Kinematics of the Broad Emission Line Region in NGC 5548

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ABSTRACT

We derive both total flux and velocity-resolved response functions for the CIV 1549 emission line from the data obtained in the 1993 NGC 5548 monitoring campaign. These response functions imply:
1.) the emission region stretches from inside 1 lt-d to outside 10 lt-d, and is probably better described as round than flat; 2.) the velocity field is dominated by a red/blue symmetric component (e.g. 2–d or 3–d random motions, or rotation in a disk) but there is also significant radial infall. Quantitative modelling indicates that the random speeds are typically a few times as large as the radial speed. However, no simple model gives a completely acceptable fit to the data. These inferences rule out numerous simple and otherwise plausible models for broad line region dynamics, including outflowing winds, radial free-fall, rotation in a disk, or collisionless orbital motion.
1. Introduction

One of the most salient characteristics of active galactic nuclei is their strong, broad emission lines. Ever since the discovery of these objects (Seyfert 1943), the origin of the large velocities implied by these widths has been the subject of much discussion and speculation (e.g. Mathews & Capriotti 1985). In addition to the intrinsic interest in answering this question, there has also long been hope that if we could learn the nature of the motions in the broad emission line region, we could use that material as diagnostics of the accretion flow generally believed to power the central engine.

Despite the many reasons to investigate the kinematics of the broad emission line gas, there remains considerable uncertainty about their true nature. Early work that tried to determine the kinematics of the BLR relied on fitting profile shapes to particular velocity models (for example, infall, outflow, random, disk etc.). However, this technique has difficulty producing unambiguous results because there is too much freedom to change the distribution of physical conditions, and hence the distribution of line emissivity (Shields 1978; Capriotti et al. 1980; Wu et al. 1981; Mathews 1982; Wilkes 1984; Kallman et al. 1993).

Velocity-resolved reverberation mapping is potentially a much more powerful method (Blandford & McKee 1982; Welsh & Horne 1991; Perez et al. 1992). This method rests on a comparatively clean theoretical base: If the emission lines are powered by photoionization due to the continuum radiated by the central engine, the emission line light curves should be delayed and smoothed replicas of the continuum light curve. This is because the light travel time from the continuum source to the emission line region \[ \sim (1 - 100) L_{44}^{1/2} \text{ lt-d} \], where \( L_{44} \) is the ionizing continuum luminosity in units of \( 10^{44} \text{ erg s}^{-1} \) is likely to be significant both on the human timescale and with respect to the timescale of
continuum variations, while the local response times within the line-emitting gas are probably many orders of magnitude shorter. In principle, then, one should be able to unfold the fluctuation histories of the continuum and the emission lines in order to map the line response onto the paraboloidal surfaces of constant delay. By doing this separately for segments of the lines with different line of sight velocities, one obtains a picture of the correlation between velocity and position. Qualitatively, whichever side of the line responds with the least delay to continuum fluctuations is the one predominantly made on the near side of the source; i.e. if the red wing leads, infall prevails, while if the blue wing leads, the flow is dominated by outward motion. If both sides move together, there is little net radial motion.

However, to fully recover a map of the velocity-resolved line response requires large amounts of high signal/noise data well-sampled on the relevant timescales. Such campaigns require immense observing effort. For this reason, the only attempt at this program made by anyone hitherto was the work by Wanders & Horne 1994, who used ground-based monitoring data on the type 1 Seyfert galaxy NGC 3516 to find the velocity-resolved response of the H\(\alpha\) line. While they were able to demonstrate that the motions could not be predominantly radial, they were hampered by limited sampling. Their data included only 18 measurements of the line profile, and these were spaced irregularly with a mean interval of \(\approx 9\)d, while the response was almost entirely more rapid than \(\approx 20\)d.

Because poor temporal sampling of the lightcurves is very common, many have pragmatically adopted a more limited goal: to recover only a weighted (and biassed: Edelson & Krolik 1988) moment of the response function (the characteristic lag) by studying the cross-correlation of the continuum fluctuations with the line fluctuations, and dividing the line profile into only two segments,
the red and blue halves. Those studies with the least sparse data also support the idea that radial motions are not the dominant velocity field, most notably in NGC4151 (Clavel et al. 1987; Gaskell 1988; Maoz et al. 1991), NGC5548 (Koratkar & Gaskell 1991a; Koratkar & Gaskell 1991b; Korista et al. 1995) and Mkn 279 (Stirpe et al. 1994). However, many of these studies (particularly the earlier ones) also suffered from inadequate temporal resolution. In addition, there have been conflicting claims regarding the same objects, most notably the subject of this paper, NGC 5548. Rosenblatt & Malkan 1990 favored radial outflow with obscuration; Crenshaw & Blackwell 1990 argued for radial infall; Koratkar & Gaskell 1991a saw evidence for primarily random motions; while Malkov 1993 interpreted the data as showing bi-conical infall.

The 1993 monitoring campaign on NGC 5548 (Korista et al. 1995) provides the best dataset so far from which to recover the full velocity-resolved response function. That campaign obtained 39 UV spectra of the nearby (z = 0.0174) type 1 Seyfert galaxy NGC 5548 with the HST, taken at 1d intervals. In coordination with the HST observations, IUE spectra were obtained every 2d, beginning 36d before the first HST spectrum, and continuing to the end of the HST campaign. Because (as we shall show) there is significant response at lags up to at least $\approx 20d$, the continuum fluxes recorded with IUE give substantial aid to the analysis of the HST emission line light curves.

Two main considerations limit the choice of emission lines for velocity-resolved reverberation mapping. First, stronger lines are preferred so that the S/N remains high even when they are divided into segments. Because stronger lines carry a larger portion of the total emission line flux, they are also more “representative” of the emission line region as a whole. Second, it is desirable that any absorption features or different lines blended into the main feature be as weak as possible. Applying these criteria to the NGC 5548 spectrum identifies
CIV 1549 as by far the best line to use. It is the strongest single line in the spectrum, exceeding even Lyα in mean flux. In addition, the only significant blending in CIV 1549 is a small contribution to its extreme red wing by HeII 1640, although it does have two weak absorption features a few hundred km/s to the blue of the systemic redshift (Korista et al. 1995). By contrast, Lyα is strongly contaminated in its blue wing by geocoronal Lyα, in its red wing by NV 1240, and is strongly absorbed. The next strongest line after Lyα is CIII] 1909, but its flux is only $\sim 0.2 \times$ that of CIV 1549 (Korista et al. 1995), and it is also blended with SIII] and AlIII. For these reasons, we concentrate exclusively on CIV 1549 in the present work.

2. The Data

In the following analysis we use two data sets: the HST data by itself, and the HST data combined with the IUE data. We use the combined data set only for the continuum lightcurve; all CIV 1549 data that we use is taken solely from HST observations.

The statistical errors associated with the HST measurements are extremely small, around $\sim 0.3\%$ rms for the total flux in CIV, and $\sim 1.7\%$ for the continuum (Korista et al. 1995). However, the known systematic errors in the repeatability of FOS short timescale (orbit to orbit) photometry are at least 1.4% rms, from analysis of well centered, standard star calibrations (Korista et al. 1995). Not all the spectra of NGC5548 are well centered, so this is a lower limit to the size of the systematic errors. The true size of the systematic error in these data is not well known: Korista et al. 1995 estimate that it lies within the range of 2 – 4.5%. As these are systematic errors they are not independent (the continuum and all the CIV velocity-resolved line components should be
affected in the same way in a given spectrum, but are uncorrelated with respect to systematic errors in other spectra). Because there is no obvious way to take account of this, we add an error of 2% in quadrature to the continuum, total CIV, and velocity-resolved lightcurves. Note that this procedure leads to a slight overestimate of the amount of correlation between the lightcurves at zero lag, but this overestimate should be independent of velocity. Errors for the IUE data are rather larger: typically $\simeq 6\%$, but with a large dispersion from spectrum to spectrum.

When we discuss the total CIV line flux, we use the figures given in Korista et al. 1995. For velocity-resolved work, we divided the CIV profile into four segments of approximately equal mean flux; we call them the blue wing, blue core, red core, and red wing. “Core” denotes the range from zero velocity relative to systemic out to 2456 km s$^{-1}$; “wing” denotes an integration from 2456 km s$^{-1}$ to 10840 km s$^{-1}$. In each case, the flux was obtained by a direct integration from the measured spectra; no special correction was made for the narrow absorption feature which appears within the blue core. This particular velocity cut has the advantage that it gives roughly equal flux in all the line segments. These data and integrations were kindly made available to us by Kirk Korista.

