Non-perturbative effects in a rapidly expanding quark-gluon plasma

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Within first-order phase transitions, we investigate the pre-transitional effects due to the nonperturbative, large-amplitude thermal fluctuations which can promote phase mixing before the critical temperature is reached from above. In contrast with the cosmological quark-hadron transition, we find that the rapid cooling typical of the RHIC and LHC experiments and the fact that the quark-gluon plasma is chemically unsaturated suppress the role of non-perturbative effects at current collider energies. Significant supercooling is possible in a (nearly) homogeneous state of quark gluon plasma.

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I. INTRODUCTION

It is possible to model the gross general features of a phase transition from a quark-gluon plasma (QGP) to a hadronic phase through a phenomenological potential with a scalar order parameter. Assuming the transition to be discontinuous, or first-order, as suggested by some recent lattice QCD simulations, the QGP is cooled to a temperature where a second minimum appears, indicating the presence of a hadronic phase. With further cooling, the two phases become degenerate at the critical temperature 

\[ T_c = \frac{m}{\sqrt{\frac{\sigma}{\lambda}}}, \]

where \( m \) is the quark mass, \( \sigma \) the surface tension, and \( \lambda \) the correlation length. This general behavior models both the cosmological quark-hadron phase transition and the production of a QGP during heavy-ion collision experiments, as those under way at the RHIC and planned for the LHC. In the latter case, the plasma generated by the collision expands and cools, relaxing back to the hadronic phase.

Recent interest has been sparked by the possibility that this relaxation process is characterized by the formation of disoriented chiral condensates (DCC), which are coherent pion condensates similar to the domains typical of quenched ferromagnetic phase transitions. The nonequilibrium properties of this relaxation process and DCC formation have also been studied as a first order chiral phase transition where the supercooled phase may naturally lead to a "quenched" initial condition.

Recent work on the dynamics of weak first-order phase transitions have shown that, in certain cases, it is possible to have nonperturbative, large-amplitude fluctuations before the critical temperature is reached, which promote phase mixing. Studies performed in the context of the cosmological electroweak phase transition and quark-hadron phase transition have indicated that, for a range of physical parameters controlling the transition, these effects are present. It is thus natural to consider if similar effects are present during heavy-ion collisions.

Whenever pre-transitional phenomena are relevant, one should expect modifications from the usual homogeneous nucleation scenario, which is based on the assumption that critical bubbles of the hadronic phase appear within a homogeneous background of the QGP phase. The dynamics of weakly first-order transitions will be sensitive to the amount of phase mixing at \( T_c \); for large phase mixing, above the so-called percolation threshold, the transition may proceed through percolation of the hadronic phase, while for small amounts of phase mixing, by the nucleation of critical bubbles in the (inhomogeneous) background of isolated hadronic domains, which grow as \( T \) drops below \( T_c \). An ideal quark gluon plasma in one dimension expands according to the Bjorken scaling, where \( T^3 t \) is constant. Assuming the initial temperature of the plasma produced at RHIC and LHC energies to be 2 to 3 times \( T_c \), scaling implies that the time \((\Delta t)\) taken by the plasma to cool from \( T_1 \) to \( T_c \) is of order a few fm/c, which could be comparable with the time scale of the subcritical hadronic fluctuations. On the other hand, the expansion rate of the early universe in the range \( T_1 \leq T \leq T_c \) is slow enough (\( \Delta t \) could be of the order of a few \( \mu \) sec), that nonperturbative thermal fluctuations may achieve equilibrium. Another difference is that collisions at RHIC and LHC energies will lead to the formation of a highly (chemically) unsaturated plasma, i.e., the initial gluon and quark contents of the plasma remain much below their equilibrium values. A chemically unsaturated plasma will cool at an even faster rate than what is predicted from Bjorken scaling. The cooling rate will also be accelerated further if expansion in three dimensions is considered. Therefore, we will show that although the equilibrium
density distribution of subcritical hadron bubbles is significant - particularly when the transition is weak - unlike the situation in cosmology, they do not contribute strongly to phase mixing. For the range of parameters we investigated, of relevance for RHIC and LHC energies, the plasma cools so rapidly that the subcritical bubbles do not have time to reach their equilibrium distribution and promote substantial phase mixing: significant supercooling is possible in a (nearly) homogeneous quark gluon state.

