Chirp effects on pair production in oscillating electric fields with spatial inhomogeneity

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Abstract

Dirac-Heisenberg-Wigner formalism is used to study chirp effects on the vacuum pair creation under inhomogeneous electric fields. For rapidly oscillating electric fields, the particle momentum spectrum is sensitive to both of the spatial scale and the chirp parameter, while the total particle number depends linearly on the spatial extent when the chirp parameter is maximally large. For slowly oscillating electric fields, chirp effects could be identified at large spatial extents and the carrier phase may play a significant role reflecting chirp effects even at small spatial scales. We also notice that, the local density approximation holds for all external field profiles considered in this work at the quasihomogeneous limit allowing one to use arguments from homogeneous scenarios to analyze inhomogeneous results with large spatial extents.

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I. INTRODUCTION

Decay of the vacuum state under intense external fields is one of the nonperturbative predictions of the quantum electrodynamics (QED) yet to be observed [1–3]. Reflecting the consequence of the quantum vacuum in the background field, detection of this nonlinear QED phenomena requires field strength of order $E_{cr} \sim 10^{16} \text{V}/\text{cm}[4]$. Although matter creation using strong external fields has been beyond the reach of experiments, electron positron creation in the laser experiments [5, 6] and the realization of the light-by-light scattering of quasi-real photons in the particle accelerators [7, 8] have strengthened the interest in further quests for pair production studies. The upcoming experiments play a major stimulus on this journey by promising laser intensities at the unprecedented level which makes it possible to observe nonperturbative vacuum pair production in the near future [9–12].

Theoretically, it is crucial to consider more realistic external field modes to provide reliable predictions for future experiments; for a review see Ref. [13]. The nonlinear nature of the vacuum pair creation causes the process to be sensitive to external field parameters, thus careful shaping of the applied field might induce special momentum signatures reflecting specific field structures and corresponding particle creation dynamics [14–18]. Moreover, the possibility of lowering the required field strength is reported for the clever combination of external fields with different frequencies [19].

Both of the characteristic momentum spectrum signatures and the enhancement mechanism are found in the pair creation under chirped laser field modes [15, 20]. Such frequency variations of the external field are crucial to be studied not only because they represent more complex field forms but for the fact that intense laser fields are realized in laboratories using the chirped pulse amplification (CPA) technique [21]. The chirp effect is considered in tunneling and multiphoton absorption modes, and the effects of chirp parameters are mostly captured in the momentum spectrum, while the total particle number is raised drastically for large chirp parameters in the multiphoton process [15, 20].

On the other hand, previous works on pair creation studies in the inhomogeneous mode indicate that the spatial dependence of the external field may have nontrivial effects. Particle selfbunching was reported in the momentum spectrum of particles in Schwinger pair creation where the total particle number also depends nonlinearly on the spatial scale of the external field [23]. For an oscillating profile, the spatial focusing of the electric field introduces ponderomotive force effects
into multiphoton process due to strong spatial variation of the electric field \[24\]. Moreover, a new kind of oscillatory pattern was noticed in Schwinger pair creation for the narrow spatial focusing \[25\]. These findings further indicate the importance of including spatial variations of the external field in pair production studies under more realistic field forms; see Ref. \[26\] for further discussions.

In this paper, we use the real time Dirac-Heisenberg-Wigner(DHW) formalism \[22, 23\] to investigate chirp effects in the spatially inhomogeneous mode using simplified model for a chirped laser pulse. A large span of spatial scales are considered to reflect the extent of spatial dependence of the external field. In the quasihomogeneous scenario, where the spatial extent is large, we compare our results with pair creation in homogeneous cases. When the spatial extent is small, we investigate results for differently oscillating external fields which may display various mechanisms due to the complex interplay between temporal and spatial parameters. Special care is given to the carrier phase effect in the slowly oscillating scenario.

