Power corrections to the pion transition form factor from higher-twist distribution amplitudes of photon

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Abstract

In this paper we investigate the power suppressed contributions from two-particle and three-particle twist-4 light-cone distribution amplitudes (LCDAs) of photon within the framework of light-cone sum rules. Compared with leading twist LCDA result, the contribution from three-particle twist-4 LCDAs is not suppressed in the expansion by $1/Q^2$, so that the power corrections considered in this work can give rise to a sizable contribution, especially at low $Q^2$ region. According to our result, the power suppressed contributions should be included in the determination of the Gegenbauer moments of pion LCDAs with the pion transition form factor.
1 Introduction

As one of the simplest hard exclusive processes, the pion transition form factor $F_{\gamma^*\gamma\to\pi^0}(Q^2)$ at large momentum transfer is of great importance in exploring the strong interaction dynamics of hadronic reactions in the framework of QCD, and to determine the parameters in the LCDAs of pion. It is defined via the matrix element

$$\langle \pi(p) | j_{\mu}^{em} | \gamma(p') \rangle = g_{em}^2 \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \epsilon^\nu(p') F_{\gamma^*\gamma\to\pi^0}(Q^2),$$

where $q = p - p'$, $p$ and $p'$ refer to the four-momentum of the pion and the on-shell photon respectively, the electro-magnetic current

$$j_{\mu}^{em} = \sum_q g_{em} Q Q \bar{q} \gamma_{\mu} q, \quad \epsilon_{0123} = -1.$$  

In collinear factorization theorem, pion transition form factor can be factorized into the convolution of the hard kernel and the leading twist pion LCDA at leading power of $1/Q^2$ [1–4], and the hard kernel has been calculated up to two-loop level [5–8]. At one-loop level, the factorization formula can be written by

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = \sqrt{2} \left( \frac{Q_u^2 - Q_d^2}{Q^2} \right) f_\pi \int_0^1 dx \left[ T_2^{(0)}(x) + T_2^{(1)}(x, \mu) \right] \phi_{\pi}(x, \mu) + \mathcal{O}(\alpha_s^2),$$

where the leading twist pion LCDA is defined as

$$\langle \pi(p) | \bar{\xi}(y) \gamma_{\mu} \gamma_5 \xi(0) | 0 \rangle = -i f_\pi p_{\mu} \int_0^1 du \, e^{iu\cdot y} \phi_{\pi}(u, \mu) + \mathcal{O}(y^2),$$

and the superscript “$\Delta$” indicates the scheme to deal with $\gamma_5$ in dimensional regularization which is a subtle problem in QCD loop diagrams [9–15]. Employing the trace technique, the $\gamma_5$ ambiguity of dimensional regularization was resolved by adjusting the way of manipulating $\gamma_5$ in each diagram to preserve the axial-vector Ward identity [6]. In a recent paper [16], the one loop calculation is revisited by applying the standard OPE technique [17–19] with the evanescent operator(s) [20,21], in both the NDR and HV schemes for $\gamma_5$ in the $D$-dimensional space. At one-loop level it has been shown explicitly that the scheme dependence of the hard kernel and the twist-two pion LCDA are cancelled out precisely, which guarantees the form factor $F_{\gamma^*\gamma\to\pi^0}(Q^2)$ is free from $\gamma_5$ ambiguity.
At leading power the pion transition form factor has also been studied with transverse momentum dependent (TMD) factorization approach at one-loop level [22–24], where the joint resummation of the large logarithms $\ln^2 k^2_\perp/Q^2$ and $\ln^2 x$ was performed in moment and impact-parameter space [25]. The prediction of joint resummation improved TMD factorization approach can accommodate the anomalous BaBar measurements [26] of $F_{\gamma^*\gamma\to\pi^0}(Q^2)$, which have stimulated intensive theoretical investigations with various phenomenological approaches as well as lattice QCD simulations (see for instance [27–29]). In ref [30,31], a leading twist pion LCDA with the non-vanishing end-point behavior was proposed to explain the anomalous BaBar data at high $Q^2$. Later it was found [32] that this method [30,31] is able to be achieved by introducing a sizable nonperturbative soft correction from the TMD pion wavefunction.

