Abstract

We present a new boosting algorithm, motivated by the large margins theory for boosting. We give experimental evidence that the new algorithm is significantly more robust against label noise than existing boosting algorithms.

1. Introduction

Since the invention of Adaboost by Freund and Schapire (1995) it has become very popular with both theoreticians and practitioners of machine learning. Many variants of the algorithm have been devised.

One of the most intriguing properties of Adaboost is the fact that it tends not to overfit. In many cases the test error of the generated classifier continues to decrease even after the training error has decreased to zero (1998; 1999; 2000). There are two main theories for explaining this behaviour. The first is the large margins theory, proposed by Schapire et al. (1999). This theory is closely related to the theory of support vector machines (SVM) (1999). The focus of large margin theory is on the task of minimizing the classification error rate on the test set. The different theory, proposed by Friedman et al. (2000), related Adaboost to logistic regression. The main focus of this theory is on maximizing the likelihood of a conditional probability distribution represented as a logistic function. The decrease in classification error is seen as a by-product of the increase in the likelihood.

One problem with Adaboost that has been realized early on is it’s sensitivity to noise (1998). The performance of Adaboost deteriorates rapidly when random label noise is added to the training set. Friedman et al. proposed a variant of Adaboost, which they named gentle Adaboost or Logitboost, which is significantly better than Adaboost at tolerating label noise. Similar algorithms to Logitboost are log-loss Boost proposed by Collins, Schapire and Singer (2000) and MAdaboost, proposed by Domingo and Watanabe (2001), as these algorithms are very similar to Logitboost, we will refer only to Logitboost from now on. While Adaboost puts unbounded large weights on mislabeled examples, the weight placed on any example by Logitboost is bounded. This decreases the penalty on mislabeled examples and increases the ability of the algorithm to tolerate noise.

All of these algorithms can be described as methods for minimizing a potential function using gradient descent (2000). Moreover, the potential function used by Adaboost, Logitboost, Logloss Boost and MAdaboost are all convex. The minimum of convex potential functions can be computed efficiently, which is the reason boosting is an efficient algorithm. However, Long and Servedio (2001) prove that any boosting algorithm that is based on a convex potential function can be defeated by random label noise. They present a simple construction of a distribution that cannot be learned using such algorithms.

In this paper we present a new boosting algorithm, which we call Robustboost, which is significantly more robust against label noise than either Adaboost or Logitboost. The new algorithm is based on the Freund’s Boost-by-Majority algorithm (1999) and Brownboost (2001). The algorithm is a potential based algorithm. However, the potential function is not convex and it changes during the boosting process.

The paper is organized as follows. In Section 2 we describe the potential based approach for learning linear discriminators and the problems associated with label noise and convex potential functions. In Section 3 we discuss the margin based explanation for Adaboost’s resistance to overfitting and how to apply it to problems in which the data is not linearly separable. In Section 4 we present the new boosting algorithm. In Section 5 we give the experimental evidence that the new algorithm has superior robustness against label noise. We conclude in Section 6.
2. Learning Linear Discriminators under noise

To simplify this explanation, we fix the set of base classifiers and assume that the weak learner picks the base classifier with the smallest weighted error at each iteration. In this section we focus on the problem of minimizing the number of mistakes that the combined classifier makes on the training set. The performance of the learned classifier on examples outside the training set will be discussed in Section 3.

We assume that we are given a training set \( S = \{(x_1, y_1), \ldots, (x_N, y_N)\} \) where \( x_i \) are the feature vectors and \( y_i \in \{-1, +1\} \) are the binary labels. The classification rule generated by Adaboost is of the form \( c(x) = \text{sign}(\sum_{i=1}^{d} \alpha_i h_i(x)) \), where \( \alpha_1, \ldots, \alpha_d \) are real numbers and \( h_1, \ldots, h_d \) is the fixed set of base rules. We assume that the range of the base classifiers is \( \{-1, +1\} \). As the base classifiers are fixed, we can represent the feature vector \( x \) by the \( d \) dimensional binary vector \( \vec{h} = \langle h_1(x), \ldots, h_d(x) \rangle \). This reduces the problem to that of learning a weights vector \( \vec{\alpha} = \langle \alpha_1, \ldots, \alpha_d \rangle \) that defines a good linear discriminator of the form \( \text{sign}(\vec{\alpha} \cdot \vec{h}) \).

