Quantum Foam, Gravitational Thermodynamics, and the Dark Sector

Y. Jack Ng
Institute of Field Physics, Department of Physics & Astronomy, University of North Carolina, Chapel Hill, NC 27599-3255, USA
E-mail: yjng@physics.unc.edu

Abstract.
Is it possible that the dark sector (dark energy in the form of an effective dynamical cosmological constant, and dark matter) has its origin in quantum gravity? This talk sketches a positive response. Here specifically quantum gravity refers to the combined effect of quantum foam (or spacetime foam due to quantum fluctuations of spacetime) and gravitational thermodynamics. We use two simple independent gedanken experiments to show that the holographic principle can be understood intuitively as having its origin in the quantum fluctuations of spacetime. Applied to cosmology, this consideration leads to a dynamical cosmological constant of the observed magnitude, a result that can also be obtained for the present and recent cosmic eras by using unimodular gravity and causal set theory. Next we generalize the concept of gravitational thermodynamics to a spacetime with positive cosmological constant (like ours) to reveal the natural emergence, in galactic dynamics, of a critical acceleration parameter related to the cosmological constant. We are then led to construct a phenomenological model of dark matter which we call “modified dark matter” (MDM) in which the dark matter density profile depends on both the cosmological constant and ordinary matter. We provide observational tests of MDM by fitting the rotation curves to a sample of 30 local spiral galaxies with a single free parameter and by showing that the dynamical and observed masses agree in a sample of 93 galactic clusters. We also give a brief discussion of the possibility that quanta of both dark energy and dark matter are non-local, obeying quantum Boltzmann statistics (also called infinite statistics) as described by a curious average of the bosonic and fermionic algebras. If such a scenario is correct, we can expect some novel particle phenomenology involving dark matter interactions. This may explain why so far no dark matter detection experiments have been able to claim convincingly to have detected dark matter.

1. Introduction
This talk is based on several loosely related pieces of work I did mostly with various collaborators. I will start with one aspect of John Wheeler’s spacetime foam or quantum foam by which he was referring to a foamy structure of spacetime due to quantum fluctuations. So how large are those fluctuations? I will briefly discuss (in section 2) a gedanken experiment to measure a distance $l$ and deduce the intrinsic limitation $\delta l$ to the accuracy with which we can measure that distance, $|l|/\delta l \approx 1$ for that distance undergoes quantum fluctuations. To gain more insight I

1 I will further show that the scaling of $\delta l \sim l^{1/3}r_p^{2/3}$, as deduced from the gedanken experiment, is exactly what the holographic principle [4, 5] demands, according to which the maximum amount of information stored in a
will use (in section 2) an argument in mapping out the geometry of spacetime [7] to arrive at the same result. When I generalize the argument to the case of an expanding Universe, we will see that something akin to a (positive) cosmological constant emerges [8] — an effective dynamical cosmological constant that has its origin in the quantum fluctuations of spacetime. This dynamical cosmological constant will be shown [9, 10] (in section 3) to have the same magnitude as the one deduced by using unimodular gravity [11, 12] in combination with causal-set theory [13].

Next I switch gear to discuss (in section 4) gravitational thermodynamics /entropic gravity, inspired by the work of Ted Jacobson [14] and Eric Verlinde [15]. By generalizing their work to our spacetime with positive cosmological constant Λ, we will be led to a critical acceleration parameter $a_c$ of the same magnitude as the one introduced by Milgrom by hand in his formulation of MOND (modified Newtonian dynamics) to explain flat galactic rotation curves. But I will argue that $a_c$ actually is a manifestation of the existence of dark matter of a specific mass profile. My collaborators and I call that model of dark matter “modified dark matter” (MDM) to distinguish it from cold dark matter (CDM). [16, 17] Recently my collaborators and I have sucessfully tested MDM (see Section 5) with galactic rotation curves and galactic clusters. [18]

The take-home message from this talk is this: It is possible that the dark sector (viz., dark energy and dark matter) has its origin in quantum gravity. If so, then we can perhaps understand why the dark sector is really so different from ordinary matter. And if the scenario to be sketched in Section 6 is correct, then we can expect some rather novel particle phenomenology, for the quanta of the dark sector obey not the familiar Bose-Einstein or Fermi-Dirac statistics, but an exotic statistics that goes by the name infinite statistics [19, 20, 21, 22] or quantum Boltzmann statistics. [23, 17] However, it is known that theories of particles obeying this exotic statistics are non-local — meaning that we cannot use conventional quantum field theories to describe these particles’ interactions. On the positive side, this non-locality may explain why so far dark matter detection experiments have failed to definitively detect dark matter. Furthermore we expect that the extended nature of the quanta of the dark sector may connect them to certain global aspects of spacetime such as the cosmological constant and the Hubble parameter (as will be shown in Section 4).

