Implications of Graviton-Graviton Interaction to Dark Matter.

A. Deur\textsuperscript{1*}.

\textsuperscript{1}University of Virginia, Charlottesville, VA 22904

(Dated: May 6, 2009)

Abstract

Our present understanding of the universe requires the existence of dark matter and dark energy. We describe here a natural mechanism that could make exotic dark matter and possibly dark energy unnecessary. Graviton-graviton interactions increase the gravitational binding of matter. This increase, for large massive systems such as galaxies, may be large enough to make exotic dark matter superfluous. Within a weak field approximation we compute the effect on the rotation curves of galaxies and find the correct magnitude and distribution without need for arbitrary parameters or additional exotic particles. The Tully-Fisher relation also emerges naturally from this framework. The computations are further applied to galaxy clusters.

PACS numbers: 95.35.+d,95.36.+x,95.30.Cq

* Present address: Thomas Jefferson National Accelerator Facility, Newport News, VA23606, USA
Cosmological observations appear to require ingredients beyond standard fundamental physics, such as exotic dark matter [1] and dark energy [2]. In this Letter, we discuss whether the observations suggesting the existence of dark matter and dark energy could stem from the fact that the carriers of gravity, the gravitons, interact with each others. In this Letter, we will call the effects of such interactions “non-Abelian”. The discussion parallels similar phenomena in particle physics and so we will use this terminology, rather than the one of General Relativity, although we believe it can be similarly discussed in the context of General Relativity. We will connect the two points of view wherever it is useful.

Although massless, the gravitons interact with each other because of the mass-energy equivalence. The gravitational coupling $G$ is very small so one expects $G^2$ corrections to the Newtonian potential due to graviton-graviton interactions to be small in general. However, gravity always attracts (gravitons are spin even) and systems of large mass $M$ can produce intense fields, balancing the smallness of $G$. Indeed, $G^2$ corrections have been long observed for the sun gravity field since they induce the precession of the perihelion of Mercury. Such effects are calculable for relatively weak fields, using either the Einstein field equations (the non-linearity of the equations is related to the non-Abelian nature of gravity [3]), or Feynman graphs in which the one-graviton exchange graphs produce the Newtonian (Abelian) potential and higher order graphs give some of the $G^2$ corrections. Gravity self-coupling must be included in these calculations to explain the measured precession [3]. The non-Abelian effects increase gravity’s strength which, if large enough, would mimic either extra mass (dark matter) or gravity law modifications such as the empirical MOND model [4].

Galaxies are weak gravity field systems with stars moving at non-relativistic speeds. For weak fields, the Einstein-Hilbert action can be rigorously expanded in a power series of the coupling $k$ ($k^2 \propto G$) by developing the metric $g_{\mu \nu}$ around the flat metric $\eta_{\mu \nu}$. This is known (see e.g. refs. [5], [6]) but we recall it for convenience: $g_{\mu \nu}$ is parametrized, e.g. $g_{\mu \nu} = (e^{k \psi})_{\mu \nu}$, and expended around $\eta_{\mu \nu}$. It leads to:

$$\frac{1}{16\pi G} \int d^4 x \sqrt{-g} g_{\mu \nu} R^{\mu \nu} = \int d^4 x (\partial \psi \partial \psi + k \psi \partial \psi \partial \psi + k^2 \psi^2 \partial \psi \partial \psi + ...) + k \psi_{\mu \nu} T^{\mu \nu} \quad (1)$$

Here, $g = \text{det} g_{\mu \nu}$, $R^{\mu \nu}$ is the Ricci tensor, $\psi^{\mu \nu}$ is the gravity field, and $T^{\mu \nu}$ is the source (stress-energy) tensor. Since our interest is $\psi$ self-interactions, we will not include the source term in the action. (We note that it does not mean that $T^{\mu \nu}$ is negligible: we will use later the fact that $T^{00}$ is large. It means that $T^{\mu \nu}$ is not a relevant degrees of freedom.
in our specific case. This will be further justified later.) A shorthand notation is used for the terms \( \psi^n \partial \psi \partial \psi \) which are linear combinations of terms having this form for which the Lorentz indexes are placed differently. For example, the explicit form of the shorthand \( \partial \psi \partial \psi \) is given by the Fierz-Pauli Lagrangian \[7\] for linearized gravity field.

