The Computational Model of Firing Error and Transfer Range During Naval Gun Implementing Transfer Fire at Sea Target

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Abstract. According to the characteristic for naval gun to implement transfer fire at sea target, the causes of transfer fire errors are analysed, and the computational model of transfer fire errors is put forward, then the calculating methods of transfer value and transfer range are proposed. This paper provides strong technical support for naval gun to implement transfer fire.

1. Introduction
In the process of naval battle, naval gun can implement continuous attack and defense, and it can complete the tasks of firing at sea target, defending from air attack and so on. So naval gun is one type of shipborne weapon which has strong comprehensive combat capability [1].

When naval gun is firing at sea target, if there are more than two targets to attack, the method of transfer fire is needed. The diagram of transfer fire is as Figure 1.

![Figure 1. The diagram of transfer fire.](image)

As far as transfer fire is concerned, the variation of distance and direction will bring new firing errors, and the new errors are related to angle and distance of transfer fire. In order to ensure that firing precision of transfer fire is appropriate, this paper analyses the causes of transfer fire errors, then put forward the calculating methods of transfer value and transfer range.
2. The causes of transfer fire errors

When naval gun implement transfer fire at sea target, the ballistic and meteorology factors may be considered as unchanged. To get accuracy fire adjustment, each error of ballistic and meteorology should be found out. However, we can get total fire deviation by trial fire, it is hard to isolate the certain firing error which is caused by one of ballistic and meteorology factors separately [2].

In order to simplify calculation, we may suppose that distance deviation which can be obtained by trial fire is totally caused by error of muzzle velocity [3]. And the value of distance correction can be converted to error of muzzle velocity $\Delta V_0 \%$ which is corresponding to trial fire point, then distance correction value for aiming at target can be calculated. At the same time, direction correction value can keep invariant.

There is error between distance correction value which is calculated under transfer fire principle and true distance correction value. In other words, it is distance error caused by transfer fire, and the calculating formula is as follows:

$$\Delta D_s = K_1 \cdot \Delta D_S - \Delta D_M$$ (1)

In the formula, $K_1$ is coefficient of transfer fire, $\Delta D_S$ means distance correction value which is corresponding to trial fire point, $\Delta D_M$ means distance correction value for true target.

3. The calculating model of transfer fire errors

In the process of transfer fire, transfer fire errors can be divided into distance errors and direction errors, then we will discuss these two parts of errors separately.

3.1. The calculating model of distance errors

If we want to analyze distance errors of transfer fire, the focal point is to analyze errors which are contained by correction value $\Delta D_S$ and $\Delta D_M$ [4].

3.1.1. The calculating model of correction errors for trial fire

When we conduct trial fire, distance correction value $\Delta D_S$ is including following errors:

(1) Errors of ballistic and meteorology [5]. When naval gun shoot at trial fire point, $\delta V_0$ is accidental error to confirm deviation of muzzle velocity; $\delta \rho$ is accidental error to confirm air density; $\delta W_x$ is accidental error to confirm longitudinal wind; $\delta X_{as}$ is accidental error for firing calculation.

$$\frac{\partial X}{\partial V_{0s}}$$ means the distance change corresponding to trial fire point when $\frac{\delta V_0}{1\%}$, $\frac{\partial X}{\partial \rho_s}$ means the distance change when $\frac{\delta \rho}{1\%}$, $\frac{\partial X}{\partial W_{xs}}$ means the distance change when $\delta W_x = 1\text{ m/s}$. (2)

(2) The accidental error $\delta D_{0s}$ to confirm present distance of trail fire point.

(3) When we use the deviation between mean impact point and trial fire point to replace deviation between scattering center and trail fire point, accidental error $X_c$ will emerge. It is caused by three parts of errors. The distance correction value is mean value of $N$ salvo fire deviation:

$$X_c = \frac{\sum_{i=1}^{N} (\delta X_p + \delta \theta \cdot \frac{\partial X}{\partial \theta} + \delta X_c)}{N}$$ (2)

Therefore, distance errors $\Delta D_S$ of correction value can be expressed as follows:
\[
\Delta D_s = \delta V_0 \cdot \frac{\partial X}{\partial V_{os}} + \delta \rho \cdot \frac{\partial X}{\partial \rho_s} + \delta W_x \cdot \frac{\partial X}{\partial W_{xs}} + \delta X_{as} + \delta D_{os} + \frac{\sum \delta X_{r} \cdot \hat{\theta} \cdot \frac{\partial X}{\partial \hat{\theta}} + \delta X_c}{N},
\]

(3)

