The algebraic structure of
Galilean superconformal symmetries

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Abstract

The semisimple part of \( d \)-dimensional Galilean conformal algebra \( \mathfrak{g}(d) \) is given by \( \mathfrak{h}(d) = O(2, 1) \oplus O(d) \), which after adding via semidirect sum the \( 3d \)-dimensional Abelian algebra \( \mathfrak{t}(d) \) of translations, Galilean boosts and constant accelerations completes the construction. We obtain Galilean superconformal algebra \( \mathfrak{G}(d) \) by firstly defining the semisimple superalgebra \( \mathfrak{H}(d) \) which supersymmetrizes \( \mathfrak{h}(d) \), and further by considering the expansion of \( \mathfrak{H}(d) \) by tensorial and spinorial graded Abelian charges in order to supersymmetrize the Abelian generators of \( \mathfrak{t}(d) \). For \( d = 3 \) the supersymmetrization of \( \mathfrak{h}(3) \) is linked with specific model of \( \mathcal{N} = 4 \) extended superconformal mechanics, which is described by the superalgebra \( D(2, 1; \alpha) \) if \( \alpha = 1 \). We shall present as well the alternative derivations of extended Galilean superconformal algebras for \( 1 \leq d \leq 5 \) by employing the Inönü-Wigner contraction method.

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1 Introduction

Many applications of the relativistic AdS/CFT correspondence \cite{1, 2, 3} brings the question of its nonrelativistic limit. For such purpose we should define the nonrelativistic conformal field theory and the algebra of nonrelativistic conformal symmetries. If we recall that the relativistic AdS/CFT correspondence has its best justification after the supersymmetric extension (e.g. $D = 5$ supergravity versus $N = 4$ $D = 4$ SYM theory) it appears important to study as well the nonrelativistic superconformal symmetries.

In seventies \cite{4, 5, 6, 7} the Galilean symmetries were extended to the Schrödinger symmetries, by supplementing the Galilean Lie algebra by two additional generators: $D$ (dilatations) and $K$ (extensions). Because of conformal nature of the generators $D$, $K$ the Schrödinger symmetries were named as the nonrelativistic conformal symmetries. However soon appeared better candidate for such symmetries, described by the Galilean conformal algebra (GCA)\footnote{Galilean conformal algebra (GCA) was considered in \cite{8, 9, 10, 11}: the case $D = 2 + 1$, unique dimension permitting central charge, was considered in \cite{12}. The GCA-invariant mechanics models with $d \geq 1$ were studied recently in \cite{13}; for the review of geometric techniques related with nonrelativistic conformal algebras see \cite{14}.} the $c \to \infty$ contraction limit of the relativistic conformal algebra $\mathfrak{g}_{\mathfrak{o}(2, 1)}$. The Galilean conformal symmetries describe massless nonrelativistic systems – contrary to the case of Schrödinger algebra the central extension of Galilean conformal algebras which introduces the nonrelativistic mass parameter is not allowed. It was further realized (for recent review see \cite{14}) that if we relate nonrelativistic conformal symmetries with Newton-Cartan structure of nonrelativistic space and time, one gets families of infinite-dimensional Virasoro-like conformal algebras (see e.g. \cite{15, 16}). In particular, there was introduced the notion of generalized Schrödinger algebras $\mathfrak{g}_{\mathfrak{g}(d)}$ which do depend on the numerical parameter $z$ called dynamical exponent, characterizing the scaling properties of space and time coordinates. If $z = 1$ we obtain GCA, and for $z = 2$ one gets the Schrödinger algebra; recently it has been shown $\cite{17}$ that for rational choices of $z = \frac{N}{N}$ ($N = 1, 2, ...$) all the generalized Schrödinger algebras are finite-dimensional.

In this paper we restrict our studies to the Galilean conformal algebras and their supersymmetric extensions. Superalgebras of the Galilean superconformal symmetry has been constructed in \cite{18, 19} (see also \cite{20, 21}) from the relativistic conformal superalgebras by the contraction methods. In this paper we argue how the structure of the Galilean superconformal algebras (SUSY GCA) is related with the superalgebras describing the models of extended superconformal mechanics. We show that the supersymmetrization of the Abelian sector of translations, Galilean boosts and constant accelerations requires the addition of suitable sector of graded Abelian fermionic supercharges and possibly additional Abelian bosonic charges.

Let us consider the Galilean superconformal algebras as enlargement of simple standard superalgebras by the bosonic and fermionic graded Abelian charges\footnote{For the enlargements of Lie (super)algebras by extension and expansion procedure see e.g. \cite{22}.} Taking into account that the Galilean superconformal algebras contain as its semisimple part the conformal algebra $\mathfrak{g}_{\mathfrak{o}(2, 1)}$ of conformal mechanics \cite{23, 24} and the algebra of spatial rotations $O(d)$\footnote{We will denote the algebras by capital letters for convenience, making no distinction in the notation of the algebras and corresponding group. This will not lead below to any misunderstanding.} we firstly introduce the simple superalgebras $G_{\mathfrak{g}(d)}$ which contain as bosonic subalgebra $O(2, 1) \oplus O(d)$. Further we extend the superalgebra $G_{\mathfrak{g}(d)}$ by graded Abelian superalgebra $G_{\mathfrak{g}(d)}^{-} = (\mathcal{B}_{\mathfrak{g}(d)}, \mathcal{Q}_{\mathfrak{g}(d)})$ where

\[ G_{\mathfrak{g}(d)}^{-} : \quad [\mathcal{B}_{\mathfrak{g}(d)}, \mathcal{B}_{\mathfrak{g}(d)}] = 0, \quad [\mathcal{B}_{\mathfrak{g}(d)}, \mathcal{Q}_{\mathfrak{g}(d)}] = 0, \quad \{\mathcal{Q}_{\mathfrak{g}(d)}, \mathcal{Q}_{\mathfrak{g}(d)}\} = 0. \] (1.1)

We recall that the conformal Galilean algebra $g_{\mathfrak{g}(d)}$ is described as the following semidirect
sum
\[ g^{(d)} : \quad (O(2, 1) \oplus O(d)) \in \mathcal{A}^{(3d)} \]
with the real generators \( R_r \ (r = 0, 1, 2), \ J_{ij} = -J_{ji} \ (i, j = 1, \ldots, d), \ A_{r,i} = (P_i, B_i, F_i) \)
\[ O(2, 1) : \quad [R_r, R_s] = i \epsilon_{rs}^t R_t, \]
\[ O(d) : \quad [J_{ij}, J_{kl}] = i (\delta_{ik} J_{jl} + \delta_{jl} J_{ik} - \delta_{il} J_{jk} - \delta_{jk} J_{il}), \]
\[ \mathcal{A}^{(3d)} : \quad [A_{r,i}, A_{s,j}] = 0 \]
where we denote by \( P_i \) \( d \)-dimensional momenta, by \( B_i \) the Galilean boosts and by \( F_i \) the generators of constant accelerations. The generators \( A_{r,i} \) satisfy the following covariance relations
\[ [R_r, A_{s,i}] = i \epsilon_{rs}^t A_{t,i}, \quad [J_{ij}, A_{r,k}] = i (\delta_{ik} A_{r,j} - \delta_{jk} A_{r,i}). \]
We shall postulate that the generators \( A_{r,i} \) belong to \( G^{(d)}_+ \) \( (A_{r,i} \in \mathcal{B}^{(d)}_-) \). In our SUSY GCA which we shall denote by \( G^{(d)} \) the graded Abelian generators of \( G^{(d)}_+ = (\mathcal{B}^{(d)}_-, \mathcal{Q}^{(d)}_-) \) satisfy (1.1) and
\[ [G^{(d)}_+, G^{(d)}_-] \subset G^{(d)}_-. \]
We arrive therefore at the semidirect sum structure of SUSY GCA
\[ G^{(d)} = G^{(d)}_+ \oplus G^{(d)}_- \]
It should be added that the structure (1.8) also follows from the Inönü-Wigner (IW) supercoset contraction procedure \([25, 20, 21, 19]\). In this paper we shall show as well how to arrive at the structure of SUSY GCA \( G^{(d)} \) by suitable enlargement of the semisimple superalgebra \( G^{(d)}_- \). The simple \( (N=1) \) SUSY GCA will be defined as minimal supersymmetrization which contains bosonic sector \( h^{(d)} = O(2, 1) \oplus O(d) \); for the extended \( (N > 1) \) SUSY GCA we should add also to the bosonic sector of the superalgebra \( G^{(d)}_+ \) the Galilean internal sector (e.g. for \( d=3 \) it will be given by \( U(N; \mathbb{H}) \cong USp(2N) \)).

