Stagnation point flow on bioconvection nanofluid over a stretching/shrinking surface with velocity and thermal slip effects

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Abstract. Nanofluid containing nanometer sized particles has become an ideal thermal conductivity medium for the flow and heat transfer in many industrial and engineering applications due to their high rate of heat transfer. However, swimming microorganisms are imposed into the nanofluid to overcome the instability of nanoparticles due to a bioconvection phenomenon. This paper investigates the stagnation point flow on bioconvection heat transfer of a nanofluid over a stretching/shrinking surface containing gyrotactic microorganisms. Velocity and thermal slip effects are the two conditions incorporated into the model. Similarity transformation is applied to reduce the governing nonlinear partial differential equations into the nonlinear ordinary differential equations. The transformed equations are then solved numerically. The results are displayed in the form of graphs and tables. The effects of these governing parameters on the skin friction coefficient, local Nusselt number, local Sherwood number and the local density of the motile microorganisms are analysed and discussed in details.

1. Introduction

Flow and heat transfer in the boundary layer is frequently encountered in many industrial and engineering applications. Nanofluid that contains colloidal suspensions of nanometer-sized particle is first proposed by [1]. Despite all other properties, the thermal conductivity of a nanofluid is familiar and crucial in enhancing the process of flow and heat transfer applications. Hence, flow and heat transfer in the field of nanofluid have gained numerous attention from researchers in recent years.

Since [2] initiated the study of the boundary layer flow over a flat surface with a constant speed, many researchers successively investigated various kinds of the boundary layer flow across different surfaces. Examples are the model of the natural convective boundary layer flow of a nanofluid in a porous medium [3] and a model of passing a vertical plate [4]. On the other hand, various aspects of bioconvection problems in a suspension have been carried out. Bioconvection flow in a suspension of solid particles has been discussed by [5]. While [6] investigated bioconvection flow in a suspension of gyrotactic microorganisms over a stretching sheet. The study established the concept by introducing slip parameter that could reduce the fluid properties such as heat and mass transfer rate.

In this paper, the authors aim to extend the work done by [7] taking into account velocity and thermal slips effects. Similarity transformation is applied and the differential equations are solved numerically using Maple 18 software.
2. Mathematical Formulation

A steady flow of an incompressible fluid in the region of \( y > 0 \) driven by a stretching/shrinking surface with a fixed stagnation point at \( x = 0 \) is considered. The stretching/shrinking velocity, the ambient fluid velocity and the mass flux velocity are denoted as \( u_w(x), v_w(x) \) and \( v_w(x) \) respectively. It is also assumed that \( T \) is the uniform temperature, \( C \) is the uniform nanofluid volume fraction and \( N \) is the uniform concentration of microorganisms. The subscripts \( w \) and \( \infty \) denote the corresponding values at the surface and far from the surface, respectively \([4]\). The model presented is based on the Buongiorno model of a suspension of gyrotactic microorganism \([7]\). Under these assumptions, the governing equations representing the conservation of mass, momentum, thermal energy, nanoparticle concentration and microorganisms density are formulated, as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 ,
\]

\[
u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial y} + \nu \frac{\partial^2 u}{\partial y^2} ,
\]

\[
u \frac{\partial T}{\partial x} + u \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{D_b}{\alpha} \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_l}{T_o} \right) \frac{\partial^4 T}{\partial y^4} ,
\]

\[
u \frac{\partial C}{\partial x} + u \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_l}{T_o} \right) \frac{\partial^4 T}{\partial y^4} ,
\]

\[
u \frac{\partial N}{\partial x} + u \frac{\partial N}{\partial y} + \frac{\partial N}{\partial y} = \frac{\partial^2 N}{\partial y^2} .
\]

Here, \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) directions, \( \nu = \left( \frac{b W}{\Delta C} \right) \left( \frac{\partial C}{\partial y} \right) \) and \( \nu \) is the kinematic viscosity. The boundary conditions are:

\[
v=v_w(x), \quad u=u_w + N \frac{\partial u}{\partial y}, \quad T = T_w + D_1 \frac{\partial T}{\partial y}, \quad C = C_w, \quad N = N_w \quad \text{at} \quad y = 0 ,
\]

\[
u=u_w(x), \quad T = T_w, \quad C = C_w, \quad N = N_w \quad \text{as} \quad y \to \infty ,
\]

where, \( u_w(x) = cx^m, \) \( u_w(x) = \alpha x^m \) and \( v_w(x) \) represents suction while \( v_w(x) = \alpha x^m \) represents injection. \( S_1 \) is the mass flux velocity parameter with \( S_1 > 0 \) for suction and \( S_1 < 0 \) for injection.