We call the argument of the signum function the score function \( s(x) = \vec{\alpha} \cdot \vec{h} \). The (un-normalized) margin of an example \((x, y)\) is defined to be the product of the score function and the label \( m(x, y) = y s(x) \). Clearly \( m(x, y) > 0 \) if and only if the classification \( c(x) \) is correct. Thus the indicator function \( 1[m(x_i, y_i) \leq 0] \) is 1 if \( c(x) = y \) and 0 otherwise, we call this indicator function the “error step function”.

Our goal in this section is to find the linear discriminator which minimizes the training error, which can be expressed as

\[
P_S[c(x) \neq y] = \frac{1}{N} \sum_{i=1}^{N} 1[m(x_i, y_i) \leq 0]
\]

If there exists a linear classifier whose training error is zero we say that the training data is linearly separable. In this case there are several provably efficient algorithms for finding the separating hyperplane (for example, the perceptron algorithm). On the other hand, when the training data is not linearly separable there is no known efficient algorithm for finding the vector \( \vec{\alpha} \) that minimizes the training error. In fact, it is known that this is an NP-hard, and that it is also NP-hard to find the linear classifier with the minimal number of disagreements or to approximate it within a factor of \( 2 - \epsilon \) (?).

As finding good linear separators is a problem of great practical importance algorithms have been developed that can find the optimal separating hyperplane under particular assumptions regarding the distribution of the examples. Two prominent examples are linear discriminant analysis, which assumes that the two classes are normally distributed with equal covariance matrices and logistic regression which assumes that the conditional probability of the label given the input is described by the logistic function. Friedman et al (?) show that Logistic regression is closely related to Adaboost and to Logitboost.

Logitboost and Adaboost can be represented using a potential function. The potential function \( \Phi \) is a decreasing function of the margin \( m \) which upper bounds the error step function \( \Phi(m) \geq 1 \{m \leq 0\} \). As the average potential is an upper bound on the classification error, decreasing it is a good heuristic for decreasing the classification error. The potential function for Adaboost is \( \exp(-m) \) and the potential function for Logitboost is \( \ln(1 + \exp(-m)) \). These functions are very close when \( m > 0 \) but diverge when \( m \) is negative because for \( m \leq 0 \) the potential for Logitboost is approximately linear \( \ln(1 + \exp(-m)) \approx -m \). Minimizing these potential functions can be done very effectively using gradient descent methods. In particular using the chain rule we get a simple expression for the derivative of the average potential w.r.t. \( \alpha_i \). Denoting \( m(x_j, y_j) \) by \( m_j \) we get

\[
\frac{d}{d\alpha_i} \frac{1}{N} \sum_{j=1}^{N} \phi(m_j) = \frac{1}{N} \sum_{j=1}^{N} \frac{d m_j}{d\alpha_i} \frac{d\phi(m)}{dm} \bigg|_{m=m_j}
\]

\[
= \frac{1}{N} \sum_{j=1}^{N} y_j h_i(x_j) \frac{d\phi(m)}{dm} \bigg|_{m=m_j}
\]

It is therefore natural to define a weight function \( w(m) \) that is (minus) the derivative of the potential function with respect to \( m \). Using this notation we get the
expression
\[
\frac{d}{d\alpha_i} \frac{1}{N} \sum_{j=1}^{N} \phi(m(x_j, y_j)) = -\frac{1}{N} \sum_{j=1}^{N} y_j h_i(x_j) w(m(x, y))
\]

which has the attractive interpretation that the derivative of the average potential w.r.t. \(\alpha_i\) is equal to correlation between \(h_i(x)\) and \(y\) over the training examples, weighted by \(w(m(x, y))\). The weights represent the relative importance of different examples in reducing the average potential. The weight function for Adaboost is \(w(m) = \exp(-m)\) and the weight function for Logitboost is \(w(m) = 1/(1+\exp(m))\). Note that the weights assigned by Adaboost rapidly increase to infinity when \(m < 0\) while the weights assigned by Logitboost are at most 1.

