Measurement of topological charges of optical vortices by antiphased semicircular slit pair

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Abstract

Determination of orbital angular momentum (OAM) states is a subject of crucial importance for their applications in areas ranging from classical physics to quantum information. Here, we propose the antiphase semicircular slit pair (ASSP) as a novel approach to determine the topological charge of OAM states. The ASSP contains two semicircular slits with a diameter increment and symmetrically arranged in upper and lower circle. It converts an incident OAM state into light field with two bright spots, of which the relative shift is twice as spot shift for a semicircular slit. Physically, we introduce the two models of equivalent spiral slit and the Young’s-like interference, obtaining two approximate linear relations between the shift and the incident topological charge. Analytically, the antiphase of the diffracted fields for the two semicircular slits cancels a main Bessel vortex term, and doubles the complement fields contained in that for a single semicircular slit, realizing the field with two bright intensity spots with the relative shift doubled. The diffracted field is fundamentally approximated as the weighted superposition of finite Bessel vortex eigenstates. Using shift between the bright spots, the determination of topological charge of OAM states becomes a feasible and convenient, and the experimental measurement conforms to the theory with satisfying accuracy.

1. Introduction

The optical vortex of a twisted phase wavefront exp(i\(m\theta\)), with \(\theta\) the azimuth angle and \(m\) the topological charge, carries orbital angular momentum (OAM) \(m\hbar\) per photon [1], and it has a doughnut intensity distribution with dark core at the phase singularity. In the past years, the studies of optical vortices have attracted great interests and remarkable advances have been achieved in a variety of research areas.

Regarding the applications of optical vortices, the classical applications include super diffraction-limit imaging [2], precision metrology [3], optical tweezing [4], particle acceleration [5], etc. Forming the eigen states of OAM in an infinite dimensional Hilbert space, optical vortices have also been used as the information carrier for classical optical communications [6, 7], quantum entanglements [8], quantum communications [9]. Regarding the generation and manipulation of the optical vortices, various approaches and devices have been developed, including the well familiar spiral phase plate [10], q-plate [11] and fork-shaped grating [12]. In more fundamental aspects, optical vortices have been extended to regimes of different dimensions and different degrees of freedom recently. The nanoscale optical vortices have been flexibly controlled via spin–orbit conversion in metasurfaces for classical [13] and quantum light [14]; generation of vortex beams of wavelength from visible [15] to x-ray light [16], ultra-broadband tunable OAM states [17] have been realized; OAM states of extremely high quanta and quantum entanglement of
these states have been performed [8]. More interestingly, the recent development in topology of the knots and links of the vortex cores has drawn great attention [18, 19].

In the last years, versatile methods have been developed to detect the OAM states and to discriminate topological charge of the vortex beams, and exemplified implementations include the sort of intensity distributions diffracted from the different apertures [20], the Mach–Zehnder interferometer [21] and double-slit interference [22], and the log-polar-based azimuthal mode-sorter [23]. Moreover, the determination of nanoscale vortex states has become the intensely studied topic, and most of the performances are conducted using the SPP waves excited by metallic nanostructures, such as arrays of orthogonal nanoslits [24, 25], metagratings [26], nanowire [27] and plasmonic nanoslit [28, 29] and nanohole [30]. Particularly, the semicircular plasmonic nanoslit, which has been initially used to ease the measurement of photonic spin Hall effect [31], are proposed as an efficient tool for the on-chip discrimination of OAM [32], and for OAM nanometry with the slit fabricated in topological insulator film [33]. Using such semicircular slits, an OAM state is focused into subwavelength spot with an on-chip spatial shift of constant interval between neighboring states [32, 33] and thus it is distinguished. In fact, the spin–orbit interaction [34] in the nanoscale slit causes an illuminating circularly polarized OAM beam to become the superposition of two OAM states of orthogonal circular polarizations [35], while the geometric phase imposed only to the OAM state of the conversed polarization leads to complicated mechanism of the nanoscale OAM measurement, which has been rarely explicitly considered.

Despite this, the straightforward mapping the OAM states to the spot shifts in a nanoscale system would inspire the exploration of semicircular slit for the measurement of non-nanoscale OAM states in far-field zone in the optical systems of conventional scale. However, some issues need to be treated: first, whether the shift of the focused spot takes place in the far-field needs to be verified. Next, for the non-nanoscale semicircular slit without spin–orbit interactions [34], the behaviors of the diffracted OAM states need to be understood. Also, the difficulty in determining unsymmetrical shift of a single focused spot without an obvious reference needs new route of solution. In this work, we propose the antiphased semicircular slit pair (ASSP) for measurement of topological charges of optical vortices in the far-field zone. The slit pair contains the upper and lower semicircular slits symmetrically arranged in a circle with a diameter increment to offer antiphase for the passing beam. The vortex beam is diffracted as two focused spots lying symmetrically aside the origin of the far-field observation plane, and the input OAM state is mapped to, and is discriminated with, the shift between the two spots, which is twice the spot shift for a semicircular slit. Start from verifying formation of focused spot in the intensity distributions due to the diffraction of the vortex beams by a single semicircular slit, we derive analytically the diffracted light field to be the superposition of the main Bessel vortex beam of topological charge \( m \) and a complement field. We introduce two models of the equivalent spiral slit and the equivalent Young’s double slits of extended arc segments, to achieve the approximate linear relations of the spot shift with the topological charge. For the designed the ASSP, the antiphase of the fields diffracted by the two semicircular slits cancels the main Bessel vortex term, but doubles the complement fields, and thereby diffracted field includes two bright intensity spots with the relative doubled shift. More fundamentally, the output field are expanded as the weighted superposition of infinite eigen Bessel vortex states, and the field can be approximately substituted by the superposition of finite eigen states depending on the input topological charges. With the approximate linear relation, the robust map of shift between the two bright spots to input topological charge provide a feasible means for determination of OAM states. The experimental measurement conforms well with the theoretical results. The work would be of significance to the areas related to the fundamentals and application of vortex beams, such as classical and quantum communications [6–9], precision metrology [3], topology of the vortex-core links [18, 19].

