The generalized second law of gravitational thermodynamics on the apparent and event horizons in FRW cosmology

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Abstract
We investigate the validity of the generalized second law (GSL) of gravitational thermodynamics on the apparent and event horizons in a non-flat Friedmann–Robertson–Walker (FRW) universe containing dark energy interacting with dark matter. We show that for the dynamical apparent horizon, the GSL is always satisfied throughout the history of the universe for any spatial curvature and it is independent of the equation of state parameter of the interacting dark energy model. On the other hand, for the cosmological event horizon, the validity of the GSL depends on the equation of state parameter of the model.

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1. Introduction

The present acceleration of the universe expansion has been well established through numerous and complementary cosmological observations [1]. A component which is responsible for this accelerated expansion is usually dubbed ‘dark energy’ (DE). However, the nature of DE is still unknown, and people have proposed some candidates to describe it (for review see [2, 3] and references therein).

One of the important questions in cosmology concerns the thermodynamical behavior of the accelerated expanding universe driven by DE. It was shown that the Einstein equation can be derived from the first law of thermodynamics by assuming the proportionality of entropy and the horizon area [4–6]. In the cosmological context, attempts to disclose the connection between Einstein gravity and thermodynamics were made. It was shown that the differential form of the Friedmann equation in the Friedmann–Robertson–Walker (FRW) universe can be written in the form of the first law of thermodynamics on the apparent horizon [7]. Further studies on the equivalence between the first law of thermodynamics and the
Friedmann equation have been conducted in various gravity theories such as Gauss–Bonnet, Lovelock and braneworld scenarios [7–9].

Besides examining the validity of the thermodynamical interpretation of gravity by expressing the gravitational field equations into the first law of thermodynamics in different spacetimes, it is also of great interest to investigate the validity of the generalized second law (GSL) of thermodynamics in the accelerating universe driven by DE. The GSL of thermodynamics is as important as the first law, governing the development of the nature [10–19].

Here our aim is to investigate the validity of the GSL of gravitational thermodynamics for the interacting DE model with dark matter (DM) in a non-flat FRW universe enclosed by the dynamical apparent horizon and the cosmological event horizon. Note that in the literature, people usually have studied the validity of the GSL for a specific model of DE with a special kind of interaction term. But we would like to extend it to any DE model with a general interaction term. This paper is organized as follows. In section 2, we study the DE model in a non-flat FRW universe that is in interaction with DM. In section 3, we investigate the validity of the GSL of gravitational thermodynamics for the universe enclosed by the apparent horizon and the event horizon which is in thermal equilibrium with the Hawking radiation of the horizon. Also, we give an example for the case of the cosmological event horizon. In section 4, we investigate the effect of the interaction term between DE and DM on the GSL. Section 5 is devoted to conclusions.

2. Interacting DE and DM in FRW cosmology

The first Friedmann equation in FRW cosmology takes the form

\[ H^2 + \frac{k}{a^2} = \frac{8\pi}{3} (\rho_\Lambda + \rho_m), \]  

(1)

where we take \( G = 1 \) and \( k = 0, 1, -1 \) represent flat, closed and open FRW universes, respectively. Also, \( \rho_\Lambda \) and \( \rho_m \) are the energy densities of DE and DM, respectively.

From (1), we can write

\[ \Omega_m + \Omega_\Lambda = 1 + \Omega_k, \]  

(2)

where we have used the following definitions:

\[ \Omega_m = \frac{8\pi \rho_m}{3H^2}, \quad \Omega_\Lambda = \frac{8\pi \rho_\Lambda}{3H^2}, \quad \Omega_k = \frac{k}{a^2 H^2}. \]  

(3)

Since we consider the interaction between DE and DM, \( \rho_\Lambda \) and \( \rho_m \) do not conserve separately. Hence, the energy conservation equations for DE and DM are

\[ \dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \]  

(4)

\[ \dot{\rho}_m + 3H\rho_m = Q, \]  