Consider now what happens when we apply Adaboost or Logitboost to a linearly separable dataset to which we added independent label noise. In other words, we take a training set \(S\) which is perfectly classified with the a hyper-plane defined by \(\vec{\alpha}\) and flip the label of each example independently at random with probability \(q < 1/2\). The training set is no longer linearly separable. However, the classifier \(\vec{\alpha}\) which previously separated the two classes is still almost optimal as its error rate is approximately \(q\) which is the lowest achievable error rate. On the other hand \(\alpha\) is unlikely to be the point at which the average potential of Adaboost or Logitboost achieves it’s minimum. Long and Servedio (?) give a rigorous proof of this fact. Here we give a short intuitive explanation.

If there are examples in the clean dataset whose margin \(m\) with respect to \(\vec{\alpha}\) is large, then a fraction of about \(q\) of these examples now have a margin \(m\) that is a large negative number. If the margin of such a noisy example is \(m \ll 0\) then the potential assigned to it by Adaboost is \(\exp(-m)\) and the potential assigned to it by Logitboost is about \(-m\). Both potentials are larger than the error step function which they bound (see Figure [1]). Moving away from \(\vec{\alpha}\) can decrease the distance between the misclassified examples and the decision hyperplane, i.e. increase the corresponding negative margins, thereby decreasing the potential of those examples and therefor the overall average potential. The potentials and the weights assigned to mislabeled examples by Adaboost are much larger than those assigned by Logitboost. Therfore Logitboost is much more robust against label noise than Adaboost. However, Long and Servedio show that random label noise is a problem for any boosting algorithm that uses a convex potential function. The algorithm we propose in this paper is based on a non-convex potential function. It thus even more robust the Logitboost against label noise. In Section [5.1] we give an experimental evidence that our algorithm is much more robust than Logitboost and Adaboost for the classification problem suggested by Long and Servedio.

3. The large margins theory

In the previous section we focused on minimizing the training error. However, it is clear that Adaboost and Logitboost are doing something more than minimizing training error. In many experiments (??; ??; ??) the test error of the generated strong classifier continues to decrease for many boosting iterations after the training error becomes zero. Characterizing the criterion that Adaboost optimizes which is a better predictor of the test error than the training error is an important step towards finding a better boosting algorithm. To this end we use the large margins theory of Schapire et al (??).

We use the following terminology. Recall that the definition of the un-normalized margin is \(m(x, y) = y \text{sign}(\sum_i \alpha_i h_i(x))\). We define the normalized margin to be \(\hat{m}(x, y) = (y \text{sign}(\sum_i \alpha_i h_i(x))) / (\sum_i |\alpha_i|)\).

The generalization error of a the classifier \(c(x)\) is the probability that \(c(x) \neq y\) when \((x, y)\) is generated by the underlying distribution \(D\). Using the margin notation we express the generalization error as \(P_D[\hat{m}(x) \leq 0]\). We denote the optimal classification error by \(c^*\).

The margin theory posits that large positive margins on training examples are predictive of small generalization error. Specifically, Theorem 1 in (??) states that for any \(\theta > 0\), with probability \(1 - \delta\) over the random choice of the training set

\[
P_D[\hat{m}(x) \leq 0] \leq P_S[\bar{m}(x) \leq \theta] + O\left(\frac{1}{\theta} \sqrt{\frac{\log N \log d + \log(1/\delta)}{N}}\right).
\]

where \(P_D[\hat{m}(x) \leq 0]\) is generalization error, i.e. the probability of making a mistake with respect to the underlying distribution, \(P_S[\bar{m}(x) \leq \theta]\) is the fraction of the examples in the training set for which the margin is smaller than \(\theta\), \(d\) is the number of base rules and \(N\) is the size of the training set.

The interpretation of this theorem is that to minimize the generalization error we should find linear classifier that minimizes the number of training examples for which \(\bar{m}(x) \leq \theta\) for a large value of \(\theta\). Note that varying \(\theta\) has opposite effects on the two term in the bound. Increasing \(\theta\) causes the first term to increase to 1 while decreasing \(\theta\) towards 0 causes the second...
term to blow up.

Schapire et al show that Adaboost will tend to decrease the bound given in Equation \([2]\) by increasing the value of \(\theta\) for which the term \(P_S[\bar{m}(x) \leq \theta]\) is equal to zero. In other words, by maximizing the minimal margin. Breiman (?) and Grove and Schuurmans (?) give experimental evidence against this explanation for why Adaboost does not overfit. They maximized the minimal margin directly and showed that this does not tend to decrease the generalization error.

