WHITE HOLES, BLACK HOLES
AND CPT IN TWO DIMENSIONS

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ABSTRACT: It is argued that a unitarity-violating but weakly CPT invariant superscattering matrix exists for leading-order large-$N$ dilaton gravity, if and only if one includes in the Hilbert space planckian “thunderpop” excitations which create white holes. CPT apparently cannot be realized in a low-energy effective theory in which such states have been integrated out. Rules for computing the leading-large-$N$ superscattering are described in terms of quantum field theory on a single multiply-connected spacetime obtained by sewing the future (past) horizons of the original spacetime with the past (future) horizons of its CPT conjugate. Some difficulties which may arise in going beyond leading order in $1/N$ are briefly discussed.
1. INTRODUCTION

It has been forcefully argued [hawk] that the laws of quantum mechanics, when applied to black holes, imply the breakdown of unitarity, at least for observers outside the black hole who cannot tell whether the the information has been destroyed at a singularity, carried away by a baby universe, or inaccessibly stored in a remnant. The $S$-matrix must then be replaced by a superscattering matrix $\mathcal{S}$ which in general maps pure states into mixed states (but conserves probability). Detailed analyses of two-dimensional toy models over the last year and a half[CGHS,emod,rst,gine,lowe,pira] have greatly fortified the arguments of [hawk] – although there are some new wrinkles [bos]. At least in some models information is effectively lost. Thus the role of unitarity as a cornerstone of physics is in serious jeopardy.

CPT is also considered a sacred principle of physics. However, its validity is also seriously threatened in the context of quantum black hole processes. Indeed, it is easy to see [page] that unitarity violations imply a breakdown of strong CPT. Strong CPT states that if a given in density matrix $\rho_{in}^A$ is mapped to a given out density matrix $\rho_{out}^B$:

$$\rho_{out}^B = \mathcal{S} \rho_{in}^A,$$

then the CPT reverse of $\rho_{out}^B$ is mapped to the CPT reverse of $\rho_{out}^A$:

$$\Theta \rho_{in}^A = \mathcal{S} \Theta^{-1} \rho_{out}^B,$$

where $\Theta(\Theta^{-1})$ is the CPT map from in (out) to out (in) density matrices. Multiplying both sides of (gtwo) by $\Theta^{-1}$ and inserting it into the right hand side of (gone) one learns that

$$\mathcal{S}^{-1} = \Theta^{-1} \mathcal{S} \Theta^{-1}$$

is the inverse of $\mathcal{S}$. However, it is easily seen [page], that if pure states go to mixed states, information is lost and $\mathcal{S}$ therefore can not have an inverse.

Put another way, the $\mathcal{S}$-matrix for black hole processes contains an intrinsic arrow of time: it takes pure states to mixed states but not mixed states to pure states. This is inconsistent with strong CPT.

However Wald [wald] has pointed out that strong CPT violation is really a consequence of our CPT non-invariant description of the dynamics, rather than of a fundamental arrow of time. To see this consider [sork] a $\mathcal{S}$-matrix which maps any state of a spin 1/2 particle
to either an up or down state, both with probability one half. Clearly this involves no fundamental arrow of time: one could not discern whether a movie of the scattering process was being run forward or backward. Physical discernable violations of CPT must violate the weak CPT condition. This states that the probability to scatter from a given pure in-state $\rho_{in}^P$ to a pure out-state $\rho_{out}^Q$ is equal to the probability of scattering from $\Theta^{-1}\rho_{out}^Q$ to $\Theta\rho_{in}^P$:

$$\rho_{out}^Q S \rho_{in}^P = (\Theta \rho_{in}^P) S (\Theta^{-1}\rho_{out}^Q).$$

Equivalently,

$$S^T = \Theta^{-1} S \Theta^{-1}.$$  \hspace{1cm} (wcpt)

Note that (wcpt) and (scpt) are different. (wcpt) is obeyed in the given example of the spin $1/2$ particle.

Wald has speculated that the $S$-matrix for black hole formation and evaporation obeys the weak CPT condition. An immediate consequence of this would be that the CPT reverse of the final state of black hole evaporation (consisting mostly of outgoing Hawking radiation) usually collapses to form a black hole [wald].

In this paper we will show that this is not the case in a two dimensional model. CPT reversed Hawking radiation is below threshold for black hole formation: it is reflected through the origin. We strongly suspect that this is also the case in four dimensions.

