Mirror fermions and LEP precision data

C. Csáki and F. Csikor
Institute for Theoretical Physics, Eötvös University, Budapest
February, 1993

Abstract

The three generation mirror fermion model is compared with LEP precision and low energy data. While for zero mixing of ordinary and mirror fermions the model is not favoured, for non zero mixing very low $\chi^2$ fits have been obtained, in particular when right handed leptonic mixings are small and left handed leptonic mixings are large (of the order of 0.1 radians.)

The validity of the standard model (SM) has been successfully tested at the one loop level by the LEP data [1]. The precise experimental data severely limit the possibility of any kind of new physics. It is thus compulsory to confront any hypothetical extension of the standard model with the precise LEP data. In this note we consider the possible existence of mirror fermions. The very simple extension of the SM of [2] is to enlarge only the fermion content by introducing mirror (i.e. opposite chirality property) fermions to each fermion of the SM (i.e. to each ordinary fermion), preserving the $SU(2) \otimes U(1)$ group structure. Ordinary and mirror fermions are allowed to mix. In fact mixing is necessary in order to avoid stable mirror fermions. Present experiments directly exclude mirror fermions with masses below roughly half of the $Z^0$ mass. More massive mirror fermions are still allowed. In the following we consider heavy mirror fermions with masses exceeding the weak vector boson masses. Many of the phenomenological consequences of such a model have been worked out in [3], [4], where motivations for the model are also discussed.

To compare the mirror model with LEP data one has to calculate the one loop corrections to the physical observables. This is particularly easy in the case of no mixing between ordinary and mirror fermions. Since LEP observables refer only to ordinary fermions it is easy to see that the effects
of mirror fermions in the loops are the same as those of heavy sequential fermions. This kind of new physics has been considered in the literature (e.g. in [5], [6], [7], [8]). The modification of the SM predictions can be expressed in terms of the familiar $S$, $T$, $U$ (or the equivalent $\epsilon_i$) variables. The heavy fermions modify only the vector boson self energies. The latest analysis by Ellis et al. [8] (including LEP data, $W$ mass and low energy data) yields:

$\tilde{\epsilon}_1 = (0.09 \pm 0.25) \times 10^{-2}$ ($\tilde{T} = 0.01233 \pm 0.3425$), $\tilde{\epsilon}_2 = (0.09 \pm 0.72) \times 10^{-2}$ ($\tilde{U} = -0.1125 \pm 0.90$), $\tilde{\epsilon}_3 = -(0.24 \pm 0.38) \times 10^{-2}$ ($\tilde{S} = -0.3024 \pm 0.48$), where the tilde refers to the deviation of the variables from the SM reference point (i.e. $m_{top} = 130 GeV$, $M_{Higgs} = M_Z$.) The nonzero values may represent the new physics. The contribution of a heavy fermion family consisting of degenerate doublets to $T$ and $U$ is vanishing, while $S$ gets a positive contribution of $2/3\pi$. The possibility of new heavy (ordinary or mirror) unmixed fermion generations is thus severely constrained. (The $2\sigma$ upper bound on $\tilde{S}$ is only slightly larger than the mirror contribution.)

Though the above scenario is quite discouraging, it does not yet exclude the possibility of three heavy generations, because of the possibility of mixing. The mixing schemes of sequential and mirror fermions are different. In the following we are only concerned with the possibility of mirror fermions. The obvious modification to the case of zero mixing is that tree level couplings change. As a consequence one loop corrections to LEP observables can not be discussed in the usual way considering the experimental determinations of $\tilde{S}$, $\tilde{T}$, $\tilde{U}$.