However, setting the first term in Equation \([2]\) to zero and minimizing the second term is often not the best way to minimize the bound. If the training set is not linearly separable it is impossible to set the term \(P_S[\bar{m}(x) \leq \theta]\) to zero for any \(\theta \geq 0\). We can still get meaningful bounds from Equation \([2]\) but in order to do that we need an algorithm for finding a weight vector \(\vec{a}\) for which \(\bar{m}(x_i, y_i) > \theta\) for most but not all of the training examples.

We therefor redefine the goal of the boosting algorithm, instead of minimizing the number of mistakes on the training set, we define the goal as minimizing the number of examples whose normalized margins is smaller than some value \(\theta > 0\). Stated using our defined notation, our goal is to minimize

\[
\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}[\bar{m}(x_i, y_i) \leq \theta].
\]

(3)

In the next section we describe the potential-based boosting algorithm for minimizing this target function.

4. Robustboost

Our proposed algorithm, which we call Robustboost, is a variation on Brownboost algorithm proposed by Freund (?) which, in turn, is based on Freund’s Boost-by-Majority (BBM) algorithm (?). We give a brief description of Boost-by-majority and Brownboost and then describe Robustboost.

BBM is based on the idea of finite horizon. The number of boosting iterations is set in advance based on an error goal parameter \(\epsilon\) which is given to BBM as input. While all boosting algorithm give small weight to examples with large positive margins, BBM gives small weight to examples with large negative margins on the later iterations. Intuitively, it “gives up” on examples which are so far on the incorrect side of the boundary that they are unlikely to be classifier correctly at the end. These examples become part of the training error, which is quantified by \(\epsilon\). As the weight for a given margin is the derivative of the potential, this implies that the potential has decreasing slope for large positive margins as well as large negative margins. In other words, the potential function is non-convex.

One deficiency of BBM is that it is not adaptive. In other words, the base classifier added at each iteration is assigned a weight of one regardless of its accuracy (a base classifier is assigned a larger weight if it is added in multiple iterations). Brownboost is the adaptive version of BBM. It assigns based classifiers with small error larger weight than base classifiers with error close to 1/2. Brownboost uses a real valued variable called “time” and denoted by \(t\). Before the first iteration \(t = 0\) and it is increased in each iteration. While BBM terminates after a pre-defined number of iterations, Brownboost terminates when \(t\) reaches a pre-defined value \(c\). The parameter \(c\) defines the horizon of the boosting process and is pre-computed according to the target error \(\epsilon\). As \(\epsilon \to 0\), \(c \to \infty\) and Brownboost becomes equivalent to Adaboost. In other words, Adaboost is a special case of Brownboost where the target error is zero.

The algorithm we propose here is very similar to Brownboost. The main difference is that instead of minimizing the training error it’s goal is to minimize the margin-based cost function defined in Equation \([3]\). In order to minimize this cost function it needs to use a normalized weight vector. Designing an algorithm that would keep the \(L_1\) norm of the weight vector bounded proved difficult. Our solution is to normalize the weight vector so that the variance of the scores is bounded. In other words, the algorithm controls the weight vector \(\vec{\alpha}\) so that \(\text{Var}(\vec{\alpha} \cdot h)\) is small. This is achieved by adding to the drift in the underlying Brownian motion process a component which pushes the examples towards zero. This makes the underlying process equivalent to the mean-reverting Ornstein-Uhlenbeck process (see page 75 in (?)).

We now describe the details of Robustboost. In our setup the range of the time variable is \(0 \leq t \leq 1\). Denoting the margin by \(m\) we define the potential function to be

\[
\Phi(m, t) = 1 - \text{erf}\left(\frac{m - \mu(t)}{\sigma(t)}\right).
\]

(4)

Where \text{erf} is the error function

\[
\text{erf}(a) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{a} e^{-x^2} dx
\]

\(\mu(t)\) and \(\sigma(t)\) are defined by the equations

\[
\sigma^2(t) = (\sigma_0^2 + 1)e^{2(1-t)} - 1
\]

(5)

and

\[
\mu(t) = (\theta - 2\rho)e^{1-t} + 2\rho
\]

(6)
Robust Boosting

**Figure 2.** The potential functions used by Robustboost as a function of $t$. The potential function at $t = 1$ is a close approximation of the margins error function $1 [\hat{m}(x_i, y_i) \leq \theta]$.

