Relativistic hydrodynamics with momentum dependent relaxation time

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A second order relativistic hydrodynamic theory has been derived using momentum dependent relaxation time in the relativistic transport equation. In order to do that, an iterative technique of gradient expansion approach, namely Chapman-Enskog (CE) expansion of the particle distribution function has been employed. The key findings of this work are, (i) momentum dependent relaxation time in collision term results in an extended Landau matching condition for the thermodynamic variables, (ii) the result from numerical solution of Boltzmann equation lies somewhere in between the two popular extreme limits: linear and quadratic ansatz, indicating a fractional power of momentum dependence in relaxation time to be appropriate, (ii) an equivalence has been established between the iterative gradient expansion method like CE and the well known moment approach like Grad’s 14-moment method.

In last few decades of exploring the deconfined quark-gluon plasma (QGP) at heavy-ion experimental facilities like RHIC and LHC, relativistic dissipative hydrodynamics has offered itself to be a trusted theory that considerably explains the experimental observables. It can be best outlined as an effective long-wavelength theory which describes the dynamics of conserved, macroscopic quantities. However, in order to derive a formalism of this macroscopic theory, a microscopic theory is essential to begin with that captures the dynamical interaction of the system. Boltzmann transport equation is serving the purpose for quite some time which describes the evolution of the single particle distribution function by a collision term that essentially includes the microscopic interactions of the system. The hydrodynamic equations so obtained provides the space-time evolution of thermodynamic and dissipative quantities, in which the microscopic interactions enter through transport coefficients.

However, there are several existing methods of extracting hydrodynamic equations from transport theory. The two popularly used competing methods are, (i) an iterative technique of successive gradient expansion (order by order) of the out of equilibrium distribution function - Chapman-Enskog (CE) method and (ii) taking moment integral directly from the transport equation - Grad’s 14-moment method. Analysis from both the methods along with entropy maximization technique present a vast deal of work in the existing literature. Clearly, a correspondence between these different approaches is certainly desirable in order to have a unique theory.

The difficulty in solving the transport equation comes from the non-linearity of the collision term (it includes product of the distribution functions). Different approximations are made to linearize the theory among which relaxation time approximation of the particle distribution function proposed in [11], is one of the simple yet efficient methods in near equilibrium situations. However, [11] as well as most of the works concerning relaxation time approach consider it to be independent of particle momenta. This assumption has two serious drawbacks. First it does not consider the microscopic momentum anisotropies (i.e., assumes that equilibrium restoration times of phase space distribution functions belonging to all particle momenta are same), and secondly, it results in same relaxation time for microscopic particle distributions and macroscopic fields like viscous flow, while the later is expected to have a slower relaxation rate with respect to the former depending on the length and time scales of the concerning system.

In this present work a second order relativistic hydrodynamic theory has been developed including momentum dependent relaxation time ($\tau_R$) of particle distribution function for the first time, using CE method for a conformal system with no conserved charges. The out of equilibrium distribution function derived in this manner shows serious consequences on the Landau matching condition of macroscopic variables. An interesting finding of this work turns out to be the possibility of fractional power of momentum dependence of $\tau_R$ between two accepted extreme limits - linear and quadratic ansatz, as predicted by [12]. Finally, the derived hydrodynamic equation has been compared with the same obtained from moment method and an equivalence between the two approaches has been attempted thereafter.

To construct the necessary formalism, we begin with the relativistic transport equation in the following form,

$$p^\mu \partial_\mu f(x,p) = C[f] = -\frac{p^\mu u_\mu}{\tau_R} \delta f. \quad (1)$$

Here $f$ is the single particle momentum distribution which is function of particle 4-momenta $p^\mu$ and space-time variable $x$. $C[f]$ is the collision term expressed in terms of relaxation time $\tau_R$ and the out of equilibrium part of the distribution function $\delta f = f^{(0)} \phi$, with $f^{(0)}$ being the equilibrium distribution function and $\phi$ being the deviation from it.

