Neutrino Masses: Shedding Light on Unification and Our Origin*

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In the first part of the talk, three key ideas proposed in the 1970s, and in particular their combined role in providing an understanding of the neutrino-masses as well as of the baryon-asymmetry of the universe, are expounded. The ideas in question include: (i) The symmetry $SU(4)$-color, which introduces the right-handed neutrino as an essential member of each family and also provides (rather reliably) the Dirac mass of the tau-neutrino by relating it to the top quark mass; (ii) SUSY grand unification together with the scale of the meeting of the three gauge couplings, which provides the scale for the superheavy Majorana masses of the RH neutrinos; and (iii) the seesaw mechanism, which combines the Dirac and the superheavy Majorana masses of the neutrinos obtained as above to yield naturally light LH neutrinos and in particular the right magnitude for $m(\nu^L_\tau)$. In the second part, an attempt is made, based in part on recent works, to show how a set of diverse phenomena including (a) fermion masses, (b) neutrino oscillations, (c) CP and flavor violations, and (d) baryogenesis via leptogenesis can fit together neatly within a single predictive framework based on an effective symmetry group $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ or $SO(10)$, possessing supersymmetry. CP and flavor violations arising within this framework include enhanced rates (often close to observed limits) for $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ and also measurable electric dipole moments of the neutron and the electron. Expectations arising within the same framework for proton decay are summarized at the end. It is stressed that the two notable missing pieces of this framework, which is otherwise so successful, are supersymmetry and proton decay. While the search for supersymmetry at the LHC is eagerly awaited that for proton decay will need the building of a next-generation megaton-size underground detector.

1. Introduction

Since the discoveries (confirmations) of the atmospheric [11] and solar neutrino oscillations [23], the neutrinos have emerged as being among the most effective probes into the nature of higher unification. Although almost the feeblest of all the entities of nature, simply by virtue of their tiny masses, they seem to possess a subtle clue to some of the deepest laws of nature pertaining to the unification-scale as well as the nature of the unification-symmetry. In this sense, the neutrinos provide us with a rare window to view physics at truly short distances. As we will see, these turn out to be as short as about $10^{-30}$ cm. Furthermore, it appears most likely that the origin of their tiny masses may be at the root of the origin of matter-antimatter asymmetry in the early universe. In short, the neutrinos may well be crucial to our own origin!

The main purpose of my talk here today will be to present a unified picture, in accord with observations, of

- Fermion masses and mixings
- Neutrino oscillations
- CP and flavor violations
- Baryogenesis via leptogenesis, and
- Proton decay

The goal will be to exhibit the intimate links that exist between these different phenomena. Each of these features, or a combination of some of

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them (though not all in conjunction with each other), have been considered widely in the literature. My main theme will be to exhibit how the first four, on which we have much empirical data, hang together within a single predictive framework based on the gauge symmetry $G(224) = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)^c$ or $\text{SO}(10)$, leaving proton decay and supersymmetry as the two missing pieces of this picture. The crucial ingredients of the picture turn out to be:

1. **Existence of the right-handed neutrino ($\nu_R$)** — that has been a compelling prediction of the symmetry $\text{SU}(4)^c$ as well as of $\text{SU}(2)_L \times \text{SU}(2)_R$ from the 1970s. It is now needed to implement the seesaw mechanism as well as baryogenesis via leptogenesis.

2. **The observed meeting of the three gauge couplings** at a scale $M_U \approx 2 \times 10^{16}$ GeV. This observation on the one hand provides a strong evidence in favor of the ideas of both grand unification and supersymmetry; on the other hand it sets the scale for the superheavy Majorana masses of the RH neutrinos, which figure prominently in the seesaw formula for the masses of the LH neutrinos.

3. **The gauge symmetry $\text{SU}(4)$-color**, which introduces three characteristic features — of direct relevance to neutrino physics: (a) the RH neutrinos (mentioned above); (b) B-L as a local symmetry, which protects the RH neutrinos from acquiring a Planck or string-scale mass; and (c) two simple mass-relations for the third family:

\[
\begin{align*}
    m_b(M_U) & \approx m_\tau \\
    m(\nu_{\text{Dirac}}) & \approx m_{\text{top}}(M_U)
\end{align*}
\]

4. **The seesaw mechanism** which combines the Dirac and the superheavy Majorana masses of the neutrinos obtained as above to yield naturally light LH neutrinos and in particular the right magnitude for $m(\nu_L^e)$.

To set the background for a discussion along these lines I will first recall in the next section the salient features of certain unification ideas based on $\text{SU}(4)$-color, which developed in the early 1970s, their inter-relationships as well as their relevance in the present context. In a subsequent section, I will present a predictive framework based on previous works on fermion masses and neutrino oscillations, and in the following sections I will discuss the topics of (a) CP and flavor violations, (b) leptogenesis, and (c) proton decay, as they arise within the same framework. In the last section I will present a summary and make some concluding remarks.

2. **Unification with $\text{SU}(4)$-color: Neutrino Masses and the Seesaw Mechanism**

Going back to the days of Pauli, neutrinos have been rather special from the day he postulated their existence in the 1930s. They were distinct then and are distinct even now from all other known elementary particles in that they are essentially massless and almost interactionless. Up until the 1990s, prior to the discoveries of neutrino oscillations, many (perhaps even most in the 1970s) in fact believed, given that the upper limits on neutrino masses were already known to be so small ($m(\nu_e)/m_e \lesssim 10^{-6}$ and, after the “discovery” of $\nu_\tau$, $m(\nu_\tau)/m_{\text{top}} < 10^{-9}$), that the neutrinos will turn out to be exactly massless. This is in fact what the two component theory of the neutrino or the standard electroweak model of particle physics, possessing only left-handed neutrinos ($\nu_L$’s), would naturally suggest. If neutrinos indeed existed only in

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1. The extent to which this belief was ingrained among many (even in the 1990’s) may be assessed by an interesting remark by C. N. Yang at the recent Stony Brook Conference on neutrinos.

2. This is barring, of course, possible contributions to the Majorana mass of $\nu_L$ from lepton-number violating quantum gravity effects $\sim (v_{\text{EW}}^2/M_P)$. 

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the LH form without a RH counterpart (a feature that would suggest itself if neutrinos were truly massless), that would have implied that nature is manifestly and intrinsically left-right asymmetric, parity violating. Many in the 1970s believed that that may indeed be the case. In fact the minimal grand unification symmetry $SU(5)$ is built on such a belief.

There were, however, theoretical ideas of quark-lepton unification based on the symmetry $SU(4)$-color and the concomitant idea of left-right symmetry based on the commuting gauge symmetry $SU(2)_L \times SU(2)_R$, proposed in the early 1970s [4], purely on aesthetic grounds, which professed that nature is intrinsically quark-lepton and simultaneously left-right symmetric — that is parity-conserving. Within this picture, the symmetries $SU(4)$-color and $SU(2)_R$ are assumed to be broken spontaneously at high energies [17], such that $SU(3)$-color and $SU(2)_L$ remain unbroken; this stage of symmetry breaking marks the onset of quark-lepton distinction and parity violation, as observed at low energies.

Now the minimal symmetry containing $SU(4)$-color as well as the SM symmetry, and simultaneously providing a compelling reason for the quantization of electric charge is given by the symmetry group [4]:

$$G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$$  \hspace{1cm} (2)

Either one of the symmetries $SU(4)$-color or $SU(2)_R$ implies, however, that there must exist the right-handed counterpart ($\nu_R$) of the left-handed neutrino ($\nu_L$). This is because the RH neutrino ($\nu_R$) is the fourth color partner of the RH up quark and it is also the $SU(2)_R$ doublet partner of the RH electron. Thus the symmetry $G(224)$ necessarily had to postulate the existence of an unobserved new member in each family — the right-handed neutrino. This requires that there be sixteen two-component fermions in each family as opposed to fifteen for the SM. Subject to left-right discrete symmetry ($L \leftrightarrow R$) which is natural to $G(224)$, all 16 members of the electron family now became parts of a whole — a single left-right self-conjugate multiplet $F = \{F_L \oplus F_R\}$, where

$$F_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^c \end{bmatrix}_{L,R}.$$  \hspace{1cm} (3)

The multiplets $F_L^c$ and $F_R^c$ are left-right conjugates of each other transforming respectively as $(2,1,4)$ and $(1,2,4)$ of $G(224)$; likewise for the muon and the tau families. The symmetry $SU(2)_{L,R}$ treat each column of $F_{L,R}$ as a doublet; while the symmetry $SU(4)$-color unifies quarks and leptons by treating each row $F_L^c$ and $F_R^c$ as a quartet; thus lepton number is treated as the fourth color. As mentioned above, because of the parallelism between $SU(2)_L$ and $SU(2)_R$, and because $SU(4)$-color is vectorial, the symmetry $G(224)$ naturally permits the notion that the fundamental laws of nature possess a left $\leftrightarrow$ right discrete symmetry (i.e. parity invariance) that interchanges $F_L \leftrightarrow F_R^c$ and $W_L \leftrightarrow W_R$; the observed parity violation is then interpreted as being a low-energy phenomenon arising entirely through a spontaneous breaking of the L $\leftrightarrow$ R discrete symmetry [17]. I will return in just a moment to the relevance of having the RH neutrinos for an understanding of the neutrino masses. First, it is worth noting a few additional features of the symmetry $G(224)$ and its relationship to still higher symmetries.

The symmetry $G(224)$ introduces an elegant charge formula: $Q_{em} = I_{3L} + I_{3R} + (B-L)/2$, that applies to all forms of matter (including quarks and leptons of all six flavors, Higgs and gauge bosons). Note that the quantum numbers of all members of a family, including the weak hypercharge $Y_W = I_{3R} + (B-L)/2$, are now completely determined by the symmetry group $G(224)$ and the tranformation-property of $(F_L \oplus F_R)$. This is in contrast to the case of the SM for which the 15 members of a family belong to five disconnected multiplets, with unrelated quantum numbers. Quite clearly the charges $I_{3L}$, $I_{3R}$, and $B-L$ being generators of $SU(2)_L$, $SU(2)_R$, and $SU(4)^c$ respectively are quantized; so also then is
the electric charge $Q_{em}$.

At this point, an intimate link between $SU(4)^c$ and $SU(2)_L \times SU(2)_R$ is worth noting. Assuming that $SU(4)^c$ is gauged and demanding an explanation of the quantization of electric charge as above leaves one with no other choice but to gauge minimally $SU(2)_L \times SU(2)_R$ (rather than $SU(2)_L \times U(1)_{1R}$). Likewise, assuming $SU(2)_L \times SU(2)_R$ and again demanding a compelling reason for the quantization of electric charge dictates that one must minimally gauge $SU(4)^c$ (rather than $SU(3)^c \times U(1)_{B-L}$). The resulting minimal gauge symmetry is then $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ that simultaneously achieves quantization of electric charge, quark-lepton unification and left-right symmetry [1]. In short, the concepts of $SU(4)$-color and left-right gauge symmetry (symbolized by $SU(2)_L \times SU(2)_R$) become inseparable from each other, if one demands that there be an underlying reason for the quantization of electric charge. Assuming one automatically implies the other.

In brief, the symmetry $G(224)$ brings some attractive features to particle physics. These include:

1. Unification of all 16 members of a family within one left-right self-conjugate multiplet;
2. Quantization of electric charge;
3. Quark-lepton unification through $SU(4)$-color and the consequent mass relations given in Eq. [1];
4. Conservation of parity at a fundamental level [17];
5. Existence of the right-handed neutrinos ($\nu_R$’s) as a compelling feature;
6. $B - L$ as a local symmetry; and
7. Just the right set-up (as was realized in the late 70’s and 80’s) for implementing both the seesaw mechanism and leptogenesis.

As I will discuss, the three features (3), (5), and (6) — that distinguish symmetries possessing $SU(4)$-color from alternative symmetries like $SU(5)$ — are now needed to provide the set-up mentioned in 7 and thereby gain an understanding of the neutrino masses and the baryon-excess of the universe.

Now one can retain all the advantages (1)–(7) of the symmetry group $G(224)$ and in addition achieve gauge coupling unification, if one extends the symmetry $G(224)$ (which is isomorphic to $SO(4) \times SO(6)$) minimally into the simple group $SO(10)$ [2]. As a historical note, it is worth noting, however, that all the attractive features of $SO(10)$, which distinguish it from $SU(5)$, in particular the compelling need for the RH neutrino, the L-R discrete symmetry, and SU(4)-color symmetry, which are now relevant to an understanding of the neutrino masses and baryon asymmetry, were introduced entirely through the symmetry $G(224)$ [1], long before the $SO(10)$ papers appeared. This is because these features arise already at the level of the symmetry $G(224)$. The symmetry $SO(10)$ of course fully preserves these features because it contains $G(224)$ as a subgroup. It is furthermore remarkable that $SO(10)$ preserves even the left-right conjugate 16-plet multiplet structure of $G(224)$ by using the set $F = (F_L \oplus (F_R)^c)$ to represent the members of a family. The 16-plet now constitutes the sixteen-dimensional spinorial representation of $SO(10)$. Thus $SO(10)$ does not need to add any new matter-fermions beyond those of $G(224)$. By contrast, if one extends $G(224)$ to $E_6$ [19], the advantages (1)–(7) are retained but in this case one must extend the family structure from a 16 to a 27-plet by postulating additional fermions.

