3D Jet Tomography of Twisted Strongly Coupled Quark Gluon Plasmas

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The triangular enhancement of the rapidity distribution of hadrons produced in p+A reactions relative to p+p is a leading order in $(A^{1/3}/\log s)$ violation of longitudinal boost invariance at high energies. In A+A reactions this leads to a trapezoidal enhancement of the local rapidity density of produced gluons. The local rapidity gradient is proportional to the local participant number asymmetry, and leads to an effective rotation in the reaction plane. We propose that three dimensional jet tomography, correlating the long range rapidity and azimuthal dependences of the nuclear modification factor, $R_{AA}(\eta,\phi,p_{T}; b > 0)$, can be used to look for this intrinsic longitudinal boost violating structure of $A + A$ collisions to image the produced twisted strongly coupled quark gluon plasma (sQGP). In addition to dipole and elliptic azimuthal moments of $R_{AA}$, a significant high $p_{T}$ octupole moment is predicted away from midrapidity. The azimuthal angles of maximal opacity and hence minima of $R_{AA}$ are rotated away from the normal to the reaction plane by an ‘Octupole Twist’ angle, $\theta_3(\eta)$, at forward rapidities.

I. INTRODUCTION

The experimental discovery of a new form of strongly interacting Quark Gluon Plasma matter (called the sQGP) at the Relativistic Heavy Ion Collider (RHIC) has raised many new questions about the physics of ultra-dense matter produced in relativistic nuclear collisions. Striking new phenomena observed thus far include jet quenching, fine structure of elliptic flow, nuclear gluon saturation, and the baryon-meson anomaly. The primary experimental parameters available at the RHIC are the beam energy, $\sqrt{s} = 20 - 200$ AGeV, and the variation of nuclear geometry through different $B + A$ combinations. Variations of the global nuclear geometry via impact parameter or centrality cuts adds another set of important experimental methods and instruments with which the geometric configuration of the sQGP can be systematically varied. In non central collisions of nuclei, the transverse area of the participant interaction region has an elliptic shape with an aspect ratio or eccentricity that can be varied in a controlled way via global multiplicity cuts. In this paper we explore a more subtle aspect of the 3D geometry of sQGP produced in non central A+A collisions that can be explored by extending jet tomography techniques to full 3D taking the third “slice” dimension to be the rapidity variable.

Strong elliptic transverse flow resulting from transverse elliptic asymmetry provides the key barometric probe of the sQGP equation of state. A common simplifying assumption in most hydrodynamic and transport analyses of the data thus far is that the initial conditions are longitudinally boost invariant. However, the rapidity dependence of elliptic flow data clearly demonstrate important deviations of the global collective dynamics from boost invariance away from the midrapidity region. Even the most advanced 3+1D hydrodynamic calculations, which relax the ideal 2+1D Bjorken initial conditions, have not yet been able to reproduce the observed rapidity dependence of elliptic flow. The solution may require better understanding of the geometry of the initial sQGP in addition to better theoretical control over the growing dissipative hadronic corona at high rapidities.

As we show below, the dimensionless parameter that controls violations of longitudinal boost invariance in A+A reactions at finite energies is the local relative rapidity slope of the low transverse momentum partons

$$\delta = \frac{A^{1/3} - 1}{2Y},$$

where $2Y = \log s$ is the rapidity gap between the projectile and target. In $Au + Au$ at RHIC $\sqrt{s} = 200$ AGeV, $\delta \approx 0.45$ cannot be ignored. Even in $Pb+Pb$ at the Large Hadron Collider (LHC) with $\sqrt{s} = 5500$ AGeV, $\delta \approx 0.28$ will not be much smaller.

In this paper, we explore some observable consequences of such violations of longitudinal boost invariance. We predict that there is an intrinsic rotation of the initially produced matter in the reaction plane in non-central $A + A$ that can induce an octupole moment of the nuclear modification factor $R_{AA}(\eta \neq 0, \phi, p_{T}; b > 0)$ at moderate
to high $p_\perp$ away from mid-rapidities. This effect arises from the combination of two basic properties of high energy hadron-nucleus reactions with jet quenching dynamics in $A + A$.

