The Pairwise Velocity Distribution Function of Galaxies in the LCRS, 2dF, and SDSS Redshift Surveys

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ABSTRACT

A comparison of the galaxy pairwise velocity distribution functions determined from the three largest publicly available galaxy redshift surveys is presented. This is the first direct comparison of this function across these surveys using an identical method. It is found that the two r-band selected surveys, the LCRS (Las Campanas Redshift Survey) and the SDSS (Sloan Digital Sky Survey), are in excellent agreement with $363\pm44$ km sec$^{-1}$ and $357\pm17$ km sec$^{-1}$ respectively. The b-band selected 2dF survey gives slightly lower results with $331\pm19$ km sec$^{-1}$. This difference is expected given the sampling biases in the different surveys although it is not highly significant. These results and analysis of subsets of data indicate that the method is stable to singular features in each survey such as the prevalence of rich clusters. The technique utilizes a Fourier-space deconvolution of the redshift-space distortions in the correlation function in the $(r_p, \pi)$ basis, which has been previously described. This method returns the entire distribution function rather than just the second moment. In all cases the distribution function is well-characterized by an exponential.

Subject headings: large-scale structure of the universe—cosmology: observations—methods: analytical—methods: data analysis—galaxies: distances and redshifts—cosmological parameters

1. Introduction

If, as generally accepted, clustering evolves through gravitational instability then the properties of galaxy clustering and the distribution of galaxy peculiar velocities contain fundamental information on the global properties of the Universe and provide important constraints on cosmological models. For a given initial mass fluctuation spectrum, clustering evolution and the generation of galaxy peculiar velocities is strongly dependent on the value of
the mass density parameter $\Omega_m$. For example, in a low $\Omega_m$ Universe mostly devoid of matter, it is very difficult to generate high peculiar velocities since the Hubble flow will dominate over gravitational instability from early times unless the mass fluctuation amplitude is very high. Consequently, low values of $\Omega_m$ generate relatively smaller peculiar velocities than high values for a given fluctuation amplitude. Once both the present galaxy clustering amplitude and peculiar velocity distribution functions are well-determined, cosmologists will have made significant progress in determining the initial conditions and evolution of the mass density in the Universe.

Unfortunately, neither galaxy positions nor peculiar velocities are presently measurable with significant accuracy to make robust, direct measurements of these two quantities. Rather, most of the information available today is contained in galaxy redshift surveys. As is well-known, the redshift to a galaxy represents the sum of the galaxy’s ‘distance’ and its radial peculiar velocity. Therefore, measurements of the galaxy two-point correlation function are contaminated by these redshift distortions and what is measured is appropriately called the redshift-space correlation function (see also Juszkiewicz et al. 2000).

It is possible, however, to exploit the anisotropies created by the peculiar velocities in the redshift-space correlation function (see Peebles 1980). In this method, the redshift space correlation function is represented in two-dimensions, where the axes correspond to the directions parallel ($\pi$) and perpendicular ($r_p$) to the line-of-sight. The resultant correlation function $\xi_z(r_p, \pi)$ is then anisotropic since the the peculiar velocities distort the correlation function principally along the line-of-sight. Since the galaxy-galaxy correlation function is a two-point estimator, the anisotropies being generated actually reflect the value of the pairwise galaxy peculiar velocity distribution function. Measurement of this distribution function is the focus of this Letter.

In a seminal paper, Davis and Peebles (1983) using redshift information from the first Center for Astrophysics Redshift Survey (CfA1) measured a value for the second moment of the pairwise velocity distribution function (hereafter PVD) of $\sigma_{12} = 340 \pm 40 \text{ km sec}^{-1}$ and characterized the distribution as an exponential. Subsequent numerical work predicted a much larger value of approximately 1000 km sec$^{-1}$ for a standard $\Omega h = 0.5$ Cold Dark Matter (CDM) model (Davis et al. 1985). However, later work has questioned the accuracy of the original CfA1 result (Mo, Jing, & Börner 1993, Somerville, Davis & Primack 1997).

