Transient Performance of Power Systems with Distributed Power-Imbalance Allocation Control

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Abstract—We investigate the sensitivity of the transient performance of power systems controlled by Distributed Power-Imbalance Allocation Control (DPIAC) on the parameters of the control law. We model the disturbances at power loads as Gaussian white noises and measure the transient performance of the frequency deviation and control cost by the $H_2$ norm. For a power system with a communication network of the same topology as the power network, analysis shows that the transient performance of the frequency can be greatly improved by accelerating the convergence of the frequencies to the nominal value through a single control gain coefficient. However, the control cost increases linearly as this control gain coefficient increases. Hence, in the feedback control law, DPIAC, there is a trade-off between the frequency deviation and control cost which is determined by this control gain coefficient. By increasing another control gain coefficient, the control cost can be decreased with an accelerated consensus of the marginal costs during the transient phase. Furthermore, the behavior of the state approaches that of a centralized control law when the consensus of the marginal costs is accelerated.

Index Terms—Secondary frequency control, transfer matrix, $H_2$ norm, control gain coefficient.

I. INTRODUCTION

In power systems, the frequency deviation is caused by the imbalance between the power demand and supply. The power system is expected to keep the frequency within a small range around the nominal value to avoid damages to electrical devices. This is accomplished by regulating the active power injection of sources.

Three forms of frequency control can be distinguished from fast to slow timescales, i.e., primary control, secondary control, and tertiary control [4], [13]. Primary frequency control has a control objective to keep the frequency of the power systems at the nominal frequency, based on local feedback at each power generator. However, the frequency of the entire power system with primary controllers may still deviate from its nominal value. These deviations are caused primarily because of the interaction of the local power systems via the power network. Secondary frequency control restores the system frequency to its nominal value and is operated on a slower time scale than primary control. The primary and secondary frequency control are operated on-line. Based on the predicted power demand, tertiary control determines the set points for both primary and secondary control over a longer period than used in secondary control. The operating point is usually the solution of an optimal power flow problem. However, as more and more renewable power sources, such as wind turbines and solar panels, are integrated into the power system, the power supply cannot be precisely predicted in advance because of the uncertainties of the weather.

Considering the on-line economic power dispatch between all the controllers in the power system [5], various control methods are proposed for the secondary frequency control. These includes passivity based centralized control methods such as Automatic Generation Control, Gather-Broadcast Control [4], and distributed control methods such as the Distributed Average Integral (DAI) [22], [3], primal-dual algorithm based distributed method such as the Economic Automatic Generation Control (EAGC) [11], Unified Control [23] et al. However, the primary design objectives of these methods focus on the steady state only. As investigated in our previous study in [20], the closed-loop system for any of the above control laws has a poor transient performance even though the control objective of reaching the steady state is achieved. For example, from the global perspective of the entire power system, the passivity based methods, e.g., the AGC, GB and DAI methods, are actually a form of integral control. A drawback of integral control is that large integral gain coefficients may result in extra oscillations of the system because of the overshoot of the control input while small gain coefficients result in a slow convergence speed towards a steady state. Slow convergence speed usually results in a large frequency deviation which is the main concern of the system integrated with a large amount of renewable energy. To address the large frequency deviation problem, a fast convergence to a steady state is critical for the power system. It has been analyzed that the transient performance of the primal-dual method is also poor [15].

The way to improve the transient performance of the traditional methods is to tune the control gain coefficients either by obtaining satisfactory eigenvalues of the linearized closed-loop system or by using a control law based on $H_2$ or $H_{\infty}$ control synthesis [2], [13]. However, besides the complicated computations, these methods focus on the linearized system only and the improvement of the transient performance is still limited because it also depends on the structure of the control laws. Concerning the transient performance of the system and the balance of the advantages of the centralized and distributed control structure, the authors have proposed a multilevel control method, Multi-Level Power-Imbalance Allocation Control (MLPIAC) [21] for the secondary frequency control. There are two special case of MLPIAC, a centralized control called Gather-Broadcast Power-Imbalance

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Allocation Control (GBPIAC), and a distributed control called Distributed Power-Imbalance Allocation Control (DPIAC). It has been analyzed that in MLPIAC the convergence of the frequency and the consensus of the marginal costs can be both accelerated by increasing the corresponding control parameters respectively, thus improving the transient performance of the frequency. However, the analysis is not sufficient without quantifying the impact of these control parameters on the transient performance.

In this paper, we focus on the distributed control law, DPIAC, and analyze the impact of the control parameters on the transient performance of the frequency deviation and the control cost after a disturbance. In order to compare the performance of DPIAC, we also investigate the transient performance of the centralized control method GBPIAC. Without communications between the controllers, DPIAC reduces to a decentralized control method, called Decentralized Power-Imbalance Allocation Control (DecPIAC), in which the transient performance of the system will also be studied for comparison with that of DPIAC. For a linearization of the nonlinear power system, we model the disturbances at power loads (or generation) as Gaussian white noises, and measure the transient performance by the $H_2$ norm for a linear input/output system with Gaussian white noise. We show that there is a trade-off between the transient performance of frequency deviation and control cost, which is determined by the control parameters.

The main contributions of this paper are,

1) analytic formulas for how the transient performance of the frequency deviation and of the control cost depend on the parameters of the control law, where the transient performance are quantified by $H_2$ norm;
2) advice on how to select proper parameters for the control law with a trade-off between transient performance and control cost;
3) a comparison of the performance of the distributed control method DPIAC and the centralized control method GBPIAC.

