CP VIOLATION IN B DECAYS

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The role of $B$ decays in the study of CP violation is reviewed. We treat the interactions and spectroscopy of the $b$ quark and then introduce CP violation in $B$ meson decays, including time-dependences, decays to CP eigenstates and non-eigenstates, and flavor tagging. Additional topics include studies of strange $B$'s, decays to pairs of light pseudoscalar mesons, and the roles of gluonic and electroweak penguin diagrams, and final-state interactions.

1 Introduction

Discrete symmetries such as time reversal (T), charge conjugation (C), and space inversion or parity (P) have provided both clues and puzzles in our understanding of the fundamental interactions. The realization that the charge-changing weak interactions violated P and C maximally was central to their formulation in the $V-A$ theory. The theory was constructed in 1957 to conserve the product CP, but within seven years the discovery of decay of the long-lived neutral kaon to two pions showed that even CP was not conserved. Nearly twenty years later, Kobayashi and Maskawa (KM) proposed that CP violation in the neutral kaon system could be explained in a model with three families of quarks, at a time (1973) when no evidence for the third family and not even all evidence for the second had been found. The quarks of the third family, now denoted by $b$ for bottom and $t$ for top, were subsequently discovered in 1977 and 1994, respectively.

Decays of hadrons containing $b$ quarks now appear to be particularly fruitful ground for testing the KM hypothesis and for displaying evidence for any new physics beyond this “standard model” of CP violation. A meson containing a $b$ quark will be known generically as a $B$ meson, in the same way as a $K$ meson contains an (anti-) strange quark $s$. The present lectures are devoted to some tests of CP violation utilizing $B$ meson decays. (Baryons containing $b$ quarks also may display CP violation but we will not discuss them here.)

We first deal with the spectroscopy and interactions of the $b$ quark. In Section 2 we describe the discovery of the charmed quark, the tau lepton, the $b$ quark, and $B$ mesons. Section 3 is devoted to the spectroscopy of hadrons containing the $b$ quark, while Section 4 treats its weak interactions. Neutral mesons containing the $b$ quark can mix with their antiparticles (Section 5),
providing important information on the weak interactions of \( b \) quarks.

We then introduce CP violation in \( B \) meson decays. After general remarks and a discussion of decays to CP eigenstates (Section 6) we turn to decays to CP-noneigenstates (Section 7) and describe various methods of tagging the flavor of an initially-produced \( B \) meson (Section 8). Some specialized topics include strange \( B \)'s (Section 9), decays to pairs of light mesons (Section 10), and the roles of penguin diagrams (Section 11), and final-state interactions (Section 12).

Topics not covered in detail in the lectures but worthy of mention in this review are noted briefly in Section 13. The possibility that the Standard Model of CP violation might fail at some future time to describe all the observed phenomena is discussed in Section 14, while Section 15 concludes. Other contemporary reviews of the subject\(^5\)\(^6\) may be consulted.

2 Discovery of the \( b \) quark

2.1 Prelude: The charmed quark

During the 1960’s and 1970’s, when the electromagnetic and weak interactions were being unified by Glashow, Weinberg, and Salam\(^7\) it was realized\(^8\) that a consistent theory of hadrons required a parallel\(^9\) between the then-known two pairs of weak isodoublets of leptons, \((\nu_e, e^-), (\nu_\mu, \mu^-)\), and a corresponding multiplet structure for quarks, \((u, d), (c, s)\). The known quarks at that time consisted of one with charge 2/3, the up quark \( u \), and two with charge \(-1/3\), the down quark \( d \) and the strange quark \( s \). The charmed quark \( c \) was a second quark with charge 2/3 and a proposed mass of about 1.5 to 2 GeV/\( c^2 \).\(^10\)\(^11\)

The parallel between leptons and quarks was further motivated by the cancellation of anomalies\(^8\)\(^12\) in the electroweak theory. These are associated with triangle graphs involving fermion loops and three electroweak currents. It is sufficient to consider the anomaly for the product \( I_{3L} Q^2 \), where \( I_{3L} \) is the third component of left-handed isospin and \( Q \) is the electric charge. The sum \( \sum_i (I_{3L})_i Q^2 \) over all fermions \( i \) must vanish. If a family of quarks and leptons consists of one weak isodoublet of quarks and one of leptons, this cancellation can be implemented within a family, as illustrated in Table 1.

The first hints of charm arose in nuclear emulsions\(^13\) and were recognized as such by Kobayashi and Maskawa.\(^2\) However, more definitive evidence appeared in November, 1974, in the form of the \( ^3S_1 \) \( c\bar{c} \) ground state discovered simultaneously on the East\(^14\) and West\(^15\) Coasts of the U. S. and named, respectively, \( J \) and \( \psi \). The East Coast experiment utilized the reaction \( p + Be \rightarrow e^+e^- + \ldots \) and observed the \( J \) as a peak at 3.1 GeV/\( c^2 \) in the effective \( e^+e^- \) mass. The West Coast experiment studied \( e^+e^- \) collisions in the SPEAR storage ring and
Table 1: Anomaly cancellation in the electroweak theory.

| Family             | 1   | 2   | 3   | Contribution per family |
|--------------------|-----|-----|-----|-------------------------|
| Neutrino           | νe  | νμ  | ντ  | (1/2)(0)² = 0           |
| Charged lepton     | e⁻  | μ⁻  | τ⁻  | (−1/2)(−1)² = −1/2      |
| Q = 2/3 quark      | u   | c   | t   | 3(1/2)(2/3)² = 2/3      |
| Q = −1/3 quark     | d   | s   | b   | 3(−1/2)(−1/3)² = −1/6   |

saw a peak in the cross section for production of $e^+e^-, \mu^+\mu^-$, and hadrons at a center-of-mass energy of 3.1 GeV. Since the discovery of the $J/\psi$ the charmonium level structure has blossomed into a richer set of levels than has been observed for the original “onium” system, the $e^+e^-$ positronium bound states.

The lowest charmonium levels are narrow because they are kinematically unable to decay to pairs of charmed mesons (each containing a single charmed quark). The threshold for this decay is at a mass of about 3.73 GeV/c². Above this mass, the charmonium levels gradually become broader. The charmed mesons, discovered in 1976 and subsequently, include $D^+ = c\bar{d}$ (mass 1.869 GeV/c²), $D^0 = c\bar{u}$ (mass 1.865 GeV/c²), and $D_s = c\bar{s}$ (mass 1.969 GeV/c²). These mesons were initially hard to find because the large variety of their possible decays made any one mode elusive. For example, the two-body decay $D^0 \to K^-\pi^+$ has a branching ratio of only about 3.8%; higher-multiplicity decays are somewhat favored.

2.2 Prelude: The $\tau$ lepton

About the same time as the discovery of charm, another signal was showing up in $e^+e^-$ collisions at SPEAR, corresponding to the production of a pair of new leptons: $e^+e^- \to \gamma^* \to \tau^+\tau^-$. The $\tau$ signal had a number of features opposite to those of charm: lower- rather than higher-multiplicity decays and fewer rather than more kaons in its decay products, for example, so separating the two contributions took some time.

The mass of the $\tau$ is 1.777 GeV/c². Its favored decay products are a tau neutrino, $\nu_\tau$, and whatever the charged weak current can produce, including $e\bar{\nu}_e, \mu\bar{\nu}_\mu, \pi, \rho$, etc. It thus contributes somewhat less than one unit to

$$R \equiv \sum_i Q_i^2 = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

which would have risen from the value of 2 for $u, d, s$ quarks below charm threshold to 10/3 above charm threshold if charm alone were being produced, but was seen to rise considerably higher.
One problem with accepting the \( \tau \) as a companion of the charmed quark was that the neat anomaly cancellation provided by the charmed quark, mentioned above, was immediately upset. The anomaly contributed by the \( \tau \) lepton would have to be cancelled by further particles, such as a pair of new quarks \((t, b)\) with charge \(2/3\) and \(-1/3\). Such quarks had indeed already been utilized two years before the \( \tau \) was established, in 1973 by Kobayashi and Maskawa in their theory of CP violation. The names “top” and “bottom” were coined by Harari in 1975 in analogy with “up” and “down.”

2.3 Dilepton spectroscopy

One reason for the experiment which discovered the \( J \) particle was an earlier study, also at Brookhaven National Laboratory, by L. Lederman and his collaborators, of \( \mu^+\mu^- \) pairs produced in proton-uranium collisions. The \( m(\mu^+\mu^-) \) spectrum in this experiment displayed a shoulder around 3.5 GeV/\( c^2 \). It was not recognized as a resonant peak and was displaced in mass from the true \( J/\psi \) value because of the poor mass resolution of the experiment.

After the discovery of the \( J/\psi \), Lederman’s group continued to pursue dilepton spectroscopy. In 1977 a search with greater sensitivity and better mass resolution turned up evidence for peaks at 9.4, 10.0, and possibly 10.35 GeV/\( c^2 \). These were candidates for the 1S, 2S, and 3S levels of a new \( Q\bar{Q} \) system. Several pieces of evidence identified the heavy quark \( Q \) as a \( b \) quark.

1. The \( \Upsilon(1S) \) and \( \Upsilon'(2S) \) were produced in 1978 by the electron-positron collider DORIS at DESY and their partial widths to \( e^+e^- \) pairs were measured. It was shown that if the \( QQ \) system was bound by the same quantum chromodynamic force as as the \( cc \) (charmonium) system, one could use the \( cc \) states to gain some idea about the details of the \( QQ \) binding. Since \( \Gamma(Q\bar{Q}) \propto e_Q^2 \), where \( e_Q \) is the charge of the quark \( Q \), it was possible to conclude from the data that \( |e_Q| = 1/3 \) was favored over \( |e_Q| = 2/3 \).

2. The Cornell \( e^+e^- \) ring CESR began operating in 1979 reaching a fourth \( \Upsilon(4S) \) peak and finding it broader than the first three. This indicated that the meson pair threshold lay below \( M(\Upsilon(4S)) = 10.58 \text{ GeV}/c^2 \). Farther above this threshold, wiggles in the total cross section for hadron production averaged out to indicate a step in \( R \) of 1/3, confirming that \( |e_Q| = 1/3 \).

3. The possibility that \( Q \) was an isosinglet quark of charge \(-1/3\), and thus not the partner of some quark \( t \) with charge \(2/3\), was ruled out by the absence of significant flavor-changing neutral current decays such as \( b \to s\mu^+\mu^- \).

The structure of the \( \Upsilon \) levels is remarkably similar to that of the charmonium levels except for having more levels below flavor threshold. For example, the fact that the 3S level is below flavor threshold allows it to decay to the
2P levels via electric dipole transitions with appreciable branching ratios; the transitions between the S and P levels are well described in potential models which reproduce other aspects of the spectra. Several reviews treat the fascinating regularities of the spectroscopy of these levels.

2.4 Discovery of B mesons

The lightest meson containing a b quark and each flavor of light antiquark is expected to decay weakly. The allowed decays of b are (c or u) + (virtual $W^-$), with the c giving rise to lots of strange particles while the u gives few strange particles. The virtual $W^-$ can decay to $\bar{u}d$, $\bar{e}s$, $e^-\bar{\nu}_e$, $\mu^-\bar{\nu}_\mu$, and $\tau^-\bar{\nu}_\tau$.

In $e^+e^-$ collisions above $BB$ threshold, several signals of $B$ meson production were observed by the CLEO Collaboration starting around 1980:

- Prompt leptons (signals of semileptonic decay)
- An abundance of kaons (a signal that $b \to c + W^-_{\text{virt}}$ is preferred over $b \to u + W^-_{\text{virt}}$)
- “Daughter” (lower-momentum) leptons from $c$ semileptonic decays.

These indirect signals were followed by reconstruction of $B^+$ and $B^0$ decays, e.g.,

$$B^+ = \bar{b}u \to \bar{c}ud \to \bar{D}^0\pi^+ \quad \text{and} \quad B^0 = \bar{b}d \to \bar{c}udd = D^-\pi^+ .$$

Typical branching ratios for these final states are $(5.3 \pm 0.5) \times 10^{-3}$ for $D^0\pi^+$ and $(3.0 \pm 0.4) \times 10^{-3}$ for $D^-\pi^+$. $B^0 \to D^0\pi^0$ is also allowed but not yet observed. These small branching ratios mean that reconstruction of exclusive final states is even harder for $B$ mesons than for charmed particles.

3 The known B hadrons

3.1 B mesons

The nonstrange ground-state $B$ (pseudoscalar) and $B^*$ (vector) mesons are compared with the corresponding charmed mesons in Table 2. Evidence for the $B^*$ exists in the form of a photon signal for the decay $B^*_s \to B_s\gamma$. The photon energy, 46 MeV, is expected to be the same as that seen in $B^{*0}\to B^{0}\gamma$. Since the $B^*$ and $B$ states are separated by only 46 MeV, a $B^*$ should always decay to a $B$ of the same flavor and a photon. This is in contrast to the case of the $D^*$ and $D$ states, whose separation is just about a pion mass. The electromagnetic mass splittings are such that $D^{*+} \to D^0\pi^+$, $D^{*0} \to$
Table 2: Ground-state heavy-light ($Q\bar{q}$) pseudoscalar mesons and the corresponding vector mesons. Here the spectroscopic notation $2L+1L_J$ is used to denote the spin, orbital, and total angular momenta of the $Q\bar{q}$ state.

| Quark content | Pseudoscalar ($^1S_0$) meson— | Vector ($^3S_1$) meson— |
|---------------|--------------------------------|-------------------------|
|               | Name  | Mass (GeV/$c^2$) | Name  | Mass (GeV/$c^2$) |
| $c\bar{u}$    | $D^0$ | 1864.5 ± 0.5   | $D^{*+}$ | 2066.7 ± 0.5 |
| $c\bar{d}$    | $D^+$ | 1869.3 ± 0.5   | $B^{*+}$ | 2010.0 ± 0.5 |
| $c\bar{s}$    | $D_s$ | 1968.6 ± 0.6   | $D_s^*$ | 2112.4 ± 0.7 |
| $b\bar{u}$    | $B^+$ | 5279.0 ± 0.5   | $B^{*+}$ | 5325.0 ± 0.6 |
| $b\bar{d}$    | $B^0$ | 5279.4 ± 0.5   | $B^{*0}$ | 5325.0 ± 0.6 |
| $b\bar{s}$    | $B_s$ | 5369.6 ± 2.4   | $B_s^*$ | ≃ 5416 |

$D^+\pi^0$, and $D^{*0} \to D^0\pi^0$ are just barely allowed, while $D^{*0} \to D^+\pi^-$ is forbidden. The low-momentum $\pi^+$ in $D^{*+} \to D^0\pi^+$ acts as a “tag,” useful both for signalling the production of a charmed meson and, by its charge, distinguishing the $D^0$ from a $\bar{D}^0$. Since $B^*$ decays are not useful for this type of “flavor” tag, one must resort to the decays of heavier excited $b\bar{q}$ states (Section 8).

