Group delay time of fermions in graphene through tilted potential barrier

Youssef Fattasse¹, Miloud Mekkaoui¹, Ahmed Jellal¹,²,ᵃ, and Abdelhadi Bahaoui¹

¹ Laboratory of Theoretical Physics, Faculty of Sciences, Chouaib Doukkali University, PO Box 20, 24000 El Jadida, Morocco
² Canadian Quantum Research Center, 204-3002 32 Ave, Vernon, BC V1T 2L7, Canada

Received 20 June 2022 / Accepted 27 July 2022 / Published online 12 August 2022
© The Author(s), under exclusive licence to EDP Sciences, SIF and Springer-Verlag GmbH Germany, part of Springer Nature 2022

Abstract. The group delay time of Dirac fermions subjected to a tilting barrier potential along the x-axis is investigated in graphene. We start by finding the eigenspinor solution of the Dirac equation and then relating it to incident, reflected, and transmitted beam waves. This relationship allows us to compute the group delay time in transmission and reflection by obtaining the corresponding phase shifts. We discovered that the barrier width, incident energy, and incident angle can all be used to modify the group delay time, and that the particles travel through the barrier at the Fermi velocity $v_F$. Our findings also show that the transmission group delay might be controlled, and that gate voltage control could be useful in graphene-based tilting barriers.

1 Introduction

The ability to change particle behavior by adjusting the gate bias voltage has prompted a lot of interest in quantum tunneling [1–3]. Electrical conduction in gapless graphene cannot be turned off using the control voltages required for conventional transistor operation [4]. This problem can be solved in graphene by establishing an energy gap in the energy spectrum of particles. The desired gap is a measurement of the threshold voltage and the on–off ratio of the field effect transistors [5,6]. However, significant progress has been made in understanding quantum phenomena in graphene systems, with the group delay time being one of the most important parameters connected to the dynamic aspect of the tunneling process [7,8]. By studying the behavior of wave packets, Hartman demonstrated that the group delay may be described in terms of the derivative of the phase shift with respect to the energy [7]. The Hartman effect states that the effective group velocity of a particle can become superluminal for sufficiently large barriers [8,9].

We previously investigated the quantum tunneling for Dirac fermions in graphene scattering by a linear vector potential [14,15]. In [14], it was demonstrated that the infinite mass boundary condition discretizes the transverse momentum. As a result, an effective massive 1D Dirac equation is derived in which the quantized transverse momentum behaves as an effective mass. In [15], the Goos–Hänchen shifts (GH shifts) were studied using the solutions of the energy spectrum of graphene in a linear barrier potential. The procedure begins with the transmission and reflection probabilities being used to determine the appropriate phase shifts. According to a numerical analysis, incident energy, barrier height, and width have a significant impact on GH shifts, which can change positively or negatively under particular conditions.

We investigate the group delay time in transmission and reflection for Dirac fermions in graphene subjected to a tilting barrier potential based on our prior work [14,15]. We illustrate how to calculate the group delay time as a function of different physical parameters based on phase shifts and GH shifts in transmission using the energy spectrum solution. We propose a numerical investigation under various conditions to provide a better understanding of our findings. In particular, the tilting barrier is shown to be able to manage the group delay time.

The following is a breakdown of the current paper’s structure. We define our problem in Sect. 2, and write out the corresponding Hamiltonian and energy spectrum solutions for different regions. We then calculate the transmission and reflection probabilities from which the phase shifts are determined. As a result, we determine the group delay time in terms of the physical parameters that define our system using traditional definitions in section 3. We numerically examine and highlight the basic aspects of the group delay time in Sect. 4. In the concluding section, we summarize our findings.

*a-e-mail: a.jellal@ucd.ac.ma (corresponding author)
2 Hamiltonian and energy spectrum

As seen schematically in Fig. 1, massless Dirac fermions in graphene are scattered by tilted barrier potentials $V_0$ and $V_1$. The current system is divided into three zones designated by the numbers $j = 1, 2, 3$, each of which has a distinct potential.

