EMMI Rapid Reaction Task Force on
“Thermalization in Non-abelian Plasmas”

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Abstract

Recently, different proposals have been put forward on how ther-
malization proceeds in heavy-ion collisions in the idealized limit of very
large nuclei at sufficiently high energy. Important aspects of the para-
metric estimates at weak coupling may be tested using well-established
classical-statistical lattice simulations of the far-from-equilibrium gluon
dynamics. This has to be confronted with strong coupling scenarios in
related theories based on gauge-string dualities. Furthermore, closely
related questions about far-from-equilibrium dynamics arise in early-
universe cosmology and in non-relativistic systems of ultracold atoms.
These were central topics of the EMMI Rapid Reaction Task Force
meeting held on December 12-14, 2011, at the University of Heidel-
berg, which we report on.

1 Motivation

The topic of “Thermalization in expanding Non-abelian Plasmas” is one of
the most pressing issues in the physics of ultra-relativistic heavy ion colli-
sions. Heavy ion experiments involve at least initially far-from-equilibrium
systems of strongly interacting matter described by quantum chromodynam-
ics (QCD). The data suggest a rapid apparent thermalization of the produced
matter, with robust collective phenomena, whose theoretical understanding
from QCD still represents a great challenge. Non-abelian plasmas out of equilibrium are also important in the early universe. A central question there is the reheating after inflation, a period of strongly accelerated expansion which dramatically dilutes all matter and radiation. The subsequent rapid transition to a thermal Hot-Big-Bang cosmology seems to be crucial for our current understanding of the evolution of the universe.

For a sufficiently weakly coupled plasma close to equilibrium much progress has been achieved in recent years. The dynamics in this case may be efficiently described in terms of kinetic theory including elastic and inelastic processes. However, much less is known about the relevant dynamics far from equilibrium. This concerns even the (a priori) theoretically clean case of very large nuclei at sufficiently high energy. In this idealized limit, the matter formed shortly after the collision is believed to be described by the “color glass condensate”, with a characteristic “saturation” momentum scale $Q_s$ that grows with the energy and the size of nuclei. When $Q_s$ is large, the coupling constant is small. Yet the gluons at saturation appear strongly coupled because their low momentum modes have very high occupancy.

Recently, different groups have put forward scenarios of how thermalization proceeds in this framework. In contrast to earlier works, there are now detailed proposals covering the dynamics from immediately after the collision until well after the plasma becomes nearly equilibrated. Plasma instabilities, which have been extensively studied in recent years, seem to dominate the early-time dynamics. Initially they drive the system close to isotropy with a gluon density that is parametrically large when compared to a system in thermal equilibrium with the same energy density. Different scenarios of what happens next have been proposed. In one scenario, it is argued that Bjorken expansion is strong enough to prevent the system to become isotropic on short time scales. In another scenario, it is argued that scattering, while probably not sufficient to fully restore isotropy, may nevertheless maintain the system in a state of fixed anisotropy for a long time. For instance, if particle number conserving processes dominate, the over-population of low-momentum modes may lead to the dynamical generation of a Bose-Einstein condensate, corresponding to a large occupation of the zero momentum mode. This may be accompanied or preceded by the formation of turbulent cascades at intermediate stages. Since the parametric estimates that have been presented allow for different types of solutions, it is crucial to test the analytic predictions against lattice simulations of the far-from-equilibrium gluon dynamics. Here it is important that the dynamics is, to a large extent, classical, so that well-established classical-statistical simulation techniques can be employed.
The special situation of having very detailed analytical proposals and the prospect of testing them with state-of-the-art simulation techniques led us to expect that a significant advance can be achieved by bringing together leading scientists working on this very specific set of questions. Therefore, a particular emphasis was to confront weak-coupling parametric estimates with classical lattice gauge theory results for the far-from-equilibrium dynamics. This had to be compared to strong coupling scenarios in related theories based on gauge-string duality. Furthermore, closely related questions about far-from-equilibrium dynamics in early-universe cosmology and in non-relativistic systems of ultracold atoms were taken into account. This very focused meeting was organized as the first EMMI Rapid Reaction Task Force, a new instrument of the ExtreMe Matter Institute. The agenda of the meeting and the list of participants are available from the website http://www-aix.gsi.de/conferences/emmi/tnp2011/.

