A Study of Multi-$\Lambda$ Hypernuclei within Spherical Relativistic Mean-field Approach

Asloob A. Rather1, M. Ikram1, A. A. Usmani1, B. Kumar2,3, S. K. Patra2,3

1 Department of Physics, Aligarh Muslim University, Aligarh-202002, India.
2 Institute of Physics, Bhubaneswar-751 005, India.
3 Homi Bhabha National Institute, Anushakti Nagar, Mumbai - 400094, India.

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Abstract. This research article is a follow up of earlier work by M. Ikram et al., reported in International Journal of Modern Physics E 25, 1650103 (2016) wherein we searched for $\Lambda$ magic numbers in experimentally confirmed doubly magic nucleonic cores in light to heavy mass region (i.e. $^{16}\text{O} - ^{208}\text{Pb}$) by injecting $\Lambda$'s into them. In present manuscript, working within the state-of-art relativistic mean field theory with inclusion of $\Lambda N$ and $\Lambda \Lambda$ interaction in addition to nucleon-meson NL3* effective force, we extend the search of lambda magic numbers in multi-$\Lambda$ hypernuclei using the predicted doubly magic nucleonic cores $^{292}$120, $^{304}$120, $^{320}$120, $^{320}$132, $^{336}$138, $^{396}$138 of elusive superheavy mass regime. In analogy to well established signatures of magicity in conventional nuclear theory, the prediction of hypernuclear magicity are made on the basis of one-, two- $\Lambda$ separation energy ($S_\Lambda$, $S_{2\Lambda}$) and two lambda shell gaps ($\delta_{2\Lambda}$) in multi-$\Lambda$ hypernuclei. The calculations suggest that the $\Lambda$ numbers 92, 106, 126, 138, 184, 198, 240, and 258 might be the $\Lambda$ shell closures after introducing the $\Lambda$'s in elusive superheavy nucleonic cores. The appearance of new lambda shell closures other than the nucleonic ones predicted by various relativistic and non-relativistic theoretical investigations can be attributed to the relatively weak strength of spin-orbit coupling in hypernuclei compared to normal nuclei. Further, the predictions made in multi-$\Lambda$ hypernuclei under study resembles quite closely with the magic numbers in conventional nuclear theory suggested by various relativistic and non-relativistic theoretical models. Moreover, in support of $\Lambda$ shell closure the investigation of $\Lambda$ pairing energy and effective $\Lambda$ pairing gap has been made. We noticed a very close agreement of the predicted $\Lambda$ shell closures with the survey made on the pretext of $S_\Lambda$, $S_{2\Lambda}$ and $\delta_{2\Lambda}$ except for the appearance of magic numbers corresponding to $\Lambda = 156$ which manifest in $\Lambda$ effective pairing gap and pairing energy. Also, lambda single-particle spectrum is analyzed to mark the energy shell gap for further strengthening the predictions made on the basis of separation energies and shell gaps. Lambda and nucleon spin-orbit interactions are analyzed to confirm the reduction in magnitude of $\Lambda$ spin-orbit interaction compared to the nucleonic case, however interaction profile is similar in both the cases. Lambda and nucleon density distributions have been investigated to reveal the impurity effect of $\Lambda$ hyperons which make the depression of central density of the core of superheavy doubly magic nuclei. Lambda skin structure is also seen.

Key words. Hypernuclei, magic number, separation energy, spin-orbit interaction, relativistic mean field theory

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1 Introduction

Hypernuclear physics has become an exciting area of research due to the epoch-making strides in the experimental arena. The primary goal of strangeness nuclear physics is to understand the baryon-baryon interaction which is fundamental and is very important for nuclear physics studies. The two-body scattering are quite useful for elucidating the nature of baryon-baryon interactions. Pursuing this line of thought many NN scattering experiments have been performed and the total number of NN scattering are more than 4000. However, because of the difficulty encountered in carrying out the nucleon-hyperon and hyperon-hyperon scattering experiments, the YN data available is very scarce. To be more precise, the number of differential cross-section data is nearly about 40 and there is no YY scattering data. Further, the YN and YY potential models put forth so far have large ambiguities in their predictions. Thus, it is imperative and wise to study the structure of single-$\Lambda$, double-$\Lambda$ and multi-$\Lambda$ hypernuclei rather than constraining the various theoretical
models employing YN and YY potentials. As the number of data points in hyperon-nucleon sector is few, it is quite difficult to extract the information regrading scattering lengths even. This necessitated to analyze this small data using symmetry of SU(3)\_f models of baryon-baryon interaction which establishes close connections with these data in NN sector. However, realistic models must include the symmetry breaking terms as it is badly broken owing to the difference in mass between s and (u,d) quark. Keeping in view above consideration, various YN potential models have been developed along these lines for hypernuclear studies. The well known models are, the Nijmegen models including the hard core models D \cite{1} and F \cite{2}, the soft core models NSC89 \cite{3}, NSC97 \cite{4} and the extension of soft core models like ESC04 \cite{5}, ESC08 \cite{6} which consider two-meson exchanges and other short range contributions in addition to one-boson exchange. In particular, these models enable extension to hyperon-hyperon potentials, where there is no scattering data available \cite{7}, thus implying an unfortunate increase in model dependence. The other type of models known by Bonn-Julich multi-meson exchange models \cite{8,9,10} which are based on SU(6) symmetry of quark-model. However, the short range nature of YN interaction in these models and Nijmegen models are governed by the way scalar meson interactions are introduced and thus imperactively has got model dependence. The Effective theory chiral model of Leading Order LO \cite{11} and Next Leading Order NLO \cite{12} are another class of model used for hypernuclear studies that makes use of regularized PS Goldstone-boson exchange YN potentials and adding zero-range contact terms for the parameterization of YN coupled-channel interactions and details are reported in Ref. \cite{13}. Further, a quark-model baryon-baryon potential which respects SU(6) symmetry \cite{14} can be used for the construction of hyperon-nucleus potentials \cite{15}.

