ON PARTICIPANTS NUMBER FLUCTUATIONS FOR GIVEN CENTRALITY AA-INTERACTIONS IN THE CLASSICAL GLAUBER APPROACH

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Abstract
In the framework of the classical Glauber approach the exact analytical expression for the variance of the number of participants (wounded nucleons) for given centrality AA interactions is presented. It’s shown, that in the case of nucleus-nucleus collisions along with the optical approximation term the additional ”contact” term appears. The numerical calculations for PbPb collisions at SPS energies show that at intermediate values of the impact parameter the ”optical” and ”contact” terms contributions to the variance of the participants number are of the same order and their sum is in a good agreement with the results of independent MC simulations of this process. The correlation between the numbers of participants in colliding nuclei is taken into account. In particular it’s demonstrated that in PbPb collisions at SPS energies the variance of the total number of participants approximately three times exceeds the Poisson one in the impact parameter region 10-12 Fm. The fluctuations of the number of collisions are also discussed.

1 Variance of participants number in one nucleus

At first we consider the variance \( V[N_{w}^{A}(b)] \) of the number of participants (wounded nucleons) in one of the colliding nuclei \( N_{w}^{A}(b) \) at a fixed value of the impact parameter \( b \). In the framework of pure classical, probabilistic approach to nucleus-nucleus collisions, formulated in [1], we find for the mean value and for the variance of \( N_{w}^{A}(b) \):

\[
\langle N_{w}^{A}(b) \rangle = AP(b) ,
\]

\[
V[N_{w}^{A}(b)] = AP(b)Q(b) + A(A - 1)[Q^{(12)}(b) - Q^2(b)] ,
\]

where \( P(b) = 1 - Q(b) \). For \( Q(b) \) and \( Q^{(12)}(b) \) we have:

\[
Q(b) = \int da_{1}T_{A}(a_{1})[1 - \sigma_{1}(a_{1})]^{B} ,
\]

\[
Q^{(12)}(b) = \int da_{1}da_{2}T_{A}(a_{1})T_{A}(a_{2})[1 - \sigma_{1}(a_{1}) - \sigma_{1}(a_{2}) + \sigma^{(12)}(a_{1}, a_{2})]^{B} ,
\]

where

\[
\sigma_{1}(a_{1, 2}) \equiv \int db_{1}T_{B}(b_{1})\sigma(a_{1, 2} - b_{1} + b) ,
\]

\[
\sigma^{(12)}(a_{1}, a_{2}) \equiv \int db_{1}T_{B}(b_{1})\sigma(a_{1} - b_{1} + b)\sigma(a_{2} - b_{1} + b) .
\]

Here \( T_{A} \) and \( T_{B} \) are the profile functions of the colliding nuclei \( A \) and \( B \); the \( \sigma(a) \) is the probability of inelastic interaction of two nucleons at the impact parameter value \( a \).
\((f \sigma(da) = \sigma_{NN}^{\text{in}} \equiv \sigma)\) and all integrations imply the integration over two-dimensional vectors in the impact parameter plane.

The formula (1) and the first term in (2) correspond to the naive picture (so-called optical approximation) implying that in the case of \(AB\)-collision at the impact parameter \(b\) one can use the binomial distribution (10) for \(N^A_w(b)\) with some averaged probability \(P(b)\) of inelastic interaction of a nucleon of the nucleus \(A\) with nucleons of the nucleus \(B\). At that the \(P(b)\) is considered the same for all nucleons of the nucleus \(A\).

The whole expression (2) is the result of more accurate calculation (see appendix A), when one uses probabilistic considerations taking into account the impact parameter plane positions of nucleons in the nuclei \(A\) and \(B\) and averaging then over these positions:

\[
V[N^A_w(b)] = \langle N^A_w(b)^2 \rangle - \langle N^A_w(b) \rangle^2 ,
\]

where

\[
\langle X \rangle \equiv \langle \langle X \rangle \rangle_A \equiv \int X \prod_{k=1}^{B} T_B(b_k) db_k \prod_{i=1}^{A} T_A(a_i) da_i.
\]

Here \(X\) means average of some variate \(X\) at fixed positions of all nucleons in \(A\) and \(B\); \(\langle \rangle_A\) and \(\langle \rangle_B\) mean averaging over positions of these nucleons.

In the limit \(r_N \ll R_A, R_B\) formulae (1) reduce to

\[
\sigma_1(a_{1,2}) \approx \sigma T_B(a_{1,2} + b) ,
\]

\[
\sigma^{(12)}(a_1, a_2) \approx I(a_2 - a_1) \cdot T_B(a_1 + b) , \quad \text{with} \quad I(a) \equiv \int ds \sigma(s) \sigma(s + a) .
\]

Note that in this limit the \(Q(b)\) and hence the first term of (2) and (1) depend only on the integral inelastic \(NN\) cross-section \(\sigma \equiv \sigma_{NN}^{\text{in}}\), but the \(Q^{(12)}(b)\) entering the second term of (2) depends also on the shape of the function \(\sigma(b)\) through the integral \(I(a)\) (see equation (7))

Note also that using of the approximation \(\sigma(b) = \sigma \delta(b)\) for \(NN\) interaction gives the same result (as taking of the limit \(r_N \ll R_A, R_B\)) only for naive part of the answer, which is expressed through \(Q(b)\). If one will try to use the approximation \(\sigma(b) = \sigma \delta(b)\) for to calculate \(Q^{(12)}(b)\), then one gets \(I(a) = \sigma^2 \delta(a)\) and \(\sigma^{(12)} = \sigma^2 \delta(a_2 - a_1) \cdot T_B(a_1 + b)\), which leads to infinite \(Q^{(12)}(b)\) at \(B \geq 2\). Meanwhile, for any correct approximation of \(\sigma(b)\), when \(\sigma(b) \leq 1\) in correspondence with its probabilistic interpretation, we find definite finite answer for \(Q^{(12)}(b)\).

