The orbital decay and tidal disruption of a star cluster in a galaxy is studied in an analytical manner. Owing to dynamical friction, the star cluster spirals in toward the center of the galaxy. Simultaneously, the galactic tidal field strips stars from the outskirts of the star cluster. Under an assumption that the star cluster undergoes a self-similar evolution, we obtain the condition and timescale for the star cluster to reach the galaxy center before its disruption. The result is used to discuss the fate of so-called intermediate-mass black holes with \( \gtrsim 10^3 \, M_\odot \) found recently in young star clusters of starburst galaxies and also the mass function of globular clusters in galaxies.

Subject headings: galaxies: star clusters — galaxies: starburst — stellar dynamics

1. INTRODUCTION

Tremaine, Ostriker, & Spitzer (1975) studied the circular motion of a star cluster around the center of a galaxy (see also Binney & Tremaine 1987, p. 427). The star cluster interacts with background stars of the galaxy via dynamical friction, loses its orbital kinetic energy, and thereby spirals in toward the galaxy center. Tremaine et al. (1975) approximated the mass density \( \rho_b \) of the background stars with that of a singular isothermal sphere:

\[
\rho_b(R) = \frac{v_b^2}{4\pi GR^2},
\]

where \( R \) is the galactocentric radius, \( v_b \) is the circular velocity of the background stars, and \( G \) is the gravitational constant. The cluster mass \( M \) was set to be constant. The circular velocity of the star cluster was set to be \( v_b \). Then, the rate of change of the galactocentric radius of the star cluster was obtained as

\[
R \frac{dR}{dt} = -0.4276 \ln \Lambda_b \frac{GM}{v_b},
\]

where \( \ln \Lambda_b \) is the Coulomb logarithm. Solving equation (2) with the initial condition \( R = R_i \) at \( t = 0 \), Tremaine et al. (1975) found that the star cluster reaches the galaxy center at the time \( t = \tau_{\text{decay}} \):

\[
\tau_{\text{decay}} = \frac{1.169 R_i^2 v_b}{\ln \Lambda_b \frac{GM}{v_b}}.
\]

This timescale \( \tau_{\text{decay}} \) has been often used to study the evolution of a star cluster.

However, in practice, the star cluster is subject to the galactic tidal field. Stars belonging to the cluster have to lie within the tidal radius \( r_t \) from the cluster center (Spitzer 1987, p. 101; see also Capriotti & Hawley 1996):

\[
r_t = \frac{2R}{3} \left[ \frac{M}{3M_b(R)} \right]^{1/3} \left[ 1 - \frac{R}{3M_b(R)} \frac{dM_b(R)}{dR} \right]^{-1/3},
\]

where \( M_b(R) \) is the total mass of the background stars within the cluster’s galactocentric radius \( R \), i.e.,

\[
M_b(R) = 4\pi \int_0^R \rho_b R^2 dR.
\]

If equation (1) is used for the mass distribution \( \rho_b \), equation (4) becomes

\[
r_t = \frac{2}{3} \left( \frac{G}{2v_b^2} \right)^{1/3} M^{1/3} R^{2/3}.
\]

Thus, with a decrease of the galactocentric radius \( R \), the tidal radius \( r_t \) becomes small. Stars are accordingly stripped from the outskirts of the cluster and become part of the background stars. Since the cluster mass becomes small, the dynamical friction becomes inefficient. In addition, if the star cluster is not sufficiently massive, it dissolves before it reaches the galaxy center.

This effect of tidal stripping was incorporated in some past studies of the orbital decay of a star cluster, but they were numerical studies with limited ranges of parameters. A more general analytical study is desirable.

Here we conduct an analytical calculation for the first time. The star cluster is assumed to evolve in a self-similar manner (§ 2). Then, the rate of change of the cluster mass is obtained as a function of the galactocentric radius of the star cluster (§ 3, eq. [14]). Using this mass-loss rate together with equation (2) for the rate of change of the galactocentric radius of the star cluster, we derive the condition and timescale for the star cluster to reach the galaxy center before it dissolves (§ 4, eqs. [27] and [28]). We discuss the validity of our assumptions and the astrophysical applications of our result (§ 5). The latter discussion focuses on the fate of so-called intermediate-mass black holes (BHs) with mass \( \gtrsim 10^3 \, M_\odot \) found in young star clusters of starburst galaxies and also on the mass function of globular clusters in galaxies.