In nuclear physics, the nuclei with magic numbers have been a hot topic since the birth of this subject \cite{16,17}. With the advent of state-of-art facility radioactive nuclear ion beams (RNBs) the quenching of traditional magic numbers and appearance of new magic numbers have been observed in nuclei with exotic isospin ratios. The explanation of unusual stability attributed to nuclei by neutron(proton) numbers: 2, 8, 20, 28, 50, 82 and 126 commonly referred to as magic numbers in literature, was made in non-relativistic shell model by 3D harmonic oscillator central potential in conjunction with a very strong spin orbit interaction added manually \cite{16,17,18,19,20}. Contrary to this, the models which come under the aegis of relativistic framework say the relativistic mean-field (RMF) \cite{21,22,23,24} in which the strong spin-orbit interaction is a natural outcome of the interplay between strong scalar and vector potentials, which are indispensable for reproducing the saturation properties of matter. The shell structure is obtained due to proper setting of scalar and vector potentials without incorporating any additional parameter for spin-orbit splitting. The precise location of shell and subshell closures in nuclear landscape has provided significant inputs needed for the development of nuclear structure models and theory. However, the position of shell closures is not static particularly for exotic nuclei where there are large number of neutrons relative to protons. The ongoing experimental endeavours are striving for the location of these shell closures in previously unknown nuclei and trace the movement of single-particle orbitals which are responsible for this dynamic shell structure. However, we are lucky enough to have a number of experimental observables that show variation at shell closures across the nuclear chart. The nucleon magicity can be inferred from the discontinuity in these observables like separation energy (one- and two-neutron separation energies), two nucleon shell gap, the energies of α and β transitions (nuclear ground-state moments), shell correction energy, pairing correction, effective pairing gap, binding energy and excitation energy of low-lying vibrational states. Further, nuclear quadrupole moment provides a measure of nuclear charge distribution and hence the nuclear shape. The ground-state quadrupole moment goes to zero at the shell closures and thus shape of nuclei is expected to be spherical; however far away from shell closures the nuclei may be axially deformed and might possess large value of quadrupole moment. The early theoretical investigations employing macro-microscopic approach predicted the spherically doubly magic nucleus \cite{29,30,114}. However, the existence analysis of magicity in nuclear landscape using different models revealed that showed volatility (variation) in the prediction of shell closures and many reports of investigation were successful in establishing the fact that nuclear magicity is dependent on both model and effective force employed. The inference that can be drawn from these studies that reliability of different parameterizations of the models and the accuracy with which the single-particle energies are estimated plays an indispensable role in heavy and superheavy nuclei owing to the large level density in such nuclei. Thus, it can be concluded that magic numbers in nuclear landscape are fragile/localized and search-space dependent implying that different search space would possibly result in the emergence of different magic numbers.

