The SQCD vacuum coupled to supergravity and string theory moduli

Thomas Dent*
Centre for Theoretical Physics, University of Sussex, Brighton BN1 9QH, U.K.

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Abstract
We calculate the scalar potential of supersymmetric QCD (in the regime \( N_f < N_c \)) coupled to \( \mathcal{N} = 1 \) supergravity with moduli-dependent gauge kinetic function and masses. The gauge dynamics are described by the Taylor-Veneziano-Yankielowicz superpotential for composite effective fields. The potential can be expanded about the “truncated” point in the gaugino and matter condensate directions in order to find corrections to the globally supersymmetric minimum. The results are relevant to the phenomenology of supersymmetry-breaking in string-inspired supergravity models, and also to recent work on domain walls in SQCD.

1 Introduction

Supersymmetric gauge theory with matter representations plays a central role in many models of supersymmetry-breaking relevant to string phenomenology. In \( \mathcal{N} = 1 \) supergravity models based on perturbative string constructions, the gauge dynamics in a “hidden sector” [2] can induce a hierarchically small vacuum expectation value (v.e.v.) for the gaugino bilinear operator \( \text{Tr} \lambda^a \lambda_a \) (the “gaugino condensate”). If there are hidden matter multiplets [3, 4], there may also be nonzero v.e.v.’s for the squark bilinears \( \tilde{q}_a \tilde{q}^a \) where \( a \) is the gauge group representation index and \( i, j \) are flavour indices. When coupled to supergravity the gaugino condensate is a source of supersymmetry-breaking [4] which can in principle explain the ratio of the electroweak scale, and the notional masses of supersymmetric partners, to the mass scale of string theory. The scale of SUSY-breaking can be parameterized by the gravitino mass, \( m_{3/2} \), which is proportional to \( \langle \text{Tr} \lambda^a \lambda_a \rangle / M_P^2 \) in the case of a single confining group. By using the different gauge groups and matter contents which appear in string constructions it is possible to reach phenomenologically reasonable values of \( m_{3/2} \), of order 1 TeV [2].

*E-mail address: t.e.dent@sussex.ac.uk
It has usually been assumed that the structure of the gauge theory vacuum is well described by globally supersymmetric QCD, since the strong-coupling scale is well below $M_P$. In the limit of global supersymmetry $M_P \to \infty$ the condensates do not break supersymmetry [6, 7, 8]. The condensate values may be found by instanton calculations [8] or by constructing a supersymmetric effective action [6, 7] in terms of gauge invariant composite superfields $U \propto \text{Tr} W^\alpha W_\alpha$ and $V_{ij} \propto \bar{Q}_a Q^a_j$, where $W^\alpha$ is the supersymmetric gauge field strength and $Q, \bar{Q}$ are the quark and antiquark chiral superfields respectively. The supersymmetric zero-energy vacuum satisfies the conditions $F_U = -\partial W_{np}^*/\partial U^* = 0$, $F_V = -\partial W_{np}^*/\partial V_{ij}^* = 0$, where $W_{np}$ is the nonperturbative superpotential generated by the gauge dynamics. The gaugino condensate value is then determined (up to a discrete symmetry) in terms of the renormalisation group invariant scale $\Lambda$ and the squark mass matrix $M_{ij}$. In string-inspired supergravity models the scale $\Lambda$ and the squark masses are functions of the dilaton and moduli scalars of the underlying theory. Hence the supersymmetry-breaking has a non-trivial dependence on the “flat directions” of string theory and it becomes possible to stabilize the moduli [10, 11] and (under certain assumptions!) the dilaton [12], while inducing supersymmetry-breaking of the right magnitude [5, 13].

Recently, SQCD has also been used in the construction of (supersymmetric) domain walls [14], motivated by cosmological issues or by the “brane world” scenario in which the observable matter fields are confined to a region localized in one or more directions within a higher-dimensional spacetime. The discrete set of degenerate vacua that is required is naturally realised in SU($N$) SQCD, since the vacuum with nonzero gluino and matter condensates transforms under a non-anomalous $Z_N$ symmetry. When supersymmetry is broken by an explicit gluino mass term [15], the degeneracy is also broken, which raises doubts as to the stability of domain walls when gravitational corrections to the scalar potential are included. A non-zero v.e.v. for $\text{Tr} \lambda^\alpha \lambda_\alpha$ will in general break (local) SUSY and give the gluinos, including hidden sector gauginos, a mass. We will find, however, that the vacuum degeneracy is unbroken when the source of SUSY-breaking is the gaugino condensate itself.

