QCD corrections to $e^+e^- \rightarrow J/\psi gg$ at B Factories

Yan-Qing Ma (a), Yu-Jie Zhang (a), and Kuang-Ta Chao (a,b)

(a) Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
(b) Center for High Energy Physics, Peking University, Beijing 100871, China

In heavy quarkonium production, the measured ratio $R_{c\bar{c}} = \sigma(J/\psi + c\bar{c} + X)/\sigma(J/\psi + X)$ at B factories is much larger than existing theoretical predictions. To clarify this discrepancy, in nonrelativistic QCD (NRQCD) we find the next-to-leading-order (NLO) QCD correction to $e^+e^- \rightarrow J/\psi + gg$ can enhance the cross section by about 20%. Together with the calculated NLO result for $e^+e^- \rightarrow J/\psi + c\bar{c}$, we show that the NLO corrections can significantly improve the fit to the ratio $R_{c\bar{c}}$. The effects of leading logarithm resummation near the end point on the $J/\psi$ momentum distribution and total cross section are also considered. Comparison of the calculated cross section for $e^+e^- \rightarrow J/\psi + gg$ with observed cross section for $e^+e^- \rightarrow J/\psi + non(c\bar{c})$ is expected to provide unique information on the issue of color-octet contributions.

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In recent years a number of challenging problems in heavy quarkonium production have appeared[1]. Aside from the $J/\psi$ production cross sections and polarizations in hadron-collisions at the Tevatron, charmonium production in $e^+e^-$ annihilation at B factories[2, 3] also conflicted with theoretical predictions. The observed double charmonium production cross sections for $e^+e^- \rightarrow J/\psi\chi_c(1S, 2S)$ were larger than the LO calculations in NRQCD[4] by an order of magnitude[5], and later it was found that these discrepancies could be largely resolved by the NLO QCD corrections (see[6, 7] for $J/\psi\chi_c$ and[8] for $J/\psi\chi_0$) with relativistic corrections[9, 10]. For the $J/\psi$ production associated with an open charm pair $e^+e^- \rightarrow J/\psi + c\bar{c}$, the NLO QCD correction[11] was also found to significantly enhance the cross section (see also[12]), and reduce the large gap between experiment and the LO calculations[13].

Another important issue concerns the ratio

$$R_{c\bar{c}} = \frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \rightarrow J/\psi + X)}.$$  (1)

Belle found first $R_{c\bar{c}} = 0.59^{+0.15}_{-0.13} \pm 0.12[2]$, and later $R_{c\bar{c}} = 0.82 \pm 0.15 \pm 0.14[14]$. On the contrary, LO NRQCD[13, 15] and light-cone pQCD predictions[16] for the ratio are only about 0.1–0.3. The color evaporation model gives a value of only 0.06[17].

In NRQCD, $\sigma(J/\psi + X)$ includes color-singlet contributions $\sigma(J/\psi(S_1^{[1]} + c\bar{c})$ and $\sigma(J/\psi(S_1^{[1]} + gg)$, and color-octet contributions $\sigma(J/\psi(P_{g1}^{[8]} + S_0^{[8]} + g)$. Contributions of other Fock states are suppressed by either $\alpha_s$, the strong coupling constant, or $v$, the relative velocity between quark and antiquark in heavy quarkonium. $\sigma(J/\psi(P_{g1}^{[8]} + S_0^{[8]} + g)$ was calculated at LO in $\alpha_s[18]$, and an apparent enhancement at the $J/\psi$ maximum energy was predicted. But experiments did not show any enhancement at the end point. The resummations of the $v$ expansion and $log(1-z)$ where $z = E_{c\bar{c}}/E_{max}$ are considered[19], but the theoretical results rely heavily on the phenomenological shape function. It is possible that the observed end point behavior of $J/\psi$ and the large ratio $R_{c\bar{c}}$ might indicate that the color-octet matrix elements are much smaller than previously expected. To test this thought we assume the color-octet contribution to be ignored and only consider the color-singlet contributions. Under this assumption, the ratio becomes

$$R_{c\bar{c}} = \frac{\sigma(J/\psi + c\bar{c})}{\sigma(J/\psi + c\bar{c}) + \sigma(J/\psi + gg)}.$$  (2)

In the following we concentrate on $\sigma(J/\psi + gg)$ in NRQCD. Aside from the LO calculations in NRQCD (see related references in[13, 15]), Ref.[21] considered $\sigma(J/\psi + gg)$ within the framework of soft collinear effective theory (SCET), and Ref.[21] summed over the leading and next-to-leading logarithms in the end point region of $\sigma(J/\psi + gg)$. However, considering the crucial importance of the NLO QCD corrections found in many heavy quarkonium production processes[4, 7, 8, 11, 12, 22], it is necessary to carry out the calculation of NLO QCD correction to $e^+e^- \rightarrow J/\psi + gg$.

We now present this calculation. We use FeynArts[22] to generate Feynman diagrams and amplitudes, FeynCalc[24] to handle amplitudes, and LoopTools[25] to evaluate the infrared-finite scalar Passarino-Veltman integrals. Feynman diagrams for the Born, virtual correction, and real correction are shown in Fig. 1, Fig. 2 and Fig. 3. Note $e^+e^- \rightarrow J/\psi c\bar{c}$ is excluded in the real correction, because it should be included in the $J/\psi$ production associated with open charm $e^+e^- \rightarrow J/\psi + c\bar{c} + X$. Moreover, we include ghost diagrams in the real correction because we choose unphysical polarizations for the gluons in the final state.

