Heralded non-destructive quantum entangling gate with single-photon sources

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(Dated: October 29, 2020)

Heralded entangling quantum gates are an essential element for the implementation of large-scale quantum computation. Yet, the experimental demonstration of genuine heralded entangling gates with free-flying output photons in an all-optical system was hindered by the intrinsically probabilistic source and double-pair emission in parametric down-conversion. Here, by using an on-demand single-photon source based on semiconductor quantum dots and pseudo photon-number-resolving detectors, we demonstrate a heralded controlled-NOT (CNOT) operation between two single photons for the first time. To characterize the performance of the CNOT gate, we estimate its average quantum gate fidelity of $(87.8 \pm 1.2)\%$. As an application, we generated event-ready Bell states with a fidelity of $(83.0 \pm 2.4)\%$. Our results are an important step towards the development of photon-photon quantum logic gates.

Entangling gates are a crucial building block in scalable quantum computation, as it enables the construction of any quantum computing circuits when combined with single-qubit gates [1]. One of the canonical examples is the controlled-NOT (CNOT) gate, which flips the target qubit state conditional on the control qubit. Photons are generally accepted as the best candidate for a qubit due to their negligible decoherence and ease of single-qubit operation. Unfortunately, ambitions to implement optical CNOT gates are hampered as they require strong interactions between individual photons well beyond those presently available. Surprisingly, projective measurements with photodetector can induce an effective nonlinearity sufficient for the realization of entangling gates using linear optics [2,3]. Since then, many schemes to implement optical CNOT gates have been theoretically proposed [4, 6] and experimentally demonstrated [7, 18].

Early demonstrations came at the expense of destroying the output states [7, 12], thus limiting the scaling to larger systems. To be scalable, heralded CNOT gates are necessary. Specifically, a successful operation is heralded by the detection of ancillary photons. Such heralded gates are highly important as they provide information classically feed forwardable, which is crucial for a scalable architecture both in the standard circuit model [11, 22] and one-way model using cluster states [19, 21]. Implementations of heralded CNOT gates assisted with entangled [13, 17] or single ancilla photons [18] have been reported. However, all the demonstrations employed spontaneous parametric down-conversion (SPDC) [22] as photon sources. Due to the probabilistic nature of SPDC that involves multiphoton emissions, one cannot obtain a photon pair or a heralded single photon deterministically with high generation rate [23], and daunting scalability issues raise if we wish to use these photons. Also, as multiple pair events are always present and detectors without photon number resolution are commonly used, which introduce false heralding signals, post-selection is necessary in the experiments to confirm a successful gate operation [24]. For this reason, while the schemes in principle work in a heralded way, the previous experiments are actually a destructive version of the heralded CNOT gates, limiting their further applications. To overcome these issues, on-demand photon sources will be the fundamental assets.

Semiconductor quantum dots (QDs) confined in a micro-cavity are particularly appealing emitters of on-demand photons, which can deterministically emit single photons [25, 28] as well as entangled photon pairs [29, 30]. They have been shown to generate single photons simultaneously exhibiting high brightness, near-unity single-photon purity and indistinguishability [28, 31], and currently have the best all-around single-photon source-performance [32, 33]. Until now, QD single-photon sources (SPSs) have already been used to realize optical CNOT gates in a semiconductor waveguide chip [34] and bulk optics [35]. Nevertheless, none of them are heralded since they necessarily destroy the output states.

In this Letter, we demonstrate the first heralded CNOT operation between two single photons using a QD coupled in a micro-pillar cavity. Under pulse resonant excitation, the QD SPS generates high-quality single photons that are deterministically demultiplexed into four indistinguishable SPSs for implementing CNOT gates. The gate performance can be further improved using pseudo photon-number-resolving detectors (PNRDs) constructed by superconducting nanowire single-photon detectors (SNSPDs). Conditional on the coincidence detection of two single an-
In Fig. S1, two Bell states are measurement of Bell states on ancilla photons together to the coincidence between $D_1$ and $D_2$, then 1-bit trigger information will be sent to do the related Pauli operations on the outputs, and we will get the desired heralded CNOT gate. The total success probability is $1/8$ (for details, please see Ref. [18]), which can be improved to the optimal value of $1/4$ by harnessing complete Bell-state analyzer assisted with more ancilla photons and hybrid degree of freedoms [38].

