Theory of Open Quantum Systems as Applied to Spin Relaxation in Solids

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Abstract

We employ the method of the theory of open quantum systems to analyze spin relaxation and decoherence in semiconductors in the presence of a magnetic field. We derive a set of Bloch equations for electron spin with a fully microscopic determination of longitudinal and transverse relaxation times. Electron scattering from optical and acoustic phonons and random impurities is taken into account. We obtain explicit expressions for the spin relaxation times in terms of material constants and coupling strengths, exhibiting formal agreement with earlier treatments in the zero magnetic field limit with microscopic specification of their phenomenological parameters.
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I. INTRODUCTION

Modern statistical physics encompasses a wide range of scientific problems from elementary particles to the evolution of the Universe. Its methods facilitate the determination of the macroscopic characteristics of complex nonequilibrium systems without having to exactly analyze the highly complicated individual microscopic dynamics of each constituent part. In many cases the system of interest can be separated into two distinct subsystems. One of them, which we will term the ”dynamical subsystem”, has only few degrees of freedom, whereas the other, the ”bath”, has, in an ideal case, the infinite number of them. The bath is a source of random perturbations (fluctuations) for the dynamical subsystem, also functioning as an absorber of dissipative energy. This separation is at the core of the theory of open quantum systems (TOQS) based on papers of Schwinger [1], developed in Refs. [2–4] and successfully applied to radiation damping [5] and some problems in solid state physics [6].

In the present paper we demonstrate the applicability of TOQS to the problem of spin dynamics in semiconductors, which has been extensively studied over the last decade both experimentally [7] and theoretically [8] in conjunction with numerous proposals for a variety of spin-based devices [9]. The most promising materials for device purposes, the III-V and II-VI compounds, have been shown to have spin relaxation rates dominated by the D’yakonov-Perel’ (DP) mechanism [10] at moderate temperatures and low hole concentrations. In contrast to earlier treatments, we develop a fully microscopic stochastic theory of spin dynamics in the presence of an external magnetic field, taking account of pertinent scattering mechanisms. Our analysis is based on an innovative implementation of the two step relaxation process corresponding to the relaxation time hierarchy involved in (a) electron thermalization, and (b) spin relaxation. In the first stage of solution, we determine the relaxation rates and fluctuation characteristics of electron orbital motion due to coupling to optical and acoustic phonons and random impurities as a bath. Spin relaxation dynamics (the slowest process in the system) can be neglected in stage (a). The second
stage, (b), proceeds with the analysis of the spin relaxation process due to spin-orbit interaction, wherein the orbital degrees of freedom are considered as an "effective heat bath", having the characteristics determined in stage (a). The orbital motion can be considered as an intermediary transmitting fluctuations from the phonon/impurity bath to electron spin, and, also, transferring the dissipated energy flow in the opposite direction. A set of Bloch equations with two distinct relaxation times (longitudinal relaxation time, $T_1$, responsible for spin magnetic moment relaxation, and transverse relaxation time, $T_2$, responsible for decoherence) is derived in this second stage. In both stages of our analysis we employ the TOQS method.

The structure of the paper is as follows. In Section II we present a brief outline of TOQS and the principal relations used in subsequent Sections. The microscopic formulation of the spin relaxation problem is given in Section III. In Section IV we analyze the orbital electron dynamics in the presence of an external magnetic field with coupling to optic and acoustic phonons and random impurities. The spin-orbit interaction is examined in Section V, where we derive the Bloch equations on a microscopic basis. A summary of this work is given in Section VI.

II. THEORY OF OPEN QUANTUM SYSTEMS

The full Hamiltonian of the system, separated into a "dynamical subsystem" and a "bath", can be written as

$$H = H_0 + H_B + H_{int},$$

where $H_0$ is the Hamiltonian of the dynamical subsystem, $H_B$ is the Hamiltonian of the bath and $H_{int}$ describes their interaction. We can write the last term in a product form

$$H_{int} = - \sum_\alpha F_\alpha(t) Q_\alpha(t),$$

where $F_\alpha(t)$ is, in the general case, a nonlinear function of the dynamical subsystem variables, $Q_\alpha(t)$ is a function of the bath variables, and the summation index $\alpha$ can refer to either
coordinate projections or a mode index. One can see from $H_{int}$ that both the dynamical subsystem and the bath influence each other. Moreover, $F_{\alpha}$ plays a role of a generalized force conjugate to the generalized bath coordinate, $Q_{\alpha}$ and vice versa (Figure 1).

In the case of Gaussian statistics of the unperturbed bath variables, or when the coupling of the dynamical subsystem to the bath is weak, the full Heisenberg operator of the bath is given by

$$Q_{\alpha}^{h}(t) = Q_{\alpha}^{0}(t) + \int dt_{1} \varphi_{\alpha\beta}(t, t_{1}) F_{\beta}(t_{1}),$$

where

$$\varphi_{\alpha\beta}(t, t_{1}) = \left\langle \frac{i}{\hbar} [Q_{\alpha}^{0}(t), Q_{\beta}^{0}(t_{1})] \right\rangle \eta(t - t_{1})$$

is a linear response function, or the retarded Green’s function of the unperturbed bath variables. $\eta(t - t_{1})$ is the Heaviside unit step function. Another important function is the correlation function of the unperturbed bath variables

$$M_{\alpha\beta}(t, t_{1}) = \left\langle \frac{1}{2} [Q_{\alpha}^{0}(t), Q_{\beta}^{0}(t_{1})]_{+} \right\rangle = M_{\alpha\beta}(\tau).$$

Here and below we use the notation $[\ldots, \ldots]_{\pm}$ for a commutator, and $[\ldots, \ldots]_{\mp}$ for an anticommutator. The linear response function of Eq. (4), and the correlation function of Eq. (5), are related by means of the fluctuation-dissipation theorem

$$S_{\alpha\beta}(\omega) = \hbar \chi''_{\alpha\beta}(\omega) \coth \frac{\hbar \omega}{2k_{B}T},$$

where

$$S_{\alpha\beta}(\omega) = \int d\tau e^{-i\omega\tau} M_{\alpha\beta}(\tau),$$

and

$$\chi_{\alpha\beta}(\omega) = \int d\tau e^{-i\omega\tau} \varphi_{\alpha\beta}(\tau) = \chi'_{\alpha\beta}(\omega) + i\chi''_{\alpha\beta}(\omega).$$

To derive the fluctuation-dissipation theorem we employed the relation
\[ \langle \{ Q_\beta^0(0), Q_\alpha^0(\omega) \}_+ \rangle = \langle \{ Q_\beta^0(0), Q_\alpha^0(\omega) \}_- \rangle \coth \frac{\hbar \omega}{2k_BT}, \]  
(9)

based on the Gibbs distribution and, therefore, the fluctuation-dissipation theorem, in the present form, is valid only for systems in equilibrium.

