Vibration Characteristics of Graphene nano resonator as mass sensor

S H Desai¹, A A Pandya² and M B Panchal¹

¹Department of Mechanical Engineering, Institute of Technology, Nirma University, Ahmedabad, Gujarat, India- 382481.
²Department of Electronics and Communication Engineering, Institute of Technology, Nirma University, Ahmedabad, Gujarat, India- 382481.

Abstract. The decrease in the physical dimensions of the devices have sought the attention of research community due to quick response, high sensitivity and sturdiness of the devices. These devices in the form of nano resonators have been extensively used as sensors to detect the entity at submicron level as well as to identify the properties of matter at submicron level. With the process like chemical vapour deposition, lithography technique as well as mechanical exfoliation techniques, it has become possible to produce materials which are 2D in nature. The excellent mechanical and electrical properties of graphene as well as its complete plain geometry advocate it as an ideal material for the development of sensors used for identifying the object at nano level. Here an attempt is made to analyse the vibration characterization of graphene resonator in the form of membrane to understand the shift in the frequency by adsorbing the mass and change in the temperature. The tobacco mosaic virus is considered as a mass adsorbed onto the graphene nano ribbon based membrane. Along with the adsorption of the mass, the effect of variation in temperature is also introduced to observe the shift in the natural frequency of the graphene membrane based resonator.

Keywords. Graphene, Membrane, Frequency shift, Mass sensor, Temperature

1. Introduction

Now-a-days, with the state of the art manufacturing technologies available, the decrease in the physical dimensions of the devises has sought the attention of researchers. The devices used as resonators, nano-tweezers, pumps, switches have been extensively implemented for information processing, atomic level sensing and further to investigate and analyse the properties of matter at submicron level. Earlier, the devices were available in micron size but the micron sized resonator had a limitation of detecting the object at submicron level. To mitigate the problem along with the coping up of the requirements of quick response, high sensitivity and robustness of the devices, the need of nano electromechanical system (NEMS) resonator has arisen. In the typical NEMS, the prime element is the beam kind of material which oscillates under the application of the force [1, 2]. Fundamentally, NEMS can be defined as the conjunction of mechanical beam element with the electrical control system in which the a.c. or d.c. or combination of both signals are applied to perturb the mechanical beam element induce the oscillations at GHz level. Such ultrahigh frequency was challenging to attain in micron sized devices because of the physical dimensions compared to with the nano regime dimensions. The NEMS possesses motivating features which are (1) atomic level mass and force sensing capability atatto-newton level (2) Gas vapour detection (3) Virus detection (4) Ultrahigh natural frequency and Q-factor (5) sensing of heat in yoctocalorie [3-6].

In recent years, deliberated efforts have been dedicated for design the materials with planar geometry such as silicon carbide, LiNbO₃, MoS₂, Galium Arsenide, Si doped Aluminium Nitride,
Graphene, PZT and many more [7]. Out of these materials, graphene has emerged as a promising material for the construction of sensing devices at submicron level due to its planar geometry and high surface to volume ratio. Graphene possesses young’s modulus of elasticity about 1 TPa [8, 9], thermal conductivity of 5000 W/m-K [10, 11] with the surface density of 0.76 mg/m² [12]. Despite possessing low surface density, it can be deformed up to 25% higher than other materials which is useful for the tuning application of radio frequency devices. Due to its breaking strength 200 times higher than steel which is ductile material, it is used as accelerometers, pressure sensors, membranes and resonators [13]. Apart from mechanical properties, graphene possesses electron mobility ~15000 cm²/V·sec [14] which makes it a good electrical conductor. This property is conceptualized for making a nano sized electrode. In traditional piezoelectric resonator, especially for ultra-high frequency application, the loss of energy is due to bulky metal electrode attached to the active component [15] of resonant system which produce damping and interfacial strain which restrict the fundamental frequency, $f_0$ and quality factor, Q. The physical and electrical properties of the metal electrodes basically limit the volume and frequency scaling [16-18]. So arbitrary reduction in layer of metal electrode is not possible. Eliminating the physical contact between metal electrode and substrate by artificially creating nano gap is the encouraging solution for overcoming these losses. Being virtually massless (specific mass = $3.74 \times 10^{-6}$ g/µm²) [19] and chemically inert, graphene has minimum chemical interaction with underlying substrate and virtually floats over any substrate at van der Waals distances [19, 20]. The study shows that the graphene electrode is able to behave like an ideal massless electrode which enables piezoelectric NEMS devices to operate at theoretically unloaded frequency limits with enhanced electromechanical performance and reduced volume for a range of frequencies in terms of GHz. So, these properties advocate graphene a preferred candidate for the material of nano resonators. In addition to this, Graphene optical properties have drawn attention for its ability to absorb 2.3 percent of incident light over a broad range of wavelengths. In the Medical and biology sectors, the sensors made using graphene have been used for in vivo sensing of glucose in blood (University of Minnesota), detection of Escherichia coli (Indian Institute of Science, Bangalore) and detection of organic vapours (Case Western Reserve University) [21].

