SELF-SUSTAINED TURBULENCE WITHOUT DYNAMICAL FORCING: A TWO-DIMENSIONAL STUDY OF A BISTABLE INTERSTELLAR MEDIUM

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ABSTRACT

In this paper, the nonlinear evolution of a bistable interstellar medium is investigated using two-dimensional simulations with a realistic cooling rate, thermal conduction, and physical viscosity. The calculations are performed using periodic boundary conditions without any external dynamical forcing. As the initial condition, a spatially uniform unstable gas under thermal equilibrium is considered. At the initial stage, the unstable gas quickly segregates into two phases: cold neutral medium (CNM) and warm neutral medium (WNM). Then, self-sustained turbulence with velocity dispersion of $0.1–0.2 \text{ km s}^{-1}$ is observed in which the CNM moves around in the WNM. We find that the interfacial medium (IFM) between the CNM and WNM plays an important role in sustaining the turbulence. The self-sustaining mechanism can be divided into two steps. First, thermal conduction drives fast flows streaming into concave CNM surfaces toward the WNM. The kinetic energy of the fast flows in the IFM is incorporated into that of the CNM through the phase transition. Second, turbulence inside the CNM deforms interfaces and forms other concave CNM surfaces, leading to fast flows in the IFM. This drives the first step again and a cycle is established by which turbulent motions are self-sustained.

Key words: hydrodynamics – instabilities – ISM: kinematics and dynamics – ISM: structure

Online-only material: color figures

1. INTRODUCTION

It is well known that the interstellar medium (ISM) has a thermally bistable structure in the optically thin regime as a result of the balance of radiative cooling and heating due to external radiation fields and cosmic rays (Field et al. 1969; Wolfire et al. 1995, 2003). The bistable gas consists of two thermal equilibrium phases, i.e., a clumpy low-temperature phase (cold neutral medium (CNM)) and a diffuse high-temperature phase (warm neutral medium (WNM)). The CNM is observed as H$\text{I}$ clouds ($n \approx 10–100 \text{ cm}^{-3}$, $T \approx 10^2 \text{ K}$), and the WNM is observed as diffuse H$\text{I}$ gas ($n \approx 0.1 \text{ cm}^{-3}$, $T \approx 6000 \text{ K}$). In the temperature range between these phases, the gas is thermally unstable.

Linear analyses of the thermal instability (TI) have been investigated by Field (1965) for a uniform gas under thermal equilibrium and by Balbus (1986) for thermal nonequilibrium gas. They found criteria for the TI. Iwasaki & Tsuribe (2008) discovered a one-parameter family of self-similar solutions that describes the nonlinear development of the TI for various scales under a plane-parallel geometry. Their linear stability was investigated by Iwasaki & Tsuribe (2009).

The basic physics of bistable gas has been investigated by many authors. Zel’dovich & Pikel’ner (1969) investigated the steady-state structure of a transition layer connecting the CNM and WNM under a plane-parallel geometry (see also Iwasaki & Inutsuka 2012 for a larger parameter space). The thickness of the transition layer corresponds to the Field length, below which the TI is stabilized by thermal conduction (Field 1965). They found a so-called saturation pressure $P_{\text{sat}}$ at which there is a static solution. If the surrounding pressure is larger (smaller) than $P_{\text{sat}}$, the solution describes condensation (CNM$\rightarrow$WNM) (evaporation (CNM$\rightarrow$WNM)). Yatou & Toh (2009) discovered pulse-like static solutions and demonstrated that they are sustained by the balance between viscosity and the pressure gradient. Elphick et al. (1991, 1992) have investigated the interaction between multitransition layers. They found that the transition layers tend to approach and annihilate. The merging timescale is an exponentially increasing function of the separation between the transition layers. Aranson et al. (1993) investigated the nonlinear evolution of a thermally unstable phase under a plane-parallel geometry for the case of open boundaries. In the early phase, runaway condensation occurs in dense regions and rarefied parts are heated up until both reach thermal equilibria. Finally, the pressure approaches the saturation pressure (Zel’dovich & Pikel’ner 1969). Linear analysis of a plane-parallel transition layer has been done by Aranson et al. (1995) for the long-wavelength limit including curvature effects and by Inoue et al. (2006) for the long- and short-wavelength limits. They found that an evaporation front is unstable against corrugation-type fluctuations while a condensation front is stable. Recently, Kim & Kim (2013) have investigated the nonlinear development of the evaporation-front instability. Stone & Zweibel (2009) have shown that the presence of magnetic fields perpendicular to transition layers modifies their stability properties.

The multidimensional dynamics of a bistable gas is quite different from the one-dimensional case. Graham & Langer (1973) found a minimum cloud size below which clouds inevitably evaporate by investigating isobaric flows in a spherical symmetrical geometry. Elphick et al. (1991) found that a transition layer at $P = P_{\text{sat}}$ is not static in the multidimensional case. Nagashima et al. (2005, 2006) investigated the evaporation and condensation of a spherical and cylindrical CNM surrounded by a WNM under the isobaric approximation. The front velocity is proportional to the inverse of the radius at $P = P_{\text{sat}}$ and is constant in the case of much larger clouds or pressure far from $P_{\text{sat}}$. Those results indicate that the motion of a transition layer depends on its curvature, suggesting that the multidimensional structure is more complex than the one-dimensional structure.

Numerical hydrodynamical simulations are powerful tools to investigate the multidimensional evolution of bistable gas because analytic analyses are quite difficult in general situations.
Koyama & Inutsuka (2006) have investigated the nonlinear evolution of bistable gas by using two- and three-dimensional numerical simulations incorporating a realistic cooling rate with periodic boundary conditions. They used realistic thermal conduction and viscosity in their fiducial model. Interestingly, they found self-sustained turbulence in bistable gas even though they did not consider any external dynamical forcing. In contrast, Brandenburg et al. (2007) have performed similar calculations and concluded that there is no sustained turbulence. However, to resolve the thickness of the interface, they adopted an artificially large thermal conductivity so that the Field length was as large as \(\sim 0.5 \, \text{pc}\) and spatially constant. The actual Field length has \(\sim 10^3 \, \text{pc}\), it is computationally quite expensive to perform three-dimensional simulations even if the simulation box is as small as several parsecs. Thus, as a first step, the two-dimensional simulation is analyzed in a more detailed treatment (Micic et al. 2013). The thermal conductivity for neutral hydrogen \((\kappa(T) = 2.5 \times 10^3 \sqrt{T} \, \text{cm}^{-1} \, \text{K}^{-1} \, \text{s}^{-1})\) is adopted. In a neutral monatomic gas, the viscosity coefficient is given by \(\mu = 3k/(2c_p)\), where \(c_p = \gamma k_B/\mu_\text{HI}m_\text{H}(\gamma - 1)\) is the specific heat at constant pressure.