The plan of our paper is the following. In Sect. 2 we shall consider simple \( N=1 \) \( d=3 \) SUSY GCA obtained as the suitable enlargement of the simplest supersymmetrization of the algebra \( O(2, 1) \oplus O(3) = O^*(4) = U_{\alpha}(2; \mathbb{H}) \)
\[ U_{\alpha}(2; \mathbb{H}) = OSp(4^* \mid 2) \]
which contain the bosonic sector \( U_{\alpha}(2; \mathbb{H}) \oplus U(1; \mathbb{H}) \cong (O(2, 1) \oplus O(3)) \oplus O(3) \) \( 5 \) It will be shown that the possibility of supersymmetrizing the Abelian subalgebra \( \mathcal{A}^{(9)} \) selects out from
\[ \Omega = -\Omega^T \]
where the quaternionic \( N \times N \) antiHermitian metric \( \Omega \) satisfies the condition
\[ \Omega^+ = (\Omega)^T = -\Omega. \]
infinite family of superalgebras $D(2, 1; \alpha)$ describing versions of $\mathcal{N}=4$ superconformal mechanics [29, 30, 31, 32, 33, 34] only the choice $\alpha=1$, providing $D(2, 1; \alpha) \cong OSp(4^*|2)$. In Sect. 3 we shall consider the $N$-extended $d=3$ SUSY GCA and discuss the quaternionic structure of $d=3$ Galilean symmetries and supersymmetries. We shall link the IW contraction procedure with the standard nonrelativistic contraction described by $c \to \infty$ limit. In Sect. 4 we shall consider $N$-extended SUSY GCA for $d=1, 2, 4, 5$. In our final Sect. 5 we provide an outlook, in particular we comment on the link of $d$-dimensional SUSY GCA with $\mathcal{N}$-extended superconformal mechanics and mention about the generalizations of SUSY GCA to the description of nonrelativistic $p$-branes for $p \geq 1$ (see also [20, 21, 19]).

We add that there was proposed other supersymmetric extension of GCA (see [35]), outside our algebraic scheme, without supersymmetrization of all GCA generators. In [35] the GCA generators belonging to the $O(2, 1) \oplus O(d)$ are not expressed as bilinears of supercharges. Because the Hamiltonian $H$ belongs to $O(2, 1)$, the supersymmetrization in [35] does not include the basic relation $\{Q, Q\} \sim H$ of supersymmetric quantum mechanics, what we evaluate as an argument in favour of our scheme.

2 $N=1$ $d=3$ Galilean superconformal algebra from the extension of $D(2, 1; 1) \cong OSp(4^*|2)$

Let us consider firstly the physical example of $d=3$, with the GCA described by the algebra

$$g^{(3)} = (O(2, 1) \oplus O(3)) \in \mathcal{A}^{(9)} = h^{(3)} \in k^{(3)}.$$  \hspace{1cm} (2.1)

The simplest supersymmetrization of the semisimple part $h^{(3)} = O(2, 1) \oplus O(3)$ is provided by $OSp(3|2) \cong OSp(3|2; \mathbb{R})$, but it can be shown that in such a case it is not possible to enlarge such superalgebra by graded Abelian sector containing $\mathcal{A}^{(9)} = (P_i, B_i, F_i) (i = 1, 2, 3)$, where $P_i$ generate space translations, $B_i$ yield Galilean boosts and $F_i$ produce the constant nonrelativistic accelerations. The next candidate for the supersymmetrization of $h^{(3)}$ is $OSp(4|2) \equiv OSp(4|2; \mathbb{R})$, with internal sector $O(4) = O(3) \oplus O(3)$. In such a way in the process of supersymmetrization of $O(2, 1) \oplus O(3)$ we add still additional factor $O(3)$. Such bosonic symmetry describes the symmetries of $\mathcal{N}=4$ superconformal mechanics. Interestingly enough, we were also not able to show that the enlargement of superalgebra $OSp(4|2)$ permits the supersymmetrization of $g^{(3)}$ (see (2.1)) in a way described in previous section. Subsequently we shall consider the one-parameter generalization $D(2, 1; \alpha)$ of $OSp(4|2)$ superalgebra, where $D(2, 1; -\frac{1}{2}) \cong OSp(4|2)$. We shall show that the Abelian enlargement of $D(2, 1; \alpha)$ leading to the supersymmetrization of $g^{(3)}$ is possible only if $\alpha = 1$, in which case

$$D(2, 1; 1) \cong U_\alpha U(2; 1|\mathbb{H}) \cong OSp(4^*|2).$$  \hspace{1cm} (2.2)

We start from the set of the generators written in spinorial basis

$$G^{(3)}_+ = (Q^{+}_{\alpha \alpha' \beta}; R_{ab}, J_{\alpha \beta \gamma}, T_{AB}).$$  \hspace{1cm} (2.3)

odd $\Omega$ is purely quaternionic-imaginary (e.g. for $N=1$ one can choose $\Omega = e_2$). In complex notation we obtain that $U_\alpha(N) = O^*(2N)$. Further $U(N; \mathbb{H})$ describes the unitary quaternionic algebra; in complex notation we get $U(N; \mathbb{H}) = U(2N; \mathbb{H}) \cap Sp(2N; \mathbb{C}) \equiv USp(2N)$. Analogously, we get for quaternionic unitary superalgebra $U_\alpha U(M|N; \mathbb{H}) \cong SU(2M|2N) \cap OSp(2M|2N; \mathbb{C})$.  \hspace{1cm} (1.12)

The superalgebra $U_\alpha U(M|N; \mathbb{H})$ for even $N$ is equivalent to the quaternionic-valued superalgebra $OSp(N|M; \mathbb{H})$ [20, 27].
which form the $D(2, 1; \alpha)$ superalgebra (for details see for example [36, 37, 34])

$$\{Q^+_{\alpha\alpha A}, \ Q^+_{\beta\beta B}\} = 2 \left( \epsilon_{\alpha\beta} \epsilon_{AB} R_{ab} + \alpha \epsilon_{ab} \epsilon_{AB} J_{\alpha\beta} - (1 + \alpha) \epsilon_{ab} \epsilon_{\alpha\beta} T_{AB} \right),$$  \hspace{1cm} (2.4)

$$[R_{ab}, R_{cd}] = i (\epsilon_{ac} R_{bd} + \epsilon_{bd} R_{ac}) ,$$  \hspace{1cm} (2.5)

$$[J_{\alpha\beta}, J_{\gamma\delta}] = i (\epsilon_{\alpha\gamma} J_{\beta\delta} + \epsilon_{\beta\delta} J_{\alpha\gamma} ) ,$$  \hspace{1cm} (2.6)

$$[T_{AB}, T_{CD}] = i (\epsilon_{AC} T_{BD} + \epsilon_{BD} T_{AC}) ,$$  \hspace{1cm} (2.7)

$$[R_{ab}, Q^+_{\alpha\alpha A}] = -i \epsilon_{c(a} Q^+_{\beta\beta B)},$$  \hspace{1cm} (2.8)

$$[J_{\alpha\beta}, Q^+_{\beta\beta A}] = -i \epsilon_{\gamma(a} Q^+_{\alpha\alpha B)},$$  \hspace{1cm} (2.9)

$$[T_{AB}, Q^+_{\alpha\alpha C}] = -i \epsilon_{C(a} Q^+_{\alpha\alpha B)} ,$$  \hspace{1cm} (2.10)

with other commutators vanishing. All $D(2, 1; \alpha)$ generators are Hermitian, i.e. they satisfy the relations

$$\left( Q^+_{\alpha\alpha A} \right)^\dagger = \epsilon^{\alpha\beta} \epsilon^{AB} Q^+_{\beta\beta B} ,$$
$$\left( R_{ab} \right)^\dagger = R_{ab} ,$$
$$\left( J_{\alpha\beta} \right)^\dagger = \epsilon^{\alpha\gamma} \epsilon^{\beta\delta} J_{\gamma\delta} ,$$
$$\left( T_{AB} \right)^\dagger = \epsilon^{AC} \epsilon^{BD} T_{CD} .$$

Thus, the generators $R_{ab}$ form the $O(2, 1)$ algebra ($R_{11} = H$, $R_{22} = K$, $R_{12} = D$), whereas $J_{\alpha\beta}$ and $T_{AB}$ describe two $O(3)$ groups. Everywhere in this paper we take $\epsilon_{12} = \epsilon^{21} = 1$. The link between $O(3)$ and $O(2, 1)$ vectors and symmetric second-rank spinors is provided by corresponding $\sigma$-matrices: $J_{\alpha\beta} = J_{\alpha}^{(\sigma)}_{\beta}, \ R_{ab} = R_{\alpha}(\rho^\tau)^{\alpha}{}^{b}$ and $J_{\alpha\beta} = \epsilon_{\beta\gamma} J_{\alpha\gamma}, \ R_{ab} = \epsilon_{ac} R_{a}{}^{c}$. The fermionic supercharges $Q^+_{\alpha\alpha A}$ unify in one $O(2, 1)$ spinor the standard supercharges $Q^+_{\alpha\alpha A} = Q^+_{A\alpha A}$ and the generators of conformal supertranslations $S^+_{\alpha A} = Q^2_{2\alpha A}$.

