Classification ensembles for multivariate functional data with application to mouse movements in web surveys

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Summary. We propose new ensemble models for multivariate functional data classification as combinations of semi-metric-based weak learners. Our models extend current semi-metric-type methods from the univariate to the multivariate case, propose new semi-metrics to compute distances between functions, and consider more flexible options for combining weak learners using stacked generalisation methods. We apply these ensemble models to identify respondents’ difficulty with survey questions, with the aim to improve survey data quality. As predictors of difficulty, we use mouse movement trajectories from the respondents’ interaction with a web survey, in which several questions were manipulated to create two scenarios with different levels of difficulty.

Keywords: supervised learning; computer mouse movement trajectories; $d$-variate functional data; paradata; semi-metrics; stacked generalisation

1. Introduction

Paradata describes information concerning the process of responding to a survey (Couper, 2000), often auxiliary data describing the interaction of a respondent with the survey instru-
ment. For example, in a computer-assisted interview (CAI), paradata might reflect the time a participant takes to complete a question, or the number of back-ups or edits. Paradata attracted particular attention with the introduction of web surveys (Couper, 2017), as this mode is especially convenient for automatically and quickly collecting a large amount of data as a by-product of the respondent’s engagement with the survey (e.g., computer mouse clicks and positions, changes of answers in a question). See Callegaro (2013) and Kreuter and Casas-Cordero (2010).

Paradata can help researchers and practitioners better understand and improve different survey aspects, including survey design and survey data quality (Kreuter, 2013). In CAI and web survey modes, for example, response times have been one of the most popular paradata types considered for predicting break-offs in web surveys and designing interventions (Mittereder and West, 2021), studying the relationship of response times and measurement errors (Heerwegh, 2003) or detecting sources of difficulty in the survey (Conrad et al., 2007; Yan and Tourangeau, 2008). However, response times are not always reliable descriptors of the entire survey process (e.g., a long response time may not necessarily be due to the survey question but to other non-survey-related tasks such as checking or responding to emails, see Horwitz et al., 2017; Fernández-Fontelo et al., 2021). Web surveys, in particular, provide another promising source of paradata in the form of computer mouse movements, which have recently demonstrated the ability to convey additional information beyond response times (O’Hora et al., 2016; Stillman et al., 2017; Horwitz et al., 2017, 2020; Fernández-Fontelo et al., 2021).

Mouse movements have been used so far in a number of applications in different fields (Freeman, 2018; Chen et al., 2001). In survey research, mouse movement measures (i.e., features of mouse movement patterns such as the number of changes in direction or the number of times a participant is inactive for a certain period of time) have also shown promising results: Stieger and Reips (2010), for example, found that several mouse movement measures in an online questionnaire (e.g., longer inactivities or an excessive number of clicks) had a negative correlation to data quality, and Horwitz et al. (2017) demonstrated—in the controlled environment of a laboratory—that mouse movement measures other than response times were good predictors of respondents’ perceived difficulty with an online survey question. Recent studies in the field showed that several mouse movements reflected issues with items of a web survey (Horwitz et al., 2020) and further that these measures were also good predictors of such issues (Fernández-Fontelo et al., 2021). Although all previous papers have demonstrated the potential of a number of mouse movement measures as scalar-valued features to improve some survey aspects, none of them have yet considered using the entire mouse movement trajectory as a bivariate function of the mouse cursor positions on the computer screen over time, which is the most detailed representation of the available information.

Therefore, we will investigate in this paper the potential of mouse movement trajectories as bivariate functional predictors of difficulty in web survey questions. Here, difficulty is of interest as it correlates with measurement errors in responses and thus worse survey data quality (Yan and Olson, 2013). To this end, our application is based on a web survey that contained a number of questions (henceforth “target questions”) that were experimentally manipulated to create two different levels of difficulty; for each of these target questions, mouse movements were collected (more details are given in
Section 3; see also Horwitz et al., 2020; Fernández-Fontelo et al., 2021). The purpose of the application is thus to find a predictive model using bivariate mouse movement trajectories as functional predictors to classify easy and difficult question settings. Applied to future surveys, this would be especially useful for survey researchers and practitioners since respondents facing difficulties are more likely to make errors when responding, but these errors may be corrected if they are identified early enough.

A number of classification methods exist that can use functional predictors. We give below an overview of the most important approaches out of a range of methods that have been proposed for classification with either functional or time series predictors (which we can view as functional predictors), and discuss, in particular, methods for multivariate functional data as relevant for our bivariate trajectories.

Following Pfisterer et al. (2019), we can classify these methods into at least two main approaches. The first approach turns a functional classification into a non-functional one by extracting relevant features from the functions using, e.g., functional principal component analysis, wavelets, or splines (Greven and Scheipl, 2017; Pfisterer et al., 2019). Then, typical machine learning methods for classification tasks may be considered using the functional features as multivariate predictors. In the context of survey research, Fernández-Fontelo et al. (2021) used scalar-valued features of (bivariate) mouse movement trajectories as multivariate predictors in a number of machine learning models to predict question difficulty in a web survey successfully.

The second approach is genuinely functional, using the complete functional observations directly rather than summary features. It comprises a wide range of classification methods, which we grouped into methods proposed for (i) functional predictors and (ii) time series predictors. The first block includes (semi-)parametric methods such as generalised functional linear models (Marx and Eilers, 1999; James, 2002; Müller and Stadtmüller, 2003) and generalised functional additive models (McLean et al., 2014), which extend the classical generalised linear and generalised additive models, respectively. These models expand all model terms using appropriate basis representations (e.g., functional principal components, wavelets, splines, etc.) and consider corresponding regularisation (Greven and Scheipl, 2017; Srivastava and Klassen, 2016). The model parameters can be estimated, e.g., using iteratively weighted least squares, or using component-wise gradient boosting (Brockhaus et al., 2020). The first block also includes non-parametric methods for functional data classification, which are generally more flexible because they are distribution-free and allow for modelling non-linear and non-additive relationships. Methods here include the kernel-based non-parametric curves discrimination (kNCD) (Ferraty and Vieu, 2003, 2006), the functional $k$-nearest neighbours (fkNN) (Fuchs et al., 2015), and the kernel-based model proposed by Selk and Gertheiss (2021). All of these methods use semi-metrics, which were first introduced by Ferraty and Vieu (2000) in the context of functional regression with real-valued responses. See also Morris (2015) for further details on semi-parametric and non-parametric models for regression and classification tasks using functional predictors. Finally, the second block includes methods originally proposed for classification tasks with time series predictors, many of which can also be used with functional predictors (e.g., Dempster et al., 2020; Prieto et al., 2013; Górecki and Luczak, 2015; Mei et al., 2016; Wang and Wu, 2017). See also Bagnall et al. (2016) and Abanda et al. (2019) for comprehensive reviews on univariate and multivariate time series classification methods.
Despite the large number of methods for classification using functional predictors that have already been proposed, most of them are either univariate functional methods or use feature extraction techniques. To date, little attention has been paid to the extension of univariate genuinely functional methods to the multivariate case, leaving considerable room for improvement in that regard. For example, Selk and Gertheiss (2021) proposed a non-parametric model for regression and classification tasks using continuous and categorical predictors, and univariate and multivariate functional predictors. While the model seems promising, the authors measured the functions’ proximity using only the $L^2$ distance, which is sometimes too simplistic. Other examples include the methods by Prieto et al. (2013), Gorecki and Luczak (2015), Mei et al. (2016) and Wang and Wu (2017), all of which allow for multivariate time series predictors, but measure time series proximity using either DTW (Dynamic Time Warping)-type distances or feature extraction methods (e.g., shapelets) to reduce the time series dimension. Although DTW is often used to measure the proximity of functions—especially if they may likely vary in speed—there are other relevant distances (e.g., the Fréchet or Hausdorff distances) that are frequently used in curves comparison and carry relevant information. Ideally, a classification method should combine information based on different distance measures. To date, however, only a few of the existing methods combine weak classifiers to improve overall models’ predictive performance (e.g., Prieto et al., 2013; Wang and Wu, 2017) and, in these two cases, the weak classifiers are all built on the same distance (DTW).

