Nonlinear $\sigma$-model study of magnetic dephasing in a mesoscopic spin glass

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**Abstract** – We propose a nonlinear sigma-model for the description of quantum transport in a mesoscopic metallic conductor with magnetic impurities frozen in a spin glass phase. It accounts for the presence of both the corresponding scalar and magnetic random potentials. In a spin glass, this magnetic random potential is correlated between different realizations. As the strength of the magnetic potential is varied, this model describes the crossover between orthogonal and unitary universality classes of the nonlinear sigma-model. We apply this technique to the calculations of the correlation of conductance between two frozen spin configurations in terms of dephasing rates for the usual low-energy modes of weak-localization theory.

**Introduction.** – In a metallic conductor of $\mu$m size, the interplay between quantum coherence of electrons and disorder is at the origin of several remarkable phenomena at low temperature [1,2]. The interferences between different diffusive paths in the sample lead to a strong dependance of the conductance on the disorder realization through the geometry of these diffusive paths. In particular the conductance exhibits universal sample to sample fluctuations and reproducible random variations as a function of a transverse magnetic flux. These coherent transport phenomena are naturally suppressed by various perturbations which are usually referred to as dephasing sources. This includes inelastic scattering, for example from phonons, free magnetic impurities, other electrons. In this case two electrons following the same path encounter different potentials. This leads to a random relative phase between them and the suppression of interference effects. Such a dephasing source acts on the electrons themselves, and is usually taken into account phenomenologically through a dephasing rate $\gamma_\phi$ for the electrons. Elastic scattering by symmetry-breaking potentials by, e.g., random spin-orbit interactions or frozen magnetic impurities is also considered as another source of dephasing. In this case, the origin of dephasing is physically different from that in the inelastic case. Quantum corrections to transport result from interference between electrons travelling along loops of diffusive paths either in the same directions (Diffuson modes) or in the opposite directions (Cooperon modes). In the presence of magnetic disorder, the spins of counter-propagating electrons experience different sequences of rotations. As a result the corresponding interferences are gradually suppressed for longer and longer loops. This suppression is usually interpreted as dephasing of the corresponding Cooperon modes. Moreover the magnetic disorder selects out the Diffuson modes not affected by this dephasing; the relative phase of two electrons with spins forming a singlet state is insensitive to this spin rotation sequence, in contrast to that of triplet states. Hence, this symmetry-breaking elastic scattering induces a spatial decay of these particular diffusion modes, effectively accounting for this electron’s dephasing phenomenon. As a particular consequence, the magnitude of the universal conductance fluctuations is determined by symmetry properties.

This magnetic dephasing of electrons and its signature on coherent transport has been recently proposed as a promising probe of spin glass physics [3]. In the spin glass phase, which is a fascinating but poorly understood state of matter [4], impurity spins $\{\vec S_i\}$ coupled by frustrating interactions freeze below the transition temperature $T_g$. Of particular interest for its understanding are the correlations between different configurations of spins $\{\vec S_i^{(1)}\}$ and $\{\vec S_i^{(2)}\}$ corresponding to either different times [5, 6], or different quenches below $T_g$. The correlation of

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conductance between these two spin configurations (corresponding to different quenches or times) is determined by the correlation of magnetic dephasing of the electrons in these two configurations. This correlation can be expressed in terms of the dephasing rates for the diffusons and cooperons composed of electrons traveling along the same path but experiencing two different magnetic potentials corresponding to two different spin glass configurations. Hence, such measurement of correlations of conductance opens the route a direct probe of correlations between frozen spin configurations [3,7]. This naturally requires a quantitative description of this relative magnetic dephasing between different spin configurations, on which we focus in this letter.

