SMIL: Multimodal Learning with Severely Missing Modality

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Abstract

A common assumption in multimodal learning is the completeness of training data, i.e., full modalities are available in all training examples. Although there exists research endeavor in developing novel methods to tackle the incompleteness of testing data, e.g., modalities are partially missing in testing examples, few of them can handle incomplete training modalities. The problem becomes even more challenging if considering the case of severely missing, e.g., 90% training examples may have incomplete modalities. For the first time in the literature, this paper formally studies multimodal learning with missing modality in terms of flexibility (missing modalities in training, testing, or both) and efficiency (most training data have incomplete modality). Technically, we propose a new method named SMIL that leverages Bayesian meta-learning in uniformly achieving both objectives. To validate our idea, we conduct a series of experiments on three popular benchmarks: MM-IMDb, CMU-MOSI, and avM-NIST. The results prove the state-of-the-art performance of SMIL over existing methods and generative baselines including autoencoders and generative adversarial networks. Our code is available at https://github.com/mengmennn/SMIL.

Introduction

Multimodal learning attracts intensive research interest because of broad applications such as intelligent tutoring (Petrovica, Anohina-Naumeca, and Ekenel 2017), robotics (Noda et al. 2014), and healthcare (Frantzidis et al. 2010). Generally speaking, existing research efforts mainly focus on how to fuse multimodal data effectively (Liu et al. 2018; Zadeh et al. 2017a) and how to learn a good representation for each modality (Tian, Krishnan, and Isola 2020).

A common assumption underlying multimodal learning is the completeness of modality as illustrated in Figure 1. Existing methods (Ngiam et al. 2011; Zadeh et al. 2017b; Hou et al. 2019) often assume full and paired modalities are available in both training and testing data. However, such an assumption may not always hold in real world due to privacy concerns or budget limitations. For example, in social network, we may not be able to access full-modality data since users would apply various privacy and security constraints. In autonomous driving, we may collect many imaginary data but not as so for 3D point cloud because LiDARs are much less affordable than cameras.

Although there exist a bunch of research efforts (Tsai et al. 2019; Pham et al. 2019) in developing novel methods to tackle the incompleteness of testing data, few of them can handle incomplete training modalities. An interesting yet challenging research question then arises: Can we learn a multimodal model from an incomplete dataset while its performance should as close as possible to the one that learns from a full-modality dataset?

In this paper, we systematically study this problem by proposing multimodal learning with severely missing modality (SMIL). We consider an even more challenging configuration of severely missing modality in training, testing, or both. To jointly achieve both objectives, we leverage Bayesian meta-learning framework in designing a new method. The key idea is to perturb the latent feature space so that em-
beddings of single modality can approximate ones of full modality. We highlight that our method is better than typi-
cal generative designs, such as Autoencoder (AE) (Tran et al.
2017), Variational Autoencoder (VAE) (Kingma and Welling
2013), or Generative Adversarial Network (GAN) (Good-
fellow et al. 2014), since they often require a significant
amount of full-modality data to learn from, which is usu-
ally not available in severely missing modality learning. To
summarize, our contribution is three-fold:

• To the best of our knowledge, we are the first work to
systematically study the problem of multimodal learning
with severely missing modality.

• We propose a Bayesian meta-learning based solution to
uniformly achieve the goals of flexibility (missing modal-
ities in training, testing, or both) and efficiency (most train-
ing data have incomplete modality).

• Extensive experiments on MM-IMDb, CMU-MOSI, and
avMNIST validate the state-of-the-art performance of
SMIL over generative baselines including AE and GAN.

Related Work

Multimodal learning. Multimodal learning utilizes com-
plementary information contained in multimodal data to im-
prove the performance of various computer vision tasks. One
important direction in this area is multimodal fusion, which
focuses on effective fusion of multimodal data. Early fu-
sion is a common method which fuses different modalities
by feature concatenation, and it has been widely adopted in
previous studies (Wang et al. 2017; Poria et al. 2016). In-
stead of concatenating features, Zadeh et al. (Zadeh et al.
2017b) proposed a product operation to allow more interac-
tions among different modalities during the fusion process.
Liu et al. (Liu et al. 2018) utilized modality-specific factors
to achieve efficient low-rank fusion.