There is wealth of literature available regarding the theoretical attempts being made for identifying the nuclear magic numbers in superheavy region. Some are as follows: Within the parameterizing nucleon mean field in the form of Woods-Saxon potential Brack et. al. \cite{26} explored the nuclear magicity in a wide range of nuclear chart and suggested the proton magicity in the region of Z = 114, 120 and neutron magicity at N = 184. In the same vein, the proton magic number at Z = 114 and neutron magic number at N = 184 was confirmed in the works of Refs. \cite{27,28,29,30}. The microscopic self-consistent calculations in Hartree-Fock-Bogoliubov (HFB) with effective Gogny forces yielded proton magic numbers at Z = 114, 120 and N = 164, 184, 228 \cite{31}. The parameterizations SLy4 and SkP resulted the proton magicity at Z = 124 and 126 while SkO predicted at Z = 126 \cite{32,33,34}. Further, all these parameterization driven calculations predicted neutron magic number at N = 184 \cite{32,33,34}. Liran et. al. \cite{35,36,37} predicted proton magicity at Z = 126 and neutron magicity at N = 184 within the semi-empirical shell model. Calculations within the relativistic
HFB approach with finite range pairing force of Gogny effective interaction D1 and effective interaction NLSH \[^{[38]}\] established that Z = 114, and N = 160, 166, 184 combinations exhibit high stability compared to their neighbours and further doubly magic character at Z = 106 and N = 160 was predicted in Ref. \[^{[38]}\]. The non-relativistic skyrme Hartree Fock (SHF) with effective forces SkP and SLy7 predicted the nuclear magic number at Z = 126 and N = 184 and in addition presented the evidence of increased stability around N = 162 owing to the deformed shell effects \[^{[38]}\]. K. Rutz et. al. \[^{[40]}\], made a prediction about the doubly magic character of \[^{258}114, 292\] and \[^{310}120\]. Several other theoretical attempts employing relativistic mean-field models \[^{[32, 33, 41, 42]}\] yielded the magic numbers at Z = 120 and N = 172, 184 while the relativistic model in Ref. \[^{[33]}\] predicted the nuclear magicity at Z = 114, 120 and N = 172, 184, and 258 and the reconfirmation of closed shells at Z = 120 and N = 172, 184, 258 are given in Ref. \[^{[41]}\]. The exhaustive investigations performed within the framework of continuum relativistic Bogolubov theory \[^{[45]}\] using various effective forces predicted proton numbers Z = 106, 114, 120, 126, 132, and 138 and neutron numbers N = 138, 164, 172, 184, 198, 216, 228, 238, 252, 258 and 274 as possible magic numbers. The survey made by Denisov on the basis of shell correction calculations \[^{[50]}\] predicted spherical proton magic numbers at Z = 82, 114, 164, 210, 274, 354 and spherical neutron magic numbers at N = 126, 184, 228, 308, 406, 524, 644, 772. Moreover, an extensive study by Nakada and Sugiyama \[^{[46, 47]}\] over a wide range of even-even nuclei within spherical relativistic mean field approaches with semi-realistic interactions (M3Y-P6 and P7 parameter sets) has been made and they choose the proton(neutron) pairing energy correction over a wide range of even-even nuclei within spherical relativistic mean field approaches with semi-realistic interactions (M3Y-P6 and P7 parameter sets) has been made and they choose the proton(neutron) pairing energy correction and suggested the possible magicity features in the region Z = 14, 16, 34, 38, 40, 58, 46, 92, 120, 124, 126 and N = 40, 56, 90, 124, 172, 178, 164, 184 which show remarkable agreements with the results in Refs. \[^{[45, 49]}\] where Gogny DIS and DIM interactions was used respectively. Further, Ismail et.al., \[^{[50]}\] have also made more exhaustive investigation about to search of nucleonic closed shell within ultra heavy region and predicted a huge number of magic candidates. These predictions have been made on the basis of separation energy, shell gaps, pairing energy and shell correction energy etc. It has to mention that in previous work \[^{[51]}\] we have inferred lambda magicity in quite close agreement with nuclear magicity which arouse us to investigate lambda magic number in more exotic region and its predictions would give the strong evidence about the confirmation of nuclear magicity in superheavy regime. The work in present manuscript is also suppose to be another attempt in support of nuclear magicity of superheavy region. Among the different methodologies used for identifying the nuclear magicity, we adopt the calculations of one-, two-, and three- \(\Lambda\) separation energies, two \(\Lambda\) shell gaps, lambda pairing energy and effective lambda pairing gap to study lambda magicity for the considered set of multi-\(\Lambda\) hypernuclei. Thus, in this paper, our main emphasis would be to make extensive investigation to search the \(\Lambda\) magic number in multi-\(\Lambda\) hypernuclei within the spherical relativistic mean field approach and to suggest the confirmation of nucleon magic number of superheavy region within the favour of \(\Lambda\) magicity. The subject matter of manuscript is organised as follows: The theoretical framework of spherical relativistic mean field model is outlined in sect. two. Sect. three contains the results and discussion. Finally, the paper is summarized in sect. four.

2 Theoretical formalism

Relativistic mean field theory has established itself as a promising theoretical framework to study the infinite nuclear systems, finite nuclei including the superheavy mass region, hypernuclei and multi-strange systems \[^{[52, 53, 54]}\]. The suitable extension to strangeness sector for studying the hypernuclei and multi-\(\Lambda\) hypernuclei is accomplished by incorporating the lambda-baryon as well as lambda-lambda interactions Lagrangian with effective AN and AA potentials in addition to nucleon-meson NL3* force parameter and such type of attempts have already been made and further detailed discussion about the confirmation of nuclear magicity in superheavy region, hypernuclei and multi-strange systems \[^{[52, 53, 54, 55, 56, 57, 58, 59]}\]. The suitable extension to strangeness sector for studying the hypernuclei and multi-\(\Lambda\) hypernuclei is accomplished by incorporating the lambda-baryon as well as lambda-lambda interactions Lagrangian with effective AN and AA potentials in addition to nucleon-meson NL3* force parameter and such type of attempts have already been made and further detailed discussion about the confirmation of nuclear magicity in superheavy region, hypernuclei and multi-strange systems \[^{[52, 53, 54, 55, 56, 57, 58, 59]}\]. For quantitative description of the multi-\(\Lambda\) hypernuclei additional strange scalar (\(\sigma^*\)) and vector (\(\phi\)) mesons have been incorporated in the the Lagrangian which simulate the AA interaction \[^{[60, 61, 62, 63]}\]. Thus, the total Lagrangian density can be expressed as

