Fifth Force and Hyperfine Splitting in Bound Systems

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Two recent experimental observations at the ATOMKI Institute of the Hungarian Academy of Sciences (regarding the angular emission pattern of electron-positron pairs from nuclear transitions from excited states in $^4$Be and $^4$He) indicate the possible existence of a particle of a rest mass energy of roughly 17 MeV. The so-called X17 particle constitutes a virtual state in the process, preceding the emission of the electron-positron pair. Based on the symmetry of the nuclear transitions ($1^+ \rightarrow 0^+$ and $0^- \rightarrow 0^+$), the X17 could either be a vector, or a pseudoscalar particle. Here, we calculate the effective potentials generated by the X17, for hyperfine interactions in simple atomic systems, for both the pseudoscalar as well as the vector X17 hypotheses. The effective Hamiltonians are obtained in a general form which is applicable to both electronic as well as muonic bound systems.

Because of the short range of the X17-generated potentials, the most promising pathway for the observation of the X17-mediated effects in bound systems concerns hyperfine interactions, which, for $S$ states, are given by modifications of short-range (Dirac-$\delta$) potentials in coordinate space. For the pseudoscalar hypothesis, the exchange of one virtual X17 quantum between the bound lepton and the nucleus exclusively leads to hyperfine effects, but does not affect the Lamb shift. Effects due to the X17 are shown to be drastically enhanced for muonic bound systems. Prospects for the detection of hyperfine effects mediated by X17 exchange are analyzed for muonic deuterium, muonic hydrogen, true muonium ($\mu^+\mu^-$ bound system), and positronium.

I. INTRODUCTION

For decades, atomic physicists have tried to push the accuracy of experiments and theoretical predictions of transitions in simple atomic systems higher [1]. The accurate measurements have led to stringent limits on the time variation of fundamental constants [2–4], and enabled us to determine a number of important fundamental physical constants [5] with unprecedented accuracy. Yet, a third motivation, hitherto not crowned with success, has been the quest to find signs of a possible low-energy extension of the Standard Model, based on a deviation of experimental results and theoretical predictions.

Recently, the possible existence of a fifth-force particle, commonly referred to as the “X17” particle because of the observed rest mass of 16.7 MeV, has been investigated in Refs. [2–4], based on a peak in the emission spectrum of electron-positron pairs in nuclear transitions of excited helium and beryllium nuclei. Two conceivable theoretical explanations have been put forward, both being based on low-energy additions to the Standard Model. The first of these involves a vector particle (a “massive, dark photon”, see Refs. [8, 9]), and the second offers a pseudoscalar particle (see Ref. [10]), which couples to light fermions as well as hadrons.

Somewhat unfortunately, the rest mass range of 16.7 MeV makes the X17 particle hard to detect in atomic physics experiments. The observed X17 rest mass energy is larger than the binding energy scale for both electronic as well as muonic bound systems [11]. Even more importantly, the Compton wavelength of the X17 particle (about 11.8 fm) is smaller than the effective Bohr radius for both electronic as well as muonic bound systems. Because the Compton wavelength of the X17 particle determines the range of the Yukawa potential, the effects of the X17 are hard to distinguish from nuclear size-effects in atomic spectroscopy experiments [11].

We recall that the Bohr radius amounts to $a_0 \sim 5 \times 10^{-11}$ fm, while the effective Bohr radius of a muonic hydrogen atom is $a_0 \sim \hbar/(\alpha m_e c) \sim 256$ fm. It is thus hard to find an atomic system, even a muonic one, where one could hope to distinguish the effect of the X17 particle on the Lamb shift from the nuclear-finite-size correction to the energy. A possible circumvention has been discussed in Ref. [11], based on a muonic carbon ion, where the effective Bohr radius approaches the range of the Yukawa potential induced by the X17, in view of the larger nuclear charge number. However, it was concluded in Ref. [11] that considerable additional effort would be required in terms of an accurate understanding of nuclear-size effects, before the X17 signal could be extracted reliably.

The definition of the Lamb shift $\mathcal{L}$, as envisaged in Ref. [12] and used in many other places, e.g., in Eq. (67) of Ref. [13], explicitly excludes hyperfine effects. Conversely, hyperfine effects, at least for $S$ states, are induced, in leading order, by the Dirac-$\delta$ peak of the magnetic dipole field of the atomic nucleus at the origin [see Eq. (9) of Ref. [14]]. The Fermi contact interaction, which gives rise to the leading-order contribution to the hyperfine splitting

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for $S$ states, is proportional to a Dirac–δ in coordinate space, commensurate with the fact that the atomic nucleus has a radius not exceeding the femtometer scale. The effect of short-range potentials is thus less suppressed when we consider the hyperfine splitting, as compared to the Lamb shift. The aficionados of bound states thus realize that, if we consider hyperfine effects, we have a much better chance of extracting the effect induced by the X17, which, on the ranking scale of the contributions, occupies a much higher place than for the Lamb shift alone. Despite the large mass of the X17 particle, which leads to a short-range potential proportional to $\exp(-m_X r)$ (in natural units with $\hbar = c = e_0 = 1$, which will be used throughout the current paper), the effect of the X17 could thus be visible in the hyperfine splitting in muonic atoms.

Here, we shall elaborate on this idea, and derive the leading corrections to the hyperfine splitting of $nS$, $nP_{1/2}$ and $nP_{3/2}$ states in ordinary as well as muonic hydrogenlike systems, due to the X17 particle, by matching the nuclear-spin dependent terms in the scattering amplitude with the effective Hamiltonian. Anticipating some results, we can say that the relative correction (expressed in terms of the leading Fermi term) is proportional to $m_3 R / m_X$, where $m_X$ is the reduced mass of the two-body bound system, and thus enhanced for muonic in comparison to electronic bound systems.

This paper is organized as follows. In Sec. II we summarize the interaction Lagrangians for both a hypothetical X17 vector exchange \( \mathcal{L}_{X,V} \), as well as a pseudoscalar exchange \( \mathcal{L}_{X,S} \), with corresponding conventions for the coupling parameters. In Sec. III we derive the effective hyperfine Hamiltonians for both vector and pseudoscalar exchanges. In Sec. IV we evaluate general expressions for the corrections to hyperfine energies induced by the X17 particle, for \( S \) and \( P \) states. In Sec. V we derive bounds on the coupling parameters for both models in the muon sector, based on the muon $g$ factor. Finally, in Sec. VI we apply the obtained results to muonic hydrogen, muonic deuterium and true muonium (bound $\mu^-\mu^-$ system), and discuss the measurability of the X17 effects in the hyperfine structure of the mentioned atomic systems. Conclusions are reserved for Sec. VII.

## II. INTERACTION LAGRANGIANS

In the following, we intend to study both the interaction of X17 vector and pseudoscalar particles with bound leptons (electrons and muons) and nucleons (protons and deuterons). Vector interactions will be denoted by the subscript $V$, while pseudoscalar interactions will carry the subscript $S$, while pseudoscalar interactions will carry the subscript $S$. The interaction Lagrangian $L_{X}^{f}$ for the interaction of an X17 vector boson with the fermion fields $f = e, \mu$ (electron and muon) and the nucleons $N = p, n$ (proton and neutron) as follows,

\[
L_{X,V} = - \sum_{f} \varepsilon_{f} \bar{\psi}_{f} \gamma^{\mu} \psi_{f} X_{\mu} - \sum_{N} \varepsilon_{N} \bar{\psi}_{N} \gamma^{\mu} \psi_{N} X_{\mu}
\]

\[
= - \sum_{f} \varepsilon'_{f} \bar{\psi}_{f} \gamma^{\mu} \psi_{f} X_{\mu} - \sum_{N} \varepsilon'_{N} \bar{\psi}_{N} \gamma^{\mu} \psi_{N} X_{\mu} .
\]

where we follow the conventions delineated in the remarks following Eq. (1) of Ref. [8] and Eq. (10) of Ref. [9]. Here, $\varepsilon_{f}$ and $\varepsilon_{N}$ are the flavor-dependent coupling parameters for the fermions and nucleons, while $e = -\sqrt{4\pi\alpha} = -0.091$ is the electron charge. The fermion and nucleon field-operators (the latter, interpreted as field operators for the composite particles) and denoted as $\bar{\psi}_{f}$ and $\psi_{N}$, while the $X_{\mu}$ is the X17 field operator. For reasons which will become obvious later, we use, in Eq. (1), the alternative conventions,

\[
\varepsilon'_{f} = \varepsilon_{f} e, \quad \varepsilon'_{N} = \varepsilon_{N} e ,
\]

for the coupling parameters to the hypothetical X17 vector boson. Our conventions imply that for $\varepsilon_{N} > 0$, the coupling parameter $\varepsilon'_{N}$ parameterizes a “negatively charged” nucleon under the additional $U(1)$ gauge group of the vector $X$ particle.

