Abstract

The postulate of the preferred reference frame in which the signal propagation is governed by retarded causality is a must for any theory of faster-than-light particles and signals. Such a system does exist and is the comoving system of the relativistic cosmology. Restrictions imposed by the causality conservation can be expressed via a causal Θ function assumed to be acting in both, the classical and quantum field theories of tachyons. A Lorentz-covariant introduction of this Θ function, which ensures a causal behaviour of real tachyons (asymptotic tachyonic states) preventing the appearance of casual loops constructed with the use of faster-than-light particles and signals, into the tachyon quantum field operators is suggested.

Keywords: tachyons, causality conservation, preferred reference frame, tachyon vacuum

1. Introduction

During the late fifties and sixties of the last century a possibility of the introduction of a concept of faster-than-light particles into the quantum field theory was considered in several papers [1, 2, 3, 4, 5, 6]. The particles were called tachyons, from the Greek word ταχιστός meaning swift [5]. These considerations have generated a strong critical response based on the generally accepted principles of causality [7, 8, 9], vacuum stability [10] and unitarity [11, 12]. A consensus was achieved that within the special relativity and the canonical quantization procedure faster-than-light particles are incompatible with those principles.

Nevertheless, in ref. [13] an approach to a tachyon theory based on the requirement of the causality non-violation was suggested. This approach, which solves also two other problems of tachyon models mentioned above, replaces the standard demand to a tachyon theory to be Lorentz-invariant by a softer condition, requiring the theory to be formulated in a Lorentz-covariant form, which means an equivalent mathematical description of tachyon behaviour in all reference frames. Then the causality violation by tachyons is removed by an introduction of a preferred reference frame in which the events of tachyon exchange are ordered by retarded causality, which ensures the absence of causal loops in any frame. It was shown in [13] that the preferred reference frame considered there should be associated with the so called comoving frame (see the definition of the latter, for example, in [14]), being a universal reference frame in which our universe is embedded. In particular, the distribution of matter in the universe is isotropic in this frame only, the same is true for the relic black body radiation. The introduction of the preferred frame leads, as a straightforward consequence, to the concept of absolute time as the universal time acting in the preferred frame.

The causality protection formula, valid in all inertial frames, was formulated in [13] as follows:

$$Pu \geq 0,$$  \hspace{1cm} (1.1)
where $P$ is a 4-vector of particle momenta transferring a signal and $u$ is a 4-velocity of the preferred reference frame with respect to (any particular) inertial observer. It is a boundary condition which should be imposed on solutions of any tachyon equation of motion.

As a straightforward result, it turns out that the negative energy tachyons, which could be used for a construction of the causal loops, cannot appear in the preferred reference frame since the eigenvalues of the tachyon Hamiltonian, as has been shown in [13] (and will be demonstrated in Sect. 3 of this note), are restricted from below, in this frame, by zero value, which automatically solves the problem of the tachyon vacuum instability. The tachyon vacuum in the preferred frame is represented by an infinite ensemble of zero-energy, but finite-momentum, on-mass-shell tachyons propagating isotropically. Thus the space of the preferred frame is spanned by the continuous background of mass-shell zero-energy tachyons; in some respects this is the reincarnation of the ether concept in its tachyonic version. Simultaneously it turns out that in any reaction in which tachyons participate asymptotic “in” and “out” tachyonic Fock spaces are unitarily equivalent, which removes the unitarity problem.