In this paper we attempt to find the corrections to the vacuum structure of SQCD due to its coupling to $N = 1$ supergravity, including the dilaton and an overall modulus denoted by chiral superfields $S$ and $T$ respectively. The starting point will be the non-perturbative superpotential of [7] and the resulting zero-energy vacuum of the globally supersymmetric theory. It has already been shown in the case of a SYM hidden sector without matter, that the resulting value of the gaugino condensate is identical to that at the minimum of the $N = 1$ supergravity scalar potential $\mathcal{V}(U, S, T)$, when terms in $U$ of mass dimension higher than 4 (i.e. suppressed by powers of $M_P$) are discarded [16, 4]. We will show that this also holds in the case of SQCD: on discarding the terms in the supergravity potential with mass dimension > 4 in the condensate fields (note that we treat $S$ and $T$ as being dimensionless), we recover the global SQCD vacuum. The size of deviations

\footnote{This is the subgroup of the anomalous U(1)$_R$ that survives breaking by instantons: in general there will be a $Z_{c(G)}$ degeneracy.}
from the “truncated approximation” for the condensates \[3\] can be found by minimising the scalar potential in supergravity as a function of \(U\) and \(V_{ij}\), including higher-order terms. A similar approach was taken in the case of pure SYM coupled to supergravity in \[1\] \[7\] \[18\].

2 Effective action for the gaugino and squark condensates

We follow the approach of Burgess et al. \[19\] in which the gaugino condensate is described by the classical field \(U \equiv \hat{U}/S_0^3\), where \(\hat{U} = \langle \text{Tr} W^\alpha W_\alpha \rangle\) and the chiral compensator superfield \(S_0\) is introduced to simplify the formulation of supergravity coupled to matter \[20\] using the superconformal tensor calculus \[2\]. The gaugino bilinear \(\langle \text{Tr} \lambda^\alpha \lambda^\alpha \rangle\) corresponds to the lowest component of \(\hat{U}(\theta = \bar{\theta} = 0)\). Similarly the squark condensate is the lowest component of a (mass dimension 2) composite superfield \(V_{ij} = \langle \bar{Q}_i^a Q_j^a \rangle\) where \(Q_i, \bar{Q}_j\) are \(N_f\) flavours of left chiral superfields in the representation \(R\) and its complex conjugate \(\bar{R}\) respectively. The v.e.v.’s of the gaugino and squark bilinears are then given by the scalar components of \(U\) and \(V_{ij}\) at the stationary point of the effective action \(\Gamma(U, V, S, T)\) \[19\].

The effective action results from the standard \(\mathcal{N} = 1\) supergravity formula with the Taylor-Veneziano-Yankielowicz (TVY) nonperturbative superpotential in terms of \(U\) and \(V\), and an appropriate Kähler potential \(K\). The TVY superpotential, suitably amended for the case of a non-minimal gauge kinetic function and a modulus-dependent mass matrix, is

\[
W(U, V_{ij}, S, T) = \frac{1}{4} f_G(S, T) U - \frac{1}{96 \pi^2} U \ln \left( k U^{b+2c} (\det V_{ij})^{-3c/N_f} \right) - M_{ij}(T) V_{ij}.
\]

Here \(f_G(S, T)\) is the gauge kinetic function, equal to \(S\) at tree level, which in general depends on the modulus \(T\) through string loop threshold corrections \[21\]. \(b\) is the one-loop beta function coefficient such that \(b = -3c(G) + 2N_f T(R)\), and \(c = 2N_f T(R)\); \(c(G)\) is the second-order Casimir invariant for the gauge group \(G\) and \(T(R)\) is the index, equal to 1/2 in the fundamental representation. \(k\) is a constant which will be discussed shortly. It is convenient to diagonalise \(V_{ij}\) by performing unitary flavour rotations of the matter fields \(Q_i, \bar{Q}_j\) so that \(V_{ij} = V_i \delta_{ij}\). The superpotential can then be re-expressed as

\[
W = -\frac{b}{96 \pi^2} U \ln \left( k U^{1+2c/b} \prod_i V_i^{-3c/N_f} \omega(S) h(T) \right) - M_i(T) V_i
\]

where \(\omega(S) \equiv e^{-24\pi^2 S/b}\) and \(h(T)\) is a function which transforms under the target-space duality group \(\text{SL}(2, \mathbb{Z})\) \[11\] \[22\] such that the argument of the logarithm in \(1\) is invariant.