According to a remark following the text after Eq. (9) of Ref. [9], conservation of $X$ charge implies that the couplings to the proton and neutron currents fulfill the relationships

\[
\varepsilon_{p} = 2\varepsilon_{u} + \varepsilon_{d}, \quad \varepsilon_{n} = \varepsilon_{u} + 2\varepsilon_{d} ,
\]

where the up and down quark couplings are denoted by the subscripts $u$ and $d$. Numerically, one finds [see Eqs. (38) and (39) of Ref. [9]] that the electron-positron field coupling $\varepsilon_{e}$ needs to fulfill the relationship

\[
2 \times 10^{-4} < \varepsilon_{e} < 1.4 \times 10^{-3} .
\]
Furthermore, in order to explain the experimental observations [6], one needs the neutron coupling to fulfill [see Eq. (10) of Ref. [8]]

\[ |\varepsilon_n| = |\varepsilon_u + 2\varepsilon_d| \approx \frac{3}{2} \varepsilon_d \approx \frac{1}{100}. \]  

(5)

Because the hypothetical X vector particle acts like a “dark photon” which is hardly distinguishable from the ordinary photon in the high-energy domain, the proton coupling \( \varepsilon_p \) is highly constrained. According to Eq. (8) and (9) of Ref. [9], and Eq. (35) of Ref. [9], one needs to have

\[ |\varepsilon_p| = |2\varepsilon_u + \varepsilon_d| \ll 8 \times 10^{-4}. \]  

(6)

This is why the conjectured X17 vector boson is referred to as “protophobic” in Refs. [8, 9].

Following Ref. [10], we write the interaction Lagrangian for the fermions interacting with the pseudoscalar candidate of the X17 particle as follows,

\[ \mathcal{L}_{\text{X,A}} = - \sum_f \bar{h}_f \gamma^5 \psi_f A - \sum_N \bar{h}_N \gamma^5 \psi_N A, \]  

(7)

where \( A \) is the field operator of the pseudoscalar field. Inspired by an analogy with putative pseudoscalar Higgs couplings [15], the pseudoscalar couplings have been estimated in Refs. [10, 15] to be of the functional form

\[ h_f = \xi_f \frac{m_f}{v}, \quad h_N = \xi_N \frac{m_N}{v}, \]  

(8)

where \( v = 246 \text{ GeV} \) is the vacuum expectation value of the Higgs (or Englert–Brout–Higgs, see Refs. [16, 17]) field, \( m_f \) is the fermion mass, and \( m_N \) is the nucleon’s mass. Furthermore, the parameters \( \xi_f \) and \( \xi_N \) could in principle be assumed to be of order unity. Note that the spin-parity of the Standard Model Higgs boson has recently been determined to be consistent with a scalar, not pseudoscalar, particle [18], but it is still intuitively suggested to parameterize the couplings to the novel putative pseudoscalar X17 in the same way as one would otherwise parameterize the couplings to the Higgs particle.

According to Eq. (2.7) and the remark following Eq. (3.12) of Ref. [10], the nucleon couplings can roughly be estimated as

\[ h_p = \frac{m_p}{v} (-0.40 \xi_u - 1.71 \xi_d) \approx -2.4 \times 10^{-3}, \]  

(9a)

\[ h_n = \frac{m_n}{v} (-0.40 \xi_u + 0.85 \xi_d) \approx 5.1 \times 10^{-4}, \]  

(9b)

where we have assumed \( \xi_u = \xi_d = 0.3 \). For the electron-positron field, based on other constraints detailed in Ref. [10], one has to require that [see Eq. (4.2) of Ref. [10]]

\[ \xi_e > 4, \quad h_e > \frac{4 m_e}{v} = 8.13 \times 10^{-6}. \]  

(10)

Based on a combination of experimental data [19] and theoretical considerations [20, 22], one can also derive an upper bound,

\[ \xi_e < 500, \quad h_e < \frac{500 m_e}{v} = 10^{-3}, \]  

(11)

which will be used in the following.

### III. MATCHING OF THE SCATTERING AMPLITUDE

In order to match the scattering amplitude (see Fig. 1) with the effective Hamiltonian, we use the approach outlined in Chap. 83 of Ref. [23], but with a slightly altered normalization for the propagators, better adapted to natural unit system (\( \hbar = c = \epsilon_0 = 1 \)). Specifically, we use the bispinors in the representation [cf. Eq. (83.7) of Ref. [23]]

\[ u_{f,N} = \begin{pmatrix} (1 - \frac{\vec{p}^2 f,N}{8m^2}) w_{f,n} \\ \vec{\sigma} \cdot \vec{p} \frac{w_{f,N}}{2m} \end{pmatrix}, \]  

(12)
FIG. 1. The one-quantum exchange scattering amplitude for the X17 particle is matched against the effective Hamiltonian, for the vector hypothesis [diagram (a)] and the pseudoscalar hypothesis [diagram (b)].

where $f$, $N$ stands for the bound fermion, or the nucleus, and $w_{f,n}$ are the nonrelativistic spinors. The massive “dark photon” propagator (for the X17 vector hypothesis) is used in the following normalization (we may ignore the frequency of the “dark photon” in the order of approximation relevant for the current article),

$$D_{00}(q) = -\frac{1}{q^2 + m_X^2}, \quad D_{ij}(q) = -\frac{1}{q^2 + m_X^2} \left[ \delta^{ij} - \frac{q^i q^j}{q^2 + m_X^2} \right].$$

(13)

The pseudoscalar propagator is used in the normalization

$$D_A(q) = -\frac{1}{q^2 + m_X^2},$$

(14)

where we also ignore the frequency. The scattering amplitude for the X17 vector particle reads as

$$M_{f,V} = \bar{u}'_f \gamma^0 u_f \left\{ (\bar{u}'_N \gamma^0 \bar{u}_N) D_{00} + (\bar{u}'_N \gamma^i \bar{u}_N) D_{ij} \right\},$$

(15)

and

$$M_{f,A} = \bar{u}'_f \gamma^5 u_f \left\{ (\bar{u}'_N i \gamma^5 \bar{u}_N) D_A \right\},$$

(16)

for the pseudoscalar case. Here, we denote the final states of the scattering process by a prime, $u'_f = u_f(p'_f)$, $u'_N = u_N(p'_N)$, while the initial states are $u_f = u_f(p_f)$ and $u_N = u_N(p_N)$, and the bar denotes the Dirac adjoint. Analogous definitions are used for the $w_{f,n}$ in Eq. (12). Furthermore, we have $p'_f + p'_N = p'_f + p'_N$. The momentum transfer is $q = p'_f - p_f = p'_N - p_N$.