Similarity variables are introduced as below:

\[
\eta = \left( \frac{u_w(x)}{\nu x} \right)^{\frac{1}{2}} \quad \psi = \left( u_w(x) \nu x \right)^{\frac{1}{2}} \quad f(\eta), \quad \theta(\eta) = \left( \frac{T - T_w}{\Delta T} \right), \quad \phi(\eta) = \left( \frac{C - C_w}{\Delta C} \right), \quad \chi(\eta) = \left( \frac{N - N_w}{\Delta N} \right)
\]

where \( \Delta T = T_w - T_w, \) \( \Delta C = C_w - C_w, \) \( \Delta N = N_w - N_w, \) and \( \psi \) is the stream function defined as
\( u = \frac{\partial y}{\partial y} \) and \( v = -\frac{\partial y}{\partial x} \). Hence, \( u = u_0(x) f'(\eta) \) and \( v = -\frac{m+1}{2} \left[ \frac{u_0(x)}{x} \right] \left[ f(\eta) + \frac{m-1}{m+1} \eta f'(\eta) \right] \),

where prime denotes differentiation with respect to \( \eta \).

By substituting the similarity variables into the governing equations, we obtained the following ordinary differential equations:

\[
\begin{align*}
    f'' + \frac{m+1}{2} f' f'' + m - m f'' &= 0, \\
    \theta' + Pr \left( \frac{m+1}{2} f \theta' + Nb \theta' \phi' + Nt \theta' \phi' \right) &= 0, \\
    \phi' + \frac{m+1}{2} Le f \phi' + \frac{Nt}{Nb} \theta' &= 0, \\
    \chi' + \frac{m+1}{2} Sc f \chi' - Pe \left[ \phi' \chi' + (\sigma + \chi) \phi' \right] &= 0.
\end{align*}
\]

The boundary conditions (6) become

\[
f(0) = S_i, f'(0) = \lambda + \gamma f'(0), \quad \theta(0) = 1 + \beta \theta'(0), \quad \phi(0) = 1, \quad \chi(0) = 1,
\]

\[
f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad \chi(\infty) = 0.
\]

Here, \( \gamma = N_i \left( \frac{u_0(x)}{\nu x} \right)^{\frac{1}{2}} \) and \( \beta = D_i \left( \frac{u_0(x)}{\nu x} \right)^{\frac{1}{2}} \), where \( N_i \) is the Navier slip coefficient and \( D_i \) is the thermal slip factor, while \( \lambda \) denotes the stretching \((\lambda > 0)\) or shrinking \((\lambda < 0)\) parameter. \( Pr \) is the Prandtl number, \( Le \) is the Lewis number, \( Pe \) is the bioconvection Péclet number, \( Sc \) is the Schmidt number, \( Nt \) is the thermophoresis parameter and \( \sigma \) is a dimensionless constant. They are defined as:

\[
\lambda = \frac{c}{a}, \quad Pr = \frac{\nu}{\alpha}, \quad Le = \frac{\nu}{D_b}, \quad Pe = \frac{bW}{D_n}, \quad Sc = \frac{\nu}{D_n}, \quad Nb = \frac{\tau D_b \Delta C}{\nu}, \quad Nt = \frac{\tau D_i \Delta T}{\nu T_i}, \quad \sigma = \frac{N}{N_N}.
\]

The parameters of interest are the skin friction coefficient \( C_f \), the local Nusselt number \( Nu \), the Sherwood number \( Sh \) and the local density of the motile microorganisms \( N_n \), which are defined as:

\[
C_f = \frac{\tau_w}{\rho u_c}, \quad Nu = \frac{q_w}{k \Delta T}, \quad Sh = \frac{q_w}{D_b \Delta C}, \quad N_n = \frac{q_w}{D_n \Delta N},
\]

where \( \tau_w, q_w, q_m, \) and \( q_n \) are the surface shear stress, the wall heat flux, the wall mass flux and the wall motile microorganisms flux with the corresponding representative:

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_b \left( \frac{\partial C}{\partial y} \right)_{y=0}, \quad q_n = -D_n \left( \frac{\partial N}{\partial y} \right)_{y=0}.
\]

By using equations (7), (13) and (14), we obtain
where $Re = \frac{u(x)}{\nu}$ is the local Reynolds number.

3. Results and Discussion

Numerical solutions of the transformed equations are obtained using the shooting method programmed in Maple18 software. Here, the case is considered only for $m=1$ and it shows the stagnation point flow past a linearly stretching/shrinking surface and the boundary layer thickness $\eta_s$ is set to 8. The comparison values of the local skin friction coefficient, local Nusselt number, local Sherwood number and the local density of the motile microorganisms of the present paper with those obtained by [7] for the stretching/shrinking effects are presented in Table 1. It is observed that the results show a very good agreement.

Table 1. Comparison of the present results with the results from the paper [7] for the values of $Re^{1/2} C_f$, $Re^{1/2} Nu_x$, $Re^{1/2} Sh_x$ and $Re^{1/2} Nn_x$ for different values of $\lambda$ when $m=1$, $Pr = 6.2$ (water), $S_i = 3$, $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$, $Nt = Nb = 0.5$, and $\gamma = \beta = 0$.

| $\lambda$ | $Re^{1/2} C_f$ | $Re^{1/2} Nu_x$ | $Re^{1/2} Sh_x$ | $Re^{1/2} Nn_x$ |
|-----------|----------------|----------------|----------------|----------------|
| $\lambda = -1$ | 6.5350 (-4.0602) | -3.9965 (-14.0256) | 6.5350 (-4.0602) | -3.9665 (-14.0256) |
| $\lambda = 2$ | 4.1326 (8.7410) | 4.0321 (5.0543) | 4.1326 (8.7410) | 4.0321 (5.0543) |