2. Output field of vortex beam diffraction by traditional scale semicircular slit

2.1. Fundamental theory

As shown in figure 1(a), an incident beam illuminates the screen with an aperture of semicircular slit lying in the object plane \( O x_0 y_0 \). The semicircular slit is in the half-plane above \( x \)-axis with the center of the circle at origin \( O \). \( O x y \) is the observation plane at a distance \( z \) away from the object plane. From the Huygens–Fresnel principle, the light field on the observation plane \( O x y \) is written as

\[
U_{\text{om}}(x, y) = \frac{1}{i\lambda} \iint U_0(x_0, y_0) \exp(i kl) \cos \left( \hat{n} \cdot \hat{l} \right) dx_0 dy_0, \tag{1}
\]

where \( U_0(x_0, y_0) \) is the light field immediately behind the aperture, \( k \) is the wave vector, \( \lambda \) the wavelength, and \( \cos \left( \hat{n} \cdot \hat{l} \right) = z/l \) is the inclination factor. Here \( l = \sqrt{x^2 + (x - x_0)^2 + (y - y_0)^2} \) is the distance point...
The transmittance of semicircular slit aperture is given by

\[ t(r, \theta) = \begin{cases} 1, & r_0 \leq r \leq r_0 + \delta r_0; \quad 0 \leq \theta \leq \pi \\ 0, & \text{otherwise} \end{cases} \]

where \( r_0 \) and \( r_0 + \delta r_0 \) are the inner and outer radii of the slit, respectively. When a vortex beam \( U_0(r, \theta) = a(r) \exp(i m \theta) \) of integer-order \( m \) illuminates the slit, from equation (2), \( U_{um}(r, \theta) \) is derived as:

\[ U_{um}(r, \theta) = \frac{\exp(ikz)}{i\lambda z} \int \int a(r) t(r, \theta) \exp(i m \theta) \exp \left\{ ik \left[ r^2 + \rho^2 - 2r \rho \cos(\theta - \varphi) \right]/2z \right\} r \, dr \, d\theta. \]  

Since \( \delta r_0 \) is sufficiently small compared with \( r_0 \), the area element \( r \, dr \, d\theta \approx r_0 \delta r_0 \, d\theta \), and \( a(r) \approx a(r_0) = a_0 \), the above equation is written as:

\[ U_{um}(\rho, \varphi) = C \exp \left[ i k \left( r_0^2 + \rho^2 \right)/2z \right] \sum_{n=-\infty}^{\infty} (-i)^n J_n \left( kr_0 \rho/z \right) \exp(\text{i}n\varphi) \int_0^\pi \exp[\text{i} (m-n) \theta] \, d\theta \]  

where \( J_n \) represents the \( n \)th order Bessel function of the first kind, \( C = -i[a_0 r_0 \delta r_0 \exp(ikz)]/\lambda z \) is a complex constant, and the identity has been made use of:

\[ \exp[-ikr_0 \rho \cos(\theta - \varphi)/z] = \sum_{n=-\infty}^{\infty} (-i)^n J_n \left( kr_0 \rho/z \right) \exp[\text{i}n(\varphi - \theta)]. \]  

Noticing that the integrals in equation (4) are \( \pi \) for \( n = m, 2i(m-n) \) for \( n = m \) odd numbers, and 0 for \( n = m \) even numbers, respectively, we obtain:

\[ U_{um}(\rho, \varphi) = C' \exp \left[ i k \left( r_0^2 + \rho^2 \right)/2z \right] \left\{ \pi J_m \left( kr_0 \rho/z \right) \exp(\text{i}m\varphi) \right\} \]

\[ - \sum_{n_1=-\infty}^{n_1=\infty} \frac{2(-1)^n}{2n_1+1} J_{m-2n_1-1} \left( kr_0 \rho/z \right) \exp[\text{i}(m-2n_1-1)\varphi] \]

where \( C' = (-i)^m C \), and \( n_1 = (m-n-1)/2 \) with \( n = m \) odd number. \( U_{um}(\rho, \varphi) \) can be further written as:

\[ U_{um}(\rho, \varphi) = U_{bm}(\rho, \varphi) + U_{cm}(\rho, \varphi). \]
Where

\[ U_{bn}(\rho, \varphi) = C' \exp \left[ \frac{ik}{2z} \left( r_0^2 + \rho^2 \right) \right] \frac{\pi J_n(\rho r/\rho)}{z} \exp (aim\varphi), \]

\[ U_{cm}(\rho, \varphi) = -C' \left[ \frac{ik}{2z} \left( r_0^2 + \rho^2 \right) \right] \sum_{n_1=-\infty}^{n_1=\infty} \frac{2(-1)^{n_1}}{2n_1 + 1} J_{m-2n_1-1}(\rho r/\rho) \exp \left[ i(m - 2n_1 - 1) \varphi \right]. \]

