DARK MATTER: THEORETICAL PERSPECTIVES

Michael S. Turner

Departments of Physics and of Astronomy & Astrophysics
Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637-1433

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory, Batavia, IL 60510-0500

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Abstract

I both review and make the case for the current theoretical prejudice: a flat Universe whose dominant constituent is nonbaryonic dark matter, emphasizing that this is still a prejudice and not yet fact. The theoretical motivation for nonbaryonic dark matter is discussed in the context of current elementary-particle theory, stressing that: (i) there are no dark matter candidates within the standard model of particle physics; (ii) there are several compelling candidates within attractive extensions of the standard model of particle physics; and (iii) the motivation for these compelling candidates comes first and foremost from particle physics. The dark-matter problem is now a pressing issue in both cosmology and particle physics, and the detection of particle dark matter would provide evidence for “new physics.” The compelling candidates are: a very light axion ($10^{-6}$ eV – $10^{-4}$ eV); a light neutrino ($20$ eV – $90$ eV); and a heavy neutralino ($10$ GeV – $2$ TeV). The production of these particles in the early Universe and the prospects for their detection are also discussed. I briefly mention more exotic possibilities for the dark matter, including a nonzero cosmological constant, superheavy magnetic monopoles, and decaying neutrinos.
1 Overview

One of the simplest yet most fundamental questions we can ask in cosmology concerns the quantity and composition of the matter in the Universe: What is mass density, expressed as a fraction of the critical density, $\Omega_0$, and what are the contributions of the various constituents, e.g., baryons, photons, and whatever else? (The critical density $\rho_{\text{crit}} = \frac{3}{8\pi G} = 1.88h^2 \times 10^{-29} \text{g cm}^{-3} = 1.05 \times 10^4 \text{eV cm}^{-3}$ where $H_0 = 100h \text{ km sec}^{-1} \text{Mpc}^{-1}$.) The answer to this question bears upon almost every topic discussed at this Colloquium: the expansion age and fate of the Universe; the origin of structure in the Universe and CBR anisotropies; galactic disks, rotation curves, and morphology; cluster dynamics; gravitational lensing; and the distribution of light and mass. The only thing we know with great precision is the contribution of photons, $\Omega_\gamma = 2.49h^{-2} \times 10^{-4}$ (assuming $T_\gamma = 2.73 \text{K}$), and neutrinos, $\Omega_\nu = 1.70h^{-2} \times 10^{-4}$ (assuming all three species are massless); and based on primordial nucleosynthesis, we know the contribution of baryons to within a factor of two, $\Omega_B h^2 = 0.01 - 0.02$ [1].

In principle, the classic kinematic tests—luminosity-red shift, angular size-red shift, number count-red shift, and so on—can be used to determine $\Omega_0$ (provided that we know the equation of state of the Universe) [2]. To date these tests have not been successful because they require standard objects of one sort or another (luminosity, size, or number density), though hope was expressed at this Colloquium that new techniques may change this situation (e.g., K-band Hubble diagram, K-band number counts, type I or II supernovae, and so on). At present, our knowledge of $\Omega_0$ derives primarily from dynamical estimates that sample small, often atypical environments (e.g., rich clusters and bright spiral galaxies). There is an exception, the recent attempts to infer $\Omega_0$ based upon the peculiar motion of the Local Group, which interestingly enough yield a value for $\Omega_0$ of order unity and with small error estimates [3, 4]. Beyond the fact that this measurement supports theoretical prejudice, it may well come the closest to weighing a large, fair sample of the Universe.

What is clear is that most of the mass density is accounted for by dark matter (i.e., matter that emits nor absorbs any radiation) and that $\Omega_0$ is at least 0.1—and perhaps as high as order unity. Since primordial nucleosynthesis provides very convincing evidence that baryonic matter can contribute no more than 10% of critical density [1], we are left with two possibilities: (i) conclude that $\Omega_0$ lies at its lower bound, that $\Omega_B$ lies at its upper boundary,
and that $h < 0.5$, in which case $\Omega_0 \simeq \Omega_B \simeq 0.1$; or (ii) conclude that there is a “gap” between $\Omega_0$ and $\Omega_B$ and consider the consequences.

While the second possibility is the more radical, the evidence for a gap, though not yet conclusive, continues to mount. If we accept this gap as real, and make the leap all the way to a flat Universe there are important implications: By a wide margin most the Universe is comprised of nonbaryonic matter, and because there are no nonbaryonic dark-matter candidates within the standard model of the elementary particles, the dark-matter problem becomes one of pressing interest in particle physics also. Particle physics rises to the occasion: In several of the most attractive extensions of “their standard model” there are hypothetical particles, whose motivations are unrelated to cosmology, but whose relic abundance is close to the closure density. The most promising are: an axion of mass $10^{-6} \text{ eV} - 10^{-4} \text{ eV}$; a neutralino of mass $10 \text{ GeV} - 2 \text{ TeV}$; and a neutrino of mass $90 h^2 \text{ eV}$.\footnote{A massive neutrino is not considered part of the standard model because neutrino masses are not accommodated within standard model.}

Most theorists would agree that a flat Universe dominated by nonbaryonic matter is the most attractive hypothesis, so attractive it is sometimes forgotten that it is still just that. This paradigm has become an almost indispensable crutch for those who study the formation of structure. In fact, I know of no viable model of structure formation based upon a Universe with $\Omega_0 = \Omega_B \simeq 0.1$.\footnote{Peebles’ isocurvature baryon model comes close, but as I understand it, the model requires that $\Omega_B \sim 0.2$ and $h \sim 0.8$.}

That being the case, it is important that we take our theoretical beliefs seriously enough to test them! At our disposal are a host of laboratory experiments and observational tests. They include cosmological measurements of $\Omega_0$, $H_0$, the age of the Universe, CBR anisotropies, large-scale structure and so on. In the laboratory there are efforts to directly detect halo dark-matter particles, to produce new particles at high-energy accelerators, to detect dark-matter annihilation products (coming from the sun or the halo), as well as a multitude of experiments that search for evidence for neutrino masses.
2 Weighing the Universe

Measuring the mean density of the Universe is no simple task; nor is summarizing the measurements and putting them in perspective\textsuperscript{3}. Simply put, one would like to weigh a representative volume of the Universe, say $100h^{-1}\text{Mpc}$ on a side. Easier said than done. Because of the inconclusiveness of the kinematic methods, I will focus on the dynamical measurements.

