Pricing Strategy for Reproduction of Worn-out Ball and Gate Valves in Oil and Gas Industry: Using Game Theory on Four Closed-loop Supply Chain Scenarios

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Abstract
Collecting and remanufacturing worn-out products provide significant financial advantages. In this research, we examine how remanufacturing worn-out ball and gate valves, which are important pieces of equipment in the oil and gas industry, could improve the profitability of the closed-loop supply chain. In this regard, four different scenarios for collecting and remanufacturing processes are considered: (1) the manufacturer collects the worn-out product from the consumers and remanufactures them, (2) the retailer collects the worn-out products and both manufacturer and retailer remanufacture them, (3) the third party collects the worn-out products and both manufacturer and third party remanufacture them, and (4) the manufacturer collects the worn-out products without remanufacturing them. To formulate the interactions of the CLSC members under the four different scenarios, we use the Nash and Manufacturer-Stackelberg games. Accordingly, the optimal decision variables i.e., the acquisition price, wholesale price, and retail price are calculated under four scenarios. Then, the optimal solutions are compared in four scenarios. In addition, the optimal profit of the closed-loop supply chain and members are obtained under the different scenarios. The results show that in the Nash game, Scenario 2 is the best scenario. However, in the Stackelberg game, Scenario 3 is the best scenario.

Keywords: Closed-loop Supply Chain, Collection strategy, Collection option, Pricing Strategy, Worn-out Products, Game Theory.

1. Introduction
In recent decades, manufacturers have incorporated remanufacturing issues into their supply chain (SC) operations besides manufacturing new products. Accordingly, closed-loop supply chains (CLSCs) are introduced by collecting the used products in the reverse flow [1]. In today's competitive environment, many manufacturers collect the used products because of the potential of products for remanufacturing. Remanufacturing the used products not only conserves the environment but also reduces the costs of manufacturing. According to Wan & Hong [2], manufacturers could have 40-50% cost saving. In the oil and gas industry, the ball valves which
are used to cut and connect the fluid flow are suitable for remanufacturing. Because of their compact structure and the fast operation expected in controlling the fluid flow, the ball valves are subject to severe corrosion and erosion. Thus, remanufacturing these products reduces the costs and avoids more use of natural resources. Additionally, the remanufactured ball valves are used in active service satisfactorily [3]. As a result, collecting and remanufacturing the worn-out bell and gate valves are profitable for the manufacturers.

In the CLSC, the amount of the returned products directly affects the CLSC profitability. Accordingly, to increase the amount of returned products, adopting an effective strategy is of high significance [4]. One of the effective strategies is to offer an acquisition price for each used product to entice the customers to return their products. Monetary incentives for collecting the used products not only increase the amount of the returned products but also can boost the customers’ demand because of achieving public praise [4]. Therefore, the acquisition price can have a bilateral effect on both reverse and forward flows in the CLSC.

In addition, to collect the returned products from the customers, one of the significant challenges is to find a suitable reverse flow [5]. In this regard, some manufactures directly collect the used products from the customers such as Xerox and Canon [6]. The other manufacturers collect the used products through the retailers such as Kodak [7] or third parties such as Dell and Acer [5]. A suitable option for collecting significantly affects the performance of the CLSC. Thus, it is high of importance that a suitable option is selected for the collection process.

The oil and gas industry is of special importance in Iran and scholars investigate reducing costs (e.g., [8]), and increasing productivity (e.g., [9], and [10]) issues in this industry. Motivated by the significance of the oil and gas industry in Iran, in this study a CLSC with one manufacturer, one retailer, and one third party is considered. The manufacturer produces the ball and gate valves and sells them through a retailer. Furthermore, the manufacturer collects worn-out products from the customers and remanufactures the worn-out products. To entice the consumers to return the used products, an acquisition price is offered for each used item. In addition, the used products can be collected by the manufacture, retailer, or third party. In this study, four different scenarios are investigated to find a suitable collection channel in the CLSC. The Stackelberg and Nash games are used to obtain the optimal decisions such as the acquisition price, wholesale price, and retail price in each scenario. The main aims of this study are as follows: (i) examining different scenarios for the collecting and remanufacturing the ball and gate valves in the oil and gas industry to find the suitable option for the collecting process, (ii) examining the interactions of the CLSC members in each scenario by applying the Nash and Stackelberg games, (iii) evaluating the bilateral effects of the acquisition price under four different scenarios, (iv) finding the optimal acquisition price, retail price, and wholesale price in each scenario under both Stackelberg and Nash games.

The rest of this paper is as follows. In Section 2, the related literature is reviewed in three streams. The problem definition and model formulation are provided in Section 3. A set of sensitivity analyses on the key parameters is analyzed in Section 4. Finally, Section 5 represents the discussion and conclusions of this study.

2. Background

In this section, the related literature is reviewed in three streams to highlight the contributions: (1) Collection strategy in the CLSCs, (2) Collection option in the CLSCs, and (3) Game theory in the CLSCs. At the end of this section, the research gaps and contributions are provided.

2.1. Collection strategy in the CLSCs
In the recent era, many manufacturers involve themselves in remanufacturing because remanufacturing process not only reduces the manufacturing costs but also achieves environmental sustainability [5]. The remanufacturing issues have been extensively studied in the CLSC literature. In the CLSCs, the amount of returned products is an important challenge for the manufacturers. To deal with this challenge, effective strategies should be adopted by the collectors in the CLSCs.

In the CLSC literature, different strategies are adopted to collect used products. Most of the studies consider that the amount of returned products is influenced by the collection rate, which is determined by the collectors, e.g., Giri et al. [11], Modak et al. [1], Wan & Hong [2], Hosseini-Motlagh et al. [12], and Mondal & Giri [5]. Other strategies such as discounts, exchange offers, corporate social responsibility, and services are investigated by several papers. For instance, Govindan & Popiuc [13] consider a reverse SC in which the retailer’s discount influences the customers’ willingness to return the used products. Maiti & Giri [14] investigate a CLSC in which the used products are collected via exchange offers to the customers. Hosseini-Motlagh et al. [15] consider that the amount of return products is influenced by the dealers’ services. The effects of corporate social responsibility on the amount of returned products are considered by Hosseini-Motlagh et al. [16] and Hosseini-Motlagh et al. [17].

