Composite Avenue beyond the Standard Model

—— Legacy of Sakata in LHC Era ——

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Higgs boson may be a composite particle as Sakata vigorously looked for never-ending substructures of Nature. He proposed the Sakata model for hadrons, which was the prototype of the quark model and thus launched the last Revolution in particle physics continued all the way up to Kabayashi-Maskawa work which completed the Standard Model today. Inspired by the Sakata’s spirit we shall discuss composite Higgs boson in various models of our own for the dynamical symmetry breaking with large anomalous dimension: The techni-dilaton in the walking technicolor (WTC) with $\gamma_m \simeq 1$, the $\bar{t}t$ composite (“top-Higgs”) in the top-quark condensate model with $\gamma_m \simeq 2$, and their variants in the models with $1 < \gamma_m < 2$ (strong ETC Technicolor, etc.). Among others we will focus on WTC which has an approximate scale symmetry in the region relevant to the dynamical mass generation. Such a conformal gauge dynamics is characterized by the essential singularity scaling, breakdown of the Ginzburg-Landau/Gell-Mann-Levy effective theory, and also by a large anomalous dimension $\gamma_m = 1$. In contrast to the folklore that Technicolor is a “Higgsless theory”, there exists a composite Higgs, techni-dilaton, in the WTC as a composite pseudo Nambu-Goldstone boson associated with the spontaneously broken (approximate) scale symmetry, with its mass only arising from the (nonperturbative) scale anomaly and hence being much smaller than those of other techni-hadrons. The techni-dilaton has a mass typically of order $500 - 600$ GeV and can be discovered at LHC. We shall also touch upon the endeavor to discover WTC on the lattice.

§1. Introduction

Composite idea has been a main stream of modern physics, from atoms to nuclei, hadrons and eventually to quarks. The latest establishment of this idea, the quark, proposed by Gell-Mann\(^1\) and Zweig\(^2\) in 1964 had its precedence, the Sakata model\(^3\) which was born in Nagoya back in 1955 (published in 1956). Here is the phrase in Ref.\(^1\) : “A formal mathematical model based on field theory can be built up for the quarks exactly as for $p, n, \Lambda$ in the old Sakata model, \ldots For the weak current, we can take over from the Sakata model the form suggested by Gell-Mann and Lévy, \ldots”.

The influence of the Sakata model is more eminent in the ace model\(^2\) which consists of the fundamental triplet ($p_0, n_0, \Lambda_0$) mocking up the Sakata model triplet ($p, n, \Lambda$). In 2006 we had a symposium at Nagoya celebrating Jubilee of the Sakata model. The poster of the symposium\(^3\) was designed to describe the development from ($p, n, \Lambda$) into ($u, d, s$), by moving ($p, n$) (and $\Lambda$) gradually upside-down to ($u, d$) (and $s$). At Nagoya the Sakata model developed into the “Nagoya model”\(^4\) based on the correspondence of fundamental triplet ($p, n, \Lambda$) to the leptons triplet ($\nu, e, \mu$). After discovery of $\mu$-neutrino, the model was modified into the Maki-Nakagawa-Sakata model\(^5\) which proposed the neutrino mixing $\nu_e - \nu_\mu$ (so-called MNS matrix) and also a new entry $\nu'$ into the fundamental triplet so as to be extended into the fundamental quartet ($p, n, \Lambda, p'$) corresponding to the quartet of leptons ($\nu, e, \mu, \nu_\mu$). This quartet
The Origin of Mass is the most urgent issue of the particle physics today and is to be resolved at the LHC experiments. In the SM, all masses are attributed to a single parameter of the vacuum expectation value (VEV), \( \langle H \rangle \), of the hypothetical elementary particle, the Higgs boson, which triggers the spontaneous symmetry breaking (SSB). The VEV simply picks up the mass scale of the input parameter \( M_0 \) which is tuned to be tachyonic \( (M_0^2 < 0) \) in such a way that \( \langle H \rangle \approx 246 \text{ GeV} \) ("naturalness problem"). As such SM does not explain the Origin of Mass. Particle theorists looking desperately beyond the SM have been fighting on this central problem over 30 years without decisive experimental information. Now we are facing a new era that LHC experiments will tell us which theory is right.

It should be recalled that the very concept of SSB was created by the 2008 Nobel prize work of Nambu \(^8\) \(^9\) in a concrete form of the dynamical symmetry breaking (DSB) where the nucleon mass was dynamically generated via Cooper pairing of (then elementary) nucleon and anti-nucleon, "nucleon condensate", based on the Bardeen-Cooper-Schrieffer (BCS) analogue of superconductor: Accordingly, there appeared pions as massless Nambu-Goldstone (NG) bosons which were dynamically generated to be nucleon composites in the same sense as in the Fermi-Yang/Sakata model. \(^3\) Thus the SSB was born as DSB! Before advent of the concept of SSB, low energy hadron physics was well described by the effective theory of Gell-Mann-Levy (GL) linear sigma model with an elusive scalar boson, the sigma meson, which was simply assumed to have negative mass squared. Actually, the GL linear sigma model Lagrangian is a model formally equivalent to the SM Higgs Lagrangian, with the Higgs boson being the counterpart of the sigma meson. The real physical meaning of this mysterious tachyonic mode was thus revealed as the BCS instability where attractive forces (effective four-fermion interactions) between nucleon and anti-nucleon give rise to the nucleon Cooper paring (tachyonic bound state) which changes the vacuum from the original (free) one into the true one having no manifest symmetry.

The Nambu’s theory for the origin of mass of nucleon (then the “elementary particle”) was later developed into DSB in the underlying microscopic theory, QCD, where the gluonic attractive forces again generate the Cooper paring of quark and antiquark (instead of nucleon and anti-nucleon), the quark condensate \( \langle \bar{q}q \rangle \), which then gives rise to the BCS instability and the dynamical mass of quarks: Pions are now composites of quarks instead of nucleons. Hence Nambu’s idea was established in a deeper level of matter. Note that the nucleon mass in the Nambu’s theory is originated from the explicit mass scale carried by the dimensionful coupling in the four-fermion theory, while in QCD (with massless quarks) there is no mass scale at classical level: The intrinsic mass scale in QCD, \( A_{QCD} \), arises from the scale anomaly quantum mechanically, which manifests itself in the running of the gauge coupling.
This is a salient feature of the mass generation of the gauge theory.

