

\textbf{pp and pp Elastic Scattering in a Multipole-Pomeron Model}

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Abstract

We assume that the Pomeron is a sum of Regge multipoles, each corresponding to a finite gluon ladder. From a fit to the diffraction cone data of \( pp \)− and \( \bar{p}p \)− scattering we found that the triple pole is significant for the rise of the ratio \( \sigma_{el}/\sigma_{tot} \) at high energies.

1 Introduction

It has been conjectured recently [1] that the Pomeron instead of being an infinite gluon ladder [2] may appear as a finite sum of gluon ladders corresponding to a finite sum of Regge multipoles with increasing multiplicities. The first term in the \( \ln s \) series contributes to the total cross-section with a constant term and can be associated with a simple pole, the second one (double pole) goes as \( \ln s \), the third one (triple pole) as \( \ln^2 s \), etc. All Pomeron poles have unit intercepts. A new prong opens each time the available energy exceeds the threshold value. The values of threshold parameter were found [1] from a fit to the experimental data. Another important set of parameters is related to the coupling of the gluons to hadrons, giving the relative weight of various diagrams. In ref. [1] they were fitted to the forward scattering data and subsequently they were also calculated in QCD. Below we study this problem both in the forward and non-forward directions.

Because of its complexity, we do not consider here the effect of the progressively opening channels as it was done in ref. [1]. Instead we first consider a model with a simple and dipole Pomeron [DP] contribution used earlier to fit elastic hadron scattering [3], low-\( x \) structure functions [4] and photoproduction of vector mesons at HERA [5].

The DP ansatz reads [3]

\[ P(s, t) = i s g_0 \sum_{i=1}^{2} c_i R_i^2(s) e^{R_i^2(s)t}, \]

where \( c_1 = 1, c_2 = \lambda b - 1 = -\varepsilon; \) \( R_1^2(s) = \alpha'(b + \ln(-i\frac{s}{s_0})); \) \( R_2^2(s) = \alpha' \ln(-i\frac{s}{s_0}) \). Apart from the normalization factor \( g_0 \), this model contains 4 adjustable parameters: \( \lambda, b, s_0 \) and \( \alpha' \), moreover their number can be still reduced within the cone region (i.e. \( |t| \leq 1 \text{ GeV}^2 \) by setting \( \lambda = \frac{1}{b} \)). Here a linear Pomeron trajectory, \( \alpha(t) = 1 + \alpha' t \) is implied. Generalization to include nonlinear trajectory is straightforward. The model is strongly constrained by an integral relation between the residue at the simple and double poles [3], that produces the observed dip-bump structure in the differential cross-section in accordance with the experimental data.

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The Dipole Pomeron model with a unit Pomeron intercept \( \alpha_P \) was used to describe successfully \( pp^- \) and \( pp^- \) elastic scattering at the ISR and the Collider energies, however the ratio \( \sigma_{el}/\sigma_{tot} \) was found \( \sigma_{el} \) to decrease asymptotically for all physical values of the parameters. To obtain an increasing function for this ratio the authors of \( \sigma_{el} \) introduced a factor corresponding to the supercritical Pomeron behavior \( \sigma_{el} \). However in this case the unitarity is broken. It was shown \( \sigma_{el} \) that the unitarity violation occurs at energies only slightly above the Tevatron energy of 1.8 TeV, and therefore it is a problem of the present and not of the future. To avoid this problem, we consider a model containing a finite series of Pomeron terms up to \( \ln^2 s \), in accordance with the unitarity constrains.

2 The model

Our ansatz for the scattering amplitude is:

\[
A_{pp}^P = P + R_f \pm R_\omega, \tag{2}
\]

where we introduce a tripole contribution to the DP in the simplest way (see also \( \sigma_{el} \)):

\[
P(s,t) = isg_0 \left[ b + \ln \left( -\frac{i}{s_0} \frac{s}{s_0} \right) + c \ln^2 \left( -\frac{i}{s_0} \frac{s}{s_0} \right) \right] e^{\alpha_P t} e^{(\alpha_P t - 1) \ln(-\frac{s}{s_0})}. \tag{3}
\]

The Pomeron trajectory:

\[
\alpha_P(t) = \alpha_P(0) + \alpha'_P t + \gamma_P \left( \sqrt{t_P} - \sqrt{-t} \right), \tag{4}
\]

where the two-pion threshold \( t_P = 4m^2_\pi \).