In our numerical calculations we’ll use for \(NN\) interaction the “black disk” approximation:

\[
\sigma(b) = \theta(r_N - |b|) ,
\]

or Gauss approximation:

\[
\sigma(b) = \exp(-b^2/r_N^2) .
\]

In both cases \(\sigma = \pi r_N^2\). For the nuclear profile functions \(T_A\) and \(T_B\) we’ll use the standard Woods-Saxon approximation.

We would like to emphasize that nontrivial second term in (2) arises only in the case of nucleus-nucleus collisions. For \(A = 1\) or \(B = 1\) it’s equal to zero. At \(A = 1\) due to explicit factor \(A - 1\) in (2) and at \(B = 1\) due to fact that in this case \(Q^{(12)}(b) = Q^2(b)\). This corresponds to the well known fact that for nucleus-nucleus collisions the Glauber approach doesn’t reduce to the so-called optical approximation even in the limit \(r_N \ll R_A, R_B\) (see, for example, [2]).
Figure 1: The variance of the number of wounded nucleons in one nucleus for PbPb collisions at SPS energies ($\sigma \equiv \sigma_{NN}^{in}=31$ mb, $r_N=1$ Fm) as a function of the impact parameter $b$. The points $\bullet$ and $\blacksquare$ - results of numerical calculations by formulæ (2), (3) and (7) using respectively the ”black disk” (8) and Gaussian (9) approximations for $NN$ interaction; $\circ$ and $\square$ - results of independent MC calculations using for $NN$ interaction ”black disk” (8) or Gaussian (9) approximation; $\ast$ - ”optical” approximation (the first term in formula (2)); + - the Poisson variance: $V[N_w^A(b)] = \langle N_w^A(b) \rangle$. The curves are shown to guide eyes.

This additional term, which arises in (2) in the case of nucleus-nucleus collisions, depends, as we have mentioned, not only on integral value of inelastic $NN$ cross-section $\sigma_{NN}^{in} \equiv \sigma = \int \sigma(a) da$, but also on the shape of the function $\sigma(b)$, i.e. on the details of $NN$ interaction at $r_N$ distances, which are much smaller than the typical nuclear distances. In quantum Glauber approach this corresponds to the fact that in the case of $AA$ collisions, in contrast with $pA$ collisions, the loop diagrams of the type shown in Fig2 appear. The typical momentum corresponding this loop integration is much larger than the typical nuclear momenta (which corresponds to smaller than typical nuclear distances) and one encounters the ”contact” terms problem (see, for example, [2, 3, 4]). The second term in formulæ (2) is the manifestation of this problem at the classical level.

Figure 2: An example of the loop diagram in quantum Glauber approach to $AB$-collisions. 1 and 2 - nucleons of the nucleus A; 1’ and 2’ - nucleons of the nucleus B (see [2, 3, 4] for details).
Figure 3: The same as in Fig.1 but for the mean number of wounded nucleons in one nucleus, calculated by formulae (1), (3) and (4); * - ”optical” approximation, calculated using formulae (1), (3) and (7).

The numerical evaluation of the contribution of the additional - "contact" term in (2) are presented in Fig.1 for PbPb collisions at SPS energies (σ ≡ σ_{NN}^{in}=31 mb, r_N=1 Fm). For the control we also carried out independent Monte-Carlo calculations of the mean values and the variances involved presenting the results on the same figures. All calculations were done at fixed values of the impact parameter b (Δb = 0).

In Fig.1 we see that the calculated ”contact” term in (2) is essential and gives approximately the same contribution to the \( \langle N_A^A(b) \rangle \) variance for PbPb collisions at intermediate values of b, as the first ”optical” term in (2). We see also that the results of independent MC calculations of the \( \langle N_A^A(b) \rangle \) variance are in a good agreement with the results of analytical calculations by formula (2), but only with taking into account its second term.

Note that due to this ”contact” term the \( \langle N_A^A(b) \rangle \) variance is larger than the Poisson one for peripheral PbPb collisions (at \( b > 7 \) Fm). The week dependence of the results on details of \( NN \) interaction at nucleon distances is also seen. The results lay systematically slightly higher in the case of using the ”black disk” (8) approximation for \( \sigma(b) \), than in the case of using the Gaussian (9) approximation with the same value of \( \sigma \).

For the mean value \( \langle N_A^A(b) \rangle \), in contrast to the variance of \( N_A^A(b) \), the exact answer coincides with the ”optical” approximation result (see formula (1) and appendix A) and depends only on \( \sigma \). In Fig.3 we see that MC calculations also confirm this result.