This paper is organized as follows. We first introduce the model of the power system, the GBPIAC and DPIAC methods in section III, then introduce the $H_2$ norm to measure the transient performance of the system and formulate the problem of this paper in section III. We calculate the corresponding $H_2$ norms and analyze the impact of the control parameters on the transient performance of the frequency deviation, control cost and the coherence of the marginal costs in section IV, and finally conclude with remarks in section V.

II. THE SECONDARY FREQUENCY CONTROL LAWS

We use a graph $G = (\mathcal{V}, \mathcal{E})$ with nodes $\mathcal{V}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ to describe a power system, where a node represents a bus and edge $(i, j)$ represents the direct transmission line connection between node $i$ and node $j$. The buses can connect to synchronous machines, frequency dependent power sources (or loads), or passive loads. We neglect the resistance of the transmission lines and denote the susceptance by $B_{ij} = \frac{1}{s}$. The set of the buses of the synchronous machines, frequency dependent power sources, passive loads are denoted by $\mathcal{V}_M$, $\mathcal{V}_F$, $\mathcal{V}_P$ respectively, thus $\mathcal{V} = \mathcal{V}_M \cup \mathcal{V}_F \cup \mathcal{V}_P$. The dynamic of the phase angle $\theta_i$ and the frequency deviation $\omega_i$ from the nominal frequency (e.g., 50 or 60 Hz) at node $i$ of the power system is described by the following Differential Algebraic Equations (DAEs), e.g., [4],[5],

$$\dot{\theta}_i = \omega_i, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F,$$

$$M_i \omega_i = P_i - D_i \omega_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j) + u_i, \quad i \in \mathcal{V}_M, \quad (1a)$$

$$0 = P_i - D_i \omega_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j) + u_i, \quad i \in \mathcal{V}_F, \quad (1b)$$

$$0 = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j), \quad i \in \mathcal{V}_P, \quad (1c)$$

where $\{M_i > 0, \quad i \in \mathcal{V}_M\}$ are the moments of inertia of the synchronous machines, $\{D_i > 0, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ are the droop control coefficients, $\{P_i, \quad i \in \mathcal{V}\}$ are the power supply (or demands), $\{B_{ij} = B_jiV_jV_i, \quad (i, j) \in \mathcal{E}\}$ is the effective susceptance of transmission lines, $\{V_i, \quad i \in \mathcal{V}\}$ are the voltages, $\{u_i, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ are the inputs for the secondary control. The nodes in $\mathcal{V}_M$ and $\mathcal{V}_F$ are assumed to be equipped with secondary frequency controllers, denoted by $\mathcal{V}_K = \mathcal{V}_M \cup \mathcal{V}_F$. Since the control of the voltage and the frequency can be decoupled, we do not model the dynamics of the voltages and assume the voltage of each bus is a constant that can be directly derived from a power flow calculation [6].

The synchronized frequency deviation can be expressed as

$$\omega_{syn} = \frac{\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i}{\sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i}, \quad (2)$$

which states the necessity for the secondary frequency control. In fact, if the secondary controllers are absent, i.e., $u_i = 0$ for all $i \in \mathcal{V}$, it follows from (2) that $\omega_{syn} \neq 0$ if the power-imbalance $P_s = \sum_{i \in \mathcal{V}} P_i \neq 0$. The necessary condition for $\omega_{syn} = 0$ at a steady state can be satisfied by solving the following optimization problem

$$\min_{u_i \in \mathcal{R}} \sum_{i \in \mathcal{V}_K} J_i(u_i) \quad s.t. \quad P_s + \sum_{i \in \mathcal{V}_K} u_i = 0, \quad (3)$$

where $J_i(u_i) = \frac{1}{2}u^2$ represents the control cost at node $i$.

A necessary condition for the solution of the optimization problem (3) is that the marginal costs $dJ_i(u_i)/du_i$ of the nodes are all identical, i.e.,

$$\alpha_i u_i = \alpha_j u_j, \quad \forall \ i, j \in \mathcal{V}_K.$$

For a control law with the objectives of Problem (3), it is required that the total control input $u(t) = \sum_{i \in \mathcal{V}_K} u_i$ converges to $-P_s$ and the marginal costs achieve a consensus at the steady state. Various control methods have been proposed based on either passivity method, e.g., [4], [13], or primal-dual method, e.g., [11], [23], [12]. However, Problem (3) focuses on the steady state only and the transient performance is often poor, e.g., the controllers either introduce extra oscillations to the frequency or lead to a slow convergence. In order to
obtain a good transient performance where the convergence is accelerated with no extra oscillations of the frequency, the MLPIAC method has been proposed in [21] with two special cases, GBPIAC and DPIAC. In this paper, we focus on the transient performance of the DPIAC method. In order to fully understand the characteristics of the DPIAC method, we also study the transient performance of GBPIAC. In the following three subsections, we introduce the GBPIAC and DPIAC method with the following assumption of the connectivity of the communication network.

**Assumption 2.1:** For the power system $\mathbb{H}$, there exists a undirected communication network such that all the nodes in $\mathcal{V}_K$ are connected.

### A. The centralized control approach: GBPIAC

GBPIAC is defined as a centralized control law as follows.

