Why Don’t We Have a Covariant Superstring Field Theory?

Martin Cederwall

Institute for Theoretical Physics
Chalmers University of Technology and Göteborg University
S-412 96 Göteborg, Sweden
email: tfemc@fy.chalmers.se

Abstract. This talk deals with the old problem of formulating a covariant quantum theory of superstrings, “covariant” here meaning having manifest Lorentz symmetry and supersymmetry. The advantages and disadvantages of several quantization methods are reviewed. Special emphasis is put on the approaches using twistorial variables, and the algebraic structures of these. Some unsolved problems are identified.

Before going into supersymmetric strings, let me examine the situation in bosonic string theory.

The bosonic string has a well defined (Lorentz) covariant first quantized formulation as a gauge theory, preferably through BRST [1]. This formulation is the starting point and an absolute prerequisite for the second quantization, the field theory [2]. It is probable that field theory can provide a framework for posing questions about the big symmetries of string theory, including general coordinate invariance. Some aspects of background invariance have already been addressed in bosonic string theory [3].

In this perspective, what is the corresponding status of supersting theory? It is not so good. Why is this so?

To be clear about the ambitions, one would like a covariant quantum superstring theory to fulfill the following requirements:

1. It should have manifest space-time symmetry, including supersymmetry.
2. It should contain first class constraints only.

Concerning the second of these points, there may well be second class constraints present, but they have to be dealt with in a covariant manner before first quantization. There are different methods for doing this – in Dirac’s original treatment of constraints [4] second class constraints are eliminated consistently by letting remaining variables (parametrizing the second class constraint surface) obey Dirac brackets; in the method by Batalin and Fradkin [5] additional constraints are added to turn
the constraints into \(1\square\) class ones.

There are a number of quite different formulations of superstring theory. Let us examine them with respect to the requirements! The three main classes of models are:

1. Spinning string
2. Green-Schwarz superstring
3. Twistor superstrings (main subject of this talk).

In each of these approaches, there may be alternative formulations or modifications. I will briefly review the different models, and make some indications on to what extent and on which points the requirements we have set up fail to be fulfilled.

The spinning/fermionic/NSR string [6] has an \(N=1\) world-sheet supersymmetry. The action is the \(N=1\) generalization of

\[
S = \int d^{2}\sigma \sqrt{-g} g^{\alpha \beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X_{\mu}
\]

where a world-sheet gravitino and a fermionic space-time vector have been included [7]. The space-time supersymmetry of the spinning string is not present until the GSO projection [8] is performed on the spectrum and it is highly non-manifest. A functioning field theory exists for the spinning string [9], but it has of course the same drawbacks as the \(1\square\) quantized theory. The only thing that this formulation does not give us is manifest supersymmetry. The calculational power that two-dimensional conformal field theory comprises makes this approach to superstring field theory the best one so far.

Green and Schwarz found a space-time supersymmetric action for the superstring [10],

\[
S = \int d^{2}\sigma \left\{ \sqrt{-g} g^{\alpha \beta} \Pi_{\alpha \beta} - i e^{\alpha \beta} \partial_{\alpha} X^{\mu} (\bar{\theta}^{1} \gamma_{\mu} \partial_{\beta} \theta^{1} - \bar{\theta}^{2} \gamma_{\mu} \partial_{\beta} \theta^{2})
\right. \\
+ \left. i e^{\alpha \beta} \bar{\theta}^{1} \gamma_{\mu} \partial_{\alpha} \theta^{A} \bar{\theta}^{2} \gamma_{\mu} \partial_{\beta} \theta^{A} \right\}
\]

where

\[
\Pi_{\alpha}^{\mu} = \partial_{\alpha} X^{\mu} - i \bar{\theta}^{A} \gamma_{\mu} \partial_{\alpha} \theta^{A}
\]

This action has some interesting properties, that can as well be analyzed in the simpler superparticle case, containing only the first term in the action. Its constraint structure is given by

\[
L \equiv P^{2} \approx 0
\]
\[
\Phi \equiv p_{\theta} - i P_{\mu} \gamma^{\mu} \theta \approx 0
\]

There is one bosonic constraint, generating translations along the world-line, and a fermionic spinor of local supersymmetry generators. Due to \(P^{2} = 0\), the rank of

\[
\{ \Phi_{a}, \Phi_{b} \} = -2i P_{\mu} \gamma_{ab}^{\mu}
\]

is 8 (out of 16). The chiral spinor \(\Phi\) thus contains eight \(1\square\) class constraints ("\(\kappa\)-symmetry" [11]) and eight \(2\square\) class constraints. There is no way (naively) of eliminating covariantly the second
class constraints before quantization [12]. $\Phi$ is a chiral spinor, transforming in the irreducible representation 16 of $\text{Spin}(1,9)$, and cannot be decomposed without giving up Lorentz covariance. Exactly the same is true for the superstring.

The difficulty of getting rid of the second class constraints in a covariant manner is closely connected to the problems with finding covariant supersymmetric field theories in ten dimensions.

One can always choose a light-front gauge, where only the physical degrees of freedom remain (no gauge invariance). Then the step to field theory is straightforward [13].

