Testing the effects from dark radiation

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Abstract
In this paper, the effects of dark radiation (DR) are tested. Theoretically, the phase-space analysis method is applied to check whether the model is consist with the history of our universe which shows positive results. Observationally, by using the observational data (SuperNovae Legacy Survey (SNLS), Wilkinson Microwave Anisotropy Probe 9 Years Result (WMAP9), Planck First Data Release (PLANCK), baryon acoustic oscillations (BAO), Hubble parameter data ($H(z)$) and Big Bang nucleosynthesis (BBN)), the DR is found to have the effect of wiping out the tension between the SNLS data and the other data in a flat $\Lambda$CDM model. The effects of DR also make the best fit value of $N_{\text{eff}}$ slightly larger than 3.04.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The observations hint that our universe is accelerating now (e.g. [1–5]). The observations also show a nearly flat universe with roughly 72\% dark energy, 28\% matter and 0.1\% radiation (e.g. [6–10]). How to describe these observations by theories? The $\Lambda$CDM model is the simplest candidate. In the $\Lambda$CDM model, the generation of neutrino is assumed as three. And, the number of the effective neutrino species is $N_{\text{eff}} = 3.04$, where the effects from the non-instantaneous neutrino decoupling from the primordial photon–baryon plasma are taken into account. However, many theoretical models indicate the existence of extra radiation, e.g. the FRW model in the Randall–Sundrum scenario [11–14]; the Brans–Dicke theory [15, 16]; the Horava–Lifshitz theory [17, 18]; the decaying vacuum [19–21]; the negative Casimir effect [22].
Recently, the measurement of the temperature anisotropy of cosmic microwave background (CMB) has shown a less-power spectrum at a small scale, suggesting that $N_{\text{eff}}$ has a bigger value than the one predicted by the standard model of particle physics, so the existence of ‘dark radiation’ (DR). The results of WMAP7 [10], the Atacama Cosmology Telescope [23, 24] (ACT) and South Pole Telescope [25] (SPT) which give out the $1\sigma$ level of the effective neutrino number are $N_{\text{eff}} = 4.56 \pm 0.75$ (WMAP7), $N_{\text{eff}} = 2.78 \pm 0.55$ (WMAP7 + ACT), $2.96 \pm 0.44$ (WMAP7 + ACT + SPT); while the Big Bang nucleosynthesis (BBN) data show $N_{\text{eff}} = 3.24 \pm 0.6$ [26]. So many discussions on DR have already appeared (e.g. [27–30]).

In this paper, the $\Lambda$CDM model with DR will be used to test the DR effect which could be generated from an electroweak phase transition [31]. This paper is organized as follows. In section 2, the model will be introduced. In section 3, a phase-space analysis will be presented to obtain the evolution of our universe. Then in section 4, we apply the observation data to test the model parameter space, including the SNLS complication of supernova Ia parameter data [41, 42] and the BBN data [43, 44]. We will show the constraining results in section 5. Finally, a short summary will be given out in section 6.

2. The model

Here, the geometry of spacetime is assumed to be described by the FRW (Friedmann–Robertson–Walker) metric with a non-zero curvature,

$$\text{d}x^2 = -\text{d}t^2 + a^2(t) \left( \frac{\text{d}r^2}{1-kr^2} + r^2 (\text{d}\theta^2 + \sin^2\theta \text{d}\phi^2) \right),$$

where $a$ is the scale factor, and $k$ is the curvature parameter with the values of 0, $\pm 1$ representing a flat, closed and open spatial sector, respectively.

The energy density components in our universe are represented by the pressureless matter part $\rho_m$, the dark energy part $\rho_d$, the ordinary radiation part $\rho_r$, the DR part $\rho_{dr}$ and the curvature part $\rho_k$. The Friedmann equation is

$$H^2 = \frac{1}{3m_{\text{pl}}^2} (\rho_m + \rho_d + \rho_r + \rho_{dr} + \rho_k),$$

where $\rho_k = -k/a^2$ and $m_{\text{pl}}$ is the Planck mass. We call the $\Lambda$CDM model plus the DR as the flat or curved DR model. This kind of model could be derived from a quintessence scenario phenomenologically whose potential includes interactions of the field with virtual particles and the heat bath. As the potential is similar to the Higgs potential in the electroweak phase transition, a first-order phase transition at redshift $z \sim 3$ releases energy in a relativistic model (DR). After that, $\rho_d$ becomes a constant, and the DR appears [31].

