Remarks on nuclear matter: how an $\omega_0$ condensate can spike the speed of sound, and a model of $Z(3)$ baryons

Robert D. Pisarski

1Department of Physics, Brookhaven National Laboratory, Upton, NY 11973

I make two comments about nuclear matter. First, I consider the effects of a coupling between the $O(4)$ chiral field, $\vec{\phi}$, and $\omega_0$ meson, $\sim +\omega_0^2\vec{\phi}^2$; for any net baryon density, a condensate for $\omega_0$ is unavoidably generated. I assume that with increasing density, a decrease of the chiral condensate and the effective $\omega_0$ mass gives a stiff equation of state (EoS). In order to match that onto a soft EoS for quarkyonic matter, I consider an $O(N)$ field at large $N$, where at nonzero temperature quantum fluctuations disorder any putative pion “condensate” into a quantum pion liquid (QπL) [1]. In this paper I show that the QπL persists at zero temperature. If valid qualitatively at $N = 4$, the $\omega_0$ mass goes up sharply and suppresses the $\omega_0$ condensate. This could generate a spike in the speed of sound at high density, which is of relevance to neutron stars. Second, I propose a toy model of a $Z(3)$ gauge theory with three flavors of fermions, where $Z(3)$ vortices confine fermions into baryons. In $1 + 1$ dimensions this model can be studied numerically with present techniques, using either classical or quantum computers.

To determine the equation of state (EoS) for nuclear matter, effective models can be used for baryon densities, $n_B$, up to and above that for nuclear saturation, $n_{\text{sat}}$ [2–11]. Neutron stars probe densities $n_B > n_{\text{sat}}$. In the past few years astronomical observations of neutron stars with masses above two solar masses [12–33] and astronomical observations, especially of their mergers in binary systems [14–33], have provided a wealth of data. Many models apply above $n_{\text{sat}}$ [34–102], but to date, consensus is lacking.

In this paper I make two comments about nuclear matter. First, I consider the effects of a coupling between the $O(4)$ chiral field, $\vec{\phi}$, and the $\omega_0$ meson, $\sim +\omega_0^2\vec{\phi}^2$; for any net baryon density, a condensate for $\omega_0$ is unavoidably generated. I assume that with increasing density, a decrease of the chiral condensate and the effective $\omega_0$ mass gives a stiff equation of state (EoS). In order to match that onto a soft EoS for quarkyonic matter, I consider an $O(N)$ field at large $N$, where at nonzero temperature quantum fluctuations disorder any putative pion “condensate” into a quantum pion liquid (QπL) [1]. In this paper I show that the QπL persists at zero temperature. If valid qualitatively at $N = 4$, the $\omega_0$ mass goes up sharply and suppresses the $\omega_0$ condensate. This could generate a spike in the speed of sound at high density, which is of relevance to neutron stars. Second, I propose a toy model of a $Z(3)$ gauge theory with three flavors of fermions, where $Z(3)$ vortices confine fermions into baryons. In $1 + 1$ dimensions this model can be studied numerically with present techniques, using either classical or quantum computers.

To determine the equation of state (EoS) for nuclear matter, effective models can be used for baryon densities, $n_B$, up to and above that for nuclear saturation, $n_{\text{sat}}$ [2–11]. Neutron stars probe densities $n_B > n_{\text{sat}}$. In the past few years astronomical observations of neutron stars [12–53] have provided significant insight into the nuclear EoS at densities above $n_{\text{sat}}$. This includes quantities such as their mass, radius, and tidal deformability. The EoS is given by the pressure, $p$, as a function of the energy density, $\epsilon$. Analyses with piecewise polytropic EoS are useful [19,22].

However, a more sensitive probe of the EoS is given by the speed of sound squared: $c_s^2 = \partial p/\partial \epsilon$. Free, massless fermions have $c_s^2 = 1/3$, which is termed soft. In contrast, several studies of neutron stars find that it is essential for the nuclear EoS to have a region in which the EoS is stiff, where $c_s^2$ significantly larger than $1/3$ [23–33].

For example, consider the analysis of Drischler et al. [27], who extrapolate up from $n_{\text{sat}}$ using chiral effective field theory. To obtain neutron stars with masses above two solar masses, they find that there is a region of density in which the EoS is stiff: if there is a neutron star of 2.6 solar masses, at some $n_B$, $c_s^2 \sim 0.55$. To agree with small tidal deformability from GW170817, though, the EoS of nuclear matter must be soft until $n_B \sim 1.5 \sim 1.8n_{\text{sat}}$.

That is, there is a “spike” in the speed of sound, with a relatively narrow peak at a density significantly above $n_{\text{sat}}$: see, e.g., Fig. (1) of Greif et al. [24].

As the density $n_B \rightarrow \infty$, by asymptotic freedom the EoS approaches that of an ideal gas of dense quarks and gluons, and so is soft. In Quantum ChromoDynamics (QCD), corrections to the quark EoS have been computed in part up to four loop order [110–114]. However, perturbation theory is only useful down to densities much larger than $n_{\text{sat}}$. At very high densities, excitations near the Fermi surface are dominated by color superconductivity [34–35, 37].

Going down in density, nuclear matter becomes quarkyonic [49–61]. The free energy is close to that of QCD perturbation theory, but the excitations near the Fermi surface are confined, and so baryonic. A quarkyonic regime is inescapable for a $SU(N_{\text{color}})$ gauge theory as $N_{\text{color}} \rightarrow \infty$, as then quark loops are suppressed by $\sim 1/N_{\text{color}}$. This does not seem to be special to large $N_{\text{color}}$, though. In lattice gauge theory, when $N_{\text{color}} \geq 3$ the sign problem prevents classical computers from computing at zero temperature and nonzero quark density [115]. Two colors, however, is free of the sign problem, and while it has unique features - notably, since baryons are bosons there is no Fermi sea - lattice simulations find a broad quarkyonic region [116–121]. This suggests the same applies to QCD, where $N_{\text{color}} = 3$.

I assume that the quarkyonic EoS is soft. Bedaque and Steiner [122] have argued, from a variety of examples, the any quasiparticle model is soft. While some authors propose that quarks can give a stiff EoS [29–31], for simplicity I do not.

To match a nuclear onto a quarkyonic EoS, McLerran...
and Reddy take a quark EoS up to some Fermi momentum $k_{FQ}$, which is then surrounded by a baryonic shell of width $\Delta$, Fig. (1) of Ref. [55]. As the baryon density increases, $k_{FQ}$ grows, and $\Delta$ shrinks. Taking an ideal EoS for both quarks and baryons, an appropriate choice of the width $\Delta$ generates a spike in the speed of sound, Fig. (2) of Ref. [55].

At densities near $k_{FQ}$, though, neither equation of state is close to ideal. The Quantum HadroDynamics (QHD) of Serot and Walecka [3 4 6] can be used for baryons, although to be capable of modeling a confined but chirally symmetric phase, all chiral partners of the nucleons and mesons must be included in a parity-doubled QHD (PdQHD) [64–71, 76–80]. The quark EoS can be modeled by coupling quarks and gluons to a linear sigma model for mesons. Such a PdQHD was considered by Cao and Liu [79].

My purpose here is to discuss, in an entirely qualitative manner, of how an $\omega$ condensate [1, 103–109] could affect the EoS in PdQHD. My discussion is admittedly speculative, because given the wealth of experimental data, it is not easy to describe the EoS of nuclear matter both near $n_{sat}$ and at $n_B \gg n_{sat}$.

In QHD, saturation results from a balance between repulsion from the $\omega\mu$ meson and attraction from the $\sigma$ meson [3 4 6]. That the $\omega\mu$ meson could generate a stiff EoS was first noted by Zeldovich [2]. Given the coupling of the $\omega\mu$ to a nucleon $\psi$ as $\sim g_\omega \bar{\psi} \gamma^\mu \omega \mu \psi$, then at any nonzero baryon density, $\langle \bar{\psi} \gamma^0 \psi \rangle = n_B \neq 0$, a condensate for $\omega_0$ is automatically generated [129]:

$$\mathcal{L}_0 = -g_\omega n_B \omega_0 + \frac{m_\omega^2 \omega_0^2}{2} \Rightarrow \langle \omega_0 \rangle = \frac{3}{m_\omega^2} n_B.$$  (1)

If only these terms matter, then the EoS is as stiff as possible, with the speed of sound equal to that of light, $c_s^2 = 1$. Son and Stephanov showed that QCD at nonzero isospin density provides a precise example of this [124].

