Interacting six-dimensional topological field theories

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Abstract. We study the gauge-fixing and symmetries (BRST-invariance and vector supersymmetry) of various six-dimensional topological models involving Abelian or non-Abelian 2-form potentials.

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1 Introduction

Recently, L. Baulieu and P. West introduced a six-dimensional topological model of Witten-type involving 2-form potentials \([1]\). In the sequel, the gauge-fixing procedure and twist in this model have been studied in more detail in reference \([2]\); these authors also determined vector supersymmetry (VSUSY-) transformations which represent an additional symmetry of the model.

The goal of the present paper is to discuss two different generalizations of the free Abelian model \([1]\) to interacting models. The first consists of coupling the Abelian 2-form potentials to a non-Abelian Yang-Mills field by virtue of a Chern-Simons term, as suggested in reference \([1]\). The corresponding action represents a six-dimensional topological version of the Chapline-Manton term appearing in the action for ten-dimensional supergravity coupled to super Yang-Mills theory \([3]\). The second generalization consists of considering non-Abelian (charged) 2-form potentials which are coupled to a Yang-Mills connection, following the lines of reference \([4]\).

Before discussing these generalizations, we summarize the free Abelian model \([1, 2]\) while using differential forms to simplify the notation (section 2). Our paper concludes with some comments concerning possible extensions of these six-dimensional topological models. We note that all of our considerations concern the classical theory (tree-level).

2 Free Abelian model

The arena is a compact pseudo-Riemannian 6-manifold \(\mathcal{M}_6\) and the basic fields are Abelian 2-form potentials \(B_2\) and \(B_2^c\) which are independent of each other. From the associated curvature 3-forms \(G_3 = dB_2\) and \(G_3^c = dB_2^c\), one can construct the classical action \([4]\)

\[
\Sigma_{\text{cl}} = \int G_3 G_3^c.
\]

Here and in the following, the integrals are understood as integrals of 6-forms over \(\mathcal{M}_6\) and the wedge product symbol is always omitted.

2.1 Symmetries

The action \([4]\) is invariant under the ordinary gauge transformations

\[
\delta_{\lambda_1} B_2 = d\lambda_1, \quad \delta_{\lambda_1} B_2^c = 0,
\]

which represent a reducible symmetry in the present case, and it is invariant under the shift-(or topological Q-) symmetry

\[
\delta_{\lambda_2} B_2 = \lambda_2, \quad \delta_{\lambda_2} B_2^c = 0,
\]
which also represents a reducible symmetry. In equations (2), (3) and in the following, it is understood that the “c-conjugated” equations also hold, e.g. c-conjugation of equation (2) gives $\delta_{\chi^c_1}B_2^c = d\chi^c_1$, $\delta_{\chi^c_1}B_2 = 0$.

In the sequel, we will describe the infinitesimal symmetries in a BRST-framework and we will derive the BRST-transformations from a horizontality condition (Russian formula) \[5\]. Thus, we introduce a series of ghost fields associated with the reducible gauge transformations \[2\] and we collect them in a generalized 2-form,

$$\tilde{B}_2 = B_2 + V_1^1 + m^2.$$  

(4)

Here, the upper and lower indices denote the ghost-number and form degree, respectively. The total degree of a field is the sum of its ghost-number and form degree, and all commutators $[\cdot, \cdot]$ are graded with respect to this total degree. The BRST-differential $s$, which describes both the ordinary gauge transformations and the shift-transformations, is combined with the exterior derivative $d$ in a single operator,

$$\tilde{d} = d + s,$$

(5)

which is nilpotent by virtue of the relations $d^2 = s^2 = [s, d] = 0$. Thus, the generalized field strength

$$\tilde{G}_3 \equiv \tilde{d}\tilde{B}_2$$

(6)

satisfies the generalized Bianchi identity

$$\tilde{d}\tilde{G}_3 = 0.$$

(7)

