Fate of branes in external fields is reviewed. Spontaneous creation of the Brane World in $AdS_5$ with external field is described. The resulting Brane World consists of a flat 4d spatially finite expanding Universe and curved expanding "regulator" branes. All branes have a positive tension.

During the last years branes were recognized as the important ingredient of the complete picture in the string and field theories. The branes actually have the dual nature; they can be considered as the fundamental objects where the open strings can end on and as the solitons in the supergravity theories. The branes below can be viewed on as the fundamental ones or as effective ones, e.g. domain walls. Domain walls typically appear in the context of N=1 SUSY gauge theories and since they saturate BPS condition they can be treated quasiclassically. In what follows we shall focus on the behaviour of the branes in the external fields and the applications of some quasiclassical processes involving branes to the Brane World scenario.

The low-energy brane action consists of two pieces, a tension term and a charge term. Tension term reads

$$S_{\text{tension}} = T \int \sqrt{\det(g_{\text{ind}})}$$  \hspace{1cm} (1)

where the integration is over the brane world-volume, $g_{\text{ind}}$ is the metric induced on the world-volume via its embedding into the target space and $T$ is the tension of the brane. This term is an analogue of the mass term for a particle

$$m \int \sqrt{\det(g_{\text{ind}})}$$  \hspace{1cm} (2)

where the integration is over the particle world-line. 

The charge term looks as follows

$$S_{\text{charge}} = e \int B,$$  \hspace{1cm} (3)

where $B$ is a $n$-form gauge field field, corresponding curvature being $H = dB$. Branes are the sources for this field and they are affected by this field. In what follows we assume $H$ to be a top degree form.

Everything we are going to discuss here is related to the Schwinger type process - production of branes by homogeneous external field $H$. 
1 A warm up example

Consider first a warm up example: production of the particles by a homogeneous $E$ field.

For this case the effective action reads

$$S_{\text{eff}} = TL - eEA$$  \hspace{1cm} (4)

where $L$ is the length of the world-line of the particles produced, $T$ is a mass of the particle, (as usual, particle-antiparticle history looks like a closed world-line), and $A$ is the area surrounded by the world-line.

Notice, by the way, that upon the appropriate identification of the parameters, the same effective action describes false vacuum decay in $(1+1)$ scalar field theory. Particles are substituted by kinks, electric field - by the energy difference between false and true vacua.

Extremal world-line for the action Eq.(4) is a circle of the radius $\bar{r} = \frac{T}{eE}$ therefore the probability $P$ of the spontaneous process looks as follows

$$P \propto \exp\left(-\frac{\pi T^2}{eE}\right).$$ \hspace{1cm} (5)

Notice that Minkowskian evolution is obtained from the Euclidean bounce by the analitical continuation.

2 Spontaneous production of branes

The above warm-up example is easily generalized to the case of the spontaneous production of branes in a homogeneous $H$ field [4]. The effective action in the assumption of constant $H$ reads

$$S_{\text{eff}} = TA - ehV.$$ \hspace{1cm} (6)

World-volume of the brane produced forms a closed hypersurface (like the world-line of the particle produced in the above example). $A$ in Eq.(6) is the area of the world-volume, $V$ - volume of the region inside the brane. The Euclidean bounce is a $p + 1$-dimensional sphere of radius $\bar{r} = \frac{(d-1)T}{eh}$. The value of the effective action on the bounce yields the probability of the brane production:

$$P \propto \exp(-\text{const} \frac{T^d}{(eh)^{d-1}}).$$ \hspace{1cm} (7)

Analogously to the above example, The Minkowskian evolution can be obtained from the Euclidean bounce via the analytical continuation.
3 Induced brane production

Let us now consider the induced brane production [2]. To this aim we have to introduce a new ingredient - brane junctions [3]. Brane world-volumes can meet along the junction manifold. Angles at which the branes meet each other are fixed by the tension force balance condition. Of course at the junction the charge conservation condition is fulfilled. The fact of the key importance is that junctions are BPS configurations therefore they are classically stable and can be used in the quasiclassical considerations. The BPS property of the junction of the fundamental branes is more or less clear while the similar argument for the effective branes follows from the existence of the specific central charges in SUSY algebra. These central charges were found in WZ model in D=4 [4] as well as N=1 SYM theory [5]. The effective junctions have been also recognized in the M theory framework [6].

