Extended Electroweak Interactions and the Muon $g_{\mu} - 2$

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August 7, 2001

Abstract

We look at a number of simple, but representative, models of extended electroweak gauge structures, and present the general contributions to $a_{\mu}$ from the heavy $Z'$ and $W'$ electroweak gauge bosons. Of the models we have examined, none can explain the observed discrepancy between the current experimental value of $a_{\mu}$ and the Standard Model prediction if we require that the gauge fields explain the discrepancy by themselves. In the context of models with new matter fields as well as the additional gauge fields discussed here, however, the gauge field contributions to $a_{\mu}$ can be a substantial and important part of the discrepancy.

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1 Introduction

The recent measurement of the muon anomalous magnetic moment, $a_\mu$, by the Brookhaven E821 Collaboration [1] has raised the tantalizing possibility that new physics lies within the reach of current (or soon to be conducted) collider experiments. If we assume that the discrepancy between this measurement and the Standard Model prediction,

$$\delta a_\mu = a_\mu(\exp) - a_\mu(\text{SM}) = 426(165) \times 10^{-11},$$

really is due to new physics, we should consider all possible mechanisms that may generate a value of this size and sign. Many authors have weighed in with possible explanations of the discrepancy, including supersymmetric scenarios [2, 3], muon substructure [4], leptoquark models [5, 6, 7], scenarios with extra dimensions [8, 9], and exotic fermions [10].

In this letter, we would like to consider a different class of models, namely “pure gauge extensions” to the SU(2)$_L \times$ U(1)$_Y$ Standard Model electroweak gauge structure. By “pure gauge extension” we mean that we add additional gauge groups, those scalars with non-zero VEVs that are necessary to break the gauge symmetries, and those spectator fermions necessary to cancel gauge anomalies, but no other degrees of freedom. We are motivated to consider these models by the fact that the difference between experiment and the Standard Model prediction is of the same order, and has the same sign, as the Standard Model weak contribution: to two loop order, the Standard Model weak contribution is

$$a_\mu(\text{weak}) = 152(4) \times 10^{-11},$$

with the three loop contribution predicted to be negligible compared to this value (for a recent review to the theoretical state of the art, we cite [11] and the references therein). In this letter, we will consider only the one loop contributions of the extended gauge symmetries; extrapolating from the Standard Model, we might expect the two-loop expressions to reduce the contributions found here by a few percent, but the precise size of the contributions is less crucial to our explorations than are the general results we obtain.

Other authors have considered contributions to $a_\mu$ from new gauge bosons [10, 12]. We will consider a different class of models, based on the extended gauge groups SU(2)$_L \times$ SU(2)$_R \times$ U(1) and SU(2)$_L \times$ U(1)$_Y \times$ U(1)$_{\mu}$. First, we will discuss the general structure and properties of models of these types, and present relations which can be used to calculate the contributions of general electroweak $Z'$ and $W'$ bosons to the muon anomaly. We will then apply these results to three specific classes of extended electroweak models: lepton-quark non-universal models (the ununified models), left-right symmetric models, and generation non-universal models. In the most general case of models with new gauge bosons, the contributions to $a_\mu$ can be arbitrarily large. In the class of models we consider here, the extra gauge bosons decouple from the theory as their masses increase. Since we have not seen these gauge bosons directly or indirectly in the electroweak data, we would not generally expect to see contributions to $a_\mu$ as large as $\delta a_\mu$. However, there are a number of reasons we should consider the magnitude of contributions in these models. First, both the errors on the data and the standard model theoretical contributions are large compared to both the Standard Model electroweak contributions and the measured discrepancy and the final value may be considerably smaller than the current value. Second, in the context of these models, it is important to determine what fraction of the discrepancy can be accounted for by the new gauge physics, in order to determine what other types of new physics, such as new fermions or scalars, might be necessary to describe all of the available data in the context of extended electroweak gauge models. In addition to these decoupling scenarios, we will consider the effects of fermion mixing in the generation non-universal models, where the new gauge structure admits tree level, flavor changing couplings in the charged lepton sector which do not generically decouple. In all cases, we will use the measured muon anomaly to either constrain the masses of the new gauge fields, under the assumption that the new fields are responsible for all of the discrepancy, or we will use precision electroweak bounds on

\[1\] While most of the explicit models we consider are motivated by dynamical symmetry breaking, similar gauge structures arise from different motivations, in particular from models motivated by string theory and grand unification; for an overview, we direct the reader to [13] and the references therein.
the masses of the new fields to find the maximum contribution they can make to the discrepancy. We will
close this paper by drawing some general conclusions and will suggest future directions in model building in
light of our results.