The four lightcurves are generally remarkably similar to each other, showing immediately that the velocity field must be predominantly red/blue symmetric (Korista et al. 1995). Table 1 gives quantitative measures for testing just how similar the lightcurves are. For the total line flux, total continuum flux, and each of the four line segments, it shows: the mean, $<F> = (1/N)\sum_j F_j / \sigma_j^2$; total variance (including the effect of the error bars) $\sigma_{tot}^2 = 1/(N-1)\sum_j (F_j - <F>)^2$; error variance, $\sigma_{err}^2 = N/\sum_j \sigma_j^2$; the fractional intrinsic $r.m.s.$ variability relative to the mean, $\delta F / <F> = \sqrt{(\sigma_{tot}^2 - \sigma_{err}^2)/ <F>^2}$; and the signal/noise ratio.
\[ S/N = \delta F / \sigma_{err}. \]

The dilution of the core components by the (constant) narrow line flux can be estimated in two ways. Firstly, from a previous HST observation of NGC5548 in which the source was in an unusually low state so that the narrow lines were especially prominent (July 1992, Crenshaw et al. 1993). This gave an estimated narrow line flux of \( 7.2 \pm 1.2 \times 10^{-13} \text{ ergs cm}^{-2} \text{ s}^{-1} \), compared to the mean red plus blue core lightcurve flux in these data of \( 3.8 \times 10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1} \), i.e. \( 19 \pm 3\% \) of the total red plus blue core flux. Secondly, fitting the mean HST spectrum gives an (absorption corrected) estimate for the narrow line contribution of \( \sim 5.6 \times 10^{-13} \text{ ergs cm}^{-2} \text{ s}^{-1} \) on a total core flux of \( 3.75 \times 10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1} \), i.e. a 15\% contaminating flux. Systematic errors in modelling the line and absorption features dominate over statistical errors, but clearly we expect the dilution to be at least 10\%. Thus for the two core components, the fractional r.m.s. variability has been adjusted to allow for a 10\% contribution to the mean flux due to the narrow component.

There is a systematic trend for both \( \delta F / <F> \) and \( S/N \) to rise monotonically from the blue wing across the profile to the red wing. This trend is strong enough that the relative source variance of the red wing is significantly different from that of the blue wing (95\% confidence on an F test – ratio of variances – with 39 degrees of freedom). Some of the additional variance in the red wing may be due to contamination from the blue wing of HeII (Korista et al. 1993), but, as we argue in §5.1, we do not believe that this contamination can be large enough to explain much of the difference. Even allowing for only 10\% dilution for the narrow line flux on the red and blue cores, they too have significantly (\( \geq 90 \% \) confidence) more variance than the blue wing.

Thus the line lightcurves on their own show that there are subtle but significant variability differences between the velocity components, in the sense
that there is more variability in the red than the blue side of the line. We will elaborate on how to interpret that contrast in §5.3.

Finally, we note that because the variance is predominantly due to fluctuations on the longest timescales, the fluctuations seen on timescales of 1 – 2d are largely noise. For this reason, when we construct lightcurves with 2d intervals from the HST data, we do so by convolving the 1d sampling with a Gaussian kernel $\propto \exp[-(\Delta t/1d)^2]$.

3. Response Functions

3.1. Methodology

Our fundamental assumption is that the line flux fluctuations at time $t$ are due to earlier fluctuations in the continuum flux. We write the relation between the flux $F_l(t, u)$ in line $l$ at time $t$ and line of sight velocity $u$, and the continuum flux $F_c(t)$ in the form

$$F_l(t, u) = \langle F_l \rangle + \int_0^{\tau_{\text{max}}} d\tau \Psi_l(\tau, u) \left[ F_c(t - \tau) - \langle F_c \rangle(\tau) \right],$$  \hspace{1cm} (1)

where $\tau$ is the time delay, the angle brackets denote a time average, and $\Psi_l(\tau, u)$ is called the “response function” for velocity $u$ of line $l$. As explained in Krolik & Done 1995, the proper time average of the continuum flux is a function of $\tau$.

As is also explained in Krolik & Done 1995, we use the convolution equation between the line and continuum fluctuations rather than between the total line and continuum flux as the line response need not be truly linear (see also Horne 1994).

Equation 1 is an appropriate approximation when the fractional continuum fluctuations are small, i.e.

$$\frac{\delta F_c}{\langle F_c \rangle} < 1.$$  \hspace{1cm} (2)
In addition, the error in the linearized description of the line response is dominated by uncertainties in the data rather than nonlinear corrections to the model when

\[
\max \left[ \frac{\sigma_l}{\langle F_l \rangle}, \frac{\sigma_c}{\langle F_c \rangle} \frac{\partial \ln F_l}{\partial \ln F_c} \right] > \frac{1}{2} \frac{\partial^2 \ln F_l}{\partial (\ln F_c)^2} \left( \frac{\delta F_c}{\langle F_c \rangle} \right)^2 .
\] (3)

Here \( \delta F_{c,l} \) and \( \sigma_{c,l} \) are the rms fluctuations and rms error in the continuum and line, respectively. Photoionization models suggest that for CIV 1549 the first logarithmic partial derivative is \( \simeq 1 \) in the range of ionization parameters likely to apply in NGC 5548. For example, Krolik et al. 1991 found that in NGC 5548 \( \Xi \) was in the range 0.1 – 0.6, corresponding (for the specific continuum shape they chose) to \( U \simeq 0.003 – 0.04 \); their models indicated \( \frac{\partial \ln F_l}{\partial \ln F_c} \simeq 1.1 \).

Similarly, using a variety of continuum shapes, O’Brien et al. 1995, Binette et al. 1989, and Ferland & Mushotzky 1982 predicted that when \( U \sim 0.01 – 1 \), \( \partial \ln F_l/\partial \ln F_c \simeq 1 \) and the second logarithmic partial derivative is \( \simeq -0.4 – -1 \). Considering the HST data alone, \( \delta F_c/\langle F_c \rangle \simeq 0.14, \sigma_c/\langle F_c \rangle = 0.026 \), and \( \sigma_l/\langle F_l \rangle \simeq 0.02 \) for the total CIV flux; we have included the systematic error in these numbers. However, when we form the merged continuum light curve and smooth the HST CIV light curve to 2d resolution, these quantities become \( \delta F_c/\langle F_c \rangle \simeq 0.24, \sigma_c/\langle F_c \rangle = 0.05 \), and \( \sigma_l/\langle F_l \rangle \simeq 0.013 \). Thus, the first condition is easily satisfied, and the second condition is likewise (weakly) satisfied due to the relatively large systematic errors, and the large random errors of the IUE measurements. However, without the systematic errors, the second order corrections would start to dominate. When that is the case, while the linear approximation may still account for much of the system’s behavior, it is no longer possible to find purely linear response models which would fit the lightcurves to within the errors. We caution that progressively better data will therefore require more complex techniques which take into account nonlinear line response (Horne 1994; Pijpers 1994).
We solve the convolution equation by the method of regularized linear inversion (Krolik & Done 1995). Deconvolution is inherently a noise amplifying process, so merely minimizing the $\chi^2$ goodness of fit between the observed line and that predicted from convolving the continuum with some transfer function leads to a derived transfer function which is extremely “choppy”. The linear regularization constraint minimizes the sum of $\chi^2$ and a measure of how far the inferred transfer function differs from some a priori smooth form. For the results presented here, our smoothness constraint is minimal deviation from either a linear or parabolic form. To control the relative weights given to minimizing $\chi^2$ and the quantity measuring deviation from smoothness, we multiply the smoothness measure by a Lagrange multiplier we call $\lambda_s$. When $\lambda_s = 1$, equal weight is given to the two quantities to be minimized; $\lambda_s = 0$ removes the a priori term so that only $\chi^2$ is minimized, while $\lambda_s \gg 1$ gives much more weight to the a priori expectations of the transfer function shape than to the discrepancies between the light curve it predicts and the observed light curve. To gauge the quality of a solution, we compare the $\chi^2$ it predicts to the effective number of degrees of freedom, a quantity which reflects the number of independent data points adjusted by the relative S/N of the line and continuum light curves (Krolik & Done 1995).

To estimate the uncertainty in the derived transfer function due to measurement error, we resample both the line and continuum light curves within the error distribution 100 times, and then calculate the r.m.s. dispersion at each lag of the resulting 100 derived transfer functions. It is important to recognize that this gives the range of uncertainty constrained by the underlying assumptions of the model, i.e. the particular values of $\lambda_s$ and the maximum lag $\tau_{\text{max}}$, and also that the error bars so derived are not independent. For a comparison of the properties of this inversion technique with others, such as
maximum entropy and subtractive optimized local averages, see Horne 1994; Krolik 1994; Pijpers 1994.