To achieve more precise theoretical predictions, power corrections need to be taken into account especially at low $Q^2$. In [32,33], the soft correction to the leading twist contribution is evaluated with the dispersion approach and found to be crucial to suppress the contributions from higher Gegenbauer moments of the twist-2 pion LCDAs [25,34]. Alternatively, the subleading power “hadronic” photon correction can also be taken into account effectively with dispersion approach. In [16], the QCD factorization of the correlation function for the construction of the LCSRs for the hadronic photon contribution to the pion-photon form factor is established. Both the hard matching coefficient and the leading twist photon LCDAs are independent of the $\gamma_5$ prescription in dimensional regularization, and the next-to-leading logarithmic(NLL) resummation of the large logarithms was also perform by solving the renormalization group equations(RGE) in momentum space. The contribution from the twist-4 pion LCDA is also calculated at tree level in [16,35]. There is strong cancellation between this contribution and the contribution from hadronic structure of photon, which makes the overall power correction not significant. The LCDAs of photon, including both two-particle and three-particle Fock state, have been studied up the twist-4 level [36]. The higher-twist LCDAs are not suppressed in many processes such as radiative leptonic $B$ meson decay $B \to \gamma \ell \nu$ [37,38]. In this paper we will investigate the contribution from the full set the LCDAs of photon up to twist-4 to the pion transition form factor using LCSRs approach.
The outline of this paper is as follow: in Section 2 we present the analytic calculation of the pion transition form factor from the higher twist photon LCDAs within LCSRs framework. The numerical results and discussions are given in section 3. The last section is closing remark.

2 Power corrections from the hadronic structure of photon

All two- and three-particle LCDAs of photon have been defined and classified up to twist-4, and the expressions of the LCDAs have also been obtained through the conformal expansion of in presence of the background field [36]. To evaluate the power suppressed contribution to the pion-photon form factor due to the hadronic photon effect, the following correlation function is employed

\[ G_\mu(p', q) = \int d^4 z e^{-i q \cdot z} \langle 0 | T \{ j_{\mu,\perp}^{em}(z), j_\pi(0) \} | \gamma(p') \rangle = -g_{em}^2 \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta G(p^2, Q^2), \]

(5)

where pion interpolating current \( j_\pi \) is defined by

\[ j_\pi = \frac{1}{\sqrt{2}} (\bar{u} \gamma_5 u - \bar{d} \gamma_5 d). \]

(6)

The power counting rule for the external momenta

\[ |n \cdot p| \sim \bar{n} \cdot p \sim n \cdot p' \sim O(\sqrt{Q^2}), \]

(7)

will be adopted to determine the perturbative matching coefficient entering the factorization formula of \( G_\mu(p', q) \). Applying the standard definition for the pion decay constant

\[ \langle 0 | j_\pi | \pi(p) \rangle = -i f_\pi \mu_\pi(\mu), \quad \mu_\pi(\mu) \equiv \frac{m_\pi^2}{m_u(\mu) + m_d(\mu)}, \]

(8)

we can write down the hadronic dispersion relation of \( G(p^2, Q^2) \)

\[ G(p^2, Q^2) = \frac{f_\pi \mu_\pi(\mu)}{m_\pi^2 - p^2 - i0} \int_{\gamma\gamma\to\pi^0} F^{NLP}_{\gamma\gamma\to\pi^0}(Q^2) + \int_{s_0}^{\infty} ds \frac{\rho^h(s, Q^2)}{s - p^2 - i0}. \]

