Modified Chaplygin Gas as Scalar Field
and Holographic Dark Energy Model

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Abstract

We study the correspondence between field theoretic and holographic dark energy density of the universe with the modified Chaplygin gas (MCG) respectively both in a flat and non-flat FRW universe. We present an equivalent representation of the MCG with a homogeneous minimally coupled scalar field by constructing the corresponding potential. A new scalar field potential is obtained here which is physically realistic and important for cosmological model building. In addition we also present holographic dark energy model described by the MCG. The dynamics of the corresponding holographic dark energy field is determined by reconstructing the potential in a non-flat universe. The stability of the holographic dark energy in this case in a non-flat universe is also discussed.

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1 Introduction:

In the recent years a number of observations have been carried out that lead to a precise knowledge of the cosmological evolution. Cosmological observations like abundances of galaxy cluster large scale redshift surveys [1], angular power spectrum of CMBR [2], and baryon oscillations [3] suggest that the universe is nearly flat and almost 73% of the matter in the form of dark energy. Further the magnitude redshift surveys of type Ia Supernovae indicates the universe has recently entered a phase of accelerating phase of expansion [4]. On the otherhand it is generally accepted that our universe might have also emerged from an accelerating phase in the past. Thus one of the essential ingredients in modern cosmology is inflation. However, inflation cannot be accommodated in a perfect fluid assumption in the framework of Einstein general theory of Relativity (GTR). This led to explore gravitational theories with a modification of the matter sector or the gravitational sector. It is known that the early inflation may be realized in a semiclassical theory of gravity where matter is described by quantum fields [5]. Starobinsky also obtained inflationary solution considering a curvature squared term in the Einstein-Hilbert action [6] long before the advent of inflation was known. However, the efficacy of inflation is known only after the seminal work of Guth who first employed the phase transition mechanism to accommodate inflation. A large volume of literature appeared with a modification of gravitational sector in which curvature squared terms [7] were added to the Einstein-Hilbert action to realize early inflationary universe scenario. In the case of inflaton field cosmology early inflation may be realized with an equation of state \( p = \omega \rho \), where \( \omega = -1 \). However, an accelerating late universe with barotropic fluid emerges when \( \omega < -1 \). The usual matter fields in the standard model of particle physics unable to accommodate the late accelerating phase of the universe. Therefore, it is a challenging task to formulate a
consistent theoretical framework which might accommodate the observational facts. Recent astronomical data when interpreted in the context of Bigbang model have provided some interesting information about the composition of the universe. It is believed that the universe is dominated by a huge amount of dark energy. To accommodate such a huge energy various mechanism have been proposed during the last few years in order to classify the physical nature of the cosmic fluid [8]. Among the different theories proposed, the single component fluid known as Chaplygin gas [9] with an equation of state \( p = -\frac{A}{\rho} \), where \( \rho \) and \( p \) are the energy density and pressure respectively and \( A \) is a constant, has attracted large interest in cosmology [7]. The above equation of state, however, has been conceived in studies of adiabatic fields. It was used to describe lifting forces on a plane wing in aerodynamic process. In cosmology, although it admits an accelerating universe [10], fails to address structure formation and cosmological perturbation power spectrum [11]. Subsequently, a generalized form of the equation of state (in short, EOS) \( p = -\frac{A}{\rho^\alpha} \) with \( 0 \leq \alpha \leq 1 \) was considered to construct a viable cosmological model [12, 13], which is known as generalized Chaplygin gas (in short, GCG) in cosmology. It has two free parameters \( A \) and \( \alpha \). The fluid behaves initially like dust for small size of the universe, but at a later epoch the fluid may be described by an equation of state \( p = \omega \rho \). It has string connection, the above equation of state can be obtained from the Nambu-Goto action for a D-brane moving in a (D+2)-dimensional space-time in the light cone parametrization [14]. Recently a new form of equation of state \( p = f(\rho) \) has been considered to study the dark energy content of the universe [15]. Consequently a three parameter modified form of the equation of state for Chaplygin gas is more important. Therefore, we consider an equation of state of the form

\[
p = B\rho - \frac{A}{\rho^{\alpha}} \quad \text{with} \quad 0 \leq \alpha \leq 1,
\]  

