TeV gravity searches

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Abstract

In scenarios with extra dimensions the gravitational interaction may become strong at TeV energies. This could modify the $\nu N$ cross section and imply distinct signals at neutrino telescopes. In particular, cosmogenic neutrinos of $E \approx 10^9$ GeV could experience frequent interactions with matter where they lose a very small fraction of their energy. We define a consistent model of strong gravity at the TeV scale with just one extra dimension and a first Kaluza-Klein excitation of the graviton of mass around 1 GeV. We describe the collisions at transplanckian energies (multigraviton exchange, graviton emission and black hole formation) as well as the possible signature of these processes at km$^3$ telescopes and their impact in cosmogenic neutrino searches.

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1 Introduction

The hierarchy problem, namely, how to make consistent a quantum field theory that includes very different scales, has defined the model building in particle physics during the past four decades. In units of the Planck mass, $M_P = 1.2 \times 10^{19} \text{ GeV}$, the electroweak (EW) scale ($m_h^2 \approx 10^{-34} M_P^2$) and the vacuum energy density ($\Lambda \approx 10^{-120} M_P^4$) have extremely small values. These two scales are free parameters in our theory, but they include $\mathcal{O}(1)$ quantum corrections that require a large fine tuning in order to reproduce the values that we see. The usual strategy to explain naturally the fine tuning in $m_h^2$ had been to complete the standard model with new symmetries (TeV particles that cancel the quantum corrections to $m_h$) and/or new dynamics (the Higgs as a composite of a new interaction that becomes strong at the TeV). In both cases the result may be an EW scale with only logarithmic sensitivity to the ultraviolet (UV) physics.

In this context, it is difficult to overstate the impact on the community of the 1998 paper on extra dimensions and TeV gravity by Arkani-Hamed, Dimopoulos and Dvali (ADD) [1]. All the basic ingredients in their analysis were already known: there were previous proposals of compact dimensions with radius $R \approx (1 \text{ TeV})^{-1}$ to break supersymmetry [2] or with $R \approx (10^{12} \text{ GeV})^{-1}$ in M-theory to lower $M_{\text{string}}$ to $M_{\text{GUT}} \approx 10^{16} \text{ GeV}$ [3], and it had even been shown how different fields of the same theory can live in a different number of dimensions (D-branes) [4]. However, ADD realized that the fundamental scale of gravity was not necessarily $M_P$, and that it could be as low as 1 TeV. In this case, obviously, the hierarchy problem introduced by the EW scale would disappear. The model by ADD has 3 basic parameters that are related by $M_P$: the fundamental scale of gravity ($M_D$), the number of compact dimensions ($n$) and their radius ($R$). One year later Randall and Sundrum (RS) [5,6] generalized the framework with a new parameter, a higher dimensional curvature ($k$) that was zero in the ADD model.

Since then, the models with extra dimensions have revealed a very rich phenomenology. Not only they are able to accommodate hierarchies, they also seem to provide an alternative (holographic) description of strongly coupled 4-dimensional theories [7,8], which opens unlimited possibilities for model building. If the LHC confirms the absence of new physics below 1 TeV we will learn that nature does not deal with the hierarchy problem the way we assumed, but this will not diminish the relevance of extra dimensions. On one hand, they remind us that a more fundamental scale may appear anywhere, so they represent a stimulus for the exploration of higher energies. On the other hand, they may be used to explain the apparent fine tuning in the free parameters of a model (e.g., the small Yukawa couplings required in the standard model, specially if neutrinos have a Dirac nature [9]) or
Strong TeV gravity is particularly relevant for the physics of ultrahigh energy neutrinos. What makes neutrinos special is that they only have weak interactions, implying that the relative effect of the new physics could be more important than for quarks and charged leptons. In particular, cosmogenic neutrinos appear when $10^{10} - 10^{11}$ GeV cosmic rays propagate and interact inelastically with the 2.7 K cosmic microwave background radiation \[11\]. This cosmogenic $\nu$ flux is certainly there (see Fig. 1) \[12\], and it implies a few tens of neutrinos of energy around $10^9$ GeV reaching the Earth per km$^2$, unit of solid angle and year \[13,14\]. In the collision of such neutrinos with a nucleon the center of mass (c.o.m.) energy is $\sqrt{s} = \sqrt{2m_N E} \approx 45$ TeV, well above the scale explored at colliders. Since the standard model interaction length in ice for a $10^9$ GeV neutrino is around 1000 km, cosmogenic $\nu$s could (should!) be detected in the near future. Moreover, their absence in experiments like ANITA \[15\], LUNASKA \[16\] or IceCube-Gen2 \[17\] could mean that new physics may be hiding them. We will show that this could be the case if, at energies above a threshold around $10^7$ GeV, neutrinos experience transplanckian interactions with matter.

The plan of this chapter is as follows. First we will discuss in some detail the basic ideas of TeV gravity through extra dimensions and will define a consistent set up. Then we will obtain the neutrino–nucleon cross section in that framework and discuss its validity at high energies. In particular, we will argue that at $s \gg M_D^2$ the result is independent of the UV
details of the theory, \textit{i.e.}, of how gravity is embedded in a consistent quantum theory. Finally we will discuss the possible signal of these scenarios at large-scale neutrino telescopes.

