Lensing and the Warm-hot Intergalactic Medium

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Abstract

The correlation of weak lensing and Cosmic Microwave Anisotropy (CMB) data traces the pressure distribution of the hot, ionized gas and the underlying matter density field. The measured correlation is dominated by baryons residing in halos. Detecting the contribution from unbound gas by measuring the residual cross-correlation after masking all known halos requires a theoretical understanding of this correlation and its dependence with model parameters. Our model assumes that the gas in filaments is well described by a log-normal probability distribution function, with temperatures $10^5 - 10^7$ K and overdensities $\xi \lesssim 100$. The lensing-Comptonization cross-correlation is dominated by gas with overdensities in the range $\xi \approx [3-33]$; the signal is generated at redshifts $z \lesssim 1$. If only 10% of the measured cross-correlation is due to unbound gas, then the most recent measurements set an upper limit of $T_e \lesssim 10^8$ K on the mean temperature of the intergalactic Medium. The amplitude is proportional to the baryon fraction stored in filaments. The lensing-Comptonization power spectrum peaks at a different scale than the gas in halos, making it possible to distinguish both contributions. To trace the distribution of the low-density and low-temperature plasma on cosmological scales, the effect of halos will have to be subtracted from the data, requiring observations with larger signal-to-noise ratios than are currently available.

Key words: cosmic background radiation – cosmology: observations – cosmology: theory – gravitational lensing: weak – intergalactic medium

1. Introduction

A recent account of the baryon distribution in the local universe concluded that about half the baryons synthesized in the Big Bang have yet to be identified (Shull et al. 2012), confirming an earlier deficit of baryons found by Fukugita et al. (1998). Numerical simulations (Cen & Ostriker 1999; Davé et al. 1999, 2001; Cen & Ostriker 2006; Smith et al. 2011) indicated that only 10%–20% of all baryons are in collapsed objects. Baryons in the intergalactic medium (IGM) exist in a wide range of densities and temperatures. Penton et al. (2004) and Lehner et al. (2007) concluded that another ~30% reside in low-redshift Lyα absorption systems, while the rest reside in the shock-heated IGM with temperatures $10^5 - 10^7$ K and overdensities $\xi \lesssim 100$. This unbound gas is usually known as warm-hot intergalactic medium (WHIM). Identification of the WHIM phase and its spatial distribution at low redshift is an ongoing theoretical and observational effort (for a review, see McQuinn 2016). The low density makes it difficult to detect the WHIM in emission (Soltan 2006); it is more promising to use absorption lines in the far-ultraviolet to soft X-ray range, but some earlier detections remain controversial (Shull et al. 2012). Cappelluti et al. (2012) and Roncarelli et al. (2012) searched for the contribution of the WHIM to the diffuse X-ray emission, but failed to find a statistically significant result.

Since the WHIM is highly ionized, there has been an extensive search of the thermal and kinematic Sunyaev–Zel’dovich CMB temperature anisotropies (hereafter tSZ and kSZ; Sunyaev & Zel’dovich 1970, 1972) generated by this baryon component (Atrio-Barandela & Mücket 2006; Atrio-Barandela et al. 2008). Cross-correlation of CMB temperature data from WMAP or Planck with matter templates produced only marginal evidence of tSZ anisotropies due to the WHIM (Génova-Santos et al. 2013, 2015; Suarez-Velásquez et al. 2013b). Combining X-ray and tSZ observations could be a promising tool to study the WHIM (Ursino et al. 2014). The first evidence of warm-hot gas beyond the virial radius of clusters was presented in Planck Collaboration (2013), who detected a filamentary structure between the cluster pair A399-A401. The distribution of gas in a cosmic web has also been confirmed by XMM-Newton observations of the cluster Abell 2744 by Eckert et al. (2015), who found filamentary structures of gas at $10^7$ K that were coherent over a scale of 8 Mpc. At those temperatures and densities, the kSZ effect can have a contribution of similar amplitudes to the tSZ effect. The kSZ effect has been used to trace large-scale peculiar velocity fields (Kashlinsky et al. 2008; Atrio-Barandela et al. 2015) and the anisotropies due to the pairwise velocity dispersion of clusters and galaxies have been measured (Hand et al. 2012; Soergel et al. 2016; Schaan et al. 2016; De Bernardis et al. 2017). These latter observations probe baryons on cluster and galaxy scales but have not yet provided a measurement of the fraction of free electrons. A search of the kSZ anisotropies due to the WHIM found no statistically significant evidence in WMAP data (Génova-Santos et al. 2009). Recently, Hernández-Monteagudo et al. (2015) and Planck Collaboration (2016) presented evidence of the peculiar motion of extended gas on Mpc scales with a statistical significance at the $3\sigma–3.7\sigma$ level. Hill et al. (2016) measured the kSZ effect correlating WMAP and Planck data with a galaxy sample from the Wide-field Infrared Survey Explorer verifying that baryons approximately trace the Dark Matter (DM) distribution down to ~Mpc scales.