Recently, there have been a wide range of research inter-
ests in handling missing modalities for multimodal learn-
ing, such as testing-time modality missing (Tsai et al. 2019)
and learning with data from unpaired modalities (Shi et al.
2020). In this paper, we tackle a more challenging and
novel multimodal-learning setting where both training and
test data contain samples that have missing modalities.
Generative approaches, such as auto-encoders (Tran et al.
2017; Lee et al. 2019), GANS (Goodfellow et al. 2014), and
VAEs (Kingma and Welling 2013), offer a straightforward
solution to handle this setting, but these methods are neither
flexible nor efficient as SMIL.

Meta-regularization. Meta-learning algorithms focus on
designing models that are able to learn new knowledge and
adapt to novel environments quickly with only a few train-
ing samples. Previous methods studied meta-learning from
the perspective of metric learning (Koch 2015; Vinyals et al.
2016; Sung et al. 2018) Snell, Swersky, and Zemel 2017
or probabilistic modeling (Fe-Fei et al. 2003; Lawrence
and Platt 2004). Recent advances in optimization-based
approaches have evoked more interests in meta-learning.
MAML (Finn, Abbeel, and Levine 2017) is a general op-
timization algorithm designed for few-shot learning and re-
forcement learning. It is compatible with models that learn
through gradient descent. Nichol et al. (Nichol, Achiam,
and Schulman 2018) further improved the computation effi-
ciency of MAML. Other works adapted MAML for domain
generalization (Li et al. 2018; Qiao, Zhao, and Peng 2020)
and knowledge distillation (Zhao et al. 2020). In this work,
we extend MAML by learning two auxiliary networks for
missing modality reconstruction and feature regularization.

Conventional handcrafted regularization techniques (Ho-
eri and Kennard 1970; Tibshirani 1996) regularize model
parameters to avoid overfitting and increase interpretabil-
ity. Balaji et al. (Balaji, Sankaranarayanan, and Chellappa
2018) modeled the regularization function as an additional
network learned through meta-learning to regularize model
parameters. Li et al. (Li et al. 2019) followed the same
idea of (Balaji, Sankaranarayanan, and Chellappa 2018)
but learned an additional network to regularize latent features.
Lee et al. (Lee et al. 2020b) proposed a more general algo-

dithm for latent feature regularization. Other than perturbing
features, we propose to learn the regularization function fol-
lowing (Lee et al. 2020b) but regularize the feature to reduce
discrepancy between the reconstructed and true modality.

Multimodal generative models. Generative models for
multimodal learning fall into two categories: cross-modal
generation and joint-model generation. Cross-modal gen-
eration methods, such as conditional VAE (CVVAE) (Sohn,
Lee, and Yan 2015) and conditional multimodal auto-
coder (Pandey and Dukkipati 2017), learn a conditional
generative model over all modalities. On the other hand,
joint-model generation approaches learn the joint distri-
bution of multimodal data. Multimodal variational autoen-
coder (MVVAE) (Wu and Goodman 2018) models the joint

deri as a product-of-expert (PoE). Multimodal VAE (JM-
VAE) (Suzuki, Nakayama, and Matsuo 2016) learns a shared
representation with a joint encoder. With only a few modifi-
cations to the original algorithms, we show that multimodal
generative models serve as strong baselines for learning with
severely missing modalities proposed in this paper.

Proposed Method

We are interested in multimodal learning with severely miss-
ing modality, e.g., 90% of the training samples contain in-
complete modalities. In this paper, without loss of gener-
ality, we consider a multimodal dataset containing two modal-
ities. Formally, we let \( D = \{ D^f, D^m \} \) denote a multi-
modal dataset; \( D^f = \{ x^i_1, x^i_2, y_i \} \), is a modality-complete
dataset, where \( x^i_1 \) and \( x^i_2 \) represent two different modal-
ities of \( i \)-th sample and \( y_i \) is the corresponding class
label; \( D^m = \{ x^i_1, y_i \} \), is a modality-incomplete
data, where one modality is missing. Our target is to leverage both
modality-complete and modality-incomplete data for model
training. We propose to address this problem from two per-
spectives: 1) Flexibility: how to uniformly handle missing
modality in training, testing, or both? 2) Efficiency: how to
improve training efficiency when major data suffers from
missing modality?

