Transfer of optical vortices in coherently prepared media

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Abstract

We consider transfer of optical vortices between laser pulses carrying orbital angular momentum (OAM) in a cloud of cold atoms characterized by the Λ configuration of the atom-light coupling. The atoms are initially prepared in a coherent superposition of the lower levels, creating a so-called phaseonium medium. If a single vortex beam initially acts on one transition of the Λ scheme, an extra laser beam is subsequently generated with the same vorticity as that of the incident vortex beam. The absorption of the incident probe beam takes place mostly at the beginning of the atomic medium within the absorption length. The losses disappear as the probe beam propagates deeper into the medium where the atoms are transferred to their dark states. The method is extended to a tripod atom-light coupling scheme and a more general $n + 1$-level scheme containing $n$ ground states and one excited state, allowing for creating of multiple twisted light beams. We also analyze generation of composite optical vortices in the Λ scheme using a superposition of two initial vortex beams and study lossless propagation of such composite vortices.

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I. INTRODUCTION

The coherent manipulation of pulse propagation through atomic ensembles \[1\text{–}4\] leads a plethora of important phenomena, such as electromagnetically induced transparency (EIT) \[1\text{–}7\] and slow light propagation \[1\text{–}5\text{,} 8\], enhancement of optical nonlinearities \[9\text{–}12\], generation of matched pulses \[13\text{,} 14\], creation of a spinor slow light \[15\text{–}19\], formation of adiabatons \[20\text{,} 21\] and optical solitons \[22\text{,} 23\]. It has been demonstrated that due to the EIT the light pulses not only could be slowed down, but also stored by switching off the controlled beam \[5\text{–}27\]. Therefore, the atomic system can be used as an optical memory for transferring the quantum state of light to the matter and back to the light \[5\text{,} 28\text{–}29\]. By using extra energy levels and additional laser field one arrives at more complex atom-light coupling schemes \[2\text{,} 30\text{–}41\] which can provide more than one dark states and offer different directions in studying of propagation effects in coherently driven atomic media.

Light can carry an orbital angular momentum (OAM) \[42\text{,} 43\]. Such light beams have helical (twisted) wavefronts that spiral along the beam direction much like a corkscrew. The twisted light field is characterized by a phase factor \(e^{il\phi}\), where \(\phi\) is the azimuthal angle with respect to the beam axis and \(l\) denotes the vortex integer winding number (OAM number). When interacting with atoms such optical vortex beams reveal a number of interesting effects, including light-induced-torque \[44\text{,} 45\], atom vortex beams \[46\], entanglement of OAM states of photon pairs \[47\], OAM-based four-wave mixing \[48\text{,} 49\], spatially dependent optical transparency \[50\text{–}52\], and the vortex slow light \[33\text{,} 53\text{–}55\]. The twisted slow light \[18\text{,} 33\text{,} 53\text{–}57\] gives additional possibilities in manipulation of the optical information during the storage and retrieval of the slow light \[58\text{,} 59\].

The previous studies on the EIT have concentrated on a situation where the atoms are initially in their ground states, and the Rabi-frequency of the probe field is much weaker than that of the control field. It has been demonstrated that the OAM of the control vortex beam can be transferred to the probe field in the tripod atom-light coupling scheme during the storage and retrieval of the probe field \[33\text{,} 57\]. Without switching off and on of the control fields (hence without storage and retrieval of the probe field), transfer of optical vortices take place by applying a pair of weaker probe fields in the closed loop double-\(\Lambda\) \[55\] or double-tripod \[18\] schemes.

In this paper, we demonstrate that the exchange of optical vortices in non-closed loop
structures is possible under the condition of weak atom-light interaction in coherently prepared atomic media. To this end, we analyze the interaction of multi-component laser pulses carrying OAM propagating in multi-level atom-light coupling schemes with atoms prepared in a coherent superposition of lower levels. Such a medium has been named the phaseonium [60–63]. We derive the basic equations describing the propagation of the coherent laser pulses weakly interacting with atoms in multi-level configurations. To elucidate the physical situation of exchange of OAMs, we begin with a basic three-level configuration, the Λ system, containing only a pair of laser pulses. Subsequently we extend our model to more complicated schemes involving additional laser pulses and additional atomic levels. It is shown that the transfer of optical vortices is also possible for the tripod system and a more general $n + 1$-level scheme. Furthermore we show that composite optical vortices can be formed in the Λ system when both probe fields are present at entrance of the medium.

