FINE TUNING IN LATTICE SU(2) GLUODYNAMICS VS CONTINUUM-THEORY CONSTRAINTS

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Recently, it has been observed that the non-Abelian action associated with lattice monopoles and vortices is ultraviolet divergent, at least at presently available lattices. On the other hand, the total length of the monopole trajectories and area of the vortices scale in physical units. Coexistence of the two different scales, infrared and ultraviolet, for the same vacuum fluctuations represents a fine tuning. To check consistency of the newly emerging picture of non-perturbative fluctuations we consider constraints from the continuum theory on the ultraviolet behaviour of the monopoles and vortices. The constraints turn to be satisfied by the data in a highly non-trivial way. Namely, it is crucial that the monopoles populate not the whole of the four dimensional space but a two-dimensional subspace of it.

1. Introduction

By fine tuning one understands usually a particular problem arising in theory of charged scalar particles. Namely, expression for the scalar boson mass looks as

\[ m_H^2 = \delta M_{rad}^2 - M_0^2 , \]

where \( \delta M_{rad}^2 \) is the radiative correction while \(-M_0^2\) is a counter term. The problem is that \( \delta M_{rad}^2 \) diverges quadratically in the ultraviolet (UV),

\[ \delta M_{rad}^2 \sim \alpha \cdot (\text{const}) \int \frac{d^4 k}{k^2} \sim \alpha \Lambda_{UV}^2 , \]

where \( \alpha \) is the coupling and \( \Lambda_{UV} \) is an ultraviolet cut off. If one uses the Planck mass for the cut off, \( \Lambda_{UV}^2 \sim (10^{19} \text{ GeV})^2 \), then to keep the mass of the charged (Higgs) boson in the 100 GeV region one should assume that the counter term is tuned finely to the value of the radiative correction and this tuning is readjusted with each order of perturbation theory.
What is outlined above is the standard problem of the Standard Model but a priori one would bet that it has nothing to do with the vacuum state of the lattice $SU(2)$ theory and with monopoles, which is a particular kind of the vacuum fluctuations.

However, lattice measurements indicate strongly (see\(^1\) and references therein) that the monopole mass is ultraviolet divergent, the same as the radiative correction in case of a point-like particle:

$$\langle M(a)_{\text{mon}} \rangle \sim \frac{\text{const}}{a},$$

where $a$ is the lattice spacing playing the role of an ultraviolet cut off (in the coordinate space). Moreover, the mass in Eq (2) is directly related to the excess of the non-Abelian action associated with the monopoles.

Let us emphasize that (2) is a pure phenomenological observation, with no theory involved. Which makes it a solid starting point for a discussion\(^a\): some objects with property (2) are certainly there. However, interpretation remains an open question: the monopoles themselves are defined not in terms of original Yang-Mills fields but rather in terms of projected fields, for reviews see, e.g.,\(^3\). Namely, for a given configuration on replaces $SU(2)$ by the “closest” $U(1)$ field configuration. (Through projecting to the closest $Z_2$ configuration one can define vortices, for review see\(^4\).) This is a well defined algorithm and the results like (2) are unique. One could define, however, monopoles in a different way and then their properties would change, generally speaking.

Under the circumstances, we feel that the following strategy could be appropriate. We will assume that, through the projection, one detects in fact—to some accuracy and on average—gauge invariant objects\(^b\). Then one can look for other gauge invariant characteristics and, indeed, there is accumulating evidence that there exist further $SU(2)$ invariant properties of the monopoles, see, in particular,\(^7,8,9,10\). Still, this evidence is pure numerical and, at some time, it would be desirable to switch into the language of the continuum theory. Our point here is that the very fact of appearance of the ultraviolet divergence (2) allows to establish strong constraints from the continuum theory. In particular, the asymptotic freedom does not allow new particles, in apparent contradiction with (2). Closer examination reveals, however, that according to the data the monopoles are not ordinary particles indeed. Since they occupy not the whole of the $d = 4$ (Euclidean)

\(^a\)To some extent, we follow the logic of\(^2\).