**Definition 2.2 (GBPIAC):** Consider the power system $\mathbb{H}$, the Gather-Broadcast Power-Imbalance Allocation Control (GBPIAC) law is defined as [21]

$$
\dot{\eta}_s = \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i, \quad (4a)
$$

$$
\dot{\xi}_s = -k_1(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta_s) - k_2 \xi_s, \quad (4b)
$$

$$
u_i = \frac{\alpha_s \alpha_i^{-1}}{\xi_i}, \quad i \in \mathcal{V}_K, \quad (4c)
$$

$$
\frac{1}{\alpha_s} = \sum_{i \in \mathcal{V}_K} \frac{1}{\alpha_i}, \quad (4d)
$$

where $\eta_s \in \mathbb{R}$, $\xi_s \in \mathbb{R}$ are state variables of the central controller, $k_1, k_2$ are positive control gain coefficients, $\alpha_s$ is the price at node $i$ defined as in the optimization problem $\mathbb{H}$. The control architecture of GBPIAC is that of a centralized controller which interacts with all the nodes. The central controller receives from the nodes the observations of the local frequencies $\{\omega_i, i \in \mathcal{V}_K\}$, computes the states $\xi_s$ and $\eta_s$ of the controller, computes the new inputs $\{u_i, i \in \mathcal{V}_K\}$ for all nodes, and communicates to each node $i \in \mathcal{V}_K$ the new input value $u_i$.

In order to compensate the power-imbalance quickly, the control gain coefficient $k_2$ should yield $k_2 \geq 4k_1$. For details of the configuration of $k_1$ and $k_2$, we refer to [21]. In this paper, we set $k_2 = 4k_1$ for convenience in the following transient analysis. In GBPIAC, $k_2 \xi_i$ represents the sum of all new inputs at any time $t \geq 0$. 

### B. The distributed control approach: DPIAC

DPIAC is defined as a distributed control law as follows.

**Definition 2.3 (DPIAC):** Consider the power system $\mathbb{H}$, define the Distributed Power-Imbalance Allocation Control (DPIAC) law as a dynamic controller where for node $i \in \mathcal{V}_K$ [21],

$$
\dot{\eta}_i = D_i \omega_i + k_3 \sum_{j \in \mathcal{V}_K} l_{ij} (k_2 \alpha_i \xi_i - k_2 \alpha_j \xi_j), \quad (5a)
$$

$$
\dot{\xi}_i = -k_1(M_i \omega_i + \eta_i) - k_2 \xi_i, \quad (5b)
$$

$$
u_i = k_2 \xi_i, \quad (5c)
$$

$\eta_i \in \mathbb{R}$ and $\xi_i \in \mathbb{R}$ are state variables of the local controller at node $i$, $k_1, k_2$ and $k_3$ are positive gain coefficients, $\{l_{ij}\}$ defines a weighted undirected communication network with Laplacian matrix $(L_{ij})$

$$
L_{ij} = \begin{cases} 
-l_{ij}, & i \neq j, \\
\sum_{k \neq i} l_{ik}, & i = j, 
\end{cases}
$$

and $l_{ij} \in [0, \infty)$ is the weight of the communication line connecting node $i$ and node $j$. The marginal cost at node $i$ is $\alpha_i u_i = k_2 \alpha_i \xi_i$.

The control architecture of DPIAC is that of local controllers which exchange information. The local controller of node $i \in \mathcal{V}_K$ obtains the local measurement $\omega_i$ and receives from its neighbors in the communication network the marginal cost $\{\xi_j, j \in N(i)\}$. $N(i)$ denotes the sets of neighbors of node $i$. The local controller computes the controller state $\eta_i$ and $\xi_i$, and the local input $u_i$ which is then sent to the local power source. In addition, it communicates to its neighbors of the communication network its value $\xi_i$.

As in the case for the GBPIAC method, we also set $k_2 = 4k_1$ for DPIAC in this paper. In DPIAC, the marginal costs of nodes achieve a consensus at the steady state. However, different from in GBPIAC, they are not identical during the transient phase. Hence the control cost of DPIAC will be larger than that of GBPIAC. In fact, with a constant value $k_1$, the control cost of DPIAC reduces to that of GBPIAC as $k_3$ goes to infinity because the consensus speed is accelerated which reduces the control cost. This will be analytically verified in section IV.

Without the coordination on the marginal costs of the nodes, DPIAC reduces to a decentralized control method as follows.

**Definition 2.4 (DecPIAC):** Consider the power system $\mathbb{H}$, the Decentralized Power-Imbalance Allocation Control (DecPIAC) is defined as a dynamic controller where for node $i \in \mathcal{V}_K$,

$$
\dot{\eta}_i = D_i \omega_i, \quad (6a)
$$

$$
\dot{\xi}_i = -k_1(M_i \omega_i + \eta_i) - k_2 \xi_i, \quad (6b)
$$

$$
u_i = k_2 \xi_i, \quad (6c)
$$

$k_1$ and $k_2$ are positive control gain coefficients.

The control architecture of DecPIAC consists of a set of local controllers. The controller of node $i \in \mathcal{V}_L$ obtains its local frequency measurement $\omega_i$, computes the state variables of the controller $\eta_i$ and $\xi_i$, and computes the local input $u_i$. No information is received or sent to other nodes.

For the control law of MLPIAC, we refer to [21].

### III. Problem Formulation

In this paper, we investigate the transient performance of the power system with a DPIAC control law and compare it to the GBPIAC method. For the asymptotic stability of GBPIAC and DPIAC control laws, we refer to [21]. We introduce the $\mathcal{H}_2$ norm to measure the transient performance of the system $\mathbb{H}$ and formulate the problem to be studied.