Table 2. Values of $Re^{1/2} C_f$, $Re^{1/2} Nu_x$, $Re^{1/2} Sh_x$ and $Re^{1/2} Nn_x$ for different values of $\beta$ where $m=1$, $Pr = 6.2$ (water), $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$, $Nt = Nb = 0.5$, $\gamma = 2$, $S_i = 3$ and $\lambda = -1$.

| $\beta$ | $Re^{1/2} C_f$ | $Re^{1/2} Nu_x$ | $Re^{1/2} Sh_x$ | $Re^{1/2} Nn_x$ |
|---------|----------------|----------------|----------------|----------------|
| 0.6     | 0.8811 (-0.8080) | 1.3167 (1.4021) | 4.9981 (2.2237) | 12.9511 (7.3585) |
| 0.8     | 0.8811 (-0.8080) | 1.0469 (1.0951) | 5.2556 (2.6252) | 13.4687 (8.1962) |
| 1.0     | 0.8811 (-0.8080) | 0.8676 (0.8984) | 5.4271 (2.8821) | 13.8108 (8.7319) |

Table 3. Values of $Re^{1/2} C_f$, $Re^{1/2} Nu_x$, $Re^{1/2} Sh_x$ and $Re^{1/2} Nn_x$ for different values of $\gamma$ where $m=1$, $Pr = 6.2$ (water), $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$, $Nt = Nb = 0.5$, $\beta = 1$, $S_i = 3$ and $\lambda = -1$.

| $\gamma$ | $Re^{1/2} C_f$ | $Re^{1/2} Nu_x$ | $Re^{1/2} Sh_x$ | $Re^{1/2} Nn_x$ |
|----------|----------------|----------------|----------------|----------------|
| 1.0      | 1.5709 (-1.4885) | 0.8674 (0.8995) | 5.3885 (2.8982) | 13.7264 (8.1985) |
| 1.5      | 1.1293 (-1.1083) | 0.8675 (0.8989) | 5.4134 (2.8857) | 13.7809 (8.7588) |
| 2.0      | 0.8811 (-0.8984) | 0.8676 (0.8984) | 5.4271 (2.8821) | 13.8108 (8.7319) |

Figures 1 to 4 show the effects of thermal slip parameter $\beta$ on the four different profiles for the stretching case ($\lambda = 2$). It can be seen from Figure 1 that changing values of $\beta$ does not affect the velocity distribution for both solutions, while the first and second solutions for the temperature profiles from Figure 2 are seen to decrease with increasing of $\beta$. However, both plots in Figures 3 and 4 show...
an increasing pattern of nanoparticle volume fraction and density of motile microorganisms profiles for both solutions in the presence of the thermal slip parameter. The effects of the thermal slip parameter on the velocity, temperature, nanoparticle concentration and the microorganism density profiles for the shrinking case ($\lambda = -1$) show the exactly same manner performed in the stretching case ($\lambda = 2$). The numerical results are recorded in Table 2.

Figures 5 to 8 present the effects of velocity slip parameter $\gamma$ on four different profiles for the stretching case ($\lambda = 2$). It can be observed from Figure 5 that the first and second solutions for the velocity distribution increases with $\gamma$. Whereas Figures 6, 7, and 8 depict that solutions for the temperature, nanoparticle volume fraction and density of motile microorganisms distributions decreases as $\gamma$ increases, respectively. Table 3 shows the result of the effect of velocity slip parameter for the shrinking case ($\lambda = -1$). As can be seen, the first solution of the velocity profiles decreases with increasing of $\gamma$ but the opposite behavior is observed on the second solution. Next, similarity is found on the temperature, nanoparticle volume fraction and density of motile microorganism profiles. The first solution increases with the velocity slip parameter while the opposite manner is obtained for the second solution.

**Figure 1.** Effect of thermal slip parameter $\beta$ on velocity distribution for $\lambda = 2$.

**Figure 2.** Effect of thermal slip parameter $\beta$ on temperature distribution for $\lambda = 2$.

**Figure 3.** Effect of thermal slip parameter $\beta$ on nanoparticle concentration distribution for $\lambda = 2$.

**Figure 4.** Effect of thermal slip parameter $\beta$ on microorganism density distribution for $\lambda = 2$. 
4. Conclusions
A numerical study on the stagnation point flow of bioconvection nanofluid over a stretching/shrinking surface with slip effects has been carried out. It is found that suction effect increases the local skin friction coefficient, local Nusselt number, local Sherwood number and the local density of the motile microorganisms. Thermal slip parameter reduces the temperature of the flow as all the graphs converges to zero but increases the nanoparticle volume fraction and the density of motile microorganism profiles. Other than that, velocity slip parameter thickens the boundary layer, influences the velocity flow and retarded temperature, nanoparticle volume fraction and density of motile microorganism profiles. Moreover, Local Nusselt number decreases with an increase in slip parameter, indicates the low rate of heat transfer at the surface.

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