Using a novel grey model to forecast the unconventional water sources supply volume in China

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Abstract. Unconventional water resources are an important part of fresh water resources, which can help relieve pressure of water supply and further a sustainable development could be achieved, especially in China. In this paper, a novel optimize fractional order polynomial grey model is proposed, which is abbreviated as OFOPGM (1,1). The novel grey model is validated and calibrated and forecast the unconventional water sources supply volume in China. The forecast results prove the validity and practicability of the novel model.

1. Introduction

Fresh water resource, including surface water, groundwater and unconventional water sources, is one of the substances on which human beings depend for survival [1-2]. The world's new population of 3 billion will face a severe shortage of water resources by 2050, according to the United Nations research and analysis [3]. The Global Environment Outlook (GEO), issued by the United Nations Environment Programme in 2002, argues that half of the world's rivers discharge are substantially reducing or pollution aggravation [4]. China is a country with an extreme shortage of fresh water, which accounts for only of the world fresh water resource is a shortage of resources for the social and economic development in China, and has become one of the factors restricting the construction of a well-off society [5]. Unconventional water sources are an important part of fresh water resources, which is related to the sustainable development of China. So, accurate prediction unconventional water sources supply volume (UWSV) of China is important for the rational allocation and efficient utilization of fresh water resources [6-8].

Grey prediction models is the most important part of the grey system theory, which have soon become popular and appealed numerous researches, Since the Professor. Deng[9] pioneered the grey system theory. The grey prediction models are efficient to predict the original series with a few sample data, especially the GM(1,1) can be built with only four points. Although GM(1,1) model is constantly improving, it is not always satisfied with volatility sequence of applications. To dispose this challenge, many variants have been developed using the similar modelling procedures of the GM(1,1) e.g. the NGBM(1,1)[10], the optimal time response function IRGM(1,1)[11], the optimized single-variable discrete grey forecasting model OSDGM(1,1)[12], Yang et al.[13] modified optimized fractional grey model using the error feedback, and the performance is evaluated and greatly improved in modelling. Meng et al. [14] develop a discrete grey model with fractional operators, which also makes use of genetic algorithms to optimize the modelling parameter. etc, which has been successfully applied in many disciplines[15-16].
This paper presents novelties in two aspects. First, propose the OFOPGM(1,1) model, and two alternative models, including classical GM(1,1), fractional order grey model FOGM(1,1) were compared. Solutions indicate that the novel model is superior to others. Second, the fractional order of grey model was calibrated, according to the data of unconventional water sources supply volume of China form 2005 to 2016, and the optimal forecasting model is obtained.

2. The proposed of novel grey model

In this section, we will present the grey system model with optimize fractional order accumulation and the nonlinear multivariate grey system model with fractional order accumulation, including the principles and the computational steps.

2.1. Grey model with fractional order accumulation

Let the \( r \)th order accumulated generating operator of the original nonnegative sequence \( X^{(0)} \) be \( X^{(r)} \), \( r=1,2,...,n \). set \( \begin{pmatrix} \frac{p}{q} \\ 0 \end{pmatrix} = 1, \begin{pmatrix} k-1 \\ k \end{pmatrix} = 0, k=1,2,...,n \), then

\[
x^{(r)}(k) = \sum_{i=1}^{k} \begin{pmatrix} k-i+r-1 \\ k-i \end{pmatrix} x^{(0)}(i)
\]

where \( \begin{pmatrix} k-i+r-1 \\ k-i \end{pmatrix} = \frac{(r+k-i-1)(r+k-i-2)...(r+1)r}{(k-i)!} \)

Fractional derivatives accumulate the whole history of the system in weighted form. \( x^{(0)}(k) \) in Grey system theory denotes the weight of \( x^{(0)}(i) (i=1,2,...,k) \) as 1. The larger \( r \) of \( x^{(r)}(k) \) is, the larger the weight of old data is. Reducing \( r \) can reduce the weights of old data, which can put more emphasis on the newer data.

2.2. The representation of the FOPGM(1,1) model

The new form of the FOPGM(1,1) same as PGM \( \left( \frac{\tau}{\lambda} \right) (1,1) \) model

\[
x^{\left( \frac{\tau}{\lambda} \right)}(k) = x^{\left( \frac{\tau}{\lambda} \right)}(k-1) + \lambda z^{\left( \frac{\tau}{\lambda} \right)}(k)
\]

where

\[
z^{\left( \frac{\tau}{\lambda} \right)}(k) = \frac{x^{\left( \frac{\tau}{\lambda} \right)}(k) + x^{\left( \frac{\tau}{\lambda} \right)}(k+1)}{2}
\]

\( z^{\left( \frac{\tau}{\lambda} \right)}(k) \) is called the background value.