Flexibility. We aim to achieve a unified model that can
handle missing modality in training, testing, or both. Our
idea is to employ a feature reconstruction network to achieve
Figure 2: SMIL can uniformly learn from severely missing modality and test with either single or full modality. The reconstruction network \( \phi_c \) outputs a posterior distribution, from which we sample weight \( \omega \) to reconstruct the missing modality using modality priors. The regularization network \( \phi_r \) also outputs a posterior distribution, from which we sample regularizer \( r \) to perturb latent features for smooth embedding. The collaboration \((\phi_c, \phi_r)\) guarantees flexible and efficient learning.

Efficiency. We intend to train a model on the modality severely missing dataset to achieve comparable performance as the model trained on a full-modality dataset. However, the severely missing modality setting poses significant learning challenges to the feature reconstruction network. The network would be highly bias-prone due to the scarcity of modality-complete data, yielding degraded and low-quality feature generations. Directly train a model with degraded and low-quality features will hinder the efficiency of the training process. We propose a feature regularization approach to address this issue. The idea is to leverage a Bayesian neural network to assess the data uncertainty by performing feature perturbations. The uncertainty assessment is used as feature regularization to overcome model and data bias. Compared with previous deterministic regularization approaches (Balaji, Sankaranarayanan, and Chellapilla 2018; Zhao et al. 2020), the proposed uncertainty-guided feature regularization will significantly improve the generalization performance of the multimodal model for robust generalization.

Missing Modality Reconstruction

We introduce the feature reconstruction network to approximate the missing modality. For a modality-incomplete sample, the missing modality is reconstructed conditioned on the available modality. Given the observed modality \( x^1 \), in order to obtain the reconstruction \( \hat{x}^2 \) of the missing modality, we optimize the following objective for the reconstruction network:

\[
\phi^*_c = \arg \min_{\phi_c} \mathbb{E}_{p(x^1, x^2)}(-\log p(x^2|x^1; \phi_c)).
\]

However, under severely missing modality, it is non-trivial to train a reconstruction network from limited modality-complete samples. Inspired by (Kuo et al. 2019), we approximate the missing modality using a weighted sum of modality priors learned from the modality-complete dataset. In this case, the reconstruction network are trained to predict weights of the priors instead of directly generating the missing modality. We achieve this by learning a set of modality priors \( \mathcal{M} \) which can be clustered among all modality-complete samples using K-means (MacQueen 1967) or PCA (Pearson 1901).

Specifically, let \( \omega \) represent the weights assigned to each modality prior. We model \( \omega \) as a multivariate Gaussian with fixed means and changeable variances as \( \mathcal{N}(\mathbf{I}, \sigma) \). The variances are predicted by the feature reconstruction network \( \sigma = f_{\phi_c}(x^1) \). Given the weights \( \omega \), we can reconstruct the missing modality \( \hat{x}^2 \) by calculating the weighted sum of the modality priors. Then, the reconstructed missing modality can be achieved by:

\[
\hat{x}^2 = (\omega, \mathcal{M}), \quad \text{where} \quad \omega \sim \mathcal{N}(\mathbf{I}, \sigma).
\]
Uncertainty-Guided Feature Regularization

We propose to regularize the latent features by a feature regularization network. In each layer, the regularization network takes the features of the previous layer as input and applies regularization to the features of the current layer. Let \( r \) denote the generated regularization and \( h^l \) be the latent feature of the \( l \)-th layer. Instead of generating a deterministic regularization \( r = f_\phi(h^{l-1}) \), we assume that \( r \) follows a multivariate Gaussian distribution \( N(\mu, \sigma) \), where the means and variances are calculated using \( (\mu, \sigma) = f_\phi(h^{l-1}) \). Then, we can compute the regularized feature by the following equation:

\[
h^l := h^l \circ \text{Softplus}(r), \quad r \sim N(\mu, \sigma),
\]

where \( \circ \) is a predefined operation (either addition or multiplication) for feature regularization. In our experiments, we observe that directly applying regularization to latent features will prevent the feature regularization network from convergence. Hence, we adopt Softplus (Dugas et al. 2000) activation to weaken the regularization.

