CONFORMALITY AND UNIFICATION OF GAUGE COUPLINGS

P.H. FRAMPTON

Department of Physics and Astronomy,
University of North Carolina,
Chapel Hill, NC 27599-3255, USA.
E-mail: frampton@physics.unc.edu

By use of the AdS/CFT correspondence on orbifolds, models are derived which can contain the standard model of particle phenomenology. It will be assumed that the theory becomes conformally invariant at a renormalization-group fixed-point in the TeV region. A recent application to TeV unification is briefly mentioned.

1. Introduction

Conformality is inspired by superstring duality and assumes that the particle spectrum of the standard model is enriched such that there is a conformal fixed point of the renormalization group at the TeV scale. Above this scale the coupling do not run so the hierarchy is nullified.

Until very recently, the possibility of testing string theory seemed at best remote. The advent of AdS/CFTs and large-scale string compactification suggest this point of view may be too pessimistic, since both could lead to \( \sim 100 \) TeV evidence for strings. With this thought in mind, we are encouraged to build AdS/CFT models with realistic fermionic structure, and reduce to the standard model below \( \sim 1 \) TeV.

Using AdS/CFT duality, one arrives at a class of gauge field theories of special recent interest. The simplest compactification of a ten-dimensional superstring on a product of an AdS space with a five-dimensional spherical manifold leads to an N=4 SU(N) supersymmetric gauge theory, well known to be conformally invariant\(^1\). By replacing the manifold \( S^5 \) by an orbifold \( S^5/\Gamma \) one arrives at less supersymmetries corresponding to N = 2, 1 or 0 depending\(^2\) on whether \( \Gamma \subset SU(2), \ SU(3), \) or \( / \subset SU(3) \) respectively, where \( \Gamma \) is in all cases a subgroup of \( SU(4) \sim SO(6) \) the isometry of the \( S^5 \) manifold.

It was conjectured in\(^3\) that such \( SU(N) \) gauge theories are conformal
in the $N \to \infty$ limit. In $^4$ it was conjectured that at least a subset of the resultant nonsupersymmetric $N=0$ theories are conformal even for finite $N$. Some first steps to check this idea were made in $^5$. Model-building based on abelian $\Gamma$ was studied further in $^6,^7,^8$, arriving in $^8$ at an $SU(3)^7$ model based on $\Gamma = Z_7$ which has three families of chiral fermions, a correct value for $\sin^2 \theta$ and a conformal scale $\sim 10$ TeV.

The case of non-abelian orbifolds based on non-abelian $\Gamma$ has not previously been studied, partially due to the fact that it is apparently somewhat more mathematically sophisticated. However, we shall show here that it can be handled equally as systematically as the abelian case and leads to richer structures and interesting results.

In such constructions, the cancellation of chiral anomalies in the four-dimensional theory, as is necessary in extension of the standard model (e.g. $^9,^{10}$), follows from the fact that the progenitor ten-dimensional superstring theory has cancelling hexagon anomaly $^{11}$. It offers a novel approach to family unification $^{12,13}$.

2. Gauge Coupling Unification

There is not space here to describe many technical details which are, however, available in the published papers cited at the end of this talk. But I would like to emphasize one success of the approach which involves the unification of gauge couplings $^7,^{15}$. Recall that the successful such unification is one primary reason for belief in supersymmetric grand unification e.g. $^{16}$. That argument is simple to state: The RG equations are:

$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln \left( \frac{M_G}{M_Z} \right)$$

(1)

Using the LEP values at the Z-pole as $\alpha_3 = 0.118 \pm 0.003$, $\alpha_2 = 0.0338$ and $\alpha_1 = \frac{5}{3} \alpha_Y = 0.0169$ (where the errors on $\alpha_{1,2}$ are less than 1%) and the MSSM values $b_i = (6,1,-3)$ leads to $M_G = 2.4 \times 10^{16}$ GeV and the prediction that $\sin^2 \theta = 0.231$ in excellent agreement with experiment.

In the present approach the three gauge couplings $\alpha_{1,2,3}$ run up to $\sim 1$ TeV where they freeze and embed in a larger (semi-simple) gauge group which contains $SU(3) \times SU(2) \times U(1)$.

I will give two examples, the first based on the abelian orbifold $S^5/Z_7$ and the second based on the non-abelian orbifold $S^5/(D_4 \times Z_3)$.

In the first, abelian, case we choose $N=3$, $\Gamma = Z_7$ and the unifying group $^7,^8$ is therefore $SU(3)^7$. It is natural to accommodate one $SU(3)$
factor (color) into one of the seven \( SU(3) \) factors, \( SU(2)_L \) as a diagonal subgroup of two and to identify the correctly normalized \( U(1) \) as the diagonal subgroup of the remaining four \( SU(3) \) factors. This implies that \( \alpha_2/\alpha_1 = 2 \) and consequently:

\[
\sin^2 \theta = \frac{\alpha_Y}{\alpha_2 + \alpha_Y} = \frac{3/5}{2 + 3/5} = \frac{3}{13} = 0.231
\]

(2)

There is a small correction for the running between \( M_Z \) and the TeV scale but this is largely compensated by the two-loop correction and the agreement remains as good as for SUSY-GUTS. This is strong encouragement for the conformality approach.

In the second, non-abelian, example we use \( \Gamma = Z_3 \times D_4 \) and choose \( N=2 \) to arrive at a unification based on the Pati-Salam group \( SU(4)_C \times SU(2)_L \times SU(2)_R \) instead of the trinification \( SU(3)^3 \). This is possible because this non-abelian \( \Gamma \) has two-dimensional representations as well as one-dimensional ones.

