CONTINUOUS-VARIABLE TELEPORTATION:
A NEW LOOK

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Abstract

In contrast to discrete-variable teleportation, a quantum state is imperfectly transferred from a sender to a remote receiver in a continuous-variable setting. We recall the ingenious scheme proposed by Braunstein and Kimble for teleporting a one-mode state of the quantum radiation field. By analyzing this protocol, we have previously proven the factorization of the characteristic function of the output state. This indicates that teleportation is a noisy process that alters, to some extent, the input state. Teleportation with a two-mode Gaussian EPR state can be described in terms of the superposition of a distorting field with the input one. Here we analyze the one-mode Gaussian distorting-field state. Some of its most important properties are determined by the statistics of a positive EPR operator in the two-mode Gaussian resource state. We finally examine the fidelity of teleportation of a coherent state when using an arbitrary resource state.

1 INTRODUCTION

Quantum teleportation within continuous-variable (CV) settings is based on the same ideas as in the discrete case: these were put forward in the seminal work of Bennett et al. [1], who discovered the teleportation of qubits. The proposal of Braunstein and Kimble [2] was the first CV-teleportation scheme implemented experimentally. We find it useful to give here a succinct account of their protocol for teleporting a single-mode state of the quantum radiation field.
Two distant operators, Alice, at a sending station, and Bob, at a receiving terminal, share an entangled two-mode state $\rho_{AB}$. Mode $A$ is operated by Alice and mode $B$ is controlled by Bob. When the cross-correlations between modes are strong enough, Alice and Bob can exploit the non-local character of the bipartite state $\rho_{AB}$ as a quantum resource for teleporting an unknown one-mode state $\rho_{in}$. The inseparable state $\rho_{AB}$ that connects the two parties is usually called an Einstein-Podolsky-Rosen (EPR) state. Without going into details, we briefly recall the successive steps of the Braunstein-Kimble (BK) teleportation protocol.

Alice performs a von Neumann measurement of a pair of commuting continuous variables. She combines two optical operations on her modes: mode mixing and homodyne detection. More specifically, Alice mixes the input mode whose state $\rho_{in}$ is to be teleported with her $A$-mode of the EPR state $\rho_{AB}$ by employing a balanced lossless beam-splitter. As a result, she is ready to detect quadratures of the output modes. By applying convenient projectors, Alice chooses to measure simultaneously the pair of commuting quadratures $\hat{q}_A := \frac{1}{\sqrt{2}}(\hat{q}_{in} - \hat{q}_1)$, $\hat{p}_A := \frac{1}{\sqrt{2}}(\hat{p}_{in} + \hat{p}_1)$.

She conveys to Bob, through a classical channel, the result $\{q, p\}$ of her homodyne measurement as a complex amplitude, $\mu := q + ip$. Any individual CV measurement performed by Alice is accompanied by a collapse of the initial tripartite state $\rho_{in} \otimes \rho_{AB}$. This results into a modified reduced $B$-mode state. Bob employs the value $\mu$ transmitted by Alice via the classical channel to perform a suitable displacement of the new reduced one-mode state at his side, $\rho_B(\mu) \to \rho'_B(\mu)$.

The outcome $\mu$ is a continuous random variable. Therefore, Alice has to repeat her measurement under identical conditions in order to obtain a significant ensemble of results. She sends to Bob all these results, one by one, by successive classical communications. Every time, Bob operates the suitable displacement on the mode $B$ at his hand. He can thereby infer the distribution function $P(\mu) := P(q, p)$ of the random variable $\mu$. Bob is eventually able to build an imperfect replica of the initial state $\rho_{in}$ by averaging on the above-mentioned ensemble with the corresponding distribution function:

$$\rho_{out} = \int d^2\mu P(\mu)\rho'_B(\mu).$$  \hspace{1cm} (1.2)

Here we have denoted $d^2\mu := dq \, dp$.

Had we summarized the key steps of the BK protocol, this is explained in the framework of quantum mechanics, in Section 2, in terms of measurements and operations. Our main tool is the Weyl expansion of the density operators.
of the states involved. For subsequent use, we introduce a non-local positive operator that we call EPR operator. Section 3 deals with two-mode Gaussian EPR states. We first prove the existence of a one-mode distorting-field state that is entirely determined by the EPR state. Its properties are then carefully analyzed and we show that a Gaussian teleportation channel does exist. The accuracy of the CV teleportation, measured either by the amount of added noise or by the fidelity of teleporting a coherent state, is investigated in Section 4.

