The graceful exit from the anomaly-induced inflation: 
Supersymmetry as a key

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Abstract. The stable version of the anomaly-induced inflation does not need a fine tuning and leads to sufficient expansion of the Universe. The non-stable version (Starobinsky model) provides the graceful exit to the FRW phase. We indicate the possibility of the inflation which is stable at the beginning and unstable at the end. The effect is due to the soft supersymmetry breaking and the decoupling of the massive sparticles at low energy.

Introduction

The inflation solves many problems of the Early Universe and there are not so much doubts that it really took place (see, e.g. [1]). On the other hand, the conventional inflaton-based approach requires an exact fine-tuning of the form of the inflaton potential or initial data. Some people believe that there must be a natural mechanism for inflation, which should originate from the vacuum quantum effects of matter fields. The desired solution would not require a fine-tuning neither for initial conditions nor for the graceful exit to the Friedmann-Robertson-Walker (FRW) power-low expansion phase. The purpose of this article is to suggest a qualitative version of such a mechanism. Our approach is based on the Starobinsky model [2, 3, 4, 5, 6] and supersymmetry (SUSY). We shall also make use of the fact that SUSY is not seen in the low-energy phenomena.

Consider the vacuum quantum effects in the Early Universe, when the typical energy of quantum processes is very high but below the Planck scale. Then, the appropriate framework is not the string theory but some quantum field theory. Furthermore, at the energies greater than the masses of the particles one can apply an approximation in which the masses of the fields are negligible. The matter filling the Universe is characterized by pressure $p$ and energy density $\rho$. The standard relation $\rho = 3p$ holds in the ultra-relativistic limit, consequently, the matter decouples from the conformal factor of the metric. This means local conformal (Weyl) invariance for the quantum fields [7]. Then, the vacuum quantum effects of the matter fields can be taken into account through the conformal anomaly [2, 3, 4, 5, 6] and may lead to inflation [3]. On the other hand, the masses of particles should be relevant at lower energies. At the intermediate scales, the anomaly-induced action can serve as a model for the mass-independent part of the effective action [4, 5, 6] while the leading effect of particle masses would be the renormalization of the Einstein-Hilbert term in the vacuum action. Taking this renormalization into account, we arrive at the tempered form of inflation [8].
The paper as organized as follows. In the next section we present the qualitative description of the inflation and the graceful exit mechanism. In section 3 some numerical estimates will be done, in particular we establish the relation between gravitational scale and the radiation temperature. The last section contains brief general discussion of the anomaly-induced approach.

2. Quantum effects, inflation and graceful exit

Consider the gauge theory which has $N_0$ real scalars, $N_{1/2}$ Dirac spinors, and $N_1$ vectors. We shall be mainly interested in the vacuum effects and suppose that the interaction between quantum fields is weakened at high energies due to the asymptotic freedom. Than the one-loop vacuum contributions play the leading role. Notice that the vacuum quantum effects originate from the virtual particles, therefore the numbers $N_{0,1/2,1}$ do not describe the real matter which might fill the Universe.

The classical action of vacuum can be very complicated and include infinitely many local and non-local terms. However, in order to meet a renormalizability requirement there must be, at least, the following three terms [9, 10]:

$$S_{\text{vac}} = \int d^4x \sqrt{-g} \left\{ l_1 C^2 + l_2 E + l_3 \Box R \right\}.$$  (1)

Here, $l_{1,2,3}$ are some parameters, $C^2$ is the square of the Weyl tensor and $E$ is the integrand of the Gauss-Bonnet topological invariant. Action (1) consists of the conformal invariant and surface terms. Therefore, if we add (1) to the Einstein-Hilbert action, then the usual (homogeneous and isotropic) FRW solution remains unaltered.

