Exploring Short-GRB afterglow parameter space for observations in coincidence with gravitational waves

M. Saleem1, L. Resmi2, Kuntal Misra3, Archana Pai4 and K. G. Arun5,6
1Indian Institute of Science Education and Research Thiruvananthapuram, CET Campus, Trivandrum 695016
2Indian Institute of Space Science and Technology, Trivandrum.
3Aryabhatta Research Institute of Observational Sciences, Nainital.
4Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai 400076
5Chennai Mathematical Institute, Siruseri, 603103 Tamilnadu.
6Institute for Gravitation and the Cosmos, Pennsylvania State University, State College, PA 16802.

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ABSTRACT
Short duration Gamma Ray Bursts (SGRB) and their afterglows are among the most promising electro-magnetic (EM) counterparts of Neutron Star (NS) mergers. The afterglow emission is broadband, visible across the entire electro-magnetic window from γ-ray to radio frequencies. The flux evolution in these frequencies is sensitive to the multi-dimensional afterglow physical parameter space. Observations of gravitational wave (GW) from BNS mergers in spatial and temporal coincidence with SGRB and associated afterglows can provide valuable constraints on afterglow physics. We run simulations of GW-detected BNS events and assuming all of them are associated with a GRB jet which also produces an afterglow, investigate how detections or non-detections in X-ray, optical and radio frequencies can be influenced by the parameter space. We narrow-down the regions of afterglow parameter space for a uniform top-hat jet model which would result in different detection scenarios. We list inferences which can be drawn on the physics of GRB afterglows from multi-messenger astronomy with coincident GW-EM observations.

Key words: Gravitational Waves – Double neutron star mergers – Gamma Ray Bursts – Multi-messenger astronomy.

1 INTRODUCTION
The most favoured progenitor model for short duration Gamma Ray Bursts (SGRBs) are mergers of two neutron stars (NS) or a black hole and a neutron star. The recent observation of gravitational waves from BNS merger GW170817 (Abbott et al. 2017d) in spatial and temporal coincidence with GRB170817A has confirmed Binary Neutron Star (BNS) mergers to be one of the progenitors of SGRBs. The absence of an associated supernova in SGRBs was the most supportive evidence for the binary merger hypothesis (Levan et al. 2008; Fong et al. 2016). There were further additional observations consistent with the binary merger hypothesis. For example, the diversity in SGRB host galaxies, including both early and late type galaxies, were indicative of the progenitors belonging to old stellar populations (Fong et al. 2013). The GRB positions showed a statistically larger offset from the photosphere of the host galaxies, naturally explained from the natal kicks of Neutron Stars (Fong et al. 2010). In 2013, the observation of a nearby short GRB 130603B showed an excess emission in near-infrared, consistent with the expectations of a kilonova emerging from the r-process nucleosynthesis of neutron rich material ejected from the merger (Li & Paczynski 1998; Berger et al. 2013; Tanvir et al. 2013).

Direct and indirect evidences associate GRBs to relativistic outflows (Goodman 1986; Paczynski 1986; Meszaros & Rees 1993; Frail et al. 1997; Taylor et al. 2004) collimated.
to narrow opening angles of a few degrees (Rhoads 1999; Sari et al. 1999; Harrison et al. 1999). As a consequence of the relativistic bulk motion, the flux is expected to be heavily reduced if an observer’s line of sight is not aligned within the jet. Therefore, there may be less chances to detect prompt γ-ray emission from GRB jets directed away from us. However, Gamma Ray Bursts are followed by long lasting afterglow emission from γ-ray to radio frequencies (Costa et al. 1997; Frontera et al. 1998; van Paradijs et al. 1997; Frail et al. 1997). During the afterglow phase, the jet decelerates considerably and the doppler de-boost is alleviated making the emission visible to even observers oriented away from the jet cone (Miodonski et al. 2000; Rossi et al. 2002; Dalal et al. 2002; Granot et al. 2002). This makes afterglows to be potential candidates for electromagnetic (EM) follow-up observations. The joint event GW170817 and GRB170817A was followed-up by various EM observatories (Abbott et al. 2017c). An optical/IR/UV transient consistent with predictions from kilonovae models (Pian et al. 2017; Smartt et al. 2017; Arcavi et al. 2017) and X-ray/radio transient potentially from the GRB jet (Troja et al. 2017; Alexander et al. 2017; Hallinan et al. 2017; Margutti et al. 2017) were detected.

With this observation, it became clear that neutron star mergers observed in the gravitational wave window may be accompanied with Gamma Ray Burst prompt emission and subsequent longer wavelength afterglows. Here, we focus on the joint detection of a GW event and the afterglow of the associated GRB. In an associated paper (Saleem et al. 2017) we report the rate of afterglow detections in X-ray, optical, and radio wavelengths in such a scenario.

