THE ALL-GENUS STRING EFFECTIVE ACTION

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Abstract

We use the off-shell string effective action method developed by E.S. Fradkin and A.A. Tseytlin to obtain the formula for all-genus string effective action with and without compactification at the low-energy approximation in the massless background fields. We find that for the bosonic string, one can determine the dilaton vacuum expectation value from the all-genus effective action because of the nontrivial dependence of potential energy on dilaton. For compactified four-dimensional string models, if one requires that the target-space dilaton field lie on a Kähler manifold, we obtain a constraint which will specify the worldsheet dilaton in terms of the constant background fields. We also show that under this constraint, the tree-level kähler potential

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and superpotential are not changed by the higher-genus effect. This proves again the non-renormalization theorem for a string moving in massless background fields in the low-energy approximation.
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In this paper, we use the off-shell string effective action method developed by E.S. Fradkin and A.A. Tseytlin [1] to derive the form of all-genus effective action in the background fields with and without compactification at the low-energy approximation. We will first review their derivation of the bosonic string effective action using this method. Then we will show that this method can be easily generalized to all genus case and we will derive the form of all-genus effective action for bosonic string, superstring and heterotic string with or without compactification. We find that the dependence of coupling constants on dilaton remains unchanged with a redefinition of dilaton, but the kinetic energy of dilaton is modified. For a bosonic string, from all-genus effective action one can determine the vacuum expectation value (VEV) of world-sheet dilaton field. For four-dimensional string models, we find that the requirement of target-space dilaton field on a Kähler manifold constrains the worldsheet dilaton to take a specific value determined by the constant string background fields. We also find that with a redefinition of dilaton fields, the all-genus Kähler potential and superpotential remain the same as the tree-level ones. This gives another proof of the non-renormalization theorem for effective string theory with massless background fields in the low-energy approximation.

One of the big goals of string phenomenology is to determine the low energy effective theory from string dynamics. To achieve this, one needs to study string theories in a curved background (i.e. interactive string theories). So far there are three methods of studying the interactive string theory, namely the S-matrix method [2, 3], the beta function method [4] and the string effective action method [1]. The S-matrix approach is to construct an effective field theory that yields the same scattering amplitudes as those given by the full string theory at the low-energy limit. In the beta function method, one derives the background field equations of motion and the effective lagrangian from the conformal invariance of string theory. The off-shell string effective action method, developed by E.S. Fradkin and A.A. Tseytlin, directly deduces the effective action from the path integral formulation of
interactive string theory.

The starting point of E.S. Fradkin and A.A. Tseytlin’s method is the covariant effective action

$$\Gamma[C, G_{ij}, B_{ij}...],$$

which is defined through the string path integral to be:

$$\Gamma[C, G_{ij}, B_{ij}...] \equiv \sum_{\chi} e^{\sigma \chi} \int_{M^2} Dg_{\alpha\beta} DX^i e^{-I}. \quad (1)$$

Here the path integral is over all maps \(X(z)\) of the worldsheet \(M^2\) into the spacetime manifold \(M^D\), and over all two-dimensional metrics \(g_{\alpha\beta}\) on the worldsheet, and \(\chi = (4\pi)^{-1} \int d^2 z \sqrt{g} R = 2 - 2n\) is the Euler number of \(M^2\). The constant \(\sigma\) is related to the dimensionless string theory coupling constant \(g_c = e^{-\sigma}\). Note that \(\sigma\) can be absorbed into a constant part of dilaton field \(C\) and is fixed when the vacuum value of dilaton field \(C\) is determined by the full theory. So the worldsheet dilaton field \(C\) can also be thought of as the string coupling constant. \(I\) is the string action including the coupling of string to massless string excitations (or for a string moving in the manifold with the background fields). For the closed bosonic string, in which the dilaton \(C\), graviton \(G_{ij}\) and antisymmetric tensor \(B_{ij}\) form the massless level of string spectrum, the string action including these “excited” modes is:

$$I = \int d^2 \sigma [(4\pi)^{-1} \sqrt{g} R C(X) + (4\pi \alpha')^{-1} \sqrt{g} g_{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j B_{ij}(X) + i e^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j B_{ij}(X)]. \quad (2)$$

