About development of numerical methods of analysis of spatial plate-shell reinforced concrete structures with allowance for non-linearities

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Abstract. The distinctive paper is devoted to development of numerical methods of spatial plate-shell reinforced concrete structures analysis with allowance for non-linearities. Immediacy of the problem and its contemporary state (within Russia and without) are presented. Particularly types of diagrams for reinforced concrete structures modeling, construction of reinforced concrete general deformation models, strength criteria for reinforced concrete structures and methods of structural analysis, main objectives and scientific novelty of study, its theoretical and practical relevance are under consideration.
1. Introduction. Immediacy of the problem
The modern stage of modelling of reinforced concrete structures behavior is associated with the widespread use of numerical methods [1]. Substantial progress in the computer industry and computational mathematics, which lasted for the last decades, led to a change in the ratio of analytical, experimental (model and full-scale) and numerical approaches to the complex structures analysis, buildings and facilities. Practice puts forward the multivariate research problems of two-dimensional and three-dimensional systems. Correct and adequate solution to these problems can often be obtained only numerically. Normally it is impossible to find a closed analytical solution for the majority of problems, and experimental studies are often very expensive, and sometimes incomplete. This, in particular, explains the certain prevalence of numerical methods in structural analysis. Generally at all stages of modeling of structure behavior, mathematical theory, research by analytical and experimental methods, and numerical analysis should be applied jointly and in coordination. The mathematical modeling of the solid mechanics considering problems and structural mechanics is normally based on sophisticated numerical methods (with corresponding discretization in space and time), namely, the finite element method.

Thus, universal and specialized finite-element software systems are used widely in structural analysis. It should be noted that the dependencies for a solid elastic isotropic body are often used as physical relationships between forces and deformations. Meanwhile, application of such approach for reinforced concrete structures can lead to significant errors due to crack formation and development of inelastic deformations under the action of operational loads. This causes a decrease in stiffness (deformative) parameters, redistribution of forces in structural members and increase in deflections and displacements. In this regard, further improvement and modifications of reinforced concrete models and their integration in software systems for structural analysis remain very important.

2. About types of diagrams for reinforced concrete structures modeling
There are three main types of diagrams which are normally used in theory of reinforced concrete structures analysis: analytical form; piecewise linearization of the analytical dependencies of the diagrams in the process of step loading; ladder polygonal line (due to multi-point setting of stresses and relative deformations). The diagram of reinforcement deformation in elements with cracks is under special consideration. Analytic dependencies, which define diagrams of concrete and reinforcement deformation were developed in the research works of specialists all over the world. We should note here in particular V.Ya. Bachinsky, V.N. Baykov [2], V.M. Bondarenko [3, 4], S.V. Bondarenko [5], V.P. Chayka, P. Desayi [6–8], P.F. Drozdov [2], V.S. Fedorov, K. Gerstle [9, 10], A.B. Golyshev, Yu.P. Gushcha, E.A. Guzeev [11], N.I. Karpenko [12, 13], S.F. Klovanich, V.I. Kolchunov [3, 4, 14], S. Krisman, V.M. Kruglov, S.A. Madatyan, L.R. Mailyan [15, 16], S.I. Merkulov, V.M. Mitasov, G.V. Murashkin, Ya.M. Nemirovsky, V.G. Nazarenko, L.L. Panshin, A.N. Petrov [13], A.B. Piradov, B.S. Rastorguev [11], L. Saennz, R.S. Sanzharovsky [5], E.N. Shcherbakov, B. Sinha, V.I. Travush [14, 17–21], L. Tulin etc. Two methods of diagrams definition are used: definition in the form of stress-strain curves (direct method), or definition in the analogous curves form with the use of secant modules. The first method is the most popular. Particularly, it was used by T.A. Balan and S.F. Klovanich for development of concrete model in incremental form based on a diagram recommended by the European Committee for Concrete (ECC). Secant modules were used in the research works of N.I. Karpenko, T.A. Mukhamediev, A.N. Petrov et al (modules were expressed through stress levels). It should be noted that discontinuities of the derivatives entering into the analytic functions dependencies are possible in the diagrams of reinforcement in reinforced concrete elements at the crack formation moment. This characteristic property requires special methods that complicate the structural analysis. This disadvantage is deprived of the above-mentioned methods of relationships definition between stress and strain increments by piecewise linearizing the diagrams with respect to stepwise loading. The development of this direction was laid down by the research works of S.N. Karpenko. The third methods of diagrams definition is the most universal. Due to rapid development of new effective types of concrete and reinforcement, it is not always possible to quickly
define the deformation diagrams by analytic dependences. In these cases, we can take a multipoint form of diagrams defining with linear segments between points. It should be noted that the real diagrams replacement with two or three segments of the polygonal line was considered in many research works (particularly by A.A. Gvozdev [22], N.I. Karpenko [12,13,17], O.A. Kokovin, A.S. Zalesov et al. V.M. Bondarenko [3,4] and S.V. Bondarenko [5] proposed to use piecewise-linear diagrams of general form for the linearization of systems of nonlinear differential equations. This idea was developed later in the research works of S.N. Karpenko [12,13,17].