2. SELF-SIMILAR EVOLUTION

The description of a star cluster requires at least two independent internal parameters, e.g., the cluster mass \( M \) and the half-mass radius \( r_h \), in addition to external parameters such as the galactocentric radius \( R \). To simplify our calculation, we consider a self-similar evolution, in which one of the two internal parameters is exceptionally determined by the other. Here is an idealized self-similar model for a star
cluster in a tidal field, which originates in the work of Spitzer (1987, p. 59; see also Capriotti & Hawley 1996).

Recall that the crossing timescale \( \tau_{\text{cross}} \approx (r_h^3/GM)^{1/2} \) is much less than the other timescales relevant to the evolution of a star cluster. The star cluster is always nearly in virial equilibrium (Binney & Tremaine 1987, p. 211):

\[
E = K + W = -K = \frac{W}{2}, \tag{6}
\]

where \( E \) is the total energy, \( K \) is the kinetic energy, and \( W \) is the potential energy of the star cluster. Their definitions are

\[
K = \frac{M \langle v^2 \rangle}{2}, \quad W = \frac{M \langle \Phi \rangle}{2}, \tag{7}
\]

where \( v \) is the velocity of a star, \( \Phi \) is the gravitational potential at the position of a star, and \( \langle \ldots \rangle \) denotes an average over stars in the cluster. Spitzer (1969) observed that various spherical clusters approximately have the relation

\[
\langle v^2 \rangle = \frac{2GM}{5r_h}. \tag{8}
\]

Then we have \( E = -GM^2/5r_h \), which yields

\[
\frac{dE}{dt} = \frac{GM^2}{5r_h^3} \frac{dr_h}{dt} - \frac{2GM}{5r_h} \frac{dM}{dt}. \tag{9}
\]

This change in total energy \( dE \) is due to mass loss across the tidal radius \( r_t \) and is thereby equal to the change in potential energy associated with the displacement of \( dM \) from the tidal radius to infinity. Thus, we also have

\[
\frac{dE}{dt} = -\frac{GM}{r_t} \frac{dM}{dt}. \tag{10}
\]

Equations (9) and (10) combine to yield

\[
\frac{d\ln r_h}{dt} = 3 \left( 2 - \frac{r_h}{r_t} \right) \frac{d\ln r_t}{dt}, \tag{11}
\]

where we have used \( M \propto r_h^3 \) (eq. [5]). If \( r_h/r_t = 1/3 \), this ratio remains the same. Since the tidal radius \( r_t \) is determined by the cluster mass \( M \) and the galactocentric radius \( R \) (eq. [5]), the evolution of the star cluster is also determined by these two quantities.

### 3. MASS-LOSS RATE

The escape of the individual stars from the cluster across the tidal radius is due to two-body relaxation. This process is thereby represented by removing a fraction \( f_{\text{esc}} \) of the stars every relaxation timescale \( \tau_{\text{relax}} \) (Binney & Tremaine 1987, pp. 490, 523; Spitzer 1987, p. 52):

\[
\frac{dM}{dt} = -\frac{f_{\text{esc}}}{\tau_{\text{relax}}} M. \tag{12}
\]

The half-mass relaxation timescale is used for \( \tau_{\text{relax}} \) (Spitzer & Hart 1971; Binney & Tremaine 1987, p. 514; Spitzer 1987, p. 40):

\[
\tau_{\text{relax}} = \frac{0.1386 M}{\ln \Lambda \ m} \left( \frac{r_h^3}{GM} \right)^{1/2}, \tag{13}
\]

where \( \ln \Lambda \) is the Coulomb logarithm for the star cluster and \( m \) is the mean stellar mass. Using equation (5), we rewrite equation (13) as

\[
\tau_{\text{relax}} = (0.05335/\ln \Lambda)(r_h/r_t)^{3/2} (MR/mv_h). \tag{14}
\]

Substituting this into equation (12), we obtain

\[
\frac{dM}{dt} = -18.74 \ln \Lambda \left( \frac{r_h}{r_t} \right)^{-3/2} f_{\text{esc}} mv_h \frac{R}{R}. \tag{15}
\]

If \( r_h/r_t \) is constant, the mass-loss rate in equation (14) becomes larger with a decrease of the galactocentric radius \( R \). This is because the tidal radius \( r_t \), the half-mass radius \( r_h \), and hence the relaxation timescale \( \tau_{\text{relax}} \), become smaller.

