Surface spin waves in superconducting and insulating ferromagnets

V. Braude and E. B. Sonin
Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel
(Dated: January 3, 2022)

Surface magnetization waves are studied on a semi-infinite magnetic medium in the perpendicular geometry. Both superconducting and insulating ferromagnets are considered. Exchange and dipole energies are taken into account, as well as retardation effects. At large wave vectors, the spectrum for a superconductor and insulator is the same, though for the former the branch is terminated much earlier than for the latter due to excitation of plasmons. At small wave vectors, the surface wave is more robust in the superconductor since it is separated from the bulk continuum by a finite gap.

Studies of spin waves began long ago and proved to be a powerful method of investigation of magnetically ordered materials [1]. These studies can be divided into two categories. In the first category, problems are included regarding spin waves in the bulk of an infinite medium, where long-range geometric effects do not arise, while the spin-exchange stiffness is taken into account. The second category deals with spin-wave modes in finite-size samples of various geometries giving rise to ferromagnetic resonance (FMR). These modes were usually treated in the magnetostatic approximation, in which only the long-range dipole energy is retained in the dynamical equations, while the exchange energy, as well as well as the retardation effects, are neglected [1, 2].

A special class of restricted-geometry spin modes is a surface magnetic wave, which propagates along the surface of a magnetically ordered material. In addition to a great physical interest of surface modes, they can be important for various technological applications [3, 4], since for these waves the surface serves as a waveguide, which allows effective spin transport. The latter attracts now a lot of attention because of the prospects of spintronics.

The most famous example of a magnetic surface wave is the Damon-Eshbach wave [2], which exists in slabs with the equilibrium magnetization parallel to the surface (parallel geometry). While the existence of this mode can be revealed within the magnetostatic approximation alone, it is not sufficient to obtain the correct spectrum. Thus, within this approximation, the surface mode lies above the bulk modes. It has been demonstrated [1] that inclusion of the exchange energy can modify the results dramatically. Moreover, in the perpendicular geometry, a surface mode can exist only if the exchange energy is included. However, the analysis in Ref. [1] was only carried out numerically for some chosen parameter values. Ref. [2] considered the effect of displacement currents (for the parallel geometry), with the conclusion that they also can be important, but the exchange energy was disregarded in that work.

This state of affairs, when it is clear that the magnetostatic approximation is insufficient for the description of the surface wave spectrum, while no inclusion of the neglected parts has been done systematically, is part of the motivation for this work. Another goal of this work is to study the surface-wave spectrum for a superconducting ferromagnet (SCFM) and to compare its behavior to that of an insulating ferromagnet and a nonmagnetic superconductor. SCFM’s have attracted a lot of attention during the last decade in the context of unconventional superconductivity. The existence of spin modes can serve as a clear proof of magnetic order in these materials, where detection of ferromagnetism is hindered by screening Meissner currents [6]. Knowledge of the surface-wave properties, while interesting by itself, might be also useful for a design of new experimental techniques to study coexistence of ferromagnetism and superconductivity.

This Letter studies the spectrum of surface magnetization waves on a semi-infinite medium in the perpendicular geometry, taking into account the dipole and spin-exchange energies and the retardation effects. Two types of materials are considered: SCFM and an insulating ferromagnet. In the SCFM, (longitudinal) plasmons are excited together with the transverse magnetic modes in the surface wave; however, their presence is only important for determination of the upper termination point of the branch. On the other hand, in the insulator, plasmons are absent, and the surface wave survives to much larger wave vectors. The retardation effects are important both for very small and very large wave vectors, the latter case being relevant only for the insulator, since in the SCFM the branch does not survive to this regime. The spectrum is calculated for both small and large wave vectors. For large wave vectors, the spectrum is the same for both types of ferromagnets, apart from the difference in the branch termination points. At small wave vectors, the spectra are different for the two types of materials. The surface branch for the SCFM survives down to zero wave vector and is separated from the bulk-mode continuum by a finite gap. The branch for the insulator is terminated at a very small wave vector, where it collides with the bulk-continuum bottom.