2 QUANTUM-MECHANICAL DESCRIPTION

We present the BK protocol in the Schrödinger picture. The initial three-mode state is the product $\rho_{in} \otimes \rho_{AB}$ of the one-mode state to be teleported,

$$\rho_{in} = \frac{1}{\pi} \int d^2 \lambda \ \chi_{in}(\lambda) D(-\lambda), \quad (2.1)$$

and of the two-mode EPR state,

$$\rho_{AB} = \frac{1}{\pi^2} \int d^2 \lambda_1 d^2 \lambda_2 \ \chi_{AB}(\lambda_1, \lambda_2) D_1(-\lambda_1) D_2(-\lambda_2). \quad (2.2)$$

Equations (2.1) and (2.2) exhibit the Weyl expansions of the corresponding density operators. We have denoted here by $D(\alpha) := \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$ a Weyl displacement operator on a single-mode Hilbert space: $\hat{a}$ is the mode annihilation operator. The states (2.1) and (2.2) are described in terms of their characteristic functions (CFs) $\chi_{in}(\lambda)$ and $\chi_{AB}(\lambda_1, \lambda_2)$, which are particular cases of the multimode definition

$$\chi(\lambda_1, \lambda_2, \cdots, \lambda_n) := \text{Tr}[\rho D(\lambda_1) D(\lambda_2) \cdots D(\lambda_n)]. \quad (2.3)$$

The compatible observables (1.1) have continuous spectra. Their common eigenfunction for an outcome $\{q, p\}$ of the homodyne measurement,

$$|\Phi(q, p)\rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} d\eta e^{i\sqrt{2} \mu \eta} |\sqrt{2}q + \eta\rangle_{in} \otimes |\eta\rangle_A, \quad (2.4)$$

satisfies the orthonormality condition

$$\langle \Phi(q', p') | \Phi(q, p) \rangle = \delta(q' - q) \delta(p' - p).$$

The distribution function of the continuous random variable $\mu$ is

$$P(q, p) = \text{Tr}_{in,AB} [M(\mu)], \quad (2.5)$$
where \(M(\mu)\) is an operator on the Hilbert space \(\mathcal{H}_m \otimes \mathcal{H}_A \otimes \mathcal{H}_B\):

\[
M(\mu) := [[\Phi(q, p)] \langle \Phi(q, p) | \otimes I_B] (\rho_m \otimes \rho_{AB}).
\]

As a result of the projective measurement performed by Alice, the initial product state \(\rho_m \otimes \rho_{AB}\) collapses, so that the after-collapse \(B\)-mode reduction \(\rho_B(\mu)\) can be written by tracing out the three-mode operator (2.6) on the Hilbert space \(\mathcal{H}_m \otimes \mathcal{H}_A\):

\[
\rho_B(\mu) = \frac{1}{\mathcal{P}(\mu)} \text{Tr}_{in,A} [M(\mu)].
\]

The information provided by Alice allows Bob to perform a suitable displacement, \(\rho'_B(\mu) = D_2(\mu) \rho_B(\mu) D_2^\dagger(\mu)\). Then the ensemble averaging (1.2) yields the state emerging by CV teleportation from the unknown input state \(\rho_m\):

\[
\rho_{\text{out}} = \int d^2\mu \mathcal{P}(\mu) D_2(\mu) \rho_B(\mu) D_2^\dagger(\mu).
\]

Note that, owing to the CV-teleportation protocol itself, the output single-mode state \(\rho_{\text{out}}\), eq. (2.8), is always a mixed one.

Our main previous result is a very simple formula connecting the one-mode states \(\rho_m\) and \(\rho_{\text{out}}\). It is expressed in terms of their normally-ordered CFs and a remnant of the CF \(\chi_{AB}(\lambda_1, \lambda_2)\) of the EPR state [3, 4, 5]:

\[
\chi_{\text{out}}^{(N)}(\lambda) = \chi_{in}^{(N)}(\lambda) \chi_{AB}(\lambda^*, \lambda).
\]

The factorization formula (2.9) shows that CV teleportation is a noisy process, which always alters the input state \(\rho_m\). It is equivalent to the identity

\[
\chi_{\text{out}}(\lambda) = \chi_{in}(\lambda) \chi_{AB}(\lambda^*, \lambda).
\]

