Angle-Multiplexing Nonlinear Holography for Controllable Generations of Second-Harmonic Structured Light Beams

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Nonlinear multiplexing holography emerges as a powerful tool to produce structured lights at new wavelengths. In this work, we propose and experimentally demonstrate an angle-multiplexing nonlinear holography in an angular noncritical phase-matching configuration. In experiment, various types of structured light beams, such as vortex beam, Airy beam and Airy vortex beam, are simultaneously output at second-harmonic waves along different paths. Because of the large angular acceptance bandwidth of noncritical phase-matching, one can achieve high conversion efficiency of angle-multiplexing nonlinear holography. Our method has potentially applications in high-capacity holographic storage and security encryption.

Keywords: nonlinear holography, noncritical phase matching (NCPM), multiplexing, beam shaping, second harmonic generation

INTRODUCTION

Structured light has attracted widespread attentions because of its spatial distributions of amplitude, phase, and polarization [1]. For example, vortex beam has a unique spiral phase exp(iϕ), featuring a phase singularity at the center and a donut-shaped intensity profile [2–8]. Airy beam is capable to remain its transverse profile during propagation, and it also has self-accelerating and self-healing characteristics [9, 10]. Airy vortex beam has been realized experimentally [11–14], which combines the propagation dynamics of Airy beam and the singularity of vortex beam [15–17]. The rapid development of structured light beam significantly boosts the applications in optical manipulation, quantum communications, and super-resolution microscopy [18, 19].

Holography has many important applications in numerous areas [20–26], including data storage [20], optical encryption [21], holographic interferometry [22], microscopy [23] and dynamic holography [24–26]. Because of its powerful wavefront shaping capability, holography has recently been used in nonlinear optics to enable the generations of various structured light beams at new optical frequencies [27–34]. By utilizing the orthogonal physical dimensions of light, nonlinear holography is capable to reconstruct multiple wavefront information from a single hologram, i.e., nonlinear multiplexing holography [28, 35]. Generally, only one wavefront channel can be output at a time when the corresponding phase matching condition is satisfied in nonlinear holography. It is still difficult for nonlinear holography to output various types of structured light beams efficiently, controllably, and simultaneously.
In this paper, we propose and experimentally demonstrate an angle-multiplexing nonlinear holography in an angular noncritical phase-matching (A-NCPM) configuration. By encoding proper holograms into the fundamental waves, multiple second-harmonic (SH) beams with the desired wavefronts can be simultaneously output along different paths. High conversion efficiency is guaranteed by the large angular acceptance bandwidth of A-NCPM. In experiment, we use the angle-multiplexing nonlinear holography to generate vortex beam, Airy beam and Airy vortex beam at SH waves for example.

METHODS AND EXPERIMENT

We consider two fundamental fields, i.e., $E_1(\omega) = E_1 \exp(i\phi_1)$ and $E_2(\omega) = E_2 \exp(i\phi_2)$, which have the same optical frequencies ($\omega$) but carry different holograms ($\phi_1$ and $\phi_2$). These fields pass through a nonlinear crystal, which experience three birefringence phase matching (BPM) processes. First, 2$\omega$ harmonic generation (SHG) processes happen, in which each fundamental field is frequency-doubled with itself. Second, the two fundamental fields interact with each other through a sum-frequency-generation (SFG) process. Therefore, three fields at SH
waves are generated, which can be calculated by using nonlinear three-wave mixing equation,

\[
E_i(2\omega) = C(E_i)^2 \exp(2i\phi_i) \exp(-i\Delta k_i(z)z) \quad (1)
\]

\[
E_2(2\omega) = C(E_2)^2 \exp(2i\phi_2) \exp(-i\Delta k_2(z)z) \quad (2)
\]

\[
E_3(2\omega) = C(E_3)^2 \exp[i(\phi_3 + \phi_2)] \exp(-i\Delta k_3(z)z) \quad (3)
\]

Here, \(E_i(2\omega)\) with \(i = 1, 2, 3, \ldots\) represents the generated SH fields. \(C\) is a constant that is proportional to nonlinear coefficient of the nonlinear crystal. \(z\) is the propagation direction. The phase mismatch between the interacting fields is defined as \(\Delta k_1 = k_1(2\omega) - 2k_1(\omega)\), \(\Delta k_2 = k_2(2\omega) - 2k_2(\omega)\), and \(\Delta k_3 = k_3(2\omega) - k_1(\omega) - k_2(\omega)\). When the BPM conditions are all satisfied, i.e., \(\Delta k_1(z) = \Delta k_2(z) = \Delta k_3(z) = 0\), high conversion efficiency of nonlinear multiplexing holography can be achieved.

