New Results of Fuzzy Sampled-Data Control for Nonlinear Time-Delay Systems

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ABSTRACT In the manuscript, synthesis and stability analysis for fuzzy H2/H∞ sampled-data control of dynamical systems with delay behavior are studied by employing the input delay method. By using Lyapunov theory, a new H2/H∞ standard is derived and the fuzzy sampled-data controller is proposed to ensure H∞ performance and H2 performance simultaneously. Meanwhile, the control design is verified by two practical examples. Furthermore, experimental results also indicate that the H2/H∞ sampled-data control has a better performance.

INDEX TERMS Dynamical systems, delay behavior, fuzzy sampled-data control, H2/H∞ control.

I. INTRODUCTION

Recently, fuzzy control method has always been a hot topic in the field of control and can be widely employed. Because of its logical reasoning abilities and superior approximation performance, T-S fuzzy model was adopted to capture the nonlinear dynamical system in [1]. Subsequently, fuzzy controllers were also designed to guarantee the system stability in [2]–[6]. Generally, in the field of control, the role of computer is to control the controlled plants as a digital controller. When the continuous-time measurement signal is processed by the digital computer, the measurement signal is sampled and quantized firstly. Next, the discrete-time signal is generated by a zero-order holder, and then transmitted back to the continuous-time control input signal.

By now, there have been a lot of reports on the synthesis and analysis of sampled-data control with fuzzy form [7]–[37]. Among these existing literatures, [7], [9], [12], [21], [23], [24], [28]–[31] had analyzed the stability of the system. [18], [19], [23], [32], [33] had carried on the stabilization, [8], [10], [13], [14], [16], [25], [27] had discussed the H∞ control, [15], [21] had studied the H2 GC control, [11], [17], [26] had discussed the tracking control, and [20], [22] had considered the filtering. Meanwhile, delay phenomena often occur in some engineering systems, such as manual control, aircraft stability, ship stability and nuclear reactor. Moreover, in practical systems, the phenomenon of time delay leads to oscillations and instability frequently. In [9], [10], [14], [16], [34], [35], some fuzzy sampled-data control schemes were designed for nonlinear dynamical systems with delay behavior. The control performance design was described clearly in [9]. In [10], the issue of reliable non-uniform H∞ sampling fuzzy control was analyzed, in which the generalized model transformation and input delay method were employed. Meanwhile, H∞ control was also investigated by utilizing Leibniz-Newton formula and Lyapunov-Krasovskii functional [14]. Furthermore, a distinctive delay-dependent stabilization standard was proposed in [16]. For T-S fuzzy systems with time delay and parametric uncertainties, a robust guaranteed cost sampled-data fuzzy control design method was proposed in [34]. In [35], the stabilization of a T-S fuzzy system with time delay was explored, in which the designed fuzzy controller of sampled-data contained both the current and delayed state information. In addition, if the H2 performance and H∞ performance can be optimized simultaneously, the control system will show a better performance. Nevertheless, fuzzy H2/H∞ sampled-data control issue of dynamical systems with delay behavior has not been explored yet.

H2 performance focuses on the system state and the system input. H∞ performance is mainly reflected in the gain between the system state and the external disturbance.
However, $H_2$ control or $H_{\infty}$ control can only achieve a single control performance. In the existing literatures, fuzzy sampled-data $H_2$ control and fuzzy sampled-data $H_{\infty}$ control have been explored respectively. But, the dimension of LMIs is very large, which can bring about the conservatism of the results. Therefore, for some engineering systems, how to design a more convenient and easier to implement sampled-data controller with larger sampling interval is still an unsolved problem.

Based on the above researches, fuzzy $H_2/H_{\infty}$ sampled-data control issue is proposed for dynamical systems with delay behavior. A new $H_2/H_{\infty}$ criterion is constructed and formulated as the linear matrix inequalities (LMIs) form. The feasibility of sampled-data control scheme is tested by simulation results. Moreover, by considering both $H_{\infty}$ performance and $H_2$ performance, the global optimization algorithm is achieved and a fuzzy $H_2/H_{\infty}$ sampled-data control scheme is firstly investigated for dynamical systems with delay behavior. The presented control algorithm is less conservative, which reduces the dimension of the LMIs and shortens the implementation time.