To investigate the potential of mouse movement trajectories as functional predictors of web survey difficulty—and in light of the want of genuinely functional methods capable of dealing with multivariate functional predictors—this paper introduces new ensemble models for multivariate functional data classification as extensions of the non-parametric methods by Ferraty and Vieu (2003, 2006) and Fuchs et al. (2015) for univariate functional data classification. In particular, we propose two types of ensemble methods that are optimal combinations of weak learners and these weak learners are built based either on the fkNN classifier (Fuchs et al., 2015) or kNCD classifier (Ferraty and Vieu, 2003). Both fkNN and kNCD use semi-metrics as proximity measures between two functional observations. Semi-metrics as scalar quantities always summarise the information in these infinite-dimensional functions, and each can thus be viewed as capturing a specific aspect of the functional observations. Therefore, a challenging point is to choose the semi-metrics such that they capture the relevant functional characteristics well: Ferraty and Vieu (2003, 2006), for example, constructed parametric families of semi-metrics (e.g., based on functional principal component analysis), and Fuchs et al. (2015) combined a whole set of different semi-metrics in the so-called $k$–nearest neighbour ensemble, using a linear combination of class probabilities that are estimated with the fkNN method. The linear combination weights show the importance of the corresponding semi-metrics in the ensemble. In this paper, we build on Fuchs et al. (2015)’s approach, contributing two extensions: First, while both Fuchs et al. (2015) and Ferraty and Vieu (2003) classify univariate curves, we generalise this to multivariate ($d$-variate) functional observations. The proposed collection of semi-metrics comprises both extensions of some of those in Fuchs et al. (2015) to the multivariate case (e.g., $L^p$ distance, mean, DTW, different versions of global maximums, minimums, etc.) as well as new proposals such as the Fréchet distance and Hausdorff
distances (Alt and Guibas, 2000), edit-type distances such as the Levenshtein and Hamming distances (Navarro, 2001), a correlation-type distance (Szekely et al., 2007), and the Aitchison distance (Filzmoser et al., 2018), among others. We also incorporate a set of semi-metrics based on scalar or vectorial features of the functions. Second, we provide alternatives for combining predictions across weak learners into an ensemble model. A good ensemble combines the weak learners optimally and can considerably reduce the weak learners’ prediction error rates. Instead of using linear combinations as in Fuchs et al. (2015), we propose to construct our ensemble models using more flexible alternatives as super-learners in the stacked generalisation methods (Wolpert, 1992; Breiman, 1996; Leblanc and Tibshirani, 1996).

The paper is organised as follows. Section 2 presents our ensemble models, including different semi-metrics for multivariate functional data and several approaches for weak learners combination. Section 3 describes our models’ performance in our application, detecting difficulty in survey items based on mouse movement trajectories. Section 4 closes with a discussion.

2. Multivariate functional ensembles for classification

Consider the learning sample \((x_i, y_i), i = 1, \ldots, n\), where \(y_1, \ldots, y_n\) are values of a categorical (not ordinal) random variable \(Y\) with \(L\) mutually exclusive and exhaustive classes \(L = \{1, \ldots, L\}\). Let \(x_1, \ldots, x_n\) be realisations over \(t \in T \subset \mathbb{R}\) of independent copies \(X_1, \ldots, X_n\) of a multivariate \((d\text{-variate})\) functional random variable \(X \in \mathcal{F}\), where \(\mathcal{F}\) is a suitable functional (infinite dimensional) space such as the \(L^2(T)\) (Ferraty and Vieu, 2006). Hence, \(x_i(t) \in \mathbb{R}^d\) for each point \(t \in T\) with \(d \in \mathbb{N}\); we focus here on \(d \geq 2\). Where we talk about the \(a\)-th derivative \(x^{(a)}\) of \(x\), we assume that it exists and is in \(L^p(T)\) for appropriate \(p \in \mathbb{N}\) if required. In practice, functions are observed on a finite discrete grid in \(T\), where the grid may differ between observed functions and dimensions within a multivariate function. For simplicity, we focus here on the case of regularly spaced grids per univariate function and consider \(t \in T\) as time.

We address the following classification problem: for a new observation \(x_\ast\), infer the unknown class membership \(y_\ast\) from the learning sample of multivariate functions with known class memberships.

2.1. Semi-metrics

Semi-metrics are proximity measures between general mathematical objects and can be used in particular for functional observations taking values in an infinite-dimensional space (Ferraty and Vieu, 2000). We here use semi-metrics to measure distances between multivariate functional observations (e.g., between bivariate mouse trajectories).

Formally, let \(D(x, x_\ast)\) be the semi-metric between the multivariate functions \(x\) and \(x_\ast\), which thus fulfills: \(D(x, x_\ast) \geq 0, D(x, x) = 0\) and \(D(x, x_\ast) \leq D(x, \bar{x}) + D(\bar{x}, x_\ast), \forall x, x_\ast, \bar{x} \in \mathcal{F}\). Although semi-metrics are similar to metrics, they differ from metrics in that \(D(x, x_\ast) = 0 \iff x = x_\ast\). Thus, different curves can have a “distance” of zero, which is particularly relevant when defining semi-metrics via the functions’ derivatives \(x^{(a)}, a \in \mathbb{N}\) (e.g., the \(L^2\)-distance between the first derivatives).
We extend several semi-metrics proposed in Fuchs et al. (2015) and propose a broad collection of semi-metrics that capture different aspects of multivariate functional observations. More specifically, we propose five different families of semi-metric (cf., Table 1), which differ in whether they are computed over the raw trajectories or summaries thereof, and in whether they preserve the ordering as well as the spatial and/or temporal information in the trajectories. For other applications, this framework is extendable, allowing for the inclusion of additional user-defined semi-metrics.

The first family of semi-metrics—the lock-step family according to Mori et al. (2016)—includes metrics that compare function values at the same time points \( t \), keeping either the functions’ ordering or treating them as unordered multivariate random vectors. This family includes the Manhattan (\( L^1 \)) and Euclidean (\( L^2 \)) metrics as well as further \( L^p \) metrics for \( p \geq 1 \) for function comparison, and a correlation-type distance proposed by Székely et al. (2007) for comparison of unordered multivariate random vectors (see also Rizzo and Székely, 2016). Extending the idea of the univariate correlation coefficient, Székely et al. (2007) defined the distance correlation, \( R \), for two multivariate random vectors with finite first moments, satisfying \( 0 \leq R \leq 1 \) and \( R = 0 \) if and only if the two random vectors are independent. We use as distance \( 1 - R \) (cf. Table 1), which is 0 if two trajectories are perfectly correlated and 1 if these trajectories are independent. In practice, we use the empirical distance correlation in place of \( R \).

