THE UNIVERSALITY OF INITIAL CONDITIONS FOR GLOBULAR CLUSTER FORMATION

Sergey Mashchenko1 and Alison Sills

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ABSTRACT

We investigate a simple model for globular cluster (GC) formation. We simulate the violent relaxation of initially homogeneous isothermal stellar spheres and show that it leads to the formation of clusters with radial density profiles that match the observed profiles of GCs. The best match is achieved for dynamically unevolved clusters. In this model, all the observed correlations between global GC parameters are accurately reproduced if one assumes that all the clusters initially had the same value of the stellar density and the velocity dispersion. This suggests that the gas that formed GCs had the same values of density and temperature throughout the universe.

Subject headings: globular clusters: general — methods: N-body simulations

1. INTRODUCTION AND MOTIVATION

Globular clusters (GCs) are massive, dense clusters of stars, and in many galaxies they are the oldest datable objects. They contain important clues to early star formation in the universe and to the formation history of our Milky Way and other galaxies. The dynamical evolution of GCs has been studied for many decades and is considered to be well understood. However, the formation of GCs is still an open question.

The internal structure and dynamics of Galactic GCs are well described by a family of lowered isothermal models (so-called King models; King 1966). The radial profiles of King models have a flat core (characterized by a core radius \( r_c \)) and a sharp cutoff at a tidal radius \( r_t \). GCs are currently in dynamical equilibrium. Initially they could have formed in some nonequilibrium configuration that relaxed to a King model through violent relaxation (Meylan & Heggie 1997) followed by a slow evolution in the Galactic tidal field.

Currently no models of GC formation explain all the observed properties of these star clusters (Djorgovski & Meylan 1994; McLaughlin 2000). Correlations between parameters that are not significantly affected by the dynamical evolution of a cluster (such as half-mass radius, central velocity dispersion, and binding energy) are thought to reflect the initial conditions for GC formation. Therefore, any model of cluster formation should aim to address the observed correlations.

In this Letter, we investigate the simplest possible model for GC formation: the dynamical evolution of a homogeneous isothermal stellar sphere. Simulations of the collapse of homogeneous stellar sphere with constant velocity dispersion were performed 20 years ago (van Albada 1982) with the intention of reproducing the \( r^{1/4} \) density distribution of elliptical galaxies. Those efforts were not very successful—the collapse produced instead a core-halo structure, somewhat like a GC. In this Letter, we compare a family of such models with the observed density profiles of GCs and attempt to use the family of such models to reproduce correlations between parameters of GCs in the Milky Way.

2. MODEL

We propose a homogeneous isothermal stellar sphere as an initial nonequilibrium configuration for GCs. To test this idea, we evolve stellar spheres with a total mass \( M \), an initial density \( \rho_{ini} \) and an initial velocity dispersion \( \sigma_{ini} \) for an initial crossing time \( t_{cross} = (R_{ini}/M)^{1/2} \), where \( R_{ini} = (3M/4\pi\rho_{ini})^{1/3} \) is the initial radius of the system. (In this Letter we assume a system of units in which the gravitational constant \( G = 1 \).) We use the parallel N-body tree code GADGET (Springel, Yoshida, & White 2001) to run the simulations. The stars in the cluster are represented by \( N \sim 10^5 \) equal mass particles. The velocities of the particles have a Maxwellian distribution. The gravitational potential is softened with a softening length of \( \epsilon = 0.77R_eN^{-1/3} \), where \( R_e \) is the initial half-mass radius of the cluster. (Gravity is softened to minimize numerical noise due to two-body interactions between the particles.) The individual time steps are equal to \((2n/\epsilon a)^{1/2} \), where \( a \) is the acceleration of a particle, and the parameter \( \eta \) is made small enough to ensure the total energy conservation to better than 1%. This configuration is the simplest realization of possible initial conditions for GCs and yet results in remarkably good agreement with current cluster parameters.

