Bidirectional Controlled Quantum Teleportation Using Eight-Qubit Quantum Channel in Noisy Environments

Moein Sarvaghad-Moghaddam1 · Zeinab Ramezani2 · I. S. Amiri3,4

Received: 31 March 2020 / Accepted: 6 August 2020 / Published online: 15 August 2020
© Springer Science+Business Media, LLC, part of Springer Nature 2020

Abstract
In this work, a novel protocol is proposed for bidirectional controlled quantum teleportation (BCQT) in which a quantum channel is used with the eight-qubit entangled state. Using the protocol, two users can teleport an arbitrary entangled state and a pure two-qubit state (QBS) to each other simultaneously under the permission of a third party in the role of controller. This protocol is based on the controlled-not operation, appropriate single-qubit (SIQ) UOs, and SIQ measurements in the $Z$ and $X$-basis. Also, in this paper, a new criterion of merit named as (predictability of the controller’s qubit (QB) by the eavesdropper) is introduced, and the protocol is improved based on it. Then, the proposed protocol is investigated in two typical noisy channels, the amplitude-damping noise (ADN) and the phase-damping noise (PDN). The analysis of the protocol in the noisy environment shows that it only depends on the amplitude of the initial state and the decoherence noisy rate (DR).

Keywords Bidirectional controlled teleportation · Two-qubit state · Entangled state · Eight-qubit channel · Amplitude-damping noise · Phase-damping noise

I. S. Amiri
irajsadeghamiri@tdtu.edu.vn

Moein Sarvaghad-Moghaddam
moeinsarvaghad@aut.ac.ir

Zeinab Ramezani
ramezaniz@ymail.com

1 Quantum Design Automation Lab, Amirkabir University of Technology, Tehran, Iran
2 Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115, USA
3 Computational Optics Research Group, Advanced Institute of Materials Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam
4 Faculty of Applied Sciences, Ton Duc Thang University, Ho Chi Minh City, Vietnam
1 Introduction

One of the most outstanding results of the quantum information theory in theoretical and experimental is Quantum teleportation (QT) [1]. The original QT protocol was proposed by Bennett et al. [2]. Many theoretical and experimental papers are attributed to the study of QT and bidirectional QT (BQT) (With adding the ability to transmit information of each two users simultaneously) protocols so far [3–15]. Karlsson and Bourennane [16] firstly presented a Controlled QT (CQT) protocol In 1998. After that, many CQT protocols with quantum channels consists of various types of entangled states (ENSs) have been introduced [17–22]. In 2013, Zha et al. [23] proposed a Bidirectional CQT (BCQT) with a five-QB cluster state. Up to now, various BCQT protocols have been presented by using different multi-particle ESs, say five-QB EN [24], six-QB EN [25], and seven-QB EN [26].

Recently, two BCQT protocols using quantum channels composed of seven-QB ENs have been presented by Hong [27] and Sang [28]. In these schemes, Alice (A) can teleport an arbitrary two-qubit state (QBS) to Bob (B), and B can teleport an arbitrary single-QBS to A under the control of the third person (named Charlie (C)). After that, Li and Jin [29] presented a BCQT scheme using a nine-QB EN quantum channel. In this protocol, users can teleport an unknown two-QBS to each other, simultaneously. In the same year, Li et al. [30] introduced a BCQT protocol using a quantum channel with the six-QB cluster state. In this scheme, A can teleport an arbitrary two-QBS to B and B can teleport an arbitrary single-QBS to A under the control of C. In [31], Binayak et al. presented a protocol for transmitting two and three qubits entangled states with nine qubit cluster state. Besides, the implementation of QT has been presented in some of the works experimentally [32–36] in various quantum systems like the cavity QED system, optical and photonic systems, and ion-trap system.

In this study, a novel BCQT protocol using as a quantum channel with the eight-QB EN is proposed by which the users can teleport an arbitrary EN and a pure two-QBS to each other concurrently under permission of a controller. Also, a new criterion of merit named as (predictability of the controller’s qubit (QB) by the eavesdropper) is introduced, and the protocol is improved based on it. Also, in the proposed protocol, there is an improvement in reducing quantum resources used for preparing the quantum channel into previous works. Then, the proposed protocol is investigated in normal noisy channels, including the amplitude-damping noise (ADN) and the phase-damping noise (PDN). Finally, the fidelities of the BCQT process illustrate which they depend on the decoherence noisy rate (DNR) and the amplitude parameter of the initial state (IS). In our scheme, SIQ measurement, controlled-not operation, and appropriate UOs are necessary.

In the following, In Section 2, the proposed BCQT protocol is described. Section 3 presents the preparation and the circuit of the quantum channel. In Section 4, the effect of noise on the proposed protocol is discussed in detail. The comparison between our protocol with previous BCQT works is presented in Section 5. Finally, Sect. 6 concludes the paper.

1.1 Scheme for the Presented Protocol

In this protocol, A and B want to transmit an arbitrary EN and a pure two-QBS to each other under the permission of the controller, simultaneously described by Eq. (1).

\[ |\Phi\rangle_{A_0A_1} = \alpha_0|00\rangle + \alpha_1|11\rangle, \]
\[ |\Phi\rangle_{B_0B_1} = \beta_{00}|00\rangle + \beta_{01}|01\rangle + \beta_{10}|10\rangle + \beta_{11}|11\rangle. \] (1)

Where \( |\alpha_0|^2 + |\alpha_1|^2 = 1 \) and \( |\beta_{00}|^2 + |\beta_{01}|^2 + |\beta_{10}|^2 + |\beta_{11}|^2 = 1 \). This protocol includes the following steps:
The state of the whole system can be expressed by Eq. (3).

\[ |\phi\rangle_{a_0b_0c_1} = \frac{1}{\sqrt{8}} |\psi\rangle_{a_0b_0c_1} \otimes |\Phi\rangle_{A_0B_1} \otimes |\Phi\rangle_{B_0B_1}. \]  

**Step2.** In this step, A and B make a CNOT operation with A_0, B_0 and B_1 as control QBS and a_0, b_2 and b_3 as target QBS, respectively. The next state is as Eq. (4).