The cross-correlation of gravitational lensing maps with tSZ anisotropies is another potential probe of the relation between the hot, ionized gas and the matter density field. Hill & Spergel (2014) determined the cross-power spectrum of weak lensing of the CMB with the tSZ anisotropies measured by Planck at the $6\sigma$ confidence level, obtaining a constraint on the bias between the hydrostatic mass and the true mass of clusters and groups at redshifts $z \lesssim 2.5$. These authors interpreted their...
signal as being produced by baryons in halos. In parallel, Van Waerbeke et al. (2014) found a detection of the cross-correlation between the tSZ signal from Planck and the galaxy lensing convergence from the Canada–France–Hawaii Telescope Lensing Survey (CFHTLenS) with the same level of significance. Originally, the data were interpreted as the signal from warm and diffuse baryons. Since the distribution of galaxies in the survey peaks at $z = 0.37$, this result suggested that a large fraction of the missing baryon population had been identified. New studies and numerical simulations demonstrated that the majority of the signal came from a small fraction of baryons within halos (Battaglia et al. 2015; Hojjati et al. 2015; Ma et al. 2015). On large angular scales the simulations showed a correlation slightly above that of the halo model prediction, pointing to a 10%–15% contribution from unbound gas. The latter contribution is degenerate with respect to cosmological and physical parameters and the data did not permit a robust inference (Battaglia et al. 2015). Hojjati et al. (2016) improved the statistical significance of the lensing–tSZ cross-correlation using a larger weak lensing map derived from the Red Sequence Cluster Lensing Survey (RCSLenS) and found that their signal was best interpreted if AGN feedback removed a large quantity of hot gas from galaxy groups.

To estimate the contribution of unbound gas to the tSZ–lensing cross-correlation results described above requires an analytical model that correctly predicts the amplitude and shape of the expected signal. In Atrio-Barandela & Mücke (2006) we described the unbound gas in the weakly nonlinear filaments by means of the log-normal probability distribution function (PDF). In this article we use this description of the unbound gas to predict the cross-correlation of the lensing convergence due to the large-scale matter distribution and the tSZ temperature anisotropies. The outline of this paper is as follows: In Section 2 we describe the model and compute the tSZ-convergence cross-correlation; the derived expressions are solved numerically and the results are presented in Section 3; finally, our conclusions are summarized in Section 4.

2. Lensing–tSZ Correlation in the Filament Model

The WHIM generates temperature anisotropies on the CMB via the tSZ. If $n_e$ and $T_e$ are the electron density and temperature along the line of sight, then the anisotropy generated by the free electrons residing in the potential wells of the WHIM filaments in units of the current CMB blackbody temperature $T_0$ is $\Delta T_{tSZ} / T_0 = Y_C G(\nu)$. The Comptonization parameter measures the integrated electron pressure along the line of sight, $Y_C = k_B \sigma_T / m_e c^2 \int n_e T_e \text{adv} \, dr$, with $\alpha$ as the scale factor, $w$ as the comoving radial distance, $m_e c^2$ as the electron annihilation temperature, $k_B$ as the Boltzmann constant, and $\sigma_T$ as the Thomson cross-section; $G(\nu) = (x \coth(x/2) - 4)$ gives the frequency dependence of the tSZ effect being $x = h \nu / k_B T_0$ the reduced frequency, $h$ being the Planck constant, and $\nu$ being the frequency of observation. This frequency dependence is different from that of any other known foreground, making the tSZ anisotropy possible to distinguish from other CMB anisotropies given sufficient multi-frequency coverage. The data are usually expressed in terms of the Comptonization parameter $Y_C$ instead of the temperature anisotropy.

The intrinsic CMB temperature anisotropies are lensed by the large-scale structure traced by galaxy catalogs. The tSZ anisotropies are themselves generated by the ionized gas within the same large-scale structure. The two-point correlation function of the lenses and the spatial variations of the electron pressure along the line of sight are the weighted average of the lensing kernel $\Delta \kappa_{eff}$ due to the large-scale structure traced by galaxy catalogs and the anisotropies generated by the tSZ effect of the ionized gas.

$$C(\theta) = \left\langle \kappa_{eff} Y_C \right\rangle \theta$$

$$= \int_0^{\theta_{\text{max}}} \int_0^{\theta_{\text{max}}} \left( \Delta Y_C(\hat{x}_1, w_1) \Delta \kappa_{eff}(\hat{x}_2, w_2) \right) d\theta_1 d\theta_2,$$  \hspace{1cm} (1)

where $\theta$ is the angle between the directions $\hat{x}_1$ and $\hat{x}_2$, i.e., $\cos \theta = \hat{x}_1 \cdot \hat{x}_2$ and $w_1, w_2$ are the comoving radial distances (for notation and definitions, see Bartelmann & Schneider 2001). The integration extends out to the redshift of the surface of the last scattering, $z_H$. The average $\left\langle \cdots \right\rangle$ takes into account the distribution of the WHIM filaments and that of the lenses and their correlation. Let us briefly summarize our WHIM model and the effect of a population of lenses before discussing their statistics.