(1)
where \( B \) is an equation of state parameter and \( A \) is a constant, which is termed as modified Chaplygin gas (in short, MCG) \([16]\). An interesting feature of the MCG equation of state is that in the early universe when the size of the universe \( a(t) \) was small, it behaved like a barotropic fluid (if one considers \( B = \frac{1}{3} \) it corresponds to radiation and \( B = 0 \) it corresponds to matter) but at a later epoch it behaves as a cosmological constant which can be fitted to a \( \Lambda \)CDM model. Recently, the thermal equation of state of the MCG is studied \([17]\) and it is found that the MCG may cool down through thermodynamic processes without facing any critical point or phase transition. They noted the following constraints for a realistic solution on the values of the parameters: (i) for \( B \sim 0 \), \( 0 < \alpha < 1 \), (ii) for \( B = \frac{1}{3} \), \( 0 < \alpha < \frac{1}{2} \), (iii) for \( B = 1 \), \( \alpha \sim 0 \). Barrow \([18]\) has outlined a method to fit the Chaplygin gas in the FRW universe. In a flat Friedmann model it is shown \([10]\) that the generalized Chaplygin gas may be equivalently described in terms of a homogeneous minimally coupled scalar field \( \phi \). Gorini et al. \([19]\) using the above scheme obtained the corresponding homogeneous scalar field \( \phi(t) \) and the corresponding potential \( V(\phi) \) which can be used to obtain a viable cosmological model with the generalized Chaplygin gas. Thus it is important to look for the field and the relevant potential for the modified Chaplygin gas, which will be discussed in the section 3.

Recently, another interesting topic, namely holographic principle \([20, 21]\) is incorporated in cosmology \([22-25]\) to track the dark energy content of the universe following the work of Cohen et al. \([26]\). Holographic principle is a speculative conjecture about quantum gravity theories proposed by G’t Hooft \([27]\). The idea has been subsequently promoted by Susskind and his coworkers \([20]\) claiming that all the information contained in a spatial volume may be represented by a theory that lives on the boundary of that space. For a given finite region of space it may contain matter and energy within it. If this energy suppresses a critical density then the region collapses to a black hole. A black hole is
known theoretically to have an entropy which is proportional to its surface area of its
event horizon. A black hole event horizon encloses a volume, thus a more massive black
hole have larger event horizon and encloses larger volume. The most massive black hole
that can fit in a given region is the one whose event horizon corresponds exactly to the
boundary of the given region under consideration. The maximal limit of entropy for an
ordinary region of space is directly proportional to the surface area of the region and not
to its volume. Thus, according to holographic principle, under suitable conditions all the
information about a physical system inside a spatial region is encoded in the boundary.
The basic idea of a holographic dark energy in cosmology is that the saturation of the
entropy bound may be related to an unknown ultra-violet (UV) scale $\Lambda$ to some known
comological scale in order to enable it to find a viable formula for the dark energy which
may be quantum gravity in origin and it is characterized by $\Lambda$. The choice of UV-Infra
Red (IR) connection from the covariant entropy bound leads to a universe dominated by
blackhole states. According to Cohen et al. [26] for any state in the Hilbert space with
energy $E$, the corresponding Schwarzschild radius $R_s \sim E$, may be less than the IR cut off
value $L$ (where $L$ is a cosmological scale). It is possible to derive a relation between the
UV cutoff $\rho_{\Lambda}^{1/4}$ and the IR cutoff which eventually leads to a constraint $\left(\frac{8\pi G}{c^2}\right)^3 \left(\frac{\rho_{\Lambda}}{3}\right) \leq L$
[26] where $\rho_{\Lambda}$ is the energy density corresponding to dark energy characterized by $\Lambda$, $G$ is
Newton’s gravitational constant and $c$ is a parameter in the theory. The holographic dark
ergy density is
\begin{equation}
\rho_{\Lambda} = 3c^2 M_P^2 L^{-2},
\end{equation}
where $M_P^2 = 8\pi G$. It is known that the present acceleration may be described if $\omega_{\Lambda} =
\frac{\rho_{\Lambda}}{\rho_{\Lambda}} < -\frac{1}{3}$. If one considers $L \sim \frac{1}{H}$ it gives $\omega_{\Lambda} = 0$. A holographic cosmological constant
model based on Hubble scale as IR cut off does not permit accelerating universe. It is also
examined [22] that the holographic dark energy model based on the particle horizon as the
IR cutoff even does not work to get an accelerating universe. An alternative model of dark energy using particle horizon in closed model is also proposed [26]. Recently, Li [23] has obtained an accelerating universe considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Thus to have a model consistent with observed universe one should adopt the covariant entropy bound and choose \( L \) to be the event horizon.

