Some semiclassical structure constants for $\text{AdS}_4 \times \text{CP}^3$

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Abstract: We compute structure constants in three-point functions of three string states in $\text{AdS}_4 \times \text{CP}^3$ in the framework of the semiclassical approach. We consider HHL correlation functions where two of the states are “heavy” string states of finite-size giant magnons carrying one or two angular momenta and the other one corresponds to such “light” states as dilaton operators with non-zero momentum, primary scalar operators, and singlet scalar operators with higher string levels.

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1 Introduction

The AdS/CFT duality between string/M theories on Anti-de Sitter (AdS) background and conformal field theories (CFTs) on its boundary has been a most productive research direction ever since it was proposed [1–3]. As a strong-weak coupling duality, integrability has played crucial roles in non-perturbative computations [4]. It has been applied to compute conformal dimensions of the CFTs and energies of corresponding string states. A natural next challenge is to utilize integrability to compute structure constants which determine three-point functions.

A promising recent progress is so-called hexagon amplitude approach [5]. The structure constants are given by sums of hexagon amplitudes, which can be determined exactly in all orders of ’t Hooft coupling constant $\lambda$. This approach has proved effective in the weak coupling limit [6–8]. However, technical difficulties such as summing up all intermediate states, finite-size effects, etc. become substantial in the strong coupling limit [9].

For two-heavy and one-light operators in the semiclassical limit $\lambda \gg 1$, the “HHL” three-point functions can be obtained from explicit evaluation of light vertex operator with the heavy string configurations [10–12]. In spite of limited applicability, this method
is useful to obtain structure constants when the heavy operators have large but finite $J \gg \sqrt{\lambda}$ values. The resulting HHL functions show exponential corrections $e^{-J/\sqrt{\lambda}}$, which can be related to exact $S$-matrix, hence the integrability [13].

Type IIA string theory on $AdS_4 \times CP^3$ background is dual to $\mathcal{N} = 6$ super Chern-Simons theory in three space-time dimensions, known as ABJM theory [14]. Classical integrability [15, 16] and giant magnon solutions have been studied in [17]–[20]. The HHL 3-point functions have been computed for various string states in $AdS_4 \times CP^3$ [21]–[25]. In this paper, we will focus on finite-size effects of some normalized structure constants in $AdS_4 \times CP^3$ in semiclassical limit where the heavy string states are finite-size giant magnons. We also consider various different light string states, such as dilaton operators with non-zero momentum, primary scalar operators, and singlet scalar operators on higher string levels.

The paper is organized as follows. In section 2, we introduce preliminary contents along with various giant magnons on $CP^3$. We present the HHL functions of two giant magnons with dilaton operator with non-zero momentum in section 3, with primary scalar operators in section 4, and with singlet scalar operators on higher string levels in section 5. We conclude the paper in section 6.

2 Preliminaries

2.1 Structure constants

It is known that correlation functions of CFTs can be determined in principle in terms of the basic conformal data $\{\Delta_i, C_{ijk}\}$, where $\Delta_i$ are the conformal dimensions defined by two-point normalized correlation functions

$$\langle O_i(x_1)O_j(x_2) \rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta_i}}$$

and $C_{ijk}$ are structure constants of three-point correlation functions

$$\langle O_i(x_1)O_j(x_2)O_k(x_3) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1 - x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}}.$$

The HHL three-point functions of two heavy operators and a light operator can be approximated by a supergravity vertex operator evaluated at the heavy classical string configuration [12]:

$$\langle V_H(x_1)V_H(x_2)V_L(x_3) \rangle = V_L(x_3)_{\text{classical}}.$$

For $|x_1| = |x_2| = 1, x_3 = 0$, the correlation function reduces to

$$\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(0) \rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the structure constants can be given by

$$C_{123} = c_\Delta V_L(0)_{\text{classical}},$$

where $c_\Delta$ is the normalization constant of the corresponding light vertex operator.
2.2 The string Lagrangian and Virasoro constraints

String theory moving on certain background can be described by the Polyakov action

\[ S^P = -T \int d^2 \xi \sqrt{-\gamma} \gamma^{mn} G_{mn}, \]

\[ G_{mn} = g_{MN} \partial_m X^M \partial_n X^N, \]  \hfill (2.2)

where \( T \) is the string tension. We choose to work in conformal gauge \( \gamma_{mn} = \eta_{mn} = \text{diag}(-1, 1) \), in which the Lagrangian and the Virasoro constraints take the form

\[ \mathcal{L}_s = T(G_{00} - G_{11}), \]  \hfill (2.3)

\[ G_{00} + G_{11} = 0, \]  \hfill (2.4)

\[ G_{01} = 0. \]  \hfill (2.5)

The background metric \( g_{MN} \) for \( AdS_4 \times CP^3 \) is given by

\[ ds^2 = g_{MN} dx^M dx^N = R^2 (ds^2_{AdS^4} + ds^2_{CP^3}), \]

where \( R \) is related to the string tension \( T \) and the ’t Hooft coupling constant \( (\alpha' = 1) \) by

\[ TR^2 = \sqrt{2\lambda}. \]

There are also dilaton and RR-forms, which do not influence the motion of the classical strings. Further on, we set \( R = 1 \).

The metric of \( CP^3 \) space can be written as

\[ ds^2_{CP^3} = d\theta^2 + \sin^2 \theta \left( \frac{1}{2} \cos \vartheta_1 d\varphi_1 - \frac{1}{2} \cos \vartheta_2 d\varphi_2 + d\varphi_3 \right)^2 \]

\[ + \cos^2 \frac{\theta}{2} \left( d\vartheta_1^2 + \sin^2 \vartheta_1 d\varphi_1^2 \right) + \sin^2 \frac{\theta}{2} \left( d\vartheta_2^2 + \sin^2 \vartheta_2 d\varphi_2^2 \right), \]  \hfill (2.6)

where \( \theta \in [0, \pi], \vartheta_1, \vartheta_2 \in [0, \pi], \varphi_1, \varphi_2 \in [0, 2\pi], \varphi_3 \in [0, 4\pi] \). The angular coordinates in (2.6) can be expressed also by the following complex coordinates

\[ z_1 = \cos \frac{\theta}{2} \cos \frac{\vartheta_1}{2} \exp \left[ \frac{i}{2} (\varphi_3 + \varphi_1) \right] = r_1 \exp (i\varphi_1), \]  \hfill (2.7)

\[ z_2 = \sin \frac{\theta}{2} \cos \frac{\vartheta_2}{2} \exp \left[ -\frac{i}{2} (\varphi_3 - \varphi_2) \right] = r_2 \exp (i\varphi_2), \]

\[ z_3 = \cos \frac{\theta}{2} \sin \frac{\vartheta_1}{2} \exp \left[ \frac{i}{2} (\varphi_3 - \varphi_1) \right] = r_3 \exp (i\varphi_3), \]

\[ z_4 = \sin \frac{\theta}{2} \sin \frac{\vartheta_2}{2} \exp \left[ -\frac{i}{2} (\varphi_3 + \varphi_2) \right] = r_4 \exp (i\varphi_4), \]

\[ \sum_{a=1}^{4} r_a^2 = 1, \quad \sum_{a=1}^{4} r_a^2 \partial_m \phi_a = 0. \]
2.3 Giant magnons

