New limits on $\Omega_A$ and $\Omega_M$ from old galaxies at high redshift

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ABSTRACT

The ages of two old galaxies (53W091, 53W069) at high redshifts are used to constrain the value of the cosmological constant in a flat universe (ΛCDM) and the density parameter Ω_M in Friedmann-Robertson-Walker (FRW) models with no Λ-term. In the case of ΛCDM models, the quoted galaxies yield two lower limits for the vacuum energy density parameter, Ω_Λ ≥ 0.42 and Ω_Λ ≥ 0.5, respectively. Although compatible with the limits from statistics of gravitational lensing (SGL) and cosmic microwave background (CMB), these lower bounds are more stringent than the ones recently determined using SNe Ia as standard candles. For matter dominated universes (Ω_Λ = 0), the existence of these galaxies imply that the universe is open with the matter density parameter constrained by Ω_M ≤ 0.45 and Ω_M ≤ 0.37, respectively. In particular, these results disagree completely with the analysis of field galaxies which gives a lower limit Ω_M ≥ 0.40.

Subject headings: cosmology: theory – dark matter – distance scale
In the last few years, the positive evidences against the standard FRW models are accumulating in a slow but increasing rate. Recent observations from a large sample of type Ia supernovae are in contradiction with a universe closed by ordinary matter ($\Omega_M \geq 1$), thereby appearing ruling out with great confidence closed and flat FRW matter dominated universes. Indeed, even open models, or more generally, any model with positive deacceleration parameter seems to be in disagreement with these data (Riess et al. 1998). In the near future, one may expect that continued observations based on type Ia supernovae as standard candles may determine the magnitude of the deceleration parameter with great accuracy, as well as the value of the vacuum energy density.

Another important piece of data is provided by the conflict between the expanding age of the universe, as inferred from measurements of the Hubble parameter, and the age of the oldest stars in globular clusters. Recent measurements of the Hubble parameter from a variety of techniques are now converging into the range (one standard deviation) $h = (H_o/100\text{kms}^{-1}\text{Mpc}^{-1}) = 0.7 \pm 0.1$ (Freedmann et al. 1997, Mould et al. 1997, Nevalainen and Roos 1997). In particular, the current weighted value from the HST Key Project is $H_o = 73 \pm 6$ (statistical)+8(systematic) $\text{kms}^{-1}\text{Mpc}^{-1}$ (Friedmann 1998). This means that the expansion age for a FRW flat matter dominated universe ($t_o = \frac{2}{3}H_o^{-1}$) falls within the interval $8.1\text{Gyr} \leq t_o \leq 10.8\text{Gyr}$, while the ages inferred from globular clusters ($t_{gc}$) lie typically in the range $t_{gc} \sim 13 - 15\text{Gyr}$, or even higher (Bolton and Hogan 1995; Pont et al. 1998). Since the age of the universe for closed models is even smaller than in the flat case, though some recent determinations of $t_{gc}$ (based on the Hipparcos distance scale) have decreased this value by approximately 2 Gyr (Chaboyer et al. 1998), the unique possible conclusion is that the “age crisis” continues for closed, and at least moderately, for flat FRW models. In this connection assuming a reasonable incubation time for globular clusters ($\sim 1\text{Gyr}$), Pont et al. (1998) concluded that the minimum age of the Universe is 14 Gyr, thereby ruling out a flat Universe, unless $h < 0.48$. All these results are also in
line with the latest constraints from SNe Ia, which provides \( H_0 = 65 \pm 7\text{km}s^{-1}\text{Mpc}^{-1} \) with the uncertainty dominated by the systematic errors commonly present in the calibration of the SN Ia absolute magnitude (Riess et al. 1998). Actually, from the original matter dominated FRW class with no cosmological constant, only extremely open universes may be old enough to solve (beyond doubt) the expanding age problem.