2.3. The solution of the FOPGM(1,1) model

Within a given original sequence \( X^{(0)} = \{x^{(0)}(1), x^{(0)}(2),..., x^{(0)}(n)\} \), the least squares criteria for the NMFOGM(1,1) model can be described as the following unconstrained optimization problem:
The solution of this optimization problem can be written as the following linear system

\[
[B] = (B' B)^{-1} B' Y
\]

Where

\[
B = \begin{bmatrix}
-\bar{x}_{\tau_{1}}(2) & 5/2 & 3/2 & 1 \\
-\bar{x}_{\tau_{2}}(3) & 9 & 4 & 1 \\
\vdots & \vdots & \vdots & \vdots \\
-\bar{x}_{\tau_{n}}(n) & \sum_{i=2}^{n} n^2 + (n-1)^2 & \sum_{i=2}^{n} 2n-1 & 1
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
\bar{x}_{\tau_{1}}(2) \\
\bar{x}_{\tau_{2}}(3) \\
\vdots \\
\bar{x}_{\tau_{n}}(n)
\end{bmatrix}
\]

The linear differential equation

\[
\frac{dx_{\tau}(t)}{dt} + \lambda_i x_{\tau}(t) = \lambda_2 \sum_{i=2}^{n} \tau^2 + \lambda_3 \sum_{i=2}^{n} \tau + \lambda_4
\]

The solution of the whitenization Eq. (6) for FOPGM(1,1) is given by

\[
\bar{x}_{\tau_{1}}(k) = x^{(0)}(1) e^{-\lambda_1 k} + \sum_{i=2}^{k} \frac{1}{2} \left[ e^{-\lambda_1 (k-i)} f(t) + e^{-\lambda_1 (k-i-1)} f(t-1) \right]
\]

Where

\[
f(t) = \lambda_2 \sum_{i=2}^{n} \tau^2 + \lambda_3 \sum_{i=2}^{n} \tau + \lambda_4
\]

Within the initial condition \(x^{(0)}(1) = x^{(0)}(1)\), by applying the trapezoid formula, we can obtain the discrete response function as

\[
\bar{x}_{\tau_{1}}(k) = x^{(0)}(1) e^{-\lambda_1 k} + \sum_{i=2}^{k} \frac{1}{2} \left[ e^{-\lambda_1 (k-i)} f(t) + e^{-\lambda_1 (k-i-1)} f(t-1) \right]
\]

The response function (9) is used to compute the values of the series \(\bar{x}_{\tau_{1}}(k)\), and the predicted values of the original series \(x^{(0)}(k)\) can be obtained using the first order inverse accumulative generation operation as follows:
\[ \hat{x}^{(0)} = X^{(1)}(k) \]
\[ = \left\{ x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \right\} \quad (10) \]

2.4. Fractional order optimization

Fractional order optimization of the nonlinear multivariate GM(1,1) under the condition of minimum mean relative error, which is solving optimization equation (11) with the particle swarm optimization algorithm (PSOA).

\[ \min F(r) = \frac{1}{n} \sum_{k=1}^{n} \left| x^{(0)}(k) - \hat{x}^{(0)}(k) \right|, r \in R^* \quad (11) \]

The particle swarm optimization algorithm steps is described detail as follows:

Step 1: Initialize randomly position and velocity of particles in particle swarm, \( p_{Best} = 1 \).

Step 2: Set \( g_{Best} \) to the current position, and Set \( g_{Best} \) to the best position.

Step 3: Calculate the average relative error (ARE), \( F(p_{Best}) \) of fractional order, \( r = p_{Best} \) of the nonlinear multivariate GM(1,1).

If \( F(p_{Best}) - F(g_{Best}) \leq \delta \), \( r = g_{Best} \).

If \( F(p_{Best}) - F(g_{Best}) > \delta \), continue calculation.

Step 4: Update position and velocity of particles in particle swarm.

\[ V = aV + c_1 \times \text{rand} \times (p_{Best} - \text{Present}) + c_2 \times \text{rand} \times (g_{Best} - \text{Present}) \]
\[ \text{Present} = \text{Present} + V \quad (12) \]
\[ \omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{\text{run}_{max}} \quad (13) \]

If the particle’s fitness better than that of \( p_{Best} \), \( p_{Best} \) is the new position.

If the particle’s fitness better than that of \( g_{Best} \), \( g_{Best} \) is the new position.

Step 5: Calculate fitness variance \( \sigma^2 \) of the particle swarm, and \( F(g_{Best}) \)

\[ \sigma^2 = \frac{\sum_{i=1}^{s} \left( F_i - F_{avg} \right)^2 \cdot F \quad (15) \]

where

\[ F = \begin{cases} \max \left| F_i - F_{avg} \right|, & \max \left| F_i - F_{avg} \right| \geq 1 \\ 1, & \text{others} \end{cases} \quad (16) \]

Step 6: Calculate variation probability \( p_n \)

\[ p_n = \begin{cases} k, & \sigma^2 < \sigma_d^2 \text{ and } F(g_{best}) > F_d \\ 0, & \text{others} \end{cases} \quad (17) \]

Step 7: Generate random number \( \varepsilon \). if \( \varepsilon < p_n \)

\[ g_{Best} = g_{Best_i} \times (1 + 0.5 \times \eta) \quad (18) \]

Step 8: if \( \varepsilon \geq p_n \), judge whether the algorithm satisfaction the convergence criterion. If not satisfied the convergence criterion, repeat step 3.

