Toward Determination of $m_t$

at 50 MeV Accuracy*

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Abstract

The top quark mass will be determined to high accuracy from the shape of the $t\bar{t}$ total production cross section in the threshold region at a future linear $e^+e^-$ collider. Presently the estimated statistical error in the measurement of $m_t^{\text{MS}}(m_t^{\text{MS}})$ is $\sim 50$ MeV, while the estimated theoretical error is 150–200 MeV. In order to reduce the theoretical uncertainty to below 50 MeV, we have recently computed an important part of the higher-order corrections. We demonstrate the significance of the calculated correction.

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1 Introduction

In the last Linear Collider Workshop held in Sitges (LCWS '99), significant theoretical developments were reported concerning the top quark physics in the threshold region: the next-to-next-to-leading order corrections to the $e^+e^- \to t\bar{t}$ threshold cross section have been computed \cite{1, 2}, and understanding of the renormalon cancellation mechanism in this cross section led to an improvement of convergence of the perturbative expansion \cite{3}. These developments enabled a precise determination of the \( m_{t \bar{t}} \equiv m_{t \bar{t}}^{\overline{\text{MS}}} \) mass of the top quark, in the future experiment at an $e^+e^-$ linear collider in the $t\bar{t}$ threshold region.

The top quark mass will be determined to high accuracy from the shape of the $t\bar{t}$ total production cross section in the threshold region \cite{4}. The location of a sharp rise of the cross section is determined mainly from the mass of the lowest-lying (1S) $t\bar{t}$ resonance. Since the mass of the resonance can be calculated as a function of the top quark mass and $\alpha_s$ from perturbative QCD, we will be able to extract the top quark mass from the measurement of the 1S peak position of the cross section.

In the last workshop, uncertainties in the determination of the top quark mass were also estimated. The statistical error in the measurement of the 1S peak position of the $t\bar{t}$ threshold cross section was estimated to be $\sim 50$ MeV corresponding to a moderate integrated luminosity of 30 fb$^{-1}$ \cite{5}, while the theoretical uncertainty in the relation between the peak position and the top quark mass was estimated to be 150–200 MeV \cite{6}.

The motivation of the present study is to reduce the theoretical uncertainty to the level of the experimental error (i.e. $\lesssim 50$ MeV). We have computed an important piece of the higher-order corrections in order to achieve this goal. More precisely, Kiyo and the present author have computed in \cite{7} the $\mathcal{O}(\alpha_s^5 m)$ correction to the quarkonium 1S spectrum in the large-$\beta_0$ approximation. We clarify the significance of the correction in the context of the measurement of the top quark mass.

2 Renormalon Cancellation in the Quarkonium Spectrum

The theoretical prediction of the quarkonium mass spectrum is given as a series expansion in $\alpha_s$. There seemed to be some confusions among theoreticians in identifying the order of accuracy of the theoretical prediction. These originated from the fact that the renormalon cancellation in the perturbation series of the quarkonium spectrum is realized in a slightly non-trivial way. When the pole mass and the binding energy are given as series expansions in $\alpha_S$, renormalon cancellation takes place between the terms whose orders in $\alpha_S$ differ by one \cite{9}:}

\begin{align}
2m_{\text{pole}} &= 2\overline{m} \left( 1 + A_1 \alpha_S + A_2 \alpha_S^2 + A_3 \alpha_S^3 + A_4 \alpha_S^4 + \cdots \right), \\
E_{\text{bin}} &= 2\overline{m} \left( B_2 \alpha_S^2 + B_3 \alpha_S^3 + B_4 \alpha_S^4 + \cdots \right),
\end{align}

\( \star \) The same result was obtained later in \cite{8}.

Intuitively this may be seen by comparing the diagrams shown in Fig. 1. One way to understand...
the shift in the order counting is to regard that an extra power of $\alpha_S$ in the binding energy is provided by the inverse of the Bohr radius $\langle r^{-1} \rangle \sim \alpha_S m$,

i.e. \[ \left< C_F \frac{\alpha_s^n}{r} \right> \sim \alpha_s^{n+1} m. \] (2)

Another way to understand this is as follows. When the binding energy is expanded in $\alpha_s$,

\[ E_{\text{bin}} = 2 \bar{m} \sum_{n=2}^{\infty} P_n(\log \alpha_s) \alpha_s^n, \] (3)

the coefficients $P_n(\log \alpha_s)$ are polynomials of $\log \alpha_s$, which is a characteristic feature of the perturbative expansion of the boundstate spectrum. For large $n$, the polynomials behave as

\[ P_n(\log \alpha_s) \sim \alpha_s^{-1}, \] (4)

which effectively decrease the power of $\alpha_s$ by one.

It is legitimate to consider that the present perturbative calculation of the quarkonium spectrum, when expressed in terms of the quark $\overline{\text{MS}}$-mass, has a genuine accuracy at $O(\alpha_S^3m)$. In fact, in formal power countings, the last known term in the relation between the $\overline{\text{MS}}$-mass and the pole-mass of a quark is $O(\alpha_S^3m)$ [10], while the last known term of the binding energy (measured from twice of the quark pole-mass) is $O(\alpha_S^4m)$. The former term includes in addition to a genuine $O(\alpha_S^3m)$ part the leading renormalon contribution which does not become smaller than $O(\Lambda_{\text{QCD}})$ [11]. This renormalon contribution is cancelled [3] against the renormalon contribution [12] contained in the latter term. Therefore, after cancellation of the leading renormalons, the genuine $O(\alpha_S^3m)$ part of the mass relation determines the accuracy of the present perturbation series relating the quark $\overline{\text{MS}}$-mass and the quarkonium spectrum.