\[ |\phi\rangle_{a_0b_0c_1} = \frac{1}{\sqrt{8}} |\alpha_0\beta_0\langle[0000000000] + [00011010] + [00010010] + [00011111] + [11100000] + [11011010] + [11100100] + [11111111] \rangle 0000 \rangle + \alpha_0\beta_0\langle[00000000] + [00011000] + [00010010] + [00011110] + [11001000] + [11101100] + [11110010] + [11111111] \rangle 0001 \rangle + \alpha_0\beta_0\langle[00000000] + [00011000] + [00010010] + [00011110] + [11001000] + [11101100] + [11110010] + [11111111] \rangle 0010 \rangle + \alpha_0\beta_0\langle[00000000] + [00011000] + [00010010] + [00011110] + [11001000] + [11101100] + [11110010] + [11111111] \rangle 0011 \rangle + \alpha_0\beta_0\langle[00000000] + [00011000] + [00010010] + [00011110] + [11001000] + [11101100] + [11110010] + [11111111] \rangle 1000 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1001 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1010 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1011 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1100 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1101 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1110 \rangle + \alpha_0\beta_0\langle[10000000] + [01001000] + [00011111] + [11001000] + [11110110] + [11011010] + [01101100] + [01110011] \rangle 1111 \rangle. \]
Table 1 different channels created using of distribution of C’s QB

| Encoding the various distributions of C’s QB | The proposed channel $|\Psi\rangle_{a_0b_0a_1b_1a_2b_2a_3b_3} = \begin{cases} \frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
\frac{1}{\sqrt{8}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \end{cases}$. |
Step 3. A and B do a SIQ measurement in the $Z$-basis on $a_0$, $b_2$ and $b_3$ QBs. According to Table 2, the unmeasured QBs may decrease into one of the eight possible states with the same probability.

Step 4. In this step, A and B notify the $Z$-basis measurement results to each other. Then they apply $X$ UO on QBs $b_0$, $b_1$, $a_1$ and $a_2$ as shown in Table 3. The state of the unmeasured QBs is converted to the following form:

$$
\alpha_0 \beta_{00} |00000000\rangle + \alpha_0 \beta_{01} |00110000\rangle + \alpha_0 \beta_{10} |00100010\rangle + \alpha_0 \beta_{11} |00110011\rangle \\
+ \alpha_1 \beta_{00} |11000110\rangle + \alpha_1 \beta_{01} |11011111\rangle + \alpha_1 \beta_{10} |11100110\rangle + \alpha_1 \beta_{11} |11110111\rangle
$$

(5)

Step 5. SIQ measurements are applied in the $X$-basis on sending QBs ($A_0$, $A_1$, $B_0$ and $B_1$) by A and B. As shown in Table 4, the other QBs collapsed to one of 16 possible states with the same probability.

Step 6. A and B notify their measurement results to each other. Then, they apply $Z$ UO to their unmeasured QBs ($b_0$, $b_1$, $a_1$ and $a_2$), as shown in Table 5. The state of the unmeasured QBs is converted to the bellow form (6).

$$
\alpha_0 \beta_{00} |0000\rangle + \alpha_0 \beta_{01} |0011\rangle + \alpha_0 \beta_{10} |0100\rangle + \alpha_0 \beta_{11} |0111\rangle + \alpha_1 \beta_{00} |1100\rangle \\
+ \alpha_1 \beta_{01} |1101\rangle + \alpha_1 \beta_{10} |1110\rangle + \alpha_1 \beta_{11} |1111\rangle
$$

(6)

Step 7. C notifies the distribution status of his QB to A and B with three classical bits, as shown in Table 1. Then, he measures her QB in $X$-basis and tells to A and B his result. If C’s measured result is $|+\rangle$ ($|-\rangle$), then, the state of other QBs is as (7) or (8), respectively. The measurement results of C’s QB with corresponding unitary operations (UOs) applied by A and B are shown in Table 6 for the different channels in Table 1.

$$
\alpha_0 \beta_{00} |0000\rangle + \alpha_0 \beta_{01} |0001\rangle + \alpha_0 \beta_{10} |0010\rangle + \alpha_0 \beta_{11} |0011\rangle + \alpha_1 \beta_{00} |1100\rangle \\
+ \alpha_1 \beta_{01} |1101\rangle + \alpha_1 \beta_{10} |1110\rangle + \alpha_1 \beta_{11} |1111\rangle
$$

(7)

$$
\alpha_0 \beta_{00} |0000\rangle - \alpha_0 \beta_{01} |0001\rangle + \alpha_0 \beta_{10} |0010\rangle - \alpha_0 \beta_{11} |0011\rangle + \alpha_1 \beta_{00} |1100\rangle - \alpha_1 \beta_{01} |1101\rangle \\
+ \alpha_1 \beta_{10} |1110\rangle - \alpha_1 \beta_{11} |1111\rangle
$$

(8)

Step 8. According to C’s results, A and B apply $Z$ UO, as shown in Table 6. In final, A and B can reconstruct the transmitted states again as (9) and (10). And the BCQT is successfully finished.

$$
|\Phi\rangle_{A_0A_1} = \beta_{00} |00\rangle + \beta_{01} |01\rangle + \beta_{10} |10\rangle + \beta_{11} |11\rangle
$$

