Nano-gap planar metal electrodes: fabrication and I-V characteristics

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Abstract
The nanowires/bars and nano-gap electrodes are vital components for emerging electronics and have wide ranging applications in flat-panel displays, sensors, sub-100 nm transistor circuits, and miniaturized computers/devices. Focused ion beam (FIB) has emerged as a powerful and unique tool for nanofabrication. The research work described here is concerned with (a) the FIB fabrication of planar metallic (copper and gold) nanostructures, (b) their current-voltage (I–V) measurements in situ, and (c) a viable method for extracting the realistic values of emission parameters. The planar electrodes with gap of 80–100 nm are realized by FIB milling of thin metal films. The difficulties faced in objective interpretation of their I-V data (based on known mechanisms) are highlighted. For determining the parameters (namely, effective emission area $\alpha_{\text{eff}}$, apparent work function $\phi$, and the field enhancement factor $\beta$), Fowler-Nordheim $[\ln(I/V^2) \text{ versus } 1/V]$ plots showing a minimum with straight line of negative slope can be used. The striking findings demonstrated are (i) occurrence of emission from a tiny region ($<1 \text{ nm}^2$) vis-à-vis physical area (400 $\mu \text{m} \times 200 \text{ nm}$), (ii) significant lowering of barrier height, and (iii) enhancement of local field due to protrusions present. Typical values of $\alpha_{\text{eff}}$, $\phi$, and $\beta$ deduced are 52.3 $\text{Å}^2$, 1.62 eV, and 39.3, respectively for copper planar electrodes (gap $\sim 100 \text{ nm}$); the corresponding data for the case of gold (gap $\sim 80 \text{ nm}$) are 29.1 $\text{Å}^2$, 1.97 eV, and 12.1, respectively. Moreover, $\beta$ lowering observed with bias is accompanied by increase in the emission area due to progressive smoothening of protrusions at the cathode surface. The electrodes are found rough/rocky at the nanoscale with protrusions and varying separations at places. These features make the electron emissive region small and pointed with an enhanced local electric field and effectively of a lower barrier height. The current discrepancy in the Child-Langmuir’s space charge regime is attributed to the emission occurring from a restricted area only. These findings are important for futuristic nano-devices like thermo-tunnelling refrigerator, energy harvester, etc.

1. Introduction

Quantum mechanical tunnelling plays a crucial role in many nanodevices (such as tunnel diodes, Josephson junctions, etc) and forms the basis for the futuristic thermo-tunnelling refrigerator and ambient energy harvesting [1–4]. Tunnelling of electrons across a nanoscale metallic gap at a small bias is used in scanning tunnelling microscopes and flash memory devices [5, 6]. The phenomenon involves passage of electrons (having less energy than the work function of the cathode material) through the potential barrier and gives rise to a current via cold field emission [7]. The barrier shape is near rectangular at low field and the current evolves is termed as direct tunnelling [8]. With increase of electric field, the barrier assumes a triangular (via a trapezoidal) shape, reduces the barrier width, and delivers higher current as Fowler-Nordheim (F-N) emission [9, 10]. Thus, the current shows transition from direct tunnelling to FN emission due to field induced shape changes in the barrier [8, 11]. Since the nature of barrier (height, width, and shape) plays a vital role in determining the overall
current at a given field, its knowledge becomes crucial in applications like thermo-tunnelling cooling and energy harvesting\[12\].

The nano-wires/bars and nano-gap electrodes are also important for emerging electronics and have enormous potential applications in high brightness as well as contrast/fast response/low power flat-panel displays, sensors, high power microwave amplifiers, sub-100 nm transistor circuits, field emission electron sources, and miniaturized computers/devices with high speed and large storage capacity\[13–16\]. Further, the reliable electrical contacts are vital at the nanoscale for future electronics, i.e., Nanoelectronics, optoelectronics and photonics, particularly in light emitting source, tunable Esaki effect, tunnel field-effect transistors, etc\[17–21\]. They can bridge individual nanostructure with the macroscopic system and form devices for detecting nanoparticles\[22, 23\]. The nano-bar structures can be obtained by electron beam lithography\[24\], shadow evaporation\[25\], mechanically controllable break junction\[23\], electrochemical plating\[26\], and electromigration\[13, 27\]. Another quick and flexible technique based on focused ion beam (FIB)-direct lithography enables mask-less fabrication of not only nanogap electrodes but also a variety of three-dimensional prototype nanostructures (namely, pillars, cantilevers, and springs), useful for electromechanical systems, sensors, and actuators\[28–30\].

The interpretation of I-V data of nanoscale electrodes is quite challenging. In fact, numerous emission parameters deduced and reported in the literature are physically unrealistic. These resulted due to widespread use of faulty elementary FE equations for local emission current density (LECD) and discussed in detail by Forbes recently\[31\]. It is imperative therefore to have standard equations that describe the FE current density reliably and devise a scheme for consistent analysis of the measured data. All formulations in use lead to uncertainty in the quantitative prediction of emission current density (ECD). Nevertheless, their verification can only be made by making careful comparison with the experiments. With the inherent complications just beginning to realize/unfold in FE, the exercise has proven to be difficult in making a precise comparison of theory and experiment. Consequently, a technological subject of field electron emission is turning out to be more scientific (i.e., a FE-science problem). This manuscript focuses on its limited but a crucial part and aim to provide a comprehensive scientific description of FE by concentrating on different methodologies used for the interpretation of current-voltage data. For this, planar electrodes (cross section 400 μm × 200 nm with a gap of ~80–100 nm) are fabricated over a substrate by FIB milling of thin metal films and their I-V characteristics measured in situ to examine suitability of the well-known bulk emission processes/mechanisms in

