Chargino contributions to $\epsilon'/\epsilon$ in the left-right supersymmetric model

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We analyze the chargino contributions to the CP violating ratio $\epsilon'/\epsilon$ in the left-right supersymmetric model. We study the possibility that these contributions alone can saturate the experimental value of $\epsilon'/\epsilon$. We derive conservative bounds on supersymmetric flavor violation parameters in the up squark LL, RR, LR and RL sectors, using the mass insertion approximation. While the LL bounds are found to be consistent with the MSSM values, the LR constraints are new and much stronger.

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1 Introduction

CP violation is still a basic problem to answer in particle theory and a good probe of new physics beyond the Standard Model (SM). CP violation was first discovered in the kaon system [1], and recently confirmed in the B-system [2, 3], where assumptions about the gauge structure would be verified. There are two CP violation parameters in the kaon system: the indirect CP violation parameter $\epsilon$, which follows from the mass eigenstates of $K^0$ and $\bar{K}^0$ and is given by the imaginary parts of the diagrams leading to the mass difference $\Delta M_K$, and the direct CP violation parameter $\epsilon'$, which describes the decay of $K \rightarrow 2 \pi$.

The direct CP violation had been confirmed by [4], the world average value for the CP violating ratio $\epsilon'/\epsilon$ is [5]

$$\text{Re}(\epsilon'/\epsilon) = 1.8 \pm 0.4 \times 10^{-3}.$$  \hspace{1cm} (1)

Even there are large theoretical uncertainties and experimental errors associated with it, $\epsilon'/\epsilon$ can put stringent constraints on extensions of the SM, as for instance general supersymmetric models [6], models with anomalous gauge couplings [7], four-generation models [8] and models with additional fermions and gauge bosons [9].

In Ref. [10] we studied $\Delta S = 2$ processes in the kaon system and evaluated $\Delta M_K$ and $\epsilon$ in the left-right supersymmetric model (LRSUSY) as a scenario for new physics. In this paper we extend the analysis to $\Delta S = 1$ processes and put more constraints on the LRSUSY parameter space and CP violation by calculating $\epsilon'/\epsilon$.

There are two kinds of diagrams leading to $\Delta S = 1$ processes: box and penguin diagrams. (As opposed to $\Delta S = 2$ processes, which are mediated by box diagrams alone.) The exchange particles in general supersymmetric models can be gluinos, neutralinos and charginos. The gluino contributions have been studied widely in literature [11], while chargino contributions have not received much attention, mainly due to its strong model-
dependence [12]. The gluino and neutralino contributions put constraints on squark mixings in the down sector; there gluino contributions are dominant and therefore the bounds are sensitive to the QCD sector in the model. Studying the chargino contributions would constrain the mixing in the up squark sector, independent of the down sector, and thus test models with different electroweak symmetries than the MSSM, in particular the LRSUSY. This is the goal of the present article.

Our paper is organized as follows: in section II, after a short description of the model, we give complete analytical results for the chargino contributions to $\epsilon'/\epsilon$. We follow by presenting numerical analysis in section III. We reach our conclusions in section IV, and in the appendix we give a summary of the chargino mixing in the LRSUSY, as well as the loop and vertex functions used, for self-sufficiency of the paper.

2 The analytic formulas

2.1 Effective Hamiltonian for $\Delta S = 1$ processes in the LRSUSY

The left-right extension to the supersymmetric standard model is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [13]. The model has chiral superfields in left and right handed doublets. The Higgs sector consists of two bidoublet and four triplet Higgs superfields. The bidoublet Higgs superfields exist in all versions of LRSUSY and break the symmetry group to $SU(2)_L \times U(1)_Y$. Additional Higgs representations needed to break $SU(2)_R$ symmetry are commonly chosen to be triplets which support the seesaw mechanism. One needs to double the number of Higgs fields with respect to the non-supersymmetric version to ensure anomaly cancellations in the fermionic sector. The superpotential involving these superfields is

$$W = Y_Q^{(i)} Q^T \Phi_i i\tau_2 Q^c + Y_L^{(i)} L^T \Phi_i i\tau_2 L^c + i(Y_{LR} L^T \tau_2 \Delta_L L + Y_{LR} L^c \tau_2 L^c) + \mu_{LR} [Tr(\Delta_L \delta_L + \Delta_R \delta_R)] + \mu_{ij} Tr(i\tau_2 \Phi_i^T i\tau_2 \Phi_j).$$ (2)
In addition, flavor and CP violating effects arise from the the soft supersymmetry breaking terms