The escape probability \( f_{\text{esc}} \) is approximated with the fraction of stars in a Maxwellian distribution that have velocities exceeding the rms escape velocity \( \langle v_{\text{esc}}^2 \rangle^{1/2} \) (Binney & Tremaine 1987, p. 490; Spitzer 1987, pp. 51, 57):

\[
f_{\text{esc}} = \frac{4\pi}{(2\pi)^{3/2}} \int_0^\infty \exp \left( -\frac{v^2}{2} \right) v^2 dv. \tag{16}
\]

The escape velocity \( v_{\text{esc}} \) is defined at a position in the star cluster as

\[
\frac{v_{\text{esc}}^2(r)}{2} + \Phi(r) = -\frac{GM}{r}. \tag{17}
\]

Thus, the escape probability is constant in a self-similar model. The ratio \( r_h/r_t = 1/3 \) leads to \( \langle v_{\text{esc}}^2 \rangle \langle v^2 \rangle = 7/3 \), which in turn leads to \( f_{\text{esc}} = 0.07190 \).

### 4. ANALYTICAL SOLUTION

With the initial conditions \( R = R_i \) and \( M = M_i \) at \( t = 0 \), the behavior of a self-similar star cluster is studied by solving equations (2) and (14). The Coulomb logarithms \( \ln \Lambda_h \) and \( \ln \Lambda \), the radius ratio \( r_h/r_t \), and the escape probability \( f_{\text{esc}} \) are set to be constant. We introduce the friction timescale \( \tau_{\text{fric}} \) and the mass-loss timescale \( \tau_{\text{loss}} \):

\[
\tau_{\text{fric}} = \frac{R}{-dR/dt}_{t=0} \left|_{(18)} \right. \frac{R^2 v_h}{0.4276 \ln \Lambda_h GM_i},
\]

\[
\tau_{\text{loss}} = \frac{M}{-dM/dt}_{t=0} \left|_{(19)} \right. \frac{\left( r_h/r_t \right)^{3/2} M R_i}{18.74 \ln \Lambda v_{\text{esc}} mv^2_h R_i}. \tag{19}
\]

We also introduce the ratio between \( \tau_{\text{fric}} \) and \( \tau_{\text{loss}} \):

\[
\alpha = \frac{\tau_{\text{fric}}}{\tau_{\text{loss}}} = 43.83 \ln \Lambda \frac{v_h ^{3/2}}{r_t \ln \Lambda_h} \frac{v_{\text{esc}} mv^2_h}{GM_i^2} \tag{20}
\]

The galactocentric radius of the cluster \( R \), the cluster mass \( M \), and the time \( t \) are normalized as

\[
\frac{R}{R_i} = \tilde{R}, \quad \frac{M}{M_i} = \tilde{M}, \quad \frac{t}{\tau_{\text{fric}}} = \tilde{t}. \tag{21}
\]

Equations (2) and (14) are accordingly rewritten as

\[
\frac{d\tilde{R}}{d\tilde{t}} = -\tilde{t} \frac{\tilde{M}}{\tilde{R}}, \quad \frac{d\tilde{M}}{d\tilde{t}} = -\alpha \frac{\tilde{R}}{\tilde{M}}. \tag{22}
\]

The initial condition is \( \tilde{R} = \tilde{M} = 1 \) at \( \tilde{t} = 0 \).
Equation (22) yields $\dot{M} d\dot{M}/dt = \alpha d\dot{R}/dt$. The solution of this equation is
\[
\dot{M}^2 = 2\alpha \dot{R} + 1 - 2\alpha .
\] (23)

Using equation (23), we eliminate $\dot{M}$ from equation (22):
\[
\frac{d\dot{R}}{d\dot{R}} = -\frac{\dot{R}}{(2\alpha \dot{R} + 1 - 2\alpha)^{1/2}} .
\] (24)