\[
\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Lambda + \mathcal{L}_{\Lambda\Lambda},
\]

where \(\psi_i\) denote the Dirac spinors for nucleon and \(\Lambda\) hyperon, whose masses are \(m_N\) and \(m_{\Lambda}\), respectively. Due to the isoscalar nature of \(\Lambda\) hyperon, it does not couple to \(\rho\)-mesons. The quantities \(m_{\sigma}, m_{\rho}, m_{\omega}, m_{\phi}, m_{\sigma^*}\) represent the masses of \(\sigma, \rho, \omega, \phi, \sigma^*\) mesons and \(g_{\sigma}, g_{\rho}, g_{\omega}, g_{\phi}, g_{\sigma^*}\) are their coupling constants, respectively. The nonlinear self-coupling of-\(\sigma\)-mesons is designated by \(g_2\) and \(g_3\). The total energy of the system is given by

\[
E_{total} = E_{part}(N, \Lambda) + E_{\rho} + E_{\omega} + E_{\phi} + E_{\sigma} + E_{\sigma^*} + E_{\pi\pi} + E_{\pi\rho} + E_{\pi\omega} + E_{\pi\phi} + E_{\pi\sigma} + E_{\pi\sigma^*} + E_{\pi\rho\omega} + E_{\pi\rho\phi} + E_{\pi\rho\sigma} + E_{\pi\rho\sigma^*},
\]

where \(E_{part}(N, \Lambda)\) is the sum of the single-particle energies of the nucleons (\(N\)) and hyperons (\(\Lambda\)). The energies parts \(E_{\rho}, E_{\omega}, E_{\phi}, E_{\sigma}, E_{\sigma^*}, E_{\pi\pi}, E_{\pi\rho}, E_{\pi\omega}, E_{\pi\phi}, E_{\pi\sigma}, E_{\pi\sigma^*}\) are the contributions of meson fields, Coulomb field, pairing energy and the center of mass energy, respectively. There is wealth of available literature about RMF parameterizations for predicting the nuclear ground state properties. In the present manuscript, NL3* parameterization is employed for meson-baryon coupling constant throughout.
3 Results and discussions

In present manuscript, we made an attempt to look for magic candidates in multi-$\Lambda$ hypernuclei under inspection on the basis of one- and two-$\Lambda$ separation energies, two lambda shell gaps and pairing energy/gaps etc by performing calculations within the self-consistent relativistic mean-field model with NL3$^*$ parameterization with inclusion of $AN$ and $\Lambda\Lambda$ interactions. Our quest of hunt for magic numbers in multi-$\Lambda$ hypernuclei with doubly magic superheavy nucleonic cores receives the motivation from the seminal work of Zhang et. al., [45] wherein they made an extensive investigation by performing calculations within relativistic continuum Hartree-Bogoliubov theory and established that $Z = 120, 132, 138$ and $N = 172, 184, 198, 228, 238$ and 258 might be the possible nucleonic magic numbers. Further, Ismail et.al., [50] have made more exhaustive investigation about to search of nucleonic closed shell within ultra heavy region and predicted a huge number of magic candidates of ultra heavy region. We extend the thought of nuclear magicity towards strangeness sector with more exotic lambda region and try to predict a triple magic multi-$\Lambda$ system having doubly magic superheavy nucleonic core.

3.1 Lambda separation energy and two lambda shell gaps

The separation energy is an important physical quantity and is of paramount importance for locating and identifying the magic numbers in nuclei as well as hypernuclei. The magic character is manifested by the presence of large shell gaps in the single-particle spectrum of nuclei(hypernuclei). This in turn implies that the nucleons(hyperons) in the lower energy levels has got more binding energy than the nucleons(hyperons) in upper energy levels. The reliable measures of one- and two-nucleon separation energies for marking the shell closures in conventional nuclear theory can be conveniently carried out to strangeness sector and may be expressed as follows.

