A Proposal on the Topological Sector of 2d String

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The field content of the two dimensional string theory consists of the dynamical tachyon field and some nonpropagating fields which consist in the topological sector of this theory. We propose in this paper to study this topological sector as a spacetime gauge theory with a simple centrally extended $w_\infty$ algebra. This $w_\infty$ algebra appears in both the world sheet BRST analysis and the matrix model approach. Since the two dimensional centrally extended Poincaré algebra is naturally embedded in the centrally extended $w_\infty$ algebra, the low energy action for the metric and dilaton appears naturally when the model is truncated at this level. We give a plausible explanation of emergence of discrete states in this formulation. This theory is again the effective theory at zero slope limit. To include higher order $\alpha'$ corrections, we speculate that the whole theory is a gauge theory of a deformed $w_\infty$ algebra, and the deformation parameter is just $\alpha'$. 

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1. Introduction

Despite much progress has been made in the last few years in the study of the two dimensional string theory [1], many issues remain open. The matrix model approach provides us a very powerful tool in calculating scattering amplitudes of the tachyon [2], sometimes even nonperturbatively [3]. Just how the perturbatively unitary S-matrix [3] can be understood in a conventional spacetime picture is evading our grip so far. There are many reasons for one to go beyond what we have achieved. Although discrete states discovered in both the conformal approach [4] and the matrix model approach [5] are non-propagating, they play an important role in providing more backgrounds such as a black hole background and also in the tachyon S-matrix. The latter is unitary with a tachyon background, thus discrete states should neither appear in the intermediate channels and nor serve as external scattering states, at least in this background. Nevertheless a recent calculation [6] showed that the conventional spacetime effective action for the tachyon is nonlocal, this nonlocality may be understood by integrating out nonpropagating modes. Precisely in this place one need to understand how the tachyon is coupled to other topological modes and how these modes are integrated out to result in the effective action for tachyon.

To study the topological sector in this theory and its coupling to the tachyon, a full formulation such as a string field theory is needed. Unfortunately although much progress in that field has been made recently [7], we are still lacking a tractable string field theory. Our only guide here is the fact that the theory for the topological sector must be topological, so its form can not be random. A natural formulation of a topological theory is a gauge theory of some group or algebra. Within such a formulation, the $w_\infty$ symmetries found in the 2d string in different ways [8] [9] should be the residual symmetries of gauge symmetries after a certain gauge fixing.

A somewhat straightforward approach is to gauge the $w_\infty$ algebra in spacetime, this is what to be done in this paper. In the matrix model approach, $w_\infty$ algebra arises due to the free fermion nature of the theory, and it also appears as an infinite algebra commuting with the collective field hamiltonian. This very same algebra arises on the world sheet as the algebra of spin one vertex operators, having direct relation to discrete states in the BRST cohomology. Spacetime momenta are discrete for these currents. In both approaches, spacetime origin of the $w_\infty$ algebra is missing. To motivate our formulation, we should look for such a spacetime interpretation. There is a ready interpretation, which is just to
interpret the $w_{\infty}$ as the area-preserving diffeomorphism algebra of the plane, the spacetime. 2d Poincaré algebra (Euclidean algebra in case of Euclidean spacetime) is a subalgebra of this $w_{\infty}$ algebra. This subalgebra should play a peculiar role in the story. Indeed H. Verlinde in [10] proposed to study the low energy action of the dilaton and the metric as a gauge theory of the 2d Poincaré algebra. The topological nature of the theory is obvious in this formulation. Cangemi and Jackiw later reformulated this theory as a gauge theory of the centrally extended Poincaré algebra [11]. The cosmological constant term has a natural origin in this formulation. This gauge formulation is another motivation for our search for a gauge theory of the whole topological sector, in which the dilaton and metric are just the lowest modes. It turns out that the central extension of Poincaré algebra can be naturally embedded into the corresponding central extension of $w_{\infty}$ algebra, and the center element has a natural position in the latter algebra.