## 2 Extra dimensions: circles, orbifolds and curvature

We have mentioned that the usual strategy to solve the hierarchy problem had been to search for a mechanism that keeps $m_h$ much smaller than $M_P$. Extra dimensions provide the opposite approach: they explain why the Planck mass is so much larger than the EW scale. Let us see how it works. The Planck mass is defined by Newton’s law for the gravitational force between two masses, $M$ and $m$, separated by a distance $r$ ($c = 1 = \hbar$): \[ F(r) = -G_N \frac{M m}{r^2} \] with $G_N = 1/M_P^2 \equiv 1/(8\pi \bar{M}_P^2)$. This dependence with the distance reflects Gauss’ law: the flux of field lines through a Gaussian surface of radius $r$ around $M$ is constant, so its density \textit{dilutes} like $1/r^2$. If gravity were propagating in just two spacial dimensions instead of three, then Newton’s law would go like $1/r$, whereas 1-dimensional gravity would imply a constant ($r$ independent) force. Now suppose that gravity propagates in 2 dimensions, but that the second one is compact and has a length $L$, as given in Fig. 2. At distances much shorter than $L$ the field lines are insensitive to the fact that this dimension is compact, and gravity will be purely 2-dim. At larger distances, however, the field lines do not dilute any longer and gravity becomes 1-dim, \textit{i.e.}, there is a constant density of field lines. If we consider the usual 4-dimensional space plus one extra dimension compactified on a circle $S^1$ of radius $R$, at $r \gg R$ we will verify the usual Newton’s law, while at $r \ll R$ we will have \[ F(r) = -G_5 \frac{M m}{r^3}. \] It is important to notice that the 5-dimensional gravitational constant has now dimensions of $M^{-3}$: \[ G_5 \equiv \frac{1}{8\pi M_5^3}. \]
\( \bar{M}_5 \) is the fundamental scale in this (4+1)-dimensional theory, whereas \( \bar{M}_P \) is an effective scale that appears in long-distance interactions. Matching (1) and (2) at \( r \approx L = 2\pi R \) we find how these two scales relate:

\[
\frac{G_5}{L} = G_N \quad \text{or} \quad \bar{M}_P^2 = \bar{M}_5^3 L .
\]  

(4)

For \( n \) extra dimensions (\( D = 4 + n \)) defining a torus \( T^n = (S^1)^n \) with (common) radii \( R \), an analogous argument gives

\[
\frac{G_D}{V} = G_N \quad \text{or} \quad \bar{M}_P^2 = \bar{M}_D^{2+n}(2\pi R)^n ,
\]  

(5)

where \( G_D = 1/(8\pi \bar{M}_D^{2+n}) \).* The fact is that the gravitational interaction grows at short distances \( r < R \) faster than in \( D = 4 \), and it becomes strong at a scale \( \bar{M}_D \) that (varying \( R \)) may take values between \( m_h \) and \( M_P \).

### 2.1 Kaluza-Klein modes

Another important concept in higher-dimensional theories is that of Kaluza-Klein (KK) mode. It may be instructive to see how KK excitations appear in the simplest set up, a 5-dimensional complex scalar field with the extra dimension compactified on a circle. Let us label the coordinates \( x^M = (x^\mu, x^5) \) with \( x^5 = y \) and use the metric \( \eta_{MN} = \text{Diag}(-1,1,1,1,1) \). The action for the free field \( \Phi(x^M) \) is

\[
S_5 = - \int d^5x \partial_M \Phi^\dagger \partial^M \Phi ,
\]  

(6)

where \( d^5x = d^4x \, dy \) and \([\Phi] = E^{3/2} \). Since in \( S^1 \) we identify \( y \) with \( y + 2\pi R \), we can expand the \( y \) dependence

\[
\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \phi^{(n)}(x^\mu) e^{iny} .
\]  

(7)

Using this expansion in (6) and integrating over \( y \) we get \( S_5 = S_4^{(0)} + S_4^{(n)} \), with

\[
S_4^{(0)} = - \int d^4x \partial_\mu \phi^{(0)\dagger} \partial^\mu \phi^{(0)\dagger} ,
\]

\[
S_4^{(n)} = - \int d^4x \sum_{n \neq 0} \left( \partial_\mu \phi^{(n)\dagger} \partial^\mu \phi^{(n)} + \left( \frac{n}{R} \right)^2 \phi^{(n)\dagger} \phi^{(n)} \right) .
\]  

(8)

We have traded the field dependence on \( y \) by a tower of 4-dim KK modes of mass a multiple of \( m_c \equiv 1/R \). This mass is nothing but the quantized momentum \( p_y \) of \( \Phi \) along the compact

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*Warning: the scale most frequently used in the literature is \( M_D = \bar{M}_D(2\pi)^{\frac{n}{2+n}} \) [19], which does not coincide with the original \( M_5 \) in [1] either.
dimension: notice that $\partial_5 \Phi^i \partial^5 \Phi$ becomes the mass term in the 4-dimensional action and that each KK level includes two modes, reflecting that $p_y$ may be positive or negative. The KK masses for compactification on a $n$-dimensional torus are $m_{n_1, n_2, \ldots}^2 = (n_1^2 + n_2^2 + \ldots)/R^2$, which correspond to a momentum $\pm n_i/R$ along each extra dimension.

The KK expansion for the graviton is a bit more involved. The metric is a symmetric tensor, and in 5 dimensions its fluctuations ($g_{MN} = \eta_{MN} + h_{MN}$) have 15 independent components:

$$h_{MN} = h_{\mu\nu} \oplus h_{\mu 5} \oplus h_{5 5}.$$  \hbox{(9)}

The 5-dimensional Einstein-Hilbert action admits local transformations that may be used to eliminate ten of them, leaving five physical polarizations: two in $h_{\mu\nu}$, two in $h_{\mu 5}$ and $h_{5 5}$. The zero modes of these 5-dimensional fields will define the 4-dimensional graviton plus a vector and a real scalar field. As for the massive ($n \neq 0$) modes, $h_{\mu 5}^{(n)}$ and $h_{5 5}^{(n)}$ are eaten by $h_{\mu\nu}^{(n)}$ to define a KK tower of spin-two massive fields, each one with 5 physical polarizations. In more than five dimensions there are additional KK towers of scalar and vector fields [19].