2 Variance of the total number of participants

Now we pass to the calculation of the variance of the total number of participants \( V[N_A^A(b) + N_B^B(b)] \) at a fixed value of the impact parameter b. Clear, that we simply
have for the mean value
\[ \langle N_w^A(b) + N_w^B(b) \rangle = \langle N_w^A(b) \rangle + \langle N_w^B(b) \rangle \] (10)
and by (5) for the variance
\[ V[N_w^A(b) + N_w^B(b)] = V[N_w^A(b)] + V[N_w^B(b)] + 2\{\langle N_w^A(b)N_w^B(b) \rangle - \langle N_w^A(b) \rangle \langle N_w^B(b) \rangle \} . \] (11)

In naive approach ("optical approximation") there are no correlations:
\[ \langle N_w^A(b)N_w^B(b) \rangle - \langle N_w^A(b) \rangle \langle N_w^B(b) \rangle = 0 . \]

More accurate calculations (see appendix B), based on formulae (5) and (6), lead to
\[ \langle N_w^A(b)N_w^B(b) \rangle - \langle N_w^A(b) \rangle \langle N_w^B(b) \rangle = AB[Q^{(11)}(b) - Q(b)\tilde{Q}(b)] , \] (12)
where
\[ Q(b) = \int da_1 T_A(a_1)[1 - \sigma_1(a_1)]^B , \quad \tilde{Q}(b) = \int db_1 T_B(b_1)[1 - \tilde{\sigma}_1(b_1)]^A , \]
\[ Q^{(11)}(b) = \int da_1 db_1 T_A(a_1)T_B(b_1)[1 - \sigma_1(a_1)]^{B-1}[1 - \tilde{\sigma}_1(b_1)]^{A-1}[1 - \sigma(a_1 - b_1 + b)] . \] (13)

Here
\[ \sigma_1(a_1) \equiv \int db_1 T_B(b_1)\sigma(a_1 - b_1 + b) \approx \sigma T_B(a_1 + b) , \]
\[ \tilde{\sigma}_1(b_1) \equiv \int da_1 T_A(a_1)\sigma(a_1 - b_1 + b) \approx \sigma T_A(b + b_1) \] (14)

Note that $Q(b)$ and $\sigma_1(a_1)$ are the same as in formulae (5), (6) and (7).
The results of numerical calculations of the correlator (12) by formulae (13) and (14) together with the results of independent Monte-Carlo calculations for PbPb collisions at SPS energies are presented in Fig.4.

Comparing Fig.4 with Fig.1 we see that the contribution of this correlator to the variance of the total number of participants at intermediate values of \(b\) is about half of the variance for one nucleus \(V[N_w(b)]\) and is about the contribution of first ”optical” term in (2). (Note that the relative contribution of the correlator to \(V[N_w^A(b) + N_w^B(b)]\) even greater at large values of \(b\), \(b \geq 10\).) The results are again in a good agreement with the results of MC calculations.

In Fig.5 we present the final results for the variance of the total number of participants in PbPb collisions at SPS energies, taking into account the contribution of this correlator. We see in particular that now the calculated variance of the total number of participants \(V[N_w^A(b) + N_w^B(b)]\) is approximately three times larger than the Poisson one in the impact parameter region 10-12 Fm.

### 3 Discussion and conclusions

In the framework of the classical Glauber approach the exact analytical expression for the variance of the number of participants (wounded nucleons) in AA collisions at a fixed value of the impact parameter is presented. It’s shown, that along with the optical approximation contribution (which depends only on the total NN cross-section) in the case of nucleus-nucleus collisions there is the additional ”contact” term contribution,
depending on the integral of the overlap of two inelastic NN cross-sections in the impact parameter plane.

In the classical Glauber approach under consideration this "contact" contribution arises at correct taking into account the interactions between two pairs of nucleons in colliding nuclei (a pair in one nucleus with a pair in another). It's found, that the interactions of higher order, than between two pairs of nucleons, don't contribute to the variance. At that the expression for the mean number of participants proved to be exact already in optical approximation, based on taking into account only the averaged interaction between a nucleon in one nucleus with a nucleon in another. The same is also valid for the mean value and the variance of the number of NN-collisions in AA-interactions (see appendix C).

These results are obtained in the framework of the pure classical (probabilistic) Glauber approach \[1\]. However it's possible to suppose, that in the quantum case the one loop expression for the variance and the "tree" expression for the mean number of participants and NN-collisions will be exact.

Using the formulae obtained, the numerical calculations of the variance of the participants number in PbPb collisions at SPS energies are done. It’s demonstrated that at intermediate values of the impact parameter the "optical" and "contact" term contributions are of the same order and their sum is in a good agreement with the results of independent MC simulations of this process.

When calculating the variance of the total (in both nuclei) number of participants the correlation between the numbers of participants in the colliding nuclei is taking into account. The exact analytical expression for the correlator at a fixed value of the impact parameter is obtained. The results of numerical calculations of the correlator for PbPb collisions at SPS energies show that at intermediate and large values of the impact parameter its contribution to the variance of the total number of participants is about half of

![Figure 6](image_url)

Figure 6: The same as in Fig.5 but for the normalized variance $V[N_w(b)]/\langle N_w(b) \rangle$ of the total number of wounded nucleons in colliding nuclei, $N_w(b) \equiv N^A_w(b) + N^B_w(b)$. 
the variance in one nucleus, again in good agreement with independent MC simulations.