Let us consider the graded Abelian enlargement of the $D(2, 1; \alpha)$ superalgebra. We add the following generators

$$G^{(3)}_- = (Q^-_{\alpha\alpha A}; A_{ab,\alpha\beta}, A_0) ,$$  \hspace{1cm} (2.13)

where three 3-vectors $A_{r,i} \equiv (P_i, B_i, F_i)$ are described in spinorial notation as $A_{ab,\alpha\beta}$. The set of generators (2.13) forms the Abelian subalgebra

$$\{Q^+_{\alpha\alpha A}; Q^-_{\beta\beta B}\} = [Q^-_{\alpha\alpha A}; A_{bc,\beta\gamma}] = [Q^-_{\alpha\alpha A}; A_0] = [A_{ab,\alpha\beta}, A_{c\delta,\gamma\delta}] = [A_{ab,\alpha\beta}, A_0] = 0 .$$  \hspace{1cm} (2.14)

The crossed (anti)commutators between $G^{(3)}_-$ and $G^{(3)}_+ = D(2, 1; \alpha)$ describe the following covariance relations

$$\{Q^+_{\alpha\alpha A}, Q^-_{\beta\beta B}\} = \beta \epsilon_{AB} A_{ab,\alpha\beta} + \gamma \epsilon_{ab} \epsilon_{\alpha\beta} \epsilon_{AB} A_0 ,$$  \hspace{1cm} (2.15)

$$[Q^+_{\alpha\alpha A}, A_{bc,\beta\gamma}] = -4i \epsilon_{a(b} \epsilon_{\alpha(\beta} Q^-_{c)\gamma\gamma A} ,$$
$$[Q^+_{\alpha\alpha A}, A_0] = i Q^-_{\alpha\alpha A} ,$$  \hspace{1cm} (2.16)

$$[R_{ab}, Q^-_{\alpha\alpha A}] = -i \epsilon_{c(a} Q^-_{\beta\beta B)} ,$$
$$[J_{\alpha\beta}, Q^-_{\beta\beta A}] = -i \epsilon_{\gamma(a} Q^-_{\alpha\alpha B)} ,$$
$$[T_{AB}, Q^-_{\alpha\alpha C}] = -i \epsilon_{C(a} Q^-_{\alpha\alpha B}) ,$$  \hspace{1cm} (2.17)

$$[R_{ab}, A_{cd,\alpha\beta}] = -i \epsilon_{c(a} A_{b)d,\alpha\beta} - i \epsilon_{d(a} A_{b)c,\alpha\beta} ,$$
$$[J_{\alpha\beta}, A_{\gamma\delta}] = -i \epsilon_{\gamma(a} A_{\alpha\delta\beta} - i \epsilon_{\delta(a} A_{\alpha\beta\delta}) ,$$  \hspace{1cm} (2.18)
\[\begin{align*}
[T_{AB}, A_{\alpha \beta}] &= 0, \\
[R_{ab}, A_0] &= [J_{\alpha \beta}, A_0] = [T_{AB}, A_0] = 0.
\end{align*}\] (2.19)

Reality properties of the generators (2.13) are described by
\[\begin{align*}
(Q_{\alpha \beta A}^+) \dagger &= \epsilon^{\alpha \beta} \epsilon^{A B} Q_{\alpha \beta B}^-, \\
(A_{\alpha \beta A}^+) \dagger &= \epsilon^{\alpha \gamma} \epsilon^{\beta \delta} A_{\alpha \beta \gamma \delta}^-, \\
(A_0^+) \dagger &= A_0.
\end{align*}\] (2.20)

The commutators (2.17)-(2.19) represent the standard form of \(SO(2, 1)\) and \(SO(3)\) representations, realized on corresponding spinor indices. On the right hand of the commutators (2.15) we left undetermined two numerical coefficients which remain not determined after all possible redefinitions of the generators \(A_{\alpha \beta A}\) and \(A_0\).

Let us check the consistency of the \(D(2; 1; \alpha)\) superalgebra (2.4)-(2.10) and the relations (2.14)-(2.19). Already the Jacobi identities \((Q^+, Q^+, Q^-)\) leads to unique values of the constants \(\beta\) and \(\gamma\) in (2.15)
\[\beta = \gamma = 1.\] (2.21)

Moreover, these identities fix the value \(\alpha = 1\) of the parameter \(\alpha\) characterizing the \(D(2, 1; \alpha)\) superalgebra. All other Jacobi identities are satisfied if \(\alpha = 1\) and the relations (2.21) are valid.

The \(D(2, 1; \alpha=1)\) superalgebra defines the \(OSp(4^*|2)\) superalgebra (see \[36, 37, 34\]). Thus, we obtain that the \(d=3\) SUSY GCA is given by the semidirect sum of the \(OSp(4^*|2)\) superalgebra \(G_+^{(3)}\) (see (2.3)) and the graded Abelian superalgebra (2.13).

We see that the superextension of \(d=3\) GCA selects uniquely out of one-parameter family of \(D(2, 1; \alpha)\) supersymmetries only one representative (2.2), which is endowed with quaternionic structure. It appears that in \(d=3\) such quaternionic structure has its origin in quaternionic structure of nonrelativistic \(d=3\) \(SU(2)\) spinors, due to the relation \(U(1; \mathbb{H}) \cong SU(2)\).

The \(N\)-extended \(d=3\) SUSY GCA will be derived in next Section as the result of the IW contraction of \(2N\)-extended relativistic \(D=3+1\) superconformal algebra \(SU(2, 2|2N)\) (see also [19]) and compared with the derivation by physical contraction \(c \to \infty\) given in [18]. The quaternionic structure of \(d=3\) SUSY GCA implies, that one should contract the \(D=4\) relativistic complex conformal superalgebra to the superalgebra with the quaternionic structure.

## 3 \(N\)-extension of \(d=3\) Galilean superconformal algebra and quaternionic structure

We established in Sect. 2 that the \(N=1\) supersymmetrization of semisimple part of \(d=3\) GCA is given by the semisimple superalgebra (2.2). In order to obtain the \(N\)-extended supersymmetrization we extend the formula (2.2) as follows:

\[G_+^{(3)} = U_\alpha U(2; N|\mathbb{H}) = OSp(4^*|2N)\] (3.1)

with the following basis
\[G_+^{(3)} = \{Q_{\alpha \beta A}^+, R_{ab}, J_{\alpha \beta}, T_{AB}^+\}\] (3.2)

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7 We can exchange the role of two \(SU(2)\) algebras described by the generators \(J_{\alpha \beta}\) and \(T_{AB}\) in (2.4) and consider other Abelian enlargement by the operators \(A_{\alpha \beta A}\) in (2.13). Then, we shall obtain the condition \(-(1 + \alpha) = 1\), i.e. \(\alpha = -2\). But \(D(2, 1; \alpha=1) \cong D(2, 1; \alpha=-2) \cong OSp(4^*|2)\) [36, 34] and we obtain the same superalgebra.

8 If we describe nonrelativistic spinors by a pair of complex variables (nonrelativistic counterpart of Weyl spinors) the unitarity condition leads rather to the group \(U(2)\) instead of \(SU(2)\).
where the $O^*(4)$ generators $R_{ab}$, $J_{\alpha\beta\mu}$ describe in spinorial notation the quantum-mechanical conformal algebra $O(2, 1)$ and $O(3)$ space rotations; the generators $T^\mu_B$ ($A, B = 1, \ldots, 2N$) describe the quaternionic internal algebra $U(N|\mathbb{H}) \cong USp(2N)$. The $8N$ supercharges $Q_{\alpha A}^+$ are complex and satisfy the quaternionic (or Majorana-symplectic) reality condition (see e.g. [26]). In order to obtain the SUSY GCA we can decompose the $2N$-extended relativistic conformal superalgebra in the following way

$$SU(2, 2|2N) = OSp(4^*|2N) \cong SU(2, 2|2N) \cong H^{(3)}_N \in K^{(3)}_N$$

(3.3)

where $H^{(3)}_N = OSp(4^*|2N)$ and the IW contraction is obtained if we rescale the supercoset generators

$$H^{(3)}_N = \hat{H}^{(3)}_N, \quad K^{(3)}_N = \lambda \hat{K}^{(3)}_N; \quad \hat{H}^{(3)}_N = G^{(3)}_+, \quad \hat{K}^{(3)}_N = G^{(3)}_-$$

(3.4)

and perform the limit $\lambda \to \infty$.