The Heisenberg equation of motion for an arbitrary operator of the dynamical subsystem, \( A(t) \), is given by

\[ \dot{A}(t) = \frac{1}{i\hbar} [A(t), H_0]_+ - \frac{1}{i\hbar} [A(t), F_\alpha(t)]_- Q^h_\alpha(t), \]  
(10)

where \( Q^h_\alpha(t) \) is a Heisenberg bath operator including the influence of the dynamical subsystem. Substituting the expression of Eq. (3) for this operator, and taking account of the fact that the operator of the dynamical subsystem is commutative with \( Q^h_\alpha(t) \) at the same moment of time (only), we obtain

\[ \dot{A}(t) = \frac{1}{i\hbar} [A(t), H_0]_+ - \frac{1}{2} \{ Q^0_\alpha(t), Y_\alpha(t) \}_+ - \frac{1}{2} \int dt_1 \varphi_{\alpha\beta}(t,t_1) [Y_\alpha(t), F_\beta(t_1)]_+, \]  
(11)

where

\[ Y_\alpha(t) = \frac{1}{i\hbar} [A(t), F_\alpha(t)]_- . \]

Considering \( Y_\alpha(t) \) as a function of the unperturbed bath variables and employing the quantum analog of the Furutsu-Novikov theorem \[ \text{[3]} \], we find that

\[ \langle \frac{1}{2} \{ Q^0_\alpha(t), Y_\alpha(t) \}_+ \rangle = \int dt_1 M_{\alpha\beta}(t,t_1) \langle \frac{\delta Y_\alpha(t)}{\delta Q^0_\beta(t_1)} \rangle, \]  
(12)

and eliminate the appearance of the bath variables using functional derivatives as described by Efremov and Smirnov \[ \text{[3]} \]

\[ \langle \frac{\delta Y_\alpha(t)}{\delta Q^0_\beta(t_1)} \rangle = \langle \frac{i}{\hbar} [Y_\alpha(t), F_\beta(t_1)]_- \rangle \eta(t - t_1). \]  
(13)

Introducing the fluctuation source, \( \xi(t) \),

\[ \xi(t) = -\frac{1}{2} \{ Q^0_\alpha(t), Y_\alpha(t) \}_+ + \int_{-\infty}^{t} dt_1 M_{\alpha\beta}(t,t_1) \frac{i}{\hbar} [Y_\alpha(t), F_\beta(t_1)]_- , \]  
(14)
with zero mean value ($\langle \xi(t) \rangle = 0$), we find that the equation for the arbitrary operator $A(t)$
of the dynamical subsystem has the form

$$\dot{A}(t) = \frac{1}{i\hbar} [A(t), H_0] - \frac{1}{2} \int_{-\infty}^{\infty} dt_1 \varphi_{\alpha\beta}(t, t_1) [Y_\alpha(t), F_\beta(t_1)]_+ - \int_{-\infty}^{t} dt_1 M_{\alpha\beta}(t, t_1) \frac{i}{\hbar} [Y_\alpha(t), F_\beta(t_1)]_- + \xi(t). \quad (15)$$

In the case of weak coupling between the dynamical subsystem and the bath, the correlator
of the fluctuation force is given by

$$\left\langle \frac{1}{2} [\xi(t), \xi(t_1)]_+ \right\rangle = M_{\alpha\beta}(t, t_1) \left\langle \frac{1}{2} [Y_\alpha(t), Y_\beta(t_1)]_+ \right\rangle + R_{\alpha\beta}(t, t_1) \left\langle \frac{1}{2} [Y_\alpha(t), Y_\beta(t_1)]_- \right\rangle, \quad (16)$$

where

$$R_{\alpha\beta}(t, t_1) = \left\langle \frac{1}{2} [Q_0^\alpha(t), Q_0^\beta(t_1)]_- \right\rangle = \frac{\hbar}{2\ell} (\varphi_{\alpha\beta}(t, t_1) - \varphi_{\beta\alpha}(t_1, t)). \quad (17)$$

### III. FORMULATION OF SPIN DYNAMICS

We start from the model Hamiltonian

$$\hat{H} = H_{\text{orbital}} + H_{\text{spin}} + U_{e-ph} + U_{e-i} + U_{DP} + H_{ph}, \quad (18)$$

describing an electron with spin in the presence of a magnetic field directed along the $z$-axis,
where the magnetic field and its vector potential are given by

$$\mathbf{B} = (0, 0, B), \quad \mathbf{A} = (-\frac{By}{2}, \frac{Bx}{2}, 0). \quad (19)$$

The first term in the Hamiltonian (18) is responsible for kinetic electron orbital motion,

$$H_{\text{orbital}} = \frac{mV_x^2}{2} + \frac{mV_y^2}{2} + \frac{mV_z^2}{2}, \quad (20)$$

where the velocity component operators of the electron in a magnetic field can be written as

$$V_x = \frac{1}{m} \left( p_x - \frac{m\omega_c y}{2} \right), \quad V_y = \frac{1}{m} \left( p_y + \frac{m\omega_c x}{2} \right), \quad V_z = \frac{p_z}{m}, \quad (21)$$

$$[V_x, V_y]_- = -\frac{i\hbar\omega_c}{m}. \quad (22)$$
The next term, describing spin motion in the presence of a magnetic field (Zeeman term), is given by

\[ H_{\text{spin}} = \frac{1}{2} g \mu_B (\vec{\sigma} \cdot \mathbf{B}) = \frac{\hbar \omega_B}{2} \sigma_z, \]  

(22)

where \( g \) is the crystal \( g \)-factor (in particular, \( g = -0.44 \) for GaAs), \( \mu_B = |e| \hbar / 2m_0c \) is the Bohr magneton (\( m_0 \) is the mass of a free electron, the effective mass, \( m \), is 0.067\( m_0 \) for GaAs), \( \omega_B = g \mu_B B / \hbar \) is the frequency of the spin precession induced by the magnetic field, and \( \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is the standard set of Pauli matrices.