(9)
The form factor $F_{\gamma^*\gamma\rightarrow\pi^0}(Q^2)$ can be extracted after the correlation function being calculated by OPE in deep Euclidean region. Employing dispersion relation, subtracting the continuum state contribution with the help of quark hadron duality assumption, and performing Borel transformation, the LCSRs for the subleading power contribution to the $\pi^0\gamma^*\gamma$ form factor can be derived as

$$F_{\gamma^*\gamma\rightarrow\pi^0}(Q^2) = -\frac{\sqrt{2}}{f_\pi} \frac{(Q^2 - Q^2_0)}{Q^2} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_0^{s_0} ds \exp \left[ -\frac{s - m^2_\pi}{M^2} \right] \times \left[ \rho^{(0)}(s, Q^2) + \frac{\alpha_s CF}{4\pi} \rho^{(1)}(s, Q^2) \right] + \mathcal{O}(\alpha^2_s). \quad (10)$$

where the magnetic susceptibility of the quark condensate $\chi(\mu)$ contains the dynamical information of the QCD vacuum, and the spectral functions $\rho^{(0,1)}(s, Q^2)$ can be found in [16].

Now we will proceed to investigate the contribution from higher twist LCDAs of photon. Up to twist-4, the two-particle LCDAs of photon are defined as

$$\langle 0| \bar{q}(x)[x, 0] \sigma_{\alpha\beta} \, q(0)|\gamma(p) \rangle = i \, g_{em} \, Q_q \langle \bar{q}q \rangle(\mu) \left( p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta \right) \int_0^1 dz \, e^{iz^\mu x} \chi(\mu) \phi_\gamma(z, \mu) + \frac{x^2}{16} A(z, \mu) + \frac{i}{2} \, g_{em} \, Q_q \langle \bar{q}q \rangle(\mu) \left( x_\beta \epsilon_\alpha - x_\alpha \epsilon_\beta \right) \int_0^1 dz \, e^{iz^\mu x} h_\gamma(z, \mu).$$

$$\langle 0| \bar{q}(x)[x, 0] \gamma_\alpha \, q(0)|\gamma(p) \rangle = g_{em} \, Q_q \, f_{3\gamma}(\mu) \epsilon_\alpha \int_0^1 dz \, e^{iz^\mu x} \psi_\gamma(z, \mu).$$

$$\langle 0| \bar{q}(x)[x, 0] \gamma_\alpha \gamma_5 \, q(0)|\gamma(p) \rangle = \frac{g_{em} Q_q f_{3\gamma}(\mu)}{4} \epsilon_{\alpha\beta\rho\sigma} \, p^\rho \, x_7 \epsilon_\beta \int_0^1 dz \, e^{iz^\mu x} \psi_\gamma^{(s)}(z, \mu), \quad (11)$$

where $\psi_\gamma(z, \mu), \psi_\gamma^{(s)}(z, \mu)$ are twist-3 and $A_\gamma(z, \mu), h_\gamma(z, \mu)$ are twist-4. Employing the light-cone expansion of the $u, d$-quark propagator and keeping the subleading-power contributions to the correlation function (5) leads to

$$G_\mu(p', q) \supset \frac{1}{\sqrt{2}} \int \frac{d^4k}{(2\pi)^4} \int d^4x \, e^{i(k-q)\cdot x} \frac{k^\nu}{k^2} \sum_{q=u,d} \delta_q q_{qem} \langle 0| \bar{q}(x) \sigma_{\mu\nu} \gamma_5 q(0)|\gamma(p') \rangle \, - (q \leftrightarrow -p)$$

$$= \frac{i}{2\sqrt{2}} \epsilon_{\mu\rho\sigma} \int \frac{d^4k}{(2\pi)^4} \frac{k^\nu}{k^2} \int d^4x \, e^{i(k-q)\cdot x} \sum_{q=u,d} \delta_q q_{qem} \langle 0| \bar{q}(x) \sigma^{\rho\sigma} q(0)|\gamma(p') \rangle$$

$$- (q \leftrightarrow -p), \quad (12)$$

where $\delta_u = 1, \delta_d = -1$. The above equation indicates that only twist-2 and twist-4 two-particle LCDAs can contribute to pion transition form factor in the LCSRs approach, which is different
Figure 1: Diagrammatical representation of the tree-level contribution to the QCD amplitude $\widetilde{G}_\mu$ with the contribution from two-particle photon LCDAs.

