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Abstract

We report preliminary measurements of the first (mean) and second moment (variance) of the inclusive photon energy spectrum in $B \to X_s \gamma$ decays, for threshold values of the photon energy in the range 1.8–2.3 GeV as measured in the rest frame of the $B$-meson. These results are obtained from the Belle measurement of the spectrum, which used a data set consisting of 152 million $B\bar{B}$ pairs collected by the Belle detector at the KEKB asymmetric-energy electron-positron collider operated on the $\Upsilon(4S)$ resonance.

PACS numbers: 11.30.Er,13.20.He,12.15.Ff,14.40.Nd
INTRODUCTION

The first (mean) and second (variance) moments of the inclusive photon energy spectrum in $B \to X_s\gamma$ decays can be used to determine the Heavy Quark Effective Theory (HQET) parameters $m_b$ and $\mu^2$ (A and $\lambda_1$) [1, 2, 3, 4]. These parameters play a crucial role in the extraction of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ [5, 6, 7, 8, 9]. Furthermore, the measurements can be compared with predictions made using different theoretical treatments [1, 2, 4, 10, 11].

Belle reported moment measurements using an energy threshold of 1.8 GeV in Ref [12]; in this note we report on a further analysis of the measured spectrum yielding moment measurements at photon energy thresholds in the range 1.8–2.3 GeV as measured in the rest frame of the $B$-meson. Correlation coefficients between the measurements are also given; these are necessary for inclusion in an analysis to determine HQET parameters [8].

REVIEW OF THE BELLE MEASUREMENT

The $B \to X_s\gamma$ photon energy spectrum measurement at Belle is briefly reviewed. The analysis used data samples amounting to 140 fb$^{-1}$ and 15 fb$^{-1}$ of integrated luminosity taken at (ON) and 60 MeV below (OFF) the $\Upsilon(4S)$ resonance energy respectively.

The analysis procedure involved reconstructing photon candidates with energy greater than 1.5 GeV as measured in the $\Upsilon(4S)$ rest frame. Photon candidates were vetoed if they had a high likelihood of originating from $\pi^0$ or $\eta$ decays to two photons. The likelihood, modelled in Monte Carlo (MC), was calculated as a function of the combined invariant mass of the photon candidate paired with another photon reconstructed in the event, and the energy and polar angle of that other photon in the laboratory frame.

In general, the background of photons from the $e^+e^- \to q\bar{q}$ continuum dominates. It is suppressed through use of event shape variables, which are used as the inputs to two Fisher discriminants [13]. The first discriminant is used to distinguish spherically-shaped $B\bar{B}$ events from jet-like continuum events and includes the Fox-Wolfram moments [14], the thrust calculated using all particles detected in the event including and excluding the candidate photon, and the angles of the corresponding thrust axes with respect to the beam and candidate photon directions, respectively. The second discriminant is designed to exploit the topology of $B \to X_s\gamma$ events by utilising the energy sum of detected particles measured in three angular regions, $\alpha^* < 30^\circ$, $30^\circ \leq \alpha^* \leq 140^\circ$, and $\alpha^* > 140^\circ$, where $\alpha^*$ is the angle to the candidate photon. After cuts the remaining continuum background is removed by subtracting scaled OFF data yields from those of ON data. The scaling factor is obtained by taking the ON-to-OFF ratio of measured luminosities corrected for the difference in cross section between ON and OFF resonance centre of mass energies.

Backgrounds from $B$ decays are estimated from MC and scaled according to studies using data wherever possible and then subtracted from the data. Their contributions include:

- photons from $\pi^0$ and $\eta$ (veto leakage);
- other real photons, mainly from $\omega$, $\eta'$, and $J/\psi$;
- clusters in the calorimeter not due to single photons (mainly electrons interacting with matter, $K_L^0$ and $\bar{n}$).
• beam background.

The photon spectra for ON and scaled OFF data samples along with the results of subsequent background subtractions are plotted in Fig. 1(a). The $B \rightarrow X_s \gamma$ photon energy spectrum that has been corrected for efficiency is shown in Fig. 1(b). The analysis measured the branching fraction,

$$B(B \rightarrow X_s \gamma) = (3.55 \pm 0.32^{+0.30+0.11}_{-0.31-0.07}) \times 10^{-4},$$

where the errors are statistical, systematic and theoretical, respectively. This result agreed with the latest theoretical calculations [15, 16], as well as with previous measurements made by CLEO [17] and Belle [18].

![Figure 1](image)

FIG. 1: From [12]. (a) Photon energy spectra in the $\Upsilon(4S)$ frame. (b) Efficiency-corrected photon energy spectrum. The two error bars show the statistical and total errors.