In particular as a result it’s found that the calculated variance of the total number of participants in PbPb collisions at SPS energies approximately three times larger than the Poisson one in the impact parameter region 10-12 Fm. (See Figs.6 and 10 for the normalized variance of the number of wounded nucleons and NN-collisions.)

Note that the good agreement of the analytical and MC calculations ensures the reliability both of them and enables to use the developed MC algorithm in future experimental setup motivated calculations.

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Appendixes

A Calculation of the participants variance for one nucleus

The geometry of $AB$-collision is shown in Fig.7. All $a$ and $b$ are the two-dimensional vectors in the impact parameter plane. In the framework of the classical (probabilistic) approach [1] the dimensionless $\sigma(b)$ is the interaction probability of two nucleons at the impact parameter value $b$:

$$\int \sigma(b)db = \sigma_{NN}^m \equiv \sigma$$

(all integrations imply the integration over two-dimensional vectors in the impact parameter plane). This probabilistic interpretation means that $\sigma(b) \leq 1$, so even in the limit $r_N \ll R_A, R_B$ we can’t use approximation: $\sigma(b) = \sigma \delta(b)$. The examples of valid approximations are as follows: the "black disk" approximation:

$$\sigma(b) = \theta(r_N - |b|) , \quad \sigma = \pi r_N^2 ,$$

the "grey disk" approximation:

$$\sigma(b) = \gamma \theta(r_N - |b|) , \quad \gamma < 1 , \quad \sigma = \gamma \pi r_N^2 ,$$

Gauss approximation:

$$\sigma(b) = Ce^{-\frac{r^2}{r_N^2}} , \quad C \leq 1 (!) , \quad \sigma = C \pi r_N^2 .$$

$T_A$ and $T_B$ are the profile functions of the colliding nuclei $A$ and $B$. We’ll imply the factorization takes place:

$$T_A(a_1, ..., a_A) = \prod_{i=1}^{A} T_A(a_i) ,$$

which is sound approximation for heavy nuclei. Let us introduce also some shorthand notations:

$$\hat{da}_i \equiv T_A(a_i)da_i , \quad \int \hat{da}_i = \int T_A(a_i)da_i = 1 .$$
\[ b_{jk} = a_j - b_k + b \]

Figure 7: AB-collision.

\[
\langle X \rangle \equiv \langle \langle X \rangle \rangle_B = \int \overline{X} \prod_{k=1}^{B} db_k \prod_{i=1}^{A} da_i = \int \overline{X} \prod_{k=1}^{B} T_B(b_k) db_k \prod_{i=1}^{A} T_A(a_i) da_i \tag{21}
\]

\[ V[X] \equiv \langle X^2 \rangle - \langle X \rangle^2 \tag{22} \]

Here \( \overline{X} \) means average of some variate \( X \) at fixed positions of all nucleons in \( A \) and \( B \); \( \langle \rangle_A \) and \( \langle \rangle_B \) mean averaging over positions of these nucleons.

We introduce now the set of variates \( X_1, ..., X_A \) (each can take on a value equal only to 0 or 1) by the following way:

\( X_j = 1 \) if \( j \)-th nucleon of the nucleus \( A \) interacts with some nucleons of the nucleus \( B \)
\( X_j = 0 \) if \( j \)-th nucleon of the nucleus \( A \) doesn’t interact with any nucleons of the nucleus \( B \)

Then the number of participants (wounded nucleons) in the nucleus \( A \) in the given event can be simply expressed through these variates:

\[ N_w^A(b) = \sum_{j=1}^{A} X_j . \tag{23} \]

So we have for the mean value:

\[ \langle N_w^A(b) \rangle = \sum_{j=1}^{A} \langle X_j \rangle = \sum_{j=1}^{A} \langle \langle X_j \rangle \rangle_B \tag{24} \]

and for the variance of \( N_w^A(b) \):

\[ V[N_w^A(b)] \equiv \langle N_w^A(b)^2 \rangle - \langle N_w^A(b) \rangle^2 , \quad \langle N_w^A(b)^2 \rangle = \langle (\sum_{j=1}^{A} X_j)^2 \rangle . \tag{25} \]

Let us start our calculations from (21). Clear that for given configuration \( \{a_i\} \) and \( \{b_k\} \):

\[ \overline{X_j} = 0 \cdot P(X_j = 0) + 1 \cdot P(X_j = 1) = p_j = 1 - q_j , \tag{26} \]
where
\[ P(X_j = 0) \equiv q_j = \prod_{k=1}^{B} (1 - \sigma_{jk}) , \]
\[ P(X_j = 1) \equiv p_j = 1 - q_j = 1 - \prod_{k=1}^{B} (1 - \sigma_{jk}) , \]
\[ \sigma_{jk} \equiv \sigma(a_j - b_k + b) . \]

