Adaptive fast fixed-time three-dimensional guidance law with acceleration saturation constraints

XIAOJING LI\(^1\), JIANWEI MA\(^1\), and JIWEI GAO\(^1\)

\(^1\)School of Information Engineering, Henan University of Science and Technology, Luoyang 471000, China

Corresponding author: Jiwei Gao (email: jwgao2012@163.com).

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ABSTRACT This paper investigates continuous anti-saturation three-dimensional guidance law against maneuvering target at the desired line-of-sight (LOS) angles. Firstly, in the light of fixed-time stability, fast nonsingular terminal sliding mode is designed without the limitation of odd-ratio fractional order. Then, with the limits of lateral acceleration saturation, continuous guidance law is proposed to guarantee that desired LOS angles are reached and maintained after the specified time, while adaptive law is implemented to estimate the information on maneuver accelerations of the attacking missile. Finally, numerical simulations are presented to validate the effectiveness and rationality of the designed scheme.

INDEX TERMS Fixed-time convergence, fast terminal sliding mode, acceleration saturation, nonlinear continuous guidance.

I. INTRODUCTION

Owing to the need of modern war, various missiles have developed rapidly. In the meantime, the theory and technology, which are related to missiles or other aircrafts, have attracted the close attention of scientific researchers. Therefore, several results are also achieved about guidance laws for intercepting coming targets. However, with requirement improvements of missile performance, it is still challenging to design guidance laws.

Among all guidance laws, the proportional navigation (PN) guidance law and its variants [1–4] have been widely investigated and implemented, because of their generality and simplicity. However, the engagement systems of intercepting maneuvering target can be essentially classified as uncertain nonlinear systems, and therefore various PN guidance laws, which do not possess the ability of suppressing disturbances, can not effectively solve this problem. In recent years, with the development of nonlinear control theory, various guidance laws have been proposed to deal with this issue, such as adaptive control [6–7], sliding mode control (SMC) [8–10], \(H_\infty\) control [10], backstepping control [11]. Herein, SMC system, which has fast response and robustness against external noise and parameter perturbation, has been widely implemented and studied in the field of guidance laws.

Apart from robustness, finite-time convergence of LOS rates is important for terminal guidance of ballistic missile or air-to-air combat situation. Therefore, finite-time control schemes have been gradually proposed for guidance laws. Switching control was used to design robust guidance law for steering LOS angular rates to the neighborhood of the origin in the planar and three-dimensional (3D) environments [12], and then new finite-time guidance law [13] was developed to alleviate the cross coupling effect, which was ignored in [12]. Lateral acceleration was built to enforce nonsingular sliding mode on the switching hypersurface, and then the desired impact angle was achieved in finite time [14]. Similarly, SMC guidance law [15] was derived to guarantee that the flight-path angle can meet the desired impact angle with finite-time convergence. With regard to non-maneuvering target, continuous finite time control laws were developed to achieve high-accuracy guidance process, and furthermore one observer was united to detect target acceleration [17]. Based on the slightly complex and detailed three-dimensional engagement model, continuous finite-time guidance laws [16] were structured to intercept maneuvering target with the impact angles. These guidance laws could significantly reduce the miss distance and improve strike effect against maneuvering target.
However, the settling time of finite-time control system depends on the initial values of system state, and therefore fixed-time control algorithms, which remove this restriction, are subsequently investigated to design guidance laws. With impact angle constraints, robust guidance laws [18] were proposed to nullifying the LOS rates without decoupling cross couplings, and the convergent time can be set beforehand. Adaptive smooth fixed-time guidance law [19] was derived for intercepting maneuvering target based on fast stable system and nonsingular terminal SMC (NTSMC) in planar homing engagement geometry. Fixed-time 3D guidance law was designed to intercept maneuvering target in the light of fast NTSM, saturation function and adaptive law [20]. By two-order agreement and fixed-time SMC, distributed 3D cooperative guidance law [21] was presented to capture maneuvering target with desired impact angles. NTSMC was implemented to design fixed-time guidance law [22], while unknown target maneuver could be estimated by adaptive law. These results promote the study of guidance law to one new stage.

Saturation constraint is another important restrictive factor for practical second-order system. Until now, many researchers have studied this problem of the saturation constraint for missiles or other flight vehicles. Auxiliary system was constructed to tackle the effects of actuator magnitude constraints [23], while robust adaptive control scheme was proposed for air-breathing hypersonic vehicle by neural approximation and minimal-learning parameter technique. Integral sliding mode, adding power integrator and adaptive law are combined to solve spacecraft attitude tracking problem with actuator saturation, faults and misalignment [24]. In [25], approximate/adaptive dynamic programming was extended to broader nonlinear dynamic systems with asymmetry constraints. With actuator faults and input constraint, fixed/finite time guidance laws [26] were investigated to intercept maneuvering target, but unfortunately it was not deduced and proved in strict accordance with the concept of fixed-time stability. Backstepping and nonlinear disturbance observer are fully applied to guarantee robust tracking of altitude and velocity reference trajectories [27], and additional system was exploited to cope with actuator saturation.

Inspired by the above literatures, robust 3D fixed-time guidance law is designed to hit maneuvering target in this paper. The main contributions of this article are provided as follows: (1) fast fixed-time system is analyzed, and NTSM is established to solve control problem of nonlinear systems; (2) novel fixed-time guidance law is proposed against incoming maneuvering missile, where the upper bound is estimated for the convergence time of LOS rates by Lyapunov stability theory; (3) the constraints of accelerate saturation and desired impact are considered, while adaptive law and boundary layer are utilized to eliminate the requirement of target maneuver information and chattering phenomenon, respectively.

