Radiation pressure supported stars in Einstein gravity: eternally collapsing objects

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ABSTRACT

Even when we consider Newtonian stars, that is, stars with surface gravitational redshift \( z \ll 1 \), it is well known that, theoretically, it is possible to have stars supported against self-gravity almost entirely by radiation pressure. However, such Newtonian stars must necessarily be supermassive. We point out that this requirement for excessively large \( M \) in the Newtonian case is a consequence of the occurrence of low \( z \ll 1 \). However, if we remove such restrictions, and allow for the possible occurrence of a highly general relativistic regime, \( z \gg 1 \), we show that it is possible to have radiation pressure supported stars (RPSSs) at an arbitrary value of \( M \). Since RPSSs necessarily radiate at the Eddington limit, in Einstein gravity, they are never in strict hydrodynamical equilibrium. Further, it is believed that sufficiently massive or dense objects undergo continued gravitational collapse to the black hole (BH) stage characterized by \( z = \infty \). Thus, late stages of BH formation, by definition, would have \( z \gg 1 \), and hence would be examples of quasi-stable general relativistic RPSSs. It is shown that the observed duration of such Eddington limited radiation pressure dominated states is \( t \approx 5 \times 10^8 (1 + z) \) yr. Thus, \( t \to \infty \) as BH formation \( (z \to \infty) \) takes place. Consequently, such radiation pressure dominated extreme general relativistic stars become eternally collapsing objects (ECOs) and the BH state is preceded by such an ECO phase. This result is also supported by our previous finding that trapped surfaces are not formed in gravitational collapse and the value of the integration constant in the vacuum Schwarzschild solution is zero. Hence the supposed observed BHs are actually ECOs.

Key words: black hole physics – gravitation – stars: fundamental parameters.

1 INTRODUCTION

Stars may be defined as objects with an intrinsic self-luminosity which is generally sustained by the grip of self-gravity. Most of the stars that we know of are primarily supported by gas pressure \( p_g \) rather than by radiation pressure \( p_r \). The usually insignificant role of \( p_r \) is indicated by the parameter (Chandrasekhar 1967):

\[
\beta = \frac{p_g}{p},
\]

where \( p = p_g + p_r \) is the total pressure. For most of the known luminous (Newtonian) stars, \( \beta \approx 1 \). The actual role of \( p_r \) is seen more clearly from the adjoint parameter:

\[
1 - \beta = \frac{p_r}{p},
\]

The relative role of \( p_r \) may even be more directly expressed through the parameter

\[
x \equiv \frac{p_r}{p_g} = \frac{1 - \beta}{\beta}.
\]

For real astrophysical objects, \( x \) would be highest at the centre and in general decrease significantly towards the surface. The central value of \( x^* = 0.006 \) for the Sun and the same for a 4-M\( \odot \) main-sequence star is 0.012 (\( \odot \) denotes solar values). It may be borne in mind that the mean value of \( (1 - \beta) \ll (1 - \beta^*) \). It is known, however, that the value of \( (1 - \beta^*) \) would rise significantly if the value of \( M \) were very high though we hardly know of any single Newtonian star with \( M > 100 M_\odot \). In fact, it may be pointed out that in any given stellar model, it is the value of \( (1 - \beta^*) \) which determines the mass of the star (Chandrasekhar 1967). Suppose one makes stellar models with a fixed composition (\( \mu \)). Then under the assumption of fixed chemical composition, a stellar mass of \( M \) will be defined from the equation (Chandrasekhar 1967, see p. 82):