2 Thermalization of the quark-gluon plasma

2.1 Phenomenological perspectives

As recalled in the previous section, the problem of thermalization in ultra-relativistic heavy ion collisions proceeds from important and robust empirical evidence, obtained both from RHIC and LHC [1]:

- Matter produced in heavy ion collisions exhibits fluid behavior from very early time on (collective flow is visible in several moments of the azimuthal distributions, and is sensitive to fluctuations of the initial geometry);
- The fluid has very special transport properties, in particular a small value of the shear viscosity to entropy density ratio, $\eta/s$.

These results are usually interpreted in terms of (viscous) hydrodynamics, but this raises at least two questions. Fluid-like behavior requires some degree of local equilibration or at least isotropization. How is this achieved? The small value of $\eta/s$ points to strong interactions (short mean free path). What is the origin of these strong interactions in a system whose dynamics is governed by a coupling constant that is not very large? We shall get back to this second issue below. First we focus on the issue of local equilibrium.

Note that what the data tell us is not completely unambiguous. Thus, as was emphasized by M. Strickland [2] during the workshop, and as is well known to some hydro practitioners (see e.g. [3]), collective flow measurements provide in fact little evidence (if any) that the longitudinal pressure is identical to the transverse pressure. Let us recall that the energy momentum tensor that enters the hydrodynamical equations, $\partial_\mu T^{\mu\nu} = 0$, has, in
the local rest frame of the fluid, the generic form $T^{\mu \nu} = \text{diag}(\epsilon, P_T, P_T, P_L)$, where $P_T$ and $P_L$ are the transverse and the longitudinal pressure, respectively, and $\epsilon$ is the energy density. For massless partons, $T^{\mu \mu} = 0$ (up to corrections due to the trace anomaly, that are sizable only near $T_c$), and $\epsilon = 2P_T + P_L$. In case of complete local equilibrium, $P_T = P_L$, and $\epsilon = 3P$.

However, complete local equilibrium is not necessary to get hydrodynamic behavior. This was in particular demonstrated by R. Janik [4] who showed, within the AdS/CFT framework, that viscous hydro starts to be valid already at times where $P_L/P_\perp \sim 0.5$, that is well before $P_L/P_\perp \sim 1$.

The main message here is that we should perhaps not conclude too hastily that the data provide clear evidence for local thermal equilibrium. Some significant anisotropy of the momentum distributions (i.e., incomplete local equilibrium) may still be compatible with viscous hydrodynamic behavior. The issue of what is meant by thermalization time should perhaps be re-examined within this context. Furthermore, an interesting open question is whether there are observables sensitive to a difference between the longitudinal and the transverse pressures, $P_L$ and $P_T$?

2.2 Weak or strong coupling?

We turn now to the second empirical observation, namely the apparent smallness of $\eta/s$. This is taken as evidence for short mean free path and strong coupling. Taken to the extreme, this observation motivates the infinite coupling approach based on the AdS/CFT correspondence. This approach, reviewed by R. Janik, provides a powerful framework to derive successive viscous hydrodynamical approximations, and specify their regimes of validity. As already mentioned, a particularly striking result of such calculations is the indication that viscous hydrodynamics starts to be valid well before the pressures are equilibrated.

However, while AdS/CFT provides a very elegant framework for describing the collective, fluid-like behavior of the quark-gluon plasma, it does not answer the question of how this behavior emerges from a system of quarks and gluons whose elementary interactions are controlled by a small coupling constant. In the regimes that we are considering the strong coupling constant is not huge, typically $\alpha_s \sim 0.3 - 0.4$ at RHIC or LHC energy. Thus, although the small value of $\eta/s$ argues against the validity of strict perturbation theory, it would be surprising if weak coupling approaches would be completely misleading. For instance, such weak coupling techniques (involving appropriate resummations) provide a quantitative understanding of the thermodynamics of lattice QCD for temperatures higher than 2 to 3 $T_c$. 

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Perhaps more to the point, our present understanding of the initial stages of heavy ion collisions is based on weak coupling (for large nuclei and large energies). The initial gluon distribution is characterized by the saturation momentum $Q_s$, which grows as the energy increases and as the size of the nuclei grows. In the limit of large energies and large nuclei, $Q_s$ becomes large, and since this is the scale that controls the running coupling, $\alpha_s$ is small. However, even in this weak-coupling case the gluons at saturation appear strongly coupled because they reach high occupancies $\sim 1/\alpha_s$.