II. SUB-CRITICAL BUBBLE FORMALISM

To study the dynamics of a first order phase transition, we use a generic form of the potential in terms of a real scalar order parameter \( \phi \) given by

\[
V(\phi) = a(T)\phi^2 - bT\phi^3 + c\phi^4. \tag{1}
\]

The parameters \( a, b \) and \( c \) are determined from physical quantities, such as the surface tension \( (\sigma) \) and the correlation length \( (\xi) \) of the fluctuations, and also from the requirement that the second minimum of the above potential should be equal to the pressure difference between the two phases. The bag equation of state is used to calculate the pressure in the two phases. The potential \( V(\phi) \) has a minimum at \( \phi = 0 \) and a metastable second minimum at

\[
\phi_+ = \frac{3bT + \sqrt{9b^2T^2 - 32ac}}{8c} \tag{2}
\]

below \( T \leq T_1 \). In the thin wall approximation, \( b, c \) and \( T_1 \) can be written as

\[
b = \frac{1}{\sqrt{6\sigma\xi T_c}}; \quad c = \frac{1}{12\xi^3\sigma}; \quad T_1 = \left[ \frac{BT_c^4}{B - \frac{27}{16}V_b} \right]^{1/4}, \tag{3}
\]

where \( B \) is the bag constant and

\[
V_b(\phi_m) = \frac{3\sigma}{16\xi(T_c)} \tag{4}
\]

is the height of the degenerate barrier at \( T = T_c \) or at \( a(T_c) = b^2T_c^2/4c \). A wide spectrum of first-order phase transitions, ranging from very weak to strong, can be studied by either changing \( \sigma \) or \( \xi \) or both. For example, for a fixed value of \( \xi \), the strength of the transition is controlled by \( \sigma \), becoming very weak first order or second order when \( \sigma \to 0 \).

We follow Ref. [6] to obtain the equilibrium number density of subcritical bubbles. Let \( n(R, t) \) be the number density of bubbles with a radius between \( R \) and \( R + dR \) at time \( t \) that satisfies the Boltzmann equation

\[
\frac{\partial n}{\partial t} = -v |\partial n/\partial R| + (1 - \gamma)\Gamma_0 - \gamma \Gamma_+. \tag{5}
\]

The first term on the right-hand side is the shrinking term with velocity \( v = \partial R/\partial t \). The term \( \Gamma_0 \) is the rate per unit volume for the thermal nucleation of a bubble of radius \( R \) of phase \( \phi = \phi_+ \) (hadron phase) within the phase \( \phi = 0 \) (QGP phase). Similarly, \( \Gamma_+ \) is the corresponding rate of the phase \( \phi = 0 \) within the phase \( \phi = \phi_+ \). The factor \( \gamma \) is defined as the volume fraction in the hadron phase. Assuming \( \Gamma_0 \approx \Gamma_+(= \Gamma) \) for a degenerate potential at \( T = T_c \), we write for the rate

\[
\Gamma = AT^4 \exp \left[ -\frac{F(\phi_+)}{T} \right]. \tag{6}
\]

where \( A \) is a constant of order unity. Using the Gaussian ansatz for subcritical configurations;

\[
\phi(r) = \phi_+ \exp \left( -\frac{r^2}{R^2} \right), \tag{7}
\]

the free energy functional

\[
F(\phi) = 4\pi \int r^2 dr \left[ \frac{1}{2} \frac{\partial \phi}{\partial r}^2 + V(\phi, T) \right] \tag{8}
\]

can be written as

\[
F(\phi_+) = \alpha R + \beta R^3, \tag{9}
\]

where

\[
\alpha = \frac{3\sqrt{2\pi^{3/2}\phi_+^2}}{8} \tag{10}
\]

and

\[
\beta = \pi^{3/2}\phi_+ \left[ \sqrt{2\alpha} - \frac{\sqrt{3bT\phi_+ + c\phi_+^2}}{9} \right]. \tag{11}
\]