The free phonon Hamiltonian is given by

\[ H_B = \sum_k \hbar \omega_k (b_k^{+} b_k + \frac{1}{2}), \]  

(23)

where \( \hbar \omega_k \) is the phonon energy and \( b_k^{+} \) and \( b_k \) are the creation and annihilation operators of a phonon in the \( k \)-mode, respectively. The interaction of an electron with random impurities is described by the potential

\[ U_{e-i}(\mathbf{r}) = -\frac{1}{L^{3/2}} \sum_k U_k e^{i\mathbf{kr}}, \]  

(24)

where \( U_k \) are the spatial Fourier components of the impurity potential, and \( L^3 \) denotes the volume of the crystal. The electron-phonon interaction is given by

\[ U_{e-ph}(\mathbf{r}, t) = -\frac{1}{L^{3/2}} \sum_k i \zeta (b_k(t) - b_k^{+}(t)) e^{i\mathbf{kr}}, \]  

(25)

where \( \zeta \) presents the strength of the electron-phonon coupling. As mentioned above, the principal mechanism of spin-orbit interaction is the D’yakonov-Perel’ term, \( U_{DP} \). In semiconductor crystals lacking inversion symmetry, the effective mass Hamiltonian includes terms cubic in electron quasimomentum [10]. The presence of these terms induces a random magnetic field \( \Omega \) given by

\[ \Omega = \frac{\alpha \vec{\mathbf{x}}}{\hbar m^{3/2} \sqrt{2E_g}}, \]  

(26)

where
\[ \begin{align*}
x_x &= m^3 V_x (V_y^2 - V_z^2), \quad x_y = m^3 V_y (V_z^2 - V_x^2), \quad x_z = m^3 V_z (V_x^2 - V_y^2),
\end{align*}\]

\(E_g\) is the energy gap, and \(\alpha\) is a coefficient representing the strength of the spin-orbit coupling (\(\alpha \approx 0.07\) in GaAs). The random magnetic field induced by the orbital motion interacts with electron spin through the interaction Hamiltonian

\[ U_{DP} = \hbar \left( \mathbf{\overrightarrow{\sigma}} \cdot \mathbf{\Omega} \right) \]

It is evident from the Hamiltonian (18) that there is no direct coupling of electron spin degrees of freedom to phonons and impurities. Therefore, to analyze spin relaxation dynamics, we consider the orbital degrees of freedom as an intermediary that (a) transfers fluctuations from the phonon/impurity bath to electron spin, and (b) transfers energy from spin degrees of freedom to heat bath. This two-step process is illustrated in Figure 2. Correspondingly, we address the analysis in two stages, and in the first stage we determine the statistical characteristics of orbital motion dictated by the phonon/impurity bath, neglecting the presence of the electron spin. The pertinent first-stage Hamiltonian, \(H^{(I)}\), is given by

\[ H^{(I)} = H_{\text{orbital}} + H_{\text{int}}^{(I)} + H_{\text{ph}}, \]

where the first-stage interaction Hamiltonian, \(H_{\text{int}}^{(I)}\), has the form

\[ H_{\text{int}}^{(I)} = U_{e-ph} + U_{e-i} = -L^{-3/2} \sum_k (i\zeta(b_k(t) - b_{-k}^+(t)) + U_k) e^{ikr}. \]

Comparing this expression to Eq. (4), we take the summation index \(\alpha\) of Eq. (4), as the mode number, \(k\), and the function \(L^{-3/2}e^{ikr}\) plays the role of \(F_\alpha(t)\) of Eq. (4), whereas the bath variable, \(Q_\alpha(t)\), is \((i\zeta(b_k(t) - b_{-k}^+(t)) + U_k)\).

In the second stage of analysis we employ orbital motion as an "effective heat bath" having the characteristics determined in the first stage. The second-stage Hamiltonian, \(H^{(II)}\), can be written as

\[ H^{(II)} = H_{\text{spin}} + H_{\text{int}}^{(II)} + H_{\text{bath}}, \]
where \( H_{\text{int}}^{(II)} = U_{DP} \) and \( H_{\text{bath}} \) may be taken as the remainder of the Hamiltonian \((18)\). To compare \( U_{DP} \) to Eq. \((2)\), we rewrite \( U_{DP} \) as

\[
U_{DP} = -\sigma_x Q_x(t) - \sigma_y Q_y(t) - \sigma_z Q_z(t),
\]

where the "effective heat bath" variables are given by

\[
Q_x(t) = -\frac{\alpha m^{3/2}}{2\sqrt{2\varepsilon g}} V_x(t) \left( V_y^2(t) - V_z^2(t) \right),
\]

\[
Q_y(t) = -\frac{\alpha m^{3/2}}{2\sqrt{2\varepsilon g}} V_y(t) \left( V_x^2(t) - V_z^2(t) \right),
\]

\[
Q_z(t) = -\frac{\alpha m^{3/2}}{2\sqrt{2\varepsilon g}} V_z(t) \left( V_x^2(t) - V_y^2(t) \right).
\]

In this, there is summation over coordinate projections and the "dynamic subsystem" variables are the electron spin projections.

**IV. STAGE 1: ORBITAL DYNAMICS.**

In this step we employ TOQS to determine the statistical characteristics of electron orbital motion described by the operator equations \((34)\)

\[
\left( \frac{d}{dt} + \gamma_0 \right) V_x(t) + (\omega_c + \delta) V_y(t) = \xi_x(t),
\]

\[
\left( \frac{d}{dt} + \gamma_0 \right) V_y(t) - (\omega_c - \delta) V_x(t) = \xi_y(t),
\]

and

\[
\left( \frac{d}{dt} + \gamma_z \right) V_z(t) = \xi_z(t),
\]

where \( V_x(t), V_y(t), V_z(t) \) are electron velocity operator components, and \( \omega_c = |e| B/mc \) is the cyclotron frequency. The electron-bath interaction determines the relaxation rates, \( \gamma_0, \gamma_z \), the frequency shift, \( \delta \), and the fluctuation sources, \( \xi_x(t), \xi_y(t), \xi_z(t) \), involved in Eq. \((34)\), \((35)\). The Fourier transforms of the velocity correlation functions are given by

\[
\left< \frac{1}{2} [V_x(\omega); V_x]_+ \right> = \left< \frac{1}{2} [V_y(\omega); V_y]_+ \right> = \frac{K_1(\omega)}{2} \left( \frac{1}{(\omega - \omega_c)^2 + \gamma_0^2} + \frac{1}{(\omega + \omega_c)^2 + \gamma_0^2} \right),
\]

where \( K_1(\omega) = \frac{1}{(\omega - \omega_c)^2 + \gamma_0^2} - \frac{1}{(\omega + \omega_c)^2 + \gamma_0^2} \).
\[ \left\langle \frac{1}{2} [V_x(\omega); V_y]_+ \right\rangle = - \left\langle \frac{1}{2} [V_y(\omega); V_x]_+ \right\rangle = \frac{K_\perp(\omega)}{2i} \left( \frac{1}{(\omega - \omega_c)^2 + \gamma_0^2} - \frac{1}{(\omega + \omega_c)^2 + \gamma_0^2} \right), \tag{36} \]

and

\[ \left\langle \frac{1}{2} [V_z(\omega); V_z]_+ \right\rangle = \frac{K_z(\omega)}{\omega^2 + \gamma_z^2}, \tag{37} \]

where

\[ K_\perp(\omega) = \int d(t - t_1) e^{i\omega(t-t_1)} \left\langle \frac{1}{2} [\xi_x(t), \xi_x(t_1)]_+ \right\rangle = \int d(t - t_1) e^{i\omega(t-t_1)} \left\langle \frac{1}{2} [\xi_y(t), \xi_y(t_1)]_+ \right\rangle, \tag{38} \]

and

\[ K_z(\omega) = \int d(t - t_1) e^{i\omega(t-t_1)} \left\langle \frac{1}{2} [\xi_z(t), \xi_z(t_1)]_+ \right\rangle. \tag{39} \]

