Stable Gravastars of Anisotropic Dark Energy

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Abstract

Dynamical models of prototype gravastars made of phantom energy are constructed, in which an infinitely thin spherical shell of a perfect fluid with the equation of state $p = (1 - \gamma)\sigma$ divides the whole spacetime into two regions, the internal region filled with a dark energy (or phantom) fluid, and the external Schwarzschild region. It is found that in some cases the models represent the “bounded excursion” stable gravastars, where the thin shell is oscillating between two finite radii, while in other cases they collapse until the formation of black holes or normal stars. In the phase space, the region for the “bounded excursion” gravastars is very small in comparison to that of black holes, but not empty, as found in our previous papers. Therefore, although the existence of gravastars can not be completely excluded from current analysis, the opposite is not possible either, that is, even if gravastars exist, they do not exclude the existence of black holes.

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I. INTRODUCTION

As alternatives to black holes, gravastars have received some attention recently [1], partially due to the tight connection between the cosmological constant and a currently accelerating universe [2], although very strict observational constraints on the existence of such stars may exist [3].

The pioneer model of gravastar was proposed by Mazur and Mottola (MM) [4], consisting of five layers: an internal core $0 < r < r_1$, described by the de Sitter universe, an intermediate thin layer of stiff fluid $r_1 < r < r_2$, an external region $r > r_2$, described by the Schwarzschild solution, and two infinitely thin shells, appearing, respectively, on the hypersurfaces $r = r_1$ and $r = r_2$. The intermediate layer is constructed in such a way that $r_1$ is inner than the de Sitter horizon, while $r_2$ is outer than the Schwarzschild horizon, eliminating the apparent horizon. Configurations with a de Sitter interior have long history which we can find, for example, in the work of Dymnikova and Galaktionov [5]. After this work, Visser and Wiltshire [6] pointed out that there are two different types of stable gravastars which are stable gravastars and “bounded excursion” gravastars. In the spherically symmetric case, the motion of the surface of the gravastar can be written in the form [6],

$$\frac{1}{2} \dot{a}^2 + V(a) = 0,$$

(1)

where $a$ denotes the radius of the star, and $\dot{a} \equiv da/d\tau$, with $\tau$ being the proper time of the surface. Depending on the properties of the potential $V(a)$, the two kinds of gravastars are defined as follows.

**Stable gravastars:** In this case, there must exist a radius $a_0$ such that

$$V(a_0) = 0, \quad V'(a_0) = 0, \quad V''(a_0) > 0,$$

(2)

where a prime denotes the ordinary differentiation with respect to the indicated argument. If and only if there exists such a radius $a_0$ for which the above conditions are satisfied, the model is said to be stable. Among other things, VW found that there are many equations of state for which the gravastar configurations are stable, while others are not [6]. Carter studied the same problem and found new equations of state for which the gravastar is stable [7], while De Benedictis et al [8] and Chirenti and Rezzolla [9] investigated the stability of the original model of Mazur and Mottola against axial-perturbations, and found that
gravastars are stable to these perturbations, too. Chirenti and Rezzolla also showed that their quasi-normal modes differ from those of black holes with the same mass, and thus can be used to discern a gravastar from a black hole.

"Bounded excursion" gravastars: As VW noticed, there is a less stringent notion of stability, the so-called “bounded excursion” models, in which there exist two radii $a_1$ and $a_2$ such that

$$V (a_1) = 0, \quad V' (a_1) \leq 0, \quad V (a_2) = 0, \quad V' (a_2) \geq 0,$$

with $V(a) < 0$ for $a \in (a_1, a_2)$, where $a_2 > a_1$.

Lately, we studied both types of gravastars [10, 11], and found that, such configurations can indeed be constructed, although the region for the formation of them is very small in comparison to that of black holes.

Based on the discussions about the gravastar picture some authors have proposed alternative models [12]. Among them, we can find a Chaplygin dark star [13], a gravastar supported by non-linear electrodynamics [14], a gravastar with continuous anisotropic pressure [15] and recently, Dzhunushaliev et al. worked on spherically symmetric configurations of a phantom scalar field and they found something like a gravastar but it was unstable [16]. In addition, Lobo [17] studied two models for a dark energy fluid. One of them describes a homogeneous energy density and the other describes an ad-hoc monotonically decreasing energy density, although both of them are with anisotropic pressure. In order to match an exterior Schwarzschild spacetime he introduced a thin shell between the interior and the exterior spacetimes.

