I. INTRODUCTION

Quantum teleportation is one of the most fascinating possibilities offered by quantum information processing. In the standard protocol, Alice is to transfer an unknown quantum state to Bob using, as the sole resources, some previous shared entanglement (quantum channel) and a classical channel capable of communicating measurement results. Often the quantum channel for continuous variable (CV) teleportation is realized by using two-mode squeezed (TMS) states which may mimic non-classical Einstein-Podolsky-Rosen (EPR) correlations. The concept of quantum channel can be generalized to Gaussian channels in Section III. Hence, we examine the two situations above in Sections IV and V. Section VI is devoted to establishing the possibility to teleport Gaussian modes from a quantum state onto the vibrational mode of a movable mirror [5]. In doing so we have exploited an intense elastic carrier mode, with the same frequency \( \omega_0 \), and two additional weak anelastic sideband modes with frequencies \( \omega_0 \pm \Delta \). The physical process is very similar to a stimulated Brillouin scattering, even though in this case the Stokes and anti-Stokes component are back-scattered by the acoustic wave at reflection, and the optomechanical coupling is provided by the radiation pressure. Treating classically the intense incident beam (and the carrier mode), the quantum system is composed by three interacting bosonic modes, i.e. the vibrational mode and the two sideband modes. In our description, vibrational, Stokes and anti-Stokes modes are back-scattered by the acoustic wave at reflection, and the EPR correlations, which can be distilled for certain groupings of two modes, simply tracing out the remaining remaining mode or measuring it by heterodyne detection and communicating the result. In both situations the distilled channel comes out as a (nonzero mean) Gaussian bipartite state. The model is presented in Section III. Then, we report the general teleportation protocol through a Gaussian channel in Section III. Hence, we examine the two situations above in Sections IV and V. Section VI is devoted to conclusions.

II. EFFECTIVE HAMILTONIAN AND SYSTEM DYNAMICS

We consider a perfectly reflecting mirror and an intense quasi-monochromatic laser beam impinging on its surface (see Fig. 1). The laser beam is linearly polarized along the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite the mirror surface and focused in such a way as to excite

Gaussian acoustic modes of the mirror. These modes describe small elastic deformations of the mirror along the direction orthogonal to its surface and are characterized by a small waist, a large quality factor and a small effective mass \( m \). It is possible to adopt a single vibrational mode description limiting detection bandwidth to include a single mechanical resonance of frequency \( \Omega \). In this description the incident laser beam, with frequency \( \omega_0 \), is reflected into an elastic carrier mode, with the same frequency \( \omega_0 \), and two additional weak anelastic sideband modes with frequencies \( \omega_0 \pm \Delta \). The physical process is very similar to a stimulated Brillouin scattering, even though in this case the Stokes and anti-Stokes component are back-scattered by the acoustic wave at reflection, and the optomechanical coupling is provided by the radiation pressure. Treating classically the intense incident beam (and the carrier mode), the quantum system is composed by three interacting quantized bosonic modes, i.e. the vibrational mode and the two sideband modes. In our description, vibrational, Stokes and anti-Stokes modes are back-scattered by the acoustic wave at reflection, and their EPR correlations, which can be distilled for certain groupings of two modes, simply tracing out the remaining remaining mode or measuring it by heterodyne detection and communicating the result. In both situations the distilled channel comes out as a (nonzero mean) Gaussian bipartite state. The model is presented in Section III. Then, we report the general teleportation protocol through a Gaussian channel in Section III. Hence, we examine the two situations above in Sections IV and V. Section VI is devoted to conclusions.

\[
\hat{H}_{\text{eff}} = -i\hbar \chi (\hat{a}_1 \hat{a}_0^\dagger - \hat{a}^\dagger_1 \hat{a}^\dagger_0) - \hbar \theta (\hat{a}_2 \hat{a}_1^\dagger - \hat{a}_1 \hat{a}_2^\dagger),
\]

where \( \chi \) and \( \theta \) are couplings constants whose ratio \( r \equiv \theta / \chi = (\omega_0 + \Omega)/(\omega_0 - \Omega) \). The system dynamics is satisfactorily reproduced by the Hamiltonian of Eq. 1 as long as the dissipative coupling of the mirror vibrational mode with its environment is negligible. This happens if the interaction time is much smaller than the relaxation time of the vibrational mode (which can be \( \sim 1 \text{ s} \)) and therefore means having a high-Q vibrational mode (typically \( \Omega \sim \text{MHz} \)).