Obviously, \( U_{bn}(\rho, \varphi) \) given in equation (8) is the \( m \)th-order Bessel vortex beam with helical phase front \( \exp(aim\varphi) \), and equation (7) demonstrates that the diffracted field \( U_{cm}(\rho, \varphi) \) is the addition of Bessel vortex beam \( U_{bn}(\rho, \varphi) \) and \( U_{cm}(\rho, \varphi) \), which is referred to as complement field for the upper semicircular slit.

The \( U_{cm}(\rho, \varphi) \) expressed in equation (9) is the linear superposition of Bessel vortex beams of order \( m - 2n_1 - 1 \) with the parity opposite to \( m \) and the weight coefficient values 2 and \( 0 \), respectively, with the largest magnitude. For moving \( n_1 \), the weight coefficient for vortex beam of order \( m - 2n_1 - 1 \) in the superposition is symmetric with respect to center order \( m \), and the closer the order of the vortex beam is to \( m \), the greater its contribution to \( U_{cm}(\rho, \varphi) \).

To look at the diffracted intensity distribution, we numerically calculate the light field \( U_{bn}(\rho, \varphi) \) based on equation (4), with the distance set at \( z = 1 \) m, the amplitude of incident vortex beam \( a_0 = 1 \) and wavelength \( \lambda = 632.8 \text{ nm} \). The radius \( r_b \) of the semicircular slit is set \( r_b = 3.56 \text{ mm} \), which provides for the point on the slit an extra optical path \( \Delta l = (z^2 + r_b^2)^{1/2} - z \) of \( 10 \lambda \) over the distance \( z \). Figures 1(b1)–(b6) also show the obtained intensity maps for the incident vortex beams with topological charges \( m = 0, 1, 2, 3, 5 \) and 6, respectively. We see that bright spot that shifts from the center of the observation plane is formed in the intensity distribution, and the shift increases the topological charges \( m \), with the shift for \( m = 0 \) being zero. These maps intuitively manifest the characteristics of the bright spots and the correlation between the shifts of the spots and the topological charge of the incident vortex beam.

2.2. Models of spiral slit and semicircular slit of enlarged diameter

For a physical analysis, we compare three slits and consider the equivalence based on the phase variations, as demonstrated in figure 2(a). For the semicircular slit \( S_0 \) in the black solid curve illuminated by vortex beam of topological charge \( m = 1 \), the phase is uniformly varied with azimuth angle \( \theta \) by \( \delta\varphi_{S0} = \theta(2\pi) \), as shown in color map. We consider an equivalent slit \( S_1 \) represented by the blue dashed curve, it has an azimuthally varying radius \( r_0 + \delta r \) to introduce an additional optical path \( \Delta l = [z^2 + (r_0 + \delta r)^2]^{1/2} - [z^2 + r_0^2]^{1/2} \approx r_0\delta r/z \) and an azimuthal phase shift \( \delta\varphi = 2\pi \Delta l/\lambda \) to the diffracted wave at the center of the observation plane. Specifically, when \( \delta r \) varies in \( \delta r = \lambda m\theta/2\pi r_0 \) with \( m = 1 \), or equivalently slit \( S_1 \) takes the curve of Archimedes spiral with \( r = r_0 + \lambda m\theta/2\pi r_0 \), it will provide a phase shift \( \delta\varphi = a_0\theta(2\pi) \) with \( a_0 = 2\pi m \) under illumination of plane wave light, and the diffracted field would be the same as that produced by the semicircular slit \( S_0 \) under the vortex beam illumination. Next, we consider a semicircular slit \( S_2 \) as shown by pink dot curve in figure 2(a), of which the two endpoints of its diameter coincide with starting point and ending point of \( S_1 \), respectively. Thus the diameter of \( S_2 \) is enlarged as \( D = 2r_0 + \Delta D \), with \( \Delta D = \lambda m r_0 \) and the center \( O_2 \) shifts \( \Delta x = \Delta D/2 = \lambda m r_0/2 \) leftward with respect to the center of \( S_0 \). Obviously, the semicircular slit \( S_2 \) is very close to spiral slit \( S_1 \). The wavefields produced by \( S_1 \) can be replaced by that produced by \( S_2 \), though difference may exist, it may be insignificant owing to the small radius difference \( \delta r_{2-1} = r_2(\theta) - r_1(\theta) \), where \( r_1(\theta) \) and \( r_2(\theta) \) are the radius of \( S_1 \) and \( S_2 \). The radius difference \( \delta r_{2-1} \) is numerically calculated and is shown as the inset in figure 2(a), with the maxima and minima 9.17 \( \mu \text{m} \) and \(-9.32 \mu \text{m} \), respectively, exhibiting a range of \( S_2 \) deviating in radius from \( S_1 \) within about one tenth of \( \Delta D \) (\(-88.94 \mu \text{m} \)). Equivalent to the semicircular slit \( S_2 \) illuminated by a plane wave, the semicircular slit \( S_0 \) under the illumination of vortex beam \( m = 1 \) will cause the focused bright spot to shift a distance \( \Delta D/2 \) from the center. For comparison, the value of \( \Delta D/2 \) is calculated as \( \Delta x_{m=1} = 44.4 \mu \text{m} \), while the shift of the bright spot in the intensity map obtained by numerical calculations is \( \Delta x = 36.0 \mu \text{m} \), close to the value of \( \Delta D/2 \).

