**FairMod: Fair Link Prediction and Recommendation via Graph Modification**

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As machine learning becomes more widely adopted across domains, it is critical that researchers and ML engineers think about the inherent biases in the data that may be perpetuated by the model. Recently, many studies have shown that such biases are also imbied in Graph Neural Network (GNN) models if the input graph is biased. In this work, we aim to mitigate the bias learned by GNNs through modifying the input graph. To that end, we propose **FairMod**, a **Fair Graph Modification** methodology with three formulations: the Global Fairness Optimization (GFO), Community Fairness Optimization (CFO), and Fair Edge Weighting (FEW) models. Our proposed models perform either microscopic or macroscopic edits to the input graph while training GNNs and learn node embeddings that are both accurate and fair under the context of link recommendations. We demonstrate the effectiveness of our approach on four real-world datasets and show that we can improve the recommendation fairness by several factors at negligible cost to link prediction accuracy.

CCS Concepts: • Computing methodologies → Neural networks.

Additional Key Words and Phrases: graph embedding, group fairness, demographic parity, graph neural network

1 INTRODUCTION

The rapid development and widespread application of machine learning (ML) models in the day-to-day life of individuals highlights the pressing need to incorporate fairness in the model formulation and development. An increasing number of such ML models operate on graph data. Of these ML models, graph neural networks (GNNs) [13] have been shown to be effective on several machine learning tasks on graphs such as node classification [21], link prediction [8], and graph visualization [26]. Due to their effectiveness, GNNs have been utilized in various applications such as recommendation engines [28], drug discovery [29], and predicting social network relations [14].

Given the prevalence of GNNs in many domains, it becomes important to study and rectify the biases these models learn from the implicit or explicit biases present in the input graph. Recently, many studies have shown that standard GNNs learn node representations that perform poorly on fairness measures such as dyadic fairness [17] and counterfactual fairness [1]. In our experiments, we also observe that GNNs learn unfair node representations under the context of fair link recommendation. We present a teaser of this experimental result in Figure 1, where we see that as the number of training epochs of for a standard GNN autoencoder increases, the utility measure decreases (left) while the unfairness measure increases (right). The utility measure and the unfairness measure here are captured through reconstruction and link divergence loss, respectively (explained later). Due to the presence of such problems, it becomes critical to develop fair models that are unbiased and agnostic to sensitive attributes within the data.

To address the bias in graph learning models, several models have been proposed [1, 3, 11, 17]. These fair graph learning models can be categorized into three groups [16] – pre-processing, in-processing, and post-processing – depending on where fairness constraints are incorporated in the model training pipeline. Pre-processing models incorporate fairness by modifying the graph structure (such as [17]), in-processing algorithms incorporate specific fairness constraints into the training goal of the model ([20]), and post-processing algorithms debias the output of
Moreover, diverse measures of fairness can be incorporated into GNN models. Given the proliferation of graph-based recommendation models [28], we focus on demographic parity [5] as a fairness measure. Informally, demographic parity seeks to ensure that each group with a particular sensitive attribute receives the positive outcome at the same rate as other groups with different values for the same sensitive attribute [5]. For example, a professional networking website can utilize a GNN-based recommendation engine to recommend a job opening to certain number of individuals. Demographic parity based recommendations could ensure that no individuals of a particular race, ethnicity, or gender are less likely to receive such a recommendation.

We incorporate demographic parity in GNN-based recommendation models in an model-agnostic manner by proposing a Fair Graph Modification (FairMod) methodology. FairMod consists of a series of graph modification methods that aim to modify the graph structure to better facilitate the learning of fair graph node embeddings for downstream tasks of link prediction and recommendation. FairMod is a collection of three methods, each of which perform either microscopic or macroscopic edits to the input graph. The Global Fairness Optimization (GFO) method introduces a fairness-oriented bias to every node in the dataset, which is optimized for fairness separately from the link prediction task. The Community Fairness Optimization (CFO) method introduces a set number of new nodes connected to every other node in the graph, which are similarly optimized for fairness without regard to the link prediction task. Finally, the Fair Edge Weighting (FEW) method introduces edge weights to existing edges in the graph, and optimizes the edge weights to debias the input graph.

We demonstrate the three methods on four real world networks, training graph neural networks for the link prediction task while simultaneously optimizing Link Divergence to balance the impact of different populations on each node and satisfy the fairness constraints. We find that FairMod is competitive on the link prediction task when compared to baseline graph learning methods while also enhancing demographic parity on the link recommendation task.

2 RELATED WORK

Graph Representation Learning. Various methods for graph representation learning for node embeddings have been proposed, such as Node2Vec [8], DeepWalk [21], matrix factorization approaches [7], and graph neural network (GNN)
approaches [13]. Methods such as Node2Vec and DeepWalk construct embeddings using techniques inspired by word embeddings in natural language processing; these methods first generate random walks throughout the graph structure, forming sequences of nodes. These sequences are treated as “sentences”, with the nodes as “words” in the sentence, allowing for the use of traditional word embedding algorithms to learn node embeddings. In contrast, matrix embedding approaches operate on the adjacency matrix of the graph $A$, hoping to learn a set of embeddings $\Phi$ such that $\Phi \Phi^T = A$.