After defining $\Omega_i = \rho_i/(8\pi G H^2)$, $\Omega_m$, $\Omega_d$, $\Omega_r$, $\Omega_{dr}$ and $\Omega_k$ could represent the fractional energy densities for matter, dark energy, ordinary radiation, DR and curvature, respectively. The energy components are assumed to be conserved separately. Specially,

$$\Omega_r + \Omega_{dr} = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^4 N_{\text{eff}} \right] \Omega_\gamma,$$

where $h = H_0/100$ Mpc.km.s$^{-1}$, the index ‘0’ denotes the present value of parameter and $\Omega_\gamma$ is the energy density of the CMB photons background at temperature $T_\gamma = 2.728$ K. To represent the DR, we use the symbols $f = \Omega_{dr0}/\Omega_{\gamma0}$ which represents the ratio of today’s DR and dark energy. Then, the Friedmann equation could be rewritten as below:

$$H^2 = H_0^2[\Omega_{\gamma0} + \Omega_{\delta0}a^{-2} + \Omega_{\sigma0}a^{-3} + \Omega_{\gamma0}a^{-4} + f \Omega_{\delta0}a^{-4}].$$

2
If treating the DR as a signal of dark energy, DR leads to a characteristic time dependence in the effective equation of state (EoS) parameter of dark energy,

\[
\omega(z) = \frac{p_d + p_r}{\rho_d + \rho_r} = \frac{f(1 + z)^4}{f(1 + z)^4 + 1},
\]

where \(z\) is the redshift with the definition \(z = a^{-1} - 1\). And, the time derivative of the EoS parameter is

\[
\omega' = \frac{16}{3} \frac{f(1 + z)^4}{[f(1 + z)^4 + 1]^2},
\]

where a prime means the derivative with respect to ln \(a\). Based on equations (5) and (6), the relations of \(\omega - \omega'\), \(\omega - z\) and \(\omega' - z\) are listed in figure 1. The curves show the deviations from the \(\Lambda\)CDM model are tiny with small \(f\). Specifically, the knowledge of \(f\) is sufficient to know the present value of EoS parameter where \(1 + \omega_0 = +4f/3(1 + f)\) and \(\omega_0' = -16f/3(1 + f)^2\). If \(f\) is at the order of \(10^{-5}\), it is not surprising that the EoS parameter is very close to \(-1\) and the derivative of the EoS parameter is tiny.

3. The phase-space analysis

To do the phase-space analysis in our model, three dimensionless parameters are defined firstly,

\[
u = \sqrt{\frac{H_0^2 \Omega_{m0} a^{-3}}{H^2}},
\]

\[
v = \sqrt{\frac{H_0^2 (\Omega_{r0} + \Omega_{d0}) a^{-4}}{H^2}},
\]

\[
w = \sqrt{\frac{H_0^2 \Omega_{k0} a^{-2}}{H^2}}.
\]

Based on the Friedmann equation and the conserved ones\(^5\), the evolutions of \(u\), \(v\), \(w\) are

\[
u' = u\left(-\frac{3}{z} + \frac{1}{2} u^2 + 2v^2 + w^2\right),
\]

\(^5\) The conserved equation for each component is \(\dot{\rho}_i + 3H \rho_i (1 + \omega_i) = 0\) where \(\omega_\rho = 1/3, \omega_d = 1/3, \omega_k = 0\) and \(\omega_f = -1\).
Table 1. We list the properties of the critical points: the physical meaning of the phases, the value of the phases, the existing condition, the eigenvalues of the points and the stability of the phases.