Note that when the total number of photons in path 2 and path 3 is not two, unwanted coincidences might happen between $D_1$ and $D_2$, which will degenerate the performance of CNOT gates. Fortunately, these cases can be excluded by using PNRDs [39, 40]. Also, thanks to the PBS3 in $|R⟩/|L⟩$ basis, cases that there are two photons altogether in either path 2 or path 3 and no photon in path 1 or path 4, are not possible. Because the two photons – either one in $H$ and the other in $V$ or one in $+⟩$ and the other in $-⟩$ – will go to the same output when they pass through PBS3 due to photon bunching effect [41].

In our experiment described in Fig. S2, we use the state-of-the-art self-assembled InAs/GaAs QDs embedded inside a micro-pillar cavity [26] to create single photons of near-perfect purity, indistinguishability and high brightness. To reach the best QD-cavity coupling with optical resonance $∼893$nm, the whole sample wafer was mounted in an ultra-stable cryogenic-free bath cryostat and cooled down to 4K. Under pulse resonant excitation with a repetition rate $∼76$MHz, $∼16$MHz polarized resonance fluorescence single photons are maximally registered by a SNSPD with a detector efficiency $∼80%$. The measured second-order correlation function at zero-time delay is $0.63(1)$, yielding single-photon purity of $∼97%$. The photon indistinguishability is measured by a Hong-Ou-Mandel interferometer, yielding a visibility of $0.91(1)$ between two photons separated by $∼6.5$µs [12]. Such a semiconductor source of polarized, high-brightness, high-purity and near-transform-limited single photons is the key resource for high-quality entangling gates.

The single-photon stream is then deterministically demultiplexed into four spatial modes using three fast optical switches, where each switch consists of a PBS and a Pockels cell (PC). These PCs, synchronized to the laser pulses and operated at a repetition rate $∼0.76$MHz, actively control the photon polarization. Thanks to the high transmission efficiency (>99%) and high single-mode fiber coupling efficiency ($∼85%$), we can reach the average optical switches efficiency $∼83%$. By using single-mode fibers of different lengths and mounting each fiber output on a translation stage, we can precisely compensate time delays of the four single photons that are fed into the CNOT gate.

Our heralded CNOT operation depends on the ancilla measurement outcome and their related Pauli operators.

![Figure S1. An optical heralded CNOT gate [18]. Polarizing beam splitters (PBSs) are used, where PBS1 (PBS2, PBS3) transmit $|H⟩ (|+, |R⟩)$ states and reflect $|V⟩ (|−⟩, |L⟩)$ ones. The heralded CNOT operation depends on the ancilla measurement outcome and related Pauli operators.](image-url)
operators. As described in Fig S1, unwanted coincidences between heralded detectors can be excluded by PNRDs which are composed of SNSPDs in our experiment. We perform jointly projective measurement of Bell state $\ket{\Phi^+}$ on auxiliary photons, which means that the output state is exactly the outcome of a CNOT gate. $\ket{\Phi^+}$ corresponds to the coincidence between $D_{H1}$ and $D_{H2}$ or between $D_{V1}$ and $D_{V2}$ (without show). Thus, we can conclude that a successful heralded CNOT gate has been demonstrated if there is a coincidence between PNRDs $D_1$ ($D_{H1}$ or $D_{V1}$) and $D_2$ ($D_{H2}$ or $D_{V2}$).

To experimentally evaluate our CNOT operation, we exploit an efficient approach proposed by Hofmann [44]. We demonstrate it by preparing the control input in $\ket{+}/\ket{–}$ basis and target input in $\ket{H}/\ket{V}$ basis, and performing the measurement on the output state in $\ket{R}/\ket{L}$ basis. Our experimental result is shown in Fig S3(c), which gives fidelity $F_3 = (87.0 \pm 2.2)\%$. The average gate fidelity of $F_1$, $F_2$ and $F_3$ is $(87.8 \pm 1.2)\%$, obviously exceeding the boundary of $2/3$.

As an application of our heralded CNOT gate, we produce event-ready entangled states by preparing a separable state $\ket{–}_c\ket{V}_t$ at the inputs. Corresponding to the CNOT operation, we expect an entangled output state of $1/\sqrt{2}(\ket{HV} – \ket{VH})$, which is a maximally entangled Bell state $\ket{\Psi^-}$. To verify that the expected Bell state was implemented successfully, we measured the correlation between the polarizations of control and target photons in different basis, shown in Fig S3(d). The average fidelity of the produced state is $F_{f_{eg}} = (83.0 \pm 2.4)\%$, which clearly surpassed the 0.71 limit of Bell’s inequality [46] and ensures...
the entanglement for Bell state $|\Psi^-\rangle$. Moreover, our CNOT gate allows generating such event-ready Bell states with high count rates. Specifically, we can get $\approx 85$ CNOT operations per minute for the input state $|\rightarrow\rangle_c|V\rangle_h$, defined as output four-fold coincidence counts over the collected time for the input state), which is increased by at least an order of magnitude compared to previous experiments in Table I.