As “toy” models, the Lorentz-covariant quantum field models of scalar tachyons were considered in ref. [13]. They are based on Lorentz-invariant Lagrangians with spontaneously broken Lorentz symmetry, so the Lorentz invariance violation appears to be restricted to the tachyon sector only, affecting the asymptotic tachyon states and leaving the sector of ordinary particles within the Standard Model untouched, at least up to presumably small radiative corrections. The basic element of those tachyon models is the Lorentz-covariant causal $\Theta$-function, required by the boundary condition (1.1), which ensures the causal behaviour of tachyonic fields and, subsequently, the other gains of the models. For example, the Hermitian tachyon field operator with this $\Theta$-function, $\Theta(ku)$, reads as follows:

$$\Phi(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^4k \left[ a(k) \exp(-ikx) + a^+(k) \exp(ikx) \right] \delta(k^2 + \mu^2) \Theta(ku),$$

where $k$ is a tachyon four-momentum, $a(k), a^+(k)$ are annihilation and creation operators with bosonic commutation rules, annihilating or creating tachyonic states with 4-momentum $k$, and $\mu$ is a tachyon mass parameter. As can be seen, the expression (1.2) is explicitly Lorentz-covariant. This covariance includes the invariant meaning of the creation and annihilation operators defined in the preferred frame; thus, for example, an annihilation operator $a(k)$ remains an annihilation operator $a(k')$ in the boosted frame, even if the zero component of $k'$ may become negative.

When calculating the tachyon production probabilities and cross-sections the confining $\Theta$ functions will accompany the production amplitudes as factors restricting the reaction phase space, so the expressions for these probabilities can be displayed as follows:

$$W = \int |M|^2 d\tau \prod_i \Theta(k_i u),$$

where $M$ is a matrix element of the reaction (which has to be representable in a Lorentz-invariant form), $d\tau$ is a reaction phase space element, and the product of $\Theta$ functions includes all free tachyons (having 4-momenta $k_i$) participating in the reaction.

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1It was argued in [13] that a realistic model of a tachyon theory should be built upon the infinite-dimensional unitary irreducible representations of the Poincaré group (so called “infinite spin” tachyons). Within the conjecture that elementary particles are realizations of the unitary irreducible representations of the Poincaré group the only alternative to the infinite spin tachyon models is a scalar tachyon model. However the latter cannot represent tachyons at a fundamental level since it possesses several diseases; in particular, such a model would lead to the instability of photons via their decay to tachyon-antitachyon pairs [13] (note that decays of photons to the infinite spin tachyon-antitachyon pairs are forbidden by the angular momentum conservation combined with kinematic restrictions imposed on this process).

However all individual components of an infinite-dimensional wave equation must satisfy the Klein-Gordon equation. Therefore the whole argumentation below concerning the introduction of the causal $\Theta$-function into the tachyon field operator and its applications would be also valid for infinite-dimensional tachyon models.
During discussions of the models above the author was asked to formalize the introduction of the causal Θ function into the tachyon field operator. This note is a realization of this recommendation on the base of a scalar tachyon field model, presenting a development of the approach to a tachyon theory sketched in this Introduction.

In formulae used in the note the velocity of light $c$ and the Planck constant $\hbar$ are taken to be equal to 1.

2. Causal tachyon field operator

Let us consider a Lorentz-invariant Lagrangian of a free scalar tachyon field

$$L = \frac{1}{2} \int d^3x \left[ \dot{\Phi}^2(x) - \nabla \Phi(x) \nabla \Phi(x) + \mu^2 \Phi^2(x) \right]$$  \hspace{1cm} (2.1)

from which the Klein-Gordon equation with the negative mass-squared term $-m^2 = \mu^2$ follows:

$$\left( \frac{\partial^2}{\partial t^2} - \partial_i \partial^i - \mu^2 \right) \phi(x) = 0 , \quad i = 1, 2, 3$$  \hspace{1cm} (2.2)

It has well-known solutions in the form of plane waves, so the wave function of a free particle with a given 4-momentum $k = (\omega, \mathbf{k})$ must be

$$\text{const} \times \exp(-i(\omega t - \mathbf{k} \mathbf{x}))$$  \hspace{1cm} (2.3)

with the dispersion relation

$$\omega = \pm \sqrt{k^2 - \mu^2}$$  \hspace{1cm} (2.4)

and with the restriction on the particle 3-momentum $k$:

$$|\mathbf{k}| \geq \mu .$$  \hspace{1cm} (2.5)

