Tunguska Dark Matter Ball

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September 29, 2018

Abstract

It is suggested that the Tunguska event in June 1908 was due to a cm-large ball of a condensate of bound states of 6 top and 6 anti-top quarks containing highly compressed ordinary matter. Such balls are supposed to make up the dark matter as we earlier proposed. The expected rate of impact of this kind of dark matter ball with the earth seems to crudely match a time scale of 200 years between the impacts. The main explosion of the Tunguska event is explained in our picture as material coming out from deep within the earth, where it has been heated and compressed by the ball penetrating to a depth of several thousand km. Thus the effect has some similarity with volcanic activity as suggested by Kundt. We discuss the possible identification of kimberlite pipes with earlier Tunguska-like events. A discussion of how the dark matter balls may have formed in the early universe is also given.

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1 Introduction

In 1908 a cosmic body is supposed to have fallen down in the Tunguska region of Siberia \cite{1}, where its explosion caused the trees to fall down in an area of 2000 km$^2$. The cosmic body is commonly assumed to have been a comet or possibly a meteorite. But no genuine piece of a comet or a meteorite has been definitely identified\footnote{However indirect evidence from a magnetic and seismic-reflection study \cite{2} of Lake Cheko located about 8 km NW of the inferred explosion epicentre, is compatible with the presence of a rocky object 10 m below the lake. It has also been reported \cite{3} that ground penetrating radar studies of the Suslov crater near the epicentre, by V. Alexeev and his team from the Troitsk Innovation and Nuclear Research Institute, indicate the presence of a huge piece of ice buried deep inside the crater.} in Tunguska. Many hypotheses about what really happened have been put forward, but the question is not totally settled.

In the present article we shall propose that what happened in Tunguska was that a $10^8$ kg piece of dark matter ran into the earth at Tunguska. Despite the weight being suggested to be $10^8$ kg in our proposal, the size of the corresponding dark matter ball is only expected to have been about a centimetre, $R \approx 0.67$ cm.

It is generally believed that dark matter requires some new physics, so that it could e.g. consist of the susy-partner of some known neutral particle. In such models the mass of the dark-matter-particles is of the order of 100 GeV typically. But really the mass of dark matter particles is extremely badly determined, because on the large distance scale the behaviour of the dark matter is independent of how small or how big are the dark matter particles into which it is divided. In fact Frampton et al. \cite{4} proposed that the dark-matter particles could be black holes. In the present article we propose the existence of dark-matter particles with masses of the order of $10^8$ kg. Contrary to the new physics explanations of dark matter, a most important feature of our model is that we get it formally with the Standard Model alone. Not even the forces responsible for non-zero neutrino masses, whatever they may be, play any significant role in our model. The only new physics introduced in our model is our requirement that the parameters of the Standard Model should be fine-tuned in a special way, but formally it is just the Standard Model.

We have in fact earlier proposed \cite{5} that, by imposing a certain principle of fine-tuning, there is the possibility for a model of dark matter consisting of pea-size or hand-size balls of a new type of vacuum in which certain bound states of 6 top and 6 anti-top quarks are Bose condensed. Inside their region of the new\footnote{Strictly speaking this phase is only the new one in the sense that we have known about the outside phase much longer. We should also note that a model for dark matter using an alternative phase of QCD has been proposed by Oaknin and Zhitnitsky \cite{6}.} vacuum these balls contain some highly compressed ordinary matter, which is kept compressed by the skin separating the two phases of the vacuum (the one with the condensate and the one without).

Our balls are supposed to come out of the early universe development before the temperature had fallen to about 2.3 MeV (as we discuss in section \ref{section8.5}) and would therefore not significantly effect standard Big Bang Nucleosynthesis. Their whole existence is strongly based on the hypothesis that there exists at least the two types of vacua mentioned above. In this scenario, at temperatures above the weak scale $\approx 100$ GeV, both phases of the vacuum would exist together randomly mixed over space. However, as the temperature lowers, the walls along the border of the one phase to the other will contract and presum-
ably at the end only one phase will survive, unless some mechanism can stabilise small balls of the surrendering phase. It is of crucial importance that we assume that in the phase with the bound state condensate the Higgs field is somewhat smaller than in the usual phase without the bound state condensate. This, in turn, makes the masses of the nucleons somewhat smaller in the phase with the bound state condensate, so that they get attracted to that phase. Then it becomes possible for a ball, having the vacuum with the bound state condensate inside it, to be filled with nucleons or just matter of the ordinary type and be kept from collapsing. In this way some bit of the otherwise surrendering phase, which we take to be the one with the bound state condensate, can survive in balls. It is these balls we want to take to be the dark matter. The occurrence of the above two phases of the vacuum, which are so crucial for our model, is supposed to be due to our postulate of the “Multiple Point Principle”, according to which the parameters of any given model should be fine-tuned to ensure the existence of two or more phases with the same energy density.

In we applied this multiple point principle (MPP) to the Standard Model, requiring the effective potential for the Higgs field to possess two degenerate minima taken to be at the weak and Planck scales respectively. This vacuum degeneracy requirement led to the postdiction of \( M_t = 173 \pm 4 \text{ GeV} \) for the top quark mass and the prediction of \( M_H = 135 \pm 9 \text{ GeV} \) for the Higgs mass. According to MPP, the values of the top quark and Higgs masses should lie on the Standard Model vacuum stability curve. There have recently been recalculation of this vacuum stability curve at next-to-next-to leading order. Using a top quark pole mass of \( M_t = 173 \text{ GeV} \) as input leads to an updated MPP prediction for the Higgs mass of \( M_H = 129 \text{ GeV} \), with a theoretical uncertainty of \( \pm 1 \text{ GeV} \). However this result is rather sensitive to the value of the top quark pole mass: a change of \( \Delta M_t = \pm 1 \text{ GeV} \) gives a change in the MPP predicted Higgs mass of \( \Delta M_H = 2 \Delta M_t = \pm 2 \text{ GeV} \). Furthermore there is considerable ambiguity in the extraction of the top quark pole mass from experimental measurements using top quark decay products and Monte Carlo simulations. Thus, within the uncertainty, the MPP prediction for the Higgs mass is in agreement with the 126 GeV mass of the particle recently discovered at the LHC.

As a further application of MPP, we have suggested that the parameters of the Standard Model, in particular the observed top quark Yukawa coupling constant, are also fine-tuned to give yet another degenerate phase of the Standard Model vacuum. This third vacuum has a weak scale Higgs VEV somewhat smaller than the usual 246 GeV and contains a Bose condensate of a strongly bound 6 top + 6 anti-top quark state. It is only the two low (weak) scale vacua (with and without the bound state condensate) that are relevant for the model of dark matter balls, which we use in this paper.

We expect there to be a wall or brane-like structure between these two phases - a solitonic wall - with a tension given by the weak interaction scale, which is a very high tension compared to the nuclear physics scale. Especially there must be such a brane-like structure or skin around the balls that make up the dark matter. Because of the high
tension in the skin the dark matter balls must have a relatively big size, in order that they shall not totally quench and allow the ordinary matter inside to spit out into the outside vacuum. Therefore balls below some size limit will contract and have their with condensate vacuum inside disappear, while balls bigger than this limit will remain as stable balls.

In our scenario a major part of the nuclear matter, with its accompanying electrons, is strongly compressed into our balls before the temperature fell to 2.3 MeV. These nucleons inside the balls would avoid effectively participating in the standard Big Bang Nucleosynthesis. Thus the standard fitting of the ratios of the abundances of the light (helium lithium, deuterium,...) elements being composed in this era would lead to a ratio of baryon number to photon number \( \eta \approx 10^{-9} \) corresponding to the number of baryons being outside the balls in this era. The point is, of course, that the baryons inside the balls are effectively being kept aside from the processes going on in the standard Big Bang Nucleosynthesis time.

It should also be borne in mind that, even though these balls will still interact somewhat with ordinary matter at the surface, their supposed masses of order \( 10^8 \text{ kg} \) for the rather small pea-sized balls are so large compared to their size that the gravitational interaction comes to dominate. In this way they function as dark matter.

It is of course important that our dark matter balls indeed have the right properties for dark matter:

1. Our dark matter balls must be stable on cosmological time scales. Since they actually consist of ordinary matter just strongly compressed, they are stable simply because of baryon number conservation. So stability is more natural in our model than in the usual WIMP models where a special conserved quantum number, such as R-parity conservation, has to be invented and added to the theory just to ensure the wanted stability. As mentioned above the dark matter balls have to be larger than the critical size of order 0.67 cm, in order to avoid collapse under the pressure from the skin around the ball. We should therefore estimate the lifetime of a typical dark matter ball due to the skin tunneling into the ball and increasing the pressure so much that the matter inside gets spit out. Let us consider, for example, a ball with a radius 1 mm larger than the critical radius. There will be a mass of order \( 10^7 \text{ kg} \) outside the critical radius and each nucleon will have an effective binding energy of order 1 MeV. So the energy violated by the skin tunneling to the critical radius will be of order the Einstein energy of \( 10^4 \text{ kg} \). The corresponding tunneling time is greater than or equal to the time taken for light to travel 1 mm. It is clear that the tunneling probability for such a process is completely negligible in the lifetime of the universe.

2. The interaction of a dark matter ball with normal matter will, in order of magnitude, be similar to that of an ordinary matter ball of the same size. But, compared to the mass of the dark matter ball, this is a very small interaction. An important feature, revealing the lack of interaction between dark matter balls, is the question of whether their interaction with say light can cool down a gas of dark matter so that it would contract and form stars much like atoms. Now the typical distance between the dark matter balls is of order an astronomical unit. So the dark matter balls only
very rarely come close to one another and, consequently, are not accelerated and thus do not radiate. Therefore a halo of dark matter balls will not flatten into a disk.

3. Due to their enormous mass the dark matter balls are extremely cold in the sense of being non-relativistic, even if having a high temperature. The dynamics of dark matter on a large scale is independent of the mass of the constituents, provided there are many constituents in the region under consideration. This means that, over distances large compared to an astronomical unit, our dark matter balls act just like the dark matter in the standard $\Lambda CDM$ cosmological model.

4. Although dark matter balls have a mass of order $10^8$ GeV, they are not heavy enough to be detected in microlensing searches. However such a ball would only hit the earth once every two hundred years or so, causing a Tunguska-like event, and will not be seen in the current dark matter direct detection experiments. So a positive signal for dark matter in these searches would falsify our model. Similarly dark matter balls would not combine with known elements to form anomalously heavy isotopes.

The main point of the present article is now to discuss how these balls might be observed via their relatively smaller non-gravitational interactions. In fact we suggest that one of these balls hit the earth in Tunguska in 1908 causing the famous Tunguska event [1].

In the following section 2 we shall estimate the mass of a dark matter particle (ball) needed, in order that a ball should hit the earth around once every hundred years or so, as one must imagine the rate of Tunguska-like events to be. For this estimation we shall assume that the density of our dark matter balls reproduces the phenomenological value of the dark matter density. In section 3 we shall discuss the minimal size of the ball of the vacuum with the condensate needed to prevent its collapse and the liberation of its contained baryons. We observe that this minimal weight or size, which probably gives a typical dark matter ball mass, matches well with the mass needed to reproduce the rate of Tunguska events.

In section 4 we estimate, although we can only do so rather crudely, the attractive potential pulling the nucleons into the phase with the condensate. Our estimate for the energy gained by a nucleon going into the with condensate phase comes out to be around 10 MeV.

In section 5 we then explain what in our picture should have happened in Tunguska.

In section 6 we discuss the possibility that the geological funnels known as kimberlite pipes, through which kimberlite magmas from deep within the earth erupt at the earth’s surface, were actually created by earlier Tunguska-like events [20, 21]. In section 7 we shall then show that indeed the fall of a ball like ours would penetrate very deep into the earth, making the event more reminiscent of a volcanic eruption than the impact of a comet or a meteor. In section 8 we discuss how our dark matter balls might really have been produced in the early universe before the temperature reached down to 2.3 MeV.

In section 9 we resume and conclude that our picture of dark matter provides a viable explanation of the Tunguska event.
Finally, in two appendices, we put forward a series of smaller detailed points about the properties of the bound state condensate and the cosmological production of our dark matter balls.

2  Ball mass from Tunguska event time statistics

Here we assume that the dark matter in the universe consists of balls of significant weight and that the Tunguska event was the result of a “fall down” of one such dark matter particle. We can then estimate a mass for these balls from the presumed time scale between Tunguska events.

From astronomical studies [22] the density of dark matter in the halo of our galaxy is

$$\rho_{\text{halo}} = 0.3 \text{ GeV/cm}^3 = 0.3 \times 10^6 m_N \text{ m}^{-3},$$

where \( m_N \) is the nucleon mass.

There are reasons though to believe that, at the solar system and in the disk of the galaxy [23], the density of dark matter will be a bit increased compared to the halo density by something like a factor of 2. So, in the following, we shall take the dark matter density in the solar system to be \( \rho_\odot = 2\rho_{\text{halo}} \).

We assume that the halo dark matter particles have a typical speed of 220 km/s and that the dark matter particles which are attached rather to the disk or the solar system have a typical speed of 60 km/s. So we take a typical speed of

$$v = \sqrt{\frac{1}{2}220^2 + \frac{1}{2}60^2} \text{ km/s} = 1.6 \times 10^5 \text{ m/s}$$

for the balls making up the dark matter. The two contributions are taken crudely to be equal, in correspondence with our estimate of the density near the earth being just double that of the halo proper. Now the cross-section of the earth is

$$A_\oplus = \pi R_{\text{earth}}^2 = \pi \times (6.4 \times 10^6)^2 \text{ m}^2 = 1.29 \times 10^{14} \text{ m}^2,$$

and so the volume of space that is tested per time unit for the presence of dark matter by collisions with the earth is thus

$$V_{\text{check}} = v \times A_\oplus = 2.1 \times 10^{19} \text{ m}^3/\text{s}.$$  

Therefore on average the mass of dark matter hitting the earth per unit time is

$$V_{\text{check}} \times 2\rho_{\text{halo}} = 2.2 \times 10^{-2} \text{ kg/s} = 6.9 \times 10^5 \text{ kg/y}.$$ 

Now the time which has elapsed since the Tunguska-event is close to 100 years. If we want to be a bit more precise and say that the fact that the Tunguska event occurred 100 years ago rather means that we are approximately in the middle between two successive events, then we should take the rate of fall of dark matter on the earth to be

$$r_B = \frac{1}{2 \times 100 \text{ y}} = 1.5 \times 10^{-8} \text{ s}^{-1}.$$  
The mass of the dark matter balls falling on the earth must thus be estimated to be

\[ m_B = r_B^{-1} \cdot V_{\text{check}} \cdot 2 \rho_{\text{halo}} \]
\[ = 1.4 \cdot 10^8 \text{ kg} = 1.4 \cdot 10^5 \text{ Tons}. \]  

(7)

This corresponds to the ball containing

\[ m_B/m_N = 1.4 \cdot 10^8/1.67 \cdot 10^{-27} = 0.84 \cdot 10^{35} \text{ nucleons}. \]  

(8)

The total kinetic energy of the ball on entrance into the earth’s atmosphere will thus be

\[ T_v = \frac{1}{2} m_B \cdot v^2 = \frac{1}{2} \cdot 1.4 \cdot 10^8 \text{ kg} \cdot (1.6 \cdot 10^5 \text{ m/s})^2 = 1.8 \cdot 10^{18} \text{ J}, \]

(10)

which is equivalent to 430 megaton of TNT.

The estimated energy output observed in the Tunguska event is of the order of 10 to 30 megaton of TNT, which is smaller by a factor of 20 than our above estimate \([10]\). As we shall see in more detail, dark matter in our model consists of cm-large balls of very high mass for their size, say of the order of our estimate \(m_B = 1.4 \cdot 10^8 \text{ kg} \) in equation \((8)\).

This means that the particle in our model penetrates through the atmosphere and deep inside the earth. Thus a major part of the kinetic energy of the in-falling dark matter ball will be dissipated underground, while only a minor fraction may come out of the earth similarly to volcanic activity. That the fraction of energy coming out of the earth should be about 1/20 of the total impact energy seems not at all unreasonable.

### 2.1 Space between dark matter balls

We here estimate the volume of space which on average contains just one dark matter ball, using our above estimate for its typical mass. In the solar system, where the dark matter density is \(\rho_\odot = 2 \rho_{\text{halo}} = 0.6 \text{ GeV/cm}^3\), the volume of space containing just one dark matter ball on average is given by \((4)\) and \((6)\) to be

\[ \text{“Volume per ball”}|_\odot = V_{\text{check}} \cdot r_B^{-1} = 1.3 \cdot 10^{29} \text{ m}^3. \]  

(11)

However the present average dark matter density over the whole universe is \([21]\)

\[ \rho_{\text{universe}} = 1.2 \cdot 10^{-6} \text{ GeV/cm}^3. \]  

(12)

Therefore the average volume containing one dark matter ball in the universe today is

\[ \text{“Volume per ball”}|_{\text{universe}} = V_{\text{check}} \cdot r_B^{-1} \cdot \rho_\odot/\rho_{\text{universe}} = 6.8 \cdot 10^{34} \text{ m}^3. \]  

(13)

The radius of a sphere with this volume has the length

\[ l = 2.5 \cdot 10^{11} \text{ m}, \]  

(14)

which gives the order of magnitude of the typical distance between the balls. This distance corresponds to the present temperature of the universe \(T_0 = 2.725 \text{ K} = 2.35 \cdot 10^{-4} \text{ eV}\). To the approximation that the linear size (=scale factor) of the universe would vary like the inverse temperature \(T^{-1}\) the corresponding distance at temperature \(T\) will be

\[ l_T = 2.5 \cdot 10^{11} \text{ m} \cdot (T_0/T) = 6 \text{ m} \cdot \left(\frac{10 \text{ MeV}}{T}\right). \]  

(15)

So, for example, at the 10 MeV temperature era the typical distance between the balls would be of the order of 6 m.
3 Our ball-parameters

It is the basic idea of the present work that the dark matter is made up of small pieces of another type of vacuum with a \(6t + 6\bar{t} \) bound state condensate into which the nuclei are pulled, because the Higgs field is lower in this other vacuum inside these pieces. Following the “Multiple Point Principle” [2], we shall assume that the vacua inside and outside the balls have the same energy density. There is, however, still an appreciable energy density on the surface between the two different phases. We denote the tension or energy density per unit area in this surface by \(S\). If we now have a spherical ball of radius \(R\) surrounded by such a surface, it will provide a pressure \(P\) on the interior of the ball (relative to the pressure in the outside, which in the absence of any matter is negligibly different from the inside pressure, due to the assumed equality of the energy densities)

\[
P = 2S/R. \tag{16}
\]

3.1 Introduction to pressure estimation

In order to prevent the surface of the ball from contracting and quenching the ball, it is necessary for the contents of the ball to provide a pressure which supports the ball. This pressure is dominantly provided by the degeneracy pressure of the essentially normal matter filling the interior and under enormous pressure from the surface.

The crucial parameter for estimating the pressure obtainable is the change in energy per nucleon by passage through the surface of the ball

“potential shift” = \(\Delta V\). \tag{17}

In section 4 we shall discuss the estimation of \(\Delta V\), but here we take as a reasonable value

\[
\Delta V = 10 \pm 7 \text{ MeV}. \tag{18}
\]

We imagine that the balls, which are supposed to be formed in the early universe, say around \(1/10\) of a second after the start, are just barely stable against collapsing and thereby spitting out their nucleons (and their electrons). At least the balls must be able to support themselves from collapsing and so the stability border against collapse must function as a lower limit for the size of the balls.

In section 8 and appendix B we shall discuss our scenario for the formation of the balls together with their nucleon content in the early universe. However let us here just suppose that the balls started out at the weak scale temperature \(T \approx 100 \text{ GeV}\) as very extended objects, containing the vacuum with a bound state condensate. We argue that they will tend to carry the neighbouring plasma along with them. Thus, in first approximation, one would expect them to follow the Hubble expansion. But, of course, the tension in the wall tends to make them diminish in size relative to this Hubble expansion. However, in appendix B.3 we put forward an effect which makes sufficiently big balls expand even more than the Hubble expansion. Furthermore we estimate, in appendix B.5, that the inertia of the balls is very important and that the balls can easily collide with each other, thereby causing a rather complicated motion (at least for temperatures less than 2 GeV).
At the end, the stability of the ball depends on whether or not it collects sufficiently many nucleons as it contracts to about the present day size. If there are too few nucleons inside, then at some point the radius of the ball becomes small enough that the pressure from the skin of the ball forces the nucleons out and the ball collapses completely. The number of nucleons inside the ball of course mainly depends on the original size of the ball, although it could collect some of the nucleons spit out by collapsing smaller balls. We expect the balls containing more nucleons than those of the critical size on the borderline of stability to survive\(^3\) as balls. In any case there must be some reasonably smooth distribution of ball sizes. There is a cut-off at small size given by borderline stability and at large size \(10\) given by the Hubble radius \(1/H\) at the time when the nucleons start to be collected inside corresponding to a temperature of \(T \sim 10\ MeV\). Suppose we make a power law ansatz for the smooth mass distribution for the number density of balls \(N_{\text{balls}}(M)\). Then, in order for the expression for the total amount of mass per unit volume to converge at the small and large size cut-offs, we require that

\[
dN_{\text{balls}}(M) \propto \frac{dM}{M^2}. \tag{19}\]

This distribution gives

\[
MdN_{\text{balls}} \propto \frac{dM}{M} \tag{20}\]

and the expression for the total amount of mass diverges only logarithmically at the upper and lower limits for the ball size. It follows from (20) that the typical mass for a ball will be close to the lower limit, which is just the mass on the stability borderline.