The dihedral group \( D_4 \) consists of eight rotations which leave a square invariant: two of the rotations are flips about two lines which bisect the square and the other four are rotations through \( \pi/2, \pi, 3\pi/2 \) and \( 2\pi \) about the perpendicular to the plane of the square.

In this case the low energy gauge group is thus \( SU(4)^3 \times SU(2)^{12} \). We embed \( SU(3)_{\text{color}} \) in \( r \) of the \( SU(4) \) groups where \( r = 1 \) or 2 because \( r = 3 \) leads to loss of chirality. At the same time the \( SU(2)_L \) and \( SU(2)_R \) are respectively embedded in diagonal subgroups of \( p \) and \( q \) of the twelve \( SU(2) \) factors where \( p + q = 12 \).

Since \( p \) and \( q \) are necessarily integers it is not at all obvious \textit{a priori} that the value of \( \sin^2 \theta \) can be consistent with experiment.

The values of the respective couplings at the conformality/unification scale are now:

\[
\alpha^{-1}_{2L}(M_U) = p\alpha^{-1}_U
\]

(3)

\[
\alpha^{-1}_{2R}(M_U) = q\alpha^{-1}_U
\]

(4)

\[
\alpha^{-1}_{4C}(M_U) = 2r\alpha^{-1}_U
\]

(5)

The hypercharge coupling is related by

\[
\alpha^{-1}_1 = \frac{2}{5}\alpha^{-1}_{4C} + \frac{3}{5}\alpha^{-1}_{2R}
\]

(6)
Defining \( y = \ln (M_U/M_Z) \) we then find the general expression for \( \sin^2 \theta_W (M_Z) \) to be:

\[
\sin^2 \theta_W (M_Z) = \frac{p - (19/12\pi)y\alpha_U}{p + q + \frac{7}{4} r + (11/6\pi)y\alpha_U}
\]  

Here

\[
\alpha_S^{-1} (M_Z) = 2r\alpha_U^{-1} - \frac{7}{2\pi} y
\]

Using these formulas and \( \alpha_S (M_Z) \sim 0.12 \) we find for the natural choices (for model building) \( p = 4 \) and \( r = 2 \) that

\[
\sin^2 \theta_W (M_Z) \simeq 0.23
\]

again in excellent agreement with experiment.

It is highly non-trivial that again the gauge coupling unification works in this case which, according to the lengthy analysis in the second paper of \(^{14}\), is the unique accommodation of the standard model with three chiral families for all non-abelian \( \Gamma \) with order \( g \leq 31 \).

The successful derivation of \( \sin^2 \theta_W (M_Z) \simeq 0.23 \) from both the abelian orbifold (based on 333-trinification) and the non-abelian orbifold (based on 422-Pati-Salam unification) is strong support for further investigation of the detailed phenomenology arising from the approach.

3. TeV Unification

As one example of this approach arrived at shortly after, but inspired by, the Oxford conference, let me mention an example of strong-electroweak unification at a relatively low (\( \sim 4 \) TeV) scale\(^{17}\). It was motivated partly by bottom-up considerations which could be matched to the above top-down idea.

In the standard model, the three couplings are well measured at the Z-pole, particularly at LEP. The electroweak mixing angle \( \sin^2 \theta (M_Z) = 0.231 = \alpha_Y (\alpha_2 + \alpha_Y)^{-1} \) is close to 1/4 and as the energy scale is raised it increases going through 1/4 at a scale of about 4 TeV. This scale played a role in the 3-3-1 model\(^{10}\) and in the more recent study by Dimopoulos and Kaplan\(^{18}\).
The strong coupling $\alpha_3(\mu)$ relative to the $SU(2)_L$ coupling has a ratio $r(\mu) = \alpha_3(\mu)/\alpha_2(\mu)$ which is $r > 3$ at $\mu = M_Z$ and goes through $r=3$ at $\mu \sim 400$ GeV then $r=2$ at $\mu \sim 140$ TeV. The value $r = 5/2$ is attained at a scale impressively close to the $\sim 4$ TeV scale where $\sin^2 \theta = 1/4$.

We therefore adopt a gauge group $SU(3)^{12}$ at $\mu = 4$ TeV and identify the trinification gauge groups $SU(3)_C$, $SU(3)_W$ and $SU(3)_H$ with 2, 5, and 5 of the SU(3) factors respectively.

Assuming all the gauge couplings are equal at this unification scale there are two predictions: the correct value of $\sin^2 \theta$ and of $\alpha_3$ at the Z-pole. This is interesting because usual GUTs predict only one of these two quantities.

One could stop with such a bottom-up unification but the theory becomes more interesting when we marry it to string orbifolding, this time using $AdS_5 \times S^5/\mathbb{Z}_{12}$. We take $N=3$ 3-branes to achieve $SU(3)^{12}$. One must then specify the embedding of $\mathbb{Z}_{12}$ in SU(4) such that the scalars are adequate to allow spontaneous breaking to the standard gauge group. This leads to the choice $4 = (\alpha, \alpha^2, \alpha^3, \alpha^6)$ where $\alpha$ is the 12th root of unity.

The chiral fermions can now be deduced by drawing the dodecagonal quiver (the nodes are arrange exactly like the numbers on a clock face) and one finds that there are three chiral families. Actually there are five families and two antifamilies, and although there is insufficient space here to go into technical detail it is possible to relate the reason for three families to the difference between the numerator and denominator in the minimalized ratio $r = 5/2$, mentioned earlier. As a final merit of the model it has no GUT hierarchy because there is no scale above 4 TeV. It is a non-gravitational theory where the Planck scale is infinite.

It is hoped to pursue the phenomenology of such an approach further in the future, as well as to investigate the robustness of the predictions with respect to the input unification scale.

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