Subsequent redshift surveys have given somewhat discrepant results. The IRAS 1.2 Jy survey (Fisher et al. 1994) was in good agreement with a value of $\sigma_{12} = 317^{+40}_{-49} \text{ km sec}^{-1}$ while Marzke et al. (1995) using the second Center for Astrophysics Redshift Survey (CfA2) together with the Southern Sky Redshift Survey (SSRS) found $\sigma_{12} = 540 \pm 180 \text{ km sec}^{-1}$. Similar analysis applied to the Las Campanas Redshift Survey, hereafter LCRS, has found
\[ \sigma_{12} = 452 \pm 60 \text{ km sec}^{-1} \] (Lin et al. 1995). Guzzo et al. (1997) find \[ \sigma_{12} = 345^{+95}_{-65} \text{ km sec}^{-1} \] for late-type galaxies using the Pisces-Perseus Redshift Survey and report this to be a fair estimate for field galaxies. Other recent measurements include Small et al. (1999) using the Two Norris Redshift Surveys who report \[ 326^{+67}_{-52} \text{ km sec}^{-1} \] and Ratcliffe et al. (1998) using the Durham/UKST Galaxy Redshift Survey with an estimate of \[ 416 \pm 36 \text{ km sec}^{-1} \].

Numerical work and re-analysis of existing surveys have shown the sensitivity of the measurement of the pairwise velocity dispersion to the idiosyncracies of the data using standard techniques. Mo et al. (1993) found that the estimated dispersion is extremely sensitive to the presence of rich clusters in a sample, \[ \sigma_{12} = 300 \text{ to } 1000 \text{ km sec}^{-1} \] for subsets of the same data. Zurek et al. (1994) using high resolution CDM simulations found large variations in the value of the dispersion on CfA1 size scales. Using real data, Marzke et al. (1995) found that by excluding the rich clusters from their survey, \( R \geq 1 \), the measured velocity dispersion dropped to \[ \sigma_{12} = 295 \pm 99 \text{ km sec}^{-1} \].

The sensitivity of the standard methods to the presence of rich clusters is primarily due to the fact that they generally only estimate the second moment of the distribution, which is highly sensitive to the existence of hot, rich clusters in the data. Recognizing this problem, several authors have invented new statistics that are less sensitive to rich clusters (see Kepner, Summers & Strauss 1997, Davis, Miller & White 1997, Strauss, Ostriker & Cen 1998, and Baker, Davis & Lin 2000). While new statistics can be a powerful approach, their measures are often somewhat indirect and their results less intuitive.

To circumvent this problem and with the intent to determine the entire peculiar velocity distribution function rather than just the second moment, Landy, Szalay & Broadhurst (1998), hereafter LSB98, developed a method based on a Fourier-space deconvolution of the redshift-space distortions in the correlation function in the \((r_p, \pi)\) basis. Using this approach with the LCRS data, a value of \[ 363 \pm 44 \text{ km sec}^{-1} \] was obtained. Further, by recovering the Fourier transform of entire distribution function, it was directly shown that the PVD is well-characterized by an exponential.

Other recent measurements of the PVD using the same LCRS data but different techniques have also been reported. Jing, Mo, & Börner (1998) measured the PVD of the LCRS data and obtained \[ 570 \pm 80 \text{ km sec}^{-1} \]. The discrepancy with the result of LSB98 was ascribed to a failure of LSB98 to account for infall effects (see Jing & Börner 1998). Another analysis of the LCRS data was presented in Jing & Börner (2001), using a method closely related to that of LSB98, with a PVD value of \[ 510 \pm 70 \text{ km sec}^{-1} \]. The discrepancies between these two latter results and those in this Letter will be presented in the discussion.

Recently, Peacock et al. (2001) have published a measurement of the PVD using a
larger sample of the 2dF survey than publicly released as an adjunct to their estimation of the cosmological parameter $\beta$. They obtain a value of 385 km sec$^{-1}$ but do not report standard errors.

2. Data

The three data sets used in this analysis are all publicly available. The LCRS data is described in Shectman et al. 1996. The results for that data published here are identical to those reported in LSB98 and the reader is directed there for further information.

The 2dF data is the release of June 2001 (see www.mso.anu.edu.au/2dFGRS/). This data has been cut to $0.05 < z < 0.2$ (150 to 570 $h^{-1}$Mpc assuming $\Omega_m = 0.3, \Omega_k = 0.7$) and is naturally broken up into two subsets toward the north and south galactic caps. These sets are disjoint on the scales under consideration. The north and south data consisted of 34805 and 42309 galaxies respectively. Random catalogs for this data were generated using the publicly available routines supplied by the 2dF Survey.

