Some Features of the Production of Heavy-Quark-Containing Baryons in Electron-Positron Collisions

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Abstract

The production of various heavy-quark-containing baryons in electron-positron annihilation is considered. On the basis of exact formulas that we obtained previously within full perturbation theory, new numerical calculations of the respective cross sections are performed, and simple approximate expressions are then constructed for the results of these calculations. The dependence of the total cross sections on the masses of constituent quarks is discussed. The application of the Peterson fragmentation function and a Reggeon-type fragmentation function to describing differential cross sections is analyzed.

1. Introduction

Investigation of the mechanisms responsible for the production of hadrons containing heavy quarks is of interest from the theoretical point of view since this provides the possibility for further testing QCD – more precisely, our understanding of it. In this way, one tests both its perturbative aspect used to describe the simultaneous production of several quark pairs and nonperturbative models constructed on the basis of QCD for bound states. We recall that, even in cases that are the simplest at first glance, the results of calculations appear to be in an unexpected contradiction with experimental data, as was, for example, in the hadronic production of $J/\psi$ particles. At the same time, derivation of theoretical estimates for relevant cross sections is of importance for practical purposes, such as those associated with planning searches for such particles and investigations of their properties.

Available calculations of the cross sections for the production of baryons containing heavy $c$ and $b$ quarks rely, as a rule, on considering the production of respective diquarks, this corresponding to the fourth order of perturbation theory. A detailed review of the results obtained in this way and an exhaustive list of relevant references can be found in [1]. In the sixth order of perturbation theory $[O(\alpha^2\alpha_s^4)]$, the total and differential cross sections for the production of multiply heavy baryons $\Omega_{scb}$ and $\Omega_{ccc}$ at the $Z$ pole in electron-positron collisions were calculated in our previous studies [2, 3]. For the squares of relevant matrix

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elements, we obtained exact analytic expressions, which, as might have been expected, are very cumbersome (they exist only in the form of computer codes). As a result, numerical calculations with these matrix elements would be extremely time-consuming.

By using these expressions and performing a series of new numerical calculations of various cross sections for the production of baryons containing three nonidentical quarks, we try here to impart, to the emerging results, a broader content, simplicity, and adaptability in the possible future application to constructing estimates for planning experiments. Specifically, we find, first of all, a simple approximate dependence of the total cross section for baryon production in electron-positron collisions on the mass of each constituent quark. On the basis of the concept of fragmentation, we then approximate the differential cross sections with the aid of the Peterson function for various sets of quark masses.

In this study, we also analyze some aspects of the description of the differential cross sections for the production of $\Omega_{scb}$ and $\Omega_{ccc}$ baryons in terms of a Reggeon-type fragmentation function.

2. Dependence of the total cross sections for baryon production on consistent-quark masses

Let us consider some properties of the production of a $q_1q_2q_3$ baryon consisting of three nonidentical quarks $q_1$, $q_2$ and $q_3$ and having a mass $M$, a momentum $p$, and an energy $E$ at the Z pole in electron-positron collisions. We assume that the quark masses $m_1$, $m_2$ and $m_3$ differ markedly from each other; that is,

$$m_1^2 \ll m_2^2 \ll m_3^2. \quad (1)$$

In QCD, the elementary process corresponding to the production of such a baryon is

$$e^+ + e^- \rightarrow q_1(p_1) + q_2(p_2) + q_3(p_3) + \bar{q}_1(p_4) + \bar{q}_2(p_5) + \bar{q}_3(p_6), \quad (2)$$

where the quark and antiquark 4-momenta are indicated in parentheses.

The formation of the baryon from three quarks is described in the well-known nonrelativistic approximation [4-6], whose details required for our purposes are as follows. First, the velocities of the quarks forming the baryon are assumed to be identical. Second, the differential cross section for baryon production is obtained from the standard differential cross section for the process in (2) by replacing the phase space of three quarks by an expression proportional to the baryon phase space; that is,

$$\frac{d^3p_1}{(2\pi)^3 \cdot 2E_1} \frac{d^3p_2}{(2\pi)^3 \cdot 2E_2} \frac{d^3p_3}{(2\pi)^3 \cdot 2E_3} \rightarrow \frac{|\psi(0)|^2}{M^2} \frac{d^3p}{(2\pi)^3 \cdot 2E}, \quad (3)$$

where $\psi(0)$ is the value of the wave function at the point where relative coordinates of all three quarks are zero.