Let us recall that $D = 4$ $2N$-extended relativistic conformal algebra is obtained by adding to the superPoincaré Weyl supercharges $\tilde{Q}_A^\alpha$, $\bar{Q}_{\dot{A}}$ $= (Q_{\alpha A}^A)^+$ ($A = 1, \ldots, 2N$) satisfying the following basic SUSY relations

$$\{Q_{\alpha A}, \tilde{Q}_{\beta B}\} = 2(\sigma^\mu)_{\alpha\beta} P_\mu \delta^A_B, \quad \{Q_{\alpha A}, Q_{\beta B}\} = \{\tilde{Q}_{\dot{A}}^\alpha, \bar{Q}_{\dot{B}}^\beta\} = 0,$$

(3.5)

the $2N$ conformal Weyl supercharges $S_{\alpha A}$, $\bar{S}_{\dot{A}}$ $= (S_{\alpha A})^\dagger$, supersymetrizing the conformal momenta $K_\mu$

$$\{S_{\alpha A}, \bar{S}_{\dot{B}}^\beta\} = 2(\sigma^\mu)_{\alpha\beta} K_\mu \delta^A_B, \quad \{S_{\alpha A}, S_{\beta B}\} = \{\bar{S}_{\dot{A}}^\alpha, \bar{S}_{\dot{B}}^\beta\} = 0.$$  

(3.6)

Besides we have the relations

$$\{Q_{\alpha A}, S_{\beta B}\} = -\delta^A_B(\sigma^\mu)_{\alpha\beta} M^\mu - 4i \delta^\beta_\alpha T^A_B - 2i \delta^\beta_\alpha \bar{\delta}^A_B (D + i A),$$

$$\{\tilde{Q}_{\dot{A}}^\alpha, \bar{S}_{\dot{B}}^\beta\} = -\delta^B_A(\tilde{\sigma}^\mu)_{\dot{\alpha}\dot{\beta}} M^\mu - 4i \delta^{\dot{\alpha}}_{\dot{\beta}} T^A_B + 2i \delta^{\dot{\alpha}}_{\dot{\beta}} \bar{\delta}^A_B (D - i A),$$

$$\{Q_{\alpha A}, \bar{S}_{\dot{B}}^\beta\} = \{\tilde{Q}_{\dot{A}}^\alpha, S_{\beta B}\} = 0.$$  

(3.7)

Under the D=4 conformal generators $(M_{\mu\nu}, P_\mu, K_\mu, D)$ the supercharges $Q_{\alpha A}, S_{\alpha A}$ transform as follows

$$[M_{\mu\nu}, Q_{\alpha A}^A] = -\frac{i}{2} (\sigma_{\mu\nu})_{\alpha\beta} Q_{\beta B}^A, \quad [M_{\mu\nu}, \tilde{Q}_{\dot{A}}^\alpha] = \frac{i}{2} (\tilde{\sigma}^\mu_{\dot{\alpha}\dot{\beta}} M^\nu - \bar{\delta}^\beta_\alpha T^A_B - 2i \delta^\beta_\alpha \bar{\delta}^A_B (D + i A),$$

$$[M_{\mu\nu}, S_{\alpha A}] = -\frac{i}{2} (\sigma_{\mu\nu})_{\alpha\beta} S_{\beta B}^A, \quad [M_{\mu\nu}, \bar{S}_{\dot{A}}^\alpha] = \frac{i}{2} (\tilde{\sigma}^\mu_{\dot{\alpha}\dot{\beta}} S_{\dot{B}}^A + 2i \delta^{\dot{\alpha}}_{\dot{\beta}} \bar{\delta}^A_B (D - i A),$$

$$[P_\mu, Q_{\alpha A}^A] = [P_\mu, \tilde{Q}_{\dot{A}}^\alpha] = 0, \quad [P_\mu, S_{\alpha A}] = (\sigma_{\mu\alpha})_{\alpha\beta} \tilde{Q}_{\dot{A}}^\beta, \quad [P_\mu, \bar{S}_{\dot{A}}^\alpha] = -(\sigma_{\mu\dot{A}})_{\dot{\alpha}\dot{\beta}} Q_{\alpha A}^\beta,$$

$$[K_\mu, Q_{\alpha A}^A] = (\sigma_{\mu\alpha})_{\alpha\beta} \tilde{S}_{\dot{B}}^\dot{\beta} A, \quad [K_\mu, \tilde{Q}_{\dot{A}}^\alpha] = -(\sigma_{\mu\dot{A}})_{\dot{\alpha}\dot{\beta}} S_{\dot{B}}^\dot{\beta} A, \quad [K_\mu, S_{\alpha A}] = [K_\mu, \bar{S}_{\dot{A}}^\alpha] = 0,$$

$$[D, Q_{\alpha A}^A] = i \frac{1}{2} Q_{\alpha A}^A, \quad [D, \tilde{Q}_{\dot{A}}^\alpha] = i \frac{1}{2} \tilde{Q}_{\dot{A}}^\alpha, \quad [D, S_{\alpha A}] = -i \frac{1}{2} S_{\alpha A}, \quad [D, \bar{S}_{\dot{A}}^\alpha] = -i \frac{1}{2} \bar{S}_{\dot{A}}^\alpha.$$

(3.8)

The generators $T_B^A$ describe $SU(2N)$ algebra

$$[T^A_B, T^C_D] = i (\delta^A_D T^C_B - \delta^C_D T^A_B), \quad T^A_A = 0, \quad (T^A_B)^\dagger = -T^B_A$$

(3.12)
and transform supercharges as follows

\[ [T_B^A, Q_C^A] = \delta_B^C Q_A^A - \frac{1}{2N} \delta_B^C Q_A^C , \quad [T_B^A, \tilde{Q}_{\alpha C}] = -\delta_B^A \tilde{Q}_{\alpha B} + \frac{1}{2N} \delta_B^A \tilde{Q}_{\alpha C} , \]
\[ [T_B^A, S_{\alpha C}] = -\delta_B^C S_{\alpha B} + \frac{1}{2N} \delta_B^C S_{\alpha C} , \quad [T_B^A, S_C^\alpha] = \delta_B^C S_A^\alpha - \frac{1}{2N} \delta_B^C S_D^\alpha . \] (3.13)

The \( U(1) \) axial charge \( A = A^1 \) satisfies the relations

\[ [A, Q_A^A] = -2 \frac{N}{2N} Q_A^A , \quad [A, \tilde{Q}_{\alpha A}] = 2 \frac{N}{2N} \tilde{Q}_{\alpha A} , \quad [A, S_{\alpha A}] = 2 \frac{N}{2N} S_{\alpha A} , \quad [A, \tilde{S}^A_{\alpha}] = -2 \frac{N}{2N} \tilde{S}^A_{\alpha} , \] (3.14)

and commutes with all other bosonic generators forming \( O(4,2) \oplus SU(2N) \) algebra. We see from (3.14) that for \( N = 2 \) i.e. for \( SU(2,2|4) \) the axial charge \( A \) becomes a scalar central charge.

We rewrite the \( D=4 \) relativistic superconformal algebra in different fermionic Weyl basis 10

\[ Q_{\alpha A}^{\pm} = \frac{1}{\sqrt{2}} \left( Q_{\alpha A}^A \pm \epsilon_{\alpha \beta} \Omega^{AB} Q_{\beta B}^B \right) , \quad Q_{\alpha A}^{\pm} = \frac{1}{\sqrt{2}} \left( Q_{\alpha A}^A \pm \epsilon_{\alpha \beta} \Omega^{AB} Q_{\beta B}^B \right) , \quad (3.15) \]
\[ S^{\pm}_{\alpha A} = \frac{1}{\sqrt{2}} \left( S_{\alpha A} \pm \epsilon_{\alpha \beta} \Omega^{AB} S_{\beta B}^B \right) , \quad S^{\pm}_{\alpha A} = \frac{1}{\sqrt{2}} \left( S_{\alpha A} \pm \epsilon_{\alpha \beta} \Omega^{AB} S_{\beta B}^B \right) . \] (3.16)

where the real matrix \( \Omega = (\Omega^{AB}) \) \( (\Omega^{AB}) = (\Omega^{AB}) \) is a \( 2N \times 2N \) symplectic metric \( (\Omega^T = -\Omega) \) (see also eq. (1.11) in footnote 5)). The relations (3.15), (3.16) break Lorentz symmetry \( O(3,1) \) to \( O(3) \), and the internal symmetry \( U(2N) \) to \( U(N) \) (\( \text{Im} \) \( \cong \text{USp}(2N) \)).

The Weyl supercharges (3.15), (3.16) satisfy the subsidiary symplectic-Majorana conditions

\[ (Q_{\alpha A}^{\pm})^\dagger = \tilde{Q}_{\alpha A}^{\pm} = \pm \epsilon_{\alpha \beta} \Omega^{AB} Q_{\beta B}^B , \quad (S_{\alpha A}^{\pm})^\dagger = \tilde{S}_{\alpha A}^{\pm} = \mp \epsilon_{\alpha \beta} \Omega^{AB} S_{\beta B}^B . \] (3.17)

The subsidiary conditions (3.17) describing quaternionic structure permits to choose as independent fermionic charges Hermitian basis \( (Q_{\alpha A}^{\pm}, \tilde{Q}_{\alpha A}^{\pm}, S_{\beta B}^{\pm}, \tilde{S}_{\beta B}^{\pm}; A = 1, \ldots, 2N) \) or equivariantly, using Hermitian basis of fermionic charges,

\[ \{Q_{\alpha A}^{\pm}, \tilde{Q}_{\beta B}^{\pm}\} = \mp 2 \Omega^{AB} \epsilon_{\alpha \beta} P_0 , \quad \{Q_{\alpha A}^{\pm}, Q_{\beta B}^{\pm}\} = 2 \Omega^{AB} (\sigma_i)_{\alpha \beta} P_i , \] (3.18)
\[ \{S_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = \mp 2 \Omega^{AB} \epsilon_{\alpha \beta} K_0 , \quad \{S_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = -2 \Omega^{AB} (\sigma_i)_{\alpha \beta} K_i , \] (3.19)

where \( (\sigma_i)_{\alpha \beta} = (\sigma_i)_{\beta \alpha} = (\epsilon_{\alpha \gamma} (\sigma_i)_{\beta \gamma}) \) or equivalently, using Hermitian basis of fermionic charges,

\[ \{Q_{\alpha A}^{\pm}, \tilde{Q}_{\beta B}^{\pm}\} = 2 \delta_{\beta A} \delta_{\alpha B} P_0 , \quad \{Q_{\alpha A}^{\pm}, Q_{\beta B}^{\pm}\} = 2 \delta_{\beta A} (\sigma_i)_{\alpha \beta} P_i , \] (3.20)
\[ \{S_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = -2 \delta_{\beta A} \delta_{\alpha B} K_0 , \quad \{S_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = -2 \delta_{\beta A} (\sigma_i)_{\alpha \beta} K_i . \] (3.21)

