THE COMPARISON BETWEEN SELLING AND LEASING FOR NEW AND REMANUFACTURED PRODUCTS WITH QUALITY LEVEL IN THE ELECTRIC VEHICLE INDUSTRY

Kai Li\textsuperscript{1,2}, Tao Zhou\textsuperscript{1,*} and Bohai Liu\textsuperscript{1,3}

\textsuperscript{1}School of Management, Hefei University of Technology, Hefei, China
\textsuperscript{2}Key Laboratory of Process Optimization and Intelligent Decision-making Ministry of Education, Hefei, China
\textsuperscript{3}Research Center of Industrial Transfer and Innovation Development Hefei University of Technology, Hefei, China

(Communicated by Gerhard-Wilhelm Weber)

Abstract. Process uncertainty makes remanufacturing operations management sophisticated. To reduce the uncertainty of the timing, quality and quantity of product returns in remanufacturing, motivated by the selling and leasing of electric vehicle batteries, we consider a monopolist vendor who markets her products by adopting two models: (1) a single leasing model, and (2) a single selling model. We first investigate the firm’s marketing model with remanufacturing and analyze the impact of the quality level of the returned products on the firm’s marketing and remarketing models. Then we compare selling and leasing models. We first find that only when the quality level of returned sold products is relatively high will the vendor choose to remanufacture under the single selling model. Conversely, only when the quality level of returned leased products is relatively low will the vendor decide to remanufacture under the single leasing model. Secondly, we show that the space of remanufacturable quality level under the single selling model is bigger than the space under the single leasing model. Thirdly, selling is more profitable than leasing when the quality level of returned sold products is sufficiently high. These results are further demonstrated by a numerical study. Our study provides firms with guidance on how to optimally adopt remanufacturing and marketing strategies that take into account the quality level of the returned products.

1. Introduction. Landfilling the end-of-life or end-of-lease products directly increases the burden on the environment. How to manage these used products is the focus of academic and business domain. One of firms’ options is to remanufacture these products before they are disposed of as waste. Remanufacturing has focused on operational issues that involve disassembling, inspecting, repairing, replacing, and reassembling the components of a part or product to as-new condition, and selling them again [4, 20, 32]. Compared with the production of new parts or products, remanufacturing can save costs for the firm, and reduce the consumption of resources and environmental impact [35].

2010 Mathematics Subject Classification. Primary: 90B60; Secondary: 91B16, 91B24.

Key words and phrases. Selling, leasing, remanufacturing, quality level, electric vehicle battery.

The first author is supported by National Natural Science Foundation of China under grants 71871076, 71521001, 71531008, 71690235.

* Corresponding author: Tao Zhou.
Product returns for remanufacturing are based on previous marketing models including leasing and selling. However, product returns for remanufacturing under selling may be viewed as highly uncertain about quantity, quality, and timing of used products [13]. On the contrary, leasing is an incentive for customers to return used products, and a strategy for firms to control the process uncertainty in remanufacturing. For example, Xerox adopts a leasing strategy to ensure its used photocopiers are returned [33]. Many firms, besides Xerox, Caterpillar, GE, IBM, and Pitney-Bowes (P-B), lease products, and then remanufacture them to capture the residual value of leased items [12]. Under leasing, customers only have the right to use rather than ownership. The ownership is owned by the firms [24]. Hence, customers are more willing to take the initiative to return the off-lease items to firms.

Leasing is changing traditional business model, selling a service rather than a product. Many firms provide leasing arrangements for sizeable proportion of their products. For example, Michelin has sold tyre use “by the mile” to operators of vehicle fleets. Worn tyres are sent to Michelin’s regional plants to retread and reuse [27]. Xerox sells the services of imaging equipment rather than the physical product. Hence, Xerox can continuously incorporate technological and other changes that improve the customer experience through field service or more extensive overhauls by maintaining ownership of the equipment [1]. Renault who is an electric car manufacturer chooses to lease batteries to customers instead of selling them [35]. When the batteries no longer work, the manufacturer can re-engineer or recycle them for future use.

Standridge and Corneal [28] estimate that the number of post-vehicle application battery packs would rise from 1.4 million to 6.8 million by 2035. In addition, according to data of Germany’s nonprofit Centre for Solar Energy and Hydrogen Research Baden-Württemberg (ZSW), the sales of new energy automobiles in China reached 1.2 million in 2018, accounting for more than 50% of the global market in the same period [15]. Generally, the battery life is 5 to 8 years, which means that new energy automobiles put into the market around 2012 will gradually face the “retired” period of batteries from 2018. Both theoretical and practical data suggest that the rapid development of new energy automobiles has led to management problems of battery degradation. If these end-of-life or end-of-lease batteries are improperly disposed of, it will cause economic loss of firms and worse environmental impact.

Standridge and Corneal [29] propose three possibilities for managing post-vehicle-application batteries: (1) remanufacturing for intended reuse in vehicles, (2) repurposing for non-vehicle, stationary storage applications, (3) recycling, extracting the precious metals, chemicals and other byproducts. Even if the battery degrades, about 80% of the battery initial capacity is still retained [41]. High capacity retained enables firms to remanufacture the battery after first life cycle. Remanufacturing of the battery makes it to be in “like-new condition” for reuse. This way can lead to a 25% reduction in demand for new batteries and create economic value [29]. Due to the difference in battery degradation, the qualities of returned batteries are also different. Thus, the issue that firms face is within what range of the quality level of the returned products is it more advantageous for the firm to undertake remanufacturing?

Many countries, including 27 Member States of the European Union [38] have introduced legislation based on the concept of extended producer responsibility (EPR)
to prompt product recovery. *Interim Measures for the Management of Recycling and Utilization of Power Battery for New Energy Automobiles* stipulate that in order to implement EPR, automobile manufacturers shall undertake the main responsibility for the recovery of the battery. Automobile manufacturers now have to concern themselves with battery recovery. For most firms, when they market new energy automobiles, they only sell the ownership of the automobiles that do not include the batteries. Recently battery leasing has been introduced by automobile manufacturers and power suppliers to reduce the cost of both the operators and customers [17]. The cost of the battery cannot be ignored because it is expensive [6]. Therefore, firms are faced with the choice of product marketing model, i.e., whether selling or leasing their products?

Motivated by the remanufacturing and marketing problems faced by firms and our recent interactions with a group of managers at electric vehicle vendors that offer remanufactured versions of their products, in this paper, we consider a monopolist firm marketing electric vehicles which belong to the category of new energy automobiles. Except for the battery, other ownership of the car is directly sold to customers. The battery is marketed through two marketing models. One is to directly sell the ownership, and the other is to sell the use of a product. The latter is a leasing model. Both the new products sold and leased at the end of their service life are returned from customers for remanufacturing, and then for remarketing. The returned products have heterogeneous quality levels. Our goal is to find under which conditions that the firm would sell its products, or lease its products, or choose a combination of the both, and under which conditions that the firm would make a decision to remanufacture. We also calculate the optimal quantity of selling or leasing for new and remanufactured products. Further, we derive optimal marketing strategies with heterogeneous quality levels of returned products.

The basic structure of our model parallels previous research in the marketing literature. We assume the firm markets a product in a two-period model. In period 1, the firm sells only the battery, or leases only the battery, or sells and leases the battery together. In period 2, used products (sold or leased in the previous) require a remanufacturing operation before being remarkeved. The supply of used products depends on the previous sales or leases of new products. In our model, we assume the returned products are partially remanufactured. The intuition behind this assumption is as follows. In the case of leases, not all returned products can be remanufactured due to improper use by customers. In the case of sales, not all used product sold in the previous period can be bought back because some customers choose to continue using the product and are not willing to return them. In particular, we consider three marketing models for new and remanufactured products: (1) The vendor sells new products in period 1, and sells the new product and simultaneously the sold remanufactured product in period 2. (2) The vendor leases new products in period 1, and sells the new product and simultaneously the leased remanufactured product in period 2. (3) The vendor sells and leases new products in period 1, and sells the new product and simultaneously the remanufactured product in period 2. The questions of under what conditions does the vendor choose to remanufacture and whether leasing model or selling model of the vendor is optimal for new and remanufactured products still remains to be answered.

We assume that if customers choose to use the product, they can do so by either buying or leasing the new product or the remanufactured product based on their maximum utility. No matter which way they choose to use the product, they
replace the products periodically. We consider customers differ in their valuations of different products. We analyze these scenarios and provide insights regarding firm’s decisions about remanufacturing and marketing strategies.

We now summarize our key findings: Under the single leasing model, the firm will choose to remanufacture when the quality level of the returned product falls into a moderate range. In this case, the firm can control the quality level of the returned products to promote profitability and reduce the use of resources by remanufacturing. Under the single selling model, the firm will choose to remanufacture when the quality level of the returned product falls into a high range. In this case, the firm should encourage customers to regularly maintain products to improve the quality of the returned sold product, therefore increasing sales of remanufactured products and profits.