![Figure 1. (a)–(d) Schematic diagram of (a) an l-shaped metal thin film deposited on a glass substrate by thermal evaporation, cross-sectional views (b) before and (c) after forming a trench (width ~100 nm) by milling with a focused gallium ion beam, (d) electrodes of cross section 400 μm × 200 nm with a gap of ~100 nm, and (e) A typical scanning electron micrograph of the electrodes having a gap of 100 nm.](image)
understanding the structures at the nanoscale. The difficulties encountered in the interpretation of I-V data are highlighted and a scheme suggested for extracting the parameters (namely, effective emission area $\alpha_{\text{eff}}$, apparent work function $\phi$, and the field enhancement factor $\beta$) from the $\ln(I/V^2)$ versus $1/V$ plots, taking copper and gold as examples. The striking features of nanoscale electrodes are critically analyzed to find explanation for discrepancies, viz., (i) exceeding small emission area, (ii) non-linearity in F-N plot, and (iii) apparent lowering of work function.

2. Fabrication of planar electrodes

The steps used for realizing planar electrodes (with a gap of $\sim$100 nm) are given schematically in figure 1. First, a thin film (I-shape, thickness $\sim$200 nm) of copper is deposited on a clean glass substrate by thermal evaporation in vacuum ($\sim$10$^{-6}$ mbar) using a mask (figure 1(a)). To ensure its purity, initial vapor condensation on the substrate is prevented with a shutter assembly. A trench of length 400 $\mu$m, width 100 nm, and depth 200 nm is then created by milling of the copper film with a focused gallium ion beam (30 keV, size 10–20 nm) at a current of $\sim$100 pA in a FEI Nova NanoLab 600 (figures 1(b), (c)). The planar electrode structure of cross-section 400 $\mu$m $\times$ 200 nm with a gap of 100 nm thus created (figure 1(d)) is placed on a sample holder and inserted in the FIB chamber for measuring the I-V characteristics in vacuum ($\sim$10$^{-6}$ mbar) by employing a Keithley Source Meter model 6430.
3. Results and discussion

3.1. Current-voltage characteristics

Figure 1(e) shows a scanning electron micrograph of copper electrodes having a cross-section of 400 μm × 200 nm and gap of ~100 nm. Its current-voltage (I-V) characteristics measured in vacuum ~10^-6 mbar are depicted in figure 2(a). Note that the current increases slowly in the beginning but quite rapidly beyond a certain voltage. The corresponding ln I versus ln V plot up to ~8 V presented in figure 2(a) (inset) demonstrates V^1/2, V, and V^3/2 dependence in the voltage range 0.2–1.0 V, 1.3–2.4 V, and 3.2–8.0 V following the modified Child-Langmuir relation, direct tunnelling within the Simmon’s approximation, and classical Child-Langmuir (CL) law, respectively [32–34]. The sharp increase in current above ~8 V results possibly due to initiation of Fowler-Nordheim tunnelling caused by sufficiently high electric field developed across the electrodes. The ln(I/V^2) versus 1/V plot given in figure 2(b) contains a minimum and a straight line with negative slope above 8.3 V. The inflection point suggests transition from V^3/2 dependence to field emission. Also, ln(I/V^2) vs ln(1/V) plot in the inset (b2, figure 2(b)) shows a straight line with positive slope (as unity) in the bias range of 1.3–2.4V and represents direct tunnelling. All these features are discussed below in detail.

3.1.1. Planar electrode model

The maximum current density J_{CL} across two parallel planar electrodes separated by a gap (d) at a voltage (V) in one dimension is given by the classical Child-Langmuir law as [34]

\[ J = \frac{2e\mu d^2}{9\varepsilon_0^2} \left( \frac{V}{d} \right)^{3/2} \]

\[ J = \frac{2e\mu d^2}{9\varepsilon_0^2} \left( \frac{V}{d} \right) \]

\[ J = \frac{2e\mu d^2}{9\varepsilon_0^2} \left( \frac{V}{d} \right)^2 \]

Figure 3. Current density versus voltage (J-V) plots on semi-log scale of metal electrodes with a gap of (a) 0.1 nm, (b) 0.5 nm, (c) 1 nm, (d) 10 nm, (e) 50 nm, and (f) 100 nm obtained using quantum space charge limited current (QM) and classical Child-Langmuir law (CL) with V^1/2 and V^3/2 dependence, respectively.
Figure 4. (a) I-V characteristics of copper electrodes of cross-section 400 μm × 200 nm with a gap of 100 nm (measured as well as calculated using Child-Langmuir $V^{3/2}$ law) showing difference in current of about six orders, (b) schematic diagram of metal electrodes separated by a gap (d) depicting nature of the barrier (b1) rectangular at $V = 0$, (b2) trapezoidal at $V < \phi_m/e$ and (b3) triangular at $V > \phi_m/e$ under different conditions. $\phi_m$ being the metal work function and $E_f$ denotes the Fermi level, and (c) current density ($J$) versus $B_1/V$ plot as drawn using equation (16) with $\phi = 4.8$ eV for copper. $(-B_1)$ is the slope of straight line fitted in ln($I/V^2$) versus $1/V$ plot (figure 2(b)) for copper electrodes (cross section 400 μm × 200 nm, gap ~ 100 nm).

$$J_{CL} = \frac{4e_0}{9} \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2}$$

(1)
where \( \varepsilon_0 \) is the permittivity of free space, \( e \) is the electronic charge and \( m \) is the mass of the electron. For finite electrodes (say, strip of thickness ‘w’ and infinite length), electron emission follows the Child-Langmuir’s relation in 2-dimension, such that the current density \( J_{CL} \) is given by \([35]\)

\[
J_{CL} = J_{CL}[1 + 0.3145(d/w)]
\]