The dynamics can be easily studied in terms of the symmetrically ordered characteristic function \( \Phi(\mu) \), where \( \mu_k \equiv (\mu_k, \mu_1, \mu_2) \), is a complex variable corresponding to the operator \( \hat{a}_k \) (\( k = 0, 1, 2 \)). The relation between the density operator and the corresponding characteristic function is given by \( \rho_{012} = tr(\rho \hat{D}_k(\mu_k) \hat{D}^\dagger_k(\mu_k)) \), where \( \hat{D}_k(\mu_k) = \exp\{i \mu_k \hat{a}_k^\dagger - \mu_k^* \hat{a}_k \} = \exp\{i \sqrt{2} \mu_k^\dagger \hat{X}_k - i \mu_k^\dagger \hat{P}_k \} \) and

\[
\hat{X}_k = \frac{1}{\sqrt{2}} (\hat{a}_k + \hat{a}^\dagger_k), \quad \hat{P}_k = \frac{i}{\sqrt{2}} (\hat{a}_k - \hat{a}^\dagger_k).
\]
\( \mu_k^{(R)} \hat{F}_k \). Notice that the Fourier transform defined as above through the displacement operators \( \hat{D}_k(\mu_k) \), creates the correspondence \( \mu_k^{(1)} \leftrightarrow \hat{X}_k, -\mu_k^{(R)} \leftrightarrow \hat{P}_k \). From Eq. (4) we can deduce the dynamical equation for \( \Phi(\mu, t) \). If we assume the initial condition \( \Phi(\mu, t = 0) = \exp \left[ -\pi \mu_0^2 - \sum_{k=0}^{2} \mu_k^2 / 2 \right] \), corresponding to a vacuum state for modes 1, 2, and to a thermal state with mean thermal number of excitations \( \bar{n} \) for mode 0, the total state at time \( t \) is Gaussian, with characteristic function

\[
\Phi(\mu, t) = \exp[-Q_0|\mu_0|^2 - Q_1|\mu_1|^2 - Q_2|\mu_2|^2 + T_0(\mu_1 \mu_2 + \mu_1^* \mu_2^*) + T_1(\mu_0 \mu_2^* + \mu_0^* \mu_2) + T_2(\mu_1 \mu_0 + \mu_1^* \mu_0^*)],
\]

where: \( Q_0 \equiv B + 1/2, Q_1 \equiv A + 1/2, Q_2 \equiv E + 1/2, T_0 \equiv F, T_1 \equiv D, T_2 \equiv C \) and the coefficients \( A, B, C, D, E \) and \( F \), explicitly given in (7), depend upon \( \bar{n} \), \( r \), and the scaled time \( t' \equiv t \sqrt{\theta^2 - \chi^2} \). Since the state of Eq. (4) is a zero-mean Gaussian state, it can be fully described by its correlation matrix (CM) \( V \), defined by \( V_{lm} = \langle \Delta \hat{X}_l \Delta \hat{X}_m + \Delta \hat{X}_m \Delta \hat{X}_l \rangle / 2 \) \((l, m = 1, \ldots, 6)\), where \( \Delta \hat{X}_l \equiv \hat{X}_l - \langle \hat{X}_l \rangle \) and \( \hat{\xi} \) denotes the vector of quadratures: \( \hat{\xi} \equiv (\hat{X}_0, \hat{P}_0, \hat{X}_1, \hat{P}_1, \hat{X}_2, \hat{P}_2) \). In fact, introducing the real vectors \( \hat{\mu}_k \) \((k = 0, 1, 2)\), defined by

\[
\mu_k = \mu_k^{(R)} + i \mu_k^{(1)} \leftrightarrow (\hat{\mu}_k^{(1)}, -\hat{\mu}_k^{(R)}) \equiv \hat{\mu}_k \in \mathbb{R}^2,
\]