2. Literature Review

There have been efforts put on investigating the properties of the graphene to use as nano resonator based sensors. Bunch et al [22] prepared resonators varying from mono layer thickness to multilayer thickness and reported that graphene possesses high young’s modulus of elasticity in TPa, very light weight material and high surface to volume ratio make it suitable for nano resonator with high sensitivity for mass and displacement. Initially Poncharal et al [23] suggested the use of nano resonator mass sensors. In the work, minimum amount of mass ranging from $10^{-15}$ to $10^{-18}$ kg by observing the dynamic response of multiwall carbon nano tubes with cantilever configuration using the TEM (transmission electron microscopy) technique. This ability of attaining the capability of graphene as a mass sensor with mass resolution at an atomic level sensing motivated Arash et al. Arash et al [24] examined the potentiality of graphene based nano resonator as a sensor for the detection of noble gas. Their investigation reported the mass sensitivity of $10^{24}$ kg. Along with the experimental studies for determining the capability of graphene as mass sensor, attempts were made to identify the vibrational characteristics by considering the physical dimension ranging from micro level to submicron level with the different configurations of the resonators. In this regard, Dual et al [25] examined the effect of non-local parameter on the vibrational characteristics of CNT. They proposed a nonlocal beam model for thin beam to examine the free vibration of DWCNTs. Mahadvi et al[26] premeditated the non-linear behaviour of vibration for SWCNTs entrenched in polymer matrices. They modelled the vander-wall forces between CNT and matrix as a non-linear function of deflection of nano-tube. Further, they derived the relation between deflection amplitudes and natural frequencies of the nano tubes and observed very sensitive to boundary conditions and aspect ratios of the tube. Similarly Ansari et al [27] and Duan et al [25] performed the molecular dynamics simulation to investigate the free vibration of SWCNTs and DWCNTs at ambient temperature considering different end conditions. From their results, it is found that the resonant frequency of CNTs intensely relies on the configurations. Ansari et al [27] investigated
the free vibration of single layered graphene sheets considering different sizes and boundary conditions. The use of membrane made of graphene is now-a-days widely used for desalination of water. Surwade et al.[28]studied transport phenomena of ions and water through a suspended monolayer graphene membrane consists of stable nano scale pore produced by oxygen plasma etching and evaluated the effectiveness of the graphene membrane for desalination purpose.

Motivated by the information, an attempt is made to analyse the vibration characterization of graphene resonator in the form of membrane to understand the shift in the frequency by adsorbing the mass and change in the temperature.

3. Methodology

In the present work, the size of graphene nano ribbon based membrane (GNR) is considered as follows breadth, \( b = 600 \text{ nm} \) and Thickness of monolayer, \( t = 0.335 \text{ nm} \) and Length, \( L = 1200 \text{ nm} \). The surface density of the graphene is 0.76 mg/m\(^2\) and modulus of elasticity considered is 1 TPa. The graphene membrane is supported on silicon substrate and the membrane fixed from all edges is considered for nano sized resonator to analyse the natural frequencies and their mode shapes. The membrane has negligible resistance to shear or bending forces and the restoring forces arises from in-plane stretching or tensile forces. For the membrane working as piezoelectric sensor, it is important to know the natural frequency. There are various approaches available like Rayleigh-Ritz method, Hamilton’s principle, Euler-Bernoulli thin beam theory, S. P. Timoshenko thick beam theory etc. As for graphene nano ribbon, the lateral dimensions are very small compared to longitudinal dimension and effect of rotary inertia and shear are neglected due to mono layer resonator, an equilibrium approach is considered for the graphene nano ribbon based membrane. The application of newton’s second law of motion yields the equation of motion for the forced vibration of the membrane as [32]

\[
P \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + f = \rho \frac{\partial^2 w}{\partial t^2} \quad (1)
\]