The function

\[
\chi_{D}^{(N)}(\lambda) := \chi_{AB}(\lambda^*, \lambda)
\]

leads us to introduce the related one,

\[
\chi_D(\lambda) := \exp \left( -\frac{1}{2} |\lambda|^2 \right) \chi_{D}^{(N)}(\lambda),
\]

which enters the one-mode Weyl expansion

\[
\rho_D := \frac{1}{\pi} \int d^2\lambda \chi_D(\lambda) D_2(-\lambda).
\]

Here \(\rho_D\) is a self-adjoint Hilbert-Schmidt operator of unit trace, on the single-mode Hilbert space \(\mathcal{H}_B\) at Bob’s side.
Unless the two-mode EPR state \( \rho_{AB} \) is Gaussian, one could only conjecture the positivity of the operator \( \rho_D \). However, in the Gaussian case, we will prove this property in the next section. Accordingly, for the special class of the two-mode Gaussian EPR states, the function \( \chi_D(\lambda) \), eq. (2.12), is the CF of a B-mode state \( \rho_D \) that we have termed *distorting-field state* [5]. The multiplication rule (2.9) displays therefore the fact that \( \rho_{out} \) is the B-mode state of a superposition of two single-mode fields: the input one and a remote distorting field in the state \( \rho_D \), due to the imperfect character of CV teleportation.

We conclude this section by introducing some operators on the Hilbert space \( \mathcal{H}_A \otimes \mathcal{H}_B \) that prove to be useful when analyzing the non-locality features of the two-mode resource state \( \rho_{AB} \). The usual EPR observables are two commuting linear combinations of the single-mode canonical operators: the relative coordinate \( \hat{Q} := \hat{q}_1 - \hat{q}_2 \) and the total momentum \( \hat{P} := \hat{p}_1 + \hat{p}_2 \). In terms of them we define the EPR operator

\[
\hat{\Delta} := \frac{1}{2} \left( \hat{Q}^2 + \hat{P}^2 \right).
\]

(2.14)

For later convenience, let us introduce a non-local normal operator,

\[
\hat{A} := \hat{a}_1 - (\hat{a}_2)^\dagger = \frac{1}{\sqrt{2}}(\hat{Q} + i\hat{P}),
\]

(2.15)

as a suitable amplitude of the positive EPR operator:

\[
\hat{\Delta} = \hat{A}^\dagger \hat{A}.
\]

(2.16)

3 GAUSSIAN DISTORTING-FIELD STATE

In what follows we adopt a shorthand notation concerning the operator \( \rho_D \), eq. (2.13): we denote by \( \mathcal{H} \) its one-mode Hilbert space and by \( \hat{a} \) the corresponding annihilation operator. Moreover, in order to simplify the subsequent discussion, we assume for the moment that a distorting-field state \( \rho_D \) does exist whatever EPR state \( \rho_{AB} \). We eventually prove that this assumption is true for any two-mode Gaussian EPR state.

To start on our analysis, substitution into eq. (2.11) of the Taylor expansions

\[
\chi_{D}^{(N)}(\lambda) = \sum_{l,m=0}^{\infty} \frac{1}{l!m!} \lambda^l (-\lambda^*)^m \langle \hat{a}^l \hat{a}^m \rangle_D
\]

(3.1)

and

\[
\chi_{AB}(\lambda^*, \lambda) = \sum_{l,m=0}^{\infty} \frac{1}{l!m!} \lambda^l (-\lambda^*)^m \langle \hat{A}^l \hat{A}^m \rangle_{AB}
\]

(3.2)
yields the correlation functions in the distorting-field state:

\[ \langle \hat{a}^\dagger_l \hat{a}^m \rangle_D = \langle \hat{A}^\dagger_l \hat{A}^m \rangle_{AB}. \]

(3.3)

We will omit subsequently the pair of indices \( AB \) when writing expectation values in the EPR state \( \rho_{AB} \):

\[ \langle \ldots \rangle := \langle \ldots \rangle_{AB}. \]

In particular, the \( l \)th-order correlation function is non-negative for any \( l \):

\[ \langle \hat{a}^\dagger_l \hat{a}^l \rangle_D = \langle \hat{\Delta}^l \rangle \geq 0. \]

(3.4)

The identity (3.3) can be employed to evaluate the \( 2 \times 2 \) covariance matrix (CM) of the distorting-field state \( \rho_D \) [6]:

\[ \mathcal{V}_D = \begin{pmatrix} \sigma_D(q,q) & \sigma_D(q,p) \\ \sigma_D(p,q) & \sigma_D(p,p) \end{pmatrix} \]