2 General Results

Any extended electroweak gauge model must have the experimentally well verified Standard Model SU(2)\(_L\) \(\times\) U(1)\(_Y\)
gauge structure at low energy. Despite the strong experimental constraints on the properties of the electroweak sector, there are still numerous gauge extensions that can both satisfy the constraints and permit
interesting, relatively low-energy (that is, sub-TeV) phenomenology. We consider the general properties of
the following electroweak gauge extensions:

\[
\begin{align*}
&\text{SU}(2)\_1 \times \text{SU}(2)\_2 \times \text{U}(1)\_Y \to \text{SU}(2)\_L \times \text{U}(1)\_Y \to \text{U}(1)\_\text{em} \\
&\text{SU}(2)\_L \times \text{SU}(2)\_2 \times \text{U}(1)\_3 \to \text{SU}(2)\_L \times \text{U}(1)\_Y \to \text{U}(1)\_\text{em} \\
&\text{SU}(2)\_L \times \text{U}(1)\_1 \times \text{U}(1)\_2 \to \text{SU}(2)\_L \times \text{U}(1)\_Y \to \text{U}(1)\_\text{em} .
\end{align*}
\]

In each case, the extended symmetry is broken by the vacuum expectation value (VEV) of some scalar
object \(\Sigma\) (which may be fundamental or composite) at energy scale \(u\), followed by the Standard Model
breakdown by \(\Phi\) at energy scale \(v\). We assume that the higher breaking scale, \(u\), is large compared to the
Standard Model breaking scale \(v\) (such that \(u^2/v^2 > 1\)); this ensures that the new contributions to
\(a_\mu\) will be dominated by the additional heavy gauge fields and not the small shifts in the couplings of the Standard
Model gauge fields. Current limits from precision electroweak data ensure that this is true in the specific
models we examine later.

In order to review the structure of the couplings that arise in the models above, we generalize our notation
for the groups

\[
\begin{align*}
G_1 \times G_2 \times G_3 &\to \text{SU}(2)\_L \times \text{U}(1)\_Y \to \text{U}(1)\_\text{em} ,
\end{align*}
\]

which gives rise to the covariant derivative (displaying for simplicity only the neutral sector)

\[
D_\mu = \partial_\mu - i \sum_{i=1}^{3} g_i A^i_\mu \tau_i ,
\]

with diagonal generators \(\tau_i\). At the scale \(u\), a first stage of breaking occurs. Two of these groups (take \(G_2\) and
\(G_3\)) mix, leaving a diagonal, unbroken group \(G_2'\), and a massive gauge field, \(\tilde{Z}\) which couples approximately
to

\[
\tilde{g} \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \begin{pmatrix} 7_3 \\ -7_2 \end{pmatrix} ,
\]

(2.1)

where \(\phi\) is the mixing angle between the unbroken and broken gauge fields, and \(\tilde{g} = g_3 \sin \phi = g_2 \cos \phi\). At
the scale \(v\), a second stage of breaking occurs, and the remaining two unbroken groups (\(G_1\) and \(G_2'\)) mix,
leaving an unbroken \(\text{U}(1)\_\text{em}\) and a second massive (but lighter) gauge boson, \(Z\). The remaining, unbroken
gauge group gives rise to an exactly massless photon, with generator

\[
e (\tau_1 + \tau_2 + \tau_3) ,
\]

(2.2)

while the generator of the second massive gauge field is

\[
\frac{g}{\cos \theta_W} (T^2_L - \sin^2 \theta_W Q) .
\]

(2.3)
The generators of the $Z^0$ and $Z'$ mass eigenstates differ from those above (the $Z$ and $\tilde{Z}$) by order one terms, multiplied by powers of $v^2/u^2$; the differences are negligibly small to the order we are working, and we will not consider them here. Any charged gauge fields in the model obtain similar generators, but these are more model dependent and will not be discussed in detail here.

