Conformally coupled dark matter.

Mark Israelit

Department of Physics, University of Konstanz, PF 5560 M678, D-78434 Konstanz, Germany. On leave from: Department of Physics, University of Haifa, Oranim, Tivon 36006, Israel.

Abstract

Dark matter is obtained from a scalar field coupled conformally to gravitation; the scalar being a relict of Dirac’s gauge function. This conformally coupled dark matter includes a gas of very light ($m \approx 2.25 \times 10^{-34} \text{eV}$) neutral bosons having spin 0, as well as a time-dependent global scalar field, both pervading all of the cosmic space. The time-development of this dark matter in the expanding F-R-W universe is investigated, and an acceptable cosmological behaviour is obtained.

Keywords: General Relativity; Cosmology; Dark Matter.

\footnote{e-mail: Marc.Israelit@uni-konstanz.de, and: israelit@physics.technion.ac.il, permanent e-address}
1 Introduction.

There is a great deal of dark matter in the universe, the dark matter having a much greater mean density than visible matter, (Trimble 1987), (Turner 1991), (Tremain 1992). It is believed (Tyson 1992), (Priester, Hoell, & Blome 1995) that the density of dark matter exceeds that of luminous by a factor of about 10. At present there is a number of popular non-baryonic dark matter candidates: axions, neutrino, gravitino etc. etc. However, these candidates suffer from a number of shortcomings: their evolutionary behavior in the expanding universe is not clearly known, their origin is not related to the geometry of the universe, they are inserted into the space-time framework from the outside.

Some years ago it was suggested (Israelit, & Rosen 1992) that at least a part of the dark matter in the universe consists of massive bosons of spin 1. These particles, named weylons were created by a local Weyl vector field. The idea of such a field was based on the Weyl geometry (Weyl 1919) as modified by Dirac (1973). It was also shown (Israelit, & Rosen 1994) that dark matter consisting of heavy weylons (with mass greater then 10 Mev), may be fitted into the standard cosmological model (Weinberg 1972), as well as into the singularity-free prematter cosmological model (Israelit, & Rosen 1989). Later a global dark matter effect was obtained (Israelit, & Rosen 1995) from a time-dependent scalar field, the scalar being identical with the gauge function introduced by Dirac (1973) in his modification of the Weyl geometry. The above mentioned dark matter forms are derived from the Weyl geometry. In this paper an additional geometrically based generator of cosmic dark matter is presented.

The purpose of the present work is to consider the possibility that a scalar field, coupled conformally to gravity, can generate cosmic dark matter. The behavior of this dark matter in the expanding universe will be investigated. It will also be shown that unifying the Einstein-Hilbert action with the (reduced) action of the Weyl-Dirac theory, one obtains the action of a scalar field coupled conformally to gravitation. In this procedure the coupled scalar field appears as a relict of the Dirac gauge function.

In the present work general relativistic (geometrical) units are used, so that $G = 1$, $c = 1$, the Hubble constant is given in $cm^{-1}$, and the energy density of matter in $cm^{-2}$. However, in order to discuss physical results we turn sometimes to conventional units, noting this fact in the text. The conversion rules may be found in (Synge 1966).

2 The Weyl-Dirac theory and conformally coupling.

A detailed description of the Weyl-Dirac theory and a discussion of its physical aspects may be found in Rosen’s work (1982). A brief discussion of the theory in view of the dark matter problem is given in (Israelit, & Rosen 1992). In this section we consider the relation between the Weyl-Dirac theory and a scalar field coupled conformally to gravity. The notations of (Rosen 1982), and of (Israelit, & Rosen 1992) will be used.

The field equations of the Weyl-Dirac theory may be derived from a variational principle (Dirac 1973), (Rosen 1982)

$$\delta I_D = 0,$$

with the action

$$I_D = \int L_D (-g)^{1/2} d^4 x,$$

and the Lagrangian density $L_D$ given by

$$L_D = W_{\mu\nu}W^{\mu\nu} - \beta^2 R^\sigma_\sigma + k\beta_\mu\beta^\mu + (k-6)(2\beta w^\mu\beta_\mu + \beta^2 w_\mu w^\mu) + 2\Lambda\beta^4.$$

