Statistical Characterizers of Transport in Communication Networks

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We identify the statistical characterizers of congestion and decongestion for message transport in model communication lattices. These turn out to be the travel time distributions, which are Gaussian in the congested phase, and log-normal in the decongested phase. Our results are demonstrated for two dimensional lattices, such as the Waxman graph, and for lattices with local clustering and geographic separations, gradient connections, as well as for a 1 − d ring lattice with random assortative connections. The behavior of the distribution identifies the congested and decongested phase correctly for these distinct network topologies and decongestion strategies. The waiting time distributions of the systems also show identical signatures of the congested and decongested phases.

PACS numbers: 89.75.Hc

Investigations of traffic flows on substrates of various topologies have been a topic of recent research interest. Congestion effects can occur in real networks like telephone networks, computer networks and the Internet due to various factors like capacity, band-width and network topology. These lead to deterioration of the service quality experienced by users due to an increase in network load. Statistical characterizers which can identify the state of the network, whether congested or decongested, can be of practical utility. In this paper, we identify statistical characterizers which carry the signature of the state of congestion or decongestion of the network.

The statistical characterizer which carries the signature of the congested or decongested phase, is identified to be the travel time distribution of the messages. The travel time distribution has been studied earlier in the context of vehicular traffic, server traffic and the Internet. Hence the travel time distribution can be regarded as an useful statistical characterizer of transport. In our model networks, the travel time is defined to be the time required for a message to travel from source to target, including the time spent waiting at congested hubs. This distribution turns out to be normal or Gaussian in the congested phase, and log-normal in the decongested phase.

We demonstrate that the travel time distribution is able to identify correctly the congested/decongested state in the case of two dimensional model networks, such as the Waxman topology network, a popular model for Internet topology, as well as for a network with local clustering, and its variants with gradient connections. The same characterizer is able to distinguish between the congested and decongested phases in a network with a one dimensional ring geometry. Thus, the travel time distribution is a robust characterizer of the congested/decongested phase.

We first consider models based on 2 − d lattices. We note that communication networks based on two-dimensional lattices have been considered earlier in the context of search algorithms and of network traffic with routers and hosts and have been observed to reproduce realistic features of Internet traffic.

The first network based on a 2 − d geometry is the Waxman graph, which incorporates the distance dependence in link formation which is characteristic of real world networks and has been widely used to model the topology of intra-domain networks. We consider the case where the Waxman graphs are generated on a rectangular coordinate grid of side L with the probability $P(a,b)$ of an edge from node $a$ to node $b$ given by

$$P(a,b) = \beta \exp\left(-\frac{d}{\alpha M}\right) \quad (1)$$

where the parameters $0 < \alpha, \beta < 1$, $d$ is the Euclidean distance from $a$ to $b$ and $M = \sqrt{2} \times L$ is the maximum distance between any two nodes. Large values of $\beta$ result in graphs with larger link densities and small values of $\alpha$ increase the density of short links as compared to the longer ones. A topology similar to Waxman graphs is generated by selecting randomly a predetermined number $N_w$ of nodes in the 2 − d lattice for generating the edges. Additionally, each node of the lattice has a connection to its nearest neighbors (See Fig.1(a)).

The second network that we study is a model which incorporates local clustering and geographic separations developed in Ref. As shown in Fig.1(b), this network consists of a 2 − d lattice with nodes and hubs, where the hubs are randomly located on the lattice, and are connected to all nodes inside their given area of influence.

A given number of messages $N_m$ are allowed to travel on these lattices between fixed source target pairs by a distance based routing algorithm by which a node which holds a message looks for a hub in the direction of the target which is nearest to itself, and routes the message...
After the implementation of the gradient mechanism, the distance between $Y$ shown for the hub $X$ is an ordinary node with nearest neighbor connections. Each hub has a square influence region (as shown for the hub $Y$). A typical path from the source $S$ to the target $T$ is given by the path $S-1-2-3-\ldots-7-P-8-\ldots-11-Q-12-\ldots-T$. After the implementation of the gradient mechanism, the distance between $G$ and $F$ is covered in one step as shown by the link $g$ and a message is routed along the path $S-1-2-3-G-g-F-4-5-6-T$. (c) A 1-$d$ ring lattice of ordinary nodes $(X)$ with nearest neighbor connections and randomly distributed hubs $(Y)$. Rough the hub $Y$.

We can now identify the statistical characteriser of the congested and decongested phase for these two networks. This turns out to be the travel time distribution. Here the travel time is the total travel time of messages including the time each message waits on all the nodes to be delivered to adjacent node along the path of their journey to respective targets. For the Waxman topology network if messages are fed on the system at a constant rate of $N_m = 100$ messages at every 120 time steps for a total run time of 90000, messages are not delivered to their targets and the network is in the congested phase. For the locally clustered 2-$d$ network we allow $N_m = 100$ messages be deposited at every 120 time steps in a 100 $\times$ 100 lattice and $D_{st} = 142$, for 100 hubs and total run time of 60000. At this value of $N_m$ many messages remain undelivered in the lattice due to the onset of traps and the system is in the maximal congested regime. The travel time distribution for this congested phase for both these networks is shown in Fig. 3. The travel time distribution can be fitted by a Gaussian of the form

$$P(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right)$$

If 100 messages are fed continuously at every 200 time steps all the messages get delivered to their targets for both cases, and the data for the travel time distribution can be fitted by a log-normal distribution of the form

$$P(t) = \frac{1}{t\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln t-\mu)^2}{2\sigma^2}\right)$$

The reasons for trapping include the opposing movement of messages from sources and targets situated on different sides of the lattice, as well as edge effects.