$$S_\Lambda(N, Z, A) = BE(N, Z, A) − BE(N, Z, A − 1) \quad (5)$$

and,

$$S_{2\Lambda}(N, Z, A) = BE(N, Z, A) − BE(N, Z, A − 2) \quad (6)$$

These quantities are plotted in Fig. 1. Moreover, two-$\Lambda$ separation energy is more efficient marker of magic numbers than one-$\Lambda$ separation energy due to the absence of odd-even staggering. For the considered lambda chain ($A = 90 - 265$), the binding energy increases up to a certain limit by increasing the number of injected $\Lambda$’s in considered doubly magic nucleonic cores. However, for a fixed $N$, $Z$ combination, the $S_\Lambda$ and $S_{2\Lambda}$ show a gradual decreasing trend with the number of injected $\Lambda$’s. A sudden fall in $S_\Lambda$ and $S_{2\Lambda}$ in analogy to neutron and proton chains indicates the occurrence of $\Lambda$ shell closure. The abrupt decrease in the value of $S_{2\Lambda}$ at $A = 92, 106, 126, 132, 138, 198, 228, 240, 258$ can be easily seen in the multi-$\Lambda$ hypernuclei under investigation thus revealing their magic character. Thus, these numbers correspond to $\Lambda$ magic numbers in the multi-$\Lambda$ hypernuclei and form the triply magic system with doubly magic superheavy nucleonic cores. In order to provide a clear presentation of magicity features, the shell effects are also quantified in terms of two lambda shell gaps which is a carry over from the conventional nuclear physics. The two lambda shell gap is expressed as

$$\delta_{2\Lambda}(N, Z, A) = 2BE(N, Z, A) − BE(N, Z, A + 2) − BE(N, Z, A − 2)$$

$$= S_{2\Lambda}(N, Z, A) − S_{2\Lambda}(N, Z, A + 2) \quad (7)$$

The two lambda shell gap is considered to be strong signature for identifying the magic numbers. Two lambda shell versus number of injected lambdas is plotted in Fig. 2 for all the considered multi-$\Lambda$ hypernuclei under investigation. A peak in the two lambda shell gap indicates the drastic change in two-$\Lambda$ separation energies. A peak at certain $A$ number hints towards the $A$ shell closure. However, the quality of $A$ shell gap is dictated by the sharpness and magnitude of the peak. From figure 2 the peaks at $A = 92, 106, 126, 138, 184, 198, 240$ and 258 are evident and these numbers are suppose to be $\Lambda$ magic numbers.
3.2 Lambda pairing energies and pairing gaps

Pairing is an important ingredient in a mean-field description of nuclear structure calculations. It is worthy to mention that the pairing correlation is significant for open shell nuclei with a large density of almost degenerated states, which offers an opportunity to residual two-body interaction to mix these states in order to produce a unique ground-state. This means the pairing contribution in total energy is very less or almost zero for closed shell nuclei. The two mostly used methods to treat pairing are HFB and BCS approximations. In HFB, pairing correlations are taken into account by introducing the concept of quasi-particles defined by Bogoliubov transformation while as BCS approximation is just a simplification of HFB for time-reversal invariant systems.

In both of these schemes, each single nucleon state of HFB for time-reversal invariant systems. In both of these schemes, each single nucleon state is associated with an occupation amplitude $v_\alpha \in [0,1]$ where the complementing non-occupation amplitude is $u_\alpha = \sqrt{1-v_\alpha^2}$.

This refers to that pairing energy may serve as a reliable source of information for identifying the closed shell nuclei or hypernuclei. In BCS approximation, the pairing energy is defined in terms of effective pairing gap $\Delta$ and occupations as given by:

$$E_{\text{pair}} = -\Delta \sum_\alpha \sqrt{w_\alpha (1-w_\alpha)}.$$  \hfill (8)

In order to ensure successful calculation of every desired nucleus occupation numbers $w_\alpha$ are allowed to vary between 0 and 1, whose values are determined using schematic pairing within the constant gap approach using single-particle energy spectrum.

$$w_\alpha = \left( \frac{1}{2} + \frac{\epsilon_\alpha - \epsilon_{\text{Fermi}}}{\sqrt{(\epsilon_\alpha - \epsilon_{\text{Fermi}})^2 + \Delta^2}} \right)^{1/2}.$$  \hfill (9)

where

$$\Delta = \frac{11.2 \text{MeV}}{\sqrt{A}}$$  \hfill (10)

and the Fermi energy is determined by

$$\sum_{n=1}^\Omega W_\alpha = \left\{ \begin{array}{ll} \text{number of protons} & \text{number of neutrons} \end{array} \right\}$$  \hfill (11)

where $\Omega$ is the number of shell model states incorporated for protons, or for neutrons respectively and further, we always include one shell above the last closed shell.

The outcome of $\Lambda$ pairing energy and effective pairing gaps for considered multi-$\Lambda$ hypernuclei is plotted in Fig. 3. The Lambda pairing gap is always shown in lower panel. The $\Lambda$ pairing energies and effective pairing gaps are in close agreement with other signatures of magicity discussed earlier in the manuscript except $\Lambda = 156$ which make their appearances in effective $\Lambda$ pairing gap and pairing energy respectively for the chosen set of multi-$\Lambda$ hypernuclei.

