Quantum-like model for unconscious-conscious interaction and emotional coloring of perceptions and other conscious experiences

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Abstract

Quantum measurement theory is applied to quantum-like modeling of coherent generation of perceptions and emotions and generally for emotional coloring of conscious experiences. In quantum theory, a system should be separated from an observer. The brain performs self-measurements. To model them, we split the brain into two subsystems, unconsciousness and consciousness. They correspond to a system and an observer. The states of perceptions and emotions are described through the tensor product decomposition of the unconscious state space; similarly, there are two classes of observables, for conscious experiencing of perceptions and emotions, respectively. Emotional coloring is coupled to quantum contextuality: emotional observables determine contexts. Such contextualization reduces degeneration of unconscious states. The quantum-like approach should be distinguished from consideration of the genuine quantum physical processes in the brain (cf. Penrose and Hameroff). In our approach the brain is a macroscopic system which information processing can be described by the formalism of quantum theory.

keywords: Quantum-like model, unconscious-conscious interaction, emotional coloring, perceptions, quantum measurement theory

1 Introduction

We start the paper with citation from [1]: “Although emotions, or feelings, are the most significant events in our lives, there has been relatively little contact between theories of emotion and emerging theories of consciousness in cognitive
We want to formalize this contact in the quantum-like framework for cognition by generalizing the sensation-perception model based on quantum information theory.

In the present paper, the quantum theory of measurement is applied to quantum-like modeling of coherent generation of perceptions and emotions and generally for emotional coloring of conscious experiences. In quantum theory, a system should be separated from an observer (or a measurement apparatus) in the quantum framework, this separation is very important – although establishing the sharp boundary between a system and a measuring apparatus is a difficult problem. The brain, as a physical system, performs so to say self-measurements. To model such self-measurement, we split the brain, as an information processor, into two subsystems, unconsciousness $\mathcal{UC}$ and consciousness $\mathcal{C}$. The former plays the role of a system under observation and the latter of an observer (see [7]-[10] for mathematical modeling of join functioning of unconsciousness and unconsciousness based on treelike geometry of the brain). To model the cooperation of perceptions and emotions, the state space of $\mathcal{UC}$ is decomposed into the tensor product of corresponding state spaces,

$$\mathcal{H}_{\mathcal{UC}} = \mathcal{H}_{\text{per}} \otimes \mathcal{H}_{\text{em}}.$$ 

Similarly, there are considered two classes of observables $A$ and $O_{\text{sup}}$, for conscious experiencing of perceptions and emotions, respectively.

This paper is concentrated on modeling of emotional coloring of perceptions. But, we present the very general scheme of coloring of one class of conscious experiences with another, “basic experiences” are colored with “supplementary experiences”. The aim and the origin of such coloring will be discussed below.

Emotional coloring is modeled in the framework of quantum contextuality - emotional observables determine contexts for perceptions and other basic conscious experiences. We highlight that contextualization is a way to reduce degeneration of unconscious states. Such contextual reduction is very important, since the state space of unconsciousness $\mathcal{H}_{\mathcal{UC}}$ has huge dimension and each conscious experience $x$ is based on multidimensional subspace $\mathcal{H}_x$ of $\mathcal{H}_{\mathcal{UC}}$.

We stress that quantum-like modeling of brain’s functioning should be sharply distinguished from theories based on consideration of genuine quantum physical processes in the brain (cf. [11]-[20]). In the quantum-like approach, the brain is a macroscopic system which information processing can be described by the mathematical formalism of quantum theory (cf. Gunji et al. [21, 22]). The quantum-like cognition project (see, e.g., monographs [9], [23]-[29]) does not contradict to the quantum cognition project. However, we proceed without the assumption that quantum features of information processing by the brain are coupled to quantum physical processes. The main distinguishing feature of the quantum information processing

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1It becomes even more difficult if a conscious observer, experimenter, is involved into consideration of a measurement process. The latter was done by Wigner. He considered a physical system interacting with a measurement apparatus. The latter is under monitoring of an observer-experimenter whose nervous system detects and propagates the signal, e.g., visual, obtained from apparatus’ pointer. By Wigner [4, 5] (see also [6]), a measurement is completed by consciousness So, for him consciousness played the crucial role in the measurement process. The majority of the quantum community did not share Wigner’s viewpoint; for them physical measurement is terminated at the pointer of an apparatus.
is operating with superpositions of alternatives. Such operating can be physically realized with classical electromagnetic waves. Genuine quantum information features are coupled to measurements generating discrete events from superpositions.

We treat brain’s functioning in the purely informational framework, so the states are not physical (electrochemical) states, but the information states. We do not need to localize UC and C in the concrete brain’s areas. Of course, neuro-physiological studies give us coupling of UC and C to some areas in the brain, e.g., C to the prefrontal cortex. However, the distributed and physically nonlocalized information processing matches better our quantum-like model. We neither try to describe the neurophysiological mechanism of generation of perceptions and emotions and “coloring” of the former with the latter (see, e.g., [31] – [33] for details).

Of course, information processing in the brain is indivisibly coupled to electrochemical processes. However, as was emphasized by Liberman et al. [34] – [37] and Igamberdiev and Brenner [38], physical reduction of consciousness is impossible.

2 The basic theories of consciousness

As is well known, there are two basic competing theories of consciousness:

• the First Order Theory of Consciousness [41] – [45];
• the Higher Order Theory of Consciousness [46] – [48].

We characterize these theories with the following citation from article [1]:

“First-order theorists, such as Block, argue that processing related to a stimulus is all that is needed for there to be phenomenal consciousness of that stimulus [41] – [45]. Conscious states, on these kinds of views, are states that make us aware of the external environment. Additional processes, such as attention, working memory, and metacognition, simply allow cognitive access to and introspection about the first-order state. In the case of visual stimuli, the first-order representation underlying phenomenal consciousness is usually said to involve the visual cortex, especially the secondary rather than primary visual cortex. Cortical circuits, especially involving the prefrontal and parietal cortex, simply make possible cognitive (introspective) access to the phenomenal experience occurring in the visual cortex.”

“In contrast, David Rosenthal and other higher-order theorists argue that a first-order state resulting from stimulus-processing alone is not enough to make possible the conscious experience of a stimulus. In addition to having a representation of the external stimulus one also must be aware of this stimulus representation. This

2In Bohr’s works [3], such an event was called phenomenon and its individuality was emphasized (see [30] on comparison of quantum information processing in classical vs. quantum optics).

3See, e.g., paper [34]: “Biophysics cannot use the ordinary laws of physics and must take into account the influence on the phenomena to be studied, not only of a measurement but also of a calculation process in the real device predicting the future. See, also [37] “Living organisms measure many parameters in order to have orientation in the outer medium. That is why biophysics cannot use the ordinary laws of physics and must take into account the influence on the phenomena to be studied not only of a measurement but also of a calculation process in the real physical and biophysical device predicting the future.”
is made possible by a HOR, which makes the first-order state conscious. In other words, consciousness exists by virtue of the relation between the first- and higher-order states. Cognitive processes, such as attention, working memory, and metacognition are key to the conscious experience of the first-order state. In neural terms, the areas of the GNC, such as the prefrontal and parietal cortex, make conscious the sensory information represented in the secondary visual cortex.”

The Higher Order Theory distinguishes between unconscious and conscious processing of mental information in the brain. By this theory, what makes cognition conscious is a higher-order observation of the first-order processing. And in quantum theory observation is not simply inspection of the system’s state. This is a complex process of interaction between a system and a measurement device. This is the good place to cite Bohr [3]:

“This crucial point... implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled.”

This viewpoint matches better with the Higher Order Theory of Consciousness. A conscious experience is not simply introspection of the UC-state. In this paper, we do not describe the process of UC − C interaction. In particular, we do not operate with the states of C. This can be done similarly to paper [2] by using the scheme of indirect quantum measurements in the framework of open quantum systems theory. However, it would make the presentation essentially more complicated from the mathematical viewpoint and the basic issue of this paper, contextuality, would be overshadowed by technicalities. We apply straightforwardly the canonical quantum observation theory as it was made by von Neumann [49].

