Low Energy Experiments on $\pi$-$\pi$ Scattering

Dinko Počanić

Department of Physics, University of Virginia, Charlottesville, VA 22901, USA

Abstract. General interest in a precise determination of the threshold $\pi$-$\pi$ amplitudes has recently increased markedly due to a controversy regarding the size of $\langle 0|\bar{q}q|0\rangle$, the scalar quark condensate. This paper examines the current experimental information on the $\pi$-$\pi$ scattering lengths, in particular the recent low energy $\pi N \rightarrow \pi \pi N$ data from several laboratories and the related application of the Chew–Low–Goebel technique well below 1 GeV/c momentum. It appears that uncertainties related to the treatment of non-pion-exchange backgrounds in these studies do not yet allow an unambiguous resolution of the $\langle 0|\bar{q}q|0\rangle$ size. However, near-term prospects for new model-independent results of improved precision are very good.

1 Motivation

Pion-pion scattering at threshold is uniquely sensitive to the explicit chiral symmetry breaking (ChSB) portion of the strong interaction and has, for this reason, been the subject of detailed study for over thirty years, both theoretically and experimentally. After QCD gained universal acceptance as the theory of the strong interaction, long-time controversies regarding the mechanism of the explicit breaking of chiral symmetry were laid to rest and the Weinberg picture [1] was recognized as valid at the tree level, providing a firm prediction for $a(\pi\pi)$, the pion-pion scattering lengths.

However, QCD is not directly applicable at low energies, except numerically on the lattice, which has not yet been established as a practical and reliable calculational method. Thus, knowing $a(\pi\pi)$, the $\pi$-$\pi$ scattering lengths, precisely has remained an important goal, as these quantities provide a direct and sensitive constraint on parameters of the available effective low energy lagrangians. This is of particular importance for the chiral perturbation theory (ChPT) approach which provides a systematic framework for the treatment of low energy strong interactions in terms of diagrams with increasing powers of momentum and mass [2]. Consequently, improved calculations including one-loop [3] and two-loop [4] diagrams have been performed using standard ChPT.

Recently, however, a less restrictive version of ChPT was formulated by the Orsay group, referred to as the generalized chiral perturbation theory (GChPT) [5]. This approach makes fewer theoretical assumptions and consequently has more parameters than the standard ChPT for the lagrangian terms of a given power of momentum or mass. As in standard ChPT, all parameters need to be constrained by data. A particularly interesting possibility
that is allowed in GChPT concerns the very mechanism of chiral symmetry breaking, as follows.

The standard picture of ChSB assumes a strong scalar quark condensate:

\[ -\langle 0 | \bar{q}q | 0 \rangle \gg F_\pi^3, \]

where \( F_\pi \simeq 92 \text{ MeV} \) is the pion decay constant. In the standard ChPT calculation, which relies on the above assumption, the s-wave \( \pi\pi \) scattering lengths \( a_{l=0}^I(\pi\pi) \) are predicted to be (including terms with up to two loops \[4\]):

\[ a_0^0 \simeq 0.21 \mu^{-1} \quad \text{and} \quad a_0^2 \simeq -0.041 \mu^{-1}, \]

where \( I = 0, 2 \) are the allowed values of dipion isospin and \( \mu \) is the charge-independent pion mass.

The Orsay group has argued for some time that the assumption in (1) is not clearly justified by the available experimental evidence, and has claimed that a much weaker scalar quark condensate must, in principle, be allowed \[6,5\]. The consequences of such a scenario are many, not the least of which are radically different light quark mass ratios than the ones generally accepted now \[6,7\]. The only practical observables sensitive to the size of \( \langle 0 | \bar{q}q | 0 \rangle \) are the s-wave \( \pi\pi \) scattering lengths.

In particular, using the GChPT formalism and a weak scalar quark condensate, the Orsay group found that the most likely value of \( a_0^0(\pi\pi) \) (calculated including one and two loop diagrams) would be \( \sim 0.27 \mu^{-1} \) \[7,8\], about 30% higher than the standard ChPT calculation. Clearly, a measurement of the s-wave \( \pi\pi \) scattering length with about 10% precision is required in order to differentiate experimentally between the two theoretical results.

Although there have been many attempts at evaluating the \( \pi\pi \) scattering lengths from available data over the years, the result generally accepted as most reliable is based on a comprehensive phase shift analysis of peripheral \( \pi N \rightarrow \pi\pi N \) reactions and \( K_{e4} \) decays completed in 1979 \[9\]. The values reported in that work are

\[ a_0^0 = 0.26 \pm 0.05 \mu^{-1} \quad \text{and} \quad a_0^2 = -0.028 \pm 0.012 \mu^{-1}. \]

This result is clearly not precise enough to resolve the above theoretical controversy. We proceed to examine the more recent experiments and related attempts at extraction of new, more precise values of \( a(\pi\pi) \).

