\Delta_2 - s(r)| = \omega$ corresponding to $\sigma_{rr} = 1/2$. As discussed in appendix A, we see that $s(r_j)$ can be well preserved when the localized atom $j$ is placed within a short displacement $r_j$ to the beam core, i.e. if $r_j \ll R_b$. Hence it is possible to overcome the shifted energy exactly at the localized point (the core of LG beam) $r_j = 0$ while keeping a well preservation for larger $r_j$ values, which is so-called partial antiblockade effect. As a result we choose $\Delta_c = s(r_j) = 0$ (we have assumed $\Delta_p = 0$ giving to $\Delta_2 = \Delta_c$).

In Figure 3(a-b) we numerically compare the results from the cases of perfect antiblockade by $\Delta_c = s(r_j)$ (solid, same as discussed in Sec.3A) and the partial antiblockade (stars) by $\Delta_c = s(r_j) = 0$. Comparing Figs. 3(a) and (b) shows that one can get a better localization when $\omega_c = 1\mu m$. However, in this case, the partial antiblockade gives rise to the same results as the perfect antiblockade (see Fig.3(a)), as the change in $s$ is negligible so that it can be compensated by a constant $\Delta_c$. The degree of localization can be well kept by a partial antiblockade compensation, achieving the same optimal FWHM $a_r \approx 4.0nm$ when $\kappa = 500$, as in the case of perfect antiblockade. However, for a larger $\omega_c$ (e.g., $\omega_c = 5\mu m$) it is insufficient to compensate $s(r_j)$ within the whole localization regime, as the distortion and shrink of the blockade sphere caused by a broader beam waist $\omega_c$ can bring a significant shift to $s(r_j)$. Hence, we apply the partial antiblockade approach at $r_j = 0$ realized by $\Delta_c = s(r_j) = 0$. Fortunately, the

FIG. 3. The influence of $s$ on the steady Rydberg population $\sigma_{rr}(r_j)$ under different beam waists (a) $\omega_c = 1\mu m$ and (b) $\omega_c = 5\mu m$. Star points and solid curves are separately solved by considering a partial antiblockade $\Delta_c = s(r_j)$ and a perfect antiblockade $\Delta_c = s(r_j) = 0$. Plots corresponding to $\kappa = 10, 100, 500$ are denoted by red, green and blue curves, respectively.
partial antiblockade improves the localization at \( r_j = 0 \) by a faster fall of steady probability \( \sigma_{rr}(r_j) \), leading to a narrower FWHM, as compared to the case of perfect antiblockade \( \Delta_c = s(r_j) \) (Fig.3(b)).

Therefore, thanks to the hollow core laser beam we have realized an efficient scheme for the ultra-precision 2D Rydberg localization, with its resolution down to 4.0nm. Remarkably, once the partial antiblockade condition \( \Delta_c = s(r_j) = 0 \) is fulfilled, the Rydberg-Rydberg interaction counter-intuitively yields a better spatial resolution. This condition would be easier to be carried out in experiments.

IV. LONGITUDINAL PERIODIC LOCALIZATION

Getting rid of using a standing-wave (SW) optical Rabi frequency that may add to the complexity of our protocol, in the following we implement a spatial modulation to the detuning \( \Delta_c(z) \) that is directly connected to \( |\nu\rangle \). This modulation is enabled by the ac Stark effect from an external electric field to induce a periodic energy shift of \( |\nu\rangle \) [48], taking form of

\[
\Delta_c(z) = \Delta_{c0} \sin \left( \frac{2\pi}{\lambda_c} z \right) + \delta
\]

with its peak-peak amplitude \( \Delta_{c0} \) and an extra frequency shift \( \delta \). In such a condition, the partial antiblockade condition changes to \( \Delta_c = s(r_j = 0, z_j = 3\lambda_c/4) \) at the localized point, giving

\[
\delta - \Delta_{c0} = s_0,
\]

where \( s_0 = s(r_j = 0, z_j = 3\lambda_c/4) \). Once Eq.(17) is violated, \( i.e., \delta - \Delta_{c0} \neq s_0 \), both the precision and spatial resolution will be reduced significantly, as numerically demonstrated in Fig.5. The periodicity of the SW function \( \Delta_c(z) \) allows atoms to be localized at \( z_j = (3/4 + n)\lambda_c \ [n \in \text{integers}] \). Note that we have assumed \( n = 0 \) for the simulations.

