Multiworld motives by closed time-like curves

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Abstract. We propose a new model, entitled S-CTC, for description of quantum systems in the presence of CTC – closed time-like curves. The model is based on the viewpoint on any quantum state as an observer’s state of knowledge of the system preparation procedure. We compare and contrast our S-CTC model with D-CTC and P-CTC models and show that S-CTC shares special quantum features with both D-CTC and P-CTC. As far as the interaction of the quantum system with itself coming from the future concerns, S-CTC is formally equivalent to P-CTC. On the other hand, when calculating outcome probabilities for a measurement within the time interval between the entrance and exit of CTC, S-CTC becomes equivalent to D-CTC. Both these models require the concept of alternative realities (worlds) where different measurement outcomes are recorded and alternative connections of these realities by CTC.

1. Introduction

Presently, the key scientific approach to reality consists of two drastically different fundamental theories: quantum physics and geometrical physics (General Relativity). One of the main problems nowadays is incorporating these theories into a single framework. In search of such a framework, the ‘borderline’ problems, e.g. field quantization near the black hole’s event horizon\cite{1}, that require instruments of the both theories can be regarded as a kind of ‘metatheoretical colliders’ for Quantum Mechanics and General Relativity. Quantum models with closed time-like curves (CTC) focus attention on this point in perhaps the best way. The assumption of existence of particles with closed world lines is supported by the well-known solution of Einstein’s equations, e.g. the Gödel’s solution for rotating Universe\cite{2}.

However, the existence of CTC in the real world has long been a controversial issue. In general, quantum models with causal loops could potentially strengthen the scepticism if their predictions were in strong disagreement with the well-established views. The value of quantum physics as a referee regarding this question depends on unambiguity and correctness of its application to quite a non-standard problem that causes paradoxes even in its classical version\cite{3}. Quantum theory in presence of CTC is ambiguous, which is evident from existence of different approaches to the problem. The D-CTC approach\cite{4} developed by Deutsch is perhaps the most well-known. He suggested the space-time setup that became a de facto standard for dealing with quantum aspects of CTC. Fig.\[1\] shows a part of a 2D Minkowski space-time with an attached ‘corridor’ (not shown on the scheme) connecting the lower edge of the ‘entrance’ cut at $t = \tau$ with the upper edge of the ‘exit’ cut at $t = \tau$. The latter is in the absolute past (in terms of causal structure of Minkowski space) with regards to the ‘entrance’, which potentially allows closing particles’ world lines, thus forming CTC. We will further use abbreviation TM (time machine)
as a synonym of CTC. Hence, a particle entering TM at \( t = \tau \) almost instantly (according to its own clock) appears at its past at \( t = 0 \).

![Figure 1](image_url)

**Figure 1.** The space-time scheme of quantum models with CTC due to Deutsch.

The distinctive feature of the Deutsch’s approach is ontic view on the state of the quantum system. Its density matrix appears in the model as a real physical entity for which local self-consistency conditions hold. Deutsch’s model is based on a boundary condition that identifies the exit density matrix with the one at the entrance of TM. This matrix appears to be a solution of a consistency equation and depends nonlinearly on the initial density matrix of the system, \( \hat{\varrho}_{in} \), prepared in the absolute past of the TM. Consequently, \( \hat{\varrho}_{out} \), the state in the absolute future of TM, also gets a nonlinearly dependence on \( \hat{\varrho}_{in} \). This dependence, non-trivial and unusual for quantum mechanics, is due to interaction between the system and its version that moved through CTC during the time interval \((0, \tau)\). The space-time region of interaction is shown at Fig.1 (gray). Note that it is the interaction that gives rise to the classical TM paradoxes [3,5].