A Fourier representation of the general solution $\phi(x)$, up to a normalisation factor $1/\sqrt{(2\pi)^d}$, should be written as

$$\phi(x) = \int d^4k \exp(-ikx) \delta(k^2 + \mu^2) \phi(k),$$  \hspace{1cm} (2.6)

where $\delta(k^2 + \mu^2)$ ensures that the field $\phi(x)$ corresponds to particles positioned on the mass shell, thus validating (2.5), and $\phi(k)$ are Fourier amplitudes. A standard problem, appearing at such a decomposition, is related to the negative sign of the $\omega$ in (2.4) which, being interpreted in a straightforward way as a particle energy, would lead to a well-known problem related to particles with negative energies.

Our aim is a standard solution of this problem combined with a “soft” (covariant) introduction of a concept of the preferred reference frame in the tachyon field model. To do this we introduce two auxiliary scalar fields that obey the equation (2.2):

$$\phi^{(+)}(x) = \frac{1}{2\pi i} \int_{C_+} \phi(x - u\tau) \frac{d\tau}{\tau},$$  \hspace{1cm} (2.7)

$$\phi^{(-)}(x) = \frac{1}{2\pi i} \int_{C_+} \phi(x + u\tau) \frac{d\tau}{\tau},$$  \hspace{1cm} (2.8)

where $\tau$ is a “time” parameter (which will be explained below), $u$, primarily, is some 4-vector of dimension of a 4-velocity, and the contour $C_+$ is extended from $-\infty$ to $+\infty$, deformed below the singularity at $\tau = 0$. These auxiliary fields are similar to those introduced in [13] in order to define invariantly the plane waves with positive and negative “frequencies”. In [15] a 4-vector $\epsilon$, formally defined to be timelike and to have a positive time component, was used instead...
of the $u$. In our consideration the 4-vector $u$ has a physical meaning of the 4-velocity of the preferred reference frame with respect to the observer, i.e. it is automatically timelike and has a positive time component:

$$u_\mu u^\mu = 1, \quad u_0 = u^0 > 0, \quad \mu = 0, 1, 2, 3.$$ (2.9)

The physical meaning of integrals in (2.7), (2.8) implies the “collection” of all virtually allowed field phases ("trajectories") of each individual field mode dispersed over $\tau$, with a pole at $\tau = 0$. According to the decomposition (2.6),

$$\phi^{(+)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \frac{1}{2\pi i} \int_{C^+} \exp(i ku \tau) \frac{d\tau}{\tau},$$ (2.10)

$$\phi^{(-)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \frac{1}{2\pi i} \int_{C^+} \exp(-i ku \tau) \frac{d\tau}{\tau}.$$ (2.11)

Noting that

$$\frac{1}{2\pi i} \int_{C^+} \exp(i ku \tau) \frac{d\tau}{\tau} = \begin{cases} 1 & \text{if } ku > 0 \\ 0 & \text{if } ku < 0 \end{cases}$$ (2.12)

and

$$\frac{1}{2\pi i} \int_{C^+} \exp(-i ku \tau) \frac{d\tau}{\tau} = \begin{cases} 1 & \text{if } ku < 0 \\ 0 & \text{if } ku > 0 \end{cases}$$ (2.13)

the equations (2.10), (2.11) can be rewritten as

$$\phi^{(+)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \Theta(ku),$$ (2.14)

$$\phi^{(-)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(-ku).$$ (2.15)

Let us consider these equations in the preferred reference frame, where $u = (1, 0, 0, 0)$:

$$\phi^{(+)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(+)}(k) \Theta(\omega),$$ (2.16)

$$\phi^{(-)}(x) = \int d^4 k \exp(-ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(-\omega).$$ (2.17)

