Experimental observation of the spontaneous breaking of the time-reversal symmetry in a synchronously-pumped passive Kerr resonator

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We experimentally observe a spontaneous temporal symmetry breaking instability in a coherently-driven passive optical Kerr resonator. The cavity is synchronously pumped by time-symmetric pulses yet we report output pulses with strongly asymmetric temporal and spectral intensity profiles, with up to 71% of the energy on the same side of the pump center frequency. The instability occurs above a certain pump power threshold but remarkably vanishes above a second threshold, in excellent agreement with theory. We also observe a generalised bistability in which an asymmetric output state coexists with a symmetric one in the same pumping conditions.

PACS numbers: 11.30.Qc, 05.45.-a, 42.60.Da, 42.65.Pc, 42.65.Sf, 42.65.Wi
intracavity pulse is set by a balance between nonlinearity and that the temporal duration of the main peak of the asymmetric here, we would observe intracavity pulses that break up into wider pump pulses (larger values of \( \tau \)) and breaking its alignment with the center of the pump superimposed with the asymmetric intracavity pulse obtained 

\[
\text{temporal intensity profile of the pump pulse (dashed green)}
\]

Finally, \( S(\tau) = S(-\tau) \), the mean-field model Eq. (1) is symmetric under a time reversal transformation, \( \tau \rightarrow -\tau \), yet it admits asymmetric solutions. This behavior is illustrated in Fig. 2 obtained for a detuning \( \Delta = 0.92 \) and for symmetric chirp-free Gaussian pump pulses \( S(\tau) = \sqrt{X} \exp[-(\tau/\tau_0)^2] \), with \( \tau_0 = 2.3 \). Figure 2(a) represents the bifurcation diagram of the SSB instability where we have plotted the power of the intracavity pulses at \( \tau = 0 \), i.e., at a time corresponding to the center of the pump pulses, versus the pump peak power \( X \). For low pump power, the intracavity pulses are temporally symmetric (blue curve). However, above a certain pump power threshold, the symmetric state becomes unstable (dotted part) and is replaced by an asymmetric one (red curve). The corresponding drop in \( |E(\tau = 0)|^2 \) observed at that point is associated with the peak of the intracavity pulse shifting to the side and breaking its alignment with the center of the pump pulse. This is illustrated in Fig. 2(b) where we have plotted the temporal intensity profile of the pump pulse (dashed green) superimposed with the asymmetric intracavity pulse obtained for \( X = 6.4 \) (red). Of course its mirror image (not shown for clarity) is also a solution of the problem. We must point out that the temporal duration of the main peak of the asymmetric intracavity pulse is set by a balance between nonlinearity and dispersion, and is comparable with the duration of the cavity solitons that the Kerr cavity is known to support. With wider pump pulses (larger values of \( \tau_0 \)) than those considered here, we would observe intracavity pulses that break up into multiple peaks (modulational instability) of similar duration \[26, 29\]. While SSB manifests itself even in that regime, the experimental signature of the asymmetry is less pronounced, hence we have chosen to work here with values of \( \tau_0 \) of the order of unity. Finally, let us also note in Fig. 2(a) that the symmetry of the solution is restored above a second pump power threshold.

[Figure 1. (Color online) Experimental setup: BPF: bandpass filter; EDFA: Erbium-doped fiber amplifier; PC, polarization controller.]

[Figure 2. (Color online) (a) SSB bifurcation diagram with blue (red) curves representing the symmetric (asymmetric) states, respectively (dotted parts are unstable), for \( \Delta = 0.92 \) and \( \tau_0 = 2.3 \). (b) Asymmetric pulse temporal intensity profile obtained for \( X = 6.4 \) (red) superimposed with the pump pulse (dashed green).]
As temporally asymmetric pulses are usually associated with asymmetric spectral densities, we relied on spectral measurements of the cavity output as a convenient signature of the breaking of the time-reversal symmetry in our experiment. A first set of experimental measurements is presented in Fig. 3 and was obtained with a detuning $\Delta = 0.92$ identical to that used for the numerical results of Fig. 2. Output spectra were measured for monotonically increasing values of the pump peak power $X$ and are shown as a pseudocolor plot in Fig. 3(a). A clear shift of the spectral power towards the low-frequency components can be readily observed as the pump power increases. To highlight this aspect, the spectrum obtained for $X = 6.4$ is plotted along the right side of Fig. 3(a) (red) superimposed with the measured pump spectrum (dashed green) for comparison. Nearly 66% of the intracavity spectral power is located on the low frequency side of the pump center frequency. A good agreement is observed with the theoretically modeled spectrum (black), which corresponds to that of the pulse illustrated in Fig. 2(b).

