Compressional-Shear Velocity Model of "Toki" Field using Support Vector Regression, Offshore Niger Delta

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Abstract. Shear sonic log is invaluable for fluid and lithological classification. For most fields in the Niger Delta, shear sonic log are rarely acquired along with the compressional sonic log. Where acquired, they are usually very few relative to the number of wells. Hence, there is a need to derive shear velocity from the compressional sonic log. Most of the available models such as Castagna’s mud-rock model are not calibrated to suit the Niger Delta basin. Existing localized models are based on non-robust linear models such as the Ogagarue's localized compressional and shear velocity models for Niger Delta sedimentary region. These models are not reliable in the presence of hydrocarbon and anisotropy. A robust support vector regression (SVR) machine learning algorithm has been used to predict the relationship between compressional velocity and shear velocity. This study shows that in the Niger Delta, shear velocity can be predicted from compressional velocity with relatively high accuracy by using machine learning algorithms such as support vector regression. The mean-square error (MSE) obtained using Castagna’s and Ogagarue's models compared with acquired data are 1.8 and 2.3 times that of the value obtained using support vector regression respectively.

1. INTRODUCTION
The relationship between shear wave velocity and other well log derived attributes is very important in rock physics and reservoir characterization for fluid discrimination as well as lithological classification. Other uses of compressional and shear velocities include synthetic seismogram generation, determination of porosity, stratigraphic correlation, compaction, and overpressure analysis. For most oil and gas fields in the Niger Delta, shear sonic logs are rarely acquired along with the compressional sonic log. Where acquired, they are very few relative to the number of wells. The standard practice is to predict shear velocity from compressional velocity using an established empirical equation. The most popular compressional-shear velocity is Castagna’s mud-rock model [1]. Although the Castagna’s mud-rock model was derived from freshwater clastic rocks in the Gulf Coast, it is widely used for prediction in the Niger Delta basin ([2] and [3]). The Castagna’s mud-rock model was modified by Ogagarue in order to localize it to Niger Delta sedimentary basin [4]. His analysis showed that Castagna’s mud-rock model under predicted sonic velocities by 6% to 8% lower of the raw sonic values. Ogagarue's model for sonic velocity prediction was used by [5] in their computation of rock physics parameters for lithology and fluid prediction.

Recently, the application of machine-learning techniques to solving geophysical problems is growing rapidly ([6], [7] and [8]). In this study, Castagna’s mud-rock model and Ogagarue’s model are
compared with shear sonic velocity prediction using a machine learning approach. Well log data from water-saturated rocks as well as hydrocarbon-saturated rocks are investigated using cheap and effective machine learning approach. The aim of this study is to evaluate compressional-shear velocity model prediction based on a data-driven machine learning algorithm. Support vector regression (SVR) supervised machine learning technique is used to determine the relationship between compressional-shear transit times by extension compressional-shear velocities. Many recently published research work has demonstrated that support vector machines are effective and accurate at estimating the relationship between two or more data features [9]. Typically, the SVR algorithm searches for optimal hyperplane which maximizes the margin, largest separation, between two classes within an error limit which is defined by the margin of tolerance denoted by epsilon. The main advantage of support vector machine based algorithms in solving geophysical problems is that it does not suffer from non-uniqueness and always outputs a global solution. Non-uniqueness and global optimization are some of the challenges in geophysical inversion problems.

Multi-attribute SVR modelling was applied to predict shear sonic transit time without differentiating between sand and shale for gross prediction. Further analysis was carried out for lithology-specific prediction by differentiating between sand and shale which are the primary lithologies in the Niger Delta. The mean-square error (MSE) was computed as a single value metric to evaluate Ogagarue’s model, Castagna’s mud-rock model and SVR model for ‘Toki’ field. Finally, compressional-shear velocity ratio cross plots were explored for litho-fluid reservoir classification.

