Entanglement swapping in black holes: restoring predictability

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Hawking’s black hole evaporation process has led to several paradoxes, stimulating searches for new physics. Without introducing any exotic objects, we here show that after the full black hole evaporation the outgoing Hawking particles are entangled with each other and are consequently in a pure state. At any stage of the evaporation process, we carefully track the entanglement of the state of matter and Hawking particles. However, in the case of full evaporation, we show that the entanglement is transferred from particles inside the black hole to the outside particles created at different times, in a way equivalent to “entanglement swapping”. Therefore, the final state after the full black hole evaporation is pure, without violating unitarity, the monogamy theorem, or the equivalence principle.

I. INTRODUCTION

On the base of Hawking’s derivation [1], pairs of particles are created from the vacuum near the event horizon: one of these (having negative energy\(^1\)) falls into the black hole and the other flies away to future infinity (\(I^+\)). The particle of negative energy falling towards the black hole will eventually meet the black hole’s matter and annihilate, causing the black hole mass to decrease [1–3]. As time passes, more and more particles are annihilated and the black hole will finally evaporate. During the evaporation process the particle pairs created at the event horizon are in the following state [3],

\[
|\Psi\rangle = \bigotimes_{\omega > 0} c_\omega \sum_{N_\omega = 0} e^{-N_\omega \pi \omega / \kappa} |N_\omega\rangle^{\text{out}} \otimes |N_\omega\rangle^{\text{int}},
\]

where \(c_\omega = \sqrt{1 - e^{-2\pi \omega / \kappa}}\) is a normalization factor, \(N_\omega\) is the number of particles of energy \(\omega\), while “int” and “out” label the Hilbert spaces for the particles falling inside the black hole and those escaping to the future infinity respectively [3]. The state (1) is pure with the “int” modes inside the black hole being correlated with the “out” modes. However, after the black hole fully evaporates, we cannot find the “int” particles anymore, and the “out” reduced density matrix, obtained upon tracing out the “int” states, turns out to be in a mixed state. Therefore, the complete evolution is non-unitary because we start with a pure state and we end up with a mixed state [4].

However, closed quantum systems are expected to evolve unitarily [5]. Thus, the two most successful theories: General Relativity, in which gravity is described as curvature of the spacetime, and Quantum Mechanics, which describes the subatomic physics, seem to be in conflict. Enormous efforts were made to overcome this issue. In the first decade after Hawking’s famous paper, people mainly tried to question Hawking’s semi-classical approximations [6]. Later, it was hoped that the quantum gravitational corrections to Einstein’s theory of gravity could solve the problem, thus the paradox would lead the way to the correct quantum gravity theory [7]. Quantum gravity was hoped to show effects causing black holes to not completely evaporate through the Hawking process but leaving a “remnant”. In this case, we either reach a state in which the hole does not radiate anymore and all information is stored forever in its interior or the remnant allows information to get out in some other way, “hopefully” without causality violation. This is an attempt that was recently taken farther by [8–11]. Modifications on the quantum theory side were also suggested in [12–16], where nonlinear effects, nonviolent nonlocal effects, generalized probabilistic theories, and other generalizations were introduced and expected to solve the inconsistencies. In a more conservative attempt [17, 18], Don Page showed that although the outgoing radiation seems thermal because of the lack of local correlations, it

\[1\] Inside a Schwarzschild black hole, the time coordinate and the spatial radial coordinate interchange their role. Therefore, the energy and momentum change their role too, allowing for a well defined negative energy (actually spatial momentum) inside the black hole. This fact is at the heart of the Hawking process for the black holes’ evaporation.
can still be fundamentally pure, if there are enough correlations between the early emitted radiation and the late radiation, where “early” and “late” refer to radiation emitted before and after the Page time, when half of the black hole’s entropy has been radiated away. However, the Page scenario was shown to be incompatible with a fundamental property of entanglement, namely the “Monogamy theorem” [19]. Particles created near the event horizon, located at \( r_s = 2M \) (\( M \) is the black hole mass) are in the state \( |1\rangle \), which clearly shows that the “out” modes and the “int” modes are strongly entangled. However, the Page argument requires the early radiated particles to be almost maximally entangled with the late radiated particles, which is impossible as shown in [20] because we have at least two maximally entangled particles each of which is strongly entangled with another particle. All this motivated more innovative ideas like “Complementarity” and even “Firewalls” [21, 22]. Complementarity states that information simultaneously crosses the horizon to the black hole interior and is reflected on what is called the “stretched horizon”, however, no observer can ever experience both by measuring the same information outside and inside the black hole. This is guaranteed by a proposed thermalization time which information takes to be reflected at the stretched horizon [21, 22]. On the other hand, questioning the postulates of complementarity, in [22, 23] a high energy surface was introduced at the event horizon able to break the entanglement between the particle pairs that were created near the horizon, thus allowing the outgoing radiation to be again in a pure entangled state. This latter proposal appears to violate the equivalence principle which states that an observer free falling towards a black hole will feel nothing special while crossing the event horizon.