In addition, the effect of the acquisition price on the amount of returned products is studied by some scholars. For instance, Bulmuş et al. [18] investigate the effects of the acquisition price and marketing strategies on the remanufacturing and manufacturing systems. Li et al. [19] coordinate the wholesale price, retail price, and acquisition price decisions in a reverse SC. Zheng et al. [20] determine the acquisition price and collection effort under the incomplete information. Hosseini-Motlagh et al. [21] examine the effects of the acquisition price on the supply function. All the reviewed papers consider the effects of the acquisition price on the amount of returned products. However, in reality, the acquisition price not only influences the amount of returned products but also can affect the demand for the products [4]. In this study, the effects of the acquisition price are examined in both the amount of returned products and demand for products.

2.2. Collection options in the CLSCs

In the CLSC, the returned products can be collected by the manufacturer, retailer, and third party. Accordingly, different collection options for returned products are widely investigated by many scholars in the CLSC literature [5]. For example, Savaskan et al. [7] examine different structures for collecting the used products (i.e., collecting through the third party, retailer, and manufacturer) and they illustrate that the retailer’s collection is the most effective structure. Hong et al. [22] study the effect of the advertising on the performance of a CLSC under the centralized structure and three decentralized structures e.g., the third party collection, the retailer collection, and the manufacturer collection. Xu & Liu [23] investigate the reference price effects in a CLSC under the manufacturer collection, retailer collection, and third party collection structures. Modak et al. [1] examine the quality level, retail price, and collection rate decisions under the three different collection structures. Chen & Akmalul’Ulya [6] consider the manufacturer’s and the retailer’s green effort under the reward-penalty mechanism in four different structures. Mondal & Giri [5] investigate the collection rate, marketing effort, and green innovation decisions in a two-period CLSC under four different structures.

Similar to the all above reviewed papers, in this study, three collection options (i.e., manufacturer collection, retailer collection, and third party collection) are examined under the different structures. In contrast to the above papers, the retailer or third party is remanufactured a fraction of the returned products and the rest of the returned products is remanufactured by the manufacture. More precisely, both manufacturer and retailer remanufacture the used products, or
both manufacturer and third party remanufacture the used products. Furthermore, most of the reviewed papers are used the Stackelberg game under the different structures. However, in this study, both Nash and Stackelberg games are applied for each structure.

2.3. Game theory in the CLSCs
Game theory is a scientific method to analyze conflict and cooperation in different areas such as trade, management, and production [24]. In the past two decades, game theory has often been used as a prediction tool in the SC management by academics and authorities. Although applications of game theory in the SC are so varied, it is often used in strategic decision-making in areas like pricing, advertising, and services [25]. In the SC management literature, many studies have used game theory approaches. For instance, Yue & Raghunathan [26] use a Stackelberg game in which the manufacturer as the leader determines the wholesale price and the retailer as the follower determines the retail price and order quantity under the full returns policy. Xiong [27] examines the effects of quality and retail price on the market demand using the retailer-Stackelberg game. Zhang et al., [28] use both supplier-Stackelberg game and retailer-Stackelberg game to investigate the effects of the trade credit period on the performance SC. Noori-Daryan et al. [29] apply the Stackelberg and Nash games to examine the reaction of SC members. Amirtaheri et al. [30] use the Manufacturer-Stackelberg and Distributor-Stackelberg games to model the cooperative advertising.

Furthermore, the game theory approaches are used to formulate the problem in the CLSC. For example, Huang and Wang [31] examine the effect of remanufacturing of returned products under different models by applying the Stackelberg game. Nazari et al. [32] examine the pricing and ordering policies in the CLSC by using the Nash and Stackelberg games. Maiti & Giri [33] study a CLSC under five different scenarios using the Stackelberg and Nash games. Taleizadeh et al. [34] determine the level of effort, quality, and price decisions under different channel power structures. Jian et al. [35] investigate a CLSC with fairness concern using a Stackelberg game. Rezaei & Maihami [36] formulate the equal powers of the retailer and manufacture in a CLSC using the Stackelberg and Nash games. Asl-Najafi & Yaghoubi [37] coordinate the collection effort and selling price decisions in a CLSC.

Regarding the reviewed papers, the Nash and Stackelberg game approaches are applied for remanufacturing process in the CLSCs. The current study utilizes game theory approaches (i.e., Stackelberg and Nash games) to examine the collecting and remanufacturing processes for the ball and gate valves of the oil and gas industry. The ball and gate valves are very particular products and broadly are used in the oil and gas industry. Accordingly, remanufacturing these products influences the CLSC profitability.

2.4. Research gaps and contributions
In the following, the research gaps and the contributions of this study are provided. Table 1 shows a summary of reviewed literature. According to Table 1, many studies investigate the effects of the collection rate on the CLSC performance. Several papers consider the effects of the acquisition price on the collection amount in the CLSC. However, they ignore the effects of acquisition price on the marketing demand, except Hosseini-Motlagh et al. [4]. In contrast to the literature, this study examines the bilateral effects of acquisition price on both reverse and forward flows under four different scenarios. Additionally, both Stackelberg and Nash games are used to formulate the interactions of the CLSC members.

Furthermore, from Table 1, most scholars consider one collection option in the CLSC and also use the Stackelberg game to formulate the CLSC members’ interactions. However, in reality, the used products can be collected by the manufacturer, retailer, or third party. Moreover, the CLSC members’ interactions can be modeled with the Nash game or Stackelberg game.
Therefore, in this study, four different scenarios are proposed for collecting and remanufacturing processes in the CLSC. Additionally, to formulate interactions of the CLSC members, both Stackelberg and Nash games are applied under each scenario.