(5)

where \( Q \) stands for the interaction term. For \( Q > 0 \), there is an energy transfer from DE to DM. The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of DE and DM becomes a constant [20]. The dynamics of interacting DE models with different \( Q \)-classes have been studied in ample detail in [21]. Here we continue our study without considering a specific form for the interaction term.
Taking the time derivative of both sides of equation (1), and using equations (2)–(5), one can get the equation of state (EoS) parameter of DE as
\[ \omega_L = -\frac{1}{3\Omega_L} \left( 3 + \Omega_k + \frac{2H}{H^2} \right). \] (6)
For the DE-dominated universe, i.e. \( \Omega_L \to 1 \), the above equation yields
\[ \omega_L = -1 - \frac{2H}{3H^2}, \] (7)
which shows that for the phantom, \( \omega_L < -1 \), and quintessence, \( \omega_L > -1 \), dominated universe, we need to have \( \dot{H} > 0 \) and \( \dot{H} < 0 \), respectively. Note that for the de Sitter universe, i.e. \( \dot{H} = 0 \), we have \( \omega_L = -1 \) which behaves like the cosmological constant.

The deceleration parameter is given by
\[ q = -\left( 1 + \frac{\dot{H}}{H^2} \right). \] (8)
Replacing the term \( \dot{H}/H^2 \) from (6) into (8) yields
\[ q = \frac{1}{2} \left( 1 + \Omega_k + 3\Omega_L \omega_L \right). \] (9)

3. Generalized second law of gravitational thermodynamics

Here, we study the validity of the GSL of gravitational thermodynamics. According to the GSL, the entropy of matter and fluids inside the horizon plus the entropy of the horizon does not decrease with time [17]. Here, like [15], we assume that the temperatures of both dark components are equal, due to their mutual interaction. Note that Lima and Alcaniz [22] using a very naive estimate obtained the present value of DE temperature as \( T_{DE}^0 \sim 10^{-6} \) K. Also, Zhou et al [23] estimated the DM temperature as \( T_{DM}^0 \sim 10^{-7} \) K for the present time. Since the temperature of DE at the present time differs from that of DM, the systems must interact for some length of time before they can attain thermal equilibrium. Although in this case, the local equilibrium hypothesis may no longer hold [23, 24], Karami and Ghaffari [25] showed that the contribution of the heat flow between DE and DM in the GSL in non-equilibrium thermodynamics is very small, \( O(10^{-7}) \). Therefore, the equilibrium thermodynamics is still preserved. We also limit ourselves to the assumption that the thermal system including DE and DM bounded by the horizon remains in equilibrium so that the temperature of the system must be uniform and the same as that of the temperature of its boundary. This requires that the temperature \( T \) of both DE and DM inside the horizon should be in equilibrium with the Hawking temperature \( T_h \) associated with the horizon, so we have \( T = T_h \). This expression holds in the local equilibrium hypothesis. If the temperature of the system differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid, and the local equilibrium hypothesis will no longer hold [23–25]. This is also at variance with the FRW geometry. In general, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the horizon.

The entropy of the universe including DE and DM inside the horizon can be related to its energy and pressure in the horizon by Gibb’s equation [13]:
\[ TdS = dE + PdV, \] (10)
where \( V = 4\pi R_h^3 / 3 \) is the volume containing DE and DM with the radius of the horizon \( R_h \), and \( T = T_h = 1/(2\pi R_h) \) is the Hawking temperature of the horizon. Also,
\[ E = \frac{4\pi R_h^3}{3}(\rho_\Lambda + \rho_m). \]  

(11)

\[ P = P_\Lambda + P_m = P_\Lambda = \frac{3H^2}{8\pi}\omega_\Lambda\Omega_\Lambda. \]  

(12)

Taking the derivative of both sides of (10) with respect to cosmic time \( t \) and using equations (1)–(5) , (11) and (12), we obtain the evolution of the entropy in the universe containing DE and DM as

\[ \dot{S} = 3\pi H^2 R_h^2(\dot{R}_h \ddot{R}_h - H\dot{R}_h^2)(1 + \Omega_k + \Omega_\Lambda\omega_\Lambda). \]  

(13)

In addition to the entropy in the universe, there is a geometric entropy on the horizon \( S_h = \pi R_h^2 \) [13]. The evolution of this horizon entropy is obtained as

\[ \dot{S}_h = 2\pi R_h \dot{R}_h. \]  

