Scientific approach to fire resistance calculation of reinforced concrete beams and columns

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Abstract. Problems related to the fire resistance calculation of reinforced concrete beams and columns from a scientific point of view when using new design standards are considered. The first problem is the insufficient development of improved calculation methods, exactly their absence. Only basic principles and requirements are formulated for them. This article presents the principles and sequence of calculation. The second problem is the lack of an accurate mathematical nonlinear compression model of the stress-strain diagram for concrete. The article explains the imperfection of the existing one, the need to create a new model and presents the basic requirements that such a mathematical model must meet. The third problem of the discussed ones is the erroneous values of the relative ultimate deformation of concrete. The principle of formation of values of relative limiting deformation of concrete is specified and proceeding from it the additional requirement for mathematical nonlinear model of compression of the diagram of stress-strain for concrete is specified.

1. Introduction
Standards for calculating the fire resistance of reinforced concrete beams [1] and reinforced concrete columns [2] have been developed and adopted by NDIBK (State Research Institute of Building Constructions) in connection with the harmonization of national building codes with European ones. These standards apply to the design of reinforced concrete elements that are exposed to fire and in some cases complement the design of reinforced concrete elements at normal temperature. The norms describe the strength (compressive strength, tensile strength, relative deformation at maximum load, relative ultimate deformation, initial modulus of elasticity) and thermophysical (temperature elongation, specific heat, thermal conductivity) properties of concrete and reinforcing steel at elevated temperatures. The codes contain a description of tabular and simplified methods for calculating reinforced concrete beams and columns. These codes are a logical continuation of the previously adopted European standards [3], [4], but in turn they have also assimilated the shortcomings and inaccuracies [5] that took place in Eurocodes. There are a number of problems associated with the calculation of fire resistance and increasing its accuracy for these building elements when using these standards. The use of these standards is acceptable when assessing the fire resistance of reinforced concrete beams and columns as a separate structure and as part of the structure. With the accurate calculation of fire resistance and the global structural analysis when using these standards, there are a number of problems associated with improving its accuracy for these building elements.

2. Improved calculation methods
One of the main problems is the insufficient coverage of the improved calculation methods, their description and formulation of requirements to them. These standards [1] and [2] allow to carry out
calculations based on tabular data, simplified methods and improved calculation models, shown in Table 1, in accordance with the Codes [3].

It should be noted that only the improved methods allow to investigate the operation of the reinforced concrete structural element during a fire comprehensively and take into account possible options for the operation of all structural elements of the building as a whole like a part of the structural scheme of the building. Also, in real buildings reinforced concrete elements are interconnected and statically indeterminate, so the peculiarity of the structures during fire is that the calculated load effect during fire on the structural element \( E_\text{(d.fi)} \) differs from the calculated load effect on structural element at normal temperature. However, depending on the type of fire, its location, scale of the fire and structural parameters of the elements (stiffness, curvature, cross section of the element), the calculated load effect during a fire on a structural element \( E_\text{(d.fi)} \) for an individual element can both increase and decrease (Figure 1). This is due to the redistribution of forces between structural elements of the building on account of the nonlinear laws of deformation of concrete and reinforcing steel.

### Table 1. Comparative table of fire resistance calculation methods.

| Tabulated data                      | Simplified methods | Improved calculation models |
|-------------------------------------|--------------------|-----------------------------|
| Individual member analysis          | YES                | YES                         |
| The individual temperature mode only; member isolated. Indirect fire temperature modes. actions are not considered, except those resulting from thermal gradients | YES - standard fire and parametric fire, temperature profiles given for standard fire mode only, - material models that consider change of material properties and apply only to heating rates similar to standard fire. | Only the principles are given. |
| Analysis of parts of NO the structure | YES                | YES                         |
| Indirect fire actions within the subassembly are considered, but no time-dependent interaction with other parts of the structure. | YES - standard fire and parametric fire, temperature profiles given for standard fire only, - material models that consider change of material properties are applied only to heating modes similar to standard fire. | Only the principles are given. |
| Entire structural analysis          | NO                 | YES                         |
| Analysis of the entire structure. Indirect fire actions are considered throughout the entire structure. | NO Only the principles are given. | |
axes, modelling of nonlinear loads. The developed method of calculating the bearing capacity of structures and working loads in a fire [6] allows to predict changes in the condition of buildings, to develop scenarios of dangerous situations taking into account different combinations of heating, to develop proposals to ensure the required fire resistance.

Figure 1. Scheme of calculation of fire resistance of structures.

3. Mathematical nonlinear compression model of stress-strain diagram for concrete at elevated temperatures

An important problem is the lack of clear mathematical nonlinear compression model of the stress-strain diagram for concrete, taking into account the action of temperature effects [5,6]. This is necessary for accurate calculations of fire resistance of building elements in software packages [7] such as Lira, Scad, Ansys and their analogues. In the standards [1,2] the ascending branch of the stress-strain diagram is described by the formula, in turn the descending branch of the stress-strain diagram for concrete has a generalized description for the use of linear and nonlinear models (Figure 2).

Such wording is unacceptable when calculating fire resistance by specified methods and when modeling the effect of fire on the structure. First, the mathematical nonlinear compression model of the stress-strain diagram for concrete at elevated temperatures when used at normal temperatures, given in norms [1,2], does not agree well with the compression model (1) of the stress-strain diagram for concrete at normal temperatures, given in the design standards [8].

\[
\frac{\sigma_c}{f_{cm}} = \frac{K_0 - \eta^2}{1 + (k-2)\eta} \\
\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \\
k = 1.05E_{cm} \frac{\varepsilon_{c1}}{f_{cm}}
\]

(1) (2) (3)
where $\sigma_c$ – compressive stress in the concrete; $f_{cm}$ – mean value of concrete cylinder compressive strength; $\varepsilon_c$ – compressive strain in the concrete; $\varepsilon_{c1}$ – compressive strain in the concrete at the peak stress.