\[ L_{soft} = \left[ A_Q^i Y_Q^{(i)} \tilde{Q}^T \Phi_i \tau_2 \tilde{Q}^c + A_L^i Y_L^{(i)} \tilde{L}^T \Phi_i \tau_2 \tilde{L}^c + i A_{LR} Y_{LR} (\tilde{L}^T \tau_2 \Delta_L \tilde{L} + \tilde{L}^T \tau_2 \Delta_R \tilde{L}^c) \right. \\
\left. + m_{\Phi}^{(ij)2} \Phi_i \Phi_j \right] + \left[ (m_{Q_L}^2)_{ij} \tilde{Q}_{L_i}^T \tilde{Q}_{L_j} + (m_{Q_R}^2)_{ij} \tilde{Q}_{R_i}^T \tilde{Q}_{R_j} \right] + \left[ (m_{\tilde{L}_L}^2)_{ij} \tilde{L}_{L_i}^T \tilde{L}_{L_j} + (m_{\tilde{L}_R}^2)_{ij} \tilde{L}_{R_i}^T \tilde{L}_{R_j} \right] \\
- M_{LR}^2 \left[ Tr(\Delta_R \delta_R) + Tr(\Delta_L \delta_L) + h.c. \right] - [B \mu_j \Phi_i \Phi_j + h.c.]. \tag{3} \]

We parameterize all the unknown soft breaking parameters coming mostly from the scalar mass matrices using the mass insertion approximation [14]. We choose a basis for fermion and sfermion states in which all the couplings of these particles to neutral gauginos are flavor diagonal and parameterize flavor changes in the non-diagonal squark propagators through mixing parameters \((\delta^q_{ij})_{AB}\), where \(i, j = 1, 2, 3\) and \(A, B = L, R\).

The dimensionless flavor mixing parameters used are

\[
\begin{align*}
(\delta^q_{ij})_{LL} &= \frac{(m_{\tilde{q}_{i,j}}^2)_{LL}}{m_{\tilde{q}}^2}, & (\delta^q_{ij})_{RR} &= \frac{(m_{\tilde{q}_{i,j}}^2)_{RR}}{m_{\tilde{q}}^2}, \\
(\delta^q_{ij})_{LR} &= \frac{(m_{\tilde{q}_{i,j}}^2)_{LR}}{m_{\tilde{q}}^2}, & (\delta^q_{ij})_{RL} &= \frac{(m_{\tilde{q}_{i,j}}^2)_{RL}}{m_{\tilde{q}}^2}, \tag{4}
\end{align*}
\]

where \(m_{\tilde{q}}^2\) is the average squark mass and \((m_{\tilde{q}_{i,j}}^2)_{AB}\) are the off-diagonal elements which mix squark flavors for both left- and right-handed squarks with \(q = u, d\).

The contributions to \(\Delta S = 1\) processes are given by the effective Hamiltonian

\[
H_{\Delta S=1}^{eff} = \sum_i [C_i(\mu)Q_i(\mu) + \tilde{C}_i(\mu)\tilde{Q}_i(\mu)], \tag{5}
\]

where the relevant operators entering the sum are

\[
\begin{align*}
Q_3 &= \tilde{t}_L^\beta \gamma_\mu s_L^\alpha \sum_{q=u,d,s} \tilde{q}_L^\beta \gamma^\mu q_L^\alpha, \\
Q_4 &= \tilde{t}_L^\beta \gamma_\mu s_L^\alpha \sum_{q=u,d,s} \tilde{q}_L^\beta \gamma^\mu q_L^\alpha, \\
Q_5 &= \tilde{t}_L^\beta \gamma_\mu s_L^\alpha \sum_{q=u,d,s} \tilde{q}_R^\beta \gamma^\mu q_R^\alpha.
\end{align*}
\]
\[ Q_6 = T_L^\gamma \gamma^\mu s^\mu_q \sum_{q=u,d,s} T_R^\nu \gamma^\mu q_R, \]
\[ Q_7 = \frac{Q_4 e}{8\pi^2} m_s d_L^\nu \sigma^{\mu \nu} s^\alpha_R F_{\mu \nu}, \]
\[ Q_8 = \frac{q}{8\pi^2} m_s d_L^\nu \sigma^{\mu \nu} t^\alpha_{\alpha \beta} s^\beta_R C^\alpha_{\mu \nu}, \]

