Cosmological reconstruction and Om diagnostic analysis of Einstein-Aether theory

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Abstract. In this paper, we analyze the cosmological models in Einstein-Aether gravity, which is a modified theory of gravity in which a time-like vector field breaks the Lorentz symmetry. We use this formalism to analyse different cosmological models with different behavior of the scale factor. In this analysis, we use a certain functional dependence of the Dark Energy (DE) on the Hubble parameter $H$. It will be demonstrated that the Aether vector field has a non-trivial effect on these cosmological models. We also perform the Om diagnostic in Einstein-Aether gravity and we fit the parameters of the cosmological models using recent observational data.

Keywords: modified gravity, quantum gravity phenomenology, supernova type Ia - standard candles

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1 Introduction

The Lorentz symmetry is one of the most important symmetries in nature, and all particle physics experiments have demonstrated that this symmetry is not broken at the scale at which such experiments are performed. However, it is predicted from quantum gravity that the Lorentz symmetry should break down at Planck scale, where even the manifold structure of spacetime breaks down due to quantum fluctuations. In fact, almost all approaches to quantum gravity predict that the local Lorentz symmetry of spacetime only exists in some infrared (IR) limit of the theory. So, the Lorentz symmetry is expected to break in the UV limit. It has been explicitly demonstrated that such a breaking of Lorentz symmetry in the ultraviolet (UV) limit occur in the discrete spacetime [1], models based on string field theory [2], spacetime foam [3], spin-network in Loop Quantum Gravity (LQG) [4], non-commutative geometry [5, 6] and ghost condensation in perturbative Quantum Gravity [7].

As the Lorentz symmetry fixes the form of the energy-momentum dispersion relation, the breaking of Lorentz symmetry in the UV limit will also lead to a modification of the energy-momentum dispersion relation in the UV limit. In fact, there are indications from the Greisen-Zatsepin-Kuzmin limit (GZK limit) the usual energy-momentum relation will get modified in the UV limit [8, 9]. The Pierre Auger Collaboration and the High Resolution Fly’s Eye (HiRes) experiment have confirmed earlier results of the GZK cut-off [10]. So, it is possible that the Lorentz symmetry will break in the UV limit and only occur as an effective symmetry in the IR limit. Thus, it is important to construct a theory such that it will reproduce the general relativity in the IR limit, and break the local Lorentz symmetry in the UV limit. Such a theory has been constructed by using different Lifshitz scaling for space and time and this theory is known as Horava-Lifshitz gravity [11, 12]. This original proposal for the Horava-Lifshitz gravity improves the renormalization of gravity, as it differs from general relativity in the UV limit. However, there are several problems associated with this proposal, and these include the problems associated with instabilities, overconstrained evolution and strong coupling at low energies [13]–[18]. These problems occur due to a badly behaved scalar mode of gravity, which is produced by the presence of a nondynamical spatial foliation in the action. To resolve this problem, an extension of Horava-Lifshitz gravity called the BPSH theory has been proposed [19]. It has been demonstrated that the BPSH theory
is equivalent to general relativity coupled to a dynamical unit timelike vector field [20]. Here the vector is restricted in the action to be hypersurface orthogonal.

The phenomenology and observational constraints on the coupling parameters of Einstein-Aether gravity have been studied [21]. It may also be noted that constraints on Einstein-Aether gravity from binary pulsars have also been discussed [22]. In this work, the consequences of Lorentz symmetry, which occur in Einstein-Aether gravity, on the orbital evolution of binary pulsars. In the focus of this study was on the dissipative effects in such a process. It was observed that the breaking of the Lorentz symmetry modified such effects. Thus, the orbital dynamics of binary pulsars was also modified in Einstein-Aether gravity. Such a modification causes the emission of dipolar radiation, and this made the orbital separation decrease faster than in general relativity. The quadrupole component of the emission was also modified. The orbital evolution depends critically on the sensitivities of the stars, as this measure how their binding energies depend on the motion relative to the preferred frame. In this study such sensitivities have also been numerically calculated in Einstein-Aether gravity. These predictions have been compared with observations and this has been used to set constraints on Einstein-Aether gravity.

It has been demonstrated that the Einstein-Aether theory can be analysed in the framework of the metric-affine gravity [23]. Such a formalism resembles the gauge theory of theory. In this formalism, the Aether vector field is related to certain post-Riemannian nonmetricity pieces contained in an independent linear connection of spacetime. Black hole solution have also been studied in the Einstein-Aether gravity. It has been demonstrated that the deviations from the Schwarzschild metric are typically only a few percent for most of the explored parameter regions, and this makes it difficult to observe with electromagnetic probes, but they can be detected using gravitational wave detectors [24]. As gravitational wave detectors are going to be used extensively in future astronomy, it is interesting to study the implications of Einstein-Aether gravity.

Since the Einstein-Aether gravity introduces a time like vector field, it is expected to modify the cosmological evolution of the Universe. In fact, various different solutions for the accelerating Universe in the Einstein-Aether gravity have been studied [25]. These solutions have been used to analyse the inflationary behaviour of the early Universe and late-time cosmological acceleration. It has been demonstrated that the Aether field produces accelerated expansion in situations where inflation would not occur in general relativity. Hence, the Aether field can effect the inflation in a very non-trivial way. The cosmological evolution of cosmological models based on Einstein-Aether gravity with power-law potential have also been studied [26]. Cosmological models have also been studied in the Einstein-Aether gravity coupled to a Galileon type scalar field [27]. It was observed that in such models the Universe experiences a late time acceleration for pressure-less baryonic matter. The gravitational waves can be used to analyse the cosmological aspects of Einstein-Aether gravity since it has been demonstrated that, for cosmological models based Einstein-Aether gravity, a direct correspondence exists between perfect fluids carrying anisotropic stress and a modification in the propagation of gravitational waves [28]. As the anisotropic stress can be measured in a model-independent manner, the gravitational waves can be used to obtain constraints on the cosmological models in the Einstein-Aether gravity. Even though several studies have been done on the Einstein-Aether gravity, it is important to perform an extensive study of how this modification of general relativity can change cosmological models and how it fits with the current data. This is important as many aspects of Einstein-Aether gravity can be detected using gravitational waves, and in near future, it is expected
that the gravitational waves will be used to study many of these interesting phenomena. So, in this paper, we perform a detailed study on the modification of different cosmological models from Einstein-Aether gravity. These models have been studied in general relativity, and we will analyse them in the framework of Einstein-Aether gravity. We will first use the reconstruction technique to find some viable forms for Einstein-Aether gravity. We obtain the expressions of the modified Friedmann equations and from these equations, we can find the effective density and the effective pressure for the Einstein-Aether gravity. We will also fit the model with observational data by using the cosmographic analysis involving the $Om$ parametrization. We will also use the SNIa, BAO and Hubble data to find the $1\sigma$ and $2\sigma$ contours for density parameter $\Omega_m$.