The main innovation points of this paper are embodied in several aspects.

(I) The proposed control algorithm is less conservative, where the dimension of the LMIs is simplified, which adds the existence of the feedback gains, reduces the computational complexity and lowers the implementation time.

(II) The proposed method achieves a better performance, where fuzzy sampled-data controller has a larger sampling interval and shows a better attenuated level and faster state responses.

Notations: Throughout the manuscript, the notation $W > 0$ means the positivity (negativity) $W^T$ represent the transpose of a matrix $W$; and $*$ describes the symmetric term of a symmetric matrix. It is assumed that the matrices are compatible.

II. PROBLEM FORMULATION

Analyze the T–S fuzzy model capturing the dynamical system with delay behavior:

Rule $i$: IF $\sigma_1(t)$ is $N_{i1}$ and $\cdots$ and $\sigma_p(t)$ is $N_{ip}$, THEN

$$\dot{x}(t) = \tilde{A}_i x(t) + \tilde{A}_{id} x(t-d) + \tilde{B}_i u(t) + \tilde{C}_i w(t), \quad i = 1, \ldots, M \quad (1)$$

where the state, the input, and the disturbance are expressed by $x(t)$, $u(t)$, and $w(t)$, respectively; the matrices $\tilde{A}_i$, $\tilde{B}_i$, and $\tilde{A}_{id}$ are time-invariant; and time delay $d$ is a constant.

The global system is deduced as

$$\dot{x}(t) = \sum_{i=1}^{M} \xi_i(\sigma(t)) \left[ \tilde{A}_i x(t) + \tilde{A}_{id} x(t-d) + \tilde{B}_i u(t) + \tilde{C}_i w(t) \right] \quad (2)$$

where $\xi_i(\sigma(t)) \geq 0$, $i = 1, 2, \ldots, M$, and $\sum_{i=1}^{M} \xi_i(\sigma(t)) = 1$.

The $j$ th rule of fuzzy controller is as follows:

Rule $j$: IF $\sigma_j(t_k)$ is $N_{j1}$ and $\cdots$ and $\sigma_p(t_k)$ is $N_{jp}$, THEN

$$u(t) = G_j x(t_k), \quad t_k \leq t < t_{k+1}, \quad j = 1, 2, \ldots, M$$

where $G_j$ represents the gain, and $0 < t_{k+1} - t_k = h_k \leq h$.

Therefore, the global controller is shown as

$$u(t) = \sum_{j=1}^{M} \xi_j(\sigma(t_k)) G_j x(t_k). \quad (3)$$

Let $\tau(t) = t - t_k$, $t_k \leq t < t_{k+1}$. Obviously, the derivative $\dot{\tau}(t) = 1$ for $t \neq t_k$. By employing the input delay method, (3) is rewritten as

$$u(t) = \sum_{j=1}^{M} \xi_j(\sigma(t_k)) G_j x(t - \tau(t)). \quad (4)$$

By considering the system (2) and the controller (4), the closed-loop system is obtained as

$$\dot{x}(t) = \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_i(\sigma(t)) \xi_j(\sigma(t_k)) \left[ \tilde{A}_i x(t) + \tilde{A}_{id} x(t-d) + \tilde{B}_i u(t) + \tilde{C}_i w(t) \right], \quad (5)$$

where two delays $x(t-d)$ and $x(t-\tau(t))$ are independent.

Let us take into account the following $H_{\infty}$ control performance

$$\int_{0}^{\infty} x^T(t) S_1 x(t) dt \leq \varepsilon^2 \int_{0}^{\infty} \omega^T(t) \omega(t) dt \quad (6)$$

where $\varepsilon$ denotes a attenuation level and the matrix $S_1$ is positive-definite.

The $H_2$ control performance is as follows:

$$J = \int_{0}^{\infty} (x^T(t) S_2 x(t) + u^T(t) Q u(t)) dt \quad (7)$$

where $S_2$ and $Q$ are positive-definite matrices.

By analyzing a desired $H_{\infty}$ disturbance rejection constraint in (6), the suboptimal $H_2$ control performance (7) is obtained. Our control objective aims to explore a fuzzy sampled-data controller to ensure the $H_2/H_{\infty}$ performance for the closed-loop system (5).

