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Well Test Analysis for Fractured and Vuggy Carbonate Reservoirs of Well Drilling in Large Scale Cave

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Abstract: A well test analysis model for fractured and vuggy carbonate reservoir of wells drilling in large scale cave considering wellbore storage and skin factor is established in this paper. The Laplace transformation and Stehfest numerical inversion are applied to obtain the results of wellbore pressure. Through the sensitivity analysis of different parameters for the well test typical curves, it is found that the change of the well test curves is in accordance with the theoretical analysis. With the increase of skin factor, the hump of well test typical curves is steeper. The storage ratio influences the depth and width of the concave in the pressure derivative curves. The cross flow coefficient mainly affects the position of the concave occurrence in the pressure derivative curves. The dimensionless reservoir radius mainly affects the middle and late stages of the log-log pressure type curves, and the later well test curves will be upturned for sealed boundary. The duration of the early stage of the log-log curves will become longer when drilling in large scale cave. The effective well radius is increased to a certain extent, which is in full agreement with the conclusions in this paper. The size of the caves has the same effect on the well test typical curves as wellbore storage coefficient. Due to acidification, fracturing, and other reasons, the boundary of the cave will collapse. Therefore, considering the wellbore storage coefficient and skin effect is very important during well testing. However, the existing models for well testing of fractured and vuggy carbonate reservoir often ignore the wellbore storage coefficient and skin effect. For fractured and vuggy carbonate reservoirs of well drilling in large scale cave, the existing models are not applicable. Since the previous models are mostly based on the triple-porosity medium and the equivalent continuum. The well test model for well drilling in large scale cave of fracture-cavity carbonate reservoirs with wellbore storage coefficient and skin factor in this work has significant application value for oil field.

Keywords: carbonate; large scale cave; drilling cave; well test; sensitivity analysis

1. Introduction

Most of the word’s giant fields produce from fractured and vuggy carbonate reservoirs that have characteristics of complex pore system, large scale reservoir medium span, the variety of fracture-cavity connected mode, the coexistence of multiple forms of fluid flow [1–6], mainly because carbonate rocks are particularly sensitive to post-depositional diagenesis, including dissolution, dolomitization, and fracturing processes. Large scale caves, fractures and dissolution pores are highly dispersed.

Well test interpretation has been the focus of researchers both at home and abroad. At present, well test interpretation models can be divided into four types: equivalent and continuum media models, discrete fractures and caves models, numerical simulation and discontinuous media models.
Equivalent continuum models [7–13] are equivalent to the entire area as an imaginary continuum, and the caves and fractures are evenly distributed throughout the region. With the rate of flow equivalence principle, the permeability of large caves and fractures is equivalent to the whole area, and the macroscopic fluid flow characteristics are studied. The fluid flow accords with the Darcy flow law in the whole study area, and the existing seepage theory is used to study the fluid flow characteristics. The simplified models are not close to the real caves. For the fractured and vuggy type carbonate reservoirs, the fluid flow in the caves does not conform to the Darcy flow law, and it can’t be equivalent to the average permeability of the caves and fractures. So the equivalent continuum models cannot be used to study large scale caves and fractures.

In the discrete fracture cavity models, the fractures and caves are embedded in the matrix, and the large caves are regarded as equipotential bodies. The flow of the fluid in the cavity is free flow, and the flow of the fluid in the fractures and caves is described by the coupling of the seepage and the free flow. Discrete fracture cavity models [14–16] can reflect the development space of reservoirs, but it is highly dependent on the description technique of reservoirs. If considering the actual application and field staff training, the models do not have the advantages of application. The seam geological conditions cave type carbonate reservoir complex, discrete joint establishment, and solution hole model are difficult to achieve. For complex geological conditions of large scale fractured and vuggy carbonate reservoirs, the establishment and solution of discrete fracture cavity models are difficult to achieve.

Numerical simulation [17–19] is a convenient tool for well test analysis, and there are many software companies in the development at home and abroad, but the most representative is Saphir developed by KAPPA company (Paris, France), which has been widely used in oil companies and high schools around the world. There are many formation models available in Saphir, including homogeneous formation, dual porosity medium, composite reservoir, etc., but they are not suitable for well test interpretation of fractured and vuggy type carbonate reservoirs. Saphir does not involve the solution of large scale caves. Therefore, numerical simulation has great limitations in well test interpretation of fractured and vuggy carbonate reservoirs.