The effective action \(\Gamma\) contains all the information about quantum string theory. It is a generating functional for the correlation functions of the background fields: \(C, G_{ij}, B_{ij},...\). From the target space point of view, \(\Gamma\) is the quantum field theory effective action of fields \(C, G_{ij}, B_{ij},...\). So the above string path integral yields a direct derivation of the effective field theory action and the effective lagrangian from string dynamics.

To compute the effective action defined above, one first extracts the integral over a “center of mass” collective coordinate by splitting \(X^i = \ldots \).
\( x^i + \eta^i \), where \( x^i = \text{constant} \),

\[
\int [dX] = \int d^D x \int [d\eta].
\]

Then one calculates the “effective action” for a generalized \( \sigma \)-model (with the “internal” space \( M^D \)) defined on a curved world-sheet,

\[
\exp(-W[g, G, B]) = \int [d\eta] \exp(-I[x + \eta, G(x + \eta), B(x + \eta), C(x + \eta), g]).
\]

And finally one averages over all possible metrics on the world-sheet and sums over all possible topologies. Considerations based on covariance and the Weyl anomaly in an interacting theory determine the general structure of \( W[g, C, G, B] \):

\[
W = \epsilon^{-1} \beta \int d^2 x \sqrt{g} R + \gamma \int d^2 z d^2 z' (R \sqrt{g}) z \partial_{zz'} (R \sqrt{g}) z',
\]

\[
\beta = 4 a_1,
\]

\[
\gamma = a_1 + a_2 \alpha'(\mathcal{R}(x) - \frac{1}{12} H_{ijk} H^{ijk} + 4 \partial_i C \partial^i C)
+ a_3 \alpha'^{2} [\mathcal{R}... (x) + ...]^2 + ...,
\]

where \( \epsilon = d - 2 \to 0 \), \( R \) is the worldsheet curvature scalar and \( \mathcal{R} \) is the target-space curvature scalar. From the one- and two-loop calculations, one finds:

\[
a_1 = \frac{D}{96 \pi},
\]

\[
\gamma = \frac{D}{96 \pi} - (\alpha'/64 \pi)(\mathcal{R} - \frac{1}{12} H_{ijk} H^{ijk} + 4 \partial_i C \partial^i C)
+ O(\alpha'^{2} \mathcal{R}^{2}).
\]

Next one integrates \( e^{-W} \) over the metrics on a closed surface \( M_x^2 \) under the coordinate gauge:

\[
g_{\mu\nu} = e^{2\sigma} \hat{g}_{\mu\nu}.
\]

here \( \hat{g}_{\mu\nu} \) has a constant curvature. One finds that the logarithm of the ghost determinant has the same structure as the above \( W \), the total \( \gamma \) is:

\[
\gamma = (96 \pi)^{-1}[D - 26 - \frac{3}{2} \alpha' (\mathcal{R} - \frac{1}{12} H_{ijk} H^{ijk} + 4 \partial_i C \partial^i C) + O(\alpha'^{2} \mathcal{R}^{2})] .
\]
The derivation so far depends only on local geometric properties, and the result is true for arbitrary genus world-sheet. At tree level ($\chi=2$) we have:

\[
\Gamma_{\text{tree}} \sim \int d^Dx e^{-2C} \int [d\rho] \times \exp \left( -\gamma \int d^2zd^2z' \times (\hat{R}\sqrt{\hat{g}} - 2\hat{\partial}^2\rho)_{z\bar{z}}\hat{\partial}_{z\bar{z}}^2(\hat{R}\sqrt{\hat{g}} - 2\hat{\partial}^2\rho)_{z\bar{z}'}. \right) \tag{7}
\]