The comparison between selling and leasing for new products and remanufactured products generates following observations. First, the range of remanufacturable quality level under the single selling model is higher than the range of remanufacturable quality level under the single leasing model. That is, the firm is more likely to remanufacture under the single selling model. Second, we show that the lower bound of the quality level of the returned product under the selling model is higher than the upper bound of the quality level of the returned product under the leasing model. Therefore, the vendor chooses to remanufacture when the quality level of returned products is high under the single selling model, and choose to remanufacture when the quality level of returned products is low under the single leasing model. Third, we find that selling is more profitable than leasing when the quality level of returned sold products is sufficiently high compared with the quality level of returned leased products. Therefore, our results suggest that selling is more profitable and environmentally superior when the firm chooses to remanufacture if the quality level of returned sold products is sufficiently high.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes our model. In Section 4, we analyze the three marketing scenarios to see how they are affected by the quality level of the returned product. We conduct a numerical study to characterize the difference between these scenarios in Section 5. Conclusions and future research directions are presented in Section 6.

2. Literature review. Our research draws on several streams of literature, each of which we review below.

The literature on leasing versus selling is discussed. The operation characteristics of sales and leases have been investigated. Firms choosing to lease products have a highest control over product information, decreasing uncertainty in the quantity, quality and return of used products [24, 31]. Waldman [36] shows that leasing is a profitable option because it allows the monopolist to scrap the used goods, thereby lessening the competitive threat they pose to the new goods. Agrawal et al. [2] investigate leasing versus selling strategy for a firm. They find that the profitability and environmental impact of each strategy depends on product durability and consumer type. Desai and Purohit [11] study the effect of the depreciation rates of leased and sold units on the profitability of leasing and selling. They explain that the coexistence of leasing and selling is an optimal strategy for a durable goods firm. Lim et al. [18] consider four business models based on the type of battery ownership and battery charging options. They observe the impact of the range and resale anxieties on mass adoption of electric vehicles. They also contrast firm’s profit,
consumer surplus, adoption size, and emission savings in different business models by using numerical study. From the perspective of the uncertainty of customers’ purchasing behaviours, Zhang et al. [40] construct a triple-channel supply chain model to analyze the impacts of the consumers risk perception and the benefit perception on customers’ purchasing decisions and corporations’ pricing and channel selection strategies. They further explain the practical significance of the analysis through a case study. This stream of literature studies leases and sales regarding new products and used products, but does not combine remanufactured products. We extend this literature by introducing a firm to implement remanufacturing.

Firms engaged in remanufacturing face different relationships in closed-loop supply chains. Östlin et al. [24] present seven different types of closed-loop relationship between remanufacturers and customers for collecting used products for remanufacturing. Robotis et al. [25] discuss leasing both new and remanufactured products, and determine optimal leasing price and duration. Mont et al. [21] develop a product-service system incorporating a reverse logistics system with refurbishment and remanufacturing of baby prams. They present a new business model for leasing the pram and conduct financial analysis of the proposed system. Aras et al. [3] analyze a multi-period model where a profit-maximizing firm can lease new products and sell remanufactured products. They consider excess returned product inventory and insufficient product inventory scenarios. Loon et al. [19] propose a circular model where a firm can lease new products and refurbished products to customers. They construct cost components, and analyze the profitability of the manufacturer and the total cost of ownership of customers. Likewise, Steeneck and Sarin [30] investigate the part durability design of a firm who leases new products and recovers used products. Different durability levels affect the optimal quantity of the new products, the remanufactured products, and the recycled parts. Li and Xu [16] compare a trade-in model with a leasing model under stochastic technology innovation for technology products. They consider two types of value decay (functional depreciation and technological obsolescence), and analyze the impact of consumers’ willingness-to-pay and trade-in credit on consumer behavior, and interpret firm’s profitability. Yalabik et al. [39] study a monopolist’s remanufacturing decision in a new goods market and a secondary remanufactured goods market. They develop an economic model incorporating lease contracting, product design and remanufacturing volume. They compare a green firm and traditional firm in terms of their profitability and environmental performance. One of the important factors that affect the profitability of a firm in remanufacturing is cost. Different from most of the literature, we consider the remanufacturing cost is related to the quality level of the returned product rather than a constant.

These literature considering remanufacturing concentrate on return of the sold products, but the return of the leased products is rarely of concern. In the literature on product remanufacturing, the main source of product returns is the used products sold in previous period [26]. Guide et al. [14] develop a framework that determines the optimal acquisition prices and selling prices to maximize profits. Atasu and Souza [5] consider quality recovery inducing components or material, profitable material recovery inducing components or material, and costly recovery. They analyze how the effect of the three product recovery forms on quality and environmental impact. Mutha et al. [23] discuss the acquisition strategy and the impact of the lifecycle of the remanufactured product on the acquisition policy of a third party remanufacturer (3PR). They investigate a 3PR acquires used products
or cores in bulk with uncertain quality levels, or in sorted grades with known quality levels, and a 3PR acquires and remanufactures cores with planned acquisition or reactive acquisition. They develop a two-period sequential model to balance the acquisition and remanufacturing costs under an uncertain demand. Wang et al. [37] investigate a retailer adopts an in-house or outsourced strategy for remanufacturing. They consider the cost structures of the two strategies, uncertainty in the quality of the returned used products, consumer willingness-to-pay for remanufactured product, the possibility that the remanufactured product cannibalizes the sales of the new product, and the power structure in the channel. Considering uncertainty in remanufacturing, we extend that the return of used products comes from two marketing models: a single leasing model and a single selling model.

As shown in Table 1, from the perspective of marketing strategy, the existing literature combines product performance, used product management and customer behavior, however these studies do not fully reflect the residual value of the used products by selling them in the secondary market or directly discarding them. We complement this literature by considering a firm’s marketing decisions based on remanufacturing to maximize the economic value of used products. From the perspective of remanufacturing strategy, in comparison to the extant literature, we highlight the critical role of remanufacturing cost associated with quality, which inspires firms to make more favorable operational decisions.

3. Model description. In this section, we describe our assumptions regarding the product, the vendor, and customers. To motivate remanufacturing and marketing issues associated with leases and sales, we choose the new energy automobile industry as our running example. However, the reader should note that our analysis applies readily to remanufacturable and durable goods that can be leased and sold. The product that we consider is an electric vehicle battery referred to as battery hereafter. There can be three types of batteries marketed in the consumer market: new batteries, leased remanufactured batteries, and sold remanufactured batteries. Remanufactured batteries are previously used by customers. After being remanufactured, they are now offered to market by the vendor. Thus, the key difference between new and remanufactured batteries is raw material, i.e., whether the materials are new or used. Without loss of generality, the used products can only be

| Table 1 Some Key Literature on Remanufacturing and Marketing Strategies |
|--------------------------------------------------|
| **Marketing strategy** | Selling | Leasing | Combination of selling and Leasing |
| Remanufacturing strategy | | |
| Remanufacturing | Product performance (durability, depreciation, quality, etc.) | Mont et al. [21] | Steeneck and Sarin [30] |
| | Product returns (reverse logistics, acquisition policy, inventory etc.) | Guide et al. [14] | Leen et al. [19] |
| No remanufacturing | Product performance | Agrawal et al. [2] | Desai and Purohit [11] |
| | Product returns (secondary market or discarding) | Waldman [36] | |
remanufactured once. Leased (Sold) remanufactured batteries are remanufactured versions of new batteries leased (sold) in the previous period.

Therefore, we consider a two-period model. In this model, it is assumed that the battery lasts for exactly two periods. Considering the finite lifetime of the products, we adopt the model used by most researchers [2, 8, 9, 10, 11, 16]. However, the effect of time inconsistency leads to the distinction between selling and leasing. In period 1, only new batteries are provided in the market. In period 2, as mentioned earlier, there can potentially be three types of batteries available in the market in accordance with a specific scenario. Let the subscript \( k = 1, 2 \) represent the respective period 1 and period 2 throughout this paper. We use \( \gamma \in (0, 1) \) as a discount factor signifying the value of future cash flows.

We introduce a monopoly vendor selling or leasing electric vehicle batteries to maximize her profits. The firm focuses on three marketing models: single leasing, single selling, and the combination of both leasing and selling. Let \( m \in \{L, S, C\} \) represent the respective model. Under the single leasing model, customers only have the right to use. Under the single selling model, the ownership of a battery is owned by the customer. Under the combination model of both leasing and selling, the vendor offers leasing services for new products and sells new products simultaneously. Our objective is to investigate the firm’s marketing decisions that choosing single selling, single leasing, or a combination of both leasing and selling, to propose the conditions of remanufacturing, and to understand how the firm’s decisions are affected by the quality level of the returned product.

Due to inconsistent use of products by heterogeneous customers, the used products in the previous period deteriorate at different rates [2, 10, 11]. In this paper, we assume that the used products have different quality levels based on their residual values at the end of period 1, proposed by [5, 12, 34]. In different market models, the quality levels of product returns are different. Let \( v_{ij} \) be the value of a returned product at a certain quality level, \( i \in \{l, s\}, j \in \{n, r\}, v_{in} = 1, v_{ir} \in [0, 1) \). Where the subscript \( l \) and \( s \) represent the vendor’s respective decisions to lease or to sell the product. Where the subscript \( n \) and \( r \) refer to the new and remanufactured product, respectively. Where \( v_{ij} = 0 \) is total deterioration or zero residual value, and \( v_{ij} = 1 \) is the highest possible quality of a used product or a brand new. Generally, leased products deteriorate at a higher rate than sold products, because of moral hazard issues that previous users did not own the battery leading to greater damage to the product. Thus, quality level of the returned sold product is higher than the returned leased product, i.e., \( v_{sj} \geq v_{lj} \). The quality level represents the degree of substitutability among new, sold remanufactured and leased remanufactured products.