Of course in QHD, Eq. (1) is not the only term which matters. Integrating over nucleon loops at nonzero density, there is an infinite series of terms in $\omega_0$ which are generated at $n_B \neq 0$, including those $\sim \omega_0^2$, $\sim \omega_0^3$, and so on. Similarly, the nucleon couples to the $\sigma$, whose properties also change with $n_B$. These effects have been computed to one loop order [3 4 6], but even for strong $g_\sigma$, do not dramatically alter the EoS.

The $\omega\mu$ Lagrangian is

$$\mathcal{L}_\omega = \frac{F_{\mu\nu}^2}{4} + \frac{1}{2} \left( \tilde{m}_\omega^2 + \kappa^2 \tilde{\phi}^2 \right) \omega_\mu^2;$$  (2)

$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is the field strength for $\omega_\mu$, $\tilde{m}_\omega$ a mass term, and there is a quartic coupling $\sim \kappa^2$ between $\omega_\mu$ and $\tilde{\phi}$, where $\tilde{\phi}$ is the (4) chiral field for two light flavors, $\tilde{\phi} = (\sigma, \bar{\pi})$. The coupling $\kappa^2$ must be positive to ensure stability for large values of the $\omega_\mu$ and $\tilde{\phi}$ fields.

(Incidentally, while the term $\sim \tilde{m}_\omega^2$ can be written in gauge invariant, unitarity form [128, 130], that $\sim \kappa^2$ cannot [131].)

Although the coupling $\kappa^2$ violates vector meson dominance [91, 92, 132–134], this is relatively innocuous. For the $\rho_\mu$ meson, in vacuum a similar term $\sim \kappa^2 \rho_\mu^2$ just shifts the $\rho_\mu$ mass, and doesn’t alter its electromagnetic couplings.

In general the complete mass for the $\omega_\mu$ meson is

$$m_\omega^2 = \tilde{m}_\omega^2 + \kappa^2 \langle \tilde{\phi}^2 \rangle.$$  (3)

Spontaneous symmetry breaking occurs in the QCD vacuum, $\langle \tilde{\phi} \rangle = (\sigma_0, 0)$, where for two flavors, $\sigma_0 = f_\sigma$, the pion decay constant. Thus in vacuum, in mean field theory the mass squared of the $\omega_\mu$ meson is $m_\omega^2 = \tilde{m}_\omega^2 + \kappa^2 \sigma_0^2$. Large $\tilde{m}_\omega$ favors small $\kappa$, and vice versa. For massless pions, at tree level $\tilde{m}_\omega = 0$ when $\kappa^2 = m_\omega^2 / f_\sigma \sim 8.4$.

Couplings similar to $\kappa^2$ have appeared before. In Refs. [8] and [36], a chirally asymmetric term $\sim \omega_\mu^2 \sigma^2$ was added to the Lagrangian, but the generalization to a chirally symmetric term is obvious. Ref. [73] introduced mixing the $\omega_\mu$ and $\rho_\mu$ mesons, $\sim \omega_\mu^2 \rho_\mu^2$, and note the possibility of terms $\sim (\omega_\mu^2)^2$, etc. The implications of these terms for neutron stars were computed in Refs. [36, 72, 74, 97]. Refs. [70, 71, 76, 76] denote $\kappa^2$ as $h_1$, but neglect it, as $\kappa^2 \sim 1 / N_{\text{color}}$ is small for a large number of colors. Ref. [79] denote $\kappa^2$ as $g_{SV}$; for massive pions they find $\kappa \sim 9.0$, which seems large compared to the upper bound of $\kappa^2 \sim 8.4$ for massless pions.

As the density increases, chiral symmetry breaking becomes weaker, and $\sigma_0$ decreases. Thus Eq. (3) implies that $m_\omega$ decreases with $\sigma_0$, at least as long as $\langle \tilde{\phi}^2 \rangle = \sigma_0^2$. This is reminiscent of the scaling of Brown and Rho [38, 48].

For the $\tilde{\phi}$ Lagrangian I take [1]

$$\mathcal{L}_\phi = \frac{(\partial \tilde{\phi}^2)^2}{2M^2} + \frac{(\partial \tilde{\phi}_i^2)^2}{2} + \frac{Z(\partial \tilde{\phi}_i^2)^2}{2} + m_\phi^2 \tilde{\phi}^2 + \frac{\lambda(\tilde{\phi}^2)^2}{4},$$  (4)

and work in the chiral limit, so there is no term linear in $\tilde{\phi}$. Notice that the $\omega_\mu$ meson does not appear [62, 63, 70, 71, 76, 79]. This is because the $\omega_\mu$ corresponds to the $U(1)_B$ of baryon number, and with $q$ the quark fields, $\sim \bar{q} q$ is invariant under $U(1)_B$. Conversely, when $\tilde{\rho}_\mu$ and $\tilde{\sigma}_i$ mesons are added, they do appear in Eq. (4), since $\tilde{\phi}$ transforms non-trivially under $SU(2)_L \times SU(2)_R$. The $\omega_\mu$ meson does interact with pions through anomalous interactions generated by the Wess-Zumino-Witten Lagrangian, such as $\omega_\mu \to \sigma \pi \pi$. These interactions survive in the chirally symmetric phase, and typically become $\omega \to \sigma \pi \pi$, Eq. (15) of [63].

The anomalous interactions, though, all involve at least three derivatives, which for the $\omega_0$ meson, are all spatial derivatives. This is why the coupling $\sim \kappa^2$ in Eq. (2) is so important, as the only renormalizable, non-
derivative coupling which the $\omega_\mu$ has with the chiral field $\vec{\phi}$.

This assumes that processes which violate the axial $U(1)_A$ symmetry survive at densities far above $n_{sat}$, as indicated by an analysis using a dilute gas of instantons [11]. If at zero temperature and nonzero baryon density the axial $U(1)_A$ symmetry remains strongly broken by topologically nontrivial configurations even when the $O(4) = SU(2)_L \times SU(2)_R$ chiral symmetry is restored, then as the $\omega_\mu$, meson is a chiral singlet, it need not become degenerate with its parity partner, which is presumably the $f_1(1285)$ [75]. This is unlike mesons which carry flavor, such as the $\rho_\mu$ and $\omega_\mu$, which are degenerate in a chirally symmetric phase. Similarly, this is why I can restrict the chiral symmetry to be $O(4) = SU(2)_L \times SU(2)_R$ and not $U_A(1) \times SU(2)_L \times SU(2)_R$ [12].

Of course the $\omega_\mu$ meson interacts directly with nucleons [3, 4, 6, 8, 11, 12, 13, 14]. The CBELSA/TAPS experiment found that the mass of the $\omega_\mu$ does not shift significantly at nuclear densities [12, 13, 14], although its width is over thirty times larger than in vacuum [14]. This does not concur with QHD [3, 4, 6] nor Refs. [28, 29, 30], where the $\omega_\mu$ mass decreases by $n_{sat}$.

This does not exclude changes as the baryon density exceeds $n_{sat}$. For the usual analyses of QHD [3, 4, 6, Refs. 28, 29, 30], and PdQHD [63, 71, 75, 76], the $\sigma$ and $\omega_0$ masses both decrease, as the balance between $\sigma$ attraction and $\omega_0$ repulsion gives a soft EoS.

