Neutrino mass from higher than \( d=5 \) effective operators in SUSY, and its test at the LHC

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**Abstract**

We discuss neutrino masses from higher than \( d = 5 \) effective operators in a supersymmetric framework, where we explicitly demonstrate which operators could be the leading contribution to neutrino mass in the MSSM and NMSSM. As an example, we focus on the \( d = 7 \) operator \( LLH_u H_u H_d H_u \), for which we systematically derive all tree-level decompositions. We argue that many of these lead to a linear or inverse see-saw scenario with two extra neutral fermions, where the lepton number violating term is naturally suppressed by a heavy mass scale when the extra mediators are integrated out. We choose one example, for which we discuss possible implementations of the neutrino flavor structure. In addition, we show that the heavy mediators, in this case SU(2) doublet fermions, may indeed be observable at the LHC, since they can be produced by Drell-Yan processes and lead to displaced vertices when they decay. However, the direct observation of lepton number violating processes is on the edge at LHC.

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1 Introduction

From neutrino oscillations, it is evident that neutrinos are massive, see, e.g., Refs. [1, 2]. If the neutrino masses originate from physics beyond the Standard Model (SM) and are suppressed by a high energy scale, it is convenient to parameterize the impact of the heavy fields, present in the high-energy theory, by a tower of effective operators $O^d$ of dimension $d > 4$. These operators made out of the SM fields, are invariant under the SM gauge group [3, 4] (see also Ref. [5]). The operator coefficients are weighted by inverse powers of the scale of new physics $\Lambda_{NP}$:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{eff}^{d=5} + \mathcal{L}_{eff}^{d=6} + \cdots, \quad \text{with} \quad \mathcal{L}_{eff}^{d} \propto \frac{1}{\Lambda_{NP}^{d-4}} O^d. \quad (1)$$

Some of these effective operators result in corrections to the low-energy SM parameters and in exotic couplings. It is also known that there is only one possible operator at the lowest order in the expansion, $\mathcal{L}_{eff}^{d=5}$, namely, the famous Weinberg operator [3],

$$O_W = (\bar{L}c \tau^2 H)(H^\dagger \tau^2 L) \quad (2)$$

which leads, after Electroweak Symmetry Breaking (EWSB), to Majorana masses for the neutrinos. Here $L$ and $H$ stand for the SM lepton and Higgs doublets, respectively. At tree level, $O_W$ can only be mediated in three ways [6]: by a singlet fermion, a triplet scalar, or a triplet fermion, leading to the famous type I [7–10], type II [11–16], and type III [17] see-saw mechanisms, respectively (see also Ref. [18]). For recent discussions of SUSY versions see for example [19–21] and references therein. Compared to the electroweak scale, the mass of the neutrinos in all three cases appears suppressed by a factor $v/\Lambda_{NP}$, where $v$ is the Vacuum Expectation Value (VEV) of the Higgs boson. Substituting typical values, one obtains that the original see-saw mechanisms are pointing towards the Grand Unified Theory (GUT) scale.

More recently, however, scenarios in which $\Lambda_{NP} \sim \text{TeV}$ have been drawing some attention, since they are potentially testable at the LHC. In these cases, additional suppression mechanisms for the neutrino masses are required, and several possibilities open up: For example, the neutrino mass may be generated radiatively, where the additional suppression comes from loop integrals, or the smallness of the neutrino mass may be protected by lepton number, such as in the inverse see-saw. In this study, we instead argue that the $d = 5$ operator in Eq. (1) is forbidden, and neutrino masses originate from higher dimensional operators [22–33] (see also Refs. [34–39] for related discussions). Note that there may be additional suppression mechanisms at work in this approach, such as loop suppression or a small lepton number violating parameter, see Ref. [28] for an example.

There are several key ingredients to neutrino masses from higher dimensional operators [28]. Consider, for instance, the operator

$$O^\tau = (LLHH)(H^\dagger H), \quad (3)$$

where we have omitted spin, flavor, and gauge indices. In this case, the $(H^\dagger H)$ component can be closed in a loop, which means that the $d = 5$ operator is generated radiatively, and the $d = 7$ operator may not be the leading contribution to neutrino mass (depending on the new physics scale). We do not consider such operators in this work, which means that [28]
1. We have to forbid the lower dimensional operators by a U(1) or discrete symmetry.

2. We need new (scalar) fields to construct the higher dimensional operators, since \((H^\dagger H)\) is a singlet under any such symmetry.

The simplest possibilities to enhance the field content of the SM are the addition of a Higgs singlet [23, 24]

\[
\mathcal{L}_{\text{eff}}^{d=n+5} = \frac{1}{\Lambda_{\text{NP}}} (LLHH)(S)^n, \quad n = 1, 2, 3, \ldots
\]

(4)

or the addition of a Higgs doublet, leading to the Two Higgs Doublet Model (THDM) [22, 25, 28, 40]

\[
\mathcal{L}_{\text{eff}}^{d=2n+5} = \frac{1}{\Lambda_{\text{NP}}} (LLH_uH_u)(H_uH_u)^n, \quad n = 1, 2, 3, \ldots
\]

(5)

As it has been demonstrated in Ref. [28] in the framework of the THDM, the decomposition of Eq. (5) often leads to a linear or inverse see-saw structure [41–43] if two extra fermion singlet fields \(N_R\) and \(N'_L\) are involved. The neutral fermion mass matrix then reads, in the basis \((\nu^c_L, N_R, N'_L)\),

\[
M_\nu = \begin{pmatrix}
0 & (Y^T_\nu)v & \epsilon(Y^T_\nu) \\
(Y_\nu)v & \mu' & \Lambda_{\text{NP}} \\
\epsilon(Y^T_\nu) & \Lambda_{\text{NP}} & \mu''
\end{pmatrix}.
\]

(6)

Here \(\epsilon, \mu', \) and \(\mu''\) are typically introduced ad hoc as small parameters, because they are protected by lepton number (for the \(\epsilon\)-term, see also Refs. [18,44]). However, if the neutrino mass is generated by a higher than \(d = 5\) effective operator, some terms in the neutral fermion mass matrix in Eq. (6) can only originate from non-renormalizable interactions, which means that extra fields are needed and that the lepton number violating parameters are suppressed by powers of \(\Lambda_{\text{NP}}\). A general discussion and two specific examples leading to the \(\mu\)- and \(\epsilon\)-term in Eq. (6) can be found in Sec. 3 of Ref. [28].

In principle, all neutrino mass models can be supersymmetrized. In practice, however, it turns out that supersymmetry often requires additional particles for consistency, mainly due to the boundary condition that the superpotential has to be holomorphic. In this work, we apply Ref. [28] to the framework of SUSY, more specifically, we start with the minimal supersymmetric standard model (MSSM) and the next to minimal one (NMSSM). A related discussion in the NMSSM framework can be found in Ref. [24]. In Sec. 2, we systematically discuss higher dimensional effective operators including two Higgs doublets and one scalar within SUSY. Then in Sec. 3, we focus on one specific decomposition leading to a linear or inverse see-saw in the form of Eq. (6). We will show that this requires an extension of the particle content of the model, which is potentially observable at the LHC. The main features will be outlined using a specific model, where we also discuss constraints due to existing data. In Sec. 4, we show that lepton flavor mixing related to neutrino physics is potentially testable at the LHC. We also show that the cross sections for some lepton number violating processes can be significantly larger than naively expected. However, it turns out that they are on the edge of observability at the LHC. Finally, in Sec. 5 we draw our conclusions. In
Table 1: Effective operators generating neutrino mass in the MSSM up to $d = 9$. The operator numbers have been chosen in consistency with Table 2.

