Stability properties and asymptotics for $N$ non-minimally coupled scalar fields cosmology

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Abstract

We consider here the dynamics of some homogeneous and isotropic cosmological models with $N$ interacting classical scalar fields non-minimally coupled to the spacetime curvature, as an attempt to generalize some recent results obtained for one and two scalar fields. We show that a Lyapunov function can be constructed under certain conditions for a large class of models, suggesting that chaotic behavior is ruled out for them. Typical solutions tend generically to the empty de Sitter (or Minkowski) fixed points, and the previous asymptotic results obtained for the one field model remain valid. In particular, we confirm that, for large times and a vanishing cosmological constant, even in the presence of the extra scalar fields, the universe tends to an infinite diluted matter dominated era.

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I. INTRODUCTION

We have considered recently the homogeneous and isotropic solutions of the cosmological model described by the action [1]:

\[ S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ - (1 - \frac{\xi}{\kappa} \psi^2) \frac{R}{\kappa} + g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - 2V(\psi) \right\}, \]  

where \( \psi \) is the non-minimally coupled scalar field, \( \kappa = 8\pi G \), \( G \) is the Newtonian constant, \( R \) the scalar curvature of spacetime, and the self-interaction potential \( V(\psi) \) has the form:

\[ V(\psi) = \frac{3\alpha}{\kappa} \psi^2 - \frac{\Omega}{4} \psi^4 - \frac{9\omega}{\kappa^2}, \]  

with \( \alpha = \frac{\kappa m^2}{6} \), \( m \) the mass of the scalar field, \( \Omega \) is an arbitrary constant and \( -\frac{9\omega}{\kappa^2} \) is the usual cosmological constant \( \Lambda \). The nonminimal coupling term in (1) is, of course, \( \xi R \psi^2 \) where \( \xi \) is an arbitrary constant. The results previously obtained (see [1] for the motivations and references) were worked out for the case \( \xi = 1/6 \), the so called conformally coupled case, and they point toward some novel and very interesting dynamical behavior: superinflation regimes, a possible avoidance of big-bang and big-crunch singularities through classical birth of the universe from empty Minkowski space, spontaneous entry into and exit from inflation, and a cosmological history suitable for describing quintessence in principle. Through exhaustive numerical simulations and with some semi-analytical tools, the 3-dimensional phase space of the model has been constructed. The existence of a Lyapunov function for the fixed points was crucial for the study of the asymptotic behavior [2] and for precluding the appearance of any chaotic regime, confirming and shedding some light on some other previous results in this line [3].

The robustness of its predictions must be an essential feature of any realistic cosmological model. The study of some generalizations of the model (1) is, therefore, mandatory. In [4], we consider the implications of the inclusion of a second interacting massless scalar field in the dynamics of homogeneous and isotropic solutions. The corresponding phase space becomes now 5-dimensional, and richer structures could appear. In spite of this, we show

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that for a class of physically reasonable quartic interaction potentials, the neighborhoods of
the relevant fixed points are unaltered, and, in particular, the asymptotic regimes obtained
for the 1-field case [2] are preserved in the presence of an extra massless field. The relaxing
of isotropy was the next performed robustness test of the model. In fact, it appeared as
a corollary of a much more general result. In [5], we studied the singularities in the time-
evolution of homogeneous and anisotropic solutions of cosmological models described by the
action:

\[ S = \int d^4x \sqrt{-g} \left\{ F(\psi) R - \partial_a \psi \partial^a \psi - 2V(\psi) \right\}, \quad (3) \]

with general \( F(\psi) \) and \( V(\psi) \). Such a model has been recently considered also in [6]. We
showed that anisotropic solutions generically evolve toward a spacetime singularity that
corresponds to the hypersurface \( F(\psi) = 0 \) in the phase space. Such singularity is harm-
less for isotropic solutions. A second, and different, type of singularity corresponds to the
hypersurface \( F_1(\psi) = 0 \), where

\[ F_1(\psi) = F(\psi) + \frac{3}{2} (F'(\psi))^2, \quad (4) \]