II. THE THREE-LEVEL Λ SYSTEM

Let us first consider the Λ scheme for the transfer of optical vortices, illustrated in Fig. 1. Specifically, we study the propagation of two laser pulses with the Rabi-frequencies $\Omega_1$ and $\Omega_2$ (pulse pair) in a medium consisting of atoms in the three-level Λ configuration of the atom-light coupling. The two atomic lower states $|g_1\rangle$ and $|g_2\rangle$ are coupled to an excited state $|e\rangle$ via the two light fields. The Hamiltonian for such a system reads in the appropriate
rotating frame and in the interaction picture:

\[ H_\Lambda = \Omega_1 |g_1\rangle \langle e| + \Omega_2 |g_2\rangle \langle e| + \text{H.c.} \]  

(1)

The dynamics of the pulse pair \( \Omega_1 \) and \( \Omega_2 \) and two atomic coherences \( \rho_{g_1e} \) and \( \rho_{g_2e} \) are described by the Maxwell–Bloch equations (MBE) for an open system

\[ \dot{\rho}_{g_1e} = i(\delta_1 + i\gamma_{eg_1})\rho_{g_1e} - i\Omega_1(\rho_{ee} - \rho_{g_1g_1}) + i\Omega_2\rho_{g_1g_2}, \]  

(2)

and

\[ \dot{\rho}_{g_2e} = i(\delta_2 + i\gamma_{eg_2})\rho_{g_2e} - i\Omega_2(\rho_{ee} - \rho_{g_2g_2}) + i\Omega_1\rho_{g_2g_1}, \]  

(3)

and

\[ \frac{\partial\Omega_1}{\partial z} + c^{-1}\frac{\partial\Omega_1}{\partial t} = \frac{i\alpha_1\gamma_{eg_1}}{2L}\rho_{g_1e}, \]  

(4)

\[ \frac{\partial\Omega_2}{\partial z} + c^{-1}\frac{\partial\Omega_2}{\partial t} = \frac{i\alpha_2\gamma_{eg_2}}{2L}\rho_{g_2e}, \]  

(5)

where \( \alpha_1 \) and \( \alpha_2 \) are the optical depths of both laser pulses \( \Omega_1 \) and \( \Omega_2 \), \( L \) denotes the optical length of the medium, and \( \gamma_{eg_1} \) and \( \gamma_{eg_2} \) are the rates of decay from the excited state \( |e\rangle \) to lower states \( |g_1\rangle \) and \( |g_2\rangle \), respectively. We have defined the detunings as \( \delta_1 = \omega_{eg_1} - \omega_1 \) and \( \delta_2 = \omega_{eg_2} - \omega_2 \), where \( \omega_{eg_1} \) and \( \omega_{eg_2} \) are the frequencies of the transitions \( |g_1\rangle \leftrightarrow |e\rangle \) and \( |g_2\rangle \leftrightarrow |e\rangle \), respectively, while \( \omega_1 \) and \( \omega_2 \) represent the central frequencies of the probe beams. We have disregarded the diffraction terms containing the transverse derivatives \((2k_1)^{-1}\nabla_\perp^2\Omega_1 \) and \((2k_2)^{-1}\nabla_\perp^2\Omega_2 \) in the Maxwell equations (4) and (5), where \( k_1 = \omega_1/c \) and \( k_2 = \omega_2/c \) are the central wave vectors of the first and second beams. One can evaluate these terms as \( \nabla_\perp^2\Omega_{1(2)} \sim w^{-2}\Omega_{1(2)} \), where \( w \) represents a characteristic transverse dimension of the laser beams. This can be a width of the vortex core if the beam carries an optical vortex or a characteristic width of the beam if it has no vortex. Consequently the change of the phase of the probe beams due to the diffraction term after passing the medium is estimated to be \( L/2kw^2 \), where \( L \) is the length of the atomic cloud, where \( k \approx k_{1(2)} \). The phase change \( L/2kw^2 \) can be neglected when the sample length \( L \) is not too large, \( L\lambda/w^2 \ll \pi \), where \( \lambda = 2\pi/k \) is an optical wavelength. For example, by taking the length of the atomic cloud to be \( L = 100 \mu m \), the characteristic transverse dimension of the beams \( w = 20 \mu m \) and the wavelength \( \lambda = 1 \mu m \), we obtain \( L\lambda/w^2 = 0.25 \). Under these conditions the diffraction terms do not play a significant role and we can drop it out in Eqs. (4) and (5).
Let us assume that the atoms are initially in a superposition of both lower levels (the phaseonium medium)

$$|\psi(0)\rangle = c_1|g_1\rangle + c_2|g_2\rangle.$$  \hspace{1cm} (6)