2. EQUATIONS AND METHODS

2.1. Basic Equations

The Navier–Stokes equations with radiative cooling/heating and thermal conduction are solved,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_j)}{\partial x_j} = 0, \tag{1}
\]

\[
\frac{\partial (\rho v_j)}{\partial t} + \frac{\partial (P\delta_{ij} + \rho v_i v_j - \sigma_{ij})}{\partial x_j} = 0, \tag{2}
\]

\[
\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j} \left[ (E + P) v_j - \sigma_{ij} v_i - \frac{\partial T}{\partial x_j} \right] = -\rho \mathcal{L}(\rho, T), \tag{3}
\]

\[
P = \frac{k_B}{\mu_\text{HI}m_\text{H}} \rho T = n_\text{HI}k_B T, \tag{4}
\]

\[
\sigma_{ij} = \mu \left[ \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right], \tag{5}
\]

where \(E = P/(\gamma - 1) + \rho v^2/2\) is the total energy, \(\gamma = 5/3\) is the ratio of specific heats, \(n_\text{HI}\) is the number density of hydrogen nuclei, \(\mu_\text{HI} = 1.4\) is the mean molecular weight per hydrogen nucleus, \(\mu\) is the viscosity coefficient, \(\kappa\) is the thermal conductivity, and \(\mathcal{L}\) is a net cooling rate per unit mass.

In this paper, the following simple analytic formula (Koyama & Inutsuka 2002) is adopted:

\[
\rho \mathcal{L}(\rho, T) = n_\text{HI} [n_\text{HI} \Lambda(T) - \Gamma] \, \text{erg cm}^{-3} \, \text{s}^{-1} \tag{6}
\]

\[
\frac{\Lambda(T)}{\Gamma} = 10^7 \exp \left( - \frac{118400}{T + 1000} \right) + 1.4 \times 10^{-2} \sqrt{T} \exp \left( - \frac{92}{T} \right).
\]

The validity of the cooling function in multidimensional simulation is analyzed in a more detailed treatment (Micic et al. 2013). The thermal conductivity for neutral hydrogen \((\kappa(T) = 2.5 \times 10^3 \sqrt{T} \, \text{cm}^{-1} \, \text{K}^{-1} \, \text{s}^{-1})\) is adopted. In a neutral monatomic gas, the viscosity coefficient is given by \(\mu = 3k/(2c_p)\), where \(c_p = \gamma k_B/\mu_\text{HI}m_\text{H}(\gamma - 1)\) is the specific heat at constant pressure.

2.2. Thermal Properties of the ISM

Figure 1(a) shows the thermal equilibrium curve where \(\mathcal{L} = 0\) in the \((n, P)\) plane. One can see that the fluid can take two stable equilibrium states, the CNM and WNM, at constant pressure, as shown by the thin horizontal line. In a bistable fluid, an interfacial medium (IFM) connects the CNM and WNM. In this paper, the IFM is defined as the gas in the gray region in Figure 1(a) where \((\partial \mathcal{L}/\partial \rho)_P > 0\). Zel’dovich & Pikel’ner (1969) found steady solutions connecting the CNM and WNM. Figure 1(b) shows the temperature distribution of a steady solution. The gray region indicates the IFM corresponding to a transition layer as discussed in Section 1. The thickness of the IFM is characterized by the Field length.
(Begelman & McKee 1990),

$$\lambda_F = \sqrt{\frac{\kappa(T)T}{\rho|\nabla|}}. \quad (7)$$

The Field length depends on local temperatures and densities. The Field length for the CNM is as small as several \(10^{-3}\) pc, whereas for the WNM it is as large as 0.1 pc. This dependence can be seen in Figure 1(b). From the WNM, the temperature gradually declines because of the large Field length. As the temperature decreases, the Field length decreases, so the temperature rapidly drops to CNM values. Thus, the CNM and IFM are separated by a sharp discontinuity that hereafter we will refer to as an interface or CNM surface. In contrast, the WNM is smoothly connected with the IFM, as shown in Figure 1(b).

2.3. Methods and Initial Conditions

An operator-splitting technique is used for solving the basic Equations (1)–(3). For the inviscid part, a second-order Eulerian-remap Godunov scheme (van Leer 1979) is used. The cooling/heating, thermal conduction, and physical viscosity are calculated by explicit time integration. A square domain \(-L/2 < x, y < L/2\) is considered, where \(L\) is the domain length. Periodic boundary conditions are imposed in the \(x\) and \(y\)-directions.

As an initial condition, a uniform unstable gas (\(n_H = 4.3\) cm\(^{-3}\) and \(T = 423\) K) in thermal equilibrium is considered. The initial state is in the IFM phase. A random velocity fluctuation with a flat power spectrum whose minimum scale is \(L/4\) is added to the initial state. The amplitude of the velocity dispersion is \(\sim 2\%\) of the sound speed. It has been confirmed that saturation levels of turbulence do not depend on how initial fluctuations are added. As a fiducial model, a case with \(L = 2.4\) pc is considered. The box size dependence of turbulence will be investigated in Section 4.3. Koyama & Inutsuka (2004) have proposed the Field condition where the local Field length should be resolved by a few grids to obtain the converged results (see also Kim & Kim 2013). Thus, the minimum Field length of \(\sim 3 \times 10^{-3}\) pc in the CNM needs to be resolved. In the fiducial model, \(N = 2048^2\) is used, where \(N\) is the total cell number. The corresponding grid size is \(10^{-3}\) pc, which satisfies the Field condition.

3. RESULTS

3.1. Velocity Dispersion

In this section, the time evolution of the velocity dispersions and mass fractions of the three phases (CNM, IFM, WNM) are investigated. The phase of each grid cell is distinguished using Figure 1(a). The mass and velocity dispersion of each phase are given by

$$M_s = \int \rho dV_s, \quad (8)$$

and

$$\Delta v_s = \sqrt{\frac{1}{M_s} \int \rho v^2 dV_s}, \quad (9)$$

respectively, where the subscript “s” denotes the phase, \(s = (\text{cnm, ifm, wnm})\), and \(V_s\) indicates the volume occupied by the phase “s.”