so that \( \mathbb{C}^3 \ni \mu \equiv (\mu_0, \mu_1, \mu_2) \leftrightarrow (\hat{\mu}_0, \hat{\mu}_1, \hat{\mu}_2) \equiv \hat{\mu} \in \mathbb{R}^6 \), and expressing \( \Phi(\hat{\mu}, t) \) in terms of \( \hat{\mu} \), we get from Eq. (4)

\[
\Phi(\hat{\mu}, t) = e^{-\hat{\mu}^TV\hat{\mu}^T},
\]

where the CM \( V \) appears and it is explicitly given by

\[
V = \begin{pmatrix}
Q_0 & T_2 & 0 & -T_1 & 0 & 0 \\
0 & Q_0 & 0 & -T_2 & 0 & -T_1 \\
T_2 & 0 & Q_1 & 0 & T_0 & 0 \\
0 & -T_2 & 0 & Q_1 & 0 & -T_0 \\
-T_1 & 0 & T_0 & 0 & Q_2 & 0 \\
0 & -T_1 & 0 & -T_0 & 0 & Q_2
\end{pmatrix}.
\]

Since we deal with a closed system of three interacting oscillators its dynamics is periodic in \( t' \) with period \( 2\pi \). The separability properties of the tripartite system \( \rho_{012} \) and those of the bipartite reduced systems \( \rho_k^{(R)} = tr_k(\rho_{012}) \) \((k = 0, 1, 2)\), have been already studied in (7). Here we briefly recall these properties referring to (10) for the definition of entanglement classes of a tripartite CV Gaussian state. The state \( \rho_{012} \) is almost everywhere fully entangled (class 1) except for isolated times \( t' = 2m\pi \) \((m \in \mathbb{N})\) when it is fully separable (class 5) and \( t' = (2m + 1)\pi \) when it is one-mode biseparable (class 2). In the latter case, it is equal to the tensor product of a TMS state for the optical modes and of a thermal state for the vibrational mode.

The inseparability between one of the three modes and the other two parties can be exploited to implement a telecloning protocol \( \Phi \). When we trace out one of the three modes, we can distill bipartite entanglement only between modes 1 and 2 or between modes 1 and 0. In the first case (1 and 2), the entanglement between the optical modes exists at all times (except \( t' = 2m\pi \)), and it is extremely robust with respect to temperature. Such modes show also robust EPR correlations which are temperature-independent at \( t' = (2m + 1)\pi \) when the state is a TMS state \( \Phi \). In the second case, the entanglement between the optical Stokes mode and the mirror vibrational mode exists in two limited time intervals, just after \( t' = 0 \) and just before \( t' = 2\pi \), and they become narrower by increasing the temperature. In \( \Phi \) the second time interval has been exploited to realize quantum teleportation from an optical to the mirror vibrational mode.

III. CV TELEPORTATION THROUGH AN EPR CHANNEL

Suppose to have a quantum teleportation network, i.e. a quantum channel given by a truly multipartite entangled state shared among \( N \) parties. Such a situation has been studied in (4) where, from a particular \( N \)-partite entangled state, a bipartite entanglement between any two of the \( N \) parties can be distilled to enable quantum teleportation. Clearly the distillability of bipartite entanglement from the total channel is a necessary condition to make the teleportation network really quantum (i.e. not reproducible by any local classical means).

Here we consider a distilled channel consisting of a bipartite Gaussian state with a known drift. That happens for example when, from a \( N \)-partite zero-mean Gaussian state, we trace out \( N - 2 \) modes, or when we measure them by heterodyne detection (communicating the results to Bob through a classical channel). Suppose that such a distilled channel is shared in a network by Alice (mode \( i \)) and Bob (mode \( j \)) (for simplicity consider \( N = 3 \) and see Fig. 2). If their channel is separable then they cannot perform quantum teleportation but, if it is entangled, then they can try to perform it by exploiting possible EPR correlations of their channel. We define “EPR+” or “EPR−” correlations in the following way

\[
\text{EPR} \pm \leftrightarrow \langle \Delta \hat{X}_k^2 \rangle + \langle \Delta \hat{P}_k^2 \rangle < 2,
\]