A couple of approaches exist, where one tries to deal with the constraint structure of the Green-Schwarz string by modifying it. Siegel proposed [14] that only the first class constraints should be kept, by only demanding (in the superparticle version) $\Psi = P_\mu (\gamma^\mu \Phi) \approx 0$, and then introducing additional bosonic constraints removing fermionic degrees of freedom. The problem here is that the spinor $\Psi$ has an infinite level of reducibility. It is not clear how to treat the infinite tower of ghosts that arise. Similar approaches are advocated in [15].

Much of the original work in supertwistors was motivated by the observation that they have the potential of solving the problem with separation of the fermionic constraints in 1st and 2nd class parts. Let me therefore briefly describe the fundamental ideas of division algebra twistors for massless bosonic particles and superparticles, and then discuss application to string theory.

The classical dimensionalities of the superstring are $D = 3,4,6,10$. The gamma matrix identities $(\lambda_1 \gamma^\mu \lambda_2) \gamma^\alpha \lambda_3 + \text{cycl.} = 0$ needed are directly related to the existence of the (alternative) division algebras $\mathbb{K}_\nu = \mathbb{R}, \mathbb{C}, \mathbb{H}$ and $\mathbb{O}$. More specifically:

Existence of Clifford algebra $\leftrightarrow$ alternativity,

$\nu^\mu = \lambda \gamma^\mu \lambda$ lightlike $\leftrightarrow$ division property $|ab| = |a||b|.$

This opens the way to twistor transformations of the lightlikeness constraints in these dimensionalities:

$P^\mu = \frac{1}{2} \lambda \gamma^\mu \lambda \iff P^a_{\dot{a}} = \lambda^a \lambda_{\dot{a}}$

The lightlike directions form the sphere $S^{\nu}$. The spinor, modulo $\mathbb{R}_+$, lies on $S^{2\nu-1}$, where $\nu = D-2$. The spinor $\lambda$ is a two-component object $\lambda = [\lambda_1 \lambda_2]^T$ with entries in $\mathbb{K}_\nu$, transforming under $SL(2;\mathbb{K}_\nu) \approx Spin(1, \nu + 1)$. A vector is a hermitean matrix.

$v = \begin{pmatrix} v^+ & v^* \\ v & v^- \end{pmatrix}$

The twistor transform [16] from $\lambda$ to $P$ is the Hopf map $S^{2\nu-1} \rightarrow S^{\nu}$ with fiber $S^{\nu-1}$. The realization of the last (octonionic) Hopf map relies on the understanding of $S^7$ as “almost a Lie group” [17].

The twistor transform can be extended to superparticles [18], and it solves the 2nd class constraint problem! The fermionic variables become a Lorentz scalar element of the division algebra, except in D=10, where such things do not exist, and the fermion carries a vector representation, which actually can be identified as the fermionic variables of the spinning particle [19]. All phase space variables sit in a representation of $OSp(1|4;\mathbb{K}_\nu)$ ($\nu \neq 8$), which is the superconformal group [20] in D=3,4,6.
Strings are not (space-time) conformally invariant, except in the zero tension limit. A twistor transformation of the lightlikeness condition \((\partial X)^2 = 0\) as \(\partial X = \lambda \lambda^\dagger\) introduces 2nd class constraints between \(\lambda\) and its canonical momentum \(\omega\), since \(X\) already spans the entire phase space for the left-(right-)moving sector.

Simple counting of the number of degrees of freedom in D=10 gives

\[
8 \text{ (phys.)} = 2 \times 16 - 2 \times 1 \text{ (Vir.)} - 2 \times 7 \text{ (affine } S^7) - n \implies n = 8,
\]

so there are must be 8 2nd class constraints for the bosonic twistor string in D=10. These constraints are quite analogous to the fermionic ones in the space-time picture of the superstring. The problem with fermionic constraints can be solved, but it reappears in the bosonic sector! This is quite general for twistor formulation of strings. The problem is not universally recognized, but seems to be generic in the sense that it appears as soon as chiral spinors form part of phase space. It is not at all clear whether the problem can be circumvented. We are not in the position that we dare to formulate a no-go theorem.

Even though the problem I have pointed out seems to be a very severe one concerning the prospect of finding a covariant quantization scheme for superstrings, there is a lot of very interesting structure in twistor superstrings. Different versions exist, each with its own advantages.

1. N=8 superconformal algebra (based on \(S^7\)) as a gauge group [21]. This formulation is manifestly “octonionic”, which I consider as fundamental.

2. N=8 superfield formulation where the \(\kappa\) symmetry is identified as a local world-sheet supersymmetry [22]. The rôle of superconformal symmetry here is less clear.

It is likely that there exists an N=8 superfield formulation of 1., but the theory of N=8 superconformal field theory is unexplored.

A very interesting and intriguing observation is that the Green-Schwarz superstring gauge-fixed to the light-cone exhibits an N=8 superconformal symmetry [23]. The spin 2 and spin 3/2 generators are remnants of the Virasoro and local fermionic constraints, but the \(S^7\) generators can not be traced back to a symmetry in this way. It is tempting to think that it is actually a sign of a gauge symmetry in an action where the 2nd class constraints reflect a partial choice of gauge. This still remains a speculation – we have not been able to find such a formulation.

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The reference list, and also the text, gives a very fragmented rendering of the contributions to a vast subject. The author sincerely apologizes for that.