Flavored Gauge Mediation, A Heavy Higgs, and Supersymmetric Alignment

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Abstract

We show that the messenger-matter couplings of Flavored Gauge Mediation Models can generate substantial stop mixing and new contributions to the stop masses, leading to Higgs masses around 126 GeV with sub-TeV superpartners, and with some colored superpartners around 1-2 TeV in parts of the parameter space. We study the spectra of a few examples with a single messenger pair coupling dominantly to the top, for different messenger scales. Flavor constraints in these models are obeyed by virtue of supersymmetric alignment: the same flavor symmetry that explains fermion masses dictates the structure of the matter-messenger couplings, and this structure is inherited by the soft terms. We present the leading 1-loop and 2-loop contributions to the soft terms for general coupling matrices in generation space. Because of the Higgs-messenger mixing induced by the new couplings, the calculation of these soft terms via analytic continuation requires careful matching of the high- and low-energy theories. We discuss the calculation in detail in the Appendix.

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I. INTRODUCTION

The null results of direct searches for supersymmetry suggest that it does not manifest itself in the form of light, flavor-blind superpartners. Furthermore, if the recently discovered scalar at 126 GeV \[1, 2\] is indeed the Higgs boson, its relatively large mass requires either a large stop mixing or very heavy stops, at least in the context of the Minimal Supersymmetric Standard Model (MSSM) \[3\]. These results are especially problematic for Gauge-Mediated Supersymmetry Breaking (GMSB) models \[4, 5\], which predict zero $A$-terms and flavor-blind spectra at the messenger scale. Low scale gauge mediation is therefore strongly disfavored by the Higgs mass, and even high-scale models, with $A$-terms generated by the running below the messenger scale, require stop masses of around 8–10 TeV \[6, 7\] in the context of minimal gauge mediation \[5\]. Given the tight relations between squark and gluino masses in minimal GMSB (mGMSB), this implies that all superpartners are very heavy in these models, and beyond the reach of any foreseeable experiment.

From a purely theoretical point of view, however, GMSB models are very attractive, since both the breaking of supersymmetry and its mediation are described by well understood quantum field theories, as opposed to unknown Planck-scale physics. Indeed, flavor-blind extensions of gauge mediation have been extensively discussed in recent years \[8\]. These extensions too are only consistent with a 126 GeV Higgs for high messenger scales, unless the stops or the gluino are very heavy \[9\]. Here we will study instead a flavor-dependent extension of gauge mediation, specifically, the Flavored Gauge Mediation (FGM) models of \[10\]. In these models, the flavor structure of GMSB is in principle modified, due to superpotential couplings of the messengers to SM fields \[11\]. We will show that these messenger-matter couplings can yield significant stop $A$-terms\(^1\), as well as new contributions to the stop soft masses, resulting in a heavy Higgs and fairly light superpartners for a wide range of messenger scales.

The superpartner masses in FGM models are generated by both the SM gauge interactions, and by Yukawa-type superpotential couplings of the messengers to the SM matter fields. Thus, while the interactions mediating the breaking are not purely gauge interactions, they are still completely “visible” — occurring within simple field theoretic extensions of the MSSM, and potentially at low scales. Since the matter-messenger couplings are in principle flavor dependent, they are strongly constrained by the non-observation of flavor changing neutral currents. As stressed in \[10\], however, there are good reasons to consider these couplings. At the very least, given our ignorance about the origin of the SM Yukawas structure, it is conceivable that the messenger-matter couplings have some special structure, which results in an acceptable pattern of soft terms. Indeed, the structure of the SM fermion masses hints at some theory of flavor, and any such theory will necessarily also control the sizes of matter-messenger couplings. Furthermore, as superpartner masses are pushed to higher values by direct searches (as well as by the large Higgs mass), there is more room for non-degenerate spectra. From the point of view of LHC searches for supersymmetry, the assumption of Minimal Flavor Violation (MFV), which underlies many analyses, can result in reduced sensitivity to non-MFV spectra \[13, 15\]. So when searching for gauge-mediated supersymmetry, it is important to keep the possibility of flavor-dependent spectra in mind, and FGM models provide useful examples of such spectra.

\(^1\) Scenarios where non-trivial flavor structure leads to maximal flavor mixing have also been considered in the context of horizontal symmetries and MSSM, see for example \[12\]
The main new ingredient in our models will be superpotential messenger-matter interactions, with the up-type Higgs (or also the down-type Higgs) replaced by a messenger field of the same gauge charges. Since we would like to generate a large top $A$-term, the messengers need only couple to the top. As a concrete realization of this scenario, we invoke a flavor symmetry under which the Higgs and messenger field have identical charges. Flavor constraints in our models are thus satisfied by a combination of degeneracy — coming from the pure gauge contributions, and alignment \cite{10}. Unlike in the original alignment models of \cite{16}, in which the flavor symmetry controls the soft terms directly, here it controls the supersymmetric messenger-matter couplings so that they are aligned with the SM Yukawas. The soft terms therefore inherit this structure, even though they are generated at the messenger scale, which is typically much lower than the flavor-symmetry breaking scale.

We note that three other papers appeared recently \cite{17-19} which rely on messenger-SM couplings to raise the Higgs mass. The differences between our models and the models of \cite{17-19} arise due to the different choices of symmetries and, as a result, the allowed messenger-SM couplings. In \cite{17}, the messengers are chosen to have the same R-parity as the SM matter fields, so the relevant coupling is the analog of the Yukawa coupling with one SM matter field replaced by a messenger. Messengers in $5 + \bar{5}$ representations of $SU(5)$ in these models do not affect the $u'^c$ mass, and as a result can raise the Higgs mass only to around 118 GeV for stops below $1.5$ TeV. Therefore \cite{17} uses messengers in $10 + \bar{10}$. In \cite{18,19}, a coupling of the type Higgs-messenger-messenger is used, with one messenger being a SM gauge singlet. Since none of the fields involved is colored, the effect of this coupling is moderate, so that the Higgs mass is viable only at low messenger scales, where the negative one-loop contributions to the Higgs soft masses-squared are important \cite{19}. It is interesting that, although the new messenger couplings in our models do not involve the Higgs at all, they have a significant effect on the Higgs mass. The reason is that the key feature needed for getting a large Higgs mass is a modified stop spectrum, which only requires that the messengers couple to the top.

To calculate the 2-loop contributions to the soft terms we use analytic continuation \cite{20}. The messenger-Higgs mixing present in the models has led to some erroneous results in the literature (including in an earlier version of this article\textsuperscript{2}). We clarify this issue and discuss the calculation in detail in the Appendix. The key point is the correct identification of the relevant couplings and wave function renormalizations, for which one must match the high-energy and low-energy theories correctly. We give a simple and intuitive prescription for the calculation by identifying the physical messenger field, given by the heavy combination, and the physical Higgs field, given by the light combination, and their respective running couplings.

The resulting spectra are rich and quite unusual. There are essentially two different regions of the parameter space which lead to an enhanced Higgs mass. In the first region, the stop $A$-terms are substantial, while the LL and RR stop masses are largely unmodified, because they receive both positive and negative contributions from the new couplings. Since these negative contributions are dominated by 1-loop effects which are higher order in the supersymmetry breaking, this region occurs for relatively low messenger scales. The resulting spectra can be very light, with colored superpartners even well below 2 TeV. In the second region, the LL and RR stop masses are enhanced as well, so that the large Higgs mass is

\textsuperscript{2} Specifically, our results for the Higgs masses were correct, but our results for the remaining scalars were wrong. We thank M. Ibe for pointing this out to us.
driven not by the large mixing but rather by large stop masses. Thus a 125 GeV Higgs
requires heavy stops. While only the stop masses are modified at the messenger scale,
the running to the weak scale can affect the remaining masses dramatically, since the stop
masses are large. While most colored superpartners are above 2 TeV in this case, the weak
gauginos and sleptons can be light. Furthermore, the new contributions to the soft masses
can reverse the effect of the stops on the remaining sfermions through the RGEs, leading to
novel spectra with the NLSP being either the neutralino, or a L-handed slepton.

The organization of this paper is as follows. In Section II A we briefly review FGM and
introduce the symmetries and the superpotential of our models. In Section II B we give
expressions for the soft terms in the limit of third-generation dominance. In Section III
we present the Higgs mass and superpartner spectra for different choices of parameters.
Our conclusions and discussion of the results are presented in Section IV. We present the
calculation of the soft masses in the Appendix. In A 1 we derive the soft terms in a simple
toy model with Higgs-messenger mixing. We generalize this to the models of interest in A 2.
Full expressions for the soft terms for general 3 \times 3 coupling matrices in generation space
are presented in A 3. As a cross-check, we also calculated the relevant terms explicitly. We
describe this calculation in A 4.

II. MODELS

A. The models and supersymmetric alignment.

We begin by briefly reviewing FGM models [10]. The starting point in these models
is mGMSB [5]. Specifically, we will take \( N \) sets of messengers transforming as \( 5+\bar{5} \) of
SU(5), coupled to a supersymmetry-breaking singlet \( \langle X \rangle = M + F \theta^2 \). We use capital
letters to denote the messenger fields, with \( 5 = T + D \) and \( \bar{5} = \bar{T} + \bar{D} \), where \( T \) (\( \bar{T} \)) and
\( D \) (\( \bar{D} \)) are fundamentals (anti-fundamentals) of \( SU(3) \) and \( SU(2) \) respectively. The SM
gauge symmetry permits different couplings of the messengers to SM fields, and these would
generically give rise to flavor-dependent soft-terms [11, 20–24].