![Figure S3. Experimentally achieved CNOT gate and heralded generation of Bell states (collected in 6 minutes). Up: four-fold coincidences for all possible combinations of inputs and outputs are shown; Down: theoretical CNOT gate with 100% fidelity. (a): In the computation $|H\rangle/|V\rangle$ basis. (b): In the complementary $|+\rangle/|\rightarrow\rangle$ basis. (c): We measure the output qubits in $|R\rangle/|L\rangle$ basis when the control and target qubits are in $|+\rangle/|\rightarrow\rangle$ and $|H\rangle/|V\rangle$ basis respectively. (d): Bell state $|\Psi^+\rangle$ produced by the heralded CNOT gate for input state $|\rightarrow\rangle_c|V\rangle_h$. The coincidence counts (for the vertical axis, the fraction of the coincidences is adopted) are measured in mutually unbiased bases.](image_url)

Table I. A comparison of selected experimental optical CNOT gates. SPSs: single-photon sources; PNRDs: photon-number resolving detectors; $P_s$: theoretical success probability; $\eta_h$: heralding efficiency; /: not exit; -: unreported. For more experiments, please see supplement.

| experiments        | on-demand SPSs | (pseudo) PNRDs | Heralded | $P_s$ | $\eta_h$ | Operation/min |
|--------------------|----------------|---------------|----------|-------|----------|---------------|
| O’Brien et al. [14]| no             | no            | no       | 1/9   | /        | < 1           |
| Okamoto et al. [12]| no             | no            | no       | 1/16  | /        | < 2           |
| Gasparoni et al. [13]| no             | no            | no       | 1/4   | $<10^{-4}$| < 5           |
| Bao et al. [18]    | no             | no            | no       | 1/8   | $<10^{-2}$| < 1           |
| He et al. [35]     | yes            | no            | no       | 1/9   | /        | –             |
| Gazzano et al. [19]| yes            | no            | no       | 1/9   | /        | –             |
| This Work          | yes            | yes           | yes      | 1/8   | $\approx0.006$^1 | ~85           |

^1 measurement of output states for postselection due to the multiple pair emission of SPDC.

* for given input states $|+\rangle_c|H\rangle_h$: $\eta_h$ can be further improved to one in principle.
and pseudo PNRDs constructed by SNSPDs, we have for the first time implemented an optical heralded CNOT operation of high rates, high fidelity and heralding efficiency increased by at least an order of magnitude. We have demonstrated the CNOT gate with an average gate fidelity of $(87.8 \pm 1.2)\%$ and employed its entangling ability to reach an event-ready Bell state with a fidelity of $(83.0 \pm 2.4)\%$. Our results are promising for various QIP tasks such as complete Bell state analysis in quantum teleportation[17, 48] and heralded generation of multiphoton entanglement especially the cluster state[19, 49, 50], which is very important for large-scale quantum computation.

Interestingly, the single photons for our CNOT gates can be generated by separate sources that can be far away. That we can realize remote entanglement generation and quantum gates, which are useful for distributed quantum computing and will find new applications in the remote QIP and the future quantum internet. Furthermore, our system can be incorporated in a realistic fiber systems[51, 52] using QD SPSS at telecommunication band[53], thus one can further explore long-distance quantum communication and fibre-optic quantum network. Finally, we suggest that recent developments of integrated optics[57] could be particularly useful to fully realize the experiment demonstrated here for miniaturized and scalable photonic QIP.

This work was supported by the National Natural Science Foundation of China, the Chinese Academy of Sciences, and the National Basic Research Program of China (973 Program).