The dynamical measurements probe the mean density in a less than ideal way: A dynamical measurement, e.g., the virial mass of a cluster, is converted into a mass-to-light ratio which, when multiplied by the mean luminosity density (which itself has to be determined), yields an estimate of the mean mass density.\textsuperscript{4} There is an obvious drawback: One has to assume the mass-to-light ratio derived for the object, or portion thereof, is “typical” of the Universe as a whole. With that as a preface—and a warning—let me proceed.

Mass-to-light ratios derived for the solar neighborhood are very small, of order unity, and taken as a universal mass-to-light ratio imply a value of $\Omega_0$ of much less than 1%. Using instead the mass-to-light ratio inferred from the inner luminous regions of spiral and elliptical galaxies, of order ten or so, one infers a value for $\Omega_0$ of somewhat less than 1%. Based upon this evidence, most would agree that luminous matter contributes less than 1% of critical density\textsuperscript{7}.

The flat rotation curves of spiral galaxies give strong evidence that most of the mass in spiral galaxies exists in the form of dark halos; assuming that the halo material is distributed with spherical symmetry (for which there is only minimal evidence), the density of the halo dark matter decreases as $r^{-2}$\textsuperscript{8}. Many would cite the flat rotation curves of spiral galaxies as the strongest evidence that most of the material in the Universe is dark. Using the mass-to-light ratios derived from the flat rotation curves of spiral galaxies one infers values of $\Omega_0$ in the range of 3% to 10%. Since there is presently no convincing evidence for a rotation curve that falls as $r^{-1/2}$, indicating convergence of the total mass of the galaxy, one should regard these estimates as lower limits to $\Omega_0$ (again, based upon this technique).

There is some evidence for dark matter in elliptical galaxies and even dwarf galaxies, though it is much harder to come by, as one must measure velocity dispersions rather than rotation curves\textsuperscript{9}.

The oldest evidence for dark matter, dating back to the work of Zwicky\textsuperscript{3}, in the $B_T$ system the critical mass-to-light ratio is $1200h$, in solar units.
involves clusters; simply put, there isn’t nearly enough mass associated with the light to hold clusters together. The masses of clusters are derived using the virial theorem and involve certain assumptions: the distribution of galactic orbits must be specified and the clusters must be assumed to be “well relaxed.” The values for \( \Omega_0 \) deduced from cluster mass-to-light ratios range from 10% to 30%, though we should be mindful of the underlying assumptions (current observations seem to indicate that clusters are not well relaxed) and the fact that any material that is distributed spherically symmetrically outside the region where galaxies reside would not contribute to the virial masses derived. And of course, the fundamental assumption is that cluster mass-to-light ratios are typical, though less than 1 in 10 galaxies resides in a cluster. We should note too that dark is a relative term: It is now known that much, if not the majority, of the baryonic mass in clusters exists in the form of hot, x-ray emitting gas, that is “dark” to an optical telescope [1].

The virial masses of small groups and binary galaxies also provide evidence for dark matter, though the problem of interlopers is a severe one. The gravitational arcs produced by the lensing effect of clusters also indicate the presence of cluster dark matter. Evidence for dark matter in the Universe is nowhere lacking.

In my biased and very brief summary I have saved the best for last, a measurement that comes close to weighing a representative sample of the Universe of order 100\( h^{-1} \) Mpc on a side. It involves tying our well measured velocity with respect to the CBR, about 620 km sec\(^{-1}\), to the inhomogeneous distribution of matter in the nearby Universe. In effect, it is a simple problem in Newtonian physics: requiring that our velocity be produced by the inhomogeneous distribution of galaxies allows us to weigh a very large sample of the Universe. Here too assumptions are made: that the distribution of galaxies traces the mass at some level and that the bulk of our peculiar velocity arises from galaxies inside the survey volume and not outside. Using the red shift survey based upon the IRAS 1.2 Jy catalogue, two groups have inferred values of \( \Omega_0 \) that are close to unity: \( \Omega_0 \approx 0.3 \). Here \( b \equiv (\delta n_{\text{GAL}}/n_{\text{GAL}})/(\delta \rho/\rho) \) is the so-called bias factor, that in the simplest way accounts for the fact that bright galaxies may not faithfully trace the mass distribution. (I should mention that attempts to reconstruct the local density field from the measured peculiar velocity field also leads to a large value for \( \Omega_0 \).

To summarize the summary:
• Luminous matter (in the form of stars and associated material) provides at most 1% of the critical density.

• The flat rotation curves of spiral galaxies and virial masses of clusters indicate that the bulk of the mass density in the Universe is dark.

• The dark matter is less condensed than the luminous matter (as evidenced by galactic halos).

• $\Omega_0$ is at least 0.1, and the bulk of the data are consistent with $\Omega_0 = 0.2 \pm 0.1$ ($\pm 0.1$ is not might to be a statistical error flag).