In the next section we shall briefly review the gauge formulation of the metric and dilaton system, and point out an important point which becomes crucial in the $w_{\infty}$ extension, in the end of that section. We then in section 3 introduce the $w_{\infty}$ algebra as the area-preserving diffeomorphism algebra of the plane, and construct the gauge theory. In section 4, classical solutions are discussed and discrete states are explained in our theory. We present our conclusion and outlook in the final section.

2. Dilaton plus the metric as a gauge theory

We review in this section the observation made by H. Verlinde on the two dimensional dilaton and metric system [10], which is polished later by Cangemi and Jackiw [11]. We will work in the Euclidean spacetime, rotating back to the Minkowski spacetime is straightforward. The (zero slope) low energy action for the dilaton and the metric in the 2d string theory is

$$S = \int e^{-2\phi} \sqrt{g} (R - \partial \Phi \overline{\partial} \Phi + \lambda),$$

(2.1)

where we have used the complex coordinates system. $\lambda$ is the spacetime cosmological constant, and is fixed in the string theory by requiring the critical central charge on the world sheet. We shall leave the cosmological constant arbitrary. Our convention for the curvature is, in the conformal gauge,

$$\sqrt{g}R = \partial \overline{\partial} \ln g_{z \overline{z}}.$$

This model allows for a family of black hole solutions parametrized by the mass of the black hole \cite{12} \cite{13}. Any theory which admits solutions parametrized by a finite number of moduli must be topological. H. Verlinde observed that in the first order formalism, model \((2.1)\) can be interpreted as a gauge theory with the two dimensional Poincaré group as the gauge group and is obviously topological. The cosmological constant term is gauge invariant on-shell, namely when the field strength associated with the zweibein field vanishes. Cangemi and Jackiw promoted this theory to be gauge invariant off-shell by introducing a center element in the gauge algebra, thus the dilaton-metric system is a gauge theory of the centrally extended Poincaré group. In the Euclidean space time, the relevant generators are \((J, L_{-1}, \bar{L}_{-1}, I)\). Here \(I\) is the center element, \(J\) is the generator of rotations, and \(L_{-1}, \bar{L}_{-1}\) are translation generators, in the complex coordinates. Nonvanishing commutators are

\[
\begin{align*}
[J, L_{-1}] &= iL_{-1}, \\
[J, \bar{L}_{-1}] &= -i\bar{L}_{-1}, \\
[L_{-1}, \bar{L}_{-1}] &= iI.
\end{align*}
\]

In terms of the generators of the Virasoro algebra, \(J\) can be written as \(i(L_0 - \bar{L}_0)\). This fact is irrelevant for us, since we will not embed the central extension of Poincaré algebra into a sub-algebra of the Virasoro algebra. Next we introduce the gauge field

\[
A = \omega J + eL_{-1} + \bar{e}\bar{L}_{-1} + aI
\]

with the reality condition \(e^* = \bar{e}, *\) is complex conjugation. The field strength is then

\[
F = dA + A^2 = d\omega J + DeL_{-1} + D\bar{e}\bar{L}_{-1} + (da - ie \wedge \bar{e})I,
\]

where covariant derivatives are defined as \(De = de + i\omega \wedge e\) and \(D\bar{e} = d\bar{e} - i\omega \wedge \bar{e}\). Introduce lagrangian multipliers \((X^0, \bar{X}, X, \lambda)\) which transform coadjointly under the centrally extended algebra. The action

\[
\int \left( X^0 d\omega + \bar{X} De + XD\bar{e} + \lambda (da - ie \wedge \bar{e}) \right)
\]

is gauge invariant. Equations of motion for the gauge field derived from \((2.3)\) are the condition for flat connections. Equations of motion for the Lagrangian multipliers are

\[
\begin{align*}
DX + i\lambda e &= 0, \\
D\bar{X} - i\lambda \bar{e} &= 0, \\
dX^0 + i\bar{X} e - iX \bar{e} &= 0, \\
d\lambda &= 0.
\end{align*}
\]
The last equation implies that $\lambda$ is a constant. Substituting these equations into (2.3) and using the following definition

$$X^0 = e^{-2\Phi}, \quad g = e^{2\Phi} e \otimes \bar{e}$$

we recover the action (2.1) for the dilaton and the metric. Notice that the rescaled metric $\exp(-2\Phi)g$ is always flat by equation of motion. Equations in (2.4) imply that the combination $\lambda X^0 - X \bar{X}$ is a constant. Let it be $M$, then the metric and dilaton are

$$g = \frac{1}{\lambda(M + X \bar{X})} DX D\bar{X}, \quad e^{-2\Phi} = \frac{1}{\lambda}(M + X \bar{X}). \quad (2.5)$$