In addition, several papers which investigate the different scenarios for collection consider that all the used products are remanufactured by a member who collects the used products, except Huang and Wang [31]. Huang and Wang [31] assume that a fraction of used products is remanufactured by the third party or distributor and the rest of them is remanufactured by the manufacturer. Similar to Huang and Wang [31], in this study, the retailer or third party remanufacturers a fraction of the used products according to their ability, and the manufacturer remanufactures the rest of them. However, this study is different from the study of Huang and Wang [31] in examining the bilateral effects of the acquisition price and applying the Nash game to model the different scenarios.

The main contributions of this study are as follows: (i) investigating the collecting and remanufacturing of ball and gate valves in the oil and gas industry, (ii) examining the collection option in four different scenarios, (iii) evaluating the bilateral effects of the acquisition price in the CLSC under different scenarios, (iv) obtaining the optimal acquisition price, wholesale price, and retail price under different scenarios by using the Stackelberg and Nash games.

3. Problem definition and mathematical model

In this study, a CLSC with one manufacture, one retailer, and one third party is considered. The manufacturer produces the ball and gate valves and sells them to the customers through a retailer. The retailer determines the retail price of the products and the demand for the manufacture's products depends on the retailer's selling price. On the other hand, since remanufacturing the worn-out ball and gate valves can reduce the manufacturer's costs, the manufacturer remanufactures the worn-out products. In this regard, to entice the customers to return the worn-out ball and gate valves, an acquisition price is determined for each of the worn-out products. Accordingly, the amount of returned products depends on the acquisition price. Furthermore, the acquisition price can affect the market demand. Accordingly, determining an optimal acquisition price is of high importance to improve the value of collection and the market demand. In addition, the worn-out products can be collected by the manufacturer, retailer, or third party. Therefore, determining a suitable option for collecting the used products is of high significance.

3.1. Determining CLSC scenarios and modeling

To find a suitable collection option, we investigate four different scenarios. In Scenario 1, the manufacturer directly collects the worn-out products from the customers and remanufactures them. In Scenario 2, the retailer collects the worn-out products and both manufacturer and retailer remanufacture them. More precisely, a fraction of the returned products is remanufactured by the retailer and the rest of them is remanufactured by the manufacturer. In Scenario 3, the third party collects the worn-out products and both manufacturer and third party remanufacture them. In scenario 4, the manufacture collects the worn-out products without remanufacturing. Figure 1 shows the CLSC structure under four different scenarios. To model the CLSC members' interactions, the Manufacturer-Stackelberg game and Nash game are applied. In the Manufacturer-Stackelberg game, the manufacturer as the leader determines its decision to optimize its profitability. Then, the retailer and third party set their decisions by considering the manufacturer's decision. However, in the Nash game, all CLSC members simultaneously determine their decisions to maximize their profitability.
There are other methods used in the previous studies to validate scenarios. Some scholars use coordinate or non-coordinate methods. Regarding the non-coordinate method, some models have been solved using the meta-heuristic algorithms [38] or the Newton-Raphson algorithm [39]. This method helps to solve the problems such as the effect of quality on remanufactured products, collection of defective products, load limit parameter on storage of worn products, the effect of advertising on the collection of used products. For remanufacturing the ball valves, the non-coordinate method is the best way to model the issue. The most important factor for the customers of ball valves is the consistent quality of the products. Therefore, in this research, the non-coordinate methods are taken into account for modeling the issue to determine the decision variables such as the wholesale price, acquisition price, and retail price of each product.

Table 2 shows notations used for decision variables and parameters in this study.

Scenario 1 (A, E): Collecting and remanufacturing by manufacturer
In this scenario, a two-level CLSC including a manufacturer and a retailer is considered. The products are sold to the customers by the retailer. The manufacturer collects the returned products and remanufactures them. Similar to Oraiopoulos et al. [40] and Atasu et al. [41], the cost saving of remanufacturing each returned product is as follows:

\[ T_1 = C - G_M \]  

(1)

The market demand depends on the retail price and the acquisition price. Accordingly, the demand for products is formulated as:

\[ D(p, s) = \delta_0 - \delta_1 p + \delta_2 s \]  

(2)

The number of returned products, which depends on the acquisition price, is formulated in Eq. (3). Such a function is used in the related literature e.g., Larsen et al. [42].

\[ R(s) = \varphi_0 + \varphi_1 s \]  

(3)

In this scenario, the profit function of the manufacturer is formulated as follows:

\[ \Pi_M^A = (w - c)[D(p, s) - R(s)] + (w - c + T_1 - s)R(s) \]  

(4)

The profit function of the retailer is formulated as follows:

\[ \Pi_R^A = (p - w)(\delta_0 - \delta_1 p + \delta_2 s) \]  

(5)

The profit functions of the manufacturer and the retailer are linear with respect to \( w \). The first-order derivative of the profit functions with respect to \( w \) will be a constant value. Therefore, to avoid losing the optimal value of \( w \), we substitute the following relation to \( p \) [43].

\[ p = h + w \]  

(6)

By substituting Eq. (6) into Eq. (4), the manufacturer’s profit function is transformed to:

\[ \Pi_M^A = (w - c)[\delta_0 - \delta_1 h - \delta_2 w + \delta_2 s] + (T_1 - s)(\varphi_0 + \varphi_1 s) \]  

(7)

To obtain the optimal values of \( w, s, \) and \( p \) under the Nash game, the first-order and second-order derivatives of the profit functions should be calculated.

\[ \frac{\partial \Pi_M^A}{\partial p} = \delta_0 - 2\delta_1 p + \delta_2 s + \delta_1 w = 0 \]  

(8)

To prove the concavity of the retailer’s profit function, the second-order derivative of the retailer’s profit function must be negative with respect to \( p \).

\[ |H_r(p)| = \frac{\partial^2 \Pi_R^A}{\partial p^2} = -2\delta_1 < 0 \]  