One can see that in the preferred frame $\phi^{(+)}(x)$ differs from zero only at positive “frequencies” (the upper sign for $\omega$ in (2.4)), while $\phi^{(-)}(x)$ differs from zero only at negative “frequencies” (the lower sign for $\omega$ in (2.4)). Introducing the definition $E = \omega$ in (2.16) we can return to the covariant expression (2.14) for $\phi^{(+)}(x)$, where $k$ is redefined as

$$k = (E, k).$$ (2.18)

Analogously, introducing the definition $E = -\omega > 0$ in (2.17) and using the identity

$$\int d^4 k \exp(ikx) \int_{C^+} \exp[i(Eu_0 + ku)\tau] \frac{d\tau}{\tau} = \int d^4 k \exp(-ikx) \int_{C^+} \exp[i(Eu_0 - ku)\tau] \frac{d\tau}{\tau}$$ (2.19)

we can rewrite (2.15) as

$$\phi^{(-)}(x) = \int d^4 k \exp(ikx) \delta(k^2 + \mu^2) \phi^{(-)}(k) \Theta(ku)$$ (2.20)

with the definition (2.18) for $k$.

Thus we have arrived at a result that in both functions, $\phi^{(+)}(x)$ and $\phi^{(-)}(x)$, the variable $E$, which will be referred to in what follows as a particle energy, is, by the construction, always...
positive in the preferred reference frame. Keeping this in mind, we can write a general solution of (2.2) in an arbitrary frame in the form

\[ \Phi(x) = \int d^4k \left[ \exp(-ikx) a(k) + \exp(ikx) a^+(k) \right] \delta(k^2 + \mu^2) \Theta(ku), \]

(2.21)

the amplitudes \( \phi^+(k), \phi^-(k) \) in the previous expressions for \( \phi^+(x), \phi^-(x) \) being replaced, by a procedure of second quantization, by annihilation and creation operators \( a(k), a^+(k) \), respectively, annihilating and creating states with 4-momentum \( k \). As mentioned in the Introduction, the meanings of the operators \( a(k), a^+(k) \) to be the annihilation and creation operators are conserved in an arbitrary frame, even if the energy component of a 4-vector \( k' \) in a boosted frame (i.e. \( E' \)) can become negative under a suitable proper Lorentz transformation.

3. Chronology protection agency at work

Let us consider how the causal \( \Theta \)-function prevents the violation of the principle of causality by the use of tachyons.

The principle of causality states that any (tachyon) theoretical model should not admit an appearance of causal loops, i.e. the possibility of sending by an observer signals to its own past. However within special relativity such a possibility does exist for faster-than-light signals, as proved by R. C. Tolman in 1917 (long before the tachyon hypothesis was formulated, [16], see also [17, 18]). The introduction of the concept of the preferred reference frame, in which tachyon interactions are ordered by retarded causality, allows to destroy this possibility, as mentioned in the Introduction. Now we shall trace how this destruction occurs exploiting a concrete (scalar) tachyon model.

Using (2.1) and (2.21), the latter following from the former after an introduction of the concept of the preferred reference frame, one can proceed, integrating (2.21) over \( k_0 \) and after expressing canonical annihilation and creation operators \( a_k, a^+_k \), annihilating or creating tachyonic states with 3-momentum \( k \), via

\[ a_k = a(k) \Theta(ku)/\sqrt{2(ku)}, \]

(3.1)

\[ a^+_k = a^+(k) \Theta(ku)/\sqrt{2(ku)}, \]

(3.2)

with the denominators included to ensure a proper covariant normalisation of a single-tachyon wave function, to obtain an expression for the tachyon field operator in the form

\[ \Phi(t, x) = \int_{|k| > \mu, E > ku} d^3k \frac{2(E - ku)}{2E \sqrt{2\pi \sqrt{1 - u^2}}} \left[ a_k \exp(-iEt + ikx) + a^+_k \exp(iEt - ikx) \right]. \]

(3.3)

Returning to (2.21) and requiring the field to obey the translational invariance the following equation should hold:

\[ [P_\mu, \Phi(x)] = -i\partial_\mu \Phi(x), \]