• Primordial nucleosynthesis constrains the fraction of critical density contributed by baryons to be between 1% and 10% (more precisely, $0.01 \lesssim \Omega_B h^2 \lesssim 0.02$).

• There is growing evidence for a gap between $\Omega_B$ and $\Omega_0$.

A minimalist view is that we have a consistent solution: $\Omega_B = \Omega_0 \simeq 0.1$ and $h \lesssim 0.5$. The grander—and more radical—view is that there is a gap between $\Omega_B$ and $\Omega_0$, that $\Omega_0 = 1$, and that we live in a Universe dominated by nonbaryonic dark matter. From a theoretical perspective this is the most attractive scenario—and it may even be true!

Three points before we go on; as many have emphasized it may well be that there are several kinds of dark matter\textsuperscript{[2]}. Unless $h \gtrsim 1$ primordial nucleosynthesis already indicates evidence for dark baryons; moreover, baryons could in principle account for all the dark matter in galactic halos and possibly even clusters (provided $h \lesssim 0.5$). Dark baryons could exist in the form of black holes, neutron stars, or very low mass stars. Three large-scale efforts are well under way to search for dark matter in the form of low-mass stars in the halo of our galaxy using their microlensing of stars in the LMC\textsuperscript{[3]}.

While black holes may appear to the ideal dark-matter candidate, there are not. Black holes formed in the contemporary Universe ultimately trace their origins to baryons, and thus can contribute no more than about 10% of critical. While it is possible that mini black holes, holes much less massive than a solar mass, were produced in the early Universe from the primeval plasma and could today provide the critical density, a plausible mechanism for producing the right number without other deleterious consequences (e.g., black hole evaporations today producing gamma rays) is lacking\textsuperscript{[4]}.

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If $\Omega_0 = 1$, then the question arises as to where the bulk of the matter is, as most dynamical measurements indicate $\Omega_0 \sim 0.1 - 0.3$. This is the $\Omega$-problem. It could be that galactic halos are very large and that clusters sit at the center of gigantic distributions of dark matter, or that much of the material exists in low-luminosity galaxies (so-called biasing), or even that it exists in a form of smoothly distributed energy density, e.g., relativistic particles or a cosmological constant. In that regard one of the very nice features of neutrino dark matter is that neutrinos, owing to their large velocities, would likely remain smooth on scales out to several Mpc. In any case we know that the dark matter is less condensed than luminous matter, indicating that it does not have the ability to dissipate energy. This means that it could be in the form of particles that interact very weakly, or tied up in large objects made of baryons (e.g., dead stars or dwarfs).

3 The Evidence for a Flat Universe!

Before pursuing the hypothesis of a flat Universe dominated by nonbaryonic dark matter let me quickly summarize the evidence in support of it.

- There is evidence for a gap between $\Omega_B$ and $\Omega_0$.
- A dynamical explanation for our own peculiar velocity seems to indicate that $\Omega_0$ is close to unity.
- Some kinematic measurements of $\Omega_0$ based upon galaxy counts indicate that $\Omega_0$ is close to unity [15].
- Structure formation in a low-$\Omega_0$ Universe is more difficult and requires larger amplitude density perturbations and may not be consistent with the smoothness of the CBR [16].
- One of the most attractive scenarios of the early Universe, inflation, unambiguously predicts a flat Universe [17].
- The Dicke-Peebles timing argument [18]: If the Universe is not flat, then we must conclude that we live at a special time when the curvature terms and matter density terms are comparable.

Needless to say the evidence is not overwhelming; it does, however, make a case for taking the hypothesis of a flat Universe dominated by nonbaryonic dark matter seriously.
4 Nonbaryonic Dark-matter

If we adopt $\Omega_0 = 1$, then the gap between $\Omega_0$ and $\Omega_B$ is significant and necessitates that a new form of matter be the dominant constituent of the Universe. The point of this section is to emphasize that particle physicists too were pushed to nonbaryonic dark matter for reasons solely based upon particle physics: As a consequence of addressing very fundamental problems in particle physics, the existence of new particles was predicted, particles as it turned out whose relic cosmic abundance was close to the critical density. This could just be a coincidence, or it could be an important hint that we are on the right track.

4.1 The standard model of particle physics

Over the past two decades particle physicists have constructed a fundamental theory that accounts for all known phenomena at energies below about 300 GeV (down to length scales of order $10^{-16}$ cm). They call it “the standard model” [19]; mathematically, it is a nonAbelian gauge theory based upon the group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The $SU(3)_C$ part, known as quantum chromodynamics, describes the strong interactions (the interactions that bind quarks in hadrons). The $SU(2)_L \otimes U(1)_Y$ part describes the electroweak interactions. An important part of the standard model is the notion that the electromagnetic and weak interactions are not separate phenomena, rather different aspects of the unified electroweak force.

The fundamental particles of the standard models are three families of quarks and leptons ($u, d, c, t, b$ quarks and $\nu_e, e^-, \nu_\mu, \mu^-, \nu_\tau$, and $\tau^-$ leptons), and 12 gauge bosons (8 Gluons, $W^+, W^-, Z^0$, and the photon) that mediate the fundamental interactions. All the gauge bosons have been seen; the top quark remains to be discovered; and there is only indirect—but very strong indirect—evidence for the existence of the tau neutrino. All the particles participate in the electroweak interactions; only quarks carry color and participate in the strong interactions.