### 3.2 The degenerate electrons

For the stability in time of the ball, the pressure exerted by the material inside the ball must balance the pressure \((16)\) from the wall tension. Of course this pressure \(P\) must be in conformity with the material properties of the stuff inside the ball. In fact the dominant contribution to the pressure arises from highly degenerate electrons in matter much like white dwarf material. For white dwarf-like material the electrons form a Fermi-sea up to a Fermi momentum \(p_f\). Supposing that this matter is cold compared to the Fermi energy level - and that is certainly the case for our balls - the energy density of the electrons is given as

\[
n_e < E_e > = 2 \int_0^{p_f} \sqrt{p^2 + m_e^2} 4\pi p^2 \, dp/(2\pi)^3 \approx \frac{1}{4\pi^2} p_f^4, \tag{21}\]

and the density of electrons is

\[
n_e = 2 \int_0^{p_f} 4\pi p^2 \, dp/(2\pi)^3 \approx \frac{1}{3\pi^2} p_f^3. \tag{22}\]

Here \(< E_e >\) denotes the average energy of an electron in the Fermi gas and is

\[
< E_e > = 3p_f/4. \tag{23}\]

\(^3\)However we also speculate that explosive nuclear fusion in the highly compressed matter may lead to the expulsion of a fraction of the nucleons.
Now the pressure $P$ of a highly relativistic degenerate electron gas is given by one third of its energy density and hence

$$P = \frac{1}{3} \frac{\langle E_e \rangle n_e}{<E> / 3} \approx \frac{1}{12\pi^2} p_f^4 \quad (24)$$

Assuming that the nucleons occur roughly as an equal amount of neutrons and protons, there must be about twice as many nucleons as electrons,

$$n = 2n_e. \quad (25)$$

### 3.3 The Critical Ball Parameters

We now want to consider the ball size just to be on the borderline of stability against collapse because, as argued in subsection 3.1, we expect that the typical ball size produced in the early universe to be close to this border for collapsing. Consider indeed a ball just about to spit out its nucleons together with associated electrons. We may consider the energy required to release an electron and its associated proton-neutron pair as an effective potential term for the electron. For a ball on the stability borderline we require that such a formal electron, which is really an electron associated with its two nucleons, just has sufficient energy to escape and become “free” in the vacuum outside the ball. If we take the potential associated with the nucleons to be zero outside and $-2\Delta V$ inside the ball, the energy of the combined electron and proton-neutron pair outside is $m_e$. In the critical ball case, we must therefore have that the energy of the combination inside $-2\Delta V + E_e$ must be equal to $m_e$ for those electrons that can just go in and out of the ball. This means that the Fermi-surface energy minus the rest energy for the electrons must obey

$$E_{ef} - m_e = 2\Delta V. \quad (26)$$

Or crudely ignoring the rather small electron mass $m_e$ compared to $2\Delta V$ and noting that the electrons turn out here to be highly relativistic, we simply have

$$p_f = 2\Delta V. \quad (27)$$

Substituting the above value (27) for the Fermi-surface momentum into (22), we obtain

$$n_e = \frac{(2\Delta V)^3}{(3\pi^2)}, \quad (28)$$

which, from equation (24), gives

$$P = \frac{(2\Delta V)^4}{(12\pi^2)} = \frac{4}{3\pi^2} (\Delta V)^4. \quad (29)$$

Combining (29) with (16) we get

$$2S/R = \frac{4}{3\pi^2} (\Delta V)^4. \quad (30)$$
3.4 Connecting to the mass

The mass of an on borderline stable ball of ours is given as

\[ m_{B|\text{border}} = \frac{4\pi}{3} R^3 * m_N * n \]

(31)

\[ = \frac{4\pi}{3} \left( \frac{2 * 3\pi^2 S}{4(\Delta V)^4} \right)^3 * m_N \frac{2 * (2\Delta V)^3}{3\pi^2} \]

(32)

\[ = \left( \frac{S}{(\Delta V)^3} \right)^3 * m_N * \pi^5 * 3 * 2^3 \]

(33)

\[ \approx 7.3 * 10^3 \left( \frac{S}{(\Delta V)^3} \right)^3 * m_N. \]

(34)

From here we deduce

\[ m_B / m_N = 7.3 * 10^3 \left( \frac{S}{(\Delta V)^3} \right)^3. \]

(35)

Then, using equation (9) based on the expected rate of Tunguska events, we have

\[ \left( \frac{S}{(\Delta V)^3} \right)^3 = \frac{0.84 * 10^{35}}{7.3 * 10^3} = 1.14 * 10^{31} \]

(36)

leading to

\[ \frac{S^{1/3}}{\Delta V} = (1.14 * 10^{31})^{1/9} = 2.8 * 10^3. \]

(37)

Thus, with \( \Delta V = 10 \text{ MeV} \), we get the cubic root of the tension \( S \) to be

\[ S^{1/3} = 28 \text{ GeV}. \]

(38)

In appendix A we shall estimate the expected value of this tension in the border between the two vacua, as well as we can, from the properties of the bound state condensate. This estimate turns out to be \( \sim 16 \text{ GeV} \), which is in reasonable agreement with (38). The crux of the matter is that this quantity \( S^{1/3} \) is supposed to be related to the top quark mass, which determines the radius of the bound state [17], or be somehow given by the weak interaction scale, which would typically be counted as 100 GeV. In any case the order of magnitude \( S^{1/3} \sim 28 \text{ GeV} \), gotten from the rate of Tunguska events and the supposed Higgs field suppression in the vacuum inside the ball, agrees well with the estimates in appendix A.

Because of the ninth root under which the rate of Tunguska events occur in our calculation, a factor of two uncertainty in the cubic root of \( S \) would correspond to a factor of 512 in this rate. It would certainly be unreasonable to think that we got a Tunguska event a hundred years ago, if such events only occurred every 51200 years say. So the rate of Tunguska events is more accurately estimated for our purpose than the quark mass change effect \( \Delta V \) which is uncertain, according to our estimate in section 4 by about a factor of 2. Also of course the “weak scale” only makes sense to be used in dimensional arguments up to a factor of \( \sim 2 \) uncertainty.
3.5 Combined information on ball size

The agreement of our fitted value \(38\) of the tension with the weak scale provides phenomenological support for our picture and the hypothesis of the balls being on the border of collapsing. So this gives us confidence in using formulas like \((25, 28)\) with \(\Delta V = 10\) MeV to calculate the ball mass density:

\[
\rho_B = m_N * n = 2m_N * \frac{(2\Delta V)^3}{3\pi^2} = 5.08 * 10^5 \text{ MeV}^4 = 1.13 * 10^{14} \text{ kg/m}^3. \quad (39)
\]

With our mass estimate \((8)\) this gives the volume of the ball to be

\[
V_B = \frac{m_B}{\rho_B} = \frac{1.4 * 10^8 \text{ kg}}{1.13 * 10^{14} \text{ kg/m}^3} = 1.24 \text{ cm}^3. \quad (40)
\]

This ball volume corresponds to a ball radius of

\[
R = \left(3V_B/(4\pi)\right)^{1/3} = 0.67 \text{ cm}. \quad (41)
\]

4 Estimation of attractive potential of nucleons to the vacuum inside the balls

The philosophy of our model is that the region inside the balls has a different vacuum containing a \(6\bar{t} + 6\bar{t}\) bound state condensate. In this vacuum with a bound state condensate, the Higgs field is expected to be smaller than in the vacuum outside the balls by some factor of order unity, say half as large as in the usual vacuum without a bound state condensate. This means that the masses of hadrons, such as the nucleons, will be changed in going from one vacuum to the other one. Since the Higgs field expectation value is reduced inside the \(6\bar{t} + 6\bar{t}\) bound states \([17]\), we naturally must have the lower average Higgs field in the vacuum with a bound state condensate relative to the one without a bound state condensate. We refer the reader to appendix \(A.4\) for more details. The masses of the nucleons are crudely expected to have additive contributions given by the contained (valence) quarks. So we expect the mass contribution to the neutron and proton from the quarks to be

\[
m_n|\text{valence quarks}| = 2m_d + m_u = 17 \text{ MeV} \quad (42)
\]

\[
m_p|\text{valence quarks}| = m_d + 2m_u = 14 \text{ MeV} \quad (43)
\]

respectively. Here we used quark masses \([24]\) renormalised at 1 GeV

\[
m_d = 1.35 * 5.05 \text{ MeV} = 6.82 \text{ MeV}; \quad m_u = 1.35 * 2.49 \text{ MeV} = 3.36 \text{ MeV}. \quad (44)
\]

The average of these two contributions, i.e. the average valence quark contribution to the nucleon mass is about

\[
m_N|\text{valence quark} = 15 \text{ MeV}. \quad (45)
\]

This means that if, for example, we have a half as large Higgs field expectation value in the vacuum with a bound state condensate as in the vacuum without a bound state...
condensate (the one we live in), the nucleon energy inside the vacuum with a bound state condensate will be lowered by \( \frac{15}{2} \text{MeV} = 7.5 \text{MeV} \). That is to say there will be an effective negative potential for nucleons in the vacuum with a bound state of size 7.5 MeV.

Now, however, we expect that the masses of the pions, which are Nambu-Goldstone bosons, are very sensitive to the quark masses which, in turn, depend on the Higgs field VEV and vary appreciably from phase to phase (vacuum to vacuum). In the vacuum with a bound state condensate, the quark masses are smaller and so also the pion masses are smaller there. This in turn makes the binding of the nuclei stronger and thus the nuclei should be more strongly kept inside the vacuum with a bound state condensate than the nucleons when unbound. Very crudely we take it that the main change in the binding energy of nucleons into nuclei, due to the change in the Higgs field VEV and thereby the quark and pion masses, is given by the shift in the range of the pion exchange force (Yukawa potential). We obtain the relative change in this Yukawa potential by considering the Yukawa exponential factor \( \exp(-m_\pi r) \), where \( r \) is the distance between the nucleons binding to each other and \( m_\pi \) is the pion mass. The typical distance between neighbouring nucleons in a nucleus may be estimated from the semi-empirical picture of the nucleus, in which the radius \( R \) of the nucleus is related to the number density by the formula

\[
n = \frac{3A}{4\pi R^3} = 1.2 \times 10^{44} \text{ m}^{-3}.
\]

Here \( A \) is the number of nucleons in the nucleus and, thus, we get for \( A = 1 \) a radius for a single nucleon formally

\[
R|_{A=1} = (1.2 \times 10^{44} \text{ m}^{-3} \times 4\pi/3)^{-1/3} = 1.26 \times 10^{-15} \text{ m}.
\]

This means that the typical distance between neighbouring nucleons in a nucleus is about twice this quantity, i.e. \( \text{“distance”} \approx 2 \times 1.26 \text{ fm} = 2.5 \text{ fm} \). The pion Compton wavelength is \( m_\pi^{-1} = (140 \text{ MeV})^{-1} = 1.4 \text{ fm} \). We now assume that the quark masses inside the vacuum with a bound state condensate are lowered by a factor of 2, so that the pion mass \( m'_\pi \) in the phase with a condensate is reduced by a factor \( m_\pi/m'_\pi = \sqrt{2} \). It follows that the pion Compton wavelength increases from 1.4 fm to 2 fm. That is to say that the ratio of the pion mass dependent exponential factors, in the phase with a bound state condensate relative to the usual one without a bound state condensate, at this typical distance is

\[
\frac{\exp(-\text{“distance”} \times m'_\pi)}{\exp(-\text{“distance”} \times m_\pi)} = \frac{\exp(-2.5/2.0)}{\exp(-2.5/1.4)} = 1.7.
\]

Assuming that the shape of the potential is not changing too much and that the kinetic energy of the nucleons inside the nuclei is given by some sort of virial theorem (like the kinetic energy of the electron in the atom is given by a virial theorem to be just half the potential energy), we would expect the kinetic energy to be proportional to the potential energy. If the potential were pion exchange dominated, this would mean that the kinetic energy of the nucleons inside the nuclei should be increased by the same factor of 1.7. Let us assume, however, that approximately half of the binding energy is due to pion exchange. Then we obtain a correction to just half the binding energy by a factor of 1.7 in going into the phase with a condensate. Now the nuclear binding energy per nucleon
in the usual phase without a condensate is about 8 MeV. So half of this binding energy per nucleon, i.e. 4 MeV, is changed to $1.7 \times 4 \text{ MeV}$ in the phase with a condensate. In other words the binding energy per nucleon is larger by an amount $0.7 \times 4 \approx 3 \text{ MeV}$ in the phase with a condensate.

Due to quantum fluctuations, the effective typical distance for the pion exchange interaction could well be smaller, say by 25%, than 2.5 fm. This would lead to a reduction in the correction factor \((48)\) from 1.7 to 1.4, giving an extra binding energy per nucleon in the phase with a condensate of only 2 MeV.

Our estimate of 2 or 3 MeV for the extra binding energy per nucleon in the phase with a bound state condensate makes the effect of the nuclear binding energy smaller than the effect of the quark masses on the nucleon masses. So the large uncertainty in our estimate of the change in the nuclear binding effect is not so important for the estimate of the total attraction of “bound nucleons” into the vacuum with a bound state condensate.

In order to have a nice standard value for the attractive potential driving the nucleons, when bound inside nuclei, into the vacuum with a bound state condensate, we shall take $\Delta V = 10 \text{ MeV} \approx 7.5 \pm (2 \text{ or } 3) \text{ MeV}$ as a reasonable estimate. However we really only have a guess for the order of magnitude of the reduction of the Higgs field in the vacuum with a bound state condensate\(^4\). Thus we clearly cannot avoid an uncertainty, even for the changes in the nucleon masses, of the order of a factor of 2 or so. So let us, in conclusion, say that the binding energy of a nucleon is increased in the vacuum with a bound state condensate by somewhere between 4 MeV and 18 MeV, which we can write as $\Delta V = 10 \pm 7 \text{ MeV}$. A better estimation of this quantity would of course be highly desirable, since the mass of our dark matter ball \((35)\) is inversely proportional to the ninth power of $\Delta V$. Consequently an uncertainty by a factor of two in $\Delta V$ corresponds to an uncertainty by a factor of $2^9 \approx 500$ in the mass.

The above analysis of the change in the nuclear binding energy per nucleon due to a reduction in the quark mass by a factor of 2 is of course rather simplistic. A more detailed numerical analysis of the dependence of the binding energy of light nuclei on the quark mass has been made by Flambaum and Wiringa [25]. Extrapolation of their results to the case of a reduction in the quark mass by a factor of 2 leads to an increase in the binding energy per nucleon by a few MeV. However, although agreeing in order of magnitude with our crude estimate of the change in binding energy, one pion exchange is not the dominant term in their calculation. Rather they find that two pion exchange provides an appreciably bigger change in the binding due to the quark mass variation than does single pion exchange. In addition vector meson exchange and other contributions provide a change in the binding of the opposite sign and somewhat compensate the two pion exchange contribution. Combining all the contributions, Flambaum and Wiringa obtain values for the fractional change in binding energy $E$ relative to the fractional change in quark mass $K = \frac{\delta E/E}{\delta m_q/m_q}$ for nuclei having $A = 3$ to 8 in the range $K = -1.0$ to $-1.5$. These light nuclei have binding energies of order 7 MeV per nucleon and we choose to reduce the quark mass by a factor of two $\delta m_q/m_q = -1/2$. Thus from reference [25], we obtain an extra nuclear binding energy per nucleon in the vacuum with a bound state condensate

\[^4\text{In appendix A.4 we make a crude estimate of this reduction in the Higgs field using the properties of the bound state condensate and obtain the ratio of the Higgs field VEVs in the two vacua to be of order 0.3. This result is consistent with the value of 0.5 used here and throughout the paper.}\]
condensate of $\delta E \approx (-1 \text{ to } -1.5) \times (-0.5) \times 7 \text{ MeV} \approx 4 \text{ MeV}$. This result is accidentally not so far from our crude estimate of 2 or 3 MeV. In any case it is smaller than the change in the nucleon mass of the order of 7.5 MeV in the vacuum with a bound state condensate and does not significantly effect our estimate $\Delta V = 10 \pm 7 \text{ MeV}$ of the potential shift between the two vacua.

5 What happened in Tunguska?

The main idea of the present article is that what happened in Tunguska in July 1908 was that one of our dark matter balls hit the earth with a speed (2) of order 160 km/s. With its large mass $m_B = 1.4 \times 10^8 \text{ kg}$ (see (8)) and very small size - having a radius of about $R = 0.67 \text{ cm}$ (41) - the dark matter ball could not at all have been stopped near the surface of the earth like a meteorite, let alone in the air as it is speculated for a comet. Rather it would penetrate deeply into the earth and only get stopped after say a few thousand kilometres.

The kinetic energy of such a dark matter ball would be $T_v = \frac{1}{2} m_B v^2 \approx 1.8 \times 10^{18} \text{ J}$. It would deliver some small amount of this kinetic energy in the air presumably making its track visible, if somebody had looked; but by far the major part of this kinetic energy would be delivered deep inside the earth. Therefore the main effects of the fall of our ball should be caused by energy coming out of the earth rather than as if coming directly from an extraterrestrial object. Hence, to first approximation, our hypothesis would simulate a *volcanic explosion*, which Kundt [20] has suggested to be the cause of the Tunguska event rather than the fall of a meteorite or comet. To first approximation the ball will simply push and heat up enormously a tube through the earth not very large in radius at first compared to the cm-size of the ball. One must then imagine that at least part of the heat energy from this thin tube will push back heated earth material along the immediately formed tunnel. This tunnel would presumably lie in a skew direction, since it is very unlikely that the ball should have hit just head on into the earth. Thus a lot of heated “volcanic matter” would be sent out of the earth in a skew way. It is, however, quite likely that some of the heated material further down the originally formed tube would find a shorter way out and up to the surface of the earth than just back along the tube. The heated material could indeed go more vertically up towards the surface of the earth just above the place where it was formed from the passage of the ball. This could mean that several tunnels would be formed from the track of the ball deep inside the earth up to the surface, along which the earth material would be pushed up into the air above the surface of the earth. Indeed Kundt [20] has suggested that the Tunguska event corresponded to the formation of kimberlite pipes, which are tunnels of this type that become carrot shaped near the surface [26]. They are usually supposed to come from “volcanic explosions” hundreds of kilometres under the surface. However we suppose that pipes of this type were created at Tunguska by the hot tube formed by the passage of the ball. In section 6 we shall consider the possibility that all other kimberlite pipes were created by earlier Tunguska-like events.

The presumably gaseous material coming out of these tunnels would go up in the air, spread out and give rise to the observed explosion in the air above Tunguska. This would
create winds and shock waves, causing trees to fall and the branches to be ripped off some of the trees left standing to form the so-called telegraph poles. We expect only a fraction of the energy of the incoming ball to be available to participate in the observed explosion, since certainly a part of the energy must remain inside the earth. Especially if the ball penetrated so deep that its heated material cannot find a way out to the surface, an appreciable part of the energy would have to remain inside the earth.

We imagine, as our most likely picture, that the impact of our ball took place in Lake Cheko, and that some of the funnels observed by Kulik [27] are outburst places through which hot material found a shorter way out to the surface than simply going back along the first formed tube. The Suslov Crater, which is near the epicentre of the Tunguska explosion, we suppose is one of these places where outburst occurred from the earth via a shorter route.

5.1 How did the ball fall?

There are strong indications [2, 29] that Lake Cheko is the site where the extra-terrestrial object hit the solid surface of the earth, and we shall assume that to be the case. The epicentre of the explosion, as indicated by the tree falling pattern, is however 10 km to the south-east of Lake Cheko. So, if the Tunguska event were caused by a comet or a meteorite, the impact should have come from the south-east and have been directed towards north-west. We however, must have a different picture if we want to keep to Lake Cheko as the point of impact. Since our ball would cause the main explosion after already penetrating several kilometres inside the earth, the epicenter must lie later along the route of the ball in our model. That is to say that we need the ball to have come from the north-west and oriented towards the south-east (opposite to the direction needed for the comet or meteorite).