The form (12) is valid for the bispinors if the Dirac equation is solved in the Dirac representation of the Dirac matrices,

$$\gamma^0 = \begin{pmatrix} 1_{2 \times 2} & 0 \\ 0 & -1_{2 \times 2} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & 1_{2 \times 2} \\ 1_{2 \times 2} & 0 \end{pmatrix}.$$  \hspace{1cm} (17)

The scattering amplitudes are matched against the effective Hamiltonian by the relation

$$M_{f} = -(w_f^+ w_f)(w_N^+ w_N) U(p'_f, p'_N, q),$$

(18)

where $U(p'_f, p'_N, q)$ is the the effective Hamiltonian. The scattering amplitude $M_{fi}$ is a matrix element involving four spinors, two of which represent the final and initial states of the two-particle system.

The scattering amplitude, evaluated between four spinors [cf. Eq. (83.8) of Ref. [23]], must now be matched against a Hamiltonian which acts on only one wave function in the end. We need to remember that the scattering amplitude corresponds to a matrix element of the Hamiltonian, which entails, even in the one-particle setting, two wave functions, not one. Then, going into the center-of-mass frame $q = p'_f - p_f = p'_N - p'_N$ means that the wave function is written
in terms of a center-of-mass coordinate $\vec{R}$, and a relative coordinate $\vec{r}$. In the center-of-mass frame, one eliminates the dependence on the center-of-mass coordinate $\vec{R}$ and the total momentum $\vec{P} = \vec{p}_f + \vec{p}_N$. Fourier transformation under the condition $\vec{p}_f' + \vec{p}_N' = \vec{p}_f + \vec{p}_N$ leads to the effective Hamiltonian [cf. Eq. (83.15) of Ref. [23]].

For the record, we note that in the X17 vector case, the 00 component of the photon propagator leads to the leading, spin-independent term in the effective Hamiltonian,

$$H_0 = \frac{k'_f k'_N}{4\pi r} \exp(-m_X r).$$  \hspace{1cm} (19)

Under the replacements $k'_f \to e$ and $k'_N \to -e$, in the massless limit $m_X \to 0$, one recovers the Coulomb potential,

$$H_0 \to -\frac{e^2}{4\pi r} = -\frac{\alpha}{r}.$$  \hspace{1cm} (20)

One finally extracts the terms responsible for the hyperfine structure, i.e., those involving the nuclear spin operator $\vec{\sigma}_N$, and obtains the following hyperfine Hamiltonian for a vector X17 particle,

$$H_{\text{HFS}, V} = \frac{k'_f k'_N}{16 \pi m_f m_N} \left[ -\frac{8\pi}{3} \delta^{(3)}(\vec{r}) \vec{\sigma}_f \cdot \vec{\sigma}_N \\
- \frac{m_X^2}{r^3} \left( \vec{\sigma}_f \cdot \vec{r} \vec{\sigma}_N \cdot \vec{r} - r^2 \vec{\sigma}_f \cdot \vec{\sigma}_N \right) e^{-m_X r} \\
- (1 + m_X r) \frac{3 \vec{\sigma}_f \cdot \vec{r} \vec{\sigma}_N \cdot \vec{r} - r^2 \vec{\sigma}_f \cdot \vec{\sigma}_N}{r^5} e^{-m_X r} \\
- \left( 2 + \frac{m_f}{m_N} \right) \frac{m_X r}{r^3} e^{-m_X r} \right].$$  \hspace{1cm} (21)

Taking the limit $m_X \to 0$, and replacing

$$k'_f \to e, \quad k'_f \to \frac{g_N (-e)}{2} = \frac{g_N |e|}{2}, \quad e^2 = 4\pi \alpha,$$  \hspace{1cm} (22)

one recovers the Fermi Hamiltonian $H_F$ [see Eq. (10) of Ref. [14]],

$$H_F = \frac{g_N \alpha}{m_f m_N} \left[ \frac{\pi}{3} \vec{\sigma}_f \cdot \vec{\sigma}_N \delta^{(3)}(\vec{r}) + \frac{3 \vec{\sigma}_f \cdot \vec{r} \vec{\sigma}_N \cdot \vec{r} - r^2 \vec{\sigma}_f \cdot \vec{\sigma}_N}{8 r^5} + \frac{\vec{\sigma}_N \cdot \vec{L}}{4 r^3} \right],$$  \hspace{1cm} (23)

where we have ignored the reduced-mass correction proportional to $m_f/m_N$ in the $\vec{\sigma}_N \cdot \vec{L}$ term in Eq. (21). For a pseudoscalar exchange, one has

$$H_{\text{HFS}, A} = \frac{k_f k_N}{16 \pi m_f m_N} \left[ \frac{4\pi}{3} \delta^{(3)}(\vec{r}) \vec{\sigma}_f \cdot \vec{\sigma}_N \\
- \frac{m_X^2}{r^3} \vec{\sigma}_f \cdot \vec{r} \vec{\sigma}_N \cdot \vec{r} e^{-m_X r} \\
+ (1 + m_X r) \frac{3 \vec{\sigma}_f \cdot \vec{r} \vec{\sigma}_N \cdot \vec{r} - \vec{\sigma}_f \cdot \vec{\sigma}_N}{r^5} e^{-m_X r} \right].$$  \hspace{1cm} (24)

Note that the Hamiltonian given in Eq. (24) constitutes the complete Hamiltonian derived from pseudoscalar exchange, which, in view of the $\gamma^5$ matrix in the Lagrangian given in Eq. (7), contributes only to the hyperfine splitting, but not to the Lamb shift, in leading order [i.e., via the exchange of one virtual particle, as given in Fig. 1(a)]. For a deuteron nucleus, the spin matrix $\vec{\sigma}_N$ has to be replaced by $2 \vec{I}_N$, where $\vec{I}_N$ is the spin operator of the deuteron, corresponding to the spin-1 particle.
IV. HYPERFINE STRUCTURE CORRECTIONS

In order to analyze the $S$ state hyperfine splitting, we extract from Eqs. (21) and (24) the terms which are nonzero when evaluated on a spherically symmetric wave function. This entails the replacements

\[
\tilde{\sigma}_f \cdot \tilde{r} \tilde{\sigma}_N \cdot \tilde{r} \rightarrow \frac{1}{3} \tilde{r}^2 \tilde{\sigma}_f \cdot \tilde{\sigma}_N ,
\]
\[
\tilde{\sigma}_N \cdot \tilde{L} \rightarrow 0 ,
\]
\[
H_{\text{HFS},V} \rightarrow - \frac{\hbar^2}{24 \pi m_f m_N} \left[ 4\pi \delta^{(3)}(\tilde{r}) - \frac{m_N^2}{r} e^{-m_N r} \right] , \tag{25c}
\]
\[
H_{\text{HFS},A} \rightarrow \frac{\hbar^2}{48 \pi m_f m_N} \left[ 4\pi \delta^{(3)}(\tilde{r}) - \frac{m_N^2}{r} e^{-m_N r} \right] . \tag{25d}
\]

The expectation value of the Fermi Hamiltonian is

\[
E_F(nS) = \langle nS_1/2 | H_F | nS_1/2 \rangle = g_{N} \frac{\alpha (Z\alpha)^3 m_N^3}{3 n^3 m_f m_N} \langle \tilde{\sigma}_f \cdot \tilde{\sigma}_N \rangle . \tag{26}
\]

Here, $Z$ is the nuclear charge number, and $m_r = m_f m_N / (m_f + m_N)$ is the reduced mass of the system. We use the nuclear $g$ factor in the normalization

\[
\bar{\mu}_N = g_N \frac{|e|}{2m_N} \tilde{\sigma}_N , \tag{27}
\]

which can more easily be extended to more general two-body systems than a definition in terms on the nuclear magneton. For the proton, one has $g_p = 5.5856 \ldots$ as the proton’s $g$ factor \[24, 25\], while the definition (27) implies that $g_d = 1.713 \ldots$ for the deuteron \[5\]. For true muonium ($\mu^+\mu^-$ bound system) and positronium, one has $g_N = 2$ according to the definition (27).