The setup for the induced brane production is as follows. There is external homogeneous field and there is external neutral brane which can have junctions with the charged branes to be produced. The question is what is the probability of such process.

Let us again begin with a warm up example - one particle induced decay in (1+1) [7]. One has a massive particle in the false vacuum and the corresponding field has a zero mode on the wall which could separate false and true vacua. Then, if a bubble of the true vacuum inside the false vacuum is produced, it is more profitable for the particle to ride a part of its way in the form of zero mode on the wall of the bubble. The effective action describing this case,

\[ S_{\text{eff}} = TL - eEA + m\tilde{L}, \]  

(8)
differs from Eq.(4) only by the last term where \( m \) is the mass of the extra particle, \( \tilde{L} \) is the length of the extra particle world-line (only outside the bubble).
The bounce is now glued from the two segments of a circle of the same radius as in the case of the spontaneous decay. These two segments meet with the world-line of the particle (junction) at the angle $\alpha$ which is defined by the force balance condition, $m = 2T \cos \alpha$.

The "charge conservation" in the present case is equivalent to a trivial fact that when one passes through the bubble crossing its wall twice, one gets back to false vacuum. One then straightforwardly compute the probability of the false vacuum decay [7]. There are two clear limiting cases. When mass of the particle is small compared to the mass of the wall, bounce is not disturbed and the probability is the same as in spontaneous case. When mass of the particle is close to $2T$, bounce shrinks to a point and there is no exponential suppression in the induced vacuum decay.

Generalization to the case of branes has been developed in [2].
The bounce consists now from two segments of (d-1) dimensional sphere of the same radius as in the case of the spontaneous brane production, glued to the external brane along the junction manifold. The force balance condition reads $\tilde{T} = 2T \cos \alpha$, where $\tilde{T}$ is the tension of the external neutral brane. Minkowskian evolution follows from the analytical continuation of the bounce. The calculation of the probability is straightforward and two clear limiting cases are the same as in the particle case.

Let us mention the interesting application of such process [2]. Let us consider the noncommutative monopoles that is monopole solutions when the external B field is switched on. It is known that monopoles can be represented by D1 string ending on D3 branes [8]. In the noncommutative case one can
consider the exact solution to the classical equations of motion in U(1) theory \[9\]. If there is a constant \( H = dB \) then according to the process above the D1 string is exponentially unstable with respect to the decay into the dyonic strings. From the point of view of U(1) theory on D3 worldvolume it means that monopole in the case of nonconstant noncommutativity nonperturbatively decays into dyons.

4 A sketch of Brane World

Now let us make a digression to sketch an idea of the Brane World. This is an alternative to the idea of compactification. In the compactification approach extra dimensions are compact and small and thus cannot be seen at the moderate energies. In the Brane World, extra dimensions are infinite, but the matter \[10\], gauge fields \[11\] and gravity \[12\] are localized on a brane (domain wall or other topological defect in the extra dimensions).

The model considered in \[12\] involves two branes localized at different points on a circle (the 5th dimension is \( S^1 \) of arbitrarily large radius). One of the branes provides the physical worldvolume for d=4 (RS-brane), the other is a so-called regulator brane (R-brane). The gravity is localized on the RS brane. The drawback of the model is that R-brane has a negative tension (see e.g. discussion in \[13, 14\]).

Many modifications of the construction in \[12\] were studied (see e.g. \[15\] and other references to the original paper \[12\]), most of them includes the negative tension brane.