### 2.2 Orbifolds

Compactification of an extra dimension on $S^1$ faces a main problem: when we reduce fermions to four dimensions they are always vector-like (Dirac) fields; the theory does not admit chiral fermions. The basic reason is that $\gamma_5$ is now part of the Dirac algebra,

$$\{\gamma^M, \gamma^N\} = 2\eta^{MN},$$  \hbox{(10)}

and the boosts along the extra dimension will change the chirality of the fermion. The solution to this is to change the compactification space: instead of a differentiable manifold we will use an orbifold whose singularities will break Lorentz invariance along $y$. The orbifold $S^1/Z_2$ is obtained from the circle $-\pi R \leq y \leq \pi R$ by identifying $y \rightarrow -y$, as shown in Fig. 3.
What kind of 5-dimensional fields can live on this orbifold? Consider a scalar field \( \Phi(x, y) \); an obvious guess would be that only those 5-dimensional fields in \( S^1 \) with \( \Phi(x, -y) = \Phi(x, y) \) survive in \( S^1/Z_2 \). There is, however, a second possibility: fields with \( \Phi(x, -y) = -\Phi(x, y) \). The reason is that a field by itself is not physical, only the action is. If the 5-dimensional action on \( S^1 \) contains only even powers of \( \Phi \), then the action will be invariant under \( y \to -y \) even if \( \Phi(x, -y) = -\Phi(x, y) \). In other words, we may include the global \( Z_2 \) symmetry \( \Phi \to -\Phi \) in the \( Z_2 \) modding and obtain a consistent theory on the orbifold. This is more clear if one defines directly the theory on \( S^1/Z_2 \) instead of deforming the parent \( S^1 \) theory.

Take a 5-dimensional real scalar field:

\[
S_5 = -\int d^5x \left( \frac{1}{2} \partial_M \Phi \partial^M \Phi + V(\Phi) \right). \tag{11}
\]

Its motion is obtained imposing \( \delta S = 0 \), with

\[
\delta S = -\int d^5x \left( \frac{\partial L}{\partial \Phi} \delta \Phi + \frac{\partial L}{\partial (\partial_M \Phi)} \delta (\partial_M \Phi) \right) = -\int d^5x \left( \frac{\partial V}{\partial \Phi} \delta \Phi + \partial^M \Phi \partial_M (\delta \Phi) \right) = \int d^4x \int_0^{\pi R} dy \left[ (\partial_M \partial^M \Phi - \frac{\partial V}{\partial \Phi}) \delta \Phi - \partial_M (\partial^M \Phi \delta \Phi) \right]. \tag{12}
\]

The first term equal zero gives the 5-dimensional equations of motion, and the second term implies the boundary conditions (BCs) \( \Phi \to 0 \) at \( x^\mu \to \infty \) and \( (\delta \Phi \partial_5 \Phi)_{y=0,\pi R} = 0 \). At each 4-dimensional brane this can be satisfied in two different ways:

\[
\partial_5 \Phi = 0 \quad \text{Neumann},
\]

\[
\Phi = 0 \quad \text{Dirichlet}. \tag{13}
\]

The KK expansion of \( \Phi \) will be affected by these boundary conditions. If \( \Phi \) satisfies the Neumann boundary condition at \( y = 0 \) and \( y = \pi R \) we have

\[
\Phi_+(x, y) = \frac{1}{\sqrt{\pi R}} \phi_+^{(0)} + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_+^{(n)} \cos \frac{ny}{R}, \tag{14}
\]

whereas a field with Dirichlet boundary conditions must be expanded

\[
\Phi_-(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)} \sin \frac{ny}{R}. \tag{15}
\]

The KK modes above have been normalized so that upon integration of the extra dimension they have 4-dimensional canonical kinetic terms. Notice that in both cases the boundary conditions imposed by the orbifold eliminate (project out) half the KK tower living in the
circle and, most important, for a Dirichlet boundary condition there is not a zero mode. In a general theory, the compactification on $S^1/Z_2$ implies a $Z_2$ parity that can be used to break gauge symmetries or to define chiral fermions. More precisely, it is easy to see that the $A_\mu$ and $A_5$ components in a vector field or the $\Psi_L$ and $\Psi_R$ spinors in a 5-dimensional fermion have opposite parities; as a consequence the KK tower of one of those fields (e.g., $A_5$ and $\Psi_R$) will not include a massless mode.