Beyond the perturbative diagrammatic theory [2], a natural framework for describing the statistics of conductance is the field-theoretical nonlinear sigma model [8]. This powerful field-theoretical method is an essential tool in describing the weak-localization regime when computing higher moments of the conductance, or incorporating spatial or time correlations of the random potential. The aim of the present letter is to apply this technique to study the quantum electronic transport in a mesoscopic metallic glass. This amounts to incorporating the presence of both a scalar random potential and a weaker magnetic potential. We will show that this method allows for a very efficient and elegant description of the corresponding magnetic dephasing, by treating all elastic scattering potentials on the same footing. Altland showed how the inelastic dephasing effects can be phenomenologically accounted for within this nonlinear sigma-model [9]. While his approach required the introduction of a new fictitious scalar potential, in the present context the source of dephasing is already an elastic-scattering potential. Here, this dephasing accounts for the crossover behavior between different universality classes which characterize in particular universal weak-localization properties [10]. These classes encode the number and nature of independent diffusive modes contributing to the quantum correction of conductance for weak disorder. In the present case, without magnetic impurities the statistics of conductance is described by the orthogonal class with degenerate diffusive states. When adding magnetic disorder, the level degeneracy is lifted and simultaneously the universality class is restricted to the unitary class. Studying the magnetic dephasing induced by frozen magnetic impurities amounts to studying the cross-over between these two universality classes. This is achieved by considering all massless modes of the orthogonal case, and describing the different gap openings with increasing magnetic disorder, thereby extending nonperturbatively the work of [11] to the spin glass physics of interest here.

The nonlinear sigma-model. – We consider a $d$-dimensional mesoscopic metallic sample of size $L$ containing impurities inducing two different types of random scattering of the conduction electrons: i) nonmagnetic random scalar potential $V(r)$ coupled to the local fermionic density as $V(r)\psi(r)\bar{\psi}(r)$. This potential is assumed to be Gaussian with $\langle V(r) \rangle = 0$ and variance

$$\langle V(r)V(r') \rangle = \frac{1}{2\pi\nu_0\tau_{\nu}}\delta(r-r'),$$

where $\nu_0$ is the one-electron density of states and $\tau_{\nu}$ the corresponding elastic mean free time. ii) A magnetic disorder $\tilde{U}(r)$ originating from a collection of frozen magnetic impurities $\tilde{S}$, with coupling to the electron's density of spins $\tilde{U}(r)\cdot\bar{\psi}(r)\bar{\psi}(r)$ with $\tilde{\sigma}$ being Pauli's matrices. $U(r)$ is a three-dimensional field taken to be Gaussian with zero mean in the spin glass state. Here we focus on both the conductance fluctuations in a single magnetic disorder configuration, and conductance correlations between different magnetic disorder realizations corresponding to different spin glass states. For the sake of simplicity we consider correlations between different realizations of magnetic potential indexed by $u$ of the form

$$U_{u;\nu}(r)U_{u;\nu'}(r') = \frac{g_{uu'}}{3} \delta_{ij} \frac{1}{2\pi\nu_0\tau_{\nu}}\delta(r-r'),$$

where $g_{uu'}$ is the standard overlap in spin glass theory [12, 13], $g_{uu} = 1$ and $\tau_{\nu}$ is the elastic mean free time for scattering by magnetic impurities. Here we neglect possible correlations between the magnetic and nonmagnetic impurities.

The conductance of the sample at frequency $\omega$ for noninteracting electrons at zero temperature is given by Kubo formula ($h = m = e = 1$):

$$G(\omega) = \frac{1}{2\pi L^2} \mathrm{Tr} \left[ j_x G_{x}^{R}(r, s'; r, s) \partial \bar{\psi}_x G_{x'}^{A} \right].$$

where $G_{x}^{R,A}$ are the retarded and advanced Green functions, $r, s' = \uparrow, \downarrow$ the spin variables, and $j_x = i\partial/\partial x$ the probability current operator.