The giant magnon solutions in $CP^3$ can be found by the Neumann-Rosochatius (NR) integrable system with the following ansatz for the string embedding [18]

$$t(\tau, \sigma) = \kappa \tau, \quad r_a(\tau, \sigma) = r_a(\xi), \quad \phi_a(\tau, \sigma) = \omega_a \tau + f_a(\xi), \quad (2.8)$$

$$\xi = \sigma - \nu \tau, \quad \kappa, \omega_a, \nu = \text{constants}.$$  

2.3.1 $CP^1$ giant magnon

Let us start with the giant magnon living in the $R_t \times CP^1$ subspace. Such subspace can be obtained by setting $\theta = \vartheta_2 = \varphi_2 = \varphi_3 = 0$. What remains is [1]

$$ds^2 = -dt^2 + d\vartheta_1^2 + \sin^2 \vartheta_1 d\varphi_1^2. \quad (2.9)$$

Then (2.7) becomes

$$z_1 = \cos \frac{\vartheta_1}{2} \exp \left( \frac{i}{2} \varphi_1 \right) = r_1 \exp (i \phi_1), \quad (2.10)$$
$$z_2 = 0,$$
$$z_3 = \sin \frac{\vartheta_1}{2} \exp \left( -\frac{i}{2} \varphi_1 \right) = r_3 \exp (-i \phi_1),$$
$$z_4 = 0.$$

For this case, the induced metric on the string worldsheet is

$$G_{00} = -(\partial_0 t)^2 + (\partial_0 \vartheta_1)^2 + \sin^2 \vartheta_1 (\partial_0 \varphi_1)^2,$$
$$G_{11} = -(\partial_1 t)^2 + (\partial_1 \vartheta_1)^2 + \sin^2 \vartheta_1 (\partial_1 \varphi_1)^2,$$
$$G_{01} = -\partial_0 \vartheta_0 \partial_0 t + \partial_0 \vartheta_0 \partial_1 \vartheta_1 + \sin^2 \vartheta_1 \partial_0 \varphi_1 \partial_1 \varphi_1.$$

By using (2.8) and (2.9) in (2.3), one can write down the string Lagrangian in the following form (prime is used for $d/d\xi$)

$$\mathcal{L}_s = -T(1 - v^2) \left[ (\dot{\varphi}_1')^2 + \sin^2 \vartheta_1 \left( \left( \frac{\dot{f}_1'}{1 - v^2} \right)^2 - \frac{\omega_1^2}{(1 - v^2)^2} \right) \right], \quad (2.11)$$

from which the first integral for $f_1$ becomes

$$\dot{f}_1' = \frac{1}{1 - v^2} \left( \frac{C_1}{\sin^2 \vartheta_1} - \frac{v \omega_1}{1 - v^2} \right) \quad (2.12)$$

with an integration constant $C_1$.

The first Virasoro constraint (2.4) along with (2.12) becomes

$$(\dot{\varphi}_1')^2 = \frac{1}{(1 - v^2)^2} \left[ (1 + v^2)\kappa^2 - \frac{C_1}{\sin^2 \vartheta_1} - \omega_1^2 \sin^2 \vartheta_1 \right], \quad (2.13)$$

while the second constraint (2.5) determines the constant $C_1 = \frac{v \omega_1}{\kappa^2}$. We will further restrict $\omega_1 = 1$ since we can choose an appropriate unit of $\tau$. In terms of $\chi \equiv \cos^2 \vartheta_1$, (2.13) can be written as

$$\chi' = \frac{2}{1 - v^2} \sqrt{\chi(\chi_p - \chi)(\chi - \chi_m)}, \quad \text{with} \quad \chi_p = 1 - v^2 \kappa^2, \quad \chi_m = 1 - \kappa^2. \quad (2.14)$$

1This choice of $CP^1 = S^2$ subspace corresponds to “A”-type giant magnon in the ABJM theory [28]. The “B”-type magnon lives in another $CP^1$ subspace obtained by $\theta = \pi$. Here we consider only A-type.
2.3.2 $R^3P$ giant magnon

The $R^3P$ giant magnon lives in the $R_1 \times R^3P$ subspace, which can be obtained from (2.6) by setting $\vartheta_1 = \vartheta_2 = \frac{\pi}{2}$, $\varphi_3 = 0$. The resulting metric is

$$ds^2 = -dt^2 + d\theta^2 + \frac{\cos^2 \theta}{2} d\varphi_1^2 + \frac{\sin^2 \theta}{2} d\varphi_2^2.$$ 

Correspondingly, the coordinates (2.7) reduce to

$$z_1 = \frac{1}{\sqrt{2}} \cos \frac{\theta}{2} \exp \left( i \frac{\varphi_1}{2} \right) = r_1 \exp (i\varphi_1),$$

$$z_2 = \frac{1}{\sqrt{2}} \sin \frac{\theta}{2} \exp \left( i \frac{\varphi_2}{2} \right) = r_2 \exp (i\varphi_2),$$

$$z_3 = \frac{1}{\sqrt{2}} \cos \frac{\theta}{2} \exp \left( -i \frac{\varphi_1}{2} \right) = r_1 \exp (-i\varphi_1),$$

$$z_4 = \frac{1}{\sqrt{2}} \sin \frac{\theta}{2} \exp \left( -i \frac{\varphi_2}{2} \right) = r_2 \exp (-i\varphi_2).$$