This is all we need to find the truncated approximation for the condensate values: it simply follows from setting the derivatives of \(W\) with respect to \(U\)

\[
\text{The components of } S_0 \text{ are determined by gauge-fixing the superconformal symmetries so that the Einstein term in the Lagrangian is canonically normalised} \[19\].}
and make $V$ to zero, giving us

$$U^{(tr)} = e^{24\pi^2 f_G(S,T)/b_0 - (b+2c)/b_0 k\beta/b_0} \left(\frac{c}{32\pi^2 N_f}\right)^{3c/b_0} \prod_i M_i(T)^{-3c/N_f b_0} \quad (3)$$

and

$$V_i^{(tr)} = \frac{c}{32\pi^2 N_f} \frac{U^{(tr)}}{M_i(T)} \quad (4)$$

where $b_0 = -3c(G)$ is the beta-function coefficient for SYM without matter. We see that the constant $k$ simply adjusts the scale of the condensates; it has the same effect as a constant threshold correction to $f_G$.

When the condensate values (3) and (4) are substituted back into the superpotential, the “truncated superpotential” $W_{np}^{(tr)}(S,T) = b_0/(96\pi^2)U^{(tr)}$ emerges. This is the usual starting point for studying supersymmetry-breaking in string effective field theories (see for example [23]).

We take the Kähler potential for the dilaton and moduli to be

$$\tilde{K} = P(y) - 3 \ln(T + T^\dagger) \quad (5)$$

where $y = S + S^\dagger - 3/(8\pi^2)\delta_{GS}\ln(T + T^\dagger)$. The string tree-level Kähler potential for the dilaton $-\ln(S + S^\dagger)$ has been replaced by a real function $P(y)$ which parameterizes stringy nonperturbative dilaton dynamics [24, 12]. We take the Green-Schwarz coefficients $\delta_{GS}$ to be zero to simplify the calculations. The correct form of $P(y)$ is not known, however it is possible to constrain it by looking for a stable minimum in the potential for the dilaton and requiring $P''(y) > 0$ to obtain the right sign kinetic term.

The $\mathcal{N} = 1$ supergravity effective action is invariant under target-space SL(2, Z) transformations [27, 28] if the superpotential transforms as a modular form of weight $-3$. The modulus $T$ transforms as

$$T \to \frac{\alpha T - i\beta}{i\gamma T + \delta} \quad (6)$$

where $\alpha, \beta, \gamma, \delta$ are integers satisfying $\alpha\delta - \beta\gamma = 1$. Then $U$ must transform as

$$U \to \zeta(i\gamma T + \delta)^{-3} U, \quad (7)$$

where $\zeta$ is a unimodular phase which depends on $\alpha, \beta, \gamma$ and $\delta$. Since the gauge fields are inert under target-space modular transformations this is achieved by $S_0$ having a non-trivial transformation property. The transformation of the squark condensate $V$ is determined by that of the $Q, \bar{Q}$ fields, which we take to have (flavour-independent) modular weights $n_Q, n_{\bar{Q}}$: then $V_i \to \zeta V(i\gamma T + \delta)^{-n_Q + n_{\bar{Q}}} V_i$. The function $h(T)$ is determined by string threshold corrections (up to multiplication by a modular invariant function of $T$) as $h(T) = \eta(T)^{6b'/b}$, where $\eta(T)$ is the Dedekind eta-function and the coefficient $b'$ is $b' = 3(-c(G) + \sum_Q T(R_Q)(1 + 2n_Q))$ for a gauge group with twisted matter representations $R_Q$ of modular weight $n_Q$.

\footnote{For a discussion of modular forms see [27] or the appendix of [23].}
The complete Kähler potential is then taken to be
\[
K(U, V_i, S, T) = \tilde{K} - 3 \ln \left( 1 - A e^{\tilde{K}/3} (UU^\dagger)^{1/3} \right) + B (T + T^\dagger)^{\tilde{n}} (V_i V_i^\dagger)^{1/2}.
\]
(8)
where \(A\) and \(B\) are constants and \(\tilde{n} \equiv (n_Q + n_{\bar{Q}})/2\). It was shown in [19] that this expression for the Kähler potential of \(U\) has the correct dependence on \(S\) and \(T\); the \(V\)-dependent part is fixed by the requirements to respect \(U(N_f)\) flavour symmetry and modular invariance. The Kähler potential for the composite fields \(U\) and \(V\) is only determined up to constant factors, and may receive higher-order corrections (which, however, will be negligible when the field values are small). The constants \(A, B\) cannot at present be computed, due to our incomplete knowledge of supersymmetric gauge dynamics, but they are expected to be of order unity. Since the scalar potential in supergravity is a function of \(K\) as well as the superpotential \(W\) these constants will also appear in our results. In an earlier paper [18] the constant \(A\) was absorbed by rescaling the gaugino condensate, however since physical SUSY-breaking quantities are expressed in terms of the \textit{un-rescaled} gaugino condensate (and, in general, the normalisation of fields is fixed by the coefficients with which they enter into \(W\)) this will not be done here.

