Distillation of vacuum entanglement to EPR pairs

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It is shown that by means of local interactions between a quantized relativistic field and a pair of non-entangled atoms, entanglement can be extracted from the vacuum and delivered to the atoms. The resulting mixed state of the atoms can be further distilled to EPR pairs. Therefore, in principle, teleportation and other entanglement assisted quantum communication tasks can rely on the vacuum alone as a resource for entanglement.

The correlations between observables measured separately on a pair of entangled systems can be “stronger” compared to the correlations predicted by local “realistic” models. Hence quantum mechanics manifests a non-local behavior which is nevertheless not in conflict with macroscopic causality. In accordance with this non-locality, entanglement cannot be produced locally: a pair of separated systems which may communicate only via a classical channel, cannot become entangled as a result of local quantum operations done separately on each system. Nevertheless, when entanglement already exists, it may be locally redistributed or delivered from one subsystem to another. For instance, a sample of pairs of spins, described by a non-maximally entangled pure state, or an inseparable mixed state, can be “distilled” to singlets (EPR pairs) and remnants of non-entangled states.

In this Letter I introduce another process of entanglement manipulation which takes place between a relativistic quantum field, such as the electro-magnetic field, in its ground state (vacuum) and a pair of initially non-entangled atoms. In this process, each atom interacts locally for a finite duration with the field, and as a consequence the atoms evolve to an inseparable, and hence entangled, mixed state. Since the process takes place in two space-like separated spacetime regions, and since entanglement cannot be produced locally, this demonstrates that: a) entanglement exists in the vacuum between space-like separated regions, and that b) vacuum entanglement can be extracted and delivered into a space-like separated pair of atoms.

A sample of entangled pairs, generated in this fashion, may be further distilled to a smaller sample of maximally entangled EPR pairs (E-bits), and subsequently used for quantum communication tasks. Hence, I conclude that in principle, teleportation, dense coding, or other entanglement assisted communication tasks, can rely on the vacuum alone as a resource of entanglement, without having to deal with local preparation of E-bits and the subsequent physical delivery via a quantum channel.

It is known that field observables in vacuum at space-like separated points are correlated. For massless fields these correlations decay with the distance, $L$, between two points as $1/L^2$ (or as $e^{-mL}$ for massive fields of mass $m$). These correlations by themselves do not necessitate the existence of quantum entanglement, because they can in principle arise as classical correlations. However, a number of studies provide clear evidence that the vacuum is indeed entangled, and that these correlations are related to vacuum entanglement. Using the Rindler modes quantization, one can span the Hilbert space of a free field by direct product of Rindler particle number states $|n,1\rangle$ and $|n,2\rangle$ in the two complementary space-like separated wedges $x < -|t|$ and $x > t$, respectively. It then turns out that the ordinary (Minkowski) vacuum state can be expressed as an EPR-like state $\sim \sum_n a^n\langle n,1|n,2\rangle$ for each frequency. The resulting correlations in the number operators can be observed by a pair of opposite accelerated detectors, one in each such wedge. In a somewhat different framework, of algebraic quantum field theory, it has been shown that local field observables in two space-like separated regions violate Bell’s inequalities and hence must be entangled. Here I make one step further, and show that for a properly chosen fixed interaction between a pair of atoms and the vacuum, the atoms evolve to a mixed entangled state.

To motivate our approach, let us first examine qualitatively a simple problem of emission and absorption for a pair of time-like separated atoms. Suppose that initially atom $A$ is in its first excited state, denoted by $|\uparrow_A\rangle$, and atom $B$ is in its ground state $|\downarrow_B\rangle$. After some time, atom $A$ emits a photon, which may be captured by atom $B$. The system evolves to

$$|\Psi\rangle \approx \left( |\uparrow_A\downarrow_B\rangle + \alpha|\downarrow_A\downarrow_B\rangle \right)|0\rangle + \beta|\downarrow_A\downarrow_B\rangle|\gamma\rangle$$

