Properties of the Quark Gluon Plasma: 
A lattice perspective

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Abstract

We discuss results from lattice calculations for a few observables that are sensitive to different length scales in the high temperature phase of QCD and can give insight into its non-perturbative structure. We compare lattice results with perturbative calculations at high temperature obtained for vanishing and non-vanishing quark chemical potential.

Key words: QCD, Lattice Gauge Theory, Quark Gluon Plasma, Equation of State

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1 Introduction

The observation of large elliptic flow and jet quenching at RHIC [1] seems to suggest a rapid thermalization of the dense matter created in relativistic heavy ion collisions. This in turn suggests that the hot and dense medium created in these collisions is still strongly interacting. It is not at all an ideal gas as one might have expected from a straightforward extrapolation of the asymptotic high temperature behavior of QCD all the way down to temperatures in the vicinity of the transition region from hadronic to partonic matter.

Already from the early perturbative calculations it is well known that new non-perturbative scales arise at high temperature, which even may invalidate a perturbative approach to some basic physical quantities at any temperature [2]. Also the early non-perturbative studies of finite temperature QCD, eg. calculations of the equation of state in a purely gluonic SU(3) gauge theory
have shown that the high temperature phase of QCD is far from being simply an ideal quark-gluon gas; large deviations from ideal gas behavior, $\epsilon = 3p$, occur even at temperatures $T \sim (2 - 3)T_c$.

Non-perturbative length scales characterizing the screening of electric and magnetic modes at high temperature, which have been identified in perturbative calculations and have been studied quantitatively in non-perturbative lattice calculations, have been implemented in refined perturbative calculation schemes. This led to the hard thermal loop resummation scheme [4] as well as the reformulation of perturbative calculations in terms of separate effective theories for the electric and magnetic sectors of QCD [5]. This allowed to incorporate systematically non-perturbative aspects of QCD into perturbative calculations and produced for various observables impressive agreement with lattice calculations at high temperature, even at temperatures as low as $T \simeq 2T_c$.

In view of the experimental findings at RHIC and the speculations on the nature of the interactions still present in the QCD plasma phase at high temperature [6], it may be useful to recollect some of the results obtained in lattice studies of observable that are sensitive to different length scales characterizing the high temperature phase of QCD. We will discuss here results on screening lengths in the QGP as well as the QCD equation of state at non-zero temperature and baryon number density. We will compare these results with refined perturbative calculations and, in particular, want to stress that (i) most of the non-perturbative features of QCD at high temperature and density are already present in quenched QCD, i.e. the SU(3) gauge theory, and that (ii) the non-zero density sector seems to show remarkably little non-perturbative structure.

2 Thermal and non-thermal length scales at high temperature

Attempts to describe deviations from ideal gas behavior in terms of high temperature perturbation theory have clearly unrevealed the role of different non-perturbative length scales in the QGP. In addition to the thermal length scale which is controlled by the lowest non-zero Matsubara frequency, $r_{\text{therm}} \sim 1/T$, larger length scales related to the screening of electric modes (Debye screening) at distances $r_E \sim 1/gT$ and magnetic modes at distance $r_M \sim 1/g^2T$ have been identified. Moreover, non-thermal hard scales that control processes at distances smaller than $r_{\text{therm}}$ or, equivalently, energies significantly larger than the energy scale given by temperature, still play an important role, e.g. in the analysis of quarkonium bound states or jets in hot and dense matter.

The separation of thermal and non-thermal scales becomes very transparent
from an analysis of the distance and temperature dependence of heavy quark free energies. While at sufficiently short distances heavy quark free energies are \( T \) independent even at quite large \( T \), they are screened at large distances for any temperature. This is shown in Fig. 1(left) for a calculation in 2-flavor QCD [7]. Motivated by one loop perturbation theory for the singlet free energy, \( F_1(r, T) = -4 \alpha_{q\bar{q}}/3r \) with \( \alpha_{q\bar{q}} = g^2(r)/4\pi \), one may define a running coupling,

\[
\alpha_{q\bar{q}}(r, T) = \frac{3r^2}{4} \frac{dF_1(r, T)}{dr}.
\]  