While the 8 Gluons and the photon are massless, the $W^\pm$ and $Z^0$ bosons are not; this reflects the least well understood aspect of the standard: symmetry breaking. The full symmetry of the electroweak interactions is hidden;
the simplest explanation is the Higgs mechanism and involves a new class of fundamental (scalar) particles: Higgs bosons, which have not yet been seen. Hidden symmetry is analogous to the magnetization of a ferromagnet: at low temperatures, due to spin interactions the state of the ferromagnet with lowest free energy is characterized by aligned spins and a net magnetization, and thus does not exhibit rotational invariance. The ground state of the Higgs field at low temperatures, due to its self interactions, breaks the symmetry of the electroweak interactions and in so doing makes the $W^\pm$ and $Z^0$ bosons massive (and accounts for the masses of the quarks and leptons as well). The aspects of the standard model involving the gauge particles and quarks and leptons have been tested to very high precision (in many cases to better than 1%); there is no direct evidence for the Higgs mechanism, and it is possible that something else accounts for the hidden symmetry. One of the primary goals for the SSC is the elucidation of symmetry breaking, e.g., by the production of Higgs bosons.

The standard model is a neat little package; in accounting for all “known particle physics” it also explains the absence of other phenomena. For example, why are neutrinos so light (or perhaps massless)? The $SU(2)_L$ symmetry forbids a mass for the neutrinos (in the absence of righthanded neutrinos). Why is the proton stable (or at least very long-lived)? Again, in the standard model it is not possible to have proton decay without violating other symmetries of the standard model. Similar considerations forbid interactions that violate lepton number.

### 4.2 New physics beyond the standard model

The tapestry of the standard model is not without loose threads. Like the standard cosmology it has shortcomings that point to something grander; they include:

- Quantization of charge: quarks and leptons are separate families of particles, yet the charges of the quarks are to high precision an integer multiple of one-third the charge of an electron.

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5 This statement is true at the classical level; subtle quantum effects associated with instantons and the like lead to baryon-number violation. At temperatures $> 200$ GeV these processes are probably very important and may play a role in explaining the origin of the baryon asymmetry of the Universe [21].
• A related issue: why are there two kinds of matter particles (quarks and leptons) and three families of quarks and leptons? Are quarks and leptons fundamental, or are they made of “smaller” entities?

• In the standard model the fundamental forces are “patched” together, rather than truly unified.

• The standard model has more than 20 “input parameters” (\(\sin^2 \theta_W\), quark and lepton masses, mixing angles, etc.) that must be specified.

• Disparity of scales: the scale of the weak interaction \(G_F^{-1/2} \sim 300 \text{ GeV}\) is much, much less than that of gravity, \(G^{-1/2} \sim 10^{19} \text{ GeV}\) (the “hierarchy problem”).

• A related issue: how to keep the Higgs light enough to break the electroweak interaction at a scale of 300 GeV in the face of quantum corrections that should drive its mass to the highest energy scale in the theory \(10^{19} \text{ GeV}\).

• The strong \(CP\)-problem: within the standard model quantum effects (instantons again) lead to \(CP\) violation in the strong interactions and should lead to an electric-dipole moment for the neutron that is \(10^9\) times larger than the current upper limit.

• Where and how does gravity fit in?

These considerations lead most particle physicists to believe that there must be a “grander” theory. Moreover, the mathematical tools at hand—nonAbelian gauge theories, supersymmetry, superstrings, to mention three—allow very attractive and powerful theoretical speculations that address all of these issues. These speculations lead to the prediction of new particles, some of which are stable (due to new conservation laws) or are at least long-lived (due to their small masses and/or very weak interactions). Further—and this is the cosmological bonus—some of these new, long-lived particles have relic abundances that are comparable to the critical density. This didn’t have to be; the relic abundance of a particle species is determined by its mass and interactions. \textit{This is either the big hint or the grand misdirection.}

To put things in perspective here is a very brief summary of the extensions of the standard model and the dark-matter candidates they predict.
• Peccei-Quinn symmetry [22]: this is a very minimal extension of the standard model designed to solve the strong-$CP$ problem. It is considered by many to be the best solution and automatically arises in many supersymmetry and superstring models. Another consequence of PQ symmetry is the existence of a very long-lived, light (pseudoscalar) particle—the axion—which is a prime dark-matter candidate.

• Majoron models [23]: these are modest extensions of the standard model designed to accommodate neutrino mass, and thereby allow the three ordinary neutrino species to be dark matter candidates.

• Supersymmetry [24]: low-energy supersymmetry is perhaps the most well studied extension of the standard model. Supersymmetry, the symmetry that relates bosons and fermions, dictates that for every fermion there be a bosonic partner (and vice versa)—thereby doubling the particle content of the standard model. First and foremost, supersymmetry addresses the hierarchy problem, “stabilizing” the mass of the Higgs boson and putting scalar particles on a firm footing. It also paves the way for the unification of gravity (when supersymmetry is gauged it leads to general relativity). Supersymmetry must be a broken symmetry since the known particles do not have equal-mass partners; the superpartner masses are generically expected to be of order 10 GeV to 1000 GeV (of order the electroweak scale). In almost all models the lightest superpartner (or LSP) is stable and is a linear combination of the photino and higgsino, known as the neutralino. The neutralino is a prime dark matter candidate.