However we shall see in two dimensions that weak CPT invariance can be restored—at least in a sector of the Hilbert space—by including the possibility of white hole formation/evaporation*. This process has a semiclassical description which is basically the time reverse of black hole formation/evaporation. Black holes lead to unpredictability because information is lost behind the future event horizon, while white holes lead to unpredictability because one must integrate over initial conditions at the past event horizon. We will find that the integration measure is fixed by weak CPT invariance. An intriguing representation of the $S$-matrix will be given as an $S$-matrix on a (in general multiply-connected) spacetime obtained by sewing together white and black hole horizons of the original spacetime with its time reverse.

* This possibility was rejected by Wald [wald] on the basis of white hole instabilities [wbld]. These instabilities are suppressed in a $1/N$ expansion, but as shall be discussed in section 5, they may create difficulties at finite $N$. 

2. BRIEF REVIEW OF THE RST MODEL

An elegant model for two-dimensional black hole evaporation was introduced by Russo, Susskind and Thorlacius [rst], expanding on ideas introduced in [emod]. The RST model differs from the original CGHS model [CGHS] by finite counterterms which are fine-tuned to preserve a global symmetry. This enables an exact solution of the theory at large \( N \). Numerical analyses of the CGHS model [lowe,pira] indicate that the models are qualitatively similar, despite the fine-tuning.

The classical Lagrangian for the RST model is, in conformal gauge,

\[
S_{cl} = \frac{1}{\pi} \int d^2 x \left[ (2e^{-2\phi} - \frac{N}{12}\phi) \partial_+ \partial_- \rho 
+ e^{-2\phi}(\lambda^2 e^{2\rho} - 4\partial_+ \phi \partial_- \phi) + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right], \tag{cact}
\]

where \( \rho \) is the conformal factor, \( \phi \) is the dilaton and \( f_i \) are \( N \) scalar matter fields. Define*

\[
\Omega = \frac{12}{N} e^{-2\phi} + \frac{\phi}{2} + \frac{1}{4} \ln \frac{N}{48}, \tag{odef}
\]

\[
\chi = \frac{12}{N} e^{-2\phi} + \rho - \frac{\phi}{2} - \frac{1}{4} \ln \frac{N}{3}. \tag{cdef}
\]

In the large-\( N \) limit, with \( \chi \) and \( \Omega \) held fixed, the quantum effective action is then

\[
S = \frac{1}{\pi} \int d^2 x \left[ \frac{N}{12} (\partial_\chi \partial_+ \chi + \partial_+ \Omega \partial_- \Omega + \lambda^2 e^{2\chi - 2\Omega}) + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right]. \tag{nact}
\]

When rewritten in terms of \( \rho \) and \( \phi \), (nact) is seen to differ from the classical action (cact) by the term \( \frac{N}{12} \partial_+ \rho \partial_- \rho \) responsible for Hawking radiation. (The effects of ghosts may be ignored in the large-\( N \) limit.)

There is a residual conformal gauge invariance in (nact). We fix this by the "Kruskal gauge" choice

\[
\chi = \Omega, \tag{gchc}
\]

which implies

\[
\rho = \phi + \frac{1}{2} \ln \frac{N}{12}. \tag{rphi}
\]

* Our conventions differ slightly from [rst]. They are chosen so that \( \chi \) and \( \Omega \) are held fixed as \( N \) is taken to infinity.
In Kruskal gauge the equations of motion are simply
\[ \partial_+ \partial_- \Omega = -\lambda^2, \]  
(oeom)
and the constraints reduce to
\[ \partial^2_\pm \Omega = -T^M_{\pm \pm}, \]  
(cstr)
where
\[ T^M_{\pm \pm} = \frac{6}{N} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i + t_\pm. \]  
(tdef)
The functions \( t_\pm(x^\pm) \) are fixed by boundary conditions.