Recently, a model known as P-CTC was proposed [6, 7], in which the particle’s state is transferred to the past via quantum teleportation. In Fig.2 the preparation of entangled pair of particles (2,3) plays the role of the TM’s exit, while the entrance is represented by post-selection of the pair (1,3) in the same entangled state. The dashed fragments closing the world lines of the particles allow interpreting the transfer to the past upon successful post-selection (P-CTC) as a one-particle evolution – the world line of particle 3 is considered to be the world line of particle 1 moving backwards in time.

Both D-CTC and P-CTC predict non-trivial possibilities for processing quantum information. Despite the highly vague feasibility perspectives, TMs are important for information theory (both classical and quantum) by clarifying the nature of its tools [8]. For example, it was proved that the classical computer with access to the D-CTC resource is able to solve PSPACE-complete problems [9]. Other notable investigations of CTC in quantum information theory include development of novel communication protocols [10], information encoding [11], studies of state cloning [12].

The P-CTC model compared to D-CTC does not offer exceptional possibilities for quantum information processing. P-CTC demonstrates a more ‘mild’ nonlinear dependence of \( \hat{\varrho}_{out} \) on \( \hat{\varrho}_{in} \). On the other hand, as noted above, the quantum teleportation and, consequently, P-CTC, lacks real change of the space-time topology.
The fact that P-CTC is less radical than D-CTC does not give advantage to the latter. It was shown that in D-CTC quantum states cannot be purified, while in P-CTC this is possible [13]. Moreover, unlike D-CTC, P-CTC is compatible with the formalism of path integrals [14], which allows it to effectively describe quantum fields in the curved space-time.

D-CTC and P-CTC were examined in terms of the so-termed quantum causal analysis [15]. This shed light on intrinsic character of the types of self-consistency in these models. These types appeared quite different in their nature.

In the present work, we propose a novel quantum model with CTC, its features being in a certain sense in between those of D-CTC and P-CTC. The new model resembles D-CTC by assuming a non-trivial topology of space-time. However, its nonlinearity and the way it treats the effects of interaction between the system and its older version coming through CTC are similar to P-CTC. On the contrary, when describing a measurement in between the entrance and exit of the CTC loop, the new model shares attributes with D-CTC. In both these cases, one needs accounting alternative realities with different measurement outcomes and averaging over different connections of these realities via CTC.

2. Model
Two guiding principles are invoked in order to formulate the new model of quantum evolution in the presence of CTC. In the first place is the identity condition for inner state of the system entering TM at \( t = \tau \) and exiting it at \( t = 0 \). In D-CTC it is reflected in a special self-consistency equation. Its solution yields an explicit form of \( \hat{\rho}_{\text{CTC}} \) at the entrance and at the exit as well.

Our second guiding consideration is the radically different interpretation of the quantum state notion, in which the quantum state is treated as a state of knowledge possessed by a dweller of Minkowski space-time. Such an observer, who prepared the system 1, has no knowledge of the past of the system 2. These two points may serve as a foundation for a formal theoretical model.

Let us note that the Dirac notations for ket-vector \( |\Psi\rangle \) (and bra-vector \( \langle \Phi| \)) imply a certain time direction. The ket-vector symbolizes preparation of the system state, and dynamical
transformation of this knowledge is given by unitary evolution operators written to the left. This allows associating a ket-vector with the direction of time flow from right to left, and vice versa for bra-vector. A somewhat similar idea involving two mutually opposite time directions $\langle \Phi | \Psi \rangle$ forms the basis of time-symmetric quantum formalism [17].

The same considerations allow to 'juxtapose' the two opposite time directions starting from the same time moment by an arbitrary operator, for instance by the unit operator $\hat{1} = \sum_k |k\rangle\langle k|$. (1)

If the preparation procedure is completely unknown, the system state is given by the unit operator. It is then reasonable to use it for the system exiting TM at $t = 0$. The red color for ket and the blue one for bra in (1) show their correspondence to different time moments. It is convenient to represent the resulting picture (in the absence of interaction) horizontally (Fig. 3).