For the case at hand, the metric induced on the string worldsheet is given by

$$G_{00} = -(\partial_0 t)^2 + (\partial_0 \theta)^2 + \cos^2 \frac{\theta}{2} (\partial_0 \varphi_1)^2 + \sin^2 \frac{\theta}{2} (\partial_0 \varphi_2)^2,$$

$$G_{11} = -(\partial_1 t)^2 + (\partial_1 \theta)^2 + \cos^2 \frac{\theta}{2} (\partial_1 \varphi_1)^2 + \sin^2 \frac{\theta}{2} (\partial_1 \varphi_2)^2,$$

$$G_{01} = -\partial_0 t \partial_1 t + \partial_0 \theta \partial_1 \theta + \cos^2 \frac{\theta}{2} \partial_0 \varphi_1 \partial_1 \varphi_1 + \sin^2 \frac{\theta}{2} \partial_0 \varphi_2 \partial_1 \varphi_2.$$

The string Lagrangian in this case becomes

$$L_s = -T \left[ (1 - v^2)(\dot{\theta})^2 - \cos^2 \frac{\theta}{2} \left( (v f_1' - \omega_1)^2 - f_1^2 \right)^2 - \sin^2 \frac{\theta}{2} \left( (v f_2' - \omega_2)^2 - f_2^2 \right)^2 \right],$$

from which the first integrals for $f_1$ and $f_2$ become

$$f_1' = \frac{1}{1 - v^2} \left( \frac{C_1}{\cos^2 \frac{\theta}{2}} - v \omega_1 \right), \quad f_2' = \frac{1}{1 - v^2} \left( \frac{C_2}{\sin^2 \frac{\theta}{2}} - v \omega_2 \right)$$

with integration constants $C_1$, $C_2$. Since the $R^3P$ giant magnon should be well-defined at $\theta = \pi$, we impose an extra condition $C_1 = 0$.

The two Virasoro constraints (2.4) and (2.5) are combined along with (2.17) to give a parametric relation

$$v \kappa^2 = C_2 \omega_2,$$

and the first integral for $\theta$

$$\left( \frac{d\theta}{dt} \right)^2 = \frac{1}{(1 - v^2)^2} \left[ (1 + v^2) \kappa^2 - \omega_1^2 - \frac{C_2^2}{\sin^2 \frac{\theta}{2}} + (\omega_1^2 - \omega_2^2) \sin^2 \frac{\theta}{2} \right].$$
This time we fix $\omega_2 \equiv 1$ and define

$$\chi \equiv \cos^2 \frac{\theta}{2}, \quad u \equiv \frac{\omega_1}{\omega_2},$$

(2.20)

to rewrite this equation as

$$\chi' = \frac{\sqrt{1-u^2}}{1-v^2} \sqrt{\chi(\chi_p - \chi)(\chi - \chi_m)},$$

(2.21)

where $\chi_p$ and $\chi_m$ satisfy the equalities:

$$\chi_p + \chi_m = \frac{2 - (1 + v^2)\kappa^2 - u^2}{1 - u^2}, \quad \chi_p \chi_m = \frac{(1 - \kappa^2)(1 - v^2\kappa^2)}{1 - u^2}.$$  

(2.22)

3 HHL of dilaton operator with non-zero momentum

The vertex for the dilaton operator with non-zero momentum $j$, originally defined for $AdS_5 \times S^5$ in [12], is modified in the $AdS_4 \times CP^3$ case to

$$V_{ab}^d(j) = \left(\frac{x^mx_m + z^2}{z}\right)^{-\Delta_d} (z_0z_0)^j \left[ z^{-2} \left( \partial_+ x_m \partial_- x_m + \partial_+ z \partial_- z \right) + \partial_+ X_k \partial_- X^k \right] \quad (3.1)$$

$$x^m x_m = -x_0^2 + x_i x_i, \quad i = 1, 2,$$

where we denote the scaling dimension $\Delta_d = 4 + j$ and $(x^m, z)$ as the Poincaré coordinates on $AdS_4$. The coordinates on $CP^3$ are represented by angular coordinates $X_k$ or equivalently by the complex coordinates $z_a$ defined in (2.7). The choice of the indices $(a, b)$ determines the direction of the momentum in the $CP^3$ space.

The AdS part of the giant magnon solution is given by (after Euclidean rotation, $i\tau = \tau_e$, where $\tau$ is the worldsheet time)

$$x_{0e} = \tanh(\kappa \tau_e), \quad x_i = 0, \quad z = \frac{1}{\cosh(\kappa \tau_e)}. \quad (3.2)$$

Replacing (3.2) into (3.1), one finds

$$V_{ab}^d(j) = (\cosh \kappa \tau_e)^{-\Delta_d} (z_0z_0)^j \left( \kappa^2 + \partial_+ X_k \partial_- X^k \right). \quad (3.3)$$

3.1 With the $R_t \times CP^1$ giant magnons

From (2.10) it is clear that non-vanishing HHL is possible only with the vertex $V_{13}^d(j)$. Evaluating the Lagrangian on the giant magnon state,

$$\partial_+ X_k \partial_- X^k = \frac{2}{1-v^2} \left[ \chi - 1 + \frac{1}{2}(1 + v^2)\kappa^2 \right],$$

(3.4)

one finds

$$V_{13}^d(j) = \frac{(1 - \chi)^{j/2} \left( \kappa^2 + \chi - 1 \right)}{2^{j-1}(1 - v^2) (\cosh \kappa \tau_e)^{\Delta_d}}.$$  

(3.5)
The normalized structure constant can be obtained by integrating the vertex over the string worldsheet.

\[
C_{13}^{CP^1,d}(j) = \int_{-\infty}^{\infty} d\tau \int_{-L}^{L} d\sigma V_{13}^{d}(j)
\]

\[
= c_{\Delta} \frac{\sqrt{\pi}}{2^{j+1}(1-v^2)\kappa} \frac{\Gamma(\frac{\Delta+1}{2})}{\Gamma(\frac{\Delta+1}{2})} \int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi^j} (1-\chi)^{j/2} (\kappa^2 + \chi - 1),
\]

where we replaced the integration over \(\sigma\) with integration over \(\chi\) in the following way

\[
\int_{-L}^{L} d\sigma = 2 \int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi'},
\]

using eq. (2.14) for \(\chi'\). The parameter \(L\) is introduced here in order to take into account the giant magnons in the finite-size worldsheet volume.