Based on the findings in Fig.7(c) of appendix, it is safe to assume \( s(r_j = 0, z_j) \) to be a constant as \( s(z_j) \) can be well kept with negligible oscillations. In this case, by following the equality of \( \Delta_c(z) - s_0 = \omega \), the FWHM of \( \sigma_{rr}(z_j) \) can be solved analytically, leading to

\[
a_z = \left| \frac{1}{2} - \arcsin(1 - \frac{\omega}{\Delta_{c0}})/\pi \right| \lambda_c.
\]

Figure 4 shows numerical simulations for the population distribution \( \sigma_{rr}(z_j) \) against \( z_j \). The numerical results for \( a_z \) are in a good agreement with the analytical solutions given by Eq.(18). Increasing \( \kappa \) from 10 to 500 as reduces \( a_z \) significantly, yielding a tighter longitudinal confinement. Specifically, for a large control field detuning \( \Delta_{c0}/2\pi = 30\text{MHz} \) and \( \kappa = 500 \), the localization resolution can be enhanced, reaching \( a_z = 0.0046\lambda_c \approx 2.2\text{nm} \) (see Fig.4(a) (blue-solid)). However if the peak-peak amplitude \( \Delta_{c0} \) is set to be the orders of magnitude smaller, \( e.g., \Delta_{c0} = \Omega_{p0} \) as in Fig.4(b), it exhibits a dramatic broadening of \( a_z \) due to \( a_z \propto \Delta_{c0}^{-1} \) featured by Eq.(18).

V. EXPERIMENTAL FEASIBILITY

A. Ultrahigh-precision 3D localization

Our localization protocol benefits greatly not only from a hollow-core vortex beam which enables the confinement of atoms in a single site with a 100% detection probability, but also from the presence of Rydberg-Rydberg interaction which efficiently speeds up the fall of probability in space, finally improving the transverse localization resolution towards a subwavelength domain of \( \sim 4.0\text{nm} \). In addition, applying a spatially-modulated two-photon detuning instead of an SW laser, enables us for precisely localizing atom along the longitudinal direction, successfully reaching a resolution of \( \sim 2.2\text{nm} \).

Before proceeding, we now numerically estimate the relevant experimental parameters for verifying the practical implementation of our protocol. In the calculations we consider ground \(^{87}\text{Rb}\) atoms with energy levels \((|g\rangle, |e\rangle, |\nu\rangle) = (|5s_{1/2}\rangle, |5p_{3/2}\rangle, |60s_{1/2}\rangle) \) excited by a two-photon process at the temperature of \( 20\mu\text{K} \) [49]. The upper transition of \( |e\rangle \rightarrow |\nu\rangle \) is played by a vortex LG beam with \( (\Omega_{c0}, \omega_c, \lambda_c) = (2\pi \times 80\text{MHz}, 1\mu\text{m}, 480\text{nm}) \) for the peak intensity, the beam waist and the wavelength respectively. The lower transition from \( |g\rangle \) to \( |e\rangle \) is characterized by the continuous probe field with the Rabi frequency \( \Omega_{p0} = \Omega_{c0}/\kappa \) and the wavelength 780nm. We introduced a tunable \( \kappa \) which is manipulated by changing \( \Omega_{p0} \), when \( \Omega_{c0}/2\pi = 80\text{MHz} \) is fixed.
In general, we take $\Omega_{p0} = 2\pi \times (8.0, 0.8, 0.16)\text{MHz}$ providing $\kappa = (10, 100, 500)$. The average atom density is $\rho = 6 \times 10^9 \text{mm}^{-3}$, and the vdWs coefficient of state $|60s_{1/2}\rangle$ is $C_6/2\pi = 140\text{GHzmm}^6[50]$. The dissipation is dominated by a fast decay from the middle excited state $|e\rangle$, given by $\Gamma_e/2\pi = 6.05\text{MHz}[49]$ and other spontaneous decays are $\Gamma_{r,g} = 0$, leading to the dephasing rates $\gamma = \gamma_{er} = \gamma_{eg} = \Gamma_e/2$. To our knowledge the duration time for getting a steady localization is typically inversely proportional to the absolute values of Rabi frequencies $\Omega_{c0}$ and $\Omega_{s0}$. For $\Omega_{s0}/2\pi = 80\text{MHz}$, $\Omega_{p0}/2\pi = 0.16\text{MHz}$ we estimate that the longest time to be stable is about $80\mu s$ [not shown] which is at least one order of magnitude shorter than the lifetime of a Rydberg state $|r\rangle = |60s_{1/2}\rangle$, which reliably provides a stable measurement for the Rydberg probability due to its localization.

