Gauge Symmetry in Phase Space

Consequences for Physics and Spacetime

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Gell-Mann: Anything which is not forbidden is compulsory!

This paper is dedicated to Murray Gell-Mann for his 80th birthday.

Abstract

Position and momentum enter at the same level of importance in the formulation of classical or quantum mechanics. This is reflected in the invariance of Poisson brackets or quantum commutators under canonical transformations, which I regard as a global symmetry. A gauge symmetry can be defined in phase space \((X^M, P_M)\) that imposes equivalence of momentum and position for every motion at every instant of the worldline. One of the consequences of this gauge symmetry is a new formulation of physics in spacetime. Instead of one time there must be two, while phenomena described by one-time physics in 3+1 dimensions appear as various “shadows” of the same phenomena that occur in 4+2 dimensions with one extra space and one extra time dimensions (more generally, \(d+2\)). The 2T-physics formulation leads to a unification of 1T-physics systems not suspected before and there are new correct predictions from 2T-physics that 1T-physics is unable to make on its own systematically. Additional data related to the predictions, that provides information about the properties of the extra 1-space and extra 1-time dimensions, can be gathered by observers stuck in 3+1 dimensions. This is the probe for investigating indirectly the extra 1+1 dimensions which are neither small nor hidden. This 2T formalism that originated in 1998 has been extended in recent years from the worldline to field theory in \(d+2\) dimensions. This includes 2T field theories that yield 1T field theories for the Standard Model and General Relativity as shadows of their counterparts in 4+2 dimensions. Problems of ghosts and causality in a 2T space-time are resolved automatically by the gauge symmetry, while a higher unification of 1T field theories is obtained. In this lecture the approach will be described at an elementary worldline level, and the current status of 2T-physics will be summarized.

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I. SOME CONSEQUENCES OF THE GAUGE SYMMETRY

Gauge symmetry in phase space is an unfamiliar concept. What motivates it? In my own work the motivation emerged from my observation in 1995 that the 11-dimensional extended supersymmetry (SUSY) in M-theory is really a 12-dimensional SUSY with an SO(10, 2) symmetry, indicating the possibility of a 12D spacetime with two-time (2T) signature \[1\] (see also F-theory in 12D that developed soon afterwards \[2\], and S-theory in 11+2 dimensions \[1\]). However, if taken seriously, a theory in such a spacetime would be riddled with problems of ghosts and causality. If such a spacetime is more than a mathematical accident in M- or F- or S- theory, one would have to understand how to construct a physical theory that overcomes these problems. In 1995 the challenge seemed to be worthwhile in its own right, in addition to possibly providing a guide for constructing M- or F- or S- theory and explaining the origin of the SO(10, 2) or SO(11, 2) symmetry.

Yesterday, Murray Gell-Mann talked about “Some Lessons from 60 Years of Theorizing”. One of his main messages was that from time to time we should question some of the “received ideas”. Usually, he said, there are good reasons for why they exist, but they are sometimes wrong. He gave examples in many areas of “received ideas” which turned out to be wrong, including some famous ones such as the Earth being the center of the universe. Then he elaborated on how he overcame some of those received ideas in his own research, leading to some of his major successes that we are celebrating in this conference. Such refreshing words from our energetic honoree lifted my spirits and reaffirmed my admiration for his intellect and his seminal work.

Well, that spacetime has only one time coordinate is one of those received ideas. A major argument in favor of this one is that apparently insurmountable problems with ghosts and causality prevent additional timelike dimensions. I questioned this received idea in 1995 and three years later, in 1998, found how to overcome it. The key is the gauge symmetry in phase space \[3\] \[4\] that I will discuss in this talk.

The resolution of similar problems in one-time theories taught us over the past century that the solution to the ghost problem associated with the first time dimension is to have some carefully constructed gauge symmetries. Gauge symmetries in general relativity, Maxwell or Yang-Mills theories, as well as string theory are essential to remove ghosts, thus providing a physically sensible theory. A gauge symmetry has a dual role. On the one hand
it is the very reason for the existence of the fundamental forces while dictating the form of fundamental equations of physics, on the other hand it removes ghosts. It was evident to me that, to remove the ghost and causality problems, that are the stumbling blocks in a theory with two times, a much stronger gauge symmetry was needed. Furthermore, if such a thing existed it would lead to some powerful constraints on the fundamental formulation of physics. This could also be a guiding principle for constructing M-theory.

Before I describe the phase space gauge symmetry based on symplectic transformations \( \text{Sp}(2,R) \) let me highlight some of its important consequences.

- The \( \text{Sp}(2,R) \) gauge symmetry requires that all physics be reformulated in 4+2 dimensions (more generally \( d+2 \)). 2T is a consequence, not an input. Thus, for phase space \( (X^M, P_M) \), and all fields \( A_M(X), G_{MN}(X) \), etc., 2T signature is required by the symmetry, not just permitted. The underlying \( \text{Sp}(2,R) \) leads to greater gauge symmetry and constraints that remove all ghosts or causality problems. I called this 2T-physics.

- All 1T physics for which we have experimental evidence so far, at all known scales of energy or distance, fits into 2T-physics. The gauge invariant sector of 2T-physics in 4+2 dimensions, namely the ghost free physical sector, becomes effectively a one-time (1T) theory with an effective 3+1 dimensions. There remains no Kaluza-Klein type degrees of freedom at all. But the outcome is not the same as the 1T formulation of physics. Finding again 3+1 within 4+2 is not a zero sum game, because there are many ways in which 3+1 phase space is embedded in 4+2 phase space, leading to many emergent “times” and corresponding “Hamiltonians” within 4+2. I call the emergent 3+1 spacetimes and dynamical systems “shadows” of the “substance” in 4+2. This leads systematically to a large number of correct predictions by 2T-physics, in the form of hidden relations between dynamical systems and hidden symmetries in 3+1 dimensions, that the standard 1T formulation of physics (1T-physics) is not equipped to predict but can only verify. The new information in 3+1 provided by the systematic predictions from 4+2 (more generally \( d+2 \)) is the main new content of 2T-physics.