The final expression of the integral in (3.6) becomes

\[
C_{13}^{CP^1,d}(j) = c_{\Delta} \frac{\Gamma(\frac{\Delta}{2})}{\Gamma(\frac{\Delta+1}{2})} \frac{\pi^{3/2}}{2^{j+1}\kappa} \chi_p^{1/2} (1-\chi_p)^{j/2} \times
\]

\[
\times \left[ \chi_p F_1 \left( \frac{1}{2}; -1; -\frac{j}{2}; 1; 1 - \epsilon, \frac{\chi_p(\epsilon - 1)}{1 - \chi_p} \right) - (1 - \kappa^2) F_1 \left( \frac{1}{2}; \frac{1}{2}; -\frac{j}{2}; 1; 1 - \epsilon, \frac{\chi_p(\epsilon - 1)}{1 - \chi_p} \right) \right],
\]

where \(F_1(a; b_1, b_2; c; z_1, z_2)\) is a hypergeometric function of two variables (Appell \(F_1\)) and

\[
\epsilon = \frac{\chi_m}{\chi_p}.
\]

We used an integral representation for the \(F_1\) in (3.7) [32]

\[
F_1(a; b_1, b_2; c; z_1, z_2) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_{0}^{1} x^{a-1}(1-x)^{c-a-1}(1-z_1x)^{-b_1}(1-z_2x)^{-b_2} dx.
\]

The structure constants are given by the parameters \(\kappa, v\) which are eventually related to the conserved angular momentum \(J_1\) and the worldsheet momentum \(p\) (see (2.14), (3.8)) by

\[
J_1 = 2T \frac{\sqrt{1 - v^2}}{1 - v^2}\epsilon \left[ K(1 - \epsilon) - E(1 - \epsilon) \right],
\]

\[
p = 2v \frac{\sqrt{1 - v^2}}{1 - v^2} \epsilon \left[ \frac{1}{v^2} \Pi \left( 1 - \frac{1}{v^2}(1 - \epsilon) \right) - K(1 - \epsilon) \right].
\]

Here \(K\), \(E\) and \(\Pi\) are the complete elliptic integrals of the first, second, and third kinds, respectively.

### 3.1.1 Leading finite-size corrections

Since \(J_1\) and \(p\) define the heavy operators of the dual gauge theory, it is important to express the semiclassical structure constants \(C_{13}^{CP^1,d}(j)\) in terms of them. For given \(J_1\) and
p, one can solve numerically (3.9) and (3.10) to find corresponding values of \( \kappa, v \), which can be used to evaluate \( C_{13}^{CP^1,d}(j) \) from (3.6).

Explicit computations are possible for the case where \( J_1 \) is large but finite \( J_1 \gg T \), equivalently, \( \epsilon \ll 1 \). We start by rewriting (2.14) in the following form
\[
(1 + \epsilon)\chi_p = 2 - (1 + v^2)\kappa^2, \quad \epsilon\chi_p = (1 - \kappa^2)(1 - v^2\kappa^2).
\]
(3.11)

Next, we use the small \( \epsilon \)-expansions
\[
\chi_p = \chi_{p0} + (\chi_{p1} + \chi_{p2} \log \epsilon)\epsilon,
\]
(3.12)
\[
v = v_0 + (v_1 + v_2 \log \epsilon)\epsilon,
\]
\[
\kappa^2 = 1 + W_1\epsilon.
\]

Replacing (3.12) into (3.11), one finds relations
\[
\chi_{p0} = 1 - v_0^2, \quad \chi_{p1} = v_0(v_0 - v_0^3 - 2v_1), \quad \chi_{p2} = -2v_0v_2, \quad W_1 = -1 + v_0^2.
\]
(3.13)

The expressions for \( v_0, v_1, v_2 \) in terms of the worldsheet momentum \( p \) can be found from (3.9) and (3.10)
\[
v_0 = \cos \frac{p}{2},
\]
(3.14)
\[
v_1 = \frac{1}{4} \sin^2 \frac{p}{2} \cos \frac{p}{2} (1 - \log 16),
\]
\[
v_2 = \frac{1}{4} \sin^2 \frac{p}{2} \cos \frac{p}{2},
\]
along with the expansion parameter in terms of \( J_1 \) and \( p \)
\[
\epsilon = 16 \exp \left(-\frac{J_1}{T \sin \frac{p}{2}} - 2\right).
\]
(3.15)

The case of \( j = 0 \). Let us begin with the simplest case \( j = 0 \), i.e. dilaton with zero momentum, which is just the Lagrangian. The \( C_{13}^{CP^1,d}(j) \) in (3.7) simplifies to
\[
C_{13}^{CP^1,d}(0) = \frac{8\sqrt{\pi}c_d}{3\kappa\sqrt{\chi_p}} \left[ \chi_p E(1 - \epsilon) - (1 - \kappa^2)K(1 - \epsilon) \right].
\]
(3.16)

Replacing (3.14), (3.15) into the \( \epsilon \)-expansion of (3.16), we obtain (for dilaton with zero momentum \( \Delta_d = 4 \))
\[
C_{13}^{CP^1,d}(0) \approx \frac{8c_d}{3} \sin \frac{p}{2} \left[ 1 - 4 \sin \frac{p}{2} \left( \sin \frac{p}{2} + \frac{J_1}{T} \right) \exp \left(-\frac{J_1}{T \sin \frac{p}{2}} - 2\right) \right].
\]
(3.17)

This is exactly same result as [30].
The case of $j = \text{even integer}$. In this case, the third arguments $-j/2$ of the Appell $F_1$ functions in (3.7) are negative integers. With the help of Mathematica, we have found that these functions can be expressed in terms of the elliptic integrals, $K$ and $E$. (The $j = 0$ result analysed above belongs to this case too.) We list explicit functional relations for a few simple cases in the appendix.

Using small $\epsilon$-expansion of the elliptic integrals, one can find the leading finite-size corrections of the HHL for any even $j$ in principle. Here, we present explicit results for $j = 2$ ($\Delta_d = 6$) as an example:

$$
C_{13}^{CP_1,d}(2) = \frac{\sqrt{\pi} \epsilon d^d}{6 \kappa \sqrt{\pi} \epsilon d^d \Gamma \left( \frac{\Delta_d}{2} \right)} \left[ K(1-\epsilon)(3\kappa^2 + \chi_p^2\epsilon - 3) - \chi_p E(1-\epsilon)(3\kappa^2 + 2(\chi_p + \chi_p\epsilon - 3)) \right] 
\approx \frac{8\epsilon d^d}{45} \sin \frac{p}{2} \left[ 2 + \cos p \right. \\
+ \left( 2\cos p + 7\cos 2p - 9 - \frac{18J_1}{T} \left( \sin \frac{p}{2} + \frac{1}{3} \sin \frac{3p}{2} \right) \right) e^{-\frac{\kappa_j}{3} \sin \frac{p}{2} - 2} \right] .
$$

(3.18)