The second term proportional to $\alpha$, is generated by an “exchange” process wherein a photon is emitted by $A$ absorbed by $B$, and the field is left in its original vacuum state $|0\rangle$. The last term, corresponds to absorption, hence the field’s final state, $|\gamma\rangle$, contains one photon. Note that if later one observes that no photon was emitted, the last term is eliminated, and leaves $A$ and $B$ in a pure entangled state $|\uparrow_A\downarrow_B\rangle + \alpha|\downarrow_A\downarrow_B\rangle$. Otherwise, without observing the field, the atoms remain entangled to the field. Nevertheless, it can be verified (see below) that the reduced density matrix of the atoms is inseparable, and hence entangled. This comes to us as no surprise, because in the above process a single photon (qubit), has been causally exchanged between the atoms.

However what happens if the atoms are coupled to the field for a short enough time keeping the atoms causally...
Here $\sigma^\pm$ are the atom energy-levels raising and lowering operators, the window functions $\epsilon_A(t)$ and $\epsilon_B(t)$ are non-zero only for a duration $T$ such that $T < x_B - x_A$ ($c = 1$), and $x_A = -L/2$ and $x_B = L/2$ are fixed locations.

Since the interaction takes place in two causally disconnected spacetime regions, the field operators in $H_A$ and $H_B$ commute, and $[H_A, H_B] = 0$. Therefore, the evolution operator $U$ for the whole system may be written (in the interaction representation) as a product

$$U = e^{-i \int H_A(t) dt - i \int H_B(t') dt'}$$

$$= e^{-i \int H_A(t) dt} \times e^{-i \int H_B(t') dt'}$$

This ensures that $U$ does not change the net entanglement between the two wedges, it can only redistribute it between the local observables within each wedge.

Consider now the initial state where the atoms and the field are in their ground state: $|\Psi_i\rangle = |\downarrow_A\rangle |\downarrow_B\rangle |0\rangle$. To see how $|\Psi_i\rangle$ evolves, I will assume that the coupling functions $\epsilon_i(t)$ ($i = A, B$) are small, and expand $U$ to second order. Using the notation

$$|\Psi_i\rangle = \left[ (1 - T\Phi_A^+ \Phi_A^- - T\Phi_B^+ \Phi_B^-) |\downarrow\rangle - \phi_0^+ |\Phi_B^+ \downarrow\rangle \right] |0\rangle + O(\Phi^3)$$

where $T$ denotes time ordering. In the first term above the state of the atoms is unchanged. In the second term, both atoms are excited, and the field remains in the state $|X_{AB}\rangle \equiv \Phi_A^+ \Phi_B^+ |0\rangle$. Since $|X_{AB}\rangle$ contains either two or zero photons, it describes either an emission of two photons, or an exchange of a single “off-shell” photon between the atoms (Fig. 2). Finally, the last two terms describe an emission of one photon of either atom $A$ or $B$. In this case the final state of the field is $|E_A\rangle \equiv \Phi_A^+ |0\rangle$, or $|E_B\rangle \equiv \Phi_B^+ |0\rangle$, respectively.

![FIG. 1.](attachment:image1.png)

FIG. 1. The horizontal direction is space, and the vertical direction time. The atoms, $A$ and $B$, are initially in their ground state (\downarrow). They are coupled for time $T$ to the field, and then Purified (P).

![FIG. 2.](attachment:image2.png)

FIG. 2. Emission and exchange processes.

Tracing over the field degrees of freedom, and using the basis $\{i, j\} = \{|\downarrow\rangle, |\uparrow\rangle, |\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle\}$, and the notation $|X_{AB}|^2 = \langle X_{AB}|X_{AB}\rangle$, I find that to the lowest order the atoms (reduced) density matrix

$$\rho = \begin{pmatrix}
-\langle 0|X_{AB}\rangle & \langle 0|X_{AB}\rangle & 0 & 0 \\
\langle 0|X_{AB}\rangle & \langle X_{AB}|^2 & \langle E_A|^2 & \langle E_B|E_A\rangle \\
0 & 0 & \langle E_A|^2 & \langle E_B|E_A\rangle \\
0 & 0 & \langle E_B|E_A\rangle & \langle E_B|^2
\end{pmatrix}$$

(6)

Note two types of off-diagonal matrix elements. In the upper block, the amplitude $\langle 0|X_{AB}\rangle$ acts as to maintain coherence between the $|A\downarrow B\rangle$ and the $|A\uparrow B\rangle$ states. In the lower block, $\langle E_A|E_B\rangle$ acts to maintain coherence between $|A\downarrow B\rangle$ and $|A\uparrow B\rangle$. The relative magnitude of these off-diagonal terms, compared to the diagonal (decoherence) terms determines if the density matrix is separable.