where \( \hat{X}_k \equiv \langle \hat{X}_k \rangle \) and \( \hat{P}_k \equiv \langle \hat{P}_k \rangle \). Depending on the supposed EPR correlations, Alice and Bob can implement an appropriate teleportation protocol. We treat both cases in a compact form adopting the phase-space approach of (11). The distilled channel is described by a Gaussian Wigner function \( W^{\text{ch}}(\alpha, \alpha_j) \) with a known drift and supposing to possess EPR± correlations according to definition (4). The unknown input state at Alice’s station is described by \( W^{\text{in}}(\gamma) \) so that the total state before the beam-splitter is given by \( W(\gamma, \alpha_i, \alpha_j) = W^{\text{in}}(\gamma)W^{\text{ch}}(\alpha_i, \alpha_j) \). After the beam
where the first quantity $\gamma$ according to the substitutions shown in Eq. (12). If we
tocol by a suitable displacement of his mode, namely
in
where $\Phi$ to Bob by averaging over all possible results

State (8) is exactly the state at Bob's station when Bob
receives from Alice her result $\gamma_+^\prime$ through a classical channel. At this point Bob completes the teleportation protocol by a suitable displacement of his mode, namely

$$\alpha_j \rightarrow \alpha_j - \gamma_+^\prime - \delta_\pm,$$

where the first quantity $\gamma_+^\prime$ balances Alice's detection while the second one $\delta_\pm$ (to be specified) balances the drift of the channel and depends on the type of EPR correlations, too. The final state at Bob's station is then given by $W(\alpha_j - \gamma_+^\prime - \delta_\pm | \gamma_\pm)$.

In order to get the teleportation fidelity, we compute the mean state teleported to Bob by averaging over all possible results $\gamma_\pm^\prime$

$$W_{\pm}(\alpha_j; \delta_\pm) = \frac{1}{P(\gamma_\pm^\prime)} \int d^2 \gamma K(\pm | \pm) W(\alpha_j - \gamma_\pm^\prime - \delta_\pm | \gamma_\pm^\prime),$$

where

$$K(\pm | \pm) = \int d^2 \gamma W(\pm | \pm)$$

depends also on the parameter $\delta_\pm$. The input-output relation can be greatly simplified if we introduce the symmetrized ordered characteristic functions given by the Fourier transform of the Wigner functions, i.e. $\Phi^\prime(\lambda) = \int d^2 \alpha e^{\lambda \alpha - \lambda^* \alpha} W(\alpha)$ where $\lambda = \lambda^{(R)} + i\lambda^{(I)}$ is the conjugate variable of the complex amplitude $\alpha$. In fact, applying the Fourier transform to (11), we get:

$$\Phi_{\pm}^\prime(\lambda; \delta_\pm) = e^{\lambda \delta_\pm - \delta_\pm^* \lambda^*} \Phi^\prime(\lambda^*; \delta_\pm),$$

where $\Phi^\prime(\mu_\pm), \Phi^\prime(\mu_\pm)$ and $\Phi^\prime(\mu_\pm, \mu_\pm)$ are evaluated according to the substitutions shown in Eq. (12). If we consider a pure input state then the fidelity takes the form

$$F_{\pm}(\delta_\pm) = \frac{1}{\pi} \int d^2 \lambda |\Phi^\prime(\lambda)|^2 |\Phi^\prime(\pm \lambda^*, \delta_\pm)| e^{-\lambda \delta_\pm - \delta_\pm^* \lambda^*},$$

where we have used Eq. (12). In particular we consider a Gaussian pure input state and it can always be chosen with zero-mean, because of the invariance of $F_{\pm}(\delta_\pm)$ with respect to the amplitude of the input state.

Using the real variables of Eq. (13), the characteristic functions of the input and the channel are given respectively by $\Phi^\prime(\mu_\pm) = \exp(-\mu_\pm V^\prime(\mu_\pm))$ and $\Phi^\prime(\mu_\pm, \mu_\pm) = \exp[-(\mu_\pm, \mu_\pm) V^{ch}(\mu_\pm, \mu_\pm)^T + D^{ch}(\mu_\pm, \mu_\pm)^T]$, where the CMs $V^\prime$ and $V^{ch}$ are real positive matrices, while the drift vector $D^{ch}$ has pure imaginary elements.