The $\mathcal{H}_2$ norm of the transfer matrix of a linear system is defined as follows.
Consider a linear time-invariant system,

\[ \dot{x} = Ax + Bw, \quad (7a) \]
\[ y = Cx, \quad (7b) \]

where \( x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n} \) is Hurwitz, \( B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{r \times n} \), the input is denoted by \( w \in \mathbb{R}^m \) and the output of the system is denoted by \( y \in \mathbb{R}^r \). The squared \( \mathcal{H}_2 \) norm of the transfer matrix \( H \) of the map \((A, B, C)\) from the input \( w \) to the output \( y \) is defined as

\[ ||H||_2^2 = \text{tr}(B^T Q_o B) = \text{tr}(Q_o C C^T), \quad (8a) \]
\[ Q_o A + A^T Q_o + C^T C = 0, \quad (8b) \]
\[ A Q_o + Q_o A^T + B B^T = 0, \quad (8c) \]

where \( \text{tr}(\cdot) \) denotes the trace of a matrix, \( Q_o, Q_e \in \mathbb{R}^{n \times n} \) are the observability Grammian of \((C, A)\) and controllability Grammian of \((A, B)\) respectively [6, 17 chapter 2].

There are several interpretations of the \( \mathcal{H}_2 \) norm. When the input \( w \) is modeled as the Gaussian white noise such that \( w_i \sim N(0,1) \) for all \( i = 1, \cdots, m \), the matrix \( Q_o = C Q_o C^T \) is the variance matrix of the output at the steady state [8 Theorem 1.53, 10 Theorem 5.6.7], i.e.,

\[ Q_o = \lim_{t \to \infty} E[y(t)^2](t) \]

where \( E[\cdot] \) denotes the expectation. Thus

\[ ||H||_2^2 = \text{tr}(Q_o) = \lim_{t \to \infty} E[y(t)^2 y(t)]. \quad (9) \]

When the input \( w \) is modeled as Dirac impulse with \( w(t) = e_i \delta(t) \) where \( e_i \in \mathbb{R}^m \) is a vector with the \( i \)th component being one and others zero, and \( \delta(t) \) is a Dirac impulse. The corresponding output to \( e_i \delta(t) \) is denoted by \( y_i \in \mathbb{R}^r \) for \( i = 1, \cdots, m \). Then the squared \( \mathcal{H}_2 \) norm satisfies [16]

\[ ||H||_2^2 = \sum_{i=1}^m \int_0^\infty y_i(t)^2 y_i(t) dt. \quad (10) \]

which measures the sum of the \( L_2 \) norm of the outputs.

The \( \mathcal{H}_2 \) norm can also be interpreted as the expected \( L_2 \) norm of the output \( y \) with \( w = 0 \) and a random initial condition \( x_0 \) such that \( E[x_0 x_0^T] = B B^T \), or equivalently,

\[ ||H||_2^2 = \int_0^\infty E[y(t)^2 y(t)] dt. \quad (11) \]

The \( \mathcal{H}_2 \) norm has often been used to investigate the performance of secondary frequency control methods, e.g., [11, 15, 19], to study the optimal virtual inertia placement in Micro-Grids [12], and the price of the synchrony of power systems [16]. We focus on the following problem in this paper.

**Problem 3.2:** In the system (11), the disturbances of the system are modeled as Gaussian white noises. How do the coefficients \( k_1 \) and \( k_3 \) influence the frequency deviation and the control cost in the DPIAC method?

We address Problem (3.2) with the concept of \( \mathcal{H}_2 \) norm of linear time-invariant system. To simplify the analysis, we make the following assumption.

**Assumption 3.3:** For GBPIAC and DPIAC, assume that \( \mathcal{V}_F = \emptyset, \mathcal{V}_P = \emptyset \) and for all \( i \in \mathcal{V}_M, M_i = m > 0, D_i = d > 0, \alpha_i = 1 \). For DPIAC, assume that the topology of the communication network is the same as the one of the power system such that \( L = B \), or equivalently,

\[ L_{ij} = \begin{cases} -B_{ij}, & i \neq j, \\ \sum_{k \neq i} B_{ik}, & i = j. \end{cases} \]

where \( B = (B_{ij}) \in \mathbb{R}^{n \times n} \) is a Laplacian matrix.

From the practical point of view, the analysis with Assumption 3.3 is valuable because it provides us the insight on how to improve the transient behaviour by tuning the parameters. Since the phase angle differences \( \theta_{ij} = \theta_i - \theta_j \) are usually small in practice, sin \( \theta_{ij} \) can be approximated by \( \theta_{ij} \), e.g., [22]. [11]. With Assumption 3.3 rewriting (11) into a vector form by replacing sin \( \theta_{ij} \) by \( \theta_{ij} \), we obtain

\[ \dot{\theta} = \omega, \quad (12a) \]
\[ M \dot{\omega} = -B \theta - D \omega + P + u. \quad (12b) \]

where \( \theta = \mathrm{col}(\theta_i) \in \mathbb{R}^n \), \( n \) denotes the number of nodes in the network, \( \omega = \mathrm{col}(\omega_i) \in \mathbb{R}^n \), \( M = \mathrm{diag}(M_i) \in \mathbb{R}^{n \times n}, D = \mathrm{diag}(D_i) \in \mathbb{R}^{n \times n}, P = \mathrm{col}(P_i) \in \mathbb{R}^n, u = \mathrm{col}(u_i) \in \mathbb{R}^{n \times n} \). Here, \( \mathrm{col}(\cdot) \) denotes the column vector of the indicated elements and \( \mathrm{diag}(\beta) \) denotes a diagonal matrix \( \beta = \mathrm{diag}((\beta_1, \cdots, \beta_n)) \in \mathbb{R}^{n \times n} \) with \( \beta_i \in \mathbb{R} \).

When \( P_i \) is modeled as a constant with a disturbance, we can obtain the same linear system as (12) by linearizing the system (11) at an equilibrium point. So we model the power vector \( P \) as having independent components with \( P_i N(0,1) \) for all \( i = 1, \cdots, n \), in this paper.