Besides, the two-photon detuning $\Delta_2(z) = \Delta_c(z)$ (note that $\Delta_p = 0$) is modulated as a sinusoidal function with its amplitude $\Delta_{s0}/2\pi = 30\text{MHz}$ and periodicity $\lambda_c$. Importantly, the shifted energy $\delta$ should be decided accurately according to the partial antiblockade condition Eq. (17), leading to $\delta = \Delta_{s0} + s_0 = 2\pi \times (37.77, 31.15, 30.063)\text{MHz}$ for $\kappa = (10, 100, 500)$, where $s_0$ is numerically estimated from solving an integration equation (S3) [see Appendix] at $r_j = 0, z_j = 3\lambda_c/4$. If $\kappa \gg 1$, $\delta$ is closing to the value of $\Delta_{s0}$ due to $s_0/\gamma \to 0$. Therefore, in a real implementation in order to obtain an accurate $\delta$ for an optimal localization, one needs to scan the shifted frequency $\delta$ around $\Delta_{s0}$. A brief discussion for the determination of the shifted frequency $\delta$ is presented in Appendix B.

Our final results for the 3D Rydberg atom localization are summarized in Fig.5 with all practical parameters, where a global isosurface at half maxima of $\sigma_{rr}$ is represented. Panels (a-c) show the partial antiblockade case at the localized point: $\Delta_c = s(r_j = 0, z_j = 3\lambda_c/4)$, leading to an accurate relation $\delta - \Delta_{c0} = s_0$. It reveals a visible localization image whose spatial resolution will be optimized for a larger $\kappa$. However, once this accurate relation is slightly shifted e.g. $\delta - \Delta_{c0} = 2s_0$ as indicated in (d-f), the full-width at half maxima of $\sigma_{rr}(r)$ experiences a dramatic broadening while decreasing its peak value. Thus confirms our theoretical predictions that the sensitive partial antiblockade condition at the localized point is very important for realizing a high-quality atom localization, especially in the presence of strong Rydberg-Rydberg interactions. A slight shift of partial antiblockade condition would cause the breakdown of our localization scheme.

### B. Robustness of scheme against laser noises

In order to explore the robustness of our scheme against external perturbations, we introduce now a position-dependent random intensity noise to the peak amplitude $\Omega_{c0}$ of the LG beam. In this case, the amplitude $\Omega_{c0}$ turns to be a position-dependent, defined by

$$\Omega_{c0}(r) = \Omega^s_{c0} + \delta\Omega_{c0}(r)$$

with $\Omega^s_{c0}$ being the unperturbed laser amplitude. Here the random noise term $\delta\Omega_{c0}(r)$ is simulated as a normal distribution, which has a zero expectation value and a standard deviation $\sigma$. We only pay attention to the steady Rydberg distribution along $x$ axis due to the symmetry of system, in order to measure its variations under the effect of random fluctuations $\delta\Omega_{c0}(x)$. For reducing the uncertainty during single measurement, each plot is simulated by averaging over ten-times outputs.