The hyperfine splitting of $B$ mesons is smaller than that in charmed mesons because the chromomagnetic moments of the heavy quarks scale as the inverse of their masses:

$$\frac{m_{B^*} - m_B}{m_{D^*} - m_D} \approx \frac{1}{m_c} \frac{1}{m_b} \approx \frac{1}{3}.$$  \(3\)

### 3.2 The $\Lambda_b$ baryon

The lightest baryon containing a $b$ quark is the $\Lambda_b = b[ud]_{I=0}$. Its mass is $5624 \pm 9$ MeV/$c^2$. The $ud$ system must be in a color $3^*$ (antisymmetric) state, since the $b$ is a color triplet and the $\Lambda_b$ is a color singlet. The spin-zero state of $ud$ is favored over the spin-one state by the chromodynamic hyperfine interaction. By Fermi statistics, the $ud$ pair must then be in an (antisymmetric) isospin-zero state. For similar reasons, the $I = 0$ state of a strange quark and two nonstrange quarks, the $\Lambda = s[ud]_{I=0}$ with mass 1116 MeV/$c^2$, is lighter than the $\Sigma = s(uu, ud, dd)_{I=1}$ states with average mass 1193 MeV/$c^2$.

The charmed analogue of the $\Lambda_b$ is the $\Lambda_c = c[ud]_{I=0}$ with mass $2284.9 \pm 0.6$ MeV/$c^2$. The difference in mass of the two particles is $M(\Lambda_b) - M(\Lambda_c) = 3339 \pm 9$ MeV/$c^2$. This provides an estimate of $m_b - m_c$ since there are no hyperfine terms involving the heavy quark; the light-quark system has zero spin in both baryons. There will be a correction of order $m_c^{-1} - m_b^{-1}$ due to
possible differences in kinetic energies.

One can perform a similar estimate for $Q\bar{q}$ mesons by eliminating the hyperfine energy, performing a suitable average over vector ($^3S_1$) and pseudoscalar ($^1S_0$) meson masses. The expectation value of the relevant interaction term is $\langle \sigma_Q \cdot \sigma_{\bar{q}} \rangle = (1, -3)$ for ($^3S_1, ^1S_0$) states. Thus the hyperfine energy is absent in the combination $[3M(^3S_1) + M(^1S_0)]/4$. Since $(3M_{D^*} + M_D)/4 = 1973$ MeV/$c^2$ and $(3M_{B^*} + M_B)/4 = 5313$ MeV/$c^2$ (taking isospin averages), we estimate from the mesons that $m_b - m_c = 3340$ MeV/$c^2$, identical to the estimate from the baryons. (Again, kinetic energies could provide a correction to this result.) The spectroscopic assignment of the $\Lambda_b$ is thus likely to be the correct one.

Known decay modes of the $\Lambda_b$ include $\Lambda_b^+ \ell^- \bar{\nu}_\ell$ and $J/\psi \Lambda$. The fact that the corresponding quark subprocesses $b \to c\ell^- \bar{\nu}_\ell$ and $b \to c\bar{s}s$ conserve isospin leads to the requirement that the final states have zero isospin in both cases, restricting the number of additional pions that can be produced.

4 Interactions of the $b$ quark

In this Section we shall discuss the way in which the interactions of the $b$ quark provide information on the pattern of charge-changing weak interactions of quarks parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{ub}$. More details on determination of the CKM matrix are included in the lectures by Buchalla, DeGrand, Falk, Neubert, and Wolfenstein.

4.1 The $b$ lifetime: indication of small $|V_{cb}|$

The long $b$ quark lifetime ($>1$ ps) indicated that the CKM element $V_{cb}$ was considerably smaller than $|V_{us}| \approx |V_{cd}| \approx 0.22$. One can estimate $V_{cb}$ using a free-quark method.

The subprocess $b \to cW^{*-} \to c\ell^- \bar{\nu}_\ell$ has a rate

$$\Gamma(b \to c\ell^- \bar{\nu}_\ell) = \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2 f(m_b, m_c, m_\ell),$$

where $G_F = 1.16637(2) \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant. In the limit in which the lepton mass can be neglected, $f(m_b, m_c, m_\ell) = f(m_b^2/m_b^2)$, with $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$. The uncertainty in the prediction for $\Gamma(b \to c\ell^- \bar{\nu}_\ell)$ due to that in $m_b$ is mitigated by the constraint noted above on $m_b - m_c \approx 3.34$ GeV/$c^2$.

Taking a nominal range of quark masses around $m_b = 4.7$ GeV/$c^2$ (and hence a range around $m_c = 1.36$ GeV/$c^2$, $f(m_c^2/m_b^2) = 0.54$), $\tau_b = 1.6 \times 10^{-6}$...
and the branching ratio \( B(b \to c\ell\nu) \simeq 10.2\% \), one finds

\[
|V_{cb}| \simeq 0.0384 - 0.0008 \left( \frac{m_b - 4.7 \text{ GeV}/c^2}{0.1 \text{ GeV}/c^2} \right).
\]

Thus if \( m_b \) is uncertain by 0.3 GeV/\( c^2 \) (my guess), \( |V_{cb}| \) is uncertain by \( \pm 0.0024 \). Recent averages\textsuperscript{16,35} give rise to values of \( |V_{cb}| \) somewhat above 0.040 with errors of \( \pm 0.002 \) to \( \pm 0.003 \).

A new report by the CLEO Collaboration\textsuperscript{38} finds \( |V_{cb}| = 0.0462 \pm 0.0036 \) based on the exclusive decay process \( B^0 \to D^{*+}\ell^-\nu_\ell \). This new determination bears watching as it would affect many conclusions regarding predictions for CP-violating asymmetries in \( B \) decays. We shall take \( |V_{cb}| = 0.041 \pm 0.003 \) as representing a conservative range of present values.

\subsection*{4.2 Charmless \( b \) decays: indication of smaller \( |V_{ub}| \)}

Although the \( u \) quark is lighter than the \( c \) quark, its production in \( b \) decays is disfavored, with \( \Gamma(b \to u\ell\nu)/\Gamma(b \to c\ell\nu) \) only about 2\%. Since the phase-space factor \( f(m_u^2/m_b^2) \) is very close to 1, while \( f(m_c^2/m_b^2) \simeq 1/2 \), this means that \( |V_{ub}/V_{cb}|^2 \simeq 1\% \), or \( |V_{ub}/V_{cb}| \simeq 0.1 \). The error on this quantity is dominated by theoretical uncertainty\textsuperscript{35}; detailed studies\textsuperscript{39} indicate \( |V_{ub}/V_{cb}| = 0.090 \pm 0.025 \).

\subsection*{4.3 Pattern of charge-changing weak quark transitions}

The relative strengths of charge-changing weak quark transitions are illustrated in Fig. 1. Why the pattern looks like this is a mystery, one of the questions (along with the values of the quark masses) to be answered at a deeper level.

The interactions in Fig. 1 may be parametrized by a Cabibbo-Kobayashi-Maskawa (CKM) matrix of the form\textsuperscript{40}

\[
V_{\text{CKM}} = \begin{bmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \\
\end{bmatrix}.
\]

The columns refer to \( d, s, b \) and the rows to \( u, c, t \). The parameter \( \lambda = 0.22 \) represents \( \sin \theta_c \), where \( \theta_c \) is the Gell-Mann–Lévy–Cabibbo angle. The value \( |V_{cb}| = 0.041 \pm 0.003 \) indicates \( A = 0.85 \pm 0.06 \), while \( |V_{ub}/V_{cb}| = 0.090 \pm 0.025 \) implies \((\rho^2 + \eta^2)^{1/2} = 0.41 \pm 0.11 \).

Further information may be obtained by assuming that box diagrams involving internal quarks \( u, c, t \) with charge 2/3 are responsible for both the CP-violating contribution to \( K^0-\bar{K}^0 \) mixing and to mixing between neutral \( B \)
mesons and their antiparticles. The parameter $|\epsilon_K| = (2.27 \pm 0.02) \times 10^{-3}$ (see Buchalla’s lectures) then implies a constraint
\begin{equation}
\eta(1 - \rho + 0.39) = 0.35 \pm 0.12 ,
\end{equation}
where the $1 - \rho$ term in parentheses arises from box diagrams with two internal top quarks, while the correction 0.39 is due to diagrams with one charmed and one top quark. The error on the right-hand side is due primarily to uncertainty in the Wolfenstein parameter $A = |V_{cb}|/\lambda^2$, which enters to the fourth power in the $t\bar{t}$ contribution to $\epsilon_K$. A lesser source of error is uncertainty in the parameter $B_K$ describing the quark box diagram’s matrix element between a $K^0$ and a $\bar{K}^0$. We have chosen $B_K = 0.87 \pm 0.13$.

Present information on $B^0 - \bar{B}^0$ mixing, interpreted in terms of box diagrams with two quarks of charge 2/3, leads to a constraint on $|V_{td}|^2$ which implies $|1 - \rho - i\eta| = 0.87 \pm 0.21$ for the parameter range $f_B \sqrt{B_B} = 230 \pm 40$ MeV describing the matrix element of the short-distance 4-quark operator taking $\bar{q}d$ into $\bar{t}b$ between $B^0$ and $\bar{B}^0$ states. The best lower limit on $B_s^0 - \bar{B}_s^0$ mixing is $\Delta M_s > 15$ ps$^{-1}$, when compared with the corresponding value for
Figure 2: Region of $(\rho, \eta)$ specified by $\pm 1\sigma$ constraints on CKM matrix parameters. Solid semicircles denote limits based on $|V_{ub}/V_{cb}| = 0.090 \pm 0.025$; dashed arcs denote limits $|1 - \rho - i\eta|$ based on $B^0\bar{B}^0$ mixing; dot-dashed arc denotes limit $|1 - \rho - i\eta| < 1.01$ based on $B_s\bar{B}_s$ mixing; dotted lines denote limits $\eta(1 - \rho + 0.39) = 0.35 \pm 0.12$ based on CP-violating $K^0\bar{K}^0$ mixing. Rays: $\pm 1\sigma$ limits on $\sin 2\beta$ (see Sec. 6.4). The plotted point at $(\rho, \eta) \approx (0.20, 0.26)$ lies roughly in the middle of the allowed region.

$B^0\bar{B}^0$ mixing, $\Delta m_d = 0.487 \pm 0.014 \text{ ps}^{-1}$, leads to the bound

\[
\frac{f^2_{B_s} B_{B_s}}{f^2_B B_B} |V_{ts}/V_{td}|^2 > 29 .
\]  

This may be combined with the estimate $f_{B_s}\sqrt{B_{B_s}} \leq 1.25 f_B \sqrt{B_B}$ based on quark models. (Lattice gauge theories\cite{43} estimate this coefficient more precisely, generally giving values between 1.1 and 1.2.) One finds $|V_{ts}/V_{td}| \geq 4.4$ or $|1 - \rho - i\eta| < 1.01$. The constraints may be combined to yield the allowed range in $(\rho, \eta)$ space illustrated in Fig. 2. Smaller regions are quoted in other reviews\cite{46}, which view the theoretical sources of error differently.

4.4 Unitarity triangle

The unitarity of the CKM matrix implies that to the order we are considering, $V_{ub}^* + V_{td} = A\lambda^3$. If this equation is divided by $A\lambda^3$, one obtains a triangle in the $(\rho, \eta)$ complex plane whose vertices are at $(0, 0)$ (internal angle $\gamma$), $(\rho, \eta)$ (internal angle $\alpha$), and $(1, 0)$ (internal angle $\beta$).
CP-violating asymmetries in certain $B$ decays can measure such quantities as $\sin 2\alpha$ and $\sin 2\beta$. The former, measured in $B^0 \rightarrow \pi^+ \pi^-$ with some corrections due to "penguin" diagrams, may occupy a wide range, as illustrated in Fig. 2. The latter, measured in the “golden mode” $B^0 \rightarrow J/\psi K_S$ with few uncertainties, is more constrained by other observables.

The goal of measurements of CP violation and other quantities in $B$ decays will be to test the consistency of this picture and to either restrict the parameter space further, thus providing a reliable target for future theories of these parameters, or to expose inconsistencies that will point to new physics. Hence part of the program will be to overconstrain the unitarity triangle, measuring both sides and angles in several different types of processes. While we discuss such measurements based on $B$ mesons, Buchalla describes how, for example, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ constrains the combination $|1 - \rho - i\eta + 0.44|$, where the last term is a charmed quark correction to the dominant top quark contribution, and the purely CP-violating process $K_L \rightarrow \pi^0 \nu \bar{\nu}$ constrains $\eta$.

5 Mixing of neutral $B$ mesons

5.1 Mass matrix formalism

We shall work in a two-component basis utilizing the states $(B^0, \bar{B}^0)$. It is also sometimes useful to consider a basis $B_\pm = (B^0 \pm \bar{B}^0)/\sqrt{2}$.

The time-dependence of these states is described via a mass matrix $M = M - i\Gamma/2$, where $M$ and $\Gamma$ are Hermitian by definition:

$$i \frac{\partial}{\partial t} \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix} = M \begin{bmatrix} B^0 \\ \bar{B}^0 \end{bmatrix}.$$  (9)

The requirement of CPT invariance, which we shall assume henceforth, implies $M_{11} = M_{22}$, or equal transition amplitudes for $K^0 \rightarrow K^0$ and $\bar{K}^0 \rightarrow \bar{K}^0$.