In this paper, we generalize our previous works [10, 11] to the case where the equation of state of the infinitely thin shell is given by $p = (1 - \gamma)\sigma$ with $\gamma$ being a constant, the interior consists of a phantom energy fluid [17], while the exterior is still the Schwarzschild space. We shall first construct three-layer dynamical models, and then show both types of gravastars and black holes exist for various situations. The rest of the paper is organized as follows: In Sec. II we present the metrics of the interior and exterior spacetimes, and write down the motion of the thin shell in the form of Eq.(1). In Sec. III we show the definitions of dark and phantom energy, for the interior fluid and for the shell. In Sec. IV we discuss the formation of black holes from standard or phantom energy. In Sec. V we analyze the formation of gravastar or normal star from standard or phantom energy. In Sec. VI we study special cases where we can not have the "bounded excursion". Finally, in Sec. VII we
II. DYNAMICAL THREE-LAYER PROTOTYPE GRAVASTARS

The interior fluid is made of an anisotropic dark energy fluid with a metric given by

\[ ds^2_\Sigma = -f_1 dt^2 + f_2 dr^2 + r^2 d\Omega^2, \]  

where \( d\Omega^2 \equiv d\theta^2 + \sin^2(\theta) d\phi^2, \) and

\[ f_1 = (1 + br^2)^{1-\omega} (1 + 2br^2)^\omega, \]
\[ f_2 = \frac{1 + 2br^2}{1 + br^2}, \]  

where \( \omega \) is a constant, and its physical meaning can be seen from the following equation.

Since the mass is given by \( \bar{m}(r) = br^3/[2(1 + 2br^2)] \) then we have that \( b > 0. \) The corresponding energy density \( \rho, \) radial and tangential pressures \( p_r \) and \( p_t \) are given, respectively, by

\[ p_r = \omega \rho = \left( \frac{\omega b}{8\pi} \right) \left( \frac{3 + 2br^2}{(1 + 2br^2)^2} \right), \]
\[ p_t = -\left( \frac{b}{8\pi} \right) \left( \frac{\omega(3 + 2br^2)}{(1 + 2br^2)^2} \right) + \frac{b^2 r^2}{32\pi [(1 + 2br^2)^3(1 + br^2)]} \times \]
\[ \left\{ (1 + \omega)(3 + 2br^2) [(1 + 3\omega) + 2br^2(1 + \omega)] \right. \]
\[ -8\omega(5 + 2br^2)(1 + br^2) \right\}. \]  

The exterior spacetime is given by the Schwarzschild metric

\[ ds^2_+ = -f dv^2 + f^{-1} dr^2 + r^2 d\Omega^2, \]

where \( f = 1 - 2m/r. \) The metric of the hypersurface on the shell is given by

\[ ds^2_\Sigma = -d\tau^2 + R^2(\tau)d\Omega^2. \]

Since \( ds^2_- = ds^2_+ = ds^2_\Sigma, \) we find that \( r_\Sigma = r_\Sigma = R, \) and

\[ f_1 \dot{t}^2 - f_2 \dot{R}^2 = 1, \]
\[ f \dot{v}^2 - \frac{\dot{R}^2}{f} = 1, \]
where the dot denotes the ordinary differentiation with respect to the proper time. On the other hand, the interior and exterior normal vectors to the thin shell are given by

\[
\begin{align*}
n^-_\alpha &= (-\dot{R}, \dot{t}, 0, 0), \\
n^+_\alpha &= (-\dot{R}, \dot{v}, 0, 0).
\end{align*}
\]

