Design and development of miniature free piston free displacer dual opposed Stirling cryocooler

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Abstract. A miniature Free Piston Free Displacer (FPFD) Dual Opposed Stirling Cryocooler is designed for the applications in Infrared Imaging. Helium is used as a working gas, providing cooling effect at 80K operating under ambient conditions. A moving magnet linear motor drives the Cryocooler. Compressor and Expander are separate units connected via small diameter tube for minimum vibration export. The Cryocooler is modelled in Sage Software. Variation of the design parameters on the performance of the Cryocooler is studied. An optimization is done to maximize heat lift with frequency, displacer spring stiffness and negative facing area as optimization variables and work input as the constraint. Flexure spring is designed and analysed using Ansys. The variable features are the spiral angle, the number of arms and the gap width for the required stiffness and safe stress. Analysis of the moving magnet linear motor is done using Ansys Maxwell.

1. Introduction
The helium gas is charged at high pressure in a casing that includes a cylinder with two pistons at dual opposed configuration. Both the pistons are driven separately by a linear motor at an operating frequency. The motion of the pistons minimizes each other’s vibration. The compressed gas flows through a small diameter tube to the second compression space (warm side of the displacer). The displacer is located between the second compression space and the expansion space. The displacer inherits the regenerator inside it. Regenerator consists of woven wire meshes tightly fitted inside the displacer shell. Displacer consists of a rod that is going in a bounce space and connected with a spring. The displacer moves at a constant phase angle with respect to the piston by the combined effect of the fluidic force arising due to pressure difference across the displacer and the elastic force of the spring. The motion of piston and displacer causes repeated compression and expansion of the working gas. This creates a warm (compression) and a cool (expansion) space. As the gas shuttles through the regenerator from the compression space to the expansion space, it rejects the heat to the regenerator. On the other hand, the gas accepts heat from the regenerator as it shuttles through the regenerator from the expansion space to the compression space. In the expansion space, heat is absorbed from the surrounding through the heat exchanger and in the compression space, heat is rejected to the surrounding. Thus heat is transferred from the expansion space to the compression space. The space or the object to be cooled is connected to the cold tip through a heat exchanger.

2. Numerical Analysis
The modelling of the cryocooler has been done in SAGE v11. Sage builds machine simulation models, specify dimension of the components and predict the performance. All the components are connected together with suitable boundary connections.
2.1 SAGE Solution

The cryocooler is charged with Helium at the pressure of 24 bar and run at the frequency of 65 Hz. The stroke of both the piston is 4 mm. The cryocooler generates cooling effect at 80 K while the temperature of ambient considered as 300 K. The result of Sage solution gives heat lift of 575.5 mW with COP of 0.0805 and Carnot efficiency of 22.13%. The cryocooler has been optimized to enhance its performance as described in section 2.3.

2.2 Mapping in SAGE

The effect of various parameters on the heat lift of the system has been mapped. The heat lift increases linearly with charge pressure of the system as shown in figure 1. As the frequency increases heat lift increases, reaches a maximum and then decreases as shown in figure 2. The system generates maximum heat lift when it is driven near the resonant frequency. The mean flow area of the compression space 2 (= displacer area – displacer rod area) gives the diameter of displacer rod. Heat lift varies due to variation in differential pressure force and mean flow volume as diameter of displacer rod varies as shown in figure 3. Heat lift is maximum at an optimum value of displacer spring stiffness as shown in figure 4. The optimum spring stiffness is obtained from the dynamics of displacer.

2.3 Optimization in SAGE

The cryocooler is optimized with an objective function to maximize heat lift with the constraint of net work input ≤ 10 W. The optimization variables chosen were frequency, displacer spring stiffness and flow area of compression space 2.

Table 1. The parameters of the cryocooler.

| Component            | Parameter   | Value    |
|----------------------|-------------|----------|
| Piston               | Diameter    | 8 mm     |
| Compression Space 1  | Length      | 5 mm     |
|                      | Flow Area   | 50 mm²   |
| Connecting Duct      | Length      | 50 mm    |
|                      | Diameter    | 1.5 mm   |
|                      | Thickness   | 0.25 mm  |
### Table 2. The result of optimization in SAGE.

| Parameter          | Value   |
|--------------------|---------|
| Qlift              | 962.8 mW|
| Qrejected          | 11.21 W |
| Winput             | 10 W    |
| COP actual         | 0.0963  |
| COP theoretical    | 0.3636  |
| Carnot efficiency  | 26.48 % |
| Linear motor force | 13.5 N  |

#### 3. Linear Compressor

The linear compressor consists of moving magnet linear motor that drives the cryocooler. The radially magnetized NdFeB ring magnet is placed between soft iron outer and inner core of Steel 1008. The magnet is attached to the piston and transfers axial force generated due to the interaction of permanent magnetic field produced by permanent magnet and electromagnetic field produced by coil excitation.