It means that the current density \( J_{CL} \) improves progressively with increase in \( (d/w) \) ratio and can become greater than \( J_{CL} \) even. For example, \( J_{CL}/J_{CL} = 1.1572 \) and \( 1.3145 \) with \( (d/w) = 1/2 \) and 1, respectively. But \( J_{CL} \) is essentially equal to \( J_{CL} \) for \( (d/w) < 0.03 \), i.e., when electrodes separation \( d \) is much smaller than the thickness \( (w) \). When \( d \) is in nanometer range, quantum effects such as electron tunnelling, space-charge, and exchange correlation assume importance. The limiting current density \( J_{QM} \) then takes the form \([32, 36]\)

\[
J_{QM} = \frac{\varepsilon_0 e^2}{4\pi^2 e^{1/2} m^{3/2}} \left[ \frac{V^{1/2}}{d^4} \right]
\]

where \( h \) is the Planck’s constant. Obviously, the current density varies very differently with applied voltage \( (V) \) and electrodes gap \( (d) \) in the above two cases. The ratio of \( J_{CL} \) and \( J_{QM} \) reduces to

\[
\frac{J_{CL}}{J_{QM}} = \frac{16\sqrt{2}}{9} \left[ \frac{\pi^2 e m}{h^2} \right] d^2 V = 8.2 \times 10^{-6} d^2 V
\]

This \( J_{CL}/J_{QM} \) ratio increases linearly with voltage but quadratically with the gap \( (d) \). Consequently, \( J_{CL} \) is expected to be orders of magnitude higher than \( J_{QM} \) at \( d \geq 10 \text{ nm} \). In other words, quantum effects cause significant reduction in the current density. On the other hand, \( J_{QM} \) is greater than \( J_{CL} \) when \( d < 0.35/\sqrt{V} \) (nm). Obviously, quantum effect is prevalent at the sub-nanometer separation only. Figure 3 display \( J_{CL} \) and \( J_{QM} \) obtained from equations (1) and (3), respectively for planar electrodes of cross-section 400 \( \mu \text{m} \times 200 \text{ nm} \) and having gap of 0.1, 0.5, 1.0, 10, 50 and 100 nm. Accordingly, \( J_{QM} \) is insignificant in comparison to \( J_{CL} \) for the separation \( d \geq 1 \text{ nm} \).

Moreover, the current no doubt displays \( V^{3/2} \) dependence in the voltage range 3.2–8.0 \text{ V} but is several orders lower than that deduced from equation (1), taking electrode cross section as 400 \( \mu \text{m} \times 200 \text{ nm} \) (figures 2 and 4(a)). This discrepancy can however be understood if emission is believed to occur from a restricted portion [i.e., exceedingly small area \( \sim 80 \text{ nm}^2 \) or \( (1 \times 10^6) \)] of the cathode (figure 4(a)).

### 3.1.2. Simmons’ formulation

A structure comprising of two metal electrodes with a little separation and defined by a generalized trapezoidal barrier of height \( (\phi) \) gives rise to a tunnelling current \( (I) \) described by \([33]\)

\[
I = I_0 \{ \phi \exp(-\xi \phi^{1/2}) - (\phi + eV) \exp[-\xi(\phi + eV)^{1/2}] \}
\]

where

\[
I_0 = eA_{cr}/2\pi h (\Delta d),
\]

\[
\xi = \frac{4 \pi (\Delta d)}{h} \left( \frac{2m\phi}{h} \right)^{1/2}
\]

and

\[
\phi = \frac{1}{\Delta d} \int_{d_1}^{d_2} \phi(x) \, dx
\]

with \( A_{cr}, \phi, \text{ and } m \) being the electrode cross-sectional area, electronic charge, Planck’s constant, and electron mass, respectively. Also, \( \phi(x) \) stands for the potential energy of electron in between the two electrode surfaces, \( d_1 \) and \( d_2 \) are the distances from the first electrode to the surface 1 and 2, respectively (where the potential energy corresponds to the Fermi level), \( \Delta d = (d_2-d_1) \), and \( V \) is the voltage applied across the electrodes. A trapezoidal barrier turns rectangular at low voltages (i.e., \( V \sim 0 \)) with \( \Delta d = (d_2-d_1) = d \), separation between the two electrodes, and \( \phi = \phi_0 \), the metal work function. For \( V < \phi/e \), \( \phi \) becomes \( (\phi_0 + eV/2) \). In both these cases (figures 4(b1), (b2)), equation (5) yields

\[
I = \left\{ \frac{A_{cr} e^2(2m\phi_0)^{1/2}}{dh^2} \right\} V \exp \left\{ -\frac{4\pi d}{h} (2m\phi_0)^{1/2} \right\}
\]

Equation (9) can be rewritten as

\[
\ln \left( I/V^{2} \right) = \ln \left( 1/V \right) + \ln \left\{ \frac{A_{cr} e^2(2m\phi_0)^{1/2}}{dh^2} \right\} - \frac{4\pi d}{h} (2m\phi_0)^{1/2}
\]

\[
\ln (I/V^{2}) = \ln (1/V) + \ln \left\{ \frac{A_{cr} e^2(2m\phi_0)^{1/2}}{dh^2} \right\} - \frac{4\pi d}{h} (2m\phi_0)^{1/2}
\]

\[
(10)
\]
Thus, tunnelling current should vary linearly with voltage (equation (9)). Also, \( \ln(I) \) vs \( \ln(V) \) and \( \ln(I/V^2) \) versus \( \ln(1/V) \) plots ought to exhibit straight lines with positive unit slope. Incidentally, \( I \) values presented in figures 2(a) and (b) (insets) match with the above description in the range 1.3–2.4 \( V \).