For free vibration, \( f(x, y, t) = 0 \)

\[
P \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = \rho \frac{\partial^2 w}{\partial t^2} \quad (2)
\]

Where \( P \) is the surface tension per unit length and \( \rho \) is the density per unit area

Using the variable of separation method, \( w(x, y, t) = W(x, y) \times T(t) \) \quad (3)

Now \( W(x, y) = X(x) \times Y(y) \) \quad (4)

So, equation (2) can be written as

\[
P \left( \frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} + \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} \right) = \rho \frac{1}{T(t)} \frac{\partial^2 T(t)}{\partial t^2} \quad (5)
\]

Since the left hand side of the equation (5) is a function of \( x \) and \( y \) only and the right-hand side of the equation is a function of \( t \) only, each side must be equal to a constant \( k \).

\[
P \left( \frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} + \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} \right) = \rho \frac{1}{T(t)} \frac{\partial^2 T(t)}{\partial t^2} = k = -\omega^2 \quad (6)
\]

Two equations are formed from equation (6) where one equation is related to the displacement and another is related to with the time.

\[
\frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} + \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} = -\omega^2 \quad \frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} = -\omega^2 \quad (7)
\]

\[
\frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} = -\omega^2 \quad \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} = -\omega^2 \quad (8)
\]

Equation (7) can be expressed as

\[
\frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} + \omega^2 \frac{\alpha}{c^2} = -\frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} \quad (9)
\]

To find the \( X(x) \) and \( Y(y) \), equate both L.H.S. and R.H.S. of equation (9) to zero

\[
\frac{d^2 X(x)}{dx^2} + \alpha \frac{X(x)}{c^2} = 0 \quad (10)
\]

\[
\frac{d^2 Y(y)}{dy^2} + \beta \frac{Y(y)}{c^2} = 0 \quad (10.1)
\]

\[
-\frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} = 0 \quad (11)
\]

The \( Y(y) \) cannot be zero. So it is written as
\[
\frac{d^2Y(y)}{dy^2} + Y(y)\beta^2 = 0 \quad \text{where } \beta^2 = \frac{\alpha^2}{c^2} - \alpha^2
\]  
(11.1)

The solutions of the equations (10.1, 11.1) are expressed as follows:

\[
X(x) = C_1 \cos \alpha x + C_2 \sin \alpha x 
\]  
(12)

\[
Y(y) = C_3 \cos \beta y + C_4 \sin \beta y 
\]  
(13)

For determining the unknowns \( C_1 \) and \( C_2 \) in equation (12), \( C_3 \) and \( C_4 \) in equations (13), boundary conditions are applied in the form of \( X(0) = X(a) = 0 \) and \( Y(0) = Y(b) = 0 \)

So, \( X(x) = C_2 \sin \alpha x \) and

\[
Y(y) = C_4 \sin \beta y 
\]  
(14)

For non-trivial solutions of \( X(x) \) and \( Y(y) \), the conditions \( X(a) = Y(b) = 0 \) require that

\[
\sin \alpha a = 0 
\]

\[
\sin \beta b = 0 
\]  
(16)

So, the eigenvalues are,

\[
\alpha_m = m\pi \quad \text{where } m = 1, 2, \ldots
\]  
(18)

\[
\beta_n = n\pi \quad \text{where } n = 1, 2, \ldots
\]  
(19)

So, the natural frequencies of the membrane can be determined as

\[
\omega_{mn} = \pi C \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}
\]  
(20)

4. Results and Discussion

Graphene is non-piezoelectric in its pristine form due to its centro-symmetry. To induce the virtue of piezoelectricity, a mechanism in the form of adatoms, mechanical load or thermal load must be applied to break the centro-symmetry [29]. In the present work, fluorine (F) atom is considered as adsorbed onto the surface of graphene membrane uniformly and graphene membrane is lying on the silicon substrate with its all edges fixed. Because of ad-atom, the density of the F-doped graphene is 1.96 mg/m². The graphene membrane is exposed to detect the mass under the influence of variation in the temperature from 300 K to 200 K. The graphene has a tendency to expand with reduction in the temperature below room temperature because of its negative thermal expansion coefficient \(-3.75 \times 10^{-6}/K\) [30]. The object has tendency to expand in all directions under the thermal load. Because of this, there is a thermal interaction between the surfaces of mono-layer graphene and the substrate, the net effect is the elongation in the graphene nanoribbon based membrane. With the reduction in the temperature, the membrane elongates and so is the increase in the surface tension. Thus, graphene nanoribbon based membrane experiences a pre-stressed condition which increases the natural frequency of the system. In the work, the mass of single Tobacco mosaic virus (i.e. \(6.20 \times 10^{-8}\) kg)[31] is considered as adsorbed mass on the surface of F-doped graphene to determine the shift in the frequency of graphene nanoribbon based resonator with the variation in temperature from 300 K to 200 K. The Tobacco Mosaic virus is considered at the centre of nano ribbon based nano resonator. The density of the ribbon is changed to 2.06 mg/m². Simulations in Ansys has been performed for temperatures ranging from 300 K to 200 K. The model is generated using the design modeller and then imported to thermal-structural analysis followed by Modal analysis. The membrane is clamped from all the sides.