In this paper, we will show that the information paradox arises only if we deal with the annihilation process without enough care. Indeed, we will show that any Hawking particle infalling towards the black hole (under the assumption that it annihilates something inside the event horizon) will transfer to the outside radiation the entanglement of the black hole matter (or will break its entanglement with the outside if the particle it annihilates inside is not entangled with any other particle) without any violation of monogamy, causality or any other solid principle. The argument only relies on well known Quantum Mechanics and General Relativity. The assumption that the infalling particles do annihilate other particles inside the black hole leads to a conditional density matrix scheme where the black hole matter (or will break its entanglement with the outside if the particle it annihilates inside is not entangled with any other particle) without any violation of monogamy, causality or any other solid principle. The argument only relies on well known Quantum Mechanics and General Relativity. The assumption that the infalling particles do annihilate other particles inside the black hole leads to a conditional density matrix scheme where the entanglement is indeed allowed to be transferred at a distance without violating causality. In short, our resolution of the information paradox relies on the so-called “entanglement swapping” [23, 24], a phenomenon that has been repeatedly experimentally demonstrated [27–33].

II. ENTANGLEMENT SWAPPING AT WORK

In this paragraph we briefly introduce the entanglement swapping phenomenon between two EPR pairs. Let us consider two entangled pairs \((A, V_1)\) and \((B, V_2)\) each of them in an antisymmetric polarization-entangled Bell singlet state. Therefore, the state of the whole system is:

\[
|\Psi\rangle = \left( |0_A1_{V_1}\rangle - |1_A0_{V_1}\rangle \right) \otimes \left( |0_B1_{V_2}\rangle - |1_B0_{V_2}\rangle \right).
\]  

Let the particle A be with Alice, B with Bob, while Victor keeps \( V_1 \) and \( V_2 \). Now if Victor projects his states onto a Bell state, they get entangled. At the same time, the particles \((A,B)\) get entangled, despite having absolutely no communication. To see that, we write eq \[(2)\] as follows

\[
|\Psi\rangle = |1_{V_1}1_{V_2}\rangle |0_A0_B\rangle - |1_{V_1}0_{V_2}\rangle |0_A1_B\rangle - |0_{V_1}1_{V_2}\rangle |1_A0_B\rangle + |0_{V_1}0_{V_2}\rangle |1_A1_B\rangle.
\]  

We then let Victor to project his particles on (as an example) the state:

\[
\left( |0_{V_1}1_{V_2}\rangle - |1_{V_1}0_{V_2}\rangle \right) \left( |0_{V_1}1_{V_2}\rangle - |1_{V_1}0_{V_2}\rangle \right).
\]  

Therefore, the final state reads:

\[
|\Psi\rangle = \left( |0_{V_1}1_{V_2}\rangle - |1_{V_1}0_{V_2}\rangle \right) \otimes \left( |0_A1_B\rangle - |1_A0_B\rangle \right).
\]  

exactly as claimed above. Moreover, as a consequence of the monogamy principle mentioned above and as is clear in [5], the entanglement of A with \( V_1 \) is broken as well as the entanglement of B with \( V_2 \) [25, 26]. Now let us assume that Victor does this whole procedure inside a black hole while Alice and Bob are outside. The entanglement between Alice and Victor on one hand and Bob and Victor on the other hand will get swapped as we have just explained and Alice and Bob will fly away with an entangled pure state. Moreover, if Victor’s particles annihilate each other somehow, we will end up with only two entangled particles outside the black hole in a pure state. This is the key point of our approach to solve the information loss paradox that will be explained with all details in the rest of the paper. Note that such a swapping of entanglement does not allow instantaneous signaling, because Victor cannot control the outcome of his measurement. This point will be discussed in the appendix.
Figure 1: We here consider an entangled pair inside the black hole and a Hawking pair created at the event horizon, one with positive and one with negative energy (see Fig.1.a). The negative energy particle will get attracted to the black hole’s interior and it eventually reaches the particle A (see Fig.1.b). Assuming the “int” Hawking particle to annihilate the particle A, then, we end up with the “out” Hawking particle entangled with the particle B (see Fig.1.c).