A sufficient condition [10] for inseparability is that the matrix obtained by taking the partial transpose [11] of $\rho$, is non-positive. (For a $2 \times 2$ system this is also a necessary condition [12]). I find that $\rho$ is inseparable if either of the following inequalities is satisfied

$$|\langle 0|X_{AB}\rangle|^2 > |E_A|^2 |E_B|^2$$

(7)

or

$$|\langle E_B|E_A\rangle|^2 > |X_{AB}|^2$$

(8)

The first inequality, [10], amounts for the requirement that the single virtual photon exchange process, is more probable than an emission of one photon by each atom. Inseparability is then induced by states like $|A\downarrow B\rangle + \alpha |A\uparrow B\rangle$. Considering the second inequality [12], note that $\langle E_A|E_B\rangle$ measures the overlap of a photons emitted by atom $A$ and $B$. Hence it demands that this overlap is
larger than $|X_{AB}|^2$. When the second condition is met, the main contribution to the entanglement arises from states like $|\uparrow_A\downarrow_B\rangle + \beta|\downarrow_A\uparrow_B\rangle$.

So far I have considered only one possible initial state of the atoms. What happens if we start with $|\uparrow_A\downarrow_B\rangle$ or $|\downarrow_A\uparrow_B\rangle$? Repeating the calculation, and using the Cauchy-Schwartz inequality on the left-hand side of (3), I get a necessary condition for inseparability: $(0|\Phi_+^{\uparrow\uparrow}_+|0) > (0|\Phi_+^{\uparrow\downarrow}_+|0)$, where $i$ corresponds to the atom which is initially excited. But this demands that the excitation probability is larger then the de-excitation probability: $P(\uparrow\rightarrow\downarrow) > P(\downarrow\rightarrow\uparrow)$, which cannot be satisfied in vacuum. Hence eq. (7) can be satisfied only if both atoms are initially in their ground state; otherwise, decoherence effects due to single photon emissions dominates over the exchange process. (Similar conclusions can be reached for (8). Here one has to choose initial states like $|\uparrow_A\uparrow_B\rangle$.

In the following I will focus on the first inequality (9), which is simpler to evaluate. Specializing to the case $|\Phi_i\rangle = |\downarrow_A\downarrow_B\rangle|0\rangle$, substituting $\phi(x,t)$, and integrating over time eq. (9) can be re-expressed as

$$\int_0^\infty \frac{d\omega}{L} \sin(\omega L)\tilde{\epsilon}(\omega - \Omega)\tilde{\epsilon}(\omega + \Omega) > \int_0^\infty \omega d\omega|\tilde{\epsilon}(\Omega + \omega)|^2$$

(9)

where $\tilde{\epsilon}(\omega)$ is the Fourier transform of $\epsilon(t) = \epsilon_i(t)$.

The right hand side in the above inequality is independent of $L$ and tends to zero as $\Omega T \rightarrow \infty$. The left hand side, depends on both $T$ and $L$ and decays like $\sim 1/L^2$ for $L > T$. However for $\Omega L$ not too big, $\tilde{\epsilon}(\omega - \Omega)$ has a sharp peak near $\omega = \Omega$, which enhances the exchange amplitude. This suggests that there may exist a finite window of frequencies around some $\Omega^{-1} \sim T \sim L$, where (9) can be satisfied.

The following plots exhibit the ratio $X_{AB}/E_A$, for the window function (with $T = 1$)

$$\epsilon(t) = \begin{cases} \cos^2(\pi t), & \text{for } |t| \leq 1/2 \\ 0, & \text{for } |t| > 1/2 \end{cases}$$

(10)

as a function of the ground energy gap $\Omega$ and the separation $L$ between the atoms.

