Clustering in stable and unstable nuclei in $p$-shell and $sd$-shell regions

Y. Kanada-En’yo  
*Institute of Particle and Nuclear Studies,  
High Energy Accelerator Research Organization,  
Oho 1-1, Tsukuba-shi 305-0801, Japan*

M. Kimura  
*Institute of Physical and Chemical Research (RIKEN), Saitama 351-0198, Japan*

H. Horiuchi  
*Department of Physics, Kyoto University,  
Kitashirakawa-Oiwake, Sakyo-ku, Kyoto 606-01, Japan*

According to microscopic calculations with antisymmetrized molecular dynamics, we studied cluster features in stable and unstable nuclei. A variety of structure was found in stable and unstable nuclei in the $p$-shell and $sd$-shell regions. The structure of excited states of $^{12}$Be was investigated, while in $sd$-shell nuclei we focused on molecular states and deformed states. The deformed states in $^{28}$Si and $^{40}$Ca were discussed in connection with the high-lying molecular states. Appealing molecular states in $^{36}$Ar and $^{24}$Mg were suggested. The results signified that both clustering of nucleons and mean-field formation are essential features in $sd$-shell nuclei as well as $p$-shell nuclei.

I. INTRODUCTION

Clustering is one of the essential features in nuclear dynamics, as already seen in light stable nuclei, where various cluster structure has been known even in the low-energy region. Also in the physics of unstable nuclei, the cluster feature is one of the center subjects. In fact, the recent experimental and theoretical studies of neutron-rich Be isotopes [1,2,3,4,5,6,7,8,9,10,11,12,13,14] revealed that cluster states exist also in light unstable nuclei. On the other hand, needless to say, the mean-field nature is the other essential aspect, which become more important in heavier nuclei. These facts imply that a variety of structure appears in stable and unstable nuclei. In the $p$-shell region, both the cluster and mean-field aspects coexist in a nucleus such as $^{12}$C [15]. However, it is an open question whether or not the cluster features survive and are still essential in heavier nuclei. Concerning cluster aspect in stable $sd$-shell nuclei, remarkable phenomena are the molecular resonances, which have been observed in highly excited states of such nuclei as $^{24}$Mg, $^{28}$Si and $^{32}$S [16,17]. When we ascertain the possible coexistence of cluster and mean-field natures in $sd$-shell or heavier nuclei, one of the key issues is to connect the molecular resonances with the low-lying deformed states.

Our aim is to make a systematic study covering such a variety of structure in the wide mass number region, while focusing on the cluster features. The important thing in theoretical models is describing both the cluster structure and mean-field-type structure. We apply microscopic methods based on antisymmetrized molecular dynamics(AMD) [18,19], which meet the requirement because of the flexibility of the AMD wave functions. In this paper, the excited states of $^{12}$Be, the shape coexistence and molecular resonances in $^{28}$Si and $^{40}$Ca are studied. In $^{36}$Ar and $^{24}$Mg, possible molecular states in highly excited states are suggested.

II. FORMULATION

The formulation of AMD for nuclear structure study of the ground and excited states is explained in Refs. [18,19]. The wave function of a nucleus is written by AMD wave functions, $\Phi = c\Phi_{AMD} + c'\Phi'_{AMD} + \cdots$. An AMD wave function is a Slater determinant of Gaussian wave packets:

$$\Phi_{AMD}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, \cdots, \varphi_A\},$$

$$\varphi_i = \phi_{X_i}\chi_{\xi_i}\tau_i, \text{where } \phi_{X_i}(r_j) \propto \exp\left[-\nu r_j - \frac{X_i}{\sqrt{\nu}}\right]^2 \text{ and } \chi_{\xi_i} = \left(\begin{array}{c} \frac{\nu}{2} + \xi_i \\ \frac{\nu}{2} - \xi_i \end{array}\right).$$

$\chi_{\xi_i}$ is the intrinsic spin function and $\tau_i$ is the iso-spin function. An AMD wave function is given by a set of complex variational parameters $Z = \{X_1, X_2, \cdots, X_A, \xi_1, \cdots, \xi_A\}$, where $X_i$ is the center of Gaussian and $\xi_i$ defines the
orientation of the intrinsic spin for the \(i\)-th single particle wave function. The model space of AMD wave function covers various cluster structure and also shell-model-like states. We perform energy variation by the frictional cooling method with respect to the complex variational parameters to obtain an energy minimum state in the model space. For the excited states of \(^{12}\text{Be}\) isotopes, we performed energy variation after spin-parity projection (VAP). In case of \(sd\)-shell nuclei, we applied a generator coordinate method in the AMD framework (AMD\texttt{+GCM}). Harmonic-oscillator quanta of the system, \(\langle \sum_{i=1}^{A} p^2_i/2m\hbar \omega + m\omega r^2_i/2\hbar \rangle\), is adopted as the generator coordinate in the present paper.

