Are there flux tubes in quark-gluon plasma?

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We review both lattice evidences and theoretical arguments supporting the existence of electric flux tubes in quark-gluon plasma, above the deconfinement transition temperature $T > T_c$. At the end of this comment, we also point out certain questions which still need to be answered.

Let us start with two popular statements:

(i) Existence of flux tubes between two fundamental charges in QCD-like gauge theories is among the most direct manifestations of the confinement phenomenon.

(ii) Confinement is well described by the “dual superconductor” model [1, 2], relating it to known properties of superconductors via electric-magnetic duality.

In this note we discuss both of them, arguing that they are only partially correct. In fact we are going to argue that flux tubes do exist even above the deconfinement transition temperature, and that there are much better analogies to confinement than superconductors. Most facts and considerations needed to understand the phenomena discussed has in fact been in literature for some time. The purpose of these comments are simply to remind them, “connecting the dots” once again, since these questions continue to be asked at the meetings. We will also point out certain aspects of the phenomena which still need to be clarified.

I. FLUX TUBES ON THE LATTICE, AT ZERO $T$ AND NEAR $T_c$

Lattice gauge theory simulations have addressed the confinement issue from their beginning, and by now there are many works which studied the electric flux tubes between static charges. Most of those are done at zero/low $T$. The documented well the profile of the electric field and the magnetic current “coiling” around it.

The “dual superconductor” analogy leads to a comparison with Ginzburg-Landau theory (also called “the dual Higgs model”) and good agreement with it has been found. These results are well known for two decades, see e.g. the review [3].

However, recent lattice studies [4] exploring a near-deconfinement range of temperatures have found that a tube-like profile of the electric field persists even above the critical temperature $T_c$, to at least $1.5T_c$. One of the plots from this work is reproduced in Fig.1. It corresponds to pure gauge SU(3) theory, which has the first order transition, seen as a jump in the field strength. Note however that at $T > T_c$ the shape of the electric field transverse profile remains about the same, while the width is decreasing with $T$, making it even more tube-like, rather than expected near-spherical
Coulomb behavior. (Note also that the length of the flux tube remains constant, 0.76 fm for this plot.)

Such behavior clearly contradicts the “dual superconductor” model: in superconductors, the flux tubes are only observed in the superconducting phase. Why do we observe flux tubes in the “normal” phase, and do we really have any contradictions with theory here?

II. DOES THE $T_c$ INDEED REPRESENT THE MONOPOLE CONDENSATION TEMPERATURE?

Let us start by critically examining the very notion of the deconfinement transition itself, focusing on whether it is indeed the transition between the super and the normal phases.

The $T_c$ itself is defined from thermodynamical quantities, and for pure gauge theories (we will only consider in this note) its definition has no ambiguities.

At this point it is worth reminding that in fact the electric-magnetic duality relates QCD not to the BCS superconductors, but rather to Bose-Einstein condensation (BEC) of bosons, the magnetic monopoles. Multiple lattice studies did confirmed that $T_c$ does coincide with BEC of monopoles.

One such study I was involved in [5] has calculated the probability of the so called Bose (or rather Feynman’s) clusters, a set of $k$ monopoles interchanging their locations over the Matsubara time period. Its dependence on $k$ leads to the definition of the effective chemical potential, which is shown to vanish exactly at $T = T_c$. This means that monopoles do behave as any other bosons, and they indeed undergo Bose-Einstein condensation at exactly $T = T_c$.

Earlier studies by Di Giacomo and collaborators in Pisa group over the years, see e.g. [6], were based on the idea to construct (highly non-local) order parameter for monopole BEC. It calculates the temperature dependence of the expectation value of the operator, effectively inserting/annihilating a monopole, and indeed finds a jump exactly at $T_c$.

In summary, it has been shown beyond a reasonable doubt that $T_c$ does indeed separate the “super” and “normal” phases.

III. CONSTRUCTING THE FLUX TUBES IN THE “NORMAL” PHASE

Since electric-magnetic duality relates QCD not to the BCS superconductors, but rather to BEC, let us at this point emphasize a significant difference between them: the uncondenced bosons are also present in the system, both above and even below $T_c$, while the BCS Cooper pairs of superconductor exist at $T < T_c$ only.