We consider a weak atom-light interaction where $|\Omega_1|,|\Omega_2| \ll \gamma_{eg_1},\gamma_{eg_2}$. Then, to the first order one has $\rho_{ee} \approx 0$, $\rho_{g_1g_1} \approx |c_1|^2$, $\rho_{g_2g_2} \approx |c_2|^2$ and $\rho_{g_1g_2} \approx c_1c_2^*$, giving the following the steady state solutions for the coherences $\rho_{g_1e}$ and $\rho_{g_2e}$:

$$\rho_{g_1e} = -\frac{|c_1|^2\Omega_1 + c_1c_2^*\Omega_2}{\delta_1 + i\gamma_{eg_1}},$$  \hspace{1cm} (7)

$$\rho_{g_2e} = -\frac{c_1^*c_2\Omega_1 + |c_2|^2\Omega_2}{\delta_2 + i\gamma_{eg_2}}.$$  \hspace{1cm} (8)

The first-order approximation is valid when $|\rho_{g_1e}|,|\rho_{g_2e}| \ll 1$. Otherwise we cannot assume that $\rho_{g_1g_1}$ and $\rho_{g_1g_2}$ are not changing during the propagation of light.

Substituting Eqs. (7) and (8) into the Maxwell equations (4) and (5) one arrives at the following coupled equations for the propagation of the pulse pair:

$$\frac{\partial\Omega_1}{\partial z} = -i\beta_1(|c_1|^2\Omega_1 + c_1c_2^*\Omega_2),$$  \hspace{1cm} (9)

$$\frac{\partial\Omega_2}{\partial z} = -i\beta_2(c_1^*c_2\Omega_1 + |c_2|^2\Omega_2),$$  \hspace{1cm} (10)

where

$$\beta_a = \frac{\alpha_a\gamma_{eg_a}}{2L(\delta_a + i\gamma_{eg_a})},$$  \hspace{1cm} (11)

with $a = 1, 2$.

The second laser field is assumed to be zero $\Omega_2(0) = 0$ at the entrance $z = 0$, while $\Omega_1(0) = \Omega$. Under these conditions the solutions to Eqs. (9) and (10) read

$$\Omega_1(z) = \frac{\Omega}{X_2}(\beta_1|c_1|^2e^{-iX_2z} + \beta_2|c_2|^2),$$  \hspace{1cm} (12)

$$\Omega_2(z) = \frac{\Omega}{X_2}c_1^*c_2\beta_2(e^{-iX_2z} - 1),$$  \hspace{1cm} (13)

where

$$X_2 = \beta_1|c_1|^2 + \beta_2|c_2|^2.$$  \hspace{1cm} (14)
Up to now no assumption has been made concerning the spatial profile of the laser fields. We take now that the incident beam \( \Omega_1 \) has an optical vortex

\[
\Omega_1(0) = \Omega = |\Omega|e^{i\phi},
\]

(15)

where \( l \) is the orbital angular momenta along the propagation axis \( z \), and \( \phi \) is the azimuthal angle. For a doughnut Laguerre-Gaussian (LG) beam the transverse profile reads

\[
|\Omega| = \varepsilon \left( \frac{r}{w} \right)^{|l|} e^{-r^2/w^2},
\]

(16)

where \( r \) describes a cylindrical radius, \( w \) is a beam waist, and \( \varepsilon \) represents the strength of the vortex beam. According to Eqs. (12)-(15), the generated pulse beam \( \Omega_2(z) \sim e^{i\phi} \) acquires the same phase as the first vortex beam. Therefore the laser beam \( \Omega_1 \) transfers its vortex to the generated beam \( \Omega_2 \).