\[
\frac{1 - \beta^*}{\beta^*} = \frac{1 - \beta_\odot}{\beta_\odot} \left( \frac{M}{M_\odot} \right)^2.
\]
so that \( M \) slowly increases with decreasing \( \beta^* \) (increasing \( x \)). Note \( M \to 0 \) if \( \beta^* \to 1 \) or \( x^* \to 0 \) in this non-quantum case, and to have a finite mass luminous star, one must have a finite value of \( x \). Even in the case of (partially) degenerate objects, a low value of \( \beta \) or a higher value of \( x \) would raise their masses. The usual Chandrasekhar mass \( M_c \) corresponds to a perfectly degenerate object having ultimate relativistic degeneracy where the momentum of the pressure giving particles \( P \to \infty \) and further the object is absolutely cold, \( T = 0 \). Moreover, we remind the reader that the radius of Chandrasekhar's critical white dwarf is zero, \( R_c = 0! \) This shows that only an infinitely dense singular configuration could be perfectly degenerate and has \( T = 0 \), and, on the other hand, all real finite configurations, in the strict sense, must be either partially degenerate or at a finite \( T \). When the compact object has a finite temperature despite having assumed degeneracy, the mass upper limit gets modified as (Chandrasekhar 1967, see p. 437):

\[
\mathcal{M}_c = M_c \beta^{-3/2}.
\]

(5)

In order to keep the object almost ‘completely degenerate’, Chandrasekhar imposed some artificial restrictions on the value of \( \beta \) which resulted in

\[
\mathcal{M}_c = 1.156M_c.
\]

(6)

However, the fact remains that perfect degeneracy, in a strict sense, corresponds to \( \beta = 1.0 \) or \( x = 0 \). It may be noted that the radius of this almost completely degenerate white dwarf too is zero, \( R_c = 0 \) (Chandrasekhar 1967, see p. 441, equation 140). However, theoretically, the possibility of having arbitrary finite \( x \) cannot be ruled out. As the value of \( x \) rose, the system would be more and more partially degenerate and soon it would be more meaningful to call it ‘non-degenerate’. In such a case, as \( \beta \to 0 \), one may have \( \mathcal{M}_c \to \infty \), that is, the very notion of an ‘upper mass limit’ would vanish. Actually, the applicability of equation (5) would cease, once the degree of degeneracy would be significantly reduced.

As indicated by equation (4), in principle, an initially non-degenerate sequence of stars can always be turned into radiation pressure supported stars (RPSSs) by considering an appropriate higher value of \( M \) and vice versa. Thus, irrespective of whether one follows the standard non-degenerate route or the degenerate route, one may, in principle, end up with non-degenerate RPSSs. Now, we focus attention on the question of the mass range of such RPSSs in the context of both Newtonian and Einstein gravity.

### 2 GENERAL FORMALISM

In reality, even if one considers the stellar material to be pure hydrogen, at sufficiently high temperature, one would have a pressure contribution from both electrons and protons. Also there will always be a rest-mass contribution by the electrons. In the following, however, since we are interested only in seeing some new qualitative result, we will consider an idealized fluid which is assumed to always remain monatomic. The total proper-mass energy density of this fluid is

\[
\rho = \rho_0 + (\gamma - 1)^{-1} p_g + \rho_\gamma,
\]

(7)

where

\[
\rho_0 = m_p n c^2
\]

(8)

is the (baryonic) rest-mass energy density, \( n \) is the proper baryonic number density, \( m_p \) is the proton rest mass, \( c \) is the speed of light, and \( \gamma = \rho / m_p c^2 \) is the ratio of specific heats. Also

\[
p_g = n k T
\]

(9)

is the gas pressure, where \( k \) is the Boltzmann constant and

\[
\rho_\gamma = a T^4
\]

(10)

is the radiation energy density. Here \( a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4} \) is the radiation constant. As is known,

\[
p_\gamma = \frac{1}{3} a T^4.
\]

(11)

From equations (9) and (11), we may also find that

\[
x = \frac{\rho_\gamma}{p_g} = \frac{a T^3}{3nk}.
\]

(12)

Further, instead of \( \beta \), it will be more convenient to define a quantity \( \beta_\gamma = x^{-1} = p_g / p_\gamma \). One may also define a related parameter

\[
y = \frac{\rho_\gamma}{p_g} = \frac{a T^4}{m_p m_c^2}
\]

(13)

which is the ratio of the radiation energy density and the rest-mass energy density. In Newtonian gravity, one has \( \rho_\gamma \ll \rho_g \), that is, \( y \ll 1 \), and, consequently, \( \rho \approx \rho_\gamma \). In idealized models, fluids may be characterized by a polytropic equation of state (EOS):

\[
p = K \rho^{1+\gamma/m}
\]