**FIG. 3.** The ratio $X/E$, with $L = T = 1$, as a function of the energy gap $\Omega$.

**FIG. 4.** The ratio $X/E$, with $T = 1$ and $\Omega = 9.5$, as a function of the distance.

It follows from Fig. 3. that eq. (6) is satisfied for $8 < \Omega < 11$. For a different distance $L$, one has to employ atoms with appropriate $\Omega = O(1/L)$. It follows from Fig. 4. that the spatial region where entanglement persists, extends up to $L/T < 1.1$. This implies that the maximal space-like separation between the pair of space-time regions that affect the atoms can be extended up to $L - T \sim 0.1L$. These regions are generally much larger than the latter maximal space-like separation, hence the atoms may be viewed as contained in the space-like range of a single coherent vacuum fluctuation. Pictorially, the interaction with this single vacuum fluctuation, in some sense, “ties” the atoms together to an entangled state.

So far I have considered stationary atoms. However, it turns out that the problem becomes simpler to handle when the pair of atoms are uniformly accelerating along two mirror hyperbolic trajectories. A detailed description of this case will be given elsewhere [13]. Here I will use this set-up to provide an exact demonstration that the inequality eq. (9) can be satisfied.

Let the atoms A and B follow the trajectories

$$x_A = -\frac{L}{2} \cosh \frac{2\tau_A}{L}, \quad x_B = \frac{L}{2} \cosh \frac{2\tau_B}{L}$$

$$t_A = \frac{L}{2} \sinh \frac{2\tau_A}{L}, \quad t_B = \frac{L}{2} \sinh \frac{2\tau_B}{L}$$

(11)

and $y_A = y_B$, $z_A = z_B$. As can be seen, the trajectories of $A$ and $B$ are confined to the two complementary space-like wedges $x < -|t|$ and $x > t$, respectively. The acceleration along each trajectory is $a = 2/L$, and the proper times $\tau_A$ and $\tau_B$ in the rest frames of $A$ and $B$ have been used to parameterize the trajectories.

To adapt the interaction Hamiltonian (2) to the present case, one has to replace the time parameter by the proper times of each atom, hence send $t_i \rightarrow \tau_i$ and $\phi \rightarrow \phi(x(\tau), t(\tau))$. The emission term then reads

$$|E_A|^2 = \int d\tau_A \int d\tau_A e^{-\Omega(\tau_A - \tau_A)} D^+(A', A)$$

(12)

and the exchange term

$$\langle 0|X_{AB} \rangle = \int d\tau_A \int d\tau_B e^{i\Omega(\tau_A + \tau_B)} D^+(A, B)$$

(13)
where $D^+(x', x) = \langle 0 | \phi(x', t') \phi(x, t) | 0 \rangle = -\frac{1}{2\pi}(t' - t - i\epsilon)^2 - (x' - x)^2$ is the Wightman function. Substituting $x(\tau)$ and $t(\tau)$ one gets

$$D^+(A', A) = -\frac{1}{4\pi^2 L^2 \sinh^2 \left[ (\tau_A' - \tau_A - i\epsilon)/L \right]}$$

and when the points are on different trajectories

$$D^+(B, A) = \frac{1}{4\pi^2 L^2 \cosh^2 \left[ (\tau_B + \tau_A - i\epsilon)/L \right]}$$

The integral [12], for the emission probability, can be performed by complexifying $\tau_A' - \tau_A$ to a plane and closing the contour in the lower complex plane. This picks up the poles $\tau_A - \tau_A' = i\epsilon + i\pi nL$ with $n = -1, -2...$. On the other hand, the contour for the exchange integral [13] should be closed on the upper half plane. This picks up the contributions at $(\tau_A + \tau_B) = i\epsilon + i\pi(n + \frac{1}{2})L$ with $n = 0, 1, 2...$ The ratio between the two terms is then

$$\frac{\langle 0 | X_{AB} \rangle}{|E_A|^2} = \frac{e^{-\pi i L/2} \sum_{n=0}^{\infty} e^{-\pi nL} \sum_{n=1}^{\infty} e^{-\pi nL}}{\sum_{n=1}^{\infty} e^{-\pi nL}} = e^{\pi i L/2}$$