2.3 Curvature

The 5-dimensional space defined in the previous section has a non-trivial topology but a flat metric. RS found a very interesting deformation of this space: they introduced constant energy densities in the bulk ($\Lambda_5 = -6k^2\tilde{M}_5^3$) and the two 4-dimensional branes ($\Lambda_0 = -\Lambda_{\pi R} = \Lambda_5/k$),

$$S \supset \int d^4x dy \left[ \sqrt{-g} \left( \frac{1}{2} \tilde{M}_5^3 R + \Lambda_5 \right) + \sqrt{-g_0} \delta(y) \Lambda_0 + \sqrt{-g_{\pi R}} \delta(y - \pi R) \Lambda_{\pi R} \right],$$  \hspace{1cm} (16)

so that the space becomes a 5-dimensional slice of anti-de Sitter (AdS$_5$). The tuning of these three energy densities is equivalent to the requirement of a vanishing cosmological constant in a 4-dimensional theory. The Einstein equations for this action are solved by the metric\textsuperscript{†}

$$ds^2 = e^{2ky}f_{\mu\nu} dx^\mu dx^\nu + dy^2.$$ \hspace{1cm} (17)

Integrating $y$ in Eq. (16) one finds a 4-dimensional Einstein-Hilbert action with [20]

$$\tilde{M}_P^2 = \tilde{M}_5^3 \int_0^{\pi R} dy \ e^{2ky} = \frac{\tilde{M}_5^3}{2k} \left( e^{2k\pi R} - 1 \right).$$ \hspace{1cm} (18)

This relation between the fundamental scale $\tilde{M}_5$ and $\tilde{M}_P$ generalizes the one in ADD. If $k \to 0$, taking $e^{2k\pi R} \approx 1 + 2k\pi R$ we have the ADD relation $\tilde{M}_P^2 = \tilde{M}_5 L$, but if $kR > 1$ then $e^{2k\pi R}$ may be much larger than 1 (implying $\tilde{M}_5 \ll \tilde{M}_P$) even if all the scales ($k$, $R^{-1}$ and $\tilde{M}_5$) are similar. This scenario allows for exponentially different scales to coexist at different points of the fifth dimension, as the natural scale $\tilde{M}_5$ at $y = 0$ is blue shifted by the metric to $\tilde{M}_P$ at the UV ($y = \pi R$) brane.

The quantum fluctuations of the metric in Eq. (16) will include the massless graviton $h^{(0)}_{\mu\nu}$, the radion $h^{(0)}_{55}$ and a KK tower of massive gravitons $h^{(n)}_{\mu\nu}$, whereas the zero mode of

\textsuperscript{†} Contrary to the original RS model, we place the infrared (IR) brane at $y = 0$ and the UV brane at $y = \pi R$. This set up, proposed in [20], can be easily obtained from the usual one in [5] by redefining $y \to \pi R - y$. 

8
$h_{\mu 5}$ is projected out by the orbifold boundary conditions and the massive modes ($h^{(n)}_{\mu 5}$ and $h^{(n)}_{5 5}$) are eaten by the KK gravitons. The equations of motion ($\delta S = 0$) for $h_{\mu \nu}(x, y)$ on this warped orbifold are:

$$\partial_{\rho} \partial^{\rho} h_{\mu \nu} + e^{-2ky} \partial_{5} \left( e^{4ky} \partial_{5} h_{\mu \nu} \right) = 0,$$

(19)

with $\partial_{5} h_{\mu \nu} = 0$ at $y = 0, \pi R$. The KK expansion is then

$$h_{\mu \nu}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} h^{(n)}_{\mu \nu}(x) f^{(n)}(y),$$

(20)

where $f^{(n)}$ is an eigenfunction of $p_{y}^{2}$ (i.e., $\partial_{y} f^{(n)} = m_{n}^{2} f^{(n)}$) with normalization

$$\int_{0}^{\pi R} dy \ e^{2ky} f^{(n)}(y) f^{(m)} = \pi R \delta_{mn}.$$  

(21)

This means that $f^{(n)}$ satisfies

$$\left[ e^{-2ky} \frac{d}{dy} \left( e^{4ky} \frac{d}{dy} \right) + m_{n}^{2} \right] f^{(n)}(y) = 0,$$

$$\left. \partial_{y} f^{(n)}(y) \right|_{y=0,\pi R} = 0.$$  

(22)

The solutions can be given in terms of Bessel functions:

$$f^{(n)}(y) = \frac{z_{n}^{2}}{N_{n}} \left[ J_{2}(z_{n}) + c_{n} Y_{2}(z_{n}) \right],$$

(23)

where $z_{n} \equiv m_{n} e^{-ky}/k$ and the constants $N_{n}$ and $c_{n}$ are fixed by the orthonormality conditions and the boundary condition at $y = 0$ [20]. The mass eigenvalues $m_{n}$ are then obtained from the boundary condition at the UV brane. For $k < 1/R$ it results the usual KK spectrum on the circle, $m_{n} \approx n/R$, but with half the modes (the orbifold projects out the other half).

For $k > 1/R$, however, the graviton masses ($n > 0$) become

$$m_{n} \approx \left( n + \frac{1}{4} \right) \pi k.$$  

(24)

So it is the curvature $k$ and not $1/R$ what defines the mass of the first excitation and the gap between KK modes. Finally, the interaction of these gravitons with a field at the IR ($y = 0$) brane is deduced from

$$S \supset -\frac{1}{M_{5}^{3/2}} \int d^{5}x e^{4ky} h_{\mu \nu}(x, y) T_{\mu \nu}(x) \delta(y),$$

(25)

which upon integration on $y$ implies the 4-dimensional Lagrangian

$$\mathcal{L} = -T_{\mu \nu} \left( \frac{1}{M_{P}} h^{(0)}_{\mu \nu} + \sum_{n=1}^{\infty} \frac{1}{\Lambda_{n}} h^{(n)}_{\mu \nu} \right).$$

