Determination of Source’s Location of a Dipole Field using Euler Relation

Nuraddeen Usman¹, ², a*, Khiruddin Abdullah¹, and Mohd Nawawi¹

¹ School of Physics, Universiti Sains Malaysia, 11800 Pulau Pinang
² Umaru Musa Yar’adua University P.M.B 2218, Katsina, Nigeria

a nu14_phy055@student.usm.my.

Abstract: Magnetic materials such as rocks and other geological structures can be treated as an assortment of magnetic dipoles. This concept can be used to simulate magnetic response of a body if its parameters are known. Likewise, source location of a concealed body can be retrieved if its field response is acquired and adequate magnetization contrast with the surrounding material exists. In this paper, a magnetic response of dipole source is simulated from the assumed model parameters in order to test a technique for estimating source location. The technique is based on the assumption that the structural indices of some simple sources are related to the geology of the solid Earth. The effect of inclination on the magnetic field was studied. The magnetic field depends on magnetization contrast and inclination of the location, this corroborate that the simulation carried out is valid. The simulated field of the dipole was processed and inversed using a new technique which is based on Euler relation. The result indicated that the technique performed very well in the estimate of horizontal position and depth of the dipole source despite the effect of inclination of the field. The technique is fast means of source’s location estimate of concealed magnetized body. Filtering is recommended for the real field data because of the presence of so many sources and the Earth’s magnetic field.

1. Introduction

Geophysical techniques have been applied to investigate the subsurface of the earth in order to explore geological structures of economic interest (in most cases) in areas of environmental studies [1,2], hydrology, hydrocarbons, geochemical [3], engineering [4], geothermal studies [5], geo-hazard assessment, and solid minerals [6]. With the aid of techniques used for inversion, it is possible to determine the horizontal and vertical positions of concealed metallic objects in the near vicinity of the earth’s surface in addition to the delineation of deep-seated structures [7, 8, 9, 10, 11].

The pattern of magnetic anomaly depends on its position on the earth surface. The same structure placed at different geographical locations would give different anomaly’s shape because of the variation in magnetic latitude. The dipolar nature of the magnetic field causes distortion in the anomaly’s shape and as a result of this effect, error will be introduced to the data and thereby affecting the estimate of the anomaly’s location [12]. Reduction to the pole (RTP) and equator (RTE) are the traditional techniques to correct the effect of inclination. While the use of RTP is recommended to be applied on the data prior to the application of Euler deconvolution [13], other researchers [14] are of the opinion that it should not be applied. In the present work, the main objective is to estimate source’s location of a dipole field using fully automated Euler relation and to determine the effect of inclination on the technique. The significance of this research is that the unknown parameters in the Euler deconvolution
can be accurately estimated simultaneously. Moreover, the effect of inclination on the automated technique can be studied using simulated data of a dipole model. Modeling is useful in both testing of new technique, survey design and educational setting. In this case, the possibility of applying RTP or any other related technique prior to the deconvolution will be deduced.

2. Methodology
Dipole model has been frequently used in geophysical literatures to test filters, performance assessment of depth estimation methods and source parameter imaging. A dipole model can be interpreted geologically as compact object such as material buried in a ferro-metallic container/drum, boulder, void, cavity, buried metallic materials, iron piling and ancient features. A magnetic response due to a point dipole can be modeled in 3D Cartesian coordinate using the elements given as [15]:

\[ B_x = K \frac{(3Dr_x - r^2)}{r^5} \]  
\[ B_y = K \frac{(3Dr_y - r^2m)}{r^5} \]  
\[ B_z = K \frac{(3Dr_z - r^2n)}{r^5} \]  

In Equation (1, 2 and 3), Bx, By and Bz are the elements of the magnetic field due to a point dipole source, K is the dipole moment. The vector in the direction of point dipole towards the observation point (r) is given by the relation (Equation 4)

\[ r = \left( r_x^2 + r_y^2 + r_z^2 \right)^{1/2} \]  

r_x, r_y and r_z are the components of r in x, y and z directions respectively. The quantities that represent the magnetization directions (l, m, n and D) are defined as:

\[ l = \cos(a) \cos(b) \]  
\[ m = \cos(a) \sin(b) \]  
\[ n = \sin(a) \]  
\[ D = lr_x + mr_y + nr_z \]

where a and b are the inclination and declination in the direction of magnetization respectively. The magnetic declination (d) is the angle between the horizontal component and geographic north. This implies that the magnetic north is along the horizontal component (from the definition of declination).

The total magnetic anomaly due to a point dipole (ΔT) is related to the quantities representing the direction of geomagnetic field as (Equation 5):

\[ \Delta T = LB_x + MB_y + NB_z \]  
\[ L = \cos(A) \cos(B) \]  
\[ M = \cos(A) \sin(B) \]  
\[ N = \sin(b) \]

In the above equations, A and B are the inclination and declination of the geomagnetic field respectively. In this case, B is the angle between horizontal component and geomagnetic north. The geographic north is along x-axis. To simplify the derivation, B is set to be equal to zero (B = 0) and it is assumed that the magnetization is formed by induction. This implies that a = A and b = B.