The second family of semi-metrics includes a variety of elastic-type distances, either keeping the ordering of our functional observations (DTW and Fréchet distances), or ignoring the ordering by treating the functions as point clouds (Hausdorff distance). Unlike semi-metrics in the lock-step family, the elastic-type semi-metrics are more flexible in that they allow for one-to-many or one-to-none mappings (i.e., the time point \( t \) for one function can be matched to \( t \), to another time point \( t' \), to several, all remaining points or no time points of another function) (Mori et al., 2016). In particular, the DTW technique seeks a warping function \( \gamma^* : T \rightarrow T \) in the space of all monotonically increasing, onto and differentiable warping functions \( \Gamma \) over \( T \) that optimally aligns two (multivariate) functions to minimise a functional distance \( \Delta(\cdot, \cdot) \) in \( F \) (e.g., the \( L^2 \) distance), \( D_{DTW}(x, x_*) = \inf_{\gamma \in \Gamma} \Delta(x \circ \gamma, x_*) \). The trajectories may be of different lengths and vary in speed, and are in practice recorded over finite discrete grids. However, note that DTW is not a proper semi-metric since it does not satisfy the triangle inequality property. We can further consider other elastic-type distances such as the elastic distance based on the square-root-velocity (SRV) framework (Srivastava and Klassen, 2016; Steyer et al., 2021; Marron, 2021), which is a proper metric, although we do not consider this distance in our application due to computational cost. This family also includes the Fréchet and Hausdorff distances (Alt and Guibas, 2000): The Fréchet distance treats \( x \) and \( x_* \) as continuous functions \( x, x_* : T \rightarrow \mathbb{R}^d \). Let \( \Phi \) be the set of continuous and monotonically increasing functions \( T \rightarrow T \). The Fréchet distance is defined as \( D_F(x, x_*) = \inf_{\phi, \phi' \in \Phi} (\max_{t \in T} (\Delta(x(\phi(t)), x_* (\phi'(t)))))) \), where \( \Delta(\cdot, \cdot) \) is, e.g., the Euclidean distance in \( \mathbb{R}^d \). The Fréchet distance measures the largest remaining (point-wise) distance between two functions after optimal time alignment. By contrast, the one-sided Hausdorff distance from a set of points \( A \subset \mathbb{R}^d \) to another set of points \( B \subset \mathbb{R}^d \) is defined as \( h(A, B) = \max_{a \in A} (\min_{b \in B} \Delta(a, b)) \) for points \( a \in A \) and \( b \in B \), where \( \Delta(\cdot, \cdot) \) is, e.g., the Euclidean distance. It measures the largest distance from \( A \) to the nearest point in
Table 1. Overview of the used collection of semi-metrics to measure distances between two multivariate functional observations \( x = (x_1, \ldots, x_d)^T \) and \( x_\ast \), both defined over \( T \subset \mathbb{R} \) and \( d \in \mathbb{N} \). These semi-metrics can also be computed for the corresponding \( a \)-th derivatives. See Supporting Material for more details on our collection of semi-metrics, including the symbol sequence semi-metric family.

| Family          | Semi-metric | Definition                                                                                                                                 |
|-----------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| lock-step       | \( L^p \)  | \( \left( \int_T \sum_{k=1}^d |x_k(t) - x_{k\ast}(t)|^p \, dt \right)^{\frac{1}{p}} \)                                                                                   |
| (raw trajectories) |             |                                                                                                                                               |
| correlation     |             | \( 1 - \left( \mathcal{R}^2(x, x_\ast) \right)^{\frac{1}{2}} = 1 - \left( \frac{\mathcal{V}^2(x, x_\ast)}{(\mathcal{V}^2(x)\mathcal{V}^2(x_\ast))^{\frac{1}{2}}} \right)^{\frac{1}{2}}, \)  |
|                 |             | if \( \mathcal{V}^2(x)\mathcal{V}^2(x_\ast) > 0 \), \( \mathcal{V}^2(x, x_\ast) \) is the so-called distance covariance and \( \mathcal{V}^2(x) = \mathcal{V}^2(x, x) \)             |
| elastic         | DTW         | \( \inf_{\gamma \in \Gamma} \Delta(x \circ \gamma, x_\ast) \), where \( \Gamma \) is the space of all monotonically increasing,onto and differentiable warping functions \( \gamma \) and \( \Delta(\cdot, \cdot) \) is a distance in \( F \), e.g., \( L^2 \) |
| (raw trajectories) |             |                                                                                                                                               |
| Fréchet         |             | \( \inf_{\phi, \phi' \in \Phi} \max_{t \in T} \Delta \left( x \left( \phi(t) \right), x_\ast \left( \phi'(t) \right) \right) \),  |
|                 |             | where \( \phi, \phi' : T \rightarrow T \) are functions in the space of all continuous and monotonically increasing functions \( \Phi \) and \( \Delta(\cdot, \cdot) \) is a distance such as the Euclidean in \( \mathbb{R}^d \) |
| Hausdorff       |             | \( \max_{t \in T} \min_{t' \in T} \Delta \left( x(t), x_\ast(t') \right) \),  |
|                 |             | where \( \Delta(\cdot, \cdot) \) is a distance, e.g., the Euclidean in \( \mathbb{R}^d \) |
| svs             | scalar      | \[ \left| T(x) - T(x_\ast) \right| \]                                                                                                         |
| (summaries)     |             | for scalar summary \( T(x) \) of \( x \), e.g., \( T(x) = \int_T x_k(t) \, dt \) the mean, or the maximum or range for dimension \( k \). |
|                 | vector      | \( \left( \sum_{k=1}^r \left( T_k(x) - T_k(x_\ast) \right)^2 \right)^{\frac{1}{2}} \)                                                                 |
|                 |             | for vector summaries \( T(x) = (T_1(x), \ldots, T_r(x))^T \) of \( x \), e.g., \( T_k(x) = \frac{1}{\int_T} \int_T x_k(t) \, dt \) the mean, or the maximum or range for dimension \( k \), and \( r = d \). |
| composition     | Aitchison   | \( \left( \frac{1}{2r} \sum_{j=1}^m \sum_{s=1}^m \ln \left( \frac{T_j(x)}{T_j(x_\ast)} \right) - \ln \left( \frac{T_j(x_\ast)}{T_j(x)} \right) \right)^2 \) \( \frac{1}{2} \), |
| (summaries)     |             | where \( T(x) = (T_1(x), \cdots, T_m(x))^T \in \mathbb{S}^m \) is a compositional vector of \( m \) parts summarising the raw functional observation \( x \) |
The one-sided Hausdorff distance is asymmetric (i.e., generally $h(A, B) \neq h(B, A)$) and the Hausdorff distance is defined as $D_{H}(A, B) = \max \{h(A, B), h(B, A)\}$. Application to functions requires the functions to be observed on discrete grids for $t$ (possibly of different lengths) and treats $x$ and $x_t$ as sets of points in $\mathbb{R}^d$. Semi-metrics in the lock-step and elastic-type families can be computed for observed functions or their derivatives.

The third family of semi-metrics is the scalar-vector-summary (svs) family, which includes distances between scalar-valued or vector-valued summary features of the multivariate functional observations (or their derivatives). Examples are the Euclidean distance between the maximums, minimums or ranges of two multivariate functional observations in one of the dimensions (e.g., in our application, $\text{globMax}_x(y)$, $\text{globMin}_x(y)$ and $\text{globRange}_x(y)$, respectively, for the first (second) dimension). Examples for vector-valued summaries include the Euclidean distance between the vectors of maximums, minimums or ranges in each of the $d$ dimensions (e.g., in our application, $\text{globMax}$, $\text{globMin}$ and $\text{globRange}$, respectively). This family also includes the multivariate version of the mean distance by Fuchs et al. (2015) (i.e., the Euclidean distance between the multivariate mean vectors), as well as measure-based semi-metrics, which in our application are defined as the Euclidean distance between (potentially personalised) mouse movement measures considered by Fernández-Fontelo et al. (2021) and described in Section 3.