![Fig. 1: (a) The figure shows a Waxman topology network generated by connecting 55 points by the Waxman algorithm for $\alpha = 0.05$ and $\beta = 0.1$, on a $10 \times 10$ lattice. The number of links increases as the values of $\alpha$ and $\beta$ are increased. (b) A regular two dimensional lattice. $X$ is an ordinary node with nearest neighbor connections. Each hub has a square influence region (as shown for the hub $Y$). A typical path from the source $S$ to the target $T$ is given by the path $S-1-2-3-\ldots-7-P-8-\ldots-11-Q-12-\ldots-T$. After the implementation of the gradient mechanism, the distance between $G$ and $F$ is covered in one step as shown by the link $g$ and a message is routed along the path $S-1-2-3-G-g-F-4-5-6-T$. (c) A 1-$d$ ring lattice of ordinary nodes $(X)$ with nearest neighbor connections and randomly distributed hubs $(Y)$. Rough the hub $Y$.](image)

![Fig. 2: (Color online) The plot of number of messages $N(t)$ flowing on the lattice as a function of time $t$ for (a) the Waxman topology network and the baseline mechanism for (b) the locally clustered 2-$d$ lattice and (c) the 1-$d$ ring network.](image)
The travel time is defined to be the time for which the messages wait at congested nodes, also show an identical signature of the congestion/decongestion transition (Fig. 3(c) and Fig. 3(f)).

We also propose an one dimensional version of the communication network of nodes and hubs. The base network is a ring lattice of size $L$ with nearest neighbor interaction. Hubs are distributed randomly in the lattice where each hub has $2k$ nearest neighbors (Fig. 1(c)). In our simulation we have taken $k=4$, although Fig. 1(c) illustrates only $k=2$ connections. The distance between a source and target is defined by the Manhattan distance $D_{st} = |is - it|$. If a message is routed from a source $S$ to a target $T$ on this lattice through the baseline mechanism, it takes the path $S - 1 - 2 - Y - 3 - 4 - 5 - T$ as in Fig. 1(c). The routing algorithm is same as that used in the 2–d model. A given number $N_m$ of source and target pairs start sending $N_m$ messages continuously at every 100 time steps for a total run time of 30000. The plot of $N(t)$ as a function of time $t$ for this lattice (Fig. 2(c)) attains saturation for $t \approx 10^8$.

The travel time is calculated for a source-target sepa-
is log-normal as in the decongested case of the mechanisms, there is no congestion, and the leading behavior $N$ for power law correction is seen in the decongested phase. (a) Gaussian distribution in the congested phase. The standard deviation $\sigma$ for travel time distribution can be fitted by a Gaussian distribution $\mathcal{N}(0,1)$ for travel time distribution can be fitted by a Gaussian distribution $\mathcal{N}(0,1)$ for travel time distribution. Thus if the hubs are connected by assortative mechanisms, all the messages clear, and the distribution averaged over 1000 hub realizations. For the baseline mechanism, where network congests at these values, the data for travel time distribution is a robust characterizer of the congested or decongested phase. For the $1-d$ lattice, the distribution carries an additional signature of the topology in the form of an additive power law correction to the leading order. The waiting time distributions carry identical signatures of the congested and decongested phase. These results are valid for different lattice sizes and hub densities as well [18]. We note that networks which incorporate geographic clustering and encounter congestion problems arise in many practical situations e.g. cellular networks [10] and air traffic networks [20]. It would be interesting to see if our results have relevance in real life contexts.

We thank CSIR, India for support under their extramural scheme.

\begin{align}
P(t) &= \frac{1}{t \sigma \sqrt{2\pi}} \exp(-\frac{(\ln t - \mu)^2}{2\sigma^2})(1 + Bt^{-\delta}) \quad (4)
\end{align}

as shown in Fig.4(b).

Thus if the hubs are connected by assortative mechanisms, there is no congestion, and the leading behavior is log-normal as in the decongested case of the $2-d$ networks. An additive power law correction is seen due to the $1-d$ nature of the network. Due to the ring geometry of the network, some messages are not routed through the links created due to the assortative connections between hubs. These messages thus have larger travel times, and contribute the additive power-law corrections to the basic log-normal behavior in the decongested phase.

To summarize, the statistical characterizers of the communication networks studied here, viz. the travel time distributions show the characteristic signatures of the congested or decongested state of the network being normal in the congested phase and log-normal in the decongested phase. The results are true for the locally clustered communication network as well as the Waxman topology network, and also carry over to a one-dimensional lattice, to leading order. Thus the travel time distribution is a robust characterizer of the congested or decongested phase. For the $1-d$ lattice, the distribution carries an additional signature of the topology in the form of an additive power law correction to the leading order. The waiting time distributions carry identical signatures of the congested and decongested phase. These results are valid for different lattice sizes and hub densities as well [18]. We note that networks which incorporate geographic clustering and encounter congestion problems arise in many practical situations e.g. cellular networks [10] and air traffic networks [20]. It would be interesting to see if our results have relevance in real life contexts.

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