Lasing due to the excited state in quantum dot lasers

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Abstract. Quantum Dot Lasers (QDLs) are promising sources of light because of their favorable properties compared to other light sources. Emission in QDLs can access transitions in ground state (GS) and excited state (ES). Lasing due to the ES extends the spectral range and enables the laser to generate high output powers. Thus, lasing action due to the ES or to the dual lasing regime (GS and ES simultaneously) is expected to increase the applicability of QDLs in many future applications. We present a partially microscopic rate equation model that takes into account lasing action due to both the GS and the ES and distinguishes between both types of carriers (electrons and holes). Also, we present all possible steady-state solutions and we apply a stability analysis to the solutions to determine all stable lasing regimes (lasing due to the GS, lasing due to the ES and the dual lasing regime) to highlight the role of ES transitions. Specifically, we address the appearance of lasing due to the ES to the larger population of the ES and hence to the larger gain in higher injected current regimes.

1. Introduction
Quantum dot lasers (QDLs) are expected to be the new generation of lasers in many future applications such as telecommunications and biomedical applications because they possess many favourable properties compared to other sources of light. QDL is a semiconductor laser (SCL) whose active medium is a quantum dot (QD) double hetero structure. A QD hetero structure forms the ultimate case of quantization of carriers whose motion is restricted in all three spatial dimensions and, hence, the energy levels of QDLs are truly discrete. Thus, electrically pumped QDLs have lower threshold current density, higher differential gain, higher temperature stability and broader modulation bandwidth, compared to other SCLs. For many applications, QDLs should be able to generate high output powers. Emission in QDLs can access transitions in the ground state (GS), the excited state (ES), and both states. Lasing action due to the ES is expected to exhibit higher output power that is not achievable with other SCLs. Also, lasing action due to the ES extends the spectral range, which can enlarge the field of applications. Thus, it is of vital importance to highlight the role of lasing due to the ES in controlling the dynamical behaviour of QDLs [1, 2].

The two-state operation (GS and ES transitions) in QDLs has been reported experimentally and theoretically in many dynamical regimes [3-7]. In epitaxial growth techniques, QDs are grown on a substrate that is known as the wetting layer (WL). Thus, in electrically pumped QDLs we pump the WL and carriers relax from the WL to the energy levels of the QDL. In this work, we use a partially microscopic rate equation model that we presented in [3] to highlight the lasing predominantly via the
2. Partially microscopic rate equation model

The following model consists of rate equations for the electromagnetic field GS-intensity \((I_g)\), ES-intensity \((I_e)\) and for the GS and ES occupational probabilities for electrons and holes \((n_{e,h}^g,n_{e,h}^e)\) in the dot where the lower indices, \(e\) and \(h\), stand for electron and hole, respectively, while the upper indices, \(g\) and \(e\), stand for ground and excited states respectively.

\[
\begin{align*}
I'_g & = \left(2g(n_e^g + n_h^g - 1) - 1\right)I_g, \\
I'_e & = \left(4g(n_e^e + n_h^e - 1) - 1\right)I_e, \\
n'_{e^g} & = \eta \left(2BN_e^g \left(1 - n_e^e\right) - (n_e^e + n_h^e - 1)\right)I_e, \\
n'_{e^e} & = \eta \left(J(1 - n_e^e) - R_e^e n_e^e - BN_e^e \left(1 - n_e^e\right) - (n_e^e + n_h^e - 1)\right)I_e, \\
n'_{h^g} & = \eta \left(2B(n_h^g - n_h^e) - (n_e^e + n_h^e - 1)\right)I_e, \\
n'_{h^e} & = \eta \left(J(1 - n_h^e) - R_h^e n_h^e - B(n_h^e - n_h^e) - (n_e^e + n_h^e - 1)\right)I_e,
\end{align*}
\]

where the prime means differentiation with respect to time. Here, \(\eta = \tau p^{-1}\) describes slow relaxation of the carriers with respect to the photon lifetime, where \(\tau_p\) is the photon lifetime and \(\tau\) is the carrier recombination time. The factors 2 and 4 account for the degeneracy in the GS and the ES respectively. The higher degeneracy of the ES is due to the existence of two closely spaced p-levels, which has been verified experimentally [8]. Also, \(J\) stands for the pump current per dot. The terms \(2g(n_e^e + n_h^e - 1)\) and \(4g(n_e^e + n_h^e - 1)\) are the gains defined by the populations of both types of carriers in both states (GS and ES) and the effective gain factor \(g\), which is scaled to the cavity losses, and is assumed to be identical for both states. The term \(\hat{1} - n_{e,h}^e\) is the Pauli blocking factor, written in this form because the relaxation of carriers is limited by the filling of the states according to the Pauli exclusion principle. When the state is empty, the relaxation rate of carriers has a maximum value, and as the population increases, the rate of relaxation of carriers decreases. \(B\) is the ratio of the carrier recombination time to the capture time and it is an important intradot timescale that distinguishes QDLs form other SCLs. \(R_e\) and \(R_h\) represent the escape coefficients of electrons and holes from the ES to the WL, respectively. In this model, we assume direct pumping of the ES from the wetting layer. Also, we assume that electrons and holes are captured at the same rates \(B\), while escape of coefficients from the GS to the ES is ignored because it is very small compared to the capture rates [9, 10].

