Simulation of contact interaction of rough surfaces

V P Tikhomirov and M A Izmerov
The Department of Pipeline Transportation Systems, Bryansk State Technical University, Bryansk, Russia
dm-bgtu@yandex.ru, m.izmerov@yandex.ru

Abstract. Using simulation of contact interaction of rough surfaces, it is possible to determine the dependence of the convergence value on the load applied to the conjugate surfaces, which will allow more accurately solve tribological problems, as well as problems of ensuring a given degree of tightness, etc.

1. Introduction
Existing models of contact interaction are based on assumptions that are used to estimate the parameters of conjugation joint – these are the well-known G–W (Greenwood-Williamson) [1] and M–B (Majumdar-Bhushan) models [2]. G-W model became the basis for calculating the contact interaction parameters of nominally flat rough surfaces. According to G-W model, it was proposed to replace the contact between two rough surfaces with the conjugation of an equivalent rough surface and a solid flat surface without taking into account the interaction of irregularities. Statistical parameters (such as the roughness height, the slope of irregularities, and the rounding radius of the top part of irregularities) are highly dependent on the resolution of the measuring device or any form of signal filter. This circumstance leads to the conclusion that it is necessary to correct the statistical model of contact interaction in order to obtain adequate results. To overcome the drawback of the statistical contact model, Majumdar and Bhushan [1] developed a fractal contact model called M-B model, in which the measure of surface roughness is determined by the fractal dimension \(D_S (2 < D_S < 3)\) and the fractal roughness parameter \(G\). A distinctive feature of M-B model is to represent the surface as a fractal object, which is characterized by affine self-similarity. An example of a self-affine fractal object is the surface roughness profile (figure 1).

![Self-affine surface roughness profile](image)

**Figure 1.** Self-affine surface roughness profile.

As you can see from figure 1, the profile fragments highlighted by circles requires different zoom levels to obtain similarity. Increasing the profile section leads to greater profile detail – revealing the first level of sub-roughness, and then another level. The presence of small details of roughness allowed M-B model to assume that irregularities that come into contact first deform plastically, and...
then, as the contact area increases, elastically. This is due to the fact that small contact areas, having small radii of curvature, are subjected to plastic deformation. Indeed, the ultimate strain

\[ \delta_c = k \left( \frac{H}{E} \right)^2 R, \]

where \( H \) is the hardness of the less solid interface element; \( E \) is the elastic modulus, which depends on radius \( R \) of the upper part of asperities. In this case, even at a low level of load on the joint, micro irregularities are deformed plastically until the formed contact area under a certain load becomes elastic (figure 2).

Figure 2. Asperity of micro irregularities.

Here tilt angle of irregularities (the arithmetical average) at the base of a microasperity with size \( l \), is found by the formula

\[ \Delta_\alpha = \frac{1}{l} \int_0^l \left| \frac{dz}{dx} \right| dx. \]  (1)

The profile of an isotropic surface is described by Weierstrass-Mandelbrot equation. Assuming that the base of the asperity (figure 2) is on the midline \( z(x)=0 \), we use Weierstrass-Mandelbrot equation to describe the shape of the asperity

\[ z(x) = G^{(D-1)} l^{(2-D)} \cos(\pi x/l). \]  (2)

Here \( z(x) \) is the ordinate of an isotropic rough surface measured from the middle plane, \( G \) is the fractal roughness parameter, and \( D \) is the fractal dimension \( (1<D<2) \). The presented parameters can be determined (in accordance with [3]) by the following correspondences:

\[ G = \left( \frac{R_q \left\{ \sin \left[ \frac{\pi(2D-3)}{2} \right] \Gamma(2D-3) \right\}^{1/2}}{L_m^{(2-D)}} \right)^{1/(D-1)}; \]  (3)

\[ D = 1.548/R_q^{0.041}. \]

Here \( R_q \) is the average square deviation of the profile, and \( L_m \) is the basic length of the section when estimating roughness, \( \Gamma \) - gamma function. It is easy to show that for \( x = \pm l/2 \) we have \( z(x)=0 \). The maximum height of the asperity from the midline (plane) is equal to \( \delta = G^{(D-1)} l^{(2-D)} \).