In this picture then the first spit out of hot material back through the tunnel, formed as the ball passed through, would go out into the air from Lake Cheko in the opposite direction to the motion of the ball itself. That is to say the spit out material would come out in the direction towards the north-west. It is quite likely that this material would have had very high velocity and that some of it might be close to or have gone into orbit around the earth. Thus it is predicted in our model that material from the interior of the earth was pushed out with a velocity like a satellite moving in direction towards the north-west. Such material would quickly reach Britain and Denmark without any mysterious need for two-dimensional turbulent flow in the upper atmosphere. Consequently, such a blow-out in the north-west direction has a chance to explain the very rapidly occurring noctilucent night clouds in Northern Europe after the Tunguska event.

We imagine that the major part of the kinetic energy of the ball was dissipated deep in the earth, say of the order of thousands of kilometres deep. The heated material from deep within the earth then began to move backwards along the tunnel made by the passage of the ball. This material would move slower than the original speed of the ball and presumably make more noise. A major blob of material moving in this way backwards along the tunnel would cause a loud sound that would have been conceived, by the witnesses to the Tunguska event, as an object moving in the opposite direction to the original motion of our dark matter ball. So the witnesses would hear the sound moving
from the south-east to the north-west. It is claimed that they described it as moving from east to north [28].

5.2 Most important puzzles

The most remarkable fact about the presumed in-fall in Tunguska is that, in spite of the large energy dissipation in the atmosphere equivalent to 10 to 30 MTONs of TNT, there were no crater and no left over material from the large in-falling object. In our model this remarkable lack of material from the in-falling body is explained by the body being so small compared to its mass that it could not be stopped and dug itself so deep into the earth - and perhaps even slung out from the other side of the earth - that nobody could find it yet. Rather than a crater we predict the existence of funnels created by material expelled from the earth itself and indeed that is closer to what Lake Cheko seems to be, an opening of a kimberlite-like funnel.

The trees fell oriented as from a centre and not as if the velocity of the in-falling object had given the wind a main direction. In our picture the explosion came from the interior of the earth similar to what happens in Kundt’s [20] volcano-like model for the Tunguska event. Actually we have to imagine that the seeming explosion in the air came from the earth by material, gaseous or solid whatever, coming out through the funnels that Kulik [27] saw in the region. At least a picture in which the explosion is replaced by a volcano-like emission from the earth, rather than a fast moving object creating the air motion directly, has a much better chance of delivering a pattern of trees falling to different sides than a fast passing object with its strong directional effect.

Also the “telegraph poles” - trees stripped of their bark and branches but still standing - would more easily appear in an explosion having its origin as coming out of the earth than in one produced directly by something having a high speed along the earth.

The problem with the small amounts of iridium measured [30, 31] in the layers of the peat formed in 1908 in Tunguska is a major reason for not believing that it was a meteorite which fell down in Tunguska. But even for the hypothesis that it was a comet, the iridium content seems low; at least it could not be a comet with the same dust to ice composition as the Halley’s comet, which has approximately 40% dust. Really to explain the carbon to iridium ratios a comet with an almost pure ice core with admixtures of hydrocarbons and other organic compounds is needed [32]. In our model the extra material from the catastrophe year comes from the earth underneath, perhaps from the mantle of the earth since our ball likely went extremely deep. But presumably the iridium of the earth has sunk even deeper than the mantle and so, contrary to dust in comets or meteorites which are rich in iridium, the mantle or material closer to the surface of the earth will have comparably little iridium content. In this way our model, with stuff expelled from the earth, would easily explain the major mystery of why so little iridium has been found in the Tunguska peat.

A depletion of radioactive carbon 14 and an excess of carbon 13 are also observed [30, 31] in the Tunguska peat samples of 1908. In our model the event is mainly dominated by the eruption of materials already in the earth and not by extra-terrestrial materials. Some outburst from the deeper layers of the earth would be depleted in carbon 14 which is formed in the atmosphere. This could well lead to the observed drop of one or two
percent in the $^{14}C$ content of the peat. The standard measure of the ratio of the stable isotopes $^{13}C$ and $^{12}C$ is $\delta^{13}C$, which is defined as the parts per mille difference between the $^{13}C$ content of the sample and that of the so-called PDB standard. The main organic component in the peat has $\delta^{13}C \approx -26$, while the peat found in the layers around the Tunguska event shows a shift \cite{30,31} of about +2 parts per mille in $\delta^{13}C$. Now carbon with $\delta^{13}C \approx -5$ is a major component of the earth’s mantle \cite{33}. So the admixture of mantle material in the peat could lead to an increase of 2 parts per mille in $\delta^{13}C$.

6 Kimberlites

In this section we investigate the hypothesis that kimberlite pipes \cite{26} are created by Tunguska-like events.

Kimberlite pipes are funnels of rocks seemingly coming from very deep in the earth and sometimes carrying diamonds. These pipes commonly occur in clusters of less than 50 pipes of variable extent. The pipes rarely occur at distances larger than 50 km from the cluster field. The age ranges from Archean to Tertiary times, and kimberlite emplacements of different ages can occur in the same location. Based on presently available information, the kimberlites are restricted to the Continental Intraplate settings.

The idea, which can be said to be due to Kundt \cite{20}, is that a Tunguska type event is connected with the formation of kimberlite pipes and material being expelled through them. So that the Tunguska event is indeed a volcanic event. We ourselves propose that a cluster of kimberlite pipes is formed as a result of the impact of a dark matter ball. This ball penetrates into the earth and creates extremely strongly heated material deep in the earth, which then finds some ways out to give the observed explosion by forming the various pipes in the cluster. It is comfortable for our theory that the order of magnitude of the distance from the cluster of pipes where no more pipes are found - the 50 km - is much smaller than the ball penetration depth estimated in section \cite{7}. This estimated depth $\approx 1700$ km is likely to be uncertain by about an order of magnitude; so the ball would be deposited deep inside the earth or possibly even pass straight through.

The restriction to the intraplate regions is a priori not so welcome for our theory, in as far as we expect to find equally many falls of dark matter balls on any unit area of the earth surface. The position of the fall must be determined by quite random features of the orbit of the ball, having of course nothing to do with the geological features of the region being hit. However, the probability for kimberlites produced long ago having been preserved in such a way as to be observed today is not guaranteed to be independent of the stability of the area onto which the fall took place. Naturally a very stable intraplate region may keep its kimberlites undisturbed much longer than a region with much sedimentation activity. So statistically we expect to find more well conserved kimberlites in such intraplate regions, as seems to be the case.

The fact that in the same region we can find clusters of different age is of course expected in our model. In our picture the different clusters should be of completely different age, since the falls are unrelated to earthly happenings. However, peaking of the time distribution for kimberlite formation in certain epochs is \textit{not} so welcome for our theory.
6.1 On the distribution of kimberlites

It is our hypothesis that the phenomenon known as kimberlites is due to our dark matter balls. That is to say we imagine that each time a ball falls on the earth the ball digs a long tube which successively gets filled with material that comes up from deeper layers, perhaps from the mantle. Kimberlites understood as such tubes have been found in various places on the earth. But they are very far from being smoothly distributed all over the earth surface, as one would naively expect of a phenomenon coming from random hits from the galaxy halo. But that expectation is, of course, only for the distribution that would be observed provided one were able to observe all the kimberlites, even if they had been heavily covered by sediments.

In this section we shall rather attempt to make a crude estimate of how many fewer kimberlites are expected to be practically accessible and thus may have been truly found. We shall also remark that while sediments will quickly make a kimberlite practically invisible, erosion will not prevent the visibility of a kimberlite. This is because the tubes are supposed to go so deep that erosion may in practice never go sufficiently deep so as to reach the bottom point of the kimberlite (where in our point of view one might find the dark matter ball situated). Rather erosion could have eroded away some sediments, so that places where deep erosion has taken place should be the most likely places to find the kimberlites. This is indeed exactly the experience: The kimberlites are mainly found in the Archean cratons, meaning in geological provinces where there has indeed been mainly erosion so that very old geological layers are on the surface.

6.2 Statistical estimate of numbers of kimberlites

With the earth having existed for of the order of 4 milliard years and one ball falling around every two hundred years, we would naively expect to find of the order of \(4 \times 10^9/200 = 2 \times 10^7\) kimberlite clusters. With a surface area of the earth of the order of \((10000\text{ km})^2 = 10^{14}\text{ m}^2\), this would mean a typical distance from one kimberlite cluster to the next of the order of \(\sqrt{10^{14}/(2 \times 10^7)} \approx 2 \times 10^3\text{ m}\). That is to say there should be only about a km from one cluster of kimberlites to the next. But one has not at all found so many kimberlites as this would mean! So either our hypothesis is completely wrong or these many a priori expected kimberlites are largely not observed at all.

Of course it is not unlikely that some sediments have fallen on top of a kimberlite site and covered it, so as to make it unobservable. We shall now make a very primitive statistical model to estimate the number of kimberlites which we should really see.

6.3 Introduction to statistical model

The statistical model which we propose describes the geological history for a single site, meaning here just whether there is sedimentation or erosion going on at the site in question. To take definite presumably reasonable numbers we assume that for periods of the length of an ice age, taken to be of the order of 10000 years, one has either constant sedimentation or constant erosion. However whether we have sedimentation or erosion varies randomly from one period to the next.
Now the assumption is that we - the geologists - only “see” (discover) those kimberlites which are at present at sites more deeply eroded than that site has ever been eroded before. That is to say we only “see” the kimberlites on sites where all sediments ever settled have been eroded away. Otherwise of course the kimberlite would be covered by sediments and we would most likely not discover it.

On a given site the surface of the earth has been going up and down - in our model as a random walk - due to sedimentation and erosion respectively. The probability distribution of the depth into which there is total erosion at a given place is of course a gaussian, corresponding to the random walk of erosion or sedimentation. We take 2 milliard years as the typical time that has elapsed since the fall of a dark matter ball responsible for a Tunguska-like event. Then the typical random walk has $(2 \text{ milliard years})/(10000 \text{ years}) = 2 \times 10^5$ steps.

### 6.4 Estimating probability for pipes being exposed

Let us think of the sedimentation and erosion process at a site as a random walk of, say, $n$ steps of the exposed surface up and down relative to the material in the column under the site. What we think of as a step is the activity occurring in our period of 10000 years during which we assumed, for simplicity, that erosion or sedimentation went on in only one direction. The condition that one can see the kimberlite pipes on a site is that they must be exposed, meaning that any sediments that have fallen on the site - since the fall of the dark matter ball - should have been eroded away. In terms of the random walk this means that at the end time step number $n_0$ after the fall, where the level is today, this $n_0$th step shall be to the erosion side of the levels of all the previous steps. In other words this last $n_0$th step shall be the hitherto deepest. If we denote by $r$ the number of levels that this endpoint is under the starting level when the ball fell, it means that the condition for the kimberlite pipe being exposed is that it is the first time the random walk reaches that depth, i.e. to $r$ or deeper. For a random walk we denote the number of (time) steps to the first hit of a given value $r$ as $T_r$. If $S_n$ denotes the level reached after $n$ time steps in a certain random walk then for this walk $T_r$ denotes the number of time steps to the first case in which $S_n = r$. In other words

$$T_r = \min\{n \geq 1 \text{ such that } S_n = r\}.$$  \hspace{1cm} (49)

In order that the kimberlite pipes at a site, in which there has been erosion (into the material below) corresponding to a net number of levels $r$ (i.e. the number of time steps with erosion minus the number of time steps with sedimentation), be exposed, it is needed that in the random erosion-sedimentation walk this level $r$ is hit for the first time - since the fall of the dark matter particle. That is to say it is needed that $T_r = n_0$. Of course we have then for the actual walk $S_{n_0} = r$. The conditional probability, given the site, for
the kimberlite pipes being visible is then

\[ P(T_r = n_0 | S_{n_0} = r) = \frac{P(T_r = n_0)}{P(S_{n_0} = r)}. \tag{50} \]

Now, from the hitting time theorem \[34\], we find the random walk probability formula for this case to be

\[ P(T_r = n_0) = \frac{r}{n_0} * P(S_{n_0} = r) \tag{51} \]

for \( r \geq 1 \). Thus

\[ P(T_r = n_0 | S_{n_0} = r) = \frac{r}{n_0}. \tag{52} \]

It is well-known that the distribution of \( S_{n_0} \) is approximately a Gaussian with a width growing as the square root of \( n_0 \). So it is very unlikely in our model to find a site where \( r \) is more than a few times \( \sqrt{n_0} \). This in turn means that we can count the order of magnitude of \( \frac{r}{n_0} \) as being typically \( \approx \sqrt{\frac{1}{n_0}} \). While it is clear that in a site with more steps of sedimentation than of erosion (i.e. \( r < 0 \)) you can find no kimberlite pipes, the probability for finding the possibility of kimberlite pipes being exposed at a typical site (i.e. \( r \approx \sqrt{n_0} \)) is

\[ P(\text{exposed pipe}) = P(T_r = r | S_{n_0} = r) = \frac{r}{n_0} \approx \frac{1}{\sqrt{n_0}} \approx \sqrt{\frac{10000 \text{ years}}{2 \times 10^9 \text{ years}}} \approx 2.2 \times 10^{-3}. \tag{53} \]

This means that on a large scale it is only about \( 2.2 \times 10^{-3} \) of the area in which the kimberlite pipes will be visible. So, even in say the Canadian Shield, the number of kimberlite pipes expected to be detectable is this number \( 2.2 \times 10^{-3} \) multiplied by the number of dark matter particles falling on that region. Now the area of this Canadian Shield is about \( 5.5 \times 10^6 \text{ km}^2 \), while the total earth surface area is \( 500 \times 10^6 \text{ km}^2 \). So the Canadian Shield makes up \( \approx 1\% \) of the earth’s surface and, with one dark matter ball falling every 200 years for 2 milliard years giving \( 10^7 \) balls on the whole earth, we expect \( 10^5 \) balls to fall on the Canadian Shield. So the number of clusters of kimberlite pipes expected to be exposed on the Canadian Shield becomes

\[ \# \text{exposed pipes} \approx 2.2 \times 10^{-3} \times 10^5 \approx 200. \tag{54} \]

This is to be compared with the at least 30 observed clusters \[35\]. However if you count all the pipes rather than the number of clusters, the number observed in Canada is more than 770. That is to say the estimate from our model predicts about 6 times as many kimberlite pipe clusters as found, if you imagine that each Tunguska-like event

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\[ \text{Equation} \tag{50} \text{is trivial in as far as we of course can write the total (unconditional) probability} \ P(T_r = n_0) \text{for the first hit of level} \ r \ \text{being at time step number} \ n_0 \ \text{as} \]

\[ P(T_r = n_0) = P(S_{n_0} = r)P(T_r = n_0 | S_{n_0} = r) + P(S_{n_0} \neq r)P(T_r = n_0 | S_{n_0} \neq r). \]

Now hitting \( r \) the first time at \( n_0 \) of course means that it also hits \( r \) at time step \( n_0 \), so that \( P(T_r = n_0 | S_{n_0} \neq r) = 0 \).

\[ \text{The Canadian Shield is a broad region of pre-Cambrian rock that encircles Hudson bay.} \]

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would produce a whole cluster. But if you imagine that each Tunguska-like event only produces one pipe, we rather predicted a factor 4 too few. So using the ambiguity in what to count in order to obtain the number of Tunguska-like events as an uncertainty, our prediction agrees with the observed number within errors. But now our estimate is indeed very uncertain in itself and we shall make an estimate of this uncertainty in the next subsection.

### 6.5 Uncertainty estimate for number of kimberlite pipes/clusters

The most uncertain input to our estimate of the number of exposed pipes is presumably the time scale for the typical alternation of sedimentation versus erosion, which we took to be $\Delta t = 10000$ years, just referring to an ice-age. This 10000 years number could easily within uncertainty be a factor up to a 100 times bigger or smaller. So we take the logarithmic uncertainty in the typical time scale to be

$$\Delta \ln(\Delta t) \approx \ln(100) \approx 4.6. \quad (55)$$

Let us now similarly estimate the uncertainties of the other input quantities:

The time since the fall of a dark matter ball should at least be shorter than the age of the earth, which is supposedly 4.6 milliard years. We took 2 milliard years as the central value in our estimate above. Now on the other hand the times of the creation of the kimberlites in South Africa seem to be dominantly less than about 200 million years ago, although a few are indeed older. Thus effectively the upper limit for the age of the detected kimberlite pipes lies in the interval from $2 \times 10^8$ years to $4 \times 10^9$ years. We describe this as $2 \times 10^9$ years with a logarithmic uncertainty

$$\Delta \ln(t_0) \approx \ln(\sqrt{20}) \approx 1.5. \quad (56)$$

The rate $r_B$ of Tunguska-like particle falls is of course very uncertain, in as far as we have so far only seen one Tunguska event. We should therefore at least let our estimate of 200 years for the time between Tunguska-like events be uncertain with a factor $e = 2.718$. So we take

$$\Delta \ln(r_B) \approx 1. \quad (57)$$

The uncertainty in the area of the Canadian Shield may not be great, but why should some precise definition of the Canadian Shield be exactly where the kimberlite pipes remain visible? So we may take this area also to have an uncertainty of say a factor of about 2 to be realistic

$$\Delta \ln(\text{area of Canadian Shield}) \approx 0.5. \quad (58)$$

Then we shall remember that, in the calculation we did above, the end result for the number of exposed kimberlite pipes expected in the Canadian Shield depended on $t_0$ and $\Delta t$ only through their square roots, while $r_B$ and the area of the Canadian Shield

\footnote{We include the uncertainty in what should be counted as the earth surface area (i.e. whether only the land or also the sea area should be counted) in this value for $\Delta r_B$.}
came in as simple factors. Thus the logarithmic uncertainty in the final number of 200 exposed pipes obtained in the last section becomes

\[
\Delta \ln(\# \text{ exposed pipes}) \approx \sqrt{(4.6/2)^2 + (1.5/2)^2 + 1^2 + 0.5^2} = 2.7. \tag{59}
\]

So the result of our prediction for the number of kimberlite pipes expected to be found exposed on the Canadian Shield is

\[
\# \text{ exposed pipes} = 200 \times \exp(\pm 2.7) = 14 \text{ to } 3000. \tag{60}
\]

This result should be compared with the 30 or so observed clusters\(^8\) and 800 or so individual pipes. These two numbers, 30 and 800, \textit{both} lie inside the uncertainty range \((60)\) of our theoretical prediction.

So our hypothesis that Tunguska-like events are the reason for the formation of kimberlite pipes is at least not numerically excluded order of magnitudewise with respect to the number of pipes. However a major problem for our model is that kimberlites are distributed in a quite structured way in time \([36, 37]\) with the weight on the rather young kimberlite pipes being detected. In our model of random walk sedimentation versus erosion clumping of time intervals in which kimberlite pipes survive to be exposed today is also possible, but really the distribution found is for us not the obvious one expected.

7 Penetration depth

We shall here first estimate how deep our ball will penetrate into the earth using essentially dimensional arguments. Then we shall attempt to gradually improve this estimate.

A) The first very crude argument is the following:

The material in front of the ball as it passes through the earth has somehow or another to be pushed to the side, so that the ball can pass by without having the earth material penetrate into the ball (we assume that the ball is so dense and so solid that no earth material can penetrate into it). In the crude approximation of treating only the main part of the earth in front of the ball, the velocity needed for it to escape going into the ball is of order of the ball velocity. The slope of the side of the ball relative to the direction of motion of the ball corresponds to an angle of order unity, and thus the escape velocity for the material has to be of the same order as the ball velocity. Let us at first assume that the amount of material speeded up to this velocity is only a factor of order unity bigger than the absolutely necessary amount lying in the way of the ball. Then, order of magnitudewise, the kinetic energy that has to be provided to push the material away is of order of the the energy needed to give the column of material in the way of the ball the speed of the ball. Now suppose all the kinetic energy of the ball is transferred to the pushed away material in this way. This would mean that the mass of the column of material in the way of the ball should be, within our accuracy, of order unity times the mass of the ball. Taking the specific weight of the earth material in the region penetrated

\(^8\)In some cases these so-called clusters contain pipes with different ages and thus should certainly be counted as more than one event in our picture; so perhaps we should increase this number to 40 or more "independent" clusters.
by the ball to be \( \rho_{\text{earth}} = 6000 \text{ kg/m}^3 \), the penetration depth \( l \) is given in the first crude approximation by

\[
l_0 = \frac{m_B}{\pi R^2 \rho_{\text{earth}}} = \frac{1.4 \times 10^8 \text{ kg}}{\pi \times (0.67 \text{ cm})^2 \times 6000 \text{ kg/m}^3} = 1.7 \times 10^5 \text{ km}.
\]

(61)

We believe this first estimate \( l_0 \) appreciably overestimates the true penetration depth \( l \), as we shall argue below where we present a series of correction factors.