The following Dirac-like Hamiltonian can be used to describe the current system:

$$H = v_F \mathbf{\sigma} \cdot \mathbf{p} - (\beta x - V_0) \Theta(xd - x^2)I_2$$  \hspace{1cm} (1)$$

with the Heaviside step function $\Theta$, the Fermi velocity $v_F \approx 10^6 m/s$, the Pauli matrices $\mathbf{\sigma} = (\sigma_x, \sigma_y)$, $\mathbf{p} = -i\hbar(\partial_x, \partial_y)$, the $2 \times 2$ unit matrix $I_2$, $\beta = \frac{V_0 - V_1}{V_0 + V_1}$. The spinor $\Psi(x,y)$ at energy $E$ has a time-independent Dirac equation, which is given by

$$[v_F \mathbf{\sigma} \cdot \mathbf{p} - (\beta x - V_0) \Theta(xd - x^2)I_2] \Phi(x,y) = E\Phi(x,y).$$  \hspace{1cm} (2)$$

The system is considered to have a finite width $W$. The spinor meets the infinite mass boundary condition at the interfaces $y = 0$ and $y = W$ along the $y$-direction [16,17]. As a result, the transverse momentum $k_y$ is quantized

$$k_y = \frac{\pi}{W} \left( n + \frac{1}{2} \right), \quad n = 0, 1, 2 \ldots .$$  \hspace{1cm} (3)$$

The spinor is then obtained by separating the variables and, therefore, we write $\Psi_j(x,y) = (\varphi^+_j(x), \varphi^-_j(x))^\dagger e^{ik_y y}$. In region 1 ($x < 0$), we determine the two components of the eigenspinor after solving the eigenvalue equation

$$\Psi_1(x,y) = \begin{pmatrix} 1 \\ z_1 \end{pmatrix} e^{i(k_1 x + k_y y)}$$

$$+ r \begin{pmatrix} 1 \\ -z_1^{-1} \end{pmatrix} e^{i(-k_1 x + k_y y)}, \quad z_1 = s_1 e^{i\phi},$$  \hspace{1cm} (4)$$

where $r$ is the reflection coefficient and $\phi = \tan^{-1}(k_y/k_1)$ is the incident angle, while the sign function $s_1 = \text{sign}(E)$ indicates the conduction and valence bands. The corresponding dispersion relation is straightforward to obtain

$$E = s_1\hbar v_F \sqrt{k_1^2 + k_y^2}.$$  \hspace{1cm} (5)$$

In region 2 ($0 < x < d$), the parabolic cylinder function can be used to express the general solution, with the two components being [14]

$$\Xi^+ = b_1 D_{\eta-1}(\Lambda) + b_2 D_{-\eta}(\Lambda^*)$$  \hspace{1cm} (6)$$

$$\Xi^- = -b_1 \frac{v_F}{k_y} \sqrt{2\epsilon} e^{-i\pi/4} D_{\eta-1}(\Lambda)$$

$$- b_2 \frac{v_F}{k_y} \left[ 2(\epsilon - \beta x)D_{-\eta}(\Lambda^*) - \sqrt{2\epsilon} e^{i\pi/4} D_{-\eta+1}(\Lambda^*) \right],$$  \hspace{1cm} (7)$$

and we have defined $\eta = \frac{k_y^2}{2\epsilon}$, $\Lambda(x) = \sqrt{\frac{2\epsilon}{v_F}}(\epsilon - \beta x + \epsilon_0)$, $\epsilon = \frac{E - v_F^2}{2\epsilon}$, with $b_1$ and $b_2$ are two constants. The following are the components of the spinor solution of the Dirac equation (2) in region 2:

$$\varphi^+(x) = \Xi^+ + i\Xi^-, \quad \varphi^-(x) = \Xi^+ - i\Xi^-$$  \hspace{1cm} (8)$$