The momentum dependence of $\tau_R$ is expressed as a power law of the scaled particle energy $\tau_p = \frac{E_{\mu}}{T}$ much in the same line as indicated in [13] [14].
\[ \tau_R(x, p) = \tau_R^0(x) \tau_p^n , \]

where \( \tau_R^0 \) is the momentum independent part of relaxation time and \( n \) is a parameter specifying the power of scaled energy. In \([13]\) \( \tau_R^0 \) is identified as a time scale proportional to the mean-free-path of the system which is typically the microscopic time scale. \( u^\alpha \) and \( T \) are respectively the hydrodynamic four velocity and temperature of the system. \( n = 0 \) and \( n = 1 \) cases are termed as linear and quadratic ansatz with further explanations to follow.

Next, we proceed with the well known CE method of obtaining the unknown particle distribution function of \( r \)th order using its known \( (r-1) \)th order values in an iterative method \([13]\). Expanding the distribution function with the help of a parameter (typically, the Knudsen number which is a ratio between mean free path and the macroscopic length scale of the system) and comparing equal powers of this parameter from Eq. \((11)\) we have,

\[ \tau_p(Df)^{(r)} + \Pi^\mu \nabla_\mu f^{(r-1)} = -\frac{\tau_p}{\tau_R}f^{(0)}\phi^{(r)} , \quad r \geq 1 \]

where,

\[ (Df)^{(r)} = \sum_{s=0}^{r} \left[ \frac{\partial f^{(r-s)}}{\partial T}(DT)^{(s)} + \frac{\partial f^{(r-s)}}{\partial u^\nu}(Du^\nu)^{(s)} \right] \]

\[ + \sum_{s=2}^{r} \left[ \frac{\partial f^{(r-s)}}{\partial (\nabla^\nu T)} D(\nabla^\nu T)^{(s)} + \ldots + \right] \]

Here \( f^{(r)} = f^{(0)}\phi^{(r)} \) is the \( r \)th order gradient correction to \( f \). \( \Pi^\mu = \frac{\Pi^\mu}{T_p} \) is the scaled particle four-momenta. \( D = u^\mu \partial^\mu \) and \( \nabla^\mu = \Delta^\mu \partial^\mu \) are temporal and spatial counterparts of the total space-time derivative \( \partial^\mu = u^\mu D + \nabla^\mu \), defined with the projection operator \( \Delta^\mu = g^\mu\nu - u^\mu u^\nu \). Throughout the analysis the metric of the system has taken to be \( g^{\mu\nu} = (1, -1, -1, -1) \).

In order to solve Eq. \((3)\), we need to define the thermodynamic identities of the system that follow from conservation of energy momentum tensor \( T^{\mu\nu} \). With vanishing bulk viscosity and conserved currents it is expressed as,

\[ T^{\mu\nu}(x) = g \int d\Gamma p^\mu p^\nu f = \{ \epsilon u^\mu u^\nu - P \Delta^{\mu\nu} \} + \Pi^{\mu\nu} \]

with \( d\Gamma_p = \frac{dp^\mu dp^\nu}{2T_p \pi^{\mu\nu}} \) and \( g \) respectively as the phase space factor and degeneracy of the system. \( \epsilon = u^\mu u_\mu T^{\mu\nu} \), \( P = -\frac{1}{3} \Delta^{\mu\nu} T^{\mu\nu} \) and \( \Pi^{\mu\nu} = \Delta^{\mu\alpha} \Delta^{\nu\beta} \) are the energy density, pressure and shear stress tensor respectively with the traceless projection operator \( \Delta^{\mu\nu} = \frac{1}{2} \{ \Delta^{\mu}_{\alpha} \Delta^{\nu}_{\beta} + \Delta^{\nu}_{\alpha} \Delta^{\mu}_{\beta} - \frac{2}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \} \). The first two terms in the last expression of \((10)\) constitute the equilibrium part \( \tau_0^{\mu\nu} \) and the remaining last term denotes dissipative part \( \Delta T^{\mu\nu} \). The conservation equation \( \partial^\mu \Pi_{\mu\nu} = 0 \) contracted with \( u^\mu \) and \( \Delta^{\mu\nu} \) respectively gives the equation of energy density and velocity of the system as follows,

\[ D\epsilon = -\epsilon + P + \Pi^{\mu\nu} \sigma_{\mu\nu} \]

\[ Du^\mu = -\frac{1}{(\epsilon + P)} \left\{ (\nabla^\mu P - \Delta^\mu \nabla_\rho \pi^{\rho\nu} + \Pi^{\mu\nu} D u_\nu) \right\} \]

with \( \sigma_{\mu\nu} = \nabla_\nu u_\mu \). The notation \( \langle \rangle \) denotes the traceless irreducible tensors of rank-1 and 2 defined as \( A_{\mu\nu} = \Delta_{\mu\nu} A^\nu \) and \( A_{\mu\nu} = \Delta_{\mu\nu\alpha\beta} A^\alpha B^\beta \) respectively.