Suppose $g$ is nondegenerate, then $X$ and $\bar{X}$ can be chosen as a complex coordinates system. We shall explain why this is the case shortly. When this is done, the solution is precisely the Euclidean black hole solution with mass $M$ found in [12] [13].

There is a crucial point ignored in the previous discussion in [10]. The equation of motion for gauge field is the condition of flat connection. Since the topology of the plane is trivial, one expects that any flat connection can be gauge transformed into zero. If so, the metric in (2.5) would be degenerate and we lose the black hole solutions. To solve the puzzle, we should note that all solutions to the equation of motion for the gauge field are gauge equivalent to one and another. The null solution and constant solutions are particular solutions. For the latter case, we mean that components in $e = e_z dz + e_{\bar{z}} d\bar{z}$ are constant. Take, say $\omega = e_z = 0$ and $e_z$ nonvanishing, we see that following from (2.4) $X$ is proportional to $z$. This gives the black hole solution. When however all components of the gauge field are zero, solutions of the lagrangian multipliers are constant solutions. Since there are four such fields, it appears that there are solutions of four parameters. This is not the case. When we fix the gauge with vanishing gauge field, apparently there are residual gauge transformations. These gauge transformations are just constant transformations, and there are four parameters here too. It is easy to see that both $\lambda X^0 - X \bar{X}$ and $\lambda$ are invariant under a constant gauge transformation. The first quantity is just the parameter $M$ and the second, unfortunately, is fixed by hand in our formulation.

The above argument still does not give us a satisfactory answer to why we should favor a constant solution. The best way to settle this problem is to couple the gauge system to a matter field, say the tachyon in the context of 2d string theory. We leave this problem to a future work.

To reproduce discrete “states” in our extended theory, we will encounter exactly the same problem. Again the geometric origin is associated with coupling the gauge theory of $w_\infty$ to the tachyon. Put in another way, it should follow from a complete string field theory.
3. Gauge theory of the centrally extended $w_\infty$

We are going to show that the centrally extended Poincaré algebra can be naturally embedded into a centrally extended $w_\infty$ algebra. For simplicity and reason becoming clear later, we simply denote such a centrally extended algebra also by $w_\infty$. In fact generators of the central extended Poincaré algebra are the first few generators of $w_\infty$. We shall discuss a gauge theory of this infinite dimensional algebra, along the line in the previous section. This theory is again topological. When the theory is truncated to the centrally extended Poincaré algebra which is generated by the first few generators of $w_\infty$, the topological theory of the dilaton and the metric is obtained. This is the first hint that this theory might be the right theory for the topological sector of the 2d string. We then proceed to show that discrete states found previously in the conformal approach also naturally emerge as classical solutions in a certain gauge. This is the second evidence for taking this theory as a candidate theory for the topological sector. We begin with discussing the $w_\infty$ algebra as the area-preserving diffeomorphism algebra of the complex plane, i.e. the spacetime in the context of the 2d string theory, in the next subsection.

3.1. Area-preserving diffeomorphism algebra of the plane

Our motivation for discussing such an algebra is the following. First, the 2d Poincaré algebra must be a sub-algebra of the area-preserving diffeomorphism algebra, as rotations and translations obviously preserve the area element. Second, precisely such algebra appears in both world sheet analysis and the matrix model approach, where the $w_\infty$ symmetries seem to be the residual symmetries after certain gauge fixing in the full string theory. We are motivated to uncover the unknown, larger gauge symmetries and identify the known $w_\infty$ symmetries as the residual symmetries.

