Supersymmetric contribution to the quark-lepton universality violation in charged currents

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Abstract

The supersymmetric one-loop contribution to the quark-lepton universality violation in the low-energy charged current weak interactions is studied. It is shown that the recent experimental data with the 1-σ deviation from the universality may be explained by the existence of the light sleptons ($< 220\text{GeV}$) and relatively light chargino and neutralinos ($< 600\text{GeV}$) with significant gaugino components. The sign of the universality violation is briefly discussed.
The tree-level universality of the charged current weak interactions is one of the important consequences of the SU(2)_L gauge symmetry of the fundamental theory. The universality between quarks and leptons is expressed as the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, for example,

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \]  

(1)

Experimentally, these CKM matrix elements are extracted from the ratios of the amplitude of the semileptonic hadron decays to that of the muon decay. In this paper, we adopt the values quoted in refs.[1, 2, 3],

\[ |V_{ud}| = 0.9745 \pm 0.0007 \], \[ |V_{us}| = 0.2205 \pm 0.0018 \], \[ |V_{ub}| = 0.003 \pm 0.001 \]. \hspace{1cm} (2)

Their squared sum is then

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0017 \pm 0.0015. \]  

(3)

The universality is violated at 1-σ level.

In general, the universality (1) can be modified by the process-dependent radiative corrections, due to the spontaneous breaking of the SU(2)_L symmetry. The data (2), which are obtained after subtracting the known Standard Model corrections [1, 3, 4, 5], lead to a small deviation from the quark-lepton universality, although it is only at the 1-σ level. This may suggest a signal for physics beyond the Standard Model (SM).

In this paper, we study the possibility that the universality violation (3) is due to the radiative correction of the unknown new particles in the minimal supersymmetric (SUSY) standard model, MSSM [6, 7], which is one of the most interesting and promising extensions of the SM. We study the one-loop contribution of the SUSY particles to the quark-lepton universality violation in low-energy charged currents.

There are preceding works [8, 9] on this subject. In ref.[8], they studied only cases with very light sfermions which are now experimentally excluded, within the constraints on the MSSM parameter, tan β = 1. In ref.[9], the squark-loop contributions were ignored and only the range of the universality violation under the LEP-I constraints were shown. We refine these works by extending the analysis to cover the whole parameter space of the MSSM, and obtain constraints on SUSY particle masses from the 1-σ universality violation (3). In addition, implications of the sign of the universality violation to the SUSY model parameters are briefly discussed.

We first fix our framework, following the formalism given in ref.[10]. We study the decay \( f_2 \to f_1 \ell^- \bar{\nu} \) where \( f = (f_1, f_2) \) is a SU(2)_L fermion doublet, comparing the following two cases; muon decay (\( f_1 = \nu_\mu, f_2 = \mu^- \)) and semileptonic hadron decays (\( f_1 = u, f_2 = d, s, b \)). At the tree level, their decay amplitudes are identical up to the CKM matrix element, and are expressed in terms of the bare Fermi constant \( G_0 = g^2/4\sqrt{2}m_W^2 \). At the one-loop level, however, the decay amplitudes receive
Following ref.[10], the corrected decay amplitudes of $f_2 \to f_1 \ell^- \bar{v}$ are expressed as

$$G_f = \frac{g^2 W(0) + g^2 \delta_{GF}}{4 \sqrt{2} m_w^2}. \quad (4)$$

The effective coupling $g^2 W(0)$ represents the correction to the W-boson propagator and does not lead to the universality violation. The process-dependent term $\delta_{GF}$, which represents the vertex and box corrections, gives the universality violation. In the MSSM, $\delta_{GF} = \delta_{GF}(SM) + \delta_{GF}(SUSY)$, where $\delta_{GF}(SM)$ is the gauge vector loop contribution and $\delta_{GF}(SUSY)$ is the SUSY loop contribution.