One important property is that the rapidity density of produced hadrons is observed $\propto$ to be Triangular relative to the density produced in $p + p$ reactions with a slope given by $\sim \delta$. The second basic property follows from the Eikonal-Glauber reaction geometry that predicts local variations of the projectile and target participant number density in the transverse plane, $x_\perp$. We propose that the long range correlations of the rapidity and azimuthal dependence of jet quenching can be used to extend current 2D $R_{AA}(\phi)$ jet tomography to “image” the 3D geometry of the twisted sQGP initial state at forward rapidities $|\eta| \sim 1 - 3$.

In the next section we begin with a review of the $p + A$ triangle and its implications for $A + A$ initial conditions. In Section III we discuss the role of more realistic diffuse nuclear geometry on the local participant and binary collision densities. In Section IV we calculate jet opacities as a function of $\eta, \phi$. In Section V jet attenuation through the tilted bulk sQGP density in the reaction plane is studied, and in Section VI the azimuthal Fourier moments of the nuclear modification factor are computed as a function of rapidity. Our proposed new observable, the Octupole Twist angle $\theta_3(\eta)$, is computed. We conclude in Section VII.

II. THE $p+A$ TRIANGLE

In $p+A$ reactions, the ratio $R_{pA}(\eta) = \frac{dN_{pA}/d\eta}{dN_{pp}/d\eta}$ of produced low $p_\perp$ hadrons as a function of rapidity, $\eta$, was found at all energies to be “triangular” with the height near the rapidity ($-Y$) of the target nucleus proportional to $\nu_A \propto A^{1/3}$, the average number of “wounded target nucleons” $\nu_A$.

(Note that we are using the variable $\eta$ to denote the particle rapidity rather than the particle pseudorapidity, which $\eta$ is conventionally linked to. This is to avoid later confusion with the variable $y$, used to denote the transverse vertical coordinate in this paper.) Excess nuclear enhancement $\propto A^{1/3+q}$ near the target rapidity is due to intra-nuclear cascading of low relative energy hadrons produced in the target fragmentation region. We will not consider this extra nuclear modification further in this paper.

The projectile proton in the center of rapidity frame has rapidity, $Y$, and interacts at a transverse impact parameter, $b$, on the average with

$$\nu_A(b) \approx \sigma_{in} T_A(b) = \sigma_{in} \int d\rho_A(z, b) \propto A^{1/3}$$ (2)

target nucleons with rapidity $-Y$. Here $\sigma_{in}(\sqrt{s})$ is the NN inelastic cross section, and the Glauber profile function, $T_A(b)$ measures the number target nucleons per unit area of nucleus with proper density $\rho_A(r)$.

![FIG. 1: The approximate triangular form of the ratio of the observed $D + Au$ charged particle pseudorapidity distributions to the dashed HIJING $p + p$ curve from Fig. 1 are shown for two centrality cuts. The solid curves are predictions using HIJING 1.383 for $D + Au/p + p$. The ratio of UA 5 $p + \bar{p}$ data to the baseline HIJING $p + p$ predictions are shown by filled squares. (In color online)](image-url)
the HIJING v1.383 code[14]. (In color online)

A

Reactions at 200 AGeV from PHOBOS[12] are compared to

FIG. 2: Asymmetric pseudorapidity distributions of charged

hadrons from \(-Y < \eta < y^*\). The approximately tri-

FIG. 3: Schematic illustration of how local trapezoidal nuclear
eenhancements of the rapidity distributions in the reac-
tion plane \((x, \eta, y = 0)\) twist the bulk initial density about

the normal in non central \(A + A\) collisions. (see eqs.4-5) (In color online)

FIG. 4: As in Fig.3 from another perspective. The straight

segments in each rapidity slice are for emphasis only. Real-

istic diffuse spherical geometries curve these lines (see Fig.7

below). (In color online)

BGK[10] generalized this model to predict “trape-

zoidal” distributions that would arise when a row of \(\nu_B > 1\) projectile nucleons interacts inelastically with a row of \(\nu_A > 1\) target nucleons. This very basic feature of multi-nucleon interactions in pQCD is at the core of the phenomenological success of LUND string models such as FRITIOF[17] and HIJING[14]. The curves shown in Figs.1 and 2 are predictions of the HIJING model[14] using version 1.383 that includes the Hulthen distribution of nucleons in Deuterium. The magnitude and centrality dependence of the charged particle rapidity distributions are well predicted by the basic trapezoidal nuclear enhancement of multi-particle production dynamics. We note that Wang[18] studied this rapidity asymmetry also as a function of \(p_T\) out to moderately high \(p_{T\perp}\) to test Cronin versus shadowing dynamics away from midrapidity.