Figure 3. The splitting of the ket-bra structure of (1) illustrating relation between $|\psi_{in}\rangle$ and $|\psi_{out}\rangle$ in the absence of interaction.

By placing ket- and bra-parts of (1) in different sides of (3), both of the guiding principles of our theory are satisfied: on the one hand, the observer is of total ignorance with regard to the preparation of the second system’s state; on the other hand, an explicit coincidence of the entrance and exit states of TM is granted. The initially indivisible essence, (1) is now split into two parts. For this reason the new model will from now on be called S-CTC ('S' stands for 'splitting').

Quantum evolution in the presence of CTC with interaction between 1 and 2 has several peculiarities. Consider a special case of a unitary operation applied to 1 and controlled by the state of 2. Such an interaction, CONTROL-U, is typical for quantum information. It is defined in a fixed ‘interaction’ basis of 2. For simplicity, we will use the same basis for (1) when

Figure 4. The scheme of Fig. 3 added with interaction between the system and its later version (system 2) via controlled operation represented by a set of unitaries $\hat{U}_k$.

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1 We are using the visual representation of quantum processes developed in [16].
studying the scheme presented in Fig. 4 but the result is eventually insensitive to the choice of basis. It follows that

$$|\psi_{\text{out}}\rangle = \sum_k |k\rangle \langle k| \hat{U}_k |\psi_{\text{in}}\rangle.$$  (2)

Introducing operator

$$\hat{W} = \sum_k |k\rangle \langle k| \hat{U}_k,$$  (3)

we get

$$\langle \psi_{\text{in}}| \hat{W}^\dagger \hat{W} |\psi_{\text{in}}\rangle = \langle \psi_{\text{out}}| \psi_{\text{out}}\rangle.$$  (4)

This is the probability that the act of interaction will happen, or, equivalently, CTC get realised (the probability of system 2 to appear).

The result of interaction between 1 and 2 in P-CTC appears identical to (2). Indeed, quantum teleportation scenario implies preparation of entangled state of 2 and 3, in addition to

$$|\psi_{\text{in}}\rangle_1:|\Psi\rangle_2,3 = \sum_k |k\rangle_2 \otimes |k\rangle_3,$$  (5)

followed by post-selection of 1 and 3 in the same state $|\Psi\rangle_{1,3}$. In the case of successful post-selection,

$$|\psi_{\text{out}}\rangle_2 = \sum_{k,k'} (\langle k'|_1 \otimes \langle k'|_3) \left( \hat{U}_k |\psi_{\text{in}}\rangle_1 \otimes |k\rangle_2 \otimes |k\rangle_3 \right) = \sum_k (\langle \hat{U}_k |\psi_{\text{in}}\rangle \otimes |k\rangle_2,$$  (6)

which is exactly the same as (2).

In what follows, we use the simplest qubit model for systems ($k = 0, 1$) and controlled phase shift as interaction. In this case $\hat{U}_0 = \hat{1}$ and

$$\hat{U}_1 = |0\rangle_1 \langle 0| + e^{i\varphi} |1\rangle_1 \langle 1|.$$  (7)

Then $\hat{W} = \hat{U}_1$ and CTC realization probability is 1.

3. Measurement in CTC

Now there is a measurement of qubit 2 after it exits TM but before it interacts with qubit 1. Eigenstates $|\psi_{\sigma}\rangle$ ($\sigma = \pm$) of the measured observable are related to the ‘interaction basis’ via amplitudes $\alpha$ and $\beta$:

$$|\psi_+\rangle = \alpha |0\rangle + \beta |1\rangle, \quad |\psi_-\rangle = -\beta^* |0\rangle + \alpha^* |1\rangle,$$  (8)

where $|\alpha|^2 + |\beta|^2 = 1$.