Equations (12) and (13) show that both light beams experience losses during their propagation. Yet the losses appear only at the entrance of the medium before the EIT is established for both fields. To simplify the discussion, let us take \( \alpha_1 = \alpha_2 = \alpha \) and \( \gamma_{eg1} = \gamma_{eg2} = \gamma \), and consider a situation where both laser fields \( \Omega_1 \) and \( \Omega_2 \) are in an exact resonance with the corresponding atomic transitions (\( \delta_1 = \delta_2 = 0 \)). Then Eqs. (11) and (14) lead to

\[
\beta_1 = \beta_2 = X_2 = \frac{1}{2\alpha L_{abs}},
\]

where \( L_{abs} = L/\alpha \) is the absorption length. If optical density of the resonant medium is sufficiently large \( \alpha \gg 1 \), the absorption length constitutes a fraction of the whole medium \( L_{abs} \ll L \). For the distances \( z \) exceeding the absorption length \( z \gg L_{abs} \) both exponential terms vanish in Eqs. (12) and (13) and the EIT is established leading to lossless propagation of both fields. Using Eqs. (15) and (16) we get

\[
\Omega_1(z \gg L_{abs}) = \varepsilon \left( \frac{r}{w} \right)^{|l|} e^{-r^2/w^2} |c_2|^2 e^{i\phi},
\]

(17)

\[
\Omega_2(z \gg L_{abs}) = -\varepsilon \left( \frac{r}{w} \right)^{|l|} e^{-r^2/w^2} c_1^* c_2 e^{i\phi}.
\]

(18)

In this way, the beams experience no absorption loss for large propagation distances \( z \gg L_{abs} \). This is illustrated in Fig. 2 showing the dependence of the intensities \( |\Omega_1(z)|^2/|\Omega_1(0)|^2 \) and \( |\Omega_2(z)|^2/|\Omega_1(0)|^2 \) given by Eqs. (12) and (13) on the dimensionless distance \( z/L_{abs} \) for the resonance case \( \delta_1 = \delta_2 = 0 \) and \( \alpha = 20 \). Although initially at the beginning of the atomic medium losses occur, going deeper through the medium the losses disappear as the
FIG. 2. Dependence of the dimensionless intensities of the light fields $|\Omega_1(z)|^2/|\Omega_1(0)|^2$ and $|\Omega_2(z)|^2/|\Omega_1(0)|^2$ given in Eqs. (12) and (13) on the dimensionless distance $z/L_{abs}$ for $c_1 = c_2 = \frac{1}{\sqrt{2}}$, $\delta_1 = \delta_2 = 0$ and $\alpha = 20$.

FIG. 3. Schematic diagram of the four-level tripod quantum system.

atoms go to their dark state \[ D(z \gg L_{abs}) = \frac{\Omega_2(z \gg L_{abs})|g_1\rangle - \Omega_1(z \gg L_{abs})|g_2\rangle}{\sqrt{\Omega_1^2(z \gg L_{abs}) + \Omega_2^2(z \gg L_{abs})}}. \tag{19} \]

III. THE TRIPOD SYSTEM

Consider next the propagation of three light pulses through a medium consisting of atoms with a tripod level structure shown in Fig. 3. Previously the tripod scheme was employed when studying the transfer of optical vortices \[33, 57\]. It was shown that the transfer of optical vortices from a control field of larger intensity to a probe field of weaker intensity in tripod scheme was only possible through switching off and on of the control laser beams.
In the following, we demonstrate that the exchange of optical vortices in tripod scheme is possible even without switching off and on of the control beams, if one uses a coherently prepared atomic system.

In the tripod scheme an excited state $|e\rangle$ is coupled to three lower levels $|g_1\rangle$, $|g_2\rangle$ and $|g_3\rangle$ through three laser pulses $\Omega_1$, $\Omega_2$ and $\Omega_3$, respectively. The Hamiltonian for the tripod scheme reads in the interaction representation

$$H_T = \Omega_1|g_1\rangle\langle e| + \Omega_2|g_2\rangle\langle e| + \Omega_3|g_3\rangle\langle e| + \text{H.c.}.$$ (20)

The MBEs describing the evolution of system can be written as

$$\dot{\rho}_{g_1e} = i(\delta_1 + i\gamma_{eg_1})\rho_{g_1e} - i\Omega_1(\rho_{ee} - \rho_{g_1g_1}) + i\Omega_2\rho_{g_1g_2} + i\Omega_3\rho_{g_1g_3},$$ (21)

$$\dot{\rho}_{g_2e} = i(\delta_1 + i\gamma_{eg_2})\rho_{g_2e} - i\Omega_2(\rho_{ee} - \rho_{g_2g_2}) + i\Omega_1\rho_{g_2g_1} + i\Omega_3\rho_{g_2g_3},$$ (22)