However, when applied to attributed graphs, these methods are generally inferior to graph convolution networks. GCNs utilize the adjacency matrix $A$ and a layer input $H$ (usually the feature matrix $F$ in the first layer of a GCN or the output of the previous layer $H^{(\ell)}$) to learn node embeddings. Node embeddings $\Phi = H^{(\ell+1)}$ are extracted from the final layer in the GCN, and are learned by directly optimizing a weight matrix $W$ for the task such that

$$H^{(\ell+1)} = \sigma(D^{-\frac{1}{2}}AD^{-\frac{1}{2}}H^{(\ell)}W),$$

where $\sigma$ is the sigmoid function (or some other nonlinear activation function, often ReLU) and $D = \sum_j A_{jj}$ is a normalization vector for $A$. These methods have the added benefit of utilizing message-passing and semi-supervised learning in the embedding process, strengthening their performance compared to other methods on graph representation learning tasks. We will focus on GCNs as the primary embedding method for this paper.

**Fairness in Machine Learning.** Recent work in machine learning has demonstrated the capability of ML models to learn implicit biases present in the data, such as systemic racism in criminal recidivism models [2], facial recognition technology [4], and gender biases in machine translation [25]. As the usage of ML models across domains becomes increasingly common, it remains critical that researchers consider the fairness of their models.

Several formulations of fairness have been proposed in the ML community [18], [5]. The first and most basic formulation is to ignore sensitive attributes with attribute unaware fairness: if a model cannot use a sensitive attribute in its prediction, the model is fair [5]. However, this formulation is flawed due to correlations that may exist between sensitive attributes and other features in the dataset, such as zip-code when determining credit approval; due to systemic racism and historic segregation, some zip-codes are more strongly associated with specific races than others, which results in zip-code becoming a proxy for race.

This leads to the formulations of demographic parity and equalized odds [5], which propose specific constraints on the performance of a model. Demographic parity requires that members of different protected classes appear in the positive class at the same rate; the distribution of protected attributes of members in the positive class should match the population distribution. On the other hand, equalized odds is less focused on the model outcome, but rather on model performance; the true positive rates should be equal across protected attributes. This guarantees that a model achieves similar performance across protected attributes. In contrast to the prior definitions of fairness, the individual fairness [5] definition does not depend on sensitive attributes, but rather the similarity of members. Individual fairness dictates that similar individuals should have similar outcomes in the model.

**Fairness in Graph Embeddings.** The analysis of fairness in graph mining has received much attention in recent years. Bose et al. [3] propose an adversarial method to ensure graph embeddings do not contain information that can be used to discern an individual’s protected class. Rahman et al. [23] implement the FairWalk algorithm, which improves upon random walk algorithms by more fairly traversing the graph structure based upon the sensitive attributes of nodes.

More closely related work to our own includes InFoRM [11], which recognizes three approaches to implementing individual fairness constraints: debiasing the input graph, debiasing the mining model, and debiasing the mining result. To debias the input graph, Kang et al. construct an algorithm to optimize the adjacency matrix of graph data to improve
individual fairness. Comparatively, the recent work of Li et al. [17] proposes the FairAdj algorithm for group fairness metrics, which attempts to learn a fair adjacency matrix while preserving predictive accuracy for a dyadic link prediction task under structural constraints. Similarly, FairDrop [24] randomly drops edges in the graph while improving fairness based on dyadic attributes, also modifying the adjacency matrix.

Much of the previous work on debiasing the input graph focuses purely on modifying the existing adjacency matrix, in contrast to our work, which additionally considers debiasing the input graph with the addition of artificial nodes and edges. Not only is this approach unique, but it also allows for new information to be added to the network, contrasting approaches centered around optimizing the adjacency matrix. Additionally, there is opportunity to learn more about the features of the network through the analysis of added nodes: by understanding how the added information debias the input graph, we can better understand biases present in the original graph.

3 PROBLEM STATEMENT

Our formulations consider an undirected graph $G = (V, E)$ with vertex set $V$ containing $n$ nodes and edge set $E \subseteq V \times V$. We notate the self-connected adjacency matrix $A^{n\times n}$, the degree matrix $D^{n\times n}$ and the normalized adjacency matrix $\bar{A} = D^{-\frac{1}{2}} AD^{-\frac{1}{2}}$. The node feature matrix is written as $F^{n\times m}$, where $m$ is the dimensionality of the feature set, and the one-hot encoded sensitive attribute matrix as $S^{n\times k}$ where $k$ is the number of possible values the sensitive attribute can take on (note that in the current formulation, only a single sensitive attribute with $k$ possible values is considered). The GNN weight matrix for layer $i$ is notated $W^{h_{i-1}\times l_i}$, where $l_i$ is the hidden layer size and $l_{i-1}$ is the size of the previous layer. The embedding matrix $\Phi^{n\times d}$ is equal to the output of the last layer in the GNN, where the embedding dimension $d$ is equal to the hidden layer size of the final layer. We further represent a generic GNN architecture as $\mathcal{G}$.

