Helium-cluster decay widths of molecular states in beryllium and carbon isotopes

J.C. Pei\textsuperscript{1} and F.R. Xu\textsuperscript{1,2,3,4}

\textsuperscript{1}School of Physics and MOE Laboratory of Heavy Ion Physics, Peking University, Beijing 100871, China
\textsuperscript{2}Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China
\textsuperscript{3}Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Collisions, Lanzhou 730000, China
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The experimental search for molecular-type structures in light nuclei is of increasing interest. The Ikeda diagram as a guide line reveals that cluster structures are expected to appear near decay thresholds \cite{11}. Evidences for dimers in beryllium isotopes and polymers in carbon isotopes have been summarized by von Oertzen \cite{2}. It is convincing that the candidate cluster bands should have very large moments of inertia, corresponding to molecular-like shapes. Cluster states above thresholds should be understandable though it is difficult to perform the precise measurements of cluster decay widths. The model calculations of decay widths are generally dependent on the deformations and angular momenta of nuclear states. Hence, the predictions of widths can provide useful structure information about tentative cluster states.

In $^6$Be, the rotational cluster band and its decay property have been well established based on an $\alpha$-$\alpha$ structure \cite{3}. In heavier beryllium isotopes, two $\alpha$ particles and additional valence neutrons can give rise to covalent molecular binding. Recent experiments have observed the $\alpha$+$^6$He decay in $^{10}$Be \cite{4,5} and the $^6$He+$^6$He decay in $^{12}$Be \cite{6,7}, which would be the indication for molecular states. Different molecular-type structures in $^{10}$Be and $^{12}$Be have been suggested by the antisymmetrized molecular dynamics (AMD) calculations \cite{8,9}. It is also intriguing that carbon isotopes with three $\alpha$ particles can have two different configurations: triangular and linear. For $^{13}$C and $^{14}$C, cluster bands based on the $\alpha$+$^9$Be and $\alpha$+$^{10}$Be systems have been proposed \cite{2,10,11}, respectively. The experimental study of $\alpha$ decays from excited states of $^{13,14}$C is attracting interest \cite{12,13}.

Many theoretical works based on the WKB approach have shown the successful calculations of decay life-times, e.g., \cite{14,15,16,17,18,19,20}. Combining the Bohr-Sommerfeld quantization, Buck et al. has achieved the calculations of spectra of cluster states using macroscopic cluster potentials \cite{21}. Recently, we suggested a mean-field-type cluster potential for various charged-cluster decays from the ground states (g.s.) of even-even heavy nuclei \cite{22}. In the present work, we extended our calculations by including the deformations and angular momenta of cluster states. We focus the decay properties of excited states in light nuclei, particularly in beryllium and carbon isotopes, which are attracting great interest in experiments.

The cluster potential in the quantum tunneling approach can be written as (e.g., \cite{14}),

\begin{equation}
V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2}(L + 1/2)^2,
\end{equation}

that contains the nuclear potential $V_N(r)$, the Coulomb potential $V_C(r)$ and the Langer modified centrifugal potential \cite{14}. We suggested the nuclear potential $V_N(r)$ by \cite{22}

\begin{equation}
V_N(r) = \lambda[Z_c v_p(r) + N_c v_n(r)],
\end{equation}

where $\lambda$ is the folding factor; $N_c$ and $Z_c$ are the neutron and proton numbers of the cluster, respectively; $v_n(r)$ and $v_p(r)$ are the single-neutron and -proton potentials(excluding the Coulomb potential) respectively, obtained from Skyrme-Hartree-Fock calculations with the SLy4 force \cite{23}. The Coulomb potential $V_C(r)$ is well defined physically and should not be folded. We have approximated the Coulomb potential by $V_C(r) = Z_c v_c(r)$, where $v_c(r)$ is the single-proton Coulomb potential given by the mean-field calculation. In the spherical case, the folding factor $\lambda$ is determined with the Bohr-Sommerfeld quantization condition \cite{14},

\begin{equation}
\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}}[Q_L - V(r)] = (2N+1)\frac{\pi}{2} = (G-L+1)\frac{\pi}{2},
\end{equation}

where $r_1$, $r_2$ (and $r_3$ in Eq.(5)) are the classical turning points obtained by $V(r) = Q_L$ (the decay energy). The global quantum number $G$ is estimated by the Wildermuth rule, depending on the configurations of valence.
TABLE I: The calculated α-decay widths of excited states in $^8$Be and $^{20}$Ne, with comparison with the experimental data [24].