Let us consider in more detail the construction in \[10\].
It includes three branes localized on a line (the 5th dimension is $R^1$), two negative tension R-branes and one RS-brane between them. Cosmological constant outside R-branes is zero, cosmological constant between R-branes and RS-brane is negative. Notice that in this model cosmological constant is not really a constant, it jumps on the R-branes. So, in fact, there is a hidden external field in the model, and R-branes are charged with respect to it. This motivates the following construction in [17].

5 Spontaneous creation of the Brane World

In [17] a spontaneous creation of the Brane world in a homogeneous external field was described. The process considered is a sort of inverse to the induced brane production in the external field. Its particle counterpart is the spontaneous creation of the charged pair with the additional neutral particle in the
electric field. The calculation of its probability $P \propto \exp(-S)$ is straightforward and yields

$$S = \frac{2T_R^2}{eE} \left( \pi - \arcsin \left( 1 - \frac{T_{RS}^2}{4T_R^2} \right)^{1/2} \right) + \frac{2T_R T_{RS}}{eE} \left( 1 - \frac{T_{RS}^2}{4T_R^2} \right)^{1/2} \quad (9)$$

The bounce now consists of two segments of the charged branes (R-branes) which are glued along the junction manifold with the neutral brane (RS-brane) which is inside the bubble.

Remarkably, Einstein equations can be explicitly solved for the quite complicated brane configuration. The metric consists of two AdS pieces sewed with each other on the RS-brane and with outside flat metric on the R-branes. RS-brane inherits AdS metric, which however can be done as flat as one wishes. Of course, the metric of the "Euclidean" classical solution determines the "Minkowskian" evolution of the Universe. Importantly, the R-branes are not static, they accelerate as they should in the external field. RS-brane remains at rest.

After continuation into Minkowskian space our Universe is a spacially finite, growing slice of AdS space. The spacial radius of R-brane is also growing, and we are eventually left with the picture of [12], or [16], type. A peculiar property
of the process described is that RS-brane is not flat, it inherits $AdS_4$ metric so 4d cosmological constant problem remains unsolved. To cure this peculiarity, in [18] we have described the spontaneous creation of the Brane World with flat RS-brane.

6 Big Bang in $AdS_5$ with external field

The setup in [18] is as follows. There is $AdS_5$ with nonvanishing homogeneous top degree form field and charged test branes. The aim is to construct a tunneling into the Brane World such that the resulting RS-brane is a peace of the same flat section of $AdS_5$ as in [12]. Thus our 4d Universe living on RS-brane is flat and spatially finite and we can take for granted from [12] the 4d localization of gravity on the RS-brane and the validity of 4d Newton law far enough in the future. RS-brane is restricted by junction manifold where RS-brane meets with the rapidly expanding R-branes.

Brane production in this setup has been studied in [19]. The study essentially is reduced to the consideration of the minimal charged surfaces in AdS with the homogeneous field. These surfaces were (up to minor subtleties) described in [19]. They are classified into three classes: undercharged ones (saturating a sort of BPS inequality between charge and tension of the brane), overcharged ones (breaking the BPS inequality) and BPS ones. Actually, only overcharged ones were in [19] given a tunneling interpretation, the others have infinite volume. Our innovation compared to [19] is to include junctions of those surfaces. Our bounce now is glued out of three peaces of branes - of BPS one, of undercharged one and of overcharged one (see fig.5). The BPS brane plays the role of RS-brane, the others are R-branes. Notice that the overcharged branes inherit the metric of the sphere, the undercharged branes - the one of AdS, and the BPS brane - the flat one, we shall discuss this point in more details later.

7 Charged surfaces in AdS

We now describe, essentially following [19], ”Euclidean” minimal charged surfaces, that is, solutions of test brane worldsheet equations relevant for tunneling.