The solution of equation (24) gives the relation between the time $\dot{t}$ and the galactocentric radius of the star cluster $\dot{R}$:
\[
\dot{t} = \frac{(1 - 2\alpha - \alpha \dot{R})(1 - 2\alpha + 2\alpha \dot{R})^{1/2} - (1 - 3\alpha)}{3\alpha^2} .
\] (25)

The relation between the time $\dot{t}$ and the cluster mass $\dot{M}$ is obtained by substituting equation (23) into equation (25):
\[
\dot{t} = \frac{(3 - 6\alpha - \dot{M}^2)\dot{M} - 2(1 - 3\alpha)}{6\alpha^2} .
\] (26)

Figure 1 illustrates the evolution of $\dot{R}$ and $\dot{M}$ as a function of $\dot{t}$ for $\alpha = 0$ and $\alpha = \frac{1}{2}, \frac{1}{3},$ and 1.

The condition for the star cluster to reach the galaxy center before its dissolution is $\dot{M} \geq 0$ at $\dot{R} = 0$, for which equation (23) yields
\[
\alpha \leq \frac{1}{2} \text{ or } \frac{\tau_{\text{fric}}}{\tau_{\text{loss}}} \leq \frac{1}{2} .
\] (27)

The time at which the star cluster reaches the galaxy center is obtained by substituting $\dot{R} = 0$ into equation (25):
\[
\tau_{\text{decay}} = \frac{\tau_{\text{decay}}}{\tau_{\text{fric}}} = \frac{(1 - 2\alpha)^{3/2} - (1 - 3\alpha)}{3\alpha^2} .
\] (28)

If the parameter $\alpha$ is very small ($\alpha \ll \frac{1}{2}$), equation (28) is reduced to
\[
\tau_{\text{decay}} = \frac{1}{2} \text{ or } \tau_{\text{decay}} = \frac{\tau_{\text{fric}}}{2} \left(1 + \frac{\tau_{\text{fric}}}{3\tau_{\text{loss}}} \right) .
\] (29)

In the limit $\alpha \to 0$, equations (28) and (29) reproduce the solution $\tau_{\text{decay}} = \frac{1}{2}$ for no mass loss obtained by Tremaine et al. (1975; see our eq. [3]). The timescale $\tau_{\text{decay}}$ for $\alpha = \frac{1}{2}$ is $\frac{3}{2}$. Thus, mass loss lengthens $\tau_{\text{decay}}$ by a factor of $4/3$ at most because significant mass loss occurs only at the latest stage of the orbital decay, i.e., in the vicinity of the galaxy center (Fig. 1). The cluster mass at the time when the star cluster reaches the galaxy center is obtained by substituting $\dot{R} = 0$ into equation (23):
\[
\dot{M}_{\text{decay}} = (1 - 2\alpha)^{1/2} \text{ or } \dot{M}_{\text{decay}} = M_{\text{fric}} \left(1 - \frac{2\tau_{\text{fric}}}{\tau_{\text{loss}}} \right)^{1/2} .
\] (30)

On the other hand, if the star cluster dissolves before it reaches the galaxy center, the corresponding time is obtained by substituting $\dot{M} = 0$ into equation (26):
\[
\tau_{\text{dissolve}} = \frac{1}{\alpha - \frac{1}{3\alpha^2}} \text{ or } \tau_{\text{dissolve}} = \tau_{\text{loss}} \left(1 - \frac{\tau_{\text{loss}}}{3\tau_{\text{fric}}} \right) .
\] (31)

The corresponding galactocentric radius is obtained by substituting $\dot{M} = 0$ into equation (23):
\[
\dot{R}_{\text{dissolve}} = 1 - \frac{1}{2\alpha} \text{ or } \dot{R}_{\text{dissolve}} = R \left(1 - \frac{\tau_{\text{loss}}}{2\tau_{\text{fric}}} \right) .
\] (32)

Therefore, the behavior of the star cluster is determined by the parameter $\alpha = \tau_{\text{fric}}/\tau_{\text{loss}}$. If $\alpha \ll \frac{1}{2}$, the mass loss is slow ($2\tau_{\text{fric}} \ll \tau_{\text{loss}}$). The star cluster reaches the galaxy center before it loses much of its mass. If $\alpha \gg 1$, the mass loss is fast ($2\tau_{\text{fric}} \gg \tau_{\text{loss}}$). The star cluster dissolves before it moves significantly from its initial galactocentric radius.