As in [10], we will assume that the SM fermion masses are explained by a flavor symmetry.
This symmetry then also controls the messenger-matter couplings. In the models we will
consider, these coupling matrices will be aligned with the SM Yukawa matrices, so that flavor
constraints are satisfied. Naively, one would think that alignment can only be relevant for
high-scale supersymmetry breaking. This is indeed the case in the original alignment models
of [16]. In these models, a Froggatt-Nielsen flavor symmetry [25] dictates the structure of the
soft-term matrices at the supersymmetry-breaking scale. As explained above however, the
non-universal parts of the soft terms in FGM models are generated by superpotential matter-
messenger couplings. These supersymmetric coupling matrices are the ones controlled by
the flavor symmetry at high scales, and their near-diagonal structure is inherited by the
soft terms, which are generated at much lower scales. We therefore refer to this type of
alignment as “supersymmetric alignment”.

In addition to the flavor symmetry and R-parity, we impose a \( Z_3 \) symmetry with charges
given in Table I. The following superpotential is then allowed by the symmetries,

\[ W = \lambda X^2 \mathcal{H} \]

3 The model of [22] relies on an extra dimension in order to obtain MFV couplings.
4 In general, some messenger fields may be charged under the flavor symmetry.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Superfield & \textit{R}-parity & \textit{Z}_3 \\
\hline
\hline
\(X\) & even & 1 \\
\hline
\(D_1\) & even & -1 \\
\(\bar{D}_1\) & even & 0 \\
\(D_2\) & even & 0 \\
\(\bar{D}_2\) & even & -1 \\
\hline
\(T_I, \bar{T}_I, D_{I>2}, \bar{D}_{I>2}\) & even & 1 \\
\hline
\(q, u^c, d^c, l, e^c\) & odd & 0 \\
\hline
\(H_U, H_D\) & even & 0 \\
\hline
\end{tabular}
\caption{R-parity and \textit{Z}_3 charges.}
\end{table}

\[
W = X \left( X^2 + T_I\bar{T}_I + D_I\bar{D}_I \right) + H_U q Y_U u^c + H_D q Y_D d^c + H_D l Y_L e^c \\
+ D_1 q y_U u^c + D_2 q y_D d^c + D_2 l y_L e^c,
\]

(1)

where \(I = 1, \ldots, N\) runs over messenger pairs, \(y_U, y_D, y_L\) are messenger-matter Yukawa matrices, \(Y_U, Y_D, Y_L\) are the SM Yukawa matrices, and \(q, u^c, d^c, l, e^c\) are the MSSM chiral multiplets. We assume that the \(\mu\)-term(s) are forbidden by some \(U(1)\) symmetry. Note that to have messenger couplings to both up quarks and down quarks we need at least two sets of messengers [10, 22]. We also display here the term \(X^3\), required in mGMSB in order to generate the \(X\) VEVs, and motivating our choice of a \(\textit{Z}_3\) symmetry. In the following, however, we will limit ourselves to treating \(X\) as a supersymmetry breaking spurion.

At this point, \(D_2\) and \(H_D\), as well as \(\bar{D}_1\) and \(H_U\), have identical charges under all the symmetries, and therefore the following terms are allowed as well,

\[
XD_1 H_U + X H_D \bar{D}_2.
\]

(2)

However, we can set these couplings to zero without loss of generality. Consider for concreteness the \(H_U\) and \(\bar{D}_1\) couplings

\[
y_{Uij}\bar{D}_1 q_i u^c_j + Y_{Uij} H_U q_i u^c_j,
\]

(3)

where \(i, j\) are generation indices. Taking \(H_U\) and \(\bar{D}_1\) to have the same charges under the flavor symmetry, we can define the combination of \(\bar{D}_1\) and \(H_U\) that couples to \(X\) to be the messenger (indeed, this is the massive eigenstate), and the orthogonal combination to be the Higgs. A similar redefinition can be done for \(D_2\) and \(H_d\). Thus (1) is the most general superpotential and the entries \(y_{Uij}\) and \(Y_{Uij}\) are the same up to order-one coefficients\(^5\). Since the only order-one entry of \(Y_U\) is \(Y_{U33}\), the two matrices \(Y_U\) and \(y_U\) are approximately diagonal, and the soft terms inherit this structure. Inter-generational mixings are thus suppressed by supersymmetric alignment\(^6\).

\(^5\) The running between the UV scale and the messenger scale will introduce, of course, some mixing between \(H_U\) and \(\bar{D}_1\), but the only effect of this running is to modify the order-one coefficients of \(y_U\) and \(Y_U\).

\(^6\) It is also possible to choose different charges for \(H_U\) and \(\bar{D}_1\), (and similarly for \(H_D\) and \(D_2\)), such that the terms (2) are either forbidden or very suppressed. In this case, \(y_U\) and \(Y_U\) will have different textures, and these can be chosen to be compatible with flavor constraints [10].
B. A-terms and scalar masses

At leading order in \( F/M^2 \), the messenger-matter couplings of Eq. \([1]\) generate one-loop contributions to the \( A \)-terms, and two-loop contributions to the sfermion masses-squared. We present full expressions for the soft terms in Appendix A. In the case of interest, only the 3-3 entries of the coupling matrices are important and the soft terms (at the messenger scale) simplify to,

\[
A^U_{33} = -\frac{1}{16\pi^2} \left[ 3y_t^2 + y_b^2 \right] \frac{F}{M}, \\
A^D_{33} = -\frac{1}{16\pi^2} \left[ 3y_b^2 + y_l^2 \right] \frac{F}{M}, \\
A^L_{33} = -\frac{3y_l^2}{16\pi^2} \frac{F}{M},
\]

where \( Y_t \equiv Y_{U33} \) and similarly for the remaining couplings, and,

\[
\begin{align*}
\tilde{m}^2_{H_u} &= \frac{1}{128\pi^4} \left\{ -\frac{3}{2} Y_t^2 (3y_t^2 + y_b^2) + N \left( \frac{3}{4} g_3^4 + \frac{3}{20} g_1^4 \right) \right\} \left| \frac{F}{M} \right|^2, \\
\tilde{m}^2_{H_d} &= \frac{1}{128\pi^4} \left\{ -\frac{3}{2} Y_b^2 (3y_b^2 + y_l^2) - \frac{3}{2} Y^2 y_t^2 + N \left( \frac{3}{4} g_3^4 + \frac{3}{20} g_1^4 \right) \right\} \left| \frac{F}{M} \right|^2, \\
(\tilde{m}^2_q)_{33} &= \frac{1}{128\pi^4} \left\{ \left( y_t^2 + 3y_b^2 + 3y^2 + \frac{1}{2} y_l^2 - \frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{7}{30} g_1^2 \right) y_l^2 \\
&\quad + \left( 3y_t^2 + 3y^2 - \frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{13}{30} g_1^2 \right) y_t^2 \right\} \left| \frac{F}{M} \right|^2, \\
(\tilde{m}^2_u)_{33} &= \frac{1}{128\pi^4} \left\{ \left( 6y_t^2 + y_b^2 + Y^2 + 6Y^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right) y_t^2 - Y^2 y_b^2 \\
&\quad + N \left( \frac{4}{3} g_3^4 + \frac{4}{15} g_1^4 \right) \right\} \left| \frac{F}{M} \right|^2, \\
(\tilde{m}^2_d)_{33} &= \frac{1}{128\pi^4} \left\{ \left( 6y_b^2 + y^2 + y_t^2 + Y^2 + 6Y^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right) y_b^2 - y_t^2 y_b^2 \\
&\quad + 2 Y_b y_l Y_t y_t + N \left( \frac{4}{3} g_3^4 + \frac{1}{15} g_1^4 \right) \right\} \left| \frac{F}{M} \right|^2, \\
(\tilde{m}^2_l)_{33} &= \frac{1}{128\pi^4} \left\{ \left( \frac{3}{2} y_t^2 + 2y^2 - \frac{3}{2} g_2^2 - \frac{9}{10} g_1^2 \right) y_t^2 + \left( 2Y^2 y_t^2 + 3 Y_b Y_t y_t \right) + N \left( \frac{3}{4} g_3^4 + \frac{3}{20} g_1^4 \right) \right\} \left| \frac{F}{M} \right|^2, \\
(\tilde{m}^2_c)_{33} &= \frac{1}{128\pi^4} \left\{ \left( 3y_b^2 + 4y^2 - \frac{9}{5} g_3^2 \right) y_t^2 + \left( 2Y^2 y_t^2 + 2 Y_b y_l Y_t y_t \right) + \frac{3}{5} N g_4^4 \right\} \left| \frac{F}{M} \right|^2.
\end{align*}
\]

If the messenger scale \( M \) is below roughly \( 10^7 \) GeV, the one-loop \( \mathcal{O}(F^4/M^6) \) contributions \([11]\) to the soft masses may be important. In the limit of third-generation dominance,
these contributions are given by,

\[
\begin{align*}
\langle \delta \tilde{m}_q^2 \rangle_{33} &= -\frac{1}{6} \frac{1}{16\pi^2} (y_t^2 + y_b^2) \frac{F^4}{M^6} \\
\langle \delta \tilde{m}_{u^c}^2 \rangle_{33} &= -\frac{1}{3} \frac{1}{16\pi^2} y_t^2 \frac{F^4}{M^6} \\
\langle \delta \tilde{m}_{d^c}^2 \rangle_{33} &= -\frac{1}{3} \frac{1}{16\pi^2} y_b^2 \frac{F^4}{M^6} \\
\langle \delta \tilde{m}_l^2 \rangle_{33} &= -\frac{1}{6} \frac{1}{16\pi^2} y_\tau^2 \frac{F^4}{M^6} \\
\langle \delta \tilde{m}_{e^c}^2 \rangle_{33} &= -\frac{1}{3} \frac{1}{16\pi^2} y_\tau^2 \frac{F^4}{M^6} .
\end{align*}
\]

(6)

For completeness we also show the next-to-leading contribution in \( F/M^2 \) to the top \( A \)-term,

\[
\delta A_{33}^U = -\frac{1}{16\pi^2} y_t^2 \frac{F^3}{M^5} .
\]

(7)

Comparing the new contributions to the mGMSB expressions, we see that the importance of the new contributions relative to the mGMSB expressions is maximal for the smallest possible number of messengers. Thus we will take \( N = 1 \) when the only new Yukawa coupling is \( y_t \), and \( N = 2 \) if \( y_b \) and/or \( y_\tau \) are present as well.