2.1. The Log-normal Distribution of WHIM Filaments

Numerical simulations have shown that at redshifts $z > 1$ and at small scales the IGM forms filaments of mildly nonlinear overdensities, giving rise to the observed Lyα forest. At $z < 1$ most of the IGM baryons reside in shock-heated regions of low-density gas at temperatures 0.01–1 KeV (Shull et al. 2012) and sizes larger than 1 Mpc (Cen & Ostriker 2006). We model the distribution of this unbound IGM gas as a log-normal random field evolving with time. The log-normal PDF was introduced in Cosmology by Coles & Jones (1991) to describe the nonlinear distribution of matter in the universe when the peculiar velocity field was still in the linear regime. Based on the improved Wiener density reconstruction from the Sloan Digital Sky Survey, Kitaura et al. (2009) found that this distribution describes the statistics of the matter inhomogeneities on scales larger than $7h^{-1}$ Mpc. In the log-normal approximation, the number density of baryons at $x$, located at redshift $z$ and at a proper distance $|x(z)|$, is $n_B(x, z) = n_0(z) \xi$, where $\xi$ is a log-normal distributed random variable normalized to have unit mean, $\langle \xi \rangle = 1$, and $n_0(z) = f_B \rho_B (1 + z)^2 / m_p$ is the mean baryon number density, $\rho_B$ is the baryon density, $f_B$ is the fraction of baryons in the WHIM, $m_p$ is the proton mass, $\mu_B = 4/(8 - 5Y)$ is the mean molecular weight of the IGM and $Y$ is the He fraction by weight that we fixed to the value $Y = 0.24$. The nonlinear baryon density contrast $\xi$ in units of the baryon mean density should not be confused with $\delta$ or $\delta_B$, which are, respectively, the matter and IGM baryon overdensities in the linear regime; $\xi$ is given in terms of the Gaussian distributed variable $\delta_B$ (Choudhury et al. 2001; Atrio-Barandela & Mücke 2006)

$$\xi = e^{\delta_B (x, z) - \sigma_B^2 (z)/2},$$  \hspace{1cm} (2)

where $\sigma_B^2 (z) = \langle \delta_B^2 (x, z) \rangle$ is the variance of the zero-mean linear IGM baryon density field. The number density of
electrons in the IGM, \( n_e \), is obtained by assuming equilibrium between recombination and photoionization and collisional ionization. For the conditions of the IGM, temperatures in the range \( 10^5 \text{–} 10^7 \text{K} \) and density contrasts \( \xi \leq 100 \), the gas can be considered fully ionized, so \( n_e \approx n_B \).

The spectrum of density fluctuations of the baryons in the IGM is related to the DM density contrast \( \delta_{\text{DM}} \) by (Fang et al. 1993)

\[
\delta_B(k, z) = \frac{\delta_{\text{DM}}(k, z)}{1 + k^2 L_0^2(z)}. \tag{3}
\]

The cutoff length \( L_0 \) corresponds to the scale below which baryon density perturbations are smoothed due to physical processes. The variance of the baryon density field is given by

\[
\sigma_B^2(z) = \frac{D_z^2(z)}{2\pi^2} \int \frac{P_{\text{DM}}(k)}{1 + L_0^2(z)k^2} k^2 dk, \tag{4}
\]

where \( D_z(z) \) is the linear growth factor of matter density perturbations.

### 2.1.1. Baryon Damping Scales

At redshifts \( z \leq 1 \) small-scale baryon perturbations are erased by shock-heating (Klar & Mücke 2010). If \( T_{\text{IGM}} \) is the mean IGM temperature, the comoving cutoff scale \( L_0 \) is determined by the condition that the linear velocity perturbation \( v(x, z) \) averaged on a scale \( L_0 \) is equal to or larger than the IGM sound speed \( c_s = (k_B T_{\text{IGM}}(z)/m_p)^{1/2} \). The IGM temperature is determined by the evolution of the UV background. At redshifts \( z \leq 3 \), the temperature varies within the range \( T_{\text{IGM}} = [10^{5.3} \text{–} 10^7 \text{K}] \) and it is weakly dependent on redshift (Tittley & Meiksin 2007). For \( T_{\text{IGM}} = 10^5 \text{K} \) the sound speed is \( c_s \approx 10 \text{ km s}^{-1} \); in our subsequent analyses we will fix the sound speed to this value at all redshifts. In the linear regime and in comoving coordinates, \( \delta = - (1 + z) \nabla v \) and \( \delta = Hf \delta \), with \( f(z) = d \ln \delta / d \ln a \). In Fourier space, the peculiar velocity \( v(k) \) on a scale \( k = 2\pi / L_0 \) is 

\[
v(k, z) \approx (L_0 / 2\pi) H(z) \delta(k, z). \tag{6}
\]

The condition \( |v| \geq c_s \) and expressing \( \delta(k, z) = \delta_B(k)D_z(z) \), with \( \delta_B(k) \) as the current amplitude of the density contrast at wavenumber \( k = 2\pi / L_0 \), we obtain \( L_0 \approx [2\pi c_s(1 + z)/H(z)\delta_B D_z] \). This condition is valid only in the linear regime, hence the lower bound is obtained by imposing \( \delta_B \approx 1 \). Finally,

\[
L_0(z) = \frac{2\pi (1 + z)c_s H_0^{-1}}{(\Omega_\Lambda + \Omega_m (1 + z)^3)^{1/2} f(z) D_z(z)}, \tag{5}
\]

where \( H_0 \) is the Hubble constant and \( \Omega_\Lambda \) and \( \Omega_m \) are the energy density of the cosmological constant and matter density in units of critical density. In our numerical estimates, we fixed \( \Omega_m \) and \( \Omega_\Lambda \) to their concordance values. At \( z = 0 \) the comoving damping scale is \( L_0 \approx 1.7 \text{ h}^{-1} \text{ Mpc} \).