To be specific we take account of the contributions of polar optical phonons, deforma
tional acoustic phonons and random impurities to electron orbital dynamics. In a first
approximation in their (weak) coupling strengths to orbital motion, these contributions can
be treated as mutually independent and the overall values of the damping rates and the
time-Fourier transforms of the correlation functions are given by (superscripts \( OP \) and \( AP \)
denote the contributions of optical and acoustic phonons, respectively, and \( I \) denotes that
of random impurities):

\[ \gamma_0 = \gamma_0^{OP} + \gamma_0^I + \gamma_0^{AP}, \tag{40} \]

\[ \gamma_z = \gamma_z^{OP} + \gamma_z^I + \gamma_z^{AP}, \]

\[ K_\perp(\omega) = K_\perp^{OP}(\omega) + K_\perp^I(\omega) + K_\perp^{AP}(\omega), \]

\[ K_z(\omega) = K_z^{OP}(\omega) + K_z^I(\omega) + K_z^{AP}(\omega), \]

The microscopic expressions of all these quantities are determined using the method of Ref. \[ \text{[6]}, \]
and are given below:

The response and correlation functions for polar optical phonons are
\[ \varphi^O_P(\tau) = \frac{4\pi \Omega_0 e^2}{k^2 \epsilon^*} \sin(\Omega_0 \tau) \eta(\tau), \quad M^O_P(\tau) = \frac{\hbar 4\pi \Omega_0 e^2}{2k^2 \epsilon^*} \cos(\Omega_0 \tau) \coth(\frac{\hbar \Omega_0}{2k_B T}), \] (41)

resulting in

\[ \gamma^O_P = \frac{1}{2\sqrt{2\pi}} \frac{\Omega_0 e^2}{m \epsilon^*} \int_0^{+\infty} dk_\perp k_\perp^3 \int_0^{+\infty} dk_z \frac{\tau_e^3(k_\perp, k_z)}{k^2} \mathbb{S}^O_P(k_\perp, k_z) \]

and

\[ \gamma^O_z = \frac{1}{\sqrt{2\pi}} \frac{\Omega_0 e^2}{m \epsilon^*} \int_0^{+\infty} dk_\perp k_\perp \int_0^{+\infty} dk_z \frac{\tau_e^3(k_\perp, k_z)}{k^2} k_z^2 \mathbb{S}^O_P(k_\perp, k_z), \]

where

\[ \mathbb{S}^O_P(k_\perp, k_z) = \left\{ \left( \coth\left( \frac{\hbar \Omega_0}{2k_B T} \right) + 1 \right) (\omega_k + \Omega_0) \exp \left[ -\frac{1}{2} (\omega_k + \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] + \right. \]

\[ + \left( \coth\left( \frac{\hbar \Omega_0}{2k_B T} \right) - 1 \right) (\omega_k - \Omega_0) \exp \left[ -\frac{1}{2} (\omega_k - \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] \right\}. \] (42)

Furthermore, we obtain

\[ K^O_P(\omega) = \frac{1}{4\sqrt{2\pi}} \frac{\hbar \Omega_0 e^2}{m^2 \epsilon^*} \int_0^{+\infty} dk_\perp k_\perp^3 \int_0^{+\infty} dk_z \frac{\tau_e(k_\perp, k_z)}{k^2} \mathbb{K}^O_P(k_\perp, k_z, \omega) \] (43)

and

\[ K^O_z(\omega) = \frac{1}{2\sqrt{2\pi}} \frac{\hbar \Omega_0 e^2}{m^2 \epsilon^*} \int_0^{+\infty} dk_\perp k_\perp \int_0^{+\infty} dk_z \frac{\tau_e(k_\perp, k_z)}{k^2} k_z^2 \mathbb{K}^O_P(k_\perp, k_z, \omega), \] (44)

where

\[ \mathbb{K}^O_P(k_\perp, k_z, \omega) = \left\{ \left( \coth\left( \frac{\hbar \Omega_0}{2k_B T} \right) + 1 \right) \left( \exp \left[ -\frac{1}{2} (\omega + \omega_k + \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] + \right. \right. \]

\[ + \exp \left[ -\frac{1}{2} (\omega - \omega_k - \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] \right) + \]

\[ + \left( \coth\left( \frac{\hbar \Omega_0}{2k_B T} \right) - 1 \right) \left( \exp \left[ -\frac{1}{2} (\omega + \omega_k - \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] + \right. \right. \]

\[ + \exp \left[ -\frac{1}{2} (\omega - \omega_k + \Omega_0)^2 \tau_e^2(k_\perp, k_z) \right] \right\}, \] (45)

\( \Omega_0 \) is the optical phonon frequency, \( 1/\epsilon^* = 1/\epsilon_\infty - 1/\epsilon_0 \), (\( \epsilon_\infty \) and \( \epsilon_0 \) are the hf and static permittivities of the crystal, respectively).

The response and correlation functions of the acoustic phonons are given by
\[ \varphi_k^{AP}(\tau) = \frac{D^2k}{\rho u} \sin(uk\tau) \eta(\tau), \quad M_k^{AP}(\tau) = \frac{\hbar D^2k}{2 \rho u} \cos(uk\tau) \coth\left(\frac{\hbar uk}{2k_BT}\right), \quad (46) \]

which provide the contribution to the damping rate as

\[ \gamma_0^{AP} = \frac{1}{8\pi\sqrt{2\pi} m\rho u} \int_0^{+\infty} dk_{\perp} k_{\perp}^3 \int_0^{+\infty} dk_z k z^3(k_{\perp}, k_z) \mathcal{G}^{AP}(k_{\perp}, k_z), \quad (47) \]

and

\[ \gamma_z^{AP} = \frac{1}{4\pi\sqrt{2\pi} m\rho u} \int_0^{+\infty} dk_{\perp} k_{\perp} \int_0^{+\infty} dk_z k z^2(k_{\perp}, k_z) \mathcal{G}^{AP}(k_{\perp}, k_z), \quad (48) \]

where

\[ \mathcal{G}^{AP}(k_{\perp}, k_z) = \left\{ \left( \coth\left(\frac{\hbar uk}{2k_BT}\right) + 1 \right) (\omega_k + uk) \exp\left[ -\frac{1}{2} (\omega_k + uk)^2 \tau_e^2(k_{\perp}, k_z) \right] + \right. \]

\[ \left. + \left( \coth\left(\frac{\hbar uk}{2k_BT}\right) - 1 \right) (\omega_k - uk) \exp\left[ -\frac{1}{2} (\omega_k - uk)^2 \tau_e^2(k_{\perp}, k_z) \right] \right\}. \quad (49) \]