(26)
When the curvature is negligible one obtains \( \Lambda_n^{-1} = \sqrt{2}/\bar{M}_P \) (the flat result on the orbifold), whereas for \( k < 1/R \) the curvature pushes the KK modes towards the IR brane and increases their couplings to \( \Lambda_n^{-1} = e^{k\pi R}/\bar{M}_P \) [20]. It is interesting to notice that, if we define

\[
m_c \equiv \begin{cases} R^{-1} & \text{if } k < R^{-1} \\ \pi k & \text{if } k > R^{-1}, \end{cases}
\]

then we can write an expression for the scale \( \Lambda_n \) that works in both limits:

\[
\Lambda_n^2 = \frac{\bar{M}_5^3 \pi}{2 m_c} = \frac{M_5^3}{4 m_c},
\]

where we have used \( M_5^3 = 2\pi \bar{M}_5^3 \). This common expression for the scale \( \Lambda_n \) setting the strength of the massive graviton interactions lets us understand easily the gravitational potential both in the flat and the warped cases [34]. Consider first an ADD model (\( k = 0 \)) with one extra dimension compactified on the orbifold and a fundamental scale \( \bar{M}_5 = 1 \) TeV. To reproduce \( \bar{M}_P = 2.4 \times 10^{18} \) GeV we need \( L = \bar{M}_P^2/\bar{M}_5^3 = 7.4 \) AU, or \( m_c = 5.4 \times 10^{-19} \) eV. At distances \( r > 7.4 \) AU the potential created by each KK graviton is suppressed by a Yukawa factor of \( e^{-2mr} \) and we can neglect its contribution to the usual (4-dimensional) Newton potential. At \( r < L \), however, all the KK gravitons of mass \( m_n < 1/r \) will be active. Since their multiplicity is \( r^{-1}/m_c \) and they couple with the same strength, their effect in the potential is to change

\[
\frac{1}{\bar{M}_P^2} \to \frac{4 m_c}{\bar{M}_5^3} \times \frac{r^{-1}}{R^{-1}} = \frac{4}{\bar{M}_5^3 r}. \tag{29}
\]

Now consider a 5-dimensional set up with also \( \bar{M}_5 = 1 \) TeV but a non-zero value of \( k > R^{-1} \), for example, \( k = 0.3 \) GeV, which implies \( m_c = 1 \) GeV. The 4-dimensional Newton potential dictated by the massless graviton will now extend down to distances \( r \) of order \( 1/m_c = 0.2 \) fm, whereas at smaller distances the effect of the KK excitations gives

\[
\frac{1}{\bar{M}_P^2} \to \frac{4 m_c}{\bar{M}_5^3} \times \frac{r^{-1}}{\pi k} = \frac{4}{\bar{M}_5^3 r}. \tag{30}
\]

At these shorter distances the flat and the warped results coincide. The density of KK modes in the warped case is lower when \( k > R^{-1} \), but their coupling to matter is stronger and both effects compensate: a single graviton of mass \( m_n \) in the warped model creates the same potential as all the KK modes with mass between \( m_{n-1} \) and \( m_n \) in the flat case. The curvature in a RS model provides then an extra parameter that just rises the mass of the first KK mode but implies, at distances \( r < m_c \), the same gravitational interaction as the simpler ADD scenario.
3 Transplanckian collisions

A $2 \rightarrow 2$ collision between light particles with the mediator in the $t$-channel is characterized by two kinematical variables: the c.o.m energy $\sqrt{s}$ and the momentum transfer $q = \sqrt{-t}$, with $s$ and $t$ the usual Mandelstam variables. In the weakly coupled regime $q$ defines the typical impact parameter in the collision, $b \approx 1/q$, and its value will determine the scattering angle in the c.o.m frame: a forward collision corresponds to a long-distance interaction with a small value of $q$, whereas larger values of $q$ up to $\sqrt{s}$ imply short distance processes and larger angles. Take a neutrino of energy $E$ scattering off a particle of mass $m$ initially at rest. The neutrino will lose in the collision a fraction $y$ of its energy (the inelasticity), whereas the target particle will gain an energy $yE$. It is easy to see that $q$ also determines the value of $y$: $y = q^2/s$. Therefore, one may refer to a low-$q$ process as a forward or a soft collision.

In the 5-dimensional model of TeV gravity outlined in the previous section, we will be interested in transplanckian collisions of $\sqrt{s} > M_5$. An obvious objection would be whether such collisions would require a UV complete theory of gravity. This, however, is not the case: all the processes of interest will be dominated by long-distance interactions that are insensitive to the UV physics. Let us discuss this in some detail.

3.1 Eikonal amplitude

Graviton-mediated interactions are better understood in impact parameter space [21–26]. At distances $b$ larger than the inverse mass of the first KK graviton the scattering amplitude should be frozen, as the massless graviton couples with a strength suppressed by $M_P$ and the KK modes do not reach beyond $b \approx m_i^{-1}$. As the distance decreases more KK gravitons become active, and the process is described by an eikonalized amplitude $A_{\text{eik}}(s, t)$ that includes the infinite set of ladder and cross-ladder diagrams. Basically, $A_{\text{eik}}(s, t)$ is the exponentiation of the Born amplitude in impact parameter space:

$$A_{\text{eik}}(s, t) = \frac{2s}{t} \int d^2b \ e^{i\mathbf{q} \cdot \mathbf{b}} \left(e^{i\chi(s, b)} - 1\right),$$

(31)

where $\chi(s, b)$ is the eikonal phase, $b$ spans the 2-dimensional impact parameter space and $t \approx -q^2_\perp$. The eikonal process will be reliable as far as the integral is dominated by $b > r_H$ (see below), and it reduces to the Born amplitude for a small eikonal phase. In the transplanckian regime it is also independent of the spin of the colliding particles. The phase $\chi(s, b)$ can be