At redshifts \( z > 1 \), shock-heating becomes no longer so effective and the damping scale \( L_0 \) corresponds to the comoving Jeans length at the conditions of the photoionized IGM

\[
L_0(z) = H_0^{-1} \left[ \frac{2\gamma k_B T_0(z)}{3\mu m_p \Omega_m (1 + z)} \right]^{1/2}, \tag{6}
\]

where \( \gamma \) is the polytropic index and \( T_0 \) is the averaged background temperature of the IGM. This last parameter is minimally constrained by observations; Schaye et al. (2000) argues that at \( z \approx 3 \) He II reionization requires \( T_0 \) to be larger than \( 5 \times 10^4 \text{ K} \), while Viel & Haehnelt (2006) gave an upper bound of \( T \approx 2 \times 10^5 \text{ K} \). To simplify, we fix the average background temperature to the constant value \( T_0 = 10^5 \text{ K} \), within the interval allowed by observations.

### 2.1.2. The IGM Temperature

To describe the IGM distribution at all redshifts we will consider two limiting cases. At \( z > 1 \) the cutoff scale is the Jeans length given by Equation (6) and at \( z \leq 1 \) the cutoff scale is the shock-heated scale \( L_0 \) of Equation (5). To compute the tSZ contribution to CMB temperature anisotropies due to the IGM, we need to specify its temperature at each position and redshift. For the Jeans cutoff scale we assume the temperature follows a polytropic equation of state \( T(\xi, z) = T_0(\xi)^{\gamma-1} \). We take \( T_0(z) = 1.4 \times 10^4(1 + z)^{3/4} \text{ K} \), in agreement with the values obtained by Hui & Haiman (2003), with a weak dependence on redshift \( (\beta \approx 0) \). We chose \( \gamma = 1.5 \) and \( \beta = 1 \) as a conservative upper limit to WHIM tSZ anisotropies at \( z \geq 1 \). At \( z \leq 1 \), the shock-heated IGM has a complex distribution of densities and temperatures. Kang et al. (2005), hereafter K05, computes phase-space diagrams that can be fitted by the following equation of state:

\[
\log_{10}(T(\xi)/10^4 \text{ K}) = -2/\log_{10}(4 + \xi^{6+1/\beta}), \tag{7}
\]

which is valid for overdensities \( \xi \leq 100 \). Alternatively, Cen & Ostriker (2006), hereafter C06, find lower IGM temperatures; their phase-space diagram approximately corresponds to the equation of state \( \log_{10}(T(\xi)/10^4 \text{ K}) = -2.5/\log_{10}(4.0 + \xi^{6+9/\beta}) \). We have considered all equations of state to be independent of redshift, except for the polytropic one. These models are represented in Figure 1(a) as solid (black), dashed (blue), and dotted-dashed (red) lines corresponding to K05 with \( \alpha = (3, 1.5, 1) \), respectively. The triple dot-dashed (green) line corresponds to C06 and the dotted (gold) line corresponds to the polytropic model at \( z = 1 \).

The overall amplitude of the cross-correlation function is proportional to the fraction of baryons in the WHIM and of the mean temperature of the electron gas. In our numerical estimates we have assumed that this baryon fraction is the same at all redshifts and equal to \( f = 0.5 \). The overdensity-weighted temperature average \( T_e \approx [T(\xi)/\xi] \) depends on the temperature model. For the K05 and C06 models this average in the interval overdensity \( [\xi = [1, 100]] \) is \( T_e \approx [20, 7, 3, 0.7] \times 10^6 \text{ K} \), weakly dependent on redshift; for the polytropic model, whose equation of state varies with redshift, the mean temperature is in the range \( T_e \approx [0.4 - 1.7] \times 10^6 \text{ K} \). Any constraint on the amplitude of the cross-correlation will translate into an upper limit on the product \( f(T_e) \), and if \( f(T_e) \) is independently measured, then it would be a constraint on the mean temperature of the IGM, offering a direct probe onto the physical state of the WHIM.