In particular we can write:

$$V^{ch} \equiv \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

$$D^{ch} \equiv 2i (d_1, d_2, d_3, d_4)$$

with $d_k \in \mathbb{R}$. Inserting $\Phi^\prime(\mu_\pm)$ and $\Phi^\prime(\mu_\pm, \mu_\pm)$ in Eq. (13), we get:

$$F_{\pm}(\delta_\pm) = \frac{1}{\sqrt{\det(E_{\pm})}} \exp(-Q_{\pm})$$

where

$$E_{\pm} \equiv 2V^\prime + RAR + B \pm RC \pm C^T R^T$$

$$R \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$Q_{\pm} \equiv -D_{\pm}^{-1}(D_{\pm})^T \geq 0$$

$$D_{\pm} \equiv (-\delta_\pm^{(R)} + d_1 - d_3, -\delta_\pm^{(I)} + d_2 - d_4).$$

In Eq. (14) the positive matrix $E_{\pm}$ depends only upon the CMs $V^\prime$ through $C$, while the non-negative term $Q_{\pm}$ is linked also to the vector $D_{\pm}$ through $C$, which, in turn, contains both the drift of the Gaussian channel and the drift created by Bob's additional displacement $\delta_{\pm} = \delta^{(R)} + i\delta^{(I)}$. Without such a displacement (i.e. $\delta_{\pm} = 0$), the teleported state acquires a nonzero drift from the channel and the corresponding fidelity $F_{\pm}(0)$ will depend upon such a drift via a decreasing exponential. This does not happen if $D^{ch} = 0$, or, more generally, if $d_1 - d_3 = d_2 - d_4 = 0$. In the general case, the only way in which Bob can eliminate the nonzero drift is to choose an additional displacement $\delta_{\pm}$ which perfectly cancels the effects of $D^{ch}$, i.e., is such that $D_{\pm} = 0$. Using (13), that means:

$$\delta_{\pm}^{(R)} = d_1 - d_3$$

and

$$\delta_{\pm}^{(I)} = d_2 - d_4.$$
With such a displacement the teleportation becomes independent from the channel drift and the fidelity becomes

$$F_{\pm} = \frac{1}{\sqrt{\det(E_{\pm})}}$$

(20)

Suppose now that our input state is a coherent state and that our quantum channel has the CM in the standard form

$$V^{ch} = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & c' \\ c & 0 & b & 0 \\ 0 & c' & 0 & b \end{pmatrix};$$

(21)

then the fidelity is simply related to the quadrature variances:

$$F_{\pm} = \left[\left(1 + \langle \Delta \hat{X}^2_{\pm} \rangle\right)\left(1 + \langle \Delta \hat{P}^2_{\pm} \rangle\right)\right]^{-1/2},$$

(22)

where explicitly \(\langle \Delta \hat{X}^2_{\pm} \rangle = a + b \pm 2c\) and \(\langle \Delta \hat{P}^2_{\pm} \rangle = a + b \pm 2c'.\) In particular, if \(c' = c\), then \(\langle \Delta \hat{X}^2_{\pm} \rangle = \langle \Delta \hat{P}^2_{\pm} \rangle \geq 1\) and \(F_{\pm} \leq 1/2 = F_{class},\) i.e., quantum teleportation is not possible. Instead if \(c' = -c\) then we have \(\langle \Delta \hat{X}^2_{\pm} \rangle = \langle \Delta \hat{P}^2_{\pm} \rangle\) and \(F_{\pm} = \left[1 + \langle \Delta \hat{X}^2_{\pm} \rangle\right]^{-1}.\) In such a case the existence of EPR correlations is equivalent to quantum teleportation, i.e.

\(\langle \Delta \hat{X}^2_{\pm} \rangle < 1 \iff F_{\pm} > 1/2 = F_{class}.\)

(23)

In this sense the Gaussian state \(\Phi^{ch}(\mu_i, \mu_j) \leftrightarrow V^{ch}, D^{ch}\) having the CM of Eq. (21) with \(c' = -c\) is an EPR channel generalizing the two-parameter EPR channel studied in [12]. Note that the left hand side of Eq. (23) is a sufficient condition for bipartite entanglement.