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SUPPLEMENT INFORMATION

Fidelity Calculation

The fidelity calculation for our CNOT operation is based on an efficient approach proposed by Hofmann [33, 41]. The fidelity is defined as the probability of obtaining the correct output averaged over all four possible inputs, as follows:

\[
F_1 = \frac{1}{4} \left[ P(HH/HH) + P(HV/HV) + P(VH/VV) + P(VV/VV) \right],
\]

\[
F_2 = \frac{1}{4} \left[ P(+ + / + + )+ P(− − / + + )
+ P(− − / − − )+ P(+ + / − − ) \right],
\]

\[
F_3 = \frac{1}{4} \left[ P(RL/ + H) + P(LR/ + H) + P(RR/ + V) + P(LL/ + V) + P(LL/ − H) + P(RR/ − H)
+ P(RL/ − V) + P(LL/ − V) \right].
\]

Heralding Efficiency

Our heralded CNOT operation depends on the two ancilla measurement outcome and their corresponding Pauli operators, as shown in Fig.1 in the main text. For a heralded entangling gate, the heralding efficiency is a critical parameter for scalability of quantum computation. Generally speaking, the heralding efficiency \( \eta_h \) is the probability to obtain the expected gate operation when herald photons is measured, which is the ratio between the probability of expected two-fold coincidence detection for the desired operation and the total probability of two-fold coincidence detection between heralded detectors. For simplicity, we define experimental heralding efficiency \( \eta_h \), which is given by the ratio between the probability of four-fold coincidence detection \( P_4 \) (CNOT operations) and the probability of two-fold coincidence detection \( P_2 \) (heralding signals). Therefore, the heralding efficiency \( \eta_h \) is described as

\[
\eta_h = \frac{P_4}{P_2}. \tag{2}
\]

In our demonstration, we employ pseudo photon-number-resolving detectors (PNRDs) to exclude unwanted coincidence detections between heralded detectors \( D_1 \) and \( D_2 \) for further improving the heralding efficiency. Here, we calculate the experimental heralding efficiencies for implementations using PNRDs and standard detectors (Non-PNRDs).

For simplicity, we suppose that all losses in the experiment are uniform such that one could instead use a total single photon efficiency \( \eta_s \) as the overall losses at the CNOT inputs. To evaluate the heralding efficiency for experiments using PNRDs and Non-PNRDs, we assume that efficiency of photon brightness at the fiber output \( \eta_f \), optical switch efficiency \( \eta_s \), optical line efficiency \( \eta_l \) and detector efficiency \( \eta_d \) are the same. The heralding efficiency \( \eta_h \) is now related to the total single photon efficiency \( \eta_s \), which is defined as \( \eta_h = \eta_f \eta_s \eta_l \eta_d \). Then the mixed state of the source is taken to be

\[
\rho = (1 - \eta_s) |0 \rangle \langle 0| + \eta_s |1 \rangle \langle 1|. \tag{3}
\]

at the CNOT inputs, where \(|0 \rangle \) and \(|1 \rangle \) stand for vacuum Fock states and the single photon, respectively.

Now we calculate the heralding efficiency \( \eta_h \), which is given by \( \eta_h = P_4/P_2 \). Following the scheme in Fig.1, the successful probability of having four-fold coincidence detection is \( 1/8 \) [33]. Therefore, we have \( P_4 = \frac{\eta_s^4}{8} \) for both PNRDs and Non-PNRDs cases. To evaluate the probability of two-fold coincidence detection \( P_2 \), we have to specifically study the situations for PNRDs and Non-PNRDs. Note that \( P_2 \) also depends on specific polarization states in the CNOT inputs, thereby we here only consider the situation where the
CNOT input state is fixed. For simplicity, the following analysis of \( \eta_h \) is for given input states \( |+\rangle_c |H\rangle_t \), yielding a maximally entangled Bell state.

1. heralding efficiency with PNRDs

Here we evaluate the heralding efficiency \( \eta_h \) for the experiments using PNRDs. There are three different occasions:

(1) Four single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.

As PNRDs can discriminate the presence of one or more photons, when there are two-fold coincidence detection events (detectors \( D_1 \) and \( D_2 \) click), we would expect the remaining two photons are in the outputs \( c_{out} \) and \( t_{out} \). Hence, the probability \( P_2 \) equals to the probability \( P_4 \), which is \( P_2 = P_4 = \eta_s^4 / 8 \).

(2) Only three single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.

There are four possible combinations for three single photons at the CNOT inputs (i.e. \( \frac{3!}{1!1!1!} = 4 \)). Under the condition of two-fold coincidence detection events between heralded detectors (contributed by four cases \( |HH\rangle_{23}, |HV\rangle_{23}, |VH\rangle_{23}, |VV\rangle_{23} \); subscript represents the path of the photon), the remaining one photon can be one of the four cases \( |H\rangle_1, |V\rangle_1, |H\rangle_4, |V\rangle_4 \). Therefore, for each combination at the inputs, there are 16 cases. In total, we get 64 different satisfied cases, yielding a probability of \( \frac{1}{2} \) for all possible input-output states. Thus, in the end we have \( P_2 = \eta_s^4 (1 - \eta_s) / 2 \).