The Green functions $G_{x}^{R,A}$ for electrons in the random potentials (1) and (2) can be expressed with the help of a functional integration over Grassmann conjugated fields $\psi$ and $\bar{\psi}$ as follows:

$$G_{x}^{R,A} = \frac{e^{-i\omega_{\nu}/2}}{Z_{R,A}} \int D\bar{\psi} D\psi \tilde{\psi}(r)\psi(r') e^{iS_{\pm}},$$

with the actions

$$S_{\pm} = \int_{r} \bar{\psi}_{\pm}(r) \left[ \epsilon_{F} \pm \frac{1}{2} \omega_{\nu} - H_{\pm} \right] \psi_{\pm}(r),$$

where $H_{\pm}$ is the Hamiltonian, $\omega_{\nu} = \omega + i\delta$ and $\pm$ correspond to the retarded and advanced Green functions. Following ref. [11] we define the covariant and contravariant bispinors $\tilde{\eta} = (C\eta)^T = \frac{1}{2}\left(-\psi_{\uparrow}, -\psi_{\downarrow}, \psi_{\downarrow}, -\psi_{\uparrow}\right)$, which are related by the charge-conjugation matrix $C = i\sigma_1 \otimes \sigma_2$. The four components of the bispinor are the two "charge"
degrees and the two spin degrees of freedom. The auxiliary “charge” degree of freedom is introduced for taking into account on the same footing the two possible pairing \( \bar{\psi}\psi \) and \( \bar{\psi}\bar{\psi} \) which contribute to the slow part of the free energy. In order to generate products of the advanced and retarded Green functions we double the degrees of freedom introducing the index \( p = R, A \). For our purpose we also introduce 2 copies of the original system with different configurations of magnetic disorder enumerated by \( u = 1, 2 \). Then in terms of the bispinors the action for the noninteracting electrons can be written as

\[
S = \int d^4r \bar{\eta}_u(r) \left[ \tilde{\xi} + \frac{1}{2} \omega^+ \Lambda - V(r) - \bar{U}_u(r) \cdot \tilde{\Sigma} \right] \eta_u(r),
\]

where \( \tilde{\xi} = \xi_F - \frac{\hat{p}^2}{2m} \). \( \Lambda = (\sigma_3)_{p,p'} \otimes 1 \) and \( \tilde{\Sigma} = \sigma_3 \otimes \hat{\sigma} \).

To compute the dimensionless conductance correlation in a given sample with two different magnetic disorder configurations \((\Delta G)^2_{uu} = \frac{G(V(\bar{U}_u G(V, \bar{U}_u G)))}{v} \), we generalize the formalism of generating functional introduced in ref. [8]. We define the current operator \( J = i/2(\bar{\eta} \partial \tilde{\eta} - \partial \eta \tilde{\eta}) \). Then the generating functional for the conductance reads

\[
Z[A] = \int \mathcal{D}\bar{\eta} \mathcal{D}\eta \exp \left[ iS + \int \mathcal{D}(J_r, A_r) \right] / \int \mathcal{D}\bar{\eta} \mathcal{D}\eta \exp \left[ iS \right],
\]

where symbol \( \mathcal{D} \) stands for traces over all indices. To average eq. (7) over random potentials, we use the standard replica trick and introduce \( N \) copies of the original system so that the denominator in eq. (7) is suppressed in the replica limit \( N \rightarrow 0 \). The averaging over disorder generates in the action a quartic in \( \eta \) term which can be written in the form

\[
S_{\text{int}} = \frac{1}{4N \nu_0} \int_{r} \left( \bar{\eta}^{\mu}_{u} \eta^{\nu}_{u'} P^{k}(u, u') \right) P^{\mu}(u, u') \left( \bar{\eta}^{\nu}_{u'} \eta^{\mu}_{u} \right),
\]

where

\[
P^{k}(u, u') = (\tau_\epsilon/\tau_v) \delta^k \delta_{\epsilon} \tau_{\epsilon} + (\tau_\epsilon/3\tau_v) q_{uv} \bar{\Sigma}^{k}_{\epsilon} \Sigma^j_{\epsilon},
\]

where we introduced the new time scale \( \tau_v \) which is free at this stage as can seen from eqs. (8), (9). In the next section we will show that the proper normalization of the saddle point solution fixes \( \tau_v \) to the correct total mean free time.

