1 Introduction

The HERAPDF1.0 parton distribution functions (PDFs) were extracted using the combined inclusive cross section data from the H1 and ZEUS collaborations taken at the HERA collider during the HERA-I running period 1992-1997 [1]. These data come from neutral and charged current interactions from both $e^+p$ and $e^-p$ scattering. The combination of the H1 and ZEUS data sets takes into account the full correlated systematic uncertainties of the individual experiments such that the total uncertainty of the combined measurement is typically smaller than 2%, for $3 < Q^2 < 500$ GeV$^2$, and reaches 1%, for $20 < Q^2 < 100$ GeV$^2$. These PDFs have been updated to HERAPDF1.5 (see LHAPDFv6.8.6, http://projects.hepforge.org/lhapdf) by including a new preliminary combination of the HERA inclusive cross section data from the HERA-II running period 2003-2007 [2]. These data have greater accuracy at high $Q^2$ and high $x$ than the HERA-I data and thus they serve to decrease the PDF uncertainty in these kinematic regions.
In this presentation we discuss the extension of the HERAPDF1.5 analysis to include inclusive jet data. The new PDF set which results is called HERAPDF1.6 \cite{3}. The jet data included in HERAPDF1.6 are: two sets of high-$Q^2$ inclusive jet production data from ZEUS \cite{4}, low-$Q^2$ inclusive jet data from H1 \cite{5} and normalised high-$Q^2$ inclusive jet data from H1 \cite{6}.

The gluon PDF contributes only indirectly to the inclusive DIS cross sections. However, the QCD processes that give rise to scaling violations in the inclusive cross sections, namely the QCD-Compton (QCDC) and boson-gluon-fusion (BGF) processes, are observed as events with distinct jets in the final state provided that the energy and momentum transfer are large enough. The cross section for QCDC scattering depends on $\alpha_s(M_Z)$ and the quark PDFs. For HERA kinematics, this process dominates the jet cross section at large scales, where the quark densities are well known from the inclusive cross-section data, so that the value of $\alpha_s(M_Z)$ may be extracted without strong correlation to the shape of the gluon PDF. The cross section for the BGF process depends on $\alpha_s(M_Z)$ and the gluon PDF so that measurements of jet cross sections also provide a direct determination of the gluon density.

2 Analysis

Perturbative QCD predicts the $Q^2$ evolution of the parton distributions, but not the $x$ dependence. Parton distributions are extracted by performing a direct numerical integration of the DGLAP equations at NLO (NNLO fits are discussed in the presentation of Radescu). A parametrised analytic shape for the parton distributions is assumed to be valid at some starting value of $Q^2 = Q^2_0$. For the HERAPDF the value $Q^2_0 = 1.9$ GeV$^2$ is chosen such that the starting scale is below the charm mass threshold, $Q^2_0 < m_c^2$. Then the DGLAP equations are used to evolve the parton distributions up to a different $Q^2$ value, where they are convoluted with NLO coefficient functions to make predictions for the structure functions. The heavy quark coefficient functions are calculated in the general-mass variable-flavour-number scheme of \cite{7}, with recent modifications \cite{8}. The heavy quark masses for the central fit were chosen to be $m_c = 1.4$ GeV and $m_b = 4.75$ GeV and the strong coupling constant was fixed to $\alpha_s(M_Z) = 0.1176$. The predictions are then fitted to the combined HERA data sets for NC and CC $e^+p$ and $e^-p$ scattering. A minimum $Q^2$ cut of $Q^2_{min} = 3.5$ GeV$^2$ was imposed to remain in the kinematic region where perturbative QCD should be applicable.

For the jet data the NLO cross-sections are made using NLOjet++ \cite{9} but such
calculations are too slow to be used in an iterative QCD fit hence the programme FASTNLO \cite{10} is used to make these calculations by convoluting pre-calculated weights with the PDFs and $\alpha_S$.