Let us consider the metric of AdS as in [19],

$$ds^2 = R_{ads}^2 \left( \cosh^2 \rho \, d\tau^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_{d-2}^2 \right),$$

(10)
where $R_{ads}$ is the anti-de Sitter radius, and assume that the curvature form $H = dB$ of the $B$-field is proportional to the volume form with a constant coefficient (flux density). Assuming the spherical symmetry of the brane world-sheet, one reduces the effective action to:

$$S = T R_{ads}^{d-1} \Omega_{d-2} \int d\tau \left[ \sinh^{d-2} \rho \sqrt{\cosh^2 \rho + \left( \frac{d \rho}{d \tau} \right)^2} - q \sinh^{d-1} \rho \right]$$

(11)

where $\Omega_{d-2} = \frac{2 \pi^{(d-1)/2}}{\Gamma(d/2)}$ stands for the volume of a unit d-2 sphere and $q$ is a constant made out of the flux density, brane charge and the brane tension. The condition $q = 1$ is identified in [19] as the BPS one. The branes with $q < 1$ will be referred to as undercharged ones and those with $q > 1$ as overcharged.

Before describing them we would like to relate the metric Eq.(10) used in [19] to more canonical ones. Upon the change of variables, $$\tanh \tau = \tan \theta$$

(12)

the metric Eq.(11) is put to the form

$$ds^2 = R_{ads}^2 d\tau^2 + d\theta^2 + \sin^2 \theta d\Omega_{d-2}^2$$

(13)

Finally, upon the change of variables

$$z = e^\tau \cos \theta, \quad r = e^\tau \sin \theta$$

(14)

one obtains the following canonical form of AdS metric

$$ds^2 = R_{ads}^2 \frac{dz^2 + dr^2 + r^2 d\Omega_{d-2}^2}{z^2}.$$ $$(15)$$

The minimal surfaces for the case $q < 1$ look as follows:

$$\cosh \rho = \frac{\sinh \tau_m}{\sinh(\tau + a)}$$

(16)

where $\tanh \tau_m = q$.

In coordinates Eq.(12) the undercharged surfaces take the form

$$\cos \theta = \frac{\sinh(\tau + a)}{\sinh \tau_m}.$$ 

(17)
Restriction of the $AdS_{5}$ metric Eq.(13) onto the undercharged surfaces Eq.(19) gives the metric of $AdS_{4}$.

In the case $q > 1$ the relevant charged minimal surfaces look as follows:

$$\cosh \rho = \frac{\cosh \rho_{m}}{\cosh(\tau + b)}$$

(18)

where $\tanh \rho_{m} = 1/q$. In coordinates Eq(12) the overcharged surfaces take the form

$$\cos \theta = \frac{\cosh(\tau + b)}{\cosh \rho_{m}}$$

(19)

Restriction of the $AdS_{5}$ metric Eq.(13) onto the overcharged surfaces Eq.(19) gives the metric of the sphere $S_{4}$. The BPS case ($q=1$) can be obtained as a limit from either of the cases above. Corresponding surfaces look as follows:

$$\cos \theta = \frac{1}{z_{0}}e^{\tau}$$

(20)

where $z_{0}$ is a constant. Upon the inversion transformation one obtains

$$e^{\tau} \cos \theta = z_{0}$$

(21)

In terms of Eq.(15) these are the surfaces $z = z_{0}$. Apparently, restriction of the AdS metric onto these surfaces gives flat Euclidean metric.

Only overcharged surfaces admit a tunneling interpretation [19], since undercharged and BPS ones reach the boundary of AdS space and thus have infinite volume and infinite effective action.

8 The solution

We shall now construct the bounce which describes the tunneling into the Brane World [18]. It is glued out of three pieces - a piece of BPS brane located along $z = z_{0}$ section, Eq.(21), and playing a role of RS-brane in the Brane World, a piece of undercharged brane, Eq.(17), located above the BPS brane (in the fixed coordinates of the type of Eq.(15) and playing a role of one R-brane, and a piece of overcharged brane, Eq.(19), located below the BPS brane and playing the role of the other R-brane (see fig.5). All three pieces are glued along the junction manifold. The usual junction conditions are the charge conservation and the tension forces balance at the junction [3]. We shall assume that there is no junction energy contribution to the effective action, though this assumption is not crucial for the existence of solution.
Apparently, the configuration sketched above has a finite action since none of the constituting pieces reaches the AdS boundary, hence the tunneling goes with a finite probability which can be easily computed. Notice that analogously to constructions in [19] one needs a brane breaking BPS inequality in order to have a finite probability of tunneling. An example of this type of brane in string theory was given in [19].