For a more comprehensive analysis of the observed symmetry breaking bifurcation, we define the spectral asymmetry factor as the ratio of the integrated spectral density on the low frequency side of the pump center frequency to that on the high frequency side. The asymmetry factor can be simply evaluated from the data shown in Fig. 3(a) and the result is plotted in Fig. 3(b) [red circles]. At low pump power, it is initially equal to 1 (corresponding to a symmetric state) but rapidly rises to a maximum of about 1.9 obtained for a normalized pump power $X = 6.4$ and for which we observe the maximum asymmetry for this detuning. Comparison with numerical simulations (dashed light-gray) reveals a reasonable quantitative agreement. We note however that while the theory predicts a rather abrupt onset and disappearance of symmetry breaking, the experimentally observed transitions are much more progressive. We have traced this difference to the presence of a small amount of third-order dispersion in the cavity fiber with coefficient $\beta_3 = 0.1 \text{ ps}^3/\text{km}$. This weakly breaks the perfect time-reversal symmetry of the problem \[31\], and introduces an additional term \(d_3 \beta_3^2 E / \partial \tau^3\) on the right-hand side of Eq. (1), with \(d_3 = \sqrt{2\alpha / \mathcal{L} \beta_1 / (3|\beta_2|^{3/2})} \approx 0.002\). As can be seen in Fig. 3(b), the asymmetry factors calculated with this small contribution taken into account (black) better fit the experimental data. In Fig. 3(c), we also show how the SSB bifurcation diagram changes in presence of third-order dispersion. Here the light gray curves are identical to those shown in Fig. 2(a). The curves inclusive of the small $\beta_3$ contribution are color-coded as a function of the asymmetry factor to highlight that a clear distinction still exists between the “symmetric” state (blue) and the asymmetric ones. The major difference is the lifting of the degeneracy between the two original mirror-like asymmetric solutions (which now sit on two disconnected curves) and this is associated with the change from an abrupt to a progressive symmetry breaking transition. In these conditions, ramping up the pump power should allow only one enantiomer to be observed. However, the system is very sensitive to a small cavity synchronisation mismatch or equivalently to fluctuations in the pump laser repetition rate \[32\], and in practice we could readily observe both left- and right-handed asymmetric states in the experiment. We must also stress out that the decay of the asymmetry factor past the maximum [see Fig. 3(b)] and the restoration of nearly perfect symmetric conditions above $X \sim 11$ is remarkable in this context. It is as expected from the theory [Fig. 2(a)] and confirms that the observed asymmetry is tied to the temporal SSB instability of the nonlinear cavity dynamics.

A second set of measurements is presented in Fig. 4 for a higher detuning, $\Delta = 3.2$. In this regime the Kerr cavity is known to exhibit hysteresis ($\Delta > \sqrt{3}$, see Refs. \[30, 33\]). This is clearly evidenced in our recorded spectra [Fig. 4(a)], where the top (bottom) panel was obtained for monotonically increasing (respectively, decreasing) values of the pump peak power $X$. A clear hysteresis is also seen in the spectral asymmetry factors deduced from the measured spectra [Fig. 4(b)]. Theory (black) and measurements (red circles) are again in good qualitative agreement. For increasing values of $X$, we start on the lower symmetric branch (asymmetry factor of 1). At the end of this branch ($X \approx 8$), the corresponding upper symmetric state is unstable as highlighted by the bifurcation diagram [Fig. 4(c)] so the cavity abruptly switches to a stable asymmetric state instead. Here the measured asymmetry factor jumps from 1 to 2. We observe a maximum asymmetry factor of 2.5 for $X = 10$ [i.e., 71% of the spectral power is on the same side of the pump center frequency, see spectra highlighted on the right of Fig. 4(a)], above which no more stable solutions can be observed and the intracavity pulse breathes...
periodically. Upon decreasing $X$, the cavity remains in the asymmetric state until it smoothly reconnects with the stable part of the upper symmetric branch at $X \simeq 4.7$. When $X$ then goes below about 4.1, we see an abrupt change in the output spectrum (but no change in the asymmetry factor which stays equal to 1) as the cavity switches from the upper symmetric state back to the lower symmetric state. From $X = 4.1$ to $X = 8$, we note the coexistence of the stable symmetric lower branch solution with the asymmetric solution, i.e., generalized bistability [24].

To complete our study, we performed FROG measurements of the asymmetric states with the largest observed spectral asymmetry, i.e., for $\Delta = 0.92$, $X = 6.4$ and $\Delta = 3.2$, $X = 10$. The retrieved temporal intensity profiles are shown in Figs. 5(a) and (b), respectively. In each case, the two mirror-like stationary intracavity pulses (red and green) recombined at the output, providing a timing reference for the FROG measurement. These were all superimposed with the temporal profile of a pulse in the symmetric state measured at lower pump power (dashed curve). The retrieved temporal intensity profiles are shown in Figure 5. (Color online) Experimental temporal intensity profiles of the two mirror-like stationary intracavity pulses (red and green) retrieved with FROG for (a) $X = 6.4$, $\Delta = 0.92$ and (b) $X = 10$, $\Delta = 3.2$. The dashed curves are the symmetric pulses observed at low pump power level while the gray dotted curves are theoretical predictions.

To conclude, we have presented what we believe is the first experimental observation of the spontaneous breaking of the time-reversal symmetry in a nonlinear dissipative system. Our study performed in a synchronously-pumped Kerr optical fiber cavity has revealed the generation of pulses with a strongly asymmetric temporal intensity profile from a seemingly time-symmetric configuration in excellent agreement with theory. A generalized bistability between symmetric and asymmetric solutions has also been unequivocally observed. The passive Kerr resonator, with its simplicity and ubiquitous character, is already considered as the paradigm of nonlinear systems subject to instabilities. Given the important role of symmetries in physics, our study further reinforces this status. Our observations may also have implications for other systems based on nonlinear cavities including microresonators and in which temporal SSB may lead to uncontrollable timing jitter in the pulse trains generated by such devices.

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