We are interested in the probabilities $p(\sigma)$ of measurement outcomes. Let us first calculate them according to properly modified D-CTC self-consistency conditions for states $\hat{\rho}_{CTC}$ at the entrance and exit of TM [4]. We denote these states by $\hat{\rho}_{\pm}$, relating them to possible measurement outcomes:

$$p_\pm(\sigma) = \sum_{k=0,1} \langle k| \hat{P}_\sigma \hat{\rho}_{\delta}(I) \hat{P}_\sigma |k\rangle \hat{U}_k \hat{\rho}_{\text{in}} \hat{U}_k^\dagger.$$  (9)

Here $\hat{P}_\sigma = |\psi_{\sigma}\rangle \langle \psi_{\sigma}|$. We introduced the additional index $I$ which implicitly reflects the way of making the states consistent. Specifically, the consistency condition is satisfied independently for each of two alternative realities (worlds $W_{\pm}$), in which the measurement outcomes are recorded (Fig 5).
Figure 5. Two alternative worlds $W_{\pm}$ with CTC where measurement outcomes $\pm$ are recorded (connection by the variant I).

Figure 6. The same worlds connected by the variant $II$.

It appears that for the outcome probabilities $p_{I}(\pm)$ there is a problem: generally $p_{I}(+) + p_{I}(-) \neq 1$, i.e. $p_{I}(+)$ and $p_{I}(-)$ can not be viewed as probabilities. One can try a different way to satisfy consistency condition (variant $II$). In this case the corresponding equations are coupled (see Fig.6):

\begin{align}
    p_{II}(+)&\hat{\rho}_{\pm}(II) = \sum_{k=0,1} \langle k | \hat{P}_{\pm} \hat{\rho}_{\pm}(II) \hat{P}_{\pm} | k \rangle \hat{U}_{k} \hat{\rho}_{\pm} \hat{U}_{k}^\dagger, \\
    p_{II}(-)&\hat{\rho}_{\mp}(II) = \sum_{k=0,1} \langle k | \hat{P}_{\pm} \hat{\rho}_{\pm}(II) \hat{P}_{\mp} | k \rangle \hat{U}_{k} \hat{\rho}_{\pm} \hat{U}_{k}^\dagger.
\end{align}
Again, \( p_{II}(+) + p_{II}(-) \neq 1 \). However,

\[
\frac{1}{2} \left( p_I(+) + p_{II}(+) \right) + \frac{1}{2} \left( p_I(-) + p_{II}(-) \right) = 1.
\]

(11)

This suggests interpreting the two terms in (11) as measurement outcome probabilities from the point of view of the observer – dweller of Minkowski space:

\[
P_{\text{Prob}}^{(\sigma)}_{D-\text{CTC}} = \frac{1}{2} \left( p_I(\sigma) + p_{II}(\sigma) \right).
\]

(12)

The observer’s complete ignorance of the connection variant used is reflected by \( \frac{1}{2} \) probabilities.

Let us now consider outcome probabilities in S-CTC. Like in D-CTC, a similar dilemma of connecting entrances and exits of TM in the worlds \( W_+ \) and \( W_- \) arises. The scheme for the variant I is illustrated in Fig.7 and yields

\[
\hat{\rho}_{\text{out}}^{(\sigma)}(I) = \sum_{k,k'} |k\rangle \langle k| \hat{P}_\sigma \hat{U}_k \hat{\rho}_{\text{in}} \hat{U}^\dagger_k \hat{P}_\sigma |k'\rangle \langle k'|
\]

(13)

with the quasi-probabilities similar to D-CTC:

\[
\text{Tr} \hat{\rho}_{\text{out}}^{(\sigma)}(I) = p_I(\sigma).
\]

(14)

\[
\hat{\rho}_{\text{out}}^{(\sigma)}(II) = \sum_{k,k',\kappa,\kappa'} |k_+\rangle \langle k_-| \hat{P}_+ \hat{\rho}_{\text{in}} \hat{P}_- |\kappa_+\rangle \langle \kappa_-|
\]

Figure 7. The scheme of measurement in S-CTC with connection by the variant I.