(14)

where \( K \) must be uniform within the fluid. Though, in the Newtonian case, one uses \( \rho = \rho_\gamma \), in general relativity (GR), the \( \rho \) appearing in the above equation should indeed correspond to the total mass energy density indicated by equation (7). If there were to exist a strictly static self-gravitating configuration, where \( K \) and \( m \) would be the same everywhere, then one would have a polytrope of degree ‘\( m \)’. For the construction of stellar models, one looks for polytropes which have a finite boundary radius \( R \). Mathematically, this would mean that \( \rho = p = 0 \) for \( r > R \). In both the Newtonian and GR cases, in a strict sense, this vital boundary condition can be satisfied only if the object is absolutely cold \( T = 0 \) so that \( \rho_\gamma = \rho_\gamma = 0 \) for \( r > R \) and because one may indeed make \( p_g = \rho_g = 0 \) at the boundary. In Newtonian gravity, since \( \rho_\gamma = 0 \), and \( \rho = \rho_\gamma \), this boundary condition can be satisfied much more accurately if not absolutely. In Newtonian gravity, one can consider the results obtained by static polytropic models to be reasonably accurate. Now, we will attempt to qualitatively understand why Newtonian RPSSs must be of extremely high mass.

### 2.1 RPSSs in Newtonian gravity

For the best-known low-mass Newtonian star, namely the Sun, as we know, the central value of \( \beta_\gamma \approx \beta_\gamma \approx 0.003 \). Since \( T \) drops sharply as one moves away from the centre, the mean value of \( \beta^* \) must be extremely low in comparison to the above value. One may estimate the mean value of \( x \) for low-mass Newtonian stars in the following way. By using the Newtonian virial theorem for Newtonian stars, one can estimate the internal energy of the star as (Chandrasekhar 1967):

\[
U = \frac{1}{(5y - 6)} \frac{G M^2}{R},
\]

(15)

where \( y \) is the effective ratio of specific heats, \( G \) is the gravitational constant and \( R \) is the radius of the star. As long as radiation pressure is really small, one can have \( y \approx 5/3 \). Further, if the mean temperature of the assumed monatomic gas is \( T_m \), one has

\[
U = M \frac{3}{2} \frac{k T_m}{m_p}
\]

(16)

Then by combining equations (15) and (16), it follows that

\[
k T_m = \frac{2}{7} \frac{G M m_p}{R}
\]

(17)
In terms of the surface redshift or compactness \( z = GM/Rc^2 \), we can rewrite the mean temperature as

\[ kT_m = \frac{2}{7} z m_p c^2. \]  

(18)

We may recall that for the Sun, \( z \approx 2 \times 10^{-6} \). This shows that the temperatures of Newtonian stars are necessarily low because they are hardly compact, that is, \( z \ll 1 \). Alternatively, since \( kT_m \) is low in comparison to the nucleon rest-mass energy \( m_p c^2 \), radiation pressure is low. In other words, radiation pressure is relatively low (for low-\( M \) stars) because compactness is so low. One can ask then, what is the appropriate mean value of \( x \) for low-mass Newtonian stars? To see this, we invoke a relationship which shows that for \( z \ll 1 \), one has (Mitra 2006):

\[ y \approx \alpha \frac{GM}{Rc^2} \approx \alpha z, \]  

(19)

where

\[ \alpha = \frac{L}{L_{\odot}} \]  

(20)

is the luminosity of the star in terms of its Eddington luminosity \( L_{\odot} \). By combining equations (12), (13) and (19), we find that

\[ x \approx \alpha \frac{m_p c^2}{3kT_m}. \]  

(21)

Since we are interested only in the gross mean properties rather than precise central quantities, in the foregoing equation, we may consider \( T = T_m \) because the virial temperature is a reasonably good measure of the gross mean temperature. Then, we may use equation (18) in equation (21) to find that

\[ x \approx \frac{7\alpha}{6} \approx \alpha. \]  

(22)