We consider a semi-infinite uniformly magnetized medium with the spontaneous magnetization perpendicular to the surface. Physically this corresponds to a slab
whose thickness is large enough that the other surface can be neglected. Our goal will be to investigate surface waves in two cases: insulating medium vs. a superconducting one. We assume no dissipation in both cases, since only under this assumption, as well as in the semi-infinite geometry, are surface waves well-defined. The setup of the system is shown in Fig. 1. The \( \hat{z} \) axis together with the spontaneous magnetization \( \mathbf{M}_0 \) are perpendicular to the surface, while the \( \hat{x} \) and \( \hat{y} \) axes parallel to it. We also assume a magnetic anisotropy in the system of the easy-axis type which is strong enough in order to stabilize the system against the flip of the magnetization into the \( x-y \) plane. We will be looking for surface waves propagating along the \( \hat{x} \) direction with a wave vector \( k_0 \). That is, we will be interested in solutions of the dynamical equations (specified below), made of plane-wave combinations \( \exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r}) \) such that \( \mathbf{k}_i = k_0\hat{x} + q_i\hat{z} \), and \( k_0 \) is real, while \( q_i \) are complex.

The magnetization is governed by the Landau-Lifshitz equation, which gives for a frequency \( \omega \) [6]:

\[
-i\omega \mathbf{m} = -g\mathbf{M}_0 \times \mathbf{m}(\alpha + \gamma^2 k^2) + g\mathbf{M}_0 \times \mathbf{b},
\]

where \( \mathbf{m} \) is the dynamical magnetization such that \( \mathbf{M} = \mathbf{M}_0 + \mathbf{m} \) (\( \mathbf{m} \) is in the \( x-y \) plane); \( g \) is the gyromagnetic ratio; \( \alpha \) the magnetic anisotropy constant (assuming \( \alpha > 4\pi \)); \( \gamma \) the exchange stiffness constant, and, finally, \( \mathbf{b} \) is the magnetic induction excited by the oscillating magnetization \( \mathbf{m} \). This induction is related to the magnetization by the generalized London equation, which is given for plane-wave solutions by

\[
\mathbf{b} = \frac{4\pi k^2 \mathbf{m}_\perp}{k^2 + \lambda^2 - K^2}.
\]

Here \( \lambda \) is the superconducting penetration depth (for an insulator, \( \lambda \to \infty \)); \( K \equiv \omega / c \) the electromagnetic wave vector. The last term \( \sim K^2 \) in the denominator takes into account the displacement currents. The vector \( \mathbf{m}_\perp \) is the magnetization component transverse to the wave vector \( \mathbf{k} \):

\[
k^2 \mathbf{m}_\perp = k^2 \mathbf{m} - \mathbf{k}(\mathbf{k} \cdot \mathbf{m}) = q^2 \mathbf{m}_x + k^2 \mathbf{m}_y - q k_0 m_z \hat{z}.
\]

Note that when the denominator in the RHS of Eq. (2) vanishes, this equation does not specify anymore \( \mathbf{b} \), but, rather, fixes the direction of \( \mathbf{m} \), so that \( k^2 \mathbf{m}_\perp = 0 \). This situation happens for the Damon-Eshbach wave [2]. However, in our geometry this possibility cannot be realized, since it would lead to a non-zero \( m_z \) component. Substituting the last two equations into Eq. (1), we obtain a closed equation of motion for the magnetization:

\[
i\Omega \mathbf{m} = \hat{z} \times \left[ (\delta \alpha + \gamma^2 k^2 + \frac{4\pi \lambda^2}{k^2 + \lambda^2}) \mathbf{m} + \frac{4\pi k_0^2 \mathbf{m}_\perp}{k^2 + \lambda^2} \right],
\]

where \( \Omega \equiv \omega / gM_0 \) and \( \lambda^2 - K^2 \). The last term in the RHS breaks the rotational symmetry of the problem and mixes different circular polarizations. From this equation, the spectrum of different modes composing the surface wave is given by

\[
\Omega^2 = \left( \delta \alpha + \gamma^2 k^2 + \frac{4\pi \lambda^2}{k^2 + \lambda^2} \right) \left( \frac{4\pi \lambda^2}{k^2 + \lambda^2} \right).
\]

This is a complicated self-consistent equation in which the frequency enters both the LHS and the RHS through the definition of \( \lambda \). For given \( \Omega \) and \( k_0 \), it gives four magnetic modes.

In addition to these magnetic modes, there is also a plasma mode in the system, consisting of a longitudinal electric field alone. Its spectrum is given by \( \omega^2 = \omega_p^2 + c_p^2 k^2 \) where \( \omega_p \equiv c / \lambda \) and \( c_p << c \) is the plasma velocity originating from the electron gas compressibility. We will see that in the SCFM, plasmons are excited together with the magnetic modes in the surface wave. The electromagnetic (EM) field inside the sample is given by a combination of the above-mentioned five modes (or four for the insulator). The exact combination is determined by coupling to the EM field outside the sample using the appropriate boundary conditions.

