Finite temperature properties of a modified Polyakov−Nambu−Jona-Lasinio model

Abhijit Bhattacharyya* and Kinkar Saha†
Department of Physics, University of Calcutta, 92, A.P.C Road, Kolkata-700009, INDIA

Paramita Deb‡
Indian Institute of Technology Bombay, Mumbai-400076, INDIA

Sanjay K. Ghosh§, Soumitra Maity¶, Sibaji Raha∗∗ and Rajarshi Ray††
Center for Astroparticle Physics & Space Science, Block-EN, Sector-V, Salt Lake, Kolkata-700091, INDIA

Sudipa Upadhaya‡‡
Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Kolkata-700064, INDIA

Thermodynamic properties of strongly interacting matter are investigated using the Polyakov loop enhanced Nambu−Jona-Lasinio model along with some modifications to include the hadrons. Various observables are shown to have a close agreement with the numerical data of QCD on lattice. The advantage of the present scheme over a similar study using a switching function is that here no extra parameters are to be fitted. As a result the present scheme can be easily extended for finite chemical potentials.

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I. INTRODUCTION

The programme of various ongoing and upcoming relativistic heavy-ion collision experiments is to explore the properties of strongly interacting matter at finite temperatures and densities. There has been major theoretical advances in the finite temperature properties using simulations of Quantum Chromodynamics (QCD) on space-time lattices. Currently the temperature $T_c$ where a rapid cross-over from hadronic to partonic matter takes place is found to be close to the pion mass. The reported values are 155 MeV [1, 2] and 150 MeV [3] from HotQCD and Wuppertal-Budapest (WuB) collaborations respectively. At the same time various QCD inspired model frameworks have been developed to extract interesting physical insights. The Nambu-Jona-Lasinio or the NJL model [4–9] is one such model which effectively explains key features of QCD like chiral symmetry breaking and its restoration. However the effects of the gluon degrees of freedom are not adequately addressed in such models. The extensions like the Polyakov loop enhanced Nambu-Jona-Lasinio (PNJL) model [10–14] encapsulates this missing feature by including a temporal background gluon field. As a result, both chiral and deconfinement aspects are captured within a single framework. Various studies of the basic thermodynamic variables computed in the mean field framework display strong similarity to lattice results (see [15] and references therein).

With more sophisticated techniques and higher computational power, some of the observables from lattice QCD have been extrapolated to the physical continuum limit [2, 3]. Many of these results are quite different from those obtained earlier on smaller lattices. In view of this a reparametrization of the PNJL model was done [16] to obtain a quantitative agreement with the lattice QCD data. One important lacunae observed in this study is the mismatch of

*Electronic address: abphy@caluniv.ac.in
†Electronic address: saha.k.09@gmail.com
‡Electronic address: paramita.deb83@gmail.com
§Electronic address: sanjay@jbose.ac.in
¶Electronic address: soumitra.maiti1984@gmail.com
∗∗Electronic address: sibaji@jbose.ac.in
††Electronic address: rajarshi@jbose.ac.in
‡‡Electronic address: sudipa.09@gmail.com
the results for temperatures close to or below $T_c$. The reason was identified as the absence of hadronic contribution in the PNJL model. Several other attempts were going on to construct a suitable model to match the lattice data. In HRG+chiral perturbation theory \[17\], below the transition temperature, the decrease of the absolute value of chiral condensate is well described. Also HRG model is able to reproduce LQCD data on temperature dependence of the Polyakov-loop itself \[18\]. Similarly, a quark-hadron hybrid model \[19\] has been constructed by taking quark and hadron contributions simultaneously. The hadron volume fraction function is used to switch from one phase to other and hadron-quark transition temperature is defined in the view of the ratio of quark and hadron contribution. In this direction some of us studied a hybrid model by coupling the HRG model and the PNJL model via a switching function \[12\].