(9)
Eq. (9) is negative. Thus, the optimal value of \( p \) is calculated by solving Eq. (8).

\[
p^A = \frac{\delta_0 + \delta_2 s^A + \delta_1 w^A}{2\delta_1}
\]  

(10)

At the same time, the manufacturer's decision variables (i.e., \( s \) and \( w \)) are determined. To prove the concavity of the manufacturer’s profit function, the Hessian matrix is calculated as follows:

\[
H(w,s) = \begin{bmatrix}
\frac{\partial^2 \Pi^A_M}{\partial w^2} & \frac{\partial^2 \Pi^A_M}{\partial w \partial s} \\
\frac{\partial^2 \Pi^A_M}{\partial w \partial s} & \frac{\partial^2 \Pi^A_M}{\partial s^2}
\end{bmatrix} = \begin{bmatrix}
-2\delta_1 & \delta_2 \\
\delta_2 & -2\phi_1
\end{bmatrix}
\]  

(11)

\[
|H_1(w,s)| = \frac{\partial^2 \Pi^A_M}{\partial w^2} = -2\delta_1 < 0
\]  

(12)

\[
|H_2(w,s)| = \left( \frac{\partial^2 \Pi^A_M}{\partial w^2} \right)^2 - \left( \frac{\partial^2 \Pi^A_M}{\partial w \partial s} \right)^2 = 4\delta_1 \phi_1 - \delta_2^2 > 0
\]  

(13)

The first principle minor is negative and the second principle minor is positive. Thus, the manufacturer’s profit function is concave. The first-order derivatives with respect to \( w \) and \( s \) are as follow:

\[
\frac{\partial \Pi^A_M}{\partial w} = \delta_0 - \delta_1 h - 2\delta_1 w + \delta_2 s + \delta_1 c = 0
\]  

(14)

\[
\frac{\partial \Pi^A_M}{\partial s} = \delta_2 w - \delta_2 c - \phi_0 - 2\phi_1 s + \phi_1 T_1 = 0
\]  

(15)

By substituting Eq. (10) to Eqs. (14) and (15), we have:

\[
\frac{\partial \Pi^A_M}{\partial w} = \frac{\delta_0}{2} + \frac{\delta_2}{2} s - \frac{3\delta_1}{2} w + \delta_1 c = 0
\]  

(16)

\[
\frac{\partial \Pi^A_M}{\partial s} = \delta_2 w - \delta_2 c - \phi_0 - 2\phi_1 s + \phi_1 T_1 = 0
\]  

(17)

By solving Eqs. (16) and (17), the optimal values of \( w \) and \( s \) are as follows:

\[
w^A = \frac{\frac{\delta_0}{2} + \frac{\delta_2}{2} s - \frac{3\delta_1}{2} w + \delta_1 c}{\frac{\delta_2}{2}}
\]  

(18)

\[
s^A = \frac{\delta_2 w^A - \delta_2 c - \phi_0 + \phi_1 T_1}{2\phi_1}
\]  

(19)

Therefore, by substituting the optimal values of \( w^A, s^A, p^A \) in the profit functions of the manufacturer and retailer, as well as the market demand, the optimal profits and market demand are obtained under the Nash game.

To determine the optimal decisions under the Manufacturer-Stackelberg game, backward induction is applied. According to the backward induction, by substituting the optimal value of \( p \) in the manufacturer’s profit function, the optimal values of \( w \) and \( s \) can be determined. Similar to the previous scenario, the optimal retail price is obtained as follows:
\[ p^E = \frac{\delta_0 + \delta_2 s^E + \delta_1 w^E}{2\delta_1} \]  

(20)

By substituting Eq. (20) into Eq. (7), we have:

\[ \Pi^E_m(w, s) = (w - c)(\frac{\delta_0}{2} + \frac{\delta_2}{2} s - \frac{\delta_1}{2} w) + (T_1 - s)(\phi_0 + \phi_1 s) \]  

(21)

To prove the concavity of the manufacturer’s profit function, the Hessian matrix is calculated as follows:

\[ H(w, s) = \begin{bmatrix}
\frac{\partial^2 \Pi^E_m}{\partial w^2} & \frac{\partial^2 \Pi^E_m}{\partial s \partial w} \\
\frac{\partial^2 \Pi^E_m}{\partial w \partial s} & \frac{\partial^2 \Pi^E_m}{\partial s^2}
\end{bmatrix} = \begin{bmatrix}
-\delta_1 & \frac{\delta_2}{2} \\
\frac{\delta_2}{2} & -2\phi_1
\end{bmatrix} \]  

(22)

\[ |H_1(w, s)| = \frac{\partial^2 \Pi^E_m}{\partial w^2} = -\delta_1 < 0 \]  

(23)

\[ |H_2(w, s)| = \left( \frac{\partial^2 \Pi^E_m}{\partial w^2} \right) \left( \frac{\partial^2 \Pi^E_m}{\partial s^2} \right) - \left( \frac{\partial^2 \Pi^E_m}{\partial w \partial s} \right)^2 = 2\delta_1 \phi_1 - \frac{\delta^2}{4} > 0 \]  

(24)

The first principle minor is negative and the second principle minor is positive. Thus, the manufacturer’s profit function is concave. The first-order derivatives with respect to \( w \) and \( s \) are as follow:

\[ \frac{\partial \Pi^E_m}{\partial w} = \frac{\delta_0}{2} + \frac{\delta_2}{2} s - \delta_1 w^E + \frac{\delta_1}{2} \]  

(25)

\[ \frac{\partial \Pi^E_m}{\partial s} = \frac{\delta_2}{2} w^E - \frac{\delta_2}{2} c - \phi_0 - 2\phi_1 s^E + T_1 \phi_1 = 0 \]  

(26)

By solving Eqs. (25) and (26), the optimal decision variables of manufacturer are:

\[ s^E = \frac{4}{\delta^2} \frac{\delta_0 \delta_2 + \delta_4 \delta_2 c - \delta_2 \delta_1 c - \delta_0 \phi_0 + T_1 \delta_1 \phi_1}{-2\delta_1 \phi_1} \]  

(27)

\[ w^E = \frac{\frac{\delta_0}{2} + \frac{\delta_2}{2} s^E + \frac{\delta_1}{2}}{\delta_1} \]  

(28)

Therefore, by determining the optimal decision variables (i.e., \( w^E \), \( s^E \) and \( p^E \)) in Scenario E, the profits of the manufacturer and retailer, as well as the market demand, are obtained.