The equilibrium number density \( n_0 \) of subcritical bubbles is found by solving Eq. (8) with \( \partial n/\partial t = 0 \) and imposing the physical boundary condition \( n(R \to \infty) = 0 \). Using \( \gamma_0 \approx 4\pi R^3n_0/3 \), we get a coupled equation for \( \gamma_0 \), which can be solved to get

\[
\gamma_0 = \frac{I}{1 + 2I}. \tag{12}
\]

where

\[
I = \int_R^\infty \frac{4\pi}{3v} R^3 \Gamma(R', \phi_+) dR'. \tag{13}
\]

We will consider the statistically dominant fluctuations with \( R \approx \xi \) and estimate \( \gamma_0 \) integrating Eq. (12) from \( \xi \) to \( \infty \). Neglecting the shrinking term in Eq. (6), the time dependent solution of \( n(\xi, t) \) can be written as

\[
n(\xi, t) = n_0(\xi)[1 - \exp\{-q(\xi)t\}], \tag{14}
\]

where \( q(\xi) = [(8\pi^3/3)\Gamma]/n_0(\xi) = \Gamma(\xi)/q(\xi) \). Alternatively, in term of \( \gamma \), the above solution has the form
\[ \gamma(\xi, t) = \gamma_0(\xi)[1 - \exp(-q_0 t)], \]

where \( q_0 = (4\pi \xi^3/3)\Gamma/\gamma_0 \). The relaxation time \( \tau = q_0^{-1} \) depends on two factors \( \gamma_0 \) and \( \Gamma \) out of which only the \( \gamma_0 \) is affected by shrinking (if included). Since we know the complete solution of \( \gamma_0 \) that includes shrinking \( [Eq. (12)] \), Eq. (15) can also be used to estimate its time dependence. Note that the presence of a shrinking term in Eq. (16) results in a reduction of \( \gamma_0 \) and also in a faster relaxation process.

### III. RESULT AND DISCUSSION

First we consider the slow evolution of the medium as in the case of early universe \[ \xi, T \] so that the equilibrium scenario is applicable. Figure 1 shows the plot of \( \gamma_0 \) as a function of \( \sigma \) at \( T = T_c \) for a few typical values of the prefactor \( A \). We have fixed \( \xi \) at 0.5 fm, \( T_c \) at 160 MeV and \( v = 1/\sqrt{3} \). As expected, the equilibrium hadronic fraction increases with decreasing \( \sigma \) and becomes as large as 0.5 for weak transitions. Recent lattice QCD predictions \[ \xi, T \] suggest that the quark-hadron phase transition could be weakly first order with \( \sigma \) values in the range 2 - 10 MeV/fm\(^2\). Therefore, the choice of \( \sigma \) in the above range and \( A \sim 1 \) \[ \xi, T \] would imply significant amount of phase mixing at \( T = T_c \) so that homogeneous nucleation becomes inapplicable \[ \xi, T \].

![Graph](image)

**FIG. 1.** \( \gamma_0 \) versus \( \sigma \) at \( T = T_c \) for a few typical values of \( A, \xi \) is fixed at 0.5 fm and \( T_C \) at 160 MeV.

Next we consider the plasma expected to be formed at RHIC and LHC energies. Since the expansion of such plasma is much faster compared to the plasma at the early universe, it will be interesting to know the amount of phase mixing (the value of \( \gamma \)) built up by the time the plasma cools from \( T_1 \) to \( T_c \). Assuming ideal scaling, we can estimate the time \( \Delta t \) taken by the plasma to cool from \( T_1 \) to \( T_c \) as

\[ \Delta t = \frac{T_0}{T_c} \left[ 1 - \frac{T_c}{T_1} \right], \]