and no solution can avoid it. Starobinsky [7] was the first to identify the singularity cor-
responding to the hypersurfaces \( F(\psi) = 0 \), for the case of conformally coupled anisotropic
solutions. Futamase and co-workers [8] identified both singularities in the context of chaotic
inflation in \( F(\psi) = 1 - \xi \psi^2 \) theories (See also [9]). For this type of coupling, the first singu-
larity is always present for \( \xi > 0 \) and the second one for \( 0 < \xi < 1/6 \). The results of [5] are,
however, more general since the case of general \( F(\psi) \) is treated and all conclusions are based
on the analysis of true geometrical invariants. The phase space for the model (3) is also
5-dimensional, and many new structures appear. The appearance of the singularity of the
first type implies the instability of the anisotropic solutions even for the conformally coupled
case, for instance. The presence of any amount of anisotropy, no matter how small, makes
the model unstable, challenging its validity as a realistic cosmological model. The asymp-
totic behavior in the neighborhood of the fixed points far from the hypersurface \( F(\psi) = 0 \)
is, however, preserved.
We present here a new robustness test for the homogeneous and isotropic solutions for models of the type (1). We consider the case of several interacting scalar fields conformally coupled to spacetime curvature. Exact solutions for this case are much more difficult to find, in particular we could not identify the heteroclinics and homoclinics as it was done in [1]. We could, however, get a strong result about the stability of fixed points suggesting that chaotic regimes are ruled out. This is proved for a class of interaction potentials, through the construction of an explicit Lyapunov function. In spite of the unstable character of anisotropies for models of the type (1), the results reported here are expected to be valid in the neighborhood of the Minkowski fixed point, even for the anisotropic case.

II. THE LYAPUNOV FUNCTION

Let us consider the action for $N$ classical interacting scalar fields $\psi_1, \ldots, \psi_N$ conformally coupled to the spacetime curvature:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{-[1 - \frac{\kappa}{6}(\psi_1^2 + \ldots + \psi_N^2)] \frac{R}{\kappa} + \sum_{i=1}^{N} g^{\mu\nu} \partial_\mu \psi_i \partial_\nu \psi_i - 2V(\psi_1, \ldots, \psi_N)\right\} \quad (5)$$

where the potential $V$ is a polynomial in the $N$ fields variables $\psi_i$ up to fourth degree. We consider homogeneous and isotropic solutions corresponding to Robertson-Walker metrics with flat spatial section

$$ds^2 = d\tau^2 - a^2(\tau)(dx^2 + dy^2 + dz^2). \quad (6)$$

Variations of the action (1) with respect to the scalar fields $\psi_i$ yield $N$ coupled Klein-Gordon equations

$$\ddot{\psi}_i + 3H \dot{\psi}_i - \frac{1}{6}R \psi_i + \frac{\partial V}{\partial \psi_i} = 0, \quad (7)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble function and the dot denotes time derivative. The variation of $S$ with respect to the metric leads to the Einstein equations which can be cast, in an analogous way to the one field case, in the form
\[ \frac{\kappa}{2} \sum_{i=1}^{N} \dot{\psi}_i^2 + \kappa V - 3H^2 + \frac{\kappa}{2} H^2 \sum_{i=1}^{N} \dot{\psi}_i^2 + H \sum_{i=1}^{N} \psi_i \dot{\psi}_i = 0, \]  

(8)

i.e. the energy constraint, and

\[ R = -6(\dot{H} + 2H^2) = \kappa \left( -4V + \sum_{i=1}^{N} \psi_i \frac{\partial V}{\partial \psi_i} \right), \]

(9)

the trace equation. The system of ordinary differential equations (7) and (9) is defined in a \((2N + 1)\)-dimensional phase space. The solutions are, however, confined on the \(2N\)-dimensional zero energy hypersurface (8).

For physical interpretation reasons (the same ones of [1]), we assume that the potential \(V(\psi_1, \ldots, \psi_N)\) contains a mass contribution in the form of a quadratic term for each field \(\psi_i\), and a homogeneous quartic function \(f_4(\psi_1, \ldots, \psi_N)\) describing the interactions of the \(N\) scalar fields (including possible self-interactions), and a cosmological constant

\[ V(\psi_1, \ldots, \psi_N) = \frac{3}{\kappa} \sum_{i=1}^{N} \alpha_i \psi_i^2 + f_4(\psi_1, \ldots, \psi_N) - \frac{9\omega}{\kappa^2} \]  

(10)

where \(\alpha_i = \frac{2}{\kappa} m_i^2\), \(m_i\) being the mass of field \(\psi_i\).