(6)

where \( q_{R,L} = P_{R,L} q \) with \( P_{R,L} = (1 \pm \gamma_5)/2 \), and \( \alpha, \beta \) are color indices. The operators \( \tilde{Q}_i \) are obtained from \( Q_i \) by the exchange \( L \leftrightarrow R \). Because of the left-right symmetry, we must consider all contributions from both chirality operators. We neglect the higgsino couplings (proportional to \( m_s \) or \( m_d \)) and include single mass insertion only. To this order, the contributions are

\[ C_i = C^\text{box}_i + C^g_{\gamma\text{-penguin}} + C^\gamma_{\gamma\text{-penguin}} + C^Z_{\gamma\text{-penguin}}, \]

(7)

with

\[ C^\text{box}_4 = \frac{\alpha_W^2}{m_q^2} \sum_{i,j=1}^3 \sum_{a,b=1}^3 |V_{ji}|^2 |V_{ij}|^2 K^*_{a_2} K_{b_1} B_{\chi^+}(x_{\tilde{\chi}^+_i \tilde{q}}, x_{\tilde{\chi}^+_j \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^\text{box}_5 = \frac{\alpha_W^2}{2m_q^2} \sum_{i,j=1}^3 \sum_{a,b=1}^3 |V_{ji}|^2 |V_{ij}|^2 K^*_{a_2} K_{b_1} B_{\chi^+}(x_{\tilde{\chi}^+_i \tilde{q}}, x_{\tilde{\chi}^+_j \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac{-\alpha_s \alpha_W}{3m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 |V_{ii}|^2 K^*_{a_2} K_{b_1} f_g(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac{\alpha_s \alpha_W}{m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 |V_{ii}|^2 K^*_{a_2} K_{b_1} f_g(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac{-\alpha_s \alpha_W}{3m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 |V_{ii}|^2 K^*_{a_2} K_{b_1} f_g(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac{-\alpha_s \alpha W}{m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 |V_{ii}|^2 K^*_{a_2} K_{b_1} f_g(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LL}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac {-\pi \alpha W}{m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 K^*_{a_2} K_{b_1} \{ |V_{ii}|^2 F_2(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LL} + U_{1i} V_{ii} \frac{m_{\tilde{\chi}^+_i}}{m_d} F_4(x_{\tilde{\chi}^+_i \tilde{q}}) (\delta^u_{ab})_{LR} \}, \]
\[ C^g_{\gamma\text{-penguin}} = \frac {-\pi \alpha W}{m_q^2} \sum_{i=1}^3 \sum_{a,b=1}^3 K^*_{a_2} K_{b_1} \{ |V_{ii}|^2 \left[ F_1(x_{\tilde{\chi}^+_i \tilde{q}}) + Q_d F_2(x_{\tilde{\chi}^+_i \tilde{q}}) \right] (\delta^u_{ab})_{LL} \} \]
Figure 1: Leading supersymmetric box and penguin diagrams contributing to $\epsilon'$, $A, B, C = (L, R)$. 
\[ C_{3}^{\text{penguin}} = +U_{i1}V_{i1}^{\dagger} \frac{m_{\tilde{e}_{i}}}{m_{s}} \left[ F_{3}(x_{\tilde{e}_{i}}^{-q}) + Q_{u}F_{4}(x_{\tilde{e}_{i}}^{-q}) \right] (\delta_{ab})_{LR} \],

where \( x_{ab} = m_{a}^{2}/m_{b}^{2} \). There is no chargino box contribution to \( C_{3} \) and \( C_{6} \) because of the color structure. Similarly, there is no photon and \( Z_{L} \) penguin contribution to \( C_{4} \) and \( C_{6} \). The photon penguin gives zero contribution to \( C_{3} \) and \( C_{5} \) after the sum over quarks. Similarly, the \( Z_{L} \) penguin gives zero contribution to \( C_{5} \). We neglect contributions from the \( Z_{R} \) penguin since they are smaller by \( m_{Z_{L}}^{2}/m_{Z_{R}}^{2} \) than the contributions from the \( Z_{L} \) penguin. The notations of various vertices, mixing matrices and functions are defined in the appendix. The coefficients \( \tilde{C}_{i} \) are obtained from the coefficients \( C_{i} \) by the exchange \( L \leftrightarrow R \).