From (3.7) one obtains the following nonzero anticommutators

\[ \{Q_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = -\delta_{B}^A \left[ \epsilon_{ijk} (\sigma_k)_{\alpha \beta} M_{ij} - 2i \epsilon_{\alpha \beta} D \right] + 2i \epsilon_{\alpha \beta} T^{+A}_B , \] (3.22)
\[ \{Q_{\alpha A}^{\pm}, S_{\beta B}^{\pm}\} = -2 \delta_{B}^A \left[ i(\sigma_i)_{\alpha \beta} M_{ij} + \epsilon_{\alpha \beta} A \right] + 2i \epsilon_{\alpha \beta} T^{-A}_B . \] (3.23)

where operators

\[ T^{\pm A}_B \equiv T^{A}_B \pm \Omega^{AC} T^{D}_C \Omega^{DB} \] (3.24)
satisfy the relations
\[ T^{+C}_A \Omega_{BC} = 2 T^C_{(A} \Omega_{B)C}, \quad T^{-C}_A \Omega_{BC} = 2 T^C_{[A} \Omega_{B]C}; \quad T^{+A}_B = \pm \Omega^{AC} T^{+D}_C \Omega_{DB}. \] (3.25)

Further one can also add the covariance relations (3.8)–(3.14) written for supercharges (3.15), (3.16).

The internal symmetry generators \( T_B^A \) do split in relations (3.24) as follows
\[ \tilde{h}^{(3)}_N = (T^{+A}_B) = USp(2N), \quad \tilde{k}^{(3)}_N = (T^{-A}_B) = \frac{SU(2N)}{USp(2N)} \], (3.26)
and provide an example of symmetric Riemannian space \((\tilde{h}^{(3)}_N, \tilde{k}^{(3)}_N)\) with the algebraic relations
\[ [\tilde{h}^{(3)}_N, \tilde{h}^{(3)}_N] \subset \tilde{h}^{(3)}_N, \quad [\tilde{h}^{(3)}_N, \tilde{k}^{(3)}_N] \subset \tilde{k}^{(3)}_N, \quad [\tilde{k}^{(3)}_N, \tilde{k}^{(3)}_N] \subset \tilde{h}^{(3)}_N. \] (3.27)

In order to obtain the Galilean conformal superalgebra one can introduce the physical rescaling of the relativistic supercharges \[ Q^{+A}_\alpha = c^{-1/2} Q^{+A}_\alpha, \quad Q^{+A}_\alpha = c^{1/2} Q^{-A}_\alpha, \quad S^{+A}_{\alpha A} = c^{1/2} S^{+A}_{\alpha A}, \quad S^{-A}_{\alpha A} = c^{3/2} S^{-A}_{\alpha A}. \] (3.28)
The physical rescaling of the bosonic generators of \( O(4, 2) \oplus USp(2k) \), where \( (P_\mu, M_\mu, D, K_\mu) \in O(4, 2) \) and \( T^{+A}_j \in U(2N) \), is the following
\[ P_0 = c^{-1} H, \quad P_i = P_i, \quad M_{ij} = J_{ij}, \quad M_{i0} = c B_i, \quad D = D, \quad K_0 = c K, \quad K_i = c^2 F_i, \quad A = c A_0, \quad T^{+A}_B = T^{+A}_B, \quad T^{-A}_B = c T^{-A}_B. \] (3.29)
As a result of the contraction \( c \to \infty \) we obtain in Weyl basis the \( N \)-extended SUSY GCA which was presented in real Majorana basis for supercharges in \[ [18]. \]

The physical rescaling (3.28)–(3.29) is however not unique because the relativistic superalgebra \( SU(2, 2|2N) \) is invariant under the following rescaling:
\[ P'_\mu = \lambda P_\mu, \quad K'_\mu = \lambda^{-1} K_\mu, \quad Q^{+A}_\alpha = \lambda^{1/2} Q^{+A}_\alpha, \quad S^{+A}_{\alpha A} = \lambda^{-1/2} S^{+A}_{\alpha A}. \] (3.30)
If we compose rescaling (3.30) with physical rescaling (3.28)–(3.29) we obtain
\[ Q^{+A}_\alpha = \left( \frac{\lambda}{c} \right)^{\frac{1}{2}} Q^{+A}_\alpha, \quad Q^{-A}_\alpha = (\lambda c)^{\frac{1}{2}} Q^{-A}_\alpha, \quad S^{+A}_{\alpha A} = \left( \frac{\lambda}{c} \right)^{\frac{1}{2}} S^{+A}_{\alpha A}, \quad S^{-A}_{\alpha A} = \left( \frac{\lambda}{c} \right)^{\frac{1}{2}} S^{-A}_{\alpha A}, \quad P'_0 = \frac{\lambda}{c} H, \quad P'_i = \lambda P_i, \quad M'_{ij} = J_{ij}, \quad M'_{i0} = c B_i, \quad D = D, \quad K'_0 = \frac{\lambda}{c} K, \quad K'_i = c \frac{\lambda}{c} F_i, \quad A' = c A_0, \quad T^{+A}_B = T^{+A}_B, \quad T^{-A}_B = c T^{-A}_B. \] (3.31)

If we put in (3.31) that \( \lambda = c \) one can check that we obtain rescaling (3.4) of the supercoset decomposition of \( SU(2, 2|2N) \), with invariant subalgebra \( OS(p (4|2N) \), spanned by generators \( Q^{+A}_\alpha, Q^{-A}_\alpha, S^{+A}_{\alpha A}, S^{-A}_{\alpha A}, (H, K, D) \in O(2, 1), J_{ij} \in O(3), T^{+A}_B \in USp(2N) \). As a result of the contraction \( \lambda = c \to \infty \) we obtain the following fermionic bilinear relations of the \( N \)-extended SUSY GCA, with \( 8N \) complex supercharges
\[ \{ Q^{+A}_\alpha, Q^{+B}_\beta \} = 2 \delta_{\alpha \beta} \delta^B_A H, \quad \{ S^{+A}_{\alpha A}, S^{+B}_{\beta B} \} = -2 \delta_{\alpha \beta} \delta^B_A K, \]
\[ \{ Q^{+A}_\alpha, S^{+B}_{\beta B} \} = -\delta^A_B \left[ \epsilon_{ijk} (\sigma_k)_{\alpha \beta} J_{ij} - 2 i (\epsilon_{\alpha \beta} D \right] + 2 i (\epsilon_{\alpha \beta} D, T^{+A}_B \}
\[ \{ Q^{+A}_\alpha, Q^{-B}_{\beta B} \} = 2 \delta^A_B (\sigma_i)_{\alpha \beta} P_i, \quad \{ S^{+A}_{\alpha A}, S^{-B}_{\beta B} \} = -2 \delta^B_A (\sigma_i)_{\alpha \beta} F_i, \]
\[ \{ Q^{+A}_\alpha, S^{-B}_{\beta B} \} = -2 \delta^A_B \left[ i (\sigma_i)_{\alpha \beta} B_i + 2 i (\epsilon_{\alpha \beta} A_0 \right] + 2 i (\epsilon_{\alpha \beta} T^{+A}_B \}, \]
\[ \{ Q^{+A}_\alpha, Q^{-B}_{\beta B} \} = 0, \quad \{ S^{+A}_{\alpha A}, S^{-B}_{\beta B} \} = 0, \quad \{ Q^{+A}_\alpha, S^{-B}_{\beta B} \} = 0. \]
The relations in mixed bosonic-fermionic sector are the following

\[ [H, S_{\alpha A}^\pm] = \tilde{Q}_{\alpha A}^{\pm \dagger}, \quad [K, Q_{\alpha A}^\pm] = \tilde{S}_{\alpha A}^{\pm \dagger}, \quad [D, Q_{\alpha A}^\pm] = \frac{i}{2} Q_{\alpha A}^\pm, \quad [D, S_{\alpha A}^\pm] = -\frac{i}{2} S_{\alpha A}^\pm, \]

\[ [J_{ij}, Q_{\alpha A}^\pm] = -\frac{i}{2} \epsilon_{ijk} (\sigma_k)_{\beta}^\dagger Q_{\beta A}^\pm, \quad [J_{ij}, S_{\alpha A}^\pm] = -\frac{i}{2} \epsilon_{ijk} (\sigma_k)_{\beta}^\dagger S_{\beta A}^\pm, \]