As is well known [3], at one-loop, the Higgs mass is approximately given by,

\[
m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \left[ \log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} \left[ 1 - \frac{X_t^2}{12M_S^2} \right] \right] ,
\]

where \( X_t = A_t - \mu \cot \beta \) is the LR stop mixing and \( M_S \equiv (m_{1t}m_{2t})^{1/2} \) is the average stop mass. Clearly, the Higgs mass can be increased either by increasing the average stop mass, so that the log term is large, or by increasing the stop mixing, so that \( X_t/M_S \) is large, with the maximal \( m_h^2 \) obtained for \( X_t/M_S \) of around 2.4 [26].

Since our main objective is to obtain the correct Higgs mass with superpartners within LHC reach, we need a large \( X_t/M_S \), and therefore a large \( A_t \). As can be seen from equations (5), a non-zero \( y_t \), which generates \( A_t \), also gives new contributions to the stop masses, proportional to \( y_t^4 \). These contributions are positive, so that \( M_S \) is increased as well. Note that the new coupling also gives rise to negative contributions to the stop masses, proportional to \( y_t^6 g^2 \), but there is an accidental cancellation between these and the positive \( y_t^6 Y_t^2 \) contribution. At low messenger scales however, the one-loop contributions are important, and since these are negative, one can obtain large \( X_t/M_S \) with low \( M_S \).

Thus, at low messenger scales, our models can give a heavy Higgs together with light stops, while at high scales, a heavy Higgs necessitates heavy stops. The messenger-scale masses of the remaining superpartner remain unchanged, and will only be modified by the running.

III. HIGGS MASS AND SUPERPARTNER MASSES

A. The Higgs and stop masses

We first consider models with one set of messengers. With the \( Z_3 \) charges of Table I, only the \( D_1 \) messenger couplings to matter are allowed. Moreover, since we assume that \( D_1 \)
and $H_u$ have the same charge under the flavor symmetry of the model, the only significant messenger coupling is $y_t \equiv (y_{t'})_{33}$. We use SOFTSUSY [27] to calculate the Higgs mass for different choices of the GMSB parameters and $y_t$. Given the theoretical uncertainty in the calculation of the Higgs mass, it is interesting to study Higgs masses in the 124–128 GeV window.

In Fig. 1 we show contours of the Higgs and stop masses as a function of $\Lambda \equiv F/M$ and $y_t$, for a low messenger scale of $M = 900$ TeV, with $\tan \beta = 10$. For such a low messenger scale, the one-loop $O(F^4/M^6)$ corrections are not necessarily negligible and have been taken into account. Fig. 1(a) shows the Higgs mass contours for a wide range of $y_t$. The white region for intermediate values of $y_t$ is excluded. In this region, the stops are either tachyonic or too light for successful electroweak symmetry breaking. As explained above, for these values of $y_t$, the negative one-loop contribution to the stop mass-squared is comparable to the positive contributions from pure GMSB. As $y_t$ is increased, the $y_t^4$ contribution to the stop masses guarantees that the stops are non-tachyonic, but because of the partial cancellation between the 1-loop and 2-loop contributions, the $A$-term becomes appreciable compared to the stop masses and the resulting large mixing allows for a heavy Higgs. As $y_t$ is increased further, the stops become heavy relative to the other superpartners. In this regime, the heavy stops and large $A$-terms both play a crucial role in making the Higgs heavy.

In Fig. 1(b) we zoom in on the interesting range $y_t \sim 1$, and show contours of the Higgs mass together with the two stop masses, with the remaining parameters being the same as in Fig. 1(a). In Fig. 1(c) we also show contours of $\mu$ and the mixing parameter $x_t = |X_t/M_S|$. As expected, the largest values of $x_t$ are obtained close to the excluded regions where one of the stops is relatively light. Thus, appreciable $A$-terms can be obtained without a large increase in the stop squared masses. Indeed, as can be seen in Fig. 1(b), for these large values of $x_t$, the Higgs mass can be large even for low $\Lambda$’s, such that the stops are light. For $y_t \sim 1$ we can therefore find at least one stop below 2 TeV.

We can get even lighter stops by lowering the messenger scale, which allows for a lower $\Lambda$. In Fig. 2 we show the behavior of the models for $M = 400$ TeV. Note that since we only know the leading $F/M^2$ behavior of the new contributions to the soft masses, we keep $F/M^2 < 0.5$ (the pure GMSB higher-order corrections are known to be small [28]). Indeed, a Higgs mass of 125 GeV is obtained with both stops between 1.5 and 2 TeV, and stops below 1.5 TeV allow for Higgs masses above 124 GeV (as mentioned above, one should bear in mind the uncertainty in our Higgs mass calculation). The remaining squarks will be quite light too in this region, and we will give a few example spectra to illustrate this in the next Section.

For higher messengers scales, the behavior of the models is qualitatively different. To demonstrate this, we present similar plots for two other messenger scales, $M = 10^{12}$ GeV in Fig. 3 and $M = 10^8$ GeV in Fig. 4. Clearly, the tachyonic stop region for moderate $y_t$ is absent for these high scales, since the negative one-loop contribution is negligible. Thus a 125 GeV Higgs requires heavy stops, above 4 TeV. It is interesting to compare our results for high messenger scales with models of minimal gauge mediation. As is well known, with a lot of running, appreciable $A$-terms can be generated in pure GMSB models. This is, however, not sufficient—as was shown in Ref. [7], even with a high messenger scale a heavy Higgs requires very heavy stops near 8-10 TeV. For example, with $M = 7.9 \cdot 10^{12}$ GeV and $\tan \beta = 10$, a Higgs mass of 125 GeV can be achieved if $\Lambda = 1.3 \cdot 10^6$ GeV [7]. With such a high value of $\Lambda$, one of the stops is the lightest squark and has a mass of 7.9 TeV. In contrast, if we choose the same messenger scale in the FGM model with $y_t = 1.1$, a Higgs mass of 124.2 GeV can
FIG. 1: The Higgs and stop masses for $N = 1$, $M = 900$ TeV, $\tan \beta = 10$. Fig. 1(a) shows the Higgs mass for a wide range of $y_t$. The predictions of minimal gauge mediation can be read off from the line $y_t = 0$. The white region is excluded because it leads to tachyonic stops (see text). In Fig. 1(b) we show Higgs mass (solid), heavy stop mass (dotted) and light stop mass (dashed) contour lines in a smaller region of $y_t$. In Fig. 1(c) we show Higgs (solid), $\mu$ (dashed) and $x_t = |X_t/M_S|$ (dotted) contour lines in the same region.

be achieved with $\Lambda = 3.35 \cdot 10^5$ GeV. While the stops are still very heavy (around 5 TeV) due to the messenger Yukawa contributions, the remaining superpartners are significantly lighter, with the gluino and right handed up and charm squarks around 2.3–2.4 TeV, and a 461 GeV bino NLSP.

While the models with only messenger-top Yukawas are the most economical ones, viable
models with additional messenger-bottom and messenger-tau couplings may also be viable. As an example, in Fig. 5 we present Higgs mass contours in the $y_t - y_b$ plane for $N = 2$, with $M = 10^8$ GeV, $\Lambda = 230$ TeV, and $\tan \beta = 10$. For large values of $y_b$, the sleptons become tachyonic, leading to the white excluded region on the right.

Since we are turning on order-one superpotential couplings, it is interesting to ask at what scales these become non-perturbative. For example, if $N = 1$ the high scale models with $M = 10^{12}$ GeV remain perturbative even above the GUT scale. For $M = 10^8$ GeV, one loses perturbativity at around $10^{12} - 10^{13}$ GeV for $y_t \sim 1$. Finally, with $M = 900$ TeV,
the models stay perturbative up to $10^9 - 10^{10}$ GeV for $y_t$ above 1, and for a smaller $y_t \sim 0.9$ (as is the case with one of the lighter spectra we show below. See Table II) up to scales of around $10^{13}$ GeV. In the case of $N = 2$ the couplings remain perturbative for a few order of magnitude above the messenger scale.

B. Superpartner spectra and LHC signatures

To understand the phenomenology of our FGM models, we present in Table II complete superpartner spectra for several choices of the parameters at low, intermediate and high messenger scales. A detailed analysis of the experimental signatures is beyond the scope of

FIG. 5: Higgs mass contours in the $y_t - y_b$ plane with $M = 10^8$ GeV, $\Lambda = 230$ TeV, and tan $\beta = 10$. 
TABLE II: Model parameters, and resulting Higgs parameters and spectra for eight sample models. All mass parameters are given in GeV.

