GSTARI model of BPR assets in West Java, Central Java, and East Java

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Abstract. Bank Perkreditan Rakyat (BPR) is a financial institution in Indonesia dealing with Micro, Small, and Medium Enterprises (MSMEs). Though limited to MSMEs, the development of the BPR industry continues to increase. West Java, Central Java, and East Java have high BPR asset development are suspected to be interconnected because of their economic activities as a neighboring provinces. BPR assets are nonstationary time series data that follow the uptrend pattern. Therefore, the suitable model with the data is generalized space time autoregressive integrated (GSTARI) which considers the spatial and time interrelationships. GSTARI model used spatial order 1 and the autoregressive order is obtained of optimal lag which has the smallest value of Akaike information criterion corrected. The correlation test results showed that the location used in this study had a close relationship. Based on the results of model identification, the best model obtained is GSTAR(3,1)-I(1). The parameter estimation used the ordinary least squares with the selection of significant variables used the stepwise method and the normalization cross correlation weighting. The residual model fulfilled the assumption of white noise and normal multivariate, so the model was appropriate. The average RMSE and MAPE values of the model were 498.75 and 2.48%.

1. Introduction
Bank is one of the financial institution in Indonesia. Bank as a financial institution is expected to assist the state in supporting the implementation of national development. Based on its function, the bank is divided into three types, such as central bank, commercial bank, and rural bank (BPR).

BPR provides financial support and loans for development of Micro, Small, and Medium Enterprises (MSMEs). Nevertheless, the growth of BPR business continues to increase. It can be seen from the development of BPR assets in some regions. OJK represents the recording of assets in the BPR industry nationally for February 2016 is Rp 102.67 billion, grew 13.55% annually.

The assets are indicator of good financial health and how effectively the institution in managing its assets. The development of BPR assets is expected to improve the service of banking services to MSMEs. The development of BPR assets follows the time series data form. West Java, Central Java and East Java provinces are more likely to have higher annual rates of BPR asset growth compared to other provinces. According to Borovkova et al [1], time series data from several adjacent locations often have interdependent relationships, so that BPR assets at one location are suspected of having...
association with BPR assets in the previous period and among other locations. The associate model with this condition is space time model.

Ruchjana [4] stated that the space time model is a model that combines elements of time and location dependencies in a time series and location data. The first type of space time model was introduced in 1980 by Pfeifer and Deutsh [3], the autoregressive space time (STAR). The STAR model assumes that the characteristics for all observed sites are homogeneous. The generalized space time autoregressive (GSTAR) model is an expansion of the STAR model which tends to be inflexible when confronted with multiple locations with heterogeneous characteristics [1].

One of the main problem in GSTAR model is the location weight determination. Generally, there are four location weights, ie uniform weights, binary weights, inverse weights of distances, and normalization of cross correlation weighting. According to Suhartono and Subanar [5], it can provide optimum results compared to other location weights in GSTAR modelling process as it illustrates the crosscorrelation value of intercurrent events at the corresponding time lag.

According to Ruchjana [4], the procedure in determining GSTAR model is seen from the data system. The GSTAR model is less suitable when used for nonstationary data. The generalized space time autoregressive integrated (GSTARI) model is a model with parameters that vary by location and are used in non-stationary timezone data [2]. The development of BPR assets is time series data which is not stationary as it follows the uptrend pattern so that the appropriate model for the BPR asset development data is the GSTARI model.

2. GSTARI Model and Normalization of Cross Correlation Weighting

2.1 GSTARI Model
The GSTARI model is specific form of Vector Autoregressive model. It reveals linear dependences of space and time. The main difference is on the spatial dependent, that in GSTARI model, it is expressed by weight matrix. Let \( \{ Z(t) : t = 0, \pm 1, \pm 2, \ldots \} \) be a multivariate time series of \( N \) components. In matrix notation, the GSTARI model of autoregressive order \( p \) and spatial orders \( \lambda_1, \lambda_2, \ldots, \lambda_p \) can be written as