Then, the interior and exterior extrinsic curvature are given by

\[
K^-_{\tau\tau} = \frac{1}{2}(1 + b R^2)^{-\omega/2} \left\{ 4(1 + b R^2)^{\omega/2} b R^2 \dot{R}^2 + 2(1 + b R^2)^{\omega/2} \dot{R}^2 - (1 + 2 b R^2)^{\omega} \sqrt{1 + b R^2 b R^2 \dot{t}^2} - (1 + 2 b R^2)^{\omega} \sqrt{1 + b R^2 \dot{t}^2} \right\} \left(2 b R^2 \omega + 2 b R^2 + 3 \omega + 1\right) - 2(1 + b R^2)^{\omega/2}(1 + 2 b R^2) \dot{R}^2 \right\} (1 + 2 b R^2)^{-2}(1 + b R^2)^{-1} b R + \ddot{R} \ddot{t} - \dot{R} \dot{t},
\]

\[
K^-_{\theta\theta} = \frac{i(1 + b R^2) R}{1 + 2 b R^2},
\]

\[
K^-_{\phi\phi} = K^-_{\theta\theta} \sin^2(\theta),
\]

\[
K^+_{\tau\tau} = \dot{\nu}(4 m^2 \dot{v}^2 - 4 m R \dot{v}^2 - 3 R^2 \dot{R}^2 + R^2 \ddot{v}^2)(2 m - R)^{-1} m R^{-3} + \ddot{R} \ddot{v} - \dot{R} \dot{v},
\]

\[
K^+_{\theta\theta} = -\dot{\nu}(2 m - R),
\]

\[
K^+_{\phi\phi} = K^+_{\theta\theta} \sin^2(\theta).
\]

Since

\[
[K_{\theta\theta}] = K^+_{\theta\theta} - K^-_{\theta\theta} = -M,
\]

where \(M\) is the mass of the shell, we find that

\[
M = \dot{\nu}(2 m - R) + \frac{i(1 + b R^2) R}{1 + 2 b R^2}.
\]

Then, substituting equations (9) and (10) into (19) we get

\[
M = -R \left(1 - \frac{2 m}{R} + \dot{R}^2\right)^{1/2} + R \left[1 + b R^2 + \dot{R}^2(1 + 2 b R^2)\right]^{1/2} (1 + 2 b R^2)^{-(\omega+1)/2}(1 + 2 b R^2)^{(\omega+2)/2},
\]

In order to keep the ideas of MM as much as possible, we consider the thin shell as consisting of a fluid with the equation of state, \(p = (1 - \gamma)\sigma\), where \(\sigma\) and \(p\) denote, respectively, the surface energy density and pressure of the shell and \(\gamma\) is a constant. Then, the equation of motion of the shell is given by

\[
\dot{M} + 8 \pi R \dot{\rho} = 4 \pi R^2 \left[T_{\alpha\beta} u^\alpha n^\beta\right] = \pi R^2 \left(T^+_{\alpha\beta} u^\alpha n^\beta - T^-_{\alpha\beta} u^\alpha n^\beta\right),
\]

where

\[
\begin{align*}
T^-_{\alpha\beta} &= \frac{1}{2}(1 + b R^2)^{-\omega/2} \left\{ 4(1 + b R^2)^{\omega/2} b R^2 \dot{R}^2 + 2(1 + b R^2)^{\omega/2} \dot{R}^2 - (1 + 2 b R^2)^{\omega} \sqrt{1 + b R^2 b R^2 \dot{t}^2} - (1 + 2 b R^2)^{\omega} \sqrt{1 + b R^2 \dot{t}^2} \right\} \left(2 b R^2 \omega + 2 b R^2 + 3 \omega + 1\right) - 2(1 + b R^2)^{\omega/2}(1 + 2 b R^2) \dot{R}^2 \right\} (1 + 2 b R^2)^{-2}(1 + b R^2)^{-1} b R + \ddot{R} \ddot{t} - \dot{R} \dot{t},
\end{align*}
\]

\[
T^+_{\alpha\beta} = \frac{i(1 + b R^2) R}{1 + 2 b R^2},
\]

\[
T_{\phi\phi} = T_{\theta\theta} \sin^2(\theta),
\]