### 2.2 Hadronic Matrix Elements

We list here for completeness the relevant matrix elements of operators, which can be found in Ref. [15]

\[
\begin{align*}
\langle (\pi\pi)_{I=0}|Q_{3}|K \rangle &= \frac{X}{3}, \\
\langle (\pi\pi)_{I=0}|Q_{4}|K \rangle &= X, \\
\langle (\pi\pi)_{I=0}|Q_{5}|K \rangle &= -\frac{Y}{3}, \\
\langle (\pi\pi)_{I=0}|Q_{6}|K \rangle &= -Y, \\
\langle (\pi\pi)_{I=0}|Q_{8}|K \rangle &= \sqrt{\frac{3}{8}} \frac{1}{4} \sqrt{\frac{11}{2} \frac{m_{s}}{16\pi^{2}}} \frac{f_{K}^{2}}{m_{K}^{2}} m_{s} m_{d} \frac{f_{K}^{2}}{f_{\pi}^{2}} m_{K}^{2} m_{\pi}^{2}, \\
\langle (\pi\pi)_{I=2}|Q_{i}|K \rangle &= 0, \quad i = 3 \cdots 6, 8 \quad (9)
\end{align*}
\]

where

\[ X = \sqrt{\frac{3}{8}} \frac{f_{\pi}}{f_{K}} \left( m_{K}^{2} - m_{\pi}^{2} \right), \]
\[ Y = \sqrt{\frac{3}{2}} (f_K - f_\pi) \left( \frac{m_K^2}{m_s + m_d} \right)^2. \] (10)

From Ref. [16], \( \langle (\pi\pi)_{I=0,2} | Q_7 | K \rangle \) is proportional to the photon condensate, thus it must be very small and negligible. Accordingly the contribution to \( \epsilon' \) of \( C_7^{\gamma \text{-penguin}} \) can be neglected. The matrix elements of the operators \( \bar{Q}_i \) can be obtained by multiplying the corresponding matrix elements of \( Q_i \) by \((-1)\).

Putting all the above together, we can calculate the CP violating ratio \( \epsilon'/\epsilon \)

\[ \frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2|\epsilon|\text{Re}A_0}} \left( \text{Im}A_0 - \frac{1}{\omega} \text{Im}A_2 \right), \] (11)

where \( \omega = \text{Re}A_2 / \text{Re}A_0 \) and the amplitudes are defined as

\[ A_I e^{i\delta_I} = \langle (\pi\pi)_I | H_{\text{eff}}^{\Delta S=1} | K \rangle, \] (12)

with the isospin of the final two-pion state \( I = 0, 2 \), and \( \delta_I \)'s are strong phases induced by final state interactions, \( \delta_2 - \delta_0 \) is close to \( \pi/4 \).

### 3 Numerical Analysis

In this section we present the results of our analysis for the individual bounds on \( (\delta_{12}^u)_{AB} \), obtained by selecting only one source of flavor violation and neglecting interference between different sources. Setting the CKM phase to zero, therefore there is no SM contribution to \( \epsilon' \). The constraints are obtained by requiring the LRSUSY contributions alone to saturate the experimental value for \( \epsilon'/\epsilon \). Therefore the bounds are conservative.

We choose all trilinear scalar couplings in the soft supersymmetry breaking Lagrangian to be universal: \( A_{ij} = A\delta_{ij} \) and \( \mu_{ij} = \mu\delta_{ij} \), and we fix \( A \) to be 100 GeV, the higgsino mixing parameter \( \mu = 200 \) GeV, and \( \tan \beta = 5 \) throughout the analysis. We also demand that the chargino masses satisfy the current experimental bounds.
The bounds on \((\delta_{12}^u)_{LL}\) are presented in Table 1. From Eq. 8, the box, gluon-penguin and photon-penguin contributions are proportional to \(1/m_{\tilde{q}}^2\), while the \(Z_L\)-penguin contribution is proportional to \(1/m_Z^2\). Thus the \(Z_L\)-penguin contribution dominates over all other contributions as \(m_{\tilde{q}}^2\) are expected to be larger than \(m_Z^2\). Our bounds on \((\delta_{12}^u)_{LL}\) are comparable with previous bounds of Khalil and Lebedev [12], and they are also the same order of magnitude as the bounds on \((\delta_{12}^d)_{LL}\) obtained from the gluino contributions to \(\epsilon'\) [17]. The values we obtained are always of \(O(10^{-1})\) and fairly stable over a large range of chargino and squark mass parameters. What is different in this model from the MSSM is that, due to the mass parameters and symmetry of the model in the chargino sector, to a good approximation we get the same bounds on \((\delta_{12}^u)_{RR}\).