The part of \(\Gamma(U, V_i, S, T)\) relevant to finding the value of the condensate is the scalar potential, which is given as usual by
\[
\mathcal{V} = e^K \left( (W^*_I + K_I W^*_I)(K^{-1})^I_J (W^J + K^J W) - 3|W|^2 \right)
\]
where \(I\) and \(J\) range over the scalar components of \(U, V_i, S, T\), \(X^I \equiv \partial X/\partial \phi_I\) and \(X_J \equiv \partial X/\partial \phi_J^*\) for \(X\) any function of the scalars and their complex conjugates, and \(K^{-1}\) is defined by \((K^{-1})^I_J K^J_K = \delta^I_K\). The details of the inverse Kähler metric components are relegated to the Appendix.

As emphasized in [20], \(\Gamma(U, V_i, S, T)\) is not an effective Lagrangian in the sense of describing the light degrees of freedom only, (it is a functional of the heavy fields \(U\) and \(V\), rather it is the generating functional of “two-particle irreducible” correlation functions for the composite fields \(\text{Tr} W^2\) and \(\text{Tr} \bar{Q}_i Q_j\) (see also [19]). The kinetic terms in \(\Gamma\) must be understood as the first terms in a derivative expansion; there may be large higher-derivative corrections, so the effective action cannot be reliably used to determine particle interaction vertices. However the scalar potential, \(\mathcal{V}\) for constant field configurations, does not get these corrections.

After some calculation, we find the scalar potential in closed form, which for convenience is expressed in terms of the rescaled quantities \(z = e^{K/6} U^{1/3}\), \(\Pi_i = (T + T^*^\dagger)^{\tilde{n}} V_i\), \(\|\Pi\| = (\Pi_i \Pi_i^*)^{1/2}\):
\[
\mathcal{V} = \frac{e^{B\|\Pi\|}}{(1 - A|z|^2)^3} \left\{ \mathcal{V}_0 + \mathcal{V}_1 \right\}
\]
(9)
with
\[
\mathcal{V}_0 = \frac{b}{96 \pi^2} 3|z|^4 \frac{3|z|^4}{A} (1 - A|z|^2) \left| 1 + \frac{2c}{b} + \log \right|^2
\]
\[
\mathcal{V}_1 = \frac{b}{96 \pi^2} 3|z|^4 \frac{3|z|^4}{A} (1 - A|z|^2) \left| 1 + \frac{2c}{b} + \log \right|^2
\]
\[
4\text{We work in reduced Planck units with } \kappa^{-1} = 1/\sqrt{8\pi G} \text{ set to 1.}
\]
\[
\frac{2}{B||\Pi||} (\delta_{ij}||\Pi||^2 + \Pi^*_{i+} \Pi_j) \left( \frac{c}{32\pi^2 N_f} z^3 \Pi^*_{i+} - e^{\hat{K}/2}(T + T^*)^{-\hat{n}} M_i(T^*) \right) \\
\times \left( \frac{c}{32\pi^2 N_f} z^3 \Pi_{j+} - e^{\hat{K}/2}(T + T^*)^{-\hat{n}} M_j(T) \right),
\]
(10)