3.2. Interpretation

Unfortunately, response functions produce a map of line marginal emissivity not as a function of position relative to the source, but as a function of delay with respect to our line of sight. As a result, the surfaces of projection are oriented in an arbitrary direction (ours), and cut across a wide range of radii—the surface of delay $\tau$ includes matter at all radii $\geq c\tau/2$. Interpretation of the shape of the derived response function therefore involves a certain amount of model-dependence.

To see how this model-dependence enters, we begin by writing the line luminosity at line of sight velocity $u$ in terms of local properties inside the emission line region:

$$L_l(t, u) = \int d\tau \int d^3r n(r) A(r) S_l[r, F_{ion}(t - \tau)] \delta (\tau - \tau(r)) \int d^3v f(v) \delta (u - v \cdot z),$$

where $n(r)$ and $A(r)$ are the number density and surface area of the BLR clouds, $S_l$ is their surface brightness in the line, $F_{ion}$ is the local continuum flux, $f(v)$ is the clouds’ velocity distribution, and the delay as a function of position is

$$\tau(r) = \frac{r}{c} (1 - \cos \theta)$$

for clouds whose angle to the observer’s line of sight is $\theta$. The two $\delta$ functions pick out the component of velocity in the direction of the observer and the spatial positions corresponding to a given lag. This description also assumes that the clouds radiate line photons isotropically; it is not hard to modify equation 4 in order to include anisotropic radiation, but it is an unnecessary complication here.
Using the linear response approximation, and assuming that the continuum is radiated isotropically, we can approximate the line luminosity fluctuations by

\[ \delta L_l(t, u) = \int d\tau \int d^3r n(r) A(r) \frac{\partial S_l}{\partial F_{ion}} \frac{\delta L_c(t - \tau)}{4\pi r^2} \delta [\tau - \tau(r)] \int d^3v f(v) \delta(u - v \cdot z). \]  

(6)

From this form it is clear that the response function is

\[ \Psi_l(\tau, u) = \int d^3r n(r) A(r) \frac{\partial S_l}{\partial F_{ion}} \frac{\delta}{4\pi r^2} \delta (\tau - \tau(r)) \int d^3v f(v) \delta(u - v \cdot z). \]  

(7)

Integrating \( \Psi_l(\tau, u) \) over all velocities \( u \) gives the total line response.

The most important element of model-dependence is that a geometric symmetry must be assumed in order to use the response function as defined by equation 7 to infer a map of the emissivity. Because it is the simplest assumption (and also because the lines in NGC 5548 have such large equivalent widths that their sources must cover a fair fraction of \( 4\pi \) around the continuum: Krolik et al. 1991), we assume spherical symmetry. Equation 7 integrated over line of sight velocity then reduces to:

\[ \Psi_{tot}(\tau) = \int_{c\tau/2}^{r_{max}} dr n(r) A(r) \frac{\partial S_l(r)}{\partial F_{ion}} \frac{c}{2r}. \]  

(8)

Thus, for a thin shell at distance \( r_o \) from the source, the total response is constant at \( c/(2r_o) \) \( n(r_o)A(r_o)\partial S_l(r_o)/\partial F_{ion} \) for all lags \( 0 \leq \tau \leq 2r_o/c \), i.e. all segments of the shell contribute equally to the response function, but at a lag \( \tau = (1 - \cos \theta) r_o/c \).

Two important consequences follow from the fact that in this model each radial shell produces a square wave response function. First, if this model applies, the marginal emissivity distribution may be directly read off the total flux response function. Second, while different shells stop contributing to the response function at different maximum lags, they all contribute at \( \tau = 0 \). Therefore, if \( \partial S_l/\partial F_{ion} > 0 \) everywhere, the response function always peaks at
zero lag. If there is no material inside \( r = c\Delta \tau /2 \), where \( \Delta \tau \) is the minimum interval at which the response function may be found, \( \Psi_{\text{tot}}(\Delta \tau) = \Psi_{\text{tot}}(0) \); if there is some material inside that point, the zero lag point is an absolute maximum.

A few further comments apply to the interpretation of velocity-resolved response functions. While qualitative information can be gleaned from their direct comparison, quantitative measures do not follow simply from their form. Therefore, after computing the velocity-resolved response functions for these data, we will also construct parameterized models whose predictions may be directly compared with the light curves. We will then find the parameters for these models which, given the observed continuum light curve, minimize \( \chi^2 \) for the predicted velocity-resolved line light curves.

4. Total CIV Response

Considered on its own, there are \( N = 39 \) points spanning 38d in the HST data. The maximum lag length \( M \) that can be examined is then \( M = N/2 = 19 \), so we can search for response functions with a maximum lag \( \tau_{\text{max}} = 18 \)d. However, if we make this choice for \( M \), in the limit of \( \lambda_* \ll 1 \) there would be as many free parameters as line measurements, so a solution could always be found, even if there were in reality \textit{no} connection between the continuum lightcurve and the line lightcurve. On the other hand, when \( \lambda_* \gg 1 \), the response function is effectively being fit by a model with many fewer free parameters—two for a linear smoothing constraint, three for a quadratic condition. Thus the number of degrees of freedom in that limit becomes effectively 16 or 17. Even with extremely large amounts of smoothing (\( \lambda_* \sim 10^4 \)), the data always give \( \chi^2 \ll 16 \) (for \( \lambda_* = 10^4 \) and quadratic regularization, the actual reduced \( \chi^2 = 0.24 \)).
showing that the HST data alone are consistent with a very smooth response function (Fig. 1).

However, with this length of response function we are throwing away half of the line data, and the discarded data are the ones with the most variability. A more reasonable length to examine is $M = N/3 = 13$, or $\tau_{\text{max}} = 12\,\text{d}$, so that in effect the solution is tested on at least as much data as it is derived from, and some of the more rapid variability is included in the fit. This choice gives an excellent solution with $\chi^2_\nu = 0.6$ for $\lambda_* = 1$, or $\chi^2_\nu = 1$ for $\lambda_* = 750$ (Fig. 2). Even shorter response functions can give an acceptable solution, down to $M = 10$, where $\chi^2_\nu = 1$ can be found with $\lambda_* = 0$. Thus the HST data alone require that there is non-zero CIV response over lags at least from 1 to 10d. However, these shortest possible response functions are not necessarily the best estimates of the response function. In particular, as Fig. 2 illustrates, the shorter the permitted response function, the greater the effect of the correlated error at zero lag due to the systematic calibration errors.

Additional information is provided by the IUE continuum lightcurve. Because the IUE data begin 36d before the HST data, use of the IUE continuum light curve enables us to use the entire HST line light curve even while searching for response functions that extend to lags as great as 36d. The IUE data also carry another benefit: the continuum was substantially more variable during the time only IUE was monitoring NGC 5548 than during the time of the HST observations. Clearly we cannot now resolve any structure on scales shorter than 2d, but, as we have already argued, any signal in the data on such short timescales is probably noise-dominated in any case.

Using the combined data set for the continuum light curve, we find that with $\tau_{\text{max}} = 36\,\text{d}$ the response can be well fit by a quadratic function, giving $\chi^2_\nu \ll 1$ even for $\lambda_* \gg 1$ (Fig. 3). To show the non-uniqueness inherent in the
deconvolution procedure, but also demonstrate which features of the response function are robust, we present Fig. 4, which displays the derived response with \( \lambda_* = 1 \) for \( \tau_{\text{max}} \) between 16d (the smallest for which \( \chi^2_\nu \sim 1 \) with \( \lambda_* = 1 \)) and 36d. Two features are independent of detailed choices: the total flux response function peaks at zero lag, and it drops to zero by \( \approx 20 \text{d} \). Since we fix \( \lambda_* = 1 \) for this set, the \( \chi^2 \) falls as \( \tau_{\text{max}} \) increases, but this is at least partially due simply to the larger number of parameters available. For future reference, we call the solution with \( \lambda_* = 10^4 \), a quadratic regularization condition, and \( \tau_{\text{max}} = 36 \text{d} \) the “minimal structure” solution (Fig. 3), and the one with \( \lambda_* = 1 \), a linear regularization condition, and the same \( \tau_{\text{max}} \) the “maximal structure” solution (Fig. 4, solid line). The former can be viewed as the smoothest solution which produces an acceptable \( \chi^2 \), while the latter can be viewed as the solution with the greatest amount of plausible detailed structure.