2. THEORY
An isotropic elastic the medium parameters can be defined by three parameters first Lamé constant ($\lambda$), shear modulus ($\mu$) and density ($\rho$). When waves propagates in an elastic solid, the velocities of waves are defined by the ratio of appropriate elastic modulus and density of the medium. The two types of body wave velocities that travels in an elastic solid medium are primary wave velocity and a shear wave velocity. Equation 1 and 2 defines the relationship between elastic modulus and density for the primary wave velocity, $V_p$ and a shear wave velocity, $V_s$ respectively [10]. For clastic sediments, the densities of sand and shale are similar and overlaps within narrow spectrum because major clastic rock-forming minerals (quartz, mica and feldspar) have densities varying between 2.5-2.75 gram per cubic centimetre. Hence, elastic moduli of clastics have more significant impact on primary wave velocity and shear wave velocity than density. For a given rock type, the elastic modulus for primary wave and shear are different but density is approximately the same value. To eliminate density, equation 1 is divided by equation 2.

$$V_p = \sqrt{\frac{\kappa + 4\lambda}{\mu}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$  \hspace{1cm} (1)

$$V_s = \frac{\mu}{\rho}$$  \hspace{1cm} (2)

Therefore,

$$\frac{V_p}{V_s} = \sqrt{\frac{\kappa + 4\lambda}{\mu}} \mu = \sqrt{\frac{\lambda + 2\mu}{\mu}} = \sqrt{\frac{1-\sigma}{0.5-\sigma}}$$  \hspace{1cm} (3)

where, $\sigma$ is the Poisson’ ratio and $\kappa$, is the bulk modulus. Poisson’ ratio is constrained to a maximum value of 0.5 from equation 3. Generally, Poisson’ ratio for most consolidated rocks is about 0.25 [10]. This implies that,

$$V_p = \sqrt{3}V_s + V_0$$  \hspace{1cm} (4)
Equation 4 is analogous to a linear equation with $V_p$ as a dependent attribute, $V_z$ independent attribute, $m = \sqrt{3}$ and intercept, $V_{0z}$. Ideally $V_{0z}$, is the value of $V_p$ at $V_z = 0$. This represents the primary velocity of the saturating fluid and $V_{0z}$ should be zero for dry rocks.

Castagna’s mud-rock and Ogagarue’s models are based on the linear relationship between compressional and shear velocities in clastic rocks. Equation 5 and 6 are Castagna’s mud-rock and Ogagarue’s models respectively.

$$V_p = 1.16V_z + 1360$$  \hspace{0.5cm} (5)

$$V_p = 1.11702V_z + 1279.08$$  \hspace{0.5cm} (6)

Although a linear relationship exists between compressional and shear velocities for isotropic and homogeneous earth model in practice, due to anisotropy and inhomogeneity the compressional-shear velocity relationship is at best near linear. SVR machine learning approach for shear velocity prediction is more data-driven when compared to Castagna’s mud-rock and Ogagarue’s models. SVR can predict output from multi-attribute input data space. Hence, it can accommodate more relevant geologic information from other sources to generate a more robust shear velocity prediction model.

Given a training dataset: $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, $x$ denotes a space of input independent multi-attributes $(V_p, \rho, \ldots, \gamma)$ and $y$ the dependent output attribute $(V_s)$. SVR works by seeking a function $f(x)$, hyperplane, that has maximum deviation ($\varepsilon$) from observed $y$ such that,

$$f(x) = \langle w, x \rangle + b \quad w \in \mathbb{R} \quad \text{and} \quad b \in \mathbb{R}$$  \hspace{0.5cm} (7)

where, $w$ is a normalized vector to the hyperplane and $b$ is the offset, and $\mathbb{R}$ is a real number [11]. Solution to equation 7 is achieved by minimizing equation 8

$$\frac{1}{2} \|w\|^2$$  \hspace{0.5cm} (8)

subject to the following boundary conditions:

$$y_n - \langle w, x_n \rangle - b \leq \varepsilon \quad \text{and} \quad \langle w, x_n \rangle + b - y_n \leq \varepsilon$$  \hspace{0.5cm} (9)

Equations 8 and 9 have been further modified to optimize the constraints by introducing slack variables [12]. The slack variables are put in check by penalizing the insensitive loss function.

3. METHODOLOGY

The materials used for this study are a suite of well log comprising of gamma ray (GR), deep resistivity (RESD), density (DENS), compressional sonic transit time (DTC) and shear sonic transit time (DTS). This research employed the use of open source software, python integrated development environment and scikit learn python library, for modelling on Anaconda platform. The summary of the methodology adopted for this study is shown in Figure 1.
Firstly, the data were pre-processed by applying a null mask to removed unlogged depth data. The input data feature distributions were explored using python statistical analysis tools.