Definition 1 (Graph Representation Learning). Given a graph $G = (V, E)$ with feature matrix $F$ and a dimensionality $d \ll |V|$, graph representation learning aims to learn a function $h_G : V \rightarrow \mathbb{R}^d$ such that $\Phi = h_G(V)$ is a matrix of $d$-dimensional vector representations for nodes in the graph such that the similarity among nodes in the graph space is approximated by similarity between nodes in the embedding space.

The learned node embeddings can be used as latent features in various downstream tasks. In this work, we focus on the fair link prediction task. We select demographic parity [5] as our fairness criteria. Informally, demographic parity is satisfied if the output of the model is not dependent on a given sensitive attribute [18]. Formally, we define demographic parity fairness criteria on the link prediction problem as follows.

Definition 2 (Link Prediction with Demographic Parity Fairness). Given a graph $G = (V, E)$, a node $v$, and an embedding model $M$, let $L_M(v) = (u_1, ..., u_n)$ be the set of nodes that have the highest likelihood to form a link with node $v$ where $L_M(v)$ is computed using the model $M$. The link prediction problem with demographic parity fairness for node $v$ has the following constraint: $D(P_{L_M(v)}, P_S) = 0$ where $D$ is the distance metric between distributions, $P_{L_M(v)}$ is the distribution of sensitive attributes over the recommended nodes set $L_M(v)$ and $P_S$ is the distribution of sensitive attributes on the overall graph.

The link prediction problem with demographic parity fairness states that the distribution of sensitive attributes over recommended nodes ($L_M(v)$) should not be distinguishable from the distribution of sensitive attributes on the overall graph ($P_S$). Note that the model can perform link recommendation by performing either K-nearest neighbors or by using a classifier.
Problem Statement: We want to learn a graph neural network (GNN) model $M$ such that $M$ performs well on the link prediction task while satisfying the demographic parity fairness constraint (defined in Definition 2).

4 METHODOLOGY

To address the problem defined in Section 3, we train our proposed models with two objective functions. In the first objective function, we optimize the model for utility (better performance on the link prediction task), while in the second objective function, we optimize the model for fairness (demographic parity based recommendations). Our objective functions are described as follows:

Utility objective. We use matrix reconstruction loss as our utility objective function due to its effectiveness in link prediction [14] and recommendation systems [19, 22]. Specifically, following Kipf et al. [14], we use the sigmoid of the dot product between node embeddings to reconstruct the matrix. This reconstruction task acts as a proxy for the link prediction task. We minimize the difference between the reconstructed and original adjacency matrix by optimizing the node embeddings.

Mathematically, our utility objective is described as follows:

$$L_R(\Phi) = W_{pos} \circ H(A, \sigma(\Phi \Phi^T)),$$

where $H$ is the binary cross entropy function and $W_{pos}$ is an element-wise weighting term, defined as

$$W_{pos} = (A \cdot \|V\|^2 - 2\|E\| + (1 - A)).$$

$W_{pos}$ balances positive and negative edges in the graph, placing more weight on positive edges when the graph is sparse, and less weight on positive edges when it is highly connected.

Fairness objective. In order to achieve demographic parity for the link prediction task, we require each group present in the sensitive attribute set to receive the positive outcome at the same rate. In other words, the distribution of sensitive attributes in the positive outcome should match the population distribution of sensitive attributes. In this case, we define a positive outcome as a positive prediction of a link between nodes: $\sigma(\Phi_v \Phi_u^T) > 0.5$ indicates a positive outcome for node $u$ given node $v$. We first define a function $f(v)$ to evaluate the distribution of similarity scores for sensitive attributes in relation to the node $v \in V$. Specifically, $f(v)$ computes the total similarity score for all nodes $u \in V \setminus \{v\}$ grouped by a specific sensitive attribute value $S_u$ divided by the total number of nodes in that group. Based on this insight, we propose the link divergence loss function $L_D$ to measure the sum of KL-divergences between the population distribution $P_S$ of the sensitive attributes and $f(v)$ for each node:

$$L_D(\Phi) = \sum_{v \in V} D_{KL}(P_S \parallel f(v)),$$

where $D_{KL}$ is the KL-divergence function and

$$f(v) = \frac{1}{\|V\| - 1} \sum_{u \in V \setminus \{v\}} \sigma(\Phi_v \Phi_u^T) \cdot S_u,$$

where $S_u$ is a one-hot encoded vector representing the sensitive attribute of node $u$. Note that $f(v)$ is normalized before the KL-divergence is calculated.
Algorithm 1: Joint optimization procedure.

**Input:** node features $F$, normalized adjacency matrix $\hat{A}$, sensitive attribute matrix $S$, GNN utility weights $W$, GNN fairness modification weights $W_F$, learning rate $\eta$, and fairness weight $\lambda_f$.