Let \( c \) be the marginal cost of production for a new product in each period, then \( c(v_{jn}) = c(v_{jn}) = c \). Let \( c(v_{ir}) > 0 \) be the marginal cost of production for a remanufactured product. More specifically, \( c(v_{sr}) = c - cv_{sr}^2 \), where \( cv_{sr} \) represents cost savings for remanufacturing the returned sold product with quality level \( v_{sr} \), and \( 2cv_{sr} < 1 \) represent cost savings for per quality. \( c(v_{lr}) = c - cv_{lr} - cv_{lr}^2 \), where \( cv_{lr} + cv_{lr}^2 \) represents cost savings for remanufacturing the returned leased product with quality level \( v_{lr} \), and \( v_{lr} + 2cv_{lr} < 1 \) represent cost savings for per quality. The cost of remanufacturing associates with different quality levels. The better the quality, the lower the cost to remanufacture, that is, \( c(v_{ir}) \) is decreasing in \( v_{ir} \). Producing a remanufactured product by using a returned product is less costly than
manufacturing a new one. This assumption has been presented in several studies [5, 12].

Following the consumer preferences of Moorthy [22], the consumer willing-to-pay $\theta$ of quality is heterogeneous and uniformly distributed over a support. Without loss of generality, we normalize to $\theta \in [0, 1]$. Consistent with the model adopted by many scholars [7, 9], we assume that a customer can only use at most one unit of the product in a certain period. The market size is normalized to 1.

We follow the general approach that consumer utility is derived from the consumers’ valuations and the value provided by the product. Given the quality level of the product $v_{ij}$ and its price $p_{kij}^m$, where $p_{kij}^m$ denotes the one-period price for the product $j$ of decision $i$ in period $k$ under market model $m$. Depending on the marketing model decided by the vendor, we show later that $p_{kij}^m$ can be either a leasing or a selling price. A consumer of type $\theta$ receives a net utility $U_{mkij}^m = v_{ij} - p_{kij}^m$. $v_{ir}$ is also interpreted as the relative willingness to pay for the remanufactured product compared to the new product. As mentioned before, a reasonable assumption in our model is that, ceteris paribus, each customer prefers a new battery to a sold remanufactured battery to a leased remanufactured battery, i.e., $v_{in} \geq v_{sr} \geq v_{lr}$.

The price paid by a customer who buy the product in period 1 reflects the customer’s potential use of the services of the product over both periods. Thus, we make the reasonable assumption that the selling price in period 1 is equivalent to the sum of the one-period prices in periods 1 and 2. As mentioned earlier, the products lasts for two periods, thus the selling and leasing in period 2 have the same market effect. This construction leads to the following linear inverse demand functions:

\[
\begin{align*}
    p_{2 in}^m &= 1 - q_{2 in}^m - xv_{sr}q_{2 sr}^m - yv_{lr}q_{2 lr}^m, \\
    p_{2 sr}^m &= x\{v_{sr}(1 - q_{2 sn}^m - q_{2 sr}^m) - yv_{lr}q_{2 lr}^m\}, \\
    p_{2 lr}^m &= yv_{lr}(1 - q_{2 in}^m - xq_{2 sr}^m - yq_{2 lr}^m), \\
    p_{1 in}^m &= y(1 - xq_{1 sn}^m - yq_{1 ln}^m), \\
    p_{1 sn}^m &= x\{(1 - xq_{1 sn}^m - yq_{1 ln}^m) + \gamma(v_{sr}(1 - q_{2 sn}^m - q_{2 sr}^m) - yv_{lr}q_{2 lr}^m)\}.
\end{align*}
\]

$x, y \in \{0, 1\}$ are binary variables, and represent the decision of the vendor. $x = 1, y = 0$ indicates that the vendor only conducts a sales model. $x = 0, y = 1$ represents that the vendor only adopts a lease model. $x = 1, y = 1$ states that the vendor chooses a combination model of both selling and leasing. Corresponding proofs are given in each model that follows. The notation used in the article is summarized in Table 2.

| Symbol | Definition |
|--------|------------|
| $\gamma$ | A discount factor signifying the value of future cash flows |
| $v_{ij}$ | The quality level for product $j$ under decision $i$ |
| $U_{mkij}^m$ | The utility of a customer for product $j$ of decision $i$ in period $k$ under market model $m$ |
| $c(v_{ij})$ | The marginal cost of product $j$ of decision $i$ |
| $p_{kij}^m$ | The one-period price for the product $j$ of decision $i$ in period $k$ under market model $m$ |
| $q_{kij}^m$ | The period $k$ demand for product $j$ of decision $i$ under market model $m$ |
| $\pi^m_k$ | The profits under market model $m$ in period $k$ |
| $\pi^m$ | The total profits under market model $m$ |

4. Model analysis. In this section, we construct three marketing models for the new and remanufactured products and analyze how they are affected by the quality...
levels of the returned products. In each period, the vendor makes decisions about the quantity of new and remanufactured products to maximize her profits. We adopt subgame perfect equilibrium which is a common approach to solve our problem. To ensure that our solutions are time-consistent, the timing of the game is as follows. First, the firm determines the optimal quantity in period 2. Next, the firm solves the problem in period 1.

4.1. Single leasing model for new and remanufactured products. In this section, we assume that the products can only be leased (identified by superscript \( L \)). The vendor determines the optimal quantity of batteries to lease in both periods to maximize her profits. Let \( q_{L2n} \) and \( q_{L2r} \) denote the second period demands for respective new products and leased remanufactured products, and \( q_{1ln} \) denote the first period demand for new products. At the end of period 1, the leased products are returned by lessees. However, some returned leased products are overused and cannot be remanufactured. Therefore under this circumstance, all returned products can only be partially remanufactured, i.e., \( q_{L2r} < q_{1ln} \). After being remanufactured, they are again leased on the market. As mentioned before, the quality level of the returned leased product is \( v_{lr} \in [0, 1) \). Let \( p_{L2ln} \) denote the leasing price of a (new or leased remanufactured) product in period 2, \( p_{L1ln} \) denote the leasing price of a new product in period 1. In this case, following Equations (1), the inverse demand functions are as follows:

\[
p_{L1ln} = 1 - q_{1ln}, \tag{2}
\]

\[
p_{L2ln} = 1 - q_{2ln} - v_{lr}q_{2lr}, \tag{3}
\]

\[
p_{L2lr} = v_{lr}(1 - q_{2ln} - q_{2lr}). \tag{4}
\]

Proof. See Appendix A.

The profits of the vendor in period 2 can be expressed as:

\[
\max_{q_{2ln}, q_{2lr}} \pi_{2}^L = (p_{L2ln} - c)q_{2ln} + (p_{L2lr} - c(1 - v_{lr} - v_{lr}^2))q_{2lr} \tag{5}
\]

s.t. \( q_{2lr} < q_{1ln} \) \tag{6}

We use superscript \( * \) to denote the optimal value of the corresponding decision variables. We derive the optimal second-period quantity of new and leased remanufactured batteries.

\[
q_{2ln}^* = \frac{1 - v_{lr} - cv_{lr} - cv_{lr}^2}{2(1 - v_{lr})} \tag{7}
\]

\[
q_{2lr}^* = \frac{c(v_{lr}^2 + 2v_{lr} - 1)}{2v_{lr}(1 - v_{lr})}. \tag{8}
\]

After deriving optimal second-period solutions, we now calculate the optimal first-period quantity. Therefore, we maximize the vendor’s total two-period profits. The vendor’s first-period profit is \( \pi_{1}^L = (p_{1ln}^L - c)q_{1ln}^L \). The vendor’s total profits over both periods are:

\[
\max_{q_{1ln}} \pi_{1}^L = (p_{1ln}^L - c)q_{1ln}^L + \gamma \pi_{2}^L \tag{9}
\]

This yields:

\[
q_{1ln}^* = \frac{1 - c}{2} \tag{10}
\]
The impact of the quality level of the returned leased product (\(v_{1r}\)) is as follows:

- The optimal quantity of new products leased in period 2 decreases as \(v_{1r}\) increases.
- The optimal quantity of leased remanufactured products increases as \(v_{1r}\) increases.
- The optimal leasing price of the leased remanufactured product increases as \(v_{1r}\) increases.
- The optimal total profits of both periods for the firm increase as \(v_{1r}\) increases.

Proof. See Appendix D.

Optimal leasing prices and total profits can easily be calculated by substitution of \(q_{11n}^*, q_{21n}^*, q_{22r}^*\). The results are listed in Table 3.

**Lemma 4.1.** To ensure positive demands and profits, the quality level of the returned leased product \(v_{1r}\) and production cost \(c\) satisfy the following constraints:

- \(\sqrt{2} - 1 < v_{1r} < \frac{\sqrt{5} - 1}{2}\),
- \(0 < c \leq \min\left\{\frac{v_{1r} - 1}{2v_{1r} - 1}, \frac{1 - v_{1r}}{v_{1r} + v_{1r}^*}, \frac{1}{1 + 2v_{1r}}\right\}\).