Several qualitative points follow directly. First, the fact that the response function peaks at \( \tau = 0 \text{d} \) demonstrates that there is either considerable line-emitting gas on our line of sight, or that there are large amounts of gas at radii smaller than 1 lt-d, together with more extended material which gives the non–zero response at \( \sim 10 \text{lt-d} \). A possible geometry for this latter case is a disk of material with an inner radius of less than 1 lt-d (see e.g. the transfer functions of [Welsh & Horne 1991]; [Perez et al. 1992]). However, in order for a disk to produce the observed broad line profiles requires that there is a component of the velocity in the line of sight, i.e. that the disk is not viewed face on. This seems rather implausible for Seyfert 1’s in general given current Seyfert unification schemes which postulate that these objects are seen at rather small inclination angles (e.g. the review by [Antonucci 1993]), and for NGC 5548 in particular, which has a strong Compton reflection signature in its X–ray spectrum ([Nandra & Pounds 1994]; [Done et al. 1994]). A simple thin disk is also strongly ruled out
by the velocity field (see §6.2). Thus we prefer the former option, in which there is gas along our line of sight.

In order for there to be nothing special about the direction to us, we infer that the gas is probably distributed quasi-spherically. This inference is supported by the rather large equivalent widths of the lines in this object (Krolik et al. 1991). Large response from gas on the line of sight also means that the surface brightness on the outside of the clumps cannot be much less than the surface brightness on the side facing the continuum source. This is consistent with standard photoionization models, which predict that the front/back ratio for CIV 1549 even when the clouds are very optically thick in Lyα is only $\simeq 3$ (e.g. Kallman & Krolik 1986; Ferland et al. 1992).

Second, there is significant response over all lags from 1d to $> 16$d. Indeed, we have found that when the maximum lag considered falls below 12d, it is impossible to find any response function whose predicted light curve yields an acceptable $\chi^2$, even for $\lambda_* = 0$. Thus, there is significant flux generated at all radii from 1 lt-d or less (as argued in the previous paragraph) to at least 8 lt-d (the minimum radius capable of producing a 16d lag). In other words, the broad emission line region in NGC 5548 spans at least an order of magnitude in radius: to speak of a unique “lag” indicating a unique radial scale is seriously misleading. At best, the lag derived from cross-correlation analysis is no more than a peculiar weighted average over the response function which is always biased towards the smallest lags, with the degree of bias depending on the power spectrum of continuum fluctuations (Edelson & Krolik 1988). Our finding that the BLR in NGC 5548 spans a large range in radius illustrates the greater power of true reverberation mapping compared to cross-correlation analysis.

Third, the maximal structure solution shows a “shoulder” roughly from 8 to 18d lag. Its absence in the minimal structure solution demonstrates that this
feature, while possibly present, is not required by the data. In work which we saw in draft form after submitting this paper, Wanders et al. 1993, analyzing the same data set with different numerical methods, independently found a very similar feature, but did not discuss its statistical significance. They interpreted it as evidence for bi-conical structure in the emission line region. If the “shoulder” is real, this is one possible interpretation, but it is not unique. It could also be due to a radial non-uniformity in the emissivity of a perfectly spherical region (see equation 9). Which interpretation is preferable is a matter of taste.

Fourth, the fact that we can find an adequate response function to fit the lightcurve is in marked contrast to the previous 1989–1990 IUE campaign (Krolik et al. 1991; Krolik & Done 1995), although the shape of the derived response function is qualitatively the same in that it is high at zero lag and drops to zero by 20 days. It is entirely possible that the structure of the broad line region in NGC 5548 has changed significantly over the four intervening years: the crossing time for material moving at the median line of sight velocity is only $\approx 3\text{yr}$. Changes in the H$\beta$ response have also suggested that there is significant structural change on few year timescales (Peterson et al. 1994). It is also possible that there were times during the previous IUE campaign when the UV continuum was not a good tracer of the total ionizing continuum flux, perhaps during the third ‘event’ in the continuum lightcurve where the fractional line change is much larger than in the previous two ‘events’ (Maoz 1994). For example, a soft X-ray flare was observed in a ROSAT monitoring campaign on NGC5548 which was not linearly correlated with the UV continuum (Done et al. 1994; Done et al. 1995).

One problem which first appeared in the 1989 monitoring campaign still remains, however: the discrepancy between $\partial \ln F_i / \partial \ln F_{\text{ion}}$ (or equivalently $\partial \ln S_i / \partial \ln F_{\text{ion}}$) as estimated from the data and as predicted by photoionization
As we have already remarked, in the likely range of ionization parameters, it is expected that $\partial \ln F_l / \partial \ln F_{ion} \simeq 1$. However, in the earlier experiment, the empirically estimated value was $\simeq 0.4$ (Krolik et al. 1991).

Because we are able to find a response function which provides a good fit to the data for the 1993 experiment, we can estimate the logarithmic partial derivative by a more reliable method than was possible using the 1989 data. This method begins by using Newton’s Theorem applied to equation 8 to solve for the covering factor per unit radius:

$$\frac{dC}{dr} \equiv n(r)A(r) = \frac{4r}{c^2} \frac{d\Psi_{tot}}{d\tau} (\tau = 2r/c) \frac{\langle F_{ion} \rangle}{\partial \ln S_l / \partial \ln F_{ion}}. \quad (9)$$

Since the total line luminosity is

$$L_l = \int_{r_{min}}^{r_{max}} dr \frac{4\pi r^2 dC}{dr} S_l(r), \quad (10)$$

we may then solve for the emissivity-weighted mean logarithmic partial derivative:

$$\langle \frac{\partial \ln S_l}{\partial \ln F_{ion}} \rangle = \frac{\langle F_c \rangle}{\langle F_l \rangle} \int_{r_{min}}^{r_{max}} dr \frac{4r}{c^2} \left[ -\frac{d\Psi_{tot}}{d\tau} (2r/c) \right]. \quad (11)$$

Using the maximal structure solution then yields an estimated $\langle \partial \ln S_l / \partial \ln F_{ion} \rangle \simeq 0.46$, while the minimal structure solution gives 0.54.

In other words, just as in the 1989 experiment, the CIV 1549 line appears to vary significantly more weakly with changing continuum flux than photoionization models would predict.

It is possible that the photoionization models are wrong. However, it is also possible that the problem is due to this experiment’s duration having artificially imposed a maximum radius which is too small. In other words, there could be line-emitting material at such large radius that it does not respond on the relatively short timescales probed by this experiment, but does contribute to the time-averaged line flux. That the much longer 1989 campaign
encountered a similar problem argues against this interpretation, but that may be a consequence of the fact that no satisfactory solution for $\Psi_{\text{tot}}(\tau)$ could be found from that campaign’s data (Krolik & Done 1995).

5. Velocity Resolved Light Curves

5.1. Direct comparison of the lightcurve segments

Figure 5a shows the correlation between the blue core and red core lightcurves at zero lag. If both sections of the line respond in the same way to the continuum fluctuations (i.e. have the same transfer function) then there should be a linear relationship between them. This is clearly consistent with the data: a linear regression, taking into account the fact that the errors on both quantities are of roughly equal magnitude (Press et al. 1993) gives $\chi^2 = 11.8$ for 37 d.o.f. Consequently, the velocity structure is consistent with being completely red/blue symmetric in the core, ruling out pure radial motions for the BLR.

However, no such linear correlation exists between the blue wing and red wing lightcurves (Fig. 5b), where $\chi^2 = 53.4$ for 37 d.o.f. At low flux levels, the ratio between the red wing and the blue wing is much greater than at higher fluxes. In principle, the contrast between red and blue wings might be due to contamination of the CIV red wing by the blue wing of HeII 1640 because the HeII line varies much more rapidly than does CIV. The total flux response of HeII is almost entirely confined within $\simeq 6$ d, whereas the CIV 1549 response must extend to much longer lags (Fig. 4). We have tested the idea that HeII contamination accounts for the contrast between the red and blue wings by computing the red wing–blue wing correlation after having subtracted a variable fraction $f_{\text{He}}$ of the HeII 1640 light curve from the red wing of CIV 1549. We find that over the entire range $0 \leq f_{\text{He}} \leq 1$, the smallest $\chi^2$ for the red
wing–blue wing correlation is 44 for 37 degrees of freedom, and that is achieved for \( f_{He} = 0.65 \), a magnitude of contamination very hard to believe. At the largest plausible \( f_{He} \), 20%, the \( \chi^2 = 49 \), i.e. there is less than 10% probability that the red and blue wings are linearly correlated.