| Op. # | Effective interaction | Charge | Same as |
|-------|-----------------------|--------|---------|
| $d = 5$ | $1 \ LLH_u H_u$ | $2q_L + 2q_{H_u}$ | |
| $d = 7$ | $3 \ LLH_u H_u H_d H_u$ | $2q_L + 3q_{H_u} + q_{H_d}$ | |
|            | $4 \ LLH_u H_u SS$ | $2q_L - 2q_{H_d}$ | |
| $d = 8$ | $5 \ LLH_u H_u H_d H_u S$ | $2q_L + 2q_{H_u}$ | #1 |
|            | $6 \ LLH_u H_u SSS$ | $2q_L + 2q_{H_u}$ | #1 |
| $d = 9$ | $7 \ LLH_u H_u H_d H_u H_d H_u$ | $2q_L + 4q_{H_u} + 2q_{H_d}$ | |
|            | $8 \ LLH_u H_u H_d H_u SS$ | $2q_L + q_{H_u} - q_{H_d}$ | #2 |
|            | $9 \ LLH_u H_u SSSS$ | $2q_L + q_{H_u} - q_{H_d}$ | #2 |

Table 2: Effective operators generating neutrino mass in the NMSSM up to $d = 9$. Here $S$ is the NMSSM scalar, which means that its charge $q_S$ is fixed by the terms $\lambda \hat{S} \hat{H}_u \hat{H}_u$, i.e., $q_S = -(q_{H_u} + q_{H_d})$, and $\kappa \hat{S}^3$, i.e., $3q_S = 0$, which have been used to derive the charge condition (cf., Eq. (7)).

Appendix A we comment on possible fundamental theories leading to the $d = 7$ operator discussed in this paper. In Appendix B we approximate some of the couplings for the model considered in Sec. 4.

2 Neutrino masses from higher than $d=5$ operators in SUSY

We take the MSSM as a starting point for various extensions. Note that its Higgs sector has the structure of a type II–THDM with the restriction that it is CP-invariant at leading order. As in Ref. [28], we require a discrete symmetry (in the sense of a matter parity) to forbid the $d = 5$ operator as leading contribution to neutrino mass, where the simplest possibility is $\mathbb{Z}_3$ in the case of SUSY.\footnote{Note, that the requirement of forbidding the $d = 5$ operator automatically implies conserved R-parity because a $\Delta L = 1$ operator immediately implies a contribution to the $d = 5$ operator [45]. For example, the sneutrino can get a VEV if R-parity is broken. Because of the neutrino-sneutrino-neutralino interaction, an additional $d = 5$ effective operator, which contributes to the neutrino mass, is then possible [46]. Therefore, we require R-parity conservation, which also has strong constraints on the possible decompositions of the effective operators.} In Table 1, we list all possible higher dimensional operators made from lepton doublets and the two Higgs fields up to dimension nine. Note that compared to the THDM, the holomorphicity of the superpotential implies that the only possible effective $d = 7$ operator in the MSSM is $LLH_u H_u H_d H_u$, and it also limits the
number of possible decompositions further.\footnote{The holomorphicity of the superpotential implies that interactions among scalars of the form $\phi_i \phi_j \phi_k^\dagger$ can only be introduced via F-terms (another possibility to get non-holomorphic terms for the neutrino mass operator are non-canonical terms in the Kähler potential, as discussed in [37,47]). Since there are no SUSY invariant interactions with both, fermions and F fields, the only possible effective $d = 7$ operator in the MSSM is $\LL \HH_u \HH_u \HH_d \HH_d$.} In Table 1, we show in addition to the possible operators the required charge combination such that the corresponding operator respects the discrete symmetry. However, in the MSSM, the combination $\mu \HH_u \HH_d$ appears in the superpotential, which is breaking such the discrete symmetry explicitly. For this reason, we consider models with extended particle content.

A possible extension beyond the MSSM is the NMSSM, where an additional Higgs singlet $S$ is introduced, see e.g. Ref. [48, 49] for reviews. This singlet couples to the usual Higgs doublets $H_u$ and $H_d$ and obtains a non-zero vacuum expectation value. The NMSSM superpotential is

$$W_{\text{NMSSM}} = W_{\text{Yuk}} + \lambda S \HH_u \HH_d + \kappa S^3,$$

where $W_{\text{Yuk}}$ denotes the superpotential for the Yukawa couplings, i.e., the MSSM superpotential. In this case, the charge of $q_S$ is fixed by the second and third terms being uncharged. From the discrete symmetry point of view, one can easily see that the last term in Eq. (7) is invariant under the $\mathbb{Z}_3$ for any charge assignment.

In Table 2, we list all possible higher dimensional neutrino mass operators made from lepton doublets and the two extra Higgs fields up to dimension nine for the NMSSM. In the column “Charge” we also show the discrete symmetry charge using the fact that the terms in Eq. (7) have to be uncharged. In the last column “Same as”, we indicate if the same condition as for a lower dimensional operator is obtained, i.e., the lower dimensional operator cannot be avoided in this case. One can read off the table that operators #2, #3, #4, and #7 can be independently chosen as leading contribution of neutrino mass, while the lower dimensional operators are forbidden. In general one can show that the NMSSM operators of the type $LLH_u H_u (H_d H_u)^n S^k$ (with $n \geq 1$, $k \geq 1$ or $n = 0$, $k \geq 3$) always imply that other operators of lower dimension are allowed as well. This is due to the fact that one finds field products of the type $H_u H_d S$ or $S^3$, which have to be singlets under the discrete symmetry, since they appear in the superpotential Eq. (7). This means that $d > 7$ effective operators generating neutrino mass with singlet scalars will always come together with lower dimensional operators. On the other hand, the effective operators with lepton and Higgs doublets only (such as #3 and #7) are per se interesting alternatives because one can choose even higher dimensional operators $d \geq 9$ as leading contribution.

Note that operators #1, #2, and #4 have been studied in Ref. [24], whereas we focus on the $d = 7$ operator #3 in the following. In this case, a possible charge assignment for the $\mathbb{Z}_3$ symmetry is

$$q_{H_u} = 0, \quad q_{H_d} = 1, \quad q_L = 1, \quad (q_S = 2).$$

While in this case, both the MSSM and NMSSM can be used as a framework, one has to be aware of the fact that the $\mu$-term of the MSSM, $\mu \HH_u \HH_d$, explicitly breaks the discrete
symmetry. This problem is automatically circumvented in the NMSSM, as Eq. (7) respects the discrete $Z_3$ symmetry and generates the $\mu$-term when $S$ takes a VEV. In SUSY, there are some differences compared to the THDM case in Ref. [28]. For instance, the Lagrangian in Ref. [28] was invariant under a new U(1) symmetry in some cases, taking the role of the $Z_3$ symmetry here, which potentially lead to unwanted Goldstone bosons; see discussion in Sec. 3.1 of Ref. [28]. Even if the Lagrangian was invariant under such a symmetry, it is obvious that Eq. (7) would break it explicitly, while it respects $Z_3$. In addition, note that a $d = 5$ operator is inevitably generated by connecting the external $H_d$ and $H_u$ lines of a $d = 7$ operator using a discrete symmetry breaking term $m^2 H_d \cdot H_u$ (see discussion in Sec. 3.1 of Ref. [28]). The term $\mu \tilde{H}_u \cdot \tilde{H}_d$ in the superpotential corresponds to the scalar terms $|\mu|^2 H_u^2 H_u$ and $|\mu|^2 H_d^3 H_d$ in the Lagrangian here (see e.g. Sec. 16 of Ref. [50]), which means that this problem does not occur. Instead, one has a SUSY soft breaking term $B \mu H_u \cdot H_d$, which however would break the discrete symmetry and thus occurs only below $\Lambda_{NP}$. Therefore it should be sufficiently smaller than $\Lambda_{NP}$ resulting in a suppressed $d = 5$ one-loop contribution. Note however, that its value is bounded from below due to searches for the MSSM Higgs boson as it is proportional to the mass of the pseudoscalar Higgs boson.