Here we study an alternative scheme where the hadron contributions are added in a simple way except that we consider their medium dependent masses. The confinement feature through the Polyakov loop always suppresses the contribution of the constituent quarks at low temperatures and densities. The switching function was necessary to rather cut off the contribution of the hadrons at high temperatures and densities. Instead of the switching function, here the rising effective masses of the hadrons will naturally make them unfavorable in the thermodynamics.

In the next section, we briefly outline the PNJL model. The following section gives a description of how we handle the hadrons. This is followed by our results and conclusions.

II. PNJL MODEL

We now discuss the particular form of the PNJL model as discussed in Ref. \[10\], which will be employed here. The scheme in the PNJL model was to add a Polyakov loop effective potential to the NJL model \[10, 12, 20\]. The chiral properties are taken care of by the NJL part, while the confinement properties and the gluonic contributions are effectively incorporated through the Polyakov loop potential. Various studies have been carried out using PNJL model with 2 and 2+1 flavors \[12, 21–30\]. For our study we shall use the 2+1 flavor model having up to six quark interactions. The thermodynamic potential is given as \[10\],

\[
\Omega(\Phi, \bar{\Phi}, \sigma, T, \mu) = 2g_s \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2} \sigma_u \sigma_d \sigma_s - 6 \sum_{f} \int_{0}^{\infty} \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|) \\
- 2T \sum_{f} \int_{0}^{\infty} \frac{d^3p}{(2\pi)^3} \ln \left[ 1 + 3 \left( \Phi + \bar{\Phi} e^{-(E_f - \mu_f)/T} \right) e^{-(E_f - \mu_f)/T} + e^{-(E_f - \mu_f)/T} \right] \\
- 2T \sum_{f} \int_{0}^{\infty} \frac{d^3p}{(2\pi)^3} \ln \left[ 1 + 3 \left( \bar{\Phi} + \Phi e^{-(E_f + \mu_f)/T} \right) e^{-(E_f + \mu_f)/T} + e^{-(E_f + \mu_f)/T} \right] \\
+ \mathcal{U}'(\Phi, \bar{\Phi}, T). \tag{1}
\]

The first five terms on the R.H.S. are the terms of the NJL model suitably modified due to the Polyakov loop. Here $\sigma_f = \langle \bar{\psi}_f \psi_f \rangle$ correspond to the two light quark $(f = u, d)$ condensates and the strange $(f = s)$ quark condensate respectively. There is a four quark coupling term with coefficient $g_S$ and a six quark coupling term breaking the axial U(1) symmetry explicitly with a coefficient $g_D$. The corresponding quasiparticle energy is $E_f = \sqrt{p^2 + M_f^2}$, for a given flavor $f$. The dynamically generated constituent quark masses is given by,

\[
M_f = m_f - 2g_S \sigma_f + \frac{g_D}{2} \sigma_{f+1} \sigma_{f+2}, \tag{2}
\]

where, if $\sigma_f = \sigma_u$, then $\sigma_{f+1} = \sigma_d$ and $\sigma_{f+2} = \sigma_s$, and so on in a clockwise manner.

The third term on the R.H.S. of Eq. (1) gives the zero point energy, while the fourth and fifth terms are the finite temperature and chemical potential contributions of the constituent quarks and anti-quarks respectively. The latter two terms arise from the fermion determinant in the NJL model duly modified by the fields corresponding to the traces of Polyakov loop and its conjugate given by $\Phi = \frac{Tr N_c L}{N_c}$ and $\bar{\Phi} = \frac{Tr N_c L^\dagger}{N_c}$ respectively. Here $L(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^{1/T} d\tau A_4(\vec{x}, \tau) \right]$ is the Polyakov loop, and $A_4$ is the temporal component of background gluon field.

