Mechanisms with Bevel Internal Gears Having a Small Difference in Numbers of Teeth

O Crivoi¹, I Doroftei²,*

¹,²Mechanical Engineering, Mechatronics and Robotics Department, “Gheorghe Asachi” Technical University of Iasi, Romania

*E-mail: idorofte@mail.tuiasi.ro

Abstract. These mechanisms have gained prowess because of the benefits they offer, especially obtaining high reduction ratios in one reduction step. In addition, there is no longer the risk of trochoid interference that may occur in the case of the internal cylindrical gears when the teeth numbers difference is small. Since the satellite performs a nutation motion, these gears are also called nutation gears. The purpose of this paper is to propose some new nutation geared mechanisms using bevel internal gears having small difference in numbers of teeth. Based on these mechanisms, some robotic wrists will be proposed in another paper of the same journal.

1. Introduction
There are previous researches in the field of geared mechanisms, generally [1, 2], or in the field of the mechanisms with internal gears having a small difference in numbers of teeth [3, 4].

As a kind of transmission form, the nutation drive has a broad development spaces in the field of robotics wrist and aerospace craft with the advantages of low noise, higher carrying capacity, steady transmission ability, higher transmission ratio and small volume. Thus, the nutation drive mechanism has great prospect development and research significance, leading wide research of experts [5-13].

These mechanisms have gained prowess because of the benefits they offer, especially obtaining high reduction ratios in one reduction step. In addition, there is no longer the risk of trochoid interference that may occur in the case of the internal cylindrical gears when the teeth numbers difference is small. Since the satellite performs a nutation motion, these gears are also called nutation gears. The rotation movement of the driven element may be uniform or uneven, depending on the solution used to hold the satellite at rotation around its own axis.

A nutating gear drivetrain has the potential to create comparable speed reductions and torque multiplications. The advantages of a nutating gear drivetrain over a conventional planetary gear train include: a decreased number of drivetrain parts, an ability to use lower-strength gear materials, a higher overall reliability, and a decreased manufacturing cost.

In this paper some new nutation geared mechanisms using bevel internal gears having small difference in numbers of teeth are proposed. Based on these mechanisms, some robotic wrists will be proposed in another paper of the same journal.

2. Nutation motion
Nutation is a rocking, swaying, or nodding motion in the axis of rotation of a largely axially symmetric object. Nutation concept was discovered almost by accident, by Robert Davison, a
beekeeper from New Zealand [14]. He rotated a coin on the table and he noticed that as a spinning coin begins to decrease in speed, a peculiar wobble takes place. This wobble is nutation and this innovative concept can be adapted for high gear speed reductions (see figure 1).

As the coin wobbles, it traces a circle on the top of a table. Until the coin comes to rest on the table, the diameter of the circle traced on the table top is always smaller than the diameter of the coin. If we suppose that $R$ is the radius of the spinning coin and $\theta$ is the angle between the table-top axis and the coin axis, the radius of the table-top circle will be $R \cos \theta$. In the case of the internal conical gear, the coin can be one of the gear wheels, and the table's surface, the second wheel in the gear.

![Figure 1. Nutation motion.](image)

Considering now the point $A$ on the coin, as this coin wobbles $A$ comes in contact with the table at point the point $B$ (figure 1.a). As shown in figure 1.a, the coin continues to wobble and, when $A$ contacts the table a second time, the edge of the coin has traced out a path on the table equal to the circumference of the coin ($2\pi R$). Because the circle on which the path has been traced is only $2\pi R \cos \theta$ in circumference, point $A$ does not contact point $B$ at its second contact; it contacts a point $C$ further along the table-top circle (see figure 1.c). Equation (1) shows the length of the circle arc between the points $B$ and $C$:

$$s = BC = 2\cdot \pi \cdot R \cdot (1 - \cos \theta)$$  \hspace{1cm} (1)

This equation expresses the rotation of the coin around its own axis at a swing of it. The rotation angle $\varphi$ of the coin can be calculated with the next relation,

$$\varphi = \frac{s}{R},$$  \hspace{1cm} (2)

namely

$$\varphi = 2 \cdot \pi \cdot (1 - \cos \theta).$$  \hspace{1cm} (3)

Using the relation (1), the limit conditions of the nutation can be determined. Thus, if the coin is spinning and has not yet begun to topple, $\theta = 90^\circ$, the trajectory described on the table is zero or a point. If the coin is flat on the table, $\theta = 0^\circ$, the trajectory described is $2\pi R$, or the same as the coin circumference.