Our paper is organized as follows. In Sec. \[III\] we present the treatment of pair creation in 1+1 dimensions by introducing the field model to be considered in this study in Sec. \[II A\] and reviewing the key points of the DHW formalism in Sec. \[II B\] In Sec. \[III\] the numerical results obtained for various field shapes are presented with physical implications. Sec. \[III A\] and Sec. \[III B\] show the momentum distribution as well as the particle yield for fast and slow oscillations respectively. We present our conclusion in Sec. \[IV\].

Throughout this article, natural units ($\hbar = c = 1$) are used and the quantities are presented in terms of the electron mass $m$.

II. OSCILLATING ELECTRIC FIELDS AND PAIR CREATION

A. External fields

In this article, we study the electron positron pair production in 1+1 dimensions by considering the following oscillating electric field mode with space and time dependencies:

\[
E(x, t) = E_0 f(x) g(t)
= \epsilon E_{cr} \exp\left(-\frac{x^2}{2\lambda^2}\right) \exp\left(-\frac{t^2}{2\tau^2}\right) \cos(b t^2 + \omega t + \phi),
\]

where $E_{cr}$ is the critical field strength and $\lambda$ reflects the spatial scale of the external field. We choose $E_0 = 0.5E_{cr}(\epsilon = 0.5)$ and set $\omega = 0.7m$ with $\tau = 45m^{-1}$ when studying the rapidly
oscillating mode, while for the slowly oscillating field, we let \( \omega = 0.1 m \) and \( \tau = 25 m^{-1} \). In this idealized standing wave profile, we attempt to model the oscillating electric field with spatial dependency constructed by two counter propagating coherent laser fields aligned to cancel out the magnetic component. We point out that the direction of the field is along the \( x \)-axis and field strength varies in both \( x \) and \( t \). Through this simplified model, we are investigating how spatial variation and other temporal field parameters interplay with each other and affect pair creation by calculating the produced particles’ number density in the phase space.

B. DHW formalism

The DHW method was developed from the fermion density operator to explore the phase space structure of the Dirac vacuum \cite{22}, and it is an efficient formalism for the investigations on pair creation under both spatially homogeneous \cite{20,27–31} as well as inhomogeneous electromagnetic fields \cite{23–25,28}. In this paper, the DHW equations of motion are solved numerically \cite{25,32} for spatially inhomogeneous external field modes given in Eq. (1). Since the detailed calculations and numerical strategies could be found in Refs. \cite{23–25,32}, we only provide the equations and observable quantities relevant to our study in the following.

The complete set of equations of motion for the Wigner components are reduced to 4 equations in the 1+1 dimensional scenario \cite{23–25}:

\[
\begin{align*}
D_t \zeta - 2 p_x \Phi &= 0, \\
D_t \nu_0 + \partial_x \nu &= 0, \\
D_t \nu + \partial_x \nu_0 &= -2m \Phi, \\
D_t \zeta + 2 p_x \zeta &= 2m \nu,
\end{align*}
\]

with the pseudodifferential operator

\[
D_t = \partial_t + e \int_{-1/2}^{1/2} d\xi \ E_x (x + i\xi \partial_p, t) \partial_p.
\]

Among the four Wigner components in the the equations of motion \cite{2}–\cite{5} we have only two non-vanishing vacuum initial conditions \cite{32}:

\[
\begin{align*}
\zeta_{vac} &= \frac{-2m}{\omega}, & \nu_{vac} &= \frac{-2p_x}{\omega},
\end{align*}
\]
where $\omega = \sqrt{p_x^2 + m^2}$ is the energy of a particle. By explicitly subtracting these vacuum terms, the modified Wigner components are written as:

$$w^v = w - w_{\text{vac}},$$

where $w$ denotes the four Wigner components in our 1+1 formalism. Then the particle number density in the phase space could be defined via dividing the total energy of the created particles by individual particle energy [23]:

$$n(x, p_x, t) = \frac{m s^v(x, p_x, t) + p_x w^v(x, p_x, t)}{\omega(p_x)}.$$  (9)