My principal assumption is that for some $n_B > n_1 > n_{sat}$, that one enters a region dominated by the $\omega_0$ condensate. Notably, if $Z$ decreases with increasing $n_B$, the effective mass squared of the $\sigma$ increases as $\sim 1/Z$, while if $\kappa \neq 0$, the $\omega_0$ becomes light as the chiral symmetry is restored. By Eq. (1), $\mathcal{L}_\sigma^B = -\frac{\kappa^2 n_B^2}{2m_\sigma^2}$, and a heavy $\sigma$, with a light $\omega_0$, could generate a stiff EoS for $n_B > n_1$.

Assuming that a light $\omega_0$ gives a stiff EoS, then how can the $\omega_0$ condensate evaporate to match onto a soft quarkyonic EoS? Presumably the couplings of the $\omega_\mu$ with nucleons behave smoothly with density. That leaves the couplings of the $\omega_\mu$ to the chiral field $\vec{\phi}$, but as demonstrated above, these are limited. This question does assume that a hadronic phase matches onto quarkyonic matter. It is possible to simply paste a stiff hadronic EoS onto a soft quark EoS through what is presumably a strongly first order transition. This is not consistent, however, with the analyses for either $N_{color} \to \infty$ [19, 61] or lattice results for $N_{color} = 2$ [116, 121], which indicate a quarkyonic regime. Nor why the nuclear EoS appears to be soft near $n_{sat}$, and only stiff when $n_B \sim 1.5 - 1.8 n_{sat}$ [27].

I stress that reducing the contribution of the $\omega_0$ condensate at large chemical potential, $\mu$, and low temperature, $\mu \gg T$, has no analogy to the more familiar case, at nonzero temperature and low density. When $T \gg \mu$, it is easy matching the EoS of hadronic matter, with a relatively few degrees of freedom, onto a quark-gluon plasma, with many. This is precise in the limit of a large number of colors, $N_{color} \to \infty$, where the pressure in the hadronic phase is $\sim N_{color}^0$, versus $\sim N_{color}^2$ in the deconfined phase. Similarly, the contribution of the chiral condensate is only $\sim N_{color}^1$, and decreases as $T$ increases. In contrast, at $\mu \gg T$, the pressure is always $\sim N_{color}^1$, in both the hadronic and quark-gluon phases.

At nonzero density, the appearance of a condensate for $\omega_0$ is special to the $\omega_\mu$ meson: there is no other hadron which couples directly to the net baryon density. This assumes that the only net charge is for baryon number. When there is a net isospin charge, a condensate for the $\tilde{\rho}_\mu$ meson is generated, $\sim \rho_0^3$. In this case, terms such as $\vec{\phi}^2 \rho_0^2$, amongst others [70, 71, 72, 73], need to be included; further, couplings between the $\omega_\mu$ and $\tilde{\rho}_\mu$ mesons, $\sim \omega_\mu^2 \rho_0^2$, must be added [74].

It is then very difficult to fit the EoS of an $\omega_0$ condensate onto that of cold quarks: either the coupling of the $\omega_0$ becomes small, or the mass of the $\omega_0$ becomes large. Since the coupling of the $\omega_0$ is strong in vacuum, the former is most implausible. Thus the mass of the $\omega_0$ must increase, although this does not occur in mean field theory [151]. I now argue that the mass of the $\omega_0$ increases sharply due to large quantum fluctuations.

Returning to the Lagrangian in Eq. (1), it is standard except for the term quartic in the spatial derivatives, $\sim 1/M^2$ [152]. Causality implies that only terms with two time derivatives enter. With the term $\sim 1/M^2$ to ensure stability, it is possible to allow the coefficient of the term with two spatial derivatives, $Z$, to be negative.

While in vacuum $Z = 1$ by Lorentz covariance, this is not true in a medium. If $Z$ is negative, classically a condensate is generated:

$$\vec{\phi} = \sigma_0 (\cos(k_z z), \sin(k_z z), 0, 0); \quad k_z^2 = \frac{-Z M^2}{2}, \quad (5)$$

which is a pion condensate in the $z$ direction [1, 103, 109, 153]. In 1+1 dimensions, such chiral spiral condensates are ubiquitous at low temperature and nonzero density [106, 108], although in general the solutions are more involved. Given these examples, it is natural to assume that in QCD, at low temperature $Z < 0$ for some range in density above $n_{sat}$.

Most discussions of a pion condensate use a nonlinear Lagrangian, in which the $\sigma$ meson does not explicitly appear. The advantage of using a linear Lagrangian is that it is much easier studying how the symmetric phase is approached. Following Ref. 1 I generalize from $O(4)$ to $O(N)$, where the solution is direct as $N \to \infty$ [142, 154].

The solution at large $N$ is standard, and proceeds by introducing the a field $\xi = \vec{\phi}^2$, and a constraint field, $\epsilon$, $\mathcal{L}_{cons} = i\epsilon (\xi - \vec{\phi}^2)/2$. I only seek the solution for the symmetric phase, although the solution in the broken phase can also be determined [1]. Using this constraint,
the $\phi$ and $\xi$ fields are integrated out to give the effective action

$$S_{\text{eff}} = \frac{N}{2} \text{tr} \log \Delta^{-1} + \int d^4x \left( \frac{\tilde{\phi}^2}{4\lambda} + \frac{\tilde{m}_{\phi}^2\omega^2}{2} - g_\omega \omega_0 \rho_B \right); \quad (6)$$

$\Delta^{-1}$ is the inverse propagator for the $\phi$ field, which in momentum space is $\Delta^{-1}(\omega, \tilde{k}) = \omega^2 + E(k)^2$, where

$$E(k)^2 = \frac{(\tilde{k}^2)^2}{M^2} + Z \tilde{k}^2 + m_{\phi}^2 , \quad (7)$$

I expand about a stationary point in $\epsilon$ and $\omega_0$, $\epsilon = i\tilde{c} + \epsilon_q$ and $\omega_0 = \omega_0^q + \omega_0^q$, where $\epsilon_q$ and $\omega_0^q$ are quantum fluctuations. The effective mass $m_{\phi}^2 = m_0^2 + \tilde{c} + \kappa^2 \omega_0^2$. To have a well defined limit for large N, as $N \rightarrow \infty$ all terms in the action should scale as $\sim N$, so I take $\lambda, \kappa^2 \sim 1/N$, $g_\omega \rho_B, \omega_0^q \sim \sqrt{N}$, and $\tilde{m}_{\phi}^2, M^2, Z, m_0^2, \tilde{c}, m_{\phi}^2 \sim N^0$. Remember that $N$ is just a fictitious parameter, and is not related to the number of colors or flavors.

Requiring that the effective action is a stationary point in $\epsilon_q$ and $\omega_0^q$ fixes $\tilde{c}$ and $\omega_0^q$,

$$\tilde{c} = \lambda \text{tr} \Delta; \quad \omega_0^q = -\frac{g_\omega \rho_B}{m_0^2 + \kappa^2 N \text{tr} \Delta} , \quad (8)$$

The solution for general values of the parameters is involved, so to make a qualitative point I only consider the limit of $Z \rightarrow -\infty$, where classically the condensate of Eq. (4) dominates. Instead, in perturbation theory one finds that would be Goldstone bosons have a double pole at non-zero momentum, about $k_c$. Such a double pole generates a logarithmic infrared divergence at zero temperature, and a power law divergence at nonzero temperature.

The solution at large $N$ shows how these infrared divergences are avoided. As $Z \rightarrow -\infty$, at $N = \infty$ take $m_{\phi}^2 \approx -ZM/2 + \delta m_{\phi}$. About $k \approx k_c$,

$$E(k \approx k_c)^2 \approx \frac{1}{M^2} ((k^2 - k_c^2)^2 - ZM^3 \delta m_{\phi} + \ldots) . \quad (9)$$

The loop integral is dominated by $k \approx k_c + \delta k$, and to leading logarithmic order becomes

$$\text{tr} \Delta \approx \frac{\sqrt{-ZM^2}}{2^{5/2}\pi^2} \int \frac{d\delta k}{\sqrt{(\delta k)^2 + M \delta m_{\phi}^2/2}}$$

$$\approx \frac{\sqrt{-Z}}{M^2} \frac{2^{7/2}\pi}{2\sqrt{\lambda}} \log \left( \frac{\# \sqrt{-ZM^2}}{\delta m_{\phi}} \right) . \quad (10)$$

Solving Eq. (8) for $\tilde{c}$, as $Z \rightarrow -\infty$,

$$\delta m_{\phi} \approx \# \sqrt{-ZM} \exp \left( -\frac{ZM^3/\pi^2}{\lambda N} \right) , \quad (11)$$

where $\#$ is a positive, nonzero number.