The SDSS data is the release of June 2001 (see Stoughton et al. 2001). As with the 2dF data, the redshift limits were set at $0.05 < z < 0.2$ and subsets of the data towards the north and south galactic caps were analyzed independently. This data consisted of 8918 galaxy positions in the North and 7862 in the South. The random catalogs for this data were kindly supplied by Adrian Pope.

3. Method

In essence, this approach extracts the Fourier transform of the PVD from the Fourier transform of the galaxy-galaxy correlation function in the $(r_p, \pi)$ basis using a deconvolution procedure. By working in Fourier space, the deconvolution is one of simple division that directly returns the Fourier transform of the PVD. Since the method here is identical to that described in LSB98, the reader is directed there for further details. However, given the stability of the reported results, it is worth re-iterating two of the principle advantages of working in Fourier space.

Firstly, by utilizing the Fourier transform of the $\xi(r_p, \pi)$ correlation function, the measurement of the PVD is no longer a pair-weighted statistic. This reduces the contamination by rich clusters in the data. More specifically, the presence of rich clusters in the sample super-imposes a number of pairs with a higher dispersion than the thermal dispersion of field galaxies. These pairs predominately effect the tail of the correlation function and produce
an overestimation of the PVD, especially of its second moment. By determining the Fourier transform of the PVD rather than the PVD itself, the large number of bins in the tail are all compressed into the innermost few resolution elements in Fourier space. Therefore they are naturally down-weighted in the fitting procedure, which uses equal weights for every cell in $k$-space. This method does not exclude the signal from the rich clusters but rather incorporates them in a way which decreases their contribution to the variance in a robust fashion.

Secondly, in the deconvolution procedure, the weighting function is effectively proportional to $r_p^{-1}$, which emphasizes the high signal-to-noise core of the correlation function. Since most of the mass of the correlation function lies within the central core, the resulting signal will predominately reflect the value of the PVD within a scale of about $1h^{-1}\text{Mpc}$. This both reduces contamination by infall effects, which are expected to be small on these scales, and localizes the measurement to reflect the value of the PVD on small scales.

4. Results

The Fourier transform of the PVD for each redshift survey is shown in Figure 1. In all cases, this function is well-characterized by a Lorentzian. Since the Fourier transform of a Lorentzian is an exponential, the PVD itself is well-characterized by an exponential. For comparison, the best-fitting Gaussian distributions are also shown. All values reported correspond to the equivalent exponential value for a given fitted Lorentzian.

The result is very stable across all three redshift surveys. Since the 2dF is a $b$-band selected survey and the LCRS and SDSS are both $r$-band selected, it is expected that the 2dF Survey will contain a relatively greater number of field galaxies. Thus, the PVD for the 2dF survey should have a lower value as is found: $357 \pm 17 \text{ km sec}^{-1}$ for the SDSS versus $331 \pm 19 \text{ km sec}^{-1}$ for the 2dF. However, this difference is not of high significance given the standard errors for the data.

The Lorentzian in the case of the 2dF data shows additional structure around the central peak. This type of structure has also shown up in some of the analyses of subsets of the data. The effect is likely a consequence of the fact that the method necessarily constrains the value of the signal to unity at the origin. Adjusting the fit to exclude the structure in the central peak does not significantly change the measured value of the dispersion.

In order to get a better handle on the uncertainties in the results, the north and south data in the 2dF survey was also divided into two halves and an identical analysis performed. All results are reported in Table 1.
5. Discussion

As is evident in Figure 1, the PVD function for all cases is clearly well-characterized by an exponential. Furthermore, Table 1 shows that these measurements are very stable both within and between data sets.

As mentioned in the Introduction and Method sections, one of the major difficulties in the measurement of the PVD has been the contamination by rich clusters in the data. For example, Marke et al. (1995) found that by excluding rich clusters in their data that the signal fell from $\sigma_{12} = 540 \pm 180$ km sec$^{-1}$ to $\sigma_{12} = 295 \pm 99$ km sec$^{-1}$. Additionally two analyses of the LCRS data, one using a method based on the correlation function and one presented in this Letter, give $\sigma_{12} = 452 \pm 60$ km sec$^{-1}$ (Lin et al. 1995) and $363 \pm 44$ km sec$^{-1}$ (LSB98) respectively. It was also shown by Mo et al. (1993) that the estimated dispersion is extremely sensitive to the presence of rich clusters in a sample, to the order of errors of several hundred km sec$^{-1}$ for subsets of the same data.

