NMSSM neutralino dark matter for $W$-boson mass and muon $g - 2$ and the promising prospect of direct detection

Tian-Peng Tang$^{a,b}$, Murat Abdughani$^{a,c}$, Lei Feng$^{a,b,d}$, Yue-Lin Sming Tsai$^a$, Jian Wu$^{a,b}$, and Yi-Zhong Fan$^{a,b}$

$^a$Key Laboratory of Dark Matter and Space Astronomy, Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210033, China

$^b$School of Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, China

$^c$School of Physical Science and Technology, Xinjiang University, Urumqi, Xinjiang 830046, China and

$^d$Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University – Purple Mountain Observatory, Nanjing 210093, China

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Abstract

Two experiments from the Fermilab, E989 and CDF II, have reported two anomalies for muon anomalous magnetic moment ($g - 2$) and $W$-boson mass that may indicate the new physics at the low energy scale. Here we examine the possibility of a common origin of these two anomalies in the Next-to-Minimal Supersymmetric Standard Model. Considering various experimental and astrophysical constraints such as the Higgs mass, collider data, B-physics, dark matter relic density and direct detection experiments, we find that a neutralino in the mass range of $\sim 150 - 290$ GeV is a viable solution. Moreover, the favored parameter region can be effectively probed by the ongoing direct detection experiments like LZ, PandaX-4T and XENON-nT. The velocity averaged annihilation cross section of the dark matter particles, however, is suppressed.
I. INTRODUCTION

The simplest Supersymmetric model to solve the hierarchy problem and achieve gauge coupling unification is the Minimal Supersymmetric Standard Model (MSSM), which is however unable to treat the \(\mu\)-problem. The Next-to-Minimal Supersymmetric Standard Model (NMSSM), an additional gauge singlet Higgs included in the MSSM, generates the \(\mu\)-term dynamically through the singlet Higgs vacuum expectation value. Comparing with the MSSM, the additional particle content in the NMSSM are not only one scalar and one pseudoscalar Higgses, but also their superpartner Singlino. Hence, it is phenomenologically richer in both Higgs sector (3 scalars, 2 pseudoscalars, and one charged Higgses) and neutralino sector (5 neutralinos).

The experiment E989 at Fermilab reported an muon anomalous magnetic moment \((a_\mu)\) with a relative precision of 368 parts-per-billion (ppb). The combination with the Brookhaven National Lab (BNL) \([1]\) data yields a \(\delta a_\mu = (2.51 \pm 0.59) \times 10^{-9}\), which is in tension with the Standard Model (SM) prediction at a confidence level of 4.2\(\sigma\) \([2, 3]\). Very recently, the CDF collaboration has reported their precise measurement of \(W\)-boson mass \(m_{W,\text{CDF}} = 80.4335 \pm 0.0094\)\,GeV \([4]\) using approximately 4 million \(W\)-boson candidates from CDF II detector data corresponding to integrated luminosity of \(L = 8.8\)\,fb\(^{-1}\) collected in \(p\bar{p}\) collisions at a \(\sqrt{s} = 1.96\)\,TeV center-of-mass energy, which deviates from the SM prediction \(m_{W,\text{SM}} = 80.357 \pm 0.006\)\,GeV \([5]\) about 7\(\sigma\). In the NMSSM, the muon \(g - 2\) anomaly can be explained by the contributions of light electroweakinos and sleptons running in the loops \([6-21]\). Coincidentally, such light electroweakinos and sleptons can also enhance the one-loop contribution of the gauge boson self-energy \([22]\). Importantly, if the \(R\)-parity is conserved, the lightest neutralino can serve as dark matter (DM) candidate but its interactions to the SM are stringently constrained by the PLANCK relic density measurement \([23]\). However, unlike these three possible signals of beyond the standard model (BSM), the null observation from PandaX-4T \([24]\) can exclude a large part of parameter space. Thus, the remaining parameter space is highly confined and can shed valuable light on the future BSM experimental exploration.

