No static regular black holes in Einstein-complex-scalar-Gauss-Bonnet gravity

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In this brief report, we investigate the existence of 4-dimensional static spherically symmetric black holes (BHs) in the Einstein-complex-scalar-Gauss-Bonnet (EsGB) gravity with an arbitrary potential $V(\phi)$ and a coupling $f(\phi)$ between the scalar field $\phi$ and the Gauss-Bonnet (GB) term. We find that static regular BH solutions with complex scalar hairs do not exist. This conclusion does not depend on the coupling between the GB term and the scalar field, nor on the scalar potential $V(\phi)$ and the presence of a cosmological constant $\Lambda$ (which can be either positive or negative), as long as the scalar field remains complex and is regular across the horizon.

I. INTRODUCTION

Although Einstein’s general relativity (GR) has achieved a great success in physics and astronomy, especially after the recent detections of gravitational waves [1–4] and the shadow of the M87 black hole (BH) [5], the theory is still facing some challenges, such as those related to dark energy, dark matter and quantum gravity. Therefore, various modified gravitational theories have been proposed, including $f(R)$ gravity [7, 8], EinsteinÆther (æ-) theory [9–16], Hořava-Lifshitz gravity [17–20], scalar-tensor gravity [21–27], and so on. $f(R)$ gravity generalizes the Einstein-Hilbert action to an arbitrary function of the Ricci scalar. This generalization may be able to explain the accelerated expansion and structure formation of the universe without adding unknown forms of dark energy or dark matter. Einstein-Æther gravity introduces a timelike dynamical unit vector field coupled with metric. This vector field breaks the Lorentz symmetry of the theory. It is then possible to study the preferred frame effects. Hořava-Lifshitz gravity treats time and space unequally at high energy level. In scalar-tensor gravity, a scalar field and a tensor field mediate the gravitational interaction. At present, no any modified theory of gravity can replace GR, but we could still be able to catch a glimpse of dawn to resolve the open problems in theoretical physics, astrophysics and cosmology by exploring them. This also provides us an excellent opportunity to understand GR in different angles.

As a special scalar-tensor theory, the Einstein-scalar-Gauss-Bonnet (EsGB) gravity has attracted lots of attention recently. One of the main motivations is that, as we just enter the era to explore the strong field regime of gravity through the direct detections of gravitational waves and BHs, understanding the effects of higher-order curvature terms becomes more urgent and important. However, the inclusion of such terms often leads to the well-known ghost problem [28]. The existence of ghosts is closely related to the fact that higher-order curvature terms contain time-derivatives higher than two. In the quadratic case, for example, the field equations are generally fourth-orders. Then, following the powerful theorem due to Mikhail Vasilevich Ostrogradsky, who established it in 1850 [29], such a system is generically not stable, unless the degenerate conditions are satisfied [20, 30]. The Gauss-Bonnet (GB) term belongs precisely to the latter: Due to a particular combination of the Riemann and Ricci tensors, it contains no more than the second-order derivative terms, although it consists of the quadratic terms of the Riemann tensor. As an immediate result, the theory is ghost-free. However, when it is minimally coupled to the Einstein-Hilbert action, it becomes a topological term in 4-dimensional spacetimes, and has no contributions to the field equations. To make this term meaningful in 4-dimensional spacetimes, one way is to consider it coupling non-minimally with a scalar field, partially motivated from string/M-Theory [31].

Along this vein, EsGB gravity has been extensively studied in the past decade or so [32–34]. In particular, both static [35–37] and stationary [38] BH solutions have been found, through the so-called spontaneous scalarization, by which an initial GR BH spontaneously grows hairs through a tachyonic instability. The circulation of...
the well-known no-hair theorems is through the non-minimally coupling between the scalar and the GB term, which effectively produces a negative mass-squared term in the scalar field perturbations. It should be noted that spontaneous scalarization has been known for a long time in the studies of neutron stars in a particular class of scalar-tensor theories, in which the effective mass was provided by matter terms of the theory. However, in the EsGB theory, it is provided by the coupling of the GB curvature term with a scalar field.

It should be also noted that in the above studies of the BH spontaneous scalarization, the scalar field was always assumed to be real. In this short Note, we generalize these studies to a complex scalar field, and then ask ourselves if BHs can still exist. The answer is somehow a bit surprising and negative: No static regular BHs with positive temperatures exist in the Einstein-complex-scalar-Gauss-Bonnet (EcsGB) theory with an arbitrary coupling between the GB term and the scalar field and an arbitrary potential \( V(\phi) \), as long as the scalar field remains complex and regular across the horizon. Since \( V(\phi) \) is arbitrary, this also includes the case with a non-vanishing cosmological constant \( \Lambda \), which can be either positive or negative.

Before proving our above claim, we note that in the Standard Model (SM) of particle physics, the Higgs field is a complex scalar with the Standard Model (SM) of particle physics, the Higgs field either positive or negative.