\[
\begin{align*}
K^+_{\tau\tau} &= \dot{\nu}(4 m^2 \dot{v}^2 - 4 m R \dot{v}^2 - 3 R^2 \dot{R}^2 + R^2 \ddot{v}^2)(2 m - R)^{-1} m R^{-3} + \ddot{R} \ddot{v} - \dot{R} \dot{v},
\end{align*}
\]

\[
\begin{align*}
K^+_{\theta\theta} &= -\dot{\nu}(2 m - R),
\end{align*}
\]

\[
\begin{align*}
K^+_{\phi\phi} &= K^+_{\theta\theta} \sin^2(\theta).
\end{align*}
\]
where $u^\alpha$ is the four-velocity. Since the interior fluid is made of an anisotropic fluid and the exterior is vacuum, we get

$$\dot{M} + 8\pi R \dot{R}(1 - \gamma)\sigma = 0.$$  

(22)

Recall that $\sigma = M/(4\pi R^2)$, we find that Eq. (22) has the solution

$$M = k R^{2(\gamma - 1)},$$  

(23)

where $k$ is an integration constant. Substituting Eq. (23) into Eq. (20), and rescaling $m$, $b$ and $R$ as,

$$m \rightarrow mk^{-\frac{1}{\gamma - 1}},$$  

$$b \rightarrow bk^{-\frac{2}{\gamma - 3}},$$  

$$R \rightarrow Rk^{-\frac{1}{\gamma - 3}},$$  

(24)

we find that it can be written in the form of Eq. (11) with $a$ replaced by $R$, and

$$V(R, m, \omega, b, \gamma) = \frac{1}{2 R^2 b_2 \left[ b_2^{(\omega + 1)} - b_1^{(\omega + 1)/2} \right]^2} \left\{ b_2^{(\omega + 2)} R^{4(\gamma - 1)} b_1^{(\omega + 1)/2} - 2b_2^{(3\omega + 4)/2} R^{2(\gamma - 1)} b_1^{(\omega + 1)/4} \left[ b_2^{(-\omega)} b_1^{(\omega + 1)/2} R^2 - b_2^{-(\omega + 1)} b_1^{(\omega + 3)/2} R^2 \right] - 2b_2^{(-\omega)} b_1^{(\omega + 1)/2} mR + b_1 R^2 + b_2 R^2 + 2b_2 mR + b_2 R^{4(\gamma - 1)} \right\}^{1/2} + b_2^{(\omega + 2)} R^2 b_1^{(\omega + 1)/2} - b_2^{(2\omega + 3)} R^2 - 2b_2^{(\omega + 2)} mR b_1^{(\omega + 1)/2} + 2b_2^{(2\omega + 3)} mR + b_2^{(2\omega + 3)} R^{4(\gamma - 1)} - b_1^{(\omega + 2)} R^2 + b_2^{(\omega + 1)} b_1^{(\omega + 3)/2} R^2 \right\}.$$  

(25)

where

$$b_1 \equiv 1 + bR^2, \quad b_2 \equiv 1 + 2bR^2.$$  

(26)

Clearly, for any given constants $m$, $\omega$, $b$ and $\gamma$, equation (26) uniquely determines the collapse of the prototype gravastar. Depending on the initial value $R_0$, the collapse can form either a black hole, a gravastar, a Minkowski, or a spacetime filled with phantom fluid. In the last case, the thin shell first collapses to a finite non-zero minimal radius and then expands to infinity. To guarantee that initially the spacetime does not have any kind of horizons, cosmological or event, we must restrict $R_0$ to the range,

$$R_0 > 2m.$$  

(28)
where $R_0$ is the initial collapse radius. When $m = 0 = b$, the thin shell disappears, and the whole spacetime is Minkowski. So, in the following we shall not consider this case.