which results in the spinor

$$\Psi_2(x,y) = a_1 \begin{pmatrix} \chi^+(x) \\ \chi^-(x) \end{pmatrix} e^{ik_y y} + a_2 \begin{pmatrix} \xi^+(x) \\ \xi^-(x) \end{pmatrix} e^{ik_y y},$$  \hspace{1cm} (9)$$

where the functions $\chi^\pm(x)$ and $\xi^\pm(x)$ are written as follows:

$$\chi^\pm(x) = D_{\eta-1}(\Lambda) \mp \frac{1}{k_y} \sqrt{2\epsilon} e^{i\pi/4} D_{\eta}(\Lambda)$$  \hspace{1cm} (10)$$

$$\xi^\pm(x) = \pm \frac{1}{k_y} \sqrt{2\epsilon} e^{-i\pi/4} D_{-\eta+1}(\Lambda^*)$$

$$\pm \frac{1}{k_y} (2i\epsilon_0 \pm k_y + 2i\beta x) D_{-\eta}(\Lambda^*),$$  \hspace{1cm} (11)$$

$a_1$ and $a_2$ are two constants.

The following spinor is found in region 3 ($x > d$):

$$\Psi_3(x,y) = t \begin{pmatrix} 1 \\ z_1 \end{pmatrix} e^{i(k_1 x + k_y y)}$$  \hspace{1cm} (12)$$

propagating with the same wave vector $k_1$ as in region 1.

The transmission $t$ and reflection $r$ coefficients associated with phase shifts will be computed using the previous solutions.
3 Transfer matrix method and group delay time

Let us begin by defining the following abbreviations to determine the group delay time. This is about the eigenspinors’ components

\[ \chi^\pm(0) = \chi^\pm_d, \quad \chi^\pm(d) = \chi^\pm_d, \quad \xi^\pm(0) = \xi^\pm_d, \quad \xi^\pm(d) = \xi^\pm_d. \]  

(13)

Because the spinors must be consistent at all interfaces, we get a set of equations stated in terms of transfer matrices \( M_{jj+1} \) between different regions. Then, over the entire tilted barrier, the full transfer matrix can be expressed as

\[
\begin{pmatrix} 1 \\ r \end{pmatrix} = M \begin{pmatrix} t \\ 0 \end{pmatrix} = \prod_{j=1}^{4} M_{jj+1} \begin{pmatrix} t \\ 0 \end{pmatrix},
\]

(14)

where \( M_{12}, M_{23} \) are transfer matrices that connect the \( j \)-th region wavefunction to the \((j+1)\)-th region wavefunction. Explicitly, we have

\[
M_{12} = \begin{pmatrix} 1 & 1 \\ z_1 & -z_1^* \end{pmatrix}^{-1} \begin{pmatrix} \chi_{d}^+ \xi_d^+ \\ \chi_{d}^- \xi_d^- \end{pmatrix},
\]

\[
M_{23} = \begin{pmatrix} \chi_{d}^+ \xi_d^+ \\ \chi_{d}^- \xi_d^- \end{pmatrix}^{-1} \begin{pmatrix} e^{ik_1d} & e^{-ik_1d} \\ z_1 e^{ik_1d} & -z_1^* e^{-ik_1d} \end{pmatrix},
\]

(15)

and then

\[
M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}.
\]

(16)

As a result, the transmission and reflection coefficients are given by

\[
t = \frac{1}{|m_{11}|} e^{i\varphi_t}, \quad r = \frac{|m_{21}|}{m_{11}} e^{i\varphi_r},
\]

(17)

where

\[
\varphi_t = \arctan \left( \frac{t^* - t}{t + t^*} \right), \quad \varphi_r = \arctan \left( \frac{r^* - r}{r + r^*} \right).
\]

(18)

are the phase shifts of the transmission and reflection amplitudes, respectively. After some lengthy but straightforward algebra, we obtain the following coefficients:

\[
t = e^{-ik_1d} \frac{1 + z_1^2}{\partial} (\xi_d^+ \chi_d - \xi_d^- \chi_d^+), \quad r = \frac{\delta}{\partial}
\]