The renormalization of the action (1) leads to the conformal anomaly [11, 9, 10]

$$\langle T_{\mu}^{\mu} \rangle = -(wC^2 + bE + c\Box R),$$  (2)

where $w, b, c$ are the $\beta$-functions for the parameters $l_1, l_2, l_3$

$$w = \frac{N_0 + 6N_{1/2} + 12N_1}{120 \cdot (4\pi)^2}, \quad b = -\frac{N_0 + 11N_{1/2} + 62N_1}{360 \cdot (4\pi)^2}, \quad c = \frac{N_0 + 6N_{1/2} - 18N_1}{180 \cdot (4\pi)^2}. \quad (3)$$

One can consider the cosmological model directly, using the anomaly (3), however, it is useful to construct the anomaly-induced effective action. The quantum correction $\tilde{\Gamma}$ to the classical action of vacuum is related to the anomaly

$$-\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta \tilde{\Gamma}}{\delta g_{\mu\nu}} = \langle T_{\mu}^{\mu} \rangle.$$  (4)

The solution of this equation can be found explicitly in the form [12, 13]:

$$\tilde{\Gamma} = S_c[\bar{g}_{\mu\nu}] + \int d^4x \sqrt{-\bar{g}} \left\{ w\sigma C^2 + b\sigma (E - \frac{2}{3} \Box \bar{R}) + 2b\sigma \Delta \sigma \right\} - \frac{3c + 2b}{36} \int d^4x \sqrt{-\bar{g}} R^2,$$  (5)

where we use, along with the original metric $g_{\mu\nu}$, another variables: the new metric $\bar{g}_{\mu\nu}$ with the fixed determinant and $\sigma$, such that $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma}$. The effective action (5) includes an unknown conformal-invariant functional $S_c[\bar{g}_{\mu\nu}]$, which is the only indefinite component of the solution (5). This term is indeed irrelevant when we consider the cosmological (homogeneous and isotropic)
metrics. In this case (5) is the exact one-loop correction to the classical action of vacuum. The second term on the r.h.s. of (5) is non-covariant but it can be rewritten in a covariant non-local form \[12, 14\].

Now we are in a position to discuss the arbitrariness in the classical action of vacuum. It is important that this is the action of an external gravitational field and that the metric is not quantized. Then, as it was already mentioned above, one can add to the vacuum action (1) any local or non-local terms. In particular, one can introduce terms similar to those which emerge as quantum corrections (5). In this way, one can change the numerical coefficients of all terms in anomaly (2) or even cancel the anomaly completely. Of course, if we insist that the classical action of vacuum should be local, then only the last \( \int \sqrt{-g} \Box R \)-term in (2) can be modified by introducing the \( \int \sqrt{-g}R^2 \)-term into the classical action. Sometimes, this operation is called "introducing finite counterterm".

The introduction of some (\( \int \sqrt{-g}R^2 \)-type or non-local) extra terms into the classical action of gravity may be equivalent to the introduction of the inflaton-like fields (see, e.g., \[15, 16\]). From our point of view, the necessity to adjust the classical action of vacuum after the calculation of quantum corrections means that the program of "natural" inflation fails. Hence, we are not going to introduce any special vacuum terms. Neither the coefficient of the \( \int \sqrt{-g}R^2 \)-term in the classical action of vacuum will not be taken unnaturally large. On the contrary, we assume that this coefficient is essentially smaller than the one in (5) and then, for the sake of simplicity, set it to zero.

The cosmological model is based on the action

\[
S_{\text{total}} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_{\text{matter}} + S_{\text{vac}} + \bar{\Gamma},
\]

where the quantum correction \( \bar{\Gamma} \) is given by (5). It proves useful to introduce the following variables: conformal factor \( a = e^\sigma \), physical time \( t \) (where \( dt = a(\eta)d\eta \) and \( \eta \) is the conformal time) and \( H(t) = \dot{\sigma}(t) \). For the conformally flat case (similar solutions for the FRW metric with \( k = \pm 1 \) are also possible, see \[3\]) \( \bar{g}_{\mu\nu} = \eta_{\mu\nu} \), the equation for \( H(t) \) has the form \[3\]

\[
\ddot{H} + 7\dot{H}H + 4 \left( 3 - \frac{b}{c} \right) \dot{H}H^2 + 4 \dot{H}^2 - 4 \frac{b}{c} H^4 - \frac{1}{8\pi Gc} \left( 2H^2 + \dot{H} \right) = 0,
\]

There are special solutions with \( H = \text{const} \):

\[
H_0 = 0 \quad \text{and} \quad H_{1/2} = \pm \frac{1}{\sqrt{-16\pi Gb}}, \quad a(t) = a_0 \cdot \exp Ht.
\]

Indeed the positive sign in the last expression corresponds to inflation \[3\].