The first construction of the flux tube in the normal phase has been made by Liao and myself [7]. The key point is that it does not require supercurrents. Indeed, various flux tubes are found in plasmas: e.g. one can even see them in solar corona in an average telescope. What is needed for flux tube formation is in fact the presence of dual plasma, a medium including moving magnetic charges.

Their scattering on the electric flux tube schematically shown in Fig.2 does not change the monopole energy but changes direction of its momentum, thus creating a force on the flux tube. If it is strong enough able to confine the electric field, a flux tube solution can be constructed.

For further details see the original paper [7]. Let us only comment that (i) the “uncondenced’ monopoles exert a larger force than those in the condensate, as their momenta are larger; and (ii) it has in fact been predicted there that the highest $T$ at which such solution may exist is about $1.5T_c$.

![FIG. 2: A sketch of a monopole traversing the electric flux tube](image-url)
FIG. 3: Left: Free (red rhombs) energy $F(r)$ and potential (blue squares) energy $V(r)$, at $T_c$, compared to the zero temperature potential (black line). Right: Effective string tension for the free and the internal energy, from [9].

IV. TWO STATIC POTENTIALS AND THE FLUX TUBE ENTROPY ISSUE

So far there were no subtleties involved. But let us asks now the following question: Provided these flux tubes at $T > T_c$ carry some tension (energy-per-length), does it still imply existence of a linear potential between quarks, up to $T = 1.5 T_c$? And if they do, would it imply that, in a sense, confinement remains enforced there?

In order to have proper perspective on what is going on, let us look back at lattice studies of the static quark (fundamental) potentials, e.g. [8]. The key point is that there are two kinds of the potentials. At finite temperatures the natural quantity to calculate, for the observed flux tubes between static charges, is the free energy. It can be written as

$$F(r) = V(r) - TS(r), \quad S(r) = \frac{\partial F}{\partial r} \quad (1)$$

where $S(r)$ is the entropy associated with a pair of static quarks. Since it can be calculated from the free energy itself, as indicated in the r.h.s., one can subtract it and plot also the potential energy $V(r)$. The derivatives over $r$ are known as the string tensions.

These lattice calculations have shown that in certain range of $r$ the tension is constant (the tension is approximately $r$-independent). Two resulting tensions, shown in Fig.3 (left) have very different temperature dependence. The tension of the free energy shows the expected behavior: $\sigma_F(T)$ vanishes as $T \to T_c$. But the tension of the potential energy $\sigma_V(T)$ shows drastically different behavior, with large maximum at $T_c$, and non-zero value above it. This unexpected behavior was hidden in $\sigma_F(T)$, studied in many previous works, because in it a large energy and a large entropy cancel each other.

So, everything would be consistent, provided these novel flux tubes at $T > T_c$ do indeed carry the potential energy only, but no free energy tension $\sigma_V \neq 0, \sigma_F = 0$. In other words, we suggest that the potential energy detected (via electric field squared) in [4] must be canceled by the entropy associated with it, and no actual force between charges would be present in equilibrium. This conjecture can and should be checked.

Similar comment applies also to the theoretical calculation of the potential: in [7] only the mechanical stability of the tube solution was derived. The entropy associated with the flux tube still remains to be calculated.

As a parting comment, while this conjecture sounds like the well known idea of Hagedorn string transition, it cannot be exactly that. Indeed, this idea is known to suggest that at $T > T_c$ string gets to be infinitely long. If so, the tube completely delocalizes, and there would be a Coulomb field rather than what was observed by [4]. The entropy in question is perhaps related to monopoles bound to the tube rather than its multiple shapes. Also a Hagedorn transition seems to be at odds with the tension increase and size decrease as a function of $T$ observed.
(Finally, let us for clarity mention that we only discuss in this note static potentials in thermal equilibrium. We do not discuss potentials in quarkonia, in which quarks are not standing but moving. This problem is associated with certain time scales, inducing deviation from equilibrium and possible dissipation. It would therefore require a completely separate discussion.)

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