Here \( P(X_j = 0(1)) \) is the probability that the variate \( X_j \) will be equal to 0 or 1 correspondingly. We have to keep in mind that \( p_j \) and \( q_j \) are the functions of \( a_j, b_1, \ldots, b_B \) and \( b \):
\[ q_j = q_j(a_j, \{b_k\}, b) , \quad p_j = p_j(a_j, \{b_k\}, b) . \]

Straightforward calculations give:
\[ \langle N^A_w(b) \rangle = \sum_{j=1}^{A} \langle \langle X_j \rangle_B \rangle_A = \sum_{j=1}^{A} \langle \langle p_j \rangle_B \rangle_A = \sum_{j=1}^{A} \langle \langle 1 - q_j \rangle_B \rangle_A = A - \sum_{j=1}^{A} \langle \langle q_j \rangle_B \rangle_A \]
\[ \langle q_j \rangle_B = \prod_{k=1}^{B} (1 - \sigma_{jk}) = \prod_{k=1}^{B} \int dB_k (1 - \sigma_{jk}) = \prod_{k=1}^{B} \int dB_k (1 - \sigma_{jk}) = \prod_{k=1}^{B} (1 - \int dB_k \sigma_{jk}) =
\]
\[ = (1 - \sigma_j)^B , \quad \sigma_j \equiv \int dB_1 \sigma_1 = \int dB_1 T_B(b_1) \sigma(a_j - b_1 + b) \equiv \sigma_1(a_j) \]
\[ \langle q_j \rangle_B = \langle (1 - \sigma_j)^B \rangle_A = \prod_{i=1}^{A} \int da_i (1 - \sigma_j)^B = \prod_{i=1}^{A} \int da_i (1 - \sigma_j)^B \]
\[ = Q(b) , \quad Q(b) \equiv \int da_1 T_A(a_1) (1 - \sigma_1)^B , \quad \sigma_1 = \int dB_1 T_B(b_1) \sigma(a_1 - b_1 + b) \equiv \sigma_1(a_1) \]
\[ \langle N^A_w(b) \rangle = A - \sum_{j=1}^{A} \langle \langle q_j \rangle_B \rangle_A = A - \sum_{j=1}^{A} Q(b) = A - AQ(b) = A(1 - Q(b)) = AP(b) , \]

which coincides with formula (1) of the text. We see that the result for the mean value of the number of participants is the same as in an optical approximation.

Let us now calculate the variance of \( N^A_w(b) \). We start from (25):
\[ \langle N^A_w(b)^2 \rangle = \langle (\sum_{j=1}^{A} X_j)^2 \rangle = \langle \sum_{j_1,j_2=1}^{A} X_{j_1} X_{j_2} \rangle = \sum_{j_1 \neq j_2=1}^{A} \langle X_{j_1} X_{j_2} \rangle + \sum_{j=1}^{A} \langle X_j^2 \rangle . \]

So we have to calculate the following two sums:
\[ \sum_{j=1}^{A} \langle X_j^2 \rangle = \sum_{j=1}^{A} \langle \langle X_j^2 \rangle_B \rangle_A = \sum_{j=1}^{A} \langle \langle X_j \rangle_B \rangle_A \]
\[ \sum_{j_1 \neq j_2=1}^{A} \langle X_{j_1} X_{j_2} \rangle = \sum_{j_1 \neq j_2=1}^{A} \langle \langle X_{j_1} X_{j_2} \rangle_B \rangle_A = \sum_{j_1 \neq j_2=1}^{A} \langle \langle X_{j_1} \cdot X_{j_2} \rangle_B \rangle_A . \]

Note that the last expression can’t be reduced to
\[ \neq \sum_{j_1 \neq j_2=1}^{A} \langle \langle X_{j_1} \rangle_B \rangle_A \cdot \langle \langle X_{j_2} \rangle_B \rangle_A \] (!

Just in this point the optical approximation breaks.
Notations are the same as in (27)–(32). And for the first sum (34) we find:

\[
\sum_{j=1}^{A} \langle X_j^2 \rangle_A = \sum_{j=1}^{A} \langle \overline{X_j}^2 \rangle_A = \sum_{j=1}^{A} \langle \overline{X_j}^2 \rangle_A = Q = Q^A(b) = AP(b) = A(1 - Q(b)).
\] (36)

For the second sum (35) the straightforward calculations give:

\[
\overline{X_j X_j} = \overline{X_j} \cdot \overline{X_j} = p_j p_{j2} = (1 - q_{j1})(1 - q_{j2}) = 1 - q_{j1} - q_{j2} + q_{j1} q_{j2},
\]

\[
\sum_{j_1 \neq j_2 = 1}^{A} \langle X_{j_1} X_{j_2} \rangle = \sum_{j_1 \neq j_2 = 1}^{A} \langle 1 - q_{j1} - q_{j2} + q_{j1} q_{j2} \rangle_A =
\]

\[
= A(A - 1) - (A - 1) \left( \sum_{j_1 = 1}^{A} \langle q_{j1} \rangle_A + \sum_{j_2 = 1}^{A} \langle q_{j2} \rangle_A \right) + \sum_{j_1 \neq j_2 = 1}^{A} \langle q_{j1} q_{j2} \rangle_A =
\]

\[
= A(A - 1) - (A - 1)A(Q + Q(b)) + \sum_{j_1 \neq j_2 = 1}^{A} \langle q_{j1} q_{j2} \rangle_A =
\]

\[
= A(A - 1)[1 - 2Q(b) + Q^{(12)}(b)],
\]

where we introduce

\[
Q^{(12)}(b) = \frac{1}{A(A - 1)} \sum_{j_1 \neq j_2 = 1} \langle q_{j1} q_{j2} \rangle_A.
\]