For the complex reservoir characteristics of fractured and vuggy reservoirs [20,21], the conventional well test analysis methods for triple-porosity medium and fractured sandstone reservoirs are no longer applicable. So it is necessary to provide a substantial solution as a powerful tool to improve both forecasting and flow-regime interpretation for fractured and vuggy carbonate reservoirs characterization. The discontinuous medium models [22–26] establish different fluid flow equations through different medium, which solves the heterogeneous problem of fractures, caves and dissolution pores. In the discontinuous medium models, the caves are regarded as equipotential bodies. The material balance equations are used to establish the flow relationship between the caves and formation. The software established by the models can obtain well test interpretation results only by using the basic data of oilfield. Therefore, discontinuous medium models are applicable to the description of large scale fractures and caves. The study of this paper is based on the theory of discontinuous media.

With the incessant development of fractured and vuggy carbonate reservoirs, it is of great significance to study well test model with large scale caves for well testing analysis and oilfield development dynamic prediction of this typical reservoirs. During the drilling of TH (TH is an abbreviated form for “TaHe”, located in the northwest China) oilfield, the emptying of drill string and the leakage of drilling mud often occur. Through the geological conditions of the fractured and vuggy carbonate reservoir [20,21], it can be seen that large scale caves have been drilled in the drilling process, or the large scale fractures and caves near wellbore area have been connected after acidification. In view of this situation, in this paper, a well test model for drilling in large scale cave is established, and the formation parameters are obtained by well test analysis method, which provides the basis for further construction measures.
At present, the well test curves plotted by the established well test model of fractured and vuggy carbonate reservoirs do not consider wellbore storage and skin effect, and it will be difficult to obtain the reservoir characteristics of fractured and vuggy reservoirs. In this paper, a well test model for fractured and vuggy carbonate reservoirs considering wellbore storage and skin effect is established with drilled in large scale cave. This model can be used to describe the characteristics of fractured and vuggy carbonate reservoirs more truly and the well test analysis results have proper geological significance. In this work, we introduce a well test analysis model that is more general and rigorous than previous models. This new model is capable of capturing a broader range of reservoir/fracture properties and flow regime sequences than before. We also provide parameter solutions for straight-line that are applicable to each flow regime. A sensitivity study is performed to illustrate various controls on flow-regime sequences and behaviors. Finally, we demonstrate actual application of our new model to prevent misinterpretation of flow regimes, and misapplication of local solutions, by use of field example. The established model has been used for well test analysis with actual well test data, which provides significant guidance and theoretical foundation for well test analysis of fractured and vuggy carbonate reservoirs.

2. Physical Model

The well test analysis physical model for fractured and vuggy carbonate reservoir considering wellbore storage and skin effect has the following basic assumptions:

1. fluid is single-phase and micro compressible;
2. formation is circular, isotropy, horizontal, equal thickness, and the cave is surrounded by dual porosity media;
3. effects of gravity and capillary force are ignored, but the effects of wellbore storage and skin effect are considered;
4. large scale cave is regard as equipotential body. In the dual-porosity medium, the flow of fluid is Darcy flow, and the flow process is isothermal, without any special physical and chemical phenomena;
5. well is produced at a fixed volume;
6. oil reservoir is mined, and the pressure at any location in the reservoir is the initial formation pressure;
7. large scale cave is in the center of circular formation, and the cave radius is \( R \), with well drilling cave. Fractures are the main flow channel, the fluid flow into the cave through the fractures, and then through the oil well, the fluid of in large scale cave can be produced.

According to the above assumptions, the well test physical model of the fractured and vuggy carbonate reservoir for well drilling in large scale cave is established, as shown in Figure 1.

![Figure 1](image_url). The well test physical model of the fractured and vuggy carbonate reservoir for well drilling in large scale cave.
When the physical model is established, some limitations are considered, such as the fluid is single-phase and micro compressible. The effects of gravity and capillary are ignored, and production rate is fixed. The physical model is simplified to a certain extent by using the assumptions, so the description of the reservoir characteristics will be limited to a certain extent. Therefore, multi-phase fluid and complex formation conditions can be considered in future research.