III. ENTANGLEMENT SWAPPING IN BLACK HOLES

The Hawking radiation state in (1) describes all the radiated particles, but for a better exposure and analysis of the problem we can focus on one pair being created near the event horizon. Therefore, the state (1) simplifies to:

$$|\psi\rangle = \sum_\omega e^{-\frac{2\pi}{\kappa} \omega} |\omega\rangle^{\text{out}} \otimes |-\omega\rangle^{\text{int}},$$

up to a normalization factor. We now carefully look at the dynamics inside the black hole. We consider a black hole of mass $M$ as a result of the gravitational collapse of a large number of entangled particles in a pure state (we will also consider the case of particles that are entangled with nothing else). However, in this paragraph, for the sake of simplicity we consider only one entangled pair inside the BH described at a time, and later a more general state will be treated. That means we focus on the following matter state inside the black hole:

$$|\phi\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A \otimes |\omega'\rangle_B ,$$

Therefore, the initial state is given by the tensor product of (6) and (7), namely

$$|i\rangle = |\psi\rangle \otimes |\phi\rangle = \sum_\omega \sum_{\omega'} f(\omega') e^{-\frac{2\pi}{\kappa} \omega} |\omega\rangle^{\text{out}} \otimes |-\omega\rangle^{\text{int}} |\omega'\rangle_A |\omega'\rangle_B .$$

If the incident negative energy particle $| -\omega\rangle^{\text{int}}$ interacts with the particle of energy $\omega'$, either the two particles fully annihilate inside the black hole (case (i)) or they do not (case (ii)). Therefore, after the interaction has occurred the state is:

$$|f\rangle = \sum_{\omega',\omega} f(\omega') e^{-\frac{2\pi}{\kappa} \omega} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle^{\text{int}} |0\rangle_A |\omega'\rangle_B$$

$$= \sum_{\omega=\omega'} f(\omega) e^{-\frac{2\pi}{\kappa} \omega} |\omega\rangle^{\text{out}} |\omega\rangle_B |0\rangle_A^{\text{int}} + \sum_{\omega' \neq \omega} f(\omega') e^{-\frac{2\pi}{\kappa} \omega} |\omega' - \omega\rangle^{\text{out}} |\omega' - \omega\rangle^{\text{int}} |0\rangle_A |\omega'\rangle_B \equiv |f\rangle_{\text{case(i)}} + |f\rangle_{\text{case(ii)}} .$$

Note that we will continue our analysis with (9) as our initial state, but for the time being we have split the sum just to show some points before proceeding. If we focus on the case (i), which means we assume full annihilation, we explicitly see the swapping of entanglement between the Hawking pair and the pair inside the black hole (see Fig.1). In the final state $|f\rangle_{\text{case(i)}}$, the mass of the black hole is reduced to $M - \omega$ and the outside Hawking particle is entangled with one of the two black hole particles inside the event horizon (see Fig.1.c). On the other hand, in the

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2 In general the black hole’s state consists of many particles and any number of Hawking pairs, but here for the sake of simplicity we only consider one Hawking pair and two entangled matter particles. We will later consider a significantly more general state.
Figure 2: The first two figures Fig.2.a and Fig.2.b are the same as in Fig.1, but here we do not assume full annihilation of the “int” particle with the particle A. Therefore, we stay more general to end up with three entangled particles (Fig.2.c).

Figure 3: Starting from Fig.2.c, we here consider the creation of a second pair near the event horizon (Fig.3.a). In Fig.3.b the particle with negative energy $-\omega''$ crosses the horizon and scatters with the particle B with energy $\omega'$ in Fig.3.c. If the we have full annihilation inside the black hole, namely $\omega'' = \omega' = \omega$ then we end up with the situation shown in Fig.3.d when the “out” particles are entangled and the black hole mass is $M - 2\omega$.

