QUANTUM, CHAOS AND THE UNIVERSE

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Abstract

In this paper we suggest a formulation that would bear out the spirit of Prigogine's "Order Out of Chaos" and Wheeler's "Law Without Law". In it a typical elementary particle length, namely the pion Compton wavelength arises from the random motion of the $N$ particles in the universe of dimension $R$. It is then argued in the light of recent work that this is the origin of the laws of physics and leads to a cosmology consistent with observation.

1 Introduction

Although the universe is apparently governed by iron clad laws, it would be more natural to expect that the underpinning for these laws would be chaos itself, that is Prigogine's, "Order out of chaos" or in the words of Wheeler [1], we seek ultimately a "law without law." As he put it, (loc.cit), "All of physics in my view, will be seen someday to follow the pattern of thermodynamics and statistical mechanics, of regularity based on chaos, of "law without law". Specifically, I believe that everything is built higgledy-piggledy on the unpredictable outcomes of billions upon billions of elementary quantum phenomena, and that the laws and initial conditions of physics arise out of this chaos by the action of a regulating principle, the discovery and proper formulation of which is the number one task...." A step in this direction has been the considerable amount of work of Nelson [2], De Pena [3, 4], Lehr [5], Gaveau [6].

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Landau[7] and others, who have tried to derive the Schrödinger equation, the Klein-Gordon equation and even the Dirac equation from stochastic considerations, and in general develop an underpinning of stochastic mechanics and stochastic electrodynamics. The literature is vast and some of the references given cite an extensive bibliography. However, all these derivations contain certain assumptions whose meaning has been unclear.

In the spirit of the above considerations, we propose below that purely stochastic processes lead to minimum space-time intervals of the order of the Compton wavelength and time, and it is this circumstance that underlies quantum phenomena and cosmology, and, in the thermodynamic limit in which $N$, the number of particles in the universe $\rightarrow \infty$ classical phenomena as well. In the process, we will obtain a rationale for the adhoc assumptions referred to above.

2 The Emergence of Quantized Space-Time and Physics

Our starting point is the well known fact that in a random walk, the average distance $l$ covered at a stretch is given by

$$l = R/\sqrt{N}$$

where $R$ is the dimension of the system and $N$ is the total number of steps. We get the same relation in Wheeler’s famous travelling salesman problem and similar problems[8, 9]. The interesting fact that equation (1) is true in the universe itself with $R$ the radius of the universe $\sim 10^{26} cm$, $N$ the number of the particles in the universe $\sim 10^{80}$ and $l$ the Compton wavelength of the typical elementary particle, the pion $\sim 10^{-13} cm$ has been noticed[11, 12].

We mention in passing that equation (1) which has been considered by some to be accidental, may not be so at all and has been shown to arise quite naturally in a cosmological scheme based on fluctuations. As this has been discussed by the author in detail at other places, for example[13, 14] the details will not be given, but we will merely touch upon the main results shortly.

We would like to stress that the Compton wavelength given by (1) is an important and fundamental minimum unit of length. Indeed, even in the
theory of the Dirac equation\cite{[15]} every electron apparently has the velocity of light $c$ brought out by the well known Zitterbewegung or rapid oscillation effect and non-Hermitian (or complex valued) position coordinate:

$$\vec{x} = (c^2 \vec{p} H^{-1} t) + \frac{\hbar}{2} \left( \vec{a} - c \vec{p} H^{-1} \right) H^{-1}, \tag{2}$$

It is only on averaging over space-time intervals of the order of the Compton wavelength and time that we recover physical electrons. This has been the basis of the recent formulation of elementary particles as Kerr-Newman type Black Holes\cite{[16, 17]}. Though on the one hand the puzzling fact has been known that the Kerr-Newman metric describes the field of the electron accurately, including the anomalous $g = 2$ factor\cite{[18]}, on the other, it was recognized that an electron could not be treated as a Kerr-Newman Black Hole as this would lead to a naked singularity, that is a complex radius:

$$r_+ = \frac{GM}{c^2} + ib, b \equiv \left( \frac{G^2 Q^2}{c^8} + a^2 - \frac{G^2 M^2}{c^4} \right)^{1/2} \tag{3}$$

Infact, as has been discussed in detail elsewhere (cf.refs.\cite{[12, 14, 16, 17]}, \cite{[19, 20]}), (2) corresponds to (3), and becomes meaningful once the above averaging over these minimum space-time intervals is done - physics begins outside these minimum intervals. It has also been shown that this leads to a unified description of interactions - the quarks and leptons (including the neutrino) are what may be called Quantum Mechanical Kerr-Newman Black Holes. The quark description emerges at the Compton wavelength while outside we recover the leptonic description\cite{[19, 20]}.

It may be mentioned that such a minimum time interval, the chronon, has been considered earlier in a different context by several authors\cite{[21, 22, 23]}. We now come to Nelson’s derivation of the Schrodinger equation from Brownian processes (cf.ref.\cite{[2]}). In this case, the random change in the $x$ coordinate say, is given by,

$$|\Delta x| = \sqrt{\langle \Delta x^2 \rangle} = \nu \sqrt{\Delta t},$$

in an obvious notation, where the diffusion constant $\nu$ is given by

$$\nu = \frac{\hbar}{m} = lv \tag{4}$$

$l$ being the mean free path or correlation length.