- 2T-physics was initially formulated as a theory for particles moving on worldlines. In recent years the formulation was successfully extended to 2T field theory, which includes the Standard Model (SM) \[8\] and General Relativity (GR) \[9,10\] as 2T...
field theories in 4+2 dimensions. These are consistent with their 1T counterparts in 3+1 dimensions. In fact the usual SM and GR emerge as one of the shadows from 4+2, namely the “conformal shadow”. The status of further developments of the 2T approach, including SUSY, higher dimensions, string theory, will be summarized at the end. Suffice it to say that 2T-physics agrees with 1T-physics but it goes beyond by its potential to make new testable predictions, that 1T-physics misses and, which so far are consistent with known data. This additional information, namely the hidden symmetries and the systematic relationships among the emergent multiple shadows, provides a probe for discovering indirectly the properties of the extra 1+1 dimensions.

II. PHASE SPACE GAUGE SYMMETRY \( SP(2, R) \)

A clue for the fundamental principle is a position↔momentum global symmetry in classical or quantum mechanics. Specifically, position and momentum appear at the same level of importance in specifying boundary conditions or in reporting the results of any measurement. More importantly the formulation of classical mechanics in terms of Poisson brackets \( \{ X^M, P_N \} = \delta^M_N \) is invariant under all infinitesimal canonical transformations, \( \delta_\epsilon X^M = \partial_\epsilon (X, P) P_M \), \( \delta_\epsilon P_M = -\partial_\epsilon (X, P) X^M \), since \( \delta_\epsilon \{ X^M, P_N \} = 0 \) for any \( \epsilon (X, P) \). A quantum ordered version of the same symmetry holds for the fundamental quantum commutators \( [X^M, P_N] = i\hbar \delta^M_N \), since \( \delta_\epsilon [X^M, P_N] = 0 \). The symmetry under infinitesimal classical canonical transformations is also the symmetry of the first term of any action in the first order formalism \( S = \int d\tau (\dot{X}^M P_M - \cdots) \) since one gets a total derivative for

\[
\delta_\epsilon \left( \dot{X}^M P_M \right) = \frac{d}{d\tau} \left( P_M \frac{\partial_\epsilon (X, P)}{\partial X^M} - \epsilon (X, P) \right). \tag{1}
\]

This symmetry is spoiled for the action when a specific Hamiltonian \( H(X, P) \) is inserted in the action as part of the “\( \cdots \)”. However a specific Hamiltonian focusses on a specific dynamical system rather than the general formalism. We learned in special and general relativity that the notion of time and the corresponding Hamiltonian are dependent on the observer. Einstein showed us how to detach the formulation of fundamental laws from the perspective of observers by requiring equivalence of all perspectives in all spacetime frames. My idea was to take this equivalence notion one step further to all perspectives in phase space \( (X^M, P_M) \), not only perspectives in spacetime \( X^M \), by requiring a gauge symmetry
in phase space.

In this approach the Hamiltonian (and the associated time) would be regarded as an emergent concept that depends on some perspective from the point of view of phase space. Hence, I ignored the Hamiltonian and instead focused on requiring an action principle with a local symmetry in phase space.

To begin, the canonical transformation above should be regarded as a global symmetry on the worldline since the $\varepsilon (X, P)$ of canonical transformations depends on the proper time $\tau$ only through the “fields” $X(\tau), P(\tau)$. So any infinitesimal parameters included in $\varepsilon (X, P)$ are global parameters. To have a local symmetry on the worldline one needs a symmetry with parameters that depend arbitrarily on the worldline parameter $\tau$, through additional $\tau$ dependent parameters, leading to $\varepsilon (X(\tau), P(\tau), \tau)$ local on the worldline. It turns out that there is a limit on how large the symmetry can be because the system may turn out to be trivial if constrained by too much local symmetry. What worked is an $\text{Sp}(2, R)$ local symmetry\(^1\) formulated as follows.

Introduce the three generators of $\text{Sp}(2, R)$ as a symmetric $2 \times 2$ tensor $Q_{ij} (X, P)$, namely $Q_{11} (X, P), Q_{22} (X, P)$ and $Q_{12} (X, P) \equiv Q_{21} (X, P)$ and require that they form the Lie algebra of $\text{Sp}(2, R)$ under Poisson brackets. I will then require local symmetry on the worldline with arbitrary local parameters $\omega^{ij} (\tau)$ that define $\varepsilon (X, P, \tau) = \frac{1}{2} \omega^{ij} (\tau) Q_{ij} (X, P)$.

An action invariant under this local transformation can be constructed by introducing the $\text{Sp}(2, R)$ gauge potentials on the worldline $A^{ij} (\tau)$

$$\begin{align*}
S &= \int d\tau \left( \dot{X}^M P_M - \frac{1}{2} A^{ij} (\tau) Q_{ij} (X (\tau), P (\tau)) \right). 
\end{align*}$$

(2)

It can be verified that the action is invariant under the local transformations of the matter and gauge degrees of freedom $\delta_\varepsilon X^M = \frac{1}{2} \omega^{ij} (\tau) \frac{\partial Q_{ij} (X, P)}{\partial P_M}$, $\delta_\varepsilon P_M = -\frac{1}{2} \omega^{ij} (\tau) \frac{\partial Q_{ij} (X, P)}{\partial X^M}$, $\delta_\varepsilon A^{ij} = D_\tau \omega^{ij} \equiv \partial_\tau \omega^{ij} + [A, \omega]^{ij}$ (summed indices are contracted with the antisymmetric $\text{Sp}(2, R)$ metric $\varepsilon_{ij}$), provided the $Q_{ij}$ form the Lie algebra under Poisson brackets. It is possible to generalize this action by adding a term of the form $S' = -\int d\tau U (X (\tau), P (\tau))$ provided $U (X, P)$ is invariant under $\text{Sp}(2, R)$, namely $\{Q_{ij}, U\} = 0$.