It is worth contrasting this solution with that at nonzero temperature [1]. Then the integral over $\omega$ is a discrete sum, and the zero energy mode is the most important. It generates a power law divergence, with the solution $\delta m_{\phi} \approx 1/Z^4$, Eq. (58) of Ref. [1]. The statement in Ref. [1] that $\delta m_{\phi}$ vanishes at zero temperature is incorrect: it is just that $\delta m_{\phi}$ is suppressed exponentially in $1/\sqrt{-Z}$, instead of by a power. I refer to this disorder as a quantum pion liquid, QπL [155].

I have neglected the equation for $\omega_0$ in Eq. (8). While $\delta m_{\phi}^2/\delta \omega_0$ is very different at zero and nonzero temperature, though, what matters there is the value of the loop integral. Since $\tilde{c} \approx m_{\phi}^2 \approx Z^2M^2/4$, by Eq. (8)

$$\text{tr} \Delta \approx \frac{Z^2}{4\lambda N} , \quad Z \rightarrow -\infty , \quad (13)$$

As $\langle \bar{\phi}^2 \rangle = N \text{tr} \Delta$, by Eq. (8) the $\omega_0$ mass increases sharply,

$$m_{\omega}^2 = \tilde{m}_{\omega}^2 + Z^2 \frac{\kappa^2}{4\lambda} M^2 , \quad (14)$$

and by Eq. (8) suppresses the $\omega_0$ condensate, $\omega_0 \sim 1/Z^2$. Note that the presence of the coupling $\sim \kappa^2$ is essential for this to occur.

When $Z$ is negative, classically one expects a pion condensate to form, but the solution at large $N$ shows that instead a quantum pion liquid (QπL) forms. While this is rigorous at large $N$, as it arises from the double pole at $k_c \neq 0$ for the would be Goldstone modes, it is very likely that there is a QπL for all $N > 2$ [1]. I also assume that the quantum fluctuations are sufficiently strong so that a QπL forms for massive pions.

My suggestion is thus the following. For $n_B > n_1 > n_{sat}$, the theory enters a phase dominated by the $\omega_0$ condensate, which stiffens the EoS. When $n_B > n_2$, it is approximately described by a QπL; both the $\sigma$ and $\omega_0$ are heavy, which suppresses $\omega_0$. In total, the enhancement and then suppression of the $\omega_0$ condensate generates a spike in the speed of sound.

Clearly a detailed analysis is required to determine the dependence of the various parameters with density, or more properly for thermodynamics, with the baryon chemical potential, $\mu_B$. This includes the $\mu_B$ dependence of the wave function renormalization constant $Z$, the mass parameter $M$ (which is of some hadronic scale), $m_0$, $\lambda$, and so forth.

The most direct approach is to use PdQHD, with a self-consistent one loop approximation for the nucleons, the chiral fields $\phi$, and the $\omega_0$. While involved, I comment that it is far simpler to look for a QπL - which is just a non-monotonic dispersion relation, Eq. (9) - than for a pion condensate, which is not spatially homogeneous [106].

As quantum computers are (very) far from computing the properties of cold, dense QCD [115], to proceed from first principles requires the functional renormalization group (FRG) [91,93,133,140,156,159]. Ref. [91]
use a chiral effective model up to $\sim 2n_{\text{sat}}$, matching onto QCD perturbation theory with a Fierz complete FRG [137,139] at intermediate $n_B$. They find evidence for a spike in the speed of sound at $\sim 10n_{\text{sat}}$ [91], which is much higher than Ref. [27]. The ultimate goal is to use the parameters determined by the FRG in vacuum [156,158] to compute the EoS for nuclear matter. Fu, Pawlowski, and Rennecke [159] find that $Z < 0$ at rather high $T$ and $\mu_B \neq 0$, Fig. (21) of [159]. A complete FRG analysis should certainly see a quantum pion liquid, if it exists.

The pion is not an exact Goldstone boson, but I assume it is so light that the Q$\pi$L phase wins over a pion condensate. The same may not be true for strange quarks [160]. When at some $n_B$ the Fermi sea spills over to form one of strange quarks, if the pion $Z$ is negative, by SU(3) flavor symmetry that for kaons will be as well. As the strange quark is much heavier than up and down quarks, instead of a quantum kaon liquid, a kaon condensate might form [105]. This would be a crystal of real kinks, where $\langle \chi \rangle$ oscillates about a constant, nonzero value Bringoltz [153] showed that this happens for the ’t Hooft model in 1 + 1 dimensions [161].

Admittedly my analysis is merely a sketch of how a spike in the speed of sound might arise in nuclear matter. It appears inescapable, though, that the interaction of the $\omega_0$ and the chiral fields plays an essential role.

A MODEL OF Z(3) BARYONS

Some properties of nuclear matter, such as those discussed above, are surely special to QCD. It would be useful, however, to have the simplest possible model which exhibits the confinement of some type of “quarks” into baryons. A SU(N$\text{color}$) gauge theory in 1 + 1 dimensions [161] has baryons [153,162,164], but as $N_{\text{color}} \to \infty$, there are $\sim N_{\text{color}}^2$ flavors of freedom. There are also models in 1 + 1 dimensions which are soluble about the conformal limit [107,108], but these do not generalize to higher dimensions.

An understanding of confinement from $Z(N_{\text{color}})$ vortices in a SU(N$\text{color}$) gauge theory was proposed by ’t Hooft [165,166]; for recent work, see [167,169] and references therein. I suggest discarding the non-Abelian degrees of freedom in SU(N$\text{color}$) to retain just those of $Z(N_{\text{color}})$. A Z(3) gauge theory is constructed following Krauss, Wilczek, and Preskill [170,172]:

$$L_{Z(3)} = \frac{F_{\mu\nu}^2}{4} + |D_\mu \chi|^2 + m_\chi^2 |\chi|^2 + \lambda_\chi (|\chi|^2)^2$$

$$+ \sum_{i=1}^{3} \eta_i (D_\mu + m_q q_i)$$

(15)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength for an Abelian gauge field; $\chi$ is a complex valued scalar, and there are three degenerate types, or “flavors” of fermions, $q_i$, with equal mass $m_q$. The $q_i$ have unit charge, $D_\mu = \partial_\mu - ie A_\mu$, but I choose the scalar to have charge three, $D_\mu = \partial_\mu - 3ie A_\mu$.

I consider the case of 1+1 dimensions first, and assume that the fermions are heavy. (Light fermions, $m_q \ll e$, may undergo spontaneous symmetry breaking, which because of the lack of Goldstone bosons in 1+1 dimensions, complicates the analysis [107,108,162,164], and is really secondary to my desire to construct a theory for nuclear matter.) If $m_\chi^2 < 0$, spontaneous symmetry breaking occurs, and the photon becomes massive. For large distances, $\sim 1/(3e|m_\chi|)$, naively one expects that there is no interaction from the photons, and the fermions propagate freely. Besides perturbative fluctuations, there are also vortices, which in two (Euclidean) dimensions are like pseudoparticles, localized at a given point. The vacuum is a superposition of vortices, where each vortex has an action $S_v \sim (m_\chi)^4/\lambda_\chi$. If $\chi$ had unit charge, the propagation of fermions is affected only when they are near a vortex, and the vortices are relatively inconsequential.

When $\chi$ has charge three, however, a vortex can carry a Z(3) charge, which greatly affects the propagation of the fermion. If a fermion of unit charge encircles a single vortex, it picks up an Aharonov-Bohm phase of $\exp(\pm 2\pi i/3)$. With a vacuum composed of an infinite number of vortices, these phases confine [173], the fermions entirely through these random phases, exactly analogous to how Z(3) vortices in a SU(3) gauge theory confine [108].