| $J^+$ | $E_j^+$ (MeV) | $\Gamma_\alpha$(calc.) [keV] | $\Gamma_\alpha$(expt.) [keV] |
|-------|---------------|-----------------------------|-----------------------------|
| 0$^+$ | 0.00 (4, 0)   | 7.8 ev                      | 6.8±1.7 ev                  |
| 2$^+$ | 3.04 (4, 2)   | $1.6\times10^3$             | $1.5\times10^3$             |
| 4$^+$ | 11.40 (4, 4)  | $2.3\times10^3$             | $\approx3.5\times10^3$     |

$^{20}$Ne ($Q_0=-4.729$ MeV)

| $J^+$ | $E_j^+$ (MeV) | $\Gamma_\alpha$(calc.) [keV] | $\Gamma_\alpha$(expt.) [keV] |
|-------|---------------|-----------------------------|-----------------------------|
| 6$^+$ | 8.78 (8, 6)   | 0.17                        | 0.11±0.02                   |
| 8$^+$ | 11.95 (8, 8)  | 90 ev                       | 35±10 ev                    |
| 1$^-$ | 5.79 (9, 1)   | 18 ev                       | 28±3 ev                     |
| 3$^-$ | 7.16 (9, 3)   | 4.7                         | 8.2±0.3                     |
| 5$^-$ | 10.26 (9, 5)  | 54                          | 145±40                      |
| 7$^-$ | 15.37 (9, 7)  | 120                         | 110±10                      |
| 9$^+$ | 22.87 (9, 9)  | 116                         | 225±40                      |

TABLE II: The calculated $\alpha$-decay widths for the $K^\pi=0^+$ cluster band in $^{12}$Be ($Q_0=-7.413$ MeV) at the different cases of the spherical and deformed shapes.

| $J^\pi$ | $E_j^\pi$ [MeV] | $\Gamma_\alpha$(sph.) [keV] | $\Gamma_\alpha$(def.) [keV] | $\Gamma_\alpha$(z-axis) [keV] | $\Gamma_\alpha$(expt.) [keV] |
|---------|----------------|-----------------------------|----------------------------|----------------------------|-----------------------------|
| 0$^+_2$ | 6.18           | -                           | 2$^+_2$                   | 7.54 ev                    | 0.85 ev                     |
| 2$^+_2$ | 10.15          | 35 keV                      | 97 keV                     | 584 keV                    | 130 keV                     |

TABLE III: The calculated $^6$He-widths of the states belonging to the $K^\pi=0^+$ band in $^{12}$Be ($Q_0=-10.11$ MeV). The AMD predictions are also given for comparison.

| $J^\pi$ | $E_j^\pi$ [MeV] | $\Gamma_\alpha$(He)(present) [keV] | $\Gamma_\alpha$(He)(0) [keV] |
|---------|----------------|----------------------------------|-----------------------------|
| 0$^+_3$ | 10.9           | 410                              | 700                         |
| 2$^+_2$ | 11.3           | 285                              | 1                           |
| 4$^+_2$ | 13.2           | 190                              | 7                           |
| 6$^+_2$ | 16.1           | 34                               | 16                          |
| 8$^+_2$ | 20.9           | 34                               | 16                          |

nucleons [24]. The cluster decay energy from an excited state is given by,

$$Q_L^\pi = Q_0 + E_j^\pi,$$

where $Q_0$ is the decay energy from the ground state, and $E_j^\pi$ is the excitation energy of a given state with the spin $J$. The decay process can occur only if the state has a positive $Q_L^\pi$ value.

The partial decay widths can be calculated by [14],

$$\Gamma = \frac{(k^2/4\mu)\exp[-2\int_{r_1}^{r_3} dr k(r)]}{\int_{r_1}^{r_2} dr/2k(r)},$$

where $k(r) = \sqrt{\frac{2\hbar}{\mu}|Q_L^\pi - V(r)|}$ is the wave number, and $P$ is the preformation factor of the cluster. For even-even nuclei, the extreme $P=1$ assumption under using the Bohr-Sommerfeld condition can well reproduce the experimental half-lives of various cluster decays [14][22]. The decay half-life can be obtained by $T_{1/2} = \hbar n^2/\Gamma$.