For the Sun, \( L_{\odot} \) at the surface is \( \sim 1.3 \times 10^{38} \) erg s\(^{-1} \) so that \( \alpha \approx 3 \times 10^{-5} \). Therefore, the mean value of \( x \) for the Sun is also \( \sim 3 \times 10^{-5} \). Again recall that \( z \approx 2 \times 10^{-6} \) for the Sun. Note the cosmic coincidence that, for the Sun, the three apparently uncorrelated dimensionless quantities, namely, \( x, \alpha \) and \( z \), have similar values within a factor of 10. In order to become a RPSS, the mean value of \( x \) has to rise from such an extremely low value to attain a stage of \( x \gg 1 \). Since for low-mass stars \( L \sim M^{5.5}/R^{0.5} \) while \( L_{\odot} \sim M_{\odot} \), we will have \( \alpha = L/L_{\odot} \sim M^{4.5}/R^{0.5} \). For low-mass main-sequence stars, crudely, \( M \propto R \), so that \( \alpha \sim M^4 \). Thus, if one moved to higher mass stars, initially, mean \( x \) would increase very rapidly. However, as \( x \) increases, equation (15) would need significant revision as the star would move away from being a gas pressure dominated \( \gamma \approx 5/3 \) polytrope to a radiation pressure dominated \( \gamma \approx 4/3 \) polytrope. In such a case, the pattern of variation of \( x \) with \( M \) would change drastically. It appears that, in the regime of \( p_t \gg p_g \), the supposed Newtonian static RPSSs may be reasonably described by a Newtonian polytrope with \( m \approx 3 \) (Weinberg 1972). Then, the mass of the star is (Weinberg 1972):

\[ M = 18 M_{\odot} \beta_w^{-2} \mu^2 = 18 M_{\odot} x^2 \mu^2, \]  

(23)

where \( \mu \) is the mean molecular weight. For a fully ionized hydrogen plasma \( \mu = 2 \), it is interesting to invert equation (23) as

\[ x \approx 0.2 \left( \frac{M}{M_{\odot}} \right)^{1/2} \mu^{-1}, \]  

(24)

which shows that in the large-\( M \) range \( x \) increases relatively more rapidly as \( M^{1/2} \) for the Newtonian stars. And eventually, still, \( M \) has to be very high to ensure \( x \gg 1 \). To have a value of \( x \sim 10 \) (at least) one must have \( M \sim 1800 M_{\odot} \mu^2 \) which is already very high. Note that since, in principle, \( x \) can be arbitrarily high, again, there is no upper limit on the mass of self-gravitating configurations once we allow the existence of sufficiently high radiation pressure. We may also see how the ‘compactness’ of the Newtonian star would change in this very high-\( M \) range. To appreciate this point further, we consider a specific model of Newtonian RPSSs (Weinberg 1972, see equation [11.5.9]):

\[ R \propto \beta_w^{-2/3} \propto x^{2/3}. \]  

(25)

Using equations (23) and (25), we find then that in the Newtonian case

\[ z \propto \frac{M}{R} \propto x^{4/3}. \]  

(26)

Further, using equation (24) in the foregoing equation, we see that

\[ z \propto (M^{1/2})^{4/3} \propto M^{2/5}. \]  

(27)

also increases quite rapidly with mass. Thus compared to the low-mass Newtonian stars, in the high-mass range, on a relative scale, one can have quite high values of \( z \). However, this does not mean that for Newtonian stars one can ever have \( z \gg 1 \). This point has been emphasized by Weinberg.

The entire derivation leading to the concept of any Newtonian star necessarily assumes \( y \equiv p_i/p_g \ll 1 \), and this assumption must be ensured for (Newtonian) supermassive stars as well. As shown by Weinberg, in such a case, one must always have (Weinberg 1972):

\[ z = \frac{GM}{Rc^2} \approx 0.39. \]  

(28)

In summary, the fundamental reason that Newtonian RPSSs are extremely massive is that, by definition, \( z \ll 1 \), and, in particular, \( z \ll 0.39 \). It may be also pointed out that since in general \( L \propto M^d \), where \( d > 1 \) (for low-mass stars, \( d \sim 5 \)), and \( L_{\odot} \propto M, \alpha \) in general increases with \( M \). Then equation (27) would show that at least in the Newtonian regime, in general, \( \alpha \) increases with \( z \). However, the maximum value of \( \alpha = 1 \), and all the RPSSs, Newtonian or relativistic, have this maximal value of \( \alpha \) because a larger \( \alpha \) would disrupt the star due to excessive radiation pressure.