In light of the muon \(g - 2\) anomaly reported in Ref. \([3]\), the geometric mean of the masses of the electroweakinos and sleptons has to be lighter than \(\mathcal{O}(400\)\,GeV) \([25]\). Therefore, we reexamine the general behavior of the bino neutralino in the low mass region \(m_{\tilde{\chi}_1^0} < 500\)\,GeV.
In the case of that the squarks and new Higgses are as heavy as 3 TeV, the low mass bino neutralino can produce correct relic density only with the slepton-coannihilation or $Z/H$-resonance mechanism. In this work we further explore whether these two mechanisms can also successfully account for $m_{W,CDF}$.

The paper is organized as follows. In Sec. [II] we briefly review the NMSSM model setup and the free parameters chosen in this work. We then summarize the experimental constraints and measurements in Sec. [III]. Finally, we present the results and discussion in Sec. [IV].

II. THE NMSSM AT THE ELECTROWEAK SCALE

The superpotential of the scale invariant NMSSM [26, 27], which includes a $Z_3$ symmetric gauge singlet chiral superfield $\hat{S}$, can be written as

$$W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda SH_uH_d + \frac{\kappa}{3} S^3,$$

where the MSSM component is $W_{\text{MSSM}}$. We denote $\kappa$ and $\lambda$ as dimensionless couplings in the extended Higgs sector. The supersymmetry (SUSY) breaking soft Lagrangian is

$$-\mathcal{L}_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + \lambda A\lambda SH_uH_d + \frac{1}{3} \kappa A\kappa S^3 + \text{h.c.},$$

where $m_{H_u}$, $m_{H_d}$, $m_S$, $A\lambda$, and $A\kappa$ are the soft SUSY breaking parameters. By using the minimization condition for the Higgs potential, one can replace $m_{H_u}$, $m_{H_d}$ and $m_S$ with $Z$ boson mass, $\tan \beta$ and $\mu$ parameter. We therefore have in total six free parameters ($\lambda$, $\kappa$, $A\lambda$, $A\kappa$, $\mu$, $\tan \beta$) for the Higgs sector.

The DM candidate is the lightest neutralino (i.e., the lightest mass eigenstate of mixed bino, wino, up-type higgsino, down-type higgsino and singlino gauge eigenstates)

$$\chi_1^0 = Z_1 \tilde{B} + Z_2 \tilde{W} + Z_3 \tilde{H}_u + Z_4 \tilde{H}_d + Z_5 \tilde{S},$$

where $Z_i$ for $i = (1, 5)$ are coefficients determined by diagonalizing the neutralino mass
matrix,

\[
M_{\chi^0} = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W & 0 \\
M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W & 0 \\
0 & -\mu & -\lambda v_u \\
0 & -\lambda v_d \\
\frac{2\kappa}{\lambda} \mu & \\
\end{pmatrix},
\]

where \(M_1\) and \(M_2\) are the soft bino and wino mass terms, \(\mu\) is the effective \(\mu\)-term. The vacuum expectation values for \(H_u\) and \(H_d\) are denoted as \(v_u\) and \(v_d\), and their ratio is defined as \(\tan \beta \equiv v_u/v_d\). Abbreviations \(s_\beta\) and \(c_\beta\) are \(s_\beta = \sin \beta\) and \(c_\beta = \cos \beta\), respectively. The sine and cosine of Weinberg angle are \(s_W\) and \(c_W\). To account for the \(g-2\) and \(m_W\) anomalies, we have to adjust the electroweino parameters \(M_1, M_2\) and slepton mass parameter \(M_{\tilde{\ell}_1,2}\). In total, we have nine free parameters at electroweak scale and their ranges are \(10^{-4} < \lambda < 1, 10^{-4} < |\kappa| < 2, |A_\lambda| < 3000\ GeV, |A_\kappa| < 100\ GeV, 30\ GeV < M_1 < 700\ GeV, 1.5 M_1 < M_2 < 10M_1, 1.5 M_1 < |\mu| < 10M_1, 100\ GeV < M_{\tilde{\ell}_{1,2}} < 700\ GeV,\) and \(2 < \tan \beta < 65\). Other parameters \((M_Q, A_{u,d,b,t}, M_3, M_{\tilde{\ell}_3})\) are set to 3 TeV to decouple, while \(A_t = 4\) TeV to adjust SM-higgs mass.

