RRT* Combined with GVO for Real-time Nonholonomic Robot Navigation in Dynamic Environment

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Abstract— Challenges persist in nonholonomic robot navigation for dynamic environments. This paper presents a framework for nonholonomic robot navigation in dynamic environment based on the model of Generalized Velocity Obstacles (GVO). The idea of velocity obstacles has been well studied and developed for obstacle avoidance since proposed in 1998. Though proved to be successful, most studies assume equations of motion to be linear, which limit the application to holonomic robots. In addition, more attention has been paid to the immediate reaction of robots while advance planning has always been ignored. By applying GVO model to differential drive robots and combining it with RRT*, we reduce the uncertainty of robot trajectory, thus further reduce the concerned range, and save both computation time and running time. By introducing uncertainty for the dynamic obstacles by Kalman filter, we reduce the risk of considering obstacles to uniformly move along a straight line and guarantee the safety. Special concern has been given to the path generation, including curvature check, making the generated path feasible for nonholonomic robots. We experimentally demonstrated the feasibility of the framework.

I. INTRODUCTION

As robots are increasingly involved in human daily life, it is common to see robots working around humans. They are assigned with different tasks, serving as tourist guide, patrolman or autonomous car, etc. In most scenarios, these robots are required to navigate to target places with the presence of moving humans or other objects. To ensure the safety of both humans and robots and also enable robots to work efficiently, suitable control strategies applicable for the navigation tasks need to be developed. It requires robot to move towards the target in a short time and can avoid either static or dynamic obstacles observed from the sensors, which indeed involves with efficient path planning and valid obstacle avoidance. Though these two topics have been well researched, currently, there is no ideal solution dealing with the navigation problem in dynamic environment. The difficulty attributes to the uncertainty of environment, particularly for the dynamic objects.

A. Related work

Existing works that target at solving the complex obstacle avoidance problem can be roughly classified into two categories, i.e., model-based and learning based approaches. As the study of deep learning keeps heating up, learning based approaches [1][2] have been put forward. The key idea is to mimic or learn human decision policies when they are dealing with the same problem and inverse reinforcement learning [3][4] have played a important role in advancing this progress. Though these methods have already shown some results, it is hard to explain the scheme, and the training process is time and data consuming. Also, it is less stable due to the stochasticity of human’s behavior. Different from the learning methods, model-based methods rely on reasonable geometric rules and are considered more computationally efficient. One of the most representative work is Velocity Obstacle(VO)[5]. Proposed in 1998, it utilizes the collision cones to define the region of velocity that will cause collision at some time in the future. Keeping selecting velocity outside the cone ensures a safe navigation. After that, different variations are proposed trying to solve problems encountered for different scenarios. As the VO model holds the assumption that the dynamic obstacles move passively and will not react to the robot, which is not true for multiple agents, reciprocal obstacle avoidance(RVO)[6] modifies the position of collision cone by assuming every robot share half of the responsibility of collision avoidance. To solve the "reciprocal dance" problem in [6], the hybrid reciprocal velocity obstacle(HRVO)[7] enlarges the area of one side of the cone. And optimal reciprocal collision avoidance(ORCA)[8] defines the set of safe velocities to be a half plane with respect to VO and guaranteed local-free motion for large number of robots. As these methods are limited to holonomic robots, various methods have been proposed to extend them to differential-drive[9][10], car-like robots[11]. To be more general, Wilkie et al.[12] define the velocity cone as GVO and make it general and applicable for robots with different

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kinematic constraints. Though great efforts have been put in the modelling of obstacle avoidance for robots with different constraints, humans are always treated as simple moving objects and only the instantaneous speed are considered. Kim et al.[13] combined the human trajectories prediction with the obstacles avoidance model and have shown good results in simple outdoor environment.