The next step is to study the stability of inflationary solution under fluctuations of the conformal factor \( a(t) \). The analysis of stability can be performed analytically \[3\] or numerically \[3\]. Solution (8) is stable under fluctuations of \( H(t) \) if the particle content of the quantum theory satisfies the condition \( b/c < 0 \) \[3\]. By using Eq. (3), we arrive at the following criterion of stability \[17\]:

\[
N_1 < \frac{1}{3} N_{1/2} + \frac{1}{18} N_0.
\]

\(^2\)The non-local nature of the anomaly contribution has been first noticed in \[15\].
One can imagine that there was some string phase transition at the Planck scale. When all massive string modes decouple, we are left with the ultra-relativistic matter described by field theory. Then, if (9) is satisfied, the inflation starts independent on the initial conditions. The numerical analysis of [6] shows that the stabilization of inflation performs in a fraction of the Planck time and the Universe needs just about 100 Planck times (we define the Planck time as \( t_P = 1/M_P = G^{1/2} \)) to expand into necessary 65 e-folds. The main problem is that the stable inflation is eternal and no simple receipt is known for the graceful exit to the FRW phase. The possible solutions of this problem were discussed in Refs. [6, 17] using the effective field theory approach. In particular, we have supposed that when the typical energy decreases, the masses of matter particles become relevant and the deviation of \( a(t) \) from the exponential behavior might lead to the graceful exit. To some extent, this letter is devoted to the realization of this idea (see also [8]).

Let us also mention Ref. [18]. In this interesting paper the classical action of vacuum is chosen in such a way that the total \( \int \sqrt{-g} R^2 \)-term in (6) is absent. Then the inflation is stable but it can be destabilized by metric fluctuations. Indeed, this kind of solution is theoretically possible, but we will not discuss it here.

The non-stable version of the anomaly-induced inflation [3, 7, 5, 19] has another advantage: it easily breaks and one can achieve the graceful exit to the FRW phase [3, 5]. Furthermore, there can be rapid oscillations of the conformal factor after the breaking, potentially producing the reheating [3]. The effective action (6) includes the massive degree of freedom associated to \( \sigma \). The decay of this mode into matter particles has been discussed in [3, 6, 5, 19]. We remark that this massive mode is not a fundamental field but instead it is a degree of freedom induced by the quantum effects of matter fields. If some matter field decouples (say, because it has a large mass), then it does not contribute to the massive mode of \( \sigma \) anymore and the coefficients \( N_0, N_{1/2}, N_1 \) change correspondingly. At this moment the energy of the induced mode can transfer into the real (not virtual) matter sector through the creation of particles. In the time scale, the last decoupling is the one of the lightest neutrino. After that the particle content in (6) is \( N_1 = 1 \) (one photon) and \( N_0 = N_{1/2} = 0 \). It is easy to see that in this case the inflationary solution is unstable while the solution \( H_0 \) in (5) is stable. Due to this circumstance, in the present day Universe we do not have fast inflation! This simple example shows, by the way, the predictive power of the anomaly-induced inflation. The existence of more than 18 massless scalars or more than 6 Weyl fermions is completely ruled out by the condition (9).

The shortcoming of the non-stable inflation is that without some ”strong measures” it does not last long enough and the Universe does not expand sufficiently. The necessary ”strong measures” have been discussed in [3, 5]. They consist in the extremely exact fine tuning of the initial conditions and in the introduction of the \( \int \sqrt{-g} R^2 \) term into the classical action [3].

Let us summarize. We have two sorts of the anomaly-induced inflation: stable and unstable. The advantage of the stable one is that it does not depend on the initial data. However, it remains unclear how the inflation stops. On the contrary, the non-stable inflation stops immediately and one is forced to use the ”strong measures” in order to achieve the necessary expansion of the Universe. Indeed, both versions do not look perfect if they are considered separately. The best situation...

\[^{3}\text{I am very grateful to A.A. Starobinsky for explaining to me this point.}\]
would be to have an inflation which is stable at the beginning and unstable at the end. If one could
switch from one to another in a natural way, this could be the desired explanation of inflation.