In expression (3), $w^\mu(x^\lambda)$ is the connection vector of the Weylian geometry, the Weylian length curvature tensor $W_{\mu\nu}$ is defined as $W_{\mu\nu} = w_{\mu,\nu} - w_{\nu,\mu}$ (a comma denoting partial differentiation), $\beta(x^\lambda)$ is the Dirac gauge function, and $\beta_\mu \equiv \beta, \mu$. Further, $\Lambda$ is the cosmological constant, $R^\sigma_\sigma$ is the Riemannian curvature scalar, and $k$ is an arbitrary parameter. In order to get a geometrically based description of gravitation and electrodynamics, Dirac took $k = 6$. In that case $w_\mu$ may be treated as the vector potential of the electromagnetic field, and $W_{\mu\nu}$ becomes the field tensor. By Dirac’s choice one has from equation (3)

$$L_D = W_{\mu\nu}W^{\mu\nu} - \beta^2 R^\sigma_\sigma + 6\beta^2 w_\mu w^\mu + 2\Lambda\beta^4.$$

In this paper an additional geometrically based generator of cosmic dark matter is presented.
For cosmological considerations we can neglect the Maxwell term in (4), so that
\[ L_D = -\beta^2 R^\sigma_\sigma + 6\beta^\sigma \beta_\sigma + 2\Lambda \beta^4. \]  
(5)

Inserting (5) into (2) we obtain the action for a conformal space. Further, combining it with the Einstein-Hilbert action for gravitation and matter
\[ I_G = \int (R^\sigma_\sigma + L_m)(-g)^{1/2}d^4x, \]  
(6)

\((L_m\) is the Lagrangian density of matter), we get
\[ I = \int [R^\sigma_\sigma + L_m + 8\pi(\beta^\sigma \beta_\sigma - \frac{1}{6} \beta^2 R^\sigma_\sigma + \frac{1}{3} \Lambda \beta^4)](-g)^{1/2}d^4x. \]  
(7)

This is the general covariant action of a scalar field \(\beta(x^\lambda)\) coupled conformally to gravitation. In the following procedure \(L_m\) will be considered as that of ordinary (luminous) matter, and it will be assumed that \(L_m\) does not depend on \(\beta\). On the other hand, dark matter will be obtained from the conformally coupled scalar field \(\beta(x)\). We also shall consider models without the cosmological constant so that in the action (5) we shall set \(\Lambda = 0\).

### 3 The field equations.

Varying in (7) the metric tensor \(g_{\mu\nu}\) we obtain the Einstein equation
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\sigma_\sigma = -8\pi(T_{\mu\nu} + \Theta_{\mu\nu}) \]  
(8)

with the Ricci tensor \(R_{\mu\nu}\), and with the energy-momentum density tensor \(T_{\mu\nu}\) of ordinary (luminous) matter
\[ 8\pi \sqrt{-g} T_{\mu\nu} = \frac{\delta(\sqrt{-g}L_m)}{\delta g_{\mu\nu}}. \]  
(9)

The dark matter is represented in (8) by the energy-momentum density tensor of the \(\beta\)-field
\[ 6\Theta_{\mu\nu} = 4\beta_{\mu\nu} - g_{\mu\nu} \beta^\sigma - 2\beta \beta_{\mu\nu} + 2g_{\mu\nu} \beta^\sigma - \beta^2 (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\sigma_\sigma). \]  
(10)

Further, variation with respect to the field \(\beta\) yields the following field equation:
\[ \beta^\sigma_{;\sigma} + \frac{1}{6} \beta R^\sigma_\sigma = 0. \]  
(11)

Making use of (11), one obtains from (10)
\[ \Theta^\sigma_\sigma = 0, \]  
(12)

so that (8) gives
\[ R^\sigma_\sigma = 8\pi T^\sigma_\sigma. \]  
(13)