The rest of this paper is arranged as follows. Section 2 states problem formulations and preliminaries. In Section 3, fixed-time NTSM is constructed, and then continuous guidance law is designed based NTSM technique and adaptive law, while fixed-time stabilization is deduced and analyzed. To demonstrate the effectiveness of the proposed guidance law, numerical simulation is provided in Section 4. Conclusions are finally drawn in Section 5.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PROBLEM FORMULATION OF 3D GUIDANCE

For simplicity, these assumptions are made: (1) the pursuer and target are considered as point masses; (2) seeker dynamics and the autopilot of the pursuer are fast enough to be neglected; (3) the speeds of the missile and target are constant, and the angle-of-attack is small enough to be neglected. The 3D homing guidance geometry is shown in Fig. 1, where the relevant explanations are provided below. The interceptor is fixed at the origin of the reference frame. $OX, Y, Z$ denotes the inertial reference frame; $ Ox', Y', Z'$ is LOS frame; $ Ox', Y', Z'$ represents missile and target body frame, respectively. $ V_m, V_t$ denote velocity vectors of pursuer and target; $ \phi$ and $ \theta$ represent LOS angles; $ \phi_m, \theta_m$ are Euler angles from LOS frame to pursuer body frame; $ \phi_l, \theta_l$ denote Euler angles from LOS frame to target body frame.

According to Fig. 1, the kinematic engagement equations can be derived from classical principles of dynamics:

\[
\dot{r} = V_r \cos \theta \cos \phi - V_m \cos \theta_m \cos \phi_m \tag{1}
\]

\[
r \dot{\theta} = V_r \sin \theta \cos \theta_m - V_m \sin \theta_m \tag{2}
\]

\[
r \dot{\phi} = V_r \cos \phi - V_m \cos \phi_m \tag{3}
\]

\[
\dot{\theta}_m = \frac{\alpha_m}{V_m} - \dot{\theta}_l \sin \theta_m \sin \phi_m - \dot{\phi}_l \cos \phi_m \tag{4}
\]
\[
\dot{\phi}_m = \frac{a_{ym}}{V_m \cos \theta_m} + \phi_l \tan \theta_m \cos \phi_m \sin \theta_l
\]
\[
-\dot{\theta}_l \tan \theta_m \sin \phi_m - \phi_l \cos \theta_m
\]
\[
\dot{\theta}_i = \frac{a_{yi}}{V_i} - \phi_l \sin \theta_i \sin \phi_i - \dot{\theta}_l \cos \theta_i
\]
\[
\dot{\phi}_i = \frac{a_{yi}}{V_i} \tan \theta_i \cos \phi_i \sin \theta_i
\]
\[
-\dot{\theta}_i \tan \theta_i \sin \phi_i - \phi_i \cos \theta_i
\]

where \(a_{ym}\) and \(a_{ym}\) are the missile accelerations in the yaw and pitch directions; \(a_{yi}\) and \(a_{yi}\) are the target accelerations in the yaw and pitch directions. Note that, \(a_{ym}\) and \(a_{ym}\) are divided by \(\cos \theta_m\) and \(\cos \theta_i\), and therefore the LOS angles \(\theta_l\) and \(\phi_i\) should be driven under the assumptions \(|\theta_m| \neq \pi/2\) and \(|\theta| \neq \pi/2\).

The purpose of guidance law is to steer LOS angular rates \(\dot{\theta}_l\) and \(\dot{\phi}_l\) to the origin or the neighborhood of the origin, while LOS angles \(\theta_l\) and \(\phi_i\) can be driven to the desired values \(\theta_{lD}\) and \(\phi_{iD}\). Therefore, in order to design guidance law and explore other rules, the above differential equations (1)-(7) are transformed into two-order engagement dynamics (8)-(9) between the LOS angles and the angular rates.

\[
\ddot{\theta}_l = -\frac{2r \dot{\theta}_l}{\cos \theta_m} \sin \theta_l \cos \theta_l \cos \phi_m \sin \theta_l - \frac{\cos \theta_m}{r} a_{ym}
\]
\[
+ \frac{\cos \theta_1}{r} a_{yi}
\]
\[
\ddot{\phi}_l = \frac{2r \dot{\phi}_l}{\cos \theta_m} + 2 \phi_l \dot{\theta}_l \tan \theta_l + \frac{\sin \theta_m \sin \phi_m}{r \cos \theta_l} a_{ym}
\]
\[
- \frac{\cos \phi_m}{r \cos \theta_l} a_{ym} - \frac{\sin \theta_m \sin \phi_m}{r \cos \theta_l} a_{yi}
\]
\[
+ \frac{\cos \phi_1}{r \cos \theta_l} a_{yi}
\]

According to (8)-(9), it can be seen that there exist cross coupling terms between them, while acceleration components \(a_{ym}\) and \(a_{yi}\) simultaneously affect the changes of LOS angles \(\theta_l\) and \(\phi_l\). Compared with engagement dynamics [12, 21] of the spherical coordinate, the above model can express the relationship between lateral accelerations and LOS angles in more detail. Furthermore, this 3D guidance model can be also rewritten as

\[
\begin{bmatrix}
\dot{\theta}_l \\
\dot{\phi}_l
\end{bmatrix} = F + Bu + d
\]

with

\[
d = \begin{bmatrix}
d_1 \\
d_2
\end{bmatrix} = \begin{bmatrix}
\frac{\cos \theta_m}{r} a_{yi} \\
\frac{\cos \phi_m}{r \cos \theta_l} a_{ym} - \sin \theta_m \sin \phi_m \frac{a_{yi}}{r \cos \theta_l}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-\frac{\cos \theta_m}{r} & 0 \\
\frac{\sin \theta_m \sin \phi_m}{r \cos \theta_l} & -\frac{\cos \phi_m}{r \cos \theta_l}
\end{bmatrix},
\]

\[
u = \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
a_{ym} \\
a_{yi}
\end{bmatrix}
\]

**Assumption 1.** It is supposed that relative distance \(r\) and velocity \(\dot{r}\) satisfy \(r \in [r_0, r(0)]\) during the time domain of the guidance phase, where the positive constants \(r(0)\) and \(r_0\) mean the initial relative distance and miss distance, respectively.

**Assumption 2.** It is assumed that the LOS angles \(\theta_l \neq \pm \pi/2\) are fulfilled during the guidance process.

**Assumption 3.** For the disturbances \(d\), there exist relevant upper bounds \(\delta_i\) \((\delta_i > 0)\) such that they satisfy \(|d_i| \leq \delta_i\) \((i = 1, 2)\).

**Remark 1.** The trigonometric functions are bounded for heading angles \(\theta_l\) and \(\phi_i\), while lateral accelerations \(a_{yi}\) and \(a_{yi}\) of the target are also bounded. Assumption 1 means that the relative distance \(r\) is the positive scalar. In the process of interception engagement, LOS angles constraint is reasonable for Assumption 2. Therefore, from Assumptions 1-2 and the relevant analysis, it can be concluded that Assumption 3 is also reasonable.