1: while $W$ or $W_F$ has not converged do
2: Compute $L_R$ according to Eqn. 1
3: Set $g_W \leftarrow \nabla_W L_R$
4: Set $W \leftarrow W - \eta \cdot \text{Adam}(W, g_W)$
5: Compute $L_D$ according to Eqn. 3
6: Set $g_W_F \leftarrow \lambda_f \cdot \nabla_W_F L_D$
7: Set $W_F \leftarrow W_F - \eta \cdot \text{Adam}(W_F, g_W_F)$
8: end while

**Output:** Fair node embeddings $\Phi \leftarrow \text{GNN}_{W,W_F}(F, \hat{A})$

Optimization Process: In our formulations, we want to constrain optimization of the reconstruction loss and link divergence to separate weight terms. To do this, each formulation constructs two optimization problems dependent on separate weight terms which are solved using a joint optimization. The reconstruction loss weights are only changed according to the gradient of the reconstruction loss and similarly the link divergence weights are changed only according to the gradient of link divergence.

Additionally, we introduce a hyper-parameter $\lambda_f \in (0, \infty)$ to weigh the impact of the fairness loss in the above optimization process. By scaling the gradients of the link divergence by a factor of $\lambda_f$, we retain a level of control over the impact of the fairness optimizations on the model. Naturally, due to inherent biases present in data, many models suffer performance losses when fairness is highly weighted [15]. As such, $\lambda_f$ allows one to balance the importance of fairness and utility in model application. Larger values of $\lambda_f$ will scale up the gradients of the link divergence loss, placing a larger weight on the fairness optimization during training, while lower values of $\lambda_f$ will let the optimization place more emphasis on the reconstruction loss. This optimization process is outlined in algorithm 1.

4.1 Global Fairness Optimization (GFO)

We first consider a graph modification resulting from the introduction of a new partner node for each node in the graph, which is only connected to the node it is partnered with. The partner node $v^*$ is responsible for balancing out biases present in the original node $v$.

We introduce a new set of $n$ nodes $V^*$ with accompanying $n$ edges $E^* = \{(v_i, v_i^*)\}_{i=1}^n$ each with edge weight 1 and features $F^*$ initialized from a Glorot normal distribution [6]. We add $V^*$ and $E^*$ to the original graph $G$ to construct a modified graph $\hat{G} = (V \cup V^*, E \cup E^*)$ with feature matrix $\hat{F} = \begin{bmatrix} F \\ F^* \end{bmatrix}$ and adjacency matrix $\hat{A} = \begin{bmatrix} \hat{A} & A^* \\ A^* & 0 \end{bmatrix}$, where $A^*$ is the $n \times n$ matrix with edge weights on the diagonal and 0 elsewhere.

Applying the graph convolution operation on the modified graph $\hat{G}$, we obtain node embeddings

$$\Phi = \begin{bmatrix} \Phi \\ \Phi^* \end{bmatrix} = \hat{G}(\sigma(\hat{A}\hat{F}W)),$$

where $\sigma$ is a nonlinear activation and $\hat{G}$ represents subsequent GNN layers. Since we do not need to learn resulting node embeddings for the introduced artificial nodes, we can simplify Equation 5 to only calculate embeddings for nodes
We can further simplify the equation by distributing the edge weights of $\mathbf{A}^*$ with glorot normal initialization \cite{glorot2010understanding}. We add $\mathbf{W}$ weights (weights for the fairness objective): matrix of edge weights.

When $\mathbf{A}$ process in the CFO formulation. Hence, the matrices did in Equation 7. As a result, one needs to maintain the inherent constraints (rank deficiency) during the optimization process in the CFO formulation. Hence, the matrices $\mathbf{A}^*$ and $\mathbf{F}^*$ must be optimized separately to ensure $\mathbf{A}^* \mathbf{F}^*$ cannot achieve full rank. This results in the following optimization problems, dependent on $\mathbf{G}$, $\mathbf{W}$, $\mathbf{A}^*$, and $\mathbf{F}^*$:

$$
\min_{\mathbf{G}, \mathbf{W}} (L_R(\Phi)) \text{ and } \min_{\mathbf{W}_f} (L_D(\Phi))
$$

4.2 Community Fairness Optimization (CFO)

Next, we consider a generalization of the GFO approach. Here, instead of introducing a partner node for every node in the graph, we introduce a set of $c$ new nodes. The onus to correct the bias on the input graph falls on these $c$ new nodes. Each node in the graph is connected to these newly added nodes. We use the notation CFO to represent the CFO method using $c$ additional nodes.

We introduce $V^*$ with $c$ nodes, $E^* = \{(v_i, v'_j)\}_{i,j=1}^c$ with $n \cdot c$ edges, and features $\mathbf{F}^*$. The features $\mathbf{F}^*$ are initialized with glorot normal initialization \cite{glorot2010understanding}. We add $V^*$ and $E^*$ to the original graph $\mathbf{G}$ to construct a modified graph $\tilde{\mathbf{G}} = (V \cup V^*, E \cup E^*)$ with feature matrix $\tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{F} \\ \mathbf{F}^* \end{bmatrix}$ and adjacency matrix $\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{A}^* \\ \mathbf{A}^\top & 0 \end{bmatrix}$, where $\mathbf{A}^*$ is the $n \times c$ matrix of edge weights.

