COLD + HOT DARK MATTER AFTER SUPER-KAMIOKANDE

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Abstract

The recent atmospheric neutrino data from Super-Kamiokande provide strong evidence of neutrino oscillations and therefore of non-zero neutrino mass. These data imply a lower limit on the hot dark matter (i.e., light neutrino) contribution to the cosmological density $\Omega_\nu > 0.001$ — almost as much as that of all the stars in the universe — and permit higher $\Omega_\nu$. The “standard” COBE-normalized critical-matter-density (i.e., $\Omega_m = 1$) Cold Dark Matter (CDM) model has too much power on small scales. But adding to CDM neutrinos with mass of about 5 eV, corresponding to $\Omega_\nu \approx 0.2$, results in a much improved fit to data on the nearby galaxy and cluster distribution. Indeed, the resulting Cold + Hot Dark Matter (CHDM) cosmological model is arguably the most successful $\Omega_m = 1$ model for structure formation [1, 2, 3, 4]. However, other recent data has begun to make the case for $\Omega_m \lesssim 0.6$ fairly convincing. In light of all this new data, we reconsider whether cosmology still provides evidence favoring neutrino mass of a few eV in flat models with cosmological constant $\Omega_\Lambda = 1 - \Omega_m$. We find that the possible improvement of the low-$\Omega_m$ flat ($\Lambda$CDM) cosmological models with the addition of light neutrinos appears to be rather limited.

1 Evidence for Neutrino Mass

Recent articles [3, 4] conclude that, of all the currently popular cosmological models, the one whose predictions agree best with the data on the Cosmic Microwave Background (CMB) anisotropies and the large-scale distribution of galaxies and clusters in the nearby universe is the $\Omega_m = 1$ Cold + Hot Dark Matter (CHDM) model. In this model, most of the matter (70% of the total) is cold dark matter, 20% is hot dark matter, and 10% is ordinary baryonic matter. Hot dark matter is defined as particles that were still moving at nearly the speed of light at about a year after the Big Bang, when the temperature was about a keV and gravity first had time to encompass the amount of matter in a galaxy like the Milky Way; cold dark matter is particles that were moving sluggishly then. Few-eV mass neutrinos are the standard example of hot dark matter. Three species of neutrinos — $\nu_e$, $\nu_\mu$, and $\nu_\tau$ — are known to

Invited review at the Xth Rencontres de Blois meeting The Birth of Galaxies, eds. B. Guiderdoni et al, (Gif-sur-Yvette: Edition Frontieres).
exist. The thermodynamics of the early universe implies that, just as there are today about 400 CMB photons per cm$^3$ left over from the Big Bang, there are about 100 per cm$^3$ of each of the three species of light neutrino (including the corresponding anti-neutrinos). There are thus about $4 \times 10^8$ times as many of each species of neutrino as there are electrons or protons, and as a result a neutrino mass of only 4.7 eV, a mere $10^{-5}$ to $20\%$ of critical density in the CHDM model with $h = 0.5$. The relationship between the total neutrino mass $m(\nu)$ and the fraction $\Omega_\nu$ of critical density that neutrinos contribute is $\Omega_\nu = m(\nu)/(92h^2\text{eV})$, where $h = 0.5 - 0.8$ is the expansion rate of the universe (Hubble constant $H_0$) in units of 100 km/s/Mpc.

Direct measurements of neutrino masses have given only upper limits. The upper limit on the electron neutrino mass is roughly 10-15 eV; the Particle Data Group \cite{5} notes that a more precise limit cannot be given since unexplained effects have resulted in significantly negative measurements of $m(\nu_e)^2$ in recent precise tritium beta decay experiments. There is a (90\% CL) upper limit on an effective Majorana neutrino mass of 0.45 eV from the Heidelberg-Moscow $^{76}\text{Ge}$ neutrinoless double beta decay experiment \cite{5}. The upper limits from accelerator experiments on the masses of the other neutrinos are $m(\nu_\mu) < 0.17 \text{ MeV}$ (90\% CL) and $m(\nu_\tau) < 18 \text{ MeV}$ (95\% CL) \cite{5}, but since stable neutrinos with such large masses would certainly “overclose the universe” (i.e., prevent it from attaining its present age), cosmology implies a much lower upper limit on these neutrino masses.