$$\dot{\rho}_{g_3e} = i(\delta_1 + i\gamma_{eg_3})\rho_{g_3e} - i\Omega_3(\rho_{ee} - \rho_{g_3g_3}) + i\Omega_1\rho_{g_3g_1} + i\Omega_2\rho_{g_3g_2},$$ (23)

and

$$\frac{\partial \Omega_1}{\partial z} + c^{-1}\frac{\partial \Omega_1}{\partial t} = \frac{i\alpha_1\gamma_{eg_1}}{2L}\rho_{g_1e},$$ (24)

$$\frac{\partial \Omega_2}{\partial z} + c^{-1}\frac{\partial \Omega_2}{\partial t} = \frac{i\alpha_2\gamma_{eg_2}}{2L}\rho_{g_2e},$$ (25)

$$\frac{\partial \Omega_3}{\partial z} + c^{-1}\frac{\partial \Omega_3}{\partial t} = \frac{i\alpha_3\gamma_{eg_3}}{2L}\rho_{g_3e},$$ (26)

where the diffraction terms have been neglected in the Maxwell equations (24) and (26), like for the $\Lambda$ scheme.

The atoms are initially prepared in a superposition of three lower states

$$|\psi(0)\rangle = c_1|g_1\rangle + c_2|g_2\rangle + c_3|g_3\rangle.$$ (27)

For a sufficiently weak atom-light interaction, $|\Omega_j| \ll \gamma_{eg_j}$, we can approximate $\rho_{ee} \approx 0$, $\rho_{g_1g_1} \approx |c_1|^2$, $\rho_{g_2g_2} \approx |c_2|^2$, $\rho_{g_3g_3} \approx |c_3|^2$, $\rho_{g_1g_3} \approx c_1c_3^*$, $\rho_{g_1g_3} \approx c_1c_3^*$ and $\rho_{g_2g_3} \approx c_2c_3^*$, giving the following steady state equations for the atomic coherences

$$\rho_{g_1e} = -\frac{|c_1|^2\Omega_1 + c_1c_2^*\Omega_2 + c_1c_3^*\Omega_3}{\delta_1 + i\gamma_{eg_1}},$$ (28)
Substituting Eqs. (28)-(30) into the Maxwell equations (24)-(26) and assuming that at the entrance \( z = 0 \) there is a single light beam \( \Omega_1(0) = \Omega, \Omega_2(0) = 0, \) and \( \Omega_3(0) = 0 \), we obtain

\[
\Omega_1(z) = \frac{\Omega}{X_3} (\beta_1 c_1^2 e^{i X_3 z} + \beta_2 c_2^2 + \beta_3 c_3^2),
\]

\[
\Omega_2(z) = \frac{\Omega}{X_3} c_1^* c_2 \beta_2 (e^{-i X_3 z} - 1),
\]

\[
\Omega_3(z) = \frac{\Omega}{X_3} c_1^* c_3 \beta_3 (e^{-i X_3 z} - 1),
\]

where

\[
X_3 = \beta_1 c_1^2 + \beta_2 c_2^2 + \beta_3 c_3^2,
\]

and \( \beta_a \) is defined by Eq. (11), with \( a = 1, 2, 3 \). Considering again the first laser pulse \( \Omega_1 \) initially carries an optical vortex (defined by Eqs. (15) and (16)), two vortex beams \( \Omega_2 \sim e^{i l \phi} \) and \( \Omega_3 \sim e^{i l \phi} \) are generated with the same vorticity as the first laser pulse \( \Omega_1 \sim e^{i l \phi} \).

Using Eqs. (15)-(16) and (31)-(33), one gets for sufficiently large \( z \) \( (z \gg L_{abs}) \) under the resonance condition \( \delta_1 = \delta_2 = \delta_3 = 0 \) and assuming that \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha, \gamma_{eg_1} = \gamma_{eg_2} = \gamma_{eg_3} = \gamma \):

\[
\Omega_1(z \gg L_{abs}) = \varepsilon \left( \frac{r}{w} \right)^2 |l| e^{-r^2/w^2} c_2^2 e^{i l \phi},
\]

\[
\Omega_2(z \gg L_{abs}) = -\varepsilon \left( \frac{r}{w} \right)^2 |l| e^{-r^2/w^2} c_1^* c_2 e^{i l \phi},
\]

\[
\Omega_3(z \gg L_{abs}) = -\varepsilon \left( \frac{r}{w} \right)^2 |l| e^{-r^2/w^2} c_1^* c_3 e^{i l \phi}.
\]