2.3. Model of Young’s-like interference with arc segment sources

More physically, the shift of bright spot is explained based on the superposition of light waves analogous to Young’s interference. We consider two point sources \( A \) and \( B \) on the semicircular slit \( S_0 \) which are reflectively symmetric about \( y \)-axis, as shown in figure 2(a), and the azimuth angle difference and the distance between point \( A \) and \( B \) are \( \Delta \theta = \pi - 2\theta \) and \( d = 2r_0 \sin(\Delta \theta/2) \), respectively. When plane wave illuminates the slit, the interference of the waves from the two points forms bright fringe of zeroth order at the center of the observation plane, and the fringe pitch \( d_1 = \lambda d/\pi \). While the vortex beam of topological charge \( m \) illuminates the slit, a phase difference \( \Delta\varphi_{S0} = m\Delta \theta \) will be added to the waves from the two points, and the zeroth bright fringe will transversely shift a distance of \( w_2 = zm\Delta \theta/kd = zm\Delta \theta/2kr_0 \).
\[ \Delta \sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\sin(\si
On the whole, though the errors, the two models can intuitively describe the intensity distribution with the shifted spot and provide the simple approximate expressions of the spot shift with reasonable accuracy. By linearly fitting with $\Delta x_c = A_m$ to corresponding curves, we obtain the proportion coefficient $A = 35.08 \pm 0.211$, while the coefficients $A_m = 31.44$ and $A_m = 40.83$ are calculated to be $w_p = A_2 m$ and $\Delta x_m = A_{m2}m$, respectively, and they are close to $A$ with satisfactorily accuracy. Besides, with the fitting coefficient $A$ and the relation $\Delta x_c = A_m$ as the calibration, the topological charge $m$ of the incident vortex beam can be extracted by reading the spot shift from the diffracted intensity distribution, enabling the discrimination of topological charge of vortex beam and the detection of OAM states.

3. The antiphased semicircular slit pair and its diffraction for the incident vortex beams

3.1. The design of antiphased semicircular slit pair and the calculation of light field distribution

To explore method to produce light field that has two symmetrically shifted spots for easier and more accurate measurement, we start from adding the lower semicircular slit $S_d$ to the above upper one, forming a complete circle slit. Under illumination of a vortex beam, the light field $U_{cm}(\rho, \varphi)$ diffracted from the circle slit is the addition of fields $U_{um}(\rho, \varphi)$ and $U_{dm}(\rho, \varphi)$ from the upper slit $S_u$ and lower slit $S_d$, respectively. As $U_{um}(\rho, \varphi)$ is expressed in equations (6) to (9), the field $U_{dm}(\rho, \varphi)$ for the lower slit can be derived similarly and is written as:

$$U_{dm}(\rho, \varphi) = C \exp \left\{ i \left[ \frac{\rho^2}{2z} \right] - \frac{\rho \varphi}{2z} \right\} \int_0^{2\pi} \exp \left[ -ik_{0}\rho \cos(\theta - \varphi)/z \right] d\theta,$$

where the two integrals from 0 to $\pi$ and from $\pi$ to $2\pi$ are $U_{um}(\rho, \varphi)$ and $U_{dm}(\rho, \varphi)$ for the upper and lower slits, respectively. As $U_{um}(\rho, \varphi)$ is expressed in equations (6) to (9), the field $U_{dm}(\rho, \varphi)$ for the lower slit can be derived similarly and is written as:

$$U_{dm}(\rho, \varphi) = C \int \sum_{n=-\infty}^{\infty} (-i)^n J_n \left( \frac{kr_{0}\rho}{z} \right) \exp \left( i\varphi \right) \int_0^{2\pi} \exp \left[ i(m-n)\theta \right] d\theta.$$

The integrals in the above gives $U_{dm}(\rho, \varphi)$ the counterpart form of $U_{cm}(\rho, \varphi)$ in equation (6):

$$U_{dm}(\rho, \varphi) = C' \exp \left\{ i \left[ \frac{\rho^2}{2z} \right] - \frac{\rho \varphi}{2z} \right\} \left\{ \pi J_m \left( \frac{kr_{0}\rho}{z} \right) \exp \left( i\varphi \right) \right.$$  

$$+ \sum_{n=1}^{\infty} \frac{2(-1)^n}{2n+1} J_{2n+1} \left( \frac{kr_{0}\rho}{z} \right) \exp \left[ i(m-2n-1)\varphi \right] \right\}.$$  

Comparing with equation (6), we notice that summation term in the above is in positive sign, opposite to that of the counterpart term $U_{cm}(\rho, \varphi)$ in equation (6). Then in the abbreviated form, it is expressed as:

$$U_{dm}(\rho, \varphi) = U_{bm}(\rho, \varphi) - U_{cm}(\rho, \varphi),$$

where $U_{bm}(\rho, \varphi)$ and $U_{cm}(\rho, \varphi)$ are given in equations (8) and (9), respectively.