Exercise: (a) Show this. Remember time reversal is an antiunitary operator. (b) Show that a similar argument applied to $M_{12}$ or $M_{21}$ leads to no constraint. (c) Relate the result in (a) to the result quoted by Wolfenstein for the $(B^0 \pm \bar{B}^0)/\sqrt{2}$ basis.

[Answer to Part (a): Insert the unit operator $(CPT)^{-1}CPT$ before and after $M$ in $M_{11} = \langle B^0 | M | B^0 \rangle$. Note that $CPT(M - i\Gamma/2)(CPT)^{-1} = M + i\Gamma/2$. Then

$$M_{11} = \langle CPTB^0 | M + \frac{i\Gamma}{2} | CPTB^0 \rangle^* = \langle \bar{B}^0 | M + \frac{i\Gamma}{2} | B^0 \rangle^* = \langle \bar{B}^0 | M - \frac{i\Gamma}{2} | B^0 \rangle = M_{22},$$  (10)
where the antiunitarity of $T$ has been used in the first step. For a discussion of antiunitary operators see, e.g., Sakurai’s book on quantum mechanics.\[47\]

The eigenstates of $\mathcal{M}$ may be denoted by $B_H$ ("heavy") and $B_L$ ("light"):\[11\]

$$|B_{H,L}\rangle = p_{H,L}|B^0\rangle + q_{H,L}|\bar{B}^0\rangle .$$

The corresponding eigenvalues $\mu_{H,L} = m_{H,L} - \frac{i}{2} \Gamma_{H,L}$ satisfy\[12\]

$$\mathcal{M} \begin{bmatrix} p_i \\ q_i \end{bmatrix} = \mu_i \begin{bmatrix} p_i \\ q_i \end{bmatrix} \quad (i = H, L) ,$$

specifically,\[13\]

$$\mathcal{M}_{11} + \mathcal{M}_{12} \frac{q_i}{p_i} = \mathcal{M}_{21} \frac{p_i}{q_i} + \mathcal{M}_{11} = \mu_i ,$$

so that $\mathcal{M}_{12}(q_i/p_i) = \mathcal{M}_{21}(p_i/q_i)$, or $(p_i/q_i)^2 = \mathcal{M}_{12}/\mathcal{M}_{21}$.

For $B^0$-$\bar{B}^0$ mixing, in contrast to the situation for neutral kaons, the scarcity of intermediate states accessible to both $B^0$ and $\bar{B}^0$ and the presence of a large top-quark contribution to $M_{12}$ means that $|\Gamma_{12}| \ll |M_{12}|$, so that\[14\]

$$\frac{p_i}{q_i} = \pm \sqrt{\frac{M_{12}}{M_{21}}} .$$

We may choose $p_L = p_H = p$, $q_L = -q_H = q$. Normalizing $|p|^2 + |q|^2 = 1$, we then write\[15\]

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle , \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle .$$

The sign ambiguity in [14] may be resolved as follows. Since\[16\]

$$\mu_L = \mathcal{M}_{11} + \mathcal{M}_{12} \frac{q}{p} = \mathcal{M}_{11} + \mathcal{M}_{21} \frac{p}{q} ,$$

$$\mu_H = \mathcal{M}_{11} - \mathcal{M}_{12} \frac{q}{p} = \mathcal{M}_{11} - \mathcal{M}_{21} \frac{p}{q} ,$$

then $\mu_H - \mu_L = -2\mathcal{M}_{12}(q/p)$, which in the limit $|\Gamma_{12}| \ll |M_{12}|$ is $\mu_H - \mu_L = (\pm)\sqrt{M_{12}}$ for the choice of $(+, -)$ in [14]. Since $M_{21} = M_{12}^*$ ($M$ is Hermitian), we must take the $-$ sign in [14] in order that the "heavy" mass $m_H$ be greater than the "light" mass $m_L$. Then\[17\]

$$\frac{p_i}{q_i} = -\sqrt{\frac{M_{12}}{M_{21}}} .$$
Neglecting $\Gamma_{12}$ in comparison with $|M_{12}|$, we then find

$$\Delta m \equiv m_H - m_L = 2|M_{12}|, \quad \Delta \Gamma \equiv \Gamma_H - \Gamma_L \simeq 0.$$  \hspace{1cm} (19)

If one keeps $\Gamma_{12}$ to lowest order, one can show that

$$\frac{q}{p} = -\frac{M^*_{12}}{|M_{12}|} \left( 1 - \frac{1}{2} \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right).$$  \hspace{1cm} (20)

In the limit that $\Gamma_{12}$ is negligible and $\Delta \Gamma = 0$, $q/p$ is a pure phase, determined by the phase of $M_{12}$.

Now, $M_{12}$ takes $B^0 = b\bar{d}$ into $B^0 = d\bar{b}$, so its phase is that of $(V_{tb}V_{td}^*)^2$, or $e^{2i\beta}$. Thus in this limit we find $q/p \simeq e^{-2i\beta}$. More specifically, in the phase convention in which $(CP)|B^0\rangle = +|\bar{B}^0\rangle$, we find

$$M_{12} = -\frac{G^2}{12\pi^2} (V_{tb}V_{td}^*)^2 M_{12}^* m_B f_B^2 B \eta_B S \left( \frac{m_t}{M_W} \right).$$  \hspace{1cm} (21)

Here $f_B$ is the $B$ meson decay constant, $B_B$ is the vacuum saturation factor, $\eta_B = 0.55$ is a QCD correction factor, and

$$S(x) \equiv \frac{x}{4} \left[ 1 + \frac{3 - 9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^3} \right].$$  \hspace{1cm} (22)

The appropriate top quark mass for this calculation is $m_t(m_t) \simeq 165$ GeV/$c^2$. The BaBar Physics Book may be consulted for further conventions and details.

### 5.2 Time dependences

We would like to know how states which are initially $B^0$ or $\bar{B}^0$ evolve in time. The mass eigenstates evolve as $B_i \rightarrow B_i e^{-i\mu t}$ ($i = L, H$). The flavor eigenstates are expressed in terms of them as

$$t = 0: \quad |B^0\rangle = \frac{|B_L\rangle + |B_H\rangle}{2p}, \quad |\bar{B}^0\rangle = \frac{|B_L\rangle - |B_H\rangle}{2q},$$  \hspace{1cm} (23)

$$t > 0: \quad |B^0(t)\rangle = \frac{|B_L\rangle e^{-i\mu L t} + |B_H\rangle e^{-i\mu H t}}{2p},$$

$$|\bar{B}^0(t)\rangle = \frac{|B_L\rangle e^{-i\mu L t} - |B_H\rangle e^{-i\mu H t}}{2q}.$$  \hspace{1cm} (24)
Now substitute back for $B_{L,H}$:

$$|B^0(t)⟩ = |B^0⟩ f_+(t) + \frac{q}{p} f_-(t) |B^0⟩ ,$$

$$|B^0(t)⟩ = |B^0⟩ f_+(t) + \frac{p}{q} f_-(t) |B^0⟩ ,$$

$$f_+(t) \equiv e^{-imt} e^{-\Gamma t/2} \cos(\Delta \mu t/2) ,$$

$$f_-(t) \equiv e^{-imt} e^{-\Gamma t/2} i \sin(\Delta \mu t/2) ,$$

$\Delta \mu \equiv \mu_H - \mu_L = \Delta m - i(\Delta \Gamma/2)$, $\Delta m \equiv m_H - m_L$, $\Delta \Gamma \equiv \Gamma_H - \Gamma_L$, $m \equiv (m_H + m_L)/2$, $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$.

Again, for simplicity, we shall neglect $\Delta \Gamma$ in comparison with $\Delta m$. A lowest-order quark model calculation (for which QCD corrections change the answer) gives

$$\Gamma_{12} / M_{12} = -\frac{3\pi}{2} \frac{m_t^2 / m_W^2}{S(m_t^2 / m_W^2)} \frac{m_b^2}{m_l^2} \left(1 + \frac{8}{3} \frac{m_c V_{cb} V_{cd}^*}{m_b \sqrt{3} V_{tb} V_{td}^*}\right) \simeq -\frac{1}{180} ,$$

where $S(x)$ was defined in Eq. (22). The intermediate states dominating the loop calculation of $\Gamma_{12}$ have typical mass scales $m_b$, whereas loop momenta of order $m_t$ give rise to the main contributions to $M_{12}$.

Neglecting $\Delta \Gamma$ and performing time integrals, one finds

$$\Gamma \int_0^\infty dt |f_+(t)|^2 = \frac{2 + x_d^2}{2(1 + x_d^2)} ,$$

$$\Gamma \int_0^\infty dt |f_-(t)|^2 = \frac{x_d^2}{2(1 + x_d^2)} ,$$

where $x_d \equiv \Delta m_{B_d} / \Gamma_{B_d}$, and $B_d$ is another name for $B^0 = \bar{b}d$ to distinguish it from $B_s = \bar{s}s$. The sum of the two terms is 1. The first term is 1 for $x_d = 0$, approaches 1/2 for $x_d \to \infty$, and is about 0.82 for the actual value $x_d = 0.754 \pm 0.027$. The second term is 0 for $x_d = 0$, approaches 1/2 for $x_d \to \infty$, and is about 0.18 in actuality. Thus a neutral non-strange $B$ of a given flavor ($B^0$ or $\bar{B}^0$) has about 18% probability of decaying as the opposite flavor.

6 CP violation

6.1 Asymmetry: general remarks

We wish to compare $\langle f | B^0_{t=0}(t) \rangle$ and $\overline{\langle f | B^0_{t=0}(t) \rangle}$, where $f$ is a final state and $\overline{f} \equiv (CP)f$. Now define

$$x \equiv \frac{\langle f | B^0 \rangle}{\langle f | B^0 \rangle} ,$$

$$\bar{x} \equiv \frac{\overline{\langle f | B^0 \rangle}}{\overline{\langle f | B^0 \rangle}} ,$$

$$\lambda_0 \equiv \frac{q}{p} x ,$$

$$\bar{\lambda}_0 \equiv \frac{p}{q} \bar{x} .$$
Using the time evolution derived earlier for $B^0_{t=0}$ and $\overline{B}^0_{t=0}$, one then finds
\begin{align}
\langle f | B^0_{t=0}(t) \rangle &= \langle f | B^0 \rangle \left[ f_+(t) + \lambda_0(t)f_-(t) \right], \quad (33) \\
\langle \overline{f} | \overline{B}^0_{t=0}(t) \rangle &= \langle \overline{f} | \overline{B}^0 \rangle \left[ f_+(t) + \overline{\lambda}_0(t)f_-(t) \right]. \quad (34)
\end{align}

This result can be simplified under several circumstances. (a) If there is a single strong eigenchannel, final-state strong interaction phases in $x$ or $\bar{x}$ cancel, since the numerator and denominator refer to the same final state. Then $\bar{x} = x^*$, since weak phases flip sign under CP. (b) Recall that $|q/p|$ is nearly 1 for $B$ mesons. (For non-strange $B$'s, we found $q/p \approx e^{-2i\beta}$.) Combining (a) and (b), we find $\bar{\lambda}_0 = \lambda_0^*$ for these cases.

6.2 Time-dependent asymmetry

According to Eq. (33), the rates for a $(B^0, \overline{B}^0)$ produced at $t = 0$ to evolve to the respective final states $(f, \overline{f})$ at a time $t$ are
\begin{align}
\frac{d\Gamma(B^0_{t=0} \to f)}{dt} &\sim |f_+(t) + \lambda_0(t)f_-(t)|^2, \quad (35) \\
\frac{d\Gamma(\overline{B}^0_{t=0} \to \overline{f})}{dt} &\sim |f_+(t) + \overline{\lambda}_0(t)f_-(t)|^2, \quad (36)
\end{align}

with the coefficients of proportionality identical if there is a single strong eigenchannel. Now consider the case of a CP-eigenstate $f$ such that $\overline{f} = \pm f$. Then we have not only $\bar{x} = x^*$ (see above), but also $\bar{x} = x^{-1}$, so $|x| = 1$. In that case, when $|q/p| = 1$ as is the case for neutral $B$'s, we have $|\lambda_0| = 1$ and $\overline{\lambda}_0 = \lambda_0^*$. Then
\begin{align}
|f_+ + \lambda_0 f_-|^2 &= e^{-\Gamma t} \left| \cos \frac{\Delta m t}{2} + i\lambda_0 \sin \frac{\Delta m t}{2} \right|^2 \\
&= e^{-\Gamma t} \left[ 1 - \text{Im}\lambda_0 \sin \Delta m t \right], \quad (37)
\end{align}

\begin{align}
\frac{d\Gamma(B^0_{t=0} \to f)}{dt} &\sim e^{-\Gamma t} \left[ 1 - \text{Im}\lambda_0 \sin \Delta m t \right], \\
\frac{d\Gamma(\overline{B}^0_{t=0} \to \overline{f})}{dt} &\sim e^{-\Gamma t} \left[ 1 + \text{Im}\lambda_0 \sin \Delta m t \right]. \quad (39)
\end{align}

The second term in each of these equations consists of an exponential decay modulated by a sinusoidal oscillation. The time-dependent asymmetry is then
\begin{align}
\mathcal{A}_f &= \frac{\frac{d\Gamma(B^0_{t=0} \to f)}{dt} - \frac{d\Gamma(\overline{B}^0_{t=0} \to \overline{f})}{dt}}{\frac{d\Gamma(B^0_{t=0} \to f)}{dt} + \frac{d\Gamma(\overline{B}^0_{t=0} \to \overline{f})}{dt}} = -\text{Im}\lambda_0 \sin \Delta m t. \quad (40)
\end{align}

When $\Delta m/\Gamma \gg 1$, the wiggles in Eqs. (39) average out, and not much time-integrated asymmetry is possible, while when $\Delta m/\Gamma \ll 1$, the decay occurs...
before there is time for oscillations. The maximal time-integrated asymmetry occurs when $\Delta m/\Gamma = 1$.

When more than one eigenchannel is present, the condition $|\lambda_0| = 1$ need not be satisfied, so that the terms $\cos^2(\Delta mt/2)$ and $\sin^2(\Delta mt/2)$ in (37) need not have the same coefficients, and a $\cos \Delta mt$ term is generated in the rates. This is the signal of “direct” CP violation, as will be discussed below. Its presence for $B \to \pi \pi$ was pointed out by London and Peccei and by Gronau.