Technicolor (TC)\textsuperscript{12} is an attractive idea to account for the Origin of Mass without introducing ad hoc Higgs boson and tachyonic mass parameter: The mass arises \textit{dynamically} from the condensate of the techni-fermion and the anti-techni-fermion pair \( \langle \bar{F}F \rangle \) which is triggered by the attractive gauge forces between the pair analogously to the quark-antiquark condensate \( \langle \bar{q}q \rangle \) in QCD. The dynamically generated mass scale for the W/Z boson mass is characterized by the dynamical mass of the techni-fermion, \( m_F = \mathcal{O}(\text{TeV}) \) which is a universal scale of techni-hadron mass \( M_{\text{TH}} = \mathcal{O}(m_F) \) and the techni-condensate: \( \langle \bar{F}F \rangle \sim -m_F^3 \). Actually, \( m_F \) picks up the intrinsic mass scale \( \Lambda_{\text{TC}} \) of the theory (analogue of \( \Lambda_{\text{QCD}} \) in QCD) already generated by the scale anomaly through quantum effects (“dimensional transmutation”) in the gauge theory which is \textit{scale-invariant} at classical level (for massless flavors):

\[
m_F \sim A_{\text{TC}} = \mu \cdot \exp \left( -\int \frac{\alpha(\mu)}{\beta(\alpha)} \right),
\]

where \( A_{\text{TC}} \) is independent of the renormalization point \( \mu \), \( \frac{dA_{\text{TC}}}{d\mu} = 0 \), and the running (scale-dependence) of the coupling constant \( \alpha(\mu) \), with non-vanishing beta function \( \beta(\alpha) \equiv \mu \frac{d\alpha(\mu)}{d\mu} \neq 0 \), is a manifestation of the scale anomaly. Thus the \textit{Origin of Mass is eventually the quantum effect (scale anomaly)} in this picture, with the scale symmetry broken explicitly at the scale of \( \mathcal{O}(A_{\text{TC}}) \) without remnant of the scale symmetry at all. Note that the intrinsic scale \( A_{\text{TC}} \) can largely be separated from the fundamental scale, the Planck scale \( \Lambda_{\text{Pl}} \) through logarithmic running (“naturalness”):

\[
\text{Naturalness(QCD/ScaleUp)} : \quad M_{\text{TH}} = \mathcal{O}(m_F) = \mathcal{O}(A_{\text{TC}}) = \mathcal{O}(\text{TeV}) \ll \Lambda_{\text{Pl}}.
\]

However, the original version of TC\textsuperscript{12}, a naive scale-up version of QCD, was dead due to the excessive flavor-changing neutral currents (FCNC). In order to give mass to the quarks/leptons not just to the W/Z boson, we need to introduce another scale, say \( A_{\text{ETC}} \), typically through the extended TC (ETC) model\textsuperscript{13} \#1: \( m_{q/l} \sim -\langle \bar{F}F \rangle / A_{\text{ETC}}^2 \sim m_F^3 / A_{\text{ETC}}^2 \). This also induces FCNC roughly of order \( 1 / A_{\text{ETC}}^2 \), which is constrained by the experiments as \( A_{\text{ETC}} > 10^3 - 10^4 \) TeV and hence reproduces only \( 10^{-3} \) times the realistic value for \( m_{q/l} \).

It was resolved long time ago by the Walking TC (WTC)\textsuperscript{15} initially dubbed “Scale-Invariant Technicolor”, based on the SSB solution of the ladder Schwinger-Dyson (SD) equation with \textit{non-running (scale invariant/conformal)} gauge coupling, \( \alpha(p) \equiv \alpha \), which we found gives rise to the large anomalous dimension,

\[
\gamma_m = 1,
\]

when the dynamical mass generation \( m_F \neq 0 \) takes place for strong coupling \( \alpha > \alpha_{\text{cr}}(= \mathcal{O}(1)) \), thus enhancing the techni-fermion condensate \( \langle \bar{F}F \rangle |_{\mu = Z^{-1} \langle \bar{F}F \rangle |_{\mu = m_F} } \).

\#1 Such a scale can be introduced by other models, for instance a composite quark/lepton/techni-fermion model\textsuperscript{13}. 

\textsuperscript{12}Technicolor

\textsuperscript{13}Extended Technicolor

\textsuperscript{14}Composite model

\textsuperscript{15}Walking Technicolor
\[ \langle FF \rangle |_{FF} \sim -m_F^2, \] by the factor \[ Z_m^{-1} = Z_m^{-1}(\mu/m_F) = (\mu/m_F)^{\alpha_m} = \mu/m_F \] such that \[ \Lambda_{ETC}/m_F \sim 10^3 \] for \( \mu = \Lambda_{ETC} \). (A solution to the FCNC problem by the large anomalous dimension was suggested earlier by simply assuming the existence of a large anomalous dimension without any concrete dynamics and concrete value of the anomalous dimension\(^\text{[13]}\). It was noted\(^\text{[19]}\) that the coupling (> \( \alpha_{ct} \)) actually does become running slowly (“walking”) nonperturbatively a la Miransky\(^\text{[19]}\), \( \alpha = \alpha(Q) \) for \( m_F < Q < \Lambda_{ETC} \) \( (Q^2 \equiv -p^2 > 0) \), with nonperturbative beta function\(^\text{[19]}\),

\[ \beta_{NP}(\alpha) = -(2\alpha_{ct}/\pi) \cdot (\alpha/\alpha_{ct} - 1)^{3/2}, \]

yielding a non-perturbative scale anomaly at the scale \( m_F \) dynamically generated by the SSB. Subsequently, a similar FCNC solution was discussed without notion of anomalous dimension and scale invariance\(^\text{[20]}\).

The WTC also predicted a Techni-dilaton (TD)\(^\text{[13]}\) a pseudo Nambu-Goldstone (NG) boson of the approximate scale symmetry, which is a composite Higgs, a scalar \( \bar{F}F \) bound state, behaving similarly to the SM Higgs and will be most relevant to the LHC physics as we will discuss in this talk. (For reviews of WTC see Ref.\(^\text{[21]}\).)

