Application of Reliability Simulation Based on Monte Carlo Method in GNSS Receiver

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Abstract. At present, the analytical method (reliability prediction) is mainly used to calculate the equipment reliability of electronic equipment system. But reliability verification and evaluation of electronic equipment, which is highly reliable, needs a lot of time and economic costs. In this paper, a reliability simulation algorithm based on Monte Carlo method is proposed to verify and evaluate the reliability of highly reliable GNSS receiver. Which can meet the requirements and greatly reduce the project cost.

1. Introduction
With the continuous development of computer technology, numerical simulation technology plays an increasingly important role in system design and analysis, and becomes an indispensable tool for system design and analysis. Similarly, the digital simulation technology is also a very useful tool in the system reliability analysis[1].

Reliability digital simulation has been widely used in the development, production and application of all kinds of systems (especially those with maintainability). NASA used it to predict the probability of completing the Apollo mission and reliability of spacecraft. Lockheed Martin used it to predict aircraft attendance reliability. The Air Force Technical College used it to evaluate fighter reliability. Westinghouse Defense and Space Center used it as the simulation of fault and corresponding system[2].

Nowadays, reliability digital simulation is widely used in multi-failure distribution systems. When the failure distribution of each component of the system is different, it requires a great amount of computation to analyze the complex system by the common analytical method, so the application of reliability digital simulation in electromechanical equipment and complex system is relatively more.

The reliability of any system is a stochastic system problem. Digital simulation technology is easy to simulate the stochastic process[3]. So in the field of electronic equipment, reliability digital simulation also has a wide range of application space.

Furthermore, the digital simulation of system reliability can expand and deepen the role of reliability in each stage of the system. In the system demonstration stage, the reliability prediction of complex system can be carried out. In the system design stage can be used as a comparison tool, that is, through various systems and the components of the same system, with different distribution of digital simulation analysis and comparison, optimize the design scheme; After the system design is determined, it can be used to determine the reliability of the system. Therefore, it is necessary to carry out the research on electronic equipment reliability digital simulation.
2. Reliability digital simulation based on Monte Carlo method
The reliability digital simulation based on Monte Carlo method simulates the life process of the system, and analyzes the reliability of the system on the basis of the simulation. Monte Carlo method is a numerical calculation method based on the system theory and method of probability theory, also known as statistical test method or computer stochastic simulation method[4]. From these two names, it can be seen that it is a probabilistic statistical method suitable for similar tests in computer simulation of randomness problems, which is in good agreement with the key of system reliability analysis, and is very similar to the process of system reliability test verification and evaluation.

2.1. General principles of the Monte Carlo method
The basic idea of Monte Carlo method is firstly to construct a probability space, and then determine the statistic \( g(x) \) in the probability space, which depends on the random variable \( X \), its mathematical expectation is

\[
E(g) = \int g(x)dF(x)
\]  

(1)

\( F(X) \) is the distribution function of \( X \). Then generates simple sample of \( X \) as \( x_1, x_2, \ldots, x_N \) and \( g(x) \) as \( g(x_1), g(x_2), \ldots, g(x_N) \). The arithmetic mean of \( g(x) \) is

\[
G_N = \frac{1}{N} \sum_{i=1}^{N} g(x_i)
\]  

(2)

Which can be the approximate estimates of system reliability parameter, with further calculation, the approximate estimated value of system reliability is obtained.

2.2. Reliability simulation method for electronic equipment
To carry out reliability digital simulation, the simulation model and reliability logic relationship of the system should be established firstly, which can be realized by the expression form based on the system reliability block diagram.

2.2.1. Simulation Model Establishment. Assuming that the system is composed of \( n \) units and the effectiveness distribution function of each unit is known, the system can be expressed as

\[
s = \{z_1, z_2, \ldots, z_n\}
\]  

(3)

Each unit has only one state: the normal state or the failed state. The actual state of each unit is a function over time, can be defined as

\[
b_i(t) = \begin{cases} 
1, & \text{normal state} \\
0, & \text{failed state}
\end{cases}
\]  

(4)

The system state function can be expressed as

\[
\phi(t) = \{b_1(t), b_2(t), \ldots, b_n(t)\}
\]  

(5)

2.2.2. System reliability logic relationship. The logic relation of system reliability is to deduce the working state of the system by using the state of each unit according to the system reliability block diagram. The connection mode of each unit in the system reliability block diagram can be expressed as series or parallel mode. If the system is composed of \( n \) units in series, the system state function can be expressed as

\[
\phi(t) = \prod_{i=1}^{n} b_i(t)
\]  

(6)

If the system consists of 5 units in parallel, the system state function can be expressed as

\[
\phi(t) = \sum_{i=1}^{n} b_i(t)
\]  

(7)

So the value of system state function and its representative meaning are
The reliability block diagram of a GNSS receiver is shown as Figure 1.

\[ \phi(t) = \begin{cases} 0, & \text{failure} \\ > 0, & \text{normal} \end{cases} \] (8)

2.2.3. System failure time sampling. It is assumed that the system consists of n elements, and the life of each element is randomly sampled according to the Monte Carlo method, and a simple distribution of the failure time of each element is obtained. Since the life distribution of each element of electronic equipment belongs to exponential distribution, its failure distribution function can be expressed by definite analytical formula, so the simple failure time distribution of each element can be generated by direct sampling method.