(14)

Finally, the GSL due to different contributions of DE, DM and the horizon is obtained as

\[ \dot{S}_{tot} = 3\pi H^2 R_h^2(\dot{R}_h \ddot{R}_h - H\dot{R}_h^2)(1 + \Omega_k + \Omega_\Lambda\omega_\Lambda) + 2\pi R_h \dot{R}_h, \]  

(15)

where \( S_{tot} = S + S_h \) is the total entropy. Note that in \( S_{tot} \) we ignored the contribution of baryonic matter (BM) \( \Omega_{BM}^0 \sim 0.04 \) in comparison with DM and DE \( (\Omega_{DM}^0 + \Omega_{DE}^0 \sim 0.96) \). According to the recent measurements of the supermassive black hole mass function, the present entropy of BM \( S_{BM}^0 = (2.7 \pm 2.1) \times 10^8 \) is five to seven orders of magnitude smaller than that of the DM \( S_{DM}^0 = 6 \times 10^{86\pm1} \) [26].

In the following sections, we investigate the validity of the GSL given by equation (15) for the dynamical apparent and cosmological event horizons.

3.1. The dynamical apparent horizon

The dynamical apparent horizon in the FRW universe is given by [27]

\[ R_A = H^{-1}(1 + \Omega_k)^{-1/2}. \]  

(16)

For \( k = 0 \), the apparent horizon is the same as the Hubble horizon.

Recently Cai et al [27] proved that the apparent horizon of the FRW universe with any spatial curvature has indeed an associated Hawking temperature \( T_A = 1/2\pi R_A \). Cai et al [27] also showed that the Hawking temperature can be measured by an observer with the Kodoma vector inside the apparent horizon.

If we take the derivative of both sides of (16) with respect to cosmic time \( t \), then we obtain

\[ \dot{R}_A = \frac{3(1 + \Omega_k + 3\Omega_\Lambda\omega_\Lambda)}{2(1 + \Omega_k)^{3/2}}. \]  

(17)

Using equations (16) and (17), one can get

\[ R_A \dot{R}_A - H R_A^2 = \frac{(1 + \Omega_k + 3\Omega_\Lambda\omega_\Lambda)}{2H(1 + \Omega_k)^2}. \]  

(18)

Substituting equations (16)–(18) into (13) and (14) yields

\[ \dot{S} = \frac{3\pi}{2H(1 + \Omega_k)^3}(1 + \Omega_k + 3\Omega_\Lambda\omega_\Lambda)(1 + \Omega_k + \Omega_\Lambda\omega_\Lambda). \]  

(19)

\[ \dot{S}_A = \frac{3\pi}{H(1 + \Omega_k)^2}(1 + \Omega_k + \Omega_\Lambda\omega_\Lambda). \]  

(20)
Equation (19) shows that for $-\left(\frac{1+\Omega_0}{\Omega_0}\right) < \omega_\Lambda < -\frac{1}{3} \left(\frac{1+\Omega_0}{\Omega_0}\right)$, the contribution of the entropy of the universe inside the dynamical apparent horizon in the GSL is negative, i.e. $\dot{S} < 0$. For the late time or the DE-dominated universe where $\Omega_\Lambda \to 1$ and $R_\Lambda = H^{-1}$, the entropy of the universe will be a non-increasing function of time in the quintessence regime with $-1 < \omega_\Lambda < -1/3$.

Equation (20) clarifies that for $\omega_\Lambda > -\left(\frac{1+\Omega_0}{\Omega_0}\right)$, the contribution of the dynamical apparent horizon in the GSL is positive, i.e. $\dot{S}_A > 0$. For the DE-dominated universe, the entropy of the dynamical apparent horizon will be an increasing function of time in the quintessence regime with $\omega_\Lambda > -1$.

Finally, using equations (19) and (20), the GSL due to different contributions of DE, DM and the apparent horizon can be obtained as

$$\dot{S}_{tot} = \frac{9\pi}{2H(1+\Omega_k)^3}(1 + \Omega_k + \Omega_\Lambda \omega_\Lambda)^2 \geq 0. \quad (21)$$

Equation (21) shows that the GSL for the universe containing the DE interacting with DM enclosed by the dynamical apparent horizon is always satisfied throughout the history of the universe for any spatial curvature, and it is independent of the EoS parameter of the interacting DE model.