III. FUZZY $H_2/H_{\infty}$ SAMPLED-DATA CONTROL

Based on the LMIs, a fuzzy sampled-data $H_2/H_{\infty}$ control criterion is proposed as follows.

Theorem 1. Considering the closed-loop system (5), for given matrices $S_1 > 0$, $S_2 > 0$, and $Q > 0$, scalars $\varepsilon > 0$, $h > 0$, and $\lambda > 0$, the $H_2/H_{\infty}$ control performance in (6) and (7) is satisfied, simultaneously, if there exist matrices $T_1 > 0$ and $T_2 > 0$ satisfying the LMIs (8) and (9)

$$R_{ij} = \begin{bmatrix} R_{ij11} & R_{ij12} & R_{ij13} & R_{ij14} & R_{ij15} \\ * & R_{ij22} & 0 & 0 & 0 \\ * & * & R_{ij33} & R_{ij34} & 0 \\ * & * & * & R_{ij44} & R_{ij45} \\ * & * & * & * & R_{ij55} \end{bmatrix} < 0 \quad i, j = 1, 2, \ldots, M \quad (8)$$
where

\[ \begin{bmatrix}
    z_{ij11} & z_{ij12} & z_{ij13} & 0 & z_{ij15} & z_{ij16} \\
    * & z_{ij22} & 0 & 0 & 0 & 0 \\
    * & * & z_{ij33} & z_{ij34} & z_{ij35} & 0 \\
    * & * & * & z_{ij44} & 0 & 0 \\
    * & * & * & * & z_{ij55} & z_{ij56} \\
    * & * & * & * & * & z_{ij66}
\end{bmatrix} < 0 \]

(9)

Proof: By employing (13) and Leibniz-Newton formula, we get

\[
\begin{align*}
\dot{V}_2(x_t) &= x^T(t)Q_1x(t) - x^T(t-d)Q_1x(t-d) \\
&= -\left(\int_{t-d}^{t} x(s)ds\right)^T Q_2 \left(\int_{t-d}^{t} x(s)ds\right).
\end{align*}
\]

(13)

Leibniz-Newton formula is

\[
\int_{t-h}^{t} x(s)ds = x(t) - x(t-h).
\]

(14)

By employing (13) and Leibniz-Newton formula, we get

\[
\dot{V}_3(x_t) = h^2\dot{x}^T(t)Q_2\dot{x}(t) - h \int_{t-h}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds
\]

\[
\leq h^2\dot{x}^T(t)Q_2\dot{x}(t) - (x(t) - x(t-h))\int_{t-h}^{t} Q_2(x(t) - x(t-h))ds
\]

\[
= h^2\dot{x}^T(t)Q_2\dot{x}(t) - x^T(t)Q_2x(t) + x^T(t)Q_2x(t) - x^T(t)Q_2x(t-h)
\]

\[
= -\dot{x}^T(t)Q_2x(t) + x^T(t)Q_2x(t-h).
\]

(15)

It should be clarified that for a given \( \mu > 0 \), we have

\[
0 = -2\lambda\dot{x}^T(t)W\dot{x}(t) + \lambda\dot{x}^T(t)W\dot{x}(t)
\]

\[
+ x^T(t - \tau(t))G_T B_T W\dot{x}(t) + x^T(t)W\dot{A}_ix(t)
\]

\[
+ x^T(t)W\dot{A}_{id}x(t-d) + x^T(t)W\dot{B}_igx(t - \tau(t))
\]

\[
+ \frac{1}{\varepsilon^2}x^T(t)WWx(t) + \varepsilon^2\omega^T(t)\omega(t)
\]

\[
- \frac{1}{\varepsilon}Wx(t) - \varepsilon\omega(t)\left(\frac{1}{\varepsilon}(Wx(t) - \varepsilon\omega(t))\right]
\]

\[
\leq \sum_{i=1}^{M} \sum_{j=1}^{M} \varepsilon_i(\sigma(t)))\varepsilon_j(\sigma(t)) [x^T(t)\dot{A}_i^TWx(t)
\]

\[
+ x^T(t - \tau(t))G_T B_T W\dot{x}(t)
\]

\[
+ x^T(t - \tau(t))G_T B_T W\dot{x}(t) + x^T(t)W\dot{A}_ix(t)
\]