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4In terms of the metric defined in the previous section, $s = -(p_1 + p_2)^2$ and $t = -(p_1 - p_3)^2$.
deduced from the Fourier transform to impact parameter space of $A_{\text{Born}}(s, t)$:

$$\chi(s, b) = \frac{1}{2s} \int \frac{d^2q}{(2\pi)^2} e^{-i\mathbf{q} \cdot \mathbf{b}} A_{\text{Born}}(s, -q^2) .$$  (32)

Our Born amplitude comes from the $t$-channel exchange of the KK graviton tower:

$$A_{\text{Born}}(s, t) = -\frac{4m_c s^2}{M_5^3} \sum_{n=1}^{\infty} \frac{1}{t - (nm_c)^2}$$

$$= \frac{2\pi s^2}{M_5^3 q} \left( \coth \frac{\pi q}{m_c} - \frac{m_c}{\pi q} \right),$$  (33)

where $q = \sqrt{-t}$ and we have not included the contribution of the massless graviton (with a much smaller coupling than the massive modes if $m_c > R^{-1}$). At $q > m_c$ we have $\tilde{A}_{\text{Born}} \approx 2\pi^2 s^2/(M_5^3 q)$ whereas at $q \to 0$ the amplitude becomes $A_{\text{Born}}(s, 0) = 2\pi s^2/(3M_5^3 m_c)$. In our calculation of $A_{\text{eik}}$ we will use the first expression for all the values $q$ and will then correct the result for low $q$. The eikonal phase is in that case

$$\chi(s, b) = \frac{s}{2M_5^3} \int_0^\infty dq \ J_0(qb) \equiv \frac{b_c}{b},$$  (34)

with $b_c = s/(2M_5^3)$. The divergence in the eikonal phase at short distances ($b = 0$) does not affect the amplitude in Eq. (31): the contributions from the region $b \ll b_c$ are quickly oscillating and tend to cancel. This is also the basic reason why $A_{\text{eik}}$ is insensitive to the UV completion of gravity.

The eikonal amplitude can then be written

$$\tilde{A}_{\text{eik}}(s, t) = 4\pi s b_c^2 F_1(b_c q),$$  (35)

where the tilde indicates that the expression is not valid at $q \to 0$ and

$$F_1(u) = -i \int_0^\infty dv \ v \ J_0(uv) \left( e^{iv-n} - 1 \right).$$  (36)

It is easy to see that the integral defining $A_{\text{eik}}(s, t)$ in Eq. (31) is dominated by a saddle point at $b_s = (q^2 b_c)^{-1}$ if $q > b_c^{-1}$ and by $b \approx q^{-1}$ for $q < b_c^{-1}$. The modulus of the complex function above is $|F_1(u)| \approx 1/\sqrt{1.57v^3 + u^2}$, and $\tilde{A}_{\text{eik}}$ can be corrected at low $q$ by reintroducing the factor that we took from $A_{\text{Born}}$ in Eq. (33):

$$A_{\text{eik}}(s, t) = 4\pi s b_c^2 F_1(b_c q) \left( \coth \frac{\pi q}{m_c} - \frac{m_c}{\pi q} \right).$$  (37)

In Fig. 4 we provide a plot of the absolute value of these amplitudes. Notice that at $q < b_c^{-1}$ the eikonal and the Born amplitudes coincide, and that at $q > m_c$ the correction factor goes to 1 and $A_{\text{eik}} \approx \tilde{A}_{\text{eik}}$. 

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Figure 4: Absolute value of the amplitudes $A_{\text{Born}}$ (in Eq. 33), $\tilde{A}_{\text{eik}}$ (in Eq. 35) and $A_{\text{eik}}$ (in Eq. 37). We have taken $M_5 = 1$ TeV with $s = (10 \text{ TeV})^2$ (upper) or $s = (100 \text{ TeV})^2$ (lower) and $m_c = 0.1 \text{ GeV}$ (left) or $m_c = 1 \text{ GeV}$ (right).
3.2 Transplanckian collisions at shorter distances

The eikonal description will prevail as far as the dominant impact parameter $b_s$ is larger than the Schwarzschild radius $r_H$ of the system:

$$r_H(s) = M_5^{-1} \sqrt{\frac{2}{3\pi}} \left( \frac{s}{M_5^2} \right)^{1/4}.$$

As $b$ gets smaller, however, $q$ and the inelasticity $y$ grow, non-linear corrections become important and two other processes dominate: the emission of soft gravitons (bremsstrahlung) [27] and the capture of the incident particle to form a microscopic black hole (BH) [28–31]. Let us discuss these processes in some more detail.

Graviton emission appears as an imaginary contribution to the eikonal phase corrected by H diagrams ($\chi_H$). This contribution is of absorptive type, it damps the elastic cross section showing a Bloch-Nordsieck mechanism at work. For a given value of $b$, the average number $N$ of gravitons radiated during the scattering can be read directly from $\chi_H$:

$$N = \text{Im} (\chi_H) \approx \left( \frac{b_r}{b} \right)^5,$$

with $b_r = r_H (b_c/r_H)^{1/5}$. Therefore, the typical transverse momentum radiated in the process is $Q \approx N b^{-1}$. To obtain the energy lost by the incoming particle this momentum must be boosted from the c.o.m. to the target rest frame. In an eikonal scattering the dominant impact parameter is $b \approx b_s$. Both the number of gravitons $N \approx y^{5/4} (s/M_5^2)^{3/4}$ and the energy that each one carries decrease at small $y$, implying that for $y \ll 1$ the amount of gravitational radiation during the scattering is small.