Taking into account (13), one can rewrite the field equation (11) as
\[ \beta^\sigma_{;\sigma} = -\frac{4\pi}{3} \beta T^\sigma_\sigma, \]  
(14)

so that the behavior of the conformally coupled field depends on the amount and state of ordinary matter. Finally, making use of the contracted Bianchi identity and of (13), one obtains from (10)
\[ \Theta^\nu_{\mu;\nu} = 0, \]  
(15)

and from (8) one has
\[ T^\nu_{\mu;\nu} = 0, \]  
(16)

so that one obtains separated energy-momentum relations, the first, (15) for dark matter, and the second, (16) for ordinary matter.
Let us consider a homogeneous and isotropic spatially closed universe described by the F-R-W line element
\[ ds^2 = dt^2 - R^2 \left( \frac{dr^2}{1 - r^2} + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta \, d\varphi^2 \right), \]
(17)
with \( R(t) \) being the radius of the universe. We assume that the universe is filled with ordinary cosmic matter, having an energy density \( \rho(t) \), and a pressure \( P(t) \), as well as with dark matter, created by the scalar function \( \beta \). For the metric given by (17) we can write down the Einstein equation (8) with \( \mu = 0; \ \nu = 0; \)
\[ \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = \frac{8\pi}{3} (\rho + \Theta_0^0), \]
(18)
and from eq. (13) we obtain
\[ \frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = \frac{4\pi}{3} (\rho - 3P). \]
(19)
In addition we have from the energy relation (16)\[ \dot{\rho} + \frac{3\dot{R}}{R} (\rho + P) = 0. \]
(20)
The equations obtained in this section enable us to investigate the scalar field \( \beta \) coupled conformally to a F-R-W universe filled with ordinary matter. The generation of dark matter by this \( \beta \)-field may be considered either by means of a local approach, or by a global one.

4 A local approach. Scalar bosons.

From the global standpoint, in a homogeneous and isotropic universe \( \beta \) depends only on the cosmic time \( t \). However, one can assume that there exist small regions of inhomogeneity, in which \( \beta \) may depend also on spatial coordinates. Introducing local Lorentzian coordinates \( (ct, x, y, z) \), one can rewrite (14) as
\[ \Box \beta + \frac{4\pi}{3} (\rho - 3P) \beta = 0. \]
(21)
Further in small space-time regions one can neglect the dependence of \( \rho \) and \( P \) on the cosmic time \( t \), so that
\[ \frac{4\pi}{3} (\rho - 3P) \equiv \kappa^2 \simeq \text{const.}, \]
(22)
and (21) may be rewritten in the form of a Klein-Gordon equation
\[ \Box \beta + \kappa^2 \beta = 0. \]
(23)
From the point of view of quantum mechanics, eq. (23) describes spinless particles, scalar bosons, with a mass \( m \) and Compton wavelength \( \lambda_c \). In conventional units one has
\[ \kappa = \frac{mc}{\hbar} = \frac{2\pi}{\lambda_c}, \]
(24)
and by (22), and (24) one obtains for the mass in conventional units
\[ m = \frac{\hbar}{c^2} \left[ \frac{4\pi}{3} G (\rho - 3P) \right]^{1/2}, \]
(25)
with \( G \) being Newton’s gravitational constant. It will be recalled that \( \rho \), and \( P \) are the energy density and pressure of ordinary (luminous) cosmic matter.

In order to estimate the boson mass at present we will make use of the Hubble constant. Following (Peebles 1993) let us write for its present value \( H_N = H_0 \), where \( H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \div (9.78 \times 10^9 \text{ y})^{-1} \div 1.08 \times 10^{-28} \text{ cm}^{-1} \) (cf. (Weinberg 1972)), and with \( 0.5 \leq h \leq 0.8 \) (cf. (Priester, Hoell, & Blome 1995)). Adopting the value \( h \approx 0.5 \) we get
\[ H_N = 0.5 \times 10^{-28} \text{ cm}^{-1}, \]
(26)
so that for the critic density we obtain

\[ \rho_c = \left( \frac{3}{8\pi} \right) H^2_N = 0.3 \times 10^{-57} \text{cm}^{-2}. \]  

(27)

To discuss a closed universe one can take for the total (luminous and dark) matter density at present \( \rho_{\text{total, N}} \approx 0.35 \times 10^{-57} \text{cm}^{-2} > \rho_c \). Further one can assume that the luminous matter makes up 0.1 of the total one, so that the following values must be substituted into (25):

\[ P_N = 0, \quad \rho_N = 0.35 \times 10^{-57} \text{cm}^{-2}. \]  

(28)

Expressing \( \rho_N \) in conventional units one obtains the present value

\[ m_N \approx 4 \times 10^{-67} = 2.25 \times 10^{-34} \text{eV}. \]  

(29)

According to (25) in the radiation period the boson mass vanishes, whereas during the dust-dominated period it is given by

\[ m = m_N \left( \frac{R_N}{R} \right)^{1/2}, \]  

(30)

with \( R_N \) being the present value of the cosmic scale factor.