| Phase | Physical meaning | Existing | $(\lambda_1, \lambda_2, \lambda_3)$ | Stability |
|-------|------------------|----------|----------------------------------|-----------|
| $M$   | Matter dominated  | $(1, 0, 0)$ | Always $(3, -2, -1)$            | Unstable  |
| $R$   | Radiation dominated | $(0, \pm 1, 0)$ | Always $(\frac{3}{2}, 4, -1)$ | Unstable  |
| $K$   | Curvature dominated | $(0, \pm 1, 0)$ | Always $(-\frac{3}{2}, -2, 2)$ | Unstable  |
| $D$   | Dark energy dominated | $(0, 0, 0)$ | Always $(-\frac{3}{2}, -2, -1)$ | Stable    |

\[ v' = v\left(-2 + \frac{3}{2}u^2 + 2v^2 + w^2\right), \]

\[ w' = w\left(-1 + \frac{3}{2}u^2 + 2v^2 + w^2\right). \]

When $u'$, $v'$ and $w'$ are all equal to 0, the corresponding value of $u$, $v$, $w$ gives a critical point. Four points are listed in Table 1. And, we could put a small perturbation near the critical points’ neighbor. Then, the perturbation equations are obtained. If the real parts of the eigenvalues of the perturbation equations are all positive, the corresponding critical point is an unstable fixed point. In contrast, the negative real parts of the eigenvalues denotes a stable point. In particular, if the real parts of the eigenvalues are mixed with the negative one and the positive one, the corresponding critical point is an unstable saddle point \[46–48\].

Generally speaking, the model with DR could go through the unstable radiation-dominated phase ($R$), the unstable matter-dominated phase ($M$), the stable dark energy-dominated phase ($D$) and the unstable curvature-dominated phase ($K$)\[^6\]. The dark energy-dominated phase is stable which means the universe will be dominated by the cosmological constant in the future. Before that, our universe is supposed to go through these unstable phases. This process is corresponding to the history of our universe: the radiation-dominated phase at first, then matter-dominated phase and finally the dark energy-dominated phase.

4. The data and the method analysis

Once treating DR as a signal of dark energy, the observational testing method used in the dark energy model could be applied to test the DR. In this section, the data and the analytical methods will be presented separately.

4.1. The data analysis

4.1.1. The SNLS data. SNe Ia (supernovae) is used in the standard distance method which measures the expansion of our universe. For the SNLS data, \[32\] gives the apparent $B$ magnitude $m_B$, and the covariance matrix for $\Delta m = m_B - m_{mod}$, with

\[ m_{mod} = 5 \log_{10} D_L(z|s) - \alpha (s - 1) + \beta C_{SN} + M, \]

where $D_L(z|s)$ is the luminosity distance multiplied by $H_0$ for a given set of cosmological parameters $s$, $C_{SN}$ is the color measure for supernovae and $M$ is a nuisance parameter representing some combination of the absolute magnitude of a fiducial SNe Ia. The time dilation part of the observed luminosity distance depends on the total redshift $z_{hel} \[50\]

\[ D_L(z|s) \equiv c^{-1}H_0(1 + z_{hel})r(z|s), \]

\[^6\] The unstable curvature-dominated phase often represents some very early physics. Here, we ignore this phase on the discussions of the history of our universe.

\[^7\] s is the stretch measure of the SNe Ia light curve shape.
where $c$ is the color index, $z$ and $z_{\text{hel}}$ are the CMB rest frame and heliocentric redshifts of the supernovae. The correlated errors are
\[ \chi^2_{SN} = \Delta m^T \cdot C_{SN}^{-1} \cdot \Delta m, \]
where $C_{SN}$ is the $N \times N$ covariance matrix of the SNe Ia, where $N$ is the number of the components. The nuisance parameter $H_0$ is marginalized over by evaluating $\chi^2_{SN}$.