where \( \nu = 3 \) in \((1+1)\) dimensions. Since \( T_1 \) depends on \( \sigma \) [see Eq. (11)], \( \Delta t \) will also depend on \( \sigma \), being smaller the weaker the transition. In the standard scenario, we can assume the initial temperature \( T_0 \approx 320 \) MeV and the formation time \( t_0 \approx 1 \) fm. However, several perturbative-inspired QCD models \[ \xi, \nu \] suggest a very different collision scenario at RHIC and LHC energies, which lead to the formation of unsaturated plasma with high gluon content. Such a plasma will attain thermal equilibrium in a short time \( t_0 \approx 0.3 - 0.7 \) fm, but will remain far from chemical equilibrium. Since the initial plasma is gluon rich, more quark and anti-quark pairs will be needed in order to achieve chemical equilibration. The dynamical evolution of the plasma undergoing chemical equilibration was studied initially by Biro et. al. \[ \xi, \nu \] and subsequently by many others \[ \xi, \nu \] by solving the hydrodynamical equations along with a set of rate equations governing chemical equilibration. It was found that a chemically unsaturated plasma cools faster than what is predicted by Bjorken scaling, since additional energy is consumed during chemical equilibrium. Following Ref. \[ \xi, \nu \], we have studied chemical equilibration and dynamical evolution of the QGP with two sets of initial conditions, HIJING \[ \xi, \nu \] and Self Screened Parton Cascade Model (SSPM) \[ \xi, \nu \], as listed in Table I. The Perturbative QCD inspired models like Parton Cascade Model (PCM) \[ \xi, \nu \] and HIJING (Heavy Ion Jet Interaction Generator) \[ \xi, \nu \] are generally used to simulate the nuclear collisions at collider energies on the level of microscopic parton dynamics. The PCM calculations describe the space time evolution of quark and gluon distributions by Monte Carlo simulations of relativistic transport equations. The HIJING model also incorporates the perturbative QCD approach and multiple minijet productions, however, it does not incorporate a direct space time description. Early PCM calculations were done by assuming a \( p_T \) cutoff to ensure the applicability of the perturbative expansion of the QCD scattering process. In the recently formulated self screened parton cascade model (SSPM) \[ \xi, \nu \], early hard scattering produces a medium which screens the longer range color fields associated with softer interactions. The screening occurs on a length scale where perturbative QCD still applies and the divergent cross sections in the calculation of the parton production can be regulated self-consistently without an ad hoc cutoff parameter. The numerical studies based on the parton cascade model suggest that the parton plasma produced in the central region is essentially a hot gluon plasma and the dynamics is mostly dominated by gluons. Gluons thermalize rapidly reaching approximately isotropic momentum distributions in a very short time scale. The densities of quarks and antiquarks stay well...
below the gluon density and can not build up to the full equilibration values required for an ideal chemical mixture of gluons and quarks. The similar conclusions have also been drawn from the calculations based on HIJING approach. Though both PCM and HIJING are QCD inspired models, the two still differ in quantitative predictions possibly due to different treatment of multiple parton interactions and collective effects. In the following, we take the initial conditions obtained both from HIJING and SSPM calculations at the time when parton momentum distribution becomes isotropic. We consider two dominant reaction channels $q\bar{q} \leftrightarrow gg$ and $gg \leftrightarrow ggg$ that contribute to the chemical equilibrium. The fugacity $\lambda_{g(q)}(\leq 1)$ gives the measure of the deviation of the gluon (quark) density from the equilibrium value; chemical equilibrium is achieved when $\lambda_i's \rightarrow 1$. For a detail discussion on chemical equilibration, we further refer to [18].

TABLE I. Initial conditions are taken from Ref. [20] as predicted by SSPM and HIJING calculations. The fugacities $\lambda_i$'s give a measure of the deviation of the gluon or quark densities from the equilibrium values.

| CODE  | ENERGY | $t_0$ (fm/c) | $T_0$ (GeV) | $\lambda_g$ | $\lambda_q$ | $\nu$ |
|-------|--------|-------------|-------------|-------------|-------------|-------|
| SSPM  | RHIC   | 0.25        | 0.668       | 0.34        | 0.064       | 2.2   |
| SSPM  | LHC    | 0.25        | 1.02        | 0.43        | 0.082       | 2.2   |
| HIJING| RHIC   | 0.7         | 0.55        | 0.05        | 0.008       | 1.9   |
| HIJING| LHC    | 0.5         | 0.82        | 0.124       | 0.02        | 1.8   |