## 2 Experiments on Threshold \( \pi-\pi \) Scattering

As free pion targets cannot be fabricated, experimental evaluation of \( \pi\pi \) scattering observables is restricted to the study of a dipion system in a final state of more complicated reactions. Scattering lengths are especially hard to determine since they require measurements close to the \( \pi\pi \) threshold, where the available phase space strongly reduces measurement rates. Over time
several reactions have been studied or proposed as a means to obtain near-threshold $\pi\pi$ phase shifts, such as $\pi N \rightarrow \pi\pi N$, $K_{e4}$ decays, $\pi^+\pi^-$ atoms (pionium), $e^+e^- \rightarrow \pi\pi$, etc. In practice, only the first two reactions have so far proven useful in studying threshold $\pi\pi$ scattering, although there are ambitious plans to study pionium in the near future. The main experimental methods and current results are discussed below.

2.1 $K_{e4}$ Decays

By most measures, the $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay (called $K_{e4}$) provides the most suitable tool for the study of threshold $\pi\pi$ interactions. The interaction takes place between two real pions on the mass shell, the only hadrons in the final state. The dipion invariant mass distribution in $K_{e4}$ decay peaks close to the $\pi\pi$ threshold, and only two states, $l_{\pi\pi} = I_{\pi\pi} = 0$ and $l_{\pi\pi} = I_{\pi\pi} = 1$, contribute appreciably to the process. These factors, as well as the well understood $V-A$ weak lagrangian giving rise to the decay, favor the $K_{e4}$ process among all others in terms of theoretical uncertainties. Measurements are, however, impeded by the low branching ratio of the decay, $3.9 \times 10^{-5}$.

Thus, $K_{e4}$ decay data provide information on the $\pi\pi$ phase difference $\delta_0 - \delta_1$ near threshold. The most recent published $K_{e4}$ experimental result was obtained by a Geneva–Saclay collaboration in the mid-1970’s [10]. Figure 1 summarizes the $\pi\pi$ phase shift information below 400 MeV derived from all $K_{e4}$ data published to date. The curves in Fig. 1 correspond to three different values of $\alpha_0^0(\pi\pi)$, and illustrate the relative insensitivity of the data to $\alpha_0^0$ at the level of experimental accuracy achieved by Rosselet et al.

Clearly, the available $K_{e4}$ data are of insufficient accuracy. Taken alone they provide a $\sim 35\%$ constraint on $\alpha_0^0$. Only after they are combined with $\pi\pi$ phase shifts extracted from peripheral $\pi N \rightarrow \pi\pi N$ reactions (see Sect. 2.2) is it possible to reduce the uncertainties to the level of about $20\%$, as quoted in (3). However, new, substantially more precise $K_{e4}$ data are expected in the near future (see Sect. 3).

We note that $K_{e4}$ decays provide no information on the $I = 2$ $\pi\pi$ phase shifts. Hence, other reactions must be used to supplement the $K_{e4}$ data in order to study $I = 2$ $\pi\pi$ scattering.

2.2 Peripheral $\pi N \rightarrow \pi\pi N$ Reactions at High Momenta

Goebel as well as Chew and Low showed in 1958/59 that particle production in peripheral collisions can be used to extract information on the scattering of two of the particles in the final state [13]. This approach is, of course, useful primarily for the scattering of unstable particles and has been used to great advantage in the study of the $\pi\pi$ system. Applied to the $\pi N \rightarrow \pi\pi N$ reaction, the well-known Chew–Low formula,

$$
\sigma_{\pi\pi}(m_{\pi\pi}) = \lim_{t \rightarrow \mu^2} \left[ \frac{\partial^2 \sigma_{\pi\pi N}}{\partial t \partial m_{\pi\pi}} \cdot \frac{\pi}{\alpha f^2} \cdot \frac{p^2(t - \mu^2)^2}{tm_{\pi\pi}k} \right],
$$

(4)
Fig. 1. $\pi\pi$ phase shift difference $\delta_0^0 - \delta_1^1$ extracted from $K\pi\pi$ data is plotted against $m_{\pi\pi}$, the dipion invariant mass. Full circles: Rosselet et al. [10]; open squares Zylberstejn [11]; open triangles Beier et al. [10]. The three curves correspond to phase shift solutions assuming three different values of $a_0^0$, as noted.

