Baryons with Many Colors and Flavors

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Abstract

Using recently-developed diagrammatic techniques, I derive some general results concerning baryons in the $1/N$ expansion, where $N$ is the number of QCD colors. I show that the spin-flavor relations which hold for baryons in the large-$N$ limit, as well as the form of the corrections to these relations at higher orders in $1/N$, hold even if $N_F/N \sim 1$, where $N_F$ is the number of light quark flavors. I also show that the amplitude for a baryon to emit $n$ mesons is $O(1/N^{n/2-1})$, and that meson loops attached to baryon lines are unsuppressed in the large-$N$ limit, independent of $N_F$. For $N_F > 2$, there are ambiguities in the extrapolation away from $N = 3$ because the baryon flavor multiplets for a given spin grow with $N$. I argue that the $1/N$ expansion is valid for baryons with spin $O(1)$ and arbitrary flavor quantum numbers, including e.g. baryons with isospin and/or strangeness $O(N)$. This allows the formulation of a large-$N$ expansion in which it is not necessary to identify the physical baryons with particular large-$N$ states. $SU(N_F)$ symmetry can be made manifest to all orders in $1/N$, yet group theory factors must be evaluated explicitly only for $N_F = N = 3$. To illustrate this expansion, I consider the non-singlet axial currents, baryon mass splittings, and matrix elements of $\bar{s}s$ and $\bar{s}\gamma_\mu\gamma_5s$ in the nucleon.

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1. Introduction

A great deal has been learned about QCD by studying the large-$N$ limit, where $N$ is the number of colors [1]. The large-$N$ limit for baryons is more subtle than for mesons because baryons contain $N$ quarks, and the structure that emerges is much richer [2]. It has been known for some time that in the large-$N$ limit, static baryon matrix elements obey the nonrelativistic quark-model relations [3][4]. There has been a recent revival of interest in this problem is due to the results of refs. [5], which showed (for example) that many interesting large-$N$ relations have corrections starting at $O(1/N^2)$. These results have been extended using a variety of different methods [6][7][8]. In this paper, I will use the formalism of ref. [8] to derive some general results concerning the $1/N$ expansion for baryons:

- I review the argument of ref. [8] and point out that the main conclusions remain valid when $N_F \sim N$, where $N_F$ is the number of light quark flavors. Specifically, the spin-flavor relations which hold in the large-$N$ limit, as well as the form of the corrections at higher orders in $1/N$ are independent of $N_F$. This is in contrast to many large-$N$ results for mesons (such as Zwieg’s rule), which only hold if $N_F \ll N$.

- I derive some general results concerning baryon–meson interactions using diagrammatic arguments. I show that the amplitude for a baryon to emit $n$ mesons is $\lesssim O(1/N^{n/2-1})$, independent of $N_F$. (This result was obtained in ref. [2] using a Hartree–Fock description of baryons made of heavy quarks.) I also show that meson loops attached to baryon lines in an effective hadronic field theory are not suppressed by powers of $1/N$, again independent of $N_F$. This agrees with explicit calculations in chiral perturbation theory [5][6][8].

- Next, I consider the $1/N$ expansion for $N_F > 2$. In this case, the baryon $SU(N_F)$ representations grow with $N$, and there are ambiguities in how to extrapolate the physical baryons to $N > 3$. I argue that baryons with spin $O(1)$ and arbitrary flavor quantum numbers have a well-defined large-$N$ limit, including e.g. states with isospin and/or strangeness $O(N)$. The methods of ref. [8] can then be used to give a $1/N$ expansion with manifest $SU(N_F)$ symmetry which describes all the lowest-lying baryons with spin $O(1)$. In this expansion, it is not necessary to identify the physical baryons with particular large-$N$ states. In addition, the expansion is calculationally very simple, since explicit calculations of matrix elements need only be carried out for $N = N_F = 3$. I briefly discuss the relation between this expansion and the one proposed in ref. [6].

- I give several examples to show how the expansion described above works in practice. I consider matrix elements of non-singlet axial currents, baryon mass splittings due to quark current masses, and matrix elements of $\bar{s}\gamma_\mu\gamma_5 s$ and $\bar{s}s$ in nucleon states. More detailed
applications which include both $SU(N_F)$ breaking and $1/N$ corrections will be presented elsewhere (see e.g. ref. [9]).

2. Baryons for large $N$ and $N_F$

In this section, I briefly review the argument of ref. [8] and show that the main conclusions continue to hold even if $N_F \sim N$.* One might expect that this large-$N$ limit to be the most phenomenologically relevant in the real world, where $N = N_F = 3$. There is some evidence that this is so: the $\eta'$ can be regarded as a $U(1)_A$ Nambu–Goldstone boson in the large-$N$ limit with $N_F \ll N$ [10], but the explicit breaking due to the anomaly are $O(N_F/N)$, so this picture breaks down if $N_F \sim N$. In the real world, the the $\eta'$ does not look much like a Nambu–Goldstone boson: for example, $m_{\eta'} = 958$ MeV.

I begin by discussing the baryon masses in the limit of exact $SU(N_F)$ flavor symmetry. The idea is to write a perturbative expansion for the baryon masses at the quark level and use this to derive the $N$-dependence of physical quantities, working to all orders in the QCD coupling constant. It is not clear whether the perturbative expansion converges in any meaningful sense, but I will assume that the $N$-dependence of the perturbative expansion is reliable. This type of reasoning is commonly used for the meson sector [2] and is known to work in $1 + 1$ dimensions [11].