B) The material that has to go around the ball and not be carried along with it must use at least some region of the cross section outside the ball itself. Thus a larger column of material has to be speeded up to the speed of the order of that of the ball. If, for instance, this column corresponded to a doubling of the diameter of the disturbed region, then the area \( \pi R^2 \) in our calculation would have to be increased by about a factor 4. Subsequently the penetration depth \( l_0 \) would be decreased by a factor 4.

C) Also the material cannot truly escape just with exactly the same speed as that of the ball itself, because the closer one gets to the very front of the ball the faster must the material move in order to escape. However, we may imagine that, under the motion, the ball could have a cone of very strongly compressed material in front of it, which could push the material being met away as the ball progresses. Thereby the angle of the effective surface of the ball with the direction of motion could be prevented from being just 90° as it a priori is just in front of the ball. Such a 90° angle between the direction of motion and the direction along the effective ball surface would mean that the velocity needed to escape at such a place would be infinite. This fact indeed suggests that there should be some peaked material in front of the ball, so as to avoid such an infinite speed.

Let \( \theta \) denote the spherical polar angle for the points on the surface of the ball with \( \theta = 0 \) at the front of the ball in its motion with velocity \( v \). Then the escape speed needed by the earth material at the surface of the ball in the direction transverse to the direction of motion of the ball is \( v_{tr} = v \cot \theta \). Then we can formally evaluate the average of this transverse speed squared over the surface of the ball (which we need to estimate the kinetic energy of the earth material) as the integral form

\[
<v_{tr}^2> = v^2 \int_0^{\pi/2} \cot^2 \theta \sin \theta \, d\theta = v^2 \int_0^{\pi/2} \frac{\cos^2 \theta}{\sin \theta} \, d\theta.
\]

(62)

This integral is divergent at the tip of the ball where \( \theta = 0 \). It diverges logarithmically. The typical (non-divergent) part of the integral is of order unity. Presumably the logarithmic divergence will be cut off due to a nose of compressed material sitting at the front of the ball, giving the ball an effective shape with a peak at the front. This will modify the integral (62) near the front point of the ball and so remove the logarithmic divergence. We so to speak obtain the logarithm of the nose-angle.

We guess that this logarithm may be of the order of 3 or 4, giving \( <v_{tr}^2> \sim 4v^2 \). Hence our estimate of the kinetic energy deposited in the earth per unit length is increased by a factor of, say, 4. This corresponds to reducing the penetration depth by another factor of 4.

D) We may think of the motion of the material pushed out from the pathway of the ball either as a macroscopic motion of material flowing as a whole, or as ions/particles
moving essentially in a thermal motion ensuring the escape of the particles in front of the ball. While the material in front of the ball gets heated up in this way to a sufficiently high temperature so that it has the velocity needed to escape, the ball itself must actually still keep to a much lower temperature. So during the main part of the ball's motion we must consider the ball itself to be cold compared to the temperature of the material in front of it. Thus a fraction of the energy of the material pushed away from the ball will be lost by heat conduction into the ball, heating the latter up. Let us say that $K$ times the energy deposited in this material as heat due to its interaction with the ball is conducted into the ball. Then it means that the amount of energy per unit time needed to get the material pushed out with a prescribed speed - to avoid being in place where the ball comes by - gets increased by a factor $1 + K$, simply because for each joule deposited $K$ joules stream into the ball heating it up. But this extra energy proportional to $K$ has to come from the work done by the ball as it presses its way into the earth. Thus the estimated braking force acting on the ball must be made stronger by the factor $1 + K$. This means that the estimated stopping distance of the ball must be shorter and that the penetration depth should be reduced by a factor of $1 + K$.

So it becomes important to estimate if possibly $K$ could be appreciably larger than 1. When say a nucleus hits the ball, it may effectively hit a few electrons or nuclei in the ball and, after such a collision, it would still have an energy that well may have decreased but not by a dramatically big factor. Now, however, the same say Si-nucleus in front of the ball could hit the ball many times and thus could lose a much bigger fraction of its energy than by just hitting once. However, in this case, the nucleus would have hit another particle somewhat further away from the ball, which may consequently not hit the ball. So, on the average, we expect the particles in front of the ball only hit the ball of order once before they escape from the path of the ball. Thus we conclude that $K$ should be of the order of unity, although it could be somewhat bigger than 1. For definiteness we shall take $K = 2$, leading to a suppression of the penetration depth by a factor of $1 + K$.

E) So far we have only really counted the kinetic energy and have ignored the fact that some energy will be deposited in the earth as potential energy. As a rough approximation we can take the potential energy deposited to be the same as the kinetic energy deposited in the earth. This has the effect of reducing the penetration depth by a further factor of 2.

As a conclusion of this section we may claim:

Most crudely we got that the mass of the material penetrated in a column with the cross section of the ball should be just the mass of the ball. This gave the first estimate of the penetration depth to be $L_0 = 1.7 \times 10^5$ km. However there were several ways of making a somewhat more realistic estimate. All the proposed corrections were in the direction of increasing the energy needed to push material aside by the ball. Thus a bigger energy loss per unit length by the ball is needed. In fact the above four corrections B)-E) lead to an increase in the energy loss and hence a decrease in the penetration depth $l$ by a factor of order $4 \times 4 \times 3 \times 2 \sim 100$. Thus we finally end up with a penetration depth of

$$l = l_0/100 = 1700 \text{ km.} \quad (63)$$

as our best estimate. However it is of course extremely crude and could be wrong by an
order of magnitude or more.

A priori this penetration depth means that the ball would penetrate deeply into the earth and remain buried there. However, within the accuracy of our estimation, it could nonetheless be the case that the ball would pass straight through the earth. In any case the total ball energy or a large fraction of it would be deposited in the earth. Then this energy would have either to stay in the earth or come out by some tubes extending from its route through the earth or simply backward along the way the ball came in.

8 How were the balls formed in the early universe?

If, as we assume from the Multiple Point Principle, the vacuum with and the vacuum without the bound state condensate have the same energy density, there will at temperatures higher than the weak scale be approximately equal amounts of the two types of vacua. In fact a high density of the walls separating the two vacua will be present due to thermal fluctuations around in space. The amounts of space covered by each of the two vacua will be about the same.

As the temperature $T$ falls with cosmological time $t$ according to the order of magnitude formula
\[ t \times T^2 \approx 1 \text{ s MeV}^2 \approx M_{Pl}, \tag{64} \]
the walls tend to contract and there are less extensive walls. Consequently when the temperature falls below the weak scale (say 100 GeV) the local extensions of the phases/vacua become larger. The balance between the two phases/vacua becomes unstable, in the sense that due to some possibly small effect one of the phases has become a bit less copious than the other one and will tend to contract further. In the appendix B.1 we argue that it is the phase with a condensate which contracts.

At first one would estimate that the walls would quickly reach velocities of the order of the speed of light for dimensional reasons. However, at several temperatures the walls tend to brake the passage of one species of particle or the other. Consequently the plasma of the mixture of the various particles tends to be driven along with the walls.

If indeed the coupling of the walls to the plasma is sufficiently strong, one could have imagined that the temperature and pressure inside a vacuum region about to contract will rise and soon slow down the contraction (relative to the general Hubble expansion\(^1\)) until heat can be conducted out and thus again allow the contraction. However, it turns out that neutrinos can transfer heat so efficiently that the balls cannot attain sufficiently high temperatures inside to prevent collapse due to such an effect.

There is, however, an effect causing a higher inner pressure in the with condensate balls, so as to drive them towards growing rather than contracting. At different temperatures this effect originates from those Standard Model particles which have their mass scale in the range around the prevailing temperature. Basically the effect is that, due to the lower Higgs expectation value in the with condensate phase, the masses of the

\(^9\)Here we use a reduced Planck mass, which we take to be given by $M_{Pl} = 1 \text{ s MeV}^2$ for the purposes of this section.

\(^10\)In this high temperature era even the balls which are about to contract will in reality typically expand due to the Hubble expansion.
particles obtaining their mass from the Higgs field become smaller in the with condensate phase than in the without condensate phase. This in turn means that, at the various temperatures, one or the other of the particles has a higher particle density in the Planck plasma in the with condensate phase than in the without condensate phase. This pressure then acts to expand the balls consisting of the with condensate phase. This effect means that sufficiently big balls would expand further and not contract at all. We refer the reader to appendix B.3 for more details.

8.1 Discussion of survival of the balls

The crucial point for our model to work producing dark matter in the early universe is that the balls of the required size should survive - i.e. avoid collapsing - long enough that they can be stabilised by being filled with nucleons, and thus survive forever. Since we estimated in section 4 that the binding energy of a nucleon into the phase with the condensate is of the order of 10 MeV, the concentration of the nucleons into the phase with the condensate will only be truly active when the temperature has fallen to \( T = 10 \text{ MeV} \).

In appendix B.3 we estimate that the smaller masses of several Standard Model particles inside the balls, where the Higgs field VEV and thus these masses are smaller, cause an increased pressure from these particles tending to pump up the balls. This effect is estimated in (127) to be so strong in the \( T = 10 \text{ MeV} \) era as to make balls with radius \( R \) greater than \( R_{\text{crit}} \approx 2 \text{ mm} \) grow rather than contract. These balls can avoid collapse until the temperature is sufficiently low that nucleons start to collect inside them.

8.2 The phenomenologically required balls

In subsection 3.5 we have estimated that the balls we want to form should at the end get a size of around 0.67 cm. These balls should have quickly collected nucleons once the temperature fell sufficiently low for them being able to catch them - namely below about 10 MeV. In fact we estimate, in appendix B.5, that the balls in this 10 MeV era run around in a rather complicated motion and even hit each other, thereby sweeping up essentially all of the nucleons. Remember that we imagine that at the weak scale of temperature the two vacua filled comparable amounts of the total volume, so that the distance between the “balls” at that time was of the order of their typical size. This size should have then crudely followed the Hubble expansion. So the size, before the partial collapse to their final stable size, should be of the order of the distance between the balls. We earlier estimated, see equation (15), that the distance \( l_T \) between the phenomenologically wanted balls at the 10 MeV era was of the order of 6 m. So, for our model to work, we need that this wanted size of 6 m has to be bigger than the critical size below which the balls would have collapsed before they get, so to speak, rescued from total collapse by the nucleons piling up inside them. In the foregoing subsection and appendix B.3 we saw that the critical size for survival to the 10 MeV era was 2 mm. Since 6 m is appreciably bigger than 2 mm it looks, even though we have made only crude estimates, very likely that the balls, which we postulate constitute dark matter, could have survived until the era when nucleons can fill them up and prevent their total collapse.
8.3 Capturing the nucleons

Once the temperature falls down to around $T \approx \Delta V \approx 10 \text{ MeV}$, the nucleons start to collect into the phase \textit{with a bound state condensate}. This phase is then favoured by the Boltzmann distribution for the nucleons, because the Higgs field and thereby the quark masses are smaller there. However, in appendix [B.4], we have estimated the rate of diffusion of nucleons in the plasma to be too slow for them to spread throughout the volumes of \textit{with condensate} phase in this era. Rather it appears that, if the walls between the phases do not move around significantly, there will only be a pile-up of nucleons in a thin layer on the \textit{with condensate} side of the walls. This pile-up would be formed by pulling over nucleons from a thin layer on the \textit{without condensate} side of the wall. In this case many of the nucleons would remain outside the balls in the phase without a condensate.

However, in appendix [B.5] we consider the motion of the walls and find that they move rather freely at least when the temperature has fallen below 20 MeV. Equation (147) shows that, for wavelengths characteristic of typical balls, the time for the waves to die out - the survival time - is larger than the Hubble time for temperatures below 20 MeV.

For example consider the situation for a typical ball which ends up as dark matter; at a temperature of 10 MeV it has a radius of order 6 m. So we now use equation (145) to evaluate the survival time $\tau_i$ for the following parameters: “wavelength” = 6 m, $T = 10 \text{ MeV}$, $S^{1/3} = 28 \text{ GeV}$.

$$\tau_i \approx g^4 \left( \frac{28 \text{ GeV}}{10 \text{ MeV}} \right)^3 \times 6 \text{ m} \approx \begin{cases} 400 \text{ s} & \text{for} \quad g^2 = 1 \\ 0.04 \text{ s} & \text{for} \quad g^2 = 1/100. \end{cases}$$

This means that the waves of this relevant length first decay after a time interval lying in the range between 0.04 s and 400 s. However that is longer than the Hubble time at the 10 MeV era, which is only 0.01 s. This means that, in the 0.01 s era, the walls run with almost negligible friction through the plasma. They are essentially freely moving walls and their inertia becomes very important.

The walls will thus clash together and at least sometimes get thrown away from each other again so as to run on. During such processes new regions of the with condensate phase can get created. Nucleons will be caught in such regions formed by the running walls, provided the temperature has fallen to less than or about the $\Delta V = 10 \text{ MeV}$ value. Because almost all space will soon have been passed through by these running walls, it becomes likely that all the nucleons in the Universe quickly get collected into the with condensate phase once the temperature fell to the order of $\Delta V = 10 \text{ MeV}$. Thus we expect that before the contraction of the balls, at a temperature of the order of 2.3 MeV (see equation (68) in section 8.5 below), all the nucleons will already have gotten collected into the with condensate phase.

8.4 Ordinary versus Dark Matter

Depending on the details of the development of the dark matter balls, they will collect a bigger or smaller fraction of the excess nucleons in the plasma. If there happens to be an
era after the temperature has gone below $\Delta V \approx 10$ MeV in which there is a relatively free passage of nucleons in and out of the balls, then the Boltzmann distribution of the nucleons will mean that almost all the nucleons get into the dark matter balls. Really we argued above in subsection 8.3 that this is indeed the case.

So practically all the nucleons go into the balls which eventually come to make up the dark matter. If the transport of nucleons across the skin of the balls stopped at this time, there would be essentially no ordinary matter. However it is quite possible that the dark matter balls can expel some of their nucleons and thereby supply some ordinary matter. Indeed we have earlier published the idea that nucleosynthesis is likely to have occurred inside the balls during the late stages of their contraction. Emission of nucleons could have been the main way of getting rid of the excess energy, released due to the increased binding energy of the nuclei, in some of the steps in this process. In fact we suggested that this cooling by nucleon emission dominantly occurred during the transition from helium to all the heavier nuclei. It should then be understood that the formation of helium occurred at a lower nucleon density with the heat dissipated rather by the emission of gamma rays and/or electrons and positrons. We briefly discuss these nuclear physics processes in appendix B.7.

Actually the main point of our previous work was to call attention to the following numerical coincidence:

The increase in binding energy in going from helium to essentially all the heavier elements is equal to 1.4 MeV per nucleon. Now suppose that the ball would irradiate this amount of gained energy, by sending out nucleons with a small kinetic energy. Then, for each nucleon emitted, the system would have to provide its binding energy of 7.1 MeV in helium. The binding energy in heavier nuclei is 8.5 MeV per nucleon. So for each nucleon emitted from a ball on the borderline of stability, there should be $8.5/1.4 \approx 6$ nucleons becoming bound into heavy nuclei. It follows that if, under the high nucleon density inside the ball, all the nucleons remaining inside formed heavy nuclei then a fraction of about 1/6 should be emitted. It is these emitted nucleons that should be identified with the ordinary matter and those remaining inside the balls identified as dark matter. The ratio of ordinary matter to dark matter is then predicted to be about 1/5, in good agreement with astronomical data. In appendix B.5 we repeat the calculation for median size balls and obtain a prediction for this ratio of ordinary to dark matter of 1/5.6, which is to be compared with the recent Planck measurement of 1/5.44.

It is very important that such a process of creating ordinary matter, or rather separating it from the dark matter, takes place before the Big Bang Nucleosynthesis gets under way. Otherwise it could disturb the usually so successful understanding and fitting of the abundances of the light elements by Big Bang Nucleosynthesis. Especially the nucleons emitted by our balls would tend to change the neutron-proton ratio, unless the weak interactions were still active in maintaining the Boltzmann equilibrium ratio. So the separation of ordinary matter from dark matter in our model must be essentially completed before the neutrino decoupling temperature of 0.8 MeV is reached; otherwise there is a severe danger that our model would change the predictions from Big Bang

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11 Here we use the nuclear binding energies as measured in the normal phase, without a bound state condensate, and assume that the ratio of the binding energies per nucleon between two different nuclei is the same in both phases of the vacuum.
Nucleosynthesis.

8.5 Can the balls form before the Big Bang Nucleosynthesis?

When do now these balls, which made up the ones we “see” as dark matter, contract around their nucleons? The size of a ball still following the Hubble expansion will be inversely proportional to the temperature in the radiation dominated era. So the phenomenologically wanted balls would have the size

$$l_T = \frac{6 \text{ m} \times 10 \text{ MeV}}{T}.$$  \hspace{1cm} (66)

However, as explained in eq. (177) of appendix B.6 this order of magnitude formula should be corrected as follows with a few factors of order unity

$$\left(\frac{1}{2}\right)^{1/3} \times 6 \text{ m} \times \frac{10 \text{ MeV}}{T} \approx \frac{3 \times (28 \text{ GeV})^3}{n_{\text{eff}} \sigma}.$$  \hspace{1cm} (67)

Here $n_{\text{eff}}$ is the effective number of particle species times their number of spin-components weighted with their efficiency of interaction with the phase border and $\sigma = \pi^2/60$ is the Stefan-Boltzmann constant; the factor $(1/2)^{1/3}$ corrects for the fact that when the balls are big - i.e. before their collapse - they only fill half the spatial volume. This gives the result that the temperature, in the era when the typical ball considered contracts, is

$$T_{\text{contraction}} \approx \frac{2^{1/9} \times 3^{1/3} \times 28 \text{ GeV}}{(n_{\text{eff}} \sigma)^{1/3} \times (60 \text{ MeV m})^{1/3}} \approx 0.65 \text{ MeV} \approx 2.3 \text{ MeV}.$$  \hspace{1cm} (68)

Here we used the value for $n_{\text{eff}}$ as estimated in eqs. (171) and (174) of appendix B.6

$$n_{\text{eff}} \approx 4 \times 0.159 \times \frac{7 \Delta m''}{8 T} \text{ for } T \text{ around a few MeV},$$  \hspace{1cm} (69)

where only the electron and positron are important. In this formula we use the notation $(\Delta m'')^2 = m_{\text{without condensate}}^2 - m_{\text{with condensate}}^2$ for the shift in mass squared between the two phases of a particle species relevant for calculating the pressure at the temperature $T$; the factor of 4 denotes the number of particle plus antiparticle components for this species (the electron in this case). In practice we take $\Delta m'' \approx \frac{1}{4}$. This temperature (68) is to be compared to the temperature at which the weak transformations between protons and neutrons stop $T_f = 0.8 \text{ MeV}$ (the freeze-out temperature). Now there is the little caveat that the balls, which form, have of course some mass or size distribution as discussed in appendix B.2. However there is an effective cut-off size given by the Hubble distance $100 \text{ GeV}$ is $1/H \approx 1 \text{ cm}$. Such maximal size balls will collapse onto their nucleons at the lower temperature

$$T_{\text{contraction, maximal}} \approx \frac{3^{1/3} \times 28 \text{ GeV}}{(n_{\text{eff}} \sigma)^{1/3} \times (100 \text{ GeV cm})^{1/3}} \approx 0.6 \text{ MeV}.$$  \hspace{1cm} (70)
Here we have used \( \frac{\Delta n}{n} \approx \frac{2}{3} \).

It follows that, for the majority of the balls after their collapse to their final size of today, there should be sufficient time in which the neutron proton ratio for the nucleons expelled from the balls can take on its thermodynamic equilibrium value. Even for the maximal size balls it is probably also approximately true, since the neutron freeze-out temperature 0.8 MeV is of the same order as the collapse temperature for the biggest balls. Thus our balls, which are by then very small in volume, will not disturb the development of the expelled nucleons from behaving just as under the usual Big Bang Nucleosynthesis calculations \[24, 39\]. Since these calculations are very successful in predicting ratios of light isotopes in metal poor regions of the universe, it is important that we retain these predictions in our model without having to refit with extra parameters. It should be borne in mind though that of course the very important \( \eta \)-parameter giving the baryon number relative to the photon number used in the Big Bang Nucleosynthesis calculations has to be carefully identified in our model; this baryon number should be taken to be the number of baryons \textit{expelled from the balls under the fusion explosion}. That is to say that the baryons staying back inside the balls are \textit{not counted into} \( \eta \), although they are of course truly baryons and there are appreciably more of them than the ones expelled. The crux of the matter is that these baryons staying inside the balls are hiding behind each other, so that at most the ones sitting on the surface of the balls can at all be hit by the external baryons. In other words the vast majority of the matter in the balls is effectively completely inaccessible to the external nucleons and other particles. Compared to the enormous weight of each ball and thereby its gravitational interaction the balls are extremely “dark”, in as much as they almost do not interact in other ways than gravitationally.