\(^b\)Such a possibility is noticed, in particular, in Ref\(^5\).
space but rather a $d = 2$ subspace of it, for a recent review see an accompanying paper by the same author\textsuperscript{11}. We will argue that existence of such 'branes' is consistent with the asymptotic freedom. A posteriori, one can say that this is a unique way to reconcile (2) with the asymptotic freedom. The theory of the branes themselves is lacking, however.

2. SU(2) fine tuning, seen on the lattice

As is mentioned in the Introduction, monopoles in $SU(2)$ are defined through a projection on $U(1)$ fields, for review see\textsuperscript{3,4}. In more details, one starts with generating a representative set of vacuum configurations of the original Yang-Mills theory with the standard action. Then each configuration – and this is the central point – is projected into the closest configuration of $U(1)$ fields. The projection itself is in two steps. First, one uses gauge invariance to minimize, over the whole lattice the functional

$$ R = \sum_{\text{links}} \left[ (A^1_\mu)^2 + (A^2_\mu)^2 \right], \quad (3) $$

where $A^i_\mu$ is the gauge potential and $i(i = 1, 2, 3)$ are color indices. The meaning of minimizing $R$ is that ‘charged’ fields are minimized. This fixation of the gauge does not change physics, of course. However, at the next step, which is the projection itself, one sends $A^1_\mu, A^2_\mu$ to zero generating in this way effective $U(1)$ fields, $\tilde{A}^3_\mu$.\textsuperscript{13}

Finally, the monopole current, $j^{\text{mon}}_\nu$ is related to violations of the Bianchi identities in terms of the projected fields:

$$ \partial_\mu \tilde{F}_{\mu\nu} = j^{\text{mon}}_\nu, \quad (4) $$

where $F_{\mu\nu}$ now is the field strength tensor constructed on the projected fields $\tilde{A}^3_\mu$ and $\tilde{F}_{\mu\nu}$ is dual to $F_{\mu\nu}$. A non-vanishing current (4) implies projected fields to be singular in the continuum limit. However, all the singularities are regularized by the lattice and the expression (4) is well defined\textsuperscript{13}.

The vortices are defined in terms of projected, or closest $Z_2$ configurations which are matrices $\pm I$ ascribed to each link. The vortices are unification of all the negative plaquettes evaluated on the $Z_2$ projection. The corresponding surfaces are closed by definition, as boundary of a boundary.

\textsuperscript{11}The hypothesis behind is, of course, Abelian dominance in the infrared. Consistency of this hypothesis can be checked. For a recent and amusing example of the Abelian dominance in the infrared see\textsuperscript{12}.
An analysis of this type ends up with a net of monopole trajectories or central vortices for each original non-Abelian field configuration. Theoretical task is then to interpret the data on the monopole and vortex clusters. Because of the use of the projection this task turns to be very difficult.

Phenomenologically, both lattice monopoles and vortices exhibit remarkable properties. Which we will briefly summarize here. Note that we present a simplified picture, emphasizing only the main features of the data (as we appreciate them). For details, one is to consult the original papers.

- There is an excess of non-Abelian action associated with the monopoles which can be memorized as:

\[ < S_{\text{mon}} > \approx \ln 7 \cdot \frac{L}{a} , \tag{5} \]

where \( L \) is the length of the monopole trajectory. The overall constant in (5) is actually poorly known. We put it equal to “\( \ln 7 \)” since this value does not contradict the data, on one hand, and would have a simple theoretical interpretation, on the other.

- For each field configuration, there exists a single percolating cluster which extends through the whole of the lattice. The length of the corresponding trajectory per unit volume does not depend on \( a \) and scales in the physical units, see \(^7\) and references therein. The observation can be formulated also in terms of the probability of a given link on the (dual) lattice to belong to the percolating cluster:

\[ \theta(\text{link}) \sim (a \cdot \Lambda_{\text{QCD}})^3 , \tag{6} \]

for all the values of \( a \) tested.