In contrast to the extension of $G(224)$ to $SO(10)$ or $E_6$, if one wished to extend only the SM symmetry $G(213)$ to a simple group, the minimal such extension would be $SU(5)$ [9]. In the 1970s, long before the discovery of neutrino oscillations, the symmetry $SU(5)$, being the smallest simple group possessing the SM symmetry, had the virtue of demonstrating the ideas of grand unification simply. It, however, does not contain $G(224)$ as a subgroup. As such, except for quanti-
zation of electric charge (feature (2)), $SU(5)$ does not possess any of the other features (i.e. features (1), (3), (4), (5), (6) and (7)) of $G(224)$ listed above. In particular, it does not contain (a) the RH neutrino as a compelling feature, (b) $B-L$ as a local symmetry, and (c) the second mass-relation of Eq. 1 based on $SU(4)$-color. As discussed below, all three of these features play crucial roles in providing an understanding of neutrino masses and in implementing baryogenesis via leptogenesis. Furthermore $SU(5)$ splits members of a family (not including $\nu_R$ or $(\nu_R)^5$) into two multiplets: $5+10$. And it violates parity, like the SM, manifestly.

In short, the symmetries $SO(10)$ and $E_6$ possess all the advantages (1)–(7) listed above because they contain $G(224)$ as a subgroup, while $SU(5)$ does not possess them (barring feature (2)) because it does not contain $G(224)$ as a subgroup.

Having discussed some of the main ideas on higher unification, which developed in the early 1970s, I now return to a discussion of the issue of the neutrino masses, that arises in this context and the need for the seesaw mechanism. As we saw, symmetries based on either $SU(4)$-color or left-right symmetry implies that the LH neutrinos ($\nu_L$'s) must have their RH counterparts (the $\nu_R$'s). That in turn implies, however, that the neutrino should acquire at least a Dirac mass, in short it must be massive (not massless). The dilemma that faced such a theory in the early 1970s, with the RH neutrino being linked to the RH up quark through $SU(4)$-color, despite its aesthetic merits, is however this: What makes the neutrino so extra-ordinarily light ($\lesssim 1$ eV) compared to the other fermions, including even the electron? Although the resolution of this dilemma was staring at one’s face, given that the RH neutrinos are singlets of the SM and that $B-L$ is necessarily violated in a theory as proposed in [4], it waited for six years until 1979 when the seesaw mechanism was discovered [6], which fully resolved the dilemma.

The idea of the seesaw mechanism is simply this. In a theory with spontaneous breaking of $B-L$ and $I_{3R}$ at a high scale ($M$), already inherent in [4], the RH neutrinos can and generally will acquire a superheavy Majorana mass ($M(\nu_R) \sim M$) that violates lepton number and $B-L$ by two units. Combining this with the Dirac mass of the neutrino ($m(\nu_{Dirac})$), which arises through electroweak symmetry breaking, one would then obtain a mass for the LH neutrino given by

$$m(\nu_L) \approx m(\nu_{Dirac})^2 / M(\nu_R)$$

which would be naturally super-light because $M(\nu_R)$ is naturally superheavy. This then provided a simple but compelling reason for the lightness of the known neutrinos. In turn it took away the major burden that faced the ideas of $SU(4)$-color and left-right symmetry from the beginning. In this sense, the seesaw mechanism was indeed the missing piece that was needed to be found for consistency of the ideas of $SU(4)$-color and left-right symmetry.

In turn, of course, the seesaw mechanism needs the ideas of $SU(4)$-color and SUSY grand unification so that it may be quantitatively useful. Because these two ideas not only provide (a) the RH

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Footnote 2). For this reason, believing in massless neutrinos, many had expressed in the 1970s their preference for $SU(5)$ over $G(224)$ or $SO(10)$. In their opinion, the RH neutrino was an unnecessary and uneconomical luxury. One’s only defense at that time was reliance on aesthetics; the ideas of $SU(4)$-color and left-right symmetry appeared (to me) to be much prettier. As is well known now and as discussed below, with the invention of the seesaw mechanism and the discovery of neutrino oscillations, the situation has changed dramatically. The RH neutrino, introduced in the early 1970s, is no longer a luxury but a necessity.

Footnote 3). Once $B-L$ is gauged and thus coupled to a massless gauge boson, such a gauge boson must acquire a mass through SSB so as to avoid conflict with the Eötvös-type experiments. In this case, $B-L$ must be violated spontaneously [4].
neutrino as a compelling feature (crucial to seesaw), but also provide respectively (b) the Dirac mass for the tau neutrino (cf. Eq. 1), and (c) the superheavy Majorana mass of the $\nu_R^\tau$ (see Sec. 4). Both these masses enter crucially into the seesaw formula and end up giving the right mass-scale for the atmospheric neutrino oscillation as observed. To be specific, Eq. (1), based on $SU(4)$-color yields $m(\nu^\tau_{\text{Dirac}}) \approx m_{\text{top}}(M_U) \approx 120$ GeV, and the SUSY unification scale, together with the protection provided by $B-L$ that forbids Planck-scale contributions to the Majorana mass of $\nu^\tau_R$, naturally yields $M(\nu^\tau_R) \sim 10^{15}$ GeV ($1/2-2$) [cf. Sec. 4]. The seesaw formula generalized to include 2-3 family mixing then yields $m(\nu^\tau_R) \approx (2.9)(120 \text{ GeV})^2 / 10^{15} \text{ GeV}(1/2-2) \approx (1/24 \text{ eV})(1/2-2)$, where the factor 2.9 comes from 2-3 mixing (see Sec. 4). This is just the right magnitude to go with the mass scale observed at SuperK [11].

Without an underlying reason as above for at least the approximate values of these two vastly differing mass-scales — $m(\nu^\tau_{\text{Dirac}})$ and $M(\nu^\tau_R)$ — the seesaw mechanism by itself would have no clue, quantitatively, to the mass of the LH neutrino. In fact it would yield a rather arbitrary value for $m(\nu^\tau_L)$, which could vary quite easily by more than 10 orders of magnitude either way around the observed mass scale.\footnote{To see this, consider for simplicity just the third family. Without $SU(4)$-color, even if a RH two-component fermion $N$ (the analogue of $\nu_R^\tau$) is introduced by hand as a singlet of the gauge symmetry of the SM or $SU(5)$, such an $N$ by no means should be regarded as a member of the third family, because it is not linked by a gauge transformation to the other fermions in the third family. Thus its Dirac mass term given by $m(\nu^\tau_{\text{Dirac}})[\nu^\tau_L N + h.c.]$ is completely arbitrary, except for being bounded from above by the electroweak scale $\sim 200$ GeV. In fact a priori (within the SM or $SU(5)$) it can well vary from say 1 GeV (or even 1 MeV) to 100 GeV. Using Eq. (1), this would give a variation in $m(\nu^\tau_L)$ by at least four orders of magnitude if the Majorana mass $M(N)$ of $N$ is held fixed. Furthermore, $N$ being a singlet of the SM as well as of $SU(5)$, the Majorana mass $M(N)$, unprotected by $B-L$, could well be as high as the Planck or the string scale ($10^{18-10^{17}}$ GeV), and as low as say 1 TeV; this would introduce a further arbitrariness (by some fourteen orders of magnitude) in $m(\nu^\tau_L)$. Such arbitrariness both in the Dirac and in the Majorana masses, is drastically reduced, however, once $\nu_R^\tau$ is related to the other fermions in the family by an $SU(4)$-color gauge transformation (see Eq. (1) and Sec. 4).}

In short, the seesaw mechanism needs the ideas of SUSY unification and $SU(4)$-color, and of course vice-versa; together they provide an understanding of neutrino masses as observed. Schematically, one thus finds:

$$
\text{SUSY UNIFICATION} \quad \oplus \quad \text{SEESAW} \quad (5)
$$

$$
m(\nu^\tau_L) \sim 1/10 \text{ eV}.
$$

I will return to a more quantitative discussion of the mass scale and the angle associated with the atmospheric neutrino oscillations in Sec. 4.

In the next section I first briefly discuss (assuming a string-theoretic origin of the effective symmetry in four dimensions) the issue of the four-dimensional symmetry being either $G(224)$ or $SO(10)$ near the string scale.

3. The Question of a Possible Preference

For the Effective Symmetry in 4D Being $G(224)$ or $SO(10)$

We have argued in the previous section that one needs an effective symmetry in 4D like $G(224)$ or $SO(10)$ containing $SU(4)$-color to understand neutrino masses. Such a need will be further strengthened by our discussions of fermion masses, neutrino oscillations, and leptogenesis in Secs. 4 and 6. The advantages of these two symmetries — $G(224)$ and $SO(10)$ — as regards these three issues turn out to be rather identical. Here I briefly present some characteristic differences between the two symmetries — $G(224)$ versus $SO(10)$ — and discuss the question of whether one may have a good reason to choose between them, viewing each of these as an effective symmetry in 4D, that emerges from an underlying theory like the string/M theory in higher dimensions [20]. The answer depends in part on an understanding of the observed gauge coupling unification on the one hand and resolving the problem of doublet-triplet splitting for SUSY GUT-theories on the other hand.

It has been known for some time that when the three gauge couplings are extrapolated from their values measured at LEP to higher energies,
in the context of weak-scale supersymmetry \[^8\], they meet, to a very good approximation, at a scale given by:

\[
M_U \approx 2 \times 10^{16} \text{ GeV} \tag{6}
\]

This dramatic meeting of the three gauge couplings provides a strong support for the ideas of both grand unification and supersymmetry.

The most straightforward interpretation of such a meeting of the gauge couplings is that a supersymmetric grand unification symmetry (often called GUT symmetry), like SU(5) or SO(10), is operative above the scale \(M_U\) and that it breaks spontaneously into the SM symmetry \(G(213)\) at around \(M_U\), while supersymmetry breaking occurs at some high scale and induces soft masses of order one TeV.

Even if supersymmetric grand unification is a good effective theory below a certain scale \(M\) lying above \(M_U\) (\(M_U \lesssim M\)), it seems imperative, however, that it should have its origin within an underlying theory like the string or \(M\)-theory, which is needed to provide a good quantum theory of gravity and also to unify all the forces of nature including gravity.

In the context of string or \(M\)-theory defined in \(D = 10\) or \(11\), an alternative interpretation of the meeting of the three gauge couplings is, however, possible. This is because even if the effective symmetry in 4D emerging from the string theory is non-simple like \(G(224)\), string theory can still ensure familiar gauge coupling unification at the string-scale \(M_{st}\) \[^21\,22\]. With this in mind one can consider two alternative possibilities both of which would account for coupling unification and would also be equally suitable for understanding neutrino masses and leptogenesis.

First, if the effective symmetry in 4D emerging from the string/M-theory is a simple group like \(SO(10)\), which breaks into the SM symmetry at \(M_U\) by the Higgs mechanism, the observed gauge coupling unification can of course be understood simply in this case, even if \(M_{st}\) is (say) an order of magnitude higher than \(M_U\). This is because \(SO(10)\) will preserve coupling unification from \(M_{st}\) down to \(M_U\).\[^9\]

Second, even if the effective symmetry in 4D emerging from string theory is non-simple like \(G(224)\) (as in \[^23\]), as long as the string-scale is not far above the GUT-scale (suppose \(M_{st} \approx (2-3)M_U\), say\[^10\]), the couplings of \(G(224)\) unified at the string scale will remain essentially so at the GUT scale \((M_U)\) so as to match the observed coupling unification. Despite the short gap between \(M_{st}\) and \(M_U\) in this case, one would still have the benefits of \(SU(4)\)-color to understand neutrino masses (as alluded to in Sec. 2) and baryogenesis via leptogenesis (to be discussed in Sec. 6).

In short, observed coupling unification can be attributed to either a simple group like \(SO(10)\) or a string-derived non-simple group like \(G(224)\) (with \(M_{st} \approx (2-3)M_U\)) being effective in 4D above the GUT scale \(M_U\).

There is, however, a characteristic difference between a GUT (like \(SO(10)\)) versus a non-GUT (like \(G(224)\)) string solution in 4D as follows. A SUSY 4D GUT solution possessing symmetries like \(SO(10)\) would need the color triplets in the \(10\) of \(SO(10)\) (see Sec. 4) to become super-heavy, while doublets remain light, by the so-called doublet-triplet splitting mechanism, so as to avoid rapid proton decay. While such a mechanism can be constructed for a 4D theory \[^20\], it requires a rather special choice of Higgs multiplets and of their couplings, and it remains to be seen whether such a choice can in fact emerge for a string-derived \(SO(10)\) solution in 4D.