The effective potential for the $\Phi$ and $\bar{\Phi}$ fields are given by $\mathcal{U}'$, appearing as the last term in Eq. (1). Various forms of the potential exist in the literature (see e.g. \[23, 31, 34\]). We shall use the form prescribed in \[16\], which reads as,

\[
\frac{\mathcal{U}'(\Phi, \bar{\Phi}, T)}{T^4} = \frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} - \kappa \ln[J(\Phi, \bar{\Phi})]. \tag{3}
\]
Here $U(\Phi, \bar{\Phi}, T)$ chosen as a Landau-Ginzburg type potential commensurate with the global $Z(3)$ symmetry of the Polyakov loop is given as \[12\],

$$\frac{U(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{b_2(T)}{2} \Phi \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\Phi \bar{\Phi})^2$$

(4)

The coefficient $b_2(T)$ is chosen to have a temperature dependence of the form \[16\],

$$b_2(T) = a_0 + a_1 \exp(-a_2 \frac{T}{T_0})$$

(5)

and $b_3$ and $b_4$ are chosen to be constants. The term $J[\Phi, \bar{\Phi}] = (1 - 6 \Phi \bar{\Phi} + 4 (\bar{\Phi}^3 + \Phi^3) - 3 (\Phi \bar{\Phi})^2)$ is the Jacobian of transformation from the Polyakov loop to its traces. $\kappa$ is a dimensionless parameter which is determined phenomenologically.

| $m_u$ (MeV) | $m_s$ (MeV) | $\Lambda$ (MeV) | $g_s \Lambda^2$ | $g_D \Lambda^5$ |
|----------------|----------------|-----------------|----------------|----------------|
| 5.5            | 134.758        | 631.357         | 3.664          | 74.636         |

TABLE I: Parameters in the NJL model

| $T_0$ (MeV) | $a_0$ | $a_1$ | $a_2$ | $b_3$ | $b_4$ | $\kappa$ |
|--------------|-------|-------|-------|-------|-------|---------|
| 175          | 6.75  | -9.0  | 0.25  | 0.805 | 7.555 | 0.1     |

TABLE II: Parameters for the Polyakov loop potential.

The different parameter values in the NJL terms are given in Table I. And the parameter values used in the Polyakov loop potential are given in Table II.

Previously some of us \[16\] discussed that this model gives a crossover temperature of $T_c \sim 160$ MeV as well as quantitative agreement of temperature variations of pressure and various other observables commensurate with the observations in lattice QCD in the continuum limit \[16\]. However the quantitative agreement though close, was not exact in different ranges of temperatures. Significant discrepancies appeared in the low temperature region where the hadronic degrees of freedom dominate. A possible step towards removal of this lacunae was proposed by us \[15\] by coupling the PNJL model with the Hadron Resonance Gas model via a switching function. This scheme was successful in getting a much better agreement between the results from PNJL model and the lattice QCD data. Here a key role is played by the switching function that switches the hadronic or the partonic degrees of freedom. However this approach requires us to immaculately choose a form and parametrization of the switching function itself. Here we discuss an alternative scheme where PNJL model is modified such that the hadronic contributions would appear more naturally in the relevant region of the phase space and shut off in other regions, without having to use a switching by hand. In the next section we shall describe this scheme.

III. HADRONIC SECTOR

Our aim is to include all the correct degrees of freedom allowed in strong interactions in our model framework. As discussed in \[15\] (and references therein), the prominent degrees of freedom would depend on the thermodynamic conditions. This gave a scope for introducing the phenomenologically determined switching function in \[15\], and couple the PNJL model to the HRG model. Here we ask if a more natural mechanism exists to include the hadronic contributions.

As is well known, the thermodynamic potential given by Eq. 1 is obtained in the mean field approximation for the quark propagators. A consistent method to extract the thermodynamic potential beyond mean field for a quark meson plasma in the framework of the NJL model was outlined in \[35–37\]. The mesonic contributions appear in the next to leading order contributions in a $1/N_c$ expansion in the form of ring diagrams. For a meson $M$ the contribution to the thermodynamic potential is given by,

$$\delta \Omega_M = g_M \int \frac{d^3 p}{(2\pi)^3} \int d\omega \left[ \omega^2 + T \ln(1 - e^{-\omega/T}) \right] \frac{1}{\pi} \frac{d\delta M(\omega, \vec{p}, T)}{d\omega}. \quad (6)$$
Here, \( g_M \) is the internal degrees of freedom of the meson and \( \delta_M(\omega, \vec{p}, T) \) is the scattering phase shift of a quark and anti-quark in the \( M \) channel.