| Parameter | Spect. 1 | Spect. 2 | Spect. 3 | Spect. 4 | Spect. 5 | Spect. 6 | Spect. 7 | Spect. 8 |
|-----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $M_{\text{mess}}$ | 2·10^5 | 4·10^5 | 9·10^5 | 1·10^8 | 1·10^8 | 1·10^{12} | 1·10^{12} | 7.9·10^{12} |
| $\Lambda$ | 1.05·10^5 | 1.65·10^5 | 3.03·10^5 | 3.08·10^5 | 2.74·10^5 | 4.00·10^5 | 3.55·10^5 | 3.35·10^5 |
| $\tan \beta$ | 10 | 10 | 10 | 20 | 20 | 10 | 10 | 10 |
| $y_t$ | 1.45 | 1.20 | 0.92 | 1.19 | 1.18 | 1.13 | 1.14 | 1.10 |
| $\mu$ | 2606 | 3165 | 4053 | 6405 | 5648 | 7844 | 7091 | 6278 |
| $h_0$ | 124.9 | 125.4 | 126.0 | 125.0 | 124.5 | 125.0 | 124.5 | 124.2 |
| $A$ | 2686 | 3281 | 4248 | 6570 | 5792 | 8107 | 7319 | 6493 |
| $H_0$ | 2686 | 3281 | 4250 | 6567 | 5791 | 8105 | 7319 | 6493 |
| $H_\pm$ | 2687 | 3283 | 4249 | 6571 | 5792 | 8107 | 7320 | 6494 |
| $t_1$ | 1620 | 1997 | 1795 | 5698 | 4986 | 6243 | 5634 | 4899 |
| $t_2$ | 2050 | 2315 | 2623 | 7232 | 6305 | 7576 | 6855 | 5826 |
| $b_L$ | 1680 | 2069 | 2615 | 5654 | 4948 | 6195 | 5591 | 4864 |
| $b_R$ | 1119 | 1683 | 2918 | 2721 | 2439 | 3359 | 3007 | 2803 |
| $u_L$, $c_L$ | 1179 | 1780 | 3116 | 2987 | 2672 | 3616 | 3229 | 3029 |
| $u_R$, $c_R$ | 1096 | 1668 | 2950 | 2508 | 2257 | 2704 | 2401 | 2281 |
| $d_L$, $s_L$ | 1182 | 1781 | 3117 | 2987 | 2673 | 3617 | 3230 | 3030 |
| $d_R$, $s_R$ | 1133 | 1698 | 2950 | 2508 | 2257 | 2704 | 2401 | 2281 |
| $e_L$, $\mu_L$ | 305 | 525 | 1039 | 569 | 523 | 578 | 417 | 620 |
| $e_R$, $\mu_R$ | 356 | 458 | 618 | 1514 | 1333 | 2378 | 2149 | 1991 |
| $\tau_L$ | 244 | 550 | 1038 | 505 | 462 | 555 | 390 | 603 |
| $\tau_R$ | 392 | 418 | 604 | 1476 | 1303 | 2363 | 2135 | 1979 |
| $\nu_e$ | 295 | 519 | 1035 | 555 | 509 | 562 | 397 | 606 |
| $\nu_{\mu}$, $\nu_\tau$ | 295 | 519 | 1036 | 563 | 516 | 571 | 408 | 614 |
| $\chi_1$ | 151 | 233 | 425 | 426 | 378 | 552 | 488 | 461 |
| $\chi_2$ | 299 | 457 | 822 | 826 | 732 | 1056 | 935 | 880 |
| $\chi_3$ | 2642 | 3208 | 4107 | 6573 | 5793 | 8010 | 7239 | 6403 |
| $\chi_4$ | 2643 | 3209 | 4108 | 6573 | 5793 | 8010 | 7240 | 6404 |
| $\chi_4^-$ | 299 | 458 | 823 | 826 | 733 | 1056 | 935 | 880 |
| $\chi_2^\pm$ | 2643 | 3210 | 4109 | 6573 | 5794 | 8011 | 7240 | 6404 |
| $g$ | 804 | 1315 | 2240 | 2251 | 2024 | 2832 | 2540 | 2404 |

The parameter space of the models region is such that gluino and squark masses will be accessible in the 14 TeV LHC. Indeed, in many of the examples shown in this paper, and these spectra are only meant to illustrate the general features of the models. Thus for example, spectrum 1, which has a light gluino and first generation squarks, is ruled out by jets-plus-missing-energy searches like [31, 32] and possibly by specific GMSB searches [33, 34]. It is nonetheless useful to point out a few key features.

First, large regions of the parameter space yield gluino and squark masses that will hopefully be accessible in the 14 TeV LHC. Indeed, in many of the examples shown in Table II, the gluino and some first generation squarks are below 2.5 TeV, and sometimes considerably lighter.

Second, even with a single messenger pair, for which mGMSB models usually predict a neutralino NLSP, the NLSP in our models can be either a bino, a left-handed charged...
slepton, or a sneutrino, depending on the choice of parameters. This is due to the fact that the $U(1)_Y$ contributions to the RG evolution of sfermion masses contain a term proportional to $S \equiv \text{Tr}[\Sigma_j m^2_{\phi_j}]$, where the trace is taken over all the SM sfermions. Normally this contribution suppresses the right-handed sfermions masses, but in flavored GMSB the sign of $S$ changes for sufficiently large $y_t$. Furthermore, since for large $y_t$ the stop masses are much larger than the remaining soft-terms, the effect of $S$ through the running is dramatic. As a result, in some regions of the parameter space, the right-handed sleptons become heavy while the left-handed ones are relatively light.

While squarks are generically heavy in this class of models, the sleptons, charginos and neutralinos can be quite light, and may therefore be discovered even if produced directly. This depends of course on the details of the spectrum, and in particular on the identity and lifetime of the NLSP. Whether or not the NLSP decays inside the detector depends on the gravitino mass, which involves a lot of uncertainty\(^7\). In any case, the fact that the gravitino is the LSP in these models can provide additional handles for their discovery, using either prompt NLSP decays to the gravitino as in [33,34], or the long lifetime of a charged NLSP. Thus for example, the current bound on a long-lived charged slepton NLSP in mGMSB is 300 GeV [35] and the model-independent bound on Drell-Yan produced right-handed sleptons just somewhat below that. These bounds are likely to improve considerably for long-lived charged left-handed sleptons at the 14 TeV LHC.

It would also be interesting to study models with more messenger pairs. Since the pure gauge contribution to the scalar squared masses is proportional to $N$, we expect that for such models the cancellation between the negative new contributions and pure gauge contributions will be less dramatic than for the $N = 1$ models, so that very light spectra would not occur. Such models would typically give rise to gluino masses above squark masses, and to slepton or sneutrino NLSPs.

**IV. CONCLUSIONS**

We have shown that gauge-mediated models with messenger-matter couplings can give rise to an acceptable Higgs mass, with colored superpartners within LHC reach. The important new ingredient in these models is the messenger-scale top $A$-term, which gives rise to significant stop mixing, and therefore enhances the Higgs mass. While a heavy Higgs can be obtained for a wide range of messenger scales, the details of the superpartner spectra may vary significantly. For low messenger scales, the one-loop $O(F^4)$ negative contributions to the stop masses are important, so that the stops are relatively light while the Higgs mass is raised largely due to the $A$-terms. For large messenger scales, the stops become heavy due to large positive contributions proportional to the messenger Yukawas. As a result, the Higgs mass is raised both due to the $A$-terms and to the large stop masses.

The messenger Yukawas often lead to another novel effect – the $U(1)_Y$ contribution to

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\(^7\) If $F_X$ is the dominant $F$-term, the gravitino mass in our models varies between about 100 eV for $M = 900$ TeV to a GeV for the high-scale models. It is quite plausible however that there are much higher $F$ terms in the supersymmetry-breaking sector, as is often the case in calculable models, in which case $F_X$ is generated through several small couplings from a higher $F$-term. For large values of the gravitino mass, the NLSP would decay outside the detector. Heavy gravitinos from late NLSP decay could also supply warm dark matter [7,29].
sfermion RGE’s may change sign so that left-handed sleptons become light. We see that the resulting spectra can be quite diverse with either a neutralino or slepton NLSP. We leave a detailed study of the phenomenology of these models for future work. It would also be interesting to examine models with down-type messenger couplings, which may lead to rather different phenomenology. We further note that while we have concentrated on models with only a single pair of messengers and a messenger-top coupling, the results can be easily generalized to models with several messenger pairs, with or without messenger down-type couplings.

The structure of the matter-messenger couplings can naturally be the same as the structure of the usual Yukawas, since the new couplings are obtained by replacing $H_U$ and/or $H_D$ by the messenger of the same gauge charge. The models are therefore protected against flavor-violation by \textit{supersymmetric alignment}. This can be simply realized in the context of flavor symmetries if the Higgs and corresponding messenger have the same charges under the flavor symmetry.

The amount of tuning in our models is related to the tuning of the new messenger-matter coupling $y_t$. As can be seen in Table II for spectrum 1, which relies on a finely-dialed $y_t$ to obtain a light spectrum, the $\mu$-term is 2.6 TeV, while less tuned choices of $y_t$ require larger $\mu$-terms, around 6 or 8 TeV. Generating an acceptable $\mu$-term in our models is an important direction to pursue. If there is a successful mechanism for generating the $\mu$-term, the tuning question would translate into the question of how finely one needs to tune the coupling $y_t$. It is probably far from trivial to find a successful mechanism for generating the $\mu$-term, and even if it is found, the required tuning of $y_t$ is likely to be significant. Still, since $y_t$ is a superpotential coupling, the tuning involved is qualitatively different from the tuning of the Higgs mass in the standard model.

\textbf{Note added}

After posting the original version of this article on the archive, M. Ibe drew our attention to [36, 37], which use the same messenger-top coupling in order to raise the Higgs mass. There is thus some overlap between our paper and [36, 37] as concerns the implications of a heavy Higgs. The origin of the messenger-matter coupling is different however in [36, 37], with the result that these models are MFV. Also, while we calculate the new contributions of the full 3-generation coupling matrices, [36, 37] only consider third generation couplings. While completing this revised version, Ref. [38] appeared, which surveys different messenger-matter couplings. Our results for the soft masses now agree with [36, 38], but our approach to the calculation differs from theirs.

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Appendix A: Derivation of the soft terms

In this Appendix, we describe the calculation of the soft terms. As pointed out in [20], these can be extracted from the wave function renormalizations of the light fields, treating the heavy threshold as a spurion. The main advantage of this method is that the running of the wave function renormalizations, as well as of the various couplings, is only needed at one-loop. The method of [20] was used in [22] to obtain the soft terms in models with messenger-matter couplings. However, the analysis of [22], as well as the analysis in an earlier version of this article, did not treat the matter-messenger mixing correctly. In A1 we clarify this issue by discussing a simple toy model. In A2 we generalize the results to models with multiple fields and couplings. Our final results are given in A3. As a double check of our derivation, we explicitly calculate the relevant contributions, namely the mixed $y^2Y^2$ terms, in A4.