The integration over $[\rho]$ under the fixed surface area gauge condition yields a trivial factor which does not depend on $\gamma$, we get:

\[
\Gamma_{\text{tree}} \sim \int d^Dx e^{-2C} \exp(-\gamma \int d^2zd^2z'(\hat{R}\sqrt{\hat{g}})_{z\bar{z}}\hat{\partial}_{z\bar{z}}^2(\hat{R}\sqrt{\hat{g}}_{z\bar{z}}). \tag{8}
\]

For a sphere, $\int \hat{R}\hat{\partial}^2\hat{R} = 16\pi$. For a critical string in the low energy approximation, i.e., $\alpha'$ small, one gets:

\[
\Gamma[G, C, B] = -MD \int d^Dx \sqrt{G} e^{-2S} \times \left[ 1 + \frac{1}{4} \alpha'(R + 4\partial_iC\partial^iC - \frac{1}{12} H_{ijk}H^{ijk}) + \ldots. \right] \tag{9}
\]

Here $M \sim (\alpha')^{1/2}$ is a normalization mass. Making the Weyl rescaling $G_{ij} \rightarrow G_{ij} \exp[4C/(D - 2)]$, we obtain:

\[
\Gamma[G, C, B] = \int d^Dx \sqrt{G}\{-4/k^2\alpha\exp[4C/(D - 2)] + k^{-2}[\mathcal{R} + (4/(D - 2))(\partial_iC)^2 - \frac{1}{12} H_{ijk}^2 e^{x[p[-8C/(d - 2)])}\},
\]

\[
k \sim (\alpha')^{(D-2)/4}. \tag{10}
\]

This result agrees with that deduced from the S-matrix method and the beta-function method. E.S. Fradkin and A.A. Tseytlin also extend the above string effective action approach to superstring theories. In this paper, we will assume the above method can be applied to obtain the effective action for supersymmetric and heterotic strings.

In the following, we will generalize the above 0-genus result to an all-genus one. This is possible because the derivation of (3) depends only on
local geometric properties and it is true for a world-sheet with arbitrary topology. So we can easily extend the above tree-level result perturbatively to an all-genus result. For example, for an n-genus world-sheet, the string effective action is:

\[
\Gamma^{n\text{-genus}} \sim \int d^Dx e^{-2(1-n)C} \int_{M_x} dm_j \int [d\rho] \exp \left( -\gamma \int d^2z d^2z' \times (\hat{R}\sqrt{\hat{g}} - 2\hat{\partial}^2\rho)_{z\z'} \hat{\partial}_{z\z'}^2(\hat{R}\sqrt{\hat{g}} - 2\hat{\partial}^2\rho)_{z\z'} \right)
\]

\[
\sim \int d^Dx e^{-2(1-n)C} \int_{M_x} dm_j \exp(-\gamma \int d^2z d^2z' (\hat{R}\sqrt{\hat{g}})_{z\z'} \hat{\partial}_{z\z'}^2(\hat{R}\sqrt{\hat{g}})_{z\z'}). \quad (11)
\]

Here \(m_j\) are the moduli parameterizing the n-genus worldsheet. In the low-energy approximation, these expressions can be rewritten in a form similar to the tree-level results:

\[
\Gamma^{n\text{-genus}}[G, B, C, ...] = \int d^Dx \sqrt{G} e^{(2n-2)C} \left\{ -\left(\frac{4}{k^2}\alpha \right) b_n + k^{-2} d_n \left[ -\mathcal{R} - 4(\partial_i C)^2 + \frac{1}{12} H_{i\bar{j}k} H_{ij\bar{k}} \right] + \ldots \right\}, \quad (12)
\]

with \(b_n\) and \(d_n\) defined to be:

\[
b_n = \int_{M_x} dm_j, \quad (13)
\]

\[
d_n = -(64\pi)^{-1} \int_{M_x} dm_j \int d^2z d^2z' (\hat{R}\sqrt{\hat{g}})_{z\z'} \hat{\partial}_{z\z'}^2(\hat{R}\sqrt{\hat{g}})_{z\z'}. \quad (14)
\]