(19)

as well as having defined

\[
\delta = \chi_0^+ \xi_d^+ + z_1 (\chi_d^+ \xi_0^+ + \chi_0^+ \xi_d^+ - \chi_d^+ \xi_0^- + \chi_0^+ \xi_d^-),
\]

\[
- z_1^2 (\chi_d^+ \xi_0^- - \chi_0^- \xi_d^- - \chi_d^- \xi_0^+ + \chi_0^- \xi_d^+),
\]

(20)

\[
\vartheta = (\xi_0^+ + z_1 \xi_0^-) (\chi_d^+ + z_1 \chi_d^-)
\]

(21)

The transmission \( T = \frac{t}{t} \) and reflection \( R = \frac{r}{r} \) probabilities are calculated using the current of densities \( J_i, J_r, \) and \( J_t \) representing the incident, reflected, and transmitted waves, respectively. We get the current density from the Hamiltonian

\[
J = cu_F \psi^+ \sigma_x \psi
\]

(22)

leading to the probabilities

\[
T = |t|^2, \quad R = |r|^2.
\]

(23)

A transverse wave vector \( ky = k_{y0} \) and an incident angle \( \phi(k_{y0}) \in [0, \frac{\pi}{2}] \), indicated by the subscript 0, are then used to analyze Goos-Hänchen shifts (GH shifts) and group delay time. A temporal–spatial wave packet, which is the weighted superposition of plane wave spinors, can be used to describe a finite pulse electron beam. As a result, the incident, reflected at \( x = 0 \), and transmitted wave packets at \( x = d \) wave functions can be expressed as a double Fourier integral over \( k_y \) \([18,19]\)

\[
\psi_t(x,t) = \iint f(k_y,\omega) e^{-i(k_y y - \omega t)} dk_y d\omega,
\]

(24)

\[
\Psi_r(x,t) = \iint rf(k_y,\omega) e^{-i(k_y y - \omega t)} dk_y d\omega,
\]

(25)

\[
\Psi_t(x,t) = \iint tf(k_y,\omega) e^{-i(k_y y - \omega t)} dk_y d\omega,
\]

(26)

where the involved spinors are solutions of Dirac equation (2). Here, \( f(k_y,\omega) = w_y e^{-w_y^2(k_y - \omega)^2} \) is the Gaussian angular distribution, with \( \omega = E/h \) and \( w_y \) is the half beam width at waist \([20]\). The total phases of reflected and transmitted waves at \( x = 0, d \), respectively, are given by

\[
\Phi_{\nu} = \varphi_{\nu} + ky y - \omega t, \quad \nu = t, r.
\]

(27)

Using the stationary phase approximation, we may get analytical equations for group delay and lateral GH shift by assuming that the distribution \( f(k_y,\omega) \) is a smooth and steeply peaked function around the center energy/wavevector \([21,22]\). GH shifts are obtained from
\[ S_{\nu} = -\frac{\partial \varphi_\nu}{\partial k_\nu} \]  

(28)

For retaining the nice shape throughout propagation, the equation of motion is determined using the constraint \( \partial \Phi_\nu / \partial \omega = 0 \). This provides the group delay time

\[ \tau_\nu = \frac{\partial \varphi_\nu}{\partial \omega} + \frac{\partial k_\nu}{\partial \omega} S_{\nu} \]  

(29)

\[ = \tau_s^\nu + \tau_p^\nu, \]  

(30)

where \( \tau_p^\nu \) represents the time derivative of phase shifts

\[ \tau_p^\nu = \frac{\hbar}{2} \frac{\partial \varphi_\nu}{\partial E}, \]

\[ \tau_r^\rho = \frac{\hbar}{2} \frac{\partial \varphi_\rho}{\partial E}, \]

(31)

and the second \( \tau_s^\nu \) results from the contribution of \( S_{\nu} \)

\[ \tau_s^\nu = \frac{\sin \phi}{v_F} S_t, \]  

\[ \tau_s^r = \frac{\sin \phi}{v_F} S_r. \]

(32)

We shall proceed with numerical analysis after obtaining closed form equations of the group delay in various energy domains.