The general mixing scheme in the mirror fermion model has been considered in [9]. It is possible that left and right handed fields mix with different mixing angles. Denoting mixing angles by $\alpha_L^f$ and $\alpha_R^f$ the tree level couplings are easily obtained from the appropriate part of the Lagrangean. For the neutrino - electron (ordinary and mirror) doublets e.g. we have

$$\mathcal{L}_{CC} = -\frac{e_0}{2\sqrt{2}s_2} \left[ (\bar{e} \cos \alpha_L^e + \bar{E} \sin \alpha_L^e) \gamma_\mu (1 - \gamma_5)(\nu \cos \alpha_L^\nu + N \sin \alpha_L^\nu) + \right.$$  
$$\left. (-\bar{e} \sin \alpha_R^e + \bar{E} \cos \alpha_R^e) \gamma_\mu (1 + \gamma_5)(-\nu \sin \alpha_R^\nu + N \cos \alpha_R^\nu) \right] \cdot W^\mu + \text{h.c.}$$

(1)
\[ \mathcal{L}_{NC} = -\frac{e_0}{2s2c2} \]

\[
[ (\bar{e} \cos \alpha_L^e + \bar{E} \sin \alpha_L^e)\gamma_\mu (-\frac{1}{2} + s2)(1 - \gamma_5)(e \cos \alpha_L^e + E \sin \alpha_L^e) + \\
(-\bar{e} \sin \alpha_R^e + \bar{E} \cos \alpha_R^e)\gamma_\mu (-\frac{1}{2} + s2)(1 + \gamma_5)(-e \sin \alpha_R^e + E \cos \alpha_R^e) + \\
(-\bar{e} \sin \alpha_L^e + \bar{E} \cos \alpha_L^e)\gamma_\mu s2(1 - \gamma_5)(-e \sin \alpha_L^e + E \cos \alpha_L^e) + \\
(\bar{e} \cos \alpha_R^e + \bar{E} \sin \alpha_R^e)\gamma_\mu s2(1 + \gamma_5)(e \cos \alpha_R^e + E \sin \alpha_R^e) ] \cdot Z^\mu \\
- \frac{e_0}{4s2cW} \left[ (\bar{\nu} \cos \alpha'_L^\nu + \bar{N} \sin \alpha'_L^\nu)\gamma_\mu (1 - \gamma_5)(\nu \cos \alpha'_L^\nu + N \sin \alpha'_L^\nu) + \\
(-\bar{\nu} \sin \alpha'_R^\nu + \bar{N} \cos \alpha'_R^\nu)\gamma_\mu (1 + \gamma_5)(-\nu \sin \alpha'_R^\nu + N \cos \alpha'_R^\nu) \right] \cdot Z^\mu \\
- e_0 j^el_\mu \cdot A^\mu. \tag{2} \]

where \( e_0 \) is the proton charge, the \( \alpha \)'s denote the mixing angles and \( s2 = 1 - c2 \equiv \sin^2 \theta_W \). The interactions for other fermion doublets have similar forms with different mixing angles. Note that we have excluded possible intergeneration mixings and CP violation.

Due to the special structure of mass matrices (mirror masses are generated by spontaneous symmetry breaking at the electroweak scale, ordinary-mirror mixing is a consequence of direct coupling of the 'current' fields), there is an unusual relation among the masses and mixing angles of a particular doublet (ordinary and mirror) (3). For the neutrino - electron doublets e.g. we have:

\[
-m_\nu \sin \alpha'_R^\nu \cos \alpha'_L^\nu + m_N \cos \alpha'_R^\nu \sin \alpha'_L^\nu = \\
-m_e \sin \alpha'_R^e \cos \alpha'_L^e + m_E \cos \alpha'_R^e \sin \alpha'_L^e. \tag{3} \]

Such relations are very important in the calculation of loop corrections, since they play a role in the cancellation of the divergencies.

Experimental information on the mixing angles (in the form of upper bounds) is given in [10] and [11]. Mixing angles are bounded typically by 0.1 - 0.2. A different and often much stronger bound is obtained for leptonic mixing angles using the experimental constraint on the mirror fermion contribution to the anomalous magnetic moments of the electron and the muon. E.g. assuming equal mixing angles in a lepton doublet the bound is 0.02 (as given in [3]). These small mixing angles result in too small cross-sections, so that mirror fermion production at HERA would be undetectable. As
discussed in HERA cross-sections may be saved assuming $\alpha'_R \approx 0$, ensuring small anomalous magnetic moment contributions without restricting $\alpha^e_L$ and $\alpha^e_R$.