9 Conclusion

We have put forward the idea that the Tunguska-event was indeed caused by the impact of one of our dark matter balls. Such a ball consists of a small piece of a new type of vacuum, characterized by having a Bose condensate of our proposed bound state of \( 6t + 6\bar{t} \) in it, and is surrounded by a skin separating the new vacuum from the normal vacuum. Inside it is filled with ordinary matter under a high pressure (caused by the skin or wall surrounding the ball).

The picture is supported by a few remarkable agreements:

- Impact rate leads to weak scale tension in ball wall

The rate of in-falls of the proposed balls onto the earth is exceedingly crudely estimated to be \( r_B \approx 1/(200 \text{ years}) \), just from the fact that we saw only one ball about one hundred years ago. Further we assume that the balls are typically of such a size that they only \textit{barely avoid collapsing} by getting the nucleons pressed out through the wall. These two natural assumptions lead to the very nice agreement with the idea that the tension \( S \) of the wall is given by the weak interaction scale \( S^{1/3} \approx 100 \text{ GeV} \). (Really our best estimate in appendix A.2 for the condensate wall tension is \((16 \text{ GeV})^3\), while our estimate using the presumed rate of Tunguska falls and critical ball size gives the value \((28 \text{ GeV})^3\) for the tension \[38\].)
• Explosion energy is comfortably smaller than the estimated kinetic energy of the ball

The Tunguska explosion energy has been estimated to be about 10 to 30 megaton TNT. On the other hand, the mass we got from the density of dark matter, the impact rate \( r_B \) and the typical halo velocity gives a kinetic energy for the ball corresponding to 430 megatons TNT. But this kinetic energy is bigger than the Tunguska explosion energy by a factor 20. Indeed it is quite likely that 1/20th of the energy of the ball would have appeared on the surface of the earth in a short time after the impact.

• Dark to Normal Matter

We have previously speculated \(^5\) that at some moment - when the temperature of the universe was about 2.3 MeV according to \((68)\) - an explosive fusion of helium to heavier elements took place inside our balls. This would have happened at a time when the typical balls had already contracted so much that explosively expelled nucleons would no longer be captured by other balls, because by then the balls would have already made up a too tiny part of the volume. Simple energy estimates suggest, from the increase in binding energy from 7.1 MeV per nucleon in helium to 8.5 MeV in the heavier elements, that about one nucleon out of 6 nucleons were expelled. The expelled nucleons should become the ordinary matter, and thus we explained \(^5\) the ratio of ordinary to dark matter to be about 1/5 in good agreement with astronomical fits. In appendix B.5 we have repeated the calculation for median size balls and found this ratio of ordinary to dark matter predicted to be 1/5.6, while the recent Planck data \(^{38}\) are fitted by the value 1/5.44 for this ratio.

Just the idea that the Tunguska event was caused by some small object of extremely high specific density could be helpful in solving some of the mysteries of the Tunguska event. Such an object would penetrate so deep into earth that nothing from the object itself would be seen on the surface. Most obviously such a rather small but, for its size, extremely heavy particle would not be mainly observed via its track in the atmosphere but rather by hot earth material being seemingly thrown out from the earth. This material would presumably emerge from funnels, much like kimberlite pipes, such as the Suslov Crater, Lake Cheko and perhaps the funnels observed by Kulik \(^{27}\). If such small objects of high density were moving at a relativistic speed, they might not be observable astronomically and cosmologically today as dark matter since the latter should be “cold”. But suppose they are non-relativistic and with a typical, say, halo speed of the order of 160 km/s as discussed in the present article. Then if such a particle should deposit in the air above Tunguska\(^{12}\) about 1/20 of the energy deposited at all in the earth, the total amount of energy and thereby also of mass carried by these particles would have to be like that of the dark matter in the halo. So, under the assumption that the particles are in the halo of our galaxy, they would have a total mass similar to that of the dark matter there. Thus it would be hard not to identify them with the dark matter; otherwise there would no longer be place for the dark matter. Of course for such a particle of the

\(^{12}\)Here we also assume such particles cause other Tunguska-like events at a rate of approximately once every 200 years
non-relativistic type to deliver the energy suggested by the Tunguska event, we need also the particle mass to be about the one in our fit, namely about $10^8$ kg.

It is in general a bit of a mystery why the density of dark matter in the universe is so close to the density of ordinary matter, only deviating from it by a factor of 5. Supersymmetry models with WIMPs can at best obtain this result as being a bit of an accident. A model like ours, in which the dark matter at the root of it consists of ordinary matter bound into small objects, should a priori have a better chance to cope with this mystery. As already mentioned we even get just the right ratio of dark to normal matter. But then the needed high specific density requires a very strong compression or some alternative mechanism for obtaining an anomalously high specific density. It is also crucial that the part of the ordinary matter presented as dark matter gets so strongly compressed that it is effectively completely inaccessible under the Big Bang Nucleosynthesis; otherwise the good agreement with data of the Big Bang Nucleosynthesis calculations \cite{24,39} with only gravitationally interacting dark matter would be spoilt.

In this article we presented a picture of the balls being formed, by having two types of vacua with somewhat different Higgs field expectation values in the two vacua. These VEVs are assumed to deviate by a factor of order unity, with the Higgs field being smaller in the vacuum inside the dark matter balls.

Let us immediately stress the very important property of this model: the model only uses the Standard Model, so that no new physics is to be assumed. This is contrary to the majority of models for dark matter in high energy physics, which typically need for instance supersymmetry so as to get supersymmetric partners functioning as dark matter particles. It should though be admitted that we make an extra assumption. This assumption does not truly modify the Standard Model, but only provides a way of fixing some of the coupling constants. In fact we assume the so-called “Multiple Point Principle” (MPP), which postulates that there should be several vacua having the same energy density. We note that the MPP prediction \cite{8} for the Higgs mass is in remarkably good agreement with the recent LHC measurements.

For the present article the crucial assumption is that there exists one vacuum with a condensate of bound states of $6t + 6\tilde{t}$ and one without this condensate; furthermore these two vacua have the same energy density. The existence of such a condensate requires a fine-tuning of the top quark Yukawa running coupling constant $g_t$ \cite{16}. We have made a detailed analysis \cite{17} of this fine-tuning requirement and obtained the estimated value $(g_t)_{MPP} = 1.00 \pm 0.14$, which within errors is in agreement with the experimental value of the top quark mass. However this result is controversial, as it has not been confirmed by other calculations \cite{18}.

With our assumption of the Multiple Point Principle, it is of course much easier to get several phases of the vacuum realized, since they can now coexist in balance w.r.t. energy density. Indeed in our model we have two vacua, with and without the $6t + 6\tilde{t}$ bound state condensate. So, at the temperature scale of the weak interactions in the early universe, there would be a random distribution in space of these two vacuum phases. In the present article we estimated that the phase without a condensate would eventually dominate, but that balls of the phase with a condensate could be prevented from contracting for some time. This is due to an effect of the lower Higgs field inside the balls (i.e. in the with condensate phase) causing lower masses for several Standard Model particles, which in
turn cause them to provide a bigger pressure through their Planck radiation. It follows that many balls would remain essentially uncontracted until the temperature fell below 10 MeV. At this temperature a stabilisation process set in, due to the balls getting filled with nucleons, which stopped the balls from totally contracting away. In other words we actually estimated that, under the conditions derived from just the Standard Model with the MPP determined couplings, permanently stabilised balls would be formed with an excess of nucleons inside. Really it is the vacuum inside the balls having a bound state condensate which captures at first almost all the nucleons.

It is speculated that an explosive fusion of helium to heavier nuclei occurs inside these “with condensate” balls, causing about 1/6 of the nucleons to escape outside into the dominant “without condensate” vacuum. We estimated that these processes of contraction of balls and their explosive emission of nucleons would be mainly finished, at a temperature of about 2.3 MeV, just in time for not disturbing the Big Bang Nucleosynthesis. After that time the balls have their sizes stabilised by their content of ordinary matter. Then they are so small and concentrated with density around $10^{14}$ kg/m$^3$ that by far they dominantly only interact by gravity. Thus the balls can be treated as just dark matter from there on. Of course if you get really close to a single ball, like in Tunguska, its non-gravitational interactions can become relevant. In this way we managed, for practical purposes, to obtain a dark matter model from just the Standard Model extended with the extra assumption of the Multiple Point Principle!

In addition this dark matter model leads to events of the Tunguska type, one about every 200 years, and each of them would dig deep holes into the earth. It is not excluded that they would go out again at another place on the earth, since the order of magnitude for the penetration depth of a dark matter ball is similar to the earth radius. These holes would of course be filled with molten material, which is likely to be pumped up from much deeper regions in the earth. In fact they would be much like kimberlite pipes. We are therefore tempted to identify these pipes, formed by the balls falling through the earth, with kimberlite pipes. Indeed we made a model for estimating the number of such kimberlite pipes which should be exposed so as to be visible on, for example, the Canadian Shield. We found that, within the large uncertainties of our estimate, the number of kimberlite pipes actually found were compatible with our estimate of the number expected from dark matter ball impacts with the earth. Now, however, it must be admitted that we have a problem for this picture of the origin of kimberlite pipes. The times of creation of various kimberlite pipes have been estimated [36, 37] and a distribution found, even in regions where Archean pre-Cambrian rocks were exposed, with clustering in various geological eras. In particular there is a concentration of kimberlite emplacements in a couple of clusters during the Cretaceous period earlier than about 200 million years ago. This is in contradiction with our picture, because in our model we must have equally many Tunguska-like impacts per year both during the very long pre-Cambrian eras and the comparably shorter Cretaceous and Mesozoic eras. So, if Archean rocks are exposed, why should only the kimberlite pipes from a much later time be visible there? This is strange in our model, in which Tunguska-like particles fall at random and only the visibility today can be dependent on the geological conditions. There should be no connection in our model with tectonic events like the splitting up of the supercontinent Gondwanaland.
If the kimberlites should match our model too badly, there is of course the possibility that our falls of dark matter as Tunguska events have nothing to do with kimberlites. Potentially our ball falls could even be associated with one special kind of kimberlite, but there may be so few of them that they would be hidden among the “normal” kimberlites.

Apart from the problem with the time distribution of the kimberlite pipes we have a very consistent and viable picture of both dark matter and the Tunguska event. We present the values of the various parameters in our model in Table 1.

Table 1: The parameters of our model picture of the Tunguska particle as a ball of a new type of vacuum with a bound state condensate, filled with ordinary white dwarf-like matter and on the borderline of stability.

| Parameter                          | Value                                      |
|------------------------------------|--------------------------------------------|
| Time Interval of impacts           | $r_B^{-4}$ 200 years                       |
| Rate of impacts                    | $r_B 1.5 \times 10^{-8}$ s$^{-1}$          |
| Dark matter density in halo        | $\rho_{halo} 0.3$ GeV/cm$^3$              |
| Dark matter near solar system      | $\approx 2\rho_{halo} 0.6$ GeV/cm$^3$     |
| Mass of the ball                   | $m_B 1.4 \times 10^8$ kg                  |
| Estimated typical speed of ball    | $v 160$ km/s                              |
| Kinetic energy of ball             | $T_v 1.8 \times 10^{18}$ J                |
| Energy observed in Tunguska        | $E_{Tunguska} (4 - 13) \times 10^{16}$ J  |
| Potential shift between vacua      | $\Delta V 10$ MeV                         |
| Cube root of tension               | $S^{1/3} 28$ GeV                          |
| Cube root of tension               | $S^{1/3}$ 16 GeV                          |
| Ball density                       | $\rho_B 10^{14}$ kg/m$^3$                 |
| Radius of ball                     | $R 0.67$ cm                               |

All together we think that our model - apart from troubles with the time distribution of kimberlite pipes, and some dispute [17, 18] about the predicted top quark Yukawa coupling - has many features that makes it viable as a picture explaining at first sight rather separate phenomena in high energy physics, the Tunguska event, geology and cosmology.

The most favoured competing model for the explanation of the Tunguska event is the comet hypothesis. However, in order to explain the lack of iridium from the Tunguska event, the comet would have to have been especially low in dust content. Halley’s comet has 40% dust and such a comet would have given too much iridium and also presumably left more debris from the comet on the site. Really the most remarkable fact about the Tunguska event is the lack of any rudiments of the impact object itself. That is why our model, in which the cosmic body went so deep into the earth that it practically disappeared, has a good chance to be true (of course ice may also disappear).

Now what could give a hint as to whether our model is indeed true?:

- New Bound State
If the high energy physics part of our model is indeed true, there should be at least a couple of phases of the vacuum, in one of which there is a condensate of a bound state of $6t + 6\bar{t}$. This bound state is then, by our Multiple Point Principle, tuned in to be between being a tachyon and an ordinary positive mass squared particle. Hence its mass must have been fine-tuned to be rather small compared to the typical mass of 12 top quarks, which is 2 TeV. We have already suggested [16, 41] that this bound state could be produced in pairs in co-production with top-quarks in the LHC-accelerator. Indeed we have estimated the mass of this new bound state in appendix A.3 to be about $m_{NBS} \approx 260$ GeV. Hence a pair of of new bound states cannot contribute to the invisible decay width of a Higgs particle of mass 126 GeV, although they would couple strongly to it.

- **Other Dark Matter Models should be false**
  It is of course clear that a likely way to falsify our model would be to establish another model for dark matter, such as a weakly interacting massive particle (WIMP) often identified as the lightest superpartner in supersymmetric models. The DAMA collaboration [42] have claimed a positive signal for direct WIMP production from the observed annual modulation in their data over 12 annual cycles. More recently the COGENT [43] and CRESST [44] collaborations have also claimed evidence for WIMP production. However it is very difficult to reconcile these positive results with the negative results from the CDMS, EDELWEISS, XENON and LUX experiments [24, 45].

- **Lack of New Physics at LHC**
  Our model is based only on the Standard Model. Therefore the LHC gives support to our model, as long as it continues to provide no evidence for new physics with a suitable dark matter candidate.

- **Absence of MACHOs**
  Microlensing searches [24, 46] for dark matter in the form of massive compact halo objects (MACHOs) are insensitive to objects with masses less than $10^{-8}M_\odot$. Hence our dark matter balls are too light for observation by microlensing. In fact observations [46] show that MACHOs contribute less than 8% to the mass of the galactic halo and hence do not provide a significant source of dark matter.

- **Looking for effects of our balls in stars**
  It is possible that dark matter balls collect into the core of a collapsing star. Then, when the density and temperature in the interior of the star gets sufficiently big, the balls could catch the nucleons, particularly any free neutrons since they do not feel the electric potential which has to be passed to get into the vacuum with a condensate inside the ball. This electric potential is of the order of 10 MeV and lower for the balls bigger than the minimal size, which we have taken to be close to the typical size. Thus the physics of supernovae could be changed by our model, if some balls expand and finally make the resulting neutron star into what is formally really a huge piece of dark matter.
10 Acknowledgements

CDF would like to acknowledge the hospitality and support from Glasgow University and the Niels Bohr Institute. HBN would like to acknowledge the hospitality and his status as professor emeritus at the Niels Bohr Institute.

A Appendix: Properties of condensate

In order to estimate especially the surface tension, but also the attraction potential $\Delta V$ for nucleons into the vacuum with the condensate, we shall now set up a crude picture of the condensate of the bound states of 6 top + 6 anti-top quarks. The 12 quarks form a closed 1s shell in this “new bound state” (NBS). The crude assumption to initiate our estimates is to say that, instead of only thinking of the condensate as consisting of a region with the bound states present, we can also imagine that one bound state would begin to attract further top and anti-top quarks in addition to the first 12; now they should collect in the 2s and 2p states. Really our hypothesis is that the distances between the bound states in the condensate can be estimated, by assuming the nearest bound states to a given one to be at the distance from the centre of the bound state out to a top quark in a 2s or 2p orbit around the bound state. In appendix C of our previous article [17] we obtained an estimate of the radius $r_0$ of the bound state:

$$r_0 \approx \sqrt{\frac{3}{4}} \frac{1}{m_t},$$  \hspace{1cm} (71)

where $m_t$ is the top quark mass. Actually $r_0$ was defined by assuming that the single particle wave function for a top or anti-top quark in the 1s bound state takes the form $\psi \propto \exp(-r/r_0)$; so the mean square radius is given as $<r^2> = 3r_0^2$. Remembering that the Bohr radius for an orbit with principal quantum number $n$ is proportional to $n^2$, we get that the next i.e. $n = 2$ orbits have a radius of about $r_{n=2} = 4r_0$. So, with our crude assumption, the distance between “neighbouring” bound states should be about $4r_0$. In order to have the correct number of top quarks for each colour and spin in such a region at a distance $4r_0$ away from a bound state, we should have four “neighbouring” bound states at this distance of $4r_0$. This brings to mind the distribution of carbon atoms in a diamond, in which each atom is surrounded by four nearest neighbours. Now diamond has a bond-length of $1.54 \times 10^{-8}$ cm and has a density of $1.76 \times 10^{23}$ cm$^{-3}$; so in a cube, with a side of length equal to the bond-length, there are 0.64 carbon atoms. Taking our condensate to also have this property, we estimate that our condensate has a density of 0.64 bound states per cube with side of length equal to the bond-distance, which we took to be $4r_0 \approx 4\sqrt{\frac{3}{4}}/m_t$. Hence the density of bound states in the vacuum with a condensate is estimated to be

$$\rho_{\text{number}} = 0.64 \times \frac{1}{(4r_0)^3} = 0.0154 m_t^3 = (43 \text{ GeV})^3.$$  \hspace{1cm} (72)

Let us introduce an effective scalar field $\phi_{\text{NBS}}$ for the bound state. Now $\phi_{\text{NBS}}$ is a real field and one cannot define a conserved particle number. So, purely for the purpose
of introducing a number density of bound states $\rho_{\text{number}}$, we will formally treat $\phi_{\text{NBS}}$ as a complex field in the following definition of the number of bound states in a given volume:

$$N = \int \phi_{\text{NBS}}^\dagger \overleftarrow{\partial_0} \phi_{\text{NBS}} \, d^3 \vec{x}.$$  \hfill (73)

Denoting the expectation value of $\phi_{\text{NBS}}$ in the condensate by

$$v = <\phi_{\text{NBS}}> \hfill (74)$$

and the energy of a bound state in the condensate by $E_{\text{cond}} \approx m_{\text{NBS}}$, our estimate of the number density then becomes

$$\rho_{\text{number}} = 2E_{\text{cond}}v^2. \hfill (75)$$

Here $m_{\text{NBS}}$ denotes the mass of our new bound state in the vacuum with a condensate.

