Wave-particle duality is one of the most intriguing features in quantum physics. A well-known gedanken experiment that provides evidence for this is the Wheeler’s delayed-choice experiment based on a Mach-Zehnder interferometer, and many different versions of delayed-choice experiments have been conducted with both classical and quantum detecting devices. A recent proposal suggests that the delayed-choice experiment can be considered in the perspective of device-independent causal model. Here, we experimentally realize this modified delayed-choice experiment with a deterministic high purity single-photon source, which plays a key role in the Wheeler’s delayed-choice experiment. We examine that any two-dimensional nonretrocausal classical model can be excluded in a device-independent manner through our experimental results based on the violation of the dimension witness inequality. We also quantify the degree of retrocausality in our experiment and the results show that the amount of violating dimension witness inequality can be connected to the degree of quantum wave-particle superposition.

Introduction.—In the heart of quantum mechanics, there lies the wave-particle duality. Young’s double-slit interfering experiment is a celebrated example in which the concept of duality plays an important role in the famous Bohr-Einstein debate and prompted Bohr to formulate the complementarity principle. Bohr’s complementarity principle states that a single quantum object can behave as a wave or as a particle depending on the measurement apparatus. However, there also exists an alternative view of complementarity, assuming that the particle somehow knows the type of detecting device and adjust its own behavior before entering the apparatus. To examine this idea, Wheeler proposed the delayed-choice gedanken experiment where the choice of which property will be observed is made after the photon has passed the first beamsplitter of a Mach-Zehnder interferometer: “Thus one decides the photon shall have come by one route or by both routes after it has already done its travel”.

Since the Wheeler’s delayed-choice experiment (WDCE) was proposed, many modified versions of this experiment have been conducted, the original version of this experiment was first realized using a fast electronic device. Then, a quantum version of the delayed-choice experiment (QDCE) was proposed by Ioniicioiu and Terno (IT) in a particular wave-particle objective model, they suggested that the second beamsplitter in WDCE is replaced by a quantum controlled beamsplitter that can be in a superposition of being present or absent until after the photon is detected. The questions whether the beam splitter was truly in a quantum superposition state motivated entanglement-assisted QDCE which rely on the violation of a Bell inequality to rule out the IT model in a device-independent (DI) manner. What’s more, the work by Tang et al. observed the quantum wave-particle superposition through the interference fringes directly and illustrated the quantum wave-particle superposition state is distinct from the classical mixture state because of quantum interference between the wave and particle states.

Recently, a new proposal based on causal model demonstrated that a two-dimensional classical hidden variable model can explain the outcomes of WDCE and QDCE, they showed that delayed-choice experiments can be considered from the perspective of device-independent causal models in a prepare-and-measure scenario. What’s more, the proposal can exclude any two-dimensional nonretrocausal classical model in a device-independent manner based on the violation of the dimension witness inequality.

In this letter, we experimentally examine a causality assisted version of the delayed-choice experiment with a high purity deterministic single-photon source generated from hBN color-center defect. Single photon source actually is a very important resource in the delayed-choice experiment which can exclude the multi-photon pair events. With the help of our high purity single-photon source, we can exclude any two-dimensional nonretrocausal classical model in a device-independent manner through our experimental results based on the violation of the dimension witness inequality.
preparation and measurement devices are assumed to be independent, we test the dimension witness $|\text{Det}(W_2)|$ in our experiment. The results are highly robust to technical imperfections and can be used in the presence of arbitrary noise and arbitrarily low detection efficiency, which allow us to realize loophole-free experimental test of delayed-choice experiment. We can also violate the dimension witness inequality $I_{\text{DW}}$ when the preparation and measurement devices are allowed to be correlated via shared randomness. More importantly, the photons used to violate the dimension witness inequality can be used to perform the Hanbury-Brown and Twiss (HBT) \[22\] experiment at the same time, which shows the wave and particle nature simultaneously. At last, we analysis the relationship between the dimension witness inequality and the wave-particle superposition. We quantify the degree of retrocausality in our experiment and show that the amount of violating dimension witness inequality can be used to quantify the quantum wave-particle superposition state.