By contrast, in the limit $m_X \rightarrow \infty$, one verifies that

\[
\lim_{m_X \rightarrow \infty} \left\{ \frac{m_N^2}{r} e^{-m_N r} \right\} = 4\pi \delta^{(3)}(\tilde{r}) , \tag{28}
\]

and the two Hamiltonians given in Eqs. (25c) and (25d) vanish in the limit of a massless X17 particle. This implies that the expectation value of $S$ states of the Hamiltonians in Eqs. (25c) and (25d) have to carry at least one power of $m_X$ in the denominator, and in particular, that the correction to the hyperfine energy will be of order $(Z\alpha)^4$, not $(Z\alpha)^3$, as one would otherwise expect from the two individual terms in Eqs. (25c) and (25d), individually. For the vector hypothesis, one finds that $E_{X,V}(nS_1/2) = \langle nS_1/2 | H_{\text{HFS},V} | nS_1/2 \rangle$, for the leading and subleading terms in the expansion in inverse powers of $m_X$, can be expressed as

\[
E_{X,V}(nS_1/2) = \hbar^2 \frac{2(Z\alpha)^4}{3\pi n^3} \frac{m_f^4}{m_f m_N m_X} + \frac{5(Z\alpha)^5}{6\pi n^3} \left( 1 + \frac{1}{5n^2} \right) \frac{m_r^5}{m_f m_N m_X} \langle \tilde{\sigma}_f \cdot \tilde{\sigma}_N \rangle_{S_1/2,F} . \tag{29}
\]

We have neglected relative corrections of higher than first order in $\alpha m_f / m_X$ and $m_f / m_N$. For the pseudovector hypothesis, one finds that $E_{X,A}(nS_1/2) = \langle nS_1/2 | H_{\text{HFS},A} | nS_1/2 \rangle$ is given as follows,

\[
E_{X,A}(nS_1/2) = \hbar^2 \frac{2(Z\alpha)^4}{3\pi n^3} \frac{m_f^4}{m_f m_N m_X} - \frac{5(Z\alpha)^5}{6\pi n^3} \left( 1 + \frac{1}{5n^2} \right) \frac{m_r^5}{m_f m_N m_X^2} \langle \tilde{\sigma}_f \cdot \tilde{\sigma}_N \rangle_{S_1/2,F} . \tag{30}
\]

The $S$ state splitting is obtained from the following expectation values,

\[
\langle \tilde{\sigma}_f \cdot \tilde{\sigma}_N \rangle_{S_1/2,F=1} = 1 , \quad \langle \tilde{\sigma}_f \cdot \tilde{\sigma}_N \rangle_{S_1/2,F=0} = -3 . \tag{31}
\]

Expressed in terms of the leading term, given in Eq. (23), one has the following corrections due to the X17 particle,

\[
\frac{E_{X,V}(nS_1/2)}{E_F(nS_1/2)} \approx - \frac{2\hbar \hbar_N}{g_N \pi} \frac{Z m_r}{m_X} , \quad \frac{E_{X,A}(nS_1/2)}{E_F(nS_1/2)} \approx \frac{\hbar \hbar_N}{g_N \pi} \frac{Z m_r}{m_X} , \tag{32}
\]

depending on the vector ($V$) or pseudoscalar ($A$) hypothesis. One notices the different sign of the correction, depending on the symmetry group of the new particle. We observe that the relative correction to the Fermi splitting is enhanced.
for muonic bound systems, by a factor $m_e/m_X \sim m_\mu/m_X$ as compared to electronic bound systems, because the corresponding factor $m_e/m_X$ is two orders of magnitude smaller.

For $nP_{1/2}$ states, whose wave function vanishes at the nucleus in the nonrelativistic approximation, one finds for the first-order corrections $E_{X,V}(nP_{1/2}) = \langle nP_{1/2} | H_{\text{HFS},V} |nP_{1/2} \rangle$ and $E_{X,A}(nP_{1/2}) = \langle nP_{1/2} | H_{\text{HFS},A} |nP_{1/2} \rangle$,

\[
E_{X,V}(nP_{1/2}) = \frac{\hbar'_f \hbar'_N (Z\alpha)^5}{\pi n^3} \left(1 - \frac{1}{n^2}\right) \frac{m_f^5}{m_f m_N m_X} (\vec{\sigma}_f \cdot \vec{\sigma}_N)_{nP_{1/2},F},
\]

\[
E_{X,A}(nP_{1/2}) = \frac{\hbar_f \hbar_N (Z\alpha)^5}{2\pi n^3} \left(1 - \frac{1}{n^2}\right) \frac{m_e^5}{m_f m_N m_X^2} (\vec{\sigma}_f \cdot \vec{\sigma}_N)_{nP_{1/2},F}.
\]

Under the replacement $\hbar'_f \rightarrow \hbar_f$ and $\hbar'_N \rightarrow \hbar_N$, the correction, for a vector X17 particle, assumes the same form as for the pseudoscalar hypothesis, up to an additional overall factor 1/2. For $nP_{1/2}$ states, the expectation values are

\[
\langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{1/2},F=1} = -\frac{1}{3}, \quad \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{1/2},F=0} = 1.
\]

The leading term in the hyperfine splitting for $nP_{1/2}$ states is well known to be equal to

\[
E_{F}(nP_{1/2}) = -g_N \frac{\alpha (Z\alpha)^3 m_e^3}{3 n^3 m_f m_N} \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{1/2},F}.
\]

Expressed in terms of the leading term, one obtains the following corrections due to the X17 particle for $nP_{3/2}$ states,

\[
\frac{E_{X,V}(nP_{3/2})}{E_F(nP_{1/2})} \approx -\frac{3\hbar'_f \hbar'_N Z m_e}{g_N \pi m_X} \left(1 - \frac{1}{n^2}\right) \left(\frac{Z\alpha m_e}{m_X}\right),
\]

\[
\frac{E_{X,A}(nP_{3/2})}{E_F(nP_{1/2})} \approx -\frac{3\hbar_f \hbar_N Z m_e}{2g_N \pi m_X} \left(1 - \frac{1}{n^2}\right) \left(\frac{Z\alpha m_e}{m_X}\right).
\]

Parametrically, these are suppressed with respect to the results for $S$ states, by an additional factor $Z\alpha m_e/m_X$.

For the $nP_{3/2}$ states, one considers the corrections $E_{X,V}(nP_{3/2}) = \langle nP_{3/2} | H_{\text{HFS},V} |nP_{3/2} \rangle$ and $E_{X,A}(nP_{3/2}) = \langle nP_{3/2} | H_{\text{HFS},A} |nP_{3/2} \rangle$, with the results

\[
E_{X,V}(nP_{3/2}) = -\frac{(Z\alpha)^5}{12\pi n^3} \left(1 - \frac{1}{n^2}\right) \frac{m_f^5}{m_N^2 m_X} \hbar'_f \hbar'_N \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{3/2},V},
\]

\[
E_{X,A}(nP_{3/2}) = \frac{2(Z\alpha)^6}{45\pi n^3} \left(1 - \frac{1}{n^2}\right) \frac{m_e^5}{m_f m_N m_X} \hbar_f \hbar_N \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{3/2},A}.
\]

Here, the expectation values are

\[
\langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{3/2},F=2} = 1, \quad \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{3/2},F=1} = -\frac{5}{3}.
\]

The leading term in the hyperfine splitting for $nP_{3/2}$ states is well known to be equal to

\[
E_{F}(nP_{3/2}) = g_N \frac{\alpha (Z\alpha)^3 m_e^3}{15 n^3 m_f m_N} \langle \vec{\sigma}_f \cdot \vec{\sigma}_N \rangle_{nP_{3/2}}.
\]