I would like to take this opportunity to make a disclaimer on my own behalf: In a recent paper “New Constraints on Quantum Gravity from X-ray and Gamma-Ray Observations” by Perlman, Rappaport, Christiansen, Ng, DeVore, and D. Pooley [24], it was claimed that detections of quasars at TeV energies with ground-based Cherenkov telescopes seem to have ruled out the holographic spacetime foam model (with δl scaling as $l^{1/3}P^{2/3}$). But now I believe this conclusion is conceivably premature when correct averaging is carried out. The point is that these authors (including myself!) have considered the instantaneous fluctuations in the distance between the location of the emission and a given point on the telescope aperture. Perhaps one should average over both the huge number of Planck timescales during the time it takes light to propagate through the telescope system, and over the equally large number of Planck squares across the detector aperture. It is then possible that the net fluctuations are exceedingly small, but at the moment there is no formalism for carrying out such averages. [25]

2. Spacetime (Quantum) Foam and Effective Cosmological Constant Λ

Spacetime is foamy due to quantum fluctuations. To examine how large the fluctuations are, let us consider a gedankan experiment in which a light signal is sent from a clock to a mirror (at a region of space scales as the area of its two-dimensional surface, like a hologram [6].
distance $l$ away) and back to the clock in a timing experiment to measure $l$. From the jiggling of the clock’s position alone, the Heisenberg uncertainty principle yields $\delta l^2 \gtrsim \frac{h}{mc}$, where $m$ is the mass of the clock. On the other hand, the clock must be large enough not to collapse into a black hole; this requires $\delta l \gtrsim \frac{Gm}{l}$. We conclude that the fluctuations of a distance $l$ scales as

$$\delta l \approx l^{1/3} l_P^{2/3},$$

(1)

where $l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33}$ cm is the Planck length.  

One can further show that the scaling of $\delta l$ given above is exactly what the holographic principle demands. Heuristically, this comes about because a cube with side $l$ contains $\sim l^3/l_P^3$ number of small cubes with side $\delta l$. Imagine partitioning a cubic region with side $l$ into small cubes. The small cubes so constructed should be as small as physical laws allow so that intuitively we can associate one degree of freedom with each small cube. In other words, the number of degrees of freedom that the region can hold is given by the number of small cubes that can be put inside that region. A moment’s thought tells us that each side of a small cube cannot be smaller than the accuracy $\delta l$ with which we can measure each side $l$ of the big cube. This can be easily shown by applying the method of contradiction: assume that we can construct small cubes each of which has sides less than $\delta l$. Then by lining up a row of such small cubes along a side of the big cube from end to end, and by counting the number of such small cubes, we would be able to measure that side (of length $l$) of the big cube to a better accuracy than $\delta l$. But, by definition, $\delta l$ is the best accuracy with which we can measure $l$. The ensuing contradiction is evaded by the realization that each of the smallest cubes (that can be put inside the big cube) indeed measures $\delta l$ by $\delta l$ by $\delta l$. Thus, the number of degrees of freedom $I$ in the region (measuring $l$ by $l$ by $l$) is given by $l^3/\delta l^3$, which, according to the holographic principle, is

$$I \approx l^2/l_P^2.$$ (2)

It follows that $\delta l$ is bounded (from below) by the cube root of $l_P^3$, the same result as found above in the gedanken experiment argument.

We can rederive the scaling of $\delta l$ by another argument. Let us consider mapping out the geometry of spacetime for a spherical volume of radius $l$ over the amount of time $2l/c$ it takes light to cross the volume. One way to do this is to fill the space with clocks, exchanging signals with the other clocks and measuring the signals’ times of arrival. The total number of operations, including the ticks of the clocks and the measurements of signals, is bounded by the Margolus-Levitin theorem which stipulates that the rate of operations cannot exceed the amount of energy $E$ that is available for the operation divided by $\pi h/2$. This theorem, combined with the bound on the total mass of the clocks to prevent black hole formation, implies that the total number of operations that can occur in this spacetime volume is no bigger than $2(l/l_P)^2/\pi$. To maximize spatial resolution, each clock must tick only once during the entire time period. If we regard the operations as partitioning the spacetime volume into “cells”, then on the average each cell occupies a spatial volume no less than $\sim l^3/(l_P^2/l_P^2) = ll_P^2$, yielding an

\[^{2}\text{Now the amount of fluctuations in the distance } l \text{ can be thought of as an accumulation of the } l/l_P \text{ individual fluctuations each by an amount plus or minus } l_P. \text{ But note that the individual fluctuations cannot be completely random (as opposed to random-walk); actually successive fluctuations must be entangled and somewhat anti-correlated (i.e., a plus fluctuation is slightly more likely followed by a minus fluctuation and vice versa), in order that together they produce a total fluctuation less than that in a random-walk model (for which } \delta l \approx l^{1/2}l_P^{1/2}. \text{)}\]

\[^{20}\text{This small amount of anti-correlation between successive fluctuations (corresponding to what statisticians call fractional Brownian motion with self-similarity parameter } 1/2 \text{) must be due to quantum gravity effects.}\]
average separation between neighboring cells no less than $\sim l^{1/3}/l_P^{2/3}$. This spatial separation can be interpreted as the average minimum uncertainty in the measurement of a distance $l$, that is, $\delta l \sim l^{1/3}/l_P^{2/3}$, in agreement with the result found in the gedanken experiment to measure the fluctuation of a distance $l$.