The Lagrangian \( \mathcal{L} \) is a sum of \( \psi^n \partial \psi \partial \psi \). These terms can be transformed into \( \frac{1}{n+1} \psi^{n+1} \partial^2 \psi \) by integrating by part in the action \( \int d^4x \mathcal{L} \). We consider first the \( \partial \psi \partial \psi \) term. The Euler-Lagrange equation of motion obtained by varying the Fierz-Pauli Lagrangian leads to \( \frac{\partial^2 \psi}{\partial \mu \partial \nu} = -k^2(T^\mu{}^\nu - \frac{1}{2} \eta^\mu{}^\nu Tr(T)) \). Since the \( T^{00} \) component dominates \( T^\mu{}^\nu \) within the stationary weak field approximation, so too \( \partial^2 \psi^{00} \) dominates \( \partial^2 \psi^\mu{}^\nu \) and one can keep only the \( \psi^{00} \) terms in \( \psi \partial^2 \psi \), i.e. in \( \partial \psi \partial \psi \). Finally, after applying the harmonic gauge condition \( \partial^\mu \psi^\mu = \frac{1}{2} \partial^\nu \psi^\kappa \), we obtain for the first term in \( \mathcal{L} \partial \psi \partial \psi \rightarrow \frac{1}{4} \partial_\lambda \psi^{00} \partial^\lambda \psi^{00} \). Higher order terms proceed similarly since they are all of the form \( \frac{1}{n+1} \psi^{n+1} \partial^2 \psi \). The factor in front of each \( \psi^n \partial \psi \partial \psi \) may depend however on how \( g^{\mu \nu} \) is expanded around \( \eta^{\mu \nu} \). For this reason, and because the higher order terms are complicated to derive, we use a different approach to determine the rest of the Lagrangian: we build it from the appropriate Feynman graphs (see Fig. 1) using, with hindsight of previous discussion, only the \( \psi^{00} \equiv \phi \) component of the field. Each term in the Lagrangian corresponds to a Feynman graph: the terms quadratic, cubic and quartic in \( \phi \) correspond respectively to the free propagator, the three legs contact interaction and the four legs contact interactions. The forms \( \phi \partial \phi \partial \phi \) and \( \phi^2 \partial \phi \partial \phi \) (rather than \( \phi^3 \) or \( \phi^2 \partial \phi \) for example for the three legs graph) are imposed by the dimension of \( G \). (Note that in Eq. 1 the origin of the two derivatives in the generic form \( \phi^n \partial \phi \partial \phi \) is from the two derivatives in the Ricci tensor and the absence of derivative in \( g_{\mu \nu} \)).

Since we are considering the total field from all particles, then (neglecting here non-linear effects and binding energies) \( k^2 = \sum m \ 16 \pi G \) where \( m \) is the nucleon mass and \( \sum m = M \) with \( M \) the total mass of the system. This may be modeled with a space that is discretized with a lattice spacing \( d \). We are interested in the attraction between two cubes of \( d^3 \) volume filled with the gravity fields generated by \( N \) sources of similar masses and homogeneous
distribution. Since we are unable to treat \( N \) sources we consider only a global field. Under the field superposition principle, the magnitude of the total field in each cube is proportional to \( N \). As gravity always attracts, we used a global coupling \( \sum_1^N mG = MG \).

Under these simplifications and hypotheses we obtain from Eq. 1:

\[
\int d^4x L = \sum_{\mu=1}^{4} \int d^4x (\partial_\mu \phi \partial_\mu \phi + \frac{\sqrt{16\pi GM}}{3!} \phi \partial_\mu \phi \partial_\mu \phi + \frac{16\pi GM}{4!} \phi^2 \partial_\mu \phi \partial_\mu \phi + \ldots).
\]

To quantify gravity’s non-Abelian effects on galaxies, we have used numerical lattice techniques: A Monte Carlo Metropolis algorithm was employed to estimate the two-point correlation function (Green function) that gives the potential. To test our Monte Carlo, we computed the case for which the high-order terms of \( L \) are set to zero, and recovered the expected Newtonian potential or, when a fictitious mass \( m_\phi \) is assigned to the field \( \phi \), the expected Yukawa potential \( V(r) \propto (e^{-m_\phi r})/r \). We also insured the independence of our results from the lattice size and the physical system size. In our calculations, the usual circular boundary conditions cannot be used. A pathological example is the one of a linear potential, for which a simulation with such conditions would return an irrelevant constant rather than the potential. Instead of circular boundary conditions, we set the boundary nodes of the lattice to be random with an average zero value. These nodes were never updated. In addition, although we update the fields on the nodes close to the boundary nodes, we did not use them in the calculation of the Green function (in the results presented, we ignored the 4 nodes closest to the lattice boundary. We varied this number and found compatible results).