Putting \( r = 1 \) in Eq. \((3)\), employing Eq. \((4)\) and using identities \((11)\) and \((7)\), we obtain the first order correction to the particle distribution function in terms of velocity gradient,

\[ \phi^{(1)} = \frac{\tau_p}{\tau_R} \Pi_{\mu\nu} \Gamma^{\mu\nu} \sigma_{\mu\nu} \]

Clearly one can see, this first order correction is a linear and a quadratic function of particle momenta for \( n = 0 \) and \( n = 1 \) respectively and hence bears the name. Putting Eq. \((3)\) in the well known first order correction of shear viscous stress tensor,

\[ \pi^{(1)}_{\mu\nu} = g \int d\Gamma p^\rho p^\sigma f^{(0)} \phi^{(1)} = 2 \eta \sigma^{\mu\nu} \]

provides the following relation that constrains the momentum independent part \( \tau_R^0 \) by the shear viscosity \( \eta \) scaled over entropy density \( s = \frac{(\epsilon + P)}{T} \) of the system,

\[ \tau_R^0 = \frac{\eta s}{T} \frac{5!}{(n + 4)!} \]

With \( r = 2 \) in Eq. \((3)\) and doing a bit of algebra, the second order correction to the particle distribution function is obtained as the following,

\[ \phi^{(2)} = \frac{\tau_p}{\tau_R} \left[ \frac{\tau_p}{12P} \pi^{(1)}_{\mu\nu} \sigma_{\mu\nu} - \frac{\tau_p}{4P} \Pi_{\mu\nu} \pi^{(1)}_{\mu\nu} \right] \]

\[ + \frac{5!}{(n + 4)!} \int \left\{ \frac{\tau_p}{8P} \pi^{(1)}_{\mu\nu} \pi^{(1)}_{\mu\nu} \right\} \]

\[ - \frac{\tau_p}{8P} \pi^{(1)}_{\mu\nu} \pi^{(1)}_{\mu\nu} \]

\[ - \left( \frac{(n - 1)}{\tau_p} \right) \pi^{(1)}_{\mu\nu} \pi^{(1)}_{\mu\nu} \]

Putting \((11)\) into second order correction of shear viscous stress tensor,

\[ \pi^{(2)}_{\mu\nu} = g \int d\Gamma p^\rho p^\sigma f^{(0)} \phi^{(2)} \]

and combining with Eq. \((9)\) we have the evolution equation of shear stress tensor \( (\pi^{\mu\nu} = \pi^{(1)}_{\mu\nu} + \pi^{(2)}_{\mu\nu}) \) up to second order in velocity gradient,

\[ \frac{\pi^{\mu\nu}}{\tau_p} = 2 \beta_\pi \sigma^{\mu\nu} - \left( D\pi^{\mu\nu} - 2 \pi^{(1)}_{\rho\sigma} \pi^{\rho\sigma} \right) \]

\[ + \lambda \pi^{(1)}_{\rho\sigma} \pi^{\rho\sigma} + \frac{3}{5} \pi^{\mu\nu} \]

\[ \quad \left( \partial \cdot u \right) \]
with $\beta_\pi = \frac{\tau}{\tau_R}$. The second order transport coefficients with $n^{th}$ power of momentum dependence in $\tau_R$ is given by,

$$\tau_n = \tau_R \frac{(2n + 4)!}{(n + 4)!} = \frac{\eta/s}{T} \frac{5!(2n + 4)!}{(n + 4)!^2}, \quad \lambda = \frac{\tau}{2(2n + 5)}.$$  \hspace{1cm} (14)

Eq. (11) and (13) along with Eq. (14) are the main results of the current work. It can be observed from Eq. (14) that $\tau_n$ which is defined as the relaxation time of shear viscous field and the microscopic relaxation time of particle distribution can only be identical for momentum independent situation, i.e., with $n = 0$. With increasing value of $n, \tau_n$ is becoming larger at a factorial rate making the viscous field to decay at a slower rate with respect to the microscopic scale as expected. Where for linear ansatz ($n = 0$) we have $\tau_\pi = \tau_R$, for quadratic ansatz ($n = 1$) we have $\tau_\pi = 6\tau_R^2$. The transport coefficients are, for linear ansatz $\tau_\pi = \frac{3n}{2}$ and $\lambda = 10/7$ as given in [10] and with quadratic ansatz, $\tau_\pi = \frac{3n}{2}$ and $\lambda = 2$.