The position distribution or the momentum distribution of the created particles could be obtained from $n(x, p_x, t)$ by integrating out $p_x$ or $x$ respectively, and the total particle yield is calculated by integrating over the whole phase space:

$$N(t) = \int dx dp_x n(x, p_x, t).$$  (10)

The numerical treatment for Eqs. (2)-(5) follows from the techniques developed in Ref. [32], where the spectral method is employed to handle the pseudodifferential operator (6). The calculation parameters are chosen in similar way as in Refs. [25, 32] and tested for reasonable parameter space such that final results are convergent.

### III. NUMERICAL RESULTS

#### A. Rapidly oscillating electric field: $\omega = 0.7m$.

In this subsection, we obtain results for $\omega = 0.7m$ characterizing pair production in the rapidly oscillating electric fields with spatial focusing using external field model (1) for various chirp parameter values. We set $\tau = 45m^{-1}$ such that, for $b = 0$ (see Fig. 1(a)), we could obtain same results as in Fig. 3 of Ref. [24], where the temporal pulse envelope is chosen as $\cos^4(\frac{\tau}{2})$ with $\tau = 100m^{-1}$. Before we discuss the results for nonzero chirp values, we investigate in detail the pair production process for $b = 0$ in the quasihomogeneous limit so that it may assist the discussions for the rest of the calculations.
1. **Local density approximation.**

In Fig. 1(a), we recover the signatures of multiphoton absorption of the homogeneous scenario in the quasihomogeneous limit where $\lambda = 1000m^{-1}$. However, compared with the homogenous limit ($\lambda \to \infty$), the main peak at vanishing momentum seems to be more pronounced in the quasihomogeneous scenario as is noticed in Ref. [24]. Furthermore, from the position distribution of certain momenta $n(x, p = p_i, t = t_f)$ shown in Fig. 1(b), we observe that, with the departure of particle location from the origin, the heights of typical momentum peaks behave like that of pair production under homogeneous fields with decreasing field strength $\epsilon$: The related peak values would decrease for $p = 0.9m$ and $p = 1.4m$, while the vanishing momentum peak would reach maximums and minimum with decreasing $\epsilon$, see Fig. 1(c). It seems that the spatial dependency of the external field is playing the role of an effective field strength such that at different locations the field obtains different amplitudes according to the gaussian function of the position $\epsilon(x) = 0.5 \exp(-\frac{x^2}{2\lambda})$, and the pair production process seems to occur independently at each location.

By this observation, we could calculate a new momentum distribution by summing results for homogeneous fields with different field strengths given as:

$$\tilde{n}(p, t \to \infty) = \sum_x \frac{n(\epsilon(x)|p, t \to \infty)}{\lambda},$$  

(11)

where $n(\epsilon(x)|p, t \to \infty)$ is the momentum distribution for the homogeneous field $E(t)$ with effective field strengths:

$$E(t) = \epsilon(x)E_{cr} \exp\left(-\frac{t^2}{2\tau^2}\right)\cos(bt^2 + \omega t + \phi).$$  

(12)

And the spatial scale $\lambda$ in the denominator in Eq. (11) is introduced since we calculate the reduced particle number distribution. The corresponding result agrees with the quasihomogeneous calculation as shown in Fig. 1(a) where the red dot-dashed curve is the approximation result. Thus we may understand the maximum peak around $p = 0$ for $\lambda = 1000m^{-1}$ by referring to pair creation in homogeneous fields where the maximally large peak is present for smaller effective strengths, see Fig. 1(c).