**FIG. 1**: The causal model and prepare-and-measure scenario for the delayed-choice experiment. (a) The DAG representation of the causal structures for the delayed-choice experiment. Under the assumption of non-retrocausality, the variables $Y$ and $\Lambda$ should be statistically independent. When the retrocausality is allowed, there will be causal influence between the variables $Y$ and $\Lambda$. (b) Device-independent scenario for testing the delayed-choice experiment. When button $X$ is pressed, the state preparator emits a particle in a state $\rho(X)$, and it will be sent to a measurement device. When button $Y$ is pressed, the device performs measurement on the particle and the measurement produces outcome $D$.

**Brief reviewing the causal model.**—As shown Fig. 1(a), the causal relationships between $n$ random variables ($X_1,...,X_n$) can be graphically described by directed acyclic graphs (DAGs), each node in the graph represents a variable and each directed edge represents a causal relation between two variables. The causal model in Fig. 1(a) implies that any observed distribution compatible with it should factorize as $p(d|x,y) = \sum_{\lambda} p(d|x,y)p(\lambda|x)$. Under the assumption of non-retrocausality, the variables $Y$ and $\Lambda$ should be statistically independent. When the retrocausality is allowed, there will be causal influence between the variables $Y$ and $\Lambda$. The classical causal model can be ruled out in a device-independent manner in the prepare-and-measure scenario (Fig. 1(b)). When pressing button $X$, the state preparator emits a particle in the state $\rho(x)$. In order to perform the witness, the emitted particles are sent to the measurement device. When button $Y$ is pressed, the measurement device performs measurements on the incoming particles and the measurement produces the outcomes. The experiment is thus described by the probability distribution $p(d|x,y)$, giving the probability of obtaining an outcome when measurement is performed on the prepared state $\rho(x)$.

**Brief introducing the single photon source.**—The deterministic single-photon source (SPS) in our experiment is fabricated based on the $N_BV_N$ color-center defect in hexagonal boron nitride (hBN) flakes, and could maintain the stable quantum emission performance at room temperature due to the large band gap $\sim$6 eV of hBN and the deep energy level of the $N_BV_N$ defect \[23\]. To fabricate the hBN SPS, firstly the bulk hBN crystal (HQ Graphene) is prepared as the hBN flake by mechanical exfoliation onto the silicon wafer with the 285-nm SiO$_2$ layer; Then the hBN sample is irradiated by the 3 keV nitrogen ion with $10^{14}$ ion cm$^{-2}$ to generate the $N_BV_N$ defect; Finally the hBN sample is annealed for 30 min at 850 °C under 0.5 Torr of argon to enhance the color center stability and reduce the background fluorescence \[24\]. The SPS in the hBN sample after above processing is searched for by a home-made confocal microscopy, where the 532-nm continuous laser is used for excitation, the NA=0.9 objective lense is used to focus the excitation laser and collect the sample fluorescence, the scanning galvo systems (Thorlabs, GVS012/M) is used to scan the excitation point on the hBN sample and search for the quantum-emission center.

**Experimental setup and results.**—Our experimental setup for the delayed-choice experiment is illustrated in Fig. 2. The single photons generated from hBN sample worked as photon source for our causal model demonstration, which is shown in the top platform. Before the single photons are sent into the interferometer, we initialize the polarization state of the single photon as $1/\sqrt{2}(|H\rangle + |V\rangle)$ by a polarization beam splitter (PBS) and a half wave plate (HWP) followed the fiber coupler lied at the left side of the table below. Then, a motorized phase controller, which is realized by a quart-wave plate (QWP)-sandwiched electric-controlled HWP \[25\], can finish the preparation tasks required by $x_i$, and prepare the state as $1/\sqrt{2}(|H\rangle + e^{i\phi_x}|V\rangle)$. The PBS, assisted with a HWP1 at 22.5°, exists in the Mach-Zehnder (MZ) interferometer that build a quantum random switch (QRS) which can randomly decide which measurements ($y_1$ or $y_2$) that the single photon will be performed. The randomness of the choice can be guaranteed by a 2 m MZ interferometers. Since the lifetime of single photon has 1.693±0.121 ns, which indicated it will collapse after propagating around 0.5 m. Therefore, 1 m light path is long enough to make the wave-package of photon completely pass the QRS before doing the decision of which measurement and detecting. An attenuator lies on one
The path of the interferometer is used to adjust the transmittance $T_a$ (or $T_b$) artificially. The phase plate (PP) in each measurement arm can compensate the non-ideal phase disturbance induced by optical elements. Another motorized phase controllers in each measurement arm need to prepare 4 initial states and perform 2 measurements. The matrix of interest is given by