In each of the Feynman diagrams corresponding to the process in (2), one can indicate a virtual gluon $g$ such that it transforms into a quark-antiquark pair $q_i(p_i)\bar{q}_i(p_{i+3})$ without emitting a new gluon $g'$. The denominator of the propagator of the gluon $g$ – it has the form $(p_i + p_{i+3})^2$ – attains a minimum of $4m_i^2$ at $p_{i+3} = p_i$. But if a virtual gluon $g$ transforms into
two quark-antiquark pairs \( q_i(p_i)q_j(p_j)\bar{q}_i(p_{i+3})\bar{q}_j(p_{j+3}) \), the denominator of its propagator has a minimum of \( 4(m_i + m_j)^2 \). Taking into account the inequalities in (1), we can deduce from the above that the leading contribution to the amplitude of the process in (2) comes from the diagrams where the production of quark-antiquark pairs proceeds hierarchically from the heaviest to the lightest. This sequence that ends up in the formation of a baryon from the quarks \( q_1, q_2, \) and \( q_3 \) is generally referred to as the fragmentation of the quark \( q_3 \) into a \( q_1q_2q_3 \) baryon. It is often described analytically in the form

\[
\frac{d\sigma}{dz} = \sigma_{q_3\bar{q}_3} \cdot D(z),
\]

where \( \sigma_{q_3\bar{q}_3} \) is the total cross section for the process \( e^+e^- \rightarrow q_3\bar{q}_3 \) and \( D(z) \) is the respective fragmentation function. For the variable \( z \), one usually takes the quantity \( x_p = p/p_{\text{max}} \) or \( x_E = E/E_{\text{max}} \).

It is obvious that the amplitude of any process is a homogeneous function of the 4-momenta \( P_j \), the masses \( M_j \), and the widths \( \Gamma_j \) of real and virtual particles involved in the process. Therefore, the total cross section or one differential cross section or another can be represented in the form of the product of some power of the total energy \( \sqrt{s} \) and a function of the reduced 4-momenta \( P_j/\sqrt{s} \), the reduced masses \( M_j/\sqrt{s} \), and the reduced widths \( \Gamma_j/s \). From here, it follows, among other things, that the fragmentation function \( D(z) \) appearing in (4) depends parametrically on the reduced masses \( m_i/\sqrt{s} \) of the product quarks. In the following, we will write the reduced masses explicitly only in the logarithmic factors on the right-hand side of formula (6) (see below).

In comparing experimental results obtained in electron-positron collisions with some fragmentation function, attention is given primarily to its form depending on one or two parameters but not to its normalization.

For want of experimentally observed events involving the production of \( q_1q_2q_3 \) baryons in electron-positron annihilation, it is reasonable to focus on the total cross section—namely, on the dependence of the total cross section on the masses of the quarks \( q_1, q_2, \) and \( q_3 \). The choice of a simple algebraic expression representing this dependence is based on the fact that the square of the matrix element of the process being considered is similar, in some respects, to a rational function of the momenta of product particles, its denominator at the minimum involving constituent quark masses and their sums as factors. It is well known that the integral of such a function can generally include logarithmic terms.

By using relation (3) and the results of numerical calculations of the total cross section for the production of \( q_1q_2q_3 \) baryons at the \( Z \) pole in electron-positron collisions for six sets of masses \( m_1, m_2, \) and \( m_3 \) for the same set of electroweak-interaction coupling constants (such as that for the production of \( scb \) baryons), we arrive at the formulas

\[
\sigma_{\text{tot}} = \frac{|\psi(0)|^2}{(m_1 + m_2 + m_3)^2} G, \quad (5)
\]

\[
G \approx \frac{C}{m_1^2 m_2^2} \ln \left( \frac{\sqrt{s}}{4m_1} \right) \ln \left( \frac{\sqrt{s}}{m_3} \right), \quad (6)
\]

where \( \sqrt{s} = 91.2 \) GeV and \( C = (0.0407 \pm 0.0006) \) pb.
In Table 1, we present the sets of masses $m_1$, $m_2$ and $m_3$; the values of $G$ that are obtained from a Monte Carlo calculation of the integral of the square of the matrix element for the process in (2) over the phase space of four final particles (QCD column in the table); and the values of $G$ that are obtained by formula (6).