Figure 8. The scheme of measurement in S-CTC with connection by the variant II.
For connection by the variant $II$, there is a correlated state for the worlds $W_{+}$ and $W_{-}$ (Fig. 8)

$$\hat{\varrho}_{\text{out}}^{(+,-)}(II) = \sum_{k_{+},k'_{+}} \sum_{k_{-},k'_{-}} |k_{+}\rangle\langle k_{+}| \hat{P}_{+}\hat{U}_{k_{+}} \hat{\varrho}_{\text{in}} \hat{U}_{k'_{+}}^{\dagger} \hat{P}_{+} |k'_{+}\rangle\langle k'_{+}| \otimes |k_{-}\rangle\langle k_{-}| \hat{P}_{-}\hat{U}_{k_{-}} \hat{\varrho}_{\text{in}} \hat{U}_{k'_{-}}^{\dagger} \hat{P}_{-} |k'_{-}\rangle\langle k'_{-}|.$$  

(15)

By tracing, one gets the product of the same quasi-probabilities which appeared in (10):

$$Tr \hat{\varrho}_{\text{out}}^{(+,-)}(II) = p_{II}(+)p_{II}(-).$$  

(16)

One can introduce density matrices $\hat{\varrho}_{\text{out}}^{(\sigma)}(II)$ according to

$$\hat{\varrho}_{\text{out}}^{(+)}(II) = Tr_{2} \hat{\varrho}_{\text{out}}^{(+,-)}(II) p_{II}(-); \quad \hat{\varrho}_{\text{out}}^{(-)}(II) = Tr_{1} \hat{\varrho}_{\text{out}}^{(+,-)}(II) p_{II}(+)$$  

(17)

which ensures the coincidence between S-CTC and D-CTC with respect to outcome probabilities for both of the connection types.

The expression (12) can be generalised up to

$$\hat{\varrho}_{\text{out}}^{(\sigma)} = \frac{1}{2} \left( \hat{\varrho}_{\text{out}}^{(\sigma)}(I) + \hat{\varrho}_{\text{out}}^{(\sigma)}(II) \right),$$  

(18)

thus introducing the final state of the qubit. In contrast to the outcome probabilities the final states in D-CTC and S-CTC prove to be quite different.

4. Joint probability distribution

Consideration of purely quantum-mechanical effects, such as quantum entanglement, can provide an understanding of a deeper connection between the theories under study.

So, we now move to a scheme which involves two-party measurements performed by experimenters Alice ($A$) and Bob ($B$). As before, one can not dispense with the two topologies (Fig. 9, 10). The experimenters prepare a pair of particles in the entangled state $|\psi_i\rangle$, and hold one of the particles for themselves. Also, both researchers have measuring devices ($\hat{P}_{a}$ and $\hat{P}_{b}$), and Alice additionally has an access to CTC with the $\hat{U}$ interaction. It is proposed to find joint outcome probabilities in this scheme and determine their nature (whether they are full-fledged probabilities or pseudo-probabilities).
Figure 9. Physical scheme of the CTC experiment with a joint probability distribution (topology I).

Figure 10. Physical scheme of the CTC experiment with a joint probability distribution (topology II).

For certainty, we will assume that the entangled particles are in a singlet state
\[ |\psi_i\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B). \]  
(19)