$$W_2 = \begin{pmatrix}
p(0,0) - p(1,0) & p(2,0) - p(3,0) \\
p(0,1) - p(1,1) & p(2,1) - p(3,1)
\end{pmatrix},$$

we can find that $|\text{Det}(W_2)| = 0$ for the two dimensional classical hidden variable theory. In our experiment, we allow to manipulate the transmittance of one path in the interferometer ($T_a$), and then statistics is given by $p(x, y) = 1/(4(T_a^2 + 1) + (T_a/2)\cos(\varphi_x - \sigma_y))$. The experimental results of the dimension witness is shown in Fig. 4(a). For testing the dimension witness $|\text{Det}(W_2)|$, we prepare 4 initial states $\{\varphi_x, i = 1 \sim 4\}$ and perform 2 measurements $\{\sigma_y, i = 1 \sim 2\}$. Here, we choose $\varphi_x = \{0, \pi, -\pi/2, \pi/2\}$, and $\sigma_y = \{\pi/2, 0\}$ respectively. Since the nature of this dimension witness method, we can obtain the violation under any non-zero transmittance $T_a$ (and also for $T_b$ and detection efficiency $\eta$). Through varying $T_a$, we find as long as $T_a$ is larger than zero, dimension witness $|\text{Det}(W_2)| > 0$ always exist. Especially, when $T_a = 1$, we can get the maximal violation $|\text{Det}(W_2)| = 1.073 \pm 0.085$. Therefore, we can see that the prepare-and-measure (PAM) scenario can rule out
the classical hidden variable model robustly, and not be affected by noise and detection efficiency.

![Graphs and data](image)

**FIG. 4: Experimental results of the dimension witness tests.**
(a) Experimental results of the dimension witness $|\text{Det}(W)|^2$ when we change the real transmittance coefficients $T_k$ of one path of the MZ interferomether. (b) Results of the dimension witness $I_{DW}$ at the maximal violation setting. $\langle D_{00}\rangle, \langle D_{01}\rangle, \langle D_{10}\rangle, \langle D_{11}\rangle, \langle D_{20}\rangle$ are the theoretical and experimental values pairs of the term in the dimension witness $I_{DW}$ with maximal violation. The error bars are derived using Monte Carlo method (the same below). (c) Experimental results of the dimension witness $I_{DW}$ with fixed preparation $(x_i)$ and different measurement settings. The semi-transparent bar with the gray edge lies between the black-edge bar denote the error range of the data.

When the preparation and measurement devices are allowed to be correlated via shared randomness, we can also violate the dimension witness $I_{DW}$. We employ the dimension witness inequality [21]

$$I_{DW} = \langle D_{00}\rangle + \langle D_{01}\rangle + \langle D_{10}\rangle - \langle D_{11}\rangle - \langle D_{20}\rangle \leq 3. \quad (3)$$

The experimental results of the dimension witness $I_{DW}$ is shown in Fig. 4(b) and Fig. 4(c). In Fig. 4(c), we fix the 3 preparations $x_i$ as $\pi/4, 3\pi/4, -\pi/2$ respectively, and change the measurement $Y1$ and $Y2$. We select totally 25 examples from $\sigma_{g1} \in [0, \pi/2]$ and $\sigma_{g2} \in [0, \pi]$, and it can be found that only in some measurement pairs, $I_{DW}$ can be violated. The maximal violation is obtained when we set $\sigma_{g1} = \pi/2$ and $\sigma_{g2} = 0$. In this case, we can get $I_{DW} = 3.869 \pm 0.128$, and it is violated by 6.7 standard deviations. The data for each value of $\langle D_{ij}\rangle$ contained in dimension witness $I_{DW}$ is shown Fig. 4(b).