There are therefore strong motivations to pursue efforts to understand the thermalization of the quark-gluon plasma from weak-coupling considerations. The issues discussed in the remaining part of this report are to be understood within this context.

2.3 Instabilities and initial conditions

It has been recognized that the color fields that could be present in the initial stages of nucleus-nucleus collisions may exhibit instabilities that can play an important role in the early dynamics [5, 6]. As discussed by G. Moore, these instabilities, which can be of various kinds (Weibel instability, Nielsen-Olesen instability, etc.) involve primarily color magnetic fields, and their role is mainly to redistribute the momentum directions, with no change in the energy. These instabilities can isotropize the momentum distributions much more rapidly than ordinary scattering processes at sufficiently weak coupling.

Quantitative studies of the evolution at early times are typically performed either in simulations of semi-classical transport approaches such as the hard-loop framework [7] and taking into account back reactions on the momentum distribution of the hard particles [8] or in numerical simulations of the classical-statistical field theory [9, 10, 11]. Instabilities are also closely related to entropy production in the system, as was emphasized by B. Muller and A. Schafer [12, 13].

Whether momentum distributions remain isotropic or not in the presence of the strong longitudinal expansion is an important open question. In fact it was recently pointed out that, within a weak-coupling description, two scenarios could be possible [14, 15]. One in which isotropy could be maintained throughout the expansion, i.e., the anisotropy remains at most of order one. Another scenario in which the longitudinal expansion is so strong as to keep the momentum distributions anisotropic until the very late stages where thermalization occurs. In order to see whether in the idealized world of truly weak coupling, one scenario prevails over the other,
it is necessary to push the analysis beyond the simple parametric estimates. This requires not only to get the power dependence of various observables on the strong coupling $\alpha_s$, but also to get the coefficients multiplying $\alpha^n_s$. This splits into two questions: one is whether one can do this by making a careful separation of all scales in the weak-coupling limit (similar to the philosophy leading to hard-loop effective theories [7]); another is whether the same goal can maybe better achieved with methods that treat physics at more than one scale at the same time (e.g. classical-statistical lattice gauge theory [9, 10, 11], or particle plus soft-field simulations [8]). Note finally that the weak-coupling analysis of instabilities may be complicated by effects due to the overpopulation of the initial gluon distribution, while it can well be described using classical fields on the lattice, as we shall discuss below.

The physics of instabilities is influenced by the initial conditions. In particular, these determine the initial size of boost non-invariant fluctuations which are further amplified by the instability. At some point they could grow large enough for non-linear corrections to become important. These non-linearities can lead to a secondary stage of enhanced amplification of modes in a momentum region exceeding by far the initially unstable band. Nonlinear amplifications with strongly enhanced growth rates have been identified in fixed-box studies of plasma instabilities in classical Yang-Mills theory [10, 16]. Secondary instabilities have also been analyzed in longitudinally expanding systems for scalar field theories, where the evolution of the quantum theory can also be studied [17]. They could significantly speed-up the isotropization process, but their relevance depends to some extent on the initial conditions for these modes. All this leads to the important question of whether one can perform a first-principle calculation of the initial conditions that prevail at the beginning of heavy ion collisions. This is what the plasma picture is claiming to achieve as discussed by R. Venugopalan [18]: strong magnetic and electric fields are produced from highly localized sources, with the fields subsequently decaying into particles on a time scale of order $1/Q_s$.

However, one could envisage a purely partonic description, without fields, with parton-parton collisions putting gluons on-shell in a time scale of order $1/Q_s$. It would be very useful to clarify further to which degree one controls the space-time evolution of the system at the very early stages. The evolution of such a partonic system using transport theory has been discussed by C. Greiner [19, 20].
2.4 Turbulence and cascade towards the infrared

Related to the physics of instabilities is that connected with turbulence and various types of cascades in momentum space [21, 22]. These phenomena can be studied using numerical simulations of classical fields, with suitable averages over the initial conditions, as was explained by J. Berges [23, 11]. Let us recall that such classical field simulations can accurately describe the regimes of large occupation \( n(p) \ll 1 \) all the way up to the overpopulated regime \( n(p) \sim 1/\alpha_s \). They fail of course when occupations become smaller than unity, at which point quantum effects need to be taken into account. These regimes of large occupations can also be described by kinetic theory (whose validity extend toward the quantum regime), except a priori in the overpopulated regime where multiparticle processes become all of the same order of magnitude. There are, however, cases where one can resum these high order processes into an effective kinetic theory with only \( 2 \rightarrow 2 \) processes and effective (momentum dependent) vertices. In this particular case, an effective kinetic theory can also be used to describe the overpopulated regime.