The BRST-transformations of the classical and ghost fields\[\footnote{Here, “classical” fields are not opposed to quantum fields, but simply refer to the fields appearing in the classical action.} are now obtained from the horizontality condition \[1, 2\]

$$\tilde{G}_3 = G_3 + \psi_1^1 + \varphi_1^2 + \phi_3,$$

(8)

which involves a series of ghosts associated with the shift-symmetry. By inserting the field expansions \(4\) and \(8\) into relations \(3\) and \(7\), we obtain the s-variations \[1, 2\]

\[
\begin{align*}
    sB_2 &= \psi_1^1 - dV_1^1, \\
    sV_1^1 &= \varphi_1^2 - dm^2, \\
    sm^2 &= \phi_3, \\
    s\psi_1^1 &= -d\varphi_1^2, \\
    s\psi_1^1 &= -d\varphi_1^2, \\
    s\varphi_1^2 &= -d\phi_3, \\
    s\phi_3 &= 0
\end{align*}
\]

(9)

and $sG_3 = -d\psi_1^1$. Since the field expansions \(4\) and \(8\) have not been truncated, the obtained BRST-transformations are nilpotent by construction \[1\].
2.2 Gauge-fixing

Let us briefly review the gauge-fixing procedure [1, 2] while using differential forms. To start with, we consider the shift degrees of freedom for the fields $B_2$ and $B_2^c$. These are fixed by imposing a self-duality condition relating the corresponding field strengths:

$$\ast G_3 = -G_3^c.$$  

(10)

This relation is equivalent to imposing a self-duality condition on $G_- \equiv d(B_2 - B_2^c)$ and an anti-self-duality condition on $G_+ \equiv d(B_2 + B_2^c)$. Henceforth, relation (10) is analogous to the self-duality condition for the curvature 2-form $F$ in four-dimensional topological Yang-Mills theory [7].

With the help of a BRST-doublet $(\chi_3^{-1}, H_3)$, i.e.

$$s\chi_3^{-1} = H_3 , \quad sH_3 = 0 ,$$

(11)

the constraint (10) can be implemented in the gauge-fixing action:

$$\Sigma_{sd} = s \int \{ \chi_3^{-1} (\ast G_3 + G_3^c) \} .$$

(12)

Since the shift-symmetry represents a reducible symmetry, it is necessary to re-iterate the gauge-fixing procedure for the action $\Sigma_{cl} + \Sigma_{sd}$: this leads to the introduction of the anti-ghosts and Lagrange multipliers of tables 1 and 2, all of which have been arranged in Batalin-Vilkovisky pyramids. These fields again represent BRST-doublets:

$$s\phi^{-3} = \eta^{-2} , \quad s\eta^{-2} = 0$$

$$s\varphi_1^{-2} = \eta_1^{-1} , \quad s\eta_1^{-1} = 0$$

$$s\chi_1 = \eta_2^1 , \quad s\eta_2^1 = 0 .$$

(13)

In summary, the shift-invariance of the classical action is fixed by virtue of the gauge-fixing action

$$\Sigma_Q = \Sigma_{sd} + s \int \{ \varphi_1^{-2} d \ast \psi_2^1 + \phi^{-3} d \ast \varphi_1^2 + \chi_1^1 d \ast \varphi_1^{-2} - CC \} ,$$

(14)

where $CC$ stands for the c-conjugated expressions.

The reducible gauge symmetry (2) is fixed in a similar way: one considers the usual gauge condition $d \ast B_2 = 0$ and re-iterates the gauge-fixing procedure. This leads to the introduction of the series of anti-ghosts and multipliers presented in tables 3 and 4, the $s$-variations being given by

$$sm^{-2} = \beta^{-1} , \quad s\beta^{-1} = 0$$

$$sV_1^{-1} = b_1 , \quad sb_1 = 0$$

$$sn = \beta_1^1 , \quad s\beta_1^1 = 0 .$$

(15)
Thus, the ordinary gauge degrees of freedom of $B_2$ are fixed by the functional
\[ \Sigma_{og} = s \int \{ V_{1}^{-1} d \ast B_2 + m^{-2} d \ast V_1^1 + nd \ast V_1^{-1} - CC \} \]  
and the complete gauge-fixed action of the model reads
\[ \Sigma = \Sigma_{cl} + \Sigma_Q + \Sigma_{og}. \]  