We perform the scanning in the ranges summarized above with Markov Chain Monte Carlo (MCMC) method implemented in emcee [28]. We scan \(\lambda\) and \(\kappa\) with log prior, while the other parameters are linear uniform distributed. The mass spectra and decay information are generated by using NMSSMTools-5.5.2 [29]. DM relic density, \(\delta a_\mu\), B-physics observables, DM-nucleon SI and SD cross sections are obtained using package MicrOMEGAs-5.2.11 [30]. One loop corrections to mass of the \(W\)-boson is calculated via FlexibleSUSY-2.6.1 [31, 32].

III. EXPERIMENTAL CONSTRAINTS AND MEASUREMENTS

For the constraints examined in this work, unless specifically mentioned, a Gaussian distribution is assumed when the central values \((\mu)\), experimental errors \((\sigma)\), and theoretical errors \((\tau)\) are available. Our total \(\chi^2_{\text{tot}}\) is the sum of \(\chi^2_i\), where \(i\) runs over all the constraints considered in this work such as SM-like Higgs mass \(m_{h_{\text{SM}}}\), B-physics observations [including \(\text{BR}(B_s \rightarrow X_s \gamma)\), \(\text{BR}(B_s^0 \rightarrow \mu^+\mu^-)\) and \(\text{BR}(B_u \rightarrow \tau\nu)\)], DM direct detection, DM relic
density $\Omega h^2$, and muon $g - 2$. The Gaussian $\chi^2$ is defined as
\[
\chi^2 = \frac{(\theta - \mu)^2}{\sigma^2 + \tau^2},
\]
where $\theta$ is the theoretical prediction being calculated and $\mu$ is the experimental central value. The uncertainty includes both theoretical error $\tau$ and experimental error $\sigma$.

For most of collider constraints (LEP and LHC Higgs searches), we apply a $2\sigma$ hard cut likelihood function as implemented in the NMSSMTool-5.5.2. On the other hand, several theoretical conditions such as Landau Pole or tachyonic problem are also taken into account by using NMSSMTools as well. In what follows, we summarize those experimental constraints/measurements which is implemented differently or missing in NMSSMTools.

- **The charged SUSY particle mass**
  The most conservative limit of charged SUSY particle mass is from the LEP2\(^1\). The limit for chargino is $m_{\chi^\pm} > 91.9 \text{ GeV}$, but we simply require chargino and slepton to be heavier than 100 GeV. Constraints from LHC to low scale electroweak and sfermion sector are rather severe. However, the constraints can be weakened when scenarios with compressed mass spectra are considered. We examine our samples against ATLAS $\sqrt{s} = 13 \text{ TeV}$ with 139 fb\(^{-1}\) luminosity analyses including 2 leptons plus missing energy \cite{33} and 2 leptons plus missing energy with jets \cite{34}. In this work, the 95% C.L. contours in Fig. 3 of Ref. \cite{33} are implemented as a hard cut in our MCMC scans.

- **The SM-like Higgs and W-boson mass**
  The latest LHC measured Higgs masses is $m_{h_{\text{SM}}} = (125.36 \pm 0.41 \pm 2.0) \text{ GeV}$ \cite{35}. On the other hand, the $W$-boson mass is recently measured to be $m_{W_{\text{CDF}}} = 80.4335 \pm 0.009 \text{ GeV}$ \cite{4}. Hence, we apply the likelihoods of $H$ and $W^\pm$ masses as Gaussian distribution Eq. \cite{10} with the theoretical uncertainties of 2 GeV and 0.006 GeV, respectively.

- **Higgs decay**
  The SM Higgs properties can be basically determined by its invisible decay and decay to diphoton. The former constrains the Higgs decaying to new particles particularly

\footnote{\url{http://lepsusy.web.cern.ch/lepsusy/www/inoslowdmsummer02/charginolowdm_pub.html}}
DM, while the later gives an upper limit to the new charge particles involving in the loop calculations. The LHC experimental measurements of the signal strength of Higgs decaying to diphoton $R_{\gamma\gamma}$ \cite{36} and branching ratio of Higgs decay to invisible particles $R_{\text{inv}}$ \cite{37} are

$$R_{\text{inv}} < 0.17, \quad R_{\gamma\gamma} = 1.08 \pm 0.13.$$  \hspace{1cm} (6)

Signal strength of Higgs decay to diphoton is defined as

$$R_{\gamma\gamma} \equiv \frac{\sigma_{h\gamma\gamma}}{\sigma_{h\gamma\gamma}^{\text{SM}}} \simeq \frac{\text{BR}(h \to \gamma\gamma)}{\text{BR}(h \to \gamma\gamma)^{\text{SM}}}$$  \hspace{1cm} (7)

where $\text{BR}(h \to \gamma\gamma)^{\text{SM}} = 2.27 \times 10^{-3}$ \cite{5} is the branching ratio of Higgs decay to diphoton final state in the SM.