Non-GUT string solutions (based on symmetries like \(G(224)\) or \(G(2113)\)) have a distinct advantage in this regard over a SUSY GUT solution in that the dangerous color triplets that induce

\[^8\]The case of weak-scale supersymmetry is of course motivated independently by the desire to avoid unnatural fine tuning in the Higgs mass.

\[^9\]With the GUT symmetry \(SO(10)\) being intact above \(M_U\), one should still ensure that the \(SO(10)\) gauge coupling does not grow too rapidly to become non-perturbative \((a_{GUT} \gtrsim 1)\) below \(M_{st}\). (This would in fact suggest avoiding large Higgs multiplets like 126 and \(\overline{126}\).

\[^10\]The case of the string scale being rather close to the GUT scale as above can arise quite plausibly by utilizing the ideas of string duality (following E. Witten \[^23\]) which can lower the string scale below its perturbative value of \(\approx 4 \times 10^{17}\) GeV \[^24\], and/or those of semi-perturbative unification (K. S. Babu and J. C. Pati \[^25\]) which raises \(M_{GUT}\) above the conventional MSSM value of \(2 \times 10^{16}\) GeV.
rapid proton decay are often naturally projected out for these solutions\cite{27,24,28}. Furthermore, the non-GUT solutions invariably yield desired “flavor” symmetries, which help resolve certain naturalness problems of supersymmetry such as those pertaining to the issues of squark degeneracy\cite{30}. CP violation\cite{31}, and quantum gravity induced rapid proton decay\cite{32}. I should mention that promising string theory solutions yielding the $G(224)$-symmetry in 4D have been obtained (using different approaches) by a number of authors\cite{33}. And, recently there have also been several attempts based on compactifications of five and six-dimensional GUT-theories which yield the $G(224)$-symmetry in 4D with some very desirable features\cite{34}.

Weighing the advantages and possible disadvantages of both, it seems hard at present to make a priori a clear choice between a GUT $SO(10)$ solution or a non-GUT $G(224)$ solution emerging from string theory in 4D. As expressed elsewhere\cite{35}, it therefore seems prudent to keep both options open and pursue their phenomenological consequences. As mentioned in Sec. 2 and discussed further in the following sections, the advantages of both are essentially the same as regards gaining an understanding of fermion masses, neutrino oscillations, and baryogenesis via leptogenesis. In Sec. 8, distinctions between the two cases as regards proton decay will be noted.

4. Fermion Masses and Neutrino Oscillations Within a $G(224)/SO(10)$-Framework

Following Ref.\cite{29}, I now present a simple and predictive pattern for fermion mass matrices based on $SO(10)$ or the $G(224)$-symmetry.\superscript{12} One can obtain such a mass matrix for the fermions by utilizing only the minimal Higgs system that is needed to break the gauge symmetry $SO(10)$ to $SU(3)^c \times U(1)_{em}$. It consists of the set:

$$H_{\text{minimal}} = \{45_H, 16_H, 16_H, 10_H\} \quad (7)$$

Of these, the VEV of $\langle 45_H \rangle \sim M_X$ breaks $SO(10)$ in the B-L direction to $G(2213) = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^c$, and those of $\langle 16_H \rangle = \langle 16_H \rangle$ along $\langle \bar{\nu}_{RH} \rangle$ and $\langle \tilde{\nu}_{RH} \rangle$ break $G(2213)$ into the SM symmetry $G(213)$ at the unification-scale $M_X$. Now $G(213)$ breaks at the electroweak scale by the VEV of $\langle 10_H \rangle$ to $SU(3)^c \times U(1)_{em}$.\superscript{13}

The $3 \times 3$ Dirac mass matrices for the four sectors ($u, d, l, \nu$) proposed in Ref.\cite{29} were motivated in part by the notion that flavor symmetries\cite{37} are responsible for the hierarchy among the elements of these matrices (i.e., for “$33$” $\gg$ “$23$” $\gg$ “$22$” $\gg$ “$12$” $\gg$ “$11$”, etc.), and in part by the group theory of $SO(10)/G(224)$, relevant to a minimal Higgs system (see below). Up to

\superscript{11}One must still ensure in the context of a realistic $G(224)$ (or $SO(10)$) solution, capable generating CKM mixings, that only one pair of Higgs doublets ($H_u$ and $H_d$) remain light. A possible mechanism for realizing such a solution is noted in\cite{24}.

\superscript{12}I will present the Higgs system for $SO(10)$. The discussion would remain essentially unaltered if one uses the corresponding $G(224)$-submultiplets instead.

\superscript{13}Large dimensional tensorial multiplets of $SO(10)$ like $126_H, 126_H, 120_H$, and $54_H$ are not used for the purpose in part because they tend to give too large threshold corrections to $\alpha_3(m_Z)$ (typically exceeding 20%), which would render observed coupling unification fortuitous [see e.g. discussions in Appendix D of Ref.\cite{29}]. Furthermore, the multiplets like $126_H$ and $120_H$ do not seem to arise at least in weakly interacting heterotic string solutions\cite{36}. 

Because they are meant to be relatively small quantities (specified below) and unimportant for purposes of Ref. [29], they are multiplied by $\bar{\Psi}_L$ on left and $\Psi_R$ on right. For instance, the row and column indices of $M_u$ and $M_d^\nu$ and also in $M_u$ and $M_1$, it will become clear that the $\epsilon$ and $\epsilon'$ entries are proportional to $B - L$ and are antisymmetric in the family space (as shown above). Thus, the same $\epsilon$ and $\epsilon'$ occur in both ($M_u$ and $M_d$) and also in ($M_u^\nu$ and $M_1$), but $\epsilon \to -3\epsilon$ and $\epsilon' \to -3\epsilon'$ as $q \to l$. Such correlations result in enormous reduction of parameters and thus in increased predictivity. Such a pattern for the mass-matrices can be obtained, using a minimal Higgs system $45_H$, $16_H$, $16$ and $10_H$ and a singlet $S$ of $SO(10)$, through effective couplings as follows:

$$L_{\text{Yuk}} = h_{33} 16_4 16_5 10_H + h_{23} 16_2 16_3 10_H (S/M)$$

Typically we expect $M'$, $M''$ and $M$ to be of order $M_{\text{string}}$ [10]. The VEV's of $\langle 45_H \rangle$ (along $B - L$), $\langle 16_H \rangle$ (along standard model singlet sneutrino-like component) and of the $SO(10)$-singlet $\langle S \rangle$ of are of the GUT-scale, while those of $10_H$ and of the down type SU(2)$_L$-doublet component in $16_H$ (denoted by $16_R$) are of the electroweak scale $[29,41]$. Depending upon whether $M'(M'') \approx M_{\text{GUT}}$ or $M_{\text{string}}$ (see [12]), the exponent $p(q)$ is either one or zero [22]. The entries $1$ and $\sigma$ arise respectively from $h_{33}$ and $h_{23}$ couplings, while $\eta \equiv \eta - \sigma$ and $\eta'$ arise respectively from $g_{23}$ and $g_{12}$-couplings. The ($B$-$L$)-dependent antisymmetric entries $\epsilon$ and $\epsilon'$ arise respectively from the $a_{23}$ and $a_{12}$-couplings. [Effectively, with $\langle 45_H \rangle \propto B - L$, the product $10_H \times 45_H$ contributes as a 120, whose coupling is family-antisymmetric.] The small entry $\zeta_{22}^u$ arises from the $h_{22}$-coupling, while $\zeta_{22}^d$ is not. Using some of the observed masses as inputs, one obtains $|\eta'| \sim |\sigma| \sim |\epsilon| \sim O(1/10)$, $|\eta'| \approx 4 \times 10^{-3}$ and $|\epsilon'| \approx 2 \times 10^{-4}$. The success of the framework presented in Ref. [29] (which set $\zeta_{22}^u = \zeta_{22}^d = 0$) in describing fermion masses and mixings remains essentially unaltered if $|\zeta_{22}^u, \zeta_{22}^d| \leq (1/3)(10^{-2})$ (say).

Such a hierarchical form of the mass-matrices, with $h_{33}$-term being dominant, is attributed in part to flavor gauge symmetry(ies) that distinguishes between the three families, and in part
to higher dimensional operators involving for example \(\langle 45_H \rangle / M'\) or \(\langle 16_H \rangle / M''\), which are suppressed by \(M_{\text{GUT}}/M_{\text{string}} \sim 1/10\), if \(M'\) and/or \(M'' \sim M_{\text{string}}\). The basic presumption here is that effective dimensionless couplings allowed by \(SO(10)/G(224)\) and flavor symmetries are of order unity [i.e., \((h_{ij}, g_{ij}, a_{ij}) \approx 1/3-3\) (say)]. The need for appropriate powers of \(\langle S/M \rangle\) with \(\langle S \rangle/M \sim M_{\text{GUT}}/M_{\text{string}} \sim (1/10-1/20)\) in the different couplings leads to a hierarchical structure. As an example, introduce just one \(U(1)\)-flavor symmetry, together with a discrete symmetry \(D\), with one singlet \(S\). The hierarchical form of the Yukawa couplings exhibited in Eqs. (8) and (9) would follow, for the case of \(p = 1\), \(q = 0\), if, for example, the \(U(1)\) flavor charges are assigned as follows:

\[
\begin{array}{cccc}
\text{16}_3 & \text{16}_2 & \text{16}_1 & \text{10}_H \\
 a & a+1 & a+2 & -2a \\
 \text{16}_H & \overline{16}_H & 45_H & S \\
 -a -1/2 & -a & 0 & -1
\end{array}
\]

(10)

The value of \(a\) would get fixed by the presence of other operators (see later). All the fields are assumed to be even under the discrete symmetry \(D\), except for \(\text{16}_H\) and \(\overline{16}_H\) which are odd. It is assumed that other fields are present that would make the \(U(1)\) symmetry anomaly-free. With this assignment of charges, one would expect \(|\varsigma^{u,d}_{22}| \sim \langle \langle S/M \rangle^2\rangle\); one may thus take, for example, \(|\varsigma^{u,d}_{22}| \sim (1/3) \times 10^{-2}\) without upsetting the success of Ref. [29]. In the same spirit, one would expect \(|\zeta_{13}, \zeta_{31}| \sim \langle \langle S/M \rangle\rangle \sim 10^{-2}\), and \(|\tilde{\zeta}_{11}| \sim \langle \langle S/M \rangle\rangle^4 \sim 10^{-4}\) (say). where \(\zeta_{11}, \zeta_{13}, \) and \(\zeta_{31}\) denote the “11,” “13,” and “31,” elements respectively. In the interest of economy in parameters and thus greater predictivity, we drop these elements \((\zeta_{11}, \zeta_{13}, \) and even \(\zeta_{22}\)) as a first approximation, in this section, as in Ref. [29]. But these elements can in general be relevant in a more refined analysis (e.g. \(\zeta^{u,d}_{11}\), though small, can make small contributions to \(m_{u,d}\) of order few MeV without altering significantly the mixing angles, and \(\zeta_{22}\) can be relevant for considerations of CP violation).

To discuss the neutrino sector one must specify the Majorana mass-matrix of the RH neutrinos as well. These arise from the effective couplings of the form [33]:

\[
\mathcal{L}_{\text{Maj}} = f_{ij} \text{16}_i \text{16}_j \overline{16}_H / M
\]

(11)

where the \(f_{ij}\)'s include appropriate powers of \(\langle S/M \rangle\), in accord with flavor charge assignments of \(\text{16}_i\) (see Eq. (10)), and \(M\) is expected to be of order string or reduced Planck scale. For the \(f_{33}\)-term to be leading \((\sim 1)\), we must assign the charge \(-a\) to \(\overline{16}_H\). This leads to a hierarchical form for the Majorana mass-matrix [29]:

\[
M_R' = \begin{bmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{bmatrix} M_R
\]

(12)

Following the flavor-charge assignments given in Eq. (10), we expect \(|y| \sim \langle S/M \rangle \sim 1/10\), \(|z| \sim \langle \langle S/M \rangle \rangle^2 \sim (1/200)\) (1 to 1/2, say), \(|x| \sim \langle \langle S/M \rangle \rangle^4 \sim (10^{-1}-10^{-5})\) (say). The “22” element (not shown) is \(\langle \langle S/M \rangle \rangle^4\) and its magnitude is taken to be \(< |y|/3\rangle\), while the “12” element (not shown) is \(\langle \langle S/M \rangle \rangle^3\). In short, with the assumption that the “33”-element is leading, the hierarchical pattern of \(M_R'\) is identical to that of the Dirac mass matrices (Eq. (8)). We expect

\[
M_R = \frac{f_{33} \langle \overline{16}_H \rangle^2}{M} \approx (10^{15}\ \text{GeV})(1/2-2)
\]

(13)

where we have put \(\langle \overline{16}_H \rangle \approx 2 \times 10^{16}\ \text{GeV}, M \approx M_{\text{string}} \approx 4 \times 10^{17}\ \text{GeV}\) [21] and \(f_{33} \approx 1\). These lead to an expected central value of \(M_R\) of around \(10^{15}\ \text{GeV}\). Allowing for 2-3 family-mixing in the Dirac and the Majorana sectors as in Eqs. (7) and (10), the seesaw mechanism leads to [29]:

\[
m(\nu_3) \approx B \frac{m(\nu_{\text{Dirac}})^2}{M_R}
\]

(14)

The quantity \(B\) represents the effect of 2-3 family-mixing and is given by \(B = (\sigma + 3\epsilon)(\sigma + 3\epsilon - 2y)/y^2\) (see Eq. (24) of Ref. [29]). Thus \(B\) is fully calculable within the model once the parameters \(\sigma, \eta, \epsilon,\) and \(y\) are determined in terms of inputs involving some quark and lepton masses (as noted below). In this way, one obtains \(B \approx (2.9 \pm 0.5)\). The Dirac mass of the tau-neutrino is obtained by using the \(SU(4)\)-color relation (see Eq. (11)).
m(ν^\text{Dirac}_L) \approx m_{\text{top}}(M_X) \approx 120 \text{ GeV}. One thus obtains from Eq. (12) (as noted in Sec. 2):

\[
m(ν_3) \approx \frac{(2.9)(120 \text{ GeV})^2}{10^{15} \text{ GeV}}(1/2 - 2) \approx (1/24 \text{ eV})(1/2 - 2)
\]  

(15)

Noting that for hierarchical entries — i.e. for (σ, ε, and y) \sim 1/10 — one naturally obtains a hierarchical spectrum of neutrino-masses: \(m(ν_1) \ll m(ν_2) \approx (1/10)m(ν_3),\) we thus get:

\[
\sqrt{Δm_{23}^2} \approx m(ν_3) \approx (1/24 \text{ eV})(1/2 - 2)
\]  

(16)

This agrees remarkably well with the SuperK value of (\(\sqrt{Δm_{23}^2}\)SK\sim 1/20 eV), which lies in the range of nearly (1/15 to 1/30) eV. As mentioned in the introduction, the success of this prediction provides clear support for (i) the existence of νR, (ii) the notion of SU(4)-color symmetry that gives m(ν^\text{Dirac}_R), (iii) the SUSY unification-scale that gives M_R, and (iv) the seesaw mechanism.