In a collision at $b \approx r_H$ the incident particle will transfer a large fraction of its momentum to the target, changing its trajectory and losing to radiation a significant fraction of energy. At these and smaller values of $b$ one expects the formation of a microscopic BH. Notice that $r_H$ grows with the c.o.m. energy as $s^{1/4}$, i.e., the larger the energy the larger the transverse distance with the target that is sufficient to place the whole system inside the gravitational horizon. Classical (long-distance but non-perturbative) gravity is then all we need to describe these collisions. It is not that massive physics, like a $Z$ boson or a string excitation, are not to be produced in these transplanckian processes [32, 33]. The main point is that all this short-distance physics occurs inside the BH horizon, and thus it is unable to change the sequence of events that we see outside. At energies not too far from $M_5$ (e.g., $\sqrt{s} \approx 10 M_5$), however, it has been shown that a number of factors (angular momentum, charge, geometry of the trapped surface or total radiation before the collapse) make a precise estimate of BH production difficult.
3.3 Neutrino-nucleon cross section

The model under study has then two unrelated parameters: the scale \( M_5 \approx \text{TeV} \) where gravity becomes strong and the mass \( m_c \) of the first KK excitation [34]. We have learned in the previous subsection that the second parameter is only relevant in long-distance interactions, in particular, it can be used to suppress the soft contributions of \( q < m_c \).

Let us now consider a collision of a neutrino of energy \( E = 10^9 \text{ GeV} \) with a nucleus at rest for \( M_5 = 2 \text{ TeV} \). At very low momentum transfer, \( q \leq r_p^{-1} \approx 1/(0.2 \text{ GeV}) \) the proton radius, the neutrino may interact coherently with a nucleon, and at even smaller values of \( q \) it may do it with the whole nucleus. It is easy to see, however, that such collisions imply a very low inelasticity \( y \leq 2.7 \times 10^{-11} \) (i.e., energy depositions below 20 MeV) and a cross section

\[
\frac{d\sigma_{\text{eik}}}{dq^2} = \frac{1}{16\pi s^2} |A_{\text{eik}}|^2
\]

of order \( 0.1 \mu\text{b} \). Therefore, the main effect will appear at shorter distances, when the neutrinos exchange momenta \( q > 1 \text{ GeV} \) with the partons inside a nucleon. The differential \( \nu N \) cross section is then

\[
\frac{d\sigma_{\text{eik}}}{dy} = \int_{M_5^2/s}^1 dx \frac{1}{16\pi x s} |A_{\text{eik}}(x, y)|^2 \sum_{i=q,\bar{q},g} f_i(x, \mu),
\]

where \( y = q^2/(xs) \), we restrict to transplanckian collisions \( (xs \geq M_5^2) \), and the PDFs \( f_i(x, \mu) \) must be calculated at \( \mu = b_s^{-1} \) for \( q > b_c^{-1} \) and \( \mu = q \) when \( q < b_c^{-1} \). Notice that quarks and gluons interact with the same amplitude. In Fig. 5 we plot this cross section for \( E = 10^9 \text{ GeV} \), \( M_5 = 2 \text{ TeV} \) and \( m_c = 0.5, 5 \text{ GeV} \).

As for BH production in neutrino–parton interactions, the cross section (also in Fig. 5) can be estimated as

\[
\sigma_{\text{BH}} = \int_{M_5^2/s}^1 dx \frac{\pi r_H^2}{s} \sum_{i=q,\bar{q},g} f_i(x, \mu),
\]

with \( r_H(xs) \) given in Eq. (38) and \( \mu = r_H^{-1} \). The eikonal process is dominated by much larger impact parameters, and its overlapping with the (inclusive) geometrical cross section \( \sigma_{\text{BH}} \) will be negligible. The BH, of mass \( M_{BH} \approx \sqrt{xs} \) and temperature \( T = 1/(2\pi r_H) \), will evaporate almost instantly into SM particles (see [35] and references therein).

In summary, TeV gravity implies a \( \nu N \) cross section that grows fast with the energy above the threshold \( E_{\text{th}} \approx M_5^2/(2m_p) \) defining the transplanckian regime. In particular, the collision of an ultrahigh energy neutrino with the partons inside the nucleon may give events where the neutrino deposits a small fraction \( y = 10^{-7} - 10^{-3} \) of its energy and keeps going. The cross section \( \sigma_{\text{eik}} \) for these collisions can be well above the standard one mediated by
Due to their increased coupling to matter (relative to the massless graviton), in the early universe they may be in thermal equilibrium at temperatures \( T \geq m_c \). If their mass is larger than 10 MeV, however, at the time of primordial nucleosynthesis all of them will be gone and their decay products thermalized: these massive gravitons are then cosmologically safe.

In astrophysics, if their mass is below 100 MeV they may be produced abundantly in protoneutron stars during a core collapse. For \( m_c = 50–100 \) MeV and \( M_5 \geq 2 \) TeV the

\[
\tau \approx \frac{50\pi}{4n_V + n_{19}} \frac{M_5^3}{m_1^4} = 4 \times 10^{-14} \text{s} \left( \frac{1 \text{ GeV}}{m_1} \right)^4 \left( \frac{M_5}{2 \text{ TeV}} \right)^3. \quad (43)
\]

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In astrophysics, if their mass is below 100 MeV they may be produced abundantly in protoneutron stars during a core collapse. For \( m_c = 50–100 \) MeV and \( M_5 \geq 2 \) TeV the
short lifetime of these gravitons does not let them escape the core and change significantly
the dynamics of the explosion. In particular, they would not shorten the duration of the
neutrino signal produced during a supernova explosion. At lifetimes $\tau \approx 10^{-4}$ s these
massive gravitons could even play a role in the revival of the stalled shock front in a core
collapse [40–42].