In addition, boson stars in the framework of EcsGB theory were studied recently in \([54, 55]\), while boson stars in GR coupled with a (complex) scalar field but without the GB term have been extensively studied (see, for example, \([56, 58]\) and references therein), since the pioneer work of Kaup \([59]\), and Ruffini and Bonazzola \([60]\).

\section{II. NO-GO THEOREM OF STATIC BHS IN ECSGB THEORY}

The general action of the EcsGB theory takes the form,

\[ S = S_g + S_m, \tag{2.1} \]

where

\[ S_g = \int dx^4 \sqrt{-g} \left( \frac{\alpha}{2\kappa} - \frac{2\Lambda}{\kappa} + \alpha \mathcal{L}_{GB} + \mathcal{L}_\phi \right), \]

\[ S_m = \int dx^4 \sqrt{-g} \mathcal{L}_{m} \left( g_{\mu\nu}, \psi \right), \tag{2.2} \]

with \( \kappa \equiv 8\pi G \), \( \psi \) denoting collectively the matter fields, and

\[ \mathcal{L}_{GB} \equiv f(\phi^*) \mathcal{G}, \]

\[ \mathcal{L}_\phi \equiv -\nabla^\mu \phi \nabla_\mu \phi^* - V(\phi^*), \]

\[ \mathcal{G} \equiv R^2 + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu}, \tag{2.3} \]

where \( \Lambda \) is the cosmological constant, \( c_1 \) and \( \alpha \) are coupling constants, an \( R_{\mu\nu\rho\sigma} \), \( R_{\mu\nu} \) and \( R \) are the Riemann, Ricci tensors and Ricci scalar, respectively. \( \nabla_\mu \) denotes the covariant derivative with respect to the metric \( g_{\mu\nu} \), and \( g = \text{det}(g_{\mu\nu}) \). It’s worth noting that the action requires \( c_1 \geq 0 \) to avoid \( \phi \) being a phantom field. For the \( c_1 > 0 \) case, we can directly redefine the parameters so that \( c_1 = 1 \). On the other hand, the GR terms will vanish when \( c_1 = 0 \), while the effects of the scalar field vanish and the action reduces to GR when \( c_1 \to \infty \).

Before proving our above claim, we note that in the Standard Model (SM) of particle physics, the Higgs field is a complex scalar with the Standard Model (SM) of particle physics, the Higgs field with a non-minimally coupling can serve as the inflaton field \([49, 50]\), and is consistent with all the observations \([51]\). It should be also noted that the Higgs field minimally coupled to gravity usually produces matter fluctuations that are many orders of magnitude larger than those observed, unless a fine-tuned value of the Higgs self-coupling constant \( \lambda \) is invoked \([52, 53]\).

In addition, boson stars in the framework of EcsGB theory were studied recently in \([54, 55]\), while boson stars in GR coupled with a (complex) scalar field but without the GB term have been extensively studied (see, for example, \([56, 58]\) and references therein), since the pioneer work of Kaup \([59]\), and Ruffini and Bonazzola \([60]\).

In such spacetimes, we adopt the standard form of the complex scalar field, \( \phi = \Psi(r)e^{-iwt} \), so the corre-
sponding $T^{GB}_{\mu\nu}$ is real and static, as required by the field equations (2.5) and (2.6), where $w$ is real.\footnote{In the framework of GR coupled with a scalar field, it was found that stationary scalar clouds around a rotating BH can exist \cite{2}, while in the spherically symmetric case, such a cloud exists around a Schwarzschild BH only in time-dependent case \cite{62}, in which the scalar field decays slowly for a long time. If the mass of the scalar field is ultralight, the decay time can be comparable with the age of our Universe. As a result, such ultralight scalar fields can be considered as possible candidates for dark matter \cite{55,56}. For more details, see the review articles \cite{57,58}.}

Substituting the metric and scalar field into the field equations (2.4) and (2.5), we obtain three independent equations with three unknown functions $\Psi(r)$, $A(r)$ and $B(r)$:

$$
\Psi'' + \left(4 + rA' - rB'\right) \frac{\Psi'}{2r} + \left\{ \frac{w^2 - e^A V(\Psi)}{e^{A-B}} - \frac{2\alpha f(1)}{r^2 e^B} (2A'' + A^2) (e^B - 1) + 2\frac{\alpha f(1)}{r^2 e^B} (e^B - 3) A'B' \right\} \Psi = 0, \quad (2.8)
$$

$$
\left[ c_1 e^B r + 8\alpha k (e^B - 3) \Psi \Psi' f(1) \right] e^A B' - 16\alpha k e^A \Psi'' (e^B - 1) f(1) - 16\alpha k e^{A+2B} V r^2 - \kappa \left[ w^2 e^{2B} r^2 + 32\alpha e^A (e^B - 1) \Psi^2 f(2) \right] \Psi^2 + e^{A+B} \left[ c_1 (e^B - 1) - \Lambda r^2 e^B \right] - 16\alpha k e^A \left[ e^B r^2 + 16\alpha (e^B - 1) f(1) \right] \Psi^2 = 0, \quad (2.9)
$$

$$
\left[ c_1 r + 8\alpha k (1 - 3e^{-B}) \Psi \Psi' f(1) \right] e^A A' + 16\alpha k e^A (e^B - w^2 \Psi^2) + e^A \left[ c_1 + e^B (\Lambda r^2 - c_1) - 16\alpha k^2 \Psi^2 \right] = 0. \quad (2.10)
$$