\[
+ x^T(t)W\dot{A}_{id}x(t-d) + x^T(t)W\dot{B}_igx(t - \tau(t))
\]

\[
+ \frac{1}{\varepsilon^2}x^T(t)WWx(t) + \varepsilon^2\omega^T(t)\omega(t)
\]

\[
- \frac{1}{\varepsilon}Wx(t) - \varepsilon\omega(t)\left(\frac{1}{\varepsilon}(Wx(t) - \varepsilon\omega(t))\right]
\]

\[
\leq \sum_{i=1}^{M} \sum_{j=1}^{M} \varepsilon_i(\sigma(t))\varepsilon_j(\sigma(t)) [x^T(t)\dot{A}_i^TWx(t)
\]

\[
+ x^T(t - \tau(t))G_T B_T W\dot{x}(t)
\]

\[
+ x^T(t - \tau(t))G_T B_T W\dot{x}(t) + x^T(t)W\dot{A}_ix(t)
\]

\[
+ x^T(t)W\dot{A}_{id}x(t-d) + x^T(t)W\dot{B}_igx(t - \tau(t))
\]

\[
+ \frac{1}{\varepsilon^2}x^T(t)WWx(t) + \varepsilon^2\omega^T(t)\omega(t)
\]

\[
- \frac{1}{\varepsilon}Wx(t) - \varepsilon\omega(t)\left(\frac{1}{\varepsilon}(Wx(t) - \varepsilon\omega(t))\right]
\]

(12)
\[
\dot{V}(x_i) \leq \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_i(\sigma(t))\xi_j(\sigma(t)) \left[ \lambda T(t)W_{\hat{A}_i}x(t) + x^T(t)W_{\hat{A}_i}x(t) - x^T(t - d)\hat{A}_{id}^T W_{\hat{A}_i}x(t) + \lambda\omega^2(t)W_{\hat{A}_i}x(t) \right] \\
\leq -2\lambda\dot{x}(t)W_{\hat{x}}(t) + \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_i(\sigma(t))\xi_j(\sigma(t)) \left[ \lambda T(t)W_{\hat{A}_i}x(t) + x^T(t)W_{\hat{A}_i}x(t) - x^T(t - d)\hat{A}_{id}^T W_{\hat{A}_i}x(t) + \lambda\omega^2(t)W_{\hat{A}_i}x(t) \right]
\]

where
\[
W = W^{-1}, \quad \mathcal{Q}_i = W^{-1}Q_1W^{-1}, \quad \mathcal{Q}_7 = W^{-1}Q_2W^{-1}, \\
\mathcal{S}_1 = W^{-1}S_1W^{-1}, \quad \bar{G}_j = G_j\bar{W}, \quad j = 1, 2, \ldots, M.
\]

By adopting the Schur complement in (8), we have \(\mathcal{C}_{ij} < 0\). Therefore, \(\mathcal{C}_{ij} < 0\). Substituting \(\mathcal{C}_{ij}' < -\mathcal{S}\) into (17), one has

\[
\mathcal{V}(x_i) \leq -\lambda\dot{x}(t)\dot{\mathcal{S}}(t) + \varepsilon^2\omega^2(t)\omega(t). \quad (21)
\]

Owing to \(\mathcal{S} = \text{diag} \left[ (1 + \lambda)S_1 0 0 0 \right] \), we get

\[
\mathcal{V}(x_i) \leq -(1 + \lambda)\dot{x}(t)\mathcal{S}x(t) + (1 + \lambda)\varepsilon^2\omega^2(t)\omega(t). \quad (22)
\]

Integrating both sides of (22), there is

\[
\int_0^\infty x^T(t)\mathcal{S}x(t)dt \leq \varepsilon^2\int_0^\infty \omega^2(t)\omega(t)dt. \quad (23)
\]

Remark 1: Eq. (8) provides a new relaxed stability condition for the system (2). Unlike the existing works, in the proof, eq.(16) is introduced to consider the fuzzy relationship between \(\dot{x}(t), x(t), x(t - \tau(t))\) and \(x(t - d)\). By using the LMIs, it is easy to determine the feedback gains.

Now, we continue to consider the \(H_2\) control performance for the closed-loop system (5) in the absence of \(\omega(t)\).