relates $\sigma_{\pi\pi}(m_{\pi\pi})$, the cross section for pion-pion scattering, to double differential $\pi N \rightarrow \pi\pi N$ cross section and kinematical factors: $p$, momentum of the incident pion, $m_{\pi\pi}$, the dipion invariant mass, $t$, the Mandelstam square of the 4-momentum transfer to the nucleon, $k = (m_{\pi\pi}^2 / 4 - \mu^2)^{1/2}$, momentum of the secondary pion in the rest frame of the dipion, $f_\pi$, the pion decay constant, and $\alpha = 1\text{ or } 2$, a statistical factor involving the pion and nucleon charge states. The method relies on an accurate extrapolation of the double differential cross section to the pion pole, $t = \mu^2$, in order to isolate the one pion exchange (OPE) pole term contribution. Since the exchanged pion is off-shell in the physical region ($t < 0$), this method requires measurements under conditions which maximize the OPE contribution and minimize all background contributions. Thus, suitable measurements require peripheral pion production at values of $t$ as close to zero as possible, which becomes practical at incident momenta typically above $\sim 3\text{ GeV/c}$.

The essential steps of the Chew–Low–Goebel procedure are illustrated in Fig. 2. The method relies on the assumption that the dominant process in peripheral pion production (small $|t|$) is the OPE. Since the pion has the smallest mass of all hadrons, the OPE pole lies closest to the physical region ($t < 0$) of any competing terms. Thus, for small $|t|$, the non-OPE background varies much more slowly than the OPE term which, in turn, is proportional
to $t/(t - \mu^2)^2$. Hence, measured $\pi N \rightarrow \pi\pi N$ differential cross sections are plotted in the so-called Chew–Low plane, $m_{\pi\pi}$ against $t$, as shown in Fig. 2. Data points are subdivided into bins (strips) of $m_{\pi\pi}$ and for each bin the Chew–Low extrapolating function $F$, defined as

$$F(s, t, m_{\pi\pi}) = \frac{\partial^2 \sigma_{\pi\pi N}(s)}{\partial t \partial m_{\pi\pi}} \frac{\pi}{f^2_{\pi}} \frac{p^2(t - \mu^2)^2}{t m_{\pi\pi}(m^2_{\pi\pi} - 4\mu^2)^{1/2}},$$  \hspace{1cm} (5)$$

is extrapolated to the pion pole $t = \mu^2$ which lies outside of the physical domain. When angular momenta higher than $l = 0$ contribute significantly in the $\pi\pi$ system, $F(s, t, m_{\pi\pi})$ must first be decomposed into spherical harmonics and the resulting amplitudes extrapolated to the pion pole.

**Fig. 2.** Illustration of the Chew–Low extrapolation in the $m_{\pi\pi}$ vs. $t$ plane (Chew–Low plane) to the pion pole $t = \mu^2$. The physical region of the data is bounded by the closed contour in the second quadrant ($t < 0$, $m_{\pi\pi} \geq 2\mu$).

The Chew–Low method has been refined considerably over time, particularly by Baton and coworkers [14]. Crossing, Bose and isospin symmetries, analyticity and unitarity, provide dispersion relation constraints on the $\pi\pi$ phase shifts, the “Roy equations” [15,16,17]. Roy equations are indispensable in evaluating $\pi\pi$ scattering lengths due to the restricted phase space of peripheral $\pi N \rightarrow \pi\pi N$ reactions below $m_{\pi\pi} \approx 500$ MeV; dispersion relations embodied in the Roy equations make use of more accurate data available at higher $\pi\pi$ energies, compensating thus for the limitations of low-$m_{\pi\pi}$ data.
Since the Chew–Low–Goebel method relies on extrapolation in a two-dimensional space, it requires kinematically complete data of high quality, both in terms of measurement statistics and resolution—these have been the limiting factors in all analyses to date.

The data base for these analyses has not changed essentially since the early 1970’s, and is dominated by two experiments, performed by the Berkeley [18] and CERN-Munich [19] groups. The latter of the two measurements has much higher statistics (300 k events compared to 32 k in the Berkeley experiment). A comprehensive analysis of this data base, with addition of the Geneva-Saclay $K_{e4}$ data, was performed by Nagels et al. [9], as discussed in Sect. 2.1. The resulting values of $a_0^{0.2}$ are given in (3).