2. **Criterion for the transition from a plastic state to an elastic one**

Applying load \( F \) to a separate asperity, represented as a set of micro irregularities, leads to the formation of a contact spot, which growth causes the transition to an elastic state. Let asperity deformation \( w \) be defined in the following way

\[ w = a/(\pi R) \]  (4)
Here $a$ is the spot area; $R$ is the radius of the upper asperity part as a fractal object. Equating the loads that cause both the plastic and elastic states of the contact spot, we get

$$ a_c H = \frac{4}{3} E R^{1/2} \left( \frac{a_c}{\pi R} \right)^{3/2}. \quad (5) $$

Solving the obtained equation related to the critical value of the contact spot area $a_c$, we get the following

$$ a_c = \frac{9}{16 \pi^3} \left( \frac{H}{E} \right)^2 R^2. \quad (6) $$

Using fractal representations (M-B model), let us express the radius of the rounded top of the asperity as

$$ R = \frac{a^{b/2}}{G^{D-1} \pi^2}. \quad (7) $$

Substituting the radius value in the expression (6), we find the critical area

$$ a_c = \left[ \frac{16 \pi}{9} \left( \frac{E}{H} \right)^2 \right]^{\frac{1}{D-1}} G^2. \quad (8) $$

For example, for a surface with an average square deviation of the profile $R_q = 0.417 \ \mu m$, hardness $H = 3000 \ \text{MPa}$, and a reduced elastic modulus $E = 10^3 \ \text{MPa}$ (for the basic length $L_m = 800 \ \mu m$), fractal dimension of the profile will be $D = 1.605$ and the fractal roughness parameter $G = 8.214 \cdot (10^{-3}) \ \mu m$. Then the critical area of the contact spot will be $a_c = 18.89 \ \mu m^2$ and the diameter of the contact spot will be $d_n = 4.9 \ \mu m$.

### 3. Micro asperity deformation

According to figure 2 when the fractal asperity is deformed due to small radii of curvature of the sub-roughness, plastic deformation of the sub-rough layer occurs initially until the formed contact area reaches an elastic state. When the load increases, the asperity is deformed elastically, then elastically and plastically and then plastically (figure 3).

![Figure 3. Asperity deformation.](image_url)
Let us present the relationship between load and deformation for a single micro asperity under different conditions of the contact state.

- The initial state of the loaded contact is characterized by plastic deformation:
  \[ F_{p-e} = Ha, \quad 0 \leq a \leq a_c. \]  
  \[ (9) \]
  The criterion for transitioning from a plastic state to an elastic one is determined by the value of the critical area of the contact spot \( a_c \).
- Elastic contact occurs when
  \[ \frac{F_e}{F_c} = \left( \frac{a}{a_c} \right)^{3/2}, \quad 1 \leq \frac{a}{a_c} \leq 7.2. \]  
  \[ (10) \]
  here \( F_c = \frac{4\sqrt{\pi}}{3} E G \frac{D-1}{2} a_c^{(3-D)/2}. \)
- Elastic-plastic contact, taking into account the data presented in [4], is formed under the following condition
  \[ \frac{F_{e-p}}{F_c} = 1.302 \left( \frac{a}{a_c} \right)^{1.509}, \quad 7.2 \leq \frac{a}{a_c} \leq 206.1. \]  
  \[ (11) \]
- Plastic contact occurs if
  \[ F_p = H \left( \frac{a}{a_c} \right) a_c, \quad \frac{a}{a_c} > 206.1. \]  
  \[ (12) \]

4. Multiple contact

Solving the problem of contact interaction of rough surfaces requires knowing the probabilistic distribution of contact spots areas. When surfaces interact under the action of a compressive load, a contact consisting of discrete spots is formed (figure 4). Visualization was performed using the original software developed by the authors.

![Figure 4. Interface of fractal surfaces.](image)

Normal load growth leads to an increase in the actual contact area, mainly due to the introduction of new irregularities into contact. The fundamental proposition in the theory of contact interaction about the contact discreteness assumes the presence of a certain distribution of the areas of contact spots. It is considered that such a distribution follows a universal power law. For a more accurate estimation of the contact interaction parameters of rough surfaces, the presence of smaller irregularities on the asperity limits the use of the height distribution of asperities. In addition, accounting of such a distribution is difficult for the case when the number of interacting asperities is small.