Next, let us explore the possibility of a fractional value of $n$ for the hydrodynamic evolution equation (13). The motivation comes from the work [12] where in the context of radiative energy loss it has been shown that the momentum dependence of $\tau_R$ in defining the viscous correction to the phase-space distribution lies somewhere in between the two extreme limits of linear and quadratic ansatz assuming fractional powers as well. As mentioned earlier, in order to have second order viscous evolution equation, Eq. (11) has been substituted in (12). A closer inspection will reveal that the first two terms of (11) do not contribute in the corresponding moment integrals by the virtue of inner product property of irreducible tensors (inner product of two irreducible tensors with different ranks must be 0). The remaining terms gives finite contribution to moment integrals in order to produce Eq. (13). The power over the scaled energy $\tau_\pi$ in all these four terms is $(2n - \alpha), \alpha$ being an integer. Thermodynamic moment integral is defined for a $n^{th}$ rank tensor [17]. So, at most the acceptable fractional value for $n$ can be a half integer such that the total momentum power under the integral is an integer. So, taking $n = 1/2$ between limiting value of linear and quadratic ansatz, we have $\tau_\pi = 2.5\tau_R$, $\lambda = \frac{11}{7}$.

This so far straightforward formalism faces serious consequences while trying to pursue the Landau matching condition, $u_\mu u_\nu \Delta T^{\mu\nu} = 0$, such that $u_\mu u_\nu \tau_\pi = \epsilon$. This requires the integral $\int d\Gamma R^2 f(0) \phi(r)$ to be zero for every order $r$. From Eq. (8) it is evident that the first order correction to distribution function always satisfies matching condition irrespective of the value of $n$ (inner product property of irreducible tensors). However, it is the second order correction (11) which behaves differently. Defining $\Lambda = u_\mu u_\nu \Delta T^{\mu\nu}$, and replacing (11) in the expression of $\Delta T^{\mu\nu} = g \int d\Gamma R R^2 \mu^\mu \nu^\nu \delta f$, we find that,

$$\Lambda = \frac{\tau_\pi}{2\eta} \left[ (n + 1) - \frac{(n + 4)!}{4!(2n + 4)!} \right] \pi^{(1)\mu\nu} \pi^{(1)}_{\mu\nu}.$$ \hspace{1cm} (15)

It can be readily observed that $n = 0$ is the only case where $\Lambda = 0$ as also observed in [18]. For momentum dependent $\tau_R$ ($n \neq 0$), $\Lambda$ clearly retains a non-zero value, which increases with increasing power of $n$. Before proceeding further to analyze the situation, we need to go through the origin and implications of the matching condition. The matching or fitting conditions are actually constraints imposed on the dissipative part of energy-momentum tensor $T^{\mu\nu}$ and particle current $N^\mu$, in order to uniquely determine them from second law of thermodynamics. Violating matching conditions can result in thermodynamic instability (explained later) as well as altering thermodynamic equations like (6) and (7). In rescue to this situation, a recently developed formulation of dissipative hydrodynamics by extending the matching conditions [19, 21] come in useful.

As nicely explained in the Appendix of [21], if the entropy 4-current $S^\mu$ has a term proportional to $\Pi u^\mu$ as non-equilibrium contribution ($\Pi$ being bulk viscous pressure), the non-vanishing derivative $\partial (u_\mu S^\mu)/\partial \Pi |_{\Pi=0}$ makes the system thermodynamically unstable. It means the system is not in a maximum-entropy configuration although equilibrium state has been used. However, as argued in [19], a natural extension of equilibrium entropy current towards its off-equilibrium expression, may indeed contain such a term. Moreover, [21] discusses, if the out of equilibrium distribution function is taken as a combination of dissipative fluxes (ansatz for moment method), it is again natural that such a term to appear in $S^\mu$. Ref [19] treats the situation by nullifying this undesirable contribution with the extended matching condition $u_\mu u_\nu \Delta T^{\mu\nu} = \Lambda$, retaining thermodynamic stability. In [19] the conformal contribution of $\Lambda$ turns out to be proportional to $\pi^{\mu\nu} \pi_{\mu\nu}$ with a proportionality constant $\tau_\pi/2\eta$, which is quite similar (apart from some numerical factors) to what has been obtained in Eq. (15).

