Three-dimensional Heat Transfer Analysis of Two Wire Tandem Submerged Arc Welding

Degala Venkat KIRAN,1) Biswajyoti BASU,2) Arun Kumr SHAH,2) Sourav MISHRA3) and Amitava DE1)

1) Department of Mechanical Engineering, Indian Institute of Technology Bombay, India. 2) Naval Materials Research Laboratory, Thane, Mumbai, India. 3) Department of Metallurgical Engineering and Materials Science, IIT Bombay, India. (Received on December 17, 2010; accepted on January 26, 2011)

Two wire tandem submerged arc welding process facilitates high rate of joint filling with little increase in the overall rate of heat input due to the simultaneous deposition from two electrode wires. Since the lead wire is usually connected to a DC welding arc and the trail wire to a pulsed AC arc, the tandem process requires appropriate selection of a large number of process variables. A quantitative understanding of the effect of the welding conditions on weld joint dimensions and weld thermal cycle is difficult through experimental studies only. Here we present a three-dimensional heat transfer analysis based on finite element method using two independent volumetric heat sources to account for heat input from two welding arcs. The shapes of the heat sources are estimated based on the original joint geometry and welding conditions. The results show that the trail wire current pulses significantly influences the reinforcement height and weld width while lead wire current affects the depth of penetration. For a constant trail wire effective current, increase in the negative pulse time results in greater reinforcement height and reduced weld width with very little influence on the cooling rate and weld strength. In contrast, increase in trail wire negative current pulse increases both reinforcement height and weld width while reduces cooling rate and weld strength.

KEY WORDS: tandem submerged arc welding; numerical modeling; heat transfer analysis.

1. Introduction

Numerical modeling of heat transfer phenomenon has been successfully used for the estimation of peak temperature and thermal cycle in fusion welding processes. In many cases, the computed thermal cycles are used to predict the final microstructure and mechanical properties of the weld. The typical nature of the submerged arc welding (SAW) process makes it difficult to measure peak temperature and thermal cycles in the weld experimentally. In particular, the two wire submerged arc welding (SAW-T) process provides a greater difficulty due to the application of two welding arcs and use of greater amount of fluxes. A recourse is thus to develop reliable process model based on physical principles to compute peak temperature and thermal cycles in SAW-T process.

Although significant efforts are reported in modeling fusion welding processes without any material deposition, similar process models that can also include deposition of electrode material are only a few. In fusion welding processes involving electrode deposition, only a fraction of the heat is transferred from the electric arc to the weld pool that also receives superheated metal drops from the consumable electrode. Tekriwal et al. simulated the transient temperature distribution in gas metal arc welding (GMAW). The metal transfer from the consumable electrode was approximated in the form of heated elements at 2300 K, which were added to the solution domain periodically as the arc moved along the plate. Pardo et al. modeled GMAW process considering that 60% of the available arc energy was used by electrode wire and the rest by the work piece. Kumar et al. assumed a cylindrical heat source that entailed the heat energy from the welding arc and also from molten droplets in modeling GMAW process. However, the computed results were sensitive to the assumed superheated temperature of the molten electrode droplets. Fanous et al. and Mahapatra et al. used element deactivation and activation technique, available in commercial finite element (FE) software, to simulate the deposition of the electrode metal. Mandal et al. considered two independent volumetric heat sources, superimposed on each other, to simulate heat input during the peak and base current pulses for typical pulsed GMAW process. The volumetric heat sources were defined based on experimentally measured weld shapes.

In summary, although numerical process models have facilitated a better understanding of consumable electrode fusion welding processes, very little effort is made to model complex process such as SAW-T that involves two electrode wires each connected to an independent arc. The fact that one arc is connected to a direct current while the other to a pulsed alternating current polarity makes such an analysis all the more difficult. The authors present here a 3D conduction heat transfer model of the SAW-T process using the commercial finite element software. The model considers two individual volumetric heat sources to account for heat input from the lead and the trail arcs. The novelty of this
work is that each of the volumetric heat sources are approximated based on weld joint geometry and process parameters. Further, a unique attempt is made to account for the influence of variable polarity ever present in the pulsed alternating trail arc on the distribution of arc energy based on available experimental results in independent literature. The computed weld dimensions and thermal cycles in heat affected zone (HAZ) are validated with the corresponding experimental measured results. The effect of the pulsating nature of the trail arc current on the weld dimensions, cooling rate and the mechanical properties of the weld is studied.