Substitute Equation 6, 7 and 8 into Equation 5 to obtain Equation 9. Equation 9 is used to simulate the field used in this research.
\[
\Delta T = \cos A \cos B \cdot K \left( \frac{3Dr_x - r^2 l}{r^5} \right) + \cos (A) \sin (B) \cdot \frac{K}{r^5} \left( \frac{3Dr_y - r^2 m}{r^5} \right) + \sin A \cdot K \left( \frac{3Dr_z - r^2 n}{r^5} \right)
\]

\[
\Delta T = \frac{K}{r^5} \left( 3Dr_x l - r^2 l^2 \right)
\]

\[
\Delta T = \frac{K}{r^5} \left( 3Dr_x l + 3Dr_y m + 3Dr_z n - r^2 l^2 - r^2 m^2 - r^2 n^2 \right)
\]

But \( D = l_r + m_r + n_r \)

\[
\Delta T = \frac{K}{r^5} \left( 3D^2 - r^2 \right)
\]

(9)

And \( l^2 + m^2 + n^2 = 1 \) (trigonometry)

\[
2.1 \quad \text{Theory of the method}
\]

Euler deconvolution method is based on Euler’s homogeneity relationship [16] it was first initiated to solve 2D magnetic field by [13]. Euler deconvolution equation is normally used in order to find the source location \((x_0, y_0, z_0)\). Equation (10) forms the basis for the methodology used in this research and it is given as

\[
(x - x_0) \frac{\partial F}{\partial x} + (y - y_0) \frac{\partial F}{\partial y} + (z - z_0) \frac{\partial F}{\partial z} = N (B - F)
\]

(10)

where \( x, y \) and \( z \) are the observation point coordinates; \( x_0, y_0, \) and \( z_0 \) are the source locations; \( \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \) are the potential derivatives; \( N \) is the structural index; \( B \) is the background of field \( F \) [13].

2.2 Multiple Linear Regression

The Multiple Linear Regression (MLR) method has been extensively applied in statistics to establish relationships among multiple variables (more than two) by fitting a straight line to the observed data. MLR model [17] can be expressed in Equation 11:

\[
Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + \epsilon_i
\]

(11)

where \( \beta_0 \) is the intercept of \( Y_i; \beta_1, \beta_2 \) and \( \beta_k \) denote the slope of regression lines for each variable respectively. \( \epsilon_i \) is the regression error term and \( Y_i \) is the dependent variable. To apply the MLR on 3D Euler deconvolution technique, Equation (10) can be written in the following form (Equation 12) as MLR method (Equation 12): 

\[
x \frac{\partial F}{\partial x} + y \frac{\partial F}{\partial y} + z \frac{\partial F}{\partial z} = NB + x_0 \frac{\partial F}{\partial x} + y_0 \frac{\partial F}{\partial y} + z_0 \frac{\partial F}{\partial z} - NF + \epsilon_i
\]

(12)

By comparing Equation 11 and Equation 12, it could be observed that:

\[
Y_i = x \frac{\partial F}{\partial x} + y \frac{\partial F}{\partial y} + z \frac{\partial F}{\partial z}; \beta_0 = NB; \beta_1 = x_0; \beta_2 = y_0; \beta_3 = z_0; \beta_4 = N; X_1 = \frac{\partial F}{\partial x}; X_2 = \frac{\partial F}{\partial y}; X_3 = \frac{\partial F}{\partial z} \text{ and } X_4 = -F.
\]

All \( \beta \)s (\( \beta_0, \beta_1, \beta_2, \beta_3, \beta_4 \)) are the coefficients (unknown) that need to be solved and all \( X_i \) values are the independent variables, in which the values are known from the data (input). Assuming that
Equation 3 is linear, the equation could solve 5 unknowns which are horizontal positions \((x_0, y_0)\), depth \((z_0)\), SI (N) and background (B).

2.3 Dipole model
The analysis of the model was carried out using 2D and 3D forms. In 2D form, the analysis is qualitative whereas in 3D form the analysis is quantitative.

2.3.1. 2D analysis
This model was simulated with the assumption that the surrounding is sandstone and the target is granite. The susceptibility contrast was 30 and magnetization was 0.00126 A/m. It was located at 12.5 km in both x and y directions and the depth was 1 km. The radius of the cylinder was 0.4 km. It is designed to study the effect of inclination by analyzing the data obtained in profile and map forms. The profile passes directly at the centre (12.5 km along y-direction) of the dipole in x direction. Using the mentioned parameters, the model is simulated using 7 inclinations \((I = 0°, 15°, 30°, 45°, 60°, 75°, 90°)\) and a fixed declination \((D = 0)\). In each case, the data along the mentioned profile is taken and plotted.

