Shock Wave Refraction at Gas Media Interface

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Abstract

Background/Objectives: This article discusses the problem of refraction of the shock wave with finite amplitude at contact discontinuity between two ideal gases is non-linear, and can be solved numerically for both stationary two-dimensional and transient one-dimensional cases. Methods/Statistical Analysis: Here considered the problem of interaction of one-dimensional shock wave (weak and strong) with the contact surface dividing two different gases with different chemical composition and temperature. The interface surface is considered parallel to the shock wave surface. Findings: The sound speed maximal increase in the layer after the contact discontinuity by means of the gas selection with minimal molar mass and its heating seems to be the most satisfying technical solution for the shock wave suppression. Applications/Improvements: This work can be used for both solution of the problem of the intense explosion shock wave attenuation, and the problem of optimal organisation of detonation combustion, where the wave attenuation at the interface should be minimal.

Keywords: Detonation Combustion, Progressive Shock Wave, Refraction

1. Introduction

The scientific literature covers the ways of the underwater shock wave amplitude contraction by means of the gas blanket over the protected body¹, and from the air wave by means of a gas protective layer creation with small molecular weight², high temperature³,⁴ or multi-phase layer⁵–⁷ described by the Rudinger model of equilibrium pseudo-gas⁸,⁹. It is expected that attenuation of the shock wave front happens, mainly, at the interface of two substances (contact discontinuity).

Also, there is an inverse problem. The detonation engines have the oxidant-fuel mixture ignited due to the shock wave action (in cases when we do not use forced ignition by a plug or fuse). It is clearly that for organisation of the quickest fuel combustion the shock wave intensity should be maximal, and its attenuation at the two mediums interface should be minimal.

The problem of refraction of the shock wave with finite amplitude at contact discontinuity between two ideal gases is non-linear, and can be solved numerically for both stationary two-dimensional¹⁰,¹¹ and transient one-dimensional¹²,¹³ cases (with the exception of quasi-acoustical model¹⁴–¹⁶ used for interaction analysis of weak waves with multilayer coating).

2. Method

Further we will examine the shock wave refraction $s_1$ Figure 1, a-c at contact discontinuity $K$ between two perfect gases with adiabatic exponents $\gamma_1$ and $\gamma_2$ on the left and on the right from the surface of discontinuity. According to the flow compatibility conditions, the gas initial speeds and static pressure on the discontinuity sides $K$ are equal $(u_1=u_2=u, p_1=p_2=p)$. The contact discontinuity value is characterized by the only parameter: Sonic speed ratio $A=a_2/a_1$.

Collision of wave $s_1$ and the surface of discontinuity generates new (discontinuity penetrating) shock wave $s_2$, reflected isentropic depression wave $r_3$ Figure 1(a) or shock wave $s_j$ Figure 1(b), and contact discontinuity $K_2$ with more speed as compares with discontinuity $K$. As opposed to wave’s $s_1$ and $s_2$, reflected wave $s_j$ or $r_3$ goes to the left relative to the gas flow. The reflected wave type
is unknown because of the problem specification. In the borderline case, the reflected wave degenerates into weak discontinuity $v_3$ Figure 1(c).

![Figure 1](image)

**Figure 1.** Shock wave-interface interaction diagrams. (a) Reflected depression wave. (b) Reflected shock wave. (c) Reflected weak discontinuity.

Each of considered waves $s_i$ and $r_i$ is specified with its intensity $J_i=p_{in}/p_i$, where $p_i$ and $p_{in}$ – the gas pressure before the wave and after it. Flow speed change $[u]_i=U_{in}/U_i$ is specified by intensity $J_i$ and sound speeds $a_i$ in gas before appropriate waves:

$$[u]_i = \frac{\chi(1-\varepsilon_j)(J_i-1)a_i}{\sqrt{(1+\varepsilon_j)(J_i+\varepsilon_j)}}$$

$$[u]_i = \frac{\chi(1-\varepsilon_j)}{\varepsilon_j}\left(1-J_i^{2\varepsilon_j/(1+\varepsilon_j)}\right)a_i$$

for shock waves and isentropic waves accordingly. At the same time, $\chi = 1$ for “right” ($s_i$, $s_j$) and $\chi = -1$ for “left” ($r_i$, $s_j$) waves. Value $\varepsilon_j = (\gamma_j-1)/(\gamma_j+1)$ is specified by adiabatic index $\gamma_j$ ($j=1,2$) of the gas where this wave propagation. Sound speed ratio $a_{in}/a_i$ also depends on intensity $J_i$:

$$\frac{a_i}{a_i} = \frac{J_i(1+\varepsilon_j)}{(J_i+\varepsilon_j)}$$

$$\frac{a_i}{a_i} = J_i^{2\varepsilon_j/(1+\varepsilon_j)}$$

for shock and isentropic waves. At last, amplitude $\Delta p_i$ of any wave $s_i$ or $r_i$ is specified by its intensity and static pressure before it:

$$\Delta p_i = \bar{p}_i - p_i = p_i(J_i - 1)$$

Condition of equality for the gas flow pressure and speed on discontinuity sides $K_2$ leads to the Equations set

$$J_3 = J_2$$

$$[u]_1 + [u]_3 = [u]_2,$$

allowing to calculate the wave intensity $s_2$ and $r_3$ ($s_3$), and, consequently, the following gas flow property at the arriving wave known intensity $J_i$.