The resonant frequency of the compressor is given by,

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{k_g + k_s + k_m}{m}}
\]

where \(k_s\) is the stiffness of flexure bearing, \(k_g\) is the gas stiffness, \(k_m\) is the magnet stiffness caused due to spring effect of the moving magnet and \(m\) is the total reciprocating mass. Magnet stiffness is defined as the ratio of average force over the complete stroke to the stroke of the piston when no excitation current is supplied.

#### 3.1 Linear Motor Analysis in Ansys Maxwell

![Figure 5](image1.png) **Figure 5.** Variation of force with magnet position without excitation current.

![Figure 6](image2.png) **Figure 6.** Variation of force with excitation current.

![Figure 7](image3.png) **Figure 7.** Variation of force with magnet position at 380 amp-turns.

From figure 5, it is observed that when no excitation current is supplied the force variation about the mean position of the magnet is symmetrical and average force for the complete stroke is almost negligible. Hence, magnet stiffness can be neglected. Figure 6 shows the linear variation of force with current as force is directly proportional to current. In order to generate required 13.5 N of axial force,
380 amp-turns of current input is required. Figure 7 shows the variation of axial force with relative position of magnet during the stroke of the piston at 380 amp-turns using Ansys Maxwell 3D and 2D. The difference in force generated is less than 11% during the piston stroke. The variation of magnetic flux density over different components of the moving magnet linear motor at extremes and mean position of the magnet is shown in figure 8.

![Figure 8. Flux density variation at different magnet positions (my).](image)

4. Flexure Bearing
The flexure bearing has spiral cuts in the flexures that allows accurate motion of piston when the load is applied in the axial direction. The flexure bearing has very high radial to axial stiffness that gives negligible movement of flexure in radial direction. The Archimedian spiral considered as the flexure profile is generated using Matlab. The flexure is modelled using Solidworks and analysis of the flexure is done using Ansys.

The effect of various parameters of the flexure on the stress developed and stiffness are shown in the below figures 9 to 12.

![Figure 9. Variation of stress and stiffness with spiral angle.](image)

![Figure 10. Variation of stress and stiffness with thickness.](image)

![Figure 11. Variation of stress and stiffness with number of arms.](image)

![Figure 12. Variation of stress and stiffness with width of cut.](image)
Table 3. The specification of the flexure bearing.

| Flexure material | SS 301 |
|------------------|--------|
| Spiral angle     | 600°   |
| Number of arms   | 3      |
| Gap width        | 1 mm   |
| Thickness        | 0.5 mm |
| Diameter         | 42 mm  |
| Central hole diameter | 3 mm |
| Outer/fixing hole diameter | 2 mm |

![Figure 13. Result of flexure analysis in Ansys (a) Stress distribution (b) Safety factor distribution.](image)

Table 3 gives the specification of the flexure bearing. Analysis of the flexure is done using Ansys. The flexure have an axial stiffness of 1030 N/m and radial stiffness of 169 kN/m. The flexure gives large radial to axial stiffness ratio and hence the clearances are maintained. Stress and safety factor distribution are shown in figure 13. The fatigue limit of the material is 240 MPa. The maximum stress of around 168 MPa is generated over the flexure and it gives the safety factor of 1.4282.

5. Conclusions

The cryocooler driven by moving magnet linear motor with flexure bearing is designed. The variation of heat lift with various input parameters has been obtained which helps in deciding the optimization variables. Optimization of the cryocooler in Sage increased the COP and Carnot efficiency by 19.6%. Magnet stiffness for the configuration of the moving magnet linear motor can be neglected as the average force during the piston stroke is negligible when no excitation current is supplied. The difference in force obtained from Maxwell 3D and 2D is less than 11%. Variation of magnetic flux density on the linear motor has been obtained for the piston stroke of 4 mm using Ansys Maxwell. Stiffness of the flexure increases with reduction in spiral angle, reduction in number of flexure arms, reduction in width of cut and increase in thickness of flexure. Stress developed over the flexure decreases with increase in spiral angle, increase in number of flexure arms, increase in width of cut and decrease in the thickness of the flexure.

6. References

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