Measurement of the $\eta$–$\eta'$ mixing angle in $\pi^-$ and $K^-$ beams with the GAMS-4$\pi$ Spectrometer

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Abstract The $\eta$–$\eta'$ mixing angle has been measured with the GAMS-4$\pi$ spectrometer in a high statistics experiment. $\eta'$ and $\eta$ mesons were generated with charge–exchange reactions in 32.5 GeV/c $\pi^-$ and $K^-$ beams. Using both $\pi^-$ and $K^-$ beams allows the study of the of the $|\eta_8\rangle$ and $|\eta_s\rangle$ components of the meson wave function. The cross section ratio at $t' = 0$ (GeV/c)$^2$ in the $\pi^-$ beam is $R_\pi (\eta'/\eta) = 0.54 \pm 0.04$, and the mixing angle $\phi = (36.3 \pm 1.0)\circ$. For the $K^-$ beam the ratio is $R_K (\eta'/\eta) = 1.27 \pm 0.15$. It was found that the gluonium content in $\eta'$ is $\sin^2 \phi_G = 0.17 \pm 0.07$.

1 Introduction

The investigation of the structure of mesons started during the second part of the last century, when in the naive quark model frame the experimental consequence of the $SU(3)$ broken symmetry was measured. In this model the physical states $\eta$ and $\eta'$ are linear combinations of $SU(3)$ singlet and nonet states:

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = \begin{pmatrix}
\cos \theta_P & -\sin \theta_P \\
\sin \theta_P & \cos \theta_P
\end{pmatrix} \begin{pmatrix}
\eta_8 \\
\eta_1
\end{pmatrix}
$$

(1)

where

$$
|\eta_1\rangle = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle,
$$

$$
|\eta_8\rangle = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle
$$

(2)

and $\theta_P$ is the mixing angle in the singlet-octet representation. In the non-strange (NS)–strange (S) quark base

$$
|\eta_q\rangle = \frac{|u\bar{u} + d\bar{d}|}{\sqrt{2}}, \quad |\eta_s\rangle = |s\bar{s}\rangle
$$

(3)

the mixing becomes

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}
$$

(4)

The singlet-octet and flavor bases are connected by the rotation matrix $U_2(\theta_i)$,

$$
\begin{pmatrix}
\eta_8 \\
\eta_1
\end{pmatrix} = \begin{pmatrix}
\cos \theta_i & -\sin \theta_i \\
\sin \theta_i & \cos \theta_i
\end{pmatrix} \begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}
$$

(5)

and the angles $\theta_P$ and $\phi$ are related through

$$
\theta_i = \arctan \sqrt{2}, \quad \phi = \phi_P - \theta_i.
$$

(6)

However, it seems that the internal structure of the pseudoscalars is not so simple, and the above approach is simplified [1]. The experimental properties of the $\eta'$ can be explained by assuming that there is a gluonium component in the meson wave function [2–5]. A modern approach [6] includes in an $\eta$–$\eta'$ mixing scheme a pseudoscalar glueball $G$ and gluon field $g$:

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = U_3(\theta_j)U_1(\phi_G)U_3(\theta_i) \begin{pmatrix}
\eta_q \\
\eta_s \\
g
\end{pmatrix}
$$

(7)

where $U_3(\theta_i)$ is the extended rotation matrix from the flavor base to the octet-singlet base. We have

$$
U_1(\phi_G) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_G & \sin \phi_G \\
0 & -\sin \phi_G & \cos \phi_G
\end{pmatrix}
$$

(8)

and

$$
U_3(\theta) = \begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

(9)

This method used the assumptions that there is no contribution from other flavors (such as $c\bar{c}$) and that a pure glueball...
$g$ does not get mixed with $\eta_8$. Evidence for the latter assumption is that the coupling of $\eta'$ with the gluon field is $\approx 9$ times stronger than that of the $\eta$ [7].

With these simplifications we obtained the two-angle scheme which is used by KLOE [8] to describe the $\eta$ and $\eta'$ wave functions:

$$\eta' = \cos \phi_G \sin \phi_P |\eta_q\rangle + \cos \phi_G \cos \phi_P |\eta_t\rangle$$

$$+ \sin \phi_G |g\rangle,$$  \hspace{1cm} (10)

$$\eta = \cos \phi_P |\eta_q\rangle - \sin \phi_P |\eta_t\rangle.$$  

Here $\phi_P$ is the $\eta$–$\eta'$ mixing angle and $\sin^2 \phi_G$ is the gluonium fraction in $\eta'$. 