**Figure 1A** shows a collinear type-I BPM configuration, in which two horizontally-polarized fundamental fields propagate collinearly along the \(z\) axis. Three vertically-polarized SH fields [\(E_1(2\omega), E_2(2\omega),\) and \(E_3(2\omega)\)] as described in Eqs 1–3 are produced through three collinear BPM processes. Because \(E_1(\omega)\) and \(E_2(\omega)\) have the same wave vectors, it is easy to simultaneously satisfy the phase matching conditions \(\Delta k_1(z) = \Delta k_2(z) = \Delta k_3(z) = 0\). However, this leads to overlapping of these SH fields at the image plane, as shown in Figure 1A. To effectively separate the generated SH fields, we add an additional term of \(\exp(i\Delta k)\) to \(E_1(\omega)\), and an additional term of \(\exp(-i\Delta k)\) to \(E_2(\omega)\), i.e., the fundamental fields of \(E_1(\omega)\) and \(E_2(\omega)\) are noncollinear. As a result, the SH fields \(E_1(2\omega)\) and \(E_2(2\omega)\) are output along the axisymmetric direction, while the SH field \(E_3(2\omega)\) is still output along the \(z\) axis (Figure 1B). Clearly, the generated SH fields under such noncollinear configuration propagate along different paths in space, which can be well distinguished on the image plane.

This noncollinear scheme can well solve the problem of SH field overlap. However, the phase mismatch becomes \(\Delta k_1(z) = \Delta k_2(z) = \Delta k_3(z)\), which cannot be simultaneously compensated in a traditional BPM crystal. Here, we propose A-NCPM to solve this problem [36]. A-NCPM is a popular phase-matching configuration, in which the input fundamental beam generally propagates along the optical principal axis of nonlinear crystal. Under A-NCPM configuration, one can obtain a large angular acceptance bandwidth. By use of A-NCPM scheme, the conversion efficiency of the SHG process can be well maintained even the input fundamental beam is tilted by a certain angle. **Figure 1C** compares the normalized SHG conversion efficiencies of BPM and A-NCPM at different incident angles [36, 37]. In a traditional BPM process, the angular acceptance bandwidth is typically less than 0.1°. In contrast, the angular acceptance bandwidth is significantly enhanced to about 3.7° under an A-NCPM configuration in our experiment.

**Figure 2** shows the experimental setup used in this work. In the optical alignment, the fundamental wave is derived from a 1,064 nm laser with a pulse repetition frequency of 20 kHz and a pulse width of 100 ns. A polarizing beam splitter (PBS) is used to select a horizontally-polarized light. The fundamental wave is
then shaped using a 4-f system consisting of two lenses (L1 and L2 with \( f_1 = 200 \text{ mm} \) and \( f_2 = 75 \text{ mm} \)) and a pinhole. After passing through a 50:50 beam splitter (BS), the fundamental wave is equally divided into two beams. These two beams are separately modulated by using two spatial light modulators (SLM1, BNS, P1920-600-1300-HDMI; and SLM2, Holoeye, Pluto-2-NIR-011) to carry the designed holograms. Because SLM1 used in the experiment only works for vertically-polarized light, we add a half-wave plate (HWP) between BS and SLM1. After modulation, they are combined via the 50:50 BS and then shaped using another 4-f system consisting of lenses L3 and L4 (with \( f_3 = 500 \text{ mm} \) and \( f_4 = 50 \text{ mm} \)). Then, the fundamental beams are incident into an LBO crystal (type-I, \( \theta = 0, \phi = 90^\circ, 4 \times 4 \times 15 \text{ mm}^3 \)). A filter is placed after the LBO crystal to filter out the fundamental wave, and a lens L5 (\( f_5 = 200 \text{ mm} \)) is then used to perform the Fourier transform of the generated SH fields. Finally, the SH beams are recorded using a charge-coupled device (CCD) camera (Newport, LBP2-HR-VIS2). The SH vortex beams is tested by using a cylindrical lens [38].

