Abstract

We discuss power corrections to infrared safe cross sections and event shapes, and identify a nonperturbative function that governs $1/Q$ corrections to these quantities.

1 Introduction

Nonperturbative corrections to infrared safe jet cross sections and event shapes are an important issue in the study of QCD. They are the natural starting point for a unified perturbative-nonperturbative treatment of high energy cross sections, because they enter at the level of nonleading powers of the hard momentum scale, $Q$.

For example, a classic analysis\footnote{Presented at the Fifth International Workshop on Deep Inelastic Scattering and QCD, Chicago, IL, April 14-18, 1997.} of the total cross section for $e^+e^-$ annihilation to hadrons identifies terms $\alpha_s^n(Q^2)b_2^n n!$ at $n$th order, which may be attributed to the infrared (IR) behavior of the QCD running coupling, $\alpha_s(k^2) = 4\pi/[b_2 \ln(k^2/\Lambda^2)]$. Borel analysis shows that this nonconvergence is due to an ambiguity in the cross section at $Q^{-4}$ relative to leading behavior. This ambiguity is in direct correspondence with the contribution of the gluon condensate to the operator product expansion (OPE). The OPE, however, is available in only a few cases. Nevertheless, we would like to abstract from these considerations a method of “substitution”. That is, we shall assume that the contribution of any region of momentum space where the contour integrals of perturbation theory (PT) are trapped by mass-shell and IR singularities...
is a source of nonperturbative corrections, whose power suppression may be estimated from PT itself. We outline a general procedure, which we shall briefly illustrate below.

Let $\sigma$ be an IR safe cross section at large scale $Q$: (i) Identify regions $R$ in momentum space where lines are pinched on-shell in $\sigma$, by use of Landau equations or an equivalent method. (ii) Organize logarithms of momenta $k$ that occur in $R$ into: (a) $\alpha_s(f(k))$, with $f(k)$ a characteristic momentum scale, and/or (b) explicit kinematic integrals. (iii) Introduce a cutoff $\kappa$ on (some) components: $k^\nu < \kappa$, to define the contribution $\sigma_R$ from region $R$, such that $\alpha_s(f(k)) > \alpha_0$, $\alpha_0$ fixed. (iv) With the coupling fixed, evaluate the power behavior $\sigma_R \sim Q^{-2-m}$. (v) Find a “universal” matrix element $\langle O \rangle$, of dimension $m$, whose perturbative expansion is identical to that for $R$. (vi) Remove $\sigma_R$ from $\sigma$, replacing it with $\langle O \rangle$, $\sigma = \sigma^{(\text{reg})}(\kappa) + \langle O \rangle(\kappa)/Q^{-2-m}$. Of these steps, items (ii) and (v) require special treatment on a case-by-case basis. Nevertheless, this approach includes the analysis of infrared renormalons [2, 3], and represents, we believe, a somewhat more general viewpoint.

2 Power Corrections in Event Shapes

As an application, we consider the perturbative expansion for infrared safe event shapes, such as the thrust $T$, close to $T = 1$, the limit of two perfectly narrow jets. The leading behavior for a large class of such event shapes $w$ is $1/w$ (times logs of $w$) in this limit. Keeping only the $1/w$ terms, the program outlined above may be carried out explicitly. To be specific, we shall assume that to the power $1/w$, the weight factorizes into contributions from individual particles,

$$ w(k) = \sum_{\text{particles } i} w(k_i/Q) = \sum_{\text{particles } i} \frac{k_i^0}{Q} f_w(\cos \theta_i), $$

(1)

for some function $f_w$ of the cosine of the angle of particle $i$ to the two-jet axis. In the case of thrust, $f_{1-T} = (1 - |\cos \theta_i|)$.

In the two-jet limit, the differential cross section $d\sigma/dw$ factorizes into functions describing the internal evolutions of the jets convoluted in $w$ with a “two-jet” soft-gluon function $\sigma_2(Q, w)$. $\sigma_2$ is the eikonal approximation to the cross section at fixed weight $w$, in which the jets are replaced by oppositely-moving, lightlike Wilson lines [3]. For the $1/w$ contributions, the directions of the Wilson lines may be considered as fixed. We now apply our reasoning to $\sigma_2$. Suitably defined, the jet functions give nonleading power corrections.

