Electromagnetic and gravitational decay of the Higgs boson

R. Delbourgo and Dongsheng Liu

School of Mathematics and Physics, University of Tasmania,
Hobart, Australia 7001
(May 31, 2018)

Abstract

The decays of a scalar particle, of either parity, into two photons or into two gravitons are evaluated. The effective interactions are of the form $\phi F F$, $\phi F \tilde{F}$ or $\phi R R$, $\phi R \tilde{R}$; in particular, Higgs boson decay into gravitons cannot be adduced to an interaction $\phi R$, as has been recently claimed.

12.10.Gq, 12.10.Dm, 12.15.Ji, 12.25+e
In a very recent paper, Srivastava and Widom [1] have claimed that the decay width of the Higgs meson into two gravitons is given by $\sqrt{2G_F m_H^2}/16\pi$. In their result, which they say stems from an effective interaction $\phi R/\langle \phi \rangle$, Newton’s constant disappears and gets replaced by the Fermi constant, leading to a large magnitude for the process. If their result were true, it would be counterintuitive to the notion that gravitational interactions are miniscule in particle physics and it would imply that the Higgs mesons disappears very quickly into a puff of gravitational plus electromagnetic radiation! In this paper we show that their result is not right and we derive the correct magnitudes for the decay amplitudes, be the decaying particle scalar or pseudoscalar; our faith in the weakness of induced gravitational (and electromagnetic) effects is happily restored.

We consider parity-conserving decays of a $0^+$ or $0^-$ particle into two photons ($\gamma$) or into two gravitons ($h$). A simple helicity amplitude analysis [2] shows that either process is governed by just one reduced helicity amplitude, $\langle k, \lambda; k', \lambda' | S | 0 \rangle$, where $\lambda = \lambda' = 1$ or 2, because of parity conservation. The same conclusion is reached by a covariant amplitude analysis, using the twin principles of gauge and general covariance. If $\varepsilon$ represents the external wavefunction of an incoming massless particle ($\gamma$ or $h$), we may write down the unique couplings,

Scalar $\rightarrow \gamma(k, \lambda)\gamma(k', \lambda')$

$$\mathcal{L}_{\phi \gamma \gamma} = g\varepsilon^{*\mu}_{\chi}(k)\varepsilon^{\nu}_{\chi}(k')T_{\mu\nu}(k, k'); \quad T_{\mu\nu}(k, k') \equiv k'_{\mu}k'_{\nu} - \eta_{\mu\nu}k \cdot k',$$

(1)

Pseudoscalar $\rightarrow \gamma(k, \lambda)\gamma(k', \lambda')$

$$\mathcal{L}_{\phi \gamma \gamma} = g\varepsilon^{*\mu}_{\chi}(k)\varepsilon^{\nu}_{\chi}(k')k^{\alpha}k^{\beta}\epsilon_{\mu\nu\alpha\beta},$$

(2)

Scalar $\rightarrow h(k, \lambda)h(k', \lambda')$

$$\mathcal{L}_{\phi hh} = G\varepsilon^{*\mu}_{\chi}(k)\varepsilon^{\nu\sigma}_{\chi}(k')[T_{\mu\nu}(k, k')T_{\rho\sigma}(k, k') + T_{\mu\rho}(k, k')T_{\nu\sigma}(k, k')]/2;$$

(3)

Pseudoscalar $\rightarrow h(k, \lambda)h(k', \lambda')$

$$\mathcal{L}_{\phi hh} = G\varepsilon^{*\mu}_{\chi}(k)\varepsilon^{\nu\sigma}_{\chi}(k')[\epsilon_{\mu\rho\alpha\beta}k^{\alpha}k^{\beta}T_{\rho\sigma}(k, k') + (3 \text{ other perms})]/4;$$

(4)

Above, $\eta$ stands for the Minkowski metric, about which we have expanded the gravitational metric field ($g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}$) and with respect to which Lorentz scalar contractions are made.

There are four significant points about eqs (1) - (4). Firstly, we note that insistence on current and stress tensor conservation means that each photon is accompanied by one power of momentum and each graviton comes with two powers of momentum, entirely in keeping with low-energy theorems [3]. Secondly, we can encapsulate the gauge invariances by regarding the effective couplings above as corresponding to effective gauge interactions,

$$g\phi F_{\mu\nu}F^{\mu\nu}/4, \quad g_{5}\phi_{5}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}/8, \quad G\phi R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}/2\kappa^2, \quad G_{5}\phi_{5}\epsilon_{\mu\nu\rho\sigma}R_{\alpha\beta}^{\mu\nu\rho\sigma}/4\kappa^2,$$