Now, table 1 and 2 show the effect of built-in tension on the natural frequency of the graphene based nano resonator. The results depict that with increase in the temperature, graphene elongates which causes to induce a pre-stressed condition. Hence the natural frequency of the nano resonator increases. The results also depicts that with increase in mass, natural frequency of the two-dimensional resonator decreases. If mass increases, the frequency will decrease further which indicates that change in mass is inversely proportional to natural frequency of the graphene membrane. There is an error of \(\pm 6\%\) between the results obtained through analytical and simulation approach. Thus, the obtained results using both approaches are in good agreement with each other.

| Table 1. Effect of built-in tension onto the natural frequency and shift in the frequency of the graphene based nano resonator under the effect of temperature without the mass of TMV |  |  |
Temperature (°K) | Change in temperature, ΔT = T - 300 | Built-in tension | Nature Frequency of the fundamental mode (Without TMV) | Error (%) |
|-----------------|--------------------------------|------------------|-----------------------------------------------|----------|
|                 |                               |                  | Analytical                                    | Finite Element |
| 298             | -2                             | 2.5125 × 10⁻³    | 2.0906 × 10⁸                                 | 0.101048825 |
| 290             | -10                            | 1.2563 × 10⁻²    | 4.6867 × 10⁸                                 | 2.696204003 |
| 280             | -20                            | 2.5125 × 10⁻²    | 6.6279 × 10⁸                                 | 4.247788572 |
| 270             | -30                            | 3.7688 × 10⁻²    | 8.1175 × 10⁸                                 | 0.92560895 |
| 260             | -40                            | 5.0250 × 10⁻²    | 9.3733 × 10⁸                                 | 4.3466324  |
| 250             | -50                            | 6.2813 × 10⁻²    | 1.0480 × 10⁹                                 | 2.144096298|
| 240             | -60                            | 7.5375 × 10⁻²    | 1.1480 × 10⁹                                 | 2.786867218|
| 230             | -70                            | 8.7937 × 10⁻²    | 1.2399 × 10⁹                                 | 1.925379128|
| 220             | -80                            | 1.005 × 10⁻¹     | 1.3255 × 10⁹                                 | 4.449962975|
| 210             | -90                            | 1.1306 × 10⁻¹    | 1.4060 × 10⁹                                 | 0.12081024 |
| 200             | -100                           | 1.2563 × 10⁻¹    | 1.4821 × 10⁹                                 | -5.117723464|

Table 2. Effect of built-in tension onto the natural frequency and shift in the frequency of the graphene based nano resonator under the effect of temperature with the mass of TMV

Temperature (°K) | Change in temperature, ΔT = T - 300 | Built-in tension | Nature Frequency of the fundamental mode (With TMV) | Error (%) |
|-----------------|--------------------------------|------------------|-----------------------------------------------|----------|
|                 |                               |                  | Analytical                                    | Finite Element |
| 298             | -2                             | 2.5125 × 10⁻³    | 2.0444 × 10⁸                                 | 1.160898764 |
| 290             | -10                            | 1.2563 × 10⁻²    | 4.571 × 10⁸                                  | 2.771369625 |
| 280             | -20                            | 2.5125 × 10⁻²    | 6.465 × 10⁸                                  | 2.77134591 |
| 270             | -30                            | 3.7688 × 10⁻²    | 7.918 × 10⁸                                  | 3.156810463|
| 260             | -40                            | 5.0250 × 10⁻²    | 9.1429 × 10⁸                                 | -2.38996392 |
| 250             | -50                            | 6.2813 × 10⁻²    | 1.0222 × 10⁹                                 | -0.027694377|
| 240             | -60                            | 7.5375 × 10⁻²    | 1.1197 × 10⁹                                 | 2.865066468|
| 230             | -70                            | 8.7937 × 10⁻²    | 1.2095 × 10⁹                                 | 3.853098148|
| 220             | -80                            | 1.005 × 10⁻¹     | 1.2930 × 10⁹                                 | 2.862574274|
| 210             | -90                            | 1.1306 × 10⁻¹    | 1.3714 × 10⁹                                 | -0.32462843|
| 200             | -100                           | 1.2563 × 10⁻¹    | 1.4456 × 10⁹                                 | -6.410132515|