Similarly, at high voltages \( V > \phi_0/e \), the barrier assumes a triangular shape with \( \phi = \phi_0/2 \) and effective separation \( \Delta d = d/2 \phi_0/e \) (figure 4(b)). By making substitutions, equation (5) takes the form

\[
\ln(I/V^2) = \ln\left(\frac{e^2A_\alpha}{4\pi\hbar\phi_0S^2} \right) - \frac{2\sqrt{2}\pi}{5} (2m)^{1/2}S\phi_0^{3/2}\left(\frac{1}{V}\right)
\]

Hence, the \( \ln(I/V^2) \) versus \( 1/V \) plot yields a straight line with negative slope. Notice that both the slope and the intercept can determine the barrier height. Equation (11) describes the field emission and is like Fowler-Nordheim tunnelling through a triangular barrier (discussed later).

The above formulation indicates that while the \( \ln(I/V^2) \) versus \( \ln(V) \) plot depicts direct tunnelling and linearity with unit slope at low bias, \( \ln(I/V^2) \) versus \( 1/V \) plot signifies field emission and contains a straight line with negative slope at high voltages. No current should practically be observed in electrodes of significant barrier height \( \phi_0 \) and/or large width \( d \) at low voltages, prior to initiation of field emission. Nevertheless, the \( I-V \) data presented in figure 2(b) demonstrate a transition from direct tunnelling to field emission despite of the electrode work function being high (copper, \( \phi_0 = 4.8 \) eV) and the separation \( d \) large (100 nm). This discrepancy is resolved by invoking an apparent work function as shown later.

### 3.1.3. Fowler-Nordheim mechanism

The generalized Fowler-Nordheim field emission current equation is written as [37]

\[
J = \frac{a}{\phi^2}E^2 \exp\left(-\frac{b\phi^{3/2}}{E}v(y)\right)
\]

where \( J \) is the current density in \( \text{Am}^{-2} \), \( a = e^3/8\pi n \) and \( b = (8\pi/3)(2m)^{1/2}/e \) are constants equal to \( 1.54 \times 10^4 \text{ A}(eV)^{-2} \) and \( 6.83 \times 10^8 \text{ eV}^{-3/2} \text{A}(eV)^{-1} \text{m}^{-1} \), respectively, \( \phi \) is the work-function of electrode, \( E \) is the field strength, \( v(y) \) and \( t^2(y) \) are slowly varying functions of Schottky barrier lowering parameter \( y = 3.79 \times 10^{-8}E^{1/2}/\phi \). If \( \alpha \) is the emission area, the current becomes equal to \( I = J\alpha \). Also, the local field strength is represented as \( \beta = \beta(V/d) \) where \( d \) is the physical separation of electrodes and \( \beta \) is a field enhancement factor determined by the geometry of the emitting electrode [38]. By making these substitutions, equation (12) takes the form

\[
I = \frac{a\alpha}{\phi^2} \left(\frac{\beta V}{d}\right)^2 \exp\left(-\frac{b\phi^{3/2}d}{\beta V}v(y)\right)
\]

Accordingly, the \( \ln(I/V^2) \) versus \( 1/V \) plot corresponds to a straight line with a negative slope, whose magnitude (say, \( B \)) is given by

\[
B = \frac{b\phi^{3/2}d}{\beta} [v(y)]
\]

Charbonnier and Martin [39] suggested a simple way for extracting information about the field emission source in the low current density range \( (10^2–10^6 \text{ Am}^{-2}) \) from equation (13) assuming \( v(y) = s(y) - 1.062y^2 \), \( t^2(y) = 1.044 \), and \( s(y) = 0.956 \). With these substitutions and putting \( y = 3.79 \times 10^{-8}/\phi (\beta V/d)^{1/2} \), equation (13) yields

\[
I = \frac{a\alpha b^2 [sv(y)]^2V^2G(\phi)}{1.044B_1^2} \exp\left(-\frac{B_1}{V}\right)
\]

with \( B_1 = \frac{b\phi^{3/2}d}{\beta} [v(y)] \) and \( G(\phi) = \phi^2 \left(\exp(10.4/\phi^{1/2})\right) \)

It may be noted that \( B_1 \) is the slope now of the straight line observed in \( \ln(I/V^2) \) versus \( 1/V \) or F-N plot. Inserting the values of constants \( (a \text{ and } b), s(y) = 0.956 \) and rearranging, the current density \( J = (I/\alpha) \) is expressed as

\[
J = 6.29 \times 10^{13}G(\phi) \left[\frac{\exp(-B_1/V)}{(B_1/V)^2}\right]
\]

Thus, the current density \( J \) depends on the work-function \( \phi \) through \( G(\phi) \), slope \( B_1 \) of \( \ln(I/V^2) \) versus \( 1/V \) plot, and the applied voltage. The function \( G(\phi) \) departs from a fixed value of 2700 by less than \( \pm 7\% \) for \( \phi \geq 4 \text{ eV} \). They made \( J \) vs \((B_1/V)\) plots using equation (16) for a given \( B_1 \) and several \( \phi \) values and demonstrated sole dependence of \( J \) on \( (B_1/V) \) for \( \phi \geq 4 \text{ eV} \) (precisely between 5 and 9 \text{ eV}). Further, the exact value of the work function was essential to evaluate \( J \) from equation (16) below \( \phi = 4 \text{ eV} \). The current density \( J \) can be determined from equation (16) at different voltages with an accuracy of \( 7\% \) by taking \( B_1 \) as slope of \( \ln(I/V^2) \)
versus $1/V$ plot for $\phi \geq 4$ eV. The emission area $'a_{eff}'$ can then be known as measured $I$ is equal to $(I\alpha)$. Figure 4(c) depicts the current density $(J)$ versus $(B/V)$ plot drawn by deducing $B$ from the $\ln(I/V^2)$ versus $1/V$ curve in the negative slope regime and taking $\phi$ as 4.8 eV for copper planar electrodes of cross-section 400 $\mu$m × 200 nm and the separation $(d) \sim$ 100 nm. The value of $I$ at $B/V$ (=2.0) with the equivalent measured current ($I = 2.74 \times 10^{-4}$ A) gives the emission area as $2.9 \times 10^{-22}$ m$^2$. This $'a'$ value is extremely small and unrealistic vis-à-vis real cross-sectional area of electrodes $(8 \times 10^{-11}$ m$^2$). Charbonnier and Martin [39] envisaged low current densities $(10^5$–$10^6$ Am$^{-2}$) for application of the above method. In contrast, the current density values here are high (range being $10^{15}$–$10^{16}$ Am$^{-2}$, figure 4(c)) and so amount to huge discrepancy. Another drawback lies in assuming the work-function above 4 eV.