where $\kappa^2 = 8\pi G_{\text{Newton}}$. Thirdly, the mass dimensions of $g, g_5$ are $M^{-1}$ and of $G, G_5$ are $M^{-3}$. Fourthly and most importantly, the curvature tensor $\mathcal{R}$ has only two powers of derivatives, but of course contains all powers of the graviton field $h$, as befits a nonlinear theory. Thus it
is quite impossible for an interaction like $\phi R$ to lead to two-graviton decay, as it is essential to conjure up four powers of momentum in the decay amplitude. In this respect we find the Srivastava-Widom claim extremely puzzling, notwithstanding that one is dealing with a spontaneously broken gauge symmetry and a vacuum expectation value which is closely tied to the Higgs boson field.

It is straightforward to compute the decay rates ensuing from the interactions (1) to (4). Because gravitons and photons are massless the results are really quite simple in the end; if $m$ denotes the mass of the decaying scalar, one finds the partial widths,

$$
\Gamma_{\phi\gamma\gamma} = |g|^2 m^3/64\pi, \quad \Gamma_{\phi_{5}\gamma\gamma} = |g_5|^2 m^3/64\pi, \quad \Gamma_{\phi_{h}h} = 3|G|^2 m^7/512\pi, \quad \Gamma_{\phi_{5}h} = |G_5|^2 m^7/512\pi. \quad (6)
$$

The first and third of these are relevant to the Higgs boson. The only remaining question is the magnitude of the various coupling constants, $g$ to $G_5$.

Because the decays are induced by quantum effects, they first arise at one-loop level. We may determine the couplings by applying the standard Feynman rules to the graphs depicted in Figure 1. In quoting the answers below, for a source field $\psi$ of mass $m_\psi$ circulating round the loop which is either scalar or spinor and of unit charge $e$, we should point out that certain graphs of Figure 1 are absent or give vanishing contributions; for example there is no Figure 1d for photons, but it certainly arises for gravitons; similarly Figure 1c does not occur for fermions undergoing electromagnetic decay but is present for gravitons and always exists for boson sources. As a purely technical point, we have applied dimensional regularization when calculating these diagrams, in order to respect gauge invariance.

$$
g = \frac{e^2 g_{\phi\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{(4xy - 1)}{(m_\psi^2 - m^2 xy)}; \quad \text{fermion loop} \quad (7a)
$$

$$
g = \frac{e^2 g_{\phi\psi\psi}}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{xy}{(m_\psi^2 - m^2 xy)}; \quad \text{boson loop} \quad (7b)
$$

$$
g_5 = \frac{e^2 g_{\phi_{5}\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{1}{(m_\psi^2 - m^2 xy)}; \quad \text{fermion loop only} \quad (8)
$$

$$
G = \frac{\kappa^2 g_{\phi\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{xy(1 - 4xy)}{(m_\psi^2 - m^2 xy)}; \quad \text{fermion loop} \quad (9a)
$$

$$
G = -\frac{\kappa^2 g_{\phi\psi\psi}}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{x^2 y^2}{(m_\psi^2 - m^2 xy)}; \quad \text{boson loop} \quad (9b)
$$

$$
G_5 = -\frac{\kappa^2 g_{\phi_{5}\psi\psi} m_\psi}{2\pi^2} \int_0^1 \int_0^1 dx dy \theta(1 - x - y) \frac{xy}{(m_\psi^2 - m^2 xy)}; \quad \text{fermion loop only}. \quad (10)
$$

Note that $g_{\phi\psi\psi}$ is dimensionless when $\psi$ is a spinor, but acquires dimensions of mass when $\psi$ is a scalar field. Anyhow, we see that all the integrals can be written as combinations of the basic integral,
\[ I_n(\mu) \equiv \int_0^1 \int_0^1 dxdy \theta(1-x-y) (xy)^n/(1-\mu xy); \quad \mu = m^2/m_\psi^2, \quad (11) \]

which can be expressed as sums of dilog functions and the like \[4\]. As there is little to be gained by looking up tables of Spence functions, it turns out to be much easier to evaluate the \( I_n \) numerically for input values of \( \mu \).