We make two observations: [29] [23] First, maximal spatial resolution (corresponding to $\delta l \sim l^{1/3}/l_P^{2/3}$) is possible only if the maximum energy density $\rho \sim (l_P^3)^{-2}$ is available to map the geometry of the spacetime region, without causing a gravitational collapse. Secondly, since, on the average, each cell occupies a spatial volume of $l_P^3$, a spatial region of size $l$ can contain no more than $\sim l^3/(l_P^3) = (l/l_P)^3$ cells. Hence, this result for spacetime fluctuations corresponds to the case of maximum number of bits of information $l^3/l_P^3$ in a spatial region of size $l$, that is allowed by the holographic principle[4, 5].

It is straightforward to generalize [29] the above discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by the substitution of $l$ by $H^{-1}$ in the expressions for energy and entropy densities, where $H$ is the Hubble parameter. (Henceforth we adopt $c = 1 = \hbar$ for convenience unless stated otherwise.) Thus, applied to cosmology, the above argument leads to the prediction that (1) the cosmic energy density has the critical value

$$\rho \sim (H/l_P)^2,$$

and (2) the universe of Hubble size $R_H$ contains $I \sim (R_H/l_P)^2$ bits of information. (For the present cosmic epoch we have $I \sim 10^{122}$.) It follows that the average energy carried by each particle/bit is $\rho R_H^3/I \sim R_H^{-1}$. Such long-wavelength constituents of dark energy give rise to a more or less uniformly distributed cosmic energy density and act as a dynamical cosmological constant with the observed small but nonzero value

$$\Lambda \sim 3H^2.$$

3. Cosmological Constant $\Lambda$ via Unimodular Gravity and Causal-set Theory

The dynamical cosmological constant we have just obtained will be seen to play an important role in our subsequent discussions. So let us “rederive” it by using another method based on quantum gravity. The idea makes use of the theory of unimodular gravity[11, 9] (which can be regarded as the ordinary theory of gravity except for the way the cosmological constant $\Lambda$ arises in the theory). But here we will use the (generalized) version of unimodular gravity given by the Henneaux and Teitelboim action[12]

$$S_{\text{unimod}} = -\frac{1}{16\pi G} \int \left[\sqrt{g}(R + 2\Lambda) - 2\Lambda \partial_\mu T^\mu\right] (d^3x) dt. \quad (5)$$

One of its equations of motion is $\sqrt{g} = \partial_\mu T^\mu$, the generalized unimodular condition, with $g$ given in terms of the auxiliary field $T^\mu$. Note that, in this theory, $\Lambda/G$ plays the role of “momentum” conjugate to the “coordinate” $\int d^3x T_0$ which can be identified, with the aid of the generalized unimodular condition, as the spacetime volume $V$. Hence $\Lambda/G$ and $V$ are conjugate to each other. It follows that their fluctuations obey a Heisenberg-type quantum uncertainty principle,

$$\delta V \delta \Lambda/G \sim 1. \quad (6)$$

Next we borrow an argument due to Sorkin[13], drawn from the causal-set theory, which stipulates that continuous geometries in classical gravity should be replaced by “causal-sets”, the discrete substratum of spacetime. In the framework of the causal-set theory, the fluctuation in
the number of elements $N$ making up the set is of the Poisson type, i.e., $\delta N \sim \sqrt{N}$. For a causal set, the spacetime volume $V$ becomes $l_p^4 N$. It follows that

$$\delta V \sim l_p^4 \delta N \sim l_p^4 \sqrt{N} \sim l_p^4 G \sqrt{V}.$$  

(7)

Putting Eqs. (6) and (7) together yields a minimum uncertainty in $\Lambda$ of $\delta \Lambda \sim V^{-1/2}$. This cosmological constant, like the one given by Eq. (4) from a heuristic quantum mechanical consideration, is finite and is to be identified with the fully renormalized cosmological constant from a quantum field-theoretic argument given by the path integration method, to which we turn next.

Following an argument due to Baum[30], Hawking[31], and Adler[32], one can now plausibly argue[9] that, in the framework of unimodular gravity, $\Lambda$ vanishes to the lowest order of approximation and that it is positive if it is not zero. The argument goes as follows: Consider the vacuum functional for unimodular gravity given by path integrations over $T^\mu$, $g_{\mu\nu}$, the matter fields (represented by $\phi$), and $\Lambda$:

$$Z_{\text{Minkowski}} = \int d\mu(\Lambda) \int d[\phi]d[g_{\mu\nu}] \int d[T^\mu]\exp\{-i[S_{\text{unimod}} + S_M(\phi, g_{\mu\nu})]\},$$

(8)