Figure 2: Diagrammatical representation of the tree-level contribution to the three-particle photon LCDAs.

from the method based on TMD factorization \cite{39}. Making use of the definitions in Eq.(11), it is straightforward to write down

$$G_{\mu}^{2{\text{PHT}}} (p, q) = -\frac{g_{\text{em}}^2}{4}\epsilon^\nu_{\mu\alpha\beta} q^\alpha p^\beta \frac{Q_u^2 - Q_d^2}{\sqrt{Q^4}} (\bar{q}q)(\mu) \int_0^1 du \left[ \frac{A(u, \mu)}{(\bar{u} + ur)^2} + \frac{\bar{A}(u, \mu)}{(u + r\bar{u})^2} \right], \quad (13)$$

where the contribution from $h_\gamma(z, \mu)$ vanishes due to the anti-symmetric structure. The resulting LCSRs for the two-particle higher-twist hadronic photon corrections to the pion transition form factors can be further derived as follows

$$F_{\gamma^*\gamma \rightarrow \pi^0}^{2\text{PHT}} (Q^2) = -\frac{\sqrt{2}}{4f_\pi} \frac{(Q_u^2 - Q_d^2)}{\mu_\pi(\mu)} (\bar{q}q)(\mu) \left\{ \frac{1}{Q^2} \bar{A}(u_0) e^{-\frac{s_0 - m_p^2}{M^2}} \right. \right.$$  

$$+ \left. \int_{u_0}^1 \frac{du}{u^2} \frac{1}{M^2} \exp \left[ -\frac{\bar{u}Q^2 - um_p^2}{uM^2} \right] A(u, \mu), \right\} \quad (14)$$

where $u_0 = Q^2/(s_0 + Q^2)$.  

5
To compute higher-twist three-particle hadronic photon corrections to the pion transition form factors, the definition of three-particle photon LCDA is required. In the appendix we collect the definition of three-particle twist-4 photon LCDA for an incoming photon state. Keeping the one-gluon/photon part for the light-cone expansion of the quark propagator in the background gluon/photon field

$$ (0|T\{q(x), \bar{q}(0)\}|0) \ni i \int \frac{d^4k}{(2\pi)^4} \epsilon^{-ik\cdot x} \int_0^1 du \frac{ux\gamma_\mu}{k^2} \frac{k\sigma_{\mu\nu}}{2k^4} G^{\mu\nu}(ux) $$

$$ + igmQ_q \int_0^\infty \frac{d^4k}{(2\pi)^4} \epsilon^{-ik\cdot x} \int_0^1 du \frac{ux\gamma_\mu}{k^2} \frac{k\sigma_{\mu\nu}}{2k^4} F^{\mu\nu}(ux) $$

(15)

where $G^{\mu\nu} = i[D_\mu, D_\nu]$. By evaluating Fig.1b, we obtain

$$ \Pi_\mu(p,q) \ni \frac{1}{2\sqrt{2}} g^2 \sum_q \delta_q Q_q^2 \epsilon^{\mu\nu\lambda\rho} q^\nu \epsilon^\rho p^\lambda \langle \bar{q}q \rangle(\mu) \int_0^1 du \int [D\alpha_i] \frac{1}{[q-(\alpha_i + \bar{u}\alpha_g - 1)p]^4} $$

$$ \times \rho^{3PTH}(\alpha_i, u, \mu) - (q \leftrightarrow -p) $$

(16)

where

$$ \rho^{3PTH}(\alpha_i, u, \mu) = 2\{(2u-1)[T_1(\alpha_i) - T_2(\alpha_i) + T_3(\alpha_i) + T_4(\alpha_i) - \tilde{S}(\alpha_i) + T_4(\alpha_i)] $$

$$ + S(\alpha_i, \mu) + S_\gamma(\alpha_i, \mu) + T_2(\alpha_i, \mu) - T_1(\alpha_i, \mu)\} $$

(17)

and the integration measure is defined as

$$ \int [D\alpha_i] \equiv \int_0^1 d\alpha_q \int_0^1 d\bar{u} \int_0^1 d\alpha_g \delta(1 - \alpha_q - \alpha_g - \alpha_{\bar{q}}). $$