Still two points are needed to be made here. Extended matching conditions turns the energy momentum tensor to take the following expression,

$$T^{\mu\nu} = \{\epsilon u^\mu u^\nu - P \Delta^{\mu\nu}\} + \{\Lambda u^\mu u^\nu + \pi^{\mu\nu}\}.$$ \hspace{1cm} (16)

the first part being equilibrium and the later purely viscous correction. It alters the energy density of the system by an additive factor of $\Lambda$ with respect to its equilibrium value. However, $\Lambda$ is a second order correction and for a near equilibrium situation must have small contribution. It can be explained as a reflection of non-uniformity in the microscopic relaxation rate depending upon the particle momenta. Greater the dependence of equilibrium restoration on particle momenta, larger the distortion in phase space distribution which finally results in generating macroscopic fields involving velocity gradients. However, entropy being treated as a more fundamental quantity it retains the same expression as without considering this extended matching condition. Secondly, being a second order correction only, $\Lambda$ can not alter thermodynamic equations (15) and (7) within second order dissipative-the-
ory, since the consequent corrections from $\Lambda$ will be at least of third order (correction in energy density equation (6) is $(DA + \Lambda \partial \cdot u)$ and in velocity equation (7) is $\Lambda D\mu^\rho$). However, for higher order theories the situation can become non-trivial yet interesting.

To have a quantitative idea how different powers of $n$ over particle momenta in $\tau R$ affect the physical observables, Eq. (13) has been solved for a boost invariant Bjorken case for a massless Boltzmann gas with ultrarelativistic equation of state ($\epsilon = 3P$). In terms of Milne coordinates $(\tau, x, y, \eta)$ with $\eta = \tanh^{-1}(z/t)$ and $\pi = \tau \tanh(\eta/\tau)$ and considering the only independent component of $\pi^\mu\nu$ to be $\pi^{\mu m} = -\pi^{m\mu}$, we have the following equations for energy density and shear pressure respectively,

\[
\frac{de}{d\tau} = \frac{\epsilon + P}{\tau} + \frac{\pi}{\tau},
\]

\[
\frac{d\pi}{d\tau} = \frac{-\pi}{\tau} + \frac{4\beta_\pi}{3\tau} - \frac{(4 + \lambda)\pi}{3\tau}.
\]

Eq. (17) and (18) have been solved with initial time and temperature at $\tau_i = 0.4 fm$ and $T_i = 0.5 GeV$ with initial viscous pressure $\pi_i = 0$, for three values of momentum power $n$ in Eq. (2). Here, two limiting values of $n$ corresponding to linear ($n = 0$) and quadratic ($n = 1$) ansatz and one in between fractional value ($n = 1/2$) has been considered with different $\eta/s$ values. The corresponding transport coefficients $\tau_\eta$ and $\lambda$ has been obtained earlier for these three cases. Fig. (1) shows the proper time evolution of pressure anisotropy defined as $P_l/P_T = (P - \pi)/(P + \pi/2)$ for these three values of $n$ and four sets of $\eta/s$ ratio. The obtained results have been compared with a numerical solution of the Boltzmann equation based on parton cascade simulations (BAMPS) (22), shown by solid circles. The dashed lines indicate the results from linear ansatz ($n = 0$), the dot-dashed lines indicate the same for quadratic ansatz ($n = 1$) while the solid lines depict the $n = 1/2$ case. The $n = 0$ case as shown in (18) under predicts the BAMPS data which becomes prominent for large values of viscosity. $n = 1$ case clearly over predicts the data a good deal showing even larger deviation from BAMPS for high $\eta/s$. However, the $n = 1/2$ situation remarkably agrees with BAMPS results even with large viscosity like $\eta/s = 3.0$ throughout the evolution range. For small viscosity like $\eta/s = 0.2$ linear ansatz suffices to describe the dissipation, but with increasing viscous correction it is the fractional power of momentum dependence $n = 1/2$ which provides a faithful representation of BAMPS data within the scope of second order dissipative hydrodynamic theory. This reasonable agreement of numerical data with fractional power of momentum dependence is very illuminating in the context of Ref (12) which argued that most of interaction theories relevant for QGP lie between the two extreme limits of linear and quadratic ansatz and QCD kinetic theory predicts a momentum dependence within this range.