Now we come back to the condition of stability \((\mathbb{1})\), which depends on the particle content.
The known spectrum of particles fits with the Minimal Standard Model (MSM). Let us present a
few details. The SM includes 6 quarks, each of them has 2 chiralities and 3 colors. Hence, quarks
contribute \(N_{\text{quarks}} = 18\). Furthermore, there are 6 leptons. Taking all the neutrino massive, we
arrive at \(N_{\text{leptons}} = 6\). The scalar sector has one Higgs doublet \(N_0 = 4\), while the vector one
consists of 8 gluons, \(W^\pm\), \(Z\) and photon. After all, we have \(N_0 = 4, N_{1/2} = 24, N_1 = 12\). Then,
Eq. \((\mathbb{1})\) indicates to the non-stable inflation and this perfectly fits with our dream to have unstable
inflation at the end. The same concerns, obviously, the present-day \(N_1 = 1, N_0 = N_{1/2} = 0\) case
considered above.

Now, let us remember that the anomaly-induced inflation is supposed to occur at the sub-Planck
energy domain. There are many reasons to expect that the particle spectrum at this scale goes
beyond the SM. In particular, it may happen that the high-energy theory possesses supersymmetry.
Let us, for example, look at the particle content of the Minimal Supersymmetric Standard Model
(MSSM). This model has \(N_1 = 12\) as in the SM. Besides the known fermions, one needs additional
superpartners (gaugino) to all the vectors. The same concerns Higgs particles, which require
higgsino. In total, we have \(N_{1/2} = 32\) for the MSSM. Finally, the scalar sector includes two Higgs
doublets and numerous superpartners of the fermions: squarks and sleptons, such that \(N_0 = 104\).
It is easy to see that this particle content provides stability in \(\mathbb{1}\). In fact, similar result can
be expected for any realistic supersymmetric model. The supersymmetric extension of the gauge
theory implies the replacement of any vector multiplet by the \(N = 1\) vector superfield. Moreover,
one has to add superpartners to the fermions in such a way that they form chiral superfields. Both
operations increase the number of spinors and scalars in \(\mathbb{1}\) and we arrive at the stable inflation.

![Feynman Diagrams](image)

**Figure.** The samples of the one-loop Feynman diagrams which contribute to the anomaly.
The bubble of the matter field has 2, 3 or 4 external lines of the field \(\sigma\).

Thus, the situation when the anomaly-induced inflation is stable at the beginning and unstable
at the end means exactly that supersymmetry breaks at the last stage of inflation. If we suppose
that the typical energy scale decreases during the inflation, this can be associated with the supersymmetry
breaking at low energies. Let us explain the last statement in more detail. The Feynman
diagrams which contribute to the anomaly-induced action \(\mathbb{1}\) consist of a quantum bubble of matter
(non-gravitational) fields with external tails of the \(\sigma\) field (see the Figure). According to the
Appelquist and Carazzone theorem \(\mathbb{20}\), the loop of massive field decouples when the energy of
external lines becomes much smaller than the mass of the quantum field in the loop. One has to notice that if the origin of masses of quantum fields is not the Spontaneous Symmetry Breaking, the decoupling theorem applies nicely. The massive spartic les decouple when the typical energy of the external lines of the field $\sigma$ becomes smaller than the masses of these particles. Therefore, the graceful exit is realized most naturally for the soft SUSY breaking which is, also, the most acceptable from the phenomenological point of view (see, e.g. [21]). The mechanism of decreasing the typical energy of gravitons during inflation will be discussed in the next section.

3. Some numerical estimates

It is important to estimate the duration of inflation until the SUSY breaks. Since the breaking of the unstable inflation occurs in a very short time [3], this is equivalent to the two questions:

i) What is the SUSY breaking scale?

ii) How to evaluate the energy of the $\sigma$-field quanta in the external lines of the diagrams?

The first problem has been widely discussed in the literature (see, e.g. [21]). The most popular is the situation when the supersymmetry breaks softly above the electroweak scale $M_F \approx 300 \text{ GeV}$. There are no upper bounds for SUSY, and its breaking may occur at the GUT scale $M_X \approx 10^{16} \text{ GeV}$ or even at the Planck scale $M_P = G^{-1/2} = 10^{19} \text{ GeV}$, depending on the SUSY model.