The mass generation due to such a scale-invariant (conformal) dynamics takes the form of essential-singularity scaling, Miransky scaling\(^\text{[13]}\)

\[ m_F \sim \Lambda \cdot \exp \left( -\int \alpha(A) \frac{d\alpha}{\beta_{NP}(\alpha)} \right) \sim \Lambda \cdot \exp \left( -\frac{\pi}{\sqrt{\alpha_{ct}} - 1} \right) \ll \Lambda, \tag{1.4} \]

in a way to ensure a large natural hierarchy \( m_F \ll \Lambda \) \((\alpha \simeq \alpha_{ct})\). This is characterized by the “conformal phase transition”\(^\text{[22]}\). Thus the essence of the WTC is a model setting of walking (scale-invariant) coupling \( \alpha(p) \approx \text{const.} \approx \alpha_{ct} \) as an input (perturbative) coupling in the SD equation, which results in non-perturbatively walking coupling for the wide energy region \( m_F < Q < \Lambda \) \((m_F \ll \Lambda)\) for \( \Lambda = \Lambda_{ETC} \).

Such a situation is actually realized\(^\text{[23]}\)\(^\text{[24]}\)\(^\text{[25]}\) by the two-loop perturbation in the large \( N_f \) QCD which has the Caswell-Banks-Zaks (CBZ) IR fixed point\(^\text{[26]}\), \( \alpha \) in the beta function:

\[ \beta_{2\text{-loop}}(\alpha) = -b\alpha^2(1 - \alpha/\alpha_s), \]

for 0 < \( Q < \Lambda_{TC} \), while it runs asymptotically free and diminishes rapidly in the same way as the ordinary QCD in the UV region:

\[ \alpha(Q) \sim 1/\ln(Q/\Lambda_{TC}) \]

for \( Q > \Lambda_{TC} \), where \( \Lambda_{TC} \) is the intrinsic scale, a two-loop analogue of \( \Lambda_{QCD} \), defined by Eq.\(^\text{[1.1]}\) with the two-loop beta function \( \beta(\alpha) \Rightarrow \beta_{2\text{-loop}}(\alpha) \). Thus the situation is similar to the original model\(^\text{[15]}\) with \( \Lambda_{TC} \) playing a role of the UV cutoff \( \Lambda = \Lambda_{ETC} \):

\[ \Lambda_{TC} \sim \Lambda_{ETC}. \]

Salient feature of the WTC is the large hierarchy of techni-hadron masses and \( \Lambda_{TC} \): \( M_{TH} = O(m_F) \ll \Lambda_{TC} \), which is naturally realized as in Eq.\(^\text{[1.4]}\) by the scale-invariant dynamics. This is contrasted to the QCD-scale up TC where there is no hierarchy between the techni-hadron mass and the intrinsic scale \( \Lambda_{TC} \) which is a typical IR scale instead of UV scale: \( M_{TH} = O(m_F) = O(\Lambda_{TC}) \).

\#2 For \( Q > \Lambda_{TC} \) the WTC model no longer makes sense as the same theory but becomes a part of a larger model like ETC, though. Dynamical origin of the scale \( \Lambda_{ETC} \) is a separate issue which may be the scale of tumbling or further compositeness arising eventually from the scale anomaly in the more fundamental gauge theory.
Even more striking feature of the WTC is the "Techni-dilaton (TD)"\textsuperscript{15,17} a naturally light scalar $\bar{F}F$ composite Higgs as mentioned above. As a pseudo NG boson, the TD mass $M_{\text{TD}}$ is even smaller than those of all other techni-hadrons: $M_{\text{TD}} < M_{\text{TH}}$. This is against a long-standing folklore that the TC is a "Higgsless" model having no light scalar composite. Such a folklore is applied only to the original TC as a simple scale-up of the QCD where the coupling is running already at perturbative level, with the scale symmetry badly broken for all energy region and there is no remnant of the scale symmetry. Thus, besides that the hierarchy $\Lambda_{\text{TC}} \ll \Lambda_{\text{Pl}}$ as in QCD scale-up in Eq.(1.2), we have additional natural hierarchies related with the scale symmetry:

\textbf{Naturalness (WTC)} : \quad M_{\text{TD}} < M_{\text{TH}} = \mathcal{O}(m_F) \ll \Lambda_{\text{TC}} \ll \Lambda_{\text{Pl}}, \quad (1.5)

with a model setting $\Lambda_{\text{TC}} \sim \Lambda_{\text{ETC}}$. From various calculations\textsuperscript{29,30} related with the ladder approximation we suggested\textsuperscript{31} that the TD mass in a typical TC model (one-family model) is (up to large uncertainty in the ladder-like calculations):

$$M_{\text{TD}} \simeq 500 - 600 \text{ GeV}. \quad (1.6)$$

In this talk we argue\textsuperscript{32} that such a TD as a composite Higgs can be soon discovered by the LHC experiment\textsuperscript{34}. We will also describe the related composite Higgs boson in various models of dynamical symmetry breaking with large anomalous dimension (For reviews see Ref.\textsuperscript{26}), namely a class of composite Higgs models based on the walking gauge dynamics having large anomalous dimension characteristic to the conformal UV/IR fixed point: Strong-ETC TC with $1 < \gamma_m < 2$\textsuperscript{36} Top Quark Condensate Model\textsuperscript{37} with $\gamma_m \simeq 2$, and their variants. They will be tested soon in the on-going LHC experiments.

\section{Walking Technicolor}

The WTC is the model with dynamical mass generation $m_F$ with large anomalous dimension\textsuperscript{15}

$$\gamma_m \simeq 1. \quad (2.1)$$

The model was proposed based on the SSB solution of the ladder Schwinger-Dyson (SD) equation (Fig. 1) for the fermion full propagator $S_F(p)$ parameterized as $iS_F^{-1}(p) = A(p^2)\not{p} - B(p^2)$, with non-running (scale-invariant) gauge coupling $\alpha(Q) \equiv \alpha > \alpha_c$ ($Q^2 \equiv -p^2 > 0$). It was shown\textsuperscript{38} that the SSB solution exists only for strong coupling $\alpha > \alpha_{cr} = \mathcal{O}(1)$.