Extensions of this work in the PNJL model has been done in [38–42], wherein the authors have studied various effects of this additional contribution to the mean-field thermodynamic potential. On the other hand here we set out to make a detailed study of the various thermodynamic observables and contrast them to the results reported in the continuum limit in the lattice QCD framework. Here we do not try to be rigorous with the beyond mean-field calculations but carry out a simple heuristic approach. We simply add the hadronic contribution to the mean field PNJL model. The masses of such hadronic excitations may be computed from the pole condition in the respective polarizations, and would therefore depend implicitly on the mean fields and explicitly on the thermodynamic parameters. This approach is similar to the near-pole approximation \( (\omega^2 = E_M^2 = \vec{p}^2 + m_M^2) \) of the above thermodynamic potential [33]. In practice this approach is similar to our earlier approach [13] of adding the hadronic contribution to the PNJL model, but without a switching function. Here the effect of switching off/on of the hadronic contributions will rather be taken care of by the relative strength of the temperature dependent hadronic masses to the quark masses.

The temperature dependent mesonic masses are obtained from the pole condition

\[
1 - 2G_M \Pi_M(\omega = m_M, \vec{k} = 0) = 0.
\]

Here \( G_M \) is the effective vertex factor for the given flavor combination and \( \Pi_M(k^2) \) is the one-loop polarization function for corresponding mesonic channel given by the Random Phase Approximation [43] as,

\[
\Pi_M(k^2) = \int \frac{d^3p}{(2\pi)^3} Tr[\Gamma_M S(p + \frac{k}{2})\Gamma_M S(p - \frac{k}{2})],
\]

where \( S(p) \) is the quark propagator. In this work we shall only consider the lowest lying nonet mesons. The details of the calculations may be found in our earlier work [44, 45]. The final computations however consider the reparametrized PNJL model as discussed in the previous section. The mesonic contribution to the thermodynamic potential is given as [31].

\[
\delta\Omega_M = -\nu_M T \int \frac{d^3p}{(2\pi)^3} \ln(1 - e^{-\frac{E_p}{T}})
\]

where \( \nu_M \) is statistical weight factor of corresponding mesonic species and \( E_p = \sqrt{\vec{p}^2 + m_{pole}^2(T)} \), where \( m_{pole} \) is the mesonic mass obtained by solving Eq. (11).

In the baryonic sector, the lower lying window is occupied by the nucleons, protons and neutrons. They having a bare mass \( \sim 940 \) MeV contribute insignificantly to the thermodynamics. Also chiral perturbation theory results like in [46] indicate that the nucleon masses increase with temperature apart from a very small decrease in the intermediate regimes of temperature. In this study we therefore consider only the constant mass for nucleons. Role of other baryon species are left out in this exercise. The baryonic contribution to the thermodynamic potential is given by [47].

\[
\delta\Omega_B = \nu_B T \int \frac{d^3p}{(2\pi)^3} \ln(1 + e^{-\frac{E_p}{T}})
\]

where \( \nu_B \) is statistical weight factor of corresponding baryonic species. The final thermodynamic potential is the sum of the parts obtained from Eq. (11), Eq. (11) and Eq. (11). We shall refer to this as the Modified Polyakov loop enhanced Nambu–Jona-Lasinio (MPNJL) model.