Let us now calculate \(Q^{(12)}(b)\):

\[
\langle q_{j1} q_{j2} \rangle_B = \left( \prod_{k_1 = 1}^{B} (1 - \sigma_{j1 k_1}) \right) \left( \prod_{k_2 = 1}^{B} (1 - \sigma_{j2 k_2}) \right) = \int \prod_{k=1}^{B} \hat{d}b_k(1 - \sigma_{j1 k})(1 - \sigma_{j2 k}) =
\]

\[
= \prod_{k=1}^{B} \int \hat{d}b_k(1 - \sigma_{j1 k})(1 - \sigma_{j2 k}) = \left( \int \hat{d}b_1(1 - \sigma_{j1 1})(1 - \sigma_{j2 1}) \right)^B =
\]

\[
= \left( \int \hat{d}b_1(1 - \sigma_{j1 1} - \sigma_{j2 1} + \sigma_{j1 j2}) \right)^B = \left( 1 - \int \hat{d}b_1 \sigma_{j1 1} - \int \hat{d}b_1 \sigma_{j2 1} + \int \hat{d}b_1 \sigma_{j1 j2} \right)^B =
\]

\[
= (1 - \sigma_{j1} - \sigma_{j2} + \sigma^{(j1 j2)})^B,
\]

where \(\sigma_{j1}\) and \(\sigma_{j2}\) are given by (31) and

\[
\sigma^{(j1 j2)} \equiv \int \hat{d}b_1 \sigma_{j1 1} \sigma_{j2 1} = \int \hat{d}b_1 T_B(b_1)\sigma(a_{j1} - b_1 + b)\sigma(a_{j2} - b_1 + b) \equiv \sigma^{(12)}(a_{j1}, a_{j2})
\] (37)

So for \(Q^{(12)}(b)\) we find

\[
Q^{(12)}(b) \equiv \frac{1}{A(A - 1)} \sum_{j_1 \neq j_2 = 1} \langle \langle q_{j1} q_{j2} \rangle_B \rangle_A = \frac{1}{A(A - 1)} \sum_{j_1 \neq j_2 = 1} \langle (1 - \sigma_{j1} - \sigma_{j2} + \sigma^{(j1 j2)})^B \rangle_A
\]

\[
= \frac{1}{A(A - 1)} \sum_{j_1 \neq j_2 = 1} \int \prod_{i=1}^{A} \hat{d}a_i(1 - \sigma_{j1} - \sigma_{j2} + \sigma^{(j1 j2)})^B
\]

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\[ \frac{1}{A(A-1)} \sum_{j_1 \neq j_2=1}^{A} \int \tilde{d}a_{j_1} \tilde{d}a_{j_2} (1 - \sigma_{j_1} - \sigma_{j_2} + \sigma^{(12)})^B \]
\[ = \int da_1 da_2 T_A(a_1) T_A(a_2) (1 - \sigma_1 - \sigma_2 + \sigma^{(12)})^B , \]

where
\[ \sigma^{(12)} = \int \tilde{d}b_1 \sigma_{11,21} = \int db_1 T_B(b_1) \sigma(a_1 - b_1 + b) \sigma(a_2 - b_1 + b) \equiv \sigma^{(12)}(a_1, a_2) . \]

Substituting (33)–(38) into (25) we find for the variance of \( N^A_w(b) \):
\[ V[N^A_w(b)] \equiv \langle N^A_w(b)^2 \rangle - \langle N^A_w(b) \rangle^2 = \sum_{j_1 \neq j_2=1}^{A} \langle X_{j_1} X_{j_2} \rangle + \sum_{j=1}^{A} \langle X_j^2 \rangle - \langle N^A_w(b) \rangle^2 \]
\[ = A(A-1) [1 - 2Q(b) + Q^{(12)}(b)] + A(1 - Q(b)) - [A(1 - Q(b))]^2 \]
\[ = AQ(b) - A^2 Q^2(b) + A(A-1)Q^{(12)}(b) = AQ(b)[1 - Q(b)] + A(A-1)[Q^{(12)}(b) - Q^2(b)] , \]

which coincides with the formula (2) of the text.

The naive approach (optical approximation) implies the binomial distribution for \( N^A_w \):
\[ P_{opt}(N^A_w) = C_A^{N^A_w} P(b)^{N^A_w} Q(b)^{A-N^A_w} , \quad P(b) = 1 - Q(b) \]
(40)
which immediately leads to
\[ \langle N^A_w(b) \rangle_{opt} = AP(b) \]
(41)
and
\[ V[N^A_w(b)]_{opt} = AP(b)Q(b) = \langle N^A_w(b) \rangle Q(b) . \]
(42)

We see that this gives true answer only for the mean value \( \langle N^A_w(b) \rangle \). For the variance \( V[N^A_w(b)] \) the results coincide only at \( A = 1 \) due to explicit factor \( A - 1 \) in (39) or at \( B = 1 \) as in this case \( Q^{(12)}(b) = Q^2(b) \) (i.e. for pA-collisions). For nucleus-nucleus collisions, when \( A \geq 2 \) and \( B \geq 2 \) the naive result (42) for variance is not valid (see text for details).