Let us now consider a second Hawking pair created near the Event Horizon, namely $|\psi_2\rangle$. Using again (6) for $|\psi_2\rangle$ and assuming $|f\rangle$ as the initial state, the whole system is described by the tensor product $|i'\rangle = |f\rangle \otimes |\psi_2\rangle$ (see Fig.4.a-b), namely

$$|i'\rangle = \sum_{\omega''', \omega', \omega} f(\omega') e^{-\frac{\pi(\omega + \omega'')}{\kappa}} |\omega\rangle_{\text{out}} |\omega' - \omega\rangle_A^\text{int} |\omega'\rangle_B |\omega''\rangle_{\text{out}} |\omega''\rangle_A^\text{int}.$$  \hspace{1cm} (11)

Now say the new created Hawking particle interacts with the particle B, we will get (see Fig.3.c)

$$|f'\rangle = \sum_{\omega''', \omega', \omega} f(\omega') e^{-\frac{\pi(\omega + \omega'')}{\kappa}} |\omega\rangle_{\text{out}} |\omega''\rangle_{\text{out}} |\omega' - \omega\rangle_A^\text{int} |\omega' - \omega''\rangle_B^\text{int}.$$  \hspace{1cm} (12)

The resulting state consists of two particles inside the black hole partially entangled between each other and with the two Hawking particles outside. Finally, assuming full annihilation of the two particles inside, it is easy to see that one gets

$$|f\rangle_{\text{Evap}} = \sum_{\omega} f(\omega) e^{-\frac{4\pi\omega}{\kappa}} |\omega\rangle_{\text{out}} |0\rangle_A^\text{int} |0\rangle_B^\text{int},$$  \hspace{1cm} (13)

\footnote{In our treatment of the annihilation process we have labeled the states with their energies $|\omega\rangle$, although $\omega$ does not fully specify the state. We omitted the momentum label $p$ because it is not a conserved quantity and for notational simplicity.}
which is clearly an entangled pair outside the black hole (see Fig. 3d). The pure state (7) has evolved in a similar pure state (13). If we now trace out the “int” system the state (13) stays the same. In Fig. 4, the same scenario is represented in the Penrose diagram for the full black hole formation and evaporation process.

Notice that nothing changes if the second Hawking particle interacts with the particle A (instead of B). The whole process could eventually take longer but will be qualitatively the same. Moreover, it is possible that the incident Hawking particle scatters to produce more than one particle inside the black hole. In this case a multipartite entangled state is created (see section III C).

A. Colliding a pure state inside the black hole

For completeness we also study the case in which the particle inside the black hole is not entangled with any other subsystem (we call this particle “A”). Therefore, the state (7) is replaced with

$$|\phi_2\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A .$$

(14)

An analysis similar to the one in (10), gives the following final state

$$|f''\rangle = \sum_{\omega,\omega'} f(\omega) e^{-\frac{\pi}{\kappa} (\omega - \omega')^2} |\omega\rangle_{\text{out}} |\omega'\rangle_{\text{int}} |0\rangle_A$$

$$= \sum_{\omega} f(\omega) e^{-\frac{\pi}{\kappa} (\omega')^2} |\omega\rangle_{\text{out}} |0\rangle_A + \sum_{\omega' \neq \omega} f(\omega') e^{-\frac{\pi}{\kappa} (\omega - \omega')^2} |\omega\rangle_{\text{out}} |\omega'\rangle_{\text{int}} |0\rangle_A \equiv |f''\rangle_{\text{case(i)}} + |f''\rangle_{\text{case(ii)}} .$$

(15)

Assuming full annihilation, we end up with the pure state $|f''\rangle_{\text{case(i)}}$. Indeed, the initial non entangled pure state has evolved to a non entangled pure state as well. Only in the intermediate stage the created Hawking pair is entangled.

B. General state inside

For the analysis developed in the previous section we assumed the particles inside the black hole to have the same energy, but it is straightforward to generalize to an arbitrary entangled state. Let us consider again a Hawking pair in the state (6), and a particle pair inside the black hole in the state $|\chi\rangle$ defined as

$$|\chi\rangle = \sum_{\omega'} f(\omega') |g(\omega')\rangle_A |\omega'\rangle_B ,$$