The derivative of \(V\) in the absence of \(\omega(t)\) is as follows:

\[
\dot{V}_1(x) = x^T(t)W_{\hat{A}_i}x(t) + x^T(t)W_{\hat{A}_i}\dot{x}(t)
\]
\[
= \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_j(\sigma(t))\xi_j(\sigma(t)) x^T(t)\hat{A}_{id}^T W_{\hat{A}_i}x(t)
\]
\[
+ x^T(t - d)\hat{A}_{id}^T W_{\hat{A}_i}x(t) + x^T(t - \tau(t))G_j^T\hat{B}_i^T W_{\hat{A}_i}x(t)
\]
\[
+ x^T(t)W_{\hat{A}_i}x(t) + x^T(t)W_{\hat{A}_i}x(t - d)
\]
\[
+ x^T(t - d)W_{\hat{B}_i}G_jx(t - \tau(t)). \quad (24)
\]

By using (15),

\[
\dot{V}_2(x_i) \leq h^2x^2(t)\mathcal{Q}_2\dot{x}(t) - x^T(t)\mathcal{Q}_2x(t)
\]
\[
+ x^T(t - \tau(t))Q_2x(t) + x^T(t)Q_2x(t - \tau(t)) - x^T(t - \tau(t))Q_2x(t - \tau(t)). \quad (25)
\]

It is noted that for a given \(\mu > 0\), one has
\[
0 = -2\lambda\dot{x}^T(t)W_{\hat{A}_i}x(t)
\]
\[
+ \lambda\dot{x}^T(t)W\left[ \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_j(\sigma(t))\xi_j(\sigma(t)) \right] \hat{A}_{id}x(t)
\]
\[
+ \hat{A}_{id}x(t - d)
\]
\[
+ \hat{B}_iG_jx(t - \tau(t))). \quad (26)
\]
From (24)-(27), we can obtain that
\[
\dot{V}(x_t) + x^T(t)S_2x(t) + u^T(t)Qu(t) \\
\leq \sum_{i=1}^{M} \sum_{j=1}^{M} \xi_i(x(t))\xi_j(x(t))\lambda_i^2(t) \dot{\lambda}_i x(t) \\
+ \lambda_i \dot{x}(t) W \dot{\lambda}_i x(t - d) + \lambda_i \dot{x}(t) W \dot{\lambda}_i x(t - \tau(t)) \\
+ \lambda_i \dot{x}(t) \dot{A}_i W \dot{x}(t) + \lambda_i \dot{x}(t - d) \dot{A}_i W \dot{x}(t) \\
+ \lambda_i \dot{x}(t - \tau(t)) G_j \dot{B}_j W \dot{x}(t).
\]

(27)

where
\[
\lambda_i(t) = [x^T(t) x^T(t - \tau(t)) \dot{x}^T(t) x^T(t - d)]^T
\]

(30)

and
\[
\bar{\lambda}_i = \bar{A}_i W + W \bar{A}_i^T + Q_1 - Q_2 + S_2, \\
\lambda_i j_2 = W \bar{B}_j G_j + Q_2, \\
\lambda_i j_4 = \bar{A}_i W, \quad \lambda_i j_4 = W \bar{A}_i, \\
\lambda_i j_2 = -Q_2 + G_j^T Q_j, \\
\lambda_i j_2 = \lambda_j G_j \bar{B}_j W, \quad \lambda_i j_3 = -2\lambda_j W + h^2 Q_2, \\
\lambda_i j_4 = \lambda_j W \bar{A}_i, \\
\lambda_i j_4 = -Q_1.
\]

The design of this paper is formulated as the following optimization problem.

**Theorem 2:** Considering the fuzzy closed-loop system (5), if the following problem
\[
\min _{\bar{W}} \text{Trace}(U) \\
s.t. \quad \bar{W} > 0, \quad \bar{G}_j > 0, \quad \bar{G}_2 > 0, \quad (8), \quad \text{and} \quad \left[ \begin{array}{cc} U & I \\ * & \bar{W} \end{array} \right] > 0
\]

has a solution \(\bar{G}_j, j = 1, \ldots, M\) and \(\bar{W}\), then a whole optimal \(H_2/\infty\) control performance is achieved. The control gains are \(G_j = \tilde{G}_j \bar{W}^{-1}, j = 1, 2, \ldots, M\).