The already classical “age problem” becomes even more acute if we consider the measurements of the age of the universe at high redshifts. For instance, the 3.5Gyr radio galaxy (53W091) at \( z = 1.55 \) discovered by Dunlop et al. (1996) and confirmed by Spinrad et al. (1997) has been proved to be incompatible with the age estimate for a FRW flat universe with no cosmological constant unless the Hubble parameter is smaller than \( 45\text{km}s^{-1}\text{Mpc}^{-1} \) (Dunlop et al. 1996, Krauss 1997). Such a constraint is more restrictive than globular cluster age constraints. Therefore, with the exception of a very low Hubble constant variant, the conclusion that the standard FRW models seems to be inconsistent or only marginally compatible with the estimated age of the universe is inescapable.

On the other hand, a wide range of independent observations and theoretical arguments, closely related to inflationary scenarios, suggest that the more realistic model, which accommodates all the tests available at present, is a flat universe with cosmological constant (Krauss and Turner, 1995). A positive cosmological constant significant today helps to solve the “age problem” because it allows a period of cosmic acceleration at low redshifts, thereby leading to expanding ages greater than the ones computed with matter dominated universes. In particular, if \( \Omega_\Lambda = 0.8 \) and \( \Omega_M = 0.2 \), the range of the age for the above considered values of \( h \) is \( 13.2\text{Gyr} \leq t_o \leq 17.5\text{Gyr} \). However, although having several independent positive evidences, the possibility of a non-zero cosmological constant has not been proved beyond doubt and remains basically an open question. In such a state of affairs, it is interesting to obtain lower limits on the value of the vacuum energy density
using different methods. On the other hand, extremaly open models (OCDM) are at present the main competitor of the ΛCDM models (Krauss 1998). Since the age of the universe diminishes when Ω_M increases, the existence of old high redshift galaxies will provide an upper limit to the density parameter.

In the present work we discuss the constraints on Ω_Λ and Ω_M due to the existence of old high redshift galaxies (OHRG). Our study differs from Krauss’ work (1997) in two main aspects (see also Roos and Harun-or-Rashid 1998). First of all, we take into account a second OHRG recently reported (Dunlop et al. 1998). The second aspect is concerned with a more methodological reason. Instead of analysing the constraints imposed by the ages of these galaxies on the parameter space (h, Ω_M) as done by Krauss, we focus our analysis directly on the diagram t(z) as a function of Ω_Λ. Our goal is to show how the estimated lower bounds on the dimensionless “age parameter” of these two galaxies may be translated as lower limits on the cosmological constant itself. As argued above, this approach may also be applied to set upper limits for Ω_M in open universes with no cosmological constant.

The general age-redshift relation for FRW type universes with cosmological constant as a function of the observable parameters is

\[ t(z) = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\sqrt{1 - \Omega_M + \Omega_M x^{-1} + \Omega_\Lambda (x^2 - 1)}} \equiv H_0^{-1} f(\Omega_M, \Omega_\Lambda, z), \tag{1} \]

where Ω_M, Ω_Λ, are the present day matter and cosmological constant density parameters, respectively. If Ω_Λ = 0, the above expression reproduces the well known result for the standard model regardless of the value of Ω_M (Kolb and Turner 1990). For a flat universe, equation (1) reduces to

\[ t(z) = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\sqrt{(1 - \Omega_\Lambda) x^{-1} + \Omega_\Lambda x^2}}, \tag{2} \]

where the matter density parameter Ω_M has been replaced in terms of Ω_Λ using the flat condition constraint Ω_M + Ω_Λ = 1. The above equation can be readily integrated (Gradstein
and Ryzhik, 1980) yielding

\[ t(z) = \frac{2H_o^{-1}}{3\sqrt{\Omega_\Lambda}} \ln \left[ \sqrt{\frac{\Omega_\Lambda}{1 - \Omega_\Lambda}}(1 + z)^{-3} + \sqrt{\frac{\Omega_\Lambda}{1 - \Omega_\Lambda}}(1 + z)^{-3} + 1 \right], \tag{3} \]

which should be compared with equation (2) in the paper by Krauss (1997). As one may check, in the limit \( \Omega_\Lambda \to 0 \) the age-redshift relation of the standard flat universe is recovered, namely

\[ t(z) = \frac{2H_o^{-1}}{3(1 + z)^{\frac{3}{2}}} . \tag{4} \]