In this work, we systematically explore the influence of the afterglow parameter space (particularly the ranges and distributions of the parameters) on the detectability of different afterglow components, for observations in coincidence with the GW-detected BNS merger events. We have considered EM facilities in X-ray, optical, and radio bands for afterglow detections. Similarly we considered a 5-detector network of GW detectors for BNS merger detections. We recall that the first four GW detections from binary blackhole events (Abbott et al. 2016a,b, 2017a,b) were made by a 2-detector advanced LIGO network and most recently, the first ever 3-detector observation of gravitational waves was reported from a BBH system with LIGO-Virgo network (Abbott et al. 2017c) followed by the BNS merger GW170817 (Abbott et al. 2017d). With more detectors, we expect to observe more number of compact binary mergers. Our studies are done with simulated BNS sources whose physical parameters are distributed typically within their ranges inferred from observations. With simulated afterglow light curves, we investigate various components of the afterglow parameter space which could influence detections, and we specifically focus on the observer’s viewing angle with respect to the jet axis. We divide the afterglow population into two: within-jet cases where the observer’s line of sight points to within the jet cone, and outside-jet cases where the line of sight falls outside the jet cone. With different detection/non-detection scenarios in X-ray, optical, and radio bands, we identify favourable regions in the afterglow parameter space for both cases.

We observe that most within-jet afterglows are detected by X-ray and optical instruments independent of other afterglow parameters. However, only radio afterglows are expected to be detected from sources where the observer line of sight is directed far off the jet edge. Even in such cases, afterglow parameters like jet energy and ambient medium density are critical for radio observations. The paper is organized as follows. In section-2 of the paper we give a description of the multi-dimensional afterglow parameter space, and explain the basic evolution of the afterglow spectrum and lightcurve. Section-3 discusses the simulated SGRB population and their association with GW-detectable BNS merger events. We present our results and findings in section-4 explaining how the afterglow parameter space results in different multi-band detection scenarios. We summarize our results in section-5.

2 GAMMA RAY BURST AFTERGLOWS

In this section, we discuss the basics of afterglow theory and introduce the multi-dimensional afterglow parameter space. GRB afterglow emission arises from the interaction of the jet with the medium surrounding the burst (Rees & Meszaros 1992; Paczynski & Rhoeds 1993). The ultra-relativistic shock generated in the ambient medium enhances the magnetic field in the shock downstream and accelerates particles to high energies. The non-thermal electron population accelerated by the shock radiates via the synchrotron process. This radiation is seen as the afterglow emission. The shock decelerates as it encounters more material, therefore the thermal energy density it deposits in the downstream decreases with time (Blandford & McKee 1976), giving rise to a time evolving afterglow light curve in frequencies ranging from γ-rays to radio (Meszaros & Rees 1993; Meszaros & Rees 1997; Sari 1997). The GRB afterglow emission peaks in high frequencies first, followed by lower frequencies. Typically, afterglows can be observed for several days in X-ray/optical to months and years in radio. See Piran (1999); Meszaros (2006); Gehrels et al. (2009); Kumar & Zhang (2014) for reviews.

In addition to the above described forward shock which is moving in to the external medium, a reverse shock travels back to the ejecta which can produce bright early afterglow emission especially in optical and radio bands (Meszaros & Rees 1993; Akergof et al. 1999; Sari & Piran 1999). We have not considered this component in the paper.

2.1 Afterglow parameter space

There are six physical parameters intrinsic to the emitting plasma that decide the afterglow spectral evolution with time. These are: the isotropic equivalent kinetic energy $E_{iso}$ carried by the jet, initial half opening angle of the jet $\theta_j$, number density $n$ of the circumburst medium (assumed to be homogeneous), fraction $\epsilon_e$ and $\epsilon_B$ of the shock thermal energy in downstream magnetic field and non-thermal electrons respectively, and power-law index $p$ of energy-spectrum of the non-thermal electrons radiating synchrotron emission. We are not considering dust and gas absorption due to the intervening medium.

Apart from these six parameters, there are two parameters external to the emitting region, distance $D_L$ and the angle $\theta$ between the observer’s line of sight and jet axis.
Unlike in gravitational wave studies, $D_l$ is a fixed parameter as it is known sufficiently precisely (albeit with an underlying cosmological model) through redshift from optical spectroscopy in most cases. Therefore, the final afterglow parameter space is a 7-dimensional one.

In the next section, we describe how these parameters enter in the expression of the afterglow flux evolution $f_\nu(t)$ measured at the observed frequency $\nu$ and at an observer time $t$ measured from the GRB trigger time.

### 2.2 Afterglow spectral evolution and lightcurves

Synchrotron spectrum of a single electron peaks at a characteristic frequency ($\nu_{\text{iso}}$) governed by the average magnetic field ($B$) of the plasma, and the lorentz factor ($\gamma$) of the electron. Afterglow synchrotron spectrum at a given epoch from a collection of electrons in the power-law distribution is obtained by a convolution of the single electron spectrum and the electron distribution (Rybicki & Lightman 1979). It can be approximated as a combination of piecewise power-law segments separated by three break-frequencies (Sari et al. 1998; Wijers & Galama 1999); (see Fig. 1). These break-frequencies are: (i) $\nu_m$, the frequency above which synchrotron radiative losses are severe; (ii) $\nu_s$, the frequency below which the fireball is optically thick; and (iii) $\nu_e$, the characteristic synchrotron frequency of the lowest energy electrons (Sari et al. 1996; Waxman 2000; Resmi et al. 2005). The parameters used are $E_{\text{iso}}$, $n$, $\epsilon_E$, $\epsilon_B$, $\theta_0$, and $\rho$.