3. About construction of general deformation models of reinforced concrete

The construction of the general reinforced concrete models deformation was considered in the well-known research papers of T.A. Balan, O.Ya. Berg, V.M. Bondarenko [3,4], S.V. Bondarenko [5], G.A. Geniev [22–26], A.A. Gvozdev [27], N.I. Karpenko [12,13,17], V.N. Kissyuk, S.F. Klovanich, V.I. Kolchunov [3,4,14], V.M. Kruglov, A.N. Petrov [13], B.S. Sokolov [28], G.A. Tyupin, Yu.V. Zaitsev [29,30], etc. Particularly methods of analysis of plates and shells with the use of various deformation models were considered in research papers of V.N. Baykov [2], V.M. Bondarenko [3,4], Yu.V. Chinenkov, N.I. Karpenko [12,13,17], S.B. Krylov, S.M. Krylov, L.D. Lifshits, I.E. Mileikovsky, M.M. Onishchenko, S.N. Paluvina, P.A. Shagin, I.T. Timko, V.V. Shugaev [24,31], V.F. Vladimirovand others. We believe that anisotropic model of slabs deformation with cracks is the most general. It was verified in research works of A.L. Gurevich, M.I. Levy, A.N. Petrov [13], S.N. Paluvina, S.N. Karpenko [12,13,18] et al. At the same time, until recently, deformation models in increments remained undeveloped. However, research works of T.A. Balan, A.N. Donets, G.A. Geniev, S.F. Klovanich, V.M. Kruglov, L.Yu. Solovyov, G.A. Tyupin, S.A. Tikhomirov, G.V. Vasilkov, et al. based on the development of plastic flow theory applied to concrete are corresponding exceptions. Developments within the concrete theory of V.M. Kruglov, L.Yu. Solovyov and G.V. Vasilkov are the most general from this point of view. Their research works take into account the discrepancy between surface of the yield beginning and the plastic potential surface, the dilatation effect and some other features of the deformation of concrete. However, this leads to a significant complication of the computational model. Generally nonlinear models of reinforced concrete, where the properties of reinforced concrete with cracks are approximated by the properties of some continuous anisotropic body are the most widespread in practical applications. Models based on the deformation theory of the concrete plasticity and reinforced concrete of G.A. Geniev and the theory of reinforced concrete deformation with cracks of N.I. Karpenko are successfully developing.

4. About strength criteria for reinforced concrete structures and methods of structural analysis

The results of research in the field of development and application of strength criteria for reinforced concrete structures are presented in the papers of M.S. Borishansky, A.A. Gvozdeva [27], O.F. Ilyin, K. Jogansen, N.I. Karpenko [12,13,17], S.N. Karpenko [12,13,18], A.A. Kondratchik, E.N. Pankov, I.A. Titov, V.V. Tour, Kh.A. Zabinshina, A.S. Zalesov, A.I. Zvezdov, etc.

It is necessary to note V.V. Bolotin [32], R. Clough, A. Griffiths, J.L. Lions [33,34], A.I. Lurie[35,36], N. Newmark, Ya.G. Panovko [37], Yu.N. Rabotnov [38], A.R. Rzhanitsyn [39], V.I. Travush [14,17-21], S.P. Tymoshenko [40], K. Vasiliev [41], Vestergard, A.S. Volmir, A.B. Zolotov [42,43] and others are scientists-mechanics who made a significant contribution to the mathematical formulations development, analytical methods, the foundations of structural analysis numerical methods. General theoretical and applied aspects of contemporary numerical methods (finite element method, finite difference method, variational-difference method, boundary elements method) and semianalytical (numerical-analytical) methods are reflected in the papers of N.P. Abolovsky, P.A. Akimov [44–50], S.M. Aleynikov [51], N.S. Bakhalov [52], K.J. Bathe [53,54], M.V Belyi [42], C.A. Brebbia [55], V.E. Bulgakov [42], R.P. Fedorenko [56], G.J. Fix [57], P.P. Gajjurov, R.H. Gallagher [58], S.K. Godunov [59], V.A. Ilyichev [3], G.G. Kashevarova [20,21,60], L.S. Lyakhovich [37], J.T. Oden [61], A.V. Perelmuter [62,63], I.A. Rozin [64], A.A. Samarsky, V.A. Semenov, V.V.
Shaidurov [65], V.N. Sidorov [66, 67], V.I. Slivker, E.L.G. Strang, Wilson [54], O.C. Zienkiewicz [61, 68], A.B. Zolotov [42,43] and others.