### III. EXCITED STATES OF \(^{12}\text{BE}\)

\(^{12}\text{Be}\) is an interesting nucleus where the vanishing of magic number \(N = 8\) occurs. Recently, many excited states of \(^{12}\text{Be}\) have been experimentally measured \([1,2,3,20]\). We applied the VAP method to the excited states of Be isotopes by using effective interactions, MV1 force (case 1, \(m = 0.65\)) + G3RS-type LS force (\(u_1 = -u_2 = 3700\text{ MeV}\)) + Coulomb. In Fig.1, the positive-parity rotational bands are displayed in comparison with the experimental data of the spin-assigned positive-parity states. The highly excited states measured by He-He breakup reactions \([2]\) are candidates of molecular-like states. Theoretical results obtained by the VAP calculation well fit to the experimental data. We found three rotational bands \(K^\pi = 0^+_1, 0^+_2\) and \(0^+_3\) in the positive-parity states. The ground band consists of the intruder states (2\(\hbar\omega\) excited configurations), which are well-deformed states with 2\(\alpha\) core in the surrounding neutrons. On the other hand, the normal neutron-shell-closed states belong to the second band. It means that the vanishing of neutron magic number \(N = 8\) occurs in \(^{12}\text{Be}\). As a result of the inversion between the cluster states and the normal p-shell closed states, the calculation consistently agrees with the data of the \(\beta\) decay strength from the ground state into \(^{12}\text{B}\) \([12]\). The other interesting point in the results is the prediction of the \(^6\text{He}+^6\text{He}\) molecule-like structure in the third \(0^+_3\) band. The experimentally measured \(4^+\) and \(6^+\) states are the candidates of these \(^6\text{He}+^6\text{He}\) states in the \(K^\pi = 0^+_3\) band. Thus we found a variety of structures in the excited states of \(^{12}\text{Be}\). We make a point of the coexistence of cluster and mean-field natures in this light neutron-rich nucleus. In the ground states, the valence neutrons move around the 2\(\alpha\) core. In other words, they show the deformed-mean-field behaviour, which causes an enhancement of the 2\(\alpha\) clustering. On the other hand, the shell-model-like states belong to the \(K^\pi = 0^+_2\) band, while the developed \(^6\text{He}+^6\text{He}\) molecular structure was found in the \(K^\pi = 0^+_3\) band.
IV. SD-SHELL NUCLEI

In the study of cluster features in sd-shell nuclei, the subject of molecular resonances is one of the key problems. Molecular resonances have been experimentally measured in such Z = N nuclei as $^{24}$Mg, $^{28}$Si and $^{32}$S. It is a long standing issue what is the connection between the molecular resonances in the high energy region and the low-lying deformed states. In present paper, we applied AMD+GCM method for the study of sd-shell nuclei.

![Graphical representation of rotational bands of $^{28}$Si.](image)

**FIG. 2.** Rotational bands of $^{28}$Si. (a) Calculated positive-parity bands dominated by O+C cluster components (filled circles) and the ground band (open circles). Experimental data of the prolate band (the O-C molecular resonances) are demonstrated by symbols + (∗). (c) Calculations of the ground band and the positive-parity α-cluster band (circles). Cross points are the experimental ground band. Symbols (∗) denote the experimental data observed in α-transfer reactions. (d) Low-lying negative-parity $K^\pi = 3^-$ and $K^\pi = 5^-$ bands. (b) The O-C inter-cluster wave function in the band-head states of three rotational bands displayed in figure (a).