The transient performance of the frequency deviation and control cost can be measured by the \( \mathcal{H}_2 \) norm of the corresponding transfer functions with output \( y = \omega \) and \( y = u \) respectively.

For the system (12), denote the squared \( \mathcal{H}_2 \) norm of the transfer matrix of (12) with output \( y = \omega \) by \( ||H(\omega)||_2^2 \), thus following from (2), the expected value of \( \omega^T(\omega) \omega(t) \) at the steady state is

\[ ||H(\omega)||_2^2 = \lim_{t \to \infty} E[\omega(t)^T \omega(t)], \]

and denote the squared \( \mathcal{H}_2 \) norm of the transfer matrix of (12) with output \( y = u \) by \( ||H(u)||_2^2 \), thus the expected value of \( u(t)^T u(t) \) at the steady state is

\[ ||H(u)||_2^2 = \lim_{t \to \infty} E[u(t)^T u(t)]. \]

These two norms are used to measure the transient performance of the frequency deviation \( \omega(t) \) and of the control cost \( u(t) \) respectively.

**IV. THE TRANSIENT PERFORMANCE ANALYSIS**

In this section, we calculate the \( \mathcal{H}_2 \) norms of the frequency deviation and of the control cost for GBPIAC and DPIAC.

Denote the identity matrix by \( I_n \in \mathbb{R}^{n \times n} \) and the \( n \) dimensional vector with all elements equal to one by \( 1_n \). Following the stability analysis of [21], the closed-loop systems of the GBPIAC, DPIAC control laws are both asymptotically stable. For the symmetric matrix \( L \), we have the following lemma.
Lemma 4.1: For a symmetric Laplacian matrix $L \in \mathbb{R}^{n \times n}$, there exist an invertible matrix $Q \in \mathbb{R}^{n \times n}$ such that

$$Q^{-1} = Q^T,$$  
$$Q^{-1}LQ = \Lambda,$$  
$$Q_1 = \frac{1}{\sqrt{n}}1_n,$$  

where $Q = [Q_1, \ldots, Q_n]$, $\Lambda = \text{diag}(\lambda_i) \in \mathbb{R}^{n \times n}$, $Q_i \in \mathbb{R}^n$ is the unit eigenvector of $L$ corresponding to eigenvalue $\lambda_i$, thus $Q_i^TQ_j = 0$ for $i \neq j$. Because $L1_n = 0$, $\lambda_1 = 0$ is one of the eigenvalues with unit eigenvector $Q_1$.

In the following part of this section, we study the transient performance of the frequency deviations and of the control cost of GBPIAC and DPIAC in subsection IV-A and IV-B respectively by calculating the corresponding $H_2$ norm. In addition for DPIAC, we calculate a $H_2$ norm which measures the coherence of the marginal costs. The performance of GBPIAC and DPIAC will be compared in subsection IV-C.

A. Transient performance analysis for GBPIAC

By Assumption 3.3 we derive the control input $u_i = \frac{1}{n}k_1\xi_s$ at node $i$ as in (4). With the notations in section III and Assumption 3.3 and setting $k_2 = 4k_1$, we obtain from (12) and (4) the closed-loop system of GBPIAC written in a vector form as follows.

$$\dot{\theta} = \omega,$$  
$$m\dot{\omega} = -L\theta - d\omega + \frac{4k_1}{n}\xi_s n + P,$$  
$$\dot{\eta}_s = d_1^T \theta, $$  
$$\dot{\xi}_s = -k_1m\xi_s n - k_1\eta_s - 4k_1\xi_s.$$  

where $\eta_s \in \mathbb{R}$ and $\xi_s \in \mathbb{R}$.

For the transient performance of the frequency deviation $\omega(t)$ and of the control cost $u(t)$ in GBPIAC, the following theorem can be proved.

Theorem 4.2: Consider the closed-loop system (14) of GBPIAC, where power generation (or loads) $P = \text{col}(P_i)$ are modeled as Gaussian white noise such that $P_i \sim N(0, 1)$. The squared $H_2$ norm of the frequency deviation $\omega$ and of the control inputs $u$ are,

$$||H_G(\omega)||^2_2 = \frac{n - 1}{2md} + \frac{d + 5mk_1}{2m(2k_1m + d)^2},$$  
$$||H_G(u)||^2_2 = \frac{k_1^2}{2}.$$  

Proof: With the linear transform $x_1 = Q^{-1}\theta$, $x_2 = Q^{-1}\omega$ where $Q$ is defined in Lemma 4.1, we derive from (14) that

$$\dot{x}_1 = x_2,$$  
$$m\dot{x}_2 = -\Lambda x_1 - d\dot{x}_2 + \frac{4k_1}{m}\xi_s n + \frac{1}{m}Q_1^TP,$$  
$$\dot{\eta}_s = d_1^T \theta, $$  
$$\dot{\xi}_s = -k_1m\xi_s n - k_1\eta_s - 4k_1\xi_s.$$  

where $\Lambda$ is the diagonal matrix defined in Lemma 4.1. Since $1_n$ is an eigenvector of $L$ corresponding to $\lambda_1 = 0$, we obtain $Q^{-1}1_n = [\sqrt{n}, 0, \ldots, 0]^T$. Thus the components of $x_1$ and $x_2$ can be decoupled as

$$x_{11} = \hat{x}_1,$$  
$$x_{21} = -\frac{m}{d}x_{21} + \frac{4k_1}{m}\sqrt{n}\xi_s - \frac{1}{m}Q_1^TP,$$  
$$\dot{\eta}_s = d\sqrt{n}\xi_s,$$  
$$\dot{\xi}_s = -k_1m\sqrt{n}\xi_s - k_1\eta_s - 4k_1\xi_s$$  

and for $i = 2, \ldots, n$.