The correlation of dimensionless conductance can be then written as the \( N \rightarrow 0 \) limit of the expression

\[
(\Delta G)^2_{uu} = \frac{1}{4N^2l^2} \left\{ \prod_{u=v, u'} \Omega \left( \frac{\partial^2}{\partial A_{u,v} \partial A_{u',v}} \right) \right\} Z[A],
\]

where \( \text{tr} \) stands only for traces over retarded-advanced replica indices and integration over \( r \). In order to restore the correct structure of the prefactor in front of the Green functions product which is defined in eq. (3) we introduce the matrix \( \Omega = \sigma_0 \otimes y \) where \( \sigma_0 \) and \( y \) are, respectively, the \( 2 \times 2 \) unit matrix and a matrix with all entries equal to 1.

Introducing a new field \( \bar{Q} \) of the same rank and symmetry as \( \bar{\eta} \otimes \eta \) we perform the Hubbard-Stratanovich transformation on the quartic term (8). The charge-conjugation symmetry ensures the invariance with respect to the transformation \( Q = C\bar{Q}^*C^{-1} \) so that the Hermitian matrix \( Q \) can be expressed as

\[
Q = \begin{pmatrix}
\begin{array}{cccc}
 d_{1\uparrow} & d_{1\downarrow} & -c_{1\downarrow} & c_{1\uparrow} \\
 d_{2\uparrow} & d_{2\downarrow} & -c_{2\downarrow} & c_{2\uparrow} \\
 -c_{1\uparrow} & -c_{1\downarrow} & -d_{1\uparrow} & d_{1\downarrow} \\
 -c_{2\uparrow} & -c_{2\downarrow} & -d_{2\uparrow} & d_{2\downarrow}
\end{array}
\end{pmatrix},
\]

where the arrows denote the two electron states with the spins up and down. This takes automatically into account the existence of two pairing channels: i) Diffusion modes \( d \) corresponding in the standard diagrammatics to the ladder diagrams with a small transfer momentum; ii) Cooperon modes \( c \) corresponding to the ladder diagrams with a small sum momentum [2]. Instead of the \( \uparrow, \downarrow \) basis of eq. (11), we now switch to the natural spin basis of the Diffusion and Cooperon Singlets and Triplets modes defined by \( d_S = \frac{1}{\sqrt{2}}(d_{\uparrow\downarrow} + d_{\downarrow\uparrow}) \), \( d_T = \{d_{\uparrow\downarrow}, \frac{\text{Tr}}{2}(d_{\uparrow\downarrow} - d_{\downarrow\uparrow}), d_{\uparrow\downarrow} \} \), \( c_S = \frac{1}{\sqrt{2}}(c_{\uparrow\downarrow} - c_{\downarrow\uparrow}) \), and \( c_T = \{c_{\uparrow\downarrow}, \frac{\text{Tr}}{2}(c_{\uparrow\downarrow} + c_{\downarrow\uparrow}), c_{\uparrow\downarrow} \} \).

Integrating out the bispinor fields \( \eta \) we exclude all “fast” modes and obtain the free energy in terms of “slow” degrees of freedom \( Q \)

\[
\mathcal{F} = \int \left\{ \frac{\pi \nu_0}{8\tau_v} Q K K^\dagger - \frac{1}{2} \mathcal{Tr} \ln \left[ \frac{\xi + iQ/2\tau_v}{\text{[\[Q]]}_+} \right] \right\},
\]

where \( K \) satisfies \( P_{ji}^{k} K_{km} = \delta^j_{\epsilon} \delta^m_{\epsilon} \) and \( [\[Q]]_+ \) stands for an anticommutator.