The possible decompositions of $LLH_u H_u H_u H_u$ are systematically derived in Appendix A at tree level, where the mediators for different possibilities are listed in Table 6. Note that the right-handed fields listed there have to be incorporated as charge-conjugated left-handed fields in the superpotential. These decompositions can be roughly categorized as extensions of the well known $d = 5$ decompositions, the type I, II, and III see-saw scenarios. We define a decomposition as extended type II see-saw, if all mediators are scalars, i.e., decompositions #5, #6, and #21-#24 in Table 6. Therefore the only appearing fermions are the external lepton doublets. The only lepton number violating interaction is then

$$ (\bar{\ell} \gamma_2 \tau L) \tilde{\phi}, \quad (9) $$

where $\phi$ represents one of the scalar mediators. This vertex violates lepton number by $\Delta L = 2$ and therefore conserves R-Parity. All other decompositions that have fermionic mediators can be seen as extensions of the $d = 5$ type I or type III see-saw mechanism. Since we can have several combinations of scalar fields and SU(2) singlet, doublet, or triplet fields as mediators, a further distinction will not be made. Depending on the topology and the actual realization of these operators, the various decompositions have different characteristics. Note that integrating out all but two neutral fermion fields will lead to an inverse see-saw-like scenario, as in Eq. (6).

Let us now illustrate some of the complications involving extra scalars as mediators. As an example we take decomposition #1 from Table 6, shown in Fig. 1. One can easily see that the scalar $\phi$ has the same quantum numbers as the scalar of the NMSSM, and it also has the same coupling to the Higgs fields. If the terms in Eq. (7) are present in the superpotential, it can get a VEV $v_\phi$. This in turn means that the $d = 6$ operator of the type $LLH_u H_u S$ (#2 in Table 2) may be allowed, where $\phi \equiv S$, which may be the leading contribution to neutrino mass. Indeed one can see from the $\lambda H_u H \phi \phi^\dagger$-vertex in Fig. 1 that we have for the discrete symmetry charge $q_\phi = q_{H_u} + q_{H_d}$, which means that we cannot forbid the $d = 6$ operator $LLH_u H_u S$ which leads to neutrino mass if $\phi$ obtains a VEV. In summary, the MSSM extended by a scalar singlet mediator can potentially be NMSSM-like, and can potentially
Figure 1: Decomposition #1 from Table 6 of the effective $d = 7$ operator $LLH_u H_u H_d H_u$. Here $N$ and $N'$ are fermion singlets, and $\phi$ is a scalar singlet.

Figure 2: Decomposition #17 from Table 6 of the effective $d = 7$ operator $LLH_u H_u H_d H_u$. Here $N$ and $N'$ are fermion singlets, and $\xi$ and $\xi'$ are fermion doublets.

induce the $d = 6$ operator, which may dominate the neutrino mass contribution. Since the operator $LLH_u H_u S$ in the NMSSM has been studied in Ref. [24], and since we want to avoid the $d = 6$ operator genuinely, we focus on decompositions with two neutral fermions (to reproduce the inverse see-saw) and no singlet scalars in the following. One of the simplest examples is decomposition #17 in Table 6, which we will discuss in greater detail, see Fig. 2. While for neutral SM singlets as mediators the production rates of the new particles are rather low, the SU(2) doublets in #17 will lead to gauge interactions with potentially observable phenomenology at the LHC. However, note that also the fermion singlets could be replaced by triplets, which would lead to a see-saw III-type phenomenology.

3 A linear or inverse see-saw example

We have seen in the previous section that extensions of the MSSM containing NMSSM-like singlets do have some problems. Therefore we consider a model where we add two
doublets, we obtain

\[ W = W_{\text{quarks}} + Y_e e^\dagger L \cdot \hat{H}_d - Y_N \bar{N}_L \cdot \hat{H}_u + \kappa_1 \bar{N'} \hat{\xi} \cdot \hat{H}_d - \kappa_2 \bar{N'} \hat{\xi'} \cdot \hat{H}_u + m_N \bar{N} \hat{N}' \]

\[ + m_\xi \hat{\xi'} \cdot \hat{\xi} + \mu \hat{H}_u \cdot \hat{H}_d, \]

and the corresponding lepton number assignments are

\[ L(\hat{N}) = -1, \quad L(\hat{N}') = +1, \quad L(\hat{\xi}) = -1, \quad L(\hat{\xi'}) = +1 \]

implying that the interaction proportional to \( \kappa_2 \) breaks lepton number by two units. This superpotential yields the following part of the Lagrangian for the fermions carrying lepton number

\[ \mathcal{L}^{\text{fermionic}} = - Y_e (e^\dagger L \cdot H_d + e^c R L \cdot \bar{H}_d) + Y_N (N_L \cdot H_u + \bar{N}_L \cdot \bar{H}_u + \bar{N} \bar{L} \bar{H}_u) \]

\[ - \kappa_1 (N' \xi \cdot H_d + \bar{N}' \xi \cdot \bar{H}_d + N' \bar{\xi} \cdot \bar{H}_d) + \kappa_2 (N' \xi' \cdot H_u + \bar{N}' \bar{\xi}' \cdot \bar{H}_u + \bar{N}' \bar{\xi}' \cdot \bar{H}_u) \]

\[ - m_N N' N - m_\xi \xi' \cdot \xi + \text{h.c.}, \]

using 2-component spinors. For completeness, note that for the leptons we use the “\( \sim \)” for scalars, whereas for the Higgs bosons we use the “~” for the fermionic partners. After the Higgs fields get a VEV, the mass matrix for the neutral fermions reads in the basis

\[ f^0 = (\nu, N, N', \xi^0, \xi'^0) \]

\[ M^0_f = \begin{pmatrix}
0 & Y_N v_u & 0 & 0 & 0 \\
Y_N^T v_u & 0 & m_N^T & 0 & 0 \\
0 & m_N & 0 & \kappa_1 v_d & \kappa_2 v_u \\
0 & 0 & \kappa_1^T v_d & 0 & -m_\xi \\
0 & 0 & \kappa_2^T v_u & -m_\xi & 0
\end{pmatrix}. \]  

(14)

The corresponding mass eigenstates will be denoted by \( n_i \) with \( |m_i| \leq |m_j| \) for \( i < j \). The mass terms for the charged fermions are given by

\[ -v_d e^c Y_e e^c L - m_\xi \xi^+ \xi'^- . \]  

(15)

We can now determine the neutrino mass, by integrating out the mediator fields. For the sake of simplicity, let us first of all ignore the flavor structure. Integrating out the heavy doublets, we obtain

\[ \mathcal{L}_N = N^\dagger i \tilde{\sigma}^\mu \partial_\mu N + N'^\dagger i \tilde{\sigma}^\mu \partial_\mu N' - m_N N N' - Y_N N_L \cdot H_u - \frac{\kappa_1 \kappa_2}{m_\xi} N' N' H_u \cdot H_d + \text{h.c.}, \]

\[ \mathcal{L}_N = N^\dagger i \tilde{\sigma}^\mu \partial_\mu N + N'^\dagger i \tilde{\sigma}^\mu \partial_\mu N' - m_N N N' - Y_N N_L \cdot H_u - \frac{\kappa_1 \kappa_2}{m_\xi} N' N' H_u \cdot H_d + \text{h.c.}, \]  

(16)

\[ 3 \text{Note that this assignment is, to some extent, arbitrary, and that a different assignment could be chosen such that the interaction proportional to } \kappa_2 \text{ breaks lepton number by two units. However, none of our conclusions is affected by this specific choice.} \]
which reads in the basis \((\nu, N, N')\) after electroweak symmetry breaking

\[
M_f^{\nu'} = \begin{pmatrix}
0 & Y_N v_u & 0 \\
Y_N v_u & 0 & m_N \\
0 & m_N & \hat{\mu}
\end{pmatrix}
\] (17)

with \(\hat{\mu} = v_u v_d (2\kappa_1 \kappa_2) / m_\xi\). This is an inverse see-saw mass matrix, as in Eq. (6). As it is characteristic for the inverse see-saws from higher dimensional operators, the lepton number violating term is suppressed by a heavy scale. If in addition the singlet fermions are integrated out, we obtain for the neutrino mass scale

\[
m_\nu = v_u^3 v_d Y_N^2 \frac{\kappa_1 \kappa_2}{m_\xi m_N^2}.
\] (18)

For a neutrino mass \(m_\nu \approx 1\) eV and \(v \approx 177\) GeV and the heavy mass scale at 1 TeV this means couplings \(O(10^{-3})\) are required. Note that this coupling strength in not unreasonably small, although these couplings are, in addition, protected by lepton number.