A Bayesian Meta-Learning Framework

We leverage a Bayesian Meta-Learning framework to jointly optimizing all the networks. Specifically, we meta-train the main network \( f_0 \) on \( D^m \) with the help of reconstruction \( f_\phi \) network and regularization \( f_r \) network. Then, we meta-test the updated main network \( f_0^* \) on \( D^f \). Finally, we meta-update network parameters \( \{\theta, \phi, \phi_r\} \) by gradient descent.

For simplicity, we let \( \psi = \{\phi, \phi_r\} \) denote the combination of the parameters of the reconstruction and regularization network. Our framework aims to optimize the following objective function:

\[
\min_{\theta, \psi} \mathcal{L}(D^f; \theta^*, \psi),
\]

where \( \theta^* = \theta - \alpha \nabla_\theta \mathcal{L}(D^m; \psi) \).

For the above function, \( \mathcal{L} \) denotes the empirical loss such as cross entropy, and \( \alpha \) is the inner-loop step size. We use \( X \) and \( Y \) to represent all training samples and their corresponding labels, respectively. Let \( z = \{\omega, r\} \) be the collection of the generated weights and regularization. Then, inspired by (Finn, Xu, and Levine 2018; Gordon et al. 2019; Lee et al. 2020a), we define the generative process as optimizing the likelihood in a meta-learning framework:

\[
p(Y, z; X; \theta) = p(z) \prod_{i=1}^N p(y_i | x_i^1, x_i^2, z; \theta) \prod_{j=1}^M p(y_j | x_j^1, z; \theta).
\]

The goal of Bayesian Meta-Learning is to maximize the conditional likelihood: \( \log p(Y | X; \theta) \). However, solving it involves the true posterior \( p(z | X; \theta) \), which is intractable. Instead, we approximate the true posterior distribution by an amortized distribution \( q(z | X; \psi) \) (Finn, Xu, and Levine 2018; Gordon et al. 2019; Lee et al. 2020a). The resulting form of approximated lower bound for our meta-learning framework can be defined as:

\[
\mathcal{L}_{\theta, \psi} = E_{q(z | X; \theta, \psi)}[\log p(Y | X, z; \theta)] - \text{KL}[q(z | X; \psi) \| p(z | X)].
\]

We maximize this lower bound by Monte-Carlo (MC) sampling. After combining all these together, we obtain the full training objective of the proposed meta-learning framework for \( \theta \) and \( \psi \) which is defined as:

\[
\min_{\theta, \psi} \frac{1}{L} \sum_{l=1}^L -\log p(y_l | x_i^1, x_i^2, z_l; \theta) + \text{KL}[q(z | X; \psi) \| p(z | X)]
\]

with \( z_l \sim q(z | X; \psi) \).

Experiments

In this section, we analyze the results of the proposed algorithm for multimodal learning with severely missing modality on three datasets from two perspectives: efficiency under severely missing modality (Section 4.2) and flexibility to various modality missing pattern (Section 4.3).

Experiment Setting

Datasets. Totally three datasets are used in the experiment:

- The Multimodal IMDB (MM-IMDb) (Arevalo et al. 2017) contains two modalities: image and text. We conduct experiments on this dataset to predict a movie genre using image or text modality, which is a multi-label classification task as multiple genres could be assigned to a single movie. The dataset includes 25,956 movies and 23 classes. We follow the training and validation splits provided in the previous work (Vielzeuf et al. 2018).