The first subject is $^{28}$Si, where we have already known various experimental suggestions on the shape coexistence and the molecular resonances. In the low-energy region, the oblate ground band and a prolate excited band are known, while exotic shapes in the negative parity states are expected from the $\gamma$-ray transition strengths [22]. In the highly excited states, $^{16}$O+$^{12}$C molecular resonances were experimentally observed [16]. Moreover, α-cluster states are suggested [21]. There were many theoretical efforts by mean-field approaches and cluster approaches [23,24,25], but one did not yet catch a total understanding of overall these phenomena. We studied the structure of the ground, excited states of $^{28}$Si and investigated the molecular resonant behaviour with AMD+GCM method (fixed intrinsic spins version). First, we obtain energy curves as a function of the generator coordinate (GC) $\langle \sum_{i=1}^{A} p_i^2/2m\hbar + m\hbar^2 r_i^2/2\hbar \rangle$, harmonic oscillator quanta. In case of $^{28}$Si, two local minimum states are found in each value of GC. The lower one corresponds to the oblate deformed state and the higher one is the prolate deformation. In the small GC region, these states compose the oblate ground band and the prolate excited band. With the increase of the GC value, the oblate deformation varies into α-cluster($\alpha+^{24}$Mg) states, while from the prolate deformation the $^{16}$O+$^{12}$C clustering enlarges. After the superposition of the obtained basis, O-C molecular states and α-cluster states appear from the prolate band and the oblate ground band, respectively. In the final results, we chose 23 AMD wave functions as base states in the diagonalization. By analysing the amplitudes of the base states in the final states, we can classify many of the excited states into two groups. The first group principally consists of the prolate states, while the second group is a family based on the oblate states.
In the positive-parity states of the former group (prolate deformed states), we found three \( K^\pi = 0^+ \) bands dominated by \( O+C \) cluster states. We demonstrate in Fig. 3(a) the excitation energy of the three \( O+C \) cluster bands and the oblate ground band. The lowest band of the three is consistent with the normal prolate deformed band in the experimental data. In order to discuss the molecular resonant behaviour of the higher bands, we give in Fig. 3(b) the extracted inter-cluster wave functions between the \( O_{g.s.} \) and \( C_{g.s.} (SU_3 \) limit g.s. \) clusters. It is found that the lowest band has the minimum node number \( n_0 = 8 \) in Pauli allowed states, while the second and third bands are the higher nodal \( O-C \) molecular states with larger node numbers of the inter-cluster wave functions as \( n_0 = 9 \) and \( n_0 = 10 \), respectively. In the energy region near the third bands, there exist many experimentally observed \( O-C \) molecular resonances. It is interesting that the low-lying prolately band of \( ^{28}\text{Si} \) is regarded as the lowest \( O+C \) cluster state and the higher nodal molecular bands appear due to the excitation of the inter-cluster motion. In fact, the molecular resonances in \( ^{28}\text{Si} \) were often discussed in the connection with the low-lying prolately deformation in the studies with two-body cluster models \[28,29\]. Here we remark the importance of the present work that we did confirm the appearance of \( O-C \) molecular states without assuming the existence of any clusters.

In the family of the oblate states, we obtain an excited band consisting of \( \alpha \)-cluster states in addition to the oblate ground band (Fig. 3(c)). The \( \alpha \)-cluster band is about 10 MeV higher than the ground band in the present results. There are experimental levels observed in \( \alpha \)-transfer reactions \[21\] in the corresponding energy region. In the negative parity states, the calculated results indicate that exotic shapes compose \( K^\pi = 3^- \) and \( K^\pi = 5^- \) bands (Fig. 3(d)). For example, \( K^\pi = 5^- \) band is described by a pentagon shape given by \( 7\alpha \) structure. The results agree to the experimental data of the energy levels and the in-band \( E2 \) transition strengths of the \( K^\pi = 3^- \) and \( K^\pi = 5^- \) bands.