3.3 Lambda single-particle energy spectrum

The single-particle spectrum reflects any kind of change occurring in a system either normal nuclei or hypernuclei. In order to analyze the effects of hyperon addition to the predicted doubly magic nucleonic cores and to confirm or provide the support to our calculations, we have plotted the lambda single-particle energy levels as given in Figs.
The prescription for filling the lambda energy levels is same as that of nucleon energy levels filled according to shell model scheme with $\Lambda$ spin-orbit potential. However, it is evident that the single-particle energy gap between lambda levels is smaller as compared to gaps in nucleon single-particle energy scheme owing to relatively weaker strength of $\Lambda$ spin-orbit interaction. In previous article [51], we have analyzed the $\Lambda$ single-particle energy levels up to the filling of 82 lambdas and predicted the magic number from 2 to 82. Therefore, in present case we analyze the higher levels above the filling of 82 $\Lambda$’s to look for the lambda magicity in higher region. To make this things, we focused on $^{304+nA}_{120}$, $^{360+nA}_{132}$ and $^{396+nA}_{138}$ superheavy multi-$\Lambda$ hypernuclei.

After carefully analyzing the single-particle energy diagram of $^{360+nA}_{132}$ a large gap exists between $1h_{11/2}$ and $2f_{7/2}$ thus resulting in the appearance of $\Lambda = 92$ as strong magic shell closure. This is followed by a relatively smaller gap between $2f_{5/2}$ and $1i_{13/2}$ thus making $\Lambda = 106$ a relatively weak magic number compared to $\Lambda = 92$. Further, the energy gaps between $1i_{13/2}$ and $3p_{1/2}$, $3p_{3/2}$ and $1i_{11/2}$ are large compared to the previous case. This results in the emergence of relatively strong magic numbers at $\Lambda = 120$ and $\Lambda = 126$, respectively. Moreover the energy gap between $1i_{11/2}$ and $2g_{9/2}$ is of comparable magnitude to the energy gap corresponding to $\Lambda = 92$ magic number and thus makes $\Lambda = 138$ a strong magic number. In addition, two gaps of almost same magnitude between $4s_{1/2}$ to $1j_{13/2}$ and $1j_{13/2}$ to $2h_{11/2}$ which hints towards the two strong magic numbers corresponding to $\Lambda = 184, 198$. Moreover, a large energy gap is observed between $4p_{1/2}$ and $1k_{15/2}$ which results in $\Lambda = 258$ as strong magic number. The identical $\Lambda$ magic numbers are reproduced in $^{396+nA}_{138}$ except for $\Lambda = 120$ which disappears or gets quenched. The shell closure at $\Lambda = 120$ is quenched in $^{304+nA}_{120}$, however large shell gap between $4p_{1/2}$ to $1k_{15/2}$ is appeared resulting the emergence of $\Lambda = 240$ magic number. Some inversion of lambda levels is noticed in considered multi-hypernuclear candidates. For example, $3s_{1/2}$ filled faster than $1i_{11/2}$ in case of $^{304+nA}_{120}$, $^{396+nA}_{138}$ which is inverted in case of $^{360+nA}_{132}$. The lambda level, $4p_{1/2}$ is the last filled level in $^{396+nA}_{138}$ and $^{360+nA}_{132}$ while this level got inverted with $1k_{15/2}$ in $^{304+nA}_{120}$. Therefore, an inversion between $\{4p_{1/2}, 1k_{15/2}\}$ and $\{3s_{1/2}, 1h_{11/2}\}$ is observed. The inversion or filling of higher levels faster than lower one and hence this type of filling is responsible for emerging the variation in prediction of new magic number and this is due to the spin-orbit interaction. This is also seen here for example, $\Lambda = 120$ is appeared to be $\Lambda$ magic number in some systems which is quenched in others. Also, $\Lambda = 240$ is appeared only in $^{304+nA}_{120}$ and $^{292+nA}_{120}$ but missed in other systems. In general, it is quite worth to mention that pronounced shell gaps for lambda number 92, 106, 126, 138, 184, 198, 240 and 258 are noticed in considered multi-$\Lambda$ hypernuclei and being suggest to be strong $\Lambda$ shell closures as well as reflected in $\delta_{2\Lambda}$ calculations.