Following the same formulation of the graph convolution operation as the GFO method, we obtain the following output for the first GNN layer for CFO:

$$
\Phi = \mathcal{G}(\sigma((\tilde{\mathbf{A}} \mathbf{F} + \mathbf{A}^* \mathbf{F}^*) \mathbf{W})).
$$

There are now two separate formulations that can take place depending on the number of added nodes $c$. If $c$ is less than both the number of original nodes $n$ and the number of features $m$, the $n \times m$ matrix $\mathbf{A}^* \mathbf{F}^*$ is guaranteed to be rank deficient:

$$
\text{rank}(\mathbf{A}^* \mathbf{F}^*) \leq \min(\text{rank}(\mathbf{A}^*), \text{rank}(\mathbf{F}^*))
\leq \min(\min(n, c), \min(c, m))
= c < \min(n, m).
$$

Due to the rank deficiency of $\mathbf{A}^* \mathbf{F}^*$, we cannot generalize the product $(\mathbf{A}^* \mathbf{F}^*)$ into a single weight matrix $(\mathbf{W}_f)$ as we did in Equation 7. As a result, one needs to maintain the inherent constraints (rank deficiency) during the optimization process in the CFO formulation. Hence, the matrices $\mathbf{A}^*$ and $\mathbf{F}^*$ must be optimized separately to ensure $\mathbf{A}^* \mathbf{F}^*$ cannot achieve full rank. This results in the following optimization problems, dependent on $\mathbf{G}$, $\mathbf{W}$, $\mathbf{A}^*$, and $\mathbf{F}^*$:

$$
\min_{\mathbf{G}, \mathbf{W}} (L_R(\Phi)) \text{ and } \min_{\mathbf{A}^*, \mathbf{F}^*} (L_D(\Phi))
$$

In contrast, we can consider the case where $c$ is greater than or equal to either the number of original nodes $n$ or the number of features $m$. When $c \geq n$ or $c \geq m$, the rank of the matrix product $\mathbf{A}^* \mathbf{F}^*$ is limited by $\min(n, m)$. Because this
formulation is unconstrained by the value of \( c \), we can simplify Equation 9 to match the GFO solution (Equation 7) by introducing the same weight matrix, \( W_f = A^* F^* \). Thus, we observe that the GFO formulation is a special case of CFO.

### 4.3 Fair Edge Weighting (FEW)

In both GFO and CFO methods, we mitigate the bias present in the input graph by the introduction of new nodes. In the FEW method, we mitigate bias learned by the GCN by editing the existing edge weights of the graph. The edge weights in a graph act as a weighting function when learning the node embedding for a node \( v \) from its neighbors \( u \in \mathcal{N}(v) \). The edge weight for edge \((u, v)\) determines the degree to which the features of node \( u \) contribute to the embedding of node \( v \). By scaling the edge weights in the adjacency matrix, we can place more emphasis on particular edges that could correct the bias for nodes in the graph. The onus to correct the bias present in the input graph falls on the edge weights.

We introduce a edge weight matrix, \( W_f \), to modify the existing normalized adjacency matrix \( \hat{A} \) prior to the graph convolution operation:

\[
\Phi = \mathcal{G}(\sigma((\hat{A} \circ W_f)FW)).
\]  

(11)

Because the normalized adjacency matrix \( \hat{A} \) is expected to have values of 0 for non-existent edges, the element-wise multiplication operation will only introduce weights on existing edges. This results in the following optimization problems, dependent on \( \mathcal{G}, W, \) and \( W_f \):

\[
\min_{\mathcal{G}, W} (L_R(\Phi)) \quad \text{and} \quad \min_{W_f} (L_D(\Phi))
\]  

(12)

Note that in this formulation of FEW, there are no constraints on \( W_f \), so introduced weights have a range of \((-\infty, \infty)\) and are not necessarily symmetric, allowing FEW to construct a directed graph.

### 5 EXPERIMENTS

#### 5.1 Dataset

We conduct our experiments on a set of four real-world datasets (see Table 1 for details, including sensitive attribute information). Edges in the Citeseer, Cora, and Pubmed datasets (https://linqs.soc.ucsc.edu/data) represent paper citations and edges in the Facebook-1684 ego-network dataset (https://snap.stanford.edu/data/) represent Facebook friendships. Citeseer, Cora, and Pubmed have bag-of-word feature vectors for each node while Facebook-1684 has anonymized features for each node representative of various attributes of a person’s Facebook profile.

| Dataset    | Nodes | Edges  | Features | Sensitive Attribute | Clustering Coefficient |
|------------|-------|--------|----------|---------------------|------------------------|
| Citeseer   | 3,527 | 4,732  | 3,703    | Topic (6)           | 0.2407                 |
| Cora       | 2,708 | 5,278  | 1,433    | Topic (7)           | 0.1426                 |
| Facebook   | 786   | 14,024 | 317      | Gender (2)          | 0.4757                 |
| Pubmed     | 19,717| 44,327 | 500      | Topic (3)           | 0.0602                 |

Table 1. Dataset statistics for the four experimental datasets. The number \( n \) next to the sensitive attribute label indicates how many values the sensitive attribute may take on.

#### 5.2 Experimental Setup

For each dataset, we train a basic graph convolution network (GCN), a GCN with GFO optimization, two GCNs with \( CFO_{10} \) \((c = 10)\) and \( CFO_{100} \) \((c = 100)\) optimization, and a GCN with FEW optimization. All models use a two-layer
GCN autoencoder as the neural network model. Following prior work [14], each GCN has a 32-dim hidden layer and a 16-dim embedding layer. Additionally, we compare our methods to FairWalk [23] and FairAdj [17] using similar parameters.

