The Genesis of Cosmological Tracker Fields

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Abstract

The role of the quintessence field as a probable candidate for the repulsive dark energy, the conditions for tracking and the requisites for tracker fields are examined. The concept of ‘integrated tracking’ is introduced and a new criterion for the existence of tracker potentials is derived assuming monotonic increase in the scalar energy density parameter $\Omega_\phi$ with the evolution of the universe as suggested by the astrophysical constraints. It provides a technique to investigate generic potentials of the tracker fields. The general properties of the tracker fields are discussed and their behaviour with respect to tracking parameter $\epsilon$ is analyzed. It is shown that the tracker fields around the limiting value $\epsilon \simeq \frac{2}{3}$ give the best fit with the observational constraints.

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There is strong evidence, based on recent luminosity-redshift observations of Type Ia supernovae [1] and consistently low measurements of matter density [2], to suggest that the major fraction of the energy content of the observable universe consists of an 'exotic matter' with negative pressure, often referred to as 'dark energy' [3]. In cold dark matter (CDM) cosmology, the most probable candidates for dark energy are the cosmological constant $\Lambda$ and the weakly coupled scalar fields with negative pressure which might mimic $\Lambda$ to produce enough repulsive force to counter gravitational attraction and cause acceleration in the expanding universe at the present epoch. Comprehensive review reports on the observational, theoretical, physical and anthropic significance of the cosmological constant $\Lambda$ have been published by Zeldovich [4], Weinberg [5], Sahni and Starobinsky [6]. The cosmological constant seems to be a natural choice for the source of cosmic repulsion but it is hard to reconcile its constant value $\Lambda \sim 10^{-47} GeV^4$ (to be comparable with
the present energy density of the universe) with the particle physics scales $\sim 10^{56} \text{GeV}^4$ subsequent to inflation. Since $\Lambda$ has stayed constant through cosmic evolution, it demands setting up a new energy scale to explain as to why it should take 15 billion years of time for $\Lambda$ to dominate in the universe today (known as coincidence problem). To address this problem, a comprehensive study of the observational consequences of a dynamical $\Lambda$ term (representing vacuum energy) decaying with time [7] and the cosmological consequences of rolling scalar fields was undertaken [8]; subsequently, Caldwell et al [12] discussed the possibility that a significant contribution to the energy density of the universe might be from the scalar fields with an evolving equation of state, unlike radiation, matter or $\Lambda$ fields and proposed the nomenclature 'quintessence' for such scalar fields which, during the process of roll-down, acquire negative pressure and might act as $\Lambda_{\text{eff}}$. But for the scalar energy density $\rho_{\phi}$ or $\Lambda_{\text{eff}}$ to be comparable with the present energy density $\rho_n$ of the universe, the initial conditions for the quintessence fields must be set up carefully and fine tuned. To overcome the 'fine tuning' or the 'initial value' problem, the notion of tracker fields [13,14] was introduced. It permits the quintessence fields with a wide range of initial values of $\rho_{\phi}$ to roll down along a common evolutionary track with $\rho_n$ and end up in the observable universe with $\rho_{\phi}$ comparable to $\rho_n$ at the present epoch. Thus, the tracker fields can get around both the coincidence problem and the fine tuning problem without the need for defining a new energy scale for $\Lambda_{\text{eff}}$. Although tracking is a useful tool to promote quintessence as a likely source of the missing energy in the universe, the concept of tracking as given by Steinhardt et al [13,14] does not ensure the physical viability of quintessence in the observable universe. It simply provides for synchronized scaling of the scalar field with the matter/radiation field in the expanding universe in such a way that at some stage (undefined and unrelated to observations), the scalar field energy starts dominating over matter and may induce acceleration in the Hubble expansion. Since there is no control over the slow roll-down and the growth of the scalar field energy during tracking, the transition to the scalar field dominated phase may take place much later than observed. Moreover, any additional contribution to the energy density of the universe, such as quintessence, is bound to affect the dynamics of expansion and structure formation in the universe. As such, any physically viable scalar field must comply with the cosmological observations related to helium abundance, cosmic microwave background and galaxy formation, which are the pillars of the
success of the standard cosmological model. A realistic theory of tracking of scalar fields must, therefore, take into account the astrophysical constraints arising from the cosmological observations. With this perspective in mind, we have introduced the notion of ‘integrated tracking’ in this paper which implies tracking compatible with astrophysical constraints. It provides a firm and credible foundation to the quintessence theory. Most of the investigations [10,11,12,13] on the scalar fields so far have been confined to exploring scalar potentials which roll down with the desired tracking behaviour to end up with dominance of quintessence energy. The first theoretical derivation of the tracking condition was attempted by Steinhardt et al [14] who put forth different criteria for tracking under varying conditions and discussed the tracking properties of certain exponential and inverse power law potentials. In this Letter, we report our investigations towards a systematic theory of integrated tracking in which the tracking behaviour of the scalar field is closely related to the growth of its cosmological density parameter through the tracking parameter $\epsilon$, subject to astrophysical constraints as discussed in the paper. This approach provides us a powerful technique to study the general behaviour of the tracker fields with respect to $\epsilon$ and also a window to investigate the generic potentials of the tracker fields instead of dealing with isolated potentials and their properties.