Thus, the picture we have from the raw HST lightcurves alone is one in which there is a marked similarity between the red and blue line core lightcurves, indicating that the dominant velocity field is red/blue symmetric. Such a symmetric velocity field can be produced from a disk, or random orbits in 2 or 3 dimensions (Welsh & Horne 1991; Perez et al. 1992). However, at the same time there is such a strong contrast between the red and blue wings that there must also be at least some net radial flow.

5.2. Response functions

These inferences from the light curves are substantiated and elaborated by the response functions. To illustrate the range of uncertainty due to choices about our a priori model, we display in Figs 6a–d two possible response functions for each of the four velocity segments: both have \( \tau_{max} = 36d \), but one minimizes deviation from a straight line and sets \( \lambda_* = 1 \), while the other minimizes deviation from a quadratic and sets \( \lambda_* \) as large as it can be to still produce a reduced \( \chi^2 \approx 1 \) (this value ranges from \( \sim 100 \) – \( 10^4 \) over the four segments). The former choice produces response functions with the maximal amount of plausible structure, while the latter gives response functions with the least structure consistent with the data. Again, the line and continuum lightcurves have been sampled 100 times within the measurement error distribution, and the error bars in Figs 6a–d show the effect of the observational uncertainty on the minimal emissivity structure model. Error bars for the maximal emissivity
structure model are suppressed for clarity, but they are typically larger due to the smaller value of $\lambda^*$. Again we stress that these error bars are not independent, but are strongly correlated from point to point.

5.3. Qualitative interpretation

Independent of the detailed choices made for the solutions, two points stand out from these curves: the red and blue cores are nearly identical in their response; and the red wing response has a strong peak at small lags, but the blue wing response rises only very slightly towards $\tau = 0$.

The similarity between the red and blue cores at all lags suggests that, to zeroth order, there is little correlation between the local mean radial velocity of the line-emitting gas in NGC 5548 and position. In addition, the weak dependence of the core/wing ratio on lag suggests that the magnitude of the mean velocity depends only weakly on radius.

At the same time, however, the dramatic contrast between the blue and red wings at small lag indicates that there is more material near the line of sight and/or at small radii which is travelling away from us than is travelling towards us. This suggests that any net radial flow is toward the central object.

Finally, in all the maximal structure responses, there is a sharp minimum (which in some cases actually reaches negative values) between 20 and 25d and a “shoulder” at 10 to 15d. Both features are also seen in the total flux response function. As we discussed in §4, while these features are plausible, they are not required by the data. Just as for the total flux response function, similar features were also found, but not tested for statistical significance, by Wanders et al. 1995.
6. Modelling

Unfortunately, more quantitative statements are difficult to make solely on the basis of these response functions. In order to clarify our conclusions, we have embarked on a program of direct modelling. While the forms that our models take are guided qualitatively by our solutions for the velocity-resolved response functions, the parameter fits we perform are, with one exception (the emissivity as a function of radius—see the discussion below), statistically quite independent of these prior solutions.

6.1. Model construction

Equation 7 gives the general form for the velocity-resolved response function. The response function corresponding to one of our four segments is simply $\Psi(\tau, u)$ integrated over the appropriate range of $u$. In principle it would take a very large number of parameters to describe all the functions determining the four response functions, but for obvious reasons it is desirable to find models with the least number of free parameters consistent with arriving at an acceptable description of the data.

Fortunately, although the cloud numbers, distribution and response are all unknown, it is not necessary to define each of them explicitly, or even to guess a model for them. Instead, guided by the shape of the total flux response function, we make the ansatz that the CIV line emissivity is a function of radius alone (spherical symmetry), and that the material emits isotropically. As we have previously demonstrated, if these assumptions apply, the marginal emissivity as a function of radius may be simply read off the total flux response function $\Psi_{\text{tot}}(\tau)$ (see equation 9).
As we have also already discussed, it is hard to choose a form for the response function which is uniquely best. To illustrate the effects of this range of uncertainty, we have done model fits deriving the emissivity as a function of radius from two response functions we believe to span the likely range. These are the “minimal structure” response function \((\tau_{\text{max}} = 36d, \lambda_* = 10^4, \text{a quadratic regularization condition, Fig. 3})\), and the maximal plausible structure response function \((\tau_{\text{max}} = 36d, \lambda_* = 1, \text{a linear regularization condition, solid line in Fig. 4})\).

To complete the model specification all we need to do is construct a parameterized form for the velocity distribution function as a function of \(r\). We investigate two forms for this distribution, one combining radial motion with 3-d random motions, the other combining radial motion with random motions confined to the local tangent plane. By this means we test whether there is a preferred eccentricity for the random orbital motions.

The velocity distribution function for the first form may be written as:

\[
\Psi_l(\tau, u) = \int_{c\tau/2}^{\tau_{\text{max}} = c\tau_{\text{max}}/2} dr \left( -\frac{d\Psi_{\text{tot}}(\tau)}{d\tau} \right) \frac{2c^{-[u-v(r)(1-c\tau/r)]^2/\sigma(r)^2}}{\sqrt{\pi\sigma(r)}}. \tag{12}
\]

The last step in specifying such a “random + radial” model is to define \(v_r(r)\) and \(\sigma(r)\). We choose the forms

\[
v_r(r) = v_o(r/r_o)^{\alpha}, \tag{13}\]

and

\[
\sigma(r) = \sigma_o(r/r_o)^{\beta}. \tag{14}\]
where $r_o$ is a fiducial radial scale, which we set at 1 lt-d. Thus, this class of model is determined completely by only four free parameters.

Our other form restricts the random motions to the local tangent plane. In this case, rather than defining the velocity distribution in terms of its density in 3-d velocity space, it is most simply described in terms of the total magnitude of the velocity and the direction of the component in the tangent plane. That is, if we define a local set of basis vectors in the tangent plane:

\[
\begin{align*}
\hat{x}_t &= -\sin \theta \hat{z} + \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} \\
\hat{y}_t &= -\sin \phi \hat{x} + \cos \phi \hat{y},
\end{align*}
\]

we may write the tangential velocity as $v_t = v_t(r)(\cos \gamma \hat{x}_t + \sin \gamma \hat{y}_t)$. Then the 2-d random velocity distribution is simply

\[
df(v, \gamma) = \delta \left( v - \sqrt{v_r^2 + v_t^2} \right) \frac{dv \, dv \gamma}{2\pi}. \tag{17}\]

With this form for the velocity distribution, the response function becomes

\[
\Psi(\tau, u) = \frac{1}{\pi c} \int_{r_1}^{r_2} dr \left( -\frac{d \Psi_{tot}(\tau)}{d \tau} \right) \left\{ v_r^2 \left[ 2 \frac{c \tau}{r} - \left( \frac{c \tau}{r} \right)^2 \right] - \left[ v_r \left( 1 - \frac{c \tau}{r} \right) - u \right]^2 \right\}^{-1/2}. \tag{18}\]

where $r_1$ and $r_2$ are the radii limits over which the integrand exists.

Once again, the most concise parameterization of the characteristic velocities is as power-laws in radius. We retain the same notation for $v_r(r)$; we write $v_t(r)$ as

\[
v_t(r) = v_{to}(r/r_o)^\epsilon. \tag{19}\]

We integrate these equations numerically using an evenly spaced grid of 200 points in velocity space, and oversample the $\tau$ points by a factor 10, linearly interpolating for the values of $d \Psi/d \tau$ within this. For the 2-d random motions, the nature of the integrand is such that a smooth response function can be better obtained by averaging over the oversampled points either side of the desired $\tau$. 
6.2. Model Fits

The most natural way to measure the quality of a model is, of course, to compute its $\chi^2$ with respect to the data and compare that $\chi^2$ to the number of degrees of freedom present. In this application, while it is obvious how to compute $\chi^2$ (generate a response function from the model, fold it through the measured continuum light curve, and compare the results to the observed line light curves in each of the four segments), it is not quite so obvious how to compute the number of degrees of freedom. In each of the four line light curves smoothed to 2d resolution, there are 20 points. However, we are fitting to their fluctuations about the mean, so they are constrained to sum to zero. This reduces the number of degrees of freedom by one for each light curve. In addition, we have removed the mean from the continuum light curve, so on this basis one might expect that there are $4 \times (20 - 1) - 1 = 75 - n_{\text{par}}$ degrees of freedom, where $n_{\text{par}}$ is the number of parameters in the model. However, the expected error in the predicted line light curve must also include the error induced by noise in the continuum light curve. To account for this effect, we increase the effective number of degrees of freedom in each line light curve by the same factor dependent on the comparative S/N of the continuum and line light curves that we used to judge the quality of the regularized inversions (see Krolik & Done 1995). These ratios are 1.08, 1.10, 1.12, and 1.14, for the blue wing, blue core, red core, and red wing, respectively. On this basis, the total number of effective degrees of freedom becomes $83.36 - n_{\text{par}}$.