Generally, path planning aims at finding a curve starting from a start node to the target. And in this case, we are dealing with local path planning problem with a near target and are required to give a path with high resolution. Various methods have been proposed and can be divided into complete and probabilistically complete algorithms. For probabilistically complete approaches, i.e. sampling based method, the most representative methods are rapid exploring random trees(RRT)[14] and Probabilistic Road Maps(PRMs)[15]. And one of the most famous variation is RRT*[16], which adds a extra “rewiring” step to the RRT tree and converges towards an optimal solution. Other methods that guarantee optimality are based on graph search and called complete algorithms, including A*[17] which combines Best-First Search and Dijkstra algorithm[18] to find the optimal solution by searching among all possible paths. Dynamic A* Search(D*)[19] focuses on the cost updates to minimize state expansions and further reduces computational costs. Other methods including some local planning algorithms are represented by Dynamic Window Approach(DWA)[20] and Vertical Field Histogram(VFH)[21]. For DWA, it generates acceptable velocity by sampling and evaluates trace by heading angle error. As it treats all obstacles equally and shows no concern for the motion state of obstacles, it is limited to applications in static environment.

B. Contribution

In this paper, we proposed a scheme for real-time non-holonomic robot navigation in dynamic environment. As shown in Fig.1(b) by applying GVO model to differential drive robots and introducing uncertainty for the dynamic obstacles by Kalman filter, we reduced the risk of considering obstacles to move along a straight line with current speed. By combining it with RRT*(shown in Fig.1(a)), we reduced the uncertainty along robot trajectory, thus further reduced the concerned range, saving both computation time and running time. Besides, given a reference path, the navigation problem can be solved efficiently in a more clustered complex environment.

Contributions of this paper should be addressed as follows:

- We applied GVO to differential drive robots and also proved the generality of this model.
- We combined GVO with RRT*, and successfully demonstrate collision avoidance in both dynamic and static environment. The proposed approach proved to be more robust to complicated environments than GVO alone.
- We introduced uncertainty of human state by Kalman filter and guarantee safer navigation.
- Some parts of RRT* and GVO are specially designed for more applicability in this case.

C. Organization

The rest of the paper is organized as follows. Firstly, we present the framework of the navigation system. Then each part of the system will be introduced in detail, including path generator, path follower and obstacle avoidance. After that, the experimental platforms will be introduced and results will be analyzed and evaluated.

II. System

A. Framework

The whole system consists of robot trajectory generating, dynamic object status estimation, obstacle avoidance and path-following control. At the beginning, objects defined by laser points will be clustered and classified into static and dynamic obstacles. Only static points will be considered for path generation. Then a collision free and smooth path is generated by RRT* and spline interpolation. After that cluster of human will be registered and the position will be treated as the observation input of a Kalman filter. The Kalman filter is updated every cycle and both moving speed and position can be directly obtained. If the obstacles are far away from the robot, a path-following controller will take control and robot will follow the generated path. Otherwise, obstacle avoidance will dominate and controller will only offer a reference action. All the obstacles will be treated as GVOs. By estimating the status of robot and obstacles after taking a certain action within a certain time, a set of actions that satisfy the collision condition can be acquired. The final action will be decided by the similarity to the reference velocity. The framework was shown in Fig.2 where state transition happened according to the data published in different topics.

B. Path generator

The path generator was mainly based on RRT* algorithm and extra constraints have been added to it to get satisfied solutions. As shown in Algorithm 1, to begin with, static obstacles will be extracted by DBSCAN clustering because only static obstacles will be considered for path generation. Different with RRT*, informed RRT* [22] induces heuristic-biased sampling, which increases the sampling probability inside the heuristic sampling domain while reduces the
probability outside. For informed RRT*, the heuristic domain is an ellipse with its shape defined by the distance between the start point and goal as well as the given minimal distance cost. Here, similar things happened but all the samplings were done inside the ellipse as we have got the distance cost from the previous RRT* algorithm and ascertain that one solution can be obtained inside it. Sampling density of the RRT* is set to be much smaller than the latter. Besides the sampling method, collision check and curvature check were introduced to better satisfy the motion condition of nonholonomic robot. After that, spline interpolation was conducted to get a smooth path. Examples of generated path are shown in Fig.

Algorithm 1 Path generator

Classify all laser points into static obstacles $Ob_s$ and dynamic obstacles $Ob_d$ by DBSCAN clustering

Generate one trace by RRT* with low sampling density and take the distance cost as the minimal distance cost obtained $c_{best}$.