\[\mathcal{V}_1 = \frac{P'(y)^2}{P''(y)} (1 - A|z|^2) \mathcal{V}_S + \frac{1 - A|z|^2}{3(1 - \frac{A}{3}(1 - A)|z|^2)B||\Pi||} \mathcal{V}_T \]
\[+ \left( \frac{b}{96\pi^2} \right)^2 |z|^6 \left( -3 \left( 1 + \frac{2c}{b} \right)^2 - \frac{6c}{b}(\mathcal{LOG} + c.c.) + B||\Pi|| \cdot |\mathcal{LOG}|^2 \right) \]
\[+ \frac{b}{96\pi^2} (2 + B||\Pi||) \left( e^{\hat{K}/2}(T + T^*)^{-\hat{n}} z^3 \mathcal{LOG} \cdot M_i(T^*) \Pi_i + c.c. \right) \]
\[+ \frac{3b}{96\pi^2} \left( 1 - A|z|^2 \right) \left( 1 + \frac{2c}{b} \right) \left( e^{\hat{K}/2}(T + T^*)^{-\hat{n}} z^3 M_i(T^*) \Pi_i + c.c. \right) \]
\[+ e^{\hat{K}}(T + T^*)^{-2\hat{n}} (1 + 3A|z|^2 + B||\Pi||) |M_i(T)\Pi_i|^2 \]
(11)

where

\[\mathcal{LOG} = \ln(kU^{1+2c/b} \prod_i V_i^{-3c/N_f} \omega(S) h(T)),\]
(12)

\[\mathcal{V}_S = \left| \frac{b}{96\pi^2} z^3 \left( \frac{1}{P'(y)} \omega'(S) - \left( 1 + \frac{2c}{b} \right) \right) + e^{\hat{K}/2}(T + T^*)^{-\hat{n}} M_i(T) \Pi_i \right|^2,\]

\[\mathcal{V}_T = \left| \frac{b}{96\pi^2} z^3 \left( (T + T^*) \frac{h'(T)}{h(T)} + 3 \left( 1 + \frac{2c}{b} (1 + \hat{n}) \right) \right) + \right. \]
\[\left. + e^{\hat{K}/2}(T + T^*)^{-\hat{n}} \left( (T + T^*) \frac{M_i(T)}{M_i(T)} - (3 + 2\hat{n}) \right) M_i(T) \Pi_i \right|^2. \]
(13)

In this somewhat unwieldy expression, \( \mathcal{V}_0 \) includes the terms of mass dimension 4 [glossing over factors of \((1 - A|z|^2)\)], which survive in the global supersymmetry limit \( M_P \to \infty, z \to 0, \Pi \to 0 \), while \( \mathcal{V}_1 \) includes all terms of higher order in \( z \) and \( \Pi \). Note also the “modular covariant” expressions

\[\hat{G}^T \equiv (T + T^*) \frac{h'(T)}{h(T)} + 3 \left( 1 + \frac{2c}{b} (1 + \hat{n}) \right) , \quad \hat{G}^M \equiv (T + T^*) \frac{M_i(T)}{M_i(T)} - (3 + 2\hat{n})\]

appearing in \( \mathcal{V}_T \), which transform into themselves up to a phase under \( \text{SL}(2, \mathbb{Z}) \). For future convenience we also define

\[\hat{G}^S \equiv \frac{1}{P'(y)} \omega'(S) - \left( 1 + \frac{2c}{b} \right).\]

As expected, \( \mathcal{V}_0 \) has a zero-value minimum in the condensate directions\(^5\) at \( z = z^{(tr)} \equiv e^{\hat{K}/6} U^{(tr)} 1/3 \) and \( \Pi_i = \Pi_i^{(tr)} \equiv (T + T^*)^{-\hat{n}} V_i^{(tr)} \). The “truncated superpotential” result for the scalar potential is then equivalent to substituting these condensate values back into the full potential, which is then

\(^5\)Note that the principal value of the logarithm in (13) must be taken. Since the argument is real in vacuo this does not cause an ambiguity.
proportional to $V_1$, and discarding terms of dimension greater than 6 in powers of $z$ and $\Pi_i$. It is a non-trivial check on the result \([11]\) to carry out this substitution and verify that the scalar potential then coincides with previous results \([22, 4]\). In fact we obtain

$$V^{(tr)} \equiv V(S, T)\big|_{z = z^{(tr)}, \Pi = \Pi^{(tr)}} = \left(\frac{b_0}{96\pi^2}\right)^2 \left|z^{(tr)}\right|^6 \left\{ \frac{1}{P'(y)} \left|\frac{\omega'(S)}{\omega_0(S)} - P'(y)\right|^2 + \right.$$  

$$+ \frac{1}{3}\left|\frac{(T + T^*)h'_0(T)}{h_0(T)} + 3\right|^2 - 3\} + O(z^{(tr)^8})$$

where $\omega_0(S) = e^{-24\pi^2 S/b_0}$, $h_0(T) = \eta(T)^{\delta b/b_0} \prod_i M_i(T)^{3e/N_f b_0}$. Note that the unknown constants $A$ and $B$ drop out of this expression. This occurs because the truncated approximation for the condensates is implemented via the superpotential $W_{np}$ alone, and does not (at the given order of approximation in $z/M_P$) affect the Kähler potential for $S$ and $T$.

