Unification of Dark Energy and Dark Matter

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Abstract

We propose a scenario in which dark energy and dark matter are described in a unified manner. The ultralight pseudo-Nambu-Goldstone (pNG) boson, $A$, naturally explains the observed magnitude of dark energy, while the bosonic supersymmetry partner of the pNG boson, $B$, can be a dominant component of dark matter. The decay of $B$ into a pair of electron and positron may explain the 511 keV $\gamma$ ray from the Galactic Center.
1 Introduction

The cosmological constant problem is a long-standing problem. The recent astrophysical observation \[1\] has established that the expansion of the present universe is accelerating, indicating the existence of dark energy. It is not excluded logically that the cosmological constant is tuned exactly zero in the true vacuum by a yet unknown mechanism, although the anthropic explanation of the observed small vacuum energy, \( \Lambda_{\text{cos}}^4 \simeq (2 \times 10^{-3}\text{eV})^4 \), is quite natural \[2\]. If it is the case, the present cosmological constant \( \Lambda_{\text{cos}}^4 \) should be a potential energy carried by an extremely light boson called quintessence \[3\], and the magnitude of \( \Lambda_{\text{cos}} \) may be directly linked to relevant energy scales beyond the standard model.

Some years ago, it was pointed out that a pseudo-Nambu-Goldstone (pNG) boson \( A \) coupled to left-handed-lepton currents plays a role of the quintessence \[4\]. In fact, one-loop diagrams induce the pNG boson coupling to the weak SU(2)\(_L\) gauge fields. And SU(2)\(_L\) instantons generate a potential for the pNG boson as

\[
V \simeq C e^{-2\pi/\alpha_2(F_A)} m_{3/2}^3 F_A \left(1 - \cos \left( \frac{A}{F_A} \right) \right),
\]

where \( C \) is a constant of order 1 and \( F_A \) the decay constant of the pNG boson \( A \). Provided \( F_A \simeq M_{PL} \simeq 2.4 \times 10^{18} \text{ GeV} \), the observation on \( \Lambda_{\text{cos}} \) suggests the gravitino mass is \( m_{3/2} \simeq \mathcal{O}(10) \text{ MeV} \). Here, we have used \( \alpha_2(M_{PL}) \simeq 1/23 \). To be more precise we will consider a bit larger scale of the symmetry breaking, \( F_A \simeq 4\pi M_{PL} \). This is because the quintessence \( A \) has already started to roll down to the potential minimal, otherwise. We do not, however, take too large value for the decay constant \( F_A \), since the born unitality of graviton and graviton scattering processes is violated at such energy scale and our field-theory description becomes no longer valid.

In this short note, we point out that the rather small gravitino mass leads to an interesting effect on cosmology. A scalar boson \( B \) which is a bosonic supersymmetry (SUSY) partner of \( A \) has a mass of order \( m_{3/2} \simeq 10 \text{ MeV} \). Since the life time of this boson \( B \) is so long, the \( B \) density easily overcloses the present universe. However, the energy density does crucially depend on the cosmological history. If late-time entropy production \[5,6\] occurred, the \( B \) density should have been diluted. In fact it is known
that a thermal inflation \[5\] at the weak scale dilutes the energy density substantially, and renders the $B$ field to be a dominant component of the dark matter in the present universe. Therefore, in our scenario, the dark energy and dark matter are unified, in a sense that they are explained by the pNG boson $A$ and its bosonic SUSY partner $B$, respectively. See Refs. [7, 8] for different approaches to unified description of dark energy and dark matter.

We would stress that $A$ and $B$ have couplings with a pair of electron and positron as

$$-\mathcal{L} = i m_e \frac{A}{\sqrt{2} F_A} \bar{e} \gamma_5 e + m_e \frac{B}{\sqrt{2} F_A} \bar{e} e. \quad (2)$$

Thus, the dark matter $B$ decays into a pair of electron and positron. The life-time of $B$ is much longer than the age of the present universe owing to the large scale of $F_A$. It is very encouraging that the decay of $B$ into $e + \bar{e}$ may explain the 511 keV $\gamma$ ray excess from the bulge of our galaxy observed by SPI/INTEGRAL [9] (See also Refs. [10, 11, 12]).

In the next section, we provide a model which unifies dark energy and dark matter. We will give discussions in the last section.