Generally experience of leading Russian foreign research and educational centers allows us to conclude that the wide application of mathematical and computer modeling, as well as computational experiments, serves as the nearest strategic reserve for accelerating methods scientific and technological progress.

5. Main objectives and scientific novelty of study, theoretical and practical relevance

The aim of the starting work is research, development, numerical and computer realization of the reinforced concrete deformation incremental model and methods of reinforced concrete structures analysis under complex stress states with allowance for physical nonlinearity, anisotropy and constructive heterogeneity (based on a new system of physical relationships in finite increments with allowance for various factors of physical nonlinearity and anisotropy, the corresponding low-iteration computational algorithms, strength criteria, the multifactor finite-element models development of buildings and structures, including unique ones).

Thus, we have the following objectives:

- research and development, numerical and computer realization of reinforced concrete computational model deformation at various stressed states in incremental form with allowance for physical nonlinearity of reinforced concrete components, crack formation and heterogeneity and anisotropy acquired as a result of fracture formation and low iteration algorithms based on it;
- research and development, numerical and computer realization of strength criteria multifactor system for reinforced concrete elements in complex stress states;
- development and verification of spatial finite-element models of reinforced concrete structures and buildings (including unique buildings and structures) with high-precision modeling of complex structural joints of various structural elements.

An important factor in the attainability of these objectives is the availability and widespread application of well-known finite element software systems verified in the Russian Academy of Architecture and Construction Sciences. ANSYS and SIMULIA Abaqus are effective tools for verifying models, methods and algorithms.

Generally we believe that scientific novelty and technical potential of this research are quite high. The studies will be based on the latest achievements in the theory field of the reinforced concrete building structures analysis, structural mechanics, mechanics of deformable solids, computational mathematics and programming achieved by the world's leading research centers.

Scientific novelty of study is brought by the following its main components.

1. Research and numerical realization of the equations, which define relationship between the increments of stresses and deformations (incremental relationships) for reinforced concrete as a physically nonlinear material with anisotropy acquired as a result of deformation and cracking under various stress states, application and development of general methods for defining physical relationships in increments, including the following elements:

- reasonable choice of relations between the increments of stresses and deformations on the basis of various diagrams of concrete and reinforcement deformation;
- analysis of the specific definition in incremental form of physical relations for reinforced concrete elements with allowance for discontinuity of the first kind for stresses in the reinforcement and concrete at the time of crack formation;
- realization of the general method of the physical relations systems transformation between stresses and deformations in a ratio between their final increments on the basis of stepwise linearization;
- computer realization of stress-strain diagrams and their parameters with the use of data sets and the definition of physical relations in increments based on them.
2. Research and numerical realization of more perfect criteria for the strength of reinforced concrete structures with cracks under various stressed states.

3. Construction and verification of more perfect spatial finite element models of reinforced concrete structures and buildings (including unique ones) with high-precision modeling of complex structural joints for various structural elements.

Analysis Methods of prefabricated monolithic and monolithic conventional and prestressed concrete and concrete structures of industrial and civil buildings and facilities (with allowance for physical nonlinearity, anisotropy, crack formation and other factors, as well as the influence of these factors on the forces redistribution, the change in deformation and the crack resistance of structures) are under consideration. The complete diagrams of the materials deformation (concrete and reinforcement) and their transformation depending on the complexity of the stress-strain states are investigated and implemented at the numerical and program-algorithmic levels. General strength criteria and physical relations are considered with orientation on application of so-called diagrammatic methods of structural analysis, allowing evaluating the strength and serviceability of the building, proceeding from two groups of limit states by advanced computational methods and primarily by the finite element method (FEM). Application of proposing models, methods and algorithms will provide reliable information about the stress-strain state of concrete and structural reinforcement.

We believe that developing methods of reinforced concrete structures analysis will replace multi-iterative approaches to the solution of physically nonlinear problems and move from the practically possible high-precision analysis of individual structures to the complex structural systems analysis with allowance for various factors of physical nonlinearity and anisotropy. As a result reliability of design solutions will increase significantly. The strength criteria used in this way, in turn, will also eliminate a number of errors in existing methods for strength analysis.

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