Dicke and Fano effects in single photon transport

Maritza Ahumada, Natalia Cortés, M. L. Ladrón de Guevara, and P. A. Orellana

1Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile

The single photon transport through a one dimensional array of cavities is studied theoretically. Analytical expressions of the reflection and transmission are given. The transmission shows an energy spectrum with forbidden and allowed bands that depends on the detuning parameter of the system. We show that the allowed miniband is formed due to the indirect coupling between the atoms in each cavity. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We discuss the analogy between this phenomenon with the Fano and Dicke effects.

I. INTRODUCTION

In the last years the single photon transport in one dimensional waveguide scattering by atoms or quantum dots has attracted great interest both theoretically and experimentally [1–8]. The single photon transport has potential application in optical communication, optical quantum computer, quantum information, and quantum devices and as photon transistors. The possibility of implementing photon transistors in analogy to the electron transistor is a great challenge. Several kinds of waveguides have been involved, such as metal nanowires [9], silicon wires [10], quantum resonators, etc. The behavior of photons in quantum resonator arrays is in analogy to the movement of electrons in periodic potentials. The photon can feel the presence of the quantum resonators and it can be transmitted through allowed mini bands and reflected by forbidden mini bands. The single photon scattering properties in the coupled resonator array waveguide embedded with a two-level or three-level atom have also been explored recently [12–14]. In this context, recently Yue Chang et al. [13] considered new quantum devices based on a quantum resonator array. In these works, the authors have proposed a setup based on the coupled-resonator array with doped atoms, which is expected to exhibit perfect reflection within a wide spectrum of frequencies, and thus can perfectly reflect an optical pulse, or namely a single photon wave packet. They used a thick atomic mirror, which is made of an array of two-level atoms individually doped in some cavities arranged in a coordinate region of the one-dimensional coupled-cavity waveguide.

Quantum interference effects as Fano effect play a crucial role in single photon transport [15]. Very recently Bei-Bei Li et al. [16] experimentally observed Fano resonances in a single toroidal micro-resonator, in which two modes are excited simultaneously through a fiber taper. By adjusting the fiber-cavity coupling strength and the polarization of incident light, the Fano-like resonance line shape can be controlled.

In this work we report further progress along the lines indicated above. We study the single photon transport through a system of a 1D array cavity. We obtain analytical expressions of the reflection and transmission. We show that the transmission displays an energy spectrum with forbidden and allowed bands that depends on the detuning parameter of the system. We show that an allowed miniband is formed by the indirect coupling between the levels of the atom in each cavity. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We discuss the analogy between this phenomenon with the Fano and Dicke effects.

II. MODEL

The system under consideration is shown schematically in Fig. 1. This consists in a 1D coupled cavity array with an embedded N cavity-atom array with one V-type three-level atom in each cavity.

The total Hamiltonian is described by the sum of a free atom Hamiltonian of the atom $\mathcal{H}_a$, the Hamiltonian describing the propagation of a photon through the cavities $\mathcal{H}_p$, and the term describing the atom-field interaction represented by a Jaynes-Cummings Hamiltonian under the rotating wave approximation $\mathcal{H}_I$,

$$\mathcal{H} = \mathcal{H}_p + \mathcal{H}_a + \mathcal{H}_I,$$  \hspace{1cm} (1)

$$\mathcal{H}_p = \sum_{i=-\infty}^{\infty} \omega c_i^\dagger c_i + \sum_{i=-\infty}^{\infty} v(c_{i+1}^\dagger c_i + c_{i+1}^i c_i)$$  \hspace{1cm} (2)
\[ \mathcal{H}_a = \sum_{j=1}^{N} (\omega'_a |a_j\rangle \langle a_j| + \omega'_c |e_j\rangle \langle e_j|), \]

\[ \mathcal{H}_t = \sum_{j=1}^{N} [g_a (\hat{c}_j |a_j\rangle \langle g_j| + \hat{c}^+_j |g_j\rangle \langle a_j|] + g_c (\hat{c}_j |e_j\rangle \langle g_j| + \hat{c}^+_j |g_j\rangle \langle e_j|)] \]

where \( \hat{c}^+_j \) and \( \hat{c}_j \) are, respectively, the creation and annihilation operators of one photon in the \( j \)-th cavity, \( v \) is the coupling between the cavities, \( \omega'_{a,c} \) are the energy transitions and \( g_a \) (\( g_c \)) is the coupling constant between the field and transition \( |g\rangle \rightarrow |a\rangle \) (\( |g\rangle \rightarrow |e\rangle \)). In what follows we assume \( g_a = g_c = \tilde{g} \).