To answer the ii) question is not a simple problem. One can try to use the framework of renormalization group in curved space-time [10]. This version of the renormalization group links the scaling of all dimensional quantities with the one of the metric. Then, as any dimensional quantity, the typical energy of the $\sigma$-quanta should vary as $\mu_\sigma \sim 1/a$, while the metric transforms as $g_{\mu\nu} \rightarrow g_{\mu\nu} \cdot a^2$. Unfortunately, this consideration is consistent only for the constant scaling parameter $a$. In the case of Eq. (8) the dynamics of such important dimensional quantity as scalar curvature is different from the above rule, and in fact it is constant $R = -12H^2$. Since $H$ and $R$ are the most important local dimensional parameters associated to $a(t)$, we can identify the energy of the $a$-quanta with $H = \dot{a}/a$. At low energies, when the high derivative terms in (5) become negligible, this also follows from the Einstein equations which we shall apply below.

After identifying the graviton energy with $H$ we meet another problem. The decoupling mechanism described above works if the energy scale is decreasing during inflation, but in the exponential phase (8) the Hubble parameter $H(t) \equiv H_1$ does not decrease. However, this is true only if we do not take the masses of the quantum fields into account. Compared to the massless case, the fourth derivative anomaly-induced terms are still present in the effective action of massive fields. Besides, there are three other contributions. First of all, one meets the renormalization of the Newton constant $G$, and its scale dependence. In a parallel paper [8] we have shown that this effect can be taken into account and results in the following evolution of $H$:

$$H_1 = \frac{M_P}{\sqrt{-16\pi b}} \rightarrow H_1(a) = \left\{ \frac{M_P^2}{-16\pi b} - \frac{1}{3 (4\pi)^2} \sum_i N_i m_i^2 \log\left[ \frac{a(t)}{a_0} \right] \right\}^{1/2}, \quad (10)$$

where $N_i$ and $m_i$ are multiplicity and mass of the fermion of the specie $i$.

Another important effect is the renormalization of the cosmological constant. In this paper we do not consider this effect which will be discussed in details elsewhere. It is sufficient to mention...
that the cosmological constant does not change the graceful exit mechanism. Finally, there are contributions given by the infinite power series in curvature. Unfortunately, there is no available method for calculating these terms in a general form. Some existing approximations may be misleading, in particular because we can not use the weak curvature limit. Let us suppose that in the high energy region the leading effect is the renormalization of $G$ and perform our analysis using Eq. (10).

For simplicity we suppose that the SUSY breaks at GUT scale, and that the gauge group and particle content are such that

$$\sum_i \frac{1}{3(4\pi)^2} N_i m_i^2 = (10 M_X)^2 = 10^{34} GeV^2 = 10^{-4} M_P^2.$$ (11)

Since the numbers $N_0, N_{1/2}, N_1$ are much greater than for the MSSM, the magnitude of the Hubble parameter $H_1$ will be essentially smaller than $H_1^{(MSSM)} \approx M_P$. The exact relation between $H_1$ and $M_P$ requires fixing the particle content of the SUSY GUT. Without going into these details, let us suppose that in this situation $H_1 = M_P/10$. The inflation will last until the $H_1(a)$ becomes comparable to $M_X$. Then (10) and (11) immediately lead to the relation

$$a(H_1 = M_X) \approx a_0 \cdot e^{100}.$$ (12)

It is easy to see that our suppositions about the spectrum of GUT fermions were not in favor of too much expansion, but still we arrived to much more than the minimal necessary 65 $e$-folds. The lower energy scale for the SUSY breaking can increase the number of $e$-folds to enormous extent.

The next important problem is to link the energy of the $a(t)$-quanta with the temperature $T_r$ of the electromagnetic radiation. Consider the lower energy region after the SUSY breaks down and the inflation becomes unstable. Then the higher derivative terms in the effective action are irrelevant and the graviton energy $H$ can be defined from the Einstein equation with the radiation dominating the matter stress tensor. The standard estimate [1] shows that

$$T_r \approx (H \cdot M_P)^2.$$ (13)

Taking $H \sim M_P$ we obtain $T_r \sim M_P$ that corresponds to the unification of all fundamental sources and the limit of applicability of the semiclassical approximation. Taking (as in our estimate leading to (12)) the value $H = M_X$, we meet the temperature satisfying $M_X < T_r < M_P$. In order to illustrate the gravitational suppression of $T_r$, let us consider the energy when the lightest neutrino decouples from gravitons $H = m_\nu \approx 10^{-3} eV$. According to (13) the corresponding temperature is $T_r \geq 10^3 GeV$. This temperature is much higher than the QCD and EW scales, such that all vectors of the MSM should be considered massless. According to (8), the instability of inflation is guaranteed. An important consequence is that the nucleosynthesis can not be jeopardized by the anomaly-induced inflation.