The motivation of the paper is two folds (i) to explore an equivalent scalar field representation corresponding to the MCG in cosmology and (ii) to explore an equivalent holographic dark energy field, considering the event horizon as the cosmological scale in a non-flat universe. We obtain holographic description of the MCG dark energy in FRW universe and reconstruct the potential and the dynamics of the scalar field which describes the MCG cosmology. We consider both flat and non-flat universe here as it is not yet decided. The paper is organized as follows: in sec. 2, the relevant field equation with modified Chaplygin gas in FRW universe is presented; in sec. 3 we present an equivalent field theoretic representation of MCG by a scalar field, constructing the corresponding potential, in sec. 4, we suggest a correspondence between holographic dark energy fields with MCG. In sec. 5, squared speed of sound for holographic dark energy is evaluated for a closed universe i.e., \( k = 1 \) to study the stability of the field. Finally in sec. 6, a brief discussion.

2 Modified Chaplygin Gas in FRW universe:

The Einstein’s field equation is given by

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}
\]

(3)

where \( \kappa^2 = 8\pi G \), and \( T_{\mu\nu} \) is the energy momentum tensor.
We consider a homogeneous and isotropic universe given by
\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right] \] (4)
where \( a(t) \) is the scale factor of the universe, the matter is described by the energy momentum tensor \( T^\mu_\nu = (\rho, p, p, p) \) where \( \rho \) and \( p \) are energy density and pressure respectively.

Using the metric (4) and the energy momentum tensor, the Einstein’s field equation (3) can be written as
\[ H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2 \rho} \] (5)
where we use \( 8\pi G = M_p^{-2} = \kappa^2 \). The conservation equation for matter is given by
\[ \frac{d\rho}{dt} + 3H(\rho + p) = 0. \] (6)

For the modified Chaplygin gas, using the EOS given by eq. (1), the eq. (6) can be integrated to obtain the energy density which is given by
\[ \rho = \left( \frac{A}{1 + B} + \frac{C}{a^n} \right)^{\frac{1}{1+\alpha}} \] (7)
where \( C \) is an arbitrary constant and we denote \( 3(1 + B)(1 + \alpha) = n \). We define the following
\[ \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \quad \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_k = \frac{k}{a^2 H^2} \] (8)
where \( \rho_{cr} = 3M_p^2 H^2 \), \( \Omega_\Lambda, \Omega_m \) and \( \Omega_k \) represent density parameter corresponding to \( \Lambda \), matter and curvature respectively in the paper.

3 Modified Chaplygin Gas (MCG) as a scalar field:

In this section, we obtain the field theoretic representation of the modified Chaplygin gas assuming a homogeneous scalar field \( \phi(t) \). We use Barrow’s scheme [18] here and the corresponding energy density and pressure of the homogeneous field are identified as:
\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \left( \frac{B}{A + 1} + \frac{C}{a^n} \right)^{\frac{1}{1+\alpha}}, \] (9)
\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = -\frac{B}{A+1} + \frac{A C}{(B+1) + \frac{C}{a^n}} \]

which are equated with that of MCG given by eqs. (1) and (7) respectively. The scalar field potential and the corresponding kinetic energy of the field are obtained from eqs. (9) and (10), which are

\[ V(\phi) = \frac{A}{B+1} + \frac{1-B}{2} \frac{C}{a^n} \left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{1}{n+1}}, \]  

\[ \dot{\phi}^2 = \frac{(B+1)C}{(A)(B+1) + \frac{C}{a^n}} \left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{1}{n+1}}. \]  

In the next section we consider the above equations to determine the potential in the two cases (i) flat universe and (ii) non-flat universe using eq. (5).