It can be noticed that at big values of the angle $\theta$, the trajectory described is of small length, and the value of the rotation angle $\varphi$ is high, and at low values of $\theta$, the angle of rotation $\varphi$ is small.

In the case of a bevel internal gear, this means that at low values of $\theta$, the transmission ratio is high, so the difference between the teeth numbers is small.

3. Structural and kinematic aspects

Basically, nutating gears are high shaft angle bevel gear arrangements where one bevel gear is nutating instead of rotating. The rotation of one gear nutating against a fixed gear is now based upon a
difference in tooth numbers instead of a difference in circumferences as illustrated above with the coin.

When only two of the mating gears have a different number of teeth, the resulting system is called a simple nutating gear arrangement. However, in order to restrain the nutating gear from rotating relative to the output axis, a second set of gears must be added. This second set of gears not only constrains the motion of the input gear, but also provides support and rigidity to the entire drivetrain. If the tooth numbers of the added set are identical to each other, the added gears have no effect on the overall reduction ratio of the gear train.

However, in a major departure from conventional gear theory, the diametral pitches of nutating gears must be slightly different to avoid interference problems.

Let us consider the kinematics of a simple planetary mechanism with bevel internal gear (see figure 2).

\[ i_{C3}^3 = \frac{z_2}{z_3 - z_2'} \]  \hspace{1cm} (4)

For \( z_3 - z_2' = \pm 1 \), the mechanism has two reduction steps and the reduction ratio may be determined using next relation

\[ i_{C3}^1 = \pm z_3 \] \hspace{1cm} (5)

If \( z_1 \neq z_2 \) and \( z_2' \neq z_3 \), then the mechanism has also two reduction steps but the reduction ratio will be

\[ i_{C3}^1 = \frac{z_2z_3}{z_2z_3 - z_1z_2'} \] \hspace{1cm} (6)

For \( z_1 - z_2 = z_3 - z_2' = \pm 1 \) and \( z_2 = z_3 \),

\[ i_{C3}^2 = \pm \frac{z_2^2}{z_3} \] \hspace{1cm} (7)

Let now consider an original more complex mechanism with three output shafts (figure 3). If \( z_3 = z_4 \), \( z_3' = z_4' \) and \( z_3'' = z_4'' \), these gears being used to avoid the rotation of the satellites \( z_1 \), \( z_1' \) and \( z_1'' \) around their own axis, then the reduction ratios for each mechanism embranchment could be computed using the next relations

\[ i_{C3}^3 = \pm \frac{z_2z_3}{z_2z_3 - z_1z_2'} \] \hspace{1cm} (8)

For \( z_1 - z_2 = z_3 - z_2' = \pm 1 \) and \( z_2 = z_3 \),

\[ i_{C3}^2 = \pm \frac{z_2^2}{z_3} \] \hspace{1cm} (9)

\[ i_{C3}^3 = \pm \frac{z_2^2}{z_3} \] \hspace{1cm} (10)
\[ i_{C2}^1 = \frac{z_2}{z_2 - z_1} \]  
\[ i_{C2'2'}^1 = \frac{z_2'}{z_2' - z_1'} \]  
\[ i_{C2''2''}^1 = \frac{z_2''}{z_2'' - z_1''} \]

**Figure 3.** Planetary gear with three output shafts: a) mechanism kinematics; b) its spatial schematics.

Extending this idea, the mechanism may have five output shafts in a spatial structure (see its schematics in figure 4). Depending on the teeth numbers for each gear, the reduction ratio may be the same or different for each mechanism embranchment.