The linear dilaton vacuum solution
\[ \phi = -\frac{1}{2} \ln \left[ -\lambda^2 N x^+ x^- \right] \]  
(pldv)
\[ t^0_\pm = -\frac{1}{4(x_\pm)^2} \]  
(tpm)
corresponds to
\[ \Omega = -\lambda^2 x^+ x^- - \frac{1}{4} \ln[-4\lambda^2 x^+ x^-]. \]  
(oldv)
The solution corresponding to general incoming matter from \( I^- \) is
\[ \Omega = -\lambda^2 x^+(x^- + \frac{1}{\lambda^2} P_+(x^+)) + \frac{1}{\lambda} M(x^+) \]
\[ -\frac{1}{4} \ln[-4\lambda^2 x^+ x^-], \]  
(gsol)
where
\[ M(x^+) = \lambda \int_0^{x^+} d\tilde{x}^+ \tilde{x}^+(T^M_{++} - t^0_+), \]  
(mdef)
\[ P_+(x^+) = \int_0^{x^+} d\tilde{x}^+(T^M_{++} - t^0_+). \]  
(pdef)
and \( t_- = t^0_- \). By transforming back to \( \rho, \phi \) variables it can be seen for large \( M \) that this corresponds at early times to a black hole which forms and evaporates.

However, the late-time behavior of (gsol) is unphysical. Viewed as a function of \( \phi, \Omega \) has a minimum at
\[ \phi_{cr} = -\frac{1}{2} \ln \frac{N}{48}, \]
\[ \Omega_{cr} = \frac{1}{4}. \]
There is no real value of $\phi$ corresponding to $\Omega < \Omega_{cr}$. $\Omega = \Omega_{cr}$ should be regarded as the analog of the origin of radial coordinates and the end of the spacetime, rather than continuing to negative radius. Reflecting boundary conditions, consistent with energy conservation should be imposed. RST accordingly require

$$f|_{\Omega = \Omega_{cr}} = 0,$$

$$\partial_{\pm}\Omega|_{\Omega = \Omega_{cr}} = 0. \quad (rbc)$$

The line $\Omega = \Omega_{cr}$ along which the boundary conditions are imposed undergoes dynamical motion in the $x^+, x^-$ plane. Of course this boundary line could be moved to a fixed timelike coordinate line e.g. $x^+ = x^-$ by a conformal transformation. However, this would be incompatible with Kruskal gauge and does not simplify the analysis.

It follows from the equations of motion that the boundary curve $x^-(x^+)$ obeys

$$\lambda^2 \partial_+ x^-(x^+) = -\partial_+ P_+(x^+) + \frac{1}{4(x^+)^2}. \quad (ccc)$$

If $\partial_+ P_+$ is small enough, the right hand side is positive and the boundary curve is a timelike line. No black holes are formed: incoming matter is benignly reflected up to future null infinity. A similar behavior occurs in four-dimensional general relativity in that sufficiently weak scalar $S$-waves can simply pass through the origin without collapse.

On the other hand, if $\partial_+ P_+$ exceeds the critical value $1/4(x^+)^2$, the boundary curve turns to the right (towards spatial infinity) and becomes spacelike as in the shock wave geometry of figure 1. It can be seen that the scalar curvature diverges along the spacelike segments of the boundary curve. It is not possible to implement the boundary condition (rbc) along these segments. Such spacelike boundary segments necessarily bound regions of future trapped points where $\partial_+ \Omega < 0$ and $\partial_- \Omega < 0$, which is the interior of a black hole. Thus these spacelike singularities resemble in every way the singularities inside four-dimensional black holes. We regard the singularity as a boundary of the spacetime, and information which flows into this spacelike curve is lost to asymptotic observers*.

The trajectory of a spacelike segment of the boundary curve is determined, not by boundary conditions, but by the initial conditions on $I^-$. If the incoming energy is finite,

* We are essentially defining - as opposed to deriving - the model to lose information by this boundary condition. Physics at the singularity is of course not controlled by our approximations, and other boundary conditions might be possible. For example one might imagine a (causality violating) conveyor belt which moves the information along the singularity and then releases it at the endpoint. It would be of great interest to find alternate boundary conditions which consistently implement this idea.
the boundary curve will eventually revert to a timelike trajectory. This is the “endpoint” at which the future apparent horizon—the boundary dividing the regions $\partial_{+}\Omega > 0$ and $\partial_{+}\Omega < 0$—meets the singularity, and the black hole has evaporated to zero size. After the endpoint the boundary conditions (rbc) are immediately imposed[rst, AS]. This is depicted in figure 1.