- In case of vortices, the total area scales in the physical units, see \(^4\) and references therein. Numerically \(^9\):

\[ A_{\text{vort}} \approx 24 (fm)^{-2} \cdot V_4 , \tag{7} \]

where \( A_{\text{vort}} \) is the total area of the vortices in the lattice volume \( V_4 \). One can rewrite (7) in terms of probability for a particular plaquette to belong to the percolating vortex:

\[ \theta(\text{plaq}) \sim (a \cdot \Lambda_{\text{QCD}})^2 . \tag{8} \]

- The non-Abelian action associated with the vortices is ultraviolet divergent \(^9\):

\[ < S_{\text{vort}} > \approx 0.54 \cdot \frac{A}{a^2} . \tag{9} \]
It is worth emphasizing that all the properties (5), (6), (8) and (9) are perfectly gauge invariant. Thus, the data suggest that, through projections, one detects gauge invariant objects.

3. Fine tuning: well understood examples

Since singular non-Abelian fields have an infinite action in the continuum limit, common wisdom tells us that such fields would drop off by themselves and, therefore, one would assume that the singularity (4) arises as an artifact of the projection. Observations on the monopoles and vortices summarized above imply that something is missing in this standard logic. Generically, coexistence of the ultraviolet and infrared scales can be called “fine tuning”. (Relation to the fine tuning of the standard model will be clarified later.) To orient ourselves in the problem, we will start with reviewing cases when the fine tuning is well understood.

3.1. Free particle

Consider first a free particle with the classical action:

$$S_{cl} = M(a) \cdot L,$$

(10)

where $M(a)$ is a mass parameter and we reserved for its possible dependence on the lattice spacing, while $L$ is a length of a trajectory of the particle, everything in the Euclidean space.

Furthermore, define a propagator as a path integral:

$$D(x_i, x_f; a) = \sum_{paths} \exp(-S_{cl}),$$

(11)

where $x_{i,f}$ are the end points of trajectories. The summation in (11) can be performed explicitly. In the momentum space:

$$D(p; a) = \frac{1}{c^2 a^2} D_{free}(m_{ph}^2),$$

(12)

where $c$ is a constant depending on details of the ultraviolet regularization and $m_{ph}$ is the propagating mass. The relation of $m_{ph}$ to the bare mass $M(a)$ introduced in (10) is as follows:

$$m_{ph}^2 = \frac{8}{a} \left( M(a) - \frac{\ln 8}{a} \right),$$

(13)

This subsection is mostly a text-book material, see, e.g., 14.
where the constants in front of the ultraviolet factors (i.e., inverse powers of $a$) are in fact regularization dependent and hereafter we have in mind hyper-cubic lattice. Eq (13) demonstrates that to keep the physical mass fixed, i.e. independent of $a$, one should tune the bare mass to a pure geometrical factor.

One could proceed further and consider interaction as well. Here, we will use this approach only to derive a useful relation for the vacuum expectation value of the corresponding scalar field squared \(^{15,16}\). Namely, the average value of the length of the particles trajectories in the vacuum is given by:

$$\langle L \rangle = \frac{\partial}{\partial M} \ln Z,$$

(14)

where $Z$ is the partition function. Moreover, one can replace:

$$\frac{\partial}{\partial M} \rightarrow \frac{8}{a} \frac{\partial}{\partial m^2_{ph}}.$$

(15)

The derivative with respect to $m^2_{ph}$, on the other hand, is related to the vacuum expectation of the $|\phi|^2$ where $\phi$ is a (complex) scalar field entering the standard formulation of field theory. Indeed, the standard Lagrangian contains a term $m^2_{ph}|\phi|^2$.

Finally,

$$V_4 \langle 0 | |\phi|^2 | 0 \rangle = \frac{a}{8} \langle L \rangle,$$

(16)

where $\langle L \rangle$ is average length of trajectory in the volume $V_4$. Eq (16) relates quantities entering the standard and polymer representations of theory of a scalar field. For us, it is important that the lengths of the monopole trajectories are directly measurable.