where $S_M$ stands for the contribution from matter (including radiation) fields (and $d\mu(\Lambda)$ denotes the measure of the $\Lambda$ integration). The integration over $T^\mu$ yields $\delta(\partial_\mu \Lambda)$, which implies that $\Lambda$ is spacetime-independent (befitting its role as the cosmological constant). A Wick rotation now allows us to study the Euclidean vacuum functional $Z$. The integrations over $g_{\mu\nu}$ and $\phi$ give $\exp[-S_{\Lambda}(\bar{g}_{\mu\nu}, \bar{\phi})]$ where $\bar{g}_{\mu\nu}$ and $\bar{\phi}$ are the background fields which minimize the effective action $S_{\Lambda}$. A curvature expansion for $S_{\Lambda}$ yields a Lagrangian whose first two terms are the Einstein-Hilbert terms $\sqrt{g}(R + 2\Lambda)$. Note that (1) $\Lambda$ now denotes the fully renormalized cosmological constant after integrations over all other fields have been carried out; (2) the Einstein-Hilbert terms are exactly the first two terms in Eq. (5), hence the fluctuation $\delta \Lambda \sim V^{-1/2}$ we found above now applies to the renormalized $\Lambda$. Next we can make a change of variable from the original (bare) $\Lambda$ to the renormalized $\Lambda$ for the integration in Eq. (8). Let us assume that for the present and recent cosmic eras, $\phi$ is essentially in the ground state, then it is reasonable to neglect the effects of $\phi$. To continue, we follow Baum[30] and Hawking[31] to evaluate $S_{\Lambda}(\bar{g}_{\mu\nu}, 0)$. For negative $\Lambda$, $S_{\Lambda}$ is positive; for positive $\Lambda$, one finds $S_{\Lambda}(\bar{g}_{\mu\nu}, 0) = -3\pi/G\Lambda$, so that

$$Z \approx \int d\mu(\Lambda)\exp(3\pi/G\Lambda).$$

(9)

This implies that the observed cosmological constant in the present and recent eras is essentially zero (or more accurately, very small but positive). So we[9] conclude that $\Lambda$ is positive and it fluctuates about zero with a magnitude of

$$\Lambda \sim V^{-1/2} \sim R_H^{-2},$$

(10)

where, we recall, $R_H$ is the Hubble radius of the Universe, contributing an energy density $\rho$ given by: $\rho \sim \frac{1}{4\pi G R_H^2}$, which is of the order of the critical density as observed!

4. From Cosmological Constant $\Lambda$ to Modified Dark Matter (MDM)

The dynamical cosmological constant (originated from quantum fluctuations of spacetime) can now be shown to give rise to a critical acceleration parameter in galactic dynamics. The argument[16] is based on a simple generalization of E. Verlinde’s recent proposal of entropic gravity[15][14] for $\Lambda = 0$ to the case of de-Sitter space with positive $\Lambda$. Let us first review Verlinde’s derivation
(or prescription, if you like) of Newton’s second law $\vec{F} = m\vec{a}$. Consider a particle with mass $m$ approaching a holographic screen at temperature $T$. Using the first law of thermodynamics to introduce the concept of entropic force $F = T\Delta S/\Delta x$, and invoking Bekenstein’s original arguments [34] concerning the entropy $S$ of black holes, $\Delta S = 2\pi k_B mc/\hbar$, Verlinde gets $F = 2\pi k_B mbc^2 T$. With the aid of the formula for the Unruh temperature, $k_B T = \hbar a/(2\pi c)$, associated with a uniformly accelerating (Rindler) observer, Verlinde then obtains $\vec{F} = m\vec{a}$. Now in a de-Sitter space with cosmological constant $\Lambda$, the net Unruh-Hawking temperature, [35], [36], [37] as measured by a non-inertial observer with acceleration $a$ relative to an inertial observer, is

$$\tilde{T} = \frac{\hbar \tilde{a}}{2\pi k_B c^3},$$  

(11)

with [38]

$$\tilde{a} = \sqrt{a^2 + a_0^2} - a_0,$$  

(12)

where $a_0 \equiv \sqrt{\Lambda/3}$. Hence the entropic force (in de-Sitter space) is given by the replacement of $T$ and $a$ by $\tilde{T}$ and $\tilde{a}$ respectively, leading to

$$F = m\left[\sqrt{a^2 + a_0^2} - a_0\right].$$  

(13)

For $a \gg a_0$, we have $F/m \approx a$ which gives $a = a_N \equiv GM/r^2$, the familiar Newtonian value for the acceleration due to the source $M$. But for $a \ll a_0$, $F \approx m a^2/2 a_0$, so the terminal velocity $v$ of the test mass $m$ in a circular motion with radius $r$ should be determined from $ma^2/(2a_0) = mv^2/r$. In this small acceleration regime, the observed flat galactic rotation curves ($v$ being independent of $r$) now require $a \approx (2a_N a_0^3/\pi)^{1\over 4}$. But that means $F \approx m\sqrt{a_N a_0}$. This is the celebrated modified Newtonian dynamics (MoND) scaling [39], [40], [41] discovered by Milgrom who introduced the critical acceleration parameter

$$a_c = a_0/(2\pi) = cH/(2\pi)$$  

(14)

by hand to phenomenologically explain the observed flat galactic rotation curves. Thus, we have recovered MoND with the correct magnitude for the critical galactic acceleration parameter $a_c \sim 10^{-8} \text{cm/s}^2$. From our perspective, MoND is a classical phenomenological consequence of quantum gravity (with the $\hbar$ dependence in $T \propto \hbar$ and $S \propto 1/\hbar$ cancelled out in the product $TS$ for the entropic force). [15] As a bonus, we have also recovered the observed Tully-Fisher relation ($v^4 \propto M$).