Our first observation is that $\sigma_2(Q, w)$ is a convolution,

$$ \sigma_2(Q, w) = \sum_{n=0}^{\infty} \frac{1}{n!} \int dw_1 \ldots dw_n \delta \left( w - \sum_{i=1}^{n} w_i \right) \prod_{i=1}^{n} S(Q, w_i), $$

(2)

in terms of a kernel, $S(Q, w)$. The convolution fixes the weights of final states that contribute to $\sigma_2(Q, w)$. Then, if the weight function satisfies Eq. (1), the Laplace transform of $\sigma_2(Q, w)$ exponentiates (up to small corrections, which we suppress),

$$ \tilde{\sigma}_2(Q, N) = \int_0^{w_{\text{max}}} dw \ e^{-Nw} \sigma_2(Q, w) = \exp[\tilde{S}(Q, N)]. $$

(3)
By construction of \( w \), this transform is infrared finite. The perturbative expansion of
the kernel \( S \) in Eq. (2) is identical to a sum of two-particle irreducible diagrams called
“webs” long ago [4]. Webs have a number of important properties. First, they require
only a single, additive UV renormalization, corresponding to multiplicative renormal-
ization for \( \sigma_2 \). Second, they give rise to only a single overall collinear and infrared
logarithm each, aside from logarithms which may be organized into the running of
the coupling. These conditions are summarized in the integral representation,

\[
\tilde{S}(Q, N) = \int_0^Q \frac{d^2 k}{k^2} \int_0^{Q^2-k^2} \frac{d^2 k_T}{k^2 + k_T^2} \times \int \frac{dk_0}{\sqrt{k^2+k_T^2}} \frac{\gamma_w}{\sqrt{k^2-k_0^2-k_T^2}} \gamma_w \left( \frac{k}{\mu}, Q, \alpha_s(\mu), N \right),
\]

where \( k \) represents \( k_0, k_T \) and \( \sqrt{k^2} \). The combination \( \gamma_w(k, N)/[k^2(k^2+k_T^2)] \) is an
integrable distribution for \( k^2, k_T^2 \rightarrow 0 \). The two overall logarithms of \( N \) are generated
from the \( k \) integrals. In Eq. (4), we have implemented items (i) and (ii) in the method
of substitution above. To identify the explicit forms of power corrections in \( Q \), we
expand \( \gamma_w \) in \( Q \) at fixed values of \( N \). The precise form of dependence on soft regions
in (4), and the corresponding substitutions (see above), depend on the weight, but
are simplified by the web structure, leading to clear sources of power corrections.

For \( w = 1 - T \), the expansion of \( \gamma_{1-T} \) gives

\[
\frac{\gamma_{1-T}^{(1)}}{k^2} = \frac{N}{Q} \delta(k^2) 2C_F \frac{\alpha_s(k_T)}{\pi} \left( k_0 - \sqrt{k_0^2 - k_T^2} \right) + \mathcal{O}(N^2/Q^2),
\]

where we have used the renormalization-group invariance of \( \gamma_{1-T} \) to set \( \mu = k_T \). After
the \( k_0 \) integral in (4), we find an exponentiating \( 1/Q \) correction, which multiplies the
(infrared regulated) perturbative result,

\[
\tilde{\sigma}_2^{(1-T,\text{corr})}(Q, N) \sim \exp \left[ \frac{N}{Q} \frac{2C_F}{\pi} \int_0^\kappa dk_T \alpha_s(k_T) \right],
\]

where we have introduced, as above, a new scale \( \kappa \) to isolate the infrared-sensitive
region around \( k_T = 0 \). This, of course, is a typical infrared renormalon, now in the
exponent [4].

We now ask if it is possible to give an operator interpretation to this result, and
thus to “substitute” a nonperturbative matrix element for the IR region of PT, leading
to a universal quantity that controls \( 1/Q \) behavior [4]. To construct this matrix
element, we define operators that measure the energy that arrives over time at a
sphere “at infinity”,

\[
\Theta(\hat{y}) = \lim_{|\hat{y}| \rightarrow \infty} \int_0^\infty \frac{dy_0}{(2\pi)^2} |\hat{y}|^2 \hat{y}_i \theta_{0i}(y^\mu),
\]

with \( \theta_{\mu\nu} \) the energy-momentum tensor, and \( \hat{y} \) a unit vector. The energy density that
flows in direction \( \hat{y} \) for \( \sigma_2(Q, w) \) is

\[
\mathcal{E}(\hat{y}) = \langle 0 | W_{v_1 v_2}^\dagger(0) \Theta(\hat{y}) W_{v_1 v_2}(0) | 0 \rangle,
\]
where $W_{v_1v_2}(0)$ is the product of outgoing Wilson lines in the $v_i$ directions, joined by a color singlet vertex at the origin. In these terms, the leading power correction, Eq. (6), is of the general form

$$\ln \tilde{\sigma}_2^{(w, \text{corr})}(Q, N) = \frac{N}{Q} \int \frac{d\Omega_y}{2\pi} f_w(\cos \theta) \mathcal{E}(\hat{y}) ,$$

with $f_w$ the function in Eq. (4). 1/Q corrections for a wide class of event shapes are thus generated from the nonperturbative function $\mathcal{E}(\hat{y})$, the matrix element of a nonlocal operator, which describes the nonperturbative component of energy flow, associated with two-jet color flow. Generalizations to multijet cross sections are possible. We anticipate that this approach will help to unify the treatments of power corrections for a variety of infrared-safe quantities.

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