Before discussing the resulting diagrams is useful to understand the method employed here. First of all, we take for granted that the age of the universe in a given redshift is bigger than or at least equal to the age of their oldest objects. As one may conclude, in the case of models with cosmological constant, the comparison of these two quantities implies in a lower bound for \( \Omega_\Lambda \) since the age of the universe increases with the growth of the vacuum energy density. For standard matter dominated FRW models the same analysis points to the opposite direction. In fact, the age of the universe increases when \( \Omega_M \) decreases, thereby implying the existence of an upper bound. In order to quantify these qualitative arguments, it proves convenient to introduce the ratio

\[ \frac{t_z}{t_g} = \frac{f(\Omega_M, \Omega_\Lambda, z)}{H_o t_g} \geq 1 , \tag{5} \]

where \( t_g \) is the age of an arbitrary old object, say, a galaxy at the redshift \( z \), and \( f(\Omega_M, \Omega_\Lambda, z) \) is the dimensionless factor defined by (1).

Notice that for each galaxy, the denominator of the above equation defines a dimensionless age parameter \( T_G = H_o t_g \). For the galaxy discovered by Dunlop et al. (1996), the lower limit to the age of this galaxy yields \( T_G(1.55) = 3.5H_o \text{Gyr} \), which take values on the interval \( 0.21 \leq T_G \leq 0.28 \). The extreme values of \( T_G \) have been determined by the error bar of \( h \). It thus follows that \( T_G \geq 0.21 \), and from (4) we see that at this \( z \) the matter dominated flat FRW model furnishes an age parameter \( T_{FRW} \leq 0.16 \), which is far less than
the previous value of \( T_G \). Naturally, for a given value of \( h \), only models having an expanding age parameter bigger than the corresponding value of \( T_G \) at \( z = 1.55 \) will be compatible with the existence of this galaxy. In particular, the standard Einstein-de Sitter FRW model is ruled out by this test.

In principle, the confidence on the limits derived here is ensured because we always consider the under estimation age of the galaxies (Peacock et al. 1998) as well as the minimal value of “little” \( h \). We argue here that these two conditions for \( t_g \) and \( h \) give robust lower and upper limits for \( \Omega_\Lambda \) and \( \Omega_M \), respectively. The former condition is necessary because galaxies with small ages are more easily accomodated by the models, and the later because the age of the universe is inversely proportional to \( h \), thereby also favouring the model. Naturally, similar considerations may also be applied to the 4.0 Gyr old galaxy at \( z = 1.43 \). In this case we have \( T_G \geq 0.24 \) while the matter dominated flat FRW model yields \( T_{FRW} \leq 0.17 \).

In Fig. 1 we show the diagrams of the dimensionless age parameter \( T(z) = H_0 t(z) \) as a function of the redshift for several values of \( \Omega_\Lambda \). The forbidden regions in the graphs have been determined by taking the values of \( T_G \), for each galaxy, separately. We see that the minimum values of \( \Omega_\Lambda \) are, \( \Omega_\Lambda \geq 0.42 \) and \( \Omega_\Lambda \geq 0.5 \) providing a minimal total age of 12.9Gyr and 13.5Gyr, respectively. In Fig. 2, we present similar plots for matter dominated universes. As should be physically expected, instead of lower bounds as in the case of vacuum energy density, we now have upper limits to the value of \( \Omega_M \). The two galaxies lead to \( \Omega_M \leq 0.45 \) and \( \Omega_M \leq 0.37 \) and provide a minimal total age of 12.4Gyr and 12.8Gyr, respectively. As widely known, \( \Lambda \)CDM models are much more efficient in solving the “age problem”.