### 3.2. Lattice $U(1)$

Lattice $U(1)$, see, e.g., \(^{17}\), is actually close to the case of free particle just considered. A new point is that $M(a)$ is now calculable as energy of the magnetic field:

$$M(a)_{mon} = \frac{1}{8\pi} \int H^2 d^3r \sim \frac{\text{const}}{e^2a},$$

(17)

where one has to introduce an ultraviolet cut off, $a$ since $H \sim 1/r^2$ and the integral diverges at small distances. Note also that we kept explicit dependence on the electric charge $e$ which is due to the Dirac quantization condition. Finally, and might be most noteworthy, Eq (17) does not
contain contribution of the Dirac string. This is a privilege of the lattice regularization (for more details see, e.g., 19).

Note that upon substituting (17) into (13) we reproduce in fact (1). Now, if one tunes $e^2$ in such a way that $m_{ph} = 0$, where $m_{ph}$ is defined in (13) the monopoles condense. This is confirmed by the lattice data 18.

### 3.3. Percolation

Percolation is a common notion in papers on the monopoles (for a review of percolation theory see, e.g., 20). In most cases it is related to existence of an infinite cluster of monopoles, see 5 and references therein. Phenomenologically the percolating cluster is very important since the confining potential for external heavy quarks is entirely due to this infinite cluster while finite clusters do not confine.

Uncorrelated percolation is the simplest kind of percolation. In this case, one introduces a probability $p, p < 1$, for a link to be “open” and this probability does not depend on the neighbors. In our case, an open link would correspond to a link belonging to a monopole trajectory. The probability to find a connected trajectory of length $L$ is given by

$$W(L) = p^{L/a} \cdot N_L,$$

where $L/a$ is the number of steps and $N_L$ is the number of various trajectories of the same length $L$. Moreover,

$$N_L = 8^{L/a}. \quad (19)$$

Indeed, the monopoles occupy centers of cubes and at each step the trajectory can be continued to a neighboring cube. There are 8 such cubes for $d = 4$. Note that uncorrelated percolation is equivalent to a free field theory. Indeed by identification

$$M(a) = \ln p/ a,$$

the factor $p^{L/a}$ in Eq (19) reduces to the action factor for a free particle while $N_L$ represents the entropy.

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Monopoles are defined as end points of the Dirac strings and occupy centers of lattice cubes. Alternatively, one can say that on the dual lattice monopoles occupy sites and the monopole trajectories are built up on the links on the dual lattice. In most cases, we do not mention that it is the dual lattice which is implied in fact.

For charged particles, which we are considering, the factor 8 in Eq (19) is to be replaced by the factor of 7. Indeed, if one and the same link is covered by a trajectory in the both directions, then the link does not belong to a trajectory at all. In the field theoretical language this cancellation corresponds to the fact particle and anti-particle have opposite charges. For simplicity of presentation we will keep Eq (19) without change.
3.4. Supercritical phase of percolation

Clearly, there is a critical value of $p$, $p = p_c$, when any length $L$ is not suppressed. This is the point of phase transition to percolation. In the supercritical phase, $p > p_c$, there always exists a single infinite percolating cluster $\text{20}$. Most interesting, if $(p - p_c) \ll 1$ the probability for a link to belong to the percolating cluster is also small:

$$\theta(p) \sim (p - p_c)^\alpha,$$

where the critical exponent $0 < \alpha < 1$. In other words, the supercritical phase can be consistently treated as far as $p - p_c \ll 1$ $\text{20}$.

A simple effective action to describe the percolating monopole trajectory was proposed recently in $\text{21}$:

$$S_{\text{eff}} = -\mu \cdot L_{\text{perc}} + \gamma L_{\text{perc}}^2 / V^4,$$

(21)

where $\mu, \gamma$ are, generally speaking, functions of $\alpha, \Lambda_{\text{QCD}}$. Note that $\mu$ is positive so that in the (formal) limit of $V \to \infty$ the action (21) corresponds to a single tachyonic mode. Because of the tachyonic mode the theory is actually not defined at all without the $L^2$ term in (21).