It is convenient to express the mass of a cluster \( M \) in units of the virial mass \( M_{vir} \) : \( M = M_{vir}10^6 \), where \( \beta \) is a mass parameter. For a homogeneous sphere \( M = \sigma_{ini}^3/(4\pi\rho_{ini}/375)^{1/3} \). The total energy of the system becomes positive (and the system becomes formally unbound) for \( \beta < -0.45 \).

We ran a set of 17 different models (see Table 1) with the same values of \( \rho_{ini} \) and \( \sigma_{ini} \) and parameter \( \beta \) ranging from \(-0.8 \) to \( 2 \). (The corresponding initial virial parameter \( \nu \equiv 2K/W = 10^{-200} \) is \( \nu = 3.4 \pm 0.046 \), where \( K \) and \( W \) are initial kinetic and potential energy.) Models with \( \beta = -0.8 \) are completely unbound throughout the simulation, whereas models with \( \beta = -0.7 \geq -0.6 \geq -0.5 \) form a bound cluster, containing less than \( 100\% \) of the total mass, after the initial expansion phase. Models with \( \beta \geq 0 \) initially collapse to a smaller half-mass radius \( r_{min} \) (see Table 1) and then bounce. All models with \( \beta \geq -0.7 \) experience phase mixing and/or violent relaxation and within 100–1000 crossing times reach an equilibrium state with a flat core and sharply declining density at large radii. The projected surface density profiles of equilibrium clusters closely resemble the profiles of Galactic GCs. The match is the best for the GCs that have experienced little dynamical evolution (see Fig. 1).

Our models fit the observed surface density profiles of GCs very well, despite the lack of a tidal radius as present in King models. We do not include an external tidal field in our simulations, so this lack is expected. The tidal cutoff in surface
density is observed in very few GCs (Trager et al. 1995) because of the contamination with background stars at the outskirts of the clusters.

As we show in § 3, all real GCs should have experienced an adiabatic collapse (when the orbital angular momentum is conserved for individual stars). To make sure that our models are in the same collapse regime as the real clusters, all our models should satisfy the following adiabaticity criterion (Aarseth, Lin, & Papaloizou 1988):

$$\sigma_{\text{min}} \geq N^{-1/6} \left( M/R_{\text{min}} \right)^{1/2}. \quad (1)$$

In our case, this condition can be expressed as $\beta \leq (1/2) \log N - (3/2) \log 5$. The adiabatic collapse criterion is then $\beta \leq 1.5$ for $N = 10^3$ and $\beta \leq 2.0$ for $N = 10^6$. According to this criterion, all our models undergo an adiabatic collapse (see Table 1). In our models, the collapse factor $C \equiv R_i/R_{\text{min}}$ correlates well with the virial parameter $\nu$: $C \propto \nu^\gamma$. The value of the exponent $\gamma = 0.95 \pm 0.02$ is very close to the adiabatic value of $\gamma = 1$ (Aarseth et al. 1988).

From our set of models, we obtained the following correlations between the projected half-mass radius $r_j$, projected central velocity dispersion $\sigma_j$, central surface density $\Sigma_j$, core radius $r_0$, binding energy $E_0$, and mass $M$ (the uncertainties in the exponents are $\pm 0.01$):

$$\begin{align*}
r_j &\sim \text{constant}, \\
\sigma_j &\propto M^{0.57}, \\
\Sigma_j &\propto M^{1.39}, \\
r_0 &\propto M^{-0.27}, \\
E_0 &\propto M^{1.95}, \\
E_0 &\propto \sigma_j^{0.55}. \quad (2)
\end{align*}$$

These correlations are valid for $\beta \geq 0$, with the last relation being valid for $\beta \geq -0.6$. Most of correlations become nonlinear in log-log space for $\beta < 0$ (corresponding to $\nu > 1$). Although for our coldest models, $r_j$ scales (as expected) as the initial radius (and hence as $M^{1/3}$), the total variation in $r_j$ over 2 orders of magnitude in system mass ($\beta = 0$--2) is only $\sim 0.12$ dex (see Table 1; please note that $R_{\text{min}} \propto M^{1/3}$).