\[ [P_i, S_{\alpha A}^+] = - (\sigma_i)_{\alpha A}^\dagger Q_{\alpha A}^-, \quad [F_i, Q_{\alpha A}^+] = - (\sigma_i)_{\alpha A}^\dagger S_{\alpha A}^-, \]

\[ [B_i, Q_{\alpha A}^+] = \frac{i}{2} (\sigma_i)_{\alpha A}^\dagger Q_{\alpha A}^-, \quad [B_i, S_{\alpha A}^+] = \frac{i}{2} (\sigma_i)_{\alpha A}^\dagger S_{\alpha A}^-, \]

\[ [A_0, Q_{\alpha A}^+] = \frac{i}{2} (1 - \frac{2}{N}) Q_{\alpha A}^-, \quad [A_0, S_{\alpha A}^+] = \frac{i}{2} (1 - \frac{2}{N}) S_{\alpha A}^- \]

\[ \text{The covariance relations with respect to the generators } \hat{h}_N^{(3)} \text{ and } \hat{k}_N^{(3)} \text{ take the form} \]

\[ [T_{A}^+, Q_{\alpha A}^+] = (\mathcal{U}^A_B)^{CD}_{\alpha D} Q_{\alpha C}^+, \quad [T_{A}^+, S_{\alpha A}^+] = - (\mathcal{U}^A_B)^{CD}_{\alpha D} S_{\alpha C}^+, \]

\[ [T_{A}^-, Q_{\alpha A}^-] = (\mathcal{T}^A_B)^{CD}_{\alpha D} Q_{\alpha C}^-, \quad [T_{A}^-, S_{\alpha A}^-] = - (\mathcal{T}^A_B)^{CD}_{\alpha D} S_{\alpha C}^-, \]

\[ [T_{A}^-, Q_{\alpha A}^-] = 0, \quad [T_{A}^-, S_{\alpha A}^-] = 0, \]

where the $2N \times 2N$ matrix $(\mathcal{U}^A_B)$

\[ (\mathcal{U}^A_B)^{CD}_{\alpha D} = \delta_{D}^{A} \delta_{B}^{C} - \Omega^{AC} \Omega_{BD} \]

defines the representation of the $USp(2N)$ algebra and we define

\[ (\mathcal{T}^A_B)^{CD}_{\alpha D} = \delta_{D}^{A} \delta_{B}^{C} + \Omega^{AC} \Omega_{BD} - \frac{1}{N} \delta_{B}^{A} \delta_{D}^{C}. \]

In $N = 1$ case the contracted algebra \((3.32)\)-\((3.35)\) coincides with the superalgebra considered in Sect.2. We note that in $N = 1$ case, when $\Omega_{AB} = \epsilon_{AB}$, $A = 1, 2$, we have $T_{A}^+ \equiv 0$ and $(\mathcal{T}^A_B)^{CD}_{\alpha D} \equiv 0$ (see \((3.24)\) and \((3.37)\)).

From \((3.4)\) follows that the coset generators (see \((3.27)\)) are rescaled $\hat{h}_N^{(3)} \equiv \tilde{h}_N^{(3)}$, $\hat{k}_N^{(3)} \equiv \tilde{k}_N^{(3)}$ and in the limit $\lambda \to \infty$ we get

\[ [\tilde{h}_N^{(3)}, \tilde{h}_N^{(3)}] \subset \tilde{h}_N^{(3)}, \quad [\tilde{h}_N^{(3)}, \tilde{k}_N^{(3)}] \subset \tilde{k}_N^{(3)}, \quad [\tilde{k}_N^{(3)}, \tilde{k}_N^{(3)}] = 0. \]

The generators $\tilde{k}_N^{(3)}$ described in \((3.32)\) by generators $\mathbf{T}_{B}^-$ are Abelian and they describe a sort of internal complex momenta. One gets the following structure of Galilean conformal internal symmetry algebra

\[ \mathbf{T} = \tilde{h}_N^{(3)} \oplus \tilde{k}_N^{(3)}, \quad \tilde{h}_N^{(3)} = U(N)[\mathbb{H}] = USp(2N), \quad \tilde{k}_N^{(3)} = \hat{A}^{N(2N-1)} \text{ (Abelian)}, \]

i.e. general structure of $G^{(3)}$ (see \((1.8)\)) is realized by the following Galilean generators

\[ G_+^{(3)} = \left( \begin{array}{c} \tilde{Q}_{\alpha A}^{+, A}, \tilde{S}_{\alpha A}^{+, A}, \tilde{S}_{\alpha A}^{+, A} \end{array} \right) ; \quad \begin{array}{c} \mathbf{H}, \mathbf{K}, \mathbf{D} \end{array} ; \quad \begin{array}{c} \mathbf{R}_{ab} = O(2,1) \end{array} ; \quad \begin{array}{c} \mathbf{J}_{ij} \end{array} ; \quad \begin{array}{c} \mathbf{T}_{T}^{+ A} \end{array} \right) = OSp(4^*|2N), \]

\[ G_-^{(3)} = \left( \begin{array}{c} \tilde{Q}_{\alpha A}^{-, A}, \tilde{S}_{\alpha A}^{-, A}, \tilde{S}_{\alpha A}^{-, A} \end{array} \right) ; \quad \begin{array}{c} \mathbf{P}_{i, B} \end{array} ; \quad \begin{array}{c} \mathbf{B}_{i, A} \end{array} ; \quad \begin{array}{c} \mathbf{F}_{i} \end{array} ; \quad \begin{array}{c} \mathbf{A}_{0} \end{array} ; \quad \begin{array}{c} \mathbf{T}_{T}^{- A} \end{array} \right). \]
The \( N \)-extended SUSY GCA has a quaternionic structure. For \( N = 1 \) the compact internal symmetry is described by \( SU(2) \cong U(1; \mathbb{H}) \) and for arbitrary \( N \) by \( USp(2N) \cong U(N; \mathbb{H}) \). The Galilean conformal supercharges \( (Q^{\pm A}, Q^{\pm A}_S, S^{\pm}_a, S^{\pm A}_a) \) can be described as the \( N \) quadruplets of quaternionic supercharges. Indeed, the two-component Weyl spinor \( Z_\alpha = (z_1, z_2) \) can be described by a single quaternion \( q \) (see e.g. [26]) as follows

\[
\begin{align*}
  z_1 &= x_1 + iy_1, & z_2 &= x_2 + iy_2 & \leftrightarrow & & q^\mathbb{H} = x_1 + e_3 y_1 + e_2 (x_2 + e_3 y_2). 
\end{align*}
\]

We see that the \( N \)-extended \( d=3 \) SUSY GCA is generated by \( N \) quadruplets \( Q^{\pm M}, S^{\pm}_M \) of quaternionic supercharges, where the indices \( M \) span the quaternionic representation space of Galilean compact internal symmetry \( U(N; \mathbb{H}) \) group. It is a matter of quite tedious calculations (compare e.g. with [26], Sect. 6) to reexpress \( N \)-extended \( d=3 \) SUSY GCA in the quaternionic form.

4 \hspace{1em} \textit{N}-extended superconformal mechanics and their links with Galilean superconformal algebras for \( d = 1, 2, 4, 5 \)

4.1 \hspace{1em} \( d = 1 \)

The set \( G^{(1)}_+ \) of real generators is the following \( (a, b = 1, 2) \)

\[
G^{(1)}_+ = (Q^+_a; R_{ab}), \quad (Q^+_a)^\dagger = Q^+_a, \quad (R_{ab})^\dagger = R_{ab} = R_{ba}.
\]

which form the real \( OSp(1|2) \) superalgebra describing the graded symmetries of \( \mathcal{N} = 1 \) superconformal mechanics [30, 31]. The nonvanishing (anti)commutators are (2.5) and

\[
\{Q^+_a, Q^+_b\} = 2 R_{ab}, \quad [R_{ab}, Q^+_c] = -i \epsilon_{c(a} Q^+_b). \tag{4.2}
\]

The graded Abelian enlargement of the \( OSp(1|2) \) superalgebra is given by generators

\[
G^{(1)}_- = (Q^-_a; A_{ab}), \quad (Q^-_a)^\dagger = Q^-_a, \quad (A_{ab})^\dagger = A_{ab} = A_{ba}, \tag{4.3}
\]

which form the Abelian subalgebra

\[
\{Q^-_a, Q^-_b\} = [Q^-_a, A_{bc}] = [A_{ab}, A_{cd}] = 0 \tag{4.4}
\]

and transform as the \( OSp(1|2) \) representation as follows

\[
\{Q^+_a, Q^-_b\} = \beta A_{ab}, \tag{4.5}
\]

\[
[Q^+_a, A_{bc}] = 2i \epsilon_{a(b} Q^-_c), \tag{4.6}
\]

\[
[R_{ab}, Q^-_c] = -i \epsilon_{c(a} Q^-_b), \quad [R_{ab}, A_{cd}] = -i \epsilon_{c(a} A_{b)d} - i \epsilon_{d(a} A_{b)c} \tag{4.7}
\]

where we postulated in (4.5) that one real parameter \( \beta \) is not determined. It can be seen however that the Jacobi identities \( (Q^+, Q^+, Q^-) \) fix the value \( \beta = 1 \) of the constant \( \beta \). For such choice of \( \beta \) all other Jacobi identities are satisfied.
The finite-dimensional $d=1$ relativistic conformal superalgebra with $D=2$ conformal algebra $O(2,1) \oplus O(2,1) = Sp(2) \oplus Sp(2)$ is provided by the sum of the $d=0$ Galilean conformal superalgebras $OSp(1|2) \oplus OSp(1|2)$. The $d=1$ simple SUSY GCA can be obtained by IW contraction of supercoset decomposition

$$OSp(1|2) \oplus OSp(1|2) = OSp(1|2)_D \equiv \frac{OSp(1|2)_L \oplus OSp(1|2)_R}{OSp(1|2)_D}, \quad (4.8)$$

where the generators $G^{(1)}_J = OSp(1|2)_D$ are obtained by taking the diagonal sums $\frac{1}{\sqrt{2}}(\hat{g}_L + \hat{g}_R) = \hat{g}_D$ of left and right generators. The contraction of rescaled difference $\frac{1}{\sqrt{2}}(\hat{g}_L - \hat{g}_R)$ provides 3 generators $A_{ab}$.