**Crossover between the orthogonal and unitary universality classes.** The first step in elucidating the physics of low-energy excitations is to find out the classical solution corresponding to a spatially homogenous field \( Q \). The corresponding saddle point equation for \( A = 0 \) reads

\[
\pi \nu_0 K = i\left[ \xi + iQ/(2\tau_v) \right]^{-1}.
\]

Equation (13) can easily be solved in the limit of no magnetic disorder \( 1/\tau_v \rightarrow 0 \) which corresponds to the orthogonal universality class. In this limit the homogeneous solutions satisfies the standard conditions \( Q^2 = 1 \) and \( \mathcal{Tr} Q = 0 \) provided that \( \tau_v = \tau_v \). One can factorize out the spin degrees of freedom so that the saddle point manifold can be parameterized by \( Q = U^\dagger U \) where \( U \) belongs to the coset space \( Sp(4N)/Sp(2N) \oplus Sp(2N) \) with \( Sp(2N) \) being the symplectic group. This insures that \( U^\dagger U = 1 \) and that \( U \) is a real quaternion matrix, i.e. it can be expressed as \( 4N \times 4N \) matrix with elements being a linear combination of quaternions with real coefficients (see, e.g., [11]).

The presence of magnetic disorder, i.e. finite \( \tau_v \), breaks the symmetry and results in a gap for some diffusion
modes. Nevertheless, we can still enforce \( Q^2 = 1 \) and \( \text{Tr} Q = 0 \) for the classical solution that, however, leads to an additional condition

\[
K Q = Q. \tag{14}
\]

This requirement controls the lowering of the symmetry from the orthogonal class to unitary one for \( q_{uu'} = 1 \). Indeed, the matrix \( K \) has 4 different eigenvalues out of 16 that correspond to the Singlets and Triplets components of usual Diffuson and Cooperon modes

\[
\lambda^A_B = \frac{1}{\tau_\kappa} + \kappa^A_B \frac{\tau_\kappa}{\tau_\xi}, \tag{15}
\]

where \( \kappa^A_S = 1, \ k^A_D = -1/3, \ k^C_S = -1, \ k^C_D = 1/3 \). The magnetic disorder cuts all those diffusion modes whose eigenvalues are different from 1. In the unitary ensemble the only massless mode is the Diffusion Singlet so that \( \lambda^D_S = 1 \). Through this requirement we recover as expected the Matthiessen rule \( 1/\tau_e = 1/\tau_x + 1/\tau_s \). Hence this rule appears in our formalism via a proper chosen normalization of the field \( Q \).

Having determined the classical solution for finite \( \tau_s \), we can now expand (12) around it. The saddle-point manifold can be parameterized by \( Q = (1 - W/2)^\Lambda (1 - W/2)^{-\Lambda} \) where \( \Lambda = \text{diag}(18\gamma, -1 \kappa L, \gamma_\nu \nu) \) and \( W \) is an anti-Hermitian matrix anticonmuting with \( \Lambda \).

Performing a gradient expansion around the above homogeneous saddle point we rewrite the effective action as

\[
\mathcal{F}[W, 0] = -\frac{\pi \nu_0}{8} \int \tau_e \left[ \frac{1}{\tau_e} W AMW + D(\partial W)^2 \right], \tag{16}
\]