2 Einstein-Aether gravity

In this section, we review the main features of the Einstein-Aether gravity. The Einstein-Aether gravity is equivalent to the BPSH generalization of the Horava-Lifshitz gravity [20], in which the Lorentz symmetry is broken. However, the cosmology described by this theory would still be described by the standard Friedmann equations with an additional matter contribution [25]. Since the breaking of Lorentz symmetry occurs due to a time-like vector field in the Einstein-Aether theory, the cosmological effects can be obtained by analysing the correction to the standard Friedmann equations from this additional time-like vector field. The action $S$ of the Einstein-Aether gravity is given by [29, 30],

$$ S = \int d^4x \sqrt{-g} \left[ \frac{R}{4\pi G} + L_{\text{EA}} + L_m \right], \quad (2.1) $$

where the quantity $L_{\text{EA}}$ indicates the Lagrangian density of the Aether vector field while the quantity $L_m$ indicates the Lagrangian density of the usual matter fields. Moreover, $g$ indicates the determinant of the metric tensor $g^{\mu\nu}$, $R$ indicates the Ricci scalar and $G$ is the gravitational constant.

The Lagrangian density of the Aether vector field $L_{\text{EA}}$ can be expressed as follows:

$$ L_{\text{EA}} = \frac{M^2}{16\pi G} F(K) + \frac{1}{16\pi G} \lambda(A^a A_a + 1), \quad (2.2) $$

$$ K = M^{-2} K_{cd} \nabla_a A^c \nabla_b A^d, \quad (2.3) $$

$$ K_{cd} = c_1 g^{ab} g_{cd} + c_2 \delta^a_c \delta^b_d + c_3 \delta^a_d \delta^b_c, \quad a, b = 0, 1, 2, 3 \quad (2.4) $$

where $c_1$, $c_2$ and $c_3$ are three dimensionless constant parameters, $M$ is a coupling constant parameter, $\lambda$ is a Lagrangian multiplier, and $A^a$ is a contravariant vector. Here $F(K)$ is an arbitrary function of the parameter $K$, and a function of the Hubble parameter $H$.

Now using eq. (2.1), the field equations, for this theory, can be written as [29, 30]

$$ G_{ab} = T_{ab}^{\text{Einstein-Aether}} + 8\pi G T_m^{ab}, \quad (2.5) $$

$$ \nabla_a (F' J^a_b) = 2\lambda A_b, \quad (2.6) $$

where

$$ F' = \frac{dF}{dK}, \quad (2.7) $$

$$ J^a_b = -2K_{bc} \nabla_d A^e, \quad (2.8) $$
where a prime indicates a derivative with respect to $K$. Moreover, $T_{ab}^m$ represents the energy-momentum tensors for the matter field and $T_{ab}^{\text{Einstein-Aether}}$ is the energy-momentum tensor for the Aether vector field and they are defined as follows:

\begin{align}
T_{ab}^m &= (p + \rho) u_a u_b + p g_{ab}, \\
T_{ab}^{\text{Einstein-Aether}} &= \frac{1}{2} \nabla_d \left[ \left( J^d A_b - J^d_a A_b - J_{(ab)} A_d \right) F^d \right] \\
&\quad - Y_{(ab)} F' + \frac{1}{2} g_{ab} M^2 F + \lambda A_a A_b,
\end{align}

where $\rho$ indicates the energy density while $p$ is the pressure of the matter. Moreover, $u_a$ is defined as $u_a = (1, 0, 0, 0)$ and it represents the four-velocity vector of the fluid and $A^a = (1, 0, 0, 0)$, i.e., $A^a$ is represented by a time-like unitary vector. Furthermore, $Y_{ab}$ is defined as follows:

\begin{equation}
Y_{ab} = -c_1 \left[ (\nabla_d A_a) (\nabla^d A_b) - (\nabla_a A_d) (\nabla^d A^a) \right].
\end{equation}

The subscript $(ab)$ indicates a symmetry with respect to the two indices.

We consider a Friedmann-Robertson-Walker (FRW) metric, which is given by:

\begin{equation}
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right],
\end{equation}

where $a(t)$ represents the scale factor (which contains useful information about the expansion rate of the Universe), $t$ is the cosmic time, and $k$ is the curvature parameter. The values $k = -1, k = 0,$ and $k = +1$ corresponding to an open, a flat, and a closed Universe, respectively. The range of $\theta$ and $\phi$ are $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The four coordinates $(r, t, \theta, \phi)$ are also known as co-moving coordinates.

Using eqs. (2.3) and (2.4), we can easily obtain the following general expression for the parameter $K$ [29, 30]:

\begin{equation}
K = \frac{3\varepsilon H^2}{M^2},
\end{equation}

where $\varepsilon$ indicates a constant parameter while $H = \dot{a}/a$ is the Hubble parameter. This $K$ is different from the metric parameter $k$ and is a feature of Einstein-Aether gravity. So, using eq. (2.5), we obtain the Friedmann equations modified by the Einstein-Aether gravity as follows:

\begin{align}
\varepsilon \left( -F' + \frac{F}{2K} \right) H^2 + \left( H^2 + \frac{k}{a^2} \right) &= \left( \frac{8\pi G}{3} \right) \rho, \\
\varepsilon \frac{d}{dt} (HF') + \left( -2\dot{H} + \frac{2k}{a^2} \right) &= 8\pi G (p + \rho).
\end{align}

The conservation equation can now be written as follows:

\begin{equation}
\dot{\rho} + 3H (p + \rho) = 0,
\end{equation}

where an overdot indicates a temporal derivative.

We denote the effective energy density in Einstein-Aether gravity by $\rho_{EA}$ and the effective pressure in Einstein-Aether gravity by $p_{EA}$. So, we can rewrite eqs. (2.14) and (2.15) as
follows:

\[
\left( H^2 + \frac{k}{a^2} \right) = \frac{8\pi G}{3} \rho + \frac{1}{3} \rho_{\text{EA}},
\]

(2.17)

\[
\left( -2 \dot{H} + \frac{2k}{a^2} \right) = 8\pi G (\rho + \rho_{\text{EA}}) + \rho_{\text{EA}}. 
\]

(2.18)

Therefore, comparing eqs. (2.14) and (2.15) with eqs. (2.17) and (2.18), we can write

\[
\rho_{\text{EA}} = 3 \varepsilon H^2 \left( F' - \frac{F}{2K} \right),
\]

(2.19)

\[
p_{\text{EA}} = -3 \varepsilon H^2 \left( F' - \frac{F}{2K} \right) - \varepsilon \left( \dot{H} F' + H \dot{F}' \right)
\]

\[= -\rho_{\text{EA}} - \frac{\dot{\rho}_{\text{EA}}}{3H}.
\]

(2.20)

Using eq. (2.19), we obtain the following differential equation:

\[
F' - \frac{F}{2K} = \frac{\rho_{\text{EA}}}{3 \varepsilon H^2},
\]

(2.21)

which is equivalent to the following master equation (using the expression of \(3 \varepsilon H^2\) derived from eq. (2.13)):

\[
F' - \frac{F}{2K} = \frac{\rho_{\text{EA}}}{KM^2}.
\]

(2.22)

