HYDRODYNAMICAL EVOLUTION
OF A QUARK-GLUON PLASMA DROP WITH BOUNDARY

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We consider the evolution of a limited Quark-Gluon Plasma (QGP) drop in the framework of relativistic hydrodynamics. In the presence of the boundary, the expanding and cooling of QGP drop may appear as oscillating processes, which realize via multiple traveling of rarefaction and compression waves through the plasma and leads to increasing of plasma life-time. Within this picture, deflagration seems to be the main possible way of hadronization. The slow cooling mechanism also leads to a significant enhancement of photon yield in comparison with Bjorken scenario.

1. Introduction

When discussing the signals of a quark-gluon plasma produced in heavy ion collisions, particularly the ones connected with thermal photons or lepton pairs [1], it is necessary to have an adequate underlying picture of the space-time evolution of plasma. Moreover, the very characteristic details of the evolution of plasma may be considered as signals by their own [2].

In principle, it is generally accepted that relativistic hydrodynamics is able to provide the wanted description, but the exact solution of the mathematical problem is rather difficult. The approach, which is most widely exploited in this context, if not to say the only one, is known as Bjorken model [3]. This model is based on the assumption of a space-time scaling, which means that all processes in a hot strong-interacting system do only depend on the local proper time \( \tau \). The assumption of space-time scaling leads to major simplifications and allows to obtain an analytical solution of the hydrodynamical equations. At a further step, this scaling solution is often used as a ground for more sophisticated models [4, 5, 6] where the transverse motion of matter is taken into account as a correction to its scaling-like longitudinal expansion.

However, this model is only an idealization, which ignores some important features of the real world. For example, it is the fact that the QGP drop is, anyway, limited, and so there must exist a boundary, which separates two phases with different equations of state.

Our own study is based upon a direct numerical analysis of hydrodynamical equations together with the boundary conditions between plasma and
vacuum. In the present work we have considered the space-time evolution of a spherically-symmetric QGP drop and have tested various sets of initial conditions.

A similar approach, also applied to a spherically symmetric QGP drop, has been proposed in ref. [7]. The author of [7] mainly focuses his attention on the effects of pion-pion Bose-Einstein correlations and compares his predictions with Bjorken Model. This is well complementary to our present consideration, where the emission of thermal photons is revised. In addition, we include in our analysis various ways of hadronization, both equilibrium and nonequilibrium, whereas the paper [7] deals with only one type of phase transitions, the deflagration.

Having the a priori space-time scaling given up, we get a new, qualitatively different view of the evolution of plasma. Among the noticeable consequences of our analysis is the fact that supersonic phase transitions from QGP to hadrons are impossible, and this may strongly affect the eventual proportion between different particles, which appear in the course of hadronization [2, 8]. Another important observation is the enhanced yield of thermal photons, which exceeds the traditional estimations by about one order of magnitude. Our results remain valid in a wide range of model parameters.

The outline of the paper is the following. In sect. 2 we present the basic equations of hydrodynamics and the equation of state of the deconfined matter. In sect. 3 we briefly discuss the numerical scheme to integrate the dynamical equations with boundary conditions. In sect. 4 we analyze the evolution of a QGP drop as it appears in our approach and point out some essential features of the dynamics. In sect. 5 we calculate the production of thermal photons. Our main findings and conclusions are summarized in sect. 6.

2. Equations of motion and equations of state

To describe the evolution of matter created in a relativistic collision of heavy nuclei one commonly uses the hydrodynamical approach [10, 11]. The equations of motion represent the momentum and energy conservation. In the absence of dissipative processes (diffusion, viscosity, heat conductivity) they read

\[ \partial_\mu T^{\mu\nu} = 0, \]  

(1)
where the energy-momentum tensor
\[ T^{\mu \nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu \nu}. \]  

(2)

Here \( \varepsilon \) is the energy density, \( p \) is the pressure, \( u^\mu \) is the 4-velocity, and \( g^{\mu \nu} \) is the metric tensor.

