Magnetization Transfer by a Quantum Ring Device

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We show that a tight-binding model device consisting of a laterally connected ring at half filling in a tangent time-dependent magnetic field can in principle be designed to pump a purely spin current. The process exploits the spin-orbit interaction in the ring. This behavior is understood analytically and found to be robust with respect to temperature and small deviations from half filling.

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Can we transfer magnetization between two distant bodies directly through a lead, without moving any charges? We need a device that can produce a pure spin current, without any charge current associated to it. In this work we show that in principle Quantum Mechanics allows us to build a device to achieve just that, by a laterally connected ring (Fig. 1a) with half-filled band and spin-orbit interaction and a time-dependent magnetic field in the plane of the ring. Due to the spin-orbit interaction, opposite spin electrons circulate in the ring with opposite chiralities, but then the in-plane time-dependent magnetic field flips spins and pumps them out, yielding a pure spin current in both wires. A current of spin-up electrons hops to the right and an equal current of spin-down electrons hops to the left. Taking advantage of such a mechanism, one might realize a device allowing for the magnetization of systems, in a situation in which no capacitor charging can occur. Such an application of our results, if realized in a practical device, would of course offer a very good solution for the problems connected to Spintronics applications.

In the past several years there has been in the literature a growing interest in persistent as well as transient currents in quantum rings threaded by a magnetic flux, with a promising outlook in the quest for new device applications in spintronics, memory devices, optoelectronics, quantum pumping, and quantum information processing.

Aharonov-Bohm-type thermopower oscillations of a quantum dot embedded in a ring for the case when the interaction between electrons can be neglected, were investigated in the literature, showing it to be strongly flux- and experimental geometry- dependent. Also, the general subject of pumping phenomena in mesoscopic or ballistic conductors has been already addressed by several authors of theoretical papers. Although electron-electron interactions are not needed, in order to introduce the notion of pumping and study the corresponding phenomena, nonetheless including such interactions in this problem, would yield the breakdown of the Fermi liquid, thus leading to the formation of the Luttinger liquid. Within the pumping context, a distinct place was attributed to the pumping properties of a Luttinger liquid and, in particular, of a quantum ring laterally connected to open one-dimensional leads described within the Luttinger liquid model. In a laterally connected ring the external circuit is tangential to the ring and, in such a maximally asymmetric situation, a current in the wires produces a magnetic moment.

The latter is not obtained by substituting the quantum current in the semiclassical formula. Indeed, in previous works it was shown that in nanoscopic circuits containing loops, magnetic moments excited by currents are dominated by quantum effects and depend nonlinearly on the exciting bias, quite at variance from classical expectations of a linear behavior.

The creation of a magnetic dipole by a bias-induced current is a process which can be reversed, by magnetically exciting the ring in the absence of bias. Hence, ballistic rings asymmetrically connected to wires and excited by a time-dependent inner magnetic flux can produce ballistic currents in the external wires even in the absence of an external bias and thus, by the same token, they can be useful, in order to obtain charge pumping. Several methods were found to work, one method being based on the introduction of integer numbers of fluxons, another method consisting in connecting the ring to a junction. In general, by studying the real-time quantum evolution of tight-binding models in different geometries, several kinds of crucial experimental tests of these ideas can be envisaged, resulting in potentially useful devices.

As a direct consequence of the above mentioned nonlinearity, one can achieve, by employing suitable flux protocols, single-parameter nonadiabatic pumping, where an arbitrary amount of charge can be transferred from one side to the other, a phenomenon which, for a linear system, would be readily ruled out by the Brower theorem.