We note in passing that even larger gauge extensions, such as $SU(2) \times SU(2) \times U(1) \times U(1)$, are possible; see, for example, [14]. We will not consider them explicitly here because, in general, we expect that the lowest mass vectors will possess many of the properties of the vectors we study here, and the additional heavier states will make negligible contributions to $a_\mu$. In general, we expect that our general results will be independent of the precise details of such large extensions.

The gauge structure of these models assures that, to lowest order, the couplings of the new gauge fields to the photon will have the same structure as for the Standard Model electroweak gauge fields. In particular, there will be no new multi-gauge boson vertices such as $Z'Z^0\gamma$ or $W'W\gamma$. With this restriction, we can find the general one-loop contributions of new charged and neutral vectors to $a_\mu$.

The one-loop contribution in Feynman gauge from charged vectors in the narrow width approximation ($\Gamma_{W'} = 0$) is given by

$$a_\mu W' = \frac{m_\mu g_{\text{eff}}^2}{4\pi^2} \int_0^1 du \frac{m_\mu (2u + 1) (vv^\dagger + aa^\dagger) - 3m_{\text{int}} (vv^\dagger - aa^\dagger)}{(1-u)m^2_{\text{int}} + uM^2 + u(u-1)m^2_\mu}$$

$$- \frac{m_\mu}{8\pi^2} \int_0^1 du \frac{u^2 ((m_{\text{int}} - m_\mu)vv^\dagger - (m_{\text{int}} + m_\mu)aa^\dagger)}{uM^2 + (1-u)m^2_{\text{int}} + u(u-1)m^2_\mu}$$

$$- \frac{m_\mu g_{\text{eff}}^2}{8\pi^2} \frac{1}{M^2} \int_0^1 du \left( \frac{m_\mu ((m_{\text{int}} - m_\mu)^2 vv^\dagger + (m_{\text{int}} + m_\mu)^2 aa^\dagger)}{uM^2 + (1-u)m^2_{\text{int}} + u(u-1)m^2_\mu} + \frac{m_{\text{int}} ((m_{\text{int}} - m_\mu)^2 vv^\dagger - (m_{\text{int}} + m_\mu)^2 aa^\dagger)}{uM^2 + (1-u)m^2_{\text{int}} + u(u-1)m^2_\mu} \right),$$

where the first line comes from a diagram with two $W'$ bosons in the loop, the second line contains the contributions of the two diagrams where one vector is replaced by the unphysical scalar, and the final two lines are the contribution where both vectors are replaced by unphysical scalars. Most of the terms in the above expressions arise from our definition of the coupling between the gauge fields, the muon, and a neutrino\(^2\) in the Lagrangian

$$\mathcal{L} \sim \sum_i g_{\text{eff}i} \bar{\nu}_L \gamma^\mu (v^i + a^i \gamma^5) \nu_i W'_\mu + \text{h.c.} = \sum_i g_{\text{eff}i} \bar{\nu}_L \gamma^\mu C^i_L \nu_i W'_\mu + g_{\text{eff}R} \gamma^\nu C^i_R \nu_i W'_\nu + \text{h.c.,}$$

We have written the interaction both in terms of the vector, $v^i$, and axial, $a^i$, couplings and in terms of the left-, $C^i_L$, and right-handed, $C^i_R$, chiral couplings,\(^3\) where the sum indicates that we can couple the muon to any of the neutrinos, $\nu_i$. The remaining terms include the neutral fermion mass, $m_{\text{int}}$, and the vector mass $M$. In the limit that the model has no additional heavy neutral fermionic states (that is, there are only the

\(^2\)Neutrino here refers to any neutral fermion in the extended model with appropriate quantum numbers to couple to the muon.

\(^3\)The different types of couplings are related by

$$v = \frac{1}{2} (C_R + C_L) \quad a = \frac{1}{2} (C_R - C_L).$$
three light Standard Model neutrinos), that those neutrinos are massless, and that the muon mass is small compared to the vector mass, the above expression reduces to
\[
a_\mu W' = \frac{m_\mu^2 g_{\text{eff}}^2}{8\pi^2} \frac{10}{4M^2_W} (C_L^2 + C_R^2) .
\]
(2.5)

In particular, this result holds in the Standard Model case, where \( g_{\text{eff}} = g \), \( C_L = 1/\sqrt{2} \), and \( C_R = 0 \), giving a Standard Model \( W \) contribution to \( a_\mu \) of
\[
a_\mu W = \frac{m_\mu^2 G_F}{8\pi^2 \sqrt{2}} ,
\]
in agreement with the standard result.