Figures 2(a) and (b) show the early evolution for \(t < 40\) Myr of the velocity dispersion and mass fraction, \(M_s/M_{\text{tot}}\), of the three phases, respectively, where \(M_{\text{tot}}\) is the total mass. Initially, the TI causes runaway cooling in the dense regions while runaway heating in the rarefied parts keeps the pressure almost constant. During this time, the velocity dispersion of the IFM increases exponentially (see Figure 2(a)). From Figure 2(b), one can see that \(M_{\text{ifm}}\) begins to decrease while \(M_{\text{cnm}}\) quickly increases around \(t \sim 4\) Myr. This indicates that the dense parts of the IFM change into the CNM. Around \(t \sim 15\) Myr, \(M_{\text{cmn}}/M_{\text{tot}}\) reaches \(\sim 80\%\). The formation epoch of the CNM lags behind that of the CNM because the heating timescale in the rarefied parts is longer than the cooling timescale in the dense parts. Around \(t \sim 20\) Myr, a bistable fluid consisting of the CNM and WNM is formed. The IFM occupies the regions in between. The mass fraction of the three phases is \(M_{\text{cmn}}:M_{\text{ifm}}:M_{\text{wnm}} \sim 0.85:0.10:0.05\).

Before proceeding, the resolution dependence of the velocity dispersion is investigated. Figure 2(c) shows the velocity dispersion of CNM for \(N = 512^2 (\Delta x = 1.6\lambda_{\text{F, min}}), 1024^2 (\Delta x = 2\lambda_{\text{F, min}}), 2048^2 (\Delta x = 4\lambda_{\text{F, min}})\), and \(N = 4096^2\) at \(t = 20\) Myr.
0.8λ_{F,\text{min}}), 2048^2(Δx = 0.4λ_{F,\text{min}}), and 4096^2(Δx = 0.2λ_{F,\text{min}}), where λ_{F,\text{min}} = 3 \times 10^{-3} \text{ pc} is the minimum Field length. From Figure 2(c), for \( t < 10 \text{ Myr} \) when the TI develops, the velocity dispersion is independent of resolution. This is because all models resolve the maximum growth scale of the TI (∼0.04 pc). After the bistable gas is formed, the WNM and CNM are separated by the transition layers, whose thicknesses near the CNM correspond to λ_{F,\text{min}} (see Section 2.2). For the lowest resolution case (\( N = 512^2 \)), λ_{F,\text{min}} is not resolved. That is why only the result with \( N = 512^2 \) exhibits the lowest velocity dispersion and turbulence does not increases with time. On the other hand, Δ\( v_{\text{cnm}} \) for the three higher resolution models (\( N = 1024^2, 2048^2, \) and \( 4096^2 \)) reach almost the same values at \( t = 100 \text{ Myr} \), and the results appear to be converged. The two models (\( N = 2048^2 \) and \( 4096^2 \)) satisfy the Field condition whereas the model with \( N = 1024^2 \) barely resolves λ_{F,\text{min}}. These results are consistent with Koyama & Inutsuka (2004). Thus, the fiducial model produces the converged result.

Next, the long-term evolution of the bistable fluid is investigated. Figure 3 is the same as Figure 2(a) but for \( 40 < t / \text{Myr} < 530 \). It is found that turbulence is maintained until at least 530 Myr in all phases. The evolution of Δ\( v_{\text{cnm}} \) is quite similar to that of Δ\( v_{\text{ifm}} \), whereas Δ\( v_{\text{ifm}} \) has larger fluctuations. The time-averaged velocity dispersion is as large as ∼0.1 km s\(^{-1}\). In contrast, Δ\( v_{\text{wnm}} \) is smaller than the other two phases. Most of the kinetic energy of the turbulence resides in the CNM because of its large mass fraction. If there is only CNM, the turbulence is expected to decay within its crossing timescale \( ∼L/(0.1 \text{ km s}^{-1}) ∼ 23 \text{ Myr} \). Thus, maintenance of the turbulence requires a supply of kinetic energy into the CNM.

### 3.2. Density and Velocity Distributions

Figure 4(a) shows color and contour maps of the temperature at a fixed epoch. One can see that the CNM (blue) has a complicated structure in the WNM (red). To see the turbulent structure, the color map of the velocity amplitude |\( v | \) is shown in Figure 4(b). The green lines correspond to CNM/IFM interfaces and the gray lines indicate IFM/WM boundaries. Thus, the regions between these lines belong to the IFM.

Figure 4(b) shows that the CNM has a complicated fine velocity structure, whereas the WNM does not. This comes from the large difference of the Reynolds numbers, \( LΔv/\nu \), of the WNM and CNM that are given by

\[
\text{Re}_{\text{wnm}} = 60 \left( \frac{n_{\text{wnm}}}{0.5 \text{ cm}^{-3}} \right) \left( \frac{T_{\text{wnm}}}{6000 \text{ K}} \right)^{-1/2} \left( \frac{\Delta v_{\text{wnm}}}{0.06 \text{ km s}^{-1}} \right) \left( \frac{L}{2.4 \text{ pc}} \right),
\]

and

\[
\text{Re}_{\text{cnm}} = 10^5 \left( \frac{n_{\text{cnm}}}{50 \text{ cm}^{-3}} \right) \left( \frac{T_{\text{cnm}}}{50 \text{ K}} \right)^{-1/2} \left( \frac{\Delta v_{\text{cnm}}}{0.1 \text{ km s}^{-1}} \right) \left( \frac{L}{2.4 \text{ pc}} \right),
\]
respectively, where $Δv_{\text{wnm}}$ and $Δv_{\text{cnm}}$ are evaluated in Figure 3. One can see that the Reynolds number of WNM is much smaller than that of CNM by about three orders of magnitude. Thus, the turbulent CNM is embedded in the viscous WNM. This dissipative feature of the WNM is also seen in Figure 3, where the WNM has the smallest velocity dispersion. Note that fast flows are seen in the IFM near the deformed CNM/IFM interfaces.

Figure 5 shows a close-up view of Figure 4(a). There are two prominent types of strongly curved CNM surfaces. One is a deep concave CNM surface toward the WNM. The other is a pillar corresponding to an elongated convex surface toward the WNM. From Figure 5, it is seen that the fast flows in the IFM flood into the concave CNM surface, whereas the gases in the IFM stream into the WNM from the heads of the pillars.