1. Analytic continuation in the presence of mixing

To discuss the calculation of the soft terms in the presence of messenger-SM mixings, we first consider a simple toy model, with the superpotential

\[ W = X \bar{D}D + (y^0 D + Y^0 H)le. \] (A1)

Here $X = M + F \theta^2$ is the SUSY breaking spurion, $\bar{D}, D, H, l$ and $e$ are singlet fields, and we use the superscript $^0$ to denote the superpotential couplings $y^0$ and $Y^0$ in order to distinguish them from the running couplings.

Our analysis closely follows [22], which applied the method of [20] to models with multiple couplings, in which one cannot integrate the one-loop RGEs to obtain closed-form expressions for the wave function renormalizations and couplings. The necessary ingredients in the calculation are the RGE’s for the various couplings, and the boundary conditions for these couplings. In the absence of mixing between the messengers and SM fields, there is a clear distinction between the messengers and light fields. In our toy model however, $H$ and $D$ mix, and as a result, there is some ambiguity in the identification of the messenger and Higgs couplings. The key in the calculation is therefore the correct matching of the high-energy and low-energy theories, which we will perform by identifying the physical heavy and light combinations of messenger and Higgs fields, and by demanding that the physical coupling of the light combination is continuous across the threshold.

Let us first recall the main results of [20]. At leading order in $F/M^2$, the soft mass of the light field $f$ can be extracted from its wave function renormalization $Z_f$,

\[ m_f^2(\mu) = -\frac{1}{4} \frac{\partial^2 \ln Z_f(\mu)}{\partial \ln M^2} \frac{F^2}{M^2}, \] (A2)

at $\mu \leq M$. This relies on the fact that, at this order, the only threshold dependence enters through the one-loop running of $Z_f$, and therefore one can obtain the soft masses
by promoting $M$ to a superfield. Specifically, as argued in [20] based on symmetries and holomorphy, $Z_f(M) \rightarrow Z_f(\sqrt{X^\dagger X})$. Note that, since the theory is defined at a scale $\Lambda$ above $M$, the derivatives with respect to $M$ are taken while holding the physical couplings at $\Lambda$ fixed. It is therefore natural to choose a canonical Kähler potential at $\Lambda$, and to hold the superpotential couplings fixed while taking derivatives with respect to $M$.

It will be convenient to rewrite our model by defining, $\phi_1 \equiv D, \phi_2 \equiv H, y_1^0 \equiv y^0$ and $y_2^0 \equiv Y^0$. The high energy theory is then defined at the cutoff $\Lambda$, by the superpotential

$$W = XD\phi_1 + y_0^0 \phi_1 e$$

(A3)

where $i = 1, 2$. As noted above, we take the Kähler potential to be canonical at $\Lambda$.

At any scale $\mu$ below $\Lambda$ and above the messenger scale, the renormalized fields are

$$\phi_r(\mu) \equiv Z^{1/2}(\mu) \phi_j \quad l_r(\mu) \equiv Z^{1/2}(\mu) l, \quad e_r(\mu) \equiv Z^{1/2}_e(\mu) e,$$

(A4)

Here $Z^{1/2}$ is the square-root of the two-by-two wave-function renormalization matrix $Z$. The running couplings are given by

$$y_i(\mu) = Z_l^{-1/2}(\mu) Z_e^{-1/2}(\mu) Z^{-1/2}(\mu)_{ji} y_j^0.$$  

(A5)

Note that $Z$ is a real superfield. At one loop, $Z$ runs according to

$$\frac{dZ}{dt} = Z^{1/2} \gamma Z^{1/2},$$

(A6)

where $\gamma$ is the two-by-two matrix of anomalous dimensions.

At the messenger scale $\mu_X$, we have a heavy combination $\tilde{\phi}_{r1}$ and (the orthogonal) light combination $\tilde{\phi}_{r2}$,

$$\tilde{\phi}_{r1} = \left[ Z^{-1/2}(\mu_X)_{11} \phi_{r1} + Z^{-1/2}(\mu_X)_{12} \phi_{r2} \right] / C(\mu_X)$$

(A7)

$$\tilde{\phi}_{r2} = \left[ Z^{-1/2}(\mu_X)_{11} \phi_{r2} - Z^{-1/2}(\mu_X)_{21} \phi_{r1} \right] / C(\mu_X)$$

(A8)

where

$$C(\mu_X) = \sqrt{(Z^{-1/2}(\mu_X)_{11})^2 + |Z^{-1/2}(\mu_X)_{12}|^2}$$

(A9)

and where we used

$$(Z^{-1/2})_{ij} = (Z^{-1/2})_{ji}$$

(A10)

We can now find the physical couplings at the threshold. The physical coupling of the heavy combination $\tilde{\phi}_{r1}$ to $l$ and $e$ is,

$$\tilde{y}_1(\mu_X) = Z_l^{-1/2}(\mu_X) Z_e^{-1/2}(\mu_X) C(\mu_X) \left[ y_0^0 + \frac{y_2^0 Z^{-1/2}(\mu_X)_{21} (Z^{-1/2}(\mu_X)_{11} + Z^{-1/2}(\mu_X)_{22})}{C(\mu_X)^2} \right],$$

(A11)

and the physical coupling of the light combination $\tilde{\phi}_{r2}$ to $l$ and $e$ is,

$$\tilde{y}_2(\mu_X) = Z_l^{-1/2}(\mu_X) Z_e^{-1/2}(\mu_X) \frac{Z^{-1/2}(\mu_X)_{11} Z^{-1/2}(\mu_X)_{21} - |Z^{-1/2}(\mu_X)_{12}|^2}{\sqrt{|Z^{-1/2}(\mu_X)_{11}|^2 + |Z^{-1/2}(\mu_X)_{12}|^2}} y_0^0.$$  

(A12)
The physical messenger scale is
\[
\mu_X^2 = (|Z^{-1/2}(\mu_X)_{11}|^2 + |Z^{-1/2}(\mu_X)_{12}|^2) \, X^\dagger X. \tag{A13}
\]
To leading order, we can replace \( \mu_X^2 \) by \( X^\dagger X \), since the difference between the two gives a 3-loop correction to the soft masses (see also [30]). In the following we will therefore set \( \mu_X = M \). Furthermore, the expression for the soft masses (A2) involves the running of \( Z \) and the couplings at one-loop only. Thus we only need to match the couplings at the threshold at one-loop, and at this order the coupling of the heavy combination is
\[
\tilde{y}_1(M) = Z_t^{-1/2}(M) Z_e^{-1/2}(M) Z_l^{-1/2}(M)_{11} \left( y_1^0 + 2Z^{-1/2}(M)_{21} y_2^0 \right), \tag{A14}
\]
while the coupling of the light combination is,
\[
\tilde{y}_2(M) = Z_t^{-1/2}(M) Z_e^{-1/2}(M) Z_l^{-1/2}(M)_{22} y_2^0. \tag{A15}
\]
Equations (A14) and (A15) are the key results of the preceding analysis, and lead to the main difference between our results and the results of [22]. The point is that these couplings do not coincide with the running couplings \( y_i(M) \) of eqn. (A5). In particular, the coupling of the light combination at the threshold, \( \tilde{y}_2(M) \), does not involve either \( y_1^0 \) or the mixed anomalous dimension \( \gamma_{l2} \) since at one loop \( (Z^{-1/2})_{22} \) only depends on \( \gamma_{l2} \). Consequently, as we will see below, the soft mass of the light combination \( H \) does not depend on the mixed anomalous dimension \( \gamma_{l2} \). This is precisely what one would expect intuitively\(^8\). On the other hand, the coupling of the heavy combination (A14) involves both \( y_1^0 \) and \( y_2^0 \), with the latter multiplied by the mixed anomalous dimension \( \gamma_{l2} \). However, this contribution appears with a factor of 2 compared to the analogous term in the running coupling \( y_1(M) \).

The two conclusions of the above discussion, namely, the absence of \( \gamma_{l2} \) in the coupling of the light combination, and the factor of 2 multiplying \( \gamma_{l2} \) in the coupling of the heavy combination, only involve the fields \( H \) and \( D \), and are not affected by the structure of the couplings to the remaining fields \( l \) and \( e \). These conclusions therefore carry over trivially to the full 3-generation model. In other words, we only need to integrate out the heavy field once, and at one-loop, this procedure only involves the wave-function renormalizations of \( H \) and \( D \).

We can now turn to the low energy theory. Clearly, this theory can be written in terms of the fields \( l \), \( e \), and \( H \). Its coupling is defined by matching to the high scale theory at the threshold. That is, we require that the running coupling in the low-energy theory, \( Y(\mu) \) match the physical coupling of the light combination at the threshold \( M \) at one-loop,
\[
Y(M) = \tilde{y}_2(M) \tag{A16}
\]
with \( \tilde{y}_2(M) \) given by eqn. (A15). The low-energy theory is therefore defined by
\[
W = Y^0 Hle \tag{A17}
\]
with \( Z_l(M), Z_e(M) \) and \( Z_H(M) \) continuous across the threshold. More precisely, for the latter\(^9\),
\[
Z_H(M) = Z_{22}(M). \tag{A18}
\]
\(^8\) In fact, in the earlier version of this article, this intuition motivated us to ignore the contributions of \( \gamma_{l2} \) in the soft masses. This is indeed correct for \( H \), but not for the other SM fields.  
\(^9\) Note that at one loop, \( (Z_2^{-1/2})^2 = Z_2^{-1} \).
Thus, both the wave-function renormalization and the physical coupling of the light combination are continuous across the threshold as one would expect, but, as noted above, the coupling of this combination is different from the running coupling $y_2(M)$.