At this point, it is interesting to compare the results summarized in Table 1 with some determinations of \( \Omega_\Lambda \) and \( \Omega_M \) derived from independent methods. As widely known, the
current statistics of strong gravitational lenses provides powerful constraints on \( \Lambda \)CDM models. Recent studies predicted an upper limit \( \Omega_\Lambda \leq 0.66 \) at 95.4\% (2\( \sigma \)) confidence level (Kochanek 1996). This result, however, is not free of drawbacks, specially due to the presence of systematic errors, like extinction and the specific galaxy lens model. Recently, taking into account extinction, Falco, Kochanek and Muñoz (1998) estimated that \( \Omega_\Lambda \leq 0.7 \) (2\( \sigma \)). More recently, this limit has again been slightly softened to \( \Omega_\Lambda \leq 0.76 \), at the same confidence level (Waga and Miceli 1998). If extinction is absent, the corresponding limit is \( \Omega_\Lambda \leq 0.55 \) (2\( \sigma \) level), which is slightly more stringent than the earlier Kochanek’s bound. Therefore, by combining all these results with the OHRG lower bounds derived here, we find that the vacuum density parameter probably lies in a very short range, \( 0.42 \leq \Omega_\Lambda \leq 0.7 \). Assuming that the errors from extinction and galaxy lens model may be neglected or cancel out each other, the limit is more stringent, \( 0.42 \leq \Omega_\Lambda \leq 0.66 \). If the results of Waga and Miceli are considered, the corresponding ranges, with and with no extinction, are \( 0.42 \leq \Omega_\Lambda \leq 0.76 \) and \( 0.42 \leq \Omega_\Lambda \leq 0.55 \) (2\( \sigma \) level), respectively. Notice that our lower bound is less severe than \( \Omega_\Lambda \geq 0.60 \) obtained by Roos and Harun-or-Raschid (1998).

As stated before, a more satisfactory evidence for a positive \( \Omega_\Lambda \) as well as for an accelerated state of the present universe, has been obtained from distant SNe Ia (Riess et al. 1998). The combination of data from a large sample of 48 SNe Ia point (nearby and at high \( z \)) favors consistently an expanding universe dominated by a positive cosmological constant, e.g., a negative deceleration parameter (\( q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda < 0 \)). From this set of data the values of the density parameters are \( \Omega_\Lambda = 0.72^{+0.72}_{-0.48} \) and \( \Omega_M = 0.24^{+0.56}_{-0.24} \) (see Table 1). The main conclusion is that a non-negligible vacuum energy density is extremely probable. In a recent paper, Perlmutter et al. (1998) also obtained \( \Omega_\Lambda = 0.4 \pm 0.2 \), that is, \( \Omega_\Lambda \geq 0.2 \) (see Table 1). Note that both lower bounds, namely, \( \Omega_\Lambda \geq 0.24 \) and \( \Omega_\Lambda \geq 0.2 \) are much less stringent than \( \Omega_\Lambda \geq 0.42 \) derived here. There is, however, a possibility that these two lower bounds for \( \Omega_\Lambda \) (from OHRG and SNe Ia) might become compatible in the near future.
For example, suppose that due to some physical effect, which has been unnoticed so far or simply not taken into account in the estimates, the lower limits to the age of these objects are smaller than 3.0 Gyr. This means that the lower limit of $\Omega_\Lambda$ is proportionally smaller than the ones derived here. Let us now assume a more conservative lower bound than the ones provided by SNe Ia observations, say, $\Omega_\Lambda \geq 0.14$. In this case, using the analytical formula (3), one may check that the age of the galaxy 53W091 should be 2.6 Gyr. However, if the lower limit of $\Omega_\Lambda$ is 0.2 as claimed by Perlmutter et al. (1998) or 0.24 (Riess et al. 1998), the ages of this galaxy are 2.8 Gyr and 2.86 Gyr, respectively. Parenthetically, with a high level of confidence, the age of this object is bigger than 2.5 Gyr (Dunlop 1999, private communication). In the previous analysis, we have implicitly supposed that possible evolutionary effects of SNe Ia are completely under control, thereby giving rise to very robust constraints. Naturally, if the opposite viewpoint is assumed, the range allowed for $\Omega_\Lambda$ combining OHRG and SLG data is maintained, and some basic aspect of the standard SNe Ia approach deserves a closer scrutiny.