### 2.2. Lensing Kernel

The gravitational field generated by weak density perturbations lenses the radiation propagating in the universe. The deflection angle of the weakly deflected rays can be related to an effective surface-mass density \( \kappa_{\text{eff}} \), known as convergence, closely related to the mass distribution (Bartelmann & Schneider 2001). The convergence due to a population of
The redshift distribution of galaxies is modeled as
\[ \int_0^{z_m} \frac{\delta(f_k(w)\hat{x}, w)}{a(w)} \, dw, \]  
where \( \hat{x} \) is the direction in the sky into which the light ray starts to propagate, \( f_k(w) \) is the comoving angular diameter distance at \( w \), \( \delta \) is the matter density contrast along the unperturbed light ray, and \( c \) is the speed of light. The kernel weights the relative contribution of lenses along the line of sight
\[ W(w) = \int_0^{w} dw' G(w') \frac{f_k(w' - w)}{f_k(w')}. \]

The redshift distribution of galaxies is modeled as
\[ p(z) = A(z/z_0)^b \exp[-(z/z_0)^{3/2}], \]
where \( z_0 \) is the effective depth of the lens population, related to the mean redshift of the distribution as \( z_m = 1.412 z_0 \) (Smail et al. 1995); the normalization constant is fixed by setting \( \int p(w) \, dw = 1 \). The distribution of galaxies in the CFHTLenS peaks at \( z = 0.37 \) (Van Waerbeke et al. 2014), which corresponds to \( z_0 = 0.3 \). These lens distributions are represented in Figure 1(b); dashed (blue), dotted–dashed (red), and triple dotted–dashed (green) lines correspond to \( z = 0.1, 0.3, 0.5 \) respectively. The more recent analysis by Hojjati et al. (2016) uses the deeper RCSLenS catalog that includes all galaxies with \( \text{mag}_r > 18 \). These authors provide a numerical fit of their lens distribution, plotted in Figure 1(b) with a solid (black) line.

From Equation (7), the contribution from lenses on a thin shell of width \( dz \) at comoving distance \( w = w(z) \) and direction \( \hat{x} \) is
\[ \Delta \kappa_{\text{eff}} = \frac{3H_0^2 \Omega_m}{2c^2} W(z) f_k(z) \frac{\delta(f_k(z)\hat{x}, z)}{a(z)} \frac{dw}{dz}. \]

In Figure 1(c) we plot the convergence of Equation (9) for \( \delta(f_k(w)\hat{x}, w) = 1 \) as a function of redshift for the four lens distributions given in Figure 1(b), with lines following the same conventions. The integration range of Equation (7) must extend up to the horizon \( w_H \) or up to a redshift \( z_H \) high enough to include the effect of all possible lenses. We took \( z_H = 5 \), and no significant differences were found when taking \( z_H = 10 \). This is expected since the lensing kernel drops exponentially following the distribution of the lensing sources (see Figures 1(b) and (c)).

Equation (9) was derived in the thin lens approximation and only linear terms in the density contrast were retained. Higher-order terms contain products of the density field, but while the
density contrast could be large for a density perturbation crossed by a given ray, the average overdensity is $\delta \ll 1$ for most rays and higher-order terms can be safely neglected (Bartelmann & Schneider 2001). Within this approximation the PDF of the lenses is that of the linear density field and consequently is well described by a Gaussian distribution.

2.3. Lensing–TSZ Cross-correlation

To compute the correlation function of lenses and WHIM sources of TSZ anisotropies given by Equation (1), the average $\langle \cdots \rangle$ has to account for the probability distribution of the WHIM filaments and that of the lenses. Let $dP(\xi, \delta) = F(\xi, \delta) d\xi d\delta$ be the probability that a filament with overdensity $\xi$ is located at $(\hat{x}_1, z_1)$ when an overdensity $\delta$ is at $(\hat{x}_2, z_2)$, with $F(\xi, \delta)$ as the associated PDF. Then, the average in Equation (1) can be written as

$$\langle \Delta Y_C(\hat{x}_1, z_1) \Delta \kappa_{\text{eff}}(\hat{x}_2, z_2) \rangle (\theta) = \int^{\hat{z}_{\text{up}}}_{\hat{z}_{\text{up}}} d\hat{z}_1 \int^{\hat{z}_{\text{up}}}_{\hat{z}_{\text{up}}} d\hat{z}_2 \int_{0}^{100} d\xi \int_{-\infty}^{\infty} d\delta \Delta Y_C(\hat{x}_1, z_1) \Delta \kappa_{\text{eff}}(\hat{x}_2, z_2) F(\xi, \delta),$$

(10)

with $\cos \theta = \hat{x}_1 \cdot \hat{x}_2$ and $\hat{z}_{\text{up}}$ being the highest redshifts beyond which WHIM and lenses do not generate a significant cross-correlation. To complete our model we need to specify the bivariate PDF of the lens-filament distribution, $F(\xi, \delta)$. As discussed above, lensing is dominated by a large-scale structure and the lensing overdensities $\delta$ are well described by a Gaussian PDF, but the nonlinear overdensities $\xi$ of the IGM filaments are distributed according to a log-normal PDF. Since $\xi = \Delta \rho / \bar{\rho}$ is log-normal distributed, $\log(\xi)$ follows a Gaussian distribution with mean $\mu_\xi = -\sigma_\xi^2 / 2$ and variance $\sigma_\xi^2$, in terms of this variable the probability can be written as $dP = G(\log(\xi), \delta) d\log(\xi) d\delta$, where $G$ is a bivariate Gaussian and

$$F(\xi, \delta) = \frac{1}{2\pi \sigma_\delta \sigma_\xi (1 - \rho_s)^{1/2}} \exp$$

$$\times \left[ \frac{1}{2(1 - \rho_s^2)} \left( \frac{(\log \xi - \sigma_\xi^2 / 2)^2}{\sigma_\xi^2} \right) - 2\rho_s \left( \frac{\log \xi - \sigma_\xi^2 / 2}{\sigma_\delta} (\delta - \mu_\delta) \right) + \frac{(\delta - \mu_\delta)^2}{\sigma_\delta^2} \right],$$