Note that, since we only have a single combination of $\phi_1$ and $\phi_2$ in the low-energy theory, we have reverted to the original notation and replaced $y_2^0$ by $Y^0$. The running coupling below $M$ is therefore

$$Y(t) = \frac{Z_t^{-1/2}(t)}{Z_t^{-1/2}(M)} \frac{Z_e^{-1/2}(t)}{Z_e^{-1/2}(M)} \frac{Z_H^{-1/2}(t)}{Z_H^{-1/2}(M)} \bar{y}_2(M). \quad (A19)$$

### a. slepton mass

Let us use this to calculate the $l$ mass following [22]. For $\mu < M$,

$$\ln Z_l(\mu) = -\int_{M}^{\mu} \gamma_l^>(t') dt' - \int_{t}^{M} \gamma_l^<(t') dt', \quad (A20)$$

with $t = \ln \mu$. We use the superscript $> (<)$ to denote the theory above (below) $M$.

We have

$$\frac{\partial}{\partial \ln M} \ln Z_l(\mu) = \gamma_l^>(M) - \gamma_l^<(M) - \int_{t}^{M} \frac{\partial}{\partial \ln M} \gamma_l^<(t') dt', \quad (A21)$$

and

$$\frac{\partial^2}{\partial \ln M^2} \ln Z_l(M) = -\frac{\partial}{\partial \ln M} \left( \gamma_l^>(M) - \gamma_l^<(M) \right) - \frac{\partial}{\partial \ln M} \gamma_l^< (t)|_{t=\ln M}. \quad (A22)$$

The jump in the $l$ anomalous dimension is given by the contribution of the heavy field to $\gamma_l$. Therefore,

$$\gamma_l^>(M) - \gamma_l^<(M) = -\frac{2}{16\pi^2} |\bar{y}_1(M)|^2. \quad (A23)$$

The $l$ anomalous dimension at scales below $M$ is given by

$$\gamma_l^<(t) = -\frac{2}{16\pi^2} |Y(t)|^2, \quad (A24)$$

with

$$|Y(t)|^2 = \frac{Z_l(M)Z_e(M)Z_h(M)}{Z_l(\mu)Z_e(\mu)Z_H(\mu)} Y(M) = Z_i^{-1}(\mu)Z_e^{-1}(\mu)Z_H^{-1}(\mu) |y_2^0|^2. \quad (A25)$$

Thus,

$$m_l^2(M) = -\frac{1}{4} \frac{F}{M}^2 \frac{\partial^2}{\partial \ln M^2} \ln Z_l(M) = \frac{1}{4} \frac{F}{M}^2 \left[ \frac{2}{16\pi^2} \left| \frac{\partial}{\partial \ln M} |\bar{y}_1(M)|^2 \right| + \frac{\partial}{\partial \ln M} |Y(t)|^2 \right]_{t=\ln M}. \quad (A26)$$

The derivatives in the expression above involve of course the beta functions of the two couplings, which in turn are combinations of the various anomalous dimensions. The derivative of the first term can be obtained from eqn. [A14], and, as explained above, involves double the usual contribution of $\gamma_{12}$. 
In contrast, the derivative of the second term does not contain \( \gamma_{12} \) at all, since \( \gamma_{12} \) cannot appear in the theory below \( M \), and does not appear in \( Y(M) \) as we saw above. To obtain the second term of eqn. (A26), we can use first eqn. (A20) which gives at one loop,

\[
\frac{\partial}{\partial \ln M} \ln Z_l(M) = \Delta \gamma_l(M) = \gamma_l^\ast(M) - \gamma_l^\ast(M)
\]

so that

\[
\frac{\partial}{\partial \ln M} Z_l^{-1}(\mu) = -\Delta \gamma_l(M).
\]

We also need the analogous expression for \( Z_H \),

\[
\ln Z_H(\mu) = \ln Z_{22}(M) - \int_t^{\ln M} \gamma_H dt'
\]

so at one-loop

\[
\frac{\partial}{\partial \ln M} Z_H(M) = \gamma_{22}(M) - \gamma_H(M) = 0.
\]

Plugging these in eqn. (A26), we have,

\[
m_l^2(M) = \frac{1}{4} \frac{2}{16\pi^2} \left[ (\gamma_l^\ast + \gamma_c^\ast + \gamma_{11})y^2 + 2\gamma_{12}yY - [\Delta \gamma_l + \Delta \gamma_c]Y^2 \right] \left| \frac{F}{M} \right|^2
\]

with everything evaluated at the scale \( M \). Substituting in the values of the anomalous dimensions one gets

\[
m_l^2 = \frac{1}{(4\pi)^4} (4y^4 + 2y^2Y^2) \left| \frac{F}{M} \right|^2,
\]

or for a simplified model where all fields are singlets

\[
m_l^2 \bigg|_{\text{simplified}} = \frac{1}{(4\pi)^4} (3y^4 + 2y^2Y^2) \left| \frac{F}{M} \right|^2,
\]

Alternatively, we can rewrite eqn. (A31) in a way that is more similar to the expression of [22],

\[
\tilde{m}^2(M) = -\frac{1}{4} \left( \frac{d \Delta \gamma}{dy} \left[ \beta_y \right]_{\gamma_{12}=0} - \gamma_{12} \frac{d \Delta \gamma}{dy} Y - \frac{d \gamma^\ast}{dy} [\Delta \beta_Y]_{\gamma_{12}=0} \right) \left| \frac{F}{M} \right|^2.
\]

Here the various \( \beta \)'s and anomalous dimensions are the standard ones, and \([\beta_y]_{\gamma_{12}=0}\) indicates that \( \gamma_{12} \) should be set to zero in the expression for the relevant \( \beta \). One could in principle denote the couplings collectively by \( \lambda \), as in [22], but \( \Delta \gamma \) only depends on the messenger couplings \( y \), whereas \( \gamma^\ast \) only depends on the Higgs couplings \( Y \).

b. Higgs mass

To calculate the Higgs mass we again need to take two derivatives of

\[
\ln Z_H(\mu) = \ln Z_{22}(M) - \int_t^{\ln M} \gamma_H^\ast(t') dt'.
\]
Since there is no jump in the anomalous dimension of $H$, one could immediately start with the analog of eqn. (A22) and set $\Delta \gamma_H = 0$. We can also derive this result more carefully. Writing

$$Z_{22}(M) = 1 - \int_{\ln M}^{\ln \Lambda} (Z^{1/2} \gamma Z^{1/2})_{22} dt \quad (A36)$$

we find (dropping 3-loop terms)

$$\frac{\partial^2}{\partial \ln M^2} \ln Z_H(M) = \frac{\partial}{\partial \ln M} \gamma_{22}(M) + |\gamma_{12}|^2(M). \quad (A37)$$

Then

$$\frac{\partial^2}{\partial \ln M^2} \ln Z_H(M) \bigg|_{\mu=M} = \frac{\partial}{\partial \ln M} (\gamma_{22}(M) - \gamma_H(M)) + |\gamma_{12}|^2 - \frac{\partial}{\partial \ln M} \gamma_H^< (t) \bigg|_{t=\ln M}. \quad (A38)$$

Note that $\gamma_{22}(M)$ and $\gamma_H(M)$ differ at one-loop:

$$\gamma_{22}(M) = \frac{-2}{16\pi^2} |y_2(M)|^2 = \frac{-2}{16\pi^2} \left[ |y_2^0|^2 Z_{22}^{-1}(M) Z_{l}^{-1}(M) Z_{e}^{-1}(M) + \left( y_1^0 y_2^{0*} Z_{12}^{-1/2}(M) + \text{cc} \right) \right], \quad (A39)$$

whereas

$$\gamma_H(M) = \frac{-2}{16\pi^2} |\tilde{y}_2(M)|^2 = \frac{-2}{16\pi^2} |y_2^0|^2 Z_{22}^{-1}(M) Z_{l}^{-1}(M) Z_{e}^{-1}(M). \quad (A40)$$

Therefore

$$\frac{\partial}{\partial \ln M} (\gamma_{22}(M) - \gamma_H(M)) = \frac{-2}{16\pi^2} \frac{\partial}{\partial \ln M} \left( y_1^0 y_2^{0*} Z_{12}^{-1/2}(M) + \text{cc} \right) = -|\gamma_{12}(M)|^2, \quad (A41)$$

which precisely cancels the third term in (A38). We are then left with

$$\frac{\partial^2}{\partial \ln M^2} \ln Z_H(M) \bigg|_{\mu=M} = -\frac{\partial}{\partial \ln M} \gamma_H^< (t) \bigg|_{t=\ln M}. \quad (A42)$$

In this case, the soft mass only depends on the anomalous dimension in the low-energy theory, and therefore does not involve the mixed anomalous dimension $\gamma_{12}$ as explained in the previous section.

Using the results of the last section we find,

$$m_{2H}^2 = \frac{1}{4} \frac{2}{16\pi^2} \left[ \Delta \gamma_l + \Delta \gamma_e \right] |Y|^2 \left| \frac{F}{M} \right|^2 = -\frac{1}{(4\pi)^4} \left( 3 |y|^2 |Y|^2 \right) \left| \frac{F}{M} \right|^2, \quad (A43)$$

or for a simplified model where all fields are singlets

$$m_{2H}^2 = -\frac{1}{(4\pi)^4} \left( 2 |y|^2 |Y|^2 \right) \left| \frac{F}{M} \right|^2. \quad (A44)$$

Again, we can rewrite this in analogy with eq. (A34),

$$\tilde{m}_{2H}^2(M) = \frac{1}{4} \frac{d \gamma_H^<}{dY} \left[ \Delta \beta_Y \right]_{\gamma_{12}=0} \left| \frac{F}{M} \right|^2, \quad (A45)$$

where we used $\Delta \gamma_H = 0$. 