Last part of the present work deals with attempting to establish an equivalence between this iterative technique involving gradient expansion of particle distribution and the well known moment method. Following the formalism presented in (22), the out of equilibrium part of the distribution function is expanded in particle momentum basis which in the absence of bulk viscosity and conserved charges becomes $\phi = -C_{\mu\nu}\Pi^{\mu\nu}$). The unknown coefficient $C_{\mu\nu}$ can be extracted by replacing $\phi$ in the expression of shear stress $\pi^{\mu\nu} = g\int d^4k\Pi^{(\mu\nu)}f^{(0)}$ (note that this is the full expression for $\pi^{\mu\nu}$ unlike order by order expansion given by (9) and (12), which gives the total deviation $\phi = \frac{1}{n\tau}\Pi^{\mu\nu}\Pi^{(\mu\nu)}$). Next, we take the moment of relativistic transport equation (11) by multiplying it with $g_T\Pi^{(\alpha\beta)}$ and integrating over $d \Gamma_p$ ($r$ is a power over the scaled energy which must be an integer). Now one thing is to note here. For $r = -1$, we get the following equation,

\[
\int d \Gamma_p \Pi^{(\alpha\beta)} f^{(0)} = - \int d \Gamma_p \Pi^{(\alpha\beta)} \frac{\partial f}{\partial (p \cdot u)} - \int d \Gamma_p \Pi^{(\alpha\beta)} \frac{1}{p \cdot u} C[f] (19)
\]

This is same as Eq. (14) and (15) combined of (13) which boils down to the fact that $r = -1$ case represents the DNMR theory, where $r = 0$ gives the usual Grad’s 14-moment theory (21).

Performing the moment integrals (details in (22)), we obtain hydrodynamic evolution equation of shear viscous stress tensor in moment method as the following,

\[
\frac{\pi^{\mu\nu}}{\tau_\eta} = 2\beta_\pi \sigma^{\mu\nu} - D\pi^{\mu\nu} - 2\pi^{\mu\nu} + \lambda \pi^{(\mu\nu)} + \frac{4}{3} \pi^{\mu\nu}(\partial \cdot u) (20)
\]

\[
\tau_\eta = \frac{\eta}{T} (r + 6) , \quad \lambda = \frac{2}{T} (2r + 7) , \quad \beta_\pi = \frac{\eta}{T_\eta} (21)
\]

Eq. (20) is structurally exactly the same as (18) obtained.
from CE method. The corresponding transport coefficients \(^{(14)}\) and \(^{(21)}\) are observed to have identical expressions for two sets of choices of \(n\) and \(r\) values, \(n = 0, r = -1\) and \(n = 1, r = 0\). Evidently, this is the reason why linear ansatz of iterative gradient expansion \(^{(16)}\) has identical results with DNMR theory \(^{(13)}\) and the results from quadratic ansatz of CE in Fig. \(^{(1)}\) coincide with the ultrarelativistic Grad’s 14-moment results \(^{(23)}\). This equivalence is quite reassuring in acknowledging the fact that starting from the same microscopic theory (relativistic Boltzmann transport equation in this case) different methods of obtaining hydro equations finally converge with each other. Any other correspondence between the two approaches (set of \(n\) and \(r\) values giving same result other than the mentioned) is not known to the author.

To summarize, a second order relativistic hydrodynamic theory has been developed with momentum dependent relaxation time approach using Chapman-Enskog formalism of gradient expansion. The hydrodynamic evolution equation along with the transport coefficients have been estimated for two commonly used limiting cases - namely linear and quadratic ansatz as well as for a fractional power of momentum dependence. The pressure anisotropy for the fractional power of momentum dependence shows an impressive agreement with the numerical solution of Boltzmann equation indicating the system dynamics to lie somewhere in the middle of the two limiting ansatz. The anomaly in Landau matching condition has been rescued with the help of an extended matching condition recently proposed. Finally, a correspondence between the iterative technique of gradient expansion method and moment method has been established.

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