The perturbative expansion for the baryon mass closely parallels the perturbative expansion for the vacuum energy. The starting point for the expansion of the baryon mass is the state $|B_0\rangle$, which is an $N$-quark state with the same quantum numbers as a physical 1-baryon state. The state $|B_0\rangle$ plays the same role as the “no-particle” state $|0\rangle$ in the evaluation of the vacuum energy; it is defined by

$$|B_0\rangle \equiv B^{a_1\alpha_1\ldots a_N\alpha_N}e^{A_1\ldots A_N}a^\dagger_{a_1\alpha_1A_1} \ldots a^\dagger_{a_N\alpha_NA_N}|0\rangle,$$

where $a_1, \ldots, a_N = 1, \ldots, N_F$ are flavor indices, $\alpha_1, \ldots, \alpha_N = \uparrow, \downarrow$ are spin indices, and $A_1, \ldots, A_N = 1, \ldots, N$ are color indices. The operators $a^\dagger$ in eq. (1) create a quark with definite flavor, spin, and color, in a perturbative 1-quark state $|\phi\rangle$. The details of the state $|\phi\rangle$ are irrelevant for the argument. The tensor $B$ specifies the spin and flavor quantum numbers of the state $|B_0\rangle$.

The diagrammatic expansion for the baryon mass is obtained as follows:

$$\langle B_0|e^{-iHT}|B_0\rangle = \sum \text{diagrams} \quad \longrightarrow |\langle B_0|B\rangle|^2 e^{-iE_B t} \quad \text{as} \quad t \to -i\infty,$$

* I do not consider the case $N_F \gg N$, since asymptotic freedom is lost in this limit, and there is presumably no confinement or chiral symmetry breaking.
where $H$ is the hamiltonian for the system; $|B\rangle$ is the lowest-energy 1-baryon state with the same quantum numbers as $|B_0\rangle$, and $E_B$ is the energy of the state $|B\rangle$. In order to match the $t$-dependence of the right-hand side of eq. (2), the diagrams must exponentiate, so that

$$-iE_B t = \sum \text{connected diagrams.} \quad (3)$$

Because the state $|B_0\rangle$ contains quarks, the diagrams are somewhat more complicated than the ones appearing in the perturbative expansion of the vacuum energy. Typical diagrams are shown in fig. 1. Each diagram corresponds to a matrix element in the state $|B_0\rangle$ of an operator containing an equal number of creation and annihilation operators for the 1-quark state $|\phi\rangle$.

This discussion has been very sketchy, since I will only need topological properties of the diagrams in this paper. More details can be found in ref. [8].

Subtracting away the vacuum energy, one obtains an expression for the baryon mass

$$M_B = \sum_r c_r \langle B_0 | \mathcal{O}^{(r)} | B_0 \rangle, \quad (4)$$

where $\mathcal{O}^{(r)}$ is an $r$-body operator; that is, it contains $r$ creation and $r$ annihilation operators. It must be a singlet under $SU(N_F)$ as well as $SU(2)_{\text{rot}}$ spatial rotations in the $B$ rest frame.

To determine the $N$-dependence of the coefficients $c_r$ in eq. (4), one can look at typical graphs (such as those in fig. 1) to conclude that

$$c_r \lesssim \frac{1}{N r - 1}. \quad (5)$$

This result holds even if $N_F \sim N$. The reason is that each quark–gluon vertex is suppressed by $1/\sqrt{N}$, and at least $r - 1$ gluon lines are required to form a connected $r$-body operator. If $N_F \sim N$, quark loops are not suppressed; each quark loop gives a factor $O(N_F/N) \sim 1$ due to the sum over quark flavors, but eq. (5) still holds. This is the main result of this section.

A similar result holds for baryon matrix elements:

$$\langle B' | T \mathcal{O}_1(p_1) \cdots \mathcal{O}_n(p_n) | B \rangle = \sum_{\ell, r} F_{\ell, r}(p_1, \ldots, p_n) \langle B'_0 | \mathcal{O}_{\ell}^{(r)} | B_0 \rangle, \quad (6)$$

where

$$F_{\ell, r} \lesssim \frac{1}{N r - 1}. \quad (7)$$
even if $N_F \sim N$. Here, $F_{\ell,r}$ is a “form factor” depending on the kinematics and transforming under some irreducible representation (labelled by $\ell$) of $SU(N_F) \times SU(2)_{\text{rot}}$. $O^{(r)}_{\ell}$ is an $r$-body operator which also transforms according to an irreducible representation of $SU(N_F) \times SU(2)_{\text{rot}}$.

In ref. [8], it is shown that eqs. (5) and (7) give rise to the nonrelativistic quark-model relations for baryon masses and matrix elements in the large-$N$ limit, as well as a classification of corrections to all orders in $1/N$. The new feature of the above discussion is the observation that these results hold even if $N_F \sim N$.

3. Meson–baryon interactions in the $1/N$ expansion

I now consider the question of meson–baryon interactions in the $1/N$ expansion. I will use an argument similar to the one used by Witten [2] to determine the $N$-dependence of meson amplitudes for large $N$. I briefly review these arguments here, and show how they can be extended to processes involving baryons.

To study meson masses, consider the 2-point function $\langle \Omega | T J(x) J(0) | \Omega \rangle$, where $|\Omega\rangle$ is the (exact) vacuum state and $J$ is a quark bilinear operator with the right quantum numbers to create a 1-meson state. In the large-$N$ limit with $N_F \ll N$, the leading graphs contributing to the 2-point function are planar graphs containing no internal quark loops, as illustrated in fig. 2. Witten’s observation is that a cut of such a diagram contains only one color-singlet combination of quark fields $q$ and gluon fields $G$ with color indices contracted as $\sim \bar{q}_{A_1} G^{A_1} A_2 \cdots G^{A_{n-1}} A_n q^{A_n}$. The intermediate state is therefore expected to correspond to a 1-meson state due to confinement. Therefore, the spectral representation for the 2-point function can be written

$$\langle \Omega | T J(x) J(0) | \Omega \rangle = \sum_{\sigma} \int \frac{d^4q}{(2\pi)^4} e^{-iq \cdot x} \frac{|\langle \sigma | J(0) | \Omega \rangle|^2}{q^2 - m_\sigma^2 + i0+} + \text{subtractions},$$

(8)

where the sum is over 1-meson states $\sigma$; the “subtractions” are local terms required by the short-distance behavior of the 2-point function. Eq. (8) has the form of a sum of tree diagrams in a field theory of mesons; the subtractions correspond to local counterterms.