Figure 6 demonstrates that suffering from the influence of the intensity noise, the steady Rydberg popula-
The position-dependent steady Rydberg population $\sigma_r(x_j)$ starts fluctuating and broadening. When the noise amplitude characterized by $\bar{\sigma}$ is relatively small, e.g. $\bar{\sigma} = 0.2\Omega_\omega$, the population distribution is quite steady closing to the case of $\bar{\sigma} = 0.0$ (no noise). Nevertheless, if $\bar{\sigma}$ is increased to be $0.5\Omega_\omega$, $\sigma_r(x_j)$ reveals a strong fluctuation with its resolution (half-width) becoming worse. To our knowledge the realistic intensity noise can be suppressed to a very weak level under current experimental technique, which strongly supports the robustness of our localization protocol towards a new ultra-precision standard.

VI. CONCLUDING REMARKS

In conclusion, we have proposed a robust protocol for localizing the highly-excited Rydberg atoms. The periodicity of the SW field in earlier schemes was an obstacle for detecting atoms in single excitation regions. We have overcame this obstacle by applying an optical vortex, enabling a super transverse localization of Rydberg atoms solely in the vicinity of the vortex core and with a resolution down to the nanometer scale. The presence of the Rydberg-Rydberg interaction also yields a better localization when it is partially compensated by a suitable detuning. We have also demonstrated that a 3D localization is possible when applying simultaneously a vortex beam and an auxiliary SW modulation to the two-photon detuning. The SW modulation of the detuning provides a longitudinal confinement, while the vortex field localizes Rydberg atoms transversely. The vortex based approach has unique advantages that may be especially useful for Rydberg quantum computation in a nanometer-scale level, in which its robustness to the laser intensity noise revealed will offer special applications for the high-precision operation of a stable quantum logic gate.

Appendix A: Rydberg-Rydberg shifted energy $s(r)$

The position-dependent shifted energy $s(r)$ can be solved numerically. In the frame of cylindrical coordinate, from Eq. (8) $s(r_j, z_j)$ of the $j$th atom contributed by other surrounding $m$ atoms has a reduced dual-integration form, described by

$$s(r_j, z_j) = 2\pi C_6 \int_0^\infty \int_{-\infty}^\infty \bar{\sigma}_r \rho \chi \bar{r} dz dr,$$

(A1)

where the azimuthal angle $\phi$ has been dropped out due to the symmetry. An adjustable coefficient $\chi$

$$\chi = \begin{cases} 0 & r^2 + z^2 < R_b^2 \\ 1 & r^2 + z^2 \geq R_b^2 \end{cases}$$

(A2)

is introduced to control the interaction strength of adjacent atoms $m$. Hence, if the adjacent atom $m$ is placed inside the blockade sphere preventing from all Rydberg excitations, then $\chi = 0$ and $s = 0$; otherwise $\chi = 1$, leading to $s \neq 0$. The excited atom $m$ will induce a finite Rydberg shift to the atom $j$ at $(r_j, z_j)$. In the calculations, the entire integration regime contains a computational lattice with $10^4 \times 10^4$ points in $(r, z)$-directions, where the computational lengths and the lattice spacing along each dimension are $L_r = L_z = 100\lambda_c$ and $\delta r = \delta z = 0.01\lambda_c$.

In Fig. 7(a1) we study the position-dependent blockade radius $R_b(r, z)$ for $\kappa = 10, 100, 500$. Explicitly, when the localized atom $j$ is placed at the core of LG field, $R_b$ is essentially anisotropic and increases with $\kappa$. The reason is that near the beam core where $L_r \approx 0$, $R_b$ is inversely proportional to $I_p$. However, a common deep occurs at the beam waist around $r \approx 1.5\lambda_c$, arising from the fact that at this point the intensity $I_r$ arrives at a same maximal value $I_\omega$. Due to the dominant role played by the strong intensity $I_\omega$, the blockade radius tends to be the same around $r \approx 1.5\lambda_c$ no matter what $\kappa$ is. Inset shows a visual picture of a collection of superatom ensembles illuminated by the LG (red) and TW (amaranth) fields. A 3D plot of the anisotropic blockade sphere is given in Fig. 7(a2) to visualize the imaging, in which the boundary of $R_b$ as same as described in (a1) is denoted by a red-dashed curve.