\#3 After the Symposium, LHC announced some excess around 125GeV\textsuperscript{33} which happen to be consistent with the techni-dilaton\textsuperscript{35} Such a lower mass of the techni-dilaton is in fact protected by the scale symmetry: The quadratic divergence loop corrections are $\delta M_{\text{TD}}^2 \sim \mu^2/(4\pi)^2 < m_F^2/(4\pi)^2 < m_F^2$, with those from $m_F < \mu < \Lambda_{\text{TC}}$ being highly suppressed by the scale symmetry.
The asymptotic form of the SSB solution of the fermion mass function $\Sigma(Q) = B(p^2) / A(p^2)$ ($\Sigma(m_F) = m_F$) in Landau gauge ($A(p^2) \equiv 1$) reads,\textsuperscript{38, 39}

$$\Sigma(Q) \sim 1/Q \quad (m_F < Q < \Lambda),$$

(2.2)

which we found\textsuperscript{10} implies a large product value of the anomalous dimension $\gamma_m = 1$, to be compared with the operator product expansion (OPE), $\Sigma(Q) \sim 1/Q^2 \cdot (Q/m_F)^{\gamma_m}$. Accordingly, we had a linearly divergent condensate ($F F$)$_\Lambda = Z^{-1} \cdot (F F)_m \sim -(\Lambda/m_F) \cdot m_F^2$, with the (inverse) mass renormalization constant being $Z_1^m = (\Lambda/m_F)^{\gamma_m} = \Lambda/m_F > 10^3$ for $\Lambda = \Lambda_{ETC} > 10^3 m_F$, which in fact yields the desired enhancement: We actually obtained $m_{q/L} \sim m_F^2 / \Lambda_{ETC}$,\textsuperscript{15} which is compared with the QCD scale-up TC $m_{q/L} \sim m_F^2 / \Lambda_{ETC}$.

The model\textsuperscript{15} was actually formulated in terms of the Miransky’s nonperturbative renormalization\textsuperscript{13} of the SSB solution which takes the essential singularity form of Eq.(1.4) (Miransky scaling):\textsuperscript{39, 13}

$$m_F \sim \Lambda \exp \left(-\pi/\sqrt{\alpha/\alpha_{cr} - 1}\right),$$

(2.3)

where the critical value $\alpha_{cr}$ reads\textsuperscript{39}

$$C_2(F) \cdot \alpha_{cr} = \frac{\pi}{3},$$

(2.4)

in the $SU(N_{TC})$ gauge theory, where $C_2(F) = (N_{TC}^2 - 1) / 2 N_{TC}$ is the quadratic Casimir of the techni-fermion representation. Once $m_F$ is dynamically generated in such a way, the coupling does depend on $\Lambda/m_F$, and no longer remains constant but does start walking with the nonperturbative beta function\textsuperscript{19}

$$\beta^{NP}(\alpha) = \frac{\partial \alpha}{\partial \ln(\Lambda/m_F)} = -\frac{2\pi^2 \alpha_{cr}}{\ln^3(\Lambda/m_F)} = -\frac{2\alpha_{cr}}{\pi} \left(\frac{\alpha}{\alpha_{cr}} - 1\right)^{3/2},$$

(2.5)

where the critical coupling $\alpha_{cr}$ was identified with a nontrivial UV fixed point $\alpha = \alpha(\Lambda/m_F) \rightarrow \alpha_{cr}$ as $\Lambda/m_F \rightarrow \infty$. This reflects explicit breaking of the scale symmetry (nonperturbative scale anomaly) due to the generated mass scale $m_F$ which is the very origin of spontaneous breaking of the scale symmetry. Although the IR scale $m_F$ is originated from the UV scale (fundamental/intrinsic scale) $\Lambda$ of the theory as in Eq.(2.3), this scale anomaly characterized by the scale $m_F$ is persistent, even if we removed the UV scale $\Lambda \rightarrow \infty$ ("perturbatively complete scale-invariant limit"). Note that the essential singularity corresponds to multiple zero of $\beta^{NP}(\alpha)$ as seen from Eq.(1.4), which is never realized in the perturbative calculations.\textsuperscript{#4}

The essential feature of the above is precisely what happens in the modern version\textsuperscript{20, 21, 22} of the WTC based on the CBZ IR fixed point\textsuperscript{22} of the large $N_f$ QCD, the QCD-like theory with many flavors $N_f \gg N_{TC}$ of massless techni-fermions.\textsuperscript{#5}

\textsuperscript{#4} Linear zero of the perturbative beta function, $\beta(\alpha) \sim (\alpha - \alpha_{cr})^1$, never reproduces the essential singularity scaling, as is evident from Eq.(1.4).

\textsuperscript{#5} For WTC based on higher representation/other gauge groups see, e.g., Ref.\textsuperscript{20}
The intrinsic scale $\Lambda$, a two-loop analogue of the $\Lambda_{QCD}$, may be chosen as $\alpha(\Lambda_{T}) = 1/(1 + 1/e) \alpha_s \approx \alpha$, so that the two-loop coupling is almost non-running in the UV region: $\alpha(Q) \approx \alpha_s (Q \ll \Lambda_{T})$, while it runs as in the ordinary QCD in the UV region: $\alpha(Q) \approx 1/\ln(Q/\Lambda_{T}) (Q \gg \Lambda_{T})$. Thus $\Lambda_{T}$ plays a role of cutoff $\Lambda(t)$ in the original model. Note that $\alpha_s = \alpha_s(N_{f}, \Lambda_{T}) \rightarrow 0$ as $\Lambda_{T} \rightarrow 11N_{T}/2 (b \rightarrow 0)$ and hence there exists a certain range $N_{f}^c < N_{f} < 11N_{T}/2$ ("Conformal Window") satisfying $\alpha_s < \alpha_{cr}$, where the gauge coupling $\alpha(p) (\alpha_s < \alpha_{cr})$ is not strong enough to trigger the SSB. The $N_{f}^c$ such that $\alpha_s(N_{f}, N_{f}^c) = \alpha_{cr}$ may be evaluated by using the value of $\alpha_{cr}$ from the ladder SD equation Eq. (2.4).