We can similarly derive an expression for the contribution of neutral vectors at one-loop. In Feynman gauge and applying the narrow width approximation, we find
\[
a_\mu Z' = \frac{-m_\mu g^2}{8\pi^2} \int_0^1 du \frac{u(u-1)}{(1-u)M^2 + um_{\text{int}}^2 + u(u-1)m_\mu^2} \left[ 2m_\mu(u-2)(vv^\dagger + aa^\dagger) + 4m_{\text{int}}(vv^\dagger - aa^\dagger) \right] \\
+ \frac{m_\mu g^2 m_{\text{int}}}{8\pi^2 M^2} \int_0^1 du \frac{(m_{\text{int}} - m_\mu)^2 vv^\dagger - (m_{\text{int}} + m_\mu)^2 aa^\dagger}{(1-u)M^2 + um_{\text{int}}^2 + u(u-1)m_\mu^2} \\
- \frac{m_\mu g^2 m_\mu}{8\pi^2 M^2} \int_0^1 du \frac{2 \left( (m_{\text{int}} - m_\mu)^2 vv^\dagger + (m_{\text{int}} + m_\mu)^2 aa^\dagger \right)}{(1-u)M^2 + um_{\text{int}}^2 + u(u-1)m_\mu^2} (u-1) ,
\]
(2.6)

where the second and third lines are the contributions from the unphysical scalar diagram. Again, most terms arise from the Lagrangian couplings
\[
L \sim \sum_i g_{\text{eff}} \overline{\ell} \gamma_i^\mu (v^i + a^i \gamma^5) \ell_i Z' + \text{h.c.} = \sum_i g_{\text{eff}} L_i \gamma_i Z' + \text{h.c.} .
\]

We have explicitly included the possibility of flavor changing neutral couplings; although most of the gauge extensions we will look at contain a GIM-like mechanism that requires \( \ell_i = \mu \), there are extensions where this is not the case. When such tree level flavor changing couplings are allowed, the \( v \) and \( a \) terms will include the mixing factors, and we will have to sum over all possible \( \ell_i \) that can circulate inside the loop. In the limit where the \( \mu \) and \( \ell_i \) masses are small compared to the vector mass, the above expression reduces to
\[
a_\mu Z' = -\frac{m_\mu g_{\text{eff}}^2}{8\pi^2 M_Z^2} \left( m_\mu \left( C_L^2 + C_R^2 \right) - 3m_{\text{int}} C_L C_R \right) ,
\]
(2.7)

where we have explicitly retained the possibility that the new gauge physics will admit flavor changing neutral current (FCNC) couplings. In the Standard Model the GIM cancellation ensures that \( m_{\text{int}} = m_\mu \), and the gauge couplings are given by \( g_{\text{eff}} = g/\cos \theta_W \), \( C_L = -1/2 + \sin^2 \theta_W \), and \( C_R = \sin^2 \theta_W \). The Standard Model \( Z^0 \) contribution to \( a_\mu \) is then
\[
a_\mu Z^0 = -\frac{m_\mu^2 G_F}{8\pi^2 \sqrt{2}} \left( 1 + 2 \sin^2 \theta_W - 4 \sin^4 \theta_W \right) .
\]
The contributions to \( a_\mu \) from scalars with non-vanishing VEVs (i.e. \( v \neq 0 \)) are negligible, and we will not consider this issue further. Detailed derivations of all of these expressions are presented in [15] and compared to the results of the references cited therein.
3 Model Contributions to $a_\mu$

In this section, we analyze the contributions to $a_\mu$ from a number of explicit realizations of the three models presented at the beginning of the previous section. We divide this section into three subsections, devoting one to each of the following classes of models: the ununified (lepton-quark non-universal) models, the left-right symmetric models, and the generation non-universal models.

3.1 Ununified Models

In the Ununified Models, leptons and quarks are assigned charges under different gauge groups. The Ununified model of Georgi, Jenkins and Simmons [16, 17] has the unbroken gauge group

$$\text{SU(2)}^L_\ell \times \text{SU(2)}^q_L \times \text{U}(1)^Y,$$

where the left-handed leptons charged under $\text{SU(2)}^L_\ell$ and the left-handed quarks under $\text{SU(2)}^q_L$. The dominant additional contributions to $a_\mu$ in this case are from the $Z'$ and $W'$ which couple to

$$g \left( \frac{c_\phi}{s_\phi} T_q - \frac{s_\phi}{c_\phi} T_\ell \right),$$

with $T_q = T_q^3$ for the $Z'$, $T_q = T_q^1$ for the $W'$. We have also used the shorthand $s_\phi = \sin \phi$ and $c_\phi = \cos \phi$.