(11)

In this expression, $\mu_\delta$ is the mean of the matter density contrast, which in this case $\mu_\delta = 0$. The variance of the matter density field is $\sigma_\delta^2 = \langle \delta^2 \rangle / 2\pi^2 = P_{\text{DM}}(k) k^2 dk$. At small scales, $P_{\text{DM}}(k) \propto k^{-3}$ and the integral is logarithmically divergent. Therefore, we remove small-scale perturbations by filtering the density field with a top-hat window of radius $R_{\text{cut}} = 0.5 \, h^{-1} \, \text{Mpc}$. Physically this corresponds to removing the contribution from galaxies, groups, and clusters from the lensing kernel. Then,

$$\sigma_{\delta}^2 = \frac{D_2^2(\zeta)}{2\pi^2} \int P_{\text{DM}}(k) W_{\text{th}}^2(kR_{\text{cut}}) k^2 dk,$$

(12)

where $W_{\text{th}}(kR_{\text{cut}})$ is the Fourier transform of the top-hat filter. Changing the cutoff scale to $R_{\text{cut}} = 1 \, h^{-1} \, \text{Mpc}$ reduces $\sigma_\delta$ by a factor 0.85. The coefficient $\rho_s = \langle \log \xi \rangle / \sigma_\xi \sigma_\delta$ is the correlation between two Gaussian variables

$$\rho_s(r) = \frac{D_2(\zeta_1)D_2(\zeta_2)}{2\pi \sigma_\delta \sigma_\delta} \int \frac{P_{\text{DM}}(k)}{1 + L_0^2(\zeta_1)k^2} \times W_{\text{th}}(kR_{\text{cut}}) I_0(k|\bar{x}_1 - \bar{x}_2|) k^2dk,$$

(13)

with $I_0(\theta, z_1)$ being the transverse distance between two points located at the same redshift. Note that $\rho_s(0) = 1$, since the two distributions, IGM and lenses, are not fully correlated.

In Figure 1(d) we display the absolute value of the correlation coefficient for different cosmological parameters. We assume a flat universe, i.e., $\Omega_m + \Omega_\Lambda = 1$. The matter power spectrum is normalized to $\sigma_8 = 0.8$. We verified that varying the parameters within the ranges given in Figure 1(d) has an effect on the Comptonization-convergence cross-correlation that is small compared with the differences in the lens distribution or the equation of the state of the IGM temperature, so we will not discuss further variations of cosmological parameters and their effect on our results.

3. Results and Discussion

We compute the Comptonization-convergence cross-correlation using Equation (10). The integration over the lensing part extends up to the redshift of the last scattering surface. However, as Figure 1(c) indicates, the lensing kernel drops exponentially following the distribution of the lensing sources, and effectively we can stop the integration at $z_{\text{up}}$ when a similar drop factor has been reached. We verified that, as expected, extending the integration further does not increase the cross-correlation.

More delicate is deciding out to what redshift our model of the IGM is valid. At $z \gtrsim 1$ shock-heating stops being dynamically important. In Figure 2(a) we compute the amplitude of the effective lensing-Comptonization cross-correlation at zero lag, $C(0) = \langle \kappa_{\text{eff}} Y_C \rangle(0)$ as a function of the upper limit of integration $z_{\text{up}}$. The results, from top to bottom, correspond to the K05 model with $\alpha = 3, 1.5, 1$ (black solid, dashed blue, and dotted–dashed red lines) and the C06 model (triple dotted–dashed green line). In Figure 2(b) we plot the differential contribution. This figure indicates that most of the cross-correlation originates from $z \lesssim 1$.

Since the cross-correlation scales with the fraction of electrons in the IGM as $\langle \kappa_{\text{eff}} Y_C \rangle \propto (f_e/0.5)$, we need to know the fraction of electrons in the WHIM to translate constraints on $C(0)$ into constraints in the mean temperature of the gas. Although 80% of all baryons reside in $\text{Ly}_{\alpha}$ systems at redshift $z \approx 2$ and $f_e \lesssim 0.2$ at that redshift (Fukugita et al. 1998), numerical simulations indicate that $f_e \geq 0.4$ out to $z \approx 1$ (C06),
the range in redshift space that dominates the cross-correlation. Therefore, by taking $f_e = 0.5$ and constant our constraints on the mean WHIM temperature will be reasonably accurate.