(18)

Taking advantage of quark-hadron duality, we arrive at the LCSRs of the contribution from three-particle photon LCDA

$$ F^{3PTH}_{\gamma^*\gamma^*\rightarrow\pi^0}(Q^2) = -\sqrt{2} \frac{(Q_w^2 - Q_0^2)}{2f_\pi \mu_\pi(\mu)} \langle \bar{q}q \rangle(\mu) \frac{1}{Q^2} \left\{ \int_0^{s_0/(s_0+Q^2)} d\alpha_q \int_{1-\alpha_q-s_0/(s_0+Q^2)}^{1-\alpha_q} \frac{d\alpha_g}{\alpha_q} \right. $$

$$ \times \rho^{3PTH}(\alpha_q, \alpha_g, \alpha_{\bar{q}} = 1 - \alpha_q - \alpha_g, u_{s_0}, \mu) e^{-\frac{s_0-m_\pi^2}{s_0}} $$

$$ + \frac{1}{M^2} \int_0^{s_0} ds e^{-\frac{s-m_\pi^2}{s_0}} \int_0^{s/(s+Q^2)} d\alpha_q \int_{1-\alpha_q-s/(s+Q^2)}^{1-\alpha_q} \frac{d\alpha_g}{\alpha_q} $$

$$ \times \rho^{3PTH}(\alpha_q, \alpha_g, \alpha_{\bar{q}} = 1 - \alpha_q - \alpha_g, u_s, \mu) \right\} $$

(19)
where \( u_s = [s/(s + Q^2) - \alpha_q]/\alpha_g \). The overall higher-twist photon LCDAs contribution is written by

\[
F_{HT}^{\gamma\gamma\rightarrow\pi^0}(Q^2) = F_{HT}^{2\text{PHT}}(Q^2) + F_{HT}^{3\text{PHT}}(Q^2).
\]

Now we discuss the scaling behavior of our results. The power counting scheme for the sum rule parameters are given below:

\[
s_0 \sim M^2 \sim \mathcal{O}(\Lambda^2), \quad \bar{u}_0 \sim \mathcal{O}(\Lambda^2/Q^2).
\]

Employing Eq.(21), one can obtain that the contribution from leading twist LCDA of photon is suppressed by a factor \( \Lambda^2/Q^2 \) [16] compared with LP contribution. The higher twist contributions are conjectured to be also suppressed by only one power of \( \Lambda^2/Q^2 \) due to the absent correspondence between the twist counting and the large-momentum expansion [32]. For the contribution from two-particle twist-4 LCDA of photon, the result in Eq.(14) is suppressed by \( \Lambda^4/Q^4 \) compared with LP contribution as the scaling behavior of twist-4 photon LCDA is suppressed with respect to leading twist one. While for the contribution from 3-particle twist-4 LCDAs in Eq.(19), the scaling of \( \alpha_q \) is \( \mathcal{O}(\Lambda^2/Q^2) \), and \( \alpha_g \) is \( \mathcal{O}(1) \). Although there is an overall factor \( 1/Q^2 \), the result is only suppressed by \( \Lambda^2/Q^2 \) for the spectral function \( \rho_{3\text{PHT}} \) is not suppressed at endpoint region. This result confirms the conjecture in [32].

3 Numerical analysis

In the following we explore the phenomenological consequences of the hadronic photon correction to the pion-photon form factor, and the most important input is the LCDAs of photons. The models of twist-4 LCDAs of photon used in this paper are written by

\[
A_\gamma(z, \mu) = 40 z^2 \bar{z}^2 \left[ 3 \kappa(\mu) - \kappa^+(\mu) + 1 \right] + 8 \left[ \zeta_2^+(\mu) - 3 \zeta_2(\mu) \right] \left[ z \bar{z} (2 + 13 z \bar{z}) + 2 z^3 (10 - 15 z + 6 \bar{z}^2) \ln z + 2 \bar{z}^3 (10 - 15 \bar{z} + 6 z^2) \ln \bar{z} \right],
\]