For a finite volume one can readily determine the average length of the percolating cluster:

$$\langle L_{\text{perc}} \rangle = \frac{\mu}{2\gamma} V^4,$$

(22)

and adjust the coefficients $\mu, \gamma$ to reproduce the observed density of percolating monopoles. Moreover, action (21) can describe, for a finite volume, also fluctuations of $L_{\text{perc}}$ around its central value (22) $\text{21}$.

The $L^2$ term in (21) corresponds in fact to the specific heat in the language of thermodynamics and could be postulated on general grounds $\text{8}$. Basing on the general arguments one can expect that the coefficient $\gamma$ tends to zero as $p$ approaches $p_c$ from above.

3.5. Lattice $Z_2$ gauge theory

Because of space considerations, we cannot go into details of the lattice $Z_2$ gauge theory. Roughly speaking, the mechanism of the phase transition to percolation is similar to the $U(1)$ case, with replacement of trajectories with closed $d = 2$ surfaces. Namely, both the action and entropy factor are

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$\text{8}$The remark is due to L. Stodolsky, for more details see $\text{23}$. 
divergent in ultraviolet as exponents of $A/a^2$ and can be tuned to each other by choice of the coupling.

4. Data vs. theoretical constraints

4.1. Asymptotic freedom and counting degrees of freedom

We see that both in case of $U(1)$ and $Z_2$ gauge theories confinement can be ensured by tuning the corresponding couplings. Moreover, the idea that the $SU(2)$ confinement is similar to the $U(1), Z_2$ cases was the driving force to finally discover the amusing properties of the monopoles and vortices in the $SU(2)$ case. Also, Eqs. (6), (8) look typical for a percolating system in the supercritical phase, see Eq. (20).

Nevertheless, there are accumulating arguments that the fine tuning in the $SU(2)$ case is to be different. On the theoretical side and to begin with, there is no coupling to tune since it is running. Moreover, the monopoles and vortices are not intrinsic to the full $SU(2)$, but rather to its subgroups, that is $U(1)$ and $Z_2$. Namely, there is no topological definition of monopoles (vortices) in the full $SU(2)$ which would imply lower bounds on the action of the topologically non-trivial fluctuations. As a result, there is no answer to the question why the monopole or vortex action cannot go down 19. If it were allowed, however, monopole and vortices would have packed the whole of the lattice.

Now, that it has been revealed that the monopoles and vortices are associated with divergent actions, see Eqs. (5) and (9) the problems become even more acute. Indeed, Eq. (5) looks exactly the same as for a point-like particle but it is clear that in an asymptotically free theory we are not allowed to add new particles.

Let us look closer, what the actual constraints are. Consider the ‘cosmological constant’, that is density of vacuum energy:

$$\varepsilon_{\text{vac}} \approx \sum_{k} \omega(k) / 2 . \quad (23)$$

In an asymptotically free theory Eq (23) should be a valid approximation. The sum (23) diverges in the ultraviolet as $a^{-4}$. The coefficient in front of the divergence depends on the number of degrees of freedom. Moreover, all the divergences are regularized by the lattice, so that Eq (23) is well defined and can be used to predict the average value of the plaquette action $\langle P \rangle$ on the lattice: $\langle P \rangle$:

$$\langle 1 - P \rangle_{\text{pert}} \approx \frac{c_G}{a^2} , \quad (24)$$
where the coefficient $c_G$ is known, for further details and references see 28.

However, this constraint is formulated in terms of the original gluonic fields. To appreciate the meaning of the constraint in terms of the monopole trajectories, which are our basic observables now, notice that an uncorrelated percolation is equivalent to a ‘free particle’ in the tachyonic mode, see Sect. 3.4. The crucial point is then that in the percolation picture there is nothing happening to the total monopole density at $p = p_c$. Indeed the total density of ‘open’ links is simply:

$$\rho_{\text{tot}}^{\text{perc}} = p \cdot \frac{1}{a^3},$$

and there is no discontinuity or non-analyticity at $p = p_c$. It is only the density of the infinite cluster which exhibits a threshold behaviour (20).