- **B-physics**
  The measurements of $\text{BR}(B \to X_s\gamma)$ \cite{38}, $\text{BR}(B^0_s \to \mu^+\mu^-)$ \cite{39}, and $\text{BR}(B_u \to \tau\nu)$ \cite{40} can also constrain SUSY parameter space,

$$\text{BR}(B \to X_s\gamma) = (3.27 \pm 0.14) \times 10^{-4},$$

$$\text{BR}(B^0_s \to \mu^+\mu^-) = (3.0 \pm 0.6 \pm 0.3) \times 10^{-9},$$

$$\text{BR}(B_u \to \tau\nu) = (1.09 \pm 0.24) \times 10^{-4}. \hspace{1cm} (8)$$

Here, we take 10\% uncertainty as theoretical error for $\text{BR}(B \to X_s\gamma)$ and $\text{BR}(B_u \to \tau\nu)$.

- **Muon magnetic anomaly $\delta a_\mu$**
  The E989 $\delta a_\mu$ measurement \cite{3} has revealed

$$\delta a_\mu = (2.51 \pm 0.59) \times 10^{-9}. \hspace{1cm} (9)$$

In one-loop level, the Feynman diagrams in the NMSSM are identical to those in the MSSM but their relevant couplings can be different. The involving diagrams are $\chi^0_1$ and $\tilde{\mu}$ exchanging in one loop as well as $\chi^\pm_1$ and $\tilde{\nu}_\mu$ exchanging in the other loop.

- **DM relic density**
  In our analysis, we adopt the PLANCK 2018 data ($\Omega h^2 = 0.1186 \pm 0.002$) \cite{23} to
constrain the parameter space. Moreover, we have to consider the uncertainties from Boltzmann equation solver and the entropy table at the early universe. Hence, we conservatively introduce $\tau = 10\% \times \Omega h^2$ as theoretical error.

- **DM direct detection**

  When a neutralino scatters off a nucleon, a spin-independent component is resulted from a Higgs boson exchange while spin-dependent one is through a $Z$-boson exchange. In the NMSSM, the $\chi^0_1$-proton and $\chi^0_1$-neutron spin-independent couplings are more or less the same, but the spin-dependent components are not due to the iso-spin violation. The most stringent limit on the $\chi^0_1$-proton spin-independent cross section $\sigma^\text{SI}_p$ is from the PandaX-4T collaboration [24]. For the spin-dependent cross section scatterings off a proton $\sigma^\text{SD}_p$ and a neutron $\sigma^\text{SD}_n$ are from PICO60 [41] and XENON1T [42] experiments, respectively.