(9)
Table 2  The Z-basis measurement results of the corresponding collapsed state and users

| A’s results | B’s results | The collapsed state of QBs $b_y b_1 a_1 a_2 c_A a_1 b_y b_1$ |
|-------------|-------------|---------------------------------------------------------------|
| 00          | $a_0 b_0 (000000000)$ + $a_0 b_0 (001100000)$ + $a_0 b_0 (001000000)$ + $a_0 b_0 (001100010)$ + $a_0 b_0 (001000110)$ + $a_0 b_0 (110001100)$ + $a_0 b_0 (110111101)$ + $a_0 b_0 (110001110)$ + $a_0 b_0 (111111111)$ |
| 01          | $a_0 b_0 | 000110000)$ + $a_0 b_0 | 000000001)$ + $a_0 b_0 | 001110001)$ + $a_0 b_0 | 001000001)$ + $a_0 b_0 | 001110011)$ + $a_0 b_0 | 110011101)$ + $a_0 b_0 | 110001110)$ + $a_0 b_0 | 111111111)$ |
| 10          | $a_0 b_0 | 000000000)$ + $a_0 b_0 | 001110000)$ + $a_0 b_0 | 000000000)$ + $a_0 b_0 | 001110000)$ + $a_0 b_0 | 001000110)$ + $a_0 b_0 | 110001100)$ + $a_0 b_0 | 111111111)$ + $a_0 b_0 | 111111111)$ |
| 11          | $a_0 b_0 | 001110000)$ + $a_0 b_0 | 000000000)$ + $a_0 b_0 | 001110000)$ + $a_0 b_0 | 000000000)$ + $a_0 b_0 | 001110000)$ + $a_0 b_0 | 110001100)$ + $a_0 b_0 | 111111111)$ + $a_0 b_0 | 111111111)$ |
| 0           | $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 000001100)$ + $a_0 b_0 | 001000000)$ + $a_0 b_0 | 001000000)$ |
| 1           | $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 000001100)$ + $a_0 b_0 | 001000000)$ + $a_0 b_0 | 001000000)$ |
| 0           | $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 110110000)$ + $a_0 b_0 | 110000000)$ + $a_0 b_0 | 000001100)$ + $a_0 b_0 | 001000000)$ + $a_0 b_0 | 001000000)$ |
| 1           | $a_0 b_0 | 111110000)$ + $a_0 b_0 | 111010000)$ + $a_0 b_0 | 111000000)$ + $a_0 b_0 | 111010000)$ + $a_0 b_0 | 111000000)$ + $a_0 b_0 | 001000110)$ + $a_0 b_0 | 001100011)$ + $a_0 b_0 | 001100011)$ |
Φ_{ji} B_0 B_1 = α_0 |00⟩ + α_1 |11⟩

1.2 Preparation of Eight-Qubit Entangled State (QB ENS)

The proposed quantum channel (eight-QB EN) is practically feasible, as shown in Fig. 1. As shown in this figure, the quantum circuit of the proposed channel can be created by utilizing three Hadamard gates and using four to seven CNOT gates.

The steps of creating a channel are explained in details as the following:

The first, IS is prepared with zero states as Eq. 11.

After applying Hadamard gates, one CNOT gate is applied with QB b0 as a control QB and b1 as a target QB. Then, the whole state of the system is as the following:

In the next step, three CNOT gates are applied as that QBs b0, a1 and a2 are Control QBs and QBs a0, b2 and b3 target QBs, respectively. Then, the state of all the eight QBs becomes as same Eq. (13).

Finally, the controller can apply CNOT gates by U function so that he can consider each combination of QBs a0, b2 and b3 as control and QB c as the target. So, he can create different eight states, as shown in Table 1 and type of used combination can be encoded by three classical bits according to sequence a0, b2, b3. For example, if he creates a state in Eq. (2), then he needs to apply one CNOT gate with QB b3 as control and QB c as the target. So, the proposed channel can be created as Eq. (14).
Table 4  The measurement results based on $X$ and collapsed states

| A’s Result | B’s Result | The collapsed state of QBs $(b_0 | b_1 | a_1 | a_2 | c)$ |
|------------|------------|-------------------------------------------------|
| ++         | ++         | $a_\alpha b_\alpha | 00000 \rangle + a_\beta b_\alpha | 00011 \rangle + a_\alpha b_\beta | 00100 \rangle + a_\alpha b_\beta | 00111 \rangle + a_\beta b_\alpha | 11000 \rangle + a_\alpha b_\beta | 11011 \rangle + a_\beta b_\alpha | 11100 \rangle + a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\alpha b_\alpha | (00011 \rangle + a_\beta b_\alpha | (00100 \rangle - a_\alpha b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11000 \rangle - a_\alpha b_\alpha | 11011 \rangle + a_\beta b_\alpha | 11100 \rangle - a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle - a_\alpha b_\alpha | (00100 \rangle - a_\alpha b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11000 \rangle + a_\alpha b_\alpha | 11011 \rangle - a_\alpha b_\alpha | 11100 \rangle - a_\beta b_\alpha | 11111 \rangle |
| ++         | ++         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\beta b_\alpha | (00100 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11000 \rangle + a_\beta b_\alpha | 11011 \rangle + a_\beta b_\alpha | 11100 \rangle + a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\beta b_\alpha | (00100 \rangle + a_\beta b_\alpha | (00111 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11000 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11100 \rangle - a_\beta b_\alpha | (00111 \rangle - a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle - a_\beta b_\alpha | (00100 \rangle - a_\beta b_\alpha | (00111 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11000 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11100 \rangle - a_\beta b_\alpha | (00111 \rangle - a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\beta b_\alpha | (00100 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11000 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11100 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11111 \rangle |
| ++         | --         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\beta b_\alpha | (00100 \rangle - a_\beta b_\alpha | (00111 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11000 \rangle - a_\alpha b_\alpha | (00111 \rangle - a_\alpha b_\alpha | 11100 \rangle - a_\beta b_\alpha | (00111 \rangle - a_\beta b_\alpha | 11111 \rangle |
| --         | ++         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\beta b_\alpha | (00100 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11000 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11100 \rangle + a_\beta b_\alpha | (00111 \rangle + a_\beta b_\alpha | 11111 \rangle |
| --         | ++         | $a_\alpha b_\alpha | (00000 \rangle - a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | ++         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | ++         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | ++         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | --         | $a_\alpha b_\alpha | (00000 \rangle + a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
| --         | --         | $a_\alpha b_\alpha | (00000 \rangle - a_\alpha b_\alpha | (00011 \rangle + a_\alpha b_\alpha | (00100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11000 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11100 \rangle + a_\alpha b_\alpha | (00111 \rangle + a_\alpha b_\alpha | 11111 \rangle |
\[
|\psi\rangle = \frac{1}{\sqrt{8}} (|00000000\rangle + |00001101\rangle + |00010010\rangle + |00111111\rangle + |11001000\rangle + |11101101\rangle \quad (14)
\]
\[
+ |11110010\rangle + |11111111\rangle \rangle_{abcb2} \quad \text{for} \quad m = \eta_{AD}\text{ or } \eta_{PD}.
\]
As stated above, an eight-QBS can be prepared and used as a quantum channel. Until now, the SIQ UO and single QB measurements have already been used in various quantum systems [28–29–30]. So, the proposed scheme can be implemented in quantum information technology with advances in the future.