Where $\theta, \sigma_f$ and $\rho$ are parameters of the algorithm that we will describe shortly. Taking the partial derivative of $\Phi(m, t)$ with respect to $m$ we get that the weight function is

$$w(m, t) = \exp \left( -\frac{(m - \mu(t))^2}{2\sigma^2} \right)$$

(7)

The parameter $\theta \geq 0$ is the goal margin, as defined in Equation (3). It is set using cross-validation. Increasing $\theta$ decreases the difference between the performance on the training set and on the validation set. The parameter $\sigma_f > 0$ defines the slope of the step in the final potential function (see Figure 2). We set $\sigma_f = 0.1$ to avoid numerical instability when $t$ is close to 1.

Like Brownboost, Robustboost is a self-terminating algorithm. It terminates when $t \geq 1$. If the error goal $\epsilon$ is set too small then Robustboost will not terminate. Setting the right value for $\epsilon$ is done by searching for the minimal value of $\epsilon$ for which the algorithm terminates within a reasonable number of iterations. Setting $\epsilon$ determines the value of the parameter $\rho$. That value is the solution to the following equation

$$\epsilon = \Phi(0, 0) = 1 - \text{erf} \left( \frac{2(e - 1)\rho - e\theta}{\sqrt{e^2(\sigma_f^2 + 1) - 1}} \right)$$

(8)

set $\rho$ to satisfy Equation (8).

**initialize** $k = 1$, $t_1 := 0$, $H_0 \equiv 0$

$m(1) := 0, \ldots, m(N) := 0$

**repeat**

Define the distribution $D_k$ over the $N$ training examples by normalizing $w(m, t)$ defined in Equations (7, 5, 6)

$$D_k(j) = \frac{w(m(j), t_k)}{Z}, \quad Z = \sum_{j=1}^{N} w(m(j), t_k)$$

**call** base learner and get $h_k : \mathcal{X} \rightarrow \{-1, +1\}$ which is slightly correlated with the label:

$$E_{j \sim D_k} |y_j h_k(x_j)| > 0$$

**find** $\Delta m_k > 0, 1 - t_k \geq \Delta t_k > 0$ that simultaneously satisfy the following two equations:

$$\sum_{j=1}^{N} y_j h_k(x_j) w(m(j), t_k + \Delta t_k) = 0 \text{ or } \Delta t_k = 1 - t_k$$

$$\sum_{j=1}^{N} \Phi(m(j), t_k) = \sum_{j=1}^{N} \Phi(m'(j), t_k + \Delta t_k)$$

where

$$m'(j) \equiv m(j)e^{\Delta t_k} + y_j h_k(x_j) \Delta m_k$$

and $\Phi(\cdot, \cdot), w(\cdot, \cdot)$ are defined by Equations (14, 15, 16).

**update**: $t_{k+1} := t_k + \Delta t_k \forall 1 \leq j \leq N \quad m(j) := m'(j) \quad H_k = H_{k-1}e^{-\Delta t_k} + \Delta m_k h_k \quad k := k + 1$

until $t_k \geq 1$

**Output**: the final hypothesis $H_k$.

5. Experiments

We report the results of two sets of experiments using synthetic data distributions. The first distribution is taken from Long and Servedio (2013). The second is taken from Mease and Wyner (2007).

5.1. The Long/Servedio problem

Long and Servedio suggested the following challenging classification problem. The input is a binary feature vector of length $21$: $(x_1, \ldots, x_{21})$ where $x_i \in \{-1, +1\}$ and the output is $y \in \{-1, +1\}$. A random example is generated as follows. First, the label $y$ is chosen
with equal odds to be $-1$ or $+1$. Given $y$ the features $x_i$ are generated according to the following mixture distribution:

- **Large margin examples**: With probability $1/4$ all of the $x_i$ are set equal to the label $y$.
- **Pullers**: With probability $1/4$ we set the first eleven coordinates equal to the label $x_1 = x_2 = \cdots = x_{11} = y$ and $x_{12} = x_{13} = \cdots = x_{21} = -y$.
- **Penalizers**: With probability $1/2$ we do the following. Choose at random 5 coordinates out of the first 11 and 6 coordinates out of the last 10 and set those equal to the label $y$. Set the remaining 10 coordinates to $-y$.