These previous findings emphasize the importance of applying identical methods to subsets of data and across surveys. The results presented here reflect a very robust and stable technique. The small variance in the measured signal across and between data sets clearly indicate that the signal is not being dominated by rich clusters in the data.

This conclusion is also supported by the agreement between the results of the two r-band selected surveys, the LCRS and SDSS, both in terms of the value of the measured signal and the small standard errors. The standard errors of the 2DF b-band selected survey are also of a similar magnitude. Since this latter b-band survey is expected to contain relatively fewer late-type galaxies, it would be expected that its standard error would be substantially less than that of the r-band selected surveys whose signal should be more strongly contaminated by clusters in the data.

One criticism of this method (see Jing & Börner 1998) is that it does not adequately take into account infall effects and consequently underestimates the value of the PVD; compare their value of $570 \pm 80$ km sec$^{-1}$ for the LCRS data. In a complete characterization (see Peebles 1980), the galaxy peculiar velocity distribution function $f(v_{12}|r)$ is usually modeled as an exponential along one dimension with

$$f(v_{12}|r) = \frac{1}{\sqrt{2}\sigma_{12}} \exp\left(-\frac{\sqrt{2}|v_{12} - \bar{v}_{12}|}{\sigma_{12}}\right).$$ \hspace{1cm} (1)

In our analysis, the infall parameter $\bar{v}_{12}$ is not included for several reasons. Firstly, given the excellent fit of the Fourier transform of the PVD with the Lorentzian, there is no
evidence that the result is being contaminated by infall effects. Secondly, as was discussed in LSB98, since this method relies on a Fourier transform of the correlation function and most of the mass of the correlation function lies within a central core of about 1 \( h^{-1}\text{Mpc} \), the resulting signal will predominately reflect the value of the PVD within a scale of about 1\( h^{-1}\text{Mpc} \) where the infall is expected to be small.

In later work, Jing & Börner (2001) re-analyze the LCRS and obtain a result of 510 ± 70 km sec\(^{-1}\). This method very similar to that of LSB98 although the model includes a large-scale infall parameter in the linear regime characterized by \( \beta \). In our method by truncating the correlation function at approximately 30\( h^{-1}\text{Mpc} \), the approximate scale of the transition between the linear and non-linear regimes, and windowing the distribution, the signal is limited to the non-linear regime. This is very important considering that the distortions of the redshift correlation function due to the small-scale PVD and the large-scale linear velocity infall are competing effects and so their errors are positively correlated. Additionally, although Jing & Börner (2001) incorporate a model for the infall effects of the \( \beta \) distortions, they do include it in the fit but rather set it at a value of \( \beta = 0 \).

The competing effects of these two distortions can be clearly seen in Fig. 4 of Peacock et al. (2001) where a joint maximum likelihood fit of the estimation of \( \sigma_{12} \) and \( \beta \) from a larger 2dF data set is presented. The best fit values are \( \sigma_{12} = 385 \text{ km sec}^{-1} \) and \( \beta = 0.43 \pm 0.07 \). The LSB98 value of 363 ± 44 km sec\(^{-1}\) is within one standard error of this result and would imply a \( \beta \) of 0.4, while the measurement of Jing & Börner (2001) 510 ± 70 km sec\(^{-1}\) implies a best-fitting value of \( \beta \approx 0.6 \).

Davis, Miller & White (1997) also present a method for determining the thermal velocity dispersion of galaxies using a galaxy-weighted rather than the more standard pair-weighted measures. An analysis of this result on the LCRS data is reported in Baker, Davis & Lin (2000). Although it is difficult to directly compare their approach to the one presented here, the results are generally consistent.

6. Conclusion

Since galaxy distances and peculiar velocities can not be adequately directly measured at this point, every approach to measure the properties of the galaxy peculiar velocity distribution function is somewhat indirect. For example, almost every method attempts to measure the pairwise galaxy peculiar velocity distribution function rather the the single particle function itself. Also, it has been shown that due to the existence of hot clusters in any large redshift survey, that a direct fit to the correlation function itself is problematic.
The method utilized in this Letter has several advantages over other extant techniques. Firstly, it relies on a straightforward Fourier transform of the $\xi(r_p, \pi)$ correlation function, a mathematical operation that is well understood. Secondly, in extracting the Fourier transform of the PVD, the need to model the underlying $\xi(r)$ is obviated. Thirdly, by restricting the analysis to non-linear scales, the need to model for infall effects is eliminated. Fourthly, the method returns the Fourier transform of the entire PVD function and its functional form can be directly investigated. And lastly, the technique is very stable within and between independent surveys.