There have been other Chew–Low type analyses since 1979. One, performed by the Kurchatov Institute group in 1982, was based on a set of some 35,000 $\pi N \rightarrow \pi \pi N$ events recorded in bubble chambers [20]. Patarakin, Tikhonov and Mukhin, members of the same group, recently updated the 1982 analysis by including available data on the $\pi N \rightarrow \pi \pi \Delta$ reaction, as well as the published $K_{e4}$ data [21]. The resulting s-wave $\pi \pi$ scattering lengths were found to be bounded by

$$0.205 \mu^{-1} < a_0^0 < 0.270 \mu^{-1} \quad \text{and} \quad -0.048 \mu^{-1} < a_0^2 < -0.016 \mu^{-1} \quad \text{(6)}$$

Although the above limits on $a_0^0$ carry slightly smaller uncertainties than the generally accepted $a_0^0$ value of Nagels et al. listed in (3), the result of Patarakin et al. still cannot exclude one of the two competing pictures of chiral symmetry breaking (strong vs. weak scalar quark condensate, as discussed in Sect. 1). The central value, though, is lower than in (3), more in line with the conventional, strong condensate picture that leads to the standard ChPT two-loop prediction of $a_0^0 \simeq 0.21 \mu^{-1}$.

At this point it is worth to note a recent analysis by the Cracow group of old unpublished CERN-Cracow-Munich $\pi^- \bar{p} \rightarrow \pi^- \pi^+ n$ data at 17.2 GeV, measured on a transversely polarized proton target [22]. The $m_{\pi\pi}$ range of this study is from 610 to 1590 MeV. In their analysis the Cracow group used a relativistic coupled channel Lippmann-Schwinger treatment of the $\pi\pi$ and $KK$ systems. Results of the analysis of two data sets yielded values of $a_0^0$ substantially lower than any discussed above:

$$a_0^0 = \begin{cases} 
0.172 \pm 0.008 \mu^{-1} & \text{for data set 1}, \\
0.174 \pm 0.008 \mu^{-1} & \text{for data set 2}.
\end{cases} \quad \text{(7)}$$

This interesting analysis may have been affected adversely by the way the original CERN-Cracow-Munich data were preserved. Nevertheless, like the work of the Kurchatov Institute group, this work points out that peripheral $\pi N \rightarrow \pi \pi N$ data may indeed favor a lower value of $a_0^0$ than indicated by the presently available $K_{e4}$ data.

It is regrettable that new high energy ($E_\pi > 3$ GeV) peripheral $\pi N \rightarrow \pi \pi N$ measurements are not planned in the future. Therefore much attention
during the last decade has been devoted to the study of the $\pi N \rightarrow \pi \pi N$ reaction at lower energies, $p_\pi \leq 500$ MeV. These results are discussed next.

### 2.3 Inclusive $\pi N \rightarrow \pi \pi N$ Reactions Near Threshold

Weinberg showed early on [1] that the OPE graph dominates the $\pi N \rightarrow \pi \pi N$ reaction at threshold. Subsequently, Olsson and Turner constructed a soft-pion lagrangian containing only the OPE and contact terms at threshold [23]. This enabled them to introduce a simple parametrization of the relation between the $\pi \pi$ and $\pi N \rightarrow \pi \pi N$ threshold amplitudes. Although this work was superseded by the emergence and general acceptance of QCD, it did provide the impetus for a number of inclusive measurements of $\pi N \rightarrow \pi \pi N$ total cross sections near threshold. Results of these studies published before 1995 are reviewed in detail in Ref. [24]. That data base has remained unchanged, apart from small additions that are discussed below.

As in peripheral pion production at high energies, there are 5 charge channels accessible to measurement,

\[
\pi^- p \rightarrow \begin{cases} 
\pi^- \pi^+ n \\
\pi^0 \pi^0 n \\
\pi^- \pi^0 p 
\end{cases} \quad \text{and} \quad \pi^+ p \rightarrow \begin{cases} 
\pi^+ \pi^0 p \\
\pi^+ \pi^+ n 
\end{cases}.
\]

Total cross sections of the five reactions are described by only four independent isospin amplitudes $A_{2I,I_{\pi\pi}}$, namely, $A_{31}$, $A_{32}$, $A_{10}$ and $A_{11}$, where $I$ is the total ($\pi p$) isospin and $I_{\pi\pi}$ is the isospin of the dipion system. Two of the four amplitudes vanish at threshold due to Bose symmetry. Thus, the amplitudes are, in principle, overconstrained by data; this redundancy is welcome given how difficult absolute measurements near threshold are.

The amount and quality of available inclusive near-threshold $\pi N \rightarrow \pi \pi N$ data, especially that collected since 1985, is impressive and has resulted in relatively rigorous constraints on the $\pi \pi N$ isospin amplitudes. This is illustrated in Fig. 3 which shows the whole data base in the form of quasi-amplitudes obtained by removing from the angle-integrated cross sections the uninteresting but strong energy dependence due to the reaction phase space.