**Scenario 2 (B, F): Collecting by retailer and remanufacturing by both manufacturer and retailer**

In this scenario, both manufacturer and retailer remanufacture the worn-out products. In other words, a fraction of the returned products is remanufactured by the retailer, and the rest of them is remanufactured by the manufacturer. The manufacturer sells the products to the retailer at the wholesale price \( w \), then the retailer sells the product to the consumer at the retail price \( p \).

The cost saving of remanufacturing each returned product for the manufacturer is similar to Eq. (1), and the cost saving of remanufacturing each returned product for the retailer is formulated in Eq. (29). Similar cost saving is used in the literature e.g., Huang and Wang [31].
Moreover, the number of returned products is similar to Scenario 1 which is formulated in Eq. (3). Thus, in Scenario 2, the manufacturer’s profit function is as follows:

\[ \Pi^m_2(w) = (w-c)[D(p,s)-R(s)] + [(w-c+T_1-b)(1-t)]R(s) + ftR(s) \]  

(30)

By simplifying Eq. (30), the manufacturer’s profit function is:

\[ \Pi^m_2(w) = (w-c)[\delta_0 - \delta, p + \delta_2 s] + [(f-w+c) t + (T_1-b)(1-t)] (\varphi_0 + \varphi_t s) \]  

(31)

The retailer’s profit function can be formulated as:

\[ \Pi^r_2(p,s) = (p-w) D(p,s) + (w-c+T_1)_R(s) + b(1-t) R(s) - sR(s) - ftR(s) \]  

(32)

By simplifying Eq. (32), the retailer’s profit function is:

\[ \Pi^r_2(p,s) = (p-w)(\delta_0 - \delta, p + \delta_2 s) + [(w-c+T_2-b-f)t-b-r] (\varphi_0 + \varphi_t s) \]  

(33)

**Scenario 3 (C, G): Collecting by third party and remanufacturing by both manufacturer and third Party**

In this scenario, a CLSC with one manufacturer, one retailer, and one third party is considered. The manufacturer sells the products to the retailer at the wholesale price (w), and the retailer sells them to the consumer at the retail price (p). The worn-out products are collected by the third party by offering an acquisition price (s) for each worn-out product. Then, a fraction of worn-out products (t) is remanufactured by the third party and the rest of them is remanufactured by the manufacturer. The cost saving of remanufacturing each returned product for the manufacturer is similar to Eq. (1). According to Huang and Wang [31], the cost saving of remanufacturing each returned product for the third party is modeled as follows:

\[ T_3 = C - G_r \]  

(34)

Market demand and the number of returned products are similar to Eq. (2) and Eq. (3), respectively. Hence, in Scenario 3, the profit function of the manufacturer is as follows:

\[ \Pi^m_3(w) = (w-c)[D(p,s)-R(s)] + [(w-c+T_1-b)(1-t)]R(s) + ftR(s) \]  

(35)

The manufacturer’s profit function can be simplified as:

\[ \Pi^m_3(w) = (w-c)[\delta_0 - \delta, p + \delta_2 s] + [(f-w+c) t + (T_1-b)(1-t)] (\varphi_0 + \varphi_t s) \]  

(36)

The retailer’s profit function is formulated in Eq. (37).

\[ \Pi^r_3(p) = (p-w)(\delta_0 - \delta, p + \delta_2 s) \]  

(37)

In this scenario, the third party collects and remanufactures a fraction of returned products. Accordingly, the profit function of the third party is as follows:

\[ \Pi^r_3(s) = (w-c+T_3)_t R(s) - sR(s) + b(1-t) R(s) - ftR(s) \]  

(38)

By simplifying Eq. (38), the profit function of third party is:

\[ \Pi^r_3(s) = [(w-c+T_3-b-f)t-b-s] (\varphi_0 + \varphi_t s) \]  

(39)

**Scenario 4 (D, H): Collecting by manufacturer without remanufacturing**

In this scenario, a CLSC with one manufacturer and one retailer are considered. The manufacturer purchases the worn-out product from the customer at an acquisition price (s). The number of worn-out products which are collected by the manufacturer is similar to Eq. (1). Under this scenario, the profit function of the manufacturer is as follows:

\[ \Pi^m_4(w,s) = (w-c)(\delta_0 - \delta, p + \delta_2 s) - s (\varphi_0 + \varphi_t s) \]  

(40)

in which, the first term represents the revenue of selling the product and the second term shows the cost of collecting the returned products.

The retailer’s profit function is formulated in Eq. (41).

\[ \Pi^r_4(p) = (p-w)(\delta_0 - \delta, p + \delta_2 s) \]  

(41)
In this study, to model the CLSC members’ interactions, we use the Nash and Manufacturer-Stackelberg games. Accordingly, the optimal decision variables such as the acquisition price, wholesale price, and retail price are determined under Nash and Stackelberg games. In addition, the profit of the whole CLSC and members (i.e., the manufacturer, retailer, and third party) are calculated under Nash and Stackelberg games.

Under the Nash game, the closed-forms of the optimal decision variables (i.e., wholesale price, acquisition price, and retail price) in four different scenarios are illustrated in Table 3.

Under the Stackelberg game, the closed-forms of the optimal decision variables in four different scenarios are shown in Table 4.