The probability that a randomly selected spot will have an area greater than \( a \) is the following

\[ Pr(A > a) = Fa^{-\beta}. \]  
\[ (13) \]
Let us arrange the areas of spots in order of decreasing their size. Selecting one such a spot randomly with a uniform distribution means selecting one sequence number from the list. In this case, we can replace $Pr (A>a)$ with $N(A>a)$. Then the total number of contact spots is expressed as the following expression

$$N(A > a) \propto \left( \frac{a}{a_{\text{max}}} \right)^{-D/2}.$$ (14)

Since the distribution of the areas of the loaded contact is considered relative to the maximum contact area $a_{\text{max}}$, the integral function of the distribution of spots is represented as

$$F \left( \frac{a}{a_{\text{max}}} \right) \propto \left( \frac{a}{a_{\text{max}}} \right)^{B}.$$ (15)

Despite the structural similarity of the formulas above, their difference is that the number of spots is determined by integral number $\geq 1$, and the distribution function $F \left( \frac{a}{a_{\text{max}}} \right) \in [0,1]$. Let us denote $a^*$ as the relative area equal to the ratio of the spot area to the maximum area of contact $a^* = a/a_{\text{max}}$.

Figure 5 shows graphs in logarithmic coordinates of the dependence of contact spots number on corresponding areas and the integral distribution function.

**Figure 5.** Dependence of contact spots number on contact area ($A>a_{\text{min}}$; $\alpha=0.8$) and the function of spots distribution on the area ($a_{\text{min}}=10^{-4}$; $\alpha=0.8$).

Let the distribution density of the relative area of the contact spot corresponds to the power law

$$f(a^*) = C(a^*)^{-\alpha},$$ (16)

where $0 < a_{\text{min}}^* \leq a^* \leq 1$, $a_{\text{min}}^* = a_{\text{min}}/a_{\text{max}}$.

Taking the logarithm of the distribution density equation, we can write

$$\log f(a^*) = \log C - \alpha \log a^*.$$ (17)

The average size of the contact spot area is

$$\langle a^* \rangle = \int_{a_{\text{min}}^*}^{1} C(a^*)^{-\alpha} \cdot a^* \, da^*.$$ (18)

Here $a^*$ is the variable of integration.

Taking the integral we will get (up to $\alpha \neq 0$)

$$\langle a^* \rangle = \frac{C}{2-\alpha} \left[ 1 - (a_{\text{min}}^*)^{2-\alpha} \right].$$ (19)

We can find the value of $C$ from the normalization condition

$$\int_{a_{\text{min}}^*}^{1} C(a^*)^{-\alpha} \, da^* = 1. \quad \rightarrow \quad C = \frac{1 - \alpha}{1 - (a_{\text{min}}^*)^{1-\alpha}}.$$ (20)
Then the area of the average contact spot (for an infinitely large number of spots, \( N \) is the mathematical expectation) is found from the expression
\[
\langle a^* \rangle = \frac{1 - \alpha \left[ 1 - (a^*_{min})^{2-\alpha} \right]}{2 - \alpha \left[ 1 - (a^*_{min})^{1-\alpha} \right]}.
\] (21)

The integral distribution function is defined by the relation
\[
F(a^*) = \int_{a^*_{min}}^{a^*} C(a^*)^{-\alpha} \, da^* = \frac{C}{1 - \alpha} \left[ (a^*)^{1-\alpha} - (a^*_{min})^{1-\alpha} \right] = \frac{(a^*)^{1-\alpha} - (a^*_{min})^{1-\alpha}}{1 - (a^*_{min})^{1-\alpha}}.
\] (22)

Power laws of probability distribution are called distributions with "heavy tails", which are deleted if the normal distribution law is adopted. Power distribution laws, being universal, reflect the structure and processes of complex systems. For simpler systems, exponential distribution laws are used. The power law of distribution in double logarithmic coordinates has the form of a straight line, which indicates scaling behavior, i.e. the absence of selected scales and the presence of fractal behavior.

