Galactic Collapse of Scalar Field Dark Matter

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We present a scenario for galaxy formation based on the hypothesis of scalar field dark matter. We interpret galaxy formation through the collapse of a scalar field fluctuation. We find that a cosh potential for the self-interaction of the scalar field provides a reasonable scenario for galactic formation, which is in agreement with cosmological observations and phenomenological studies in galaxies.

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In the last years, the quest concerning the nature of the dark matter in the Universe has received much attention and has become of great importance for understanding the structure formation in the Universe. Some candidates for dark matter have been discarded and some others have recently appeared. The standard candidates of the Cold Dark Matter (CDM) model are axions and WIMP’S (Weakly Interacting Massive Particles), which are themselves not free of problems. Axions are massive scalar particles with no self interaction. In order for axions to be an essential component of the dark matter content of the Universe, their mass should be \( m \sim 10^{-5} eV \). With this axion mass, the scalar field collapses forming compact objects with masses of order \( M_{crit} \sim 0.6 m^2_{pl}/m \sim 10^{-6} M_\odot \), which corresponds to objects with the mass of a planet. Since the dark matter mass in galaxies is ten times higher than the luminous matter, we would need tenths of millions of such objects around the solar system, which is clearly not the case. On the other hand, there are many viable particles with nice features in super-symmetric theories, such as WIMP’S. However, since these candidates behave just like standard CDM, they can not explain the observed scarcity of dwarf galaxies and the smoothness of the galactic-core matter densities, since high resolution numerical simulations with standard CDM predict an excess of dwarf galaxies and density profiles with cusps. This is the reason why we need to look for alternative candidates that can explain both the structure formation at cosmological level, the observed amount of dwarf galaxies, and the dark matter density profile in the core of galaxies.

In a recent series of papers, we have proposed that the dark matter in the Universe is of a scalar field nature with a strong self-interaction. The scalar field has been proposed as a viable candidate, since it mimics standard CDM above galactic scales very well, reproducing most of the features of the standard Lambda Cold Dark Matter (ΛCDM) model. However, at galactic scales, the scalar field model presents some advantages over the standard ΛCDM model. For example, it can explain the observed scarcity of dwarf galaxies since it produces a sharp cut-off in the Mass Power Spectrum. Also, its self-interaction can, in principle, explain the smoothness of the energy density profile in the core of galaxies. Nevertheless, the main problem when a new dark matter candidate is proposed is the study of the final object that would be formed as a result of a gravitational collapse.

The formation of galaxies through gravitational collapse of dark matter is not an easy problem to understand. A good model for galaxy formation has to take into account all the observed features of real galaxies. For example, it seems that many disc galaxies contain a black hole in their center, but others do not. Typical galaxies are spiral, elliptical or dwarf galaxies (irregular galaxies may be galaxies still evolving). In most spiral and elliptical galaxies the luminous matter extends to \( \sim 10 – 30kpc \), and the total content of matter (including dark matter) is of the order of \( 10^{10} – 10^{12} M_\odot \), with about 10 times more dark matter than luminous one. The central density profile of the dark matter in galaxies should not be cusp. Even though the luminous matter represents only a small fraction of the total amount of matter in galaxies, it plays an important role in galaxy formation and stability. On the other hand, it is still not well established if the mass of the central black hole and the mass of the halo are correlated, etc.

There are some ideas in this respect when dealing with a scalar field. It is known that the final stage of a collapsed scalar field could be a massive object made of scalar field particles in quantum coherent states, like boson stars (for a complex scalar field) or oscillatons (for a real scalar field). It is thus important to investigate whether the scalar field would collapse to form structures of the size of galaxies and provide the correct properties of any galactic dark matter candidate, like growing rotation curves and appropriate dark mat-
We now introduce the first order variables \( \Psi = \Phi/r \) and \( \Pi = a\Phi/r/\alpha \). Using these new variables, the Hamiltonian constraint becomes

\[
\frac{a_r}{a} = \frac{1}{2}\frac{a^2}{r^2} + \frac{\kappa_0 r}{4} \left[ \Psi^2 + \Pi^2 + 2a^2 V \right],
\]

and the polar-areal slicing condition takes the form:

\[
\frac{\alpha_r}{\alpha} = \frac{a_r}{a} + \frac{a^2 - 1}{r} - \kappa_0 r a^2 V.
\]

All other components of Einstein’s equations either vanish, or are a consequence of the last two equations.