But there is mounting astrophysical and laboratory data suggesting that neutrinos oscillate from one species to another \cite{5}, which can only happen if they have non-zero mass. Of these experiments, the ones that until the new Super-Kamiokande data were regarded as probably most secure were those concerning solar neutrinos. But the experimental results that are most relevant to neutrinos as hot dark matter are the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos and the higher energy atmospheric (cosmic ray) neutrino experiments Kamiokande, Super-Kamiokande, MACRO, and Soudan 2.

Older Kamiokande data \cite{8} showed that, for events attributable to atmospheric neutrinos with visible energy $E > 1.3 \text{ GeV}$, the deficit of $\nu_\mu$ increases with zenith angle. The much larger Super-Kamiokande detector has confirmed and extended the results of its smaller predecessor \cite{8}. These data imply that $\nu_\mu \rightarrow \nu_\tau$ oscillations occur with a large mixing angle $\sin^2 2\theta > 0.82$ and an oscillation length several times the height of the atmosphere, which implies that $5 \times 10^{-4} < \Delta m^2_{\mu\tau} < 6 \times 10^{-3} \text{ eV}^2$ (90\% CL). (Neutrino oscillation experiments measure not the masses, but rather the difference of the squared masses, of the oscillating species, here $\Delta m^2_{\mu\tau} \equiv |m(\nu_\tau)^2 - m(\nu_\mu)^2|$. This in turn implies that if other data requires either $\nu_\mu$ or $\nu_\tau$ to have large enough mass ($\gtrsim 0.5 \text{ eV}$) to be a hot dark matter particle, then they must be nearly equal in mass, i.e., the hot dark matter mass would be shared between these two neutrino species. Both the new Super-Kamiokande atmospheric $\nu_e$ data and the lack of a deficit of $\bar{\nu}_e$ in the CHOOZ reactor experiment \cite{10} make it quite unlikely that the atmospheric neutrino oscillation is $\nu_\mu \rightarrow \nu_e$. If the oscillation were instead to a sterile neutrino, the large mixing angle implies that this sterile species would become populated in the early universe and lead to too much $^4\text{He}$ production during the Big Bang Nucleosynthesis epoch \cite{11}. (Sterile neutrinos are discussed further below.) It may be possible to verify that $\nu_\mu \rightarrow \nu_\tau$ oscillations occur via a long-baseline neutrino oscillation experiment. This would look for missing $\nu_\mu$ due to $\nu_\mu \rightarrow \nu_\tau$ oscillations with a beam of $\bar{\nu}_\mu$ from the Japanese KEK accelerator directed at the Super-Kamiokande detector, with more powerful Fermilab-Soudan and possibly CERN-Gran Sasso long-baseline experiments later which could look for $\tau$ appearance. However, the lower range of $\Delta m^2_{\mu\tau}$ favored by the Super-Kamiokande data will make such experiments more difficult than was hoped based on the earlier Kamiokande data.

The observation by LSND of events that appear to represent $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations followed by $\bar{\nu}_e + p \rightarrow n + e^+$, $n + p \rightarrow D + \gamma$, with coincident detection of $e^+$ and the 2.2 MeV neutron-
capture $\gamma$-ray, suggests that $\Delta m^2_{\mu\nu} > 0 \, [12]$. The independent LSND data [13] suggesting that $\nu_\mu \rightarrow \nu_e$ oscillations are also occurring is consistent with, but has less statistical weight than, the LSND signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations. Comparison of the latter with exclusion plots from other experiments allows a range $10 \text{ eV}^2 \lesssim \Delta m^2_{\mu\nu} \lesssim 0.2 \text{ eV}^2$. The lower limit in turn implies a lower limit $m_\nu \gtrsim 0.45 \, \text{eV}$, or $\Omega_\nu \gtrsim 0.02 (0.5/h)^2$. This implies that the contribution of hot dark matter to the cosmological density is larger than that of all the visible stars $\Omega_* \approx 0.004$ [14]. Such an important conclusion requires independent confirmation. The KArlsruhe Rutherford Medium Energy Neutrino (KARMEN) experiment has added shielding to decrease its background so that it can probe a similar region of $\Delta m^2_{\mu\nu}$ and neutrino mixing angle; it has not seen events attributable to $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations, but the implications are not yet clear. The Booster Neutrino Experiment (BOONE) at Fermilab could attain greater sensitivity.