Proof. See Appendix C.
the higher the total profits of both periods. This implies that when the quality level of the returned leased product is high, the vendor can achieve higher profits by remanufacturing. In this case, the vendor also reduces the use of resources due to the reduction in production of new products.

As list in Table 3, the leasing price of the new product and the quantity of the new product leased in period 1 are not affected by the quality level of the returned leased product. This is because customers’ lease behavior occurs in two separate periods and the firm implements partial remanufacturing under this model. Generally, leasing prices for new and remanufactured products are identical if the quality of the remanufactured products is “as good as new” [31]. However, the leasing price of a new product is higher than a leased remanufactured product when considering the different quality levels of returned products. The rationale behind this is that the cost savings of remanufacturing brings the price advantage of the remanufactured product.

4.2. Single selling model for new and remanufactured products. In this section, we model the case where the vendor only sells her product (identified by superscript $S$). The vendor sells $q_{1{sn}}^S$ new products in period 1, and additional $q_{2{sn}}^S$ new products, as well as $q_{2{sr}}^S$ sold remanufactured products in period 2. At the end of period 1, considering the uncertainty of customer behavior, only partial sold products are returned by users. Therefore, $q_{2{sr}}^S < q_{1{sn}}^S$. The quality level of returned sold products is $v_{sr} \in [0,1)$. Let $p_{2{sn}}^S$ denote the selling price of a (new or sold remanufactured) product in period 2, $p_{1{sn}}^S$ denote the selling price of a new product in period 1. Every customer will anticipate the second-period price for new customers and for customers who purchase the sold remanufactured products. Following Equations (1), the inverse demand functions are as follows:

\[
p_{1{sn}}^S = 1 - q_{1{sn}}^S + \gamma p_{2{sr}}^S, \\
p_{2{sn}}^S = 1 - q_{2{sn}}^S - v_{sr} q_{2{sr}}^S, \\
p_{2{sr}}^S = v_{sr} (1 - q_{2{sn}}^S - q_{2{sr}}^S).
\]

**Proof.** See Appendix A.

We first solve the vendor’s problem in period 2. The vendor’s profits in period 2 are given by:

\[
\max_{q_{2{sn}}, q_{2{sr}}} \pi_2^S = (p_{2{sn}}^S - c)q_{2{sn}}^S + (p_{2{sr}}^S - c(1 - v_{sr}))q_{2{sr}}^S
\]

s.t. $q_{2{sr}}^S < q_{1{sn}}^S$.

Hence, the vendor’s optimal second-period quantity of new batteries and sold remanufactured batteries is:

\[
q_{2{sn}}^S = \frac{1 - v_{sr} - cv_{sr}^2}{2(1 - v_{sr})}, \\
q_{2{sr}}^S = \frac{c(v_{sr}^2 + v_{sr} - 1)}{2v_{sr}(1 - v_{sr})}.
\]

The vendor’s first-period profit is $\pi_1^S = (p_{1{sn}}^S - c)q_{1{sn}}^S$. The vendor’s total profits from both periods are:

\[
\max_{q_{1{sn}}} \pi_1^S = (p_{1{sn}}^S - c)q_{1{sn}}^S + \gamma \pi_2^S
\]
This yields:

\[
q_{1sn}^S = \frac{1 - c}{2} + \frac{\gamma(v_{sr} + c(1 - v_{sr}^2))}{4}
\]  

(19)

Optimal selling prices and total profits can easily be calculated by substitution of \(q_{1sn}^S\), \(q_{2sn}^S\), and \(q_{2sr}^S\). The results are listed in Table 3.

**Proof.** See Appendix E. \[]

Similar to Lemma 4.1 and Proposition 1, Lemma 4.2 and proposition 2 are proposed as follows. The proofs are omitted.

**Lemma 4.2.** To ensure positive demands and profits, the quality level of the returned sold product \(v_{sr}\), production cost \(c\), and discount factor \(\gamma\) satisfy the following constraints:

- \(\sqrt{\frac{5}{2}} - 1 < v_{sr} < 1\),
- \(0 \leq c \leq \min\left\{\frac{1}{2v_{sr}}, \frac{1-v_{sr}}{v_{sr}}, \frac{v_{sr}-2v_{sr}^2+\gamma v_{sr}^2-\gamma v_{sr}^3}{4v_{sr}^2-2\gamma v_{sr}+\gamma^2 v_{sr}^2-\gamma^2 v_{sr}^3}\right\}\).

Lemma 4.2 shows that the vendor chooses to remanufacture if the quality level of the returned sold product is high enough. The higher the quality level of the returned sold product, the lower the space of the production cost. Therefore, the vendor can’t reduce the production cost at the expense of product quality.

**Proposition 2.** The impact of the quality level of the returned sold product \((v_{sr})\) is as follows:

- The optimal quantity of the new products sold in period 2 decreases as \(v_{sr}\) increases.
- The optimal quantity and selling price of the sold remanufactured product increase as \(v_{sr}\) increases.
- The optimal quantity and selling price of the new product sold in period 1 increase as \(v_{sr}\) increases.
- The optimal total profits of both periods for the firm increase as \(v_{sr}\) increases.

Proposition 2 indicates that a higher quality level of the returned sold product reduces demand for the new product, but increases demands for the sold remanufactured product in period 2. This is because that a higher quality level means a higher willingness to pay for sold remanufactured products, which raises the demand for sold remanufactured products. For the new product sold in period 1, although higher quality level raises the selling price, it raises sales volume. This implies that a higher customer valuation for the sold remanufactured product in period 2 can improve the sales volume of the new product in period 1. The increases in quality level of the returned sold product leads to increases in total profits of both periods. This creates an incentive for the vendor to improve the quality level of the returned sold product. The firm can make a commitment that customers can buy new products at a discount price if they return high quality used products.

According to the results recorded in Table 3, the second-period selling price of the new product is only related to the production cost if the vendor only acquires partial products sold in period 1. The selling price of the new product is higher than the selling price of the sold remanufactured product in period 2.
4.3. Leasing and selling combination model for new and remanufactured products. Under the combination marketing model (identified by superscript $C$), the vendor sells three types of products in period 2. Because period 2 is the last period, this implies that the selling price of a new product is equivalent to the leasing price of a new product, that is, $p_{1n}^C = p_{2n}^C$. Let $q_{1n}^C$ denote second period demand for new products. Let $q_{2sr}^C$ and $q_{2lr}^C$ denote second period demands for sold remanufactured products and leased remanufactured products, respectively.

In period 1, the firm markets $q_{1n}^C$, new batteries, of which $q_{1sn}^C = \omega q_{1n}^C$ are sold and $q_{1ln}^C = (1 - \omega)q_{1n}^C$ are leased. $\omega$ is the fraction of the first period sales. The leasing price of a new battery is $p_{1n}^C$. A portion of new batteries sold (leased) in period 1 becomes sold (leased) remanufactured products in period 2, i.e., $q_{1sn}^C > q_{2sr}^C$ ($q_{1ln}^C > q_{2lr}^C$).

Following Equations (1), the first-period and second-period inverse demand functions are as follows:

\[
\begin{align*}
  p_{1n}^C &= 1 - q_{1n}^C, \\
  p_{1sn}^C &= 1 - q_{1n}^C + \gamma P_{2sr}^C, \\
  p_{2n}^C &= 1 - q_{2n}^C - v_{sr}q_{2sr}^C - v_{lr}q_{2lr}^C, \\
  p_{2sr}^C &= v_{sr}(1 - q_{2sn}^C) - v_{sr}q_{2sr}^C - v_{lr}q_{2lr}^C, \\
  p_{2lr}^C &= v_{lr}(1 - q_{2sn}^C - q_{2sr}^C - q_{2lr}^C).
\end{align*}
\]

Proof. See Appendix A. \qed

The vendor’s profits in period 2 can be expressed as:

\[
\max_{q_{2n}^C, q_{2sr}^C, q_{2lr}^C} \pi_2^C = (p_{2n}^C - c)q_{2n}^C + (p_{2sr}^C - c(1-v_{sr}^2))q_{2sr}^C + (p_{2lr}^C - c(1-v_{lr}^2))q_{2lr}^C.
\]

s.t. $q_{2sr}^C < \omega q_{1n}^C$

$q_{2lr}^C < (1 - \omega)q_{1n}^C$

The second-period optimal quantity of three batteries is given by:

\[
\begin{align*}
  q_{2n}^{C*} &= \frac{1 - v_{sr} - c v_{sr}^2}{2(1 - v_{lr})} \\
  q_{2sr}^{C*} &= \frac{c(-1 - v_{sr}^2)(1 - v_{lr}) + (1 - v_{lr} - v_{sr}^2)(1 - v_{sr}) + (v_{sr} - v_{lr})}{2(1 - v_{lr})(v_{sr} - v_{lr})} \\
  q_{2lr}^{C*} &= \frac{c((1 - v_{sr}^2)v_{lr} - (1 - v_{lr} - v_{sr}^2)v_{lr})}{2v_{lr}(v_{sr} - v_{lr})}
\end{align*}
\]

The vendor’s total profits from both periods are:

\[
\max_{\pi_1, \omega} \pi^C = \omega(p_{1sn}^C - c)q_{1n}^C + (1 - \omega)(p_{1ln}^C - c)q_{1n}^C + \gamma \pi_2^{C*}
\]

This yields:

\[
\begin{align*}
  \omega^* &= 1 \\
  q_{1n}^{C*} &= \frac{1 - c}{2} + \frac{\gamma(c + v_{sr} - c v_{sr}^2)}{4}
\end{align*}
\]
Above results indicate that the firm choose to sell the entire quantity of new products in period 1 in the case of partial remanufacturing. Therefore, the combination model of both leasing and selling is reduced to the single selling model. The analysis of this combination model is consistent with the single selling model.