Focussing on the Higgs boson now, and in respect of spin 0 and spin 1/2 contributions to the decay widths \( \Gamma_{H\gamma\gamma} \) and \( \Gamma_{Hgg} \), we should remind ourselves that \( g_{H\psi\psi} = -2m_\psi^2/v \) for scalar isosinglets and \( g_{H\psi\psi} = -m_\psi/v \) for quarks, where \( v = 246 \text{ GeV} \) is the Higgs vacuum expectation value. Therefore the couplings entering the partial widths (6) equal \( g = e^2 I_1(\mu)/\pi^2 v \) and \( G = -\kappa^2 [I_1(\mu) - 4I_2(\mu)]/\pi^2 v \) from a boson loop; in particular, a massless loop particle (\( \mu \to \infty \)) reassuringly gives zero! Since the Higgs boson in the standard model couples just to the quarks/leptons and gauge bosons, apart from its self-interactions, we get the fermionic contributions,

\[ g = \frac{e^2}{2\pi^2 v} \left[ \sum_{q=u,c,t} \frac{4}{3} (I_0(m_q) - 4I_1(m_q)) + \sum_{q=d,s,b} \frac{1}{3} (I_0(m_q) - 4I_1(m_q)) + \sum_{l=e,\mu,\tau} (I_0(m_l) - 4I_1(m_l)) \right] \quad (12) \]

\[ G = -\frac{\kappa^2}{2\pi^2 v} \left[ \sum_{q=u,c,t} \frac{4}{3} (I_1(m_q) - 4I_2(m_q)) + \sum_{q=d,s,b} \frac{1}{3} (I_1(m_q) - 4I_2(m_q)) + \sum_{l=e,\mu,\tau} (I_1(m_l) - 4I_2(m_l)) \right] \quad (13) \]

To complete the calculation for electroweak theory one should add the contributions from the W and Z gauge bosons, including ghost field contributions as required by the gauge-fixing scheme in use. These have been computed previously \[4\], and are a factor of 21/4 greater than the fermionic terms. But in any event we see that the gravitational decay rate is largely determined by the top quark (and the heavy gauge bosons), with a magnitude \( G \sim G_{\text{Newton}} m_t^2/vm_H^2 \) that is fixed by the Newtonian constant. This enters eq. (6) and duly gives a miniscule result for the gravitational decay rate, of the order of \( \Gamma_{Hhh} \sim m_H (G_{\text{Newton}} m_H^3/v)^2/500 \sim m_H \times 10^{-75} \).

This is as it should be: the disappearance of the Higgs boson into gravitational radiation will never be observed.

**ACKNOWLEDGMENTS**

We wish to thank the Australian Research Council for providing financial support in the form of a large grant, #A69800907 and have benefitted from discussions with Peter Jarvis.
REFERENCES

* EMail: bob.delbourgo@utas.edu.au
** EMail: d.liu@utas.edu.au

[1] Y.N. Srivastava and A. Widom, “Gravitational decay modes of the standard model Higgs Particle”, arXiv:hep-ph/000311.

[2] This is a trivial extension to spin 2 of the analysis found, for example, in R. Delbourgo and Dongsheng Liu, Phys. Rev. 57D, 5732 (1998).

[3] S. Weinberg, Phys. Rev. 135, B1049 (1964); ibid 140, B515 (1965).

[4] In this connection, observe that the pseudoscalar interactions are the same as the axial current anomalies when the pseudoscalar field is divided out, both for electromagnetism and for gravity. See R. Kimura, Prog. Theor. Phys. 42, 1191 (1969), R. Delbourgo and A. Salam, Phys. Lett. 40B, 381 (1972), T. Eguchi and P.G.O. Freund, Phys. Rev. Lett. 37, 1251 (1976). With regard to the scalar gravitational interaction, the Gauss-Bonnet invariant sum is of little utility because of the inclusion of the $\phi$ field.

[5] The analytic results read $I_0 = [\text{dilog} z^+ + \text{dilog} z^-]/z$, $I_1 = [\text{dilog} z^+ + \text{dilog} z^- - z/2]/z^2$, $I_2 = [\text{dilog} z^+ + \text{dilog} z^- - z/2 - z^2/24]/z^3$, where $z^\pm \equiv 2/[1 \pm \sqrt{1 - 4/\mu}]$. Also worth noting are the limits $I_n(\mu = 0) = (n!)^2/(2n+2)!$, corresponding to a superheavy massive loop particle; and for $\mu \gg 1$, $I_0(\mu) \simeq \log^2(\mu)/2\mu, I_1(\mu) \simeq -1/2\mu, I_2(\mu) \simeq -1/6\mu$, corresponding to a light fermion loop.

[6] B.A. Kniehl, Phys. Rep. 240, 211 (1994) and references therein. The Higgs decay into two photons has now been computed to two-loop order in electroweak theory; see A. Djouadi, P. Gambino and B.A. Kniehl, Nucl. Phys. B523, 17 (1998).
FIGURE CAPTION

Figure 1. One-loop contributions to Higgs decay into two massless gauge particles where the circulating loop particle can be scalar, spinor or vector.
FIG. 1. R Delbourgo