Using the expressions for \(\rho_{\text{EA}}\) and \(p_{\text{EA}}\) given by eqs. (2.19) and (2.20), we obtain that the EoS parameter \(\omega_{\text{EA}}\) for the Einstein-Aether gravity is given by the following relation:

\[
\omega_{\text{EA}} = \frac{\rho_{\text{EA}}}{p_{\text{EA}}} = -1 - \frac{\left( \dot{H} F' + H \dot{F}' \right)}{3H^2 \left( F' - \frac{F}{2K} \right)}.
\]

(2.23)

### 3 Models for dark energy

The astrophysical data obtained from distant Supernovae Ia, Large Scale Structure (LSS), Baryon Acoustic Oscillations (BAO), weak lensing and Cosmic Microwave Background (CMB) radiation clearly indicate the existence of Dark Energy [31–40]. In this paper, we analyse some Dark Energy models using Einstein-Aether gravity. The effective density and the effective pressure produced by Einstein-Aether gravity can be used to generate dark energy if the condition \(\rho_{\text{EA}} + 3p_{\text{EA}} < 0\) is satisfied (i.e., if the strong energy condition is violated). Thus, we obtain:

\[
2H^2 \left( F' - \frac{F}{2K} \right) > - \left( \dot{H} F' + H \dot{F}' \right),
\]

(3.1)

which is the equation can be used to analyse Dark Energy in Einstein-Aether gravity. So, in order to analyse the effects of Dark Energy on cosmological models in the Einstein-Aether gravity, we can use \(F(K)\) and \(\omega_{\text{Einstein-Aether}}\). The modified effective Friedmann equation can be written as

\[
8\pi G \rho_{\text{dark energy}} = \sum_{\Sigma n_i = n} A(X, Y, \ldots) \frac{\partial^n f(X, Y, \ldots)}{\partial X^{n_1} \partial Y^{n_2} \ldots}.
\]

(3.2)
where \( f(X, Y, \ldots) \) is the matter part \( K \) action. For a model of Dark Energy, for example the Holographic Dark Energy, we have \( \rho_{\text{dark energy}} = \rho_{\text{dark energy}}(H, \dot{H}, \ldots) \), which can also be written as \( \rho_{\text{dark energy}} = \rho_{\text{dark energy}}(X, Y, \ldots) \). So, if we can solve the partial differential equations for \( f(X, Y, \ldots) \), we can obtain the effect of dark energy on such cosmological models.

The generalized Nojiri-Odintsov Holographic dark energy models can be used to analyze the dependence of the dark energy on the Hubble parameter \([59]\). In fact, using the Granda-Oliveros model \([41]\), which is a specific kind of Nojiri-Odintsov Holographic dark energy model, we can write the cut-off of the system as follows:

\[
L_{\text{GO}} = \left( \alpha H^2 + \beta \dot{H} \right)^{-\frac{1}{2}}, \tag{3.3}
\]

where \( H = \dot{a}/a \) is the Hubble parameter, and \( \dot{H} \) is the temporal derivative of \( H \). This model is characterized by two constant parameters, \( \alpha \) and \( \beta \). As the dark energy dominates the present cosmological epoch while its contribution to cosmological epoch as near the Big Bang was negligible (i.e. the amount of dark energy increased with the expansion of the Universe), the energy density can be assumed to be a function of the Hubble parameter \( H \) and its temporal derivative. Such a dependence is characterized by these two parameters. There are other physical reasons which motivate the Granda-Oliveros model. In fact, if the IR cut-off chosen is given by the particle horizon, the Holographic Dark Energy models are not able to produce an accelerated expansion for the present cosmological epoch. However, if the future event horizon is used as the IR cut-off, then the Holographic dark energy models have a problem with causality. It is possible to resolve both these problems by using Granda-Oliveros model \([41]\).

In the limiting case corresponding to \( \{\alpha, \beta\} = \{2, 1\} \), \( L_{\text{GO}} \) becomes proportional to the Ricci scalar curvature, i.e. \( L_{\text{GO}} \propto R \).

In this paper, we want to analyze the effect of Einstein-Aether gravity on the cosmological evolution using the Granda-Oliveros model. We will analyze this for different models for the evolution of the scale factor of the Universe, and analyze such a model for the observationally motivated values of \( \alpha \) and \( \beta \).

The Granda-Oliveros model has been generalized to a Chen-Jing model \([43]\). In this cosmological model, the Dark Energy density is a function of the Hubble parameter squared (i.e. \( H^2 \)) and of first and second temporal derivatives of \( H \), i.e. \( \dot{H} \) and \( \ddot{H} \):

\[
\rho_D = 3c^2 \left( \alpha H^2 + \beta \dot{H} + \gamma \ddot{H}^2 \right), \tag{3.7}
\]
where $\alpha$, $\beta$ and $\gamma$ represent three arbitrary dimensionless parameters. The inverse of the Hubble parameter, i.e. $H^{-1}$, is introduced in the first of the three terms of eq. (3.7) so that each of these three terms have the right dimensions.

In the limiting case corresponding to $\alpha = 0$, we recover the energy density of dark energy given by the Granda-Oliveros model [44, 45]. Furthermore, in the limiting case of $\alpha = 0$, $\beta = 1$ and $\gamma = 2$, we obtain the expression of the energy density of dark energy with the IR cut-off proportional to the average radius of the Ricci scalar curvature, $L \propto R^{-1/2}$ (when $k = 0$). We also have that, in eqs. (3.6) and (3.7), $c^2$ indicates a dimensionless parameter of the order of the unity.

In this paper we want to analyze this model for various cosmological models with different expressions of the evolution of the scale factor. We will obtain general expression for various parameters for this dark energy model and they can be compared to various observational data.

4 Cosmological models

In this section, we will analyze the behavior of various cosmological models in Einstein-Aether gravity. These will correspond to different evolution of the scale factor. We will analyze them for the Granda-Oliveros model and the Chen-Jing model using the data obtained from observations. We start considering the the power-law cosmology. This cosmological model is an interesting proposal for the evolution of the scalar factor and it has been motivated by the existence of the flatness and horizon problems in the standard cosmology [46]. In this cosmological model, it is possible to assume the following form for the evolution of the scale factor:

$$a(t) = a_0 t^m. \tag{4.1}$$

where the quantity $a_0$ indicates the present day value of the scale factor $a(t)$. It is also important to only consider $m > 0$ since only for this case the model will be able to produce an accelerating Universe [46]. Now for $m > 1$, the power-law cosmology can solve the horizon problem, the flatness problem and the problem associated with the age of the early Universe [47, 48]. The power-law cosmology has been used for analysing the cosmological behavior in modified theories of gravity [49, 50]. Now, for Einstein-Aether gravity, it is possible to analyze the Granda-Oliveros cut-off for power-law cosmological model. Thus, for a non-flat Universe, we obtain the following expression:

$$F(K) = \frac{2c^2 K (0.8824m - 0.5016)}{\varepsilon m} + C_1 \sqrt{K}. \tag{4.2}$$