Proof. See Appendix F.

4.4. The comparison between leasing and selling for new and remanufactured products. In this section, we conduct the comparison between leasing and selling with respect to prices, quantities, and profits. The differences between these models are affected by the relative quality level of returned products, respectively.

Proposition 3. The comparison between leasing and selling is as follows:

- The price of the sold remanufactured product is higher than the price of the leased remanufactured product if and only if \( v_{sr} < \sqrt{v_{lr} + v_{sr}} \).
- The quantity of leased remanufactured product is lower than the quantity of the sold remanufactured product if and only if \( \sqrt{\frac{4(2v_{lr} + v_{sr}) + 1}{2}} < v_{sr} < 1 \).
- A single selling model is more profitable than leasing model if and only if \( \sqrt{\frac{4(2v_{lr} + v_{sr}) + 1}{2}} < v_{sr} < 1 \).

Proof. See Appendix G.

The comparison between leasing and selling suggests that if the cost savings from remanufacturing under selling are smaller than the cost savings from remanufacturing under leasing, then the price of the sold remanufactured product is higher than the leased remanufactured product. The price difference between the two types of remanufactured products increases as the quality level of returned sold products, and decreases as the quality level of returned leased products. Therefore, higher quality level of returned sold products make sales of sold remanufactured products more profitable, vice versa, higher quality level of returned leased products make lease of leased remanufactured products more profitable. However, the high price of the sold remanufactured product does not make the quantity of sold remanufactured products lower than the leased remanufactured products. In addition, if the quality level of the returned sold product is sufficiently high and higher than the returned leased product, then single selling is more profitable than single leasing.

The upper bound of the quality level of the returned product under the leasing model is lower than the lower bound under the selling model. The upper (lower) bound of the quality level of the returned product means that the maximum (minimum) quality level of the returned product that the vendor is willing to remanufacture. Thus, this result suggests that the vendor is willing to remanufacture returned products from selling at a higher quality level, and remanufacture returned products from leasing at a lower quality level.

5. Numerical study and sensitivity analysis. In this section, we conduct numerical study and characterize the vendor’s optimal solution. In addition, we perform sensitivity analysis with respect to the following parameters: the quality level of the returned product (the sensitivity analysis of customers’ relative willingness to pay for a remanufactured product is consistent with the impact of quality), and production cost.
5.1. **Optimal decision and sensitivity analysis of quality under the single leasing model.** To obtain the optimal decision and understand the effect of quality level of the returned leased product (customers’ relative willingness to pay for a sold remanufactured product) under the single leasing model, as inputs, we use the parameter values $\gamma = 0.9$, and $c = 0.25$. We thus reduce the vendor’s optimal solution to a single variable $v_{lr}$ by substituting $c$ and $\gamma$. Optimal results are presented in Figs. 1a-3a.

5.1.1. **Optimal solution analysis.** Interpreted in the context of given cost and discount factor parameters, these results have the following implications. The vendor will choose to lease the new product in period 1, and to lease both the new and leased remanufactured products in period 2 if the quality level of the returned leased product meets a certain range.

Fig.1a shows that the quantity of new products leased in period 1 is higher than the quantity of leased remanufactured products leased in period 2. The rationale for this result is as follows. The vendor can only remanufacture partial returned products at the end of period 1 because of the irregular use of products by some customers. Therefore, the firm could improve the management of lease service by increasing customers’ cost. For example, the vendor can charge fees for customers’ behavior of improperly using the product. When $v_{lr}$ is relatively low, the quantity of the new products leased is higher than the quantity of leased remanufactured products leased in period 2, vice versa. In this case, the vendor should choose an appropriate quality level of the returned leased product to remanufacture for mitigating the cannibalization effect on new products.

Fig. 2a illustrates that the leasing price of the leased remanufactured product is lower than the leasing price of the new product in period 2 regardless of the value of $v_{lr}$, ceteris paribus. This is because the cost savings of remanufacturing brings the price advantage of the remanufactured product.

![Figure 1](image1.png)

**Figure 1.** Optimal quantity as a function of $v_{lr}$, given that $c$ and $\gamma$

5.1.2. **Sensitivity analysis of quality.** Figs.1a and 2a together demonstrate that the quantity and the leasing price of the new product in period 1 are the same for
different $v_{lr}$. Under the single leasing model, the production activities of the firm in each period are independent. Therefore, in the case where the remanufacturing is not subject to the first period, $v_{lr}$ will not affect the quantity and the price of the new product in period 1.

It can also be observed that, in period 2, the quantity of new products is decreasing in $v_{lr}$, and the quantity of leased remanufactured products is increasing in $v_{lr}$. The leasing price of the leased remanufactured product is increasing in $v_{lr}$.

The reason for this is as follows. The quality level of the returned leased product generates impact on the customer’s acceptability of the leased remanufactured product. If $v_{lr}$ is high, the customer’s willingness to pay for leased remanufactured products is high, which will increase the demand for the leased remanufactured product. However, the increased demand for the leased remanufactured product will cannibalize the leasing of new products, thereby reducing the quantity of new products.
products in period 2. At the same time, a high quality level of the returned leased product enables the vendor to set a high price for the remanufactured product.

In Fig.3a, we present optimal profits of leasing as a function of $v_{lr}$. The total profits of both periods are increasing in $v_{lr}$. The implication of this result is that the higher the quality level of the returned leased product, the higher the vendor’s profits, the stronger the incentive for the vendor to remanufacture. Hence, to increase the quality level of returned leased products and reduce the use of resources, the vendor can adopt some strategies to prevent customers from improperly using products during the leasing period.

5.2. **Optimal decision and sensitivity analysis of quality under the single selling model.** In order to ensure the consistency of the analysis, we still keep the parameter values $\gamma = 0.9$, and $c = 0.25$. Thus, by substituting these values, the vendor’s optimal solution under the single selling model is only affected by the quality level of the returned sold product (customers’ relative willingness to pay for a sold remanufactured product).

5.2.1. *Optimal solution analysis.* Figs.1b-3b have several implications for optimal sales of new and sold remanufactured products. For a given cost and discount factor, the vendor will choose to sell the new product in period 1, and to sell both the new and sold remanufactured products in period 2 if the quality level of the returned product falls into a certain range.

Fig.1b generates the following insights. The optimal sales volume of new products in period 1 is higher than the sales volume of sold remanufactured products in period 2. This occurs because of the lack of control over product returns under selling. The firm could provide incentives to reduce uncertainty of product returns. For example, the vendor can develop a trade-in strategy for customers to facilitate their willingness to return used products. When $v_{sr}$ is relatively high, the quantity of the sold remanufactured products is higher than the quantity of the new products sold in period 2, vice versa. In this case, the difference between the quantity of sold remanufactured products and the quantity of new products sold in period 2 increases with $v_{sr}$. Although higher returned sold product quality reduces the sales volume of new products, it also enables customers to purchase more sold remanufactured products, therefore reducing the use of raw materials. This is a resource-saving way for the firm. Consistent with the fact that customers perceive that the remanufactured product is inferior to the new product, Fig.2b shows that the selling price of the new product in period 1 is higher than the sold remanufactured product in period 2.

5.2.2. *Sensitivity analysis of quality.* In Fig.1b, we observe that optimal sales volume of new products in period 1 or sold remanufactured products in period 2 is increasing in $v_{sr}$, and optimal sales volume of new products in period 2 is decreasing in $v_{sr}$. The rationale for this result is as follows. If $v_{sr}$ is high, customers are more willing to buy sold remanufactured products. In this case, the substitution effect of sold remanufactured products on new products is enhanced, resulting in an increases in sales volume of new products and a decrease in sales volume of sold remanufactured products in period 2. The increase in sales volume of sold remanufactured products further facilitates the vendor to produce more new products in period 1.

As shown in Fig.2b, the selling price of the new product in period 1 and the sold remanufactured product in period 2 is increasing in $v_{sr}$. The selling price of the
new product in period 2 does not change with \(v_{sr}\). This is because when \(v_{sr}\) is high, customers consider the difference between the new and sold remanufactured product is not obvious, so they are willing to pay more for the sold remanufactured product. High quality level of the returned sold product increases the customer’s valuation of the new product in period 1, thereby increasing the customer’s willingness to pay for the product.

Fig.3b shows the total profits of both periods are increasing in \(v_{sr}\). As \(v_{sr}\) improves, the demand and selling price of new products in period 1 or sold remanufactured products in period 2 increase, which results in an increase in the profits generated by the two products. Although the increase in \(v_{sr}\) will decrease the profit generated by new products in period 2, the increase in profits dominates the reduction in profits, which increases the total profits.