**B. FIXED-TIME STABILITY**

Consider the nonlinear system

\[
\dot{x}(t) = f(x(t)), \quad x(0) = x_0
\]

with \(x \in \mathbb{R}^\nu\), and \(f : \mathbb{R}^n \to \mathbb{R}^n\) is an autonomous function such that \(f(0) = 0\) \((x = 0)\) is an equilibrium point of nonlinear system (11). Then, some definitions and lemmas are provided as follows.

**Definition 1[30].** The equilibrium point of system (11) is globally finite-time stable if it is globally asymptotically stable and any solution \(\xi(t, x_0)\) of this system converge to the origin within finite time, \(\forall t \geq T(x_0) : \xi(t, x_0) = 0\) where \(T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}\) and \(\mathbb{R}_+ = \{t : t \in \mathbb{R}, t > 0\}\).

**Definition 2[29].** The equilibrium point of system (11) is fixed-time stable if the settling time is bounded and independent of initial conditions, i.e., \(\exists T_{\max} > 0: \forall x_0 \in \mathbb{R}^n\) and \(T(x_0) \leq T_{\max}\).

**Lemma 1[28, 29].** For the system (11), if there exists Lyapunov function \(V(x)\), scalars \(\alpha, \beta, p, q \in \mathbb{R}_+\) \((0 < p < 1, q > 1)\) and \(0 < \beta < \infty\) such that \(\dot{V}(x) \leq -\alpha V(x)^p - \beta V(x)^q + \mathcal{G}\) holds, the trajectory of this system is
practical fixed-time stable. The residual set of the solution for system (11) is provided by
\[
\lim_{t \to \infty} V(\mathbf{x}) \leq \min \left\{ \left[ g / (\alpha (1-\bar{\nu})) \right] V_0, \left[ g / (\beta (1-\bar{\nu})) \right] V(p) \right\}
\]
with \( 0 < \bar{\nu} < 1 \). The bounded time required to reach the residual set is estimated by
\[
T(x_0) \leq 1/\left[ \beta (1-p) \right] + 1/\left[ \alpha (q-1) \right].
\]

**Lemma 2**[31]. For \( \chi_i \in \mathbb{R} \) (\( i = 1, 2, \ldots, n \)), \( 0 < \chi_i \leq 1 \) and \( \nu_i > 1 \), the following inequalities are fulfilled
\[
\left( \sum_{i=1}^{n} |\chi_i| \right)_{\bar{\nu}} \leq \sum_{i=1}^{n} |\chi_i|_{\bar{\nu}} \quad \text{and} \quad \left( \sum_{i=1}^{n} |\chi_i| \right)_{\nu} \leq n^{\nu-1} \sum_{i=1}^{n} |\chi_i|^\nu .
\]

**III. MAIN RESULTS**

In this section, fast fixed-time TSM technique is developed, and then adaptive continuous fixed-time 3D guidance law will be proposed to nullify the LOS angular rates within the given time. The specific results are provided as follows.

**A. NONSINGULAR FIXED-TIME FAST TSM**

To construct the sliding mode surface of LOS angle errors, new variable is defined as \( \mathbf{x} = \left[ x_1, x_2 \right] = \left[ \theta_L - \theta_{LD}, \phi_L - \phi_{LD} \right] \), and the derivative of this variable is \( \dot{\mathbf{x}} = \left[ \dot{x}_1, \dot{x}_2 \right] = \left[ \dot{\theta}_L, \dot{\phi}_L \right] \). A new fixed-time TSM is designed as
\[
S = \dot{\mathbf{x}} + h(\mathbf{x}) \left( \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2) + \gamma_s \mathbf{x} \right)
\]
with \( 0 < p_1 < 1 \), \( q_i > 1 \), \( \alpha_i > 0 \), \( \beta_i > 0 \), \( \gamma_i > 0 \) and
\[
h(\chi) = 1/ \left[ \chi_i + (1-\chi_i) e^{-z} \right]^{\nu}
\]
and \( S_0 \) is
\[
S_0 = \begin{cases} \text{sgn}(\chi), & \text{if } S_i = 0 \text{ or } S_i \neq 0, |\chi_i| > \epsilon \\ \ell_1 \chi_1 + \ell_2 \text{sgn}(\chi_2) & \text{if } S_i \neq 0, |\chi_i| < \epsilon \end{cases}
\]
\[
\tilde{S} = \dot{\mathbf{x}} + h(\mathbf{x}) \left( \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2) + \gamma_s \mathbf{x} \right)
\]
with \( \ell_1 = (2-p_1) e^{\nu-1} \) and \( \ell_2 = (p_1-1) e^{\nu-2} \), and \( \epsilon > 0 \) is a small constant.

**Lemma 3.** Consider guidance system (10) for fixed-time TSM satisfying \( S = \tilde{S} = 0 \). Then the equilibrium point can be reached within the determined time \( T_1 \), which is irrelevant with the initial value of the system states.

\[
T_1 \leq \frac{1}{\gamma_i (1-p_1)} \ln \left( 1 + \frac{\gamma_i}{\beta_i} \right) + \frac{\nu^2}{2 \alpha_i (q_i - 1)}
\]

Proof: If TSM occurs, it implies
\[
\dot{x} = -h(\mathbf{x}) \left( \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2) + \gamma_s \mathbf{x} \right)
\]
Design a Lyapunov function as \( V_1 = x^T x \), and its derivative can be expressed as
\[
V_1 = -2x^T h(x) \left( \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2) + \gamma_s \mathbf{x} \right)
\]
\[
= -2h(x) \left( \alpha_s \sum_{i=1}^{n} |\chi_i| + \beta_s \sum_{i=1}^{n} |\chi_i|_\nu + \gamma_s \sum_{i=1}^{n} |\chi_i|^\nu \right)
\]
(17)
\[
= -\frac{1}{\alpha_i} V_1^2 - \beta_i V_1 - \gamma_i V_1
\]
and then taking its derivative yields
\[
\Phi = \frac{1-p_i}{2} \left( \frac{\gamma_i}{\alpha_i} \right)
\]
By solving (18), the upper bound of the settling time can be computed as
\[
T_i = \int_0^{\Phi(0)} \frac{1}{1-p_i} \left( \frac{1}{2 \alpha_i \Phi^\nu + \beta_i + \gamma_i \Phi} \right) d\Phi
\]
(19)
\[
= \frac{1}{1-p_i} \int_0^{\Phi(0)} \frac{1}{2 \alpha_i \Phi^\nu + \beta_i + \gamma_i \Phi} d\Phi
\]
(20)
\[
\leq \frac{1}{1-p_i} \int_0^{\Phi(0)} \frac{1}{2 \alpha_i \Phi^\nu} d\Phi + \int_0^{\Phi(0)} \frac{1}{\beta_i + \gamma_i \Phi} d\Phi
\]
Since \( h(\mathbf{x}) > 1 \), it can be proved that \( T_i < T_i' \) is satisfied, and therefore system (10) is fixed-time stable after TSM variable is maintained on SM surface.