The Green function is shown in Fig. 2. These results are for a lattice of size \( L = 28d \), with \( d \) the lattice spacing, \( \sqrt{\frac{16\pi GM}{d}} = 4.9 \times 10^{-3} \) (\( d \) converts the coupling to lattice units), \( M = 10^{10} M_\odot \) and \( d = 1 \) kpc. For \( r \gtrsim 5 \), the Green function is roughly linear.

We now apply our calculations to the case of galaxies. For homogeneous distributions with spherical symmetry, the net field distortion from the force carrier self-interaction cancels out [11]. Similarly, a cylindrical symmetry reduces the effect. To first order we treat a spiral galaxy as a thin disk with a cylindrical symmetry and approximate \( V(r) \) in Fig. 2 as linear: the force is of constant value \( b \). We are interested in the force between the disk center and the circumference points. The field lines point evenly outward, so the force at any point on the circumference is reduced by \( 2\pi r \): the force is then \( b/(2\pi r) \) and \( V(r) = b \ln(r)/(2\pi) \). Adding back the (unaffected) “Abelian” part \( a/r \), we obtain:
\[ V(r) = -GM \left( \frac{1}{r} + \frac{b}{2\pi a} \ln(r) \right) \]  

(For a homogeneous spherical distribution, the constant force becomes \( b/4\pi r^2 \), leading to a Newtonian potential \( (a + b/4\pi)/r \propto 1/r \), as expected. We note that typically, \( a \gg b/4\pi \).)

We can now look at rotation curves for spiral galaxies. Those, shown in Fig. 3, are obtained by calculating \( a \) and \( b \) for given galaxy masses and sizes and assuming an exponential decrease of the galaxy density with its radius: 
\[ \rho(r) = \frac{M}{2\pi r_0^2} e^{-r/r_0} \]
Galaxy luminous masses and sizes being not well known, we adjusted \( M \) and \( r_0 \) to best fit the data. They can be compared to the luminosity \( L \) of the galaxies and the values of \( r_{SL} \) from Ref. 12 also given in Fig. 3. We did not use \( L \) in the simulation but report it since it indicates a lower bound for \( M \) (consequently, NGC7331 for which \( M < L \) poses a problem within our simple spiral galaxy model). The curves reproduce well the data given our simple model of galaxy. In addition to our rough approximation in of modeling galaxies, it should be emphasized that while our results should conservatively be viewed as indicating quantitatively the self-coupling effects \( 13 \) of the gravity field, there are several caveats: 1) The particular choice of boundary conditions may generate a non-physical artifact, although we checked within the means of our lattice simulation that this was not the case; 2) There are approximations inherent to a lattice calculation, in particular the cut-off on the high energy modes due to the lattice finite spacing; 3) Approximations are used to go from the Einstein-Hilbert action to the polynomial scalar action; 4) We have used an approximate magnitude for the field self-coupling of \( \sqrt{GM} \), which neglects non-linear effects and the specific distribution of sources in the studied system.
FIG. 3: Computed rotation curves (continuous lines) compared to the measurements (squares) and the curves without field self-interaction (dashed lines). We used the values of $R_{25}$ given in Ref. [12] for the galaxy radii, and the values given on each plots for the parameters $M$ and $r_0$. The luminosity $L$ of the galaxies and the values of $r_0$ (noted scale length, SL) from Ref. [12] are also given for comparison (units are $10^9 M_\odot$ for $M$ and $L$ and kpc for $r_0$).

The calculation applies similarly to dwarf galaxies. Results for galaxies DDO 170 and DDO 153 are shown in Figure 4. The results agree with the observation that the luminous mass together with a Newtonian potential contributes especially little to dwarf galaxy rotation curves.

FIG. 4: Dwarf galaxy rotation curves.