5.3. Comparison of the leasing and selling models. For a given cost and discount factor, the comparison between leasing and selling has been showed in Figs.1-3.

In Fig.1, it can be observed that the quantity of new products sold is higher than the quantity of new products leased in period 1. The difference between the quantity of new products sold and leased in period 2 depends on the comparison of the values of quality level in the two models. The difference between the sold and leased remanufactured products also depends on the comparison of the values of quality level. In the case where quality level of the returned sold product is higher than the returned leased product, customers perceive that the value of sold remanufactured products is higher than that of leased remanufactured products, so they are more willing to buy the former. This difference in customers’ willingness to pay enables the firm to generate greater incentives to sell more new products instead of leasing in period 1.

Fig.2 shows the selling price of the new product is higher than the leasing price of the new product in period 1. The selling price of the new product is the same as the leasing price of the new product in period 2. The price of the sold remanufactured product is higher than the price of the leased remanufactured product. This is because the quality level of returned sold products is higher than the quality level of returned leased products. Customers have a higher willingness to pay for higher quality. Therefore, the firm can set a higher price for the sold remanufactured product.

Fig.3 illustrates that selling is more profitable than leasing when the quality level of returned sold products is higher than the quality level of returned leased products. This is due to the fact that the firm can obtain a higher margin from the sale of the new product in period 1 and the sale of the remanufactured product in period 2 if the quality level of returned sold products is higher. Profitability in leasing model or selling model depends on the quality level of returned products. Higher quality level of the returned product leads to higher profits.

5.4. Sensitivity analysis of production cost. We analyze the effect of the production cost on the optimal values of decision variables and objective functions by keeping all parameters fixed except \(c\). Thus, we use the parameter values \(\gamma = 0.9, v_{lr} = 0.6\) and \(v_{sr} = 0.8\). Optimal results are summarized in Figs. 4-6. As can be observed in Fig.4, the vendor will decide to remanufacture only if the production cost is within a certain range. This is because the firm cannot benefit from the selling or leasing of remanufactured products when the production cost exceeds this
SELLING AND LEASING FOR NEW AND REMANUFACTURED PRODUCTS

Therefore, we only discuss the effect of production cost in the case of the firm providing both new and remanufactured products.

As indicated in Fig.4a and 4b, the optimal quantity of new products is decreasing in \( c \), and the optimal quantity of remanufactured products increasing in \( c \). Fig.5 suggests that the price is increasing in \( c \) for all product types. Fig.5 also demonstrably presents that the effect of increase in the price of new products caused by the increase of one unit of production cost is higher than the increase effect of the price of remanufactured products. Therefore, the price increase effect caused by the increase in cost makes the remanufactured product more attractive to customers than the new product. Sequentially, the optimal quantity of remanufactured products increases as \( c \) increases. On the contrary, the optimal quantity of new products

**Figure 4.** Optimal quantity as a function of \( c \), given that \( v_{ij} \) and \( \gamma \)

**Figure 5.** Optimal price as a function of \( c \), given that \( v_{ij} \) and \( \gamma \)
decreases as \( c \) increases. Fig.6 reveals that the less the production cost is, the larger the vendor’s profit can reach. This is because that the profit gain obtained from the remanufactured products is dominated by the profit loss due to the decrease in the quantity of new products. As a result, a low production cost can benefit the vendor’s profit, so she will try to reduce the production cost.

6. Conclusions. Our work is motivated by recent energy and resource issues in the new energy vehicle industry. To optimize resource utilization and reduce environmental impact, we consider a firm’s marketing strategy with remanufacturing. However, process uncertainty makes remanufacturing operations management sophisticated. To reduces the uncertainty of the timing, quality and quantity of product returns in remanufacturing, in this paper, we consider a monopolist vendor who markets her products by adopting three models: (1) she can sell them, (2) she can lease them, and (3) she can sell and lease them simultaneously. We first conduct an insight into the vendor’s marketing model with remanufacturing and analyze the impact of the quality level of the returned products and production cost on the firm’s marketing and remarketing models. Since the firm only adopt partial remanufacturing, the optimal result of the combination of leasing and selling comes down to the single selling. Then we compare selling model with leasing model based on the quality level of the returned product.

Our model analysis and numerical study generate the following managerial insights. Under each single marketing model, when the quality level of the returned product is high, the vendor can achieve higher profits and reduce the use of resources by remanufacturing. Therefore, the firm can make a commitment that customers can buy new products at a discount price if they return high quality used products under the single selling model, and adopt some strategies to prevent customers from improperly using products under the single leasing model. Furthermore, when the production cost beyond a threshold, selling or leasing of remanufactured products is not conducive to the firm’s profit. Therefore, only within a reasonable production cost range will the firm adopt a remanufacturing strategy in both marketing models.
In the comparison of different marketing models, we first find that the range of remanufacturable quality level under the single selling model is larger than that under the single leasing model. The firm has more remanufacturable space under the single selling model. Secondly, we show that the lower bound of the quality level of the returned product in the selling model is higher than the upper bound of the quality level of the returned product in the leasing model. Therefore, only when the quality level of returned sold products is relatively high will the vendor choose to remanufacture in the selling model. On the contrary, only when the quality level of returned leased products is relatively low will the vendor make a decision to remanufacture in leasing model. Thirdly, we characterize how the impact of the quality level of returned products on the firm’s profitability. Specifically, selling is more profitable than leasing when the quality level of returned sold products is sufficiently high compared with the quality level of returned leased products.

Some limitations of our work need to be discussed in the near future. First, we study a monopoly firm who processes remanufacturing activity by himself. In some case, firms need to outsource remanufacturing operation. Future research could construct the competition or game relationship between multiple players. Second, Our analysis is on the basis of partial remanufacturing and firm’s direct recovery. In reality, the return of used products can be full if the firm reinforces the management of lease or increases incentives to facilitate product returns under selling. The remanufacturable resources may come from indirect ways recycled through retailers or third-party. Therefore, full return or indirect recycle can be as a direction for further extension. Third, a potential extension for future research is to cover cases in which the quality level of the returned product under the single leasing model is higher than that under the single selling model. Last, our main interest is electric vehicle battery. It is worth noting that regulations play an vital role in this process. What impact these regulations will have on the marketing structure of the new product and the remanufactured product should be discussed.

Acknowledgments. This work was supported by the fund from National Natural Science Foundation of China under grant nos 71871076, 71521001, 71531008, 71690235.

Appendix A. Proof of inverse demand function under each model.

Proof. Under the single leasing model, a consumer of type $\theta$ receives utility $U_{2t}^{L} = \theta - p_{2t}^{L}$ from the lease of a new product but derives utility $U_{2t}^{Lr} = v_{t}\theta - p_{2t}^{Lr}$ for a leased remanufactured product in period 2. If $\theta^{L} < \theta \leq 1$, $\theta^{L} = \frac{p_{2t}^{L} - p_{2t}^{Lr}}{v_{t}}$, a consumer with valuation $\theta$ will take a lease of the new product. If $\theta^{L} \leq \theta \leq \theta^{L''}$, a consumer with valuation $\theta$ will take a lease of the leased remanufactured product. If $\theta^{L''} \leq \theta \leq \theta^{L'}$, a consumer with valuation $\theta$ will take a lease of the leased remanufactured product. If $0 \leq \theta < \theta^{L''}$, a consumer with valuation $\theta$ will not take action. The demands for new products and leased remanufactured products are given by $q_{2t}^{L} = 1 - \theta^{L}$ and $q_{2t}^{Lr} = \theta^{L} - \theta^{L''}$, respectively. Substituting $\theta^{L}$ and $\theta^{L''}$ into $q_{2t}^{L}$ and $q_{2t}^{Lr}$, and solving for $p_{2t}^{L}$ and $p_{2t}^{Lr}$, we derive second-period inverse demand functions. Similarly, we get $p_{2t}^{L} = 1 - q_{2t}^{L}$.

The inverse demand functions under the single selling model and the combination model of both leasing and selling can be proved by the same method.
Appendix B. Proof of optimal solutions under the single leasing model.

Proof. The profit function Eq.(5) is jointly concave in $q_{2ln}^L$ and $q_{2lr}^L$, because $\frac{\partial\pi^L}{\partial q_{2ln}^L} = -2 < 0$, $\frac{\partial^2\pi^L}{\partial q_{2ln}^L}\partial q_{2lr}^L = -2v_{lr} < 0$, $\frac{\partial^2\pi^L}{\partial q_{2ln}^L,\partial q_{2lr}^L} = -2v_{lr} < 0$, which means that
\[
\left| \begin{array}{cc} -2 & -2v_{lr} \\ -2v_{lr} & -2v_{lr} \end{array} \right| > 0
\]
ensuring a negative definite Hessian. The associated Lagrangian is $L(q_{2ln}^L, q_{2lr}^L, \lambda^L) = \pi^L(q_{2ln}^L, q_{2lr}^L) + \lambda^L(q_{1ln}^L - q_{2lr}^L)$. The KKT conditions are $\frac{\partial L}{\partial q_{2ln}^L} = 1 - c - 2q_{2ln}^L - 2v_{lr}q_{2lr}^L = 0$, $\frac{\partial L}{\partial q_{2lr}^L} = c - v_{lr} - q_{2lr}^L - 2v_{lr}q_{2ln}^L - 2v_{lr}q_{2lr}^L - \lambda^L = 0$. The complementary slackness conditions $\lambda^L(q_{1ln}^L - q_{2lr}^L) = 0$. Since we only consider partial remanufacturing, the multiplier can only be zero. The system of equations $\frac{\partial L}{\partial q_{2ln}^L} = 0, \frac{\partial L}{\partial q_{2lr}^L} = 0$, yields the solution $q_{2ln}^{L*}, q_{2lr}^{L*}$.