Recently, fast NTSM techniques have been applied to deal with guidance problem. In [19], fixed-time sliding mode variable can be expressed as
\[
S_{1*} = \dot{x} + \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2)
\]
Another fast fixed-time sliding mode can be seen in [20], which is provided as
\[
S_{2*} = \dot{x} + \alpha_s \text{sgn}(\chi_1) + \beta_s \text{sgn}(\chi_2)
\]
For these two fixed-time sliding mode algorithms, the power constants are $\xi_1 = (q_1 + 1)/2 + (q_1 - 1)/2 \cdot \text{sign}(\|x\| - 1)$ and $\xi_2 = (p_i + 1)/2 + (1 - p_i)/2 \cdot \text{sign}(\|x\| - 1)$, and other parameters are selected as the same as the above.

**Lemma 4.** If the parameter selection $\gamma_i \geq \alpha_i$ is fulfilled, the convergent rate of (14) is faster than (21) after system trajectories reach fixed-time sliding mode surface. When the parameters satisfy $\gamma_1 \geq \alpha_1$ and $\gamma_1 \geq \beta_1$, the convergent rate of (14) is faster than (22).

**Proof:** Choose the same Lyapunov function as $V_i = x^T x$. With regard to (22), the following inequality holds

$$
\dot{V}_i \leq -2 \frac{1 + \eta_i}{2} \alpha_i V_i^2 - 2 \beta_i V_i, \quad \text{if } V_i > 1
$$

and

$$
-2 \alpha_i V_i - 2 \beta_i V_i^2, \quad \text{if } V_i \leq 1
$$

with $\sqrt{V_i} = \|x\|$. By implementing $\Phi = V_i^{\frac{1 - p_i}{2}}$ and solving differential equation (23), the settling time $T_{s_i}$ of (22) can be estimated

$$
T_{s_i} = \frac{1}{1 - p_i} \left[ \int_{0}^{\phi^{(i)}} \frac{1}{2 \alpha_i \Phi^{\alpha_i} + \beta_i \Phi} d\Phi + \int_{\phi^{(i)}}^{1} \frac{1}{2 \alpha_i \Phi^{\alpha_i} - \beta_i \Phi} d\Phi \right]
$$

By subtracting $T_{s_i}$ from $T'_{s_i}$, it can be obtained that

$$
T'_{s_i} = \int_{0}^{\phi^{(i)}} \left( \frac{(\alpha_i - \gamma_i) \Phi - 2 \gamma_i \alpha_i \Phi}{2 \gamma_i \alpha_i \Phi^{\alpha_i} + \beta_i \Phi} \right) d\Phi + \int_{\phi^{(i)}}^{1} \left( \frac{(\beta_i - \gamma_i) \Phi - \beta_i}{2 \gamma_i \alpha_i \Phi^{\alpha_i} + \beta_i \Phi} \right) d\Phi
$$

Therefore, the conditions of three parameters $\gamma_i \geq \alpha_i$ and $\gamma_i \geq \beta_i$ are satisfied, the convergent rate of (14) is faster than (22). According to the similar approach, for the settling time $T_{s_i}$ of (21), the corresponding result can be derived to meet $T'_{s_i} - T_{s_i} < 0$ with $\gamma_i \geq \alpha_i$. That is to say, the convergent rate of (14) is faster than (21) after system trajectories reach fixed-time sliding mode surface. In addition, with the same parameters, the convergent time of (14) is less than (21) after sliding motion occurs.

**Remark 2.** In [26], fixed-time NTSM was also designed to cope with guidance problem. However, the algorithm is only one special form of the proposed NTSM (12) or (14) when the circumstances $q_i = 1 + 2/\eta_i$, $p_i = 1 - 2/\eta_i$, $h(x) = 1$ are fulfilled. On the basis of the characteristic for the function $h(x)$, it can be deduced that the stability time of system states for (14) is less than that of [26] after sliding motion on the designed hypersurface.

**Remark 3.** If system states are far from the equilibrium point, $h(x) > 1 (1/h(x) \rightarrow \chi_i)$ is fulfilled. When system states approach the equilibrium point, $h(x)$ tends to 1. Therefore, one faster convergent rate can be obtained under the action of $h(x)$, while three parameters $\chi_i (i = 1, 2, 3)$ can be adjusted. Herein, $\chi_1$ and $\chi_2$ have relatively more important impact on the convergent rate.

**B. The Design of Fixed-Time Guidance Law**

In this section, adaptive continuous anti-saturation guidance law is proposed to intercept maneuvering target with fixed-time convergence. Firstly, consider the limits of acceleration amplitudes, and 3D guidance system can be expressed as

$$
\begin{bmatrix}
\dot{\theta}_i \\
\dot{\phi}_i
\end{bmatrix} = F + Bu_i + d
$$

where $u_i = [u_{1i}, u_{2i}]^T$ is vector of actual accelerations generated by the thrusters, which can be described as $u_i = u + \bar{u}_i$. Therein, addition term $\bar{u}_i = [u_{1i}, u_{2i}]^T$, which is caused by saturation limits, is defined as

$$
\bar{u}_i = \begin{cases}
0 & |u| < u_{\text{max}} \\
\text{sign}(u_i) u_{\text{max}} - u_i & |u| \geq u_{\text{max}}
\end{cases}
$$

where $u_{\text{max}}$ is the maximum amplitude of actual acceleration.

