Quantum Gravity with the Standard Model

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Abstract

A unified theory of four-dimensional gravity together with the standard model is presented, with supersymmetry breaking of M-theory at a TeV. Masses of the known particles are derived. The cosmological constant is quantum generated to the observed value. Quantum corrections to the classical compactification are analyzed, and the scenario is stable.
1 Introduction

Supersymmetry breaking at a TeV scale has emerged as the leading theoretical candidate for the phenomenological formulation of standard model extensions.

M-theory compactifications on $S^7$ manifolds [1, 2] and Joyce manifolds [3, 4] realize closely the generation of the standard model. In this paper we examine the formulation upon compactifying eleven dimensional [5] M-theory [6] on $dS_{3,1} \times S^7$, with twisting the $S^7$; there is a four-form flux turned on to make the background a solution to the classical equations of motion. Twisting the $R_{3,1}$ is also viable, as well as incorporating the Robertson-Walker spacetime in order to describe a different era in the cosmological evolution. We consider these scenarios to model the spacetime with no inhomogeneties [7]. The local spacetime is $R_{3,1}$, but there is a cosmological constant that is quantum generated; this constant agrees with data and modifies the geometry to de Sitter space. String modes and branes are incorporated via wrapping membranes on the compact cycles of the internal geometry.

In addition to the supersymmetry breaking and gauge group generation, the question of the origin of mass is answered. The known fermion masses may be obtained via a symmetry of the fermion interactions, and the masses come from perturbative resummations or membrane configurations in the compactified theory. Breaking supersymmetry at approximately 2, or 1 to 3, TeV is ideal.

The mass pattern is apparent and linear on a log scale and follow in accordance with a symmetry of the fermions; the associated charges are in the centers of the $SO(3, 1)$ and $SU(3) \times SU(2) \times U(1)$ gauge groups. This fermionic symmetry requires the modded seven-sphere and is also made possible via modding the $R_{3,1}$. In addition, there is a scenario at the GUT scale.

The gauge couplings may be explained via running and unification up to the GUT scale, but we do not analyze this scenario in too much detail. The presented compactification scenario works in the GUT scale unified theory after adjusting some of the parameters. The three couplings $\alpha_{QED}, \alpha_{EW}$, and $\alpha_S$ also have a symmetric form that comes about from the geometry of the compactified theory.

The quantum modifications to the unified gravity/standard model are analyzed, and the corrections shown not to modify the theory, apart from the usual renormalization of the fields. The setup is well-suited for supersymmetric phenomenology below a TeV, and collider energy scales are in the range to be tested by the Large Hadron Collider. Gravity corrections are suppressed by the dimensionless parameter $\Lambda/m_{pl}$, with $\Lambda$ a TeV.

$^1$Perturbations of the underlying spacetime may be inserted to model large scale structure.
In addition to model building, the feasibility of performing the complicated quantum calculations in gravity and gauge theory has improved much; there is progress in the art of gauge and gravity calculations, but little progress has been made beyond the multi-loop level due to the complications of the integrals. Quantum chromodynamics amplitudes have been well studied to one-loop and partially to two loops using a variety of techniques. Additional progress has been in this direction through the use of the derivative expansion in three types of theories: scalar, maximally supersymmetric models, and non-supersymmetric gauge theories with matter. This work may be examined in [7].

The perturbative quantum structure of the gravity and standard model, in the derivative expansion is an infinite expansion in energies. Loop graphs are conventionally an expansion in the coupling constant; the derivative expansion enables one to perform all loop integrations. The theory becomes pseudo-classical after this reformulation, and the spectrum of quantum chromodynamics is analyzed. The derivative calculations, presented in [7], allow for a non-perturbative definition of QCD together with a perturbative formulation of gravity. This approach contains simple techniques to evaluate the quantum corrections in the gravitational theory, as well as the quantum corrections in the most general lagrangian formulation.

In subsequent work a holographic definition of QCD is given, containing gravity in one of the scenarios presented here. Previous holographies equate gravity to gauge theory; this pseudo-holographic work has both states in the theory, without one being the bound state of another.