The EM field outside the slab can be found using the Maxwell equations for the vacuum with the frequency \( \omega \) and wave vector \( \mathbf{K} = k_0 \hat{x} + k_z \hat{z} \), where \( k_z \) is negative imaginary and, as before, \( K = \omega / c \). The obtained fields should be coupled to the fields inside the slab by requiring continuity of the EM fields parallel to the slab surface. For the insulator, continuity of the normal fields is then satisfied automatically, while for the SCFM, it is an independent condition which determines the amplitude of the plasmon mode. Eliminating the latter from the equations (for the SCFM), we obtain from these conditions the following relations for the magnetic modes:

\[
\sum_i^4 \frac{m_y}{k_{i0}^2 + \lambda^2 - K^2} \left[ 1 + \frac{k_0^2}{q k_z} + K^2 \lambda^2 \left( \frac{q}{k_z} - 1 \right) \right] = 0,
\]

\[
\sum_i^4 \frac{m_x}{k_{i0}^2 + \lambda^2 - K^2} \left[ q + \frac{k_0^2}{k_z} + \frac{\lambda^2}{k_z} \right] = 0.
\]
where \( q_p = \sqrt{-\lambda^2 c^2 / \epsilon_s^2 - k_0^2} \) is the \( z \) component of the plasmon wave vector (for the insulator, the term with \( q_p \) should be dropped as the plasmons are irrelevant). In addition to these, two more conditions are needed for the amplitudes of spin wave modes. A possible choice for these can be the condition of vanishing spin currents at the slab surface \( \mathbf{1} \), which gives

\[
\sum_{i} q_i m_{x,i} = 0 \quad \text{and} \quad \sum_{i} q_i m_{y,i} = 0 . \tag{6}
\]

These four conditions together with the requirement that \( q_i \) are positive imaginary and \( k_0 \) negative imaginary specify a full system of equations whose solution yields the spectrum of the surface wave.

We have solved these equations in the limit of small and large wave vector: \( k_0 \ll \gamma^{-1} \) and \( k_0 \gg \gamma^{-1} \) and found important differences between the SCFM and the insulator in these two regimes. For a small wave vector, \( k_0 \ll \gamma^{-1} \), the plasmon contribution in the SCFM can be neglected as it is small by the parameter \((c_p / c)^{-1/2}\). Moreover, in this limit, the spin-wave modes have a definite circular polarization, and their dispersion to leading order is given by the usual dispersion of spin waves in SCFM with \( k \parallel \mathbf{z} \mathbf{2} \). \( \Omega = \pm (\delta \alpha + \gamma^2 q^2 + \alpha q^2 / (1 + q^2 \lambda^2)). \) As long as the condition \( k_0 \gg K \) holds (which is true except for very small wave vectors \( \sim 1/C \)), where \( C \equiv c / \gamma M_0 \) is the light velocity in magnetic units), retardation effects can be neglected. Then Eqs. \( \mathbf{3} \mathbf{4} \mathbf{5} \mathbf{0} \) lead to

\[
\left| \frac{1/q_1 + \lambda^2 k_0}{q_1^2 + \lambda^2} - \frac{1/q_2 + \lambda^2 k_0}{q_2^2 + \lambda^2} \right| = 0 . \tag{7}
\]

In the leading order only the positively polarized modes (1, 2) are excited, and the above equation becomes

\[
q_1^2 + q_2^2 + q_1 q_2 + \lambda^{-2} = -\lambda^2 k_x q_1 q_2 (q_1 + q_2)/2 , \tag{8}
\]

from which a linear surface-wave dispersion is found: \( \Omega = \Omega_h + s |k_0| \), where

\[
\Omega_h = \frac{\lambda (\alpha - \Omega_h)}{1 + 2\gamma \lambda^2 / \alpha - \Omega_h} \left( 1 + \gamma \lambda^2 / \alpha - \Omega_h \right) \tag{9}
\]

and \( \Omega_h = \delta \alpha - \gamma^2 \lambda^{-2} / 2 + \gamma \lambda^{-1} (4\pi + \gamma^2 \lambda^{-2} / 4)^{1/2} \). This frequency is the frequency of (nonuniform) FMR for a thick SCFM slab \( \mathbf{3} \). It is remarkable that the surface-wave spectrum contains information about it.