If we assume instead a hierarchy of the heavy particles where the isospin singlets are heavier than the doublets, we first can integrate out the singlets. The mass matrix for the remaining neutral fields \((\nu, \xi^0, \xi'^0)\) reads then

\[
M_f^{\nu''} = \begin{pmatrix}
0 & \tilde{\kappa}_1 v_d & \tilde{\kappa}_2 v_u \\
\tilde{\kappa}_1 v_d & 0 & -m_\xi \\
\tilde{\kappa}_2 v_u & -m_\xi & 0
\end{pmatrix},
\] (19)

where \(\tilde{\kappa}_{1/2} = \kappa_{1/2} Y_N^2 / m_N\). Integrating out the \(\xi\) fields afterwards we again arrive at Eq. (18) for the mass of the light neutrinos.

As in the conventional inverse see-saw, in which the \(\hat{\mu}\)-term in Eq. (17) is introduced ad hoc, there are various interesting phenomenological effects of this scenario. It is expected that non-unitarity and its CP violation can be tested at possible long-baseline neutrino oscillation experiments (see, e.g., Refs. [51, 52]). Furthermore, one may observe lepton-flavor-violating (LFV) processes such as \(\mu \to e\gamma\). Lepton-number-violation, on the other hand, is expected to be hardly testable in conventional scenarios, since the heavy Majorana neutrinos form pseudo-Dirac particles with suppressed Majorana character, see, e.g., Ref. [53].

There are several ways to realize a flavor structure that is in accordance with neutrino physics. Since there are three distinct (active) mass eigenstates, at least two of them must be massive. The straightforward approach is to add three generations of the heavy fields, which leaves, however, many unconstrained parameters. Another possibility is to generate one neutrino mass by the inverse seesaw with one generation of mediators, and the second neutrino mass at the one-loop level [57] if the flavor structures in the soft SUSY sector differs from the ones in the superpotential. A third version is the minimal inverse seesaw scenario in Ref. [58], consisting of only two generations of the heavy fields, which narrows down the number of free parameters.

We follow a similar approach, where we assume that one neutrino state is massless. We assume two generations of \(N\) and \(N'\) each, and only one generation for the other mediators.

\[^4\text{Some attempts to avoid the suppression are discussed in e.g., Refs. [54–56].}\]
Thus, compared to Eq. (18), we obtain a mass matrix
\[
(m_\nu)_{\alpha\beta} = v_u^3 v_d (Y_N)_{\alpha i} (m_N^{-1})_{ij} \mu_{jk} (m_N^{-1,T})_{kl} (Y_N^T)_{l\beta},
\]  
(20)
where
\[
\mu_{jk} = \frac{1}{m_\xi} ((\kappa_1)_j (\kappa_2)_k + (\kappa_2)_j (\kappa_1)_k).
\]  
(21)
The flavor basis can be chosen in a way that \(M_N\) (and consequently \(M_N^{-1}\)) is diagonal, without loss of generality. We choose the parameters to reproduce tri-bimaximal mixings [59]5, which depend on the mass hierarchy:

**Normal hierarchy.** A rather straightforward parameterization is:
\[
Y_N = y_N \begin{pmatrix}
\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}, \quad \kappa_1 = k_1 \begin{pmatrix}
-1 \\
1
\end{pmatrix}, \quad \kappa_2 = k_2 \begin{pmatrix}
1 \\
1
\end{pmatrix}, \quad M_N = m_N \begin{pmatrix}
1 & 0 \\
0 & \rho
\end{pmatrix},
\]  
(22)
where
\[
\rho = \sqrt{m_2/m_3}, \quad 2v_u^3 v_d y_N^2 k_1 k_2 / (m_N^2 m_\xi) \equiv m_2.
\]  
(23)
This reproduces the tri-bimaximal mixing pattern and two non-zero mass eigenvalues. In this case, the flavor structure of the neutrinos is dominantly generated by the neutrino Yukawa couplings \(Y_N\). Since we have more parameters than constraints from neutrino physics, there is a certain freedom in the parameters of the couplings. For example one can vary \(y_N\) as long as this is compensated by an according change of \(m_N\) or \(k_{1/2}\). The mass ratio \(\rho = \sqrt{m_2/m_3}\) can be generated by \(y_N, m_N\) or \(k_{1/2}\). A possible set of parameters is \(3y_N = 10^{-3}, k_1 = k_2 = 10^{-2}, \tan \beta = 10, m_N = 1070 \text{ GeV},\) and \(m_\xi = 200 \text{ GeV},\) which we will use in the next section. We have checked that this point is compatible with bounds on rare lepton decays such as \(\mu \rightarrow e\gamma\) as well as with the search for the tri-lepton signal of supersymmetric particles at the Tevatron [61] and searches for new physics in final states containing leptons at the LHC [62,63]. In order to satisfy the bounds from the rare decays, we have assumed that the scalar leptons are so heavy that their contributions are suppressed and that the leading contributions are due to loops containing fermions and the \(W\)-boson. Note that the product \(v_u^3 v_d\) in Eq. (23) peaks at about \(\tan \beta \simeq 2\), and it becomes small for large \(\tan \beta\).

**Inverted hierarchy.** The inverted hierarchy can be obtained by the parameterization
\[
Y_N = y_N \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}
\end{pmatrix}, \quad \kappa_1 = k_1 \begin{pmatrix}
-1 \\
1
\end{pmatrix}, \quad \kappa_2 = k_2 \begin{pmatrix}
1 \\
1
\end{pmatrix}, \quad M_N = m_N \begin{pmatrix}
1 & 0 \\
0 & \rho
\end{pmatrix},
\]  
(24)
where \(\rho = \sqrt{m_1/m_2}\) and
\[
2v_u^3 v_d y_N^2 k_1 k_2 / (m_N^2 m_\xi) \equiv m_1.
\]  
(25)
In the following, we only consider the normal hierarchical example.

---

5If \(\theta_{13} > 0\), as indicated by the recent T2K hint [60], a different flavor structure can be easily implemented.
Figure 3: Total cross section $\sigma(pp \rightarrow \xi^\pm \xi^0)$ as a function of the mass $m_\xi$.

4 LHC phenomenology

In many supersymmetric versions of neutrino mass models one finds traces of the underlying mechanism generating neutrino masses in the spectrum and decay properties of the supersymmetric particles, for an incomplete list see e.g. [19, 21, 64–74]. Before discussing this in more detail, let us have a look on the number of parameters related to neutrino physics in this model. Working a basis where the lepton Yukawa couplings are flavor diagonal, the following parameters contribute: $Y_N, \kappa_i, m_N$ and $m_\xi$ which amounts in our specific model into 24 real parameters if all CP violating phases are taken into account, but which gets reduced to 13 if all phases are zero. From these at most six can be determined in the near future, leaving 18 (7) parameters undetermined. Here the question arises to which extent they might be measured or at least constrained at the LHC. In principle we have sufficient many decays to determine them: six heavy neutral states decaying into the three charged leptons, in total 18 decays.

