Searching for family-number conserving neutral gauge bosons from extra dimensions

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Previous studies have shown how the three generations of the Standard Model fermions can arise from a single generation in more than four dimensions, and how off-diagonal neutral couplings arise for gauge-boson Kaluza-Klein recurrences. These couplings conserve family number in the leading approximation. While an existing example, built on a spherical geometry, suggests a high compactification scale, we conjecture that the overall structure is generic, and work out possible signatures at colliders, compatible with rare decays data.

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1. INTRODUCTION.

One reason to invoke more than four space-time dimensions is to obtain elegant solutions for several long-standing problems of particle physics [1] (see Ref. [2] for a review). In particular, in the frameworks of "large extra dimensions", models have been suggested [3, 4] and studied [5] where three generations of the Standard Model (SM) fermions appear as three zero modes localized in the four-dimensional core of a defect with topological number three. When both fermions and Higgs boson are localized on a brane, the overlaps of their wave functions may result in a hierarchical pattern of fermion masses and mixings [6]. This occurs naturally in the models under discussion [3]. If the gauge fields are not localized on a brane (localization of them is a complicated issue [7]), then their Kaluza-Klein (KK) modes mediate flavour-changing processes. For the case of the compactification of two extra dimensions on a sphere [8], and with one particular pattern of charge assignments, the constraints from a flavour violation were discussed in Ref. [9]. A distinctive feature of the models of this class is the (approximate) conservation of the family number. This note aims to discuss, without appealing to a particular model, the phenomenology of flavour-violating KK bosons in this class of theories, to be searched in future experiments.

2. DISTINCTIVE FEATURES OF "SINGLE-GENERATION" EXTRA-DIMENSIONAL MODELS

If our four-dimensional world is nothing but a core of a topological defect in (4 + D) dimensions, then specific interactions of matter fields with the defect may induce localization of massless modes of these fields inside the core of the defect. Identification of the SM fields with these (almost massless, compared to the scale of the defect) modes allows the extra dimensions to be large but unobserved (see Ref. [2] for a review and list of references). In particular, the index theorem guarantees the existence of N linearly independent chiral zero modes of each fermion field in the bosonic background with topological number N. This suggests to use N = 3 to obtain three generations of the SM fermions from a single one in extra dimensions. Quite non-trivially, the linear independence of the three modes results in their different behaviour at the origin, which may give rise to a naturally hierarchical pattern of masses of the fermions of three generations. We concentrate here on the most elaborated example of two extra dimensions (D = 2), though our qualitative results hold for a more involved case of higher dimensions as well.

With D = 2, the two extra dimensions can be parametrized in terms of one radial, r, and one angular, φ, variables. The location of the defect corresponds to r = 0; we suppose that the compactification preserves rotational invariance and allow φ to vary from 0 to 2π. The defect itself has a structure of the U(1) vortex. The definition of r and its maximal value depend on the compactification scheme. The three light four-dimensional families of particles arising from a single family in six dimensions are characterized then by different winding

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properties in $\phi$: three families enumerated by $n = 1, 2, 3$ have the following wave functions:

$$\psi_n \sim f_n(r)e^{i(3-n)\phi}.$$  

These wave functions correspond, in four dimensions, to the gauge eigenstates of the SM fermions. To the first approximation, both the theory and the background possess rotational invariance (shifts in $\phi$ supplemented by $U(1)$ transformations). The fermion mass matrix originates from a $\phi$-independent scalar field, and is thus perfectly diagonal, while the mass spectrum results from the (in principle calculable) overlap of the wave functions of the scalar and fermions, giving the usual hierarchy between families. At this level, the mass and gauge eigenstates coincide, the family number corresponds to the six-dimensional angular momentum and is thus exactly conserved (note that this still does not forbid processes where both quark and lepton flavours change oppositely, e.g. $K \to \mu\bar{\epsilon}$). Mixing between fermions of different species, leading to the desired Kobayashi-Maskawa (K-M) matrix, arises as a suppressed, second-order effect controlled by an auxiliary scalar field with winding number one, which generates transitions between adjoining generations.

We will be concerned here with the gauge interactions. The lowest mode of the gauge bosons in four dimensions left massless by the vortex localization of the fermions, eventually acquires mass by the Brout-Englert-Higgs formalism. The electrically neutral such bosons stand as usual diagonal in their interactions with the fermionic mass eigenstates. The charge universality is provided by the fact that the lowest mode of a gauge boson is constant in transverse dimensions and overlap integrals of the normalized fermionic wave functions with this mode coincide with each other. This is not the case for the higher KK modes of the vector particles; their profiles in $(r, \phi)$ are determined by

$$A_{lm}(r, \phi) = a_{lm}(r)e^{im\phi},$$

where $l = 1, 2, \ldots$ and $-l \leq m \leq l$. Non-trivial profiles $a_{lm}$ cause different overlaps with fermions of different families while non-zero windings result in transitions between generations.