The equivalence principle in phase space I outlined suggests that one should consider all possible $Q_{ij} (X, P)$ that satisfy $\text{Sp}(2, R)$ to recover all possible physical systems for a spinless

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\(^1\) This is for a spinless particle. For particles with spin and/or supersymmetry the symmetry group is larger, but it must include $\text{Sp}(2, R)$ as a subgroup in a special way \[^{16}\] \[^{20}\] \[^{3}\].
particle (for particles with spin see footnote 1). I found an infinite number of $Q_{ij}(X,P)$ that form $\text{Sp}(2,R)$ and I classified them up to canonical transformations [4][5]. I now consider some examples.

An example of the $Q_{ij}(X,P)$ that satisfy $\text{Sp}(2,R)$ is

$$Q_{11} = X \cdot X, \quad Q_{22} = P \cdot P, \quad Q_{12} = X \cdot P.$$ (3)

These special $Q_{ij}$ are constructed by using a dot product $X \cdot X = X^M X^N \eta_{MN}$ where the signature of the flat metric $\eta_{MN}$ in target space is not specified à priori. The $\text{Sp}(2,R)$ invariants that satisfy $\{Q_{ij}, U\} = 0$ are all possible functions $U(L^{MN})$ of the angular momentum generators $L^{MN} = X^M P^N - X^N P^M$. For this example the $\text{Sp}(2,R)$ transformation defined above through Poisson brackets amounts to a local linear transformation on $(X^M, P^M)$ such that these behave like a doublet for each $M$, as follows [3]

$$\begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix} = \begin{pmatrix} a(\tau) & b(\tau) \\ c(\tau) & d(\tau) \end{pmatrix} \begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix}, \quad ad - bc = 1,$$ (4)

where for the infinitesimal transformation $a(\tau) = 1 + \omega^{12}(\tau) + \cdots$, $b(\tau) = \omega^{22}(\tau) + \cdots$, $c(\tau) = -\omega^{11}(\tau) + \cdots$, $d(\tau) = 1 - \omega^{12}(\tau) + \cdots$. Furthermore, the action above can be rewritten in terms of usual Yang-Mills type covariant derivatives appropriate for the doublets $(X, P)$ [3].

One of the consequences of the general action (2) is the equation of motion for the gauge field $A^{ij}$ which acts like a Lagrange multiplier. This requires that the $\text{Sp}(2,R)$ charges should vanish

$$Q_{ij}(X,P) = 0.$$ (5)

The meaning of this equation is that only the $\text{Sp}(2,R)$ gauge invariant subspace of phase space is physical. Hence, only gauge invariant motion is allowed. The solution space for these $\text{Sp}(2,R)$ conditions are called “shadows”. I will show some examples of shadows in the next section.

It turns out that nontrivial solutions to $Q_{ij} = 0$ exist only if the target space has 2 times, no less and no more. To see why, consider the example in Eq.(3). If the metric $\eta_{MN}$ is Euclidean ($0T$) then the only solution is trivial $X^M = P^M = 0$. If the metric $\eta_{MN}$ is Minkowski ($1T$) then a solution is possible only if $X^M, P^M$ are lightlike and parallel, which means the angular momentum vanishes $L^{MN} = 0$. This is trivial because it does not describe
even a free particle. To have non-trivial solutions one must have a metric $\eta_{MN}$ with two times (2T) or more. With 2T it turns out there is just enough gauge symmetry to remove the ghosts, but with three or more times the Sp(2, R) gauge symmetry is insufficient to remove ghosts. Hence there must be two times, no less and no more. I have shown that 2T is an outcome, not an input, since it is demanded by the gauge symmetry and nontrivial physical content.

For the model of 2T-physics based on the $Q_{ij}$ in Eq. (3) there is an automatic global SO($d, 2$) symmetry. This SO($d, 2$) is the symmetry of the dot products and has generators $L^{MN}$ that are Sp(2, R) gauge invariant \{ $Q_{ij}$, $L^{MN}$ \} = 0, with $L^{MN} = X^M P^N - X^N P^M$.

The action may be modified by an additional Sp(2, R) gauge invariant term of the form $S' = - \int d\tau U (L (\tau))$, where $U (L)$ is an arbitrary function of the $L^{MN}$, which could break the global SO($d, 2$) symmetry partially or fully. The inclusion of $U$ does not change the essential point that the $Q_{ij}$ must vanish, leading to the same 1T shadows (see next section), and that spacetime is $d + 2$ at the fundamental level, even if the global SO($d, 2$) symmetry is broken.

I am often asked if it is possible to have more times by enlarging the gauge symmetry beyond Sp(2, R). My answer is that it is unlikely, but I don’t have a theorem so far. This is based on the following considerations. First, it is certainly possible to write a gauge invariant action identical in form to (2) for any Lie algebra whose generators $Q_a (X, P)$ close (assuming these can be constructed in phase space). The issue is whether the gauge invariance condition $Q_a (X, P) = 0$ has non-trivial content and also if the emerging shadows are ghost free. In all attempts so far, with specific examples $Q_a (X, P)$ for spinless particles, we have found that, such a scheme based on noncompact groups, either leads to trivial content for the solutions of $Q_a = 0$, or the emergent spaces (i.e. shadows) have ghosts because all the timelike dimensions could not be removed from $X^M, P_M$. One remarkable exception that has worked so well is Sp(2, R), which seems to indicate that 2T-physics may be special [15].