These boundary conditions have an important consequence at the endpoint†, as follows. Parameterizing the boundary curve as $\hat{x}(x^-)$, the constraints imply

$$\lambda^2 \partial_\perp \hat{x}^+ = -\partial_\perp P^- + \frac{1}{4(x^-)^2}. \quad (bcm)$$

Following the boundary curve down from above the endpoint, we see that it makes a sudden turn to the right (away from spatial infinity) at the endpoint. This is consistent with (bcm) only if $\partial_\perp P^-$, or equivalently $T^M_- \perp$ is negative and infinite at the endpoint. RST indeed find in the exact solution that $T^M_- \perp$ must have a negative delta-function. Since the $f_i$ are zero on the boundary, this implies that $t_-$ has a delta-function at the endpoint value of $x^-$. A general lesson, which will be important in the following, can be extracted from the preceding discussion. Throwing positive energy in from $I^-\perp$ causes the boundary curve to lean towards spatial infinity. Above some threshold, it actually causes it to become spacelike. Throwing negative energy at the boundary from $I^-\perp$ causes it to recede. Since it is receding, throwing more negative energy at the boundary has less effect. It can be forced to make a spacelike turn to the left only by an infinitely negative energy density.

Returning to the discussion of the endpoint, a delta function in $T^M_- \perp$ corresponds to a signal which travels from the endpoint up to $I^+\perp$. This signal is referred to as a “thunderpop”. The thunderpop is an essentially planckian object, since it involves high frequencies and emerges from a high-curvature region. Ordinarily in a $1/N$ expansion wavelengths are restricted to be greater then $1/N$, since at shorter distances energy fluctuations become large and the $1/N$ expansion breaks down. Thus thunderpop type states would not usually be included in the Hilbert space. However here we see that they generically arise from the evolution of long-wavelength states, and so must be considered. This is a breakdown of decoupling. Of course as the detailed structure of the thunderpop is sensitive to short distance physics, the description we give here is approximate. At the very least the thunderpop should be smeared over a region of size $1/N$.

† RST seem to inexplicably alter their boundary condition at the endpoint.
Since $f_i = 0$, yet $T^M_-$ (which should be understood as an expectation value) is non-zero and negative along the thunderpop, it is a “highly quantum” state of $f$-modes. Negative $T^M_-$ can arise from off-diagonal terms in the expectation value [prtr]. However there are restrictions on how negative $T^M_-$ can be: the total energy must be positive for reasonable states, and a region of negative $T^M_-$ can not be “too far” from a region of positive $T^M_-$. Thus it is not obvious that the thunderpop can really be represented as a semiclassical quantum state. In the appendix we demonstrate that it is nevertheless possible.

It is important to keep in mind that, although the RST model has an actual spacelike singularity, this will not be important in the following because the $\mathcal{S}$-matrix is insensitive to all physics behind the global horizons. One can certainly imagine altering the RST model in such a way so as to eliminate the spacelike singularity and store the information forever inside the black hole, or allow it to be carried away by a baby universe. The following discussion would apply equally to such models.

In conclusion, the RST model embodies all the features of black hole evaporation anticipated by Hawking. Black holes form and evaporate in a finite time, leaving nothing behind. Information is lost behind a global event horizon. The only unanticipated feature is the generic occurrence of a thunderpop at the evaporation endpoint. In section 4 we shall see this plays a key role in the restoration of CPT invariance.

3. THE RST $\mathcal{S}$-MATRIX

Given the quantum state on $I^+$, it is not in general possible to reconstruct the quantum state on $I^-$. Additional information, corresponding to the state swallowed by the black hole, is required. One therefore expects that only a $\mathcal{S}$-matrix, rather than an $S$-matrix, exists for the RST model. It is perhaps possible to obtain analytic formulae for the matrix elements to leading order in large-$N$ using technology developed in [gine]†. However, such explicit formulae will not be required for our considerations, and we will content ourselves in this section with a general description of the large-$N$ $\mathcal{S}$-matrix.

Incoming states are created by acting with $f$-oscillators on the $f$-vacuum. Let us first consider states obtained by acting on the vacuum with order $N^0$ creation operators. It

† This has been partially worked out for shock wave geometries by L. Thorlacius (private communication).
follows from (nact) or (tdef) that there is no back-reaction on the metric or dilaton to leading order in $1/N$. This is simply because “Newton’s constant” is taken to zero as $1/N$. Thus the scattering is simply obtained by reflecting the $f$-particles off of the boundary at $\Omega = \Omega_{cr}$, (according to (rbc)), which in the vacuum is the timelike line $4\lambda^2 x^+ x^- = -1$. Clearly a unitary $S$-matrix exists in this sector of the Hilbert space.