Eq. 1

Fig. 1(right) shows that this coupling agrees with the zero temperature perturbative form of the running coupling at short distances and follows the zero temperature behavior even in the non-perturbative regime for temperatures \( T \lesssim 1.5T_c \). Here the rapid rise of the coupling with increasing distance concurs with the fact that the singlet free energy still follows the linear rising zero temperature confinement potential also in this temperature regime above \( T_c \). This suggests that remnants of the confining force survive the transition to the high temperature phase and play an important role for physical processes (bound states) that are characterized by length scale less than \( r_{\text{screen}} \). We note that this is not related to the fact that the transition in QCD with physical quark masses is not a true phase transition but rather a rapid crossover from the hadronic regime to the quark gluon plasma. The \( T \) and \( r \)-dependence of free energies and the running coupling are similar also for a SU(3) gauge theory. In the high temperature phase the transition from non-thermal, vacuum-like behavior to thermal screening of the free energy and the running coupling occurs in quenched as well as 2-flavor QCD at a similar distance scale, \( r_{\text{screen}} \).

To a good approximation one thus finds for the running coupling,

\[
g^2(r, T) \simeq g^2(r, 0) \quad \text{for} \quad r < r_{\text{screen}} \sim 0.5 T_c/T.
\]  

Eq. 2
3 Screening of electric and magnetic modes

Debye screening: At high temperature the interaction among static quark sources, mediated by gluons, is screened. The corresponding electric screening length can be calculated in leading order perturbation theory at high temperature and for non-vanishing quark chemical potential $\mu_q$,

\[
\frac{m_D(T, \mu_q)}{g(T) T} = \sqrt{\frac{N_c}{3} + \frac{n_f}{6} + \frac{n_f}{2\pi^2} \left( \frac{\mu_q}{T} \right)^2} f_E(g^2(T), T, \mu_q)
\]

\[
= m_0(T) + m_2(T) \left( \frac{\mu_q}{T} \right)^2 + \mathcal{O}(\mu_q^4). \quad (3)
\]

Here $f_E(g^2(T), T, \mu_q)$ summarizes all higher order perturbative as well as any non-perturbative contributions. We note that $f_E$ would be a function of $g^2$ only, if only perturbative terms would contribute to $m_D$; in leading order perturbation theory we have $f_E = 1$.

In lattice calculations for the pure SU(3) gauge theory the electric screening mass has been calculated from Polyakov loop correlation functions up to fairly high temperature [8]. It has been found there that deviations from the leading order perturbative result are large in the vicinity of the phase transition temperature and that the approach to the leading order perturbative form is slow; it may approximately be realized only at temperatures as large as $10^{100} T_c$. This is in accordance with the logarithmic running of the QCD coupling, which in fact is observed to be well described by the 2-loop $\beta$-function of QCD already close to $T_c$. In Fig. 2(left) we show a comparison of the electric screening mass extracted from the long distance behavior of singlet quark-antiquark
free energies in 2-flavor QCD and the $SU(3)$ gauge theory [7]. We note that in both cases at temperatures up to a few times the transition temperature the variation of $m_D/T$ with $T$ is well accounted for by the 2-loop running of the coupling. Higher order corrections and non-perturbative contributions in this small temperature interval are well approximated by an almost flavor independent constant $f_E(g^2(T), T) \simeq A$, with $A = 1.52(2)$ for quenched QCD and $A = 1.42(2)$ for 2-flavor QCD [7].

The right hand part of Fig. 2 shows the leading order (quadratic) correction to $m_D$ as function of the quark chemical potential [9]. It is apparent that deviations from the leading order perturbative result are large in the vicinity of $T_c$. However, already at $T \simeq 1.5 T_c$ the corrections agree well with the leading order perturbative value, which is obtained from an expansion of the $\mu_q$-dependent prefactor in Eq. 3. The weak flavor dependence of $f_E(g^2(T), T)$ as well as the early onset of perturbative behavior for the ratio $m_2/m_0$ already at temperatures $T \gtrsim 1.5T_c$ suggest that non-perturbative aspects of the screening of electric modes in the high temperature phase of QCD are mainly controlled by the gluonic sector.