\[
Z_{t+k} = \sum_{k=1}^{p} \Theta_{k0} W^{(0)}(t) + \sum_{l=1}^{\lambda_1} \Theta_{kl} W^{(l)} \] 

which \( Z_{t+k} \) is \((N \times 1)\) random vector at time \( t \), \( \Theta_{kl} \) is the autoregressive and the space time parameter at time lag \( k \) and spatial lag \( l \), \( W^{(l)} \) is \((N \times N)\) the weight matrix for spatial lag \( l \) (where \( l = 0,1 \)), and \( e_{t+k} \) is the \( N \) variate white noise vector mean \( 0 \) and variance-covariance \( \sigma^2 I_N \).

2.2 Normalization of Cross Correlation Weighting

Determination of space weight by using the normalization result of cross correlation between locations at the appropriate time lag is firstly proposed by Suhartono and Atok [6]. In general, cross correlation between two variables or location \( i \) and \( j \) at the time lag \( k \), \( \text{cor} [Z_i(t), Z_j(t-k)] \), defined as

\[
\rho_{ij}(k) = \gamma_{ij}(k) / \sigma_i \sigma_j, \quad k = 0, \pm 1, \pm 2, \ldots
\]

which \( \gamma_{ij}(k) \) is cross covariance between observation in location \( i \) and \( j \) at the time lag \( k \). \( \sigma_i \) and \( \sigma_j \) is standard deviation of observation in location \( i \) and \( j \). The estimated of cross correlation in sample data is

\[
\hat{\rho}_{ij}(k) = \frac{\sum_{t=-\infty}^{\infty} \left( Z_{it} - \bar{Z}_i \right) \left( Z_{jt} - \bar{Z}_j \right)}{\left[ \sum_{t=-\infty}^{\infty} (Z_{it} - \bar{Z}_i)^2 \sum_{t=-\infty}^{\infty} (Z_{jt} - \bar{Z}_j)^2 \right]^{1/2}}
\]
Then, determination of space weight could be done by normalization of the statistical inference to the cross correlation between locations at the appropriate time lag. This process generally yields space weight

\[ w_{ij}(k) = \frac{\eta_{i,j}(k)}{\sum_{k=1}^{\infty} |\eta_{i,j}(k)|} \]

where \( i \neq j \) and satisfies \( \sum_{i,j} |w_{ij}| = 1 \).

3. Research Methodology

3.1 Research Data

This study took asset data of BPR in West Java, Central Java and East Java Provinces obtained from the website of OJK from January 2011 through December 2016. Data from January 2011 up to December 2015 is used for parameter estimation while January data up to December 2016 is used for model accuracy testing. The variables used in this study consisted of assets of BPR in the Province West Java denoted \( Z_1 \), BPR assets in Central Java Province denoted \( Z_2 \), and the assets of BPR in East Java Province are denoted \( Z_3 \).

3.2 Step Research

The first step we examine the data kestasioneran by ADF test and MACF plot. If data is not stationary, then the data is done differentiation. After that we identify autoregressive order by checking the result of MPACF plot and determine the best model based on the smallest AICC value. Next, we determine the weights of cross-correlation normalization using data after differentiation. The next step is estimating model parameters with the least squares method and selecting variables which is significant with the stepwise method using differentiation data.

After that we test the residual assumption model with white noise test and normal multivariate, forecast of data for 2016 and test of model accuracy using data in 2016. The last we forecast for BPR asset data for 2017 and draw conclusions.

4. Results and Discussion

4.1. Statistics Description

In this research, the in sample of BPR asset data are monthly data from January 2011 to December 2015. The time series plot of each variable can be seen in Figure 1.

![Figure 1. Time series plot of BPR asset data in three locations](image)

Figure 1 shows the similarity of BPR asset data patterns of the three provinces that tend to rise jointly and continuously allow interrelated effects between the provinces.
4.2. Identification Model

Testing of stationarity of data is a process needed in GSTAR model. The stationarity can be seen simultaneously through the MACF plot. The MACF plot is represented by Figure 2. Figure 2 shows that the BPR asset data is not stationary in the mean. This is indicated by the number of symbols (+) that appear on each lag, which means simultaneously the three locations have a positive correlation on each lag so it needs to do differencing to make the BPR asset data is stationary.

