Limits on Neutrino Mass from Cosmic Structure Formation

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(June 28, 2018)

We consider the effect of three species of neutrinos with nearly degenerate mass on the cosmic structure formation in a low matter-density universe within a hierarchical clustering scenario with a flat initial perturbation spectrum. The matching condition for fluctuation powers at the COBE scale and at the cluster scale leads to a strong upper limit on neutrino mass. For a flat universe with matter density parameter \( \Omega = 0.3 \), we obtain \( m_\nu < 0.6 \) eV for the Hubble constant \( H_0 < 80 \) km s\(^{-1}\) Mpc\(^{-1}\). Allowing for the more generous parameter space limited by \( \Omega < 0.4, H_0 < 80 \) km s\(^{-1}\) Mpc\(^{-1}\) and age \( t_0 > 11.5 \) Gyr, the limit is 0.9 eV.

14.60.Pq, 98.80.Es

Recent experiments for atmospheric and solar neutrino fluxes suggest that the neutrinos are massive. In particular, the atmospheric neutrino experiment indicates an almost maximal mixing between the two neutrinos, which is most naturally understood if the relevant species are nearly degenerate in mass. Nearly maximal mixing is also a viable possibility to explain the long-standing solar neutrino problem with oscillation either in vacuum or in matter, although there remains the solution that it is explained by small-angle mixing via oscillation in matter [1]. For these reasons the idea has gained popularity that the three neutrinos are massive and almost degenerate in mass (e.g., [1,2]). The degenerate neutrinos mean that neutrino mass is larger than several tenths of eV, and this means that they provide the universe with a matter density comparable to or more than that in stars, and play some role in cosmological structure formation.

There are a few authors who discussed the possibility that neutrinos have played an active role in the formation of large-scale structure of the universe, especially in giving a power at a large scale which otherwise cannot be accounted for in the standard cold dark matter scenario at the critical matter density [3]. At the time of the emergence of this idea theorists took more seriously the Einstein-de Sitter (EdS) universe of the critical matter density, so that typical compositions of the matter were assumed to be \( \Omega_{C,DM} = 0.7 - 0.8 \) and \( \Omega_\nu = 0.3 - 0.2 \) in units of the closure density, \( 10.54h^2 \text{keV} (\text{cm})^{-3} \), where \( h \) is the Hubble constant \( H_0 \) in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). This neutrino mass density corresponds to neutrino mass of \((30-20)h^2\) eV. There have been many explorations of this scenario since the proposal [4], and the current conclusion is that the neutrino density in excess of \( \Omega_\nu \geq 0.3 \) is disfavoured in the EdS universe from the viewpoint of early cosmic structure formation.

Over the last few years the evidence has been accumulated indicating a low density universe. There are also observations pointing to the dominance of the vacuum energy (cosmological constant, \( \Lambda \)) that makes universe’s curvature flat, which is also preferred from the theoretical point of view for a low matter density universe.

The list in favour of a low matter-density universe includes: (1) Hubble constant - cosmic age mismatch for the \( \Omega = 1 \) universe; (2) No positive indications for the presence of copious matter beyond the cluster scale: the mass to light ratio inferred from clusters and galaxies, \( M/L = (100 - 400)h \), corresponds to \( \Omega = 0.1 - 0.3 \) [5]; (3) consistency of the cluster baryon fraction with the field value [6]; (4) The slow evolution of the cluster abundance from redshift \( z=0 \) to 0.8 together with the abundance normalization at \( z=0 \) [7]; (5) The Hubble diagram of Type Ia supernovae [8]. The data indicate a finite cosmological constant. In view of systematic errors in various steps of the analysis, however, a zero \( \Lambda \) is probably not excluded, whereas an \( \Omega = 1 \) universe is too far away from the observations; (6) The perturbation spectral-shape parameter \( \Gamma = \Omega_\Lambda = 0.2 - 0.3 \) from large-scale structure [9]; (7) Matching of the power spectrum between COBE and galaxy clustering [10]; (8) Evolution of small scale non-linear galaxy clustering [11]; (9) Local velocity field versus density enhancement [12]. The results of (1) - (9) converge between \( \Omega_0 = 0.2 \) and 0.4.