### 2.3 Vector supersymmetry

Due to the fact that we are considering a topological model of Witten-type, one expects the complete gauge-fixed action to admit a VSUSY. At the infinitesimal level, the VSUSY-transformations are described by the operator $\delta_{\tau}$ where $\tau \equiv \tau^\mu \partial_\mu$ is a constant, $s$-invariant vector field of ghost-number zero\footnote{In order to avoid technical complications related to the global geometry, we limit the considerations of this section to flat space-time.}. The variation $\delta_{\tau}$ acts as an antiderivation which lowers the ghost-number by one unit and which anticommutes with $d$. The operators $s$ and $\delta_{\tau}$ satisfy a graded algebra of Wess-Zumino type,
\[ [s, \delta_{\tau}] = \mathcal{L}_\tau, \]  
where $\mathcal{L}_\tau \equiv [i_{\tau}, d]$ denotes the Lie derivative along the vector field $\tau$ and $i_{\tau}$ the interior product by $\tau$. We will simply refer to the relation (18) as the SUSY-algebra.
The $\delta_\tau$-variations of all fields can be determined by applying the general procedure introduced in reference [6]. To start with, we derive the VSUSY-transformations of the classical and ghost fields by expanding the so-called 0-type symmetry conditions

$$ \delta_\tau \tilde{B}_2 = 0, \quad \delta_\tau \tilde{G}_3 = \mathcal{L}_\tau \tilde{B}_2, $$

with respect to the ghost-number. We thus obtain

$$ \delta_\tau B_2 = 0, \quad \delta_\tau \psi_1^2 = \mathcal{L}_\tau B_2, \quad \delta_\tau V_1^1 = 0, \quad \delta_\tau \phi_1^2 = \mathcal{L}_\tau V_1^1, \quad \delta_\tau m^2 = 0, \quad \delta_\tau \phi^3 = \mathcal{L}_\tau m^2. $$

The $\delta_\tau$-variations of the anti-ghosts are found by requiring the $\delta_\tau$-invariance of the total action (17) and by applying the commutation relations (18). Finally, the VSUSY-transformations of all multipliers follow from the ones of the corresponding anti-ghosts by imposing the algebra (18) for all of them:

$$ \delta_\tau \chi^{-1}_3 = 0, \quad \delta_\tau H_3 = \mathcal{L}_\tau \chi^{-1}_3, \quad \delta_\tau \phi^{-3} = 0, \quad \delta_\tau \eta^{-2} = \mathcal{L}_\tau \phi^{-3}, \quad \delta_\tau \varphi^{-2}_1 = 0, \quad \delta_\tau \eta^{-1} = \mathcal{L}_\tau \varphi^{-2}_1, \quad \delta_\tau \chi^1 = -\mathcal{L}_\tau n, \quad \delta_\tau \eta^2 = \mathcal{L}_\tau \chi^1 + \mathcal{L}_\tau \beta^1, \quad \delta_\tau m^{-2} = \mathcal{L}_\tau \phi^{-3}, \quad \delta_\tau \beta^{-1} = \mathcal{L}_\tau m^{-2} - \mathcal{L}_\tau \eta^{-2}, \quad \delta_\tau V_1^{-1} = \mathcal{L}_\tau \varphi^{-2}_1, \quad \delta_\tau b_1 = \mathcal{L}_\tau V_1^{-1} - \mathcal{L}_\tau \eta^{-1}, \quad \delta_\tau n = 0, \quad \delta_\tau \beta^1 = \mathcal{L}_\tau n. $$

Thus, it is by construction that the total action is $\delta_\tau$-invariant and that the SUSY-algebra is fulfilled off-shell for all fields of the model. Our results coincide with those found in reference [2] by other methods. We refer to the latter work for the relation of VSUSY to a twist of a supersymmetric field theory.