Of particular interest is the question to which extent the new particles, which are postulated in Eq. (10), can be produced at the LHC. It turns out that except for finely tuned parameter combinations, the heavy states are either mainly $SU(2)$ singlets or mainly $SU(2)$ doublets corresponding to the electroweak states. The singlet states contained in the superfields $\hat{N}$ and $\hat{N}^\prime$ can only be produced in cascade decays. However this will happen in rare occasions only due to the smallness of the involved Yukawa couplings. The $SU(2)_L$ doublets contained in $\hat{\xi}$ and $\hat{\xi}^\prime$, on the other hand, can in principle be produced directly in Drell-Yan processes similarly to sleptons or charginos and neutralinos within the usual MSSM [75]. In Figure 3 we show the cross section for $\xi^+ \xi^0$ as a function of $m_\xi$ for $\sqrt{s} = 7$ TeV and 14 TeV assuming that the mixing with the singlet fields is small. The numbers have been obtained using the WHIZARD package [76]. If one takes for example a mass of 200 GeV for $\xi^+$ and $\xi^0$ using the numerical values for the couplings as given in the previous section we find for the total cross section $\sigma(pp \rightarrow \xi^\pm \xi^0)$ about 122 fb (417 fb) in case of 7 (14) TeV cms-energy using the WHIZARD program [76]. Note, that the states $n_4$ and $n_5$ are mainly a nearly
maximal mixed superposition of the neutral components of the $SU(2)$ doublets in this case $n_{4,5} \simeq (\xi^0 \pm \xi'^0)/\sqrt{2}$.

In the following, we concentrate on the fermionic states, as they are directly related to neutrino physics. We assume for the moment that their scalar partners are much heavier, so that the only possible decay channels are into SM fermions and Higgs bosons, or vector bosons. In this case, the main decay modes are

$$\xi^+ \rightarrow W^+ \nu_k, \ H^+ \nu_k$$  \hspace{1cm} (26)

for $\xi^+$, which is the Dirac fermion composed of the charged components of $\xi$ and $\xi'$. Note, that there are no decays into $Z$ or $h^0$ as this particle does not mix with the charged leptons at tree-level. One expects small decay widths as these decays have their origin in the mixing of $\xi^0$ with neutrinos and, thus, their widths are proportional to the corresponding mixing matrix element squared. We indeed find $\Gamma(\xi^+) = 1.42 \cdot 10^{-5} \text{ keV}$. In case of the neutral fermions $n_i$, a larger variety of decay channels is possible:

$$n_i \rightarrow W^\pm l^\mp_j, \ H^\pm l^\mp_j$$  \hspace{1cm} (27)

$$n_i \rightarrow Z\nu_k, \ h^0\nu_k, \ H^0\nu_k, \ A^0\nu_k$$  \hspace{1cm} (28)

with $l_j = e, \mu, \tau$. In these cases, the decays also originate from the mixing of the neutral states with the neutrinos and, thus, the corresponding widths are expected to be small as can be seen in Table 3 where we give the corresponding widths and branching ratios for the scenario discussed in the previous section. Obviously some of the widths are so small that one can expect sizable decay lengths at the LHC in the range of 100 $\mu$m to several mm once the boost factor is taken into account. This is an important feature because in this way one can not only suppress the SM background, but one can also identify the leptons coming from these decays and distinguish them from the ones coming from the cascade decays of supersymmetric particles. Another interesting feature is, that there are two pairs of states where within each pair the branching ratios are nearly equal: $n_6/n_7$ and $n_8/n_9$. The reason is that they form a quasi Dirac fermion. Also in case of $n_4$ and $n_5$ we have a quasi Dirac fermion resulting in difficulties to determine the branching ratios of the individual states. As a consequence at most 9 branching ratios can be related to neutrino physics in praxis.

The fact that the $n_i$ decay into $W^\pm l^\mp$ clearly proves that these particles carry lepton number and, thus, one might suspect that they are related to the generation of neutrino masses. An important question is in this context to which extent one can prove their Majorana nature by observing both lepton charges in the final states. Therefore one has to look for final states which violate lepton number by two units compared to the initial one:

$$u\bar{d} \rightarrow l^+l'^+W^-$$  \hspace{1cm} (29)

$$u\bar{d} \rightarrow l^+l'^+W^-Z$$  \hspace{1cm} (30)

$$q\bar{q} \rightarrow l^+l'^+W^-W^-, l^-l'^-W^+W^+$$  \hspace{1cm} (31)

Note that the leptons can easily be of different generations due to the large mixing angles in the neutrino sector. In the calculation of these processes we have included all possible
The results for the 2 → 3 processes are shown in Table 4. Note that in this case only final states containing a W-boson are possible, as the ξ^+ does not decay into charged leptons. The main contributions in this case are due to

$$ud \rightarrow l^+ n_i^* \quad (i = 1, \ldots 5)$$

as shown in Fig. 4. Here the $n_i$ are in this case either mainly neutrinos ($i = 1, 2, 3$), or an admixture of $ξ^0$ or $ξ^{0*}$ ($i = 4, 5$). The contributions of the neutrino-like states are suppressed because they are off-shell, whereas in case of the $ξ^0/ξ^{0*}$-like states, there are on-shell contributions. These are, however, somewhat suppressed by the small mixing elements.
with the neutrinos. We have put a cut on the invariant mass of the leptons of 10 GeV as otherwise the $e^+e^-$ final states would be enhanced by several orders of magnitude due to an nearly on-shell photon. The flavor mixed final states are of the order of a few $fb$ and thus are potentially observable if sufficient luminosity is accumulated. Note that for extracting the corresponding signal, only the hadronic final states of the $W$-boson should be considered and, thus, the cross section shown has to be multiplied by the corresponding branching ratio $BR(W \rightarrow q\bar{q}')$. In the case that the two leptons have different flavor, these processes are essentially background free. However, in case of equal flavor leptons multi $W$-production in association with a $Z$-boson or an off-shell photon will contribute. Both contributions can be suppressed by putting cuts on the invariant mass of the two leptons.

One also sees from these tables that the lepton number violating final states are strongly suppressed which is due to the appearance of the pseudo-Dirac like state $n_4/n_5$ implying that the final contribution to the cross section is proportional to $m^2_{n_5} - m^2_{n_4} \simeq O(m^2_\nu)$, and thus tiny.

In Table 5, we give cross sections for the $2 \rightarrow 4$ processes containing two $W$-bosons. Note that we do not give the corresponding ones containing a $Z$-boson, see Eq. (30), which are smaller because the corresponding contributions are those of $2 \rightarrow 3$ processes plus an additional $Z$-boson, attached to all internal and external lines in case of the lepton flavor mixing/violating final states. As expected, the cross sections of lepton flavor conserving and lepton flavor mixing final states are about two orders of magnitude smaller than the ones of the corresponding $2 \rightarrow 3$ processes. However, the cross sections for the lepton number violating processes are larger than naively expected. This can be understood as follows: in case of the $2 \rightarrow 3$ processes all lepton number violating contributions are due to the Majorana nature of the neutral fermions and are suppressed by the pseudo-Dirac like nature of the heavy states. In case of the $2 \rightarrow 4$ processes, there are additional contributions which are proportional to the momentum of the off-shell neutral particles times two powers of lepton number violating couplings, e.g. they are proportional to $|c_{d_i}d_i|^2$. Performing an approximate diagonalization of the neutral fermion mass matrix as done in Appendix B, one