We notice here that by adding equations (6) for $U_{um}(\rho, \varphi)$ and (11) for $U_{dm}(\rho, \varphi)$, i.e., we have Bessel beam field for the circle slit $U_{cm}(\rho, \varphi) = 2U_{bm}(\rho, \varphi)$: the Bessel beam $U_{bm}(\rho, \varphi)$ included in both $U_{um}(\rho, \varphi)$ and $U_{dm}(\rho, \varphi)$ has the amplitude of half of $U_{cm}(\rho, \varphi)$, i.e., $U_{bm}(\rho, \varphi) = U_{cm}(\rho, \varphi)/2$. Thus, when we subtract half of the field of the circle slit $U_{cm}(\rho, \varphi)/2$ as demonstrated in figure 3(a2), from field of the semicircular slits, say, from $U_{um}(\rho, \varphi)$ for the upper slit, as demonstrated in figure 3(a1), for $m = 6$, we have only the term $U_{cm}(\rho, \varphi)$ left. More interestingly, we also note that the following equality holds:

$$2U_{cm}(\rho, \varphi) = U_{um}(\rho, \varphi) - U_{dm}(\rho, \varphi).$$
The upper semicircular slit $S_u$, the circle slit, and antiphased semicircular slit pair, respectively. (b1)-(b3) The intensity maps diffracted from the slit as demonstrated in (a1)-(a3) for $m = 6$, respectively. (c) The light field $U_{um}(\rho, \varphi)$, $U_{cm}(\rho, \varphi)$, and $U_{spm}(\rho, \varphi)$ corresponding to intensity maps as demonstrated in (b1)-(b3), respectively.

We note that the Bessel vortex beam which provide an additional optical path difference $\Delta r_0/z = \lambda/2$ from the upper and the lower slits to area near origin $O$ on plane $Oxy$, and then is given by $\Delta r_0 = \lambda z/2r_0$. Thus the whole slit contains the two semicircular slits symmetrically arranged in upper and lower circle, the lower semicircular slit with the decreased radius converses its light field to $- U_{dum}(\rho, \varphi)$ by the additional phase shift $\pi$, and thus $2U_{cm}(\rho, \varphi)$ is obtained. Here we refer to the whole slit as ASSP owing to the $\pi$-phase shift between the constituent upper and lower slits. On the other hand, as demonstrated in figure 3(a3), $2U_{cm}(\rho, \varphi)$ has the intensity distribution with two spots symmetrically lying on the left and right sides of the center point $O$, and the separation of the two spots is twice of spot shift for a semicircular slit. This expected outcome may ease the performance and enhance the accuracy, which is realized simply by a pair of slits with a radius decrement of $\Delta r_0 = \lambda z/2r_0$.

With the ASSP is illuminated by vortex beam, referring to equations (4), (10) and (13), the light field $U_{spm}(\rho, \varphi)$ produced by the ASSP is written as:

$$U_{spm}(\rho, \varphi) = U_{um}(\rho, \varphi) + \exp (i\pi) U_{dm}(\rho, \varphi)$$

$$= C \exp [i(k r_0^2 + \rho^2)/2z] \left\{ \left( \int_0^\pi - \int_{\pi}^{2\pi} \right) \exp (im\theta) \exp [-ikr_0 \rho \cos(\theta - \varphi)/z] d\theta \right\}.$$

(15)

We note that the Bessel vortex beam $U_{bm}(\rho, \varphi)$ contained the integrals is eliminated in $U_{spm}(\rho, \varphi)$ because of the antiphase factor $\exp(i\pi)$. Using equations (14) and (9), $U_{spm}(\rho, \varphi)$ is finally written as:

$$U_{spm}(\rho, \varphi) = 2U_{cm}(\rho, \varphi)$$

$$= -2C' \exp [i(k r_0^2 + \rho^2)/2z] \sum_{n_1 = -\infty}^{\infty} \frac{2(-1)^n_1}{2n_1 + 1} J_{m - 2n_1 - 1}(kr_0 \rho/z) \exp [i(m - 2n_1 - 1) \varphi].$$

(16)

The above is the superposition form of eigen Bessel vortex states $J_n(kr_0 \rho/z) \exp(i\varphi)$ for the light field $U_{spm}(\rho, \varphi)$, under illumination of the vortex beam of topological charge $m$, as $U_{cm}(\rho, \varphi)$ that is given in equation (9).

Based on equation (2) with the aperture transmittance by the ASSP, we numerically calculate the diffracted light field $U_{spm}(\rho, \varphi)$ for different topological charge $m$ of the illuminating vortex beam, and the radii of the upper and lower semicircular slits are set at $r_1 = 3.65$ mm and $r_2 = r_0 = 3.56$ mm, respectively, which provide an additional optical path difference $\delta \lambda = (z^2 + r_1^2)^{1/2} - (z^2 + r_2^2)^{1/2} = \lambda/2$ between the upper and lower slits to a point near the center of the observation plane. Figures 4(a1)-(a7) and (b1)-(b7) show the calculated intensity and phase maps for the incident vortex beams with topological charges $m = 0$ to 3, 5, 6 and 8, respectively. We see that except for cases of $m = 0$ and 1, two bright spots with transverse
symmetric shifts from the center are formed, and the separation between the two shifted spots increases with topological charges $m$.