The case of $j = \text{odd integer}$. Since the Appell functions can not be written in terms of the elliptic integrals in this case, we express the $F_1$ in (3.7) using an infinite sum [31]

$$
F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \frac{(a)_k(b_2)_k}{(c)_k k!} \ 2F_1(a + k; b_1 + k; c + k; z_1) z_2^k, \ (a)_k = \frac{\Gamma(a + k)}{\Gamma(a)} .
$$

(3.19)

We can use small $\epsilon$-expansions for the hypergeometric $2F_1$ functions and resum afterward. The resulting structure constants (3.7) in the leading-order of $\epsilon$ are

$$
C_{13}^{CP_1,d}(j) \approx \frac{\sqrt{\pi} \epsilon d}{2^{j-1} \Gamma \left( \frac{\Delta_d}{2} \right)} \sin \frac{p}{2} \frac{\cos \frac{p}{2}}{2} \left[ 2F_1 \left( \frac{1}{2}; -j, \frac{3}{2}; -\tan^2 \frac{p}{2} \right) \right] + C \epsilon
$$

(3.20)

$$
C = \sec^j \frac{p}{2} - \frac{\pi^{3/2} \csc \frac{\pi j}{2}}{2 \Gamma \left( 1 - \frac{j}{2} \right) \Gamma \left( 1 + \frac{j}{2} \right)} - \frac{1}{2} \left( \gamma + \log 4 - \frac{J_1}{T} \csc \frac{p}{2} \right) \sec^j \frac{p}{2}
$$

(3.21)

$$
\left. - \frac{1}{4} \left[ 2F_1 \left( \frac{1}{2}; -j, \frac{3}{2}; -\tan^2 \frac{p}{2} \right) \sin \frac{p}{2} \left[ (1 + 3j) \sin \frac{p}{2} + (1 + j) \frac{J_1}{T} \right] \right] \\
+ \frac{\sqrt{\pi}}{4} \left[ 2F_{1reg}^{(0,1,0)} \left( \frac{1}{2}; -j, \frac{3}{2}; -\tan^2 \frac{p}{2} \right) + 2F_{1reg}^{(1,0,0)} \left( \frac{1}{2}; -j, \frac{1}{2}; -\tan^2 \frac{p}{2} \right) \right].
$$

Here $\gamma$ is the Euler’s constant and $2F_1(a; b; c; z)$ the Gauss hypergeometric function,

$$
2F_{1reg}(a; b; c; z) = \frac{1}{\Gamma(c)} \ 2F_1(a; b; c; z),
$$

$$
2F_{1reg}^{(0,1,0)}(a; b; c; z) = \left( \frac{\partial}{\partial b} \right) \ 2F_{1reg}(a; b; c; z),
$$

$$
2F_{1reg}^{(0,0,1)}(a; b; c; z) = \left( \frac{\partial}{\partial c} \right) \ 2F_{1reg}(a; b; c; z).
$$
3.2 With the $R_t \times RP^3$ giant magnons

From (2.15), all combinations $(a, b)$ in the dilaton vertex (3.1) are non-vanishing. However, we will focus on only $V_{13}^d(j)$ and $V_{24}^d(j)$ for which explicit expressions can be obtained. Working in the same way as the $CP^1$ case (see (3.4)), one derives

$$\partial_+ X_k \partial_- X^k = \frac{2(1-u^2)}{1-v^2} \left[ x - \frac{1}{2}(1+v^2)\kappa^2 \right],$$

which can be used to $V_{13}^d(j)$ and $V_{24}^d(j)$:

$$V_{13}^d(j) = \frac{2^{1-j}}{1-v^2} (\cosh \kappa \epsilon)^{-\Delta d} \chi^d \left[ (1-u^2)\chi - (1-\kappa^2) \right],$$

$$V_{24}^d(j) = \frac{2^{1-j}}{1-v^2} (\cosh \kappa \epsilon)^{-\Delta d} (1-\chi)^d \left[ (1-u^2)\chi - (1-\kappa^2) \right].$$

(3.22)

(3.23)

By integrating $V_{13}^d(j)$ and $V_{24}^d(j)$ over the string worldsheet coordinates, we derive the corresponding structure constants $C_{13}^{RP^3,d}$ again in terms of the $F_1$ and $2F_1$:

$$C_{13}^{RP^3,d}(j) = \frac{\epsilon^d \pi^{3/2} \sqrt{1-u^2}}{2^{j-1} \kappa} \frac{\Gamma \left( \frac{\Delta d}{2} \right)}{\Gamma \left( \frac{\Delta d+1}{2} \right)} x_p^{j+1/2} \times$$

$$\times \left[ 2F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon \right) - \frac{(1-\kappa^2)}{(1-u^2)\chi_p} 2F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon \right) \right],$$

(3.24)

$$C_{24}^{RP^3,d}(j) = \frac{\epsilon^d \pi^{3/2} \sqrt{1-u^2}}{2^{j-1} \kappa} \frac{\Gamma \left( \frac{\Delta d}{2} \right)}{\Gamma \left( \frac{\Delta d+1}{2} \right)} x_p^{j+1/2} (1-\chi_p)^j \times$$

$$\times \left[ F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon, \frac{\chi_p(\epsilon - 1)}{1-\chi_p} \right) - \frac{(1-\kappa^2)}{(1-u^2)\chi_p} F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon, \frac{\chi_p(\epsilon - 1)}{1-\chi_p} \right) \right],$$

(3.25)

where $\chi_p$ is given by (2.22).

The parameters $\kappa$, $u$, $v$ in (3.24), (3.25) are related to the conserved angular momenta $J_1$, $J_2$ and the worldsheet momentum $p$ along with (2.22) by

$$J_1 = \frac{T \sqrt{\chi_p}}{\sqrt{1-u^2}} \left[ 1 - \frac{u^2 \kappa^2}{\chi_p} K(1-\epsilon) - E(1-\epsilon) \right],$$

$$J_2 = \frac{T u \sqrt{\chi_p}}{\sqrt{1-u^2}} E(1-\epsilon),$$

$$p = \frac{2v}{\sqrt{(1-u^2)\chi_p}} \left[ \kappa^2 \frac{1}{1-\chi_p} \Pi \left( \frac{\chi_p(1-\epsilon)}{1-\chi_p}(1-\epsilon) \right) - K(1-\epsilon) \right].$$

For given $J_1$, $J_2$, $p$, one can find corresponding $\kappa$, $u$, $v$, with which the structure constants can be evaluated.