Using the couplings of Eqs. (1), (2) for all the doublets it is straightforward to calculate the $Z^0 \to f \bar{f}$ effective couplings at the scale of the $Z^0$ mass. We have used the on shell scheme. New diagrams (as compared to the SM case) occur in the vector boson propagator corrections, vertex and box corrections. The $G_\mu - M_W$ relationship and the running of $\alpha$ also changes. A precise calculation of all the diagrams is quite lengthy, though straightforward. However, since we are concerned with the effects of heavy fermions, the dominant effect comes from vector boson propagator effects. We have therefore calculated these corrections precisely and approximated the rest (i.e. vertex and box corrections) by the zero mixing angle formulae. Our results for the effective couplings are

\[
\begin{align*}
v_f &= \sqrt{\rho_f} (t_{3f} \cos^2 \alpha^f_L + \sin^2 \alpha^f_R) - 2Q_f \sin^2 \Theta_W) \\
\alpha_f &= \sqrt{\rho_f} t_{3f} (\cos^2 \alpha^f_L - \sin^2 \alpha^f_R) \\
\rho_f &= \rho^{SM} + \alpha \tilde{T} + 2\epsilon^2 \\
\sin^2 \Theta_W &= \sin^2 \Theta_W^{SM} - \frac{2c^2 s^2}{c^2 - s^2} \epsilon^2 + \frac{\alpha}{4(c^2 - s^2)} (\tilde{S} - 4c^2 s^2 \tilde{T}),
\end{align*}
\]

where the terms containing $\epsilon$ arise from the modification of $G_\mu$ in the mirror model. We have

\[
\epsilon^2 = \frac{1}{2} \left[ 1 - \left( \frac{(1 + a^2)^2 + 4a^2}{8} \right)^2 (\cos \alpha^l_L \cos \alpha^\nu_L + \sin \alpha^l_R \sin \alpha^\nu_R)^2 \right],
\]

where lepton universality (i.e. $\alpha^\mu = \alpha^e$, $\alpha^\nu = \alpha^\nu_L$) and the smallness of mixing angles (so that the 4th powers are negligible) has been assumed.

The $M_W - M_Z$ relationship changes to

\[
\begin{align*}
\cos^2 \Theta_W^{SM} \to \cos^2 \Theta_W^{SM} (1 + \frac{2s^2}{c^2 - s^2} \epsilon^2 + \\
\frac{\alpha}{4(c^2 - s^2) s^2} (4c^2 s^2 \tilde{T} - 2s^2 \tilde{S} + (c^2 - s^2) \tilde{U})),
\end{align*}
\]

where $\cos^2 \Theta_W^{SM}$ is the usual SM expression (in terms of $\alpha$, $G_\mu$ and $M_Z$.)
Evaluation of the SM correction has been performed using the ZFITTER-DIZET package [12]. Note that the ordinary fermion contributions to $s$, $t$, $u$ are not finite for nonzero mixing. Cancellation of divergencies is achieved only after including mirror fermion and mixed ordinary - mirror fermion loops. Therefore we have directly determined the complete $s$, $t$ and $u$ for non-zero mixing, subtracting the standard model (i.e. zero mixing, mirrors excluded) contribution. Eq. (3) and similar relations play a decisive role in the cancellation of divergencies.