### 3 Beyond the truncated approximation

The truncated approximation relies on the assumption that the condensate values at the minimum of $V_0$ is not much changed when the gravitationally-suppressed terms in $V_1$ are added. This is likely to be true when the condensate values $z$, $\Pi$ are small, which is also the phenomenologically interesting regime for most applications. We are able to test this assumption explicitly by expanding about the truncated values to find the shift in the minimum. As a simplifying assumption the masses $M_i(T)$ are set equal to $M(T)$ and the squark condensates $\Pi_i$ equal to $\Pi$. We define the fractional changes in the condensate values

$$\Delta z = \frac{z}{z^{(tr)}} - 1, \quad \Delta \Pi = \frac{\Pi}{\Pi^{(tr)}} - 1$$

and take the real and imaginary parts Re $\Delta z$, Im $\Delta z$, Re $\Delta \Pi$, Im $\Delta \Pi$ as independent variables. We then expand the scalar potential \([11]\) up to terms of mass dimension 6 in the fields (in fact, we would expect higher-order corrections to the Kähler potential for $U$ and $V_i$ to become significant beyond this order) and up to quadratic order in $\Delta z$ and $\Delta \Pi$. The result takes the form

$$V = V^{(tr)} + \vec{\nabla}_L \cdot \vec{\Delta} + \vec{\Delta}^T \nabla_{\vec{Q}} \vec{\Delta}$$

(14)

where $\vec{\Delta}$ is the column-vector with entries Re $\Delta z$, Im $\Delta z$, Re $\Delta \Pi$, Im $\Delta \Pi$, and $\vec{\nabla}_L$ and $\nabla_{\vec{Q}}$ are a vector and symmetric matrix of coefficients respectively. The displacements $\vec{\Delta}_{\text{min}}$ at the minimum of $V$ are given by

$$\vec{\Delta}_{\text{min}} = -\frac{1}{2} \nabla_{\vec{Q}}^{-1} \cdot \vec{\nabla}_L$$

(15)

and the change in the potential from the truncated value is

$$V_{\text{min}} - V^{(tr)} = -\frac{1}{4} \vec{\nabla}_L \nabla_{\vec{Q}}^{-1} \vec{\nabla}_L.$$

(16)
The coefficients in $\mathcal{V}_Q$ are of dimension four and six in the fields (and in the modulus-dependent mass $M(T)$) while those in $\hat{V}_L$ are dimension six only (since they originate from $V_i$). Since we are evaluating $\Delta_{\text{min}}$ and $\mathcal{V}_{\text{min}}$ to leading order in $z^{(tr)}$ we can discard dimension six terms in $\mathcal{V}_Q$, provided that the matrix does not become singular. Then we have

$$\mathcal{V}_{\text{L}}^T \cdot \Delta = \frac{|z^{(tr)}|^6}{(32\pi^2)^2} \left[ \text{Re} \Delta z \left( \frac{P'(y)^2}{P''(y)} \left( \frac{2b^2}{3}(\hat{G}^S)^2 + 2bc\hat{G}^S \right) + \frac{2b^2}{9} |\hat{G}^T|^2 \right) + \frac{2bc}{3} \text{Re} (\hat{G}^T \hat{G}^{M*}) - 2b_0b \right] + \text{Im} \Delta z \cdot \frac{2bc}{3} \text{Im} (\hat{G}^T \hat{G}^{M*}) + \text{Re} \Delta \Pi \left( \frac{P'(y)^2}{P''(y)} \left( \frac{2b^2}{3}\hat{G}^S + 2c^2 \right) + \frac{2c^2}{3} |\hat{G}^M|^2 \right) + \frac{2bc}{9} \text{Re} (\hat{G}^T \hat{G}^{M*}) + \frac{2b_0c}{3} \right] + \text{Im} \Delta \Pi \cdot \frac{2bc}{9} \text{Im} (\hat{G}^T \hat{G}^{M*}) \right] (17)$$

and

$$\Delta^T \mathcal{V}_Q \Delta = \frac{|z^{(tr)}|^4}{(32\pi^2)^2} \left[ \frac{3}{A} ((b + 2c)^2((\text{Re} \Delta z)^2 + (\text{Im} \Delta z)^2) - 2c(b + 2c)(\text{Re} \Delta z \text{Re} \Delta \Pi + \text{Im} \Delta z \text{Im} \Delta \Pi) + c^2((\text{Re} \Delta \Pi)^2 + (\text{Im} \Delta \Pi)^2)) + \frac{4(32\pi^2)cN_f^{1/2} e^{K/2} |M(T)|}{B |z^{(tr)}|(T + T^*)^n} \left( 9((\text{Re} \Delta z)^2 + (\text{Im} \Delta z)^2) + (\text{Re} \Delta \Pi)^2 + (\text{Im} \Delta \Pi)^2 \right) + \mathcal{O}(|z^{(tr)}|^6) \right] (18)$$