Since the potential (26) is so complicated, it is too difficult to study it analytically. Instead, in the following we shall study it numerically. Before doing so, we shall

III. CLASSIFICATIONS OF MATTER, DARK ENERGY, AND PHANTOM ENERGY FOR ANISOTROPIC FLUIDS

Recently [20], the classification of matter, dark and phantom energy for an anisotropic fluid was given in terms of the energy conditions. Such a classification is necessary for systems where anisotropy is important, and the pressure components may play very important roles and can have quite different contributions. In this paper, we will use this classification to study the collapse of the dynamical prototype gravastars, constructed in the last section. In particular, we define dark energy as a fluid which violates the strong energy condition (SEC). From the Raychaudhuri equation, we can see that such defined dark energy always exerts divergent forces on time-like or null geodesics. On the other hand, we define phantom energy as a fluid that violates at least one of the null energy conditions (NEC’s). We shall further distinguish phantom energy that satisfies the SEC from that which does not satisfy the SEC. We call the former attractive phantom energy, and the latter repulsive phantom energy. Such a classification is summarized in Table I.

For the sake of completeness, in Table II we apply it to the matter field located on the thin shell, while in Table III we combine all the results of Tables I and II, and present all the possibilities.

In order to consider the equations (4) and (6) for describing dark energy stars we must analyze carefully the ranges of the parameter $\omega$ that in fact furnish the expected fluids. It can be shown that the condition $\rho + p_r > 0$ is violated for $\omega < -1$ and fulfilled for $\omega > -1$, for any values of $R$ and $b$. The conditions $\rho + p_t > 0$ and $\rho + p_r + 2p_t > 0$ are satisfied for $\omega < -1$ and $-1/3 < \omega < 0$, for any values of $R$ and $b$. For the other intervals of $\omega$ the energy conditions depend on very complicated relations of $R$ and $b$. See figure 1. This provides an explicit example, in which the definition of dark energy must be dealt with great care. Another case was provided in a previous work [20]. Note that in the paper [17] where the solution is used for the first time to model dark energy star, the author presented a particular
TABLE I: This table summarizes the classification of the interior matter field, based on the energy conditions [19], where we assume that $\rho \geq 0$.

| Matter                        | Condition 1         | Condition 2         | Condition 3         |
|-------------------------------|---------------------|---------------------|---------------------|
| Normal Matter                 | $\rho + p_r + 2p_t \geq 0$ | $\rho + p_r \geq 0$ | $\rho + p_t \geq 0$ |
| Dark Energy                   | $\rho + p_r + 2p_t < 0$ | $\rho + p_r \geq 0$ | $\rho + p_t \geq 0$ |
| Repulsive Phantom Energy      | $\rho + p_r + 2p_t < 0$ | $\rho + p_r < 0$   | $\rho + p_t \geq 0$ |
| Repulsive Phantom Energy      | $\rho + p_r + 2p_t < 0$ | $\rho + p_r \geq 0$ | $\rho + p_t < 0$   |
| Repulsive Phantom Energy      | $\rho + p_r + 2p_t < 0$ | $\rho + p_r < 0$   | $\rho + p_t < 0$   |
| Attractive Phantom Energy     | $\rho + p_r + 2p_t \geq 0$ | $\rho + p_r < 0$   | $\rho + p_t \geq 0$ |
| Attractive Phantom Energy     | $\rho + p_r + 2p_t \geq 0$ | $\rho + p_r < 0$   | $\rho + p_t < 0$   |

TABLE II: This table summarizes the classification of matter on the thin shell, based on the energy conditions [19]. The last column indicates the particular values of the parameter $\gamma$, where we assume that $\rho \geq 0$.

| Matter                        | Condition 1         | Condition 2         | $\gamma$          |
|-------------------------------|---------------------|---------------------|-------------------|
| Normal Matter                 | $\sigma + 2p \geq 0$ | $\sigma + p \geq 0$ | -1 or 0          |
| Dark Energy                   | $\sigma + 2p < 0$   | $\sigma + p \geq 0$ | $7/4$            |
| Repulsive Phantom Energy      | $\sigma + 2p < 0$   | $\sigma + p < 0$   | 3                |
| Attractive Phantom Energy     | $\sigma + 2p \geq 0$ | $\sigma + p < 0$   | Not possible      |

case for $\omega = -0.5$. However, it is easy to see graphically that for this value of $\omega$ the weak and strong energy conditions are satisfied for any positive value of the parameter $b$ and the coordinate $R$. Thus, the corresponding solution does not represent a dark energy star as it is claimed by the author. Taking several values of $\omega$ in the intervals $-1 < \omega < -1/3$ and $\omega > 0$, we could not found any case where the interior dark energy exist.