4.1.2. The CMB data. The CMB data are implemented to add distance measurements at higher redshift ($z > 10$). We use the derived quantities of the WMAP9 and PLANCK measurements [60, 61]: the shift parameter $R(z^*)$, the acoustic scale $l_\text{A}(z^*)$ at the decoupling redshift and the base parameter $\omega_b$ whose definition is $\Omega_b h^2$, where $\Omega_b$ is the fractional energy densities for baryon. $\chi^2$ of CMB data is
\[ \chi^2_{\text{CMB}}(p, \Omega_b h^2, h) = \sum_{i,j=1}^{3} \Delta x_i C_{\text{CMB}}^{-1}(x_i, x_j) \Delta x_j, \]
where the three parameters are $x_i = (R(z^*), l_\text{A}(z^*), \omega_b)$, $\Delta x_i = x_i - x_i^{\text{obs}}$ and $C_{\text{CMB}}(x_i, x_j)$ is the covariance matrix for the three parameters [10, 60, 61]. The shift parameter $R$ is expressed as
\[ R(z^*) = [\sqrt{\int_{x_0}^{\infty} \sin(\sqrt{\Omega_B} f'_0 dz/E(z))]}/\sqrt{\Omega_B} = 1.710 \pm 0.019. \]
The acoustic scale is $l_\text{A}(z^*) = \pi d_l(z^*)/(1 + z^*) r_l(z^*) = 302.1 \pm 0.86$. And the decoupling redshift $z^*$ is fitted by [51] with $z^* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}[1 + g_1(\Omega_b h^2)^{0.76}] = 1090.04 \pm 0.93$, where $g_1 = 0.0783(\Omega_b h^2)^{-0.239}[1 + 39.5(\Omega_b h^2)^{0.76}]$ and $g_2 = 0.560/[1 + 21.1(\Omega_b h^2)^{1.81}]$.

4.1.3. The BAO, $H(z)$ and BBN data. To produce tightest cosmological constraint, we try to use other cosmological probes as well.

The BAO data are used as a standard rule. Due to the sound waves in the plasma of the early universe, the wavelength of BAO is related to the co-moving sound horizon at the baryon drag epoch which is $d_c = r_d(z_d)/D_V(z)$, $D_V$ is the effective distance with $D_V(z) = [zd_s^2(z)/H(z)(1 + z)^2]^{1/3}$, $z_d$ is the drag redshift defined in [49], and $r_d(z) = \int_{x_0}^{\infty} c_s(x) dx/E(x)$ is the co-moving sound horizon. For the BAO data, we use the measurements from the 6dFGS (hereafter Bao1) [40], the distribution of galaxies (hereafter Bao2) [38] and the WigleZ dark energy survey (hereafter Bao3) [39]. The 6dFGS (Bao6dF) gives
\[ \chi^2_{\text{Bao1}} = \frac{(d_{0,106} - 0.336)^2}{0.015^2}. \]
And, the distribution of galaxies (Bao2) measured the distance ratio at two redshifts $z = 0.2$ and $z = 0.35$, whose $\chi^2$ is
\[ \chi^2_{\text{Bao2}} = \sum_{i,j=1}^{2} \Delta d_i C_{dc}^{-1}(d_i, d_j) \Delta d_j, \]
where $d_i = (d_{i,0.2}, d_{i,0.35})$, $\Delta d_i = d_i - d_i^{\text{obs}}$ and the covariance matrix $C_{dc}(d_i, d_j)$ for $d_c$ at $z = (0.2, 0.35)$ is taken from equation (5) in [38]. Furthermore, the WiggleZ dark energy survey measured the acoustic parameter $A(z) = D_V(z)/\sqrt{\Omega_m H_0^2}/z$ at three redshifts $z = 0.44$, $z = 0.6$ and $z = 0.73$, and the results and their covariance matrix are listed in tables 2 and 3 of [39]. $\chi^2$ is
\[ \chi^2_{\text{Bao3}} = \sum_{i,j=1}^{3} \Delta A_i C_A^{-1}(A_i, A_j) \Delta A_j, \]
8 The sound speed is $c_s(z) = 1/\sqrt{3[1 + R_b/(1 + z)]}$, where $R_b = 3\Omega_b h^2/(4 \times 2.469 \times 10^{-5})$. \]
where $A_i = [A(0.44), A(0.6), A(0.73)]$, $\Delta A_i = A(z_i) - A(z_i)^\text{obs}$. Then, the total $\chi^2$ for BAO is

$$\chi^2_{\text{Bao}}(p, \Omega_0 h^2) = \chi^2_{\text{Bao1}} + \chi^2_{\text{Bao2}} + \chi^2_{\text{Bao3}}.$$  