While the global features of the \(p_{T\perp}\) integrated \(dN/d\eta\)

are well accounted for, we focus in this paper on the novel local implications of the trapezoidal violations of longitudinal boost invariance for the initial conditions in \(B + A\). In a non-central collision at an impact parameter \(b\), the local enhancement of bulk particle production relative to \(p + p\) at a particular transverse coordinate \(x_{\perp}\) is approximately given in the BGK model by

\[
\frac{dR_{BA}}{d^2x^\perp}(\eta, x^\perp; b) = \frac{dN^{BA}/d\eta d^2x^\perp}{dN^{pp}/d\eta}
= \nu_A(x^\perp - b/2)Y - \eta \over 2Y + \nu_B(x^\perp + b/2)Y + \eta \over 2Y . (3)
\]

Even if \(p + p\) were longitudinally boost invariant with a constant \(dN^{pp}/d\eta\), the variation of \(\nu_A\) and \(\nu_B\) with \(x^\perp\) leads to local violations of longitudinal boost invariance in non-central \(B + A\). The local deviations are controlled by

\[
\delta(x^\perp; b) = \frac{\nu_B - \nu_A}{2Y} , (4)
\]

i.e. the local participant asymmetry \(\nu_T - \nu_P\) at \(x_{\perp}\).

For very large energies, \(Y \gg 1\), Bjorken longitudinal boost invariance is restored. However for \(Au + Au\) at RHIC, \(Y \approx 5, A^{1/3} \approx 6\), and there is an order unity variation of \(\delta\) across the transverse profile of the interaction region that leads to the twisted ribbon geometry of the local density distribution as illustrated schematically in Figs. 3,4. In these figures, the coordinate normal to the reaction plane , \(y\), is set to zero, and the surface \(dR_{AA}\) is illustrated as a function of \(x = x_T\) in different \(\eta\) slices. In reality only the fixed \(x\) slices have trapezoidal shape while the \(\eta\) slices are rounded due to the nonlinear dependence of the \(\nu_T\) on transverse coordinates.

At the global \(dN/d\eta\) level, the integration over the
transverse coordinates leads to

\[ R_{BA}(\eta; b) = \frac{dN^{BA}/d\eta}{dN^{pp}/d\eta} \]

\[ \approx \left\{ \frac{1}{2}(N_A + N_B) + \frac{\eta}{2Y}(N_B - N_A) \right\}, \]

(5)

where \( N_A = \int d^2x_1 \nu_A(x_1; b) \) and \( N_B \) are the total number of wounded target and projectile nucleons interacting at impact parameter \( b \). What is remarkable about eqs. (3) and readily seen on in Figs 3.4 is that for symmetric \( A + A \) reactions, the global ratio may appear to be approximately boost invariant, but the local density retains an intrinsic rapidity asymmetry that can be characterized by a rotation twist in reaction plane

\[ T_{x\eta} = \frac{\partial^2}{\partial \eta \partial x} \log \left( \frac{dN}{dyd^2x} \right), \]

(6)

where \( x \) and \( y \) are the coordinates in and out of reaction plane. A useful property of this characterization is that it is independent of the normalization of \( dN/d\eta \) and independent of the multiplicative rapidity modulation factor \( dN^{pp}/d\eta \).

For a sharp sphere geometry with

\[ \nu \propto \sqrt{R^2 - (x \pm b/2)^2 - y^2} \]

(7)

the \( x\eta \) twist in the center \( x = y = 0 \) of the oval interaction region is independent of \( \eta \) and given by

\[ T_{x\eta}(x_\perp = 0) = \frac{1}{R} \frac{2b}{4R^2 - b^2}. \]

(8)

The twist would of course be absent if nuclei were uniformly thick. The magnitude for realistic spherical nuclei depends on the actual diffuse Wood Saxon geometry that we use for our numerical estimate below. However, eqs. (3) shows that its typical magnitude of \( T_{x\eta} \) is \( 1/(RY) \) in minimum bias reactions.