In the axially deformed case, the decay width can be approximated by averaging widths at various directions of the space as follows [23]

$$\Gamma = \int_0^{\pi/2} \Gamma(\theta) \sin(\theta) d\theta.$$

To calculate the width $\Gamma(\theta)$, a deformed cluster potential has to be employed. In the present work, the deformed potential $V(r, \theta)$ is constructed with axially deformed single-particle potentials $v_n(r, z)$, $v_p(r, z)$ and $v_c(r, z)$ that are given by the shape-constrained Skyrme-Hartree-Fock calculation [24], with the folding factor determined at the spherical case. The variables $(r, z)$ are the cylindrical coordinates, and $\theta$ is the angle between the symmetry axis and the radius for the cluster emission.

The most well-established cluster structures in light nuclei are in $^8$Be and $^{20}$Ne [24], with an $\alpha$-particle coupling to the magic cores of $^4$He and $^{16}$O, respectively. Their cluster structures can be well described by the semiclassical cluster model [3][27]. For $^8$Be, we take $G=4$ for the ground-state band, according to the Wildermuth condition [24]. For $^{20}$Ne, we take $G=8$ for the $K^\pi=0^+$ band (the ground-state band) and $G=9$ for the $K^\pi=0^-$ band, as discussed in Refs. [24][27]. The calculated results are given in Table I. It can be seen that the present calculations agree well with the experimental widths within a factor of 3. $^{20}$Ne is a particularly interesting nucleus that can have very different structures for different states. While the $K^\pi=0^-$ band with the sequence of $J^\pi = 1^-, 3^-, ..., 9^-$ has an almost pure $\alpha+^{16}$O cluster structure, the $0^+$ ground-state band has a considerable mixture of the cluster structure and the deformed mean-field structure [24]. For the $0^-$ band, both experiments and our calculations give remarkable large $\alpha$-decay widths comparable with the Wigner limit [29], indicating the significant $\alpha+^{16}$O cluster structure.

For heavier beryllium isotopes, the 2$\alpha$-cluster structures play an important role. For example, the well-known parity inversion in $^{13}$Be is related to a large par-
members have been established experimentally \[5\, 30\]. It was found that a channel radius as large as \(8 \pm 1\) fm has to be adopted to reproduce the \(\alpha\)-decay width for the 7.54 MeV state \[4\], while the \(\alpha\)-width of the 10.15 MeV state is associated with a smaller channel radius of \(5 \sim 6\) fm \[3\].

From the shell-model viewpoint, the \(^{10}\text{Be}(0^+_2)\) structure has a \((sd)^2\) configuration due to the large prolate deformation \[31\]. Hence we assume a global number \(G=6\) for the decay calculation. The calculated widths in \(^{10}\text{Be}\) are displayed in Table II. It can be seen that calculations in the spherical case give much smaller \(\alpha\)-decay widths compared with experiments. As predicted by the molecular orbital model \[31\], the \(\alpha\)-\(\alpha\) distance in the second \(0^+\) state is about 4 fm. This corresponds to an axis ratio of 2.5:1 (or a prolate deformation of \(\beta_2 \approx 1.1\)). With such a large deformation employed in Eq.(6), the \(\alpha\)-width for the 10.15 MeV state can be reproduced reasonably. We found that the \(\alpha\)-width of the \(2^+(7.54\text{ MeV})\) state is sensitive to the decay energy rather than the deformation of the cluster potential (the width can be reproduced with an increase of only 50 keV in the decay energy). This can be understood considering that, for a state near the threshold, the cluster tunnelling with a very small decay energy is dominated by the long tail of Coulomb potential (the tail is not sensitive to the deformation).