**MOMENT MEASUREMENTS**

We follow the procedure used in the published analysis [12], with some slight variations. No attempt is made to correct for the part of the spectrum that is not measured with a satisfactory precision *i.e.* from energy below 1.8 GeV. We apply lower energy threshold cuts as measured in the $\Upsilon(4S)$ rest frame ($E^*_{\text{cut}}$) to the efficiency corrected spectrum, from which we obtain truncated first and second moments. Corrections are applied to recover the moments such that the lower energy thresholds correspond to quantities measured in the $B$-meson rest frame ($E_{\text{cut}}$).

A simple procedure is used to unfold the effects of detector resolution, the small $B$-meson boost in the $\Upsilon(4S)$ frame, and that of the 100 MeV wide bins. We define the first moment as $\langle E_\gamma \rangle$ (mean) and the second moment as $\Delta E^2_\gamma \equiv \langle E^2_\gamma \rangle - \langle E_\gamma \rangle^2$ (variance). The corrections are as follows:
**B-meson boost:** The small momentum $|\vec{p}_B|$ of the $B$ meson in the $\Upsilon(4S)$ frame reduces the mean of the energy distribution on average by

$$\frac{\Delta E_{\text{boost}}}{\langle E \rangle} = 1 - \frac{2m_B}{m_{\Upsilon(4S)}} \approx 0.002$$

and adds a Doppler broadening of

$$\left( \frac{\langle E^*_\gamma \rangle |\vec{p}_B|}{\sqrt{3}m_B} \right)^2 \approx 0.006 \text{GeV}^2$$

to the second moment.

**Binning:** Using 100 MeV bins artificially adds a contribution

$$\left( \frac{1}{\sqrt{12}} 0.1 \text{GeV} \right)^2 \approx 0.0008 \text{GeV}^2,$$

to the second moment.

**Energy Resolution:** The energy measurement resolution has the effect of broadening the spectrum, adding

$$W_{\text{ECL}}^2 = (\delta_{\text{ECL}} \langle E^*_\gamma \rangle)^2 \approx 0.004 \text{GeV}^2$$

to the second moment, where $\delta_{\text{ECL}} = 2.8\%$ is the energy resolution.

**Bias correction:** The previous corrections can be classified as model independent and do not together compensate for the full effect of measurement. An additional bias correction derived from the signal MC sample is implemented. This sample, which was also used to correct the measured spectrum for acceptance, was generated as a weighted sum of $B \to K^*\gamma$ decays, where $K^*$ is any known spin-1 resonance with strangeness $S = 1$ where the relative weights are obtained by matching the total photon spectrum to a theoretical model [19]. The bias correction is calculated as the difference of the true moment and the moment measured from the signal MC sample once all aforementioned corrections have been applied. The low-energy tail of the photon energy resolution, in general, decreases the first moment but we found the correction to be model dependent and therefore preferred to classify it as a bias correction. Bias corrections for both moments are shown in Table I.

A systematic uncertainty on the moments stems from the systematic error on the binned signal yields and is referred to as the “scaling” systematic uncertainty. Systematic variations that were considered for the branching fraction measurement are implemented, namely we: vary the number of $B\bar{B}$ events simultaneously with the ON to OFF data ratio; vary within uncertainty the ON and OFF selection efficiency difference; vary the fitting functions that correct for DATA and MC differences in response to the $B\bar{B}$ background; vary by $\pm 20\%$ background from $\eta^\prime$, $\omega$ and bremsstrahlung; vary the $\eta$ veto efficiency for real $\eta$ mesons; use an “alternate” signal MC that favours high-mass resonances decaying into high-multiplicity final states and is the same sample that was used to estimate the model dependence in
TABLE I: Bias corrections as a function of $E_{\text{cut}}$ for the first and second moments.

| $E_{\text{cut}}$ (GeV) | Bias $\langle E_\gamma \rangle$ (%) | $\Delta E^2_{\gamma}$ (%) |
|------------------------|-------------------------------|-----------------|
| 1.8                    | +2.0                          | 0.0%            |
| 1.9                    | +1.6                          | -0.4%           |
| 2.0                    | +1.2                          | -7.1%           |
| 2.1                    | +0.8                          | -17.4%          |
| 2.2                    | +0.2                          | -35.3%          |
| 2.3                    | -0.3                          | -57.9%          |

the branching fraction measurement; and vary the photon detection efficiency by $\pm 2.3\%$ for both signal and backgrounds. We also implement a $\pm 50\%$ variation on the bias correction for the first moment while for the second moment the correction is re-calculated using the alternate signal MC sample. In addition for the second moment a variation, that neglects the lower energy tail in the resolution and assumes a Gaussian model, where $\delta_{\text{ECL}} = 1.9\%$, is implemented. The observed difference due to each variation listed above is assigned as a systematic uncertainty. We also assign a $\pm 100\%$ uncertainty on the binning correction for the second moment. The net systematic error is calculated from the sum in quadrature of the individual uncertainties. Table II lists the moment measurements and they are plotted in Figure 2. Table III gives a full account of the uncertainties for both the first and second moment with varying values of the lower energy threshold $E_{\text{cut}}=1.8–2.3$ GeV.