In general, the energy densities $\rho_n$ and $\rho_\phi$ scale down at different rates in the expanding universe. For a scalar field with potential $V(\phi) = \frac{1}{2}(\rho_\phi - p_\phi)$ and kinetic energy $\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(\rho_\phi + p_\phi)$, the equation of motion of the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (1)$$

leads to $\rho_\phi \sim a^{-3(1+w_\phi)}$ where $1 + w_\phi = 1 + \frac{p_\phi}{\rho_\phi} = \frac{2\eta}{\rho_\phi} = 2\eta$, $\eta$ denotes the ratio of kinetic energy to $\rho_\phi$.

For the background energy, $\rho_n \sim a^{-3(1+w_n)} \sim \frac{1}{a^n}$, $n = 4$(radiation) and $n = 3$(matter). Obviously, the scalar field has a wider range of scaling as $\rho_\phi \sim a^{-6\eta}$, $(0 \leq \eta \leq 1)$ depending upon the choice of $\eta$. When the kinetic energy is dominant, $\rho_\phi$ can scale down as steeply as $\frac{1}{a^\eta}$. It rolls down slowly as $V(\phi)$ starts dominating and the rolling reduces to a crawl as $\eta$ tends to zero. Therefore, the kinetic energy plays an important role in scaling down the energy of the scalar field. In order to solve the ‘dark energy’ problem, we want domination of the scalar field ($\rho_\phi \gtrsim \rho_n$) today but at
the same time it is imperative that $\rho_\phi < \rho_n$ during radiation and matter dominated era and grows slowly to the present state so as not to interfere with the formation of galactic structure and the success of nucleosynthesis during cosmic evolution. Therefore, tracking requires proper synchronization of the scaling of the two fields so that $\rho_\phi$ rolls down slower than $\rho_n$ (i.e. $w_\phi < w_n$) along a common evolutionary track and eventually overtakes it, causing acceleration in the cosmic expansion. This implies that $w_\phi < \frac{1}{3}$ during radiation era, $w_\phi < 0$ during matter domination and $w_\phi$ tends to $-1$ during scalar energy dominated phase, constraining $\eta$ to lie in the range $(0 \leq \eta < \frac{2}{3})$. Naturally a fixed value of $\eta$ does not lead to tracking. It must vary with the roll down of the scalar field but its variation over the wide span of the cosmic time is so small that it may be regarded as almost a constant and the time derivatives of $\eta$ may be neglected. This assumption simplifies the dynamics of evolution of the tracker fields. Using Eq. (1), the logarithmic differentiation of $V(\phi) = (1-\eta)\rho_\phi$ yields an important condition to be satisfied by the tracker fields.

$$\pm \frac{V'(\phi)}{V(\phi)} = 6\eta \frac{H}{\dot{\phi}} = \sqrt{\frac{6\eta}{M_p^2 \Omega_\phi}}$$

(2)

where $\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_n}$, $H$ is the Hubble constant given by $H^2 = \frac{\rho_\phi + \rho_n}{3M^2_p}$ and $M_p = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. According to our notation, the prime denotes derivative with respect to $\phi$, an overdot denotes time-derivative and $\pm$ sign applies to $V' > 0$ and $V' < 0$ respectively.

It is remarkable that the maintenance of the tracker condition (2) constrains the order of magnitude of the various terms involved. For instance, the middle term $\sqrt{\frac{\rho_\phi + \rho_n}{\rho_\phi}}$ in (2) must remain nearly of $O(1)$ throughout tracking. It follows, therefore, that the scalar fields with $\rho_\phi \ll \rho_n$ will remain frozen until Hubble expansion slows down to the level when $\rho_n \approx \rho_\phi$; thereafter, the tracker condition holds good and tracking takes place. Thus, the tracker condition ensures that a large class of quintessence fields with widely diverse initial conditions ($\rho_\phi \ll \rho_n$) would scale down to the same present state with $\rho_\phi \approx \rho_n$. However, the scalar fields with $\rho_\phi \gg \rho_n$ violate astrophysical constraints given below although relation (2) continues to hold good. Hence the tracker condition (2) is not a sufficient condition for tracking and it has to be supplemented with more stringent requirements based on the astrophysical...
constraints discussed below.