Existence of the percolating cluster is crucial for the confinement and that is why one usually concentrates on $\rho_{\text{perc}}$. However, now we come to the conclusion that from the theoretical point of view it is the total monopole density which is constrained by the asymptotic freedom. If the density of the percolating cluster is vanishing at the point of the phase transition (see (20)) where the total density (25) goes to? Clearly, to finite clusters. Thus, we should address theory of finite clusters.

4.2. Monopoles as a novel probe of short distances

Let us consider first free particle case. Then the simplest Feynman graph is a vacuum loop. Usually, textbooks say that this graph is not observable since all the amplitudes are normalized to the vacuum-to-vacuum transition. Similarly, one is usually saying that the vacuum energy is normalized to zero by definition, However, once the lattice regularization is introduced, the ‘cosmological constant’ turns directly observable. The same is true for the vacuum-to-vacuum transition. For free particles, vacuum loops are directly observable and one can try to evaluate them theoretically 16.

Moreover, the calculations are in fact straightforward. Since it is the trajectories that are directly observable one is invited to use the polymer representation of field theory, see Sect. 2.1. There are two basic properties of finite clusters which can be predicted starting from the assumption that the monopoles at short distances can be treated as free particles 16. First, the spectrum of the finite clusters as function of their length is given by b:

$$N(L) = \frac{\text{const}}{L^{d/2+1}} = \frac{\text{const}}{L^3},$$

bThe result can be actually read off from the equations derived, e.g., in Ref. 24.
where we substituted the number of dimensions of the space $d = 4$. Second, radius of a cluster is predicted to be:

$$R(L) \sim \sqrt{L \cdot a}.$$  \hspace{1cm} (27)

The predictions (26) and (27) are in perfect agreement with the data\textsuperscript{5,10}. Eq (26) is especially remarkable for the fact that the spectrum depends explicitly on the number of dimensions of space-time. Thus, at short distances the monopoles move as free particles in $d = 4$ space.

### 4.3. Conspiracy of the monopoles and vortices

The fact that the predictions (26) and (27) agree with the data, at first sight, is in contradiction with what we said earlier on counting degrees of freedom at short distances. However, Eqs (26), (27) do not yet allow for a direct comparison with expectations based on asymptotic freedom. Consider therefore the vacuum expectation value (16). Clearly, the asymptotic freedom requires that

$$\langle 0 | |\phi|^2 | 0 \rangle \sim \Lambda_{QCD}^2,$$  \hspace{1cm} (28)

while any ultraviolet divergence in this vacuum expectation value would imply that monopoles are to be introduced on equal footing with the gluons and this is not allowed.

Constraint (28) can be turned into a constraint on monopole densities defined as:

$$L_{\text{mon}} = L_{\text{perc}} + L_{\text{fin}} \equiv \rho_{\text{perc}}V_4 + \rho_{\text{fin}}V_4,$$  \hspace{1cm} (29)

where $L_{\text{perc}}$ and $L_{\text{fin}}$ are the lengths of the trajectories belonging to the percolating and finite clusters, respectively. Thus, Eq. (28) implies:

$$\rho_{\text{fin}} \leq \frac{\text{const}'}{a}.$$  \hspace{1cm} (30)

It is most amusing that the constraint (30) is satisfied by the data! The first indication to the $1/a$ behaviour of the total density of the monopoles was obtained in Ref\textsuperscript{7} and is now confirmed in Ref\textsuperscript{10}.

It is easy now to figure out the geometrical meaning of the relation (30)\textsuperscript{10}. Clearly, monopoles are to be associated with a $d = 2$ subspace of the whole $d = 4$ space. Taken as a prediction, this sounds bizarre. Unfortunately, no prediction of this kind was done (and our analysis is post hoc).

\textsuperscript{1}The Coulomb-like interaction in $d = 4$ leaves actually (26) and (27) unchanged\textsuperscript{16}. 

\hspace{1cm}
The fact that the vortices are populated by monopoles is rather known empirically. Thus, there exists a long-range correlation between finite clusters. As a result, the constraint (28) gets satisfied by the data. It is amusing that at short distances monopoles are ‘primary’ object and the vortices span on the trajectories, see discussion around Eq. (26). At large distances, to the contrary, the vortices are ‘primary’ and monopoles are ‘secondary’. Striking differences between the ultraviolet and infrared behaviour are well known in case of random walks, for a text-book presentation see, e.g., 14, while data on the monopole clusters can be found in Ref. 8. For the surfaces, we believe, the topic is quite new.