The Klein-Gordon (KG) equation now reads

\[
\Phi,_{tt} - \frac{1}{r^2} \left( \frac{r^2 \alpha \Phi}{a} \right),_r - \alpha \frac{dV}{d\Phi} = 0.
\]

Equations (10) and (12) form the complete set of differential equations to be solved numerically. For numerical purposes, the evolution equation for \( \Pi \) above is further transformed into the equivalent form:

\[
\Pi, = 3 \frac{d}{dr^3} \left( r^2 \alpha \Phi \right) - a \alpha \frac{dV}{d\Phi}.
\]

Notice that the first term on the right hand side of this equation includes now a first derivative with respect to \( r^3 \) (and not a third derivative). The reason for doing this

\[
\begin{align*}
\gamma \frac{\kappa_0 r}{4} \left( \Psi^2 + \Pi^2 + 2a^2 V \right),
\end{align*}
\]

and

\[
\begin{align*}
\alpha_r/\alpha = a_r/a + a^2 - 1/r - \kappa_0 r a^2 V.
\end{align*}
\]
transformation has to do with the numerical regularization near the origin of the $1/r^2$ factor in equation (11) above (see Ref. [17]).

In order to deal with non-dimensional units, we define $x = l r$. A natural scale for the potential is given by $l^{-1} = 1/\sqrt{\kappa_0 V_0} = 12 \, \text{pc} = 40 \, \text{yr}$. The parameter $l$ also gives the time scale $\tau = tl$. The scalar field has an initial Gaussian profile

$$\sqrt{\kappa_0} \Phi(x, t = 0) = Ae^{-x^2/s^2},$$

with $A$ the amplitude and $s$ the width of the Gaussian. The physical properties describing the state of the system are the energy density of the scalar field $\rho_s = (m^2_{Pl} l^2/8\pi) \rho_s$, with $\rho_s$ the dimensionless quantity

$$\rho_s = 1/2a^2 (\Psi^2 + \Pi^2) + V,$$

and the integrated mass

$$M(x) = m^2_{Pl} 2l \int_0^x \rho_s(X) X^2 dX.$$

Some details about our numerical implementation are in order. To integrate the KG equation numerically we use a method of lines with standard centered second order finite differences in space, and a third order in time integrator. At the outer boundary we impose a condition for simple outgoing radial waves. We deal with the singularity at $x = 0$ by straddling the origin and imposing the adequate parity conditions for each function on an auxiliary point at $x = -\Delta x/2$. The ordinary differential equations for $a$ and $\alpha$ are solved using a second order Runge-Kutta method. The third order in time integration has been chosen to reduce as much as possible the numerical dissipation in our code, which we have found to be crucial in order to obtain reliable results for the long time runs we have studied.

The numerical simulations suggest that the critical mass for the case considered here, using the scalar potential (10), is approximately

$$M_{\text{crit}} \simeq 0.1 \frac{m^2_{Pl}}{\sqrt{\kappa_0 V_0}} = 2.5 \times 10^{13} M_\odot.$$

The results of the numerical simulations are as follows. Essentially, we have found three different types of behavior for the scalar field collapse. In the first case, a generic feature is that scalar field distributions with an initial mass slightly larger than the critical mass collapse very violently and form a black hole. In the second type of behavior, fluctuations with an initial mass significantly smaller than the critical mass can not form stable oscillations: the scalar field is completely ejected out as the system evolves [18]. The third behavior corresponds to a case where a fraction of the initial density is spread out, leaving an oscillating object that appears to be stable. This situation happens in a narrow window of initial conditions, between $0.05 - 1 \times M_{\text{crit}}$ [18].