*Spatial string tension:* It has been pointed out already in 1980 by A. Linde that in QCD magnetic modes are screened only at $O(g^2 T)$ [10]. The proportionality factor of, eg, the magnetic screening length, is entirely non-perturbative. Nonetheless, the modern formulation of high temperature perturbation theory takes advantage of the existence of this new scale. It allows to isolated the contribution of the magnetic sector of QCD in terms of a 3-d effective theory, which is the ordinary 3-d $SU(3)$ gauge theory. A remarkable prediction of this setup is that non-perturbative contributions to observables, which at high temperature are dominated by contributions of magnetic modes, are flavor independent; a flavor dependence only enters indirectly through the flavor dependence of the running coupling. An observable that can be used to test this aspect of the dimensional reduction approach to QCD is the spatial string tension. It has, indeed, been found that spatial Wilson loops still show area law behavior in the high temperature phase of a $SU(3)$ gauge theory [11] and thus allow to define a spatial string tension, $\sigma_s$, that is sensitive to the non-perturbative magnetic screening length,

$$\sigma_s = \lim_{R,S \to \infty} \frac{1}{RS} \ln W(R, S) \sim \left[ c_M g^2(T) T f_M(g^2(T), T) \right]^2 .$$

(4)

Here $W(R, S)$ denotes a Wilson loop of size $R \times S$ in a spatial plane of a $(3+1)$-dimensional lattice. The proportionality constant $c_M$ is entirely non-perturbative. In the high temperature limit $f_M(g^2(T), T) \to 1$ and $c_M$ should agree with the proportionality factor for the string tension of a 3-d $SU(3)$ gauge theory, $\sqrt{\sigma_3} = c_3 g_3^2$. 

5
Fig. 3. Spatial string tension in a SU(3) gauge theory and (2+1)-flavor QCD (left). The right hand figure shows a comparison of the SU(3) results with a 2-loop perturbative calculation for the coupling in the effective 3-d theory for the magnetic sector of QCD [15].

It has been questioned whether the dimensional reduction scheme, which is a central concept of todays perturbative and resummed perturbative calculations at high temperature, can be applicable to QCD with light quarks [13]. Light compound fermionic (mesonic) modes might introduce new light degrees of freedom that cannot be integrated out. Moreover, large spatial Wilson loops will not show area law behavior in the presence of light quarks and similar to the free energy shown in Fig. 1 one may expect that the spatial potential from which $\sigma_s$ is extracted will show string breaking at a certain distance scale.

In Fig. 3(left) we show results from a calculation of $\sigma_s$ in quenched [3] and (2+1)-flavor QCD [14]. The solid lines show fits with the leading order ansatz ($f_M \equiv 1$) given in Eq. 4 using the perturbative 2-loop form for $g^2(T)$. The fit yields for the proportionality constant, $c_M = 0.566(13)$ for quenched QCD and $c_M = 0.587(41)$ for (2+1)-flavor QCD. This should be compared with the result in a 3-d SU(3) gauge theory [12], $\sigma_3 = 0.553(1)$. Fig. 3(right) shows a perturbative analysis of $\sigma_s$, which uses $\sigma_3$ as non-perturbative leading order input [15]. At least for this particular gluonic observable the dimensional reduction approach thus seems to work very well down to temperatures $T \gtrsim 1.7T_c$. We also note that the analysis of $\sigma_s$ yields a result for the temperature dependent running coupling $g^2(T)$. At $T \approx 2T_c$ one finds for quenched and (2+1)-flavor QCD, $g^2(2T_c) \approx 2.0$ and $g^2(2T_c) \approx 2.4$, respectively. This is consistent with values deduced from the analysis of $m_D/T$ shown in Fig. 2.