6.3 Time-integrated asymmetry

If one integrates the rates for $B_{t=0}^0 \to f$ and $B_{t=0}^0 \to \bar{f}$, one can form the time-integrated asymmetry

$$C_f \equiv \frac{\Gamma(B_{t=0}^0 \to f) - \Gamma(B_{t=0}^0 \to \bar{f})}{\Gamma(B_{t=0}^0 \to f) + \Gamma(B_{t=0}^0 \to \bar{f})}.$$  \hspace{1cm} (41)

If we consider the cases (as above) in which $|\langle f|B^0 \rangle| = |\langle \bar{f}|\bar{B}^0 \rangle|$, we just need the integral

$$\int_0^\infty dt \sin(\Delta mt)e^{-\Gamma t} = \frac{x_d}{1 + x_d^2},$$  \hspace{1cm} (42)

and we then find

$$C_f = -\frac{x_d}{1 + x_d^2} \text{Im} \lambda_0$$  \hspace{1cm} (43)

when $|x| = 1$. This is indeed maximal when $x_d = 1$; the coefficient of $-\text{Im} \lambda_0$ is $1/2$. For the actual value of $x_d \simeq 0.75$, the coefficient is 0.48 instead, very close to its maximal value.

6.4 Specific examples in decays to CP eigenstates

When $f$ is a CP eigenstate, a CP-violating difference between the rates for $B^0 \to f$ and $\bar{B}^0 \to \bar{f}$ arises as a result of interference between the direct decays and those proceeding via mixing (i.e., $B^0 \to \bar{B}^0 \to f$ and $\bar{B}^0 \to B^0 \to \bar{f}$). The second term in Eqs. (39) is the result of this mixing. As mentioned, the rate asymmetry goes to zero when $x_d \to 0$ or $x_d \to \infty$. We now illustrate the calculation for two specific examples, $B^0 \to J/\psi K_S$ and $B^0 \to \pi \pi$.

The “golden mode”: $J/\psi K_S$

The quark subprocess governing $B^0 \to J/\psi K_S$ is $b \to \bar{c}c\bar{s}$, whose CKM factor is $V_{cb}^* V_{cs}$. The $K_S$ is produced through its $K^0$ component. The corresponding decay $\bar{B}^0 \to J/\psi K_S$ proceeds via $b \to c\bar{c}s$ and involves the $\bar{K}^0$ component of $K_S$.  

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For a CP-eigenstate, we defined \( x \equiv \langle f | B^0 | f \rangle \langle f | B^0 \rangle \) and \( \lambda_0 = (q/p)x \), but what we actually calculate is \( \left( \frac{f}{B^0} \right) \langle f | B^0 \rangle \) where \( f = \eta_{CP} f \) with \( \eta_{CP} = \pm 1 \). For \( f = J/\psi K_S \), \( \eta_{CP} = -1 \). To show this, note that \( CP|K_S\rangle = |K_S\rangle \) and \( CP|J/\psi\rangle = |J/\psi\rangle \) (since \( J/\psi \) has odd C and P). The decay of the spin-zero \( B^0 \) to the spin-one \( J/\psi \) and the spin-zero \( K_S \) produces the final particles in a state of orbital angular momentum \( \ell = 1 \) and hence odd parity, introducing an additional factor of \(-1\). Then

\[
\lambda_0 = - \frac{\langle K_S | \overline{K} \rangle \langle \overline{K} | B \rangle}{\langle K_S | K \rangle \langle B | K \rangle}.
\]

(44)

We assumed a specific quark to dominate the calculation of \( M_{12} \) to illustrate the self-consistency of the expression for \( \lambda_0 \) with respect to redefinition of quark phases. Note first of all that the denominator is the complex conjugate of the numerator. Then note that each quark is represented by the same number of \( V \)'s and \( V^* \)'s in the numerator: 2 for the charmed quark and 1 each for \( d, s, b \), and \( t \). Thus any phase rotation of a quark field leaves the expression invariant. (Bjorken and Dunietz have introduced a nice representation of this invariance.) The same cancellation would have occurred if we were to say another quark dominated \( K^0 \overline{K}^0 \) mixing.

For the final state \( f = J/\psi K_S \) we thus find \( \lambda_0 = -e^{-2i\beta} \) and \( \text{Im} \lambda_0 = \sin 2\beta \), leading to the time-integrated rate asymmetry \( C_{J/\psi K_S} = -x_s\sin 2\beta/(1 + x_s^2) \). In practice the experiments often select events occurring for a proper time \( t \geq t_0 > 0 \) in order to enhance the signal/noise ratio, so that analyses are usually based on the time-dependent asymmetry mentioned earlier.

Some recent results on \( \sin 2\beta \) are quoted in Table 3 and \( \pm 1\sigma \) limits from the average are plotted in Fig. 2. While the central value is somewhat below that favored by other observables, there is no significant discrepancy.

The \( \pi^+ \pi^- \) mode and its complications

The main subprocess in \( B^0 \rightarrow \pi^+ \pi^- \) is the “tree” diagram in which \( \bar{b} \rightarrow \pi^+ \bar{u} \), with the spectator \( d \) combining with the \( \bar{u} \) to make a \( \pi^- \). Let us temporarily assume this is the only important process and compute the
Table 3: Values of $\sin 2\beta$ implied by recent measurements of the CP-violating asymmetry in $B^0 \to J/\psi K_S$.

| Experiment | Value          |
|------------|----------------|
| OPAL       | $3.2^{+1.8}_{-2.0} \pm 0.5$ |
| CDF        | $0.79^{+0.41}_{-0.44}$     |
| ALEPH      | $0.84^{+0.82}_{-1.04} \pm 0.16$ |
| Belle      | $0.58^{+0.32+0.09}_{-0.34-0.10}$ |
| BaBar      | $0.34 \pm 0.20 \pm 0.05$    |
| Average    | $0.48 \pm 0.16$            |

CP-violating rate asymmetry. We shall return in Section 11 to the important role of “penguin” diagrams.

Since the (spin-zero) $\pi^+\pi^-$ system in $B^0$ decay has even CP, we find

$$x \equiv \frac{\langle \pi^+\pi^- | B^0 \rangle}{\langle \pi^+\pi^- | B^0 \rangle} = \frac{V_{ub}V_{us}^*}{V_{ud}V_{us}^*}$$

(46)

$$\lambda_0 = \frac{\eta}{\rho} = \frac{V_{td}V_{tb}^* V_{ub}V_{us}^*}{V_{td}V_{tb}V_{ub}V_{us}} = e^{-2i\beta} e^{-2i\gamma}.$$  

(47)

**Exercise:** Check the invariance of this expression under redefinitions of quark phases.

Since $\beta + \gamma = \pi - \alpha$, we have $\lambda_0 = e^{2i\alpha}$, $\text{Im}(\lambda_0) = \sin 2\alpha$, and $C_{\pi^+\pi^-} = -x_d \sin 2\alpha(1 + x_d^2)$. [Remember that our asymmetries are defined in terms of $(B^0 - B^0)/(B^0 + B^0)$.] This result is limited in its usefulness for several reasons.

(a) Our neglect of penguin diagrams will turn out to make a big difference.

(b) The range of $\sin \alpha$ is large enough that early asymmetry measurements are unlikely to expose contradictions with the standard prediction.

(c) An even larger range of negative $\sin 2\alpha$ turns out to be allowed if $V_{cb}$ is larger than assumed in Sec. 4.

An interesting exercise (whose result would, of course, be modified by penguin contributions) is to suppose that the asymmetries in $B^0 \to J/\psi K_S$ and $B^0 \to \pi^+\pi^-$ are due *entirely* to mixing (i.e., to a “superweak”) interaction. In this case, since $J/\psi K_S$ and $\pi^+\pi^-$ have opposite CP eigenvalues, one has $C_{\pi^+\pi^-} = -C_{J/\psi K_S}$. What range of parameters in the standard CKM picture would imitate this relation? In other words, for what $\rho$ and $\eta$ would one have $\sin 2\alpha = -\sin 2\beta$? [The answer is $\eta = (1 - \rho) \sqrt{\rho/(2 - \rho)}$.]
7 Decays to CP-noneigenstates

If the final state $f$ is not a CP eigenstate, i.e. if $f \neq \pm \mathbf{f}$, then a CP-violating rate asymmetry requires two interfering decay channels with different weak and strong phases:

$$A(B \rightarrow f) = A_1 e^{i \phi_1} e^{i \delta_1} + A_2 e^{i \phi_2} e^{i \delta_2}.$$  \hspace{1cm} (48)

$$A(\bar{B} \rightarrow \bar{f}) = A_1 e^{-i \phi_1} e^{i \delta_1} + A_2 e^{-i \phi_2} e^{i \delta_2}. \hspace{1cm} (49)$$

Here the weak phases $\phi_i$ change sign under CP conjugation, while the strong phases $\delta_i$ do not. Define $\Delta \phi = \phi_1 - \phi_2$, $\Delta \delta = \delta_1 - \delta_2$, and

$$A(f) = \frac{|A(B \rightarrow f)|^2 - |A(\bar{B} \rightarrow \bar{f})|^2}{|A(B \rightarrow f)|^2 + |A(\bar{B} \rightarrow \bar{f})|^2}. \hspace{1cm} (50)$$

Then

$$A(f) = -\frac{2A_1 A_2 \sin \Delta \phi \sin \Delta \delta}{A_1^2 + A_2^2 + 2A_1 A_2 \cos \Delta \phi \cos \Delta \delta}. \hspace{1cm} (51)$$

7.1 Examples of interesting channels

$B^0 \rightarrow K^+ \pi^-$ vs. $\bar{B}^0 \rightarrow K^- \pi^+$

We illustrate two types of contribution to $B^0 \rightarrow K^+ \pi^-$ in Fig. 3. The “tree” contribution, which in this case is color-favored since the color-singlet current can produce a quark pair of any color, has weak phase $\gamma = \text{Arg}(V_{ub}^* V_{us})$ and strong phase $\delta_T$, while the “penguin” contribution has weak phase $\pi = \text{Arg}(V_{ub}^* V_{ts})$ and strong phase $\delta_P$. 

Figure 3: Contributions to $B^0 \rightarrow K^+ \pi^-$. (a) Color-favored “tree” amplitude $\sim V_{ub}^* V_{us}$; (b) “penguin” amplitude $\sim V_{ub}^* V_{ts}$. 

Even though $\delta_T - \delta_P$ is unknown, and may be small so that little CP-violating asymmetry is present in $B \to K^\pm \pi^\mp$, it will turn out that one can use rate information for several processes, with the help of flavor SU(3) (which can be tested) to learn weak phases such as $\gamma$.

**Exercise:** Identify the main amplitudes which contribute. What are the differences with respect to $B \to K^\pm \pi^\mp$?

**Answer:** There are two “tree” amplitudes, one color-favored [as in Fig. 3(a)] and one color-suppressed (Fig. 4). Both have weak phases $\gamma = \arg(V_{ub}^* V_{us})$. There is a penguin amplitude [as in Fig. 3(b)] with weak phase $\pi = \arg(V_{tb}^* V_{ts})$. Since $\pi^0 = (\bar{d}d - \bar{u}u)/\sqrt{2}$ in a phase convention in which $\pi^+ = u\bar{d}$, the color-favored tree and penguin amplitudes are the same as that in $B^0 \to K^+ \pi^-$, but divided by $\sqrt{2}$. Thus the overall rate for $B^\pm \to K^\pm \pi^0$ is expected to be $1/2$ that for $B \to K^\pm \pi^\mp$ if the penguin amplitude dominates or if the color-suppressed amplitude is negligible. In that case one expects similar CP-violating asymmetries for $B^0 \to K^+ \pi^- \text{ and } B^+ \to K^+ \pi^0$. [4,5]

**Exercise:** Show that there is no tree amplitude and hence no CP-violating asymmetry expected. This process is expected to be dominated by the penguin amplitude and thus provides a reference for comparison with other processes in which tree amplitudes participate.

Small contributions to $B^+ \to K^+ \pi^0$ and $B^+ \to K^0 \pi^+$ are possible from the process in which the $\bar{b}u$ pair annihilates into a weak current which then produces $\bar{s}u$. A $q\bar{q}$ pair is produced in hadronization, giving $K^+ \pi^0$ if $q = u$ and $K^0 \pi^+$ if $q = d$. These contributions are expected to be suppressed by a factor
of $f_B/m_B$ if the graphs describing them can be taken literally. However, they can also be generated by rescattering from other contributions, e.g., $(B^+ \rightarrow K^+\pi^0)_{\text{tree}} \rightarrow K^0\pi^+$. We shall mention tests for such effects in Sec. 12.

7.2 Pocket guide to direct CP asymmetries

We now indicate a necessary (but not sufficient) condition for the observability of direct CP asymmetries based on the interference of two amplitudes, one weaker than the other. The result is that one must be able to detect processes at the level of the absolute square of the weaker amplitude. This guides the choice of processes in which one might hope to see direct CP-violating rate asymmetries.

Suppose the weak phase difference $\Delta \phi$ and the strong phase difference $\Delta \delta$ are both near $\pm \pi/2$ (the most favorable case for detection of an asymmetry). Then the asymmetry $A$ in Eq. (51) has magnitude

$$|A| = O \left( \frac{2A_1A_2}{A_1^2 + A_2^2} \right) \simeq \frac{2A_2}{A_1} \quad \text{for} \quad A_2 \ll A_1.$$ (52)

Imagine a rate based on the square of each amplitude: $N_i = \text{const.} |A_i|^2$. Then $|A| \simeq 2\sqrt{N_2/N_1}$. The statistical error in $A$ is based on the total number of events. For $A_2 \ll A_1$, one has $\delta A \simeq 1/\sqrt{N_1}$. Then the significance of the asymmetry (in number of standard deviations) is

$$\frac{A}{\delta A} \sim O(2\sqrt{N_2}).$$ (53)

Thus (aside from the factor of 2) one must be able to see the square of the weaker amplitude at a significant level in order to see a significant asymmetry due to $A_1-A_2$ interference.