To study meson interactions, consider the $n$-point function $\langle \Omega | T J(x_1) \cdots J(x_n) | \Omega \rangle$. By considering the possible cuts of the quark diagrams contributing to the $n$-point functions in the large-$N$ limit, one finds that the only possible intermediate states are 1-meson states (see fig. 3). Since the resulting amplitudes obey unitarity and crossing, it is not hard to see that they are equivalent to the tree approximation of a field theory of mesons.

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* Eq. (7) must be modified if the representation $\ell$ is a $SU(N_F)$ singlet. See ref. [8], and section 5.3 below.
Subleading contributions in $1/N$ include diagrams involving internal quark loops, as well as non-planar diagrams. It can be shown that diagrams suppressed compared to the leading diagrams by $1/N^L$ can be cut into at most $L+1$ color-singlet combinations of quark and gluon fields, and therefore the spectral representation can contain intermediate states with at most $L+1$ color-singlet particles [2]. An example at order $1/N$ is shown in fig. 4. Again, these amplitudes must satisfy crossing and unitarity, and it is extremely plausible that they can be represented as the $L$-loop approximation to a hadronic lagrangian. (This conclusion would follow from Weinberg’s “theorem” that any consistent set of amplitudes arises from some quantum field theory [12].)

Note that if $N_F \sim N$, quark loops are no longer suppressed because of the sum over quark flavors, and hence meson interactions are no longer described by the tree approximation of a lagrangian in the large-$N$ limit. However, there are other properties of meson interactions for large $N$ which continue to hold even if $N_F \sim N$. For example, note that

$$f_\sigma \equiv \langle \sigma | J | \Omega \rangle \sim \sqrt{N},$$

(9)
since the leading contributions to the right-hand side of eq. (8) are $O(N)$ (even if $N_F \sim N$). Also, the amplitudes for meson scattering are related to the $n$-point functions discussed above by LSZ reduction:

$$\mathcal{A}(\sigma_1 \cdots \sigma_k \to \sigma_{k+1} \cdots \sigma_n) \sim \frac{1}{f_\sigma^n} \langle \Omega | T J \cdots J | \Omega \rangle \sim \frac{1}{N^{n/2-1}},$$

(10)
since the $n$-point function is $O(N)$ (even if $N_F \sim N$).

To study meson–baryon interactions, I again apply the principle that physical hadrons in intermediate states correspond to color-singlet combinations of quark and gluon fields. When counting color singlets in diagrams involving baryons, one must take into account the fact that the color lines which attach to the baryon state combine with the other color indices in $|B_0\rangle$ to form a color singlet. With this in mind, it is easy to count color singlets in the double line notation. For example, the diagram in fig. 5 is a contribution to the baryons mass which can give rise to an intermediate state containing a meson and a baryon. Evaluating fig. 5 gives

$$\text{fig. 5} \sim \frac{1}{N} \langle B_0 | \text{2-body operator} | B_0 \rangle \lesssim N.$$

(11)
(Note that the form of this contribution agrees with the general result of eqs. (4) and (5).)

It is not hard to see that one can draw diagrams which give $O(N)$ contributions to the baryons mass, and which have arbitrary numbers of intermediate color-singlet states by “iterating” fig. 5 (see fig. 6). Therefore, in a hadronic theory of baryons and mesons, loops
of meson lines attached to baryons are not suppressed by powers of $1/N$, even if $N_F \ll N$; the $N$ dependence of loop contributions is constrained only by eq. (4). This discussion has focussed on baryon masses, but similar results can easily be seen to hold for matrix elements as well. This agrees with explicit calculations in baryon chiral perturbation theory for large $N$ [5][6][8].

To determine the $N$-dependence of the amplitude for a baryon to emit $n$ mesons, consider the $n$-point function $\langle B'|T J(x_1) \cdots J(x_n)|B \rangle$. In the diagrammatic expansion, this matrix element is given as a sum of connected diagrams with $n$ insertions of the operator $J$; it is easy to see that it is $\lesssim N$ in the large-$N$ limit. (An example of a diagram which contributes to meson–baryon scattering is shown in fig. 7.) On the other hand, the matrix element is related to the desired amplitude via LSZ reduction:

$$A(B\sigma_1 \cdots \sigma_k \rightarrow B'\sigma_{k+1} \cdots \sigma_n) \sim \frac{1}{f^nn} \langle B'|T J \cdots J|B \rangle \lesssim \frac{1}{N^{n/2-1}}. \quad (12)$$

This result was derived ref. [2] using a Hartree–Fock picture. However, it is worth noting that the arguments of ref. [2] can be made precise only for heavy quarks in 3+1 dimensions, and that the argument given here extends the result to the case $N_F \sim 1$.

These results may appear paradoxical, since the amplitude to emit a single meson is $O(\sqrt{N})$, so one might expect the amplitude to emit $n$ mesons to be $O(N^{n/2})$. At the quark level, this expectation can be seen to be false because because the diagrams which contribute to the physical amplitude must be connected, and therefore the $N$ dependence arising from combinatoric factors in diagrams with extra operator insertions does not factor. In an effective field theory of mesons and baryons, the only $N$ dependence is in the coupling constants, and the breakdown of factorization occurs because of cancellations among different graphs [5][6][8]. (This has been checked explicitly only for small values of $n$.) Similar mechanisms resolve the apparent contradiction between eq. (12) and the fact that loop contributions are unsuppressed. This a nice example of the duality between the quark-level and hadronic descriptions of baryons for large $N$.