\[
h_\gamma(z, \mu) = -10 \left( 1 + 2 \kappa^+(\mu) \right) C_2^{1/2}(2z - 1),
\]

\[
S(\alpha_i, \mu) = 30 \alpha_i^2 \left\{ (\kappa(\mu) + \kappa^+(\mu)) (1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2 \alpha_g) \right\}
\]
these parameters satisfy the following relations

\[ S_\gamma(\alpha_1, \mu) = -60 \alpha_g^2 (\alpha_q + \alpha_{\bar{q}}) \left[ 4 - 7 (\alpha_q + \alpha_{\bar{q}}) \right], \]

\[ T_1(\alpha_1, \mu) = -120 \left( 3 \zeta_2(\mu) + \zeta^+_2(\mu) \right) (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_{\bar{q}} \alpha_g, \]

\[ T_2(\alpha_1, \mu) = 30 \alpha_g^2 (\alpha_q - \alpha_{\bar{q}}) \left[ (\kappa(\mu) - \kappa^+(\mu)) + (\zeta_1(\mu) - \zeta^+_1(\mu)) (1 - 2 \alpha_q) + \zeta_2(\mu) (3 - 4 \alpha_q) \right], \]

\[ T_3(\alpha_1, \mu) = -120 \left( 3 \zeta_2(\mu) - \zeta^+_2(\mu) \right) (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_{\bar{q}} \alpha_g, \]

\[ T_4(\alpha_1, \mu) = 30 \alpha_g^2 (\alpha_q - \alpha_{\bar{q}}) \left[ (\kappa(\mu) + \kappa^+(\mu)) + (\zeta_1(\mu) + \zeta^+_1(\mu)) (1 - 2 \alpha_q) + \zeta_2(\mu) (3 - 4 \alpha_q) \right], \]

\[ T_4^*(\alpha_1, \mu) = 60 \alpha_g^2 (\alpha_q - \alpha_{\bar{q}}) \left[ 4 - 7 (\alpha_q + \alpha_{\bar{q}}) \right]. \quad (22) \]

In the above equations, the conformal expansion of the photon LCDAs have been truncated up to the next-to-leading conformal spin. Due to the Ferrara-Grillo-Parisi-Gatto theorem \[40\], these parameters satisfy the following relations

\[ \zeta_1(\mu) + 11 \zeta_2(\mu) - 2 \zeta^+_2(\mu) = \frac{7}{2}. \quad (23) \]

The scale evolution of the nonperturbative parameters is given by

\[ \kappa^+(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma^+-\gamma_{qq})/\beta_0} \kappa^+(\mu_0), \quad \kappa(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma^-+\gamma_{qq})/\beta_0} \kappa(\mu_0), \]

\[ \zeta_1(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma^{Q(1)}_Q-\gamma_{qq})/\beta_0} \zeta_1(\mu_0), \quad \zeta^+_1(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma^{Q(5)}_Q-\gamma_{qq})/\beta_0} \zeta^+_1(\mu_0), \]

\[ \zeta^+_2(\mu) = \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{(\gamma^{Q(2)}_Q-\gamma_{qq})/\beta_0} \zeta^+_2(\mu_0), \quad (24) \]

where the anomalous dimensions at one loop read \[36\]

\[ \gamma^+ = 3 C_A - \frac{5}{3} C_F, \quad \gamma^- = 4 C_A - 3 C_F, \quad \gamma_{qq} = -3 C_F, \]

\[ \gamma^{Q(1)}_Q = \frac{11}{2} C_A - 3 C_F, \quad \gamma^{Q(3)}_Q = \frac{13}{3} C_F, \quad \gamma^{Q(5)}_Q = 5 C_A - \frac{8}{3} C_F. \quad (25) \]
\[
\chi(\mu_0) \quad \langle \bar{q}q \rangle(\mu_0) \quad b_2(\mu_0) \quad \kappa(\mu_0) \quad \kappa^+(\mu_0) \quad \zeta_1(\mu_0) \quad \zeta_1^+(\mu_0) \quad \zeta_2^+(\mu_0)
\]