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FIG. 1: Left panel: the predicted $m_W$ as a function of $m_{\tilde{\mu}}$. The corresponding model parameters are $M_1 = 189.30$ GeV, $M_2 = 377.11$ GeV, $\mu = 713.98$, $\lambda = 8.08 \times 10^{-3}$, $\kappa = 7.83 \times 10^{-3}$, $A_\lambda = 1625.75$ GeV, and $A_\kappa = -83.76$ GeV. Three benchmark $\tan \beta$ are given by red, blue, and green lines. For the given $m_{\tilde{\mu}}$, we find that the higher the $m_W$, the smaller the $\tan \beta$. Right panel: $\sigma_p^{SI}$ as a function of $\tan \beta$ with three benchmark points. The $Z$ (Higgs)-resonance represented as black solid line (orange dashed line) is based on the input parameters $M_1 = 45.22$ (63.70) GeV, $M_2 = 351.93$ (746.45) GeV, $\mu = 324.86$ (424.03) GeV, $M_{\tilde{\ell}_{1,2}} = 109.55$ (245.46) GeV, $\lambda = 1.69 \times 10^{-3}$ (5.76 $\times 10^{-2}$), $\kappa = 6.58 \times 10^{-2}$ (2.91 $\times 10^{-1}$), $A_\lambda = 1.45$ ($-813.48$) GeV, and $A_\kappa = -4.29$ ($-2.48$) GeV. The parameter configuration of co-annihilation is $M_1 = 189.30$ GeV, $M_2 = 377.11$ GeV, $\mu = 713.98$ GeV, $M_{\tilde{\ell}_{1,2}} = 199.67$ GeV, $\lambda = 8.08 \times 10^{-3}$, $\kappa = 7.83 \times 10^{-3}$, $A_\lambda = 1625.75$ GeV, and $A_\kappa = -83.76$ GeV. For the given $\tan \beta$, the $Z$/Higgs resonances predict a significantly larger $\sigma_p^{SI}$, which have thus been stringently constrained by the direct detection experiments.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In general, the Higgs sector contribution to $m_W$ characterized by $\tan \beta$ is rather large and negative. Hence, the correct value of $W$ boson mass predicted in the NMSSM requires a small $\tan \beta$ and $\lambda$, as pointed out in Ref. [22]. In the left panel of Fig. 1 we plot $m_W$ as a function of smuon mass $m_{\tilde{\mu}}$ for a selected model point with three benchmarks including
FIG. 2: The $2\sigma$ likelihood distributions with and without the CDF $m_W$ constraint (see the red and green points, respectively). The left panel displays the predicted value of $\delta a_\mu$ with respect to the geometric mean of the masses of $m_{\chi_1^0}$, $m_{\chi_1^\pm}$, $m_{\tilde{\mu}}$ and $m_{\tilde{\nu}_\mu}$, where the shaded light green belt represents the E989 $2\sigma$ region. Both $\delta a_\mu$ and the $m_{W,CDF}$ can be accounted for in the NMSSM. The right panel shows the expected value of $m_W$ as a function of $m_{\tilde{\mu}}$ and the shaded light blue belt is for the $2\sigma$ region of the CDF II $m_W$ measurement.

$\tan \beta = (5, 7, 9)$ marked in (red, blue, green). Qualitatively, only when smuon mass is lighter than 300 GeV, the loop contributions to $m_W$ can be amplified. Clearly, a lighter sfermion mass requires a smaller $\tan \beta$ to yield an “enhanced” $m_W$. However, $\tan \beta < 10$ boosts the spin-independent component of the DM-proton elastic scattering cross section $\sigma_p^{SI} \propto \sin^2 2\beta$, as shown in the right panel of Fig. [1]. We select three benchmark points from our numerical scan, corresponding to $Z$-resonance (black solid line), Higgs-resonance (orange dashed line), and sfermion coannihilation (purple dash-dotted line). Overall speaking, $\sigma_p^{SI}$ can be enhanced by reducing $\tan \beta$, regardless of the DM production mechanism in the early universe. Consequently, the CDF II $m_W$ measurement would suggest a promising prospect of the dark matter direct detection.

In our numerical analysis, the Profiled Likelihood method is adopted. For a two-dimensional plot, the 95% confidence ($2\sigma$) region is defined by $\delta \chi^2 < 5.99$ under the assumption of approximate Gaussian likelihood. The group in red takes into account all the likelihoods
FIG. 3: The allowed $m_{\tilde{\mu}}$ as a function of the dark matter mass $m_{\chi_1^0}$. The ATLAS sets tight bounds, and most of the surviving points are in the co-annihilation region.

summarized in the Sec. III, while the group in green excludes the $W_{m,CDF}$ constraint.

In the left panel of Fig. 2, we confirm the conclusion in [25] that the geometric mean of the masses of the electroweakinos and slepton shall be less than $\mathcal{O}(400 \text{ GeV})$ to reproduce the $\delta a_\mu$ measurement. It is also evident that $m_{W,CDF}$ plays a crucial role in narrowing down the parameter space. This is somewhat surprising since these four particles (i.e., $m_{\chi_1^0}, m_{\chi_1^\pm}, m_{\tilde{\mu}}$ and $m_{\tilde{\nu}_\mu}$) all contribute to the loop corrections of $a_\mu$ and $m_W$. However, the $a_\mu$ anomaly is only 4.2$\sigma$ but the $m_W$ measurement is 7$\sigma$ away from the SM. In addition to the left panel of Fig. 1, the right panel of Fig. 2 displays the predicted value of $m_W$ with respect to $m_{\tilde{\mu}}$ taking the constraints into account. To reproduce $m_{W,CDF}$, we find out that the $m_{\tilde{\mu}}$ clusters in the range of ~180 – 300 GeV.