There are three classes of physical processes to measure the $\eta$–$\eta'$ mixing angle [12], which include radiative vector- and pseudoscalar-meson decays and decays of $J/\psi$. The measurement of the mixing angle in $B_s$, $B_d$ decays [9, 11] is relatively new, and modern situation with these measurements are considered in [6, 12]. The $\eta$–$\eta'$ mixing angle from LHCb report is $\phi_P = (45.5 \pm 1.2^\circ)$ [10]. The most precision value for mixing angle $\phi_P = (40.4 \pm 0.6)^\circ$ and gluonium fraction in $\eta'$ $\sin^2 \phi_G = 0.12 \pm 0.04$ has been obtained KLOE [8] by measurement the ratio $R_{\phi} = BR(\phi \to \eta'\gamma)/BR(\phi \to \eta\gamma)$.

As was understood in the process of studying the meson structure, the accuracy of the mixing angle measurement can be improved by preparing known initial quark states ($|\eta_q\rangle$ or $|\eta_t\rangle$), as in $B_s$ and $B_d$ meson decays, see Fig. 1. In this process the $d$ quark probes the NS-part of the $\eta'(\gamma)$ wave function and the $s$ quark the S-part.

The production of $\eta$ and $\eta'$ mesons in the charge-exchange reaction with $\pi^-$ and $K^-$ beams can be represented by planar diagrams, Fig. 1. These diagrams dominate, and the initial state $d\bar{d}$ ($|\eta_q\rangle$) is prepared in a $\pi^-$ beam while the $s\bar{s}$ ($|\eta_t\rangle$) state dominates in a $K^-$ beam. The $\pi^-$ and $K^-$ beams act like “flavor filters” for the pseudoscalar wave function, as do $B_s$ and $B_d$ meson decays. In this approach the ratio of the $\eta'$ and $\eta$ cross sections for a $\pi^-$ beam is

$$R_\pi = \frac{\sigma(\eta')}{\sigma(\eta)} = \tan^2 \phi_P$$  \hspace{1cm} (11)

and for a $K^-$ beam is

$$R_K = \frac{\sigma(\eta')}{\sigma(\eta)} = \cot^2 \phi_P.$$  \hspace{1cm} (12)

The idea of measuring the mixing angle by cross sections ratios was first proposed in Ref. [13]. Experimental results obtained in a $\pi^-$ beam were published in [14, 15]. To remove the mixing angle dependence on kinematic factors and phase space the measurement is performed at 4-momentum-transfer squared $t' = t - t_{\text{min}} = 0$:

$$R = \frac{d\sigma(\eta')}{dt} \frac{1}{d\sigma(\eta)} \left| \frac{d\sigma(\eta)}{dt} \right|_{t=0}. \hspace{1cm} (13)$$

If the differential cross section parameters are the same, then $R$ equal (11) or (12) depending on the beam particle type.

The results of the $\eta$–$\eta'$ mixing angle measurements in $\pi^-$ and $K^-$ beams are presented in this paper. The data were taken simultaneously in a $\pi^-$ and $K^-$ beam, with the highest statistics in the neutral decay modes. The dependence of $R_\pi$ and $R_K$ on $t$ are also shown. The data analysis had been performed using a simple quark model with added gluon content.

2 GAMS-4$\pi$ detector and data analysis

The experiment was carried out at the U70 IHEP accelerator, in a secondary beam of the negative particles. The beam consists of 98% $\pi^-$ and 2% $K^-$ mesons; the admixture of other particles is negligible. The beam telescope used a coincidence of five scintillation counters, with time resolution better than 1 ns. The type of the beam particle is defined reliably by two threshold Cherenkov counters [21] with quartz optics.

The interaction point of the beam particles is measured with precision $\pm 4$ cm from the amount of Cherenkov light, emitted in the 40 cm long liquid hydrogen target. The target is surrounded by guard system, which consists of scintillation veto layer and lead glass counters to detect recoil baryons. This set of the detectors allows the identification of the charge-exchange processes with neutral final states. The main detector of GAMS-4$\pi$ setup is the lead glass electromagnetic calorimeter GAMS [16]. The central part of the calorimeter contains PWO crystals in order to provide good energy and spatial resolution [17]. A more detailed description of the performance of the experiment and data processing has been given elsewhere [18].

Fig. 1 The diagrams of the decay $B_s, \bar{B}_s$ mesons (top) and charge–exchange reaction in $\pi^-$ and $K^-$ beams (bottom)
The charge–exchange reaction with a 32.5 GeV/c beam momentum was used as the source of the monoenergetic pseudoscalar mesons

$$\pi^- (K^-) p \rightarrow M^0 n (\Lambda)$$

(14)

with the $M^0$ state decaying into photons. The analysis of the data was performed in several steps. At first the 4-momenta of the photons were reconstructed by special procedures [19, 20] and events with exactly two photons in GAMS were selected.