Guided by the anisotropy of blockade radius we further exploit the accumulated Rydberg shift $s(r_j, z_j)$ via the variation of positions, by considering $z_j = 3\lambda_c/4$ (Fig. 7(b)) and $r_j = 0$ (Fig. 7(c)), separately. Note that the atom $j$ is localized at $(r_j, z_j) = (0, (3/4 \pm n)\lambda_c)$ with $n = 0, 1, 2...$ denoting the periodic number coming from the SW modulation $\Delta_s(z)$. Here we choose $n = 0$. Based on Fig. 7(b-c), generally speaking the value of $s(r_j, z_j = 3\lambda_c/4)$ or $s(r_j = 0, z_j)$ can be robustly preserved no matter how $\kappa$ is tuned, benefiting from the tiny localization regime around $r_j = 0$ and $z_j = 3\lambda_c/4$. That preservation property gives rise to a partial antiblockade relation by $\delta - \Delta_s = s_0$ where $s_0$ means the shifted energy at the localized point, and $\delta$,
\(\Delta_{c0}\) are related to the modulation function \(\Delta_c(z)\). Beyond the localization regime an insufficient preservation due to partial antiblockade effect can counter-intuitively speed up the fall of the excited-state probability, making the atom position confined within a narrower area.

On the other hand, it is also confirmed that \(s(r_j, z_j)\) significantly decreases for a larger \(\kappa\). The reason can be understood that, if \(\kappa\) is large, the localized atoms placed around the core would suffer from a weaker interaction from other \(m\) atoms due to the sufficient size of blockade radius. An extensive plot in (b) supplementarily shows the shifted energy \(s(r_j)\) indeed changes slightly if the atom is placed far from the beam core where \(r_j\) and \(R_b\) is comparable, especially for a small \(\kappa\). Whereas the shifted energy \(s(z_j)\) keeps a long-range and stable preservation along the \(z\) axis in every period [see inset of (c)].

**Appendix B: The shifted frequency \(\delta\)**

To realize a robust 3D localization, we reveal the importance of partial blockade relation \(\delta - \Delta_{c0} = s_0\) with \(\Delta_{c0}\) the peak-peak modulation amplitude. Here \(\Delta_{c0}\) is arbitrarily chosen by \(\Delta_{c0}/2\pi = 30\text{MHz}\). So the shifted frequency \(\delta\) should be determined by \(\delta = s_0 + \Delta_{c0}\). Here \(s_0\) stands for the average Rydberg shift at the localized point \((r_j, z_j) = (0, 3\lambda_c/4)\), which can be solved by rewriting Eq. (A1) into

\[
s_0 = 2\pi C_6 \rho I_p \int_0^\infty \int_{-\infty}^\infty \frac{\chi}{[(r-r_j)^2+(z-z_j)^2]^3} B \, dz \, dr,
\]

where \(B = I_c(r) + 4\pi R_b^2 \rho I_p / 3 + (\gamma^2+2I_p/\Delta_c(z))^{1/2} / I_p+I_c(r)\). If \(s_0\) is numerically solved it is possible to apply the value \(\delta = s_0 + \Delta_{c0}\) for structuring the sinusoidal modulation \(\Delta_c(z)\). Fig. 8 plots the behavior of \(s_0\) with respect to \(\kappa\). It is clearly shown that \(s_0\) decreases significantly as \(\kappa\) grows, and \(s_0\) also preserves a tiny value if \(\kappa\) is very large. For example as \(\kappa > 100\), \(s_0\) has entered the regime below \(\sim 0.1\gamma\) or smaller. Owing to the use of a weak probe field \(I_p\), the poor Rydberg excitation probability can lead to a smaller Rydberg shift \(s_0\).

However, although \(s_0\) is very small the chosen of \(\delta\)
should be sensitive to it. Once the relation of $\delta = s_0 + \Delta_{\omega_0}$ breaks, a dramatic broadening of FWHM occurs that can greatly reduce the spatial resolution of localization [see Fig.5(d-f)]. In experiment one needs to scan the frequency $\delta$ very precisely around $\Delta_{\omega_0}$ to improve the localization quality. For example, we numerically obtain the values of $\delta/2\pi = (37,77,31,15,30,063)$ MHz for $\kappa = (10,100,500)$, confirming that $\delta$ becomes very closer to $\Delta_{\omega_0} = 30$ MHz when $\kappa$ is sufficiently large.

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