The consequences of the local worldline Sp(2, R) symmetry for local field theory (fields that depend only on $X^M$) [6][14] and an extension of these concepts to field theory in phase space (fields that depend on both $X$ and $P$) [5] have been developed, but there will not be sufficient time to discuss them in this talk. They will be described only briefly at the end of this paper.
III. SHADOWS

In this section I concentrate on the $2T$ free particle in flat spacetime described by the $Q_{ij}$ in Eq.(3). To obtain the $1T$ shadows I will make two gauge choices and solve the two constraints $X^2 = 0$ and $X \cdot P = 0$. This fixes two components of $X^M$ and two components of $P^M$ in terms of the remaining independent degrees of freedom, thus reducing the theory from $d + 2$ dimensions to various shadows in $d$ dimensions. There will remain still one gauge symmetry and one unsolved constraint that can remove the ghosts in the remaining timelike degree of freedom in the shadow.

To perform these steps it is useful to define a lightcone type basis $X^M = (X^+, X^-, X^\mu)$ so that the flat metric in $d + 2$ dimensions is expressed as $ds^2 = -2dX^+dX^- + dX^\mu dX^\nu \eta_{\mu\nu}$ with $\eta_{\mu\nu}$ the Minkowski metric in $d$ dimensions including 1 time.

As the first example of a shadow, consider the following solution [3] which I call the “conformal shadow”

\[ X^+ (\tau) = 1, \quad X^- = \frac{1}{2} x^2 (\tau), \quad X^\mu (\tau) \equiv x^\mu (\tau) \]
\[ P^+ (\tau) = 0, \quad P^- = x (\tau) \cdot p (\tau), \quad P^\mu (\tau) \equiv p^\mu (\tau), \quad p^2 = 0. \]  

(6)

Here the two gauges are $X^+ (\tau) = 1$ and $P^+ (\tau) = 0$ for all $\tau$. The solution of the constraint $X^2 = 0$ yields $X^-$ and the solution of the constraint $X \cdot P = 0$ yields $P^-$ as given above. The remaining degrees of freedom which were named as $x^\mu, p^\mu$ are still subject to the constraint

\[ P^2 = -2P^+P^- + P^\mu P_\mu = 0 \]

which takes the form $p^2 = 0$. This phase space $(x^\mu (\tau), p^\mu (\tau))$ describes the free massless $1T$ relativistic particle in $d$ dimensions. This is confirmed by inserting the gauge choices into the original action, yielding

\[ S = \int d\tau \left( \dot{x}^\mu p_\mu - \frac{1}{2} \dot{A}^2 p^2 \right), \]

which is the action for the free massless relativistic particle. One can also start from the equations of motion for $X^M (\tau), P^M (\tau)$, insert the gauge fixed configuration above, and obtain the equations of motion of the free massless relativistic particle.

The original action had an SO($d, 2$) global symmetry. The global symmetry of the action does not disappear since the action is gauge invariant. However, it becomes hard to notice the symmetry in terms of the remaining degrees of freedom because it takes a non-linear form. The generators $L^{MN} = X^M P^N - X^N P^M$ were also gauge invariant, so they can be expressed in terms of the remaining degrees of freedom by inserting the configuration in Eq.(6). This gives the following components of $L^{MN}$ in their shadow form.
\[ L^{-'+} = x \cdot p, \quad L^\mu = x^\mu p^\nu - x^\nu p^\mu, \]
\[ L^{-'\mu} = p^\mu, \quad L^{-'\mu} = \frac{1}{2} x^2 p^\mu - x \cdot px^\mu. \] (7)

This is recognized as the generators of the conformal group SO\((d,2)\)^2. Their action on the massless degrees of freedom \(x^\mu, p^\mu\) is given by computing their Poisson brackets \(\delta_\epsilon x^\mu = \frac{1}{2} \epsilon_{MN} [L^M, x^N]\) and similarly for \(\delta_\epsilon p^\mu\). Using this, one can check that these are indeed generators of symmetry for the action for the massless particle [3][22]. This is expected automatically since both \(S\) and \(L^{MN}\) are gauge invariants and one already knew that \(S\) was invariant under SO\((d,2)\).

A second example is the massive relativistic particle given by the following shadow configuration (a different looking form in [22] is gauge equivalent)

\[ X^{-'+} = \frac{1+a}{2a}, \quad X^{-'\mu} = \frac{x_2 a}{1+a}, \quad X^\mu \equiv x^\mu(\tau), \quad a \equiv \left(1 + \frac{m^2 x^2}{(x \cdot p)^2}\right)^{1/2}, \quad p^2 + m^2 = 0 \] (8)

The remaining constraint in this case \(P^2 = 0\) takes the form \(p^2 + m^2 = 0\), which says that the shadow phase space \((x^\mu(\tau), p^\mu(\tau))\) now corresponds to the 1T massive relativistic particle.

As before, this can be confirmed by computing both the action and the equations of motion. Now we get a surprise not noticed before in 1T-physics. The 2T approach leads us to expect that the action for the massive relativistic particle \(S = \int d\tau (\dot{x}^\mu p_\mu - \frac{1}{2} A^{22} (p^2 + m^2))\), which is a gauge fixed form of the original action [2], should be invariant under SO\((d,2)\), but no one suggested this before in 1T-physics. To find out how to construct the symmetry generators in this case (the massive analog of (7)) [22] all one needs to do is insert the shadow phase space of Eq.(8) into \(L^{MN} = X^M P^N - X^N P^M\).