The fourth compositional family of semi-metrics summarises the trajectory in a composition of time spent in a set of $m$ areas of interest (AOIs) in $\mathbb{R}^d$. It includes the Aitchison distance for compositional vectors in the simplex $S_m = \{(z_1, \cdots, z_m) : z_j > 0, j = 1, \cdots, m; z_1 + \cdots + z_m = 1\}$. This distance between compositions uses essentially a Euclidean distance between vectors of log-ratios of all component pairs to account for the relative nature of compositions and the structure of the $S_m$.

The last symbol sequence family includes a number of edit-type distances aimed at finding an optimal alignment between two sequences of symbols, here taken to indicate the location of the trajectory at time $t$ within a given AOI. Out of the edit-type distances, we consider in the following the Levenshtein and Hamming distances (Navarro, 2001). In particular, the Levenshtein distance counts the number of substitutions (or mismatches) and insertions and deletions (or indels) required to change one sequence into another not necessarily of the same length. The Hamming distance only counts the minimum number of substitutions (mismatches) needed to convert one sequence into another of equal length. To ensure the same length across all trajectories for the Hamming distance, the sequences of symbols in our application were constructed using the time-normalised trajectories of computer screen locations.

The families of semi-metrics can also be classified with regard to the information they maintain: In contrast to the compositional family, the symbol sequence family preserves the temporal information, and both families keep the spatial information in an aggregated fashion. The svs family extracts features of interest and thus does not keep either the spatial or temporal information in the functions. Lock-step and elastic families keep the spatial and temporal information in the functions, with the elastic family allowing for stretching and compressing of time for better matching between functions. See Table 1 for a formal definition of some of the considered semi-metrics and Section SM.1 of the Supporting Material for more details on the complete collection of semi-metrics, including details of some of them in practice.
2.2. Weak learners for multivariate functional observations

Weak learners are the basis of ensemble building, as their combination gives a more robust model (ensemble) with potentially better predictive abilities. We focus here on two methods for weak learner construction: the fkNN used by Fuchs et al. (2015) in the context of univariate functional data classification and kNCD proposed by Ferraty and Vieu (2003) in the context of scalar-on-function regression. Both methods measure the proximity of functional observations through semi-metrics, and we use here the collection of semi-metrics for multivariate functional data described in Section 2.1.

The fkNN rule predicts the class membership $y_*$ of the new observation $x_*$ with the majority class among the $k$ closest neighbours of $x_*$. We identify these neighbours by ordering the training predictors $x_1, \ldots, x_n$ given a specific semi-metric $D(\cdot, \cdot)$ such that:

$$D\left(x_*, x_{(1)}\right) \leq \cdots \leq D\left(x_*, x_{(k)}\right) \leq \cdots \leq D\left(x_*, x_{(n)}\right),$$

where $x_{(1)}, \ldots, x_{(n)}$ are the ordered training predictors from the nearest to the farthest to $x_*$ given the semi-metric $D(\cdot, \cdot)$. According to expression (1), the neighbourhood $N^k(x_*)$ of the $k$ closest training functional predictors of $x_*$ given $D(\cdot, \cdot)$ is defined as:

$$N^k(x_*) = \left\{ x_j : D\left(x_*, x_j\right) \leq D\left(x_*, x_{(k)}\right) \right\}.$$  \hspace{1cm} (2)

We finally predict the class membership $y_*$ of $x_*$ as the most frequent class of the observations in the neighbourhood $N^k(x_*)$. That is, $\hat{y}_* = \arg\max_{l \in \mathcal{L}} \left(\hat{p}_l(k)\right)$, where $\hat{p}_l(k)$ is the estimated probability of the new observation $x_*$ belonging to class $l \in \mathcal{L}$ defined as:

$$\hat{p}_l(k) = \frac{1}{k} \sum_{x_i \in N^k(x_*)} 1_{y_i = l},$$

where $1_{y_i = l}$ is the indicator function taking 1 if $y_i = l$ and 0 otherwise. Although $\hat{p}_l(k)$ is dependent on the parameter $k$, this dependence will be suppressed from now on in the notation for simplicity. The estimated probability $\hat{p}_l$ is thus the proportion of observations $x_j$ belonging to the neighbourhood $N^k(x_*)$ whose class is $y_j = l$ for $l \in \mathcal{L}$. The class $y_*$ can be predicted with the majority class among observations $x_j \in N^k(x_*)$. If there are any ties in the majority class, they are broken at random. Also, all candidates are included in the vote if the ties are in the vector of the $k$ nearest neighbours. More details are found in Fuchs et al. (2015), who focused only on the univariate case.

The kNCD proposed by Ferraty and Vieu (2003) extends the idea of fkNN such that the estimated probability $\hat{p}_l, l \in \mathcal{L}$ depends on all training trajectories rather than only on those $k$ closest to $x_*$ according to the semi-metric. The computation of $\hat{p}_l$ in kNCD depends on a kernel function (Li and Racine, 2007), which weights the training observations by the distance to the new observation given a semi-metric, with larger weights for smaller distances. The new observation is classified into the group with the highest $\hat{p}_l, l \in \mathcal{L}$. Formally, giving the training observations $x_1, \ldots, x_n$, and the new observation $x_*$, the estimated probability of $x_*$ belonging to class $l \in \mathcal{L}$ is defined as:

$$\hat{p}_l(h) = \frac{\sum_{i=1}^{n} 1_{y_i = l} K\left(D(x_i, x_*)/h\right)}{\sum_{i=1}^{n} K\left(D(x_i, x_*)/h\right)},$$

where $K$ is a kernel function, $h$ is a bandwidth parameter, and $D(x_i, x_*)$ is a distance measure between the new observation and the training observations.
where $K$ is the kernel function and $h > 0$ is the bandwidth parameter that scales the kernel function to the observed data. Note that in expression (4), we have again stressed the dependence of the probabilities on the parameter $h$, which needs to be tuned (e.g., cross-validation-based methods), but that we suppress this dependence in the notation for simplicity. Note also that if a Uniform kernel function is considered, expression (3) is very similar to expression (4). In this case, differences in the kNCD method (Ferraty and Vieu, 2003) and the fkNN (Fuchs et al., 2015) only concern the chosen neighbourhood, which has a fixed size according to distance in the first and a fixed number of neighbours in the second case. More details can be found in Ferraty and Vieu (2003, 2006).

2.3. Ensemble

We use ensembles as flexible tools that combine weak learners (e.g., using fkNN or kNCD) based on different semi-metrics chosen to capture a diverse set of characteristics of the trajectories. Ensemble models are expected to reduce error rates and improve the bias-variance trade-off compared to single weak learners (Hastie et al., 2009). We combine weak learners using the stacked generalisation method (Wolpert, 1992; Breiman, 1996; Leblanc and Tibshirani, 1996), a two-step procedure that uses the predictions of several weak learners (first step) as predictors in a new learning model (“super-learner”, second step), for a categorical output in our classification case. In particular, in addition to stacked generalisation using linear combinations of weak learners’ predictions as in Fuchs et al. (2015) (see, for example, the discussion by Leblanc and Tibshirani, 1996), we use more flexible super-learners such as tree-based random forests and gradient boosting.

2.3.1. Stacked generalisation ensembles for functional predictors

The first considered ensemble method using linear combinations of weak learner predictions was first used by Fuchs et al. (2015) for univariate functional data classification based on fkNN only. Following Maierhofer (2017), we call this ensemble LCE (Linear Combination Ensemble), and extend it to the multivariate case with fkNN and kNCD.