4 Numerical analysis

We compute the group delay time in transmission for electrons passing through a tilting barrier under various incident angle \( \phi \), potential height \( V_0 \) and width \( d \), and incident energy \( E \). Dimensionless group delay time \( \tau_{t0} \), which results in transversal time \( \tau_0 = \frac{d}{v_F} \cos \phi \), is convenient for our task. The key findings of this study are depicted in the following seven figures, each have a distinct set of physical parameters.

In Fig. 2, the group delay in transmission \( \tau_{t0} \) is shown versus the incident angle \( \phi \) by choosing different values of the remaining physical parameters. It is evident that at normal incidence, i.e., \( \phi = 0 \), the particles propagate through the barrier with the Fermi velocity \( v_F \) \( \tau_{t0} \) = 1, but that as \( \phi \) increases, \( \tau_{t0} \) begins to progressively increase until it reaches a maximum, after which it decays exponentially. It approaches zero when \( \phi = 90^\circ \), since the wave vector inside the barrier becomes imaginary, and the wave function in the barrier region becomes an evanescent wave. The behavior of \( \tau_{t0} \) is affected by incident energy \( E \), barrier width \( d \), and height \( V_1 \), as it drops as \( E \) and \( d \) increase, but increases as \( V_1 \) grows. This means that the linear potential can affect the group delay time by modulating it.

In Fig. 3, we plot the group delay time \( \tau_{t0} \) as a function of the barrier widths \( d \) for the barrier heights (a):

\[ V_1 = 0 \text{ meV and } b: V_1 = 20 \text{ meV with } E = 20 \text{ meV (blue line)} \]  

\[ E = 25 \text{ meV (red line)}, \]  

\[ E = 30 \text{ meV (green line)}. \]

\[ c: V_1 = 50 \text{ meV, } E = 30 \text{ meV (blue line)}, \]  

\[ E = 35 \text{ meV (red line)}, \]

\[ E = 40 \text{ meV (green line)}. \]

\[ d: V_1 = 50 \text{ meV, } E = 30 \text{ meV, } d = 40 \text{ nm (green line)}, \]  

\[ d = 60 \text{ nm (red line)}, d = 80 \text{ nm (blue line)} \]

Fig. 2 The group delay time in transmission \( \tau_{t0} \) as a function of the incident angle \( \phi \) for \( V_0 = 80 \text{ meV, } d = 80 \text{ nm.} \) a \( V_1 = 0 \text{ meV and b: } V_1 = 20 \text{ meV with } E = 20 \text{ meV (blue line)} \]  

\[ E = 25 \text{ meV (red line)}, \]  

\[ E = 30 \text{ meV (green line)}. \]

\[ c: V_1 = 50 \text{ meV, } E = 30 \text{ meV (blue line)}, \]  

\[ E = 35 \text{ meV (red line)}, \]

\[ E = 40 \text{ meV (green line)}. \]

\[ d: V_1 = 50 \text{ meV, } E = 30 \text{ meV, } d = 40 \text{ nm (green line)}, \]  

\[ d = 60 \text{ nm (red line)}, d = 80 \text{ nm (blue line)} \]
$V_1 = 0 \text{ meV}$, (b): $V_1 = 20 \text{ meV}$, (c): $V_1 = 50 \text{ meV}$ and (d): $V_1 = V_0$. The other computation parameters being $V_0 = 80 \text{ meV}, \phi = 30^\circ$ and for different values of $E = 20 \text{ meV}$, $E = 25 \text{ meV}$, and $E = 30 \text{ meV}$. The possibility of modulating $\tau_1/\tau_0$ by changing the height of the potential barrier through different applied gate voltages is also present in the barrier tilting structure. Particularly interesting is that if $V_0$ is kept constant and $V_1$ is modified, $\tau_1/\tau_0$ shows identical behavior in Fig. 3a for $V_1 = 0 \text{ meV}$ and Fig. 3b for $V_1 = 20 \text{ meV}$. However, if $V_1$ is increased from $V_1 = 50 \text{ meV}$ up to $V_1 = V_0$, the oscillations in $\tau_1/\tau_0$ are again observed, as illustrated in Fig. 3c, d, such that their number increases and also their peak value decreases. On the other hand, we observe $\tau_1/\tau_0$ increasing with the increase of incident energy $E$. Therefore, the incident energy modifies the period and amplitude of the oscillations by increasing them. As the barrier width $d$ is increased, the peaks show a discernible spread due to the Fabry–Pérot enhancement. The particles travel back inside the tilting barrier due to Fabry–Pérot resonances between the barrier edges, which explains $\tau_1/\tau_0$.