As the slope of all the spectral segments will be uniquely determined by the power-law index $p$, the four spectral parameters described above and $p$ together determine the flux $f_\nu(t)$. These four spectral parameters are functions of the physical parameters $E_{\text{iso}}, n, \epsilon_E, \epsilon_B$. Therefore, in general the five physical parameters ($E_{\text{iso}}, n, \epsilon_E, \epsilon_B, p$) uniquely decide $f_\nu$ at a given $t$. However, in different synchrotron spectral regimes, the dependency will be different. For example, for a fixed index of $p=2$, in $\nu_m < \nu < \nu_s$, $f_\nu \propto E_{\text{iso}}^{1/2} \epsilon_E \epsilon_B n^{-1/2}$. If the observed frequency is above $\nu_s$, flux is independent of $n$ and $f_\nu \propto E_{\text{iso}} \epsilon_E \epsilon_B$. The dependencies are more complex in the optically thick regime. See (Wijers & Galama 1999) for details. In the example, shown in figure-1, the spectrum is calculated at $t = 0.1$ day since burst (red). The parameters used are $E_{\text{iso}} = 10^{51}$ erg, $n = 1.0$ atom/cc, $\theta_0 = 5^\circ$, $\epsilon_E = 0.1$, $\epsilon_B = 0.01$, and $p = 2.5$. The observer is on the axis of the jet at 300 Mpc away. Locations of the break-frequencies $\nu_e$, $\nu_s$, and $\nu_m$ are marked. The X-ray band is above $\nu_e$, optical frequencies are between $\nu_s$ and $\nu_e$, and X-ray frequencies are below $\nu_s$ (optically thick). The blue curve shows the evolution of the spectrum at a later epoch ($t = 2$ day). The order of these frequencies can change depending on the physical parameters. For example, higher number densities can result $\nu_e > \nu_s$ and for high magnetic field values, i.e., for larger values of $\epsilon_B$, $\nu_s$ can be below $\nu_m$.

**Figure 1.** Afterglow spectrum for $t = 0.1$ day since burst (red). The parameters used are $E_{\text{iso}} = 10^{51}$ erg, $n = 1.0$ atom/cc, $\theta_0 = 5^\circ$, $\epsilon_E = 0.1$, $\epsilon_B = 0.01$, and $p = 2.5$. The observer is on the axis of the jet at 300 Mpc away. Locations of the break-frequencies $\nu_e$, $\nu_s$, and $\nu_m$ are marked. The X-ray band is above $\nu_e$, optical frequencies are between $\nu_s$ and $\nu_e$, and X-ray frequencies are below $\nu_s$ (optically thick). The blue curve shows the evolution of the spectrum at a later epoch ($t = 2$ day). The order of these frequencies can change depending on the physical parameters. For example, higher number densities can result $\nu_e > \nu_s$ and for high magnetic field values, i.e., for larger values of $\epsilon_B$, $\nu_s$ can be below $\nu_m$. With 6 physical parameters (i.e., $E_{\text{iso}}, n, \epsilon_E, \epsilon_B, p, \theta_0$), afterglow flux evolution $f_\nu(t)$ can be calculated for an observer along the axis of the jet (Panaitescu & Kumar 2001a,b; Resmi & Bhattacharya 2008).

#### 2.2.2 Lightcurves for on-axis observers

As the spectrum evolves with time, a given observed frequency moves across different spectral segments. For on-axis observers (i.e., for $\theta_0 = 0$), if the observed frequency is in the optically thin part of the synchrotron spectrum (i.e., above $\nu_s$), the light curve peaks when its frequency crosses $\nu_m$. For low frequencies like radio, which are likely to be below $\nu_s$ (i.e., for which the fireball is optically thick) the light curve peak delays till $\nu_s$ crosses the band (Panaitescu & Kumar 2000; Resmi et al. 2005).

#### 2.2.3 Lightcurves for off-axis observers

For off-axis observers, $\theta_1$, the viewing angle, enters the picture through relativistic effects. Due to the high lorentz factor of the jet, flux observed at line-of-sights which are off the jet-axis will be severely doppler de-boosted. The de-boost is relaxed at $t_0$, when the monotonically decreasing $\Gamma$ goes below $1/\theta_1$ (Moderski et al. 2000; Granot et al. 2002). Optically thin frequencies, like X-ray, will peak at $t_0$. In low radio frequencies, the peak arrives at a later epoch when the fireball becomes optically thin. See figure-2 for X-ray and radio lightcurves for on-axis and off-axis ($\theta_0 = 2\theta_1$) observers. The parameters used are same as that of figure-1, except for...
As mentioned earlier, it is believed that the BNS mergers discussed in detail in section 2. are characterised by a set of 7 populations of SGRB afterglow lightcurves where each one is associated with BNS merger events. For our study, we use the simulated radio bands detected as counterparts to the GW-detected afterglow parameters which we inferred distance can be reduced if the red-shift measured is consistent with the galaxy associated with the GW event or an optical counterpart (afterglow or kilonova). The distance estimates can in turn improve our knowledge of inclination of the galaxy. In realistic observations, the error on gravitational wave parameters can be constrained from the spectroscopy of either the galaxy associated with the GW event or an optical counterpart (afterglow or kilonova). The distance estimates can in turn improve our knowledge of inclination of the galaxy. In realistic observations, the error on gravitational wave parameters can be constrained from the spectroscopy of either the galaxy associated with the GW event or an optical counterpart (afterglow or kilonova). The distance estimates can in turn improve our knowledge of inclination of the galaxy. In realistic observations, the error on gravitational wave parameters can be constrained from the spectroscopy of either the galaxy associated with the GW event or an optical counterpart (afterglow or kilonova). The distance estimates can in turn improve our knowledge of inclination of the galaxy.
3.2 GW-detected BNS mergers