The stationary state can be written as

\[ |E\rangle = \sum_{i=-\infty}^{\infty} u_i |1_i, g\rangle + \sum_{j=1}^{N} [d_{a,j} |0, a_j\rangle + d_{e,j} |0, e_j\rangle], \]

where \( u_i \) represents the amplitude to find a photon in the \( i \)-th cavity and the corresponding atom in the ground state, and \( d_{a,j} \) \( (d_{e,j}) \) the probability amplitude to find no photon in the cavity array and the atom in the \( j \)-th cavity in the excited state \( e \), while the rest of the atoms keep in the ground state.

From the eigenvalues equation \( \mathcal{H}|E\rangle = E|E\rangle \) it is obtained the following linear difference equations

\[ (E - \omega) u_j = v (u_{j+1} + u_{j-1}), \quad \text{for} \quad j \leq 0 \quad \text{and} \quad j > N, \]

\[ (E - \omega) u_j = v (u_{j+1} - u_{j-1}) + \tilde{g} (d_{c,j} + d_{a,j}), \]

\[ (E - \omega) d_{a,j} = \tilde{g} u_j, \]

\[ (E - \omega) d_{e,j} = \tilde{g} u_j, \]

for \( j = 1, \ldots, N \). From Eqs. (7b) and (7c) we obtain

\[ d_{e,j} = \frac{\tilde{g} u_j}{E - \omega^2}, \quad d_{a,j} = \frac{\tilde{g} u_j}{E - \omega^2}, \]

which inserted in Eq. (7a) make Eqs. (7) reduce to the single equation

\[ (E - \omega - \tilde{\varepsilon}) u_j = v (u_{j-1} + u_{j+1}), \quad j = 1, \ldots, N, \]

where

\[ \tilde{\varepsilon} = \tilde{g}^2 \frac{2E - \omega'_a - \omega'_c}{(E - \omega'_a)(E - \omega'_c)} \]

is the renormalized energy. Thus, the problem reduces to that of a linear chain of \( N \) sites with effective energies \( \tilde{\varepsilon} \).

In order to study the solutions of Eq. (8), we assume that the photon is described by a plane wave incident from the far left with unity amplitude and a reflection amplitude \( r \) and at the far right by a transmission amplitude \( t \). That is,

\[ u_j^{(k)} = e^{ikj} + re^{-ikj}, \quad j < 1, \]

\[ u_j^{(k)} = te^{ikj}, \quad j > N. \]

Inserting Eqs. (11) in Eq. (8) we obtain the following dispersion relation for the incident photon

\[ E = \omega + 2r \cos(k). \]

From Eqs. (9), (10), and (11) we obtain an inhomogeneous system of equations for the probability amplitudes \( u_j \) \( (j = 1, \ldots, N) \), \( r \) and \( t \), leading to the following expression for \( t \)

\[ t = \frac{2e^{-ikN}}{\Delta} \sin k, \]

with \( \Delta \) given by

\[ \Delta = e^{-ik} \frac{\sin (N + 1)q}{\sin q} + 2 \frac{\sin Nq}{\sin q} + e^{ik} \frac{\sin (N - 1)q}{\sin q}, \]

if \( |(E - \omega - \tilde{\varepsilon})/2v| \leq 1 \) and

\[ \Delta = e^{-ik} \frac{\sinh(N + 1)\kappa}{\sinh \kappa} + 2 \frac{\sinh N\kappa}{\sinh \kappa} + e^{ik} \frac{\sinh(N - 1)\kappa}{\sinh \kappa}, \]

if \( |(E - \omega - \tilde{\varepsilon})/2v| \geq 1 \). The reflection and transmission probabilities are \( R = |r|^2 \) and \( T = |t|^2 \). Then if \( |(E - \omega - \tilde{\varepsilon})/2v| \leq 1 \) we have

\[ R = \frac{\sin^2(Nq)(\cos q + \cos k)^2}{\sin^2(Nq)(\cos k \cos q + 1)^2 + [\sin k \sin q \cos(Nq)]^2}, \]

\[ T = \frac{1}{\cos^2(Nq) + [\sin(Nq)(1 + \cos q \cos k)/\sin q \sin k]^2}, \]

where \( q = \cos^{-1}[-(E - \omega - \tilde{\varepsilon})/2v] \). We note that these probabilities oscillate as a function of both \( N \) and \( q \). On the other hand, if \( |(E - \omega - \tilde{\varepsilon})/2v| \geq 1 \) we get

\[ R = \frac{\sinh^2(N\kappa)(\cosh k + \cosh k)^2}{\sinh^2(N\kappa)(\cosh k \cosh k + 1)^2 + [\sinh k \sinh \kappa \cosh(N\kappa)]^2}, \]

\[ T = \frac{1}{\cosh^2(N\kappa) + [\sinh(N\kappa)(1 + \cosh k \cosh k)/\sinh k]^2}, \]

where \( \kappa = \cosh^{-1}[-(E - \omega - \tilde{\varepsilon})/2v] \). Then in this energy region \( T \sim e^{-2N\kappa} \) tends to zero when \( N \) is large and \( R \) tends to unity.