2. Experimental Investigation

Table 1 shows the chemical composition of base plate and filler wire of HSLA steel used in all the experiments. Table 2 depicts the welding conditions used to prepare the bead-on-groove weld samples in plates of 780 mm × 360 mm × 12 mm (thickness) with a groove angle of 45°. The weld dimensions are measured on transverse weld cross-section after polishing and etching the weld sample with 2% Nital solution. All-weld sub-size tensile specimens (length ~ 25 mm, width and thickness ~ 6 mm) are used to measure weld strength as per ASTM E8M in a pc-interfaced universal tensile testing machine at a crosshead speed of 5 mm/min. Three repeated measurements are done at each welding condition.

3. Theoretical Formulation

A three-dimensional transient heat conduction analysis is carried out to simulate the SAW-T process with the governing equation given as

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + Q = \rho C \frac{\partial T}{\partial t} \ldots (1)$$

where ρ, C and k refer respectively to density, specific heat and thermal conductivity; T and t are temperature and time variables, respectively. In Eq. (1), Q depicts rate of internal heat generation per unit volume and for the arc heat input, and estimated as

$$Q_i = 6\sqrt{3} \pi \rho P \frac{f_i P_{le}}{a_i b_i c_i} \exp \left( -\frac{3x^2}{a_i^2} - \frac{3y^2}{b_i^2} - \frac{3z^2}{c_i^2} \right) \ldots (2)$$

where the subscript i refers to either lead (i = LE) or trail (i = TR) arc. The power from the lead arc to the work piece (P_{LE}) is computed as

$$P_{LE} = \eta \alpha V_{LE} I_{LE} \ldots (3)$$

where η is the process efficiency, I_{LE} and V_{LE} are the lead arc current and voltage, respectively, and α is the fraction of the available arc energy supplied to the work piece in direct current electrode positive (DCEP) mode. Since the trail arc current is in alternating pulse mode, the effective trail wire current and trail arc power to the work piece are computed respectively as

$$I_{TR} = \left[ \frac{I_{LE}^{TR} + I_{TR}^{LE}}{t_{TR}^{LE} + I_{TR}^{TR}} \right] \ldots (4)$$

$$P_{TR} = \left[ \eta V_{TR} \left( \gamma I_{LE}^{TR} + \beta I_{TR}^{LE} \right) \right] \ldots (5)$$

where, I_{LE}^{TR} and I_{TR} are the positive and negative current pulses, respectively, and, t_{TR} and t_{LE} the corresponding time durations. In Eq. (5), γ and β refer to the fractional arc energy supplied to work piece in the positive and negative current pulses, respectively. The values of both α (in Eq. (3)) and γ (in Eq. (5)) are taken as 0.75, and of β (in Eq. (5)) is considered to be 0.65,14,18 The factor f in Eq. (2) allows partitioning of total arc energy towards the front and the rear of the heat source with respect to the arc center for a moving welding arc.19 The value of f is considered to be 0.6 for the front and 1.4 for the rear portion of the heat source based on literature.20 The value of process efficiency, η in SAW is reported to be smaller in direct current electrode negative (DCEN) polarity compared to in DCEP and varies in the range of 0.90 to 0.99.15,17 To account for the value of η for a DCEP lead arc and a pulsed AC trail arc in the present work, a ratio is defined as

$$EN = \frac{I_{LE} I_{TR}}{I_{LE}^{TR} + I_{TR}^{TR}} \ldots (6)$$

where EN refers to electrode negative ratio and can vary from 0.5 (a balanced AC) to 1.0 (complete DCEN). It is assumed that the value of η will increase linearly from 0.90 to 0.99 as EN ratio will reduce from 1.0 to 0.5 and will remain constant thereafter. The terms a_{LE}, b_{LE}, c_{LE} and a_{TR}, b_{TR}, c_{TR}, which are inherent in Eq. (2) and required to define the volumetric heat source terms corresponding to the lead and the trail arcs, are estimated analytically as shown in