Another very interesting correlation is $r_j \approx r_{\text{min}}$ within the measurement errors for $\beta \geq 0.2$ (Table 1). The above correlation can be understood if we rewrite the theoretical result $C \equiv R_i/R_{\text{min}} \propto \nu^{-1}$ (Aarseth et al. 1988) for the case of constant $\rho_{\text{ini}}$ and $\sigma_{\text{ini}}$: $r_{\text{min}} \propto M^{-1/3}$. The theoretical relation is very close to the model relation $r_j \propto M^{-0.27}$, resulting in $r_j \propto r_{\text{min}}$ as observed.

We tested the numerical convergence of our results by making two additional runs for $\beta = 1.6$. In the first one, the number of particles was reduced to $N = 10^5$, with the optimal value for the softening length of $\epsilon(R_{\text{ini}}) = -1.88$. Despite the fact that this run had the most poorly resolved "crunch" among all our models ($r_{\text{min}}/\epsilon \approx 3.6$), all the derived model parameters were identical to the original run parameters within measurement errors. In the second run, we again used a reduced number of particles $N = 10^5$, but the parameter $\epsilon$ was made much

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**TABLE 1**

| Input Parameters | Derived Parameters |
|------------------|-------------------|
| $\beta$ | $N$ | $\nu$ | $t_{\text{c},m}/t_{\text{c},m}$ | $\log (\epsilon/R_{\text{ini}})$ | $\log (r_{\text{ini}}/R_{\text{ini}})$ | $\log (r_i/R_{\text{ini}})$ | $\log (r_j/R_{\text{ini}})$ | $\log (\sigma_j/\sigma_{\text{ini}})$ | $\log (\rho_j/\rho_{\text{ini}})$ |
|------------------|-------------------|
| $-0.8$ | $10^3$ | $3.4$ | $5000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.7$ | $10^3$ | $2.9$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.6$ | $10^3$ | $2.5$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.5$ | $10^3$ | $2.2$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.4$ | $10^3$ | $1.8$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.3$ | $10^3$ | $1.6$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $-0.2$ | $10^3$ | $1.4$ | $1000$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $0.0$ | $10^3$ | $1.0$ | $200$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $0.2$ | $10^3$ | $0.74$ | $200$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $0.4$ | $10^3$ | $0.54$ | $200$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $0.6$ | $10^3$ | $0.40$ | $200$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $0.8$ | $10^3$ | $0.29$ | $50$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $1.0$ | $10^3$ | $0.22$ | $50$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $1.2$ | $10^3$ | $0.16$ | $50$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $1.4$ | $10^3$ | $0.12$ | $50$ | $-1.88$ | $...$ | $...$ | $...$ | $...$ |
| $1.6$ | $10^3$ | $0.086$ | $15$ | $-2.21$ | $...$ | $...$ | $...$ | $...$ |
| $2.0$ | $10^6$ | $0.046$ | $10$ | $-2.21$ | $...$ | $...$ | $...$ | $...$ |

**Note.**—Here $r_{\text{ini}}$, $\sigma_{\text{ini}}$, and $\rho_{\text{ini}}$ are the core parameters for relaxed models: King core radius, projected central velocity dispersion, and central density, respectively; $r_{\text{min}}$ is the minimum half-mass radius during the collapse, and $r_j$ is the projected half-mass radius for relaxed models.

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**Fig. 1.**—Surface density profile (solid line) for a model with $\beta = 0.4$. This model matches well the surface density profiles of five GCs (symbols; Trager, King, & Djorgovski 1995), which are among the least dynamically evolved ones. Surface density is normalized to 1 at the center of the cluster. The vertical short-dashed and long-dashed lines show the values of the softening length $\epsilon$ and the model core radius $r_0$, respectively.
smaller than the optimal value: \( \log (\epsilon R_{\text{in}}) = -2.67 \). In this run, the crunch is resolved very well, \( r_{\text{min}}/\epsilon = 23 \), but again we did not see significant deviations from the original run parameters. The results of the additional runs suggest that the number of particles \( N \) and the value of the softening length \( \epsilon \) that we use in our runs are adequate for resolving the violent relaxation process.