We also note that the trapezoidal distribution of low \( p_T \) multi-particle production in \( D + Au \) follows more generally if the collinear factorized QCD mini-jet dynamics of BGK is replaced by the \( k_T \) factorized gluon fusion mechanism in the Color Glass Condensate (CGC) model. Quantitative differences between these mechanisms arise at moderate \( p_T > 2 - 5 GeV \) which are currently under investigation at RHIC. They are connected with different physical approximations to Cronin enhancement and gluon shadowing.

The empirical trapezoidal form in both BGK and CGC cases differs from local equilibrium initial conditions based on firetube or firestreak models. While such models also predict an effective tilt in the \( x\eta \) plane, these corresponds to possible initial rapidity shear of the hydrodynamic fluid at fixed \( \eta \). In our approach, there is no initial rapidity shear and the collective longitudinal “fluid” velocity vanishes in the comoving \( (y = \eta) \) frame over the whole transverse region. However, there is an initial gradient of the density along the beam axis that can induce a fluid shear during later evolution. Polarization has been proposed as an observable sensitive to rapidity shear. We concentrate here on generic trapezoidal (BGK, HIJING, or CGC) initial “twisted sQGP” conditions because they follow directly from QCD parton dynamics and well constrained by \( p + A \) phenomenology.

III. DIFFUSE NUCLEAR GEOMETRY

To take the diffuse nuclear geometry into account, we recall that the local projectile \( B \) and target \( A \) participant densities is given in Glauber theory by

\[ \frac{dN^B}{dx dy} = T_B(r_+)(1 - e^{-\sigma_{in}T_A(r_-)}) \]

\[ \frac{dN^A}{dx dy} = T_A(r_-)(1 - e^{-\sigma_{in}T_B(r_+)}) \]

(9)

where \( r_\pm = \sqrt{(x \pm b/2)^2 + y^2} \) and \( \sigma \approx 42 \) mb at \( \sqrt{s} = 200 \) AGeV. Rare jets are produced on the other hand distributed in the transverse plane according to the the binary collision number distribution given by

\[ \frac{dN_{\text{bin}}}{dx dy} = \sigma_H T_B(r_+) T_A(r_-) \]

(10)

where \( \sigma_H(p_{L\perp}, \eta) \) is the pQCD jet cross section. For \( B = A \) collisions, the binary distribution is obviously symmetric in the transverse plane at any impact parameter, unlike the participant distributions. Nuclear shadowing and Cronin effects can break this symmetry, but for the present study we assume here that at high enough \( p_T \), these initial state nuclear effects can be neglected.

Jet quenching at a given rapidity in azimuthal direction \( \phi \) depends on a transverse coordinate line integral across the sQGP density profile at that rapidity. The initial sQGP density is assumed here to be proportional to the local participant density. In the BGK model this is then approximately given by

\[ \frac{dN_a}{dx dy d\eta} = C \frac{\exp[-\eta^2/\sigma^2]}{2Y} \theta(Y - |\eta|) \]

\[ \left\{ \frac{dN_{\text{Part}}}{dx dy} (Y - \eta) + \frac{dN_{\text{Part}}}{dx dy} (Y + \eta) \right\}, \]

(11)

where we have introduced a Gaussian rapidity envelope to model \( dN^{pp}/d\eta \) at RHIC. With \( Y = 5, \sigma = 3 \) and \( C \approx 24 \), the \( x_\perp \) integrated \( dN_a/d\eta \times 1/3 \) fits the BRAHMS central \( Au + Au \rightarrow \pi^+, \pi^- \) data as shown in Fig. 5.

Figures 3 and 4 show the 3D diffuse participant geometry of the twisted sQGP initial conditions in the transverse plane in several impact parameter and rapidity slices. For \( b = 0 \) there is complete azimuthal symmetry at all rapidities of course. For finite \( b \) the initial sQGP density shifts to the left at forward rapidities and to the right in backward rapidities with
our conventions. The shifts are small < 2 fm, but distinct. There is also a small increase in the eccentricity, \( \epsilon(\eta) = \langle (x - \langle x \rangle)^2 - y^2 \rangle / \langle (x - \langle x \rangle)^2 + y^2 \rangle \), of the twisted sQGP density away from midrapidity as shown in Fig. 9.