(16)

a pure bipartite entangled state can always be written in this form, $g(\omega')$ is a general function of its argument. The initial state (5) is replaced with $|i_g\rangle = |\psi\rangle \otimes |\chi\rangle$ and, if we assume the negative energy particle to interact with the particle B, the final state is:

$$|f_g\rangle = \sum_{\omega',\omega} f(\omega') e^{-\frac{\pi}{\kappa} (\omega - \omega')^2} |g(\omega')\rangle_A |\omega'\rangle_{\text{int}} |0\rangle_B$$

$$= \sum_{\omega} f(\omega) e^{-\frac{\pi}{\kappa} (\omega')^2} |g(\omega)\rangle_A |0\rangle_B + \sum_{\omega' \neq \omega} f(\omega') e^{-\frac{\pi}{\kappa} (\omega - \omega')^2} |g(\omega')\rangle_A |\omega'\rangle_{\text{int}} |0\rangle_B .$$

(17)

If we have annihilation, only the first term on the right hand side of (18) survives (Fig. 1). However, the general case (17) is again elucidated in Fig. 2.

C. Multipartite entangled black hole matter

We now consider a general multipartite entangled pure state describing a black hole resulting from a gravitational collapse. For the sake of simplicity we do not here consider initial mixed states. However, our analysis applies in that case too. This will also help to understand the previously mentioned case where the incident Hawking particle scatters inside the black hole to produce more than one particle.
Figure 4: The Penrose diagram for the formation and evaporation of a Schwarzschild black hole in this figure includes the transfer of entanglement from the particles inside to the particles outside the event horizon. A Hawking pair is created on the Cauchy surface $\Sigma_a$ and evolves to the surface $\Sigma_c$ where we see two entangled pairs: the "int" and "out" Hawking particles on the right and two entangled black hole matter particles. In $\Sigma_d$ one of the matter particles and the "int" particle interact and generate a new particle making a system of three entangled particles. On $\Sigma_e$ the remaining matter particle (of the latter three-partite entangled system) comes very close to a new Hawking particle created on $\Sigma_b$ and in $\Sigma_f$ they interact and we have an entangled system of four particles: two inside and two outside the black hole. Finally, assuming full annihilation inside the black hole we end up with two "out" entangled particles on $\Sigma_g$.

The multipartite matter state is a generalization of the simple bipartite state given in (16), namely

$$|\Psi\rangle = \sum_{\omega_1, \ldots, \omega_k} f(\omega_1, \ldots, \omega_k) |\omega_1\rangle_{A_0} |g_1(\omega_1, \ldots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \ldots, \omega_k)\rangle_{A_2} \cdots |g_k(\omega_1, \ldots, \omega_k)\rangle_{A_k},$$

(19)

where $f(\omega_1, \ldots, \omega_k)$ is a general phase factor and $A_0, \ldots, A_k$ are $k+1$ particles. Now consider an incident Hawking particle of energy $\omega$ that scatters with the particle $A_0$ to produce a particle of energy $\omega_1 - \omega$. The state of the whole system, before the interaction takes place, is the tensor product of (19) and (6), namely $|\Psi\rangle \equiv |\Psi\rangle \otimes |\psi\rangle$,

$$|\Psi\rangle = \sum_{\omega_1, \ldots, \omega_k} f(\omega_1, \ldots, \omega_k) e^{-i\frac{\pi}{\kappa} \omega_1 A_0 |\omega_1 A_0 - \omega\rangle |g_1(\omega_1, \ldots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \ldots, \omega_k)\rangle_{A_2} \cdots |g_k(\omega_1, \ldots, \omega_k)\rangle_{A_k} \otimes |\omega\rangle^\text{int} |\omega\rangle^\text{out}. $$

(20)

When the “int” particle interacts with the particle $A_0$ the state becomes:

$$|\Psi''\rangle = \sum_{\omega_1, \ldots, \omega_k} f(\omega_1, \ldots, \omega_k) e^{-i\frac{\pi}{\kappa} \omega_1 A_0 - \omega} |g_1(\omega_1, \ldots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \ldots, \omega_k)\rangle_{A_2} \cdots |g_k(\omega_1, \ldots, \omega_k)\rangle_{A_k} \otimes |\omega\rangle^\text{out}. $$