The observed deficit of solar electron neutrinos in three different types of experiments suggests that some of the $\nu_e$ undergo Mikheyev-Smirnov-Wolfenstein matter-enhanced oscillations $\nu_e \rightarrow \nu_x$ to another species of neutrino $\nu_x$ with $\Delta m^2_{e\nu} \approx 10^{-5} \, \text{eV}^2$ as they travel through the sun [15], or possibly “Just-So” vacuum oscillations with even smaller $\Delta m^2_{e\nu}$ [16]. The LSND $\nu_\mu \rightarrow \nu_e$ signal with a much larger $\Delta m^2_{\mu\nu}$ is inconsistent with $x = \mu$, and the Super-Kamiokande atmospheric neutrino oscillation data is inconsistent with $x = \tau$. Thus a fourth neutrino species $\nu_s$ is required if all these neutrino oscillations are actually occurring. Since the neutral weak boson $Z^0$ decays only to three species of neutrinos, any additional neutrino species $\nu_s$ could not couple to the $Z^0$, and is called “sterile.” This is perhaps distasteful, although many modern theories of particle physics beyond the standard model include the possibility of such sterile neutrinos. The resulting pattern of neutrino masses would have $\nu_e$ and $\nu_s$ very light, and $m(\nu_\mu) \approx m(\nu_\tau) \approx (\Delta m^2_{e\mu})^{1/2}$, with the $\nu_\mu$ and $\nu_\tau$ playing the role of the hot dark matter particles if their masses are high enough [17]. This neutrino spectrum might also explain how heavy elements are synthesized in core-collapse supernova explosions [18]. Note that the required solar neutrino mixing angle is very small, unlike that required to explain the atmospheric $\nu_\mu$ deficit, so a sterile neutrino species would not be populated in the early universe and would not lead to too much $^4\text{He}$ production.

Of course, if one or more of the indications of neutrino oscillations are wrong, then a sterile neutrino would not be needed and other patterns of neutrino masses are possible. But in any case the possibility remains of neutrinos having large enough mass to be hot dark matter. Assuming that the Super-Kamiokande data on atmospheric neutrinos are really telling us that $\nu_\mu$ oscillates to $\nu_\tau$, there are two possibilities regarding neutrino masses:

A) Neutrino masses are hierarchical like all the other fermion masses, increasing with generation, as in see-saw models. Then the Super-Kamiokande $\Delta m^2 \approx 0.002$ implies $m(\nu_\tau) \approx 0.05 \, \text{eV}$, corresponding to $\Omega_\nu = 0.0015 (m_\nu / 0.05\text{eV}) (0.6/h)^2$. This is not big enough to affect galaxy formation significantly, but it is another puzzling cosmic coincidence that it is close to the contribution to the density from ordinary matter.

B) The strong mixing between the mu and tau neutrinos implied by the Super-Kamiokande data suggests that these neutrinos are also nearly equal in mass, as in the Zee model [19] and many modern models [16, 17]. Then the above $\Omega_\nu$ is just a lower limit. An upper limit is given by cosmological structure formation. In Cold + Hot Dark Matter (CHDM) models with $\Omega_m = 1$, if $\Omega_\nu$ is greater than about 0.2 the voids are too big [20] and there is not enough early structure (cf. [2] and references therein).