IV. RESULTS

The thermodynamic potential [11] is minimized with respect to the \( \sigma, \Phi \) and \( \bar{\Phi} \) to obtain the mean fields. These are then inserted into the equations [11] and [5] to obtain the meson masses as function of temperature. In Fig. 11 we have plotted the variations of meson masses for \( \pi, \sigma, K, \eta \) and \( \eta' \) with temperature. The results are similar to some of the earlier works [11, 58]. The most significant change is the mass of the \( \pi \) which rises from about 140 MeV near the crossover temperature to 550 MeV as the temperature nears 400 MeV. The \( \sigma \) mass has the expected behavior of first a strong decrease to reach the \( \pi \) mass and then increase along with the \( \pi \) mass. In this temperature range the \( \eta' \) mass decreases by almost 300 MeV. The masses of the \( K \) and \( \eta \) vary by a relatively small amount. The various
combinations of constituent quark masses are also plotted for comparison. Obviously the signature of the chiral 
symmetry restoration in the meson sector at high temperatures is evident as the constituent masses go down.

The mean fields are then put back in Eq. (11), and the pole masses in the Eq. (9) and Eq. (10) to obtain the pressure. The scaled pressure and scaled entropy density are shown in Fig. 2(a) and Fig. 2(b) respectively. These are compared 
to the continuum extrapolated lattice QCD (HotQCD [2], and Wuppertal-Budapest [3]) data. Both the quantities in 
the MPNJL model agree with the usual PNJL model and lattice QCD data for the higher temperatures. At the lower 
temperature the MPNJL model remarkably reproduces the lattice QCD data, where the PNJL model fails. Obviously 
a similar result was obtained with the hybrid PNJL model [15]. But unlike the hybrid model, where the switching 
function had to be tuned, here we have no extra parameters, apart from those already present in the PNJL model.

Given that we have considered only a few mesons corresponding the flavor SU(3) octet and the lowest lying nucleons, 
the agreement of the bulk thermodynamics in the MPNJL model and lattice QCD data is surprising. However as 
shown in the figures, the scaled pressure and scaled entropy density obtained in the ideal Hadron Resonance Gas 
model have an excellent overlap with the MPNJL and lattice QCD data in the low temperature region. It therefore 
seems sufficient to include the limited number of hadrons for the present study.

With the MPNJL model we now obtain the specific heat at fixed volume, which includes the second derivative 
of the thermodynamic potential with respect to temperature. The variation of the scaled specific heat with temperature 
is shown in Fig. 3(a). For higher temperatures the lattice results are distinctly different. In fact the difference can 
be seen to be gradually increasing as we move from the scaled pressure to scaled entropy density to finally the scaled 
specific heat. As already mentioned in [16], we chose the parameters in the Polyakov loop potential to agree with the 
HotQCD data. In the lower temperature ranges it is difficult to conclude if the results of the MPNJL model may be
FIG. 3: (color online) Specific heat and speed of sound as functions of temperature

preferred over the PNJL model results when compared to the lattice QCD data.

To bring out the difference between the two models we therefore consider the squared speed of sound, which turns out to be the ratio of the entropy to the specific heat at fixed volume. This is shown in Fig. 3(b). Here we see a wide difference between the PNJL and MPNJL model results for the lower temperatures. The MPNJL model results indeed agrees well with both the Hadron Resonance Gas model, as well as the lattice QCD data.

With these results we demonstrated the necessity and utility of introducing the beyond mean field contributions to the PNJL model, though with quite a few assumptions. As we see that here we did not need any extra switch between the hadron and PNJL contributions. The switch between the degrees of freedom are affected by the varying masses with temperature. For lower temperatures the constituent quark masses are quite high and the meson masses comparatively low, giving rise to meson domination. This is in addition to the suppression of quark excitation by the Polyakov loop. The condition is reversed as one approaches higher temperatures, and the system becomes quark dominated.