$$\dot{x}_{1i} = \hat{x}_i,$$  
$$\dot{x}_{2i} = -\frac{m}{d}x_{2i} - \frac{d}{m}x_{2i} + \frac{1}{m}Q_1^TP,$$  
$$\dot{\eta}_i = d\sqrt{n},$$  
$$\dot{\xi}_i = -k_1m\sqrt{n}\xi_i - k_1\eta_i - 4k_1\xi_i$$

We rewrite the decoupled systems of $17$ and $18$ in the general form as in (7) with

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \eta_s \\ \xi_s \end{bmatrix},$$  
$$A = \begin{bmatrix} 0 & -dL_m & 0 & 0 \\ 0 & d\sqrt{n} & 0 & 0 \\ 0 & 0 & -k_1m\sqrt{n} & -k_1 - 4k_1 \end{bmatrix},$$  
$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{Q_2}{m} \\ 0 \end{bmatrix},$$

where $v_T = [1, 0, \ldots, 0] \in \mathbb{R}^n$. The $H_2$ norm of a state variable e.g., the frequency deviation and the control cost, can be determined by setting the output $y$ equal to that state variable. Because the closed-loop system (14) is asymptotically stable, $A$ is Hurwitz regardless the rotations of the phase angle $\theta$.

For the transient performance of $\omega(t)$, setting $y = \omega = Qx_2$ and $C = [0, Q, 0, 0]$, we obtain the observability Gramian $Q_o$ of $(C, A)$ in (35) in the form,

$$Q_o = \begin{bmatrix} Q_{o11} & Q_{o12} & Q_{o13} & Q_{o14} \\ Q_{o21} & Q_{o22} & Q_{o23} & Q_{o24} \\ Q_{o31} & Q_{o32} & Q_{o33} & Q_{o34} \\ Q_{o41} & Q_{o42} & Q_{o43} & Q_{o44} \end{bmatrix}.$$  

Thus,

$$||H_G(\omega)||^2_2 = \text{tr}(QO_o),$$

$$B = \frac{\text{tr}(QQ_{o22}v_T)}{m^2} = \frac{\text{tr}(QQ_{o22})}{m^2}.$$  

Because

$$C^T C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

the diagonal elements $Q_{o22}(i, i)$ of $Q_{o22}$ can be calculated by solving the observability Gramian $\tilde{Q}_i$ of $(C_i, A_i)$ which satisfies

$$\tilde{Q}_iA_i + A_i^T \tilde{Q}_i + C_i^T C_i = 0,$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{d}{m} & 0 & \frac{4k_1}{m\sqrt{n}} \\ 0 & d\sqrt{n} & 0 & 0 \\ 0 & -k_1m\sqrt{n} & -k_1 & -4k_1 \end{bmatrix},$$  
$$C_i^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

and

$$\tilde{Q}_iA_i + A_i^T \tilde{Q}_i + C_i^T C_i = 0, \quad i = 2, \ldots, n,$$
where
\[ A_i = \begin{bmatrix} 0 & 1 \\ \frac{-d}{m} & \frac{-a}{m} \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \]

In this case, the diagonal elements of \( Q_{o22} \) satisfy \( Q_{o22}(i,i) = \tilde{Q}_i(2,2) \) for \( i = 1, \ldots, n \). We thus derive from the observability Gramian \( \tilde{Q}_i \) that
\[
\text{tr}(B^TQ_oB) = \frac{n-1}{2m} + \frac{d + 5mk_1}{2m(2k_1m + d^2)}, \tag{20}
\]
which yields \( \text{15a} \) directly. Similarly by setting \( y = u_k(t) = 4k_1\xi_k(t) \) and \( C = [0, 0, 0, 4k_1] \), we derive the norm of \( u_k(t) \) as
\[
||H_G(u_k)||^2 = \frac{k_1m}{2}, \tag{21}
\]
With \( u_i = \frac{\omega}{n} \) for \( i = 1, \ldots, n \) and \( ||u(t)||^2 = u(t)^Tu(t) = \sum_i u_i^2(t) \), we further derive the norm \( \text{15b} \) of the control cost for \( u(t) \).

The conclusions of Theorem 4.2 for control design of power systems follow. If the control parameter \( k_1 \) increases then the frequency deviations while the control cost increases. Thus, a trade-off between the frequency deviation and control cost can be determined by \( k_1 \) in GBPIAC. The norm of the frequency deviation \( \omega(t) \) in \( \text{15a} \) includes two terms, the first term on the right-hand side describes the relative oscillations which depend on the primary control, and the second one describes the overall frequency deviation which depends on the secondary control. Since the relative oscillations described by the first term on the right-hand side of \( \text{15a} \) cannot be influenced by the secondary controllers, the frequency deviation cannot be controlled with an arbitrarily small range.