Spindt et al [40] expressed equation (13) by taking $v(y) = 0.95 - y^2$, $v'(y) = 1.1$ and $y = 3.79 \times 10^{-5}E^{1/2}/\phi$ in the form

$$I = AV^2 \exp \{-B/V\}$$

such that

$$A = \frac{a\alpha}{1.1\phi} \left(\frac{\beta}{d}\right)^2 \exp\left(\frac{b(1.44 \times 10^{-6})}{\phi^{1/2}}\right)$$

and

$$B = 0.95 b(d/\beta)\phi^{1/2}$$

On differentiating equation (17) with respect to $V$ and rearranging, one gets

$$\frac{dI}{dV} = \frac{I}{V\left(2 + \frac{B}{V}\right)}$$

Thus, by measuring $I$-$V$ characteristics and determining $(dI/dV)$ for each set $(I, V)$, the value of $B/V$ (or $B$) can be deduced from equation (20). The term $A$ can then be obtained from equation (17). Three unknown quantities namely, $A$, $\alpha$, and $\phi$ are to be found with the existing two equations (18) and (19). So, they combined equations (18) and (19) and used the values of constants $a = 1.54 \times 10^{-6}$ A(eV)$V^{-2}$ and $b = 6.87 \times 10^9$ (eV)$-3/2V$m$^{-1}$ to obtain the relation

$$AB^2 = (5.96 \times 10^{13})\alpha \{\phi^2 \exp(9.89/\phi^2)\}$$

The value of $\{\phi^2 \exp(9.89/\phi^2)\}$ was shown to lie within ±10% if fixed at 2250 over a range of $\phi = 3.4$–11.6 eV. Taking $\{\phi^2 \exp(9.89/\phi^2)\} = 2250$, equation (21) reduces to

$$\alpha = (7.46 \times 10^{-18})AB^2$$

Equations (17) and (22) give

$$(\alpha/I) = (7.46 \times 10^{-18})(B/V)^2 \exp(B/V)$$

Once $(B/V)$ is found from equation (20) using the $I$-$V$ data, emission area $(\alpha)$ can be obtained from equation (23). The above procedure enabled Spindt et al [40] to estimate the emission area of molybdenum cone cathodes (1.5 μm tall, tip radius ~50 nm), fabricated with e-beam evaporated thin films and electron beam microolithography, within ±10%. The value of $'a'$ turned out to be exceedingly small (i.e., $1.3 \times 10^{-19}$ m$^2$ or $13\AA^2$) compared to cathode tip radius of 50 nm—suggesting emission contribution essentially from a few atoms. For the case under study of copper planar electrodes, $I = 1.86 \times 10^{-6}$ A at 15 V and $(dI/dV) = 4.11 \times 10^{-7}$ (figure 3), equation (23) gives $\alpha = 8.82 \times 10^{-23}$ m$^2$ for $B/V = 1.31$. This figure is extremely small and unrealistic with no physical significance/meaning. Obviously, the above method fails to yield realistic values of the emission area.

Wong and Ingram [41] fabricated Au/Ti lateral tunnel diodes with inter-electrode separation below 50 nm on Si$_3$N$_4$ substrates using high resolution electron beam lithography and lift-off metallization process. The $I$-$V$ characteristics measured under vacuum (10$^{-6}$ Torr) and in air (760 Torr) followed Fowler-Nordheim tunnelling above a threshold voltage of 45 V. They used $\ln(I/V^2)$ versus $1/V$ plot to estimate the field enhancement factor $(\beta)$ and apparent emission area from the negative gradient and intercept at the ordinate, respectively with equations like (17–19); the values arrived at were $\beta = 2.63$ and $\alpha = 2 \times 10^{-20}$ m$^2$ (or 2 Å$^2$ only) for a diode having inter-electrode gap of 25 nm.

Forbes [42, 43] examined the issue of extracting emission area from the intercept $(\ln A)$ and negative slope $(–B)$ of a straight line drawn by linear fitting in the Fowler–Nordheim $\ln(I/V^2)$ versus $1/V$ plot and proposed a formulation with new symbols and correction factors but finally obtained relations analogous to equation (16) of Charbonnier and Martin [39] and equation (21) of Spindt et al [40] given above. Further, the emission area $(a_{eff})$ was given by

$$\alpha = AB^2/C_2\Gamma$$
where $C_2$ is a universal constant ($=7.192 \times 10^{13} \text{Am}^{-2} \text{eV}^{-2}$) and $\Gamma$ is termed as extraction parameter related to work function ($\phi$) of the emitter and the current density ($J$). $C_2 \Gamma$ is analogous to the term $6.29 \times 10^{13} \text{G(eV)}$ of equation (16). Forbes [26] expressed the logarithmic current density in terms of Nordheim parameter $y (= 3.79 \times 10^{-3} \text{E}^{1/2}/\phi)$ and its functions namely, $v(y)$ and $s(y)$. He carried out numerical calculations using a spreadsheet by varying $y$. The values of $y$ and, in turn, $\Gamma$ were then tabulated for various log $J$ and $\phi$. Since $\phi$ and $E$ are not known at the emission site, the value of $\Gamma$ can be estimated approximately. The range of $\Gamma$ is chosen by experimental conditions considering plausible values of $\phi$ and $J$ to estimate the emission area from equation (24).