The Alice’s and Bob’s measurement settings are denoted by \( \alpha \) and \( \beta \). The corresponding measurement bases are
\[ |+; \alpha\rangle = \alpha_0 |0\rangle_A + \alpha_1 |1\rangle_A, \]  
(20)
\[ |-; \alpha\rangle = -\alpha_1^* |0\rangle_A + \alpha_0^* |1\rangle_A, \]  
(21)
\[ |+; \beta\rangle = \beta_0 |0\rangle_B + \beta_1 |1\rangle_B, \]  
(22)
\[ |-; \beta\rangle = -\beta_1^* |0\rangle_A + \beta_0^* |1\rangle_B. \]  
(23)
where $\alpha_0, \beta_0, \alpha_1, \beta_1$ – complex amplitudes. The outcomes of the measurements will be denoted by $a = \pm$ and $b = \pm$. Then, the corresponding projectors are expressed in the standard way:

$$\hat{P}_{\sigma}(\varepsilon) = |\sigma; \varepsilon\rangle \langle \sigma; \varepsilon|,$$

where $\sigma = \pm$, $\varepsilon \in \{\alpha, \beta\}$. As before, we will choose the interaction as a controlled phase shift \((7)\) and will be interested in the joint outcome probabilities $p(a,b|I)(\alpha, \beta)$ and $p(a,b|II)(\alpha, \beta)$ (here the explicit dependence on the measurement settings is shown).

Equivalent computational schemes for the S-CTC model are shown in the Figures 11, 12.

**Figure 11.** Computational scheme of the S-CTC experiment with two measurements (topology I).

**Figure 12.** Computational scheme of the S-CTC experiment with two measurements (topology II).
Based on them, we can now write the final states:

\[
|\psi_{f,I}^{(a,b)}(\alpha,\beta)\rangle = \left( |k\rangle_A \langle k| \hat{P}_a(\alpha) \hat{U}_k \otimes \hat{P}_b(\beta) \right) |\psi_i\rangle, \tag{25}
\]

\[
|\psi_{f,II}^{(+,-,b)}(\alpha,\beta)\rangle = \left( |k_+\rangle_A \langle k_-| \hat{P}_-(\alpha) \hat{U}_{k_+} \otimes \hat{P}_b(\beta) \right) |\psi_i\rangle \otimes \left( |k_-\rangle_A \langle k_+| \hat{P}_+(\alpha) \hat{U}_{k_-} \otimes \hat{P}_b(\beta) \right) |\psi_i\rangle, \tag{26}
\]

where \(|\psi_{f,I}^{(a,b)}(\alpha,\beta)\rangle\) is the final two-qubit state for the first topology \(I\), \(|\psi_{f,II}^{(+,-,b)}(\alpha,\beta)\rangle\) is the final four-qubit state for the second topology \(II\), the symbols + and − refer to Alice’s outcomes.

Desired values \(p_I(a,b|\alpha,\beta)\) and \(p_{II}(a,b|\alpha,\beta)\) expressed in terms of (25) and (26), consequently:

\[
p_I(a,b|\alpha,\beta) = \langle \psi_{f,I}^{(a,b)}(\alpha,\beta) | \psi_{f,I}^{(a,b)}(\alpha,\beta) \rangle \tag{27}
\]

\[
p_{II}(+,b|\alpha,\beta) \cdot p_{II}(-,b|\alpha,\beta) = \langle \psi_{f,II}^{(+,-,b)}(\alpha,\beta) | \psi_{f,II}^{(+,-,b)}(\alpha,\beta) \rangle. \tag{28}
\]

The expression (27) is simplified to the form

\[
p_I(a,b|\alpha,\beta) = \frac{1}{2} \left[ \langle 0| \hat{P}_a(\alpha)|0\rangle \langle 1| \hat{P}_b(\beta)|1\rangle + \langle 1| \hat{P}_a(\alpha)|1\rangle \langle 0| \hat{P}_b(\beta)|0\rangle - \langle 0| \hat{P}_a(\alpha)|1\rangle \left( \langle 0| \hat{P}_a(\alpha)|0\rangle + e^{i\phi} \langle 1| \hat{P}_a(\alpha)|1\rangle \right) \langle 1| \hat{P}_b(\beta)|0\rangle - \langle 1| \hat{P}_a(\alpha)|0\rangle \left( \langle 0| \hat{P}_a(\alpha)|0\rangle + e^{-i\phi} \langle 1| \hat{P}_a(\alpha)|1\rangle \right) \langle 0| \hat{P}_b(\beta)|1\rangle \right]. \tag{29}
\]