B1 to B6, C1 to C4, E1 to E6, and H1 to H7 which are used in tables 3 and 4 are defined in Appendix 1.

4. Numerical and sensitivity analyses

The cost of repairing the sensitive ball valves is about 50-60% of a new ball valve, and the cost of repairing the insensitive ball valves is about 30-40% of a new ball valve. The prices offered for the remanufactured products depend on the customer’s order, and the prices usually decrease as the rate of manufacturing increases. Moreover, different parts are sold as worn-out and used stock, which can be reused as new parts with the same initial quality by the proper repairs.

The wholesale price, acquisition price, retail price are considered as the decision variables. Other parameters in the manufacturing and remanufacturing of ball valves have been obtained based on the statements of the specialists dealing with this industry. In order to collect the data, we visited the centers of remanufacturing and overhaul of oil and gas industries in 2019 and collected the required data by studying the documents of companies, which are as follows:

\[
\begin{align*}
\delta_0 &= 3000, \delta_1 = 8.4, \delta_2 = 2.4, c = 25, \\
G_M &= 5, G_R = 5, G_T = 5, \\
\varphi_0 &= 80, \varphi_1 = 2.2, \\
f &= 40, b = 50
\end{align*}
\]

4.1. Selecting Pricing Strategy under the Nash Equilibrium

Figure 2 illustrates the effects of retailer’s/third party’s ability for remanufacturing on the CLSC profit under the Nash equilibrium in the four scenarios (i.e., scenarios A, B, C, and D). From Figure 2, the best option under the Nash game is Scenario B, where the retailer collects the worn-out products and both manufacturer and retailer remanufacture the worn-out products. Under Scenario B, when \( t \leq 0.85 \), the CLSC profit increases, and when \( t > 0.85 \), the CLSC profit decreases. Thus, the optimal value of \( t \) is 0.85. It means that 85% of returned products must be remanufactured by the retailer and the rest of them is remanufactured by the manufacturer.

As shown in Figure 3, when \( t \geq 0.85 \), the retail price in Scenario B decreases compared to the other scenarios. Thus, under Scenario B, remanufacturing the returned products not only decreases the production costs but also reduces the retail price. Accordingly, under this scenario, the market demand and the CLSC profitability improve.

Furthermore, by increasing \( t \), the profit of the CLSC increases in Scenario C, and the optimal value of \( t \) is 0.95 in Scenario C. If Scenario 3 is applied, remanufacturing all the worn-out products by the third party is preferred in the CLSC. Table 5 shows the optimal decision
variables and profits of the manufacturer, retailer, third party, and whole CLSC under the four scenarios in the Nash equilibrium. The ability of retailer and third party for remanufacturing are considered 0.8 and 0.95, respectively.

According to Table 5, in Scenario B, by reducing production costs, the retail price decreases compared to the other scenarios, and the CLSC profit improves in comparison with the other scenarios.

4.2. Choosing a pricing strategy under the Manufacturer-Stackelberg game

In this section, the four scenarios are compared under the Manufacturer-Stackelberg game. Similar to the Nash equilibrium, a numerical example is provided to obtain the optimal decision variables and the CLSC profit in each scenario. Figure 4 illustrates the trend of CLSC profits over $t$ in each scenario, which are represented by the symbols E, F, G, and H.

From Figure 4, when $t < 0.4$, the profit of the CLSC in Scenario G is more than those of the other scenarios. This scenario, in which the manufacturer and the third party remanufacture the returned products and the retailer sells the product to the customers, leads to more profit for the CLSC. However, when $t \geq 0.4$, Scenario F is more profitable in which the manufacturer and the retailer remanufacture the returned products in the CLSC. Therefore, the best scenario from the CLSC viewpoint depends on the ability of the retailer/third party for remanufacturing ($t$). Furthermore, Figure 5 shows that when $t < 0.4$, the retail price decreases over $t$ in Scenario G.

In addition, in Scenario G, the optimal $t$ is 0.37 which maximizes the profit of the CLSC ($\Pi_C^G = 196321.1$). In Scenario F, the optimal $t$ is 0.43 which maximizes the CLSC profit ($\Pi_C^F = 191330$). According to the optimal $t$ in scenarios G and F under the Stackelberg game, Table 6 shows the optimal decision variables and the profit of whole CLSC and members in the four scenarios.

As can be seen in Table 6, in Scenario G, the CLSC profit is more than those of the other scenarios under the Stackelberg game. In addition, the retail price ($p$) in this scenario is lower than those of the other scenarios. It is concluded that under Scenario G, the worn-out ball valves can be remanufactured at a lower cost, and accordingly, the products can be sold to the consumer at a lower retail price. Thus, the CLSC achieves a higher profit compared to other scenarios.

4.3. Choosing the best pricing strategy

Figure 6 shows the profit in the different scenarios under the Stackelberg and Nash games. From Figure 6, under the Nash game, the profit is more than those of the Stackelberg game. Moreover, Scenario B is the best scenario in comparison with the other scenarios.

5. Discussions and Conclusion

This research develops four different scenarios for collecting and remanufacturing the worn-out ball and gate valves. The CLSC members’ interactions are formulated using the Nash and Manufacturer-Stackelberg games. Under both Nash and Manufacturer-Stackelberg games, the optimal decision variables such as the acquisition price, wholesale price, and retail price are determined. Furthermore, the profit of all CLSC members and whole CLSC are calculated under all different scenarios. The results of this study show that under the Nash game, Scenario 2 is the best strategy in comparison with other scenarios. Additionally, in Scenario 2, the retail price is
lower than those of the other scenarios. Under the Stackelberg game, Scenario 3 is the best in comparison with the other scenarios. In Scenario 3, the CLSC profit is more than those of the others and the wholesale and retail prices are lower than those of the other scenarios. On the other hand, under the Nash game, the CLSC profitability increases compared to the Stackelberg game. Therefore, the simultaneous interaction among the CLSC members in the ball valve industry will yield better results compared to the situation where the manufacturer is the leader in the CLSC. Since the important decisions such as the acquisition price and retail price do not determine by the manufacturer in most scenarios, the manufacturer as the leader cannot reduce costs as much as the Nash game.