To reiterate the central point: in case of uncorrelated percolation we would get

\[ \langle 0 | |\phi|^2 |0 \rangle_{\text{perc}} \sim p \cdot a^{-2} \]  

recovering the standard quadratic divergence which plagues theory of scalar elementary particles. As we emphasized above, (31) is not allowed by the asymptotic freedom. For similar reasons we could not allow for percolation of fine tuned surfaces which is relevant to the pure \( Z_2 \) case. On the other hand, percolation of ‘branes’, i.e. of \( d = 2 \) surfaces populated with monopoles is allowed by the constraint (28) and is observed on the lattice.

4.4. \textit{Gauge-invariant approaches}

As is mentioned a few times above, no gauge invariant description of the vortices has been developed. Let us still mention that it seems natural to consider determinant \( D \) constructed on three independent color magnetic fields, for a given choice of the ‘time’ slice:

\[ D(\mathbf{x}) = \| H_i^a(\mathbf{x}) \|, \]  

where \( a = 1, 2, 3 \) is a color index and \( i = 1, 2, 3 \) is a space index. Zeros of the determinant (32) define a two-dimensional surface. One could try to identify these surfaces with the vortices detected on the lattice. There is a phenomenological support for such an identification since the vortices are indeed ‘thin’ and this corresponds to color magnetic field spread tangentially on surfaces. Monopoles would correspond to a zero of second order and even more asymmetric field (at short distances).

Another way of searching for two-dimensional surfaces in an explicitly gauge-invariant fashion is to study properties of the Dirac operator, for details see 26 and references therein. Moreover, vanishing of the determinant (32) means changing of a left-handed configuration of the magnetic fields.
into the right-handed and vice versa. Thus, surfaces associated with degeneracy of the magnetic fields could well be detected through studies of the chiral modes and the vortices discussed here could be identical or closely related to the surfaces found in Ref. 26.

Finally, let us mention that combining Eq. (16) with the data on the monopole density in finite clusters one can find:

$$\langle |\phi|^2 \rangle \approx 0.8 \ (fm)^{-2}.$$  \hspace{1cm} (33)

The result (33) is perfectly gauge invariant. Moreover, the condensate has dimension 2 and one can wonder, what kind of condensate in terms of the fundamental gluon fields can be behind (33). This question could have been considered as a serious objection to (33) since, naively, the simplest gluon condensate, $$\langle (G_{\mu\nu}^a)^2 \rangle$$ has dimension 4. However, it was demonstrated recently that dimension-two condensate can well be defined in terms of gluon fields 27.

5. Conclusions

We have argued that the fine tuning exhibited by the lattice data on $SU(2)$ gluodynamics appears to be a novel phenomenon. Namely, the density of the percolating monopole and lattice clusters are similar to well known examples of the lattice $U(1)$ and $Z_2$ gauge theories near the phase transition. However, in the latter cases there exist elementary monopoles (vortices). As a result, vacuum expectation values of the corresponding fields are ultraviolet divergent. Such a divergences would be inconsistent with the asymptotic freedom in the non-Abelian case. And this constraint, at first sight, rules out an ultraviolet divergence in the monopole mass (see Eq (2)) which is similar to the point-like particle. The way out is suggested by the data. Namely, the monopoles populate not the whole $d = 4$ space but rather a $d = 2$ subspace of it. Existence of these new objects (“branes”) is consistent with the continuum theory. Which adds credibility to the lattice evidence.

Note that, combining Eqs. (7) and (9) we find that the contribution of the vortices into the average plaquette action is of order

$$\langle 1 - P \rangle_{\text{vort}} \approx 0.1 \ GeV^2 \ a^{-2}.$$  \hspace{1cm} (34)

This contribution matches the so called ultraviolet renormalon, for discussions see, in particular, 28,29. We hope to come back to this issue in a separate note.
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