**Remark 2:** Fuzzy \(H_2/\infty\) sampled-data control is firstly discussed for time-delay systems. In practical engineering systems, the \(H_2/\infty\) sampled-data control is more appealing in achieving the desired control performance. And, the focus of \(H_\infty\) performance is mainly on the gain between the system state and the external disturbance. Meanwhile, both the system state and the system input are considered in \(H_2\) performance. Furthermore, \(H_2/\infty\) control synthesizes the merits of \(H_\infty\) control and \(H_2\) control.

**Remark 3:** The proposed control algorithm simplifies the dimension of the LMI, reduces the computational complexity, and is less conservative. At the same time, it increases the existence of the feedback gain and shortens the implementation time. The designed method has better performance, in which the fuzzy sampled-data controller has a larger sampling interval, a faster state response and a better attenuation level.

**IV. SIMULATION EXAMPLES**

The proposed design is demonstrated by two practical dynamical systems with delay. In addition, the superiority is obvious in the obtained simulation results.

**Example 1:** The truck-trailer system in [39] is
\[
\dot{x}_1(t) = -\bar{a}\frac{v^2}{L_0}x_1(t) - (1 - \bar{a})\frac{v^7}{L_0}x_1(t - t_d) \\
+ \frac{v^7}{L_0}u(t) + w(t) \\
\dot{x}_2(t) = \bar{a}\frac{v^2}{L_0}x_1(t) + (1 - \bar{a})\frac{v^7}{L_0}x_1(t - t_d) \\
\dot{x}_3(t) = \frac{v^7}{L_0} \sin(x_2(t)) + \bar{a}(v^7/2L_0)x_1(t) + (1 - \bar{a})(v^7/2L_0)x_1(t - t_d)
\]

(35)

where \(l = 2.8, L = 5.5, v = -1.0\bar{a} = 0.7, \tilde{t} = 2.0, t_0 = 0.5, w(t)\) is the external disturbance.

Let \(x(t) = [x_1(t) x_2(t) x_3(t)]^T\). The truck-trailer system is represented as

\[
\dot{x}(t) = \sum_{i=1}^{2} \xi_i(x(t))\left[ \dot{A}_i x(t) + \dot{A}_i d x(t - d) + \dot{B}_i u(t) + \omega(t) \right]
\]

(33)
The maximum sampling interval with $\varepsilon = 1.0$.

| Method  | [10] | Theorem 1 |
|---------|------|-----------|
| $h_{\text{max}} (t = 0.5)$ | 0.374 | 0.432 |
| $h_{\text{max}} (t = 1)$ | 0.315 | 0.378 |
| $h_{\text{max}} (t = 2)$ | 0.251 | 0.312 |

By employing the methods of [22], [10], and Theorem 1, a lower dimension in the LMIs is obtained in this paper, see, Table 1. That is to say, the computation burden of this paper is smaller than that of the methods in [22] and [10].

Using the methods of [10] and Theorem 1, a larger sampling interval is obtained, see, Table 2.

The maximum sampling interval with $\varepsilon = 1.0$.

| Method  | [10] | Theorem 1 |
|---------|------|-----------|
| $h_{\text{max}} (t = 0.5)$ | 0.374 | 0.432 |
| $h_{\text{max}} (t = 1)$ | 0.315 | 0.378 |
| $h_{\text{max}} (t = 2)$ | 0.251 | 0.312 |

When $\varepsilon = 1.0$, Theorem 1 gives the maximum sampling interval $h = 0.432$. With $\lambda = 0.6, S_1 = \text{diag}(1, 1, 0.1) \times 10^{-8}, S_2 = \text{diag}(1, 1, 0.1) \times 10^{-8}, Q = 1 \times 10^{-5}$, fuzzy state feedback gains are

$$
\begin{align*}
G_1 &= [1.2749, -0.6031, 0.0122], \\
G_2 &= [1.2749, -0.6031, 0.0122].
\end{align*}
$$

Experimental results are depicted in FIGUREs. 1-4. The system stability is confirmed and the piecewise continuous behavior of the control input is also plotted.

When $h = 0.432$, we find that this paper shows faster state responses than those of the $H_{\infty}$ method in [22].