### 3 Abelian model with Chern-Simons term

The authors of reference [1] considered the interaction of the Abelian 2-forms $B_2$ and $B_2^c$ with a non-Abelian Yang-Mills (YM) connection $A$ by virtue of a Chern-Simons term with coupling constant $\lambda$,

$$ \Omega_3(A) = \lambda \text{tr} (AdA + \frac{2}{3}AAA). $$

The proposed action reads

$$ \hat{\Sigma}_{cl} = \int (G_3 - \Omega_3) (G_3^c - \Omega_3). $$

\[3\text{We do not include the ordinary YM-action } \int \text{tr} (F \ast F) \text{ as in reference [4] since it depends on the metric and therefore destroys the topological nature of the model.}\]
This functional represents a six-dimensional topological version of the expression \( \int \text{tr} \left( G_3 - \Omega_3 \right) \ast (G_3 - \Omega_3) \) which appears in the action for ten-dimensional supergravity coupled to super YM [3].

The equations of motion for \( A \) and \( B_2 \) (or \( B_2^c \)) have the form

\[
F(G_3 - G_3^c) = 0 \quad \text{and} \quad \text{tr} \left( FF \right) = 0,
\]

where \( F = dA + \frac{1}{2}[A, A] \) denotes the curvature 2-form associated to \( A \). The latter equations imply \( F = 0 \) (i.e. the same equation of motion as in the three-dimensional Chern-Simons theory).

### 3.1 Symmetries

The action (23) is not anymore invariant under the shift \( \delta B_2 = \psi_2^1 \). However, it is invariant under the YM-gauge transformations

\[
\begin{align*}
\delta A &= -Dc \equiv -(dc + [A, c]), \\
\delta B_2 &= \lambda \text{tr} (cdA) = \delta B_2^c,
\end{align*}
\]

which leave \( G_3 - \Omega_3 \) and \( G_3^c - \Omega_3 \) invariant.

The BRST-transformations of \( A \) and of the YM-ghost read

\[
sA = -Dc, \quad sc = -\frac{1}{2}[c, c]. \tag{25}
\]

They follow from the horizontality condition \( \tilde{F} = F \), where

\[
\tilde{F} = d\tilde{A} + \frac{1}{2}[[\tilde{A}, \tilde{A}], \tilde{A}] = A + c. \tag{26}
\]

The generalized Chern-Simons form

\[
\tilde{\Omega}_3 = \lambda \text{tr} \left( \tilde{A}d\tilde{A} + \frac{2}{3} \tilde{A}\tilde{A}\tilde{A} \right)
\]

can be expanded with respect to the ghost-number,

\[
\tilde{\Omega}_3 = \Omega_3 + \Omega_1^1 + \Omega_1^2 + \Omega_1^3, \tag{27}
\]

which provides the well-known solution of the descent equations (e.g. see [3]):

\[
\begin{align*}
\Omega_3 &= \lambda \text{tr} (AdA + \frac{2}{3}AAA), \quad s\Omega_3 + d\Omega_1^1 = 0, \\
\Omega_1^1 &= \lambda \text{tr} (cdA), \quad s\Omega_1^1 + d\Omega_2^2 = 0, \\
\Omega_1^2 &= \lambda \text{tr} (-ccA), \quad s\Omega_1^2 + d\Omega_3^3 = 0, \\
\Omega_3^2 &= \lambda \text{tr} (-\frac{1}{3}ccc), \quad s\Omega_3^3 = 0. \tag{28}
\end{align*}
\]
We now use this result to discuss the $B$-field sector. The generalized field strength of $B_2$ is defined as before (i.e. $\tilde{G}_3 = \tilde{d}\tilde{B}_2$ with $\tilde{B}_2 = B_2 + V_1^1 + m^2$), but the horizontality condition of the free model is now replaced by the horizontality condition

$$\tilde{G}_3 = G_3 + \Omega^1_2 + \Omega^2_1 + \Omega^3.$$

(29)

Expansion with respect to the ghost-number yields the $s$-variations

$$sB_2 = -dV_1^1 + \Omega^1_2$$

$$sV_1^1 = -dm^2 + \Omega^2_1$$

$$sm^2 = \Omega^3,$$

(30)

where the explicit expressions for the $\Omega^q_p(A,c)$ were given in equations (28). Furthermore, substitution of (29) in $\tilde{d}\tilde{G}_3 = 0$ leads to $sG_3 = -d\Omega^1_2$ (and reproduces some of the descent equations (28)).