Let us first note that when the scalar field is real ($w = 0$), the Schwarzschild-de Sitter solution is indeed a solution of Eqs. (2.8)-(2.10), if we further set $\Psi = \Psi_0$ and assume that the scalar field stays at its minimum, i.e., $V(\Psi_0) = 0 = f(1)(\Psi_0)$, where $\Psi_0$ is a real constant.

However, as longer as $w \neq 0$, the situation will be completely different. To show this, we first assume that a regular BH exists, and is located at $r = r_h$, so that $e^{A(r_h)} = e^{-B(r_h)} = 0$. Additionally, we assume that the temperature of BH is positive and finite, and that the scalar field is also regular across the horizon. Then, near the horizon the metric and the scalar field can be expanded as \cite{33,34},

$$
e^A = \sum_{i=1}^{\infty} F_i (r - r_h)^i,
$$

$$
e^{-B} = \sum_{i=1}^{\infty} H_i (r - r_h)^i,
$$

$$
\Psi = \sum_{i=0}^{\infty} \Psi_i (r - r_h)^i, \quad (2.11)
$$

where $F_i$, $H_i$ and $\Psi_i$ are constant coefficients. Substituting the above functions into the field equations, and then expanding them near the horizon $r_h$, we find

$$
\frac{w^2 \Psi \Psi r^2_h}{\alpha F_1 H_1 c^2(1)} + O(r - r_h) = 0,
$$

$$
\frac{\kappa w^2 \Psi^2 \Psi r^2_h}{F_1 \left( c_1 r_h + 8\alpha k \Psi_0 \Psi f(1) \right)} + O(r - r_h) = 0,
$$

$$
\frac{\kappa w^2 \Psi^2 \Psi r^2_h}{H_1 \left( c_1 r_h - 4\alpha k \Psi_0 \Psi f(1) \right)} + O(r - r_h) = 0, \quad (2.12)
$$

where $f_c(1) \equiv f(1)(\Psi_0^2)$. Thus, to the zeroth-order of $r - r_h$, we have $w \Psi r^2_h = 0$. Therefore, there are three possibilities: $r_h = 0$, $w \neq 0$, and $\Psi = \Psi_0$.\cite{43} For BHs with complex scalar hairs, the above shows that we must have $\Psi_0 \equiv \Psi(r = r_h) = 0$.

After substituting $\Psi_0 = 0$ into Eqs. (2.8)-(2.10), to the first order of $r - r_h$, we find

$$
F_1 = \frac{c_1 - (\kappa V(0) + \Lambda) r^2_h}{c_1 r_h},
$$

$$
H_1 = - \frac{c_1 r_h w^2}{c_1 - (\kappa V(0) + \Lambda) r^2_h}, \quad (2.13)
$$

from which we obtain $F_1 H_1 = - w^2 < 0$. Then, the temperatures of such BHs $T_h = \sqrt{F'(r_h) H''(r_h)}/4\pi$ become imaginary, which is physically unacceptable. In other words, static spherically symmetric BHs with a complex scalar field does not exist in the EcsGB theory.

### III. CONCLUDING REMARKS

In this short brief report, we have shown that static regular BHs in EcsGB theory do not exist, as longer as the scalar field remains complex and the BH is regular and has a positive temperature. This conclusion in particular is independent of the coupling $f(|\phi|^2)$ between the GB term $\mathcal{L}_{GB}$ [defined by Eq. (2.3)], and the complex scalar field $\phi$. It is also independent of the potential $V(|\phi|^2)$ of the complex scalar field, and the presence of a non-vanishing cosmological constant $\Lambda$, which can be either positive or negative. Therefore, the no-go theorem is quite general.

To circulate the theorem, one way is to consider time-dependent spherically symmetric BHs, similar to the case with a real scalar field studied in \cite{63,64}. Another extension is to consider BHs with rotation. Again, in the real scalar field case, such stationary rotating BHs exist \cite{33,34}. For BHs with complex scalar hairs, the above shows that we must have $\Psi_0 \equiv \Psi(r = r_h) = 0$.
In addition, when the scalar field is complex, but without its coupling to the GB term, it was also found that rotating BHs with complex scalar hairs exist [70]. Therefore, it would be very interesting to show that rotating BHs with complex scalar hairs exist [70]. We hope to report our findings in these directions soon.

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