Accordingly,

\[ K_{\perp}^{AP}(\omega) = \frac{1}{16\pi\sqrt{2\pi} m^2\rho u} \int_0^{+\infty} dk_{\perp} k_{\perp}^3 \int_0^{+\infty} dk_z k z(k_{\perp}, k_z) \mathcal{K}^{AP}(k_{\perp}, k_z, \omega), \quad (50) \]

and

\[ K_z^{AP}(\omega) = \frac{1}{8\pi\sqrt{2\pi} m^2\rho u} \int_0^{+\infty} dk_{\perp} k_{\perp} \int_0^{+\infty} dk_z k z^2(k_{\perp}, k_z) \mathcal{K}^{AP}(k_{\perp}, k_z, \omega), \quad (51) \]

where

\[ \mathcal{K}^{AP}(k_{\perp}, k_z, \omega) = \left\{ \left( \coth\left(\frac{\hbar uk}{2k_BT}\right) + 1 \right) \left( \exp\left[ -\frac{1}{2} (\omega + \omega_k + uk)^2 \tau_e^2(k_{\perp}, k_z) \right] + \right. \right. \]

\[ + \exp\left[ -\frac{1}{2} (\omega - \omega_k - uk)^2 \tau_e^2(k_{\perp}, k_z) \right] \right) + \]

\[ \left. + \left( \coth\left(\frac{\hbar uk}{2k_BT}\right) - 1 \right) \left( \exp\left[ -\frac{1}{2} (\omega + \omega_k - uk)^2 \tau_e^2(k_{\perp}, k_z) \right] + \right. \right. \]

\[ + \exp\left[ -\frac{1}{2} (\omega - \omega_k + uk)^2 \tau_e^2(k_{\perp}, k_z) \right] \right\}. \quad (52) \]

Here, \( D \) is the deformation potential, \( \rho \) is the crystal density, and \( u \) is the sound velocity.

For the case of static random impurities, there is not any response function, due to the lack of dynamics. The correlation function is given by
\[ M_k^I = \frac{2e^4 n_t^*}{\pi \varepsilon_0^2 (k^2 + r_0^{-2})^2}, \]  

(53)

where \( r_0 \) is the screening radius, \( n_t^* = \sum_\alpha n_\alpha Z_\alpha^2 \); \( n_\alpha \) is the impurity concentration for species \( \alpha \), and \( Z_\alpha \) is their charge number. The relaxation rates associated with impurities have the forms

\[ \gamma_0^I = \frac{1}{2\pi^2 \sqrt{2\pi m^2 \varepsilon_0^2}} \int_0^{+\infty} dk_\perp k_\perp^3 \int_0^{+\infty} dk_z k_z^2 \tau_c^3(k_\perp, k_z) \exp \left( -\frac{\omega_k^2 \tau_c^2(k_\perp, k_z)}{2} \right), \]  

(54)

and

\[ \gamma_z^I = \frac{1}{\pi^2 \sqrt{2\pi m^2 \varepsilon_0^2}} \int_0^{+\infty} dk_\perp k_\perp \int_0^{+\infty} dk_z k_z^2 \tau_c^3(k_\perp, k_z) \exp \left( -\frac{\omega_k^2 \tau_c^2(k_\perp, k_z)}{2} \right), \]  

(55)

Finally, the contributions to the Fourier transforms of the fluctuation force correlators can be written as

\[ K_{\perp}^I(\omega) = \frac{1}{2\pi^2 \sqrt{2\pi m^2 \varepsilon_0^2}} \int_0^{+\infty} dk_\perp k_\perp^3 \int_0^{+\infty} dk_z \tau_c(k_\perp, k_z) \left( \exp \left[ -\frac{1}{2} (\omega + \omega_k)^2 \tau_c^2(k_\perp, k_z) \right] + \exp \left[ -\frac{1}{2} (\omega - \omega_k)^2 \tau_c^2(k_\perp, k_z) \right] \right), \]  

(56)

and

\[ K_z^I(\omega) = \frac{1}{\pi^2 \sqrt{2\pi m^2 \varepsilon_0^2}} \int_0^{+\infty} dk_\perp k_\perp \int_0^{+\infty} dk_z \tau_c(k_\perp, k_z) \left( \exp \left[ -\frac{1}{2} (\omega + \omega_k)^2 \tau_c^2(k_\perp, k_z) \right] + \exp \left[ -\frac{1}{2} (\omega - \omega_k)^2 \tau_c^2(k_\perp, k_z) \right] \right), \]  

(57)

In all these formulae we have used the notation \( k = \sqrt{k_\perp^2 + k_z^2} \), \( \omega_k = \hbar k^2/2m \), \( \tau_c^{-2}(k_\perp, k_z) = k_\perp^2 \langle V^2_\perp \rangle + k_z^2 \langle V^2_z \rangle \). \( \langle V^2_\perp \rangle \) and \( \langle V^2_z \rangle \) can be determined self-consistently using \( \langle V^2_\perp \rangle = K_\perp(\omega_c)/2\gamma_0 \) and \( \langle V^2_z \rangle = K_z(0)/2\gamma_z \). All these results are obtained for relatively weak magnetic fields, \( \omega_c \tau_c << 1 \). However, the energy shift, \( \delta \), due to electron-bath coupling is even smaller, \( \delta << \omega_c \), and will be neglected.