Overall, the physical mechanisms for the link between shift of the spots and the topological charge are straightforward. As shown in figure 2(a), the azimuthally varying phase of the vortex beam illuminating the upper semicircular slit can be equivalented as the phase related to the optical path difference of the spiral slit under illumination of a plane wave, and by further approximating the spiral slit as semicircular slit with the enlarged diameter, the bright spot is formed at center of the enlarged slit, which shifts to the left of the center of upper slit, and the shift is proportional to the topological charge $m$. Similarly, with the lower slit illuminated by the vortex beam, the bright spot shifted to the right is formed. As the superposition of the light fields of the upper and the lower slits, the light field of the ASSP under the vortex beam illumination is formed to contain two bright spots shifted to the left and to the right of the center, respectively, as demonstrated in figure 3, and the distance between the two bright spots is also proportional to the topological charge $m$. Besides, we note that here the ASSP producing bright spots is at the ordinary scale much larger than the wavelength, and spin–orbit interaction can be neglected [34]. Hence the spots and their shifts are formed by conventional diffraction of slits as represented by equations (1) and (2), and the influences of the spin dependent effects such as photonic spin Hall effect and geometrical phases are negligible.

### 3.2. Analysis for the light field as superposition of eigen Bessel vortex states

In the expression of equation (16) for $U_{\text{sp}}(\rho, \varphi)$, only the eigen states of the integer order with parity opposite to $m$ are included in the summation. To examine the evolution of superposed light field with the constituent eigen states in equation (16), we calculate the light fields summed over gradually-broadened interval of topological charge including finite number of eigen states, instead of the summation over infinity. Figures 5(a1)–(a5) and (b1)–(b5) shows the results of intensity patterns and phase maps, for the superposition of eigen states with $m - 2n_1 - 1 = n$ taking odd numbers in the intervals from $[-1, 1], [-3, 3], [-5, 5], [-7, 7]$ and $[-9, 9]$, respectively, under the illumination of vortex beam of $m = 6$. The curves of the function $J_{m - 2n_1 - 1}(kr_0\rho/z)/(2n_1 + 1)$ squared also with $m = 6$ along $x$-axis, i.e., at $y = 0$, are plotted in figure 5(c); here the coefficient $1/(2n_1 + 1)$ represents the amplitude modulation for the vortex eigen state of order $m - 2n_1 - 1$ contributing to the superposition of $U_{\text{sp}}(\rho, \varphi)$, as given in equation (16).

From figure 5(c), we see that the curve for eigen state of order $m - 1 = 5$, the lower order immediately adjacent to $m = 6$, with coefficient $1/(2n_1 + 1) = 1$, has the highest profile, demonstrating that this state contributes the most to the superposition. Also, the curve for order $m + 1 = 7$, the higher order immediately adjacent to $m$, with coefficient $1/(2n_1 + 1) = -1$, has the second highest profile following order 5, contributing significantly to the superposition. Though the profiles for the other eigen states decrease dramatically with their order $m - 2n_1 - 1$ going farther away from the topological charge $m$, they may still have obvious effect on the light field when included in the superposition.

We now examine the superposed light field by gradually extending the interval of odd topological charge integers symmetrically with respect to $m = 6$, and the superposed intensity patterns and the phase maps are
Figure 5. (a1)–(a5) Intensity patterns, and (b1)–(b5) phase maps for the superposition of weighted eigen states with topological charge $n = m - 2n_1 - 1$ taking odd numbers in the intervals as given on the top title line, respectively, under the illumination of vortex beam of $m = 6$. (c) The curves for the function $J_{m-2n_1-1}^m$ squared along $x$-axis with $m = 6$.

Figure 6. (a1)–(a8) Intensity patterns and (b1)–(b8) phase maps of the superposition of the weighted eigen states in intervals as given on the top, respectively, with $m = 6$ for the incident vortex beam.

shown in figures 6(a1)–(a5) and (b1)–(b5), respectively, for the intervals of $[m - 2n_1 - 1, m + 2n_1 + 1]$ with $n_1$ from 0 to 4. For comparison, the intensity pattern shown figure 4(a6) calculated using the integral expression in equation (2) with the ASSP, is taken as the practical pattern for reference. The starting pattern in figure 6(a1), for the interval $[5, 7]$ having only the two constituent eigen states with $J_5(kr_0 \rho /z)$ and $J_7(kr_0 \rho /z)$, sets the basic form of the intensity distribution with two bright spots in crescent moon-like shape and their separation close to that between two spots in the practical pattern, as marked by the identical dashed circle in figure 6 which passes the centers of the two practical spots. With the increase of the eigen states in the intervals, the pattern of superposed field gradually evolves to practical pattern.