- CMU Multimodal Opinion Sentiment Intensity (CMU-MOSI) (Zadeh et al. 2016) consists of 2,199 opinion video clips from YouTube movie reviews. Each clip contains three modalities: the image modality includes the visual gesture, the text modality includes the transcribed speech, and the audio modality includes the automatic audio. We use the feature extraction model from Liu et al.
Table 1: Binary classification accuracy (%) and F1 Score for different methods under three text modality ratios (10%, 20%, and 100%) on the CMU-MOSI dataset.

| Method       | Accuracy (%) ↑ | F1 Score ↑ |
|--------------|----------------|------------|
|              | 10% 20% 100%   | 10% 20% 100% |
| Lower-Bound  | – – 44.8       | – – 27.7   |
| Upper-Bound  | – – 71.0       | – – 70.5   |
| MVAE         | – – 58.5       | – – 58.1   |
| AE           | 56.4 60.4 –    | 54.4 59.0 – |
| GAN          | 56.5 60.6 –    | 54.6 59.1 – |
| SMIL         | 60.7 63.3 –    | 58.0 62.5 – |

Table 2: Multi-label classification scores (F1 Samples and F1 Micro) for different methods under three text modality ratios (10%, 20%, and 100%) on the MM-IMDb dataset.

| Method       | F1 Samples ↑ | F1 Micro ↑ |
|--------------|--------------|------------|
|              | 10% 20% 100% | 10% 20% 100% |
| Lower-Bound  | – – 47.6     | – – 48.2   |
| Upper-Bound  | – – 61.7     | – – 62.0   |
| MVAE         | – – 48.4     | – – 48.6   |
| AE           | 44.5 50.9 –  | 44.8 50.7 – |
| GAN          | 45.0 51.1 –  | 44.6 51.0 – |
| SMIL         | 49.2 54.1 –  | 49.5 54.6 – |

(2018) for each modality. We conduct experiments on this dataset to predict the sentiment class of the clips, which is a binary classification task as the sentiment of video clips can be either negative or positive. There are 1,284 segments in the training set, 229 in the validation set, and 686 in the test set. In the experiment section, we only use the image and text modality.

• **Audiovision-MNIST (avMNIST)** (Vielzeuf et al. 2018) consists of an independent image and audio modalities. The images, which are digits from 0 to 9, are collected from the MNIST dataset (LeCun et al. 1998) with a size of 28 × 28, and the audio modality is collected from Free Spoken Digits Dataset[1] containing raw 1,500 audios. We use the mel-frequency cepstral coefficients (MFCCs) (Tzanetakis and Cook 2002) as the representation of audio modality. Each raw audio is processed by MFCCs to get a sample with a size of 20 × 20 × 1. The dataset contains 1,500 samples for both image and audio modalities. We randomly select 70% data for training and use the rest for validation.

**Evaluation metrics.** For MM-IMDb dataset, we follow previous works (Arevalo et al. 2017; Vielzeuf et al. 2018) by adopting the F1 Samples and F1 Micro to evaluate multi-label classification. For CMU-MOSI, we follow Liu et al. (2018) to compute the binary classification accuracy and F1 Score. For avMNIST dataset, we compute accuracy to measure the performance.

**Baseline methods.** We compare the proposed approach with the following baseline methods:

• **Lower-Bound** is a model trained using single modality of the data, i.e., 100% image, 100% text, etc. It serves as the lower bound for our method.

• **Upper-Bound** is a model trained leveraging all modalities of the data, i.e., 100% images and 100% text, etc. We regard it as the upper bound.

• **AE** (Autoencoder) (Lee et al. 2019) is a deep model used for efficient data encoding. We can use AE to preprocess the original dataset to tackle the severely missing modality problem. We now describe the procedure for preprocessing. First, we sample a dataset containing only modality-complete samples from the original dataset. Then, we assume one modality is missing and train AE to reconstruct the missing modality. Finally, we impute the missing modality of modality-incomplete data using the trained AE. After finishing the imputation, the dataset is now available for multimodal learning.

• **GAN** (Generative adversarial network) is a deep generative model composed of a generator and a discriminator. We leverage GAN to tackle our problem following the same procedure as described in AE.

• **MVAE** (Wu and Goodman 2018) is proposed for multimodal generative task. We adopt the widely used linear evaluation protocol to adapt MVAE for classification. Specifically, we first train MVAE using all the modalities. We then keep the learned MVAE frozen to train a randomly initialized linear classifier using the latent representation generated by the encoder of MVAE.

**Efficiency with Severely Missing Modality**

**Conclusion:** Our method demonstrates consistent efficiency across different datasets, when training data contains a different ratios of modality missing.