The Tully-Fisher relation [14] is readily explainable by non-Abelian effects. This empirical
law relates linearly the log of a galaxy mass to the log of its rotation velocity \( v \): 

\[
\ln(M) = \alpha \ln(v) - \beta,
\]

with \( \alpha = 3.9 \pm 0.2 \) and \( \beta \approx 1.5 \). Equating the centripetal force acting on an object of mass \( m \), 

\[-mv^2 \mathbf{u}_r / r \quad (\mathbf{u}_r \text{ is the unit vector)}
\]

to the gravitational force 

\[-GMb\mathbf{u}_r / (2\pi a r)
\]
given by the potential of Eq. 3 for large distances yields 

\[v^2 = GMb / (2\pi a).
\]

Since \( a \), the coefficient of the Newtonian potential in Eq. 3 is proportional to \( GM \) (\( a = \tau GM \)) and since \( b \) can only be a function of the field self-coupling magnitude, \( b(\sqrt{GM}) \), we have 

\[v^2 = b(\sqrt{GM}) / (2\pi \tau),
\]

which correlates \( v \) and \( M \) since \( G \) is a constant. This qualitatively explains the Tully-Fisher relation. The coefficient \( \alpha \) of the Tully-Fisher relation can be obtained by expanding \( b(\sqrt{GM}) \): 

\[b = b_0 + b_1 \sqrt{GM} + b_2 GM + \ldots.
\]

Without field self-coupling (i.e. setting \( \sqrt{GM} = 0 \) in Eq. 2), \( b = 0 \) so \( b_0 = 0 \). Since \( \sqrt{GM} \gg GM \) we have at leading order

\[\ln(M) = 4\ln(v) + \ln(2\pi \tau / \sqrt{Gb_1}).
\]

We remark that the Tully-Fisher relation is not explained in the dark matter scenario, and is a built-in feature of MOND as are the flat rotation curves.

Dark matter was first hypothesized to reconcile the motions of galaxies inside clusters with the observed luminous masses of those clusters. Estimating the non-Abelian effects in galaxy clusters with our technique is difficult: 1) the force outside the galaxy is suppressed since the binding of the galaxy components increases (this will be discuss further at the end of the Letter), but 2) the non-Abelian effects on the remaining outside field could balance this if the remaining outside field is strong enough. Since clusters are made mostly of elliptical galaxies for which the approximate sphericity suppresses the non-Abelian effects inside them, we ignore the first effect. We assume furthermore that the intergalactic gas is distributed homogeneously enough so that non-Abelian effects cancel (i.e. the gas does not influence our computation). Finally, we restrict the calculation to the interaction of two galaxies, assuming that others do not affect them. With these three assumptions, we can apply our calculations. Taking 1 Mpc as the distance between the two galaxies and \( M=40 \times 10^9 \) \( M_{\odot} \) as the luminous mass of the two galaxies, we obtain \( b = -0.012 \) in lattice units. We express this from the dark matter standpoint by forcing gravity to obey a Newtonian form:

\[V(r) = -G \frac{M}{2} \left( \frac{1}{r} - \frac{b}{a} \right) \equiv -G \frac{M'}{2} \frac{1}{r},
\]

with \( M'/M = 1 - r^2 b/a = 251 \). Gaseous mass in a cluster is typically 7 times larger than the total galaxy mass. Assuming that half of the cluster galaxies are spirals or flat ellipticals for which the non-Abelian effects on the remaining field are neglected, we obtain for the cluster
a ratio \((M'/M)_{\text{cluster}} = 18.0\), that is our model of cluster is composed of 94% dark mass, to be compared with the observed 80-95%.

Non-Abelian effects emerge in asymmetric mass distributions. This makes our mechanism naturally compatible with the Bullet cluster observation \([15]\) (presented as a direct proof of dark matter existence since it is difficult to interpret in terms of modified gravity): Large non-Abelian effects should not be present in the center of the cluster collision where the intergalactic gas of the two clusters resides if the gas is homogeneous and does not show large asymmetric distributions. However, the large non-Abelian effects discussed in the preceding paragraph still accompany the galaxy systems.

In addition to reproducing the rotation curves and cluster dynamics and to explain the Tully-Fisher relation, our approach implies several consequences that can be tested: 1) Since the Non-Abelian distortions of the field are suppressed for spherically homogeneous distributions, rotation curves closer to Newtonian curves should be measured for spherical galaxies; 2) Two spiral galaxies should interact less than a similar system formed by two spherical galaxies. 3) In a two-body system, we expect a roughly linear potential for large enough effective coupling \((\gtrsim 10^{-3})\). This may be testable in a sparse galaxy cluster; 4) The past universe being more homogeneous, and density fluctuations being less massive, the non-Abelian effects should disappear at a time when the universe was homogeneous enough; 5) Structure formations would proceed differently than presently thought since dark matter is an ingredient of the current models, and since those assume an Abelian potential. Particularly, models of mergers of galaxies using a linear potential rather than dark matter constitute another test.