The statistical uncertainty and scaling systematic uncertainty have been calculated using a toy MC study. We generated numerous random spectra according to the measured spectrum with the bin yields and their uncertainties corresponding to the mean and standard deviation of a Gaussian random variable, respectively. The moments and their fluctuations with respect to each other were measured for each generated spectrum, and finally averaged to yield the covariance matrix, from which the uncertainties due to statistics and systematics scaling were obtained. The covariance matrix was also obtained from systematic variations due to the corrections to the moments. Table IV shows the correlation coefficients calculated from the combined covariance matrix.

At $E_{\text{cut}} = 1.8$ GeV the uncertainty on the first moment is dominated by the systematic uncertainty from scaling and the bias correction. The second moment systematic uncertainty is dominated by the bias correction. In total the systematic error is larger than the statistical error. This circumstance is reversed at $E_{\text{cut}} = 2.3$ GeV where the statistical error dominates. The amount of OFF data used in the analysis limits the statistical precision. For increasing $E_{\text{cut}}$ the systematic uncertainty reduces due to decreasing $B\overline{B}$ event backgrounds. Note the statistical and systematic errors on the moments at $E_{\text{cut}} = 1.8$ GeV are slightly different to the values quoted in our publication [12]. This is due to both the use of toy MC as well as the bias correction uncertainty; the latter has increased the systematic uncertainty in the second moment.

We performed a cross check where we measured the moments using a method that utilises the analysis that determined parameters in the Kagan-Neubert prescription (KN) [19], which were found to best fit our spectrum [20] ($\mu_2^{(\text{KN})} = 4.62$ GeV/$c^2$, $\mu_\pi^2(\text{KN}) = 0.40$ GeV/$c^2$). We generated the photon spectrum in the rest frame of the $B$-meson with these parameters as input and extracted the moments for $E_{\text{cut}}=1.8–2.3$ GeV. The results are plotted in
TABLE II: Moment measurements depending on the lower energy threshold $E_{\text{cut}}$ (GeV), where the first error is statistical and the second is systematic.

| $E_{\text{cut}}$ (GeV) | $\langle E_\gamma \rangle$ (GeV) | $\Delta E^2_\gamma$ (GeV$^2$) |
|-----------------------|--------------------------------|-------------------------------|
| 1.8                   | 2.292 ± 0.027 ± 0.033          | 0.0305 ± 0.0079 ± 0.0099      |
| 1.9                   | 2.309 ± 0.023 ± 0.023          | 0.0217 ± 0.0060 ± 0.0055      |
| 2.0                   | 2.324 ± 0.019 ± 0.016          | 0.0179 ± 0.0050 ± 0.0036      |
| 2.1                   | 2.346 ± 0.017 ± 0.010          | 0.0140 ± 0.0046 ± 0.0024      |
| 2.2                   | 2.386 ± 0.018 ± 0.005          | 0.0091 ± 0.0045 ± 0.0025      |
| 2.3                   | 2.439 ± 0.020 ± 0.004          | 0.0036 ± 0.0045 ± 0.0028      |

Fig. 3 along with the measured moments. We find very good agreement between the moments measured from these independent methods for all but the first moment at $E_{\text{cut}} = 2.3$ GeV. Even for that case the results still agree. The moment measurements agree within uncertainty with the CLEO result [17] as well with preliminary measurements reported by BaBar [21, 22].

SUMMARY

We have reported preliminary measurements of the first (mean) and second (variance) moment and their correlations, of the $B \to X_s \gamma$ photon energy spectrum measured by Belle, for lower threshold values of the photon energy in the range 1.8–2.3 GeV, as measured in the $B$-meson rest frame. These can be used to determine HQET parameters $m_b$ and $\mu^2_\pi$ in the kinetic scheme or equivalently $\bar{\Lambda}$ and $\lambda_1$ in the $1S$ scheme.

FIG. 2: $B \to X_s \gamma$ photon spectrum (a) mean and (b) variance as a function of $E_{\text{cut}}$. 