The latter is slightly generalized by representation of observables by projector valued measures, instead of Hermitian operators. This generalization is motivated by two things. Starting with a Hermitian operator, it is possible to find its spectral decomposition and the projectors onto the subspaces corresponding to the eigenvalues, but the subspaces appear as secondary objects. In the POV representation of observers, subspaces are fundamental, and observer’s values are associated with them. In our model, conscious experiences are associated with concrete subspaces of HUC. Another advantage of using POV-observables is that we are not obliged to label the values of observables by real numbers (operator’s eigenvalues); any vocabulary can be used for description of conscious experiences.

We remark that, although Bohr’s viewpoint on the outcomes of quantum mea-

4In [20] Igamberdiev considered another approach to quantum measurement theory, namely non-demolition quantum measurements, as the basis of bio-information processing. Such measurements differ from the von Neumann measurements. In some sense, they are closer to classical measurements of electromagnetic waves. Although he referred to genuine physical processes, he emphasized that in biosystems these processes are lifted to the macrolevel: “The smallest details of living systems are molecular devices that realize non-demolition quantum measurements. These smaller devices form larger devices (macromolecular complexes), up to living body. The quantum device possesses its own potential internal quantum state (IQS), which is maintained for a prolonged time via reflective error-correction.
measurements dominates, a few respectable scientists claimed that these outcomes can be considered as the objective properties of physical systems. Quantum measurements are treated as just approaching the premeasurement values of observables. This position was presented in the well know paper Einstein, Podolsky, and Rosen (EPR) \[50\]; later Bell elaborated (and modified) EPR’s argument \[51, 52\], but he confronted the problem of nonlocality. This line of thought matches better to the First Order Theory of Consciousness. We shall follow the Bohr’s line of thought \[53\] and hence couple quantum measurement theory with the Higher Order Theory of Consciousness.

In application to emotions, the First Order Theory of Consciousness matches the somatic theories of emotions rooted to James \[54\]; the first of them is James-Lange theory \[55\]. Nowadays this viewpoint on emotions is advertised by some prominent neuro-physiologists, e.g. Damasio \[56\].

3 Perceptions and emotions

3.1 Perception representation of sensations

We follow to von Helmholtz \[57\] theory of sensation-perception. Perceptions are not simply a copies of sensations, not “impressions like the imprint of a key on wax”, but the results of complex signal processing including unconscious cognitive processing. In the modern science formulation, the process of creation of perception can be described as follows: “Sensory information undergoes extensive associative elaboration and attentional modulation as it becomes incorporated into the texture of cognition. This process occurs along a core synaptic hierarchy which includes the primary sensory, upstream unimodal, downstream unimodal, heteromodal, paralimbic and limbic zones of the cerebral cortex.” \[58\]

3.2 Context-representation via emotions

As is emphasized in \[1\], “Emotion schema are learned in childhood and used to categorize situations as one goes through life. As one becomes more emotionally experienced, the states become more differentiated: fright comes to be distinguished from startle, panic, dread, and anxiety.” In our terminology, each emotion-generation scheme is crystallized on of the basic life-contexts. Context-labeling is the basic function of emotions. Contextualization of surrounding environment was one of the first cognitive tasks of biosystems and this ability was developed in parallel with establishing of sensation-perception system.

Memory is heavily involved in emotional activity, both for memorizing the features of contexts and for comparison of new perceptions with these contexts. (See, e.g., \[31\] for the memory’s role in cognition). Thus, evolutionary there was designed a mental information processing system representing the basic life-contexts. This system is fixed at the level of the brain (and more generally the nervous system) hardware. But, memorizing a variety of contextual experiences is done on the basis of the experiencing various situations (see above citation from \[1\]). This context-reflection system was the root of the present emotion-system in humans. The latter has complex cognitive functions, not only contextual. However, in this paper we concern mainly contextuality.
Emotions represent adaptive reactions to environmental challenges; they are a result of human evolution; they provided optimal (from the viewpoint of computational resources) solutions to ancient and recurring problems that faced our ancestors [59].

We emphasize that in our model emotions are conscious, cf. [1]: “One implication of our view is that emotions can never be unconscious. Responses controlled by subcortical survival circuits that operate nonconsciously sometimes occur in conjunction with emotional feelings but are not emotions. An emotion is the conscious experience that occurs when you are aware that you are in particular kind of situation that you have come, through your experiences, to think of as a fearful situation. If you are not aware that you are afraid, you are not afraid; if you are not afraid, you aren’t feeling fear.”

4 Unconscious vs. conscious information processing in the brain

4.1 Unconsciousness
An essential part of information processing in the brain is performed unconsciously; the information system responsible for such processing (call it unconsciousness) is denoted by the symbol $\mathcal{UC}$. The space of its states is denoted by $\mathcal{H}_\mathcal{UC}$. In the quantum-like model, this is a complex Hilbert space (section 5).

The reader need not couple the notion of unconsciousness with the names of James [60], Freud [61], and Jung [62] (although the author of this paper was strongly influenced by them, cf. with the previous works [7] – [10]). In this paper, $\mathcal{UC}$ denotes a special information processor of the brain. It performs pre-observational processing of the mental state.

4.2 Consciousness
Perceptions and emotions are commonly treated as conscious entities. So, in our model the brain contains another information processing system generating conscious experiences; denote it by the symbol $\mathcal{C}$. In our quantum-like framework, its functioning is modeled as performing measurements on the system $\mathcal{UC}$. Introduction of two systems $\mathcal{UC}$ and $\mathcal{C}$ matches the quantum measurement scheme, $\mathcal{UC}$ is the analog of a physical system exposed to measurements and $\mathcal{C}$ is the analog of a complex of measurement apparatuses.

In the operational quantum approach, measurable quantities, observables, are represented by Hermitian operators acting in system’s state space. More advanced models within theory of open quantum systems are based on consideration of the states of a measurement apparatus and interaction between the states of the system and apparatus [63] – [65]. In our quantum-like modeling, the latter corresponds to

\[5\] Although we do not follow the James-Lange theory of emotions [55], this is the good place to mention that James [54] pointed out that “feeling of the same changes as they occur is the emotion. Here, for us the key words are “the same changes as they occur”, i.e., the complex of repeatable bodily changes - physiological encoding of a context.
consideration of the conscious states represented in complex Hilbert space $\mathcal{H}_C$. In this paper we shall not consider (cf. [2, 69] for modeling the process of $UC - C$ interactions and generation of outputs of conscious observables).

Unconscious-conscious modeling of the brain’s functioning matches well to the philosophic paradigm of the ontic-epistemic structuring of scientific theories. The ontic level is about reality (physical and mental) as it is if nobody observes it, the epistemic level describes observations (see Atmanspacher [67]). In contrast to quantum physics, in our model the epistemic level is related not to external observers, but to brain’s self-observations. In the brain, unconscious and conscious processes are closely coupled and the sharp separation between them is impossible (cf. Brenner [68] and Brenner and Igamberdiev [69]).

4.3 Unconscious and conscious counterparts of the processes of generation of perceptions and emotions

In this paper, we shall be mainly concentrated on functioning of two information processors transforming

- sensations $\rightarrow$ perceptions,
- contexts $\rightarrow$ emotions.

Both processors have conscious outputs. Their functioning is strongly correlated; in the formalism quantum theory correlations are represented by entangled states.

We denote unconscious counterparts of these processors by the symbols $UC_{\text{per}}$ and $UC_{\text{em}}$, respectively. In modeling of the emotional coloring of perception (its contextualization), we shall consider the compound information system $(UC_{\text{per}}, UC_{\text{em}})$.

This is the good place to mention the first theory of emotions, the James-Lange theory [55]. James claimed that “I am trembling. Therefore I am afraid.” He stated: “My thesis ... is that the bodily changes follow directly the perception of the exciting fact and that our feeling of the same changes as they occur is the emotion.” This way of thinking matches with the First Order Theory of Consciousness and the EPR-Bell viewpoint on quantum measurements.

Following LeDoux [31] (see also [1]), we treat emotions within the Higher Order Theory of Consciousness coupled to Bohr’s interpretation of quantum measurements.