The family of extended $d=1$ SUSY GCA is parametrized by a pair of numbers $(N, M)$ and can be defined as the suitable IW contractions of the supercoset decompositions of the superalgebras $OSp(N|2)_L \oplus OSp(M|2)_R$.

4.2 $d=2$

We start from the following set $G^{(2)}_J$ of the generators

$$G^{(2)}_J = (Q^+_a, \bar{Q}^+_a; R_{ab}, J, C), \quad (4.9)$$

which define the $SU(1,1|1) \cong OSp(2|2)$ superalgebra with one central charge (see e.e. [38, 39, 29, 30])

$$\{Q^+_a, \bar{Q}^+_a\} = 2(R_{ab} + \epsilon_{ab}J + \epsilon_{ab}C), \quad (4.10)$$

$$[R_{ab}, Q^+_a] = -i \epsilon_{c(a} Q^+_b), \quad [R_{ab}, \bar{Q}^+_a] = -i \epsilon_{c(a} \bar{Q}^+_b), \quad (4.11)$$

$$[J, Q^+_a] = -\frac{i}{2} Q^+_a, \quad [J, \bar{Q}^+_a] = \frac{i}{2} \bar{Q}^+_a, \quad (4.12)$$

where the commutators (2.5) and other (anti)commutators vanish. Hermiticity properties of the $SU(1,1|1)$ generators are the following

$$(Q^+_a)^\dagger = Q^+_a, \quad (R_{ab})^\dagger = R_{ab} = R_{ba}, \quad (J)^\dagger = -J, \quad (C)^\dagger = -C. \quad (4.13)$$

The generators $R_{ab}$ form $SO(1,2)$ algebra, whereas $J$ is the $O(2)$ generator of $d=2$ space rotations. The generator $C$ provides the central charge. Here we use the realization of the $OSp(2|2)$ algebra as in [29] with fermionic supercharges being complex $O(2)$ spinors.

We propose now the enlargement of the $SU(1,1|1)$ superalgebra defining $d=2$ SUSY GCA. We add the generators

$$G^{(2)}_J = (Q^-_a, \bar{Q}^-_a; A_{ab}, \bar{A}_{ab}), \quad (Q^-_a)^\dagger = \bar{Q}^-_a, \quad (A_{ab})^\dagger = \bar{A}_{ab}, \quad (\bar{A}_{ab})^\dagger = A_{ba}, \quad (4.14)$$

which form graded Abelian subalgebra

$$\{Q^-_a, \bar{Q}^-_b\} = \{Q^-_a, Q^-_b\} = [Q^-_a, A_{bc}] = [Q^-_a, \bar{A}_{bc}] = [A_{ab}, A_{cd}] = [A_{ab}, \bar{A}_{cd}] = 0 \quad (4.15)$$

and transform as the following $SU(1,1|1)$ representations (see [17])

$$\{Q^+_a, Q^-_b\} = 2\beta A_{ab}, \quad \{Q^+_a, \bar{Q}^-_b\} = 2\beta \bar{A}_{ab}, \quad (4.16)$$

$$[Q^+_a, \bar{A}_{bc}] = -2i \epsilon_{a(b} \bar{Q}^-_{c)}, \quad [\bar{Q}^+_a, A_{bc}] = -2i \epsilon_{a(b} Q^-_{c)}, \quad (4.17)$$
\[
\begin{align*}
[C, Q_a] &= -i \gamma Q_a^\alpha, \quad [C, Q_a^\alpha] = i \gamma Q_a^\alpha, \quad [C, A_{ab}] = -i \gamma A_{ab}, \quad [C, \bar{A}_{ab}] = i \gamma \bar{A}_{ab}, \\
[T_{ab}, Q_c] &= -i \epsilon_{(c} Q_{b)}, \quad [T_{ab}, \bar{Q}_c] = -i \epsilon_{(c} \bar{Q}_{b)}, \\
[T_{ab}, A_{cd}] &= -i \epsilon_{c(a} A_{b)d} - i \epsilon_{d(a} A_{b)c}, \quad [T_{ab}, \bar{A}_{cd}] = -i \epsilon_{c(a} \bar{A}_{b)d} - i \epsilon_{d(a} \bar{A}_{b)c}, \\
[J, Q_a] &= \frac{i}{2} Q_a, \quad [J, \bar{Q}_a] = \frac{i}{2} \bar{Q}_a, \quad [J, A_{ab}] = -i A_{ab}, \quad [J, \bar{A}_{ab}] = i \bar{A}_{ab}.
\end{align*}
\]

We introduced in (4.16), (4.18) two real parameters \(\beta\) and \(\gamma\). If we check the consistency of the \(SU(1,1)\) superalgebra (4.10) - (1.12) with the relations (4.16) - (2.21) one can show that only the Jacobi identities \((Q^+, Q^+, Q^-)\) leads to a complete fixing of the constants \(\beta\) and \(\gamma\) in (4.16), (4.18), namely \(\beta = \gamma = 1\). All other Jacobi identities are then satisfied.

For consistency it is important the presence of \(C\) in the superalgebra \(G^{(2)}_+\). This generator is the central charge in the superalgebra \(G^{(2)}_+\), but in the full superalgebra \(G^{(2)}\) it produces the \(U(1)\) transformations acting on \(G^{(2)}_+\) sector (see (1.18)).

For \(N\)-extended SUSY GCA as basic subsuperalgebra \(G^{(2)}_+\) we take the central extension of the \(SU(1,1\mid N)\) superalgebra,

\[
G^{(2)}_+ = SU(1,1\mid N) \oplus U(1),
\]

which can be introduced for any \(N \geq 2\) (see [10]). Bosonic subalgebra is \(SU(1,1) \oplus U(N) \oplus U(1)\); the \(U(1)\) factor in the decomposition \(U(N) = SU(N) \oplus U(1)\) acts nontrivially on fermionic charges, except if \(N = 2\) case when its describes a central charge [29].

In order to obtain \(N\)-extended \(d=2\) SUSY GCA we introduce supercoset decomposition of \(D=1+2\) relativistic conformal superalgebra \(OSp(4\mid 2N)\)

\[
OSp(4\mid 2N) = \left( SU(1,1\mid N) \oplus U(1) \right) \oplus \frac{OSp(4\mid 2N)}{SU(1,1\mid N) \oplus U(1)} = H^{(2)}_N \oplus K^{(2)}_N.
\]

The factor \(K^{(2)}_N\) contains product of two bosonic factor \(\frac{Sp(4)}{SU(1,1) \oplus U(1)} \cdot O(2N)\). After IW contraction \(H^{(2)}_N \subset K^{(2)}_N \rightarrow G^{(2)}_+ \subset G^{(2)}\) the first factor \(\frac{Sp(4)}{SU(1,1) \oplus U(1)}\) produces the sets of the generators \((P_i, B_i, F_i)\), \(i = 1, 2\) and from second factor \(\frac{O(2N)}{U(N)}\) one gets internal bosonic Abelian charges of \(N\)-extended \(d=2\) SUSY GCA.

### 4.3 \(d = 4\)

In this case, the simple relativistic \(D=5\) superconformal algebra is described by the exceptional superalgebra \(F(4)\) [40], or more precisely, by its real form given by the real superalgebra \(F(4) = F(4;2)\) [36], which contains the bosonic subalgebra \(O(5,2) \oplus O(3)\). We stress that this choice is different from other real superalgebra \(F(4) = F(4;0) \supset O(2,1) \oplus O(7)\) which has been used in \(N=8\) superconformal mechanics [31 [41]. The \(N>1 D=5\) superconformal algebras can be obtained (see [32 [33]) only by the dimensional reduction of \(D=6\) superconformal algebras, discussed below in Sect. 4.4.