5. DISCUSSION

5.1. Model Assumptions

We have assumed that a star cluster loses its mass via tidal stripping and two-body relaxation alone. This assumption is correct only if the orbit of the star cluster is circular and in the galactic plane. The star cluster otherwise suffers from gravitational shocks in passages through the galactic bulge or disk (Ostriker, Spitzer, & Chevalier 1972). These complicated cases are beyond the reach of our analytical calculation.

We have assumed that the star cluster always has $r_h/r_t = 1/3$. This ratio is unstable in our idealized model (eq. [11]). If $r_h/r_t > 1/3$, the ratio increases still more. The star cluster dissolves at $r_h/r_t = 4/5$, where the escape velocity $v_{\text{esc}} = 0$ (Capriotti & Hawley 1996; see our eq. [17]). If $r_h/r_t < 1/3$, the ratio decreases toward 0. The ratio typical of globular and open clusters is $r_h/r_t \simeq 0.2$ (Binney & Tremaine 1987, p. 26). Nevertheless, in practice, such star clusters
do not undergo \( r_h/r_t \to 0 \). The ratio is rather sustained to be nearly constant by heating due to binaries, a process that is not considered in our idealized model (Binney & Tremaine 1987, p. 543; Spitzer 1987, p. 148). Since the mass-loss rate for \( r_h/r_t = 0.2 \) obtained from equations (14), (15), and (17) is lower by only 12\% than that for \( r_h/r_t = 1/3 \), our model with \( r_h/r_t = 1/3 \) is of practical use.

There are other self-similar models for a star cluster in a tidal field. They are based on different assumptions. For example, Hénon (1961) assumed the presence of an energy source at the cluster center and obtained \( r_h/r_t = 0.1446 \) and \( f_{\text{esc}} = 0.045 \) (see also Spitzer 1987, p. 59).\(^1\) The energy source could represent the binary heating and sustains the small value of \( r_h/r_t \) as compared to that of our present model. It is possible to adapt these self-similar models into our calculation because the escape probability \( f_{\text{esc}} \) is constant in any self-similar model (eqs. [15] and [17]). We nevertheless favor our present model, which is simple but still sufficient for our purpose.

We have assumed that a star with velocity greater than the escape velocity immediately escapes from the cluster. With somewhat improved treatments of the escape condition, escape probabilities \( f_{\text{esc}} \) close to ours were obtained by the Fokker-Planck numerical simulations of Lee & Ostriker (1987) and Lee & Goodman (1995).

The escape probability \( f_{\text{esc}} \) is affected by two remarkable processes that are not considered in the above calculations. First, since stars with velocities slightly above the escape velocity can escape only through small holes near the Lagrangian points, they stay in the cluster for many crossing timescales. Some of the stars are scattered to lower velocities and become bound again. The escape probability is accordingly lowered (Baumgardt 2001). It might be better to consider that our \( f_{\text{esc}} \) value tends to be an upper limit. Second, when the star cluster is young, mass loss in the course of evolution of the individual stars, i.e., stellar wind and supernova explosions, causes a decrease of the tidal radius and thereby enlarges the escape probability. This process is nevertheless unimportant in our study, which applies mainly to relatively old clusters.

Taking account of the distribution of stellar masses as well as the evolution of stars and binaries, Portegies Zwart et al. (2002) conducted realistic \( N \)-body numerical simulations of a star cluster in a static tidal field. While they did not vary the total number of stars in the cluster, they varied its initial density distribution and its distance from the galaxy center. The mass-loss timescale \( \tau_{\text{loss}} \) was found to scale with the relaxation timescale at the tidal radius (eq. [13], but with \( r_t \) substituted for \( r_h \)).\(^2\) Even if this is generally the case, our approach is still of use. We only have to set \( f_{\text{esc}} = \tau_{\text{relax}}/\tau_{\text{loss}} \) with \( r_h/r_t = 1 \).