3. Mathematical Model

3.1. Establishment of Mathematical Model

According to the principle of mass conservation, the continuity equation of fluid unsteady flow is established. According to the law of Darcy’s linear flow, the equation of motion is established. Combining the state equation of liquid and rock, the basic differential equations of unsteady flow are obtained:

The basic differential equation of fracture:

\[ \frac{K_f}{\mu} \frac{\partial^2 p_f}{\partial r^2} + \frac{K_f}{\mu} \frac{1}{r} \frac{\partial p_f}{\partial r} + \frac{K_m}{\mu} (p_m - p_f) = \frac{\phi f C_f}{3.6} \frac{\partial p_f}{\partial t} \]  

The basic differential equation of dissolution pore:

\[ -\frac{K_m}{\mu} (p_m - p_f) = \frac{\phi m C_m}{3.6} \frac{\partial p_m}{\partial t} \]  

Equation of mass conservation of cave:

\[ 172.8 \pi R h \frac{K_f}{\mu} \frac{\partial p_f}{\partial r} \bigg|_{r=R} - 24 \pi R^2 h C_v \phi_v \frac{\partial p_w}{\partial t} - 24 C \frac{\partial p_w}{\partial t} = q B \]  

Connection condition:

\[ p_f \big|_{r=R} - p_w = S R \frac{\partial p_f}{\partial r} \bigg|_{r=R} \]  

Initial condition:

\[ p_f(r, t = 0) = p_i, (j = f, v, m) \]  

Outer boundary condition:

\[ \frac{\partial p_f}{\partial r} \bigg|_{r=r_e} = 0 \]  

where \( p_i \) is initial formation pressure, MPa; \( p \) is formation pressure, MPa; \( p_w \) is wellbore pressure, MPa; \( K \) is absolute permeability, \( \mu m^2 \); \( C_m \) is matrix compressibility, MPa\(^{-1}\); \( C_f \) is fractures compressibility, MPa\(^{-1}\); \( C_v \) is cave compressibility, MPa\(^{-1}\); \( \phi \) represents porosity; \( r_w \) is wellbore radius, m; \( q \) is production rate, \( \text{m}^3/\text{d} \); \( \mu \) is fluid viscosity, mPa·s; \( S \) is skin factor; Subscript \( f, v \) and \( m \) are fracture, cave and matrix respectively; \( B \) is oil formation volume factor, \( \text{m}^3/\text{m}^3 \); \( \alpha \) is shape factor; \( R \) is radius of large scale cave; \( t \) is time, h.

For large scale caves, the material balance equation is established, as shown in Equation (3). Equations (1) and (6) make up a mathematical model. In Equations (3) and (4), the influence of wellbore storage and skin factor is considered respectively.

3.2. Solution of Mathematical Model

Define dimensionless quantities:

\[ p_{iD} = \frac{K_i h}{1.842 \times 10^{-3} \eta \mu B} (p_i - p_f), (j = f, v, m), \lambda = \frac{K_m}{K_f} r_w^2, r_{eD} = \frac{r_e}{r_w}, R_D = \frac{R}{r_w}, r_D = \frac{r}{r_w} \]
where $p_{\text{ID}}$ is dimensionless formation pressure; $r_{\text{D}}$ is dimensionless reservoir radius; $R_D$ represents dimensionless cave radius; $t_\text{D}$ is dimensionless time; $C_D$ is dimensionless wellbore storage coefficient; $\omega_v$ is storage capacity ratio of cave; $\omega$ is storage capacity ratio of fracture; $\lambda$ is cross flow coefficient.

Using the defined dimensionless to Equations (1) and (6) can obtain dimensionless Equations (7) and (12):

\[
\frac{\partial^2 p_{\text{ID}}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{\text{ID}}}{\partial r_D} + \lambda (p_{\text{mD}} - p_{\text{ID}}) = \omega \frac{\partial p_{\text{ID}}}{\partial t_D} 
\]

\[
- \lambda (p_{\text{mD}} - p_{\text{ID}}) = (1 - \omega) \frac{\partial p_{\text{mD}}}{\partial t_D}
\]

\[
R_D \frac{\partial p_{\text{ID}}}{\partial r_D} \bigg|_{r_D=0} - \omega v R_D^2 \frac{\partial p_{\text{wD}}}{\partial t_D} - C_D \frac{\partial p_{\text{wD}}}{\partial t_D} = 1
\]

\[
p_{\text{ID}} \bigg|_{r_D=0} - p_{\text{wD}} = S R_D \frac{\partial p_{\text{ID}}}{\partial r_D} \bigg|_{r_D=0}
\]

\[
p_{\text{ID}}(r_D, t_D = 0) = 0, (j = f, v, m)
\]

\[
\frac{\partial p_{\text{ID}}}{\partial r_D} \bigg|_{r_D=r_{\text{D}}} = 0
\]