The current data base is increased compared to that of 1994 by the addition of new, more precise $\pi^\pm p \rightarrow \pi^\pm \pi^+ n$ cross sections very near threshold from TRIUMF [25]. The new measurements have confirmed the same group’s earlier published data [26] on the $\pi^+ p \rightarrow \pi^+ \pi^+ n$ reaction, thus definitively invalidating older data taken by the OMICRON collaboration at CERN [27] (high-lying points with large error bars in the bottom panel of Fig. 3).

In spite of the relative abundance and high accuracy of the near-threshold inclusive pion production data, their interpretation in terms of $\pi \pi$ scattering lengths has been plagued by theoretical uncertainties. This shortcoming has recently been successfully addressed within the framework of the heavy baryon chiral perturbation theory (HBChPT) [28]. Theoretical uncertainties
limited the ability of this analysis to produce a stringent constraint on the $I = 0$ \pi\pi channel. However, the HBChPT study did provide a restrictive new $I = 2$ scattering length. The two results are:

$$a_0^0 \simeq 0.21 \pm 0.07 \mu^{-1}\quad \text{and} \quad a_0^2 = -0.031 \pm 0.007 \mu^{-1}.$$  \quad (9)

The $a_0^0$ result was recently refined by Olsson who used the so-called universal curve, a model-independent relation between $a_0^0$ and $a_0^2$ due to the forward
Low Energy Experiments on $\pi$-$\pi$ Scattering

dispersion relation or, equivalently, to the Roy equations [29]. Olsson found

$$a_0^0 = 0.235 \pm 0.03 \mu^{-1}.$$  

(10)

Any analysis based on HBChPT cannot, however, be expected to result in $\pi\pi$ scattering lengths different from the standard ChPT prediction because the latter is built into the lagrangian used.

2.4 Chew–Low analysis of Low Energy $\pi N \to \pi\pi N$ Data

Given the theoretical uncertainties in the interpretation of inclusive $\pi N \to \pi\pi N$ data near threshold, it was suggested some time ago to apply the Chew–Low method to low energy $\pi N \to \pi\pi N$ data [30]. Recently several exclusive $\pi N \to \pi\pi N$ data sets suitable for such treatment have become available. These are, in the order in which they were measured:

(a) $\pi^- p \to \pi^0 p n$ data from BNL [31],
(b) $\pi^+ p \to \pi^+ n p$ data from LAMPF [32], and
(c) $\pi^- p \to \pi^- n p$ data from TRIUMF [33].

We discuss below the current results of two analyses: first, of the LAMPF E1179 data, set (b) above, by the University of Virginia group, and, second, of the CHAOS data, set (c) above, by the TRIUMF group.

**Chew–Low Analysis of LAMPF E1179 Data.** A Virginia–Stanford–LAMPF team studied the $\pi^+ p \to \pi^+ n p$ reaction at LAMPF at five energies from 190 to 260 MeV [32]. The LAMPF $\pi^0$ spectrometer and an array of plastic scintillation telescopes were used for $\pi^+$ and $p$ detection. Three classes of exclusive events were recorded simultaneously: $\pi^+ n p$ and $\pi^+ p$ double coincidences, and $\pi^+ n p$ triple coincidences. Since the acceptance of the apparatus and the backgrounds were significantly different for the three classes of events, this experiment had a strong built-in consistency check. The $\pi^+ p \to \pi^+ n p$ reaction is sensitive only to the $I = 2$ s-wave $\pi\pi$ scattering length.

Figure 4 illustrates the main source of difficulty in this analysis, namely, the relatively broad energy resolution that considerably smears the cross section data bins in a Chew–Low plot of $m_{\pi\pi}$ against $t$. Consequently, in order to obtain a physically interpretable array of double differential cross section bins, a complicated deconvolution procedure had to be implemented first [34]. Limited counting statistics presented an additional difficulty in the analysis, as it increased the uncertainties in both the deconvolution procedure and in the final Chew–Low extrapolation.

Preliminary results of this analysis for one bin of $m_{\pi\pi} = 2.26 \pm 0.18 \mu$ are shown in Fig. 5. Open circles in the figure indicate data points excluded from the Chew–Low extrapolation procedure due to large value of $|t| > 6 \mu^2$, where OPE is weak, and the smallest $|t|$ point which has a large normalization uncertainty due to the cross section deconvolution procedure. The resulting $\pi\pi$ cross section is $0.79 \pm 0.56 \text{mb}$. A proper procedure to extract $a_0^0(\pi\pi)$ would
be to include the new data point in a comprehensive dispersion-relation $\pi\pi$ phase shift analysis. Uncertainties in the current analysis do not justify such an undertaking at this time. However, the precision one might expect from this result is illustrated by evaluating $a_0^2$ from the above cross section datum directly. Doing so one obtains