Table 1. Chemical composition (in mass%) of base metal and filler wire.

| Element | C | Mn | Si | S | P | Cr | Ni | Cu | V | Al | Mo |
|---------|---|----|----|---|---|----|----|----|---|----|----|
| Base metal | 0.09 | 1.35 | 0.28 | 0.004 | 0.004 | 0.07 | 0.72 | 0.02 | 0.04 | 0.05 | — |
| Filler wire | 0.02–0.06 | 1.1–1.5 | 0.10–0.25 | <0.01 | <0.01 | 2.2–2.5 | <0.05 | <0.01 | 0.01–0.02 | <0.1 | 2.2–2.5 | <0.05 | <0.01 | 0.01–0.02 | <0.1 |

Table 2. Welding conditions used for the experimental investigations.

| Parameters | Legend |
|------------|--------|
| Lead wire current, A | I_{LE} |
| Trail wire positive current pulse, A | I_{TR} |
| Trail wire negative current pulse, A | I_{TR} |
| Trail wire negative pulse time, ms | t_{TR} |
| Welding speed, mm s⁻¹ | v |
| Inter wire distance, mm | L |
| Lead and trail wire extensions, mm | E_{LE}, E_{TR} |

© 2011 ISIJ 794
Appendix-I.

Figure 1 schematically presents the solution domain, volumetric heat sources and the electrode wire metal deposition in weld pool considered by discrete addition of new elements. A symmetrical analysis is carried out considering the weld interface along the middle of the V-groove as the symmetry plane. In Fig. 1, M1, M2 and M3 correspond to electrode material already deposited in a previous time step, yet to be deposited (i.e. deactivated elements) and deposited in the current time step (i.e. newly activated elements), respectively. Thus, elements designated as M1 and M3 are assigned with the thermophysical properties of work piece material. The M2 set of elements are assigned with very low values of thermal conductivity to mimic atmosphere (thermally insulator). A new set of elements (M3) are activated at the liquidus temperature in the beginning of each time step. The time stepping and the activation scheme of new elements are synchronised in a manner such that, two separate sets of new elements (M3) will be activated in each time step to account for the depositions from the lead and the trial wires.

4. Results and Discussion

Figures 2(a) and 2(b) show a comparison of the instantaneous current waveform of the trail wire current for two different sets of pulse parameters that numerically result to the same value (~ 618 A) of trail wire effective current ($I_{TR}$). In Fig. 2(a), the positive ($I_{TR}^+$) and the negative ($I_{TR}^-$) current pulses are 378 and 797 A, and the corresponding pulse times are 7.11 and 9.56 ms, respectively. Similarly, the positive and the negative current pulses are 516 and 654 A, and the corresponding pulse times as 4.67 and 12.0 ms, respectively, in Fig. 2(b). Increase in $I_{TR}^+$ improves the overall process efficiency, $\eta$, and increase in $I_{TR}^-$ enhances the rate of electrode melting, although the value of $\eta$ tends to reduce in the later case.

The pulsating nature of the trail arc current, as shown in Fig. 2, indicates that the numerical calculations should account for the corresponding heat input during the positive and negative current pulses for reliable prediction of weld pool dimensions. A comparison between the numerically computed weld shapes using pulsed current (i.e. $I_{TR}^+$, $I_{TR}^-$, $I_{TR}^+_{TR}$ and $I_{TR}^-$) and effective current ($I_{TR}$) for a typical welding condition is presented in Figs. 3(a) and 3(b). The corresponding experimentally measured weld fusion zone is characterized by the typical columnar grains oriented perpendicularly towards the weld center. The computed weld pool shapes are represented in terms of the temperature isotherms with the zones heated above 1745 K and in between 1745 K and 1000 K as the fusion zone and the HAZ, respectively. Table 3 shows the temperature dependent thermal properties used for the calculations. Figure 3 clearly indicates that the discrepancy between the computed and the corresponding measured weld pool shape is the minimum when pulsating nature of the trail current is used (Fig. 3(a)). For example, the computed values of the weld width, depth of penetration and reinforcement height are 17.8, 8.7 and 1.4 mm, respectively when actual trail wire current pulse is con-
considered as opposed to 18.5, 12.0 and 1.35 mm, respectively, when only ITR is used. The corresponding values of the measured weld width, depth of penetration and reinforcement height are 21.6, 8.9 and 1.7 mm, respectively. Figures 4(a) and 4(b) show the comparisons between the measured and the computed weld pool shapes, which are calculated considering the pulsating nature of trail arc current, for two different welding conditions. It is thus clear in Figs. 3(a) and 4(a–b) that the predicted weld pool shapes are in good agreement with the corresponding measured results when the pulsating nature of the trail arc current is used in the model calculations.