The process of direct sampling is to calculate the inverse function of a random variable X whose distribution function is \( F(X) \). U is a uniformly distributed random variable, and direct sampling results following \( F(X) \) can be obtained as

\[ \xi = F_X^{-1}(u) \] (10)

The life distribution of the electronic equipment unit follows an exponential distribution, and the failure rate is \( \lambda \), then the failure probability density function of the unit is

\[ f(t) = \lambda e^{-\lambda t} \] (11)

The failure distribution function is

\[ F(t) = \int_0^t f(t)dt = \int_0^t \lambda e^{-\lambda t}dt = 1 - e^{-\lambda t} \] (12)

Its inverse function is

\[ F_X^{-1}(x) = -\frac{1}{\lambda} \ln(1 - x) \] (13)

The direct sampling result of the unit's failure time can be obtained as

\[ t_F = F^{-1}(\xi) = -\frac{1}{\lambda} \ln(\xi) \] (14)

Then, the failure time sampling of the system can be obtained according to the system state function.

2.2.4. System reliability calculation. A large number of sample values of system failure time are obtained by multiple system failure time sampling, which can be used to calculate the point estimation of system reliability.

As we know, system reliability is related to system working time[5], so it is necessary to determine the maximum working time \( t_{\text{max}} \) according to the design life of the system, and divide the working time \((0, t_{\text{max}}]\) into m intervals. Each time interval is
By counting the failure times of the system in the time interval \([t_{i-1}, t_i]\), the failure times of the system in the working time \(t \leq t_i\) can be obtained as

\[
m_i = \sum_{j=1}^{r} \Delta m_j
\]  

The unreliability of the system at the end of working time is defined as

\[
F_{\hat{s}}(t_i) = P \{ t \leq t_i \}
\]  

The system unreliability is estimated as

\[
F_{\hat{s}}(t_i) = \frac{m_i}{N}
\]  

The system reliability is estimated as

\[
R_{\hat{s}}(t_i) = 1 - F_{\hat{s}}(t_i) = 1 - \frac{m_i}{N}
\]  

3. Implementation and application of simulation algorithm

According to the above method, the reliability simulation work of high reliable GNSS receiver system is carried out. The algorithm implementation process is shown in the figure 2.

![Figure 2. The algorithm flow chart](image-url)
The GNSS receiver system reliability simulation is completed by using this algorithm. The system reliability block diagram is shown in Figure 1. The maximum operating time of the receiver is 8 years, and the failure rate of each unit is shown in Table 1.

Table 1. Failure rate of each unit.

| Unit | Failure rate (fit) |
|------|--------------------|
| 1    | 54.8               |
| 2    | 62.9               |
| 3    | 98.7               |
| 4    | 265.4              |
| 5    | 65.7               |
| 6    | 479.2              |
| 7    | 113.2              |
| 8    | 98.1               |
| 9    | 86.2               |
| 10   | 63.4               |
| 11   | 54.7               |

Through calculation, the reliability at the end of the 8-year life of the receiver is 0.99168, which is basically consistent with the reliability evaluation result of 0.99234 verified by the test.

4. Conclusion
In this paper, a reliability simulation algorithm of electronic system based on Monte Carlo method is proposed and applied to the reliability simulation calculation of GNSS receiver system. The simulation results are compared with the reliability evaluation results through the test, and the two results are basically consistent. The results show that the reliability analysis of electronic products can be realized by reliability simulation calculation. This method can greatly reduce the time and economic cost of reliability evaluation test, and has a good application benefit.

References
[1] Yang Kai, Hisham Younis, A Semi-Analytical Monte Carlo Simulation Method for System’s Reliability with Load Sharing and Damage Accumulation, J. Reliability Engineering and System Safety, 2005, 87: 191~200.
[2] Ramirez Marquez Jose E, Coit David W, A Monte-Carlo Simulation Approach for Approximating Multi-State Two-Terminal Reliability, J. Reliability Engineering and System Safety, 2005, 87: 253~264.
[3] Dagang Lu, Boqiang Li, A New Hybrid Simulation Approach to Structural Reliability Analysis Using Uniform Design, ANN Meta-model, Genetic Algorithms and Form, C. The Seventh China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems, 2012.
[4] Xuegang Luo, A Reliability Prediction Method Based on Simulation Analysis, C. 2014 Asia-Pacific International Symposium on Aerospace Technology, 2014.
[5] Chowdhury A A, Bertling L, Glover B P, et al, A Monte Carlo Simulation Model for Multi-Area Generation Reliability Evaluation, C. 9th International Conference on Probabilistic Methods Applied to Power Systems, KTH, Stockholm, Sweden, 2006: 1~10.