In this case, the contributions to $a_\mu$ can be determined from Equations 2.5 and 2.7; we find

$$a_\mu^{\text{UUM}} = \frac{m_\mu^2 G_F}{8 \pi^2 \sqrt{2}} \frac{s_\phi^2}{c_\phi^2} \frac{M_Z^2}{M_W^2}, \quad (3.1)$$

where we have combined the $Z'$ and $W'$ contributions, since $M_{Z'} = M_{W'}$ to lowest order. Limits obtained on this Ununified model from precision electroweak data [18] can be used to find upper bounds on this contribution. The largest possible value of this contribution is less than $10^{-11}$ (except for very large $s_\phi$, which is precisely the region where our approximations break down and our calculations no longer apply due to the large corrections to the $Z^0$ and $W$ couplings compared to the Standard Model), and hence these vectors can not by themselves explain the observed discrepancy.

We could also consider an ununified model with gauge group

$$\text{SU(2)}_L \times \text{U}(1)^Y_L \times \text{U}(1)^q_Y,$$

with a $Z'$ coupling

$$g' \left( \frac{c_\phi}{s_\phi} Y_q - \frac{s_\phi}{c_\phi} Y_\ell \right)$$

and of course there is no $W'$. The contribution to $a_\mu$ is then given by

$$a_\mu^{Z'} = \frac{m_\mu^2 G_F}{8 \pi^2 \sqrt{2}} \frac{4}{3} \frac{M_Z^2}{M_{Z'}^2} \frac{s_\phi^2}{c_\phi^2} s_W^2. \quad (3.2)$$

We can use the experimental value of $a_\mu$ to place a limit on the value of $M_{Z'}$ necessary to fully account for the observed discrepancy

$$\frac{M_{Z'}}{t_\phi} < 26 \text{ GeV}.$$

In order not to disagree with the LEP precision observables, however, $s_\phi$ must be small (otherwise the contribution to leptonic observables near the $Z^0$ pole from the $Z'$ coupling above would be large), hence $t_\phi$ must be smaller than 1. This requirement effectively rules out a model of this type as the sole explanation for the $a_\mu$ discrepancy, as such a light $Z'$ would have been observed at CERN LEP and the Fermilab Tevatron.
3.2 Left-Right Models

In the Left-Right Models, left- and right-handed fermion doublets transform under different gauge groups. While there are many ways to build such models (for a brief overview and references, see [19]), we choose to analyze a “generic” model with the gauge group

\[ SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} , \]

where the left-handed fermions transform as doublets under SU(2)_L, the right-handed fermions transform as doublets under the SU(2)_R, and both left- and right-handed fields charged under the B – L hypercharge (baryon number minus lepton number). The coupling of the heavy neutral gauge field, Z_R, is given by

\[ g' \left( \frac{c_\phi}{s_\phi} \frac{1}{2} (B - L) - \frac{s_\phi}{c_\phi} T_R^3 \right) , \]

while the coupling to the heavy charged field, W_R, is given by

\[ \frac{g'}{c_\phi} T_R^+ . \]

We can determine \( a_\mu \) from Equation 2.7, and we obtain

\[ a_\mu = \frac{m_\mu^2 G_F}{8\pi^2 \sqrt{2}} \frac{M_{Z_R}^2}{M_{Z_R}} s_W^2 \left( \frac{10}{3} \frac{1}{s_\phi^2 c_\phi^2} - \frac{4}{3} \left( \frac{s_\phi^2}{c_\phi^2} - \frac{1}{s_\phi^2} + 1 \right) \right) . \]  

\[ (3.3) \]

Assuming a very conservative 700 GeV lower bound on the mass\(^4\) of the Z_R, we find that the Z_R/W_R contribution to \( a_\mu \) is less than \( 35 \times 10^{-11} \) for \( 0.25 < s_\phi < 0.99 \). For larger or smaller values of \( s_\phi \), the assumptions we made in deriving our results break down; in these extremes, the contributions to \( a_\mu \) will be dominated by the shifts in electroweak couplings, and not from the new gauge fields. These are precisely the regions of parameter space that should be ruled out by the electroweak data. Hence, the discrepancy can not be entirely explained by this type of model.