• Technicolor [25]: is a very attractive idea for replacing the Higgs mechanism with a mechanism akin to the BCS mechanism in the BCS theory of superconductivity. A stronger version of QCD—technicolor—leads to the formation of bound states of techniquarks, and these bound states play the role of the Higgs. Technicolor addresses the hierarchy problem as the mass of the Higgs is set by the energy scale at which technicolor becomes “strong” (just as the mass of the hadrons is set by the scale at which color becomes strong) and eliminates the need for scalar particles. However, it is an attractive idea that has been very difficult to implement: There is currently no viable model of technicolor. Whether or not it predicts the existence of dark-matter candidates remains to be seen.
• Grand unification: the basic goal of grand unified theories (GUTs) is to truly unify the strong, weak, and electromagnetic interactions within a single gauge group with one coupling constant. The simplest GUT is based upon the gauge group $SU(5)$ and predicts a proton lifetime of $10^{30}$ yrs, which, sadly, has been falsified. Other GUTs include $SO(10)$, $E_6$, $E_8$, and on and on. That unification is even possible—given that the coupling strengths of the different interactions are so different at low energies—is remarkable. In nonAbelian gauge theories coupling strengths vary (or “run”) with energy (logarithmically); the strengths of the three known interactions seem to become equal at an energy scale of about $10^{16}$ GeV or so, which sets the scale of grand unification. Among other things, GUTs predict proton decay, neutrino masses, and the existence of superheavy magnetic monopoles (masses of order the unification scale)—the last two being dark-matter candidates. In many GUTs neutrino masses arise via the “see-saw mechanism” and $m_\nu \simeq m_l^2 / \mathcal{M}$ where $m_l$ is the charged lepton mass, and $\mathcal{M}$ is an energy scale associated with unification (not necessarily the unification scale itself—perhaps orders of magnitude smaller). This explains why neutrino masses are so very small—and in many models suggests that neutrinos may have masses in “the eV range” (anywhere from $\mu$eV to tens of eV).

• Superstrings: superstring theories unify all the forces (including gravity) in a finite quantum theory (WOW!) and are most naturally formulated in ten dimensions (suggesting the existence of six extra spatial dimensions that today must be compactified). The fundamental objects of the theory are 1-dimensional string-like entities whose size is order $10^{-33}$ cm. The expectations for the superstring are high: ultimately, explanations for everything—quark/lepton masses, coupling constants, the strong-$CP$ problem, the number of families, spartner masses, the electroweak scale. The path has been more difficult than expected, and there have been few definite predictions (that are not wrong). Broadly, superstring theory provides theoretical support for the axion, supersymmetry, grand unification, and neutrino masses—providing motiva-

\footnote{About a decade ago the convergence of the coupling constants occurred in ordinary GUTs at an energy scale of about $10^{14}$ GeV; better measurements of $\sin^2 \theta_W$ indicate that such a convergence does not occur in nonSUSY GUTs, but does in SUSY GUTs at an energy scale of about $10^{16}$ GeV.}
tion for all the dark-matter candidates mentioned above.

Of course, there are other ideas that I have not mentioned because at present they do not seem viable. For example, preons, which were postulated as the constituents of quarks and leptons, and higher-dimensional analogs of superstrings, known as membranes.

4.3 Two birds with one stone

Particle dark matter is attractive because new particles that owe their existence to attempts to solve very fundamental puzzles in particle physics have a relic abundance of order the critical density! Historically, such coincidences have been a sign that one is on the right track. While there are now literally dozens of particle dark matter candidates, there are but a handful of particles whose existence owes to well motivated attempts to solve important problems in particle physics and whose relic abundance is in the right ballpark. They are:

- The neutralino [30]. In most supersymmetry models, the neutralino is the lightest supersymmetric partner and is stable (due to a new symmetry called $R$-parity). Its interactions with ordinary matter are roughly the strength of the weak interactions, and this fact ultimately explains why its relic abundance is of order the critical density. At present, supersymmetric models have many parameters that must be dialed in, and the mass of the neutralino is only known to be somewhere between 10 GeV and 2 TeV.

- The axion [31]. Peccei-Quinn symmetry seems to be the best solution to the nagging strong-$CP$ problem. The mass of the axion depends upon a single parameter: the energy scale of PQ symmetry breaking, $f_{\text{PQ}}$, and

$$m_a \sim m_{\chi}^2 / f_{\text{PQ}} \sim 10^{-5} \text{ eV} \left( 10^{12} \text{ GeV} / f_{\text{PQ}} \right).$$

The strength of the axion’s

\footnote{For a while, some believed that one could get three birds with one stone: Cosmions, dark-matter particles of mass 4 GeV to 10 GeV with scattering cross sections of order $10^{-35} \text{ cm}^2$, were proposed to solve both the solar-neutrino and the dark-matter problems. This possi- bility is all but ruled out on both theoretical—the corresponding annihilation cross section leads to a cosmion abundance that is too small in both the sun and the cosmos—and experimental grounds—cosmions should have been detected in dark-matter searches [29].}
couplings to ordinary matter are proportional to its mass. When the axion was first invented, only one scale of symmetry breaking was known: the weak scale and there seemed to be a unique prediction for its mass, around 200 keV. This idea was quickly falsified. It is now realized that there are likely to many energy scales in Nature, the GUT scale, the Planck scale, the intermediate scale and so on. The symmetry-breaking scale has been constrained, largely by astrophysical and cosmological arguments, to lie in the interval, $10^{10} \text{GeV} \lesssim f_{\text{PQ}} \lesssim 10^{13} \text{GeV}$, corresponding to an axion mass in the range $10^{-6} \text{eV}$ to $10^{-3} \text{eV}$. This also happens to be the range where the relic abundance of axions is of order the critical density.

- Light neutrino. The neutrino exists; it comes in three varieties; and we know its relic abundance to three significant figures, $113 \text{cm}^{-3}$ per species. Further, essentially all extensions of the standard model predict that neutrinos have mass, and the see-saw mechanism implies masses in the general range of eV, give or take a factor of $10^3$ or so.

- Dark horses. There are also a few well motivated long shots. They include the superheavy magnetic monopole: It is a generic prediction of GUTs; the only problem is its abundance, without inflation far too many monopoles are produced, and with inflation essentially no monopoles are produced. There is the supersymmetric partner of the axion, the axino, which arises in theories with both PQ symmetry and supersymmetry. Its mass is expected to be in the keV range, and its abundance is significantly less than neutrinos as it decouples much earlier.