Note that the quadratic terms originating from $V_0$ fall into two parts, reflecting their origins in $|F_V|^2$ and $|F_V|^2$, and there is an explicit dependence on the mass $M(T)$. Without the terms proportional to $M$, the matrix $\mathcal{V}_Q$ would be singular, reflecting the runaway instability in massless globally supersymmetric QCD. With the mass terms $\mathcal{V}_Q$ can easily be inverted and it is a matter of algebra to find $\Delta_{\text{min}}$ and $\mathcal{V}_{\text{min}}$.

If the functions $G^T$ and $G^M$ have the same phase then the deviations $\Delta z$ and $\Delta \Pi$ are real. In this case we can get an intuitive idea by plotting the dependence of the full potential $\mathcal{V}$ on the absolute values of the condensates $|z|, |\Pi|$ (given that their phases are fixed to the truncated values). Figure 1 shows the scalar potential $\mathcal{V}(|z|, |\Pi|)$ for $b_0 = -12, c = 2, |z^{(tr)}| = 0.15$ and $M = 0.07$, with $\hat{G}^S, \hat{G}^T, \hat{G}^M$ set to the (somewhat arbitrary) values of 0.5, 3.0, 0.0 respectively. Figure 2 shows the gravitationally-suppressed corrections $\mathcal{V}_1(|z|, |\Pi|)$ for the same values of parameters. The structure of these terms is much richer than in the case without matter multiplets: however along the curve specified by $\partial W/\partial \Pi = 0$, the form of the scalar potential and supergravity corrections reduces to the case of pure SYM (fig. 3) (see e.g. [22]).

There remains the question of the fate of the $N$ degenerate vacua in SU($N$) SQCD when supersymmetry is broken. Under the $\mathbb{Z}_N$ symmetry the
Figure 1: Full scalar potential $V(|z|,|\Pi|)$ ($|z^{(tr)}| = 0.15, |M| = 0.07$, other parameters as specified in the text)

Figure 2: Supergravity corrections $V_1(|z|,|\Pi|)$
condensates transform as $U \rightarrow e^{2i\pi/N}U$, $V_i \rightarrow e^{2i\pi/N}V_i$. It is known \cite{30,29} that the logarithmic term in the TVY effective action does not respect the symmetry, since the branch cut is crossed. There are in practice various ways of restoring the symmetry for the $\mathbb{Z}_N$ transformed vacua, including giving the squark condensates extra $2\pi$ phases (see \cite{14,first reference}) such that the argument of the logarithm is invariant, adding extra fields \cite{29}, or shifting the theta-angle by $2\pi$ (equivalent to taking $S \rightarrow S + i/8\pi^2$) which will exactly cancel the effect of an axial $\mathbb{Z}_N$ transformation. Taking such “fixes” into account, the scalar potential (1) is invariant under the $\mathbb{Z}_N$ symmetry by inspection, since $z^3$ and $\Pi_i$ get the same phase. While this is a reassuring result for the construction of domain walls, we must ask why it appears to contradict the reasoning that any gaugino mass must break the vacuum degeneracy. We certainly expect that the (hidden sector) gauginos will get a small (in general complex) SUSY-breaking mass in the case we have studied. However, since the source of the mass is the gaugino condensate itself, the gaugino mass is not inert under the $\mathbb{Z}_N$ symmetry. In fact the mass term will be proportional to $z^3\text{Tr}\lambda\lambda$ (see for example \cite{13,first reference}), which is invariant\cite{11}.