Taking the first order of Eq.(9) with respect to $q_{1ln}^L$, we have $\frac{\partial \pi^L}{\partial q_{1ln}^L} = 1 - c - 2q_{1ln}^L = 0$, $\frac{\partial L}{\partial q_{1ln}^L} = 1 - c - 2q_{1ln}^L$.
Setting this to zero and solving, we get $q_{1ln}^{L*} = \frac{1 - c}{2}$. The second order of Eq.(9) with respect to $q_{1ln}^L$ is $\frac{\partial^2 \pi^L}{\partial q_{1ln}^L} = -2 < 0$, which means that $q_{1ln}^{L*}$ maximizes profits. We now need to check whether $q_{1ln}^{L*} - q_{2lr}^{L*} > 0$ holds. Substituting $q_{1ln}^{L*}$ and $q_{2lr}^{L*}$ into it, we find that this condition holds if $c - 3cv_{lr} + v_{lr} - v_{lr}^2 > 0$.

Appendix C. Proof of Lemma 4.1.

Proof. Lemma 4.1 can be easily derived by calculating $q_{2ln}^{L*} \geq 0$, $q_{2lr}^{L*}$, $c(v_{lr}) > 0$, $q_{1ln}^{L*} > q_{2lr}^{L*}$, and $c + 2v_{lr} < 1$.

Appendix D. Proof of proposition 1.

Proof. Taking Lemma 4.1 as a precondition, $\frac{\partial \pi^L}{\partial q_{1ln}^L} = \frac{c(v_{lr}^2 - 2v_{lr})}{(1 - v_{lr})^2} < 0$, $\frac{\partial \pi^L}{\partial q_{1ln}^L} = \frac{c(v_{lr}^2 - 2v_{lr})}{(1 - v_{lr})^2} > 0$, $\frac{\partial \pi^L}{\partial q_{2lr}^L} = \frac{-c(v_{lr}^2 + 2v_{lr} - 1)(2v_{lr}^2 - 3v_{lr} - 1)}{4v_{lr}(1 - v_{lr})^2} > 0$.

Appendix E. Proof of optimal solutions under the single selling model.

Proof. The profit function Eq.(14) is jointly concave in $q_{2sn}^S$, and $q_{2sr}^S$, because $\frac{\partial^2 \pi^S}{\partial q_{2sn}^S} = -2 < 0$, $\frac{\partial^2 \pi^S}{\partial q_{2sr}^S} = -2v_{sr} < 0$, $\frac{\partial^2 \pi^S}{\partial q_{2sn},\partial q_{2sr}^S} = -2v_{sr} < 0$, $\frac{\partial^2 \pi^S}{\partial q_{2sr},\partial q_{2sn}^S} = -2v_{sr} < 0$, which means that
\[
\left| \begin{array}{cc} -2 & -2v_{sr} \\ -2v_{sr} & -2v_{sr} \end{array} \right| > 0
\]
ensuring a negative definite Hessian. The associated Lagrangian is $L(q_{2sn}^S, q_{2sr}^S, \lambda^S) = \pi^S(q_{2sn}^S, q_{2sr}^S) + \lambda^S(q_{1sn}^S - q_{2sr}^S)$. The KKT conditions are $\frac{\partial L}{\partial q_{2sn}^S} = 1 - c - 2q_{2sn}^S - 2v_{sr}q_{2sr}^S = 0$, $\frac{\partial L}{\partial q_{2sr}^S} = c - v_{sr} - q_{2sr}^S - 2v_{sr}q_{2sn}^S - 2v_{sr}q_{2sr}^S - \lambda^S = 0$. The complementary slackness conditions $\lambda^S(q_{1sn}^S - q_{2sr}^S) = 0$. Since we only consider partial remanufacturing, the multipliers can only be zero. The system of equations $\frac{\partial L}{\partial q_{2sn}^S} = 0, \frac{\partial L}{\partial q_{2sr}^S} = 0$, yields the solution $q_{2sn}^S, q_{2sr}^S$.

Taking the first order of Eq.(18) with respect to $q_{1sn}^S$, we have $\frac{\partial \pi^S}{\partial q_{1sn}^S} = 1 - c - 2q_{1sn}^S + \frac{\gamma(v_{sr} + c(1 - v_{sr}^2))}{2} + \gamma \lambda^S$. Setting this to zero and solving, we get $q_{1sn}^S =$.
The second order of Eq.(18) with respect to \( q^2_{1 sn} \) is \( \frac{\partial^2 \pi^*}{\partial q^2_{1 sn}} = -2 \), which means that \( q^2_{1 sn}^* \) maximizes profits. We now need to check whether \( q^2_{2 sr} - q^2_{2 sr}^* \) holds. Substituting \( q^2_{1 sn}^* \) and \( q^2_{2 sr}^* \) into it, we find that this condition holds if \( c(2 - 4v_{sr} + \gamma v_{sr} - \gamma v_{sr}^2 - \gamma v_{sr}^3 + \gamma v_{sr}^4) + 2v_{sr} - 2v_{sr}^2 + \gamma v_{sr}^2 - \gamma v_{sr}^3 > 0 \).

**Appendix F. Proof of optimal solutions under the combination model of both leasing and selling.**

**Proof.** The profit function Eq.(25) is jointly concave in \( q^2_{2n}, q^2_{2sr}, \) and \( q^2_{2tlr} \), because

\[
\frac{\partial^2 \pi^*}{\partial q^2_{2n}} = -2 < 0, \quad \frac{\partial^2 \pi^*}{\partial q^2_{2sr}} = -2v_{sr} < 0, \quad \frac{\partial^2 \pi^*}{\partial q^2_{2tlr}} = -2v_{tlr} < 0, \quad \frac{\partial^2 \pi^*}{\partial q^2_{2sr}^2} = -2 < 0, \quad \frac{\partial^2 \pi^*}{\partial q^2_{2tlr}^2} = -2 < 0, \quad \frac{\partial^2 \pi^*}{\partial q^2_{2sr} \partial q^2_{2tlr}} = -2 < 0,
\]

ensuring a negative definite Hessian. The associated Lagrangian is \( L(q^2_{2n}, q^2_{2sr}, q^2_{2tlr}, \lambda^{C1}, \lambda^{C2}) = \pi^*(q^2_{2n}, q^2_{2sr}, q^2_{2tlr}) + \lambda^{C1}((1 - \omega)q^2_{2n} - q^2_{2sr}) + \lambda^{C2}((1 - \omega)q^2_{2sr} - q^2_{2tlr}) \). The KKT conditions are

\[
\frac{\partial L}{\partial q^2_{2n}} = 1 - c - 2q^2_{2n} - 2v_{sr}q^2_{2sr} - 2v_{tlr}q^2_{2tlr} = 0, \quad \frac{\partial L}{\partial q^2_{2sr}} = v_{sr} - c(1 - v_{sr}^2) - 2v_{sr}q^2_{2sr} - 2v_{sr}q^2_{2tlr} - \lambda^{C1} = 0, \quad \frac{\partial L}{\partial q^2_{2tlr}} = v_{tlr} - c(1 - v_{tlr}^2) - 2v_{tlr}q^2_{2sr} - 2v_{tlr}q^2_{2tlr} - \lambda^{C2} = 0.
\]

The complementary slackness conditions \( \lambda^{C1}(\omega q^2_{2n} - q^2_{2sr}) = 0, \quad \lambda^{C2}((1 - \omega)q^2_{2sr} - q^2_{2tlr}) = 0 \). Since we only consider partial remanufacturing, the multipliers can only be zero. The system of equations \( \frac{\partial L}{\partial q^2_{2n}} = 0, \frac{\partial L}{\partial q^2_{2sr}} = 0, \frac{\partial L}{\partial q^2_{2tlr}} = 0 \) yields the solution \( q^2_{2n}^*, q^2_{2sr}^*, q^2_{2tlr}^* \), and \( q^2_{2sr}^* \).

Taking the first order of Eq.(31) with respect to \( q^2_{ln} \), we have \( \frac{\partial \pi^*}{\partial q^2_{ln}} = 1 - c - 2q^2_{ln} + \frac{\gamma v_{sr} + (1 - v_{sr}^2)}{4} \). Setting this to zero and solving, we get \( q^2_{ln}^* = \frac{1 - c}{\gamma} + \frac{\gamma v_{sr} + (1 - v_{sr}^2)}{4} \). The second order of Eq.(31) with respect to \( q^2_{ln} \) is \( \frac{\partial^2 \pi^*}{\partial q^2_{ln}} = -2 < 0 \), which means that \( q^2_{ln}^* \) maximizes profits. The total profits \( \frac{\partial \pi^*}{\partial \omega} = \frac{\gamma (v_{sr} + (1 - v_{sr}^2))}{8} \) hold. \( \omega^* = 1 \).