### 4.4 Why not baryons or modified gravity?

The particle dark matter hypothesis is a radical solution; are there other alternatives that are less radical or perhaps more attractive? I think not, but to convince the reader let me mention two such ideas: $\Omega_B \sim 1$ and modified Newtonian dynamics.

Primordial nucleosynthesis provides the best determination of the amount of baryonic matter in the Universe, pinning down the number density of baryons to within a factor of two. To be sure, the arguments involve assumptions about the Universe in the distant past. Over the years many have
suggested alternative scenarios of primordial nucleosynthesis that would allow one to evade the nucleosynthesis and have $\Omega_B \sim 1$. The most recent attempt involved the role of large inhomogeneities that might have been produced in the quark/hadron transition if it were strongly first order. It was hoped that such inhomogeneities would allow $\Omega_B \sim 1$. This possibility is now “doubly forbidden.” As discussed at this Colloquium inhomogeneous nucleosynthesis allows very little, if any, loosening of the standard bound $\Omega_B$; moreover, numerical simulations of the quark/hadron transition suggest that such inhomogeneities would not have arisen in the first place, as the transition is at best a weakly first-order phase transition, and perhaps not a phase transition at all (more like recombination).

Theorists are rarely criticized for their conservatism! Moreover, it seems that every theorist worth his salt has tried to find a theory of gravity to supplant Einstein’s. So one might have expected that theorists would have embraced Milgrom’s modified Newtonian dynamics (MOND). The basic idea of MOND is that the form of Newton’s second law is modified for accelerations less than about $cH_0 \sim 10^{-7}$ cm sec$^{-2}$, $F \simeq ma^2/cH_0$, thereby eliminating the need for dark matter to explain flat rotation curves. While theorists are more than ready to consider modifications to Einstein’s theory, especially in light of superstring theory, to most theorists MOND looks like a nonstarter. The reason is simple: it is purely a Newtonian theory, and attempts to formulate it in terms of a relativistic field theory have been unsuccessful. Without such a formulation one cannot construct a cosmological model or evaluate its predictions for the many tests we have of relativistic theories of gravity—bending of starlight, precession of the perihelion of Mercury, gravitational red shift, radar time delay, and the myriad of tests offered by the binary pulsar. If that were not bad enough, it has been argued that MOND can be falsified on the basis of rotation curves measured for galaxies of very different sizes.

In sum, theorists have looked hard for other explanations; I believe that it is fair to say that the particle dark matter explanation is the most attractive. Whether or not it proves to be correct is another matter.

5 Dark-matter Relics: Origins

Since an important motivation for particle dark matter is the fact that the relic abundance of these handful of promising candidates is comparable to
the critical density it is worth reviewing how a cosmological relic arises. There are several qualitatively different mechanisms for particle dark-matter production in the early Universe.

5.1 Thermal relics: hot, warm, and cold

Much—but not all—of the history of the Universe is characterized by thermal equilibrium. So long as equilibrium pertains the abundance of a massive particle relative to photons is\(^8\) of order unity for temperatures \(T \gg m/3\); of order \((m/T)^{3/2} \exp(-m/T)\) for \(T \ll m/3\). For reference, the fraction of critical density contributed by a relic species is

\[
\Omega h^2 \simeq \left(\frac{m}{25 \text{ eV}}\right) \left(\frac{n}{n_\gamma}\right). \tag{1}
\]

If equilibrium were the entire story, relic abundances would be far too small to be of any interest.

Consider, a stable, massive particle species; its abundance is necessarily regulated by annihilations and pair creations. In the expanding Universe the temperature is decreasing, \(\dot{T}/T \simeq -H\); equilibrium can be maintained only if annihilations and pair creations occur rapidly on the expansion timescale, \(H^{-1}\). Because of the temperature dependence of equilibrium number densities and of cross sections, annihilation and pair creation reactions eventually become ineffective (“freeze out”) and the abundance of a particle species relative to photons approaches a constant value (“freezes in”) \([38]\).

If freeze out occurs when the species is relativistic, then the species’ relic abundance is comparable to that of photons. Such a species is referred to as a hot relic; a light (mass \(\lesssim \text{MeV}\)) neutrino species is a hot relic.

On the other hand, if freeze out occurs when the species is nonrelativistic, then its relic abundance is significantly less than that of photons, and depends inversely upon its annihilation cross section (in thermal equilibrium the annihilation rate and pair creation rate are related by detailed balance).

The relic abundance is

\[
\left(\frac{n}{n_\gamma}\right) \sim \frac{\ln(0.01 m m_{\text{Pl}} (\sigma v)_{\text{ann}})}{m m_{\text{Pl}} (\sigma v)_{\text{ann}}} \Rightarrow \Omega \sim \frac{10^{-3}}{T_0 m_{\text{Pl}} (\sigma v)_{\text{ann}}}; \tag{2}
\]

\(^8\)The number of particles per comoving volume, \(R^3 n\), is actually proportional to the ratio of the particle number density to the entropy density, \(n/s\), where \(s \propto g_s T^3\) and \(g_s\) counts the effective number of ultrarelativistic degrees of freedom. So long as \(g_s\) is constant, \(s\) and \(n_\gamma\) are related by a constant numerical factor, today about 7.04.
where the second relation follows from the fact that $\rho_{\text{CRIT}} \sim 10^4 T_0^4$. This formula is quite remarkable: Neglecting the logarithmic factor and the overall numerical constant, it implies that the fraction of critical density contributed by a cold relic only depends upon its annihilation cross section, and, further, that $\Omega \sim 1$ obtains for $\langle \sigma v \rangle_{\text{ann}} \sim 10^{-3} / T_0 m_{\text{Pl}} \sim 10^{-37} \text{cm}^2$! This is very roughly a weak-interaction cross section ($\sim \text{GeV}^2 G_F^2$), and indicates that a stable particle with weak interactions will necessarily have a relic abundance comparable to the critical density. A stable neutrino of mass a few GeV would fit the bill were it not ruled out by experiment [39]. The neutralino fits the bill nicely, as its interactions with ordinary matter are roughly weak.