We believe that the use of the factor $\ln(\sqrt{s}/4m_1)$ in formula (6) is quite justified empirically. It seems plausible that the quantity $G$ depends only slightly on the mass $m_3$ of the heaviest quark; relying on the results of our calculations exclusively, we cannot be confident, however, that the $G$ depends on $m_3$ through the factor $\ln(\sqrt{s}/m_3)$, as follows from Eq. (6).

| $m_1$ (GeV) | $m_2$ (GeV) | $m_3$ (GeV) | QCD, $G$ (pb · GeV$^4$) | Formula (6), $G$ (pb · GeV$^4$) | Parametr $\epsilon$ from (8) |
|------------|------------|------------|----------------|----------------|----------------|
| 0.5 | 1.5 | 4.8 | $(7.85 \pm 0.16) \cdot 10^{-1}$ | $(8.15 \pm 0.14) \cdot 10^{-1}$ | $0.124 \pm 0.016$ |
| 0.3 | 1.5 | 4.8 | $(2.58 \pm 0.07) \cdot 10^{0}$ | $(2.57 \pm 0.04) \cdot 10^{0}$ | $0.098 \pm 0.012$ |
| 0.075 | 1.5 | 4.8 | $(5.59 \pm 0.20) \cdot 10^{1}$ | $(5.42 \pm 0.09) \cdot 10^{1}$ | $0.066 \pm 0.008$ |
| 0.01 | 1.5 | 4.8 | $(4.29 \pm 0.12) \cdot 10^{3}$ | $(4.12 \pm 0.07) \cdot 10^{3}$ | $0.048 \pm 0.006$ |
| 0.01 | 0.5 | 4.8 | $(3.90 \pm 0.28) \cdot 10^{4}$ | $(3.71 \pm 0.06) \cdot 10^{4}$ | $0.016 \pm 0.002$ |
| 0.01 | 0.5 | 1.5 | $(5.10 \pm 0.53) \cdot 10^{4}$ | $(5.18 \pm 0.09) \cdot 10^{4}$ | $0.24 \pm 0.08$ |

Strictly speaking, the dependence of the total cross section on the quark masses $m_1$, $m_2$ and $m_3$ is not exhausted by the explicit expressions in formulas (5) and (6), since the baryon-state wave function at the origin, $\psi(0)$, must change in response to a change in the masses. The dependence of $\psi(0)$ on the masses of constituent quarks is determined within potential quark models, which are not discussed in the present study. On the basis of the numerical values of $|\psi(0)|^2$ that are presented in [7] for six sets of quarks $q_1$, $q_2$, and $q_3$, we can nevertheless estimate cross sections by using the approximate expression

$$|\psi(0)|^2 \approx Dm_2^{1.5}m_3, \quad \text{if } m_1 \ll m_2 \leq m_3, \quad (7)$$

where $D = 0.065 \cdot 10^{-3}$ GeV$^{3.5}$.

Taken together, relations (5)-(7) indicate that the total cross section for the production of $q_1q_2q_3$ baryons in electron-positron annihilation is highly sensitive to the mass of the lightest of three quarks. This circumstance is especially important in the case where, for the quark $q_1$, one takes a $u$ or a $d$ quark, since, from the point of view of simple nonrelativistic concepts, their masses can be varied within rather wide intervals, from 50 MeV (in pions) to 300 MeV (in nucleons). In order to estimate cross sections for the production of baryons containing two heavy quarks and a $u$ or a $d$ quark, we set $m_u = m_d = 100$ MeV in Eqs. (5)-(7), bearing in mind that, according to the approximation specified by Eq. (6), these cross section are determined to within a factor of 10.

For want of something better and, to some extent, as a continuation of the strategies adopted in [7], we propose extending the procedure of the present study and of previous studies [2, 3] to the case of deriving estimates for the production of baryons containing one heavy quark $c$ or $b$, a strange quark $s$, and a light quark $u$ or $d$–namely, we propose supplementing the perturbative part of calculations (sixth order of perturbation theory) with the following nonrelativistic nonperturbative elements: the assumption of equal velocities of the quarks fusing into the baryon in question, relation (3), and the approximation specified by...
Eq. (7). What we inherit from nonrelativistic potential models reduces to the extrapolations in (3) and (7). It is reasonable to indicate here that, in contrast to $u$ and $d$ quarks, a strange quark of mass $m_s = 500$ MeV leads to an acceptable level of agreement with naive nonrelativistic expectations for the masses of the meson and baryon ground states $[s\bar{s} (\phi)]$ and $s s s (\Omega)$, respectively.