Here we address the definition of the CKM matrix elements in eq.(2). In testing $\delta_{GF}$, we show the analytic form of $G_f^{(SUSY)}$, derived from the loops with left-handed sfermions ($\tilde{\nu}_e, \tilde{e}, \tilde{f}_1, \tilde{f}_2)_L$, charginos $\tilde{C}_j (j = 1, 2)$, neutralinos $\tilde{N}_i (i = 1 - 4)$ and a gluino $\tilde{g}$. It is expressed as a sum $\delta_f^{(v)} + \delta_f^{(e)} + \delta_f^{(b)}$, where we use the abbreviations $\delta_f \equiv \delta_{GF}(SUSY)$ etc. The correction $\delta_f^{(v)}$ to the $W^+ f_1 f_2$ vertex is

$$(4\pi)^2 \delta_f^{(v)} = \frac{g^2}{2} \sum_i |V_{ii}|^2 B_1(\tilde{C}_i, \bar{f}_{1L}) + |U_{ii}|^2 B_1(\tilde{C}_i, \bar{f}_{2L})]
\begin{align*}
+ g^2 Z & |g_{N_{IL}}^{(f_2)}|^2 B_1(\tilde{N}_i, \bar{f}_{2L}) + |g_{N_{IL}}^{(f_1)}|^2 B_1(\tilde{N}_i, \bar{f}_{1L})
\nonumber \nonumber \nonumber
+ 2g^2 Z & \sum_i g_{N_{IL}}^{(f_1)} g_{N_{IL}}^{(f_2)^*} 2C_{24}(\bar{f}_{2L}, \tilde{N}_i, \bar{f}_{1L})
\nonumber \nonumber \nonumber
- 2gg Z & \sum_{i,j} U_{ij} g_{N_{IL}}^{(f_2)^*} \left[ r_{ij}^0 - 2C_{24}(\tilde{N}_i, \bar{f}_{2L}, \tilde{C}_j) - \frac{1}{2} \right] + r_{ij}^0 m_{\tilde{C}_j} m_{\tilde{N}_i} (-C_0(\tilde{N}_i, \bar{f}_{2L}, \tilde{C}_j))
\nonumber \nonumber \nonumber
+ 2gg Z & \sum_{i,j} V_{ij}^* g_{N_{IL}}^{(f_1)} \left[ r_{ij}^0 - 2C_{24}(\tilde{C}_j, \bar{f}_{1L}, \tilde{N}_i) - \frac{1}{2} \right] + r_{ij}^0 m_{\tilde{C}_j} m_{\tilde{N}_i} (-C_0(\tilde{C}_j, \bar{f}_{1L}, \tilde{N}_i))
\nonumber \nonumber \nonumber
+ Cg g_{N_{IL}}^2 [4C_{24}(\tilde{g}, \bar{f}_{2L}, \tilde{f}_{1L}) + B_1(\tilde{g}, \bar{f}_{1L}) + B_1(\tilde{g}, \bar{f}_{2L})].
\end{align*} \quad (5)
where \( C_q = 4/3 \) and \( C_\ell = 0 \). The box correction \( \delta_f^{(b)} \) is

\[
(4\pi)^2 \delta_f^{(b)} = -4 g_Z^2 \sum_{i,j} \left[ g_{NL}^{(e)} (f_{ij})^e |U_{ij}|^2 m_N^2 D_2 \left( \tilde{f}_{2L}, \tilde{e}_L, \tilde{C}_j, \tilde{N}_i \right) 
+ g_{NL}^{(e)} (f_{ij})^u |V_{ij}|^2 m_N^2 D_2 \left( \tilde{f}_{1L}, \tilde{e}_L, \tilde{C}_j, \tilde{N}_i \right) 
- 2 g_Z^2 \sum_{i,j} \left[ g_{NL}^{(e)} (f_{ij})^e |U_{ij}|^2 m_N^2 m_W^2 D_0 \left( \tilde{f}_{2L}, \tilde{e}_L, \tilde{C}_j, \tilde{N}_i \right) 
+ g_{NL}^{(e)} (f_{ij})^u |V_{ij}|^2 m_N^2 m_W^2 D_0 \left( \tilde{f}_{1L}, \tilde{e}_L, \tilde{C}_j, \tilde{N}_i \right) \right].
\]

(6)

Here the couplings of charginos \( \tilde{C}_j \) and neutralinos \( \tilde{N}_i \) are given by

\[
g_{NL}^{(e)} = I_{3f} N_{i2} \cos \theta_W + (Q_f - I_{3f}) N_{i1} \sin \theta_W,
\]

\[
r_{ij}^{0-} = N_{i2} U_{j1}^* + \frac{1}{\sqrt{2}} N_{i3} U_{j2}^*,
\]

\[
r_{ij}^{0-} = N_{i2} V_{j1} - \frac{1}{\sqrt{2}} N_{i4} V_{j2},
\]

(7)

and \( g_Z = g / \cos \theta_W \). \( U, V, N \) are the mixing matrices for charginos and neutralinos [12]. The masses of external fermions are ignored in eqs. (5,6). We adopt the notation of ref. [10] for the \( \alpha, \beta, M \) parameters are set as \( \alpha = 0.2312, \alpha (M_Z) = 1/128.72 \) and \( \alpha_s (M_Z) = 0.12 \).