Besides, the possible of retrocausality should be allowed when the above violated results are required to simulated by binary classical hidden variable model. In Ref. [19], the authors proposed a retrocausality quantifier which has a direct relationship with $I_{DW}$. This degree of retrocausality $R$ measurement is given by

$$\min R_{\rightarrow \Lambda} = \max\left[\frac{1}{4} - \frac{3}{4}, 0\right]. \quad (4)$$

Thus, according to the maximal violated $I_{DW}$ obtained above, we can find the corresponding $R_{\rightarrow \Lambda} = 0.217 \pm 0.032$. The experimental results of the retrocausality $R_{\rightarrow \Lambda}$ under different initial state $x_i$ is shown in Fig. 5. In order to exhibit the impaction of the initial state to the retrocausality, we fix the measurement basis at the optimal case found in Fig. 4(c), and change all the 3 involved preparation $x_i$, $i = 1, 2, 3$. For example, when we focus on the $x_1$, we will set $x_{2,3} = 3\pi/4, -\pi/2$ respectively. Through varying each preparation individually, we can find within a complete period, the experimental values within the corresponding error indeed below the threshold 0.207.

**Discussion.**—As shown in the proposal by Ionicioiu and Terno [12], a hidden-variable theory that can explain the delayed-choice experiment must satisfy two conditions: it should reproduce the quantum mechanical statistics, and the property of being a particle or a wave is intrinsic. The recent causal model proposed by R. Chaves et al. [19] proved that the incompatibility with the quantum predictions of any hidden-variable model whereby the photon has a definite intrinsic wave or particle nature. We can take this idea a step forward by considering the amount of violating dimension witness inequality can be used to quantify the degree of quantum wave-particle superposition. This reminds us of the work by Tang et al. [18] which observed the quantum wave-particle superposition through the interference fringes directly, and now we have a chance to describe the quantum wave-particle superposition with a more rigorous mathematical tool—the dimension witness inequality. Since the violation of dimension witness inequality shows $\Lambda$ is the qubit state rather than some hidden values with classical binary dimension, and $\Lambda$ also correspond to the wave or particle...
behavior. The violation, in fact, examines the exist of the wave-particle superposition, and its quantity can therefore be considered as the degree of wave-particle superposition. As shown in Fig. 5, the retrocasuality achieves the theoretical predicted upper bound (≈ 0.207) when we varying the initial states under the optimal measurement basis. It presents the largest retrocasuality we can get in this model, and in other word, it shows the maximal degree of wave-particle superposition which this model can exhibit.

Summary.—In conclusion, we experimentally realize a causality-assisted version of the modified delayed-choice experiment with a deterministic single-photon source (SPS) based on the hBN color-center defect. The second-order-time correlation \( g^2(0) = 0.072 \pm 0.044 \) demonstrates the remarkable quantum emission performance and the high single-photon purity at room temperature. We can exclude any two-dimensional nonretrocausal classical model in a device-independent manner through our experimental results based on the violation of the dimension witness inequality. Two kinds of dimension witness inequality were tested in our experiment, when the preparation and measurement devices are assumed to be independent. Our experimental results are highly robust to technical imperfections and any non-zero efficiency, which allow us to realize loophole-free experimental test of delayed-choice experiment. We can also violate the dimension witness inequality when the preparation and measurement devices are allowed to be correlated via shared randomness. At last, we analysis the relationship between the dimension witness inequality and the wave-particle superposition. We quantify the degree of retrocasuality in our experiment and show that the amount of violating of dimension witness inequality can be used to quantify the quantum wave-particle superposition state.

Note added. When we almost finished the manuscript of our experiment, we noticed similar works by E. Polino et al. [26] and H.-L. Huang et al. [27], which were carried out simultaneously and independently.

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