3.3 Generation Non-universal Models

In the Generation Non-universal models, the third generation fermions are charged under a different gauge group than the first and second generation fermions. We will consider two models.

Generation non-universal models arise in certain extended technicolor models [20, 21], and the topflavor models [22]. Here we examine the non-commuting extended technicolor (NCETC) scenario due to Chivukula, Simmons, and Terning [21]. This model has the gauge group

\[ SU(2)^\ell_L \times SU(2)^h_L \times U(1)_Y , \]

where SU(2)^\ell_L couples to the left-handed first and second generation fermions (the light generations) and SU(2)^h_L couples to the left-handed third generation fermions (the heavy generation). The coupling is given by

\[ g \left( \frac{c_\phi}{s_\phi} T_h - \frac{s_\phi}{c_\phi} T_\ell \right) , \]

\[ \text{ Bounds on } M_{Z_R} \text{ are usually quoted as bounds on parameters which are equivalent to placing bounds in the } M_{Z_R}-\phi \text{ plane, not as simultaneous bounds on } M_{Z_R} \text{ and } \phi \text{ as for the other models discussed in this paper. See references in [19]. It is thus not possible to provide limits on these models in the same form as for the other models considered in this paper.} \]
where again, the $T_i = T_3 (T^\pm)$ for the $Z'$ ($W'$).

Again, we can determine the contribution to $a_\mu$ from the results in Equations 2.5 and 2.7, and we obtain

$$a_\mu^{\text{NCETC}} = \frac{m_\mu^2 G_F}{8\pi^2 \sqrt{2}} \left[ \frac{s_\phi^2 M_Z^2}{c_\phi^2 M_{W'}^2} \right].$$

(3.4)

The constraints from precision electroweak data on the “light” case of NCETC in [21] imply that the largest possible contribution to $a_\mu$ is smaller than $3 \times 10^{-11}$. Thus, this extension alone can not explain the discrepancy.

There are other generation non-universal models; Topcolor Assisted Technicolor (TC2) [23], for example, contains an extend weak sector with gauge group

$$\text{SU}(2)_L \times \text{U}(1)^f_Y \times \text{U}(1)^h_Y.$$  

The coupling of the $Z'$ is given by

$$g' \left( \frac{c_\phi}{s_\phi} Y_h - \frac{s_\phi}{c_\phi} Y_L \right)$$

If we choose to assign fermionic charge for the muon under $U(1)^f_Y$ as in the Standard Model, then scaling from the Standard Model $Z^0$ contribution, we find

$$a_\mu^{Z'} = \frac{m_\mu^2 G_F}{8\pi^2 \sqrt{2}} \left[ \frac{4 M_Z^2 s_\phi^2}{c_\phi^2 M_{Z'}^2} \right] s_W^2.$$  

(3.5)

Chivukula and Terning have used precision electroweak data to constrain the parameters of this TC2 model [24] (our hypercharge assignment is their “optimal” scenario, which we label OTC2); using their results, we find that the OTC2 contribution to $a_\mu$ can be no greater than about $0.3 \times 10^{-11}$. Hence, this model can not explain the discrepancy by itself.

In a gauge theory with a larger gauge group than the Standard Model where the couplings of the fermions are not generation universal, there will arise, in the absence of additional symmetries, tree level mixings between fermion mass eigenstates at gauge-fermion-fermion vertices, even if all neutrino masses are zero.[12, 13, 22, 25] In other words, there will be no automatic GIM cancellation in the extended neutral current interactions, although the SM neutral currents will still admit an approximate GIM mechanism in these cases. If we don’t eliminate these couplings (with additional discrete flavor symmetries, for example), we have to consider the possibility that heavier fermions may propagate on the internal lines of the $Z'$ diagram. From Equation 2.7, we see that heavy internal fermions can make potentially large contributions to $a_\mu$. Let us see how this works.