The two positive indications for the presence of a cosmological constant are the Type Ia supernova Hubble diagram mentioned above and the acoustic peak distribution in the cosmic microwave background radiation (CMB) anisotropies [13], which says that the universe is close to flat whether dominated by matter or vacuum energy.

We remark that the results from large-scale velocity flow analyses are controversial; the resulting \( \Omega \) varies from analyses to analysis compounded by the uncertainty in the biasing parameter regarding the extent to which galaxies trace the mass distribution [14]. Recent analyses of galaxy peculiar velocities combined with other observations claim that they are consistent with a low density universe with a finite \( \Lambda \) [15].

There seems no extensive analysis available for the effect of neutrinos in a low matter-density universe, although a brief reference has been made in [4]. In this
paper we consider the effect of massive, three degenerate neutrinos on the cosmic structure formation in the low matter-density universe. We assume the hierarchical structure formation dominated by cold dark matter (CDM), the current standard model of the cosmic structure formation. The match of the power spectrum in the COBE scale with that in the galaxy clustering provides the most conspicuous evidence for the clustering.

While power spectrum analyses are the most common way to demonstrate the consistency of the hierarchical clustering scenario, the amplitude estimated from galaxy clustering receives unknown biasing factors associated with galaxy formation mechanisms; therefore, this is not appropriate for a quantitative analysis as presented here. We consider matching of the two normalizations of the cosmic mass density fluctuation power, the normalizations derived from COBE at a several hundred Mpc scale and the rich cluster abundance at $z=0$ which measures the power at $\approx 8 h^{-1}$ Mpc scale and most conveniently represented by the rms mass fluctuation parameter $\sigma_8$.

We do not use the $\sigma_8$ parameters derived from velocity fields or other observations, which are more susceptible to various uncertainties. The advantage of using the cluster abundance information is that it refers to the mass function that is not affected by any biasing uncertainties, and the fiducial length scale $8 h^{-1}$ Mpc is close to that of clusters before collapse. The requirement of this matching leads us to derive quantitative constraints on the neutrino contribution to cosmic structure formation.

We first consider the flat universe with low matter density, but also discuss later the case for open universes.

We assume a flat (Harrison-Zeldovich) initial power spectrum, $P(k) = A(k^3)$ with $n = 1$, which is the most natural prediction of inflation. The fluctuation spectrum receives a modification as $A(k^3)T(k)$ for a large $k$ as the fluctuations evolve [16]. The shape of the transfer function $T(k)$ depends on the assumed cosmological model, and the neutrino content. We use the computer code CMBFast [17] to calculate the transfer function for many choices of parameters. The spectrum is normalized with a fitting formula around $\ell = 10$ deduced by Bunn & White [18] from the four-year COBE-DMR data [19]. They have estimated one standard deviation error to be 7% in square root of the harmonic coefficient of CBR anisotropies $C_\ell$.

We take the baryon fraction $\Omega_B = 0.015h^2$ corresponding to $n_0 = 4$ [20]. The result is not very sensitive to this choice.

We calculate the specific mass fluctuations within a sphere of a radius of $8 h^{-1}$ Mpc by integrating the spectrum with a top hat window:

$$\sigma_8^2 = \langle (\delta \rho / \rho)^2 \rangle_{r < 8h^{-1}} = \int_{r < 8h^{-1}} dk 4\pi k^2 |\delta k|^2 \left[ 3 \sin(kr) - (kr) \cos(kr) \right]^2. \quad (1)$$

The resulting $\sigma_8$ for a given Hubble constant and the neutrino mass density $\Omega_\nu$ is presented in Fig. 1 for a flat universe (a) $\Omega = 0.3$ and $\lambda = 0.7$, and (b) $\Omega = 0.4$ and $\lambda = 0.6$. A set of curves (increasing towards the right) gives contours of constant $\sigma_8$. Another set of curves shows the neutrino mass density for given neutrino mass.