Figures 5(a) and 5(b) show a comparison between the computed and the correspondingly measured thermal cycles at two different values of ITR keeping other welding conditions same. The longitudinal, transverse and depth locations of the thermocouples for the measured thermal cycle in Figs. 5(a) and 5(b) are (300 mm, 6.0 mm, 5.5 mm) and (300 mm, 6.5 mm and 5.5 mm), respectively. The transverse distance is from the original weld interface, the longitudinal distance is from the weld start point and the depth is from the top surface. Figures 5(a) and 5(b) show that as ITR increases from 563 to 797 A, the measured cooling rate, $\Delta T_{8/5}$ (from 1 073 to 773 K), decreases from 4.5 to 3.1 K/s. The corresponding computed values of $\Delta T_{8/5}$ are 5.2 and 3.7 K/s, respectively. Increase in ITR increases heat input from the trail arc (Eq. (5)) resulting in greater weld joint dimen-

### Table 4

| Data sets | ILE (A) | ITR (A) | ITR (ms) | v (mm/s) | H (J/mm) |
|-----------|---------|---------|----------|----------|----------|
| 1         | 300     | 399     | 572      | 10.44    | 12.225   | 1398     |
| 2         | 506     | 378     | 563      | 9.56     | 14.45    | 1486     |
| 3         | 445     | 374     | 545      | 12.53    | 12.225   | 1623     |
| 4         | 445     | 399     | 572      | 10.44    | 12.225   | 1684     |
| 5         | 445     | 409     | 600      | 8.35     | 12.225   | 1711     |
| 6         | 506     | 344     | 563      | 9.56     | 10.0     | 2 110    |
| 7         | 506     | 344     | 563      | 11.32    | 10.0     | 2 133    |
| 8         | 384     | 344     | 797      | 11.32    | 10.0     | 2 163    |
| 9         | 506     | 344     | 797      | 11.32    | 10.0     | 2 427    |

Figures 5, 6 and 7 show the comparisons between the measured and the computed weld pool shapes with: (a) $I_{\text{LE}} = 344$ A, $I_{\text{TR}} = 563$ A, $\tau_{\text{TR}} = 7.11$ ms, $\tau_{\text{TR}} = 9.56$ ms, and (b) $I_{\text{LE}} = 344$ A, $I_{\text{LE}} = 797$ A, $\tau_{\text{TR}} = 5.35$ ms, $\tau_{\text{TR}} = 11.32$ s. Values of $V_{\text{TR}}, V_{\text{T}}, v$ and $I_{\text{TR}}$ are same as in Fig. 3.

Figures 6(a) to 6(c) depict the influences of several welding conditions (shown in Table 4) variables on the measured and corresponding computed values of weld bead dimensions. A few points are worth noting in Figs. 6(a) to 6(c). Firstly, the weld dimensions corresponding to the data sets #2 and #6 show that increase in welding speed reduces the weld width, depth of penetration and reinforcement height at constant lead and trail wire currents. For example, an increase in welding speed from 10 to 14.45 mm/s reduces weld width from 21.0 to 15.8 mm, penetration from 9.1 to 8.5 mm and reinforcement height from 1.3 to 0.3 mm. Secondly, increase in ILE increases the depth of penetration for a constant welding speed and effective trail wire current (data sets #8 and #9). However, increase in TR decreases the weld width and depth of penetration while increases the reinforcement height (data sets #3, #4 and #5). Increase in ILE increases the rate of heat input per unit length resulting in greater weld joint dimen-