### 1.3 Effects of Channel Noises on the Proposed Protocol

In this section, the effect of two models of environment noise includes an AD and PD noisy environment on the proposed BCQT process are discussed. These two environment noises can be determined by Kraus operators [37] shown in EQBS. (15) and (16), respectively.

\[
E^A_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta_A} \end{bmatrix}, E^A_1 = \begin{bmatrix} 0 & \sqrt{\eta_A} \\ 0 & 0 \end{bmatrix}. \quad (15)
\]

\[
E^P_0 = \sqrt{1-\eta_P} I, E^P_1 = \sqrt{\eta_P} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E^P_2 = \sqrt{\eta_P} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (16)
\]

Where \( \eta_A (0 < \eta_A \leq 1) \) and \( \eta_P (0 < \eta_P \leq 1) \) are the DRs (the error occurring probability of a quantum state (QS) in the corresponding channel when a travel QB passes through it) for the ADN and the PDN, respectively. Also, \( I \) is the identity matrix in the Hilbert space of \( C_2 \times 2 \).

The channel proposed in the previous section \( \langle \psi | \psi \rangle \) is a pure state. The corresponding density matrix can be described as \( \rho = \langle \psi | \psi \rangle \). So, the effect of the noise described by (15) or (16) on the density operator \( \rho \) can be stated as Eq. (17).

\[
\xi (\rho) = \sum_m (E^m_0)^\dagger (F^m_{ai}) (E^m_{ai})^\dagger (F^{i0}_{m}) (F^{j0}_{m}) (F^{kl}_{m}) (F^{kl}_{m})^\dagger
\]
\[
\rho (F^{i0}_{m}) (F^{j0}_{m}) (F^{kl}_{m}) (F^{kl}_{m})^\dagger. \quad (17)
\]

Where \( r = \{A, P\} \). For \( r = A \), ADN, \( m \in \{0, 1\} \), while for \( r = P \), PDN, \( m \in \{0, 1, 2\} \), and the superscripts \( ai \) and \( bi \) represent the operator \( E \) act on which QB. \( \xi \) denotes a quantum operation.
Table 6 Applying Z UO for the different channel showed in Table 1

| Coding bits to show the different channels | C’s Results | The collapsed state of QBs | UO on $b_0, b_1, a_1, a_2$ |
|-------------------------------------------|-------------|----------------------------|-----------------------------|
| 000                                       | $|+\rangle$ | $|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle$ | $I \otimes I \otimes I \otimes I$ |
|                                           | $|-\rangle$ | $|0000\rangle - |0001\rangle + |0010\rangle - |0011\rangle + |1100\rangle - |1101\rangle + |1110\rangle - |1111\rangle$ | $I \otimes I \otimes I \otimes Z$ |
| 010                                       | $|+\rangle$ | $|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle$ | $I \otimes I \otimes I \otimes Z$ |
|                                           | $|-\rangle$ | $|0000\rangle - |0001\rangle - |0010\rangle - |0011\rangle + |1100\rangle - |1101\rangle - |1110\rangle - |1111\rangle$ | $I \otimes I \otimes I \otimes Z \otimes Z$ |
| 100                                       | $|+\rangle$ | $|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle$ | $I \otimes I \otimes I \otimes I \otimes Z \otimes Z$ |
|                                           | $|-\rangle$ | $|0000\rangle - |0001\rangle - |0010\rangle + |0011\rangle - |1100\rangle - |1101\rangle - |1110\rangle - |1111\rangle$ | $I \otimes Z \otimes I \otimes Z \otimes I \otimes I \otimes Z$ |
| 110                                       | $|+\rangle$ | $|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle$ | $I \otimes I \otimes Z \otimes Z$ |
|                                           | $|-\rangle$ | $|0000\rangle - |0001\rangle - |0010\rangle + |0011\rangle - |1100\rangle - |1101\rangle - |1110\rangle - |1111\rangle$ | $I \otimes Z \otimes I \otimes I \otimes Z \otimes Z$ |
| 111                                       | $|+\rangle$ | $|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle$ | $I \otimes I \otimes Z \otimes Z \otimes Z$ |
|                                           | $|-\rangle$ | $|0000\rangle - |0001\rangle - |0010\rangle + |0011\rangle - |1100\rangle - |1101\rangle - |1110\rangle - |1111\rangle$ | $I \otimes I \otimes I \otimes Z \otimes Z \otimes Z$ |
which maps from $\rho$ to $\xi(r)$ due to the noise. In Eq. (17), We suppose only QBs belong to A and B are affected by noisy environments (NEs). Because controller (C) create the channel and these QBs are transmitted through the NE by the controller (C) to A and B. But, QB $c$ is considered without the effect of the NE due to belonging to controller (C) and is not transmitted in the channel. Also, it is considered that both the QBs sent to A and B are affected by the same Kraus operator. In these two-noise environments, the quantum channel would become a mixed state, as shown in Eq. (18) and (19).

$$
\xi_A(\rho) = \frac{1}{8} \{ [[(00000000) + (1-\eta_A)(0001101)] + (1-\eta_A)(00010010) + (1-\eta_A)^2(00011000)(0000100)] \\
+ \sqrt{(1-\eta_A)^5}(11100000) + \sqrt{(1-\eta_A)^5}(11101101) + \sqrt{(1-\eta_A)^5}(11100010) + \sqrt{(1-\eta_A)^5}(11111111)] \\
\times [(00000000) + (1-\eta_A)(00001010) + (1-\eta_A)(00001010) + (1-\eta_A)^2(00011111) + \sqrt{(1-\eta_A)^5}(11100000)] \\
+ \sqrt{(1-\eta_A)^5}(11110101) + \sqrt{(1-\eta_A)^5}(11110010) + \sqrt{(1-\eta_A)^5}(11111111]$$