Step 9: If satisfied the convergence criterion, \( r = g_{Best} \) is the best value. Calculate the prediction value \( \hat{x}^{(0)}(k) \), relative error and average relative error of the fractional order (\( r = g_{Best} \)) optimization of the nonlinear multivariate GM(1,1).
3. Application

The mean absolute percentage error (MAPE) is used to evaluate the overall forecast performance of the prediction models, which is defined as follows:

$$\text{MAPE} = \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - x^{(1)}(k)}{x^{(0)}(k)} \right| \times 100\%$$ (19)

where $x^{(0)}(k)$ is the original series, and $x^{(1)}(k)$ is the fitted or predicted series.

The raw data of the unconventional water sources supply volume ($10^9$ m$^3$) of China are collected from the official website National Bureau of Statistics of China (http://www.stats.gov.cn/tjsj/ndsj/) as shown in Table 1. The data from 2005 to 2012 are used to build the prediction models, and the data from 2013 to 2016 are used to validate the modeling accuracy.

**Table 1.** Unconventional water sources supply volume ($10^9$ m$^3$) of China from 2005 to 2016.

| Year | water volume | Year | water volume |
|------|--------------|------|--------------|
| 2005 | 22           | 2011 | 44.8         |
| 2006 | 22.7         | 2012 | 45.6         |
| 2007 | 25.7         | 2013 | 49.9         |
| 2008 | 28.7         | 2014 | 57.5         |
| 2009 | 31.2         | 2015 | 62.5         |
| 2010 | 33.1         | 2016 | 70.8         |

The numerical results of the OFOPGM(1,1) model are compared to the commonly used prediction models, including the including GM(1,1), FOGM(1,1) model, as shown in Table 2, and the predicted values are also plotted in Fig. 1.

**Table 2.** Numerical results by FOPGM(1,1), GM(1,1), FOGM(1,1).

| Year | PFOGM | Error(%) | GM | Error(%) | FOGM | Error(%) |
|------|-------|----------|----|----------|------|----------|
| 2005 | 22    | 0        | 22 | 0        | 22   | 0        |
| 2006 | 21.42 | -5.65    | 44.17 | 94.59 | 21.51 | -5.25    |
| 2007 | 24.89 | -3.15    | 25.07 | -2.46 | 24.94 | -2.96    |
| 2008 | 28.62 | -0.29    | 28.34 | -1.25 | 28.59 | -0.37    |
| 2009 | 32.54 | 4.29     | 32.04 | 2.70  | 32.47 | 4.06     |
| 2010 | 35.70 | 7.86     | 36.23 | 9.45  | 36.62 | 10.63    |
| 2011 | 41.16 | -8.13    | 40.96 | -8.57 | 41.11 | -8.24    |
| 2012 | 45.95 | 0.78     | 46.31 | 1.56  | 45.99 | 0.85     |
| MAPE | 3.77  |          | 15.07 |       | 4.05  |          |
| 2013 | 51.14 | 2.49     | 52.36 | 4.93  | 51.32 | 2.84     |
| 2014 | 56.77 | -1.27    | 59.20 | 2.95  | 57.15 | -0.62    |
| 2015 | 62.89 | 0.62     | 66.93 | 7.09  | 63.54 | 1.66     |
| 2016 | 69.56 | -1.76    | 75.67 | 6.88  | 70.56 | -0.34    |
| MAPE | 1.53  |          | 5.46  |       | 1.36  |          |

The Table 2 shows that for build-sample performance MAPE of OFOPGM(1,1) is 3.77%, respectively, lower than those of other grey models. Comprehensively, performance of MRE for the three models, in terms of Table 1, only the OFOPGM(1,1) model achieved highly accurate forecasting level, and the FOGM(1,1) model achieved good forecasting level. However, GM(1,1) model achieved weak and inaccurate forecasting level. For validate-sample performance, consumption data of 2013-2016 is used.
to check forecasting precision. The proposed model, OFOPGM(1,1), is prior to others, while results of FOGM(1,1) is more precise than GM(1,1) model, according to relative percentage error of validate-sample performance. All of the three models achieved highly accurate forecasting level.

![Figure 1](image_url)

**Figure 1.** Prediction results by OFOPGM(1,1), GM(1,1), FOGM(1,1).

### 4. Conclusions

Accurately predict the supply volume of unconventional water resources is propitious to make full use of the unconventional water sources. The OFOPGM(1,1) model have the properties of fractional order accumulation and the polynomial function, and it is not applicable for all the series according to its mathematical formulation. The priority of new information can be better reflected when the accumulation order number becomes smaller in the in-sample model. The discrete response function of the TDPFOGM(1,1) model is combination of an exponential function and a discrete integral with an exponential function and the polynomial function. Based on a series of validated and calibrated for the novel grey prediction system the OFOPGM(1,1) model is established, and it is appropriate and propitious to forecast unconventional water sources supply volume of China.

### 5. References

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