**Remark 4.** The guidance law or acceleration is bounded as it will be designed and includes system states which are bounded. Therefore, additional control term $\bar{u}_i$ can be considered bounded and reasonable.

The time derivative of the proposed TSM (12) can be obtained as

$$
\dot{S} = F + h(x) \left( \alpha_i \text{sign}(x) \right) + \gamma_i \dot{x} + h(x) \left( \alpha_i q_i \text{sign}(x) \right) + \beta_i \dot{x} + \gamma_i \dot{x}
$$

with $\bar{d} = d + B \bar{u}_i$, and $\dot{S}_c$ can be expressed as

$$
\dot{S}_c = \begin{cases}
p_i |x|^{-1} \dot{x}, & \text{if } \bar{S}_c = 0 \text{ or } \bar{S}_c 
eq 0, |x| \geq \varepsilon
\\
el \dot{x} + 2 |x| \dot{x}, & \text{if } \bar{S}_c = 0, \text{ or } |x| < \varepsilon
\end{cases}
$$

According to Assumption 3 and Remark 4, it can be got that $\|\bar{S}\| \leq \bar{S}$ is satisfied, and $\bar{S}$ is a Lipschitz constant.
On the basis of the above analysis, adaptive continuous fixed-time guidance law is proposed
\[ u = -B^T \left[ \hat{F} + \alpha_2 \text{sig}^\rho(S) + \beta_2 \text{sig}^\rho(S) + \gamma S + \frac{\hat{\rho}}{2\sigma^2} S \right] \] (30)
with
\[ \hat{F} = F + h(x) \left( \alpha_1 \text{sig}^\rho(x) + \beta S_x + \gamma x \right) \]
\[ + h(x) \left( \alpha_q \text{diag} \left( [x]^{-1} \right) \hat{x} + \beta \hat{S}_x + \gamma \hat{x} \right) \]
\[ 0 < p_2 < 1, \quad q > 1, \quad \alpha_1 > 0, \quad \beta > 0 \] and \( \gamma > 0 \). \( \hat{\rho} \) is implemented to estimate the value of \( \rho = \Delta^2 \) and updated by
\[ \hat{\rho} = c_1 \left( \frac{\|S\|^2}{2\sigma^2} - c_2 \hat{\rho} \right) \] (31)
with \( c_1 > 0 \) and \( c_2 > 0 \), and \( \sigma \in \mathbb{R} \) is a small positive scalar.

**Theorem 1.** For 3D guidance system (26), if TSM is designed as (12), while adaptive anti-saturation guidance law is derived as (30)-(31), then the sliding mode variable \( S \), LOS angular rate \( \hat{\theta}_l \) and \( \phi_s \) are practical fixed-time stability.

Proof: Select the following Lyapunov function as
\[ V_2 = \frac{1}{2} S^T S + \frac{1}{2c_1} \hat{\rho}^2 \] (32)
with \( \hat{\rho} = \rho - \hat{\rho} \). Then, differentiating \( V_2 \) with respect to time yields
\[ \dot{V}_2 = S^T \dot{S} - \frac{1}{c_1} \hat{\rho} \hat{\rho} \]
\[ = S^T \left[ F + h(x) \left( \alpha_1 \text{sig}^\rho(x) + \beta S_x + \gamma x \right) \right. \]
\[ \left. + h(x) \left( \alpha_q \text{diag} \left( [x]^{-1} \right) \hat{x} + \beta \hat{S}_x + \gamma \hat{x} \right) \right] \]
\[ + Bu + \hat{d} \] + \frac{1}{c_1} \hat{\rho} \hat{\rho} \] (33)
By using the designed guidance law (30)-(31), one has
\[ \dot{V}_2 = -S^T \left[ \alpha_2 \text{sig}^\rho(S) + \beta_2 \text{sig}^\rho(S) + \gamma S + \frac{\hat{\rho}}{2\sigma^2} S + \hat{d} \right] \]
\[ - \frac{1}{c_1} \hat{\rho} \hat{\rho} \]
\[ \leq \|S\| \|F\| - \alpha_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \beta_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \gamma S^T S \]
\[ - \frac{\hat{\rho}}{2\sigma^2} S^T S - \frac{1}{c_1} \hat{\rho} \hat{\rho} \] (34)
For arbitrary scalar \( \sigma \), it can be got that
\[ \|S\| \|\hat{d}\| \leq \|S\| \|S\|^T / (2\sigma^2) + \sigma / 2 \] based on Young’s inequality. As \( \|F\| \leq \delta \) and \( \rho = \Delta^2 \) hold, it can be obtained that
\[ \|S\| \|\hat{d}\| \leq \rho \|S\|^T / (2\sigma^2) + \sigma / 2 \] (2). Moreover, for any \( \varepsilon > 1 / 2 \), one has
\[ c_2 \hat{\rho} \hat{\rho} = -c_2 \hat{\rho} (\hat{\rho} - \rho) \leq \frac{c_2 (2\varepsilon - 1)}{2\varepsilon} \rho^2 + \frac{c_2 \varepsilon}{2} \rho^2 \] (35)
Furthermore, the following result can be provided
\[ \dot{V}_2 \leq \|S\| \|\hat{d}\| - \alpha_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \beta_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \gamma S^T S \]
\[ - \frac{\hat{\rho}}{2\sigma^2} S^T S - \frac{1}{c_1} \hat{\rho} \hat{\rho} \]
\[ \leq \|S\| \|\hat{d}\| + \frac{\sigma}{2} - \alpha_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \beta_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \gamma S^T S \]
\[ - \frac{\hat{\rho}}{2\sigma^2} S^T S - \frac{c_2 (2\varepsilon - 1)}{2\varepsilon} \rho^2 + \frac{c_2 \varepsilon}{2} \rho^2 \] (36)
\[ \leq \frac{\sigma}{2} + \frac{c_2 \varepsilon}{2} \rho^2 - \gamma S^T S - \frac{c_2 (2\varepsilon - 1)}{2\varepsilon} \rho^2 \]
\[ \leq -\kappa V_2 + \Delta \]
with \( \kappa = \min \left\{ \frac{c_2 (2\varepsilon - 1)}{2\varepsilon}, \gamma \right\} \) and \( \Delta = \frac{\sigma}{2} + \frac{c_2 \varepsilon}{2} \rho^2 \).