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures/figure4.png}
\caption{Dominant contribution to the LNV processes $u\bar{d} \rightarrow W^-e^+e^+$ and $q\bar{q} \rightarrow W^-W^-e^+e^+$.}
\end{figure}
| Process                        | $\sigma$ [fb] (7 TeV) | $\sigma$ [fb] (14 TeV) |
|-------------------------------|-----------------------|------------------------|
| $pp \to W^+ e^+ e^-$          | $(1.651 \pm 0.024) \cdot 10^2$ | $(4.161 \pm 0.023) \cdot 10^2$ |
| $pp \to W^- e^+ e^-$          | $(9.240 \pm 0.033) \cdot 10$ | $(2.671 \pm 0.042) \cdot 10^2$ |
| $pp \to W^+ e^+ \mu^-$        | $(1.068 \pm 0.099)$ | $(2.848 \pm 0.011)$ |
| $pp \to W^+ e^- \mu^+$        | $(1.057 \pm 0.013)$ | $(2.871 \pm 0.012)$ |
| $pp \to W^- e^- \mu^-$        | $(5.748 \pm 0.015) \cdot 10^{-1}$ | $(1.742 \pm 0.015)$ |
| $pp \to W^- e^- \mu^+$        | $(5.755 \pm 0.015) \cdot 10^{-1}$ | $(1.753 \pm 0.017)$ |
| $pp \to W^+ e^- \tau^-$       | $(1.058 \pm 0.096)$ | $(2.861 \pm 0.011)$ |
| $pp \to W^+ e^+ \tau^+$       | $(1.056 \pm 0.095)$ | $(2.854 \pm 0.011)$ |
| $pp \to W^- e^+ \tau^-$       | $(5.714 \pm 0.015) \cdot 10^{-1}$ | $(1.754 \pm 0.015)$ |
| $pp \to W^- e^- \tau^+$       | $(5.750 \pm 0.015) \cdot 10^{-1}$ | $(1.744 \pm 0.019)$ |
| $pp \to W^+ \mu^+ \mu^-$      | $(1.676 \pm 0.014) \cdot 10^2$ | $(4.116 \pm 0.023) \cdot 10^2$ |
| $pp \to W^- \mu^+ \mu^-$      | $(9.242 \pm 0.033) \cdot 10$ | $(2.677 \pm 0.035) \cdot 10^2$ |
| $pp \to W^+ \mu^+ \tau^-$     | $(2.668 \pm 0.024) \cdot 10^{-1}$ | $(7.092 \pm 0.028) \cdot 10^{-1}$ |
| $pp \to W^- \mu^+ \tau^+$     | $(2.652 \pm 0.026) \cdot 10^{-1}$ | $(7.187 \pm 0.029) \cdot 10^{-1}$ |
| $pp \to W^- \mu^- \tau^-$     | $(1.432 \pm 0.006) \cdot 10^{-1}$ | $(4.424 \pm 0.038) \cdot 10^{-1}$ |
| $pp \to W^+ \mu^- \tau^+$     | $(1.439 \pm 0.004) \cdot 10^{-1}$ | $(4.433 \pm 0.037) \cdot 10^{-1}$ |
| $pp \to W^+ \tau^+ \tau^-$    | $(1.665 \pm 0.023) \cdot 10^2$ | $(4.138 \pm 0.063) \cdot 10^2$ |
| $pp \to W^- \tau^+ \tau^-$    | $(9.265 \pm 0.034) \cdot 10$ | $(2.652 \pm 0.035) \cdot 10^2$ |
| $pp \to W^- e^+ e^+$          | $(4.711 \pm 0.069) \cdot 10^{-12}$ | $(4.847 \pm 0.030) \cdot 10^{-11}$ |
| $pp \to W^+ e^- e^-$          | $(1.423 \pm 0.008) \cdot 10^{-12}$ | $(1.818 \pm 0.071) \cdot 10^{-11}$ |
| $pp \to W^- e^+ \mu^+$        | $(1.017 \pm 0.014) \cdot 10^{-11}$ | $(9.869 \pm 0.054) \cdot 10^{-11}$ |
| $pp \to W^- e^- \mu^-$        | $(3.184 \pm 0.015) \cdot 10^{-12}$ | $(3.22 \pm 0.15) \cdot 10^{-11}$ |
| $pp \to W^+ e^+ \tau^+$       | $(1.169 \pm 0.015) \cdot 10^{-11}$ | $(1.050 \pm 0.054) \cdot 10^{-10}$ |
| $pp \to W^+ e^- \tau^-$       | $(4.173 \pm 0.020) \cdot 10^{-12}$ | $(4.12 \pm 0.28) \cdot 10^{-11}$ |
| $pp \to W^- \mu^+ \mu^+$      | $(5.861 \pm 0.082) \cdot 10^{-9}$ | $(2.278 \pm 0.013) \cdot 10^{-8}$ |
| $pp \to W^+ \mu^- \mu^-$      | $(2.377 \pm 0.010) \cdot 10^{-9}$ | $(1.153 \pm 0.017) \cdot 10^{-8}$ |
| $pp \to W^- \mu^+ \tau^+$     | $(1.184 \pm 0.013) \cdot 10^{-8}$ | $(4.584 \pm 0.023) \cdot 10^{-8}$ |
| $pp \to W^- \mu^- \tau^-$     | $(4.788 \pm 0.018) \cdot 10^{-9}$ | $(2.363 \pm 0.039) \cdot 10^{-8}$ |
| $pp \to W^- \tau^+ \tau^+$    | $(5.956 \pm 0.080) \cdot 10^{-9}$ | $(2.292 \pm 0.031) \cdot 10^{-8}$ |
| $pp \to W^+ \tau^- \tau^-$    | $(2.383 \pm 0.010) \cdot 10^{-9}$ | $(1.120 \pm 0.014) \cdot 10^{-8}$ |

**Table 4**: Cross-sections for the processes with $W^{\pm} \ell^{\pm} \ell^{\pm}$ as final states (lepton number violating processes in lower section). A cut on the invariant lepton mass of 10 GeV has been assumed.
Table 5: Cross-sections for the processes with $W^+\ell^-W^\pm\ell^\mp$ as final states (lepton number violating processes in lower section). A cut on the invariant lepton mass of 10 GeV has been assumed.

| Process                        | $\sigma$ [fb] (7 TeV)       | $\sigma$ [fb] (14 TeV)       |
|--------------------------------|-----------------------------|-----------------------------|
| $pp \rightarrow W^+e^-W^-e^+$  | $(3.447 \pm 0.87) \cdot 10^{-1}$ | $(1.277 \pm 0.66)$ |
| $pp \rightarrow W^+e^-W^-\mu^+$ | $(7.06 \pm 0.15) \cdot 10^{-3}$ | $(3.141 \pm 0.027) \cdot 10^{-2}$ |
| $pp \rightarrow W^+e^-W^-\mu^-$  | $(6.99 \pm 0.16) \cdot 10^{-3}$ | $(3.206 \pm 0.027) \cdot 10^{-2}$ |
| $pp \rightarrow W^+e^-W^-\tau^+$ | $(1.037 \pm 0.020) \cdot 10^{-2}$ | $(4.293 \pm 0.036) \cdot 10^{-2}$ |
| $pp \rightarrow W^+e^-W^-\tau^-$  | $(1.015 \pm 0.021) \cdot 10^{-2}$ | $(4.411 \pm 0.036) \cdot 10^{-2}$ |
| $pp \rightarrow W^+\mu^-W^-\mu^+$ | $(3.74 \pm 0.10) \cdot 10^{-1}$ | $(1.279 \pm 0.017)$ |
| $pp \rightarrow W^+\mu^-W^-\tau^+$ | $(2.913 \pm 0.048) \cdot 10^{-3}$ | $(1.096 \pm 0.007) \cdot 10^{-1}$ |
| $pp \rightarrow W^+\mu^+W^-\tau^-$  | $(2.990 \pm 0.042) \cdot 10^{-2}$ | $(1.139 \pm 0.007) \cdot 10^{-1}$ |
| $pp \rightarrow W^+\tau^-W^-\tau^+$ | $(4.27 \pm 0.10) \cdot 10^{-1}$ | $(1.606 \pm 0.017)$ |
| $pp \rightarrow W^+e^-W^+e^-$    | $(1.112 \pm 0.013) \cdot 10^{-4}$ | $(4.261 \pm 0.028) \cdot 10^{-4}$ |
| $pp \rightarrow W^+e^-W^+\mu^-$  | $(1.537 \pm 0.023) \cdot 10^{-3}$ | $(5.810 \pm 0.050) \cdot 10^{-3}$ |
| $pp \rightarrow W^+e^-W^+\tau^-$  | $(4.721 \pm 0.055) \cdot 10^{-3}$ | $(1.761 \pm 0.016) \cdot 10^{-2}$ |
| $pp \rightarrow W^+\mu^-W^+\mu^-$ | $(4.099 \pm 0.052) \cdot 10^{-3}$ | $(1.514 \pm 0.013) \cdot 10^{-2}$ |
| $pp \rightarrow W^+\mu^-W^+\tau^-$ | $(2.704 \pm 0.036) \cdot 10^{-2}$ | $(1.062 \pm 0.093) \cdot 10^{-1}$ |
| $pp \rightarrow W^+\tau^-W^+\tau^-$ | $(4.614 \pm 0.065) \cdot 10^{-2}$ | $(1.729 \pm 0.016) \cdot 10^{-1}$ |

