Heavy quark asymmetries with DELPHI

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The measurements of the forward-backward asymmetry in \(Z \rightarrow c\bar{c}\) and \(Z \rightarrow b\bar{b}\) decays are among the most precise determinations of \(\sin^2 \theta_W^{\text{eff}}\). In this paper the results obtained by the DELPHI experiment at LEP with three different analyses are reviewed together with the impact of the combined LEP result on the global Electroweak fit.

I. INTRODUCTION

As a consequence of the parity violating couplings of the Z boson to the fermions, in electron positron annihilation at the Z mass fermions are more likely produced in the forward direction, with respect the incoming electron, than in the backward. In the Electroweak Standard Model the asymmetry at the Z pole is expressed as:

\[
A_{FB}^{f,0} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f
\]

\(\mathcal{A}_e\) and \(\mathcal{A}_f\) are functions of the ratio \(x_f\) of vector \(v_f\) and axial \(a_f\) couplings of the Z boson to the fermions:

\[
\mathcal{A}_f = \frac{2x_f}{1+x_f^2}
\]

The ratio depends on the quantum numbers of the fermion, the charge \(Q_f\) and the weak isospin \(I_{3,f}\), and on the fundamental parameter of the theory the electroweak mixing angle \(\sin^2 \theta_W\):

\[
x_f = 1 - 4|Q_f| \sin^2 \theta_W.
\]

The dependence of the Born level \(\mathcal{A}_f\) on \(\sin^2 \theta_W\) for different fermion species is shown in fig 1. For \(\sin^2 \theta_W \approx 0.23\) the asymmetry for q\bar{q} final states is from 3 to 5 times larger than for leptonic final states.

As among all the q\bar{q} final states the decays of the Z boson into c\bar{c} and b\bar{b} pairs are the most easily distinguished, the experimental effort was concentrated on the measurement of \(A_{FB}^c\) and \(A_{FB}^b\).

Most of the electroweak radiative corrections can be accounted for by introducing in the Born level equations an effective electroweak mixing angle \(\sin^2 \theta_W^{\text{eff}}\) defined for each fermion family. On the contrary the sensitivity of the quark asymmetry to the final state couplings is heavily suppressed. This can be understood from the the dependence of the sensitivity on \(Q_f\) and \(x_f\):

\[FIG. 1. \quad \text{Dependence of Born level } A_f \text{ on } \sin^2 \theta_W \text{ for leptons, up-type quarks and down-type quarks.}\]

1 Throughout this paper pseudo-observables defined at the Z pole are indicated with an apex 0 while values measured at the Z peak (√s = 91.26 GeV) without.
\[
\frac{1}{A_f \sin^2 \theta_{W,\text{eff}}} \frac{\partial A_f}{\partial f} = 4|Q_f| \frac{1-x_f^2}{x_f(1+x_f^2)}
\]

The final state \(c\) and the \(b\) quarks have a smaller charge and a higher value of \(x_f\) compared with the initial state leptons. The sensitivity to the quark couplings is reduced of about a factor 50 for the \(b\) and 5 for the \(c\) asymmetry.

The main motivation for a precise measurement of the asymmetry is that \(m_H\), the mass of the still undiscovered Higgs boson, enters in the definition of \(\sin^2 \theta_{W,\text{eff}}\). Table II shows that, even if the dependence is only logarithmic, the measurement of \(A_{FB}^{b,0}\) sets currently the most stringent limits on \(m_H\): A 4\% accuracy in the determination corresponds to values of the mass of the Higgs boson in a range from 70 GeV/c\(^2\) to 1000 GeV/c\(^2\). On the contrary the ratios of partial widths of the Z boson into \(c\bar{c}\) and \(b\bar{b}\) pairs are, in practice, not sensitive to \(m_H\) at all. The current precision on \(A_{FB}^{b,0}\) is smaller than this value and the prediction on the Higgs boson mass is spoiled mainly by the uncertainty on \(\alpha_{QED}(M_Z^2)\).