Another aspect of the energy scale problem is the following. The change of the regime of expansion due to the decoupling of massive sparticles should provoke yet another quantum effects

\[ ^4 \text{This is nice, because our semiclassical approximation (no quantum gravity, string etc) becomes really consistent. At lower energies } H_1 \text{ in } (8) \text{ could be greater, but this does not matter at all, because after the SUSY breaking the exponential solution gets unstable and inflation does not occur.} \]
like particle creation. The new particles absorb the energy of the decaying massive mode of the $\sigma$ field, as it was discussed in [3, 5]. Also, during the inflation (8), the real matter filling the Universe is out of the equilibrium state and does not lose energy. Therefore, the content of the Universe after the end of inflation are both hot matter which came from the string epoch and did not completely cool down, and the hot matter created during the transitional period from the stable to the unstable inflation. After this transition is over, the Universe entered into the FRW phase and all processes proceed in a standard way.

After the transition into an unstable phase, there can be, in general, very different versions of further behaviour for $a(t)$ [2, 22, 6]. The analysis of the phase structure [3] shows that $a(t)$ has various attractors and only some of them can be associated to the graceful exit to the FRW solution. The evolution of $a(t)$ phase can be studied numerically. The result depends on the initial data $a(0), \dot{a}(0), \ddot{a}(0), \dddot{a}(0)$, which are not well defined since we do not have the detailed description of the decoupling. In this situation it is better to consider different versions of these initial data. For example, if we take the purely massless case (6) and substitute the initial data $a(k)(0) = H^k_1$ with $H_1$ of the MSSM into the equation with the MSM parameters $b$ and $c$, the system goes to the undesirable "hyperinflation" [3]. However, if we take the masses into account, the situation changes drastically. From the numerical analysis of [8] follows that at the end of the stable period the evolution goes in a power-like manner $a(t) \sim t^{1/n}$. Then the initial data for the second MSM phase must be taken such that each derivative must be much smaller than the previous one. The typical time of the stable phase is $100 t_P$. If we take, using the Planck time units [3]

$$a(0) = 1, \quad \dot{a}(0) = 10^{-2}, \quad \ddot{a}(0) = 10^{-4}, \quad \dddot{a}(0) = 10^{-6},$$

then the asymptotic behaviour of $a(t)$ is quite similar to the desirable $a(t) = t^{1/2}$. The possible small changes in (14) do not modify the qualitative behaviour of $a(t)$. Of course, these results must be accepted with the proper caution. Let us remind that by the end of the decoupling the approximation (10) is not safe and one has to take into account other contributions to the effective action. This can lead to the more precise definition of the initial data $a(i)(0)$ for the unstable inflation. The subsequent progress in the anomaly-induced inflation requires quantitative description of the decoupling process (unfortunately, this is a difficult problem similar to the one people meet in QCD) and of the transition to the FRW phase. After that the metric and density perturbation analysis (see, e.g., [3, 23, 17, 19]) should provide useful information for the phenomenological investigations.

4. Discussions

It is interesting to discuss the general status of the anomaly-induced inflation. Theoretically, the phenomena of anomalous breaking of the local conformal symmetry is well established (see, e.g [11]). The same concerns the existence of the inflationary solution [8]. Of course, there is no guarantee that this kind of inflation really took place in the Early Universe. However, there is only one way to avoid the anomaly-induced inflation: to introduce the classical action of vacuum in such a way that the inflation becomes unstable at all scales. As an example we mention the $\sqrt{-g} R^2$-term [3] which could provide inequality $b/c > 0$ independent on the particle content $N_0, N_{1/2}, N_1$. Then, the inflation must have another origin. But, in this paper we have shown that the absence of extra
vacuum terms may produce the natural inflation and the natural graceful exit due to the SUSY breaking. At this level, there are no contradictions neither with the present-day Universe nor with the nucleosynthesis mechanism.

Further investigations of the density and metric perturbations are necessary. They can bring us to the point when experiments and observations can show whether the anomaly induced inflation really took place.

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