B. Transient performance analysis for DPIAC

With Assumption 3.3 and setting \( k_3 = 4k_1 \), we derive the closed-loop system of DPIAC from \( \text{12} \) and \( \text{13} \) as
\[
\dot{\theta} = \omega, \tag{22a}
\]
\[
mI_\omega \dot{\omega} = -L\theta - dI_\omega \omega + 4k_1\xi + P \tag{22b}
\]
\[
\dot{\eta} = D\omega + 4k_3L\xi, \tag{22c}
\]
\[
\dot{\xi} = -k_1I_\omega - k_1\eta - 4k_1\xi. \tag{22d}
\]
where \( \eta = \text{col}(\eta_i) \in \mathbb{R}^n \) and \( \xi = \text{col}(\xi_i) \in \mathbb{R}^n \). Note that \( L = (L_{ij}) \in \mathbb{R}^{nxn} \) is the Laplacian matrix of the power network and also of the communication network. Because the differences of the marginal costs can be fully represented by \( 4k_1L\xi(t) \), we use the squared norm of \( (4k_1L\xi(t)) \) to measure the coherence of the marginal costs in DPIAC. Denote the squared \( H_2 \) norm of the transfer matrix of \( \text{22} \) with output \( y = 4k_1L\xi \) by \( ||H_D(4k_1L\xi)||^2 \), thus the expected value of \( (4k_1L\xi)^T(4k_1L\xi) \) at the steady state is
\[
||H_D(4k_1L\xi)||^2 = \lim_{t \to \infty} E[ (4k_1L\xi)^T(4k_1L\xi) ]. \tag{23}
\]

In this subsection, we also calculate the squared \( H_2 \) norm of \( 4k_1L\xi \) as an additional metric of the influence of \( k_3 \) on the control cost.

The following theorem states the properties of the \( H_2 \) norm of the frequency deviation, control cost and the coherence of the marginal costs in DPIAC.

**Theorem 4.2:** Consider the closed-loop system \( \text{22} \) where power generation (or loads) \( P = \text{col}(P_i) \) are modeled as the Gaussian white noise with \( P_i = N(0,1) \) for \( i = 1, \ldots, n \), the squared \( H_2 \) norm of the frequency deviation \( \omega(t) \), control inputs \( u(t) \) and \( 4k_1L\xi \) are
\[
||H_D(\omega)||^2 = \frac{d + 5mk_1}{2m(2k_1m + d^2)} + \frac{1}{2m} \sum_{i=2}^{n} b_{ii}, \tag{24a}
\]
\[
||H_D(u)||^2 = \frac{k_1}{2} + \sum_{i=2}^{n} \frac{b_{ii}}{e_i}, \tag{24b}
\]
\[
||H_D(4k_1L\xi)||^2 = \sum_{i=2}^{n} \frac{\lambda_i^2b_{ii}}{m^2e_i}. \tag{24c}
\]

where
\[
b_{ii} = \lambda_i^2(4k_1^2k_3m - 1)^2 + 4dk_1^3
\]
\[+ k_1(d + 4k_1m)(4d^2k_1^4k_3 + 5\lambda_i + 4dk_1),
\]
\[
b_{ii} = 2dk_1^2(4d + 2k_1m)^2 + 2k_1^4m^2(4k_1k_3 + 4).
\]
\[
e_i = dk_1^2(4k_1^2k_3m - 1)^2 + 16d^2k_1^4k_3m^2 + d^2\lambda_i k_1
\]
\[+ 4k_1(d + 2k_1m)^2(dk_1 + \lambda_i) + d\lambda_i k_1 k_3).
\]

**Proof:** Let \( Q \in \mathbb{R}^{nxn} \) be defined as in Lemma 4.1 and let \( x_1 = Q^{-1}\theta, x_2 = Q^{-1}\omega, x_3 = Q^{-1}\eta, x_4 = Q^{-1}\xi \), we obtain the closed-loop system in the general form as \( \text{17} \) with
\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{L}{m} \end{bmatrix} \begin{bmatrix} I \\ -\frac{d}{m} \end{bmatrix} \begin{bmatrix} I_n \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 4k_1I_n \end{bmatrix} \begin{bmatrix} 0 \\ -k_1mI_n \\ -k_1I_n \\ -4k_1I_n \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
\[
B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]
where \( A \) is the diagonal matrix defined in Lemma 4.1 Each of the block matrices in the matrix \( A \) are either the zero matrix or a diagonal matrix, so the components of the vector \( x_1 \in \mathbb{R}^n, x_2 \in \mathbb{R}^n, x_3 \in \mathbb{R}^n, x_4 \in \mathbb{R}^n \) can be decoupled.

With the same method for obtaining \( \text{20} \) in the proof of Theorem 4.2 setting \( \gamma = \omega = Qx_2, \quad C = [0, Q, 0, 0] \), we derive \( \text{24a} \) for the norm of the frequency deviation \( \omega(t) \). Then for the norm of the control cost \( u(t) \), setting \( \gamma = u(t) = 4k_1\xi(t) \) and \( C = [0, 0, 0, 4k_1Q] \), we derive \( \text{24b} \). Finally for the coherence measurement of the marginal cost, \( ||4k_1L\xi(t)||^2 \), setting \( y = 4k_1L\xi \) and \( C = [0, 0, 0, 4k_1LQ] \), we derive \( \text{24c} \).

Based on Theorem 4.3 we analyze the impact of \( k_1 \) and \( k_3 \) on the norms in DPIAC by focusing on 1) the frequency deviation, 2) control cost and 3) coherence of the marginal cost.

1) The frequency deviation: We first pay attention to the influence of \( k_1 \) when \( k_3 \) is fixed. From \( \text{24a} \), we derive
\[
\lim_{k_1 \to \infty} ||H_D(\omega)||^2 = \frac{1}{2m} \sum_{i=2}^{n} \frac{\lambda_i^2k_3^2}{dL^2k_3^2 + d(1 + 2k_1k_3)},
\]
which indicates that even with a large gain coefficient \( k_1 \), the frequency deviations cannot be decreased anymore when \( k_3 \) is a nonzero constant. This is because a trade-off between the minimal control cost and the frequency deviation has to be resolved. So similar to the GBPIAC control law, if the economic power dispatch is considered, the frequency
deviation cannot be controlled to an arbitrarily small range in DPIAC. However, when \( k_3 = 0 \), DPIAC reduces to the DecPIAC method \((6)\) and we derive
\[
||H_D(\omega)||_2^2 \sim O(k_1^{-1}),
\]
which states that, with a sufficiently large \( k_1 \), the frequency deviation can be controlled within an arbitrarily small range. Thus, the frequency deviations are locally balanced, which however results in a high control cost for the entire network.

