Stability and collapse of a coupled Bose-Einstein condensate

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Abstract

The dynamics of a coupled Bose-Einstein condensate involving trapped atoms in two quantum states is studied using the time-dependent Gross-Pitaevskii equation including an interaction which can transform atoms from one state to the other. We find interesting oscillation of the number of atoms in each of the states. For all repulsive interactions, stable condensates are formed. When some of the atomic interactions are attractive, the possibility of collapse is studied by including an absorptive contact interaction and a quartic three-body recombination term. One or both components of the condensate may undergo collapse when one or more of the nonlinear terms are attractive in nature.

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I. INTRODUCTION

Recent experimental success [1–5] in Bose-Einstein condensation (BEC) at ultralow temperature in dilute trapped alkali metal and hydrogen atoms has intensified theoretical studies of the inhomogeneous, weakly interacting dilute Bose gas and its condensation [6–17]. At zero temperature such a system is supposed to be fully condensed and described by the mean-field Gross-Pitaevskii (GP) equation [12,18,19].

Two most interesting phenomena in BEC are the collapse for attractive interaction and the formation of coupled atomic condensate. For attractive atomic interaction [4–7], the condensate is stable for a maximum critical number of atoms. In the presence of an external source of atoms when the number increases beyond the critical number, due to interatomic attraction the radius of BEC tends to zero and the central density tends to infinity. Consequently, the condensate collapses emitting atoms until the number of atoms is reduced below the critical number and a stable configuration is reached. Then the condensate can grow again and a series of collapses can take place, which was observed experimentally in the BEC of $^7$Li with attractive interaction [4–7].

There has also been experimental realization of BEC involving atoms in two different quantum states [20–22]. In one experiment $^{87}$Rb atoms formed in the $|F = 1, m = -1\rangle$ and $|F = 2, m = 1\rangle$ states by the use of a laser served as two different species, where $F$ and $m$ are the total angular momentum and its projection [20,21]. In another experiment a coupled BEC was formed with the $^{87}$Rb atoms in the $|F = 1, m = -1\rangle$ and $|F = 2, m = 2\rangle$ states [12,22]. It is possible to use the same magnetic trap to confine atoms in two quantum states and this makes these experiments technically simpler compared to a realization of BEC with two different types of atoms requiring two independent trapping mechanisms.

Here we study theoretically the dynamics of a coupled BEC composed of two quantum states 1 and 2 of an atom using the coupled time-dependent GP equation. We include an interaction term in the Hamiltonian that allows for a coherent boson-number-conserving nondissipative transition of atoms from one quantum state to another. This interaction
simulates the action of a laser which can been used experimentally to transform one species of atoms to another. This model exhibits rich and novel phenomenology which we study using the numerical solution of the coupled GP equation, which can generate oscillation in the number of atoms in a specific state in the presence of the interaction that transforms one species of atoms to another. Also, depending on the nature of the atomic interactions one can have three possibilities of BEC: (i) stable condensate of both components for all repulsive interactions, (ii) collapse of both components for all attractive interactions, and (iii) a stable condensate for one component and collapse in the other.

The collapse for attractive interaction(s) in a coupled condensate is studied by introducing absorptive contact interactions responsible for a growth in the atomic numbers from external source in addition to imaginary three-body quartic interactions leading to recombination loss \[3,4\]. In the presence of these imaginary interactions the GP equation does not conserve the overall number of atoms. If the strengths (and signs) of these imaginary interactions are properly chosen, one can have collapse in one or both components of the condensate. The solution of the GP equation could produce a growth of the condensate with time for atom numbers less than the critical numbers for collapse. Once they increase past the critical numbers, the three-body interaction takes control and the numbers suddenly drop below the critical level by recombination loss signaling a collapse \[3,7\]. Then the absorptive term takes over and the individual numbers start to increase. This continues indefinitely showing an infinite sequence of collapses for one or both components.