This local density approximation that we find to be valid in this multiphoton absorption process is first mentioned in Ref. [23] in the tunneling mode of the Schwinger pair production. In both tunneling and multiphoton absorption processes, we see the summed momentum from homogeneous approach gives the same result as in the quasihomogeneous case performed in the $1+1$ dimensional...
treatment. Moreover, we have noticed that, not only the typical momentum peak height at various locations is captured by the homogeneous result with corresponding effective field strength (as shown in Fig. 1(b)), the complete momentum distribution at each location almost overlaps with the corresponding homogeneous calculations. The momentum spectrum extracted from the phase space distribution for a certain location $x_i$, denoted as $n(x = x_i, p, t = t_f)$, overlaps with the homogeneous result with field strength $e(x_i)$ in Fig. 1(c) where we have omitted $n(x = x_i, p, t = t_f)$ curves for the sake of readability.

2. Chirp effects.

When we introduce the nonzero chirp, the momentum distributions at different spatial scales change drastically with the increase of chirp values as shown in Fig. 2. For $\lambda = 1000m^{-1}$, the main features of chirp effects at homogeneous set up is preserved with the slight difference because of the addition of momentum spectrum at each location that separately is related to the spectrum for homogeneous fields with effective field strengthes. For instance, the main peak at $p_x = 0$, present in the small chirp cases, would split into two peaks for large chirp (see Fig. 2(d)) as is presented in Ref. [20] in the linearly polarized case where the pattern is more obvious. For $\lambda = 10m^{-1}$, the momentum spectra gradually loose symmetry about $p_x = 0$ with the increasing chirp yet still differ from the chirp signatures for $\lambda = 1000m^{-1}$ for small $b$. However, we could observe momentum spectrum displaying the same complicated oscillating pattern as in the quasihomogeneous case for large chirp parameters, see Fig. 2(d).

Finally, at the extremely small spatial focusing with $\lambda = 2.5m^{-1}$, the peak splitting is present till the chirp parameter reaches $b = 0.5\omega/\tau$ where we loose such effect in the momentum spectrum. This could be related to the highly nonuniform oscillation caused by the large chirp parameter inhibiting the formation for ponderomotive force which pushes particles towards low field intensity regions in space [24].

We have also calculated the result for maximally large chirp $b = 1.0\omega/\tau$, so that it could reflect the overall tendency of the effects for an increasing chirp. At this extreme case, the momentum distribution looks similar for all spatial scales and the limited range of the spectrum also expands to very large positive momentum regions with side peaks, see Fig. 3. The spread in the momentum might be caused by the vanishing effective frequency $\omega^* = \omega + bt$ getting closer to the maximum field strength regions with the increase of $b$ where the particle acceleration would be possible at
FIG. 1: Reduced momentum spectrum (top), the position distribution for typical momentum peaks (middle) for spatially focused oscillating electric fields with $\omega = 0.7\, \text{m}$ and $b = 0$. Other field parameters are $\epsilon = 0.5$, $\phi = 0$, and asymptotic final time is chosen as $t_f = 6.5\tau$ for the position distribution. Note that, the effective field strength approximation labeled as $\Sigma_x \epsilon(x)$ (red dot-dashed curve) overlaps with the quasihomogeneous result. In the bottom, the homogeneous calculation results are given.

such large effective pulse lengths which may even induce the tunneling process. In addition, note that the enhancement due to large chirp value is even observable in the momentum spectrum.

The effects of chirp parameter at various spatial extents could also be seen in the total particle yield. As is observed in the momentum spectrum, with the increase of chirp value, the spatial dependency of pair creation would become trivial such that for maximally large chirp, the reduced total yield curve almost becomes a straight line which is raised significantly due to the enhancement by maximal chirp, see Fig. 4. Such enhancement due to large chirp values is reported in Ref. [20], where even greater enhancement factor is achieved using smaller field strength $\epsilon = 0.1$. Moreover, the enhancement at small focusing for $b = 0$ is lost for nonzero chirp values similar to Ref. [24], where this increasing effect at small spot size is lost for larger frequency. Also, for
FIG. 2: Reduced momentum spectrum for spatially focused oscillating electric fields (1) with nonzero chirp values. Other field parameters are same as zero chirp case in Fig. 1(a).