Observationally, the cosmological density parameter $\Omega_\phi$ of the scalar field is an important quantity since it measures the relative magnitude of the energy densities $\rho_\phi, \rho_n$ during cosmic evolution as given by

$$\Omega_\phi = (1 + a^{-3\epsilon})^{-1} \quad (3)$$

where $\epsilon \equiv w_n - w_\phi, \ 0 < \epsilon \leq 1$. It may be used to regulate the tracking behaviour according to the following astrophysical constraints:

I. $\Omega_\phi < 0.13 - 0.2$ at the nucleosynthesis epoch [10] around redshift $z = 10^{10}$
II. $\Omega_\phi < 0.5$ during galaxy formation epoch [3] around $z = 2$ to 4
III. $\Omega_\phi = 0.5$ at the onset of acceleration in cosmic expansion at redshift $z (0 \leq z < 2)$
IV. $\Omega_\phi \simeq 0.65 \pm 0.05$ with $w_\phi \leq -0.4$ at the present epoch ($z = 0$) [15]

As stated above, the tracker condition given by Eq.(2) ensures slow rolling of the scalar potential $V(\phi)$ and may be regarded as a necessary condition for tracking but not as a criterion for tracker fields. To lay down a physical criterion for tracker fields, we take a clue from the astrophysical constraints I - IV which require progressive growth of $\Omega_\phi$ during tracking and postulate that $\dot{\Omega}_\phi > 0$ for tracker fields. This can be monitored by a single parameter $\epsilon$ (known as tracking parameter) since it reveals a clear picture of scaling of $\rho_\phi$ vs. $\rho_n$ throughout the range of tracking. The limiting value of $\epsilon$, as derived from Eq.(8) ensures the transition from matter to the scalar dominated phase.

By logarithmic differentiation of Eq. (2), $\frac{\dot{\Omega}_\phi}{\Omega_\phi}$ may be expressed in terms of $V(\phi)$ and its derivatives as

$$\frac{\dot{\Omega}_\phi}{\Omega_\phi} = \mp 12\eta H (\Gamma - 1) \quad (4)$$

where $\Gamma \equiv \frac{V''}{V^2}$. Again the time derivative of Eq.(3) gives

$$\dot{\epsilon} \ln a = \frac{1}{3} \frac{\dot{\Omega}_\phi}{\Omega_\phi} - \epsilon H. \quad (5)$$
The choice of $V(\phi)$ may be further restricted by setting a stronger condition for tracking i.e $\dot{\epsilon} \geq 0$ which requires
\[
\dot{\Omega}_\phi \geq 3\epsilon H \Omega_\phi \Omega_n > 0.
\] (6)

Eqs. (4) and (6) lead to the final criterion for tracker fields
\[
\mp (\Gamma - 1) \geq \frac{\epsilon \Omega_n}{4\eta}.
\] (7)

The above inequality implies that a given scalar potential $V(\phi)$ will give rise to a tracker field if $\Gamma \leq 1 - \frac{\Omega_n}{4\eta}$ in case of increasing potential ($V > 0$) and $\Gamma \geq 1 + \frac{\Omega_n}{4\eta}$ in case of decreasing potential ($V < 0$) where $\frac{\Omega_n}{4\eta} < 1$ and $\epsilon$ conforms to the astrophysical constraints I - IV.

The tracking criterion for the quintessence potentials derived above is based on the restrictive assumption that $\Omega_\phi$ increases monotonically through most of the cosmological history. In fact, this assumption is motivated by the astrophysical Constraints I - IV listed above which demand the progressive increase in the fractional magnitude of $\Omega_\phi$ from nucleosynthesis epoch around $z \simeq 10^{10}$ through galaxy formation era to the present day ($z=0$). This assumption, although restrictive, seems to be quite natural and consistent with the thermal history of the universe. Dodelson et al [17] have suggested an alternative scenario for tracking with oscillating energy under which the scalar potential with sinusoidal modulation oscillates about the ambient energy density. The oscillating tracker potentials may satisfy the astrophysical constraints I - IV provided the magnitude and frequency of the oscillations are fine-tuned to comply with the specific requirements listed under the constraints. Similar existence conditions for the tracker fields have been obtained by Steinhardt et al [14] but they hold under the restriction $\Omega_n = 1$. The above results not only lay down the criterion for the existence of tracker fields but they also emphasize the importance of the tracker parameter $\epsilon$ which may be used with advantage to derive generic potentials for tracking fields as discussed below. Again, the astrophysical constraints during the phase transition from matter to scalar dominated phase, when $\Omega_n$ is significantly less than 1, may be utilized to limit the range of $\epsilon$ as shown below. This is to ensure that the roll down of the scalar field is not too slow and the universe must enter the phase of accelerating expansion at the right epoch at red shift $z_0$ ($0 < z_0 < 2$) after the galactic structure has formed.
The general behaviour of tracker fields, regardless of the form of \( V(\phi) \), may be outlined by deducing the value of \( \epsilon \) at the onset of acceleration subject to astrophysical constraints III and IV. Using the Friedmann equation during matter dominated era