4.4 Basic and supplementary conscious experiences, application to decision making

Although we are mainly interested in emotional coloring of perceptions, the formalism under consideration can be applied to the very general class of compound information processing systems, $(UC_{\text{bas}}, UC_{\text{sup}})$. The latter is used for determining stable repeatable and evolutionary fixed contexts for the former. Simplest generalization of the perception-emotion scheme is emotional contextualization of decision making which modeling is based the compound system $(UC_{\text{dm}}, UC_{\text{em}})$.

Generation of conscious experiences (basic and supplementary) is modeled with POV-observables; denote the corresponding classes by the symbols $O_{\text{bas}}$ and $O_{\text{sup}}$. In particular, we shall consider the pairs $(O_{\text{per}}, O_{\text{em}})$ and $(O_{\text{dm}}, O_{\text{em}})$.
An expert in cognition may suggest other pairs of basic and supplementary conscious experiences. One of the properties of the supplementary mental experiences is their rapid processing. They should not inhibit processing of the basic mental experiences. We remark that emotion’s generation is characterized by high speed (see [31]).

5 Quantum formalism: states and observables

Denote by $\mathcal{H}$ a complex Hilbert space. For simplicity, we assume that it is finite dimensional. Pure states of a system $S$ are given by normalized vectors of $\mathcal{H}$. Later we shall consider mixed states (section ), but, for the moment, we proceed with only pure states and call them simply states.

Physical observable $A$ is represented by a Hermitian operator denoted by the same symbol. Consider an operator with discrete spectrum; its spectral decomposition has the form:

$$\hat{A} = \sum_x x E^A_x,$$

(1)

where $E^A_x$ is the orthogonal projector onto the subspace of $H$ corresponding to the eigenvalue $x$, i.e., $\mathcal{H}^A_x = E^A_x \mathcal{H}$. We recall that the spectral family of orthogonal projectors satisfies the normalization condition

$$\sum_x x E^A_x = I,$$

(2)

where $I$ is the unit operator, and the mutual orthogonality condition

$$E^A_x \perp E^A_y, x \neq y,$$

(3)

or equivalently

$$E^A_x E^A_y = \delta(x - y) E^A_x.$$

(4)

The probability to get the answer $x$ for a pure initial state $\psi$ is given by the Born rule

$$\text{Pr}\{A = x \| \psi\} = \|E^A_x \psi\|^2 = \langle \psi | E^A_x \psi \rangle.$$

(5)

and according to the projection postulate (von Neumann [49]) the post-measurement state is generated by the map:

$$\psi \rightarrow \mathcal{T}^A_x \psi = E^A_x \psi/\|E^A_x \psi\|. $$

(6)

This state transformation is generated by observation’s feedback to the system which initially was in the state $\psi$, i.e., observations disturb systems’ states.

The quantum state update is the basis of quantum generalization of classical Bayesian inference . The projection update has properties which crucially differ from the classical probability update. In particular, it generates violation of the law of total probability (playing the important role in Bayesian inference) [70,9,21] and the order effect which is absent in the classical probability theory (see Wang and Busemeyer [72], Ozawa and Khrennikov [66] for its quantum-like modeling). The
author of the present paper have stressed many times that non-Bayesian character of the quantum state and probability update is one of the distinguishing features of the quantum-like modeling of the brain’s functioning \cite{73} (cf. Gunji et al. \cite{21,22}).

In physics, the values of observables are labeled by real numbers and it is convenient to represent observables by linear operators. But, in fact, the basic formulas \eqref{5}, \eqref{6} contain only the orthogonal projectors \((E^A_x)_{x \in X}\) encoding an observable \(A\) should satisfy two constraints \eqref{2}, \eqref{4}. They guarantee that, for any state \(\psi\), quantity \(\Pr\{A = x \mid \psi\}\) determined by Born’s rule \eqref{5} is a probability measure. (For simplicity, we consider only discrete sets of labels.) Such family of projectors \((E^A_x)_{x \in X}\) is the simplest example of a positive operator valued measure \(\text{(POVM)}\), namely, the projector-valued measure \(\text{(POV)}\). The state transformation map \(\psi \rightarrow I^A_x \psi\), determines the simplest quantum instrument \cite{63}–\cite{65}. The symbol \(A\) just labels an observable (not Hermitian operator), \(A = (E^A_x)\). In principle, the same scheme can be realized with general quantum instruments and POVMs \(\text{(see \cite{63}–\cite{65})}\), but for simplicity we proceed with POVs.

6 Incompatible conscious observables

In quantum physics, observables \(C_1\) and \(C_2\) are called compatible if they can be jointly measurable and the joint probability distribution given by Born’s rule \eqref{5} is defined on the set \(X\). The family of orthogonal projectors \((E^{C_1}_{x_1}E^{C_2}_{x_2})_{x_1,x_2 \in X}\) encoding an observable \(A\) should satisfy two constraints \eqref{2}, \eqref{4}. They guarantee that, for any state \(\psi\), quantity \(\Pr\{A = x \mid \psi\}\) determined by Born’s rule \eqref{5} is a probability measure. (For simplicity, we consider only discrete sets of labels.) Such family of projectors \((E^{C_1}_{x_1}E^{C_2}_{x_2})_{x_1,x_2 \in X}\) is the simplest example of a positive operator valued measure \(\text{(POVM)}\), namely, the projector-valued measure \(\text{(POV)}\). The state transformation map \(\psi \rightarrow I^{C_1}_{x_1}I^{C_2}_{x_2} \psi\), determines the simplest quantum instrument \cite{63}–\cite{65}. The symbol \(A\) just labels an observable (not Hermitian operator), \(A = (E^{C_1}_{x_1}E^{C_2}_{x_2})\). In principle, the same scheme can be realized with general quantum instruments and POVMs \(\text{(see \cite{63}–\cite{65})}\), but for simplicity we proceed with POVs.

Incompatible conscious observables

In quantum physics, observables \(C_1\) and \(C_2\) are called compatible if they can be jointly measurable and the joint probability distribution \(p_{\psi}(C_1 = x_1, C_2 = x_2)\) is well defined; observables which cannot be jointly measurable and, hence, their JPD cannot be defined are called incompatible. In the mathematical formalism, compatibility and incompatibility are formalized through commutativity and noncommutativity, respectively. If observables are described as Hermitian operators \(C_1, C_2\), compatibility is encoded as \([C_1, C_2] = 0\); if they are POV-observables, then compatibility is encoded as

\[
[C^{C_1}_{x_1}, C^{C_2}_{x_2}] = 0, \quad \text{for all } x_1, x_2. \tag{7}
\]

Incompatibility is encoded as \([C_1, C_2] \neq 0\) or, for POV-observables, as violation of \eqref{7} at least for one pair \((x, y)\).

For compatible observables, JPD is given by the following extension of the Born’s rule:

\[
p_{\psi}(C_1 = x_1, C_2 = x_2) = |\langle E^{C_1}_{x_1}E^{C_2}_{x_2} \psi, \psi \rangle|^2 = |\langle E^{C_2}_{x_2}E^{C_1}_{x_1} \psi, \psi \rangle|^2. \tag{8}
\]

By using JPD compatible observables can be modeled in the classical probabilistic formalism. The formula \eqref{9} can be generalized for an arbitrary number of observables \(C_1, \ldots, C_m\), as

\[
p_{\psi}(C_1 = x_1, \ldots, C_m = x_m) = |\langle E^{C_1}_{x_1}E^{C_2}_{x_2} \ldots E^{C_m}_{x_m} \psi, \psi \rangle|^2 \tag{9}
\]

and this expression is invariant w.r.t. permutations.

We stress that the space of observables \(O_{\text{per}}\) can contain incompatible perceptions as well as \(O_{\text{em}}\) can contain incompatible emotions. In physics incompatibility is often seen as the exotic property of quantum theory - comparing with classical
physical theory. Philosophically it is represented in the Bohr’s complementarity principle which is difficult for understanding and was many times reformulated by Bohr [3]. However, in the mental framework the notion of incompleteness can be interpreted very naturally: there exist say emotions which can be experienced simultaneously; say happiness and sadness, pride and shame; in the same way it is evident that there exist incompatible, i.e., jointly unobservable perceptions and other conscious experiences. We note that mental observations are brain’s self-observations. May be this self-observational property simplifies the incompleteness issue.