The final case is warm dark matter. If a species decouples while it is still relativistic, but very early on ($T \gg 1 \text{ GeV}$), then after it decouples its abundance relative to photons will be diminished as various species disappear and transfer their entropy to the photons (and other species). In this case, its abundance is less than that of photons, but not exponentially less, and so closure density obtains for masses in the keV range; plausible warm dark matter candidates include the axino [34] and a light gravitino [40]. (This dilution by “entropy transfer” is precisely what makes the relic neutrino temperature and abundance less than that of photons.)

### 5.2 Skew relics

Implicit in the previous discussion is the assumption that the particle and its antiparticle were equally abundant. If there is an asymmetry between particle and antiparticle and net particle number is conserved, then the relic abundance can become no smaller than the net particle number per photon [41]. Provided that annihilations can reduce the particle’s abundance to this level, the relic abundance is determined by the particle-antiparticle asymmetry.

Baryons are an example of a skew relic; were it not for the asymmetry between baryons and antibaryons, the relic abundance of each would be about $10^{-18}$ that of photons [42]. The mass density contributed by a skew relic is

$$\Omega_X h^2 \sim \left( \frac{\eta_X}{10^{-10}} \right) \left( \frac{m_X}{250 \text{ GeV}} \right);$$

where $\eta_X$ is the particle-antiparticle asymmetry relative to photons. A stable neutrino species with mass of order 100 GeV and asymmetry of order the
baryon asymmetry could provide closure density. (Neither the precision measurements of the width of the $Z^0$ boson nor nucleosynthesis preclude such a fourth neutrino species; dark matter searches employing ionization detectors do unless the mass exceeds a TeV or so.)

5.3 Nonthermal relics

The magnetic monopole and axion are examples of particles whose relic abundance involves coherent, nonthermal processes. Monopoles are produced as (point-like topological) defects in the GUT symmetry breaking phase transition [33]. On the basis of causality considerations one expects of the order of one monopole per horizon volume (at the time of the phase transition), which leads to a relic abundance of order $n/n_\gamma \sim (T/m_{Pl})^3$. For the GUT phase transition, $T \sim 10^{15}$ GeV or so, which results in a gross overabundance of monopoles (very crudely, “$\Omega \sim 10^{12}$”). This is the monopole problem. Inflation can solve the monopole problem provided that the GUT phase transition occurs before inflation, so that monopoles are diluted by the massive entropy production. This being said, it appears that monopoles are a terrible dark matter candidate; however, scenarios have been proposed where their relic abundance can be close to critical [33].

Axions arise not only as thermal relics, but also due to two nonthermal processes, the misalignment process and the decay of axionic strings [32]. For the interesting axion masses, $10^{-6}$ eV to $10^{-4}$ eV, their thermal relic abundance cannot come close to closure density. Since there is some disagreement as to the importance of the axionic-string decay process [44] and it is impotent in an inflationary Universe I will focus on the misalignment process [31].

It is the $\Theta$ parameter of QCD that leads to the strong-$CP$ problem; $\Theta$ is an angular parameter that controls the strength of the offending instanton effects. In the PQ solution $\Theta$ becomes a dynamical variable whose value is anchored at the $CP$-conserving value of zero by the instanton effects themselves. However, at temperatures much greater than 1 GeV these effects are impotent and the value of $\Theta$ is left undetermined by dynamical considerations. Thus, one expects the value of $\Theta$ to be randomly distributed in different causally independent regions of the Universe. When the QCD instanton effects do become important $\Theta$ will in general be “misaligned” — i.e., not at $\Theta = 0$ — and will evolve toward $\Theta = 0$; as it does, $\Theta$ overshoots and is left oscillating. These cosmic harmonic oscillations correspond to a
condensate of very nonrelativistic axions, whose relic density is roughly

$$\Omega h^2 \simeq \left( \frac{m}{10^{-5} \text{eV}} \right)^{-1.2}.$$  \hspace{1cm} (4)

The energy associated with the misalignment of $\Theta$ is converted into an enormous number of axions, about $10^9 \text{cm}^{-3}$ for $m_a = 10^{-5} \text{eV}$.

5.4 Significant-other relics

While our first interest is in elucidating the nature of the ubiquitous dark matter, it is possible that there are a number of particle relics in our midst. Needless to say, a particle relic that contributes significantly less than closure could still be interesting—both from the point of view of cosmology and of particle physics—moreover, it could be detectable. The CBR provides such an example: $\Omega_\gamma \sim 10^{-4}$. Until it was ruled out by a telescope search for its decays, an eV-mass axion provided another possibility [45]. If Nature is supersymmetric and the lightest supersymmetry particle is stable, it is difficult to avoid a supersymmetric relic that contributes less than about $10^{-3}$ of closure density. Magnetic monopoles provide yet another example. If the earliest history of the Universe is as interesting as many think, there may be many relics whose abundance is far from critical, but are still potentially detectable.