(18)

And

$$
\xi_P(\rho) = \frac{1}{8} \{ (1-\eta_P)^7[(00000000) + (0001101) + (00010010) + (00011111) \\
+ (11100000) + (11101101) + (11100010) + (11111111)] \times [(00000000) \\
+ (00001101) + (00010010) + (00011111) + (11100000) + (11101101] \\
+ (11110010) + (11111111) + \eta_P^2[00000000)(00000000) \\
+ \eta_P^7[11111111)](11111111)] \}
$$

(19)
According to A’s, B’s and C’s measured results, A and B can make apt operation on their QBs to recover the original state. Then, the final state can be represented as a density matrix, as shown in Eq. (20)

$$
\rho_{\text{out}} = Tr_{A_0B_0B_1a_0c_b2b_3} \left\{ U \left[ \rho_{A_0B_1} \otimes \rho_{B_0B_1} \otimes \xi \left( \rho \right) \right] U^\dagger \right\},
$$

(20)

Where $Tr_{A_0B_1} A_1 B_0 B_1 a_0c_b2b_3$ is the partial trace over QBs $(A_0, A_1, B_0, B_1, a_0, c, b_2, b_3)$ and $U$ is a UO to explain the BQCT process, which is written by

$$
U = \left\{ I_{A_0A_1} \otimes I_{B_0B_1} \otimes I_{a_0b_2} \otimes I_{b_1c_1} \otimes \xi_{\text{in noqpsat}} \right\}
$$

$$
\left\{ I_{A_0A_1} \otimes I_{B_0B_1} \otimes \xi_{a_0b_2} \right\}
$$

$$
\left\{ \phi_{A_0}^{\beta_{1}} \phi_{A_1}^{\alpha_{0}} \phi_{B_0}^{\eta_{b}} \phi_{B_1}^{\beta_{b}} \phi_{a_0}^{\alpha_{0}} \phi_{b_2}^{\beta_{b}} \phi_{b_1}^{\alpha_{1}} \phi_{c_1}^{\beta_{c}} \phi_{d_2}^{\eta_{d_2}} \right\}
$$

(21)

Where $l, m, n, o, p, q, s, t \in \{1, 2\}$, with $U_{l\alpha_0}$ stands for controlled operations on the QBs $A_0, B_0$ and $B_1$ as control and QBs $a_0, b_2$ and $b_3$ as target, $|\phi\rangle_{n_{a_0}}^{\beta_{1}} |\phi\rangle_{n_{b_2}}^{\alpha_{0}} |\phi\rangle_{n_{b_1}}^{\eta_{b}} |\phi\rangle_{n_{c_1}}^{\beta_{b}}$ denote A’s single-QBS measurement results, $|\phi\rangle_{b_1}^{\beta_{1}} |\phi\rangle_{b_2}^{\alpha_{0}} |\phi\rangle_{b_3}^{\eta_{b}} |\phi\rangle_{b_4}^{\beta_{b}}$ show B’s single-QBS measurement results and $|\phi\rangle_{c_1}^{\beta_{c}} |\phi\rangle_{d_2}^{\eta_{d_2}}$ is A’s and B’s recovery operation depending on A’s and B’s and C’s measurement results.

Given that the choice, we may have to take the final QS $\rho_{\text{out}}$ which is the product of the QBs generated on the side of the receivers A and B in a NE. On the other hand, where A would have QB $(\beta_{00}^{00} |00\rangle + \beta_{01}^{01} |01\rangle + \beta_{10}^{10} |10\rangle + \beta_{11}^{11} |11\rangle)_{a_2}$ in her possession and B would have $(\alpha_{0}^{00} + \alpha_{1}^{11} |00\rangle + \alpha_{0}^{01} |01\rangle + \alpha_{1}^{10} |10\rangle + \alpha_{1}^{11} |11\rangle)_{b_0}$ b_1 in his possession, the expected final state in the absence of noise is a product state. As we know, in the ideal situation would be $|\Phi\rangle = (\beta_{00}^{00} |00\rangle + \beta_{01}^{01} |01\rangle + \beta_{10}^{10} |10\rangle + \beta_{11}^{11} |11\rangle)_{a_2} \otimes (\alpha_{0}^{00} + \alpha_{0}^{01} |00\rangle + \alpha_{1}^{10} |01\rangle + \alpha_{1}^{11} |11\rangle)_{b_0} b_1$.

By comparing the QS $\rho_{\text{out}}$ in the NE with the state $|\Phi\rangle$ by using fidelity the effect of noise can be shown as follow:

$$
F = \langle \Phi | \rho_{\text{out}} | \Phi \rangle.
$$

(22)

According to (20) and (21), get the resultant state $\rho_{\text{out}}^r$ from (18) and (19) is easy and the $\rho_{\text{out}}^r$ is related to three participants’ measurement results $l, m, n, o, p, q, s, t$. However, after calculation, we get that output state $\rho_{\text{out}}^r$ is independent of three participants’ measurement results. And the $\rho_{\text{out}}^r$ is shown as follows.

$$
\rho_{\text{out}}^r = \{ (1-\eta_{\alpha A}) \alpha_{0}^{00} |0000\rangle + (1-\eta_{\alpha A}) \alpha_{0}^{01} |0001\rangle + (1-\eta_{\alpha A}) \alpha_{0}^{10} |0010\rangle
$$

$$
+ (1-\eta_{\alpha A})^2 \alpha_{0}^{11} |0011\rangle + \sqrt{(1-\eta_{\alpha A})^3} \alpha_{0}^{11} |0100\rangle |1000\rangle + \sqrt{(1-\eta_{\alpha A})^5} \alpha_{0}^{11} |1011\rangle
$$

$$
+ \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1000\rangle + \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1011\rangle
$$

$$
+ \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1100\rangle + \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1111\rangle
$$

$$
+ \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1111\rangle
$$

$$
+ \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |1111\rangle + \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |0000\rangle |0000\rangle + \sqrt{(1-\eta_{\alpha A})^2} \alpha_{0}^{11} |0000\rangle |0000\rangle
$$