The outline of this work is as follows. In Section 2 we examine the fundamental constants of masses and gauge couplings. Charges and symmetries are introduced, and the quantized masses are found from these charges. In Section 3 we build the compactification scenarios containing the standard model and generate the masses and couplings of the standard model. In section 4, the electroweak model and QCD are discussed in relation to the former. In Section 4 we examine the quantum corrections to the gravity theory. Cosmology and its pertinence to the model in this work is in section 5. Discussions conclude.

2 Mass and Coupling quantizations

The masses of the fermions, and bosons, are examined and derived via the geometry of the underlying gravity theory. A feature of having geometry break the supersymmetry as well as generate the fermion masses that we do not require a chiral theory initially. QCD is believed to have a chiral phase transition at $T = 0$ in $d = 4$ and this conjecture
is not required; furthermore, anomaly cancellation is avoided and leaves more room for further generations.

The geometry, and contributing M2 and M5 branes, generate a symmetric mass pattern agrees with observed phenomenology of the standard model; in fact, this work via extrapolating the fermionic symmetry would of predicted the top quark mass. The masses are labeled in accord with the integers,

\[ \Lambda \left( \frac{\Lambda}{m_{pl}} \right)^{n_j/16} \equiv \Lambda \rho^{n_j/16} \] \hspace{1cm} (2.1)

and follow from the representations of \( SU(3)/Z_3 \cdot U(2)/Z_2 \cdot SO(3,1) \), the standard model gauge group in four dimensions.

The masses of the observed fermions fall into the pattern for the leptons, with masses measured in electron Volts,

\[ m_e = \alpha_e \Lambda \rho^{5/16} \quad 0.5 \times 10^6 \] \hspace{1cm} (2.2)
\[ m_{\mu} = \alpha_{\mu} \Lambda \rho^{3/16} \quad 1.1 \times 10^8 \] \hspace{1cm} (2.3)
\[ m_{\tau} = \alpha_{\tau} \Lambda \rho^{3/16} \quad 1.7 \times 10^9 \] \hspace{1cm} (2.4)

and the quarks

\[ m_t = \alpha_t \Lambda \rho^{1/16} \quad 1.7 \times 10^{11} \] \hspace{1cm} (2.5)
\[ m_b = \alpha_b \Lambda \rho^{2/16} \quad 4.7 \times 10^9 \] \hspace{1cm} (2.6)
\[ m_c = \alpha_c \Lambda \rho^{3/16} \quad 1.5 \times 10^9 \] \hspace{1cm} (2.7)
\[ m_s = \alpha_s \Lambda \rho^{4/16} \quad 1.5 \times 10^8 \] \hspace{1cm} (2.8)
\[ m_u = \alpha_u \Lambda \rho^{5/16} \quad 8 \times 10^6 \] \hspace{1cm} (2.9)
\[ m_d = \alpha_d \Lambda \rho^{6/16} \quad 4 \times 10^6 . \] \hspace{1cm} (2.10)

The parameters \( \alpha \) are of order unity, and the \( \rho \) variable is

\[ \rho = \Lambda/m_{pl} \quad m_{pl} = 2.1 \times 10^{28} \text{ eV} . \] \hspace{1cm} (2.11)
The parameters are obtained from the classical breaking of supersymmetry, at the TeV scale. All of these numbers, except two that miss be a factor of 2 and 3, match the fermions if we have $\Lambda = 10^{12}$ ev; of course this is not a prediction for supersymmetry at a TeV, but it is suggestive in the $1 - 3$ range. The $u$ and $d$ quarks are almost degenerate and we may interchange them with the caveat that quantum corrections are the cause.

It is important to note that chirality does not enter into the description of the fermions as they are massive, and neither does $SU(3)$ anomaly cancellation (although the content cancels the anomaly). We find the group theory origin of these quantum numbers in the following. The pattern is quantization of the masses, and there is a sum rule,

$$\sum_l n_l = \frac{4}{7} \sum_q n_q,$$

(2.12)

from

$$\sum_q m_q = 21, \quad \sum_l m_l = 12.$$

(2.13)