4. Baryons with large quantum numbers

For $N_F = 2$ and $N \gg 1$, the physical baryon states have quantum numbers $I = J = \frac{1}{2}, \frac{3}{2}, \ldots$, and it is clear that one should identify the physical baryons with the large-$N$ states with the same isospin and spin quantum numbers. For $N_F > 2$, the size of the $SU(N_F)$ representations associated with a given spin $J$ grows with $N$ (see fig. 8); for example, for $N_F = 3$, there are baryons with spin $J$ and strangeness $S = 0, -1, \ldots, -\left(\frac{1}{2}N + J\right)$. It is therefore not obvious how to extrapolate the physical baryon states to $N > 3$. For
example, the Ξ baryons have maximal strangeness for $N = 3$, and one could reasonably extrapolate them to either $S = -2$ or $S = -\frac{1}{2}(N + 1)$.

In this section, I argue that the expansion can be formulated so that there is no need to identify the physical baryons with particular large-$N$ states. The key point is the observation that the $1/N$ expansion holds for baryons with spin $J \sim 1$ and all possible flavor quantum numbers, including e.g. $S \sim -N$.

I now use the arguments of the previous section to show that the $1/N$ expansion for meson–baryon scattering amplitudes can break down only if the amplitude is sensitive to the physical process of a baryon decaying to a single meson.* For example, the width for a baryon of spin $J \sim N$ to decay to a baryon of spin $J - 1$ plus a meson is $O(N)$, since the amplitude is $O(\sqrt{N})$ and all kinematic invariants are $O(1)$. (The mass difference between the baryons is at least $O(1)$ in this case [8].) Therefore, baryons with spin $J \sim N$ do not have a well-defined large-$N$ limit. On the other hand, the width for a baryon of spin $J \sim 1$ to decay to a baryon of spin $J - 1$ plus a pion is $O(1/N^2)$ (in the chiral limit), because the mass difference between the baryons is $O(1/N)$, reducing the phase space for the decay. Baryons with spin $J \sim 1$ are therefore expected to be narrow states in the large-$N$ limit.

Applying these criteria, one can see that the $1/N$ expansion can be applied to baryons with $J \sim 1$ and arbitrary quantum numbers. For example, one can consider decays of a baryon with strangeness $S$ to a baryon with strangeness $S + 1$ plus a meson with strangeness $-1$. If the initial and final baryons are in the same $SU(N_F)$ multiplet, then the decay will be kinematically forbidden near the chiral limit, since the mass difference between the baryons is $O(m_s)$ (see below) while the lightest mesons with strangeness $-1$ are kaons with mass $O(m_s^{1/2})$. If the initial and final baryons differ in spin by $O(1)$, the width for this decay is $O(1/N^2)$ (in the chiral limit). One can generalize these considerations to processes involving more mesons to show that baryons with arbitrary strangeness are well-defined narrow states for large $N$. Similar arguments can be given for other quantum numbers. Thus, scattering amplitudes for all baryons with $J \sim 1$ have a well-defined $1/N$ expansion.

Similar arguments also hold for matrix elements. These can grow as powers of $N$, but this simply means that they must be rescaled to have a finite large-$N$ limit. (For example,

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* In this paper, I consider only processes involving momentum transfers which are $O(1)$ in the large-$N$ limit. If the momentum transfer grows with $N$, there is additional $N$ dependence in the quark-level diagrams arising from the kinematics. For example, quarks with momenta $O(N)$ have vanishing overlap with the states $|B_0\rangle$ in the large-$N$ limit. Effects such as these tend to suppress exclusive processes involving momentum transfer $O(N)$ by factors of $e^{-N}$ [2].

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the electric charge operator must be multiplied by $O(1/N)$ in order that the baryon have charge $O(1)$ in the large-$N$ limit.)

4.1. The $1/N$ expansion at $N = 3$

I now show how the $1/N$ expansion can be carried out for physical quantities at $N = 3$ without ambiguities arising from the extrapolation of physical baryons to large $N$. To understand the idea, consider the $1/N$ expansion of a static operator matrix element:

$$\langle \mathcal{B}'|J(0)|\mathcal{B}\rangle = \sum_{\ell,r} \frac{a_{\ell,r}}{N^{r-1}} \langle \mathcal{B}_0'|\mathcal{O}_{\ell}^{(r)}|\mathcal{B}_0\rangle,$$

where the coefficients $a_{\ell,r}$ are order-1 coefficients that depend on strong interaction dynamics. The operators $\mathcal{O}_{\ell}^{(r)}$ which appear on the right-hand side are $r$-body operators written in terms of the quark creation and annihilation operators $a^\dagger$ and $a$ for the 1-quark state $|\phi\rangle$ used to define $|\mathcal{B}_0\rangle$. Each of the operators $\mathcal{O}_{\ell}^{(r)}$ on the right-hand side of eq. (13) must have the same transformation properties under $SU(N_F) \times SU(2)_{rot}$ as the operator $J(0)$ on the left-hand side.

The $1/N$ expansion is carried out in three steps: (i) Classify the possible operators $\mathcal{O}_{\ell}^{(r)}$ that can appear in eq. (13). This classification is explained in ref. [8], and I will give some examples below. (ii) Evaluate the $N$-dependence of the contribution of each operator $\mathcal{O}_{\ell}^{(r)}$ to determine whether it contributes at the order one is working. Of course, the $N$-dependence of an operator depends on the states in which it is evaluated. In the expansion considered here, all of the baryon states with $J \sim 1$ are included, and so one must keep an operator if its matrix element in any pair of states $|\mathcal{B}\rangle$ and $|\mathcal{B}'\rangle$ is of sufficiently high order in $N$. (iii) Evaluate the surviving terms on the right-hand side of eq. (13) for $N = 3$ to obtain the physical results. Note that $SU(N_F)$ symmetry is manifest at each step, since the operators $\mathcal{O}_{\ell}^{(r)}$ have well-defined $SU(N_F)$ quantum numbers.