$$a_0^2 = -0.055 \pm 0.021 \mu^{-1},$$

(11)

which shows that the current status of this analysis does not provide a strong new constraint of the $\pi\pi$ phase shifts. In comparison, the BNL $\pi^- p \rightarrow \pi^0 \pi^0 n$ data, while having much higher event statistics, are characterized by an even
Fig. 5. Chew-Low function $F(s, t, m_{\pi\pi})$ constructed from $\pi^+p \to \pi^+\pi^0p$ exclusive cross sections at 260 MeV is plotted as a function of $t$ along with a linear fit (preliminary). Full circles: data points included in the fit. Open circles: data points excluded from the fit. The extrapolated value of the $\pi\pi$ total cross section at $m_{\pi\pi} = 2.26 \pm 0.18 \mu$ is indicated.

broader energy resolution and poorer coverage of the low $|t|$ region critical for the Chew–Low extrapolation.

Chew–Low Analysis of the CHAOS $\pi^-p \to \pi^+\pi^-n$ Data. The most significant development in this field in the past few years has been the construction and operation of the Canadian High Acceptance Orbit Spectrometer (CHAOS), a sophisticated new detector at TRIUMF [35]. This impressive device, composed of a number of concentric cylindrical wire chamber tracking detectors and total energy counters mounted between the poles of a large bending magnet, provides nearly 360° of angular coverage for in-plane events, with excellent acceptance for multi-particle events. It is no surprise that the CHAOS collaboration has very quickly measured the most comprehensive set of exclusive in-plane $\pi^-p \to \pi^+\pi^-n$ cross sections below 300 MeV.

The CHAOS $\pi^-p \to \pi^+\pi^-n$ data set covers four incident beam energies between 223 and 284 MeV. Unlike the LAMPF and BNL measurements,
these data have an excellent energy resolution of $\sigma \simeq 4.8\text{MeV}$. In order to carry out a Chew–Low analysis, the CHAOS collaborators binned their data into an acceptance-corrected $10 \times 10 \times 10$ lattice of $m_{\pi\pi}^2$, $t$ and $\cos \theta$. The $\cos \theta$ dimension was integrated out, resulting in double-differential cross sections $d^2\sigma/dm_{\pi\pi}^2 dt$, which were used to construct the Chew–Low extrapolating function $F(s, m_{\pi\pi}, t)$, as given in (5). A linear fit over a carefully selected interval in $t$ was made for every bin of $m_{\pi\pi}^2$. The resulting fits and linear extrapolation are shown in Fig. 6.

![Fig. 6. Plots of the Chew–Low extrapolation function $F(s, m_{\pi\pi}, t)$ produced by the CHAOS group [33]. The points at $t = +\mu$ are deduced from extrapolation and yield the $\pi\pi$ cross section. Solid circles: data points used in the linear fit; crosses: data points not used in the fit.](image)

From the extrapolated values of $F(s, m_{\pi\pi}, t)$ the authors extracted $\pi\pi$ cross sections at six $\pi\pi$ energies in the range $m_{\pi\pi}^2 = 4.15-5.65\mu^2$ with uncertainties ranging from about 16% at the lowest energy to 63% at the highest. These $\pi\pi$ cross section data were then added to the data base of Ref. [21], and a Roy equation constrained phase shift analysis was performed following
the same procedure as in Ref. [21]. One parameter, $a_0^2$, was left free to vary in the analysis. Minimizing the $\chi^2$ of the fit, the authors obtained

$$a_0^2 = 0.206 \pm 0.013 \mu^{-1},$$

which would strongly confirm the validity of the standard ChPT and the strong scalar quark condensate implied therein, at the same time ruling out the possibility of the weak scalar quark condensate proposed by the Orsay group [6,5].

### Problems with the Chew–Low–Goebel Method at Low Energies?

Bolokhov et al. of the Sankt Petersburg State University have recently performed a detailed study of the reliability of the Chew-Low method at low energies using sets of synthetic $\pi N \rightarrow \pi \pi N$ “data” between 300 and 500 MeV/c [36]. In this work the authors constructed data sets with: (a) the OPE contribution only, (b) OPE + other allowed mechanisms, (c) all mechanisms without the OPE. Both linear and quadratic Chew-Low extrapolation were used. The authors found 25–35 % deviations in the reconstructed OPE strength in case (a), 100–300 % deviations under (b), and large “OPE amplitude” without any pion pole in the synthetic data under (c). This led the authors to conclude that “...noncritical application [of the Chew–Low method] results in 100 % theoretical errors, the extracted values being in fact random numbers ...”