For the spherically symmetric case considered here it is convenient to rewrite the above equations (1),(2) in an explicitly symmetric form:
\[ \partial_t u + \frac{1}{r^2} \partial_r (r^2 f(u)) = 0, \quad u \equiv \begin{pmatrix} w \gamma^2 - p \\ w \gamma^2 v_r \end{pmatrix}, \quad f(u) \equiv \begin{pmatrix} (w \gamma^2 - p)v_r + pv_r \\ w \gamma^2 v_r^2 + p \end{pmatrix}, \]  

(3)

where \( w = \varepsilon + p \) is the enthalpy density, \( v_r \) is the radial component of the velocity, and \( \gamma = (1 - v_r^2)^{-1/2} \).

The dynamics of a QGP drop is now completely defined by the initial conditions, the boundary conditions and the equation of state, which relates \( p \) and \( \varepsilon \).

We adopt the MIT-motivated equation of state of plasma \[ [12] \] and get for massless quarks and gluons
\[ p = \alpha T^4 - B, \]
\[ \varepsilon = 3\alpha T^4 + B, \]  

(4)

where \( T \) is the temperature and \( B \) is a phenomenological constant corresponding to the energy density and the pressure of the QCD vacuum. The parameter \( \alpha \) defines the number of degrees of freedom in QGP and is taken \( \alpha = \pi^2/90(2 \times 8 + 2 \times 2 \times 3 \times 3 \times N_f \times \frac{7}{8}) \approx 23\pi^2/45, \) where \( N_f = 2.5 \) for taking into account u-, d-, s-quarks.

Since the QGP drop is assumed to be of limited size it is surrounded by the hadronic vacuum with the equation of state
\[ \varepsilon_{vac} = p_{vac} = 0. \]  

(5)

We will recall this equation when formulating the boundary conditions.

3. Numerically integrating the hydrodynamical equations

Numerical algorithms aimed at the solution of the hydrodynamical problems are being developed for many years. Among the various approaches described in the literature we prefer the nonlinear monotonic transport
scheme FCT (Flux-Corrected Transport) [13, 14, 15] developed by J.P.Boris and D.L.Book in the framework of Eulerian grid methods. It has been shown to apply to a wide class of physical systems and is found quite accurate and reliable.

Let us remind the guideline of this method.

Consider one-dimensional continuity equation for the mass density flux:

$$\partial_t \rho + \partial_x (\rho v) = 0.$$  \hspace{1cm} (6)

Let $\rho^0_i$ denote the density in the $i^{th}$ point of the grid at a given time. An explicit scheme to calculate the density $\tilde{\rho}_i$ at the next time step may be presented as follows:

$$\tilde{\rho}_i = \rho^0_i - \epsilon_{i+1/2}(\rho^0_{i+1} + \rho^0_i) + \epsilon_{i-1/2}(\rho^0_{i-1} + \rho^0_i) +$$

$$\nu_{i+1/2}(\rho^0_{i+1} - \rho^0_i) + \nu_{i-1/2}(\rho^0_{i-1} - \rho^0_i).$$  \hspace{1cm} (7)

This form observes the mass conservation. Here the quantities with half-integer indices $i \pm 1/2$ correspond to arithmetic-mean values of the ones taken in the points $i$ and $i \pm 1$, and $\epsilon_{i\pm1/2} \simeq (1/2) v_{i\pm1/2} (\delta t/\delta x)$ with $\delta t$ being the time step and $\delta x$ the space constant of the grid. The terms with $\nu$ stand for an artificial diffusion and are introduced to guarantee the positivity of mass density, which demands

$$\nu_{i\pm1} \geq |\epsilon_{i\pm1}| \quad \text{for all } i.$$  \hspace{1cm} (8)