The expression \(p_{II}(a,b|\alpha,\beta)\) is a decomposition of (28), finding which in the general case is not a trivial task. However, it is possible to notice some regularity in the pseudo-probabilities obtained earlier and ‘guess’ the proper structure, using (29):

\[
p_{II}(a,b|\alpha,\beta) = \frac{1}{2} \left[ \langle 0| \hat{P}_a(\alpha)|0\rangle \langle 1| \hat{P}_b(\beta)|1\rangle + \langle 1| \hat{P}_a(\alpha)|1\rangle \langle 0| \hat{P}_b(\beta)|0\rangle - \langle 0| \hat{P}_a(\alpha)|1\rangle \left( \langle 1| \hat{P}_a(\alpha)|1\rangle + e^{i\phi} \langle 0| \hat{P}_a(\alpha)|0\rangle \right) \langle 1| \hat{P}_b(\beta)|0\rangle - \langle 1| \hat{P}_a(\alpha)|0\rangle \left( \langle 1| \hat{P}_a(\alpha)|1\rangle + e^{-i\phi} \langle 0| \hat{P}_a(\alpha)|0\rangle \right) \langle 0| \hat{P}_b(\beta)|1\rangle \right]. \tag{30}
\]

A bit longer evaluation in D-CTC model confirms this result. One can also check that the said joint distributions are probabilities indeed:

\[
\sum_{a,b} p(a,b|\tau)(\alpha,\beta) = 1 \tag{31}
\]

for \(\tau = I, II\) and arbitrary choice of \(\alpha\) and \(\beta\), thus being free from inconsistencies discussed above.

From the joint probabilities distributions the one-party (Bob) probability distributions are readily obtained:
\[ \pi_I(b|\alpha, \beta) = \sum_a p_r(a, b|\alpha, \beta), \]  
\[ \pi_II(b|\alpha, \beta) = -\frac{1}{2} \langle 1 | \hat{P}_b(\beta) | 0 \rangle (|\alpha_0|^2 - |\alpha_1|^2) \alpha_0 \alpha_1^* (1 - e^{i\varphi}) - \]  
\[ -\frac{1}{2} \langle 0 | \hat{P}_b(\beta) | 1 \rangle (|\alpha_0|^2 - |\alpha_1|^2) \alpha_1 \alpha_0^* (1 - e^{-i\varphi}) + \frac{1}{2}, \]  
\[ \pi_{II}(b|\alpha, \beta) = -\frac{1}{2} \langle 1 | \hat{P}_b(\beta) | 0 \rangle (|\alpha_1|^2 - |\alpha_0|^2) \alpha_0 \alpha_1^* (1 - e^{i\varphi}) - \]  
\[ -\frac{1}{2} \langle 0 | \hat{P}_b(\beta) | 1 \rangle (|\alpha_1|^2 - |\alpha_0|^2) \alpha_1 \alpha_0^* (1 - e^{-i\varphi}) + \frac{1}{2}. \]  
Expressions (33) and (34) indicate the presence of the effect of ‘superluminal telegraph’ – Alice, choosing the type of her measurement, has the opportunity to influence the outcome of Bob’s measurement. Due to the non-physical nature of such an effect, it is necessary to find the conditions for ‘restoring physicality’. And this turns out to be possible when taking into account two topologies with equal weights:

\[ \frac{1}{2} \pi_I(b|\alpha, \beta) + \frac{1}{2} \pi_{II}(b|\alpha, \beta) = \frac{1}{2}. \]  

That is, the randomness of the implementation of a particular topology deprives Alice of the opportunity to influence the statistics of measurement outcomes in Bob’s laboratory.