**IV. AZIMUTHAL DEPENDENCE OF THE TWISTED SQGP OPACITY**

Unlike the participant scaling bulk sQGP density, the binary jet distribution, [14], does not manifest a rapidity asymmetry at least at enough high \( p_T \) where shadowing is expected to be small. The jet production points are therefore not rotated away from the beam direction in this limit. This is illustrated in Fig. 9.

Jet quenching due to energy loss in the medium depends on the opacity and hence density of the plasma in the direction of propagation [3, 26]. The mismatch between the distribution of jet production points and the rotated distribution of the bulk sQGP matter therefore induced a peculiar azimuthal dependence of the weighed

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**FIG. 5:** A Gaussian fit of the \( \pi^\pm \) rapidity density in central \( Au + Au \) at 200 AGeV is shown compared to BRAHMS data. (In color online)

**FIG. 6:** Contours of the twisted sQGP initial density in the transverse plane \((x, y)\) in different rapidity \( \eta = -3, 0, 3 \) and impact parameter \( b = 0, 6, 9 \) fm slices. Note the opposite transverse shifts at \( \eta \) and \(-\eta\). (In color online)

**FIG. 7:** Transverse profile of the twisted sQGP at \( b = 6 \) Fm as a function of transverse coordinate \( x \) at \( y = 0 \) for \( \eta = \pm 2, \pm 4 \). The normalization is to the local transverse participant density \( fm^{-2} \). The overall decrease of the density away from midrapidity is due to the approximate Gaussian envelope in Fig. 5. (In color online)

**FIG. 8:** The rapidity dependence of the elliptic eccentricity of the twisted sQGP. (In color online)
opacity

\[ \chi_\alpha(x_0, y_0, \eta, \phi, b) = c_\alpha \int_0^\infty dt \, t^\alpha \]

\[ \frac{dN_f}{dx dy d\eta}(x_0 + t \cos(\phi), y_0 + t \sin(\phi), b). \quad (12) \]

Different \( \alpha = -1, 0, 1 \) weights correspond to different mechanisms of energy loss. Elastic scattering energy loss through longitudinally expanding\( ^4 \) (or static) sQGP matter corresponds to \( \alpha = -1 \) (0). Inelastic radiative energy loss through longitudinally Bjorken expanding (or static) sQGP matter corresponds, on the other hand, to \( \alpha = 0 \) (1). The results turned out numerically to be within 10% in all cases. Therefore, we drop from now on the \( \alpha \) index and show results only for \( \alpha = 0 \).

We shown in Figs. 10 11 the \( \phi \) dependence of the opacity for different \( b \) and \( \eta \). In all cases the opacity is normalized to the maximal \( x_0 = y_0 = b = \eta = 0 \) opacity

\[ \chi_0 \equiv \chi_0(0, 0, 0, 0) \quad (13) \]

In Fig. 10 \( \chi_0(0, 0, \eta, \phi)/\chi_0 \) is shown for a jet produced at the origin \( x = y = 0 \) for \( b = 3, 6, 9 \) fm. For each \( b \), we consider the azimuthal variation at \( \eta = 0, \pm 2, \pm 4 \). First, note that the magnitude of the opacity is proportional to the decreasing central density as either \( b \) or \( |\eta| \) increases. Second, the amplitude of the elliptic second moment \( \langle \cos(2\phi) \rangle \) increases as the eccentricity of the sQGP increases with \( b \) as seen in Fig. 8. Third, the maximum opacity at \( \eta = 0 \) is in directions perpendicular to the impact parameter vector where there is more matter and minimum parallel to it. These features and trends are the familiar ones in the conventional standard Bjorken scenario.24