We note that alternative symmetries such as SU(5) would have no compelling reason to introduce the νR's. Even if one did introduce ν^\text{Dirac}_R by hand, there would be no symmetry to relate the Dirac mass of ν_τ to the top quark mass. Thus m(ν^\text{Dirac}_R) would be an arbitrary parameter in SU(5), which, as noted in footnote 5, could well vary from say 1 GeV to 100 GeV. Furthermore, without B-L as a local symmetry, the Majorana masses of the RH neutrinos, which are singlets of SU(5), can well be as high as the string scale \sim 4 \times 10^{17} \text{ GeV} (say), and as low as say 1 TeV. Thus, as noted in footnote 5, within SU(5), the absolute scale of the mass of ν_3, obtained via the familiar seesaw mechanism [29], would be uncertain by more than ten orders of magnitude.

Other effective symmetries such as [SU(3)]^4 [41] and SU(2)_L × SU(2)_R × U(1)_{B-L} × SU(3)^C [43] would give ν_\nu and B-L as a local symmetry, but not the desired SU(4)-color mass-relations: \(m(ν^\text{Dirac}_R) \approx m_t(M_X)\) and \(m_b(M_X) \approx m_\tau,\) Flipped SU(5) \times U(1) [10] on the other hand would yield the desired features for the neutrino system, but not the empirically favored b-τ mass relation (Eq. (1)). Thus, combined with the observed b/τ mass-ratio, the SuperK data on atmospheric neutrino oscillation seems to clearly select out the effective symmetry in 4D being either G(224) or SO(10), as opposed to the other alternatives mentioned above. It is in this sense that the neutrinos, by virtue of their tiny masses, provide crucial information on the unification-scale as well as on the nature of the unification-symmetry in 4D, as alluded to in the introduction.

Ignoring possible phases in the parameters and thus the source of CP violation for a moment, and also setting ζ^d = ζ^\tau = 0, as was done in Ref. [29], the parameters (σ, η, ε, η', η^0, η^0', y) can be determined by using, for example, \(m_\tau^{\text{phys}} = 174 \text{ GeV}, \) \(m_\tau(m_\tau) = 1.37 \text{ GeV}, \) \(m_\tau(1 \text{ GeV}) = 110 - 116 \text{ MeV} , \) \(m_\tau(1 \text{ GeV}) = 6 \text{ MeV},\) the observed masses of e, μ, and τ and m(ν_2)/m(ν_3) \sim 1/(6 ± 1) (as suggested by a combination of atmospheric and solar neutrino data, the latter corresponding to the LMA MSW solution, see below) as inputs. One is thus led, for this CP conserving case, to the following fit for the parameters, and the associated predictions [29]. In this fit, we leave the small quantities x and z in M_R^\nu undetermined and proceed by assuming that they have the magnitudes suggested by flavor symmetries (i.e., x \sim (10^{-4} - 10^{-5}) and z \sim (1/200)(1 to 1/2) (see remarks below Eq. (12)):

\[
\begin{align*}
σ & \approx 0.110 \\
η & \approx 0.151 \\
ε & \approx -0.095 \\
|η'| & \approx 4.4 \times 10^{-3} \\
ε' & \approx 2 \times 10^{-4} \\
M^0_t & \approx m_t(M_X) \approx 120 \text{ GeV} \\
M^0_b & \approx m_b(M_X) \approx 1.5 \text{ GeV} \\
y & \approx -1/17.
\end{align*}
\]  

(17a) - (17h)

These output parameters remain stable to within 10% corresponding to small variations (\lesssim 10%) in the input parameters of \(m_t, m_c, m_s,\) and \(m_u,\) These in turn lead to the following predictions for
the quarks and light neutrinos \[29,47\]:

\[
\begin{align*}
    m_d(m_u) & \approx (4.7-4.9) \text{ GeV}, \\
    \sqrt{\Delta m^2_{23}} & \approx m(\nu_3) \approx (1/24 \text{ eV})(1/2-2), \\
    V_{cb} & \approx \begin{vmatrix} m_u \frac{\eta+\epsilon}{\eta-\epsilon} - m_d \frac{\eta-\epsilon}{\eta+\epsilon} \end{vmatrix} \\
    & \approx 0.044, \\
    \theta_{\nu_e \nu_r}^{\text{osc}} & \approx \begin{vmatrix} \frac{m_e}{m_\tau} \frac{\eta-\epsilon}{\eta+\epsilon}^{1/2} + \frac{m_\tau}{m_\mu} \end{vmatrix} \\
    & \approx 0.437 + (0.378 \pm 0.03),
\end{align*}
\]

\[\text{(18)}\]

Thus, \(\sin^2 2\theta_{\nu_e \nu_r}^{\text{osc}} \approx 0.993,\)

\(V_{us} \approx \begin{vmatrix} m_{us} \end{vmatrix} \approx 1/6,\)

\(V_{cb} \approx \begin{vmatrix} m_{cb} \end{vmatrix} \approx 0.20,\)

\(m_d(1 \text{ GeV}) \approx 8 \text{ MeV}.\)

It is rather striking that all seven predictions in Eq. \[13\] agree with observations, to within 10%. Particularly intriguing is the (B-L)-dependent group-theoretic correlation \(\text{between the contribution from the first term in } V_{cb} \text{ and that in } \theta_{\nu_e \nu_r}^{\text{osc}},\)

which explains simultaneously why one is small (\(V_{cb}\)) and the other is large (\(\theta_{\nu_e \nu_r}^{\text{osc}}\)).\[13\] That in turn provides some degree of confidence in the pattern of the mass-matrices.

The Majorana masses of the RH neutrinos (\(N_{iR} \equiv N_i\)) are given by \[17\]:

\[
\begin{align*}
    M_3 & \approx M_R \approx 10^{15} \text{ GeV (1/2-1)}, \\
    M_2 & \approx |y|^2 M_3 \approx (2.5 \times 10^{14} \text{ GeV})(1/2-1), \\
    M_1 & \approx |x-z|^2 M_3 \approx (1/2-2) \times 10^{-5} M_3 \\
    & \approx 10^{10} \text{ GeV (1/4-2)}. 
\end{align*}
\]

Note that we necessarily have a hierarchical pattern for the light as well as the heavy neutrinos (see discussions below on \(m_{\nu_1}\)).

As regards \(\nu_e-\nu_\mu\) and \(\nu_e-\nu_\tau\) oscillations, the standard seesaw mechanism would typically lead to rather small angles (e.g. \(\theta_{\nu_e \nu_\mu}^{\text{osc}} \approx \sqrt{m_\mu/m_\tau} \approx 0.06\)), within the framework presented above \[29\]. It has, however, been noted recently \[47\] that small intrinsic (non-seesaw) masses \(\approx 10^{-3} \text{ eV}\) of the LH neutrinos can arise quite plausibly through higher dimensional operators of the form \[49\]: \(W_{12} \supset \kappa_{12} \mathbf{16}_d \mathbf{16}_d \mathbf{16}_d \mathbf{16}_d \mathbf{10}_H \mathbf{10}_H \mathbf{10}_H \mathbf{M}_3^{\text{eff}},\)

without involving the standard seesaw mechanism \[6\]. One can verify that such a term would lead to an intrinsic Majorana mixing mass term of the form \(m_{\nu_\mu}^{\text{eff}} = 3/4, \) with a strength given by \(m_{\nu_e}^{\text{eff}} \approx \kappa_{12} (16_H)^2 (175 \text{ GeV})^2 / M_3^{\text{eff}} \approx (1.5-6) \times 10^{-3} \text{ eV},\) for \(\kappa_{12} \approx 1,\) if \(M_3^{\text{eff}} \approx M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \[40\]. Such an intrinsic Majorana \(\nu_\mu-\nu_\mu\) mixing mass \(\approx 10^{-3} \text{ eV},\) though small compared to \(m(\nu_3),\) is still much larger than what one would generically get for the corresponding term from the standard seesaw mechanism \(\text{as in Ref.} \[29\].\)

Now, the diagonal \((\nu_\mu-\nu_\mu)\) mass-term, arising from standard seesaw can naturally be \(\approx (3-8) \times 10^{-3} \text{ eV for } |y| \approx 1/20\),\[29\]. Thus, taking the net values of \(m_{\nu_\mu}^{\text{eff}} \approx (6 - 7) \times 10^{-3} \text{ eV,}\) \(m_{\nu_\mu}^{\text{eff}} \approx 3 \times 10^{-3} \text{ eV,}\) as above and \(m_{\nu_\mu}^{\text{eff}} \approx 1 \times 10^{-3} \text{ eV,}\) which are all plausible, we obtain \(m_{\nu_\mu}^{\text{eff}} \approx (6 - 7) \times 10^{-3} \text{ eV,}\) \(m_{\nu_\mu}^{\text{eff}} \approx 1 \times 10^{-3} \text{ eV,}\) so that \(\Delta m^2_{12} \approx (3.6-5) \times 10^{-5} \text{ eV}^2\) and \(\sin^2 2\theta_{\nu_e \nu_r}^{\text{osc}} \approx 0.6-0.7.\)

These go well with the LMA MSW solution of the solar neutrino puzzle.

Thus, the intrinsic non-seesaw contribution to the Majorana masses of the LH neutrinos can possibly have the right magnitude for \(\nu_e-\nu_\mu\) mixing so as to lead to the LMA solution within the G(224)/SO(10)-framework, without upsetting the successes of the seven predictions in Eq. \[13\]. [In contrast to the near maximality of the \(\nu_\mu-\nu_\mu\) oscillation angle, however, which emerges as a compelling prediction of the framework \[29\], the LMA solution, as obtained above, should, be regarded as a consistent possibility, rather than as a compelling prediction, within this framework.]

It is worth noting at this point that in a theory leading to Majorana masses of the LH neutrinos as above, one \textit{would of course expect the neutrinoless double beta decay process (like } n + n \rightarrow p p^- e^- \text{), satisfying } |\Delta L| = 2 \text{ and } |\Delta B| = 0, \text{ to occur at some level.} \) The search for this process is most important because it directly tests a fundamental conservation law and can shed light on the Majorana nature of the neutrinos, as well as on certain CP violating phases in the neutrino-system (assuming that the process is dominated by neutrino-exchange). The crucial parameter which controls the strength of this process is given by \(m_{ee} = |\sum_{i} m_{\nu_i} U^2_{ei}|.\) With a non-seesaw contribution leading to \(m_{\nu_\mu} \sim few \times 10^{-3} \text{ eV,}\)
\( m_{\nu_2} \approx 7 \times 10^{-3} \text{ eV}, \quad \sin^2 \theta_{12} \approx 0.6 - 0.7, \) and an expected value for \( \sin^2 \theta_{13} \sim m_{\nu_3}^0 / m_{\nu_1}^0 \sim (1 - 5) \times 10^{-3} \text{ eV} / (5 \times 10^{-2} \text{ eV}) \sim (0.02 - 0.1), \) one would expect \( m_{\nu_e} \approx (1 - 5) \times 10^{-3} \text{ eV}. \) Such a strength, though compatible with current limits, would be accessible if the current sensitivity is improved by about a factor of 50–100. Improving the sensitivity to this level would certainly be most desirable.