Applying this approach to the present case of copper electrodes (cross section $400 \mu m \times 200 \text{ nm}$, gap $\sim 100 \text{ nm}$) and taking $\phi = 4 \text{ eV}$, lower bound of $\Gamma$ turns out to be 564 for current density $\sim 3.2 \times 10^{13} \text{A m}^{-2}$. With values

### Table 1.

| Voltage range (V) | $\phi$ (eV) | $\alpha_{\text{eff}}$ ($\mu^2$) | $\beta$ | $E \times 10^{-9}$ (V m$^{-1}$) | $\Delta \phi$ (eV) | $\phi + \Delta \phi$ (eV) |
|------------------|-------------|-----------------------------|---------|-----------------------------|------------------|-----------------|
| 12–17.5          | 0.50        | 44.3                        | 14.2    | 1.7                         | 1.57             | 2.07            |
|                  | 1.00        | 11.1                        | 40.3    | 4.8                         | 2.64             | 3.64            |
|                  | 1.37        | 5.9                         | 64.3    | 7.7                         | 3.33             | 4.70            |
|                  | **1.40**    | **5.6**                     | **66.8**| **8.0**                     | **3.39**         | **4.80**        |
|                  | 1.45        | 5.4                         | 70.4    | 8.4                         | 3.48             | 4.93            |
| 18.0–20.9        | 0.50        | 533.4                       | 6.8     | 1.2                         | 1.32             | 1.82            |
|                  | 1.00        | 133.3                       | 19.1    | 3.4                         | 2.22             | 3.22            |
|                  | 1.35        | 73.2                        | 30.0    | 5.4                         | 2.79             | 4.14            |
|                  | 1.58        | 33.8                        | 37.8    | 6.8                         | 3.13             | 4.70            |
|                  | **1.62**    | **52.3**                    | **39.3**| **7.1**                     | **3.19**         | **4.80**        |
|                  | 1.65        | 50.1                        | 40.6    | 7.3                         | 3.24             | 4.89            |

Figure 5. Scanning electron micrographs of gold planar electrodes (a1) having cross-section $400 \mu m \times 200 \text{ nm}$ with a gap of $\sim 80 \text{ nm}$ and (a2) showing rough edges/protrusions from a small portion, (b1) Current–voltage ($I-V$) characteristics of the gold planar electrodes with a gap of $\sim 80 \text{ nm}$ with inset showing the ln($I$) versus ln($V$) plot, exhibiting $V^{1/2}$-dependence and the $V^{3/2}$-law regime, and (b2) ln($I/V^2$) versus $1/V$ plot depicting a minimum and a curve (instead of straight line) with negative slope above $\sim 22 \text{ V}$; The non-linear portion is fitted with three straight lines in the voltage range of 30–34 V, 37–43 V and 45–53 V.
of ‘A’ and ‘B’ obtained from the ln(I/V^2) versus 1/V plot (figure 4), equation (24) gives effective emission area \( \alpha_{\text{eff}} \) as 1.92 \times 10^{-21} m^2. This is too small and not of physical significance.

Singh and Kumar [44] revisited this issue and expressed the Fowler-Nordheim (F-N) field emission current (I) at a voltage (V) by equation (17) with A approximated as

\[
A = [\alpha_{\text{eff}} a]^{1/1.1}\phi(\beta/d)^2
\]

and B = \{0.95 (d/\beta) \phi^{1/2}\} as before by equation (19). The constant a is 1.54 \times 10^{-6} A(eV)^{-1} V^{-2}, b is 6.83 \times 10^{9} (eV)^{-3/2} Vm^{-1}, \phi is now the apparent work function, \( \alpha_{\text{eff}} \) is the effective area, ‘d’ is the inter-electrode separation, and \( \beta \) is the field enhancement factor. Thus, the ln(I/V^2) versus (1/V) plot shows a straight line with negative slope of magnitude (B) and intercept at the ordinate \((\ln A)\). A procedure was suggested to determine the emission parameters like \( \alpha_{\text{eff}} \) and \( \phi \) using the relation

\[
\alpha_{\text{eff}} \phi^2 = 1.7 \times 10^{-14}(\text{Slope})^2 \{\exp(\text{intercept})\}
\]

\( \ln(I/V^2) \) versus 1/V plot of copper planar electrode structure presented in figure 2(b) corresponds to a straight line with negative slope above 8.3 V. This straight line is split into two covering voltage ranges 12.0–17.5 V and 18.0–20.9 V with slopes (magnitude B = 16.1 and 33.9) and intercepts on the y-axis (ln A) = −17.5 and −16.5, respectively (inset of figures 2(b), (b1)). Equation (26) gives the values of \( \alpha_{\text{eff}} \phi^2 \) product as 11.3 and 136.5 Å^2, respectively. With each product, the possible combination of \( \alpha_{\text{eff}} \) and \( \phi \) can be determined (table 1). Now taking \( d = 100 \) nm, the value of \( \beta \) can be deduced from equation (19). The local electric field \((E)\) is equal to \[(\beta(V/d))\] at the onset bias \((V)\) of 12 and 18 V for straight line 1 and 2 respectively. The Schottky barrier lowering is expressed as \( \Delta \phi = [3.79 \times 10^{-5} \text{ (eV)}^{1/2}] \) and the work function of the electrode material is given by \((\phi + \Delta \phi)\). The emission parameters so deduced are summarized in table 1. Accordingly, values are (i) the apparent work function 1.40 eV and 1.62 eV; (ii) the effective emission area 5.6 Å^2 and 52.3 Å^2; and (iii) the enhancement factor \((\beta)\) 66.8 and 39.3 for the voltage range of 12–17.5 and 18–20.9 V, respectively for the bulk work function of copper as 4.8 eV. The increase in \( \alpha_{\text{eff}} \) is caused by disappearance of protrusions due to local heating as indicated with decrease in enhancement factor.