Although the consequences of non-Abelian effects in gravity for galaxies are not familiar, similar observations (increases of a force’s strength at large distance) are well known in sub-nuclear physics. Those, closely related to the confinement of non-relativistic quarks inside hadrons, are fully explained by the theory of the strong nuclear force (Quantum Chromodynamics, QCD). First, QCD is the archetype non-Abelian theory. Second, although the quark color charge is only unity, the QCD effective coupling \(\alpha_{\text{eff}}\) is large at the scale of the nucleon size (about 1 at \(10^{-15}\) m \([16]\)) so the overall force’s intensity is large, as for massive systems in gravity. These are the two ingredients needed to confine quarks: The gluons emitted by the non-relativistic quarks strongly interact with each other and collapse into string-like flux tubes. Those make the strong force to be constant for distances \(r \approx 10^{-15}\)
m rather than displaying an $\alpha_s^{\text{eff}}(r)/r^2$ dependence, see e.g. [17]. We also remark that a relation akin to the Tully-Fisher one exists for the strong force in the confinement regime: the angular momenta and squared masses of hadrons are linearly correlated. These “Regge trajectories” are at the origin of the string picture of the strong force. Lattice techniques are a well developed tool to study gluon-gluon interactions at large distances. Hence, it was a ready-to use tool for our purpose. The simplest lattice QCD calculations displaying quark confinement are done in the “gluonic sector”, that is without dynamical quark degrees of freedom). Similarly, our calculation excluded $T^\mu\nu$, the sources of $\psi$ in the Lagrangian $\mathcal{L}$. We also note that the QCD Lagrangian has a similar structure as $\mathcal{L}$ in Eqs. [1] and [2]. The close analogy between gravity and QCD is the reason we used the particle physics terminology in this Letter. This analogy has been already noticed and discussed, see e.g. [18].

Before concluding, we exploit further the QCD-gravity analogy, now on a qualitative level. The confinement of gluons inside a hadron not only changes the $1/r$ quark-quark potential into an $r$ potential, but also causes two hadrons to not interact through the strong force [19] since there is no strong force carriers outside the hadrons. Similarly, the increased binding inside a galaxy would weaken its interaction with outside bodies. Such reduction of the strength of gravity is opposite to what we would conclude by explaining galaxy rotation curves with halos of exotic dark matter or with gravity modifications, and may be relevant to the fact that the universe expansion is accelerating rather than decelerating. This is currently explained by the repulsive action of a dark energy, see e.g. [2]. However, if gravity is weakened, the difference between the assumed Abelian force and the actual strength of the force would be seen as an additional repulsive effect. Such effect would not explain a net repulsion since it would at most suppress the force outside of the mass system (as for QCD). Thus, it would not be directly responsible for a net acceleration of the universe expansion. Nevertheless, it may reduce the need for dark energy. To sum up, the gravity/QCD parallel propounds that dark energy may be partly a consequence of energy conservation between the increased galaxy binding energy vs. the outside effective potential energy. This would implies a quantitative relation between dark energy and dark matter, which might explain naturally the cosmic coincidence problem [2].

To conclude, the graviton-graviton interaction suggests a mechanism to explain galaxy rotation curves and cluster dynamics. Calculations done within a weak field approximation agree well with observations involving dark matter, without requiring arbitrary parameters
or exotic particles. The Tully-Fisher relation arises naturally in our framework. Our approach hints that dark energy could partly be a consequence of energy conservation between the increased galaxy binding energy and the outside potential energy.

**Acknowledgments:** We thank P.-Y. Bertin, S. J. Brodsky, V. Burkert, G. Cates, F.-X. Girod, B. Mecking, A. M. Sandorfi and Xiaochao Zheng for useful discussions. We are grateful to the reviewer of this Letter for pointing out the Tully-Fisher relation.