5.2. Astrophysical Applications

The Coulomb logarithms \( \ln \Lambda_b \) and \( \ln \Lambda \) are of the order of \( 10^4 \), and the circular velocity \( v_b \) is of the order of \( 10^2 \) km s\(^{-1} \) (Binney & Tremaine 1987, pp. 423, 427). Thus, equations (18)–(20) are written as

\[
\tau_{\text{esc}} = \left( 5.316 \times 10^9 \right) \left( \frac{\ln \Lambda_b}{10} \right)^{-1} \left( \frac{v_b}{100 \ \text{km s}^{-1}} \right) \times \left( \frac{M_i}{10^6 \ M_\odot} \right)^{-1} \left( \frac{R_i}{1 \ \text{kpc}} \right)^2, \tag{33}
\]

\[
\tau_{\text{loss}} = \left( 1.397 \times 10^{11} \right) \left( \frac{\ln \Lambda}{10} \right)^{-1} \left( \frac{r_h/r_t}{1/3} \right)^{3/2} \left( \frac{f_{\text{esc}}}{0.07190} \right)^{-1} \times \left( \frac{m}{1 \ M_\odot} \right)^{-1} \left( \frac{v_b}{100 \ \text{km s}^{-1}} \right)^{-1} \times \left( \frac{M_i}{10^6 \ M_\odot} \right) \left( \frac{R_i}{1 \ \text{kpc}} \right), \tag{34}
\]

\[
\alpha = \frac{\tau_{\text{esc}}}{\tau_{\text{loss}}} = 0.03807 \left( \frac{\ln \Lambda}{\ln \Lambda_b} \right) \left( \frac{r_h/r_t}{1/3} \right)^{3/2} \left( \frac{f_{\text{esc}}}{0.07190} \right) \times \left( \frac{m}{1 \ M_\odot} \right) \left( \frac{v_b}{100 \ \text{km s}^{-1}} \right)^2 \left( \frac{M_i}{10^6 \ M_\odot} \right)^{-2} \left( \frac{R_i}{1 \ \text{kpc}} \right). \tag{35}
\]

These formulae allow us to obtain the actual values of \( \tau_{\text{decay}}, M_{\text{decay}}, \tau_{\text{dissolve}}, \) and \( R_{\text{dissolve}} \) (eqs. [28], [30], [31], and [32]). The condition for the star cluster to reach the galaxy center before its dissolution, \( \alpha \leq 1/3 \) (eq. [27]), is written as

\[
M_i \geq \left( 2.759 \times 10^5 \ M_\odot \right) \left( \frac{\ln \Lambda}{\ln \Lambda_b} \right)^{1/2} \times \left( \frac{r_h/r_t}{1/3} \right)^{-3/4} \left( \frac{f_{\text{esc}}}{0.07190} \right)^{1/2} \times \left( \frac{m}{1 \ M_\odot} \right)^{1/2} \left( \frac{v_b}{100 \ \text{km s}^{-1}} \right)^{1/2} \left( \frac{R_i}{1 \ \text{kpc}} \right)^{1/2}. \tag{36}
\]

Figure 2 illustrates this condition as well as the values of \( \tau_{\text{decay}} \) and \( \tau_{\text{dissolve}} \) as a function of the initial mass \( M_i \) and the initial galactocentric radius \( R_i \).

5.2.1. Fate of Intermediate-Mass Black Holes

Our result is used to discuss the fate of massive BHs found in young star clusters of starburst galaxies. At the 2 \( \mu \)m secondary peak of the starburst galaxy M82, i.e., an active site of star formation, there is a source of compact X-ray emission (Matsushita et al. 2000; Kaares et al. 2001; Matsumoto et al. 2001). The observed strong variability implies that the source is an accreting BH. The observed luminosity of \( 10^{34} \) ergs s\(^{-1} \) implies that the mass is greater than \( 10^3 M_\odot \) if the emission is isotropic and its luminosity is below the Eddington limit. BHs of similar masses have also been found in other starburst galaxies, e.g., NGC 3628 (Strickland et al. 2001). They are called intermediate-mass BHs (Taniguchi et al. 2000) because their masses are in between those of stellar-mass BHs arising from stellar evolution and supermassive BHs found as the central engines of active galactic nuclei. The likely origin is successive mergers of massive stars and stellar-mass BHs that had sunk into the core of a star cluster (Taniguchi et al. 2000; Ebisuzaki et al. 2001; Portegies Zwart & McMillan 2002). Under a favorable

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1 Hénon’s model was numerical. If we use our analytical approximation in eqs. (15) and (17), \( r_h/r_t = 0.1446 \) yields \( f_{\text{esc}} = 0.02006 \).