To compensate this overstrong diffusion one applies an antidiffusion correction:

$$\rho^1_i = \tilde{\rho}_i - \mu_{i+1/2}(\tilde{\rho}_{i+1} - \tilde{\rho}_i) - \mu_{i-1/2}(\tilde{\rho}_{i-1} - \tilde{\rho}_i),$$  \hspace{1cm} (9)

with $\mu_{i\pm1/2}$ being positive coefficients. This step removes extra smoothing but no longer guarantees the positivity of the newly computed densities $\rho^1_i$. To restore the positivity and the stability of the eventual solution the antidiffusion fluxes $f_{i\pm1/2} \equiv \mu_{i\pm1/2}(\tilde{\rho}_{i\pm1} - \tilde{\rho}_i)$ need to be corrected. In the simplest form one can take $\mu_{i\pm1/2} = \nu_{i\pm1/2} - |\epsilon_{i\pm1/2}|$. Eliminating the residual diffusion ($\nu - \mu$) is only possible with nonlinear corrections adjusted to the local behaviour of the solution.

The fundamental requirement of the FCT method is that the antidiffusion step must neither generate new extrema in the solution nor enhance the ones already existing. This is observed with the following prescription:

$$\rho^1_i = \tilde{\rho}_i - f^\text{cor}_{i+1/2} - f^\text{cor}_{i-1/2},$$

$$f^\text{cor}_{i\pm1/2} = S \max \{0, \min [S(\tilde{\rho}_{i\pm2} - \tilde{\rho}_{i\pm1}), |f_{i\pm1/2}|, S(\tilde{\rho}_i - \tilde{\rho}_{i\mp1})]\},$$

$$|S| = 1, \quad \text{sign } S = \text{sign}(\tilde{\rho}_{i\pm1} - \tilde{\rho}_i).$$  \hspace{1cm} (10)
The condition (10) is constructed in such a way that it takes into account all possible combinations of the signs of local gradients of the physical solution. A detailed description of various transport, antidiffusion and correction schemes may be found in [15]. Like it is done in the original papers [14, 15], we also use the time-splitting procedure to calculate the velocities.

An essential point in our analysis is the presence of the moving phase boundary, which greatly complicates the consideration. In general, Lagrangian grids are better suited for this kind of problems. Since the grid cells are made moving together with the matter, one can identify the intercell boundary with the phase boundary, thus avoiding the ambiguous situation of mixing two different equations of state in one cell. To fit in with the FCT method we have adopted a combined scheme.

All the internal cells of our grid are of Eulerian type and have definite unchangeable size (it will be referred to as ”standard size”). On the contrary, the cell adjacent to the phase boundary is of Lagrangian type, with its outer face moving together with the boundary. Under the evolution of the QGP drop the size of this cell changes continuously. If, due to the expansion of the drop, the length of the surface cell becomes larger than 1.5 standard lengths, the cell splits into two new cells. The one, which does not touch the phase boundary, gets the full standard size. It now belongs to the Eulerian part of the grid. The other cell, which is adjacent to the phase boundary, takes the rest of the total length and continues its life as Lagrangian cell. In the opposite case, if the length of the surface cell reduces to a quantity less than 0.5 standard lengths, it fuses with the neighboring Eulerian cell, thus forming a Lagrangian cell of extended size.

All the internal Eulerian cells are treated according to the standard FCT algorithm. The parameters of the boundary Lagrangian cell are found from the momentum and the energy conservation. The incoming and outcoming fluxes of the momentum and the energy for the Lagrangian cell are known from the parameters of the neighboring Eulerian cells on the inner side and from the equation of state of vacuum (3) on the outer side. Given these fluxes, we calculate the new values of the momentum and the energy densities. Then we extract from these quantities the velocity of plasma. This velocity is assumed to be the velocity of the boundary of the QGP drop. Given the velocity, we calculate the new size of the Lagrangian cell and recalculate the momentum and the energy densities according to the changed size.
The calculated values of the densities and the volume of the Lagrangian cell may then be corrected in such a way that to achieve an exact conservation of the energy. This means