The contribution of the IGM comptonization parameter to the cross-correlation from $z > 1$ is less than 10%. In fact, this correction is overestimated. First, the fraction of baryons in the WHIM drops with redshift. Second, and as mentioned in Sec 2.1.2, the IGM behaves as a polytrope and, on average, its temperature is smaller than the K05 shock-heated models and is similar to C06 (see also Figure 1(a)). In the interval $z > 1$ the cross-correlation with the polytropic equation of state and the damping scale of Equation (6) is $\langle \kappa_{\text{eff}} Y e \rangle \sim 1 - 10 \times 10^{-12}$ for the different lens distributions. This is a very small contribution and essentially we could have stopped our calculation at $z = 1$ or extended the shock model out to $z = 3$ since it would have introduced an error smaller than 10%. We adopted this latter

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**Figure 2.** Amplitude of the convergence-Comptonization cross-correlation at the origin, $C(0) = \langle \kappa_{\text{eff}} Y e \rangle(0)$ as a function of the upper limit of integration $z_1^{up}$. The results, from top to bottom, correspond to the K05 model with $\alpha = 3, 1.5, 1$ (black solid, dashed blue and dotted–dashed red lines) and the C06 model (triple dotted–dashed line). The lens distribution is the numerical fit to the RCSLenS sources. The baryon cutoff scale is given by Equation (5).

**Figure 3.** Comptonization parameter-lensing convergence cross-correlation (upper panels) and power spectra (lower panels) for lens source catalogs of different scale $z_0$. The cutoff scale is given by Equation (5). From top to bottom, lines correspond to the K05 model with $\alpha = 3, 1.5, 1$ (solid black, dashed blue, and dotted–dashed red, respectively) and the C06 model (triple dotted–dashed line). The corresponding power spectra are shown in the bottom panels, with lines following the same conventions. In (b) the correlation data and error bars were taken from Van Waerbeke et al. (2014).
optical and by not including the Jeans cutoff scale (Equation (6)) and its corresponding polytropic equation of state, we simplify the parameter space of our model and the physical interpretation of our results.

Figures 3 and 4 constitute our main result. In Figure 3 we plot the $\langle \kappa_{\text{eff}} Y_C \rangle$ cross-correlation for the three lens distributions with $z_0 = 0.1$, 0.3, 0.5 (upper panels) and their corresponding power spectra (lower panels). The power spectrum is computed from the correlation function integrating the quadrature

$$C_{\ell} = 2\pi \int \langle \kappa_{\text{eff}} Y_C \rangle P_{\ell} (\cos \theta) d \cos \theta,$$

with $P_{\ell}$ as the $\ell$th Legendre polynomial. Hence, we are required to compute the correlation function over the range $\theta = [0, \pi]$ rad. To simplify our calculation we have assumed the sky is flat (Equation (14)) and although this approximation limits the accuracy of the low-$\ell$ multipoles, it should be accurate for multipoles where data are available, $\ell \geq 100$.

In the panels of Figure 3, from top to bottom, the solid (black), dashed (blue), and dotted-dashed (red) lines correspond to the K05 model with $\alpha = 3$, 1.5, 1 and the triple dotted-dashed (green) line corresponds to the C06 model. The distribution of the CFHTLenS is well approximated by $z_0 = 0.3$, then in Figure 3(b) we also plot the data from van Waerbeke et al. (2014) and their respective error bars. In Figure 4 we plot the correlation function and the power spectrum for the same shock-heating temperature models but for the RCSLenS sources with mag$_r > 18$. Lines follow the same conventions as in Figure 3. The data are taken from Hojjati et al. (2016).

To analyze the contribution of the different IGM overdensities we divide the integration of Equation (10) in four intervals with equal logarithmic spacing: $\xi = ([1 - 3.3], [3.3 - 10], [10 - 33], [33 - 100])$. We computed the contribution in each interval for the K05 model with $\alpha = 1.5$ and for lens distributions $z_0 = 0.3$ (Figure 3(b)) and RCSLenS sources (Figure 4(a)). The fractional contribution to the correlation at the origin, $\langle \kappa_{\text{eff}} Y_C \rangle(0)$, was (0.08, 0.45, 0.41, 0.06) for the first case and (0.1, 0.39, 0.46, 0.05) in the second. Similar results occur for other lens distributions and temperature models: most of the correlation comes from overdensities in the range $\xi \approx [3-33]$. The numerical simulations of Davé et al. (2001) found that this is the density range where most of the WHIM is stored. In this respect, if our log-normal model were to be accurate only at these intermediate overdensities, integration of Equation (10) would still provide very accurate results.

The comparison of the measured data with the theoretical predictions already offers some insights into the nature of the IGM. For the CFHTLenS sources shown in Figure 3(b), all temperature models are allowed by the data. As indicated in Section 2.1.2, the shock-heating model with $\alpha = 3$ corresponds to an average temperature of $T_r = 2 \times 10^7$ K and is still compatible with the measured correlation. However, our results do not include the contribution due to clusters and galaxy groups. Since at most 15% of the measured signal comes from unbound gas, if we restrict the overall IGM contribution to be that fraction of the overall signal, then only models with $\alpha \leq 1.5$ are compatible with the data. In other words, the mean temperature of the IGM free electron gas would be $T_e \leq 7 \times 10^6$ K.