Instead, for a flat Universe, we obtain the following relation:

$$F(K) = \frac{2c^2 K (0.8502m - 0.4817)}{\varepsilon m} + C_1 \sqrt{K}. \tag{4.3}$$

It is also possible to analyze the power-law cosmology using the Chen-Jing model, we derive this expression for $F(K)$:

$$F(K) = \frac{2c^2 K [2\alpha + m(m\beta + \gamma)]}{\varepsilon m^2} + C_2 \sqrt{K}. \tag{4.4}$$

In this section, $C_i$ will denote integration constants.
It is also possible to consider a different kind of power law given by the following relation [51, 52]:

\[ a(t) = a_0(t_s - t)^{-n}, \] (4.5)

where \( n > 0 \) and \( t < t_s \). Such models have a future singularity at finite time and this is denoted by \( t_s \). The Granda-Oliveros cut-off for such model can be analyzed for both a flat Universe and a non-flat Universe. For a non-flat Universe, we obtain the following relation:

\[ F(K) = \frac{2c^2K(0.8824n + 0.5016)}{\varepsilon n} + C_3\sqrt{K}. \] (4.6)

Instead, for a flat Universe, we obtain the following relation:

\[ F(K) = \frac{2c^2K(0.8502n + 0.4817)}{\varepsilon n} + C_3\sqrt{K}. \] (4.7)

Moreover, using the Chen-Jing model, we obtain that \( F(K) \) is given by the following expression:

\[ F(K) = \frac{2c^2K[2\alpha + n(\beta + \gamma)]}{\varepsilon n^2} + C_4\sqrt{K}. \] (4.8)

It is also possible to analyze the intermediate inflation in Einstein-Aether gravity. The intermediate inflation has been used to obtain exact analytic solutions for a given class of potentials for the inflation. The scale factor for intermediate inflation can be expressed as follows [53, 54]:

\[ a(t) = e^{B_t^\theta}, \] (4.9)

where \( B > 0 \) and \( 0 < \theta < 1 \). For the Granda-Oliveros mode, we have that, for a non-flat Universe, \( F(K) \) can be expressed as follows:

\[
F(K) = 2(B\varepsilon\theta)^{-1}c^2K \left( \frac{KM^2}{3B^2\varepsilon\theta^2} \right)^{\frac{-\theta}{2(1+\theta)}} \\
\times \left[ -0.5016(-1 + \theta)^2 + 0.8824B\theta \left( \frac{KM^2}{3B^2\varepsilon\theta^2} \right)^{\frac{-\theta}{2(1+\theta)}} \right] \\
+ \sqrt{KC_7}. \] (4.10)

Instead, for a flat Universe, we have that:

\[
F(K) = 2(B\varepsilon\theta)^{-1}c^2K \left( \frac{KM^2}{3B^2\varepsilon\theta^2} \right)^{\frac{-\theta}{2(1+\theta)}} \\
\times \left[ -0.4817(-1 + \theta)^2 + 0.8502\theta \left( \frac{KM^2}{3B^2\varepsilon\theta^2} \right)^{\frac{-\theta}{2(1+\theta)}} \right] \\
+ \sqrt{KC_7}. \] (4.11)
Moreover, for the Chen-Jing model, we obtain the following relation:

\[
F(K) = \varepsilon^{-1} 2^{-\theta} (\frac{1}{-1+\theta})^{-2\theta} - 2^{1-\theta} (\frac{1}{-1+\theta})^\theta c^2 K \left( \frac{KM^2}{B^2 \varepsilon \theta^2} \right)^{4\theta} \left( \frac{-1}{-1+\theta} \right)^{-2\theta} \\
\times \left\{ \frac{2^\theta \beta}{1+\theta} \left( \frac{KM^2}{B^2 \varepsilon \theta^2} \right)^{2\theta} \left( \frac{-1}{-1+\theta} \right)^{1+2\theta} + 4 \left( \frac{-1}{-1+\theta} \right)^{3\theta} + \frac{2^\theta 3^{-\theta} (\frac{-1}{-1+\theta})^\theta \gamma}{1+\theta} \left( \frac{1}{1+\theta} \right)^{-1+2\theta} (\frac{KM^2}{B^2 \varepsilon \theta^2})^{2\theta} \left( \frac{-1}{-1+\theta} \right)^{\theta} + \frac{2^\theta B}{1+2^\theta} \left( \frac{1}{1+\theta} \right)^{2\theta} + 2^\theta B \left( \frac{1}{1+\theta} \right)^{2\theta} \left( \frac{-1}{-1+\theta} \right)^{\theta} \right\} \\
+ \frac{3^{2\theta} (\frac{-1}{-1+\theta})^\theta \alpha}{1+2^\theta} \left( \frac{1}{1+\theta} \right)^{-2\theta} \left( \frac{-1}{-1+\theta} \right)^{\theta} \\
\left[ 1 + 2 \left( \frac{1}{1+\theta} \right)^{-2\theta} \left( \frac{-1}{-1+\theta} \right)^{\theta} \right] \right\} \\
+ C_8 \sqrt{K}.
\]

(4.12)

It is possible to analyze models of emergent Universe, and such model do not contain the big bang singularity [55, 56]. The scale factor for such cosmological models can be expressed as follows [55, 56]:

\[
a(t) = A (B + e^{nt})^\lambda,
\]

(4.13)

where \(A, B, n\) and \(\lambda\) are four positive constant parameters. In order to avoid singularities, we have to use \(B > 0\). Furthermore, for the positivity of scale factor, we have to use \(A > 0\). In this model, for \(a < 0\) or \(\lambda < 0\), a singularity exists. So, for the expanding model, we have to only consider \(a > 0\) and \(\lambda > 0\). Using the Granda-Oliveros cut-off for this model, we can analyze a flat Universe and a non-flat Universe. So, for a non-flat Universe, we obtain the following expression for \(F(K)\) for the Granda-Oliveros model:

\[
F(K) = \frac{c^2 K \left( 1.7648\lambda - 1.0032 + 0.5016 \sqrt{\frac{3\varepsilon n^2 \lambda^2}{KM^2} \log K} \right)}{\varepsilon \lambda} \\
+ \sqrt{KC_5}.
\]

(4.14)

Instead, for a flat Universe, we have that:

\[
F(K) = \frac{c^2 K \left( 1.7004\lambda - 0.9634 + 0.4817 \sqrt{\frac{3\varepsilon n^2 \lambda^2}{KM^2} \log K} \right)}{\varepsilon \lambda} \\
+ \sqrt{KC_5}.
\]

(4.15)

We can also use the Chen-Jing model, obtaining the following relation:

\[
F(K) = c^2 (\varepsilon M^2 \lambda^2)^{-1} \left[ -6 \varepsilon n^2 \alpha \lambda^2 + 2 KM^2 (2\alpha - \gamma \lambda + \beta \lambda^2) \right. \\
\left. + \sqrt{3} KM^2 \sqrt{\frac{\varepsilon n^2 \lambda^2}{KM^2} (-3\alpha + \gamma \lambda) \log K} \right] \\
+ C_6 \sqrt{K}.
\]