Expressed in terms of the leading term, one obtains the following corrections due to the X17 particle for $nP_{1/2}$ states,

\[
\frac{E_{X,V}(nP_{3/2})}{E_F(nP_{3/2})} \approx -\frac{5\hbar'_f \hbar'_N Z m_e}{4g_N \pi m_X} \frac{m_f}{m_N} \left(1 - \frac{1}{n^2}\right) \left(\frac{Z\alpha m_e}{m_X}\right),
\]

\[
\frac{E_{X,A}(nP_{3/2})}{E_F(nP_{3/2})} \approx \frac{2\hbar_f \hbar_N Z m_e}{3g_N \pi m_X} \left(1 - \frac{1}{n^2}\right) \left(\frac{Z\alpha m_e}{m_X}\right)^2.
\]

Parametrically, in comparison to $nP_{1/2}$ states, the correction for $nP_{3/2}$ states is suppressed for a vector X17 by an additional factor $m_f/m_N$, while for a pseudoscalar X17, the suppression factor is $Z\alpha m_e/m_X$. For electrons bound to protons and other nuclei, both suppression factors are approximately of the same order-of-magnitude, while for muonic hydrogen and deuterium, the vector contribution dominates over the pseudoscalar one.
FIG. 2. The X17 particle induces vertex corrections to the anomalous magnetic moment of the muon.
For the X17 vector hypothesis, one obtains diagram (a), while the pseudoscalar hypothesis leads to
diagram (b). The interaction with the external magnetic field is denoted by the zigzag line.

V. X17 PARTICLE AND MUON ANOMALOUS MAGNETIC MOMENT

One aim of our investigations is to explore the possibility of a detection of the X17 particle in the hyperfine splitting
of muonic bound systems. To this end, it is instructive to derive upper bounds on the coupling parameters
$h'_{\mu}$ and $h_{\mu}$, for the muon. The contribution of a massive pseudoscalar loop to the muon anomaly [see Fig. 2(b)] has been
studied for a long time [26–30], and a recent update of theoretical contributions [31] has confirmed the existence of
the 3.7σ discrepancy of theory and experiment. The contribution of a massive vector exchange [see Fig. 2(a)] has
recently been revisited in Ref. [30]. Specifically, the experimental results for the muon anomaly $a_\mu$ [see Eqs. (1.1) and
(3.36) of Ref. [31]] are as follows,

$$a_\mu^{(\text{exp})} = 0.00116592091(54)(33), \quad a_\mu^{(\text{thr})} = 0.001165918204(356).$$

The 3.7σ discrepancy $a_\mu^{(\text{exp})} - a_\mu^{(\text{thr})} \approx 2.7 \times 10^{-9}$ needs to be explained.

According to Eq. (41) of Ref. [30], we have the following correction to the muon anomaly due to the vector $X$ vertex
correction in Fig. 2(a),

$$\Delta a_\mu = \frac{(h'_{\mu})^2 m_{\mu}^2}{8\pi^2 m_X^2} \int_0^1 \frac{dx x^2 (2-x)}{(1-x) [1 - (m_\mu/m_X)^2] + (m_\mu/m_X)^2 x} \ dx \int_0^1 \frac{dx x^3}{(1-x) [1 - (m_\mu/m_X)^2] + (m_\mu/m_X)^2 x},$$

where we have used the numerical value $m_X = 16.7$ MeV. The following numerical value

$$h'_{\mu} = (h'_{\mu})_{\text{opt}} = 5.6 \times 10^{-4}$$

is “optimum” in the sense that it precisely remedies the discrepancy described by Eq. (41) and will be taken as
the input datum for all subsequent evaluations of corrections to the hyperfine splitting in muonic bound systems.
Note that, even if the vector X17 particle does not provide for an explanation of the muon anomaly discrepancy, the
order-of-magnitude of the coupling parameter $h'_{\mu}$ could not be larger than the value indicated in Eq. (43), because
otherwise, the theoretical value of $a_\mu$ would increase too much beyond the experimental result.

According to Eq. (20) of Ref. [30], the vertex correction due to a virtual pseudoscalar X17 particle to the following
correction,

$$\Delta g_\mu = - \frac{(h_{\mu})^2 m_{\mu}^2}{4\pi^2 m_X^2} \int_0^1 \frac{dx x^2}{(1-x) [1 - (m_\mu/m_X)^2] + (m_\mu/m_X)^2 x}$$

Here, because the correction is negative and decreases the value of $a_\mu$, the experimental-theoretical discrepancy given
in Eq. (41) can only be enhanced by the pseudoscalar X17 particle. If we demand that the discrepancy not be increased
beyond $6\sigma$, then we obtain the condition that $|\delta \mu|$ could not exceed a numerical value of $3.8 \times 10^{-4}$. In the following, we take the maximum permissible value of

$$\delta \mu = (\delta \mu)_{\text{max}} = 3.8 \times 10^{-4}, \quad (45)$$

in order to estimate the magnitude of corrections to the hyperfine splitting in muonic bound systems, induced by a hypothetical pseudoscalar X17 particle.

VI. NUMERICAL ESTIMATES AND EXPERIMENTAL VERIFICATION

A. Overview

The relative corrections to the hyperfine splitting due to the X17 particle, expressed in terms of the leading Fermi interaction, for $S$ and $P$ states, are given in Eqs. (32), (36a), (36b), (40a), and (40b). All of the formulas involve at least one factor of $m_r/m_X$, and so, experiments appear to be more attractive for muonic rather than electronic bound systems. Furthermore, parametrically, the corrections are largest for $S$ states, which is understandable because the range of the X17 potential is limited to its Compton wavelength of about $11.8 \text{ fm}$, and so, its effects should be more pronounced for states whose probability density does not vanish at the origin, i.e., for $S$ states. This intuitive understanding is confirmed by our calculations. Note that the formulas for the corrections to the hyperfine splitting, given in Eqs. (29), (30), (33a), (33b), (37a), and (37b), are generally applicable to bound systems with a heavy nucleus, upon a suitable reinterpretation of the nuclear spin matrix $\vec{\sigma}_N$ in terms of a nuclear spin operator.

B. Muonic Deuterium

In view of a recent theoretical work [32] which describes a $5\sigma$ discrepancy of theory and experiment for muonic deuterium, it appears indicated to analyze this system first. Indeed, the theory of nuclear-structure effects in muonic deuterium has been updated a number of times in recent years [32–34]. Expressed in terms of the Fermi term, the discrepancy $\delta E_{\text{HFS}}(2S)$ observed in Ref. [32] can be written as

$$\frac{\Delta E_{\text{HFS}}(2S_{1/2})}{E_F(2S_{1/2})} = 0.0094(18). \quad (46)$$

In order to evaluate an estimate for the correction due to the X17 vector particle, we observe that the interaction is protophobic [8, 9]. Hence, we can assume that the coupling parameter of the deuteron is approximately equal to that of the neutron. We will assume the opposite sign for the coupling parameter of the deuteron (neutron), as compared to the coupling parameter in Eq. (43). This choice is inspired by the opposite charge of the muon and nucleus with respect to the $U(1)$ gauge group of quantum electrodynamics. In view of Eq. (10) of Ref. [8] and Eq. (5) here, we thus have the estimate

$$\delta h'_d \approx \delta h'_n = -\frac{1}{100} \sqrt{4\pi\alpha} = -3.02 \times 10^{-3}. \quad (47)$$

Because of the numerical dominance of the proton coupling to the pseudoscalar particle over that of the neutron [see Eq. (29)], we estimate the pseudoscalar coupling of the deuteron to be of the order of

$$h_d \approx h_p = -2.4 \times 10^{-3}. \quad (48)$$

In view of Eq. (32), we obtain the estimates

$$\frac{E_{X,V}(nS_{1/2})}{E_F(nS_{1/2})} \approx 3.8 \times 10^{-6}, \quad \frac{E_{X,A}(nS_{1/2})}{E_F(nS_{1/2})} \approx -1.0 \times 10^{-6}, \quad (49)$$

where the symbol $\approx$ is used to denote an estimate for the quantity specified on the left, including its sign, based on the estimates of the coupling parameters of the hypothetical vector and pseudoscalar X17 particle, as described in the current work. As already explained, the modulus of our estimates for the coupling parameters is close to the upper end of the allowed range; the same thus applies to the absolute magnitude of our estimates for the X17-mediated corrections to hyperfine energies. Note that the vector X17 contribution slightly decreases the discrepancy noted in
Ref. [32], while the hypothetical pseudoscalar effect slightly increases the discrepancy, yet, on a numerically almost negligible level.