[1] B. Sadoulet, Rev. Mod. Phys. **71**, S197 (1999)
[2] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003)
[3] See e.g. R. P. Feynman, F. B. Morinigo and W. G. Wagner, The Feynman Lectures on Gravitation, Westview Press (1995)
[4] M. Milgrom, Astrophys. J. **270**, 365 (1983)
[5] See e.g. A. Salam IC/74/55 (1974) or A. Zee, Quantum Field Theory in a Nutshell, Princeton University Press (2003).
[6] See e.g. C. Kiefer, Quantum Gravity. Oxford Science Publications (2004).
[7] M. Fierz and W. Pauli; Proc. R. Soc. A **173** 211 (1939)
[8] The $\psi^{\mu\nu} |_{\mu\neq0,\nu\neq0}$ terms that are neglected in the operations leading to $\partial\psi\partial\psi \rightarrow \partial_{\lambda}\psi^{00}\partial_{\lambda}\psi^{00}$ can be ignored in front of the $\psi^{00}\partial_{\lambda}\psi^{00}\partial_{\lambda}\psi^{00}$ term: The $\partial\psi\partial\psi$ do not contribute to the field self-interaction so they are irrelevant to the effects we are studying. $\partial_{\lambda}\psi^{00}\partial_{\lambda}\psi^{00}$ dominates these other $\partial_{\lambda}\psi^{\mu\nu}\partial_{\lambda}\psi^{\mu\nu}$. In our results, these should be a correction to a normalization factor (a). Because of the simplicity of our galaxy and cluster models, we ignore such corrections. Likewise, the corrections coming from the modification of the Euler-Lagrange equation of motion enter only at the level of the terms $\psi^{n+1}\partial^2\psi$, with $n \geq 2$.
[9] The additive form of the global coupling is supported by the $G^2$ correction to the precession of the perihelion of the orbit of two bodies of masses $m_1$ and $m_2$, that is given by $\frac{Gm_1m_2}{r^3}(\frac{G(m_1+m_2)}{2r})$, or in dominant one-loop correction to the Newtonian potential established in quantum gravity (i.e. the first order graviton self-interaction): $V(r) = \frac{Gm_1m_2}{r^3}(1 + \frac{3G(m_1+m_2)}{2r})$. Comparison of these corrections to our global coupling are relevant because they stem in part from the field-self interaction. Clearly, our estimate of the magnitude of the
global coupling is naive. In particular, the use of the field superposition principle is inadequate when large non-linear effects are present.

[10] The factors 3! and 4! cancel the combinatorial factors that arise during the calculation of the green function when a Wick contraction is done. Hence, each Feynman graph associated with a $\phi^n \partial \phi \partial \phi$ term has a coupling $G^{(n-2)/2}$ at an $n$-legs vertex rather than $n!G^{(n-2)/2}$.

[11] This can be best pictured in the Quantum Chromodynamics (QCD) case whose similarities with gravity are discussed at the end of the letter. For a non-relativistic two-quark system, gluon field lines attract each other and, if the QCD coupling is large enough, collapse in a flux tube (flux tubes are familiar objects for QCD in its confinement regime). For spherical symmetry, attractions compensate, and there is no net effect.

[12] K. G. Begeman, A. H. Broeils and R. H. Sanders, Mon. Not. R. astr. Soc. 249 523 (1991)

[13] Strictly, it is improper to talk of “non-Abelian effects” with our scalar field $\varphi$ since the gauge parameter would be a (commuting) scalar, as in Quantum Electrodynamics (QED). To underline the origin of the effect discussed here and to avoid confusion with other non-linear effects (e.g. of the type of the small non-linearities of QED, an Abelian theory, or the non-linearity of general relativity), we will still refer to “non-Abelian effects” or “self-coupling” (or “field self-interaction”) in the general discussion but only to “self-coupling” when specifically discussing the results using $\varphi$. Furthermore, we do not discuss any gauge symmetry for the field $\varphi$ since it would be irrelevant (it is relevant for the field $\psi^{\mu
u}$, not its approximation $\varphi$, just as e.g. the fact that the Newtonian potential is not Lorentz invariant). This is another reason why using “non-Abelian” would not be rigorous with $\varphi$. The gauge symmetry for general relativity is the general coordinate transformation, as can be seen since gravity couples to 4-momentum, the conserved quantities stemming from space-time translation invariance. The general coordinate transformation group is non-Abelian. See e.g. B. S. Dewitt Phys. Rev. 162, 1195 (1967).

[14] R. B. Tully and J. R. Fisher, Astronomy and Astrophysics, 54, 661 (1977)

[15] D. Clowe et al., Astro. J. Lett. 648 109 (2006)

[16] A. Deur, V. Burkert, J.P. Chen and W. Korsch; Phys. Lett. B650 244 (2007); Phys. Lett. B665 349 (2008)

[17] N. Brambilla and A. Vairo, [hep-ph/9904330]

[18] M. Reuter and H. Weyer; JCAP 12 001 (2004)

[19] Except for a residual Yukawa force at long distance and quark interchange at shorter distance.