While a state such as $q_1^3$ is neutral under Z(3), this vanishes, as $q_1$ is a fermion field which anti-commutes with itself. This is different from QCD, where the antisymmetric tensor in color space can be used to form a baryon with one flavor, $\sim e^{\epsilon^{ab}} q_1^a q_2^b q_3^c$. Consequently, in a Z(3) model to obtain (simple) baryons it is necessary to take three flavors, so the baryon $\sim q_1 q_2 q_3$, is neutral under Z(3). The mesons form an octet in flavor, which is (presumably) lighter than the singlet meson (plus higher excitations, of course).

In weak coupling the action of a single vortex is small, vortices are dilute, and confinement occurs over large distances, $\sim \exp(-S_v)$. The fermions interact over distance $\sim 1/m_\chi$, but at long distances, only interact through the Z(3) phases generated by the vortex ensemble in vacuum. These Z(3) baryons are weakly bound over large distances, so that in any scattering experiment, it would be obvious that they have composite substructure. This is in contrast to QCD, where baryons have weak attraction at large distances, but a strong repulsive core at short distances.

That is, in QCD it is hard prying the quarks out of a baryon. This would occur if the density of vortices is large. In the effective model above, this requires strong coupling, which cannot be studied analytically. However, this limit can be studied on the lattice, and just produces
a $Z(3)$ gauge theory [174] coupled to three flavors of degenerate fermions.

In $1 + 1$ dimensions, as for the $U(1)$ gauge theory [175–177], the $Z(N)$ gauge theory confines. On a lattice, classical computers have been used to study the properties in vacuum of $Z(2)$ [177, 181] and $Z(3)$ [178, 181] gauge theories with a single flavor. The behavior of a $U(1)$ theory with two flavors was computed at nonzero density in Ref. 182. Thus classical computers can be used to compute the properties of a $Z(3)$ gauge theory with three degenerate flavors at nonzero density. This can then provide a benchmark to compare against computing the free energy at nonzero density using quantum computers. The great advantage of a $Z(3)$ gauge group is that only two qubits are needed to describe a group element, as opposed to many more for any continuous gauge group.

In $2 + 1$ dimensions the vortices sweep our lines in space-time, and cylinders in $3 + 1$ dimensions. Assuming (3) vortices confine in QCD, these models should exhibit confinement as well. It would be interesting analyzing the behavior of $Z(3)$ nuclear matter at strong coupling as a counterpoint to that in QCD.

I thank S. Carignano, L. Classen, V. Dexheimer, T. Iizubuchi, Y. Kikuchi, Y.-L. Ma, L. McLerran, F. Rennecke, M. Rhö, D. Rischke, F. Tartaglia, A. Tsvelik, and A. Tomiya for discussions; also Brian Serot, albeit some time ago, about the ineluctable virtues of cockatiels and QHD; and S. Reddy, for discussions on the role of an $\omega$ condensate. This research was supported by the U.S. Department of Energy under contract DE-SC0012704; the Lab Directed Research and Development program 18-036; the work in Sec. II, by the U.S. Department of Energy, Office of Science National Quantum Information Science Research Centers under the award for the “Co-design Center for Quantum Advantage”. After this work was completed, the nature of the $\Omega_{QL}$ regime was developed further with A. Tsvelik [183]; the signatures of a $\Omega_{QL}$ regime in heavy ion collisions were analyzed with F. Rennecke [184].

References

[1] R. D. Pisarski, A. M. Tsvelik, and S. Valgushev, How transverse thermal fluctuations disorder a condensate of chiral spirals into a quantum spin liquid, Phys. Rev. D 102, 016015 (2020), arXiv:2005.10259 [hep-ph]
[2] Y. Ze’’dovich, The equation of state of ultrahigh densities and its relativistic limitations, Sov. Phys. JETF 14, 1143 (1962).
[3] B. D. Serot and J. D. Walecka, The Relativistic Nuclear Many Body Problem, Adv. Nucl. Phys. 16, 1 (1986).
[4] J. Walecka, Theoretical nuclear and subnuclear physics (Imperial College Press, London, 1995).
[5] G. A. Lalazissis, J. König, and P. Ring, A New parametrization for the Lagrangian density of relativistic mean field theory, Phys. Rev. C 55, 540 (1997).
[6] B. D. Serot and J. D. Walecka, Recent progress in quantum hadrodynamics, Int. J. Mod. Phys. E 6, 515 (1997), arXiv:nucl-th/9701058
[7] A. Akmal, V. Pandharipande, and D. Ravenhall, The Equation of state of nuclear matter and neutron star structure, Phys. Rev. C 58, 1804 (1998), arXiv:nucl-th/9804027.
[8] J. Carriere, C. J. Horowitz, and J. Piekarewicz, Low mass neutron stars and the equation of state of dense matter, Astrophys. J. 593, 463 (2003), arXiv:nucl-th/0211015.
[9] P. Danielewicz, R. Lacey, and W. G. Lynch, Determination of the equation of state of dense matter, Science 298, 1592 (2002), arXiv:nucl-th/0208016.
[10] E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Modern Theory of Nuclear Forces, Rev. Mod. Phys. 81, 1773 (2009), arXiv:0811.1338 [nucl-th].
[11] R. Machleidt and D. Entem, Chiral effective field theory and nuclear forces, Phys. Rept. 503, 1 (2011), arXiv:1105.2919 [nucl-th].
[12] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Shapiro Delay Measurement of A Two Solar Mass Neutron Star, Nature 467, 1081 (2010), arXiv:1010.5788 [astro-ph.HE].
[13] J. Antoniadis et al., A Massive Pulsar in a Compact Relativistic Binary, Science 340, 1235232 (2013), arXiv:1304.6875 [astro-ph.HE].
[14] A. L. Watts et al., Colloquium: Measuring the neutron star equation of state using x-ray timing, Rev. Mod. Phys. 88, 021001 (2016), arXiv:1602.01081 [astro-ph.HE].
[15] F. Özel and P. Freire, Masses, Radii, and the Equation of State of Neutron Stars, Ann. Rev. Astron. Astrophys. 54, 401 (2016), arXiv:1603.02608 [astro-ph.HE].
[16] B. Abbott et al. (LIGO Scientific, Virgo), GW170817: Measurements of neutron star radii and equation of state, Phys. Rev. Lett. 121, 161101 (2018), arXiv:1805.11581 [gr-qc].
[17] M. C. Miller, C. Chirenti, and F. K. Lamb, Constraining the equation of state of high-density cold matter using nuclear and astronomical measurements.10.3847/1538-4357/ab4ef9 (2019), arXiv:1904.08907 [astro-ph.HE].
[18] L. Baiotti, Gravitational waves from neutron star mergers and their relation to the nuclear equation of state, Prog. Part. Nucl. Phys. 109, 103714 (2019), arXiv:1907.08534 [astro-ph.HE].
[19] E. Annala, T. Gorda, J. Nättiälä, and A. Vuorinen, Evidence for quark-matter cores in massive neutron stars, Nature Phys. 10.1038/s41567-020-0914-9 (2020), arXiv:1903.09121 [astro-ph.HE].
[20] L. L. Oster, A. Windisch, F. J. Llano-Estrada, and M. Alford, nEoS: Neutron Star Equation of State from hadron physics alone, J. Phys. G 46, 084001 (2019), arXiv:1901.05271 [gr-qc].
[21] D. Drischler, R. Furnstahl, J. Melendez, and D. Phillips, How well do we know the neutron-star matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, (2020), arXiv:2004.07232 [nucl-th].
[22] D. Greif, K. Hebler, J. Lattimer, C. Pethick, and A. Schwenk, Equation of state constraints from nuclear physics, neutron star masses, and future moment of
[23] T. Tews, J. Carlson, S. Gandolfi, and S. Reddy, Constraining the speed of sound inside neutron stars with chiral effective field theory interactions and observations, Astrophys. J. 860, 149 (2018) arXiv:1801.01923 [nucl-th].