The quark-lepton universality violation (3) is now expressed as

\[
\delta q^\ell \equiv \frac{\delta G_q}{G_q} - \frac{\delta G_\mu}{G_\mu} = \left( |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right)^{1/2} - 1
= \delta q - \delta \ell
= (\delta q^{(v)} + \delta \ell^{(v)} + \delta q^{(b)}) - (2\delta \ell^{(v)} + \delta \ell^{(b)})
= (\delta q^{(v)} + \delta q^{(b)}) - (\delta \ell^{(v)} + \delta \ell^{(b)})
= -0.0009 \pm 0.0008.
\]

(8)

In the following numerical estimates of \( \delta q^\ell \), we assume the generation independence of the sfermion masses, so that the relevant SUSY particle masses and couplings are parameterized in terms of the seven variables \( (M_{\tilde{q}}, M_{\tilde{L}}, M_2, M_1, \mu, \tan \beta, M_{\tilde{g}}) \) [6, 7, 12]. We further impose the following mass relations suggested by the minimal supergravity model with grand unification [3]

\[
M_1 = \frac{5}{3} M_2 \tan^2 \theta_W, \quad M_{\tilde{g}} = \frac{g_2^2}{g_1^2} M_2, \quad M_{\tilde{Q}}^2 = M_{\tilde{L}}^2 + 9 M_2^2,
\]

(9)

to reduce the number of independent parameters to four. The standard model parameters are set as \( m_W = 80.24 \) GeV, \( \sin^2 \theta_W = 0.2312, \alpha (M_Z) = 1/128.72 \) and \( \alpha_s (M_Z) = 0.12 \).
In Fig.1, the quark-lepton universality violation $\delta_{q\ell}$ is shown in the $(M_2, \mu)$ plane for several values of $(\tilde{m}_\nu, \tan \beta)$. The solid lines are contours for constant $\delta_{q\ell}$'s. The regions below the thick solid lines ($\delta_{q\ell} = -0.0001$) are consistent with the 1-$\sigma$ universality violation (3). The regions below the thick dashed lines are excluded by LEP-I experiments. Therefore the regions under the thick solid lines and above the thick dashed lines are favored by the present data. For the $(m_{\tilde{\nu}} = 200\text{GeV}, \tan \beta = 2)$ case (Fig.1c), however, the favorable regions lie in the $|\mu| > 500\text{GeV}$ region, outside of the frame of the figure. As seen in the figure, the SUSY parameters which satisfy the universality violation (3) and the LEP-I bound tend to lie in the $M_2 \lesssim |\mu|$ region, where the lighter chargino and neutralinos are gaugino-like. On the other hand, when $M_2 \gtrsim |\mu|$, $\delta_{q\ell}$ tends to be positive and disfavors the negative deviation (3). We find that the 1-$\sigma$ allowed region in the $(M_2, \mu)$ plane reduces with increasing $m_{\tilde{\nu}}$, while the $\tan \beta$ dependence is not significant. For example, we find that the favored regions for the $(m_{\tilde{\nu}} = 100\text{GeV}, \tan \beta = 10)$ case are similar to those for the $(m_{\tilde{\nu}} = 100\text{GeV}, \tan \beta = 2)$ case (Fig.1b). Note, however, that under the LEP-I kinematical constraints, the SUSY contribution to $\delta_{q\ell}$ cannot reach its present central value of eq.(8), $-0.0009$.