Here, we assume that the complete network of ground based advanced detectors is functioning. Thus, we consider a 5-detector network LHVKI which includes LIGO-Livingstone(L), LIGO-Hanford(H), Virgo(V) and the two upcoming detectors Kagra(K) and LIGO-India(I). For convenience, we assume that all the detectors have achieved similar sensitivity as that of LIGO’s designed sensitivity.

We simulate $3 \times 10^5$ non-spinning BNS sources with component masses $1.4 M_\odot$ each, uniformly distributed in comoving volume between 100-740Mpc. Inclination($\theta$, or $i$) of the population is distributed as uniform in cos $i$. For each source, we simulate GW signal using the analytical 3.5 order post-Newtonian TaylorF2 waveform (Blanchet 2006; Blanchet et al. 2004, 1995) and compute the network SNR(Signal-To-Noise Ratio) (Pai et al. 2001). We consider all sources with minimum network SNR of 8 as being detected. Applying this criterion, we obtain $\sim 5 \times 10^3$ sources with LHVKI up to a maximum distance $\sim 730$Mpc.

Figure 3 shows the distribution of distance($D_L$) and inclination ($i$) of the detected BNS sources. For the actual simulated population which is uniformly distributed in volume, the distribution of distance follows $P(D_L) \propto D_L^2$ such that the larger distance move the number of sources. In addition, due to the antenna pattern functions, the sensitivity of GW detector networks is not isotropic, rather varies for different directions. Hence the sources located at highly sensitive regions in sky can be detected even if they are deep in distance whereas only nearby sources can be detected from less sensitive regions in sky. On an average, this results in less number of detectable sources at larger distances as shown in the left panel of Figure 3. Similarly, in the simulated population, $i$ is distributed as uniform in cos $i$, (ie, $P(\cos i) \propto U(-1,1)$ which translates as $P(i) \propto \sin i$). However, as seen in the right panel of Figure 3, the inclination distribution of detected sources is biased towards face-on sources ($i \to 0$ or 180 degree). Thus, face-on sources are detectable to much larger distances than the edge-on sources ($i \to 90$ degree). We have drawn 50,000 BNS sources from the detections and associated them to the 50,000 SGRB sources in population-1&2 described below.

3.3 Short-GRB population choices

We have two populations namely population-1 and population-2, with each of them containing 50,000 SGRB afterglow events(sources). The two populations differ in the choice of priors on parameters $E_{iso}, n, \epsilon_B$. For both the populations, we consider $E_{iso}$ ranging in the typical limits between $10^{49}$ erg and $10^{52}$ erg, number density between $10^{-4}$ – $10^{-1}$ and energy fraction in the magnetic field $\epsilon_B$ ranges between $10^{-2}$ – $10^{-1}$. In population-1, we have drawn $E_{iso}, n$ and $\epsilon_B$ within the ranges described above as uniform in log $E_{iso}$, log $n$ and log $\epsilon_B$ respectively and in population-2, as uniform in $E_{iso}, n$ and $\epsilon_B$ respectively. The two types of priors are the two limiting distributions we consider here and expect that they would capture the essence of variations in distributions. Moreover, the log prior in energy also reflects the luminosity function of short GRBs, which is believed to be of a power-law nature (Guetta & Piran 2005). Log priors in $E_{iso}$ and $n$ ensure that there is a considerable number of bursts with lower energies and number densities. The jet half opening angle $\theta_j$ is uniformly distributed between 3 – 30 degrees for both the populations. We fix the upper limit to 30 degrees following the values of jet collimation angles from a numerical simulation of binary NS merger by Rezzolla et al. (2011). We fixed $\epsilon_k$ and $p$ at fiducial typical values of 0.1 and 2.5 respectively. While the flux is not highly sensitive to the value of $p$, radio afterglow observations indicate that $\epsilon_k$ is confined to a narrow range (Beniamini & van der Horst 2017).

The remaining two parameters $D_L$ and $\theta_j$ are the ones which are used to associate the afterglows and GW signal, as discussed in section 3.1. Since our population should be representative of EM follow-up observations of BNS merger detections, we draw these two parameters from a simulated distribution of GW-detectable BNS population as discussed in section 3.2. The complete set of parameters along with the priors are given in Table 3.2.