![Graph showing the effect of temperature on frequency with and without TMV](image)
Figure 1. Frequency–>Temperature for graphene membrane bases two-dimensional nano resonator

The plot of frequency versus temperature depicts that the frequency increases with decrease in temperature. This is due to negative thermal expansion coefficient of graphene. Below the room temperature, the graphene expands and strain is induced. As the temperature decreases, the strain increases so is the increase in the built-in stress. Due to this, the graphene membrane based nano resonator experiences a pre-stressed condition which results in the rise in the natural frequency with decrease in temperature. So, it can be concluded that the temperature plays a vital role on the vibration characteristics of the graphene membrane based nano resonator.

4.1 Mode-shape analysis
(k). At 250 K

(l). At 250 K

(m). At 240 K

(n). At 240 K

(o). AT 230 K

(p). AT 230 K

(q). At 220 K

(r). At 220 K

(s). At 210 K

(t). At 210 K

(u). At 200 K

(v). At 200 K
Conclusion
In the present analysis, the natural frequency of the graphene based nano resonator is obtained using membrane theory and finite element analysis for with and without considering the single tobacco mosaic virus. From the results, it can be concluded that as the mass deposits onto the surface of the nano resonator, the natural frequency reduces. Also it can be depicted from the results that the natural frequency increases with increase in the built-in tension. This is due to the attribute of negative thermal expansion coefficient of graphene and change in the temperature. The graphene membrane based resonator expands with reduction in the temperature which results in the form of surface tension. Due to the surface tension, the pre-stressed condition occurs in the graphene membrane which tends to increase the natural frequency of the graphene membrane based resonator. Regarding this, the mode shape analysis has been performed for the temperature varying from 300 K to 200 K. The obtained results for both the cases are close with each other with an error of ±6 %. The present analysis can be extended for developing the graphene based nano resonator as mass sensor and the study can be extended to investigate the effect of location of foreign mass onto natural frequency of the nano resonator.

References
[1]. Verbiest G J, Kirchhof J N, Sonntag J, Goldsche M, Khodkov T, Stampfer C 2018 Detecting ultrasound vibrations with graphene resonators Nano letters. 18(8), pp. 5132-7.
[2]. Craighead H G 2000 Nanoelectromechanical systems Science 290(5496), pp. 1532-5.
[3]. Cleland A N and Roukes M L 1996 Fabrication of high frequency nanometer scale mechanical resonators from bulk Si crystals Applied Physics Letters, 69(18), p. 2653-2655.
[4]. Roukes M L 1999 Yoctocalorimetry: phonon counting in nanostructures Physica B: Condensed Matter (263), pp. 1-15.
[5]. Rueckes T, Kim K, Joselevich E, Tseng G Y, Cheung C L, Lieber C M 2000 Carbon nanotube-based nonvolatile random access memory for molecular computing. 289(5476), pp. 94-7.
[6]. Ilic B, Craighead H G, Krylov S, Senaratne W, Ober C, Neuzil P J 2004 Attogram detection using nanoelectromechanical oscillators Journal of applied physics 95(7) pp. 3694-703.
[7]. Rais-Zadeh M, Gokhale V J, Ansari A, Faucher M, Théron D, Cordier Y, Buchaillot L 2014 Gallium nitride as an electromechanical material. Journal of Microelectromechanical Systems 23(6) pp.1252-71.
[8]. Akita S, Nakayama Y, Mizooka S, Takano Y, Okawa T, Miyatake Y, Yamanaka S, Tsuji M, Nosaka T (2001) Nanotweezers consisting of carbon nanotubes operating in an atomic force microscope Applied Physics Letters 79(11) pp.1691-3.
[9]. Kim P and Lieber C M 1999 Nanotube nanotweezers Science, 286(5447) pp.2148-2150.
[10]. Guo Z, Zhang D, Gong XG 2009 Thermal conductivity of graphene nanoribbons Applied physics letters 95(16) p. 163103.
[11]. Kinaret J M, Nord T, Viefers S 2003 A carbon-nanotube-based nanorelay Applied Physics Letters 82(8) pp.1287-9.
[12]. Pandya A, Jha P K 2017 Electron transport parameters study for transition metal-doped armchair graphene nanoribbon via acoustical phonon interactions Journal of Electronic Materials 46(4) pp. 2340-6.
[13]. Lee C, Wei X, Kysar J W, Hone J 2008 Measurement of the elastic properties and intrinsic strength of monolayer graphene science 321(5887) pp. 385-8.
[14]. Ke C, Espinosa H D 2004 Feedback controlled nanocantilever device Applied Physics Letters 85(4) pp.681-3.
[15]. Ke C, Espinosa H 2005 *Handbook of theoretical and computational nanotechnology* (American Scientific Publishers), 1 pp. 1-38.