| \chi(\mu_0) | \langle \bar{q}q \rangle(\mu_0) | b_2(\mu_0) | \kappa(\mu_0) | \kappa^+(\mu_0) | \zeta_1(\mu_0) | \zeta_1^+(\mu_0) | \zeta_2^+(\mu_0) |
|------------|-----------------|---------|---------|---------|---------|---------|---------|
| (3.15 \pm 0.3) GeV^{-2} | -(246^{+28}_{-26} \text{ MeV})^3 | 0.07 \pm 0.07 | 0.2 \pm 0.2 | 0 | 0.4 \pm 0.4 | 0 | 0 |

Table 1: Numerical values of the nonperturbative parameters entering the photon LCDAs at the scale \(\mu_0 = 1.0 \text{ GeV}\) [36, 41].

Numerical values of the input parameters entering the photon LCDAs up to twist-4 are collected in Table 1, where for the estimates of the twist-4 parameters from QCD sum rules [42] 100 % uncertainties are assigned.

Now we are in the position to investigate the phenomenological significance of the contribution of higher twist photon LCDAs. For the factorization scale in the evaluation of the contribution of higher-twist photon LCDAs, we will take the value \(\mu^2 = \langle x \rangle M^2 + \langle \bar{x} \rangle Q^2\) as widely employed in the sum rule calculations [32]. The Borel mass \(M^2\) and the threshold parameter \(s_0\) can be determined by applying the standard strategies described in [43, 44],

\[
M^2 = (1.25 \pm 0.25) \text{ GeV}^2, \quad s_0 = (0.70 \pm 0.05) \text{ GeV}^2.
\]  

It has been checked that the contribution of higher-twist photon LCDAs are almost independent of the Borel mass and the threshold parameter in the intervals in Eq.(26). In Fig.3 the \(Q^2\) dependence of the relevant power suppressed contributions is presented. Compared with the contribution from leading-twist photon LCDA, the two-particle twist-4 contribution is obviously suppressed as the curve declines more quickly and approaches zero at large \(Q^2\). While for the contribution from three-particle twist-4 LCDAs of photon, the result is comparable with that from leading twist photon LCDA, as they are at the same power. As mentioned in [16], there exists strong cancellation effect between the contribution from leading twist photon LCDA and the twist-4 pion LCDA, thus the overall power correction is mainly from the
contribution from twist-4 LCDAs of photon. The total result including power suppressed contributions are shown in Fig. 4. It can be seen that the higher power photon LCDAs manifestly modify the LP power result especially at small $Q^2$ region.

We present our final predictions for $Q^2 F_{\gamma^*\gamma\rightarrow\pi^0}(Q^2)$ with both LP contribution and power corrections included in Fig. 5, where the combined theory uncertainties are due to the variations of the input parameters $a_2, a_4$ in BMS model of pion LCDA, $\xi, \langle \bar{q}q \rangle, b_2$ in twist-2 photon LCDAs, $\kappa, \zeta_1, \zeta_2$ in twist-4 photon LCDAs, quark mass, and factorization scale, etc. The most important uncertainty comes from the shape parameters $a_2, a_4$ of leading twist pion LCDA. The power suppressed contributions are able to give sizable correction to low $Q^2$ region, and only slightly modify the behavior at large $Q^2$, but it is hard to accommodate BaBar and Belle data after including the power corrections. Taking power corrections into account, the predicted $Q^2 F_{\gamma^*\gamma\rightarrow\pi^0}(Q^2)$ from BMS model is larger than the experimental data, thus the power suppressed contributions should play an important role in the determination of the Gegenbauer moments of pion LCDAs. The pion transition form factor is still sensitive to the Gegenbauer moments of leading twist pion LCDA after the power suppressed contributions.
considered. Thus this process proves a good platform to determine the parameters in the LCDAs of pion, which can also be compared with the future lattice simulation with the help of quasi parton distribution amplitude [47]. It is noted that the NLO QCD corrections are not considered in this paper, which might provide sizable numerical corrections and improve the scale dependence behavior.