To alleviate the double integration of the EoS parameter $\omega(z)$, we also apply the measurements of the Hubble parameter $H(z)$ which could better constrain $w(z)$ at high redshift. We use the $H(z)$ data at 11 different redshifts obtained from the differential ages of red-envelope galaxies in [41], and three more Hubble parameter data $H(z = 0.24) = 76.69 \pm 2.32$, $H(z = 0.34) = 83.8 \pm 2.96$ and $H(z = 0.43) = 86.45 \pm 3.27$ determined by [42]. Then, $\chi^2$ of Hubble parameter data is

$$\chi^2_H(p, h) = \sum_{i=1}^{14} \frac{[H(z_i) - H_{\text{obs}}(z_i)]^2}{\sigma_{h_i}^2},$$

where $\sigma_{h_i}$ is the 1σ uncertainty in the $H(z)$ data.

Furthermore, the constraint data from BBN are added for this DR model. $\chi^2$ of the BBN data [43–45] is

$$\chi^2_{\text{BBN}} = (\Omega_{0b} h^2 - 0.022)^2, \quad 0.002^2$$

where $\Omega_{0b} = 0.02253h^2$ is the present value of dimensionless energy density for baryon.

4.1.4. Data discussion. To use the data properly, the SNLS data will be used individually at first and be denoted as ‘data 1’. To utilize the WMAP9 and PLANCK data separately, we treat WMAP9 $+$ BAO $+$ $H(z)$ $+$ BBN and PLANCK $+$ BAO $+$ $H(z)$ $+$ BBN as ‘data 2’ and ‘data 3’. If all the three data are consistent, then we combine them as ‘data 4: SNLS $+$ WMAP9 $+$ PLANCK $+$ BAO $+$ $H(z)$ $+$ BBN’.

4.2. The analysis method

Monte Carlo Markov Chain (MCMC) method is used to compute the likelihood for the parameters in the model. By using the Metropolis–Hastings algorithm, the MCMC method randomly chooses values for the parameters, evaluates $\chi^2$ and determines whether to accept or reject the set of parameters.

4.2.1. The AIC principle. After combining different data, the total $\chi^2$ could be obtained by adding all the observation’s $\chi^2$ together. The model parameters are determined by minimizing $\chi^2$. To value the goodness of fitting, the Akaike information criterion (AIC) principle will be used which is very popular in Mathematics and Physics [62, 63],

$$\text{AIC} = \chi^2_{\text{min}} + 2n,$$

where $n$ is the number of parameters and $\chi^2_{\text{min}}$ is the minimum value of $\chi^2$. The smaller the AIC value is, the better the constraint is. If $\chi^2$ difference between two models is in a range of $0 < \Delta (\text{AIC}) < 2$, the constraints of the two models are considered to be equivalent.

4.2.2. The Om diagnostic. The $\text{Om}$ diagnostic [52, 53] is proposed to distinguish dynamical dark energy from the cosmological constant both with and without the matter density. In another saying, it is on the basis of observations of the expansion history. The $\text{Om}$ diagnostic could be characterized by

$$\text{Om}(x) = \frac{H^2(x)/H_0^2 - 1}{x^2 - 1}, \quad x = 1 + z.$$
Table 2. The maximum likelihood values with 1σ and 2σ confidence ranges are presented for the flat $\Lambda$CDM model, the curved $\Lambda$CDM model, the flat $\Lambda$CDM model with DR (flat DR model) and the curved $\Lambda$CDM model with DR (flat DR model).