The supersymmetrization \(G^{(4)}_+\) of the bosonic semisimple algebra \(O(2,1) \oplus O(4)\) leads as in \(d=3\) case to the superalgebra \(G^{(3)}_+ = OSp(4\mid 2)\) defined in (2.4)-(2.8). If we introduce supercoset decomposition

\[
F(4;2) = OSp(4\mid 2) \oplus \frac{F(4;2)}{OSp(4\mid 2)} = H^{(4)}_N \oplus K^{(4)}_N
\]

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and perform the IW contraction $H_N^{(4)} \in K_N^{(4)} \to G_+^{(4)} \in G^{(4)}$ we shall obtain $d=4$ SUSY GCA. The supercoset $K_N^{(4)}$ becomes graded Abelian superalgebra $G_+^{(4)}$ with 8 fermionic graded Abelian supercharges $Q_{\alpha A}$ and 15 bosonic Abelian ones, obtained from the contraction of $O(5,2)@O(3)/O(2,1)@O(4)$, which include $d=4$ generators ($P_i, B_i, F_i$).

### 4.4 $d=5$

In order to get $d=5$ SUSY GCA we can use again the method of IW contraction. Firstly let us introduce the D=6 superconformal algebra, which should contains as a factor the D=6 conformal algebra $O(6,2) \cong U_a(4; \mathbb{H})$. We arrive at the following quaternionic $N$-extended D=6 superconformal algebra [40, 44, 26]

$$U_a U(4|N; \mathbb{H}) \cong OSp(8^*|2N)$$

(4.25)

with 16N real supercharges and bosonic sector $O(6,2) \oplus OSp(2N)$.

In order to define the superalgebra $G_+^{(5)}$ one should supersymmetrize the bosonic sector $g^{(5)} = O(2,1) \oplus O(5)$, where $O(2,1) \cong Sp(2) \cong SU(1,1)$ and $O(5) \cong U(2; \mathbb{H}) \cong OSp(4)$. In supersymmetrization procedure we should therefore embed $O(2,1)$ algebra into an algebra with quaternionic structure. Using $U_a(2; \mathbb{H}) = O(2,1) \oplus O(3)$ we arrive at the following minimal superalgebra $G_+^{(5)}$

$$G_+^{(5)} = U_a U(2|2; \mathbb{H}) \cong OSp(4^*|4)$$

(4.26)

with 16 real supercharges and bosonic subalgebra $U_a(2; \mathbb{H}) \oplus U(2; \mathbb{H}) \cong O(3) \oplus O(2,1) \oplus O(5)$.

The $N=1$ $d=5$ SUSY GCA is obtained by IW contraction of the following coset decomposition of $D=6$ $N=2$ relativistic conformal superalgebra

$$OSp(8^*|4) = OSp(4^*|4) \oplus \frac{OSp(8^*|4)}{OSp(4^*|4)} = H^{(5)} \in K^{(5)}.$$ 

(4.27)

After IW contraction $H^{(5)} \in K^{(5)} \to G_+^{(5)} \in G^{(5)}$ the supercoset $K^{(5)}$ becomes graded Abelian superalgebra $G_+^{(5)}$ with 16 fermionic graded Abelian charges and 22 bosonic Abelian ones which include $d=5$ generators ($P_i, B_i, F_i$) obtained from the contraction of $O(6,2)/O(2,1)@O(3)$.

For $N>1$ one should consider the contraction of $D=6$ superconformal algebra [41,25] with even $N = 2n$. The extended $d=5$ superalgebra $G_+^{(5)}$ will take a form

$$G_+^{(5)} = U_a U(2|2n; \mathbb{H}) \cong OSp(4^*|4n)$$

(4.28)

and the $N$-extended $d=5$ SUSY GCA is given by the IW contraction of the following supercoset

$$OSp(8^*|4n) = OSp(4^*|4n) \oplus \frac{OSp(8^*|4n)}{OSp(4^*|4n)} = H_N^{(5)} \in K_N^{(5)}.$$ 

(4.29)

The graded superalgebra $G_+^{(5)}$ obtained from $K_N^{(5)}$ contains $16n$ fermionic anticommuting charges and 22 bosonic Abelian charges. We see that the number of bosonic generators in the subalgebra $G_-^{(5)}$ does not depend on $n$. 

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5 Final remarks

The aim of this paper is the description of explicit algebraic structures of SUSY GCA in $1 \leq d \leq 5$ space dimensions. In the dimensions $d = 1, 2, 3, 5$ for $D = d + 1$ space-time do exist $N$-extended relativistic conformal superalgebras with arbitrary $N$ and we present also in these dimensions the $N$-extended SUSY GCA; for $d = 4$ exist only unique relativistic $D = 5$ conformal superalgebra \[40, 36\] and corresponding $d = 4$ SUSY GCA. As it was discussed in detail for $d = 3$ in Sect. 2, the SUSY GCA can be also obtained as an enlargement of semisimple superalgebra $G_{+}^{(d)}$ which supersymmetrizes the bosonic semisimple part $h^{(d)}$ of full SUSY GCA. We observe that our choices of simple (nonextended) superalgebras $G_{+}^{(d)}$ are linked with the superalgebras describing the $N$-extended supersymmetries in the models of superconformal mechanics. In particular,

$$G_{+}^{(d)} = \begin{cases} 
OSp(1|2) & \text{for } d = 1 \\
SU(1, 1|1) \oplus U(1) & \text{for } d = 2 \\
OSp(4^*|2) & \text{for } d = 3 \text{ and } d = 4 \\
OSp(4^*|4) & \text{for } d = 5 
\end{cases} \quad \cong \begin{cases} 
N = 1, \\
N = 2, \\
N = 4, \\
N = 8. 
\end{cases} \quad (5.1)$$

Note that in cases $d = 3$ and $d = 4$ the superalgebras $G_{+}^{(3)}$ and $G_{+}^{(4)}$ are the same, with the bosonic sector of $OSp(4^*|2)$ equal to $h^{(4)} = O(3) \oplus h^{(3)}$, i.e. we see that the $d = 3$ internal symmetry sector $O(3)$ is incorporated in $d = 4$ into the space symmetry sector, $O(4) = O(3) \oplus O(3)$.

An important task is to apply the present results for the models with a physical interpretation. We mention here the following possibilities:

i) In \[45\] we introduced the fundamental spinorial realization of the subalgebra $h^{(3)} = O(2, 1) \oplus O(3)$ of $d = 3$ GCA as introducing nonrelativistic counterpart of $D = 4$ relativistic twistors. Analogously, the superalgebras $G_{+}^{(3)}$ can be used as the ones which define by its fundamental graded representations the nonrelativistic counterpart of $D = 4$ relativistic supertwistors \[46\].

ii) One can extend the coset techniques providing in \[13\] the geometric $\sigma$-models of GCA-invariant classical mechanics to the supersymmetric $\sigma$-models invariant under SUSY GCA. In particular by keeping on SUSY Galilean conformal group manifold the time $t$ and space coordinates $x_i$ as independent parameters we can promote remaining SUSY GCA parameters to the $D = (d+1)$-dimensional nonrelativistic Goldstone fields. Using for $d = 3$ scalar four-forms as action densities \[47\] one can arrive at the nonrelativistic field-theoretical models invariant under SUSY GCA.

iii) In the known nonrelativistic versions of AdS/CFT correspondence \[48, 49\] the Schrödinger (super)algebras \[1\] are used as describing nonrelativistic (SUSY) CFT. It is interesting to look for other nonrelativistic limit of AdS/CFT, with contracted AdS symmetries (see e.g. \[52\]) corresponding to the Galilean nonrelativistic CFT with GC symmetries. For $D = 1 + 1$ ($d = 1$) the relativistic CA as well as its nonrelativistic GCA limit are infinite-dimensional \[53, 10\]. For $d \geq 2$ the relativistic CA is finite-dimensional but in nonrelativistic theory there are there are the infinite-dimensional versions of GCA describing the sets of conformal isometries of Galilean space-time \[11, 14, 16\]. In such a context it is interesting to observe the $D = 3$ correspondence \[15\] between the infinite-dimensional nonrelativistic conformal isometries and Bondi-Metzner-Sachs group \[54\] which describes asymptotic isometries of flat Minkowski space at null infinity. We

\[11\] For the description of superSchrödinger symmetry see \[50, 51\].
point out that it is interesting to consider the supersymmetrized versions of all these conformal symmetry schemes.

iv) In this paper we considered the rescaling \( \vec{x}' = \vec{x}, \ x_0 = c t \) of space and time coordinates before applying the \( c \to \infty \) contraction limit. Such redefinition introducing nonrelativistic time \( t \) is natural and well justified if we consider the classical mechanics models with distinguished role of time as the evolution parameter. If we wish to study the nonrelativistic contraction of the \( p \)-brane dynamics in \( D = d + 1 \) space-time we should split the total space-time \( (x_0, x_1, ..., x_d) \) into the \( p + 1 \) coordinates \( (x_0, x_1, ..., x_p) \) describing the worldvolume of \( p \)-brane (inner “generalized time” manifold) and the remaining outer coordinates transversal to \( p \)-brane. If we rescale differently the “inner” and “outer” coordinates of (super)\( p \)-brane one obtains in the contraction limit more general nonrelativistic (super)symmetries [20, 21, 55, 19] described by the corresponding so-called semi-Galilean (super)algebras. We can add that semi-Galilean CA are useful as well if we wish to generalize the BMS/GCA correspondence to \( D > 3 \) [15].

Many issues presented above are now under our consideration.

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