We continue to use complex coordinates. The area element is $\frac{1}{i}dz \wedge d\bar{z}$. Any diffeomorphism preserving this area element can be generated by $\delta z = i\delta f$, $\delta \bar{z} = -i\partial f$, $f$ is a regular real function. Any such function can be expanded into series

$$ f = \sum_{n,m \geq 0} f_{m,n} z^m \bar{z}^n $$

with reality condition $f_{m,n}^* = f_{n,m}$, in other words, $f = (f_{m,n})$ is a hermitian matrix. Note that a constant function results in a trivial diffeomorphism. Nevertheless precisely
this generator can be interpreted as a center element, as explained later. For a pair of functions \( f \) and \( g \), we define the Poisson bracket associated to our area element:

\[
\{f, g\} = i(\partial f \overline{\partial g} - \overline{\partial f} \partial g)
\]

For the basis \( V_{m,n} = z^m \overline{z}^n \), we have a closed algebra

\[
[V_{m_1,n_1}, V_{m_2,n_2}] = i(m_1 n_2 - m_2 n_1)V_{m_1+m_2-1,n_1+n_2-1},
\]

where we have replaced the Poisson bracket by commutator, for later convenience. This algebra is just the \( w_\infty \) algebra, as becomes familiar in a new basis \( Q_{s,l} = V_{(s+l)/2,(s-l)/2} = r^s \exp(i l \theta) \). In the second expression we used the polar coordinates \( z = r \exp(i \theta) \). Note that for nontrivial generators \( s \geq 1 \), the range of integer \( l \) for a given \( s \) is \( -s, -s+2, \ldots, s \).

Commutators in this basis are

\[
[Q_{s_1,l_1}, Q_{s_2,l_2}] = \frac{i}{2}(l_1 s_2 - l_2 s_1)Q_{s_1+s_2-2,l_1+l_2}.
\]

This is precisely the \( w_\infty \) algebra found in the 2d string theory. It is easy to identify the translation generators and the rotation generator, from our discussion. Indeed \( L_{-1} = Q_{1,-1}, \bar{L}_{-1} = Q_{1,1} \) and \( J = Q_{2,0} \). These are first few generators in \( w_\infty \). The Euclidean subalgebra is

\[
[Q_{2,0}, Q_{1,-1}] = iQ_{1,-1},
\]

\[
[Q_{2,0}, Q_{1,1}] = -iQ_{1,1},
\]

(3.3)

to be compared with commutators in (2.2). Remarkably enough, we can extend the algebra (3.2) to include the element \( Q_{0,0} \). This element might be explained as the trivial generator corresponding to the constant function \( f \). Commutation relation in (3.2) extended to include \( Q_{0,0} \) is consistent, and we have

\[
[Q_{1,1}, Q_{1,-1}] = iQ_{0,0}.
\]

Compared to (2.2) we soon find the identification \( Q_{0,0} = I \). Note that, according to (3.2) this generator commutes with all other generators. We thus find that the center element in the centrally extended Euclidean (Poincaré) algebra can be very naturally included in the \( w_\infty \) algebra. We shall denote this extended algebra again by \( w_\infty \).

Finally, we note that the \( w_\infty \) algebra is different from the algebra of area-preserving diffeomorphism on a cylinder or a punctured plane [14]. A copy of Virasoro algebra can be embedded in the latter, but not in the \( w_\infty \) algebra under discussion.
3.2. The gauge theory

It is straightforward to generalize the gauge theory discussed in section 2 to the whole $w_\infty$ algebra introduced in the previous subsection. The gauge field is

$$A = \sum_{s,l} A^{s,l} Q_{s,l}. \quad (3.4)$$

Reality condition $(A^{s,l})^* = A^{s,-l}$ is imposed. By embedding of the centrally extended Poincaré algebra, we identify the first few components

$$A^{0,0} = a, \quad A^{2,0} = \omega, \quad A^{1,-1} = e, \quad A^{1,1} = \bar{e}. \quad (3.5)$$

The field strength, calculated from the structure of the algebra, is

$$F = dA + A^2 = \sum_{s,l} F^{s,l} Q_{s,l},$$

$$F^{s,l} = dA^{s,l} + \sum_{s',l'} i 4 (l's - ls' + 2l') A^{s',l'} \wedge A^{s-s'+2,l-l'}. \quad (3.6)$$