|               | Flat $\Lambda$CDM | Curved $\Lambda$CDM | Flat DR model | Curved DR model |
|---------------|-------------------|-------------------|---------------|-----------------|
| **Data 1: SNLS** |                   |                   |               |                 |
| $\Omega_m$    | 0.226$^{+0.019}_{-0.037}$ | 0.174$^{+0.136}_{-0.154}$ | 0.297$^{+0.306}_{-0.295}$ | 0.145$^{+0.832}_{-0.435}$ |
| $f$           | $^{+−0.007}$     | $^{+−0.016}$     | $^{+−0.019}$   | $^{+−0.016}$     |
| $\Omega_b$    | $^{+0.150}_{-0.036}$ | $^{+0.410}_{-0.056}$ | $^{+0.515}_{-0.122}$ | $^{+0.515}_{-0.122}$ |
| $N_{\text{eff}}$ | $^{+−0.01}$     | $^{+−0.01}$     | $^{+−0.01}$   | $^{+−0.01}$     |
| $\chi^2$      | 420.10            | 419.77            | 418.60         | 419.78          |
| AIC           | 422.10            | 423.77            | 422.60         | 425.78          |
| **Data 2: WMAP9 + BAO + $H(z) + BBN$** |                   |                   |               |                 |
| $\Omega_m$    | 0.280$^{+0.029}_{-0.025}$ | 0.280$^{+0.034}_{-0.026}$ | 0.286$^{+0.030}_{-0.026}$ | 0.287$^{+0.049}_{-0.050}$ |
| $f$           | $^{+−0.065}$     | $^{+−0.065}$     | $^{+−0.065}$   | $^{+−0.065}$     |
| $\Omega_b$    | $^{+0.099}_{-0.091}$ | $^{+0.107}_{-0.092}$ | $^{+0.118}_{-0.092}$ | $^{+0.118}_{-0.092}$ |
| $H_0$         | 70.19$^{+0.21}_{-0.29}$ | 70.43$^{+0.26}_{-0.29}$ | 70.73$^{+0.30}_{-0.30}$ | 70.74$^{+0.31}_{-0.31}$ |
| $N_{\text{eff}}$ | $^{+−0.09}$     | $^{+−0.09}$     | $^{+−0.09}$   | $^{+−0.09}$     |
| $\chi^2$      | 10.17             | 9.95              | 9.55           | 9.54            |
| AIC           | 12.17             | 13.95             | 13.55          | 15.54           |
| **Data 3: PLANCK + BAO + $H(z) + BBN$** |                   |                   |               |                 |
| $\Omega_m$    | 0.290$^{+0.024}_{-0.026}$ | 0.290$^{+0.024}_{-0.026}$ | 0.296$^{+0.030}_{-0.026}$ | 0.292$^{+0.047}_{-0.050}$ |
| $f$           | $^{+−0.065}$     | $^{+−0.065}$     | $^{+−0.065}$   | $^{+−0.065}$     |
| $\Omega_b$    | $^{+0.099}_{-0.091}$ | $^{+0.107}_{-0.092}$ | $^{+0.118}_{-0.092}$ | $^{+0.118}_{-0.092}$ |
| $H_0$         | 69.46$^{+1.75}_{-1.81}$ | 70.35$^{+2.76}_{-2.90}$ | 70.31$^{+2.95}_{-2.90}$ | 70.46$^{+3.34}_{-3.34}$ |
| $N_{\text{eff}}$ | $^{+−0.09}$     | $^{+−0.09}$     | $^{+−0.09}$   | $^{+−0.09}$     |
| $\chi^2$      | 10.73             | 9.84              | 9.71           | 9.62            |
| AIC           | 12.73             | 13.84             | 13.71          | 15.62           |
| **Data 4: SNLS + WMAP9 + PLANCK + BAO + $H(z) + BBN$** |                   |                   |               |                 |
| $\Omega_m$    | $^{+0.029}_{-0.027}$ | $^{+0.031}_{-0.035}$ | $^{+0.033}_{-0.032}$ | $^{+0.039}_{-0.050}$ |
| $f$           | $^{+−0.007}$     | $^{+−0.016}$     | $^{+−0.019}$   | $^{+−0.016}$     |
| $\Omega_b$    | $^{+0.150}_{-0.036}$ | $^{+0.410}_{-0.056}$ | $^{+0.515}_{-0.122}$ | $^{+0.515}_{-0.122}$ |
| $H_0$         | $^{+−0.01}$     | $^{+−0.01}$     | $^{+−0.01}$   | $^{+−0.01}$     |
| $N_{\text{eff}}$ | $^{+−0.01}$     | $^{+−0.01}$     | $^{+−0.01}$   | $^{+−0.01}$     |
| $\chi^2$      | 433.62            | 433.86            | 433.66         | 433.66          |
| AIC           | 433.62            | 437.86            | 439.66         |                 |