Thus the lossless propagation of generated vortex beams takes place at distances exceeding the absorption length \( L_{abs} \) as illustrated in Fig. 4. In that case the atomic systems goes to a superposition of two dark states \[30, 31, 64, 65\]

\[
D_1(z \gg L_{abs}) = \frac{\Omega_3(z \gg L_{abs})|g_3 \rangle - \Omega_1(z \gg L_{abs})|g_3 \rangle}{\sqrt{\Omega_1^2(z \gg L_{abs}) + \Omega_2^2(z \gg L_{abs})}},
\]
IV. THE MULTI-LEVEL SYSTEM

Let us now extend our model by considering the propagation of \( n \)-component light pulses through an \((n+1)\)-state atomic medium with \( n \) lower atomic states and one excited state shown in Fig. 5. Denoting the excited state by \(|e⟩\), the lower levels by \(|g_1⟩, |g_2⟩, ..., |g_n⟩\) and the Rabi-frequency of laser pulses by \(Ω_m (m = 1, 2, ..., n)\), the interaction Hamiltonian for such a multi-level atom reads

\[
H_M = \sum_{m=1}^{n} Ω_m |g_m⟩⟨e| + \text{H.c.} .
\]

The MBEs describing the dynamics of the system are given by

\[
\dot{ρ}_{g_m e} = i(δ_m + iγ_{eg_m})ρ_{g_m e} - iΩ_m (ρ_{ee} - ρ_{g_m g_m}) + i \sum_{j=1,j\neq m}^{n} Ω_j ρ_{g_m g_j} . \tag{41}
\]
and
\[
\frac{\partial \Omega_m}{\partial z} + c^{-1} \frac{\partial \Omega_m}{\partial t} = i \frac{\alpha_m \gamma_{eg}}{2L} \rho_{gm} \mbox{e} (m = 1, 2, \ldots, n),
\]
with \(m = 1, 2, \ldots, n\). As before, the diffraction terms are ignored in the Maxwell equations (42).

The atoms comprising the system are initially in the superposition of \(n\) ground states:
\[
|\psi(0)\rangle = \sum_{m=1}^{n} c_m |g_m\rangle.
\]
Assuming the atom-light interaction to be sufficiently weak, \(|\Omega_j| \ll \gamma_{egj}\), one can approximate \(\rho_{ee} \approx 0, \rho_{g_sg_s} \approx |c_s|^2, \rho_{g_st} \approx c_s c_t^*\) to the first order in all laser fields, giving
\[
\rho_{gm} = -\sum_{j=1}^{n} \frac{c_m c_j^* \Omega_j}{\delta_m + i \gamma_{egm}}, \quad m = 1, 2, \ldots, n
\]
(44)

If all the laser pulses except the first one are zero at the entrance \((\Omega_1(0) = \Omega\) while \(\Omega_2(0) = 0, \Omega_3(0) = 0, \ldots, \Omega_n(0) = 0\), the solutions to the MBEs (41) and (42) are
\[
\Omega_1(z) = \frac{\Omega}{X_{n}} (\beta_1 |c_1|^2 e^{-iX_{n}z} + \sum_{N=2}^{n} \beta_N |c_N|^2),
\]
(45)
\[
\Omega_N(z) = \frac{\Omega}{X_{n}} c_1^* c_N \beta_N (e^{-iX_{n}z} - 1), \quad (N = 2, \ldots, n),
\]
(46)
with
\[
X_{n} = \sum_{m=1}^{n} \beta_m |c_m|^2.
\]

From Eqs. (15) and (16) it follows then that if the first light pulse photons carry an OAM of \(hl\) along the propagation direction, \(n - 1\) optical vortices are generated in the medium with
the same vorticity as the first laser beam \( \Omega_1 \). In the vicinity of the vortex core the generated vortex beams look like a LG beam with their intensity vanishing at the core \( r \to 0 \).

Calling on Eqs. (15) and (16), Eqs. (45) and (46) provide the following solutions for the distances exceeding the absorption length \( z \gg L_{abs} \)

\[
\Omega_1(z \gg L_{abs}) = \varepsilon\left(\frac{r}{w}\right)^{|l|} e^{-r^2/w^2} |c_2|^2 e^{i l \phi},
\]

\[
\Omega_N(z \gg L_{abs}) = -\varepsilon(\frac{r}{w})^{l|} e^{-r^2/w^2} c_1 c_N e^{i l \phi},
\]

where we have assumed that \( \alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha, \gamma_{eg_1} = \gamma_{eg_2} = \cdots = \gamma_{eg_n} = \gamma, \) and \( \delta_1 = \delta_2 = \cdots = \delta_n = 0 \). Equations (48)-(49) demonstrate the lossless propagation of the \( n \)-component optical vortices because for \( z \gg L_{abs} \) the multi-level model goes to a linear superposition of \( n - 1 \) dark states.