Note that for peripheral AA collisions (at large \( b \)), when \( P(b) \) becomes small \( (P(b) \ll 1, Q(b) \approx 1) \), the naive distribution (40) and the variance (42) reduce to the Poisson ones: \( V[N^A_w(b)]_{opt} = \langle N^A_w(b) \rangle \) (see Fig.1).

**B Correlation between the numbers of wounded nucleons in colliding nuclei at fixed centrality**

The calculations are similar to ones in the appendix A (we use the same notations). Along with the set of variates \( X_1, ..., X_A \) we introduce in the symmetric way also the set of variates \( \tilde{X}_1, ..., \tilde{X}_B \) (each can again take on a value equal only to 0 or 1) by the following way:
\[ \tilde{X}_k = 1 \text{ if } k\text{-th nucleon of the nucleus } B \text{ interacts with some nucleons of the nucleus } A \]
\[ \tilde{X}_k = 0 \text{ if } k\text{-th nucleon of the nucleus } B \text{ doesn't interact with any nucleons of the nucleus } A \]
Then similarly to (23) the number of participants (wounded nucleons) in the nucleus $B$ in the given event can be simply expressed through these variates:

$$N_w^B(b) = \sum_{k=1}^{B} \bar{X}_k .$$  \hspace{1cm} (43)

Then

$$\langle N_w^A(b) N_w^B(b) \rangle = \sum_{j=1}^{A} \sum_{k=1}^{B} \langle X_j \bar{X}_k \rangle = \sum_{j=1}^{A} \sum_{k=1}^{B} \langle \langle X_j \bar{X}_k \rangle_B \rangle_A $$ \hspace{1cm} (44)

and

$$X_j \bar{X}_k = 1 \cdot 1 \cdot P_{jk}(1, 1) + 0 \cdot P_{jk}(1, 0) + 0 \cdot 1 \cdot P_{jk}(0, 1) + 0 \cdot 0 \cdot P_{jk}(0, 0) = P_{jk}(1, 1).$$  \hspace{1cm} (45)

Here $P_{jk}(0(1), 0(1))$ is the probability that the variates $X_j$ and $\bar{X}_k$ will be equal to 0 or 1 correspondingly. We have

$$P_{jk}(1, 1) = \sigma_{jk} + (1 - \sigma_{jk})\rho_{jk} \bar{\rho}_{jk} ,$$  \hspace{1cm} (46)

where $\sigma_{jk}$ is the probability of the interaction of the $j$-th nucleon of the nucleus $A$ with the $k$-th nucleon of the nucleus $B$:

$$\sigma_{jk} = \sigma(a_j - b_k + b) ,$$  \hspace{1cm} (47)

and $\rho_{jk}$ is the probability of the interaction of the $j$-th nucleon of the nucleus $A$ with at least one nucleon of the nucleus $B$ except the $k$-th nucleon (correspondingly $\bar{\rho}_{jk}$ is the probability of the interaction of the $k$-th nucleon of the nucleus $B$ with at least one nucleon of the nucleus $A$ except the $j$-th nucleon):

$$\rho_{jk} = 1 - \prod_{k'=1}^{B} (1 - \sigma_{jk'}) , \hspace{1cm} \bar{\rho}_{jk} = 1 - \prod_{j'=1}^{A} (1 - \sigma_{j'k}) .$$  \hspace{1cm} (48)

Combining (44)–(48) and acting as in the appendix A we find the formulae (12)–(14) of the text.

C On fluctuations of the number of collisions

In this appendix we discuss briefly the fluctuations of the number of NN-collisions in AA-interactions at fixed value of centrality in the framework of the approach under consideration.

To calculate the number of collisions we define the set of $A$ variates $Y_1, ..., Y_A$ (each can take on a value from 0 to $B$) by the following way:

$Y_j = 0$ if $j$-th nucleon of the nucleus $A$ doesn’t interact with any nucleons of the nucleus $B$

$Y_j = 1$ if $j$-th nucleon of the nucleus $A$ interacts with one nucleon of the nucleus $B$

$Y_j = 2$ if $j$-th nucleon of the nucleus $A$ interacts with two nucleons of the nucleus $B$

...
\[ Y_j = B \text{ if } j\text{-th nucleon of the nucleus } A \text{ interacts with all nucleons of the nucleus } B \]

Then \( N_c(b) \) (the number of NN-collisions in the given event with impact parameter \( b \)) can be expressed through these variates as follows:

\[ N_c(b) = \sum_{j=1}^{A} Y_j \quad (49) \]

Clear that again (see appendix A):

\[ P(Y_j = 0) = P(X_j = 0) = q_j = \prod_{k=1}^{B} (1 - \sigma_{jk}) \quad (50) \]