### III. RESULTS

To avoid the profusion of free parameters, for the sake of clarity we set \( \Omega = E - \omega \), the energies of the atoms as, \( \omega'_a = \omega_0 - \Delta \omega \), and \( \omega'_c = \omega_0 + \Delta \omega \) and we express all energies in units of \( \gamma \) with \( \gamma = g^2/2v \), hereafter.
In what follows, we consider the degenerate case, \( \Delta \omega = 0 \), for different values of \( N \). Simple expressions for the transmission and reflection can be readily obtained for \( N = 1 \). For \( N = 1 \), the reflection reduces to a Breit-Wigner line shape of semi-width \( \gamma \),

\[
R = \frac{\gamma^2}{(\Omega - \omega_0)^2 + \gamma^2},
\]

and the transmission takes the form of a symmetrical Fano line shape,

\[
T = \frac{(\Omega - \omega_0)^2}{(\Omega - \omega_0)^2 + \gamma^2} = \frac{(\epsilon + q)^2}{\epsilon^2 + 1},
\]

with \( q = 0 \) and \( \epsilon = (\Omega - \omega_0)/\gamma \). Notice that \( T \) vanishes just at \( \Omega = \omega_0 \). This result can be interpreted as follows. The photon has two paths when going from left to right, a direct and indirect one. In the latter the photon is absorbed and emitted by the atoms. The destructive interference between these two paths gives rise to the Fano effect in this case.

Figure 2 shows the transmission and reflection probabilities versus the detuning \( \Omega - \omega_0 \) for different values of \( N \). As \( N \) grows (Fig 2b-d) a forbidden mini band (gap) is formed in \( T \). It is apparent that \( R \) (\( T \)) tends to unity (zero) within a range \([-2\gamma, 2\gamma]\), and the system behaves as a quantum mirror within this interval of energies.

![Figure 2](image1)

**FIG. 2.** (Color online) Transmission (blue solid line) and reflection (red dash line) as a function of the detuning \( \Omega - \omega_0 \) for \( \Delta = 0 \) and a) \( N = 1 \), b) \( N = 3 \), c) \( N = 5 \) and d) \( N = 7 \).

Now, let us consider the situation with \( \Delta \omega \neq 0 \). Figure 3 displays the reflection (red dash line) versus detuning for \( \Delta = 0.5 \), for a) \( N = 1 \), b) \( N = 3 \), c) \( N = 5 \) and d) \( N = 7 \).

![Figure 3](image2)

**FIG. 3.** (Color online) Transmission (blue solid line) and reflection (red dash line) versus detuning for \( \Delta = 0.5 \), for a) \( N = 1 \), b) \( N = 3 \), c) \( N = 5 \) and d) \( N = 7 \).

![Figure 4](image3)

**FIG. 4.** (Color online) Transmission versus \( \Omega \) for \( \Delta = 1 \) (black solid line) \( \Delta = 0.5 \) (red dash line) and \( \Delta = 0.25 \) (blue dotted line), for \( N = 1 \) (upper panel) \( N = 7 \) (lower panel).

Figure 4 displays a zoom of the transmission at the center of the band for \( \Delta \omega = 0.5\gamma \), for two values for \( N \), \( N = 1 \) (upper panel) and \( N = 7 \) (lower panel). We note the dramatic change of the width of the miniband as \( \Delta \omega \) decreases. In fact, for \( N = 1 \), it is straightforward to show that for \( \Delta \omega \ll \gamma \) the width of this allowed transmission band is \( \Delta \omega^2/2\gamma \), the reflection can be written as a difference between two Breit-Wigner line-shapes with \( \gamma \) and \( \delta \), respectively, and the transmission can be written as a superposition of a symmetrical Fano line-shape and a Breit-Wigner line-shape,

\[
R \approx \frac{\gamma^2}{(\Omega - \omega_0)^2 + \gamma^2} - \frac{\delta^2}{(\Omega - \omega_0)^2 + \delta^2},
\]

\[
T \approx \frac{(\Omega - \omega_0)^2}{(\Omega - \omega_0)^2 + \gamma^2} + \frac{\delta^2}{(\Omega - \omega_0)^2 + \delta^2},
\]

with \( \delta = \Delta \omega^2/2\gamma \).
The above phenomenon resembles the Dicke effect, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wavelength much larger than the separation between them. The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. The former state, coupled weakly to the electromagnetic field, is called subradiant, and the latter, strongly coupled, superradiant state. In the present case this effect is due to indirect coupling between the atom states through the common cavity. The states strongly coupled to the continuum give a forbidden miniband with width $4\gamma$ and the states weakly coupled to the continuum give an allowed Dicke miniband with width $\delta$. This effect is a special case the Fano-Feshbach resonances in the systems exhibiting more than one resonance.