Unfortunately, $n_{\text{par}}$ is not altogether unambiguous, either. In each model for the velocity distribution examined below, there are (depending on the model) between 1 and 4 free parameters. If one were to assume that the radial emissivity derived from our fit to the total line flux lightcurve is correct, $n_{\text{par}}$ would include only those. However, another interpretation would be to include
the radial emissivity as part of the model being tested. In that case, one would 
count an additional 2 free parameters when the minimal structure emissivity is 
used (a parabolic fit with $\Psi_{tot}(\tau_{\text{max}})$ fixed at zero), or possibly as many as 19 
(the total number of elements in the solution vector) in the limit of $\lambda_* \to 0$! 
Simulations show that $\lambda_* = 1$ lies approximately half way between these two 
extremes (Krolik & Done 1995) in terms of the effective number of degrees of 
freedom, so that there are an additional $\sim 10$ free parameters from the radial 
emissivity in the maximal structure solution. However, since this latter number 
is rather poorly defined, and so as to have a fixed standard of comparison, we 
will adopt the definition that the radial emissivity introduces 2 additional free 
parameters, but we stress that changes in reduced $\chi^2$ are more important for 
our argument than the absolute value of the reduced $\chi^2$. With that definition, 
the models have between 77.4 and 80.4 effective degrees of freedom.

We begin by demonstrating that the three simplest models can be easily 
rulled out, no matter how the free parameters are counted (the data for the 
models we describe here and in subsequent paragraphs are summarized in Tables 
2 and 3.) If we fix $\sigma_o = \beta = 0$ (i.e. a model with purely radial motion), then 
the best fit model (using either form for the total flux response) has a reduced 
$\chi^2 \simeq 7$! Less trivially, a purely random model can also be eliminated. With 
$v_o = \alpha = 0$, the best fit has a reduced $\chi^2 = 2.20$ for the “minimal structure” 
response, or 1.88 for the “maximal structure” form. The fit qualities for the best 
purely tangential models are very similar to the best purely random models. 
Making any allowance for additional implicit free parameters in the maximal 
structure solution would only increase the least reduced $\chi^2$.

However, as we have already inferred from the qualitative character of the 
velocity-resolved response functions, a combination of either 3-d or 2-d random 
motion plus inward radial flow does much better. Combined 3-d random and
radial velocity fields give a best reduced $\chi^2 \simeq 1.34$ for the maximal structure emissivity when $v_o \simeq -790 \text{ km s}^{-1}$, $\alpha \simeq 0.35$, $\sigma_o \simeq 5600 \text{ km s}^{-1}$, and $\beta = -0.14$. The four light curves predicted by this model are compared to the observed light curves in Figs. 7.

There is no simple description for the uncertainty ranges of the fit parameters because there are strong mutual correlations between the parameters, and their acceptable ranges depend on the form of the model. In order to give some sense of this uncertainty, we discuss the effects of altering various model assumptions in the following paragraphs.

The formal reduced $\chi^2$ of the combined random + radial model depends on the description of the emissivity: using the minimal structure emissivity increases the least reduced $\chi^2$ to 1.71. Most of this increase can probably be attributed to the greater $\chi^2$ of the total flux response function which defines the emissivity in this model. In addition, the minimal structure emissivity leads to rather different (and somewhat surprising) best-fit parameters: $v_o \simeq -0.04 \text{ km s}^{-1}$, $\alpha \simeq 4.3$, $\sigma_o \simeq 22,000 \text{ km s}^{-1}$, and $\beta \simeq -0.8$. Nonetheless, the reduction in reduced $\chi^2$ due to adding radial motions to the model is nearly independent of the emissivity model: in the maximal structure case, reduced $\chi^2$ falls by 0.54, while in the minimal structure case, it is diminished by 0.49. We take this to indicate that a combination of radial and random motion is required for any total flux emissivity consistent with the data.

2-d and 3-d random motions are indistinguishable in this regard. With the maximal structure emissivity, the least 2-d reduced $\chi^2$ for a random + radial model is 1.37, and this is achieved for parameters very similar to those found in the 3-d solution: $v_o \simeq -820 \text{ km s}^{-1}$, $\alpha \simeq 0.35$, $v_{o_1} \simeq 5200 \text{ km s}^{-1}$, and $\epsilon = -0.13$. 2-d motions without any radial velocity lead to a reduced $\chi^2$ larger by $\simeq 0.9$. We therefore conclude that these data do not discriminate between
orbits of differing eccentricity.

The magnitude of the random velocity in the middle of the emission region is nearly model-independent; at 5 lt-d it is between 4200 and 5400 km s\(^{-1}\) in essentially all the random + radial models. This is, of course, because they must match the characteristic width of the velocity profile. Similarly, the magnitude of the radial velocity at 5 lt-d is in the range \(-1400\) to \(-2000\) km s\(^{-1}\) for all the successful maximal structure models, although significant radial velocities tend to be reached only at larger radii in the minimal structure models.

The degree to which we may constrain the magnitudes of the velocity gradients (\(\alpha\), \(\beta\), and \(\epsilon\)) can depend on one’s choice of model as well as the choice of emissivity. For example, while fixing \(\beta = -0.5\) in the maximal structure 3-d random + radial model raises reduced \(\chi^2\) by \(\simeq 0.25\), the increase in reduced \(\chi^2\) when \(\epsilon\) is set to -0.5 in the maximal structure 2-d random + radial model is only 0.08, while the same exercise applied to the minimal structure models increases \(\chi^2\) by even smaller amounts.

Not surprisingly, it is even harder to constrain gradients in the radial velocity. In minimal structure models, the best-fit values of \(\alpha\) are generally 3 – 4; forcing \(\alpha\) to be as small as 0 increases reduced \(\chi^2\) by \(\simeq 0.1 - 0.2\). That is, with this description of the emissivity, \(\alpha\) is likely to be fairly large and positive, but smaller gradients are not strongly ruled out. On the other hand, in the maximal structure models, the best-fit values are \(\simeq 0.3 - 0.4\), and forcing \(\alpha = 0\) has only a trivial effect on \(\chi^2\) in the 3-d case, but can be ruled out if the motions are 2-d. Thus, while the best-fit value of \(\alpha\) is generally positive, it has a large range of uncertainty.

We conclude by pointing out the one possible escape from our conclusion that both radial infall and random motions must be present in the BLR of NGC
5548: the difficulty all models have in fitting the red wing. Because the red wing lightcurve consistently gives a worse fit than any of the other velocity segments, one might suspect that there is an unidentified problem in the red wing data. We have already argued against any significant contamination from HeII 1640, but there may be some other problem which disproportionately affects this segment of the line. To test this hypothesis, we have also examined models fit to the three other line segments, but ignoring the red wing. Dropping the red wing does improve the fit: a maximal structure emissivity random + radial model with $\alpha = \beta = 0$ then gives $\chi^2 = 1.07$ for $v_o = -1500$ km s$^{-1}$ and $\sigma_o = 4200$ km s$^{-1}$; similarly, without the red wing the minimal structure emissivity permits an analogous model with reduced $\chi^2 = 1.41$. However, even without the red wing, the maximal structure emissivity still requires radial motion: if we force $v_o = 0$ while ignoring the red wing, the least reduced $\chi^2$ for the maximal structure emissivity increases by 0.2 even allowing $\beta$ to vary. Only if one restricts attention to the minimal structure emissivity model and ignores the red wing does the omission of radial motion not significantly increase $\chi^2$.

Comparing the best-fit model red wing response with the curve derived directly from the data suggests a more likely explanation for our problems in fitting the red wing: the relatively short duration of the experiment. Adopting our ansatz of spherically symmetric, isotropically radiated emission means that the largest radius whose emissivity is revealed by the total flux response function is 18 lt-d; our models therefore have zero response at larger radii. When there is some radial infall, however, the lag in the red wing at $\tau > 20d$ depends largely on material at radii $> 20$ lt-d. Consequently, the absence of data on longer lags cripples the ability of our models to account for the red wing’s response beyond $\approx 20d$. 
7. Discussion: Comparison with Previous Work, and Once Popular Models Now Ruled Out

On the basis of the arguments presented in the previous sections, we have now arrived at a clearly-established qualitative picture of the kinematics in the BLR of NGC 5548: Significant CIV 1549 emission stretches over at least a factor of ten in radius, from 1 lt-d or closer, out to 10 lt-d or farther. At any particular location within that region, the velocity of the line-emitting gas exhibits a fairly broad distribution, but there is a net tendency toward inward motion. The magnitude of the mean inward speed is perhaps a few times smaller than the characteristic spread in velocities, and that characteristic spread may vary rather slowly with radius, but is not absolutely required to do so.