4 Equation of state and perturbation theory

Probably the most direct manifestation of strong interactions in the high temperature phase of QCD is given by the temperature dependence of basic bulk thermodynamic observables, eg, the energy density ($\epsilon$) and pressure ($p$). Al-
ready the early calculations in quenched QCD have shown that up to a few times $T_c$ $\epsilon$ and $p$ show large deviations from ideal gas behavior. E.g. one has for the pressure in QCD with massless quarks

$$\frac{p}{T^4} = \left( \frac{8\pi^2}{45} + \sum_{f=u,d} \left[ \frac{7\pi^2}{60} + \frac{1}{2} \left( \frac{\mu_f}{T} \right)^2 + \frac{1}{4\pi^2} \left( \frac{\mu_f}{T} \right)^4 \right] \right) f_p(g^2(T), T, \mu_f), \quad (5)$$

with $f_p \equiv 1$ and $\epsilon = 3p$ in the infinite temperature limit. Deviations from ideal gas behavior are parametrized here by the function $f_p$. Its structure is apparently dominated by non-perturbative contributions arising already in the gluonic sector of QCD. This is apparent from Fig. 4(left), which shows the pressure in quenched as well as 2 and 3 flavor QCD with (moderately) light quarks at vanishing quark chemical potential [16] and normalized to the corresponding ideal gas value given by the prefactor in Eq. 5. The temperature dependence of this ratio shows only little flavor dependence. Recent studies of $\epsilon/T^4$ and $p/T^4$ with smaller quark masses and closer to the continuum limit [17] as well as the (isentropic) equation of state for non-zero quark chemical potential [18] (Fig. 5(left)) show that this pattern is a generic feature of QCD thermodynamics. In the temperature interval $T_c \leq T \leq 3T_c$ thermodynamics thus is characterized by large deviations from the conformal ideal gas limit, which also results in large deviations of the velocity of sound, $v_s^2 = dp/d\epsilon$, from the asymptotic infinite temperature value, $v_s^2 \to 1/\epsilon$ (Fig. 5(right)).

The QCD pressure for $\mu_q \geq 0$ has been analyzed to all orders that are calculable in perturbation theory [19]. Non-perturbative contributions at $O(g^6)$ have been analyzed [20] and the screening of electric modes has also been implemented in self-consistent calculations [21]. We show in Fig. 4(right) results from an analysis of the pressure that uses HTL-resummed gluon self energies in the gluon propagator [21]. This suggests that the structure of the pressure,
Fig. 5. The equation of state \( p(\epsilon) \) in 2-flavor QCD at different ratios of the entropy per net baryon number (left) [18] and the velocity of sound (right).

in particular the deviations from ideal gas behavior and the slow approach to the ideal gas limit visible for \( T > 3T_c \), is well understood in the perturbative context, by taking into account non-perturbative screening effects.

The confrontation of refined perturbative calculations with lattice results suggest that at least for temperatures \( T > (2 - 3)T_c \), the high temperature behavior of bulk thermodynamic observables indeed can be understood in terms of the basic partonic degrees of freedom, quarks and gluons. Similar conclusions can also be drawn from the analysis of hadronic fluctuations [22], eg. fluctuations of the baryon number and correlations between baryon number and strangeness fluctuations.

5 Conclusions

We have discussed some basic results on the separation between thermal and non-thermal length scales in the high temperature phase of QCD as well as electric and magnetic screening lengths. We have shown that at short distances, \( r < r_{\text{screen}} \sim 0.5T_c/T \), the QCD coupling constant is not yet modified by temperature effects. On the other hand, at large distance the running of the coupling is controlled by temperature; large distance observables like the electric and magnetic screening lengths are controlled by a temperature dependent running coupling, \( g^2(T) \), which in magnitude is comparable to couplings characterizing perturbative vacuum physics, i.e. \( \alpha < 0.25 \) or \( g^2 \lesssim 3 \).

Bulk thermodynamic observables, eg. energy density and pressure, show strong deviations from ideal gas behavior even at (2-3) times the transition temperature and approach the asymptotic ideal gas limit only slowly. The good agreement between lattice results and refined perturbative calculations that take into account some non-perturbative effects, arising from the screening of electric and magnetic modes, suggests that for \( T > (2 - 3)T_c \) bulk thermodynamics
can be described in terms of quasi-particles that carry the quantum numbers of quarks and gluons. This is further supported by the analysis of fluctuations of conserved hadronic charges, eg. baryon number and strangeness.

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