A proper determination of the state of strongly interacting matter at finite temperatures and chemical potentials requires knowledge of the fluctuations of conserved charges \[ \chi^X \]. They also act as indicators of phase transition or crossover through which the system passes \[ \chi^X \]. At a given temperature and arbitrary chemical potentials, the pressure of the system may be expanded as a Taylor series around zero chemical potentials, where the coefficients are directly related by the fluctuation-dissipation theorem \[ \chi^X \]. The n-th order Taylor expansion coefficient \[ c^X_n \] of scaled pressure can be written in terms of fluctuations \[ \chi^X_n \] as,

\[
c^X_n(T) = \frac{1}{\mu^X} \frac{\partial^n \left( P/T^4 \right)}{\partial \mu^X/T^n} = \frac{T^{n-4}}{n!} \chi^X_n(T)
\]

where the expansion is carried out around \( \mu_B = \mu_Q = \mu_S = 0 \). In Fig. 4 we present our results for the second order fluctuations of the conserved charges along with a comparison of the continuum data from lattice QCD \[ \chi^X \]. In the model, these fluctuations are obtained by a suitable Taylor series fitting as discussed in detail in \[ \chi^X \].

The baryon number fluctuation \( c^B_2 \) obtained in the PNJL and the MPNJL model are very close to each other in the whole range of temperatures studied. The only hadrons that contribute additionally in the MPNJL model are the nucleons with a mass \( \sim 1 \) GeV, which is much heavier than the corresponding constituent mass of the quarks. So the difference between the two model results is insignificant. There is a possible concern for overcounting the baryons in the MPNJL model – as the constituent quarks and as the nucleons. Obviously this would be of concern as more and more baryons are included. But in the present case the constituent quarks overwhelm the system due to both their lower masses as well as larger degrees of freedom. In view of these discussions, it is surprising to find a reasonable agreement of the results from the PNJL model with the lattice QCD data even for temperatures below 150 MeV. It seems that the partonic fluctuations manifest themselves strongly in the baryon susceptibilities.

This is not the case for the electric charge fluctuations \( c^Q_2 \). The PNJL and MPNJL model results differ significantly for temperatures close to 200 MeV. The MPNJL model has a very good agreement with the lattice QCD results. The dominant hadrons in this sector are the pions. For very low temperatures the pion mass is almost half the mass of
the constituent quarks. With increase in temperature the pion mass increases and the quark mass decreases such that the combination nicely reproduces the lattice QCD data.

The strangeness fluctuation $c_s^2$ in the PNJL and MPNJL models differ for low temperatures by a smaller amount when compared to the charge fluctuations. For the lowest temperatures in the lattice QCD data, the MPNJL model seems to agree. But thereafter the two models merge and they deviate from the lattice QCD data. In the hadronic sector the dominant contributors are the $K$ and $\eta$. Their masses, though almost half of the constituent masses $M_u + M_s$, are still quite large. Moreover the $K$ mass is almost constant over the whole temperature range. On the other hand the decrease in the constituent mass of the strange quark with temperature is not fast enough. So their contributions to strangeness fluctuations above temperatures of 150 MeV is not enough to agree with the lattice QCD data.

V. CONCLUSION

Numerous attempts are being made to predict the correct EoS for the strongly interacting system. Lattice QCD is the most robust abinitio technique. However effective models that are much easier to handle and suitable enough to extract interesting physical outcomes are regularly employed. The reliability of such models in quantitative estimates have often come under review. In this regard we are investigating the various possible improvements for the PNJL model so that it can serve as an effective tool for quantitative analysis of strong interactions in chemical equilibrium.

In an earlier work [16], the Polyakov loop potential was reparametrized to bring various thermodynamic quantities in reasonable agreement with the lattice QCD data. Among the issues pointed out in that work was the insufficiency of the PNJL model to reproduce the correct results for temperatures close to and below the crossover transition. The relevance of the hadronic degrees of freedom was realized and a hybrid model was built [15] with the HRG model and PNJL model coupled via a switching function. The method worked well, but a more natural framework was
sought. The existing literature on the beyond-mean field calculations in the NJL and PNJL models led us to propose the present modified PNJL model, where the hadronic contributions are additively included. There is no switching function, but the hadrons are given medium modified masses. The relative variation of the hadron and constituent quark masses with temperature effectively selects the dominant degrees of freedom. The best utility of this scheme over the scheme using switching function is for finite chemical potentials. The parameters in the switching function being additional parameters had to be fixed at various temperatures and chemical potentials. Here on the other hand there are no extra parameters and the effect of temperature and chemical potentials are taken care of through the respective hadronic distribution functions.