However, the new twists caused by the intrinsic longitudinal boost violating \( \delta = O(1) \) initial conditions illustrated in Fig. 9 are (1) the emergence of a long range rapidity asymmetry between +\( \eta \) and −\( \eta \) with a 180° phase shifted azimuthal dependence and (2) an azimuthal pattern that has odd harmonics in addition to quadrupole \( v_2 = \cos(2\phi) \) moment. It is clear that there is a significant high \( p_T \) \( v_1 = \cos(\phi) \) dipole moment away from midrapidity. This is manifest in the difference between \( \chi(\phi = 0, \eta) \) and \( \chi(\pi, \eta) \) as expected from Fig. 9.
The observable is the nuclear modification factor 
\[ R_{AA}(\eta, \phi; b) = \frac{1}{N_{\text{Bin}}(0)} \int d^2x_0 \frac{dN_{\text{Bin}}(x_0, b)}{d^2x} e^{-\kappa \chi(x_0, \eta, \phi, b)} \] (16)

We simplify the computation of \( \kappa \) by simply fitting the observed approximately \( p_T \) independent central collision \( R_{AA} \approx 0.2 \) in \( A + Au \) at \( \sqrt{s} = 200 \) AGeV. (This is achieved if \( \kappa \approx 0.25 \) in units where the central opacity for \( b = 0 \) is \( \chi_0 = 11 / fm^2 \).

The computed jet survival probability, \( R_{AA} \), as a function of azimuthal angle for different rapidities and impact parameters can be seen in Figure 12. The solid black line in the figure is the \( R_{AA} = 0.2 \) independent of \( \phi \) is baseline fit for \( \eta, b = 0 \). The main features to note are that as in Figs. 10,11 but with opacity maxima replaced by slightly shifted \( R_{AA} \) minima. First, the magnitude of \( R_{AA} \) increases with both increasing impact parameter as well as \( \eta \). Next, note that at mid rapidity, \( \eta = 0 \), there is a symmetry between \( \phi = 0 \) and \( \phi = \pi \) as usual. The elliptic distribution at \( b > 0 \) but \( \eta = 0 \) leads to the usual minimum of the survival probability at \( \phi = \pi / 2, 3\pi / 2 \).

However, the reflection symmetry is broken away from \( \eta < 0 \) by the jet source and bulk sQGP are shifted relative to each other. As expected from Figs. 10,11, the minimum survival probability is rotated away from the reaction plane normal in opposite directions in the forward and backward rapidity region. To better quantify this effect we introduce in the next section the octupole twist observable.
quantify the magnitude of these long range rapidity correlations, we normalize Figure 13. For each fixed rapidity slice and impact parameter, compare the jet survival probability in forward and backward rapidity on the same polar plot as shown in Fig. 9. This leads to the highest survival probability in the longest axis oriented normal to the reaction plane as seen in Fig. 13. The long range twisted sQGP initial conditions lead on online) to the unit circle independent of $\phi$, $\eta$, $b$. For $b = 6$ fm, the minima are at $\phi = 0, \pi$, and maxima are at $\phi = 0, \pi$. However, for both $b > 0$ and $|\eta| > 0$, twisted sQGP initial conditions lead to an octupole “pear” deformation with the minima shifted to $\phi = 0, \pi, 2\pi - \eta, 2\pi + \eta$. The octupole twist angle, $\theta_3(\eta)$, is a measure of the long range in rapidity correlations caused by intrinsic local $O(A^{1/3}/Y)$ violations of longitudinal boost invariance in $A+\bar{A}$. (In color online)

VI. THE OCTUPOLE TWIST AND FOURIER DECOMPOSITION

One way to visualize the azimuthal asymmetry of $R_{AA}$ is to compare the jet survival probability in forward and backward rapidity on the same polar plot as shown in Figure 13. For each fixed rapidity slice and impact parameter, we normalize $R_{AA}(\eta, \phi; b)$ by its minimum $R_{AA}^{\text{min}}$ over $\phi$ and plot the resulting $R_{AA}/R_{AA}^{\text{min}}$ as a function of $\phi$.

For $b = 0$ collisions, this relatively normalized ratio is a unit circle independent of $\phi$ and $\eta$. It is shown as a thin circle. For $b > 0$, but $\eta = 0$, the transverse density profile has an elliptic shape centered at $x = y = 0$ with its longest axis oriented normal to the reaction plane as seen in Fig. 9. This leads to the highest survival probability in the reaction plane $\phi = 0, \pi$, as shown by the thick solid black line in Fig. 13.