3. COMPARISON WITH OBSERVATIONS

We assume that the mass-to-light ratio \( M/L \) is a constant, which is well established for observed GCs \( \log (M/L) = 0.16 \pm 0.03 \) [McLaughlin 2000]. Then we derive the same correlations for observed clusters as in equation (2) using the online version of the Milky Way GC catalog of W. E. Harris (Harris 1996). We used data for 109 non-core-collapsed GCs. Forty-five of these clusters have a measured \( \sigma_0 \). Binding energies were calculated using equations (5a)–(5c) of McLaughlin (2000). The observed correlations are

\[
\begin{align*}
  & r_h \sim \text{constant}, \quad \sigma_0 \propto M^{0.43 \pm 0.05}, \quad \Sigma_0 \propto M^{1.31 \pm 0.10}, \\
  & \rho_0 \propto M^{1.53 \pm 0.17}, \quad r_0 \propto M^{-0.28 \pm 0.07}, \quad E_b \propto M^{2.05 \pm 0.08}, \quad (3)
\end{align*}
\]

The theoretical exponents differ from the observational values by 1 \( \sigma \) or less [with the exception of the \( \sigma_0 (M) \) correlation, where the difference is 3 \( \sigma \)]. The closeness of the model correlations (eq. [2]) to the observed correlations (eq. [3]) can be understood if we assume that in their initial nonequilibrium configuration, all Galactic GCs had the same values of stellar density \( \rho_{\text{in}} \) and velocity dispersion \( \sigma_{\text{in}} \).

To estimate the values of the universal parameters \( \rho_{\text{in}} \) and \( \sigma_{\text{in}} \), one can use in principle any two or more pairs of model/observational correlations from equations (2) and (3). However, one has to keep in mind that few GC parameters are well suited for comparing the model and observed correlations. Most Galactic GCs are in advanced stages of their dynamical evolution. As GCs evolve, some of their parameters deviate from initial equilibrium values. This process affects mainly the central core region of a cluster where encounters between stars are the most frequent. Analytical and numerical calculations show that at some point a GC should experience a runaway core collapse due to gravothermal instability (Spitzer 1987). Around 20% of Galactic GCs are believed to be in a post–core-collapse state (Harris 1996). The analytical theory of core collapse (Spitzer 1987) can be used to find GC parameters that are least affected by dynamical evolution. We obtained the following relations:

\[
\begin{align*}
  & \rho_0 \propto r_{\text{in}}^{-2.21}, \quad \Sigma_0 \propto r_{\text{in}}^{-1.21}, \quad \sigma_0 \propto r_{\text{in}}^{-0.10}. \quad (4)
\end{align*}
\]

To obtain an analogous relation for binding energy \( E_b \), we used equations (5a)–(5c) of McLaughlin (2000). Assuming that during core collapse the tidal radius \( r_h \) is also a very slowly evolving parameter (Spitzer 1987).

