Temperature Dependent Structure Function of Nucleon

A. Bhattacharya ‡, A. Sagari, B. Chakrabarti and S. Mani

Department of Physics, Jadavpur University
Calcutta 700032, India.

PACS No.s: 12.39.-x, 12.39.Jh, 12.39.Pn, 14.20Dh,

Abstract

A relation between temperature, Fermi momentum and radius of the nucleon has been derived. The percentage increase in radii of the nucleon with temperature and the temperature dependent $F_2$ structure function of the nucleon have been investigated. An estimate of the Fermi momentum of the nucleon has been made. The results are found to be in agreement with existing theoretical and experimental suggestions.

‡ E-mail pampa@phys.jdvu.ac.in
Introduction:-

The collision of particles provide useful information about their structure and properties. This method is widely used in nuclear and high energy physics. There has been several attempts towards the understanding and estimation of the structure function of the nucleons [1]. In relativistic heavy-ion collision, high temperature may be produced by the transforation of a great deal of energy. So it is important to study the properties of hadrons at finite temperature. It has been suggested that the pion cloud contribution plays a crucial role in the long range structure of the nucleons. Due to its small mass, the pion plays a special role in the dynamics of hot hadronic matter. Therefore it is quite important to understand the temperature behaviour of the pion’s Green function. While the pion is (hadronically) stable at T=0, it is expected to develop a width (imaginary part of the Green function) at non zero temperature, such width being interpreted as a damping coefficient which should diverge at the critical temperature for de-confinement. This follows from a proposal [2,3] to consider the width of a hadron as a phenomenological order parameter for the de-confinement phase transition. In fact as the temperature is increased and the hadron melts, its width should increase until it becomes infinite at T=T_C ensuring no resonance peaks in the hadronic spectral function. The latter should become a smooth function of the energy and coincide with its perturbative QCD value. These properties have been confirmed by detailed calculation in the framework of the Linear Sigma Model [4] and the virial expansion [5]. Since, the discovery of the EMC effect [6], there have been a great number of discussions on how nuclear structure function vary with the environment. Most of these discussions relate the structure functions of a nucleon in a nucleus to the mass number. As the structure function of a nucleon plays an important role in some collision process, it is useful to discuss the structure function of a nucleon at finite temperature. Zhen et al [7] has investigated the influence of
temperature on structure function, volume energy, radius and mass of the nucleon in the context of quantum field theory.

In this present work we have derived a relation between temperature, Fermi momentum and radius of the nucleon in the framework of the Statistical Model [8]. The increase in radii of the nucleon with the increase of temperature has been estimated. The effect of increase in temperature on $F_2$ structure function of the nucleon has been studied. It has been observed that the Gaussian width of Fermi momentum $p_F=0.54$ GeV yields reasonable value of structure function as predicted by experiment [6,7].

The Model

In statistical model [8] the probability density for a nucleon is obtained as:

$$|\Psi(r)|^2 = \frac{315}{64\pi r_0^9}(r_0 - r)^{3/2}\theta(r_0 - r)$$  \hspace{1cm} (1)

where $r_0$ is the radius parameter of the nucleon and $\theta$ is usual step function. The momentum space wave function $\psi(k)$ can be represented as:

$$\psi(k) = \frac{c}{k} \int_0^\infty r\psi(r)dr.sinkr$$  \hspace{1cm} (2)

where $c$ is appropriate normalisation constant. The normalised momentum space wave function with (1) is obtained as:

$$\psi(k) = 2\sqrt{3\pi r_0}k^{-1}j_1(kr_0)$$  \hspace{1cm} (3)

It is to be noted that $\psi(k)$ depends only on the corresponding size parameter of the nucleon.

The free nucleon $F_2$ structure function in the non-linear limit runs as:

$$F(x) = \frac{M}{8\pi^2} \int_{k_{min}}^\infty |\psi(k)|^2dk^2$$  \hspace{1cm} (4)

where $M$ is the mass of the nucleon and $k_{min} = M |x - \frac{1}{3}|$.

In the statistical model [8] we come across a distribution of ensemble of identical indistinguishable Fermions (q and $\bar{q}$) as both the valance and virtual quark are treated at the same footing [8]. The distribution formula for the
Fermions, when the temperature is small, (in the first order approximation) leads to [9]

\[
n_q(r) = \frac{N}{V} = \frac{1}{\pi^2 h^3} \int_0^\infty \frac{p^2 dp}{1 + e^{(\epsilon - \mu)/T}} \tag{5}
\]

where the chemical potential \( \mu = ar \) and \( \epsilon = \frac{p^2}{2m} \), \( n_q(r) \) = number density of quarks and \( T \) is expressed as function of \( r \). The temperature gradient may be considered as a manifestation of color concentration gradient from \( r=0 \) to \( r=r_0 \) representing the non-equilibrium process in the context of the model [8]. However we have assumed that the average state is approximately described by the equations of state independent of the gradients. To make an estimate of order of the temperature \( T \), we note that at \( r=r_0 \), \( n_q(r_0)=0 \) and at \( r=0 \), \( n_q(0)=\frac{315}{64\pi r_0} \) so that the average \( \overline{n_q}=\frac{315}{128\pi r_0} \) and the average chemical potential \( \overline{\mu}=\frac{ar_0}{2} \) as \( \mu(0)=0 \) and \( \mu(r_0)=ar_0 \). Now integrating the equation (5) between \( p=0 \) to \( p=p_F \), radius of the Fermi sphere in momentum space and retaining first term we obtain:

\[
r_0^4 - \frac{6p_F^2 r_0^3}{10am} - \frac{945\pi h^3 T}{64ap_F^3} = 0 \tag{6}
\]