(3) Only two single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.

There are six possible combinations for two single photons at the CNOT inputs (i.e. \( \frac{3!}{2!1!} = 6 \)). For each input combination, conditional on two-fold coincidence detection events between heralded detectors, we only have the four contributed cases \( |HH\rangle_{23}, |HV\rangle_{23}, |VH\rangle_{23}, |VV\rangle_{23} \). In total, we get 24 satisfied cases, yielding a probability of 1/2 for all possible input-output states. In the end, we have \( P_2 = \eta_s^4 (1 - \eta_s)^2 / 2 \).

With the above situations for PNRDs, we have the total probability \( P_2 \) of two-fold coincidence detection events

\[
P_2 = \frac{\eta_s^4}{8} + \frac{\eta_s^2 (1 - \eta_s)}{2} + \frac{\eta_s^2 (1 - \eta_s)^2}{2}
\]

In the end, we obtain the heralding efficiency \( \eta_h \)

\[
\eta_h = \frac{P_1}{P_2} = \left( \frac{\eta_s}{2 - \eta_s} \right)^2
\]

2. heralding efficiency with Non-PNRDs

Here we evaluate the heralding efficiency \( \eta_h \) for the experiments using Non-PNRDs. There are also three different occasions:

(1) Four single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.

As non-PNRDs can only discriminate zero and non-zero photons, there will be more cases contributing to two-fold coincidence detection events between heralded detectors. When the two-fold coincidence detection events require one of the four cases \( |HH\rangle_{23}, |HV\rangle_{23}, |VH\rangle_{23}, |VV\rangle_{23} \), there are three different situations:

• If the remaining two photons are in output paths \( c_{out} \) and \( t_{out} \), we will have the same result as the situation in PNDRs. Thus, we get \( P_2 = P_4 = \eta_s^4 / 8 \).

• One of the remaining two photons is in output path \( c_{out} \) or \( t_{out} \) (four possibilities) and the other one is in heralded path (two possibilities). In this situation, we will have 32 satisfied combinations, yielding a probability of 1/16 for all possible input-output states. Thus, in the end we have \( P_2 = \eta_s^2 / 16 \).

Overall, we have the probability \( P_2 \) of two-fold coincidence detection events between heralded detectors \( P_2 = \eta_s^4 / 8 + \eta_s^2 / 16 + 0 = 3\eta_s^4 / 16 \).

(2) Only three single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.

For two-fold coincidence detection events between heralded detectors, the trigger outputs have to contain one of the four terms \( |HH\rangle_{23}, |HV\rangle_{23}, |VH\rangle_{23}, |VV\rangle_{23} \). There are two situations:

• The remaining one photon is in output path \( c_{out} \) or \( t_{out} \). In this case, we have the same result as the situation in PNDRs, thus we get \( P_2 = \eta_s^4 (1 - \eta_s) / 2 \).

• The remaining one photon is in heralded paths. For two-fold coincidence detection events from \( |HH\rangle_{23} \), other cases such as \( |HHHH\rangle_{2223}, |HH\rangle_{23} \) can also contribute the coincidence detection. In the total, we have 48 satisfied combinations, yielding a probability of 1/8 for all possible input-output states. Thus, we have \( P_2 = \eta_s^4 (1 - \eta_s) / 8 \).

Overall, we get the probability \( P_2 \) of two-fold coincidence detection events \( P_2 = \eta_s^4 (1 - \eta_s) / 2 + \eta_s^2 (1 - \eta_s) / 8 = 5\eta_s^4 (1 - \eta_s) / 8 \).

(3) Only two single photons are produced at the CNOT inputs and there are two-fold coincidence detection events.
This situation is the same as that for PNDRs, we have the probability 
\[ P_2 = \eta_s^2(1 - \eta_b)^2/2 \.