(3.4)

where \( P_\mu \) is an operator of a 4-momentum of the field. Its solution for \( P_\mu \) is:

\[ P_\mu = \frac{1}{2} \int \frac{d^4k}{(2\pi)^3} \frac{k_\mu}{2E} \left[ a^+(k)a(k) + a(k)a^+(k) \right] \delta(k^2 + \mu^2) \Theta(ku) \]

(3.5)

when choosing the bosonic commutation relations for \( a, a^+ \) operators:

\[ [a(k), a(k')] = 0, \quad [a^+(k), a^+(k')] = 0. \]

(3.6)
\[
[a(k), a^+(k')] \delta(k^2 + \mu^2) \delta(k'^2 + \mu^2) \Theta(\kappa u) \Theta(k'u) = \delta^4(k - k') \delta(k^2 + \mu^2) \Theta(ku).
\] (3.7)

In particular, the field Hamiltonian is
\[
H \equiv P_0 = \frac{1}{2} \int \frac{d^4k}{(2\pi)^3} k^0 \left[ a^+(k)a(k) + a(k)a^+(k) \right] \delta(k^2 + \mu^2) \Theta(\kappa u).
\] (3.8)

Passing again to a 3-dimensional integral with the use of (3.1, 3.2), followed by dropping as usually the infinite \(c\)-number related to zero-point oscillations, we arrive at the Hamiltonian
\[
H = \int_{|k|>\mu, E>\kappa u} \frac{d^3k}{(2\pi)^3} \frac{E - \kappa u}{\sqrt{1 - u^2}} a_k^+ a_k,
\] (3.9)

whose eigenvalues are bounded from below. This results, in particular, in a possibility of the standard operator definition of the invariant vacuum state \(|0\rangle\) via the annihilation operators \(a(k), a(k)|0\rangle = 0\) for all \(k\) such that \(|k| > \mu\), with the vacuum energy boundaries to be defined by the tachyon vacuum gauge \(P_0 = 0\). (3.10)

For example, in the frames moving with respect to the preferred frame the energy boundaries of the tachyon vacuum are given by expressions
\[
E_0^+ = \frac{\mu|u|}{\sqrt{1 - u^2}}
\] (3.11)

for the direction of motion of the preferred frame coinciding with that of the tachyon velocity \(v\), and by
\[
E_0^- = -\frac{\mu|u|}{\sqrt{1 - u^2}}
\] (3.12)

for the opposite direction. Thus the tachyon vacuum energy boundaries are rotationally invariant in the preferred reference frame only. In this frame
\[
H = \int_{|k|>\mu, E>0} \frac{d^3k}{(2\pi)^3} E a_k^+ a_k
\] (3.13)

having non-negative eigenvalues.

Just the equation (3.11) prevents the construction of causal loops with the use of tachyons: to build such a loop one needs to send tachyons having velocities \(|v| > 1/|u|\) along the direction of motion of the preferred reference frame \(u\), but such velocities, as can be easily seen, would correspond to tachyon energies below the allowed energy limit in this direction (3.11), i.e. they are forbidden. One can say that acasual tachyons get confined within the tachyon vacuum.

It is interesting to note that formula (1.1) works in the case of ordinary particles also, destroying the possibility of having negative energies of these particles. This suggests the idea that this formula has a general application and can be considered as a realization of “the chronology protection agency”, the term being primarily introduced by S. W. Hawking [19] to protect the causality in some general relativity applications. Thus, the great efforts undertaken to resolve the problem of particle negative energies appearing in relativistic quantum theory may be considered, formulating an alternative point of view, as equivalent to the introduction of the concept of the preferred reference frame (and hence “absolute time”) in the philosophy of that theory, which trivially solves the problem, analogously to the prescriptions (2.7), (2.8) for a scalar tachyon field.