7.3 Interesting levels for charmless $B$ decays

Typical branching ratios for the dominant $B$ decays to pairs of light pseudoscalar mesons are in the range of 1 to 2 parts in $10^5$. Some recent data are summarized in Table 4. Here the average between a process and its charge conjugate is quoted. These data are based on results by CLEO, Belle, and BaBar. The averages are my own.

The relative $K\pi$ rates are compatible with dominance by the penguin amplitude, which predicts the rates involving a neutral pion to be half those
Thus in order to see a significant CP-violating rate asymmetry in $B^+$ decays to pairs of light pseudoscalar mesons, one needs to be able to see branching ratios of a few parts in $10^{-6}$, or

$$\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+\pi^-) \simeq \left( \frac{f_K}{f_\pi} \right)^2 \left| \frac{V_{us}}{V_{ud}} \right|^2 \times 10^{-5} .$$

With $f_K = 161$ MeV, $f_\pi = 132$ MeV, $f_K/f_\pi = 1.22$, $V_{us}/V_{ud} = \tan \theta_c = 0.22/0.975 = 0.226$, the coefficient of $10^{-5}$ on the right-hand side is 0.076. Thus in order to see a significant CP-violating rate asymmetry in $B \rightarrow K\pi$ one needs at least 13 times the sensitivity that was needed in order to see all the $B \rightarrow K\pi$ modes. This would correspond to about 100 fb$^{-1}$ at $e^+e^-$ colliders, or samples of about $10^8$ identified $B$'s at hadron machines. In other words, one needs to be able to see branching ratios of a few parts in $10^7$ with good statistical significance. This is within the capabilities of experiments just now getting under way.

8 Flavour tagging

8.1 States of $B\bar{B}$ with definite charge-conjugation

The process $e^+e^- \rightarrow B\bar{B}$ is typically studied at the mass of the $\Upsilon(4S)$ resonance, $E_{\text{c.m.}} = 10.58$ GeV, above the threshold of $2M_B = 10.56$ GeV for $B\bar{B}$ production but below the threshold for production of one or two $B^*$'s:

Table 4: Branching ratios, in units of $10^{-6}$ for $B^0$ or $B^+$ decays to pairs of light pseudoscalar mesons.

| Mode       | CLEO | Belle | BaBar | Average |
|------------|------|-------|-------|---------|
| $\pi^+\pi^-$ | $4.3^{+1.6}_{-1.4} \pm 0.5$ | $5.6^{+2.3}_{-2.0} \pm 0.4$ | $4.1 \pm 1.0 \pm 0.7$ | $4.4 \pm 0.9$ |
| $\pi^+\pi^0$ | $5.4 \pm 2.6$ | $7.8^{+3.8}_{-3.2} \pm 1.2$ | $5.1^{+2.0}_{-1.8} \pm 0.8$ | $5.6 \pm 1.5$ |
| $K^+\pi^-$ | $17.2^{+2.5}_{-2.4} \pm 1.2$ | $19.3^{+3.4}_{-3.2} \pm 1.5$ | $16.7 \pm 1.6^{+1.2}_{-1.7}$ | $17.4 \pm 1.5$ |
| $K^0\pi^+$ | $18.2^{+4.6}_{-4.5} \pm 1.6$ | $13.7^{+5.7}_{-4.8} \pm 1.9$ | $18.2^{+3.3}_{-3.0} \pm 1.6$ | $17.3 \pm 2.4$ |
| $K^+\pi^0$ | $11.6^{+3.0}_{-2.7} \pm 1.3$ | $16.3^{+3.5}_{-3.3} \pm 1.8$ | $10.8^{+2.1}_{-1.9} \pm 1.2$ | $12.2 \pm 1.7$ |
| $K^0\pi^0$ | $14.6^{+5.9}_{-5.1} \pm 2.4$ | $16.0^{+7.2}_{-5.9} \pm 2.7$ | $8.2^{+3.1}_{-2.7} \pm 1.1$ | $10.4 \pm 2.6$ |
| $K^+\eta'$ | $80^{+10}_{-9} \pm 7$ | $62 \pm 18 \pm 8$ | $75 \pm 10$ |
| $K^0\eta'$ | $89^{+10}_{-9} \pm 9$ | $78 \pm 9$ (a) |

(a) Average for $K^+\eta'$ and $K^0\eta'$ modes.
MB + MB* = 10.605 GeV, 2MB* = 10.65 GeV. Now, the Υ(4S) has C = −1. It is produced via a virtual photon γ∗ (which has odd C). It is a 3S1 b̅b state, where the superscript 2Sb̅b + 1 denotes the total spin Sb̅b = 1. The b̅b pair has orbital angular momentum L = 0 and total angular momentum J = 1. A QQ state 2S+1LJ in general has C = (−1)L+S.

The BB pair produced at the Υ(4S) thus has a definite eigenvalue of charge-conjugation, C(BB) = −1, correlating the flavor of the neutral B whose decay (e.g., to J/ψKs) is being studied with the flavor of the other B used to “tag” the decay, e.g., via a semileptonic decay b → cℓ−νℓ or ¯b → ¯cℓ+νℓ.

Let B0B0 be in an eigenstate of C with eigenvalue ηC = ±1. (To get a state with ηC = +1 it is sufficient to utilize the reaction e+e− → B0B0 or B0B0∗ → B0B0γ just above threshold.) In the B0B0 center-of-mass system, the wave function of the pair,

\[ Ψ_C \equiv \frac{1}{\sqrt{2}} \left[ B^0(\hat{p})B^0(\hat{-p}) + \eta_C B^0(\hat{p})B^0(\hat{-p}) \right] \]  

may be expressed in terms of the mass eigenstates BL,H in order to study its time evolution. Since

\[ B^0 = \frac{1}{p\sqrt{2}} [B_L + B_H] \] \hspace{1cm} \[ \overline{B}^0 = \frac{1}{q\sqrt{2}} [B_L - B_H] \]

we have

\[ Ψ_C \equiv \frac{1}{\sqrt{2}pq} \left\{ [B_L(\hat{p}) + B_H(\hat{p})][B_L(\hat{-p}) - B_H(\hat{-p})] \right\} + \eta_C [B_L(\hat{p}) - B_H(\hat{p})][B_L(\hat{-p}) + B_H(\hat{-p})] \]  

For ηC = −1 the LL and HH terms cancel (this is also a consequence of Bose statistics) and one has

\[ Ψ_C(ηC = −1) = \frac{1}{\sqrt{2pq}} [B_H(\hat{p})B_L(\hat{-p}) - B_L(\hat{p})B_H(\hat{-p})] \]  

For ηC = +1 the HL and LH terms cancel and one has

\[ Ψ_C(ηC = +1) = \frac{1}{\sqrt{2pq}} [B_L(\hat{p})B_L(\hat{-p}) - B_H(\hat{p})B_H(\hat{-p})] \]  

Define t and \( \overline{t} \) to be the proper times with which the states \( \hat{p} \) and \( \hat{-p} \) evolve, respectively:

\[ B_{L,H}(\hat{p}) \rightarrow B_{L,H}(\hat{p})e^{-i\mu_{L,H}t} \] \hspace{1cm} \[ B_{L,H}(\hat{-p}) \rightarrow B_{L,H}(\hat{-p})e^{-i\mu_{L,H}\overline{t}} \]
Project the state with $\hat{\rho}$ into the desired decay mode (e.g., $J/\psi K_S$) and the state with $-\hat{\rho}$ into the tagging mode (which signifies $B^0$ or $\bar{B}^0$ at time $\bar{t}$, e.g., $\ell^- \leftrightarrow \bar{B}^0$, $\ell^+ \leftrightarrow B^0$). Then, for a CP-eigenstate, it is left as an Exercise to show in the limit $\Delta \Gamma = 0$ that

$$
\frac{d^2 \Gamma[f(t), \ell^-(\bar{t})]}{dt \ d\bar{t}} \bigg|_{\eta_C = \mp 1} \sim e^{-\Gamma(t + \bar{t})} \left[ 1 - \sin \Delta m(t + \bar{t}) \text{Im} \lambda \right] , \quad (61)
$$

$$
\frac{d^2 \Gamma[f(t), \ell^+(\bar{t})]}{dt \ d\bar{t}} \bigg|_{\eta_C = \mp 1} \sim e^{-\Gamma(t + \bar{t})} \left[ 1 + \sin \Delta m(t + \bar{t}) \text{Im} \lambda \right] . \quad (62)
$$

(Hints: Recall that $\lambda = (q/p)\langle f|B^0\rangle/\langle f|B^0\rangle$, $\bar{\lambda} = (p/q)\langle f|\bar{B}^0\rangle/\langle f|\bar{B}^0\rangle$, $|\lambda| = 1$, $\bar{\lambda} = \lambda^*$, $\Delta m = m_H - m_L$. For $\eta_C = -1$, write the decay amplitude as a function of $t$ and $\bar{t}$. It will have two terms, one $\sim e^{-i(m_H t + m_L \bar{t})}$ and the other $\sim e^{-i(m_L t + m_H \bar{t})}$, whose interference in the absolute square of the amplitude gives rise to the $\sin \Delta m(t - \bar{t})$ terms.) These results have some notable properties.

1. For either value of $\eta_C$, the sum of the $\ell^+$ and $\ell^-$ results is as if one didn’t tag, and the oscillatory terms cancel one another.

2. For $\eta_C = -1$, note the antisymmetry with respect to $t - \bar{t}$. This is a consequence of the Bose statistics and the $C = -1$ nature of the initial state. If one integrates over all times, the CP-violating asymmetry vanishes. Thus in order for the tagging method to work in a C-odd state like $\Upsilon(4S)$ one must know whether $t$ or $\bar{t}$ was earlier. An asymmetric $B$-factory like PEP-II or KEK-B permits this by spreading out the decay using a Lorentz boost.

**Exercise:** Show for $\eta_C = -1$ that if one subdivides the $t, \bar{t}$ integrations according to $t < \bar{t}$ or $t > \bar{t}$, then

$$
\int \int dt \ d\bar{t} \left( \frac{d^2 \Gamma}{dt \ d\bar{t}} \right) \left[ (\ell^-, t > \bar{t}) - (\ell^-, t < \bar{t}) - (\ell^+, t > \bar{t}) + (\ell^+, t < \bar{t}) \right] = -\frac{x_d}{1 + x_d^2} \text{Im} \lambda . \quad (63)
$$

In practice the BaBar and Belle analyses will probably fit the time distributions rather than simply subdividing them, since background rejection and signal/noise ratio are functions of $t - \bar{t}$.

3. For $\eta_C = +1$ the oscillatory term behaves as $\sim \Delta m(t + \bar{t})$, so it is not necessary to know whether $t$ or $\bar{t}$ was earlier, and the asymmetric collision geometry is not needed. However, as shown above, in order to produce a $B^0\bar{B}^0$ state with $\eta_C = +1$ in $e^+e^-$ collisions one must work at or above $BB^*$ threshold, thereby losing the cross section advantage of the $\Upsilon(4S)$ resonance.
8.2 Uncorrelated B̅B pairs

Pairs of B’s produced in a hadronic environment are likely to arise from independent fragmentation of b and ¯b quarks, so that it is unlikely that they are produced in a state of definite ηC. (The interesting case of partially-correlated B-¯B pairs can be attacked by density-matrix methods.) Thus, one must resort to either the fact that a b is always produced in association with a ¯b by the strong interactions (“opposite-side tagging”), or the fact that the fragmentation of a b into a B0 favors one particular sign of charged pion close to the B0 in phase space (“same-side tagging”).

“Opposite-side” methods

The strong interactions q̅q → b¯b or g g → b¯b (g = gluon) conserve beauty, so that a b can be identified if it is found to be produced in association with a ¯b. The opposite-side ¯b can be identified in several ways.

(1) The jet-charge method makes use of the fact that a jet tends to carry the charge of its leading quark, since the average charge of the fragmentation products is zero in the flavor-SU(3) limit. (There is some delicacy if strange quark production is suppressed, since Q(u) ≠ −Q(d).)

(2) The lepton-tag method uses the charge of the lepton in the semileptonic decays b → cℓ− ¯ν and b → cℓ+ ν to signify the flavor of the decaying opposite-side b quark. The signal is diluted since semileptonic decays occur after the b quark has been incorporated into a meson. Sometimes this meson is a neutral B, in which case information on its flavor is nearly completely lost if it is a strange B and partially lost if it is a nonstrange B. An initial Bs will decay half the time as a B, while an initial B0 will decay about 18% of the time as a B0. (See Section 5.2.)

(3) The kaon-tag method uses the fact that the signs of kaons produced in b decays are correlated with the flavor of the decaying b. A b gives rise to c, whose products have more K− than K+. This method is subject to the same dilution as the lepton-tag method.

In order to utilize the above methods, one needs the relative probabilities of production of B0, B+, Bs, and A. The CDF Collaboration has measured these in high-energy hadron collisions to be in the ratios 0.375:0.375:0.3:0.09, while LEP Collaborations find 0.40:0.40:0.097:0.104 for Z0 → b¯b decays. Taking account of P(B0 → B̅0) ≃ 18% and P(Bs → B̅s) ≃ 1/2, the probability of a “wrong” tag is (3/8)(0.18) + (0.16)(1/2) ≃ 0.15, which dilutes the efficacy of the tag by a factor (right − wrong)/(right + wrong) ≃ 0.70.

For an extensive study of the first two tagging methods, see recent papers by the CDF Collaboration. These methods were a key ingredient in obtaining
the CP-violating asymmetry in $B^0 \to J/\psi K_S$ mentioned in Sec. 6.4.

"Same-side" methods: Fragmentation and $B^{**}$ resonances

The fragmentation of a $b$ quark into a neutral $B$ meson is not charge-symmetric. This was noted quite some time ago in the context of strange $B$'s. A $B_s$ contains a $\bar{b}$ and an $s$. This $s$ must have been produced in association with a $\bar{s}$. If that $\bar{s}$ is incorporated into a charged kaon, the kaon must be a $K^+ = u\bar{s}$.