At very small wave vectors, \( k_0 \sim K \sim 1/C \), retardation is important. When it is taken into account, the spectrum gets modified, and instead of the linear expression, it is given by \( \Omega = \Omega_h + s (p_z - K^2 / p_z) \) where \( p_z^2 \equiv k_0^2 - K^2 \). Solving this, we obtain the SCFM surface-wave spectrum for \( k_0 \ll \gamma^{-1} \):

\[
k_0^2 = \frac{\Delta \Omega^2}{2s^2} + 2K^2 + \frac{\Delta \Omega}{s} \sqrt{\frac{\Delta \Omega^2}{4s^2} + K^2} . \tag{10}
\]

where \( \Delta \Omega = \Omega - \Omega_h \). Its form is given in Fig. 2(a).

As \( k_0 \to 0 \), only the long-wavelength mode contributes to the surface wave, and the surface-mode spectrum becomes \( \Omega = Ck_0(1 + \zeta_0^2 / 2) \), where \( \zeta_0 \) is the surface impedance of SCFM in the zero-frequency limit \( \mathbf{3} \): \( \zeta_0 = -iK\lambda \sqrt{m} \).

\[
\mu = \frac{\alpha (\delta \alpha + \gamma^2 \lambda^{-2} + 2 \gamma \lambda^{-1} \sqrt{\alpha} / \sqrt{\alpha})}{(\delta \alpha + \gamma \lambda^{-1} \sqrt{\alpha})^2} . \tag{11}
\]

This is just the surface impedance of a usual superconductor with magnetic permeability \( \mu \), so the magnetic order plays no role in this regime except for adding some effective permeability. The corresponding mode, existing on the surface of a conductor, is known as the Zenneck wave \( \mathbf{1} \). If the medium is not a superconductor, but a metal, then the wave is damped. Note that the bottom of the conduction band, given at \( k_0 \to 0 \) by \( \Omega_m = \delta \alpha - \gamma^2 \lambda^{-2} + 2 \sqrt{4\pi \gamma \lambda^{-1}} \), is separated from the surface-wave branch by a finite gap.

For the insulator at \( k_0 \ll \gamma^{-1} \), the surface-wave frequency is \( \Omega \approx \delta \alpha \). Around this frequency the modes have the following wave vectors: \( k_{1,2} \sim \gamma^{-1} k_0 \); \( k_3 \sim \gamma^{-1} \), and \( k_4 \sim K \sim 1/C \). To obtain the surface wave, it is enough to keep the first two modes and use them in the two equations for \( m_y \) from Eqs. \( \mathbf{5} \mathbf{6} \mathbf{2} \). The surface-wave spectrum is found from the condition \( q_1 = -q_2 \), which is also the condition for the bottom of bulk mode continuum. Thus by leading order in \( \gamma k_0 \), these two coincide and are given by

\[
\Omega = \delta \alpha + \sqrt{8\pi \gamma^2 (k_0^2 - 2|\delta \alpha / C|^2)} . \tag{12}
\]

The limit \( C \to \infty \) and \( k_0 \to 0 \) (while \( k_0 \gg 1/C \)) produces \( \Omega = \delta \alpha \), which corresponds to FMR frequency for the insulator.

Taking into account the next order, it is found that the surface wave is situated slightly below the bottom of bulk mode continuum, with a small separation \( \Omega_{su} - \Omega_{bulk} = \gamma^2 (K^2 - k_0^2) \approx -\gamma^2 (k_0^2 - \delta \alpha^2 / C^2) \). For very small wave vectors, \( k_0 \sim K \), the retardation effects become important. At this region the branch is terminated, not surviving to \( k_0 = 0 \). The reason for this is that the bulk-wave continuum at very small \( k_0 < K \) [Fig. 2(b)] is different from that for the superconductor [Fig. 2(a)], since photons with velocity \( c / \sqrt{\mu} \) can propagate in an insulating ferromagnet, where \( \mu = \alpha / \delta \alpha \). Namely, when \( k_0^2 = K^2 (1 + \alpha / \delta \alpha) \) the mode 4 starts propagating. At this point, given by \( k_0 = \mu_{\min} = (\alpha + \delta \alpha \gamma^{1/2} / C)^1 / 2 \), the continuum bottom has a cusp, going down steeply for smaller \( k_0 \), where it is no more given by \( q_1 = -q_2 \), but by \( q_4 = 0 \). The surface-wave branch collides with the continuum bottom at \( k_{\min}^2 \) and is terminated there. Note that near this point the penetration depth of the wave diverges, so the other surface might become important. The behavior of the branch for the insulator at small \( k_0 \) is shown in Fig. 2(b).
Next we consider the behavior of the surface wave branch for large \( k_0 \gg \gamma^{-1} \). To leading order in \( \gamma k_0 \), the spectrum can be found using the spin-wave approximation, in which only the exchange stiffness energy is retained, so that the frequency is given by \( \Omega = \gamma^2 k^2 \) (the spin-wave mode obtained in this approximation will be called mode 1). Then the surface-wave spectrum for both the insulator and SCFM is found from the condition \( q_1 = 0 \), leading to