### TABLE I. Dependence of the electroweak pseudo observables in the heavy flavours sector on radiative correction (from [1]).

| \(m_t\) = 175 GeV/c\(^2\) | \(m_H\) = 300 GeV/c\(^2\) | \(1/\alpha_{QED}(M_Z^2) = 128.896\) |
|--------------------------------|-----------------|-------------------|
| \(A_{FB}^{b,0}\) | 0.0998±0.0010 | ±0.0013 |
| \(A_{FB}^{c,0}\) | 0.0711±0.0008 | ±0.0028 |
| \(R_0\) | 0.2158±0.0002 | ±0.0010 |
| \(R_0\) | 0.1722±0.0001 | ±0.0010 |

### II. EXPERIMENTAL TECHNIQUES.

All the analyses of the asymmetry follow three steps: The tag of the flavour of the decay of the Z boson, the determination of the direction of the primary quark and the separation of quark and antiquark hemispheres. The analyses described differ on how the tag is performed: Exploiting the long flight distance of the heavy flavoured hadron, the presence of a high momentum lepton or finally a reconstructed D meson in the final state. This choice also how the assignment of quark-antiquark hemispheres is done: Using the charge flow in the event or the charge of the identified lepton or the charge state of the D meson. Common to the analyses is the determination of the direction of the initial \(q\bar{q}\) pair done with the thrust axis of the event conventionally oriented to form an angle \(\theta_T < 90^\circ\) with the direction of the incoming electron beam. The observed asymmetry is finally extracted from a \(\chi^2\) fit to the differential asymmetry \(\frac{N_i^+ - N_i^-}{N_i^+ + N_i^-}\), where \(N_i^+ (N_i^-)\) is the number of forward (backward) events in bin \(i\) of \(\cos \theta_T\).

#### A. \(A_{FB}^{b,0}\) using jet charge technique.

Methods exploiting the long fly distance of \(b\)-hadrons \((\gamma/\beta c t_b \sim 2\ mm)\) and the accurate resolution on secondary vertex reconstruction of silicon vertex detectors provide the most efficient way to select a sample of \(b\bar{b}\) events. In order to maximize the efficiency versus purity performance the \(b\)-tag algorithm used in the DELPHI experiment combines the informations from the reconstructed secondary vertex (its effective mass, the rapidity of the tracks associated to it and the fraction of the jet energy carried by them) with the lifetime information.

\(^2\)If not explicitly mentioned charge conjugate states are implicitly included.
In the measurement of $A_{FB}^{b,b}$, the fraction $P_b(cut)$ of $b\bar{b}$ events in the sample after a cut in the $b$-tag variable is determined from the data themselves using the relation:

$$P_b(cut) = \frac{F(cut) - R_c \times \epsilon_c(cut) - (1 - R_c - R_b) \times \epsilon_{uds}(cut)}{F(cut)}$$

where $F(cut)$ is the fraction of the data after the cut, $\epsilon_{uds}(cut)$ and $\epsilon_c(cut)$ the efficiencies for light ($q = u, d, s$) and charm quark taken from the simulation and $R_c$ and $R_b$ the partial widths of the $Z$ into $c\bar{c}$ and $b\bar{b}$ pairs from the Standard Model. Fig. 2 shows the distribution of the $b$-tag variable in data and in simulation. The value of the tag chosen for the analysis corresponds to $\epsilon_b = 75\%$ and $P_b = 92\%$.

The separation of $b$ from $\bar{b}$ quark relies on the hemisphere charge $Q_{\text{hem}}$, defined as the sum of the charges $q_i$ of the tracks in each hemisphere, as defined by the thrust axis $\vec{T}$, weighted by the projection of their momenta $\vec{p}_i$ along $\vec{T}$ itself, to some power $\kappa$:

$$Q_{\text{hem}} = \frac{\sum_i q_i |\vec{p}_i \cdot \vec{T}|^\kappa}{\sum_i |\vec{p}_i \cdot \vec{T}|^\kappa}$$

This estimator is based on the fact that, due to the electric charge conservation, the particles produced in the hadronization of the primary quark retain some information of its charge. In the analysis done by DELPHI $\kappa = 0.8$. This choice minimizes the total statistical and systematic error.

In each event the total charge $Q_{TOT} = Q_F + Q_B$ and the charge flow $Q_{FB} = Q_F - Q_B$ are measured. A part from reinteractions $\langle Q_{TOT} \rangle \simeq 0$, while the average charge flow is directly related to the asymmetries:

$$\langle Q_{FB} \rangle = \sum P_q \eta_q \delta_q A_{FB}^q$$  \hspace{1cm} (1)

$P_q$ is the fraction of events of flavour $q$ in the sample, $\eta_q$ is an acceptance correction factor and $\delta_q = \langle Q_q - Q_{\bar{q}} \rangle$ is the charge separation: It would be twice the charge of the quark if quarks would be observed directly. As for $P_b$, the charge separation for $b$ quark is measured directly in the data. The principle of the method is sketched in fig. 3. For a pure $b$ sample the charge separation leads to an increase of the spread $\sigma_{FB}$ of the distribution of $Q_{FB}$ compared to $\sigma_{TOT}$ from the distribution of $Q_{TOT}$ so that $\delta_q^2 \simeq \sigma_{FB}^2 - \sigma_{TOT}^2$. As for the sample composition $\delta_{u,d,s,c}$ are taken from the simulation, carefully tuned in order to reproduce the measured distributions of hadronic event shapes and charged particle inclusive quantities.