IV. TRACING OUT ONE MODE

We can consider our optomechanical system (Fig. 1) as a 3-mode teleportation network where we distill a quantum channel between two arbitrary modes \(i\) and \(j\) tracing out the remaining mode \(k\) (Fig. 2). We denote with \(\rho^{(k)} = tr_k(\rho_{012})\) the reduced state. The corresponding characteristic function is given by \(\Phi^{(k)}(\mu_i, \mu_j) = \Phi(\mu_i, \mu_j, \mu_k = 0),\) and using Eq. (3), we obtain \(\Phi^{(k)}(\mu_i, \mu_j) = \exp(-\langle \mu_i, \mu_j \rangle V^{(k)}(\mu_i, \mu_j) \rangle^T),\) where:

\[
V^{(k)} = \begin{pmatrix} Q_i & 0 & (-)^k T_k & 0 \\ 0 & Q_i & 0 & -T_k \\ (-)^k T_k & 0 & Q_j & 0 \\ 0 & -T_k & 0 & Q_j \end{pmatrix}
\]

(24)

In other words the distilled channel is a zero-mean Gaussian state with CM in the standard form. It follows that Alice \((i)\) and Bob \((j)\) can arrange the teleportation protocol of Section III without any additional displacement \(\delta_{\pm}\) and with fidelity \(F_{\pm}^{(k)}\) given by Eq. (22) for a coherent input. It is easy to see from Eq. (24) that for \(k = 1\) we have \(F_{\pm}^{(1)} \leq 1/2\) while for \(k = 0, 2\) we have an EPR channel with fidelity \(F_{\pm}^{(k)} = [1 + Q_i + Q_j \pm 2T_k]^{-1}.

In Fig. 5 we report \(F_{\pm}^{(0)}, F_{\pm}^{(2)}\) versus \(n = 0, 1^3\) and choosing \(r = 1 + 2.5 \times 10^{-7}\) as in Ref. [3]. We have \(F_{\pm}^{(0)}(2\pi) = 1/2\) but while \(F_{\pm}^{(0)} \leq 1/2\) \(\forall t',\) we have \(F_{\pm}^{(0)} \geq 1/2\) \(\forall t'\) (at least up to \(n = 10^5,\) see also Fig. 5). In particular, \(F_{\pm}^{(0)}\) is approximately constant near \(2\pi\) and has the expansion \(F_{\pm}^{(0)} \sim [2 - (r - 2)^2/2]^{-1}.\) Analyzing \(F_{\pm}^{(2)}\), we see that the quantum channel distilled for the Stokes mode \((1)\) and the mirror mode \((0)\) has EPR correlations before \(2\pi\) (exploited in [3]) and EPR correlations after \(2\pi\). The existence of this symmetric region is important not only because it allows to perform an additional quantum teleportation just after \(t' = 2\pi\) \((\Rightarrow t' = 0)\) but also because in the time interval delimited by the intersections between \(F_{\pm}^{(2)}\) and \(F_{\pm}^{(0)}\) (see Fig. 5) these latter fidelities are strictly greater than \(1/2.\) This means that a region exists after \(2\pi\) where a telecloning protocol can be arranged using the Stokes mode \((1)\) as “port” and the anti-Stokes \((2)\) and the mirror \((0)\) modes as “receivers.”