Case I: For a flat universe \((k = 0)\), using eq. (5) and (9) we get

\[ H^2 = \frac{1}{3M_P^2} \left(\frac{B}{A+1} + \frac{C}{a^n}\right)^{\frac{1}{n+1}}. \]  

Using eqs. (12) and (13) we get

\[ \phi - \phi_o = \pm \frac{2M_P}{\sqrt{n(1+\alpha)}} sinh^{-1} \left[ \sqrt{\frac{(B+1)^2}{A}} \right] \]  

and the corresponding potential is

\[ V(\phi) = \frac{A}{1+B} + \frac{A(1-B)}{2(1+B)} sinh^2 \left(\frac{\sqrt{\frac{a(1+\alpha)}{2M_P}}(\phi - \phi_o)}{A}\right) \left(\frac{A}{1+B}\right)^{\frac{1}{2(1+B)}} cosh^{\frac{2\alpha}{1+\alpha}} \left(\frac{\sqrt{\frac{a(1+\alpha)}{2M_P}}(\phi - \phi_o)}{2}\right) \]  

The potential asymptotically approaches a constant as \(\phi \to \phi_o\), however, it increases with increasing value of the field if \(\phi \neq \phi_o\). The potential obtained by Gorini et al. [19] can be recovered here if one puts \(\alpha = 1\) and \(B = 0\).
Case II : For a non flat \((k \neq 0)\) universe, the evolution of the scalar field is obtained using eqs (5), (9) and (12), which is
\[
\phi - \phi_o = \pm \int \sqrt{\frac{12 M_P^2 (B + 1)}{n^2}} \frac{dz}{\sqrt{(\mu^2 + z^2) - \frac{3 M_P^2 k}{C^{2/n}} z^{2/n}(\mu^2 + z^2)^{\alpha + 1}}} \tag{16}
\]
where \(k = +1\) for closed universe (\(k = -1\) for open universe) and we denote \(\mu^2 = \frac{A}{B + 1}\), \(z = \sqrt{\frac{c}{a^n}}\). The above integration is not simple so as to express the potential in terms of the field. However, for some special choice of the parameters the potential may be obtained which can be expressed in terms of the field \(\phi\). We choose the following :

- \(B = -\frac{1}{3}\), \(A = 0\), the scalar field evolves as
  \[
  \phi_{\pm} = \phi_o \pm \sqrt{\frac{2 M_P^2}{n^2(1 - \frac{3 M_P^2 k}{c^{2/n}})}} \ln \left( \frac{C}{a^n} \right), \tag{17}
  \]
the corresponding scalar field potential is given by
  \[
  V(\phi) = \frac{2}{3} E x p \left[ \sqrt{\frac{1}{\frac{2 M_P^2 (\alpha + 1)^2}{n^2(1 - \frac{3 M_P^2 k}{c^{2/n}})}}} \left( \phi - \phi_o \right) \right]. \tag{18}
  \]
It is an exponential potential which increases (decreases) depending on the evolutionary behaviour of the scalar field \(\phi_+\) (\(\phi_-\)) as given in (17). We note that the potential is positive definite if \(C > (3 M_P^2 k)^{n/2}\). In the case of closed universe the above inequality gives a lower bound on the values of \(C\) which is a positive number. But for an open universe \(C\) picks up both positive and negative values for an even integral values of \(n\).

- \(B = \frac{1}{3}\), \(A \neq 0\), in this case the scalar field evolves as
  \[
  \phi - \phi_o = \pm \sqrt{\frac{16 M_P^2}{n^2(1 - \frac{3 M_P^2 k}{c^{2/n}})}} \sinh^{-1} \left( \frac{4 C}{3 A a^{n/2}} \right), \tag{19}
  \]
and the corresponding potential is given by
  \[
  V(\phi) = \text{sech}^2 \left( \frac{n^2(1 - \frac{3 M_P^2 k}{c^{2/n}})}{16 M_P^2} \right) (\phi - \phi_o) + \frac{1}{3} \tanh^2 \left( \frac{n^2(1 - \frac{3 M_P^2 k}{c^{2/n}})}{16 M_P^2} \right) (\phi - \phi_o). \tag{20}
  \]
Figure 1: shows the plot of $V$ versus $\phi$ with the parameter \( \frac{n^2(1-\frac{3M_P^2 k}{2})}{16M_P^2} = 1 \).