The neutrino content is believed to be approximately .01 ev. We split the three neutrinos as

$$m_{\nu_e} = \alpha_{\nu_e} \Lambda \rho^{15/16} .001 \quad (2.14)$$

$$m_{\nu_\mu} = \alpha_{\nu_\mu} \Lambda \rho^{14/16} .01 \quad (2.15)$$

$$m_{\nu_\tau} = \alpha_{\nu_\tau} \Lambda \rho^{13/16} .1 \quad (2.16)$$

The netrinos may also be generated via a perturbative gravity calculation,

$$\alpha \frac{\Lambda^2}{m_{pl}} = .01 \text{ eV},$$

(2.17)

with $\alpha \sim 10$ from the one-loop factors. This corresponds to a one-loop gravity calculation. The sum rule for these neutrinos is

$$\sum_\nu m_\nu = 2 \sum_q m_q = 7/2 \sum_l m_l,$$

(2.18)

with

$$\sum_\nu m_\nu = 42.$$

(2.19)
Neutrinos don’t couple to photons, and barring the SU(3) couplings, this one-loop gravity calculation would generate the neutrino masses; the higher-loop contributions in gravity mixed with matter are suppressed by $\Lambda/m_{pl} = 10^{-16}$. This is the second mechanism for generating the observed values of the neutrino masses, without the mass splittings to first order.

The gauge bosons have masses

$$m_{W^\pm} = \Lambda \rho^{1/16} \quad 8 \times 10^{10}$$

(2.20)

$$m_Z = \Lambda \rho^{1/16} \quad 9.2 \times 10^{10}.$$  

(2.21)

$$m_\gamma = m_{\text{glue}} = m_{\text{graviton}} = 0$$

(2.22)

Scalars generically get perturbative masses on the order of a TeV; if the mass were generated gravitationally then it would be in the 100 GeV range.

The remaining scales are:

$$\Lambda_{QCD} = \Lambda \left( \frac{\Lambda}{m_{pl}} \right)^{6/16}$$

(2.23)

$$\Lambda_{EW} = \Lambda \left( \frac{\Lambda}{m_{pl}} \right)^{1/16}.$$  

(2.24)

Breaking the supersymmetry at TeV scale generates the observed mass pattern of the standard model, and the quantum gravitational sector is incorporated in a direct fashion.

The gauge couplings at the strong scale point are

$$\alpha_{QED} = \frac{1}{137} \sim \frac{1}{(23)^6},$$

(2.25)

$$\alpha_{EW} = \frac{1}{(23)^3} \left( \frac{2}{3} \right)^{1/2},$$

(2.26)

$$\alpha_S \sim 1,$$

(2.27)

with the coupling relation,

$$\alpha_{QED} = \left( \frac{8}{3} \right)^{1/2} \alpha_{EW}.$$  

(2.28)
These factors may be interpreted via the underlying geometry. This completes the review of the masses and couplings of the standard model, with the exception of the Higgs mass.

The final number we examine is the cosmological constant; the experimental value is

$$\Lambda^4 \left( \frac{\Lambda}{m_{pl}} \right)^4,$$

with $\Lambda$ at 2 TeV. This number is derivable if we break a maximally supersymmetric gravity theory in such a way that there are eight independent scales. The breaking pattern is generated by this work, and as such, the value of the cosmological constant in this era is generated. The corrections are $O(\Lambda/m_{pl})$ and are suppressed. We turn to deriving the mass quantizations.

### 3 Geometry of Masses

The following model is contained in M-theory, with supersymmetry breaking at a around a TeV (LHC). We begin with a $S^7/Gamma \times R^{3,1}_3$ background in eleven dimensions; in subsequence we examine a $S^7 \times R_{3,1}/\Gamma$ to be used in a holographic formulation of QCD. The seven sphere in the first case contains four independent parameters,

$$\lambda z_i \bar{z}_{i,a} + \lambda_7 X_7^2 + \lambda X_8^2 = R_7^2,$$  \hspace{1cm} (3.1)

with the $a$ index labeling the components of the complex number $z_i$. To begin we take a quotient $X_7 \leftrightarrow -X_7$ and $X_8 \leftrightarrow -X_8$ to eliminate the extraneous fields generating along these cycles, and denote this by $\Gamma_2$. The geometry contains an $SU(3) \times SU(2)$ (the index $a = 1, 2, 3$) group of isometries. One may think of this as coming from an $SU(5)$ group, via a complexified SO(5), from the metric

$$\lambda z_i z_i + \bar{\lambda} z^i z^i + \lambda_7 X_7^2 + \lambda X_8^2 = R_{S^7}^2.$$  \hspace{1cm} (3.2)