## 2 Unification Model

We denote the pNG chiral multiplet as $S$ whose bosonic component $s$ is consist of the pNG boson $A$ and the scalar boson $B$;

$$s = \frac{B + iA}{\sqrt{2}}. \quad (3)$$

The global $U(1)$ symmetry is represented by a shift symmetry,

$$S \rightarrow S + i\alpha F_A, \quad (4)$$

where $\alpha$ is a real constant. We see that the Kähler potential $K(S + S^\dagger)$ is invariant under the symmetry. The Kähler potential is written as

$$K = F_A^2 \left[ \frac{1}{2} \left( \frac{S}{F_A} + \frac{S^\dagger}{F_A} \right)^2 + \kappa_3 \left( \frac{S}{F_A} + \frac{S^\dagger}{F_A} \right)^3 + \kappa_4 \left( \frac{S}{F_A} + \frac{S^\dagger}{F_A} \right)^4 + \cdots \right] \quad (5)$$

Here, we have absorbed the linear term of $S + S^\dagger$ into the definition of the field $S$. We further impose that the left-handed lepton doublets $\ell_i (i = 1 \sim 3)$ have the $U(1)$ charge
and they transform under the global symmetry as
\[ \ell_i \rightarrow e^{-i\alpha} \ell_i. \]  
(6)

Then, we have an invariant superpotential,
\[ W = f_i e^{S/F_A} e_i \ell_i H_d, \]  
(7)

which leads to the following interaction in the electroweak-symmetry breaking vacuum \( (\langle H_d^0 \rangle \neq 0) \);
\[ - \mathcal{L} = i m_e \frac{A}{\sqrt{2} F_A} \bar{e} \gamma_5 e + m_e \frac{B}{\sqrt{2} F_A} \bar{e} e + \cdots. \]  
(8)

The above global symmetry has a SU(2)\(_L\) gauge anomalies and one-loop diagrams of the internal left-handed leptons induce
\[ \mathcal{L}_{\text{kin}} = \int d^2 \theta \left[ \frac{3}{32\pi^2} \frac{S}{F_A} W^{a\alpha} W^a_{\alpha} \right] + \text{h.c.}, \]  
(9)

where \( W^{a}_{\alpha} \) is the gauge kinetic function of the weak SU(2)\(_L\) gauge multiplet, and \( a = 1 \sim 3 \) and \( \alpha = 1, 2 \) run over the SU(2) generators and the components of spinors, respectively.

This gives the anomaly interaction of the pNG boson \( \mathcal{A} \) as
\[ \mathcal{L} = - \frac{3}{32\pi^2} \frac{A}{\sqrt{2} F_A} F^{a}_{\mu\nu} \tilde{F}^{a\mu\nu}. \]  
(10)

The potential of \( \mathcal{A} \) is generated by SU(2) instanton \( [4] \):
\[ V \simeq \Lambda_A^4 (1 - \cos(\mathcal{A}/F_A)) \]  
(11)

with
\[ \Lambda_A^4 \simeq C e^{-2\pi/\alpha_2(F_A)} m_3^3/2 F_A, \]  
(12)

where \( C \) is a constant of order unity. Here let us consider the dynamics of \( \mathcal{A} \), assuming \( \Lambda_A \sim (10^{-3} \text{eV})^4 \). If \( F_A \sim M_{PL} \), the mass of \( \mathcal{A} \) becomes roughly equal to the present Hubble parameter, which means that \( \mathcal{A} \) has already started rolling down to the potential minimum. To circumvent this we adopt slightly larger value of \( F_A \sim 4\pi M_{PL} \). Interestingly enough, such value of \( F_A \) turns out to be favorable in the context of explaining 511.

\(^1\)The theory with such large cut-off has been proposed in another context \[13\]. In this scheme, due to the large cut-off scale, inflation tends to predict almost scale invariant spectrum. The little hierarchy between \( M_{PL} \) and \( F_A \) might be ascribed to the anthropic explanation \[13\].
keV γ line, as we will see later. The potential energy of A accounts for the present dark energy, if the gravitino mass $m_{3/2}$ is $O(10)\text{MeV}$:

$$\Lambda_A^4 \simeq C \left(1 \times 10^{-3}\text{eV}\right)^4 \left(\frac{m_{3/2}}{15\text{MeV}}\right)^3 \left(\frac{F_A}{4\pi M_{PL}}\right),$$

(13)

where we have used $\alpha_2^{-1}(F_A) \simeq \alpha_2^{-1}(M_{PL}) - 1/(2\pi) \cdot \ln (4\pi)$.