(4.16)
It is possible to analyze matter dominated Universe and the accelerated phase of the Universe using a single formalism [57, 58]. In such cosmological models, the Hubble constant is given by [59, 60]: 

\[ H(t) = H_0 + \frac{H_1}{t}, \quad (4.17) \]

where \( H_0 \) and \( H_1 \) are two constant parameters. In this case, for the non-flat Universe, we obtain 

\[ F(K) = -2c^2(9\varepsilon M^2 H_1)^{-1}[-9.0288\varepsilon^2 KM^2 H_0 - 13.5431\varepsilon H_0^2 + KM^2(0.5016\varepsilon^3 KM^2 - 7.9416H_1)] + \sqrt{KC_{10}}. \quad (4.18) \]

Furthermore, for the flat Universe, we obtain 

\[ F(K) = -2c^2(9\varepsilon M^2 H_1)^{-1}[-8.6706\varepsilon^2 KM^2 H_0 - 13.0059\varepsilon H_0^2 + KM^2(0.4817\varepsilon^3 KM^2 - 7.5618H_1)] + \sqrt{KC_{10}}. \quad (4.19) \]

Now using the Chen-Jing model, we obtain 

\[ F(K) = \sqrt{K} \left[ C_{11} + \frac{2c^2M^2\alpha K^{3/2}}{\epsilon H_1^2} - \frac{c^2M^2\beta K^{3/2}}{\epsilon H_1} \right. \\
+ \frac{c^2M^2\gamma K^{3/2}}{\epsilon} - \frac{9c^2\sqrt{2}M H_0\alpha K}{\sqrt{\epsilon}H_1^2} + \frac{3c^2\sqrt{2}M H_0\beta K}{\sqrt{\epsilon}H_1} \\
+ \frac{18\sqrt{2}c^2 H_0^2 H_1^2 M \ln (K)}{H_1^2 M} - \frac{3\sqrt{2}c^2 H_0^2 H_1^2 H_1^2 \ln (K)}{H_1 M} \\
- \frac{6\sqrt{2}c^2\alpha H_0^3 \ln (K)}{H_1^2 M} \left. \right]. \quad (4.20) \]

The quantum deformed de Sitter (\( q \)-de Sitter) solution has been obtained by a quantum deformation of the quantum deformation of the conformal group [61]. In fact, the \( q \)-deformed de Sitter solution has also been used in the analyzing of dS/CFT correspondence and the entanglement entropy for such a solution has also been obtained [61]. The \( q \)-de Sitter has also been used for analyzing cosmology [62]. Now we will analyze this model for the \( q \)-de Sitter scale factor [62],

\[ a(t) = e_q(H_0 t) = \left[ 1 + (q - 1)H_0 t \right]^\frac{1}{q-1}. \quad (4.21) \]

In this model, it is possible to interpolate between the cosmological model based on a power-law and the cosmological model involving de Sitter spacetime. In fact, for early times, \( H_0 t \gg 1 \), we obtain:

\[ a_{\text{early}}(t) \sim \left[ H_0 t \right]^\frac{1}{q-1} = t^p. \quad (4.22) \]

It is possible to have an accelerated expansion when \( p > 1 \) and \( q < 2 \). Therefore, we can write:

\[ a_{\text{early}}(t) \preceq e_q(H_0 t) \preceq a_{\text{dS}}(t). \quad (4.23) \]
This inequality produces interesting cosmological evolution in the $q$-de Sitter model. The $q$-de Sitter can be used to smoothly connect the early cosmological epoch to late time evolution Universe. Now we can analyze such a model for a non-flat Universe as

$$F(K) = 6c^2 H_0^2 \left( \frac{K}{\epsilon H_0^2} \right)^{1 + \frac{1}{-2 + q}} \left[ \left( \frac{K}{\epsilon H_0^2} \right)^{-2 + \frac{1}{-2 + q}} \right]^{-\frac{1}{1 + q}}$$

$$\times \epsilon \left[ q \left( -2 + q \right) \log(K) + \log \frac{K}{\epsilon H_0^2} + 2(-2 + q) \log \left[ \frac{K}{\epsilon H_0^2} \right]^{-\frac{1 + q}{2(-2 + q)}} \right]$$

$$\times K^{-\frac{q}{4 - 6q + 2q^2}} \left( \frac{K}{\epsilon H_0^2} \right)^{-\frac{1 + 2q}{2(-2 + q)^2(-1 + q)}} 0.8824$$

$$\times - \frac{(-2 + q)^2 \cdot 0.5016}{-1 + q} M^{-2} + \sqrt{KC_{12}}. \tag{4.24}$$

We can also analyze such a model for the flat Universe, obtaining the following expression:

$$F(K) = 6c^2 H_0^2 \left( \frac{K}{\epsilon H_0^2} \right)^{1 + \frac{1}{-2 + q}} \left[ \left( \frac{K}{\epsilon H_0^2} \right)^{-2 + \frac{1}{-2 + q}} \right]^{-\frac{1}{1 + q}}$$

$$\times \epsilon \left[ q \left( -2 + q \right) \log(K) + \log \frac{K}{\epsilon H_0^2} + 2(-2 + q) \log \left[ \frac{K}{\epsilon H_0^2} \right]^{-\frac{1 + q}{2(-2 + q)}} \right]$$

$$\times K^{-\frac{q}{4 - 6q + 2q^2}} \left( \frac{K}{\epsilon H_0^2} \right)^{-\frac{1 + 2q}{2(-2 + q)^2(-1 + q)}} 0.8502$$

$$\times - \frac{(-2 + q)^2 \cdot 0.4817}{-1 + q} M^{-2} + \sqrt{KC_{12}}. \tag{4.25}$$

Considering the Chen-Jing model, we obtain:

$$F(K) = \sqrt{KC_{13}} + \frac{3c^2 (2 \alpha q^2 - 4 \alpha q + 2 \alpha - \beta q + \beta + \gamma)}{\epsilon} K. \tag{4.26}$$

Thus, we can see the Einstein-Aether gravity modifies the cosmological evolution in various different models with different evolution of the scale factor. Hence, this deformation is almost a universal feature of Einstein-Aether gravity. We have calculated $F(K)$ for these different cosmological models. Thus, these cosmological model directly depend on the Aether vector field. It is also possible to calculate $L_{GO}$, $\omega_{EA}$, $\rho_{EA}$ and $p_{EA}$ for these various different cosmological models. It can be argued that these quantities will also depend on the Aether field. Hence, the breaking of Lorentz symmetry by the introduction of a time-like Aether vector field can modify the cosmological dynamics in a non-trivial way. Here we explicitly calculated such a modification for a large number of cosmological models.
5 Om diagnostic analysis