Example 2: The stirred tank reactor system in [40] is

$$
\begin{align*}
\dot{x}_1(t) &= -\frac{1}{\nu} x_1(t) + D_x (1-x_1(t)) e^{\frac{x_2(t)}{\nu}} + \left(\frac{1}{\nu} - 1\right) x_1(t - \tau) \\
\dot{x}_2(t) &= \left(\frac{1}{\nu} + \beta\right) x_2(t) + HD (1-x_1(t)) e^{\frac{x_2(t)}{\nu}} \\
&\quad + \left(\frac{1}{\nu} - 1\right) x_2(t - \tau) + \beta u(t) + \beta w(t)
\end{align*}
$$

(36)
where $\gamma_0 = 20$, $H = 8$, $D_e = 0.072$, $v = 0.8$, $\beta = 0.3$, $0 \leq x_1(t) \leq 1$, and $w(t)$ is the external disturbance.

Let $x(t) = [x_1(t), x_2(t)]^T$. The stirred tank reactor system is described as

$$\dot{x}(t) = \sum_{i=1}^{3} \xi_i(\sigma(t))[\hat{A}_i x(t) + \hat{A}_{id} x(t - \tau) + \hat{B}_i u(t) + \omega(t)]$$

with

$$\hat{A}_1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix}, \quad \hat{A}_{1d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6268 \end{bmatrix}, \quad \hat{A}_{2d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{A}_3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9387 \end{bmatrix}, \quad \hat{A}_{3d} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, \quad \hat{B}_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$$

$\xi_1(x_2(t)) = \begin{cases} 1, & x_2 \leq 0.8862 \\ \frac{x_2(0.8862) - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2 < 2.7520 \\ 0, & x_2 \geq 2.7520 \end{cases}$

$\xi_2(x_2(t)) = \begin{cases} 1 - \xi_1(x_2(t)), & x_2 \leq 2.7520 \\ 1 - \xi_3(x_2(t)), & x_2 > 2.7520 \end{cases}$

$\xi_3(x_2(t)) = \begin{cases} 0, & x_2(2.7520) - 2.7520 \\ \frac{x_2(2.7520) - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2 < 4.7052 \\ 1, & x_2 \geq 4.7052 \end{cases}$

By using the methods of [22], [10] and Theorem 1, a lower dimension in the LMIs is achieved, see, Table 3.

That is to say, the computation burden in this paper is smaller than that of the methods in [22] and [10].

When $h = 0.1$, there is $\tau_{\min} = 0.0497$ under Theorem 1. The minimum disturbance attenuation is 0.0497, which is ineffective in the LMI conditions of $H_\infty$ control [22]. This implies that the proposed fuzzy $H_2/H_\infty$ sampled-data control method is superior to the existing $H_\infty$ control method.

When $\varepsilon = 1.0$, Theorem 1 gives $h_{\max} = 0.222$. The maximum sampling interval is 0.222, which is infeasible in the LMI conditions of $H_2$ control [10]. This implies that the proposed fuzzy $H_2 / H_\infty$ sampled-data control method is superior to the existing $H_2$ control method.

According to the design procedure, we get the optimal design parameters $\varepsilon = 0.0603, h = 0.12, \lambda = 0.2$. With $S_1 = \text{diag}(9.8 \times 10^{-6}, S_2 = \text{diag}(2.6 \times 10^{-4}, Q = 1 \times 10^{-6}$, Theorem 1 gives state feedback gains

$G_1 = [30.2128 - 17.8191], \quad G_2 = [30.2128 - 17.8191]$

$G_3 = [30.2128 - 17.8191]$

Simulation results under $\tau = 5$ are shown in FIGUREs. 5-6. The system stability is confirmed and the piecewise continuous behavior of the control input is also portrayed.
V. CONCLUSION
Fuzzy $H_2/H_\infty$ sampled-data control for dynamical systems with delay behavior is addressed by using the input delay method. Based on Lyapunov theory, a fuzzy sampled-data controller is proposed, which can guarantee the $H_\infty$ performance and $H_2$ performance concurrently. Moreover, a new $H_2/H_\infty$ standard is derived. Both stability analysis and simulation results demonstrate the proposed design and the superiority have also been verified through comparative analysis. In the future, this method will be applied to uncertain dynamical systems and provide theoretical support for further research. Furthermore, we will pay attention to more developments on the sampled-data control, the adaptive neural control, the adaptive fuzzy tracking control, the event-triggered scheme and the stochastic control in the literatures.