Even more surprising (for 1T-physics) in this regard is the shadow for the massive non-relativistic particle given by [22]

\[ X^{+'} = t(\tau), \quad X^{-'} = \frac{\vec{r} \cdot \vec{p} - mH}{m}, \quad X^0 = \pm \left| \vec{r} - \frac{1}{m} \vec{p} \right|, \quad X^i = \vec{r}^i(\tau), \]
\[ P^{+'} = m, \quad P^{-'} = H(\tau), \quad P^0 = 0 \quad P^i = \vec{p}^i(\tau) \] (9)

\(^2\)Dirac [31] was the first to use a 6 dimensional space to describe conformal symmetry SO\((4,2)\). His approach, which was further developed [32][?] had faded away when I discovered my approach, as a gauge symmetry of phase space, without being aware of Dirac’s different formalism or reasoning. We now know that Dirac’s work is automatically part of 2T-physics, since it coincides with my “conformal shadow” for the special case of \(Q_{ij}\) in [3]. But this is just an example of a particular shadow within the larger scope of 2T-physics.
where $t(\tau)$ and $H(\tau)$ are shadow canonical variables just like $(\vec{r}(\tau), \vec{p}(\tau))$ as described by the action $S = \int d\tau \left( -H \partial_\tau t + \partial_\tau \vec{r} \cdot \vec{p} - \frac{1}{2} A^{22} (-2mH + \vec{p}^2) \right)$, which follows from (2) by inserting the shadow configuration above. In this case the remaining constraint $P^2 = 0$ takes the form $-2mH + \vec{p}^2 = 0$ which shows that $H$ is the non-relativistic Hamiltonian when the constraint is solved and the final gauge choice is made $t(\tau) = \tau$. The corresponding completely gauge fixed action in $(d - 1)$ space dimensions is $S = \int d\tau \left( \partial_\tau \vec{F} \cdot \vec{p} - \frac{\vec{p}^2}{2m} \right)$. Evidently it describes the massive nonrelativistic particle. However, surprisingly (for 1T-physics) it is invariant under SO($d, 2$), which is realized by rather complicated non-linear and $\tau$ dependent transformations generated by $L^{MN}$, which are the analogs [22] of Eq.(7).

Another remarkable property is the emergence of the mass parameter $m$ in Eqs.(8) and (9) as a modulus in the embedding of the shadow phase space $(x, p)$ in the higher dimensional phase space $(X, P)$. I still believe that mass is very likely explained by the Higgs particle as hopefully will be confirmed at the LHC. But the alternative mass generation mechanism I have just displayed must also mean something in Nature and I hope to find its meaning some day. If the Higgs gets into trouble at the LHC it might be a good idea to investigate seriously for the origin of mass in this alternative direction.

Fig.1: Some 1T shadows of the 2T free particle in flat spacetime.
cles in 1T-physics. However, there are also all sorts of shadows of the same “substance” that behave like particles subject to a variety of forces, as shown with some examples in Fig.1. These include some non-relativistic potentials (Hydrogen atom, harmonic oscillator), and some curved spaces, such as the Robertson-Walker expanding universe, any conformally flat space \( \text{AdS}^{d-n} \times S^n \), maximally symmetric space, and even some singular spaces. Furthermore for all of these shadows new twistor formulations provide an alternative expression of the shadow phase space \([18][20]\), as indicated on the figure. The mathematical expressions for these shadows (similar to Eqs.(6,8,9)) were developed non-systematically \([22]\) over several years and some of them are summarized in tables I,II,III in \([13]\).

So, parameters, such as mass, coupling, curvature, emerge from the moduli for embedding \((x,p)\) into \((X,P)\). All of these shadows have the hidden \(\text{SO}(d,2)\) symmetry, which is realized in terms of non-linear realizations of \(L^{MN}\). When these systems are quantized, the Casimir operators can be evaluated (they are all zero at the classical level) and show that they all give the same quantized value, such as \(C_2 = \frac{1}{2}L_{MN}L^{MN} = 1 - d^2/4\). This says that this is the singleton representation of \(\text{SO}(d,2)\). All shadows are in the same representation, but each shadow is realized in unitarily equivalent bases of this symmetry.

Evidently there is a lot of information in the hidden relationships among these systems. This information resides in the gauge invariant properties of the 2T “substance” in \(d + 2\) dimensions which is captured holographically by each 1T shadow in \(d\) dimensions. 1T-physics treats all the shadows as different from each other and gives no clues that they may be related. By contrast 2T-physics makes the prediction that observers in \(d\) dimensions will discover the predicted relationships and hidden symmetries if they look hard enough.

The relationship between the shadows is similar to duality transformations, which in the present case amount to \(\text{Sp}(2,R)\) gauge transformations from one fixed gauge to another. These transformations involve not only change of coordinates and momenta but also parameters such as mass, coupling, curvature, etc. All the relations among shadows amount to the fact that \(L^{MN}\) is gauge invariant and therefore any function \(F(L)\) must have the same gauge invariant value in all the shadows as expressed in terms of the phase space for that shadow. This is the key for all the expected duality relations derivable from the free 2T particle in flat spacetime.

A complete classification of the possible shadows that emerge from the set of \(Q_{ij} = (X^2, P^2, X \cdot P)\) is not known. Other forms of \(Q_{ij}(X,P)\) will produce their own set of
shadows. Similarly the corresponding 2T field theories \[6\] \[12\] produce shadows in the form of 1T field theories \[13\] \[14\]. This rich set of dualities is likely to be useful for developing computational tools. So far this has remained largely unexplored due to lack of time and other pressing priorities.