[24] M. M. Forbes, S. Bose, S. Reddy, D. Zhou, A. Mukherjee, and S. De, Constraining the neutron-matter equation of state with gravitational waves, Phys. Rev. D 100, 083010 (2019) arXiv:1904.04233 [astro-ph.HE].

[25] B. Reed and C. Horowitz, Large sound speed in dense matter and the deformability of neutron stars, Phys. Rev. C 101, 045803 (2020) arXiv:1910.05463 [astro-ph.HE].

[26] C. Drischler, S. Han, J. M. Lattimer, M. Prakash, S. Reddy, and T. Zhao, Limiting masses and radii of neutron stars and their implications, (2020), arXiv:2009.06441 [nucl-th].

[27] R. Essick, T. Tews, P. Landry, S. Reddy, and D. E. Holz, Direct Astrophysical Tests of Chiral Effective Field Theory at Supranuclear Densities, Phys. Rev. C 102, 055803 (2020) arXiv:2004.07744 [astro-ph.HE].

[28] C. Xia, Z. Zhu, X. Zhou, and A. Li, Sound velocity in dense stellar matter with strangeness and compact stars, (2019), arXiv:1906.00826 [nucl-th].

[29] S. Han and M. Prakash, On the Minimum Radius of Very Massive Neutron Stars, Astrophys. J. 899, 164 (2020) arXiv:2006.02207 [astro-ph.HE].

[30] A. Kanakis-Pegios, P. Koliogiannis, and C. Moustakidis, Speed of sound constraints from tidal deformability of neutron stars, (2020), arXiv:2007.13399 [nucl-th].

[31] T. Kojo, QCD equations of state and speed of sound in neutron stars (2020) arXiv:2011.10940 [nucl-th].

[32] K. Fukushima and C. Sasaki, The phase diagram of neutron stars (2020) arXiv:2007.08098 [nucl-th].

[33] C. Drischler, S. Han, J. M. Lattimer, M. Prakash, B. Reed and C. Horowitz, Large sound speed in dense stellar matter and the deformability of neutron stars, Phys. Rev. D 100, 103022 (2019) arXiv:1906.04095 [astro-ph.HE].

[34] J. W. Holt, M. Rho, and W. Weise, Chiral symmetry and effective field theories for hadronic, nuclear and stellar matter, Phys. Rept. 621, 2 (2016) arXiv:1411.6681 [nucl-th].

[35] H. Pais and C. Providência, Vlasov formalism for extended relativistic mean field models: The crust-core transition and the stellar matter equation of state, Phys. Rev. C 94, 015808 (2016) arXiv:1607.05899 [nucl-th].

[36] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, and T. Takatsuka, From hadrons to quarks in neutron stars: a review, Rept. Prog. Phys. 81, 056902 (2018) arXiv:1707.04966 [astro-ph.HE].

[37] G. Brown and M. Rho, Scaling effective Lagrangians in a dense medium, Phys. Rev. Lett. 66, 2720 (1991)

[38] T. Hatsuda and S. H. Lee, QCQ sum rules for vector mesons in the nuclear medium, Phys. Rev. C 46, 34 (1992).

[39] K. Saito, K. Tsushima, and A. W. Thomas, Variation of hadron masses in finite nuclei, Phys. Rev. C 55, 2637 (1997) arXiv:nucl-th/9612001.

[40] R. S. Hayano and T. Hatsuda, Hadron properties in the nuclear medium, Rev. Mod. Phys. 82, 2949 (2010) arXiv:0812.1702 [nucl-ex].

[41] P. Gubler and D. Satow, Recent Progress in QCD Condensate Evaluations and Sum Rules, [Prog. Part. Nucl. Phys. 106, 1 (2019) arXiv:1812.00385 [hep-ph].

[42] J. Holt, T. T. Kuo, K. Phua, M. Rho, and I. Zahed, eds., Quarks, Nuclei and Stars: Memorial Volume Dedicated to Gerald E Brown (World Scientific, Singapore, 2017).

[43] M. Rho and Y.-L. Ma, Going from Asymmetric Nuclei to Neutron Stars to Tidal Polarizability in Gravitational Waves, Int. J. Mod. Phys. E 27, 1830006 (2018) arXiv:1807.02670 [nucl-th].

[44] Y.-L. Ma and M. Rho, Sound velocity and tidal deformability in compact stars, Phys. Rev. D 100, 114003 (2019) arXiv:1811.07071 [nucl-th].

[45] Y.-L. Ma and M. Rho, Towards the hadron–quark continuity via a topology change in compact stars, Prog. Part. Nucl. Phys. 113, 103791 (2019) arXiv:1909.05889 [nucl-th].

[46] Y.-L. Ma and M. Rho, What’s in the core of massive neutron stars?. (2020) arXiv:2006.14173 [nucl-th].

[47] M. Rho and Y.-L. Ma, Manifestation of Hidden Symmetries in Baryonic Matter: From Finite Nuclei to Neutron Stars, (2021), arXiv:2101.07121 [nucl-th].

[48] L. McLerran and R. D. Pisarski, Phases of cold, dense quarks at large N(c), Nucl. Phys. A796, 83 (2007) arXiv:0706.2191 [hep-ph].

[49] A. Andronic et al., Hadron Production in Ultrarelativistic Nuclear Collisions: Quarkyonic Matter and a Triple Point in the Phase Diagram of QCD, Nucl. Phys. A 837, 65 (2010) arXiv:0911.4806 [hep-ph].

[50] T. Kojo, Y. Hidaka, L. McLerran, and R. D. Pisarski, Quarkyonic Chiral Spirals, Nucl. Phys. A843, 37 (2010) arXiv:0912.3800 [hep-ph].

[51] T. Kojo, R. D. Pisarski, and A. M. Tsvelik, Covering the Fermi Surface with Patches of Quarkyonic Chiral Spirals, Phys. Rev. D82, 074015 (2010) arXiv:1007.0248 [hep-ph].

[52] T. Kojo, Y. Hidaka, K. Fukushima, L. D. McLerran, and R. D. Pisarski, Interweaving Chiral Spirals, Nucl. Phys. A875, 94 (2012) arXiv:1107.2124 [hep-ph].

[53] K. Fukushima and T. Kojo, The Quarkyonic Star, Astrophys. J. 817, 180 (2016) arXiv:1509.00356 [nucl-th].

[54] L. McLerran and S. Reddy, Quarkyonic Matter and Neutron Stars, Phys. Rev. Lett. 122, 122701 (2019) arXiv:1811.12503 [nucl-th].

[55] K. S. Jeong, L. McLerran, and S. Sen, Dynamically generated momentum space shell structure of quarkyonic matter via an excluded volume model, Phys. Rev. C 101, 035201 (2020) arXiv:1908.04799 [nucl-th].

[56] D. C. Duarte, S. Hernandez-Ortiz, and K. S. Jeong, Excluded Volume Model for Quarkyonic Matter II: Three-flavor Shell-like Distribution of Baryons in Phase Space, (2020) arXiv:2007.08099 [nucl-th].

[57] D. C. Duarte, S. Hernandez-Ortiz, and K. S. Jeong, Excluded-volume model for quarkyonic Matter: Two-flavor baryon-quark Mixture, Phys. Rev. C 102, 023203 (2020) arXiv:2003.02362 [nucl-th].