**V. STAGE 2: SPIN DYNAMICS.**

In this section we examine the spin relaxation process in the presence of the "effective bath" with variables given by Eq. (33). Their response functions and correlation functions have the form \((i, j = x, y, z)\)
\[ M_{ij}(t, t_1) = \left( \frac{1}{2} [Q_i(t), Q_j(t_1)]_+ \right) \]  \hspace{1cm} (58) \\
\[ \varphi_{ij}(t, t_1) = \left( \frac{i}{\hbar} [Q_i(t), Q_j(t_1)]_- \right) \eta(t-t_1) \]  \hspace{1cm} (59)

It is evident that these functions are of the sixth order of the electron velocity projections. Using the quantum analog of the Furutsu-Novikov theorem for Gaussian variables \[3\], they can be represented in terms of pair correlators. The resulting expressions for the spectral functions are given by

\[ S_{ij}(\omega) = \int d(t-t_1) e^{i\omega(t-t_1)} M_{ij}(t, t_1), \]  \hspace{1cm} (60)

\[ S_{xx}(\omega) = S_{yy}(\omega) = \lambda^2 \left\langle \frac{1}{2} [V_x(\omega), V_x]_+ \right\rangle \left( (\langle V_x^2 \rangle - \langle V_x^2 \rangle)^2 + 4 \langle V_x V_y \rangle \langle V_x V_y \rangle \right) + \]  \hspace{1cm} (61) \\
\[ + \lambda^2 \int \frac{\frac{d\omega_1}{2\pi}}{2\pi} \int \frac{\frac{d\omega_2}{2\pi}}{2\pi} \Xi(\omega_1, \omega_2, \omega - \omega_1 - \omega_2) \cdot \] \\
\[ \cdot \left\{ 2 \left\langle \frac{1}{2} [V_x(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_x(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_x(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} + \quad (61a) \\
\[ 2 \left\langle \frac{1}{2} [V_x(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_x(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_x(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} - \]  \hspace{1cm} (61b) \\
\[ -4 \left\langle \frac{1}{2} [V_x(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_x(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_x(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} \right), \] \\
and

\[ S_{xy}(\omega) = -S_{yx}(\omega) = \lambda^2 \left\langle \frac{1}{2} [V_y(\omega), V_x]_+ \right\rangle \left( (\langle V_x^2 \rangle - \langle V_x^2 \rangle)^2 + 4 \langle V_x V_y \rangle \langle V_x V_y \rangle \right) + \]  \hspace{1cm} (62) \\
\[ + \lambda^2 \int \frac{\frac{d\omega_1}{2\pi}}{2\pi} \int \frac{\frac{d\omega_2}{2\pi}}{2\pi} \Xi(\omega_1, \omega_2, \omega - \omega_1 - \omega_2) \cdot \] \\
\[ \cdot \left\{ 2 \left\langle \frac{1}{2} [V_y(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_y(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_y(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} + \quad (62a) \\
\[ 2 \left\langle \frac{1}{2} [V_y(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_y(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_y(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} - \]  \hspace{1cm} (62b) \\
\[ -4 \left\langle \frac{1}{2} [V_y(\omega_1), V_x]_+ \right\rangle \left\langle \frac{1}{2} [V_y(\omega_2), V_x]_+ \right\rangle + \left\langle \frac{1}{2} [V_y(\omega - \omega_1 - \omega_2), V_x]_+ \right\rangle \right\} \right), \] \\
and

\[ S_{zz}(\omega) = \lambda^2 \int \frac{\frac{d\omega_1}{2\pi}}{2\pi} \int \frac{\frac{d\omega_2}{2\pi}}{2\pi} \Xi(\omega_1, \omega_2, \omega - \omega_1 - \omega_2) \cdot \]  \hspace{1cm} (63) \\
\[ \cdot \left\{ 4 \left\langle \frac{1}{2} [V_z(\omega_1), V_z]_+ \right\rangle \left\langle \frac{1}{2} [V_z(\omega_2), V_z]_+ \right\rangle + \left\langle \frac{1}{2} [V_z(\omega - \omega_1 - \omega_2), V_z]_+ \right\rangle \right\} - \]  \hspace{1cm} (63a) \\
\[ - \left\langle \frac{1}{2} [V_z(\omega_1), V_z]_+ \right\rangle \left\langle \frac{1}{2} [V_z(\omega - \omega_1 - \omega_2), V_z]_+ \right\rangle \right\}, \]
where we defined an auxiliary function

\[
\Xi(\omega_1, \omega_2, \omega_3) = 1 + \tanh \left( \frac{\hbar \omega_1}{2T} \right) \tanh \left( \frac{\hbar \omega_2}{2T} \right) + \\
+ \tanh \left( \frac{\hbar \omega_1}{2T} \right) \tanh \left( \frac{\hbar \omega_3}{2T} \right) + \tanh \left( \frac{\hbar \omega_2}{2T} \right) \tanh \left( \frac{\hbar \omega_3}{2T} \right)
\]

which appears due to the replacement of the velocity commutators by their anticommutators in accordance with Eq. (9). In Eqs. (61-63) we use the shortened notation

\[
\langle V_j^2 \rangle = \int \frac{d\omega}{2\pi} \langle \frac{1}{2} [V_j(t), V_j(t)]_+ \rangle, \quad \langle V_x V_y \rangle = \int \frac{d\omega}{2\pi} \langle \frac{1}{2} [V_x(t), V_y(t)]_+ \rangle.
\]

Employing a second application of the TOQS method, we obtain equations for the average spin projections as follows:

\[
\frac{d}{dt} \langle \sigma_x(t) \rangle = -\Gamma_{xx} \langle \sigma_x(t) \rangle - (\omega_B + \Gamma_{xy}) \langle \sigma_y(t) \rangle + \Gamma_{xz} \langle \sigma_z(t) \rangle + \Gamma_0^x,
\]

\[
\frac{d}{dt} \langle \sigma_y(t) \rangle = (\omega_B + \Gamma_{yx}) \langle \sigma_x(t) \rangle - \Gamma_{yy} \langle \sigma_y(t) \rangle + \Gamma_{yz} \langle \sigma_z(t) \rangle + \Gamma_0^y,
\]

\[
\frac{d}{dt} \langle \sigma_z(t) \rangle = \Gamma_{zz} \langle \sigma_z(t) \rangle + \Gamma_{zy} \langle \sigma_y(t) \rangle - \Gamma_{xz} \langle \sigma_x(t) \rangle + \Gamma_0^z,
\]

where the coefficients are given by

\[
\Gamma_{xx} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{yy} (t_1) \cos \omega_B (t - t_1) + M_{yB} (t_1) \sin \omega_B (t - t_1) + M_{zz} (t, t_1) \},
\]

\[
\Gamma_{xy} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{yy} (t_1) \sin \omega_B (t - t_1) - M_{yx} (t_1) \cos \omega_B (t - t_1) \},
\]

\[
\Gamma_{xz} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{zz} (t_1) \cos \omega_B (t - t_1) - M_{zy} (t_1) \sin \omega_B (t - t_1) \},
\]

\[
\Gamma_0^x = \frac{2}{\hbar} \int_{-\infty}^{t} dt_1 \{ \varphi_{xx} (t_1) \sin \omega_B (t - t_1) + \varphi_{zy} (t_1) \cos \omega_B (t - t_1) - \varphi_{yz} (t, t_1) \},
\]

\[
\Gamma_{yx} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{xx} (t_1) \sin \omega_B (t - t_1) + M_{xy} (t_1) \cos \omega_B (t - t_1) \},
\]

\[
\Gamma_{yy} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{xx} (t_1) \cos \omega_B (t - t_1) - M_{xy} (t_1) \sin \omega_B (t - t_1) + M_{zz} (t, t_1) \},
\]