In summary, quite generally, 2T-physics defines a “substance” that has many “shadows” in 1T-physics. Each one of them inherits holographically the gauge invariant properties of the “substance” (i.e. the theory defined by $Q_{ij} (X, P)$, and corresponding generalization in field theory, including spin, etc.) in $d + 2$ dimensions, but the shadows themselves are effective systems in $(d - 1) + 1$ dimensions with only 1T. Many possible 1Ts emerge from phase space in $d + 2$ dimensions, so the 1T in a given shadow is not the same 1T in another shadow. For this reason each shadow is described by a different Hamiltonian in the usual language of 1T-physics. Automatically, these 1T dynamical systems are related to each other by their gauge invariance properties. But 1T-physics is not equipped to display those hidden relationships among the shadows, because for 1T-physics they seem like unrelated dynamical systems with separate Hamiltonians. This is how 1T-physics misses the systematic predictions of 2T-physics.

### IV. GRAVITY AND STANDARD MODEL IN 2T-PHYSICS

A particle moving in arbitrary backgrounds, including electromagnetic, gravitational or other general fields in $d + 2$ dimensions is formulated in terms of more general $Q_{ij} (X, P)$ \[4\]. This formulation treats the Maxwell-type gauge symmetries, general coordinate transformations and more general cases of gauge symmetries in a unified way, all as special forms of canonical transformations \[4\]. I consider here just the gravitational background given by \[4\] \[9\]

$$Q_{11} = W (X), \quad Q_{12} = V^M (X) P_M, \quad Q_{22} = G^{MN} (X) P_M P_N.$$  \hfill (10)

Contrast this to the flat background in Eq.(3) to understand the significance of the background fields, noting that one specializes to the flat case with $G^{MN}_{\text{flat}} (X) = \eta^{MN}$, $W_{\text{flat}} (X) = X^2$, and $V^M_{\text{flat}} = X^M$. There is one further requirement for this to be compatible with the Sp(2, $R$) gauge symmetry of the action in Eq.(2), that is, these $Q_{ij}$ must close into the Sp(2, $R$) Lie algebra under Poisson brackets. Consequently the background fields $W (X), V^M (X), G^{MN} (X)$ must satisfy certain equations which I have called the “kinematic
equations”. There is no space to discuss them here, but they can be found in ref. [9].

Next, to construct a 2T field theory for gravity, a dilaton field $\Omega (X)$ is also needed in addition to the fields $W (X), G_{MN} (X)$ (the field $V_M$ can be solved as $V_M = \frac{1}{2} \partial_M W$, so it is not independent). The field theory action must be such that the “kinematic equations” mentioned in the previous paragraph must emerge as some of the equations of motion through the variational principle $^3$. Furthermore, the $\text{Sp}(2, R)$ constraints $Q_{ij} \sim 0$ (gauge invariant physical states) must also be satisfied as dynamical or kinematical field equations when the field interactions are turned off. These requirements, combined with general coordinate invariance in $d + 2$ dimensions, are so strong that they lead to a unique theory for 2T gravity as a field theory. The action for 2T gravity is [9]

$$S_{\text{grav}} = \gamma \int d^{d+2} X \sqrt{G} \left\{ \delta (W) \left[ \Omega^2 R (G) + \frac{1}{2a} \partial \Omega \cdot \partial \Omega - V (\Omega) \right] \right. + \delta' (W) \left[ \Omega^2 (4 - \nabla^2 W) + \partial W \cdot \partial \Omega^2 \right] \right\}. \quad (11)$$

Here $R (G)$ is the Riemann curvature scalar, $a$ is the special constant $a \equiv \frac{d - 2}{8 (d - 1)}$, while the potential $V$ can only have the form $V (\Omega) = \lambda \Omega^{2d}$ with a dimensionless coupling $\lambda$. Other than $\lambda$ there are no parameters at all. The field $W (X)$ appears in a delta function and its derivative $\delta (W), \delta' (W)$, as well as in additional terms. This unusual and unique structure emerged from the underlying properties of the $\text{Sp}(2, R)$ gauge symmetry on the worldline theory as outlined above. In particular the delta functions are consistent with one of the $\text{Sp}(2, R)$ physical state requirements $Q_{11} = W (X) = 0$, while the others $Q_{12}, Q_{22} \sim 0$ emerge from the equations of motion that follow from this action. This field theoretic structure has a bunch of unusual gauge symmetries of its own, which I called 2T gauge symmetries [9]. These are just strong enough gauge symmetries to eliminate all ghosts from the 2T fields and yield shadows in two lower dimensions (analogs of Fig.1) that are ghost free physical interacting 1T field theories in $d$ dimensions. Dualities must relate these shadow 1T field theories to each other.

This theory of gravity has no dimensionful constants, in particular there is no Newton’s constant $G$. This emerges from the condensate of the dilaton (and other scalars, see below) in the conformal shadow. The action above yields a shadow 1T General Relativity in $d$

$^3$ This is analogous to string theory, where background fields are restricted by worldsheet local conformal symmetry, while the field theory must be constructed to reproduce these field equations as equations of motion derived from the field theory action.
dimensions in the form \( S_{\text{grav}} = \int d^d x \sqrt{-g} \left\{ \phi^2 R(g) + \frac{1}{2a} \partial \phi \cdot \partial \phi - V(\phi) \right\} \), where \( \phi, g_{\mu\nu} \) are the shadows of their counterparts. In this shadow, due to the special constant \( a \), there is an emergent local scaling (Weyl) symmetry which is a remnant of general coordinate transformations in the extra 1+1 dimensions [10]. Since the coefficient of \( R(g) \) is positive, the dilaton must have the wrong sign kinetic energy to satisfy the Weyl symmetry, so \( \phi \) is a ghost. Using the Weyl gauge symmetry the shadow dilaton is gauge fixed to a constant \( \phi_0 \) (thus eliminating the ghost which would also have been a Goldstone boson after condensation), yielding precisely Einstein’s General Relativity \( S_{\text{grav}} = \int d^d x \sqrt{-g} \phi_0^2 R(g) \) where the condensate \( \phi_0^2 \) must be interpreted as Newton’s constant.