\[ \varepsilon_N V_N = \sum_{i=1}^{N} \varepsilon_i V_i - \sum_{i=1}^{N-1} \varepsilon_i V_i \equiv E(t=0) - \sum_{i=1}^{N-1} \varepsilon_i V_i, \]  

where \( E(t=0) \) is the initial total energy of the system. This equality may be used either to correct the volume \( V_N \) of the Lagrangian cell (if to think the energy density \( \varepsilon_N \) is known correctly) or to correct the energy density (if to think the volume is known) or to make a weighted correction for both these quantities. The latter case is realized in our paper. This scheme was tested in 1-dimensional case with very simple Lagrangian method [10].

4. The dynamics of a QGP drop

In the present section we consider the dynamical properties of the evolution of a QGP drop. The initial conditions for this problem must be derived from the dynamics of nucleus-nucleus collisions. In fact it leaves a large piece of freedom. We do not pretend to give precise and detailed predictions on the behaviour of plasma in real experimental conditions. Our purpose is rather to give a qualitative description, which reveals the characteristic features of a new mechanism of expansion and cooling.

For illustrative purposes, we consider a spherically symmetric baryonless drop of quark-gluon plasma. The main conclusions derived from our analysis would neither depend on the exact shape of a QGP drop nor on the details of the equation of state such as nonzero baryon density. Instead, it is the presence of the boundary, which is the origin and the reason of the distinctive properties of our model.

So we consider a spherical drop at a constant temperature \( T_0 \) and pressure \( p_0 \) in the initial moment. The initial velocity is assumed to rise linearly from \(|v| = 0\) at the center of the drop to \(|v| = v_0\) at the boundary. We have tried widely different values of \( p_0 \) and \( v_0 \). We have also tested the sensitivity of the results on the choice of the bag constant \( B \) in the equations of state and to the strength of the surface tension, which determines the mechanical stability of the drop against breaking.

Consider first the case of zero initial velocity \( v_0 = 0 \) (fig. 1) and following parameters: radius \( r_0 = 3 \, fm \), \( p_0 = 6.5 \, GeV/fm^3 \), \( B = 0.2 \, GeV/fm^3 \).
The expansion of a QGP drop starts from the nearby-surface regions. The overfall of pressure (from plasma to vacuum) leads to the formation of a rarefaction wave, which propagates from the surface of the drop to its center (fig. 1 a,b,c). The velocity of this wave is the speed of sound. This wave puts the plasma in motion, thus causing the process of expansion. The continuous expansion of the QGP drop during all this time results in a monotonic decrease of the pressure and the temperature in the whole of the drop (fig. 1 a-h).

If the initial velocity is high \( v_0 \geq v_s \), \( v_s \) being the speed of sound, the evolution starts from the creation of excess pressure region at the boundary of the QGP drop. This leads to a formation of a compression wave, which also moves from the surface of the drop to its center with the speed of sound. In the same time the QGP drop expands as a whole, so that the pressure and the temperature decrease as in the previous case. When the compression wave reaches the center of the drop and reflects from it, the evolution of the drop becomes very similar to the case of zero initial velocity.

The expansion of a QGP drop may be induced either by the rarefaction wave (if \( v_0 = 0 \)) or the initial motion (\( v_0 > 0 \)), or both these reasons, but the behaviour of plasma looks the same in the sense that the expansion and cooling are always accompanied by the propagation of waves.

Depending on the initial amount of energy (or the initial temperature) the expansion may take different time. The compression waves may pass through the drop several times, reflecting successively from the center and from the boundary. The expansion and cooling of plasma carry on until the pressure falls below zero or the conditions for a phase transition are reached. What happens next depends on the mechanical and thermodynamical stability of the plasma. For the time, let us forget about phase transitions and concentrate on a purely hydrodynamical problem.