![Figure 2. MACF plot of BPR asset data](image)

The model identification can be seen from MPACF plot of stationary data.

![Figure 4. MPACF plot of differencing BPR asset data](image)
Table 1. AICC of differencing BPR asset data

| Lag | Mh 0   | Mh 1   | Mh 2   | Mh 3   | Mh 4   | Mh 5   |
|-----|--------|--------|--------|--------|--------|--------|
| AR 0| 29.90155| 30.24918| 30.16321| 30.15223| 30.490401| 30.405073|
| AR 1| 29.209021| 29.672095| 29.03416| 29.982093| 30.377028| 30.104971|
| AR 2| 29.17654| 29.613905| 29.815468| 30.052323| 30.387494| 30.069457|
| AR 3| 28.925163| 29.402991| 29.623825| 29.739614| 30.111427| 30.258642|
| AR 4| 29.039888| 29.376816| 29.869716| 30.072654| 30.741639| 31.338793|
| AR 5| 29.319587| 29.878978| 30.019394| 30.549318| 31.585072| 31.803624|

Based on Table 1, it appears that the smallest AICC value in AR (3) is 28,925, so the corresponding model is GSTAR (3) - I(1).

4.3. Normalization of Cross Correlation Weighting

The normalization of cross correlation weighting is the location weighting by using the normalization of cross correlation inter location on the corresponding lag. The GSTARI model that applied in the BPR asset data has an autoregressive order of 3. Therefore, the weighted matrix of the cross-correlation normalization location is expressed as:

\[
W^1 = \begin{bmatrix}
0 & -0.645 & -0.355 \\
-0.5 & 0 & -0.5 \\
-0.273 & 0.927 & -0.073
\end{bmatrix},
W^2 = \begin{bmatrix}
0 & 0.32 & 0.68 \\
0.883 & -0.117 & 0
\end{bmatrix},
W^3 = \begin{bmatrix}
0 & 0.316 & 0.684 \\
-0.581 & 0 & 0.419 \\
-0.74 & -0.26 & 0
\end{bmatrix}
\]

4.4. Parameter Estimation

Table 2 shows the significant parameter estimation using stepwise method. It is also known that the parameter values $\phi_{10}, \phi_{20}, \phi_{30}, \phi_{10}, \phi_{21}, \phi_{31}$ are significant against $\alpha = 0.05$.

Table 2. The results of parameter estimation using stepwise method

| Parameter | Estimation | Statistic-t | p-value | Parameter | Estimation | Statistic-t | p-value |
|-----------|------------|-------------|---------|-----------|------------|-------------|---------|
| $\phi_{10}$ | -0.187 | -2.019 | 0.045 | $\phi_{20}$ | 0.595 | 3.685 | 0.000 |
| $\phi_{20}$ | 0.463 | 5.155 | 0.000 | $\phi_{31}$ | 1.232 | 2.668 | 0.008 |
| $\phi_{30}$ | 0.413 | 4.341 | 0.000 | $\phi_{21}$ | 0.615 | 4.111 | 0.000 |
| $\phi_{30}$ | 0.503 | 4.791 | 0.000 | $\phi_{31}$ | 0.611 | 3.400 | 0.001 |

The equation of model GSTAR (3) - I(1) that can be used to forecast BPR assets in West Java, Central Java and East Java provinces as follows:

\[
Z_1^*(t) = -0.187Z_1^*(t - 1) + 0.463Z_1^*(t - 2) + 0.394Z_2^*(t - 2) + 0.838Z_3^*(t - 2) \\
+ 0.193Z_2^*(t - 3) + 0.418Z_3^*(t - 3)
\]

\[
Z_2^*(t) = 0.413Z_2^*(t - 2) + 0.503Z_3^*(t - 3)
\]

\[
Z_3^*(t) = 0.595Z_3^*(t - 3) + 0.543Z_1^*(t - 2) - 0.072Z_2^*(t - 2)
\]
4.5. Residual Assumption
GSTARI model can be suitable if the residual fulfill two assumptions. The assumptions that have to be fulfilled are having the white noise and the multivariate normalization distribution.