Alice \((1)\) measures \((\hat{x}_i, \hat{p}_i)\) and classically sends the result \(\gamma_\rightarrow \) to Bob \((0)\) and Charlie \((2),\) who perform the displacements \(\alpha_j \rightarrow \alpha_j - \gamma_\rightarrow, j = 0, 2.\) The clones created at the receivers are both quantum, but the asymmetry of the total channel is such that \(F_{\pm}^{(0)}\) is only slightly greater than \(1/2\) in all the “telecloning interval”, while \(F_{\pm}^{(2)}\) takes its maximum \(\left[F_{\pm}^{(2)}\right]_{\text{max}} = (1 + r)^2/[1 + 2r(1 + r)] \simeq 0.8,\) \(\forall n,\) and for \(t'_{\text{max}} = \varsigma/2 + 2\pi\) with \(\varsigma \equiv \cos^{-1}(2r^{-2} - 1).\) Since \(F_{\pm}^{(2)}(t'_{\text{max}} - \varsigma/2) = F_{\pm}^{(2)}(t'_{\text{max}} + \varsigma/2) = (2n)^{-1},\) the telecloning interval becomes narrower by increasing the temperature. On the other hand, the behavior of \(F_{\pm}^{(0)}\) (see \(F_{\pm}^{(0)}\) in Fig. 5) shows the robustness with respect to temperature of the EPR correlations relative to the distilled channel between the two optical modes: the curves for \(n = 0\) and \(n = 10^5\) are almost indistinguishable. Such EPR channel, which reduces to a TMS state at \(t' = \pi\) (it is in particular \(F_{\pm}^{(0)}(\pi) = 1/2 + r/(r^2 + 1) \simeq 1)\), enables quantum teleportation even at high temperatures and points out the present optomechanical system as an alternative source of two-mode squeezing. From the \(r\)-dependence of the maximum values \(\left[F_{\pm}^{(2)}\right]_{\text{max}}\) and \(F_{\pm}^{(0)}(\pi),\) we can see that the optimal values, achieved for \(r \rightarrow 1,\) are 4/5 and 1 respectively.

V. HETERODYNING ONE MODE

Consider now our optomechanical system as a 3-mode teleportation network where we distill a quantum channel
between two arbitrary modes $i$ and $j$ heterodyning the remaining mode $k$ and sending the result to Bob through a classical channel (Fig. 2). We denote with $\alpha$ the measurement result and again with $\Phi^{(k)}$ the state of the reduced system involving modes $i$ and $j$. The corresponding characteristic function is given by $\Phi^{(k)}(\hat{\mu}_i, \hat{\mu}_j) = N_k(\alpha) \int d^2 \mu_k (\alpha| \hat{D}_k^\dagger(\mu_k) \alpha \rangle \langle \alpha|^{\Phi(\mu_k)}$ with $N_k(\alpha) = \pi^{-1}(Q_k + 1/2)^{-\frac{1}{2}} \exp[(Q_k + 1/2)^{-1}|\alpha|^2]$. Using Eq. (8), we obtain $\Phi^{(k)}(\hat{\mu}_i, \hat{\mu}_j) = \exp[-\langle (\hat{\mu}_i, \hat{\mu}_j) V^{(k)}(\hat{\mu}_i, \hat{\mu}_j) \rangle^T + D^{(k)}(\hat{\mu}_i, \hat{\mu}_j)]^T$, where the CM $V^{(k)}$ is given by (24) except for the replacements

\[ Q_{i(j)} \rightarrow Q_{i(j)} - \frac{T^2_{j(i)}}{Q_k + 1/2}, \quad T_k \rightarrow T_k + \frac{T_i T_j}{Q_k + 1/2} \quad (25) \]

and the drift $D^{(k)}$ is given by

\[ D^{(k)} = \frac{2i}{Q_k + 1/2} \times \left( \alpha^{(R)} T_j, -\alpha^{(I)} T_j, (-\delta^{(k)} + \alpha^{(R)} T_i, -\alpha^{(I)} T_i) \right) \quad (26) \]

[Notice that $D^{(k)}$ depends on $\alpha$ and on the $i - j$ ordering too]. It is evident that the distilled channel is still a Gaussian state with CM in the standard form but with a non-zero drift. It follows that Alice ($i$) and Bob ($j$) can arrange the teleportation protocol of Section [III] with a suitable additional displacement $\delta^{(k)}$ and with fidelity $F^{(k)}_{\pm}$ given by (22) for a coherent input. For $k = 1$ we have again $F^{(1)}_{\pm} \leq 1/2$, while for $k = 2$ we have an EPR channel with a modified fidelity $F^{(2)}_{\pm} = [1 + Q_i + Q_j \pm 2T_k - (T_i + T_j)^2(Q_k + 1/2)^{-1}]^{-1}$. In such a case ($k = 0, 2$) we have $\delta^{(k)} = \alpha(T_i + T_j)(Q_k + 1/2)^{-1}$. In Fig. we report $F^{(2)}_{\pm}$ versus $t'$ and for $\tilde{n} = 0, 1, 10^7 (r = 1 + 2.5 \times 10^{-7})$. When $\tilde{n} = 0$ we have $F^{(2)}_{\pm} > 1/2$ for $\pi < t' < 2 \pi$ and $F^{(2)}_{\pm} > 1/2$ for $0 < t' < \pi$. However, when $\tilde{n} \neq 0$, the quantum character survives only in small intervals before $t' = 2\pi$ and after $t' = 0$ (or equivalently after $t' = 2\pi$).