In particular, the lowest components are identified with those of the central extension of Euclidean algebra

$$F^{0,0} = da - ie \wedge \bar{e}, \quad F^{1,-1} = de + i\omega \wedge e + 2i\bar{e} \wedge A^{2,-2},$$

$$F^{1,1} = d\bar{e} - i\omega \wedge \bar{e} - 2ie \wedge A^{2,2}, \quad F^{2,0} = d\omega - ie \wedge A^{3,1} + i\bar{e} \wedge A^{3,-1} + 4iA^{2,2} \wedge A^{2,-2}. \quad (3.7)$$

Two notable new features appear in these components. First, the spin connection is not simply related to zwei-bein fields when condition $F^{1,-1} = F^{1,1} = 0$ is imposed, because of new components in the gauge field. This implies that the spin connection has torsion. Second, again due to new components, the spin connection is no longer flat when the gauge field is a flat connection. One is attempting to interpret this as back-reaction of high rank fields to the spin connection and zwei-bein fields.

Introducing lagrangian multipliers $(X_{s,l})$ which transform coadjointly under gauge transformations and imposing the reality condition $(X^{s,l})^* = X^{s,-l}$, we generalize the action presented in section 2:

$$S = \int \sum_{s,l} X_{s,l} F^{s,l}. \quad (3.8)$$
This is a topological theory too. The equation of motion for the gauge field is simply \( F^{s,l} = 0 \). For a component \( F^{s,l} \), there are only finitely many components of the gauge field involved. The identification of lowest \( X_{s,l} \) with those discussed in section 2 is

\[
\begin{align*}
X_{0,0} &= \lambda, & X_{1,-1} &= X, \\
X_{1,1} &= X, & X_{2,0} &= X^0.
\end{align*}
\]

The equation of motion for lagrangian multipliers is

\[
dX_{s,l} + \sum_{s',l'} (l's - ls' + 2l)X_{s',l'}A^{s'+2-s',l'-l} = 0. \tag{3.9}
\]

Note that for a given \((s,l)\), there are infinitely many components of \(X\) and \(A\) involved in (3.9). We cannot derive simple relations among \(X_{s,l}\) as integrations of the equation of motion. In addition, it is not possible to use the equation of motion to eliminate the lagrangian multipliers in the action (3.8).

4. Classical solutions

The equation of motion for the gauge field is the condition of flat connection. Just as in the theory discussed in section 2, any such connection is gauge equivalent to the vanishing one, as we choose spacetime as the 2-plane. Does this imply that the content of our theory is trivial? The answer is no. Suppose we are allowed to gauge transform the gauge field to zero, the residual gauge transformations are then constant gauge transformations. The equation of motion for \(X_{s,l}\) in this gauge is just \(dX_{s,l} = 0\). So solutions of \(X_{s,l}\) are constant solutions, and the space of solutions is infinite dimensional. Since the residual gauge transformations are constant, the space of gauge inequivalent solutions is the space of coadjoint orbits of the 2d area-preserving diffeomorphism group in the infinite dimensional space of constant solutions of \((X_{s,l})\). Note that, the central element has no impact in a gauge transformation.

Under gauge transformation with gauge parameter \(\epsilon = \sum \epsilon_{s,l} Q_{s,l}\), the gauge field transforms according to

\[
\delta A^{s,l} = d\epsilon^{s,l} + \sum_{s',l'} \frac{i}{2} (l's - ls' + 2l')A^{s',l'}\epsilon^{s+2-s',l-l'},
\]
and the lagrangian multipliers transform as

$$\delta X_{s,l} = -\sum_{s',l'} \frac{i}{2} (l's - ls' + 2l) X_{s',l'} \epsilon^{s'+2-s,l'-l}. \quad (4.1)$$

The gauge transform of $X_{s,l}$ is essentially different from that of the gauge field in that there are infinitely many terms in (4.1).