The $R_{3,1}$ term with a quantum induced constant becomes $dS^4$. We begin with the topology,

$$\sum_{j=1}^4 \lambda_j X_j^2 = R_{dS}^2.$$  \hspace{1cm} (3.3)
possessing five independent parameters, and a weak curvature. The quantum gravity with eight parameters of supersymmetry breaking generates the value in eqn. (2.29) as a two-loop effect. We work with the geometry in eqn. (3.3) stabilized around this value, which is almost the flat $R_{3,1}$. This is the simplest case, and it is easy to analyze the early universe with a supermassive black hole and the Robertson-Walker geometry.

In order to achieve the mass pattern we require a symmetry of the fermions such that only the gravitational instantonic terms, and their quantum corrections, are allowed to contribute to the particle’s masses. This could be achieved in conjunction with a monodromy of the quantum theory with respect to the supersymmetry breaking scale, $\Lambda \rightarrow e^{2\pi i} \Lambda$.

The Lagrangian for the four components of fermions is,

$$\mathcal{L} = \bar{\psi}^\alpha \bar{i} \partial^\alpha \psi_\alpha + \bar{\chi}^\alpha \bar{i} \partial^\alpha \chi_\alpha + m \left( \psi^\alpha \chi_\alpha + \bar{\psi}^\alpha \bar{\chi}_\alpha \right) \quad (3.4)$$

The simplest symmetry is

$$\psi_\alpha^a \rightarrow e^{-2\pi in/30} \psi_\alpha^a \quad \chi_\alpha^a \rightarrow e^{-2\pi in/30} \chi_\alpha^a \quad (3.5)$$

and

$$\bar{\psi}_\alpha^a \rightarrow e^{2\pi in/30} \bar{\psi}_\alpha^a \quad \bar{\chi}_\alpha^a \rightarrow e^{2\pi in/30} \bar{\chi}_\alpha^a \quad , \quad (3.6)$$

combined with the mass term rotating due to the monodromy,

$$m \rightarrow m \ e^{2\pi in/15} \quad . \quad (3.7)$$

In four-component notation we have,

$$\psi \rightarrow e^{-2\pi in/30} \psi \quad , \quad \bar{\psi} \rightarrow e^{-2\pi in/30} \bar{\psi} \quad . \quad (3.8)$$

With these transformations the mass terms for the fermions are invariant when,

$$\Lambda \left( \frac{\Lambda}{m_{pl}} \right)^{n/15} \bar{\psi} \psi, \quad (3.9)$$

which is the monodromy of $\Lambda \rightarrow e^{2\pi i} \Lambda$. The charge assignments for the fermions follows from the compactification on the $S^7/\Gamma$ and its quotient.
We list the appropriate quotient $\Gamma$ of the $S^7$. The masses are generated through the symmetry generated by the twist of the $S^7$ coordinates:

\[
\begin{pmatrix}
  z_1 & -1 = e^{\pi i} & z_1 \\
  z_2 \rightarrow & 1 & z_2 \\
  z_3 & -1 = e^{-\pi i} & z_3
\end{pmatrix}
\]

(3.10)

with the six coordinates labeled as $z^a_j$ (pertaining to the SU(2) and SU(3)); this action is $G_a$.

The second action is,

\[
\begin{pmatrix}
  z^1 & -1 = e^{5\pi i} & z^1 \\
  z^2 \rightarrow & 1 = e^{-2\pi i} & z^2
\end{pmatrix}
\]

(3.11)

and,

\[
\begin{pmatrix}
  z^1 \rightarrow e^{2i\pi} & z^1 \\
  z^2 & -1 = e^{7\pi i} & z^2
\end{pmatrix}
\]

(3.13)

which we label $G_b$. These $Z_3 \times Z_2$ actions are in the center of $SU(2) \times SU(3)$ and have a $U(1)$ component. The $Z_3$ pertains to $z_i$ picking one of three factors, and the $Z_2$ to the two values given a single $z_i$ coordinate. There are six possibilities of the quotient, three $z_i$ and two $z^a$, this corresponds to the sextet structure, or the doublet-triplet, of all of the fermions. We give the charges below. The commutation of $G_a$ and $G_b$ requires a $Z_3$ action.