The necessity to operate with various incompatible entities is the main roots of the use of the quantum(-like) information representation. In the absence of in-compatibility, i.e., if, for the same mental state, the brain were able to construct the consistent probabilistic representation (in the form of JPD) of all possible combinations of say emotions, the quantum state formalism would be unnecessary.

\section{Resolution of degeneration of states and quantum contextuality}

Consider an observable $A$ which is mathematically described as $POV (E_A^k)_{k \in X}$. For the discrete set of observable’s values $X = (x_k)$, we use notation $E_A^k = E_{x_k}^A$. Suppose now that some of its projections $E_A^k$ are degenerate; $\dim \mathcal{H}_A^k > 1$, where $\mathcal{H}_A^k = E_A^k \mathcal{H}$. Moreover, for some outcomes, degeneration is very high, $\dim \mathcal{H}_A^k >> 1$. In this case, a huge set of states corresponds to the same outcome $x_k$.

Observer may be unsatisfied by such a situation; observer wants to refine his observation to split (at least partially) the states corresponding to the fixed outcome. How can it be done? The quantum measurement formalism presents the very natural and simple procedure for refinement of states’ structure.

Consider another quantum $POV$-observable $B = (E_B^m)$ compatible with the original observable $A$, i.e., $[E_A^k, E_B^m] = 0$ for all indexes $k, m$. We remark that $B$-observable has its own set of outcomes, $Y = (y_m)$, which need not coincide with $X$.

For any (pure) quantum state $\psi$, the $POV$-observables $A$ and $B$ can be jointly measurable with outcomes $(x_k, y_m)$ and the join probability distribution

$$p_{AB}(x_k, y_m|\psi) = ||E_A^k E_B^m \psi||^2 = ||E_B^m E_A^k \psi||^2.$$ (10)

The projection postulate and commutativity of projectors imply that the post-measurement state is the same for the joint measurement with outcome $(x_k, y_m)$ and sequential measurements, first $A = x_k$ and then $B = y_m$ or vice versa:

$$\psi \rightarrow \mathcal{T}_x^A \mathcal{T}_y^B \psi = \frac{E_A^x E_B^y \psi}{||E_A^x E_B^y \psi||} = \frac{E_B^m E_A^x \psi}{||E_B^m E_A^x \psi||} = \mathcal{T}_y^B \mathcal{T}_x^A \psi.$$ (11)

The state space $\mathcal{H}_A^k$ is reduced to the space $\mathcal{H}_{km}^{(A,B)} = E_A^k E_B^m \mathcal{H} = E_B^m E_A^k \mathcal{H}$; so

$$\mathcal{H}_A^k = \oplus_m \mathcal{H}_{km}^{(A,B)}.$$ (12)
spectra. Let all projectors $T_{m}^{A}$ be one dimensional, $\dim \mathcal{H}_{m}^{A} = 1$ and let $e_{m}^{B}$ be the corresponding basis vector, i.e., $E_{j}^{B} e_{m}^{B} = \delta(j - m)e_{m}^{B}$. In this case, the outcome $(x_{k}, y_{m})$ completely determines the post-observation mental state: $\psi \rightarrow e_{m}^{B}$.

The correspondence between labels $(x_{k}, y_{m})$ and post-measurement states is one-to-one.

Consider now another observable $C$ which is also compatible with $A$, mathematically this is expressed as $[E_{k}^{A}, E_{m}^{C}] = 0$ for all $k, n$. The state spaces corresponding to $A$-outcomes can also be refined w.r.t. $C$-outcomes, i.e.,

$$\mathcal{H}_{k}^{A} = \oplus_{m} \mathcal{H}_{km}^{(A,C)},$$

where $\mathcal{H}_{km}^{(A,C)} = E_{k}^{A}E_{m}^{C}\mathcal{H} = E_{m}^{C}E_{k}^{A}\mathcal{H}$. In particular, if, for all $n$, $\dim \mathcal{H}_{n}^{C} = 1$ with the basis vector $e_{m}^{C}$, i.e., $E_{j}^{C} e_{m}^{C} = \delta(j - m)e_{m}^{C}$, then the outcome $(x_{k}, z_{n})$ completely determines the post-observation mental state: $\psi \rightarrow e_{n}^{C}$.

In quantum measurement theory, selection of observables co-measurable with $A$ is considered as specification of measurement context of $A$-measurements; the $A$-value in the $B$-context can differ from the $A$-value in the $C$-context, for the same premeasurement state $\psi$. This is the essence of contextuality playing so important role in quantum information theory [74].

**Definition 1.** If $A$, $B$, $C$ are three quantum observables, such that $A$ is compatible with $B$ and $C$, a measurement of $A$ might give different result depending upon whether $A$ is measured with $B$ or with $C$.

We note that contextual behavior corresponds to the case of incompatible quantum observables $B$ and $C$, i.e., there exist indexes such that $[E_{m}^{B}, E_{n}^{C}] \neq 0$. If all observables are pairwise commute, i.e., for all indexes $[E_{k}^{A}, E_{m}^{B}] = [E_{k}^{A}, E_{m}^{C}] = [E_{m}^{B}, E_{n}^{C}] = 0$, then, for any state $\psi$, it is possible to construct the noncontextual model of measurement based on the joint probability distribution for triple outcomes

$$p_{ABC}(x_{k}, b_{m}, c_{n}|\psi) = \|E_{k}^{A}E_{m}^{B}E_{n}^{C}\psi\|^2 = \|E_{k}^{A}E_{n}^{C}E_{m}^{B}\psi\|^2 = \|E_{k}^{A}E_{m}^{B}E_{n}^{C}\psi\|^2.$$

If $B$ and $C$ are incompatible, such a model is impossible. This is the contextuality scenario. However, contextuality formalized via Definition 1 cannot be tested experimentally, since it involves counterfactual reasoning. The only possibility is test contextuality (based on Definition 1) indirectly with the aid of Bell-type inequalities (see appendix).

If the sets of observables’ outcomes coincide with subsets of the real line, then the above considerations can be essentially simplified - with the Hermitian linear operators representing observables. The contextuality scenario is related to observables satisfying the commutation relations $[A, B] = [A, C] = 0$, $[B, C] \neq 0$.

However, the operator language is misleading and not only because representation of outcomes by real numbers is too special for coming cognitive applications. The main problem is that in the linear-operator approach the basic entities are outcomes, eigenvalues of the operator-observable. The subspace $\mathcal{H}_{m}^{A}$ is constructed as the space of the eigenvectors of the operator $A$ corresponding to the eigenvalue $x$.

In our coming modeling, we proceed another way around. The basic structures are
mutually orthogonal subspaces $\mathcal{H}_k$ such that

$$\mathcal{H} = \oplus_k \mathcal{H}_k.$$  \hspace{1cm} (15)

Then each of these subspaces is identified with some value of the POV-observable $A$ given by projectors on these subspaces. Roughly speaking, first states then values.

8 Resolution of degeneration of the states of consciousness via contextual coloring

We start with the presentation of the general scheme for the resolution of degeneration of the $\mathcal{C}$-states. In this scheme $\mathcal{C}$, operates with two classes of observables $O_{\text{bas}}$ and $O_{\text{sup}}$ representing the "basic and supplementary conscious experiences", respectively.

In the quantum-like model of generation of conscious experiences developed by the author [2], consciousness $\mathcal{C}$ is modeled as a system performing observations over unconsciousness $\mathcal{UC}$. As in section 4.1, the symbol $\mathcal{H}$ denotes the space of $\mathcal{UC}$-states. A conscious observable $A$ is represented as a POV-observable $E^A_x$, $x \in X$, where $X$ is the set of conscious experiences. The latter can be, for example, the set of language expression or visual images. The values of an observable are determined by the subspaces $\mathcal{H}^A_x$. If consciousness $\mathcal{C}$ detects a state belonging to $\mathcal{H}^A_x$, then it generates the conscious experience $x \in X$.

The space of unconscious states $\mathcal{H}$ has very high dimension. In reality, it is infinite dimensional, since this is the quantum information representation of electrochemical waves in the brain (see [75] for details). We restrict modeling to the finite dimensional case for state spaces of high dimension, $\dim \mathcal{H} \gg 1$.