We see \(b_n\) and \(d_n\) are some constants which depend only on the topology of a worldsheet. Summing over all-genus worldsheets, we get:

\[
\Gamma[G, B, C] = \int d^Dx \sqrt{G} \left\{ -\left(\frac{4}{k^2}\alpha \right) \sum_{n=0,1,...} e^{-(2n-2)C} b_n + k^{-2} \left[ -\mathcal{R} - 4(\partial_i C)^2 - \frac{1}{12} H_{i\bar{j}k} H_{ij\bar{k}} \right] \sum_{n=0,1,...} d_n e^{-(2n-2)C} \right\}, \quad (15)
\]

With the Weyl rescaling \(G_{ij} \rightarrow G_{ij} h^{\frac{-2}{(2-\mathcal{R})}}(C)\),

\[
h(C) = \sum_{n=0,1,...} d_n e^{-(2n-2)C},
\]
and

\[ f(C) = \sum_{n=0,1,...} b_n e^{-(2n-2)C}, \]

we obtain:

\[
\Gamma[G, B, C] = \int d^D x \sqrt{G} \{- (4/k^2 \alpha) f(C) h^{-\frac{D}{2}} + k^{-2}[-\mathcal{R} - (4 - \frac{D-1}{D-2} h^2(C))(\partial_i C)^2 - \frac{1}{12} h^{-\frac{8}{D-2}} H_{ijk} H^{ijk}]}. \tag{16}
\]

We see that the dilaton field is modified in the all-genus case. We express the effective action in terms of the modified dilaton \( C \rightarrow h \):

\[
\Gamma[G, B, C] = \int d^D x \sqrt{G} \{- (4/k^2 \alpha) f(C) h^{-\frac{D}{2}}(C) + k^{-2}[-\mathcal{R} - (\frac{4}{h^2(C)} - \frac{D-1}{D-2})(\partial_i h)^2 - \frac{1}{12} h^{-\frac{8}{D-2}} H_{ijk} H^{ijk}], \tag{17}
\]

with

\[ h'(C) = \frac{\partial h(C)}{\partial C}. \]

We see that the high-genus effect modifies the tree-level kinetic energy of dilaton. But the coupling constants, for example the coupling constant in front of \( H_{ijk} \) and, if gauge fields exist, the gauge coupling constant have the same dilaton dependence under the redefined dilaton field. Another interesting result from the all-genus effective action is that for bosonic string because of the nontrivial dependence of vacuum energy on the dilaton field, one can determine the worldsheet dilaton (VEV) from:

\[
\frac{\partial}{\partial C} [f(C) h^{-\frac{D}{2}}(C)] = 0 \tag{18}
\]

Next we generalize the tree-level result to an all-genus one for the supersymmetric string or heterotic string. In these cases, the above derivation still applies except now \( b_n = 0, n = 0,1,... \). This is because the integration of supermoduli over a constant yields zero. So the all-genus effective action for
the supersymmetric strings is of the form:

$$
\Gamma[C, G_{ij}, B_{ij}] \equiv \int d^Dx \mathcal{L} = \int d^Dx \mathcal{L}^{tree} \left( C, G_{ij}, B_{ij}, A_{i} \ldots \right) h(C),
$$

$$
= \int d^Dx \sqrt{G} \left\{ k^{-2} [-R - \left( \frac{4}{h^2(C)} - \frac{D-1}{D-2} \right) (\partial h)^2 \right. \\
+ \frac{1}{12} h^{-\frac{4}{D-2}} H_{ijk} H^{ijk} \bigl] + \frac{1}{4} h^{-\frac{2}{D-2}} F_{ij} F^{ij} \bigr\}. \quad (19)
$$

Here again, the coupling constants remain unchanged under the redefined dilaton but the dilaton kinetic energy is modified. Notice for superstring, the determination of dilaton VEV is not possible.