In Fig. 3 we show the favored regions of $m_{\tilde{\mu}}$ versus $m_{\chi_1^0}$. For $m_{\tilde{\mu}} > 180$ GeV, there is a tight correlation between $m_{\chi_1^0}$ and $m_{\tilde{\mu}}$, which suggests a co-annihilation origin. In Fig. 4, the group in blue includes the likelihood from $m_{W,CDF}$ but not $\delta a_\mu,E989$. The key SUSY parameters for $\delta a_\mu$ and $m_W$ loop contributions are given in the left panel of Fig. 4. Clearly, the lighter sparticles (i.e., $m_{\chi_1^0}, m_{\chi_1^\pm}, m_{\tilde{\mu}}$ and $m_{\tilde{\nu}_\mu}$) and smaller tan $\beta$ are needed to account for the $m_{W,CDF}$ data rather than $\delta a_\mu,E989$ data. Still, a small tan $\beta$ confronts the tight bounds from the direct detection experiments (see the right panel of Fig. 1). While the ATLAS data...
FIG. 4: The left panel displays the \( \tan \beta \) as the function of the geometric mean of the sparticle masses. The right panel shows the predicted value of \( \sigma_p^{SI} \) with respect to the dark matter mass \( m_{\chi_1^0} \). The inclusion of \( m_{W,CDF} \) has stringently narrowed down the parameter space and the rest will be further effectively probed by the ongoing direct detection experiments like PandaX-4T, XENONnT/LZ in the near future. The two solid lines represent the current constraints set by XENON1T [42] and PandaX-4T (i.e., the first result reported in [24]), while the two dashed lines are the projected sensitivities of XENONnT/LZ [43, 44] and DRAWIN [45].

has set stringent constraints on the light slepton (see the Fig.3). As a result, the majority of surviving points are located at the co-annihilation region. We would like to also remind that the resonance region is marginally excluded by PandaX-4T in 2\( \sigma \). Strikingly, the parameter space favored by the current data can be entirely probed by the ongoing DM direct detection experiments such as the PandaX-4T and XENONnT [43]. Note that in the right panel of Fig.4, the tiny \( \sigma_p^{SI} \) value in the blue region owes to the negative \( \mu \), which causes the negative \( \delta a_\mu \). The blue solid line is the upper limits set by the first result of PandaX-4T [24] while the pink and green dashed line represents the projected sensitivity of XENONnT/LZ [43, 44] and DRAWIN [45].

Finally, we would like to summarize our findings. The CDF \( m_W \) anomaly is in favor of the light SUSY. Together with the recent \( a_\mu \) anomaly, we attempt to strike these two anomalies with a solution from the NMSSM. In this work, we only compute one-loop correction to
the $m_W$ via **FlexibleSUSY-2.6.1**. Motivated by the favored region to explain the recent $a_\mu$ anomaly, we only focus on a lighter slepton sector without mass splitting between the left and right sparticles. We successfully find the solution to explain both anomalies without being in conflict with other experimental constraints. Although three regions such as $Z$-resonance, Higgs-resonance, and the slepton co-annihilation are still viable, they can be soon tested in the future DM direct detection due to a small tan $\beta$ required by a rather heavy $W$ boson. Indeed, the PandaX-4T has already set tight constraints on the regions of $Z$-resonance and Higgs-resonance (see the right panel of Fig.4). Moreover, the DM annihilation cross sections at the present time are less than $10^{-27}$ cm$^3$s$^{-1}$ due to the $p$-wave suppression or co-annihilation, which is hence challenging for the indirect detection experiments. We note that two days after the submission of this manuscript to arXiv, several works (i.e., [46–53]) appeared online to interpret the CDF II $W$ mass measurement with other models.

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