In the case of scalar theory, detailed calculations can be performed in these various regimes, using both classical field simulations, as well as large-\( N \) \( 2\)PI techniques (that allow for a quantum treatment) [24]. What emerges from these calculations is that the dynamics is dominated by a dual cascade, one towards the infrared, associated to the approximate conservation of particle number, the other towards the ultraviolet, associated with approximate conservation of the energy. For overpopulated initial conditions, a Bose condensate develops and remains present in the system until eventually inelastic scattering depletes it. Exponents associated with these two cascades are under analytical control and are well reproduced in the simulations. In particular, the value of the exponent for the UV cascade can be understood as arising from the existence of the condensate leading to an effective cubic interaction.

The extension of these results to the case of non-abelian gauge theories offers a number of challenges that were very much discussed. It was emphasized by A. Mueller [25] that cascades can be very different in scalar theories and in gauge theories. In scalar theories, the fact that the interactions are nearly local in momentum space typically plays a crucial role to explain the flow of momenta. This is not the case in gauge theories (where we can go from hard to soft momenta in one collision). In the case of scalar theories, one can control to a large extent analytically what is going on. In the case of gauge theories, the rigorous determination of the scaling exponents
is complicated by questions of convergence of the relevant integrals whose scaling behavior one wants to study. Remarkably, classical Yang-Mills theory simulations starting from over-populated initial conditions indicate the same weak wave turbulence exponent as for scalars \([24]\). This is also what one gets from an analytical determination of turbulent scaling exponents ignoring questions of convergence. Although the present gauge theory results exhibit behavior reminiscent of the scalar case, more work is needed in order to get conclusive results, in particular, about the infrared behavior. Finally similar approximations as the large-\(N\) 2PI expansion, which give analytical control for the scalar case, are much more difficult to implement in gauge theories.

It should also be kept in mind that most simulations so far ignore the longitudinal expansion, which is an essential aspect of the dynamics of heavy ion collisions.

### 2.5 Bose-Einstein condensation

The standard picture of heavy ion collisions assumes that the gluons that contribute dominantly to the energy density of the produced matter are freed over a time scale of order \(\tau_0 \sim Q_s^{-1}\), and have typical transverse momenta of order \(Q_s\). Their contribution to the energy density is \(\epsilon_0 \sim Q_s^4/\alpha_s\), while their number density is \(n_0 \sim Q_s^3/\alpha_s\). One may characterize this initial distribution of gluons by the dimensionless combination \(n_0 \epsilon_0^{-3/4}\). In comparison, in an equilibrated system of gluons at temperature \(T\), \(\epsilon_{\text{eq}} \sim T^4\), \(n_{\text{eq}} \sim T^3\), so that \(n_{\text{eq}} \epsilon_{\text{eq}}^{-3/4} \sim 1\). There is therefore a mismatch, by a large factor \(\alpha_s^{-1/4} \gg 1\) (in weak coupling asymptotics, \(\alpha_s \ll 1\)), between the value of \(n\epsilon^{-3/4}\) in the initial condition and that in an equilibrated system of gluons. This mismatch can be interpreted as an “overpopulation” of the initial distribution.

Gluons in a plasma become “massive” as a result of their interactions, and the number of massive gluons can be controlled by a chemical potential. It is easy to show, however, that a chemical potential will not help decreasing the overpopulation if the system is driven to equilibrium by elastic collisions alone. In this case, Bose condensation will occur, leading to an equilibrium state in which most of the particles are to be found in the condensate, while most of the energy density remains carried by thermal particles. Note that inelastic, particle number changing processes preclude the possibility that the true equilibrium state be a Bose condensate, but it leaves open the possibility that a transient condensate develops during the evolution of the
The question of how the system evolves towards its equilibrium state was addressed in [26], using a simple kinetic equation of the form \((\partial_t + \mathbf{v} \cdot \nabla)f(k, X) = C_k[f]\), where \(C_k[f]\) is the usual collision integral for 2 to 2 processes (mean field contributions are left out in the left hand-side: such terms are responsible for the instabilities discussed earlier, and it was just assumed in [26] that their main role is to maintain some degree of isotropy). One finds then, in the small angle approximation for the collision integral, that the overpopulated plasma is driven towards Bose condensation. Remarkably, in the regime where \(f \gg 1\) (\(f \sim 1/\alpha_s\)), all dependence on \(\alpha_s\) disappears, so that the system seems strongly coupled (a property that is also manifest in the classical field description). Inelastic particle production or annihilation processes modify the collision integral, but in the overpopulated regime, the inelastic scattering does not change the basic time scales in the problem. Whether the condensate forms or not can then only be answered after a detailed numerical analysis of the solution of the transport equation. Here it would be very useful to implement suitable resummations to extend the validity of the transport equation to the overpopulated regime, similar to what is achieved with large-\(N\) 2PI techniques for the scalar case.