We now proceed to show the existence of a Lyapunov function for the dynamical system (7)-(9) under some well defined conditions on the potential (10). We focus on the sub-system of Klein-Gordon equations (7). By multiplying each equation by \(\psi_i\) and summing on the \(N\) fields we get

\[ \frac{d}{d\tau} \left[ \sum_{i=1}^{N} \dot{\psi}_i^2 + V \right] - \frac{1}{6} R \sum_{i=1}^{N} \psi_i \dot{\psi}_i = -3H \sum_{i=1}^{N} \dot{\psi}_i^2 \]  

(11)

Assuming that \(-\frac{1}{6} R \psi_i\) derives from a potential

\[ -\frac{1}{6} R \psi_i = \frac{\partial U}{\partial \psi_i}, \]

(12)

we would obtain:

\[ \frac{d}{d\tau} \left[ \sum_{i=1}^{N} \dot{\psi}_i^2 + V + U \right] = -3H \sum_{i=1}^{N} \dot{\psi}_i^2. \]

(13)

If the function between square brackets has a minimum at a given fixed point, it will be a candidate for a Lyapunov function for \(H > 0\). Equations (7)-(9) have, obviously, many
fixed points, but we are concerned only with de Sitter ($\omega < 0$) or Minkowski ($\omega = 0$) fixed points, for which $\psi_i = \dot{\psi} = 0$ and $H = \pm \sqrt{-3\omega/\kappa}$. We are restricted, therefore, to $\omega \leq 0$ (non-negative cosmological constants, as in [1,2]). A closer analysis of the energy constraints (8) reveals that the system can cross the $2N$-hypersurface $H = 0$ only on a $2N-1$ submanifold of the $2N+1$ original phase space. In a neighborhood of the fixed point $(H = \sqrt{-3\omega/\kappa}, \psi_i = 0)$ where $V$ is non-negative, the system can reach $H = 0$ only at the point where $V$ vanishes, otherwise the system cannot reach $H = 0$. Hence, we will have for $H > 0$

$$\frac{d}{d\tau} \left[ \frac{\dot{\psi}_i^2}{2} + V + U \right] \leq 0 \quad (14)$$

in such a neighborhood and, provided that $V + U$ has a minimum at the origin,

$$L = \sum_{i=1}^{N} \frac{\dot{\psi}_i^2}{2} + V + U \quad (15)$$

is a Lyapunov function, ensuring, thereby, the stability of the fixed point $(H = \sqrt{-3\omega/\kappa}, \psi_i = 0)$.

The relevant question here is the validity of the hypothesis (12). It is valid, for instance, if all the fields have the same mass, $\alpha_1 = \alpha_2 = \ldots = \alpha_N$. This is a strong constraint that, as we will see below, can be somehow relaxed. From equation (9), with the potential given by (10), we have

$$-\frac{1}{6} R \psi_i = \psi_i \sum_{j=1}^{N} \alpha_j \psi_j^2 - \frac{6\omega}{\kappa} \psi_i \quad (16)$$

Condition (12) requires

$$\frac{\partial (R \psi_i)}{\partial \psi_k} = \frac{\partial (R \psi_k)}{\partial \psi_i}, \quad (17)$$

implying, from Eq. (9), that

$$\alpha_k \psi_i \psi_k = \alpha_i \psi_i \psi_k \quad (18)$$

for all couples $(i, k)$ with $i \neq k$. The equal masses case is, obviously, a particular solution of (18). For this case, the potential $U$ is readily shown to be
\[
U(\psi_1, \psi_2, \ldots, \psi_N) = U_0 + \frac{\alpha}{4}|\psi|^4 - \frac{3\omega}{\kappa}|\psi|^2
\]  
(19)

where \( U_0 \) is an arbitrary constant and \(| \cdot |\) stands to the usual Euclidean norm

\[
|\psi|^2 = \psi_1^2 + \ldots + \psi_N^2.
\]  
(20)

In this case, the Lyapunov function is given explicitly by

\[
L = \frac{1}{2}|\dot{\psi}|^2 + \frac{3}{\kappa}(\alpha - \omega)|\psi|^2 + f_4(\psi_1, \ldots, \psi_N) - \frac{9\omega}{\kappa^2} + U_0 + \frac{\alpha|\psi|^4}{4}
\]  
(21)

for arbitrary homogeneous functions of degree four \( f_4 \). In summary, the function \( L \) has a minimum on the fixed point and there is a neighborhood (the attraction basin) with \( H \geq 0 \) where \( L \) has non-positive time derivative. As the system can reach the hypersurface \( H = 0 \) only in the fixed point, this provides a stability proof for the fixed point \( \dot{\psi}_i = \psi_i = 0 \), \( H = +\sqrt{3|\omega|/\kappa} \). An analysis of the time-reversed system reveals that the symmetric fixed point (with \( H = -\sqrt{3|\omega|/\kappa} \)) must be repulsive. The trajectories starting in the vicinity of the last point will leave this region and some of them will cross the hypersurface \( H = 0 \). Some of the crossing trajectories can be eventually trapped in the half-space \( H > 0 \) and will tend asymptotically toward the stable fixed point. This phase portrait suggest a regular behavior, and it is very reminiscent of the behavior exhibited by the solutions of the one-field model, where chaotic regimes were ruled out.