![Fig. 4. Lambda single-particle energy levels for $^{304+nA}_{120}$ multi-$\Lambda$ hypernuclei for $\Lambda = 92, 106, 126, 138, 172, 184, 198, 240$ and 258. Filled levels are represented by solid lines whereas dashed line represents the unoccupied levels of the given system.](image)

![Fig. 5. Same as Fig. 4 but for superheavy mass $^{304+nA}_{132}$ multi-$\Lambda$ hypernuclei.](image)

3.4 Lambda magicity

The impetus for searching the magic numbers in hypernuclear physics earns its motivation from the ever evolving search of magic numbers in conventional nuclear physics. This encourages us to look for the magicity in strangeness sector. We conducted a study on the multi-$\Lambda$ hypernuclei under investigation within the powerful framework of spherical relativistic mean field with NL3* nucleon-meson parameterization in inclusion of $\Delta N$ as well as $\Delta \Lambda$ interactions. Prediction of magic numbers in superheavy region within various theoretical models is a debated subject which arouse our interest to look for magicity in strangeness sector. After analyzing the $S_{\Lambda}$ and $S_{2\Lambda}$ for the multi-$\Lambda$ hy-
In order to make a stringent test to our findings and to confirm our calculations based on observed that the predictions made on the basis of investigation on the basis of two lambda shell gap which is considered to be the strong signature of magicity. We observed at 184, 240 and 258 in the two lambda shell gap signaling the presence of strong shell closure. The peak at 106, 198 are of slightly less magnitude than the others. Moreover, in favour of \( \Lambda \) predictions the investigation of \( \Lambda \) pairing energy and effective \( \Lambda \) pairing gap has been made and its outcome are found to be in very close agreement with the survey made on the pretest of \( S_A \) and \( S_{2A} \). A pronounced peak is observed for \( \Lambda = 92, 126, 138 \) at \( \sim 100 \) MeV and \( \sim 198 \) MeV, 240 and 258 in the two lambda shell gap signaling the presence of strong shell closure. The peak at \( \Lambda = 106, 198 \) of are of slightly less magnitude than the others. Moreover, in favour of \( \Lambda \) predictions the investigation of \( \Lambda \) pairing energy and effective \( \Lambda \) pairing gap has been made and its outcome are found to be in very close agreement with the survey made on the pre-test of separation energy and shell gaps. Further, in order to support in these calculation and to reveal the magic nature of these numbers, we analyzed the lambda single-particle energy spectrum of \( ^{304+n}_{-120}, ^{360+n}_{-132} \) and \( ^{396+n}_{-138} \). Inspection of the single-particle energies for these chosen multi-\( \Lambda \) hypernuclei reveals a large shell gaps at \( \Lambda = 92, 106, 126, 138, 184, 198, 240 \) and 258. The shell gaps at \( \Lambda = 120 \) do not appear in all investigated candidates and make its appearance only in \( ^{360+n}_{132} \). This leads us to the conclusion that \( \Lambda = 120 \) is not a strong magic number in strangeness physics and might be considered as very feeble magic number; however, it is predicted to be a strong proton magic number supported by many theoretical investigations. The predicted \( \Lambda \) magic number in present calculations quite resembles with the nuclear magic number of superheavy region. Therefore, it can be concluded that \( \Lambda \) magicity resembles quite closely with the predicted magic numbers in enigmatic superheavy valley. The investigations about magic numbers might serve as a significant input to reveal important information regarding new nucleonic shell closures.

### Table 1. Lambda magic number produced in various considered multi-\( \Lambda \) hypernuclei are tabulated here.

| Hypernuclei | Lambda magic numbers |
|-------------|----------------------|
| \( ^{292+n}_{-120} \) | 92 106 - 126 138 184 198 240 258 |
| \( ^{304+n}_{-120} \) | 92 106 - 126 138 184 198 240 258 |
| \( ^{360+n}_{-132} \) | 92 106 120 126 138 184 198 - 258 |
| \( ^{370+n}_{-132} \) | 92 106 - 126 138 184 198 - 258 |
| \( ^{396+n}_{-138} \) | 92 106 - 126 138 184 198 - 258 |
| \( ^{396+n}_{138} \) | 92 106 - 126 138 184 198 - 258 |