Therefore (10) is always satisfied. Unlike the previous stationary case, this ratio can become arbitrary large, while $X_{AB}$ and $E_A$ becoming exponentially small. The reason for that is that for the hyperbolic trajectories we can have $\tau \Omega \rightarrow \infty$ while keeping $\Omega L$ finite. By increasing $L$, the emission probability decreases like $|E_A|^2 \sim e^{-\pi i L}$. However $\langle 0 | X_{AB} \rangle \sim e^{-\pi i L/2}$ decreases slower, hence the ratio (11) increases exponentially.

I have so far discussed methods for extracting vacuum entanglement into a pair of atoms. However, can one further distill identically prepared pairs to a maximally entangled state? It turns out that for our case, of a $2 \times 2$ system, the answer is positive for any inseparable mixed state [15]. Hence, two observers can preset a definite procedure, (e.g. fix $\epsilon_i$ and $L$), so that the resulting mixed state is known to both. After generating a sample of entangled pairs they will employ classical communication and local quantum operations to distill a certain number of E-bits. The maximal number of E-bits which can be distilled, per given space-like separated regions, constitutes a lower bound for the “density” of vacuum entanglement. This number is bounded; once too many atoms are used, mutual disturbances will presumably destroy the pairwise inseparability.

In conclusion, I have shown that by means of local interactions between a quantized massless field and a pair of atoms, entanglement can be extracted from the vacuum and delivered to the atoms. This applies for any separation $L$ provided that one sets $\Omega \sim 1/L$. (For massive fields the exponential decay of the correlations demands $L \sim 1/m$.) This process may be interpreted either as an exchange of a (superluminal) off-shell virtual photon, or as an interaction with a single vacuum fluctuation whose coherence extends over space-like regions. The question of how the net amount of entanglement is conserved in this processes is still open. By turning off and on the interaction, energy (and photons) are fed into the vacuum. This, presumably, acts as to lower the entanglement in the field. A detailed study of this “balance” of entanglement may provide new insights into the question of “entanglement thermodynamics” [13].

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[1] J. S. Bell, Physics 1, 195 (1964).
[2] A density matrix is said to be inseparable if it cannot be expressed as $\sum_i \rho_i^{AB} \rho_i^{AB}$, where $\rho_i^{AB}$ are density matrices for the two subsystems $A$ and $B$.
[3] C.H. Bennett, H.J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A 53 2-46 (1996).
[4] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin and W. K. Wootters, Phys. Rev. Lett. 72, 722 (1996).
[5] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[6] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
[7] W. G. Unruh, Phys. Rev. D 14, 870 (1989).
[8] S. J. Summers and R. Werner, Phys. Lett. 110, 257 (1985), J. Math. Phys. 28 2440 (1987); 28, 2448 (1987).
[9] M.G.L. Redhead, Found. Phys. 25, 123, (1995), M.G.L. Redhead and F. Wagner Found. Phys. Lett. 11, 111, (1998), R. Clifton, D. Feldman, M.L.G Redhead, and A. Wilce, Phys. Rev. A58 .135 (1998).
[10] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
[11] Let $\rho_{ij,kl}$ be the matrix elements with respect to the basis $|i\rangle |j\rangle$ of the two atoms. The partial transposition takes $\rho_{ij,kl} \rightarrow \rho_{ij,kl}.$
[12] M. Horodecki, P. Horodecki and R. Horodecki, Phys. Lett. A 223, 1 (1996).
[13] B. Reznik, in preparation.
[14] N. D. Birrell and P.C. W. Davies, Quantum Fields in Curved Space”, Cambridge Univ. Press, Cambridge, 1982, page 53.
[15] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. 78, 574 (1997).
[16] S. Popescu and D. Rohrlich, Phys. Rev. A 56, 3219 (1997).