All estimates that can be obtained on the basis of formulas (5)-(7) by setting $m_b = 4.8$ GeV, $m_s = 0.5$ GeV, and $m_u = m_d = 0.1$ GeV are given in Table 2 (the symbol $q$ in the subscripts there stands for a $u$ or a $d$ quark). The factor $C$ in (6) is set to $0.0407 \text{ pb}$ in calculating cross sections for the production of $\Omega_{scb} , \Xi_{qcb}$, and $\Xi_{qsb}$ baryons and to $C = 0.0407 \text{ pb} \cdot \frac{(g_V^c)^2 + g_A^c}{(g_V^b)^2 + g_A^b}$ = 0.0317 pb in dealing with $\Xi_{qsc}$ baryons.

| Baryon       | $10^3 \cdot |\psi(0)|^2$, GeV$^6$ | $\sigma_{\text{tot}}$, fb  |
|--------------|-----------------------------|-------------------|
| $\Omega_{scb}$ | 0.57                        | 0.0097            |
| $\Xi_{qcb}$   | 0.57                        | 0.40              |
| $\Xi_{qsb}$   | 0.11                        | 0.98              |
| $\Xi_{qsc}$   | 0.034                       | 2.2               |

We would like to bring to the attention of the reader that, in [8], the quantity $|\psi(0)|^2$ was calculated at the following values of the constituent quark masses: $m_b = 5.29$ GeV, $m_c = 1.905$ GeV, $m_s = 0.6$ GeV, and $m_u = m_d = 0.3$ GeV. In our previous studies [2, 3], we employed the values of $|\psi(0)|^2$ from [8] without introducing any corrections. In the present study, all cross-section values, including those that are given in the figures, are calculated by using Eq. (7).

It should be emphasized that the approximate formula (6) was obtained for the constituent quark masses obeying the inequalities in (1). But if these inequalities are not satisfied and if, in addition, there are quarks identical in flavor among the quarks in question, formula (6) cannot be used even for a rough estimate of cross sections. Indeed, it can be seen that, for the production of an $\Omega_{ccc}$ baryon in electron-positron annihilation, a direct numerical calculation of the quantity $G$ on the basis of formula (5) gives the value of 2.27 pb GeV$^4$, while expression (6) yields 0.090 pb GeV$^4$. The reasons for so significant a discrepancy between the values of $G$ are rather obvious: if the inequalities in (1) hold, the main contribution to the cross section comes only from a few Feynman diagrams, but, if all three masses $m_1 , m_2$, and $m_3$ are close to one another, all diagrams make approximately equal contributions (for the production of an $\Omega_{ccc}$ baryon, there are 504 such diagrams); additionally, interference effects arise if there are identical quarks.

3. Differential cross section and Peterson fragmentation function

Let us now proceed to consider a simple algebraic description of the differential cross sections for the production of $q_1 q_2 q_3$ baryons in electron-positron collisions on the basis of the fragmentation approach. Experimental data on the production of heavy-quark hadrons in electron-positron annihilation are usually approximated in terms of the Peterson function [9]

$$D(z) \sim \frac{1}{z[1 - (1/z) - \varepsilon/(1 - z)]^2}$$

(8)
where \( \varepsilon \) is a phenomenological parameter. We used this function in [2] to describe approximately the production of \( \Omega_{scb} \) baryons in electron-positron collisions.

We performed complete numerical calculations of the differential cross sections for the production of \( q_1q_2q_3 \) baryons for six sets of constituent quark masses (see Table 1) and then determined the values of the parameter \( \varepsilon \) of the Peterson function (8) that ensure the best agreement between the form of this fragmentation function and the form of the differential cross sections \( d\sigma/dx_p \). The resulting values of the parameter are given in the last column of Table 1. The results of numerical calculations of \( d\sigma/dx_p \) and their approximation in terms of the Peterson function are shown in Fig. 1.