Then distance probability error of correction value are as following:

\[
E_{\Delta OS} = \left( \frac{\partial X}{\partial V_{os}} \right)^2 \cdot E_{V_{os}}^2 + \left( \frac{\partial X}{\partial \rho_s} \right)^2 \cdot E_{\rho_s}^2 + \left( \frac{\partial X}{\partial W_{xs}} \right)^2 \cdot E_{W_{xs}}^2 + E_{X_{as}} + E_{D_{os}} + E_{X_c}
\]

(4)

### 3.1.2. The calculating model of correction errors for transfer fire

When operating effective fire at sea target, the true distance correction value \( \Delta D_M \) is as follows [6]:

\[
\Delta D_M = \delta V'_0 \cdot \frac{\partial X}{\partial V_{OM}} + \delta \rho' \cdot \frac{\partial X}{\partial \rho_M} + \delta W'_x \cdot \frac{\partial X}{\partial W_{XM}} + \delta X_{BM} + \delta D_{OM}
\]

(5)

In above formula, when naval gun fire at the target, \( \delta V'_0 \) is accidental error to confirm deviation of muzzle velocity; \( \delta \rho' \) is accidental error to confirm air density; \( \delta W'_x \) is accidental error to confirm longitudinal wind. Because the ballistic and meteorology factors are unchanged, value of \( \delta V'_0 \) and \( \delta \rho' \) is the same with trail fire point. But the direction between trial fire point and target is different, \( \delta W'_x \) and \( \delta W_x \) is not the same.

The probability error of correction \( \Delta D_M \) is as follows:

\[
E_{\Delta DM} = \left( \frac{\partial X}{\partial V_{OM}} \right)^2 \cdot E_{V_{OM}}^2 + \left( \frac{\partial X}{\partial \rho_M} \right)^2 \cdot E_{\rho_M}^2 + \left( \frac{\partial X}{\partial W_{XM}} \right)^2 \cdot E_{W_{XM}}^2 + E_{X_{BM}} + E_{D_{OM}}
\]

(6)

### 3.1.3. Analysis distance errors of transfer fire

After transferring fire, we put the value \( \Delta S \) and \( \Delta D_M \) into formula (1), then we may get:

\[
\Delta D_s = K_s \left[ \delta V_0 \cdot \frac{\partial X}{\partial V_{as}} + \delta \rho \cdot \frac{\partial X}{\partial \rho_s} + \delta W_x \cdot \frac{\partial X}{\partial W_{as}} + \delta X_{as} + \delta D_{as} + \frac{\sum \delta X_r \cdot \hat{\theta} \cdot \frac{\partial X}{\partial \hat{\theta}} + \delta X_c}{N} \right]
\]

(7)

\[
\delta W_x - \delta W'_x \text{ is determined by transfer angle } \beta, \text{ and its probability error is changed with } \beta [7].
\]

When vector error of wind speed is subject to circular distribution, we may get following formula by calculation:

\[
E_{\Delta W_x} = 2 E_{W_x} \sin \frac{\beta}{2}
\]

(8)
Totally, after implementing transfer fire, the distribution of distance errors is as follows:

\[
C_x = \left( \frac{\partial X}{\partial W_{ds}} \right)^2 + \left( \frac{\partial X}{\partial W_{dM}} \right)^2 E_x^2 + \left( K_1 \frac{\partial X}{\partial W_{ds}} - \frac{\partial X}{\partial W_{dM}} \right)^2 E_{x1}^2 + \left( K_2 \frac{\partial X}{\partial W_{dM}} - \frac{\partial X}{\partial W_{dM}} \right)^2 E_{x2}^2 \\
+ \left( 2E_{wz} \sin \frac{\beta}{2} \frac{\partial X}{\partial W_{dM}} \right)^2 + \left( K_1 E_{xRS} - E_{xSM} \right)^2 + \frac{B_{sk}}{K} + \left( E_{wz}^2 \frac{\partial X}{\partial \theta_0} + E_D^2 \right) + E_{wz}^2
\]

(9)

3.2. The calculating model of direction errors

As in the above case, we can obtain direction errors of transfer fire, the formula is as follows:

\[
C_x = \left( \frac{\partial Z}{\partial W_{zS}} \right)^2 + \left( \frac{\partial Z}{\partial W_{zM}} \right)^2 E_{wz}^2 + \left( 2E_{wz} \sin \frac{\beta}{2} \frac{\partial Z}{\partial W_{z}} \right)^2 + \frac{B_{sk}}{0.1D\sqrt{K}} + \frac{E_{wz}^2 + E_{\beta}^2}{N} + E_{z0}^2
\]

(10)

In above formula, \( 2E_{wz} \sin \frac{\beta}{2} \) is probability error of \( \delta_{WX} - \delta_{W_z} \), \( E_{wz} \) is probability error of crosswind, \( E_\beta \) is probability error to measure direction deviation of burst point, \( E_{z0} \) is probability error of \( \delta_{Z0S} - \delta_{Z0M} \), which is determined by property of trial fire point, aiming method after transfer fire is implemented.