**Remark 4.4:** By the decentralized control, it follows from \((27)\) that if all the nodes are equipped with the secondary frequency controllers, the frequency deviation can be controlled to any prespecified range. However, \( k_1 \) cannot be arbitrarily large in practice, which depends on the response time of the control devices.

**Remark 4.5:** This analysis is based on Assumption \(3.3\) which requires that each node in the network is equipped with a secondary frequency controller. However, for the power systems without all the nodes equipped with the controllers, the disturbance from the node without a controller must be compensated by some other nodes with controllers. In that case, the equilibrium of the system is changed and oscillation can never be avoided even when the controllers of the other nodes are sufficiently sensitive to the disturbances.

We next analyze the influence of \( k_3 \) on the frequency deviation when \( k_1 \) is fixed. It can be easily observed from \((24a)\) that the order of \( k_3 \) in the term \( b_{1i} \) is two which is the same as in the term \( e_i \), thus \( k_3 \) has little influence on the frequency deviation.

2) **The control cost:** We first analyze the influence of \( k_1 \) on the cost and then the influence of \( k_3 \). For any \( k_3 \geq 0 \), we derive from \((24b)\) that
\[
||H_D(u)||_2^2 \sim O(k_1),
\]
which indicates that the control cost increases as \( k_1 \) increases. From Remark 4.4, we conclude that minimizing the control cost always conflicts with minimizing the frequency deviation. Hence, a trade-off should be determined to obtain the desired frequency deviation with an acceptable control cost.

Next, we analyze how \( k_3 \) influences the control cost. From \((24b)\), we obtain that
\[
||H_D(u)||_2^2 \sim \frac{k_1}{2} + O(k_1k_3^{-1}),
\]
where the second term in the right hand side is positive. It shows that the control cost decreases as \( k_3 \) increases. This is because the consensus speed of the marginal cost is accelerated. This will be further discussed when studying the coherence of the marginal costs in the next subsection. Note that as analyzed in subsection \(IV-B1\) \( k_3 \) has little influence on the frequency deviation. Hence the control cost can be decreased by \( k_3 \) without increasing the frequency deviation much.

3) **The coherence of the marginal cost in DPIAC:** We study the coherence of the marginal cost which is measured by the norm of \( ||H_D(4k_1L\xi)||^2 \). From \((24c)\), we obtain
\[
||H_D(4k_1L\xi)||_2^2 = O(k_3^{-1})
\]
which indicates that the difference of the marginal cost decreases as \( k_3 \) increases. Hence, this analytically confirms that the consensus speed can be increased by increasing \( k_3 \).

**Remark 4.6:** In practice, similar to \( k_1 \), the coefficient \( k_3 \) depends on the communication devices and cannot be arbitrarily large. In addition, the communication delay also influences the transient performance of the marginal cost, which still needs a further investigation.

So we have confirmed, for the power system controlled by DPIAC with Assumption \(3.3\) that:

(i) as \( k_1 \) increases, the frequency deviation decreases while the control cost increases.

(ii) as \( k_3 \) increases, the consensus speed of the marginal cost is accelerated and the control cost is decreased, which however does not influence so much to the frequency deviation.

Hence, the trade-off between the frequency deviation and the control cost can be determined by \( k_1 \) and \( k_3 \) in DPIAC. We remark that the analysis is only for the power system satisfying Assumption \(3.3\) For a general power system, similar results were shown in simulations of \(21\).

**C. Comparison of the GBPIAC and DPIAC control laws**

In this subsection, we compare the performance of GBPIAC and DPIAC control laws.

With a positive \( k_1 \), we can easily obtain from \((15b)\) \((24b)\) that
\[
||H_G(u)|| < ||H_D(u)||, \quad (30)
\]
which is due to the differences of the marginal costs. The difference of the control cost between these two control laws can be decreased by accelerating the consensus speed of the marginal cost as explained in the previous subsection. From \((24a)\) and \((24b)\) we derive that
\[
\lim_{k_3 \to \infty} ||H_D(\omega)||_2^2 = \frac{n-1}{2md} + \frac{d + 5mk_1}{2m(2k_1m + d)} = ||H_G(\omega)||^2, \quad \lim_{k_3 \to \infty} ||H_D(\omega)||_2^2 = \frac{k_1}{2} = ||H_G(\omega)||^2.
\]
Hence, as \( k_3 \) goes to infinity, the transient performance of the frequency deviation in DPIAC converges to the one in GBPIAC.

**V. Conclusion**

For the power system controlled by DPIAC, we investigated the performances of the frequency deviation and control cost during the transient phase. The convergence of the total control input and the consensus of the marginal cost can be accelerated by increasing the corresponding gain coefficients respectively. The accelerated convergence of the total control input leads to smaller frequency deviations which however requires a higher control cost, while the accelerated consensus of the marginal costs decreases the control cost. Hence, a trade-off between the control cost and frequency deviations can be determined by the parameters. In addition, the performance of the distributed control converges to a centralized control law.
with a sufficiently large coefficient for the consensus of the marginal costs.

The analysis and conclusion are based on Assumption 3.3. Further investigation is needed for the general cases where the assumption is not satisfied. In addition, there usually are noises and delays in the measurement of the frequency and communications in practice, which are neglected in this paper. How these factors influence the transient behaviors of the frequency deviation and control cost will be investigated later.

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