In order to investigate black hole dynamics, one must consider states with incoming energies (i.e., $\langle T_{++}^M \rangle$) of order one. Semiclassical configurations corresponding to collapsing matter are typically described by specifying $c$-number initial data $\rho^c(x^+), \phi^c(x^+), f^c_i(x^+)$ and $t_+(x^+)$ on $\mathcal{I}^-$. This description is redundant because of conformal gauge invariance. The semiclassical quantum state corresponding to this initial data has the simplest description in the gauge $t_+ = 0$ (the gauge transformation properties of $t_+$ are described in the appendix). In this gauge it can be represented on $\mathcal{I}^-$ as a coherent state of the form:

$$|f^c_i\rangle = A : e^{-\frac{i}{4} \sum_{i=1}^{N} \int dx^+ \partial_+ f^c_i(x^+ f^c_i(x^+)) : |0\rangle, \quad (chst)$$

where the normal ordering and the vacuum are defined with respect to $t_+ = 0$ coordinates and $A$ is a normalization factor. Its time evolution is then given - to leading order in $N$ - by classical evolution of $f^c_i$. In general the coherent states are an overcomplete basis, but in the semiclassical large-$N$ limit $A$ becomes small and they become orthogonal.

In $t_+ = 0$ coordinates the Kruskal condition (gchc) will of course not hold in general. Rather one has

$$\Omega = \chi + \omega, \quad (tgc)$$

where $\omega$ is a $c$-number solution of the free wave equation. In this $t_+ = 0$ gauge the operator constraints become

$$-\partial_+^2 \omega - \partial_+ \omega \partial_+ \omega + 2 \partial_+ \omega \partial_+ \Omega + \partial_+^2 \Omega = -\frac{6}{N} \sum_{i=1}^{N} \partial_+ f^c_i \partial_+ f^c_i. \quad (tcs)$$

$\Omega$ is then found in terms of the physical degrees of freedom $f_i$ by integrating this equation, with some integration constants fixed by (oeom)*.

If the $c$-number field configurations $f^c_i$ in (chst) are such that

$$\langle T_{++}^M \rangle = \frac{6}{N} \sum_{i=1}^{N} \partial_+ f^c_i \partial_+ f^c_i \quad (texp)$$

* This is the analog of light-cone gauge in string theory. A DDF-like description, in which $\chi$ and $\Omega$ are independent operators, is also possible[verl,deal].
is order $N^0$, there is a leading order back reaction on the geometry, and $\rho$ and $\phi$ are non-zero. If $\rho$ is non-zero, then the representation (chst) of the incoming state will not correspond to the natural Fock basis of particles which might be seen by asymptotic inertial observers. The Bogoliubov transformation associated to the coordinate transformation from the Kruskal gauge (gchc) to $\rho = 0$ gauge can be used to reexpress the state $|f_i^c\rangle$ in this basis. It will not in general be a simple coherent state in this asymptotic Fock basis.

If $\langle T^{++}_M \rangle$ is below threshold for black hole production, the semiclassical geometry is obtained by solving the equations (oeom) with initial conditions determined by the expectation value of (tcs) and boundary conditions of the form (rbc) suitably modified to conform to the gauge (tgc). The outgoing coherent state on $I^+$ into which $|f_i^c\rangle$ scatters is found by reflection off of this dynamically determined boundary. This will naturally lead to an expression in terms of oscillators which are positive frequency with respect to $t^+ = 0$ coordinates on $I^+$. However, in the process of solving for the semiclassical geometry, the conformal factor $\rho$ is altered on $I^+$. Thus a Bogoliubov transformation is required to obtain the outgoing state in a natural Fock basis on $I^+$.

Let us now consider further acting on the state $|f_i^c\rangle$ in (chst) with order $N^0 f$ creation operators. To leading order in $1/N$, this will not affect the stress tensor (texp) and the semiclassical geometry and boundary curve will be unaltered*. The outgoing state is then determined by boundary reflection and Bogoliubov transformation. In this manner the $S$-matrix for states obtained by acting with order $N^0$ oscillators on (chst) can be constructed.