Here we are interested in the SSB phase slightly outside of the conformal window, $0 < \alpha_s - \alpha_{cr} \ll 1$ ($N_{f} \approx N_{f}^c$). We may use the same equation as the ladder SD equation with $\alpha(Q) \approx \alpha_s$, yielding the same result, Eqs. (2.1, 2.2, 2.3):

$$\Sigma(Q) \sim 1/Q, \quad \gamma_m \approx 1, \quad (m_F < Q < \Lambda_{T} (= \Lambda_{ETC})).$$

where the cutoff $\Lambda(t)$ was identified with $\Lambda_{T}$.

Eq. (2.8) implies a nonperturbative beta function $\beta^{NP} (\alpha)$ of the form Eq. (2.5) for $\alpha = \alpha_s = \alpha(\Lambda_{T}/m_F)$, with $\alpha_{cr}$ regarded as a UV fixed point in the limit $\Lambda_{T}/m_F \rightarrow \infty$. Such a beta function has a multiple zero corresponding to the essential singularity as noted before. Thus the two-loop beta function $\beta^{(2\text{-loop})}(\alpha)$ is to be replaced by the nonperturbative one, $\beta(\alpha) \approx \beta^{NP} (\alpha)$, as in the original model whereas it is essentially operative, $\beta(\alpha) \sim \beta^{(2\text{-loop})}(\alpha)$, in the UV region $Q > \Lambda_{T} (\alpha < \alpha_{cr})$.

Hence the full beta function within the SD equation analysis may be depicted as in Fig. 2. This should be tested by the fully nonperturbative studies like lattice simulations. A possible phase diagram (Fig 3 of Ref.22) of the large $N_f$ QCD on the lattice is also waiting for the test by simulations.

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Footnote: The value should not be taken seriously, since $\alpha_s = \alpha_{cr}$ is of $O(1)$ and the perturbative estimate of $\alpha_{cr}$ is not so reliable. Lattice simulations still suggest diverse results as to $N_{f}^c$. 

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§3. S Parameter Constraint

Now we come to the next problem of TC, so-called $S,T,U$ parameters measuring possible new physics in terms of the deviation of the LEP precision experiments from the SM. The most straightforward computation of the $S$ parameter for the large $N_f$ QCD is that based on the on-shell equation and (inhomogeneous) Bethe-Salpeter (BS) equation in the ladder approximation. The results show that there is a tendency of the $S$ getting reduced when approaching the conformal window $\alpha_s/\alpha_{cr} \approx 1$ ($N_f \rightarrow N_f^\gamma$), and at the present technical computational limit $\alpha_s/\alpha_{cr} \approx 1.13$ it reads $S \sim 0.056(N_{TC}, N_D)$ for $N_D$ doublets, about a half of that in the ordinary QCD, $S_{QCD} \sim 0.11 N_c$ ($N_D = 1$). The result is barely consistent with the experiments for $N_{TC} = 2, N_D = 1$. Highly desirable is the computation closer to $\alpha_s/\alpha_{cr} = 1$.

As another approach the reduction of $S$ parameter in the WTC has been argued in a version of the hard-wall type bottom up holographic QCD deformed to WTC by tuning a parameter to simulate the large anomalous dimension $\gamma_m \approx 1$. We find $S/N_D \approx 8\pi(F_\pi/M_\rho)^2 = 4\pi/g^2$ for $N_{TC} = 3$, where $M_\rho^2 = aF_\pi^2 g^2$ ($a \approx 2$) and $g$ is the gauge coupling of the hidden local symmetry. Although the results contain full contributions from an infinite tower of the vector/axial-vector Kaluza-Klein modes (gauge bosons of hidden local symmetries) of the 5-dimensional gauge bosons, the resultant value turned out close to that of a single $\rho$ meson dominance. This implies that as far as the pure TC dynamics (without ETC dynamics, etc.) is concerned, the $S$ parameter can be reduced only by tuning $F_\pi/M_\rho$ very small, namely techni-$\rho$ mass very large to several TeV region. Curiously enough, when we calculate $F_\pi/M_\rho$ from the $S/N_D$ $= 3$ lies on the line of the holographic result.

§4. Techni-dilaton

Now we come to the discussions of Techni-dilaton (TD). Existence of approximate scale invariance for for wide region $m_F < Q < \Lambda_{TC}$ between two largely separated scales, $m_F \ll \Lambda_{TC}$, is the most important feature of WTC near conformality, in sharp contrast to the ordinary QCD (in the chiral limit) where there is no scale invariant system and all the mass parameters $M$ are of order of a single scale parameter of the theory $\Lambda_{QCD}$, $M = O(\Lambda_{QCD})$. The intrinsic scale $\Lambda_{TC}$ is related with the scale anomaly corresponding to the perturbative running effects of the coupling $\alpha(Q)$ for $Q > \Lambda_{TC}$, with the ordinary two-loop beta function $\beta^{(2-loop)}$ in the same sense as in QCD. $\langle \partial^\mu D_\mu \rangle_{pert} = \langle \partial_\mu \rangle_{pert} = \langle \partial_\mu \rangle_{pert} = -\frac{2}{3}\langle aG_{\mu\nu}^2 \rangle_{pert} = -O(\Lambda_{TC})$.

In the WTC, on the other hand, there exists another scale $m_F \ll \Lambda_{TC}$ in addition to $\Lambda_{TC}$, where the largely separate scale $m_F$ is related with a totally different scale anomaly associated with the nonperturbative running of the coupling due to the dynamical generation of $m_F$ as in Eq. (20), which does exist even in the idealized case with perturbatively scale-invariant coupling $\alpha^{pert}(Q) = \alpha(> \alpha_{cr})$ for entire
region \(0 < Q < \Lambda_{TC} \to \infty\). Note that there is no idealized limit where the TD becomes exactly massless to be a true NG boson, in sharp contrast to the chiral symmetry breaking. The SSB of the scale symmetry as well as the chiral symmetry is triggered by the dynamical generation of \(m_F\), which however is the very cause of the nonperturbative running of the coupling, namely the nonperturbative scale anomaly:

The scale symmetry is always broken explicitly as well as spontaneously.