With the above situations for Non-PNRDs, we have the total probability \( P_2 \) of two-fold coincidence detection events

\[ P_2 = \frac{3\eta_s^4}{16} + \frac{5\eta_s^2(1 - \eta_s)}{8} + \frac{\eta_s^2(1 - \eta_b)^2}{2} \quad (6) \]

In the end, we get the heralding efficiency \( \eta_h \)

\[ \eta_h = \frac{P_4}{P_2} = \frac{2\eta_s^2}{\eta_s^2 - 6\eta_s + 8} \quad (7) \]

3. PNDRs VS Non-PNRDs

![Figure S4. For given input states \(|+\rangle_c|H\rangle_t\), heralding efficiency \( \eta_h \) depends on the total single photon efficiency \( \eta_s \) for the experiments using PNDRs and Non-PNRDs. Here we study the heralding efficiency \( \eta_h \) derived from Eq.5 and 7 when one varies the total single photon efficiency \( \eta_s \), as shown in Fig. S4. In our current demonstration, the experimental parameters are \( \eta_s \approx 16M/0.8/76M = 0.263 \), \( \eta_{d_{\text{det}}} \approx 0.83 \), \( \eta_{d_{\text{det}}} \approx 0.80 \), \( \eta_d \approx 0.80 \), \( \eta_{d_{\text{det}}} \approx 0.80 \), we get \( \eta_h = \eta_s\eta_w\eta_l\eta_d \approx 0.14 \). Bring these numbers into Eq.5 and 7, we have experimentally measured heralding efficiency \( \eta_h = \eta_s^2/(2 - \eta_s)^2 \approx 0.006 \) and \( \eta_h = 2\eta_s^2/(\eta_s^2 - 6\eta_s + 8) \approx 0.006 \) for PNDRs and Non-PNRDs, respectively. However, by coupling QDs to an asymmetric micro-cavity [31], the output brightness of our SPS will hopefully reach to near-unity. Moreover, the switches efficiency can gradually increase to one in principle, which also help improving \( \eta_s \). Thus, \( \eta_h \) in principle can reach to one and \( \eta_h \) can then be greatly improved with PNDRs.
A comparison of selected experimental linear optical CNOT gates

Table II. A comparison of selected experimental optical CNOT gates. SPSs: single-photon sources; PNRDs: photon-number resolving detectors; $P_s$: theoretical success probability; $\eta_h$: heralding efficiency; /: not exit; –: unreported.

| experiments         | on-demand SPSs | (pseudo) PNRDs | Heralded | $P_s$ | $\eta_h$ | Operation/min |
|---------------------|----------------|----------------|----------|-------|----------|---------------|
| Pittman et al. [8]  | no             | no             | no       | 1/4   | /        | < 1          |
| Pittman et al. [9]  | no             | no             | no       | 1/4   | /        | ~ 6          |
| O’Brien et al. [17] | no             | no             | no       | 1/9   | /        | < 1          |
| Kiesel et al. [55]  | no             | no             | no       | 1/9   | /        | < 1          |
| Langford et al. [10] | no            | no             | no       | 1/9   | /        | < 1          |
| Okamoto et al. [11] | no             | no             | no       | 1/9   | /        | < 1          |
| Okamoto et al. [12] | no             | no             | no       | 1/16  | /        | < 2          |
| Gasparoni et al. [13]| no             | no             | no$^*$   | 1/4   | $< 10^{-4}$ | < 5          |
| Gao et al. [10]     | no             | no             | no$^*$   | 1/9   | $< 10^{-4}$ | < 1          |
| Zhao et al. [14]    | no             | no             | no$^*$   | 1/4   | $< 10^{-6}$ | < 1          |
| Bao et al. [18]     | no             | no             | no$^*$   | 1/8   | $< 10^{-8}$ | < 1          |
| He et al. [35]      | yes            | no             | no       | 1/9   | /        | –            |
| Gazzano et al. [45] | yes            | no             | no       | 1/9   | /        | –            |
| Pooley et al. [34]$^c$ | yes         | no             | no       | 1/9   | /        | –            |
| Politi et al. [56]$^c$ | no           | no             | no       | 1/9   | /        | –            |
| Crespi et al. [57]  | no             | no             | no       | 1/9   | /        | –            |
| Zhang et al. [58]$^c$ | no            | no             | no       | 1/9   | /        | –            |
| Zeuner et al. [17]$^c$ | no          | no             | no$^*$   | 1/4   | $< 10^{-4}$ | ~ 6          |
| Ours                | yes            | yes            | yes      | 1/8   | ~0.006$^*$ | ~85          |

notes: $^a$ for controlled-Phase gate, $^b$ for controlled-Z gate, $^c$ for integrated optical CNOT gates
$^*$ measurement of output states for postselection due to the multiple pair emissions of SPDC
$^1$ for given input states $|+\rangle_H|H\rangle_i$; $\eta_h$ can be further improved to one in principle.