It is easy to check that the majority vote rule $\text{sign}(\sum x_i)$ is a perfect classifier for this data. To learn this classifier using boosting we define the base classifiers to be single coordinates, i.e. $h_i(\vec{x}) = x_i$. Clearly, using these base classifiers the target classifier can be represented exactly. As this is a linearly separable distribution, both AdaBoost and LogitBoost can learn it perfectly.

However, if we add label noise to this problem, i.e. if we flip each label with probability 0.1 then the performance of both AdaBoost and LogitBoost degrades severely. The optimal rule remains the majority vote rule, but the average potential of this rule is very large and the minimal potential is achieved by a sub-optimal classifier. Long and Servedio show that any potential based boosting algorithm that uses a convex potential function will fail to find a classifier whose training error is close to that of the majority vote rule. It is important to note that the failure of these learning algorithms demonstrates itself already on the training set and is a problem of under-fitting, not overfitting, the training data.

We compare the performance of RobustBoost, LogitBoost and AdaBoost on this problem. We run each boosting algorithm for 300 iterations. We generated 10 datasets, each consisting of 800 training examples. We run each algorithm on each of the data sets and compute the training error for each case. We also compute the error with respect to the clean labels, which have not been corrupted by noise. We report the average and standard deviation of these errors and their relative order for individual datasets.

We set the parameters of RobustBoost to be $\sigma_f = 0.1$ and $\epsilon = 0.14$. The value of $\epsilon$ was chosen so that RobustBoost terminates after 200-300 iterations for most datasets. We tried two settings $\theta$. In one setting $\theta = 0$, i.e. the goal of the algorithm is to minimize the training error. In the other setting $\theta = 0.2$ which means that the goal of the algorithm is to minimize the number of training examples whose margin is less than 0.2. The numbers in this and the following tables correspond to percent errors (i.e. there are numbers between 0 and 100).

|        | Ada  | Logit | Robust $\theta = 0$ | Robust $\theta = 0.2$ |
|--------|------|-------|----------------------|-----------------------|
| Err    | $28.1 \pm 1.5$ | $26.6 \pm 1.5$ | $13.2 \pm 1.3$     | $10.0 \pm 1.3$     |
| Clean | $24.3 \pm 1.7$ | $22.6 \pm 1.7$ | $5.5 \pm 2.5$      | $0.9 \pm 1.0$      |

The relative order of the errors was always the same. The error of RobustBoost is far smaller than that of LogitBoost, which is slightly better than that of AdaBoost. Surprisingly, the error of RobustBoost is further improved when we set $\theta = 0.2$. The difference is even more pronounced when comparing the predictions of the classifier to the noiseless labels. In particular, the error of RobustBoost is about 1.0% even though the data on which it was trained has 10% error with respect to the clean data. In other words, RobustBoost is able to detect and correct most of the mislabeled examples.

We can gain insight into the reasons that RobustBoost succeeds while AdaBoost and LogitBoost fail by comparing the evolution of the score distributions for the different algorithms. In figure 3 we show the score distribution to which LogitBoost converges after 100 iterations, this distribution changes very little from iteration 100 to iteration 300. What we see is that the algorithm converged to a minimal potential vector $\vec{\alpha}$ in which the large margin examples and the pullers are well separated, but the penalizers are distributed more or less randomly. The reason is that the mislabeled large margins and pullers have relatively large
weights (the derivative of he potential is close to one) while the weight of each individual penalizer is small. As the penalizers are sparse, they cannot “pull” $\vec{a}$ from the direction suggested by the pullers and large margin examples and so about half of them are mislabeled, contributing about 25% to the training error.

Contrast this with the score distributions shown in Figure 3. After 100 boosting iterations the potential is such that the weight of large margin examples is close to zero whether or not they are mislabeled and the weight of mislabeled pullers is smaller than it was with Logitboost. This means that the algorithm ignores the large margin examples and concentrates on the pullers and the penalizers, without giving the pullers too much weight. The result is that after 200 iterations many of the penalizers are classified correctly and the pullers are mixed in with the penalizers. Note that for the ideal solution the margins of pullers and penalizers are mixed in with the penalizers. Note that for many of the penalizers are classified correctly and the large margin examples and concentrates on the pullers and the penalizers, without giving the pullers too much weight. The result is that after 200 iterations many of the penalizers are classified correctly and the pullers are mixed in with the penalizers. Note that for the ideal solution the margins of pullers and penalizers that are not mislabeled is equal to $11/21 − 1/2 = 1/42$. Our main point is that Robustboost avoids the incorrect mini ma that trap Adaboost and Logitboost by ignoring examples with large negative margins.