The Laplace transformation of dimensionless time is performed on Equations (7) and (12):

\[
\frac{\partial^2 \tilde{p}_{\text{ID}}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \tilde{p}_{\text{ID}}}{\partial r_D} + \lambda (\tilde{p}_{\text{mD}} - \tilde{p}_{\text{ID}}) = \omega s \tilde{p}_{\text{ID}}
\]

\[
- \lambda (\tilde{p}_{\text{mD}} - \tilde{p}_{\text{ID}}) = (1 - \omega)s \tilde{p}_{\text{mD}}
\]

\[
R_D \frac{\partial \tilde{p}_{\text{ID}}}{\partial r_D} \bigg|_{r_D=0} - \omega v R_D^2 \tilde{p}_{\text{wD}} - C_D \tilde{p}_{\text{wD}} = \frac{1}{s}
\]

\[
\tilde{p}_{\text{ID}} \bigg|_{r_D=0} - \tilde{p}_{\text{wD}} = S R_D \frac{\partial \tilde{p}_{\text{ID}}}{\partial r_D} \bigg|_{r_D=0}
\]

\[
\tilde{p}_{\text{ID}}(r_D, s = 0) = 0, (j = f, v, m)
\]

\[
\frac{\partial \tilde{p}_{\text{ID}}}{\partial r_D} \bigg|_{r_D=r_{\text{D}}} = 0
\]

where $s$ is Laplace’s variables on dimensionless time $t_D$.

To solve Equations (13) and (18), gets the solution of wellbore pressure in Laplace’s space, and the solution is shown in Equation (19):

\[
\tilde{p}_{\text{wD}} = \frac{s[A I_0(R_D \sqrt{f(s)}) + B K_0(R_D \sqrt{f(s)})] + s}{S \cdot s^2 \cdot R_D^2 \cdot w_v + s + C_D s^2}
\]

(19)

where $A$ and $B$ are undetermined constants; $K_0$ and $I_0$ are second kind Bessel functions of zero order.

\[
A = \frac{K_1(\sqrt{f(s)r_{\text{D}}})}{I_1(\sqrt{f(s)r_{\text{D}}})} B
\]

(20)
\[ f(s) = s^{\lambda + \omega s(1 - \omega)} / (\lambda + (1 - \omega)s)^{\omega} \]  

(21)

By the Stehfest numerical inversion method, the wellbore pressure solution of Laplace space is transformed into the solution [27] of the real space, therefore the wellbore pressure \( p_{wD} \) is obtained finally.

4. Analysis on Well Test Curves

4.1. Characteristics Analysis of the Well Test Typical Curves

The typical curves of well test are plotted by using the values of the corresponding wellbore pressure \( p_{wD} \) at different time. A visual interface is programmed using the software, and the dimensionless wellbore pressure curves and the dimensionless wellbore pressure derivative curves can be displayed through the interface. The well test typical curves of well drilling in large scale cave with wellbore storage effect and skin factor are obtained, as shown in Figure 2.

![Figure 2](image-url)

**Figure 2.** The well test typical curves of well drilling in large scale cave with wellbore storage effect and skin factor.

The basic parameters of well test typical curves in Figure 2: \( \omega = 0.01, \lambda = 1 \times 10^{-7}, r_{eD} = 1000, \omega_v = 0.1, R_D = 30, C_D e^{2S} = 100, S = 1.0. \)