where \( D = \nu_0^2 \tau_e / d \) is the usual diffusion constant and we introduced the matrix \( M^A_B(u, u') = K^A_B(u, u') - \frac{1}{2}(K^A_B(u, u) + K^B_B(u, u)) \). Note that the free energy (12) is invariant under the gauge transformation \( Q \rightarrow U^\dagger QU \) and \( A \rightarrow U^\dagger AU + U^\dagger QU \) [8]. This gauge symmetry ensures that the dependence of (12) on \( A \) enters only in the combination \( \partial Q - [A, Q] \), and thus, this dependence can be easily restored in eq. (16). The matrices \( W \) are of the form \( W = \text{offdiag}(B, -B^T) \) with \( B \) satisfying the charge-conjugation symmetry \( B = C B^* C^T \). To parametrize \( B \), we introduce a generalization of the standard quaternion basis to the bispinor space [11] by defining the following basis \( \phi_{\mu \nu} = e_{\mu \nu} \sigma_\mu \otimes \sigma_\nu \), where \( c = i \), for \( 0 \leq \mu \leq 2 \) \( \leq 3 \) and for \( 3 \leq \mu \leq 0 \), and 1, otherwise. These matrices \( \phi_{\mu \nu} \) form a complete set and satisfy the charge-conjugation symmetry and relation \( \text{Tr}[\phi_{\mu \nu} \phi_{\mu ' \nu '}] = 4 \delta_{\mu \mu '}. \) In the resulting decomposition \( B = \sum b_{\mu} b^\dagger_{\nu} \) and \( B^T = \phi^T b^\dagger b^T \), the \( b_{\mu} \) are now real matrices. Using this decomposition of matrices \( B \), we obtain the following quadratic part of the free energy in terms of real variables \( b_{\mu} \):

\[
\mathcal{F}[b, 0] = \pi \nu_0 \sum \{ \gamma_{\mu} + D q^2 \} b_{\mu}^\dagger b_{\nu}^\dagger b_{\nu}^\dagger b_{\mu}, \tag{17}
\]

In deriving this expression, we used that \( \phi_{\mu} \) naturally diagonalize the matrix \( M \), and satisfy the identity \( \phi_{\mu}^T M \phi_{\nu} = 4 \tau_e \delta_{\mu \nu} \gamma_{\mu} \). As expected, among all masses \( \gamma_{\mu} \) there are only 4 different values

\[
\gamma^A_B(u, u') = \gamma_{\mu} + 1 - \kappa^A_B q_{uu'}, \tag{18}
\]

which are the dephasing rates of Singlets and Triplets components of Diffuson and Cooperon modes. These rates are computed here beyond the previous first-order expansion in \( \tau_e/\tau_x \) [3] and valid as long as the above classical solution is stable, implying at least \( \tau_x/\tau_s < 1 \) as follows from expression (18). Note that in a single disorder realization \( (q_{uu'} = 1) \), the above results identify with results from diagrammatic theory even beyond the first perturbative order in \( \tau_e/\tau_s \), as can be inferred from [2].

The expressions (18) generalizes these results to dephasing rates between different magnetic disorder configurations \( (q_{uu'} \neq 1) \), which are now all finite.

**Correlation of conductance.** – As an example of application of our formalism we derive the correlation of conductance between two different magnetic disorder configurations. The necessary terms of order \( \nu_0^2 \) in the generating functional read to one loop order: \( \mathcal{Z} = \int [\nu_0 D]^2 [\nu_0 F_11 + F_22 + 2F_12] \) with \( \nu_0 = \int \text{Tr}[W W A A] \) and \( F_12 = \int \text{Tr}[W W A A] \). Expressing \( W \) in the basis of \( \phi_{\mu} \), performing Wick’s contractions of \( b_{\mu} \) and substituting in eq. (10) we obtain the correlation of conductance. The result can be conveniently written in terms of the four dephasing lengths given by \( L^A_B(u, u') = \sqrt{D/(2\gamma^A_B)} \) and \( L_s = \sqrt{D/\tau_s} \):

\[
(\Delta G)^2_{uu'} = \left( G(V, \bar{U}_0)G(V, \bar{U}_0) - G(V, \bar{U}_0) \right)^2 = f \left( \frac{L_{\tau_x}}{L_s} \right) + \frac{f \left( \frac{L_{\tau_x}}{L_s} \right)}{3} + f \left( \frac{L_{\tau_x}}{L_s} \right) \tag{19}
\]