(21)
Therefore, the resulting particle of energy \( \omega_1 A_0 - \omega \) is entangled with the black hole matter and the Hawking “out” particle too. If more Hawking pairs are created, we have more “out” particles entangled with the black hole matter and the state is:

\[
\left| \Psi^{(k)} \right| = \sum_{\omega_1, \ldots, \omega_k} f(\omega_1, \ldots, \omega_k) e^{-\frac{2(\omega_1 + \ldots + \omega_k)}{r}} \left| \omega_1 A_0 - \omega \right| g_1(\omega_1, \ldots, \omega_k)_{A_1} - \omega' \right| g_2(\omega_1, \ldots, \omega_k)_{A_2} - \omega'' \right| \ldots g_k(\omega_1, \ldots, \omega_k)_{A_k} - \omega^{(k)} \right|_{\text{BH}}
\]

\[
\otimes \left| \omega, \omega', \ldots, \omega^{(k)} \right|^{\text{out}}, \tag{22}
\]

where the sum above is on all the frequencies \( \omega_1, \ldots, \omega_k, \omega', \omega'', \ldots, \omega^{(k)} \). Now we have an entangled state involving all the particles inside and outside. If we assume full evaporation\(^4\) of the black hole, the entanglement is swapped to the outside radiation and the state reads:

\[
\sum_{\omega_1, \ldots, \omega_k} f(\omega_1, \ldots, \omega_k) e^{-\frac{2(\omega_1 + \ldots + \omega_k)}{r}} \left| 0 \right|_{\text{BH}} \otimes \left| \omega_1 A_0 \right|_{\text{out}} \left| g_1(\omega_1, \ldots, \omega_k)_{A_1} \right|_{\text{out}} \left| g_2(\omega_1, \ldots, \omega_k)_{A_2} \right|_{\text{out}} \ldots g_k(\omega_1, \ldots, \omega_k)_{A_k} \right|^{\text{out}}; \tag{23}
\]

where we labelled the states also with the index \( A_i \) to keep track of the “int” particles that have been annihilated with the particles \( A_1, \ldots, A_k \).

The state \( \left| 23 \right| \) is clearly an entangled pure state of Hawking’s “out” particles after the black hole has fully evaporated. Notice that the state \( \left| 22 \right| \) is a superposition of all energy’s eigenstates. Therefore, the projection to the particular final state \( \left| 23 \right| \) is only due to the black hole full evaporation and not to an intrinsic unitarity violation.

The outcome of this section can be summarized as follows. The pure entangled state describing matter inside the black hole (19) evolves into the pure entangled state at \( I^+ \) (23). We here only assumed annihilation inside the black hole between negative and positive energy particles.

**IV. THE SINGULARITY ISSUE**

There are reasons to believe that our solution of the information loss problem seems to work regardless of whether the spacetime is singular or singularity-free [8, 9, 34–39]. In the previous sections we never mention the spacetime singularity issue at \( r = 0 \). Indeed, our analysis is based on the natural and commonly made assumption that particles inside the black hole annihilate. As long as the “int” particles interact with the matter inside the black hole that have not reached \( r = 0 \) yet, as in Fig. 4 for \( v < v_s \), the dynamics (more technically the S-matrix) is well defined and the scattering takes place without violating unitarity. On the other hand, for \( v > v_s \) the “int” particles must annihilate with matter particles that have already reached the singularity. In this paper as well as most others in the literature, it is assumed that the annihilation takes place regardless of the singularity\(^5\). Therefore, we are entitled to believe that also entanglement is gently transferred to and/or from the matter at the singularity. On the other hand, if there is no annihilation at the singularity we do not have evaporation and thus any information loss problem because there are correlations between the matter inside and/or particles outside the black hole, purifying the state of the whole system.

However, the case of a singular spacetime will surely require further investigations.

Finally, in any singularity-free black hole our proof is *a priori* expected to apply and there is no information loss problem because in this case the spacetime is geodesically complete and the needed interactions for \( v > v_s \) can happen smoothly. In a future project, we will carefully work out which black hole’s geometries allow our process of entanglement transfer and which ones (if any) do not. This analysis could eventually support some quantum gravitational theories over some others.

\(^4\) This is equivalent to saying that an observer at infinity makes a measurement of the black hole mass.