4 SIMULATION RESULTS

For all the SGRB sources in population-1 and 2, we carry out BoxFit simulations and compute the afterglow light...
Figure 4. Afterglow peak Flux distributions in X-ray, Optical and Radio bands for 50,000 sources detected in GW detector network LHVKI. The black vertical lines in each panel are the detection thresholds of XRT, LSST and JVLA respectively. X-ray and optical peak flux distributions are made up of two bell-curves. The smaller one at the right are for within-jet sources (i.e., $\theta < \theta_j$) and the larger one at the left are outside-jet sources ($\theta > \theta_j$). Radio has a symmetric peak-flux distribution due to reduction in doppler beaming at the time of typical radio peaks (see text for a detailed description). These figures are made for population-1. Please note that the legends in 3rd panel apply to all 3 panels.

Figure 5. Afterglow peak flux distributions in X-ray for populations with different ranges of $\theta_j$ where other parameters are distributed as in population-1. Given the $\theta_i$ distribution, for smaller ranges of $\theta_j$ (for eg, left panel where $\theta_j : 3 - 15^\circ$), less sources are within-jet and majority are outside-jet whereas for larger ranges of $\theta_j$ (for eg, right panel where $\theta_j : 3 - 90^\circ$) relatively more sources are within-jet. This reflects in the statistical properties of the two bell shaped curves in each panel.

Figure 6. Same as figure-4 but for population-2. Changes in the distribution of $E_{\text{iso}}$, $n$, and $\epsilon_B$ (though the ranges of these parameters are the same) has influenced the distribution of peak fluxes. See text for details.

curves in X-ray, optical and radio frequencies. Specifically, we compute lightcurves at frequencies $2.4 \times 10^{18}$, $4.5 \times 10^{14}$, and $15 \times 10^9$ Hz for X-ray, optical, and radio bands respectively.

We consider three representative instruments, the Swift-XRT in X-ray, the Large Synoptic Survey Telescope (LSST) in optical (R-band), and the Jansky Very Large Array (JVLA) in radio. The XRT sensitivity of $10^{-14}$ erg cm$^{-2}$ sec$^{-1}$ for 10$^4$ sec integration in 0.3–10 keV is converted to mJy by assuming a flat spectrum. The corresponding XRT threshold at $2.4 \times 10^{18}$ Hz comes out to be $4.37 \times 10^{-7}$ mJy. We adopt a single visit R-band sensitivity of 24.5 AB-magnitude for LSST. The corresponding detection threshold will be

\[ \text{https://swift.gsfc.nasa.gov/about_swift/xrt_desc.html} \]
5.75 \times 10^{-4} \text{ mJy}$. We considered 50\mu Jy to be the 3\sigma limiting flux required for radio detections at 15 GHz.

If at least one point in the simulated lightcurve is above the threshold, we consider it as a detection. Since the lightcurves are sampled logarithmically ($\delta t/t \sim 1$), this condition is sufficiently conservative. We use 5 hours since burst as the start time of the observations. This particularly ensures that the above mentioned detection criteria is consistent with the 10$^4$ sec integration time required for XRT.

In addition, this criteria also helps us stay clear of the early afterglow phase which could be influenced by delayed flares from the central engine. However, in the companion paper (Saleem et al. 2017), we demonstrate how cadence can affect the detection statistics.

Below, we summarize our results and findings of the synthetic joint observations of GW from BNS and the associated afterglow detection in the EM window. We discuss various plausible joint observation scenarios for the same.

### 4.1 Peak flux distribution

As discussed above, if at least one point in the simulated lightcurve is above the threshold, we call it a detection. In other words, if the peak flux of a given lightcurve is below the threshold, it will not get detected. Therefore, the distribution of peak flux is a proxy to understand the influence of the GRB parameter space on detection.

We compute the peak flux for the 50,000 simulated SGRB lightcurves (counterparts to GW-detected BNS mergers). Defining $\theta_j \equiv \theta_0/\theta_j$, first of all we divide the afterglows in two cases: 1. *within-jet* ($\theta_j < 1$) cases where the observer’s line of sight aligned within the jet cone and 2. *outside-jet* ($\theta_j > 1$) cases where the line of sight directs somewhere off the jet cone. We investigate the lightcurve behaviour for these two cases separately. Figure-4 shows the peak flux distributions in X-ray, optical and radio bands from population-1. The black vertical lines in each panel are the detection thresholds considered for XRT, LSST and JVLA.

#### 4.1.1 Characteristic features

The peak flux distribution shown in Figure-4 is a combination of two bell-shaped curves, corresponding to the within-jet and outside-jet cases together. The area under the blue(green) curve represents the total number of within(outside)-jet cases and the red curve shows the entire sample. We see clearly that the total number of within-jet cases is smaller than that of the outside-jet cases. This is a reflection of our original simulated afterglow sample (only 15% are within-jet cases) and is not a consequence of any detection criterion. Typically, *within-jet* cases have higher flux as the doppler boost enhances their flux as opposed to the *outside-jet* cases which are heavily de-boosted.

The *within-jet* and *outside-jet* cases appear prominently distinct in the combined histogram (red) of the X-ray and the optical afterglow observations while the radio show a smooth resultant distribution. This is because the radio peaks are delayed compared to higher frequencies and the doppler de-boost is well relaxed by their peak time (see section-2.2.3).