### 3. Discussion

#### 3.1 The Shock Wave Amplification and Attenuation at the Contact Discontinuity

Equation (1) shows that penetration wave gets stronger than arriving wave ($J_i>1$, $\Delta p_i>\Delta p_j$) if the reflected wave is shock wave ($J_i>1$), and gets weaker otherwise ($J_i<1$). So, the reflected wave form ($r_3$ with intensity $J_i<1$ or $s_3$ with intensity $J_i>1$) testifies the fact of the shock wave attenuation or boost by the two gases interface, and the very value $J_3$ shows the boost/attenuation level. Criteria specifying the reflected wave and its limiting intensity are the study main subject. Besides intensity $J_3$, the shock wave amplitude change at the contact discontinuity is characterized with the value

$$K = \Delta p_2/\Delta p_1 = (J_2 - 1)/(J_1 - 1) = (J_3/J_1 - 1)/(J_1 - 1),$$

which, hereinafter, is called as the amplitude coefficient change.

The system (1-2) comes to the only six-degree equation relative to one of sought intensities (e.g., $J_3$), if the reflected wave is shock wave ($s_3$). If the isentropic wave $r_3$ is reflected, the wave intensity $s_2$ is calculated from the transcendental Equation. This equation has one real root satisfying the problem physical meaning. As the reflected disturbance type is originally unknown, the criterion of its change is of keen interest for the calculation method selection.

#### 3.2 The Weak Wave Amplification and Attenuation at the Contact Discontinuity

For the weak shock wave refraction ($J_i>1$), system solution (1-2) tends to a trivial one ($J_i>1$, $J_i>1$ – penetrating and reflected waves degenerate into weak discontinuities).
Amplification or attenuation of shock wave $s_2$ as compared with wave $s_1$ is characterized by amplitude change coefficient $C$. Value of coefficient $C$, even for weak wave refraction, depends on the reflected disturbance type. Particularly, for refraction of shock wave $s_1$ this value makes up

$$C = \frac{2\gamma_2}{(A\gamma_1 + \gamma_2)}$$

(3)

for equality of indexes of two gases adiabat ($\gamma_1 = \gamma_2$), the amplitude change is evaluated very simply: $C=2/(A+1)$.

In case of reflection of depression wave $r_3$, attenuation of the transmitted wave amplitude is expressed by:

$$C = \frac{3\gamma_2}{(A\gamma_1 + 2\gamma_2)}.$$  

(4)

and, particularly, $C = 3/(A+2)$ for $\gamma_1 = \gamma_2$. Figure 2 shows the interface of domains of applicability of Equations (3) and (4): domain 1 over shown straight line $A = \gamma_2/\gamma_1$ meets the reflection of depression wave $r_3$, when the shock wave going through the contact discontinuity gets weaker. For inadequate sound speed, shock wave $s_3$ is reflected in the gas layer following the contact discontinuity (small values $A$ meeting domain 2), and the wave going through the two mediums interface gets stronger. So, the shock wave gets attenuated, first of all, for high sound speed in the gas after the contact discontinuity.

3.3 Sound Speed Effect in Gas Following the Contact Discontinuity for the Weak Wave Amplification and Attenuation at Contact Discontinuity

Sound speed influence in gas following the contact discontinuity onto amplification or attenuation of air-shock wave ($\gamma_1 = 7/5$) front by the two gases interface is shown in Figure 3. Like the previous case, application of the problem parameters situated over curves 1-6 meets the depression wave reflection, and under them – the shock wave reflection. Accordingly, the original shock wave gets weaker or stronger.

The reflected disturbance type changes when the incident wave intensity meets ratio

$$I_1 = \frac{\gamma_1 A^2(y_1 -1) - \gamma_2(y_2 -1)}{\gamma_1 A^2(y_1 +1) - \gamma_2(y_2 +1)}$$

(5)

or, another form

$$A = \frac{\sqrt[\gamma_2]{[\gamma_2 + 1]h_1 - \gamma_2 - 1]}{\sqrt[\gamma_1]{[\gamma_1 + 1]h_1 - (\gamma_1 - 1)}}$$

Figure 2. Domains of amplification (2) and attenuation (1) of the weak shock wave amplitude.

If the two gases adiabatic indexes are equal, Equation (5) cannot be satisfied (except for trivial case $I_1 = 1$ for $A=1$). This means that the type of disturbance reflected by the contact discontinuity does not depend on the incident wave intensity: For $A > 1$ the depression wave is always reflected, and for $A < 1$ – the shock wave.