The symmetry of the $S^7$ quotient with the generators above affect the fields in $R_{3,1}$ via the compactification. The net quantum four-dimensional field $\psi$ is $\psi \chi(Y) \times \phi^a[Y]$, with $\phi^a$ carrying the $SU(3)/Z_3 \times SU(2)/Z_2 \times U(1)$ indices. The $X$ and $Y$ are coordinates of the base and compactified spaces. The group action on the compact fields generates a projection, by definition,

\[
\phi(G \cdot Y) = G \cdot \phi(Y).
\]

(3.14)

In this manner there is an action on $\psi$, the phase because the group pertaining to the quotient action is diagonal, so that the net field $\phi(Y) \times \psi(X)$ is invariant. The kinetic term,
\[ \int d^4x \sqrt{g} \bar{\psi} \partial \psi \]  
\hspace{1cm} (3.15)

is invariant, and the four-dimensional symmetry allows a mass term only if there is a compensating transformation on the mass. This is via the monodromy of \( \Lambda \), the supersymmetry breaking scale.

The fermion assignments due the group action follow from the quotient, or rather from the center of the gauge group \( SU(2) \times SU(3) \) together with the \( U(1) \). The phases are for the sextet of the quarks,

\[
\begin{pmatrix}
1 \\
0 \\
-1
\end{pmatrix}
\]  
\hspace{1cm} (3.16)

with six actions,

\[
\begin{pmatrix}
t & 6 \\
b & 5 \\
c & 4 \\
s & 3 \\
d & 2 \\
u & 1
\end{pmatrix}
\]  
\hspace{1cm} (3.17)

The same for the leptons and quark

\[
\begin{pmatrix}
1 \\
0 \\
-1
\end{pmatrix}
\]  
\hspace{1cm} (3.18)

with six actions,

\[
\begin{pmatrix}
e & 5 \\
\nu_e & 15 \\
\mu & 4 \\
\nu_\mu & 14 \\
\tau & 3 \\
\nu_e & 13
\end{pmatrix}
\]  
\hspace{1cm} (3.19)
The fermions transform in the way described, and these combinations allow the classical mass term exhibited earlier. There are corrections, of course, allowed by the symmetry, which include both perturbative and non-perturbative terms; the former comes from the usual loop graphs and the latter with M2 and M5 branes wrapped on the $S^7$ for example.

The charge assignments follow naturally with the doublet-triplet structure of the fermions. The fermion doublets and triplets are

\[
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix},
\begin{pmatrix}
  u \\
  c \\
  t
\end{pmatrix}
\] (3.20)

and,

\[
\begin{pmatrix}
  e \\
  \nu_e
\end{pmatrix},
\begin{pmatrix}
  \mu \\
  \nu_\mu
\end{pmatrix},
\begin{pmatrix}
  \tau \\
  \nu_\tau
\end{pmatrix}.
\] (3.21)

The U(1) charge assignment for the quarks and lepton/neutrino doublets are $(2, -1)/3$ and $(-1, 0)$. One may think of the $\pm 1$ in the SU(3) as $3(\frac{1}{3})$.

An alternate charge assignment, in conjunction with a mass displacement, potentially due to a singularity is the following. Flip the SU(3) assignment to $(-1, 0, 1)$. One may extract the $(5, 2)$ and $(4, 14)$ to, $(2 - 1) + 3$ and $(-10, 0) + 14$; the two factors are a $U(1)$ charge and a uniform constant for all of the fermions. The charge assignments then have the interpretation of a SU(3) charge, $U(1)$ charge, and a constant. The constant may have the interpretation of the contribution to the mass from a resolved singularity. (Another phase split is to write $(5, 2) = (3/2, -3/2) + 7/2$ and $(4, 14) = (3/2, 21/2) + 7/2$.) This procedure can be carried through on $dS_{3,1} \times S^7$ with a quotient in and singularity of the $dS_{3,1}$; in this case the interpretation is that of $N = 8$ supergravity with a singularity, and resolution to a black hole.