Before giving some examples of the expansion proposed above, I briefly contrast this expansion to the $1/N$ expansion proposed in ref. [6]. In ref. [6] the baryons at $N = 3$ are extrapolated to baryons with strangeness $O(1)$ in the large-$N$ limit. In a world where the physical value of $N$ is very large, this expansion would work well only for states with small strangeness ($|S| \ll \frac{1}{2}N$). In the real world, where $N = 3$, it is not clear where to draw the line between “large” and “small” strangeness, and it is consistent to assume that all baryons which arise at $N = 3$ have “small” strangeness. In the language of eq. (13), this expansion would (in general) differ from the one proposed above in which operators to keep at a given order in the $1/N$ expansion, since the expansion of ref. [6] requires that the operators on the right-hand side be tested only on states with strangeness of order 1.
expansion advocated here is therefore more conservative, in the sense that (in general) more operators are kept at a given order in the expansion. The “extra” terms in this expansion are suppressed by $1/N$ according to the philosophy of ref. [6], and experiment can in principle be used to decide whether these terms are in fact suppressed for $N = 3$.

5. Examples

In this section, I give several examples of the expansion described above. More detailed phenomenological applications will appear elsewhere (see e.g. [9]).

For specific applications, it is convenient to use the effective lagrangian described in ref. [8], which I briefly review. Because the baryon mass is of order $N \Lambda_{QCD}$, the baryons can be described using a heavy-particle effective field theory [13]. The baryon momentum is written $P = M_0 v + k$, where $M_0 \sim N$ is a baryon mass and $v$ is a 4-velocity ($v^2 = 1$) which defines the baryon rest frame. The effective field theory is then written in terms of baryon fields whose momentum modes are the residual momenta $k$. This effective field theory gives an expansion in $1/M_0$ around the static limit.

For $N$ large, the baryon $SU(N_F)$ representations are large, and it is convenient to use a Fock-space notation to keep track of baryon quantum numbers: the baryons fields are written

$$|B(x)\rangle \equiv B^{a_1\alpha_1 \cdots a_N\alpha_N}(x) \alpha^\dagger_{a_1\alpha_1} \cdots \alpha^\dagger_{a_N\alpha_N} |0\rangle.$$ (14)

The $\alpha^\dagger$’s are bosonic creation operators which create a “quark” with definite flavor and spin, and $|0\rangle$ is the Fock “vacuum” state; $a_1, \ldots, a_N$ are $SU(N_F)$ flavor indices and $\alpha_1, \ldots, \alpha_N = \uparrow, \downarrow$ are spin indices in the rest frame defined by $v$. The baryon kinetic term can then be written

$$\mathcal{L}_{\text{eff}} = (B|iv^\mu \partial_\mu |B) + \cdots.$$ (15)

The couplings of PNGB’s to baryons can be introduced using standard techniques from chiral perturbation theory [8]. (Note that if $N_F \sim N$, the $\eta'$ is not light and is not present in the effective lagrangian.)

5.1. Axial Currents

As a first application, I consider the non-singlet axial currents. Matrix elements of the axial currents are defined by coupling a source $A^\mu$ to the axial currents in the QCD lagrangian:

$$\delta \mathcal{L}_{\text{QCD}} = \bar{q} A^\mu \gamma_\mu \gamma_5 q; \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix},$$ (16)
The terms in the effective lagrangian linear in the source can be written

\[ \delta L_{\text{eff}} = (\mathcal{B}) \left[ g \{ A^\mu \sigma_\mu \} + \frac{h}{N} \{ A^\mu \} \{ \sigma_\mu \} + \frac{h'}{N^2} \{ \sigma_\mu \} \{ A_\nu^\sigma \sigma_\nu \} \{ \sigma_\mu \} + \cdots \right] \mathcal{B}. \tag{17} \]

Here \( \sigma_\mu \equiv (\psi \gamma_\mu - v_\mu) \gamma_5 \) is the spin matrix, and “quark-model” operators are defined by

\[ \{ A^\mu \sigma_\mu \} \equiv \alpha_\alpha^\dagger (A^\mu)^a_b \sigma_\mu^{a b \beta} \tag{18} \]

etc. The general rule for the \( N \)-dependence of coefficients in the effective lagrangian is obtained by imposing eq. (7). The result is that \( r \)-body operators in the effective lagrangian have coefficients \( O(1/N^{r-1}) \). In particular, \( g, h, h' \sim 1 \) in the large-\( N \) limit.

In order to determine the \( N \)-dependence of the axial current matrix elements, one must evaluate the matrix elements of quark-model operators such as the one in eq. (18). In appendix A, it is shown that \( \{ A^\mu \sigma_\mu \} \) has matrix elements which are \( O(N) \) in some baryon states with \( J \sim 1 \). (A \( r \)-body operator can have matrix elements at most \( O(N^r) \).) This means that \( \{ A^\mu \sigma_\mu \} \) counts as being \( O(N) \) in the expansion described in the previous section. The operator \( \{ \sigma_\mu \} \) simply measures the total spin of the baryon, and therefore has matrix elements which are \( O(1) \) on all states with \( J \sim 1 \). Therefore, the contributions to axial current matrix elements proportional to \( g \) are \( O(N) \), those proportional to \( h \) are \( O(1) \), and all others are suppressed by additional powers of \( 1/N \).

At leading order in \( 1/N \), the the axial current matrix elements are determined by a single coupling constant \( g \). The matrix elements of \( \{ A^\mu \sigma_\mu \} \) are exactly what one would compute in the static quark model, and one obtains the relations

\[ D = g, \quad F = \frac{2}{3} g, \quad C = -2g, \tag{19} \]

with corrections \( O(1/N) \). Here \( D \) and \( F \) are the usual \( SU(3) \) axial couplings, and \( C \) is the decuplet-octet axial coupling defined in ref. [14]. (In the static quark model, \( g = 1 \). In the \( 1/N \) expansion of QCD, determining \( g \) requires a non-trivial dynamical calculation.) At subleading order in \( 1/N \), the axial current matrix elements are determined by two coupling constants \( g \) and \( h \). In appendix A, it is shown that these terms are independent at this order in the \( 1/N \) expansion. For \( N_F = 3 \),

\[ D = g, \quad F = \frac{2g + h}{3}, \quad C = -2g, \tag{20} \]

with corrections \( O(1/N^2) \).