**Table 3. LB Residual Test Model GSTAR (31) - I(1)**

| Lag | LB value | p-value | Conclusion | Lag | LB value | p-value | Conclusion |
|-----|----------|---------|------------|-----|----------|---------|------------|
| 1   | 0.5247   | 0.4688  | white noise| 6   | 11.2632  | 0.08057 | white noise|
| 2   | 1.1779   | 0.5549  | white noise| 7   | 12.1655  | 0.09525 | white noise|
| 3   | 3.9126   | 0.2711  | white noise| 8   | 13.4794  | 0.09638 | white noise|
| 4   | 4.634    | 0.327   | white noise| 9   | 13.5373  | 0.1398  | white noise|
| 5   | 7.474    | 0.1877  | white noise| 10  | 13.5765  | 0.1932  | white noise|

According to Table 3, the GSTAR model (31) - I(1) with the normalized weights of correlation has shown that residual white noise.

![Figure 5. Plot of Normal Distribution Multivariate Residual Model](image)

Based on Figure 5, visually the residual distribution of the GSTAR model (31) - I(1) approaches a straight line, so it can be said that the residual follows a multivariate normal distribution. The formal testing of normal multivariate normal distributed residual was performed by Kolmogorov-Smirnov test. The residual GSTAR model (31) - I(1) with cross-normalization weights cross-sectional has a value of less than and the p-value value so it is not rejected which means the residual model is multivariate normal distribution.

4.6. Model Validation
The RMSE and MAPE calculations of the GSTAR model (31) - I(1) with normalization of cross correlation weighting in the three locations are represented in Table 4.

**Table 4. RMSE and MAPE values**

| Location     | RMSE   | MAPE  |
|--------------|--------|-------|
| West Java    | 898.92 | 4.26% |
| Central Java | 134.27 | 0.44% |
| East Java    | 463.04 | 2.77% |
| Average      | 498.75 | 2.48% |
4.7. Forecasting
After we obtained the appropriate model, then we can forecast using the next can be forecasting using one step forecast by restoring the differencing data.

Table 5. Forecasting

| Month | West Java | Central Java | East Java | Month | West Java | Central Java | East Java |
|-------|-----------|--------------|-----------|-------|-----------|--------------|-----------|
| Jan/2017 | 18692.2   | 25135.49     | 12324.5   | Jul/2017 | 19375.1    | 26345.91     | 13479.47  |
| Feb/2017 | 18987.4   | 25335.56     | 12636.39  | Aug/2017 | 19446.66   | 26524.44     | 13547.99  |
| Mar/2017 | 19008.76  | 25555.9      | 12728.25  | Sep/2017 | 19458.35   | 26701.43     | 13670.3   |
| Apr/2017 | 19086.1   | 25769.05     | 12983.91  | Oct/2017 | 19635.55   | 26868.9      | 13834.9   |
| Mei/2017 | 19260.34  | 25960.69     | 13165.23  | Nov/2017 | 19638.16   | 27031.8      | 13869.28  |
| Jun/2017 | 19214.23  | 26159.55     | 13264.53  | Dec/2017 | 19733.05   | 27189.99     | 14026.22  |

5. Conclusion
Based on the results and discussion can be obtained the best model that can be used for forecasting BPR asset data in West Java, Central Java, and East Java provinces is GSTAR $\left(3_4\right)$ - I(1) with normalization of cross correlation weighting as it fulfilled the white noise and normal multivariate assumptions with the average of RMSE is 424.929 and MAPE 2.38%.

The GSTAR $\left(3_4\right)$ - I(1) explains that the BPR asset data in Central Java is only influenced by the previous data in the province itself, and it isn’t influenced by other provinces but can affect other provincial BPR assets. While the assets of BPR in West Java and East Java affect each other.

Acknowledgments
The authors would like to thank to Universitas Sebelas Maret for providing financial support through Grant of Fundamental Research 2017.

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