3 RADIATION PRESSURE SUPPORTED STARS IN EINSTEIN GRAVITY

The general definition of ‘compactness’ in GR may be given in terms of the surface redshift

\[ z = \left( 1 - \frac{2GM}{Rc^2} \right)^{-1/2} - 1. \]  

(29)

One then easily finds that when \( GM/Rc^2 \ll 1 \) (i.e. in the truly Newtonian regime), one has

\[ z \approx \frac{GM}{Rc^2}. \]  

(30)

It also follows that the event horizon (EH) of a black hole (BH), defined by \( R = \gamma = 2GM/c^2 \), corresponds to \( z = \infty \), and this explains why gravitation is extremely strong for BHs. As is well known, very massive objects undergo continued collapse to become BHs. Thus, by definition, very massive objects, during their continued collapse, must pass through stages having \( z > any finite number \). Although it is not necessary, we may note the less mention that the exterior space-time of any contracting self-gravitating object is represented by a radiating Vaidya metric (Vaidya 1951) which allows the...
possibility that $z \to \infty$. Further, it has been shown that, whenever self-luminous objects have $z \gg 1$ (Mitra 2006), one will have

$$y \approx \alpha \left( \frac{z}{2} \right).$$

Then by combining equations (12) and (13), equation (22) gets modified, in the extreme GR case, as

$$x \approx \alpha \left( \frac{z}{2} \right) \frac{m_{\odot} c^2}{3 \pi T}.$$  \hspace{1cm} (32)

In all physically realistic cases of self-luminous objects, $\alpha$ is always finite. In the Newtonian case of $z \ll 1$, we found that $\alpha$ increases rapidly with $M$ and hence with $z$. Since an increase of $z$ implies stronger self-gravity, we may say that as if stronger gravity churns out more radiation from self-luminous self-gravitating objects. Also, as $z \to \infty$, the entire object becomes a ball of radiation/pure energy ($\rho_\gamma / \rho_p \to \infty$). Note that this happens before the formation of any EH which indicates that the EH is actually synonymous with the central singularity, and the integration constant of the vacuum Schwarzschild solution has the unique value of zero (Mitra 2005a,b). Actually, one can have continued gravitational collapse for arbitrary (small) $M$ too, provided $\rho$ is suitably high. In contrast to the Newtonian case, where $\alpha$ (relatively) higher $z$ demands higher $M$ (equation 27), in GR, the very fact that there could be continued gravitational collapse at any mass scale means that one can have high $z \gg 1$ for arbitrary mass and hence one may have $\alpha \approx 1$, at a suitably high value of $z$, at arbitrary mass scale. For instance, for the Sun, $\alpha \approx 3 \times 10^{-3}$ and $z \approx 2 \times 10^{-6}$. However, if we consider a collapsing hot protoneutron star in its final stage when it is giving birth to a hot neutron star, $\alpha \approx 10^{-3}$ as $z \approx 10^{-1}$ (Mitra 2006). Note the rise in the value of $\alpha$ with $z$ even at a low mass scale of $M \approx 1 \text{M}_\odot$ as we are about to enter the regime of Einstein gravity.