3. Results and discussion

The three nonzero intensity steady state solutions are: lasing due to the GS only \((I_g \neq 0 \text{ and } I_e = 0)\), lasing due to the ES only \((I_g = 0 \text{ and } I_e \neq 0)\) and the dual lasing regime \((I_g \neq 0 \text{ and } I_e \neq 0)\). We examine the stability of these solutions by applying Routh-Hurwitz conditions [11] at different values of \(g\). For \(g = 0.75\), we find that lasing starts at a low threshold injected current (first threshold) due to the GS transitions and the lasing action remains stable up to the appearance of a second threshold at which a dual lasing stable regime emerges and the single state (GS) regime loses stability. For \(g = 0.65\), we find that the only stable regime is lasing due to the ES only and for \(g = 0.7056\), we find that the dual lasing...
regime is the only stable regime. From the intensity expressions and occupational probabilities for both stable regimes, and by checking the solutions for different capture rates, we can define a critical effective gain factor ($g_c$) at which lasing action takes place first due to both GS and ES simultaneously. By plotting the second threshold current density versus $g$ for a large range of capture rates, we find that all curves emerge from the same point $g_c$. At large capture rates, $g_c$ depends on the escape rates of electrons and holes from the dot to the WL, through

$$g_c = \left(3R_e + \sqrt{9R_e^2 + 8R_e(R_h - R_e)}\right)/8R_e,$$

(7)

which yields the value $g_c = 0.7056$ for InAs/GaAs QDL operating at 1.3 µm, and where $R_e = 1$ and $R_h = 0.75$.

Depending on the value of $g$, we can achieve stability for all three mentioned solutions. For $g > g_c$, lasing takes place first due to the GS and this regime tracks in stability up to the appearance of the second threshold at which lasing takes place due to the dual lasing regime. In this case, lasing due to the ES only is absent (figure 1 (a)). For $g = g_c$, lasing takes place due to the dual lasing regime which is the only stable regime in this case (figure 1 (b)). For $g < g_c$, lasing takes place due to the ES only and lasing due to the GS is absent (figure 1 (c)).

![Figure 1](image)

**Figure 1:** Bifurcation diagrams of the stable GS and ES steady-state intensities. The blue dots stand for the intensity of the GS and the red dashed line stands for the intensity of the ES. $\eta = 0.01$, $\tau = 1$ ns, $\tau_p = 10$ ps, $R_h = 0.75$, $R_e = 1$ and $B = 100$. (a) $g = 0.75$ (b) $g = 0.7056$ (c) $g = 0.65$.

In QDLs, lasing action takes place as a result of the efficient energy relaxation into the lowest available energy states. Thus, for $g > g_c$, lasing due to the GS takes place first for QDLs operating at low level of injection and increasing of the injected current leads to a larger population of the ES, which, in turn, leads to the appearance of the ES threshold, and to dual lasing in both the GS and the ES. GS lasing takes place first due to the higher gain and further increasing the injected current results in lasing predominantly via the ES. In lasing due to ES and in dual lasing regime the intensity of ES increases with injected current because the hole levels are closely spaced in both states and as lasing due to ES takes place, ES is preferred because of the higher degeneracy of ES compared to GS.

Either GS threshold or ES threshold may appear first, or both GS and ES may start lasing at the same injected current. From the steady-state solutions, we find that lasing threshold of the ES is directly proportional to $g$ while lasing threshold of the GS is inversely proportional to $g$. Thus, lasing action takes place due to the ES only for $g$ less than $g_c$.

4. Conclusion

QDLs operate at both GS and ES transitions. For higher injected currents, lasing due to ES is dominant because of the higher degeneracy of the ES. ES emission has its origin in the slow band relaxation of
carriers and the discrete-nature of density of states for QD Structures. Also, the effective gain factor plays an important role in determining the possible stable lasing regimes.

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