2 Hot Dark Matter and Structure Formation

A little hot dark matter can have a dramatic effect on the predicted distribution of galaxies [21]. In the early universe the free streaming of the fast-moving neutrinos washes out any
inhomogeneities in their spatial distribution on the scales that will later become galaxies. If these neutrinos are a significant fraction of the total mass of the universe, then although the density inhomogeneities will be preserved in the cold dark matter, their growth rates will be slowed. As a result, the amplitude of the galaxy-scale inhomogeneities today is less with a little hot dark matter than if the dark matter is only cold. (With the tilt \( n \) of the primordial spectrum fixed — as we discuss below is not necessarily reasonable — the fractional reduction in the power on small scales is \( \Delta P/P \approx 8\Omega_\nu/\Omega_m \) \[24\].) Since the main problem with \( \Omega_m = 1 \) cosmologies containing only cold dark matter is that the amplitude of the galaxy-scale inhomogeneities is too large compared to those on larger scales, the presence of a little hot dark matter could be just what is needed. And, as was mentioned at the outset, this CHDM model is perhaps the best fit to the data on the nearby universe of any cosmological model.

But this didn’t take into account the new high-z supernova data and analyses \[23\] leading to the conclusion that \( \Omega_\Lambda - \Omega_{\text{matter}} \approx 0.2 \), nor the new high-redshift galaxy data. As Somerville, Primack, and Faber \[24\] recently found, none of the \( \Omega_m = 1 \) models with a realistic power spectrum (e.g., CHDM, tilted CDM, or \( \tau \)CDM) makes anywhere near enough bright \( z \sim 3 \) galaxies. But we found that \( \Lambda CD\)M with \( \Omega_m \approx 0.4 \) makes about as many high-redshift galaxies as are observed \[24\]. This value is also suggested if clusters have the same baryon fraction as the universe as a whole: \( \Omega_m \approx \Omega_b/f_b \approx 0.4 \), using for the cosmological density of ordinary matter \( \Omega_b = 0.019h^{-2} \) \[25\] and for the cluster baryon fraction \( f_b = 0.06h^{-3/2} \) \[26\]. Also a new analysis of the cluster abundance as a function of redshift is compatible with \( \Omega_m \approx 0.4 - 1 \), with X-ray temperature data favoring \( n \approx 0.5 \) \[27\] and ENACS and CNOC velocity dispersion data consistent with higher \( \Omega_m \) \[29\]. Thus most likely \( \Omega_m \approx 0.4 \) and there is a cosmological constant \( \Omega_\Lambda \approx 0.6 \). In the 1984 paper that helped launch CDM \[27\], we actually considered two models in parallel, CDM with \( \Omega_m = 1 \) and \( \Lambda \)CDM with \( \Omega_m = 0.2 \) and \( \Omega_\Lambda = 0.8 \), which we thought would bracket the possibilities. It looks like an \( \Lambda \)CDM intermediate between these extremes may turn out to be the right mix.

The success of \( \Omega_m = 1 \) CHDM in fitting the CMB and galaxy distribution data suggests that low-\( \Omega_m \) cosmologies with a little hot dark matter be investigated in more detail. We have done this in order to choose “best” models of this \( \Lambda \)CHDM type to simulate, and we report the results of this investigation here. We have used CMBFAST \[30\] to examine \( \Lambda \)CHDM models with various \( h \), \( \Omega_m \), and \( \Omega_\nu \), assuming \( \Omega_b = 0.019h^{-2} \), and we have adjusted the amplitude and tilt \( n \) of the primordial power spectrum \( P(k) \propto k^n \) in order to match the COBE amplitude and the ENACS abundance of clusters \[29\]. (We checked the CMBFAST calculation of \( \Lambda \)CHDM models against Holtzman’s code used in our earlier investigation of \( \Lambda \)CHDM models \[1\]. Our results are also compatible with those of a recent study \[31\] in which only \( n = 1 \) models were considered. But we find that some \( \Lambda \)CDM and \( \Lambda \)CHDM models require \( n > 1 \), called “anti-tilt”, and it is easy to find cosmic inflation models that give \( n > 1 \) — cf. \[32\].) The Figures show results for \( h = 0.6 \), where the simultaneous fit to CMB and APM data is the best of the cases we considered. Here the neutrino mass is shared between \( N_\nu = 2 \) equal-mass species — as explained above, this is required by the atmospheric neutrino oscillation data if neutrinos are massive enough to be hot dark matter. (This results in slightly more small-scale power compared to \( N_\nu = 1 \) massive species \[1\], but the \( N_\nu = 1 \) curves are very similar to those shown.)