There are generally ultraviolet(UV), infrared(IR), and Coulomb singularities. Conventional Dimensional Regularization (CDR) with $D = 4 - 2\epsilon$ is adopted to
FIG. 2: Feynman diagrams for the virtual correction to \( e^+e^- \to J/\psi gg \).

FIG. 3: Feynman diagrams for the real correction to \( e^+e^- \to J/\psi gg \).
the differential cross section (and other quantities) as \([21]\). Using a similar approach, we define and 

\[ R_{\mu} = \frac{\sigma_{\text{LO}}(\mu)}{\sigma_{\text{NLO}}(\mu)} \]

where \((d\sigma_{\text{resum}})_{\text{LO(NLO)}}\) means expanding \(d\sigma_{\text{resum}}\) in \(\alpha_s\) to LO (NLO). To be consistent with our previous calculation, we choose \(\mu_c = 2(1 - z)\mu_H\), and \(\mu_H = \mu = 2m\) in \([21]\). In Fig. 6 we show the cross sections of \(e^+e^- \rightarrow J/\psi gg\) as functions of the J/\(\psi\) momentum \(P_{J/\psi}\). The correction of LL resummation to the total cross section is about \(-6.6\%\) at LO and \(0.5\%\) at NLO. But it becomes large at the end point region when \(z = 1\), suppressing the LO cross section and enhancing the NLO cross section. The LL resummation changes the J/\(\psi\) momentum distribution near the end point, but has only a little effect on the total cross section. It is interesting to note that with the NLO correction, the J/\(\psi\) momentum spectrum becomes much softer than the LO result.

In summary, we find that by considering all NLO virtual and real corrections, and factoring the Coulomb singular term into the c\(\bar{c}\) bound state wave function, we get an ultraviolet, infrared, and collinear finite cross section near the end point, but has only a little effect on the total cross section. It is interesting to note that with the NLO correction, the J/\(\psi\) momentum spectrum becomes much softer than the LO result.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & \(\mu = 2.8\text{GeV}\) & \(\mu = 2.8\text{GeV}\) & \(\mu = 5.3\text{GeV}\) & \(\mu = 5.3\text{GeV}\) \\
 & LO & NLO & LO & NLO \\
\hline
\(\sigma(gg)\) & 0.57 & 0.67 & 0.36 & 0.53 \\
\(\sigma(c\bar{c})\) & 0.38 & 0.71 & 0.24 & 0.53 \\
\(R_{\mu}\) & 0.40 & 0.51 & 0.40 & 0.50 \\
\hline
\end{tabular}
\caption{Cross sections of prompt (feeddown included) \(J/\psi gg\) (this Letter) and \(J/\psi c\bar{c}\) (Re.\([11]\)) production in \(e^+e^-\) annihilation at B factories in units of pb.}
\end{table}

strong. A reasonable choice should be between \(\mu = 2m_c\) and \(\mu = \sqrt{s}/2\), and more preferably the latter.

Finally, we note that the large logarithms of \(\log(1 - z)\) appear at the endpoint in NLO calculation, where \(z = E_{J/\psi}/E_{J/\psi}^{\text{max}}\). The leading logarithms (LL) have been resummed in \([20, 21]\). Using a similar approach, we define the differential cross section (and other quantities) as \([21]\)

\[
d\sigma_{\text{LO(NLO)+LL}} = d\sigma_{\text{LO(NLO)}} + P[r, z]d\sigma_{\text{resum}} - P[r, z](d\sigma_{\text{resum}})_{\text{LO(NLO)}},
\]

where \(P[r, z]\) is the probability that the c\(\bar{c}\) pair is not produced at a given rapidity \(y\). The result significantly reduces the discrepancy between theory and experiment. The effect of the leading logarithm resummation near the end point on the \(J/\psi + gg\) total cross section is found to be small.

Note. Very recently Belle reported a new measurement with higher statistics \([20]\):

\[
\sigma(e^+e^- \rightarrow J/\psi + c\bar{c}) = (0.74 \pm 0.08^{+0.09}_{-0.08})\text{pb},
\]

\[
\sigma(e^+e^- \rightarrow J/\psi + \text{non}(c\bar{c})) = (0.43 \pm 0.09 \pm 0.09)\text{pb}.
\]

The observed cross section of \(e^+e^- \rightarrow J/\psi + \text{non}(c\bar{c})\) and \(R_{\mu}\) are displayed in Fig. 4 and Fig. 5 with central values.
and error bands in comparison with theoretical predictions. We see that, our predictions (NLO with feeddown) for $\sigma(e^+e^- \rightarrow J/\psi + gg)$ are consistent with the new measurement of $\sigma(e^+e^- \rightarrow J/\psi + non(c\bar{c}))$ within certain uncertainties. Moreover, the predicted $J/\psi$ momentum spectrum also agrees with the experiment\cite{29}. Importantly, our result of $\sigma(e^+e^- \rightarrow J/\psi + non(c\bar{c}))$ indicates that the calculated $\sigma(e^+e^- \rightarrow J/\psi + gg)$ has already saturated the observed $\sigma(e^+e^- \rightarrow J/\psi + non(c\bar{c}))$, hence leaving little room for the color-octet contributions. These are also confirmed by a similar study\cite{30}, which agrees with ours.

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