If the “conscious-experience vocabulary” $X$ (for observable $A$) is not so large, i.e., number of points in set $X << \dim \mathcal{H}$, the same conscious experience $x$ is generated by huge variety of unconscious states. This degeneration is not good for cognitive behavior - reactions to external and internal stimuli and communications with other humans, especially for the latter.

How can the brain, as the self-observable, reduce this mental state degeneration? The answer is known from quantum theory. Consciousness $\mathcal{C}$ has to complete the $A$-observation, $A \in O_{\text{bas}}$, with the observation of a compatible observable $B \in O_{\text{sup}}$. The latter plays the role of context for the $A$-observation. The value $A = x$ is contextualized with the value $B = y$. In each outcome $(x, y)$ of the joint measurement of $(A, B)$, the value $x$ represents the basic conscious experience and $y$ its contextual coloring.

The crucial point is that such contextualization-observable $B$ should not carry conscious meaning which is directly related to the $A$-meaning, otherwise the joint observation $(A = x, B = y)$ can essentially modify the meaning of the outcome $A = x$. It is also preferable that $B$-observation can be combined not only with $A$-observation, but with observation of any $A' \in O_{\text{bas}}$.

We proceed with POV-observables; in particular, to enrich selection of outcomes’ vocabularies. However, the reader use Hermitian operators and restrict all vocabularies to subsets of the real line. In the operator-observables framework, the condition of compatibility is formulated simply as $[A, B] = 0$ for any $A \in O_{\text{bas}}$ and
any $B \in O_{\text{sup}}$. In our POV-observables framework, we proceed with the condition

$$[E^A_x, E^B_y] = 0 \text{ for observables } A = (E^A_x), B = (E^B_y),$$

or symbolically

$$[O_{\text{bas}}, O_{\text{sup}}] = 0.$$  \hfill (17)

In general two observables $A, A' \in O_{\text{bas}}$ do not commute, i.e., there can exist two projectors such that $[E^A_x, E^{A'}_{x'}] \neq 0$. Thus, an arbitrary $A' \in O_{\text{bas}}$ cannot be used for refinement of $A \in O_{\text{bas}}$.

As was mentioned, for the outcome $A = x$ the co-outcome $B = y$ can be considered as coloring of the experience $A = x$. Cognitive $O_{\text{bas}}$-representation can be compared with black-white pictures of houses in a town, the $O_{\text{sup}}$-representation adds colors: the house $A = x$ is “colored” with the color $B = y$. (Here we use “coloring” metaphorically.) Conscious experiences of $(x, y)$ and $(x, \tilde{y})$, where $x \in X$ and $y, \tilde{y} \in Y$ (the value-sets for $A$ and $B$) can differ essentially.

Thus, appeal to supplementary conscious experiences given by the set of observables $O_{\text{sup}}$ enriches tremendously the set of basic conscious experiences. At the level of mental states, it makes correspondence between states and experiences less degenerate. The latter helps a lot in social communication between individuals.

In principle, $O_{\text{bas}}$ also can contain compatible observables, say $A, A'$ such that $[A, A'] = 0$. In such a case, the outcomes of $A'$ might be used by $C$ for “coloring” of the outcomes of $A$ and vice versa. However, the $O_{\text{sup}}$-coloring is preferable. The set of observables $O_{\text{sup}}$ is specified by $C$ at the level of hardware and software; $C$ need not to check whether an observable from $O_{\text{sup}}$ can be used or not for coloring of an arbitrary observable $A \in O_{\text{bas}}$. Another problem with mutual coloring of observables $A, A'$ is that both carry important cognitive meanings and coloring of $A$ by $A'$ (or vice versa) can modify the cognitive meaning of $A$, since $C$ should process simultaneously two basic conscious functions.

And finally, we repeat that generation of outcomes of observables belonging to $O_{\text{bas}}$ is slower (in some situations, e.g., for emotional coloring, essentially slower [31]) than generation of $O_{\text{sup}}$-observables. So, by attempting to color $A \in O_{\text{bas}}$ with $A' \in O_{\text{bas}}$, $C$ would consume more time and even time scale inconsistency can be a problem.

### 8.1 Emotional coloring of perceptions

In the above scheme, we set $O_{\text{bas}} \equiv O_{\text{per}}$ and $O_{\text{sup}} \equiv O_{\text{em}}$.

As was stated in section 4.3 the information processing system $UC$ contains the following two subsystems. One is involved in processing of sensations into perceptions – still unconscious processing; denote it as $UC_{\text{per}}$. Another system $UC_{\text{em}}$ processes unconscious emotional states. The corresponding state spaces denote by the symbols $H_{\text{per}} \equiv H_{UC_{\text{per}}}$ and $H_{\text{em}} \equiv H_{UC_{\text{em}}}$, respectively.

Generally $UC$ is not reduced to $UC_{\text{per}}$ and $UC_{\text{em}}$. But, for simplicity, for modeling emotional coloring of perceptions, we assume that $UC$ is compound solely of these subsystems. This is really oversimplified picture of functioning of unconsciousness which is not reduced to transformation of sensations into perceptions and generation of emotional states. But, we proceed in this framework.
9 Tensor product formalism

Now we turn again to the general scheme of modeling of basic and supplementary conscious experiences. Before we have operated with the corresponding sets of observables \( O_{bas} \) and \( O_{sup} \), now we would like to proceed in the dual framework by operating with states.

In quantum information theory, contextuality is typically presented within the tensor product formalism.\(^6\)

Let, as above, \( \mathcal{H} \) be the state space of \( UC \). Suppose that it is factorized in the tensor product \( \mathcal{H} = \mathcal{H}_{bas} \otimes \mathcal{H}_{sup} \), where \( \mathcal{H}_{bas} \) and \( \mathcal{H}_{sup} \) are state spaces coupled to basic and supplementary conscious experiences which are represented at the conscious level by observables belonging to the sets \( O_{bas} \) and \( O_{sup} \). These state spaces are generated by two different unconscious information processing systems, say \( UC_{bas} \) and \( UC_{sup} \). These systems are concentrated in different brain’s areas created at the different stages of the brain’s evolution. However, concentration is not sharp, processing is distributed and the processing areas have overlap. The complex Hilbert space \( \mathcal{H} \) is the state space of the compound system \( (UC_{bas}, UC_{sup}) \). In the present model, it can be identified with unconsciousness, \( UC \). (Generally \( UC \) has more complex structure.)

Observables belonging to \( O_{bas} \) and \( O_{sup} \) represent measurements which are performed by \( C \) on the subsystems \( UC_{bas} \) and \( UC_{sup} \), respectively. If observables are mathematically described as Hermitian operators, then each operator \( A : \mathcal{H}_{bas} \to \mathcal{H}_{bas} \) is represented as the operator \( A = A \otimes I : \mathcal{H} \to \mathcal{H} \) and each operator \( B : \mathcal{H}_{sup} \to \mathcal{H}_{sup} \) is represented as the operator \( B = I \otimes B : \mathcal{H} \to \mathcal{H} \). For POV-observables, we use the same procedure: for \( A = (E^A_x) \) and \( B = (E^B_y) \), we set \( A = (E^A_x \otimes I) \) and \( B = (I \otimes E^B_y) \). The joint measurement of two POV-observables is represented by POV \( A \otimes B = (E^A_x \otimes E^B_y) \).

The state space \( \mathcal{H} = \mathcal{H}_{bas} \otimes \mathcal{H}_{sup} \) is generated by tensor products of the form \( \psi_{bas} \otimes \psi_{sup} \). Measurements of observables on such separable states are reduced to independent measurements on the subsystems \( UC_{bas} \) and \( UC_{sup} \) (for \( A, B \) of aforementioned classes). The real quantum information effects become visible for non-separable, entangled, states, those states which cannot be represented in the form of the tensor product.