The gravitational mass energy of the star is defined as

$$M c^2 = \int_0^R 4 \pi r^2 \rho \, dr.$$  \hspace{1cm} (33)

The mean value of the mass energy density is thus

$$\rho = \frac{3 M c^2}{4 \pi R^3}.$$  \hspace{1cm} (34)

From equation (29), note that in the $z \gg 1$ range, the radius of the contracting body would be hovering around its instantaneous Schwarzschild radius, that is, $R \approx R_s = 2GM/c^2$. Using this fact in the foregoing equation, we have

$$\rho = \frac{3c^4}{32 \pi a^3 G^3 M^2}.$$  \hspace{1cm} (35)

Note that equation (31) implies that, for $z \gg 1$, $\rho_\gamma \gg \rho_p$ so that the total $\rho = \rho_\gamma + \rho_p \approx \rho_\gamma$. Further recalling that $\rho_\gamma = aT^3$, from equation (35), we obtain

$$T = \frac{(3c^4)}{32 \pi a G^3 M^2}^{1/4}.$$  \hspace{1cm} (36)

Numerically, one finds that, for such a state, the mean temperature of the body in this phase is

$$T \approx 600 \left( \frac{M}{15 \text{M}_\odot} \right)^{-1/2} \text{ MeV}.$$  \hspace{1cm} (37)

By substituting equation (37) in equation (32), we obtain

$$x \approx \alpha \left( \frac{M}{15 \text{M}_\odot} \right)^{1/2}. \hspace{1cm} (38)$$

Note that, even in the extreme GR case, $x$ increases with $M$, and in particular the behaviour $x \sim M^{1/2}$ is just what we found for Newtonian RPSSs. However, unlike the Newtonian case, in the GR case, one can have $z \gg 1$. Equation (38) shows that, with $\alpha > 0$, and $z \to \infty$, one would be able to satisfy the condition $x > 1$ at arbitrary mass scale for an appropriately high value of $z \gg 1$. As mentioned earlier, the occurrence of $x \gg 1$ means $\alpha = 1$ (rather than $\alpha \gg 1$, which would disrupt the star). For isolated bodies, radiation pressure and energy density are directly associated with outward radiation/heat flux. This outward radiation flow has a repulsive action on the plasma. As radiation pressure tends to grow unabated with an unabated increase of $z$, the repulsive effect on the plasma must be able to counterbalance the pull of gravity at some stage, and this stage corresponds to the attainment of $L = L_\odot$ or $\alpha = 1$. In fact, the domination of radiation pressure is a much less demanding phenomenon than the domination of radiation energy, because recall that though the Newtonian supermassive stars have $\rho_\gamma \gg \rho_p$, they still have $\rho_\gamma \gg \rho_p$. On the other hand, since equation (31) demands $\rho_\gamma \gg \rho_p$, at appropriately high $z$, such an occurrence automatically denotes domination of radiation pressure. Therefore, for $x \gg 1$, we may rewrite equation (38) as

$$x \approx z \left( \frac{M}{15 \text{M}_\odot} \right)^{1/2} \approx 0.25 z \left( \frac{M}{M_\odot} \right)^{1/2} z \gg 1. \hspace{1cm} (39)$$

The absence of $z$ in equation (24) demands that one can have $x \gg 1$ only for very large values of $M$. On the other hand, the presence of $z$ in equation (39) shows that, in the extreme relativistic case, one can have RPSSs for an arbitrary stellar mass, high or low. Hence as one proceeds towards $z \to \infty$ during continued gravitational collapse, one must obtain radiation pressure supported configurations ($x \gg 1$) at arbitrary mass rather than finite mass BHs. Although we have worked only in the regions of $z \ll 1$ (equation 24) and $z \gg 1$ (equation 39), the close similarity in the forms of equations (24) and (39) strongly suggests that equation (39) may be valid for RPSSs in the entire relativistic range of $z > 1$.

4 DISCUSSION

Newtonian stars are defined by $z \ll 1$ as well as $\rho_\gamma / \rho_p$, that is, $y \ll 1$. It is this self-imposed constraints which cause $x \ll 1$. However, even in the Newtonian regime, that is, despite having $y \ll 1$ and $z \ll 1$, one can have RPSSs ($x > 1$) for masses above $7200 \text{M}_\odot$. Hence, we find that the non-occurrence of low-mass RPSSs in the Newtonian regime is intricately linked with the occurrence of $z \ll 1$ and $y \ll 1$ in such cases. On the other hand, GR is unleashed in full glory during continued gravitational collapse when one can have $z \gg 1$ and $y \gg 1$ for an arbitrary mass, low or high. It naturally follows then that, in the extreme GR case, one can have a RPSS even at arbitrary low $M$. While the occurrence of Newtonian supermassive stars may be only a theoretical possibility, Einstein RPSSs must be a reality because of the following simple reason.