Matter fields can be added, including Klein-Gordon scalars \( S_i(X) \), Dirac or Weyl spinors \( \Psi_\alpha(X) \) and Yang-Mills type vectors \( A_M(X) \), all in \( d + 2 \) dimensions. There are special restrictions on each one of these, on the form of their kinetic energies, and the forms of permitted interactions among themselves and with the gravitational triplet \( (W, \Omega, G_{MN}) \). These restrictions emerge from the underlying \( \text{Sp}(2, R) \) gauge symmetry and the corresponding physical state conditions at the worldline level.

Within these restrictions I constructed the 2T field theory for the Standard Model in 4+2 dimensions [8]. It is a perfectly consistent, ghost free 2T field theory because of the new 2T gauge symmetries [8] satisfied by these new field theoretic structures. In the conformal shadow it yields the usual Standard Model in 3+1 dimensions which is in exquisite agreement with experiment. This shadow Standard Model has some additional constraints on the scalar sector (Higgs and others) and their interaction with the dilaton. The new features are consistent with known phenomenology, but may help shed some light on Higgs physics when more data becomes available, and perhaps the absence of axions [8] which awaits further clarification until the quantum version of 2T field theory is better understood.

In the coupling of gravity to matter there is another interesting physics prediction to be emphasized. Every scalar \( S_i \) in the complete field theory must couple to the curvature term just like the dilaton in Eq.(11), but with the opposite sign and standard normalization in

\[4\text{ This is in spirit similar to the Higgs phenomenology talk we heard from J. Gunion in this conference (see also [25][28]) because the Higgs in the 2T Standard Model is required to couple to at least one additional scalar [8], which may be the dilaton, or another electroweak neutral scalar. Note that historically the importance of this neutral sector for phenomenology was pointed out based on the prediction from the 2T Standard Model [8] prior to the (less theoretically motivated) recent phenomenological studies were undertaken.} \]
the kinetic term. Then in the conformal shadow the curvature term is predicted to take the form $(\phi^2 - a \sum_i s_i^2)R(g)$ with a required relative minus sign! Hence the gravitational constant must emerge from the condensates of all the scalars, not only the dilaton’s. This predicts a physical effect, that the effective gravitational constant $G \sim (\phi^2 - a \sum_i s_i^2)^{-1}$ is not really a constant, rather it must increase after every phase transition of the universe as a whole (since the dominant part of each field is the condensate after the phase transition, this quantity is approximately a constant in between the phase transitions). Thus the Newton constant we measure today cannot be the same as the analogous constant before the various transitions occurred, such as inflation, grand unification, SUSY breaking, electroweak symmetry breaking. Of course the earlier ones are the dominant condensates in the sum. There is also the curious possibility that $G \sim (\phi^2 - a \sum_i s_i^2)^{-1}$ could turn negative if the other scalars dominate over the dilaton in some regions of the universe, or in the history of the universe, thus producing antigravity in those parts of spacetime. In fact, the Big Bang may be related to the vanishing of $(\phi^2 - a \sum_i s_i^2)$ at which point the effective $G$ blows up. The effects of this idea on cosmology is currently under investigation [24].

V. PROGRESS IN 2T-PHYSICS

Here I will list various points with only very brief comments due to lack of space.

1) Local $Sp(2,R)$ on the worldline, as a gauge symmetry in phase space, has proven to be a physically correct general principle in both classical and quantum mechanics. The advantages of this new principle include the unification of various 1T dynamical systems under a new unifying umbrella which I called 2T-physics (as summarized in the example of Fig.1). As compared to 1T-physics, 2T-physics reveals much more correct information on physical phenomena which is systematically missed in the usual formalism of physics at all scales of distance or energy.

2) The principles of 2T field theory in $d+2$ dimensions have been established. New types of gauge symmetries eliminate all ghosts and produce a physical sector which effectively is in two lower dimensions. As in the corresponding worldline theory, the 2T field theory produces shadow 1T field theories with duality relations among them (field analogs of Fig.1). Within these principles I have constructed various physically relevant 2T field theories. These include the Standard Model, General Relativity, Grand Unified theories, in 4+2 dimensions,
whose conformal shadows are basically the same as the familiar corresponding theories in 3 + 1 dimensions, except for some additional restriction (mainly on scalar fields) which so far are consistent with phenomenology, and may even lead to measurable signals at the LHC or in cosmology.

3) Both the worldline and field theory approaches to 2T-physics have been generalized to supersymmetry. In particular, the general $N = 1, 2, 4$ SUSY 2T field theories in $4 + 2$ dimensions have been constructed \[11\] \[12\]. Consequently, the SUSY generalizations of the Standard Model or GUTS in $4 + 2$ dimensions are already available. So, if SUSY phenomenology becomes relevant at the LHC, the constraints from 2T-physics could become interesting.