| \(M_L / m_{\tilde{q}}\) (GeV) | 300 | 500 | 700 | 900 |
|--------------------------|-----|-----|-----|-----|
| 150                      | 0.10| 0.14| 0.16| 0.18|
| 250                      | 0.18| 0.29| 0.42| 0.57|
| 350                      | 0.23| 0.30| 0.38| 0.46|
| 450                      | 0.29| 0.38| 0.42| 0.50|

Table 1 Limits on \(|\text{Im}(\delta_{12}^u)_{LL}|\) from \(\epsilon'\) for different values of \(M_L = M_R\) and \(m_{\tilde{q}}\), with \(\tan \beta = 5\) and \(\mu = 200\) GeV. The bounds are insensitive to \(\tan \beta\) in the range of 2 – 30 and \(\mu\) in the range of 200 – 500 GeV.

The bounds on \((\delta_{12}^u)_{LR}\) are presented in Table 2. These bounds come from the gluon and photon penguins; as before we would obtain approximately the same bounds on \((\delta_{12}^u)_{LR}\) as on \((\delta_{12}^u)_{RL}\). As there is a large factor \(m_{\tilde{\chi}^-}/m_s\) in the penguin LR contributions, the bounds on \((\delta_{12}^u)_{LR}\) are tighter by 1-2 orders of magnitude than the bounds on \((\delta_{12}^u)_{LL}\). These bounds are also less stable than the previous ones, and can vary by a factor of \(10^2\) over the range of chargino and squark masses explored. Both sets of bounds are rather insensitive to other parameters, such as \(\tan \beta\) or the higgsino mixing parameter \(\mu\), over a low-intermediate range of values.
Table 2  Limits on $|\text{Im}(\delta_{12}^{u})_{LR}|$ from $\epsilon'$ for different values of $M_L = M_R$ and $m_{\tilde{q}}$, with $\tan \beta = 5$ and $\mu = 200$ GeV. The bounds are insensitive to $\tan \beta$ in the range of 2 – 30 and $\mu$ in the range of 200 – 500 GeV.

4 Conclusions

We have studied the chargino contributions to the CP violating ratio $\epsilon'/\epsilon$ in the LRSUSY. Assuming the CP violation to arise from the supersymmetric contributions only, we derived bounds on the imaginary parts of the supersymmetric flavor violation parameters in the LL, RR, LR and RL sectors of the up squark mass matrix, under the assumption that only one such insertion dominates. The bounds in the LL sector are of $\mathcal{O}(10^{-1})$, they agree with the corresponding bounds obtained in the MSSM, and are weak compared to down squark mass insertion bounds. The bounds on the LR mass insertions are much stronger (of $\mathcal{O}(10^{-4} - 10^{-2})$) and new to this analysis: no similar bounds exist for the MSSM. Comparing this analysis with the previous results coming from the up squark sector of the $K^0 - \bar{K}^0$ parameter $\epsilon$ [10], we see that the constraints from $\epsilon'/\epsilon$ are weaker by one or more orders of magnitude. Therefore, even if more than one mass insertion would drive the CP violation, the constraints cannot be made to coincide. Thus, in the LRSUSY, as in the MSSM, supersymmetric contributions coming from complex mass insertions in the squark mass matrices fail to saturate both $\epsilon$ and $\epsilon'/\epsilon$, and thus cannot be responsible alone for the CP violation.

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Appendix

For self-sufficiency, we list the mass-squared matrices for charginos and neutralinos, relevant Feynman rules and functions used for this calculation.