FIG. 3: Reduced momentum spectrum for spatially focused oscillating electric fields (1) with chirp value $b = 1.0 \omega / \tau$. Other field parameters are same as in Fig. 1(a). Spatial scales seem to be insignificant yet the oscillation is missing for $\lambda = 2.5 m^{-1}$ where the focusing size is too narrow to retain signatures from temporal pulse structure.

In the small chirp parameter region, chirp parameter behaves differently for different $\lambda$ such that no common increase or decrease can be seen, which might call for detailed analysis in optimization searches.
FIG. 4: Reduced particle yield at various spatial extents for chirped oscillating electric fields with frequency $\omega = 0.7m$. The blue dashed line for the maximally large chirp $b = 1.0\omega/\tau$ shows the overall tendency of the total particle numbers with the increasing chirp at different spatial scales.

B. Slowly oscillating electric field: $\omega = 0.1m$.

Now we consider the slowly oscillating electric field mode by choosing $\omega = 0.1m$ and try to make comparison to similar cases in the homogeneous studies by splitting the results for $\phi = 0$ and $\phi = \pi/2$. The temporal pulse length is chosen as $\tau = 25m^{-1}$, so that we achieve numerical convenience while retaining the main features of pair creation in electric fields with subcycle structure [14, 15]. In this tunneling dominated regime, we have also recovered the quasihomogeneous results by applying the local density approximation, see red dot-dashed lines shown in Fig. 5 and Fig. 6.

1. $\phi = 0$

For $\phi = 0$ shown in Fig. 5, we could identify the main signatures from the homogeneous studies at the quasihomogeneous limit $\lambda = 1000m^{-1}$. When the chirp parameter takes a small value, we would see the appearance of the interference patterns in the momentum spectrum that is shifted by the larger chirp values followed by the appearance of enhanced peaks for certain momenta. The interference pattern emerges for the small chirp similar to the effect of a phase change while the shift for large chirp could be related to the large effective pulse lengths caused by chirp parameter,
see also Ref. [15] for the explanations using turning point analysis.

At the strong focused scenario, where $\lambda$ is small, chirp effects are not as pronounced as in the large spatial extent case. This implies that it is not straightforward to identify the chirp effects, such as strong oscillations and shift in the spectrum, at highly inhomogeneous cases. In other words, when spatial extent is small, it is hard to extract information on the external field structure from produced particles’ momentum distribution. However, chirp parameters have nontrivial consequences in the $\phi = \pi/2$ case as is presented in the second part of this subsection.

2. $\phi = \frac{\pi}{2}$

For $\phi = \pi/2$ shown in Fig. 6, the coherent interference signature in the momentum spectrum for zero chirp parameter would be violated with the introduction of nonzero chirp values when $\lambda = 1000 m^{-1}$ as is observed in the homogeneous study [15]. Compared with the homogeneous case, where asymptotic minimum in the momentum spectrum would reach the bottom, the quasi-homogeneous result gives a weaker interference pattern, which could be understood in the light of the local density approximation where one adds individual momentum spectrum with complete
interference patterns for different effective field strengths.

FIG. 6: Reduced particle yield at various spatial extents for chirped oscillating electric fields with frequency \( \omega = 0.1m \) and \( \phi = \pi/2 \). Chirp effect is noticeable even for \( \lambda = 10m^{-1} \).

Interestingly, compared with the results for \( \phi = 0 \) shown in Fig. 5, chirp effects are seen clearly in the momentum spectrum for \( \lambda = 10m^{-1} \) in Fig. 6. With the increase of \( b \), we observe the merging of two major momentum peaks finally forming an oscillatory structure similar to the complex oscillation for corresponding quasihomogeneous case.