4 Closing remark

In this paper we performed a study on the power suppressed contributions from higher-twist LCDAs of photon within the LCSRs. The twist-3 LCDAs cannot contribute for their Lorentz structure, thus the contributions from two-particle and three-particle twist-4 LCDAs of photon are considered in this work. According to the power analysis, the three-particle twist-4 contribution is not suppressed compared with the leading twist photon LCDA result, so that the power corrections considered in this work can give rise to sizable contribution, especially at low $Q^2$ region. In addition, there exists strong cancellation between the contrition from lead-
Figure 5: Comparison between the theoretical predictions in this paper and the experimental data. Points from CLEO [45] (purple squares), BaBar [26] (orange circles) and Belle [46] (brown spades) are displayed here.

The importance of the twist-4 photon LCDA contribution is further highlighted. The numerical result indicates that after including power corrections, the predicted $Q^2 F_{\gamma \gamma \rightarrow \pi^0}(Q^2)$ from BMS model is no longer consistent with the experimental data, thus the power suppressed contributions should be included in the determination of the Gegenbauer moments of pion LCDAs. Note that for the higher-twist photon LCDAs contribution, we only presented a tree level calculation, the NLO QCD corrections might modify the current result to some extent and stabilize the factorization scale dependence. A more systematic study based on effective theory is necessary for a thorough understanding of the next-to-leading power corrections to the pion transition form factor.

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A Definition of three-particle twist-4 LCDAs of photon

In the following, we present the definition of the three-particle photon LCDAs up to twist-4.

$$\langle 0 | \bar{q}(x) g_s G_{\alpha \beta}(u \ x) \ q(0) | \gamma(p) \rangle$$

$$= i \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ (p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta) \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ S(\alpha_i, \mu)$$

$$= i \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ (p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta) \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ S(\alpha_i, \mu)$$

$$= \ g_{em} \ Q_q \ f_{3\gamma}(\mu) \ p_\rho \ (p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta) \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ A(\alpha_i, \mu)$$

$$= \ g_{em} \ Q_q \ f_{3\gamma}(\mu) \ p_\rho \ (p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta) \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ V(\alpha_i, \mu)$$

$$= i \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ (p_\beta \epsilon_\alpha - p_\alpha \epsilon_\beta) \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ S(\alpha_i, \mu).$$

$$\langle 0 | \bar{q}(x) \ A_{\rho \tau} \ g_s G_{\alpha \beta}(u \ x) \ q(0) | \gamma(p) \rangle$$

$$= - \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ [p_\rho \epsilon_\alpha g_{\tau \beta}^\perp - p_\tau \epsilon_\alpha g_{\rho \beta}^\perp - (\alpha \leftrightarrow \beta)] \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ T_1(\alpha_i, \mu)$$

$$- \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ [p_\rho \epsilon_\alpha g_{\tau \beta}^\perp - p_\tau \epsilon_\alpha g_{\rho \beta}^\perp - (\rho \leftrightarrow \tau)] \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ T_2(\alpha_i, \mu)$$

$$- \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ \frac{(p_\alpha x_\beta - p_\beta x_\alpha)(p_\rho \epsilon_\tau - p_\tau \epsilon_\rho)}{p \cdot x} \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ T_3(\alpha_i, \mu)$$

$$- \ g_{em} \ Q_q \langle \bar{q} q \rangle(\mu) \ \frac{(p_\rho x_{\alpha} - p_{\alpha} x_\rho)(p_\alpha \epsilon_\beta - p_\beta \epsilon_\alpha)}{p \cdot x} \ \int [D\alpha_i] \ e^{i(\alpha_\gamma + \bar{u} \alpha_\gamma - 1) p \cdot x} \ T_4(\alpha_i, \mu).$$
Note that we have employed the following notations for the dual field strength tensor and the integration measure

\[
\tilde{G}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\rho\sigma} G^{\rho\sigma}, \quad \int [\mathcal{D} \alpha_i] \equiv \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_g \delta (1 - \alpha_q - \alpha_{\bar{q}} - \alpha_g) \quad (34)
\]

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