Assume robot position as $P_{robot} = (P_{x}, P_{y})$, goal position as $P_{goal} = (P_{xg}, P_{yg})$, the ideal minimal distance: $c_{min} = \sqrt{(P_{xg} - P_{x})^2 + (P_{yg} - P_{y})^2}$

for numnodes <= area * density * numnodes do

Generate one potential node $A$ by sampling inside a ellipse:

$$
\begin{bmatrix}
    x \\
    y \\
    \theta
\end{bmatrix}
= \begin{bmatrix}
    P_{xg} - P_{x} \\
    P_{yg} - P_{y}
\end{bmatrix}^{-1} \begin{bmatrix}
    x_{0} \\
    y_{0}
\end{bmatrix} + \begin{bmatrix}
    \frac{P_{yg} - P_{y}}{c_{min}} \\
    \frac{P_{xg} - P_{x}}{c_{min}}
\end{bmatrix} \begin{bmatrix}
    \text{rand} \\
    0
\end{bmatrix}
\sqrt{\frac{1}{2} c_{best} - c_{min}}$$

where $x_{0}^2 + y_{0}^2 = 1$

Adjust $A$ to ensure that $A$ is close to at least one node in the accepted node set.

Get the parent node of $A$, marked as node $B$ and the parent node of $B$, marked as node $C$

if min(dis($AB, Ob_s$)) > dis$_{th}$ and $|\text{angle}(AB) - \text{angle}(BC)| < \text{angle}_{th}$ then

Add new node $A$

end if

Do curvature check and collision check during rewire

end for

Track back from the closest node to goal to start point

Generate trace points by spline interpolation

C. Path follower

After path generating, trace will be passed to a path follower. It was a close-loop controller proposed by Klancar et al.[23] to make the robot move along a reference path. The robot architecture can be seen in Fig.4. For differential drive robot, the motion equations are described in Eq(1) where $v$ and $\omega$ are forward and angular velocities, $\theta$ is the forward direction of robot in the world frame. The error between the real pose $(x, y, z)$ and the reference pose $(x_r, y_r, \theta_r)$ in the frame of the real robot can be calculated by Eq(2). Multiplying the error by gain matrix $K$, we can get the feedback $(u_{e1}, u_{e2})$ (shown in Eq(3)). The final output actions $(u_1, u_2)$ can be obtained from reference actions $(u_{e1}, u_{e2})$ and $(u_1, u_2)$. The $K$ matrix depends on the reference actions. And $\xi$ and $g$ have a large influence on the result. In experiments, we found that with large $g$ values, robot will go in a zigzag, as the controller became too sensitive to the error. In this case, error was defined by the closest distance from the current robot position to the spline, and the reference actions were given by looking several steps forward. If all the obstacles are out of collision range, this controller will take the control and send the velocity command $(v, \omega)$. Otherwise, output of the controller will serve as reference actions and be passed to obstacle avoidance part.

\[
\begin{bmatrix}
    \dot{x} \\
    \dot{y} \\
    \dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
    \cos\theta & 0 & 0 \\
    \sin\theta & 0 & 1 \\
    0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix}
\] (1)

\[
\begin{bmatrix}
    e_1 \\
    e_2 \\
    e_3
\end{bmatrix}
= \begin{bmatrix}
    \cos\theta & -\sin\theta & 0 \\
    \sin\theta & \cos\theta & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x - x_r \\
    y - y_r \\
    \theta - \theta_r
\end{bmatrix}
\] (2)

\[
\begin{bmatrix}
    u_{e1} \\
    u_{e2}
\end{bmatrix}
= \begin{bmatrix}
    -k_1 & 0 & 0 \\
    0 & -\text{sign}(u_{e1}) & k_2 \\
    0 & k_3 & 0
\end{bmatrix}
\begin{bmatrix}
    e_1 \\
    e_2 \\
    e_3
\end{bmatrix}
\] (3)

\[
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix}
= \begin{bmatrix}
    \cos\theta & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    u_{r1} \\
    u_{r2} \\
    u_{r3}
\end{bmatrix}
\] (4)

\[
k_1 = k_3 = 2\xi \sqrt{u_{r1}(t)^2 + g u_{r1}(t)^2}
\] (5)