FIG. 2. Distributions of the $b$-tag variable in data and simulation for 1994 data.

FIG. 3. Sketch of the principle of the $\langle Q_{FB} \rangle$ and the $\delta_f$ measurement for a single (down type) flavour f.
The observable \( \langle Q_{FB} \rangle \) in data and simulation as a function of increasing \( b \)-purity of the sample is shown in fig 4. The observed difference is due to different values of both \( \delta_b \) and the input asymmetry between data and simulation.

![Graph showing \( \langle Q_{FB} \rangle \) as a function of b-purity](image)

**FIG. 4.** \( \langle Q_{FB} \rangle \) as obtained from data and simulation for \( \kappa = 0.8 \) and for 1994 data.

To exploit the angular dependence of the asymmetry itself, \( A_{FB} \) is extracted from a \( \chi^2 \) fit of equation 1 to \( \langle Q_{FB} \rangle \) distribution in 4 bins of \( \cos \theta_T \), the acceptance being limited to \( \theta_T > 35^\circ \) by the actual angular coverage of the vertex detector.

The results for the Z peak asymmetry for 1992-95 sample is \( A_{FB}^b = 0.0982 \pm 0.0047 \) (stat). The main contributions to the systematic error are listed in the top part of table II. The largest contribution (\( \pm 0.0011 \)) comes from the hemisphere-hemisphere charge correlation which is due to charge conservation, the common thrust axis and the particle crossovers between hemispheres. This correlation is checked between data and simulation by means of the variable \( H \) defined such that differences in the description of the secondary interactions mostly cancel out: \( H = \langle Q_F Q_B \rangle - \langle Q_{TOT} \rangle^2/4 \). The systematic error accounts for 20% discrepancy between data and simulation found in the distribution of \( H \).

Effects related to the description of the physical processes which could effect the \( b \)-tag performance (gluon splitting into \( c \bar{c} \) and \( b \bar{b} \), \( K^0 \) and \( \Lambda \) content in light quark events, lifetimes, spectra and production fraction of D mesons) are smaller than \( \pm 0.0003 \) each.

**TABLE II.** Main contributions, in units of \( 10^{-3} \), to the systematic error to \( A_{FB}^c \) and \( A_{FB}^b \) for Z peak data.

| Source | \( \delta A_{FB}^c \) | \( \delta A_{FB}^b \) |
|--------|----------------|----------------|
| Jet Charge tag | | |
| total | | 1.6 |
| leptons tag | | |
| \( c \) decay model | 1.8 | 1.4 |
| \( b \rightarrow c \rightarrow \ell \) | 1.9 | 1.3 |
| bgd asymmetry | 4.7 | 1.1 |
| total | 6.3 | 2.7 |
| D mesons tag | | |
| MC stat | 2.5 | 3.5 |
| fit method | 1.7 | 2.8 |
| bgd asymmetry (\( b, c \) quark) | 1.3 | 3.6 |
| \( \chi_{eff} \) | | 5.8 |
| total | 3.5 | 8.5 |

**B.** \( A_{FB}^{c,0} \) and \( A_{FB}^{b,0} \) using prompt leptons.

Electrons and muons are produced in the decay of heavy flavoured hadrons mainly by 3 processes with a branching ratio of about 10% each:

- primary semileptonic \( b \) decays, \( b \rightarrow \ell^- \)
- weak cascades of \( b \) hadrons, \( \bar{b} \rightarrow \bar{c} \rightarrow \ell^- \)
- primary semileptonic \( c \) decays, \( c \rightarrow \ell^- \)
These leptons can be used to select on statistical basis \( c\bar{c} \) or \( b\bar{b} \) events as they have momentum \((p)\) spectrum harder than fragmentation particles, \( \langle x_E \rangle_c \sim 0.5 \) and \( \langle x_E \rangle_b \sim 0.7 \), and high transverse momentum \((p_\perp)\) with respect the direction of the jet \(^1\) because of the high mass of the parent hadron.