\[
\Omega = \gamma^2 q_0^2,
\]

which is identical to the continuum bottom. In order to find a small separation between the continuum bottom and the surface wave, the full spin-wave dispersion relation, Eq. (11) must be used. Then the wave vector \( \mathbf{k}_1 \) has a small imaginary component perpendicular to the surface, \( q_1 = i \pi \gamma^{-2} k_0^{-1} \), so that \( \Omega_{sw} - \Omega_{bulk} = \gamma^{-2} q_1^2 = \pi^2 \gamma^{-2} k_0^{-2} \). These expressions are correct as long as \( k_0 \gg K \), which is satisfied for the SCFM, as discussed below. For the insulator, this condition loses its validity near the (upper) termination point, and there

\[
q_1 = i \pi \gamma^{-2} (k_0^2 - 2 K^2)/(k_0^2 - K^2)^{3/2}
\]

so that \( \Omega_{sw} - \Omega_{bulk} = -\pi^2 \gamma^{-2} (k_0^2 - 2 K^2)/(k_0^2 - K^2)^3 \). While the surface-wave dispersion is the same for the two considered types of materials at large \( k_0 \), the branches are terminated at points which are very different. Namely, for the SCFM, the branch termination point is determined by the condition that plasmons start propagating. This happens at the plasma frequency, when \( \lambda^{-2} \rightarrow 0 \), that is, \( K^2 \rightarrow \lambda^{-2} \). At this point, given by \( k_0 = k_{0\text{max}}^{SC} = \gamma^{-1} \Omega_p^{1/2} \), the continuum bottom collides with the surface-wave branch and terminates it. We see that for \( k_0 < k_{0\text{max}}^{SC} \) the condition \( K \leq \lambda^{-1} \ll k_0 \) is satisfied, as was stated above. In the insulator, on the other hand, the surface wave branch survives to much larger wave vectors. It is terminated when the mode 1 starts propagating, that is, when \( q_1 = 0 \), which gives \( k_0 = k_{0\text{max}}^{ins} = \sqrt{2} K = 2^{-1/2} \gamma^{-2} C \).

In conclusion, we have considered a surface wave in superconducting and insulating semi-infinite ferromagnets, taking into account the dipole and the exchange-stiffness energies, as well as the displacement currents. The spin-exchange stiffness is crucial for all regimes. The displacement currents are important at very small wave vectors for both types of materials and very large wave vectors, near the branch termination point, for the insulator. The presence of plasmons in the SCFM does not modify the spectrum, but cuts off the branch at the plasma frequency. For the insulator, the surface-wave spectrum at very small \( k_0 \) approaches the continuum band bottom, whereas in the SCFM, the branch remains separated from the continuum bottom by a finite gap (which depends on \( \gamma, \lambda \) and \( M_0 \)). Hence the wave in the latter is more robust against smearing by dissipation. The magnetic order in the SCFM makes a profound influence on the surface wave (except at very small wave vectors \( k_0 \ll K \)). Hence the surface wave spectrum might provide information about magnetic order in superconductors, in particular, in unconventional superconductors with broken time-reversal symmetry.

We acknowledge discussions with M. Golosovsky. This work has been supported by the grant of the Israel Academy of Sciences and Humanities.

[1] A. I. Akhiezer, V. G. Bar’yakhtar, and S. V. Peletminskii, Spin Waves (North-Holland, Amsterdam, 1968).
[2] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids 19, 308 (1960).
[3] R. Bhandari and Y. Miyazaki, Jpn. J. Appl. Phys., Part 1 39, 3076 (2000); R. Bhandari and Y. Miyazaki, Jpn. J. Appl. Phys., Part 1 40, 3768 (2001).
[4] R. E. De Wames and T. Wolfram, Appl. Phys. Lett. 15, 297 (1969); R. E. De Wames and T. Wolfram, J. Appl. Phys. 41, 987 (1970).
[5] A. Hartstein, E. Burstein, A. A. Maradudin, R. Brewer, and R. F. Wallis, J. Phys. C 6, 1266 (1973).
[6] V. Braude and E. B. Sonin, Phys. Rev. Lett. 93, 117001 (2004).
[7] T. K. Ng and C. M. Varma, Phys. Rev. B 58, 11624 (1998).
[8] V. Braude and E. B. Sonin, in preparation.
[9] A. Sommerfeld, Partial Differential Equations in Physics (Academic Press, N.Y., 1964).