---

*Fig. 4.* Comparison of experimentally measured and corresponding computed weld pool shapes with: (a) $I_{\text{TR}} = 344$ A, $I_{\text{TR}} = 563$ A, $\tau_{\text{TR}} = 7.11$ ms, $\tau_{\text{TR}} = 9.56$ ms, and (b) $I_{\text{TR}} = 344$ A, $I_{\text{TR}} = 797$ A, $\tau_{\text{TR}} = 5.35$ ms, $\tau_{\text{TR}} = 11.32$ s. Values of $V_{\text{TR}}, V_{\text{T}}, v$ and $I_{\text{TR}}$ are same as in Fig. 3.

*Fig. 5.* Comparison of experimentally measured and corresponding computed thermal cycles for two different welding conditions given in Table 4.
sions. An increase in welding speed reduces the rate of heat input and weld joint dimensions. For a constant trial wire effective current ($I_{TR}$), increase in EN ratio i.e. increase in either $I_{LE}$ or $I_{TR}$ primarily increases the rate of electrode melting leading to greater reinforcement height. An increase in $I_{TR}$ also increases the overall rate of heat input resulting in greater weld width. In contrast, increase in $I_{TR}$ reduces the arc heating of workpiece resulting in reduced weld width and depth of penetration. A fair agreement between the computed and the corresponding measured weld dimensions is also apparent in Figs. 6(a) to 6(c).

Figures 7(a) and 7(b) show the influence of the welding conditions (data sets #1, #3, #4, #7 and #9 in Table 4) on the cooling rates and weld joint tensile mechanical properties. Table 4 shows the heat input per unit length ($H$) corresponding to each welding condition, which is computed as $H = (P_{LE} + P_{TR})/v$ where $P_{LE}$ and $P_{TR}$ are lead and trail arc powers, respectively. Figure 7(a) depicts that increase in $I_{LE}$ from 300 to 445 A reduces cooling rate, $\Delta T_{KS}$, from 9.5 to 6.8 K/s in weld pool and from 8.7 to 5.2 K/s in HAZ (data sets #1 and #4). Consequently, the ultimate tensile strength (UTS) and yield strength (YS) of the weld joint reduce from 713 to 650 MPa and from 570 to 535 MPa, respectively while the percent elongation increases from 23.0 to 25.4 as shown in Fig. 7(b). Figures 7(a) and 7(b) also show that increase in $I_{TR}$ reduces cooling rate ($\Delta T_{KS}$) both in weld pool and HAZ and the weld joint strength (data sets #7 and #9). An increase in either $I_{LE}$ or $I_{TR}$ increases the heat input per unit length resulting in larger weld pool and reduced cooling rate that is likely to suppress the formation of strengthening phases e.g. acicular ferrite in weld.\(^{13}\)

Direct measurement of the peak temperature and thermal cycles in the weld is difficult, in particular, during SAW-T process as the weld pool is completely hidden within large volume of flux. Moreover, realizing the influence of manifold welding conditions on weld attributes is a challenging task in SAW-T process. The present work demonstrates that a heat transfer analysis considering two independent volumetric heat sources for the lead and the trail arcs can predict the weld pool shape, thermal cycle and cooling rate fairly accurately in SAW-T process. A novel approach is presented to define the geometry of the volumetric heat sources based on specific welding condition and the original joint geometry that does not need any a-priori knowledge of the final weld geometry, which has been a common limitation for the use of such source term. Furthermore, the model calculations have shown the need to consider the pulsating nature of trail arc current in estimating heat input for reliable prediction of weld geometry. The computed values of weld dimensions and thermal cycles have shown fair agreement with the corresponding measured results for a range of welding conditions.