#### 4.1.2 Radio vs. higher frequencies

As explained in section-2.2.1, the peak time $t_{\text{peak}}$ (time at which the light curve peaks) for an on-axis ($\theta_0 = 0$) burst is decided solely by the physical parameters $E_{\text{iso}}, n, \epsilon_p, \epsilon_e$. For 15 GHz radio band, this ranges from a few hours to a few days while X-ray and optical lightcurve peaks are already in decline for the $t_{\text{start}}$ we use (5 hrs since burst). See the example figure-2. As mentioned in section-2.2.3, for outside-jet cases, the peak is delayed till $t_b$. As can be seen from figure-2, for a typical radio lightcurve, $t_{\text{peak}}(\theta_0 = 0) > t_{\text{start}}$ and in both $t_{\text{peak}}(\theta_0 = 0)$ and $t_b$ are roughly of the same order.

Therefore, the doppler de-boost is not as severe in radio as it is in the higher frequencies. Moreover, the peak flux of within-jet cases and outside-jet cases therefore are not very different from each other in radio, leading to smooth peak flux distribution we observe in the figures.

It has to be noted that for the detection thresholds we used, in within-jet afterglows, the detection fraction in X-ray and optical are higher than that in radio (area of the blue curve right of the black vertical line). This is because of two factors: (i) the X-ray and optical thresholds are deeper than radio; and (ii) the flux in both these bands are enhanced by doppler boost at their peak as opposed to radio which peaks later when the Lorentz factor and boost are relatively of lesser magnitude.

#### 4.1.3 Factors affecting the peak flux distribution

We note that the peak flux distribution follows the bell curve for within jet and outside jet cases. Here, we investigate the factors affecting the properties of the bell curve. Each bell curve can be broadly approximated as a Gaussian. Thus, we can define a mean value ($\langle x \rangle$), width($\sigma$), and height ($y$) of the curve.

Obviously, the height of the curves are strongly sensitive to the value of $\theta_j$ for a given distribution of $\theta_0$. Currently we have only 15% of within-jet cases and this is because of the fact that we have let $\theta_j$ vary upto 30°. If $\theta_j$ is limited to a much lesser value around 10 – 15°, typically considered in the literature, we will have even lesser number of within-jet cases leading to a reduction in $y$ of the bell-curve of the right side. Instead, if we consider $\theta_j$ upto larger angles such as 60° or 90°, fraction of within-jet cases as well as the height of the right bell-curve will increase. This is an obvious effect, however is illustrated in Figure-5 by taking X-ray afterglows as an example.

In addition to the viewing angle effects described above, peak flux distribution is affected by the other afterglow parameters as well. Ranges of the parameters as well as their prior distributions influence the peak flux distribution. As an example, peak flux is proportional to $E_{\text{iso}}$. A higher range of $E_{\text{iso}}$ will lead to an increase in the $x$ and $y$ of the blue Gaussian provided the $\theta_j$ and $\theta_0$ distributions remain the same. In addition to that, if the nature of prior distribution is changed to $P(E_{\text{iso}}) \propto U$ instead of $P(\log(E_{\text{iso}})) \propto U$, fraction of sources with higher energy will increase leading to an increase in $x$ as well as $y$. The change in distribution obviously

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$f_r = 10^{\frac{-5 \times 10^{-4}}{U}} \text{ mJy}$

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will also affect the width of the Gaussians. This feature can be observed in the peak flux distribution of population-2 (see figure-6) which has a large fraction of high energy sources as compared to population-1.

Next, we move on to analyzing the afterglow physical parameter space that favours detections in various bands.

### 4.2 Detections of X-ray, optical and radio afterglows and favourable afterglow parameter space

If SGRB observations so far have to be considered as a reference, the detection probabilities are not the same in all bands. Our aim is to understand the role of the afterglow physical parameters in detection (or non-detection) in the three different bands. For this, we use population-1 because results from population-1 appear more in agreement with short GRB afterglow observations than population-2.

First we consider all afterglows detected in each band and analyse the parameter distributions favouring each of them as shown in the corner plots in Figure. 7. For clarity, we are focusing only on the effect of $E_{\text{iso}}, n, \theta_j$, and $\theta_v$ and not displaying $\epsilon_B$ which was varied in a narrow range of $0.01-0.1$. However, it has to be noted that for several afterglows $\epsilon_B$ is found to be of a lower value (Gao et al. 2009). A smaller $\epsilon_B$ corresponds to a lower magnetic field of the emission region thereby the flux in all bands will be reduced. The parameters $\epsilon_E$ and $p$ were kept fixed.