In this section, we perform the Om diagnostic analysis of various different cosmological models. The cosmological parameters like the Hubble parameter $H$, the deceleration parameter $q$ and the Equation of State (EoS) parameter $\omega$ are important to understand the behavior of cosmological models. It is theoretically and observationally known that different Dark Energy models produce a positive Hubble parameter and a negative deceleration parameter i.e., $H > 0$ and $q < 0$, for the present cosmological epoch. So, $H$ and $q$ cannot be used to effectively differentiate between the different dark energy models. Therefore, a higher order of time derivatives of $a(t)$ is required to analyze the dark energy models [65, 66]. Thus, a third order temporal derivative of $a(t)$ can be used to resolve the problem that most Dark Energy models produce $H > 0$ and $q < 0$ for the present cosmological epoch. The statefinder parameters $\{r, s\}$ can be expressed as follows:

$$r = \frac{\dot{a}}{aH^3},$$  \hspace{1cm} (5.1)

$$s = \frac{r - 1}{3(q - 1/2)},$$  \hspace{1cm} (5.2)

where $q$ represents the deceleration parameter, which is given by the following general expression:

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a}.$$  \hspace{1cm} (5.3)

An alternative way to write $r$ and $s$ is as

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3},$$  \hspace{1cm} (5.4)

$$s = -\frac{3H\dot{H} + \ddot{H}}{3H \left(2\dot{H} + 3H^2\right)}$$

$$= -\frac{3H + \ddot{H}/H}{3 \left(2\dot{H} + 3H^2\right)}.$$  \hspace{1cm} (5.5)

The statefinder parameters $\{r, s\} = \{1, 0\}$ represents the point where the flat $\Lambda$CDM model exists in the $r - s$ plane [67]. So, the departure of dark energy models from this fixed point can be used to obtain the distance of these models from the flat $\Lambda$CDM model, taken as reference model.

We also note that in the $\{r, s\}$ plane, a positive value of the parameter $s$ (i.e. $s > 0$) indicates a quintessence-like model of dark energy and a negative value of the parameter $s$ (i.e. $s < 0$) indicates a phantom-like model of dark energy. Furthermore, the evolution from phantom to quintessence is obtained by crossing of the point $\{r, s\} = \{1, 0\}$ in the $\{r, s\}$ plane [68].

So, different cosmological models, like the models with a cosmological constant $\Lambda$, braneworld models, Chaplygin gas and quintessence models, have been studied using such an analysis [66]. In this study, it was argued that $\{r, s\}$ can be used to differentiate between different models. An analysis based on $\{r, s\}$ has also been used to differentiate between dark energy and modified gravity [68, 69].

An important geometrical diagnostic which can be used to for such analysis is called the Om diagnostic analysis [70]. Usually, in the study of the statefinder parameters $r$ and $s$, \ldots
higher order temporal derivatives of $a(t)$ are used. However, in the $Om$ diagnostic analysis only first order temporal derivative are used since it only involves the Hubble parameter, and the Hubble parameter depends on a single time derivative of $a(t)$. So, the $Om$ diagnosis can be considered as a simpler diagnostic than the statefinder diagnosis [71]. The $Om$ diagnosis has also been applied to Galileons models [72, 73]. This set of parameters can now be represented as follows:

$$Om(z) = \frac{\left[\frac{H(z)}{H_0}\right]^2 - 1}{(1 + z)^3 - 1}.$$  

(5.6)

For a constant EoS parameter $\omega$, the expression for $Om(z)$ is given by

$$Om(z) = \Omega_{m0} + (1 - \Omega_{m0})\frac{(1 + z)^{3(1+\omega)} - 1}{(1 + z)^3 - 1}.$$  

(5.7)

Thus, we observe that we have different values of $Om(z) = \Omega_{m0}$ for the $\Lambda$CDM model, quintessence and phantom cosmological models. In figure 1, we plot the cosmological parameters $r - s$, $r - q$, and $m - \omega_{\text{Einstein-Aether}}$ for the power-law scale factor in the redshift range $0.07 \leq z \leq 2.3$. For $r - s$, we observe that as $r$ increases, $s$ decreases monotonically, and never vanishes. A similar pattern is repeated but for the negative values of $q$. The value for $q \approx -0.67$ exists in this model. In figure 2, we plot $Om(z)$ for the power-law scale factor in the redshift range $0.07 \leq z \leq 2.3$. We observe that as the redshift $z$ increases within the interval $0.07 \leq z \leq 2.3$, the $Om(z)$ decreases monotonically. We also observe that for all $m \neq 2$, $Om(z) > \Omega_{m0}$. So, this model mimics a quintessence model with effective EoS $w > -1$.

In figure 3, we plot the cosmological parameters $r - s$, $r - q$, and $\omega_{\text{Einstein-Aether}}$ for a cosmological model with a future singularity. For $r - s$, we observe that as $r$ increases, $s$ decreases monotonically, and remains negative. A similar pattern is repeated for the negative values of $r, q$. The value $q \approx -0.67$ does not exist in this model. In figure 4, we plot $Om(z)$ for models with a future singularity, in the redshift range $0.07 \leq z \leq 2.3$. We observe that as the redshift $z$ increases within the interval $0.07 \leq z \leq 2.3$, the $Om(z)$ decreases monotonically, and satisfies $Om(z) > \Omega_{m0}$. So, the models with future singularity mimics a quintessence with effective EoS $w > -1$.

In figure 5, we first plot the cosmological parameters $r - s$, $r - q$, and $\omega_{\text{Einstein-Aether}}$ for models of an emergent Universe, in the redshift range $0.07 \leq z \leq 2.3$. For $r - s$, we observe that as $r$ increases, $s$ also increases monotonically. It remains positive for this whole range. A similar pattern is repeating for $r, q$. We observe, the value $q \approx -0.67$ does not exist in this model. In figure 6, we plot $Om(z)$ for emergent Universe in the redshift range $0.07 \leq z \leq 2.3$. We observe that as the redshift $z$ increases within the interval $0.07 \leq z \leq 2.3$, the $Om(z)$ decreases monotonically, and satisfies $Om(z) < \Omega_{m0}$. So, this model mimics a phantom model with effective EoS $w < -1$.

In figure 7, we plot the cosmological parameters $r - s$, $r - q$, and $\omega_{\text{Einstein-Aether}}$ for intermediate inflation, in the redshift range $0.07 \leq z \leq 2.3$. For $r - s$, we observe that as $r$ increases, $s = -1$ remains constant. For $r, q$ as $r$ increases, $q$ decreases, and satisfies $-1 < q < 0$. The value $q \approx -0.67$ does not exist in this model. In figure 8, we plot $Om(z)$ for intermediate inflation in the redshift range $0.07 \leq z \leq 2.3$. We observe that as the redshift $z$ increases within the interval $0.07 \leq z \leq 2.3$, and the $Om(z)$ also monotonically increases. We also observe that in this interval $Om(z) < \Omega_{m0}$. So, the intermediate inflation mimics a phantom model with effective EoS $w < -1$. 


Figure 1. Plots of $r - s$, $r - q$, $m - \omega_{\text{Einstein-Aether}}$ for the case with power-law scale factor in the redshift range $0.07 \leq z \leq 2.3$. In this graph horizontal axis is redshift and vertical axis represents $r - s$, $r - q$, $m - \omega_{\text{Einstein-Aether}}$.