\[
k_2 = g \cdot |u_{r1}|
\] (6)
D. Obstacle avoidance

For obstacles avoidance, we learned from the generalized velocity obstacle model and applied it to the differential drive robot. GVO model was proposed to solve the real-time navigation problem in dynamic environments of car-like robots. The key idea is to find out the acceptable actions that will not cause any collision in the near future. Different with most of the VO models, it has no requirement for linear motion of robots, which makes it convenient to extend to various nonholonomic robots. Although the paper only focuses on dynamic obstacles, in theory it is also applicable for static environment, which make it possible for applications in complex environment. As shown in Algorithm 3, firstly, the laser data was divided into dynamic and static obstacles. Provided with a reference path, the range of static obstacles considered can be suppressed. After getting the human pose as observation, Kalman filter will be updated and then both position and velocity of human \( [p_x, p_y, v_x, v_y]^T \) can be obtained. Given a sampling space, which is mainly confined by maximum forward velocity and angular velocity of robot, one potential action was generated. Then robot pose at time \( t \) can be derived as shown in Eq[7] and Eq[8]. Different obstacles were handled differently. Relative position of static obstacles \( P_{ob_s}(t) \) is certain if error of the robot position can be omitted and time threshold \( t_{s_{th}} \) is small (will not induce large odom error). So the minimal distance between robot and obstacles given \( t \in [0, t_{s_{th}}] \) can be derived easily. For human, as there exist both pose uncertainty and velocity uncertainty, it can not be reduced to a simple linear motion model. In this paper, human pose at time \( t \), \( P_{human}(t) \), was treated as a sum of two Gaussian and was also normally distributed with a distribution of \( N(\mu_p + \mu_s t, \Sigma_p + t^2 \Sigma_s) \). It is straightforward as a longer time will increase the uncertainty of predicted human pose. To ensure the safety of human, we set a threshold and when the probability goes high enough for robot to collide, time will be record and the action will be rejected. And after sampling for \( n \) times, we will get two sets. If accepted actions set is not empty, difference between desired actions and proposed actions will be the rule of choosing the final action. The most common one is 2-norm, it was also used in the paper[12]. If there is no good choice, instead of stopping and waiting for next loop, the corresponded actions with maximum time will be chosen. When the time to collide is less than \( t_{c_{th}} \), the robot will stop. And all the thresholds in this model depend on the kinematic constraints of robot.

\[
x(t) = \frac{\theta}{\omega} \sin(\theta + \omega t) - \frac{\theta}{\omega} \sin(\theta) \tag{7}
\]

\[
y(t) = -\frac{\theta}{\omega} \cos(\theta + \omega t) + \frac{\theta}{\omega} \cos(\theta) \tag{8}
\]

III. EXPERIMENTAL RESULTS

A. Platform

To show the performance of the planner, navigation in both virtual environment and real environment were tested. The virtual environment was built in V-rep[24]. Walls and blocks were put inside as the static obstacles and a walking man was regarded as the dynamic obstacles. It walked towards a random target with simple path planning and can not avoid obstacles timely. For both environments, we used Turtlebot as the mobile ground platform, equipped with a SICK TiM56[1] 2D laser range finder(LRF). The experiment configuration was shown in Table[1]. To capture the pose of

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Algorithm 2 GVO model

Get laserscan and human pose data
Get desired actions \( u^* \) from path follower
Classify laser points into \( Ob_s \) and \( Ob_d \)
Update Kalman filter and get estimated human pose \( (p_x, p_y) \) and velocity \( (v_x, v_y) \)
Get static obstacles \( \{Ob_{s1}, Ob_{s2},...\} \) within distance \( dis_{sta} \)
for \( i = 0 \) to \( n \) do
Sample one action \( u = (w, v) \) from action space
Get estimated robot pos \( P_{robot}(t) \) at \( t \)
for all dynamic obstacles \( Ob_d \) and static obstacles \( Ob_s \) do
Let \( D_s(t) \) be the distance between \( Ob_s \) and robot at time \( t \), \( t < t_{s_{th}} \)
\( t_{min} = \text{min}(\text{argmin}(D_s(t)), t_{min}) \)
if \( D_s(t_{min}) < \text{radius}_{robot} \) then
\( free = False \)
e end if
Let \( f_d(t) \) be the normalized PDF value of human position distribution at \( P_{robot}(t) \), \( t < t_{d_{th}} \)
while \( t < t_{d_{th}} \) do
if \( f_d(t) > p_h \) then
\( free = False \)
\( t_{min} = \text{min}(t_{min}, t) \)
break
end if
\( t = t + \delta t \)
end while
end for
if \( free \) then
Add \((w, v)\) to accept actions set \( A \)
else
Add \((w, v)\) to reject actions set \( R \)
end if
end for
if \( A \neq \emptyset \) then
\( u = \text{argmin}(f(u - u^*)) \)
else
\( u = \text{argmax}(t_{min}(u)), u \in R \)
if \( t_{min} < t_{c_{th}} \) then
\( u = 0 \)
ed if
end if
Return \( u \)