Angular excitation of, for example, the first KK mode of $Z$-boson behaves in six dimensions as

$$Z' \sim e^{\pm i\phi}.$$  

After integration in extra dimensions we obtain an effective four-dimensional Lagrangian with complex vector field $Z'$, which generates "horizontal" transitions between families in which the generation number changes by one unit.

Such transitions are of course severely limited by the high mass of the excitations, but also, in the first approximation (neglecting of the K-M mixing), they do conserve the family number. For instance, the following processes are possible:

$$s + \bar{d} \Rightarrow Z' \Rightarrow s + \bar{d},$$

$$s + \bar{d} \Rightarrow Z' \Rightarrow \mu + \bar{\tau},$$

$$s + \bar{d} \Rightarrow Z' \Rightarrow \tau + \bar{\mu}$$

The first process in the first order in $Z'$ exchange thus conserves strangeness (and only small corrections linked to Cabibbo mixing would affect this), but the second, while conserving "family number", is a typical flavour-changing neutral current (FCNC) interaction, violating both strangeness and electron number. While the last reaction is only possible in collisions, the study of rare $K_L$ decay puts strong limits on the mass and coupling constant of the $Z'$ [9] (similar relations hold for the photon and gluon angular excitations).

For the time being, we wish to retain these main characteristics of the model: families are associated to some "winding number", conserved in excited boson exchanges up to small Kobayashi-Maskawa corrections. The detailed spectrum and strength of coupling of the gauge boson excitations will depend on the exact geometrical implementation. A fully worked-out example was presented in details in Ref. [9], leading however to a particularly high mass spectrum.

We conjecture that the same structure would remain intact in other implementations. In Ref. [9] we supposed that the wave functions of fermions and the first KK mode of the gauge boson overlap strongly. Then the effective Lagrangian for the interaction between fermions and flavour-changing bosons contains the same coupling constant, as interaction with the lowest KK-modes, i.e. the usual gauge bosons. However, in particular models the profiles of fermionic wave functions can be shifted, which means more freedom in couplings. Let us denote the absolute value of the overlap integral in extra dimensions between the wave functions $\psi_i, \psi_j$ of the fermions of generations $i, j$ and the wave function $\psi_{Z'}$ of the $Z$ excitation as

$$\left| \int \psi_{Z'}\psi_i\psi_j d^2x \right| = \kappa_{ij}.$$  

Then $(\bar{e}, \mu)$-interaction through $Z'$ is described by:

$$\frac{g_{EW}\kappa_{12}}{2\cos\theta_W}Z'_\mu \left[ \frac{1}{2}\bar{e}\gamma_\mu\gamma_5\mu - \left( \frac{1}{2} - 2\sin^2\theta_W \right) \bar{e}\gamma_\mu\gamma_5\mu \right].$$

The structure of this term coincides with the interaction of $\bar{e}, e$ and $Z$ in SM with the strength $g = g_{EW}\kappa_{12}$. In-
terations of other leptons and quarks arise in a similar way.

The main restriction on the mass scale of the model with \( \kappa_{ij} \approx \delta_{i,i+1} \) arises from the limit on the branching ratio for the process \( K_{L} \to \mu e \). Taking into account that \( \kappa_{12} \) can be different from 1, the strongest restriction from the rare processes gives

\[
M_{Z'} \gtrsim \kappa_{12} \cdot 100 \text{TeV}.
\]

In the simplest case when all \( \kappa_{ij} \sim \kappa' \cdot \delta_{i,i+1} \),

\[
\kappa' \lesssim \frac{M_{Z'}}{100 \text{TeV}}.
\]

The decay width of the excited Z and photon results mainly from their decay into fermions (with the possibility of model-dependent additional scalar decay channels), and, by simple counting of modes, is estimated as

\[
\Gamma(Z') = \kappa^2 \cdot \frac{M_{Z'}}{M_{Z}} \cdot 12.5 \cdot \Gamma_{Z \to \mu e} \approx \kappa^2 \cdot \frac{M_{Z'}}{M_{Z}} \cdot 1.8 \text{GeV}.
\]

Similarly, the width of the first photon angular excitation is given by

\[
\Gamma(\gamma') = \frac{16}{3} \kappa^2 \sin^2 2\theta_W \cdot \frac{M_{\gamma'}}{M_{Z}} \cdot \Gamma_{Z \to \mu e} \approx \kappa^2 \cdot \frac{M_{\gamma'}}{M_{Z}} \cdot 1.3 \text{GeV}.
\]

The first KK excitation of the gluon is wider due to the large coupling constant,

\[
\Gamma(G') \approx \kappa'^2 \frac{M_{G'}}{M_{Z}} \cdot 7.2 \text{GeV}.
\]

A typical value of \( \Gamma \) is of order \( 10^{-3} \text{GeV} \) for \( \kappa \approx 10^{-2} \) and \( M_{Z'} = 17 \text{TeV} \). In what follows we will assume that the masses of all the FCNC bosons are equal:

\[
M_{Z'} = M_{\gamma'} = M_{G'} = M,
\]

as in the case of spherical model of Ref [9].