As may be seen in Figure 2, it is clear that there are six stages for fluid flow in fractures and vugs medium of carbonate reservoir. The first stage is the influence of wellbore storage and skin effect, which is affected by dimensionless wellbore storage coefficient, skin factor and radius of cavity. And the dimensionless pressure and pressure derivative curves show a straight line with a slope of 1. At early time, a double logarithmic plot of pressure drop vs time data yield parallel straight lines. As time goes on, the straight lines obviously change into curves, so there will be the first curvature. The second stage is the flow of fluid in the cave to the wellbore. And in this stage, the formation pressure will drop as the fluid continues to be produced. At the end of the second stage, the pressure wave is transmitted to the boundary of the cave. The first concave appears on the pressure derivative curves, and the depth of the concave is related to the storage capacity ratio of the cave. The third stage is the fracture flow stage, and the dimensionless wellbore pressure curves and the dimensionless wellbore pressure derivative curves will appear in the straight line of the slope of 1/4 or 1/2. It shows that the pressure wave arrives in the surrounding dual-porosity media formation. And in the log-log plot, pressure and pressure derivatives curves will become more and more approaching. If the difference of the storage ratio between the fractures and the caves is not big, the phenomenon of the approach will be more obvious. The fourth stage is the “cavity flow”, and fluid of in large scale cave moves into the radial flow of the formation. Therefore, the dimensionless wellbore pressure derivative curves fall off. The fifth stage is the stage that the fluid of in the matrix flow to fracture, and the second
The concave will performance in the dimensionless pressure derivative curves. The depth, width, position of the concave is related to the fracture storage ratio and cross flow coefficient. The physical meaning represents the channeling process of the fluid in the dissolution pores into the large scale fractures. The sixth stage is the boundary affection stage, because the mathematical model which is established is based on the sealed boundary, for this reason, at the later stage of the dimensionless pressure curves and the dimensionless pressure derivative curves will appear rise phenomenon.

By changing the parameter values in the model, the wellbore pressure data corresponding to different parameter values can be calculated. The dimensionless pressure derivative of the wellbore is solved, and the log-log curves of wellbore pressure are plotted. Therefore, the typical curves of well test can be in-depth studied.

4.2. Sensitiveness Studies

Sensitivity analysis of well test typical curves for drilling large scale cave is carried out, and the influence of parameters on typical curves of well test such as skin factor $S$, radius of cave $R_D$, storage ratio of cave $\omega_v$, cross flow coefficient $\lambda$ and reservoir radius $r_e$, wellbore storage $C_D e^{2S}$ and storage ratio of fractures $\omega$ are illustrated. According to the basic parameters used in Figure 2, a reasonable change of different values will lead to different log-log curves.

(1) The influence of skin factor on well test typical curves

When other parameters ($r_e = 1000, \omega_v = 0.1, \omega = 0.01, C_D e^{2S} = 100, R_D = 30, \lambda = 1 \times 10^{-7}$) remain the same, by changing the value of the skin factor $S$, the wellbore pressure corresponding to different values is calculated, and log-log curves can be plotted. Effect of skin factor on the well test typical curves is shown in Figure 3.

![Figure 3. Effect of skin factor on the well test typical curves.](image)

As can be seen from Figure 3, the skin factor mainly affects the middle section of the well test typical curves, the size of the skin factor $S$ affects the hump of the typical curves of the well test: with the increase of skin factor ($S = 1.0, 4.0, 8.0$), the hump position is higher in the wellbore pressure derivative curves, and the shape of the hump becomes steeper and steeper. The corresponding pressure curves will be at the position higher. This is because the greater skin factor, the greater the resistance of fluid flow around the wellbore, and the higher the pressure required for fluid flow.

(2) The influence of cave radius on well test typical curves

When other parameters ($r_e = 1000, \omega_v = 0.1, \omega = 0.01, C_D e^{2S} = 100, \lambda = 1 \times 10^{-7}, S = 1.0$) remain the same, the wellbore pressure corresponding to different values is calculated by changing the value
of dimensionless radius of the cave. Log-log curves can be plotted. Effect of radius of cave on the well test typical curves is shown in Figure 4.

![Figure 4](image-url)

**Figure 4.** Effect of radius of cave on the well test typical curves.

Figure 4 shows that the dimensionless radius of the cave mainly affects the front section of the well test typical curves: with the increase of radius of cave \( R_D = 20, 30, 40 \), the wellbore storage stage is longer, and the slope of the earlier curves become greater. It shows that the larger the radius of the cave, the more obvious the increase of the wellbore storage coefficient, the more obvious the affection of the cave. The position of pressure and pressure derivatives curves in the early stage will be on the right. The first concave will become wider and shallower. This indicates that the capacity of storage will be greater with the decease of dimensionless radius of cave.