As it was already noted, this reasoning breaks down for \( N \) scalar fields with distinct masses. Nevertheless, we now prove that, under rather weak conditions relating the masses and the parameters for some potentials of the form (10), another Lyapunov function exists and ensures the stability of the fixed point. We will consider here the special case where the homogeneous function \( f_4 \) has the form (10)

\[
f_4(\psi_1, \ldots, \psi_N) = -\sum_{l=1}^{N} \frac{\Omega_{l}}{4} \psi_l^4 + \sum_{k,l=1(k\neq l)}^{N} \left( \alpha_{lk} \psi_l \psi_k^3 + \frac{\beta_{lk}}{4} \psi_l^2 \psi_k^2 \right)
\]  
(22)

with \( \beta_{lk} = \beta_{kl} \). The Klein-Gordon equations (7) become

\[
\ddot{\psi}_i + 3H \dot{\psi}_i + \frac{6}{\kappa}(\alpha_i - \omega)\psi_i + (\alpha_i - \Omega_i)\psi_i^3 + F_i = 0
\]  
(23)
where
\[ F_i = \psi_i \sum_{j=1(j \neq i)}^{N} (\alpha_j + \beta_{ij}) \psi_j^2 + \sum_{j=1(j \neq i)}^{N} \alpha_{ij} \psi_j^3 + 3 \psi_i^2 \sum_{j=1(j \neq i)}^{N} \alpha_{ij} \psi_j \] (24)

Let us now rescale the field variables by a constant factor \( \psi_i = \gamma_i \tilde{\psi}_i \). Klein-Gordon equations are now
\[ \ddot{\tilde{\psi}}_i + 3H \dot{\tilde{\psi}}_i + \frac{6}{\kappa} (\alpha_i - \omega) \tilde{\psi}_i + \gamma_i^2 (\alpha_i - \Omega_i) \tilde{\psi}_i^3 + \tilde{F}_i = 0 \] (25)
with
\[ \tilde{F}_i = \tilde{\psi}_i \sum_{j=1}^{N} \gamma_j^2 (\alpha_j + \beta_{ij}) \tilde{\psi}_j^2 + \frac{1}{\gamma_i} \sum_{j=1}^{N} \gamma_j \tilde{\psi}_j^3 + 3 \gamma_i \tilde{\psi}_i^2 \sum_{j=1}^{N} \gamma_j \alpha_{ij} \tilde{\psi}_j \] (26)

We again multiply both sides of (25) by \( \tilde{\psi}_i \) and sum over \( i \) leading to
\[ \frac{d}{d\tau} \left[ \sum_{i=1}^{N} \frac{\dot{\tilde{\psi}}_i^2}{2} + V_1 \right] + \sum_{i=1}^{N} \tilde{F}_i \dot{\tilde{\psi}}_i = -3H \sum_{i=1}^{N} \tilde{\psi}_i^2 \] (27)
where
\[ V_1 = \sum_{i=1}^{N} \left[ \frac{3}{\kappa} (\alpha_i - \omega) \tilde{\psi}_i^2 + \frac{\gamma_i^2}{4} (\alpha_i - \Omega_i) \tilde{\psi}_i^4 \right] \] (28)

Let us determine the conditions under which the term \( \sum_{i=1}^{N} \tilde{F}_i \dot{\tilde{\psi}}_i \) is a total derivative \( \frac{dV_2}{d\tau} \). This requires that \( \tilde{F}_i \) derives from a potential \( V_2 \), that is
\[ \frac{\partial \tilde{F}_i}{\partial \tilde{\psi}_k} = \frac{\partial \tilde{F}_k}{\partial \tilde{\psi}_i} \] (29)

Conditions (29) lead to the following algebraic relations between the masses, the coefficients of the potential \( V \) ad the scaling factor \( \gamma_i \),
\[ \gamma_i^2 (\alpha_i + \beta_{ik}) = \gamma_k^2 (\alpha_k + \beta_{ik}), \] (30)
and \( \alpha_{ik} = 0 \). The conditions (30) provides sign conditions
\[ \frac{\alpha_i + \beta_{ik}}{\alpha_k + \beta_{ik}} \geq 0 \] (31)
and homogeneous algebraic linear equations for the $\gamma_i^2$ whose compatibility conditions relate directly the masses $\alpha_i$ with the interaction coupling constant $\beta_{ik}$. For $N = 2$ (two scalar fields) the condition (30) are rather weak

\[
\frac{\alpha_1 + \beta_{12}}{\alpha_2 + \beta_{12}} \geq 0.
\]  