**Setting of missing modality.** We evaluate the efficiency of our algorithm on two datasets: MM-IMDb and CMU-MOSI. In both datasets, modalities are incomplete for some samples. We define the text modality ratio as \( \eta = \frac{N}{M} \), where \( M \) is the number of samples with text modality and \( N \) is the size of overall samples. \( \eta \) indicates the severity of modality missing. The smaller of \( \eta \), the severer the modality is missing. For both datasets, we assume image modality to be complete, and the text modality to be incomplete. We express all available data points in the form of 100% Image + \( \eta \% \) Text for both datasets.

**Implementation details.** CMU-MOSI. We follow Liu et al. (2018) to get features for the image and text modality. We use three fully-connected (FC) layers with dimension 16 to get the embedding of image modality. One layer LSTM (Hochreiter and Schmidhuber 1997) extracts the embedding for text modality. The concatenated feature of two modalities is then fed to FC layers for classification. For training process, we use Adam (Kingma and Ba 2014) optimizer with a batch size of 32 and train the networks for 5,000 iterations with a learning rate of \( 10^{-4} \) for both inner-loop and outer-loop of meta-learning. MM-IMDB. For im-
age and text modalities, we adopt the feature extraction models from Arevalo et al. (2017). We feed the feature from each modality to a FC layer to align their output dimension. On top of it, we fuse the feature together and send it to FC layers to conduct multi-label classification. We apply Adam optimizer with a batch size of 128. We train the models for 10,000 iteration with a learning rate of $10^{-4}$ for inner-loop and $10^{-3}$ for outer-loop. Besides, we follow previous work (Vielzeuf et al. 2018) to add a weight of 2.0 on the positive label to balance the precision and recall since the labels are unbalanced.

**Different ratios of modality missing.** The results on CMU-MOSI are shown in Table 1. As can be seen, our approach significantly outperforms all baselines among all ratios of modality missing, which showcases the efficiency of our approach in the missing modality problem. The results also show that the severer the missing modality is, the more efficient our approach is. More specifically, when $\eta$ is 20%, our approach outperforms AE and GAN around 5.0%, while the improvements increase to 7.6% and 7.4%, respectively, when $\eta$ decreases to 10%. Moreover, our improvements are also consistent on MM-IMDb, as shown in Table 2. The improvement increases as the modality ratio decreasing. From Table 2, we see that our approach performs better than all baseline method under different text ratio. Our method outperforms Lower-Bound and MVAE by a large margin, and quite close to Upper-Bound.

We further show the effect of multimodal learning for different classes of MM-IMDb when $\eta = 20\%$ in Figure 3. First, our method (shown as red bars) can largely improve the model performance even on the tangled genres, such as Sports and Film-Noir, while the model trained only using images (shown as blue bars) can hardly predict the classes with less training samples. Second, an interesting phenomenon in Figure 4 is that text modality will slightly decrease the performance of movie genres like Family and Animation. The possible reason is that there is a large overlap between genres of family and animations. As a result, text modality may enforce the model to learn the shared knowledge between these two genres, which reduces the discrepancy and decrease the accuracy.

**Visualization of embedding space.** We visualize the embedding space of three genres in MM-IMDb in Figure 5 and observed that our approach can effectively disentangle the latent embedding of the three genres, while the model learned only from image modality cannot. Besides, Our method is efficient when modality is severely missing. Form Figure 5, we see that our model trained using only 10% text modality is comparable to a model trained using 100% text modality.

**Justification of symbol ‘-’ used in Table 1, 2.** We use the ‘-’ symbol for two reasons. First, not applicable. Lower-Bound only requires image modality for training, so it is not applicable to report a Lower-Bound result trained using both image and text. Second, not necessary. For example, in Table 1, MVAE trained without missing modality (100% image + 100% text) achieves $\text{acc} = 58.5\%$. In comparison, our model trained with severely missing modality (100% image + 10% text) achieves acc $= 60.7\%$. So it is not necessary to train MVAE under severely missing modality.

**Flexibility with Different Missing Patterns**

**Conclusion:** Our method shows flexibility in handling various missing patterns: (1) full or missing modality at training; and (2) full or missing modality at test time.