The fig. 1 is a plot of the potential in terms of the field. It may be noted that in the case of a flat universe \((k = 0)\), one obtains a potential different from that obtained in Ref. [19] as \(B \neq 0\). It is a new and interesting potential, which has a shape similar to that one obtains in the case of tachyonic field [29]. The potential attains to a constant value at a large time leading to late acceleration of the universe. We also note that an oscillatory scalar field results for \(C < (3M_P^2 k)^{n/2}\) in a sinusoidal potential in a closed universe. However, in the case of an open universe a realistic solution is permitted for an even integer values of \(n\) only.

4 MCG as Holographic Dark Energy:

In a FRW universe we now consider a non-flat universe with \(k \neq 0\) and use the holographic dark energy density as given in (2) which is

\[
\rho_\Lambda = 3c^2 M_P^2 L^{-2},
\]  

(21)
where $L$ is the cosmological length scale for tracking the field corresponding to holographic dark energy in the universe and $c$ is a parameter. The parameter $L$ is defined as

$$L = a r(t).$$ (22)

where $a(t)$ is the scale factor of the universe and $r(t)$ is relevant to the future event horizon of the universe. Using Robertson-Walker metric one gets [24]

$$L = \frac{a(t)}{\sqrt{|k|}} \sin \left[ \sqrt{|k|} R_h(t)/a(t) \right] \text{ for } k = +1,$$

$$= R_h \text{ for } k = 0,$$

$$= \frac{a(t)}{\sqrt{|k|}} \sinh \left[ \sqrt{|k|} R_h(t)/a(t) \right] \text{ for } k = -1. \quad (23)$$

where $R_h$ represents the event horizon which is given by

$$R_h = a(t) \int_t^\infty \frac{dt'}{a(t')} = a(t) \int_o^{r_i} \frac{dr}{\sqrt{1 - kr^2}}. \quad (24)$$

Here $R_h$ is measured in $r$ direction and $L$ represents the radius of the event horizon measured on the sphere of the horizon. Using the definition of $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2H^2$, one can derive [25]

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}}. \quad (25)$$

Using eqs. (23)-(26), one determines the temporal rate of change of $L$ which is

$$\dot{L} = \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} \cos \left( \frac{\sqrt{|k|} R_h}{a(t)} \right) \text{ for } k = +1,$$

$$= \frac{c}{\sqrt{\Omega_\Lambda}} - 1 \text{ for } k = 0,$$

$$= \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} \cosh \left( \frac{\sqrt{|k|} R_h}{a(t)} \right) \text{ for } k = -1. \quad (26)$$

Using eqs. (21)-(26), it is possible to construct the required equation for the holographic energy density $\rho_\Lambda$, which is given by

$$\frac{d\rho_\Lambda}{dt} = -2H \left[ 1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X) \right] \rho_\Lambda, \quad (27)$$
where we use the notation,

\[ f(X) = \frac{1}{\sqrt{|k|}} \cos \left( \sqrt{|k|} x \right) = \cos(X) \ [1, \cosh(X)] \text{ for } k = 1 [0, -1], \quad (28) \]

where \( X = \frac{R_i}{a(t)} \). The energy conservation equation is

\[ \frac{d\rho_\Lambda}{dt} + 3H(1 + \omega_\Lambda)\rho_\Lambda = 0 \quad (29) \]

which is used to determine the equation of state parameter

\[ \omega_\Lambda = -\left( \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X) \right). \quad (30) \]

Now we consider the holographic dark energy density which is assumed to be equivalent to the modified Chaplygin gas energy density. The corresponding energy density is taken from (7), which is obtained using the equation of state given by (1). The equation of state parameter corresponding to EOS (1) can be written as :

\[ \omega = \frac{p}{\rho} = B - \frac{A}{\rho^{\alpha+1}}. \quad (31) \]

Let us now establish the correspondence between the holographic dark energy and modified Chaplygin gas energy density. In this case from eqs. (7) and (21), we get

\[ C = a^n \left[ (3c^2 M_p^2 L^{-2})^{\alpha+1} - \frac{A}{B+1} \right]. \quad (32) \]