\(^5\) As proved in the paper [40], explicitly titled “The energy-momentum tensor of a black hole, or what curves the Schwarzschild geometry?”, the source of the Ricci flat solutions (in vacuum) has a well defined meaning in the space of distributions and the energy-momentum tensor is proportional to the Dirac’s delta, namely \( T \propto M \delta(r) \) (this is also proved in many other textbooks like Landau-Lifshitz, etc). After the black hole formation, the matter is localized at \( r = 0 \) and can be reached in finite time (or finite value of the affine parameter in the massless case) by the Hawking’s “int” particles. Therefore, all the “int” particles annihilate for \( r > 0 \) in the first stage of the evaporation process or in \( r = 0 \) afterwards to finally end up with zero Bondi-Sachs mass. Notice, that if there was no source at \( r = 0 \) then the spacetime would be Minkowski and not Schwarzschild.
Comments and Conclusions

Let us now summarize our result and make some comments on the usual unitarity loss problem. Assuming no annihilation inside the black hole, the pure state (1) describes “int” and “out” radiation. Once we trace out the “int” subsystem, we find the “out” radiation in a mixed state. However, this does not imply any unitarity violation because the “int” particles still exist in the black hole interior. If we now assume that some “int” particles annihilate, then we must take into account that the entanglement is transferred to other particles inside and/or outside the event horizon. Commonly, people do not consider such swap of entanglement and unitarity is lost. On the base of Fig.3, the mistake is to trace out the interior of Fig.3.c to end up with two non-entangled particles in Fig.3.d, and of course the “out” radiation is then in a mixed state. Similarly, at the end of the black hole’s evaporation process (full annihilation of “int” particles with the black hole’s matter), one has to trace out the “int” states to end up (using the usual treatment) with “out” particles in a mixed state. In contrast, throughout our analysis we keep track of the entanglement transfer at any step of the evaporation process and we finally get a pure entanglement state outside (see (23)).

In short, in this paper we have shown that in the Hawking evaporation process the entanglement is generically transferred from the infalling negative energy particles to other particles inside and/or outside the black hole. After full evaporation all entanglement is transferred to the “out” particles, there is no more black hole, and the particles at future infinity are in a pure entangled state without any violation of the monogamy theorem, the unitarity of evolution, or the equivalence principle.

Finally, we would like to underline that the entanglement is transferred at distance in a way standard in quantum information theory. Indeed, this is very similar to the entanglement swapping, which has been observed in many physical experiments [27][33].

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Appendix: Swapping Does Not Allow Signaling

In this section we describe how the entanglement swapping does not allow instantaneous signaling because the observer that makes the measurement at the side A (we here consider a system made of two sides, A and B) cannot control the outcome of the measurement. More concretely, one can consider a bipartite system AB described by the density matrix $\rho_{AB}$. An observer can make measurements on A with different possible outcomes described by the following set of projections:

$$\{ |1\rangle\langle1|_A, \ldots, |d\rangle\langle d|_A \}.$$  

If the observer knows the measurement outcome at A, then the sub-normalized post-measurement state is:

$$\rho_{AB}^{(i)} = \left( |i\rangle\langle i|_A \otimes I_B \right) \rho_{AB} \left( |i\rangle\langle i|_A \otimes I_B \right).$$  

However, if one does not know the measurement outcome then he has to sum over all possible outcomes, and the post-measurement state will be:

$$\rho''_{AB} = \sum_i \left( |i\rangle\langle i|_A \otimes I_B \right) \rho_{AB} \left( |i\rangle\langle i|_A \otimes I_B \right).$$

The density matrix $\rho_{AB}^{(i)}$ is called “conditional density matrix”, and it is used by an observer who knows the outcome of a measurement on the subsystem A to describe the whole system AB. Notice that

$$\rho''_B = \text{Tr}_A \rho''_{AB} \neq \rho_B = \text{Tr}_A \rho_{AB},$$

which means that a measurement on A seems to change the state of B. Therefore, one might think that we could send information to B by making a measurement on A. However, for an observer who does not know the measurement.
outcome, the reduced density matrix describing the system B reads:

\[
\rho_B'' = \text{Tr}_A \sum_i \langle i | A \rangle \langle i | A \rangle I_B \rho_{AB} (|i\rangle A \otimes I_B) = \sum_{i,k} \langle k | A \rangle \rho_{AB} (|i\rangle A \otimes I_B) |k\rangle A \\
= \sum_k \rho_{AB} (|k\rangle A \otimes I_B) = \rho_B,
\]

(28)

where the second last equation is the known definition of the partial A-trace of \(\rho_{AB}\).

Let us now connect this analysis to the section II in the main text. If Victor makes the projection (4) to get his pairs entangled, Alice and Bob need a classical signal from Victor to realize that their particles are entangled. Without this classical signal they have to sum over all possible outcomes to describe the system with \(\rho_{AB}'\). As we have shown this has no observable effect because the reduced density matrix of their part will not be changed (see 28).