The terms relevant to the masses of charginos in the Lagrangian are

\[ \mathcal{L}_C = -\frac{1}{2}(\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.} \],

(13)

where \( \psi^+ = (-i\lambda_L^+, -i\lambda_R^+, \tilde{\phi}^+_{u1}, \tilde{\phi}^+_{d1})^T \) and \( \psi^- = (-i\lambda_L^-, -i\lambda_R^-, \tilde{\phi}^-_{u2}, \tilde{\phi}^-_{d2})^T \), and

\[
X = \begin{pmatrix}
M_L & 0 & g_{L\kappa_u} & 0 \\
0 & M_R & g_{R\kappa_u} & 0 \\
0 & 0 & -\mu & 0 \\
g_{L\kappa_d} & g_{R\kappa_d} & -\mu & 0
\end{pmatrix},
\]

(14)

where we have taken, for simplification, \( \mu_{ij} = \mu\delta_{ij} \). The chargino mass eigenstates \( \tilde{\chi}_i \) are obtained by

\[
\tilde{\chi}_i^+ = V_{ij}\psi_j^+, \quad \tilde{\chi}_i^- = U_{ij}\psi_j^-, \quad i, j = 1, \ldots, 4,
\]

(15)

with \( V \) and \( U \) unitary matrices satisfying

\[
U^*XV^{-1} = M_D,
\]

(16)

The diagonalizing matrices \( U^* \) and \( V \) are obtained by computing the eigenvectors corresponding to the eigenvalues of \( X^†X \) and \( XX^† \), respectively.

The vertices of \( Z_L \)-chargino-chargino are given by

\[
(Z_L)_L^{ij} = V_{i1}V_{j3}^* + \frac{1}{2}V_{i3}V_{j3}^* + \frac{1}{2}V_{i4}V_{j4}^* - \sin^2\theta_W\delta_{ij},
\]

\[
(Z_L)_R^{ij} = U_{i1}^*U_{j1} + \frac{1}{2}U_{i3}^*U_{j3} + \frac{1}{2}U_{i4}^*U_{j4} - \sin^2\theta_W\delta_{ij}.
\]

(17)

(18)

The relevant functions are listed in the following

\[
B^{(1)}_{\tilde{\chi}^-}(x, y) = \frac{1}{8(x-y)} \left[ -3x^2 + 4x - 1 + 2x^2 \log x \right] \left( \frac{1}{(x-1)^3} \right) - (x \to y),
\]

(19)
\[ B^{(2)}_\chi(x,y) = \sqrt{xy} \frac{1}{2(x-y)} \left[ \frac{-x^2 + 1 + 2x \log x}{(x-1)^3} - (x \to y) \right], \quad (20) \]

\[ f_g(x) = \frac{1 - 6x + 18x^2 - 10x^3 - 3x^4 + 12x^3 \log x}{18(x-1)^5}, \quad (21) \]

\[ f_\gamma(x) = \frac{22 - 60x + 45x^2 - 4x^3 - 3x^4 + 3(3 - 9x^2 + 4x^3) \log x}{27(x-1)^5}, \quad (22) \]

\[ F_1(x) = \frac{-1 + 9x + 9x^2 - 17x^3 + 18x^2 \log x + 6x^3 \log x}{12(x-1)^5}, \quad (23) \]

\[ F_2(x) = \frac{-1 - 9x + 9x^2 + x^3 - 6x \log x - 6x^2 \log x}{6(x-1)^5}, \quad (24) \]

\[ F_3(x) = \frac{1 + 4x - 5x^2 + 4x \log x + 2x^2 \log x}{2(x-1)^4}, \quad (25) \]

\[ F_4(x) = \frac{-x^2 - 5 - 4x - x^2 + 2 \log x + 4x \log x}{2(x-1)^4}, \quad (26) \]

\[ f_Z(x) = \frac{-1 + 4x - 3x^2 + 2x^2 \log x}{8(x-1)^3}, \quad (27) \]

\[ f_Z^{(1)}(x,y) = \frac{1}{2(x-y)} \left[ \frac{x^2 \log x}{(x-1)^2} - \frac{1}{x-1} - (x \to y) \right], \quad (28) \]

\[ f_Z^{(2)}(x,y) = \sqrt{xy} \frac{1}{(x-y)} \left[ \frac{-x \log x}{(x-1)^2} + \frac{1}{x-1} - (x \to y) \right]. \quad (29) \]

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