(3) The influence of storage ratio on well test typical curves

When other parameters \( r_{cD} = 1000, \omega = 0.01, C_D e^{25} = 100, R_D = 30, \lambda = 1 \times 10^{-7}, S = 1.0 \) remain the same, by changing the value of storage ratio of the cave \( \omega_v \), the wellbore pressure corresponding to different values is calculated, and log-log curves can be plotted. Effect of storage radio of cave on the well test typical curves is shown in Figure 5.

![Figure 5](image-url)

**Figure 5.** Effect of the storage radio of cave on the well test typical curves.

As can be seen from Figure 5, the dimensionless cavity storage capacity ratio \( \omega_v \) mainly affects the first concave of the dimensionless pressure derivative: the storage ratio determines the depth and width of the concave in the pressure derivative curves. With the increase of storage ratio \( \omega_v = 0.1, 0.2, 0.4 \), transition section is shorter, and this shows that the storage capacity of caves is greater. When the
well drills into the large scale cave, it will make the storage stage of the wellbore become longer in the
early stage. The concave depth is shallower, and the width of the dimensionless pressure derivative
curves is narrower.

(4) The influence of cross flow coefficient on well test typical curves

Let other parameters \( r_{eD} = 1000, \omega_v = 0.1, \omega = 0.01, C_{De2S} = 100, R_D = 30, S = 1.0 \) be unchanged. By changing the value of cross flow coefficient \( \lambda \), the wellbore pressure corresponding to different values is calculated, and log-log curves also can be plotted meanwhile. Effect of cross flow coefficient on the well test typical curves is shown in Figure 6.

\[
\begin{align*}
\text{Figure 6. Effect of the cross flow coefficient on the well test typical curves.}
\end{align*}
\]

Figure 6 shows that the cross flow coefficient mainly affects the time of the concave occurrence in the pressure derivative curves, namely the position of the concave: with the increase of cross flow coefficient \( (\lambda = 1 \times 10^{-7}, 2 \times 10^{-7}, 3 \times 10^{-7}) \), it indicates that the cross flow function is greater, and the cross flow occurs earlier. The position of concave in the dimensionless pressure derivative curves is on the left.

(5) The influence of dimensionless storage ratio of fractures on well test typical curves

The effect of dimensionless storage ratio of fractures on the well test typical curves is plotted in log-log plot in Figure 7. And there are basic parameters: \( r_{eD} = 1000, \omega_v = 0.1, C_{De2S} = 100, R_D = 30, \lambda = 1 \times 10^{-7}, S = 1.0 \).

\[
\begin{align*}
\text{Figure 7. Effect of dimensionless storage ratio of fractures on the well test typical curves.}
\end{align*}
\]
It can be seen from Figure 7 that the dimensionless storage ratio of fractures mainly influences the middle and late stages of the pressure and pressure derivative curves. The storage ratio of fractures affects the width and depth of the concave of the dimensionless pressure derivative curve, and the smaller the storage ratio of the fractures, the wider and deeper the concavity is. Therefore, from the Figure 7, with the increase of dimensionless storage ratio of fractures ($\omega = 0.005, 0.01, 0.015$), there will be a downward trend in the middle of the pressure curve. This shows that the greater the storage ratio of fractures, the easier the fluid flows and the smaller the pressure consumption is. On the pressure derivative curves, $\omega$ represents the capacity of storage in the surrounding double porosity formation, and it influences the late stage of pressure derivative. The greater the storage ratio, the deeper the depth of the second concave is, and the narrower the width will be.

(6) The influence of dimensionless wellbore storage on well test typical curves

Other parameters ($\omega_v = 0.1, \omega = 0.01, r_{eD} = 1000, R_D = 30, \lambda = 1 \times 10^{-7}, S = 0.0$) remain the same. If the value of dimensionless wellbore storage is changed, the log-log curves can be plotted about the wellbore storage. Effect of dimensionless wellbore storage on the well test typical curves is shown in Figure 8.

![Figure 8](image-url)

**Figure 8.** Effect of dimensionless wellbore storage on the well test typical curves.

In Figure 8, there are three different values: $C_D = 10, C_D = 100, C_D = 1000$. By comparing different well test curves, the influence of wellbore storage on well test curves is found. It can be seen from the log-log plot that the bigger the wellbore storage coefficient is, and the more serious the well storage influence is. So the longer the duration of the well storage stage is of the pressure derivative curve in the well storage period. Combining Figures 4 and 8, it can also be found that wellbore storage and radius of caves have the same effect on the log-log curves. In this paper, the well test model for fractured and vuggy carbonate reservoir is established when well drilling in large scale cave. So if well drills in cave, we can think that, to some extent, it is equivalent to the increase of wellbore storage effect. As the scale of cave when well drilling increases, the effect of wellbore storage is more obvious, and the longer the wellbore storage stage is.