In summary, given the bizarre pattern of masses and mixings of the quarks, charged leptons, and neutrinos, it seems truly remarkable that the simple pattern of fermion mass matrices, motivated in large part by the group theory of the G(224) or SO(10) symmetry and the minimality of the Higgs system, and in part by the assumption of flavor symmetry (of the type defined in Eq. (10)), leads to seven predictions in agreement with observations. Particularly significant are the predictions for \( m(\nu_L^2) \) (to within a factor of 2 or 3, say), together with that of \( m_b / m_t, \) which help select out the route to higher unification based on G(224) or SO(10) as the effective symmetry in 4D, as opposed to other alternatives. So also are the predictions for the extreme smallness of \( V_{tb} \) together with the near maximality of \( \theta_{\nu_L\nu_R}. \) I now proceed to present briefly, in the next two sections, the results of some recent works on CP and flavor violations and on baryogenesis via leptogenesis, all treated within the same framework as presented here.

5. CP and Flavor Violations Within the SUSY G(224)/SO(10)-Framework

In this section I will present briefly some recent works by K. S. Babu, Parul Rastogi, and myself which will appear in the form of two papers. At the outset I need to say a few words about the origin of CP violation within the G(224)/SO(10)-framework presented above. The discussion so far has ignored, for the sake of simplicity, possible CP violating phases in the parameters \( \sigma, \eta, \epsilon, \eta', c, \zeta_{uu}^d, y, z, \) and \( x \) of the Dirac and Majorana mass matrices [Eqs. 5 and 12]. In general, however, these parameters can and generically will have phases. Some combinations of these phases enter into the CKM matrix and define the Wolfenstein parameters \( \rho_W \) and \( \eta_W, \) which in turn induce CP violation by utilizing the standard model interactions.

Our procedure for dealing with CP and flavor violations may be summarized by the following set of considerations:

1. Since the model is supersymmetric, CP and flavor violations naturally arise also through s-fermion/gaugino loops involving scalar (mass)\(^2\)-transitions which can preserve as well as flip chirality, such as \( (\tilde{Q}_{L,R} \rightarrow \tilde{Q}_{L,R})_{\neq j} \) and \( (\tilde{G}_{L} \rightarrow \tilde{G}_{R})_{\neq j} \) respectively. These transitions (including their phases) get determined within the model in terms of the fermion mass-matrices and the SUSY-parameters as follows.

2. We assume that SUSY breaks at high scale \( M^* \sim M_{GUT} \) such that the soft parameters are flavor-blind-and thus they are family-universal at the scale \( M^* \). A number of well-motivated models of SUSY-breaking-e.g. those based on the ideas of msugra, gaugino-meditation, anomaly-meditation, dilaton-meditation or family-universal anomalous U(1) D-term contribution or (preferably) on a combination of some of these mechanisms-can induce such a breaking. In an extreme version (such as CMSSM) such a universal model would involve only five parameters \( (m_0, m_{1/2}, a_0, \tan \beta \) and sgn \( (\mu)) \), and in some cases (as in \( \text{mSUGRA} \)) \( a_0 \) would be zero at \( M^* \). While for most purposes we will use this restricted version of SUSY-breaking, including \( a_0 = 0, \) as a guide, we will not insist for example on Higgs-squark universality.

3. Although the squarks and sleptons of the three families have a common mass \( m_{0} \) and the off-diagonal (mass)\(^2\)-transitions (such as \( b \rightarrow s \) etc.) vanish at the scale \( M^* \), SUSY flavor violations arise in the model as follows. Owing to flavor-dependent

\(^{16}\)One still needs to understand the origin of flavor symmetries, for example, of the type proposed here, in the context of the ground state solution of an underlying theory like the string/M theory.
Yukawa couplings, with $h_{\text{top}}$ being dominant, renormalization group running from $M^*$ to $M_{\text{GUT}}$ in the context of $SO(10)$ or $G(224)$ makes $b_{L,R}$ and $t_{L,R}$ lighter than $(\tilde{d}, \tilde{s})_{L,R}$\[50].

Now, following common practice, we analyze SUSY-contributions in the so-called SUSY-basis, in which gluino-interactions are flavor-diagonal, with the quarks being in their physical or mass basis (likewise for leptons). Let the quark mass-matrices, defined originally in the gauge-basis, be diagonalized by the matrices $X_{L,R}^{(q)}$ at the GUT-scale (where $q = u$ or $d$) so that the CKM-matrix has the Wolfenstein-form. The squark (mass)$^2$ matrices $M_{LL}^{(0)}$ and $M_{RR}^{(0)}$, also defined in the gauge-basis, must then be transformed by the same matrices to the forms $X_{L}^{(q)} M_{LL}^{(0)} X_{L}^{(q)}$, and likewise for $L \rightarrow R$.

Since $M_{LL}^{(0)}$ and $M_{RR}^{(0)}$, though diagonal, are not proportional to unit matrices at the GUT-scale (for reasons explained above), their transformations to the SUSY-basis (as above) would then induce flavor-violating transitions such as $\tilde{b}_{L,R} \rightarrow \tilde{d}_{L,R}$, or $\tilde{s}_{L,R}$ and $\tilde{d}_{L,R} \rightarrow \tilde{s}_{L,R}$ etc., that too with phases, depending upon $X_{L,R}^{(q)}$. Note that these phases and the associated CP violation arise entirely from the quark mass-matrices, as also the CKM CP violation.

4. Additional flavor-violations arise through RG-running of the $\tilde{b}_L$-mass from $M_{\text{GUT}}$ to the electroweak scale in the context of MSSM involving the top Yukawa coupling.

5. Furthermore, even if we start with $a_0 = 0$ at the high scale $M^*$, RG running from $M^*$ to $M_{\text{GUT}}$ in the context of $SO(10)$ or $G(224)$ still induces the A-parameters at the GUT-scale utilizing the gaugino-masses and the Yukawa couplings \[50]. These get determined within the model as well, for a given choice of $m_0$, $m_{1/2}$ and $M^*/M_{\text{GUT}}$. These A-terms induce chirality-flipping transitions such as $\tilde{s}_L \rightarrow \tilde{d}_R$, $\tilde{b}_L \rightarrow \tilde{s}_R$, $\tilde{\mu}_L \rightarrow \tilde{e}_R$, $\tilde{d}_L \rightarrow \tilde{d}_R$ and $\tilde{e}_L \rightarrow \tilde{e}_R$, which can be important for $\epsilon_K$, $B_d \rightarrow \Phi K_S$, $\mu \rightarrow e\gamma$ and edm’s of the neutron and the electron.

6. The interesting point is that the net values of the off-diagonal squark-mixings including their phases, and thereby the flavor and CP violations induced by them, are entirely determined within our approach by the entries in the quark mass-matrices and the choice of $(m_0, \; m_{1/2}, \; a_0, \; \tan\beta \; \text{and} \; \text{sgn}(\mu))$; similarly for the leptonic sector. Within the $G(224)/SO(10)$ framework presented in sec. 4, the quark mass-matrices are however tightly constrained by our considerations of fermion masses and neutrino-oscillations. This is the reason why, within our approach, SUSY CP and flavor violations get intimately linked with fermion masses and neutrino-oscillations \[57,58].

With this to serve as a background, since we wish to introduce CP violation by introducing phases into at least some of the entries in the quark (and thereby lepton) mass-matrices, the question arises:

Can observed CP and/or flavor-violations in the quark and lepton sectors (including the empirical limits in some of these) emerge consistently within the $G(224)/SO(10)$-framework, for any choice of phases in the fermion mass-matrices of Eq. \[59], while preserving all its successes with respect to fermion masses and neutrino oscillations?

This is indeed a non-trivial challenge to meet within the $SO(10)$ or $G(224)$-framework, since the constraints from both CP and flavor violations on the one hand and fermion masses on the other hand are severe.

Turning to experiments, there are now four well-measured entities reflecting flavor and/or CP violations in the quark-sector which confront theoretical ideas on physics beyond the standard
model. They are:  
\[\Delta m_K, \epsilon_K, \Delta m_{B_d} \text{ and } S(B_d \rightarrow J/\Psi K_s).\]  \hspace{1cm} (20)

It is indeed remarkable that the observed values including the signs of all four entries as well as the lower limit on \(\Delta m_{B_s}\) can consistently be realized (allowing for uncertainties in matrix elements of up to 20%) within the standard CKM-model for a single choice of the Wolfenstein parameters.

\[\bar{\rho}_W = 0.178 \pm 0.046; \bar{\eta}_W = 0.341 \pm 0.028.\]  \hspace{1cm} (21)

In particular, using the observed values of \(\epsilon_K = 2.27 \times 10^{-3}\), \(|V_{ub}| = (3.55 \pm 0.36) \times 10^{-3}\), \(|V_{cb}| = 4.1 \pm 1.6) \times 10^{-2}\), \(\Delta m_{B_d} = (3.3 \pm 0.6) \times 10^{-13}\) GeV, and the upper limit on \(\Delta m_{B_d}/\Delta m_{B_s} > 0.035\), one can phenomenologically determine \(\bar{\rho}_W\) and \(\bar{\eta}_W\) in the SM and predict the asymmetry parameter \(S(B_d \rightarrow J/\Psi K_s)\) to be \(\approx 0.70 \pm 0.1\) \cite{21}. This agreement remarkably well with the observed value \(S(B_d \rightarrow J/\Psi K_s)_{\text{expt}} = 0.734 \pm 0.054\) \cite{21}. This agreement of the SM in turn poses a challenge for physics beyond the SM especially for SUSY GUT-models possessing CP and flavor-violations as described above. The question is: If such a GUT model is constrained, as in our case, by requiring that it should successfully describe the fermion and neutrino oscillations, can it still yield \(\bar{\rho}_W\) and \(\bar{\eta}_W\) more or less in accord with the values given above? In particular, adding contributions from the standard model interactions as well as from SUSY-graphs, can such a constrained \(SO(10)\) or \(G(224)\) model account for the observed values of the four entities listed in Eq. \(20\)?

First of all, one might have thought, given the freedom in the choice of phases in the parameters of the mass-matrices, that it ought to be possible to get \(\bar{\rho}_W\) and \(\bar{\eta}_W\) in accord with the SM-values given in eq. \(21\), within any \(SO(10)\)-model. It turns out, however, that in general this is indeed not possible without running into a conflict with the fermion masses and/or neutrino-oscillation parameters\(^{18}\). In other words, any predictive \(SO(10)\)-model is rather constrained in this regard.

Second, one might think that even if the derived values of \(\bar{\rho}_W\) and \(\bar{\eta}_W\), constrained by the pattern of fermion masses and neutrino oscillations, are found to be very different in signs and/or in magnitudes from the SM-values shown in Eq. \(21\), perhaps the SUSY-contributions added to the new SM-contributions (based on the derived values of \(\bar{\rho}_W\) and \(\bar{\eta}_W\)) could possibly account for all four entities listed in Eq. \(20\).

\[\text{It seems to us, however, that this is simply not a viable and natural possibility. This is because the SUSY contributions combine in different ways with the SM-contributions for the four different entities listed in Eq. } \hspace{1cm} (20)\]

Suppose the derived values of \(\bar{\rho}_W\) and \(\bar{\eta}_W\) are very different from the values given in Eq. \(21\). The SM-contributions to each of \(\epsilon_K, \Delta m_{B_d}\) and \(S(B_d \rightarrow J/\Psi K_s)^{19}\) would in this case be in gross conflict with observations. That means that the SUSY-contribution would have to be sizable for each of these three entities, for this case, so as to possibly compensate for errors in the SM-contributions. Now, the magnitude of the SUSY-contribution can perhaps be adjusted (by choosing e.g s-fermion mass) so that the combined contribution from (SM+SUSY) would give the right value for one of the three entities, but it would be rather impossible that the SM and SUSY contributions would add in just the right way in sign and magnitude so that the net contribution for the other two entities, as well as that for \(\Delta m_K\), would agree with observations.

Of course by introducing a completely arbitrary set of soft SUSY-breaking parameters in the gauge-basis, including intrinsic phases (in general there are some 105 parameters for MSSM), perhaps an agreement can be realized with respect to all four entities listed in Eq. \(20\), even if the derived values of \(\bar{\rho}_W\) and \(\bar{\eta}_W\) are very different from the SM values given in Eq. \(21\). This would, however, be a rather unnatural solution, with many arbitrary parameters. And the question would still arise: If, in the true picture SUSY-contribution is so important, why does the SM (with zero SUSY contribution) provide such an

\(^{17}\)\(\epsilon_L\), reflecting direct CP violation is well-measured, but its theoretical implications are at present unclear due to uncertainties in the matrix element

\(^{18}\)For a discussion of the difficulties in this regard within a recently proposed \(SO(10)\)-model, see e.g. Ref. \(92\).

\(^{19}\)\(\Delta m_K\) depends primarily on \((V_{es}V_{ec}^*)^2\) and thus, to a good approximation, is independent of \(\bar{\rho}\) and \(\bar{\eta}\)
excellent description for all four entities with the right prediction for $S(B_d \to J/\Psi K_s)$ in the first place?