Admitting that \( a^*_{min} = 0 \), we will find
\[
F(a^*) = (a^*)^{1-\alpha}.
\] (23)

The average area of the contact spot depends on the parameters of the mating surfaces, as well as on the area of the maximum contact spot at a certain level of convergence. In this case, a quantitative estimate of the average area of contact spots can be represented in the following way
\[
\langle a \rangle = \frac{1 - \alpha}{2 - \alpha} \, a_L,
\] (24)

where \( a_L = a_{max} \) is the area of the maximum contact spot.

5. Implementation of the simulation algorithm
Using the software developed we can present the results of computer modeling. Figure 6 shows the estimation of contact interaction parameters.

![Figure 6. Law of contact spots distribution of milled and polished surfaces.](image)
For example, the interface of milled and polished surfaces (figure 6) gives the law of contact spots distribution \( F(\alpha^*) = (\alpha^*)^{1-\alpha} = (\alpha^*)^{0.486} \). Then \( \alpha = 0.514 \), and the average area of contact spots will be \( \langle a \rangle = 0.327 a_L \).

Let us present the simulation procedure as follows.

- Modeling the contact interaction of digital copies of 3D rough surfaces obtained by three-dimensional strip chart recording, or created fractal models that are adequate to the studied surfaces, for example, as presented in the paper [5]. At this stage, we need to get a set of random variables \( a^*_m \)-relative areas of contact spots. This can also be done by knowing the law of contact spots distribution for the corresponding pair of conjugate surfaces, for example, using the following dependence (here \( x_R \propto R_{av}[0.1] \) is a random variable evenly distributed over the segment from zero to one):

\[
a^*_m = x_R^{1/(1-\alpha)}.
\]

- Define to which state each contact spot area should be assigned from the array of areas obtained at the previous stage of modeling, and using the corresponding relationship between the load and the spot area, we find the value of load \( F_i \), which represents the investigated microasperity. In this case, the total load on the conjugate surfaces \( F_R \) will be equal to the sum of the forces perceived by the contact spots in all states:

\[
F_R = \sum_{i=1}^{N_p} F_p + \sum_{i=1}^{N_{p-e}} F_{p-e} + \sum_{i=1}^{N_e} F_e.
\]

- If the specified load \( F \) applied to the conjugate surfaces exceeds the calculated \( F_R \), this means that the convergence will increase (the load capacity of the rough layer support surface at this convergence is not able to withstand the specified load), and we should increase the convergence and get a new set of contact spot areas. If the specified load is lower, the convergence should be reduced on the contrary. We will repeat the procedure (machine experiment) until the condition (\( \varepsilon \) is a specified error) is met

\[
\left| F_M - F \right| / F \leq \varepsilon.
\]

The initial load at which the area of the maximum spot does not exceed \( a_c \), we can define using the data given in M-B model by the formula

\[
F_{p-e} = H A_r \rho(p-e) = H \frac{D}{2-D} a_c.
\]

For example, using the data given above we can get the following

\[
F_{p-e} = 3000 \frac{1.605}{2-1.605} 125.5 \cdot 10^{-6} = 1.53 H.
\]

Thus, the initial nominal pressure corresponding to the nominal area \( Aa = 0.64 \text{ mm}^2 \) is equal to \( p_a = 2.39 \text{ MPa} \).

To test the presented algorithm, the simulation results were compared with the experimental data obtained by the authors in [3], who conducted research using the Leica DCM 3D 3D profilometer. Figure 7 shows a graph comparing the dependence of the contact pressure (in MPa) on the convergence (in \( \mu \text{m} \)) for the parameters \( Rq = 0.417 \text{ \mu m} \), \( D_i = 2.6045 \text{ m} \), \( G_i = 2.65 \cdot 10^{-9} \text{ m} \). The upper curve is the model data; the lower curve is the experiment.
Thus, it is possible, using simulation modeling of contact interaction of rough surfaces, to find the dependence of the convergence value on the load applied to the conjugate surfaces. The simulation results can be used for solving various tribological problems, evaluating sealing, modeling the behavior of tribosystems under load, etc.

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