To make this story complete let us remark that the need of an introduction of the concept of the preferred reference frame in the quantum theory in order to ensure the conservation of causality was noticed long time ago by P. H. Eberhard in [20], the idea being shared also by J. S. Bell [21, 22].
4. Conclusion

The introduction of the causal $\Theta$ function $\Theta(\kappa u)$ into the tachyon field operator, where $k$ is a tachyon 4-momentum and $u$ is a 4-velocity of the preferred reference frame in which tachyon interactions are ordered by retarded causality, aimed at solving several serious problems of a theory of faster-than-light particles, including the violation of causality by tachyons, is formalized in this note within a Lorentz-covariant approach.

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References

[1] H. Shmidt, Über die invarianten und kausalen Lösungen der Differentialgleichungen $(\pm \kappa^2)\Psi(r, t) = \delta^4(r, t)$, Z. Phys. 151 (1958) 365-374.
[2] S. Tanaka, Theory of matter with super light velocity, Progr. Theor. Phys., 24 (1960) 171-200.
[3] O.M.P. Bilaniuk, V.K. Deshpande, E.C.G. Sudarshan, “Meta”-relativity, Amer. J. Phys. 30 (1962) 718-723.
[4] E.P. Wigner, Invariant quantum mechanical equations of motion, in: Theoretical Physics, I.A.E.A., Vienna, 1963, pp. 59-82.
[5] G. Feinberg, Possibility of faster-than-light particles, Phys.Rev. 159 (1967) 1089-1105.
[6] D. Korff, Z. Fried, Consequences of an unreasonable mass spectrum on relativistic field theory, Nuovo Cim. A52 (1967) 173-191.
[7] R.G. Newton, Causality effects of particles that travel faster than light, Phys. Rev., 162 (1967) 1274.
[8] W.B. Rolnick, Implication of causality for faster-than-light matter, Phys. Rev. 183 (1969) 1105-1108.
[9] J.A. Parmentola, D.D.H. Lee, Peculiar properties of tachyon signals, Phys. Rev. D 4 (1971) 1912-1915.
[10] H.B. Nielsen, Tachyons in field theory, in: E. Recami (Ed.) Tachyons, Monopoles and Related Topics, North-Holland Publishing Company, Amsterdam, 1978, pp. 169-174.
[11] D.G. Boulware, Unitarity and interacting tachyons, Phys. Rev. D 1 (1970) 2426-2427.
[12] T. Jacobson, N.C. Tsamis, R.P. Woodard, Tachyons and perturbative unitarity, Phys. Rev. D 38 (1988) 1823-1834.
[13] V.F. Perepelitsa, Looking for a theory of faster-than-light particles, arXiv:gr-qc/14073245.
[14] L.D. Landau, E.M. Lifshitz, in: Theoretical Physics, vol.2, Classical Theory of Fields, Addison-Wesley, New York, 1955, Sect. 112.
[15] J. Schwinger, Quantum Electrodynamics. II. Vacuum polarization and self energy, Phys.Rev. 75 (1949) 651-679.
[16] R.C. Tolman, in: The Theory of Relativity of Motion, Univ. of California Press, Berkeley, 1917, p. 54.
[17] C. Møller, in: The Theory of Relativity, Oxford Univ. Press, Oxford, 1952, p. 52.
[18] D. Bohm, in: The Special Relativity, Addison-Wesley, New York, 1965, p. 155.
[19] S.W. Hawking, The chronology protection conjecture, Phys. Rev. D 46 (1992) 603-611.
[20] P.H. Eberhard, Bell’s theorem and the different concepts of locality, Nuovo Cim. B46 (1978) 392-419.
[21] J.S. Bell, Bertlmann’s socks and the nature of reality, in: Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 2004, pp. 139-158.
[22] J.S. Bell, The measurement theory of Everett and de Broglie’s pilot wave, in: ibid, pp. 93-99; also in: M. Plato, Z. Marie, A. Milojevic, D. Sternheimer, J.P. Vigier (Eds.), Determinism, Causality, and Particles, D. Reidel Publishing Company, Dordrecht-Holland/Boston-U.S.A., 1976, pp. 11-17.