The BRST-transformations (25) and (30) leave the classical action (23) invariant.

### 3.2 Gauge-fixing

In the YM-sector, the gauge symmetry is fixed in the standard way,

$$\Sigma^A_{gf} = s \int \text{tr} \left\{ \bar{c} \, \ast \, A \right\} = \int \text{tr} \left\{ b \ast A - \bar{c} \ast Dc \right\},$$

(31)

where we made use of a BRST-doublet ($s\bar{c} = b$, $sb = 0$).

In the $B$-sector, the local symmetry $\delta B_2 = -dV_1^1$ is fixed as for the free model, i.e. by introducing the gauge-fixing functional $\Sigma^B_{gf} \equiv \Sigma_{og}$ given by equation (16).

In summary, the complete action of the interacting model reads $\Sigma_{int} = \hat{\Sigma}_{cl} + \Sigma^B_{gf} + \Sigma^A_{gf}$.

### 3.3 VSUSY

Due to the absence of shift-symmetries in the present model, the only possible choice for VSUSY-transformations is given by the so-called $\emptyset$-type symmetry conditions. However, the derivation of $\delta_r$-variations for all fields is substantially more complicated in the present case since the SUSY-algebra only closes on-shell. Therefore, we will not further elaborate on this point here.
4 Non-Abelian model

Consider a YM-connection $A$ and a 2-form potential $B_2$, both with values in a given Lie algebra. The field strength of $B_2$ is now defined by

$$G_3 = DB_2 = dB_2 + [A, B_2]$$

and it satisfies the second Bianchi identity $DG_3 = [F, B_2]$, where $F = dA + \frac{1}{2}[A, A]$ denotes the YM-curvature.

A natural generalization of the action (1) for the Abelian potentials is given by

$$\Sigma_{cl} = \int \operatorname{tr} \{G_3 G_3^c - F [B_2, B_2^c]\}.$$  (33)

Neither $B_2$ nor $A$ propagate in this model (very much like $A$ in four-dimensional topological YM-theory).

4.1 Symmetries

Following reference [4], we now spell out all local symmetries of the functional (33). As in the previously discussed models, one considers the generalized gauge fields

$$\tilde{A} = A + c, \quad \tilde{B}_2 = B_2 + V_1^1 + m^2$$  (34)

and the associated generalized field strengths

$$\tilde{F} = \tilde{d}A + \frac{1}{2}[\tilde{A}, \tilde{A}], \quad \tilde{G}_3 = \tilde{D}\tilde{B}_2 = \tilde{d}\tilde{B}_2 + [\tilde{A}, \tilde{B}_2].$$  (35)

The BRST-transformations in the YM-sector can be summarized by the following horizontality condition which involves ghost fields for the shifts of $A$:

$$\tilde{F} = F + \psi_1^1 + \phi^2.$$  (36)

From this relation and the generalized Bianchi identity $\tilde{D}\tilde{F} = 0$, we obtain

$$sA = \psi_1^1 - Dc, \quad s\psi_1^1 = -D\phi^2 - [c, \psi_1^1],$$

$$sc = \phi^2 - \frac{1}{2}[c, c], \quad s\phi^2 = -[c, \phi^2]$$  (37)

and $sF = -D\psi_1^1 - [c, F]$.

In the $B$-sector, the $s$-variations follow from the horizontality condition

$$\tilde{G}_3 = G_3 + \psi_1^2 + \phi_1^2 + \phi^3.$$  (38)
and the generalized Bianchi identity \( \tilde{D}\tilde{G}_3 = [\tilde{F}, \tilde{B}_2] \): they read

\[
\begin{align*}
sB_2 &= \psi_2^1 - D\psi_1^1 - [c, B_2], \\
sV_1^1 &= \varphi_1^2 - Dm^2 - [c, V_1^1], \\
sm^2 &= \phi^3 - [c, m^2], \\
sG_3 &= -D\psi_1^2 - [c, G_3] + [F, V_1^1] + [\psi_1^1, B_2].
\end{align*}
\]

(39)

and \( sG_3 = -D\psi_1^2 - [c, G_3] + [F, V_1^1] + [\psi_1^1, B_2] \). The action (33) is inert under the BRST-transformations (37),(39) which are nilpotent by construction.