5. Discussion and Conclusion

The comparative study of outcome probabilities in CTC models can not be complete without considering the case of P-CTC. Assume that in Fig.2 immediately after preparation of \(|\Psi\rangle_{2,3}\) at \(t = 0\), the qubit 2 is subject to the measurement in the basis \(\{\rangle\). In P-CTC there is neither need nor possibility to take into account the type II of connecting \(W_{\pm}\). For this reason \(\hat{\varrho}^{(\sigma)}_{\text{in}}\) relates to \(\hat{\varrho}_{\text{out}}\) by (13). Outcome probabilities are given by the ABL rule (18). Since the trace of (13) is equal to \(p_I(\sigma)\), one gets

\[ Pr_{\text{ob}}^{(\sigma)}_{\text{P-CTC}} = \frac{p_I(\sigma)}{p_I(+)+p_I(-)}. \]  

Dissimilarity from (12) is evident.

As previously noted, the quantities \(p(\sigma|I)\) and \(p(\sigma|II)\) are not probabilities in general. We suppose, however, that they may enter Bayesian expressions. \textit{A priori}, the variants \(I\) and \(II\) are equiprobable for the external observer. Upon learning the measurement outcome \(\sigma\), the probabilities get updated:

\[ p(I|\sigma) = \frac{p_I(\sigma)}{p_I(\sigma) + p_{II}(\sigma)}, \quad p(II|\sigma) = \frac{p_{II}(\sigma)}{p_I(\sigma) + p_{II}(\sigma)}. \]  

The new probabilities find application if the same TM is used in experiment with another qubit B along with the measured qubit A. The B system is not measured, but it interacts with its later version as pictured in Fig.1. Let it be a controlled phase shift \(\hat{V}\) as for the qubit A. The final state of B in S-CTC model is

\[ \hat{\varrho}_{B-\text{out}}(I) = \sum_{k,k'} |k\rangle\langle k| \hat{V} \hat{k} \hat{\varrho}_{B-\text{in}} \hat{V}_{\dagger k'} \langle k'| \]  

(38)
for the first variant and
\[ \hat{\varrho}_{B-out}(II) = \sum_{k,k',l} |k\rangle\langle k| \hat{V}_l \hat{\varrho}_{B-in} \hat{V}_l^\dagger |k'\rangle\langle k'| \cdot (l|\hat{V}_k \hat{\varrho}_{B-in} \hat{V}_k^\dagger |l\rangle \] (39)

for the second one. If measuring the first qubit results in outcome \( \sigma \), the final state of \( B \) is
\[ \hat{\varrho}_{B-out}^{(\sigma)} = p(I|\sigma)\hat{\varrho}_{B-out}(I) + p(II|\sigma)\hat{\varrho}_{B-out}(II). \] (40)

Note that \( A \) and \( B \) did not interact directly, but \( \hat{\varrho}_{B-out}^{(\sigma)} \) is sensitive to the fact of measurement on \( A \) and depends on its outcome. Therefore, in S-CTC (as well as in D-CTC) a system can influence the evolution of another one by means of additional measurement-induced modification of space-time topology.

Finally, let us outline the similarities and differences in D-CTC, P-CTC and S-CTC. In absence of measurement, but in the presence of interaction between different versions of the system, P-CTC and S-CTC predict the same final states \( \hat{\varrho}_{out} \) given the same \( \hat{\varrho}_{in} \). In case of measurement, apart from final state \( \hat{\varrho}_{out}^{(\sigma)} \) for the outcome \( \sigma \) one needs to know its probability. Despite the radical difference, D-CTC and S-CTC give identical probabilities. In both cases, for obtaining physically meaningful result, it has proved the necessity to account different realities \( W_\sigma \) attached to different outcomes. This necessity appears not in the context of interpretation issues, but as an indispensable technical element of a formal construction. The probabilities predicted by P-CTC are different from D-CTC and S-CTC. The final states are different in all three models.

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