(7) The influence of dimensionless reservoir radius on well test typical curves

When other parameters ($\omega_v = 0.1, \omega = 0.01, C_{Dpe}^{2S} = 100, R_D = 30, \lambda = 1 \times 10^{-7}, S = 1.0$) remain the same, by changing the value of dimensionless wellbore radius $r_{eD}$, the wellbore pressure corresponding to different values is calculated, and log-log curves can be plotted. Effect of dimensionless wellbore radius $r_{eD}$ on the well test typical curves is shown in Figure 9.
The test layer is Ordovician, and the lithology analysis result is limestone. There are caves, large scale vuggy carbonate reservoirs.

Effect of dimensionless reservoir radius on the well test typical curves is shown in Figure 9. Figure 9 shows the log-log plot of the wellbore pressure change and its derivative in the well once reservoir radius is changed. As is shown that the dimensionless reservoir radius mainly affects the middle and late stages of the log-log pressure curves: in this paper, the boundary of reservoir is a sealed boundary, and the later well test curves will be upturned. With the increase of the dimensionless reservoir radius ($r_{eD} = 600, 800, 1000$), the dimensionless pressure and pressure derivative curves are lower. The main reason is that the larger the dimensionless reservoir radius, the greater the energy of the later formation system is, and the later time the boundary influences. With the decrease of reservoir radius, the fluid cross flow between the formation medium decreases, which makes the pressure and pressure derivative curves tend to increase in the medium term, so the two curves will become more and more close in the middle.

5. Field Example

A well analysis of the field example is located in a fractured and vuggy carbonate reservoir. The test layer is Ordovician, and the lithology analysis result is limestone. There are caves, large scale fractures and corrosion pores in the core, and the characteristics of fractured cavity reservoir are obvious. Total drilling depth is 5890 m. Wellbore radius is 0.13 m. Porosity is 0.55. Thickness is 15.8 m. Oil viscosity is 1.152 mPa·s. Oil formation volume factor is 1.31 m$^3$/m$^3$. Total compressibility is 0.00301 MPa$^{-1}$. According to the actual data, it can be seen that the drilling pipe vent and a large number of drilling fluid leakage occurred during the drilling, indicating that the drilling in a large scale cave.

The well is produced at 125.4 m$^3$/day for 35 day and then shut in 672 h for a buildup test. Leakage is 965 m$^3$. By the actual well test data and the well test model considering the wellbore storage and skin factor of fractured and vuggy carbonate reservoir when well drilling large scale cave, it can make the well test analysis. The well test typical curves matching with well test actual curves of a certain well are shown in Figure 10. Figure 10 shows the actual data and model responses in log-log plot, the model solution being based on the mathematical model, which is established and its solution of wellbore pressure. In this filed example, the pressure test is carried out to get the parameters of well test analysis. In this case, we use our method to study the pressure response of a well for fractured and vuggy carbonate reservoirs.

Well test analysis results of established model in this paper: $C = 0.192$ m$^3$/MPa, $S = 3.6$, $\omega = 5 \times 10^{-5}$, $\omega_v = 0.01$, $R_D = 30$, $\lambda = 1 \times 10^{-6}$. It is important to determine whether the pressure and pressure derivative curves are completely mathed. And it is related to the rationality of well test analysis parameters. According to the matching results in Figure 10, the well test analysis results coincide with the geological static and production dynamic data, accord with the field practice, and the well test analysis result is reasonable. We believe that the model and the analytical solution presented...
here provide valuable information on fractured and vuggy reservoir that could not be obtained from core or log data alone.

Figure 10. Log-log plot showing the matching result of the practical curves and theoretical curves. Blue Line is the practical pressure curve, which is measured by high precision pressure gauge. Grey line is the theoretical pressure curve, which is calculated by the model established in this paper. Orange line is the derivative curve of practical pressure. Yellow line is the derivative curve of theoretical curve. During the well test interpretation, the practical pressure and pressure derivative curves should be matched with the theoretical pressure and pressure derivative curves. In other words, make the gray and blue lines, yellow and orange lines match separately.