4) SUSY in higher dimensions with 2T has also been achieved. In particular the super Yang-Mills theory in $10 + 2$ dimensions has been constructed \[29\]. This 12-dimensional field theory is the first one to ever go beyond 11 dimensions. It yields many interesting shadows as well as compactifications that unify various theories, from M(atrix)-theory to the hotly pursued $N = 4$ super Yang-Mills theory in 3+1 dimensions, and thus may lead to possible new insights in these 1T theories. These connections are outlined in Fig.2, and are the basis for considering the new theory as the parent of all those mentioned in the figure. Finally a
connection between 2T-physics and 12D and 13D S-theory, where it all started in 1995 [1], is beginning to emerge in a clearer way.

5) Supergravity in 2T-physics has not been constructed yet, but seems to be around the corner as the path to follow is now clear. The expected maximal SUSY theory in 13 dimensions should yield 11-dimensional supergravity as the conformal shadow.

6) For strings, branes, and more generally for M-theory in 2T-physics, there is only old partial progress for tensionless strings and branes, and more recent progress on the 2T version of the twistor string [30]. The usual tensionful string has historically resisted 2T-physics and the reason for this may have now become clear: it must be the fact that the tension is dimensionful, but as I explained above 2T gravity does not allow any dimensionful constants. The tension will have to emerge from some condensate. This new insight has not yet been implemented.

7) There is the potential of developing powerful new computational tools that take advantage of the dualities of the shadows. As in other examples of dualities, a given theory may be more easily solvable in one dual version as compared to another. Since the gauge invariant physical content is holographically captured by every shadow, it may be possible to study physical effects more easily in some shadows and transform the result to the shadow of interest (which may be the conformal shadow in the case of field theory). The shadow phenomena is much more easily handled in the worldline formalism, while in field theory so far there is limited progress because only some shadows are easy to study but others seem to be more difficult [13][14]. In any case, due to lack of time, very little has been done so far to take advantage of this feature of 2T-physics, but I think it is where 2T-physics may become most useful to 1T physicists as well as where most tests of 2T-physics can be developed.

8) As a final comment, I should mention that I consider all the encouraging progress in 2T field theory to be only a stepping stone toward a more comprehensive 2T theory which is based on fields in phase space, not just position space. I expect the field theory to be non-commutative along the lines initiated in ref.[5]. When this approach can be connected to the successful 2T field theories that work at the present, I expect much more dramatic insight and progress.
I would like to conclude this talk with an allegory which may be helpful to convey the basic idea of 2T-physics and its role as a completion of 1T-physics. I should warn that, the allegory is not perfect and is not a substitute for the equations. So it should not be taken too far without the corresponding equations.

As in Fig.3, consider an object in a room. In the allegory this represents phase space $X^M, P_M$ in $d + 2$ dimensions, with the associated Sp$(2, R)$ generators $Q_{ij}(X, P)$. Then consider the many shadows of this object on the surrounding walls which could be formed by shining light on it from different directions. In the allegory the shadows represent the many emergent physical phase spaces $(x^\mu, p_\mu)$ in two lower dimensions which solve $Q_{ij} = 0$, as in Eqs.(6,8,9) and Fig.1.

An essential point is that one single object in the room (in the allegory a specific set of $Q_{ij}$, such as Eq.(3)) has many shadows. To observers that are stuck on the wall (like we are stuck in 3+1 dimensions) the various shadows appear like different “beasts” performing different unrelated motions. However, an observer in the room immediately knows that the many shadows coming from the same object must be related. These relationships among shadows can in principle be discovered by careful observers who live on the wall, but who have no privilege of being in the room.

The gauge choices in phase space that create the shadows are the analogs of the many perspectives for observing the object in the room. The relationships among the emergent dynamical systems in Fig.1 comes from the many forms in which the same gauge invariant information in $d + 2$ dimensions is encoded holographically in each shadow in $d$ dimensions. The 2T-physics formulation makes the relationships between the “shadows” evident and predicts them to the 1T physicists on the “wall” who can study them and verify them. The information in these relationships, which I called hidden “dualities” and hidden symmetries, is information about the perspectives, and therefore it is information that relates to the properties of the larger space-time in $d + 2$ dimensions. By interpreting this data correctly, and recognizing its relation to the higher dimensions, the 1T physicists on the “wall” have a probe for studying the extra dimensions, albeit indirectly.

In this way 2T-physics provides the privilege of being in the room. It gives us the ability to recognize that certain systems are indeed related, and that the predicted relationships
are interpreted as perspectives in a higher space-time.

Fig.3 - An allegory. “Room” = 4+2, “Walls” = 3+1. Substance versus shadows.

Colleagues that have followed my work have generally been in agreement with my results. Usually I receive encouragement and never criticism. However, sometimes I am asked: “All of this is quite nice, but do we really need 2T, can’t we do everything with 1T anyway?”.

This attitude is probably part of the reason for having doubts on whether to invest effort in 2T-physics.

My answer to this question is a definite, yes you do need 2T, you cannot do everything with 1T! I have already displayed examples of systems, as in Fig.1, where new information, not available in 1T-physics is obtained in 2T-physics. This shows definitely that in principle 1T-physics systematically misses information, while 2T-physics makes it accessible with definite predictions. 2T-physics opens new avenues and provides new information not available before. 1T-physics is clearly incomplete. Apparently this has not been fully appreciated yet by many of my colleagues.

Given that this is a fact, I would guess that, besides seeking practical applications in 1T-physics, in seeking the unified theory for everything we may find that the additional information of the extra 1+1 dimensions may lead to the “Holy Grail”.

To conclude, I would like to come back to the quotation from Gell-Mann: “Anything which is not forbidden is compulsory!”.

Murray used this phrase in connection to the properties of the strong interactions. But I would like to adopt it to the fundamental laws
of physics.

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