The current case study of ball valves in the oil and gas industry shows that Scenario 2 is the most profitable one since it brings about the most reduction in the costs of collecting, remanufacturing worn-out products, and production. Thus, it results in overall good performance of CLSC. As the consumer in this active numerical example concerns with the oil and gas outputs of the industry, the reduction in production costs will also reduce the cost of the final product that is oil and gas itself. Thus, this strategy will also have other benefits such as increased demand and sales of oil and gas in the competitive market. To conclude, a congregated effort among the manufacturer, retailer, and third party can result in the much-needed reduction in the collecting and remanufacturing costs of ball valves and can pay off with more benefits in this product’s retail market along with cross sectorial benefits for the sales of oil and gas products.

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Table 2. A brief view of the reviewed literature.

| References                  | Collection strategy | Collection option | Game theory | SC structure | Bilateral effect of acquisition price |
|-----------------------------|---------------------|-------------------|-------------|--------------|--------------------------------------|
|                             | Acquisition price   | Collection rate   | Manufacturer | Retailer     | Third party | Stackelberg | Nash | CLSC | Reverse | Bilateral effect of acquisition price |
| Savaskan et al. [7]         | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Giri et al. [11]            | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Maiti & Giri [33]           | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Li et al. [19]              | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Xu & Liu [23]               | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Huang and Wang [31]         | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Zheng et al. [20]           | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Taleizadeh et al. [34]      | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Modak et al. [1]            | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Hosseini-Motlagh et al. [21]| ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Chen & Akmal'Ulya [6]       | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Wan & Hong [2]              | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Hosseini-Motlagh et al. [4] | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Mondal & Giri [5]           | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Hosseini-Motlagh et al. [12]| ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Asl-Najafi & Yaghoubi [37]  | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |
| Proposed model              | ✓                   | ✓                 | ✓           | ✓            | ✓          | ✓           | ✓    | ✓    | ✓       | ✓                                    |

✓: cover

Table 2. Decision Variables and Parameters.

| Parameter | Value |
|-----------|-------|
| $\delta_0$ | Initial demand |
\( \delta_1 \) Demand sensitivity to the retail price

\( \delta_2 \) Demand sensitivity to the acquisition price

\( \varphi_0 \) Initial number of returned products

\( \varphi_1 \) Collection amount sensitivity to the acquisition price

\( R \) Number of returned products

\( C \) Manufacturing cost from raw materials

\( G_M \) Remanufacturing cost of the returned product for the manufacturer

\( G_R \) Remanufacturing cost of the returned product for the retailer

\( G_T \) Remanufacturing cost of the returned product for the third party

\( T_1 \) Cost saving by remanufacturing each returned product for the manufacturer

\( T_2 \) Cost saving by remanufacturing each returned product for the retailer

\( T_3 \) Cost saving by remanufacturing each returned product for the third-party

\( f \) Licensing fee paid by the retailer/third party to the manufacturer

\( b \) Transfer price of the returned product paid by the manufacturer to the retailer/third party

\( t \) Ability of the retailer and third party to remanufacturer the returned products

\( w \) Wholesale price for each product

\( s \) Acquisition price for each returned product

\( p \) Retail price

\( \Pi \) Profit function

\( D \) Market demand

\( h \) Retailer's revenue for selling each product

M,R,T,C These suffixes represent the manufacturer, retailer, third-party, and CLSC, respectively.

A,B,C,D These prefixes represent scenarios 1-4 under the Nash game, respectively.

E,F,G,H These prefixes represent scenarios 1-4 under the Stackelberg game, respectively.

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**Table 3. Closed-form of the optimal decision variables in different scenarios under the Nash game.**

| Scenario 1 (A) | Scenario 2 (B) | Scenario 3 (C) | Scenario 4 (D) |
|----------------|----------------|----------------|----------------|
| \( W \) | \( \phi_1\delta_0 + 2\phi_2\phi_3 - \frac{\delta_1^2 - \delta_2^2}{2} - \frac{\delta_3^2 - \delta_1^2}{2} \) | \( \frac{\phi_1\delta_0 - \phi_2\phi_3 - \delta_3^2 - \delta_1^2 + \delta_0\phi_0}{\delta_1 - \phi_0} \) | \( \frac{\delta_1^2 + \phi_0^2 + 2\phi_1\phi_3}{\delta_2} \) |
| \( s \) | \( \frac{\delta_1^2 w^2 - \delta_2\delta_0 - \phi_0 + \phi_1 T}{2\phi_0} \) | \( \frac{B_B_1 - B_B_3}{B_B_1 + B_B_3} \) | \( \frac{-c}{\delta_1 - \phi_0 T} \) |
| \( p \) | \( \frac{-s - \delta_2 - \delta_0 + \phi_0 + \phi_1 T}{2\phi_1} \) | \( \frac{\gamma_1(\phi_0 - \phi_1)}{\delta_1 - \phi_0} - 2\gamma_0 + \gamma_2\delta_1 \phi_1 \) | \( \frac{\delta_1^2 \delta_0 - \delta_1 \phi_0 - \delta_0 \phi_1 + \delta_1 \phi_0}{2\phi_1 \delta_1 - \delta_2^2} \) |

**Table 4. Closed-form of the optimal decision variables in different scenarios under the Stackelberg game.**

| Scenario 1 (E) | Scenario 2 (F) | Scenario 3 (G) | Scenario 4 (H) |
|----------------|----------------|----------------|----------------|
| \( W \) | \( \frac{\delta_1^2 + \delta_2^2 + \delta_3^2}{2\delta_1} \) | \( -H_s + H_e \left( \frac{f + c}{T(1 - b)(1 - t)} \right) + H_f \) | \( \frac{E_2 - E_c - E_d + E_T}{2E_s - 2E_f} \) |
| \( s \) | \( \frac{B_B_1 - B_B_3}{B_B_1 + B_B_3} \) | \( \frac{B_B_1 - B_B_3 - B_R_1 + B_R_3 - B_L_1}{B_L_1} \) | \( \frac{\delta_1^2 \phi_0 + \delta_1 \phi_1 - \delta_0 \phi_1 - \delta_1 \phi_0}{4 - 2\phi_1 \delta_1} \) |