Next, a series of criteria were applied in order to suppress instrumental and physical backgrounds without losing detection efficiency of the reactions under study (14):

- The distance between the shower centers in GAMS is larger than 55 mm.
- The distance between photon’s impacts in GAMS and the beam axis is larger than 50 mm. This requirement provides suppression of the background in the central part of calorimeter where the beam is most intense.
- The total energy deposited in GAMS is limited to the range 29–35 GeV.
- The energy of each photon is larger than 0.5 GeV.

The source of physical background in $\pi^-$ beam is provided by intense processes $\pi^- p \rightarrow \pi^0 \eta n$, $\pi^- p \rightarrow \pi^0 \pi^0 n$ with two undetected photons. To suppress this background the events with $\cos \theta^* < 0.8$ were selected, where $\theta^*$ is the angle of the meson decay in CM system.

The experimental mass spectra are shown in Fig. 2. The $0.48 \times 10^6$ events of the decay $\eta \rightarrow 2\gamma$ and $14.56 \times 10^3 \eta' \rightarrow 2\gamma$ in the $\pi^-$ beam were selected. For the $K^-$ beam the numbers of events are $2.15 \times 10^3$ and $0.25 \times 10^3$, respectively.

Kinematical fits are based on a least-squares minimization of the $\chi^2$ function which consists of the beam (momentum and directed cosines, three values) parameters, the vertex parameters (coordinates $X_v, Y_v, Z_v$, also three values), and the coordinates and energies of the photons measured by GAMS, six items for the two photons. For a 1C fit we included the constraint on the recoil baryon mass, and for a 2C fit we added the meson mass, $M_{\gamma\gamma} = M_\eta$ or $M_{\gamma\gamma} = M_{\eta'}$, as the constraint. The 2C fit was used for the $t$-dependent study to improve the resolution. The errors in the $\chi^2$ function were adjusted with a pull-variable analysis. The linearized matrix equation is solved by iteration method [22]. Events with confidence level $CL(2C) > 0.1$ are retained for further analysis.

The study of $\eta$ and $\eta'$ production in the same simplest $2\gamma$ decay mode has obvious advantages compared to other modes, such as $\eta \rightarrow 3\pi^0$ and $\eta' \rightarrow 3\pi^0 \pi^0$. The Monte Carlo efficiency calculation is straightforward, and does not require an elaborate study of the reconstruction performance in the case of overlapping or heavily distorted showers [20].

Therefore the $t$ dependencies of the detection efficiency are smooth and practically flat for all four reactions, Fig. 3.

Other factors, such as photon conversion in the liquid target or in the veto counters are the same for the four reactions and make minimal contribution in an uncertainties in Eqs. (11) and (12). We estimated that the contribution of these factors are less than 10 % in final error. This approach significantly reduced the systematic errors in the cross section ratios, and the precision of our measurements is limited mainly by statistics.

3 Mixing angle measurement

A phenomenological function, according to Ref. [23], is used to describe the differential cross section:
The ratio $S/(S + B)$ with approximation function in $\pi^-$ beam for $\eta'$ meson, where $S$ is the number of the event the decay $\eta' \to 2\gamma$ and $B$ is the number of the background events in the mass interval (0.9–1.02) GeV

$$d\sigma = \left. \frac{d\sigma}{dt} \right|_{t=0} (1 - gct)e^{ct} \quad (15)$$

where $g = \sigma_-/\sigma_+$ is the ratio of flip and non-flip cross sections of the reaction (14).

The experimental histograms were fitted with the function (15) taking into account the detection efficiency. For the case $\eta'$ in $\pi^-$ beam we include also the Signal/Background relationship, see Fig. 4. The fit results are shown in Table 1 and in Fig. 5. The low-$t$ region is presented separately in Fig. 6. It is interesting to see that in the $K^-$ beam the non-flip amplitude dominates. Taking into account $BR(\eta' \to \gamma\gamma') = 0.0218$ and $BR(\eta \to \gamma\gamma) = 0.393$ [25] the cross section ratio in the $\pi^-$ beam at $t' = 0$ is

$$R_\pi = \left. \frac{d\sigma(\eta')}{dt} \right|_{t=0} / \left. \frac{d\sigma(\eta)}{dt} \right|_{t=0} = 0.54 \pm 0.04, \quad (16)$$

and the mixing angle in the quark basis equals

$$\phi_P = (36.3 \pm 1.0)^\circ \quad (17)$$

corresponding to $\theta_P = -18.4 \pm 1.0$ in the singlet-octet representation. The result can be compare with the NICE experiment value $R_\pi(\eta'/\eta) = 0.55 \pm 0.06$ [15].