Table 5 shows that this combination of couplings does not vanish in the limit of vanishing neutrino masses as they are roughly proportional to

$$
\frac{(a_i\kappa_1 + b_i\kappa_2)^4}{M_{\kappa}^2m_{\kappa}^4}
$$

(37)

e.g. they only vanish in the limit where either one of the heavy masses goes to infinity or both couplings, $\kappa_1$ and $\kappa_2$, to zero. This is a consequence of the fact that $\xi$ and $\xi'$ form a vector-like representation of SU(2).

In summary we find that one should be able to detect the SU(2) doublets up to masses of about 1 TeV and show that they carry lepton number. However, it turns out that the cross sections for the processes violating total lepton number are on the edge to be discovered, as they would require at least a luminosity of the order of $ab^{-1}$ in the most optimistic cases, e.g., by considering at least 10 events without any background considerations due to detector effects.
5 Summary and conclusions

In this work we have studied neutrino mass generation from higher than $d = 5$ effective operators in supersymmetric models. While the $d = 5$ operator typically points towards the GUT scale, higher dimensional operators may be generated by mediators observable at the LHC. If any $d = 5$ contribution is to be forbidden, a discrete symmetry is needed, which can be used to control the dimension of the effective operator generically dominating neutrino mass. While this discrete symmetry is to be softly broken by the $\mu$-term of the MSSM, the $Z_3$ symmetry, the SUSY Lagrangian is invariant under, can be naturally used in the NMSSM. We have also taken into account that in the NMSSM, higher than $d = 5$ effective operators leading to neutrino mass may include the NMSSM scalar and the two Higgs doublets. While the NMSSM scalar can be used in $d = 6$ and $d = 7$ effective operators, for $d > 7$, only Higgs doublets are allowed in the effective operators, since otherwise lower dimensional effective operators are generated as well. Therefore, we have focused on the $d = 7$ operator $LLH_u H_u H_u H_u$ as the simplest possible example in the following, which respects this line of argumentation.

For this operator, we have derived the list of possible decompositions at tree level systematically. The results have been similar to an earlier work [28], with the exception that some topologies have been forbidden by the holomorphicity of the superpotential. Many of the derived decompositions can be regarded as extensions of the usual type I, II, or III see-saw mechanisms because of a similar field content. Models with two extra heavy fermion singlets, for example, lead to inverse see-saw scenarios if the additional mediators are integrated out, where the lepton number violating term is naturally suppressed by the mediator mass. As a peculiarity of supersymmetry, we have identified that singlet scalars are potentially harmful because they may induce lower dimensional operators dominating neutrino mass if similar to the NMSSM scalar. Therefore, we have chosen an example with two extra fermion singlets and heavy lepton doublets which are vector-like under $SU(2)$. We have also demonstrated how the flavor structure for normal and inverted hierarchy can be easily implemented using two generations of the heavy fermion singlets.

Focusing on the new fermions, we have demonstrated that parts of the model can already be tested with the current LHC run at 7 TeV by displaced vertices, and at 14 TeV we expect that it can be tested up to masses of several hundred GeV for the $SU(2)$ doublets. We have also seen that the cross sections of some of the lepton number violating processes are larger than naively expected, but still on the edge of observability at the LHC.

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Possible topologies for the effective $d = 7$ operator $LLH_uH_dH_u$. Topologies 3 and 4 cannot be realized in SUSY. Solid lines are either fermions or scalars, dashed lines are always scalars.

A Possible decompositions for the operator $LLH_uH_uH_dH_u$

In this appendix, we systematically discuss underlying, more fundamental models leading to operator #3 in Table 1 or Table 2 at tree level. The results are similar to Ref. [28] in the THDM. The possible topologies for the decompositions can be found in Fig. 5. In SUSY, however, topologies 3 and 4 can be excluded. This is due to the fact that scalar couplings in SUSY have to be of the type $\phi^\dagger \phi$, $\phi^\dagger \phi \phi$, $\phi^\dagger \phi^\dagger \phi$, or $\phi^\dagger \phi^\dagger \phi \phi$, since they are generated by F-terms and D-terms, as a consequence of holomorphy. In topology 3, the scalar four-vertex has to be of the type $HHX^*X^*$, where $X$ is a heavy virtual scalar field. The three-vertex must be $HHX^*$. These two vertices can not be connected by a propagator $\Delta_X$. Hence topology 3 can not be realized in SUSY. In topology 4, the four-scalar vertex can only be of the type $H_uH_uH_dH_u$ or $H_uH_uH_uH_u$ in order to produce an effective operator of the type $LLH_uH_uH_dH_u$. The only possible scalar four couplings allowed by SUSY, however, are of the type $\phi^\dagger \phi^\dagger \phi \phi$. Hence also topology 4 is not possible. The most economical extensions of the (N)MSSM may use the superpartners of the SM fields as mediators. However, at least at tree level and with R-Parity conservation, this is not possible. As all external fields, $L$, $H_u$ and $H_d$, have $R = +1$, a mediator with $R = -1$ would cause vertices where R-Parity is violated. As a consequence, we have to introduce additional fields as mediators, and we also obtain superpartners for them. The possible decompositions of the $d = 7$ operator $LLH_uH_uH_dH_u$ at tree level are shown in Table 6, where the brackets refer to the vertices with external fields for any given topology. If $\bar{\tau}$ appears, the fields couple to a triplet mediator; if not, they couple to a singlet. The mediators are denoted by $X^L_Y$, where

- $X$ denotes the SU(2) nature, i.e., singlet $1$, doublet $2$, or triplet $3$.
- $L$ refers to the Lorentz nature, i.e., scalar (s), vector (v), left-handed (L) or right-handed (R) chiral fermion.
- $Y$ refers to the hypercharge $Y = Q - I^W_3$.