This is why it seems to us that the only visible and natural solution for any SUSY G(224) or SO(10)-model for fermion masses and neutrino-oscillations is that, the model, with allowance for phases in the fermion mass-matrices, should not only yield the masses and mixings of all fermions including neutrinos in accord with observations (as in Sec. 4), but it should yield $\hat{\rho}_W$ and $\hat{\eta}_W$ that are close to the values shown in Eq. (21). This would be a major step in the right direction. One then needs to ask: how does the combined (SM + SUSY) contributions fare for such a solution as regards its predictions for the four entities listed above [52]. In all these cases, the SUSY-contribution turns out to be rather small ($\lesssim 5\%$ in amplitude), except however for $\epsilon_K$, which is sizable ($\approx 0.2$) in accord with observations (within $10\%$). The spectrum of $(m_{sq}, m_{s})$ considered above can be realized, for example for a choice of $(m_q, m_{1/2}) \approx (600, 220)$ GeV. Other choices of SUSY-parameters — for example $(m_q, m_{1/2}) = (60, 260)$ GeV, or $(100, 440)$ GeV, or $(1000, 250)$ GeV — which would be in accord with the WMAP-constraint [63] in the event that the LSP is the cold dark matter (see remarks later), also lead to quite acceptable values for all four entities listed above [52]. In all these cases, the SUSY-contribution turns out to be rather small ($\lesssim 5\%$ in amplitude), except however for $\epsilon_K$, which is sizable ($\approx 20 - 30\%$) and has opposite sign, compared to the SM-contribution.

We thus see that the SUSY G(224) or SO(10)-framework (remarkably enough) has met the challenges so far in being able to reproduce the observed features of both CP and quark-flavor violations as well as fermion masses and neutrino-oscillations!
Owing to introduction of four phases, the number of parameters has increased compared to that in Sec. 4, but the number of observable entities involving CP and flavor violations including those in the $B_s$ and lepton-systems, only some of which will be presented here, has increased manifold. Thus the framework will be thoroughly testable as regards its predictions for CP and flavor-violations, especially once the SUSY-parameters are determined by (hopefully successful) SUSY-searches.

3. As noted in [57,58], the mass-parameter $\delta_{RR}^{23}(b_R \rightarrow s_R)$ gets enhanced both due to (a) the SUSY flavor-violation arising from RGE running from $M^*$ to $M_{GUT}$ in the context of $SO(10)$, and equally important (b) large $\nu_\mu - \nu_\tau$ oscillation angle. This enhancement is found to be insufficient, however, within our model, to produce a large deviation in $S(B_d \rightarrow \Phi K_s)$ from the SM-prediction. This situation is found to prevail even after the inclusion of the chirality-flipping A-term contribution to $\delta_{RR}^{23}(b_R \rightarrow s_L)$, where the A-term is induced (as mentioned in 5) through RG running from $M^*$ to $M_{GUT}$. As a result, we obtain:

$$S(B_d \rightarrow \Phi K_s) \approx 0.65.$$  \hspace{1cm} (24)

Thus the framework predicts that $S(B_d \rightarrow \Phi K_s)$ will be close to the SM-prediction ($\approx 0.70 \pm 0.10$) and certainly not negative in sign. At present BaBar and BELLE data yield widely differing values of $(0.45 \pm 0.43 \pm 0.07)$ and $(-0.96 \pm 0.50^{+0.09}_{-0.07})$ respectively for $S(B_d \rightarrow \Phi K_s)$ [61]. It will thus be extremely interesting from the viewpoint of the $G(224)/SO(10)$-framework presented here to see whether the true value of $S(B_d \rightarrow \Phi K_s)$ will turn out to be close to the SM-prediction or not.

4. Lepton Flavor Violations: As regards lepton flavor-violations ($\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ etc.) we get contributions from three sources: (i) $(\delta m^2)_{LL}^{\mu\tau}$ arising from RG-running from $M^*$ to $M_{GUT}$, in the context of $SO(10)$ or $G(224)$, involving the large top Yukawa coupling; (ii) $(\delta m^2)_{LR}^{\mu\tau}$ arising from RG-running from $M_{GUT}$ to the RH neutrino mass-scales $M_{Ri}$ involving $\nu_R$ Yukawa-couplings (corresponding to Eq. 3); and (iii) chirality-flipping $(\delta m^2)_{LR}^{\mu\tau}$ arising from A-terms, induced through RG-running from $M^*$ to $M_{GUT}$ in the context of $SO(10)$ or $G(224)$, involving gaugino-masses and Yukawa couplings. Note that all three contributions are tied to our fermion mass-matrices including those of the neutrinos whose successes are discussed in Sec. 4. They are thus fixed in our model for a given choice of $m_0$, $m_{1/2}$, and $M^*/M_{GUT}$.

There is a vast literature on the subject of lepton flavor violation (LFV). (For earlier works see Ref. [53]; and for a partial list of references including recent works see Ref. [65]). Most of the works in the literature have focused on the contribution from the second source (involving the Yukawa couplings of the RH neutrinos) which is proportional to $\tan \beta$ in the amplitude. It turns out, however, that the contribution from the first source $(\delta m^2)_{LL}^{\mu\tau}$ arising from $SO(10)$-running from $M^*$ to $M_{GUT}$ (which is proportional to $\tan \beta$) and that from the third source arising from the induced A-terms ($\propto 1/\tan \beta$) are in fact the dominant ones for $\tan \beta \lesssim 10$, as long as $\ln(M^*/M_{GUT}) \gtrsim 1$. We consider the contribution from all three sources by summing the corresponding amplitudes, and by varying $(m_0, m_{1/2}, \tan \beta$ and $\text{sgn}(\mu))$. Details of these results will appear in Ref. [53]. Here I present the results for a fixed value of $\tan \beta = 10, \ln(M^*/M_{GUT}) = 1$, and four sample choices for $(m_0, m_{1/2}) = (500, 200)$ GeV (Case I), $(700, 300)$ GeV (Case II), $(100, 440)$ GeV (Case III), and $(1000, 250)$ GeV (Case IV):

$$B(\mu \rightarrow e\gamma) \quad B(\tau \rightarrow \mu\gamma)$$
$$\quad (\mu > 0, \mu < 0) \quad (\mu > 0, \mu > 0)$$
$$I \quad (0.4, 1.9) \times 10^{-11} \quad (7.4, 8.4) \times 10^{-9}$$
$$II \quad (1.3, 5.5) \times 10^{-12} \quad (1.7, 2.0) \times 10^{-9} \quad (25)$$
$$III \quad (1.4, 1.4) \times 10^{-8} \quad (8.8, 9.0) \times 10^{-8}$$
$$IV \quad (2.1, 7.4) \times 10^{-13} \quad (8.6, 8.2) \times 10^{-10}$$

Of the four cases exhibited above, Case III (low $m_0 \approx 100$ GeV, with high $m_{1/2} \sim 4.4 m_0$) and
Case IV (high \(m_0 \sim 1\) TeV, with low \(m_{1/2}\)) are in accord with the WMAP-constraint, assuming that the lightest neutralino is the LSP and it represents cold dark matter (see e.g. Ref. [63] for a recent analysis that allows for uncertainties in \((m_1, m_2)\), while Cases I and II per say are not, if we stick to the assumption stated above. It seems to me that Cases like I and II can well be consistent with the WMAP-data, however, under a variety of circumstances including the possibility that R-parity is broken mildly say by a bilinear term \((\kappa L H_u)\) in the superpotential so that the lightest neutralino (LSN) decays with a lifetime \(\sim 10^{-4} - 10^{-5}\) sec, say, to ordinary particles long before nucleosynthesis [66], and that some other particle like the axion provides CDM. An alternative possibility, considered for example in [67], is that the axino is the LSN and provides cold dark matter (subject to R-parity conservation). I do not wish to enter into a detailed discussion of this issue here, except to say that it seems prudent to keep an open mind at present as regards all four choices of \((m_0, m_{1/2})\) exhibited above, and their variants, and study their phenomenological consequences. Given the current limits of \(B(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}\) [88] and \(B(\tau \rightarrow \mu\gamma) \leq 3.11.2 \times 10^{-7}\) [69], we see that while Case III (low \(m_0 \approx 100\) GeV, and high \(m_{1/2} \sim 4.4 m_0\)) is clearly excluded\(^{20}\) by the limit on \(\mu \rightarrow e\gamma\)-decay (taking ln \(M^*/M_{GUT} \geq 1\)), the other three cases are fully compatible with the present limits. But they clearly imply that \(\mu \rightarrow e\gamma\)-decay should be observed with an improvement in the current limit by a factor of 10 – 100. Thus the \(G(224)/SO(10)\)-framework for fermion masses, neutrino oscillations and CP-violation [82] presented here, will have its stringent tests once the current limit especially on the branching ratio for \(\mu \rightarrow e\gamma\)-decay is improved by such a factor.

Electric Dipole Moments in the \(G(224)/SO(10)\)-Model

As regards CP violation, another important prediction of the model is on the edm’s for the neutron and the electron. As mentioned above, RG running from \(M^*\) to \(M_{GUT}\) induces the A-term which in turn generates chirality-flipping transitions such as \(\tilde{q}_L \rightarrow \tilde{q}_R\) and \(\tilde{l}_L \rightarrow \tilde{l}_R\). These having phases through the fermion mass-matrices induce edm’s. Given the fermion mass-matrices as in Sec. 4 and the phases determined by our analysis of CP and flavor violations in the quark-system (as discussed in the beginning of this section), all the relevant s-fermion mass-parameters — i.e. \(\text{Im}(\delta_{LR}^{ij})\), \(\text{Im}(\delta_{LR}^{i})\), and \(\text{Im}(\delta_{LR}^{ij})\) are completely known within our model, for a given value of \(M^*/M_{GUT}\) and that of \(\tan \beta\). These in turn allow us to predict the edm’s for a given choice of \(m_0, m_{1/2}, \tan \beta\) and \(M^*/M_{GUT}\). Details of the results as a function of these SUSY-parameters will be presented in Ref. [52].

The predictions for a specific choice \((m_0 = 660\) GeV, ln \((M^*/M_{GUT}) = 1)\) is given bellow\(^{21}\):

\[
d_n = \begin{cases} 
5.8 \times 10^{-26} e\text{ cm} & (\tan \beta = 5) \\
3.8 \times 10^{-26} e\text{ cm} & (\tan \beta = 10) 
\end{cases} \tag{26}
\]

\[
d_e = \frac{(2.67 \times 10^{-28} e\text{ cm})}{\tan \beta} \quad (m_i = 500 \text{ GeV}). \tag{27}
\]

Note that the A-term contribution is larger for smaller \(\tan \beta\) (For many reasons, including constraints from proton lifetime, small \(\tan \beta \leq 10\) is preferred).

Given the experimental limits \(d_n < 6.3 \times 10^{-26}\) e cm [10] and \(d_e < 4.3 \times 10^{-27}\) e cm [11], we see that the predictions of the model especially for the edm of the neutron is in an extremely interesting range suggesting that it should be discovered with an improvement of the current limit by about a factor of 10.

In summary for this section, we see that \(G(224)/SO(10)\)-framework provides a phe-

\(^{20}\)The sharp increase in \(B(\mu \rightarrow e\gamma)\) for Case III (low \(m_0\) and high \(m_{1/2}\)) is entirely because of the strong enhancement of the induced A-term contribution which is very roughly proportional to \((m_{1/2})^3/(m_0^2 m_{1/2}^2)\) in the amplitude. Thus it increases sharply both because of large \(m_{1/2}\) and small \(m_0\) for case III. As noted above, this induced A-term contribution which arises from SO(10)-running from \(M^*\) to \(M_{GUT}\) has invariably been omitted in the literature. It should, however, be present in any SUSY \(SO(10)\) or \(G(224)\)-model, if \(M^* > M_{GUT}\).

\(^{21}\)Intrinsic SUSY-phases such as that in \((\mu m_3^2)\), if present, can make additional contributions to edm’s which should be added to the contribution shown above. This contribution would increase with \(\tan \beta\). In a theory where such intrinsic phases are naturally zero or small, these contributions can of course be dropped.
nomenologically viable and a unified picture of fermion masses, neutrino oscillations as well as CP and flavor violations. One question on the framework is that it does not provide (e.g. by symmetry-arguments including flavor symmetries) any guidance on the phases-their magnitude and signs. This remains a challenge for the future. On the positive side, the framework not only provides a consistent and predictive picture as regards a vast set of phenomena noted above but also presents several crucial tests including those on the asymmetry parameter for \( (B_d \rightarrow \Phi K_s) \), branching ratio for \( \mu \rightarrow e\gamma \)-decay and edm's of the neutron.

I next discuss the issue of baryogenesis within the same framework.

6. Baryogenesis Via Leptogenesis Within the \( G(224)/SO(10) \)-Framework

The observed matter-antimatter asymmetry provides an important clue to physics at truly short distances. Given the existence of RH neutrinos, as required by the symmetry \( SU(4) \)-color or \( SU(2)_R \), possessing superheavy Majorana masses which violate B-L by two units, baryogenesis via leptogenesis \[72\] has emerged as perhaps the most viable and natural mechanism for generating the baryon asymmetry of the universe. The most interesting aspect of this mechanism is that it directly relates our understanding of the light neutrino masses to our own origin. The question of whether this mechanism can quantitatively explain the magnitude of the observed baryon-asymmetry depends however crucially on the Dirac as well as the Majorana mass-matrices of the neutrinos, including the phases and the eigenvalues of the latter-i.e. \( M_1, M_2 \) and \( M_3 \) (see Eq. \[19\]).