We motivate this study by considering two possible atomic states 1 and 2 of \(^7\)Li (attractive interaction) and of \(^{87}\)Rb (repulsive interaction) whenever possible. In the case of \(^7\)Li the interaction in state 1 is taken to be attractive which is responsible for collapse. We study the collapse with different possibilities of attraction and repulsion between atoms in states 1 and 2 of \(^7\)Li.
II. COUPLED GROSS-PITAEVSKII EQUATION

We consider the following Hamiltonian to describe the coupled BEC allowing for the possibility of transforming atoms from one state to another [12]

\[
\hat{H} = \hat{H}_0 + \hat{V}_1 + \hat{V}_2,
\]

(1)

where the kinetic and potential energies are

\[
\hat{H}_0 = \int d\vec{r} \sum_{j=1}^{2} \hat{\psi}_j^{\dagger}(\vec{r}) \left[ -\frac{\hbar^2}{2m} \nabla_j^2 + V^{(j)}(\vec{r}) \right] \hat{\psi}_j(\vec{r}),
\]

(2)

\[
\hat{V}_1 = \sum_{l,j} \frac{g_{lj}}{2} \int d\vec{r} \hat{\psi}_l^{\dagger}(\vec{r}) \hat{\psi}_j^{\dagger}(\vec{r}) \hat{\psi}_l(\vec{r}) \hat{\psi}_j(\vec{r}),
\]

(3)

\[
\hat{V}_2 = \chi \int d\vec{r} [\hat{\psi}_1^{\dagger}(\vec{r}) \hat{\psi}_2(\vec{r}) + \hat{\psi}_2^{\dagger}(\vec{r}) \hat{\psi}_1(\vec{r})],
\]

(4)

where \(\hat{\psi}_j(\vec{r})\) and \(\hat{\psi}_j^{\dagger}(\vec{r})\) are the field operators for annihilation and creation of the bosonic atom of mass \(m\) in state \(j = 1, 2\), \(V^{(j)}(\vec{r})\) is external trapping potential, \(\hat{V}_1\) is the usual nonlinear potential between the atoms interacting via a contact interaction of strength \(g_{lj} = 4\pi\hbar^2a_{lj}/m\) between two atoms in states \(l\) and \(j\), \(a_{lj}\) is the scattering length of atoms in states \(l\) and \(j\), and \(\hat{V}_2\) is the potential which allows for the laser-induced transition of an atom from state 1 to 2 and vice versa via a contact interaction of strength \(\chi\).

Coupled mean-field GP equations are obtained by replacing the field operators \(\hat{\psi}_j(\vec{r})\) by the wave functions \(\psi_j(\vec{r})\), which are the expectation values of the field operators, in the Heisenberg equation [12]:

\[
i\hbar \frac{\partial}{\partial \tau} \hat{\psi}_j(\vec{r}, \tau) = \left[ \hat{\psi}_j(\vec{r}, \tau), \hat{H} \right],
\]

(5)

where the time (\(\tau\)) dependence is explicitly shown and the result is [23–25]

\[
\left[ -\frac{\hbar^2}{2m} \frac{1}{r} \frac{\partial^2}{\partial r^2} + \frac{1}{2} c_j m \omega^2 r^2 + \sum_{l=1}^{2} g_{jl} n_l |\psi_l(r, \tau)|^2 
- i\hbar \frac{\partial}{\partial \tau} \right] \psi_j(r, \tau) + \sum_{l=1}^{2} \chi(1 - \delta_{jl}) \psi_l(r, \tau) = 0,
\]

(6)
where $\delta_{jl}$ is the Kronecker delta and $n_l$ is the number of condensed atoms in state $l$. Here we have specialized to the radially symmetric case and have used the harmonic oscillator trapping potential $V^{(j)}_{tr}(\vec{r}) = c_j m \omega^2 r^2 / 2$ where $\omega$ is the trap frequency and $c_j$ is a parameter introduced to vary independently the trap potential in each quantum state. The normalization condition for $\chi = 0$ is given by

$$4\pi \int_0^\infty r^2 dr |\psi_j(r, \tau)|^2 = 1.$$

(7)