In the mirror fermion model we have a large number of free parameters. For each doublet there are 4 mixing angles and the two masses of the mirror partners. Taking into account Eq. (3) we have 5 parameters. To make a meaningful comparison with experimental data the number of free parameters should be limited by assuming different physical scenarios, specifying the various parameters [14]. Since for zero mixing and large mirror masses (larger than 100 GeV) three mirror generations are already excluded by LEP data, it is satisfactory if consistency of (the three generation) mirror fermion model can be demonstrated for acceptable non zero mixing angle and mass values. Though the number of parameters is large it is by no means trivial that such parameters do indeed exist. It is reasonable to start with degenerate mirror doublets, so that $t$ and $u$ are small. The reasonable range for the mirror masses is from 100GeV to 500 GeV. The upper limit is suggested by the well known perturbative tree unitarity mass bounds worked out in [4] for the mirror model. These bounds are particularly strong for the case of degenerate multiplets. The following cases have been studied

I. $\alpha^l_L = \alpha^l_R = \alpha^\nu_L = \alpha^\nu_R; \alpha^q_L = \alpha^q_R$

II. $\alpha^\nu_R = 0, \alpha^l_L = \alpha^l_R = \alpha^\nu_L = \alpha^q_L = \alpha^q_R$

III. $\alpha^R = \alpha^\nu_R = 0, \alpha^l_L = \alpha^\nu_L; \alpha^q_L = \alpha^q_R$, where $\alpha^l$ refers to charged leptons, $\alpha^\nu$ to neutrinos and $\alpha^q$ to u,d,c or s quarks. Assuming lepton universality we take equal leptonic mixing angles for the three generations. The top-bottom doublet clearly plays a special role. Therefore top and bottom mixing angles are taken different from other quark’s mixing angles, while the latter are assumed to be equal. Also masses of mirror top and mirror bottom should be larger than other mirror masses and in any case larger than the top mass. We take $M_{mirror} = 100$ GeV (200 GeV) for the isospin 1/2 mirrors except that $M_{m.top} = M_{m.bottom} = 200$ GeV (300GeV). Masses of isospin -1/2 mirrors (except for m.bottom) and $\alpha^l = \alpha^R$ are determined from Eq. (3). It turns out that our fits are very insensitive to the actual values of masses (the cases of the lower and higher masses are practically indistinguishable), the important assumption
is degeneracy.

Case I has a very simple meaning, namely at tree level only the axial vector parts of the weak currents are modified by a mixing angle factor. Case II differs only slightly from case I, namely mixing of right handed neutrinos is not allowed (therefore ordinary neutrinos are purely left handed.) In these two cases leptonic mixing angles are small (less than 0.02) due to the constraints arising from the anomalous magnetic moments of electron and muon as discussed in [9]. Case III is not restricted by these limits (the mirror fermion contributions to the anomalous magnetic moments being zero), therefore the leptonic mixing angles may be larger than 0.02 and are bounded from above by $\approx 0.1$ [10], [11].

The LEP data (and W mass) we used in the fit are given in [1] together with the correlation matrix given in [3]. We use $\Gamma_{\text{total}}$, $g_\nu^2$, $g_\alpha^2$, $A_{\text{pol}}^\tau$, $\sigma_{\text{hadron}}^\text{pole}$, $A_{FB}^h$ and $1 - M_W^2/M_Z^2$. We could not find a clear interpretation of the $q\bar{q}$ charge asymmetry in the mirror fermion model, so we excluded it from the fit. For comparison we quote [1] which gives for the standard model fit (case A) $\chi^2$/d.o.f.=$2.6/5$, for the central values $m_{\text{top}}=139$ GeV, $M_{\text{Higgs}}=300$ GeV and $\alpha_s=0.135$. In our fits we fix the leptonic mixing angles, the mirror masses (except for those which are determined by Eq. (3)), top and Higgs masses as well as $\alpha_s$. Thus we fit the quark and bottom quark mixing angles only, with the number of degrees of freedom equal to 5.

