Nonminimal GUT inflation after Planck results

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In the present work we study GUT Coleman-Weinberg inflation with a nonminimal coupling to gravity. In this kind of model one usually finds that either the nonminimal coupling to gravity is large $\xi \gg 1$ or the inflaton self-coupling is unnaturally small $\lambda \sim 10^{-13}$. We have shown that the model is in agreement with the recent results from Planck for natural values of the couplings.

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Inflation [1] has become the standard paradigm for the early Universe, because it solves some outstanding problems present in the standard hot big-bang cosmology, such as the flatness and horizon problems, the problem of unwanted relics, such as magnetic monopoles, and produces the cosmological fluctuations for the formation of the structure that we observe today. The spectacular CMB data, first from the WMAP satellite [2] and recently from Planck [3], have strengthen the inflationary idea, since the observations indicate an almost scale-free spectrum of Gaussian adiabatic density fluctuations, just as predicted by simple models of inflation. However, inflation is not a theory yet, as we don’t know how to integrate it with ideas from particle physics.

The Brout-Englert-Higgs mechanism [4] in the framework of the Standard Model breaks electroweak symmetry, gives masses to the charged fermions and the massive gauge bosons, and predicts the existence of the Higgs boson. The recent LHC discovery of the Higgs boson [5] completed the particle spectrum predicted by the Standard Model, and indicates for the first time that fundamental scalar particles exist in nature. Therefore, it is a natural thing to assume that inflation is driven by the Higgs boson. Although the Higgs potential is not suitable for a viable inflationary model, the presence of a Higgs non-minimal coupling to gravity can change things to the better. A large value of the non-minimal coupling $\xi \sim 10^4$ is required, and given the uncertainties in the top quark mass and the strong coupling constant the model is still allowed [6]. Sadly, that large value of $\xi > 1$ questions the validity of the scenario in the SM Higgs inflation, as the inflationary scale exceeds the effective ultraviolet cut-off scale [7].

As it was realized long ago, radiative corrections can be the origin of electroweak symmetry breaking [8]. The Coleman-Weinberg mechanism cannot work in the Standard Model due to the large value of the top Yukawa coupling, but it can work in models beyond the Standard Model [9]. This is an interesting possibility given the Higgs naturalness problem, in the following sense. If heavy particles are coupled to the Higgs boson, like in GUT models, then the Higgs receives large radiative corrections that bring its mass close to the GUT mass scale. Supersymmetry at the TeV scale can solve the problem, but given the severe experimental constraints on the masses of the superpartners, it was proposed recently the flatland scenario [10], according to which electroweak symmetry is broken radiatively a la Coleman-Weinberg in the infrared region starting from a flat scalar potential in the ultraviolet region.

Regarding inflation, the Coleman-Weinberg type of potential is a simple and well motivated one, since it naturally arises when loop corrections are taken into account, and it is typical for the new inflation scenario [11] where inflation takes place near the maximum. Recently it has been studied in [12] in a B-L extension of the Standard Model, and a few years ago in [13] in a GUT inflationary model. As shown in [14] in the context of the effective theory of inflation, Cosmic Microwave Background together with Large Scale Structure data prefer double-well inflaton potentials. The 2013 Planck data confirmed this result, and not surprisingly the Coleman-Weinberg potential considered here belongs to this class, and succeeds in explaining the recent data from Planck, and even more importantly the r bound from the 2013 Planck release. However, in [12, 13] the inflaton was minimally coupled to gravity. Setting $\xi = 0$, although it is a popular choice, is often unacceptable as was pointed out in [15]. Non-minimal couplings are generated by quantum corrections even if they are absent in the classical action [16], and as a matter of fact the coupling is required if the scalar field theory is to be renormalizable in a classical gravitational background [17]. For early works on non-minimal inflation see for example [18] and references therein.

In the present Brief Report we imagine a scenario where inflation is driven by the scalar sector of some particle physics model beyond the Standard Model in which electroweak symmetry breaking takes place radiatively a la Coleman and Weinberg. Since the scale of inflation is close to the GUT scale, we consider a GUT model rather than a low energy one, and we also allow for a non-vanishing non-minimal coupling, as required by the quantum corrections already needed to give rise to the Coleman-Weinberg type of the inflaton potential. We find that the model is viable as it is in agreement with the recent data from Planck, and even more importantly for natural values of the parameters of the model.

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We start defining the model by the action

$$ S = \int d^4x \sqrt{-g} \left( \frac{1 + \kappa^2 \xi \phi^2}{2 \kappa^2} R - (1/2) \phi_{,\mu} \phi^{,\mu} - V(\phi) \right) $$

(1)

where $\xi$ is the non-minimal coupling and $\kappa^{-1} = M_p = 2.4 \times 10^{18} \text{GeV}$ with a Coleman-Weinberg type of potential of the standard form $^{12, 13}$

$$ V(\phi) = A \phi^4 \left( \log \left( \frac{\phi}{M} \right) - \frac{1}{4} \right) + \frac{A}{4} M^4 $$

(2)

where $A$ is related to the inflaton quartic self-coupling $^{12, 13}$, and $M$ is the inflaton vacuum expectation value (vev) at the minimum, $M = M_{\text{GUT}} \sim 10^{16} \text{GeV}$. The vacuum energy density at the origin is given by the constant term $V_0 = AM^4/4$ so that $V(\phi = M) = 0$, and the shape of the potential can be seen in Figure 1, and the inflaton mass is given by $m_\phi = 2\sqrt{A}M$. At this point we stress the fact that the Coleman-Weinberg potential considered in the present work is the standard one obtained from one-loop corrections in Minkowski spacetime. The one-loop corrections in de Sitter spacetime (which approximates an inflationary background) lead to a very different effective potential as shown in $^{19}$.