As in Refs. [11] it is convenient to use dimensionless variables defined by $x = \sqrt{2} r / a_{ho}$, and $t = \tau \omega$, where $a_{ho} \equiv \sqrt{\hbar / (m \omega)}$, and $\phi_j(x, t) = x \psi_j(r, \tau) (\sqrt{2} \pi a_{ho}^3)^{1/2}$. In terms of these variables Eq. (6) becomes [11]

$$\left[ - \frac{\partial^2}{\partial x^2} + \frac{c_j x^2}{4} + \sum_{l=1}^{2} n_{jl} \frac{|\phi_l(x, t)|^2}{x^2} - i \xi_j \frac{|\phi_j(x, t)|^4}{x^4} \right]$$

$$+ i \gamma_j - \frac{1}{i \frac{\partial}{\partial t}} \phi_j(x, t) + \sum_{l=1}^{2} \eta (1 - \delta_{jl}) \phi_l(x, t) = 0,$$

(8)

where $\eta = \chi / (\hbar \omega)$ and $n_{jl} \equiv 2 \sqrt{2} n_1 a_{jl} / a_{ho}$ could be negative (positive) when the corresponding interaction is attractive (repulsive). In Eqs. (8) we have introduced a diagonal absorptive $i \gamma_j$ and a quartic three-body term $-i \xi_j |\phi_j(x, t)|^4 / x^4$ appropriate to study collapse [6,7]. A nonzero $\gamma_j$ allows the possibility of the absorption of atoms from an external source into the condensate, whereas a nonzero $\xi_j$ allows for the possibility of ejection of atoms from the condensate due to three-body recombination. The total number of atoms in the condensate is not conserved for nonzero $\gamma_j$ and $\xi_j$.

The fluctuation of the number of atoms in the two states of the condensate is best studied via the quantities

$$N_j \equiv \int_0^\infty |\phi_j(x, t)|^2 dx, \quad j = 1, 2.$$

(9)

In the presence of a general absorptive interaction $\eta \neq 0, \gamma_j \neq 0$, and $\xi_j \neq 0$, the reduced number for the two components of the condensate are given by $n_{11}N_1$ and $n_{22}N_2$, the quantities $N_1$ and $N_2$ carry the information about time evolution of the number of the two components. The actual number of atoms in the two components are $n_j N_j, \quad j = 1, 2$. 

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For $\gamma_j = \xi_j = \eta = 0$, the number of atoms in the two components are conserved separately. This condition leads to $N_j = 1$. For $\gamma_j = \xi_j = 0$ and $\eta \neq 0$, the numbers of atoms in the two components are not conserved separately $N_j \neq 1$; but the total number is conserved. In the presence of absorptive interaction $\gamma_j \neq 0$ and/or $\xi_j \neq 0$, there is no conservation of even the total number of atoms.

III. NUMERICAL RESULTS

A. Computational Procedure

To solve Eqs. (8) numerically the proper boundary conditions as $x \to 0$ and $\infty$ are needed. For a confined condensate the asymptotic form of the physically acceptable solution is given by $\lim_{x \to \infty} |\phi_j(x,t)| \sim \exp(-x^2/4)$. At $x = 0$ one has the regularity condition $\phi_j(0,t) = 0$.

Next we discretize Eqs. (8) in both space and time by using a space step $h$ and time step $\Delta$ with a finite difference scheme using the unknown $(\phi_j)_p^k$ which are approximation to the exact solution $\phi_j(x_p, t_k)$ where $x_p = ph$ and $t_k = k\Delta$. After discretization we obtain a set of algebraic equations which could then be solved by using the known asymptotic boundary conditions [6,7,11]. The time derivative in these equations involves the wave functions at times $t_k$ and $t_{k+1}$. To discretize Eqs. (8) we express the wave functions and their derivatives in terms of the average over times $t_k$ and $t_{k+1}$ [26] and the result is the following Crank-Nicholson-type scheme [26]

\[
\frac{i}{\Delta} \left( (\phi_j)_p^{k+1} - (\phi_j)_p^k \right) = \frac{1}{2h^2} \left[ (\phi_j)_p^{k+1} - 2(\phi_j)_p^k + (\phi_j)_p^{k-1} \right] \\
+ \left( (\phi_j)_p^{k+1} + (\phi_j)_p^{k-1} - 2(\phi_j)_p^k + (\phi_j)_p^{k-2} \right) \\
+ \frac{1}{2} \left[ \frac{c_jx_p^2}{4} + \sum_{l=1}^2 n_{jl} \left( (\phi_l)_p^k \right)^2 \frac{x_p^2}{x_p^2} - i\xi_j \frac{|(\phi_j)_p^k|^4}{x_p^4} + i\gamma_j \right] \\
\times \left[ (\phi_j)_p^{k+1} + (\phi_j)_p^k + \sum_{l=1}^2 \eta(1 - \delta_{lj})(\phi_l)_p^k \right],
\]

where $j = 1, 2$. 