Figure 5(a1) shows a typical scanning electron micrograph (in the SE mode) of a gold diode having a cross-section of 400 \( \mu \)m \times 200 nm and inter-electrode separation of \( \sim 80 \) nm. Its current-voltage (I-V) characteristics measured in vacuum \((\sim 10^{-6} \text{ mbar})\) reveal \( V^{1/2} \)-dependence and \( V^{3/2} \) Child-Langmuir (CL) law in the voltage range 11–16 V and 18–22 V, respectively (figure 5(b1)). The \( \ln(I/V^2) \) versus 1/V plot contains a minimum and a curve (instead of a straight line) with negative slope above 22 V, suggesting transition to the Fowler-Nordheim field emission. The non-linear portion of continuously varying negative slope can be split into three straight lines with each having a slope B and an intercept ln A on the ordinate such that B = 33.4, ln A = −21.4; B = 67.9, ln A = −20.4; and B = 119.2, ln A = −19.2 for the voltage ranges 30–34 V, 37–43 V and 45–53 V, respectively.

### Table 2. The effective emission area \( (\alpha_{\text{eff}}, \phi) \), field enhancement factor \((\beta)\), local field strength \((E)\), Schottky barrier lowering \((\Delta \phi)\) and work function \((\phi + \Delta \phi)\) for gold electrodes corresponding to the straight lines 1, 2 and 3 of figure 5(b2).

| Voltage range (V) | \( \phi \) (eV) | \( \alpha_{\text{eff}}(\text{Å}^2) \) | \( \beta \) | \( E \times 10^{-15}\text{(Vm}^{-1}) \) | \( \Delta \phi \) (eV) | \( \phi + \Delta \phi \) (eV) |
|-------------------|----------------|------------------|-----|------------------|----------------|------------------|
| 30–34             | 0.50           | 3.9              | 5.5 | 2.1              | 1.72           | 2.22             |
|                   | 1.00           | 1.0              | 15.5 | 5.8          | 2.89           | 3.89             |
|                   | 1.20           | 0.7              | 20.4 | 7.7          | 3.32           | 4.52             |
|                   | 1.30           | 0.6              | 23.0 | 8.6          | 3.52           | 4.82             |
|                   | 1.36           | 0.5              | 24.6 | 9.2          | 3.64           | 5.00             |
|                   | 1.39           | 0.5              | 25.5 | 9.6          | 3.71           | 5.10             |
|                   | 1.45           | 0.5              | 27.1 | 10.2         | 3.82           | 5.27             |
| 37–43             | 0.50           | 44.3             | 2.7 | 1.3          | 1.34           | 1.84             |
|                   | 1.00           | 11.1             | 7.6 | 3.5          | 2.25           | 3.25             |
|                   | 1.50           | 4.9              | 14.0 | 6.5          | 3.05           | 4.55             |
|                   | 1.60           | 4.3              | 15.5 | 7.2          | 3.21           | 4.81             |
|                   | 1.68           | 3.9              | 16.6 | 7.7          | 3.32           | 5.00             |
|                   | 1.72           | 3.7              | 17.2 | 7.9          | 3.38           | 5.10             |
|                   | 1.75           | 3.6              | 17.7 | 8.2          | 3.43           | 5.18             |
| 45–53             | 0.50           | 453.6            | 1.5 | 0.87         | 1.12           | 1.62             |
|                   | 1.00           | 113.4            | 4.4 | 2.45         | 1.88           | 2.88             |
|                   | 1.50           | 30.4             | 8.0  | 4.50         | 2.54           | 4.04             |
|                   | 1.80           | 35.0             | 10.5 | 5.92         | 2.91           | 4.71             |
|                   | 1.90           | 31.4             | 11.4 | 6.42         | 3.04           | 4.94             |
|                   | 1.93           | 30.5             | 11.7 | 6.56         | 3.07           | 5.00             |
|                   | 1.97           | 29.1             | 12.1 | 6.79         | 3.12           | 5.10             |
|                   | 2.00           | 28.4             | 12.3 | 6.93         | 3.15           | 5.15             |
These results yield the emission parameters as listed in table 2. For determining the local field $E = \beta V/d$, $d$ is taken as 80 nm and the voltage as 30 V, 37 V and 45 V for the straight lines 1, 2 and 3, respectively (figure 5(b2)). For gold, the bulk work function is 5.1 eV but reduces substantially due to Schottky barrier-lowering (induced by high fields $\sim 10^9$ V m$^{-1}$) and protrusions present (figure 5(a2)). A progressive rise in the emission area results due to smoothening of protrusions by local heating with current at increasing bias. These findings on copper and gold planar diodes fabricated by FIB milling of thin films amply demonstrate (i) significant decrease in the work function and (ii) an exceedingly small area of the electrode contribution to the field emission. The main reason is the local enhancement of the electric field due to protrusions and decrease in the work function of the emitter caused by pronounced lowering of the Schottky barrier at high local electric fields.