The couplings \( D \) and \( F \) are measured in semileptonic decays, and the coupling \( C \) is measured in decuplet strong decays. A fit ignoring explicit chiral symmetry breaking gives
\(D \simeq 0.8, F \simeq 0.5\) and \(C \simeq 1.5\) [15], so that both the lowest-order relations in eq. (19) and the “corrected” relations in eq. (20) appear to work well. (Using the best-fit values for the axial couplings, \(h \simeq -0.1\).) However, chiral symmetry breaking due to the strange quark mass is expected to be sizable both for the semileptonic decays [14] and for the strong decuplet decays [16]. The validity of the \(1/N\) expansion including explicit chiral symmetry breaking will be considered in detail in a future publication.

5.2. Baryon Mass Differences

Next, I consider the \(SU(N_F)\)-breaking baryon mass differences induced by \(m_s \neq 0\). I assume that \(O(m_s)\) and \(O(1/N)\) corrections are both \(O(\epsilon) \sim 30\%\), and expand consistently in powers of \(\epsilon\).

The dependence on \(m_s\) occurs through the quark mass spurion

\[
m = m_s S, \quad S = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix},
\]

which transforms as \(m \mapsto U m U^\dagger\) under \(SU(N_F)\). The leading terms giving rise to baryon mass splittings are

\[
\delta \mathcal{L} = a \langle B|\{m\}|B \rangle + b N \langle B|\{\sigma^\mu\}\{\sigma_\mu\}|B \rangle + c N \langle B|\{m\sigma^\mu\}\{\sigma_\mu\}|B \rangle + d_0 \langle B|\{m^2\}|B \rangle + d_1 N \langle B|\{m\}\{m\}|B \rangle + d_2 N \langle B|\{m\sigma^\mu\}\{m\sigma_\mu\}|B \rangle + \cdots.
\]

Using the rules given above, all of the coefficients in this lagrangian are \(O(1)\) in the large-\(N\) limit. The matrix elements of \(\{S\}\) and \(\{S\sigma^\mu\}\) can be \(O(N)\) in the large-\(N\) limit, so the maximal \(N\)-dependence of the matrix elements relevant for the baryon masses is

\[
\{m\} \sim N m_s \sim N \epsilon, \quad \frac{1}{N} \{\sigma^\mu\}\{\sigma_\mu\} \sim \frac{1}{N} \sim N \epsilon^2, \quad \frac{1}{N} \{m\sigma^\mu\}\{\sigma_\mu\} \sim m_s \sim N \epsilon^2, \quad \{m^2\} \sim N m_s^2 \sim N \epsilon^2, \quad \frac{1}{N} \{m\}\{m\} \sim N m_s^2 \sim N \epsilon^2, \quad \frac{1}{N} \{m\sigma^\mu\}\{m\sigma_\mu\} \sim N m_s^2 \sim N \epsilon^2.
\]
It is easily checked that all other operators are at most $O(N\epsilon^3)$.

Loops of pseudo Nambu–Goldstone bosons can induce non-analytic dependence on $m_s$ of the form $m_s^{3/2}$ and $m_s \ln m_s$ [17]. These nonanalytic terms must be multiplied by $r$-body operators with coefficients $O(1/N^{r-1})$ in accordance with eqs. (4) and (5). The breaking of $SU(N_F)$ in these operators must be proportional to powers of the spurion $S$ defined in eq. (21), and it is easy to check that the operators which can contribute at $O(Nm_s^{3/2}) = O(N\epsilon^{3/2})$ and $O(Nm_s \ln m_s) = O(N\epsilon \ln \epsilon)$ are of the same form as those appearing in eq. (22). Therefore, relations which are independent of the coefficients in eq. (22) are not invalidated by the nonanalytic corrections. The leading corrections to these relations are then $O(Nm_s^{5/2}) = O(N\epsilon^{5/2})$.

The operators appearing in eq. (22) are not all independent: it is easily checked that

$$\{S^2\} = \{S\}, \quad \{S\sigma^\mu\}\{S\sigma_\mu\} = -\{S\}\{S\} + 2\{S\}\{S\}. \quad (24)$$

Therefore, there are 4 independent operators which determine the 7 independent mass differences of the octet and decuplet isospin multiplets. This gives 3 independent relations valid to $O(\epsilon^2)$,* which may be written as

$$\begin{align*}
(M_\Omega - M_\Xi) - (M_\Sigma - M_\Sigma^*) & = (M_\Sigma - M_\Sigma^*) - (M_\Sigma^* - M_\Delta), \\
3M_\Lambda + M_\Sigma - 2M_N - 2M_\Xi & = (M_\Sigma - M_\Delta) - (M_\Omega - M_\Xi) \\
M_\Xi - M_\Sigma^* & = M_\Xi - M_\Sigma.
\end{align*} \quad (25)$$

Eq. (25) is an “improved” version of the decuplet equal-spacing rule, which holds to $O(m_s^2)$ independently of the $1/N$ expansion [18]. Eqs. (26) and (27) are non-trivial predictions of the $1/N$ expansion; eq. (26) relates the deficit of the Gell-Mann–Okubo relation to that of the decuplet equal-spacing rule. The same relations are derived in ref. [6] under rather different assumptions: the strangeness of physical baryons was assumed to be $O(1)$ in the large-$N$ limit, and $m_s$ was not assumed to be a small parameter. Under these assumptions, operators proportional to high powers of $\{S\}$ are suppressed by powers of $1/N$, and the mass splittings to $O(1/N^2)$ are determined by the operators in eq. (22).