### 3.2. The cosmological event horizon

For the cosmological event horizon, defined as

$$R_E = a \int_0^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}, \quad (22)$$

one can obtain

$$R_E = H R_E - 1. \quad (23)$$

For a de Sitter spacetime where $H$ is a constant, equations (22) and (23) show that the cosmological event horizon radius is $H^{-1}$ and $\dot{R}_E = 0$. Therefore, in a spatially flat de Sitter universe, the event horizon and the apparent horizon, given by equation (16), of the universe coincide with each other, and there is only one cosmological horizon [11]. For the de Sitter universe, from equation (6), we have $\omega_\Lambda = -1/\Omega_\Lambda$; hence, equation (15) shows that $\dot{S}_E = \dot{S} = 0$, which corresponds to a reversible adiabatic expansion.

Substituting equation (23) into (13) yields the entropy of the universe inside the cosmological event horizon as

$$\dot{S}_E = -3\pi H^2 R_E^2 (1 + \Omega_k + \Omega_\Lambda \omega_\Lambda). \quad (24)$$

Equation (24) clarifies that for $\omega_\Lambda < -\left(\frac{1+\Omega_0}{\Omega_0}\right)$, the contribution of the cosmological event horizon in the GSL is positive, i.e. $\dot{S}_E > 0$. For the late-time universe, the entropy of the universe will be an increasing function of time in the phantom regime with $\omega_\Lambda < -1$.

Following [28], for the quintessence and phantom universes, $\dot{R}_E > 0$ and $\dot{R}_E < 0$, respectively. Therefore, from equation (14) one can conclude that for the quintessence universe $\dot{S}_E = 2\pi R_E \dot{R}_E > 0$ and for the phantom universe $\dot{S}_E < 0$.

Substituting equation (23) into (15) yields

$$\dot{S}_{tot} = \frac{2\pi R_E}{3H^2 R_E^2 \Omega_\Lambda} \left[ R_E - \frac{1}{2}(1 + \Omega_k + \Omega_\Lambda \omega_\Lambda) H^2 R_E^2 \right], \quad (25)$$

which shows that the GSL is satisfied, i.e. $\dot{S}_{tot} \geq 0$, when

$$\omega_\Lambda \leq \frac{2R_E}{3H^2 R_E^2 \Omega_\Lambda} - \left(1 + \frac{\Omega_k + \Omega_\Lambda \omega_\Lambda}{\Omega_\Lambda}\right). \quad (26)$$
The above constraint on the EoS parameter of the interacting DE model presents that for the late-time universe in the quintessence and phantom regime, we have

$$\omega/\lambda_1 \leq \frac{2}{3H^2 R_E},$$

and

$$\omega/\lambda_1 \leq -\frac{2|\dot{R}_E|}{3H^2 R_E},$$

respectively.

Substituting equation (23) into (26) yields

$$\omega/\lambda_1 \leq f(H R_E),$$

where

$$f(H R_E) = \frac{2(H R_E - 1)}{3H^2 R_E} - \left(1 + \frac{\Omega_k}{\Omega_\Lambda}\right).$$

In figure 1, the solid line shows $f(H R_E)$ versus $H R_E$ for the DE-dominated universe where $\Omega_\Lambda \to 1$ and the dotted line presents $f(H R_E) = -1/3$. Note that for a universe enclosed by the event horizon, we have always $\omega_\Lambda < -1/3$. Figure 1 demonstrates that for $f(H R_E) < \omega_\Lambda < -1/3$ the GSL is not satisfied on the cosmological event horizon and it remains valid only for $\omega_\Lambda \leq f(H R_E)$.

Therefore, for the non-flat FRW universe containing the DE interacting with DM enclosed by the cosmological event horizon, the GSL is satisfied for the special range of the EoS parameter of the DE model. In contrast to the case of the apparent horizon, the validity of the GSL for the cosmological event horizon depends on the EoS parameter of the interacting DE model.