The data from Hojjati et al. (2016) shown in Figure 4(a) are even more restrictive. These authors compared their measurements against the predictions of the halo model and from numerical simulations that included diffuse gas. The simulations showed a very good agreement with the observed cross-correlation from RCSLenS galaxies, with about 10%-15% contributions coming from unbound gas. The amplitude of the correlation (Figure 4(a)) in the range $\theta = [40-120]$ arcmin is that of the $\alpha = 1.5$ model. In that interval, only the C06 temperature model predicts an amplitude 10% of the measured correlation. That would imply that the average temperature of the IGM is $T_e \sim 10^6$ K, a stricter bound than that derived from the van Waerbeke et al. (2014) data.

There is a caveat when translating the results on the cross-correlation onto an upper limit on the average temperature of the IGM. Hydro-simulations consistently show that unbound gas is not well characterized by a single equation of state; more accurately, the gas coexists in different phases and there is a large spread in temperature within regions with the same overdensity. Since our temperature models fail to encode the full complexity of the temperature–density phase diagram, our upper bounds on the average temperature must be understood as an order of magnitude estimate not as a strict upper limit.

The constraints that can be derived from the measured power spectrum shown in Figure 4(b) are not as tight as those derived from the correlation function. Only the measurement at $\ell \sim 1800$...
is well below the prediction for the K05 models. What is more relevant is that the overall shape is very different. Hojjati et al. (2016) found that the shape of the correlation function and power spectrum was strongly dependent on physical processes undergone by baryons in halos, such as radiative cooling, star formation, supernovae winds, and AGN feedback. For instance, AGNs expel gas to large distances from the center of halos, lowering the signal at small scales. The properties of the hot gas in our model are rather simplified. No effects of specific physical processes are considered and only the density and temperature distributions are important. More realistic models would require detailed numerical simulations including the most relevant processes in low-density regions. Physical effects could remove power at $\ell \gtrsim 1000$, modifying the overall shape of the power spectrum and bringing it in closer agreement with the data. While a detailed discussion on this point is beyond the scope of the current paper, if the shape were to be independent of the physics of baryons, the power spectrum could be a useful discriminant between halo and unbound gas contributions.

An alternative approach to detect the WHIM contribution would be to remove known galaxies down to a given magnitude to eliminate the contribution of their halos to the Comptonization-convergence correlation. When removing fainter galaxies does not produce a further decrement of the cross-correlation, we have reached the level when the signal is due to gas outside halos. A similar approach was used by Kashlinsky et al. (2005) and Helgason et al. (2016) to isolate Cosmic Infrared Background fluctuations due to the first stars at the epoch of reionization from those of known galaxy populations in deep Spitzer data.

4. Conclusions

Models of galaxy formation predict that a significant fraction, close to half the total number of baryons, could be stored in the WHIM. The low densities and temperatures ($10^{5.3}$ K) of this medium make it difficult to detect. Searches for absorption lines and SZ contributions have provided preliminary evidence of its existence. The $\kappa Y_c$ cross-correlation measured by Van Waerbeke et al. (2014) and Hojjati et al. (2016) probes the fluctuations on the electron pressure along the line of sight and its distribution, but it is not yet a detection of the missing baryon component. As indicated by Hojjati et al. (2015), about 50% of the signal comes from the small fraction of baryons within massive halos; at most, 15% of the cross-correlation power at $\ell \sim 500$ could come from unbound gas. In this article we have shown that the contribution from the unbound gas in filaments could be of this order of magnitude, depending on model parameters. In particular, if the unbound gas is well described by a log-normal distribution and the gas is shock-heated out to a mean temperature $T_c \sim 10^6$ K, then about half the baryons in the universe could be stored in the WHIM, producing a signal that is at least one order of magnitude smaller than the measured amplitude. We have considered two different baryon cutoff lengths: the Jeans length given by Equation (6) that would better describe the physical state of the IGM at $z > 1$, and the shock-heated cutoff scale given by Equation (5) that provides a better description at $z < 1$. We have shown that $\sim 90\%$ of the contribution to the $\kappa Y_c - Y_c$ cross-correlation and to its power spectrum originates at $z \leq 1$ and at overdensities in the range $\xi \sim [3-33]$. The overall amplitude depends on the depth of the source catalog probing the convergence due to the large-scale structure, and the average electron temperature, and is proportional to the fraction of baryons in the IGM.

The tSZ–lensing cross-correlation could be a potentially powerful technique for tracing the distribution of baryons at large scales. The shape of the measured Comptonization-convergence power spectrum and the theoretical prediction for IGM gas show maxima at different scales. The difference could be due to not having included the physical effects that are relevant to the evolution of the IGM gas; but if the differences in shape are real, they could be used to separate the contribution of unbound gas from that of gas in halos. In real space, the cross-correlation is also dominated by halos. To detect the contribution due to the WHIM would require masking galaxy populations with increasing magnitude down to the level where further masking does not reduce the residual correlation. This would require extending the measurement to larger areas and to deeper lens surveys, as done by Hojjati et al. (2016), to increase the signal-to-noise by a factor of 5–10. Then, masking the halo contribution down to 10% of its original amplitude would still leave a statistically significant signal.

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