Having generalized Newton’s 2nd law, we [16] can now follow the second half of Verlinde’s argument [15] to generalize Newton’s law of gravity $a = GM/r^2$. Verlinde derives Newton’s law of gravity by considering an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature $T$, and by invoking the equipartition of energy $E = 1/2 N k_B T$ with $N = A c^3/(Gh)$ being the total number of degrees of freedom (bits) on the screen, as well as the Unruh temperature formula $k_B T = \hbar a/(2\pi c)$, and the fact that $E = Mc^2$. The generalization of Newton’s law of gravity (for the case of de-Sitter space) is obtained by the replacement of $T$ and $M$ by $\tilde{T}$ and $\tilde{M}$ respectively, so that we get

$$2\pi k_B \tilde{T} = G \tilde{M}/r^2,$$  

(15)

where

$$\tilde{M} = M + M_d$$  

(16)
represents the total mass enclosed within the volume \( V = 4\pi r^3/3 \), with \( M_d \) being some unknown mass, i.e., dark matter. For \( a \gg a_0 \), consistency with the Newtonian force law \( a = a_N \) implies \( M_d \approx 0 \). But for \( a \ll a_0 \), consistency with the condition \( a \approx (2a_N a_0^3/\pi)^{1/3} \) requires

\[
M_d \approx \frac{1}{\pi} \left( \frac{a_0}{a} \right)^2 M \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r.
\]

This yields the dark matter mass density \( \rho_d \) profile given by \( \rho_d(r) \sim M^{1/2}(r_v)(\sqrt{\Lambda}/G)^{1/2}/r^2 \), for an ordinary (visible) matter source of radius \( r_v \) with total mass \( M(r_v) \).

Thus dark matter indeed exists! And the MoNDian force law derived above, at the galactic scale, is simply a manifestation of dark matter! \[16, 42\] Dark matter of this kind can behave as if there is no dark matter but MoND. Therefore, we used to call it “MoNDian dark matter” which, to some people sounds like an oxymoron. Now we call it “modified dark matter”. Note that the dark matter profile we have obtained relates, at the galactic scale, dark matter (\( M_d \)), dark energy (\( \Lambda \)) and ordinary matter (\( M \)) to one another.

5. Observational Tests of MDM

In order to test MDM with galactic rotation curves, we fit computed rotation curves to a selected sample of Ursa Major galaxies given in [43]. The sample contains both high surface brightness (HSB) and low surface brightness (LSB) galaxies. The rotation curves, predicted by MDM as given above by

\[
F = m\left[\sqrt{a^2 + a_0^2} - a_0\right] = ma_N\left[1 + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right],
\]

along with \( F = mv^2/r \) for circular orbits, can be solved for \( a(r) \) and \( v(r) \). We [18] fit these to the observed rotation curves as determined in [43], using a least-squares fitting routine. As in [43], the mass-to-light ratio \( M/L \), which is our only fitting parameter for MDM, is assumed constant for a given galaxy but allowed to vary between galaxies. Once we have \( a(r) \), we can find the MDM density profile by using \( M_d \approx \frac{1}{\pi} \left( \frac{\sigma}{a} \right)^2 M \) to give \( \rho_d(r) = \left( \frac{\rho}{\sigma} \right)^2 \frac{d}{dr} \left( \frac{M}{r^3} \right) \).

Rotation curves predicted by MDM for NGC 4217, a typical HSB galaxy, and NGC 3917, a typical LSB galaxy in the sample are shown in Fig. 1 and Fig. 2 respectively. (See Ref. [18] for the rotation curves for the other 28 galaxies.)

In these figures, observed rotation curves are depicted as filled circles with error bars, and for the two curves at the bottom, the dotted and dashed-dotted lines show the stellar and interstellar gas rotation curves, respectively. The solid lines and dashed lines are rotation curves predicted by MDM and the standard cold dark matter (CDM) paradigm respectively. For the CDM fits, we use the Navarro, Frenk & White (NFW) [44] density profile, employing three free parameters (one of which is the mass-to-light ratio.) It is fair to say that both models fit the data well;

---

3 Actually the two acceleration limits have little to say about the intermediate regime; thus we expect that a more generic dark mass profile is of the form \( M_d = \left[ \xi \left( \frac{a}{a_0} \right) + \frac{1}{\pi} \left( \frac{a}{a_0} \right)^2 \right] M \) with positive parameter \( \xi \sim 1 \). See discussions in the next section about the dark matter mass profile.

4 This result can be compared with the distribution associated with an isothermal Newtonian sphere in hydrostatic equilibrium (used by some dark matter proponents): \( \rho(r) = \sigma(r^2 + r_0^2)^{-1} \). Asymptotically the two expressions agree with \( \sigma \) identified as \( \sim M^{1/2}(r_v)(\sqrt{\Lambda}/G)^{1/2} \).
but while the MDM fits use only 1 free parameter, for the CDM fits one needs to use 3 free parameters. Thus the MDM model is a more economical model than CDM in fitting data at the galactic scale.

Shown in Fig. 3 and Fig. 4 are the dark matter density profiles predicted by MDM (solid lines) and CDM (dashed lines) for the HSB galaxy NGC 4217 and the LSB galaxy NGC 3917 in the sample respectively. (See Ref. [18] for details.)