To ascertain the elemental composition, x-ray analysis of planar electrodes assembly was undertaken using an energy dispersive spectrometer. Besides, elemental distribution maps were observed in each case. Accordingly, metal was found distributed outside the trenched (or gap) region only. x-ray line scans of a typical platinum planar electrode assembly (gap $\sim 200$ nm) fabricated over the SiO$_2$ substrate with FIB and (b) elemental distribution map indicating presence of platinum outside the milled trench (gap) region only.

Figure 6. (a) X-ray line scans of various elements (C, O, Ga, Si, and Pt) across a typical platinum planar electrode assembly (gap $\sim 200$ nm) fabricated over the SiO$_2$ substrate with FIB and (b) elemental distribution map indicating presence of platinum outside the milled trench (gap) region only.

The fabricated copper and gold electrodes discussed here are of the same thickness (i.e., 200 nm). The difficulty arises when the current density ($J$) is regarded as emission occurring from the entire cathode geometrical cross-sectional area. The approach gives erroneous values of ($J$) and causes problem in accounting them vis-à-vis current equations developed for bulk, even though the latter described the voltage dependence reliably for the nanoscale electrodes too. This is what precisely demonstrated here. The actual current density is determined by the preferential emission sites (i.e., protrusions present in the fabricated electrodes due to lack of smoothness at the nanoscale) rather than the geometrical area. Also, the number of protrusions and their
geometry depends on the fabrication process itself. The local field becomes high at such sites and lead to preferential emission from an infinitesimal small area. This feature is clearly revealed in figure 4(a) using the I-V data. But the emission area increases with rise in applied voltage caused by local burn out (or removal) of protrusions - leading to flattening of electrode tip at high fields (tables 1 and 2). The details of analysis procedure are given in section 3.1.1.

The I-V characteristics of yet another copper planar electrode assembly of cross-section 400 μm × 1000 nm with a gap of ~200 nm is shown in figure 7(a). They also reveal a sharp increase in current (or initiation of F-N tunnelling) across the electrodes above 175 V with a minimum and a straight line having negative slope above ~175 V in the inset.

Figure 7. (a) Current-voltage (I-V) characteristics of the copper planar electrodes of cross section 400 μm × 1000 nm with a gap of ~200 nm and (b) Ln(I/V^2) versus 1/V plot depicting a minimum and a straight line with negative slope above ~175 V in the inset.

Table 3. The effective emission area (α_eff), apparent work function (ϕ), field enhancement factor (β), local field strength (E), Schottky barrier lowering (Δϕ) and work function (ϕ + Δϕ) for copper electrodes corresponding to a straight line with negative slope in inset of figure 7(b).

| Voltage range (V) | ϕ (eV) | α_eff (Å^2) | β | E × 10^{-9} (V m^{-1}) | Δϕ (eV) | ϕ + Δϕ (eV) |
|------------------|--------|-------------|---|-------------------------|---------|-------------|
| 175-210          | 1.00   | 8.07 × 10^8 | 0.5 | 39.7                   | 0.75    | 1.75        |
| 2.00             | 2.02 × 10^8 | 1.3 | 11.2 | 1.27                   | 3.27    |             |
| 3.0              | 8.96 × 10^3 | 2.4 | 2.1 | 1.72                   | 4.72    |             |
| 3.05             | 8.67 × 10^5 | 2.4 | 2.1 | 1.74                   | 4.79    |             |
| 3.10             | 8.39 × 10^5 | 2.5 | 2.2 | 1.76                   | 4.86    |             |
are summarized in Table 3 along with the values of $(\beta)$ and local electric field (E) at $d = 200$ nm and onset bias (V) = 175 V. Accordingly, the apparent work function, the effective emission area the enhancement factor $(\beta)$ are 3.05 eV, $8.67 \times 10^{5} \text{Å}^2$ and 2.4, respectively corresponding to bulk copper (work function $\sim$4.8 eV). Notice that the emission area is much larger than listed in Table 1 but for higher applied voltages (above 175 V). This happened presumably due to electrodes having larger gap (200 nm) and higher thickness as the smoothness is attained in wide area above 175 V (a relatively higher bias).

4. Conclusions

- A 30 keV focused gallium ion beam (size 7–10 nm) combined with a scanning electron microscope in a FEI Nova Nanolab 600 can be successfully employed for controlled milling of thin metal films (thickness say, 200 nm) to fabricate planar electrode with gap of $\sim$100 nm.

- The current–voltage characteristics of copper planar electrodes structure (cross-section 400 μm × 200 nm, gap $\sim$100 nm) follow Child-Langmuir’s $V^{1/2}$ law at low bias (3.2–8.0 V) and Fowler–Nordheim (F-N) field emission above 8.3 V. However, the current density values conform to an exceedingly small emission area of $\sim$8 $\times$ 10$^{-17}$ m$^2$ compared to geometrical area of $8 \times 10^{-11}$ m$^2$.

- Different approaches adopted for the estimation of the emitter area in case of field emission from the Fowler-Nordheim [ln(I/V$^2$) versus 1/V] plot give exceeding small and unrealistic values of no physical significance, e.g., 10$^{-22}$–10$^{-20}$ m$^2$ (or 10$^{-22}$–1 Å$^2$) against the actual value of 8 $\times$ 10$^{-11}$ m$^2$ for copper planar electrodes (cross section being 400 μm × 200 nm).

- A simple method developed using the negative slope and intercept of ln (I/V$^2$) versus 1/V plot enables extraction of realistic emission area ($\alpha_{eff}$) and the apparent work function ($\phi$) of the emitter. Typical values of $\alpha_{eff}$ and $\phi$ being (i) 5.23 Å$^2$ and 1.62 eV, respectively for copper and (ii) 29.1 Å$^2$ and 1.97 eV, respectively for gold planar electrode.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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