(3.5)

Explicitly, the CM \( \mathcal{V}_D \) has the following entries [5]:

\[ \sigma_D(q,q) = \frac{1}{2} + \langle \hat{Q}^2 \rangle, \quad \sigma_D(q,p) = \langle \hat{Q}\hat{P} \rangle, \quad \sigma_D(p,p) = \frac{1}{2} + \langle \hat{P}^2 \rangle. \]

(3.6)

Owing to the Schwarz inequality for a quasi-inner product,

\[ \langle \hat{Q}\hat{P} \rangle^2 \leq \langle \hat{Q}^2 \rangle \langle \hat{P}^2 \rangle, \]

(3.7)

the Robertson-Schrödinger uncertainty relation holds:

\[ \mathcal{V}_D + \frac{i}{2} J \geq 0, \quad (J := i\sigma_2), \]

(3.8)

with \( \sigma_2 \) denoting the complex Pauli matrix. Condition (3.8) is necessary for all single-mode states, but it is sufficient only for the Gaussian ones. By virtue of definition (2.11), the function \( \chi_D(\lambda) \), eq. (2.12), is Gaussian if and only if the two-mode EPR state \( \rho_{AB} \) is Gaussian too. In this case, \( \chi_D(\lambda) \) is indeed the CF of a one-mode Gaussian state \( \rho_D \): our assertion is therefore proven.

From now on, in this section, we deal only with Gaussian CV teleportation. The existence of a Gaussian distorting-field state \( \rho_D \) allows us to read eq. (2.10) as a multiplication rule of normally-ordered CFs:

\[ \chi^{(N)}_{\text{out}}(\lambda) = \chi^{(N)}_{\text{in}}(\lambda)\chi^{(N)}_D(\lambda). \]

(3.9)

Note that if the Gaussian resource state \( \rho_{AB} \) is undisplaced, so is the distorting-field state \( \rho_D \). Further, according to eqs. (3.6), \( \rho_D \) is a mixed state, since
det \( V_D > 1/4 \), unless the random EPR variables \( \hat{Q} \) and \( \hat{P} \) are constants, meeting thus the ideal EPR demand.

Moreover, eqs. (3.6) show that a matrix inequality holds, \( V_D \geq \frac{1}{2} I_2 \), pointing out that \( \rho_D \) is a classical state. Therefore, the Gaussian distorting-field state \( \rho_D \) has a regular Glauber-Sudarshan \( P \) representation,

\[
P_D(\alpha) = \frac{1}{\pi} \int d^2 \lambda \exp(\alpha \lambda^* - \alpha^* \lambda) \chi_D^{(N)}(\lambda),
\]

which is a Gaussian distribution function. The multiplication rule (3.9) is equivalent to the existence of a mapping

\[
\rho_{\text{out}} = \int d^2 \beta P_D(\beta) D(\beta) \rho_{\text{in}} D^\dagger(\beta)
\]

between one-mode states on the Hilbert space \( \mathcal{H} \). We call such a mapping a teleportation channel. In sum, Gaussian CV teleportation is described by a Gaussian channel (3.11).

It is instructive to evaluate the Glauber \( R \) function of the state \( \rho_D \) [7],

\[
R_D(\beta^*, \beta') := \exp \left( \frac{1}{2} (|\beta|^2 + |\beta'|^2) \right) \langle \beta | \rho_D | \beta' \rangle,
\]

as an integral [8]:

\[
R_D(\beta^*, \beta') = \frac{1}{\pi} \int d^2 \lambda \chi_D^{(N)}(\lambda) \exp (-|\lambda|^2 - \beta^* \lambda + \beta' \lambda^*). \tag{3.13}
\]

Making use of the eqs. (3.1) and (2.3), we find:

\[
R_D(\beta^*, \beta') = \langle \exp (-\hat{\Delta} + \beta' \hat{A}^\dagger + \beta^* \hat{A}) \rangle. \tag{3.14}
\]

It follows that the Husimi function (≡ the Glauber \( Q \) function) reads:

\[
Q_D(\beta) := \frac{1}{\pi} \langle \beta | \rho_D | \beta \rangle = \frac{1}{\pi} \exp(-|\beta|^2) \langle \exp (-\hat{\Delta} + \beta \hat{A}^\dagger + \beta^* \hat{A}) \rangle. \tag{3.15}
\]