If $\langle T^{++}_M \rangle$ exceeds the critical threshold, then the boundary curve will have a singular spacelike segment. However, the incoming state on $I^-$ will still scatter (given the endpoint prescription of section 2) to a definite out-state on $I^+ \cup \mathcal{H}^+$, where $\mathcal{H}^+$ is the future event horizon. States obtained by acting on the incoming coherent state with order $N^0 f$-oscillators will scatter to a definite pure out-state on $I^+ \cup \mathcal{H}^+$. These out-states are of the general form

$$\Psi_{out} = \sum_{x,\alpha} \Psi_{x\alpha} |x\rangle |\alpha\rangle$$

(sdec)

where $|x\rangle(|\alpha\rangle)$ is a state on $I^+$ ($\mathcal{H}^+$). The “$S$-matrix” for such states has the form

$$\langle x|\langle \alpha|S|a\rangle = S_{x\alpha}^a.$$  

(smat)

* Energy conservation should be restored by including the back reaction at the next order in $1/N$. 

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In this expression, $|a\rangle$ denotes an incoming state which collapses to form a black hole, $|\alpha\rangle$ denotes a state on $\mathcal{H}^+$, and $|x\rangle$ denotes a state on $\mathcal{I}^+$ which results from black hole evaporation—including a thunderpop.

The RST dollar matrix can now be constructed by tracing over states on $\mathcal{H}^+$

$$(S_{rst})_{x}^{y} = S_{x}^{+y\alpha}S_{x\alpha}^{a}.$$  \hfill (drst)

$S_{rst}$ conserves probability

$$(S_{rst})_{x}^{b} = \delta_{b}^{a}.$$  \hfill (pcon)

However, it is not the product of two unitary matrices, and pure states will evolve into mixed ones.

4. CPT INVARIANCE

It is easy to see that $S_{rst}$ is not, on its own, weakly CPT invariant. The CPT transform of the incoming states which collapse to form a single black hole will never appear as an out-state on $\mathcal{I}^+$:

$$\langle x|\Theta|a\rangle = 0.$$  \hfill (hrth)

This is because the out-states all contain planckian thunderpops where $\langle T_{M}^{M} \rangle$ diverges. Also $\langle T_{M}^{M} \rangle$ always exceeds the critical value somewhere on $\mathcal{I}^-$ (otherwise a black hole would not form), but $\langle T_{M}^{M} \rangle$ will generically not exceed the critical value on $\mathcal{I}^+$ (otherwise a white hole would be in the past of $\mathcal{I}^+$). The orthogonality condition (hrth) between in and out Hilbert spaces immediately precludes the possibility of weak CPT invariance[wald].

Stated in this manner, the remedy is obvious. One must include in the incoming Hilbert space thunderpop states which create white holes, as illustrated in figure 2. One may then construct the large-$N$ $S$-matrix

$$S_{cpt} = S_{rst} + \Theta S_{rst}^{T} \Theta.$$  \hfill (sccpt)

The first term acts on states which collapse to form black holes, while the second acts on thunderpop states which create white holes. It is easy to check, in bases for which $\Theta^{-1} = \Theta^{T}$, that $S_{cpt}$ obeys the weak CPT condition (wcpt) as well as probability conservation (pcon).
Physically, the second term in (sccpt) can be viewed as follows. A thunderpop carrying negative, divergent $\langle T^M_{++} \rangle$ arrives at the boundary curve, forcing it to turn left (away from spatial infinity) and become spacelike. The future of this point will then depend on boundary conditions along the null surface $\mathcal{H}^-$. One must sum over all possible states on $\mathcal{H}^-$, and construct a $\mathcal{S}$-matrix by tracing over these states. The measure for this trace is fixed by weak CPT invariance.

Note that the existence of the thunderpop appears crucial to the restoration of CPT invariance. If the black holes just quietly disappeared into the vacuum, it is hard to see how the CPT reverse of the outgoing state could form a white hole. The thunderpop is required to nucleate the white hole into which the CPT-reversed Hawking radiation subsequently falls. This makes it difficult to see how CPT could be realized in a long-wavelength effective theory in which thunderpops are integrated out. It would certainly be interesting to understand the analog of thunderpops in four dimensions. CPT would seem to require that there is something which can nucleate a white hole. Perhaps planckian negative energy densities are necessary for four as well as two-dimensional white hole nucleation.