The correlation between the charge of the lepton and the one of the quark is exploited to determine the quark hemisphere. The correlation is different in the three classes, resulting in a dilution of the measured asymmetry:

\[
A_{FB}^{ob} = (1 - 2\chi)(f_b - f_{bc})A_{FB}^b - f_cA_{FB}^c + f_{bgd}A_{FB}^{bgd}
\]

where the \( f_i \)'s are the composition of the sample, \((1 - 2\chi)\) is the dilution factor due to the \( B^0\bar{B}^0 \) mixing and the last term accounts for a non zero asymmetry of the misidentified hadrons (mainly punch through hadrons and photon conversions) and non prompt leptons from \( K \) and \( \pi \) decays.

DELPHI has recently introduced in the analysis of the data collected in 1994-95 the use of lifetime based \( b \)-tag variable which is used to remove the decays of the \( Z \) into light quarks final states. This allows an estimation of the background level less dependent on the momentum of the lepton compared to the previous procedure. This consisted first in the selection of a high \((p, p_\perp)\) region, highly pure in the lepton content, which fixed the lepton identification efficiency and then in the evaluation of the background level in a low \((p, p_\perp)\) region. Moreover, the use of the \( b \)-tag enriches the sample in \( c\bar{c} \) events increasing the sensitivity of the measurement to \( A_{FB}^b \). On the contrary the reduction of the statistical error for the \( b \) asymmetry is smaller. This is because the useful sample is limited to the region of \( p_T > 1 \) GeV/c as the lower \( p_T \) region is equally populated of primary \( b \) and cascade leptons giving opposite sign contributions to the observed asymmetry: The \( b \)-tag variable has little effect in separating these two classes of events.

The results for the \( Z \) peak asymmetry for 1991-95 statistics are \( A_{FB}^c = 0.0770 \pm 0.0113 \) (stat) and \( A_{FB}^b = 0.0998 \pm 0.0065 \) (stat). The main contributions to the systematic error are listed in the central part of table \(^4\). The various contributions can be separated into two categories: The terms arising from the model actually used to simulate the semileptonic decay processes and the ones related to the description of detector effects. In case of the \( b \) asymmetry they both amount at about \( \pm 0.002 \) the largest effect coming from the description of the \( c \) decay \((\pm 0.0014)\).

\( C. \ A_{FB}^{c,0} \) and \( A_{FB}^{b,0} \) using reconstructed \( D \) mesons.

A reconstructed \( D \) meson uniquely tags a hadronic decay of the \( Z \) boson into a heavy flavour \( q\bar{q} \) pair as it can be produced only in the hadronization of a primarily produced \( c\bar{c} \) pair or in the decay of a \( b \)-hadron in a \( b\bar{b} \) event. The charge state of the reconstructed \( D \) meson is correlated with the charge of the parent quark. Therefore the values of \( A_{FB}^{c,0} \) and \( A_{FB}^{b,0} \) are extracted form a \( \chi^2 \) fit to the distribution of \( \cos \theta_T \) in \( D \) meson events.

The analysis done by DELPHI is based on the investigation of 8 decays channels:

\[
\begin{align*}
D^{*-} & \rightarrow D^0 \pi^{*-}_{sl} \\
& \rightarrow (K^- \pi^+)\pi^+_{sl} \\
& \rightarrow (K^- \pi^+\pi^- \pi^+)\pi^+_{sl} \\
& \rightarrow (K^- \pi^+(\pi^0))\pi^+_{sl} \\
& \rightarrow (K^- \mu^+\nu_{\mu})\pi^+_{sl} \\
& \rightarrow (K^- e^+\bar{\nu}_e)\pi^+_{sl} \\
D^0 & \rightarrow K^- \pi^+ \\
D^0 & \rightarrow K^- \pi^+(\pi^0) \\
D^+ & \rightarrow (K^- \pi^+)\pi^+
\end{align*}
\]

The first step in the analysis is the reconstruction of \( D^0 \) and \( D^+ \) mesons. In case of \( D^0 \) a low momentum pion \( \pi_{sl} \) \((p_{\pi} = 40 \text{ MeV}/c)\) with the correct charge correlation with the meson is searched for the \( D^{*-} \) reconstruction. The most relevant characteristics of the DELPHI detector for this part of the analysis are the particle identification provided by the RICH and the specific energy loss \( dE/dx \) measured by the TPC \(^3\). These informations are combined into a

\(^3\)The axis of the jet is defined excluding the lepton momentum.
pion veto used in the $D^{0/+}$ channels to reduce the combinatorial background which mainly consists of misidentified pions.

