Model of an inductive sensor of cardiac activity attached to patient

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Abstract. An electric circuit model describing an inductive sensor of cardiac mechanical activity in its working condition has been developed. The sensor comprises a single-turn coil which is fed by 7.7 MHz constant current and induces probing eddy currents in the body. The inductor is considered to be attached to the thoracic surface of a normal human male, in front of the heart. A simple axial-symmetric model of the thorax, formed of tightly packed circular current tubes, has been used to calculate the resistances, self-inductances and mutual inductances within the human body. Then the inductor and the eddy currents in the body were reduced to a system of two inductively coupled coils; estimates of the parameters and frequency response of the system have been found; the active and reactive contributions of the human body to the resulting impedance of the inductor were calculated.

1. Introduction
In induction methods for electrical impedance measurement and tracking [1], [2], the probing currents are attached in the form of Foucault or eddy currents, using inductors. The same inductors are applied as signal receivers, thus forming sensors for inductive measurement. To improve the induction methods, identified models of such sensors are necessary. The present study is based on a concrete example of the Foucault cardiographic sensor which is applied to track heart mechanical activity [2], [3]. The objective of the present work was to develop a realistic model of the sensor loaded with a patient and, using the model, to study the factors which determine the origination of signal in the induction method for tracking the heart.

2. The inductive sensor
The example sensor comprises a shielded single-turn coil. The inductor coil is 13.5 cm in diameter and made of copper wire with a rectangular 10 mm × 2 mm cross-section. Together with a capacitor it forms a resonance circuit with a resonance frequency of 7.7 MHz. A single-frequency Foucault cardiograph incorporating the sensor operates at the resonance frequency of this resonance circuit. The inductor is electrically shielded by a grounded box-like surrounding. The shielding box (see figure 1) is made of 1 mm-thick foil-clad laminate that has been extra engraved to disable eddy currents in it. The sensor is assumed to be attached to the thoracic surface of a normal human male just in front of the heart.
3. Model of the induction features in the system

A static model with cylindrically symmetric geometry (similar to the one used in [2], [3], see figure 3) was applied to calculate the inductor-eddy currents interaction. The eddy currents region of the body had to be modeled as a distributed system. To calculate the interaction, the cylinder representing the body region of eddy currents was approximated by a dense-packed set of $n$ co-axial current tubes or coils that had equal small square cross-sections. The situation in figure 2 was further interpreted in terms of two inductive-coupled coils or circuits, in which there was a single-turn primary one (the inductor) and a secondary equivalent one. The impedance of the latter was considered to consist of resistive and inductive components.

At a fixed topography, a distribution of individual currents is established in the eddy currents region. It has an influence on the equivalent characteristics of the secondary circuits. Starting from equal power loss requirement, the total loss resistance of the secondary circuit $R_{II}$ has to be calculated from the individual resistances of the secondary coils $R_i$:

$$R_{II} = \sum_{i=1}^{n} K_i^2 \cdot R_i, \quad \text{where} \quad K_i = I_i \cdot \left(\sum_{k=1}^{n} I_k \right)^{-1}. \quad (1)$$

Figure 3. The model of the eddy currents region of the body. (a) and (b) – lateral and frontal cross-sections of the model, (c) – the region of calculation: a half of the section of the model (case of 60×72 secondary coils).

I – the inductor, W – thoracic wall, C – a secondary coil, P – pulmonary tissue, H – the heart.
Similarly, the equivalent self-inductance of the secondary circuit and the equivalent mutual inductance between the inductor and the secondary circuit were calculated. Before composing a single equivalent for the multiple secondary coils, the actual values of self-inductances $L_i$ and mutual inductances between the secondary coils $M_{ij}$ had to be calculated. Under the condition of diversity of the secondary (eddy) currents, the equal magnetic flux requirement has led to formulae, which express $L_i$ and $M_{ij}$ through the corresponding values $\tilde{L}_i$ and $\tilde{M}_{ij}$ for stand-alone coils:

$$L_i = nK_i \cdot \tilde{L}_i, \quad M_{ij} = nK_i K_j \cdot \tilde{M}_{ij}.$$  

(2)

The stand-alone values $\tilde{L}_i$ and $\tilde{M}_{ij}$ were calculated using available formulae from [4]. A similar approach was used to calculate the mutual inductances between the secondary coils and the inductor coil.

Then the equivalent total self-inductance [4] of the secondary circuit $L_{II}$ was expressed as

$$L_{II} = \frac{1}{n^2} \left( \sum_{i=1}^{n} L_i + \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} \right), \quad i \neq j.$$  

(3)

Skin effect causes concentration of the inductor currents into the opposite narrow sides of the inductor coil’s surface. Therefore, the inductor is modeled as two equal threads of current. The equivalent mutual inductance between the primary and the secondary circuit $M_{I,II}$ was expressed as

$$M_{I,II} = \frac{1}{n} \sum_{i=1}^{n} (M_{i,prox} + M_{i,dist}),$$  

(4)

where $M_{i,prox}$ and $M_{i,dist}$ are the mutual inductances between the secondary coils and either proximal or distal current threads in the inductor coil, calculated according to the formula (2).