The $\mathcal{S}$-matrix is naturally represented in the language of functional integrals. The scattering amplitude from a coherent state $|f(I^-)\rangle$ on $I^-$ which collapses to form a black hole to an out-state $|f''(\mathcal{H}^+), f'(I^+)\rangle$ is given by

$$S(f''(\mathcal{H}^+), f'(I^+); f(I^-)) = \int_{f(I^-)} f'(I^+), f''(\mathcal{H}^+) \mathcal{D}f e^{-iS}, \quad (sfnt)$$

where the boundary conditions are those appropriate to coherent states. The primes on the boundary data $f$ indicate that they are generally different functions. The matrix element $\mathcal{S}_{rst}$ between an initial coherent density matrix $|f\rangle \langle \tilde{f}|$ and a final coherent density matrix $|\tilde{f}'\rangle \langle f'|$ is then obtained by multiplying (sfnt) by its complex conjugate, and functionally integrating over $f''(\mathcal{H}^+)$ (i.e., tracing over states on the horizon)

$$\mathcal{S}[f'(\mathcal{I}^+), f'(I^+); f(I^-), f(I^-)]$$

$$= \int \mathcal{D}f'' \int_{f(I^-)} f'(I^+), f''(\mathcal{H}^+) \mathcal{D}f e^{-iS} \int_{\tilde{f}(\mathcal{I}^-)} f'(\tilde{I}^+), f''(\tilde{\mathcal{H}}^+) \mathcal{D}\tilde{f} e^{iS}. \quad (ssfnt)$$

The integration over common values of boundary data $f''$ on $\mathcal{H}^+$ and $\tilde{\mathcal{H}}^+$ sews together the two path integrals along the horizons. This may be represented as a single functional integral on the unusual semiclassical geometry depicted in figure 3 (corresponding to white
and black hole spacetimes sewn together at the horizons):

$$\mathcal{S}[\tilde{f}'(\tilde{I}^+), f'(I^-); \tilde{f}(\tilde{I}^-), f(I^+)] = \int_{f(I^-), \tilde{f}(\tilde{I}^+)}^{f'(I^+), \tilde{f}'(\tilde{I}^-)} \mathcal{D} f e^{-iS}. \quad (ggg)$$

It should be understood in (ggg) that the time flows forward from $\tilde{I}^+$ to $\tilde{I}^-$ and the sign of the action is reversed in the double of the original spacetime.

Let us summarize the rule for computing the large-$N$ dollar matrix element $\mathcal{S}[\tilde{f}'(\tilde{I}^+), f'(I^-); \tilde{f}(\tilde{I}^-), f(I^+)]$ for the case in which $|f\rangle$ and $|\tilde{f}\rangle$ both collapse to form a black hole, and $|f'\rangle$ and $|\tilde{f}'\rangle$ are both outgoing thunderpop states. The first step is to determine the semiclassical geometry. To do so, construct the stress tensor corresponding to $f(I^-)$, and determine the incoming geometry by solving the constraints. Then solve the semiclassical equations of motion with this initial data. This will determine the full semiclassical spacetime geometry and field configurations, including $f''(H^+)$ and $f'(I^+)$. The outgoing state is (in $t_+ = 0$ gauge) the coherent state associated to this semiclassical data. The large-$N$ approximation to the $\mathcal{S}$-matrix vanishes unless the semiclassical field configuration $f'(I^+)$ so obtained agrees with its argument because coherent states constructed with different $c$-number functions are orthogonal at large $N$. Next, solve the constraints using $\tilde{f}'(\tilde{I}^+)$ to get (CPT reversed) initial data for $\tilde{I}^+$. This initial data will produce a white hole. Use the CPT reverse of the final data $f''(H^+)$ obtained previously on $H^+$ as initial data on the white hole horizon on $\tilde{H}^+$. One may then compute $\tilde{f}(\tilde{I}^-)$ as the CPT reverse of the final data in this spacetime. The semiclassical coherent state $\mathcal{S}$-matrix element vanishes if the semiclassical data thereby obtained does not agree with its argument.

$\mathcal{S}$-matrix elements of states obtained by acting on these coherent states with order $N^0$ oscillators are now simply $S$-matrix elements (remembering to perform appropriate Bogoliubov transformations) in the sewn spacetime of figure 3.