6. Models Comparison

The well test analysis model established in the paper shows that the log-log curves matching results are good. The well test analysis results are in line with the practical situation of field site. There is no corresponding model for Saphir for fractured and vuggy carbonate reservoirs of well drilling in large scale cave. However, based on the reliability of commercial software for the interpretation of parameters, the practical well test data are used. The interpretation results of the two models are compared.

The well test interpretation of the same well is carried out by using commercial software, as shown in Figure 11.

Figure 11. Comparison of commercial software to our model for sealed boundary. Blue Line is the practical pressure curve, which is measured by high precision pressure gauge. Navy blue line is the theoretical pressure curve, which is calculated by the Saphir. Orange line is the derivative curve of practical pressure. Yellow line is the derivative curve of Saphir. During the well test interpretation, the blue and navy blue lines, yellow and orange lines must match separately.
Several models can be chosen in the Saphir software, the well test interpretation will be best if the two porosity model is chosen and the boundary is parallel faults. Figure 11 shows that matching results of practical curves and theoretical curves are not good. The well test interpretation suggests that the parameters do not conform to the actual well test data in the Saphir, there are several models can be chosen, but the well test interpretation will be best results if the two porosity model. And the boundary is parallel faults. As can be seen from Figure 11, matching results of practical curves and theoretical curves are not good. It can be analyzed from the well test interpretation that the parameters does not conform to the well test data of the actual.

Results of well test analysis of commercial software are: \( C = 0.203 \, \text{m}^3/\text{MPa}, \, S = 2.98, \omega = 8.89 \times 10^{-6}, \lambda = 2.3 \times 10^{-7}. \) The radius of the cave is not given, but through the analysis of geological data, a lot of drilling mud is lost in the drilling process, and production rate is higher, but the fractures system is not developed, so there is a large scale cave.

Based on the results and comparison of commercial software to our model in this paper, established model represents a highly effective and cost-efficient solution to properly tackle fractured and vuggy carbonate reservoirs from an industrial viewpoint; the methodology can then be considered mature for full-field reservoirs studies. This will benefit from a full integration of model inside everyday geological/engineering tools/software by means of proper proprietary plug-ins and/or packages. Eventually, all these ingredients should become an important foundation to feed a next generation high performance well test analysis.

7. Conclusions

In this paper, a well test analysis model for fractured and vuggy carbonate reservoirs when well drilling large scale caves considering wellbore storage and skin factor is established. The model is formulated using Laplace transform and Stehfest numerical inversion functions, and the actual well test analysis is made by the model. The results show that the model of well test analysis can be used to explain this kind of reservoir. Based on the results presented in this study, the following conclusions are warranted.

For the fractured and vuggy carbonate reservoir, the duration of the early stage of the log-log curves will become longer when well drilling in large scale cave. Through the matching of log-log curves with the well test model and practical test data, the wellbore storage coefficient will be greater. When drilling a large scale cave, the effective well radius is increased to a certain extent. This is equivalent to increase the actual volume of the wellbore. So, in theory, \( C (= V \cdot C_t) \) will be bigger. That is, the volume of the wellbore increases which is in full agreement with the conclusion obtained in this paper. Through the sensitivity analysis of different parameters for the well test typical curves, it is found that the effect of the well test curves is in accordance with the theoretical analysis.

1. The skin factor shows the magnitude of the fluid flow resistance, and the larger the skin coefficient, the steeper the hump.
2. The size of the cave for well drilling and the size of the wellbore storage have the same effect on the typical curves of the well test. The larger the radius of the cave is, the bigger the well storage is, and the longer the wellbore storage stage is.
3. The storage ratio influences the depth and width of the concave in the pressure derivative curves. The smaller the storage ratio is, the wider and deeper the concave is.
4. The cross flow coefficient mainly affects the position of the concave occurrence in the pressure derivative curves. The smaller of cross flow coefficient is, and it indicates that the cross flow function is smaller. The position of concave in the dimensionless pressure derivative curves is on the right.
5. The dimensionless reservoir radius mainly affects the middle and late stages of pressure curves in the log-log plot. For the sealed boundary reservoirs, the later well test curves will be upturned.
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