5.5 Truly exotic relics

Other more complicated explanations for the dark-matter problem involving early Universe relics have been suggested. Two suggestions have been made that would reconcile a flat Universe with the observational data that the amount of matter that clusters contributes only 20% or so of critical density: a “relic cosmological constant” and dark-matter that decays a modest red shift into relativistic debris which necessarily remains unclustered [46]. In either case, dynamical measurements of $\Omega_0$ would not reveal the unclustered energy density—vacuum energy or relativistic particles—and would yield values of order 20%. On the other hand, kinematic measurements could reveal the presence of the unclustered energy density [47]. In either case, a new cosmic coincidence comes into play: a cosmological constant that becomes dynamically important in the current epoch, or a particle whose lifetime is comparable to the age of the Universe.
A relic cosmological constant provokes further discussion. Historically, cosmologists have turned to the cosmological constant when faced with a crisis. In the context of quantum-field theory it is actually the absence of an enormous ($\Lambda \sim 10^{122}G^{-1}$) cosmological constant associated with the zero-point energy of quantum fluctuations of the fundamental fields that is a mystery. To confuse the situation further, several authors have argued that a Universe with a cosmological constant, cold dark matter and baryons is currently the best-fit Universe, in terms of the age of the Universe, dynamical measurements of $\Omega_0$, and the formation of structure [48].

Other puzzles have motivated suggestions for “specialized relics.” Sciama and others have argued for an unstable neutrino species whose radiative decays would lead to efficient re-ionization of the Universe [49]. Recently, “cocktails” of two particle relics—30% neutrinos and 70% cold dark matter—have been advocated to make the cold dark matter scenario for structure formation better agree with observations [50].

5.6 A new cosmic ratio

If the bulk of the mass density is in the form of nonbaryonic dark matter, then cosmologists—and particle physicists—have a new dimensionless number to explain: The ratio of ordinary matter to exotic matter. Why is it of the order of unity and not say $10^{-20}$ or $10^{20}$? The value of this ratio has important consequences for the evolution of the Universe, and the fact that it is of order unity is at the heart of many cosmological observations—e.g., the halo/disk conspiracy in rotation curves, the stability of galactic disks, and even the formation of stars.

While there is presently no good explanation for why this ratio is of order unity, it necessarily involves fundamental physics. For example, consider a skew relic whose asymmetry is comparable to the baryon asymmetry; then the ratio is just that of the exotic particle’s mass to the mass of a baryon. For other relics, requiring that this ratio be of order unity implies special relationships between fundamental energy scales in physics [51].

6 Detection

The nonbaryonic dark-matter hypothesis is a very bold one—and fortunately it is testable. While no cosmological experiment or observation is easy, es-
especially the search for a particle whose interactions could be as different as those of an axion and a neutralino, thanks to the creative efforts of many there are manifold approaches to the problem of detection [24].

First, there are the direct schemes, where the halo dark-matter particles in our local neighborhood (density $5 \times 10^{-25}$ g cm$^{-3}$) are sought out. For axions, the approach is based upon a very clever idea of Sikivie that takes advantage of the axion’s coupling to two photons [53]. A microwave cavity is immersed in a very strong magnetic field which causes halo axions to be converted to photons and excite resonant modes of the cavity; several “proof of principle experiments” have been built and operated and a new generation of Sikivie detectors with sufficient sensitivity to detect halo axions are being built [54]. Neutralino detectors exploit the neutralino’s roughly weak interactions with ordinary matter: When a multi-GeV mass neutralino scatters off a nucleus it deposits an energy of order a keV. The annihilation cross section and elastic cross section are related by “crossing” and thus the scattering cross section too should be of order $10^{-37}$ cm$^2$; this implies an event rate of the order of 1 per day per kg. A new generation of low-background, low-threshold cryogenic detectors are being developed to search for neutralinos in our halo [55]. While the magnetic monopole must considered a long shot dark-matter candidate, a football-field sized detector called MACRO is just coming on line and will achieve a sensitivity of about $10^{-16}$ cm$^{-2}$sr$^{-1}$sec$^{-1}$ [56].

Next, there are indirect searches, which involve seeking out the decay or annihilation products of dark-matter particles. For example, dark-matter annihilations in the halo of our galaxy can produce high-energy positrons that can be detected [57]. The most promising idea involves annihilations of dark matter particles that accumulate in the sun and the earth [58]; the annihilation products include high-energy neutrinos that can be detected in large, underground earth-based detectors, such as MACRO [56]. A sizable portion of the neutralino parameter space can be explored by searching for high-energy neutrinos from the sun and the earth [59].

Finally, there are numerous laboratory and astrophysical experiments that bear on the existence of particle dark matter. Searches for the supersymmetric partners of the known particles are taking place at every accelerator in the world; the discovery of even one superpartner would not only provide strong evidence for the existence of the neutralino, but would also help to narrow the parameter space. There are a host of experiments that bear on the issue of neutrino masses: experiments designed to measure the electron-neutrino mass; neutrino oscillation/mixing experiments; solar
neutrino experiments; and searches for neutrinos from type II supernovae.

7 Concluding Remarks

The theorists’ prejudice of a flat Universe dominated by nonbaryonic dark matter is at present just that! However, I hope that I have convinced the reader that: (1) the dark matter question is a most pressing one which now involves both cosmologists and particle physicists; (2) the theorists’ prejudice is well motivated by both theoretical and observational considerations; and (3) most importantly, the particle dark matter hypothesis can and is being tested. While cosmological experiments are inherently difficult and we cannot test every dark-matter candidate, I am optimistic. The most promising dark-matter candidates are detectable and the dark-matter problem has attracted the attention of many of the most talented experimentalists from both cosmology and particle physics. While this is no guarantee that we will have an answer soon, what more could one ask? And if that isn’t enough, there is the payoff: Identifying and quantifying the primary substance of the Universe and discovering “new physics” in the process!

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