2 Within a static tidal field, the cluster mass decreases linearly with time (Spitzer 1987, p. 58). This behavior is reproduced by our mass-loss rate (eq. [14]) if the galactocentric radius of the star cluster is set to be constant.
Young star clusters in starburst galaxies have masses up to $10^6 \, M_\odot$ (e.g., Mengel et al. 2002). The largest mass clusters contain the largest numbers of massive stars and thus are the most advantageous for forming intermediate-mass BHs. Such clusters are also the most advantageous for bringing the BHs to the galaxy center. Star-forming regions of starburst galaxies generally have galactocentric radii of $R \approx 10^2 - 10^3$ pc (e.g., Telesco, Dressel, & Woltzencroft 1993). If the mass is $10^6 \, M_\odot$, a star cluster reaches the galaxy center within the timescale $10^8 - 10^9$ yr (Fig. 2). The timescale is $10^8$ yr for the host cluster of the intermediate-mass BH in M82, which lies 170 pc from the galaxy center. These timescales are very small as compared to the lifetime of a galaxy, $10^{10}$ yr.

5.2.2. Mass Function of Globular Clusters

Our result is also used to discuss the mass function of globular clusters in galaxies. The globular clusters have a preferred mass scale. Their mass function has a sharp peak at $(1-2) \times 10^5 \, M_\odot$ (e.g., Harris 1991). A plausible explanation is that the mass function was initially wide but later modified by dynamical friction, tidal stripping, gravitational shocks, and so on (e.g., Fall & Rees 1977; Okazaki & Tosa 1995; Fall & Zhang 2001). Although the shape of the initial mass function is uncertain, it is possible to demonstrate that a cluster mass of $10^5 \, M_\odot$ is advantageous for a star cluster to survive dynamical friction and tidal stripping (see also Lee & Ostriker 1987; Capriotti & Hawley 1996). Suppose that star clusters were formed $\tau_{\text{age}}$ yr ago over ranges of the mass $M_i$ and the galactocentric radius $R_i$. On our $M_f$-$R_f$ diagram (Fig. 2), the star clusters that have survived up to now are those above the line for the timescale $\tau_{\text{age}}$. This line is concave. If $R_i$ is fixed, star clusters with too large masses disappear into the galaxy center because of dynamical friction. Star clusters with too small masses dissolve because of tidal stripping. There is an initial mass $M_{\text{survive}}$ with which star clusters survive for the widest range of $R_i$. This mass corresponds to $\alpha \approx 1/2$ (Fig. 2, solid line), where the net effect of dynamical friction and tidal stripping is minimal. Eliminating $R_i$ from equations (33) and (35) with $\tau_{\text{age}} = \tau_{\text{decay}} = \tau_{\text{dissolve}} = 2 \tau_{\text{fric}}/3$ and $\alpha = 1/2$, we obtain

$$M_{\text{survive}} = (2.539 \times 10^5 \, M_\odot) \left( \frac{\ln \Lambda}{10} \right)^{-1/3} \left( \frac{\ln \Lambda}{10} \right)^{2/3} \times \left( \frac{\tau_{\text{fric}}}{10^4 \, \text{yr}} \right)^{1/3}.$$  

Thus, at the present age of globular clusters $\tau_{\text{age}} \approx 10^{10}$ yr, those with $M_i \approx 10^5 \, M_\odot$ are relatively abundant. Their present masses are also of the order of $10^5 \, M_\odot$ in most cases because mass loss is significant only at the latest stage of the orbital decay.

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3 Two remarks: First, a single intermediate-mass BH is sufficient to explain the lowest luminosity active galactic nuclei, where the mass of the central engine is as low as $10^6 \, M_\odot$ (e.g., NGC 4395; Filippenko & Sargent 1989). Second, even if there remains no available gas when the intermediate-mass BH reaches the center, the galaxy has a chance to emerge with an active galactic nucleus in the future, since the supply of a large amount of gas to the nuclear region is repetitive (e.g., Mouri & Taniguchi 2002a).