In Fig.2, the 1-$\sigma$ allowed region of masses of the sneutrino $\tilde{\nu}$ and the lighter chargino $\tilde{C}_1$ from the quark-lepton universality violation (3) is shown. Although the allowed regions in Fig.1 are unbounded in the directions of $|\mu| \to \infty$, $m_{\tilde{\nu}}$ is bounded from above. The 1-$\sigma$ (67% C.L.) upper bounds are roughly $m_{\tilde{\nu}} < 220\text{GeV}$ and $m_{\tilde{C}_1} < 600\text{GeV}$, respectively. Therefore, the 1-$\sigma$ deviation (3) from the quark-lepton universality tends to favor light sleptons and relatively light chargino and neutralinos with significant gaugino components. It is interesting that the upper bound of $m_{\tilde{\nu}}$ increases with increasing $m_{\tilde{C}_1}$ for $50\text{GeV} < m_{\tilde{C}_1} < 100\text{GeV}$, similar to the case of $\delta_{\ell}$ that has been studied in ref.[15].

Finally, we examine implications of the sign of $\delta_{q\ell}$ for the SUSY model. As seen in Fig.1, $\delta_{q\ell}$ takes both signs, contrary to the result of ref.[8] where only cases with very light sfermions ($M_{\tilde{L}} < m_Z/2$, $M_{\tilde{Q}} < m_Z$) were studied and only negative $\delta_{q\ell}$ was found. In fact, it is a cancellation between the vertex and box corrections that causes the sign change and the non-monotonically decreasing behavior of $\delta_{q\ell}$ as mentioned above. In Fig.3, each term in the third line of eq.(8) is shown with their sums. Individual corrections $\delta^{(v)}$'s, $\delta^{(b)}$'s take definite signs, namely $\delta^{(v)}_{\ell,q} < 0 < \delta^{(b)}_{\ell,q}$, while the sum $\delta_{\ell}$ takes both signs. We can see that the universality violation $\delta_{q\ell}$ is basically determined by $\delta^{(v)}_{\ell} + \delta^{(b)}_{\ell}$. The reason is that the squarks are heavier than the sleptons in our assumption (4). We find that if we set $M_{\tilde{Q}} = M_{\tilde{L}}$ instead of (4), the magnitude of $|\delta_{q\ell}|$ is much reduced. We also find that the gluino contribution to $\delta_{q\ell}$ is completely negligible, less than $O(10^{-6})$, for our parameter choice.

In conclusion, we have studied the SUSY contribution to the quark-lepton universality violation in low-energy charged current interactions, for the whole parameter space of the MSSM. We have shown that the 1-$\sigma$ universality violation (3) in recent data may be a signal of the light sleptons and relatively light chargino and neutralinos.
with significant gaugino components. We have also briefly discussed the sign of the
SUSY contribution to $\delta_{q\ell}$. Although it is a hard task to reduce the uncertainty of the
CKM matrix elements in eq.(2), further improvements in the determination of the
matrix elements ($V_{ud}, V_{us}$) will give important informations on the SUSY particles.

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Figure Captions

Fig.1 The SUSY contribution to the quark-lepton universality violation parameter $\delta_{q^*}$ in the $(M_2, \mu)$ plane for $(m_{\tilde{\nu}}(\text{GeV}), \tan \beta) = (50,2)(a), (100,2)(b), (200,2)(c)$ and $(100,1)(d)$. The SUSY contribution explains the universality violation (3) at the 1-$\sigma$ level in regions below the thick solid lines. In Fig.1c, the allowed regions are outside of the frame. The regions below the thick dashed lines are excluded by LEP-I experiments.

Fig.2 The 1-$\sigma$ allowed region of $(m_{\tilde{C}_1}, m_{\tilde{\nu}})$ for explaining the universality violation (3) as the SUSY contribution for $\tan \beta = (1, 2, 10)$.

Fig.3 $\delta_{q}^{(v)}, \delta_{l}^{(v)}, \delta_{q}^{(b)}, \delta_{l}^{(b)}$ and their sums as functions of $\mu$ for $M_2 = 200\text{GeV}, m_{\tilde{\nu}} = 100\text{GeV}$ and $\tan \beta = 2$. 

Fig. 1
\[ \tan \beta = 1 \]
\[ \tan \beta = 2 \]
\[ \tan \beta = 10 \]

allowed region (1-\(\sigma\))

excluded region (1-\(\sigma\))

Fig. 2
\[ \delta q = \delta q^{(v)} + \delta l^{(v)} + \delta q^{(b)} + \delta l^{(b)} \]

\[ \tan \beta = 2 \]

\[ M_2 = 200 \text{ GeV} \]

\[ m_{\nu} = 100 \text{ GeV} \]

**Fig. 3**