Similar considerations, based on Eqs. [36a] and [36b], lead to the following results for $P$ states,

$$E_{X,V}(nP_{1/2}) / E_F(nP_{1/2}) \approx 2.5 \times 10^{-7} \left(1 - \frac{1}{n^2}\right), \quad E_{X,A}(nP_{1/2}) / E_F(nP_{1/2}) \approx 6.6 \times 10^{-8} \left(1 - \frac{1}{n^2}\right),$$

which might be measurable in future experiments. Results for the Sternheim [35] weighted differences $|n^3E_{X,V}(nS_{1/2}) - E_{X,V}(1S_{1/2})|/E_F(1S_{1/2})$ and correspondingly $|n^3E_{X,A}(nS_{1/2}) - E_{X,A}(1S_{1/2})|/E_F(1S_{1/2})$ are of the same order-of-magnitude as for the individual $P_{1/2}$ states.

C. Muonic Hydrogen

The considerations are analogous to those for muonic deuterium. However, the coupling parameter for the nucleus, for the protophobic vector model, is constrained by Eq. [39],

$$\hbar' \approx -8 \times 10^{-4} \sqrt{4\pi \alpha} = -2.42 \times 10^{-5},$$

which is much smaller than for the deuteron nucleus. The coupling parameter of the proton, for the pseudoscalar model, can be estimated according to Eq. [39]. One obtains

$$E_{X,V}(nS_{1/2}) / E_F(nS_{1/2}) \approx 8.8 \times 10^{-9}, \quad E_{X,A}(nS_{1/2}) / E_F(nS_{1/2}) \approx -2.9 \times 10^{-7}$$

for the $S$ state effects, and

$$E_{X,V}(nP_{1/2}) / E_F(nP_{1/2}) \approx 5.8 \times 10^{-10} \left(1 - \frac{1}{n^2}\right), \quad E_{X,A}(nP_{1/2}) / E_F(nP_{1/2}) \approx 1.8 \times 10^{-8} \left(1 - \frac{1}{n^2}\right)$$

for $P_{1/2}$ states. Results of the same order-of-magnitude as for individual $P_{1/2}$ states are obtained for the Sternheim difference of $S$ states. The effects, in muonic hydrogen, for the vector model, are seen to be numerically suppressed. The contributions of the X17 particle need to be compared to the proton structure effects, which have recently been analyzed in Refs. [36–40]. According to Ref. [39], the numerical accuracy of the theoretical prediction for the 2S hyperfine splitting in muonic hydrogen is currently about 72 ppm [$E_{HFS}(2S) = 22.8108(16)$ meV].

D. True Muonium ($\mu^+\mu^-$) System

Coupling parameters for the muon have been estimated in Eqs. [43] and [44] for the vector and pseudoscalar models, respectively. A quick calculation shows that the relative correction to the hyperfine splitting, for $S$ states, remains the same for the system of particle and antiparticle, despite the existence of the annihilation channel, provided the reduced mass of the system in properly taken into account. One obtains the estimates

$$E_{X,V}(nS_{1/2}) / E_F(nS_{1/2}) \approx -3.2 \times 10^{-7}, \quad E_{X,A}(nS_{1/2}) / E_F(nS_{1/2}) \approx 7.1 \times 10^{-8}$$

for the $S$ state effects, and

$$E_{X,V}(nP_{1/2}) / E_F(nP_{1/2}) \approx -2.0 \times 10^{-8} \left(1 - \frac{1}{n^2}\right), \quad E_{X,A}(nP_{1/2}) / E_F(nP_{1/2}) \approx -4.4 \times 10^{-9} \left(1 - \frac{1}{n^2}\right)$$

for $nP_{1/2}$ states. For $S$ states, the contribution of hadronic vacuum polarization in the annihilation channel is known to 20 ppm [see Eq. (41) of Ref. [41, 42] as well as Refs. [41, 43, 44]]. For $nP_{1/2}$ states, the corrections are numerically small, but perhaps not immeasurable. In general, for true muonium, one has the advantage that the nuclear structure correction is not present, but the disadvantage that the coupling parameter for muons suffers from much tighter bounds as compared to nucleons, due to the muon $g$ factor [see Eqs. [43] and [44]], as well as from the uncertainty due to hadronic vacuum polarization loops.
E. Positronium

Quite considerable efforts have recently been invested in the calculation of the $m \alpha^7$ corrections to the positronium hyperfine splitting, 45–57. In positronium, effects of the X17 particle are suppressed in view of the smaller reduced mass of the bound system. For the coupling parameters, we use the maximum allowed value for the electron in the vector model [see Eq. (9)],

$$\hbar' \approx 1.4 \times 10^{-3} \sqrt{4\pi \alpha} = 4.2 \times 10^{-4},$$

and for the pseudoscalar model [see Eq. (11)],

$$\hbar_e \approx 500 \frac{m_e}{\alpha} = 1.0 \times 10^{-3}.$$  

Under these assumptions, the estimates for $S$ states are as follows,

$$\frac{E_{X,V}(nS_1/2)}{E_F(nS_1/2)} \approx -8.7 \times 10^{-10}, \quad \frac{E_{X,A}(nS_1/2)}{E_F(nS_1/2)} \approx 2.6 \times 10^{-9}.$$  

The effects are thus numerically smaller than the $m \alpha^7$ effects currently under study 45–57.

VII. CONCLUSIONS

The conceivable existence of the X17 particle 6, 7 provides atomic physicists with a long-awaited opportunity to detect a very serious candidate for a low-energy (fifth force) addition to the Standard Model. The energy range of about 17 MeV provides for a certain challenge from the viewpoint of atomic physics; the range of the X17-induced effects in Sec. VI, for a number of simple atomic systems. The results can be summarized as follows.

- We show in Sec. V that the pseudoscalar model 10 enhances the experimental-theoretical discrepancy in the muon anomaly, while the vector model 8, 9 could eliminate it. Stringent bounds on the magnitude of the muon coupling parameters to the X17 particle can be derived based on the muon anomaly. Note that the order-of-magnitude of the maximum permissible coupling to the pseudoscalar, given in Eq. 45, also leads to a tension with the parameterization $h_f = \xi_f(m_f/v)$ given in Ref. 10 (applied to the case $f = \mu$, i.e., to the muon). Namely, the parameterization could be read as suggesting a likely increase of the pseudoscalar coupling constant with the mass of the particle. While $\xi_e$ is bound from below by the condition $\xi_e > 4$ [see Eq. (10)] the corresponding parameter in the muon sector must fulfill $\xi_\mu < 0.9$ [see Eq. (15)].

- The relative correction to the hyperfine splitting for both $S$ and $P$ states is enhanced in muonic as compared to electronic bound systems by two orders of magnitude, in view of the scaling of the relative corrections with $m_\tau/m_X$, where $m_\tau$ is the reduced mass of the two-body bound system.

- In muonic deuterium, the correction, for $S$ states, is of order $3.8 \times 10^{-6}$ (vector X17) and $-1.0 \times 10^{-6}$ (pseudoscalar X17) in units of the Fermi energy, while the experimental accuracy for the S state hyperfine splitting is of order $10^{-3}$, and there is a 5σ discrepancy of theory and experiment, in view of a recent calculation of the nuclear polarizability effects 32. One concludes that the experimental accuracy would have to be improved by three orders of magnitude before the effects of the X17 become visible, and the understanding of the nuclear effects would likewise have to be improved by a similar factor.