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TABLE III: Uncertainties contributing to the moment measurements depending on the lower energy threshold $E_{\text{cut}}$ (GeV).

| Source of systematic | $\delta(\langle E_\gamma \rangle)$ (GeV) | $\delta(\Delta E^2_\gamma)$ (GeV^2) |
|----------------------|----------------------------------------|-----------------------------------|
|                      | 1.8 1.9 2.0 2.1 2.2 2.3               | 1.8 1.9 2.0 2.1 2.2 2.3            |
| Scaling              | 0.021 0.012 0.006 0.003 0.002 0.001   | 0.0060 0.0027 0.0009 0.0003 0.0001 0.0001 |
| Energy resolution    |                                        | 0.0020 0.0020 0.0021 0.0022 0.0023 0.0024 |
| Binning              | 0.0022 0.018 0.014 0.009 0.002 0.003   | 0.0008 0.0008 0.0008 0.0008 0.0008 0.0008 |
| No. of $B\bar{B}$    | 0.004 0.001 0.001 0.002 0.002 0.002   | 0.0018 0.0010 0.0006 0.0005 0.0005 0.0004 |
| ON/FF eff.           | 0.000 0.000 0.000 0.000 0.000 0.000   | 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 |
| MC/DATA eff.         | 0.005 0.005 0.003 0.002 0.001 0.000   | 0.0010 0.0004 0.0001 0.0000 0.0000 0.0000 |
| Other $B\bar{B}$ $\gamma$ | 0.010 0.004 0.002 0.000 0.000 0.000 | 0.0024 0.0008 0.0002 0.0000 0.0000 0.0000 |
| $\eta$ veto on $\eta$ | 0.001 0.010 0.000 0.000 0.000 0.000 | 0.0003 0.0001 0.0000 0.0000 0.0000 0.0000 |
| Signal MC            | 0.004 0.004 0.004 0.004 0.004 0.004   | 0.0007 0.0005 0.0004 0.0003 0.0002 0.0000 |
| $\gamma$ detection eff. | 0.001 0.001 0.000 0.000 0.000 0.000 | 0.0003 0.0001 0.0000 0.0000 0.0000 0.0000 |
| Total systematic     | 0.033 0.023 0.016 0.010 0.005 0.004   | 0.0099 0.0055 0.0036 0.0024 0.0025 0.0028 |
| Statistical error    | 0.027 0.023 0.019 0.017 0.018 0.020   | 0.0079 0.0060 0.0050 0.0046 0.0045 0.0045 |
| Total error          | 0.043 0.032 0.025 0.020 0.019 0.021   | 0.0126 0.0081 0.0062 0.0052 0.0051 0.0053 |

TABLE IV: Correlation coefficients between the moment measurements. The calculation takes into account both statistical and systematic uncertainties.

| $E_{\text{cut}}$ (GeV) | $\langle E_\gamma \rangle$ | $\Delta E^2_\gamma$ |
|------------------------|----------------------------|---------------------|
|                        | 1.8 1.9 2.0 2.1 2.2 2.3   | 1.8 1.9 2.0 2.1 2.2 2.3 |
|                        |                           |                     |
|                        | 1.8                       | 1.00 | 0.79 | 0.68 | 0.56 | 0.38 | 0.22 | -0.46 | -0.18 | -0.01 | 0.04 | 0.01 | -0.01 |                     |
|                        | 1.9                       | 1.00 | 0.82 | 0.70 | 0.52 | 0.33 | -0.06 | -0.21 | 0.05  | 0.12 | 0.10 | 0.07  |                     |
|                        | 2.0                       | 1.00 | 0.86 | 0.67 | 0.47 | -0.14 | 0.15  | 0.12 | 0.23  | 0.20 | 0.17  |                     |
|                        | 2.1                       | 1.00 | 0.84 | 0.65 | -0.27 | 0.37  | 0.43  | 0.42  | 0.39  | 0.34 |                     |
|                        | 2.2                       | 1.00 | 0.86 | 0.43 | 0.63  | 0.79  | 0.91  | 0.88  | 0.79  |        |                     |
|                        | 2.3                       | 1.00 | 0.72 | 0.63 | 0.49  | 0.39  | 0.30  |        |        |        |                     |
|                        |                           |                     |                     |        |        |        |                     |
ACKNOWLEDGEMENTS

We thank the KEKB group for the excellent operation of the accelerator, the KEK cryogenics group for the efficient operation of the solenoid, and the KEK computer group and the National Institute of Informatics for valuable computing and Super-SINET network support. We acknowledge support from the Ministry of Education, Culture, Sports, Science, and Technology of Japan and the Japan Society for the Promotion of Science; the Australian Research Council and the Australian Department of Education, Science and Training; the National Science Foundation of China under contract No. 10175071; the Department of Science and Technology of India; the BK21 program of the Ministry of Education of Korea and the CHEP SRC program of the Korea Science and Engineering Foundation; the Polish State Committee for Scientific Research under contract No. 2P03B 01324; the Ministry of Science and Technology of the Russian Federation; the Ministry of Higher Education, Science and Technology of the Republic of Slovenia; the Swiss National Science Foundation; the National Science Council and the Ministry of Education of Taiwan; and the U.S. Department of Energy.

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