### 3.3. Driving Mechanism of Fast Flows in the IFM

It is well known that in a bistable gas, flows can be driven by thermal processes, i.e., thermal conduction and radiative cooling or heating. This relates to the phase transition (Zel’dovich & Pikel’ner 1969). The time evolution of the enthalpy is given by

$$\frac{d}{dt}(c_pT) = -\frac{1}{\rho} \frac{dP}{dt} + \frac{1}{\rho} \nabla \cdot (\kappa \nabla T) - \mathcal{L},$$

where the viscous heating term is implicitly neglected because it is much smaller than the other terms. Furthermore, the first term on the right-hand side of Equation (12) is negligible compared with the left-hand-side term.

Figures 6(a) and (b) show color maps of the net cooling term, $-\mathcal{L}$, and the thermal conduction term, $\nabla \cdot (\kappa \nabla T) / \rho$, respectively. Figure 6(c) shows the sum of the two terms, corresponding to $d(c_pT)/dt$. In each panel, the gas in the red (blue) region is heated (cooled). Figure 6(a) shows that most of the volume of the IFM is heated by the external radiation. Although the gases cool in very thin layers just outside the CNM, they are too thin to be seen in the figure. In contrast, the thermal conduction term has a complicated distribution, as shown in Figure 6(b). The thermal conduction term is positive (negative) in the IFM near the convex (concave) CNM surfaces. Figure 6(c) shows that the distribution of thermal conduction is preserved in that of $d(c_pT)/dt$, indicating that thermal conduction dominates the thermal process.

The distribution of thermal conduction reflects the complicated temperature distribution (see Figure 4(a)). We now consider an individual temperature contour. The unit vector parallel to the gradient vector $\nabla T$ at a point on the contour is defined as $\mathbf{n} = \nabla T / |\nabla T|$. The vector $\mathbf{n}$ is oriented in the direction from the CNM to the WNM. Using $\mathbf{n}$, one can write $\nabla T = (\partial_n T) \mathbf{n}$, where $\partial_n \equiv \mathbf{n} \cdot \nabla$. The thermal conduction term can be rewritten as

$$\nabla \cdot (\kappa \nabla T) = \partial_n (\kappa \partial_n T) + \kappa K \partial_n T,$$

where $K \equiv \nabla \cdot \mathbf{n}$ is the curvature of the contour line. The first term on the right-hand side of Equation (13) corresponds to the contribution from the component parallel to $\mathbf{n}$. The second term comes from the curvature effect that corresponds to the
shown in Figure 4(a). The parallel term is small in narrow valleys in the interface modifies the temperature distribution in the narrow valleys where the parallel term is small. The parallel term is important on both sides of the valleys. The parallel term is determined by that of $K\kappa\partial_n T/\rho$. By comparing Figures 4(a) and 7(b), one can see that the curvature term is dominated near the pillars where the thermal conduction term is positive. In regions with negative thermal conduction, the curvature term is important in the narrow valleys where the parallel term is small. The parallel term is important on both sides of the valleys.

The typical IFM flow velocity driven by thermal conduction is estimated. If a quasi-steady state is assumed, Equation (12) becomes

$$c_p(\partial_n T)v_n \simeq \frac{1}{\rho}K\kappa\partial_n T,$$

where $v_n$ is the gas velocity parallel to $n$, the net cooling rate is neglected, and only regions where the curvature term is dominated are considered. From Equation (14), the typical velocity $v_n$ is estimated by

$$|v_n| \sim \frac{\kappa(T_{\text{ifm}})|K|}{c_p\rho_{\text{ifm}}} = 0.2\ \text{km s}^{-1} \times \left(\frac{T_{\text{ifm}}}{2000\ \text{K}}\right)^{1/2} \left(\frac{|K|}{200\ \text{pc}^{-1}}\right)^{-1} \left(\rho_{\text{ifm}}^{-1}\ \text{cm}^{-3}\right),$$

where physical values typical of the IFM are used and a typical curvature value is derived from Figure 7(b). This typical velocity, $v_n$, is consistent with that found in Figure 4(b).

Note that the fast flows driven in the IFM are accompanied by a fast phase transition between the CNM and IFM/WNM. If the kinetic energy of the fast flows in the IFM is carried into the CNM through the phase transition, it can act as a driving force of turbulence in the CNM.

### 3.4. Mechanism of Driving and Dissipation of Kinetic Energy in Each Phase

The turbulence seems to show saturation for $t > 50$ Myr (see Figure 3). In the saturated state, driving of kinetic energy is expected to balance dissipation of kinetic energy in each phase. In this section, we investigate what mechanism drives the kinetic energy and what mechanism dissipates it in each phase. We consider the evolution equation for the total kinetic energy of phase $s = (\text{cnm, ifm, wnm})$, given by

$$\frac{\partial}{\partial t} \int_{V_s} \left(\frac{1}{2}\rho v^2\right) dV = W_{T,s} + W_{P,s} + W_{V,s} \equiv W_{\text{tot},s},$$

where $W_{P,s}$ and $W_{V,s}$ indicate the powers due to the pressure gradient and viscous force, respectively, and are given by

$$W_{P,s} = -\int_{V_s} dV v \cdot \nabla P,$$

and

$$W_{V,s} = \int_{V_s} dV v \cdot \nabla \sigma_{\mu\nu}.$$
Figure 8. Power contributions to the time evolution of the kinetic energy for the (a) CNM, (b) IFM, and (c) WNM. In each panel, the red, green, and blue lines correspond to \( W_T \), \( W_P \), and \( W_V \), respectively, and the black line indicates the total power, \( W_{\text{tot}} \). The vertical coordinates are divided by \( 10^7 \).

(A color version of this figure is available in the online journal.)

that describe shock waves unavoidably contain numerical viscosity. In Godunov’s method adopted in this paper, the result from the nonlinear Riemann solver is used in the evaluation of the pressure gradient. Thus, if a simple difference form is used in calculating the pressure gradient at the cell center, the pressure at the cell boundary is required. Godunov’s method evaluates the pressure \( P^* \) by using the result from the nonlinear Riemann solver, where the left- and right-hand-side states are derived by an interpolation from cells. The numerical viscous flux is evaluated approximately with \( (P_L + P_R)/2 \), where \( P_L \) and \( P_R \) are the pressures in the left- and right-hand-side states of the Riemann solver. The powers due to numerical viscosity are evaluated from the numerical viscous flux. The power \( W_{P,s} \) is calculated from \( (P_L + P_R)/2 \). Then, the power due to numerical viscosity is included in \( W_{V,s} \). The other caution is that \( W_{T,s} \) is difficult to estimate directly using the finite-volume method. Thus, \( W_{T,s} \) is derived indirectly by subtracting \( W_{P,s} \) and \( W_{V,s} \) from the time derivative of the total kinetic energy of the phase “s” (see Equation (16)).