5. Conclusions

A three-dimensional heat transfer model with two independent volumetric heat sources to simultaneously account for a direct current lead arc and a pulsed alternating current trail arc in SAW-T process is reported in the present work. For a given welding speed, the depth of penetration is primarily influenced by the lead wire current, whereas the weld width and reinforcement height are mainly affected by the trail wire current pulses. Increase in trail wire negative current pulse leads to greater reinforcement height, larger weld pool, reduced cooling rate and joint strength with little improvement of percent elongation characteristics. For a given welding speed, lead wire current and effective trail wire current, increase in trail wire negative pulse time reduces weld width and increases the reinforcement height with little influence on the cooling rate and weld joint tensile properties. The influence of the trail wire current pulses in estimating heat input distribution onto workpiece and electrode wire should be considered for reliable prediction of weld pool geometry.

Acknowledgement

The authors gratefully acknowledge the financial support provided by the Government of India (grant no. NMRL/PP&C/1207/MISC) to carry out the present research work.

REFERENCES

1) S. A. David and T. DebRoy: Sci., 257 (1992), 497.
2) T. DebRoy and S. A. David: Rev. Mod. Phys., 67 (1995), 85.
3) T. Zacharia, J. M. Vitek, J. Goldak, T. DebRoy, M. Rappaz and H. K. D. H. Bhadeshia: Modell. Simul. Mater. Sci. Eng., 3 (1995), 265.
4) H. K. D. H. Bhadeshia, L. E. Svensson and B. Grettlof: Acta Metall., 33 (1985), 1271.
5) A. A. B. Sugden and H. K. D. H. Bhadeshia: Metall. Trans. A, 19 (1988), 1597.
6) A. De, C. A. Walsh, S. K. Matti and H. K. D. H. Bhadeshia: Sci. Technol. Weld. Join., 8 (2003), 361.
7) P. Tekriwal and J. Mazumdar: Weld. J., 67 (1988), 150s.
8) E. Pardo and D. C. Weckman: Metall. Trans. B, 20 (1989), 937.
9) S. Kumar and S. C. Bhaduri: Metall. Trans. B, 25 (1994), 435.
10) I. F. Z. Fanous, M. Y. A. Younan and A. S. Wifi: ASME J. Pressure Vessel Technol., 125 (2003), 144.
11) M. M. Mahapatra, G. L. Datta, B. Pradhan and N. R. Mandal: Int. J. Press. Vess. Pipin., 83 (2006), 721.
12) A. Mandal and R. S. Parmar: ISIJ Int., 47 (2007), 1485.
13) D. V. Kiran, B. Basu, A. K. Shah, S. Mishra and A. De: Sci. Technol. Weld. Join., 15 (2010), 111.
14) W. G. Essers and R. Walter: Weld. J., 60 (1981), 37s.
15) N. Christensen, V. L. Davies and K. Gjermundsen: Br. Weld. J., 12 (1965), 54.
16) R. S. Chandel: Mater. Manuf. Process., 13 (1998), 181.
17) A. Sharma, N. Arora and B. K. Mishra: Int. J. Adv. Manuf. Technol., 38 (2008), 1114.
18) J. F. Lancaster: The Physics of Welding, Pergamon Press, New York, (1984), 204.
19) J. Goldak, A. Chakravarti and M. Bibby: Metall. Trans. B, 15 (1984), 299.
20) S. Bag and A. De: Metall. Trans. A, 39 (2008), 2698.
21) P. L. Harrison and R. A. Farrar: Met. Constr., 19 (1987), 392R.
22) K. Easterling: Introduction to Physical Metallurgy of Welding, Butterworths-Heinemann, London, (1992), 104.
23) SYSWELD example manual, ESI Group, Paris, (2008).
Appendix A