A few comments are in order. The structure constants (3.24) and (3.25) correspond to finite-size dyonic giant magnons living in the $RP^3$ subspace of $CP^3$. The case of finite-size giant magnons living in the $RP^2$ subspace can be obtained by setting $u = 0$. For the
infinite size case, one can take a limit of $\kappa = 1$ and $\epsilon = 0$. The above results reduce to
the zero momentum dilaton with $j = 0$. Small $\epsilon$-expansions for $RP^3$ are straightforward
for these cases since they are either in terms of the hypergeometric $\text{}_2\text{F}_1$ functions or the
Appell $\text{F}_1$ functions with special arguments which can be expressed in terms of the elliptic
integrals. We will not present detailed expressions here.

4 HHL of primary scalar operators

The vertex for primary scalar operators is given by [12]

$$V_{ab}^{pr}(j) = \left(\frac{x^m x_m + z^2}{z}\right)^{-\Delta^{pr}} (z_\alpha z_\beta)^j \left[ z^{-2} (\partial_+ x_m \partial_- x^m - \partial_+ z \partial_- z) - \partial_+ X_k \partial_- X^k \right]. \tag{4.1}$$

The scaling dimension is $\Delta^{pr} = j$. This reduces for giant magnons (4.1) to

$$[\cosh (\kappa \tau_e)]^{-\Delta^{pr}} (z_\alpha z_\beta)^j \left[ \kappa^2 \left( \frac{2}{\cosh^2 \kappa \tau_e} - 1 \right) - \partial_+ X_k \partial_- X^k \right]. \tag{4.2}$$

In the case of the $CP^1$ giant magnons, we consider $V_{13}^{pr}$ with

$$(z_1 z_3)^j = \frac{1}{2} (1 - \chi)^j/2, \tag{4.3}$$

$$\kappa^2 + \partial_+ X_k \partial_- X^k = \frac{1}{2(1 - v^2)} (\kappa^2 + \chi - 1).$$

By using (4.2) and (4.3), we can integrate the vertex over the string worldsheet to obtain
the corresponding structure constants as follows:

$$C_{13}^{CP^1, pr}(j) = c_{13}^{pr} \frac{\pi^{3/2}}{2\kappa} \frac{\Gamma \left( \frac{\Delta^{pr}}{4} \right)}{\Gamma \left( \frac{\Delta^{pr} + 2}{4} \right)} \chi_p^{-1/2} (1 - \chi_p)^j/2 \times \tag{4.4}$$

$$\times \left\{ \left[ \frac{\kappa^2 \Delta^{pr}}{\Delta^{pr} + 1} (1 - v^2) + (1 - \kappa^2) \right] F_1 \left( \frac{1}{2}; \frac{1}{2}; \frac{j}{2}; 1; 1 - \epsilon, \chi_p (\epsilon - 1) \right) \right.$$

$$- \chi_p F_1 \left( \frac{1}{2}; -\frac{1}{2}; \frac{j}{2}; 1; 1 - \epsilon, \chi_p (\epsilon - 1) \right) \left\}.$$  

The Appell $F_1$ functions in $C_{13}^{CP^1, pr}(j)$ have the same arguments as in $C_{13}^{CP^1, d}(j)$ in (3.7). Therefore, a similar analysis can be done for the leading finite-size corrections
for $C_{13}^{CP^1, pr}(j)$. We present here $j = 2$ which is the simplest case of even integer ($j = 0$ is trivial) after converting $F_1$ functions into the elliptic functions:

$$C_{13}^{CP^1, pr}(2) \approx c_{13}^{pr} \frac{\pi}{6} \csc \frac{p}{2} \left\{ 2 J_1 \sin \frac{p}{2} - (1 - \cos p) \right.$$

$$+ \left[ \cos 2p + 4 \left( 1 + \frac{J_1^2}{T^2} \right) \cos p + 4 J_2^2 T^2 - 5 + 4 J_1 \left( \sin \frac{p}{2} + \sin \frac{3p}{2} \right) \right] \exp \left( -\frac{J_1}{T \sin \frac{p}{2}} - 2 \right) \right\}. \tag{4.5}$$
For the HHLs with two $RP^3$ giant magnons, we consider again two primary scalar operators $V_{pr}^{13}$ and $V_{pr}^{24}$ and the results are given by

$$C_{RP^3,pr}^{13}(j) = c_{pr}^{13} \frac{\pi^{3/2} \sqrt{1 - u^2}}{2j-1 \kappa} \frac{\Gamma \left( \frac{\Delta_{pr}}{2} \right)}{\Gamma \left( \frac{\Delta_{pr} + 2}{4} \right)} \chi_p^{-1/2} \times$$

$$\times \left\{ \left[ \frac{\kappa^2 \Delta_{pr} - 1 - u^2 + 1 - \kappa^2}{\Delta_{pr} + 1 - u^2 + 1 - u^2} \right] 2F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon \right) - \chi_p \ 2F_1 \left( \frac{1}{2}; \frac{1}{2} - j; 1; 1 - \epsilon \right) \right\},$$

$$C_{RP^3,pr}^{24}(j) = c_{pr}^{24} \frac{\pi^{3/2} \sqrt{1 - u^2}}{2j-1 \kappa} \frac{\Gamma \left( \frac{\Delta_{pr}}{2} \right)}{\Gamma \left( \frac{\Delta_{pr} + 2}{4} \right)} \chi_p^{-1/2}(1 - \chi_p)^j \times$$

$$\times \left\{ \left[ \frac{\kappa^2 \Delta_{pr} - 1 - u^2 + 1 - \kappa^2}{\Delta_{pr} + 1 - u^2 + 1 - u^2} \right] F_1 \left( \frac{1}{2} \right) \frac{1}{1 - \epsilon}, 1 - \epsilon, \chi_p(\epsilon - 1) \right\} \chi_p^{-1/2}.$$

These results are quite similar to those of the dilaton vertex in (3.24) and (3.25). As before, the case of $RP^2$ giant magnons can be obtained by taking $u = 0$ limit. One can make small $\epsilon$-expansion by either using series expansions of $2F_1$ for $V_{pr}^{13}$ or the Appell functions which can be expressed by elliptic integrals for $V_{pr}^{24}$.

5 HHL of singlet scalar operators with string levels

The vertex for singlet scalar operators is given by [12]

$$V_q = [\cosh (\kappa \tau)]^{-\Delta_q} \left( \partial_+ X_k \partial_- X^k \right)^q,$$

where $q$ is related to the string level $n$ by $q = n + 1$ and

$$\Delta_q = 2 \left( \sqrt{(q - 1) \sqrt{\lambda} + 1} - \frac{1}{2} q(q - 1) + 1 \right),$$

with the 't Hooft coupling $\lambda$.