Now using eqs. (30)-(32), we determine the parameters as

\[ A = (3c^2 M_p^2 L^{-2})^{\alpha+1} \left[ B + \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} f(X) \right], \quad (33) \]

\[ C = (3c^2 M_p^2 L^{-2})^{\alpha+1} a^n \left[ 1 - \frac{3B + 1}{3(B+1)} - \frac{2\sqrt{\Omega_\Lambda}}{3(B+1)c} f(X) \right]. \quad (34) \]

The corresponding potential for the holographic dark energy field becomes

\[ V(\phi) = 2c^2 M_p^2 L^{-2} \left[ 1 + \frac{\sqrt{\Omega_\Lambda}}{2c} f(X) \right], \quad (35) \]
and the corresponding kinetic energy of the field is given by
\[ \dot{\phi}^2 = 2c^2 M_P^2 L^{-2} \left[ 1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X) \right]. \tag{36} \]

Considering \( x = \ln a \), we transform the time derivative to the derivative with logarithm of the scale factor, which is the most useful function in this case. We get
\[ \phi' = M_P \sqrt{2 \Omega_\Lambda \left( 1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X) \right)} \tag{37} \]

where \( ()' \) prime represents derivative with respect to \( x \). On integrating the above equation the evolution of the scalar field corresponding to holographic dark energy is evaluated which is given by
\[ \phi(a) - \phi(a_o) = \sqrt{2} M_P \int_{\ln a_o}^{\ln a} \Omega_\Lambda \left( 1 - \frac{\sqrt{\Omega_\Lambda}}{c} f(X) \right) dx. \tag{38} \]

The above result is obtained for a non-flat universe \( (k \neq 0) \). The flat case is not considered here as the holographic dark energy in a flat FRW universe is unstable \cite{30}. We study the stability of the holographic dark energy model for a non-flat universe by calculating the squared speed in the next section.

### 5 Squared speed for Holographic Dark Energy:

We consider a closed universe model \( (k = 1) \) in this case. The dark energy equation of state parameter given by eq. \( \text{(30)} \) reduces to
\[ \omega_\Lambda = -\frac{1}{3} \left( 1 + \frac{2}{c} \sqrt{\Omega_\Lambda} \cos y \right) \tag{39} \]

where \( y = \frac{R_H}{a(t)} \). The minimum value it can take is \( \omega_{\text{min}} = -\frac{1}{3} (1 + 2\sqrt{\Omega_\Lambda}) \) and one obtains a lower bound \( \omega_{\text{min}} = -0.9154 \) for \( \Omega_\Lambda = 0.76 \) with \( c = 1 \). Taking variation of the state parameter with respect to \( x = \ln a \), we get \cite{25}
\[ \frac{\Omega_\Lambda'}{\Omega_\Lambda^2} = (1 - \Omega_\Lambda) \left( \frac{2}{c} \frac{1}{\Omega_\Lambda} \cos y + \frac{1}{1 - a^\gamma} \frac{1}{\Omega_\Lambda} \right). \tag{40} \]
and the variation of equation of state parameter becomes

$$\omega'_\Lambda = -\frac{\sqrt{\Omega_\Lambda}}{3c} \left[ \frac{1 - \Omega_\Lambda}{1 - \gamma a} + \frac{2\sqrt{\Omega_\Lambda}}{c} \left( 1 - \Omega_\Lambda \cos^2 y \right) \right],$$

(41)

where \(\gamma = \frac{\Omega_o}{\Omega_m}\). We now introduce the squared speed of holographic dark energy fluid as

$$v^2_\Lambda = \frac{dp_\Lambda}{d\rho_\Lambda} = \frac{\dot{p}_\Lambda}{\rho_\Lambda} = \frac{\rho_\Lambda'}{\rho_\Lambda},$$

(42)

where variation of eq. (31) w.r.t. \(x\) is given by

$$p_\Lambda' = \omega'_\Lambda \rho_\Lambda + \omega_\Lambda \rho_\Lambda'. \tag{43}$$