5. The fitting results

5.1. The flat and curved $\Lambda$CDM Model

To a certain model, different data may give out very different constraining results which is called tension. One reason of tension is the system error of different data. The other reason could be traced to the model which may not represent the true physics.

For the flat $\Lambda$CDM model, table 2 shows the 1σ upper bound of $\Omega_m$ given by the SNLS data is 0.265 which is incompatible with the 1σ lower bound of $\Omega_m$ given by data 3, where $\Omega_m = 0.270$. In another saying, the constraining parameter range from the two datasets are not overlapped at 1σ level. For data 2, this situation are slightly better where the lower bound
of $\Omega_{m0}$ is 0.252. Anyway, that is just slightly overlapped with the SNLS data. Tension exists between the SNLS data and the other data (including WMAP9, PLANCK, $H(z)$ and BBN). And, we could not combine all the data together for the flat $\Lambda$CDM model.

Fortunately, after adding the curvature, table 2 shows that all the data are consistent. The 1σ range values of $\Omega_{m0}$ are overlapped and the tensions between the SNLS data and the other data are disappeared. Thus, it is reasonable to combine data 1–3 to obtain tighter constraints for the curved $\Lambda$CDM model. Comparing the flat and curved $\Lambda$CDM model, $\Delta(AIC) = 1.67 < 2$, so the constants of the two models are considered to be equivalent. This tension resolution hints that the system error of the data may not be the reason. The authors of [58, 59] show that the assumption of a flat universe induces critically large errors in reconstructing the dark energy equation of state even if the true cosmic curvature is very small. As the DR is also a small component, in the following, we will try to answer the question that whether the DR part could alleviate the tension problem or is not based on observations9.

5.2. The flat and curved $\Lambda$CDM model with dark radiation

After adding the DR to the $\Lambda$CDM model, table 2 shows SNLS data which give a loose parameter range. As we expected, this tension problem is disappeared. Therefore, it is reasonable to use the combined SNLS + WMAP9 + PLANCK + BAO + $H(z)$ + BBN data. It gives out the tightest constraints. Then, what results could we obtain if we add both the curvature and the DR to the $\Lambda$CDM model? As shown in figure 2, the parameter ranges of the curved one are slightly enlarged compared to the flat one.

Generally speaking, the SNLS data give very poor constraints on the model parameter compared to other data. Data 2–4 present $\Omega_{m0} \sim 10^{-2}$ and $f \sim 10^{-5}$ which denote the price we paid for the disappeared tension is reasonable. The AIC analysis also shows the constraints on both the flat and curved ones are equal because the $\Delta(AIC)$ is less than 2.

9 The effect that extra radiation can smash off the data tension is reported for other observational data, e.g. [54–57].
Anyway, the PLANCK + BAO + $H(z)$ + BBN data give a tighter constraint than the WMAP9 + BAO + $H(z)$ + BBN data.