The fit parameters are those necessary to specify the input analytic shape. Just as for HERAPDF1.0 and 1.5 the valence quark distributions $xu_v, xd_v$, and the $u$-type and $d$-type anti-quark distributions $x\bar{u}, x\bar{d}$ ($x\bar{u} = xu, x\bar{d} = x\bar{d} + x\bar{s}$) are parametrised at the input scale $Q_0^2 = 1.9$GeV$^2$ by the generic form

$$xf(x) = Ax^B(1 - x)^C(1 + Dx + Ex^2).$$ \hfill (1)

The parametrisation for the gluon distribution $xg$ is extended, in comparison to the HERAPDF1.5 NLO parametrisation, to include a term $-A_g'x^{B_g'}(1 - x)^{C_g'}$, such that the NLO gluon may become negative at low $x$ and low $Q^2$ (however it does not do so in the kinematic region of the HERA data). A further significant extension of the parametrisation is that the low-$x$ valence parameters, $B_u$ and $B_d$, are no longer set equal. Otherwise the choice of central parametrisation is as for the published HERA-PDF1.0(1.5) fits. Briefly, the normalisation parameters, $A_g, A_u, A_d$, are constrained by the quark number sum-rules and momentum sum-rule. The $B$ parameters $B_{\bar{u}}$ and $B_{\bar{d}}$ are set equal, $B_{\bar{u}} = B_{\bar{d}}$, such that there is a single $B$ parameter for the sea distributions. The strange quark distribution is expressed as $x$-independent fraction, $f_s$, of the $d$-type sea, $x\bar{s} = f_s x\bar{D}$ at $Q_0^2$. The value $f_s = 0.5$ would render the $s$ and $d$ quark densities the same, but the value $f_s = 0.31$ is chosen to be consistent with determinations of this fraction using neutrino-induced di-muon production. The further constraint $A_{\bar{u}} = A_{\bar{d}}(1 - f_s)$, together with the requirement $B_{\bar{u}} = B_{\bar{d}}$, ensures that $x\bar{u} \to x\bar{d}$ as $x \to 0$. For the central fit only the $xu_v$ PDF has non-zero $E$ and $D$ parameters.

The experimental uncertainties on the HERAPDF are determined using the conventional $\chi^2$ tolerance, $\Delta\chi^2 = 1$, for $68\%$C.L. However model uncertainties and parametrisation uncertainties are also considered. The choice of the heavy quark masses is varied in the ranges, $1.35 < m_c < 1.65$ GeV and $4.3 < m_b < 5.0$ GeV. The choice of $Q^2_{\text{min}}$ is varied in the range, $2.5 < Q^2_{\text{min}} < 5.0$, and the choice of the strangeness fraction is varied in the range, $0.23 < f_s < 0.38$. The difference between the central fit and the fits corresponding to model variations of $m_c, m_b, f_s, Q^2_{\text{min}}$ are added in quadrature, separately for positive and negative deviations, to represent the model uncertainty of the HERAPDF1.6 set.

Parametrisation variations for which the $E$ and the $D$ parameters for all the PDFs are freed one at a time are considered and variation of the starting scale $Q_0^2$ in the range, $1.5 <$
$Q_0^2 < 2.5\text{GeV}^2$, is also considered as parametrisation variation. The difference between these parametrisation variations and the central fit is stored and an envelope representing the maximal deviation at each $x$ value is constructed to represent the parametrisation uncertainty.

3 Results

Fig. 1 compares the HERAPDF1.6 NLO fit including the jet data (right) to a the HERAPDF1.5f NLO fit which does not include these data (left). Note that the HERAPDF1.5f fit differs from the publically available HERAPDF1.5 NLO fits in that it has the same flexible parametrisation for the central fit as HERAPDF1.6: there are extra parameters for the gluon PDF and the low-x behaviour of $u$-valence and $d$-valence are not required to be the same. The blue-line on the left-hand figure shows the HERAPDF1.5 central fit.

Figure 1: The parton distribution functions $xu_v, xd_v, xS = 2x(\bar{U} + \bar{D}), xg$, at $Q^2 = 10 \text{ GeV}^2$, from HERAPDF1.5f (left) and HERAPDF1.6 (right). Fractional uncertainty bands are shown below each PDF. The experimental, model and parametrisation uncertainties are shown separately.

Comparing this to the HERAPDF1.5f PDF shows that the extra flexibility has not changed the PDFs significantly. The blue line on the right-hand figure shows the central HERAPDF1.5f fit. Comparing this to the HERAPDF1.6 PDF shows that the addition of jets has not moved the PDFs outside their error bands, however the high-x Sea has
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become a little softer. Comparing the fractional uncertainty bands of the left and the right-hand plots shows that the addition of jets produces a marginal decrease in the high-$x$ gluon uncertainty. Fig. 2 compares the HERAPDF1.5f and HERAPDF1.6 fits in the form of summary plots of $xu,v,xd,v,xS = 2x(U + \bar{D}), xg$, at $Q^2 = 10$ GeV$^2$ from HERAPDF1.5f and HERAPDF1.6. The experimental, model and parametrisation uncertainties are shown separately. The gluon and sea distributions are scaled down by a factor 20.