When \( \phi = 0 \), there is one major peak in the electric field strength, while for \( \phi = \pi/2 \), there are two peaks with opposite signs for zero chirp and multiple peaks with similar strengths for nonzero chirps. The interference pattern in Fig. 6(a) could be understood as the interference between temporally separated pair creation events and the complex oscillation for large chirp as the consequence of pair creation events from multiple sources. When the spatial extent is finite(\( \lambda = 10m^{-1} \)), the signatures from temporal pulse structure is only preserved for large effective oscillations where particles stay within the finite electric field region for reasonable external field cycles to display the oscillatory signature. Therefore, in Fig. 6(a), particles leave in the opposite directions for two major field peaks with large temporal durations, while for large chirp parameter shown in Fig. 6(d), the oscillation caused by multiple peaks with shorter temporal duration can be seen in the finite focused scenario.
In Fig. 7 we display the reduced total numbers of created particles for various spatial extents. The shape of the curves for all chirp parameters are similar, which reflects the total yield features for tunneling mode where the narrow spatial extent causes a sharp drop in particle yield \[23\]. The chirp effect is playing different role at large spatial extents: for \( \phi = \pi/2 \), chirp seems to decrease the total yield, however it increase the total number for \( \phi = 0 \) in Fig. 7 which reflects the change in the temporal field shape as in the homogenous cases. Unlike for the rapid oscillating mode where pair creation is dominated by the multiphoton process, chirp parameter dose not play significant role in the enhancement of the particle yield in this tunneling profile except for the narrow spatial focusing regions.

![FIG. 7: Reduced particle yield at various spatial extents for chirped oscillating electric fields (1) with frequency \( \omega = 0.1m \). The blue lines, which are below the black lines at the large spatial scales, correspond to \( \phi = \pi/2 \) results.](image)

**IV. CONCLUSION**

We have considered two oscillating frequencies and calculated the particle number densities to show the chirp effects at various spatial scales of the external field. When the spatial scale \( \lambda = 1000m^{-1} \), the external field form \( E(t, x) = E_0 \exp(-\frac{x^2}{2\lambda^2})g(t) \) could be considered as the generalization of the homogeneous form \( E(t) = E_0 g(t) \) such that two field forms have direct connection via local density approximation. Thus the pair production features in the momentum spectrum
could be analyzed using similar arguments from the homogeneous case at this quasihomogeneous limit. Moreover, in the multiphoton absorption scenario, we notice the one to one correspondence between the momentum spectrum at individual positions and the corresponding homogenous results. This profoundness of the local density approximation maybe due to the adiabatic nature of multiphoton absorption process where particle creation happens by photon absorption instead of the work done by the external field.

The strong spatial focusing reflects chirp effects differently for various oscillating modes. In the rapidly oscillating field, with the increase of chirp parameter momentum spectrum looses symmetry and finally displays similar patterns as in the quasihomogeneous case. When the electric field oscillates slowly, the temporal carrier phase effect is found to be crucial even for $\lambda = 10\,m^{-1}$ where spatial variation is strong and the signatures from the corresponding homogeneous field is lost. The chirp effect is not obvious for $\phi = 0$ at this scale, however, for $\phi = \pi/2$ we see merging and the formation of oscillatory pattern in the momentum spectrum with the increase of chirp value which is quite different from the homogeneous instance where we see coherent interference followed by complex oscillations. This indicates that the finite spatial scale renders effects of the temporal pulse structure differently according to the length of the temporal duration of individual subcycles complicating the nonMarkovian nature of the process [33] with the additional dimension.

These results also suggest that introducing finite spatial scales of the external fields may have crucial consequences to the homogenous results, thus one needs to be more cautious when calculating multidimensional external field results to provide more accurate predictions. Also, the shape of the spatial pulse may have nontrivial effects on particle momentum spectrum and even on the total number enhancement. Furthermore, analytic tools needs to be generalized to broader parameter ranges so that it is possible to better understand nontrivial features of pair creation in inhomogeneous fields.

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