Of the \( \Lambda \)CHDM models shown, for \( \Omega_m = 0.5(0.6) \) the best simultaneous fits to the small-angle CMB and the APM galaxy power spectrum data \[33\] are obtained for the model with \( \Omega_\nu/\Omega_m = 0.1(0.2) \), and correspondingly \( m(\nu_\mu) \approx m(\nu_\tau) \approx 0.8(2.0) \) eV for \( h = 0.6 \). For \( \Omega_m \lesssim 0.4 \), smaller or vanishing neutrino mass appears to be favored. Since as mentioned above, the high-z supernovae and other data favor \( \Omega_m \approx 0.4 \), we have run a supercomputer simulation of \( \Lambda \)CHDM with \( \Omega_\nu/\Omega_m = 0.1 \), which is in agreement with the estimated nonlinear power spectrum in Figure 2. Note that the anti-tilt permits some of the \( \Lambda \)CHDM models to give a
Figure 1: CMB anisotropy power spectrum vs. angular wave number for 12 ΛCHDM models with $N_\nu = 2$ massive neutrino species and Hubble parameter $h = 0.6$, plus the two best-fitting models from Ref. [3]. The data plotted are from COBE and two recent small-angle experiments [36, 37, 38]. Note that the CHDM model only has $N_\nu = 1$ massive neutrino species because that is what [3] used.
Figure 2: Nonlinear dark matter power spectrum vs. wavenumber for the same models as in Figure 1. Note that we “nonlinearized” all the model power spectra [29], to allow them all to be compared to the APM data (the “wiggles” in the low-$\Omega_m$ power spectra are an artifact of the nonlinearization procedure). Note also that the last two models differ from the others in that they are normalized differently and the APM bias is determined over a smaller range in $k$. The bias chosen for the $\Lambda$CHDM models is that which minimizes $\chi^2$ over the entire range of available APM data.
better fit to the COBE plus small-angle CMB data than the two $n = 1$ models plotted, which are the best-fitting models according to Ref. [3]. Of these, the CHDM model is clearly the best fit to the APM data. But the $\Lambda$CDM model has too much power at small scales ($k \gtrsim 1h^{-1}\text{Mpc}$), as is well known [34] (although recent work [35] suggests that the distribution of dark matter halos in the $\Omega_m = 0.3$, $h = 0.7$ $\Lambda$CDM model may agree well with the APM data). On the other hand, the $\Lambda$CDM models may have too little power on small scales (high-resolution $\Lambda$CHDM simulations may be able to clarify this). Thus, adding a little hot dark matter to the low-$\Omega_m$ $\Lambda$CDM models may improve somewhat their simultaneous fit to the CMB and galaxy data, but the improvement is not nearly as dramatic as was the case for $\Omega_m = 1$.

Let us end with a further note of caution: all $\Lambda$CDM and $\Lambda$CHDM models that are normalized to COBE and have tilt compatible with the cluster abundance are a poor fit to the APM power spectrum near the peak. The $\Lambda$CHDM models all have the peak in their linear power spectrum $P(k)$ higher and at lower $k$ than the currently available data (e.g., from APM). (The $\Lambda$CDM and pure CHDM in the Figures are those in [3], which are normalized differently.) Thus the viability of $\Lambda$CDM or $\Lambda$CHDM models with a power-law primordial fluctuation spectrum (i.e., just tilt $n$) depends on this data/analysis being wrong. The new large-scale surveys 2dF and SDSS will be crucial in giving the first really reliable data on this, perhaps as early as next year.

Acknowledgements. JRP acknowledges support from NASA and NSF grants at UCSC, and thanks Bruno Guiderdoni for the invitation to present this talk at Blois and Avishai Dekel for hospitality at Hebrew University where this paper was finished. MAKG is grateful to the NASA High Performance Computing & Communications project for support and the use of Goddard’s T3E and “Beowulf” supercomputers.

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