Table 1: Input well log statistical summary

| Feature | Distribution |
|---------|--------------|
|          |              |

3.1. Feature Distribution
The probability distributions of the various log attributes were created using a bin size of 50. The gamma ray is a tri-modal distribution with a characteristic edge peak distribution bar. Similarly, the density distribution shows an edge peak distribution bar at the upper limit of the distribution. All other features are essentially unimodal. None of the attribute distribution is symmetrical. All the attribute distributions are nearly normal or centered apart from resistivity (Figure 2).
The skewness values are 0.50, -0.66, 0.30, 10.64 and 0.39 for gamma ray, density, compressional sonic transit time, resistivity and shear sonic transit time respectively. The largest skew occurred in resistivity distribution. Only the density distribution is negatively skewed. Based on kurtosis analysis, the most infrequent value deviation occurred in the resistivity distribution. Table 2 shows the kurtosis for the attributes.
### Table 2: Attributes kurtosis

| S/N | Feature                        | Kurtosis |
|-----|--------------------------------|----------|
| 1   | Gamma ray                      | 1.87     |
| 2   | Density                        | 0.74     |
| 3   | Compressional sonic transit time| -0.48    |
| 4   | Resistivity                    | 138.51   |
| 5   | Shear sonic transit time       | -0.50    |

#### 3.2. SVR Predictive Modelling

The input data was separated into dependent and independent features. The multi-attribute dependent data comprising gamma ray, density, compressional sonic transit time and Shear sonic transit time features were re-scale to values between 0 and 1 to remove data bias to high feature values. The resistivity attribute was not used in the final predictive modelling due to data bias even after normalization and standardization. Thirty percent (30%) of the input data was reserved for model deployment and evaluation since only one well in this field contain shear sonic transit time while seventy percent (70%) was used for modelling and validation.

Some of the methods than be employed for splitting between the training set and test set includes: train-test sets, k-fold cross-validation, leave one out cross-validation and repeated random test-train splits. For this study, k-fold cross-validation was adopted because it works by dividing the training set into k number of partitions, performing training and testing on different splits data of k-1 and k-9 several times. Hence, this technique is more reliable in making predictions on a new set of data and helps to prevent the problem of model overfitting. A 10-fold cross-validation was applied to the training and testing dataset without differentiating sand from shale. The model obtained from this gross SVR training was then used to predict shear sonic transit time for the new dataset. The predicted shear sonic transit times were converted to shear sonic velocities. The corresponding compressional sonic velocities were converted to shear sonic velocities using Ogagagure’s model and Castagna’s model. After this, the three predicted shear sonic velocities were compared for accuracy using a single metrics – squared mean error (MSE).

Furthermore, the data were differentiated into sand and shale using a shale cut-off of 86.6 GAPI determined from the histogram distribution plot. The modelling and prediction process is repeated from cross-validation splitting to model comparison for sand and shale separately.

#### 3.3. SVR Hyper-Parameter Tuning

Using a linear kernel, a three-parameter grid search for SVR optimization was set-up. The first parameter is gamma, γ, of a Gaussian function with a search space ranging from 1e-9 to 1e-4. The second is the penalty parameter of the error term, C, with a search space between 1 and 100. The penalty parameter determines the margin relation limit [13]. The final parameter is the error sensitivity parameter (ε), epsilon, ranging between 0.1 and 0.5. The cross-validation (CV) parameter was set to the default. Optimal SVR parameters of C = 1000, epsilon = 0.5 and gamma = 1e-09 was obtained. The optimized parameters were used to refine the initial SVR prediction models.

#### 4. RESULTS AND DISCUSSION

The contoured cross plot of compressional sonic slowness versus shear sonic slowness shows a good relationships between these two features (Figure 3). The relationships on the contoured plot can be considered as a series of straight lines with varying slopes and intercepts. In order words, the contoured plot describes the relationship between compressional velocity and shear velocity in an inhomogeneous
Figure 3: Contoured plot of compressional slowness versus shear slowness

The contoured plot shows that there is a very strong positive correlation between the two well log attributes as evident in the correlation coefficient value of 0.97 and the highly significant p-value of 0. The p-value of 0 implies there is 100% probability that all the data points are related.