Now let us turn to B, the bosonic SUSY partner of A. The B field obtains a soft SUSY breaking mass, $m_\beta$, which is of order $m_{3/2} = O(10)\text{MeV}$. The position of B during inflation is generally displaced from the potential minimum after inflation by $M_{PL}$, unless we impose additional symmetry. After inflation, B starts oscillating around the minimum when the Hubble parameter becomes comparable to its mass, $H \simeq m_\beta$. Since the life time of B is so long due to the large scale $F_A$, the energy density $\rho_B$ may overclose the universe. However, the present value of $\rho_B$ crucially depends on the thermal history of the universe; if the B density is diluted by late-time entropy production, B can be a dominant component of dark matter. Here we estimate the requisite amount of entropy production. Before the late-time entropy production, the primordial B density is

$$\frac{\rho_B}{s_i} \sim 10^7\text{GeV} \left(\frac{m_\beta}{10\text{MeV}}\right) \left(\frac{\mathcal{B}_i}{M_{PL}}\right)^2,$$

(15)

where $\mathcal{B}_i$ denotes the initial amplitude of B, $s_i$ the entropy density and we have assumed the reheating is completed when B starts oscillating. Thus, in order to render B to the dark matter, it must be off at least by $10^{16}$.

\footnote{In Ref. [4], the gravitino mass was fixed to the weak scale. That is why the authors introduced flavor symmetry to suppress the potential. In this paper, instead, we determine the gravitino mass from the magnitude of the cosmological constant $\Lambda_4^4$.}

\footnote{The initial displacement of B from the potential minimum can be suppressed by imposing a symmetry. For instance, let us take the following superpotential;}

$$W = X(\Phi \bar{\Phi} - \mu^2),$$

(14)

where $X(0), \Phi(+1)$ and $\Phi(-1)$ are scalar chiral superfields with the charges of the global $U(1)$ symmetry shown in the parentheses, and $\mu$ is the breaking scale of the $U(1)$. In this model, B corresponds to the difference $\Phi - \bar{\Phi}$, and B is lifted by a soft SUSY breaking mass. Assuming a symmetry interchanging $\Phi$ with $\bar{\Phi}$, the potential of B has a minimum at the origin, $B = 0$. If $\Phi$ and $\bar{\Phi}$ acquire Hubble-induced masses during inflation, B rolls down to the origin, which should coincide with the potential minimum after inflation. Then the B density due to the initial misalignment is exactly zero. Therefore it is possible to set the energy density of B to the right amount of dark matter by introducing a tiny violation of the interchanging symmetry. Such violation may naturally arise from the fact $\Phi$ and $\bar{\Phi}$ interact with matter fields in a different way due to $U(1)$ charge assignment.
The thermal inflation is able to generate the needed entropy. According to Refs. [14], the thermal inflation is driven by the potential energy of the flaton, $\phi$, with the potential

$$V(\phi) = V_0 + (T^2 - m_0^2)\phi^2 + \frac{\phi^6}{M_{PL}^2},$$

(16)

where $T$ denotes the cosmic temperature, and $V_0 \sim m_0^3 M_{PL}$. The thermal inflation lasts between $T \sim m_0^{3/4} M_{PL}^{1/4}$ and $T \sim m_0$. After the thermal inflation, the flaton oscillates around the minimum $\phi \sim \sqrt{m_0 M_{PL}}$ and decays, producing large entropy. The entropy dilution factor is

$$\Delta_{TI} \equiv \frac{s_f}{s_i} \simeq 10^{20} \left(\frac{10\text{MeV}}{T_d}\right),$$

(17)

where $s_f$ is the entropy density after thermal inflation, $T_d$ is the decay temperature of the flaton $^4$. The primordial $B$ density after thermal inflation is then

$$\Omega_B^p = O(10^{-3}) \left(\frac{m_B}{10\text{MeV}}\right)^{\frac{1}{2}} \left(\frac{B_i}{M_{PL}}\right)^2 \left(\frac{T_d}{100\text{MeV}}\right).$$

(18)

On the other hand, the thermal inflation itself displaces the potential minimum of $B$ from that in the vacuum, re-generating $B$ density at the end of the thermal inflation. The $B$ density generated in this way is estimated as

$$\Omega_B^{TI} \simeq O(0.1) \left(\frac{10\text{MeV}}{m_B}\right)^2 \left(\frac{m_0}{100\text{GeV}}\right)^3 \left(\frac{T_d}{100\text{MeV}}\right).$$

(19)

The total $B$ density is given by the sum of (18) and (19). Therefore $B$ can constitute a dominant part of the dark matter.