Now we proceed to calculate the all-genus effective action with some dimensions of spacetime compactified. In this case, the coordinate $X^I$ on compactified space decomposes into three parts: $X^I = x^I + \eta^I + X^I_c$, i.e. the center of mass part $x^I$, the excitation part $\eta^I$ and the zero modes from compactification $X^I_c$. So now we get:

$$
\int \mathcal{D}X^I = \int \mathcal{D}x^I \int \mathcal{D}\eta^I \int \mathcal{D}X^I_c. \quad (20)
$$

Here $\int \mathcal{D}X^I_c$ is the summation over the zero-modes coming from compactification. The effective action becomes:

$$
\Gamma[C, G_{ij}, B_{ij}] = \sum_{\chi=2,0,-2,\ldots} e^{-\sigma \chi} \int d^Dx \int \mathcal{D}'X \mathcal{D}g e^{-\frac{1}{2} \int \mathcal{D}X^I_c e^{-A_0}} \quad (21)
$$

Here we define $A_0$ to be the action of the zero-modes coming from compactification. Notice that the oscillation mode contribution is the same for the compactified and uncompactified coordinates. So the effective action with compactification will be the same as the one we derived before, except for one extra factor coming from the summation of compactification zero-modes. For n-genus worldsheets, we define this factor to be:

$$
Z_n(G_{IJ}, B_{IJ}, A_I, m_j) = \int_{M_n} \mathcal{D}X^I_c e^{-A_0}, \quad (22)
$$

with $A_0$ representing the n-genus worldsheet action of the zero-modes from compactification and the $m_j$ are the moduli parameterizing the n-genus
world-sheet. For torus or orbifold compactification, $Z_n(G_{IJ}, B_{IJ}, A_I, m_j)$ can be calculated. For example, for torus compactification, at the tree level, $Z_0(G_{IJ}, B_{IJ}, A_I, m_j)$ is trivial: 

$$Z_0(G_{IJ}, B_{IJ}, A_I, m_j) = 1/V_{ol}(\Lambda),$$

where $V_{ol}(\Lambda)$ is the volume of the compactified space. At the one-genus level, this has been calculated for heterotic string [5] to be:

$$Z_1(G_{IJ}, B_{IJ}, A_I, \tau, \bar{\tau}) = \int_{\chi=0}^\chi D X^I e^{-A^{1-loop}}$$

$$= \sum_{L \in L} \sum_{P \in \Lambda^*} \sum_{W \in spin(32)} q^p_l/2 q^{p_2}/2. \quad (23)$$

Here $q = e^{2i\pi \tau}$, $\tau$ is the modulus parameterizing the worldsheet torus and

$$p^I_{L,R} = 1/2 G^{IJ} P_J \mp L^i + G^{IK} B_{KL} L^L + 1/2 G^{IJ} A^a_J W^a + 1/4 G^{IJ} A_I A^a_J L^K. \quad (24)$$

$$p^I_L = p^I + A^I_{K} n^K. \quad (25)$$

$G_{IJ}, B_{IJ}$ and $A_I$ are the constant string background fields that parameterize string vacua. We see that for high-genus string worldsheet, $Z_n(G_{IJ}, B_{IJ}, A_I, \tau, \bar{\tau})$ has a nontrivial dependence on these constant background fields. With the all-genus result derived above, the formula for effective action with compactification is:

$$\Gamma[C, G, B, A] = \sum_{\chi=2,0,-2,\ldots} e^{-\sigma_\chi} \int d^p x \int d^2 X D g e^{-\sigma} Z_n(G_{IJ}, B_{IJ}, A_I, m_j)$$

$$= \int d^p x L^{tree}_{10} (C, G, B) n(C, G_{IJ}, B_{IJ}, A^a_I). \quad (26)$$

Now the function $h$ depends on the worldsheet dilaton $C$ and other constant string background fields $G_{IJ}, B_{IJ}$, and $A_I$. It is defined to be:

$$h(C, G_{IJ}, B_{IJ}, A^a_I) = \sum_{n=0,1,\ldots} d_n e^{(2-2n)C}, \quad (27)$$