In order to fulfill the energy condition $\sigma + 2p \geq 0$ of the shell and assuming that $p = (1 - \gamma)\sigma$ we must have $\gamma \leq 3/2$. On the other hand, in order to satisfy the condition $\sigma + p \geq 0$, we obtain $\gamma \leq 2$. Hereinafter, we will use only some particular values of the parameter $\gamma$ which are analyzed in this work. See Table II.
FIG. 1: In this figure we show the intervals of $\omega$ for which the weak and strong energy conditions are independent of the coordinate $R$ and the parameter $b$. The condition $\rho + p_r > 0$ is violated for $\omega < -1$ and fulfilled for $\omega > -1$, for any values of $R$ and $b$. The conditions $\rho + p_t > 0$ and $\rho + p_r + 2p_t > 0$ are satisfied for $\omega < -1$ and $-1/3 < \omega < 0$, for any values of $R$ and $b$. For the others intervals of $\omega$ the analysis of the energy conditions depends on a complex relation of $R$ and $b$.

In the next sections we will discuss three physical possibilities for the type of system that can be formed from the study of the potential $V(R, m, \omega, b, \gamma)$: (a) Black hole or dispersion of the matter, (b) Gravastar or normal star and (c) Black hole or phantom gravastar.

IV. BLACK HOLE OR DISPERSION OF THE MATTER

For $m > m_c$ the potential $V(R)$ is strictly negative as shown in figures 4, 5, 8, 9, 12, 13, 16, 17, 20, 21, 24, 25, 28, 29, 32, 33, 36, 37, 38 and 39. Then, the collapse always forms black holes. For $m = m_c$, there are two different possibilities, depending on the choice of the initial radius $R_0$. In particular, if the star begins to collapse with $R_0 > R_c$, it will approach to the minimal radius $R_c$. Once it reaches this point, the shell will stop collapsing. However, this point is unstable and any small perturbations will lead the star either to expand for ever and leave behind a flat spacetime, or to collapse until $R = 0$, whereby a Schwarzschild black hole is finally formed. On the other hand, if the star begins to collapse with $2m_c < R_0 < R_c$ as shown in these figures, the star will collapse until a black hole is formed. For $m < m_c$, the potentials $V(R)$ for each case have a positive maximal, and the equation $V(R, m < m_c) = 0$ has two positive roots $R_{1,2}$ with $R_2 > R_1 > 0$. There are also two possibilities here, depending on the choice of the initial radius $R_0$. If $R_0 > R_2$, the star will first contract to its minimal radius $R = R_2$ and then expand to infinity, whereby a Minkowski spacetime is finally formed. If $2m < R_0 < R_1$, the star will collapse continuously.
until $R = 0$, and a black hole will be finally formed.

V. GRAVASTAR OR NORMAL STAR

In this case the potential takes the shape given by figures 2, 3, 6, 7, 10, 11, 14, 15, 18, 19, 22, 23, 26, 27, 30, 31, 34 and 35, from which we can see that $V(R) = 0$ now can have one, two or three real roots, depending on the mass of the shell. For $m > m_c$ we have, say, $R_i$, where $R_{i+1} > R_i$. If we choose $R_0 > R_3$ (for $m = m_c$ we have $R_2 = R_3$), then the star will not be allowed in this region because the potential is greater than the zero. However, if we choose $R_1 < R_0 < R_2$, the collapse will bounce back and forth between $R = R_1$ and $R = R_2$. Such a possibility is shown in these figures. This is exactly the so-called ”bounded excursion” model mentioned in [6], and studied in some details in [10, 11]. Of course, in a realistic situation, the star will emit both gravitational waves and particles, and the potential shall be self-adjusted to produce a minimum at $R = R_{\text{static}}$ where $V(R = R_{\text{static}}) = 0 = V'(R = R_{\text{static}})$ whereby a gravastar or a normal star is finally formed [6, 10, 11], although in [10, 11] the potential tends to $-\infty$ when $R$ tends to $\infty$. Here it is completely different since the potential now tends to $+\infty$ when $R$ tends to $\infty$. Thus, in the cases studied here we do not have situations where the star expands leaving behind a flat spacetime, as in [10, 11].