Link predictions models are trained using 20 randomized train-test splits. Following prior works [9] for link prediction tasks, we split the dataset into training and test data using the following procedure. We first randomly sample 20% of edges and add them to the test set. From the subgraph consisting of the remaining edges, we extract its largest connected component as the training data. Finally, we remove the nodes that are not in the training data but are present in the test set from the test set.

Training hyperparameters are chosen separately for each model based on reconstruction loss optimization. All values of the fairness weight \( \lambda \) are set to 1. Losses are optimized with an Adam optimizer [12] with \( \beta_1 \) and \( \beta_2 \) kept at the default values of 0.9 and 0.999, respectively, while the learning rate \( \alpha \) and number of training epochs are tuned according to the datasets. Hyperparameters for each dataset are listed in the Table 2. Any hyperparameters not listed are kept at their default values from the source code.

Experiments for the Pubmed dataset are run on a high-performance computer with Intel Xeon E5-2680 v4 CPUs (128GB memory) and NVIDIA Tesla P100 GPUs (16GB memory). All other datasets are run on a laptop with Intel Core i7-9750H CPUs (16GB memory) and an NVIDIA GeForce GTX 1650 GPU (4GB memory).

### 5.3 Metrics

**Loss:** We evaluate all methods based on reconstruction loss \( L_R \) and link divergence \( L_D \), as discussed previously.

**Quality:** To evaluate embedding quality on the link prediction task, we use the AUROC and F1-Score metrics following the procedures described in [9]. We train a logistic regression model as a classifier for positive/negative edges using an equal number of positive and negative edges randomly selected from the training set. The AUROC and F1-Score are then recorded using the logistic regression predictions for positive edges and an equal number of randomly selected negative edges from the test set.

**Fairness:** To evaluate performance on the demographic parity fairness task for link recommendation, we compute DP\( @k \) as follows: for each node \( u \) in graph \( G \), we utilize the learned \( d \)-dimension embedding vector to find the \( k \)-nearest nodes (denoted as \( k\text{NN}(u) \)) in the embedding space using the sigmoid of the dot-product similarity score. From the \( k \)-nearest nodes \( k\text{NN}(u) \), we calculate the distribution \( \pi(k\text{NN}(u)) \) of the observed sensitive attributes and compare to the global distribution \( P_S \) of sensitive attributes of the dataset. The metric is defined for a node \( u \) as follows:

\[
\text{DP}@k(u) = D_{KL}(P_S \parallel \pi(k\text{NN}(u)))
\]

where \( \pi(k\text{NN}(u)) \) represents the normalized distribution of sensitive attribute values in the nearest neighbors of \( u \) and \( P_S \) is the distribution of the sensitive attributes in the overall dataset. The final overall metric DP\( @k \) is the average

| Dataset  | GCN Models Learning Rate | GCN Models Epochs | FairAdj Learning Rate | FairAdj T2 | FairAdj Epochs | FairWalk Learning Rate | FairWalk Epochs |
|----------|--------------------------|-------------------|-----------------------|------------|----------------|------------------------|-----------------|
| Citeseer | 0.0001                   | 300               | 0.005                 | 10         |                | 0.01                   | 1               |
| Cora     | 0.0001                   | 300               | 0.001                 | 10         |                | 0.1                    | 1               |
| Facebook | 0.0001                   | 300               | 0.01                  | 10         |                | 0.1                    | 1               |
| Pubmed   | 0.001                    | 200               | 0.005                 | 10         |                | 0.1                    | 1               |

Table 2. Model hyperparameters for the four experimental datasets.
AUROC and F1-scores are slightly better for FairAdj compared to the GFO, CFO reverse is true for Cora, Facebook, and Pubmed, which demonstrate the fair autoencoder models scoring significantly lower reconstruction loss for all datasets, the AUROC and F1-scores do not behave similarly. For the Citeseer dataset, the state-of-the-art levels for link prediction while additionally optimizing for link divergence. While FairAdj reports a

\[ \Delta S_k \]

Ideally, this value should be as close to zero as possible. When \( S \) is a binary sensitive attribute, the DP\@\( k \) metric is

\[ \text{DP}\@\( k (u) \) for all nodes \( u \) in the dataset: \]

\[ \text{DP}\@\( k = \frac{1}{|V|} \sum_u \text{DP}\@\( k (u) \).} \quad (14) \]

5.4 Results

Results for link prediction are reported in Table 3. Across the board, our methods consistently perform near or above state-of-the-art levels for link prediction while additionally optimizing for link divergence. While FairAdj reports a lower reconstruction loss for all datasets, the AUROC and F1-scores do not behave similarly. For the Citeseer dataset, the AUROC and F1-scores are slightly better for FairAdj compared to the GFO, CFO\(_{10}\), CFO\(_{100}\) and FEW models, while the reverse is true for Cora, Facebook, and Pubmed, which demonstrate the fair autoencoder models scoring significantly
higher on AUROC, particularly on the Cora and Pubmed datasets. We further note that the GFO, CFO10, CFO100, and FEW models do not consistently perform significantly better or worse than the base model they are built on, indicating that the fairness optimizations made by the models seem to not significantly impact embedding performance.