For example, let both state spaces be two dimensional (qubit spaces), and let \( |j_a \rangle \) and \( |j_b \rangle \), \( j_a, j_b = 0, 1 \), be orthonormal bases in \( \mathcal{H}_{bas} \) and \( \mathcal{H}_{sup} \), let observables \( A \) and \( B \) represented by one dimensional POV corresponding to these bases, i.e., \( E^A_{j_a} = |j_a \rangle \langle j_a| \) and \( E^B_{j_b} = |j_b \rangle \langle j_b| \), where \( j_a, j_b = 0, 1 \). Set \( |j_a j_b \rangle = |j_a \rangle \otimes |j_b \rangle \); this is an orthonormal basis in the state space \( \mathcal{H} \) of the compound system \( S = \)
Then the states of $S$ can be expanded w.r.t. this basis.

$$\psi = \sum_{ji} c_{ji} |ji\rangle, \quad \sum_{ji} |c_{ji}|^2 = 1.$$  \hspace{1cm} (18)

By the Born’s rule $p_\psi(A = a_j, B = b_i) = |c_{ji}|^2$ is the probability that $C$ observes the perception-emotion pair $(a_j, b_i)$.

For example, the state

$$\psi = (|00\rangle + |11\rangle)/\sqrt{2}$$  \hspace{1cm} (19)

is entangled. This state illustrate the correlation meaning of entanglement - in fact, the maximal entanglement. Suppose that observables $A$ and $B$ yield the real values $a_0, a_1$ and $b_0, b_1$, respectively. Consider quantum-like modeling of emotional coloring of perception, i.e., for $\mathcal{H}_{bas} = \mathcal{H}_{per}, \mathcal{H}_{sup} = \mathcal{H}_{em}$. Perception $A = a_j$ is firmly associated with emotion $B = b_j, j = 0, 1$. Thus, this state represents the perfect correlations between the $A$-perception and the $B$-emotion.

For example, consider Russia or France in 19th century, decision maker was an officer who was participating in some social event and conflicting with another officer; $A$ is the decision observable, “to challenge ($A = a_1$) or not ($A = a_0$) to a duel”; $B$ the emotion observable, “angry ($B = b_1$) or not ($B = b_0$). If an officer is in the (unconscious) mental state (19), then the emotion “angry” matches perfectly with challenge to a duel.

The general two qubit state given by superposition (18) encodes correlation

$$\langle AB\rangle_\psi \equiv \langle AB\psi, \psi \rangle = \sum_{ji} a_j b_i |c_{ji}|^2 = \sum_{ji} a_j b_i p_\psi(A = a_j, B = b_i),$$

w.r.t. state $\psi \in \mathcal{H}$. The last sum is the classical probabilistic expression for correlation. Thus, each concrete pair (perception, emotion) or (decision, emotion) can be described in the classical probabilistic framework. In particular, by experimenting with just one pair we would detect quantum-like effects.

Consider, for the same perception $A \in O_{per}$, another emotion $B' \in O_{em}$ and correlation $\langle AB'\rangle_\psi$; it can be expressed with the coefficients with respect to the basic ($|ja\rangle \otimes |j'b'\rangle$),

$$\langle AB'\rangle_\psi \equiv \langle AB'\psi, \psi \rangle = \sum_{ji} a_j b'_i |c'_{ji}|^2 = \sum_{ji} a_j b'_i p_\psi(A = a_j, B' = b'_i).$$

This is also the classical expression for the correlation.

10 Bell type inequalities and experimental testing of emotional contextuality

Now let us consider the correlation-expressions (20), (21) jointly. If emotional-observables $B = (E^B_j)$ and $B' = (E'^B_j)$ are incompatible, i.e., in the mathematical terms, there exist projectors such that $[E^B_j, E'^B_j] \neq 0$, then generally it is impossible to combine these two classical correlations in the single classical probabilistic model. This is the complex foundational problem which is formalized with the aid
of Bell type inequalities [51, 52, 74]. We are not able to go deeper into basics of quantum mechanics. We finish with the following foundational remark. In quantum physics, violation of these inequalities is coupled to violation of at least one of the followings two assumptions:

- a) realism, i.e., the possibility to assign the values of observables before measurement;
- b) locality, the absence of action at a distance.

The combination of a)+b) is known as local realism. In quantum physics, one does not distinguish the two components of local realism (but, see [76, 77] for the claim that the key issue is violation of a) and that quantum theory is local; see also Plotnitsky [78] – [80] for foundational analysis of the interplay incompatibility-nonlocality). As was argued in section 6 “mental realism”, i.e., the assumption that say all possible emotions peacefully coexist in any mental state, is hardly acceptable. Therefore we consider violation of “mental realism” as the root for violation of the Bell type inequalities.

In quantum physics, experimental testing of the Bell type inequalities is the hot topic (see, e.g., [81, 82, 83] for the recent experiments). In psychology and decision making, they have been tested by a few authors [84] – [88]. This paper can stimulate such experimenting in consciousness studies with joint measurements of the pairs \((A, B) = (\text{perception, emotion})\) or (decision making, emotion).

As in physics and the previous psychological experiments, it is natural to test the CHSH inequality [89]. To proceed with it, it is not enough to consider just one perception (decision making) \(A\) and two incompatible emotions \(B, B'\). One has to work as well with two incompatible perceptions \(A, A'\) and form cyclically their correlations. The CHSH correlation function is given by the following combination of correlations:

\[
C_{\text{CHSH}} = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle
\]

and, for dichotomous observables yielding values \(\pm 1\) the following inequality holds:

\[
|C_{\text{CHSH}}| \leq 2.
\]

Finally, we stress that the psychological studies demonstrated that the humans behavior differs from the behavior of quantum physical systems w.r.t. so-called signaling problem [87, 88]. It would be interesting to check whether experiments with emotions would lead to signaling or not. My conjecture that, for some mental states, emotional coloring (contextualization) can be performed without signaling. I stress that the proposed emotion-experiments differ from the previously performed psychological experiments in which experimenters operated with pairs composed of two decisions, i.e., one decision played the role of context for another.

11 Concluding remarks

We presented the quantum-like model of emotional coloring of perceptions and other conscious experiences, including decision making. The brain, as the information processor, is decomposed into two sub-processors, unconsciousness \(UC\) and

\footnote{We repeat that this model has no direct coupling to study of genuine quantum physical processes in the brain.}
consciousness \( \mathcal{C} \). The later plays the role of an observer on the former. This is mental realization of the quantum measurement scheme for self-observations performed by the brain. The state space of \( \mathcal{UC} \) is mathematically described as a complex Hilbert space of very high dimension. In this paper we do not model the process of \( \mathcal{UC} - \mathcal{C} \) interaction. Conscious observables are represented in the operator formalism. Perceptions and emotions are described by two classes of POV-observables, \( O_{\text{per}} \) and \( O_{\text{em}} \). Perceptions are compatible with emotions, i.e., they can be jointly observed by \( \mathcal{C} \). In the mathematical formalism, compatibility is encoded as commutativity.

Both perceptions and emotions are treated as conscious experiences. Quantum measurement formalism matches perfectly to the Higher Order Theory of Consciousness. Emotions correspond to repeatable contexts and they contextualize perceptions and other conscious experiences. Emotional coloring reduces state degeneration for them and makes the information processing less diffuse. Context-matching is also important for social communication.

One of the main distinguishing features of the quantum measurement theory is the presence of incompatible, i.e., jointly unobservable entities. In particular, the presence of incompatible observables makes impossible the use of the classical probability model (axiomatic of Kolmogorov [90]). The existence of incompatible perceptions or emotions is evident even from our personal experience. This motivates the use of the mathematical formalism to quantum theory for modeling brain’s self-observations.

To model emotional contextuality, we explore the tensor product formalism by factorizing the unconscious state space into the tensor product, one of its components describes the states of emotions. As is well known from quantum physics, the direct test of contextuality is impossible due to counterfactual nature of its formulation. The indirect tests of contextuality are based on the Bell type inequalities. We discuss the possibility of such tests for pairs (perception, emotion) or (decision making, emotion). As in physics, the problem of the interpretation of the violation of Bell inequalities is very complex. Following [76, 77], we couple violation and observables incompatibility, i.e., the existence of incompatible perceptions and emotions.

We hope that our model can stimulate the applications of the quantum formalism in conscious-studies, especially in modeling perception-emotion and decision-emotion correlations.

12 Appendix: Commutator representation of CHSH-correlation function

Here we present the purely mathematical result supporting our incompatibility interpretation of violation of the Bell type inequalities.