After this introduction and background description, we next proceed to state the model Hamiltonian (see Fig. 1a) as:

\[
H = H_D + H_B
\]
where $H_D$ is the device Hamiltonian and $H_B$ the magnetic term.

$$H_D = H_{\text{wires}} + H_{\text{ring}} + H_{\text{ring-wires}}. \quad (2)$$

the $N$-sided ring is represented by

$$H_{\text{ring}} = t_{\text{ring}} \sum_\sigma \exp[i\sigma \alpha] \hat{c}_{r,1+1,\sigma}^{\dagger} \hat{c}_{r,1,\sigma} + \text{h.c.} \quad (3)$$

Here, following A. A. Zvyagin, we included the spin-orbit interaction as a phase shift $\alpha$ for up-spin and $-\alpha_{SO}$ for down-spin electrons. Both wires are modeled by $H_{\text{wires}} = H_L + H_R = t_h \sum_{n,\sigma} c_{n,\sigma}^{\dagger} c_{n+1,\sigma}$, and the ring-wires contacts are modeled in $H_{\text{ring-wires}}$ whereby the ring is connected to the leads via a tunneling Hamiltonian with hopping $t_{lr}$ connecting two nearest-neighbor sites of the ring denoted with A and B with the ending sites of lead L and R, respectively (Fig. 1a). Below we assume for the sake of definiteness that $t_h = t_{lr} = t_\nu = 1$ eV. At equilibrium the occupation of the system is determined by the spin-independent chemical potential $\mu$, which is assumed to be zero (i.e. we assume half filling). The ring is taken in the x-y plane and the spin-polarized current is excited by a time-dependent external magnetic field $B(t)$ along the x axis. The magnetic interaction is

$$H_B = V(t) \sum_{i \in \text{ring}} (c_{i,\uparrow}^{\dagger} c_{i,\downarrow} + c_{i,\downarrow}^{\dagger} c_{i,\uparrow}) \quad (4)$$

where $V = \mu B$ with the Bohr magneton $\mu = 5.7937510^{-22} \text{eV/Tesla}$. Initially the system is in the ground state with $V = 0$. In order to describe its evolution we need the retarded Green’s function matrix elements on a spin-orbital basis:

$$g_{i,j}^\alpha(t) = \langle i|U_i(t,0)|j \rangle \quad (5)$$

where $U_i(t,0)$ is the evolution operator in the interaction representation; the number current is

$$J_{n,\sigma}(t) = -2 \frac{t_h}{\hbar} \text{Im} (G_{n,\sigma,n-1,\sigma}^<) \quad (6)$$

where

$$G_{i,j}^<(t) = \sum_\mu n^0_{\mu} g_{i,\mu}(t,0) g_{j,\mu}^*(t,0). \quad (7)$$

where $\mu$ denotes the ground state spin-orbitals for $B = 0$ and $n^0_{\mu}$ is the Fermi function.

The analysis of the time evolution is enormously simplified and can be carried out with generality for any $V(t)$ since the model is bipartite (i.e. bonds connect sites of two disjoint sublattices), and can be mapped on a spin-less model which is also bipartite (Fig. 1b). A Dirac monopole (the star in Fig. 1b) ensures the spin-orbit interaction, imparting opposite chirality to the two sub-clusters. Due to the spin-orbit interaction the parity $P : x \rightarrow -x$ and the reflection $\Sigma$ which sends each sub-cluster to the other fail to commute with $H$, but the product $P\Sigma$ is a symmetry. Considering sites at the same distance from the ring on both leads and using the correspondence $a \rightarrow \text{left}$, spin up; $\beta \rightarrow \text{left}$, spin down; $\gamma \rightarrow \text{right}$, spin up, and $\delta \rightarrow \text{right}$, spin down, at any time the currents are constrained by $J_{\alpha} = -J_{\delta}$ and $J_{\beta} = -J_{\gamma}$ and the charge densities obey $\rho_\alpha = \rho_\beta$, $\rho_\beta = \rho_\gamma$.

The study is simplest in the equivalent lattice of the instan-
a pure spin current in the half filled system, we change to an analysis of Ref. 23. We are interested in the non-adiabatic evolution, the beautiful phenomenon in which the Hamiltonian changes very rapidly but the Hamiltonian is conserved. Changing sign to the perturbation treatment in the small parameter \( V \), we may conclude that the current is pure spin current, \( \mu \text{.d} \).