(23)
And

\[
\rho_{\text{out}}^P = \{(1-\eta_P)^7[\alpha_0\beta_{00}|0000\rangle + \alpha_0\beta_{01}|0001\rangle + \alpha_0\beta_{10}|0010\rangle + \alpha_0\beta_{11}|0011\rangle \\
+ \alpha_1\beta_{00}|1100\rangle + \alpha_1\beta_{01}|1101\rangle + \alpha_1\beta_{10}|1110\rangle + \alpha_1\beta_{11}|1111\rangle\} \times [\alpha_0\beta_{00}|0000\rangle \\
+ \alpha_0\beta_{01}|0001\rangle + \alpha_0\beta_{10}|0010\rangle + \alpha_0\beta_{11}|0011\rangle \\
+ \alpha_1\beta_{00}|1100\rangle + \alpha_1\beta_{01}|1101\rangle + \alpha_1\beta_{10}|1110\rangle + \alpha_1\beta_{11}|1111\rangle \\
+ \eta_P^7\alpha_1^2\beta_{00}^2|0000\rangle \langle 0000| + \eta_P^7\alpha_1^2\beta_{11}^2|1111\rangle \langle 1111|\}. \tag{24}
\]

Using (22) and (23), the fidelity of the QS teleportation obtained by utilizing the proposed BQCT under ADN as below:

\[
F^A = \{[\alpha_0^2\beta_{00}^2 + (1-\eta_A)\alpha_0^2\beta_{01}^2 + (1-\eta_A)\alpha_0^2\beta_{10}^2 + (1-\eta_A)\alpha_0^2\beta_{11}^2 + \sqrt{(1-\eta_A)^3 \alpha_1^2\beta_{00}^2} \\
+ \sqrt{(1-\eta_A)^5 \alpha_1^2\beta_{01}^2 + \sqrt{(1-\eta_A)^5 \alpha_1^2\beta_{10}^2 + \sqrt{(1-\eta_A)^7 \alpha_1^2\beta_{11}^2}} + \eta_A\alpha_0^2\beta_{00}^2\beta_{11}^2\} \tag{25}
\]

Similarly, by using (22) and (24), the fidelity of the QS teleportation obtained by utilizing the proposed BQCT scheme under PDN as below:

\[
F^P = (1-\eta_P)^7 + \eta_P^7\alpha_0^4\beta_{00}^4 + \eta_P^7\alpha_1^4\beta_{11}^4 \tag{26}
\]

From (25) and (26), It is inferred that the fidelities only depend on the amplitude parameter of the IS and the DR. Specifically, Fig. 2 a-c, d-f) clearly shows the effect of amplitude-damping (phase-damping) noise on the fidelity $F^A$ ($F^P$) and variation of the fidelity with amplitude parameter of the IS and the DR $\eta_A$ the fidelity $F^A$ and $F^P$ always decrease with decoherence $\eta_A$ and $\eta_P$, respectively, as can be seen easily(Fig. 2 a, b, d, e).

Also, it is important to note which effect of DR on $F^A$ and $F^P$ is difference for the various amplitudes of transmitted states. E.g. Figure 3 show the rate of changes of ADN with PDN by assuming $\eta_A = \eta_P = \eta$ and $\alpha_0 = \beta_{00}$ and $\beta_{01} = \beta_{10} = 0$ (Here $\alpha_0$, $\beta_{00} \in [0.2, 0.4, 0.6, 0.8, 1]$). In this situation, for the same value of DR and $\alpha_0 = \beta_{00} \in [0.8, 1]$, the fidelity of the amplitude damping channel is always more than that of the phase-damping channel. Therefore, as can be seen, for this particular choice of the amplitude parameter of the IS, loss of information is less when the travel QBs are transferred through the amplitude damping channel as compared to the phase-damping channel. However, for the situation of $\alpha_0 = \beta_{00} = \in [0.2, 0.4, 0.6]$ can be seen that for $\eta > [0.5, 0.6, 0.8]$ the effect of phase-damping channel on fidelity is less than that of the amplitude-damping channel, respectively.

So, the fidelity decreases by increased noise in a real physical scenario. However, BQCT may be implemented with maximum fidelity, i.e., as prefect, if the states with particular $\alpha_i$ and $\beta_j$ are teleported, even in a NE. This fact is shown in the peaks of Fig. 3 c, f.

### 1.4 Comparison

In this section, the proposed protocol is compared with the best presented BCQT/BQT protocols in terms of the type of protocol, the number of QBs sent by A and B, the number of QBs used in quantum channel, efficiency, Auxiliary QB (The number of used additional qubits), GHZMs (GHZ measurements), BSMs (Bell-state measurements), SMs (single QB
measurements), Prob. (i.e. the probability of guessing C’s QB by an eavesdropper), and global operations (which for transmitting these gates, we need to add resources of entanglement states and QBs) as shown in Table 7. In this table, the efficiency is defined as the same [38], the ratio of the number of sending QBs to the number of channel QBs. Also, in this table, QCPG stands for (Quantum Controlled Phase Gate). As shown in Table 7, the proposed protocol only uses a single QB measurement basis, which is more efficient than two-QB measurements (Bell state measurements) and GHZ measurements (GHZMs), a three-qubit measurement, with more complexity than Bell-state measurements. It is well known that Bell-state measurements can be decomposed into an ordering combination of a SIQ Hadamard operation and a two-QB CNOT
operation as well as two SIQ measurements. In the other hand, in our scheme, the users can teleport an arbitrary EN and a pure two-QBS to each other, simultaneously with the permission of a third party as supervisor or controller while in some of the works [27, 30], the controller can only control one of the users. Also, in the works [28, 30], users need to apply global operations include of global CNOT gate and global QCPG between A and B. For transmitting these gates, as stated above, we need additional quantum and classical resources [39, 40]. So, these protocols are not optimal. Work [31] has been designed for a particular type of state, two, and three QBs entangled states and can only work for transmitting these states. For their work, two GHZ state and one Bell state measurements have used.