4. The calculating model of transfer range

After we have obtained transfer fire errors, we need to analyze the transfer range, and to decide whether transfer fire is used properly.

4.1. The calculating model of allowable distance interval and direction interval

According to accuracy requirements of fire adjustment, the allowable transfer fire interval contains distance interval and direction interval. The formula to determine allowable distance interval is as follows [8]:

\[
\max(\Delta x_1, \Delta x_2) \\
\text{st.} \left\{ \begin{array}{l} f(x + \Delta x_1) \geq (1-r) \cdot f' \\ f(x + \Delta x_2) \geq (1-r) \cdot f' \end{array} \right. 
\]

(11)

And allowable distance interval is: \([x - \Delta x_1, x + \Delta x_2]\).

In above formula, \([x - \Delta x_1, x + \Delta x_2]\) is allowable transfer distance interval, when \(x\) is zero, it stands for allowable transfer distance deviation; when \(x\) is distance of trial fire point, it stands for allowable transfer distance value. \(r\) is allowable declining ratio of fire effect.

Similarly, we may get the formula to determine allowable direction interval:

\[
\max(\Delta c_1, \Delta c_2) \\
\text{st.} \left\{ \begin{array}{l} f(c - \Delta c_1) \geq (1-r) \cdot f' \\ f(c + \Delta c_2) \geq (1-r) \cdot f' \end{array} \right. 
\]

(12)

And allowable direction interval is: \([c - \Delta c_1, c + \Delta c_2]\).
In above formula, \([c - \Delta c_1, c + \Delta c_2]\) is allowable transfer direction interval, when \(c\) is zero, it stands for allowable transfer direction deviation; when \(c\) is direction of trial fire point, it stands for allowable transfer direction value.

4.2. The calculating model of allowable transfer range

When implementing transfer fire, we may compare probability error of transfer fire to probability error of trial fire at target directly. In order to satisfy direction and distance transfer range at the same time, the comprehensive calculating formula is as follows:

\[
f(\Delta x, \Delta c) = r' \cdot f'
\]  

(13)

In above formula, \(f()\) is probability error of correction errors which caused by transfer fire, \(f'\) is probability error of trial fire at target directly, \(r'\) is probability error ratio of transfer fire.

Then formula (13) can be expressed as:

\[
f(\Delta x, \Delta c) = E_{\Delta d_x} = r' \cdot f' = r' \cdot E_{\Delta d_x}
\]  

(14)

4.3. The method to calculate transfer range

Supposed that \((x, c)\) is center point, we may gradually increase or decrease the value of \((\Delta x, \Delta c)\) according to interval \(x_i\) and interval \(c_i\), then find appropriate \((\Delta x, \Delta c)\) which meet the requirement of transfer fire, the formula is as follows:

\[
(\Delta x, \Delta c) = \begin{cases} 
  f(\Delta x, \Delta c) \leq r' \cdot f' \\
  \max(\Delta x, \Delta c)
\end{cases}
\]  

(15)

5. Application analysis

If a certain type of naval gun is firing at sea target, we have finished firing at trial fire point. For the sake of improving fire reaction speed, it is important for us to judge whether the condition of transfer fire is satisfied.

We may take the warship as reference, distance deviation between trial fire point and target is \(\Delta x = 2360 \text{ m}\), direction deviation is \(\Delta c = 28^\circ\). According to true shooting condition, errors of ballistic and meteorology, and measuring errors and so on, we may make use of the calculating model of transfer fire errors, then get the value of distance errors \(C_x\) and direction errors \(C_z\) by formula (9) and formula (10).

After probability error \(C_x, C_z\) of transfer fire and probability error of trial fire at target directly are worked out, we may put them into formula (15). Because \(\Delta x = 2360 \text{ m}, \Delta c = 28^\circ\) meet the requirement of formula (15), then transfer fire can be implemented.

6. Conclusion

Transfer fire can improve the abruptness and accuracy for naval gun to fire at sea target. By putting forward the computational model of firing error and transfer range, this paper can offer a strong technology support to correctly implement transfer fire [9]. In the process of transfer, it is necessary for operator to rational confirm the method of transfer fire according to different environmental condition, so as to ensure shooting accuracy of transfer fire.
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