Calculation of conditions for maintaining an ICRF-plasma using a self-consistent model

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Abstract. The article proposes a new approach to calculating the strength of the magnetic field on the inner wall of the discharge chamber, which is necessary to maintain a steady state of a low-pressure ICRF discharge. The model is treated as a nonlinear eigenproblem. The influence of the third type boundary conditions for electron density as well as and the nonlinear boundary conditions for electrical strength is considered. This approach makes it possible solving two problems of designing ICRF plasma torches: for a given electron density in the discharge find the magnetic field strength that ensures the maintenance of the discharge, or, conversely, at a given magnetic field strength, determine the value of the electron density that can be created in the discharge. In addition, the radial distributions of the electric and magnetic fields and the electron concentration can be determined.

1. Introduction

Plasma treatment in a low-pressure (13.3-133 Pa) inductively coupled radio-frequency (LP ICRF) discharge is an effective method for modifying various materials [1]. The LP ICRF plasma has the following properties: ionization degree \(10^{-7}-10^{-5}\), electron density \(10^{16}-10^{19}\) m\(^{-3}\), the temperature of atoms and ions in a plasma bunch (3-4)\(\times10^4\) K while in a plasma jet (3.5–10) \(\times10^2\) K, electron temperature 1-4 eV [1, 2]. The operating parameters of the LP ICRF plasma torch can be obtained using mathematical modeling methods.

2. Mathematical model for LP ICRF-plasma

A mathematical model of a steady-state LP ICRF discharge is considered. To simplify the model, several approximations are introduced: the inductor is infinite (neglecting the effects at the ends of the inductor), the plasma is quasineutral and has the azimuthal symmetry, thus the problem is reduced to one-dimensional (see Figure 1).

The LP ICRF plasma model includes Maxwell's equations which is rearrangement to a system of elliptic equations for the squares of the moduli of intensities [1-2], and the electron balance equation with the corresponding boundary conditions:

\[
\frac{1}{r} \frac{d}{dr} \left( \frac{r dH^2}{dH} \right) = 2\sigma E^2, \quad \left. \frac{dH}{dr} \right|_{r=0} = 0, \quad H^2|_{r=R} = H_0^2, \tag{1}
\]

\[
\frac{1}{r} \frac{d}{dr} \left( \frac{1}{r} \frac{d(r^2E^2)}{dr} \right) = 2\mu_0\omega H^2, \quad E|_{r=a} = 0, \quad \left. \frac{d(r^2E^2)}{dr} \right|_{r=R} = R^2\sqrt{E^2H^2}, \tag{2}
\]
Figure 1. Model of an “infinite” inductor. The radius vector \( r \) is drawn from the center of the plasma torch to its wall

\[
\frac{d}{dr} \left( r D_a \frac{dn_e}{dr} \right) + \nu_i n_e = 0, \quad \frac{dn_e}{dr} \bigg|_{r=0} = 0, \quad D_a \frac{dn_e}{dr} \bigg|_{r=R} = -\alpha n_e.
\]  

(3)

Here \( E(r) \) is the modulus of the electric field, \( H \) is the modulus of the magnetic field, \( \sigma \) is the conductivity, \( D_a = D_a(E) \) is the coefficient of ambipolar diffusion, \( n_e \) is the electron density, \( \nu_i = \nu_i(E) \) is the ionization frequency, \( \alpha \) is a coefficient of electron reflection from a potential barrier created by a double layer on the wall of the discharge chamber [3], \( r \) is the distance from the center of the plasma torch \( (r = 0) \) to its walls at \( r = 0.012 \) m, the conductivity is calculated using the following formula:

\[
\sigma = \frac{e^2 n_e \nu_c}{m_e (\nu_c^2 + \omega^2/4 \pi^2)}
\]

(4)

Here \( \nu_c \) is the collision frequency. The values of the distributions \( D_a, \nu_i \), and \( \nu_c \) are calculated by the Bolsig+ program [4].