Figure 8 shows the time evolution of the three powers, \( W_T \), \( W_P \), and \( W_V \) for the CNM, IFM, and WNM for \( 200 < t/\text{Myr} < 250 \). In each panel, the black line corresponds to the total power, \( W_{\text{tot},s} \). When \( W_{T,P,V,s} > 0 \) (< 0), the power increases (decreases) the kinetic energy. To evaluate quantitatively which power is dominant in each of the driving and dissipation mechanisms, the time average of \( W_{T,P,V,s} \) is calculated in the temporal range \( 100 < t/\text{Myr} < 530 \). The results are shown in Table 1, for which the values have been normalized by \( 10^7 \mu_1 L^2/\text{Myr} \), where \( L = 2.4 \text{pc} \). In each phase, the time average of the total powers, \( \langle W_{\text{tot},s} \rangle \), during \( 100 < t/\text{Myr} < 530 \) is almost zero, indicating that the turbulence reaches a quasi-steady state.

Because the CNM provides the dominant contribution to the total kinetic energy of the system, the driving mechanism of turbulence in the CNM is crucial. Figure 8(a) shows that \( W_{T,\text{cnm}} \) always takes a positive value. In contrast, \( W_{P,\text{cnm}} \) fluctuates with a large amplitude around zero. This indicates that there are compressive waves inside the CNM. As will be described later, the compressive waves are driven by the pressure decrement caused by conductive cooling in the IFM. Figure 8 shows that \( W_{T,\text{cnm}} \) is balanced with viscous dissipation \( \langle W_{V,\text{cnm}} \rangle \) whereas \( W_{P,\text{cnm}} \) is almost zero. Thus, it is confirmed that the main driver of turbulence in the CNM is kinetic energy injection from the IFM to CNM, as mentioned at the end of Section 3.4.

The kinetic energy in the CNM comes from the IFM through the phase transition. Let us see the driving and dissipation

| Phase | \( \langle W_T \rangle \) | \( \langle W_P \rangle \) | \( \langle W_V \rangle \) | \( \langle W_{\text{tot}} \rangle \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| CNM   | 1.1             | -0.045          | -1.1            | -0.009          |
| IFM   | -1.2            | 9.0             | -7.8            | -0.00035        |
| WNM   | -0.13           | 0.27            | -0.14           | -0.0005         |

Note. The values are normalized by \( 10^7 \mu_1 L^2/\text{Myr} \), where \( L = 2.4 \text{pc} \).
mechanisms of the kinetic energy in the IFM. Figure 8(b) shows that only the pressure gradient force increases the kinetic energy (also see Table 1). This pressure gradient arises from conductive cooling and heating near deformed CNM/IFM interfaces, and it accelerates the fluid. In contrast, kinetic energy is dissipated mostly by the viscosity when \(|\langle W_{V,\text{imf}}\rangle|\) is slightly smaller than \(|\langle W_{P,\text{ifm}}\rangle|\) (see Table 1). Because \(W_{T,\text{ifm}}\) is negative and \(|\langle W_{T,\text{ifm}}\rangle| \sim -\langle W_{T,\text{cnm}}\rangle\), the phase transition transports the small remaining kinetic energy from the IFM to the CNM. This kinetic energy injection drives turbulence in the CNM.

Figure 8(c) and Table 1 show that the powers in the WNM are much smaller than those of the other two phases. As mentioned in Section 3.2, the WNM is dissipative (\(\text{Re} < 1\)) for \(L = 2.4\) pc. Thus, the WNM is not expected to contribute to the driving of turbulence. The WNM is passively entrained by the fluid motion in the IFM.

Figure 8 and Table 1 suggest that turbulence in the CNM is mainly driven by kinetic energy injection through the phase transition from the IFM to the CNM. To show this process more clearly, time sequences of temperature and velocity are plotted for a rectangular region of size 0.4 pc in Figure 9. The velocity distribution in Figure 9(a) shows that there are many vortices inside the CNM that correspond to circular structures with central holes. The turbulent motion in the CNM begins to pull its interface leftward around the center in Figure 9(a). Figure 9(b) shows that the interface is largely stretched toward the CNM, and a prominent concave CNM surface is formed. As mentioned before, in the IFM enclosed by the concave CNM surface, thermal conduction cools the gas. This decreases the pressure in the IFM toward the CNM. The resultant pressure gradient drives fast flows streaming into the concave CNM surface, as shown in Figure 8(b). In the concave CNM surface, one can see two parallel interfaces facing each other across the IFM, and one side of them is connected. The distance between them is about 0.1 pc. It is well known that two parallel interfaces approach and eventually annihilate (Elphick et al. 1991; also see Appendix B). Figure 9(c) shows the merging epoch. During this merging process, the IFM between the two interfaces changes into the CNM or the phase transition. The conduction-driven flow is almost parallel to the two interfaces. This means that the conduction-driven flow is perpendicular to the approaching direction of the two interfaces. Thus, this flow velocity in the IFM is almost preserved during the phase transition. Finally, the conduction-driven flow in the IFM is incorporated into the kinetic energy of the CNM.

The variation of the pressure can be clearly seen in Figure 10, which shows the probability distribution in the \((n, P)\) plane averaged over the time range \(200 \leq \tau/\text{Myr} \leq 220\). One can see that most of the gas resides at a constant pressure of \(\log_{10}(P/k_B) \sim 3.35\). However, low-pressure regions are found in the IFM. This corresponds to the conductively cooled regions enclosed by concave CNM surfaces. Note that the CNM distributes along the thermal equilibrium curve shown by the black line. This is because the cooling/heating timescale of the CNM is so short that the CNM evolves along the thermal equilibrium curve. Moreover, the pressure distribution of the CNM extends downward in the \((n, P)\) plane. As mentioned above, this large pressure variation in the CNM is attributed to conductive cooling in the IFM enclosed by the concave CNM surfaces. This drives compressive waves in the CNM.
4. DISCUSSION

4.1. Self-sustaining Mechanism of Turbulence

From the findings in Section 3, the following self-sustaining mechanism is realized in bistable turbulence. The self-sustaining mechanism can be divided into three parts, as shown in Figure 11: (1) turbulence inside the CNM deforms its CNM surface and (2) creates concave CNM surfaces. In the IFM enclosed by the concave CNM surfaces, (3) thermal conduction drives flows that stream toward the concave CNM surfaces. The kinetic energy of the IFM flows is transported into the CNM through the phase transition. In this manner, the cycle in Figure 11 is self-sustained in a bistable system.