Let us consider the energy-momentum density of the dark matter. For small regions, in local Lorentzian coordinates, one obtains from (10) the following non-vanishing components:

\[ \Theta_0^0 = \frac{1}{2c^2} \beta^2 + \frac{1}{6} \left( \beta^2_x + \beta^2_y + \beta^2_z \right) - \frac{1}{3} \beta \left( \beta_{xx} + \beta_{yy} + \beta_{zz} \right) + \frac{1}{2} \beta^2 \left( \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right), \]  

(31)

\[ \Theta_0^1 = \frac{2}{3c} \beta \beta_x + \frac{1}{3c} \beta \beta_{xx}, \]  

(32)

\[ \Theta_1^1 = -\frac{1}{2} \beta^2 + \frac{1}{6} \left( \beta^2_y + \beta^2_z - \frac{1}{c^2} \beta^2_x \right) - \frac{1}{3} \beta \left( \beta_{yy} + \beta_{zz} - \frac{1}{c^2} \beta_{xx} \right) + \frac{1}{6} \beta^2 \left( 2 \frac{\dot{R}}{R} \frac{\dot{R}}{R^2} + \frac{1}{R^2} \right), \]  

(33)

and similar expressions for \( \Theta_0^2, \Theta_0^3, \Theta_0^4, \Theta_0^5 \). In order to calculate the energy density and pressure of dark matter, let us consider a plane wave traveling in the \( x \)-direction

\[ \beta = B \cos(\omega \tau - kx) ; \quad (B, \omega, k = \text{const.}) \]  

(34)

Making use of (34) we get from (23)

\[ \frac{\omega^2}{c^2} = k^2 + \kappa^2. \]  

(35)

In the quantum-mechanical representation one writes

\[ \varepsilon = h\omega, \quad p = h\kappa. \]  

(36)

where \( \varepsilon, p \) are the energy and momentum of the particle, so that with (24) relation (33) may be written as

\[ \varepsilon^2 = c^2 p^2 + m^2 c^4. \]  

(37)

For the plane wave given by (34) we obtain from (31)

\[ \Theta_{x-wave}^0 = (1/2) \beta^2 \left[ \frac{\omega^2}{c^2} + k^2 + \left( \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right) \right]. \]  

(38)
To describe an ensemble of particles having spatial isotropy, we can take a combination of six plane waves: the wave (34) moving in the positive direction along the $x$-axis, one moving in the opposite direction, two waves moving along the $y$-axis in both directions, and two moving along the $z$-axis. If we take the time average, we obtain for the combined system of waves

$$\Theta_0 = \frac{3}{2} B^2 \left( \frac{\omega^2}{c^2} + k^2 + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right).$$  \hspace{1cm} (39)$$

Making use of (33), we obtain for the same system of six plane waves

$$\Theta_0 = \Theta_0^2 = \Theta_0^3 = 0.$$  \hspace{1cm} (40)$$

Finally, making use of (18), (22), and (35), we get from equation (33)

$$\Theta_0 = \Theta_0^2 = \Theta_0^3 = -\frac{1}{3} \Theta_0^0.$$

The expression (39) is actually the energy density of our dark matter $\rho_D$. One can consider it as consisting of two parts, the first being the contribution $\rho_B$ of scalar bosons, the second, $\rho_G$, representing a global effect.

$$\rho_B = \frac{3}{2} B^2 \left( \frac{\omega^2}{c^2} + k^2 \right), \quad \rho_G = \frac{3}{2} B^2 \left( \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right), \quad \rho_D = \Theta_0^0 = \rho_B + \rho_G.$$  \hspace{1cm} (42)$$

Up to here a monoenergetic ensemble of bosons is discussed. According to (25) the boson has zero mass in the radiation era, whereas in the dust period it is extremely light (cf. (29), and (30)). These particles are discribed by the Bose-Einstein statistics, and there is no interaction with ordinary matter, so that the total number of bosons in the universe is conserved. In order to get a realistic scenario, one can turn to a gas of scalar bosons in thermal equilibrium, and fit it into the cosmological model. This procedure was already carried out in (Israelit & Rosen 1992, 1994) for a gas of weylons.