The mechanical stability of plasma depends on the strength of its surface tension. The estimations of this parameter are uncertain, and hence we will consider two opposite extreme cases.

First, assume that the surface tension is always strong enough to prevent the QGP drop from breaking into smaller droplets. Then the expansion of plasma by inertia may lead to states with negative pressure. If the initial pressure was small compared to the bag constant \( p_0 < B/2 \), the negative pressure appears for the first time at the moment when the rarefaction wave reaches the center of the QGP drop. Since that moment the region of
negative pressure spreads with the reflected wave towards the drop boundary. The moment when it reaches the surface of the drop corresponds to the maximal expansion of plasma and to its deepest overcooling. At this time the pressure is almost uniformly negative in all the volume of the drop and its value is approximately equal to the initial pressure taken with the opposite sign: \( p_0 \simeq -p_0 \). After that the drop starts to squeeze. At the end of the period of squeezing plasma returns to the state, which is similar to the initial one, both in its pressure (or temperature) and the occupied volume.

If the initial pressure is high (\( p_0 > B/2 \)), the process of expansion and cooling takes a longer time. As result of reflection of a plasma flow from the drop boundary, some excess of pressure is formed near the boundary. It creates the compression wave passing through the plasma drop many times reflecting successively from its center and from its surface at the background of the more strong rarefaction process. This compression wave slows down the expansion of plasma and eventually stops it. It is remarkable that even at the moment of maximal overcooling, the negative pressure can never fall below a definite value, which depends on the bag constant \( B \) but does not depend on the initial pressure \( p_0 \): \( p > p_{\text{min}}, \quad p_{\text{min}} \simeq -B/2 \) (fig. 1 j). The subsequent squeezing of plasma restores positive pressure in the whole of the drop. However, the value of this positive pressure \( p'_0 \) is not equal to the initial value \( p_0 \), but is nearly the minimal (negative) pressure taken with the opposite sign: \( p'_0 \simeq -p_{\text{min}} \) (fig. 1 l). Neither does the occupied volume contract to the initial size.

If to allow a further evolution of plasma, one would see a second period of expanding and squeezing. It essentially resembles the first period, but corresponds to a reduced initial pressure, i.e. \( p'_0 \) instead of \( p_0 \) (figs. 1 l). At the end of this period plasma comes to a state with \( p''_0 \simeq p'_0 \), as is typical for processes starting from relatively low pressure. All the subsequent periods are simply a repetition of this picture.

On the other hand, if the plasma surface tension is weak the evolution of the drop comes to end as soon as the pressure falls to zero. Then the creation of vacuum bubbles (or breaks) comes into play. The breaks prevent the temperature and the pressure from further decreasing and ”freeze” the state with \( p \simeq 0 \).

Most probably, the vacuum bubbles will break the QGP drop into smaller droplets, but their summary volume must conserve. The volume cannot increase because this would lead to further decreasing pressure and will
simply end up with new breaks. The volume cannot decrease because there is no force, which could cause squeezing of plasma, while the motion by inertia tends to move the drops away from each other. Hence, the primordial QGP drop will transform into a sponge-like object or into a cloud of smaller droplets. The pressure in this system freezes at the $p \simeq 0$ level.

It is important to point out that the dynamical limit for overcooling $p_{\text{min}} \simeq -B/2$ remains valid in all the cases. The existence of this limit restricts the possible ways of hadronization. An explanatory overview of all types of phase transitions may be found in the textbook [10], and so we will not repeat it here. According to the estimations made in [8, 9] the supersonic condensation and the detonation may only occur if $p < -B/2$. Since this degree of the overcooling can never be achieved, no matter what kind of initial conditions are chosen, we are left with slow processes like equilibrium phase transition and deflagration. The deflagration must be considered the most realistic scenario for the hadronization of plasma.

Now let us take the deflagration into account. By the definition [10], the deflagration is a slow process, which spreads with a velocity much less than the speed of sound. There are no reasons to accept that the plasma can convert into hadrons in a moment, once the pressure has fallen down to the equilibrium threshold. Moreover, the deflagration takes place only at the surface of plasma drop, but not in its inner regions.