VI. BLACK HOLE OR PHANTOM GRAVASTAR

In this case the potential takes the shape given by figures 40, 41, 42 and 43, from which we can see that $V(R) = 0$ now has four real roots, say, $R_i$, where $R_{i+1} > R_i$. If we choose $R_0 > R_4$, then again the star will not be allowed in this region because the potential is greater than zero. However, if we choose $R_3 < R_0 < R_4$, the collapse will bounce back and forth between $R = R_3$ and $R = R_4$, as in the previous case. But, if we choose $R_2 < R_0 < R_3$, we can note that this region is forbidden because either the potential is imaginary or greater than zero. However, if we choose $R_1 < R_0 < R_2$, the collapse will bounce back and forth between $R = R_1$ and $R = R_2$. If $R_0 < R_1$ the system will collapse until $R = 0$, whereby a Schwarzschild black hole is finally formed.
TABLE III: This table summarizes all possible kind of energy of the interior fluid and of the shell. The boldface figure numbers represent stable structures.

| Case | Interior Energy | Shell Energy | Figures | Structures          |
|------|-----------------|--------------|---------|---------------------|
| A    | Standard        | Standard     | 2, 3, 6, 7, 10, 11, 14, 15, 18, 19 | Normal Star |
| B    | Standard        | Dark         | 4, 8, 12, 16, 20 | Black Hole/Dispersion |
| C    | Standard        | Repulsive Phantom | 5, 9, 13, 17, 21 | Black Hole/Dispersion |
| D    | Dark            | Standard     |         | Interior not found  |
| E    | Dark            | Dark         |         | Interior not found  |
| F    | Dark            | Repulsive Phantom |         | Interior not found  |
| G    | Repulsive Phantom | Standard   | 22, 23, 26, 27, 30, 31, 34, 35 | Gravastar |
| H    | Repulsive Phantom | Dark       | 24, 28, 32, 36 | Black Hole/Dispersion |
| I    | Repulsive Phantom | Repulsive Phantom | 25, 29, 33, 37 | Black Hole/Dispersion |
| J    | Attractive Phantom | Standard | 41, 43, 40, 42 | Gravastar or Black Hole |
| K    | Attractive Phantom | Dark | 38 | Black Hole/Dispersion |
| L    | Attractive Phantom | Repulsive Phantom | 39 | Black Hole/Dispersion |

VII. CONCLUSIONS

In this paper, we have studied the problem of the stability of gravastars by constructing dynamical three-layer models of VW [6], which consists of an internal phantom fluid, a dynamical infinitely thin shell of perfect fluid with the equation of state $p = (1 - \gamma)\sigma$, and an external Schwarzschild space.

It must be noted that, although phantoms have been used widely to explain the late cosmic acceleration of the universe, microscopic models of them, either for an anisotropic or a perfect fluid, has not been constructed, yet, if there is any.

We have shown explicitly that the final output can be a black hole, a "bounded excursion" stable gravastar, a Minkowski, or a phantom spacetime, depending on the total mass $m$ of the system, the parameter $\omega$, the constant $b$, the parameter $\gamma$ and the initial position $R_0$ of the dynamical shell. All these possibilities have non-zero measurements in the phase space of $m, b, \omega, \gamma$ and $R_0$, although the region of gravastars is very small in comparison with that of black holes. All the results can be summarized in Table III. An interesting result that we
can deduce from Table III is that we can have black hole formation even with an interior phantom energy for any given $\gamma$. The results obtained in this paper further confirm our previous conclusion: *even though the existence of gravastars cannot be completely excluded in these dynamical models, our results do indicate that, even if gravastars indeed exist, they do not exclude the existence of black holes.*
FIG. 3: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 0.1$, $b = 0.000001$ and $m_c = 0.0087291$. Case A

FIG. 4: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 0.1$, $b = 0.000001$ and $m_c = 0.84374$. Case B
FIG. 5: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 0.1$, $b = 0.000001$ and $m_c = 0.51205$. Case C