The equation (3) can be rewritten in dimensionless quantities, as:

\[
-\frac{1}{\rho} \frac{d}{d\rho} \left( \rho \frac{d\tilde{n}}{d\rho} \right) = \frac{R^2 \nu_{i0}}{D_{a0}} \tilde{\lambda} \tilde{n} = \tilde{\lambda} \tilde{n}.
\]

(5)

where \( \rho = r/R, \tilde{n} = n/n_{eo}, \tilde{D} = D_a/D_{a0}, \nu_{i0} = \nu_{i}/\nu_{i0} \) is dimensionless value, \( n_{eo}, D_{a0} = D_a[E(R)], \nu_{i0} = \nu_{i}[E(R)] \) is the maximal value of the associated functions. The boundary conditions for the equation (5) are like the boundary conditions in (3). The equation (3) with the uniform boundary conditions like in (3) is an eigenproblem, and \( \lambda \) is the eigenvalue. Function \( \tilde{n}(\rho) \) must be non-negative \( (\tilde{n} \geq 0) \) at any \( 0 \leq \rho \leq 1 \), therefore \( \lambda \) is the smallest eigenvalue \( \tilde{\lambda} = \tilde{\lambda}_0 = \tilde{\lambda}_{min} \). The eigenfunction corresponding to the smallest eigenvalue can be found by the inverse iteration method [5] while the eigenvalue \( \lambda_0 \) can be found by the Rayleigh-Ritz relation [5]. For the original equations, it takes the form:

\[
\lambda_0 = \frac{D_{a0}}{R^2 \nu_{i0}} \tilde{\lambda}_0 = \frac{\int_0^R D_a(E) \left( \frac{dn_e}{dr} \right)^2 rdr}{\int_0^R \nu_i(E) \left( \frac{dn_e}{dr} \right)^2 rdr} = \frac{\lambda_0 (E_R/p)}{p}.
\]

(6)

where \( E_R = E(R), p \) is the gas pressure, \( n_e \) is the eigenfunction which is found by inverse iteration. But there is no an eigenvalue in equation (3). Therefore, the relationship \( \lambda_0(E_R/p) = 1 \) must be realized. This relationship is the nonlinear equation in unknown \( E_R \), which depends on the \( H_R \) and \( n_{eo} \) due to equation (1), (2) and relationship (4). Thus, we can find \( H_R \) at the given \( n_{eo} \), and conversely, we can find \( n_{eo} \) at the given \( H_R \).
3. Program realization of the ICRF-plasma model
An iterative method [6] for solving the problem was developed and a program was written in Python. A difference scheme is derived for each equation in the system. The eigenvalue problem is solved for the electron density. The program solves the equation for $n_e$ at given radial distributions of $E$ and $H$, then in this way $E$ and $H$ is calculated using parameters which are found on the preceding step. From these parameters, the magnetic field strength on the wall of the inductor is calculated, as well as the inductor coil current $I = 2H_R R$ under the assumption of a uniform distribution of the magnetic field strength along the radius in the inductor without load. The block diagram of the program is shown in Fig. 2a. Also, the calculation can be done in the reverse order: giving the value of the inductor coil current, we can calculate the distribution of electrons in the plasma torch (Fig. 2b).

4. Results and discussion
Parameters of an 1D model of ICRF plasma in argon at a gas pressure of 133 Pa and a field frequency of 1.76 MHz are calculated. As a result, it is found that the modulus of the magnetic field strength reaches maximum value 5670 A/m at the chamber wall. The magnetic field decreases towards the center of the plasma torch because of shielding by plasma (Fig. 3a). The modulus of the electric field strength increases almost linearly with $r$ up to 934 V/m (Fig. 3b). The maximum of electron density is reached at zero, in the center of the RF plasma torch (Fig. 4a), which qualitatively agrees with the experimental data [1]. Current density $j = \sigma E$ is shown in figure 4b; it found the inductor coil current on the wall equals to 136.08 A.

Figure 2. Algorithm for solving equations (1), (2), (3). (a) finding $n_e$ from $H_R$, (b) finding $H_R$ from $n_e$.

Figure 3. Modulus of the magnetic field (a) and modulus of the electric field along the radius of the RF plasma torch ($p = 133$ Pa, $f = 1.76 \times 10^6$ Hz, $I = 136.08$ A).
5. Conclusion
1. A self-consistent model of LP ICRF discharge has been developed, in which either boundary value of the magnetic field strength $H_R$ can be specified at a given value $n_{e0}$ which is the electron density at the discharge center, or on the contrary, $n_{e0}$ can be specified at a given $H_R$.
2. This model can be used to obtain initial approximation for the 2D model of the plasma torch.

6. Acknowledgments
The research was carried out with the financial support of the Russian Science Foundation (project No. 19-71-10055).

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