Figure 4 shows the group delay time $\tau_1/\tau_0$ as a function of the barrier width $d$ for different values of the incident angle $\phi = 5^\circ$ (red line), $\phi = 10^\circ$ (blue line), $\phi = 15^\circ$ (green line), $E = 30 \text{ meV}$, and we choose the remaining parameters as in Fig. 3. With the Fermi velocity $v_F$, the particles pass through the barrier, but when $d$ increases, the group delay time in transmission $\tau_1/\tau_0$ starts to oscillate with peak increasing. Notice that for $V_1 = 50 \text{ meV}$ and $V_1 \approx V_0 = 80 \text{ meV}$ as presented, respectively, Fig. 4c, d, $\tau_1/\tau_0$ oscillates twice and the peak value decreases compared to Fig. 4a, b, where $\tau_0$ is the time it would take a particle to travel the distance $d$ if the barrier did not exist. Here, we observe that the particles propagate through the barrier with the Fermi velocity $v_F$ ($\tau_1/\tau_0 = 1$). When $d$ increases, one sees that $\tau_1/\tau_0$ begins to oscillate and the associated amplitude increases with the increase of the incident angle $\phi$, while the peaks did not get influenced and they are still in the same positions. When $v_F$ is equivalent to the speed of light $c$ in optics, the group delay time $\tau_5$ may be smaller than $\tau_0$ ($\tau_1/\tau_0 < 1$), indicating superluminality. This phenomena faster than light is relevant for the Hartman effect in the tunneling process. On the other hand, we discover that particles propagate past the barrier at speeds greater than the Fermi velocity $v_F$.

In Fig. 5, we plot the group delay time in transmission $\tau_1/\tau_0$ (blue line) and transmission probability $T$ (red line) as a function of the incident energy $E$ for two values of the incident angle $\phi = 15^\circ, 30^\circ$ in Fig. 5a, b with $V_0 = 60 \text{ meV}, V_1 = 20 \text{ meV}$ and $d = 80 \text{ nm}$. It can be seen that when $E$ increases, both quantities exhibit closed behavior and they can be modulated by modifying the incident angle. Additionally, we see oscillating behavior in the group delay time and transmission probability. This is due to the overlapping of the reflected and incident waves, which causes self-interference delay. Furthermore, we see a peak in the $\tau_1/\tau_0$ behavior, which increases in tandem with the increase in the incident angle. As shown in Fig. 5a, when $E < V_0 - V_1$, $T$
Fig. 4 The group delay time in transmission \( \tau_t/\tau_0 \) as a function of the barrier width \( d \) for \( V_0 = 80 \) meV, \( E = 30 \) meV, \( \phi = 5^\circ \) (red line), \( \phi = 10^\circ \), (blue line), \( \phi = 15^\circ \) (green line). (a): \( V_1 = 0 \) meV, (b): \( V_1 = 20 \) meV, (c): \( V_1 = 50 \) meV, (d): \( V_1 \approx V_0 = 80 \) meV.

Decays exponentially due to the wave function in the region of the barrier becoming an evanescent wave, and goes down to a minimum, then starts increasing again. However, \( T \) approaches zero before starting to increase again when \( E > V_0 - V_1 \), as depicted in Fig. 5b.