Using this method, it has been shown that the galaxy pairwise velocity distribution function can be well-represented by an exponential for the three largest existing redshift surveys. The two r-band selected surveys, the LCRS and the SDSS, give values of with $363 \pm 44 \, \text{km sec}^{-1}$ and $357 \pm 17 \, \text{km sec}^{-1}$ respectively. The b-band selected 2dF survey gives slightly lower results with $331 \pm 19 \, \text{km sec}^{-1}$ and this difference is expected given the different selection effects of r-band and b-band selected surveys. Further analysis of subsets of the data has shown that the signal is not sensitive to idiosyncracies in the data sets such as the prevalence of rich clusters.

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REFERENCES

Baker, J. E., Davis, M. & Lin, H. 2000, ApJ, 536, 112

Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371

Davis, M., Miller, A., & White, S. D. M. 1997, ApJ, 490, 63

Davis, M., & Peebles, P. J. E. 1983, ApJ, 267, 465

Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994, MNRAS, 266, 50

Guzzo L., Fisher K., Strauss M., Giovanelli R., & Haynes M. 1997, ApJ, 489, 37

Jing, Y. P. & Börner, G. 1998, ApJ, 503, 502

Jing, Y. P. & Börner, G. 2001, MNRAS, 325, 1389

Jing, Y. P., Mo, H. J., & Börner, G. 1998, ApJ, 494, 1

Juszkiewicz, R., Ferreira, P. G., Feldman, H. A., Jaffe, A. H., & Davis, M. 2000, Science, 287, 109

Kepner, J. V., Summers, F. J. & Strauss, M. A. 1997, New Astronomy, 2, 165

Landy, S. D., Szalay, A. S., & Broadhurst, T. J. 1998, ApJ, 494, L133

Lin, H. 1995, Ph.D. Thesis, Harvard University

Marzke, R. O., Geller, M. J., da Costa, L. N., & Huchra, J. P. 1995 AJ, 110, 477

Mo, H. J., Jing, Y. P., & Börner, G. 1993, MNRAS, 264, 825

Peacock, J. A. et al. 2001, Nature, 410, 169

Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton:Princeton University Press)

Ratcliffe, A., Shanks, T., Parker, Q. A. & Fong, R. 1998, MNRAS, 296, 191

Shectman, S. A., Landy, S. D., Oemler, A., Tucker, D. L., Lin, H., Kirshner, R. P., & Schechter, P. L. 1996, ApJ, 470, 172

Small, T. A., Ma, C. P., Sargent, W. L. & Hamilton, D. 1999, ApJ, 524, 31

Somerville, R. S., Davis, M., & Primack, J. R. 1997, ApJ, 479, 616
Stoughton, C. et al. 2001, ApJ, submitted

Strauss, M. A., Ostriker, J. P., & Cen, R. 1998, ApJ, 494, 20

Zurek, W. H., Quinn, P. J., Salmon, J. K., & Warren, M. S. 1994, ApJ, 431, 559
Fig. 1.— Figure 1 shows the best Lorentzian and Gaussian fits to the Fourier transform of the galaxy pairwise velocity dispersion functions for the LCRS, 2dF, and SDSS Galaxy Redshift Surveys. In all cases, a Lorentzian is a much better fit to the data. A Lorentzian is the Fourier transform of an exponential distribution. The value of the associated exponential is reported.
Table 1. Pairwise Peculiar Velocity Dispersion Measurements (km sec\(^{-1}\))

| Survey | \(\sigma_{12}\) | \(\Delta\sigma_{12}\) | North | South | North 1 | North 2 | South 1 | South 2 |
|--------|-----------------|-------------------|-------|-------|---------|---------|---------|---------|
| LCRS   | 363             | ±44               |       |       |         |         |         |         |
| SDSS   | 357             | ±17               | 329   | 353   |         |         |         |         |
| 2dF    | 331             | ±19               | 314   | 340   | 299     | 328     | 354     | 311     |

Note. — For all surveys the mean correlation function of the north and south data was used to determine the reported dispersion. The standard errors for the LCRS were calculated from the values for the six separate slices, which are reported in LSB98. The 2dF and SDSS standard errors were determined from the values for the north and south subsets. The values for the 2dF data broken up in four subsets are also shown.