with

$$d_n = -\frac{1}{64\pi} \int d m \int d^2 x (R \sqrt{g})_z \hat{R} \hat{g}_{22} (R \sqrt{g})_{zz}. \quad (28)$$
The four-dimensional lagrangian \( \mathcal{L} \) is specified by three functions: Kähler potential \( K \), superpotential \( W \) and \( f \) function. To derive these functions, here we use the dimensional reduction method introduced by E. Witten \[1\]. We get:

\[
\Gamma[C, G_{ij}, B_{ij}, A_i] = \int d^4x \left\{ \frac{1}{2} R^{(4)} - 3 \partial_\mu \sigma \partial^\mu \sigma + 2 \partial_\mu C \partial^\mu C \\
- \frac{9}{16h^2} \partial_\mu h \partial^\mu h - \frac{3}{2} e^{-2\sigma} h^{1/2} e^{-2\sigma} (\partial_\mu a - \frac{1}{\sqrt{2}} ik \phi_x D_\mu \phi^x)^2 \\
- \frac{3}{4} h^{1/2} e^{3\sigma} H^{\mu\rho} H_{\mu\rho} - \frac{1}{4} h^{1/2} e^{3\sigma} Tr F_{\mu\nu} F^{\mu\nu} - 3 e^{-\sigma} h^{1/2} D_\mu \phi_x D^\mu C^x \\
- \frac{8}{3} g^2 h^{3/4} e^{-5\sigma} \left| \frac{\partial W}{\partial \phi_x} \right|^2 - \frac{9}{2} \left( g^2 / f \right) h^{1/2} e^{-5\sigma} \sum_i \left( \phi^i, \lambda_i \phi \right)^2 - 8k^2 g^2 h^{3/4} e^{-6\sigma} |W|^2 \right\}. \tag{29}
\]

Here \( \sigma \) is a scalar field related to “breathing mode”, \( \phi_x \) are some charged scalar fields in a string model. The axion “D” is defined by:

\[
h^{3/4} e^{3\sigma} H^{\mu\rho} = \epsilon_{\mu\rho\sigma} \partial^\sigma D.
\]

\( W \) is the cubic “superpotential”:

\[
w = 8\sqrt{2} d_{x,y,z} \phi^x \phi^y \phi^z.
\]

Identifying the gauge coupling constant with dilaton field, we find that dilaton and moduli in four-dimension, \( S \) and \( T \), should be defined by:

\[
S = e^{3\sigma} h^{1/2} + 3i \sqrt{D}, \\
T = e^{\sigma} h^{-1/2} - i \sqrt{2} a + \bar{\phi}_x \phi^x.
\tag{30}
\]

To have a Kähler geometry, the coefficients in front of the \( \partial_\mu D \partial^\mu D \) term and \( \partial_\mu S_r \partial^\mu S_r \) term (here \( S_r \) is the real part of dilaton) should be the same. This requires that:

\[
2 \partial_\mu C \partial^\mu C - \frac{9}{16h^2} \partial_\mu h \partial^\mu h = - \frac{1}{16h^2} \partial_\mu \partial^\mu h \tag{31}
\]

or:

\[
\left[ \frac{\partial h(C, G_{IJ}, B_{IJ}, A_I)}{h \partial C} \right]^2 = 4. \tag{32}
\]

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It is interesting that the requirement of a Kähler manifold for the four-dimensional target-space dilaton field leads to the determination of the worldsheet dilaton $C$ in terms of the constant string background fields in a string model. But the breathing mode $\sigma$ still remains undetermined.

It is easy to see that, under the above constraint, the all-genus superpotential and Kähler potential remain the same as the tree-level ones. This can be viewed as another proof of string non-renormalization theorem [7, 8, 9]. Notice that from our calculation the all-genus $f$ function, which is the gauge coupling function, also remains the same as the tree-level one. But this result does not contradict the result of Kaplunovsky [10]. This is because in our derivation we take into account only massless background fields, while the string threshold correction comes from the integration over massive string modes. This is a limitation of our result.

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