4 QCD and Electroweak sector

The QCD and Electroweak sector follow in a direct sense from the previous calculations. There are scalars arising from the 7 and 8 directions of the $S^7$ and the matter content for the gauge theory is already present. The Lagrangian for the gauge sector is,

\[
\mathcal{L}_g = \int d^4x \sqrt{g} \left( \text{Tr} F_{SU(3)}^2 + \text{Tr} F_{SU(2)}^2 - \frac{1}{4} F_{U(1)}^2 \right)
\] (4.1)
which is generated from the $S^7$ because the group action is taken in the center. The action for the gravitational part is,

$$\mathcal{L}_{gr} = \int d^4 \sqrt{g} \ (R + V),$$

(4.2)

and the fermionic components are,

$$\mathcal{L}_f = \sum_j \int d^4 x \sqrt{g} \left( \bar{\psi}_j (i \partial - \omega) \psi_j \right),$$

(4.3)

with $\omega$ the total spin connection from the gauge degrees of freedom.

The Higgs is required for the conventional mass generation of the gauge bosons and fermions; the scalars present may generate the masses for the $SU(2)$ and arise from the fields on the $z^a_i$ sector of the $S^7/\Gamma$.

$$\mathcal{L} = \int d^4 x \sqrt{g} \left( \nabla_{U(2)} \phi \nabla^\mu \bar{\phi} + \frac{1}{2} m^2 \phi \bar{\phi} \right) + \lambda \phi^4 + \ldots$$

(4.4)

the remaining sector of scalars do not couple directly to the $U(2)$ and the potential term is due to the non-supersymmetry of the background. The mass term is generically at the TeV scale due to quantum effects to the scalar propagator. The minimal coupling to the gauge field is sufficient to generate the mass term for the $SU(2)$ gauge fields. The gamma$_5$ couplings have not been examined in detail.

There are eight scalar modes in $N=8$ supersymmetric theory on $AdS_4 \times S^7$ with all radii different. As there are three couplings dictating the shape of the $S^7$, there are four massless moduli describing the zero mode perturbations. Only one complex scalar is charged under the $U(2)$ and the other two, pertaining to the seven and eighth direction do not couple; this combination is sufficient to generate the masses of the W’s and Z’s through the standard Higgs mechanism. The scalar content is in agreement with the MSSM. There are also towers of KK modes that may couple, and their masses are larger than a TeV.

5 Quantum Corrections

The masses and low-energy amplitudes are stable under quantum corrections. In detail to one loop the matter fields do not correct the mass forms as only the Z-factors are relevant to renormalize to this order; in general to higher loops the matter and gravitational degrees of freedom do not alter the masses a substantial amount,
but might explain the $d$ and $u$-quark splittings. The scenario is preserved under the logs and power corrections.

There are two types of quantum corrections to the background and fields: log corrections that modify the exponentiated term,

$$\Lambda \left[ \left( \frac{\lambda}{m_{\text{pl}}} \right)^{n/16} \right]^{m} \times (\ln \Lambda, \ln \ln \Lambda, \ln^p \Lambda) ,$$

(5.1)

and combinations. Second, we have further exponentiated terms that arise from wrapping M2 and M5 branes on the compact space, and other non-perturbative corrections in the $dS_{3,1}$; these generate

$$\Lambda \left[ \left( \frac{\lambda}{m_{\text{pl}}} \right)^{n/16} \right]^{m} ,$$

(5.2)

These terms may arise in a manner that respects the symmetry of the fermionic kinetic terms. Usual renormalization theory takes care of large corrections in the Z-factor or other parameters in the standard model. The $\Lambda \rightarrow \Lambda e^{2\pi i}$ symmetry is restrictive in the same sense as the $\Gamma_5$ chiral structure prohibits masses in chiral theories. Note also that log terms may exponentiate via,

$$x^{n/16} = e^{n/16 \ln x} = \sum_{j=1}^{\infty} \left( \frac{n}{16} \ln x \right)^j .$$

(5.3)

Such a term appears order by order to violate this symmetry, but the infinite summation is in accord. Perturbation theory must obey the symmetry, and to one loop order it does without summing.