As $z$ tends to increase indefinitely during continued collapse, the strong gravity almost completely traps the collapse-generated neutrinos and photons within the body of the star. The density of trapped radiation also increases because of stellar matter–radiation interaction, that is, the diffusion of the internal radiation (Mitra 2006), and gravitational trapping (Mitra & Glendenning 2006). As a result, the trapped radiation pressure and energy density increase at least as fast as $\rho_\gamma \approx R^{-1} (1 + z)^2$.  

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increases

The radiation pressure associated with $L_{\text{ed}}$ is

$$p_{\text{ed}} \sim \frac{1}{3} \frac{L_{\text{ed}}}{\pi R_{\text{ed}}^4} \sim R^{-2}(1 + z).$$

Therefore, $p_t/p_{\text{ed}} \sim R^{-1}(1 + z)$. Thus as $R$ decreases and $1 + z$ increases, $p_t \to p_{\text{ed}}$ even though, initially, $p_t \ll p_{\text{ed}}$ and $L \ll L_{\text{ed}}$. It is at this stage that $L \to L_{\text{ed}}$ and $\alpha \to 1$. Simultaneously, one will have both $p_t \gg p_\beta$ and $p_t \gg p_\beta$ at this stage. Specific models of GR continued collapse which do not restrict $\rho$, by any means do clearly show that the repulsive effects of the unabated increase of $p_t$ and $p_t$ may not only counterbalance the inward pull of gravity but also even cause the fluid to bounce back (Herrera & Santos 2004; Herrera, Prisco & Barreto 2006). Such studies are fully consistent with the generic picture of formation of quasi-static GR RPSSs at arbitrary mass scale discussed in this paper. The observed luminosity of such RPSSs will, however, be lower by a factor of $(1 + z)^2$ than the local value because of the joint effect of gravitational redshift and gravitational time-dilation:

$$L_{\text{ed}} = \frac{4\pi GMc}{\kappa(1 + z)},$$

Consequently, even if the system were assumed to be at a given value of redshift, $z$, its observed duration as seen by a distant astronomer would be

$$t(z) = \frac{Mc^2}{L_{\text{ed}}} = \frac{kc}{4\pi G}(1 + z).$$

Since in principle, during continued collapse, $z \to \infty$, clearly, the observed time-scale for depletion of mass energy becomes infinite for an arbitrary value of the opacity $\kappa$: $t = \infty$. Hence the RPSSs tend to collapse for infinite duration in order to attain the BH ($z = \infty$) state and, therefore, may be called 'eternally collapsing objects' (ECOs) (Mitra 2000, 2002). It has also been shown that since the eventual BH mass would be zero, the comoving proper time for its formation would also be infinite (Mitra 2005a,b). Since the observed BH candidates must be formed in gravitational collapse and of finite age, they must be ECOs ($z \gg 1$) rather than true BHs ($z = \infty$).

In retrospect, long ago, the RPSSs were suggested as the central engine of quasars (Hoyle & Fowler 1963; Fowler 1966). However, this attempt failed because of the following reasons: (i) such RPSSs were considered to be either Newtonian or post-Newtonian objects with low temperatures and (ii) the basic source of energy liberation was considered to be of nuclear origin. In contrast, the relativistic RPSSs considered here are fed by energy release due to secular gravitational contraction and the source of energy is the entire mass energy ($E = Mc^2$). Even if they were momentarily unstable, the contraction-generated luminosity would ensure that they pass from one quasi-static state of $z = z_1$ to another with $z_2 > z_1$.

ACKNOWLEDGMENTS

The author thanks Stanley Robertson, Darryl Leiter, Norman Glendenning and P. S. Negi for various discussions. The author is also grateful to Felix Aharonian for persistent encouragement. The anonymous referee is also thanked for some constructive suggestions.

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