In (2) the \( R_f, R_\omega \) contains the subleading Reggeon contributions (\( f \) and \( \omega \)) to the scattering amplitude:

\[
R_j(s,t) = g_j \left( -\frac{i}{s_0} \frac{s}{s_0} \right)^{\alpha_j(t)} e^{b_j t}, \alpha_j(t) = 1 + \alpha'_j t, j = f, \omega; s_0 = 1 GeV^2. \tag{5}
\]

Recently in paper \( \sigma_{el} \) the contribution of truncated BFKL Pomeron series to the \( \sigma_{tot} \) of \( pp^- \) and \( pp^- \) scattering was studied and it was shown that a reliable description can be obtained by using two orders in this series. As a by-product, the elastic differential cross section was obtained for the diffraction cone at low and high energies with a qualitative description of the experimental data. Contrary to \( \sigma_{el} \), in our model we performed a simultaneous fit to the \( \sigma_{tot} \) and \( d\sigma/dt \) data as follows.

3 Comparison with experiment. Conclusion

In order to determine the parameters that control the \( s \)-dependence of \( A(s,0) \) in a wide energy range \( 4GeV \leq \sqrt{s} \leq 1800GeV \), we used the available data for total cross sections \( \sigma_{tot} \). A total of 66 experimental data has been included for \( t = 0 \). For the differential cross sections we selected the data at the energies \( \sqrt{s} = 19; 23; 31; 44; 53; 62GeV \) (for \( pp^- \)-scattering) and \( \sqrt{s} = 31; 53; 62; 546; 1800GeV \) (for \( pp^- \)-scattering). The squared 4-momentum has been limited by \( 0.05GeV^2 < |t| < 0.5GeV^2 \), because at larger \( |t| \) the influence of the dip region becomes visible (in particular, it can be seen quite clearly at the Collider energy \( \sqrt{s} = 546GeV \)). The total of 729 experimental points have been used in the overall fit.
Figure 1: Predictions of the model with the parameters from Table 1 compared to the experimental data on $\sigma_{tot}$

Figure 2: Diffraction cone of $pp$–scattering (a factor $10^{-2}$ between successive curves is present). The solid curves are fits of the model.
Figure 3: The same as in the Fig. 3 for $\bar{p}p$--scattering.

Figure 4: Calculated ratio of $\sigma_{el}/\sigma_{tot}$ for $\bar{p}p$--scattering compared with experiment.
In the calculations we use the following normalization for the dimensionless amplitude:

\[
\sigma_{\text{tot}} = \frac{4\pi}{s} f m A(s, t = 0), \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2.
\]

The resulting fits for \(\sigma_t\), \(\frac{d\sigma}{dt}\) are shown in Figs. 1-3 with the values of the fitted parameters quoted in Table 1. From these figures we conclude that the multipole Pomeron model corresponding to a sum of gluon ladders up to two rungs fits the data perfectly well in a wide energy region within the diffraction cone. As result, the model gives a good behavior for the ratio \(\sigma_{el}/\sigma_{\text{tot}}\) for \(pp\)–scattering (see Fig. 4).

The rapid decrease of the coefficients of the first three terms (approximately as \(1 : \frac{1}{10} : \frac{1}{100}\)) in the Pomeron series in (3) provides the fast convergence of the series and ensures the applicability of this approximation at still much higher energies.

| \(g_0\) | \(b\) \((GeV^{-2})\) | \(c\) | \(\alpha'_P\) \((GeV^{-2})\) | \(\gamma_P\) \((GeV^{-1})\) | \(\alpha_f(0)\) | \(b_f\) \((GeV^{-2})\) | \(g_f\) | \(\alpha_\omega(0)\) | \(b_\omega\) \((GeV^{-2})\) | \(g_f\) |
|------|----------------|------|----------------|----------------|----------------|----------------|------|----------------|----------------|----------------|------|
| 0.253 | 7.46           | 0.180| 0.266          | 0.137          | 0.777          | 3.04           | 13.3 | 0.524          | 6.86           | 7.97           |

Table 1: Values of the fitted parameters in the model.

In this paper we have explored only the simplest extension of the dipole Pomeron to a tripole. In fact, the scattering amplitude is much more complicated than just a simple power series in \(\ln s\).

Earlier a comprehensive analysis of the \(pp\)– and \(\bar{p}p\)– diffraction cone scattering using the dipole and supercritical Pomeron models was done [11].

On the one hand, though we used just a simplified \(t\)-dependence in the model reasonably good results were obtained. Because the slopes of secondary Reggeons do not influence the fit sufficiently, we have fixed them at \(\alpha'_f = 0.84 \text{ (GeV}^{-2}\) and \(\alpha'_\omega = 0.93 \text{ (GeV}^{-2}\), which correspond to the values of Chew-Frautschi plot. On the other hand, we included the curvature of the Pomeron trajectory that cannot be negligible. It should be also taken into account that the calculated slope of the Pomeron value is close to the generally accepted value \(\alpha'_P = 0.25 \text{ (GeV}^{-2}\).

The quality of our fits \(\chi^2/dof = 2.25\) has not reached the best fit \(\chi^2/dof = 1.3\) obtained in [11]. Nevertheless, we believe this difference can be significantly reduced after adding the data of Coulomb interference region \(|t| < 0.05GeV^2\) to the global fit.

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