The work presented in [29] and the proposed protocol, both can teleport an EN and a pure two-QBS each to others simultaneously with the permission of a third party as supervisor or controller. However, the efficiency in our protocol is higher than [29]. Also, as stated above, the proposed protocol only uses eight SIQ measurements. But, work [29] used four two-QB measurements and one SIQ measurement. So, this work used a nine SIQ measurement that is not optimal. Besides, our protocol reduces the probability of guessing C’s QB by an eavesdropper to $\frac{1}{8}$. It is defined as the number of possible separate states obtained after the measurement by C (Charlie’s QB), as shown in Table 6. Also, in our protocol, the supervisor has more freedom and can control one of the users or both according to his conditions. In Table 8, we compare the proposed protocol and protocol presented in [29] in resources used for preparing quantum channels in terms of the number of CNOT and Hadamard operations. As shown in this table, our protocol needs fewer resources (four to seven CNOT operations and three Hadamard operations) compared to the previous work [29] in the same conditions.

In Table 7, some of BQT protocols are compared with our method. It is important to note that these protocols have not a controller, and as a result, they place at a lower level of security. In [14] Method 2, the protocol needs to a local Toffoli gate, a complex three-QBs gate, for decoding transmitted qubits.

2 Conclusion

In this paper, a novel protocol was proposed for BCQT using the eight-QB EN as the quantum channel by which the users can teleport an arbitrary EN and a pure two-QBS to each other.

Fig. 3 The fidelities of ADN and PDN. The blue lines stands for the ADN, the red lines stands for the PDN
### Table 7 Comparison of BCQT protocols

| Reference  | Type of protocol | B’s QB | A’s QB | Quantum channel | Efficiency | Auxiliary QB | GHZMs | BSMs | SMs | Prob. | Global Operations |
|------------|------------------|--------|--------|-----------------|------------|--------------|-------|------|-----|-------|-------------------|
| [13]       | BQT              | 2(EPR)| 1      | 5 QB            | 0          | 0            | 2     | 0    | 0   | 0     |                    |
| [14] Method 1 | BQT          | 2(EPR)| 2(EPR)| 6 QB            | 0          | 2            | 0     | 0    | 0   | 0     |                    |
| [14] Method 2 | BQT          | 3(EPR)| 2(EPR)| 6 QB            | 1          | 2            | 0     | 0    | 0   | 0     |                    |
| [27]*      | BCQT            | 2     | 1      | 7 QB ENs        | 0          | 0            | 3     | 1    | 1   | 0     |                    |
| [28]       | BCQT            | 2     | 1      | 7 QB ENs        | 0          | 0            | 3     | 1    | 1   | 1 CNOT          |                    |
| [29]       | BCQT            | 2     | 2      | 9QB             | 0          | 0            | 4     | 1    | 1   | 0     |                    |
| [30]*      | BCQT            | 1     | 2      | 6 QB cluster    | 0          | 0            | 2     | 2    | 1   | 1 QCPG          |                    |
| [31]       | BCQT            | 2(EPR)| 3(EPR)| 9 QB cluster    | 0          | 2            | 1     | 0    | 0   | 0     |                    |
| Proposed Method | BCQT        | 2(EPR)| 2      | 8-QB ENs        | 0          | 0            | 0     | 0    | 8   | 0     |                    |

*In these protocols controller can only control one of users*
simultaneously under the permission of the supervisor. This protocol was based on the controlled-not operation, appropriate SIQ UOs and SIQ measurements in the Z and X-basis, which are more efficient than two-QB measurements [25, 29, 41]. Also, in this paper, a new criterion of merit named as (predictability of the controller’s qubit (QB) by the eavesdropper) was introduced, and the protocol was improved based on it. Besides, the supervisor could select control of one of the users or both. Also, in the proposed protocol, used quantum resources for preparing the quantum channel, and also, the number of measurements were fewer than previous works. Next, we reviewed the proposed protocol in typical noisy channels, including the ADN and the PDNs. We analytically derived the fidelities of the BCQT process and showed that the fidelities only depend on the amplitude parameter of the IS and the DNR. We hope that such BCQT protocol can be realized experimentally in the future.

Compliance with Ethical Standards

Declarations of Interest none.