There are the arguments that the ratio of the flip-amplitude cross section can measure the quark content better because the flip amplitude dominates [24]. Following [14] the ratio for flip amplitude in $\pi^-$ beam is

$$R_F = \left( \frac{e_{\eta'}}{e_\eta} \right)^2 \left( 1 + \frac{g_{\eta'}}{1 + g_\eta} \right) \frac{\sigma_{\eta'}}{\sigma_\eta} \quad (18)$$

and we estimate $R_F = 0.5 \pm 0.1$.

| $\eta$ | $\pi^-$ | $K^-$ | $\chi^2$/NDF |
|---|---|---|---|
| $a$ | $(2.21 \pm 0.02) \times 10^6$ | $(2.98 \pm 0.13) \times 10^4$ | $18.3/18$ |
| $c$, (GeV/c)$^{-2}$ | $7.8 \pm 0.1$ | $8.9 \pm 2.0$ | |
| $g$ | $2.6 \pm 0.1$ | $0.0 \pm 0.60$ | |

| $\eta'$ | $\pi^-$ | $K^-$ | $\chi^2$/NDF |
|---|---|---|---|
| $a$ | $(6.57 \pm 0.3) \times 10^4$ | $(2.10 \pm 0.2) \times 10^3$ | $13.5/18$ |
| $c$, (GeV/c)$^{-2}$ | $8.6 \pm 0.2$ | $6.3 \pm 1.3$ | |
| $g$ | $2.2 \pm 0.2$ | $0.0 \pm 0.3$ | |

**Fig. 4** The ratio $S/(S + B)$ with approximation function in $\pi^-$ beam for $\eta'$ meson, where $S$ is the number of the event the decay $\eta' \to 2\gamma$ and $B$ is the number of the background events in the mass interval (0.9–1.02) GeV.

**Fig. 5** Differential cross section $\eta$ and $\eta'$ mesons with fitted function (15) in the $\pi^-$ beam (histograms $a$ and $b$) and in the $K^-$ beam ($c$ and $d$).

**Fig. 6** Differential cross section $\eta$ and $\eta'$ mesons in low $t$ region with fitted function (15) in the $\pi^-$ beam (histograms $a$ and $b$) and in the $K^-$ beam ($c$ and $d$).
In the $K^-$ beam the ratio is

$$R_K = \frac{d\sigma(\eta')}{dt}/\frac{d\sigma(\eta)}{dt} \bigg|_{t=0} = 1.27 \pm 0.15. \tag{19}$$

From the naïve quark model we can expect

$$R_\pi \cdot R_K = \tan^2 \phi_P \cdot \cot \phi_P = 1 \tag{20}$$

but the experimental data give

$$R_\pi \cdot R_K = 0.69 \pm 0.09. \tag{21}$$

This discrepancy can be explained if the internal structure of the pseudoscalar mesons contains a component not considered in the model. If we follow the KLOE approach \cite{10} then the following relations are obvious:

$$R_\pi = \cos^2 \phi_G \tan^2 \phi_P, \quad R_K = \cos^2 \phi_G \cot^2 \phi_P \tag{22}$$

and $R_\pi \cdot R_K = \cos^4 \phi_G$. The gluonium content of the $\eta'$ mesons from our data is

$$\sin^2 \phi_G = 0.17 \pm 0.07. \tag{23}$$

The KLOE result, based on radiative meson decays, is $\sin^2 \phi_G = 0.12 \pm 0.04$. The $R_\pi$ and $R_K$ as the function of $t$ are shown in Fig. 7. These figures require further study.

4 Conclusion

The ratio of the differential cross sections at $t' = 0$ (GeV/c)$^2$ in the $\pi^-$ beam is $R_\pi(\eta'/\eta) = 0.54 \pm 0.04$ and the mixing angle is $\phi_P = (36.3 \pm 1.0)\degree$ in the quark base. For the $K^-$ beam the ratio $R_K(\eta'/\eta) = 1.27 \pm 0.15$ corresponds the mixing angle $\phi_P = (41.6 \pm 1.8)\degree$, which is incompatible with pion beam result at $\approx 2.5\sigma$ confidence level. We assume that the reason of this difference is the gluon content in the $\eta'$ meson. In a two-angle mixing scheme with gluon mixing angle $\phi_G$ the gluon content is $\sin^2 \phi_G = 0.17 \pm 0.07$ for the $\eta'$ meson.

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