Considering the \(\alpha\)\(2n\)\(\alpha\) molecular structure of the \(0^+_2\) band \[5\], we estimated the \(\alpha\)-width by assuming that the \(\alpha\) particle emits along the \(z\) axis with the \(\beta_2 = 1.1\) deformed potential. The calculated widths are given in Table II. It can be seen that the \(\alpha\)-width for the 7.54 MeV state is improved significantly. However, the \(\alpha\)-width for the \(4^+\) (10.15 MeV) state is overestimated. It was pointed out that the \(0^+_2\) band states could contain the significant mixture of the \(^5\text{He}^+\)\(^5\text{He}\) configuration \[31\]. The mixture is particularly remarkable for the 10.15 MeV state because it is almost on the \(^5\text{He}^+\)\(^5\text{He}\) decay threshold, which could result in a reduced \(\alpha\)-decay width.

For the neutron-rich \(^{12}\text{Be}\) nucleus, some resonance states were recently observed, decaying to \(\alpha^+\)\(^8\text{He}\) and \(^4\text{He}^+\)\(^8\text{He}\) channels \[6\, 7\]. A rotational band with a \(^9\text{He}^+\)\(^8\text{He}\) structure has been suggested to have a very large momentum of inertia \((\hbar^2/2\mathbf{\Omega} = 0.14\text{ MeV})\) \[5\]. The AMD calculations interpreted that resonances happen in molecular states built on the \(0^+_3\) configuration \[5\]. Considering that the highest state of this band would be the \(J^\pi = 8^+\) member \[6\, 8\], we take \(G=8\) in the \(^9\text{He}^+\)\(^8\text{He}\) decay calculation. A spherical cluster potential is assumed. In Table III, the obtained widths are compared with the AMD predictions \[5\]. As stated in \[5\], the \(^9\text{He}\)-width of the \(0^+_3\) state is very large due to the lack of centrifugal barrier. It is shown that the present widths significantly decrease with increasing angular momenta. In these states, \(\alpha^+\)\(^8\text{He}\) decays have also been observed \[7\] but their description is beyond the cluster tunnelling model because of too large decay energies.

Carbon isotopes heavier than \(^{12}\text{C}\) can have three-centre cluster structures with different geometric shapes. As suggested by von Oertzen \[10\, 11\], there are possible prolate(linear) and oblate(triangle) cluster states in carbon isotopes. The \(K^\pi = 0^+_1\) and \(3^+_1\) bands in \(^{12}\text{C}\) are considered to have triangular configurations. The \(0^+_2\) state \(^{12}\text{C}\) has been predicted to be an analogue of Bose condensate \[33\]. Recently, \(\alpha\) decays from the excited states of \(^{13}\text{C}\) and \(^{14}\text{C}\) have attracted intense experimental studies (see e.g., \[12\, 13\]).

The oblate and prolate configurations certainly lead to different calculated decay widths. As shown in Fig.1, the cluster potential radius in the oblate case \((\theta = \pi/2)\) is smaller than in the prolate case \((\theta = 0)\). Therefore, decays in oblate case would be suppressed. In \(^{12}\text{C}\), the \(4^+(14.08\text{ MeV})\) and \(3^- (9.64\text{ MeV})\) states belong to the \(K^\pi = 0^+_1\) and \(3^+_1\) oblate bands, respectively. The decay channels of these low-lying states are nearly pure \(\alpha\) decays \((\Gamma_\alpha \simeq \Gamma)\) \[28\, 34\], providing a good chance to study the deformation effect. In Table IV, with a deformed cluster potential of \(\beta_2 = 0.8\) (corresponding to an axis ratio of 2:1), the obtained \(\alpha\)-widths of the \(3^-\) and \(4^+\) states at \(\theta = \pi/2\) direction (the oblate case) agree well with the experimental widths. However, their \(\alpha\)-widths at \(\theta = 0\) direction (the linear case) are calculated to be 243 keV for the \(3^-\) state and 2.2 MeV for the \(4^+\) state, which are about one order of magnitude larger than the oblate calculations. This indicates that \(\alpha\) decays are strongly hindered in triangular configurations compared with linear configurations.