The second subject is \( ^{40}\text{Ca} \), where, the rotational band of the superdeformation has been recently observed in the experimental data of \( \gamma \)-ray measurements \[24\]. It is very surprising that many-particle many-hole states lie in such a low-energy region as the excitation energy of the band-head state of this superdeformation is only 5.2 MeV. The low-lying excited states of \( ^{40}\text{Ca} \) were studied by the AMD+GCM method in Ref. \[27\], where many low-lying rotational bands were found in the results. By analysing the single-particle orbits of the positive-parity states, it was found that the ground band, the normal deformation and the superdeformation are dominated by \( 0p-0h, 4p-4h \) and \( 8p-8h \) configurations, respectively. It is consistent with the analysis based on shell-model calculations \[21\]. What is exciting in our results is that superdeformation has a parity-asymmetric shape which is composed by the \( ^{28}\text{Si}+^{12}\text{C} \)-like cluster core. It is unique that, in the superdeformation, the parity-asymmetric shape like a pear arises keeping the \( 8p-8h \) configurations dominant. With respect to the parity-asymmetric shape, we predicted the parity-doublet states in the negative parity states, which contain the mixing of \( 7h-7p \) and \( 5h-5p \) configurations \[27\]. Moreover, it is naturally expected that the \( ^{28}\text{Si}+^{12}\text{C} \) molecular states may arise from the superdeformation. From this point of view, we studied the highly excited states of \( ^{40}\text{Ca} \) in the present paper. As we increase the generator coordinate (harmonic oscillator of the system) from the superdeformed state, the \( ^{28}\text{Si}+^{12}\text{C} \) clustering enhances. After the diagonalization, we found the higher rotational bands with the remarkable \( ^{28}\text{Si}+^{12}\text{C} \) molecular states. Namely, the \( ^{28}\text{Si}+^{12}\text{C} \) molecular states appear due to the excitation of inter-cluster motion based on the superdeformation. In the other words, the low-lying superdeformation is regarded as the lower member of the \( ^{28}\text{Si}+^{12}\text{C} \) molecular states. This is a similar situation to the \( ^{28}\text{Si} \) system, where the low-lying deformed states are connected with the \( O-C \) molecular resonances. In the present work, the energy intervals between the superdeformed and the second bands, the second and the third bands are predicted to be about 10 MeV. It is concluded that we suggested the possible \( ^{28}\text{Si}+^{12}\text{C} \) molecular resonances and their connection with the low-lying superdeformation in \( ^{40}\text{Ca} \).

As mentioned above, the molecular states in the highly excited states in \( ^{28}\text{Si} \) and \( ^{40}\text{Ca} \) were discussed in the connection with the low-lying deformed states. Here, we briefly report the possibility of molecular states in other \( sd \)-shell nuclei. With the same framework of AMD+GCM, we study the excited states of \( ^{36}\text{Ar} \) and \( ^{28}\text{Mg} \). In the highly excited states of \( ^{36}\text{Ar} \), we predict an eccentric cluster states with the 3-carbon-like linear structure. To our surprise, the main component of the \( 3\)-,12C linear-chain state is a \( 8p-12h \) configuration. It means that the linear-chain state is explained also by the many-particle many-hole state in the picture of single-particle orbits. It is similar to the case of the pear-like shape of the superdeformation of \( ^{40}\text{Ca} \), which is dominated by the \( 8p-8h \) configuration. The linear-chain structure in \( ^{36}\text{Ar} \) is understood also by the two-body cluster picture. Considering that two of the three \( C \) clusters compose a \( 24\text{Mg} \) cluster, the linear-chain is regarded as the excited \( ^{12}\text{C}+^{24}\text{Mg} \) cluster states, where the \( C \) cluster contacts to the \( ^{24}\text{Mg} \) from the longitudinal direction. The rotational band of the linear structure is predicted in the energy region \( E_{\gamma} \approx 30 \text{ MeV} \) in the present calculations. Concerning \( ^{28}\text{Mg} \) nucleus, various cluster channels may appear due to the valence neutrons and they interplay to each other. A superdeformation with \( 2h\omega \) configuration is theoretically obtained in the highly excited states of \( ^{28}\text{Mg} \). The results imply the possibility of \( ^{14}\text{C}+^{14}\text{C} \) and \( ^{18}\text{O}+^{16}\text{Be} \) molecular states and \( \alpha \)-cluster states above the superdeformation. The coupling of the different channels, the fragmentation and the widths of the states should be investigated in detail.
V. SUMMARY

Based on microscopic calculations with the AMD method, we discussed the excited states of $^{12}$Be, $^{28}$Si, $^{40}$Ca, $^{36}$Ar and $^{28}$Mg while focusing on the cluster features. Both the cluster aspect and mean-field aspect are essential in the systematic study of these nuclei. These two natures coexist, compete, interplay or overlap to each other even in sd-shell nuclei as well as p-shell nuclei. The largely deformed states are described by many-particle many-hole configurations, while the exotic shapes in the deformed states arise from the cluster structure. We succeeded to connect the high-lying molecular states with the low-lying deformed states. Namely, the low-lying deformed states are regarded as lower members of the high-lying molecular states. In other words, the molecular states arise due to the excitation of inter-cluster motion and the higher molecular states start from the low-lying deformed states.