REFERENCES
[1] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” IEEE Trans. Syst., Man, Cybern., vol. SMC-15, no. 1, pp. 116–132, Jan/Feb. 1985.
[2] K. Tanaka and M. Sugeno, “Stability analysis and design of fuzzy control systems,” Fuzzy Sets Syst., vol. 45, no. 2, pp. 135–156, Jan. 1992.
[3] C. Lin, Q. G. Wang, T. H. Lee, Y. He and B. Chen, “Observer-based $H_\infty$ fuzzy control design for T-S fuzzy systems with state delays,” Automatica, vol. 44, no. 3, pp. 866–874, Mar. 2008.
[4] K. Tanaka, H. Ohtake, and H. Wang, “ Guaranteed cost control of polynomial fuzzy systems via a sum of squares approach,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 39, no. 2, pp. 561–567, Apr. 2009.
[5] H. Lam and M. Narimani, “Quadratic-stability analysis of fuzzy-model-based control systems using staircase membership functions,” IEEE Trans. Fuzzy Syst., vol. 18, no. 1, pp. 125–137, Feb. 2010.
[6] J. Dong and G.-H. Yang, “Control synthesis of T-S fuzzy systems based on a new control scheme,” IEEE Trans. Fuzzy Syst., vol. 19, no. 2, pp. 323–338, Apr. 2011.
[7] N. Ishikawa, H. Katayama, J. Yoneyama, and A. Ichioka, “Design of output feedback controllers for sampled-data fuzzy systems,” Int. J. Syst. Sci., vol. 31, no. 4, pp. 439–448, Jan. 2000.
[8] H. Katayama and A. Ichioka, “$H_\infty$ control for sampled-data nonlinear systems described by Takagi–Sugeno fuzzy systems,” Fuzzy Sets Syst., vol. 148, no. 3, pp. 431–452, Dec. 2004.
[9] H. K. Lam and F. H. F. Leung, “Sampled-data fuzzy controller for time-delay nonlinear systems: Fuzzy-model-based LMI approach,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 37, no. 3, pp. 617–629, Jun. 2007.
[10] D. Y. Yang and K. C. Y. Cai, “Reliable $H_\infty$ non-uniform sampling fuzzy control for nonlinear systems with time delay,” IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 38, no. 6, pp. 1606–1613, Dec. 2008.
[11] H. Lam and L. Seneviratne, “Tracking control of sampled-data fuzzy-model-based control systems,” IET Control Theory Appl., vol. 3, no. 1, pp. 56–67, Jan. 2009.
[12] D. W. Kim and H. J. Lee, “Stability connection between sampled-data fuzzy control systems with quantization and their approximate discrete-time model,” Automatica, vol. 45, no. 6, pp. 1518–1523, Jun. 2009.
[13] J. Yoneyama, “Robust control of uncertain fuzzy systems under time-varying sampling,” Fuzzy Sets Syst., vol. 161, no. 6, pp. 859–871, Mar. 2010.
[14] C.-H. Lien, K.-W. Yu, C.-T. Huang, P.-Y. Chou, L.-Y. Chung, and J.-D. Chen, “Robust control for uncertain T-S fuzzy time-delay systems with sampled-data input and nonlinear perturbations,” Nonlinear Anal., Hybrid Syst., vol. 4, no. 3, pp. 550–556, Aug. 2010.
[15] J. Yoneyama, “Robust guaranteed cost control of uncertain fuzzy systems under time-varying sampling,” Appl. Soft Comput., vol. 11, no. 1, pp. 249–255, Jan. 2011.
[16] C. Peng, Q.-L. Han, D. Yue, and E. Tian, “Sampled-data robust control for T-S fuzzy systems with time delay and uncertainties,” Fuzzy Sets Syst., vol. 179, no. 1, pp. 20–33, Sep. 2011.
[17] S. Hu, D. Yue, Z. Du, and J. Liu, “Reliable $H_\infty$ non-uniform sampling tracking control for continuous-time non-linear systems with stochastic actuator faults,” IET Control Theory Appl., vol. 6, no. 1, p. 120, 2012.
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