The quoted study is the first one to date to address theoretically the validity of the Chew–Low–Goebel method in the low energy regime where this technique has not been traditionally applied. Given the complex nature of the issue, it would be premature to write off using the method at low energies altogether. Clearly, a critical examination of the problem is strongly called for. However, before the matter is finally resolved, we cannot accept the CHAOS result in (12) as definitive, in spite of the high precision of the new CHAOS data, and of the elegance of the analysis.

Further grounds for caution regarding the Chew–Low–Goebel method at low energies are found in the pronounced pion beam energy dependence of the extrapolated $\pi \pi$ cross sections in the lowest $m_{\pi \pi}$ bin, a possible indication of a residual non-OPE background not properly removed by the analysis. It must be pointed out, however, that the authors found that their result in (12) did not change significantly when the lowest energy $\pi \pi$ cross section was dropped from the analysis. On the other hand, the same group’s $\pi^+ p \rightarrow \pi^+ \pi^+ n$ data were incompatible with linear fits in terms of $F(s, m_{\pi \pi}, t)$, indicating a strong dominance of non-OPE processes in that reaction channel.

### 3 Summary of Current Results and Future Prospects

Theoretical predictions and experimental results on the $\pi \pi$ scattering lengths published to date are plotted in Fig. 7 in the $a_0^2$ against $a_0^0$ plane.
Fig. 7. Summary of $\pi\pi$ scattering length predictions: Weinberg’s tree-level result [1] (full circle), ChPT one-loop calculation [3] (full square), ChPT two-loop calculation [4] (full triangle), and analyses of experimental data: Nagels et al. [9] (oval contour), Patarakin et al. [21] (oblique quadrangular contour), HBChPT analysis of Bernard et al. [28] (solid rectangle), and Olsson’s dispersion-relation constraint of the HBChPT result [29] (dashed lines).

We note that the current analyses of the available $K_{e4}$ and $\pi N \rightarrow \pi\pi N$ data (excluding the not yet fully established low energy application of the Chew–Low method) are not sufficiently accurate to distinguish between the two scenarios of chiral symmetry breaking, i.e., between the standard strong scalar quark condensate picture and the one with a weak $\langle 0|\bar{q}q|0\rangle$.

At the same time the available analyses seem to favor slightly higher values of both $a_0^0$ and $a_0^2$ than the values predicted by standard ChPT.

The threshold $\pi\pi$ scattering experimental data base will improve significantly in the near future as several new experiments, listed below, bear fruit. The same experiments are discussed in more detail elsewhere in these Proceedings.
**K_{e4} Data from BNL E865.** Recently completed measurements carried out by the E865 collaboration at BNL have resulted in more than $3 \times 10^5 K_{e4}$ decay events on tape [37]. Since the analysis of these data had not progressed far at the time of this writing, the final event statistics after the appropriate cuts are applied remains to be determined. For comparison, the data base of Rosselet et al. consisted of 30,000 events, so a significant improvement is expected from the BNL E865 work.

**K_{e4} Data from DAΦNE.** The KLOE detector at the Frascati $\phi$ factory DAΦNE will be used in an ambitious program of measurement of the $K_{e4}$ decay. The expected accuracy of the $\pi \pi$ phase shift difference $\delta^0_0 - \delta^1_1$ to be extracted from this work is 5% [38], i.e., almost an order of magnitude improvement over the current result.

**Lifetime of the $\pi^+\pi^-$ Atom (CERN).** The DIRAC experiment at the SPS at CERN [39] grew out of the first observation of the $\pi^+\pi^-$ atom (pionicium) at the Serpuhov laboratory [40]. The DIRAC project relies on the Lorentz boost of relativistic pionicium to measure the lifetime of the pionicium atom to 10% accuracy. This, in turn, will constrain the $\pi\pi$ scattering length difference $|a_0^0 - a_2^0|$ with 5% accuracy. In this respect, the pionicium and $K_{e4}$ decay experiments are complementary, as the latter provide no direct information on $a_2^0$.

As has been noted, further theoretical work is required to make use of the existing $\pi N \rightarrow \pi \pi N$ data, in particular to clarify the applicability of the Chew–Low–Goebel method at low energies. Additionally, better understanding of the electromagnetic corrections will be necessary in order to take full advantage of the forthcoming $K_{e4}$ and pionicium data. Thus, the next few years will be interesting on both the experimental and theoretical fronts.

The author wishes to thank A. A. Bolokhov, E. Frlež, O. O. Patarakin, M. E. Sevior and G. R. Smith for substantive discussions and for graciously providing access to results of their ongoing work. This work has been supported by a grant from the U.S. National Science Foundation.