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Considering that the wave function components $\phi_j$ are known at time $t_k$, Eqs. (10) involve the unknowns $-(\phi_j)_{k+1}^p, (\phi_j)_p^{k+1}$ and $(\phi_j)_{p-1}^{k+1}$ at time $t_{k+1}$. In a lattice of $N$ points Eqs. (10) represent a tridiagonal set for $p = 2, 3, \ldots, (N-1)$ for a specific component $\phi_j$. This set has a unique solution if the wave functions at the two end points $(\phi_j)^{k+1}_1$ and $(\phi_j)^{k+1}_N$ are known [26]. These values at the end points are provided by the known asymptotic boundary conditions.

To solve Eqs. (10) we employ space step $h = 0.0001$ with $x_{\text{max}} \leq 15$ and time step $\Delta = 0.05$. After some experimentation we find that these values of the steps give good convergence. The iteration is started with the known normalized (harmonic oscillator) solution of Eqs. (8) obtained with $n_{jl} = \gamma_j = \xi_j = \eta = 0$. The nonlinear parameters $n_{jl}$ are increased by equal amounts over 1000 time iterations starting from zero at $t = -50$ until the desired final value is reached at $t = 0$. During the iteration we keep $\xi_j = \gamma_j = 0$ and attribute a small numerical value to $\eta (< 0.2)$. The resulting solution is the ground state of the condensate corresponding to the specific nonlinearity and built up with this particular value of $\eta$. Then to study the dynamics we allow the system to evolve by continuing the iteration with $\Delta = 0.05$ and $h = 0.0001$ for positive $t$, but maintaining $\eta$ and $n_{ij}$ at the constant final values attained at $t = 0$. To study the dynamics for positive $t$ for attractive atomic interaction, the absorptive contact ($\gamma_j$) and the three-body interactions ($\xi_j$) are switched on at $t = 0$. This will clearly show the possible collapse of the system.

### B. Stable Condensate: Repulsive Interaction

First we consider the stationary solution of Eqs. (8) with $\gamma_j = \xi_j = 0$ in the repulsive case. We now choose the following two sets of of parameters before the actual calculation:

- ($a$) $n_{11} = n_{22} = 10, n_{12} = n_{21} = 5, c_1 = 1, c_2 = 0.25, (a)\eta = 0.075$ and (b) $\eta = 0.1$. This will show the effect of the feedback between the two atomic states. In this case all interactions are repulsive corresponding to positive sign of all $n_{jl} \equiv 2\sqrt{2}n_{jl}/a_{\text{ho}}$.

Although the parameters above are in dimensionless units, it is easy to associate them
with a physical problem of experimental interest in the case \( \eta = 0 \). For example, for the mixture of \( |F = 1, m = -1\rangle \) and \( |F = 2, m = 1\rangle \) states of \( ^{87}\text{Rb} \), the ratio of scattering lengths \( a_{|1, -1\rangle}/a_{|2, 1\rangle} = 1.062 \) \(^{21}\). If we label state \( |1, -1\rangle \) by 1 and \( |2, 1\rangle \) by 2, and consider \( a_{11}/a_{\text{ho}} \approx a_{22}/a_{\text{ho}} \approx 0.002 \), then \( n_{11} = n_{22} = 10 \) corresponds to \( n_1 \approx n_2 \approx 1770 \) for \( \eta = 0 \). Hence these models can simulate the actual experimental situation composed of two states of \( ^{87}\text{Rb} \).