The \(\alpha\) decay from the \(0^+_2\) state \(^{12}\text{C}\) has been studied with the triple cluster model \[34\]. This state has a decay energy of 288 keV to \(^8\text{Be}(g.s.)^+\)\(\alpha\) and an energy of 380 keV to \(3\alpha\) particles. For the decay to \(^8\text{Be}^+\)\(\alpha\), the present calculation with a linear structure \((\beta_2 = 0.8)\) gives an \(\alpha\)-width of 5.9 ev that agree with the observed value of 8.5 ev. The calculations assuming a triangular configuration leads to a small width of 1.6 ev. For the decay to the \(3\alpha\) particles with a triangular structure, our model estimates a width of 36 ev that is close to the predictions of the AMD \[28\] and the triple cluster model \[34\]. However, the contribution of the direct \(3\alpha\)-decay process to the total width is less than 4% obtained in the experiment \[36\]. Hence, the linear cluster decay to \(^8\text{Be}^+\)\(\alpha\) should dominate the decay of the \(0^+_2\) state \(^{12}\text{C}\).
TABLE IV: The calculated $\alpha$-widths of excited states in carbon isotopes. The experimental total decay width $\Gamma$ are taken from $^{12,13}$C and from $^{11}$ for $^{14}$C.

| $J^\pi$ | $E^+_J$(MeV) | $(G, L)$ | $\Gamma_\alpha$(calc.) | $\Gamma$(expt.) |
|---------|--------------|----------|-----------------------|---------------|
| $^{12}$C ($Q_0 = -7.366$ MeV) |              |          |                       |               |
| 0$_2^+$ | 7.654        | (6, 0)   | 5.9 ev                | 8.5 ev        |
| 3$_1^-$ | 9.641        | (5, 3)   | 17                    | 34            |
| 4$_1^+$ | 14.08        | (4, 4)   | 158                   | 258           |
| $^{13}$C ($Q_0 = -10.647$ MeV) |              |          |                       |               |
| $K^\pi = 3/2^-$ |              |          |                       |               |
| 5/2$^-$ | 10.82        | (6, 2)   | 0.2 ev                | 24            |
| 7/2$^-$ | 12.44        | (6, 2)   | 84                    | 140           |
| 9/2$^-$ | 14.13        | (6, 4)   | 32                    | 150           |
| 11/2$^-$| 16.08        | (6, 4)   | 316                   | 150           |
| $K^\pi = 3/2^+$ |              |          |                       |               |
| 3/2$^+$ | 11.08        | (7, 1)   | 41 ev                 | $\leq$4       |
| 5/2$^+$ | 11.95        | (7, 1)   | 100                   | 500           |
| 7/2$^+$ | 13.41        | (7, 3)   | 112                   | 35            |
| 9/2$^+$ | 15.28        | (7, 3)   | 888                   |               |
| 11/2$^+$| 16.95        | (7, 5)   | 125                   | 330           |
| $^{14}$C ($Q_0 = -12.011$ MeV) |              |          |                       |               |
| 5$^-$  | 14.87        | (7, 5)   | 0.4                   | 35            |
| 6$^+$  | 16.43        | (6, 6)   | 0.12                  | 35            |
| 3$^-$  | 12.58        | (7, 3)   | 22 ev                 | 95            |
| 5$^-$  | 15.18        | (7, 5)   | 31                    | 50            |
| 7$^-$  | 18.03        | (7, 7)   | 14                    | 70            |
| 6$^+$  | 14.67        | (6, 6)   | 0.21                  | 40            |

TABLE V: The calculated $\alpha$-widths of the $^{14}$C excited states decaying to $^{10}$Be(g.s.)+$\alpha$ and $^{10}$Be$(2^+)+\alpha$ channels.

| $J^\pi$ | $^{10}$Be(g.s.) | $^{10}$Be$(2^+)$ | $^{10}$Be(g.s.) | $^{10}$Be$(2^+)$ |
|---------|----------------|-----------------|----------------|----------------|
| 18.5 MeV |                |                 |                |                |
| 4$^+$   | 1.2 MeV       | 0.8 MeV         | 2.7 MeV        |                 |
| 5$^-$   | 0.28 MeV      | 0.39 MeV        | 0.7 MeV        | 1.2 MeV        |
| 6$^+$   | 11 keV        | 30 keV          | 41 keV         | 230 keV        |
| 7$^-$   | 0.8 keV       | 2.3 keV         | 3.5 keV        | 25 keV         |
| 19.8 MeV |                |                 |                |                |