Results for demographic parity in link recommendation with the DP@$k$ metrics are presented in Table 4. We observe that for the Citeseer dataset, the GFO method offers the greatest improvement in the DP@$k$ metrics compared to the base GCN model while maintaining similar AUROC and F1 scores. All of the fair autoencoders improve upon the base GCN method. This is similarly true for the Cora, Facebook, and Pubmed datasets, where the GFO method consistently ranks among the top models for the DP@$k$ metrics, though is arguably out-shined by the CFO100 method for the Cora dataset. We additionally note that as $k$ increases, the DP@$k$ metrics decrease for all methods, indicating that as more of the nearest nodes are considered, the distribution of sensitive attributes becomes fairer and more representative of the population. Additionally, our models perform better than Fairwalk and FairAdj for the Citeseer, Cora, and Facebook datasets, and better than FairAdj for the Pubmed dataset. Fairwalk achieves generally stronger DP@$k$ scores on the

| Dataset | Model | DP@10 ↓ | DP@20 ↓ | DP@40 ↓ |
|---------|-------|---------|---------|---------|
| Citeseer | Base   | 4.88 ± 1.79 | 3.01 ± 1.69 | 1.85 ± 1.4 |
|         | FairWalk | 6.96 ± 0.79 | 5.31 ± 0.742 | 2.9 ± 0.557 |
|         | FairAdj  | 6.54 ± 0.747 | 4.55 ± 0.762 | 2.16 ± 0.445 |
|         | GFO     | 2.3 ± 1.23 | 0.706 ± 0.506 | 0.172 ± 0.201 |
|         | CFO10   | 1.94 ± 0.791 | 0.879 ± 0.527 | 0.234 ± 0.285 |
|         | CFO100  | 2.34 ± 1.17 | 0.71 ± 0.692 | 0.271 ± 0.516 |
|         | FEW     | 4.17 ± 1.35 | 2.2 ± 1.28 | 1.17 ± 0.937 |
|         | Base    | 3.79 ± 1.06 | 1.89 ± 0.934 | 0.914 ± 0.652 |
|         | FairWalk | 7.35 ± 0.317 | 5.4 ± 0.208 | 3.37 ± 0.312 |
|         | FairAdj  | 7.09 ± 0.189 | 4.73 ± 0.175 | 2.51 ± 0.167 |
|         | GFO     | 3.47 ± 1.826 | 1.05 ± 0.348 | 0.132 ± 0.0486 |
|         | CFO10   | 3.38 ± 0.84 | 1.23 ± 0.536 | 0.239 ± 0.123 |
|         | CFO100  | 3.32 ± 0.808 | 0.96 ± 0.424 | 0.147 ± 0.0713 |
|         | FEW     | 3.81 ± 0.874 | 1.87 ± 0.618 | 0.719 ± 0.42 |
|         | Base    | 0.136 ± 0.161 | 0.0418 ± 0.0204 | 0.0226 ± 0.0102 |
|         | FairWalk | 0.239 ± 0.0764 | 0.0451 ± 0.00856 | 0.0226 ± 0.00254 |
|         | FairAdj  | 0.411 ± 0.114 | 0.0802 ± 0.0264 | 0.0329 ± 0.00335 |
|         | GFO     | 0.0661 ± 0.0484 | 0.0191 ± 0.0114 | 0.0076 ± 0.00446 |
|         | CFO10   | 0.17 ± 0.329 | 0.0226 ± 0.00984 | 0.0072 ± 0.00366 |
|         | CFO100  | 0.101 ± 0.0643 | 0.0246 ± 0.0133 | 0.00783 ± 0.00441 |
|         | FEW     | 0.217 ± 0.391 | 0.0413 ± 0.0252 | 0.0171 ± 0.00914 |
|         | Base    | 2.43 ± 1.36 | 0.64 ± 1.09 | 0.185 ± 0.0859 |
|         | FairWalk | 1.12 ± 0.0701 | 0.368 ± 0.0405 | 0.176 ± 0.0184 |
|         | FairAdj  | 5.72 ± 1.088 | 4.46 ± 1.36 | 3.3 ± 1.148 |
|         | GFO     | 1.52 ± 1.56 | 0.327 ± 0.72 | 0.196 ± 0.0851 |
|         | CFO10   | 2.61 ± 1.27 | 0.174 ± 0.0407 | 0.203 ± 0.0694 |
|         | CFO100  | 2.06 ± 1.51 | 0.182 ± 0.0522 | 0.168 ± 0.0906 |
|         | FEW     | 2.31 ± 1.38 | 0.328 ± 0.697 | 0.196 ± 0.0686 |

Table 4. Fairness in Link Recommendations: DP@$k$ metrics for all datasets. The highest performing model for each dataset and metric is bolded.
Fig. 2. Top left: Reconstruction loss of the CFO\(c\) model for varying values of \(c\). Top right: Link divergence of the CFO\(c\) model for varying values of \(c\). Bottom left: AUROC metric of the CFO\(c\) model for varying values of \(c\). Bottom right: DP@40 metric of the CFO\(c\) model for varying values of \(c\). All models were trained for 300 epochs with 5-fold cross validation on the Cora dataset. Values of \(c\) taken into consideration were 1, 2, 4, 8, 16, 30, 40, ..., 340.