Figure 2. Plot of $\Omega_m(z)$ for the case with power-law scale factor in the redshift range $0.07 \leq z \leq 2.3$. In this graph horizontal axis is redshift and vertical axis represents $\Omega_m(z)$.

6 Observational constraints

In this section, we apply observational data from Supernovae Ia, Baryonic Acoustic Oscillations (BAO) and data of the Hubble parameter $H$ to obtain constraints on the parameters of the different cosmological models we are considering. The total $\chi^2$ for joint data set which
Figure 3. Plots of $r - s$, $r - q$, $\omega_{\text{Einstein–Aether}}$ for model with future singularity in the redshift range $0.07 \leq z \leq 2.3$. In this graph horizontal axis is redshift and vertical axis represents $r - s$, $r - q$, $\omega_{\text{Einstein–Aether}}$.

Figure 4. Plot of $\Omega_m(z)$ for model with future singularity in the redshift range $0.07 \leq z \leq 2.3$. In this graph horizontal axis is redshift and vertical axis represents $\Omega_m(z)$.

we use is defined as follows:

$$\chi^2_{\text{tot}} = \chi^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{Hub}},$$  \hspace{1cm} (6.1)
Figure 5. Plots of $r - s, r - q, \omega_{\text{Einstein-Aether}}$ for the emergent Universe case in the redshift range $0.07 \leq z \leq 2.3$ with values of the parameters $A = 1; B = 1; \lambda = 2; n = 1$. In this graph horizontal axis is redshift and vertical axis represents $r - s, r - q, \omega_{\text{Einstein-Aether}}$.

Figure 6. Plot $\Omega_m(z)$ for the emergent Universe case in the redshift range $0.07 \leq z \leq 2.3$ with values of the parameters $A = 1; B = 1; \lambda = 2; n = 1$. In this graph horizontal axis is redshift and vertical axis represents $\Omega_m(z)$.

where the $\chi^2_i$ for each set of data is evaluated. To compute it, we need the luminosity distance $D_L(z)$, which is defined as follows:

$$D_L(z) = (1 + z) \int_0^z \frac{H_0 dz'}{H(z')}.$$  \hfill (6.2)
Figure 7. Plots of $r - s, r - q, \omega_{\text{Einstein-Aether}}$ for the intermediate inflation in the redshift range $0.07 \leq z \leq 2.3$ with values of the parameters $\theta = 2; B = 1$. In this graph horizontal axis is redshift and vertical axis represents $r - s, r - q, \omega_{\text{Einstein-Aether}}$.

Figure 8. Plot of $\Omega_m(z)$ for the intermediate inflation in the redshift range $0.07 \leq z \leq 2.3$ with values of the parameters $\theta = 2; B = 1$. In this graph horizontal axis is redshift and vertical axis represents $\Omega_m(z)$.

We use the distance modulus $\mu$, which is given by the following relation:

$$\mu = m - M = 5 \log D_L + \mu_0,$$

(6.3)

where $m$ and $M$ are the apparent and the absolute magnitudes of the Supernovae. Here
\[\chi^2_{SN}(\mu_0, \theta) = \sum_{i=1}^{580} \frac{[\mu_{\text{th}}(z_i, \mu_0, \theta) - \mu_{\text{obs}}(z_i)]^2}{\sigma_\mu(z_i)^2}, \quad (6.4)\]

where \(\mu_{\text{obs}}, \mu_{\text{th}}\) and \(\sigma_\mu\) indicate the observed distance modulus, the theoretical distance modulus and the uncertainty in the distance modulus, respectively. Furthermore, the parameters in the cosmological models are indicated by \(\theta\). For example, for the power law reconstruction scheme it is given by \(m\), in the \(q\)-de Sitter it is given by the non-extensivity parameter \(q\).

Now we can write:

\[\chi^2_{SN}(\theta) = A(\theta) - \frac{B(\theta)^2}{C(\theta)}, \quad (6.5)\]

where:

\[A(\theta) = \sum_{i=1}^{580} \frac{[\mu_{\text{th}}(z_i, \mu_0 = 0, \theta) - \mu_{\text{obs}}(z_i)]^2}{\sigma_\mu(z_i)^2}, \quad (6.6)\]

\[B(\theta) = \sum_{i=1}^{580} \frac{\mu_{\text{th}}(z_i, \mu_0 = 0, \theta) - \mu_{\text{obs}}(z_i)}{\sigma_\mu(z_i)^2}, \quad (6.7)\]

\[C(\theta) = \sum_{i=1}^{580} \frac{1}{\sigma_\mu(z_i)^2}. \quad (6.8)\]

If we use BAO data of \(\frac{d_A(z)}{D_V(z_{\text{BAO}})}\), we have \(z_* \approx 1091\) as the decoupling time, \(d_A(z) = \int_0^z \frac{dz'}{H(z')}\) as the co-moving angular-diameter distance and \(D_V(z) = \left[d_A(z)^2 \frac{z}{H(z)}\right]^{1/3}\) as the dilation scale. Using this data set, \(\chi^2_{\text{BAO}}\) can be defined as follows:

\[\chi^2_{\text{BAO}} = X^T C^{-1} X, \quad (6.9)\]
where $X$ is defined as follows:

$$X = \begin{pmatrix}
\frac{dA(z_1)}{D_V(0.106)} & -30.95 \\
\frac{dA(z_2)}{D_V(0.2)} & -17.55 \\
\frac{dA(z_3)}{D_V(0.35)} & -10.11 \\
\frac{dA(z_4)}{D_V(0.43)} & -8.44 \\
\frac{dA(z_5)}{D_V(0.6)} & -6.69 \\
\frac{dA(z_6)}{D_V(0.73)} & -5.45
\end{pmatrix}, \quad (6.10)$$

Furthermore, $C^{-1}$ is the inverse covariance matrix. Finally, we use the observational data on Hubble parameter as recently compiled by [63] in the redshift range $0.07 \leq z \leq 2.3$. In this data set, the Hubble constant $H_0$ is taken from the PLANCK 2013 results [64].

In table 2, we provide the $H(z)$ measurements (in unit $[\text{km s}^{-1}\text{Mpc}^{-1}]$) and their errors [63].

The the normalized Hubble parameter is defined by $h = H/H_0$. In this data set, the $\chi^2$ for the normalized Hubble parameter is computed as

$$\chi^2_{\text{Hub}}(\theta) = \sum_{i=1}^{20} \left[ \frac{h_{\text{th}}(z_i, \theta) - h_{\text{obs}}(z_i)}{\sigma_h(z_i)} \right]^2,$$

where $h_{\text{obs}}$ is the observed value of the normalized Hubble parameter, and $h_{\text{th}}$ is theoretical values of the normalized Hubble parameter. The error can now be estimated as

$$\sigma_h = \left( \frac{\sigma_H}{H} + \frac{\sigma_{H_0}}{H_0} \right) h,$$

where $\sigma_H$ is the error of $H$ while $\sigma_{H_0}$ is the error of $H_0$.