Elements of this picture had been hinted at in several previous studies. Suggestive evidence for a combination of random motion and radial infall has been noted by Koratkar & Gaskell 1989 in the case of Fairall 9, and by Maoz et al. 1991 in NGC 4151. Wanders & Horne 1994 advocated predominantly random motion in NGC 3516, but commented that the contrast between core and wing predicted by Keplerian motions was not present (although they made no specific test to see whether velocities scaling as \( r^{-0.5} \) were inconsistent with the data). While previous studies of NGC 5548 have produced contradictory results, some suggested pieces of what we find: Koratkar & Gaskell 1991a contended (on the basis of data with a mean sampling interval of 97d) that the motion is predominantly random, while Crenshaw & Blackwell 1990 argued for strong radial infall. The superior quality of the data provided by the HST monitoring experiment has allowed for the first time a truly quantitative approach to this problem, so that the relative contributions of random and radial motion can at last be quantitatively assessed.
Although significant error bars remain for many of the interesting parameters, the data in hand already now allow us to rule out several of the historically most popular models for the broad line region’s dynamics.

Almost all wind models, whether propelled by radiation pressure (Mathews 1974, Blumenthal & Mathews 1973, Mathews 1986), thermal winds (Weymann et al. 1982, Begelman et al. 1990), or rotating magnetic fields (Emmering et al. 1992) can be immediately discarded because they predict net outflow, not net infall. Those wind models invoking a fluid substrate are ruled out with special force because they would also find it hard to accommodate significant random velocities. The only exception to this conclusion is that class of models in which obscuration prevents us from seeing the far side of the source. Because the center of the line profile is now made by material that is actually moving towards us, the simple identification of red and blue with near and far sides which allowed us to so easily rule out simple wind models is destroyed. However, these models are still in contradiction with the data if, as they frequently do (e.g. Murray et al. 1995), they predict the greatest speeds to occur at the smallest radii. This is because in that case the blue wing would still tend to respond at the smallest lags.

Randomly oriented gravitational orbits, whether of clouds (Kwan & Carroll 1982) or stars (Penston 1988, Norman & Scoville 1988, Kazanas 1989, Alexander & Netzer 1994), and having any distribution of eccentricities, are also strongly ruled out by the fact that there is net radial infall. The model of Carroll & Kwan 1985, in which randomly oriented orbits are combined with drag against an external wind is still viable. In this model one does expect a combination of random velocities and net infall.

Emission from the surface of accretion disks (Jones & Raine 1980, Gerbal & Pelat 1981, Mathews 1982, van Groningen 1983) is ruled out by the same
arguments which eliminate randomly oriented orbits. They would also be subject to the requirement that the disk must be highly inclined (close to edge on) in order for the total flux response to peak at zero lag, which is in conflict with the detection of a strong Compton reflection spectrum in the X-ray spectrum (Nandra & Pounds 1994). The velocity field from a disk is also red/blue symmetric (Welsh & Horne 1991; Perez et al. 1992), and so a disk geometry alone cannot explain the significant differences between the blue and red wing lightcurves. Some sort of combination of a disk with infalling gas would be required in order to begin to match the kinematics.

Smooth radial accretion (Krolik & London 1983) also cannot explain these data. Models of this variety cannot allow substantial random velocities.

The fundamental reason why these data are in conflict with all simple theoretical models is that the real situation appears to be an intermediate case: not entirely collisionless, but definitely not fluid-like. No one would begin by proposing such a complicated hybrid. However, the behavior of line variations in this experiment appears to require just such a model. While the single largest component of the motion is random (whether fully 3-d or restricted to the tangent plane), there also seems to be a secondary component of radial infall whose origin must presumably lie in some sort of dissipative process.

We close with one final comment. One frequently finds in the literature attempts to estimate the central mass in AGN by multiplying a “characteristic” radius by the square of a “characteristic” velocity. The analysis we have presented in this paper demonstrates clearly that there is at least an order of magnitude dynamic range between the inner and outer radii bounding the zone of substantial broad line emission in NGC 5548. This fact alone calls into question these simple estimates. If, in addition, the hints we have seen that $\beta$ or $\epsilon$ are significantly different from -0.5 are confirmed, the whole physical basis of
the exercise would be severely undermined.

A better procedure would be to work as follows: First, construct a model for the emissivity as a function of position in which the orbital speeds are consistent with the assumed shape of the potential, and check whether this model provides an adequate description of the data. For example, the maximal structure emissivity with $\beta$ or $\epsilon$ set to -0.5 yields a not completely satisfactory reduced $\chi^2 \simeq 1.5$ (the best-fit $\sigma_o$ and $v_t$ using the minimal structure emissivity are the same as for the maximal structure emissivity, but give larger $\chi^2$). Second, relate the velocities and the central mass through the physical model. In the 3-d + random model, this would mean that $M = 3r_o v_o^2 / G$, while in the 2-d + random model $M = r_o v_o^2 / G$. Thus, with Keplerian orbital dynamics forced on the data, the central mass might be either $8 \times 10^7 M_\odot$ (for 3-d random motions) or $2 \times 10^7 M_\odot$ (for 2-d random motions). Given the large uncertainty in both procedures, it is quite surprising that these estimates are almost identical to those made in [Krolik et al. 1991] on the basis of the correlation between profile width and characteristic response time for eight different emission lines in the spectrum of NGC 5548. The only point of contact between the two estimates is that both must be consistent with the characteristic velocity width and response time for CIV 1549 in this object. Despite this striking agreement, in view of the crudeness of the quality of fit in both procedures, and the possible inapplicability of the Keplerian model, we remain reluctant to ascribe too much reality to these estimates of the central mass.

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line. We also thank Ignaz Wanders for sending us a draft of his group’s work on the same topic.
Table 1. Velocity resolved CIV line lightcurves from HST with 2% systematic
errors

| line         | $< F >$  | $\sigma_{tot}^2$ | $\sigma_{err}^2$ | $\delta F/ < F >$ | S/N |
|--------------|----------|------------------|------------------|-------------------|-----|
| total continuum | $3.49 \times 10^{-14}$ | $2.61 \times 10^{-29}$ | $8.40 \times 10^{-31}$ | 0.144 | 5.48 |
| total line    | $6.53 \times 10^{-12}$ | $3.75 \times 10^{-25}$ | $1.78 \times 10^{-26}$ | 0.091 | 4.48 |
| bw           | $1.37 \times 10^{-12}$ | $1.37 \times 10^{-26}$ | $9.05 \times 10^{-28}$ | 0.083 | 3.76 |
| bc           | $1.83 \times 10^{-12}$ | $2.74 \times 10^{-26}$ | $1.48 \times 10^{-27}$ | 0.097$^a$ | 4.23 |
| rc           | $1.96 \times 10^{-12}$ | $3.54 \times 10^{-26}$ | $1.67 \times 10^{-27}$ | 0.10$^a$ | 4.52 |
| rw           | $1.37 \times 10^{-12}$ | $2.27 \times 10^{-26}$ | $9.24 \times 10^{-28}$ | 0.11  | 4.85 |

$^a$Calculated allowing for a 10% contribution to the mean flux from the narrow
line component.

Note. — All fluxes are in ergs s$^{-1}$ cm$^{-2}$ except for the continuum for which the
units are ergs s$^{-1}$ cm$^{-2}$ A$^{-1}$. All variances are in ergs$^2$ s$^{-2}$ cm$^{-4}$ except for the
continuum for which the units are ergs$^2$ s$^{-2}$ cm$^{-4}$ A$^{-2}$. 