Since the evaluation of $\partial_+ X_k \partial_- X^k$ for the giant magnons have been given in previous sections, we just present the results here.

For the $CP^1$ case,

$$C^{CP^1}(q) = c_{CP^1} q^{3/2} \frac{\Gamma \left( \frac{\Delta_{n+1}}{2} \right)}{2\sqrt{(1 - v^2) q-1} \kappa} \Gamma \left( \frac{\Delta_{n+1}}{2} \right) \chi_p^{-1/2} \times$$

$$\times \sum_{k=0}^{q} \frac{q!}{k!(q-k)!} \left[ \frac{1}{2} (1 + v^2) \kappa^2 - 1 \right]^{q-k} \chi_p^{k} 2F_1 \left( \frac{1}{2}; \frac{1}{2} - k; 1; 1 - \epsilon \right).$$
For the $RP^3$ case,

$$C^{RP^3}(q) = \frac{c^q \pi^{3/2}(1 - v^2)}{\sqrt{(1 - u^2)\kappa^2}} \frac{\Gamma\left(\frac{\Delta + 1}{2}\right)}{\Gamma\left(\frac{\Delta}{2}\right)} \left[\frac{2(1 - u^2)}{1 - v^2}\right]^q \chi_p^{-1/2} \times$$

$$\times \sum_{k=0}^q \frac{q!}{k!(q-k)!} \left[\frac{\frac{1}{2}(1 + v^2)\kappa^2 - 1}{1 - u^2}\right]^{q-k} \chi_p^k \frac{\Gamma\left(\Delta q\right)}{\Gamma\left(\Delta q + 1\right)} \left[\frac{1}{2} - k; 1 - \epsilon\right].$$

Again, the $RP^2$ giant magnons correspond to $u = 0$.

For concrete results, we present small $\epsilon$-expansions for the $CP^1$ giant magnon with a few lower levels as follows:

$q = 1$ (level $n = 0$)

$$C^{CP^1}(1) = \frac{c^q \sqrt{\pi} \Gamma\left(\frac{\Delta + 1}{2}\right)}{\Gamma\left(\frac{\Delta}{2}\right)} \left\{ \sin \frac{p}{2} - J_1 \frac{2}{T} - \left[\frac{J_1}{T}\left(5 - \cos p + 2J_1\csc \frac{p}{2}\right) \right. \right. \right.$$

$$- 2\sin \frac{p}{2} \left[1 + \cos p + \frac{J_1}{T}\right] \exp \left(-\frac{J_1}{T\sin \frac{p}{2}} - 2\right) \right\},$$

$q = 2$ (level $n = 1$)

$$C^{CP^1}(2) = \frac{c^q \sqrt{\pi} \Gamma\left(\frac{\Delta + 1}{2}\right)}{24 \Gamma\left(\frac{\Delta + 1}{2}\right)} \left\{ 3J_1 - 2\sin \frac{p}{2} + 2 \left[\frac{J_1}{T}\left(31 + 13\cos p + 6\frac{J_1}{2T}\csc \frac{p}{2}\right) \right. \right. \right.$$

$$- 2\left(33 + 5\cos p + 3\frac{J_1^2}{T^2}\right) \sin \frac{p}{2} \exp \left(-\frac{J_1}{T\sin \frac{p}{2}} - 2\right) \right\},$$

$q = 3$ (level $n = 2$)

$$C^{CP^1}(3) = \frac{c^q \sqrt{\pi} \Gamma\left(\frac{\Delta + 1}{2}\right)}{480 \Gamma\left(\frac{\Delta + 1}{2}\right)} \left\{ 38\sin \frac{p}{2} - 15\frac{J_1}{T}\left[12 \left(49 + 57\cos p - 5\frac{J_1^2}{T^2}\cot^2 \frac{p}{2}\right) \right. \right. \right.$$

$$\left. - 2\frac{J_1}{T}(187 + 97\cos p)\csc \frac{p}{2}\right] \exp \left(-\frac{J_1}{T\sin \frac{p}{2}} - 2\right) \right\}.$$

6 Concluding remarks

We obtained here some semiclassical normalized structure constants for strings in $AdS_4 \times CP^3$ dual to $\mathcal{N} = 6$ super Chern-Simons-matter theory in three space-time dimensions (ABJM theory). We considered the cases where the two “heavy” string states are finite-size giant magnons living in the $R_t \times CP^1$, $R_t \times RP^2$ and $R_t \times RP^3$ subspaces and the “light” states are the dilaton operators with non-zero momentum, primary scalar operators, and singlet scalar operators on higher string levels.

Our results can be compared with other computations based on integrability. One of them is to formulate the HHL structure constants as diagonal form factors of the light operators with respect to the on-shell particle states corresponding to the heavy operators [13].
This approach is particularly useful to understand the finite-size corrections in the HHL in terms of the underlying integrability structure like the world-sheet $S$-matrix. In this sense, various HHL functions and their finite-size corrections which we have analysed in this paper can be used to understand new aspects of integrability in the planar limit of $AdS_4/CFT_3$.

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A Appell $F_1$ functions with special arguments

We need to evaluate Appell $F_1$ functions $F_1(1/2; \pm 1/2, -j; 1; x, y)$. Using Mathematica, we have found their relations to the elliptic functions $E, K$:

$$F_1\left(\frac{1}{2}; \pm \frac{1}{2}, -j; 1; x, y\right) = e_j^\pm(x, y)E(x) + k_j^\pm(x, y)K(x),$$

where the coefficients $e_j^\pm$ and $k_j^\pm$ ($j = 1, 2$) are given by

$$e_1^+(x, y) = \frac{2y}{\pi x}, e_1^-(x, y) = \frac{2(y + 3x - 6xy)}{3\pi x}, k_1^+(x, y) = \frac{2(x - y)}{\pi x}, k_1^-(x, y) = \frac{2y(x - 1)}{3\pi x},$$

$$e_2^+(x, y) = \frac{4y(3x - y - xy)}{3\pi x^2}, e_2^-(x, y) = \frac{2(10x y - 3xy^2 - 2y^2 + 15x^2 - 20y x^2 + 8x^2 y^2)}{15\pi x^2},$$

$$k_2^+(x, y) = \frac{2(3x^2 - 6x + 2y^2 + xy^2)}{3\pi x^2}, k_2^-(x, y) = \frac{2y(x - 1)(y - 5x + 2xy)}{15\pi x^2}.$$

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References

[1] J.M. Maldacena, The large-$N$ limit of superconformal field theories and supergravity, *Int. J. Theor. Phys.* 38 (1999) 1113 [hep-th/9711200] [insPIRE].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* 428 (1998) 105 [hep-th/9802109] [insPIRE].