For both $b > 0$ and $\eta > 0$, the bulk matter is shifted toward $\phi = \pi$ (in our conventions) leading to a smaller survival probability than in the $\phi = 0$ direction as shown by the solid (blue shifted) pear shape curve in the figure. The long range twisted sQGP initial conditions lead on the other hand for $\eta = -|\eta| < 0$ to an azimuthal pattern (red shifted dashed) that is simply the reflected version of the $\eta > 0$ curve about the $y$ axis. The pair of minima of both curves are indicated by the arrows.

We define an “octupole twist” angle, $\theta_3(\eta, b)$ to help quantify the magnitude of these long range rapidity correlations. To do this we use the fact that azimuthal harmonics are orthogonal polynomials.

\begin{equation}
\theta_3(\eta, b) = \phi(R_{AA}^{\text{min}}(\eta, b)) - \phi(R_{AA}^{\text{min}}(-\eta, b))
\end{equation}

The minima in the forward region are tilted to $(\pi - \theta_3)/2$ and $(3\pi + \theta_3)/2$ while in the backward region they are tilted in the opposite direction.

Note that the reaction plane orientation can be determined unambiguously experimentally via standard low $p_T$ bulk directed and elliptic flow $v_1, v_2$ [6]. Jet tomography of high $p_T$ hadrons provides a complementary short wavelength probe of the reaction dynamics and can also be decomposed into azimuthal harmonics as

\begin{equation}
R_{AA}(\eta, \phi, p_\perp; b) = a(1 + 2 \sum_n v_n \cos(n\phi))
\end{equation}

The numerical evaluation of the first four moments, $v_n(\eta, b)$, are shown in Figures 15 and 16. In addition to the usual $v_1, v_2$ and even $v_4$ moments, the new feature shown here is the existence of an octupole $v_3 = \langle \cos(3\phi) \rangle$ harmonic that is responsible for the pear shape deformation of $R_{AA}$ in Fig. 13. The magnitude is relatively small though, on the order of $v_4$. However, $v_4$ for low $p_T$ has
already been successfully measured, and with a large luminosity upgrade and advanced detectors, a measurement of \( v_3 \) at \( \eta = \pm 2 \) may thus become possible at a future RHIC II [29].

The predicted peak of the high \( p_T \) \( |v_1| \) at \( \eta \sim 2 \), (with opposite sign relative to the spectator \( v_1 \)) is another characteristic feature that can be used to test twisted sQGP initial conditions via 3D jet tomography.

VII. CONCLUSIONS

In this paper we applied the local trapezoidal multiparticle production BGK model to predict the magnitude of intrinsic longitudinal boost violating features of the bulk matter produced in \( A + A \) collisions. We noted that this is a generic feature of both local mini-jet based dynamics as encoded in LUND string and HIJING models as well as \( k_T \) factorized production models such as the CGC. The resulting geometrical picture is that the sQGP initial conditions has the expected transverse elliptical shape for non central collisions, but the bulk density is twisted away from the beam axis in the reaction \((x, \eta)\) plane.

In contrast, the binary collision distribution of hard pQCD jet sources remains reflection symmetric in the transverse plane oriented parallel to the beam axis at least in the approximation that Cronin and shadowing can be neglected at high enough \( p_T \). These approximations will be relaxed in a subsequent paper to explore further how the \( p_T \) dependence 3D jet tomography of twisted sQGP initial conditions can be used to further test different models of gluon saturation.

The generic displacement of the jet sources and bulk matter away from central rapidities was predicted to produce a novel jet absorption pattern that deviates from what is now known at mid-rapidities. The most intriguing is the predicted “octupole twist” of the minimum of \( R_{AA} \) away from the normal of the reaction plane and a peak of the jet \( v_3 \) near \( \eta \sim \pm 2 \). These can be further quantified in terms of a non-vanishing octupole harmonic of the azimuthal distribution. Even though the predicted \( v_3 \) is small, on the order of \( v_4 \) it may be measurable at RHIC II [29].

The long range in rapidity \( \Delta \eta \sim 4 - 6 \) correlations in-
duced by twisted sQGP initial conditions are expected to persist even at LHC energies. This is because the dimensionless parameter, $\delta \sim A^{1/3}/\log(s)$, that controls this phenomenon depends on the logarithm of the collision energy. Logarithms are very slowly varying functions of the energy and thus the rate of reduction in $\delta$ with energy will be concomitantly slow.

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