This question has been considered in a recent work \[72\] in the context of a realistic and predictive framework of fermion masses and neutrino oscillations, based on the symmetry \( G(224) \) or \( SO(10) \), as discussed in Sec. 4, with CP violation treated as in Sec. 5. It has also been discussed in a recent review \[14\]. Here I will primarily quote the results and refer the reader to Ref. \[72\] for more details especially including the discussion on inflation and relevant references.

The basic picture is this. Following inflation, the lightest RH neutrinos \( (\tilde{N}_1)'s \) with a mass \( \approx 10^{10} \text{ GeV} (1/3 - 3) \) are produced either from the thermal bath following reheating \( (T_{RH} \approx \text{ few } \times 10^9 \text{ GeV}) \), or non-thermally directly from the decay of the inflaton \[22\] (with \( T_{RH} \) in this case being about \( 10^7 - 10^8 \text{ GeV} \)). In either case, the RH neutrinos having Majorana masses decay by utilizing their Dirac Yukawa couplings into both \( l + H \) and \( \bar{l} + H \) (and corresponding SUSY modes), thus violating B-L. In the presence of CP violating phases, these decays produce a net lepton-asymmetry \( Y_L = (n_L - n_{\ell})/s \) which is converted to a baryon-asymmetry \( Y_B = (n_B - n_{\bar{B}})/s = CY_L \) (\( C \approx -1/3 \) for MSSM) by the EW sphaleron effects. Using the Dirac and the Majorana mass-matrices of Sec. 4, with the introduction of CP-violating phases in them as discussed in Sec. 5, the lepton-asymmetry produced per \( N_1 \) (or \( \tilde{N}_1 + \tilde{N}_1 \)-pair) decay is found to be \[72\]:

\[
\epsilon_1 \approx \frac{1}{8\pi} \left( \frac{M_0}{v} \right)^2 |(\sigma + 3\epsilon) - y|^2 \sin (2\phi_{21}) \\
\times (-3) \left( \frac{M_1}{M_2} \right) \\
\approx -(2.0 \times 10^{-6}) \sin (2\phi_{21}) \\
\times \left[ \frac{(M_1/M_2)}{5 \times 10^{-3}} \right] \\
\]

Here \( \phi_{21} \) denotes an effective phase depending upon phases in the Dirac as well as Majorana mass-matrices (see Ref. \[22\]). Note that the parameters \( \sigma, \epsilon, y \) and \( (M_0/v) \) are already determined within our framework (to within 10%) from considerations of fermion masses and neutrino oscillations (see Sec. 4 and 5). Furthermore, from Eq. \[19\] we see that \( M_1 \approx (1/3 - 3) \times 10^{10} \text{ GeV} \), and \( M_2 \approx 2 \times 10^{12} \text{ GeV} \), thus \( M_1/M_2 \approx (5 \times 10^{-3})(1/3 - 3) \). In short, leaving aside the phase factor, the RHS of Eq. \[28\] is pretty well deter-

\[22\]In this case the inflaton can naturally be composed of the Higgs-like objects having the quantum numbers of the RH sneutrinos \( (\tilde{\nu}_{RH} \text{ and } \tilde{\nu}_{RH}^c) \) lying in \( (1, 2, 4)_{\mu} \) and \( (1, 2, 4)_{\mu} \) for \( G(224) \) (or \( 16_{\mu} \) and \( 16_{\mu} \) for \( SO(10) \)), whose VEV's break B-L and give Majorana masses to the RH neutrinos via the coupling shown in Eq. \[19\].
mined within our framework (to within about a factor of 5), as opposed to being uncertain by orders of magnitude either way. This is the advantage of our obtaining the lepton-asymmetry in conjunction with a predictive framework for fermion masses and neutrino oscillations. Now the phase angle \( \phi_{21} \) is uncertain because we do not have any constraint yet on the phases in the Majorana sector (\( M'_{\nu} \)). At the same time, since the phases in the Dirac sector are relatively large (see Sec. 3 and Ref. [52]), barring unnatural cancelation between the Dirac and Majorana phases, we would naturally expect \( \sin(2\phi_{21}) \) to be sizable - i.e. of order 1/10 to 1 (say).

The lepton-asymmetry is given by \( Y_L = \kappa(\epsilon_1 / g^*) \), where \( \kappa \) denotes an efficiency factor representing wash-out effects and \( g^* \) denotes the light degrees of freedom (\( g^* \approx 228 \) for MSSM). For our model, using recent discussions on \( \kappa \) from Ref. [25], we obtain: \( \kappa \approx (1/18 - 1/60) \), for the thermal case, depending upon the \( 3^{11} \) entries in the neutrino-Dirac and Majorana mass-matrices (see Ref. [22]). Thus, for the thermal case, we obtain:

\[
(Y_B)_{\text{thermal}} / \sin(2\phi_{21}) \approx (10 - 30) \times 10^{-11} \tag{29}
\]

where, for concreteness, we have chosen \( M_1 \approx 4 \times 10^9 \) GeV and \( M_2 \approx 1 \times 10^{12} \) GeV, in accord with Eq. [13]. In this case, the reheat temperature would have to be about few \( \times 10^9 \) GeV so that \( N_1 \)'s can be produced thermally. We see that the derived values of \( Y_B \) can in fact account for the recently observed value \( (Y_B)_{\text{WMAP}} \approx (8.7 \pm 0.4) \times 10^{-11} \tag{24} \), for a natural value of the phase angle \( \sin(2\phi_{21}) \approx (1/3 - 1) \). As discussed below, the case of non-thermal leptogenesis can allow even lower values of the phase angle. It also typically yields a significantly lower reheat temperature \((\sim 10^{-7} - 10^{-8}) \) GeV which may be in better accord with the gravitino-constraint.

For the non-thermal case, to be specific one may assume an effective superpotential \( W_{\text{eff}}^{\text{infl}} = \lambda S(\Phi \Phi - M^2)^+ \) (non-ren. terms) so as to implement hybrid inflation; here \( S \) is a singlet field and \( \Phi \) and \( \bar{\Phi} \) are Higgs fields transforming as (1, 2, 4) and (1, 2, 4) of \( G(224) \) which break B-L at the GUT scale and give Majorana masses to the RH neutrinos. Following the discussion in [15, 12], one obtains:

\[
m_{\text{infl}} = \sqrt{2} \lambda M, \quad \text{where} \quad M = (1, 2, 4)_H \approx 2 \times 10^{16} \text{ GeV; } T_{RH} \approx (1/7)(\Gamma_{\text{infl}}M_{Pl})^{1/2} \approx (1/7)(M_1/M)(m_{\text{infl}}M_{Pl}/8\pi)^{1/2} \quad \text{and} \quad Y_B \approx -(1/2)(T_{RH}/m_{\text{infl}})\varepsilon_1. \tag{25}
\]

Taking the coupling \( \lambda \) in a plausible range \((10^{-5} - 10^{-6}) \) (which lead to the desired reheat temperature, see below) and the asymmetry-parameter \( \varepsilon_1 \) for the \( G(224)/SO(10) \)-framework as given in Eq. [25], the baryon-asymmetry \( Y_B \) can then be derived. The values for \( Y_B \) thus obtained are listed in Table 1.

The variation in the entries correspond to taking \( M_1 = (2 \times 10^{10} \text{ GeV})(1 - 1/3) \) with \( M_2 = (2 \times 10^{12}) \) GeV in accord with Eq. [11]. We see that for this case of non-thermal leptogenesis, one quite plausibly obtains \( Y_B \approx (8 - 9) \times 10^{-11} \) in full accord with the WMAP data, for natural values of the phase angle \( \sin(2\phi_{21}) \approx (1/3 - 1/10) \), and with \( T_{RH} \) being as low as \( 10^7 \) GeV \((2 - 1/2) \). Such low values of the reheat temperature are fully consistent with the gravitino-constraint for \( m_{3/2} \approx 400 \text{ GeV} - 1 \text{ TeV} \) (say), even if one allows for possible hadronic decays of the gravitinos for example via \( \gamma \gamma \)-modes [76].

In summary, I have presented two alternative scenarios (thermal as well as non-thermal) for inflation and leptogenesis. We see that the \( G(224)/SO(10) \)-framework provides a simple and unified description of not only fermion masses, neutrino oscillations (consistent with maximal atmospheric and large solar oscillation angles) and CP violation, but also of baryogenesis via leptogenesis, in either scenario. Each of the following features - (a) the existence of the RH neutrinos, (b) B-L local symmetry, (c) \( SU(4) \)-color, (d) the SUSY unification scale, (e) the seesaw mechanism, and (f) the pattern of \( G(224)/SO(10) \)

| \( \lambda \) | \( 10^{-9} \) | \( 10^{-8} \) |
|---|---|---|
| \( m_{\text{infl}} \text{ GeV} \) | \( 3 \times 10^{11} \) | \( 3 \times 10^{10} \) |
| \( T_{RH} \text{ GeV} \) | \( (5.3 - 1.8) \times 10^9 \) | \( (17 - 5.6) \times 10^6 \) |
| \( Y_B \times 10^{-4} / \sin(2\phi_{21}) \) | \((100 - 10) \) | \( (300 - 33) \) |

Table 1

Baryon Asymmetry For Non-Thermal Leptogenesis
mass-matrices allowed in the minimal Higgs system (see Sec. 4)-have played crucial roles in realizing this unified and successful description. Before concluding, I now turn to a brief discussion of proton decay in the next section.

7. Proton Decay

Perhaps the most dramatic prediction of grand unification is proton decay. I have discussed proton decay in the context of the SUSY $SO(10)/G(224)$-framework presented here in some detail in recent reviews [34,47] which are updates of the results obtained in [29]. Here, I will present only the salient features and the updated results. In SUSY unification there are in general three distinct mechanisms for proton decay.

1. The familiar $d=6$ operators mediated by $X$ and $Y$ gauge bosons of $SU(5)$ and $SO(10)$ As is well known, these lead to $e^+\pi^0$ as the dominant mode with a lifetime $\approx 10^{35.3\pm 1}$ yrs.

2. The “standard” $d=5$ operators [77] which arise through the exchange of the color-triplet Higgsinos which are in the $5_H$ of $SU(5)$ or $10_H$ of $SO(10)$. These operators require (for consistency with proton lifetime limits) that the color-triplets be made superheavy while the EW-doublets are kept light by a suitable doublet-triplet splitting mechanism (for $SO(10)$, see Ref. [26,35]. They lead to dominant $\bar{\nu}K^+$ and comparable $\bar{\nu}\pi^+$ modes with lifetimes varying from about $10^{29}$ to $10^{34}$ years, depending upon a few factors, which include the nature of the SUSY-spectrum and the matrix elements (see below). Some of the original references on contributions of standard $d=5$ operators to proton decay may be found in [78,79,80,81,82,29,47,83,84].

3. The so called “new” $d=5$ operators [85,35] (see Fig. 1)\(^\text{23}\) which can generally arise through the exchange of color-triplet Higgsinos in the Higgs multiplets like $(16_H + \bar{16}_H)$ of $SO(10)$. Such exchanges are possible by utilizing the joint effects of (a) the couplings given in Eq. (11) which assign Majorana masses to RH neutrinos, while the lower right vertex utilizes the $g_{ij}$ couplings in Eq. (9) which are needed to generate CKM mixings.

\(^\text{23}\)Note that in the presence of a second $SO(10)$-singlet field $S'$ carrying flavor charge of $2a+1/2$, an effective coupling of the form $16_H \bar{16}_H S'$ would be allowed preserving the $U(1)$-flavor symmetry introduced in Sec. 4. With $S'$ having a VEV of GUT-scale, such a coupling would generate a mass-term $16_H \bar{16}_H$ that enters into Fig. 1. The presence of this second singlet $S'$ does not in any way affect the hierarchical pattern of effective couplings exhibited in Eqs. (9) and (11). I thank Qaisar Shafi for raising this point.
been invariably omitted in the literature.

One distinguishing feature of the new $d = 5$ operator is that they directly link proton decay to neutrino masses via the Majorana masses of the RH neutrinos. The other, and perhaps most important, is that these new $d = 5$ operators can induce proton decay even when the $d = 6$ and standard $d = 5$ operators mentioned above are absent. This is what would happen if the string theory or a higher dimensional GUT-theory would lead to an effective $G(224)$-symmetry in 4D (along the lines discussed in Sec. 3), which would be devoid of both $X$ and $Y$ gauge bosons and the dangerous color-triplets in the $10_H$ of $SO(10)$. By the same token, for an effective $G(224)$-theory, these new $d = 5$ operators become the sole and viable source of proton decay leading to lifetimes in an interesting range (see below). And this happens primarily because the RH neutrinos control the strength of the standard unification and thereby restricts the source of proton decay leading to lifetimes in an interesting range (see below).