Consider $M$ independent weak learners built on either fkNN or kNCD, each based on a different semi-metric. The first step for LCE building obtains the estimated probabilities $\hat{p}_{lm}$ of the new observation $x^*$ belonging to class $l \in L$ for the $m$-th weak learner for all $m$; see expressions (3) and (4). Note that the $\hat{p}_{lm}, \forall l,m$ implicitly depend on either the parameter $k$ (if fkNN is used) or $h$ (if kNCD is used) and that both parameters can be tuned with cross-validation methods; see Section 2.3.2. The second step finds an optimal convex combination of these probabilities. In particular, the probability $\hat{\alpha}_l$ of the new observation $x^*$ belonging to class $l \in L$ is estimated as:

$$\hat{\alpha}_l = \sum_{m=1}^{M} \omega_m \hat{p}_{lm},$$  \hspace{1cm} (5)
where the unknown coefficients $\omega_m$ to be estimated satisfy:

$$\omega_m \geq 0 \quad \forall m, \quad \sum_{m=1}^{M} \omega_m = 1. \quad (6)$$

The coefficient $\omega_m$ is the contribution (weight or importance) of the $m$-th weak learner (or semi-metric) to the estimated probabilities $\hat{\alpha}_l, \forall l$. The constraint $\omega_m \geq 0, \forall m$ ensures that the estimated probabilities $\hat{\alpha}_l, \forall l$ are always positive. Also, the constraint $\sum_{m=1}^{M} \omega_m = 1$ ensures all estimated probabilities $\hat{\alpha}_l, \forall l$ sum up to 1 (for more details see Gertheiss and Tutz, 2009). Note that in expression (5) each weak learner has the same weight for each class as this is the only solution that ensures probability constraints (6) are also satisfied for the $\hat{\alpha}_l$ (see the proof of Proposition 1 by Gertheiss and Tutz, 2009).

Coefficients $\omega_m, m=1,\cdots,M$ are estimated following Fuchs et al. (2015) on the training sample by minimising the Brier score (Brier, 1950) subject to constraints in (6):

$$S(\omega) = \frac{1}{n} \sum_{i=1}^{n} \sum_{l=1}^{L} (\mathbb{1}_{y_i=l} - \hat{\alpha}_l)^2 = \frac{1}{n} \sum_{i=1}^{n} \sum_{l=1}^{L} \left( \mathbb{1}_{y_i=l} - \sum_{m=1}^{M} \omega_m \hat{p}_{ilm} \right)^2, \quad (7)$$

where $\hat{\alpha}_l$ is the estimated probability in (5) for the $i$-th observation. The Brier score is a strictly proper scoring rule, and the optimiser is thus unique (Gneiting and Raftery, 2007). The constraints in expression (6) impose a positive lasso-type penalty (Tibshirani, 1996) on the coefficients, hence the LCE has the advantage of allowing the selection of weak learners by setting coefficients to zero (Fuchs et al., 2015).

However, the LCE is restricted to convex linear combinations and equal weights of weak learners across the different classes. We thus propose to extend the LCE to more flexible combinations of weak learners’ predictions using stacked generalisation with more flexible super-learners. Many methods would be possible as super-learners. We focus here on tree-based random forests and tree-based gradient boosting. For univariate functional data classification, the former have been considered before in the Master’s thesis of Maierhofer (2017). Depending on the used super-learner, we will call these ensembles either RFE (Random Forest Ensemble) or GBE (Gradient Boosting Ensemble).

Tree-based random forests combine a number of decision trees –weak learners– tuned on bootstrapped training samples. To reduce between-tree correlation, only a subset of the complete set of predictors is considered in each split in each tree, ensuring that weaker predictors are also present in some of the trees. As random forest predictions are based on a collection of trees with (ideally) low correlation, such predictions are more reliable –less variable– than predictions of a single tree. The number of combined trees and the number of predictors to be considered in each tree split are two parameters that need to be tuned using, e.g., cross-validation methods. Tree-based gradient boosting models also combine trees, which in this case are constructed sequentially, with the output of the next tree being the information (pseudo-residuals) that was not explained in the previously grown tree. The number of trees, the number of splits in each tree, and a shrinkage parameter that controls how the gradient boosting model learns are parameters that need to be tuned. In classification tasks, both random forest and gradient boosting models assign a new observation to the majority class predicted across all tuned trees.
We use here both models as super-learners with the estimated probabilities from the selected weak learners as inputs. See Hastie et al. (2009); James et al. (2013) for more details on random forests and gradient boosting, and Sections 2.3.2 and 3 for super-learners tuning details.

RFE and GBE are more flexible methods than LCE: First, they are non-linear classifiers that allow for more complex relationships between the output and the inputs than the linear ones in LCE. Second, random forest and gradient boosting are especially well-suited to dealing with complex interactions between predictors. Because our predictors in this paper are the estimated probabilities given by a collection of weak learners (or semi-metrics), RFE and GBE are better than LCE at capturing potential interactions between semi-metrics. Third, unlike for LCE, RFE and GBE allow different semi-metrics to be important for predicting different classes, which can be important in practice. Another key aspect of RFE and GBE is that they allow including scalar covariates as predictors in addition to the weak learners’ estimated probabilities and can even capture interactions between semi-metrics and these covariates. For interpretability, we can use variable importance techniques (e.g., using permutational methods) to measure the overall relevance of each semi-metric for prediction, or class-specific variable importance to identify the most relevant semi-metrics for predicting each of the classes.

2.3.2. Stacked generalisation ensembles in practice

Weak learners rely on parameters $k$ or $h$ (cf. Section 2.2). We tune them using nested cross-validation (Stone, 1974), which splits the sample into training and validation (inner loop) and testing (outer loop) sub-samples to base model selection (inner loop) and model performance (outer loop) on independent samples, controlling model over-fitting and giving a more honest model performance assessment.

We use the same sample splits in both the inner and outer loops for all weak learners and all ensembles to ensure fair comparison, with the proportion of correctly classified observations (accuracy) used to measure performance. We first tune the $M$ considered weak learners based either on $f$kNN or $k$NCD and a given semi-metric in the inner loop, and evaluate the predictive performance of the tuned learners in the outer loop. This gives estimated probabilities $\hat{p}_{ilm}$, $\forall i, l, m$ for all learners $m$, classes $l$ and observations $i$.

We then build the ensembles. For the LCE, linear combination coefficients $\omega_m$ for the $\hat{p}_{ilm}$ are estimated in the inner loop by solving the optimisation problem in expression (5). Due to the lasso-type penalty, some of the coefficients $\omega_l$ are set to 0 and drop out for prediction evaluation in the outer loop.

To tune the super-learners random forest for RFE and gradient boosting for GBE, we add a forward selection step of weak learners to improve predictive performance. First, the two best weak learners are combined in an ensemble with the super-learner tuned on the same inner loop sample splits than for weak learners and LCE. Second, the third best weak learner is included in the ensemble, and the super-learner is re-tuned. It is kept in the ensemble if it improves the inner loop accuracy, then the next best weak learner is considered for inclusion and the process iterated recursively. The final ensemble is then evaluated in terms of outer loop accuracy.
3. Application

3.1. Survey design and sample description

The data used in this application were collected from an online survey conducted by the Institute for Employment Research (IAB) between September and October 2016 (Horwitz et al., 2020; Fernández-Fontelo et al., 2021). 1627 participants from a previous survey wave in 2014 who had consented to future contact were sent an invitation; of these, 1527 received a 5 Euros incentive, while the remaining 100 were contacted for a pretest of the mouse-tracking software shortly before the beginning of the survey. Finally, 1250 opened the survey and, of these, 1213 completed the questionnaire. Section 4.1 of Horwitz et al. (2020) provides more details on the participants’ recruiting process.

The survey included four sections with a total of 36 questions of a variety of formats (see Horwitz et al., 2020 for more details); we here focus on a subset of questions for which difficulty was manipulated experimentally, namely “employment detail”, “employee level” and “education level” (target questions) from sections “employment” and “demography”. Of the 1213 participants who responded and completed the survey, 886 (73%) used a computer mouse, and 853 (70%) had continuous mouse-tracking data available. Thus, our sample consists of these 853 participants, or a subset depending on the survey question. The sample is gender-balanced (51-49% female-male), with an average age of 51 years. When the survey took place, most of the individuals were employed.