![V(\phi)](image)

FIG. 1: The Coleman-Weinberg potential as a function of the scalar field.

The non-minimal coupling can be eliminated going to the Einstein frame (in which the scalar field is denoted by $\sigma$ and the metric by $\hat{g}_{\mu\nu}$) through a conformal trans-

formation

$$ \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} $$

(3)

$$ U = \frac{V}{\Omega^4} $$

(4)

$$ \frac{d\sigma}{d\phi} = \sqrt{1 + \kappa^2 \xi \phi^2(1 + 6\xi)} $$

(5)

where $\Omega^2 = 1 + \kappa^2 \xi \phi^2$.

Assuming that in the Jordan frame inflation takes place near the maximum of the potential, $\phi < M < M_p$, the potential is dominated by the constant term $V_0 = AM^4/4$, and the model at hand is a small-field model where the slow-roll parameters are as follows $\eta < 0, \eta < -\epsilon$ $^{20}$. Requiring that $\xi < 1$ we can make the approximation that $\sigma \sim \phi$ and therefore in the Einstein frame the potential takes the form

$$ U(\sigma) \simeq V_0(1 - 2\kappa^2 \xi \sigma^2) $$

(6)

and it is of the form

$$ V(\phi) = \Lambda^4(1 - (\phi/\mu)^2) $$

(7)

with $\mu = M_p/\sqrt{2\xi}$. One can easily see that for this model indeed $\eta < -\epsilon$, and the spectral index and tensor-to-scalar ratio are given by $^{20}$

$$ n_s = 1 - 4(M_p/\mu)^2 $$

(8)

$$ r = 8(1 - n_s)\exp(-1 - N_s(1 - n_s)) $$

(9)

and they are independent of $V_0 = \Lambda^4$. Here we take the number of e-foldings to be $N_s = 60$, for which the tensor-to-scalar ratio is computed to be $0.0089 < r < 0.0146$. The model agrees very well with the recent data from Planck $^3$ for $9 < \mu/M_p < 11$, or

$$ 0.004 < \xi < 0.006 $$

(10)

At this point we should check the validity of our approximation. We have neglected the second order term $\kappa^4 \xi^2 \sigma^4$, so now we need to show that the ratio

$$ z = \frac{\kappa^4 \xi^2 \sigma^4}{\kappa^2 \xi \sigma^2} $$

(11)

is indeed small. Using the definitions it is easy for someone to show that $r = 128\xi z$, and therefore $z \simeq 0.03$ at most.

Finally the amplitude of the curvature perturbation $\Delta_R = 4.9 \times 10^{-5}$ is given by

$$ \Delta_R = \frac{U^{3/2}}{2\sqrt{3\pi}|U|} $$

(12)

from which we find a relation between the couplings $\xi$ and $\lambda$, which is the following

$$ A(\xi) = \frac{480\pi^2}{e} \left( \frac{M_p}{M} \right)^4 \Delta_R^2 \xi^2 \exp(-8N_s \xi) $$

(13)
and can be seen in Figure 2 for $M = 0.01 M_p = 2.4 \times 10^{16} \text{ GeV}$. If we take $M$ to be slightly lower, $M = 1.1 \times 10^{16} \text{ GeV}$, $A$ becomes of the order $\sim 0.01$, and for $M = 6.7 \times 10^{15} \text{ GeV}$ the coupling $A \sim 0.1$. Therefore the model is in agreement with the recent data from Planck, but even more importantly it manages to be a viable model for natural values of the couplings $\xi \sim 10^{-3}$ and $A \sim 10^{-4} - 10^{-1}$ depending on the precise value of $M = M_{\text{GUT}}$. Contrary to the results found in other works [6, 12, 13, 21] neither the non-minimal coupling is large, $\xi \sim 10^{4}$, nor the inflaton self-coupling is tiny $A \sim 10^{-14}$. In particular, in [6] successful inflation requires that the non-minimal coupling $\xi$ and the SM Higgs quartic self-coupling $\lambda$ satisfy the relation $\xi \simeq 44700 \lambda$. But since $\lambda$ is determined by the Higgs boson mass, $\lambda \simeq 0.5$, it turns out that $\xi \sim 10^{4}$. Similarly, in the Figure 6 of [21] one can see that $\lambda$ increases with $\xi$, and therefore either $\xi$ is large or $\lambda$ is unnaturally small. Finally, in [12, 13] (where $\xi = 0$) the spectral index requires a transplanckian Higgs vev, $M \simeq 10 M_p$, and then for this value of $M$ the coupling $A$ has to be of the order of $A \sim 10^{-14}$. In our work, the non-vanishing non-minimal coupling helps in increasing $A$ keeping at the same time the Higgs vev at the GUT scale, $M = M_{\text{GUT}} \sim 10^{16} \text{ GeV}$.

In summary, in this Brief Report we have considered a GUT inflationary model where inflation is driven by the scalar sector of the model with a Coleman-Weinberg type of potential for the inflaton and a non-minimal coupling of the inflaton to gravity. As is known, at tree level the $\lambda \phi^4$ potential cannot trigger electroweak symmetry breaking, and in addition the corresponding chaotic inflationary model $\phi^4$ is ruled out by the WMAP and Planck data. However, quantum loop corrections modify the inflaton potential and at the same time generate the non-minimal coupling term. The model is characterized by two dimensionless parameters, namely the non-minimal coupling $\xi$ and the inflaton quartic self-coupling $\lambda$. We have shown that the model leads to predictions in agreement with the 95 per cent CL contours in the r-n$_s$ plane for natural values of the parameters of the model, both for $\xi$ and $\lambda$ of the order of $10^{-3}$.

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