## 5 A global approach.

Above a F-R-W universe containing small regions of local inhomogeneities was considered. In the present section we consider a completely homogeneous and isotropic universe, with $\beta$ depending only on the cosmic time $t$. With the metric given by (17), we can rewrite eq. (14) as

$$\left( \dot{\beta} R^3 \right)_{,t} = -\frac{4\pi}{3} (\rho - 3P) R^3 \beta.$$  \hspace{1cm} (43)$$

Further, making use of (17), and (43), we obtain from (10) the following non-vanishing components of the energy-momentum density tensor of the global scalar field

$$\Theta_0^0 = \frac{1}{2} \beta^2 \left( \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right),$$  \hspace{1cm} (44)$$

and

$$\Theta_0^1 = \Theta_0^2 = \Theta_0^3 = -\frac{1}{3} \Theta_0^0.$$  \hspace{1cm} (45)$$

One can consider $\beta$ as a function of the cosmic scale factor $R$, so that

$$\dot{\beta} = \dot{R} \beta, \quad \text{with} \quad \beta, R \equiv \frac{d\beta}{dR},$$  \hspace{1cm} (46)$$

and (14) may be rewritten as

$$\rho_D = \Theta_0^0 = \frac{\dot{R}^2}{2R^2} (\beta + R \dot{\beta})^2 + \frac{\beta^2}{2R^2}.$$  \hspace{1cm} (47)$$
It is remarkable that in this approach, like in the previous one (cf. 39), the energy density $\rho_D$ of the scalar field is positive, so that one obtains a dark matter effect.

Making use of (40) we can rewrite eq. (43) as follows

$$R^3 \dot{R} \beta_{,R,R} + R^2 (R \ddot{R} + 3 \dot{R}^2) \beta_{,R} + \frac{4\pi}{3} R^3 (\rho - 3P) \beta = 0 \quad .$$

(48)

For a moment, let us go back to the relations (12), and (15). Making use of (17) we obtain for the energy density of dark matter

$$\frac{8\pi}{3} \rho_D \equiv \frac{8\pi}{3} \Theta_0 = \frac{D}{R^3} \quad ; \quad (D = \text{const.}) .$$

(49)

Let us consider the dust-dominated period. Here $P \ll \rho$, and the energy relation (20) gives

$$\frac{8\pi}{3} \rho = MR^3 \quad ; \quad (M = \text{const.}) .$$

(50)

Substituting (18), and (19) into (48), and taking into account (49), (50), we obtain

$$[MR^2 + DR - R^3] \beta_{,R,R} + \left[ \frac{5}{2} MR + 2D - 3R^2 \right] \beta_{,R} = -\frac{1}{2} M \beta \quad ,$$

(51)

and turning to the dimensionless variable

$$z = \frac{R}{R_N} \quad ,$$

(52)

with $R_N$ being the present value of the cosmic scale factor, we get

$$[MR_N z^2 + Dz - R_N^2 z^3] \beta_{,z,z} + \left[ \frac{5}{2} MR_N z + 2D - 3R_N^2 z^2 \right] \beta_{,z} + \frac{1}{2} MR_N \beta = 0 \quad .$$

(53)

The constants $M$, $D$, and $R_N$ appearing in (53) may be expressed in terms of two observable cosmological quantities, the present value of the Hubble "constant" $H_N$, and that of the energy density of ordinary (luminous) matter $\rho_N$. It is convenient to introduce a parameter $\eta$, given by

$$R_N = \frac{\eta}{H_N} \quad .$$

(54)

A second helpful parameter is the ratio

$$\chi = \frac{\rho_{\text{total}}}{\rho} \equiv 1 + \frac{\rho_D}{\rho} \quad ,$$

(55)

so that for the present state of the universe we have

$$\rho_{\text{total},N} = \rho_N + \rho_{D,N} = \chi_N \rho_N \quad .$$

(56)

On the other hand, eq. (18) at present may be written as

$$H_N^2 + \frac{1}{R_N^2} = \frac{8\pi}{3} (\rho_N + \rho_{D,N}) \quad .$$

(57)

With (27), and (54) this takes on the form

$$\rho_{\text{total}} = \rho_N + \rho_{D,N} = \rho_C \left( 1 + \frac{1}{\eta^2} \right) \quad .$$