Most of the corrections from gravity are minor, being suppressed by $\Lambda/m_{\text{pl}}$. However, because the neutrinos are so light, the question is raised if a gravity correction might modify its mass substantially. A one-loop contribution to the fermions masses, which is compatible with the fermion symmetry, is

$$\beta \frac{\Lambda^2}{m_{\text{pl}}} = .0001 \beta ,$$

(5.4)

and is a factor smaller than the lightest neutrino mass in this scenario. Of course there is a factor $\beta$ from the loop integral but to first approximation this result is smaller, or on the order of the lightest candidate neutrino mass.
The gravity sector is trivial because of the $\Lambda/m_{pl}$ factor; being an effective theory up to a TeV scale, in a string-inspired regulator through the M-theory formulation, the gravitational corrections are a power series,

$$A_n = \sum_{j=0}^{\infty} A_n^{(j)} \left( \frac{\Lambda}{m_{pl}} \right)^j$$

(5.5)

with $\Lambda/m_{pl} = 10^{-16}$, a very small parameter. Expansions in loops (or $\hbar/2\pi$) is the most traditional means to generating the perturbative expansion describing amplitudes in a quantum field or string theory. However, such expansions are typically problematic in that multi-loop diagrams are difficult to compute (in the exception of ladder diagrams or certain infinite subsets of diagrams). In [6] the initiation of derivative expansions in the context of gravity theories was initially explored; the derivative expansion for QCD, scalar, and $N = 4$ supersymmetric gauge theories was recently developed and the same techniques are useful in the perturbative gravitational context. Integrals may be eliminated and perturbative gravitational amplitudes may be constructed through a recursive and algebraic approach. We refer the reader to these papers.

6 Cosmology

The model here has a cosmological constant,

$$\int d^4x \sqrt{g} V(\Lambda, m_{pl}), \quad V(\Lambda, m_{pl}) = \Lambda^4 \sum_{n=2}^{\infty} \left( \frac{\Lambda}{m_{pl}} \right)^{2n}.$$  

(6.1)

The first term is, with $\Lambda = 1$ TeV, $10^{-15}$ erg/cm$^3$ and is in agreement with cosmological data. The curvature of space induced by this constant is

$$R_{dS}^2 = \frac{1}{\Lambda^2} \left( \frac{\Lambda}{m_{pl}} \right)^{-4}$$

(6.2)

and is roughly 15 billion light-years. This value of the cosmological constant, and its small corrections, are in accord with the large scale structure of the universe.

The de Sitter space solution may be regarded at late times from the big bang, in models of the early universe. We may replace the de Sitter space with the Robertson-Walker topology. Its metric is,
\[ ds^2 = -dt^2 + a^2(t)[\frac{1}{1 + kr^2} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)] . \] (6.3)

The warp factor is,

\[ H_0 = \frac{\dot{a} |_0}{a |_0} \quad q_0 = -H_0^{-2} \left( \frac{\dot{a}}{a} \right) \] (6.4)

in terms of present day Hubble and deceleration parameters. At late times, \( a^2(t) \to 1 \), and the spacetime is flat. A modification of the Robertson-Walker metric is

\[ ds^2 = -\lambda t dt^2 + a^2(t) \left( \lambda_i dx_i dx_i \right) , \] (6.5)

and contains four free-parameters, pertinent to breaking supersymmetry with the maximal number of free parameters. At late times, this metric may be replaced with the de Sitter space solution considered in this paper,

\[ \sum_{j=1}^{4} \lambda X_j^2 = R_{dS}^2 . \] (6.6)

Perturbations due to massive objects are expected to change this idealic geometry.

7 Discussion

The standard model content, masses and couplings, have been examined in detail in the context of M-theory compactified on a \( S^7 \). The masses arise from classical gravitational configurations, from M2 and M5 branes, and the couplings from the geometry of the \( S^7 \).

The masses of the standard model are quantized and fit experimental data. The quarks and leptons are have these masses in line with the high energy symmetries of M-theory and/or supergravity. The origin of mass is gravitational from this point of view, and they are derived in this work. The couplings suggest grand unification at \( 10^{24} \) eV, but this is not required.

Supersymmetry breaking in the model considered here occurs at 2 TeV with some flexibility. With this breaking the cosmological constant comes out to the experimental value also. The quantum corrections, including gravitational, are examined and do not substantially alter the predictions made at the classical level. In addition to the M-theory on \( S^7 \), there is a pure 4-D background solution leading to the standard model parameters.
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