We used two following correlations to estimate the values of \( \rho_{\text{in}} \) and \( \sigma_{\text{in}} \): \( E_b (\sigma_0) \) and \( r_h = \text{constant} \). The \( \chi^2 \) fitting gave the following estimates: \( \log \rho_{\text{in}} = 1.14 \pm 0.26 \) and \( \log \sigma_{\text{in}} = 0.28 \pm 0.11 \). (The units for \( \rho_{\text{in}} \) and \( \sigma_{\text{in}} \) are \( M_{\odot} \) and \( \text{pc}^{-1} \) and \( \text{km} \text{s}^{-1} \)). The relation between the masses of real GCs and the model mass parameter \( \beta \) is then \( M = 3.5 \times 10^{\beta + 3/2} M_{\odot} \). The corresponding minimum initial cluster mass resulting in a bound cluster (the model with \( \beta = -0.7 \)) is \( M_{\text{in}} \approx 6900 M_{\odot} \), with a 1 \( \sigma \) interval of 3000–16,000 \( M_{\odot} \). The criterion of an adiabatic collapse of Aarseth et al. (1988; our eq. [1]) can be reexpressed as \( M \leq 3a_{\text{in}}/(4\pi\rho_{\text{in}} m_1) \), where \( m_1 \) is a typical stellar mass in the cluster. For our values of \( \rho_{\text{in}} \) and \( \sigma_{\text{in}} \), a collapse is adiabatic if \( M \leq 1.6 \times 10^7 M_{\odot} \) (we assumed \( m_1 = 0.6 M_{\odot} \)). As one can see, our adiabatic collapse models are adequate for the whole range of GC masses.

The simplest physical interpretation of the universal initial density value is to assume that the major burst of star formation in a contracting proto-GC molecular cloud occurs when the gas density reaches a certain critical value \( \rho_{\text{c}} \) (the non-equality sign is to account for less than a 100% efficient star formation and mass loss due to stellar evolution). The initial velocity dispersion \( \sigma_{\text{in}} \) can be assumed to be commensurable with the sound speed in the turbulent proto-GC cloud, and hence with its temperature \( T \), at the moment when the star burst occurs. For a purely molecular gas of primordial composition \( (Y_e = 0.291) \), we obtained the following estimates of the universal gas density \( \rho_{\text{c}} \) and temperature \( T \) for star-forming gas in proto-GCs: \( \rho_{\text{c}} > 230 \text{ cm}^{-3} \) and \( T \sim 1000 \text{ K} \). The corresponding gas pressure is \( P > 2.3 \times 10^5 \text{ K cm}^{-3} \).

In Figures 2–4 we compare three different observed correlations for Galactic GCs with the model predictions rescaled to \( \log \rho_{\text{in}} = 1.14 \) and \( \log \sigma_{\text{in}} = 0.28 \): \( E_b (\sigma_0) \), \( E_b (M) \), and \( \Sigma_0 (\rho_0) \). In Figures 2 and 3 (showing the correlations between the least evolved cluster parameters) the agreement between the model and data is excellent, with a low statistical significance suggestion that the most evolved clusters, shown as open circles, tend to deviate from the model \( E_b (M) \) relation (Fig. 3). Fig-
Fig. 3.—Cluster binding energy as a function of total cluster mass for 109 non-core-collapsed GCs (circles) and our model (solid line). The open circles are those clusters with log \( t \) < 8.3 (as in Fig. 2).

Fig. 4.—Central surface density as a function of central density for 109 non-core-collapsed GCs (circles) and our model (solid line). The open circles are more dynamically evolved clusters with log \( t \) > 9.2.

ure 4 shows a correlation between the most evolved cluster parameters: central surface density \( \Sigma_c \) and central density \( \rho_c \) (see eq. [4]). The situation with Figure 4 is in accord with our model predictions: all clusters deviate from the model "zero-age" relation in a systematic fashion, with more evolved clusters located farther from the model line. (Please note that the correlations from Figs. 3 and 4 were not used to fit the model to the data.)

4. SUMMARY

We have shown that a simple model of violent relaxation of a constant density, constant velocity dispersion stellar sphere produces clusters that are remarkably similar to the observed Galactic GCs. They have the correct surface density profiles, and a large number of correlations between cluster parameters are accurately reproduced by our models as long as we assume a constant initial density and initial velocity dispersion. This result implies that all GCs were formed from gas with a universal value of density and temperature. In reality, the constant density, constant velocity dispersion initial setup of our models may correspond to isothermal turbulent cores of giant molecular clouds with a Gaussian density profile. Our simple picture of GC formation suggests that the conditions in the cluster-forming gas must be quite uniform across a large portion of the early universe.

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