The HERAPDF1.6 fit including jet data has a total $\chi^2$ of 811.5 for 780 data points (766 degrees of freedom). For the inclusive data the $\chi^2$ is 730.2 for 674 data points and for the jet data the $\chi^2$ is 81.3 for 106 data points. For the HERAPDF1.5f fit without the inclusion of jet data the $\chi^2$ is 729 for 674 data points (660 degrees of freedom). Thus there is no tension between the jet data and the inclusive data.

In most PDF analyses the strong coupling constant, $\alpha_S(M_Z)$, is fixed. However it can also be a parameter of the fit in addition to the PDF parameters. The $\chi^2$ for the HERAPDF1.6 fit with free $\alpha_S(M_Z)$ is 807.6 for 765 degrees of freedom. The sub-$\chi^2$ for the inclusive data has barely changed but the sub-$\chi^2$ for the jet data decreases to 77.6 for 106 data points. Fig. 3 shows the summary plots for HERAPDF1.5f and HERAPDF1.6, each with $\alpha_S(M_Z)$ left free in the fit. It can be seen that without jet data the uncertainty on the gluon PDF at low $x$ is large. This is because there is a strong correlation between the low-$x$ shape of the gluon PDF and $\alpha_S(M_Z)$. However once jet data are included the
extra information on gluon induced processes reduces this correlation and the resulting uncertainty on the gluon PDF is not much larger than it is for fits with $\alpha_S(M_Z)$ fixed.

![Graph showing parton distribution functions](image)

**Figure 3:** The parton distribution functions $xu_v, xd_v, xS = 2x(\bar{U} + \bar{D}), xg$, at $Q^2 = 10$ GeV$^2$, from HERAPDF1.5f and HERAPDF1.6, both with $\alpha_S(M_Z)$ treated as a free parameter of the fit. The experimental, model and parametrisation uncertainties are shown separately. The gluon and sea distributions are scaled down by a factor 20.

The value of $\alpha_s(M_Z)$ extracted from the HERAPDF1.6 fit is:

$$\alpha_s(M_Z) = 0.1202 \pm 0.0013(exp) \pm 0.0007(model/param) \pm 0.0012(had) + 0.0045/ -0.0036(scale)$$

We estimate the model and parametrisation uncertainties for $\alpha_S(M_Z)$ in the same way as for the PDFs and we also add the uncertainties in the hadronisation corrections applied to the jets. The scale uncertainties are estimated by varying the renormalisation and factorisation scales chosen in the jet publications by a factor of two up and down. The dominant contribution to the uncertainty comes from the jet renormalisation scale variation. Fig. 3 shows a $\chi^2$ scan vs $\alpha_S(M_Z)$ for the fits with and without jets, illustrating how much better $\alpha_S(M_Z)$ is determined when jet data are included. The model and parametrisation errors are also much better controlled.

Finally in Fig. 5 we show plots of the gluon-gluon (right) and quark-antiquark (left) luminosities, as a function of the fraction of centre of mass energy taken by the subprocess, for the LHC at 7 TeV, for all the NLO HERAPDF sets in ratio to the corresponding luminosities extracted for the MSTW2008 PDFs. In general the HERAPDF1.5 and 1.6 PDF
sets are in better agreement with the MSTW2008 luminosities than the HERAPDF1.0 set. In particular the use of a larger value of $\alpha_s(M_Z) = 0.1202$, for the HERAPDF1.6 free $\alpha_s(M_Z)$ PDF, brings the gluon-gluon luminosity into better agreement at low $x$ as well as at high $x$.

Figure 5: The quark-anti-quark (left) and gluon-gluon (right) luminosities for the HERAPDFs in ratio to those for the MSTW2008 PDFs, as a function of the fraction of centre of mass energy taken by the sub-process, for the LHC at 7 TeV
4 Summary

HERA jet data have been used in addition to HERA inclusive data in order to determine the parton distributions in the proton in an NLO QCD fit. There is no tension between the jet data and the inclusive data. The PDF set including the jet data, HERAPDF1.6, is similar to the publically available HERAPDF1.5 set both in PDF central values and in uncertainties. However, the advantage of using jet data is clearly seen when the strong coupling constant $\alpha_S(M_Z)$ is allowed to be a free parameter of the fits. The uncertainty on the low-$x$ gluon PDF due to the correlation with $\alpha_S(M_Z)$ is much reduced when jet data are used and an accurate value of $\alpha_S(M_Z)$ can be obtained: $\alpha_S(M_Z) = 0.1202 \pm 0.0019$, excluding scale error.

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