5.2.1. The dark radiation. Again, as table 2 hints, the SNLS data are not sensitive to the effective neutrino number. In contrast, the constraints from other data are at much smaller orders. For concise, we only discuss the tightest constraints from data 4. The combined data favors a positive $f$ which denotes the new produced DR. Based on our definition, the data give out $N_{\text{eff}} = 3.25^{+0.74+1.00}_{-0.68-0.88}$ in the flat DR model and $N_{\text{eff}} = 3.09^{+1.17+1.53}_{-0.97-1.18}$ in the curved one. DR makes the best fit of $N_{\text{eff}}$ slightly larger than 3.04. We compare the flat and curved cases by drawing the contours of $H_0$ and $N_{\text{eff}}$ in figure 3, where the ranges of $H_0$ are nearly the same, but the range of $N_{\text{eff}}$ is larger in the curved case.

5.2.2. The Om diagnostic. The Om diagnostic is used to distinguish the DR effect. For our model,

$$Om(x) = \Omega_{d00} + \frac{(\Omega_{m0} + \Omega_{d0}) (x^2 + 1) (x + 1)}{x^2 + x + 1} + \frac{\Omega_{d0} (1 + x)}{x^2 + x + 1}.$$  \hspace{1cm} (25)
Figure 5. The 1σ parameter contours given by different data are presented.

Generally, the effect of today’s DR makes \( \delta \Omega_{m0} < 4 \Omega_{d0}/3 \). Meanwhile, the effect of today’s curvature makes \( \delta \Omega_{m0} < \Omega_{d0}/2 \). As \( f \) (or \( \Omega_{d0} \)) are relatively small, \( \delta \Omega_{m0} \) is smaller than \( \delta \Omega_{m0} \). Robustly, \( \Omega_{d0} \) (\( \sim 10^{-5} \)) is four orders smaller than \( \Omega_{m0} \) (10\(^{-1} \)) and three order smaller than \( \Omega_{d0} \) (10\(^{-2} \)). The relation of \( \Omega m - z \) are drawn out in figure 4 for the flat and
curved DR cases. In the flat one, the best fitting value of $\Omega_m$ is nearly constant, so does its $1\sigma$ and $2\sigma$ ranges. But the behavior of $\Omega_m$ in the curved $\Lambda$CDM model and the curved DR model shows dynamical signals. Considering the flat $\Lambda$CDM model which gives a constant $\Omega_m$ [52] as well, the flat DR model could not be distinguished from the flat $\Lambda$CDM model while the curved DR model can.

5.2.3. Parameter degeneration. Figure 5 presents the contours of $\Omega_{m0} - f$, $\Omega_{d0} - f$ and $\Omega_{m0} - \Omega_{d0}$ of the flat and curved DR models given by different datasets. As we mentioned above, the SNLS data give out loose constraints. Meanwhile, the three data (data 2–4) give much tighter constraints which also have the same contour directions. For the contour of $\Omega_{m0} - \Omega_{d0}$, all the data give the same constrain direction. In contrast, for $\Omega_{m0} - f$ and $\Omega_{d0} - f$, the SNLS data and the other data give contours with different directions. Obviously, the degeneration between the DR parameter $f$ and the other parameters need more data to break.

6. The summary

Theoretically, after adding DR, the phase-analysis method proved that the universe derived from the DR model could go through the radiation-dominated phase, the matter-dominated phase and the dark energy-dominated phase sequentially. In a conclusion, the model is compatible with the history of our universe.

Observationally, we use the SNLS, WMAP9, PLANCK, BAO, $H(z)$ and BBN data to constrain the DR part. As expected, the DR wiped out the tension between the SNLS data and the other data in the flat $\Lambda$CDM model. And, the constraining results are at a reasonable level, e.g. $f \sim 10^{-5}$. The small DR parameter $f$ gives a small deviation of $\omega_0$ and $\omega'_0$. And, the effect of DR makes the best fit value of $N_{\text{eff}}$ slightly larger than 3.04. Anyway, the $\Omega_m$ diagnostic could extract the curved DR from the flat $\Lambda$CDM model, but it has no effect on the flat DR model. And, more data are needed for DR because of parameter degenerations.

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