Various molecular states were suggested in sd-shell nuclei. To our impression, the carbon is the key cluster in sd-shell region because the carbon clusters are often found in the present results. Needless to say, the α cluster is the most important cluster. In addition to the α cluster we propose that the second key cluster is carbon at least in sd-shell nuclei.

The coexistence of the cluster and mean-field natures may become increasingly important in unstable nuclei. The further studies on unstable sd-shell nuclei, $^{20}$Ne and $^{32}$S were done by Kimura et al. with the extended version of AMD method(Ref. [19]).

[1] A.A. Korsheninnikov, et al., Phys. Lett. B 343, 53(1995).
[2] M. Freer, et al., Phys. Rev. Lett. 82, 1383 (1999); M. Freer, et al., Phys. Rev. C 63, 034301 (2001).
[3] A. Saito, et al., Proc. Int. Sympo. on Clustering Aspects of Quantum Many-Body Systems, eds A. Ohnishi, N. Itagaki, Y. Kanada-En’yo and K. Kato, (World Scientific Publishing Co.).
[4] M. Seya, M. Kohno, and S. Nagata, Prog. Theor. Phys. 65, 204 (1981).
[5] W. von Oertzen, Z. Phys. A 354, 37 (1996); 357, 355(1997).
[6] K. Arai, Y. Ogawa, Y. Suzuki and K. Varga, Phys. Rev. C 54, 132 (1996).
[7] A. Doté, H. Horiuchi, and Y. Kanada-En’yo, Phys. Rev. C 56, 1844 (1997).
[8] Y. Kanada-En’yo, H. Horiuchi and A. Doté, Phys. Rev. C 60, 064304(1999).
[9] N. Itagaki and S. Okabe, Phys. Rev. C 61, 044306 (2000); N. Itagaki, S. Okabe and K. Ikeda, Phys. Rev. C 62, 034301 (2000).
[10] Y. Ogawa, K. Arai, Y. Suzuki, and K. Varga, Nucl. Phys. A673 122 (2000).
[11] Y. Kanada-En’yo, Phys. Rev. C 66, 011303(2002).
[12] Y. Kanada-En’yo and H. Horiuchi, Phys. Rev. C 68, 014319 (2003), and references therein.
[13] M. Ito, Y. Sakuragi, Y. Hirabayashi, Phys.Rev.C 63, 064303(2001).
[14] P. Descouvemont and D. Baye, Phys. Lett. B 505, 71(2001).
[15] Y. Kanada-En’yo, Phys. Rev. Lett. 81, 5291 (1998).
[16] D. R. James et. al, Nucl. Phys. A274, 177 (1976); P.Charles et. al, Phys. Lett. 62B, 289 (1976); H. Frölich, et. al, Phys.Lett. 64B, 408 (1976).
[17] S. Ohkubo and K. Yamashita, Phys.Rev. C66, 021301 (2002).
[18] Y. Kanada-En’yo and H. Horiuchi, Prog. Theor. Phys. Suppl.142, 205(2001).
[19] Y. Kanada-En’yo, M. Kimura and H. Horiuchi, Comptes rendus Physique Vol.4, 497(2003), and references therein.
[20] S. Shimoura, et al., Phys.Lett. 560B, 31 (2003).
[21] K. Artemov, et. al, Sov. J. Nucl. Phys. 51 777 (1990).
[22] F. Glatz, et. al, Z. Phys. A 303 239 (1981).
[23] D. Baye and P.-H. Heenen, Nucl.Phys. A283, 176 (1977), and references in.
[24] W. Bauhoff, H. Schultheis and R. Schultheis, Phys. Rev. C26, 1725 (1982).
[25] K. Kato, S. Okabe and Y. Abe, Prog.Theor.Phys. 74, 1053 (1985).
[26] E. Ideguchi, et al, Phys. Rev. Lett. 87, 222501(2001).
[27] Y. Kanada-En’yo, Prog. Theor. Phys. Suppl. 146,190-200 (2003); Y. Kanada-En’yo, M. Kimura, H. Horiuchi, AIP Conf.Proc. 644, 188-195(2003).