Our vacuum with the condensate is supposed to be a relativistic invariant vacuum - rather than an ether-like state representing a special frame. Thus, strictly speaking, we must imagine that our diamond model is moving with a superposition of velocities having a Lorentz invariant distribution. That might give divergences, if considered seriously, because of the infinite Haar-volume of the non-compact Lorentz group. But let us hope the divergences will cancel out and consider the case when the speed of the crystal of bound states (more realistically one should of course treat the diamond-like structure as a fluid) has an associated $\gamma$. Then the energy of the bound state $E_{\text{cond}} = \gamma \ast m_{\text{NBS}}$, but also the whole “crystal” gets Lorentz contracted and its thickness in the direction of motion is diminished so as to be $1/\gamma$ times as thick as without the motion. The number density we calculated in (72) i.e. $\rho_{\text{number}} = (43 \text{ GeV})^3$ would, if boosted by this Lorentz contraction, increase its value to $\gamma \ast \rho_{\text{number}} = \gamma \ast (43 \text{ GeV})^3$. Allowing for this boosting and using (72), the correct form for (75) becomes rather

$$\gamma \rho_{\text{number}} = \gamma \ast (43 \text{ GeV})^3 = \gamma 2m_{\text{NBS}}v^2 \hfill (76)$$

or equivalently

$$\rho_{\text{number}} = (43 \text{ GeV})^3 = 2m_{\text{NBS}}v^2. \hfill (77)$$

### A.1 Effective potential for the effective field $\phi_{\text{NBS}}$

In order to facilitate the estimation of the surface tension in the surface between the two phases of the vacuum, we shall now introduce an ansatz for the effective potential for the effective bound state field $\phi_{\text{NBS}}$. The basic assumption underlying our model is the existence of these two degenerate vacua in the Standard Model. Hence the effective potential for $\phi_{\text{NBS}}$ must have two degenerate minima. We shall take the effective potential to be a function of the squared field $\phi_{\text{NBS}}^2$. The philosophy for only taking the squared field in the effective potential is that it should be easier to produce a pair of bound state out of the vacuum than to make a single bound state, so to speak, from its complicated 12 quark substructure. Thus we expect to get pairs of bound states, as if they had an approximately conserved quantum number taking values in $Z_2$ (the integers modulo 2). In any case we hope that the choice of ansatz for the effective potential should not make
so great a difference order of magnitudewise. We shall use a polynomial ansatz and take the lowest order polynomial compatible with having two degenerate minima. So we must take the sixth order polynomial (meaning third order in $\phi_{NBS}^2$):

$$V_{\text{eff}}(\phi_{NBS}) = \frac{1}{M^2} \phi_{NBS}^2 (\phi_{NBS}^2 - v^2)^2.$$  \hspace{1cm} (78)

This potential has been arranged to have two degenerate minima, namely for $\phi_{NBS} = 0$ and for $\phi_{NBS} = \pm v$ (in a way even three minima if you consider $-v$ and $v$ as different.) The potential has a maximum for $\phi_{NBS} = \pm v/\sqrt{3}$. The idea, of course, is that in the phase in which we live and there is no condensate we have the VEV $<\phi_{NBS}> = 0$, while in the phase inside the balls where there is a condensate of the bound states we have say $<\phi_{NBS}> = v$. The two values of the mass squared $m_{NBS}^2$ of the bound state, as would respectively be observed by observers living in these two phases, are obtained as the second derivatives of the effective potential $V_{\text{eff}}(\phi_{NBS})$ at the two minima:

$$\frac{\partial^2 V_{\text{eff}}}{\partial \phi_{NBS}^2} \bigg|_{\phi_{NBS}=0} = \frac{2v^4}{M^2} = m_{NBS}^2 |_{\text{without condensate}} \hspace{1cm} (79)$$

$$\frac{\partial^2 V_{\text{eff}}}{\partial \phi_{NBS}^2} \bigg|_{\phi_{NBS}=v} = \frac{8v^4}{M^2} = m_{NBS}^2 |_{\text{with condensate}}. \hspace{1cm} (80)$$

Thus, with our ansatz (78), we obtain

$$m_{NBS} |_{\text{with condensate}} = 2m_{NBS} |_{\text{without condensate}}. \hspace{1cm} (81)$$

Substituting the value of the mass of the bound state in the “with condensate phase” (80) for $m_{NBS}$ in formula (77) gives us

$$\rho_{\text{number}} = (43 \text{ GeV})^3 = 2m_{NBS} |_{\text{with condensate}} v^2 = \frac{4\sqrt{2}v^4}{M}. \hspace{1cm} (82)$$

### A.2 The solitonic wall and surface tension

At the borderline between the two phases the field must, over a rather short distance, go from $\phi_{NBS} = 0$ to $\phi_{NBS} = v$ as a soliton. Our surface tension or surface energy per unit area $S$ is given by the Hamiltonian density for a static field configuration

$$\mathcal{H}_{\text{static}} = \frac{1}{2} \left( \frac{\partial \phi_{NBS}}{\partial x} \right)^2 + V_{\text{eff}}(\phi_{NBS}) \hspace{1cm} (83)$$

evaluated for the static soliton solution. The equation for the soliton solution considered in only one dimension $x$, say, perpendicular to the wall can be written

$$\frac{1}{2} \left( \frac{\partial \phi_{NBS}}{\partial x} \right)^2 - V_{\text{eff}}(\phi_{NBS}) = C. \hspace{1cm} (84)$$

The constant $C$ is easily seen to be zero, by considering the field $\phi_{NBS}$ a long away inside the phases. So a crude typical value for the slope $\frac{\partial \phi_{NBS}}{\partial x}$ inside the solitonic wall
is determined by the square root of the maximum of the effective potential between the two degenerate minima. This maximum value occurs at the peak of the potential, where $\phi_{NBS} = v/\sqrt{3}$,

$$V_{\text{eff}}|_{\text{peak}} = V_{\text{eff}}(v/\sqrt{3}) = \frac{4}{27} M^2 v^6. \quad (85)$$

At the peak of the effective potential the gradient of the soliton solution $\frac{\partial \phi_{NBS}}{\partial x}$ is equal to $\sqrt{2}$ times the square root of $V_{\text{eff}}|_{\text{peak}}$:

$$\frac{\partial \phi_{NBS}}{\partial x} |_{\text{peak}} = \sqrt{2} \cdot \sqrt{\frac{8}{27} M^2 v^3}. \quad (86)$$

A crude estimate of the thickness $d$ of the solitonic wall is given by multiplying the inverse of this gradient by the difference between the values of $\phi_{NBS}$ in the two vacua, which is just $v$. So we get the thickness to be

$$d = v \sqrt{\frac{27}{8} \cdot \frac{M}{v^3}} = \sqrt{\frac{27}{8} \cdot \frac{M}{v^2}}. \quad (87)$$

Now the energy density inside the soliton is given by (83), in which the two terms are equal to each other. This follows from (84) with the constant $C = 0$. Averaged crudely over the soliton, we can take the soliton energy density to be half the peak value of the Hamiltonian density $\frac{1}{2} H_{\text{static}}|_{\text{peak}} = V_{\text{eff}}|_{\text{peak}}$, which is given by (85). So the energy per unit area of the surface or wall becomes this density multiplied by the thickness $d$,

$$S = d \cdot V_{\text{eff}}|_{\text{peak}} = \frac{\sqrt{2} v^4}{3 \sqrt{3} M} = \frac{1}{12 \sqrt{3}} \cdot \rho_{\text{number}} = (16 \text{ GeV})^3, \quad (88)$$

where we have used (72).

### A.3 Mass of bound state NBS

In order to also extract the mass of the bound state, we need one more relationship between the parameters in the effective potential $V_{\text{eff}}$. We do this by estimating the thickness of the solitonic wall, using the properties of the condensate. We again suppose the condensate has a density of bound states given by taking the distance between neighbouring bound states to be of the order of the radius of the $n = 2$ orbit. The thickness of the wall must be of the order of the thickness of the atomic layers in the assumed diamond-like crystal structure. We shall take this layer thickness to be given by half the bond-length between two nearest neighbours, which is supposed to be $r_{n=2} = 4r_0$. Then we obtain the following “physical” estimate of the wall thickness $d$,

$$d \approx 2r_0 = \frac{\sqrt{3}}{m_t} = \frac{1}{100 \text{ GeV}}. \quad (89)$$

Combining this thickness estimate with the formula (87) we get

$$\frac{1}{100 \text{ GeV}} \approx \sqrt{\frac{27}{8} \cdot \frac{M}{v^2}}. \quad (90)$$
Combining this with (88),

$$\frac{\sqrt{2}v^4}{3\sqrt{3}M} = (16 \text{ GeV})^3,$$

we get the following values for the parameters in our effective potential:

$$M = 0.45 \text{ GeV}$$

$$v = 9.1 \text{ GeV}.$$  

Then, using (79, 80), we obtain an estimate for the mass of the bound state in our vacuum and in the vacuum with a condensate:

$$m_{NBS|\text{without condensate}} \approx 260 \text{ GeV}$$

$$m_{NBS|\text{with condensate}} \approx 520 \text{ GeV}.$$  

**A.4 Estimate of Higgs field in the condensate**

In section 4 we estimated the value of the parameter $\Delta V$ to be 10 MeV, by assuming that the value of the Higgs field $<\phi_h>$ in the condensed phase is just 1/2 of the usual Higgs field VEV in the no-condensate phase. However there is a large uncertainty on the value of the ratio of the VEVs of the Higgs field in the two vacua. Here we want to make an estimate of this ratio of VEVs using the properties of the condensed phase conceived of as having a diamond-like structure, although really being a fluid.

First notice that we have a number density $\rho_{\text{number}} = (43 \text{ GeV})^3$ for our new bound states (NBS) present in the condensed phase. Around each of these bound states, we have the negative Yukawa potential contribution to the Higgs field

$$\Delta \phi_h |_{\text{one NBS}} = \frac{12g_t/\sqrt{2}}{4\pi r} \exp(-mr),$$

where $r$ is the distance from the centre of the NBS to the point at which we want the contribution $\Delta \phi_h$ to the Higgs field. The effective or true Higgs mass is here denoted as $m$. Averaging over space of course means

$$av(...)=\frac{\int...d^3\vec{x}}{V}=\frac{\int...d^3\vec{x}}{\int d^3\vec{x}},$$

where $V$ is the infrared cut-off volume of the universe. Now we want to sum over the contributions from all the $V\rho_{\text{number}}$ bound states in the infrared cutoff region $V$. Then we calculate the average $\Delta \phi_h$ summed over all the bound states, so as to obtain the total reduction of the ordinary Higgs field expectation value $<\phi_h>_{\text{no condensate phase}} = 246 \text{ GeV}$ in the condensed phase. This average becomes

$$av(\Delta \phi_h|_{total}) = \frac{1}{V} \int \sum_{\text{bound states}} \frac{6\sqrt{2}g_t}{4\pi r} \exp(-mr) \, d^3\vec{x}$$

$$= \frac{6\sqrt{2}g_t}{4\pi} \frac{1}{V} \sum_{\text{bound states}} \int_0^\infty \frac{1}{r} \exp(-mr) \, 4\pi r^2 \, dr$$

$$= \rho_{\text{number}} \frac{6\sqrt{2}g_t}{4\pi} \int_0^\infty r \exp(-mr) \, dr.$$
The Higgs field expectation value in the condensate phase then becomes

\[ < \phi_h >_{\text{condensate phase}} = < \phi_h >_{\text{no condensate phase}} - \text{av}(\Delta \phi_h|_{\text{total}}). \] (101)

Note that the expression (100) is convergent due to the Yukawa exponential factor \( \exp(-mr) \). This means that the contribution to the change in the Higgs field from each single NBS bound state is only important over a region cut-off by the Higgs mass exponential factor \( \exp(-mr) \). Since the Higgs mass effectively depends on the strength of the Higgs field, we would prefer to interpret this factor \( \exp(-mr) \) as symbolic for a correction factor to the zero Higgs mass approximation, which we now discuss more carefully. In our previous paper [17] we estimated that, inside of a distance \( 1.58r_0 \) from the centre of the bound state, the effective Higgs mass even becomes complex. This is because the second derivative of the Higgs field effective potential is negative, for the values of the Higgs field in that region, and thus the mass squared of the effective Higgs vibrations are also negative there. In such a region one gets a sine or a cosine factor rather than the exponential factor \( \exp(-mr) \) and we think it is best to take the Higgs mass to be zero there. Thus the Yukawa potential solution to the static Klein Gordon equation, for the Higgs field contribution due to the NBS bound state under consideration, first begins its usual exponential decrease at a distance \( r = 1.58r_0 \) from the center. So we should rather make the replacement

\[ \exp(-mr) \rightarrow \exp(-m(r - 1.58r_0)). \] (102)

for the exponential factor in the Yukawa potential. Even for \( r \geq 1.58r_0 \) the mass parameter \( m \) should rather be some effective Higgs mass corresponding to the second derivative of the Higgs potential for the value of the field \( \phi_h \) present there. Such an effective Higgs mass must at least be smaller than the measured Higgs mass of 126 GeV. Really it interpolates between values starting from being zero at \( r = 1.58r_0 \) and ending by being at most 126 GeV [13].

We have already assumed an approximate diamond-like structure for the condensate, with a separation distance of \( 4r_0 \) between neighbouring NBS bound states. So we are led to a crude estimate of the most important region in the integral (100) as being around \( r \approx 3r_0 \). Let us now identify our modified exponential factor (102) at this typical value of \( r = 3r_0 \) with the original Yukawa form, but with an effective Higgs mass \( m_{\text{eff}} \) inserted for \( m \):

\[ \exp(-m(r - 1.58r_0)) = \exp(-m_{\text{eff}}r) \quad \text{for} \quad r = 3r_0. \] (103)

The value of this effective Higgs mass is then

\[ m_{\text{eff}} \approx \frac{3 - 1.58}{3} \times 126 \text{ GeV} = 60 \text{ GeV}. \] (104)

So we should get a roughly correct estimate for the reduction in the Higgs field \( \text{av}(\Delta \phi_h|_{\text{total}}) \), by inserting the value (104) of 60 GeV for \( m \) into (100). Inserting the mathematical evaluation

\[ \int_0^\infty r \exp(-m_{\text{eff}}r) \, dr = \frac{1}{m_{\text{eff}}^2}, \] (105)
into (100) we get
\[
\text{av}(\Delta \phi_h|_{\text{total}}) = \rho_{\text{number}} \cdot 6\sqrt{2} g_t \cdot \frac{1}{m_{\text{eff}}} 
\]
\[
= \frac{(43 \text{ GeV})^3 \cdot 6\sqrt{2} g_t}{(60 \text{ GeV})^2} 
\]
\[
= 174 \text{ GeV}.
\]

Here we used \( g_t = 0.93 \) for the top quark running Yukawa coupling constant. Hence we obtain the ratio of the Higgs field VEVs in the two vacua to be
\[
\frac{\langle \phi_h \rangle_{\text{condensate phase}}}{\langle \phi_h \rangle_{\text{no condensate phase}}} = \frac{246 - 174}{246} \approx 0.3,
\]
which only differs by a factor of \( 3/5 \) from the value \( 1/2 \) we used in section 4 and throughout the paper.

We will now check the self-consistency of our assumption that, for a general point, the typical distance to the nearest bound state is \( 3r_0 \) and that the integral (100) is dominated by the region around \( r \approx 3r_0 \). This dominant region should correspond to where the integrand of (105) has its maximum value. This integrand \( r \exp(-m_{\text{eff}}r) \) has its maximal value for \( r = r_{\text{max}} = \frac{1}{m_{\text{eff}}} = 1/(60 \text{ GeV}) \), which agrees well with \( 3r_0 = 3 \cdot \sqrt{3/4}/m_t = 1/(67 \text{ GeV}) \).

\section*{B Appendix: Production of balls in early universe}

In this appendix we shall go through a few detailed points about the cosmological development of our balls in the early universe. First we shall discuss whether we can arrange the normal vacuum without the condensate to become dominant. Secondly we discuss the distribution of ball sizes. As the third subject we present our estimates of the effects of the lower Higgs field inside the balls, which provide a pressure that pumps up the balls and allows them to survive long enough to collect nucleons. Fourthly we must discuss the nuclear physics inside the balls and whether or not the fusion of nucleons to helium, and then from helium to heavier nuclei, can occur on the right time scales for our model to work. In particular we consider whether the appropriate number of nucleons could be expelled from the balls, in the explosive fusion of helium to the heavier nuclei, to form what is now seen as ordinary matter.

\subsection*{B.1 Which vacuum takes over?}

At temperatures high compared to the weak scale, meaning \( T \approx 100 \text{ GeV} \), there must be about equal volumes of the vacuum \textit{with} and the vacuum \textit{without} the condensate. It is at the time, when the temperature is of the order of the weak scale, that walls between the regions with different vacua - “with condensate” or “without condensate” - begin to contract. This contraction diminishes the local pieces of the vacuum with the larger free energy density at the weak scale. At higher temperatures the walls were present by having
effectively zero free energy density. Now the zero temperature energy density difference between the two types of vacuum is, by our Multiple Point Principle assumption, very small - essentially zero. Hence the free energy density difference comes from those species of particles which behave with different mass in the two different vacua. For various temperature scales most particles have a mass either bigger or smaller by an order of magnitude or more than the temperature. If that is the case for a certain species and the mass difference in the two vacua is only of the order of the mass itself, then that species is either effectively so heavy as to almost not be present or it effectively has zero mass. In both cases the difference in free energy density will be very small between the two phases from such a species. So, in order to get an important difference in free energy density, we must call attention to just that or those species having their mass of the order of the temperature. The most important choice of species, for determining which of the two vacua shall contract, is taken close to the beginning of contraction at the weak scale temperature era. The particle that has the right mass to be important for the free energy density difference in this era is the bound state (NBS) responsible for the condensate itself. This is because the mass \( m_{\text{NBS}} \) of the bound state is connected to the same effective potential that gives the wall its energy density, which is of course really the scale that decides when the walls between the vacua begin to contract. We have considered an ansatz for this effective potential \( V_{\text{eff}}(\phi_{\text{NBS}}) \) in appendix A.1, where we found (81) that the mass in the vacuum with the condensate \( m_{\text{NBS}}|_{\text{with condensate}} \) is larger than the mass in the normal vacuum without a condensate \( m_{\text{NBS}}|_{\text{without condensate}} \). So, when the temperature is in between the two masses \( m_{\text{NBS}}|_{\text{without condensate}} \) and \( m_{\text{NBS}}|_{\text{with condensate}} \), there is approximately Planck radiation of this bound state in the lower mass vacuum - being the normal one without a condensate - but virtually absent in the higher mass vacuum. Now such Planck radiation gives a negative free energy density. Thus finally the free energy is minimised by having as much as possible of the normal phase without a condensate. This is supposed to initiate the contraction in the direction of diminishing the amount of vacuum with a condensate. That should be the reason why we today have ended up with dominance of the without condensate phase and only find the with condensate phase inside tiny balls making up the dark matter.

### B.2 Distribution of ball sizes

In principle the initial distribution of ball sizes is given by the Boltzmann distribution of the walls at the effective starting time of the weak scale temperature. A priori the situation could even be that the two phases penetrate each other as two connected regions (with only small pieces of isolated balls). However, at least after one of the phases has taken over even if only by of order unity in relative volume, there will be disconnected balls of the minority phase. To obtain the distribution of the sizes - volumes or radii as if the balls were already spherical (which they are not at first) - of these minority phase balls a computer simulation should in principle be applicable. In any case it must be so that the total volume of the minority phase balls per volume of space must be less than half the space volume. Letting the number of (minority phase) balls per unit volume with radius
between $R$ and $R + dR$ be denoted by $\rho_{\text{balls}}(R) \, dR$ this requirement comes to mean
\[
\int_0^{1/H} \rho_{\text{balls}}(R) \frac{4\pi}{3} R^3 \, dR < 1/2. \tag{110}
\]
Here the upper limit on the size of the balls \[40\] at the weak scale is given by the Hubble distance at that time $1/H \approx 1 \text{ cm}$. This integral should converge. So $\rho_{\text{balls}}(R)$ must at least fall off asymptotically as $1/R^4$ for large $R$ and for small $R$ it cannot grow towards $R = 0$ faster than also $1/R^4$. Under for instance a simple Hubble expansion, the total density of balls per volume must decrease as $T^3$ but at the same time the radius of a given ball will increase proportional to $1/T$. Under an expansion corresponding to the radiation domination temperature ratio $T_1/T_2$, we therefore have the following change in the just defined ball density $\rho_{\text{balls}}(r)$
\[
\rho_{\text{balls}}^T_1(R) \rightarrow \rho_{\text{balls}}^T_2(R) = \rho_{\text{balls}}^T_1(R \cdot \frac{T_2}{T_1}) \cdot \frac{T_3^3}{T_1^3} \cdot \frac{dR_1}{dR_2} = \rho_{\text{balls}}^T_1(R \cdot \frac{T_2}{T_1}) \cdot \frac{T_3^3}{T_1^3}. \tag{111}
\]

The most crude just barely converging power law behaviour $\rho_{\text{balls}}(R) \propto 1/R^4$, which we shall use, would not change with time, as is easily seen, under such a Hubble expansion. So, as the Hubble expansion goes on, we do not expect the distribution of ball sizes to change much apart from the possible collapses discussed in appendix \[B.3\]. The Kibble upper radius bound \[40\] given by the Hubble distance of course expands. As cosmological time progresses the smaller balls contract first and then the bigger and bigger balls successively contract. So the $\rho_{\text{balls}}(R) \propto 1/R^4$ distribution gets cut off at the small $R$ side by a time dependent cut-off.