To test MDM with astronomical observations at a larger scale, we compare dynamical and observed masses in a large sample of galactic clusters.

\[ M_d = \frac{1}{\pi} \left( \frac{\alpha}{a_0} \right)^2 M \]

5 We should point out that the rotation curves predicted by MDM and MOND have been found [18] to be virtually indistinguishable over the range of observed radii and both employ only 1 free parameter.

6 The comparison is made in some unpublished work by D. Edmonds et al. [18]
when $a \gg a_0$. For the more general profile, the entropic force expression is replaced by

$$F = ma_N \left[ 1 + \xi \left( \frac{a_0}{a} \right) + \frac{1}{\pi} \left( \frac{a_0}{a} \right)^2 \right].$$  \quad (19)$$

Sanders [45] studied the virial discrepancy (i.e., the discrepancy between the observed mass and the dynamical mass) in the contexts of Newtonian dynamics and MOND. We [18] have adapted his approach to the case of MDM. For his work, Sanders considered 93 X-ray-emitting clusters from the compilation by White, Jones, and Forman (W JF) [46]. He found the well-known discrepancy between the Newtonian dynamical mass ($M_N$) and the observed mass ($M_{\text{obs}}$):

$$\left( \frac{M_N}{M_{\text{obs}}} \right) \approx 4.4.$$ And for the sample clusters, he found

$$\left( \frac{M_{\text{MOND}}}{M_{\text{obs}}} \right) \approx 2.1.$$

For MDM, the observed (effective) acceleration is given by $a_{\text{obs}} = \sqrt{a^2 + a_0^2} - a_0$. Using the more general expression for the MDM profile, we have

$$a_{\text{obs}} = GM_{\text{MDM}}/r^2 \left\{ 1 + \xi \left( \frac{a_0}{a} \right) + \frac{1}{\pi} \left( \frac{a_0}{a} \right)^2 \right\}.$$ Recalling that $a_{\text{obs}} = GM_N/r^2$ for Newtonian dynamics, we get

$$M_{\text{MDM}} = \frac{M_N}{1 + \xi \left( \frac{a_0}{a} \right) + \frac{1}{\pi} \left( \frac{a_0}{a} \right)^2},$$  \quad (20)

for the dynamical mass for MDM, using $\xi$ as a universal fitting parameter. With $\xi \approx 0.5$, we get

$$\left( \frac{M_{\text{MDM}}}{M_{\text{obs}}} \right) \approx 1.0.$$ In Fig. 5 and Fig. 6 we show the MOND and MDM dynamical masses respectively against the total observed mass for the 93 sample clusters compiled by WJF. The virial discrepancy is eliminated in the context of MDM! Recalling that Sanders found $\left( M_{\text{MOND}}/M_{\text{obs}} \right) \approx 2.1$, we conclude that, at the cluster scale, MDM is superior to MOND.

For completeness we mention that previously we have used $\xi = 0$ when fitting galactic rotation curves. But since now the galaxy cluster sample in our current study implies $\xi \approx 0.5$, we (in unpublished work [18]) refit the galaxy rotation curves using $\xi = 0.5$ and find the fits are nearly identical with a reduction in mass-to-light ratios of about 35%.
6. The Dark Sector and Infinite Statistics

What is the essential difference between ordinary matter and dark energy from our perspective? To find that out, let us recall our discussions in Section 2, and liken the quanta of dark energy to a perfect gas of $N$ particles obeying Boltzmann statistics at temperature $T$ in a volume $V$. For the problem at hand, as the lowest-order approximation, we can neglect the contributions from matter and radiation to the cosmic energy density for the recent and present eras. Thus let us take $V \sim R_H^3$, $T \sim R_H^{-1}$, and $N \sim (R_H/l_P)^2$. A standard calculation (for the relativistic case) yields the partition function $Z_N = (N!)^{-1} (V/\lambda^3)^N$, where $\lambda = (\pi)^{2/3}/T$. With the free energy given by $F = -T \ln Z_N = -NT [\ln (V/\lambda^3) + 1]$, we get, for the entropy of the system,

$$S = -(\partial F/\partial T)_{V,N} = N [\ln (V/\lambda^3) + 5/2]. \quad (21)$$

The important point to note is that, since $V \sim \lambda^3$, the entropy $S$ in Eq. (21) becomes nonsensically negative unless $N \sim 1$ which is equally nonsensical because $N$ should not be too different from $(R_H/l_P)^2 \gg 1$. But the solution is obvious: the $N$ inside the log in Eq. (21) somehow must be absent. Then $S \sim N \sim (R_H/l_P)^2$ without $N$ being small (of order 1) and $S$ is non-negative as physically required. That is the case if the “particles” are distinguishable and nonidentical! For in that case, the Gibbs $1/N!$ factor is absent from the partition function $Z_N$, and the entropy becomes $S = N [\ln (V/\lambda^3) + 3/2]$.

Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor (recall that the Fermi statistics and Bose statistics give similar results as the conventional Boltzmann statistics at high temperature) is infinite statistics (sometimes called “quantum Boltzmann statistics”) [19, 20, 21]. Thus we have shown that the “particles” constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics [23].

To show that the quanta of modified dark matter also obey this exotic statistics, we [17] first reformulate MoND via an effective gravitational dielectric medium, motivated by the analogy between Coulomb’s law in a dielectric medium and Milgrom’s law for MoND. Ho, Minic and I then find that MONDian force law is recovered if the quanta of MDM obey infinite statistics.