The scheme is found to satisfactorily reproduce the lattice QCD results for a range of observables including the pressure, entropy, specific heat, speed of sound and the baryon number and electric charge fluctuations. The results from the model however deviated significantly from the lattice data for strangeness fluctuations. To address this shortfall it is necessary to revisit the strangeness sector of the PNJL model, which we hope to address in future.

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[1] A. Bazavov et al. (HotQCD collaboration), Phys. Rev. D 85, 054503 (2012).
[2] A. Bazavov et al. (HotQCD collaboration), Phys. Rev. D 90, 094503 (2014).
[3] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B 730, 99 (2014).
[4] Y. Nambu, and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).
[5] T. Kunihiro and T. Hatsuda, Phys. Lett. B 206, 385 (1988).
[6] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).
[7] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[8] T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221 (1994).
[9] M. Buballa, Phys. Rep. 407, 205 (2005).
[10] K. Fukushima, Phys. Lett. B 591, 277 (2004).
[11] R. D. Pisarski, Phys. Rev. D 62, 115101 (2000); A. Dumitru and R. D. Pisarski, Phys. Lett. B 504, 282 (2001); Phys. Rev. D66, 096003 (2002).
[12] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D 73, 014019 (2006).
[13] K. Fukushima, Phys. Rev. D 77, 114028 (2008).
[14] H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi and C. Ratti, Phys. Rev. D75, 065004 (2007).
[15] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha, S. Samanta and S. Upadhyaya Phys. Rev. C 99, 045207 (2019).
[16] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha and Sudipa Upadhyaya, Phys. Rev. D 95, 054005 (2017).
[17] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010).
[18] E. Megias, E. R. Arriola, and L. L. Salcedo, Phys. Rev. Lett. 109, 151601 (2012).
[19] A. Miyahara, Y. Torigoe, H. Kouno and M. Yahiro, arXiv : 1604.05002.
[20] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B379, 163 (1996); Nucl. Phys. B, Proc. Suppl. 47, 519 (1996).
[21] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D 73, 114007 (2006).
[22] S. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D 75, 094015 (2007).
[23] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D 77, 094024 (2008).
[24] M. Ciminale, R. Gatto, N. D. Ippolito, G. Nardulli, and M. Ruggieri, Phys. Rev. D 77, 054023 (2008).
[25] A. Bhattacharyya, P. Deb, S. K. Ghosh, and R. Ray, Phys. Rev. D 82, 014021 (2010).
[26] G. Y. Shao, Z. D. Tang, M. Di Toro, M. Colonna, X. Y. Gao, and N. Gao, Phys. Rev. D 94, 014008 (2008).
[27] G. Y. Shao, Z. D. Tang, M. Di Toro, M. Colonna, X. Y. Gao, N. Gao, and Y. L. Zhao, Phys. Rev. D 92, 114027 (2015).
[28] C. A. Islam, S. Majumder, N. Haque, and M. G. Mustafa, J. High Energy Phys. 1502 (2015) 011.
[29] S. Ghosh, T. C. Peixoto, V. Roy, F. E. Serna, and G. Krein, Phys. Rev. D 93, 045025 (2016).
[30] J. Moreira, B. Hiller, A. A. Osipov, and A. H. Blin, Int. J. Mod. Phys. A 27, 1250060 (2012).
[31] C. Ratti, S. Robner, and W. Weise, Phys. Lett. B 649, 57 (2007).
[32] K. Fukushima, Phys. Rev. D 77, 114028 (2008); 78, 039902(E) (2008).
[33] G. A. Contrera, A. G. Grunfeld, and D. B. Blaschke, Phys. of Part. and Nucl. Lett., 2014, Vol 11, No. 4, pp 342-351.