The dependence of the parameter \( \varepsilon \) on the mass \( m_1 \) of the lightest quark can be approximated by a linear function,

\[
\varepsilon \approx a + bm_1
\]  

(9)

where \( a = 0.046 \) and \( b = 0.17 \) GeV\(^{-1} \). As the middle mass \( m_2 \) is decreased, the maximum of the distribution \( d\sigma/dx_p \) is shifted toward the largest possible relative momentum value, \( x_p = 1 \), while the parameter \( \varepsilon \) becomes smaller. The shift of the maximum of the cross section \( d\sigma/dx_p \) in response to the change in the mass \( m_3 \) of the heaviest quark is opposite to that in response to the analogous change in \( m_1 \) and \( m_2 \); with decreasing \( m_3 \), the extremal value of \( x_p \) decreases substantially, while the parameter \( \varepsilon \) in the function given by (8) grows.

The value \( \varepsilon \) that we found here for at the mass values of \( m_1 = 0.01 \) GeV, \( m_2 = 0.5 \) GeV, and \( m_3 = 1.5 \) GeV is very close to the values of \( \varepsilon \) that were obtained in experiments that studied the production of \( \Lambda_c, \Sigma_c, \) and \( \Xi_c \) baryons in electron-positron annihilation: \( \varepsilon = 0.236^{+0.068}_{-0.048} \) for \( \Lambda_c \) [10], \( \varepsilon = 0.29 \pm 0.06 \) for \( \Sigma_c \) [11], \( \varepsilon = 0.24 \pm 0.08 \) for \( \Xi_c \) [12].

For baryons containing quarks of identical flavor (for example, \( \Omega_{ccc} \)), it is difficult to take into account effects of interference between identical particles; therefore, the fragmentation mechanism cannot be so well justified theoretically in this case as for \( q_1q_2q_3 \) baryons under the conditions in (1). At the same time, it remains quite useful in the phenomenological aspect. For example, it was found in [3] that the transverse-momentum distribution of \( \Omega_{ccc} \) baryons that is obtained from direct numerical calculations cannot be adequately approximated with the aid of the Peterson function. However, the shape of this distribution can be faithfully reproduced by using the so-called Lund function [13]

\[
D(z) \sim \frac{1}{z}(1 - z)^a \exp(-c/z),
\]  

(10)

where the parameters are set to \( a = 2.4 \) and \( c = 0.70 \).

4. Differential cross sections and Reggeon-type fragmentation function

Let us now consider the representation of numerical results for the production of heavy-quark-containing baryons in electron-positron collisions in terms of the Reggeon-type fragmentation function [14]

\[
D(z) \sim z^\beta (1 - z)^\gamma.
\]  

(11)
This function was employed in [15, 16] in discussing experimental data on the production of $D$ and $B$ mesons in electron-positron annihilation, the parameters $\beta$ and $\gamma$ not being related there to Regge trajectories.

An approximation of the numerical results obtained here for the production of $\Omega_{scb}$ and $\Omega_{ccc}$ baryons in electron-positron annihilation with the aid of the fragmentation function in (11) was performed for the differential cross sections $d\sigma/dx_E$ and is displayed in Fig. 2.

It is of interest to compare the values obtained for the parameters of the function in (11) by approximating the numerical results of our perturbative approach to determining cross sections and the values deduced from the expressions for these parameters in terms of the intercepts of the trajectories of appropriate hadrons. If the quark $i$ fragments into a baryon consisting of the quarks $i, j$, and $k$, then [17]

$$\beta = 1 - \alpha_{ii}, \quad \gamma = \alpha_{q\bar{q}} - 2\alpha_{jkq},$$

(12)

where the symbol $q$ in the subscripts stands for a light quark $u$ or $d$, while the quantities $\alpha_{ii}$ and $\alpha_{jkq}$ are the intercepts of the meson (of $\alpha_{ii}$ quark content) and baryon (of $\alpha_{jkq}$ quark content) trajectories. An approximate linear relation between the intercepts of trajectories corresponding to different quark contents was obtained within the model of quark-gluon strings [18]:

$$2(\alpha_{ijk} - \alpha_{ijl}) = \alpha_{kk} - \alpha_{ii}.$$  (13)