[58] S. Sen and N. C. Warrington, Finite-Temperature Quarkyonic Matter with an Excluded Volume Model,
D. Zschiesche, L. Tolos, J. Schaffner-Bielich, and R. D. J. Eser, M. Grahl, and D. H. Rischke, Functional Renormalization Group Study of the Chiral Phase Transition, Phys. Rev. D 102, 023021 (2020). [arXiv:2004.08293 [nucl-ph]]

R. D. Pisarski, Where does the rho go? Chirally symmetric vector mesons in the quark-gluon plasma, Phys. Rev. D 52, 3773 (1995). [hep-ph/9503328]

R. D. Pisarski, Anomalous mesonic interactions near a chiral phase transition, Phys. Rev. Lett. 76, 3084 (1996). [hep-ph/9601316]

C. E. Detar and T. Kunihiro, Linear σ Model With Parity Doubling, Phys. Rev. D 39, 2805 (1989).

D. Jido, M. Oka, and A. Hosaka, Chiral symmetry of baryons, Prog. Theor. Phys. 106, 873 (2001). [hep-ph/0110005]

D. Zschiesche, L. Tolos, J. Schaffner-Bielich, and R. D. Pisarski, Cold, dense nuclear matter in a SU(2) parity doublet model, Phys. Rev. C 75, 055202 (2007). [arXiv:nucl-th/0608044]

S. Gallas, F. Giacosa, and D. H. Rischke, Vacuum phenomenology of the chiral partner of the nucleon in a linear sigma model with vector mesons, Phys. Rev. D 82, 045004 (2010). [arXiv:0907.5084 [hep-ph]]

J. Steinheimer, S. Schramm, and H. Stocker, The hadronic SU(3) Parity Doublet Model for Dense Matter, its extension to quarks and the strange equation of state, Phys. Rev. C 84, 045208 (2011). [arXiv:1108.2596 [hep-ph]]

S. Gallas, F. Giacosa, and G. Pagliara, Nuclear matter within a dilatation-invariant parity doublet model: the role of the tetraquark at nonzero density, Nucl. Phys. A 872, 13 (2011). [arXiv:1105.5003 [hep-ph]]

J. Eser, M. Grabl, and D. H. Rischke, Functional Renormalization Group Study of the Chiral Phase Transition Including Vector and Axial-vector Mesons, Phys. Rev. D 92, 096008 (2015). [arXiv:1508.06928 [hep-ph]]

P. Lakaschus, J. L. Mauldin, F. Giacosa, and D. H. Rischke, Role of a four-quark and a glueball state in including Vector and Axial-vector Mesons, Phys. Rev. D 87, 096008 (2013). [arXiv:1306.1092 [nucl-th]]

F. Giacosa and G. Pagliara, Neutron stars in the large-Nc limit, Nucl. Phys. A 968, 366 (2017). [arXiv:1707.02644 [nucl-th]]

Z. Khaidukov and Y. A. Simonov, Speed of sound, breaking of conformal limit and instabilities in quark-gluon plasma at finite baryon density, (2018). [arXiv:1811.08970 [hep-ph]]

T. Kojo, P. D. Powell, Y. Song, and G. Baym, Phenomenological QCD equation of state for massive neutron stars, Phys. Rev. D 91, 045003 (2015). [arXiv:1412.1108 [hep-ph]]

G. Baym, S. Furusawa, T. Hatsuda, T. Kojo, and H. Togashi, New Neutron Star Equation of State with Quark-Hadron Crossover, Astrophys. J. 885, 42 (2019). [arXiv:1903.08963 [astro-ph.HE]]

Y. Song, G. Baym, T. Hatsuda, and T. Kojo, Effective repulsion in dense quark matter from nonperturbative gluon exchange, Phys. Rev. D 100, 034018 (2019). [arXiv:1905.01005 [astro-ph.HE]]

K. Fukushima, T. Kojo, and W. Weise, Hard-core deconfinement and soft-surface delocalization from nuclear to quark matter, (2020). [arXiv:2008.08436 [hep-ph]]

M. Leonhardt, M. Pospiech, B. Schallmo, J. Braun, C. Drischler, K. Hebeler, and A. Schwenk, Symmetric nuclear matter from the strong interaction, Phys. Rev. Lett. 125, 142502 (2020). [arXiv:1907.05814 [nucl-th]]

K. Otto, M. Oertel, and B.-J. Schaefer, Hybrid and quark star matter based on a nonperturbative equation of state, Phys. Rev. D 101, 103021 (2020). [arXiv:1910.11929 [hep-ph]]

K. Otto, M. Oertel, and B.-J. Schaefer, Nonperturbative quark matter equations of state with vector interactions, (2020). [arXiv:2007.07394 [hep-ph]]

M. Shahrbaf, D. Blaschke, A. Grunfeld, and H. Moshfeh, First-order phase transition from hypernuclear matter to deconfined quark matter obeying new constraints from compact star observations, Phys. Rev. C 101, 035209 (2020). [arXiv:1908.10509 [nucl-th]]
[129] R. Amorim and J. Barcelos-Neto, BV quantization of a vector - tensor gauge theory with topological coupling, Mod. Phys. Lett. A 10, 917 (1995) arXiv:hep-th/9505093.

[130] M. Henneaux, V. Lemes, C. Sasaki, S. Sorella, O. Ventura, and L. Vilar, A No go theorem for the non-Abelian topological mass mechanism in four-dimensions, Phys. Lett. B 410, 195 (1997) arXiv:hep-th/9707129.

[131] This is true with either the Stuckelberg [125] or BF [120][130] formalisms. With the BF formalism, auxiliary two-index gauge potentials $B_{\alpha \beta}^\nu$ could be introduced, and defined to transform under form gauge transformations $\lambda^\nu_{\alpha}$ as $B_{\alpha \beta}^\nu \rightarrow B_{\alpha \beta}^\nu - \partial_\lambda \lambda^\nu_{\alpha} + \partial_\beta \lambda^\nu_{\alpha}$. Adding to the action a 3-index field strength tensor for $B_{\alpha \beta}^\nu$, integration over $B_{\alpha \beta}^\nu$ generates the coupling $\sim \kappa^2$ if the BF term is chosen as $\sim \kappa \epsilon^{\alpha \beta \mu \nu} \Phi^1 B_{\alpha \beta}^\nu \partial_\mu \partial_\nu$. However, such a BF term is gauge invariant only for constant $\phi'$, and not for a dynamical field, where $\partial_\lambda \phi' \neq 0$. This is unremarkable, given that masses for non-Abelian fields, such as the $\rho$ and $\omega$, also cannot be introduced in a form which respects gauge invariance and unitarity [125][130].

[132] H. B. O’Connell, B. Pearce, A. W. Thomas, and A. G. Williams, $\rho-\omega$ mixing, vector meson dominance and the pion form-factor, Prog. Part. Nucl. Phys. 39, 201 (1997) arXiv:hep-ph/9501251.

[133] F. Rennecke, The Chiral Phase Transition of QCD, Ph.D. thesis U. Heidelberg (main) (2015).

[134] F. Rennecke, Vacuum structure of vector mesons in QCD, Phys. Rev. D 92, 076012 (2015) arXiv:1504.03585 [hep-ph].

[135] C. Jung, F. Rennecke, R.-A. Tripolt, L. von Smekal, and J. Wambach, In-Medium Spectral Functions of Vector- and Axial-Vector Mesons from the Functional Renormalization Group, Phys. Rev. D 95, 036020 (2017) arXiv:1610.08754 [hep-ph].

[136] R.-A. Tripolt, B.-J. Schaefer, L. von Smekal, and J. Wambach, Low-temperature behavior of the quark-meson model, Phys. Rev. D 97, 034022 (2018) arXiv:1709.05991 [hep-ph].

[137] J. Braun, M. Leonhardt, and M. Pospiel, Fierz-complete NJL model study: Fixed points and phase structure at finite temperature and density, Phys. Rev. D 96, 076003 (2017) arXiv:1705.00074 [hep-ph].

[138] J. Braun, M. Leonhardt, and M. Pospiel, Fierz-complete NJL model study II. Toward the fixed-point and phase structure of hot and dense two-flavor QCD, Phys. Rev. D 97, 076010 (2018) arXiv:1801.08335 [hep-ph].

[139] J. Braun, M. Leonhardt, and M. Pospiel, Fierz-complete NJL model study III: Emergence from quark-gluon dynamics, Phys. Rev. D 101, 036004 (2020) arXiv:1909.06298 [hep-ph].