\[
\frac{2\ddot{a}}{a} = -\frac{\rho_\phi [1 + 3w_\phi + (\rho_n/\rho_\phi)]}{3M_p^2},
\]

the condition for onset of acceleration i.e. \( \ddot{a} \geq 0 \) when \( \frac{\dot{\rho}_n}{\rho_n} \simeq 1 \), leads to the limiting value \( \epsilon_0 \gtrsim \frac{2}{3} \) around the transition to the scalar dominated phase. The relation \( \frac{\dot{\rho}_n}{\rho_n} = 2(1 + z)^{-3\epsilon} \) involving redshift \( z \), enables us to find \( \rho_\phi \) at various landmark epochs in cosmic evolution for different values of \( \epsilon \) and examine which of these lead to desired tracking behaviour. By choosing \( \epsilon = 0.2, 0.6 \) and 0.7 successively in the redshift relation, we have shown by the indexed curves in the figure as to how the relative scaling of \( \rho_r, \rho_m \) and \( \rho_\phi \) takes place during different phases of evolution in the expanding universe. In fact, \( 0 < \epsilon < \epsilon_0 \) during matter and radiation dominated era; as such \( \rho_\phi \) would track down closer to \( \rho_n \) than shown in the figure. It is found that the models with \( \epsilon = 0.6 \) and 0.7 satisfy all the requisite astrophysical constraints whereas \( \epsilon = 0.2 \) violates II and III. Therefore, the best-fit quintessence models correspond to \( \epsilon \approx 0.66 \) as also indicated by the limiting value derived above. This is also consistent with the concordance analysis [15], based on a comprehensive study of observational constraints on spatially flat cosmological models containing a mixture of matter and quintessence.

For the known functional values of \( \epsilon(\phi) \), the generic potentials for the tracker fields can be found by putting the tracker criterion in the form

\[
\pm \frac{V''V - V'^2}{V'^2} \geq \frac{\epsilon(1 - \Omega_\phi)}{2(1 + w_n - \epsilon)}.
\]

Inserting the value of \( \Omega_\phi \) from Eq.(2), we get on simplification

\[
\pm \frac{\zeta'}{\zeta^2 - k^2} \geq f(\phi) \equiv \frac{\epsilon}{2(1 + w_n - \epsilon)}
\]

where \( \zeta \equiv V'/V \) and \( k^2 = \frac{6\eta}{M_p^2} \). The above equation yields generic potential \( V(\phi) \) for suitable choice of \( f(\phi) \). For example, \( \epsilon = 0 \) corresponds to the generic potentials of the form \( V(\phi) \sim exp[\beta\phi] \) for which \( \Omega_\phi \) remains constant.
Figure 1: Scaling of energy densities $\rho_r$ (radiation), $\rho_m$ (matter) and $\rho_\phi$ (tracker field) vs. red shift $z$ in the expanding universe. The curves are plotted by taking values of $\epsilon$ at the point of transition to the scalar dominated phase.
throughout. Astrophysical constraints I and II may be satisfied by choosing $\Omega_\phi < 0.15$ as discussed in [14] but the tracking remains incomplete since there is no onset of acceleration as required by the constraint III. If $\epsilon = \text{constant}$ throughout, there is limited tracking since the ratio of the kinetic energy to the potential energy of the scalar field remains stationary throughout rolling with a ceiling fixed by the constraint II; further $V(\phi)$ never attains a constant value to play the role of $\Lambda$ in the universe. The generic potentials are of the hyperbolic form; in the particular case of matter dominated universe ($\Omega_n \simeq 1$), the potentials are of the inverse power law form. Power law potentials have been discussed extensively by several authors [8,10,11,13,14,16].

The detailed investigations involving mathematical theory of tracker fields, the stability and astrophysical consequences of tracker solutions will be published under a separate communication elsewhere.

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