Those balls which are sufficiently big to get their collapse halted by the nucleons being pressed together inside them finally end up \[13\] having masses $M$ proportional to the volume before the collapse. Thus we have $M \propto R^3$, where $R$ is taken at the time when the temperature is $T = 10 \text{ MeV}$. This leads for the $\rho_{\text{balls}}(R) \propto 1/R^4$ distribution to a mass distribution $\propto dM/M^2$ and the total average mass becomes a logarithmically divergent expression cut off by the Kibble upper bound. So indeed it is not so bad an approximation to consider it that the balls making up the dark matter have a typical well defined mass, as we did in our calculations. We can in fact estimate the median size of a dark matter ball, using the power law distribution $\rho_{\text{balls}}(R) \propto 1/R^4$ cut-off at small $R$ by the stability borderline radius $R_{\text{border}}$ and at large $R$ by the Kibble radius $R_{\text{Kibble}} = 1/H$. Then the median ball radius is determined by the equation:
\[
\int_{R_{\text{border}}}^{R_{\text{median}}} \rho_{\text{balls}}(R) \, dR = \frac{1}{2} \int_{R_{\text{border}}}^{R_{\text{Kibble}}} \rho_{\text{balls}}(R) \, dR \tag{112}
\]
which gives
\[
\frac{1}{R_{\text{border}}^3} - \frac{1}{R_{\text{median}}^3} = \frac{1}{R_{\text{Kibble}}^3} - \frac{1}{R_{\text{Kibble}}^3}. \tag{113}
\]
Since $R_{\text{Kibble}} \gg R_{\text{border}}$, it is a good approximation to take the limit $1/R_{\text{Kibble}} \rightarrow 0$ giving
\[
R_{\text{median}} = 2^{1/3} R_{\text{border}}. \tag{114}
\]
\[13\] Here we ignore the nucleons being pumped in from smaller totally collapsing balls.
That is to say the typical or median ball radius $R_{\text{median}}$ is just a factor of $2^{1/3}$ larger than the radius $R_{\text{border}}$ of the minimal stable ball. In this formula (114) the radii involved were really the radii before the balls collapsed, so denoting that by an assigned bracket would make this formula read

$$R_{\text{median}}^{(\text{before})} = 2^{1/3} R_{\text{border}}^{(\text{before})}. \tag{115}$$

Before the contraction we assume the density of baryon number and thereby at the end of mass per unit volume inside the balls to be the same independent of the size of the ball, so that the mass of a ball $m_B \propto R^{(\text{before})3}$. From (10), (22), (24) and (25) we get the number density of nucleons inside the ball after the contraction

$$n = \frac{2}{3\pi^2} \left( \frac{24\pi^2 S}{R^{(\text{after})}} \right)^{3/4}. \tag{116}$$

So

$$m_B \propto n R^{(\text{after})3} \propto R^{(\text{after})9/4}. \tag{117}$$

But we also have of course

$$m_B \propto R^{(\text{before})3}. \tag{118}$$

Hence

$$R^{(\text{after})9/4} \propto R^{(\text{before})3}, \tag{119}$$

and thus (114) leads to

$$\frac{R_{\text{median}}^{(\text{after})}}{R_{\text{border}}^{(\text{after})}} = \left( \frac{R_{\text{median}}^{(\text{before})}}{R_{\text{border}}^{(\text{before})}} \right)^{4/3} = (2^{1/3})^{4/3} = 2^{4/9}. \tag{120}$$

I.e.

$$R_{\text{median}}^{(\text{after})} = 2^{4/9} R_{\text{border}}^{(\text{after})}. \tag{121}$$

When we ask for what fraction of the mass of the balls is in a given range of sizes for a mass distribution $\propto dM/M^2$, we shall find that in each order of magnitude range you have equally much mass. But if you ask for the number of balls, the majority will have masses close to the lower bound given by the borderline for stability against collapse. Thus a ball responsible for a random Tunguska-like event would statistically have its mass not far from the stability borderline.

**B.3 Survival of Balls**

After the era of the NBS which suppressed the size of balls of the with condensate vacuum, as discussed in appendix [B.1] a quite analogous effect sets in based successively on the various quark, lepton, Higgs and gauge particle species. These particles get their masses from the Higgs field and thus somewhat different masses in the two phases (vacua). For all the quark, lepton species and massive gauge particles and even the Higgs\(^{14}\), it is

\(^{14}\)While, for the quarks, leptons and gauge particles, it is well-known that their masses are proportional to the Higgs field expectation values, in the case of the Higgs itself a moment more of contemplation is needed to see this. But indeed, in going from one phase to the other, the Higgs self coupling coefficient $\lambda$ will stay practically constant and thus even for the Higgs we have such a formula.
however so that the masses are smallest in the phase with NBS condensation, because the Higgs field expectation value is the smallest in that vacuum. (It is at least expected that in the condensate phase with its high density of NBS states, inside which are highly reduced Higgs fields, there will be a smaller Higgs field VEV. See appendix A.4.) Thus the effects from these quarks and leptons etc., oppositely to the effect from the NBS itself, go in the direction of making the numerically largest but negative free energy density contribution in the with condensate phase. It follows that the effect of these Standard Model particles is to provide a pressure expanding balls made from the with condensate phase. As temperature falls with time one flavour of quark or lepton or gauge particle after the other first gets less and less relativistic and at the end essentially disappears totally. But, for each flavour or massive gauge particle, there is an interval of temperatures around the mass of that particle in which there are significantly more particles of this type in the with condensate phase (inside our balls) than outside. This is because the mass is smaller in this inside phase due to the smaller Higgs field expectation value there. So when the temperature is in one of these intervals there occurs, due to the particle in question, a more negative free energy contribution in the inside or with condensate phase. Then the minimisation of the free energy tends to expand the balls due to this effect. We assume there are sufficiently many Standard Model species distributed smoothly enough in mass, so that the intervals of activity providing the pressure to expand the balls this way worked all along the temperature scale from the weak scale to 10 MeV. Clearly the expansion pressure is of the order of $T^4$, when in the main region of the mentioned time intervals. Roughly we could imagine that the species in question were already essentially extinct in the without condensate phase. It would only be present inside the ball with an approximately massless Planck radiation pressure resulting.

Let us study the force per unit area on the wall between the two phases divided by $T^4$ and give it the name

$$\sum_m F(T, m) = \frac{\text{"Planck pressure" with condensate} - \text{"Planck Pressure" without condensate}}{T^4}.$$  \hfill (122)

This is the contribution from all the degrees of freedom. The expressions for the “Planck pressure” from massless degrees of freedom are given in equations (162) and (163) of appendix B.6. The symbol $\sum_m$ is supposed to mean the sum over the degrees of freedom symbolized by the mass of the represented particles. The division by $T^4$ is of course really to extract a number, which essentially counts the number of degrees of freedom contributing to the difference in force on the two sides of the wall.

The range of Standard Model masses extends over a logarithmic factor of $\ln m_t/m_e \simeq 12.7$. These particles have 10 bosonic and 84 fermionic component degrees of freedom. So there are $94/12.7 = 7.4$ components per $e$-factor in the mass range. The pre-factor of $7/8$ for fermions and 1 for bosons in the expressions (162) and (163) for the “Planck pressure” averaged over the 94 components is 0.89. We can now estimate the value of the expression (122), by replacing the sum by an integral over $\ln m$:

$$\sum_m F(T, m) \approx 7.4 \int F(T, m) \, d\ln m \approx 0.89 \times \frac{2}{3} \times 7.4\sigma \ln \frac{v_{\text{without}}}{v_{\text{with}}}.$$  \hfill (123)
Here $\sigma = \pi^2/60$ is the Stefan-Boltzmann constant, while $v_{\text{with}}$ denotes the Higgs field VEV in the \textit{with} condensate phase, and $v_{\text{without}}$ is the Higgs-VEV in the \textit{without} condensate phase. The ratio of these two expectation values of course leads to the same ratio for the masses between the two phases for all the particles obtaining their masses from the Higgs field.

We now explain the origin of the factor $\ln \frac{v_{\text{without}}}{v_{\text{with}}}$ in equation (123). Once we divide the $T^4$ factor out of the pressures, the pressure on one side of the wall divided by the $T^4$ only depends on the ratio of $T$ to the mass in the phase in question. The pressure from one species on one side of the wall has, after the $T^4$ division out, the shape of an almost flat curve in the high temperature region compared to the mass and is essentially zero on the low temperature side. In order to construct (123) we need the difference between two graphs of this type. For the average of this difference the exact form of the transition between the high temperature and low temperature regions does not matter, since it is the same for both the mass in the with condensate phase and in the without condensate phase. The logarithmic range between the low temperature cut-offs for a certain species, in the without condensate phase and the with condensate phase respectively, is of course $\ln \frac{v_{\text{without}}}{v_{\text{with}}}$.

Assuming a Higgs field ratio of $v_{\text{without}}/v_{\text{with}} = 2$ we get

$$7.4 \int F(T, m) \, d\ln m \approx 0.89 \times 2/3 \times 7.4 \times \left(\frac{\pi^2}{60}\right) \ln 2 = 0.50.$$  \hfill (124)

Approximating the form of $F(T, m)$ by a delta function in $\ln(m/T)$ and, assuming the function $T^4$ is smooth for this purpose, we get the total force per unit area at the temperature $T$ to be

$$\sum_m T^4 F(T, m) = 7.4 \int T^4 F(T, m) \, d\ln m = 0.50 T^4.$$  \hfill (125)

In order that this pressure (125) from the Standard Model particles should prevent a ball from collapsing, it must be greater than the pressure $2S/R$ due to the surface tension. Hence the radius $R$ of a ball, surviving (before being stabilised by collecting nucleons) to a temperature $T$ without collapse, must be greater than a critical radius $R_{\text{crit}}(T)$ given by

$$\frac{2S}{R_{\text{crit}}(T)} = 0.50 T^4,$$  \hfill (126)

leading to

$$R_{\text{crit}}(T) = \frac{4.0}{T} \left(\frac{28 \text{ GeV}}{T}\right)^3.$$  \hfill (127)

For example at $T = 10$ MeV you obtain

$$R_{\text{crit}}(10 \text{ MeV}) \approx \frac{4.0}{10 \text{ MeV}} 2800^3 \approx 2 \text{ mm}.$$  \hfill (128)

This means that even at the temperature scale $T = 10$ MeV, when we can get balls supported from collapse by their baryon content, the balls that can contract at all are smaller than our “observed” ball, whose size at this stage is of order 6 m. Thus a ball of the “observed” size really expands at this temperature rather than contracting!
B.4 “Nucleon conduction”

Our main suggestion is that at present the balls with an NBS-condensate are spanned out by an excess of baryon number density inside the balls. Thus it is crucial for the functioning of our picture that in a temperature interval around $T \approx 10$ MeV, when the Boltzmann distribution can be hoped to arrange to get the baryon number concentrated in the inside of the balls, such a concentration really happens. Such a concentration can be imagined to occur at least in two ways:

- 1. There is enough “conductivity” for this baryon number, so that the baryon number distributes itself over the full interior of the ball.

- 2. The walls run around so much in the $\Delta V = 10$ MeV era that they sweep up all the nucleons and get them into the with condensate phase.

For the purpose of possibility 1, we must estimate the “baryon conductivity” $\kappa_b$ for the spreading of baryon number density in the (Planck) plasma.

We define a “baryon number conductivity” $\kappa_b$ so that the flow of the baryon number through the plasma $\vec{j}_b(\vec{x})$ is given as

$$\vec{j}_b = \kappa_b \nabla \rho_b(\vec{x}).$$

(129)

Here $\rho_b(\vec{x})$ denotes the number of baryons per cubic metre minus the number of anti-baryons in the same volume around the point $\vec{x}$ (really we shall use it mainly here in the situation where there are just nucleons and no anti-baryons). The constant coefficient $\kappa_b$ can be estimated in terms of the mean free path $\lambda_b$ and the typical velocity $v_b$ for a nucleon in the plasma.

A nucleon hitting a typical plasma particle, taken here to be an electron or a positron, only gets scattered by a small angle. The momentum transfer in such a small angle collision of the nucleon (proton) will only be of the order of the momentum of an electron or positron in the plasma at the temperature $T$. However a scattering by an angle of order unity of say the proton requires a momentum transfer of the order of $\sqrt{m_N \ast T}$. Using the hypothesis of the small angles adding up like a random walk, we require that the momentum transferred in the single collisions of order $T$ with random signs add up to $\sqrt{m_N \ast T}$. That is to say we need of the order of

$$n_{coll} \approx \left(\frac{m_N}{T}\right)^2 = m_N/T$$

(130)

small angle scatterings. We take the nucleon transport cross section [47] to be

$$\sigma_b \approx \frac{g^4 \ast 2\pi \ln(1/\theta_{cut})}{(4\pi)^2 T^2} \approx \frac{g^4}{8\pi} T^{-2}$$

(131)

Here $\ln(1/\theta_{cut})$ is the Coulomb logarithm[15] in which $\theta_{cut}$ is the order of magnitude of the forward scattering angle below which, due to Debye screening, the scattering can no

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[15] We shall ignore the Coulomb logarithm here. Its effect would be to decrease our estimate of the baryon number conductivity $\kappa_b$ and would, thereby, strengthen our conclusion that the diffusion of nucleons in the plasma is too slow for them to collect inside the balls.
longer be regarded as Coulomb scattering. For the density of particles in the plasma we take

\[ n_{\text{plasma}} \approx n_s \frac{3}{4} \left( \zeta(3)/\pi^2 \right) T^3 \approx 0.1 n_s T^3. \]  

(132)

So we end up with the following estimate of the mean free path for a nucleon in the plasma:

\[ \lambda_b \approx \frac{n_{\text{coll}}}{\sigma_b n_{\text{plasma}}} \approx \frac{n_{\text{coll}} 8\pi}{n_s g^4 T^{-2} \times 0.1 \times T^3} \approx \frac{250 m_N}{n_s g^4 T^2}. \]  

(133)

The typical velocity of a nucleon giving rise to the baryon “conduction” is \( v_b \approx \sqrt{T/m_N} \) and we note that it moves about a distance \( \lambda_b \) in a random direction between collisions. So we estimate

\[ \kappa_b \approx \frac{v_b \lambda_b}{3} \approx \frac{80 \sqrt{m_N}}{n_s g^4 T^{3/2}}. \]  

(134)

On dimensional grounds, the time scale \( t_b \) for getting the baryon number smoothed out over a region, say a ball, of size \( R \) will then be of order

\[ t_b \approx \frac{R^2}{\kappa_b} \approx \frac{n_s g^4 T^{3/2} R^2}{80 \sqrt{m_N}}. \]  

(135)

If we here insert the relation \( R \approx l_T = (6 \text{ m} \times 10 \text{ MeV})/T = 3 \times 10^{14}/T \) for the ball size in the era of temperature \( T \) (before the ball collapsed) from equation (15) valid for the typical ball being present today in dark matter, we get

\[ t_b \approx \frac{n_s g^4 (3 \times 10^{14})^2 T^{-1/2}}{80 \sqrt{m_N}} \approx \frac{n_s g^4 \times 7.5 \times 10^5}{\sqrt{m_N T}} \text{ s MeV.} \]  

(136)

For a temperature \( T = 10 \text{ MeV} \) and say \( n_s \approx 4 \) this diffusion time for the baryon number becomes

\[ t_b \approx \left\{ \begin{array}{ll}
30000 \text{ s} & \text{for } g^2 = 1 \\
3 \text{ s} & \text{for } g^2 = 1/100.
\end{array} \right. \]  

(137)

Comparing these numbers with the Hubble time at \( T = 10 \text{ MeV} \) being 0.01 s, we see that there is not sufficient time to have the baryon number just distribute over the uncontracted balls by diffusion. So we seriously need the alternative 2 of wall motion, which we consider in appendix [B.5] in order to ensure the collection of the nucleons into the with condensate phase.

**B.5  Fading out of wall motion**

Let us estimate the decay rate for a wave of definite wave number \( \vec{k} \) of vibration running along an otherwise imagined flat wall between the two phases of vacuum. We are interested in how fast the inertia of the motion of the wall associated with such a vibration gets damped out. Now, in the neighborhood of the wall, there will appear a motion of the surrounding plasma partly following the motion of the wall itself. In the approximation valid for our case the equations describing the effective two dimensional motion of the plasma caused by viscosity, due to the wave on the wall, are of the form that leads to
harmonic functions. At least one may easily estimate that the major layer of moving plasma must extend away from the wall a distance of the order of magnitude of the wavelength of the wave on the wall. Suppose, for example, that the plasma is only disturbed in a thin layer (compared to the wavelength) along the wall. Then, thinking of the plasma as an incompressible fluid, this plasma material has to move much faster parallel to the wall than perpendicular to the wall (in which direction it is enforced to move to follow the wall). But this is intuitively unrealistic. Also, if the extension of the disturbed plasma away from the wall were much longer than the wavelength, it would mean that much bigger amounts of plasma would have to move than needed if the penetration depth were only of the order of the wavelength. Thus it follows that the motion of the plasma extends away from the wall by a distance of the order of the wavelength. We assume - but it follows from very reasonable estimates about the viscosity of the primordial plasma - that the thickness of the plasma layer crudely following the wall vibration is of the order of the wavelength of the wave on the wall.

Letting $\mu$ denote the viscosity for the plasma, the flow of momentum through the plasma is given by

$$\vec{T}(\vec{x}) = \mu \nabla \vec{v}(\vec{x}),$$

(138)

where $\vec{v}(\vec{x})$ is the velocity field and thus a function of the position $\vec{x}$. If the local velocity of the wall is $v_w$, the nearby plasma will of course follow it by having $\vec{v}(\vec{x}) = \vec{n} v_w$ near this point, where $\vec{n}$ is a unit vector normal to the wall. This co-motion of the plasma will die out over a distance of the order of the wavelength $\approx 1/k$ and thus the taking of the gradient $\nabla$ essentially means multiplication by a factor $k$. So, as time passes, the wall has to deliver to the plasma a flow of momentum - that of course will damp its own momentum density - per unit area of the order

"rate of loss of momentum per unit area" $\approx -\mu|k| * v_w.$  

(139)

Since the mass density on the wall is $S$, this means an acceleration of the wall (locally) given by

$$S \dot{v}_w \approx -\mu|k| v_w.$$  

(140)

It follows that the motion of the wall $v_w$ relative to the bulk (far away) plasma is being damped out with a decay rate $-\dot{v}_w/v_w$. Thus the survival time for such an excess velocity $\tau_i$ or the survival time for inertia of the wall is of the order

$$\tau_i = -v_w/\dot{v}_w \approx S/|\mu| \approx \text{"wavelength"} * S/\mu$$

(141)

Now the viscosity $\mu$ of the plasma is given as

$$\mu \approx \rho * \frac{1}{2} \bar{u} \lambda$$

(142)

where $\rho = n_s * \pi^2 / 30 * T^4$ is the energy density, $\bar{u} \approx 1$ is the root mean square speed of the particles in the plasma and $\lambda$ is the mean free path in the plasma. The density of particles in the plasma $[19]$ is $n = n_s * \zeta(3) / \pi^2 T^3$ where $n_s$ is the number of active species weighted with their number of polarisations and a factor $3/4$ for fermions. The cross section for such
a particle of the effectively (relative to $T$) massless type and interacting with a coupling constant $g$ behaves for dimensional reasons as $\sigma \approx g^4 T^{-2}$. Thus the mean free path in the plasma becomes

$$\lambda \approx 1/(n\sigma) \approx \frac{\pi^2}{n_s \zeta(3) g^4 T}$$

and we obtain the following expression for the viscosity of the plasma

$$\mu \approx n_s \frac{\pi^2}{60} T^4 \frac{\pi^2}{n_s \zeta(3) g^4 T} = \frac{\pi^4 T^3}{60 \zeta(3) g^4}.$$  (144)

The survival time (141) of the inertial motion for a wave number $k$ thus becomes

$$\tau_i \approx S \frac{60 \zeta(3) g^4}{\pi^4 T^3 |k|} \approx \left(\frac{S^{1/3}}{T}\right) \frac{g^4}{|k|} \approx \text{“wavelength”} \ast g^4 \left(\frac{S^{1/3}}{T}\right)^3.$$  (145)

As an example let us consider waves on a typical ball (15) for which the relevant “wavelength” is

$$R \approx l_T = (6 \text{ m} \ast 10 \text{ MeV})/T$$  (146)

and estimate whether the waves would survive the Hubble time $t$. The decay time due to being stopped by the plasma for the waves obeying (15) relative to the Hubble time is given by

$$\tau_i = \frac{l_T \ast g^4 S}{\text{MeV}^2 s/T^2} = g^4 \left(\frac{2 \text{ GeV}}{T}\right)^2.$$  (147)

We see that if $g^2 \approx 1$, i.e. for a large $g^2$, the stopping power of the plasma is sufficient to brake the wall movements in a Hubble time for temperatures above 2 GeV. But, if the coupling squared is rather $g^2 \approx 1/100$, then the braking power will be sufficient to stop the wall in a Hubble time for temperatures above 20 MeV.