4.1. Shear Sonic Slowness Prediction Comparison

The gross lithology shear velocity plot indicates that there are at least three major velocity-depth trends: 2600 -2900 m, 2900 -3100 m and 3100 m - 3400 m. The comparison of the SVR model, Ogagagure’s model and Castagna’s model in undifferentiated sand and shale sequences show that the three models overlap in some zones to a high degree. However, Ogagagure’s model tends to overestimate shear velocities while Castagna’s model underestimates shear velocities. SVR model prediction suffers the least deviation when compared with the Ogagagure and Castagna’s models (Figure 4).
The sand shear velocity plot indicates at least two major velocity-depth trends: 2600 - 2950 m and 3150 m - 3400 m. The comparison of the SVR model, Ogagagure’s model and Castagna’s model in differentiated sand sequences show that the three models overlap significantly in some zones. However, Ogagagure’s model tends to overestimate shear velocities while the Castagna’s model underestimates shear velocities similar to the undifferentiated sand and shale sequences. SVR model is robust and suffer the least deviation from the observed data when compared with the Ogagagure and Castagna’s model (Figure 5).
Figure 5: Observed vs predicted sonic velocity in the sand only

The shale shear velocity plot indicates at least two major velocity-depth trends: 2600 - 2900 m and 2950 m - 3400 m. The comparison of the SVR model, Ogagure’s model and Castagna’s model shear velocity predictions in differentiated shale sequences are similar to that of sand and undifferentiated sand and shale. (Figure 6).
Figure 6: Observed vs predicted sonic velocity in shale only

The shale shear velocity plot indicates at least two major velocity-depth trends: 2600 -2900 m and 2950 m - 3400 m. The comparison of the SVR model, Ogagure’s model and Castagna’s model shear velocity predictions in differentiated shale sequences are similar to that of sand and undifferentiated sand and shale. (Figure 6).

For gross lithology shear velocity prediction, the mean-square error (MSE) obtained using
Castagna’s and Ogagarue's models and SVR models are 96.79, 125.71 and 52.19 respectively. This implies that the mean-square error (MSE) obtained using Castagna’s and Ogagarue's models compared with acquired data are 1.8 and 2.3 times that of the value obtained using support vector regression respectively. Generally, for differentiated and undifferentiated lithologies SVR models have the smallest mean-square error. The best shear velocity prediction is that of SVR model in shale while the worst shear velocity prediction is obtained from Ogagarue's model in shale. Ogagarue's model is better in differentiated sand than Castagna’s model while Castagna’s model is better in differentiated shale than Ogagarue's model (Table 3).

Table 3: Mean Square Error for all models

| Lithology               | Mean Square Error | Mean Square Error | Mean Square Error |
|-------------------------|-------------------|-------------------|-------------------|
| SVR Model               | 52.92             | 125.71            | 96.79             |
| Ogagarue's Model        | 57.19             | 61.61             | 145.22            |
| Castagna’s Model        | 50.55             | 147.94            | 79.30             |

4.2. Compressional - Shear Velocity Ratio for litho-fluid Analyses

The compressional-shear ratio is very useful for lithology as well as fluid discrimination. Figure 7 shows that for any given depth, shale tends to have slightly higher compressional-shear ratio than corresponding sand. Figure 8 shows clusters of sand and shale on compressional-shear velocity gamma ray cross plot.

Figure 7: $V_p / V_s$ - Depth Trend
Figure 8: $V_p / V_s$ ratio lithology cross plot

This $V_p / V_s$ is sensitive to change in saturating fluid type because compressional wave velocity decreases with change in saturating fluid from brine to light hydrocarbon while shear wave velocity increase with change is saturating fluid from brine to light hydrocarbon [14]. Figure 9 shows the fluid trends in sand clusters from brine saturated sand to gas saturated sand. In this field, the $V_p / V_s$ for all sand sequences varies between 1.5 and 2.2. The $V_p / V_s$ for brine saturated sand varies between 1.8 and 2.2 while $V_p / V_s$ for oil saturated sand varies between 1.6 and 1.8. The $V_p / V_s$ for gas saturated sands are between 1.5 and 1.6.

Figure 9: $V_p / V_s$ - depth trend for litho-fluid
5. CONCLUSION
Well log data have been investigated using cheap and effective open source machine learning algorithm. Shear slowness (velocity) was predicted from multi-attribute well log features using support vector regression (SVR) machine learning algorithm for a field, offshore Niger Delta. The shear velocity predictions were carried out for gross undifferentiated sand-shale sequences as well as differentiated sand and shale sequences. Also, shear velocity was computed from compressional velocities using established Ogagagure’s and Castagna’s empirical models. The comparison of SVR, Ogagagure’s and Castagna’s empirical models show that for the three case studies the SVR model predicted shear velocity with the lowest error. Hence, SVR is more accurate than other models. For undifferentiated sand-shale sequences, the mean-square error (MSE) obtained using Castagna’s and Ogagarue's models compared with acquired data are 1.8 and 2.3 times that of the value obtained using support vector regression respectively.

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