In a large class of industrial robots, the orientation of the end effector is provided by a spherical wrist mechanism. Planetary bevel-gear trains can be adopted to derive the configurations of spherical wrist mechanisms. The bevel-gear spherical wrist mechanisms have been shown to have the following advantages: the closed form solution for the inverse kinematics is available; the workspace is easy to determine; the actuators can be mounted remotely from the wrist center to reduce the dynamic effect, and the design requirements, such as stiffness, dexterity, and heavy payloads, are provided.

The mechanisms using bevel internal gears having small difference in numbers of teeth proposed in this paper are using a single motor for their actuation. These mechanisms may be used in the design of spherical wrist mechanisms for industrial robots. Based on these mechanisms, some robotic wrists will be proposed in another paper of the same journal.
4. Conclusion
The mechanisms using bevel internal gears having small difference in numbers of teeth have gained prowess because of the benefits they offer, especially obtaining high reduction ratios in one reduction step. In addition, there is no longer the risk of trochoid interference that may occur in the case of the internal cylindrical gears when the teeth numbers difference is small. For these reasons, these mechanisms may be used in the design of spherical wrist mechanisms for industrial robots. In this paper some new nutation geared mechanisms using bevel internal gears having small difference in numbers of teeth have been proposed. These mechanisms are using a single motor for their actuation. Also, their structure and kinematics have been discussed. Based on these mechanisms, some robotic wrists will be proposed in another paper of the same journal.

5. References
[1] Atanasiu V and Doroftei I 2008 Dynamic contact loads of spur gear pairs with addendum modifications European journal of mechanical and environmental engineering 2 pp 21-26.
[2] Atanasiu V, Doroftei I, Iacob M R and Leohchi D 2011 Nonlinear dynamics of steel/plastic gears of servomechanism structures Mater. Plast. 48(1) pp 98-103.
[3] Crivoi O and Doroftei I 2018 Some Mechanisms Using Internal Gears with Small Difference in Numbers of Teeth In: Doroftei I., Oprisan C., Pislă D., Lovasz E. (eds) New Advances in Mechanism and Machine Science 57 pp. 357-366 (Springer, Cham).
[4] Macovei S and Doroftei I 2015 An overview on internal geared mechanisms with small difference between teeth number IOP Conference Series: Materials Science and Engineering 95(1) pp 012053.
[5] Zhang H L and Guo H 2010 Research on manufacturing method of nutation driving gear Mechanical Research & Application 6 pp 027.
[6] Dong G J and Wang M 2014 Modeling of Error Analysis Simulation of Normal Circular Arc Bevel Gear Transmission Key Engineering Materials 589 pp 606-610.
[7] Yao L, Gu B Haung S, Wei G and Dai J 2010 Mathematical Modeling and Simulation of the External and Internal Double Circular-Arc Spiral Bevel Gears for the Nutation Drive Journal of Mechanical Design 132(2) pp 210081-210080.
[8] Hong J, Yao L, Ji W and Huang Z 2015 Kinematic Modeling for the Nutation Drive Based on

Figure 4. Spatial schematics of a planetary gear with five output shafts.
Screw Theory *Procedia CIRP* **36** pp 123-128.

[9] Wang G and Guan T 2009 Modeling of Nutation Drive with Rolling Teeth *Applied Mechanics and Materials* **16-19** pp 708-712.

[10] Huang D, Yao L, Li W and Zhang J 2017 Geometric modeling and torque analysis of the magnetic nutation gear drive *Forsch Ingenieurwes* **81** pp 101-108.

[11] Ji W, Yao L and Zhang J 2016 Mathematical modeling and characteristics analysis for the nutation gear drive based on error parameters *Journal of Chongqing University* **15(4)** pp 149-158.

[12] Lin Z and Yao L G 2012 General mathematical model of internal meshing spiral bevel gears for nutation drive *Applied Mechanics and Materials* **101-102** pp 708-712.

[13] Lin Z and Yao L G 2013 Mathematical model and 3D modeling of involute spiral bevel gears for nutation drive *Advanced Materials Research* **694-697** pp 503-506.

[14] Kedrowski K and Slimak P 1993 Wobbling Gear Drivetrain for Cordless Screwdriver *Honors Theses. 1921*. http://scholarworks.wmich.edu/honors_theses/1921.