In collider experiments they may introduce displaced vertices or rare decays (e.g., $K \to \pi g^{(n)}$), but their production cross section is always suppressed by inverse powers of $M_5$ and several TeV may suffice to evade all constraints.

Finally, there may also be bounds from air shower experiments. Ultrahigh energy cosmic
rays have strong interactions with matter, so gravity may only introduce order 1 corrections
that do not seem excluded. We are left with airs showers produced by high energy neutrinos.
The AUGER observatory, in particular, looks for inclined events that start deep into the
atmosphere, setting constraints on a neutrino flux at $10^8$–$10^{11}$ GeV [43]. Strong TeV gravity
will not change the neutrino interactions at $E_\nu < 1$ PeV, but it could multiply by hundred
the cross section of cosmogenic neutrinos at higher energies. An obvious observation is then,
would this large cross section introduce a signal in air shower experiments detectable at
AUGER? Not necessarily. The reason is that, although large, the gravitational cross section
is very soft: a $10^9$ GeV neutrino will typically start a TeV–PeV atmospheric shower, which
is below the energy threshold at AUGER. The same argument would apply to ANITA or
LUNASKA: they search for a single $10^{10}$ GeV energy deposition from a cosmogenic neutrino,
but the typical inelasticity in an eikonal collision is just $y \approx 10^{-5}$.

A cosmogenic neutrino would leave a very characteristic signal in a km³ telescope [34];
let us briefly discuss its main features.

- First, as we have already mentioned, the typical energy of an event is not the energy of
  the cosmogenic neutrino; eikonal collisions are very soft and translate into TeV–PeV
  energy depositions.

- Second, the signal will always come from downgoing or near-horizontal directions,
ever from upgoing directions. The reason is that $10^8$–$10^{11}$ GeV neutrinos are unable
to cross the Earth. In Fig. 6 we plot the probability that a high-energy neutrino reaches
IceCube from different zenith angles without experiencing a standard interaction with
matter. Hard gravitational interactions, which may reduce further the reach of these
neutrinos, have not been included.

- In addition, the signal will only introduce shower events, never tracks. Notice that
  the eikonal amplitude describing multigraviton exchange does not change the incident
Figure 6: Probability $P_{\text{surv}}$ that a neutrino reaches IceCube from a zenith angle $\theta_z$ for several energies $E_\nu$ (we have used the $\nu N$ cross section in [44]).

• Finally, the eikonal scattering will not stop the cosmogenic neutrino: after the first collision the neutrino keeps going with basically the same energy and can interact multiple times with matter, possibly inside the detector, before it has a harder (standard or gravitational) interaction that reduces its energy.

The last point is specially interesting, as there are no standard events giving double bangs of TeV–PeV energy. Tau neutrinos may indeed produce this topology, but at much higher energies: we need $E_\nu \approx 10^8$ GeV to expect 100 meters between the creation and the decay points of a tau lepton. If, for example, the eikonal cross section is $\sigma_{\text{eik}} = 4 \mu\text{b}$, then the mean free path between interactions in ice would be around $\lambda_{\text{eik}} = 4$ km. It is easy to deduce that the probability to have $N$ interactions (bangs) along a length $L$ is [26]

$$P_N(L) = e^{-L/\lambda_{\text{eik}}} \frac{(L/\lambda_{\text{eik}})^N}{N!}.$$  \hspace{1cm} (44)

In that case the neutrino would have a 19.5% probability for a single interaction when crossing 1 km of ice or a 2.4% probability to produce a double bang within the same distance.

5 Summary and discussion

In models with extra dimensions the fundamental scale $M_D$ of gravity may take any value between $10^3$ and $10^{19}$ GeV, depending on the details (topology, length, curvature) of the
compactification space. Due to the spin 2 of the graviton, in the transplanckian regime ($s > M_D^2$) any collision is dominated by classical (long-distance but non-perturbative) gravity, as all the short-distance physics is trapped inside a BH horizon. At impact parameters larger than $r_H$ one expects graviton mediated interactions with a large cross section but of very small inelasticity. In particular, a cosmogenic neutrino of $10^8$–$10^{11}$ GeV could scatter off matter and produce a TeV–PeV shower. This scenario is actually a particular realization of a more general one where the UV completion of the SM occurs (at the TeV or at a higher scale) through *classicalization* [45].

The signal suggested by TeV gravity at large neutrino telescopes consists then of an excess of TeV–PeV shower events from downgoing directions. Indeed, the IceCube’s excess of high energy starting events [46] exhibits a preference for downgoing versus upgoing directions and for showers versus tracks, so there may be room for this type of physics in the current data.

There are two kind of observations that could clearly favor these scenarios. First, double-bang events of TeV–PeV energy. Second, the *absence* of cosmogenic neutrinos. If $10^8$–$10^{11}$ GeV neutrinos do not appear in current and future searches, the reason could be that at those ultrahigh energies they do not look like neutrinos anymore. Instead of an invisible particle able to penetrate 2 km of ice and deposit $10^{10}$ GeV, TeV gravity could turn them into a particle that interacts frequently (*e.g.*, every 0.1–1 km of ice) but deposits a much smaller amount of energy. Its detection may then require a different strategy. At any rate, these are two questions that the next generation of neutrino telescopes should be able to answer.

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