In the adiabatic limit the system is in the instantaneous ground state at each time. Then, no charge current is allowed, since it would alter the occupation of the sites. The transformation of any bond goes as follows: \( t_{n,m}^\sigma c_{n,\sigma}^\dagger c_{m,\sigma} \rightarrow t_{n,m}^\sigma b_{-\sigma}^\dagger b_{-\sigma}^\dagger \). Since \( t^\sigma \) is Hermitian this is the same as \( t_{n,m}^\sigma b_{n,-\sigma}^\dagger b_{m,-\sigma}^\dagger \). On the other hand, in the ring the sites coupled by \( V \) belong to opposite sublattices and so the transformation gives:

\[
H_B \rightarrow V(t) \sum_{i \in \text{ring}} (b_{i,\uparrow}^\dagger b_{i,\downarrow} + b_{i,\downarrow}^\dagger b_{i,\uparrow}).
\]

In this way, at every time \( t \) the transformed hole Hamiltonian \( H(h, b^\dagger) \) depends on \( b \) operators exactly as the original Hamiltonian \( H(c, c^\dagger) \) depends on the \( c \) operators. In both pictures the evolution starts in the ground state at half filling and evolves in the same way. Therefore, at any time and for any site \( n \),

\[
\langle b_{n,\sigma}^\dagger(t) b_{n,\sigma}(t) \rangle_b = \langle c_{n,\sigma}^\dagger(t) c_{n,\sigma}(t) \rangle_c.
\]

Here the l.h.s. is the average at time \( t \) in the \( b \) picture while the r.h.s. is averaged at time \( t \) in the \( c \) picture. Hence, operating the canonical transformation on the l.h.s.,

\[
(1 - n_{n,-\sigma}(t))_c = \langle n_{n,\sigma}(t) \rangle_c,
\]

which implies that the mean total occupation of each site is conserved. This cannot be true if charge currents exist. Indeed, let us consider the operators straddling each bond: since at each time

\[
\langle b_{n+1,\sigma}^\dagger b_{n,\sigma} \rangle_b = \langle c_{n+1,\sigma}^\dagger c_{n,\sigma} \rangle_c
\]

we may conclude that

\[
\langle c_{n,-\sigma}^\dagger c_{n+1,-\sigma} \rangle_c = \langle c_{n+1,\sigma}^\dagger c_{n,\sigma} \rangle_c.
\]

Hence the current is pure spin current, q.e.d.

An analytic formula for the current is desirable, but the perturbation treatment in the small parameter \( V/t \) although elementary, is too involved to be enlightening. In order to achieve a simple estimate of the effect, and capture the essential mechanism producing the spin current, we replace the ring by a renormalised bond, with hopping \( t_b \rightarrow \tau \exp(i\beta_\sigma(t)) \) with \( \tau \sim t_b \), which implies an effective potential drop across the bond which produces the current. Indeed, in terms of the Peierls prescription, this implies a spin-dependent electric field \( \vec{E}_\sigma \) such that \( \beta_\sigma = \frac{\tau}{\hbar} \int e \vec{E}_\sigma \cdot d\vec{l} \). The phase and the effective potential are spin-dependent and produce the spin current.

As a preliminary, in order to motivate the renormalised bond idea, let us consider the simpler problem of spinless electrons in the same device, but with a normal magnetic field producing a flux in the ring. Such a model was studied previously, it was shown that by suitable protocols
one can insert an integer number of fluxons in the ring in such a way that the electronic system in the ring is not left charged and is not excited, while charge is pumped in the external circuit. Writing the number current in units of $\frac{e}{\hbar}$ it turns out that $\int J \, dt$ is of order unity for every fluxon. In this case, the ring is equivalent to an effective bond with hopping $t_h \rightarrow t_h \exp(i\beta(t))$. The time dependent vector potential entails the effective bias across the bond is $e\phi_{eff} = \hbar \beta$. The quantum conductivity of the wire was discussed elsewhere, at small $\phi_{eff}$, the number current is $J = -\frac{e\phi(t)}{2\hbar}$. Integrating over time, one finds that the total charge pumped when a fluxon is swallowed by the ring is $Q = \int J \, dt = \frac{\pi}{2} \sim 1$. In other terms, inserting a flux quantum in the ring we shift an electron in the characteristic hopping time of the system. This simple argument is in good agreement with the numerical results.