### 4.2 Gauge-fixing

The shift- and ordinary gauge symmetry in the \( B \)-sector are fixed as in the free Abelian model, except that all fields are now Lie algebra-valued. Thus, the total gauge-fixing action in the \( B \)-sector is given by

\[
\Sigma^B = \Sigma^B_Q + \Sigma^B_{og},
\]

with

\[
\begin{align*}
\Sigma^B_Q &= s \int \text{tr} \left\{ \chi^{-1}_3 (G_3 + G_3^c) + [\varphi_1^{-2}d * \psi_2^1 + \phi^{-3}d * \varphi_1^2 + \chi_1^1d * \varphi_1^{-2} - CC] \right\}, \\
\Sigma^B_{og} &= s \int \text{tr} \left\{ V_1^{-1}d * B_2 + m^{-2}d * V_1^1 + nd * V_1^{-1} - CC \right\},
\end{align*}
\]

(41)

The BRST-transformations of the anti-ghost and multiplier fields are given by equations (11),(13) and (15).

In the YM-sector, the gauge-fixing can be done along the lines of four-dimensional topological YM-theory. However, the familiar four-dimensional self-duality condition for the curvature form \( F \) does not make sense in six dimensions and it has to be generalized by introducing a 4-form \( T_4 \) which is invariant under some maximal subgroup of \( SO(6) \) [10, 11]: the self-duality condition can then be written as

\[
*F = \Omega_2 F \quad \text{with} \quad \Omega_2 \equiv *T_4.
\]

(42)

This constraint is implemented by the gauge-fixing action

\[
\Sigma^A_{sd} = s \int \text{tr} \left\{ \chi^{-1}_3 (F - \Omega_2 F) \right\},
\]

(43)

where \( \chi^{-1}_3 \) belongs to a BRST-doublet (\( s\chi^{-1}_3 = H_4, sH_4 = 0 \)). The residual gauge symmetries can then be fixed as in topological YM-theory by using a linear gauge-fixing term [12, 6],

\[
\Sigma^A = \Sigma^A_{sd} + s \int \text{tr} \left\{ \bar{\phi}^{-2}d * \psi_1^1 + \bar{\phi}d * A \right\},
\]

(44)

which involves the BRST-doublets (\( \bar{\phi}^{-2}, \eta^{-1} \)) and (\( \bar{c}, b \)). In summary, the total gauge-fixed action is given by \( \Sigma = \Sigma_{cd} + \Sigma^B + \Sigma^A \).
4.3 VSUSY

In order to derive the VSUSY-transformations, one considers 0-type symmetry conditions in the A- and B-sectors:

\[
\begin{align*}
\delta_\tau  \bar{A} &= 0, & \delta_\tau  \bar{F} &= \mathcal{L}_\tau  \bar{A}, \\
\delta_\tau  \bar{B}_2 &= 0, & \delta_\tau  \bar{G}_3 &= \mathcal{L}_\tau  \bar{B}_2.
\end{align*}
\]

By expanding with respect to the ghost-number and substituting the horizontality conditions (36),(38), one finds

\[
\delta_\tau  (A, c) = 0, \quad \delta_\tau  (B_2, V^1_1, m^2) = 0.
\]

and

\[
\begin{align*}
\delta_\tau  \psi_1^1 &= \mathcal{L}_\tau  A, & \delta_\tau  \psi_2^1 &= \mathcal{L}_\tau  B_2, \\
\delta_\tau  \varphi_1^2 &= \mathcal{L}_\tau  c, & \delta_\tau  \varphi_2^1 &= \mathcal{L}_\tau  V^1_1, \\
\delta_\tau  \phi^3 &= \mathcal{L}_\tau  m^2.
\end{align*}
\]