According to the boundedness theorem, \( S \) and \( \hat{\rho} \) are uniformly ultimate bounded. Therefore, it is reasonable to assume that \( |\hat{\rho}| \leq \bar{\rho} \) holds, where \( \bar{\rho} \) is a positive constant. In view of the above analysis, (36) can be rewritten as
\[ \dot{V}_2 \leq -\alpha_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \beta_2 \sum_{i=1}^{\lambda} |S_i|^{1+\rho} - \gamma S^T S \]
\[ - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} + \Delta_2 \]
with \( \Delta_2 = \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} \]
\[ - \frac{\sigma}{2} + \frac{c_2 \varepsilon}{2} \rho^2 + \frac{c_2 \varepsilon}{2} \rho^2 \]
and \( \ell = c_1 c_2 \left( \frac{2\varepsilon - 1}{2\varepsilon} \right) (\lambda \in \mathbb{R}) \). If \( \frac{\ell}{2c_1} \bar{\rho}^2 < 1 \) is fulfilled, it has
\[ \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} \]
\[ \leq \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} < 1 \] (38)
For the case \( \frac{\ell}{2c_1} \bar{\rho}^2 \geq 1 \) , it follows that
\[ \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} \]
\[ \leq \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} - \left( \frac{\ell}{2c_1} \bar{\rho}^2 \right)^{1+\rho} \] (39)
Based on the boundedness analysis of adaptive parameter \( \hat{\rho} \), it can be inferred that there exists a compact set \( D \) such that \( D = \{ \hat{\rho} \|\hat{\rho}\| \leq \rho^* \} \) holds. Then, it yields
Thus, the relevant result can be provided as

\[
V_2 \leq -\alpha_2 V_2^2 - \beta_2 V_2^2 + \Delta_2
\]

where \( \Delta_2 = \min \left\{ \beta_2, \frac{1}{\alpha_2} \right\} \). \( \Delta_2 = \frac{\alpha_2^2}{\beta_2} \) and \( \zeta = \max \left\{ 1, \left( \frac{\ell}{2c_i} \right)^2 - 1 \right\} \).

According to Lemma 1, TSM variable can converge to the set

\[
D = \left\{ |S| \leq \sqrt{2} \min \left\{ \frac{\Delta_2}{\beta_2 ((1-\lambda)}, \frac{1}{\alpha_2 (1-\lambda)} \right\} \right\}
\]

(0 < \lambda < 1) within the settling time

\[
T_2 = \frac{1}{\alpha_2 (q_2-1)} + \frac{1}{\beta_2 ((1-\lambda))}. \] LOS angular rates can be steered to small neighborhoods of the equilibrium point in fixed time. That is to say, guidance system is practically fixed-time stable.

After TSM variables are steered to the set \( D \), three relevant cases should be analyzed.

Case 1: If \( S = 0 \) is reached, it can be obtained that \( S = 0 \) is fulfilled. From Theorem 1, LOS angles and angular rates can be stabilized to the equilibrium points within fixed time.

Case 2: If \( S \neq 0 \) and \( |x_i| \leq \varepsilon \) are satisfied, one has

\[
\dot{x}_i + h(x)\left[ \alpha_i \sin^h(x_i) + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i x_i \right] = S_i. \] Since nonsingular TSM variables (12) reach the set after fixed time \( T_2 \), it can be obtained that\( |x_i| \leq |S| + h(x)\left[ \alpha_i |x_i|^h + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i |x_i| \right] \leq \Theta_i \) holds for \( t \geq T_2 \) with

\[
\Theta_i = \Theta_3 + \alpha_i \Theta_3^h + \beta_i \Theta_3^h + \gamma_i \Theta_3
\]

\[
\Theta_3 = \max \left\{ \varepsilon, \min \left\{ \left( \frac{\Theta_3}{\alpha_i}, \frac{\Theta_3^h}{\beta_i}, \frac{\Theta_3}{\gamma_i} \right) \right\} \right\}
\]

\[
\Theta_3 = \sqrt{2} \min \left\{ \frac{\Delta_2}{\alpha_2 (1-\lambda)}, \frac{\Delta_2}{\beta_2 ((1-\lambda))} \right\}
\]

Case 3: If \( S \neq 0 \) and \( |x_i| > \varepsilon \) are fulfilled, it has

\[
\dot{x}_i + h(x)\left[ \alpha_i \sin^h(x_i) + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i x_i \right] = S_i, \] which can be expressed as

\[
\dot{x}_i + h(x)\left[ \alpha_i \sin^h(x_i) + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i x_i \right] = 0 \] (45)

\[
\dot{x}_i + h(x)\left[ \alpha_i \sin^h(x_i) + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i x_i \right] = 0 \] (46)

\[
\dot{x}_i + h(x)\left[ \alpha_i \sin^h(x_i) + \beta_i \left( \ell_i x_i + \ell_i \sin^h(x_i) \right) + \gamma_i - \frac{S_i}{\sin^h(x_i)} \right] = 0 \] (47)

Select \( \alpha_i, \beta_i \) and \( \gamma_i \) so that \( \alpha_i - \frac{S_i}{\sin^h(x_i)} > 0 \) , \( \beta_i - \frac{S_i}{\sin^h(x_i)} > 0 \) and \( \gamma_i - \frac{S_i}{\sin^h(x_i)} > 0 \) ; it can be got that LOS angular rates \( \dot{\theta}_l \) and \( \dot{\phi}_l \) can converge to the equilibrium points after the certain time. From \( \dot{\theta}_l = 0 \) and \( \dot{\phi}_l = 0 \) , it can be concluded that \( D_2 = \{ x_i \| x_i \leq \Theta_i \} \) within the specified time.

Therefore, it can be concluded that guidance system is practically fixed-time stable.

Through the analysis of the above three cases, it can be inferred that LOS angles and angular rates will reach the set after fixed time, respectively.

Remark 5. It should be noted that it is necessary to prove the fixed-time stability of the closed-loop guidance system by two steps. Because there is not the boundedness proof for the adaptive parameters in the first step, the following proof is not logically rigorous.