For $^{13}$C, the $K^\pi = 3/2^\pm$ bands constructed on the $\alpha+^9$Be$(3/2^-, \text{g.s.})$ structure were suggested experimentally $^{10}$. The parity doublet bands are related to the reflection asymmetric chain configurations, corresponding to a very large prolate deformation ($\hbar^2/2I = 0.19$ MeV) $^{10}$. We assume that the core (i.e., $^9$Be) has a similar deformation to that of $^{10}$Be with $\beta_2 \approx 0.6$ experimentally $^{37}$. In the $\alpha$-width calculations, we take $G=6$ and 7 for the $K^\pi = 3/2^-$ and $3/2^+$ bands, respectively, considering the different parity of the bands. The angular momenta $L$ that the $\alpha$ particle carries are obtained by the angular momentum selection among the parent, daughter and $\alpha$ particle. Furthermore, for the odd nucleus $^{13}$C, we take the $\alpha$ preformation factor $P=0.6$ that was adopted in the systematical $\alpha$-decay calculations of odd nuclei $^{14}$. The calculated $\alpha$-widths of the $K^\pi = 3/2^\pm$ band states in $^{13}$C are listed in Table IV. The experimental $\alpha$-widths have not been available, but we give the experimental total widths $\Gamma$ $^{10, 28}$ of the states for comparison.

The calculated $\alpha$-widths for $^{13}$C are in general smaller than experimental total widths as expected. However, calculated values for the 11/2$^-$ (16.08 MeV) and 7/2$^+$ (13.41 MeV) states $^{10}$ are larger than experimental total widths. In Ref. $^{28}$, the two states were assigned with (7/2$^+$) and (9/2$^-$), respectively. The recent experiment $^{12}$ suggests that the 13.41 MeV state is connected to an oblate structure and the 16.08 MeV state has a positive parity. Calculations can be changed significantly due to the different assignments of spins.

The predicted $\alpha$-widths for $^{14}$C decaying to the ground state of $^{10}$Be are also presented in Table IV. The bands built on the 0$_2^+$ (6.589 MeV) and 3$_2^-$ (9.801 MeV) states were suggested to have oblate (triangular) structures, while bands built on the 0$_3^+$ (9.746 MeV) and 1$_1^-$ (11.395 MeV) states have prolate (chain) structures $^{11}$. In Table IV, the 5$^-$ (14.87 MeV) and 6$^+$ (16.43 MeV) states have oblate structures and other states are prolate $^{11}$. The calculated width of the 3$^-$ (12.58 MeV) state is remarkably smaller than the experimental total width. This would indicate that the neutron emission channel dominates the decay of the state that is near the $\alpha$-decay threshold. The decay channel of $^{14}$C(16.43 MeV)$\rightarrow^{10}$Be$(2^+, 3.37$MeV)$+\alpha$ has also been observed though it is very weak $^{28}$. The decay width is estimated to be 5 ev, giving a small branching ratio compared with the $^{10}$Be(g.s.)+$\alpha$ decay channel.

It is interesting that, in $^{14}$C, the 18.5 MeV and 19.8 MeV states have strong channels decaying to both the ground state and the first 2$^+$ state of $^{10}$Be $^{12, 38}$. No experimental assignments of spins and parties have been available for the two states of $^{14}$C. We estimated their $\alpha$-widths with assuming $J^\pi = 5^-, 7^-(G=7)$ and $J^\pi = 4^+, 6^+(G=6)$. The calculations are performed in the simple spherical shape. As shown in Table V, the calculated $\alpha$-widths are sensitive to the assignments of angular momenta. With the spin of 4$\sim$5, broad widths are obtained for both channels as observed in experiments $^{14, 38}$.

In summary, we have calculated decay widths for the interesting cluster states in beryllium and carbon isotopes, such as the $\alpha+^6$He, $^6$He+$^6$He and $^8, ^9, ^{10}$Be+$\alpha$ systems, using a uniform folding cluster potential that has good descriptions for various cluster radioactivities in heavy nuclei. It has been shown that the model can well reproduce the $\alpha$-widths of $^8$Be, $^{12}$C and $^{20}$Ne. The discrepant decay properties of the 7.54 MeV and 10.15 MeV states in $^{10}$Be have been discussed. The $^8$He-widths of possible molecular states in $^{12}$Be and $\alpha$-widths of excited states in $^{13, 14}$C are predicted.

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