\[
\Gamma_{yz} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \{ M_{xx} (t_1) \sin \omega_B (t - t_1) + M_{zy} (t_1) \cos \omega_B (t - t_1) \},
\]

\[
\Gamma_0^y = \frac{2}{\hbar} \int_{-\infty}^{t} dt_1 \{ \varphi_{yy} (t_1) \sin \omega_B (t - t_1) - \varphi_{xx} (t, t_1) \cos \omega_B (t - t_1) + \varphi_{xz} (t, t_1) \},
\]

and
\[ \Gamma_{zx} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 M_{xz}(t, t_1), \] (68)

\[ \Gamma_{zy} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 M_{yz}(t, t_1), \]

\[ \Gamma_{zz} = \frac{4}{\hbar^2} \int_{-\infty}^{t} dt_1 \left\{ (M_{yy}(t, t_1) + M_{xx}(t, t_1)) \cos \omega_B (t - t_1) + (M_{yx}(t, t_1) - M_{xy}(t, t_1)) \sin \omega_B (t - t_1) \right\}, \]

\[ \Gamma_0^z = \frac{2}{\hbar} \int_{-\infty}^{t} dt_1 \left\{ (\varphi_{yx}(t, t_1) + \varphi_{xy}(t, t_1)) \cos \omega_B (t - t_1) - (\varphi_{xx}(t, t_1) + \varphi_{yy}(t, t_1)) \sin \omega_B (t - t_1) \right\}. \]

Only six of the above twelve coefficients, \( \Gamma_{ij} \), do not vanish, and we may identify relaxation times in terms of the nonvanishing \( \Gamma_{ij} \) as follows:

\[ \Gamma_{xx} = \Gamma_{yy} = \frac{1}{T_2}, \quad \Gamma_{zz} = \frac{1}{T_1}, \] (69)

where \( T_1, T_2 \) are the longitudinal and transverse relaxation times, responsible for the relaxation of the magnetic moment and decoherence respectively. Furthermore, direct calculation shows that

\[ \Gamma_{xy} = \Gamma_{yx} = \delta_s, \quad \Gamma_0^z = \sigma_0^z = -\tanh \left( \frac{\hbar \omega_B}{2T} \right), \] (70)

where \( \delta_s \) may be identified as a frequency shift due to ”effective bath” fluctuations, and \( \sigma_0^z \) is the equilibrium population difference between spin up and spin down states. With these calculated results and identifications, the spin equations take the usual Bloch form,

\[ \frac{d}{dt} \langle \sigma_x(t) \rangle = -\frac{\langle \sigma_x(t) \rangle}{T_2} - (\omega_B + \delta_s) \langle \sigma_y(t) \rangle, \] (71)

\[ \frac{d}{dt} \langle \sigma_y(t) \rangle = (\omega_B + \delta_s) \langle \sigma_x(t) \rangle - \frac{\langle \sigma_y(t) \rangle}{T_2}, \]

\[ \frac{d}{dt} \langle \sigma_z(t) \rangle = \sigma_0^z - \frac{\langle \sigma_z(t) \rangle}{T_1}. \]

In terms of the spectral functions, \( S_{ij}(\omega) \), the relaxation rates (inverse relaxation times), may be expressed as follows:

\[ \frac{1}{T_1} = \frac{4}{\hbar^2} \left( S_{xx}(\omega_B) + iS_{xy}(\omega_B) \right), \] (72)
and
\[
\frac{1}{T_2} = \frac{2}{\hbar^2} (S_{xx}(\omega_B) + iS_{xy}(\omega_B) + S_{zz}(0)). \tag{73}
\]

Although Eqs. (72), (73) are relatively simple and yield some interesting qualitative conclusions, a quantitative analysis of them is difficult, because of the complexity of the expressions for the spectral functions, \(S_{ij}(\omega)\). This complexity is relieved under the prevailing assumption of weak coupling, which permits the replacement of the Lorenzians involved in the integrands of Eqs. (61-63) by
\[
\Lambda(x; \varepsilon) = \frac{\varepsilon}{\pi x^2 + \varepsilon^2} \rightarrow \delta(x), \quad \text{when} \quad \varepsilon \rightarrow 0, \tag{74}
\]
where \(\delta(x)\) is the Dirac delta-function, and, consequently,
\[
\int dx f(x)\Lambda(x; \varepsilon a)\Lambda(x - y; \varepsilon b) = \frac{1}{2} \Lambda(y; \varepsilon (a + b)) (f(y) + f(0)) + \int \frac{1}{2} \Lambda(y; \varepsilon (a - b)) (f(y) - f(0)) \tag{75}
\]
(In this, we note that in Eqs. (35), (36) and (37), we have \(K_\perp(\omega), K_z(\omega), \gamma_0, \gamma_z\) all proportional to coupling strength \(\sim \varepsilon\)). Having employed Eqs. (74), (75) in the integrals of Eqs. (61-63) representing \(S_{xx}(\omega), S_{xy}(\omega),\) and \(S_{zz}(\omega)\), we obtain the results of integration as:
\[
S_{xx}(\omega) = \alpha^2 m^3 \frac{\pi}{8E_g} \left\{ \frac{1}{2} [V_x(\omega), V_y]_+ \right\} \left[ (\langle V_x^2 \rangle - \langle V_x^2 \rangle)^2 + 4 \langle V_x V_y \rangle \langle V_x V_y \rangle \right] + \tag{76}
\]
\[
+ \alpha^2 m^3 \frac{\pi}{8E_g} \left\{ R_1(\omega; \omega_c) + Y_1(\omega; \omega_c) \right\},
\]
\[
S_{xy}(\omega) = -\alpha^2 m^3 \frac{\pi}{8E_g} \left\{ \frac{1}{2} [V_x(\omega), V_y]_+ \right\} \left[ (\langle V_x^2 \rangle - \langle V_x^2 \rangle)^2 + 4 \langle V_x V_y \rangle \langle V_x V_y \rangle \right] + \tag{77}
\]
\[
+ \alpha^2 m^3 \frac{\pi}{8E_g} \left\{ R_2(\omega; \omega_c) + Y_2(\omega; \omega_c) \right\},
\]
\[
S_{zz}(\omega) = \alpha^2 m^3 \frac{\pi}{32E_g} \frac{\pi}{\gamma_0^2 \gamma_z} K_\perp(\omega_c) \tag{78}
\]
\[
\{ \Lambda(\omega - 2\omega_c; 2\gamma_0 + \gamma_z) (K_z(0)K_\perp(\omega - \omega_c)\Xi(\omega - \omega_c; \omega_c; 0) + \}
\]
\[
+ K_z(\omega - 2\omega_c) K_\perp(\omega_c)\Xi(\omega - 2\omega_c; \omega_c; \omega_c)) +
\]