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1https://www.sick.com/de/en/detection-and-ranging-solutions/2d-lidar-sensors/tim5xx/tim561-2050101/p/p369446
human, we test the whole system inside a motion capture system OptiTrack\(^2\). The test platform and scene can be seen in Fig.5(a) and Fig.5(b).

| TABLE I | TEST CONFIGURATION |
|---------|---------------------|
| Components | Parameters | Comments |
| Turtlebot | \(v_{\text{max}} = 1.6\text{m/s}, \omega_{\text{max}} = \pi\text{rad/s}\) | |
| Sick 2D LRF | FOV=270\(^\circ\), angle resolution=0.33\(^\circ\), ranges=[0.05m,10m], scan freq=15HZ | can be used outdoor |
| Computer | Intel NUC Kit NUC5i7RYH | |
| Lithium battery | 12 V and 19V output | Power supply |

Fig. 5. Test platform and test environment. To use the motion capture system, some ball reflectors are attached to the platform and human for localization. The right picture shows the test scene for dynamic obstacle avoidance.

B. Evaluation

To measure the performance of the navigation system, we compare the performance of methods in terms of average navigation time, successful rate, etc. Here, GVO model without path planning were chosen for comparison. It aims at evaluating the effect of including a path generator, as in most cases, the VO model has no plan for the velocity control with its reference velocity simply set towards the target. Here, the compared GVO model was also combined with state estimation of moving obstacles and the controller mentioned in the paper for velocity control. Four different scenes are evaluated in virtual environment. For better visualization, both goals of robot and human were shown as columns. And to test the robustness of the scheme, extreme cases where human will definitely collide with robot if no strategy was adopted were tested, including the scene where both the start point and goal of human and robots are in the same line and the cross scenario. Complexity of the task was increased from scene1 to scene4(shown in Fig.6). We tested 10 times for each scene with two different methods. We also test the algorithm in real environment and record the robot trace for analysis.

C. Results

The results were shown in Fig.7 and Fig.8. The bar graph shows that both GVO and GVO combined with RRT* finished the task successfully in a dynamic environment given state estimation of human by Kalman filter. And increasing the complexity of the static scene will greatly influence the performance of GVO model. As it is shown in Fig.8 both finishing time and time fluctuation will increase. Though RRT* took some time to generate the path, which makes the time a little bit longer in simple environments. It was stable while increasing the number of obstacles. Since human in the simulation environment can not react to collision immediately, the results are considered to have shown the worst situation.

![Fig. 6. Test scenes in virtual.](image)
![Fig. 7. Successful rate in different scenes for GVO only and GVO+RRT*.](image)
![Fig. 8. Average time spent in different scenes for GVO only and GVO+RRT*.](image)

We also test the whole strategy in real environment. As shown in Fig.9 trajectories are smooth in a simple dynamic environment. When more static obstacles appeared, GVO+RRT* keeps a smooth trajectories, while robot controlled by GVO kept rotating and changing direction to avoid collision. Snapshots of the collision avoidance process were shown in Fig.10.

\(^2\)http://optitrack.com/products/prime-41/
Fig. 9. Robot navigation test in real environment. Robot trace1 stands for robot navigation based on GVO + RRT* and human trace1 stands for the human activity during the navigation. Robot trace2 represents robot navigation based on GVO and target driven planning and the corresponding human trace is human trace2.

Fig. 10. Snapshots of the collision avoidance process of robot running GVO+RRT*. The red arrow indicates the moving direction of robot while the green one indicates the moving direction of human.

IV. CONCLUSIONS

In this paper, we have demonstrated a scheme for real-time nonholonomic robot navigation in dynamic environment, which combined RRT* and GVO to deal with path planning and obstacle avoidance. We also induced a Kalman filter to model human motion. The navigation scheme was proved to be more robust to complicated environments than GVO alone.

Future work includes integrating human 3D pose estimation by RGB images or 3D lidar. As it is applicable for car-like robots, further extending to autonomous driving is promising and has great significance.

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