These fidelities not only have a temperature-independent maximum value $[F^{(2)}_{\pm}]_{\text{max}} \approx 0.85$ greater than that of the traced out mode case, but also the “quantum time intervals” are larger and more robust with respect to temperature. Moreover, as in the traced out mode case, the distilled channel shared by the Stokes mode (1) and the mirror mode (0) has both types of EPR correlations and therefore one can perform a further quantum teleportation just after $t' = 2\pi$ ($\leftrightarrow t' = 0$) in addition to the one suggested in 3. Finally, in Fig. we compare $F^{(0)}$ versus $t'$ when the vibrational mode is detected ($F^{(0, \text{het})}_{\pm}$) with the corresponding fidelity when the same mode is traced out ($F^{(0, \text{tr})}_{\pm}$), for $\tilde{n} = 0, 10^3$. The improvement of the fidelity brought by the additional heterodyne measurement ($F^{(0, \text{het})}_{\pm} \sim 1$ for $\tilde{n} = 0$, except very close to $2m\pi$) is impressive, meaning that heterodyning the mirror mode (and communicating the result) allows to distill more EPR correlations between the two optical sidebands. In other words, the heterodyne measurement allows to improve the efficiency of the present scheme as an alternative source of two-mode squeezing 5.

VI. CONCLUSIONS

In conclusion, we have presented an optomechanical system as a paradigm of three-mode teleportation network. We have provided a thorough study of entanglement and teleportation capabilities of this quantum channel. The teleportation fidelities result improved by using heterodyne measurement at the remaining mode and using the acquired information to distill a finer channel.

On one hand, our results could be useful to extend quantum information processing towards macroscopic domain, by using e.g. micro-opto-mechanical-systems 14. On the other hand, they could be applied as well to all optical systems described by the Hamiltonian of Eq. 10 15.

FIG. 1: Scheme of the optomechanical system.

FIG. 2: Three-mode teleportation network. In one case mode $k$ is simply traced out while in the other case it is heterodyned and the measurement result $\alpha$ sent to Bob. From a zero-mean Gaussian state for $i, j, k$ we always distill a bipartite Gaussian state. In the first case the quantum channel has zero drift and Bob performs the displacement $\alpha_j \rightarrow \alpha_j - \gamma_{k, \pm}$. In the second case, the channel has a nonzero drift, known from the knowledge of $\alpha$, and Bob performs a modified displacement $\alpha_j \rightarrow \alpha_j - \gamma_{\pm} - \delta_{\pm}$ with $\delta_{\pm}$ depending on the drift (see text).
FIG. 3: Traced out mode case. Fidelities $F^{(0)}_\pm$ and $F^{(2)}_\pm$ versus $t'$ (around $t' = 2\pi$) for $\bar{n} = 0$ and $\bar{n} = 10^3$. We have set $r = 1 + 2.5 \times 10^{-7}$. The telecloning time interval has been marked.

FIG. 4: Heterodyne detection case. Fidelities $F^{(2)}_\pm$ versus $t'$ (around $t' = 2\pi$) for $\bar{n} = 0$ (a), $\bar{n} = 1$ (b) and $\bar{n} = 10^7$ (c). We have set $r = 1 + 2.5 \times 10^{-7}$.

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FIG. 5: Fidelity $F^{(0)}_-$ versus $t'$ in the traced out mode case $F^{(0,rr)}_-$ and in the detected mode case $F^{(0,het)}_-$, for $\bar{n} = 0$ (a) and $\bar{n} = 10^5$ (b). We have set $r = 1 + 2.5 \times 10^{-7}$.

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