We take advantage of the Taylor expansion of the \( R \) function [7],

\[
R_D(\beta^*, \beta') = \sum_{l,m=0}^\infty (\rho_D)_{lm} \frac{1}{\sqrt{l!m!}} (\beta^*)^l(\beta')^m, \tag{3.16}
\]

to write down the density matrix

\[
(\rho_D)_{lm} = \frac{1}{\sqrt{l!m!}} \langle \exp (-\hat{\Delta}) \hat{A}^l(\hat{A}^\dagger)^m \rangle. \tag{3.17}
\]
The corresponding photon-number distribution, 

\[ (\rho_D)_{ll} = \frac{1}{l!} \langle \hat{\Delta}^l \exp(-\hat{\Delta}) \rangle, \]  

(3.18)

has the generating function \( G_D(s) := \sum_{l=0}^{\infty} s^l (\rho_D)_{ll}, \) \(|s| \leq 1\):

\[ G_D(s) = \langle \exp((s - 1)\hat{\Delta}) \rangle. \]  

(3.19)

This distribution is entirely determined by the statistics of the EPR operator \( \hat{\Delta} \), eq. (2.14), in the two-mode Gaussian EPR state \( \rho_{AB} \). It is worth mentioning that eqs. (3.9)–(3.19) hold also for any classical non-Gaussian distorting-field state \( \rho_D \); in fact, classicality is required only to ensure the validity of eqs. (3.10) and (3.11), playing no role for the other ones.

Let us specialize the above discussion to a zero-mean Gaussian resource state frequently used, namely, a two-mode squeezed vacuum state (SVS): \( \rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}| \). The Schmidt decomposition of such a pure state in the standard Fock basis,

\[ |\Psi_{AB}\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |n\rangle_A \otimes |n\rangle_B, \]

is parametrized with the squeezing factor \( r > 0 \). The corresponding distorting-field state is thermal,

\[ \chi_D^{(N)}(\lambda) = \exp(-e^{-2r}|\lambda|^2), \]  

(3.20)

and has the \( P \) representation

\[ P_D(\alpha) = \frac{e^{2r}}{\pi} \exp(-e^{2r}|\alpha|^2). \]  

(3.21)

With eqs. (3.1) and (3.3), we get the correlation functions

\[ \langle (\hat{a}^\dagger)^l \hat{a}^m \rangle_D = \delta_{lm} l!(e^{-2r})^l = \langle (\hat{A}^\dagger)^l \hat{A}^m \rangle. \]  

(3.22)

Note the mean photon number, \( \langle \hat{a}^\dagger \hat{a} \rangle_D = \exp(-2r) = \langle \hat{\Delta} \rangle \), and the \( l \)th-order correlation function, \( \langle (\hat{a}^\dagger)^l \hat{a}^l \rangle_D = l!(\hat{\Delta})^l = \langle \hat{\Delta}^l \rangle \).

For the sake of completeness, we write down further the \( R \) function,

\[ R_D(\beta^*, \beta') = \frac{1}{1 + e^{-2r}} \exp \left( \frac{e^{-2r}}{1 + e^{-2r}} \beta^* \beta' \right), \]  

(3.23)

the Husimi function,

\[ Q_D(\beta) = \frac{1}{\pi} \frac{1}{1 + e^{-2r}} \exp \left( - \frac{|\beta|^2}{1 + e^{-2r}} \right), \]  

(3.24)
the density matrix,

\[(\rho D)_{lm} = \delta_{lm} \frac{1}{1 + e^{-2r}} \left( \frac{e^{-2r}}{1 + e^{-2r}} \right)^l, \tag{3.25}\]

and the generating function of the photon-number distribution,

\[G_D(s) = \left[ 1 + (1 - s)e^{-2r} \right]^{-1}. \tag{3.26}\]

All the above formulae are specific for a single-mode thermal state. Therefore, when using a SVS as the resource state, teleportation is described by a thermalization channel.