4.2. Typical Time and Length Scales of Kinetic Energy Injection

From Section 3.4, it is found that kinetic energy injection from the IFM to the CNM occurs in concave CNM surfaces. To investigate the typical timescale of kinetic energy injection, the power spectrum of $W_{T,cnm}$ is plotted in Figure 12. The vertical axis denotes the power spectrum multiplied by the frequency. From Figure 12, $f \times P_{W_{T,cnm}}(f)$ peaks around $f/2\pi \sim 1 \text{ Myr}^{-1}$ and has a power law $\propto f^{-0.32}$ in the high frequency limit. Thus, kinetic energy injection with a timescale of $2\pi/f \sim 1 \text{ Myr}$ provides a dominant contribution to $W_{T,cnm}$.

The timescale of energy injection is expected to be comparable to that of annihilation of two parallel interfaces, given by $t_{\text{merge}} = 1.47(e^{d/0.14 \text{ pc}} - 1) \text{ Myr}$, where $d$ is the distance between two interfaces (see Appendix B). From this fitting equation, $d$ is about 0.1 pc for $t_{\text{merge}} = 1 \text{ Myr}$. Assuming that the timescale of kinetic energy injection is determined by the annihilation of two parallel interfaces, the typical scale of the concave CNM surfaces is expected to be 0.1 pc. From Figures 4, 5, and 9, the typical scale of the prominent concave CNM surfaces appears to be consistent with $\sim 0.1 \text{ pc}$. Interestingly, the typical scale of 0.1 pc is comparable to the thickness of the IFM in a plane-parallel geometry (see Figure 1).

4.3. Dependence of Saturation Level on Simulation Box Size

In this section, the box size dependence of turbulence is considered. The simulations are performed under the same initial condition while keeping the grid size constant for various box sizes. Figure 13(a) shows the time evolution of the velocity dispersion for $L = 0.3, 0.6, 1.2, 2.4,$ and $4.8$ pc. Only for the smallest box size ($L = 0.3$ pc) does the turbulence decay. For larger box sizes, the velocity dispersion initially increases and saturates around $t \sim 100 \text{ Myr}$. Figure 13(a) reveals that the saturation levels increase with $L$. This trend can be understood from the driving mechanism of turbulence. If the simulation box is too small, the effect of conductive cooling can traverse the simulation box size within the survival time of curved interfaces, and a quasi-isobaric state is quickly established. As a result, driving of fast flows is limited, leading to a small turbulent velocity.
Figure 13(b) shows the box size dependence of the saturation level. The vertical axis indicates the time-averaged velocity dispersion for $100 < t/{\text{Myr}} < 300$. The averaged velocity dispersion seems to saturate for $L \geq 4.8$ pc. This saturation can be explained from the typical timescale of the energy injection (~1 Myr) found in Section 4.2. The sound crossing length within 1 Myr is as large as ~7 pc, where the sound speed of the WNM (~7 km s$^{-1}$) is used. Thus, for $L > 7$ pc, the velocity dispersion is expected to be independent of the box size. This is consistent with the fact that $(\Delta v)_b$ begins to saturate around $L = 2.4 \sim 4.8$ pc. The saturation level is expected to be $0.14 \sim 0.15$ km s$^{-1}$ by extrapolation of Figure 13(b). Furthermore, from the driving mechanism, the velocity dispersion is not expected to be larger than the conduction-driven flow. The expected saturation level is consistent with the typical velocity shown in Equation (15).

4.4. Comparison with Previous Works

Koyama & Inutsuka (2006) have investigated the dependence of the saturation level on the simulation box and found that the velocity dispersion saturates for $L > 35$ pc, and the saturation level is ~0.3 km s$^{-1}$. This conclusion appears to contradict our result in Section 4.3. This is attributed to the fact that they adopted an artificially large thermal conductivity and viscosity for calculations with larger boxes (Koyama & Inutsuka 2006). We have done the same calculation for the case with $L = 144$ pc. After 100 Myr, when they terminate the calculation, we find that the turbulence decays because of artificially large viscosity. The saturation found by Koyama & Inutsuka (2006) may come from the initial growth of the TI.

Brandenburg et al. (2007) performed similar calculations with a different analytic net cooling rate. As mentioned in Section 1, they used an artificially large thermal conductivity and viscosity that are proportional to density. If we use the same conditions in our two-dimensional calculations, we also observe decaying turbulence. To drive turbulence, it is important that turbulence in the CNM deforms the interface. However, because of the large viscosity, the turbulence quickly decays. Another difference between our work and theirs is the dimensionality. This will be discussed in the next section.

4.5. Three-dimensional Case without Magnetic Field

In this paper, we focus on the two-dimensional evolution of a bistable gas. It is well known that the evolution of vorticity strongly depends on the dimensionality. In two dimensions, enstrophy is conserved. Thus, the vortex filaments behave like particles. In contrast, in three dimensions, a vortex cascade into smaller vortices through stretching. Note that the deformation of CNM surfaces requires relatively large vortices in the CNM. For three dimensions, it is expected that the deformation of CNM surfaces becomes inefficient. Thus, it is possible that the driving mechanism proposed in this paper cannot maintain turbulence in three dimensions. However, no one has performed three-dimensional simulations with sufficient resolution to resolve the thickness of the CNM/IFM interface (Koyama & Inutsuka 2004) because the required computational cost is enormous. To simulate the three-dimensional simulation efficiently, one of the promising methods is the adaptive mesh refinement technique, for example, with a refinement criterion based on the local Field length. The effect of three dimensions is beyond the scope of this paper, but it should be investigated in future work.