Further reduction of the background is achieved with cuts in the helicity angle of the kaon candidate with respect the $D^{0/+}$ flight direction and in the 3D decay length of the meson. These cuts are applied in function of the scaled energy of the meson $X_E(D) = 2E_D/\sqrt{s}$ to account for the energy spectrum of charged particles in hadronic $Z$ decays, peaked at low momentum. Finally candidates are selected either in a $\Delta m = m_{D^*} - m_{D^0}$ region for $D^{*+}$ channels, or in a mass interval for a $D^{0/+}$ decay. The range of $\Delta m$ values goes from 160 MeV/$c^2$ to 250 MeV/$c^2$ accordingly to the invisible energy in the final state, while for $D^{*0}$ a mass region $\pm 200$ MeV/$c^2$ around the nominal mass is selected. The reconstructed mass spectra for two channels are shown in fig. 5.

To extract $A^{c,0}_{FB}$ and $A^{b,0}_{FB}$ the contributions of $D$ mesons from $c\bar{c}$ and $b\bar{b}$ events should be separated. This is achieved fitting together $A^{c,0}_{FB}$ and $A^{b,0}_{FB}$ in bins of $\cos \theta_T$, scaled momentum $X_E(D)$ and $b$-tag variable $P_{ev}$ (fig. 6).

In fig. 6 one can notice that, in addition to the pure combinatorial background, in the signal region there are two other components: Partially reconstructed $D^{*+}$ mesons and reflections from other decays mode. Because of the charge correlation with the primary quark, these two categories have to be treated like signal in the fit.

Finally in case of $b\bar{b}$ events an effective mixing is introduced to take into account the $B^0\bar{B}^0$ mixing for $B \rightarrow D$ produced mesons and the so-called “upper vertex” production of the charm in $b \rightarrow D\bar{D}$ decays. This was computed
from data collected at the \( \Upsilon(4s) \) resonance by the CLEO and the ARGUS collaborations. The values obtained are \( \chi_{\text{eff}} = 0.222 \pm 0.033 \) for \( D^+ \) and \( D^{*-} \) modes and \( \chi_{\text{eff}} = 0.170 \pm 0.030 \) for the \( D^0 \) channels, both different from the average \( \bar{\chi} = 0.1214 \pm 0.0043 \) measured at LEP for \( b \)-hadrons.

At the Z peak about \( 62 \times 10^3 \) \( D \) decays were reconstructed and the corresponding value of the asymmetries are 

\[ A_{\text{cFB}}^{\text{c}} = 0.0659 \pm 0.0094 \]  

(stat) and 

\[ A_{\text{bFB}}^{\text{b}} = 0.0762 \pm 0.0194 \]  

(stat). The main contributions to the systematic error are listed in the bottom part of table II. Besides the limited MC statistics the main contributions are related to the fit method itself and to the residual asymmetry of \( D \) mesons from pure combinatorial background in \( c\bar{c} \) and \( b\bar{b} \) events.

### III. LEP COMBINED RESULTS.

The currently available determinations of \( A_{\text{cFB}}^{\text{c}} \) and \( A_{\text{bFB}}^{\text{b}} \) from LEP experiments are shown in figure 7. The precision on the measurement of \( A_{\text{cFB}}^{\text{c}} \) is currently 7% and the average is dominated by the measurements performed with the \( D \) meson tag. On the contrary, it should be noted that the most precise single determination comes from the lepton analysis tag from OPAL. This indicates that some improvements in the measurement of \( A_{\text{cFB}}^{\text{c}} \) can still be achieved.

The precision on the measurement of \( A_{\text{bFB}}^{\text{b}} \) is currently 2% with equal weights from jet charge and lepton tag analyses.

The impact of these measurements on the determination of \( \sin^2 \theta_{W,\text{eff}}^{\text{c}} \) is

\[ \sin^2 \theta_{W,\text{eff}}^{\text{c}} = 0.2322 \pm 0.0010 \]  

for \( A_{\text{cFB}}^{\text{c}} \) and 

\[ \sin^2 \theta_{W,\text{eff}}^{\text{b}} = 0.2325 \pm 0.00038 \]  

for \( A_{\text{bFB}}^{\text{b}} \).

\( A_{\text{bFB}}^{\text{b}} \) provides together \( A_{\text{LR}} \) from SLD the most precise value available at the moment. On the contrary the two measurements are more than 2 sigmas apart from each other.

![Table and figure showing LEP combined results for \( A_{\text{cFB}}^{\text{c}} \) and \( A_{\text{bFB}}^{\text{b}} \)]

**FIG. 7.** Results for \( A_{\text{cFB}}^{\text{c}} \) and \( A_{\text{bFB}}^{\text{b}} \) shown at the ICHEP 98 Vancouver Conference.

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