Figure 6 presents the group delay time \( \tau_t/\tau_0 \) and the transmission probability \( T \) as a function of the barrier height \( V_0 \) for \( d = 80 \) nm, \( E = 50 \) meV, and \( V_1 = 0 \) meV, with Fig. 6a, b illustrate the case of \( \phi = 15^\circ \) and \( \phi = 30^\circ \), respectively. For small values of \( V_0 \), one sees that the particles propagate through the tilted barrier at the Fermi velocity \( v_F \), which they can transmit perfectly, \( T = 1 \). When \( V_0 \) is increased, \( \tau_t/\tau_0 \) and \( T \) begin to rapidly fall toward a constant value that is independent of \( V_0 \). We observe that the incident angle has an effect on \( \tau_t/\tau_0 \) and \( T \)'s behaviors, since they decrease as it increases. For \( \phi = 30^\circ \), \( T \) stabilizes at zero regardless of the value of \( V_0 \geq 80 \) meV, resulting in a total reflection.

Figure 7a depicts the group delay time in transmission \( \tau_t \) as a function of the barrier width \( d \) for tilting barrier \( (V_0 = 40 \) meV, \( V_1 = 20 \) meV) as well as barrier square \( V_1 = V_0 = 40 \) meV with \( E = 40 \) meV, \( \phi = 30^\circ \). One notices that \( \tau_t \) rapidly grows as \( d \) increases, eventually stabilizing at a maximum. We get the same behavior shape with \( V_1 = V_0 \), but with a decline. As a result, Fig. 7a shows that the Hartmann effect exists at \( V_1 = V_0 = 40 \) meV, because \( \tau_t \) saturates at a constant when the barrier width \( d \) is increased [23]. Figure 7b illustrates \( \tau_t \) as a function of incident energy \( E \) for for \( V_0 = 40 \) meV, \( V_1 = 20 \) meV, and \( \phi = 30^\circ \),
Fig. 6 The group delay time $\tau_t/\tau_0$ (blue line) and transmission probability $T$ (red line) as a function of the barrier height $V_0$ at incident angles a $\phi = 15^\circ$ and b $\phi = 30^\circ$, with $E = 50$ meV, $V_1 = 0$ meV, and $d = 80$ nm.

Fig. 7 a The group delay time in transmission $\tau_t$ as a function of the barrier width $d$ for $V_0 = 40$ meV, $E = 40$ meV, $\phi = 30^\circ$, with two values $V_1 = 20$ meV (red line) and $V_1 \approx V_0 = 40$ meV (green line). b $\tau_t$ as a function of the incident energy $E$ for $V_0 = 40$ meV, $V_1 = 20$ meV, $\phi = 30$, with three values $d = 90$ nm (blue line), $d = 80$ nm (red line), $d = 70$ nm (green line).

We investigated the group delay time in transmission $\tau_t/\tau_0$ for Dirac fermions in graphene scattered along the $x$-axis by a linear barrier potential. The group delay time has been demonstrated to oscillate in response to several physical parameters such as barrier width $d$, incident angle $\phi$, incident energy $E$, and two barrier heights ($V_0, V_1$). When the barrier width $d$ becomes large enough, our theoretical investigation supports the existence of group delay time saturation. It also proves that quantum interference has a significant impact on particle tunneling in graphene via a tilted barrier.

Also, we demonstrated that the physical parameters that characterize our system can be used to modify the behavior of $\tau_t/\tau_0$. Furthermore, we discovered that the group delay time in transmission equals unity at certain critical values of incidence energy, incident angle, and barrier width, i.e., $\tau_t/\tau_0 = 1$, implying that particles travel across the barrier with the Fermi velocity $v_F$. Finally, we expect that all of the findings will be valuable not only for the theoretical research of the tunneling effect but also for graphene’s technological applications.