[140] H. Zhang, D. Hou, T. Kojo, and B. Qin, Functional renormalization group study of the quark-meson model with $\omega$ meson, Phys. Rev. D 96, 114029 (2017) arXiv:1709.05654 [hep-ph].

[141] R. D. Pisarski and F. Rennecke, Multi-instanton contributions to anomalous quark interactions, Phys. Rev. D 101, 114019 (2020) arXiv:1910.14052 [hep-ph].

[142] For an arbitrary number of flavors, $N_f$, under a chiral rotation the chiral field $\Phi$ transforms as $\Phi \rightarrow e^{i\theta_A} e^{i\alpha_L} \Phi e^{-i\alpha_R}$, where $\theta_A$, $\alpha_L$, and $\alpha_R$ are elements of the Lie algebra for $U_A(1) \times SU(N_f)_{L} \times SU(N_f)_{R}$, respectively. For arbitrary $N_f$ the coupling $\sim \kappa^2$ in Eq. (2) generalizes trivially to $\omega_0^2 \tr \Phi \Phi$.

[143] D. Trnka et al. (CBELSA/TAPS), First observation of in-medium modifications of the omega meson, Phys. Rev. Lett. 94, 192303 (2005) arXiv:nucl-ex/0504010.

[144] M. Nanova et al. (CBELSA/TAPS), In-medium omega mass from the gamma+Nb $\rightarrow$ p0 gamma+X reaction, Phys. Rev. C 82, 035209 (2010) arXiv:1005.5694 [nucl-ex].

[145] M. Kotulla et al. (A2), In-medium modifications of the $\omega$-Meson Lifetime in Nuclear Matter, Phys. Rev. Lett. 100, 192302 (2008) Erratum: Phys.Rev.Lett. 114, 199903 (2015) arXiv:0802.0989 [nucl-ex].

[146] V. Metag, M. Thiel, H. Bergh¨auser, S. Friedrich, B. Lemmer, U. Mosel, and J. Weil (A2), Experimental approaches for determining in-medium properties of hadrons from photo-nuclear reactions, Prog. Part. Nucl. Phys. 67, 530 (2012) arXiv:1111.6004 [nucl-ex].

[147] M. Thiel et al., In-medium modifications of the $\omega$ meson near the production threshold, Eur. Phys. J. A 49, 132 (2013).

[148] S. Friedrich et al. (CBELSA/TAPS), Experimental constraints on the $\omega$-nuclear real potential, Phys. Lett. B 736, 26 (2014) arXiv:1407.0899 [nucl-ex].

[149] V. Metag, Determining the meson-nucleus potential - on the way to mesic states, Hyperfine Interact. 284, 25 (2015).

[150] V. Metag, M. Nanova, and E. Y. Paryev, Meson–nucleus potentials and the search for meson–nucleus bound states, Prog. Part. Nucl. Phys. 97, 199 (2017) arXiv:1706.09654 [nucl-ex].

[151] This is implicit in the analysis of Ref. [73]. I thank V. Dexheimer for discussions on the difficulty of eliminating the contribution of the $\omega_0$ energy in mean field theory.

[152] There are also two other terms with dimensions of inverse mass, Eq. (2) of Ref. [1], but as these only involve two spatial derivatives, they do not have a dramatic effect.

[153] B. Bringoltz, Solving two-dimensional large-N QCD with a nonzero density of baryons and arbitrary quark mass, Phys. Rev. D79, 125006 (2009) arXiv:0901.4035 [hep-lat].

[154] For arbitrary $N_f$, $\Phi$ is a complex, $N_f \times N_f$ matrix. While in general matrix models are not easily solved at large $N_f$, for the ansatz where $SU(N_f)_L \times SU(N_f)_R$ breaks to $SU(N_f)_V$, which is $\Phi = \sigma_0 1$, the solution follows directly from the vector model, with $N \rightarrow 2N_f^2$

[155] In Ref. [1] we used the term pionic quantum spin liquid ($\pi$QL), in this paper I adopt the more accurate term quantum pion liquid ($Q\pi L$), as the phenomenon has nothing to do with spin. I thank L. Classen and F. Rennecke for pointing this out and suggesting the term.

[156] M. Mitter, J. M. Pawlowski, and N. Strodthoff, Chiral symmetry breaking in continuum QCD, Phys. Rev. D 91, 054035 (2015) arXiv:1411.7978 [hep-ph].

[157] A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, and N. Strodthoff, Landau gauge Yang-Mills correlation functions, Phys. Rev. D 94, 054005 (2016) arXiv:1605.01856 [hep-ph].

[158] A. K. Cyrol, M. Mitter, J. M. Pawlowski, and N. Strodthoff, Nonperturbative quark, gluon, and meson correlators of unquenched QCD, Phys. Rev. D 97, 114029 (2018) arXiv:1803.09626 [hep-ph].
[159] W.-j. Fu, J. M. Pawlowski, and F. Rennecke, QCD phase structure at finite temperature and density, Phys. Rev. D 101, 054032 (2020) [arXiv:1909.02991 [hep-ph]].

[160] L. Tolos and L. Fabbietti, Strangeness in Nuclei and Neutron Stars, Prog. Part. Nucl. Phys. 112, 103770 (2020) [arXiv:2002.09223 [nucl-ex]].

[161] G. ’t Hooft, A Two-Dimensional Model for Mesons, Nucl. Phys. B 75, 461 (1974).

[162] P. J. Steinhardt, Baryons and Baryonium in {QCD} in Two-dimensions, Nucl. Phys. B 176, 100 (1980).

[164] G. ’t Hooft, On the Phase Transition Towards Permanent Quark Confinement, Nucl. Phys. B 138, 1 (1978).

[165] G. ’t Hooft, A Property of Electric and Magnetic Flux in Nonabelian Gauge Theories, Nucl. Phys. B 153, 141 (1979).

[166] J. Greensite, An introduction to the confinement problem, Vol. 821 (2011).

[167] J. Greensite, Confinement from Center Vortices: A review of old and new results, EPJ Web Conf. 137, 01009 (2017), arXiv:1610.06221 [hep-lat].

[168] J. C. Biddle, W. Kamleh, and D. B. Leinweber, Visualization of center vortex structure, Phys. Rev. D 102, 034504 (2020) [arXiv:1912.09531 [hep-lat]].

[169] M. de Wild Propitius and F. Bais, Discrete gauge theories in Continuum Theories, Phys. Rev. Lett. 62, 1221 (1989).

[170] L. M. Krauss and F. Wilczek, Discrete Gauge Symmetry in Quantum Mechanical Hair, Nucl. Phys. B 341, 50 (1990).

[171] M. C. Banuls, K. Cichy, J. I. Cirac, K. Jansen, and S. Kuhn, Density Induced Phase Transitions in the Schwinger Model: A Study with Matrix Product States, Phys. Rev. Lett. 118, 071601 (2017), arXiv:1611.00705 [hep-lat].

[172] J. Frank, E. Huffman, and S. Chandrasekharan, Emergence of Gauss' law in a Z_n lattice gauge theory in 1 + 1 dimensions, Phys. Lett. B 806, 135484 (2020), arXiv:1904.05414 [cond-mat.str-el].

[173] My language is common but imprecise: since the fermions carry 2\( \frac{3}{2} \) charge, any flux string can always break by the production of fermion anti-fermion pairs. By confinement I mean that there is a mass gap for all particles in the spectrum, which by necessity are gauge singlets.

[174] D. Horn, M. Weinstein, and S. Yankielowicz, Hamiltonian approach to Z(N) lattice gauge theories, Phys. Rev. D 19, 3715 (1979).

[175] S. R. Coleman, R. Jackiw, and L. Susskind, Charge Shielding and Quark Confinement in the Massive Schwinger Model, Annals Phys. 93, 267 (1975).

[176] E. Abdalla and M. Abdalla, Updating QCD in two-dimensions, Phys. Rept. 265, 253 (1996), arXiv:hep-th/9503002.

[177] G. ’t Hooft, A Property of Electric and Magnetic Flux in Nonabelian Gauge Theories, Nucl. Phys. B 153, 141 (1979).