Distribution of the currents in the secondary coils, which is necessary in formula (1) to find the coefficients $K_i$, was estimated ignoring the backward influence of the magnetic field of the induced currents. Calculation of the corresponding electromotive forces is presented in [5]. The currents were obtained using the Ohm’s Law.

An inductive sensor transforms the secondary circuit impedance into additional series impedance $\Delta \Omega = \Delta R + \omega \Delta X$ in the primary circuit coil. This feature is described by the following formulae [6]:

$$\Delta R(\omega) = -\frac{\omega^2 M_{I,II}^2}{R_{II}^2 + \omega^2 L_{II}^2} \cdot R_{II}, \quad \Delta X_1(\omega) = -\frac{\omega^2 M_{I,II}^2}{R_{II}^2 + \omega^2 L_{II}^2} \cdot \omega L_{II}.$$  

(5)

For the actualization of the model, the size dimensions of the inductor, as well as its electric properties ($R_i = 33 \, \text{m}\Omega, L_i = 228 \, \text{nH}, C_i = 1.80 \, \text{nF}$) were predetermined by (or measured in) the example sensor. The capacitances from the shielding to the inductor and to the body, $C_S \approx 40 \, \text{pF}, C_B \approx 140 \, \text{pF}$, respectively, were measured in the example sensor. Due to its relatively small value, $C_S$ in the present sensor only weakly influences the primary circuit’s parallel capacitance. The other one, $C_B$, in a geometrically symmetric model should not act at all. Though in real asymmetric cases it would cause a certain amount of noise if changed, in the present model this is not the case.

The dimensions of the body disk and its contents (Table 1) were roughly derived from a normal adult person. Determination of the specific impedances for the regions of the body disk is an intricate task. Combining the values from various literature data, a fairly realistic approximation with pure active resistivities has been taken.

| Parameters | Values |
|------------|--------|
| Diameters of the body disk, inductor coil, the heart | 40 cm, 13.4 cm, 12 cm |
| Thicknesses of the body disk, thoracic wall | 24 cm, 2 cm |
| Distance between the inductor and thoracic surface | 0.7 cm |
| Resistivities of the heart, thoracic wall, pulmonary region | 136 $\Omega \cdot \text{cm}$, 150 $\Omega \cdot \text{cm}$, 325 $\Omega \cdot \text{cm}$ |
4. Study of the model. Discussion

Calculations of the model with MATLAB gave the following properties of the system for the working frequency $f_w = 7.686$ MHz: $R_{II} = 65 \ \Omega$, $L_{II} = 45 \ \text{nH}$ ($\omega_w L_{II} = 2.2 \ \Omega$), $M_{I,II} = 39 \ \text{nH}$ ($\omega_w M_{I,II} = 1.9 \ \Omega$) that lead to $\Delta R_I = 0.054 \ \Omega$, $\Delta X_I = -0.0018 \ \Omega$. Calculation of the input impedance $Z_{in}$ of the loaded inductor, using these results, gave the frequency response shown in figure 4(a), the middle curve. It is a typical resonance curve with a quality factor $Q = 125$, max $|Z_{in}| = 1.40 \ \text{k}\Omega$. The latter agrees with the values measured using the example sensor on normal male volunteers in experiment [3], which ranged in region of $1…2 \ \text{k}\Omega$.

![Figure 4](image)

**Figure 4.** The frequency responses of the loaded sensor.

- (a) – for the modulus of the input impedance $Z_{in}$: middle curve – the actual case of the sensor, with $C_1 = 1.80 \ \text{nF}$; the shadow side curves – with $C_1$ increased or decreased 10 times.
- (b) – for the additional primary circuit resistance $\Delta R_I$ and reactance $\Delta X_I$.

Both the $\Delta R_I$ and $\Delta X_I$ are strongly frequency-dependent (figure 4(b); at the working frequency the slopes are $40 \ \text{dB/decade}$ and $60 \ \text{dB/decade}$, respectively). Thus to improve the signal magnitude, higher resonance frequency would be instrumental. This way is limited by the parasitic capacitance $C_S$.

In the sensor under review, which is attached to the patient, of the object-related contributions to the inductor coil impedance, the resistive one prevails over the reactive one up to the frequencies which are $30$ times above the present working frequency. Thus, from the signal taken with the sensor, detection of either the in-phase component or the modulus of the signal is reasonable. Though at the higher frequencies the prevalence would change, working at these frequencies is rendered impossible by the $C_S$.

We are going to use the developed model as an instrument for analyzing further the capabilities of the induction-based cardiac electrical bioimpedance recording.

5. References

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