Table 2 shows the comparative calculations of the two models of concrete compression. Second, the arbitrary interpretation of the descending branch of the diagram leads to different results of calculations, and it is difficult to assess their reliability and accuracy. Third, the formula of the reduced nonlinear compression model of the stress-strain diagram for concrete does not contain the initial modulus of elasticity of concrete, in contrast to the model given in the design standards [8].

| Compressive strain in the concrete, ppm | Compressive stress in the concrete by formula (1) in the standard [8], MPa | Compressive stress in the concrete by mathematical nonlinear compression model in the standards [1,2], MPa | Difference, % |
|----------------------------------------|------------------------------------------|------------------------------------------|--------------|
| 0                                      | 0.0                                      | 0                                        | 28.5         |
| 0.2                                    | 5.9                                      | 4.2                                      | 23.5         |
| 0.4                                    | 10.9                                     | 8.4                                      | 18.4         |
| 0.6                                    | 15.2                                     | 12.4                                     | 13.6         |
| 0.8                                    | 18.8                                     | 16.3                                     | 9.2          |
| 1                                      | 21.8                                     | 19.8                                     | 5.6          |
| 1.2                                    | 24.1                                     | 22.7                                     | 2.9          |
| 1.4                                    | 25.9                                     | 25.1                                     | 1.2          |
| 1.6                                    | 27.1                                     | 26.8                                     | 0.2          |
| 1.8                                    | 27.8                                     | 27.7                                     | 0.0          |
| 2                                      | 28.0                                     | 28.0                                     | 0.0          |

Figure 2. Mathematical model of the stress-strain diagram of compressed concrete at elevated temperatures.
Mathematical nonlinear compression model of the stress-strain diagram for concrete must be developed taking into account the following requirements:

- be a continuous mathematical function;
- have the extremum \( f_{c,\theta} \) at the point \( \varepsilon_{c1,\theta} \);
- at the starting point \( \varepsilon = 0 \) the differential of the function is equal to the modulus of deformation \( E_c \);
- be a nonlinear mathematical function;
- include the temperature variable \( \theta \).

In generalized form it will be obtained the following:

\[
\sigma = F\left(f_{c,\theta}, \varepsilon, \varepsilon_{c1,\theta}, \theta\right) \tag{4}
\]

\[
\frac{d}{d\varepsilon}\sigma = 0 \quad \text{at} \quad \varepsilon = \varepsilon_{c1,\theta} \tag{5}
\]

\[
\frac{d}{d\varepsilon}\sigma = E_c \quad \text{at} \quad \varepsilon = 0 \tag{6}
\]

where \( \sigma \) – compressive stress of concrete.

4. The value of relative ultimate deformation of concrete

Faultiness of values of relative ultimate deformation of concrete \( \varepsilon_{c1,\theta} \) at elevated temperatures [9] in these standards [1, 2] should be noted. The increased value of the relative ultimate deformation of concrete \( \varepsilon_{c1,\theta} \) at elevated temperatures leads to overestimated values of displacement of structural elements without their destruction [10]. This in turn increases the calculated value of the fire resistance of the structural element and the fire resistance of the building as a whole. This makes possible situations to arise when an element of the building should perform its functions [11], but in fact it is already destroyed according to the calculation.

To determine the relative ultimate deformation of concrete \( \varepsilon_{c1,\theta} \) it is possible to use the criterion of specific potential deformation energy of deformation (fourth strength theory) [12]. As a criterion of strength in this case, the amount of specific potential energy of deformation accumulated by the deformed element is chosen. According to this theory, a dangerous state in the general case of a stress state occurs when the specific potential energy of deformation reaches its limit value.

Thus, this adds an additional requirement to the stress-strain compression diagram for concrete at elevated temperatures. The stress-strain diagram for concrete should give an accurate definition of the relative ultimate deformation of concrete \( \varepsilon_{c1,\theta} \) as an extremum of the potential deformation energy at a given point (Figure 3).

\[
\frac{d}{d\varepsilon}U = 0 \quad \text{at} \quad \varepsilon = \varepsilon_{cu1,\theta} \tag{7}
\]

where \( U \) – potential deformation energy of concrete compression.

![Figure 3](image_url)
Determination of real values of relative limit deformation of concrete $\varepsilon_{c,\lambda,\theta}$ at elevated temperatures is possible from experimental test data of samples (prisms, cylinders) of all classes of concrete at different values of elevated temperatures using a real stress-strain compression diagram [13,14]. Obtaining a real diagram of stress-strain with a descending branch is possible only in special presses that have a high rigidity of the structure compared to conventional ones [15]. In such installations the accumulation of potential deformation energy in the power elements of the unit (rods, traverses, frames) is minimized, which allows to fully investigate the descending branch of the stress-strain diagram [16].

5. Conclusions
1. The problems of fire resistance calculation of reinforced concrete beams and columns are scientifically grounded; they arise when the current standards are used.
2. The requirements and sequence of calculation are formulated applying the specified calculation methods.
3. The disadvantages for the existing model of compression of the stress-strain diagram for concrete at elevated temperatures have been stated. Requirements for formulating a more accurate chart model have been specified.
4. The faultiness of values of the relative ultimate deformation of concrete at elevated temperatures, which is given in the current standards, has been determined. An additional requirement for the stress-strain compression diagram for concrete at elevated temperatures has been formulated.

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