The \(\delta_\tau\)-variations for the BRST-doublets occurring in the gauge-fixing action of the B-sector are given by equations (21) and those in the YM-sector read

\[
\begin{align*}
\delta_\tau  \bar{c} &= -\mathcal{L}_\tau  \bar{\phi}^{-2}, & \delta_\tau  b &= \mathcal{L}_\tau  \bar{c} + \mathcal{L}_\tau  \eta^{-1} \\
\delta_\tau  \chi_4^{-1} &= 0, & \delta_\tau  H_4 &= \mathcal{L}_\tau  \chi_4^{-1} \\
\delta_\tau  \bar{\phi}^{-2} &= 0, & \delta_\tau  \eta^{-1} &= \mathcal{L}_\tau  \bar{\phi}^{-2}.
\end{align*}
\]

The total action is invariant under the given VSUSY-transformations which satisfy the VSUSY-algebra off-shell.

5 Concluding remarks

A possible generalization of the non-Abelian model consists of the addition of a BF-term \(\int \text{tr} (B_4 F)\) (see \[\text{[8]}\] and references therein for such models in arbitrary dimensions): such a term breaks the invariance under shifts of \(A\). Other possible extensions \[\text{[4]}\] are the inclusion of the topological invariant \(\int \Omega_2 \text{tr} (F F)\) (where \(\Omega_2\) is a closed 2-form), which leads to “nearly topological” field theories \[\text{[11]}\], or the addition of a term \(\int \text{tr} (F DZ_3)\) involving a 3-form \(Z_3\). The resulting models can be discussed along the lines of the present paper \[\text{[9]}\].

References

[1] L. Baulieu and P. West, “Six-dimensional TQFTs and twisted supersymmetry,” Phys. Lett. B436 (1998) 97, hep-th/9805200.
[2] H. Ita, K. Landsteiner, T. Pisar, J. Rant, and M. Schweda, “Remarks on topological SUSY in six-dimensional TQFTs,” JHEP 11 (1999) 35, hep-th/9909166.

[3] G.F. Chapline and N.S. Manton, “Unification of Yang-Mills theory and supergravity in ten dimensions, Phys.Lett. 120B (1983) 105;
M. Green, J. Schwarz and E. Witten, Superstring Theory Vol. 2 (Cambridge University Press, 1997).

[4] L. Baulieu, “On forms with non-Abelian charges and their dualities,” Phys. Lett. B441 (1998) 250, hep-th/9808055.

[5] R. A. Bertlmann, Anomalies in quantum field theory (Clarendon Press, Oxford 1996).

[6] F. Gieres, J. Grimstrup, T. Pisar and M. Schweda, “Vector supersymmetry in topological field theories”, hep-th/0002167.

[7] E. Witten, “Topological quantum field theory,” Commun. Math. Phys. 117 (1988) 353;
L. Baulieu and I. M. Singer, “Topological Yang-Mills Symmetry,” Nucl. Phys.B (Proc.Suppl.) 5B (1988) 12;
D. Birmingham, M. Rakowski and G. Thompson, “Topological field theory”, Phys.Rep. 209 (1991) 129.

[8] O. Piguet and S. P. Sorella, Algebraic Renormalization (Springer Verlag, 1995).

[9] T. Pisar, Supersymmetric structures in topological field models (Ph.D. Thesis, TU Wien, June 2000).

[10] E. Corrigan, C. Devchand, D.B. Fairlie and J. Nuyts, “First-order equations for gauge fields in spaces of dimension greater than four”, Nucl.Phys. B214 (1983) 452;
H. Kanno, “A note on higher dimensional instantons and supersymmetric cycles”, Prog.Theor.Phys.Suppl. 135 (1999) 18.

[11] L. Baulieu, H. Kanno and I.M. Singer, “Special quantum field theories in eight and other dimensions”, Commun.Math.Phys. 194 (1998) 149, hep-th/9704167.

[12] A. Brandhuber, O. Moritsch, M. W. de Oliveira, O. Piguet, and M. Schweda, “A Renormalized supersymmetry in the topological Yang-Mills field theory,” Nucl. Phys. B431 (1994) 173–190, hep-th/9407105.