5.2. The Mease/Wyner problem

In this section we report results of experimental comparisons using synthetic distributions analyzed by Mease and Wyner (?). In this case a majority vote over the base classifier can only approximate the target classifier which significantly complicates the problem.

The input to this classification problem is a $d = 20$ dimensional vector $\vec{x}$ where each coordinate of $x$ is chosen IID according to the uniform distribution over the segment $[0, 1]$. The label $x$ is 1 if $\sum_{i=1}^{d} x_i \geq 2.5$ and −1 otherwise. The base classifiers we tested are decision stumps, i.e. rules of the form $\text{sign}(x_i − a_i)$ or a 2 level decision tree made out of these decision rules. Unlike the Long and Servedio distribution, a finite number of these base classifiers cannot exactly represent the target rule. Mease and Wyner use this distribution to compare the effects of random label noise on Adaboost and Logitboost. We add Robustboost to the comparison.

In each experiment we use 2000 training examples and 2000 test examples (we tried 200 examples but the between-experiment variation was too large to draw significant conclusions). We repeat each experiment 15 times and report the mean and standard deviation of the error on the test set. We tried two levels of random noise $q = 0.1$ and $q = 0.2\%$. The boosting algorithms are run for (at most) 500 iterations. For Robustboost we use: $\theta = 1.0$, $\sigma = 0.1$, for $q = 0.1$ $\epsilon = 0.15$, for $q = 0.2$ $\epsilon = 0.25$. For these settings of $\epsilon$

Robustboost terminates after 100-300 iterations.

| base | q | Ada | Logit | Robust |
|------|---|-----|-------|--------|
| stump | 10 | 19.3 ± 0.1 | 15.9 ± 0.9 | 13.5 ± 0.8 |
| 2tree | 10 | 21.4 ± 1.2 | 18.7 ± 1.1 | 14.8 ± 1.0 |
| stump | 20 | 29.4 ± 1.2 | 26.7 ± 1.3 | 23.8 ± 1.1 |
| 2Tree | 20 | 32.3 ± 1.0 | 29.3 ± 0.8 | 25.3 ± 0.9 |

As in the previous section Robustboost performs significantly better than Logitboost which is better than Adaboost. The relative performance of the algorithms is consistent with this table in all 15 repeats of the experiment. Using 2 level decision trees is consistently worse than using stumps. While significant, the difference between Robustboost and Logitboost is smaller than it was in the previous section, we conjecture that this is because the distribution here is much more symmetric which decreases the biasing effect of the examples with large negative margins.

Continuing only with stumps, we report the error of
the generated classifiers relative to the uncorrupted labels. The relative performance here is the same, with Robustboost leading the way.

| q  | Ada       | Logit     | Robust    |
|----|-----------|-----------|-----------|
| 10 | 11.5 ± 1.1| 7.1 ± 0.7 | 4.3 ± 0.4 |
| 20 | 15.6 ± 1.2| 11.2 ± 1.0| 6.5 ± 1.0 |

A potentially more important aspect of the classifier generated by Robustboost is that it’s predictions that are given with large margins are very trustworthy. In the following table we report the fraction of the test set on which the absolute value of the score is smaller than θ (low margin examples) and the error rate on the remaining examples relative to the uncorrupted test data.

| q  | low margin | clean err |
|----|------------|-----------|
| 10 | 10.5 ± 0.6 | 0.9 ± 0.2 |
| 20 | 10.2 ± 0.7 | 2.4 ± 0.6 |

Once more, we see that Robustboost is capable of detecting most of the incorrect labels for the examples with large margins.

6. conclusions

We present evidence that Robustboost is more robust against label noise than either Logitboost or Ada-boost. More experiments using synthetic and real-world datasets are needed to verify this claim.

The effectiveness of Robustboost suggest that after an approximate classifier has been learned it can be beneficial to down-weight examples that are far from the decision boundary regardless of their label. This suggests new directions for active learning which we are currently investigating.

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