A similar argument applies to non-strange neutral $B$'s and charged pions. A $B^0$ is then found to be associated more frequently with a $\pi^+$ nearby in phase space, while a $\bar{B}^0$ tends to be associated with a $\pi^-$. This correlation is the same as that found in resonance decays: $B^0$ resonates with $\pi^+$ but not $\pi^-$, while $\bar{B}^0$ resonates with $\pi^-$ but not $\pi^+$.

The fragmentation of a $\bar{b}$ or $b$ quark is illustrated in Fig. 5. If one cuts the diagrams to the left of the pion emission, one finds either a $\bar{b}u$ or a $b\bar{u}$ state. Thus, a positively charged resonance can decay to $B^0\pi^+$, while a negatively charged one can decay to $\bar{B}^0\pi^-$. Recall the case of $D^{*+} \to \pi^+ D^0$ mentioned in Sec. 3.1. The soft pion in that decay may be used to tag the flavor of the neutral $D$ at the time of its production, which is useful if one wants to study $D^0$-$\bar{D}^0$ mixing or Cabibbo-disfavored decays. The difference between $M(D^{*+})$ and $M(\pi^+) + M(D^0)$ (the "$Q$-value") is only about 5 MeV, so the pion is nearly at rest in the $D^{*+}$ c.m.s., and hence kinematically very distinctive. In the case of $B$'s, however, the lowest vector meson $B^{*+}$ cannot decay to $\pi^+ B^0$ since the $B^* - B$ mass difference is only about 46 MeV. One must then utilize the decays of higher-mass $B^*$'s, collectively known as $B^{**}$'s. The lightest such states consist of a $b$ quark and a light quark ($u,d,s$) in a $P$-wave.
Table 5: Quantum numbers of \( B^{* *} \) resonances (P-wave resonances between a \( \bar{b} \) quark and a light quark \( q \)).

| \( j_q \) | \( J \) | Decay prods. | Part. wave | Width |
|-------|------|-------------|------------|-------|
| 1/2   | 0    | \( B\pi \)  | S wave     | Broad |
| 1/2   | 1    | \( B^{*}\pi \) | S wave     | Broad |
| 3/2   | 1    | \( B^{*}\pi \) | D wave     | Narrow|
| 3/2   | 2    | \( B\pi, B^{*}\pi \) | D wave     | Narrow|

Many key features of the spectroscopy of a heavy quark and a light one were first pointed out in the case of charm\(^8\) and codified using heavy-quark symmetry\(^9\). The heavy quark spin degrees of freedom nearly decouple from the light-quark and gluon dynamics, so it makes sense to first couple the relative angular momentum \( L = 1 \) and the light-quark spin \( s_q = 1/2 \) to states of total light-quark angular momentum \( j_q = 1/2 \) or 3/2 and then to couple \( j_q \) with the heavy quark spin \( S_{\bar{Q}} = 1/2 \) to form total angular momentum \( J \). For \( j_q = 1/2 \) one then gets states with \( J = 0, 1 \), while for \( j_q = 3/2 \) one gets states with \( J = 1, 2 \). The \( j_q = 1/2 \) states decay to \( B\pi \) or \( B^{*}\pi \) only via S-waves, while the \( j_q = 3/2 \) states decay to \( B\pi \) or \( B^{*}\pi \) only via D-waves. These properties are summarized in Table 5.

The \( j_q = 3/2 \) resonances, decaying via D-waves, have been seen, with typical widths of tens of MeV and masses somewhere between 5.7 and 5.8 GeV/c\(^2\) in all analyses\(^10\). The \( j_q = 1/2 \) resonances are expected to be considerably broader. There is no unanimity on their properties, but evidence exists for at least one of their charmed counterparts\(^11\). More information on \( B^{* *} \)'s would enhance their usefulness in same-side tagging of neutral \( B \) mesons.

9 The strange \( B \)

9.1 \( B_s - \bar{B}_s \) mixing

The limit\(^12\) \( \Delta m_s > 15 \) ps\(^{-1}\) mentioned in Sec. 4.4 was a significant source of constraint on the \((\rho, \eta)\) plot. What range of mixing is actually expected? One may place an upper bound by noting that

\[
\frac{\Delta m_s}{\Delta m_d} = \frac{|V_{ts}|^2}{B_{B_s}} \frac{B_{B_s}}{B_B} \left( \frac{f_{B_s}}{f_B} \right)^2.
\]

Let us review the estimate\(^13\) \( f_{B_s}/f_B \leq 1.25 \). The upper limit (larger than the lattice range\(^14\)) comes from the nonrelativistic quark model, which implies\(^15\) \( |f_M|^2 = 12|\Psi(0)|^2/M_M \) for the decay constant \( f_M \) of a meson \( M \) of
mass $M_M$ composed of a quark-antiquark pair with relative wave function $\Psi(\vec{r})$. One estimates the ratios of $|\Psi(0)|^2$ in $D$ and $D_s$ systems from strong hyperfine splittings. Since $M(D^{*+}) - M(D^+) \approx M(D_s^{*+}) - M(D_s^+)$, one expects $|\Psi(0)|^2_{Q_d}/m_d \approx |\Psi(0)|^2_{Q_s}/m_s$ for mesons containing a heavy quark $Q$. In constituent-quark models $m_d/m_s \approx 0.64$, so $f_{Q_d}/f_{Q_s} \approx \sqrt{0.64} = 0.8$. An upper limit on $|V_{ts}/V_{td}|$ (see Sec. 4.3) is then $|\lambda(0.66)|^{-1} = 6.9$, implying $\Delta m_s/\Delta m_d \lesssim 74$ or $\Delta m_s \lesssim 36 ps^{-1}$. A recent prediction based on lattice estimates for decay constants is $\Delta m_s = 16.2 \pm 2.1 \pm 3.4 ps^{-1}$; there is a hint of a signal at $\sim 17 ps^{-1}$.

9.2 $B_s$ lifetime

The mass eigenstates of the strange $B$ are expected to be nearly CP eigenstates. Their mass splitting is expected to be correlated with their mixing; large values of $\Delta m_s$ imply large values of $\Delta \Gamma_s$. (In the calculation of $\Delta \Gamma/\Delta m$, the values of $|f_M|^2$ cancel.)

The value of $\Delta \Gamma_s$ is much greater than that of $\Delta \Gamma_d$ because of the shared intermediate states in the transition $\bar{B}_s = b\bar{s} \rightarrow s\bar{c}\bar{c} \rightarrow s\bar{b} = B_s$, illustrated in Fig. 6. Specific calculations imply that this quark subprocess may be dominated by $CP = +$ intermediate states, implying a shorter lifetime for the even-CP eigenstate of the $\bar{B}_s - B_s$ system by $\mathcal{O}(10\%)$.

The imaginary part of the diagram in Fig. 6 is due to on-shell states, which do not include those involving the top quark. This is the reason for the factor of $(m_t^2/m_W^2)/S(m_t^2/M_0^2)$ in Eq. (30). The lowest-order ratio, $\Delta \Gamma_s/\Delta m_s \approx -1/180$, implies that if $|\Delta m_s/\Gamma_s| = (20 ps^{-1})(1.6 ps) = 32$ (a reasonable value), then $|\Delta \Gamma_s/\Gamma_s| \approx 1/6$. Recent calculations predict $\Delta \Gamma_s/\Gamma_s = (9.3^{+3.4}_{-4.6})\%$ or $(4.7\pm 1.5\pm 1.6)\%$. The latter group also finds $f_{B_s}\sqrt{\Gamma_{B_s}} = 206(28)(7)$ MeV, $f_{B_s}/(f_{B}\sqrt{\Gamma_{B}}) = 1.16(7)$, $f_{B_s}/\sqrt{\Gamma_{B_s}} = 237(18)(8)$ MeV.
9.3 Measuring $\Delta \Gamma_s/\Gamma_s$

The average decay rate of the two mass eigenstates $B_H$ and $B_L$ can be measured by observing a flavor-specific decay, e.g.,

$$B_s(= \bar{s}b) \to D_s^- (= \bar{c}s) \ell^+ \nu_\ell \quad \text{or} \quad B_s \to D_s^- \pi^+ , \quad D_s^- \to \phi \pi^-.$$

(65)

The flavor of $D_s$ labels the flavor of the $B_s$ and then we note that

$$|B_s\rangle \simeq \frac{1}{\sqrt{2}} (|B_L\rangle + |B_H\rangle).$$

(66)

Such a flavor-specific decay then gives a rate $\bar{\Gamma} = (\Gamma_L + \Gamma_H)/2$.

One can also look for a decay in which the CP of the final state can be easily identified. $B_s \to J/\psi \phi$ is such a final state; one can perform a helicity analysis to learn its CP eigenvalue (or whether it is a mixture). Since $J(J/\psi) = J(\phi) = 1$ and $J(B_s) = 0$, the final state can have orbital angular momenta $\ell = 0, 1, 2$. Exercise: Show that $\ell = 0, 2$ corresponds to even, and $\ell = 1$ to odd CP.

9.4 Helicity analyses of $B_s \to J/\psi \phi$ and $B \to J/\psi K^*$

It is convenient to re-express the three partial wave amplitudes for decay of a spin-zero meson into two massive spin-1 mesons in terms of transversity amplitudes. These are most easily visualized by analogy with the method originally used to determine the parity of the neutral pion through its decay to two photons.

A spinless meson $M$ can decay to two photons with two possible linear polarization states: parallel and perpendicular to each other. If they have parallel polarizations, the interaction Lagrangian is $\mathcal{L}_{\text{int}} \sim M F_{\mu \nu} F^{\mu \nu} \sim M(E^2 - B^2)$, while if they have perpendicular polarizations, $\mathcal{L}_{\text{int}} \sim M F_{\mu \nu} \tilde{F}^{\mu \nu} \sim M(E \cdot B)$. Now, $E^2 - B$ is CP-even, while $E \cdot B$ is CP-odd. The observation that the two photons emitted by the $\pi^0$ had perpendicular polarizations then was used to infer that the pion had odd CP and hence (since its $C$ was even as a result of its coupling to two photons) odd $P$.

One can then identify two of the decay amplitudes for a spinless meson decaying to two massive vector mesons as $A_\parallel$ (parallel linear polarizations, even CP) and $A_\perp$ (perpendicular linear polarizations, odd CP). A third decay amplitude is peculiar to the massive vector meson case: Both vector mesons can have longitudinal polarizations (impossible for photons). Since there must be two independent CP-even decay amplitudes by the partial-wave exercise given above, this amplitude, which we call $A_0$, must be CP-even.

29
There are two recent experimental studies of decays of strange $B$’s to pairs of vector mesons. (1) The CDF Collaboration finds the results quoted in Table 6. The decays $B_s \rightarrow J/\psi \phi$ and $B^0 \rightarrow J/\psi K^{*0}$ are related to one another by flavor SU(3) (the interchange $s \leftrightarrow d$ for the spectator quark) and thus should have similar amplitude structure. We have adopted a normalization in which $|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1$. The CDF result says that $B_s \rightarrow J/\psi \phi$ is dominantly CP-even. No significant $\Delta \Gamma/\Gamma$ has been detected, but the sensitivity is not yet adequate to reach predicted levels. These conclusions are supported by results from CLEO (|$A_0|^2 = 0.52 \pm 0.08$, |$A_\parallel|^2 = 0.09 \pm 0.08$) and BaBar (|$A_0|^2 = 0.60 \pm 0.06 \pm 0.04$, |$A_\perp|^2 = 0.13 \pm 0.06 \pm 0.02$).

| Table 6: Amplitudes in the decays $B_s \rightarrow J/\psi \phi$ and $B^0 \rightarrow J/\psi K^{*0}$. |
|---------------------------------|-----------------|-----------------|
| Amplitudes                      | $B_s \rightarrow J/\psi \phi$ | $B^0 \rightarrow J/\psi K^{*0}$ |
| $|A_0|$                          | 0.78 ± 0.09 ± 0.01 | 0.77 ± 0.04 ± 0.01 |
| $|A_\parallel|$                  | 0.41 ± 0.23 ± 0.05 | 0.53 ± 0.11 ± 0.04 |
| Arg($A_\parallel/A_0$)          | 1.1 ± 1.3 ± 0.2   | 2.2 ± 0.5 ± 0.1   |
| $|A_\perp|$                      | 0.48 ± 0.20 ± 0.04 | 0.36 ± 0.16 ± 0.08 |
| Arg($A_\perp/A_0$)              | −0.6 ± 0.5 ± 0.1  | −0.6 ± 0.5 ± 0.1  |

(2) A recent ALEPH analysis finds that the decay to pairs of vector mesons occurs in mostly even partial waves, so that the lifetime in this mode probes that of the CP-even mass eigenstate, which turns out to be $B_L \simeq (B_s + B_s)/\sqrt{2}$, giving $\tau_L = 1.27 \pm 0.33 \pm 0.07$ ps. A similar study of the flavor eigenstate finds $\tau(B_s) = 1.54 \pm 0.07$ ps. Comparing the two values, one finds $\Delta \Gamma/\Gamma = (25^{+10}_{−7})\%$. This is just one facet of a combined analysis of results from CDF, LEP, and SLD that concludes $\Delta \Gamma/\Gamma = (16^{+8}_{−5})\%$, or $\Delta \Gamma/\Gamma < 31\%$ at 95\% c.l.

10 $B$ decays to pairs of light mesons

We have already noted in Table 6 some branching ratios for $B$ decays to pairs of light pseudoscalar mesons. Here we discuss these and related processes involving one or two light vector mesons in more detail.

10.1 Dominant processes in $B \rightarrow \pi \pi$ and $B \rightarrow K \pi$

The decays $B \rightarrow \pi \pi$ and $B \rightarrow K \pi$ are rich in possibilities for determining fundamental CKM parameters. The process $B^0 \rightarrow \pi^+ \pi^-$ could yield the angle
\( \alpha \) in the absence of penguin amplitudes, whose contribution must therefore be taken into account. The process \( B^0 \to K^+\pi^- \) and related decays can provide information on the weak phase \( \gamma \).

In order to discuss such decays in a unified way, we shall employ a flavor-SU(3) description using a graphical representation. This language is equivalent to tensorial methods. The graphs are shown in Fig. 7. They constitute an over-complete set; all processes of the form \( B \to PP \), where \( P \) is a light pseudoscalar meson belonging to a flavor octet, are described by only 5 independent linear combinations of these.