In Fig. 1 we plot the wave functions \( |\phi_j(x, 0)|/x \) for the two components calculated with two nonzero values of \( \eta \) above. In Figs. 2 (a) and (b) we plot the quantities \( N_1 \) and \( N_2 \) for these models. When \( \eta = 0 \), \( N_1 = N_2 = 1 \). As a nonzero value of \( \eta \) is taken, the quantities \( N_j \) oscillate with time as can be seen from Figs. 2 (a) and (b). When \( N_1 \) increases, \( N_2 \) decreases and this denotes a net transformation of atoms from state 2 to 1 and vice versa. These transformations lead to interesting oscillation in \( N_j \) and consequently, in the actual number of atoms in the two states. For a small \( \eta \) these oscillations are small and as \( \eta \) increases these oscillations increase in amplitude. These oscillations clearly show the continued dynamical conversion of one species of atoms to another in the otherwise stable condensate.

C. Collapse: Attractive Interaction

Now we study the simplest case of collapse by taking only the interaction between the atoms in state 1 to be attractive corresponding to a negative \( a_{11} \). All other scattering lengths \( -a_{22} \) and \( a_{12} (= a_{21}) \) – are taken to be positive. Quite expectedly, here the first component of the condensate could experience collapse. We also consider the collapse in both components when all scattering lengths are negative.

The calculation is performed in these cases with the following two sets of parameters (a) \( n_{11} = -3.0346, n_{22} = 4, n_{12} = n_{21} = 1, c_1 = 0.25, c_2 = 4, \eta = 0.2 \) and (b) \( n_{11} = -0.67, n_{22} = -1.4, n_{12} = -0.42, n_{21} = -0.41, c_1 = 4, c_2 = 0.25, \eta = 0.1 \). The wave function components at \( t = 0 \) in these cases are plotted in Fig. 3.
The above parameters are in dimensionless units and one can associate them with a problem of experimental interest for \( \eta = 0 \) in terms of two states of \(^7\text{Li}\). In case of model (a) above we consider the state 1 to be the ground state of \(^7\text{Li}\) with attractive interaction as in the actual collapse experiment with \( |a_{11}|/a_{\text{ho}} \simeq 0.0005 \) \([4,5]\). As \( n_{11} = 2\sqrt{2}n_1|a_{11}|/a_{\text{ho}} \), this corresponds to a boson number \( n_1 \simeq 2145 \). This number is larger than the maximum number of atoms permitted in the BEC of a single component \(^7\text{Li}\) which is about 1400 \([4,5]\). The state 2 could be one of the excited states of \(^7\text{Li}\) with repulsive atomic interaction. The presence of the second component with repulsive interaction allows for the formation of a stable BEC with more \(^7\text{Li}\) atoms in quantum state 1 than allowed in the single-component BEC. We find from Fig. 3 that \( \phi_1 \) (full line) is very much centrally peaked compared to \( \phi_2 \) (dashed line) in this case. This corresponds to a small radius and large central density for the first component denoting an approximation to collapse. If the number \( n_1 \) is slightly increased beyond 2200 the first component of the condensate wave function becomes singular at the origin and could experience collapse.

In case of model (b) we take the atomic interaction in both states of \(^7\text{Li}\) to be attractive corresponding to negative scattering lengths. For \( |a_{ii}|/a_{\text{ho}} \simeq 0.0005 \) as in the actual experiment \([4,5]\), one has \( n_1 = 470 \) and \( n_2 \simeq 990 \) for \( \eta = 0 \). The total number of particles in this case is roughly 1460, which is close to the critical number 1400 found in the actual experiment of collapse in \(^7\text{Li}\). Both wave-function components could become singular in this case as all possible interactions are attractive. The small extension of the wave-function components in space in this case as in Fig. 3 denote an approximation to collapse.