### 3.3. A pole-like type phantom universe enclosed by the event horizon

Here we give an example to study the GSL in the case of the cosmological event horizon. Following [28], we consider a phantom DE model of the universe described by a pole-like type scale factor as

$$a(t) = a_0(t_s - t)^{-n}, \quad t \leq t_s, \quad n > 0;$$

then one can get

$$H = \frac{n}{t_s - t},$$

Figure 1. $f(H R_E) = \frac{2(H R_E - 1)}{3H^2 R_E} - 1$ versus $H R_E$ for the DE-dominated universe. The dotted line shows $f(H R_E) = -1/3$. 

The above constraint on the EoS parameter of the interacting DE model presents that for the late-time universe in the quintessence and phantom regime, we have $\omega_\Lambda \leq \frac{2R_E}{3H^2 R_E} - 1$ and $\omega_\Lambda \leq -\frac{2|\dot{R}_E|}{3H^2 R_E},$ respectively.
and also
\[ H = \frac{n}{(t_s - t)^2} > 0. \]  
(30)

The cosmological event horizon can be obtained as
\[ R_E = a \int_t^{t_s} \frac{dt}{a} = \frac{t_s - t}{n + 1}, \]  
(31)

also
\[ \dot{R}_E = -\frac{1}{n + 1} < 0. \]  
(32)

Equations (30) and (32) confirm that the model (28) corresponds to a phantom-dominated universe.

The EoS and deceleration parameters of the model (28) are obtained by the help of equations (6) and (8), respectively, as
\[ \omega_\Lambda = -\frac{1}{3\Omega_\Lambda} \left( 3 + \Omega_k + \frac{2}{n} \right), \]  
(33)
\[ q = -1 - \frac{1}{n} < -1. \]  
(34)

Equation (33) shows that for the late-time universe, we have \( \omega_\Lambda = -1 - \frac{2}{3n} < -1 \) which is the EoS parameter of the phantom DE model.

From equations (14), (24), (29), (31)–(33), the entropy of the event horizon and that of the universe inside the event horizon can be obtained as
\[ \dot{S}_E = 2\pi R_E \dot{R}_E = -\frac{2\pi}{(n + 1)^2} (t_s - t) \leq 0, \]  
(35)
\[ \dot{S} = \frac{2\pi n^2}{(n + 1)^3} (t_s - t) \left( \frac{1}{n} - \Omega_k \right). \]  
(36)

Equation (35) shows that the entropy of the event horizon for the model (28) has a negative contribution in the GSL throughout the history of the universe. Equation (36) shows that the entropy of the universe has a positive contribution in the GSL only when \( \Omega_k \leq 1/n \). Finally, for the model (28), the GSL yields
\[ S_{\text{tot}} = S + S_E = -\frac{2\pi (t_s - t)}{(n + 1)^3} (1 + n^2 \Omega_k) < 0, \]  
(37)
which shows that for the positive spatial curvature, compatible with the present observations [29], the GSL breaks down.

4. The effect of the interaction term between DE and DM on the GSL

In our previous analysis, the interaction term between DE and DM did not appear explicitly in the GSL. To see how the DE–DM interaction influences the GSL, we need to incorporate a specific form of the DE model in our analysis. To do this, we consider the holographic DE (HDE) model, which is motivated by the holographic principle [30]. Following [20], the HDE density in a closed universe is given by
\[ \rho_\Lambda = 3c^2 M_P^2 L^{-2}, \]  
(38)
where $c$ is a positive constant and $M_{p}$ is the reduced Planck mass $M_{p}^{-2} = 8\pi$. Recent observational data, which have been used to constrain the HDE model, show that for the non-flat universe $c = 0.815^{+0.179}_{-0.139}$ [31]. Also, $L$ is the IR cut-off defined as

$$L = \frac{a}{\sqrt{k}} \sin y,$$

(39)

where $y = \sqrt{k} R_{E}/a$. Note that $R_{E}$ is the radial size of the event horizon measured in the $r$ direction and $L$ is the radius of the event horizon measured on the sphere of the horizon [20]. For the flat universe $L = R_{E}$. For a specific form of the interaction term between DE and DM as $Q = 3b^{2}H(\rho_{\Lambda} + \rho_{m})$, with $b^{2}$ being the coupling constant [32], the EoS parameter for the interacting HDE with DM in a non-flat FRW universe is obtained as [33]

$$\omega_{\Lambda} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\Lambda}}}{3c} \cos y - b^{2} \left( \frac{1 + \Omega_{k}}{\Omega_{\Lambda}} \right),$$