The survey was only available via the web, and participants were told to answer it using either laptop or desktop computers, as mouse-tracking data could not be collected otherwise. In addition, participants were asked to use a computer mouse if possible. The study was implemented using the SoSci Survey software (Leiner, 2019), and we used the mousetrap-web library—a platform-independent, open-source JavaScript library created by Henninger and Kieslich (2016)—as a tool to collect mouse-tracking data, including the mouse cursor x- and y-coordinates, timestamps and a representation of the mouse movement trajectory over the computer screen to link the location of the mouse cursor at each time to the question content. We collected mouse movement trajectories from survey respondents who agreed to it at the beginning of the questionnaire for each survey question. Thus raw trajectories between participants within the same question were not necessarily of the same length.

3.2. Target questions

We analysed three different target questions: The first is “employment detail” (i.e., type of employment), which was manipulated, producing two versions with response options using simple or complex language. The second and third were two ordinal-type questions on the maximum degree of responsibility at work (“employee level”) and the degree of education achieved (“education level”), which were manipulated, producing one version with a natural ascending response options’ ordering, and another with the options randomly ordered. One version of each target question was randomly assigned to each participant, ensuring that roughly half of the sample received each version of the question. See the Supporting Material for details on each question’s layout.

Employment detail had nine response options, and the majority of the sample was employed in the civil service at the municipality level. Employee level and education
detail had respectively four and eleven response options, and most of the respondents carried out a qualified occupation, either following instructions (e.g., accountant) or with some independent activities and responsibilities (e.g., scientific employee), and roughly half of the respondents obtained qualification for university entrance. See the Supporting Material for more information on the response distribution for all three target questions.

3.3. Mouse movement trajectories

We applied several pre-processing steps to the mouse movement trajectories. First, coordinates $x$ and $y$ at each timestamp depended on each participant’s computer browser; therefore, we standardised them such that each bivariate coordinate pair was comparable across trajectories within the same target question. Second, we computed the first and second derivatives over our raw trajectories separately for each dimension to allow usage of curves ($a = 0$), their velocity ($a = 1$), and their acceleration ($a = 2$). The mouse movement trajectories and their first and second derivatives were then time-normalised to the $[0, 1]$ interval. In particular, we used linear interpolation based on the timestamps separately for each trajectory dimension; we used 101 equidistant time steps for the time-normalisation using the R package mousetrap (Kieslich et al., 2021). We then applied additional filter criteria detailed in Fernández-Fontelo et al. (2021): For example, we removed trajectories from participants who took an exceptionally long time to respond (more than 7 minutes). We also excluded trajectories from respondents who did not provide an answer for gender or who selected “other” for gender since only a few participants selected this category or from participants who selected a free-form text input answer option for education level. After these pre-processing steps, 551, 501, and 548 bivariate mouse movement trajectories remained for employment detail (simple vs. complex language), and employee level and education level (sorted vs. unsorted response options), respectively, each of length 101. These were used as functional predictors of difficulty for each target question. See the Supporting Material for some images of the raw trajectories for each of the three target questions.

Before time normalisation, ten different mouse movement measures grouped in five categories according to their typology were extracted from the raw trajectories with the R package mousetrap: (i) time-type measures such as time from page load until response submission (response time, RT) and time from page load until the first mouse movement occurred (initiation time); (ii) distance-type measures such as the total Euclidean distance travelled by the computer mouse (total distance) (iii) derivative-type measures such as maximum velocity and maximum acceleration; (iv) hover-type measures such as the number of periods without movement exceeding a minimum duration threshold (hovers) and the total time of all periods without movement exceeding a minimum threshold duration (hovers time), and (v) flip-type measures such as the number of changes in movement direction along the horizontal axis (x-flips) and the vertical axis (y-flips). These measures summarising the mouse movement trajectories were previously used for difficulty prediction in Fernández-Fontelo et al. (2021). As they also showed that personalisation of measures by correcting for individual baseline behaviours and location of the given answer improved classification, we use personalised measures as defined there. We also considered in our application the total length of the raw mouse movement trajectories (i.e., the total number of timestamps), which is related to the non-personalised
response time as the longer the response time, the longer the trajectory.

### 3.4. Ensemble models for mouse movement trajectory classification

We considered several ensemble model candidates. First, to combine weak learner predictions, we considered both LCE (Fuchs et al., 2015) and the more flexible proposals RFE and GBE (cf. Section 2.3). Second, we considered two approaches for including personalised mouse movement measures (see Fernández-Fontelo et al., 2021) into the ensemble models: (i) measure-based semi-metrics (ii) using measures as additional vector-valued predictors in the corresponding super-learner. As this last alternative is only possible for RFE and GBE, this yields five combinations each for both types of weak learners built on fkNN or kNCD—their Gaussian kernel. These ten combinations were tuned and evaluated using nested cross-validation, as described above, using the same 10 outer (testing sample) and 5 inner (training and validation samples) folds for all fkNN, kNCD, and ensemble models to ensure a fair comparison.

The average accuracy across the 10 testing samples (outer loop) indicated the weak learner’s predictive performance. We then selected weak learners with an outer accuracy equal to or greater than 0.55 for potential inclusion in the ensemble. We selected 24 and 18 weak learners for fkNN and kNCD, respectively, for employment detail, 12 and 12 for employee level, and 6 and 10 for education level. Between 2 and 6 weak learners were built on measure-based semi-metrics (if ensembles are built based on approach (i)).

We then constructed ensembles based on either LCE, RFE or GBE. For LCE, following Fuchs et al. (2015), we estimated the linear combination coefficients $\omega_m, m = 1, \cdots, M$, using the function lsei from the R package limSolve to solve the optimisation problem with constraint (6), resulting in a lasso-type penalty (cf. Section 2.3.2). Note that the probabilities $\hat{p}_{ilm}$ for the different weak learners in (7) were computed on the same inner splits used to estimate the LCE coefficients, and the LCE’s accuracy was then evaluated on the testing sample. For RFE and GBE construction, we had to tune super-learners. For the random forest in RFE, we used the estimated probabilities $\hat{p}_{ilm}$ for observations in the inner loop sample as random forest predictors, and tuned the random forest parameters (number of trees and predictors in each tree) using 10-fold cross-validation and maximising the inner loop’s accuracy. RFE’s predictive performance was evaluated with the average accuracy over the 10 testing samples (outer loop). Note that for nested cross-validation, super-learners are differently tuned for each sample split in the outer loop. We used an analogous procedure for tuning (number of trees, shrinkage parameter, and a parameter related to the level of interaction between predictors) and performance evaluation of GBE.