It follows that the motion of the walls relative to the plasma will be stopped at the weak scale. However they will be stopped in a curled up state and start moving again at a lower temperature of $T = 2$ GeV for $g^2 = 1$ or $T = 20$ MeV for $g^2 = 1/100$. So at these later times, especially in the 10 MeV era relevant for catching the nucleons, we have rather freely moving walls.

This situation of rather freely moving walls is very important for the capturing of the nucleons into the balls. The point is that, because of the clashing of the walls against each other and crossing each other or being reflected against each other, almost any region of space will get passed over by some wall. Thus the nucleons anywhere will meet a wall and get trapped on the side of the wall with the condensate phase. Thereby alternative 2 of appendix B.4 is justified to work, and all the nucleons indeed get collected into the with condensate phase (before the collapse of the balls).

### B.6 Effective number of degrees of freedom $n_{\text{eff}}$

In this subsection we shall estimate the value of the critical size $R_{\text{crit}}(T)$ (127) more accurately. Especially we have to find out the value of the coefficient $n_{\text{eff}}$ representing the number of active field components, which can interact with the borderline wall and
thereby stop the contraction of the ball. For that calculation we must have in mind that the wall between the two phases only interacts via the variation in the Higgs field across the borderline surface. Remember that throughout this paper we have assumed that there is half as big a Higgs VEV in the condensate phase as in the “without” condensate phase (although our crude estimate (109) in appendix A.4 gave a ratio of 0.3). But to the majority of particles - the ordinary quarks and leptons - this Higgs field only couples via the mass. So the interaction with the wall comes only via the mass squared being different in the two phases. When a particle tends to cross the wall it can quantum mechanically risk to be reflected rather than, as happens under the typical conditions of ultra-relativistic particles just passing through the wall, being deflected only by a small angle. As already explained, the parameter on which the reflection rate for a particle \( f \) depends is the change in mass squared of the particle

\[
\Delta m^2_{\text{at wall}} = g_f^2 (<\phi_h>_{\text{no condensate phase}}^2 - <\phi_h>_{\text{condensate phase}}^2) 
\]

(148)

\[
= g_f^2 <\phi_h>_{\text{no condensate phase}}^2 (1 - (1/2)^2) = \frac{3}{4} m_f^2. 
\]

(149)

Consider now a particle of type \( f \) in an energy and momentum eigenstate scattering on a wall, in say the xy-plane perpendicular to the z-direction. Then the wave function for the situation in which this scattering goes on continuously is of the form

\[
\psi_{\text{in}}(\vec{r}) = \exp i\vec{k} \cdot \vec{r} + R \exp i(k_x x + k_y y - k_z z) 
\]

(150)

\[
\psi_{\text{out}}(\vec{r}) = T \exp i\vec{k'} \cdot \vec{r}, 
\]

(151)

on the incoming and outgoing sides of the wall respectively. Here of course \( k'_x = k_x \) and \( k'_y = k_y \). The coefficients \( R \) for the reflected wave and \( T \) for the transmitted wave are to be determined from the continuity conditions

\[
1 + R = T 
\]

(152)

\[
ik_z - ik'_z R = ik'_z T, 
\]

(153)

and the condition of the energy eigenstate being the same on both sides of the wall

\[
E = \sqrt{m_f^2 + \vec{k}^2} = \sqrt{m'_f^2 + \vec{k'}^2}. 
\]

(154)

This of course implies

\[
m_f^2 + \vec{k}^2 = m'_f^2 + \vec{k'}^2, 
\]

(155)

and thus

\[
\Delta m^2 = (k_z - k'_z)(k_z + k'_z). 
\]

(156)

Here of course say \( m_f = g_f <\phi_h>_{\text{condensate phase}} \) and \( m'_f = g_f <\phi_h>_{\text{no condensate phase}} \) or oppositely. We easily find that the continuity equations transform to give

\[
k_z - k'_z = (k_z + k'_z)R, 
\]

(157)

from where we get

\[
R = \frac{\Delta m^2}{(k_z + k'_z)^2}. 
\]

(158)
Thus the reflection probability in such a scattering is given as

\[ |R|^2 = \frac{(\Delta m^2)^2}{(k_z + k'_z)^4}. \] (159)

We see here that the rate of reflection cuts off rather quickly - by a fourth power - as the \( k_z \) or \( k'_z \) momentum becomes bigger than \( \Delta m'' = \sqrt{\Delta m^2} \). On the other hand, at least if the \( k_z \) for a particle on the way towards scattering on the wall is so small that it is kinematically impossible to enter the other phase, the reflection rate must of course be \( |R|^2 = 1 \). So when \( k_z \) or \( k'_z \) is of the order of \( \Delta m'' \) we expect to get reflection rates of order unity. For large values of \( k_z \approx k'_z \), our above estimate for the reflection probability becomes

\[ |R|^2 \approx \frac{(\Delta m^2)^2}{16|k_z|^4}, \] (160)

an approximation that of course cannot be true unless \( \frac{\Delta m''}{2|k_z|} < 1 \). If this inequality is not fulfilled, we may rather take it that the reflection probability \( |R|^2 \) is of order unity.

It follows that the pressure exerted by a gas of such particles \( f \) on the wall will appear as if only the fraction of the particles in this gas with

\[ |k_z| < \frac{\Delta m''}{2} \] (161)

were present. We namely ignore the very rapidly falling off tail of the distribution with a reflection probability \( |R|^2 \) going as the inverse fourth power of \( |k_z| \). In fact we may crudely hope this may compensate for another of our approximations being erroneous to the opposite side: we take the reflection probability to be unity when \( k_z \) obeys (161), although of course there is a chance for the particle not to be reflected.

Suppose now that the temperature \( T \) is large compared to the “mass difference” \( \Delta m'' \). Then, if all the particles were unable to penetrate through the wall but got reflected, the gas would act like Planck radiation and each polarisation component would exert a pressure equal to \( 1/3 \) of the Planck energy density for one state of polarisation. This “Planck pressure” is given as follows for bosons and fermions respectively:

\[ \text{“Planck pressure”}_b = \frac{2}{3} \sigma T^4 \text{ for one-component bosons} \] (162)

\[ \text{“Planck pressure”}_f = \frac{2}{3} \frac{7}{8} \sigma T^4 \text{ for one-component fermions,} \] (163)

where \( \sigma \) is the Stefan-Boltzmann constant

\[ \sigma = \frac{2k^4 \pi^5}{15c^2 h^3} = \frac{\pi^2}{60}. \] (164)

But now only the particles with \( p_z \) in a narrow interval around zero of width \( \Delta m'' \) contribute effectively to this pressure. So, in the fermion case, the true effective pressure
for each component is reduced by the factor

$$\frac{\text{pressure}_f}{\text{Planck pressure}_f} = \frac{\langle \Delta m'' \delta(k_z) \rangle_f}{\langle \Delta m''(k_z) \rangle} \text{ energy density weighted}$$  \hspace{1cm} (165)

$$= \frac{\Delta m'' \int \delta(k_z)(\exp(kz/T) + 1)^{-1} * k |dE_k|}{\int (\exp(kz/T) + 1)^{-1} * k |dE_k|}$$  \hspace{1cm} (166)

$$= \frac{\Delta m'' \int_0^\infty (\exp(kz/T) + 1)^{-1} 2\pi k^2 dk}{\int_0^\infty (\exp(kz/T) + 1)^{-1} 4\pi k^3 dk}$$  \hspace{1cm} (167)

$$= \frac{\Delta m'' \int_0^\infty (\exp(kz/T) + 1)^{-1} 2\pi k^2 dk}{\int_0^\infty (\exp(kz/T) + 1)^{-1} 4\pi k^3 dk}$$  \hspace{1cm} (168)

$$= \frac{\Delta m'' \int_0^\infty (\exp(kz/T) + 1)^{-1} 2\pi k^2 dk}{\int_0^\infty (\exp(kz/T) + 1)^{-1} 4\pi k^3 dk}$$  \hspace{1cm} (169)

The corresponding reduction factor for bosons is given by

$$\frac{\text{pressure}_b}{\text{Planck pressure}_b} = \frac{\Delta m'' \zeta(3) \Gamma(3)}{2T \zeta(4) \Gamma(4)} = \frac{\Delta m''}{T} \ast 0.185.$$  \hspace{1cm} (170)

We can now define the effective number of degrees of freedom at a temperature $T$ inside the ball for fermions

$$(n_{eff})_f = \sum_{\text{relevant particles}} g_f \frac{\Delta m''}{T} \ast 0.159$$  \hspace{1cm} (171)

and for bosons

$$(n_{eff})_b = \sum_{\text{relevant particles}} g_b \frac{\Delta m''}{T} \ast 0.185.$$  \hspace{1cm} (172)

Here the sum over relevant particles means that only those particles, which have sufficiently low mass as to be present in appreciable amounts at the temperature $T$ in question, are to be counted. The degeneracy factors $g_f$ and $g_b$ are simply equal to the number of spin states for each particle (including the spin states of the antiparticle if it is distinct from the particle). So finally we obtain the expression for the total pressure exerted on the wall by the plasma inside to be:

$$\text{Pressure} = \left( (n_{eff})_f \ast \frac{7}{8} + (n_{eff})_b \right) \ast \frac{2}{3} \sigma T^4 = n_{eff} \ast \frac{2}{3} \sigma T^4,$$  \hspace{1cm} (173)

where

$$n_{eff} = \left( (n_{eff})_f \ast \frac{7}{8} + (n_{eff})_b \right)$$  \hspace{1cm} (174)

is the total effective number of degrees of freedom.

For example, if we consider a temperature of $T = 2$ MeV, the heavy quarks and leptons are not relevant particles. Even the lightest family $u$ and $d$ quarks are presumably effectively confined inside nucleons and are not truly present. Also, apart from the massless photon, the gauge bosons are no longer appreciably present. Thus only electrons and
positrons contribute to the effective number of degrees of freedom at \( T = 2 \) MeV. The degeneracy factor for the electron and positron is \( g_e = 2 \times 2 = 4 \) and so we obtain the effective number of degrees of freedom at \( T = 2 \) MeV to be:

\[
(n_{\text{eff}})_f \approx \frac{4^{\Delta m''}}{2 \text{ MeV}} \times 0.159 \approx \frac{4 \times \sqrt{3/4} \times m_e}{2 \text{ MeV}} \times 0.159 \approx 0.14. \tag{175}
\]

Here \( m_e \) is of course the electron mass.

Finally we can write down an improved formula for the critical size \( R_{\text{crit}}(T) \) conceived of as a ball radius,

\[
2S/R_{\text{crit}}(T) = \text{Pressure} = n_{\text{eff}} \times \frac{2}{3} \sigma T^4. \tag{176}
\]

Thus we get

\[
R_{\text{crit}}(T) = \frac{3S}{n_{\text{eff}} \sigma T^4}. \tag{177}
\]

### B.7 Nuclear physics inside the ball

As the ball contracts after the plasma has gotten so thin that it cannot carry the pressure of the wall any longer the nucleons will, if the temperature is under about 10 MeV, be kept inside the ball and their density will increase significantly. That will of course change the balance between bound and free nucleons. In thermodynamic equilibrium the temperature \( T_{\text{NUC}} \) at which some bound state having nucleon number \( A \) - say \( ^4\text{He} \) - with binding energy \( B_A \) occurs in similar amounts as the nucleons is given by \[19\]

\[
T_{\text{NUC}} = \frac{B_A/(A-1)}{\ln \eta^{-1} + 1.5 \ln(m_N/T_{\text{NUC}})}. \tag{178}
\]

Here \( \eta \) is defined as the number of nucleons divided by the number of photons at the relevant temperature, i.e \( \eta = n/n_\gamma \approx n/T^3 \).

Before the contraction of the balls, when the total volume inside the balls was of the same order as the outside ball volume, the value of the nucleon photon ratio \( \eta \) should have been of the same order as obtained in standard Big Bang Nucleosynthesis fits. However those baryons which at the end make up our dark matter balls were also present at this early stage. So rather than the usual fitted value from Big Bang Nucleosynthesis \( \eta \approx 6 \times 10^{-10} \), there would have been a factor of 7 bigger value \( \eta \approx 4 \times 10^{-9} \). But now when the balls contracted and the nucleons were kept inside, while the photons could rather escape, the value of \( \eta \) inside the ball increased drastically. If the temperature inside the ball got in balance with the outside even after the contraction of the ball radius, say from 6 m to 0.67 cm, the \( \eta \)-parameter would go up by a factor of \( 7 \times 10^8 \). It would then end up with the value \( \eta \approx 3 \). This is what would happen, if the collapse happened just at the 10 MeV era. However if for example the contraction first happened, as we rather expect from \[68\], at 2.3 MeV there would be a stronger contraction by a factor of 4.3 in distance and the final value of \( \eta \) would be a factor \( 4.3^3 \) higher say \( \eta \approx 240 \). But before reaching the full contraction of today’s dark matter, we expect that nuclear reactions could set in and make first mainly helium nuclei which then a little later fuse to heavier elements.
For instance at a temperature scale of 2.3 MeV, which is our above estimate for the typical temperature at contraction of the balls, we achieve the critical relative density $\eta$ of nucleons to photons from equation (178) with $T_{NUC}$ put equal to 2.3 MeV. This gives us the following equation for the value of $\eta$ at our supposed contraction era

$$2.3 \text{ MeV} = \frac{(28 \text{ MeV})/(4 - 1)}{-\ln \eta + 1.5 \ln(m_N/2.3 \text{ MeV})}.$$  

Here we have used the helium binding energy $B_4 = 28$ MeV. Hence helium gets thermodynamically favoured at a temperature of 2.3 MeV for the critical value of $\eta$ given by

$$\ln \eta = 1.5 \times 6.0 - \frac{28 \text{ MeV}}{2.3 + 3 \text{ MeV}} = 5.0.$$  

So the helium formation gets thermodynamically favoured once the contraction has gone so far that $\eta$ becomes about $\exp(5.0) = 150$. This value is indeed reached during the contraction in as far as the contraction ends up having increased $\eta$ to 240. A priori the thermodynamical favouring of the helium does not necessarily mean that the helium is formed immediately. However we are here talking about a rather high temperature situation and it may not take long for the helium to form. (This question of the rate of the transition to the various elements such as helium deserves further study.)

Helium is just an example of a nucleus that will be formed, although it can be seen to be the first one getting thermodynamically favoured as $\eta$ increases. In fact we find the crucial number $B_A/(A - 1)$ in the numerator of formula (178) to be $28/3 = 9.4$ MeV for helium, while for the competing candidate carbon one gets $92.2/11 = 8.4$ MeV. So, although it is not a convincingly big difference, the suggestion is that helium formed first and then the heavier nuclei like carbon later. Outside a ball, in the ordinary big bang nucleosynthesis story, you hardly get any carbon at all; carbon is rather formed much later in the stars. For example, we find that the critical value of $\eta$ needed to thermodynamically favour the formation of carbon becomes $\ln \eta = 5.4$, which is to be compared with the value of $\ln \eta = 5.0$ for helium. This means that the critical $\eta$ is $\exp(0.4) \approx 1.5$ times bigger for carbon than for helium.

Although the limits for thermodynamical favouring of the formation of different size nuclei is not so great measured in terms of the critical $\eta$, the moments during the contraction when the formations truly take place could be more different. The point is that the Coulomb barrier for the formation of heavier nuclei is larger than for helium. Indeed the height of the Coulomb barrier is in the range of a few MeV, so that it can be quite significant at our estimated contraction temperature of 2.3 MeV.

So it is highly suggested that helium forms first and then later on the heavier elements. If indeed there is some bottle-neck, due e.g. to the Coulomb barrier, appreciably delaying the formation of the heavier elements compared to the helium, then when finally the heavy elements get formed the process will happen explosively. Also by that time the contraction of the balls may have almost come to the end and thus they may have the sizes closer to those of today. At such a late time, with all the balls being already small,

\footnote{For simplicity we shall use the nuclear binding energies in the normal vacuum without a bound state condensate.}
there will no longer be big balls around to take up emitted nucleons. Under the explosive
fusion of helium to heavier nuclei, the excess energy from the higher binding energy in
the heavier nuclei compared to that in helium is likely to be mainly transferred to freely
moving nucleons, which are more loosely bound into the ball than the nuclei. These
nucleons may then escape from the ball due to their explosively increased temperature.
Once the nucleons are outside and all the balls have contracted to be small in size, they
will no longer find any ball that can capture them. Thus these expelled nucleons will
stay around forever in the outside of the balls (i.e. in the phase without condensate) and
actually become the ordinary matter.

In this argument we talked as if neutrons and protons felt completely the same forces
and we just said that nucleons were expelled. One may, however, become worried that,
especially if the ball is already almost contracted to today’s size, there is a Fermi sea
of degenerate electrons extending a bit outside the skin. Thereby a region of electric
field is arranged outside the skin in which the protons are pulled outward due to the
excess of protons inside the skin region. One might then think that only the protons
but not the neutrons would mainly be expelled. This is, however, not true because there
will develop in the interior of the ball, at whatever stage in size the explosion occurs, a
situation corresponding to a chemical potential for which the relative density of neutrons
and protons in the regions outside the ball is about unity. This will mean that the nuclei
inside the ball are more neutron rich statistically than if they were in free space. This
effect of a chemical potential, which if you did not count the electric potential would be
much higher for neutrons than for protons inside the ball, is stronger the more contracted
the ball is at the moment we consider. Really we may just say that, due to the positive
electric potential inside the ball and the Boltzmann distribution, there will be fewer
protons than neutrons compared to what would happen without this potential. It is of
course also anyway expected that, if the degenerate electrons became very copious, the
whole inside material would have to be almost pure neutrons like in a neutron star. So
the inside matter will be more neutron rich than in free space. But this fact then means
that, also following still the effect of the electric potential, the nucleons released during
the explosive fusion of helium and running among the nuclei are more often neutrons
than protons. Actually this excess of neutrons over protons corresponds to a chemical
potential difference just equivalent to the electric potential for the protons inside the ball.
Consequently when the nucleons run out of the ball they will reach a neutron proton
ratio close to unity. So the nucleons coming out of the balls will not deviate much from
the equilibrium distribution usually assumed in estimating the next step of standard Big
Bang Nucleosynthesis.

Actually at an earlier stage of the development of our present dark matter model [5],
we based a calculation of the dark matter to ordinary matter ratio on the hypothesis
that all ordinary matter was expelled from dark matter balls in an explosive fusion of
helium nuclei into heavier nuclei. We simply used the difference in binding energy per
nucleon in helium, which is 7.1 MeV, with that in the heavier elements, which is rather
8.5 MeV. Thus during the fusion process there is an excess energy released per nucleon
of $8.5 \text{ MeV} - 7.1 \text{ MeV} = 1.4 \text{ MeV}$, which is just sufficient to release from the 8.5 MeV
binding one sixth of the nucleons. This then means that there is sufficient energy to
release approximately one sixth of the original nucleons. It then follows that the ordinary
matter would make up one sixth of the total amount of matter (both dark and ordinary matter together). Correspondingly the amount of ordinary matter would be one fifth of the amount of dark matter. It should be remarked that we here ignored the energy needed for the nucleon to escape though the ball-skin into the outside. However, since we assume that the typical balls are on the borderline of expelling their nucleons and collapsing totally away, the total escape potential - composed from the electric potential, the potential for crossing the wall and the chemical potential - will add up to zero.

This would be exactly true for a ball just on the stability borderline. However, according to appendix B.2, the median sized ball has after the collapse a radius $R_{\text{median}}^{(\text{after})}$ that is larger than the radius $R_{\text{border}}^{(\text{after})}$ of the borderline ball by a factor of $2^{4/9}$. Now the degenerate electron pressure inside the ball is related to the Fermi energy of the electrons by \(^{(21)}\), meaning the pressure is proportional to the fourth power of the electron Fermi-momentum $p_f$. This pressure is inversely proportional to the radius of the ball and so the Fermi momentum of the median sized ball is $2^{-1/9}$ times the Fermi momentum for the borderline ball. Now the effective binding energy of the nucleons into a borderline ball is essentially zero. Hence the nucleons in a median sized ball will have an effective binding energy of $(1 - 2^{-1/9})\Delta V \approx 0.74$ MeV. So the energy needed per nucleon to escape from the median sized ball to the outer space is increased from 7.1 MeV to $(7.1 + 0.74)$ MeV = 7.84 MeV. Thus with this correction we rather predict that the amount of ordinary matter relative to all the matter (ordinary plus dark) is 1.4 MeV/ $(8.5$ MeV $+ 0.74$ MeV) $= 1/6.6$. Similarly the amount of ordinary matter counted relative to the dark matter is now predicted to be $1.4/(7.1 + 0.74) = 1/5.6$. This to be compared with the recent Planck satellite result \(^{(38)}\) of 4.9%/26.4% $= 1/5.44$. Accidentally the agreement is better than the uncertainty in our calculation.

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