In the case with spin, the above approach leads us to change the equivalent model of Figure 1b) to the simplified model of Figure 2, where the vertical bonds again stand for $V(t)$ and the effective bond bears a spin-orbit induced static phase factor $\alpha$ which produces no effect at all for $V = 0$. When $V(t)$ is on, however, the electron wave function in the upper wire can interfere with a time dependent contribution from the opposite spin sector where the phase shift is opposite. Effectively this works like a time dependent phase drop across the upper bond, and an opposite phase drop across the lower one. In first order perturbation theory the amplitude to go from $k_{1+}$ in the upper wire to $k_{2-}$ in the upper wire reads

$$c_{\alpha}(k_1,k_2,t) = -\frac{i}{\hbar} \int_0^t d\tau e^{i\omega(k_2,k_1)\tau}[1 + e^{-2i\alpha + i\Delta_{12}}]V(\tau) \tag{13}$$

where $\omega(k_2,k_1) = 2(\cos(k_2) - \cos(k_1))$ and $\Delta_{12} = k_1 - k_2$. Since the graph of Fig.2 is also bipartite, the current $J$ is spin-dependent and site-independent, and $J\psi_2 = \frac{2\hbar}{\pi} \psi_2$. Therefore the mean current on the upper wire is obtained by summing over occupied states: $\langle J_\alpha \rangle = \frac{2\hbar}{\pi} \sum_{s_1} \sum_{s_2} \sin(k_2)|c_\alpha(k_1,k_2,t)|^2$. The spin current is $J_s(\alpha) = J_\alpha - J_{-\alpha}$. The calculation is completed most simply by taking $V(t) = V \theta(t) \theta(T-t)$ with $V = \mu B$ and then letting $T = \frac{2\hbar}{\alpha}$ (short rectangular spike) with the result that

$$J_s(\alpha) \sim \frac{l_h}{\hbar} \sin(\alpha) \left(\frac{VT}{h}\right)^2. \tag{14}$$

The numerical results of Fig.3 were computed for the full model according to Equation (7) by evolving the quantum state by a time-slicing integration of the Schrödinger equation. They strikingly illustrate the above analytic findings. For any time dependence of $V(t)$ the charge current vanishes identically at half filling. Here we present the results for the case of a sudden switching of $B$. Our codes calculate number currents taking $t_h = 1$. If this is interpreted to mean that $t_h = 1eV$, which corresponds to the frequency $2.42 \times 10^{14}$ s$^{-1}$, a current $J = 1$ from the code means $2.42 \times 10^{14}$ electrons per second, which corresponds to a charge current of $3.87 \times 10^{-5}$ Ampere.

We also tested the validity of the simple approximation of Eq. (14) compared to the full model. For $B = 100$ Tesla and $\alpha = 1$ one finds $J_s = 5 \times 10^{-6} \frac{\mu B}{\hbar}$ and the quadratic dependence on $B$ is fully confirmed. So the simple approximation works for the full model. Finite temperatures do not change significantly the results up to $K_B T \sim 0.025$ eV. This is interesting since up to now, strongly spin-polarized currents have been created and detected in ultra-cold atomic gases only. Instead, the results are sensitive to the filling, but for concentrations of the order of 0.51 one gets a spin current with a small charge current while the ring gets charged.

The present model neglects electron-electron interactions, but it is clearly physical that adding to the Hamiltonian a correlation term like $U(\hat{n}_\uparrow + \hat{n}_\downarrow - 1)^2$ would tend to reinforce the charge confining effects described here; in the Hartree approximation, however, it would change nothing since its average at half filling vanishes strictly during the evolution of the system.

In conclusion, we presented the theoretical analytical description of a tight-binding model device consisting of a laterally connected ring at half filling in a tangent time-dependent magnetic field that can in principle be designed to pump a purely spin current. The process exploits the spin-orbit interaction in the ring, without which the effect would not occur. This behavior is found by our calculations to be robust with respect to temperature and small deviations from half filling. Our analytical treatment revealed unusual physical properties, with potential applications to spintronics.

A wealth of experimental results have been already obtained since the first small quantum rings were fabricated by self-assembled growth of InAs on GaAs but maybe the best is yet to come!

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