As stated, for the binding energy the calculation including the genuine $O(\alpha_S^4m)$ corrections has already been completed. Also, the “large-$\beta_0$ approximation” [13] is known to be a pragmatically feasible and empirically successful estimation method of the leading renormalon
contributions. Taking these into account, one finds that it is sufficient to calculate further the following two corrections in order to improve the accuracy of the spectrum by one order and to achieve a genuine accuracy at $O(\alpha_s^4m)$: (I) the $O(\alpha_s^4m)$ relation between the $\overline{\text{MS}}$-mass and the pole-mass, and (II) the binding energy at $O(\alpha_s^5m)$ in the large-$\beta_0$ approximation. This is because the leading renormalon contribution in the full $O(\alpha_s^5m)$ correction to the binding energy will be incorporated by the large-$\beta_0$ approximation and the remaining part is expected to be irrelevant at $O(\alpha_s^4m)$.

Of these two corrections we have calculated (II) analytically for the $1S$-state in [7]. In the next section we will examine the series expansion of the “toponium” $1S$ spectrum. Also we will check validity of the above general argument explicitly at $O(\alpha_s^3m)$ where we know the exact result.

3 “Toponium” $1S$ State Spectrum

Taking the input parameter as $m_{t,\text{pole}} = 174.79$ GeV$/m_t = 165.00$ GeV and setting $\mu = m_t$ [i.e. expansion parameter is $\alpha_s(m_t) = 0.1092$], we obtain the series expansions of the mass spectrum of the toponium $1S$ state:

$$M_{1S} = 2 \times (174.79 - 0.46 - 0.39 - 0.28 - 0.19^*) \text{ GeV} \quad (\text{Pole-mass scheme}) \quad (5)$$
$$= 2 \times (165.00 + 7.21 + 1.24 + 0.22 + 0.052^*) \text{ GeV} \quad (\overline{\text{MS}}\text{-scheme}). \quad (6)$$

The numbers with stars (*) are calculated in the large-$\beta_0$ approximation (in the case of $\overline{\text{MS}}$-scheme the $O(\alpha_s^4m)$ term of the pole-$\overline{\text{MS}}$ mass relation is replaced by the large-$\beta_0$ approximation as well). The series expansion converges very slowly when the $1S$ spectrum is expressed using the pole mass. The series converges much faster when the spectrum is expressed by the $\overline{\text{MS}}$ mass.

As we argued in the previous section, parametric accuracy of the last term in (5) is $O(\alpha_s^4m)$ and we need to know further only the exact $O(\alpha_s^4m)$ term of the pole-$\overline{\text{MS}}$ mass relation to make a perturbative evaluation accurate up to this order (the exact form of the binding energy at $O(\alpha_s^5m)$ is not necessary). In order to verify validity of this argument, we replace the binding energy at $O(\alpha_s^4m)$ by its value in the large-$\beta_0$ approximation. Then the $O(\alpha_s^3m)$ term of (5) changes to 0.23. Thus, we do not lose accuracy at this order by the replacement. On the other hand, if we replace in addition the $O(\alpha_s^3m)$ term of the pole-$\overline{\text{MS}}$ mass relation by its values in the large-$\beta_0$ approximation, the same term changes to 0.31. Thus, we lose the accuracy at $O(\alpha_s^3m)$. These aspects are consistent with our general argument. Also, they suggest that the last term of (5) would be a reasonable estimate of the order of magnitude of the exact $O(\alpha_s^4m)$ term. Then, when the $O(\alpha_s^4m)$ term of the pole-$\overline{\text{MS}}$ mass relation is calculated, the remaining theoretical uncertainty is expected to be below 50 MeV.

Finally let us examine the dependence of the $1S$ spectrum $M_{1S}(\alpha_s(\mu), m_t)$ on the scale $\mu$. As shown in Fig. 2 the scale dependence decreases as we include more terms of the series expansion. The scale dependence of the sum of the series up to $O(\alpha_s^4m)$, where the last term is evaluated in the large-$\beta_0$ approximation, is also consistent with the estimate of the theoretical uncertainty below 50 MeV.
4 Discussions

There are corrections, other than the 4-loop pole-\overline{MS} mass relation, that should be computed before we achieve 50 MeV accuracy ultimately. These are:

1. The final-state interaction corrections (non-factorizable corrections) to the $e^+e^- \rightarrow t\bar{t}$ total cross section.

2. The electroweak corrections to the cross section. These include the contributions from the tree-level non-resonant-type diagrams and the one-loop resonant-type corrections.

In addition, the uncertainty originating from the large next-to-next-to-leading order corrections to the normalization of the cross section affects the measurement of the top quark mass. This problem should be solved as well. An attempt has been given recently in [14].

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