Figure 2 shows a typical example of the effect of chemical equilibration on the cooling rate for SSPM initial conditions at RHIC energy ($\lambda_{g0} = 0.34$, $\lambda_{q0} = 0.064$, $t_0 = 0.25$ fm and $T_0 = 0.668$ GeV). The dotted curve (marked as $T_B$) shows the cooling rate as a function of time which obeys Bjorken's scaling ($T^3 t=\text{constant}$) corresponding to the case of an equilibrated plasma ($\lambda_g = \lambda_q = 1.0$). In case of a chemically unsaturated plasma for which the values of initial fugacities are much less than unity, the hydrodynamical expansion of the plasma proceeds along with chemical equilibration. As a result, both $\lambda_g$ and $\lambda_q$ increase with time as well as the temperature (shown by dashed curve) drops at a faster rate as compared to the Bjorken's scaling. The solid circles show the temperature given by ($T^\nu t=\text{constant}$ for $\nu = 2.2$). In this work, since we are interested only in the cooling rate, we skip the details of the calculation and parameterize the cooling rate in terms of $\nu$ in the range $T_1 < T < T_c$ (i.e. $T^\nu t=\text{const}$). In Table I, $\nu$ has been listed for two sets of initial conditions obtained using HIJING and SSPM models at RHIC and LHC energies. Note that $\nu < 3$ implies a faster cooling. Figure 3 shows the plot of $\Delta t$ as a function of $\sigma$ as obtained from Eq. (16) for different $\nu$ values. The time $\Delta t$ depends on the initial values of the temperature $T_0$, formation time $t_0$ and also on the cooling rate $\nu$. However, except for the SSPM initial conditions at LHC energies, values of $\Delta t$ obtained with other initial conditions have nearly similar values.

FIG. 2. The temperature $T$ and fugacity $\lambda$ as a function of time $t$. The description of the various curves are given in the text.
Next we proceed to estimate the density of subcritical hadron bubbles built up at \( t = \Delta t \). Figure 4 shows \( \gamma(t)/\gamma_0 \) as a function of \( t \) at three different \( \sigma \) values. The equilibration rate of the subcritical hadron bubbles of a given radius depends on the ratio \( \Gamma/\gamma_0 \). Although both \( \Gamma \) and \( \gamma_0 \) are larger for weaker transitions, their ratio decreases with decreasing \( \sigma \). Therefore, as can be seen, equilibration is faster for a stronger transition as compared to the weak one.

Figure 5 shows the fraction of the density built up at time \( t = \Delta t \) as a function of \( \sigma \) with different initial conditions. Although the equilibrium density distribution of subcritical hadron bubbles increases with decreasing \( \sigma \), the time \( \Delta t \) decreases with decreasing \( \sigma \). As a result of these two competing effects, \( \gamma \) at \( t = \Delta t \) shows a peak at around \( \sigma \approx 20 \text{ MeV/fm}^2 \). The equilibrium fraction \( \gamma_0 \) depends on the ratio \( A/v \), which increases either due to increase in \( A \) or decrease in \( v \). However, the variation in \( A \) and \( v \) act differently on \( q_0 \) as the nucleation rate \( \Gamma \) depends only on \( A \). Therefore, we study the effect of \( A \) and \( v \) on \( \gamma_0 \) and \( \gamma \) separately. Figure 6(a) shows the plot of \( \gamma_0 \) (upper curves) and \( \gamma(t) \) (lower curves) as a function of \( \sigma \) at \( A=5,10 \) and 20 respectively. Other parameters are \( v = 1/\sqrt{3} \), \( T_c = 160 \text{ MeV} \) and \( \xi=0.5 \text{ fm} \). As expected, \( \gamma_0 \) goes up as \( A \) increases. The increase in \( \gamma_0 \) for \( A \) from 5 to 20 is about 1.5 to 2 times, but the nucleation rate \( \Gamma \) goes up by a factor of 4. Therefore, the ratio \( \Gamma/\gamma_0 \) also goes up resulting in a faster equilibration. The net consequence is both \( \gamma_0 \) and \( \gamma(t) \) go up with increasing \( A \). For the calculation of \( \gamma(t) \), we have used SSPM and RHIC initial conditions. Further, we would like to mention here that although we have varied \( A \) up to 20, the value of \( A \) more than unity is unrealistic. A recent work by us [22] and also studies in ref [23] suggest \( A << 1 \). However, the ratio \( A/v \) can also go up with decrease in \( v \), which we study in figure 6(b). Figure 6(b) shows \( \gamma_0 \) and \( \gamma \) for \( v = c = 1 \) (upper limit), 1/4 and 1/12. This corresponds to a \( A/v \) ratio of 5, 20 and 60 respectively. Therefore, \( \gamma_0 \) goes up with decreasing \( v \) as expected. Since \( A \) is fixed, \( \Gamma \) does not change, but \( q_0 \) decreases with increasing \( \gamma_0 \) resulting in slower equilibration. As a result, \( \gamma(t) \) does not build up at all. It is also interesting to note that \( \gamma(t) \) does not get affected much by the choice of \( v \) although \( \gamma_0 \) has a strong dependence on it. The \( \gamma(t) \) only depends on parameter \( A \). This aspect is interesting.
From the above studies (from figures 5 and 6), we can conclude that, the fraction in the range $2\text{MeV/fm}^2 \leq \sigma \leq 10\text{MeV/fm}^2$ does not build up to a significant level due to rapid cooling of the plasma, although the equilibrium concentration is fairly large. It may be mentioned here that we have considered expansion only in (1+1) dimensions. Inclusion of transverse expansion, significant at RHIC and LHC energies, will accelerate the cooling rate further, reducing the amount of phase mixing considerably. Since phase mixing at $T = T_c$ is negligible, the plasma will supercool and the phase transition may proceed by the nucleation of critical-size hadron bubbles within a (nearly) homogeneous background of the metastable QGP phase.