with \( f(x) = 3 \sum_{\nu \neq 0} [L_{\nu}^2 + x^2/2]^{-2} \). Specifying this result to the case \( d = 1 \) of a diffusive wire [14], we use \( f(x) = 3\nu_0^2 [x^2 \cosh (x/2)^2 + \sqrt{2}x \coth (x/2^2) - 4] \). For \( q_{uu'} = 1 \) the scaling function (19) shown for \( d = 1 \) in fig. 1 gives the sample to sample conductance fluctuations and describes the crossover between the orthogonal and unitary universality classes. This function extrapolates between the magnitudes of the universal conductance fluctuations in the systems without and with frozen magnetic impurities. The inset shows that all four terms contribute to the scaling function but the correction to the unitary fixed point is dominated by Diffuson and Cooperon Triplets. In fig. 2 we plot the average correlation between conductances for different magnetic disorder realizations, experimentally measurable through correlation of magnetococonductances [3]. In the typical regime of wire length \( L \) large compared to magnetic dephasing lengths of order \( L_s \), the correlations of conductance decay as we lower the overlap between the corresponding spin configurations.
the different Cooperon/Diffuson modes for late between the usual universal values 8/15 (orthogonal class) perturbative dephasing results of [3]. These functions extrapolate between the usual universal values 8/15 (orthogonal class) and 1/15 (unitary class). The inset shows the contributions of the 4 different Cooperon/Diffuson modes for $\tau_c/\tau_s = 0.2$.

Moreover, this decay as a function of $1 - q$ is dominated by the dephasing of the Diffuson Singlet contribution, while other modes almost compensate for each other (see the left side of fig. 2). Note that in the opposite regime $L \ll L_s$, anomalous behavior can appear with a $q_{12}$ dependence of $\langle \Delta G \rangle_{12}^2$ dominated by the Diffuson Triplet and Cooperon Singlet contributions leading to a (small) increase as a function of $1 - q_{12}$.

Let us also stress that this monotonous decrease of $\langle \Delta G \rangle_{12}^2$ in the experimental regime of interest [15] $L \gtrsim L_s$ allows for an interesting and unique test of a spin glass mean-field theory. Indeed, this theory predicts the ultrametricity of the spin glass phase space in the thermodynamic limit. According to this prediction, if we consider three spin configurations (or three $\hat{U}_n$), and sort their mutual overlap according to $1 - q_{12} \geq 1 - q_{13} \geq 1 - q_{23}$, then $1 - q_{12} = 1 - q_{13}$. This condition easily translate into the practical test $\langle \Delta G \rangle_{12}^2 = \langle \Delta G \rangle_{13}^2$ if $\langle \Delta G \rangle_{12}^2 \leq \langle \Delta G \rangle_{13}^2 \leq \langle \Delta G \rangle_{23}^2$.

Finally, we would like to note that the above results are valid in the regime of coherent transport, i.e. for samples with $L < L_0$, where $L_0 = \sqrt{D/(2\gamma u)}$ is the inelastic scattering length. This $L_0$ includes in particular contributions from possible rare spin flips in the spin glass phase. As was shown experimentally in ref. [15] one can indeed observe the coherent electronic transport below the spin glass transition temperature $T_{SG}$ in CuMn which has relatively large $D$. This experimental observation justifies the present study of coherent transport in the spin glass phase.

Conclusions. – We have shown how to account naturally for the magnetic dephasing of diffusing electrons within the usual nonlinear sigma-model. Motivated by the study of mesoscopic spin glass wires, we have used this formalism to study the relative dephasing rates between different magnetic disorder configurations, and the corresponding correlations of conductance fluctuations amenable to direct experimental measures. Let us finally stress that an advantage of this field-theoretical method is its flexibility, allowing for interesting extensions including the incorporation in our approach of more complex statistical correlations of spin configurations, along the lines of [5], as well as higher moments of the conductance correlations.

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