Table 2. Fitting the velocity resolved CIV line segments: minimal structure

| Model    | $v_o$ km s\(^{-1}\) | $\alpha$ | $\sigma_o$; $v_t$\(^{b}\) | $\beta$; $\epsilon$\(^{b}\) | $\chi^2$(bw) | $\chi^2$(bc) | $\chi^2$(rc) | $\chi^2$(rw) | $\chi^2$(Total) |
|----------|----------------------|----------|-----------------------------|-----------------------------|----------------|----------------|----------------|----------------|-----------------|
| R        | -4700                | 0\(^{c}\) | 171.4                       | 60.2                        | 58.1            | 252.3          | 542.1          |                 |                 |
| R        | -2100                | 0.42     | 231.6                       | 63.2                        | 58.3            | 175.8          | 529.0          |                 |                 |
| 3D       | 4500                 | 0\(^{c}\) | 36.5                        | 24.3                        | 29.3            | 93.6           | 183.6          |                 |                 |
| 3D       | 11,000               | -0.44    | 44.8                        | 22.6                        | 31.6            | 76.0           | 174.9          |                 |                 |
| 2D       | 4100                 | 0\(^{c}\) | 42.9                        | 21.2                        | 29.2            | 86.9           | 180.2          |                 |                 |
| 2D       | 5600                 | -0.15    | 43.7                        | 18.8                        | 30.3            | 79.3           | 172.0          |                 |                 |
| 3D+R     | -1100                | 0\(^{c}\) | 4400                        | 0\(^{c}\)                   | 39.8            | 23.8           | 30.3           | 75.3           | 169.2           |
| 3D+R     | -1100                | 0\(^{c}\) | 13,000                      | -0.54                       | 39.6            | 23.5           | 30.1           | 64.7           | 157.9           |
| 3D+R     | -0.4                 | 3.4      | 4400                        | 0\(^{c}\)                   | 35.8            | 22.2           | 29.2           | 61.0           | 148.2           |
| 3D+R     | -0.04                | 4.30     | 22,000                      | -0.81                       | 33.7            | 22.2           | 27.6           | 48.7           | 132.2           |
| 3D+R     | -0.6                 | 3.31     | 12,000                      | -0.50\(^{c}\)               | 33.1            | 21.7           | 27.8           | 51.3           | 133.9           |
| 2D+R     | -570                 | 0\(^{c}\) | 4100                        | 0\(^{c}\)                   | 29.7            | 20.9           | 28.5           | 77.0           | 156.1           |
| 2D+R     | -820                 | 0\(^{c}\) | 5500                        | -0.16                       | 31.3            | 20.0           | 27.4           | 75.2           | 153.9           |
| 2D+R     | -3.4                 | 2.53     | 4000                        | 0\(^{c}\)                   | 32.3            | 20.4           | 27.9           | 61.9           | 142.5           |
| 2D+R     | -0.4                 | 3.23     | 8000                        | -0.33                       | 29.5            | 18.3           | 29.6           | 49.0           | 126.5           |
| 2D+R     | -0.5                 | 3.06     | 12,000                      | -0.50\(^{c}\)               | 38.3            | 18.5           | 25.2           | 51.8           | 133.8           |

\(^{a}3\text{-}d\) random motions

\(^{b}2\text{-}d\) random motions

\(^{c}\)Parameter fixed

Note. — The model coding is: R for purely radial; 3D for 3-d random motions; 2D for 2-d random motions. The effective total number of degrees of freedom is $\simeq 77 - 80$. 
Table 3. Fitting the velocity resolved CIV line segments: maximal structure

| Model | $v_o$ km s$^{-1}$ | $\alpha$ | $\sigma_v^2; v_t^b$ | $\beta^a; \epsilon^b$ | $\chi^2$(bw) | $\chi^2$(bc) | $\chi^2$(rc) | $\chi^2$(rw) | $\chi^2$(Total) |
|-------|------------------|--------|-------------------|---------------|-----------|-----------|-----------|-----------|----------------|
| R     | -5100            | 0$^c$  |                   |               | 86.8      | 261.8     | 62.1      | 248.0     | 658.7         |
| R     | -1900            | 0.52   |                   |               | 172.5     | 135.1     | 88.1      | 178.6     | 574.2         |
| 3D    | 4500             | 0$^c$  | 39.9              | 22.0          | 20.2      | 71.1      | 153.3     |           |               |
| 3D    | 6100             | -0.16  | 43.9              | 20.3          | 20.1      | 65.1      | 149.4     |           |               |
| 2D    | 4200             | 0$^c$  | 48.6              | 29.8          | 32.8      | 90.5      | 201.7     |           |               |
| 2D    | 5300             | -0.12  | 40.5              | 27.5          | 25.1      | 81.1      | 174.3     |           |               |
| 3D+R  | -2000            | 0$^c$  | 4200              | 0$^c$         | 24.8      | 19.2      | 20.6      | 46.1      | 110.7         |
| 3D+R  | -1800            | 0$^c$  | 5600              | -0.14         | 23.0      | 18.4      | 18.9      | 47.2      | 107.5         |
| 3D+R  | -910             | 0.33   | 4300              | 0$^c$         | 24.5      | 18.1      | 19.8      | 44.4      | 106.9         |
| 3D+R  | -790             | 0.35   | 5600              | -0.13         | 23.7      | 17.3      | 18.2      | 44.6      | 103.8         |
| 3D+R  | -390             | 0.50   | 12,000            | -0.50$^c$     | 33.7      | 23.0      | 18.5      | 47.6      | 122.9         |
| 2D+R  | -700             | 0$^c$  | 4000              | 0$^c$         | 37.7      | 33.1      | 24.6      | 56.6      | 152.1         |
| 2D+R  | -750             | 0$^c$  | 5200              | -0.13         | 25.8      | 25.2      | 18.3      | 59.8      | 130.0         |
| 2D+R  | -830             | 0.38   | 3900              | 0$^c$         | 38.3      | 18.9      | 18.8      | 45.1      | 121.2         |
| 2D+R  | -820             | 0.35   | 5200              | -0.13         | 25.5      | 16.0      | 18.3      | 46.3      | 106.0         |
| 2D+R  | -81              | 0.79   | 12,000            | -0.50         | 27.4      | 18.4      | 19.3      | 47.3      | 112.4         |

$^a$3–d random motions

$^b$2–d random motions

$^c$Parameter fixed

Note. — The model coding is: R for purely radial; 3D for 3-d random motions; 2D for 2-d random motions. The effective total number of degrees of freedom is $\approx 77 - 80$. 
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Figure Captions

Figure 1  The CIV total flux response function making use of only \textit{HST} data, setting $\tau_{\text{max}} = 18\text{d}$, and $\lambda_* = 10^4$. The error bars show the uncertainty in the derived response due to the measurement errors and are correlated from point to point. These errors do not include the inherent non-uniqueness in the deconvolution due to the particular parameter choices of $\tau_{\text{max}}$ and $\lambda_*$.  

Figure 2  The CIV total flux response function making use of only \textit{HST} data, setting $\tau_{\text{max}} = 12\text{d}$. The smooth dotted curve shows the result of setting $\lambda_* = 750$, the largest value of $\lambda_*$ for which reduced $\chi^2 \leq 1$; the jagged solid curve with error bars (derived as for Fig. 1) is the solution for $\lambda_* = 1$. The larger response at $\tau = 0$ compared to Fig. 1 likely reflects the influence of the systematic calibration errors.

Figure 3  The smoothest possible CIV total flux response function using the combined data set for the continuum light curve. Here $\lambda_* = 10^4$, but the solution is virtually independent of $\lambda_*$ for any value greater than $\sim 100$. Error bars are derived as in Fig. 1.

Figure 4  The combined \textit{IUE} + \textit{HST} continuum light curve permits acceptable solutions with $\tau_{\text{max}} \geq 16\text{d}$. All the response function solutions shown here used $\lambda_* = 1$ and have reduced $\chi^2 \leq 1$. Error bars are suppressed for clarity.

Figure 5a  The correlation between the blue core and red core lightcurves. The fluxes are in units of $10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$. The ratio between the blue core’s fluctuation about its mean and the red core’s fluctuation about its mean is clearly constant. However, the extrapolation of this relation to zero flux in both segments does require some curvature. The best fit linear correlation (not constrained to pass through zero) has $\chi^2 = 12$ for 37 degrees of freedom.
Figure 5b  The correlation between the blue wing and red wing lightcurves. The fluxes are in units of $10^{-12}$ erg cm$^{-2}$ s$^{-1}$. Their fluctuations are clearly inconsistent with any simple linear relation, as evinced by the best fit $\chi^2$ of 53.4 for 37 degrees of freedom.

Figure 6a  The response function for the red wing. In this and the other three panels of Fig. 6, the solid line with error bars (derived as in Fig. 1) is the minimal structure response function, and the dotted line is the maximal structure response function. Error bars are shown only for the former solution for clarity.

Note that this response function has large amplitude at small lag, in sharp contrast to the blue wing response function (Fig. 6d).

Figure 6b  The response function for the red core. It is consistent with being identical to the blue core.

Figure 6c  The response function for the blue core.

Figure 6d  The response function for the blue wing.

Figure 7a – d  Predicted vs. observed lightcurves for: (a) the red wing; (b) the red core; (c) the blue core; and (d) the blue wing in the best fit combined radial and random motion model. In all four panels the solid line shows the predicted lightcurve and the points with error bars are the observations.