(58)

Finally, making in (58) use of (27), (49), (54), and (54), we obtain

$$(1 + \eta^2) R_N^2 = MR_N + D \quad .$$

(59)

In a similar way we obtain from (54)

$$\chi_N MR_N = MR_N + D \quad .$$

(60)
From (61), and (60) one obtains
\[ M = \frac{1 + \eta^2}{\chi_N} R_N, \] (61)
and
\[ D = \frac{\chi_N - 1}{\chi_N} (1 + \eta^2) R_N^2. \] (62)
Inserting (61), and (62) into eq. (53) we get:
\[
\left[ z^3 - \frac{1 + \eta^2}{\chi_N} z^2 - \frac{\chi_N - 1}{\chi_N} (1 + \eta^2) z \right] \beta_{zz} + \left[ 3 z^2 - \frac{2(1 + \eta^2)}{2 \chi_N} z - \frac{2(\chi_N - 1)}{\chi_N} (1 + \eta^2) \right] \beta_z - \frac{1 + \eta^2}{2 \chi_N} \beta = 0. \] (63)
For given values of parameters \( \eta \), and \( \chi_N \) one can obtain the dark matter function \( \beta(z) \) from equation (63). It is believed (Tyson 1992), (Blome, Priester, & Hoell 1995) that at present the total density of matter exceeds that of luminous by a factor of \( \sim 10 \). This justifies taking
\[ \chi_N = 10. \] (64)
From (54), (and 58) we see that large values of \( \eta \) describe a nearly flat universe. Below will be considered models with \( \eta = 1 \), 4, 20, combined with \( \chi_N = 10 \).
Let us take the Einstein equation (18). Making use of (49), and (50), we can rewrite it as
\[ \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} = \frac{M}{R^2} + \frac{D}{R^4}. \] (65)
A closed universe achieves its maximum radius at \( \dot{R} = 0 \), so that from (65) we can calculate \( R_{max} \). Further, substituting (49) into (57), we obtain
\[ \frac{\dot{R}^2}{R^2} (\beta + R \beta_R)^2 + \beta_R^2 = \frac{3}{4 \pi} \frac{D}{R^4}. \] (66)
Making use of the values of \( R_{max} \) obtained from (65), one can calculate \( \beta_{max} = \beta (R_{max}) \), the value at the state of maximum expansion. Finally, taking into account (49), and (50), as well as (61), and (62), we have for the ratio (53) during the dust-dominated period
\[ \chi(R) = 1 + \frac{D}{MR} = 1 + \frac{(\chi_N - 1)R_N}{R}, \] (67)
so that
\[ \chi_{max} = 1 + \frac{(\chi_N - 1)R_N}{R_{max}}. \] (68)
Taking the values of \( R_{max} \) obtained from eq. (65), we can from (68) calculate the values of \( \chi_{max} \). For a moment let us go back to (53). At the state of maximum expansion the first bracket in that equation vanishes (cf. (63)). Thus, making use of (61), and (62), and of the above mentioned values of \( \beta_{max} \), one can calculate the value of the derivative \( (\beta_R)_{max} \), for the moment when \( R = R_{max} \). By the procedure described above, we get from equations (51), (55), (65), (66), (68), for chosen values of \( \chi_N \) and \( \eta \), the results listed below:
\[
\begin{align*}
\chi_N &= 10; \quad \eta = 1; \quad R_{max} = 1.4 R_N; \quad \beta_{max} = 0.45; \quad \chi_{max} = 7.4; \quad (\beta_R)_{max} = 2.3 \times 10^{-2} H_N \\
\chi_N &= 10; \quad \eta = 4; \quad R_{max} = 4.8 R_N; \quad \beta_{max} = 0.39; \quad \chi_{max} = 2.9; \quad (\beta_R)_{max} = 4.3 \times 10^{-4} H_N \\
\chi_N &= 10; \quad \eta = 20; \quad R_{max} = 48 R_N; \quad \beta_{max} = 0.19; \quad \chi_{max} = 1.2; \quad (\beta_R)_{max} = 1.2 \times 10^{-4} H_N 
\end{align*}
\] (69)
From (65) one can readily see that the first line in (65) belongs to a well closed universe, whereas the third line describes an almost flat model. It must be noted that we have made use of two equations, (51), and (57), both for the same function \( \beta(R) \). However differentiating (51) we can verify that these two equations are coexistent. Now, if one wants, he can obtain \( \beta(R) \) from the linear differential equation.
(51) (or alternatively from (53)) for a chosen pair of parameters \( \chi, \eta \). In this procedure the above given values of \( \beta_{\text{max}} \equiv \beta(R_{\text{max}}) \), and \( (\beta, \tau)_{\text{max}} \), serve as boundary conditions.