Our main concern is to see whether or not nonzero mixing angles allow for a better fit for the mirror model than for zero mixing angles. Fig. 1 shows the equal $\chi^2$ curves for case I and the fixed parameter values given in the figure caption. The minimum $\chi^2$ is 6.125 (minimized with respect to the top and Higgs masses too.) For the same parameters and zero mixing angle $\chi^2=8.$, so the fit is worse. We do not show a figure for case II since it is very close numerically to case I. Fig. 2 shows the equal $\chi^2$ curves for case III and the fixed parameters given in the figure caption. The minimum $\chi^2$ is 1.95 (minimized with respect to the top and Higgs masses too.) It is interesting that $\alpha_q=0$ is excluded from the very low $\chi^2$ region. For the same parameters but zero mixing angles $\chi^2=14.65$, so our fit gives a dramatic improvement.

We see that acceptable fits have been obtained for all the cases, while case III gives a very good fit. The range of parameters giving comparable fits in case III is: $\alpha_L^f \in (0.08 - 0.11)$, $m_{\text{top}} \in (100,160)$ GeV, $M_{\text{Higgs}} \in (70,500)$. The favoured top mass is 110-120 GeV, while the Higgs mass dependence is very small.

It is reasonable to add low neutrino scattering and atomic parity violation data to the fit, since they have a nonnegligible effect [14].
The results of a model independent determination of the couplings are given in [15]. We have used all the nine observables determined experimentally, i.e. $g_L^2, g_R^2, \theta_L, \theta_R, g_A^e, g_A^u, C_{1u}, C_{1d}$ and $C_{2u} - \frac{1}{2} C_{2d}$ together with the correlation matrix given in [15]. In the fit it is assumed that neutrino couplings are pure left handed so we may compare the mirror model with data only in case II and III.

To estimate the one loop corrections we have made the same approximation as for the LEP case, i.e. included nonzero mixing angles at the tree level as well as in the vector boson propagators. The equal $\chi^2$ curves for cases II, III are shown in Figs. 3 and 4. The no. of degrees of freedom of the fits is 14. The fits are good for essentially the same parameters and the confidence levels are increased as compared to the fits of only LEP (and $M_W$) data.

All the above fits have been made assuming degenerate mirror doublets. It is an interesting question to ask how much this condition can be relaxed. We have found that a 50 GeV mass splitting in the mirror top-bottom doublet does not spoil the good fit of case III, a 100 GeV splitting is still acceptable ($\Delta \chi^2 \approx 4$), while 150 GeV splitting is excluded. The surprisingly small sensitivity to the doublet mass splitting is due to Eq. (3), which correlates the mass changes with an appropriate change in the mixing angles.

In conclusion the mirror fermion model with three heavy mirror generations and no mixing is ruled out by LEP data at the 90% confidence level. Allowing for nonzero mixing the model is still alive. Since nonzero mixing is necessary in order to allow for decay of mirror particles, we think that the mirror model has survived the test of LEP data. Due to the large number of parameters a determination of the model parameters is not possible. We have found particular values of the model parameters which allow for a favourable comparison of the model predictions with data. In particular zero right leptonic mixing angles and large (of the order of 0.1 radians) left leptonic mixing angles lead to excellent fits of experimental data.

Acknowledgements

We thank G. Fogli and D.Schildknecht for discussions. F. Cs. thanks M. Bilenky for advice on how to use data and help with interpretation of ZFITTER output.
References

[1] A recent analysis is: L. Rolandi, CERN-PPE/92-175, Talk given at the XXVI ICHEP 1992, Dallas

[2] I. Montvay, Phys. Lett. B205 (1988) 315

[3] F. Csikor, I. Montvay, Phys. Lett. B231 (1989) 503

[4] F. Csikor, Z. Phys. C43 (1991) 129 and Erratum ibid. C52 (1991) 710

[5] D. C. Kennedy, P. Langacker, Phys. Rev. Lett. 65 (1990) 2967 and Phys. Rev. D44 (1991) 1591

[6] G. Bhattacharyya, S. Banerjee and P. Roy, Phys. Rev. D45 (1992) 729

[7] G. Altarelli and R. Barbieri, Phys. Lett. B253 (1991) 161, G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. B269 (1992) 3