The volume of the semi-ellipsoidal heat source corresponding to the lead arc or the trail arc can be given as

\[ \Omega_i = \frac{2}{3} \pi a_i b_i c_i \]  
(A1)

where \(a_i\) and \(b_i\) are the semi-major and semi-minor axes, respectively, \(c_i\) is the depth of the semi-ellipsoid, and the subscript \(i\) refers to either lead (\(i \equiv \text{LE}\)) or trail (\(i \equiv \text{TR}\)) arc. Thus, \(\Omega_{\text{LE}}\), corresponding to the lead arc heat source will include molten workpiece volume under the arc and the unfilled V-groove volume. In contrast, \(\Omega_{\text{TR}}\), corresponding to the trail arc heat source will include molten workpiece volume under the arc and the partially filled V-groove due to the deposition from the lead wire. Hence, \(\Omega_i\) can be expressed as

\[ \Omega_i = M_i + U_i \]  
(A2)

where \(U_i\) is the unfilled volume of the V-groove and \(M_i\) is the molten workpiece volume beneath either the lead or the trail arc. The term, \(M_i\), can be analytically estimated as

\[ P_t \eta_m t_s = \rho M_i [C_p(T_l - T_A) + L] \]  
(A3)

where \(P_t\), the available power from either the lead or the trail arc, \(\eta_m\) the melting efficiency, \(\rho\) the density of workpiece material, \(C_p\) the specific heat, \(L\) the latent heat, \(t_s\) the time duration, and, \(T_l\) and \(T_A\) the liquidus and ambient temperatures, respectively. Substituting Eqs. (A1) and (A2) in Eq. (A3), we can write

\[ \frac{2}{3} \pi a_i b_i c_i = \frac{P_t \eta_m t_s}{\rho[C_p(T_l - T_A) + L]} + U_i \]  
(A4)

The unfilled V-groove volumes under lead and trail arcs, i.e. \(U_{\text{LE}}\) and \(U_{\text{TR}}\), are estimated as

\[ U_{\text{LE}} = d^2 \tan \theta \times t_s \times v \]  
(A5)

\[ U_{\text{TR}} = (d^2 - h_{\text{LE}}^2) \tan \theta \times t_s \times v \]  
(A6)

where \(d\) is the depth of the V-groove, \(\theta\) is the half-groove angle, \(v\) is the weld speed, and \(h_{\text{LE}}\) is the height of the V-groove that is filled by the deposition from the lead wire. Neglecting the loss of electrode material due to spatter, the volume of V-groove, \(G_i\), filled by either the lead or the trail wire in time \(t_s\) can be estimated as

\[ G_i = \frac{\pi \phi_i^2 e_i t_s}{4} \]  
(A7)

where \(\phi_i\) is the diameter and \(e_i\) is the feed rate of either the lead or the trail electrode wire. Next, \(h_{\text{LE}}\) is estimated by equating Eqs. (A5) and (A7) as

\[ h_{\text{LE}} = \frac{\phi_{\text{LE}}}{2} \sqrt{\frac{\pi e_{\text{LE}}}{v \tan \theta}} \]  
(A8)

Equations (A4) to (A6) and (A8) provide one single relation to estimate the three geometric shape parameters for each volumetric heat source and hence, further simplification is required. Hence, the major and the minor axes of the volumetric heat-sources are assumed to be equal i.e. \(a_i = b_i\). The depth \(c_i\), corresponding to the heat sources under the lead and the trail arcs are considered to be equal to \(d\) and \((d - h_{\text{LE}})\), respectively. Following these assumptions, the shape parameters for the lead and the trail arc volumetric heat sources are expressed as

\[ a_{\text{LE}} = b_{\text{LE}} = \left[ \frac{3}{2\pi d} \frac{P_{\text{LE}} \eta_m t_s}{\rho [C_p(T_l - T_A) + L]} + U_{\text{LE}} \right]^{0.5} \]  
(A9)

\[ a_{\text{TR}} = b_{\text{TR}} = \left[ \frac{3}{2\pi (d - h_{\text{LE}})} \frac{P_{\text{TR}} \eta_m t_s}{\rho [C_p(T_l - T_A) + L]} + U_{\text{TR}} \right]^{0.5} \]  
(A10)

The reinforcement width, \(w_R\), and height, \(h_R\), are estimated considering \(w_R = 2b_{\text{TR}}\), and assuming that the reinforcement shape confirms to a parabolic shape leading to

\[ h_R = \left[ \frac{3}{2 w_R t_s v} (G_{\text{TR}} - U_{\text{TR}}) \right] \]  
(A11)