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where we have introduced the notation

\[ R_1(\omega, \omega_c) = \frac{\pi}{16\gamma_0^3} (K_\perp(\omega_c))^2 \]
\[ \{ 3\Lambda(\omega - 3\omega_c; 3\gamma_0)K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c) + \]
\[ +\Lambda(\omega - \omega_c; 3\gamma_0)K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c; \omega_c) + \]
\[ +2\Lambda(\omega - \omega_c; \gamma_0) \right) (K_\perp(\omega) \Xi(\omega; \omega_c; -\omega_c) - K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c; \omega_c)) + \]
\[ +2\Lambda(\omega + \omega_c; \gamma_0) \right) (K_\perp(\omega) \Xi(\omega; \omega_c; -\omega_c) - K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c)) + \]
\[ +\Lambda(\omega + \omega_c; 3\gamma_0)K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c) + \]
\[ +3\Lambda(\omega + 3\omega_c; 3\gamma_0)K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c) \} , \]

\[ R_2(\omega, \omega_c) = -i \frac{\pi}{16\gamma_0^3} (K_\perp(\omega_c))^2 \]
\[ \{ 3\Lambda(\omega - 3\omega_c; 3\gamma_0)K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c) - \]
\[ -\Lambda(\omega - \omega_c; 3\gamma_0)K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c; \omega_c) - \]
\[ -2\Lambda(\omega - \omega_c; \gamma_0) \right) (K_\perp(\omega) \Xi(\omega; \omega_c; -\omega_c) - K_\perp(\omega - 2\omega_c)\Xi(\omega - 2\omega_c; \omega_c; \omega_c)) + \]
\[ +2\Lambda(\omega + \omega_c; \gamma_0) \right) (K_\perp(\omega) \Xi(\omega; \omega_c; -\omega_c) - K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c)) + \]
\[ +\Lambda(\omega + \omega_c; 3\gamma_0)K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c) - \]
\[ -3\Lambda(\omega + 3\omega_c; 3\gamma_0)K_\perp(\omega + 2\omega_c)\Xi(\omega + 2\omega_c; -\omega_c; -\omega_c) \} , \]

\[ Y_1(\omega, \omega_c) = \frac{\pi}{8\gamma_z^2\gamma_0} K_z(0) \]
\[ \{ \Lambda(\omega - \omega_c; 2\gamma_z + \gamma_0) \right) (K_\perp(\omega_c)K_z(\omega - \omega_c) \Xi(\omega - \omega_c; \omega_c; 0) + K_\perp(\omega)K_z(0) + \]

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\[ Y_2(\omega, \omega_c) = \frac{i\pi}{8\gamma_z^2\gamma_0} K_z(0) \]

These results are very general, providing explicit expressions for the spin relaxation times in terms of the material constants and coupling strengths. The magnetic field has been treated as relatively weak \( (\omega_c\tau_c << 1) \), but our estimates show that this range of fields includes those used in most experiments.

To make contact with earlier theories that did not take account of the magnetic field and used a phenomenological momentum scattering time \([10]\), we put \( \omega_c = 0 \) and \( \omega_B = 0 \) in Eqs. (76-79), which immediately yields

\[ S_{xx}(\omega) = S_{xx}(\omega) = S_{zz}(\omega) = \frac{\alpha^2 m^3 \pi [K(0)]^2}{8E_g \gamma_0^3} K(\omega) \Lambda(\omega; 3\alpha\gamma_0), \]

where

\[ K_\perp(\omega) = K_\perp(\omega) = K(\omega), \quad \gamma_0 = \gamma_z, \]

and

\[ S_{xy}(\omega) = 0. \]

In this zero field limit the relaxation times are equal and determined by

\[ \frac{1}{T_1} = \frac{1}{T_2} = \frac{4}{\hbar^2} S_{xx}(0). \]
In equilibrium, the rate of velocity fluctuations is defined only by temperature, and we obtain the result

\[ \langle V_x^2 \rangle = \frac{K(0)}{2\gamma_0} = \frac{k_B T}{m}, \]  

(84)

where \( T \) is the Kelvin temperature. This yields the common relaxation rate as

\[ \frac{1}{T_{1,2}} = \frac{4\alpha^2}{3\hbar^2 E_g \gamma_0} \left( k_B T \right)^3, \]  

(85)

which may be brought into coincidence with the expression usually used for the DP mechanism \([7,8,10]\) by the change of notation \( q\tau_p \to \frac{4}{3\gamma_0} \), with \( \tau_p \) a phenomenological momentum scattering time. It should be noted that the choice of coefficient \( q \) is unclear, with various authors assigning values in the range \( q = 0.8 - 2.7 \), with \( q \approx 2.7 \) for scattering by deformational acoustic phonons and \( q \approx 0.8 \) for scattering by polar optical phonons \([8,10]\). On the other hand, our microscopic analysis provides a clearly defined value with account of all scattering mechanisms. Moreover, we provide results for finite magnetic field derived on a microscopic basis, rather than relying on the ansatz of Ref. \([10]\), which has \( T_1(B) = T_2(B) \),

\[ \frac{1}{T_{1,2}(B)} = \frac{1}{T_{1,2}(B = 0)} \frac{1}{1 + \left( \omega_B \tau_p \right)^2}. \]  

(86)

In contradistinction to this, our analysis in the presence of a magnetic field shows that the transverse and longitudinal spin relaxation times are not equal (Eqs. \((72,73)\)).

VI. SUMMARY

In Summary, we have analyzed electron spin relaxation dynamics and decoherence in bulk semiconductors with an applied external magnetic field. In the absence of direct coupling of spin degrees of freedom to the phonon/impurity heat bath, we have used the orbital motion as an intermediary, transferring fluctuations from the bath to electron spin and, transmitting the dissipated energy from spin to the bath. To accomplish this, the two-stage procedure of solution has been employed: In the first stage, the fluctuation characteristics of the orbital motion were determined and the orbital degrees of freedom were used in the second stage as
an "effective heat bath" for spin dynamics. In both stages of the analysis, we employed the method of the theory of open quantum systems, obtaining a set of Bloch equations having longitudinal and transverse spin relaxation times, $T_1$ and $T_2$, determined on a microscopic basis. Our results provide analytic definition of the phenomenological parameters employed in earlier zero-field theories, in terms of material constants and coupling strengths. Moreover, this analysis of electron spin dynamics in finite magnetic field yields explicit formulae for the longitudinal and transverse relaxation times.

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FIGURES

FIG. 1. Schematic of "dynamical subsystem" - "heat bath" interaction.

FIG. 2. Schematic of the two-stage procedure for spin relaxation analyses
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