Let us first focus on the diagonal entries from all 3 corner plots of Figure. 7 which are the 1-D histograms of the detected afterglows. It will be useful to remember that the corresponding prior distributions of $E_{\text{iso}}, n$, and $\theta_v$ will be a horizontal line for population-1 for which these figures are made. See Figure-3 for the prior distribution of $\theta_v$. As expected, low energy afterglows fail to cross the detection threshold (upper panel in left most column). Ambient density is an important factor for radio afterglows while is not very significant for X-rays. This is because, in the synchrotron spectral regime $\nu > \nu_c$ which most X-ray afterglows are likely to occupy, $f_\nu$ is insensitive to $n$. The higher probability of larger $\theta_j$ values is again a selection effect in our population, i.e., for a given $\theta_v$ distribution, sources with larger values of $\theta_j$ are more likely to be within-jet and hence are more likely to be detected compared to sources with smaller values of $\theta_j$. Distribution

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**Figure 7.** Corner plots showing the distribution of detected sources in X-ray (top left), optical (top right) and radio (bottom) for population-1. Each corner plot contains multiple panels showing the 2D histograms of different sub-spaces of afterglow parameters $E_{\text{iso}}, n, \theta_j, \theta_v$ with the panels along the diagonal showing their histograms. In the 2D histograms, the dark region indicates higher probability. Though we have considered a range of $0.01 < \epsilon_B < 0.1$, we have not included $\epsilon_B$ in the plots.
of \( \theta \), extends to relatively larger values for radio detections as expected, because the radio peaks are delayed and doppler de-boosting is alleviated even for extreme off-axis cases by the time of their peak. (see description in section-4.1.2).

Next we explore different combinations of multi-band afterglow detection.

### 4.3 Parameter space constrained for different detection scenarios

Constraints on the afterglow parameter space can be drawn from the detections and non-detections in various bands. Here we ignore the role of cadence and field of view for non-detection in a certain band and explore the possibility that the non-detection is a consequence of afterglow parameter space and the sensitivity of instruments. For example, X-ray afterglows are detected while optical and radio afterglows are not detected for several short GRBs (Fong et al. 2015).

The possible combinations are:

1. **X**: Detection in X-ray with non-detections in optical and radio
2. **O**: Detection in optical with non-detections in X-ray and radio
3. **R**: Detection in radio with non-detections in optical and X-ray
4. **XO**: Detections in both X-ray and optical with non-detection in radio
5. **XR**: Detections in both X-ray and radio with non-detection in optical
6. **OR**: Detections in both optical and radio with non-detection in X-ray
7. **XOR**: Detections in all 3 bands - X-ray, optical and radio.

We explore all combinations of observational scenarios but are presenting corner plots for a selected list which can provide insightful constraints on the afterglow parameter space.

Further, much more stringent constraints can be arrived by using the detected flux value, which we plan to explore in future.
4.3.1 Detection in a single band alone

The top two corner plots of figure-8 shows the constrained regions in afterglow parameter space which favour detection in a single band alone while non-detection in the remaining two bands.

The top left corner plot shows the X scenario where it is seen that $\theta_v < 1$ for majority of the sources while a small fraction of outside-jet sources are also captured in this scenario (see figure-4.2). In addition to that such a scenario arises for low energy bursts mainly, since high $E_{\text{iso}}$ leads to both radio and optical detections. This can be seen by comparing the very first panels (distributions of $E_{\text{iso}}$) of all 3 corner plots of Figure-7 where we see that for low energy bursts, X-ray detections are more likely than optical and radio. No constraint can be arrived on the number density because X-ray is not very sensitive to $n$ (see discussion in section 4.2).

Detection in optical alone (O scenario) is nearly difficult for the instrument thresholds we considered, i.e., the physical parameters which ensure an optical detection by LSST always ensure an X-ray detection by XRT also.

A case of detection in radio band alone (R scenario) happens only if a high energy burst occurs in a high density ambient medium but viewed extremely off the jet cone ($\theta_v > 1$) (see figure-8 (top right)). Because, as we have seen in figure-7, high energy and high medium density are essential for radio detections. Further, being well off the jet cone ($\theta_v > 1$) favours non-detection in X-ray/opt.

4.3.2 Detections in multiple bands

Here, we try to identify parameter space regions which favour detections in more than one bands.

For a handful of observed short GRBs, afterglows are detected in X-ray and optical while not detected in radio (Fong et al. 2015). With our simulated population, we see that such a scenario (XO scenario) is most favoured when $E_{\text{iso}}$ is around $10^{49.5}$ and $n < 10^{-2}$ as shown in the bottom left panel of figure-8. The reason is that a non-detection in radio indicates that the energy and number density are lower while optical detection requires the energy to be not too low. Further, to fulfill this criteria, all these sources definitely should have $\theta_v < 1$ which is essential to ensure detection in optical band as seen in figure-7.

An interesting scenario is when a non-detection happens in optical band alone. For this, the observer should be nearly aligned along the edge of the jet, i.e., $\theta_v < 1$ (see lower right corner plot of figure-8). This ensures that both X-ray and radio are detected and optical is not detected. Or in other words, if $\theta_v < 1$, optical is likely to be detected making it XOR and if $\theta_v > 1$, the X-ray flux is likely to be below XRT threshold making it an R alone case.

The most unlikely scenario is to not to detect X-ray alone (OR). Only a handful of sources in our population have satisfied this criteria. Therefore, it is impossible to obtain any meaningful constrains on the parameters from our current simulations for this condition. As discussed in section 4.3.1, this is due to the sensitivities of X-ray and optical instruments we considered. It requires a much more sensitive optical instrument than the one we considered now to enable this to be a likely scenario.

However, for a large number of within-jet cases, we get detection in all wavelengths. This is contrary to the existing short GRB observations where the radio detections are very poor. A major difference is the lower distances that our simulation is considering for GW-detected NS mergers, allowing radio fluxes to be within the VLA threshold.