The important feature of the $B$ dark matter is that it decays into a pair of electron and positron through the interaction (8). The decay rate is $^5$

$$\Gamma_{B \rightarrow e^+e^-} = \frac{m_B^2 m_\beta}{16\pi F_A^2},$$

$$\simeq \left(10^{25} \text{sec}^{-1}\right) \left(\frac{m_\beta}{10\text{MeV}}\right)^2 \left(\frac{4\pi M_{PL}}{F_A}\right)^2.$$

(20)

$^4$For successful BBN, $T_d$ cannot be much smaller than $10\text{MeV}$. See also Refs. [15].

$^5$B can decay into two $A$s as well, if $\kappa_3$ is nonzero. Therefore it is model-dependent whether this decay channel becomes important or not. In fact, the dominant decay channel would be $B \rightarrow A + A$ if $\kappa_3$ is order unity. However, even in this case, the life time of $B$ is still much longer than the age of the present universe, therefore the predicted 511keV $\gamma$ ray flux (21) remains valid, since it is determined by only $\Gamma_{B \rightarrow e^+e^-}$, not the total decay rate.
The produced positrons will annihilate mostly by forming positroniums [9], a quarter of which produce 511keV line γ-ray. It is amazing that thus obtained decay rate explains the 511keV γ-ray from the Galactic Center observed by SPI/INTEGRAL [9] as pointed out in Refs. [11, 12]. The γ-ray flux is estimated as

$$\Phi_{511}^B \sim 10^{-3} \text{ph cm}^{-2} \text{sec}^{-1} \left( \frac{10\text{MeV}}{m_\beta} \right) \left( \frac{10^{25} \text{sec}}{\Gamma_{S\rightarrow\bar{e}+e^-}} \right),$$

while the observed flux is [9]

$$\Phi_{511}^{\text{obs}} = (1.05 \pm 0.06) \times 10^{-3} \text{ph cm}^{-2} \text{sec}^{-1}. \quad (22)$$

In deriving Eq. (21), we have assumed the following dark matter density function [16] with $\alpha = 0.1$,

$$\rho_{\text{DM}}(r) = \rho_0 \exp \left[ -\frac{2}{\alpha} \left( \left( \frac{r}{r_0} \right)^\alpha - 1 \right) \right]$$

where $r_0 = 20h^{-1}\text{kpc}$, $\rho_0$ is normalized so that $\rho_{\text{DM}}(r = 8.5\text{kpc}) = 0.3\text{GeV/cm}^3$.

### 3 Discussions

In the previous section we have concentrated on the bosonic part of the pNG chiral multiplet $S$; the imaginary component $A$ explains the dark energy, while the real one $B$ becomes dark matter. Here let us consider the fermionic component, $\tilde{s}$. Its mass is of order $m_{3/2}$ and the cut-off scale of the interactions with leptons is $F_A$. Therefore the $\tilde{s}$-abundance is always smaller than the gravitino abundance. In addition, the late-time entropy production dilutes the $\tilde{s}$ density, so $\tilde{s}$ does not play any important role in the history of the universe.

Since the late time entropy production dilutes any pre-existing baryon asymmetry, we need to either (i) generate large enough baryon asymmetry before entropy production; or (ii) generate baryon asymmetry after entropy production. The former does not seem to work since the requisite entropy dilution is very huge [17]. In the latter case, the promising baryogenesis is the Affleck-Dine leptogenesis after thermal inflation [18, 19]. It should be noted, however, that such constraints on the baryogenesis mechanism disappear if the $B$ density is suppressed by a symmetry discussed in footnote [3].
Very recently, Ref. [20] has appeared and put a stringent bound on the injection energies of positrons $\lesssim 3\text{MeV}$, by using the observed Galactic gamma-ray data. This bound corresponds to $m_B \lesssim 6\text{MeV}$ in our scenario, which can be easily satisfied since $m_B$ does not necessarily coincides with $m_{3/2} \sim 10\text{MeV}$, and may be a few times smaller.

Our model has another observational implication; $B$ decays into two photons through one-loop diagrams, producing the line gamma rays with energy $m_B/2$, as pointed out in Ref. [12]. The line gamma ray flux is so small that it is below the bound from present data [12]. Therefore future observation on the gamma ray background may be able to support or refute our scenario.

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