Using the eqs. (42) and (43) we get

$$v^2_\Lambda = \omega'_\Lambda \frac{\rho_\Lambda}{\rho_\Lambda'} + \omega_\Lambda$$

which now becomes

$$v^2_\Lambda = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda} \cos y + \frac{1}{6c} \sqrt{\Omega_\Lambda} \left[ \frac{1 - \Omega_\Lambda}{1 - \gamma a} + \frac{2}{c} \sqrt{\Omega_\Lambda} \left( 1 - \Omega_\Lambda \cos^2 y \right) \right].$$

(44)

The variation of \(v^2_\Lambda\) with \(\Omega_\Lambda\) is shown in fig. 2 for different \(y\) values. It is found that for a given value of \(c, a, \gamma\), the model admits a positive squared speed for \(\Omega_\Lambda > 0\). Thus for a stable model we require \(\Omega_\Lambda\) positive and bounded from below. We also note that for \(\frac{(2n+1)\pi}{2} < y < \frac{(2n+3)\pi}{2}\), (where \(n\) is an integer) no instability develops. We took a few cases e.g., \(n = 0\) in fig. 2, which shows that for \(y \leq \frac{\pi}{2}\) and \(y \geq \frac{3\pi}{2}\), the squared speed for holographic dark energy becomes negative which leads to instability. But, the squared speed is positive for the region \(\frac{\pi}{2} < y < \frac{3\pi}{2}\) with \(n = 0\), which implies stability. It is also found that for \(y = 0\) i.e., in flat case the holographic dark energy model is always unstable [30].
Figure 2: shows the plot of $v^2\Lambda$ versus $\Omega\Lambda$ for different values of $y$ with $c = 1$, $\gamma = 1/3$ and $a = 1$, in the first array the figures are for $y = \frac{\pi}{3}$ and $y = \frac{\pi}{2}$, in the second array for $y = \frac{1.5\pi}{2}$, $y = \pi$ and in the third array for $y = \frac{2.5\pi}{2}$, $y = \frac{3\pi}{2}$. 
6 Discussions:

In this paper we explored two aspects (i) an equivalent representation of MCG with a scalar field and (ii) a holographic dark energy model with MCG in FRW universe. In sec. 2 the equivalent scalar field potential corresponding to the fluid described by the MCG is obtained both in a flat and in non-flat universe. We note that the potential in Ref. [19] for a generalised Chaplygin gas is recovered here for $\alpha = 1$ and $B = 0$. However, in a modified Chaplygin gas we obtain a new potential determined by the parameters $A$ and $\alpha$ introduced in the equation of state. The potential asymptotically approaches to a constant when (i) $\phi \to 0$ and (ii) $\phi \to \pm \infty$ for $\alpha = 1$. In the non-flat case although it is not so simple to obtain an analytic function for the potential in terms of the field we discuss here two special cases in which potentials are expressed as a function of the field $\phi$. For example, $B = \frac{1}{3}$, $A \neq 0$, one gets a new scalar field potential, the shape of the potential is similar to that one obtains for a rolling tachyon [29]. It admits a small positive effective cosmological constant at a late epoch. Thus the MCG is useful to describe an accelerating universe at late epoch. In sec. 4 we obtain the evolution of the field corresponding to the holographic dark energy which is taken in the form of MCG and the corresponding potential in a non flat universe is determined. Thus it is important to study a closed or open universe to account for the the observational facts. The equation of state for the holographic dark energy considered by Setare [31] in the case of generalized Chaplygin gas is recovered here for $B = 0$ and $\alpha = 1$. We note that in the closed model of the universe, the holographic dark energy is stable for a given domain of the values of $\Omega_\Lambda$. It is also observed that the inclusion of a barotropic fluid in addition to the generalized Chaplygin gas (which is MCG) does not change the shape of the potential. The evolution of the holographic dark energy field is determined in terms of the parameters $A$ and $C$, which in turn depends on the
parameter $\alpha$ that appears in the equation of state for MCG. Thus it is found that the
form of the potential does not change even if one considers a barotropic fluid in addition
to GCG. However, there is a change in the overall holographic dark energy density because
of the extra barotropic term in MCG. The holographic dark energy is found to be stable
for a restricted domain of the event horizon $R_H$ determined by $n = 3(1 + B)(1 + \alpha)$ for a
positive $\Omega_\Lambda$ in a closed universe, which is shown here.

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