Our study of proton decay carried out in Ref. [29] and updated in [34] and [47] has a few distinctive features: (i) It is based on a realistic framework for fermion masses and neutrino oscillations, as discussed in Sec. 4; (ii) It includes the new $d = 5$ operators in addition to the standard $d = 5$ and $d = 6$ operators; (iii) It restricts GUT-scale threshold-corrections to $\alpha_3(m_Z)$ so as to be in accord with the demand of “natural” coupling unification and thereby restricts $M_{\text{eff}}$ that controls the strength of the standard $d = 5$ operators; and (iv) It allows for the ESSM extension [25] of MSSM motivated on several grounds (see e.g. [25] and [47]), which introduces two vector-like families in $16 + \bar{16}$ of $SO(10)$ with masses of order 1 TeV, in addition to the three chiral families.

The study carried out in [29] and its updates, based on recently reported values of the matrix elements $\beta_H$ and $\alpha_H$ and the renormalization factors $A_L$ and $A_S$ for $d = 5$ [85], and $A_R$ for $d = 6$ operators have been discussed in detail in [34] and [47]. In these reviews, I had used the latest lattice-result available at the time which gave $\beta \approx 0.014 \text{ GeV}^3$ [57]. This result was based, however, on quenching and finite lattice spacing, which could introduce sizable systematics. I had, therefore, allowed an uncertainty by a factor of two either way and taken $\beta_H = (0.014 \text{ GeV}^3)(1/2 - 2)$ and likewise $\alpha_H = (0.015 \text{ GeV}^3)(1/2 - 2)$. A very recent calculation based on quenched lattice QCD in the continuum limit yields: $|\beta_H| = 0.0096(09) (+_{2.0}^{-6.2}) \text{ GeV}^3$ and $|\alpha_H| = 0.0090(09) (+_{13}^{-5}) \text{ GeV}^3$ [88]. Allowing still for an uncertainty by $\sqrt{2}$ either way (due to quenching), I now take $|\beta_H|$ and $|\alpha_H|$ $\approx$ 0.0068. This value however nearly coincides with the value of $\beta_H$ and $\alpha_H$ at the lower end $\approx$ 0.007 GeV$^3$ used in previous estimates [34,47].

As a result the upper limits on proton lifetime presented before remain practically unaltered. The values of the parameters now used are as follows: $|\beta_H| \approx |\alpha_H| \approx (0.009 \text{ GeV}^3)(1/\sqrt{2} - \sqrt{2})$; $m_{\tilde{b}} \approx m_{\tilde{t}} \approx 1.2 \text{ TeV} (1/2 - 2)$; $(m_{\tilde{W}}/m_{\tilde{q}}) = 1/6(1/2 - 2)$; $M_{H_c}(\text{min}SU(5)) \leq 10^{16} \text{ GeV}$; $A_L \approx 0.32$, $A_S \approx 0.93$, $\tan \beta \leq 3$; $M_X \approx M_Y \approx 10^{16} \text{ GeV}$ (1 ± 25%), and $A_R(d = 6, e^+\pi^0) \approx 3.4$.

The theoretical predictions for proton decay for the cases of minimal SUSY $SU(5)$, SUSY $SO(10)$ and $G(224)$-models developed in Secs. 3 and 4, are summarized in Table 2. They are obtained by following the procedure as in [35,47] and using the parameters as mentioned above.\footnote{\textbf{24}The chiral Lagrangian parameter $(D+F)$ and the renormalization factor $A_R$ entering into the amplitude for $p \rightarrow e^+\pi^0$ decay are taken to be 1.25 and 3.4 respectively.}

It should be stressed that the upper limits on proton lifetimes given in Table 2 are quite conservative in that they are obtained (especially for the top two cases) by stretching the uncertainties in the matrix element and the SUSY spectra to their extremes so as to prolong proton lifetimes. In reality, the lifetimes should be shorter than the upper limits quoted above.

Now the experimental limits set by SuperK studies are as follows [89]:

\begin{align*}
\Gamma^{-1}(p \rightarrow e^+\pi^0)_{\text{expt}} & \geq 6 \times 10^{33} \text{ yrs} \\
\Gamma^{-1}(p \rightarrow \bar{\nu}K^+)_{\text{expt}} & \geq 1.6 \times 10^{33} \text{ yrs}
\end{align*}

(35)

The following comments are in order.

1. By comparing the upper limit given in Eq. (30) with the experimental lower limit, we see that the \textbf{minimal} SUSY $SU(5)$ with the conventional MSSM spectrum is clearly
excluded by a large margin by proton decay searches. This is in full agreement with the conclusion reached by other authors (see e.g. Ref. [84]).

2. By comparing Eq. (31) with the empirical lower limit, we see that the case of MSSM embedded in $SO(10)$ is already tightly constrained to the point of being disfavored by the limit on proton lifetime. The constraint is of course augmented by our requirement of natural coupling unification, which prohibits accidental large cancelation between different threshold corrections (see [29]).

3. In contrast to the case of MSSM, that of ESSM [25] embedded in $SO(10)$ (see Eq. (32)) is fully compatible with the SuperK limit. In this case, $\Gamma^{-1}(p \to \bar{\nu}K^+) \approx 2 \times 10^{34}$ yrs (1–50)% (Fully Compatible with SuperK) (33) with $\bar{\nu}K^+ + \bar{\nu}\pi^+$ being the dominant modes and quite possibly $\mu^+ K^0$ being prominent.

4. We see from Eq. (33) that the contribution of the new operators related to the Majorana masses of the RH neutrinos (Fig. 1) (which is the same for MSSM and ESSM and is independent of $\tan \beta$) is fully compatible with the SuperK limit. These operators can quite naturally lead to proton lifetimes in the range of $10^{33} - 10^{34}$ yrs with an upper limit of about $2 \times 10^{34}$ yrs.

In summary for this section, within the $SO(10)/G(224)$ framework and with the inclusion of the standard as well as the new $d = 5$ operators, one obtains (see Eqs. (31)–(35)) a conservative upper limit on proton lifetime given by:

$$\tau_{\text{proton}} \lesssim (1/3 - 2) \times 10^{34} \text{ yrs}$$

with $\bar{\nu}K^+$ and $\bar{\nu}\pi^+$ being the dominant modes and quite possibly $\mu^+ K^0$ being prominent.

The $e^+\pi^0$-mode induced by gauge boson-exchanges should have an inverse decay rate in the range of $10^{34} - 10^{36}$ years (see Eq. (35)). The implication of these predictions for a next-generation detector is noted in the next section.

8. Concluding Remarks

The neutrinos seem to be as elusive as revealing. Simply by virtue of their tiny masses, they
provide crucial information on the unification-scale, and even more important on the nature of the unification-symmetry. In particular, as argued in Secs. 4 and 6, (a) the magnitude of the superK-value of \( \sqrt{\delta m^2_{23}}(\approx 1/20 \text{ eV}) \), (b) the \( b/\tau \) mass-ratio, and (c) baryogenesis via leptogenesis, together, provide clear support for (i) the existence of the RH neutrinos, (ii) the existence of B-L as a local symmetry and a value for \( m(\nu^\text{Dirac}_\tau) \), (iv) the familiar SUSY unification-scale which provides the scale of \( M_R \), and last but not least, (v) the seesaw mechanism. In turn this chain of argument selects out the effective symmetry in 4D being either a string-derived \( G(224) \) or \( SO(10) \)-symmetry, as opposed to the other alternatives like \( SU(5) \) or flipped \( SU(5)' \times U(1) \).

It is furthermore remarkable that the tiny neutrino-masses also seem to hold the key to the origin of baryon excess and thus to our own origin!

In this talk, I have tried to highlight that the \( G(224)/SO(10) \)-framework as described here is capable of providing a unified description of fermion masses, neutrino oscillations, CP and flavor violations as well as of baryogenesis via leptogenesis. This seems non-trivial. The framework is also highly predictive and can be further tested by studies of CP and flavor violations in processes such as (a) \( B_d \to \phi K_S \)-decays, (b) \( B_S, B_S' \)-decays, (c) edm of neutron, and (d) leptonic flavor violations as in \( \mu \to e\gamma \) and \( \tau \to \mu \gamma \)-decays.

To conclude, the evidence in favor of supersymmetric grand unification, based on a string-derived \( G(224) \)-symmetry (as described in Sec. 3) or \( SO(10) \)-symmetry, appears to be strong. It includes:

- Quantum numbers of all members in a family,
- Quantization of electric charge,
- Gauge coupling unification,
- \( m^0_\nu \approx m^0_\tau \)
- \( \sqrt{\delta m^2(\nu_2 - \nu_3)} \approx 1/20 \text{ eV} \)
- A maximal \( \Theta^c_{23} \approx \pi/4 \) with a minimal \( V_{cb} \approx 0.04 \), and
- Baryon Excess \( Y_B \approx 10^{-10} \).

All of these features and more including (even) CP and flavor violations hang together neatly within a single unified framework based on a presumed string-derived four-dimensional \( G(224) \) or \( SO(10) \)-symmetry, with supersymmetry. It is hard to believe that this neat fitting of all these pieces can be a mere coincidence. It thus seems pressing that dedicated searches be made for the two missing pieces of this picture-that is supersymmetry and proton decay. The search for supersymmetry at the LHC and possible future NLC is eagerly awaited. That for proton decay will need a next-generation megaton-size underground detector.

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39. For G(224), one can choose the corresponding sub-multiplets – that is (1, 1, 15)H, (1, 2, 4)H, (1, 2, 4)H, (2, 2, 1)H – together with a singlet S, and write a superpotential analogous to Eq. (9).

40. If the effective non-renormalizable operator like 16H,16H,10H,45H/M′ is induced through exchange of states with GUT-scale masses involving renormalizable couplings, rather than through quantum gravity, M′ would, however, be of order GUT-scale. In this case ⟨45H⟩/M′ ∼ 1, rather than 1/10.

41. While 16H has a GUT-scale VEV along the SM singlet, it turns out that it can also have a VEV of EW scale along the “νL” direction due to its mixing with 10H, so that the Hd of MSSM is a mixture of 10H and 16H. This turns out to be the origin of non-trivial CKM mixings (See Ref. [29]).

42. The flavor charge(s) of 45H(16H) would get determined depending upon whether p(q) is one or zero (see below).

43. These effective non-renormalizable couplings can of course arise through exchange of (for example) 45 in the string tower, involving renormalizable 16H,45H couplings. In this case, one would expect M ∼ Mstring.

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47. J. C. Pati, “Probing Grand Unification Through Neutrino Oscillations, Leptonogenesis and Proton Decay”; hep-ph/0305221, Proceedings Erice School, Sept. 2002, Ed. by A. Zichichi (Publ. World Scientific), p. 194-236.

48. Note that the magnitudes of η, ε and σ are fixed by the input quark masses. Furthermore, one can argue that the two contributions for θϕmϕ [see Eq. (18)] necessarily add
to each other as long as $|y|$ is hierarchical ($\sim 1/10$) \[29\]. As a result, once the sign of $\epsilon$ relative to $\eta$ and $\sigma$ is chosen to be negative, the actual magnitudes of $V_{cb} \approx (0.044)$ and $\sin^2 2\theta_{\mu\tau} \approx 0.92 - 0.99$ emerge as predictions of the model \[29\].

49. Note that such an operator would be allowed by the flavor symmetry defined in Eq. (10) if one sets $a = 1/2$. In this case, operators such as $W_{23}$ and $W_{33}$ that would contribute to $\nu^\nu_\tau$ and $\nu^\nu_\tau$ masses would be suppressed relative to $W_{12}$ by flavor symmetry.

50. A term like $W_{12}$ can be induced in the presence of, for example, a singlet $S$ and a tenplet $(10)$, possessing effective renormalizable couplings of the form $a_1 A_1 A_1 A_1 S$ and mass terms $M_S S S$ and $M_{10} 10 10$. In this case $\kappa_{12}/M^3_{eff} \approx a_1 A_1 A_1 S$. Setting the charge $a = 1/2$, and assigning charges $\pm 1/2$ to $(10, S)$, the couplings $a_1$, and $b$ would be flavor-symmetry allowed, while $a_2$ would be suppressed but so also would be the mass of $(10)$ compared to the GUT-scale. One can imagine that $S$ on the other hand acquires a GUT-scale mass through for example the Dine-Seiberg-Witten mechanism, violating the U(1)-flavor symmetry. One can verify that in such a picture, one would obtain $\kappa_{12}/M^3_{eff} \sim 1/M^3_{GUT}$.

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54. For instance, consider the superpotential for $45_H$ only: $W(45_H) = M_{45} 45_H + \lambda 45_H/M$, which yields (setting $F_{45_H} = 0$), either $\langle 45_H \rangle = 0$, or $\langle 45_H \rangle^2 = -2M_{45}/\lambda$. Assuming that “other physics” would favor $\langle 45_H \rangle \neq 0$, we see that $\langle 45_H \rangle$ would be pure imaginary, if the square bracket is positive, with all parameters being real. In a coupled system, it is conceivable that $\langle 45_H \rangle$ in turn would induce phases (other than “0” and $\pi$) in some of the other VEV’s as well, and may itself become complex rather than pure imaginary.

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