V. COMPOSITE VORTICES

Let us next consider a situation where the \( \Lambda \) scheme is initially prepared in a superposition state given by Eqs. (6), but both fields \( \Omega_1 \) and \( \Omega_2 \) are incident on the medium. With the initial conditions where both incident fields are the vortex beams \( \Omega_1(0) = \Omega_{10} = \varepsilon(\frac{r}{w})^{l|} e^{-r^2/w^2} e^{i l_1 \phi} \) and \( \Omega_2(0) = \Omega_{20} = \varepsilon(\frac{r}{w})^{l_2|} e^{-r^2/w^2} e^{i l_2 \phi} \), the solutions to the Eqs. (9) and (10) take the form

\[
\Omega_1(z) = \frac{1}{X_2} \left[ \varepsilon_1(\frac{r}{w})^{l_1|} e^{-r^2/w^2} \left( \beta_1 |c_1|^2 e^{-i X_{2z}} + \beta_2 |c_2|^2 \right) e^{i l_1 \phi} + \varepsilon_2(\frac{r}{w})^{l_2|} e^{-r^2/w^2} c_1 c_2^* e^{i l_2 \phi} \right],
\]

\[
\Omega_2(z) = \frac{1}{X_2} \left[ \varepsilon_1(\frac{r}{w})^{l_1|} e^{-r^2/w^2} c_1 c_2^* e^{i l_1 \phi} + \varepsilon_2(\frac{r}{w})^{l_2|} e^{-r^2/w^2} \left( |c_2|^2 \beta_2 e^{-i X_{2z}} + |c_1|^2 \beta_1 \right) e^{i l_2 \phi} \right].
\]

In this way by applying two incident vortex beams \( \Omega_1(0) \) and \( \Omega_2(0) \) one produces two new beams \( \Omega_1(z) \) and \( \Omega_2(z) \) which may contain different vortices depending on the relative amplitude and phase of the incident beams.

Various situations can appear for the beams created in this way. If the winding numbers of the incident pulses are the same, \( l_1 = l_2 = l \), the resulting beams \( \Omega_1(z) \) and \( \Omega_2(z) \) have
the same vorticity \( l \), and the vortex width increases with increasing the winding number \( l \), as illustrated in Figs. 6(a), 6(c), 6(e). Figures 6(b), 6(d), 6(f) show the corresponding phase profile of the beams. If \(|l_1| < |l_2|\), the resulting composite twisted beam contains a vortex of charge \( l_1 \) located at the beam center which is surrounded by \(|l_1 - l_2|\) peripheral vortices (Fig. 7). In this case, two light vortices with different winding numbers \( l_1 \) and \( l_2 \) around same axis result in formation of vortices with shifted axes. In particular, for \( l_1 = -l_2 = l \) we superimpose two optical vortices with opposite topological charges and equal intensity, and the azimuthal dependence is given by \( e^{il\phi} + e^{-il\phi} = 2 \cos(l\phi) \). This corresponds to the flower-like “petals” intensity structures demonstrated in Fig. 8. Note that such a flower-like structure is not called as vortex, although it has a zero intensity at the center [66, 67].

Let us know assume \( \alpha_1 = \alpha_2 = \alpha, \ \gamma_{eg1} = \gamma_{eg2} = \gamma, \) and \( \delta_1 = \delta_2 = 0 \). At propagation distances exceeding the absorption length \( z \gg L_{abs} \) all exponential terms vanish in Eqs. (50) and (51) and we get

\[
\Omega_1(z \gg L_{abs}) = e^{-r^2/w^2} \left[ |c_2|^2 \varepsilon_1 \frac{R}{w} |l_1| e^{il_1\phi} - c_1 c_2^* \varepsilon_2 \frac{R}{w} |l_2| e^{il_2\phi} \right],
\]

\[
\Omega_2(z \gg L_{abs}) = e^{-r^2/w^2} \left[ -c_1^* c_2 \varepsilon_1 \frac{R}{w} |l_1| e^{il_1\phi} + |c_1|^2 \varepsilon_2 \frac{R}{w} |l_2| e^{il_2\phi} \right].
\]