4.4 Summary of the corner plots

The corner plots contain information of the multi-dimensional afterglow parameter space which was concealed in the peak flux histograms Figure-4. Corner plots of XO

Figure 9. Distributions of ratio $\theta_v/\theta_j$ in various scenarios. The distributions show the probability for a given observer to encounter a certain scenario. For example, XO (i.e, radio alone is not detected) detection indicates that the observer is very likely to be within the jet cone. If GW observations give an estimates of $\theta_v$, a prediction on $\theta_j$ can be made using this.
and XOR scenarios are populated by within-jet cases. This can be seen also in the histograms where majority of within-jet cases are above detection limit in both X-ray and optical bands (see the blue bell-curve in Figure-4). Therefore, for within-jet cases, for the standard ranges of afterglow physical parameters afterglow detection by XRT and LSST (or any such deep reaching optical telescopes) is ensured. Viewing from off the jet cone ($\theta_j > 1$) is required to have either X-ray or optical flux to be below the corresponding sensitivity limits we have considered. However, it is difficult to have the X-ray lightcurve below the XRT threshold, if physical parameters are ensuring an optical detection even for outside-jet cases. Hence we do not have any optical-alone cases in our simulations. For outside-jet, in most cases, radio afterglow will get detected. The physical parameter space is complimentary between the R-alone and XO cases in Figure-8. While XO is frequent in short GRB observations, R-alone has to wait for an NS merger triggered GRB where the jet is viewed far off from its axis.

As a summary focusing on effects due to the viewing angle, we are presenting the different scenarios as a function of $\theta_j$ in Figure-4.2.

5 CONCLUSIONS AND FUTURE DIRECTIONS

In this work, we have explored the afterglow parameter space for coincident observations of gravitational waves and SGRB afterglows generated from BNS mergers. The detection of $\gamma$-ray emission from GRB170817A in association with gravitational waves from GW170817 along with several other EM counterparts in longer wavelengths has firmly established the association between BNS mergers and SGRBs. For BNS mergers observed in GW window, afterglows are potential EM counterparts to be followed up. In our study, we have explored the detectability and detection scenarios of afterglows in various bands irrespective of the detection of SGRB in prompt $\gamma$-ray emission.

We simulate 50,000 GRB afterglow lightcurves, assuming them to be the Electromagnetic (EM) counterparts of Neutron Star mergers. We use the numerical hydrodynamic code BozFit to systematically explore the multi-dimensional afterglow parameter space. Using flux limits of three instruments, Swift-XRT, LSST, and JVLA, operating in three different bands of the EM spectrum, we explore how the afterglow parameter space results in different observational scenarios. We use the distribution of the peak flux in a given afterglow lightcurve to understand the rate of detections in that particular band, which is explored further in a companion paper (Saleem et al. 2017). Our study focuses only on the standard forward shock driven by the GRB jet. EM counterparts from other components like reverse shock, central engine powered forward shock, merger ejecta etc. are not considered.

We divide the afterglow population based on the ratio ($\theta_j = \theta/\theta_j$) of the observer’s viewing angle to the jet opening angle. Within-jet sources are the ones where the observer’s line of sight is within the jet cone ($\theta_j < 1$) and for outside-jet sources, the line of sight is beyond the edge of the jet ($\theta_j > 1$). We find that the detection scenarios are sensitive to the ranges and distributions of the physical parameters.

We notice that most within-jet sources are detected by XRT and LSST (or similar deep imaging optical instruments). A non-detection in radio for within-jet sources implies relatively low jet energy and ambient medium density (roughly, $E_{\text{iso}} < 10^{51}$ ergs and $n < 0.01$ atom/cc). X-ray and optical afterglows are not likely to be detected if $\theta_j >> 1$. However, if the jet energy and the ambient density are high enough radio afterglow alone could be detected.

In arriving at these conclusions, we have ignored the effects of field of view and cadence. In addition, the constraints on the physical parameter space are sensitive to the instruments used and detection thresholds considered.

Here we have only considered a detection or a non-detection, but not used the detected flux value. More detailed multi-messenger astronomy can be attempted by including flux measurements.

Please note that the recent joint BNS merger and associated SGRB event (GW170817 and GRB170817A) was observed at a distance around 40 Mpc while the studies in this paper have considered BNS sources uniformly distributed in comoving volume above 100 Mpc. Our choice of 100 Mpc as the lower distance limit was well consistent with the existing SGRB observations until the discovery of GRB170817A. However, in the context of detection of GW170817+GRB170817A, given the chances of detecting nearby joint events, we have ensured that the results and interpretations of our studies in this paper are not sensitive to this choice. This primarily is because the binary sources are uniformly distributed in comoving volume, among the GW detections by LHVIK, only ~1.4% sources come from within the 100 Mpc sphere. We have explicitly tested how the EM detectability (detection fractions of X-ray, optical and radio afterglows) changes if we use a lower distance cutoff at 20 Mpc instead of 100 Mpc. In all the bands, the detection fractions changes by only less than 1% and hence our interpretations about the detection scenarios and the associated constraints on afterglow parameter space will remain unaffected.

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