Though the numbers of the eigen states included in the intervals $[-3, 15]$ and $[-9, 9]$ are both 10, respectively, for the superposed intensity patterns in figure 6(a5), and in figure 5(a5), we notice that the pattern in figure 5(a5) is closer to the practical pattern in figure 4(a6). Besides, in figures 6(a6)–(a8) and (b6)–(b8), the intensity and phase patterns are shown for the intervals $[-5, 15]$, $[-7, 15]$, and $[-9, 15]$, and we observe that with lower limit of the intervals going down, the intensity distributions inside the dashed circles become closer to pattern in figure 5(a5), for interval $[-9, 9]$ and that in figure 4(a6), and the pattern for $[-9, 15]$ is the most similar. From the evolutions of the intensity patterns in figures 5 and 6, we
deduce that the eigen states of lower orders, say, within the interval \([-9, 9]\) for \(m = 6\), referred to as low order interval with limits \(\pm(m + 3)\), contributes significantly to the superposition. This result is inherently determined by the amplitude-modulated Bessel functions with their profile curves shown in figure 5(b), as detailed above.

In the above process, with the starting superposed intensity of the eigen states \(m - 1 = 5\) and \(m + 1 = 7\) for \(m = 6\), the addition of more eigen states to the superposition finally causes each of the two starting bright spots to split into 3 spots, i.e., the primary spot and two secondary spots lying on the upper and lower sides, as shown in figure 6(a8). More generally, when the incident vortex beam of very high topological charge \(m\) is diffracted from the ASSP, the introduction of more eigen states to the starting superposition of eigen states \(m - 1\) and \(m + 1\) will cause each of the two bright spots to split into a primary spot and more secondary spots on the upper and lower sides. The varied sizes, intensities and the increased numbers of the spots result in a complicated intensity distribution, and it becomes difficult for the primary spot to be distinguished from the secondary spots. As a result, possible errors can be brought about when the topological charges are measured from the distance of two primary spots. Further calculations demonstrate that when the topological charge is less than or equal to 20, the two bright spots are easily distinguishable, so that the topological charge may be measured with satisfying accuracy. Though the results with certain accuracy may be achieved for slightly higher topological charges, we conclude that the topological charges less than or equal to 20 can be measured with very good accuracy.

Besides the intensity distributions, the vortices and phase singularities in light field of vortex state superposition is particularly interesting, because of the related isolated vortex loops in form of knots and links using algebraic topology [18, 19]. From the phase maps in figures 6(b1)–(b8), we can see the evolutions of the phase distributions with the broadened interval. In figure 6(b1), the phase map for the superposition of the vortex states of orders 5 and 7 presents a vortex of order 5 inside the dashed circle and 2 (\([5–7]\)) singularities outside the circle, exhibiting the characteristic of two superposed vortex states [13, 36]. In figure 6(b2), the phase map manifests the evident characteristic of the vortex superposition of orders 3 and 5. Though the vortex of order 9 is also included in the superposition, it has almost no influence on the properties of phase singularities inside the circle. With the vortex states of orders 1 and 11 added to the superposition, as shown in the phase map of figure 6(b3), the phase map demonstrates characteristic of the superposition of the vortices of orders 3 and 1, splitting the central vortex of order 3 into a vortex of order 1 and two singularities aside; thus, the map has a central vortex and 4 singularities inside the dashed circle. Starting from figure 6(b4), with introduction of more eigen vortex states, the phase distributions do not change significantly; this demonstrates that the main features of the phase distribution depending on the superposition of the eigen states within the interval from 1 to 7. Combining the deduction obtained in the former discussions for the evolution of the intensities, we again conclude that with very good accuracy, the practical light field produced by ASSP can be approximately substituted by the superposition of the odd number eigen states in the topological charge interval \([-m - 3, m + 3]\) for \(m\) being the even number of 6.

### 3.3. The spot shifts by the antiphased semicircular slit pair

Essentially, the light field diffracted from the ASSP is the interference of the fields from upper and lower parts of the slit, and the shift between the symmetrical spots may differ from the sum of the spot shifts in the individual intensity maps. To examine the difference, we plot the spot shifts versus topological charge \(m\) in figure 7(a); therein the curve of the spot shift \(\Delta x_p\) for ASSP is obtained from the intensity distributions calculated based on equation (2); the curve of \(2\Delta x_c\) shows the two multiples of the corresponding spot shift of the upper semicircular slit. From these curves, we see that for the values of \(m\), exclusive of \(m = 0\) and 1, the shifts the two slits are nearly equal as expected. We use the linear function to fit these two curves, and obtain the slopes are 71.76 ± 0.956 and 68.61 ± 0.790 for the curves of \(\Delta x_p\) and \(2\Delta x_c\), respectively. Because \(\Delta x_c\) is the shift of the single spot for the semicircular slit, with the approximate relation \(\Delta x_c \approx w_2\) in the Young’s-like model, where \(w_2 = zm\theta/2kr_0\sin(\theta/2) = zm\sqrt{2}/8r_0\) with \(\Delta \theta = \pi/2\), as shown in figure 2(c), the distance \(\Delta x_p\) between two bright spots can be approximated as \(\Delta x_p \approx 2\Delta x_c \approx z\sqrt{2}/4r_0\). It indicates that the distance \(\Delta x_p\) can be controlled by changing the observation distance \(z\) and the ring radius \(r_0\) in experimental implementations. Obviously, either increasing \(z\) or decreasing \(r_0\) will increase the distance \(\Delta x_p\). We noted that in such performance, the radius difference \(\Delta r_0\) of the ASSP should be adjusted accordingly based on \(\Delta r_0 = \lambda z/2r_0\), so as to satisfy the optical path difference of half wavelength \(\Delta \approx r_0\Delta r_0/\lambda = \lambda/2\), as mentioned in previous text of subsection 3.1. When the shifts are used to recognize topological charges, the ASSP is evidently advantageous over the semicircular slit owing to its shift twice as large. In particular, for \(m = 0\), the intensity becomes the specific two-lobe like pattern, and according to the above analysis, such pattern is formed by the superposition of the two significant states adjacent to \(m = 0\), which are of topological charges \(+1\) and \(-1\), respectively. While for \(m = 1\), the spot shifts for the upper and
Figure 7. (a) The curves of spot shift $\Delta x_p$ for ASSP and $2\Delta x_c$ for the upper semicircular slit obtained from the calculated intensity distributions. (b) Curves for the experimental spot shift $\Delta x_e$ obtained using the ASSP and its comparison with $\Delta x_p$.