In figure 9, we plot the $1\sigma$ (dark regions) and $2\sigma$ (light regions) likelihood contours for these cosmological models, Using the joint data (SNIa+Hubble+BAO), we observe that the best fit value of the parameters which are found to be $\Omega_{m0} = 0.319$. Thus, for models with a power-law the best fit occurs for $m = 3.218_{-0.0564}^{+0.0763}(1\sigma) +0.2134_{-0.0197}^{+0.0214}(2\sigma)$. Furthermore, it is possible to have analyze certain models with a future singularity after finite time, and for these models, the best fit occurs for $n = 4.017_{-0.0453}^{+0.0765}(1\sigma) +0.2341_{-0.0876}^{+0.2341}(2\sigma)$. The best fit for emergent Universe occurs for $n = 2.054_{-0.0312}^{+0.0364}(1\sigma) +0.1268_{-0.0654}^{+0.0654}(2\sigma)$, $\lambda = 6_{-0.0976}^{+0.0131}(1\sigma) +0.1354_{-0.0584}^{+0.0584}(2\sigma)$, and the best fit for intermediate inflation occurs for $B = 2.036_{-0.0211}^{+0.0184}(1\sigma) +0.1287_{-0.0465}^{+0.0465}(2\sigma)$, $\theta = 0.756_{-0.0765}^{+0.0123}(1\sigma) +0.1254_{-0.0512}^{+0.1254}(2\sigma)$. Thus, we have analyzed different cosmological models in Einstein-Aether gravity, and used observational data to analyze the value of parameters in these cosmological models.

7 Conclusions

In this paper, we analyzed various different cosmological models based on the Einstein-Aether gravity. In Einstein-Aether gravity, a time-like vector field couples the usual Einstein Lagrangian, and this time-like vector field breaks the Lorentz symmetry of the theory. In this paper, we have analyzed various different cosmological models using Einstein-Aether gravity. It was demonstrated that the Aether field modifies the cosmology in a non-trivial way. We also obtained explicit expressions for such a modification to various different cosmological models.
| $z$  | $H(z)$ | $\sigma_H$ |
|------|--------|-----------|
| 0.070| 69     | 19.6      |
| 0.100| 69     | 12        |
| 0.120| 68.6   | 26.2      |
| 0.170| 83     | 8         |
| 0.179| 75     | 4         |
| 0.199| 75     | 5         |
| 0.200| 72.9   | 29.6      |
| 0.270| 77     | 14        |
| 0.280| 88.8   | 36.6      |
| 0.350| 76.3   | 5.6       |
| 0.352| 83     | 14        |
| 0.400| 95     | 17        |
| 0.440| 82.6   | 7.8       |
| 0.480| 97     | 62        |
| 0.593| 104    | 13        |
| 0.600| 87.9   | 6.1       |
| 0.680| 92     | 8         |
| 0.730| 97.3   | 7.0       |
| 0.781| 105    | 12        |
| 0.875| 125    | 17        |
| 0.880| 90     | 40        |
| 0.900| 117    | 23        |
| 1.037| 154    | 20        |
| 1.300| 168    | 17        |
| 1.430| 177    | 18        |
| 1.530| 140    | 14        |
| 1.750| 202    | 40        |
| 2.300| 224    | 8         |

Table 2. $H(z)$ measurements (in unit [km s$^{-1}$Mpc$^{-1}$]) and their errors [63].

It was observed that the vector field in the Einstein-Aether gravity has a non-trivial effect on the behavior of different cosmological models. In this paper, it was observed that an effective pressure can be produced by the Einstein-Aether gravity, and it can be used to produce dark energy if the strong energy condition is violated. In fact, we used the Granda-Oliveros model, which is a specific kind of Nojiri-Odintsov Holographic dark energy model, for this study. As the Granda-Oliveros model has been generalized to a Chen-Jing model, we also analyzed this model in our paper. We used these models, and analyzed various different cosmological models for Einstein-Aether gravity for such models.
Figure 9. This figure shows the $1\sigma$ (plotted in dark) and $2\sigma$ (plotted in light) likelihood contours for different cosmological models with joint data (SnIa+Hubble+BAO).

The Hubble parameter cannot be used to differentiate between the different models of dark energy. This is because it is known that different Dark Energy models produce a positive Hubble parameter and a negative deceleration parameter. So, in this paper, we used higher order temporal derivatives of $a(t)$ to differentiate between various dark energy models. In fact, we used third order temporal derivatives of $a(t)$ for such a study. The found the corresponding state-finder parameters, and analyzed the effects of Einstein-Aether gravity on such different cosmological models. It is observed that models with future singularity, and models with a power-law scale factor mimics a quintessence model with effective EoS $w > -1$. It is also observed that an emergent Universe and intermediate inflation mimics a phantom model with effective EoS $w < -1$.

We also compared the cosmological models based on Einstein-Aether gravity with observational data. This was done by using the cosmographic analysis involving the $Om$ parametrization. Thus, the SnIa, BAO and Hubble data was used to obtain the $1\sigma$ and $2\sigma$ contours for density parameter $\Omega_m$ arising from the Sne Ia + BAO. The best fit values for models with a power-law, models with a future singularity after finite time, emergent Universe and for intermediate inflation were obtained, using this study. Thus, we used the cosmological data to fix the parameters for various cosmological models, in the Einstein-Aether gravity.

It is important to perform such an analysis as it is expected that gravitational waves can be used to test Einstein-Aether gravity, and as gravitational wave will be used to test several of the predictions of Einstein-Aether gravity, in near future, it is important to analyze the effect of Einstein-Aether gravity on cosmology. In fact, it has been predicted that gravitational wave detectors can be used to test Einstein-Aether gravity [24]. Thus, it becomes important to analyze various different cosmological models using Einstein-Aether gravity. As the Einstein-Aether gravity modifies the cosmological models in a non-trivial way, it would also be interesting to analyze quantum cosmology using these modified cosmological models. It would be possible to calculate the Wheeler-DeWitt equation for
these cosmological models, and the wave function of the Universe can then be obtained as a solution to the Wheeler-DeWitt equation. We would like to mention, that such an analysis would be very interesting and important. Furthermore, as the time-like vector field breaks the time-reparametrization symmetry, it would modify the Wheeler-DeWitt equation in a very non-trivial way. It might be possible to use this time-like Aether vector field to obtain a direction of time, even in the Wheeler-DeWitt equation. Thus, it might be possible that this formalism can be used as a solution to the problem of time. It would be interesting to perform such an analysis, for these cosmological models.

The Horava-Lifshitz gravity has been used for analyzing type IIA string theory [75], type IIB string theory [76], AdS/CFT correspondence [77–80], dilaton black branes [81, 82], and dilaton black holes [83, 84]. As the Horava-Lifshitz gravity is related to the Einstein-Aether gravity [20], it would be interesting to analyze these systems using Einstein-Aether gravity. In fact, it has been demonstrated that Einstein-Aether gravity can be related to the noncritical string [74]. Thus, it would be interesting to analyze this connection further, and also study various cosmological models motivated from string theory in Einstein-Aether gravity. The Einstein-Aether gravity has been demonstrated to be equivalent to generalization of Horava-Lifshitz gravity.

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