Table 2 provides the outer loop accuracies (those greater than 0.55) of the weak learners built on kNCD for two of our target questions (employment detail: simple vs. complex language, and employee level: ordered vs. unordered response options). For employment detail, chosen semi-metrics included most notably those in the scalar-vector-summary (svs) family, such as different types of maximums and ranges as well as a mean-type semi-metric (some based on the first ($a = 1$) or the second ($a = 2$) derivatives), and semi-metrics based on personalised measures such as the Euclidean distance between response times (RT), hovers and hover time and vertical flips ($y$-flips), as well as the Euclidean distance between the bivariate vectors of flips ($x$- and $y$-flips) (bivariate flips).
We also included the Hausdorff distance (elastic family), and semi-metrics from the lock-step family such as different versions of $L^p$ distances and a correlation-type distance (based on the second derivative of the raw trajectory, $a = 2$). The elastic family was more important for employee level than employment detail since DTW, Fréchet and Hausdorff distances were relevant for difficulty prediction in this second target question. Also, semi-metrics for employee level included different types of minimums and ranges, semi-metrics based on personalised measures such as the Euclidean distance between initiation times and the number of hovers between trajectories (svs family), and a correlation-type distance (lock-step family). Finally, for education level, we selected semi-metrics mainly from the svs family, including distances between means, maximums and minimums in both $x$ and $y$-direction. We also considered different versions of the $L^p$ distance from the lock-step family, DTW (elastic family) and Hamming distance (symbol sequence type family). If weak learners were instead based on fkNN, we selected similar semi-metrics than for kNCD for each target question. Although other elastic-type distances have been considered in Section 2.1 (e.g., Steyer et al., 2021; Srivastava and Klassen, 2016), these have not been explored in this application due to their computing cost.

Table 3 shows the inner loop accuracies for weak learner selection in kNCD ensemble construction and the outer loop accuracies for evaluating the predictive capacities of the selected kNCD ensembles for employment detail and employee level questions. Results for fkNN ensembles were similar (although slightly less good and/or consistent) and are given in the Supporting Material together with similar results for education level.

The best kNCD ensemble model for employment detail used gradient boosting as a super-learner and included mouse movement measures in the ensemble via measure-based semi-metrics. Ensembles added new weak learners in a forward step-wise fashion if they improved the inner loop accuracy (cf. Section 2.3.2). Accuracies in bold indicate the inclusion of the corresponding weak learner; the ensemble model for employment detail contained measure-based semi-metrics for scalar measures such as response times (RT), trajectory lengths (length), number of hovers, time hovering and vertical flips, and the $R^2$ Euclidean distance between vectors of horizontal and vertical flips (bivariate flips). The ensemble model also included other semi-metrics from the svs family, such as the maximum distance in the $y$ (globMax-y) and $x$ (globMax-x) coordinates separately computed on the time-normalised trajectories ($a = 0$), the mean distance computed on the time-normalised first derivative of the raw trajectories ($a = 1$, velocity), and from the lock-step family such as the correlation-type distance computed on the time-normalised trajectories. We selected this ensemble model with an inner loop accuracy of 0.7057 and an outer loop accuracy –in italics and bold– of 0.6825.

Keeping the splits in the inner and outer loops constant across methods not only allows a fair comparison of accuracies in Tables 2 and 3, but also with our previous work (Fernández-Fontelo et al., 2021), where we analysed the predictive performance of a set of classifiers based only on scalar mouse movement measures as predictors. In particular, Fernández-Fontelo et al. (2021) found that the best model for employment detail was a tree-based gradient boosting with personalised measures, obtaining an outer loop accuracy of 0.6587. The three most relevant mouse movement measures in that model were response time, y-flips and x-flips. While these were also selected for potential inclusion in the ensemble (Table 2), the outer loop accuracy 0.6825 showed better predictive performance of the ensemble model than Fernández-Fontelo et al. (2021).
The best kNCD ensemble model for employee level was a stacked-generalisation-type ensemble model with a gradient boosting model as a super-learner. This ensemble model was smaller, combining only three weak learners with semi-metrics from the elastic (DTW) and svs (globMin-y and globRange-y) families and personalised mouse movement measures as scalar-vector predictors in the super-learner. We selected this ensemble model with an inner loop accuracy of 0.7788 and an outer loop accuracy of 0.7664. In comparison, Fernández-Fontelo et al. (2021) found that the best model for employee level was gradient boosting with personalised measures, obtaining an outer loop accuracy of only 0.5909. In that model, the three most important measures were initiation time, y-flips, and the number of hovers, and two of these were also relevant here. However, the outer loop accuracy of 0.7664 for the ensemble model significantly improves the outer-loop accuracy of 0.5909 of Fernández-Fontelo et al. (2021). For this second target question, the distance measure that worked best for prediction was DTW, commonly used to compare trajectories with different speeds.

In general, it appears that semi-metrics from the svs family worked best for predicting difficulty in each of the two target questions. In particular, semi-metrics such as differences in maximum values in either the first or second dimension separately and several measure-based semi-metrics worked particularly well for predicting difficulty in employment detail (easy vs. complex language). Another semi-metric that performed quite well for prediction in this target question was the difference in the trajectories’ mean velocities. Unlike for employment detail, in employee level, the best semi-metric for difficulty prediction (ordered vs. unordered answering options) was DTW, an elastic-type distance. In terms of prediction, the distance between minimums and ranges in the second dimension was likewise important for this target question. Differences in question manipulations and resulting types of question difficulty can explain the differences in best-performing semi-metrics between both target questions. For employment detail, for example, longer response times and lower velocities are expected in the difficult setting with complex language. Trajectories for employee level, on the other hand, are expected to have more accelerations and decelerations for the unordered setting, given that participants with difficulties are expected to go up and down with the mouse to find the correct answer more frequently, and this is well caught by distances such as DTW.

Outer loop accuracies show better performance of the more flexible ensembles RFE and GBE than the LCE. For employment detail, the outer loop accuracies of the four ensembles with tree-based super-learners in Table 3 are much higher than those of the LC ensemble, which is even worse than that obtained in Fernández-Fontelo et al. (2021). For employee level, the advantage of the tree-based super-learners over the LCE is similar but not as pronounced. Overall, it seems that gradient boosting worked better than random forest as a super-learner as it was selected for each of the three target questions in kNCD and for education level in fkNN; see Table 3 and the Supporting Material.

The Supporting Material gives examples of the weak learners’ outer loop accuracies for the three target questions for fkNN and kNCD. The Supporting Material also contains inner loop accuracies for weak learners (fkNN or kNCD) selection in RF (type I and II) and GB (type I and II) ensembles and the outer loop accuracies for the LCE, RFE, and GBE. Finally, the Supporting Material also provides details and the link to a GitHub repository, which contains further information on the inner and outer loops accuracies for the weak learners, LCE, RFE, and GBE, and the estimated LCE coefficients for the
Table 2. Accuracies measuring predictive performance (outer loop) of our selection of weak learners built on kNCD for two of the three target questions (employment detail and employee level). Weak learners were chosen if outer-loop accuracy was equal to or greater than 0.55, and in this case used as candidates for ensemble construction (see Table 3).

| Family | Semi-metric | a   | accuracy | Family | Semi-metric | a   | accuracy |
|--------|-------------|-----|----------|--------|-------------|-----|----------|
| svs    | RT          | 0.6135 | 0.7286   | elastic | DTW         | 0.55 | 0.5707   |
|        | globMax-y   | 0.6044 | 0.7085   | svs     | globMin-y   | 0.55 | 0.5699   |
|        | length      | 0.6008 | 0.7005   | globMin | 0.55      |
|        | globMax-x   | 0.5952 | 0.6029   | globRange-y | 0.55 |
|        | hovers      | 0.5863 | 0.5927   | globRange | 0.55 |
|        | y-flips     | 0.5843 |          | initiation time | 0.55 |
| elastic| Hausdorff   | 0.5753 | 0.5807   | lock-step | $L^1$  | 0.55 |
| svs    | mean        | 1.5699 | 0.5749   | elastic | Fréchet     | 0.55 |
|        | hover time  | 0.5699 | 0.5707   | lock-step | correlation | 1.55 |
|        | globMax     | 0.5697 |          | elastic | Hausdorff   | 0.55 |
| lock-step | correlation | 0.5681 | 0.5651   | svs     | globMax     | 2.55 |
| svs    | globMax-y   | 1.5572 | 0.5629   | hovers  | 0.5528     |
|        | globRange-y | 1.5572 |          |         |            |
| lock-step | $L^4$       | 0.5555 |          |         |            |
|        | $L^1$       | 0.5554 |          |         |            |
| svs    | globMax-y   | 2.5535 |          |         |            |
|        | bivariate flips | 0.5534 |          |         |            |
|        | globMax     | 2.5518 |          |         |            |

three target questions.