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Table 5. Optimal decision variables and profits under the Nash game (Output Data).

|   | A            | B            | C            | D            |
|---|--------------|--------------|--------------|--------------|
| w | 95.7         | 46.10        | 29.21        | 161.2        |
| s | 48.7         | 31.14        | 0.44         | 98.85        |
| p | 108.8        | 83.86        | 146.40       | 170.8        |
| Π\text{M} | 34269.70    | 15840.68    | 8693.67      | 216125.45    |
| Π\text{R} | 17193.72    | 229971.73   | 202727.84    | 17289.95     |
| Π\text{T} | -            | -            | 184.565      | -            |
| Π\text{C} | 51463.43    | 245812.42   | 211606.09    | 23415.41     |

Table 6. Optimal decision variables and profits under the Stackelberg game (Output Data).

|   | E            | F            | G            | H            |
|---|--------------|--------------|--------------|--------------|
| w | 196.2        | 189.3        | 181.1        | 195.1        |
| s | 36.01        | 92.1         | 69.8         | 28.20        |
| p | 281.8        | 286.4        | 279.1        | 280.15       |
| Π\text{M} | 119776.9    | 116065.4    | 115654.7     | 117517.3     |
| Π\text{R} | 61561.4     | 75264.4     | 80631.1      | 60762.3      |
| Π\text{T} | -            | -            | 35.27104     | -            |
| Π\text{C} | 181338.4    | 191329.9    | 196321.1     | 178279.7     |

Appendix 1

Defined fixed parameters

\[ B_1 = \frac{\delta_1^2}{\delta_1 - \phi_0 t} - 2\delta_1 \]  \hspace{1cm} (A-1)

\[ B_2 = \delta_1 + \frac{(\phi_0 t - \delta_2)\delta_1}{\delta_1 - \phi_0 t} \] \hspace{1cm} (A-2)

\[ B_3 = \delta_1 + \frac{\delta_1 c + \delta_0}{\delta_1 - \phi_0 t} \] \hspace{1cm} (A-3)

\[ B_4 = \delta_1 + \frac{(\phi_0 t - \delta_2)\delta_1}{\delta_1 - \phi_0 t} \] \hspace{1cm} (A-4)

\[ B_5 = \frac{(\phi_0 t - \delta_2)\delta_1}{\delta_1 - \phi_0 t} + 2\phi_1 \] \hspace{1cm} (A-5)

\[ B_6 = \frac{(\phi_0 t - \delta_2)\delta_1 c + \delta_0}{\delta_1 - \phi_0 t} - \phi_0 + \phi_1 [(T_2 - c - b - f)t + b] \] \hspace{1cm} (A-6)
\[ C_1 = \frac{\delta_1^2}{\delta_1 - \varphi_0 t} - 2\delta_1 \]  \hfill (A-7) \\
\[ C_2 = \delta_2 + \frac{\delta_1 (\varphi_0 t - \delta_2)}{\delta_1 - \varphi_0 t} \]  \hfill (A-8) \\
\[ C_3 = \delta_0 - \frac{\delta_1^2 c - \delta_0 \delta_1}{\delta_1 - \varphi_0 t} \]  \hfill (A-9) \\
\[ C_4 = -\varphi_0 - \frac{(\delta_1^2 + \delta_0 \delta_1) \varphi_0 t}{\delta_1 - \varphi_0 t} - \frac{C_3 \delta_1 \varphi_0 t}{(\delta_1 - \varphi_0 t) C_1} + \varphi_1 [(T_3 - c - b - f) t + b] \]  \hfill (A-10) \\
\[ H_1 = \varphi_1 [(T_2 - c - b - f) t + b] \]  \hfill (A-11) \\
\[ H_2 = \varphi_1 t - \delta_2 \]  \hfill (A-12) \\
\[ H_3 = \delta_1 \delta_2 + 2\delta_1 H_2 \]  \hfill (A-13) \\
\[ H_4 = -\frac{\delta_1}{2} - \frac{\delta_2 H_3}{2(\delta_2^2 - 4\varphi_1 \delta_1)} \]  \hfill (A-14) \\
\[ H_5 = \frac{\delta_0 - \delta_2 (\delta_0 \delta_1 + 2\delta_1 H_1)}{2(\delta_2^2 - 4\varphi_1 \delta_1)} \]  \hfill (A-15) \\
\[ H_6 = -\frac{H_4 \varphi_1}{\delta_2^2 - 4\varphi_1 \delta_1} \]  \hfill (A-16) \\
\[ H_7 = \varphi_0 - \frac{\varphi_1 (\delta_0 \delta_1 + 2\delta_1 H_1)}{2(\delta_2^2 - 4\varphi_1 \delta_1)} \]  \hfill (A-17) \\
\[ E_1 = (T_2 - c - b - f) t + b \]  \hfill (A-18) \\
\[ E_2 = \frac{\delta_0 - \delta_2 (\varphi_0 - E_1)}{4\varphi_1} \]  \hfill (A-19) \\
\[ E_3 = \frac{\delta_1 \varphi_0 t}{4\varphi_1} - \frac{\delta_1}{2} \]  \hfill (A-20) \\
\[ E_4 = \frac{\varphi_0}{2} + \frac{E_1}{2} \]  \hfill (A-21) \\
\[ E_5 = \frac{\varphi_1 t}{2} \]  \hfill (A-22) \\
\[ E_6 = E_5 [(f + c) t + (T_1 - b)(1-t)] \]  \hfill (A-23)