(40)

where $\cos y = \sqrt{1 - c^{2}\Omega_{k}/\Omega_{\Lambda}}$. Therefore, the coupling constant $b^{2}$ due to interaction appears explicitly in the EoS parameter of the HDE. For the dynamical apparent horizon, equation (21) shows that the GSL is always satisfied throughout the history of the universe regardless of the specific form of the DE model and interaction term $Q$. But for the cosmological event horizon, the story is different. The GSL for the interacting HDE with DM in a non-flat universe enclosed by the event horizon measured from the sphere of the horizon $L$ is obtained from equation (15) as

$$\dot{S}_{\text{tot}} = 3\pi H^{2}L^{2}(L \dot{L} - H L^{2})(1 + \Omega_{k} + \Omega_{\Lambda} \omega_{\Lambda}) + 2\pi L \dot{L},$$

(41)

where the necessary expressions for $L$ and $\dot{L}$ are given by equations (15) and (16) in [20], respectively, as

$$L = \frac{c}{H} \sqrt{\Omega_{\Lambda}},$$

(42)

$$\dot{L} = \frac{c}{\sqrt{\Omega_{\Lambda}}} - \cos y.$$

(43)

Substituting equations (40), (42) and (43) into (41) yields

$$\dot{S}_{\text{tot}} = \frac{\pi c^{3}}{H \Omega_{\Lambda}^{3/2}} \left\{ 2 \sqrt{\Omega_{\Lambda}} \left( 1 + \Omega_{\Lambda} \cos^{2} y \right) + \left( 1 - \frac{2}{c^{2}} \right) \Omega_{\Lambda} \cos y + 3(b^{2} - 1)(1 + \Omega_{k}) \cos y \right\},$$

(44)

which is the same as the result given by equation (1.6) in [18]. Equation (44) shows that the coupling constant $b^{2}$ of the interaction term $Q$ does affect the GSL on the radius of the event horizon $L$. For instance, for $\cos y = 0.99$, $\Omega_{\Lambda} = 0.73$, $\Omega_{k} = 0.01$ and $c = 1$ given by [34] for the present time, we get

$$\dot{S}_{\text{tot}} = \frac{4.809\pi}{H} (b^{2} - 0.264),$$

(45)

which shows that if $b^{2} \geq 0.264$ then $\dot{S}_{\text{tot}} \geq 0$, and the GSL is satisfied.

5. Conclusions

Here the GSL of gravitational thermodynamics for the interacting DE with DM in a non-flat FRW universe is investigated. Some experimental data have implied that our universe is not a perfectly flat universe and it possesses a small positive curvature [29]. Although it is believed
that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [20]. The boundary of the universe is assumed to be enveloped by the dynamical apparent horizon and the cosmological event horizon. We assumed the universe to be in thermal equilibrium with the Hawking temperature on the horizon. We found that for the dynamical apparent horizon, the GSL is satisfied throughout the history of the universe for any spatial curvature, and it is independent of the EoS parameter of the interacting DE model. But for the cosmological event horizon, the GSL is satisfied for the special range of the EoS parameter of the model.

The above results show that the dynamical apparent horizon, in comparison with the cosmological event horizon, is a good boundary for studying cosmology, since on the apparent horizon there is a well-known correspondence between the first law of thermodynamics and the Einstein equation [35]. In other words, the Friedmann equations describe local properties of spacetimes and the apparent horizon is determined locally, while the cosmological event horizon, equation (22), is determined by global properties of spacetimes [7]. Besides, in the dynamic spacetime, the horizon thermodynamics is not as simple as that of the static spacetime. The event and apparent horizons are, in general, different surfaces. The definition of thermodynamical quantities on the cosmological event horizon in the nonstatic universe is probably ill defined [17].

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