4.6. Effect of Magnetic Field

In this paper, for simplicity we focus on the two-dimensional hydrodynamical evolution of a bistable fluid without magnetic fields. However, in realistic situations, magnetic fields play important roles in the dynamics of a bistable fluid because the typical magnetic field strength of H$^1$ gas is about a few micro-Gauss (Heiles & Troland 2005). The one-dimensional evolution of magnetized bistable gas has been investigated by Inoue et al. (2007) and Stone & Zweibel (2010). They show that ambipolar diffusion efficiently transports a magnetic field across a transition layer, leading to a flat magnetic field strength profile. However, the multidimensional evolution is still unclear. The plasma beta of H$^1$ gas is less than or comparable to unity. Therefore, the turbulent velocity is less than the Alfvén speed of ~1 km s$^{-1}$ for the CNM and ~10 km s$^{-1}$ for the WNM. This corresponds to weak Alfvénic turbulence where the energy preferentially cascades in directions perpendicular to the mean magnetic field in the ideal MHD limit (Sridhar & Goldreich 1994). Thus, the outcome can be similar to two-dimensional hydrodynamic turbulence. The effects of magnetic fields will be investigated in a forthcoming paper.

4.7. Implications for Interstellar Turbulence

The self-sustaining mechanism drives turbulence in the CNM at the level of about 0.1–0.2 km s$^{-1}$. It is well known that the velocity dispersion (~1 km s$^{-1}$ at 1 pc) in the real ISM is much larger than that found in this paper (e.g., Larson 1981; Hennebelle & Falgarone 2012). Thus, the turbulence analyzed in this paper cannot alone explain the interstellar turbulence quantitatively.

The difference between turbulence in this paper and the interstellar turbulence could be a consequence of the oversimplified setup in the paper. In turbulence under periodic boundary conditions without any dynamical forcing, the two stable phases (WNM and CNM) are completely separated, and the unstable region resides only in the interfaces between the WNM and CNM. In this case, as shown in Figure 6, thermal conduction is the most important thermal process; it drives only weak turbulence with $\Delta v \sim 0.1–0.2$ km s$^{-1}$.

In realistic astrophysical environments, on the other hand, the ISM is frequently disturbed and compressed by energetic phenomena the length scales of which are larger than a few parsec, such as expansions of H II regions and supernova explosions with a timescale on the order of 1 Myr (McKee & Ostriker 1977), which is comparable to the typical timescale of the kinetic energy injection derived in Section 4.2. Thus, the large-scale disturbances should be important for the interstellar turbulence. Koyama & Inutsuka (2002) have demonstrated that the shock compression of the ISM induces turbulence that is composed of shocked warm gas and cold cloudlets. The velocity dispersion is comparable to the observed values. Recently, similar calculations have been performed by many authors (Audit & Hennebelle 2005; Hennebelle & Audit 2007; Heitsch et al. 2005, 2006, 2009; Vázquez-Semadeni et al. 2006, 2007; Inoue & Inutsuka 2008, 2009; Banerjee et al. 2009; Vázquez-Semadeni et al. 2011; Inoue & Inutsuka 2012). However, the detailed mechanism to create turbulent structure is not fully analyzed because of its complexity.

One important difference between turbulence in shocked ISM and this paper is the physical properties of the unstable gas (IFM). The shock-heated gases are thermally unstable and subject to strong radiative cooling by which the CNM is formed.
This cooling drives gas flows that reach a velocity as large as $\sim$several km \, s$^{-1}$. Since this typical velocity is large, the Reynolds number becomes high enough for turbulence to be driven. Unlike the turbulence in this paper, thermal conduction is less important because of the strong radiative cooling. In addition to the TI (strong radiative cooling), turbulence in the unstable gas is driven by vortex generation due to the baroclinic effect, the Richtmyer–Meshkov instability (Inoue et al. 2009, 2012; Inoue & Inutsuka 2012; Sano et al. 2012).

The results in Sections 3.4 show that the kinetic energy transfer from the unstable gas to the CNM through the phase transition is the main driving source of turbulence in the CNM. The same process can be expected in realistic situations with strong shocks, in addition to other possible driving mechanisms of turbulence, such as the Kelvin–Helmholtz and Rayleigh–Taylor instabilities. Because the velocity dispersion of the unstable gas is larger ($\sim$several km \, s$^{-1}$), the transferred turbulent kinetic energy is expected to be larger. The resultant velocity dispersion inside individual CNM clouds may be supersonic (sound speed of CNM $\sim$0.2 \, km \, s$^{-1}$). This mechanism may also affect molecule formation inside the CNM because of the mixing between fresh CNM and preexisting CNM. These subjects will be discussed in a forthcoming paper.

5. SUMMARY

In this paper, we have investigated the turbulent structure of the bistable ISM by using two-dimensional hydrodynamic simulations with a realistic cooling rate, thermal conduction, and physical viscosity. Our results are summarized as follows.

1. It is confirmed that turbulence is sustained for at least 500 Myr without any dynamical forcing. The velocity dispersion of the IFM is comparable to that of the CNM whereas that of the WNM is only half of the other two phases. The dominant contribution to the velocity dispersion is provided by the CNM because of its large mass fraction.

2. Fast flows are observed in the IFM near strongly deformed CNM/IFM interfaces. There are two prominent flows. First, gas in the IFM stream into concave CNM surfaces. Second, gas in the IFM flows toward the WNM from pillars of the CNM. It is found that these fast IFM flows are driven by thermal conduction.

3. The mechanisms of driving and dissipation of kinetic energy are investigated in the saturation state of the three phases. In the CNM, the dominant driving mechanism is kinetic energy injection from the IFM through the phase transition. This injected kinetic energy comes from the fast flows driven by strong conductive cooling near concave CNM surfaces. Because the IFM near concave CNM surfaces is surrounded by the CNM, the pressure drop due to conductive cooling in the IFM induces relatively large pressure fluctuations in the CNM.

4. A self-sustaining mechanism of bistable turbulence is summarized in Figure 11. Turbulence inside the CNM creates concave CNM surfaces. Fast flows driven by thermal conduction in the IFM stream into the concave CNM surfaces and their kinetic energy is transported into the CNM through the phase transition. In this way, the deformation of CNM surfaces by turbulence eventually enhances its kinetic energy. The free energy of this driving mechanism originally comes from the external heating that maintains the temperature difference between the CNM/WMN. This temperature difference drives flows in the IFM that become the driving source of turbulence in the CNM.