In experiment, we first demonstrate the generations of multiple SH vortex beams. First, vortex holographic phase holograms with \( l = -4 \) and \( l = 4 \) are loaded onto SLM1 and SLM2, respectively. Here, the hologram is designed according to binary computer-generated-hologram (CGH) theory [39]. Besides, a blazed grating phase is superimposed on each hologram to introduce the additional spatial phase \( \exp(i\Delta k) \) or \( \exp(-i\Delta k) \). Here, \( \Delta k \) is chosen to ensure the incident angle lies in the acceptance angular bandwidth of A-NCPM. In our experiment, the incident angle of the fundamental wave is measured to be 0.78°. Then, the two fundamental waves \( E_1(\omega) \) and \( E_2(\omega) \) interact in the LBO crystal to produce three SH beams. The conversion of the orbital-angular momentum (OAM) in nonlinear optical process obeys \( l(2\omega) = l(\omega) + \hat{l}(\omega)[40, 41] \). Figure 3A shows the experimental result. The two donut-shaped SH beams on the sides correspond to the SHG processes in which \( E_1(\omega) \) (or \( E_2(\omega) \)) is frequency-doubled with itself. The generated \( E_1(2\omega) \) (or \( E_2(2\omega) \)) carries an OAM of \( l = -8 \) (or \( l = 8 \)), which is measured by using a cylindrical lens, as shown in Figure 3C. The central Gaussian spot results from the SFG process between \( E_1(\omega) \) and \( E_2(\omega) \). Because the interacting fundamental waves have topological charges of opposite sign, the spiral phase is cancelled in the generated SH wave. In addition, we replace the hologram loaded on SLM2 to a \( l = 1 \) vortex
holographic phase hologram while keeping the hologram on SLM1 unchanged. As shown in Figure 4A, three SH vortex beams of \( l = -8 \), \( l = -3 \), and \( l = 2 \) present from left to right on the image plane. Their OAMs are also measured by a cylindrical lens as shown in Figure 4C, which is consistent with the OAM conservation law. Figures 3B, 4B show the numerical simulations, which are in good agreement with the experimental results in Figures 3A, 4A, respectively.

Next, we produce various types of spatial light beams simultaneously. In this experiment, we load the holograms for the generations of a 2D Airy beam and a \( l = 1 \) vortex beam on SLM1 and SLM2, respectively. Notably, if a fundamental Airy beam is directly frequency-doubled with itself in the nonlinear crystal, one achieves the product of two Airy beams rather than the SH Airy beam [42]. In the experiment, we use the Fourier transform (FT) of the Airy beam as the hologram on SLM1, which is imaged into the nonlinear crystal and performs SHG. Then, the generated SH field is converted to an SH Airy beam after FT through a lens. Under such experimental configuration, three different types of SH beams, i.e., SH vortex beam, SH Airy beam, and SH Airy vortex beam, are produced along different paths. The experimental results are shown in Figure 5A, which agree well with the simulated results as shown in Figure 5B. Notably, the SH intensities in Figures 3–5 are not the same because the conversion efficiencies of SHGs involving various structured light beams are different.

CONCLUSION

In conclusion, we propose an angle-multiplexing nonlinear holography to produce multiple structured light beams simultaneously under an A-NCPM configuration. In experiment, we demonstrate a three-channel output of various SH beams, which can be further extended to more output channels within the angular acceptance bandwidth of A-NCPM. The angle-multiplexing nonlinear holography can also be applied in nonlinear photonic crystals and nonlinear metasurfaces [43–47]. Our work provides a feasible solution to enhance the capacity of nonlinear holography for multi-wavelength display, multi-dimensional optical storage, optical encryption, all-optical diffractive neuron networks [48], and optical communications.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

WY, CZ, TW, and PC performed the experiments under the guidance of YZ and MX, WY, and YZ wrote the manuscript with contributions from all

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