Although the collapse of the coupled condensates could be inferred from the shape of the stationary wave functions of Fig. 3 (sharply peaked centrally with small radii), we also study the dynamics of collapse from a time evolution of the full GP equation \([8]\) in the presence of an absorption and three-body recombination, e.g., for \( \gamma_j \neq 0 \) and \( \xi_j \neq 0 \) as in the uncoupled case \([3]\). The general nature of time evolution is independent of the actual values of \( \gamma_j \) and \( \xi_j \) employed provided that a small value for \( \xi_j \) and a relatively larger one for \( \gamma_j \) are chosen \([3]\).
First we consider model (a). For a dynamic description of this problem we take for \( t > 0 \)
\( \gamma_1 = \gamma_2 = 0.04, \xi_1 = 0.01 \) and \( \xi_2 = 0.002 \) and allow the solution of the GP equation to evolve in time using Eq. (10). In the presence of these absorptive interactions the total number of atoms is not a constant of motion. The time evolution of \( N_1 \) and \( N_2 \) is shown in Fig. 4 (a). There is small oscillation in \( N_j \) for \( t < 0 \) signifying a transfer of one type of atoms to the other. For \( t > 0 \) these oscillations are almost undetectable. Because of the attractive interaction, the component 1 could experience a succession of collapse. This is manifested in the successive growth and decay of \( N_1 \). The condensate corresponding to the second component controlled by the repulsive interaction keeps on growing, manifested by the continuous growth of \( N_2 \) with time.

Next we consider the time evolution of the GP equation for model (b) with the following absorptive parameters: \( \gamma_1 = 0.04, \gamma_2 = 0.03, \xi_1 = \xi_2 = 0.005 \). In this case as all interactions are attractive, both components can experience collapse. In Fig 4 (b) we plot the evolution of \( N_1 \) and \( N_2 \). The oscillation of \( N_1 \) and \( N_2 \) is clearly visible for \( t < 0 \). For \( t > 0 \), the absorptive interactions take control and the total number of atoms start to increase. Consequently, the dynamics of collapse is more visible than the oscillation in \( N_1 \) and \( N_2 \). Both \( N_1 \) and \( N_2 \) is found to exhibit a succession of growth and decay corresponding to collapse. Hence we find that one can have a dynamical collapse of one or both components of the BEC in the case of a coupled condensate.

IV. CONCLUSION

To conclude, we studied the collapse in a trapped BEC of atoms in states 1 and 2 using the GP equation when some of the atomic interactions are attractive. We include an interaction in the Hamiltonian which allow for a spontaneous transformation of atoms from state 1 to 2 and vice versa. Experimentally, this can be achieved by a laser and makes the present study richer. In case of all repulsive atomic interactions one can have a stable condensate of both components. When some of the atomic interactions are attractive one
or both components of the condensate could experience collapse. The collapse could be predicted from a stationary solution of the GP equation. The time evolution of collapse is studied via the time-dependent GP equation with absorption and three-body recombination. The number of particles of the component(s) of BEC experiencing collapse alternately grows and decays with time. With the possibility of a detailed study of a coupled BEC, the results of this study could be verified experimentally in the future.

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Figure Caption:

1. Wave function components $|\phi_1(x,0)|/x$ (full line) and $|\phi_2(x,0)|/x$ (dashed line) vs. $x$ for two coupled GP equations with $n_{11} = n_{22} = 10$, $n_{12} = n_{21} = 5$, $c_1 = 1$, $c_2 = 0.25$, (a) $\eta = 0.075$ and (b) $\eta = 0.1$.

2. $N_1$ (full line) and $N_2$ (dashed line) vs. $t$ for models (a) and (b) of Fig. 1.

3. Wave function components $|\phi_1(x,0)|/x$ (full line) and $|\phi_2(x,0)|/x$ (dashed line) vs. $x$ for two coupled GP equations with (a) $n_{11} = -3.0346$, $n_{22} = 4$, $n_{12} = n_{21} = 1$, $c_1 = 0.25$, $c_2 = 4$, $\eta = 0.2$ and (b) $n_{11} = -0.67$, $n_{22} = -1.4$, $n_{12} = -0.42$, $n_{21} = -0.41$, $c_1 = 4$, $c_2 = 0.25$, $\eta = 0.1$.

4. $N_1$ (full line) and $N_2$ (dashed line) vs. $t$ for models (a) and (b) of Fig. 3.
Figure 1
Figure 2 (a)

Number (Arbitrary units)

$\eta = 0.075$

$N_1$

$N_2$
Figure 2 (b)

Number (Arbitrary units)

N_1

N_2

\( \eta = 0.1 \)
Figure 3
Figure 4 (a)

Number (Arbitrary units) vs. t

- $N_1$
- $N_2$
Figure 4 (b)

![Graph showing the number over time with two distinct curves labeled \(N_1\) and \(N_2\).]