4 Conclusions

Given an explicit form for the Kähler potential of the composite superfields representing the condensates, the scalar potential in a string-inspired $\mathcal{N} = 1$ supergravity model can be calculated; this allows us to find the minimum in the condensate directions without any assumptions about the presence

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\textsuperscript{6}To see this, it is necessary to use the formulas for the $\mathcal{N} = 1$ supergravity action in terms of $W$ and $K$ separately, rather than the Kähler function $\mathcal{G} = K + \log|W|^2$, since in going from one formulation to the other the gaugino fields are redefined by a complex phase and the phase of $W$ is eliminated \cite{11}.\footnote{To see this, it is necessary to use the formulas for the $\mathcal{N} = 1$ supergravity action in terms of $W$ and $K$ separately, rather than the Kähler function $\mathcal{G} = K + \log|W|^2$, since in going from one formulation to the other the gaugino fields are redefined by a complex phase and the phase of $W$ is eliminated \cite{11}.}
or absence of supersymmetry-breaking. The deviation from the “truncated” globally supersymmetric vacuum of SQCD have been found in terms of the dilaton and string moduli. The phenomenological implications of deviations from the truncated approximation were discussed in an earlier paper [18]. In the case with hidden matter, there is another possible cause for $\Delta z$, $\Delta \Pi$ to be large, namely if $\hat{G}^M$ becomes large due to singular behaviour of the mass $M(T)$ at some points in moduli space. Since $\det V_Q$ is proportional to $M$, the deviations may also be large if $M$ becomes small. However, in the limit $M \ll z$ the SQCD vacuum undergoes a phase transition and the charged matter fields themselves (rather than the composite operator $\text{Tr} \bar{Q}Q$) can get v.e.v’s, so our effective action will not be valid.

While the particular model from which this potential is derived, inspired by orbifolds of the heterotic string, is quite restrictive in its dependence on the dilaton and moduli, it would be relatively simple to extend the result to the case when the gauge kinetic function and mass matrix depend on several moduli, or the Kähler potential for the moduli is modified from its tree-level form (e.g. by introducing functions $P_i(T_i + T_i^\dagger)$, where $T_i$ is the $i$’th modulus). So our approach should be applicable to supergravity effective theories based on heterotic M-theory [32] or Type IIB string models [33, and references therein]. In particular we expect that the supergravity corrections will preserve the discrete symmetry of degenerate vacua in SYM and SQCD.

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Appendix: Inverse Kähler metric components

The inverse Kähler metric components are as follows:

$$
(K^{-1})^U_U = \frac{|z|^4(1 - A|z|^2)}{Ae^K} \left( 3 + \frac{\bar{n}AB|z|^2(1 - A|z|^2)\|\Pi\|}{1 - \frac{2}{3}B(1 - A|z|^2)\|\Pi\|} + \frac{A|z|^2P'(y)^2}{P''(y)} \right),
$$

$$(K^{-1})^U_i = \frac{-2\bar{n}z^3(1 - A|z|^2)\Pi_i}{e^{K/2}(T + T^*)^\bar{n}(1 - \frac{2}{3}B(1 - A|z|^2)\|\Pi\|)},
$$

$$(K^{-1})^U_S = \frac{-z^3(1 - A|z|^2)P'(y)}{e^{K/2}P''(y)},
$$

$$(K^{-1})^T_T = \frac{(T + T^*)z^3(1 - A|z|^2)}{e^{K/2}(1 - \frac{2}{3}B(1 - A|z|^2)\|\Pi\|)},
$$

$$(K^{-1})^V_i = \frac{2}{B(T + T^*)^2\|\Pi\|} \left( \delta_{ij}\|\Pi\|^2 + \Pi_i\Pi_j \frac{1 - \frac{2}{3}(1 - 2\bar{n})(1 - A|z|^2)B\|\Pi\|}{1 - \frac{2}{3}(1 - A|z|^2)B\|\Pi\|} \right),
$$

$$(K^{-1})^V_S = 0 = (K^{-1})^S_T,
$$

$$(K^{-1})^V_i = -\frac{2\bar{n}(T + T^*)(1 - A|z|^2)\Pi_i}{3(T + T^*)^\bar{n}(1 - \frac{2}{3}B(1 - A|z|^2)\|\Pi\|)}.
$$
\[ (K^{-1})_S^S = \frac{1 - A|z|^2}{P^n(y)}, \quad (K^{-1})_T^T = \frac{(T + T^*)^2(1 - A|z|^2)}{3(1 - \frac{A}{3} B(1 - A|z|^2)||\Pi||)} \]

where \( z = e^{\bar{K}/6}U^{1/3}, \) \( \Pi_i = (T + T^*)^{\bar{a}}V_i, ||\Pi|| = (\Pi_i\Pi_i^{*})^{1/2}. \)

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