[3] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* 2 (1998) 253 [hep-th/9802150] [insPIRE].

[4] N. Beisert et al., Review of $AdS/CFT$ integrability: an overview, *Lett. Math. Phys.* 99 (2012) 3 [arXiv:1012.3982] [insPIRE].

[5] B. Basso, S. Komatsu and P. Vieira, Structure constants and integrable bootstrap in planar $N = 4$ SYM theory, *arXiv:1505.00745* [insPIRE].
[6] B. Eden and A. Sfondrini, Three-point functions in $N = 4$ SYM: the hexagon proposal at three loops, *JHEP* **02** (2016) 165 [arXiv:1510.01242] [INSPIRE].

[7] B. Basso, V. Goncalves, S. Komatsu and P. Vieira, Gluing hexagons at three loops, *Nucl. Phys. B* **907** (2016) 695 [arXiv:1510.01683] [INSPIRE].

[8] B. Basso, V. Goncalves and S. Komatsu, Structure constants at wrapping order, *JHEP* **05** (2017) 124 [arXiv:1702.02154] [INSPIRE].

[9] Y. Jiang, S. Komatsu, I. Kostov and D. Serban, Clustering and the three-point function, *J. Phys. A* **49** (2016) 454003 [arXiv:1604.03575] [INSPIRE].

[10] K. Zarembo, Holographic three-point functions of semiclassical state s, *JHEP* **09** (2010) 030 [arXiv:1008.1059] [INSPIRE].

[11] M.S. Costa, R. Monteiro, J.E. Santos and D. Zoakos, On three-point correlation functions in the gauge/gravity duality, *JHEP* **11** (2010) 141 [arXiv:1008.1070] [INSPIRE].

[12] R. Roiban and A.A. Tseytlin, On semiclassical computation of 3-point functions of closed string vertex operators in $AdS_5 \times S^5$, *Phys. Rev. D* **82** (2010) 106011 [arXiv:1008.4921] [INSPIRE].

[13] Z. Bajnok and R.A. Janik, Classical limit of diagonal form factors and HHL correlators, *JHEP* **01** (2017) 063 [arXiv:1607.02830] [INSPIRE].

[14] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, $N = 6$ superconformal Chern-Simons-matter theories, $M_2$-branes and their gravity duals, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [INSPIRE].

[15] G. Arutyunov and S. Frolov, Superstrings on $AdS_4 \times CP^3$ as a coset $\sigma$-model, *JHEP* **09** (2008) 129 [arXiv:0806.4959] [INSPIRE].

[16] B. Stefanski, jr, Green-Schwarz action for type IIA strings on $AdS_4 \times CP^3$, *Nucl. Phys. B* **808** (2009) 80 [arXiv:0806.4948] [INSPIRE].

[17] G. Grignani, T. Harmark and M. Orselli, The $SU(2) \times SU(2)$ sector in the string dual of $N = 6$ superconformal Chern-Simons theory, *Nucl. Phys. B* **810** (2009) 115 [arXiv:0806.4959] [INSPIRE].

[18] C. Ahn, P. Bozhilov and R.C. Rashkov, Neumann-Rosochatius integrable system for strings on $AdS_4 \times CP^3$, *JHEP* **09** (2008) 017 [arXiv:0807.3134] [INSPIRE].

[19] S. Ryang, Giant magnon and spike solutions with two spins in $AdS_4 \times CP^3$, *JHEP* **11** (2008) 084 [arXiv:0809.5106] [INSPIRE].

[20] M.C. Abbott and I. Aniceto, Giant magnons in $AdS_4 \times CP^3$: embeddings, charges and a Hamiltonian, *JHEP* **04** (2009) 136 [arXiv:0811.2423] [INSPIRE].

[21] D. Arnaudov and R.C. Rashkov, On semiclassical calculation of three-point functions in $AdS_4 \times CP^3$, *Phys. Rev. D* **83** (2011) 066011 [arXiv:1011.4669] [INSPIRE].

[22] S. Hirano, C. Kristjansen and D. Young, Giant gravitons on $AdS_4 \times CP^3$ and their holographic three-point functions, *JHEP* **07** (2012) 006 [arXiv:1205.1959] [INSPIRE].

[23] A. Bissi, C. Kristjansen, A. Martirosyan and M. Orselli, On three-point functions in the $AdS_4/CFT_3$ correspondence, *JHEP* **01** (2013) 137 [arXiv:1211.1359] [INSPIRE].

[24] B. Gwak, B.-H. Lee and C. Park, Correlation functions of the Aharony-Bergman-Jafferis-Maldacena model, *Phys. Rev. D* **87** (2013) 086002 [arXiv:1211.5838] [INSPIRE].
[25] D. Arnaudov, *Three-point functions of semiclassical string states and conserved currents in $AdS_3 \times CP^3$*, Phys. Rev. D 87 (2013) 126004 [arXiv:1302.2119] [inSPIRE].

[26] G. Arutyunov, S. Frolov and M. Zamaklar, *Finite-size effects from giant magnons*, Nucl. Phys. B 778 (2007) 1 [hep-th/0606126] [inSPIRE].

[27] D.M. Hofman and J.M. Maldacena, *Giant magnons*, J. Phys. A 39 (2006) 13095 [hep-th/0604135] [inSPIRE].

[28] C. Ahn and R.I. Nepomechie, *$N = 6$ super Chern-Simons theory S-matrix and all-loop Bethe ansatz equations*, JHEP 09 (2008) 010 [arXiv:0807.1924] [inSPIRE].

[29] C. Ahn and P. Bozhilov, *Finite-size effects for single spike*, JHEP 07 (2008) 105 [arXiv:0806.1085] [inSPIRE].

[30] C. Ahn and P. Bozhilov, *Three-point correlation functions of giant magnons with finite size*, Phys. Lett. B 702 (2011) 286 [arXiv:1105.3084] [inSPIRE].

[31] The Wolfram functions site webpage, http://functions.wolfram.com.

[32] A.P. Prudnikov, Yu.A. Brychkov and O.I. Marichev, *Integrals and series, volume 3: more special functions*, Gordon and Breach, New York U.S.A., (1990).