In the present consideration we have simulated the deflagration as a process, which starts at the outer surface of the drop and spreads towards the center with a constant velocity $v_{\text{def}} = 0.1$ c [9]. The deflagration proceeds during the time when the necessary condition $0 < p < p_{\text{th}}$ is satisfied, where the threshold value $p_{\text{th}}$ has been estimated in [8, 9]: $p_{\text{th}} = 30$ $\text{MeV}/\text{fm}^3$.

Fig. 2 illustrates the evolution of a QGP drop with the account of deflagration and $p_0 = 2.56\text{GeV}/\text{fm}^3$ (assuming the negative pressure is allowed). Hadronization reduces the mass of plasma, but proceeds with a relatively small velocity. One can see several oscillations of the drop before the termination of the phase transformation. If the initial drop would break into smaller droplets (assuming the negative pressure is unstable) the deflagration would proceed faster because of enlarged total surface of the system. However, it takes a rather long time to reach even the state, where the deflagration is possible. This fact has a direct consequence in the enhanced yield of thermal photons and lepton pairs.
5. The yield of thermal photons

To estimate the yield of photons from QGP we recall the analytical results of ref. [17]. The emission of photons is mainly due to two elementary processes, i.e. the gluon Compton scattering and the annihilation of quarks:

\[ q(p_1) + g(p_2) \rightarrow \gamma(k) + q(k'), \]
\[ q(p_1) + q(p_2) \rightarrow \gamma(k) + g(k'). \] (12) (13)

Here the letters in the parentheses denote the momenta of the particles.

In the lowest QCD order, the spin- and color-averaged matrix elements for these processes read:

\[ |T_{qg}|^2 = -\frac{16\pi^2}{3} Q_f^2 \alpha_s \left( \frac{s}{(t - m^2)} + \frac{t}{s} \right) \] (14)
\[ |T_{qq}|^2 = \frac{128\pi^2}{9} Q_f^2 \alpha_s \left( \frac{u}{(t - m^2)} + \frac{t}{(u - m^2)} \right) \] (15)

where \( Q_f \) and \( m \) are the electric charge and the mass of a quark \((f = u, d, s)\), and \( s, t \) and \( u \) are the standard Mandelstamm variables: \( s = (p_1 + p_2)^2 \), \( t = (p_1 - k)^2 \), \( u = (p_1 - k')^2 \).

The total number of photons produced in unit volume of plasma per unit time is given by:

\[
2\omega \frac{dW^\gamma}{d^3k} = \frac{N_1 N_2}{(2\pi)^8} \int \frac{d^3k'}{2E'} \tilde{n}(E') \int \frac{d^3p_1}{2E_1} n(E_1) \int \frac{d^3p_2}{2E_2} n(E_2) \times \\
\delta^4(p_1 + p_2 - k - k') \sum_{a,b} |T_{ab}|^2
\] (16)

where \( \omega \) is the photon energy, \( E_1, E_2, E' \) are the energies of the particles with momenta \( p_1, p_2, p' \), respectively; \( N_i \) is the number of degrees of freedom (spin, color) in the initial state of the reaction, \( n(E) \) stand for Fermi or Bose distributions, and \( \tilde{n}(E) = 1 - n(E) \). If to use an approximate Maxwell-Boltzmann formula \( n(E) = \exp(-E/T) \), \( \tilde{n}(E) = 1 \) instead of exact Fermi and Bose distributions, the multidimensional integration in (16) may be performed analytically. The yield of photons per unit volume and unit time is then

\[ N = \frac{\pi}{3} \left( \sum_f Q_f^2 \right) \alpha_s(T) \ln\left( \frac{2T}{m} \right) T^4. \] (17)
Here the running QCD coupling constant is

$$\alpha_s(T) = \frac{6\pi}{(33 - n_f) \ln(\kappa T/\Lambda_{MS})}. \quad (18)$$

According to lattice calculations, $\kappa \simeq 4$ and $T_c/\Lambda_{MS} \simeq 1.8$.