With \( a=0.2 \) [10], we obtain

\[
r_0^4 - 25p_F^2 r_0^3 - \frac{231.82T}{p_F^3} = 0 \tag{7}
\]

Using the value of \( p_F \) suggested by various models [11] the structure function of the nucleon has been computed. The value of \( \frac{\bar{F}^T}{\bar{F}^0} \) with \( x \) at \( T=0.5T_C \) (with \( T_C=200 \) MeV [7,12] and \( p_F=0.54 \) GeV ) has been computed and displayed in figure-I. The ratio of the structure function \( \frac{\bar{F}^T}{\bar{F}^0} \) of the nucleon corresponding to different temperature has been estimated and displayed in figure-II. The dependence of the structure function on \( p_F \) has been shown in the figure-III.
Results and Discussions:- In the present work we have investigated the influence of temperature on the radius of the nucleon and consequently the effect on the structure function has been computed. In the context of the present model the structure function has been obtained as a function of Fermi momentum. It has been observed that as the temperature increases, the radius of the hadron increases until the critical temperature but the increase is not very prominent. We have observed that at T=0.5T\_c the increase in radius of the nucleon is about 4.56 percent and decrease in structure function is 9.46 percent of that at zero temperature for x=0.5 whereas Zhen et al [7] obtained the increase in radius as 16 percent and 5 percent decrease in structure function at x= 0.3 and T=0.5T\_c. It has been observed that the ratio of the \( \frac{F_T}{F_2} \) is less than unity at comparatively large x (≥0.2) and tends to rise above unity at x (∼ 0.14) at \( p_F = 0.54 \text{GeV} \) and T = 0.5\( T_c \). It is pertinent to point out here that similar observation has been made by EMC [6] which confirmed that the nucleon structure function
for bound and quasi free behaves differently. They have observed that the structure function ratio falls below unity at $x \geq 0.25$, rises above unity at $x \sim 0.15$ and again falls below unity at small $x$. The variation of the ratio is not very prominent with the variation of the temperature whereas Fermi momentum effects the ratio to a considerable extent.

It is interesting to note that Fermi momentum $p_F=0.54$ GeV yields the percentage increase in radius and decrease in structure function from $T = 0$ reasonably. It also reproduces the ratio of the structure functions in agreement with the EMC [6] predictions. It may be mentioned that $p_F$ is a very important parameter for extraction of the value of CP-violating ratio $|\frac{V_{ub}}{V_{cb}}|$. The precise value of $p_F$ is yet to be determined. Hwang et al [11] have extracted the value as the 0.54 GeV in the context of the relativistic quark model whereas commonly used value is 0.3 GeV. We have observed that $p_F = 0.54$ GeV yields reasonable results in the context of the present model. However further investigations would be done in our future works.
Figure 1: Behaviour of $F_2(T)/F_2(0)$ with Temperature at $x = 0.5$; Line 1: $p_T = 0.25$ GeV, Line 2: $p_T = 0.35$ GeV, Line 3: $p_T = 0.54$ GeV, Line 4: $p_T = 0.3$ GeV

Figure 2:
Figure 3: Variation of $F_2$ Structure Function with Fermi momentum $p_F$ in GeV at $x = 0.5$, $T = 0.5 T_c$.

References:
[1] K.Hagiwara et al., Phys Rev. D 66(2002) 010001 ; P.V.Pobyliitsa et al., Phys. Rev. D 59(1999) 034024 ; D.Diakonov et al., Nucl.Phys. B 480(1996) 341;
[2] C.A.Dominguez and M.Loewe, Phys.Lett.B 233(1989)201; C.A.Dominguez, Nucl.Phys.B(Proc.Suppl.)15(1990)225; C.A.Dominguez and M.Loewe, Nucl.Phys.B(Proc.Suppl.)16(1991)225;
Z.Phys.C, Particles and Fields,49(1991)423;
[3] R.D.Pisarski, Phys.Lett.B 110(1982)155;
[4] C.A.Dominguez, M.Loewe and J.C.Rojas, Z.Phys.C, Particles and Fields,59(1993)63;
[5] H.Leutwyler and A.V.Smilga, Nucl.Phys.B 342(1990)302;
[6] J.Ashman et al.(EMC collaboration) Phys.Lett.B 202 (1988)603;
[7] B.P.Zhen, J.Phys.G: Nucl.Part.Phys 16 (1998)1653;
[8] A.Bhattacharyya et al., Prog.Theor.Phys. 77 16(1987), Eur.Phys.J.C. 2671(1998), Int.J.Mod.Phys. A 16201(2001); S.N.Banerjee et al., Ann.Phys.(N.Y.)150 (1983);
[9] B. Chakraborty et al., J. Phys. G 15 (1989) L 13;
[10] W. Lucha et al., Phys. Rep. 200, 4 (1991) 240;
[11] D. S. Hwang et al., Z. Phys. C 69 (1995) 112 and references therein;
[12] D. H. Boal et al., Phys. Rev. D 26 (1982) 3285;
Acknowledgement:- Authors are thankful to Department of Science and Technology (DST), New Delhi, for financial assistance.

Figure Captions:-

Figure-I : Behaviour of $\frac{F_T}{F_2}$ with $x$ at $T=0.5T_C$, $p_F=0.54$ GeV

Figure -II : Calculated values of $\frac{F_T}{F_2}$ with Temperature (T in GeV) at $x=0.5$ ; Line 1 : $p_F=0.25$ GeV , Line 2 : $p_F=0.3$ GeV , Line 3 : $p_F=0.35$ GeV, Line 4 : $p_F=0.54$ GeV

Figure -III : Variation of $F_2$ Structure Function with Fermi Momentum ($p_F$ in GeV) at $x=0.5$, $T=0.5T_C$