We claim that the space of gauge inequivalent solutions is also infinite dimensional. To see this, consider a solution in which all components of $X$ are zero except for $X_{0,0}$, $X_{s,l}$ and $X_{s,-l} = (X_{s,l})^*$. $X_{0,0}$ is the cosmological constant $\lambda$ and we have to fix it in the string theory. It is invariant under gauge transformation (4.1) for any configuration of $X$. Now we show that the above particular configuration can not be transformed into zero. We have $\delta X_{s,l} = -i\lambda \epsilon^{2-s,-l} - i\lambda X_{s,l} \epsilon^{2,0}$. If $s$ is greater than 1, the first term is zero. The second term does not change the absolute value of $X_{s,l}$, since $\epsilon^{2,0}$ is real so that the second term is a pure phase shift. It is easy to see that any such two distinct configurations with different pairs $(s,l)$ and $(s',l')$ are not gauge equivalent. This shows that the space of gauge inequivalent solutions is infinite dimensional.

Recall that in the conformal analysis, there are infinitely many new physical “states” found in [9], and each state is assigned with a pair of $(s,l)$. Each state of this kind has pure imaginary energy in Minkowski space in which the Liouville dimension is spatial, therefore it is nonpropagating. However, these states can be used to deform the world sheet conformal field theory, thereby provide new backgrounds. Given that the space of solutions in our theory is infinite dimensional, and the solution $(\lambda, X_{s,l}, X_{s,-l})$ is a nontrivial solution, we conjecture that this space is actually the moduli space of all possible physically inequivalent backgrounds in the 2d string, when the tachyon is absent. In particular, the solution $(\lambda, X_{s,l}, X_{s,-l})$ may be identified with the moduli corresponding to deformation generated by corresponding pair of discrete states. This is encouraged by the simplest case of the black hole solution, which is gauge equivalent in the present theory to $(\lambda, X_{2,0} = X^0)$.

As we discussed in section 2, one should not be satisfied with the degenerate solution, although in the pure topological theory it is equivalent to the nondegenerate solution, namely the black hole solution. All solutions we discussed above are degenerate, as the gauge field is zero. Nevertheless it is tempting to transform a degenerate solution into

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The new states found in the third paper in [9] are argued in [15] to be physically trivial, as can be eliminated by gauge transformations in string field theory.
a nondegenerate one. As suggested by the constant solution considered in section 2, a particularly attractive nondegenerate solution is the following. We still assume the zweibein field \( e = e_z dz \), and \( e_z \) is a nonvanishing constant. Thus \( e_z \) is scalar under rotations, this corresponds to asking the gauge field \( A \) be scalar under rotations, since the generator \( L_{-1} \) transforms in the opposite way as \( dz \) does. Now consider component \( A^{s, -l} \) with \( l > 0 \). We assume the gauge \( A^{s, -l} = A_z^{s, -l} dz \). To require also that \( A \) is scalar under rotations, then \( A_z^{s, -l} \) takes a form \( f(r) \exp(i(l - 1)\theta) \) in the polar coordinates. This form is very suggestive, if we take a special function \( f(r) = r^s \). For large \( r \), the dilaton field \( X^0 = \exp(-2\Phi) \to r^2 \). In the linear dilaton region, we identify \( r = \exp(-\phi) \), where \( \phi \) is the Liouville dimension. Using this identification we find \( A_z^{s, -l} = \exp(-s\phi + i(l - 1)\theta) \). This form is the same as the momentum factor in the vertex operator of a discrete state. Note that, the oscillator part of the vertex operator depends on coordinates of the world sheet, therefore has nothing to do with a spacetime field.

We point out that when \( A^{s,l} \) is nonzero, some higher rank components must be nonzero too. Suppose \( A^{s,l} \) is the first nonvanishing component of the gauge field, except for \( e \) and \( \bar{e} \), then the equation of motion for \( A^{s,l} \) is

\[
dA^{s,l} - \frac{i}{2}(s + l + 2)e \wedge A^{s+1,l+1} + \frac{i}{2}(s - l + 2)\bar{e} \wedge A^{s+1,l-1} = 0.
\]

If \( A^{s,l} \) takes the above suggested form, we find that the last two terms in the above equation can not be zero simultaneously. This can be simply understood. The vertex operator corresponding to \( A^{s,l} \) is not exactly marginal on the world sheet, so must be supplemented with other terms on the world sheet.