### 3. COLLIDER SEARCHES

The vector bosons discussed here can, in principle, be observed at colliders due to the flavour-changing decay modes into \((\mu e)\) and \((\tau \mu)\) pairs. The corresponding process is very similar to the Drell-Yan pair production. A typical feature of the latter is the suppression of the cross section with increasing of the resonance mass at a fixed center-of-mass energy.

The flavour-changing decays of this kind have a distinctive signature: antimuon and electron (or their antiparticles) with equal and large transverse momenta in the final state.

We estimate the number of events for the case of \( pp \) collisions with the help of the CompHEP package [10]. For our calculation we use the expected LHC value of 100fb\(^{-1}\) for luminosity and \( \sqrt{s} = 14\text{TeV} \). The number of \((\mu^+ e^-)\) events is presented at Fig.1 for different values of the vector bosons mass \( M \) and \( \kappa \) adjusted to \( \kappa = M/(100 \text{TeV}) \). The same plot for \((\mu^- e^+)\) pairs is given at Fig.2

Note that production of \((\mu^+ e^-)\) pairs is more probable than \((\mu^- e^+)\) because the former process can use valence \( u \) and \( d \)-quarks in the proton, while the second only involves partons from the sea. The same numbers are representative also for the \((\mu^- \tau^+)\) channel.

There are also other signatures of FCNC effects, in particular, with hadronic final states, when \((\bar{t},s)\) or \((\bar{b},s)\) jets are produced. The dominant contribution to these processes arises from the interactions with higher KK modes of gluons, which have large coupling constant. For the mass of \( M_{G'} = 17 \text{TeV} \) we estimate the number of events as \( N = 1.2 \cdot 10^3 \). But potentially large
SM backgrounds should be carefully considered for such channels.

4. CONCLUSIONS

We have considered FCNC effects in models with approximate family-number conservation, mediated by the heavy vector bosons in a class of models. From our estimations, there is a reason for searching for such FCNC bosons with masses of order 1 TeV at LHC. The main signature is the production of \((\mu^+ e^-)\) or \((\mu^- \tau^+)\) pairs with equal and large transverse momenta of leptons. Production of \((\bar{t}c)\) quarks is more probable, but less clear-cut due to the large background from SM processes.

On the other hand, the models with heavy vector bosons, whose interactions conserve the family number, can be tested in experiments studying rare processes. The strongest and the least model-independent limit on the mass of these bosons arises from the limit on \(K_L \to \mu^+ e^-\) branching ratio (in this process, the family number does not change). Discovery of this decay without signs of rare processes which violate the generation number (such as \(\mu - e\)-conversion) would support significantly the models discussed here. Future experiments on the search of lepton-flavour violating kaon decays are thus of great importance (see Ref. [11] for relevant discussion).

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1. N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys.Lett. B429, 263 (1998), hep-ph/9803315.
2. V. A. Rubakov, Usp.Fiz.Nauk 171, 913 (2001), hep-ph/0104152.
3. M. V. Libanov and S. V. Troitsky, Nucl.Phys. B599, 319 (2001), hep-ph/0011095.
4. J. M. Frere, M. V. Libanov and S. V. Troitsky, Phys.Lett. B512, 169 (2001), hep-ph/0012306.
5. J. M. Frere, M. V. Libanov and S. V. Troitsky, JHEP 0111, 025 (2001), hep-ph/0110045.
6. J. M. Frere, M. V. Libanov and S. V. Troitsky, JHEP 0204, 055 (2002), hep-ph/0201162.
7. G. Dvali, M. Shifman, Phys.Lett. B475, 295 (2000), hep-ph/0001072.
8. J. M. Frere, M. V. Libanov, E. Y. Nugaev and S. V. Troitsky, JHEP 0306, 009 (2003), hep-ph/0304117.
9. J.-M. Frere, M. V. Libanov, E. Y. Nugaev and S. V. Troitsky, JHEP03(2004)001, hep-ph/0309014.
10. A. Pukhov et al., arXiv:hep-ph/9908288.
11. L. G. Landsberg, Phys. Atom. Nucl. 65, 1749 (2002), Yad. Fiz. 65, 1795 (2002).