4 ACCURACY OF TELEPORTATION

Originally, the quality of the teleportation protocol was quantified by the input-output overlap for pure states \[2\], or by use of the Uhlmann fidelity for mixed Gaussian states \[3, 9\]. For a clear survey on the progress in CV teleportation we refer the reader to Ref. \[10\]. More recently \[4, 5\], in analyzing CV teleportation, the present authors have introduced the distorting-field state \(\rho_D\) and focused on its properties. We point out here the conspicuous role of the EPR operator \(\hat{\Delta}\), eq. (2.14). Its expectation value in the resource state \(\rho_{AB}\), called EPR uncertainty \[11\], quantifies the non-locality of this state. As the EPR uncertainty \(\langle \hat{\Delta} \rangle\) decreases, the non-local character of the two-mode state \(\rho_{AB}\) becomes stronger. In particular, the inequality \(\langle \hat{\Delta} \rangle < 1\) is a criterion of inseparability of the bipartite state \(\rho_{AB}\).

We start by assuming first the existence of a one-mode remote-field state \(\rho_D\): we have shown that this effectively happens at least for the class of the two-mode Gaussian EPR states. The quality of teleportation can be evaluated in terms of the mean photon number \(\langle \hat{a}^\dagger \hat{a} \rangle_D\) in the one-mode state \(\rho_D\). For any undisplaced Gaussian EPR state, this can be seen as the amount of noise added by teleportation: it distorts the features of the input field state \(\rho_{in}\). The smaller this noise, the higher the quality of the CV teleportation. According to eq. (3.3), the added noise is equal to the EPR uncertainty:

\[\langle \hat{a}^\dagger \hat{a} \rangle_D = \langle \hat{\Delta} \rangle. \tag{4.1}\]

Second, we make a conjecture that extends this result to an arbitrary undisplaced two-mode EPR state: The amount of noise distorting the properties of the input field state is equal to the EPR uncertainty \(\langle \hat{\Delta} \rangle\).

A remarkable theorem proven by Giedke et al. \[12\] states that among all equally entangled pure two-mode states, the SVS has the minimal EPR uncertainty \(\langle \hat{\Delta} \rangle\), i.e., the strongest non-local character. This theorem regarding
the ranking of pure-state entanglement at a given EPR uncertainty enables us to notice an interesting property of CV teleportation: The SVS adds the minimal noise in teleportation with pure two-mode resource states having the same entanglement.

We finally give a new expression of another quantity that is widely employed to measure the teleportation accuracy: the fidelity of teleporting a coherent state, hereafter denoted by $F_{coh}$. Recall that a coherent state $\rho_{in} = |\alpha\rangle\langle\alpha|$ has a Gaussian CF:

$$\chi_{in}(\lambda) = \exp(\alpha^* \lambda - \alpha \lambda^*) \exp\left(-\frac{1}{2}|\lambda|^2\right).$$ (4.2)

The fidelity of teleporting a coherent state is the probability of the transition $\rho_{in} \rightarrow \rho_{out}$:

$$F_{coh} = \langle\alpha|\rho_{out}|\alpha\rangle.$$ (4.3)

When writing this quantity in terms of the CFs of the states involved, eq. (2.10) provides an expression that is independent of the input coherent state:

$$F_{coh} = \frac{1}{\pi} \int d^2\lambda \exp\left(-|\lambda|^2\right)\chi_{AB}(\lambda^*, \lambda).$$ (4.4)

Making use of the Taylor series (3.2), we find:

$$F_{coh} = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \langle\hat{\Delta}^l\rangle = \langle\exp(-\hat{\Delta})\rangle.$$ (4.5)

Equation (4.5) is valid for any two-mode EPR state $\rho_{AB}$. It displays to what extent the EPR operator $\hat{\Delta}$ is involved in the structure of the fidelity of teleporting a coherent state. If a distorting-field state $\rho_D$ exists, then an inspection of eqs. (3.15) and (4.5) gives the identity

$$F_{coh} = \pi Q_D(0).$$ (4.6)

For instance, let us consider again the case of a SVS chosen as a two-mode resource state. Then, by use of eqs. (3.21) and (4.6), we recover the formula

$$F_{coh} = \frac{1}{1 + \exp(-2r)},$$ (4.7)

in agreement with previous results [2, 9].

To sum up, in this paper we have examined further the CF description of the BK teleportation protocol. For the class of the two-mode Gaussian EPR states, we have been able to identify a remote distorting field mode superposed on the input one. This originates in the noisy character of the
CV teleportation. We have pointed out the main properties of a one-mode Gaussian distorting-field state. They are connected with the non-local features of the two-mode EPR state. The accuracy of the CV teleportation is measured either by the amount of added noise or by the fidelity of teleporting a coherent state. Both quantities depend on the degree of non-locality of the bipartite resource state expressed in terms of the EPR uncertainty.

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