FIG. 6: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 0.1$, $b = 0.000002$ and $m_c = 0.0012876042$. Case A
FIG. 7: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 0.1$, $b = 0.000002$ and $m_c = 0.0115153$. Case A

FIG. 8: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 0.1$, $b = 0.000002$ and $m_c = 0.843415$. Case B
FIG. 9: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 0.1$, $b = 0.000002$ and $m_c = 0.51211$. Case C

FIG. 10: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 0.1$, $b = 0.0001$ and $m_c = 0.0120041542$. Case A
FIG. 11: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 0.1$, $b = 0.0001$ and $m_c = 0.0546980$. Case A

FIG. 12: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 0.1$, $b = 0.0001$ and $m_c = 0.84269$. Case B
FIG. 13: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 0.1$, $b = 0.0001$ and $m_c = 0.51206$. Case C

FIG. 14: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 0.1$, $b = 0.05$ and $m_c = 0.3271379593$. Case A
FIG. 15: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 0.1$, $b = 0.05$ and $m_c = 0.4923412$. Case A

FIG. 16: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 0.1$, $b = 0.05$ and $m_c = 0.67527$. Case B
FIG. 17: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 0.1$, $b = 0.05$ and $m_c = 0.50247$. Case C

FIG. 18: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 0.2$, $b = 0.05$ and $m_c = 0.3357901746$. Case A
FIG. 19: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 0.2$, $b = 0.05$ and $m_c = 0.499554$. Case A

FIG. 20: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 0.2$, $b = 0.05$ and $m_c = 0.66938$. Case B
FIG. 21: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 0.2$, $b = 0.05$ and $m_c = 0.50195$. Case C

FIG. 22: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 1.5$, $b = 0.01$ and $m_c = 0.2097639045$. Case G
FIG. 23: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 1.5$, $b = 0.01$ and $m_c = 0.3775$. **Case G**

FIG. 24: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 1.5$, $b = 0.01$ and $m_c = 0.75153$. **Case H**
FIG. 25: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 1.5$, $b = 0.01$ and $m_c = 0.50833$. Case I

FIG. 26: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 3$, $b = 0.01$ and $m_c = 0.2552945103$. Case G
FIG. 27: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 3$, $b = 0.01$ and $m_c = 0.426756$. **Case G**

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FIG. 28: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 3$, $b = 0.01$ and $m_c = 0.72185$.  

Case H

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FIG. 29: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 3$, $b = 0.01$ and $m_c = 0.50661$. **Case I**

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FIG. 30: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 5$, $b = 0.01$ and $m_c = 0.3033168157$. Case G

FIG. 31: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 5$, $b = 0.01$ and $m_c = 0.472267$. Case G
FIG. 32: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 5$, $b = 0.01$ and $m_c = 0.68998$. Case H

FIG. 33: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 5$, $b = 0.01$ and $m_c = 0.50436$. Case I
FIG. 34: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = 10$, $b = 0.000001$ and $m_c = 0.0025477672$. Case G

FIG. 35: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = 10$, $b = 0.000001$ and $m_c = 0.018559$. Case G
FIG. 36: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = 10$, $b = 0.000001$ and $m_c = 0.84370$. Case H

FIG. 37: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = 10$, $b = 0.000001$ and $m_c = 0.51208$. Case I
FIG. 38: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = 3$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.51211$. Case K

FIG. 39: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = 7/4$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.83759$. Case L
FIG. 40: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.50574$. Case J

FIG. 41: The potential $V(R)$ and the energy conditions $EC1 \equiv \rho + p_r + 2p_t$, $EC2 \equiv \rho + p_r$ and $EC3 \equiv \rho + p_t$, for $\gamma = -1$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.163176025$. Case J
FIG. 42: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.51735$. Case J

FIG. 43: The potential $V(R)$ and the energy conditions $EC_1 \equiv \rho + p_r + 2p_t$, $EC_2 \equiv \rho + p_r$ and $EC_3 \equiv \rho + p_t$, for $\gamma = 0$, $\omega = -1.5$, $b = 0.01$ and $m_c = 0.150217075$. Case J