Therefore if the two incident vortex fields are nonzero, for \( z \gg L_{abs} \) both vortex beams experience no absorption as the atoms comprising the medium are converted to their dark states defined by Eq. (19). It is noteworthy that there is an even more favorable scenario for the lossless propagation of both vortex beams. Assuming that \( \Omega_1(0) = \Omega_2(0) = \Omega = \varepsilon(R/w)^{|l|} e^{-r^2/w^2} e^{il\phi} \) and choosing the values of \( c_1 \) and \( c_2 \) such that \( c_1 = -c_2 = \frac{1}{\sqrt{2}} \), one arrives at \( \Omega_1(z) = \Omega_2(z) = \Omega(z) \). Under this condition the atoms are in their dark states from the very beginning, the medium becomes completely transparent to both vortex beams, and the fields propagate without losses as in free space. Such an analysis for generation and propagation of composite optical vortices can be extended to the \( n + 1 \)-level schemes when all \( n \) laser fields are present at the entrance to the atomic medium.

VI. CONCLUDING REMARKS

We have analyzed the propagation dynamics of two (three) component laser pulses with OAM interacting with atoms in the \( \Lambda \) (tripod) atom-light coupling schemes. The quantum
FIG. 6. (a), (c), (e) Intensity distributions in arbitrary units as well as (b), (d), (f) the corresponding helical phase patterns of the beam $\Omega_1(z)$ defined by Eq. (50) generated by combining two vortex beams with (a), (b) $l_1 = l_2 = 1$, (c), (d) $l_1 = l_2 = 5$ and (e), (f) $l_1 = l_2 = 8$. Here, the parameters are $z = L/2$, $c_1 = c_2 = \frac{1}{\sqrt{2}}$, $\delta_1 = \delta_2 = 0$, $\varepsilon_1 = \varepsilon_2$ and $\alpha = 20$. The intensity distribution and phase pattern of the field $\Omega_2(z)$ are identical to the intensity distribution and phase pattern of the field $\Omega_1(z)$ shown in this figure.

The system is initially prepared in a coherent superposition of two (three) lower levels. If a vortex beam acts on one transition of the Λ (tripod) system, an extra light beams can be generated with the same OAM number as the initially injected structured light. We have also extended the analysis to a $n + 1$-level phaseonium medium for the $n$-component generation of the twisted light beams. The lossless propagation of generated vortex light beams has been also considered. It has been shown that at the propagation distances exceeding the absorption length the system goes to a linear superposition of $n - 1$ dark states leading to the transparency of the medium to the $n$-component optical vortices.

It has been recently shown that a double-Λ scheme can be employed for the exchange of
optical vortices based on EIT \cite{55}. In the double-Λ scheme there should be two additional control lasers of larger intensity to assure the exchange of optical vortices. On the other hand, in the current proposal one does not need the strong atom-light interaction as we are dealing with small intensities (|Ω_i| \ll \gamma_{eg_i}). It is only needed that a medium is initially coherently prepared in a superposition of atomic lower levels. The losses in both schemes are similar and take place mostly at beginning of the medium within the absorption length. The losses disappear when the light pulses propagate deeper through the medium.

We have also considered a situation where both vortex beams Ω_1 and Ω_2 are present at beginning of the medium of the Λ-type atoms. When the two vortex beams are incident on
FIG. 8. Intensity distributions in arbitrary units for the beam $\Omega_1(z)$ defined by Eq. (50) generated by combining two vortex beams with (a) $l_1 = 1$, $l_2 = -1$, (b) $l_1 = 2$, $l_2 = -2$, (c) $l_1 = 3$, $l_2 = -3$, and (c) $l_1 = 4$, $l_2 = -4$. The selected parameters are the same as Fig. 6. The intensity distribution of the field $\Omega_2(z)$ is identical to the intensity distribution of the field $\Omega_1(z)$ shown in this figure.

In the medium, they can create two composite beams with new vortices. Different cases for the appearance of composite vortices have been explored, and the situations for absorption-less propagation of composite vortices are discussed. We have also extended the model for generation of composite optical vortices to the $n + 1$-level structures. The coherent superposition of the ground states employed in this paper can be realized experimentally using the fractional or partial Stimulated Raman adiabatic passage (STIRAP) in which only a controlled fraction of the population is transferred to the target state.

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