What is infinite statistics? Succinctly, a Fock realization of infinite statistics is provided by a $q = 0$ deformation of the commutation relations of the oscillators: $a_k a_k^\dagger - q a_k^\dagger a_k = \delta_{kl}$ with $q$ between -1 and 1 (the case $q = \pm 1$ corresponds to bosons or fermions). States are built by acting on a vacuum which is annihilated by $a_k$. Two states obtained by acting with the $N$ oscillators in different orders are orthogonal. It follows that the states may be in any representation of the permutation group. The statistical mechanics of particles obeying infinite statistics can be obtained in a way similar to Boltzmann statistics, with the crucial difference that the Gibbs $1/N!$ factor is absent for the former. Infinite statistics can be thought of as corresponding to the statistics of identical particles with an infinite number of internal degrees of freedom, which is equivalent to the statistics of nonidentical particles since they are distinguishable by their internal states.

It has been shown that a theory of particles obeying infinite statistics cannot be local [22, 21]. For example, the expression for the number operator,

$$n_i = a_i^\dagger a_i + \sum_k a_k^\dagger a_k + \sum_l \sum_k a_k^\dagger a_i a_i a_k a_l + ..., \quad (22)$$

Using the Matrix theory approach, Jejjala, Kavic and Minic [37] have also argued that dark energy quanta obey infinite statistics.
is both nonlocal and nonpolynomial in the field operators, and so is the Hamiltonian. The lack of locality may make it difficult to formulate a relativistic version of the theory; but it appears that a non-relativistic theory can be developed. Lacking locality also means that the familiar spin-statistics relation is no longer valid for particles obeying infinite statistics; hence they can have any spin. Remarkably, the TCP theorem and cluster decomposition have been shown to hold despite the lack of locality [21].

According to the holographic principle, the number of degrees of freedom in a region of space is bounded not by the volume but by the surrounding surface. This suggests that the physical degrees of freedom are not independent but, considered at the Planck scale, they must be infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. Since the holographic principle is believed to be an important ingredient in the formulation of quantum gravity, the lack of locality for theories of infinite statistics may not be a defect; it can actually be a virtue. Perhaps it is this lack of locality that makes it possible to incorporate gravitational interactions in the theory. Quantum gravity and infinite statistics appear to fit together nicely, and nonlocality seems to be a common feature of both of them [23].

7. Summary and Conclusion

The dark sector in the concordant model of cosmology \( \Lambda \)CDM has two components: dark energy and dark matter. We have argued that quantum fluctuations of spacetime give rise to dark energy in the form of an effective cosmological constant \( \Lambda \) of the correct magnitude as observed – a result also expected for the present and recent cosmic eras in (generalized) unimodular gravity and causal-set theory. In a spacetime with positive \( \Lambda \), gravitational thermodynamics arguments then show that dark matter (i.e., modified dark matter) necessarily exists whose mass profile is intimately related to \( \Lambda \) and ordinary matter, with an emergent acceleration parameter related to \( \Lambda \) and the Hubble parameter \( H \), of the magnitude required to explain flat galactic rotation curves. Thus the dark sector in our Universe may indeed have its origin in quantum gravity.

Pursuing this line of argument further, we find that the quanta of the dark sector appear to obey an unfamiliar statistics, viz, infinite statistics (or quantum Boltzmann statistics). This indicates that the dark sector is made up of extended quanta. As a result, we expect novel particle phenomenology for interactions involving dark matter, thereby “explaining” why so far dark matter detection experiments have not yet convincingly detected dark matter. The extended nature of the MDM quanta may also explain why the mass profile of MDM depends on such global aspects of spacetime as \( \Lambda \) and \( H \).

MDM has passed observational tests at both the galactic and cluster scales. We can also mention that preliminary examinations have demonstrated (see Ref. [16]) that the cosmology with MDM is well described by the usual Friedmann’s equations. We anticipate that this fact will allow MDM to predict the correct cosmic microwave background (CMB) spectrum shapes as well as the alternating peaks. And as briefly explained in Ref. [19], the MDM mass distribution as found appears to be consistent with the observed strong gravitational lensing.

We conclude by listing a few items on our lengthy to-do list. We plan to study concrete constraints from gravitational lensing and the bullet cluster on MDM. And we would like to answer these questions: Can we distinguish MDM from CDM? How strongly coupled is MDM to baryonic matter? How does MDM self-interact? We will also test MDM at cosmic scales by studying the acoustic peaks in the CMB and by doing simulations of structure formation. Last but not least, if the quanta of MDM indeed obey infinite statistics as we found, can quantum
gravity be the origin of particle statistics and can the underlying statistics be infinite statistics such that ordinary particles obeying Bose or Fermi statistics are actually some sort of collective degrees of freedom of more fundamental entities obeying infinite statistics? And if so, what are the implications for grand unification?

Acknowledgments
This talk is partly based on work done in collaboration with (the late) H. van Dam, S. Lloyd, M. Arzano, T. Kephart, C. M. Ho, D. Minic, D. Edmonds, D. Farrah, and T. Takeuchi. I thank them all. The work reported here was supported in part by the US Department of Energy, the Bahnsen Fund, and the Kenan Professorship Research Fund of UNC-CH.