References

1. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2001)
2. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895–1899 (1993)
3. Shi, B.-S., Tomita, A.: Teleportation of an unknown state by W state. Phys. Lett. A. 296, 161–164 (2002)
4. Ursin, R., Jennewein, T., Aspelmeyer, M., Kaltenbaek, R., Lindenthal, M., Walther, P., Zeilinger, A.: Communications: quantum teleportation across the Danube. Nature. 430, 849–849 (2004)
5. Agrawal, P., Pati, A.: Perfect teleportation and superdense coding with W states. Phys. Rev. A. 74, 062320 (2006)
6. Muralidharan, S., Panigrahi, P.K.: Perfect teleportation, quantum-state sharing, and superdense coding through a genuinely entangled five-qubit state. Phys. Rev. A. 77, 032321 (2008)
7. Tang, S.-Q., Shan, C.-J., Zhang, X.-X.: Quantum teleportation of an unknown two-atom entangled state using four-atom cluster state. Int. J. Theor. Phys. 49, 1899–1903 (2010)
8. Nie, Y.-y., Li, Y.-h., Liu, J.-c., Sang, M.-h.: Perfect teleporation of an arbitrary three-qubit state by using W-class states. Int. J. Theor. Phys. 50, 3225–3229 (2011)
9. Yuan-Hua, L., Yi-You, N.: Quantum information splitting of an arbitrary three-atom state by using W-class states in cavity QED. Commun. Theor. Phys. 57, 995 (2012)
10. Luo, M.-X., Deng, Y.: Quantum splitting an arbitrary three-qubit state with χ-state. Quantum Inf. Process. 1–12 (2013)
11. Li, Y.-h., Li, X.-l., Sang, M.-h., Nie, Y.-y.: Splitting unknown two-qubit state using five-qubit entangled state. Int. J. Theor. Phys. 53, 111–115 (2014)
12. Nandi, K., Mazumdar, C.: Quantum teleportation of a two qubit state using GHZ-like state. Int. J. Theor. Phys. 53, 1322–1324 (2014)
13. Sang, M.-h.: Bidirectional quantum teleportation by using five-qubit cluster state. Int. J. Theor. Phys. 55(3), 1333–1335 (2016)
14. Zhou, R.-G., Li, X., Qian, C., Ian, H.: Quantum bidirectional teleportation 2↔ 2 or 2↔ 3 Qubit teleportation protocol via 6-Qubit entangled state. Int. J. Theor. Phys. 59(1), 166–172 (2020)
15. Koochaki, F., Sharifi, I., Talebi, H.A.: A novel architecture for cooperative remote rehabilitation system. Comput. Electr. Eng. 56, 715–731 (2016)
16. Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A. 58, 4394–4400 (1998)
17. Yang, C.-P., Han, S.: A scheme for the teleportation of multiqubit quantum information via the control of many agents in a network. Phys. Lett. A. 343, 267–273 (2005)
18. Deng, F.-G., Li, C.-Y., Li, Y.-S., Zhou, H.-Y., Wang, Y.: Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. Phys. Rev. A. 72, 022338 (2005)
19. Wang, Y.-H., Song, H.-S.: Preparation of partially entangled W state and deterministic multi-controlled teleportation. Opt. Commun. 281, 489–493 (2008)
20. Song-Song, L., Yi-You, N., Zhi-Hui, H., Xiao-Jie, Y., Yi-Bin, H.: Controlled teleportation using four-particle cluster state. Commun. Theor. Phys. 50, 633–636 (2008)
21. Wang, X.-W., Su, Y.-H., Yang, G.-J.: Controlled teleportation against uncooperation of part of supervisors. Quantum Inf. Process. 8, 319–330 (2009)
22. Tian-Yin, W., Qiao-Yan, W.: Controlled quantum teleportation with bell states. Chinese Physics B. 20, 040307 (2011)
23. Zha, X.-W., Zou, Z.-C., Qi, J.-X., Song, H.-Y.: Bidirectional quantum controlled teleportation via five-qubit cluster state. Int. J. Theor. Phys. 52, 1740–1744 (2013)
24. Shukla, C., Banerjee, A., Pathak, A.: Bidirectional controlled teleportation by using 5-qubit states: a generalized view. Int. J. Theor. Phys. 52, 3790–3796 (2013)
25. S. Hassanpour and M. Houshmand: Bidirectional quantum controlled teleportation by using EPR states and entanglement swapping, arXiv preprint arXiv:1502.03551, (2015)
26. Amiri, I.S., Alavi, S.E., Bahadoran, M., Afrozeh, A., Ahmad, H.: Nanometer bandwidth Soliton generation and experimental transmission within nonlinear Fiber optics using an add-drop filter system. J. Comput. Theor. Nanosci. 12, 221–225 (Feb 2015)
27. Hong, W.-q.: Asymmetric bidirectional controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. 55, 384–387 (2016)
28. Sang, M.-h.: Bidirectional quantum controlled teleportation by using a seven-qubit entangled state. Int. J. Theor. Phys. 55, 380–387 (2016)
29. Li, Y.-h., Jin, X.-m.: Bidirectional controlled teleportation by using nine-qubit entangled state in noisy environments. Quantum Inf. Process. 15, 929–945 (2016)
30. Li, Y.-h., Nie, L.-p., Li, X.-l., Sang, M.-h.: Asymmetric bidirectional controlled teleportation by using six-qubit cluster state. Int. J. Theor. Phys. 55, 3008–3016 (2016)
31. Choudhury, B.S., Samanta, S.: Asymmetric bidirectional 3↔ 2 qubit teleportation protocol between Alice and bob via 9-qubit cluster state. Int. J. Theor. Phys. 56(10), 3285–3296 (2017)
32. Zheng, S.-B.: Scheme for approximate conditional teleportation of an unknown atomic state without the bell-state measurement. Phys. Rev. A. 69, 064302 (2004)
33. Riebe, M., Häffner, H., Roos, C., Hänsel, W., Benhelm, J., Lancaster, G., et al.: Deterministic quantum teleportation with atoms. Nature. 429, 734–737 (2004)
34. Bouwmeester, D., Pan, J.-W., Mattle, K., Eibl, M., Weinfurter, H., Zeilinger, A.: Experimental quantum teleportation. Nature. 390, 575–579 (1997)
35. Wang, X.-L., Cui, X.-D., Su, Z.-E., Chen, M.-C., Wu, D., Li, L., Liu, N.L., Lu, C.Y., Pan, J.W.: Quantum teleportation of multiple degrees of freedom of a single photon. Nature. 518, 516–519 (2015)
36. Metcalf, B.J., Spring, J.B., Humphreys, P.C., Thomas-Peter, N., Barbieri, M., Kolthammer, W.S., Jin, X.M., Langford, N.K., Kundys, D., Gates, J.C., Smith, B.J., Smith, P.G.R., Walmsley, I.A.: Quantum teleportation on a photonic chip. Nat. Photonics. 8, 770–774 (2014)
37. Xian-Ting, L.: Classical information capacities of some single qubit quantum noisy channels. Commun. Theor. Phys. 39, 537–542 (2003)
38. Zadeh, M.S.S., Houshmand, M., Aghababa, H.: Bidirectional teleportation of a two-Qubit state by using eight-Qubit entangled state as a Quantum Channel. Int. J. Theor. Phys. 1–2 (2017)
39. N. Nickerson: Practical Fault-Tolerant Quantum Computing. (2015)
40. Eisert, J., Jacobs, K., Papadopoulos, P., Plenio, M.: Optimal local implementation of nonlocal quantum gates. Phys. Rev. A. 62, 052317 (2000)
41. Chen, Y.: Bidirectional quantum controlled teleportation by using a genuine six-qubit entangled state. Int. J. Theor. Phys. 54(1), 269–272 (2015)