The dynamical formation of a condensate was demonstrated, in the case of scalar field theories, by J. Berges and D. Sexty [24] who showed that condensation occurs as a consequence of an inverse particle cascade with a universal power-law spectrum. This particle transport towards low momenta is part of a dual cascade, in which energy is also transferred by weak wave turbulence towards higher momenta as mentioned above. Condensation was also discussed by T. Epelbaum et al. [27, 28] who presented results of their simulations. Further evidence was given in the talks by I. Tkachev [29] on the early universe based on the value of the exponent for the UV cascade, and by T. Gasenzer [30] in the context of cold atoms emphasizing the role of non-trivial topological configurations for the dynamics. Whereas a Bose condensate develops and remains present in the relativistic system until eventually inelastic scattering depletes it, for the non-relativistic theory with conserved particle number no decay of the condensate due to number changing processes is observed [24].

Whether condensation can occur in non-abelian gauge theories is an issue that was very much debated. One may indeed wonder whether the notion of a condensate makes sense in the context of non-abelian gauge theories, as this is seemingly in conflict with gauge invariance. However, it was pointed out that, in analogy with the parton model (which, strictly speaking, only makes sense in the Light Cone gauge), Bose condensation
could be a phenomenon that becomes manifest only in a specific gauge (Coulomb gauge?). In other gauges, the description of the effect of the large population of low momentum modes might be more subtle. It was pointed out, in view of past experience with pion condensation, that it would be important to estimate the size of the domains where condensation could take place, since if these domains are too small, the phenomenon would have very little impact on the dynamics. The role of topological defects, which play an important role in cosmology and in the dynamics of cold atoms was also discussed. Overall, the topic has triggered many interesting discussions, which may be expected to lead to further fruitful studies.

3 Conclusions

The particular workshop format as a Rapid Reaction Task Force of the ExtreMe Matter Institute EMMI allowed for in-depth discussions of many important issues related to the far-from-equilibrium behavior of non-abelian gauge theories: plasma instabilities, various cascading phenomena, the amplification of low momentum modes leading possibly to Bose-Einstein condensation, etc. Classical-statistical lattice gauge simulations appear as an efficient tool to explore some of these questions, while analytical methods should be able to handle the weak coupling regimes, in particular the regime of large occupation numbers where strong correlations can appear. This could be confronted with the impressive progress in the understanding of the dynamics of non-trivial field theories in the infinitely strong coupling limit based on gauge-string dualities.

It is important to work out further the relevant differences and similarities between gauge and scalar degrees of freedom out of equilibrium. It is, for instance, remarkable that classical simulations starting from over-populated initial conditions indicate for Yang-Mills theory the same turbulence exponents as for scalars. The question of how to generalize the notion of Bose condensation to gauge theories and to devise suitable gauge-invariant measures of condensation is a non trivial task. Furthermore, closely related questions about far-from-equilibrium dynamics in early-universe cosmology and in non-relativistic systems of ultra-cold atoms give valuable insight and point to universal aspects of far-from-equilibrium phenomena.

Many issues remain to be explored before we have a realistic scenario of how thermalization proceeds in heavy ion collisions. Some of the phenomena expected from the weak coupling analysis may involve too long time scales to be directly relevant. The role of the longitudinal expansion needs to be
further clarified: This should allow one to distinguish between the scenario where isotropy is maintained throughout the expansion, and the situation in which the longitudinal expansion is so strong as to keep the momentum distributions anisotropic until the very late stages where complete thermalization occurs. There remains also the important question of whether one can perform a first-principle calculation of the conditions that prevail at the beginning of heavy ion collisions, as in the glasma picture.

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