**References**

[1] S. Weinberg, Phys. Rev. Lett. 17, 616 (1966); *ibid.* 18, 188 (1967).
[2] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, 142 (1984); Nucl. Phys. B250, 465 (1985).
[3] J. Gasser and H. Leutwyler, Phys. Lett. B125, 325 (1983).
[4] J. Bijnens, et al., Phys. Lett. B374, 210 (1996); Nucl. Phys. B508, 263 (1997).
[5] J. Stern, H. Sazdjian and N. H. Fuchs, Phys. Rev. D 47, 3814 (1993); M. Knecht, B. Moussallam and J. Stern, Nucl. Phys. B429, 125 (1994).
[6] N. H. Fuchs, H. Sazdjian and J. Stern, Phys. Lett. B238, 380 (1990)
[7] M. Knecht, et al., Nucl. Phys. B457, 513 (1995).
[8] M. Knecht, et al., Nucl. Phys. B471, 445 (1996).
[9] M. M. Nagels, et al., Nucl. Phys. B147, 189 (1979).
10. L. Rosselet, et al., Phys. Rev. D 15, 574 (1977).
11. A. Zylberstein, Ph.D. thesis, University of Paris, Orsay, 1972.
12. E. W. Beier, et al., Phys. Rev. Lett. 29, 511 (1972); ibid. 30, 399 (1973).
13. C. J. Goebel, Phys. Rev. Lett. 1, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
14. J. B. Baton, G. Laurens and J. Reignier, Phys. Lett. 33B, 525 (1970).
15. S. M. Roy, Phys. Lett. 35B, 353 (1971).
16. J. L. Basdevant, J. C. Le Guillou and H. Navelet, Nuovo Cim. 7A, 363 (1972).
17. J. L. Basdevant, C. G. Froeggatt and J. L. Peterson, Nucl. Phys. B72, 413 (1974).
18. S. D. Protopopescu et al., Phys. Rev. D 7, 1279. (1973)
19. G. Grayer et al., Nucl. Phys. B75, 189 (1974).
20. E. A. Alekseeva et al., Zh. Eksp. Teor. Fiz. 82, 1007 (1982) [Sov. Phys. JETP 55, 591 (1982)].
21. O. O. Patarakin, V. N. Tikhonov and K.N. Mukhin, Nucl. Phys. A598, 335 (1996).
22. R. Kamiński, L. Leśniak and J. P. Maillet, Phys. Rev. D 50, 3145 (1994); R. Kamiński and L. Leśniak, Phys. Rev. C 51, 2264 (1995).
23. M. G. Olsson and L. Turner, Phys. Rev. Lett. 20, 1127 (1968); Phys. Rev. 181, 2141 (1969).
24. D. Počanić, in “Chiral Dynamics, Theory and Experiment”, A. M. Bernstein and B. R. Holstein, eds., Lect. Notes in Phys. Vol. 452, (Springer Verl., 1995) 95.
25. J. B. Lange, et al., TRIUMF preprint (1997).
26. M. E. Sevior, et al., Phys. Rev. Lett. 66, 2569 (1991).
27. G. Kernel et al., Z. Phys. C48, 201 (1990).
28. V. Bernard, N. Kaiser and Ulf G. Meissner, Nucl. Phys. B457, 147 (1995).
29. M. G. Olsson, Phys. Lett. B410, 311 (1997).
30. D. Počanić et al., proposal for LAMPF experiment E1179 (1989).
31. J. Lowe, et al., Phys. Rev. C 44, 956 (1991).
32. D. Počanić, et al., Phys. Rev. Lett. 72, 1156 (1994); E. Frlež, Ph. D. Thesis, Univ. of Virginia, 1993 (Los Alamos Report LA-12663-T, 1993).
33. M. Kernani, et al., TRIUMF preprint (1997).
34. S. E. Bruch, M.Sc. Thesis, Univ. of Virginia (1995).
35. G. R. Smith, et al., Nucl. Instrum. Meth. A362, 349 (1995).
36. A. A. Bolokhov, M. V. Polyakov and S. G. Sherman, e-print hep-ph/9707406 (1997).
37. J. Lowe, contribution to this Proceedings (1997).
38. M. Baillargeon and P. J. Franzini, “Accuracies of $K_\ell^4$ Parameters at DAΦNE”, in the Second DAΦNE Handbook, L. Maiani, N. Paver and G. Pancheri, eds., 1995.
39. B. Adeva et al., “Lifetime measurement of $\pi^+\pi^-$ atoms to test low energy QCD predictions”, proposal to the SPSLC, CERN/SPSLC 95-1 (1995).
40. L. G. Afanasev, et al., Phys. Lett. B308, 200 (1993).