Let us consider equation (52). Integrating it one obtains

\[
 t + C = - (D + MR - R^2)^{1/2} - \frac{M - 2R}{2} \sin^{-1} \frac{M - 2R}{\sqrt{M^2 + 4D}},
\]

with \( C \) being a constant. Inserting (51), and (52), turning to \( z \) according to (52), and making use of (54), one can rewrite (51) as follows

\[
 t + C = - \frac{\eta}{H_N} \left\{ \left[ 1 + \frac{\eta^2}{\chi_N} (\chi_N - 1 + z) - z^2 \right]^{1/2} + \frac{1 + \eta^2}{2\chi_N} \sin^{-1} \frac{1 + \eta^2 - 2\chi_N z}{\left(1 + \eta^2\right)\sqrt{1 + \frac{4(\chi_N - 1)\chi_N}{1 + \eta^2}}} \right\}. \tag{71}
\]

With the help of (71) one can estimate the age \( \tau_N \) of the universe. For this purpose one neglects the duration of the radiation, and preradiation periods (cf. Weinberg 1973), (Israelit, & Rosen 1989)). One also must take into account the fact that the radius of the universe at the beginning of the dust-dominated period \( R_0 \) is much smaller than its present value \( R_N \). Further, making use of the values of \( R_{\text{max}} \) quoted in (58), one can also estimate the moment \( \tau_{\text{max}} \), when the universe reaches its maximum radius. Straightforward calculations give the following results.

\[
\begin{align*}
\chi_N = 10; \quad \eta = 1; \quad \Rightarrow \quad \tau_N &= 0.42H_N^{-1}; \quad \tau_{\text{max}} = 1.13H_N^{-1}, \\
\chi_N = 10; \quad \eta = 4; \quad \Rightarrow \quad \tau_N &= 0.50H_N^{-1}; \quad \tau_{\text{max}} = 14.2H_N^{-1}, \\
\chi_N = 10; \quad \eta = 20; \quad \Rightarrow \quad \tau_N &= 0.51H_N^{-1}; \quad \tau_{\text{max}} = 144H_N^{-1}.
\end{align*}
\]

Let us take for the present value of the Hubble constant (cf. (26))

\[
H_N = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \equiv (2 \times 10^{10} \text{ y})^{-1} \equiv 0.5 \times 10^{-28} \text{ cm}^{-1}.
\]

Then we obtain from (72) for the models considered above

\[
0.84 \times 10^{28} \text{ cm} \leq \tau_N \leq 1.02 \times 10^{28} \text{ cm}
\]

and turning to time units we can rewrite this as

\[
8.9 \times 10^3 \text{ y} \leq \tau_N \leq 10.2 \times 10^3 \text{ y}.
\]

6 Discussion.

In the present work dark matter is obtained from a conformally coupled scalar field. It is shown that this field may be treated as a source of scalar bosons and that it can also cause a global dark matter effect. In the process of creating dark matter the role of the conformally coupled scalar field is similar to that of the Dirac gauge function. However, whereas it was possible to choose the latter almost arbitrarily (cf. (Israelit, & Rosen 1995)), the behavior of the conformally coupled field depends on the density and pressure of ordinary (luminous) matter in the universe (cf. equations (14), (21), (43))

Two approaches are considered, a local, and a global one. In the first case we assumed that the globally homogeneous and isotropic universe contains small regions of local inhomogeneities, wheer the scalar function \( \beta \) may depend on spatial coordinates. This dependence leads to a Klein-Gordon equation for \( \beta \) (eq. (23)), so that from the quantum mechanical standpoint one obtains an ensemble of neutral and spinless bosons. During the radiation period these particles have zero mass, and in the dust-dominated universe they are extremely light, so that these particles constitute dark matter pervading all of the cosmic space.