Remark 6. Adaptive continuous anti-saturation guidance law (30)-(31) is in essence NTSM control algorithm, and therefore this technique consists of two stages. Firstly, NTSM surface is constructed according to fixed-time performance criterion for steering system trajectories to the equilibrium point or the neighborhood of the equilibrium point. Then, fixed-time reaching law is designed to force the system state to reach NTSM surface such that sliding mode occurs on this hypersurface. Therefore, the settling time of the closed-loop guidance can be estimated as \( T_x = T_e + T_2 \) based on NTSM control principle and fixed-time stability theory.

Remark 7. When the designed fixed-time guidance law is applied to engagement geometry, control parameters should be selected appropriately to accomplish guidance time, control energy consumption and miss distance. However, there is not one standard procedure to choose these parameters, which can be selected by trial and error until the specified indicators are satisfactorily acquired.

Remark 8. If the initial conditions are far from the equilibrium point, it may need a big control action to guarantee the fast fixed-time convergence. So if there exits
strong constraints on control action, the settling time can be extended by selecting appropriate parameters to guarantee fixed-time convergence with respect to guidance system. In the actual engagement process, the acceleration constraint, the predetermined convergent time and the miss distance should be considered at the same time in order to achieve reasonable balance.

IV. SIMULATIONS

In this section, numerical simulations are carried out to illustrate the effectiveness of the presented guidance law to intercept one maneuvering target. Firstly, inertial reference system is fixed and centered at launch site at the instant of the launch, as shown in Fig.1. In this system, the X-axis is considered to be in the horizontal plane and points to the direction of launch with the positive Z-axis in the vertical plane, while the Y-axis is selected such that the reference frame forms a right-handed reference frame.

The missile’s initial position is set at the origin of the inertial reference frame. Its initial velocity is \(V_0 = 450\text{m}/\text{s}\), and its initial elevation and azimuth angles are \(\theta_L(0) = 60^\circ\) and \(\phi_L(0) = 30^\circ\). The initial flight-path and heading angles are \(\theta_m(0) = \phi_m(0) = 2^\circ\). On other hand, the velocity of the target is \(V_T = 240\text{m}/\text{s}\); the initial flight-path and heading angles are \(\theta_t(0) = 10\) and \(\phi_t(0) = 135\); the initial value of the relative distance is \(r(0) = 10\) km between the missile and the target, while the desired LOS angles are given as \(\theta_{LD}(0) = 65^\circ\) and \(\phi_{LD}(0) = 40^\circ\).

The parameters of the proposed guidance law are chosen as \(a_\alpha = 0.3\), \(\beta_1 = 0.2\), \(\gamma_1 = 0.1\), \(q_1 = 2.1\), \(p_1 = 0.6\), \(\chi_2 = 0.8\), \(\chi_3 = 9\), \(\chi_4 = 2\), \(\varepsilon = 0.01\), \(\alpha_2 = 1.2\), \(\beta_2 = 2.1\), \(\gamma_2 = 1.4\), \(q_2 = 1.8\), \(p_2 = 0.6\), \(c_1 = 0.3\), \(c_2 = 1.6\), and \(\sigma = 0.08\). The maximum lateral accelerations, which are imposed to the interceptor in both directions, are provided as \(u_{max} = 400\text{m}/\text{s}^2\). Two cases of target manoeuvring modes are test and analyzed.

Case 1: \(a_{\alpha} = 8\text{m}/\text{s}^2\) and \(a_{\alpha} = 8\text{m}/\text{s}^2\);

Case 2: \(a_{\alpha} = 10\text{cos}(0.3\tau)\text{m}/\text{s}^2\) and \(a_{\alpha} = 12\text{sin}(0.5\tau)\text{m}/\text{s}^2\).

In practical simulation application, fixed convergent time of closed-loop guidance system can be expressed by the time required for system trajectories or variables entering the neighborhood of the equilibrium point, while fine guidance effect is achieved. If the guidance accuracy is set as 0.005, the settling time \(T_{\rho_1}\) of NTSM variables can be denoted by the time it takes them to ensure that \(|S_i| \leq 5 \times 10^{-3}\) (\(i = 1, 2\)) is realized. The stabilization time \(T_{\rho_1}\) of can be described by the time the LOS angular rates and tracking errors spend on entering \(|x_i| \leq 5 \times 10^{-3}\) and

\[|x_i| \leq 5 \times 10^{-3}\] (\(i = 1, 2\)), \([x_1, x_2] = [\theta_L - \theta_{LD}, \phi_L - \phi_{LD}]\) and \([x_1, x_2] = [\theta_L, \phi_L]\). In theory, the settling time \(T_{\rho_1} = 14.573\text{s}\) and \(T_{\rho_1} = 5.614\text{s}\) can be determined on the basis of the provided control parameters and fixed-time convergent formulas. According to the practical setting fixed-time convergent index, the settling time \(T_{\rho_2}\) of NTSM variables are 1.14 s and 1.23s under Case 1 and 2, respectively. In addition, the LOS angular rates and tracking errors can converge to the neighborhood of the origin after the time interval, and simulation results show that the stabilization time \(T_{\rho_1}\) are 4.58s and 4.67s under Case 1 and 2, respectively. Through the analysis \((T_{\rho_1} < T_{\rho_2}\), it can be concluded that fixed-time convergence of NTSM variable, LOS angles and angular rates satisfies theoretical analysis of fixed-time stabilization.

After applying the controller (30)-(31) to engagement geometry model (1)-(7), and the simulation results are shown in Figures 2-5. Figure 2 shows curvesub-5 of engagement trajectories and relative distance under Case 1. It can be seen that the missile can precisely intercept maneuvering target with the desired LOS angles, and miss distance is 0.0364m for this case. Figure 3 (a) describes the varying profiles of LOS angles \(\theta_L\) and \(\phi_L\), and Figure 3 (b) displays LOS angular rates \(\dot{\theta}_L\) and \(\phi_L\), and it can be obtained that they can converge to the desired values or the equilibrium point within 4.58s. The convergent curves are shown about fast TSM variables \(S_1\) and \(S_2\) in Figure 2, and continuous lateral accelerations are provided without chattering phenomenon in Figure 4(b), which initially exceeds the maximum limits until TSM variables \(S_1\) and \(S_2\) reach sliding surfaces, and then gradually converge to the small certain constants along with the convergence of LOS angular rates. Heading angles of missile and target are plotted in Figure 5; due to the designed guidance law, the curves of \(\theta_m\) and \(\phi_m\) are sharply changed about 5s ago; the curves of \(\theta_t\) and \(\phi_t\) are uniformly changed all the time, owing to the action of maneuvering acceleration.