The neutrino mass that concerns us is in the range $< 1$eV for most cases. The core radius of neutrino clustering allowed from the phase space argument [21] is

$$R_\nu = 3.2 Mpc (m_\nu/1eV)^{-2} (v/1000 km s^{-1})^{1/2}, \quad (2)$$

which is large compared with the core radius of rich clusters $R_c \approx (0.12 \pm 0.02) h^{-1}$ Mpc$^{-1}$ [22] for velocity dispersion $v \approx 10^3$ km s$^{-1}$. Together with small neutrino mass density, we can ignore the neutrino component in integrating the cluster mass. We have estimated the contribution from neutrinos to the cluster mass within linear perturbation theory. The inclusion changes the result at most by a few percent, which can safely be ignored in the present argument.

![Fig. 1. Contours of constant $\sigma_8$ derived from the COBE normalization in the $H_0 - \Omega_\nu$ plane for flat universes ($\Omega + \lambda = 1$). The region that satisfies the matching condition with the cluster abundance is indicated by a shade. Another set of curves indicates neutrino mass density $\Omega_\nu = (3 m_\nu/93.8 eV) h^{-2}$. (a) $\Omega = 0.3$, (b) $\Omega = 0.4$.](image)

This calculated $\sigma_8$ is compared with the value estimated from the rich cluster abundance. The estimate of $\sigma_8$ has been made by a number of authors [23-25,6]. The most ambiguous in such analyses is the estimate of the cluster mass, but the modern results are well converged among the authors, at least for $z \approx 0$ clusters.

A summary is presented in Table 1. We take the values given by Eke et al. [24] which agree with other estimates within the error: $\sigma_8 = 0.93 \pm 0.07$ for $\Omega = 0.3$ and $\sigma_8 = 0.80 \pm 0.06$ for $\Omega = 0.4$. Adaption of Viana & Lidde's [25] value makes the derived limit on neutrino mass slightly tighter. If we add the normalization error of the
CBR anisotropies in quadrature, the errors become 0.10 and 0.09, respectively. The allowed range is shown by shadows in the figure.

We see from Fig. 1 that one can obtain the limit on neutrino mass if the Hubble constant is set. For $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ we obtain 0.21 eV ($r = \Omega_\nu/\Omega \leq 5\%$) for $\Omega = 0.3$ and 0.91 eV ($r \leq 15\%$) for $\Omega = 0.4$. For $H_0 = 80$ km s$^{-1}$ Mpc$^{-1}$ our limits are 0.62 eV ($r \leq 10\%$) for $\Omega = 0.3$ and 1.8 eV ($r \leq 22\%$) for $\Omega = 0.4$. Our limit is summarized by a fitting formula:

$$m_\nu < [5.20h(\Omega/0.3)^{2.03} - 3.20(\Omega/0.3)^{1.32}]h^2.$$  \hspace{1cm} (3)

Allowing for conservative parameter space, $\Omega \leq 0.4$, $H_0 \leq 80$ km s$^{-1}$ Mpc$^{-1}$ and $t_0 > 11.5$ Gyr, the upper limit is 0.87 eV, which corresponds to $r \leq 13\%$ of the total mass density.

![Fig. 2. Same as Fig. 1, but for open universes.](image)

A similar figure is given in Fig. 2 for zero-Λ universes, (a) for $\Omega = 0.3$ and (b) for $\Omega = 0.4$. The normalization from the cluster abundance is $\sigma_8 = 0.76 \pm 0.09$ and 0.87$\pm$0.09 including the CBR normalization error. There is no consistent parameter range for $\Omega = 0.3$ for $H_0 < 100$ km s$^{-1}$ Mpc$^{-1}$ with or without neutrinos. A consistent parameter range appears for $\Omega = 0.4$, but only with a relatively high $H_0$. No-neutrino models are consistent for $70 < H_0 < 80$ km s$^{-1}$ Mpc$^{-1}$. Requiring $H_0 \leq 80$ leads to $m_\nu < 0.5$ eV, which is significantly stronger than the one for the flat universe.