3.4 Strong Shock Wave Refraction

For the strong shock wave refraction ($I_1 \rightarrow \infty$), the reflected wave intensity makes up the cubic Equation mean root

$$\sum_{n=0}^{3} B_n (\varepsilon_1, \varepsilon_2, A) f_3^n = 0,$$

(6)

if shock wave $s_3$ is reflected, and the only root of transcendent Equation

$$(1-\varepsilon_1)(\varepsilon_2 + \varepsilon_1)I_1 + (1-\varepsilon_2)\sqrt{1+\varepsilon_1} \left[f_3^{2*}\sqrt[\varepsilon_2]{(1+\varepsilon_2)^2-1}\right] = \varepsilon$$

(7)

for reflection of depression wave $r_3$. Amplitude change coefficient, like the reflected wave intensity, tends to the same values (7-8). For adiabatic indexes equality ($\varepsilon_1 = \varepsilon_2 = \varepsilon$) coefficients $B_n$ from Equation (6) and ratio from Equation (7) are written very simply:
Shock Wave Refraction at Gas Media Interface

Figure 3. Amplification and attenuation domain boundary for shock wave at refraction into gas with adiabatic indexes: 1. $\gamma_2 = 9/7$. 2. $\gamma_2 = 1.3$. 3. $\gamma_2 = 7/5$. 4. $\gamma_2 = 1.5$. 5. $\gamma_2 = 5/3$. 6. $\gamma_2 = 2$.

$$B_0 = -4\varepsilon^2 + A_0^2 - 2A_0^2(3 - 2\varepsilon) + (1 + 2\varepsilon)(1 + 2\varepsilon),$$

$$B_2 = A_2^2 - 2\varepsilon^2(1 - \varepsilon)^2 - 2\varepsilon(1 + 2\varepsilon), B_2 = (A_2^2 - \varepsilon)^2,$$

$$A\sqrt{\varepsilon^3} + \sqrt{1 + \varepsilon^3}^{2\varepsilon/(1 + \varepsilon)} + \varepsilon + \sqrt{1 + \varepsilon} = 0.$$

In the extreme case of the shock wave reflection from the interface of “extremely cold” gas ($J_1 \to \infty$, $A \to 0$) the amplitude ratio of the penetration and incident wave depends on adiabatic indexes $\gamma_1$:

$$\tilde{N} = (1 + 2\varepsilon_1)/\varepsilon_1.$$

Shock wave intensity $s_1$ tends to the same limit ($J_1 \to \infty$ for $\gamma_1 = 7/5$) which also meets the reflection case from the solid surface. The “extremely cold” gas interface actually serves as a rigid wall which, as we know, increases excess pressure for weak shock wave reflection by two times, and for strong wave reflection – by eight times.

Analytic description (6-8) of the strong shock wave refraction is limited with development of the gas imperfect properties from the intensity range $J_1 = 25\div30$.

3.5 The Gas Adiabatic Indexes Influence after the Contact Discontinuity onto the Shock Wave Amplification or Attenuation

Figure 4 shows the dependence of the amplification/attenuation coefficient of the shock wave at the contact discontinuity at different relations the speed of sound in the respective environments and at different adiabatic index. Curves 1-5 Figure 4 show the air-shock wave ($\gamma_1 = 7/5$) amplitude change for refraction into gases with dif-

Figure 4. Values of the amplitude coefficient change for shock wave $C = J_2/J_1$, for adiabatic indexes of the layer following contact discontinuity. (a) $\gamma_2 = 2$. (b) $\gamma_2 = 5/3$. (c) $\gamma_2 = 7/5$. (d) $\gamma_2 = 1.3$. Sound speed ratios $A$: 1. $A = 5$. 2. $A = 2$. 3. $A = 1$. 4. $A = 0.5$. 5. $A = 0.2$. 

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ferent adiabatic indexes and ratios of sound speeds. It is obviously that the gas layer adiabatic index $\gamma_2$ after contact discontinuity has significantly weaker influence onto the penetration wave amplitude as compared with sound speed $a_2$.

4. Conclusion

We have considered the problem of interaction of one-dimensional shock wave (weak and strong) with the contact surface dividing two different gases. The interface surface is considered parallel to the shock wave surface.

The given analytic and numerical results do not depend on the original speed and contact discontinuity $K$, which shows invariance of solutions in relation to the inertial coordinate system selection.

The sound speed maximal increase in the layer after the contact discontinuity by means of the gas selection with minimal molar mass and its heating seems to be the most satisfying technical solution for the shock wave suppression.

Accordingly, for the shock wave amplification, the fuel-air mixture after the contact discontinuity must be cooled.

This analysis can be used for solutions of the explode load protection, and for detonation engine designing.

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6. References

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