The non-perturbative scale anomaly reads

\[
\langle \partial^\mu D_\mu \rangle^{NP} = \langle \theta^{\mu \nu} \rangle^{NP} = \beta^{NP}(\alpha) \langle \alpha G_{\mu \nu}^2 \rangle^{NP} = -m_F^4 \cdot \kappa_V \frac{N_f N_{TC}}{2 \pi^2}, \tag{4.1}
\]

where \(\langle \cdots \rangle^{NP}\) is the quantity without perturbative contributions \(\langle \cdots \rangle^{NP} \equiv \langle \cdots \rangle^{pert.}\). Here \(\kappa_V = 0.76\) for the two-loop coupling \((N_f \simeq N_{cr}^f)\), while \(\kappa_V = 8/\pi^2 \simeq 0.81\) for the non-running coupling in the SD equation. All the technifermion bound states have mass \(M_{TH} = O(m_F)\), while there are no light bound states in the symmetric phase (conformal window) \(\alpha_* < \alpha_{cr}\), a characteristic feature of the conformal phase transition.

To be concrete we estimate \(m_F\) related to \(v = 246\) GeV as

\[
F_\pi^2 = N_D v^2 = \kappa_F^2 \frac{N_{TC}}{4 \pi^2} m_F^2, \tag{4.2}
\]

where \(\kappa_F \simeq 1.5\) from the Pagels-Stokar formula for the mass function at ladder-like criticality \(N_f \simeq N_{cr}^f \simeq 4 N_{TC}\) and \(N_D\) is the number of weak doublets \((N_D = 4\) for one-family model and \(N_D = 1\) for one-doublet model).\[28\] We take \(N_f = 2 N_D + 2 N_{EW-singlet} - N_{EW-singlet}\), where \(N_{EW-singlet}\) is the number of electroweak singlet techni-fermions which only contribute to the walking coupling. Thus we have rough estimate

\[
m_F \simeq 1\text{TeV}/\sqrt{N_D N_{TC}}. \tag{4.3}
\]

4.1. Calculation from Gauged NJL model in the ladder SD equation\[22\]

More concretely, the mass of TD was estimated\[31\] by reinterpretation of the old results on the scalar bound state mass by various methods: The first method\[20\] was based on the ladder SD equation for the gauged NJL model which well simulates the conformal phase transition in the large \(N_f\) QCD. We find:

\[
M_{TD} \simeq \sqrt{2} m_F. \tag{4.4}
\]

4.2. Straightforward Calculation from Ladder SD and BS equation\[20\]

Also a straightforward calculation of the mass of TD, the scalar bound state, was made in the vicinity of the CBZ-IR fixed point in the large \(N_f\) QCD, based on the coupled use of the ladder SD equation and (homogeneous) BS equation. All the bound states masses \(M_{TH}\) as well as \(F_\pi\) are of order \(O(m_F)\) and \(M_{TH}/\Lambda_{TC}, F_\pi/\Lambda_{TC} \to 0\), when approaching the conformal window \(\alpha_* \to \alpha_{cr} (N_f \to N_{cr}^f)\) such that \(m_F/\Lambda_{TC} \to\)
Near the conformal window ($N_f \gg N_f^{TC}$) the calculated values are $M_\rho/F_\pi \simeq 11, M_{a_1}/F_\pi \simeq 12$ (near degenerate!). On the other hand, the scalar mass sharply drops near the conformal window, $M_{TD}/F_\pi \sim 4.3^4$ $M_{TD} \sim 1.5m_P \sim \sqrt{2}m_\pi (< M_\rho, M_{a_1})$, which is consistent with Eq.(4.1) and is contrasted to the ordinary QCD where the scalar mass is larger than those of the vector mesons (“higgsless”) within the same framework of ladder SD/BS equation approach. Note that in this calculation $M_{TD}/F_\pi \rightarrow \text{const.} \neq 0$ and hence there is no isolated massless scalar bound state even in the limit $N_f \rightarrow N_f^{TC}$. The result would imply a typical value

$$M_{TD} \simeq 4F_\pi \simeq 500\,\text{GeV} \quad (M_\rho \simeq M_{a_1} \simeq 12F_\pi \simeq 1.5\,\text{GeV}),$$

in the case of the one-family TC model with $F_\pi \simeq 125\,\text{GeV}$.

4.3. Holographic Techni-dilaton$^{[27]}$

The mass of TD was also calculated in a hard-wall-type bottom-up holographic TC with $\gamma_m = \frac{43}{4}$ by including effects of (techni-) gluon condensation through the bulk flavor/chiral-singlet scalar field $\Phi_X$, in addition to the conventional bulk scalar field $\Phi$ dual to the chiral condensate. The TD, a flavor-singlet scalar bound state of techni-fermion and anti-techni-fermion, will be identified with the lowest KK mode coming from the bulk scalar field $\Phi$, not $\Phi_X$. Our model with $\gamma_m = 0$ and $N_f = 3$ well reproduces the real-life QCD.

We consider a couple of typical models of WTC with $\gamma_m \simeq 1$ and $N_{TC} = 2, 3, 4$ based on the CBZ-IRFP in the large $N_f$ QCD. The TD mass can be calculated in terms of $S$ parameter and the techni-gluon condensate normalized by the QCD value: $\Gamma \equiv \left(\frac{1}{2N_f} G^2_{\mu\nu}/F_\pi^4\right)^{1/4}$ normalized. For a fixed $S$ the TD mass decreases as $\Gamma$ increases: $M_{TD}/m_F \sim 0$ as $\Gamma \rightarrow \infty$. Using the non-perturbative scale anomaly Eq.(4.11) together with the non-perturbative beta function Eq.(2.5), we can estimate $\Gamma$ in terms of $A_{ETC}/m_F$: $\Gamma \rightarrow \infty$ as $A_{ETC}/m_F \rightarrow \infty$ ($\beta_N^\text{NP}(\alpha) \rightarrow 0$). In the case of $N_{TC} = 3$ ($N_f = 4N_{TC}, N_D = 4$) and $S(\simeq N_D \cdot 8\pi(F_\pi/M_\rho)^2 \simeq 8\pi(\sqrt{3}/M_\rho)^2) = 0.1$, we have $\Gamma \simeq 7$ for $A_{ETC}/m_F = 10^{-3} - 10^{-4}$ (FCNC constraint), which yields

$$M_{TD} \simeq 600\,\text{GeV},$$

relatively light compared with $M_\rho \simeq M_{a_1} \simeq 3.8\,\text{TeV}(\simeq \nu\sqrt{8\pi/S})$. $^{[77]}$