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APPENDIX A

DERIVATION OF EVOLUTION EQUATION FOR TOTAL KINETIC ENERGY OF THREE PHASES

From Equations (1) and (2), the evolution equation for the kinetic energy is given by

$$\frac{\partial}{\partial t} \left( \frac{1}{2} \mathbf{v} \cdot \mathbf{v} \right) + \nabla \cdot (\mathbf{v} P + \mathbf{v} \rho \mathbf{\eta}) \mathbf{v} = 0. \tag{A1}$$

In this appendix, the evolution equation for the total kinetic energy is derived for each of the three phases. The procedure of the derivation is the same for all three phases. Thus, the phase “s” is considered, where “s” denotes the label of the phase (CNM, IFM, and WNM).

The whole domain is divided into two subdomains: the phase “s” and the other phases. We introduce a scalar field $\psi_s$ given by

$$\psi_s(t, \mathbf{x}) = \begin{cases} 1 & \text{inside the phase “s”,} \\ 0 & \text{elsewhere.} \end{cases} \tag{A2}$$

Using $\psi_s$, one can define a normal unit vector at the interface pointing outward with respect to the phase “s,”

$$\mathbf{n}_{\text{int}} = - \left( \frac{\nabla \psi_s}{|\nabla \psi_s|} \right)_{\text{int}}, \tag{A3}$$

where the subscript “int” denotes the value at the interface. For an observer moving with the interface, $\psi_s$ does not change in time. Thus, $\psi_s$ obeys

$$\frac{\partial \psi_s}{\partial t} + \mathbf{v}_{\text{int}} \cdot \nabla \psi_s = 0, \tag{A4}$$

where $\mathbf{v}_{\text{int}}$ is the velocity of the interface and is zero everywhere except at the interface.

The time evolution of the total kinetic energy of the phase “s” is given by

$$\frac{\partial}{\partial t} \int dV \psi_s E_{\text{kin}} = \int dV \psi_s \frac{\partial E_{\text{kin}}}{\partial t} + \int dV \frac{\partial \psi_s}{\partial t} E_{\text{kin}}, \tag{A5}$$

where $E_{\text{kin}} = \rho \mathbf{v}^2 / 2$. The first term on the right-hand side of Equation (A5) is considered. Using Equation (A1), one gets

$$\int dV \psi_s \frac{\partial E_{\text{kin}}}{\partial t} = - \int dV \psi_s \nabla \cdot (E_{\text{kin}} \mathbf{v})$$

$$- \int dV \psi_s \mathbf{v} \cdot \nabla P$$

$$+ \int dV \psi_s v_{\mu} \nabla v_{\nu} \sigma_{\mu \nu}. \tag{A6}$$
The integrand of the first term on the right-hand side of Equation (A6) can be rewritten as

\[ \psi_s \nabla \cdot (E_{\text{kin}} \mathbf{v}) = \nabla \cdot (\psi_s E_{\text{kin}} \mathbf{v}) - E_{\text{kin}} \mathbf{v} \cdot \nabla \psi_s = \nabla \cdot (\psi_s E_{\text{kin}} \mathbf{v}) + E_{\text{kin}} \mathbf{v} \cdot n_{\text{int}} |\nabla \psi_s|, \]  

(A7)

where Equation (A3) is used in the last line. From Gauss’s theorem, the volume integral of the first term on the right-hand side of Equation (A7) vanishes because of the periodic boundary conditions. From Equations (A6), (A7), and (A4), Equation (A5) becomes

\[ \frac{\partial}{\partial t} \int dV \psi_s E_{\text{kin}} = - \int dV E_{\text{kin}} (\mathbf{v} - \mathbf{v}_{\text{int}}) \cdot n_{\text{int}} |\nabla \psi_s| 
- \int dV \psi_s \mathbf{v} \cdot \nabla P + \int dV \psi_s \mathbf{v}_p \mathbf{v} \cdot \sigma_{\mu \nu}. \]  

(A8)

Because $|\nabla \psi_s|$ is a delta function that is infinity at the interface and zero elsewhere, Equation (A6) becomes

\[ \frac{\partial}{\partial t} \int dV \psi_s E_{\text{kin}} = W_{T,s} + W_{P,s} + W_{V,s} \]  

(A9)

where

\[ W_{T,s} = - \int_{\text{int}} dS E_{\text{kin}} (\mathbf{v} - \mathbf{v}_{\text{int}}) \cdot n_{\text{int}}, \]  

(A10)

\[ W_{P,s} = - \int dV \psi_s \mathbf{v} \cdot \nabla P, \]  

(A11)

\[ W_{V,s} = \int dV \psi_s \mathbf{v}_p \mathbf{v} \cdot \sigma_{\mu \nu}, \]  

(A12)

\[ \int_{\text{int}} dS \] denotes the surface integral at the interface, and $W_{T,s}$ represents the kinetic energy transport across the interface relating to the phase transition. $W_{P,s}$ and $W_{V,s}$ correspond to powers due to the pressure gradient and viscous force, respectively. The integral $\int dV \psi_s$ is the same as $\int dV \mathbf{v}$ used in Section 3.4.

APPENDIX B

ANNIHILATION OF TWO PARALLEL INTERFACES IN PLANE-PARALLEL GEOMETRY

In this appendix, to derive the merging timescale of two parallel interfaces as a function of the distance between them, a one-dimensional simulation is performed. The temperature distribution of the static solution at $P = P_{\text{Int}}$ (Zel’dovich & Pikel’ner 1969) is denoted by $T_{ZP}(x)$, that is $T_{\text{inn}}$ for $x \to -\infty$ and $T_{\text{wnm}}$ for $x \to \infty$. The origin of $T_{ZP}$ is defined at the point

\[ T(x) = T_{ZP}(x + d/2) + T_{ZP}(-x + d/2) - T_{\text{wnm}}, \]  

(B1)

where $d$ is the distance between two interfaces. Figure 14 shows a schematic picture of the initial condition. The WNM is sandwiched by two CNM phases. The velocity is assumed to be zero and the pressure is constant.

The interfaces approach each other and eventually merge. The merging time is estimated using the one-dimensional simulations for various $d$ and is plotted in Figure 14. It is seen that the merging time increases with $d$. From perturbation theory, Elphick et al. (1991) derived an analytic formula ($t_{\text{merge}} = t_0 (\exp(d/\lambda_0) - 1)$), where $\lambda_0$ is the thickness of the transition layer and $t_0$ is a typical timescale. The simulation points in Figure 15 are fitted by this analytic formula quite well. It is found that $t_0 = 1.47$ Myr and $\lambda_0 = 0.14$ pc, which is consistent with the thickness of the IFM (0.1 pc).

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