On the basis of relation (13) and the standard notation $\alpha_{u\bar{u}} = \alpha_{d\bar{d}} = \alpha_\rho$, $\alpha_{s\bar{s}} = \alpha_\phi$, $\alpha_{c\bar{c}} = \alpha_\psi$, $\alpha_{g\bar{g}} = \alpha_T$, and $\alpha_{uud} = \alpha_{udd} = \alpha_N$ the equalities in (12) can be recast into the form

$$\beta = 1 - \alpha_T, \quad \gamma = 3\alpha_\rho - \alpha_\phi - \alpha_\psi - 2\alpha_N,$$

(14)

for $\Omega_{scb}$ baryons and into the form

$$\beta = 1 - \alpha_\psi, \quad \gamma = 3\alpha_\rho - 2\alpha_\psi - 2\alpha_N,$$

(15)

for $\Omega_{ccc}$ baryons.

For some intercepts, we will take the quite reliably established values of $\alpha_N = -0.4$, $\alpha_\rho = 0.5$, and $\alpha_\phi \approx 0$ [17], while, for the others, we will use the following estimates: $\alpha_\psi = -2.2$ [19] and $\alpha_T = -8.0$ [20]. These estimates differ only slightly from those that were previously obtained in [7, 21].

Substituting the above values into relation (14) for $\Omega_{scb}$ baryons, we obtain $\beta = 9.0$ $\gamma = 4.5$, but these results are in a glaring contradiction with the values of $\beta = 3.3$ and $\gamma = 1.48$ if $m_s = 500$ MeV and with the values of $\beta = 3.2$ and $\gamma = 1.18$ if $m_s = 300$, which we obtained from a comparison of the fragmentation function in (11) with the results of direct numerical calculations of the cross section $d\sigma/dx_E$. In turn, relation (15) for the production of $\Omega_{ccc}$ baryons yields $\beta = 3.2$ and $\gamma = 6.7$, which can be thought to be in very rough agreement with the values $\beta = 2.6$ and $\gamma = 4.6$ resulting from the approximation of our numerical results for the cross section $d\sigma/dx_E$.

This suggests the simple conclusion that the perturbative and the nonperturbative (Reggeon) approach do not reduce to each other and, depending on the process being considered, they can lead either to close or to strongly different results.
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Fig. 1. Differential cross section $d\sigma/dx_p$ for the production of $q_1q_2q_3$ baryons in electron-positron collisions: (crosses) results of Monte Carlo calculations along with the errors in them and (solid curves) results of the calculation by formula (4) with the Peterson fragmentation function (8). The values of the masses of the quarks $q_1$, $q_2$, and $q_3$ (in GeV) and of the parameter $\varepsilon$ in the Peterson function in Figs. 1a–1f are the following: (a) $m_1 = 0.5$ GeV, $m_2 = 1.5$ GeV, $m_3 = 4.8$ GeV, $\varepsilon = 0.124$; (b) $m_1 = 0.3$ GeV, $m_2 = 1.5$ GeV, $m_3 = 4.8$ GeV, $\varepsilon = 0.098$; (c) $m_1 = 0.075$ GeV, $m_2 = 1.5$ GeV, $m_3 = 4.8$ GeV, $\varepsilon = 0.066$; (d) $m_1 = 0.01$ GeV, $m_2 = 1.5$ GeV, $m_3 = 4.8$ GeV, $\varepsilon = 0.048$; (e) $m_1 = 0.01$ GeV, $m_2 = 0.5$ GeV, $m_3 = 4.8$ GeV, $\varepsilon = 0.016$; (f) $m_1 = 0.01$ GeV, $m_2 = 0.5$ GeV, $m_3 = 1.5$ GeV, $\varepsilon = 0.24$. 
Fig. 2 Differential cross section $d\sigma/dx_E$ for the production of $\Omega_{scb}$ baryons [(a) $m_s = 0.5$ GeV and (b) $m_s = 0.3$ GeV] and $\Omega_{ccc}$ baryons (c) in electron-positron annihilation. The crosses show the results of Monte Carlo calculations and their errors. The solid curves correspond to the approximation of the cross sections with the aid of a Reggeon-type fragmentation function (11) with the parameters (a) $\beta = 3.3$ and $\gamma = 1.48$, (b) $\beta = 3.2$ and $\gamma = 1.18$, and (c) $\beta = 2.6$ and $\gamma = 4.6$. 