2.3.2. 3D analysis
This model is designed to simulate magnetic response of a granite boulder buried in sandstone environment. The magnetization contrast was 1 A/m. The depth to the centre and radius of the boulder were 1 and 0.5 km respectively. The horizontal coordinates in positive x directions was 12.4 km while that of y was 12.5 km. The anomalous magnetic field response using different inclinations \((I = 0°, 15°, 30°, 45°, 60°, 75° and 90°)\) have been computed. The data was inverted using the window size of 9×9 grid points. The maximum distance to accept from the centre of convolution window and SI deviation were 9 units and 0.1 respectively. The maximum accepted regression error was 2%. The derivatives of the field (in x, y and z directions) were computed. The computed derivatives were used as an input to inverse the data using Equation 14. The estimated parameters were subjected to filtering and the final solutions are obtained. The mean and standard deviation of each of the parameter of the dipole source were tabulated. The source location was plotted and superimposed on analytic signal (AS).

3. Result and Discussion
The results obtained from the inversion of the simulated data is presented in both profile and map analysis.

3.1 2D Analysis
The anomalous field simulated using different inclinations are given in Figure 1. It could be observed that at 90°, the field is symmetrical directly above the position of the target \((x = 12.4km)\). At this inclination, the field is positive (similar to the sign of magnetization). When the inclination is 0°, the field is also symmetrical, but the field takes negative value at peak. Other angles of inclination show different curve pattern along the profile. The pattern obtained in Figure 1, coincided with the ones obtained in the literature and this validates the modeling carried out in this study. Different pattern obtained from different inclination affect geophysical interpretation negatively and this effect needs to be corrected. In the case of techniques that make use of Euler relation, the field and its derivatives are used.
3.2. 3D Analysis

The anomalous magnetic field of isolated sphere model was simulated using different values of inclination ($I = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ and $90^\circ$). The anomaly is positioned appropriately when the inclination is $0^\circ$ and $90^\circ$, while for the rest of the values of $I (15^\circ, 30^\circ, 45^\circ, 60^\circ$ and $75^\circ)$ the positions have been slightly deviated from the true position. The most prominent effect of inclination could be observed at $45^\circ$, the centre of the anomaly is deviated from the true centre of the sphere. This effect of inclination is caused by the dipolar nature of the geomagnetic field [12] and it depends on the inclination as it has been observed in the plots (Figure 1). The mean estimate of each parameter together with its corresponding deviation are given in Table 1. It could observe from Table 1 that the parameters are estimated with good accuracy. The mean estimated values are about the same with the true parameters and the deviations of the estimates are minimum.

| I      | x km | ŷ km | y km | ŷ km | z km | ŵ km | N     | Ŵ     |
|--------|------|------|------|------|------|------|-------|-------|
| $0^\circ$ (375) | 12.5 | 12.5 ± 0.1 | 12.4 | 12.4 | 1 | 1.01 ± 0.03 | 3 | 3 ± 0.1 |
| $15^\circ$ (375) | 12.5 | 12.5 ± 0.1 | 12.4 | 12.4 | 1 | 1.005 ± 0.035 | 3 | 3 ± 0.1 |
| $30^\circ$ (367) | 12.5 | 12.5 | 12.4 | 12.4 | 1 | 1 ± 0.03 | 3 | 3 ± 0.1 |
| $45^\circ$ (334) | 12.5 | 12.5 | 12.4 | 12.4 | 1 | 1.005 ± 0.035 | 3 | 3 ± 0.1 |
| $60^\circ$ (303) | 12.5 | 12.5 | 12.4 | 12.4 | 1 | 1.005 ± 0.035 | 3 | 3 ± 0.1 |
| $75^\circ$ (219) | 12.5 | 12.5 | 12.4 | 12.5 | 1 | 1 ± 0.03 | 3 | 3 ± 0.1 |
| $90^\circ$ (189) | 12.5 | 12.5 | 12.4 | 12.4 | 1 | 1 ± 0.03 | 3 | 3 ± 0.1 |

4. Conclusion

A new approach of solving Euler relation that compute depth, horizontal position, structural index and background level has been tested using theoretical modeling. The simulated field of the dipole was processed and was inversed using a new technique that based on Euler relation. The result indicated that the technique performed very well in the estimate of horizontal position and depth of the dipole source despite the effect of inclination of the field. The mean values of the estimated parameters were about the same with the true parameters, the highest deviation obtained is ±0.1. The technique was fully
automated, unlike the conventional technique that was semi-automated. The technique was fast means of source’s location estimated of concealed magnetized body. Filtering is recommended for the real field data because of the presence of so many sources and the Earth’s magnetic field. Many theoretical models shall be used to ascertain the effect of inclination.

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