The FEW method does not appear as capable of improving fairness as the GFO and CFO methods. This is further observed in Figure 1, which documents the reconstruction and link divergence losses during training for the various methods on the Pubmed dataset. As the FEW model continues training, Link Divergence asymptotically converges to a higher value than the GFO and CFO methods. In contrast, the GFO and CFO methods are better able to optimize Link Divergence in the same number of epochs. Comparing the CFO\(_{10}\) and CFO\(_{100}\) methods, we observe that the CFO\(_{100}\) method consistently achieves a lower \(L_D\) loss and generally performs better than the CFO\(_{10}\) method in regards to the fairness metrics. This comparison suggests that higher values of \(c\) (the number of added nodes in CFO) are more capable of optimizing for fairness; this claim is further supported by the GFO performance, which can be considered a bound on the performance of CFO models due to the relation between the solution spaces for CFO and GFO fair weight matrices discussed in section 4.2. To investigate this further, we run the CFO method with a wider range of values of \(c\).

5.5 Impact of the parameter \(c\) in CFO

Results for a wider variety of CFO\(c\) models are shown in Figure 2 for the Cora dataset. As \(c\) increases, we see a slight increase in reconstruction loss; however, this does not correlate with an increase in AUROC, indicating models with higher \(c\) still perform well on the link prediction task. However, higher \(c\) models generally exhibit stronger performance on the fairness-related metrics. As the number of introduced nodes \(c\) increases, the Link Divergence loss decays exponentially, indicating CFO models with higher \(c\) are more capable of optimizing for fairness. This is paralleled in the observed DP@40 metric, which similarly decreases as \(c\) increases. After \(c \approx 40\), Link Divergence and DP@40 stop.
decreasing significantly, indicating the number of introduced nodes has saturated and any further increases in \( c \) will offer minimal returns for the Cora dataset.

### 5.6 Efficiency of Approaches

We first acknowledge that the time complexity of training a base \( L \)-layer GCN model is \( O(L(|E|d + nd^2)) \) \[27\], where \(|E|\) is the number of edges, \( d \) is the size of the embedding dimension, and \( n \) is the number of nodes. In each layer, GCN performs \( O(|E|) \) convolution operations with a sparse implementation then followed by a non-linear transformation, which takes \( O(nd^2) \) time.

The time complexity of our GFO approach remains \( O(L(|E|d + nd^2)) \). In each layer, GFO introduces an additive bias term into each node’s representation after the convolution, which takes \( O(nd) \) time. Since \( n \ll |E| \), this computation does not increase the time complexity.

For the CFO approach, the time complexity is \( O(\max(|E|, cn)d + (L - 1)|E|d + Lnd^2) \). CFO adds an additional \( n \cdot c \) edges into the first layer of the graph adjacency matrix, where \( c \) is the number of additional nodes. With a sparse implementation of GCN, the additional \( cn \) convolutions in the first layer takes \( O(ncd) \) time. When \( c \) is not large, the total time complexity varies little compared to the base GCN.

Similar to the GFO approach, the FEW method does not affect the overall efficiency of the graph convolution algorithm. The FEW method introduces a weight for each edge in the input graph, which scale the adjacency matrix before the graph convolution step. This does not affect the big-\( O \) runtime, which will remain \( O(L(|E|d + nd^2)) \) for a sparse matrix implementation.

### 6 CONCLUSION

While many works have expanded the graph representation learning domain, few have considered fair graph representations and the graph modifications necessary to construct such embeddings. Existing works such as InFoRM \[11\], FairWalk \[23\], and FairAdj \[17\] focus on introducing fairness by changing the input adjacency matrix; in contrast, our work utilizes three unique graph modification methods, including methods that introduce new nodes into the graph, along with a novel loss function to address demographic parity in the graph embedding domain.

Notably, all three of our fairness methods demonstrate significant ability to improve demographic parity on the link recommendation task with negligible loss (and in some cases small gains) in link prediction performance. We show that for a set of four datasets, each formulation is able to increase graph fairness under the definition of demographic parity through the introduction of new nodes or edge weights. Additionally, our methods are separable from the base GCN architecture used in the embedding learning, allowing users to extract the graph modifications from the embedding model. In the future, we hope to demonstrate the flexibility of our formulations on a wider variety of GCN architectures, such as VGAE \[14\].

We additionally hope to investigate the types of modifications our methods learn, so that we may better understand how bias is represented and remedied in these datasets. We also plan to explore ways we might impose constraints on our fair learning methods to introduce more realistic nodes and edges; while our current methods significantly improve demographic parity, introduced nodes are free to take on whatever features will do the job, regardless of if those features could exist. Finally, we intend to construct new losses for optimization in order to utilize our graph modification techniques to improve fairness on other embedding tasks, such as node classification, or for other forms of fairness, such as equalized odds or individual fairness.
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