The modification of power spectra with inclusion of massive neutrinos has been discussed by Hu et al. [26]. The change is about by a factor of two for $m_\nu = 1$ eV when $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$. The scatter among the various data and our ignorance of the biasing factor make it difficult to exclude 1 eV neutrinos using the current power spectrum data. A strong constraint, such as $m_\nu < 0.4$ eV, would be obtained only when the power spectrum is derived from a homogeneous galaxy sample with statistics as high as that would be expected in the Sloan Digital Sky Survey [26]. This would provide us with an alternative mean to set a limit on the neutrino mass, although we must still assume the biasing factor being scale independent. The reason we obtained a strong limit in this paper is ascribed to the advantage of using the cluster mass function, which is directly related to the mass fluctuation, as well as using the information spanning a very large baseline in the length scale.

Let us discuss possible uncertainties or loop-holes of our argument. We have ignored the contribution from gravitational wave perturbations to CBR spectrum. Its inclusion only makes the limit on neutrino component more stringent. A possible loop-hole in our argument is the possibility that the index of the power spectrum is significantly larger than one. The COBE data alone do not exclude an index in the range $0.9 < n < 1.5$, but the range is reduced to $1 < n < 1.2$ if supplemented by other CBR data on small scales [12]. With an index, $n < 1$, which can be easily realized with inflation models, the constraints become tighter. If $n > 1$, the excess large-scale power generated by neutrino perturbations are cancelled by the intrinsically small large-scale power and more massive neutrinos become viable. For $n = 1.2$, the limit for $\Omega = 0.3$ and $h = 0.7$ is loosened from 0.2 to 0.7 eV, and for $h = 0.8, 0.6$ to 1.4 eV. For our ($H_0, \Omega, t_0$) range discussed above the limit is 1.8 eV, still quite strong. We remark that we need some tricky tuning to give $n > 1$ in inflation models [27].

The limit we derived in this paper is quite strong. It is 5-20 times stronger than would be obtained from a straightforward mass density consideration $\leq 93.8h^2$ eV. Ellis and Lola [2] have recently developed an argument for neutrinos with a degenerate mass as large as 5 eV with interesting physics. Such neutrinos, however, bring a large mismatch into the fluctuation power between the very large scale and the cluster scale, causing a disaster to currently accepted cosmic structure formation models.

Let us finally compare our limits with those obtained from experiments or other cosmological considerations. A direct limit on electron neutrino mass from tritium beta decay is $\leq 4.4$eV (95% CL)[28] allowing for some systematic effects that make the measured $m^2_\nu$ negative. Additional limits are available if the neutrinos are of the Majorana type. The limit of lifetime for double beta decay of $^{76}$Ge has now increased to $5.7 \times 10^{25}$ year (90% CL)[29], which leads to 0.2$-$1.5 eV depending on the nuclear matrix element used. We also refer to a limit from cosmological baryon excess: the condition for baryon asymmetry left-over leads to the Majorana neutrino to be $\leq 1 \pm 2$ eV [30]. The limit obtained in this paper does not depend on neutrino types. If one would consider only one species of neutrinos being massive, the mass limit simply becomes weaker by about a factor of three.
Acknowledgements

We thank George Efstathiou and Craig Hogan for useful comments on the draft manuscript. MF and NS are supported by Grants-in-Aid of the Ministry of Education. MF thanks the Newton Institute and Institute of Astronomy in Cambridge and NS the Max Planck Institut für Astrophysik for their hospitality while this work was completed.

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