4. Discussion

Computer mouse movements as a source of paradata have been used in survey research to improve different survey aspects, including survey data quality (Kreuter, 2013; Horwitz et al., 2017). For web surveys, it has been shown that a number of features of these mouse movements (mouse movement measures) were statistically related to question difficulty (Horwitz et al., 2017, 2020) and that several measures were good predictors of question difficulty (Fernández-Fontelo et al., 2021). In terms of web survey data quality, the detection of difficulty in, e.g., specific survey questions or participants, may help identify potential sources of measurement errors in responses as well as data quality issues (Yan and Olson, 2013). However, to date, the potential of using the entire mouse movement trajectory as bivariate functional predictors has not yet been fully explored. To this end, we analysed here data from a web survey that included a number of experimentally manipulated questions to create two levels of difficulty, and collected mouse movement trajectories for each of these questions (target questions). We thus investigated the potential of the trajectories as functional predictors to identify when respondents faced difficulties with specific questions, showing that mouse movement curves as a whole contain more information for prediction than mouse movement measures alone.
Table 3. Inner-loop accuracies for kNCD ensembles for which one weak learner (row-wise) was added at a time and included if it improved the accuracy (in bold). Type I ensembles included personalised measures following Fernández-Fontelo et al. (2021) in the ensemble using measure-based semi-metrics, and type II included these measures as scalar-vector covariates in the super-learner. Used super-learners were either a random forest (RF), gradient boosting (GB), or linear combinations (LC). Accuracies in italics indicate outer loop predictive performance of the total ensemble, including all semi-metrics with bolded rows. Accuracies in both bold and italics are the outer loop accuracies for the selected ensembles for each target question.

| Target question | Family | Semi-metric | \(a\) | type I | type II |
|-----------------|--------|-------------|------|--------|--------|
| Employment detail | svs    | RT          | -    | -      | -      |
|                  | globMax-y | .6437    | .6472| -      | -      |
|                  | length  | .6427      | .6548| -      | -      |
|                  | globMax-x | .6657    | .6879| .6820  | .6924  |
|                  | hovers  | .6708      | .6885| -      | -      |
|                  | y-flips | .6765      | .6937| -      | -      |
|                  | elastic | Hausdorff  | 0    | .6755  | .6901  | .6786  | .6877  |
|                  | svs     | mean       | 1    | .6741  | .6946  | .6824  | .6923  |
|                  | hover time | .6793    | .7020| -      | -      |
|                  | globMax | 0          | .6751| .6986  | .6832  | .6918  |
|                  | lock-step | correlation | 0   | .6799  | .7049  | .6884  | .6978  |
|                  | svs     | globMax-y  | 1    | .6801  | .7038  | .6896  | .7033  |
|                  | globRange-y | 1       | .6794| .7032  | .6880  | .7030  |
|                  | lock-step | \(L^4\)    | 0    | .6768  | .7033  | .6853  | .7002  |
|                  | svs     | globMax-y  | 2    | .6829  | .6982  | .6900  | .6986  |
|                  | bivariate flips | .6793    | .7057| -      | -      |
|                  | globMax | 2          | .6812| .6980  | .6884  | .6984  |
|                  | .6806  | .6825      | .6878| .6951  | .5972  |
| Employee level   | elastic| DTW        | 0    | .7567  | .7664  | .7767  | .7773  |
|                  | svs     | globMin-y  | 0    | .7576  | .7611  | .7762  | .7737  |
|                  | globMin | 0          | .7576| .7611  | .7762  | .7737  |
|                  | globRange-y | 0      | .7667| .7664  | .7743  | .7788  |
|                  | globRange | 0       | .7625| .7656  | .7725  | .7764  |
|                  | lock-step | \(L^1\)    | 0    | .7658  | .7727  | .7753  | .7753  |
|                  | elastic | Fréchet    | 0    | .7627  | .7727  | .7738  | .7782  |
|                  | lock-step | correlation | 1   | .7654  | .7720  | .7753  | .7755  |
|                  | elastic | Hausdorff  | 0    | .7653  | .7687  | .7729  | .7755  |
|                  | svs     | globMax    | 2    | .7667  | .7720  | .7696  | .7786  |
|                  | hovers  |            |      | .7718  | .7758  | -      | -      |
|                  | .7605  | .7784      | .7924| .7664  | .7205  |
Many existing methods that use functional predictors can be classified into approaches that reduce the dimension by extracting relevant features of the functions and using these in machine learning models (e.g., Fernández-Fontelo et al., 2021), and into genuinely functional approaches using whole functions as predictors. The main limitation of the first approach is that it summarises the functional observations, which may result in a loss of valuable information. Methods in the second class address this point, but are most often designed for univariate functional predictors. The small fraction of these methods that also allow for multivariate functional predictors rarely use ensemble methods to improve models’ predictive performance, and if they do combine weak learners only based on a single semi-metric such as $L^2$ or DTW, thus capturing only certain kinds of information contained in these functions.

Due to the limitations of existing classification methods for multivariate functional predictors, and in light of our applications’ goal of predicting question difficulty in web surveys using mouse movement trajectories (bivariate functions), we introduced here new ensemble models for multivariate functional predictors as extensions of the non-parametric models fkNN (Fuchs et al., 2015) and kNCD (Ferraty and Vieu, 2003, 2006). In particular, our ensemble models combine a broad collection of weak learners built on fkNN or kNCD, which are in turn based on different semi-metrics. These weak learners are combined using stacked generalisation techniques, using as super-learners either random forest (RFE) or gradient boosting (GBE).

RFE and GBE improve on existing methods as follows: First, they are genuinely functional methods that allow for both univariate and multivariate functional predictors. For example, when compared to Fernández-Fontelo et al. (2021), which used only features of the trajectories as predictors in a set of typical machine learning methods, our models show significantly better results as they consider information in the entire bivariate function. Second, RFE and GBE are built on a broad collection of semi-metrics, each of which captures a different aspect of our functional observations; thus, they are more comprehensive than methods considering a single semi-metric (e.g., Selk and Gertheiss, 2021; Prieto et al., 2013; Górecki and Luczak, 2015; Mei et al., 2016; Wang and Wu, 2017). In fact, our results (Tables 2 and 3) show that neither $L^2$ nor DTW are always the most important semi-metric in our application. Third, they combine different sources of information using stacked generalisation methods, which can improve the predictive performance compared to non-ensemble-based models, but have rarely been used in the functional classification setting so far (e.g., Prieto et al., 2013; Wang and Wu, 2017).

In our application, we analysed three target questions in a web survey conducted by the IAB and found that our functional approach significantly improved predictions on all questions, with up to 18 percentage point improvement (59% to 77%) in predictive accuracy (on the employee level item) compared to machine learning methods using only scalar summary measures. Gradient boosting based on trees was the best super-learner in our application. The most important semi-metrics were based on scalar summaries and elastic-type distances, but other semi-metrics also added information. We also saw differences in the best-performing semi-metrics depending on the difficulty manipulation for a given question, likely related to the different reactions caused by these difficulties.

Although our models performed well in prediction, some potential improvements remain yet unexplored. For instance, while we include elastic semi-metrics that allow stretching and compressing time, these still focus on the whole function and keep the
temporal ordering. Therefore, it may also be useful in future research to develop methods that look at more local characteristics, subsections, and interactions of trajectories, or to include other weak learners than fkNN and kNCD.

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