5.3. Strangeness in the Nucleon

Next, I consider the matrix elements $\langle p|\bar{s}\gamma_\mu\gamma_5s|p\rangle$ and $\langle p|\bar{s}s|p\rangle$, which give information on the strange content of the nucleons. These matrix elements have been considered in the context of the Skyrme model [19][20], and using the Hartree–Fock picture [7].

* Note that mass splittings between states differing in strangeness by 1 unit induced by the operators in eq. (22) are smaller than the estimates in eq. (23) by $O(1/N)$.  

The leading term in the baryon effective lagrangian which can give rise to a nonzero value for \( \langle p|\bar{s}\gamma_\mu\gamma_5s|p \rangle \) is

\[
\delta L = \frac{c}{N} \text{tr}(A^\mu) \langle B|\{\sigma_\mu\}|B \rangle, \tag{28}
\]

where \( c \approx 1 \). The reason for the explicit factor of \( 1/N \) in the coefficient is that the leading quark-level diagrams which can give rise to a flavor trace involve a quark loop (see fig. 9). This factor is there even if \( N_F \sim N \), since the \( N_F \) dependence is correctly accounted for by the flavor trace in eq. (28). Since \( \{\sigma_\mu\} = O(1) \) on any baryon states with \( J \sim 1 \), \( \langle p|\bar{s}\gamma_\mu\gamma_5s|p \rangle = O(1/N) \).

Matrix elements of \( ss \) can be defined by differentiating with respect to \( m_s \) in the QCD lagrangian. The leading term which can give rise to a nonzero value for \( \langle p|\bar{s}s|p \rangle \) is

\[
\delta L = \frac{d}{\sqrt{N}} \text{tr}(m) \langle B|\{1\}|B \rangle, \tag{29}
\]

where \( d \sim 1 \). (This term was not considered above because it does not give rise to baryon mass differences.) Since \( \{1\} = N \) on any baryon state, \( \langle p|\bar{s}s|p \rangle = O(1) \).

These results disagree with refs. [19][20]; I do not understand the arguments in these papers well enough to explain this disagreement.

6. Conclusions

Using the methods of ref. [8], I have derived some general results concerning the \( 1/N \) expansion for baryons. I showed that the form of the \( 1/N \) expansion is unchanged if \( N_F \sim N \) and gave a simple method to carry out a simultaneous expansion in \( 1/N \) which keeps \( SU(N_F) \) symmetry manifest for arbitrary \( N \) and \( N_F \). These ideas were illustrated with some simple examples. Clearly, more work remains to be done.

7. Acknowledgements

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Appendix A. Calculation of matrix elements

The purpose of this appendix is to illustrate the determination of the $N$ dependence of the matrix elements discussed in the main text. For this purpose, it is sufficient to consider spin-$\frac{1}{2}$ baryons for $N_F = 3$, although the methods and results discussed here are clearly more general.

For $N_F = 3$, the most general spin-$\frac{1}{2}$ baryon states can be written

$$|B⟩ = B_{b_1…b_n}^a \chi^α α_{aa}^† A^{b_1^†}…A^{b_n^†}|0⟩,$$

where

$$A^{α^†} ≡ \frac{1}{2} e^{abc} e^{βγ} α_{bβ}^† α_{cγ}^†$$

creates a pair of quarks in a spin-singlet, flavor 3 state, and

$$n ≡ \frac{N - 1}{2}.$$

Here, $a, b_1, … = u, d, s$ are flavor indices and $α, β, … = \uparrow, \downarrow$ are spin indices. Because the $A^†$ operators commute, $B^a_{b_1…b_n}$ is totally symmetric in $b_1…b_n$.

The multiplet defined by eq. (30) contains states which have the same isospin and strangeness as the octet baryons:

$$|N⟩ ≡ C_N I^a α^{\alpha α_{aa}^†} (A^{α^†})^n|0⟩,$$

$$|Σ⟩ ≡ C_Σ I^a b^α α_{aa}^† A^{b^†} (A^{α^†})^{n-1}|0⟩,$$

$$|Ξ⟩ ≡ C_Ξ I^a α^{α_{aa}^†} A^{α^†} (A^{α^†})^{n-1}|0⟩,$$

$$|Λ⟩ ≡ C_Λ α^{α_{aa}^†} (A^{α^†})^n|0⟩,$$

where $χ (I)$ is the appropriate spin (isospin) tensor. One can identify these states with the large-$N$ limit of the physical states, as done in ref. [6]. However, there are also other natural candidates. For example, the states

$$|Ξ′⟩ ≡ C_{Ξ′} I^a a_{a_n} α^{α_{aa}^†} A^{α^†}…A^{α_n^†}|0⟩$$

have $I = n/2 = O(N)$, $S = -(n + 1) = O(N)$, the same quantum numbers as the $Ξ$ baryons for $N = 3$. The $1/N$ expansion discussed in the main text does not require that

* The methods used in this appendix are the result of a collaboration with J. March–Russell.
the physical states be identified with specific large-$N$ states, but the states in eqs. (33)–(36) will be used for illustrative purposes.