As suggestive as it appears, the above form of the gauge field \( A^{s, -l} \), if has anything to do with the known discrete state, must be justified in a more complete theory. As \( e \) must be nondegenerate if the tachyon is coupled to the gauge system, the form of a higher rank component may also be determined in a similar way. Finally, we note that although we argued that the space of physical solutions is infinite dimensional, we have not probed its structure extensively. As the space of coadjoint orbits of the 2d area-preserving diffeomorphism group, it can be studied in a mathematically rigorous way.
5. Conclusion and outlook

We have proposed in this paper a simple topological theory as the theory of the topological sector in the 2d string theory. This is a straightforward and natural generalization of the gauge theory for the system of the dilaton and metric. Our analysis in the last section is especially encouraging, since discrete states in principle can be understood in this formulation. There are many things remaining to do. The most pressing one is to couple this system to the tachyon and study the whole system both at classical and quantum levels.

Another issue we leave open in this paper is that of higher order corrections in $\alpha'$. This parameter finds no place in the present theory. It is certainly important to incorporate these corrections into the theory, in order to understand some genuine stringy phenomena. For example when the curvature becomes large, these corrections become significant. It is especially interesting to understand the exact black hole solution suggested in [16]. We believe that the topological sector remains topological, when higher order $\alpha'$ corrections are included. The theory may also be formulated as a gauge theory. There is a ready choice, namely to use some deformation of $w_\infty$ algebra as the gauge algebra in which $\alpha'$ is the deformation parameter. This possibility is currently under investigation.

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References

[1] For a review see I.R. Klebanov, “Strings in two dimensions”, Lectures at ICTP Spring School on String Theory and Quantum Gravity, Trieste, April 1991.

[2] K. Demeterfi, A. Jevicki and J.P. Rodrigues, Nucl. Phys. B362 (1991) 173; J. Polchinski, Nucl. Phys. B362 (1991) 125; G. Moore, Nucl. Phys. B368 (1992) 557; G. Mandal, A.M. Sengupta and S.R. Wadia, Mod. Phys. Lett. A6 (1991) 1465.

[3] G. Moore, R. Plesser and S. Ramgoolam, Nucl. Phys. B377 (1992) 143.

[4] A.M. Polyakov, Mod. Phys. Lett. A6 (1991) 635; B. Lian and G. Zuckerman, Yale preprint YCTP-P18-91; P. Bouwknegt, J. McCarthy and K. Pilch, CERN preprint CERN-TH.6162/91.

[5] D.J. Gross, I.R. Klebanov and M.J. Newman, Nucl. Phys. B350 (1991) 621.

[6] M. Li, Brown preprint BROWN-HET-887, Dec. 1992.

[7] B. Zwiebach, IAS preprint IASSNS-HEP-92/41.

[8] J. Avan and A. Jevicki, Phys. Lett. B266 (1991) 35; D. Minic, J. Polchinski and Z. Yang, Nucl. Phys. B369 (1992) 324; S. Das, A. Dhar, G. Mandal and S. Wadia, Mod. Phys. Lett. A7 (1992) 71.

[9] E. Witten, Nucl. Phys. B373 (1992) 187; I.R. Klebanov and A.M. Polyakov, Mod. Phys. Lett. A6 (1991) 3273; E. Witten and B. Zwiebach, Nucl. Phys. B377 (1992) 55.

[10] H. Verlinde, in Sixth Marcel Grossmann Meeting on General Relativity, M. Sato ed. (World Scientific, Singapore, 1992).

[11] D. Cangemi and R. Jackiw, MIT preprint CTP#2147, 1992.

[12] S. Elitzur, A. Forge and E. Rabinovici, Nucl. Phys. B359 (1991) 581; G. Mandal, A. Sengupta and S.R. Wadia, Mod. Phys. Lett. A6 (1991) 1685.

[13] E. Witten, Phys. Rev. D44 (1991) 314.

[14] For a review see X. Shen, Int. J. Mod. Phys. A7 (1992) 6953.

[15] S. Mahapatra, S. Mukherji and A.M. Sengupta, Mod. Phys. Lett. A7 (1992) 3119.

[16] R. Dijkgraaf, H. Verlinde and E. Verlinde, Nucl. Phys. B371 (1992) 269.