A physical realization of our setup may be made in a metal nanowire coupled to quantum dots. In this realization the metal nanowire plays the role of 1D continuum and the region where the quantum dots are coupled to the nanowire.

IV. SUMMARY

In this work we have studied the transport of a single photon through a system of a 1D array of cavities. We have obtained analytical expressions of the transmission and reflection. We have shown that the transmission displays an energy spectrum with forbidden and allowed bands that depend on the detuning parameter of the system. We have shown that the allowed miniband is formed by the indirect coupling between the levels of the atom in each cavity. In addition, the band edges can be controlled by the degree of detuning of the level of the atoms. We have discussed the analogy between this phenomenon with the Fano and Dicke effects. This setup seems a suitable system to study these effects in experiments on single photon transport.

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[1] Nam-Chol Kim, Jian-Bo Li, Zhong-Jian Yang, Zhong-Hua Hao, and Qu-Quan Wang, App. Phys. Lett. 97, 061110 (2010).
[2] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf, Nature 436 87 (2005).
[3] I. I. Smolyaninov, A. V Zayats, A. Gungor and C.C Davis, Phys. Rev. Lett. 88, 187402 (2002).
[4] Abram L. Falk, Frank H. L. Koppens, Chun L. Yu, Kibum Kang, Nathalie de Leon Snapp, Alexey V. Akimov, Moon-Ho Jo, Mikhail D. Lukin and Hongkun Park, Nature Phys. 5 807 (2009)
[5] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup and H. J. Kimble, Nature 431 163 (2005).
[6] Darrick E. Chang, Anders S. Sorensen, Eugene A. Demler and Mikhail D. Lukin Nature Phys. 3 807 (2007).
[7] J. Kim1, O. Benson, H. Kan and Y. Yamamoto, Nature 397 500 (1999).
[8] Barak Dayan, A. S. Parksins, Takao Aoki,E. P. Ostby, K. J. Vahala, H. J. Kimble, Science 319 1062 (2007).
[9] Ditlbacher, H.; Hohenau, A.; Wagner, D.; Kreibig, U.; Rogers, M.; Hofer, F.; Aussenegg, F.R.; Krenn, J.R.
Phys. Rev. Lett. 2005, 95, 257403.
[10] Zhu, S.; Fang, Q.; Yu, M.B.; Lo, G.Q.; Kwong, D.L. Opt. Express 2009, 17, 20891.
[11] Lan Zhou, Z. R. Gong, Yu-xi Liu, C. P. Sun and Franco Nori, PRL 101 100501 (2008)
[12] Z.R. Gong, H. Han, Lan Zhou and C.P. Sun, Phys. Rev. A 78, 053806 (2008).
[13] Yue CHang, Z.R. Gong and C.P. Sun, Phys. Rev. A 83, 013825 (2011).
[14] Lan Zhou, Z.R. Gong and C.P. Sun, Phys. Rev. A 83, 100501 (2011).
[15] U. Fano, Phys. Rev. 124, 1866 (1961).
[16] Bei-Bei Li,1 Yun-Feng Xiao,Chang-Ling Zou, Yong-Chun Liu, Xue-Feng Jiang, You-Ling Chen, Yan Li and Qi-huang Gong, Appl. Phys. Lett, 5098, 021116 (2011).
[17] S. E. Harris, Phys. Today 50, 36 (1997).
[18] Qiang Li, Ziyang Zhang, Jing Wang, Min Qiu, and Yikai Su, Opt. Expr. 17 933 (2009).
[19] R. H. Dicke, Phys. Rev. 89, 472 (1953).
[20] Jun Pan, Sunil Sandhu, Yijie Huo, Norbert Stuhrmann, Michelle L. Povinelli, James S. Harris, M. M. Fejer, and Shanhui Fan, Phys. Rev. B 81, 041101(R) (2010)
[21] P. A. Frasert and S. K. Burley, Eur. J. Phys. 3 230 (1982).