#### 3.5 Lambda and nucleon spin-orbit interaction potentials

It has already been mentioned that YN data available is very scanty. Moreover, a very little know about the spin-dependent interaction such as spin-orbit force and hence the spin-orbit splitting in AN interaction has become a subject of study in strangeness physics. Lambda spin-orbit coupling has been found to be smaller than that of nucleons and no definite value is produced by the experiment. The smallness of \( \Lambda \) spin-orbit splitting in AN interaction was first suggested in the \((K^-, \pi^-)\) reaction on \( ^{16}O \). The smaller magnitude of \( \Lambda \) spin-orbit potential is encountered due to the presence of antisymmetric part of spin-orbit force in AN interaction and also by the energy difference of single-particle states of \( j = 1 - 1/2 \) and \( j = 1 + 1/2 \). The spin-orbit force in AN interaction has an antisymmetric part \( \{S_A - S_N\} \) in addition to the usual force. These two parts of spin-dependent force tend to cancel in AN interaction which leads to a small amount of lambda spin-orbit splitting in \( \Lambda \) single particle states. However, for quantitative description of spin-orbit force, various theoretical relativistic and non-relativistic models with suitable effective force have been employed. In non-relativistic model, the spin-orbit force is included manually while it is naturally developed in relativistic approaches such as RMF and spin-orbit potential emerges automatically with the empirical strength. This coupling is very crucial for quantitative description of nuclear or hypernuclear structure as well as in order to reproduce the empirical magic numbers. In present case, we analyzed the nucleon as well as \( \Lambda \) spin-orbit interaction potentials for considered multi-\( \Lambda \) hypernuclei as shown in Fig. 4, where we found a consistency in nucleonic and hyperonic splitting as previous predictions that implies reduction of \( \Lambda \) spin-orbit is about 30 - 40% of nucleonic case. For example; for \( ^{292+n}_{-120} \) at \( r = 3 \) fm the \( V_{\Lambda \Lambda}^{so} \) is about of 120 MeV and this value reduced to 45 MeV for \( V_{\Lambda \Lambda}^{so} \) that comes out to be around 40% of nucleonic potentials. It is quite evident that both the potentials are different by their depth but have the similar interaction profile. Further inspection reveal that injection of \( \Lambda \)'s significantly affects the nucleon as well as \( \Lambda \) spin-orbit potential to a large extent.
It is worthy to mention that the injected \( \Lambda \) hyperon affects the every piece of physical observables such as size, shape, binding energy, density distribution etc. Therefore, it is quite interesting as well as important to study the effects of large number of \( \Lambda \) on the nuclear density and hence the distribution of nucleons and \( \Lambda \) hyperons in considered multi-\( \Lambda \) systems is analyzed by means of density profile as shown in Figure 8. From this figure, we noticed that the successive addition of \( \Lambda \) hyperons reduces the magnitude of central nuclear density resulting increases the matter radii of the system. This is happened because injected \( \Lambda \)'s pushes the lambda itself towards the periphery of the system and this kind of behaviour of \( \Lambda \) distribution form a \( \Lambda \) skin structure.

4 Summary and conclusions

In present work, we performed spherical relativistic calculations with \( \Lambda N \) and \( \Lambda A \) interactions in multi-\( \Lambda \) hypernuclei and postulated the possible \( \Lambda \) magic number i.e., 92, 106, 120, 126, 138, 184, 198, 240, 258. These predictions have been made by the survey employing various signatures of magicity in strangeness sector like one- and two-\( \Lambda \) separation energy and two lambda shell gaps. We witnessed prominent peaks as well as single-particle energy gaps for lambda numbers 92, 126, 184 and 258. Furthermore, 

lambda numbers 92, 126, 184 and 258. Further, 

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lambda numbers 92, 126, 184 and 258. Further,
the predictions made in multi-$\Lambda$ hypernuclei under study resembles quite closely with the magic numbers in conventional nuclear theory suggested by various relativistic and non-relativistic theoretical models and these $\Lambda$ predictions also support the confirmation of nucleon magicity of superheavy regime. Thus, it can be inferred that $YN$ interaction is of similar nature as $NN$ interaction except its weaker strength compared to nucleonic case. The appearance of new lambda shell closures other than nucleonic one those predicted by various theoretical approaches in superheavy mass regime can be attributed to the relatively weak strength of spin-orbit interaction in strange sector. Further, to lend a support to our calculations, we explore the occurrence of spherical shell closures by doing survey on the basis of lambda pairing energy and effective lambda pairing gap and the results are quite consistent with the predictions made using $S_\Lambda$, $S_{\Sigma\Lambda}$ and $\delta_{\Sigma\Lambda}$ except for the appearance of magic number corresponding to $\Lambda = 156$ which emerges in lambda effective pairing gap and pairing energy within the considered multi-$\Lambda$ hypernuclei. Lambda single-particle spectrum is also analyzed to mark the energy shell gap for further strengthening the predictions made on the basis of separation energy and shell gaps. Lambda and nucleon spin-orbit interactions are analyzed to confirm the reduction in magnitude of $\Lambda$ spin-orbit interaction to the nucleonic case, however interaction profile is similar in both the cases. It is also concluded that addition of $\Lambda$'s significantly affects the nucleon as well as $\Lambda$ spin-orbit potential. Lambda and nucleon density distributions have been made to further confirm the impurity effect of $\Lambda$ hyperons which make the reduction in magnitude of central density of the cores. Lambda skin structure is also reported. The stability attributed to the doubly magic superheavy cores by addition of hyperons may serve as an impetus to the experimentalists for synthesizing these triply magic system in near future, might provide significant input and thereby may shed some light pertaining to novel features of new nucleon shell closures. Further, superheavy nucleonic cores (doubly magic) are seen to have more affinity to absorb large number of hyperons. Hence, it can be said that such systems replicate the strange hadronic matter containing multiple degrees of strangeness (multi-strange) such as $\Sigma'$, $\Xi'$ and hyperons which have got huge importance in nuclear astrophysics like neutron stars, hyperon stars etc.

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