Consider Hermitian operators \( A, A' \) and \( B, B' \). In our model, the first pair belongs to \( O_{\text{per}} \) and the second one to \( O_{\text{em}} \). The considered perceptions and emotions are assumed to be compatible, i.e., \( [A, B] = 0 \),..., \( [A', B'] = 0 \). It is also assumed that the observables are dichotomous and yield the values \( \pm 1 \). In the operator terms, the latter is expressed as \( A^2 = (A')^2 = B^2 = (B')^2 = I \), where \( I \) is the unit operator.
For quantum observables, the CHSH correlation function $C_{\text{CHSH}}$ can be represented as the average of the corresponding Hermitian operators (see [77] for details):

$$C_{\text{CHSH}} = \langle \Gamma_{\text{CHSH}} \psi | \psi \rangle,$$

(24)

where

$$\Gamma_{\text{CHSH}} = A(B + B') + A'(B - B').$$

(25)

The CHSH inequality has the form:

$$|\langle \Gamma_{\text{CHSH}} \psi | \psi \rangle| \leq 2.$$  

(26)

By straightforward calculation, one can derive at the Landau identity:

$$\Gamma_{\text{CHSH}}^2 = 4I - [A, A'][B, B'],$$

(27)

where, for two operators $Q_1, Q_2$, $[Q_1, Q_2] = Q_1Q_2 - Q_2Q_1$ is their commutator. Thus, if at least one pair of observables $(A, A')$ or $(B, B')$ is compatible, i.e., at least one of commutators $[A, A']$, $[B, B']$ is equal to zero, then $\Gamma_{\text{CHSH}} = 2I$ and the CHSH inequality cannot be violated. If both commutators are nonzero, then, for some state $\psi$, it can be violated (see [1]).

The reader can see that this is purely commutativity-nocommutativity game.

References

[1] J. E. LeDoux and R. Brown, A higher-order theory of emotional consciousness. PNAS, 2017 114 (10) E2016-E2025; https://doi.org/10.1073

[2] Khrennikov A. Quantum-like model of unconscious-conscious dynamics. Frontiers in Psychology 6, 2015, 997-1010. https://www.frontiersin.org/article/10.3389/fpsyg.2015.00997

[3] Bohr, N. The Philosophical Writings of Niels Bohr; Ox Bow Press: Woodbridge, UK, 1987.

[4] Wigner, E.P., 1961. Remarks on the mind-body question. In: Good, L.J. (Ed.), The Scientist Speculates – An Anthology of Partly Baked Ideas. Heinemann, London. pp. 284-302.

[5] Wigner, E.P., 1963. The problem of measurement. Am. J. Phys. 31, 6-15.

[6] F. H. Thaheled, Does consciousness really collapse the wave function? A possible objective biophysical resolution of the measurement problem. https://arxiv.org/ftp/quant-ph/papers/0509/0509042.pdf.

[7] A.Yu. Khrennikov, Human subconscious as the $p$-adic dynamical system. Journal of Theoretical Biology, 193, 179-196 (1998).

[8] D. Dubischar, M. Gundlach, O. Steinkamp, A.Yu. Khrennikov, A $p$-adic model for the process of thinking disturbed by physiological and information noise. Journal of Theoretical Biology, 197, 451-467 (1999).

[9] A.Yu. Khrennikov, Information Dynamics in Cognitive, Psychological, Social and Anomalous Phenomena, Springer-Science + Business Media, B.Y., Dordrecht, NL, 2004.
[10] Albeverio S, Khrennikov A and Kloeden P E Memory retrieval as a p-adic dynamical system *BioSystems* 49 105–115 (1999).

[11] E. A. Liberman, S. V. Minina, N. E. Shklovsky-Kordi. Quantum molecular computer model of the neuron and a pathway to the union of the sciences. Biosystems 22, 1989, 135-154.

[12] Penrose, R. (1989). *The Emperor’s new mind*, Oxford Univ. Press: New-York.

[13] Minina S. V., Liberman E. A. Input and output channels of quantum biocomputers. Biofizika. 1990 35(1), 132-136.

[14] Igamberdiev, A.U., 1993. Quantum mechanical properties of biosystems: a framework for complexity, structural stability and transformations. *BioSystems* 31, 65–73.

[15] Umezawa, H. (1993). *Advanced field theory: micro, macro and thermal concepts*, AIP: New York.

[16] Hameroff, S. (1994). Quantum coherence in microtubules. A neural basis for emergent consciousness? *J. Cons. Stud.*, 1, 91–118.

[17] E.A.Liberman, S.V.Minina, Molecular quantum computer of neuron. *BioSystems* 35, 1995, 203-207.

[18] Vitiello, G. (1995). Dissipation and memory capacity in the quantum brain model, *Int. J. Mod. Phys.*, B9, 973.

[19] Vitiello, G. (2001). *My double unveiled: The dissipative quantum model of brain*, Advances in Consciousness Research, John Benjamins Publishing Company.

[20] A. U. Igamberdiev. Quantum computation, non-demolition measurements, and reflective control in living systems. *Biosystems* 77, 2004, 47-56.

[21] Gunji YP, Shinohara S., Haruna T., Basios V. Inverse Bayesian inference as a key of consciousness featuring a macroscopic quantum logical structure. *Biosystems*. 2017, ;152:44-65.

[22] Gunji YP, Sonoda K, Basios V. Quantum cognition based on an ambiguous representation derived from a rough set approximation. *Biosystems*. 2016 Mar;141:55-66.

[23] Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances*. Berlin-Heidelberg-New York: Springer.

[24] Busemeyer, J. & Bruza, P.(2012). *Quantum models of cognition and decision*. Cambridge: Cambridge Univ. Press.

[25] Haven, E. and Khrennikov, A. (2013). *Quantum social science*; Cambridge University Press.

[26] Asano, M., Khrennikov, A., Ohya, M., Tanaka, Y., Yamato, I. *Quantum adaptivity in biology: from genetics to cognition*, (Springer, Heidelberg-Berlin-New York, 2015).

[27] Haven, E., Khrennikov, A. and Robinson, T. R. (2017). *Quantum Methods in Social Science: A First Course*; WSP: Singapore.
[28] Bagarello, F. (2019). Quantum concepts in the social, ecological and biological sciences. Cambridge: Cambridge Univ. Press.

[29] Khrennikov, A. (2020). Social laser. Jenny Stanford Publ., Singapore.

[30] A. Khrennikov, Quantum versus classical entanglement: eliminating the issue of quantum nonlocality. Found Phys 50, 1762–1780 (2020).

[31] J. E. LeDoux, Emotional colouration of consciousness: how feelings come about. In: Frontiers of Consciousness. Chichele Lectures. L. Weiskrantz and M. Davies (eds). Oxford Univ. Press, Oxford, 2008.

[32] LeDoux, J.E. (1984). Cognition and emotion: processing functions and brain systems. In Gazzaniga, M.S. (ed.) Handbook of Cognitive Neuroscience, pp. 357–368. New York: Plenum.

[33] LeDoux, J.E. (1987). Emotion. In Plum, F. (ed.) Handbook of Physiology. I: The Nervous System. Vol. V. Higher Functions of the Brain, pp. 419–460. Bethesda, MD: American Physiological Society.

[34] E. A. Liberman, Analog-digital molecular cell computer. Biosystems 11, 1979, 111-124.

[35] M. Conrad, E.A.Liberman, Molecular computing as a link between biological and physical theory. J. Theor. Biology, 98, 1982, 239-252.

[36] E. A. Liberman, S. V. Minina, N. E. Shklovski-Kordi, Biological information and laws of nature. Biosystems 46, 1998, 103-106.

[37] Liberman, E. A. Cell molecular computer. VII. Cell biophysics and realistic or information physics. Biofizika, 1975, 20(3), 432-436.

[38] A. U. Igamberdiev, J. E. Brenner, Mathematics in biological reality: The emergence of natural computation in living systems Biosystems 204, 2021, 104395.

[39] Brenner, J. E. Logic in Reality. Dordrecht: Springer, 2008.

[40] Brenner, J. E.; A. Igamberdiev, Philosophy in Reality; A New Book of Changes. Berlin-Heidelberg-New York, 2021.