In the case of a matter dominated universe ($\Omega_\Lambda = 0$), some authors have recently argued that an undercritical density with $\Omega_M \sim 0.5$ should be adopted (Willick et al. 1997; Tammann 1998; da Costa et al. 1998). These claims are also in contradiction with the upper limits for $\Omega_M$ from old galaxies at high $z$ determined in the present paper (see Table 1). In this connection, as pointed out recently, the observational properties of 53W091 and the findings of galaxy candidates at $z \gg 5$ strongly suggest that the Universe contains more collapsed galaxies than the modified CDM model would predict (Kaslinsky and Jimenez 1998).

These results reinforce the interest in searching for old galaxies at high redshifts. Special attention should be given for those galaxies where the stellar population was born at once, that is, with no further starburst formation. By definition these are the best
clocks, and hopefully, this kind of object may be identified in the near future. The overall
tendency is that if more and more galaxies are discovered, the relevant statistical studies in
connection with the “age problem” may provide very restrictive constraints for any realistic
cosmological model. Conversely, given a population of galaxies at high $z$, the ages of which
are not well known, the method discussed here may readily be inverted yielding upper
limits to the age of the galaxies once a reasonable model of the universe is adopted.
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| Method      | Autor(s)                | $\Omega_M$  | $\Omega_\Lambda$(flat) |
|-------------|-------------------------|-------------|------------------------|
| SNe Ia      | Perlmutter et al. (1998)| 0.2 ± 0.4   | 0.4 ± 0.2              |
| SNe Ia      | Riess et al. (1998)     | 0.24$^{+0.56}_{-0.24}$ | 0.72$^{+0.72}_{-0.48}$ |
| FG          | Tammann (1998)          | $\sim$ 0.5  | -                      |
| SGL         | Kochanek (1996)         | > 0.15      | < 0.66                 |
| CMB         | Lineweaver (1998)       | 0.24 ± 0.1  | 0.62 ± 0.16            |
| OHRG ($z = 1.43$) | this paper          | $\leq 0.37$ | $\geq 0.5$            |
| OHRG ($z = 1.55$) | this paper          | $\leq 0.45$ | $\geq 0.42$            |

Table 1: Limits to $\Omega_M$ and $\Omega_\Lambda$
Fig. 1.— The dimensionless age parameter as a function of the redshift for some selected values of $\Omega_\Lambda$. All curves crossing the dark shading rectangle yield an age parameter less than the minimal value required by the galaxies discovered by Dunlop et al. (1996 and 1998). As expected, this class includes the FRW flat universe with $\Omega_\Lambda = 0$ (solid curve). The curve intersecting the corner of the rectangle corresponds to $\Omega_\Lambda = 0.42$ (53W091) and $\Omega_\Lambda = 0.5$ (53W069) and fix the minimal value of $\Omega_\Lambda$ allowed by these two galaxies.
Fig. 2.— The dimensionless age parameter as a function of the redshift for FRW universes with no cosmological constant. As before, all curves crossing the dark shading rectangle yield an age parameter less than the minimal value required by the galaxy discovered by Dunlop et al. (1996 and 1998). This subset includes the standard FRW flat universe (solid curve). Clearly, $\Omega_o = 0.45$ is the maximum value of $\Omega_o$ allowed by 53W091 and $\Omega_o = 0.37$ by 53W069.