We have also studied the effect of other parameters like $T_c$ and $\xi$ on $\gamma$. Figure 7(a) shows the plots for various $T_c$ values at $A=5$. The nucleation rate decreases with decreasing $T_c$ [see Eq. (6)] resulting in a decrease in $\gamma_0$. On the other hand, smaller $T_c$ will result in larger $\Delta t$, which may increase $\gamma(t)$. However, as shown in figure 7(a), the variation in $\gamma(t)$ with $T_c$ is not very significant although $\gamma_0$ depends on it. Similarly, figure 7(b) shows the plots at various $\xi$ (0.5, 1.0 and 2.0). Increasing $\xi$ suppresses $\gamma_0$ and $\gamma(t)$ particularly when the transition is strong. Therefore, the effect of other parameters like $v$, $T_c$ and $\xi$ on $\gamma$ are not very significant to promote phase mixing. The prefactor $A$ is the only sensitive parameter on which $\gamma(\Delta t)$ depends. While the choice of $A \approx 1$ is quite reasonable, we have also varied $A$ from 1 to 20 and did not find significant phase mixing particularly when $\sigma$ is small.

In conclusion, we have studied the effect of phase mixing promoted by thermal subcritical hadron bubbles during a first-order quark-hadron phase transition as predicted to occur during heavy-ion collisions. Although the equilibrium density distribution of these subcritical bubbles can be quite large, their equilibration time-scale is larger than the cooling time-scale for the QGP. As a consequence, for RHIC and LHC energies, they will not build up to a level capable of modifying the predictions from homogeneous nucleation theory. The phase transition may proceed either through the nucleation of critical-size hadron bubbles in a (nearly) homogeneous background of the supercooled quark-gluon plasma or through spinodal decomposition if nucleation rate is not significant [21]. This situation is to be contrasted with the cosmological quark-hadron transition, where substantial phase mixing may occur, altering the dynamics of the phase transition. We would also like to add here that even though our calculations rule out the role of subcritical bubbles, it is possible that impurities may increase the decay rate.
scale and no real supercooling will be measured, as is the case with many condensed matter systems. The question, however, remains as to what these impurities, if any, might be in this context. One possibility – ruled in this work – is that the subcritical bubbles, being seeds for nucleation, may act as impurities [24]. However, other possibilities, as the presence of condensates, may exist and should be considered in the near future. If there is supercooling there will be an extra entropy production which will reflect on the final hadron multiplicities. In this case, subcritical bubbles are not present, or are irrelevant. On the other hand, if the transition is first order and no extra entropy is observed, subcritical bubbles (or unknown impurities...) do play a role.

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[12] The solution of Einstein’s relativistic field equation yields a relation between the age of the universe and the temperature $[11]$, $t = \sqrt{\frac{\alpha}{G}}T^{-2}$ where $\alpha = 9/(164\pi^3)$. Since the Newton’s constant $G$ is very small, the above relation would imply $\Delta t = \sqrt{\frac{\alpha}{G}}(T_{c}^{-2} - T_{i}^{-2}) \sim \text{few} \mu \text{sec}$.