To calculate \( P(Y_j = n) \) for \( n = 1, \ldots, B \) let us introduce \( \{k_1, \ldots, k_n\} \) - the random sampling from the set \( \{1, \ldots, B\} \) and \( \{k_{n+1}, \ldots, k_B\} \) - the rest after sampling. Then

\[ P(Y_j = n) = \sum_{\{k_1, \ldots, k_n\}} \sigma_{jk_1} \cdots \sigma_{jk_n} (1 - \sigma_{jk_{n+1}}) \cdots (1 - \sigma_{jk_B}) \quad (51) \]

We can calculate the mean value of the number of collisions:

\[ \langle N_c(b) \rangle = \sum_{j=1}^{A} \langle Y_j \rangle = \sum_{j=1}^{A} \langle \langle Y_j \rangle \rangle_A \quad (52) \]

For the given configuration \( \{a_j\} \) and \( \{b_k\} \) we have:

\[ \sum_{n=0}^{B} n P(Y_j = n) \quad (53) \]
Figure 9: The variance of the number of NN-collisions in PbPb interactions at SPS energies as a function of the impact parameter $b$; * - "optical" approximation, calculated using formulae (57) and (60); + - the Poisson variance: $V[N_c(b)] = \langle N_c(b) \rangle$. The notations are the same as in Fig.1.

and

$$\langle Y_j \rangle_B = \sum_{n=0}^{B} n \langle P(Y_j = n) \rangle_B = \sum_{n=0}^{B} n \left\langle \sum_{\{k_1, ..., k_n\}} \sigma_{j1} ... \sigma_{jn} (1 - \sigma_{jn+1}) ... (1 - \sigma_{jB}) \right\rangle_B =$$

$$= \sum_{n=0}^{B} n \int \prod_{k=1}^{B} \hat{db}_k \sum_{\{k_1, ..., k_n\}} \sigma_{j1} ... \sigma_{jn} (1 - \sigma_{jn+1}) ... (1 - \sigma_{jB}) =$$

$$= \sum_{n=0}^{B} n C_B^n \sigma_j^n (1 - \sigma_j)^{B-n} = B \sigma_j .$$

We use the same notations (see (31)) as in appendix A

$$\sigma_j \equiv \int \hat{db}_1 \sigma_{j1} = \int db_1 T_B(b_1) \sigma(a_j - b_1 + b) \approx \sigma T_B(a_j + b) .$$

Finally we find:

$$\langle N_c(b) \rangle = \sum_{j=1}^{A} \langle Y_j \rangle = \sum_{j=1}^{A} \langle Y_j \rangle_B = \sum_{j=1}^{A} \langle B \sigma_j \rangle_A = B \sum_{j=1}^{A} \langle \sigma_j \rangle_A = B \sum_{j=1}^{A} \int \hat{da}_i \sigma_{j1} =$$

$$= B \sum_{j=1}^{A} \int \hat{da}_j \sigma_j = AB \int \hat{da}_1 \sigma_1 \equiv AB \chi(b) ,$$

where

$$\chi(b) \equiv \int \hat{da}_1 \sigma_1 = \int \hat{da}_1 \hat{db}_1 \sigma_{11} = \int da_1 db_1 T_A(a_1) T_B(b_1) \sigma(a_1 - b_1 + b)$$
Figure 10: The same as in Fig.9 but for the normalized variance, $V[N_c(b)]/\langle N_c(b) \rangle$, of the number of NN-collisions.

and at $r_N \ll R_A, R_B$

$$\chi(b) \approx \sigma \int da_1 T_A(a_1) T_B(a_1 + b) ,$$

which coincides with the optical approximation result.

Really, assuming the binomial distribution for $N_c(b)$ with the averaged probability $\chi(b)$ of NN-interaction:

$$P_{opt}(N_c) = C_{AB}^{N_c} \chi(b)^{N_c} [1 - \chi(b)]^{AB-N_c} ,$$

we have

$$\langle N_c(b) \rangle_{opt} = AB\chi(b)$$

and

$$V[N_c(b)]_{opt} = AB\chi(b)[1 - \chi(b)] = \langle N_c(b) \rangle[1 - \chi(b)] .$$

Note that for heavy nuclei $\chi(b)$ is small even for central collisions ($\chi(b) \sim r_N^2/R_A^2 \ll 1$), and the distribution (58) and the variance (60) practically coincide with the Poisson ones: $V[N_c(b)]_{opt} \approx \langle N_c(b) \rangle$.

Comparing (55) and (59) we see that the optical approximation gives true answer for the mean value $\langle N_c(b) \rangle$. Although we could not find the closed formula for the variance $V[N_c(b)]$, which would be similar to formulae (2), (11) and (12) for the variance of the number of participants $V[N^A_w(b)]$ and $V[N^A_w(b) + N^B_w(b)]$, our calculations show that the expression for the variance of $N_c(b)$ will again differ from the optical approximation one (60). The results of our separate Monte-Carlo simulations confirm these conclusions.

In Figs.8,9 and 10 we present the results of our calculations of the mean value $\langle N_c(b) \rangle$ and the variance $V[N_c(b)]$ in PbPb collisions at SPS energies at different values of the impact parameter $b$. 
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