4.4. Discovering Walking Technicolor at LHC$^{[22]}$

Now we come to discussions on the discovery signatures of TD at the ongoing LHC. Here we take the TD mass as a free parameter in the region $200\,\text{GeV} < \rho \sim M_\rho \simeq M_{a_1} \sim 3.8\,\text{TeV}(\simeq \nu\sqrt{8\pi/S}).$$^{[77]}

Note that largeness of $M_\rho$ and $M_{a_1}$ is essentially determined by the requirement of $S = 0.1$ fairly independently of techni-gluon condensation (Ladder result $M_\rho \simeq 1.5\,\text{TeV}$ corresponds to $S \simeq 0.056N_{TC}N_D \simeq 0.67$ for $N_{TC} = 3, N_D = 4$). The calculated $S$ parameter here was from the TC dynamics alone and the actual $S$ parameter could be drastically changed by the ETC-like dynamics. For instance, the fermion delocalization$^{[23]}$ in the Higgsless models as a possible analogue of certain ETC effects in fact can cancel large positive $S$ arising from the 5-dimensional gauge sector which corresponds to the pure TC dynamics. If it is the case in the explicit ETC model, then the large $S$ value from WTC sector would still be viable and the overall mass scale of techni-hadrons (and hence TD mass also) would be much lower than the above estimate.
$M_{TD} < 1000 \text{GeV}$, since the explicit calculations so far based on the ladder-like approximation $M_{TD} \approx 500 - 600 \text{GeV}$ may have large uncertainty.

The coupling of TD ($\phi$) to the SM particles are all through the loop of the techni-fermions ($F$) via the Yukawa coupling of the TD to the techni-fermions, since there is no direct coupling of TD to the SM particles. The Yukawa coupling to the techni-fermions and to the quarks/leptons ($f$) were obtained long time ago:

$$g_{TDFF} = \frac{m_F}{v_{TD}}, \quad g_{TDff} = \frac{m_f}{v_{TD}}, \quad \left( v_{TD} = \frac{F_{TD}}{3 - \gamma_m} \right) \quad (4.7)$$

where $3 - \gamma_m \simeq 2$ for the WTC, and the TD decay constant $F_{TD}$ differs from that of the SM Higgs $v = 246 \text{GeV}$. The coupling to the SM gauge bosons $L_{TDW\gamma\gamma} = g_{TDW}\phi_{W}\phi_{W} + \frac{1}{2}(W \rightarrow Z)$, $L_{TDgg/\gamma\gamma} = -g_{TDgg}\phi_{tr}[G_{\mu\nu}^2] - g_{TD\gamma\gamma}\phi_{F_{\mu\nu}}$, are given by Fig. 3.

$$g_{TDW/Z} = \frac{2m^2_{W/Z}}{v_{TD}}, \quad g_{TDgg} = \frac{1}{v_{TD}} \frac{\beta_F(\alpha_s)}{2\alpha_s}, \quad g_{TD\gamma\gamma} = \frac{1}{v_{TD}} \frac{\beta_F(\alpha_{EM})}{4\alpha_{EM}}, \quad (4.8)$$

where $\beta_F$ stands for the one-loop beta function solely due to the techni-fermions (For the actual calculations we will also include loop contributions of the $W/Z$ bosons and the top quark, which are higher loop effects numerically non-negligible (not a substantial effects, though).) (Somewhat different estimation was made.

Thus the TD couplings differ from those of the SM Higgs by the characteristic factor $1/v_{TD}$ versus $1/v = 1/(246 \text{GeV})$, and also by $\beta_F/v_{TD}$ versus $\beta_{SM}/v$, where $\beta_{SM}$ is the one-loop beta function of only the SM particle contributions. Once we fix the TD mass $M_{TD}$, we may estimate $F_{TD}$ (and hence $v_{TD}$) through the Partially Conserved Dilatation Current (PCDC) hypothesis for the non-perturbative anomaly Eq.(4.1):

$$F_{TD}^2 M_{TD}^2 = -4(g_{\mu\nu}^{EM})_{\mu
u} = 4m_F^4 \cdot \kappa_{V} N_f N_{TC} \frac{\kappa_F}{2\pi} \frac{1}{\alpha_{EM}}, \quad \kappa_{V} \sim 1, \quad (\kappa_{F}, \kappa_{V}) \approx (1.5, 0.76).$$

Combining this with Eq.(4.2), we may have the overall ratio of the TD coupling to that of the SM Higgs $g_{TD}/g_H = v/v_{TD}$:

$$g_{TD} = \frac{v}{v_{TD}} \simeq \frac{1}{4\sqrt{2\pi}} \frac{\kappa_F^2}{\kappa_{V}} N_f M_{TD} \frac{M_{TD}}{v} \simeq 1.4 \left( \frac{N_f}{4} \right) \left( \frac{M_{TD}}{600 \text{GeV}} \right), \quad (4.9)$$

where the TD coupling is roughly the same as the SM Higgs. The predicted signatures of Higgs-like particles searched at LHC of the 7 TeV run is depicted in Fig. 4 and 5 for the mass range $200 \text{GeV} < M_{TD} < 1000 \text{GeV}$.

Fig. 3. The TD couplings to $W\gamma$, $Z\gamma$ induced from techni-fermion loops.
Fig. 4. Left panel: The TD LHC production cross sections at $\sqrt{s} = 7$ TeV times the $WW/ZZ$ branching ratio in the 1DMs with $N_{TC} = 2, 3$ normalized to the corresponding quantity for the SM Higgs. Also shown is the comparison with the 95\% C.L. upper limits from the ATLAS and CMS. Right panel: The same as the left panel for the 1FMs.

In the simplest model, one-doublet model (1DM, $N_D = 1$), all the couplings are suppressed relative to the SM Higgs and hence invisible at the present LHC search. In the one-family model (1FM), on the other hand, what is dramatically different from the SM Higgs is the 2-gluon and 2-photon couplings which have an extra factor of beta function $\beta_F$: There are many techni-fermions having color and electric charges, so that we have a big enhancement of gluon fusion production of TD (by the factor $31(87)$ for $N_{TC} = 2(3)$) in comparison with the SM Higgs, which boosts overall scale of the all decay channels including a typical searched channel $WW$ at LHC (Fig.4). Also enhanced are the branching ratios of 2-gluons (by $16(44)$ for $N_{TC} = 2(3)$) and 2-photon channels (by $3.2(11)$ for $N_{TC} = 2(3)$) in 1FM relative to the SM Higgs, while other channels are roughly the same as in the SM Higgs.