(32)

For $N = 3$, these conditions provide the inequalities (31) and one strict equality :

\[
\frac{(\alpha_1 + \beta_{12})(\alpha_2 + \beta_{23})(\alpha_3 + \beta_{31})}{(\alpha_1 + \beta_{13})(\alpha_2 + \beta_{21})(\alpha_3 + \beta_{32})} = 1.
\]  

(33)

It is important to remind that $\beta_{ij} = \beta_{ji}$. For $N = 4$, there are four independent conditions

\[
\frac{(\alpha_1 + \beta_{12})(\alpha_2 + \beta_{23})(\alpha_3 + \beta_{31})}{(\alpha_1 + \beta_{13})(\alpha_2 + \beta_{21})(\alpha_3 + \beta_{32})} = 1
\]

\[
\frac{(\alpha_2 + \beta_{23})(\alpha_3 + \beta_{34})(\alpha_4 + \beta_{42})}{(\alpha_2 + \beta_{24})(\alpha_3 + \beta_{32})(\alpha_4 + \beta_{43})} = 1
\]

\[
\frac{(\alpha_1 + \beta_{12})(\alpha_2 + \beta_{24})(\alpha_3 + \beta_{31})(\alpha_4 + \beta_{43})}{(\alpha_1 + \beta_{13})(\alpha_2 + \beta_{21})(\alpha_3 + \beta_{34})} = 1
\]

\[
\frac{(\alpha_1 + \beta_{14})(\alpha_2 + \beta_{12})(\alpha_3 + \beta_{34})(\alpha_4 + \beta_{43})}{(\alpha_1 + \beta_{13})(\alpha_2 + \beta_{24})(\alpha_3 + \beta_{32})} = 1
\]

(34)

There are four conditions and eight parameters $\alpha_i, \beta_{ij}$. More generally, the total number of parameters $\alpha_i, \beta_{ij}$ is $\frac{N^2}{2}$ and the total number of conditions (strict equalities) is $\frac{N(N-1)(N-2)}{6}$.

We, thus, can determine the maximum number of interacting scalar fields above which the above conditions are overdetemined

\[
\frac{N^2}{2} < \frac{N(N-1)(N-2)}{6}
\]  

(35)

which yields the solution $N > 5$. Finally, the explicit expression of the Lyapunov function in the original variables $\psi_i$ is

\[
L = \frac{1}{2} \sum_{i=1}^{N} \frac{1}{\gamma_i^2} \psi_i^2 + \frac{3}{\kappa} \sum_{i=1}^{N} \left( \frac{\alpha_i - \omega}{\gamma_i^2} \right) \psi_i^2 + \frac{1}{4} \sum_{i=1}^{N} \left( \frac{\alpha_i - \omega_i}{\gamma_i^2} \right) \psi_i^4
\]

\[
+ \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1(i\neq j)}^{N} N \left( \frac{\alpha_i - \beta_{ij}}{\gamma_j^2} \right) \psi_i^2 \psi_j^2
\]

(36)
with the conditions (30) over the parameters $\alpha_i, \beta_{ij},$ and $\gamma_i^2$.

Again, this function vanishes at the fixed points and on the $H$-axis. It is non-negative definite in a finite domain of the phase space around that axis and its time derivative is given by

$$\frac{dL}{d\tau} = -3H \sum_{i=1}^{N} \frac{1}{\gamma_i^2} \dot{\psi}_i^2$$

which is non-positive in the half-space $H \geq 0$. This proves the stability of the fixed point $\dot{\psi}_i = \psi_i = 0, H = \sqrt{\frac{3|\omega|}{\kappa}}$. All the solutions tend to this point or go to infinity. Furthermore, both Lyapunov functions may be global, i.e.

$$\lim_{|\psi| \to \infty} L = +\infty$$

when the parameters satisfy some inequalities. In that case, the Lyapunov stability theory shows that solutions are bounded and accumulate on one of the fixed points, depending on the initial conditions.

**III. CONCLUSION**

We have shown under which conditions some $N$ non-minimally scalar fields homogeneous and isotropic cosmological models admit a Lyapunov function for their de Sitter (or Minkowski) fixed point. The physical interpretation of such conditions is still unclear.

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