[41] Dretske F. (1995) Naturalizing the Mind (MIT Press, Cambridge, MA).

[42] Tye M. (2000) Consciousness, Color, and Content (MIT Press, Cambridge, MA).

[43] Lamme V. A. F. (2005) Independent neural definitions of visual awareness and attention. Cognitive Penetrability of Perception: Attention, Action, Strategies, and Bottom-Up Constraints, ed Raftopoulos A (Nova Science, New York), pp 171–191

[44] Block N. (2011) Perceptual consciousness overflows cognitive access. Trends Cogn. Sci. 15(12):567–575.

[45] Block N. (2007) Consciousness, accessibility, and the mesh between psychology and neuroscience. Behav. Brain Sci 30(5-6):481–499, discussion 499–548.

[46] Rosenthal DM (2005) Consciousness and Mind (Oxford Univ Press, Oxford).

20
[47] Lau H, Rosenthal D (2011) Empirical support for higher-order theories of conscious awareness. Trends Cogn Sci 15(8):365–373.

[48] Carruthers P (2005) Consciousness: Essays from a High-Order Perspective (Oxford Univ Press, Oxford).

[49] Von Neumann, J. Mathematical Foundations of Quantum Mechanics; Princeton Univ. Press: Princeton, NJ, USA, 1955.

[50] Einstein, A.; Podolsky, B.; Rosen, N. Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 1935, 47, 777–780.

[51] Bell, J.S. On the Einstein Podolsky Rosen paradox. Physics 1964, 1, 195–200.

[52] Bell, J.S. Speakable and Unspeakable in Quantum Mechanics, 2nd ed.; Cambridge University Press: Cambridge, UK, 2004.

[53] Bohr, N. Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 1935, 48, 696–702.

[54] James W (1884). What Is an Emotion?. Mind. 9 (34): 188–205.

[55] Cannon, W. (1927). The James-Lange Theory of Emotions: A Critical Examination and an Alternative Theory. Amer. J. Psych. 39 (1/4): 106–124.

[56] Aziz-Zadeh L, Damasio A (2008). Embodied semantics for actions: findings from functional brain imaging. J. Physiol., Paris. 102 (1–3): 35–9.

[57] von Helmholtz, H. (1866). Treatise on Physiological Optics. Transl. by Optical Society of America in English. New York, NY: Optical Society of America.

[58] Mesulam M. M. From sensation to cognition. Brain. 1998, 121:1013-52.

[59] Ekman P (1992). An argument for basic emotions. Cognition and Emotion. 6 (3): 169–200.

[60] James, W. (1890). The Principles of Psychology (New York: Henry Holt and Co.), Reprinted 1983 (Boston: Harvard Univ. Press).

[61] Freud, S. (1957). The Standard Edition of Complete Psychological Works of Sigmund Freud, Edited and Translated by J. Strachey, Vols. I-XXIV (The Hogarth Press, London).

[62] Jung, C. G. (2001). On the Nature of the Psyche. Routledge Classics.

[63] Davies, E. B. Quantum theory of open systems, (Academic Press, London, 1976).

[64] Ozawa, M. Quantum perfect correlations. An. Phys. 321, 744-769 (2006).

[65] Ozawa, M. Probabilistic interpretation of quantum theory. New Generation Computing 34, 125-152 (2016).

[66] Ozawa, M. & Khrennikov, A. (2020). Application of theory of quantum instruments to psychology: Combination of question order effect with response replicability effect. Entropy, 22(1), 37, 1-9436.

[67] Atmanspacher, H. Determinism is ontic, determinability is epistemic, in H. Atmanspacher and R. C. Bishop (eds.), Between Chance and Choice: Inter-disciplinary Perspectives on Determinism (Imprint Academic, Thorverton UK), 2002, pp. 49–74.
[68] Brenner, J. E. Logic in Reality. Dordrecht: Springer, 2008.
[69] Brenner, J. E.; A. Igamberdiev, Philosophy in Reality; A New Book of Changes. Berlin-Heidelberg-New York, 2021.
[70] Khrennikov, A. (2004). On quantum-like probabilistic structure of mental information, Open Systems and Information Dynamics 11 (3), 267–275.
[71] Busemeyer, J. R., Wang, Z. and Townsend, J. T., (2006). Quantum dynamics of human decision making, J. Math. Psych., 50, 220–241.
[72] Wang, Z. & Busemeyer, J. R. (2013). A quantum question order model supported by empirical tests of an a priori and precise prediction. Topics in Cognitive Science, 5, 689–710.
[73] Khrennikov A. (2016). Quantum Bayesianism as the basis of general theory of decision-making. Philosophical transactions. Series A, Mathematical, physical, and engineering sciences, 374(2068), 20150245.
[74] Bell, J.S. On the problem of hidden variables in quantum theory. Rev. Mod. Phys. 1966, 38, 450.
[75] Khrennikov, A., Basieva, I., Pothos, E. M. and Yamato, I. (2018). Quantum probability in decision making from quantum information representation of neuronal states, Scientific Reports 8, Article number: 16225.
[76] A. Khrennikov, Bohr against Bell: complementarity versus nonlocality. Open Phys. 15, 734–73 (2017).
[77] A. Khrennikov, Get rid of nonlocality from quantum physics. Entropy, 21(8), 806 (2019).
[78] A. Plotnitsky, “Without in any way disturbing the system”: Illuminating the issue of quantum nonlocality. [arXiv:1912.03842] (quant-ph).
[79] A. Plotnitsky, Reality, Indeterminacy, Probability, and Information in Quantum Theory. Entropy 2020, 22(7), 747; https://doi.org/10.3390/e22070747
[80] A. Plotnitsky, The Unavoidable Interaction Between the Object and the Measuring Instruments: Reality, Probability, and Nonlocality in Quantum Physics. Found Phys (2020). https://doi.org/10.1007/s10701-020-00353-5.
[81] Hensen, B.; Bernien, H.; Drea, A.E.; Reiserer, A.; Kalb, N.; Blok, M.S.; Ruitenberg, J.; Vermeulen, R.F.; Schouten, R.N.; Abellan, C.; et al. Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km. Nature 2015, 526, 682.
[82] Giustina, M.; Versteegh, M.A.; Wengerowsky, S.; steiner, J.; Hochrainer, A.; Phelan, K.; Steinlechner, F.; Kofler, J.; Larsson, J.A.; Abellan, C.; et al. A significant-loophole-free test of Bell’s theo-rem with entangled photons. Phys. Rev. Lett. 2015, 115, 250401.
[83] Shalm, L.K.; Meyer-Scott, E.; Christensen, B.G.; Bierhorst, P.; Wayne, M.A.; Stevens, M.J.; Gerrits, T.; Glancy, S.; Hamel, D.R.; Allman, M.S.; et al. A strong loophole-free test of local realism. Phys. Rev. Lett. 2015, 115, 2504.
[84] Conte, E., Khrennikov, A., Todarello, O. and Federici, A. (2008). A preliminary experimental verification on the possibility of Bell inequality violation in mental states, Neuroquantology, 6(3), 214–221.
[85] Asano, M., Khrennikov, A., Ohya, M., Tanaka, Y. and Yamato, I. (2014). Violation of contextual generalization of the Leggett-Garg inequality for recognition of ambiguous figures, Physica Scripta, T163, 014006.

[86] Asano, M., Khrennikov, A., Ohya, M., Tanaka, Y. and Yamato, I. (2015). Quantum adaptivity in biology: from genetics to cognition, Springer: Heidelberg-Berlin-New York.

[87] Basieva, I., Cervantes, V.H., Dzhafarov, E.N., Khrennikov, A. (2019). True contextuality beats direct influences in human decision making. J. Exp. Psych.: General 148, 1925-1937.

[88] Cervantes, V.H., & Dzhafarov, E.N. (2020). Contextuality analysis of impossible figures. Entropy 22:981; doi.org/10.3390/e22090981.

[89] Clauser, J.F.; Horne, M.A.; Shimony, A.; Holt, R.A. Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. 1969, 23, 880.

[90] Kolmolgoroff, A. N. (1933). Grundbegriffe der Wahrscheinlichkeitsrechnung, (Springer-Verlag, Berlin),