20
2. Multiple couplings

We can now generalize these results to models with multiple fields and couplings. Specifically, we will take the superpotential to be

\[ W = X \bar{D}D + (y_{aa}^0 D + Y_{aa}^0 H) l_a e_a, \]  

(A46)

where the different fields can have different multiplicities.\(^{10}\) As before we define \( \phi_1 \equiv D, \phi_2 \equiv H, y_{1a}^0 \equiv y_{a}^{0} \) and \( y_{2a}^0 \equiv Y_{a}^{0}. \) The various wave-function renormalizations are now all matrices, and the expressions for the soft masses at 2-loops generalize to

\[ m^2_l(\mu) = -\frac{1}{4} \left[ \frac{\partial^2 Z(\mu)}{\partial \ln M^2} - \left( \frac{\partial Z(\mu)}{\partial \ln M} \right)^2 \right] \frac{F}{M}^2, \]  

(A47)

and similarly for \( e. \) Using the RGE for the matrix \( Z_l \) (in analogy with eqn. (A6)) this can be written as (at \( \mu = M)\),

\[ m^2_l(M) = -\frac{1}{4} \left[ \frac{\partial}{\partial \ln M} \Delta \gamma_l(M) - \frac{\partial}{\partial \ln M} \gamma^<_{l}(\mu) \right] \frac{F}{M}^2, \]  

(A48)

and similarly for \( e. \) For completeness we display again the expression for the Higgs mass,

\[ m^2_H(M) = \frac{1}{4} \frac{\partial}{\partial \ln M} \gamma^<_{H}(\mu) \left| \frac{F}{M} \right|^2. \]  

(A49)

Note that the second term of (A48) is common to all the SM fields including \( H \) and \( \gamma^< \) is given by the square of the low-energy coupling \( Y. \) On the other hand the first term of (A48) does not appear in the \( H \) mass (since its anomalous dimension is continuous across the threshold), and \( \Delta \gamma(M) \) is given by the square of \( \tilde{y}(M). \)

It is now easy to evaluate these expressions. Let us do this explicitly for the \( l \) mass. The first term of (A48) is then

\[ \frac{\partial}{\partial \ln M} \Delta \gamma_l(M)_{ba} = \left( -\frac{2}{16\pi^2} \right) \left[ \tilde{y}_{ba}^* \frac{\partial}{\partial \ln M} \tilde{y}_{aa} + cc \right] \]  

(A50)

\[ = -\frac{1}{2} \left( -\frac{2}{16\pi^2} \right) \left[ y_{ba}^* \left( \gamma_{11} y_{aa} + \gamma_{ba}^* y_{ba} + \gamma_{1a} y_{a} + 2 \gamma_{12} Y_{aa} \right) + cc \right]. \]

where in the second line we omitted the tildes because the expression is already of two-loop order. The second term of (A48) is,

\[ \frac{\partial}{\partial \ln M} \gamma^<_{l}(\mu) \left|_{\mu=M} \right. = \left( -\frac{2}{16\pi^2} \right) \left[ \tilde{y}_{ba}^* \frac{\partial}{\partial \ln M} \tilde{y}_{aa} + cc \right] \]  

(A51)

\[ = -\frac{1}{2} \left( -\frac{2}{16\pi^2} \right) \left[ y_{ba}^* \left( \Delta \gamma_{ba} + \Delta \gamma_{a} y_{a} + cc \right) \right]. \]

\(^{10}\) This covers also the models with both down-quark and lepton couplings, with \( a = 1 \ldots 3 \) running over quarks and \( a = 4 \ldots 6 \) over leptons etc.
Finally we need to substitute,
\[ \Delta \gamma_{ba} = \left( -\frac{2}{16\pi^2} \right) (yy^\dagger) \]  
(A52)
and the analogous expression for \( e \). Here again we used the fact that \( \Delta \gamma \sim \bar{y}(M)^2 \), but to leading order \( \bar{y} = y \).

3. Soft terms in the three generation model

We are now ready to present the soft terms resulting from general coupling matrices \( y_U \), \( y_D \) and \( y_L \).

Note that our couplings \( Y_U \) are actually the complex conjugates of the commonly used standard-model Yukawas, which we denote by \( y_u \). To conform with the standard notation we will therefore express the soft terms in terms of \( Y_u \) and \( y_u \) (and similarly for the down and lepton couplings with
\[ Y_u = Y_u^*, \quad Y_d = Y_D^*, \quad Y_l = Y_L^*, \quad y_u = y_u^*, \quad y_d = y_D^*, \quad y_l = y_L^*. \]  
(A53)

The 2-loop soft squared masses at \( \mu = M \) are
\[ \tilde{m}_a^2 = \frac{1}{(4\pi)^3} \left| \frac{F}{M} \right|^2 \left\{ \left( 3Tr \left( y_u^\dagger y_u \right) - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right) y_u y_u^\dagger \right. 
+ \left( Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) - g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \right) \left. y_d y_d^\dagger \right\} 
+ 3y_u y_u^\dagger y_u y_u^\dagger + 3y_d y_d^\dagger y_d y_d^\dagger + y_u y_u^\dagger y_d y_d^\dagger + y_d y_d^\dagger y_u y_u^\dagger 
+ 2y_u Y_u^\dagger Y_u + 2y_d Y_d^\dagger Y_d - 2y_u y_d Y_u Y_d - 2y_d y_u Y_u Y_d 
+ y_u Y_u^\dagger Tr \left( 3y_u^\dagger y_u \right) + Y_u y_u^\dagger Tr \left( 3y_u^\dagger y_u \right) 
+ y_d Y_d^\dagger Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) + Y_d y_d^\dagger Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) 
+ 2N_5 \left( \frac{4}{3} g_3^4 + \frac{3}{4} g_2^4 + \frac{1}{60} g_1^4 \right) _{3\times3} \} \]  
(A54)

\[ \tilde{m}_{u_R}^2 = \frac{1}{(4\pi)^3} \left| \frac{F}{M} \right|^2 \left\{ 2 \left( 3Tr \left( y_u^\dagger y_u \right) - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right) y_u y_u^\dagger \right. 
+ 6y_u^\dagger y_u y_u^\dagger + 2y_u^\dagger Y_u Y_u^\dagger y_u + 2y_u^\dagger Y_d Y_d^\dagger y_u + 2y_u^\dagger y_d Y_u^\dagger y_u 
- 2Y_u^\dagger y_u y_u^\dagger Y_u - 2Y_u^\dagger y_d y_d^\dagger Y_u + 2y_u^\dagger Y_u^\dagger y_u 
+ 2Y_u^\dagger y_u Tr \left( 3y_u^\dagger Y_u \right) + 2 \left( \frac{4}{3} g_3^4 + \frac{4}{15} \right) N_5 g_1^4 _{1\times3} \} \]  
(A55)
\[
\begin{align*}
\tilde{m}^2_{d_R} &= \frac{1}{(4\pi)^2} |F_M|^2 \left\{ 2 \left( Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^4 \right) y_d^\dagger y_d \\
&+ 6y_d^\dagger y_d y_d^\dagger y_d + 2y_d^\dagger Y_u^\dagger y_d + 2y_d^\dagger y_u y_d^\dagger y_u + 2y_d^\dagger Y_d^\dagger y_d \\
&- 2Y_d^\dagger y_d y_d^\dagger y_d - 2Y_d^\dagger y_d y_d^\dagger Y_d \\
&+ 2y_d^\dagger Y_d Tr \left( 3Y_d^\dagger y_d + Y_l^\dagger y_l \right) + 2Y_d^\dagger y_d Tr \left( 3y_d^\dagger Y_d + y_l^\dagger Y_l \right) \\
&+ 2N_5 \left( \frac{4}{3} g_3^4 + \frac{1}{15} g_1^4 \right)_{13\times3} \right\} \\
\tilde{m}^2_L &= \frac{1}{(4\pi)^2} |F_M|^2 \left\{ \left( Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) - 3 g_2^2 - \frac{9}{5} g_1^2 \right) y_l^\dagger y_l \\
&+ 3y_l^\dagger y_l y_l^\dagger y_l + 2Y_l y_l^\dagger Y_l y_l^\dagger y_l - 2Y_l^\dagger y_l y_l^\dagger Y_l \\
&+ y_l^\dagger Y_l Tr \left( 3y_d^\dagger Y_d + y_l^\dagger Y_l \right) + Y_l^\dagger y_l Tr \left( 3Y_d^\dagger y_d + Y_l^\dagger y_l \right) \\
&+ 2N_5 \left( \frac{3}{4} g_2^4 + \frac{3}{20} g_1^4 \right)_{13\times3} \right\} \\
\tilde{m}^2_{e_R} &= \frac{1}{(4\pi)^2} |F_M|^2 \left\{ 2 \left( Tr \left( 3y_d^\dagger y_d + y_l^\dagger y_l \right) - 3 g_2^2 - \frac{9}{5} g_1^2 \right) y_l^\dagger y_l \\
&+ 6y_l^\dagger y_l y_l^\dagger y_l + 2Y_l^\dagger Y_l y_l^\dagger y_l - 2Y_l^\dagger y_l y_l^\dagger Y_l \\
&+ 2y_l^\dagger Y_l Tr \left( 3Y_d^\dagger y_d + Y_l^\dagger y_l \right) + 2Y_l^\dagger y_l Tr \left( 3y_d^\dagger Y_d + y_l^\dagger Y_l \right) \\
&+ \frac{6}{5} N_5 g_1^4_{13\times3} \right\} \\
\tilde{m}^2_{Hu} &= \frac{1}{(4\pi)^2} |F_M|^2 \left\{ -3Tr \left( Y_u^\dagger y_u y_u^\dagger Y_u + Y_u^\dagger y_d y_d^\dagger Y_u + 2Y_u^\dagger Y_u y_u^\dagger y_u \right) \\
&+ 2N_5 \left( \frac{3}{4} g_2^4 + \frac{3}{20} g_1^4 \right) \right\} \\
\tilde{m}^2_{Hd} &= \frac{1}{(4\pi)^2} |F_M|^2 \left\{ -3Tr \left( Y_d^\dagger y_u y_u^\dagger Y_d + Y_d^\dagger y_d y_d^\dagger Y_d + 2Y_d^\dagger Y_d y_d^\dagger y_d \right) \\
&- Tr \left( Y_l^\dagger y_l y_l^\dagger Y_l + 2Y_l^\dagger Y_l y_l^\dagger y_l \right) + 2N_5 \left( \frac{3}{4} g_2^4 + \frac{3}{20} g_1^4 \right) \right\} \\
\end{align*}
\]
In addition, we give here the fully-flavored 1-loop contributions to the soft masses,

\[
\begin{align*}
\delta m^2_{qL} & = -\frac{1}{16(4\pi)^2} \frac{1}{6} \left(y_u y_u^\dagger + y_d y_d^\dagger\right) \frac{F^4}{M^6} \\
\delta m^2_{uR} & = -\frac{1}{16(4\pi)^2} \frac{1}{3} \left(y_u^\dagger y_u\right) \frac{F^4}{M^6} \\
\delta m^2_{dR} & = -\frac{1}{16(4\pi)^2} \frac{1}{3} \left(y_d^\dagger y_d\right) \frac{F^4}{M^6} \\
\delta m^2_{l} & = -\frac{1}{16(4\pi)^2} \frac{1}{6} \left(y_l^\dagger y_l\right) \frac{F^4}{M^6} \\
\delta m^2_{e} & = -\frac{1}{16(4\pi)^2} \frac{1}{3} \left(y_l^\dagger y_l\right) \frac{F^4}{M^6}.
\end{align*}
\]