The adhoc identification of $\nu$ in (4) has been the troublesome feature. This
has been discussed elsewhere\[24\] (cf. also ref. [16]), but the point is that (4) gives us the Compton wavelength again.

The relativistic generalization of the above to the Klein-Gordon equation has been even more troublesome\[5\]. In this case, there are further puzzling features apart from the luminal velocity as in the Dirac equation, brought out by equation (2). For Lorentz invariance, a discrete time is further required. Interestingly, Snyder had shown that discrete space-time is compatible with Lorentz transformations\[24\]. Here again, the Compton wavelength and time cut off as discussed after (2) and (3) makes the whole picture transparent. The stochastic derivation of the Dirac equation introduces a further complication\[6\].

There is a spin reversal with the frequency $mc^2/\hbar$. This again is readily explainable in the above context in terms of the Compton time. Interestingly the resemblance of such a Weiner process to the Zitterbewegung of the electron was noticed by Ichinose\[26\].

Thus in all these cases once we recognize that the Compton wavelength and time are minimum cut off intervals, the obscure or adhoc features become meaningful.

We would like to point out that the origin of the Compton wavelength is the random walk equation (1)! One could then argue that the Compton time (or Chronon) automatically follows. This was shown by Hakim\[27\]. We would like to point out the simple fact that a discrete space would automatically imply discrete time. For, if $\Delta t$ could $\rightarrow 0$, then all velocities, $\lim_{\Delta t \rightarrow 0} |\Delta x/\Delta t|$ would $\rightarrow \infty$ as $|\Delta x|$ does not tend to 0!. So there is a maximal velocity and this in conjunction with symmetry considerations can be taken to be the basis of special relativity.

In fact one could show that quantized space-time is more fundamental than quantized energy and indeed would lead to the latter\[12\]. To put it simply the frequency is given by $c/\lambda$, where $\lambda$ the wavelength is itself discrete and hence so also is the frequency. One could then deduce Planck's law. This of course, is the starting point of Quantum Theory itself.

Equally interesting is the fact that a coordinate shift on Minkowski space-time actually leads to the Dirac equation\[28\] on the one hand, and a quantized picture of electromagnetism on the other\[29\].

At this stage we make the following remark which throws further light on the origin of the Compton wavelength (cf. ref. [14]). In the case of the Dirac electron whose position is given by (2), the point electron has the velocity of light and is subject to Zitterbewegung or rapid oscillation within the
Compton wavelength. The thermal wavelength for such a motion is given by
\[ \lambda = \sqrt{\frac{\hbar^2}{mkT}} \sim \text{Compton wavelength} \]
by virtue of the fact that now \( kT \sim mc^2 \) itself.

In fact, it has been shown in [16, 14] and [17], that it is this circumstance that leads to inertial mass, gravitation and electromagnetism (as for example brought out by the Kerr-Newman metric). Other interactions also can be accommodated within this framework (cf.ref. [12]).

3 Cosmological Considerations

Given the minimum space-time intervals of the order of Compton wavelength and time taking as usual the pion to be a typical elementary particle and using the fact that given \( N \) particles, there is a fluctuational creation of particles of the \( \sim \sqrt{N} \), a cosmological scheme has been discussed in detail elsewhere, (cf.refs. [14] and [13] amongst others for details). We merely report the fact that it deduces from theory the age, radius and mass of the universe as also Hubble’s law and other hitherto inexplicable, purely adhoc relations like the mysterious relation between the pion mass and the Hubble constant and the so called Dirac Large Number relations including equation (1). The model also predicts an ever expanding universe as indeed has since been confirmed [30]. This cosmology shares certain features of the Prigogine, Dirac and Steady State cosmologies.

For example, we have
\[ \frac{dN}{dt} = \sqrt{N}/\tau \]
, where \( \tau \) is the pion Compton time and this leads to
\[ T = \tau \sqrt{N} \]  \hspace{1cm} (5)

\( T \) is the age of the universe. Other relations also follow as mentioned above. The interesting point is, that (5) provides an arrow of time, whereas the usual laws of physics, eg.electromagnetism are arrowless. As is now generally recognized an arrow emerges from a thermodynamic (or irreversible) basis. The point to be made here is that all this is very much in the spirit and letter
of the foregoing considerations. \( N \), the number of particles in the universe, typically pions, is the sole cosmological or large scale parameter, while the microphysical parameter, the Compton wavelength (and time) also follow from (1) - a circumstance that has been called stochastic holism (cf. ref. [19]).

4 Discussion

We have attempted to realize Wheeler’s “law without law” referred to in the introduction. Wheeler (loc.cit.) speaks of a “regularity principle” which added to a totally chaotic universe leads to law. For example, he considers in the example of the travelling salesman, a “minimum distance” travelled, but qualifies it by calling it a practical man’s minimum (rather than a strict mathematical minimum). In the above considerations, on the other hand, we use the “average” distance (or time) in the sense of the “root mean square” average.

So the ”iron-clad” laws of the universe, have a stochastic underpinning. Strictly speaking, these rigid laws emerge in the thermodynamic limit, \( N \to \infty \) (cf.ref. [31]). Of course, the laws are a good approximation because \( N \sim 10^{80} \). This should be true of space and time reversal symmetries also, in view of their discrete structure, though the continuum is a good approximation.

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