[8] J. Ellis, G. L. Fogli and E. Lisi, Phys. Lett. B285 (1992) 238, J. Ellis, G. L. Fogli and E. Lisi, Phys. Lett. B292 (1992) 497

[9] F. Csikor and Z. Fodor, Phys. Lett. B287 (1992) 358

[10] P. Langacker, D. London, Phys. Rev. D38 (1988) 886

[11] E. Nardi, E. Roulet, D. Tommasini, Nucl. Phys. B386 (1992) 239

[12] D. Bardin et al., Z. Phys. C44 (1989) 493; Nucl. Phys. B351 (1991) 1; Phys. Lett. B255 (1991) 290

[13] The LEP Collaborations, ALEPH, DELPHI, L3 and OPAL, Phys. Lett. B276 (1992) 247

[14] G. L. Fogli and E. Lisi, Phys. Lett B228 (1989) 389

[15] Particle Data Group, K. Hikasa et al., Phys. Rev. D45 (1992) S1
Figure captions

**Fig. 1.** Equal $\chi^2$ curves of a fit to LEP data. The fixed parameters are $m_{\text{top}}=150\text{ GeV}$, $M_{\text{Higgs}}=70\text{ GeV}$, $\alpha_s=0.14$, $\alpha_L^t = \alpha_R^t = \alpha_L^\nu = \alpha_R^\nu = 0.02$, $\alpha_L^q = \alpha_R^q$, $\alpha_L^b = \alpha_R^b$ and $\alpha_L^{\text{top}} = \alpha_R^{\text{top}}$. The area below the indicated curve is the lowest $\chi^2$ area. The next curves correspond to $\chi^2$’s increasing by steps of 0.5.

**Fig. 2.** Equal $\chi^2$ curves of a fit to LEP data. The fixed parameters are $m_{\text{top}}=110\text{ GeV}$, $M_{\text{Higgs}}=200\text{ GeV}$, $\alpha_s=0.14$, $\alpha_R^l = \alpha_R^\nu = 0.0$, $\alpha_L^l = \alpha_L^\nu = 0.09$, $\alpha_L^q = \alpha_R^q$, $\alpha_L^b = \alpha_R^b$ and $\alpha_L^{\text{top}} = \alpha_R^{\text{top}}$. The area between the indicated curves is the lowest $\chi^2$ area. The next curves correspond to $\chi^2$’s increasing by steps of 0.5.

**Fig. 3.** Equal $\chi^2$ curves of a fit to LEP and low energy data. The fixed parameters are $m_{\text{top}}=150\text{ GeV}$, $M_{\text{Higgs}}=70\text{ GeV}$, $\alpha_s=0.14$, $\alpha_R^l = 0$, $\alpha_L^l = \alpha_R^\nu = 0.02$, $\alpha_L^q = \alpha_R^q$, $\alpha_L^b = \alpha_R^b$ and $\alpha_L^{\text{top}} = \alpha_R^{\text{top}}$. The area below the indicated curve is the lowest $\chi^2$ area. The next curves correspond to $\chi^2$’s increasing by steps of 0.5.

**Fig. 4.** Equal $\chi^2$ curves of a fit to LEP and low energy data. The fixed parameters are $m_{\text{top}}=110\text{ GeV}$, $M_{\text{Higgs}}=300\text{ GeV}$, $\alpha_s=0.14$, $\alpha_R^l = \alpha_R^\nu = 0.0$, $\alpha_L^l = \alpha_R^\nu = 0.09$, $\alpha_L^q = \alpha_R^q$, $\alpha_L^b = \alpha_R^b$ and $\alpha_L^{\text{top}} = \alpha_R^{\text{top}}$. The area between the indicated curve is the lowest $\chi^2$ area. The next curves correspond to $\chi^2$’s increasing by steps of 0.5.