[16]. Qian Z, Hui Y, Rinaldi M, Liu F, Kar S 2013 *Single transistor oscillator based on a Graphene-Aluminum Nitride nano plate resonator* In2013 Joint European Frequency and Time Forum & International Frequency Control Symposium (EFTF/IFC) pp. 559-561 IEE.

[17]. Rinaldi M, Zuniga C, Zuo C, Piazza G 2009 *Super-high-frequency two-port AlN contour-mode resonators for RF applications* IEEE transactions on ultrasonics, ferroelectrics, and frequency control 57(1) pp. 38-45.

[18]. Chen C, Lee S, Deshpande V V, Lee G H, Lekas M, Shepard K, Hone J 2013 *Graphene mechanical oscillators with tunable frequency* Nature nanotechnology, 8(12) pp. 923-927.

[19]. Qian Z, Liu F, Hui Y, Kar S, Rinaldi M 2015 *Graphene as a massless electrode for ultrahigh-frequency piezoelectric nanoelectromechanical systems* Nano letters 15(7) pp. 4599-4604.

[20]. Aitken Z H, Huang R 2010 *Effects of mismatch strain and substrate surface corrugation on morphology of supported monolayer graphene* Journal of Applied Physics 107(12) p. 123531.

[21]. Bogue R 2014 *Graphene sensors: a review of recent developments* Sensor Review.

[22]. Bunch J S, Van Der Zande A M, Verbridge S S, Frank I W, Tanenbaum D M, Parpia J M, Craighead H G, McEuen P L 2007 *Electromechanical resonators from graphene sheets* Science 315(5811) pp. 490-3.

[23]. Poncharal P, Wang Z L, Ugarte D, De Heer W A 1999 *Electrostatic deflections and electromechanical resonances of carbon nanotubes* science 283(5407) pp. 1513-6.

[24]. Arash B, Wang Q, Varadan V K 2011 *Carbon nanotube-based sensors for detection of gas atoms* Journal of Nanotechnology in Engineering and Medicine 2(2).

[25]. Duan W H, Wang C M, Zhang Y Y 2007 *Calibration of nonlocal scaling effect parameter for free vibration of carbon nanotubes by molecular dynamics* Journal of applied physics 101(2) p. 024305.

[26]. Mahdavi M H, Jiang LY, Sun X 2009 *Nonlinear vibration of a single-walled carbon nanotube embedded in a polymer matrix aroused by interfacial van der Waals forces* Journal of Applied Physics 106(11) p. 114309.

[27]. Ansari R, Sahmani S, Arash B 2010 *Nonlocal plate model for free vibrations of single-layered graphene sheets* Physics Letters A 375(1) pp. 53-62.

[28]. Surwade S P, Smirnov S N, Vlassiouk I V, Unocic R R, Veith G M, Dai S, Mahurin S M 2015 *Water desalination using nanoporous single-layer graphene* Nature nanotechnology 10(5) pp. 459-64.

[29]. Ong M T and Reed E J 2012 *Engineered piezoelectricity in graphene* ACS nano. 6(2) pp. 1387-94.

[30]. Pop E, Varshney V, Roy A K 2012 *Thermal properties of graphene: Fundamentals and applications* MRS bulletin 37(12), 1273-1281.

[31]. Bionumbers.hms.harvard.edu. 2020. *Molecular Weight - Tobacco Mosaic Virus (TMV) - BNID 105958,* Available at: <https://bionumbers.hms.harvard.edu/bionumber.aspx?y=id=105958&lnsh=1> [Accessed 2 September 2020].

[32]. Rao S S 2007 *Vibration of continuous systems* (New York: Wiley)