Equation (18) leads to the following pseudorapidity and the transverse momentum distributions of photons:

$$\frac{d^2N}{dk^2 dy} = \frac{4}{\pi^3} (\sum_f Q_f^2) \alpha \alpha_s(T) T^2 \ln\left(\frac{4k\perp T}{m^2}\right) \exp\left(-\frac{k\perp T}{T}\right) \cosh(y - y'). \quad (19)$$

To calculate the total yield of photons from plasma we first integrate the hydrodynamical equations of motion and find the temperature at every time at every space point. Then we use eqs. (17) and (19).

For comparison, the prediction of Bjorken model for the evolution of temperature with time is

$$T = T_0 (\tau_0/\tau)^{nv_s^2}, \quad (20)$$

where $n = 1, 2, 3$ refer to one-, two- or three-dimensional expansion of plasma.

The results of our calculations are seen in fig. 3. Fig. 3a shows the total rate of photon emission as a function of time. Figs. 3b,c show the distribution of the produced photons on the transverse momentum for $T_0 = 258\text{ MeV}$ and $T_0 = 319\text{ MeV}$ respectively. Solid curves represent integrated spectra from the beginning till the moment when the pressure reaches zero level in the whole volume of the drop. Dashed curves correspond to the emission of photons in accord to Bjorken scenario. Dotted curves refer to photons originating from decays of final-state hadrons, as is estimated in [18].

Variations in the model parameters may shift the presented numbers but cannot smash the general picture. The conclusion on the long lifetime of plasma and on the enhanced yield of photons remains valid within a wide range of model assumptions.

6. Conclusions

We have established a novel mechanism of expansion and cooling of QGP, which consists in multiple propagation of rarefaction and compression waves trough the plasma. Since the speed of these waves is almost
constant and equal to the speed of sound, the time needed to cool plasma increases with the size of QGP drop.

Even if to allow the existence of states with negative pressure, the hydrodynamical evolution of plasma cannot lead the pressure below a definite limit, which is determined by the bag constant in the equations of state: 
\[ p_{\text{min}} \simeq -B/2. \]

Since this degree of overcooling is insufficient for supersonic phase transitions, hadronization may only occur due to rather low processes like deflagration or equilibrium transition. As a consequence, the lifetime of plasma extends considerably.

Both the slow cooling mechanism and slow hadronization process result in a longer period of photon radiation. The total yield of thermal photons gets comparable with or even exceeds the contribution from hadron decays.

The enhanced yield of photons is a fundamental result, which may be proven by a direct experimental measurement. The impossibility of supersonic phase transitions from plasma to hadrons puts some restrictions on the proportion between different hadrons and, hopefully, may also be experimentally tested.
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Figure captions

**Fig. 1**
Pressure as a function of the coordinate $r$ along the radius of a QGP drop, plotted for different times ($t$). The initial radius of the drop, the pressure and the velocity are: $r_0 = 3 \text{ fm}$, $p_0 = 6.5 \text{ GeV/fm}^3$, $v_0 = 0$ and $B = 0.2 \text{ GeV/fm}^3$. Mind the different ordinate scale in different plots (a-l).

**Fig. 2**
The process of expansion and cooling of a QGP drop with the account of deflagration.

**Fig. 3**
- **a** – Total rate of photon emission, $T_0 = 258 \text{ MeV}$.
- **b** – Transverse momentum distribution of photons.
  Solid curve - calculations within the present model,
  Dashed curve - calculations according to Bjorken model,
  Dotted curve - background from decays of hadrons.
  The initial temperature $T_0 = 258 \text{ MeV}$.
- **c** – The same as in **b**, but for $T_0 = 319 \text{ MeV}$. 