Consider the extended neutral current Lagrangian with the fermions in the “gauge basis” (where the Lagrangian is diagonal in the gauge basis flavor space, but where the coupling matrix is not necessarily a multiple of the identity)

$$\mathcal{L} = \Psi_G^i i \gamma^\mu \{ D_{\mu}^{\text{SM}} - i g_{Z'} Z_{\mu} C_{Z'} \} \Psi_G,$$

where $\Psi_G$ is a vector of charged fermions, and $C_{Z'}$ is the vertex operator matrix in the gauge basis. It is important to note that $C_{Z'}$ is diagonal, but is not a multiple of the identity; $C_{Z'} \neq \alpha I$. We now perform a rotation to the gauge basis $\Psi_G = A_{GM} \Psi_M$. Inserting this rotation, we find the Lagrangian in the fermion mass basis

$$\mathcal{L} = \Psi_M^i i \gamma^\mu \{ D_{\mu}^{\text{SM}} - i g_{Z'} Z_{\mu} L_{Z'} \} \Psi_M,$$

(5)

Our notation for $s_\phi$ and $c_\phi$ are the opposite of those used in [21].
\[ L_{Z'} = \Lambda_{GM}^{\dagger} C_{Z'}^{GG} \Lambda_{GM} \], which may not even be diagonal, permitting tree level flavor changing couplings in the extended neutral current sector.

What couplings do we find for these flavor changing interactions? Consider the diagonal (flavor conserving) elements in a three generation model, for example the \( L_{\tau\tau}^{Z'} \) element

\[ L_{\tau\tau}^{Z'} = \sum_{G=1}^{3} \Lambda_{G\tau}^{\dagger} C_{Z'}^{GG} \Lambda_{G\tau} \].

Assume that \( C_{Z'}^{11} = C_{Z'}^{22} \neq C_{Z'}^{33} \). Now, applying three-generation unitarity and rearranging, we find

\[ L_{\tau\tau}^{Z'} = C_{Z'}^{11} + (C_{Z'}^{33} - C_{Z'}^{11}) \Lambda_{3\tau}^{\dagger} \Lambda_{3\tau} \].

In the limit of small off-diagonal mixing, this expression simplifies to

\[ L_{\tau\tau}^{Z'} \approx C_{Z'}^{33} \],

as we might have assumed. For the off-diagonal (flavor violating) terms, for example the \( L_{\tau\mu}^{Z'} \) coupling, we find

\[ L_{\tau\mu}^{Z'} = \sum_{G=1}^{3} \Lambda_{G\tau}^{\dagger} C_{Z'}^{GG} \Lambda_{G\mu} \].

Applying three-generation unitarity and rearranging, we find

\[ L_{\tau\mu}^{Z'} = (C_{Z'}^{33} - C_{Z'}^{11}) \Lambda_{3\tau}^{\dagger} \Lambda_{3\mu} \].

We can now consider toy flavor mixing extensions to the generation non-universal models studied above (NCETC and OTC2). We assume these toy models have the following properties:

1. There are only three generations of fermions.
2. The left- and right-handed flavor rotations that diagonalize the fermion mass matrix are the same; this is certainly not required, but greatly simplifies the calculations.
3. Since constraints on processes such as \( \mu \to e\gamma \) and \( \mu \to eee \) are rather stringent, we assume that there are no \( e\mu Z' \) or \( e\tau Z' \) vertices; that is, only the \( \tau \) and \( \mu \) mix.
4. The \( \mu\tau \) mixing is small.

With these assumptions, we can calculate the additional contributions using Equations 2.5 and 2.7. For NCETC, we find that \( a_\mu \) is given by

\[ a_{\mu}^{NCETC} \approx \frac{m_{\mu}^{2}G_{F}}{8\pi^{2}\sqrt{2}} \frac{10}{3} \frac{s_{\phi}^{2}}{c_{\phi}} \frac{M_{W}^{2}}{M_{W}'^{2}} - \frac{m_{\mu}^{2}G_{F}}{8\pi^{2}\sqrt{2}} \frac{4}{3} \frac{s_{\phi}^{2}}{c_{\phi}^{2}} \frac{M_{W}^{2}}{M_{W}'^{2}} \left| \Lambda_{4\mu}^{1} \Lambda_{2\mu}^{1} \right|^{2} - \frac{m_{\mu}^{2}G_{F}}{8\pi^{2}\sqrt{2}} \frac{4}{3} \left( \frac{c_{\phi}}{s_{\phi}} - \frac{s_{\phi}}{c_{\phi}} \right) \left( \frac{c_{\phi}}{s_{\phi}} - \frac{s_{\phi}}{c_{\phi}} \right) \frac{M_{W}^{2}}{M_{W}'^{2}} \left| \Lambda_{3\tau}^{1} \Lambda_{3\mu}^{1} \right|^{2}. \] (3.6)