5 Conclusion

We investigated the group delay time in transmission $\tau_t/\tau_0$ for Dirac fermions in graphene scattered along the $x$-axis by a linear barrier potential. The group delay time has been demonstrated to oscillate in response to several physical parameters such as barrier width $d$, incident angle $\phi$, incident energy $E$, and two barrier heights ($V_0, V_1$). When the barrier width $d$ becomes large enough, our theoretical investigation supports the existence of group delay time saturation. It also proves that quantum interference has a significant impact on particle tunneling in graphene via a tilted barrier.

Also, we demonstrated that the physical parameters that characterize our system can be used to modify the behavior of $\tau_t/\tau_0$. Furthermore, we discovered that the group delay time in transmission equals unity at certain critical values of incidence energy, incident angle, and barrier width, i.e., $\tau_t/\tau_0 = 1$, implying that particles travel across the barrier with the Fermi velocity $v_F$. Finally, we expect that all of the findings will be valuable not only for the theoretical research of the tunneling effect but also for graphene’s technological applications.

Author contributions

All authors have contributed equally to the paper.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data that support the findings of this study are available on request from the corresponding author].

References

1. K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, A.A. Firsov, Nature 438, 197 (2005)
2. Y.B. Zhang, Y.W. Tan, H.L. Störmer, P. Kim, Nature 438, 201 (2005)
3. J. Nilsson, A.H. Castro Neto, F. Guinea, N.M.R. Peres, Phys. Rev. B 76, 165416 (2007)
4. M.I. Katsnelson, K.S. Novoselov, A.K. Geim, Nat. Phys. 2, 620 (2006)
5. Y. Lin, K.A. Jenkins, A. Valdes-Garcia, J.P. Small, D.B. Farmer, P. Avouris, Nano Lett. 9, 422 (2009)
6. J. Kedzierski, P. Hsu, P. Healey, P.W. Wyatt, C.L. Keast, M. Sprinkle, C. Berger, W.A. de Heer, IEEE Trans. Electron. Dev. 55, 2078 (2008)
7. T.E. Hartman, J. Appl. Phys. 33, 3427 (1962)
8. Z. Wu, K. Chang, J.T. Liu, X.J. Li, K.S. Chan, J. Appl. Phys. 105, 043702 (2009)
9. V.S. Olkovsky, E. Recami, Phys. Rep. 214, 339 (1992)
10. F. Goos, H. Hänchen, Ann. Phys. 436, 333 (1947)
11. X. Chen, J.-W. Tao, Y. Ban, Eur. Phys. J. B 79, 203 (2011)
12. Y. Song, H.-C. Wu, Y. Guo, Appl. Phys. Lett. 100, 253116 (2012)
13. X. Chen, P.-L. Zhao, X.-J. Lu, L.-G. Wang, Eur. Phys. J. B 86, 223 (2013)
14. H. Bahlouli, E.B. Choubabi, A. El Mouhafid, A. Jellal, Solid State Commun. 151, 1309 (2011)
15. M. Mekkaoui, R. El Kinani, A. Jellal, Mater. Res. Express 6, 085013 (2019)
16. M.V. Berry, R.J. Modragon, Proc. R. Soc. Lond. Ser. A 412, 53 (1987)
17. J. Tworzydlo, B. Trauzettel, M. Titov, A. Rycerz, C.W.J. Beenakker, Phys. Rev. Lett. 96, 246802 (2006)
18. X. Chen, C.-F. Li, Y. Ban, Eur. Phys. J. B 62, 453 (2008)
19. Yue Ban, Lin-Jun. Wang, Xi. Chen, J. Appl. Phys. 115, 173703 (2014)
20. C.W.J. Beenakker, R.A. Sepkhanov, A.R. Akhmerov, J. Tworzydlo, Phys. Rev. Lett. 102, 146804 (2009)
21. A.M. Steinberg, R.Y. Chiao, Phys. Rev. A 49, 3283 (1994)
22. C.-F. Li, Phys. Rev. A 65, 066101 (2002)
23. Yue Ban, Lin-Jun. Wang, Xi. Chen, J. Appl. Phys. 117, 164307 (2015)