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Figure 2. Response curves of Case 1: (a) three dimensional trajectories of missile and target, (b) relative distance.

Figure 3. Response curves of Case 1: (a) LOS angles, (b) LOS angular rates.

Figure 4. Response curves of Case 1: (a) fixed-time SM variables, (b) lateral accelerations of the missile. Therein, the comparison results can be seen in Table 1 in detail.

| Case | Guidance time/s | Miss distance /m | The settling time of LOS rates |
|------|-----------------|------------------|-------------------------------|
| 1    | 16.95           | 0.0364           | 4.58                          |
| 2    | 18.20           | 0.0786           | 4.67                          |
In order to verify the performance of the proposed guidance law (30)-(31), the simulation comparison under Case 2 with the continuous NTSM guidance law [16] is carried out, and control parameters are the same as the above mentioned constants. Moreover, the maximum limits of missile acceleration remain unchanged. Figure 4 (a) and (b) describe the curves of three-dimensional engagement trajectory and the relative distance. It can be seen that guidance law can guarantee that the missile effectively hits maneuvering target with miss distance 0.4226 m. The LOS angles and angular rates are shown in Figure 7 (a) and (b). Finite convergent time is $T_p = 4.76$ s after all variables get into $|\dot{x}| \leq 5 \times 10^{-3}$ and $|\dot{\theta}| \leq 5 \times 10^{-3}$. NTSM variables and missile accelerations are provided in Figure 8 (a) and (b). The stabilization time of NTSM variables is $T_{p2} = 3.06$ s, while it can be obtained that the accelerations are almost at the saturation state before $T_{p2} = 3.06$ s. Therefore, it can be inferred that energy consumption are more that of the proposed guidance law. Heading angles of missile and target are plotted in Figure 9 under Case 2. Compared with that of the proposed algorithm, it can be seen that missile heading angles change greatly. Other corresponding comparisons are shown in Table 2.

### TABLE II

| Guidance law | Fixed convergent time $T_f$ (s) | Miss distance (m) | Guidance time (s) |
|--------------|---------------------------------|-------------------|------------------|
| Continuous   | 7.82                            | 0.4226            | 18.62            |
| NTSM [16]    |                                |                   |                  |
| The proposed law | 5.90                         | 0.0786            | 18.20            |

Figure 5. Response curves of Case 1: (a) missile heading angles, (b) target heading angles.

Figure 6. Response curves of Case 2 [16]: (a) three dimensional trajectories of missile and target, (b) relative distance.
Figure 7. Response curves of Case 2 [16]: (a) LOS angles, (b) LOS angular rates.

Figure 8. Response curves of Case 2 [16]: (a) fixed-time SM variables, (b) lateral accelerations of the missile.

Figure 9. Response curves of Case 2 [16]: (a) missile heading angles, (b) target heading angles.

The simulation results by the designed guidance method (30)-(31) are shown for Case 2 under varying maneuver in Figures 10-13. Figure 10 shows curves of engagement trajectories and relative distance under Case 2. It can be seen that the missile can also hit maneuvering target. However, guidance time has been increased, and miss distance is 0.0786m for this case. Figure 11 shows the varying profiles of LOS angles and angular rates, and it can be seen that the difference is relatively small compared with Figure 3. Fast nonsingular TSM variables $S_1$ and $S_2$ are described in Figure 12(a), continuous lateral accelerations are shown in Figure 12(b). Due to the varying maneuver, from Figure 12(b), guidance accelerations also change all the time after LOS angular rates are stabilized. Heading angles of missile and target are plotted in Figure 13 under Case 2; compared with the result of Case 1, there is little difference before 5 seconds; however, the fluctuation changes of missile heading angles occur along with periodical change of target heading angles after 5 seconds.
Figure 10. Response curves of Case 2: (a) three dimensional trajectories of missile and target, (b) relative distance.

Figure 11. Response curves of Case 2: (a) LOS angles, (b) LOS angular rates.

Figure 12. Response curves of Case 2: (a) fixed-time SM variables, (b) lateral accelerations of the missile.
Furthermore, simulation results indicate that the proposed algorithm can achieve excellent performance in intercepting maneuvering targets. The effectiveness of the algorithm is verified by the convergence of the LOS angular rates and the target heading angles.

V. CONCLUSION

In this study, continuous fixed-time guidance law is designed with new NTSM technique and adaptive law, and prior information is not required. Furthermore, simulation results show LOS angular rates and NTSM variables can reach the neighborhood of the origin within the settling time, and the excellent performance is finally demonstrated for this algorithm. In the future, the balance between energy consumption, saturation restriction, guidance time and performance will be deeply investigated, while other guidance laws will be constructed to avert violation with respect to the constraint of LOS angular rates.

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XIAOJING LI was born in Luoyang, China, in 1988. She received the B.S. degree in mathematics and applied mathematics from Henan Normal University, Xinxiang, China, in 2010, and the M.E. degree in Basic Mathematics from Lanzhou University of Technology, Lanzhou, China, in 2013.

She is currently pursuing the Ph.D. degree in control science and engineering with Henan University of Science and Technology, Luoyang. Her current research interests include sliding mode control, cooperative guidance and cooperative control theory of multiple agents.

JIANWEI MA was born in Luoyang, China, in 1965. He received the Ph.D. degree in control science and engineering from Nanjing University of Science and Technology, Nanjing, China, in 2005.

He is currently a Professor with the School of Information Engineering, Henan University of Science and Technology. His current research interests include Aircraft control, navigation guidance and simulation, robot control and intelligent control theory and application.

JIWEI GAO received the B.S. degree in automation from the Henan University of Science and Technology, Luoyang, China, in 2010, and the Ph.D. degree in control science and engineering from Xi’an Jiaotong University, Xi’an, China, in 2016.

Since 2016, he has been a Lecturer with the School of Information Engineering, Henan University of Science and Technology. His main research interest includes nonlinear control theory (sliding mode control and adaptive control) with applications to mechatronic systems, including spacecrafts, unmanned aerial vehicles, and manipulators.