References
[1] Ng Y J and van Dam H 1994 Mod. Phys. Lett. A 9 335
[2] Ng Y J and van Dam H 1995 Mod. Phys. Lett. A 10 2801
[3] Karolyhazy F 1966 Il Nuovo Cimento A 42 390
[4] 't Hooft G 1993 Dimensional Reduction in Quantum Gravity (Preprint gr-qc/9310026)
[5] Susskind L 1995 J. Math. Phys. 36 6377
[6] Ng Y J 2002 Int. J. Mod. Phys. D 11 1585
[7] Lloyd S and Ng Y J 2004 Scientific American 291 #5, 52
[8] Ng Y J 2008 Entropy 10 441
[9] Ng Y J and van Dam H 1990 Phys. Rev. Lett. 65 1972; 2001 Int. J. Mod. Phys. D 10 49
[10] Ng Y J 2003 Mod. Phys. Lett. A 18 1073
[11] van der Bij J J, van Dam H and Ng Y J 1982 Physica A 116 307
[12] Henneaux M and Teitelboim C 1989 Phys. Lett. B 222 195
[13] Sorkin R D 1991 Relativity and Gravitation: Classical and Quantum ed J. C. D’Olivo et al (Singapore: World Scientific); 1997 Int. J. Th. Phys. 36 2759
[14] Jacobson T 1995 Phys. Rev. Lett. 75 1260
[15] Verlinde E 2011 JHEP 1104 029
[16] Ho C M, Minic D and Ng Y J 2010 Phys. Lett. B 693 567
[17] Ho C M, Minic D and Ng Y J 2012 Phys. Rev. D 85 104033
[18] Edmonds D, Farrah D, Ho C M, Minic D, Ng Y J and Takeuchi T 2014 ApJ 793 41; 2016 Preprint arXiv:1601.00662 [astro-ph.CO]; and 2015 unpublished work
[19] Doplicher S, Haag R and Roberts J 1971 Commun. Math. Phys. 23 199; 1974 Commun. Math. Phys. 35 49
[20] Govorkov A B 1983 Theor. Math. Phys. 54 234
[21] Greenberg O W 1990 Phys. Rev. Lett. 64 705
[22] Fredenhagen K 1981 Commun. Math. Phys. 79 141
[23] Ng Y J 2007 Phys. Lett. B 657 10
[24] Perlman E S et al 2015 ApJ 805 10
[25] Perlman E S et al to appear in Proc. 14th Marcel Grossmann Meeting on General Relativity (July 2015 Rome) ed R Ruffini et al (Singapore: World Scientific)
[26] Ng Y J 2005 Quantum Foam and Quantum Gravity Phenomenology (Preprint arXiv: gr-qc/0405078)
Proc 40th Karpacz Winter School on Theoretical Physics (“Planck Scale Effects in Astrophysics and Cosmology”), Lect. Notes Phys. 669 321 ed. J. Kowalski-Glikman and G. Amelino-Camelia (Berlin Heidelberg: Springer)
[27] Ng Y J and van Dam H 2000 Phys. Lett. B 477 429
[28] Margolus N and Levitin L B 1998 Physica (Amsterdam) D 120 188
[29] Arzano M, Kephart T W and Ng Y J 2007 Phys. Lett. B 649 243
[30] Baum E 1983 Phys. Lett. B 133 185
[31] Hawking S W 1984 Phys. Lett. B 134 403
[32] Adler S L 1982 Rev. Mod. Phys. 54 729
[33] Ng Y J and van Dam H 2001 Int. J. Mod. Phys. D 10 49
[34] Bekenstein J D 1973 Phys. Rev. D 7 2333
[35] Unruh W G 1976 Phys. Rev. D 14 870
[36] Davies P C W 1975 J. Phys. A 8 609
[37] Hawking S W 1975 Comm. Math. Phys. 43 199
[38] Deser S and Levin O 1997 Class. Quant. Grav. 14 L163
[39] Milgrom M 1983 Astrophys. J. 270 365, 371, 384
[40] Famaey B and McGaugh S S 2011 Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions (Preprint arXiv:1112.3960)
[41] Milgrom M 1999 Phys. Lett. A 253 273
[42] For an earlier attempt to relate MoND scaling to dark matter, see Kiplinghat M and Turner M S 2002 Astrophys. J. 569 L19
[43] Sanders R H and Verheijen M A W 1998 ApJ 503 97
[44] Navarro J F, Frenk C S and White S D M 1996 ApJ 462 563
[45] Sanders R H 1999 ApJ Lett. 512 L23
[46] White D A, Jones C and Forman W 1997 MNRAS 292 419
[47] Jejala V, Kavic M and Minic D 2007 Adv. High Energy Phys. 2007 21586
[48] Blanchet L Preprint arXiv:astro-ph/0605637
[49] Ng Y J, Edmonds D, Farrah D, Minic D, Takeuchi T and Ho C M 2016 Modified Dark Matter (Preprint arXiv:1602.00055) to appear in Proc. 14th Marcel Grossmann Meeting on General Relativity (July 2015 Rome) ed R. Ruffini et al (Singapore: World Scientific)