In the global approach (section 5.) the universe is completely homogeneous and isotropic, and the scalar function \( \beta(t) \) generates a global dark matter effect. In order to specify cosmological models, two parameters are introduced, the first, \( \eta \) characterizing the degree of flatness of the universe, and a second, \( \chi \) being the ratio of the total matter density to that of ordinary (luminous) matter. It is assumed that at present \( \chi_N = 10 \), and for various values of \( \eta \) the present age of the universe, its maximum radius, as
well as $\tau_{\text{max}}$, the moment when the expanding universe will achieve its maximum radius are calculated. From (69), (72-75) one can conclude that the model with $\chi_N = 10$ and $\eta = 20$ (an almost flat universe) agrees with nowadays cosmological observations (Bolte, & Hogan 1995), (Chaboyer, Demarque, Kernan, & Krauss 1996). For this model we have: $R_{\text{max}} = 48 R_N$; $\tau_N = 10.2 \times 10^9 y$; $\tau_{\text{max}} = 282 \tau_N$. In the expanding universe, during the dust-dominated period, the ratio $\eta$ (cf. (67)) decreases by a factor of about 1000, hence this dark matter form will not play an essential role in the late stages of the expansion phase, but it was important in the early stages of the dust universe. So it might have affected growing processes of galactic formations at the beginning of the dust era (Israelit, Rose, & Dehnen 1994,a,b).

The conformally coupled dark matter considered in the present paper is to be regarded as a dark matter form connected closely to the Weyl geometry. Assuming that the space-time of the universe has Weylian properties (in addition to the well known Riemannian properties) we can derive several geometrically based dark matter forms (cf. (Israelit 1995)): a) the dark matter considered in this work, b) a gas of heavy weylons (Israelit, & Rosen 1992, 1994), c) a global dark matter effect from the Dirac gauge function (Israelit, & Rosen 1995). All these forms may be fitted into cosmological models, giving agreement with observation.

Acknowledgments

The author takes this opportunity to express his cordial thanks to Professor HEINZ DEHNEN for interesting discussions.

REFERENCES.

Blome, H.J., Priester, W., & Hoell, J.: 1995, Currents in High Energy Astrophysics, p.-p. 301-312,

Sharpiro et al. (eds), (Kluver Academic Publisher).

Bolte, M, & Hogan, C., J.: 1995, Nature, 376, 399.

Chaboyer, B., Demarque, P., Kernan, P.J., & Krauss, L., M.: 1996, Science, 271, 957.

Dirac, P., M., A.: 1973, Proc. R. Soc. London A 333, 403.

Israelit, M., 1995, Proceedings of the Third Alexander Friedman International Seminar on Gravitation and Cosmology, Gnedin et al. (eds) (Friedmann Laboratory Publishing, St. Petersburg) p.-p. 126-143.

Israelit, M., Rose, B., & Dehnen, H.: 1994 a, Astrophys. Space Sci, 219, 171.

Israelit, M., Rose, B., & Dehnen, H.: 1994 b, Astrophys. Space Sci, 220, 39.

Israelit, M., & Rosen, N.: 1989, Astrophys. J., 342, 627.

Israelit, M., & Rosen, N.: 1992, Found. Phys., 22, 555.

Israelit, M., & Rosen, N.: 1994, Found. Phys., 24, 901.

Israelit, M., & Rosen, N.: 1995, Found. Phys., 25, 763.

Peebles, P., J., E., 1993, Principles of Physical Cosmology, (Princeton University Press, Princeton, New Jersey).

Priester, W., Hoell, J., & Blome, H., J.: 1995, Comments Astrophys., 17, No.6, 327.

Rosen, N.,: 1982, Found. Phys., 12, 213.

Synge, J., L.: 1966, Relativity, the General Theory, (North-Holland Publ. Comp., Amsterdam)

CH. IV-5, and Appendix B.

Tremain, S.: 1992, Phys. Today, 45, No.2, 28.

Trimble, V.: 1987, Ann. Rev. Astron. Astrophys., 25, 425.

Turner, M., S.: 1991, Physica Scripta, T 36, 167.

Tyson, A.: 1992, Phys. Today, 45, No.6, 24.

Weiberg, S.: 1972, Gravitation and Cosmology, (Wiley, New York).

Weyl, H.: 1919, Ann. Phys. (Leipzig), 59, 101.