**Implementation details.** Our network contains two modality-specific feature extractors and a few FC layers. We use LeNet-5 to extract features for image modality, and a modified LeNet-5 to extract audio features. Extracted features are then fused through concatenation and sent into FC layers to perform classification. For the training process, we use Adam optimizer with a batch size of 64 and train the networks for 15,000 iterations with a learning rate of $10^{-3}$ for both inner- and outer-loop of meta-learning.

**Setting of missing pattern.** For the avMNIST dataset, the missing modality problem only happens to audio modality.
We are interested in two different missing patterns: (1) training with 100% Image + η% Audio and testing with Image Only; (2) training with 100% Image + 20% Audio and testing with Image + Audio. In this section, we show that our approach can flexibly handle these two missing patterns.

Missing pattern 1: testing with image only. Figure (left) shows the classification accuracy under different audio ratio. We see that our approach can successfully handle testing with image modality only, but baseline methods such as AE and GAN fail in this scenario. We can see that, when η = 20%, SMIL is 6.7% higher than the generative-based method, and 3.3% higher than the lower bound. We argue that the failure of baseline methods is mainly due to the bias of the reconstructed missing modality. In single modality testing, the method is required to generate the missing modality conditioned on the available modality. The baseline method does not consider the bias of the reconstructed missing modality. In contrast, our method can leverage learned meta-knowledge to generate an unbiased missing modality. Besides, in situations where audio modality is missing severely (i.e., η = 5%), the classification accuracy of our method is 1.10% higher than the lower bound. The improvement demonstrates clear advantages of our model under severely missing modality.

Missing pattern 2: testing with image and audio. Figure (right) shows the result of our approach dealing with full modality testing. We observe that our method still performs the best. It outperforms the Lower-Bound by 4.3% and the generative-based method (AE and GAN) by 2.1%. Moreover, under different missing patterns, SMIL is consistently better than AE and GAN. When switching testing patterns from two modalities to a single modality, AE and GAN have a 5.6% performance drop, while SMIL only has a 1.0% performance drop.

Ablation Study
We conduct the ablation analysis on the MM-IMDb dataset to evaluate the effectiveness of the missing modality reconstruction, feature regularization, and Bayesian Inference. We show the results in Table 3.

Effectiveness of missing modality reconstruction. In Section , we use reconstruction network to generate weights for missing modality reconstruction. Here we denote the method that uses the reconstruction network to directly generate the feature of missing modality as SMIL w/o K-means, which has worse performance and proves the necessity of K-Means for reconstruction.

Effectiveness of feature regularization. In Section , we introduce feature regularization. Here we denote the method without feature regularization as SMIL w/o Regularization. The performance of SMIL w/o Regularization is inferior to SMIL (Full), which verifies conducting multimodal learning on D without regularization leads to a sub-optimal model. The superior performance of the regularized model is essential to the explicit objective of reducing discrepancy.

Effectiveness of Bayesian inference. In Section , we introduce the Bayesian Meta-Learning Framework. In this section, we compare it with two variants. SMIL w/ Fixed Gaussian: We fix the distribution of feature regularization to a Gaussian distribution, which is $\mathcal{N}(\mathbf{0}, \mathbf{I})$; SMIL w/ Deterministic: The missing modality construction and feature regularization is deterministic so the sampling in Eqn. 7 is removed. These two variants are inferior to Bayesian inference. The results show the superiority of Bayesian Meta-Learning framework.

Conclusion
In this paper, we address a challenging and novel problem in multimodal learning: multimodal learning with severely missing modality. We further propose a novel learning strategy based on the meta-learning framework. This framework tackles two important perspectives: missing modality reconstruction (flexibility) and feature regularization (efficiency). We apply the Bayesian meta-learning framework to infer the posterior of them and propose a variational inference framework to estimate the posterior.

In the experiments, we show that our model outperforms the generative method significantly on three multimodal datasets. Further analysis on the results shows that involving modality reconstruction and feature regularization can effectively handle the missing modality problem and flexible to various missing patterns. We believe that our work makes a meaningful step towards the real-world application of multimodal learning where partial modalities are missing or hard to collect.
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