Calculations involving these states can be carried out efficiently using an occupation number representation. Define the occupation number states

$$|n_1, \ldots, n_6\rangle \equiv (\alpha^+_1)^{n_1} \cdots (\alpha^+_6)^{n_6}|0\rangle,$$

using the abbreviation $u^\uparrow, u^\downarrow, d^\uparrow, d^\downarrow, s^\uparrow, s^\downarrow = 1, \ldots, 6$. Note that the occupation number states defined here are not unit-normalized:

$$(m_1, \ldots, m_6|n_1, \ldots, n_6) = n_1! \cdots n_6! \delta_{m_1 n_1} \cdots \delta_{m_6 n_6}.$$

In this notation, $A^{s\uparrow} = \alpha^+_1 \alpha^+_4 - \alpha^+_2 \alpha^+_3$, so that

$$|p, \uparrow\rangle = C_N \alpha^+_1(\alpha^+_1 \alpha^+_4 - \alpha^+_2 \alpha^+_3)^n|0\rangle$$

$$= C_N \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!} |n-k+1, k, k, n-k, 0, 0\rangle.$$

Using this notation, it is easy to compute matrix elements. For example, the operator

$$\{\tau_3\sigma_3\} = \alpha^+_1 \alpha^+_1 - \alpha^+_2 \alpha^+_2 - \alpha^+_3 \alpha^+_3 + \alpha^+_4 \alpha^+_4,$$

has matrix element

$$(p, \uparrow |\{\tau_3\sigma_3\}|p, \uparrow\rangle = C_N \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!} (n-k+1-k-k+n-k)$$

$$\times (p, \uparrow |n-k+1, k, k, n-k, 0, 0)$$

$$= C_N^2 \sum_{k=0}^{n} \left[ \frac{n!}{k!(n-k)!} \right]^2 (2n-4k+1)(n-k+1)! (k!)^2 (n-k)!$$

$$= (n!C_N)^2 \sum_{k=0}^{n} (2n-4k+1)(n-k+1).$$

$C_N$ can be determined by normalizing the state $|p, \uparrow\rangle$ by a similar calculation. Combining these results, one obtains

$$(p, \uparrow |\{\tau_3\sigma_3\}|p, \uparrow\rangle = \frac{2n+3}{3}.\quad (43)$$

For $N = 3$, the standard quark-model result is recovered; note that the matrix element is proportional to $N$ for large $N$. This is because the number of $u$ and $d$ quarks in the proton is of order $N$, and their contribution to this matrix element adds coherently.
Similar calculations can be used to compute the matrix elements shown in table 1. This table substantiates some of the claims made in the main text; for example, one can see that the operators \( \{ \tau_3 \sigma_3 \} \) and \( \{ \tau_3 \} \{ \tau_3 \} \) are independent, and that matrix elements of the operator \( \{ S \sigma_j \} \{ \sigma_j \} \) are of order the strangeness of the state. It is also amusing to compare the matrix elements of the states \( |\Xi\rangle \) and \( |\Xi'\rangle \) in table 1: the matrix elements are equal for \( N = 3 \), but they are very different for \( N \gg 3 \).

|       | \{ \tau_3 \sigma_3 \} | \{ \tau_3 \} \{ \sigma_3 \} | \{ S \} | \{ S \sigma_j \} \{ \sigma_j \} |
|-------|---------------------|---------------------|-------|---------------------|
| \( p \) | \( \frac{2n+3}{3} \) | 1                   | 0     | 0                   |
| \( \Lambda \) | 0                   | 0                   | 1     | 3                   |
| \( \Sigma^+ \) | \( \frac{2n+2}{3} \) | 3                   | 1     | -1                  |
| \( \Xi^0 \) | \( \frac{2n+1}{9} \) | 1                   | 2     | 4                   |
| \( \Xi' \) | \( -\frac{n^2}{n+2} \) | \( n \)             | \( n+1 \) | \( \frac{3n^2+7n+6-4\delta_{n,1}}{n+2} \) |

Table 1: Assorted “quark-model” matrix elements. The states \( p, \Lambda, \Sigma^+ \), and \( \Xi^0 \) are defined in eqs. (33)–(36); \( \Xi' \) is defined in eq. (37), and has \( I_3 = n/2 \). All states have \( J_3 = +1/2 \).
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Figure Captions

Fig. 1. Typical diagrams contributing to the baryon mass. Double-line notation is used to keep track of color flow: single lines denote quarks and double lines denote gluons. Each diagram corresponds to a term in the expansion of eq. (3), consisting of an operator matrix element in the state $|B_0\rangle$. The external quark lines ending in a “×” denote creation and annihilation operators $a$ and $a^\dagger$, while the internal lines are given by the Feynman rules described in ref. [8].

Fig. 2. A typical planar diagram contributing to the 2-point function $\langle \Omega | TJJ | \Omega \rangle$ at order $N$. The cuts of this diagram contain only a single color-singlet state, and so this contribution can only give rise to 1-meson states, as shown on the right.

Fig. 3. A typical planar diagram contributing to the 3-point function $\langle \Omega | TJJJ | \Omega \rangle$ at order $N$. The cuts of this diagram contains only a single color-singlet state, and so this contribution can only give rise to tree-level meson processes such the one shown on the right.

Fig. 4. A diagram contributing to $\langle \Omega | TJJ | \Omega \rangle$ at order 1 involving a quark loop. The cut of the diagram shown contains two color-singlet states, and so can contribute to 1-loop meson processes such as the one shown on the right.

Fig. 5. A diagram contributing to the baryon mass. Counting factors of $N$ from the gluon vertices and color loops, it is easy to see that this graph is $O(1/N)$ times a 2-body operator, and can therefore be $O(N)$. The cut of the diagram shown can contains 2 color-singlet intermediate states: the “valence” color lines which attach to the baryon state form a color singlet with the remaining quarks in the baryon, and the quark-antiquark pair at the top of the cut can be a color singlet. This graph can therefore contribute to 1-loop meson-baryon processes such as the one shown on the right.

Fig. 6. A diagram contributing to the baryon mass obtained by “iterating” the diagram of fig. 5. The cut shown contains 3 color-singlet intermediate states.

Fig. 7. A diagram contributing to the matrix element $\langle B | TJJ | B \rangle$. This graph is $O(1)$ times a 1-body operator, and can therefore be $O(N)$. It clearly has the right intermediate-state structure to contribute to the baryon-meson scattering process shown on the right.

Fig. 8. The Young tableaux corresponding to the $SU(N_F)$ representation for baryons of spin $J$.

Fig. 9. A leading diagram contributing to the matrix elements $\langle p | \gamma_\mu \gamma_5 s | p \rangle$ or $\langle p | s | p \rangle$. 
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Fig. 4

Fig. 5
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