The A-terms, i.e. the coefficients of the Lagrangian terms \(L \supset (A_u)_{i,j} \bar{q}_{Li} \bar{u}_{Rj} H_U + (A_d)_{i,j} \bar{q}_{Li} \bar{d}_{Rj} H_d + (A_l)_{i,j} \bar{L}_{Li} \bar{e}_{Rj} H_d\) at the scale \(M\) are,

\[
\begin{align*}
A_u^* & = -\frac{1}{16\pi^2} \left[ (y_u y_u^\dagger + y_d y_d^\dagger) Y_u + 2Y_u (y_u^\dagger y_u) \right] \frac{F}{M} \\
A_d^* & = -\frac{1}{16\pi^2} \left[ (y_u y_u^\dagger + y_d y_d^\dagger) Y_d + 2Y_d (y_d^\dagger y_d) \right] \frac{F}{M} \\
A_l^* & = -\frac{1}{16\pi^2} \left[ (y_l y_l^\dagger) Y_l + 2Y_l (y_l^\dagger y_l) \right] \frac{F}{M}.
\end{align*}
\]
4. Explicit 2-loop Calculation

A cross-check of the calculation described above, we have also calculated the mixed $y^2Y^2$ terms explicitly. Since we are only interested in verifying the two loop contributions, which are only known to leading order in $F/M^2$, we work in the limit $F \ll M^2$, treating $F$ as an insertion.

The scalar interaction Lagrangian is

$$L_{scalar} \supset -F \bar{D}D - F^* \bar{D}^* \bar{D} - |F_D|^2 - |F_D^*|^2 - |F_Y|^2 - |F_Y^*|^2$$

$$= -F \bar{D}D - F^* \bar{D}^* \bar{D} - |F_H + yDe|^2 - |MD|^2 - |Yle|^2$$

$$-|YHe + yDe|^2 - |YHl + yDl|^2$$

$$= -F \bar{D}D - F^* \bar{D}^* \bar{D} - |M|^2 \bar{D}^* \bar{D}$$

$$- M^* \bar{D}^* \bar{D} - M^* y \bar{D}^* \bar{D} e$$

$$- (|Y|^2 H^* H + |y|^2 D^* D + Y^* y \bar{D}^* \bar{D} + Y^* \bar{D}^* \bar{D})$$

$$- (|Y|^2 H^* H + |y|^2 D^* D + Y^* y \bar{D}^* \bar{D} + Y^* \bar{D}^* \bar{D}) e^* e$$

$$- (|Y|^2 H^* H + |y|^2 D^* D + Y^* y \bar{D}^* \bar{D} + Y^* \bar{D}^* \bar{D}) l^* l$$ \hspace{1cm} (A69)

and the fermion Lagrangian is

$$-L_{fermion} = M \bar{\psi}_D \psi_D + Y (H \bar{\psi}_e \psi_e + e \bar{\psi}_H \psi_e + l \bar{\psi}_H \psi_e) + y (D \bar{\psi}_l \psi_e + e \bar{\psi}_l \psi_D + l \bar{\psi}_D \psi_e) + c.c \hspace{1cm} (A70)$$

For the sake of brevity we will define

$$\int d\varphi = \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \hspace{1cm} (A71)$$

While we are working in the $F/M^2 \ll 1$ limit, it is important to remember that the sfermions obtain small soft mass (of order $F^4/M^2$) already at one loop. Indeed, loops of massless scalars in the calculation presented below lead to spurious IR divergences which manifest themselves in the fact that the results of the calculation appear to depend on the order of integration. The presence of non-vanishing scalar masses cuts off these divergences leading to a finite result independent of the order of integration. For the sake of brevity, below we choose the order of integration which gives the correct result even when light scalars are treated as massless.
5. \( H \) field soft mass squared

The four 2-loop diagrams with two insertions of the SUSY breaking spurion and their contributions are given by,

\[
H^{\prime} = i |F|^2 |yY|^2 \int \frac{d\varphi}{p^4(k^2 - M^2)^3(p - k)^2} \frac{M^2}{p^4(k^2 - M^2)^3(p - k)^2} \tag{A72}
\]

\[
H^{\prime} = i |F|^2 |yY|^2 \int d\varphi \frac{1}{p^4(k^2 - M^2)^3} \tag{A73}
\]

\[
H^{\prime} = -i |F|^2 |yY|^2 \int d\varphi \frac{2p \cdot (p - k)}{p^4(p - k)^2(k^2 - M^2)^3} \tag{A74}
\]

\[
H^{\prime} = i |F|^2 |yY|^2 \int d\varphi \frac{1}{(k^2 - M^2)^3(p - k)^2} \tag{A75}
\]

The integrals on the right-hand side represent contributions of either \( l \) or \( e \) propagating in the loop. Multiplying the results by a factor of two to account for the number of fields in the loop and summing the diagrams we obtain

\[
I = 2i |Y y|^2 |F|^2 \int d\varphi \frac{1}{p^2(k^2 - M^2)^3} \left( \frac{M^2}{p^2(p - k)^2} + \frac{1}{p^2} + \frac{1}{(p - k)^2} - \frac{2p \cdot (p - k)}{p^2(p - k)^2} \right)
\]

\[
= 2i |Y y|^2 |F|^2 \int d\varphi \frac{k^2 + M^2}{p^4(k^2 - M^2)^3(p - k)^2} \tag{A76}
\]

As discussed above, to avoid spurious IR divergences we will choose to perform the \( k \) integral, associated with the massive messenger loop momentum, first followed by the \( p \) integral.

The resulting Higgs mass squared is

\[
m_H^2 = -\frac{2 |Y y|^2}{(4\pi)^4} \left| \frac{F}{M} \right|^2 \tag{A77}
\]

consistent with the results obtained using analytic continuation in Equation (A44).
The diagrams contributing to the $l$-field soft mass squared which contain a $|yY|^2$ term are

\[ \mathcal{L} = i|F|^2|y|^2(|y|^2 + |Y|^2) \int d\varphi \frac{|M|^2}{(k^2 - M^2)^3(p - k)^2p^4} \]  

(A78)

\[ \mathcal{L} = i|F|^2|y|^2(|y|^2 + |Y|^2) \int d\varphi \frac{1}{(k^2 - M^2)^3p^4} \]  

(A79)

\[ \mathcal{L} = -2i|F|^2|yY|^2 \int d\varphi \frac{p \cdot (p - k)}{(k^2 - M^2)^3(p - k)^2p^4} \]  

(A80)

\[ \mathcal{L} = 2i|F|^2|yY|^2 \int d\varphi \frac{1}{(k^2 - M^2)^3(p - k)^2p^2} \]  

(A81)
We note that contributions of Feynman diagrams A79–A81 are identical to the diagrams contributing to the Higgs mass. The last of these diagrams, A81, has an additional factor of two due to two possible choices of “chirality” for $D$ and $H$ propagators. The $|yY|^2$ contribution to the $l$ soft squared mass in the diagrams of Eqs. A79–A86 can be written as

\[
= i|F|^2 |yM|^2(|y|^2 + 2|Y|^2) \int d\varphi \frac{1}{(k^2 - M^2)^3 k^2 p^2} \quad (A82)
\]

\[
= -2i|F|^2 |M|^2 |yY| \int d\varphi \frac{p \cdot (p - k)}{(k^2 - M^2)^3 k^4 (p - k)^2 p^2} \quad (A83)
\]

\[
= 2i|F|^2 |y|^2(|y|^2 + |Y|^2) \int d\varphi \frac{|M|^2}{(k^2 - M^2)^3 k^2 (p - k)^2 p^2} \quad (A84)
\]

\[
= -4i|F|^2 |yY|^2 \int d\varphi \frac{p \cdot (p - k)}{(k^2 - M^2)^3 k^2 (p - k)^2 p^2} \quad (A85)
\]

\[
= 4i|F|^2 |yY|^2 \int d\varphi \frac{1}{(k^2 - M^2)^3 k^2 p^2} \quad (A86)
\]
a sum of three integrals,

\[ I = \int d\phi \frac{k^2 + M^2}{(k^2 - M^2)^3(p - k)^2} = \frac{1}{(4\pi)^4 M^2} \]

\[ II = 3 \int d\phi \frac{k^2 + M^2}{k^2(k^2 - M^2)^3(p - k)^2} = -\frac{3}{(4\pi)^4 M^2} \]

\[ III = \int d\phi \frac{(p - k)^2 - p^2}{(k^2 - M^2)^3(p - k)^2} \left( \frac{2}{k^2} + \frac{M^2}{k^4} \right) = 0. \]

(A87)

Summing all contributions, the \(|y Y|^2\) part of the \(l\) soft squared mass reads

\[ m_l^2 \bigg|_{|y Y|^2} = \frac{2 |y y|^2}{(4\pi)^4} \frac{|F|^2}{M} \]  

(A88)

consistent with the results obtained in the revised analytic continuation in Equation (A33).

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