- In muonic deuterium, for the hyperfine splitting of $P_{1/2}$ states, the X17-mediated correction to the hyperfine splitting is of order $2.5 \times 10^{-7}$ for the vector model, and of order $6.6 \times 10^{-8}$ for the pseudoscalar model. These
effects are not suppressed by challenging nuclear structure effects and could be measurable in the future. The same applies to the Sternheim weighted difference of the hyperfine splitting of $S$ states, where the effect induced by the X17 particle is of the same order-of-magnitude as for the $P_{1/2}$ splitting.

- For muonic hydrogen, because of the protophobic character of the vector model, effects of the vector X17 are suppressed (order $10^{-9}$ for the $S$ state splitting and order $10^{-10}$ for the $P$ state splitting and the Sternheim weighted difference). For the pseudoscalar model, the $S$ state splitting is affected at relative order $10^{-7}$, and the $P$ state splitting as well as the Sternheim difference are affected at order $10^{-8}$. These effects could be measurable in the future.

- For positronium, the effects of the X17 are suppressed by the small reduced mass of the system. They are bound not to exceed the level of $10^{-10}$ for the vector model, and $10^{-9}$ for the pseudoscalar model. Thus, even taking into account all the $ma^2$ corrections currently under study, the detection of an X17-induced signal in the hyperfine splitting appears to be extremely challenging in positronium.

One concludes that the most promising approach toward a conceivable detection of the X17 in high-precision atomic physics experiments would probably concern the hyperfine splitting of $S$ states, where the effect induced by the X17 particle is of the same order-of-magnitude as for the $P_{1/2}$ splitting.

\[ \left| P \right| = \frac{m_p}{m_e} \text{ for Comparisons of Yb}^+ \text{ and Cs Atomic Clocks}, \text{Phys. Rev. Lett. 113, 210802 (2014)}. \]

\[ \left| m_e \right| / m_e \text{ corrections currently under study [45–57], the detection of an X17-induced signal in the hyperfine splitting appears to be extremely challenging in positronium.} \]

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[1] T. W. Hänsch, “Nobel Lecture: Passion for precision,” Rev. Mod. Phys. 78, 1297–1309 (2006).
[2] M. Fischer, N. Kolachevsky, M. Zimmermann, R. Holzwarth, T. Udem, T. W. Hänsch, M. Abgrall, J. Grüner, I. Maksimovic, S. Bize, H. Marion, F. Pereira Dos Santos, P. Lemonde, G. Santarelli, P. Laurent, A. Clairon, C. Salomon, M. Haas, U. D. Jentschura, and C. H. Keitel, “New Limits on the Drift of Fundamental Constants from Laboratory Measurements,” Phys. Rev. Lett. 92, 230802 (2004).
[3] R. M. Godun, P. B. R. Nisbet-Jones, J. M. Jones, S. A. King, L. A. M. Johnson, S. A. Margolis, K. Szymaniec, S. N. Lea, K. Bongs, and P. Gill, “Frequency Ratio of Two Optical Clock Transitions in $^{171}$Yb$^+$ and Constraints on the Time Variation of Fundamental Constants,” Phys. Rev. Lett. 113, 210801 (2014).
[4] N. Huntemann, B. Lipphardt, C. Tamm, V. Gerginov, S. Weyers, and E. Peik, “Improved Limit on a Temporal Variation of $m_p/m_e$ from Comparisons of Yb$^+$ and Cs Atomic Clocks,” Phys. Rev. Lett. 113, 210802 (2014).
[5] P. J. Mohr, D. B. Newell, and B. N. Taylor, “CODATA Recommended Values of the Fundamental Physical Constants: 2014,” Rev. Mod. Phys. 88, 035009 (2016).
[6] A. J. Krasznahorkay, M. Csatlós, L. Csigi, Z. Gácsi, J. Gulyás, M. Hunyadi, I. Kutí, B. M. Nyako, L. Stuhl, J. Timár, T. G. Tornyi, Zs. Vajta, T. J. Ketel, and A. Krasznahorkay, “Observation of Anomalous Internal Pair Creation in $^6$Be: A Possible Indication of a Light, Neutral Boson,” Phys. Rev. Lett. 116, 042501 (2016): A. J. Krasznahorkay, M. Csatlós, L. Csigi, J. Gulyás, T. J. Ketel, A. Krasznahorkay, I. Kutí, A. Nagy, B. M. Nyako, N. Sas, and J. Timár, “On the creation of the 17 MeV X boson in the $^{17}$MeV M1 transition of $^6$Be,” Eur. Phys. J. Web of Conferences 142, 01019 (2017).
[7] A. J. Krasznahorkay, M. Csatlós, J. Gulyás, M. Kosztka, S. Szilas, J. Timár, D. S. Fírak, A. Nagy, N. J. Sas, and A. Krasznahorkay, “New evidence supporting the existence of the hypothetic X17 particle,” e-print [arXiv:1910.10450 [nucl-ex]].
[8] J. L. Feng, B. Fornal, I. Galon, S. Gardner, J. Smolinsky, T. M. P. Tait, and P. Tanedo, “Protophobic Fifth-Force Interpretation of the Observed Anomaly in $^6$Be Nuclear Transitions,” Phys. Rev. Lett. 117, 071803 (2016).
[9] J. L. Feng, B. Fornal, I. Galon, S. Gardner, J. Smolinsky, T. M. P. Tait, and P. Tanedo, “Particle Physics Models for the 17 MeV Anomaly in Beryllium Nuclear Decays,” Phys. Rev. D 95, 035017 (2017).
G. Aad et al., “Possible explanation of the electron positron anomaly at 17 MeV in $^8$Be transitions through a light pseudoscalar,” J. High Energy Phys. **1116**, 039 (2016).

U. D. Jentschura and I. Nándori, “Atomic physics constraints on the X boson,” Phys. Rev. A **97**, 042502 (2018).

J. Sapirstein and D. R. Yennie, “Theory of Hydrogenic Bound States,” in *Quantum Electrodynamics*, Advanced Series on Directions in High Energy Physics, Vol. 7, Vol. 7, edited by T. Kinoshita (World Scientific, Singapore, 1990) pp. 560–672.

U. Jentschura and K. Pachucki, “Higher-order binding corrections to the Lamb shift of $2P$ states,” Phys. Rev. A **54**, 1853–1861 (1996).

U. D. Jentschura and V. A. Yerokhin, “Quantum electrodynamical corrections to the hyperfine structure of excited $S$ states,” Phys. Rev. A **73**, 062503 (2006).

H.-Y. Cheng and C.-W. Chiang, “Revisiting Scalar and Pseudoscalar Couplings with Nucleons,” J. High Energy Phys. **1207**, 009 (2012).

F. Englert and R. Brout, “Broken Symmetry and the Mass of Gauge Vector Mesons,” Phys. Rev. Lett. **13**, 321–323 (1964).

F. Englert and R. Brout, “Broken Symmetries and the Masses of Gauge Bosons,” Phys. Rev. Lett. **13**, 508–509 (1964).

G. Aad et al. [ATLAS Collaboration], "Study of the spin and parity of the Higgs boson in diboson decays with the ATLAS detector," Eur. Phys. J. C **75**, 476 (2015).