Besides the fixed sign of the scalars’ hypercharges and the forbidden topologies 3 and 4, the decompositions are similar to the THDM case in Ref. [28]. Note that $R$ and $L$ indicate right-
| #  | Operator                                                                 | Top. | Mediators                           |
|----|--------------------------------------------------------------------------|------|-------------------------------------|
| 1  | \((H_u \iota^2 \bar{L}^c)(H_u \iota^2 L)(H_d \iota^2 H_u)\)              | 2    | \(1_R^0, 1_L^1, 1_s^0\)          |
| 2  | \((H_u \iota^2 \bar{L}^c)(H_u \iota^2 L)(H_d \iota^2 \bar{H}_u)\)      | 2    | \(3_R^0, 3_L^1, 1_R^1, 1_L^1, 3_s^0\) |
| 3  | \((H_u \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_d \iota^2 H_u)\)      | 2    | \(3_R^0, 3_L^0, 1_s^0\)          |
| 4  | \((-\iota e_{abc})(H_u \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_d \iota^2 \bar{H}_u)\) | 2    | \(3_L^0, 3_L^0, 3_L^0\)          |
| 5  | \((L \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_d \iota^2 \bar{H}_u)\)   | 2    | \(3_L^1, 3_L^0, 1_s^0\)          |
| 6  | \((-\iota e_{abc})(L \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_d \iota^2 \bar{H}_u)\) | 2    | \(3_L^1, 3_L^0, 1_s^0\)          |
| 7  | \((H_u \iota^2 \bar{L}^c)(L \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_d \iota^2 \bar{H}_u)\) | 2    | \(3_R^0, 3_R^0, 1_s^0\)          |
| 8  | \((-\iota e_{abc})(H_u \iota^2 \bar{L}^c)(L \iota^2 \bar{L}^c)(H_u \iota^2 \bar{H}_u)\) | 2    | \(3_R^0, 3_R^0, 1_s^0\)          |
| 9  | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(1_R^0, 1_L^1, 2_R^1, 2_L^1, 1_s^0\) |
| 10 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 2_R^1, 2_L^1, 1_s^0\) |
| 11 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 12 | \((H_u \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 13 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 14 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 15 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 16 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 17 | \((H_u \iota^2 \bar{L}^c)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 18 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 19 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 20 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{H}_u)\)                   | 1    | \(3_R^0, 3_R^0, 3_R^0\)          |
| 21 | \((T \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 22 | \((T \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 23 | \((T \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 24 | \((T \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 25 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 26 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 27 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 28 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 29 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 30 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 31 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 32 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 33 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |
| 34 | \((H_u \iota^2 \bar{L}^c)(L)(H_u \iota^2 \bar{L})(H_u \iota^2 \bar{H}_u)\) | 1    | \(3_s^1, 2_s^1, 3_s^1\)          |

**Table 6:** Decompositions of the \(d = 7\) operator \(LLH_uH_uH_u\) at tree level.
and left-handed fermions, respectively, where the right-handed ones can also be represented by left-handed Weyl spinors after charge conjugation. All charged scalar fields must have an additional partner of opposite charge (not listed) to make a mass term possible in the superpotential.

B Approximate diagonalization of neutral fermion mass matrix

In our model the complete mass matrix including the flavor structure is given by

\[
\begin{pmatrix}
0 & 0 & 0 & v_u Y_{N,11} & v_u Y_{N,12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v_u Y_{N,21} & v_u Y_{N,22} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v_u Y_{N,31} & v_u Y_{N,32} & 0 & 0 & 0 & 0 \\
v_u Y_{N,11} & v_u Y_{N,21} & v_u Y_{N,31} & 0 & 0 & M_N & 0 & 0 & 0 \\
v_u Y_{N,12} & v_u Y_{N,22} & v_u Y_{N,32} & 0 & 0 & 0 & M_N & 0 & 0 \\
0 & 0 & 0 & M_N & 0 & 0 & 0 & -k_1 v_d & k_2 v_u \\
0 & 0 & 0 & 0 & M_N & 0 & 0 & k_1 v_d & k_2 v_u \\
0 & 0 & 0 & 0 & 0 & -k_1 v_d & k_1 v_d & 0 & -m_\xi \\
0 & 0 & 0 & 0 & 0 & k_2 v_u & k_2 v_u & -m_\xi & 0 \\
\end{pmatrix}
\]

(38)

Using the fact, the the left-handed neutrinos are essentially massless compared to the heavy states we can exploit the usual seesaw formulas to obtain approximate formulas for the entries responsible for the mixing of the light states with the heavy states. The mass matrix of the heavy states is given by

\[
M_H = \begin{pmatrix}
0 & 0 & M_N & 0 & 0 & 0 \\
0 & 0 & 0 & M_N & 0 & 0 \\
M_N & 0 & 0 & 0 & -k_1 v_d & k_2 v_u \\
0 & M_N & 0 & 0 & k_1 v_d & k_2 v_u \\
0 & 0 & -k_1 v_d & k_1 v_d & 0 & -m_\xi \\
0 & 0 & k_2 v_u & k_2 v_u & -m_\xi & 0 \\
\end{pmatrix}
\]

(39)

Neglecting the elements proportional to \(k_i\) (\(i = 1, 2\)) this matrix is diagonalized by

\[
R_H = \begin{pmatrix}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\end{pmatrix}
\]

(40)

The part of the mixing matrix connecting the heavy states with the light states is given by

\[
U' = m M_H^{-1} R_H
\]

\[
= \begin{pmatrix}
D_1 Y_{N,11} & D_2 Y_{N,12} & D'_1 Y_{N,11} & D'_1 Y_{N,12} & \frac{v_u v_d (k_2^2 Y_{N,12} - k_1^2 Y_{N,11})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2^2 Y_{N,12} - k_1^2 Y_{N,11})}{\sqrt{2} M_N m_\xi \rho} \\
D_1 Y_{N,21} & D_2 Y_{N,22} & D'_1 Y_{N,21} & D'_1 Y_{N,22} & \frac{v_u v_d (k_2^2 Y_{N,22} - k_1^2 Y_{N,21})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2^2 Y_{N,22} - k_1^2 Y_{N,21})}{\sqrt{2} M_N m_\xi \rho} \\
D_1 Y_{N,31} & D_2 Y_{N,32} & D'_1 Y_{N,31} & D'_1 Y_{N,32} & \frac{v_u v_d (k_2^2 Y_{N,32} - k_1^2 Y_{N,31})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2^2 Y_{N,32} - k_1^2 Y_{N,31})}{\sqrt{2} M_N m_\xi \rho} \\
\end{pmatrix}
\]
with

\[
m = \begin{pmatrix}
    v_u Y_{N,11} & v_u Y_{N,12} & 0 & 0 & 0 \\
    v_u Y_{N,21} & v_u Y_{N,22} & 0 & 0 & 0 \\
    v_u Y_{N,31} & v_u Y_{N,32} & 0 & 0 & 0
\end{pmatrix}
\]  \tag{42}

\[
D_1 = \frac{v_u (M_N m_\xi + 2k_1 k_2 v_d v_u)}{\sqrt{2} M_N^2 m_\xi}
\]  \tag{43}

\[
D_2 = \frac{v_u (\rho M_N m_\xi - 2k_1 k_2 v_d v_u)}{\sqrt{2} \rho^2 M_N^2 m_\xi}
\]  \tag{44}

\[
D'_1 = -\frac{v_u (\rho M_N m_\xi + 2k_1 k_2 v_d v_u)}{\sqrt{2} \rho^2 M_N^2 m_\xi}
\]  \tag{45}

\[
D'_2 = -\frac{v_u (M_N m_\xi - 2k_1 k_2 v_d v_u)}{\sqrt{2} M_N^2 m_\xi}
\]  \tag{46}

\[
k'_1 = k_1 - k_2 \tan \beta
\]  \tag{47}

\[
k'_2 = k_1 + k_2 \tan \beta
\]  \tag{48}

Here we have the following correspondence to the couplings in Section 4, Eq. (33):

\[
c_i = U'_{i5}, \quad d_i = (U'_{ia})^*
\]  \tag{49}

which are the dominating ones for the lepton number violating processes.

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