**Appendix G. Proof of proposition 3.**

**Proof.** The comparison between leasing and selling can be done by calculating the difference in the vendor’s optimal decisions obtained by both models. We have

\[
\begin{aligned}
p_{2sr} - p_{2slr}^* &= \frac{v_{sr} - v_{sr}^* + c(v_{sr} + v_{sr}^*)}{2} > 0 \quad \text{if and only if} \quad v_{sr} < \sqrt{v_{tlr} + \frac{1}{2}}, \quad q^2_{2sr} - q^2_{2sr}^* = \frac{c(v_{sr}^2 + v_{sr} - 1)(v_{sr} - v_{sr}^2) - (v_{sr}^2 + 2v_{sr} - 1)(v_{sr} - v_{sr}^2)}{2v_{sr} + (1 - v_{sr})^2} > 0 \quad \text{if and only if} \quad \sqrt{4v_{sr}^2 + (1 - v_{sr})^2} > 1. \\
v_{sr} < 1, \quad \sqrt{2} - 1 < v_{tlr} < \sqrt{\frac{1}{2}}, \quad \pi^* - \pi^{L*} = \frac{\gamma c + v_{sr} - c v_{sr}^2}{16} (4 + 4c - c - \gamma v_{sr}^2 - v_{sr}) + \frac{\gamma c^2 (v_{sr} - v_{sr} - 1)^2 (v_{sr} - v_{sr}^2) - (v_{sr}^2 + 2v_{sr} - 1)^2 (v_{sr} - v_{sr}^2)}{4v_{sr} + (1 - v_{sr})^2} > 0 \quad \text{if and only if} \quad \sqrt{4v_{sr}^2 + (1 - v_{sr})^2} > 1.
\end{aligned}
\]
REFERENCES

[1] J. D. Abbey, V. D. R. Guide Jr. and G. C. Souza, Delayed differentiation for multiple lifecycle products, Prod. Oper. Manag., **22** (2013), 588–602.

[2] V. V. Agrawal, M. Ferguson, L. B. Toktay and V. M. Thomas, Is leasing greener than selling?, Manage. Sci., **58** (2012), 523–533.

[3] N. Aras, R. Güllü and S. Yüriılmaz, Optimal inventory and pricing policies for remanufacturable leased products, Int. J. Prod. Econ., **133** (2011), 262–271.

[4] A. Atasu, V. D. R. Guide Jr. and L. N. Van Wassenhove, So what if remanufacturing cannibalizes my new product sales?, Calif. Manage. Rev., **52** (2010), 1–21.

[5] A. Atasu and G. C. Souza, How does product recovery affect quality choice?, Prod. Oper. Manag., **22** (2013), 991–1010.

[6] B. Avci, K. Girotra and S. Netessine, Electric vehicles with a battery switching station: Adoption and environmental impact, Manage. Sci., **61** (2015), 772–794.

[7] S. R. Bhaskaran and S. M. Gilbert, Implications of channel structure and operational mode upon a manufacturer’s durability choice, Prod. Oper. Manag., **24** (2015), 1071–1085.

[8] S. R. Bhaskaran and S. M. Gilbert, Implications of channel structure for leasing or selling durable goods, Market. Sci., **28** (2009), 918–934.

[9] J. I. Bulow, Durable-goods monopolists, J. Polit. Econ., **90** (1982), 314–332.

[10] P. Desai and D. Purohit, Competition in durable goods markets: The strategic consequences of leasing and selling, Market. Sci., **18** (1999), 42–58.

[11] P. Desai and D. Purohit, Leasing and selling: Optimal marketing strategies for a durable goods firm, Manage. Sci., **44** (1998), S19–S34.

[12] M. Ferguson, V. D. R. Guide Jr., E. Koca and G. C. Souza, The value of quality grading in remanufacturing, Prod. Oper. Manag., **18** (2009), 300–314.

[13] G. Ferrer and J. M. Swaminathan, Managing new and remanufactured products, Manage. Sci., **52** (2006), 15–26.

[14] V. D. R. Guide Jr., R. H. Teunter and L. N. Van Wassenhove, Matching demand and supply to maximize profits from remanufacturing, Manuf. Serv. Oper. Manag., **5** (2003), 303–316.

[15] E. Huang, China buys one out of every two electric vehicles sold globally, Centre for Solar Energy and Hydrogen Research Baden-Württemberg, Germany, 2019. Available from: https://qz.com/1552991/china-buys-one-out-of-every-two-electric-vehicles-sold-globally/.

[16] K. J. Li and S. H. Xu, The comparison between trade-in and leasing of a product with technology innovations, Omega, **54** (2015), 134–146.

[17] Z. Li and M. Ouyang, A win-win marginal rent analysis for operator and consumer under battery leasing mode in China electric vehicle market, Energ. Policy, **39** (2011), 3222–3237.

[18] M. K. Lim, H. Y. Mak and Y. Rong, Toward mass adoption of electric vehicles: Impacts of the range and resale anxieties, Manuf. Serv. Oper. Manag., **17** (2015), 101–119.

[19] P. V. Loon, C. Delagarde and L. N. Van Wassenhove, The role of second-hand markets in circular business: A simple model for leasing versus selling consumer products, Int. J. Prod. Res., **56** (2018), 960–973.

[20] P. Majumder and H. Groenevelt, Competition in remanufacturing, Prod. Oper. Manag., **10** (2001), 125–141.

[21] O. Mont, C. Dalhammar and N. Jacobsson, A new business model for baby prams based on leasing and product remanufacturing, J. Clean. Prod., **14** (2006), 1509–1518.

[22] K. S. Moorthy, Product and price competition in a duopoly, Market. Sci., **7** (1988), 141–168.

[23] A. Mutha, S. Bansal and V. D. R. Guide Jr., Managing demand uncertainty through core acquisition in remanufacturing, Prod. Oper. Manag., **25** (2016), 1449–1464.

[24] J. Östlin, E. Sundin and M. Björkman, Importance of closed-loop supply chain relationships for product remanufacturing, Int. J. Prod. Econ., **115** (2008), 336–348.

[25] A. Robotis, S. Bhattacharya and L. N. Van Wassenhove, Lifecycle pricing for installed base management with constrained capacity and remanufacturing, Prod. Oper. Manag., **21** (2012), 236–252.

[26] G. C. Souza, Closed-loop supply chains: A critical review, and future research, Decision Sci., **44** (2013), 7–38.

[27] W. R. Stahel, The Circular Economy, Routledge, London, 2019.

[28] C. R. Standridge and L. Connel, Remanufacturing, repurposing, and recycling of post-vehicle-application lithium-ion batteries, Mineta Transportation Institute, San José, 2014. Available from: https://rosap.ntl.bts.gov/view/dot/27425.
[29] C. R. Standridge, L. Corneal and N. Baine, Advances in repurposing and recycling of post-vehicle-application lithium-ion batteries, Mineta Transportation Institute, San José, 2016. Available from: http://transweb.sjsu.edu/project/1238.html.

[30] D. W. Steeneck and S. C. Sarin, Product design for leased products under remanufacturing, Int. J. Prod. Econ., 202 (2018), 132–144.

[31] M. Thierry, M. Salomon, J. Van Nunen and L. N. Van Wassenhove, Strategic issues in product recovery management, Calif. Manage. Rev., 37 (1995), 114–135.

[32] B. K. Thorn and P. Rogerson, Take it back, IIE Solutions, 34 (2002), 34–40.

[33] M. W. Toffel, Strategic management of product recovery, Calif. Manage. Rev., 46 (2004), 120–141.

[34] L. B. Toktay and D. Wei, Cost allocation in manufacturing-remanufacturing operations, Prod. Oper. Manag., 20 (2011), 841–847.

[35] T. Tse, M. Esposito and K. Soufani, How businesses can support a circular economy, Harvard. Bus. Rev., 2 (2016), 1–6.

[36] M. Waldman, Eliminating the market for secondhand goods: An alternative explanation for leasing, J. Law. Econ., 40 (1997), 61–92.

[37] L. Wang, G. Cai, A. A. Tsay and A. J. Vakharia, Design of the reverse channel for remanufacturing: Must profit-maximization harm the environment? Prod. Oper. Manag., 26 (2017), 1585–1603.

[38] WEEE Forum, 2012 Annual Report, European Association of Electric and Electronic Waste Take-Back Systems, 2013. Available from: http://www.weee-forum.org/what-is-the-weee-forum/annual_report_2012_final.pdf.

[39] B. Yalabik, D. Chhajed and N. C. Petruzzi, Product and sales contract design in remanufacturing, Int. J. Prod. Econ., 154 (2014), 299–312.

[40] W. Zhang, Q. Zhang, K. J. Mizgier and Y. Zhang, Integrating the customers’ perceived risks and benefits into the triple-channel retailing, Int. J. Prod. Res., 55 (2017), 6676–6690.

[41] Y. Zhang, C. Y. Wang and X. Tang, Cycling degradation of an automotive LiFePO4, lithium-ion battery, J. Power Sources, 196 (2011), 1513–1520.

Received May 2019; 1st revision August 2019; 2nd revision September 2019.

E-mail address: hfutlk@163.com
E-mail address: hfutzhoutao@163.com
E-mail address: liubohai@hfut.edu.cn