Comparing the result with the present LHC data (see also footnote 33) shown in Fig.4, we see a big chance to discover the 1FM TD for the mass range $M_{TD} > 600$ GeV, although TD for $200 \text{ GeV} < M_{TD} < 600$ GeV of the 1FM is excluded by the present LHC data. Also we can expect in future a prominent signal of 2-photon channel on $0.1-1.0$ fb level at $M_{TD} > 600$ GeV as shown in Fig.5.

4.5. Discovering Walking Technicolor on the Lattice

Since the WTC models near conformality are strong coupling theories and the ladder approximation/holographic calculations would be no more than a qualitative hint, more reliable calculations are certainly needed, including the lattice simulations, before drawing a definite conclusion about the physics predictions. Recently there have been many lattice studies of large $N_f$ QCD. Besides the phase diagram including the TC-induced/ETC-driven four-fermion couplings on the lattice, more reliable calculations of the spectra, particularly the TD, as well as the anomalous dimension, non-perturbative beta function, $S$ parameter, etc. are highly desired. For that end
we have started to do lattice simulations at KMI (LatKMI collaboration)\(^{33}\) on the large \(N_f\) QCD as a possible candidate for the WTC.

§5. Top Quark in Walking Theories

The top quark is very special in the ETC, since the top mass is too large to be accounted for by the WTC with \(\gamma_m \simeq 1\) in the usual ETC-like scenario: 

\[
m_{t/b} \sim \frac{1}{\Lambda_{\text{ETC}}} \langle \bar{F}F \rangle_{\Lambda_{\text{ETC}}} \sim m_F (m_F/\Lambda_{\text{ETC}})^{2-\gamma_m}.
\]

Moreover it would require large isospin violation in the condensate \(\langle \bar{U}U \rangle_{\Lambda_{\text{ETC}}} \gg \langle \bar{D}D \rangle_{\Lambda_{\text{ETC}}}\) in order to produce large mass splitting \(m_t \gg m_b\), which would in general contradict the \(T\) parameter constraint, unless we have \(\langle \bar{U}U \rangle_{m_F} \approx \langle \bar{D}D \rangle_{m_F}\).

A possible way-out would be the Strong-ETC TC\(^{36}\) which has an anomalous dimension larger than that of the WTC as in the gauged NJL model; \(1 < \gamma_m < 2\) only for \(\langle \bar{U}U \rangle\). This can boost the top mass as large as \(m_t = \mathcal{O}(m_F) (\gamma_m \simeq 2)\), where \(m_F \simeq 300\text{ GeV} \cdot \sqrt{4/N_D}/(3/N_{\text{TC}})\). There would be no conflict with the \(T\) parameter as far as we have \(\langle \bar{U}U \rangle_{m_F} \approx \langle \bar{D}D \rangle_{m_F}\). The TD in this case would also have a mass \(\sim 500-600\text{ GeV}\) in the ladder approximation, whereas the coupling to the top quark pair will be scaled as \((3-\gamma_m)/F_{\text{TD}} \simeq 1/F_{\text{TD}}\) instead of \(2/F_{\text{TD}}\) of WTC with \(\gamma_m \simeq 1\), suppressing the \(t\bar{t}\) channel relative to others by roughly a factor \(1/4\).\(^{51}\)

Another possibility to have large top mass is the Top Quark Condensate (Top-Mode Standard Model, TMSM)\(^{37}\) which introduced the \(\langle \bar{t}t \rangle\) based on the dynamics of gauged NJL model having \(\gamma_m \simeq 2\)\(^{36}\). The model predicted (long before the discovery of the top with mass of this large) the qualitative reality that among other quarks only the top (as well as \(W, Z\)) has mass on the order of weak scale. The model also has a composite Higgs boson as a bound state of \(t \bar{t}\) ("Top-Higgs") \(m_t < m_{H_t} < 2m_t\), which seems to be ruled out by the LHC experiments. A viable Top-mode model involving the top quark condensate would be the top-seesaw model\(^{55}\) and its combination with WTC ("top-seesaw-assisted technicolor")\(^{56}\).

§6. Conclusion

In the spirit of Sakata we have developed possible composite Higgs models with large anomalous dimension, Walking Technicolor (WTC), Strong ETC Technicolor, Top-Mode models, etc.

Particularly the Techni-dilaton (TD) was predicted as a pseudo Nambu-Golodstone boson of the approximate scale symmetry inherent to the WTC. This is a composite Higgs similar to the SM Higgs boson and hence will be the target of the most urgent Higgs search at LHC. In the typical one-family model having four doublets corresponding to techni-quarks and techni-lepton, mass \(M_{\text{TD}}\) was estimated to be around 500-600 GeV in the ladder-like computation and/or its combination with the holographic method, although the result should have large uncertainty (The simplest model, one-doublet model, will give a doubled value of mass). Treating \(M_{\text{TD}}\) as a free parameter, we estimated the coupling in terms of \(M_{\text{TD}}\) through PCAC relation,
based on the ladder estimation of the vacuum energy and techni-pion decay constant $F_\pi$, see Eq. (4.9). Crucial difference of one-family model from the SM Higgs comes from the enhanced 2-gluon and 2-photon couplings in Eq. (4.8), which have big contributions from the loops of the techni-fermions carrying color and electric charges in the form of the beta functions of color and electromagnetic couplings. One-doublet model lacking these enhancement as well as smaller coupling by factor $1/4 = 4/N_D$ in Eq. (4.9) will give a very small signals and would not be visible at the present setting of LHC.

The actual analysis was performed for the mass range $200 \text{ GeV} < M_{TD} < 1,000 \text{ GeV}$. See Fig 4 and Fig 5. The result implies that the one-family TD in the mass region $200 \text{ GeV} < M_{TD} < 600 \text{ GeV}$ is excluded. There will be a big chance to discover the one-family TD in the region $M_{TD} > 600 \text{ GeV}$ because of the large excess which will be detected in $WW$ channel and 2-photon channel at LHC in near future.

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