In Fig. 1 we show the evolution in time of the maximum value of the energy density. The parameters for the initial configuration are $A = 0.01, s = 2.0$ (see (12)), which correspond to an initial mass of $M_i = 3.25 \times 10^{12} M_\odot$. See also Fig. 3.

![FIG. 1. Temporal evolution of the maximum value of the energy density. The parameters for the initial configuration are $A = 0.01, s = 2.0$ (see (13)), which correspond to an initial mass of $M_i = 3.25 \times 10^{12} M_\odot$. See also Fig. 3.](image1)

In Fig. 2 we show the evolution in time of the maximum value of the energy density for an initial configuration that results in the formation of an oscillaton. In this case, we have taken an initial configuration with $A = 0.01, s = 2.0$, which implies an initial mass of $M_i = 3.25 \times 10^{12} M_\odot$. For this run we used $\Delta x = 0.005, \Delta t = \Delta x/10$, and 10,000 grid points, which puts the outer boundaries at $x = 50$. The run was followed until $t = 2000$, some 40 light crossing times.

![FIG. 2. Energy density of the object at $t = 2000$. In the plot we show $x^2 \rho_s(x)$ using a logarithmic scale. The solid line corresponds to a run with $\Delta x = 0.005$ and the boundaries at $x = 50$, the dashed line to a run with $\Delta x = 0.01$ and boundaries at $x = 50$, and the dotted line to a run with $\Delta x = 0.01$ and boundaries at $x = 100$.](image2)

In Fig. 3 we plot the energy density $x^2 \rho_s(x)$ of the object at $t = 2000$, using a logarithmic scale, for three different runs: the run mentioned above with $\Delta x = 0.005$ and boundaries at $x = 50$, and two runs with half the resolution ($\Delta x = 0.01$) and with the boundaries at $x = 50$ and $x = 100$ respectively. Notice how the the two runs with
the boundaries at the same location coincide very well throughout the computational domain. The run with the boundaries twice as far agrees well with the other runs for \( x < 20 \), but differs significantly outside where \( x^2 \rho_s < 10^{-6} \). This indicates that at such low levels, the solution is dominated by boundary effects. Our boundary condition is clearly introducing numerical noise at such late times.

Figure 3 shows the integrated mass for the initial and final \( (t = 2000) \) stages of the evolution. A small drop of \( \sim 0.5\% \) in the total integrated mass can be observed, but convergence tests suggest that most (if not all) of this mass loss is caused by a small amount of numerical dissipation still present in our numerical method. This implies that the system does not radiate any significant amount of energy during the time of the simulation, which indicates that the object is very stable.

From the cosmological point of view, the narrow window of initial conditions means that not all fluctuations will collapse into stable objects. Moreover, the collapsed objects will have masses of the same order of magnitude \( M_{\text{final}} \sim 10^{12} M_\odot \), as it seems to be precisely the case for galaxies.

Summarizing, from the results of the numerical simulations of the collapse of the real scalar field with a cosh potential we find many similarities with the structure of the halos of galaxies. The scalar field density profile is not singular at the center. This fact, and the values of the final masses obtained using the cosmological values for the parameters of the self-interaction potential, could correspond to objects like realistic galaxies. Moreover, it is in agreement with the observational constraints related to the phenomenological maximum galactic mass pointed out by Salucci and Burkert [13]. Therefore, we expect that fluctuations of this scalar field, due to Jeans instabilities, will in general collapse to form objects of the order of the mass of the halo of a typical galaxy.

We have shown before [15] that the SFDM model could be a good model for the universe at cosmological level, here we see that the scalar field could also be a good candidate for the dark matter content of individual galaxies (as suggested in [9,16]).

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