The graphical technique allows one to check a result for \( B \to \pi\pi \) which can be obtained using isospin invariance. The subprocess \( \bar{b} \to \bar{u}ud \) can change isospin by \( 1/2 \) or \( 3/2 \) units. The \( J = 0 \) \( \pi\pi \) final state, by virtue of Bose statistics, must have even isospin: \( I = 0, 2 \). Thus there are only two invariant amplitudes in the problem, one with \( \Delta I = 1/2 \) leading to \( I_{\pi\pi} = 0 \) and one with \( \Delta I = 3/2 \) leading to \( I_{\pi\pi} = 2 \). Hence the amplitudes for the three decays \( B^0 \to \pi^+\pi^- \), \( B^+ \to \pi^+\pi^0 \), and \( B^0 \to \pi^0\pi^0 \) obey one linear relation. In the graphical representation they are

\[
\begin{align*}
A(B^0 \to \pi^+\pi^-) &= - (T + P) , \\
A(B^+ \to \pi^+\pi^0) &= -(T + C) / \sqrt{2} , \\
A(B^0 \to \pi^0\pi^0) &= (P - C) / \sqrt{2} ,
\end{align*}
\]

leading to the relation 
\[
A(B^0 \to \pi^+\pi^-) = \sqrt{2} A(B^+ \to \pi^+\pi^0) - \sqrt{2} A(B^0 \to \pi^0\pi^0).
\]

Measurement of the rates for these processes and their charge-conjugates allows one to separate the penguin and tree contributions from one another and to obtain information on the CKM phase \( \alpha \). The only potential drawback of this method is that the branching ratio for \( A(B^0 \to \pi^0\pi^0) \) is expected to be small: of order \( 10^{-6} \).

One can use \( B \to K\pi \) and flavor SU(3) to evaluate the penguin contribution to \( B \to \pi\pi \). The decay \( B \to \pi\pi \) appears to be dominated by the tree amplitude while \( B \to K\pi \) appears to be dominated by the penguin:

\[
\frac{\text{(Tree)}_{K\pi}}{\text{(Tree)}_{\pi\pi}} \approx \frac{f_K V_{us}}{f_\pi V_{ud}} \approx \frac{\text{(Pen)}_{\pi\pi}}{\text{(Pen)}_{K\pi}} \approx \frac{V_{td}}{V_{ts}} \approx \frac{1}{4} .
\]

Many other applications of flavor SU(3) to \( B \to K\pi \) decays have been made subsequently. We shall discuss the results of one relatively recent example.

**10.2 Measuring \( \gamma \) with \( B \to K\pi \) decays**

The Fermilab Tevatron and the CERN Large Hadron Collider (LHC) will produce large numbers of \( \pi^+\pi^- \), \( \pi^0 K^\mp \), and \( K^+K^- \) pairs from neutral non-
strange and strange $B$ mesons. Each set of decays has its own distinguishing features.

The processes $B^0 \to K^+ K^-$ and $B_s \to \pi^+ \pi^-$ involve only the spectator-quark amplitudes $E$ and $PA$, and thus should be suppressed. They are related to one another by a flavor SU(3) “U-spin” reflection $s \leftrightarrow d^{108}$ and thus the ratio of their rates should be the ratio of the corresponding squares of CKM elements.

The decays $B^0 \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ also are related to each other by a U-spin reflection. Time-dependent studies of both processes allow one to separate strong interaction and weak interaction information from one another and to measure the angle $\gamma$. This appears to be a promising method for Run II at the Fermilab Tevatron.

The decays of non-strange and strange neutral $B$ mesons to $K^\pm \pi^\mp$ provide another source of information on $\gamma$ when combined with information on $B^+ \to K^0 \pi^+$. The rate for this process is predicted to be the same as that for $B^- \to \bar{K}^0 \pi^-$, providing a consistency check. We consider the amplitudes $T$ and $P$ with relative weak phase $\gamma$ and relative strong phase $\delta$, neglecting the amplitudes $E$, $A$, and $PA$ which are expected to be suppressed relative to $T$ and $P$ by factors of $f_B/m_B$. Then we find (letting $T$ and $P$ stand for...
magnitudes)

\[ A(B^0 \to K^+\pi^-) = -[P + T e^{i(\gamma + \delta)}] \tag{71} \]
\[ A(B^+ \to K^0\pi^+) = P \tag{72} \]
\[ A(B_s \to \pi^+K^-) = \tilde{\lambda}P - \frac{1}{\lambda}Te^{i(\gamma + \delta)} \tag{73} \]

with amplitudes for the charge-conjugate processes given by \( \gamma \to -\gamma \). Here \( \tilde{\lambda} \equiv |V_{us}/V_{ud}| = |V_{cd}/V_{cs}| = \tan \theta_c \simeq 0.226 \). In the penguin amplitude the top quark has been integrated out and unitarity used to replace \( V_{tb}^* V_{tq} \) by \(-V_{cb}^* V_{cq} - V_{ub}^* V_{uq} \). The term \(-V_{cb}^* V_{cq}\) is the dominant contribution to \( P \), while \(-V_{ub}^* V_{uq}\) is incorporated into \( T \).

We define the charge-averaged ratios:

\[ R \equiv \frac{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\bar{B}^0 \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to K^0\pi^-)} \tag{75} \]
\[ R_s \equiv \frac{\Gamma(B_s \to K^-\pi^+) + \Gamma(\bar{B}_s \to K^+\pi^-)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to K^0\pi^-)} \tag{76} \]

and CP-violating rate (pseudo-)asymmetries:

\[ A_0 \equiv \frac{\Gamma(B^0 \to K^+\pi^-) - \Gamma(\bar{B}^0 \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to K^0\pi^-)} \tag{77} \]
\[ A_s \equiv \frac{\Gamma(B_s \to K^-\pi^+) - \Gamma(\bar{B}_s \to K^+\pi^-)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to K^0\pi^-)} \tag{78} \]

and let \( r \equiv T/P \). We find

\[ R = 1 + r^2 + 2r \cos \delta \cos \gamma \tag{79} \]
\[ R_s = \tilde{\lambda}^2 + (r/\tilde{\lambda})^2 - 2r \cos \delta \cos \gamma \tag{80} \]
\[ A_0 = -A_s = -2r \sin \gamma \sin \delta \tag{81} \]

The relation \( A_0 = -A_s \) may be used to test the assumption of flavor SU(3) symmetry, while the remaining three equations may be solved for the three unknowns \( r, \gamma, \) and \( \delta \). An error of 10° on \( \gamma \) seems feasible. (A small correction associated with the above approximation to the penguin graph also may be applied.)
10.3 Decays with \( \eta \) and \( \eta' \) in the final state

The physical \( \eta \) and \( \eta' \) are mixtures of the flavor octet state \( \eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6} \) and the flavor singlet \( \eta_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \). This mixing is tested in many decays, such as \((\eta, \eta') \rightarrow \gamma\gamma\), \((\rho, \omega, \phi) \rightarrow \eta\gamma\), \(\eta' \rightarrow (\rho, \omega, \phi)\gamma\), etc. The result is that the \( \eta \) is mostly an octet and the \( \eta' \) mostly a singlet, with one frequently-employed approximation corresponding to an octet-singlet mixing angle of 19°:

\[
\eta \simeq \frac{1}{\sqrt{3}}(s\bar{s} - u\bar{u} - d\bar{d}) \quad , \quad \eta' \simeq \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + 2s\bar{s}) \quad , \quad (82)
\]

(A single mixing angle may not adequately describe the \( \eta-\eta' \) system for a meson with a flavor singlet component, an amplitude in addition to those depicted in Fig. 8 corresponds to the “singlet” penguin diagram, as shown in Fig. 8.)

The CLEO Collaboration’s large branching ratios for \( B \rightarrow \eta'K \):

\[
B(B^+ \rightarrow \eta'K^+) = (80^{+10}_{-9} \pm 7) \times 10^{-6} \quad , \quad B(B^0 \rightarrow \eta'K^0) = (89^{+18}_{-16} \pm 9) \times 10^{-6} \quad , \quad (83)
\]

with only upper limits for \( K\eta \) production, indicate the presence of a substantial “singlet” penguin contribution, and constructive interference between nonstrange and strange quark contributions of \( \eta' \) to the ordinary penguin amplitude \( P \), as suggested by Lipkin. The corresponding decays to nonstrange final states, \( B^+ \rightarrow \pi^+\eta \) and \( B^+ \rightarrow \pi^+\eta' \), are expected to have large CP-violating asymmetries, since several weak amplitudes in these processes are of comparable magnitude. Moreover, CLEO sees

\[
B(B^+ \rightarrow \eta K^{*+}) = (26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6} \quad , \quad (84)
\]
\[ \mathcal{B}(B^0 \to \eta K^{*0}) = (13.8^{+5.5}_{-4.6} \pm 1.6) \times 10^{-6}, \]

with only upper limits for \( K^*\eta' \) production. These results favor the opposite signs for nonstrange and strange components of the \( \eta \), again in accord with predictions.15

Much theoretical effort has been expended on attempts to understand the magnitude of the “singlet” penguin diagrams,116 but they appear to be more important than one would estimate using pertubative QCD.

10.4 One vector meson and one pseudoscalar

The decays \( B \to V P \), where \( V \) is a vector meson and \( P \) a pseudoscalar, are characterized by twice as many invariant amplitudes of flavor SU(3) as the decays \( B \to PP \), since either the vector meson or the pseudoscalar can contain the spectator quark. We can label the corresponding amplitudes by a subscript \( V \) or \( P \) to denote the type of meson containing the spectator. A recent analysis within the graphical framework uses data to specify amplitudes.70 Alternatively, one can incorporate models for form factors into calculations based on factorization.117,118 An interesting possibility suggested in both these approaches is that the large branching ratio \( \mathcal{B}(B^0 \to K^{*+}\pi^-) \) may suggest constructive tree-penguin interference, implying \( \gamma \geq 90^\circ \).

The tree amplitude in \( B^0 \to K^{*+}\pi^- \) is proportional to \( V_{ub}^* V_{us} \), with weak phase \( \gamma \), while the penguin amplitude is proportional to \( V_{tb}^* V_{ts} \), with weak phase \( \pi \). The relative weak phase between these two amplitudes is then \( \gamma - \pi \), which leads to constructive interference if the strong phase difference between the tree and penguin amplitudes is small and if \( \Gamma > \pi/2 \). This could help explain why \( \mathcal{B}(B^0 \to K^{*+}\pi^-) \) seems to exceed \( 2 \times 10^{-5} \) while the pure penguin process \( B \to \phi K \) corresponds to a branching ratio of only \( (6.2^{+2.0}_{-1.8-1.7}) \times 10^{-6} \).119

A similar tree-penguin interference can occur in \( B^0 \to \pi^+\pi^- \). The tree amplitude is proportional to \( V_{ub}^* V_{ud} \), with weak phase \( \gamma \), while the penguin is proportional to \( V_{tb}^* V_{td} \), with weak phase \( -\beta \). The relative weak phase is then \( \gamma + \beta = \pi - \alpha \). One expects destructive interference if the final strong phase difference is small and \( \alpha < \pi/2 \). This could help explain why \( \mathcal{B}(B^0 \to \pi^+\pi^-) \) is \( (4.4 \pm 0.9) \times 10^{-6} \) while the tree contribution alone, estimated (for example) from \( B \to \pi\ell\nu \),120 would lead to a branching ratio around \( 10^{-5} \).

A global fit of \( B \to PP \) and \( B \to VP \) data117 based on factorization and models for form factors leads to \( \gamma = (114^{+25}_{-26})^\circ \), which just barely clips the corner of the allowed \( (\rho, \eta) \) region. If valid, this result implies that we should see \( \Delta m_s \) near its present lower bound.
10.5 Two vector mesons

No modes with pairs of light vector mesons have been identified conclusively yet. The existence of three partial waves (S, P, D) for such processes as $B^0 \to \phi K^{*0}$ means that helicity analyses can in principle detect the presence of final-state interactions (as in the case of $B \to J/\psi K^*$). It is not clear, however, whether such final-state phases are relevant to the case of greatest interest, in which two different channels are “fed” by different weak processes such as $T$ and $P$ amplitudes. Some further information obtainable from angular distributions in $B \to VV$ decays has been noted.\[122]

10.6 Testing flavor SU(3)

The asymmetry prediction $A_s = -A_0$ for $B_s \to \bar{K}\pi$ vs. $B \to K\pi$, mentioned above, is just one of a number of U-spin relations[108] testable via $B_s$ decays, which will first be studied in detail at hadron colliders. One expects the assumption of flavor SU(3), and in particular the equality of final-state phases for non-strange and strange $B$ final states, to be more valid for $B$ decays than for charm decays, where resonances still are expected to play a major role.

11 The role of penguins

11.1 Estimates of magnitudes

Perturbative calculations of penguin contributions to processes such as $B \to K\pi$, where they seem to be dominant, fall short of actual measurements.\[122\] Phenomenological fits indicate no suppression by a factor of $\alpha_s/4\pi$ despite the presence of a loop and a gluon. One possible explanation is the presence of a $c\bar{c}$ loop with substantial enhancement from on-shell states, equivalent to strong rescattering from such states as $D_s \bar{D}$ to charmless meson pairs. If this is indeed the case, penguin amplitudes could have different final-state phases from tree amplitudes, enhancing the possibility of observing direct CP violation.

11.2 Electroweak penguins

When the gluon in a penguin diagram is replaced by a (real or virtual) photon or a virtual $Z$ which couples to a final $q\bar{q}$ pair, the process $b \to (\bar{d} \text{ or } \bar{s}) q\bar{q}$ is no longer independent of the flavor ($u,d,s$) of $q$. Instead, one has contributions in which the $u\bar{u}$ pair is treated differently from the $d\bar{d}$ or $s\bar{s}$ pair. A color-favored electroweak penguin amplitude $P_{EW}$ [Fig. 9(a)] involves the pair appearing in the same neutral meson (e.g., $\pi^0$), while a color-suppressed amplitude $P_{cEW}$ [Fig. 9(b)] involves each member of the pair appearing in a different meson.
One may parametrize electroweak penguin (EWP) amplitudes by contributions proportional to the quark charge, sweeping other terms into the gluonic penguin contributions. One then finds that the EWP terms in a flavor-SU(3) description may be combined as follows with the terms $T$, $C$, $P$, and $S$ (the “singlet”) penguin:

\begin{align}
T & \rightarrow t \equiv T + P_{EW}^c, \\
P & \rightarrow p \equiv P - \frac{1}{3} P_{EW}^c, \\
C & \rightarrow c \equiv C + P_{EW}, \\
S & \rightarrow s \equiv S - \frac{1}{3} P_{EW}.
\end{align}

The flavor-SU(3) description holds as before, but weak phases now can differ from their previous values as a result of the EWP contributions.