Figure 8. Experimental setup. Laser: He–Ne laser (632.8 nm); SPF: the spatial pinhole filter; L: convex lens with a focal length $f = 18$ cm; A: circular aperture; SPP: spiral phase plate; ASSP: the antiphased semicircular slit pair screen.

the lower half of the semicircular slit are smaller spot size, so shifted spots are actually overlapped and the interfered pattern in figure 4(a1) appears to have horizontally broadened spot of zero shift, which is also the superposition of the two adjacent significant states of $m = 0$ and $m = 2$.

4. The experimental demonstrations

In the experiment, the ASSP screen is first home-fabricated by ablating dense pinholes on a carefully flatted and fixed aluminum foil with a femtosecond laser (Spitfire, Spectral Physics, wavelength 800 nm, 1 KHz, 4 mJ/pulse). The fixed foil is mounted on a computer-controlled 3D motorized stage (resolution of 1 $\mu$m), which runs following a written program with the pinhole positions encoded and with shuttering of the incident femtosecond laser synchronized. By adjusting the number of ablating laser pulses, the defocusing distance, and the working laser power, we successfully realize a good control of the pinhole sizes, and fabricate practically usable screens of slit. The diameter of the pinhole is about 49.00 $\mu$m, and the radii of the upper and the lower halves of the slit are 3.65 mm and 3.56 mm, respectively.

Figure 8 shows the schematic of the experimental setup for measurement of the intensity patterns with the shifted spots by the ASSP diffracting the incident vortex beam. The light beam of wavelength 632.8 nm emits from a He–Ne laser. It is expanded and filtered by a spatial filter and is collimated by a convex lens (L) with a focal length $f = 18$ cm into a parallel light monochromatic planar light. A circular aperture (A) is used to adjust the beam to an appropriate cross section size, and then the light uniformly illuminates the combined spiral phase plate (C-SPP), which includes eight individual spiral phase plates (SPP) of topological charges 1 to 8. The SPP aligned to the illuminating beam outputs the optical vortex with the corresponding topological charge. The semicircular slit or the ASSP screen 12 cm behind the SPP is illuminated by the vortex beam and it is mounted on a three-dimensional stage for the fine adjustment for
its center to be coincide with vortex core. The observation plane is 1 m away from ASSP screen, where the EM-CCD (ProEM-HS 1024BX3) is placed to record the diffracted intensities.

Figures 1(c1)–(c6) show the experimental intensity maps for the optical vortices of topological charges $m = 0, 1, 2, 3, 5$ and 6 diffracted through the semicircular slit, and figures 4(c1)–(c7) show the experimental intensity maps for the vortices of topological charges $m = 0, 1, 2, 3, 5, 6$ and 8 diffracted through the ASSP. In figures 1(c1)–(c6), we see that the experimental intensity maps for the semicircular slit are well consistent with corresponding maps by the theoretical calculations in figures 1(b1)–(b6), in both the features of the bright spot and shifts from the center on the observation plane, though the determination of the center is somewhat tough work. In figures 4(c1)–(c7), the maps also conform with the theoretical results in figures 4(a1)–(a7), and two bright spots clearly appear and shift symmetrically aside the center. We have measured shifts between the two symmetrical bright spots from the experimental patterns for the incident vortices of different topological charges; the measured shifts are shown as curve of $\Delta x_e$ in figure 7(b), and the corresponding data $\Delta x_p$ is also shown for convenience of comparison. These data demonstrate the topological charges of an incident vortexes may be well discriminated from the corresponding shift between the spots. On the whole, the results validate the feasibility of the proposed method with the ASSP for both the principles and the experimental performances.

5. Conclusion

We have analytically and experimentally demonstrated the determination of OAM beams based on the ASSP consisting of upper and lower semicircular slits with an increment of diameters. The two models of the equivalent spiral slit and equivalent Young’s double slits of extended arc segments are proposed to obtain the linear relation between the spot shift and the topological charge. Base on the derived result for a single semicircular slit as the addition of the main Bessel vortex beam and a complement field, the antiphase from the ASSP cancels the main vortex and leaves the doubled complement field as the diffracted field, which has two bright spots with relative shift doubled. Further analysis indicates that diffracted field can be approximated as weighted superposition of the finite Bessel vortex eigen states with interesting evolution of phase singularities. The proposed method has been validated by experimental determination of OAM states with convenient performance and good accuracy. We believe that the functionality of the ASSP and the properties of the diffracted field may enable broadened applications in generation and detection of OAM states, and this work would be of significance to the related area such as optical communications, quantum information and topology of light field.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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