Lacking experimental data on these mixing matrices, we will have to make some assumptions to obtain numerical predictions. If we take, for example, \( \left| \Lambda_{3\tau} \right|^{2} = \left| \Lambda_{2\mu} \right|^{2} = 1 - \left| \Lambda_{3\mu} \right|^{2} = 1 - \left| \Lambda_{2\tau} \right|^{2} = 0.99, \right|1_{1e}| = 1 \), and all others zero, we find a limiting contribution that is almost unchanged from the no-mixing case, with contributions less than \( 3 \times 10^{-11} \) over the whole parameter space. The result is unchanged because there
are no right-handed couplings to the $Z'$, and hence no possibility of enhancement from the larger internal fermion (see Equation 2.7).

For the OTC2 model, we find that the dominant new contributions are further enhanced by $m_\tau/m_\mu$ compared to the NCETC values, plus potential gauge group mixing angle enhancements,

$$a_\mu^{\text{OTC2}} = \frac{m_\tau^2 G_F 4 M_{Z'_0}^2 s_\phi^2 s_W^2}{8\pi^2 \sqrt{2}} \left| A_{2\mu}^1 \Lambda_{2\mu} \right|^2 + \frac{m_\mu m_\tau G_F 8 M_{Z'_0}^2}{8\pi^2 \sqrt{2}} \left( \frac{c_\phi}{s_\phi} - \frac{s_\phi}{c_\phi} \right)^2 s_W^2 \left| \Lambda_{3\mu}^1 \Lambda_{3\mu} \right|^2 . \quad (3.7)$$

Using the same mixing angle parameters as above, we find that the mixing angle enhancement at small $s_\phi$ overlaps with a small window of low $Z'$ mass in the precision data, allowing contributions of up to $35 \times 10^{-11}$; however, over most of the mixing angle parameter space the contributions are two orders of magnitude smaller. This large enhancement is due the existence of right-handed couplings which result in a large $m_\tau/m_\mu$ enhancement when the tau circulates in the loop. It is obviously possible to obtain even larger enhancements if there are additional heavy fermionic states with the appropriate quantum numbers to mix with the muon at the $Z'$ vertex.

### 4 Conclusion

We have presented general expressions for the contributions of neutral and charged vector bosons to the anomalous magnetic moment of the muon. We then looked at a number of simple, but representative, models with extended electroweak gauge structures, and calculated their contributions to $a_\mu$. We found that, in general, models with different gauge interactions for the leptons and quarks (the ununified models), and models with different gauge interactions for the heavy and light fermions (the generation non-universal models) with flavor diagonal couplings, are both constrained by precision electroweak data and can make only very small contributions to $a_\mu$, of order $10^{-11}$. Models with simple extended left-right symmetries can generally provide only small contributions to $a_\mu$, of order $10^{-11}$, in those regions of parameter space where they are not expected to disturb the precision electroweak data. Interestingly, the generation non-universal models, as they admit the possibility of flavor changing tree level couplings, can provide potentially large contributions to $a_\mu$; even with very small mixing between the muon and the tau, some of these models can generate contributions of up to $35 \times 10^{-11}$. If such charged sector mixing is observed, one might be able to indirectly observe the new gauge bosons at the next generation of electron-positron and hadron collider experiments.

However, of the models we have examined, none can fully explain the observed discrepancy between the current experimental value of $a_\mu$ and the Standard Model prediction, even if we assume that the masses of the new gauge bosons are as small as those allowed by precision electroweak data. In the context of models with additional matter fields as well as the additional gauge fields discussed here, we have shown that some of the gauge field contributions to $a_\mu$ can be substantial and important, in particular those involving non-universal lepton couplings. The final results of the E821 Collaboration along with the results of high energy collider experiments will soon be able to tell us much more about the possible existence and properties of these extended electroweak interactions.

### Acknowledgments

I thank E. H. Simmons and T. Rador for insightful discussions and comments on the manuscript. This work was supported in part by the Department of Energy under grant DE-FG02-91ER40676, the National Science Foundation under grant PHY-0074274, and by the Radcliffe Institute for Advanced Study.
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