One early application of flavor SU(3) which turns out to be significantly affected by EWP contributions is the attempt to learn the weak phase $\gamma$ from information on the decays $B^+ \rightarrow K^+\pi^0$, $B^+ \rightarrow K^0\pi^+$, $B^+ \rightarrow \pi^+\pi^0$, and the corresponding charge-conjugate decays. The amplitude construction is illustrated in Fig. 10. The primes on the amplitudes refer to the fact that they describe strangeness-changing ($|\Delta S| = 1$) transitions. The corresponding $\Delta S = 0$ amplitudes are unprimed.

The amplitudes in Fig. 10 form two triangles, whose sides labeled $C' + T'$ and $\bar{C}' + \bar{T}'$ form an angle $2\gamma$ with respect to one another. (There will be a discrete ambiguity corresponding to flipping one of the triangles about its base.) One estimates the lengths of these two sides using flavor SU(3) from the amplitudes $A(B^+ \rightarrow \pi^+\pi^0) = (C+T)/\sqrt{2}$ and $A(B^- \rightarrow \pi^-\pi^0) = (\bar{C}+\bar{T})/\sqrt{2}$.

In the presence of electroweak penguin contributions this simple analysis must be modified, since there are important additional contributions when we replace $C' + T' \rightarrow \bar{c}' + t'$ and $\bar{C}' + \bar{T}' \rightarrow \bar{c}' + \bar{t}'$. The culprit is the $C'$. 

Figure 9: Electroweak penguin (EWP) diagrams. (a) Color-favored ($P_{EW}$); (b) Color-suppressed ($P_{EW}^c$).
Figure 10: Amplitude triangles for determining the weak phase $\gamma$. These are affected by electroweak penguin contributions, as described in the text.

amplitude, which is associated with a color-favored electroweak penguin. It was noted subsequently\textsuperscript{24} that since the $C' + T'$ amplitude corresponds to isospin $I(K\pi) = 3/2$ for the final state, the strong-interaction phase of its EWP contribution is the same as that of the rest of the $C' + T'$ amplitude, permitting the calculation of the EWP correction. The result is that

$$A[I(K\pi) = 3/2] \sim \text{const} \times (e^{i\gamma} - \delta_{EW}), \quad (88)$$

where the phase in the first term is $\text{Arg}(V_{ub}^* V_{us})$ and the second term is estimated to be $\delta_{EW} = 0.64 \pm 0.15$ when SU(3)-breaking effects are included.

Any deviation of the ratio $2\Gamma(B^+ \to K^+\pi^0)/\Gamma(B^+ \to K^0\pi^+)$ from 1 can signify interference of the $C' + T'$ amplitude with the dominant $P'$ amplitude and hence can provide information on $\gamma$. The present value for this ratio, based on the branching ratios in Table 4, is $1.27 \pm 0.47$, compatible with 1.

The triangle construction (with its generalization to include EWP contributions) avoids the need to evaluate strong phases. Other studies of calculable electroweak penguin effects have been made.\textsuperscript{25} Good use of these results will require 100 fb\textsuperscript{-1} at an $e^+e^-$ collider or $10^8$ produced $B\bar{B}$ pairs.

12 Final-state interactions

It is crucial to understand final-state strong phases in order to anticipate direct CP-violating rate asymmetries and to check whether assumptions about the smallness of amplitudes involving the spectator quark are correct. The decay $B^+ \to K^0\pi^+$ in the naive diagrammatic approach is expected to be dominated
by the penguin diagram with no tree contribution. The penguin weak phase would be \( \arg(V_{ub}^*V_{us}) = \pi \). The phase of the annihilation amplitude \( A \), which is expected to be suppressed by a factor of \( \lambda^2 f_B/m_B \) and hence should be unimportant, should be \( \arg(V_{ub}^*V_{us}) = \gamma \). This implies a very small CP-violating rate asymmetry between \( B^+ \to K^0\pi^+ \) and \( B^- \to \bar{K}^0\pi^- \), much smaller than in cases where \( T \) and \( P \) amplitudes can interfere such as \( B^0 \to K^+\pi^- \).

| Mode  | Signal events | \( A_{CP} \)           |
|-------|---------------|-------------------------|
| \( K^+\pi \) | 80^{+14}_{-11} | -0.04 \pm 0.16          |
| \( K^+\pi^0 \) | 42.1^{+10.9}_{-9.9} | -0.29 \pm 0.23          |
| \( K_S\pi^+ \) | 25.2^{+6.4}_{-5.6} | 0.18 \pm 0.24           |
| \( K^+\eta' \) | 100^{+14}_{-12} | 0.03 \pm 0.12           |
| \( \omega\pi^+ \) | 28.5^{+8.2}_{-7.3} | -0.34 \pm 0.25          |

The current data do not exhibit significant CP asymmetries in any modes. In Table 7 we summarize some recently report CP asymmetries, defined as

\[
A_{CP} \equiv \frac{\Gamma(B \to \bar{f}) - \Gamma(B \to f)}{\Gamma(B \to f) + \Gamma(B \to \bar{f})} .
\]

The asymmetry in the mode \( K_S\pi^\pm \) is no more or less significant than in other modes where \( A_{CP} \neq 0 \) could be expected. How could we tell whether the amplitude \( A \) is suppressed by as much as we expect in the naive approach?

### 12.1 Rescattering

Rescattering from tree processes (such as those in Fig. 3 or Fig. 4 contributing to \( B^+ \to K^+\pi^0 \)) could amplify the effective \( A \) amplitude in \( B^+ \to K^0\pi^+ \), removing the suppression factor of \( f_B/m_B \). The tree amplitude for \( B^+ \to K^+\pi^0 \) should be proportional to \( V_{ub}^*V_{us} \) (as in the \( A \) amplitude), but the magnitude of the rescattering amplitude for \( K^+\pi^0 \to K^0\pi^+ \) (in an S-wave) is unknown at the center-of-mass energy of \( m_Bc^2 = 5.28 \text{ GeV} \).

### 12.2 A useful SU(3) relation

A sensitive test for the presence of an enhanced \( A \) amplitude has been proposed,\(^{28}\) utilizing the U-spin symmetry \( d \leftrightarrow s \) of flavor SU(3). Under this transformation, the \( \bar{b} \to \bar{s} \) penguin diagram contributing to \( B^+ \to K^0\pi^+ \).
is transformed into the $\bar{b} \rightarrow \bar{d}$ penguin contributing to $B^+ \rightarrow \bar{K}^0 K^+$, suppressed by a relative factor of $|V_{td}^* V_{tb}/V_{ts}^* V_{ub}| \simeq \lambda$, while the annihilation diagram contributing to $B^+ \rightarrow K^0 \pi^+$ is transformed into that contributing to $B^+ \rightarrow \bar{K}^0 K^+$, enhanced by a relative factor of $|V_{td}^* V_{tb}/V_{ts}^* V_{ub}| \simeq 1/\lambda$. Thus the relative effects of the “annihilation” amplitude should be stronger by a factor of $1/\lambda^2$ in $B^+ \rightarrow \bar{K}^0 K^+$ than in $B^+ \rightarrow K^0 \pi^+$. Even if these effects are not large enough to significantly influence the decay rate, they could well influence the predicted decay asymmetry.

12.3 The process $B^0 \rightarrow K^+ K^-$

A process which should be dominated by interactions involving the spectator quark is $B^0 \rightarrow K^+ K^-$. Only the exchange (E) and penguin annihilation (PA) graphs in Fig. 7 contribute to this decay. The exchange (E) amplitude should be proportional to $(f_B/m_B) V_{ub}^* V_{ud}$, and the penguin annihilation amplitude should be suppressed by further powers of $\alpha_s$, in a naïve approach. The expected branching ratio if the E amplitude dominates should be less than $10^{-7}$. However, if rescattering is important, the $K^+ K^-$ final state could be “fed” by the process $B^0 \rightarrow \pi^+ \pi^-$, whose amplitude is proportional to $T + P$. Present experimental limits place only the 90% c.l. upper bound $\mathcal{B}(B^0 \rightarrow K^+ K^-) < 1.9 \times 10^{-6}$.

12.4 Critical remarks

Some estimates of rescattering are based on Regge pole methods. These may not apply to low partial waves at energies of $m_b c^2 = 5.28$ GeV. Regge poles have proven phenomenologically successful primarily for “peripheral” partial waves $\ell \sim k_{c.m.} \times (R \sim 1 \text{ fm})$.

13 Topics not covered in detail

13.1 Measurement of $\gamma$ using $B^{\pm} \rightarrow D K$ decays

The self-tagging decays $B^{\pm} \rightarrow D^0 K^\pm$, $B^{\pm} \rightarrow D^0 K^\mp$, and $B^{\pm} \rightarrow D_{CP} K^\pm$, where $D_{CP}$ is a CP eigenstate, permit one to perform a triangle construction very similar to that in Fig. 10 to extract the weak phase $\gamma$. However, the interference of the Cabibbo-favored decay $D^0 \rightarrow K^- \pi^+$ and the doubly-Cabibbo-suppressed decay $D^0 \rightarrow K^+ \pi^-$ introduces an important subtlety in this method, which has been addressed.
13.2 Dalitz plot analyses

The likely scarcity of the decay $B^0 \rightarrow \pi^0 \pi^0$ (see Sec. 10) may be an important limitation in the method proposed to extract the weak phase $\alpha$ from $B \rightarrow \pi\pi$ decays using an isospin analysis. It has been suggested that one study instead the isospin structure of the decays $B \rightarrow \rho\pi$, since at least some of these processes occur with greater branching ratios than the corresponding $B \rightarrow \pi\pi$ decays. One must thus measure time-dependences and total rates for the processes ($B^0$ or $\bar{B}^0$) $\rightarrow (\rho^\pm \pi^\mp, \rho\pi^0)$. A good deal of useful information, in fact, can be learned just from the time-integrated rates.

13.3 CP violation in $B^0$–$\bar{B}^0$ mixing

The standard model of CP violation predicts that the number of same-sign dilepton pairs due to $B^0$–$\bar{B}^0$ mixing should be nearly the same for $\ell^+\ell^+$ and $\ell^-\ell^-$. By studying such pairs it is possible to test not only this prediction, but also the validity of CPT invariance. The OPAL Collaboration parametrizes neutral non-strange $B$ mass eigenstates as

$$|B_1\rangle = \frac{(1 + \epsilon_B + \delta_B)|B^0\rangle + (1 - \epsilon_B - \delta_B)|\bar{B}^0\rangle}{\sqrt{2(1 + |\epsilon_B + \delta_B|^2)}}$$

$$|B_2\rangle = \frac{(1 + \epsilon_B - \delta_B)|B^0\rangle - (1 - \epsilon_B + \delta_B)|\bar{B}^0\rangle}{\sqrt{2(1 + |\epsilon_B - \delta_B|^2)}}$$

and finds (allowing for CPT violation) $\text{Im}(\delta_B) = -0.020 \pm 0.016 \pm 0.006$, $\text{Re}(\epsilon_B) = -0.006 \pm 0.010 \pm 0.006$. Enforcing CPT invariance, they find $\text{Re}(\epsilon_B) = 0.002 \pm 0.007 \pm 0.003$. A recent CLEO study finds $\text{Re}(\epsilon_B)/(1 + |\epsilon_B|^2) = 0.0035 \pm 0.0103 \pm 0.0015$ under similar assumptions. The standard model predicts $|p/q| \approx 1$ and hence $\text{Re}(\epsilon_B) \approx 0$.

14 What if the CKM picture doesn’t work?

14.1 Likely accuracy of future measurements

It’s useful to anticipate how our knowledge of the Cabibbo-Kobayashi-Maskawa matrix might evolve over the next few years. With $\sin(2\beta)$ measured in $B^0 \rightarrow J/\psi K_S$ decays to an accuracy of $\pm 0.06$ (the BaBar goal with 30 fb$^{-1}$), errors on $|V_{ub}/V_{cb}|$ reduced to 10%, strange-$B$ mixing bounded by $x_s = \Delta m_s/\Gamma_s > 20$ (the present bound is already better than this!), and $\mathcal{B}(B^+ \rightarrow \tau^+\nu_\tau)$ measured to $\pm 20\%$ (giving $f_B|V_{ub}$, or $|V_{ub}/V_{td}|$ when combined with $B^0$–$\bar{B}^0$ mixing), one finds the result shown in Fig. 11.
The anticipated \((\rho, \eta)\) region is quite restricted, leading to the likelihood that if physics beyond the standard model is present, it will show up in such a plot as a contradiction among various measurements. What could be some sources of new physics?

### 14.2 Possible extensions

Some sources of effects beyond the standard model which could show up first in studies of \(B\) mesons include:

- Supersymmetry (nearly everyone's guess) (see Murayama’s lectures\(^{142}\));
- Flavor-changing effects from extended models of dynamical electroweak symmetry breaking (mentioned, e.g., by Chivukula\(^{143}\));
- Mixing of ordinary quarks with exotic ones, as in certain versions of grand unified theories\(^{144}\).

Typical effects show up most prominently in mixing (particularly \(K^0 - \bar{K}^0\) and \(B^0 - \bar{B}^0\)\(^{144}\) but also could appear in penguin processes such as \(B \to \phi K\).\(^{145}\)
15 Summary

We are entering an exciting era of precision B physics. With experimental and theoretical advances occurring on many fronts, we have good reason to hope for surprises in the next few years. If, however, the present picture survives such stringent tests, we should turn our attention to the more fundamental question of where the CKM matrix (as well as the quark masses themselves!) actually originates.

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