Junction manifold is of the type $\theta = \text{const}, \, \tau = \text{const}$, and since overall rescaling of the solution is not important (the effective action is invariant under total rescaling, or, equivalently, under total shift in $\tau$-coordinate), we take it as follows:

$$\theta = \theta_c, \, \tau_c = 0 \quad (22)$$

From Eqs. (21), (17), (19) one immediately obtains

$$\cos \theta_c = z_0,$$
\[
\begin{align*}
\sinh a &= \sinh \tau_m \cos \theta_c, \\
\cosh b &= \cosh \rho_m \cos \theta_c.
\end{align*}
\]

The Eqs. (21), (17), (19), (22), (23) specify the geometry of the Big Bang bounce. However we still have to define charges of the branes involved and to verify that the junction conditions are fulfilled.

The charge conservation, with appropriate choice of orientation of the branes, reads
\[
Q_0 = Q_- - Q_+.
\]
Hereafter subscripts "0", "-", and "+" indicate the BPS brane, the undercharged brane and the overcharged brane. All \(Q\)'s are assumed to be positive.

The force balance condition obviously reads (we take projections onto \(\partial/\partial \theta\) and onto \(\partial/\partial \tau\) directions):
\[
\begin{align*}
T_0 \cos \alpha_0 + T_- \cos \alpha_- &= T_+ \cos \alpha_+ \\
T_0 \sin \alpha_0 + T_+ \sin \alpha_+ &= T_- \sin \alpha_- 
\end{align*}
\]
where the angles are defined on fig.5. From geometry of the picture and using Eqs. (19), (17), (22), (23) one obtains
\[
\tan \alpha_0 = \frac{\theta_c}{\sinh \tau_m}, \\
\tan \alpha_- = \frac{\sin \theta_c \sinh \tau_m}{\cosh a}, \\
\tan \alpha_+ = \frac{\sin \theta_c \cosh \rho_m}{\sinh b}.
\]

According to the above definition of \(q\) (see Eq.(11)) we take the following parametrization of the tensions of the three pieces of the bounce:
\[
T_0 = Q_0 T, \quad T_- = \frac{Q_-}{q_-} T, \quad T_+ = \frac{Q_+}{q_+} T
\]
where \(q_- = \tanh \tau_m, \quad q_+ = 1/ \tanh \rho_m\).

Substituting Eq.(27) into the force balance condition Eq.(25) and using the charge conservation condition Eq.(24) one obtains a linear system for the charges of R-branes:
\[
\begin{align*}
Q_- \left( \cos \theta_c + \frac{\cos \alpha_-}{q_-} \right) - Q_+ \left( \cos \theta_c + \frac{\cos \alpha_+}{q_+} \right) &= 0 \\
Q_- \left( \sin \theta_c - \frac{\sin \alpha_-}{q_-} \right) - Q_+ \left( \sin \theta_c - \frac{\sin \alpha_+}{q_+} \right) &= 0
\end{align*}
\]
Using Eqs. (23, 26) one can straightforwardly verify that determinant of this linear system is equal to zero, so the system is compatible and defines the ratio of charges of the R-branes at which the force balance condition Eq. (24) is satisfied. This completes our construction of the Big Bang bounce.

9 Conclusion

Our conclusions are as follows:
1. We have described induced brane production in the external field;
2. We have found the probability of the tunneling into the Brane World (a sort of Big Bang);
3. It was shown that there is no need for the negative tension branes to be included in the scenario with the external field;
4. The direct consequence of our approach is the 5d early Universe;
5. It was shown that the simple brane configuration in the $\text{AdS}_5$ with the external field yields the flat 4d Universe.

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