A. Anastasi, D. Babusci, G. Bencivenni, M. Berlowski, C. Bloise, A. Budano, L. Caldeira Balkestahl, B. Cao, F. Ceradini, P. Ciaramboli, F. Curciarello, E. Czerwinski, G. D’Agostini, E. Dane, V. De Leo, E. De Lucia, A. De Santis, P. De Simone, A. Di Cicco, A. Di Domenico, R. Di Salvo, D. Domenici, A. D’Uffizi, A. Fantini, G. Felici, S. Fiore, A. Gajos, P. Gauzzi, G. Giardina, S. Giovannella, E. Graziani, F. Happecher, Heijkenskjöld, W. Ikegami Andersson, T. Johansson, D. Kaminskia, W. Kramen’a, A. Kupsc, S. Loffredo, G. Mandaglio, M. Marić, R. Messi, S. Miscetti, G. Morello, D. Moricciani, P. Moskal, A. Passeri, V. Patera, E. Perez del Rio, A. Ranieri, P. Santangelo, I. Sarra, M. Schioppa, M. Silarski, F. Sirghi, L. Tortora, G. Venanzoni, W. Wilslick, and M. Wolke, “Limit on the production of a low-mass vector boson in $e^+e^-$ with the KLOE experiment,” Phys. Lett. B **750**, 633–637 (2015).

Daniele S. M. Alves and Neal Weiner, “A viable qcd axion in the mev mass range,” J. High Energy Phys. **1807**, 092 (2018).

J. Liu, C. E. M. Wagner, and X.-P. Wang, “A light complex scalar for the electron and muon anomalous magnetic moment,” J. High Energy Phys. **1903**, 008 (2019).

U. Ellwanger, private communication (2020).

V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics, Volume 4 of the Course on Theoretical Physics*, 2nd ed. (Pergamon Press, Oxford, UK, 1982).

J. DiSciaccia and G. Gabrielse, “Direct Measurement of the Proton Magnetic Moment,” Phys. Rev. Lett. **108**, 153001 (2012).

A. Mooser, S. Ulmer, K. Blaum, K. Franke, H. Kracke, C. Leitertz, W. Quint, C. de Carvalho Rodegheri, C. Smorra, and J. Walz, “Direct high-precision measurement of the magnetic moment of the proton,” Nature (London) **509**, 596–599 (2014).

W. A. Bardeen, R. Gastmans, and B. Lautrup, “Static Quantities and Weinberg’s Model of Weak and Electromagnetic Interactions,” Nucl. Phys. B **46**, 319–331 (1972).

J. R. Primack and H. R. Quinn, “Muon $g – 2$ and Other Constraints on a Model of Weak and Electromagnetic Interactions Without Neutral Currents,” Phys. Rev. D **6**, 3171–3178 (1972).

J. P. Leveille, “The Second-Order Weak Correction to $(g – 2)$ of the Muon in Arbitrary Gauge Models,” Nucl. Phys. B **137**, 63–76 (1978).

H. E. Haber, G. L. Kane, and T. Sterling, “The Fermion Mass Scale and Possible Effects of Higgs Bosons on Experimental Observables,” Nucl. Phys. B **161**, 493–532 (1979).

P. S. Queiroz and W. Shepherd, “New physics contributions to the muon anomalous magnetic moment: A numerical code,” Phys. Rev. D **89**, 095024 (2014).

A. Keshavarzi, D. Nomura, and T. Teubner, “Muon $g – 2$ and $\alpha(M_Z^2)$: A new data-based analysis,” Phys. Rev. D **97**, 114025 (2018).

M. Kalinowski, K. Pachucki, and V. A. Yerokhin, “Nuclear-structure corrections to the hyperfine splitting in muonic deuterium,” Phys. Rev. A **98**, 062513 (2018).

K. Pachucki and J. Komasa, “Gerade-ungerade mixing in the hydrogen molecule,” Phys. Rev. A **83**, 042510 (2011).

K. Pachucki and A. Wienczek, “Nuclear Structure in Deuterium,” Phys. Rev. A **91**, 040503(R) (2015).

M. M. Sternheim, “State-dependent mass corrections to hyperfine structure in hydrogenic atoms,” Phys. Rev. **130**, 211–222 (1963).

A. P. Martynenko, “2S Hyperfine splitting of muonic hydrogen,” Phys. Rev. A **71**, 022506 (2005).

R. Pohl, R. Gilman, G. A. Miller, and K. Pachucki, “Muonic hydrogen and the proton radius puzzle,” Annu. Rev. Nucl. Part. Sci. **63**, 175–204 (2011).

O. Tomalak, “Two-photon exchange correction to the hyperfine splitting in muonic hydrogen,” Eur. Phys. J. C **77**, 858 (2017).

O. Tomalak, “Hyperfine splitting in ordinary and muonic hydrogen,” Eur. Phys. J. A **54**, 3 (2019).

O. Tomalak, “Two-photon exchange correction to the Lamb shift and hyperfine splitting of S levels,” Eur. Phys. J. A **55**, 64 (2019).

D. A. Owen and W. W. Repko, “Vacuum-polarization corrections to the hyperfine structure of the $\mu^+\mu^–$ bound system,” Phys. Rev. A **5**, 1570–1572 (1972).
U. D. Jentschura, G. Soff, V. G. Ivanov, and S. G. Karshenboim, “Bound $\mu^+\mu^-$ system,” Phys. Rev. A 56, 4483–4495 (1997).

S. G. Karshenboim, U. D. Jentschura, V. G. Ivanov, and G. Soff, “Next-to-leading and higher-order corrections to the decay rate of dimuonium,” Phys. Lett. B 424, 397–404 (1998).

S. J. Brodsky and R. F. Lebed, “Production of the Smallest QED Atom: True Muonium ($\mu^+\mu^-$),” Phys. Rev. Lett. 102, 213401 (2009).

M. W. Heiss, G. Wichmann, A. Rubbia, and P. Crivelli, “The positronium hyperfine structure: Progress towards a direct measurement of the $2^3S_1 \rightarrow 2^1S_0$ transition in vacuum,” J. Phys. Conf. Ser. 1138, 012007 (2018).

A. Czarnecki, K. Melnikov, and A. S. Yelkhovsky, “Positronium Hyperfine Splitting: Analytical Value at $O(m\alpha^6)$,” Phys. Rev. Lett. 82, 311–314 (1999).

V. Barger, L. Fu, J. G. Learned, D. Marfatia, S. Pakvasa, and T. J. Weiler, “Glashow resonance as a window into cosmic neutrino sources,” Phys. Rev. D 90, 121301(R) (2014).

G. S. Adkins and R. N. Fell, “Positronium hyperfine splitting at order $m\alpha^7$: Light-by-light scattering in the two-photon-exchange channel,” Phys. Rev. A 89, 062518 (2014).

M. I. Eides and V. A. Shelyuto, “Hard nonlogarithmic corrections of order $m\alpha^7$ to hyperfine splitting in positronium,” Phys. Rev. D 89, 111301(R) (2014).

G. S. Adkins, C. Parsons, M. D. Salinger, R. Wang, and R. N. Fell, “Positronium energy levels at order $m\alpha^7$: Light-by-light scattering in the two-photon-annihilation channel,” Phys. Rev. A 90, 042502 (2014).

M. I. Eides and V. A. Shelyuto, “Hard three-loop corrections to hyperfine splitting in positronium and muonium,” Phys. Rev. D 92, 013010 (2015).

G. S. Adkins, C. Parsons, M. D. Salinger, and R. Wang, “Positronium energy levels at order $m\alpha^7$: Vacuum polarization corrections in the two-photon-annihilation channel,” Phys. Lett. B 747, 551–555 (2015).

G. S. Adkins, M. Kim, C. Parsons, and R. N. Fell, “Three-Photon-Annihilation Contributions to Positronium Energies at Order $m\alpha^7$,” Phys. Rev. Lett. 115, 233401 (2015).

G. S. Adkins, L. M. Tran, and R. Wang, “Positronium energy levels at order $m\alpha^7$: Product contributions in the two-photon-annihilation channel,” Phys. Rev. A 93, 052511 (2016).

M. I. Eides and V. A. Shelyuto, “Hyperfine splitting in muonium and positronium,” Int. J. Mod. Phys. A 31, 1645034 (2016).

M. I. Eides and V. A. Shelyuto, “One more hard three-loop correction to parapositronium energy levels,” Phys. Rev. D 96, 011301(R) (2017).