So far we have discussed processes involving one black hole or one white hole. The construction of the $\mathcal{S}$-matrix for two black holes or the CPT reverse, two white holes proceeds similarly. Now there are two spacelike segments along which the two spacetimes must be sewn together.

Clustering however requires that we include initial states with both black holes and white holes. This can result, after sewing, in a spacetime with closed timelike loops as in figure 4. New features, which we have not yet analyzed, may arise in computing the $\mathcal{S}$-matrix from $S$-matrix elements on such spacetimes.
This prescription for computing the $S$-matrix formally generalizes to four dimensions: Form the double of a spacetime by sewing its black (white) hole horizons to the white (black) hole horizons of its CPT conjugate. The $S$-matrix is then obtained from the $S$-matrix on the doubled spacetime. This picture suggests a natural way of including the effects of virtual black holes: they are simply wormholes which connect the original spacetime to its double. For the usual reasons [worm], this will not lead to quantum incoherence, but rather will turn coupling constants into dynamical variables. The only slight difference is that the wormhole $\alpha$-parameters label $S$-matrix rather than $S$-matrix superselection sectors. This leads us to question the assertion [hwktwo, bps] that information loss from real black holes inevitably implies that virtual black holes lead to tiny unitarity violations in a long distance effective theory.

5. SOME PROBLEMS

In this paper we have described the black and white hole superscattering matrix to leading order in a $1/N$ expansion. Going beyond leading order may not be straightforward for several reasons.

The first is that the RST boundary condition - which entered crucially into our discussion - has only been fully defined at the semiclassical level. Heuristically one might hope to define the full quantum theory by restricting the range of functional integration of the field $\Omega$. However it is not clear what this means in practice, or if the resulting theory will conserve energy. Furthermore virtual fluctuations of $\Omega$ about its vacuum value will lead to internal boundaries, or holes in spacetimes. These difficulties will certainly arise at the non-perturbative level, and perhaps even in $1/N$ perturbation theory.

A second problem is the white hole instabilities discussed in [wbld]. This problem has also been stressed - in a slightly different form - in more recent papers [brt,verl]. Consider a particular white hole formation/evaporation process obtained as the time reverse of black hole formation/evaporation, projected on to some particular pure state on $\mathcal{H}^-$ and $\mathcal{I}^-$. Now consider adding a single incoming $f$ quanta to the state on $\mathcal{I}^-$ somewhere near or shortly after the incoming thunderpop. It is straightforward to show that the energy (i.e., $T^M_{++}$) of this state is blueshifted by an amount of order $e^{M/\lambda}$, where $M$ is the white hole mass. The energy of the quanta is then of order $e^{M/\lambda}/N$.

In the large-$N$ limit the back reaction of this quanta can be neglected. However if $N$
is large but finite and $M$ is greater than $\ln N$, the back reaction is not small. The $1/N$ expansion presumably breaks down for such matrix elements at such large values of $M$.

Physically one expects, for sufficiently large $M$, that the blueshifted quanta will lead to a black hole after the white hole. Thus beyond the $1/N$ expansion one does not expect in general to always be able to determine the numbers of black and white holes from the state on $\mathcal{I}^-$ alone: knowledge of the state on $\mathcal{H}^-$ is also required. This will bring new features into the analysis, and we do not know how one develops a systematic perturbation expansion in this situation.

In conclusion, the inclusion of white holes restores CPT to leading order in a $1/N$ expansion for processes with one white hole or one black hole. The general case remains unresolved. We hope to have convinced the reader that it is an interesting problem.

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APPENDIX

In this appendix we will describe the RST thunderpop \cite{rst} as a quantum state of $f$-particles.

For an incoming shock wave $\lambda(T^M_{++} - t_0^0) = M\delta(x^+ - 1)$, the field $\Omega$ in Kruskal gauge is given by

$$\Omega = -\lambda^2 x^+(x^- + M^2) + \frac{M}{\lambda}$$

$$-\frac{1}{4}\ln[-4\lambda^2 x^+ x^-],$$

in the region above the shock wave at $x^+ = 1$, where $\Omega > \Omega_{cr}$ and prior to the thunderpop.
The thunderpop originates at the black hole endpoint

\[(x_s^-, x_s^+) = \left(- \frac{M}{\lambda^3(1-e^{-4M/\lambda})}, \frac{\lambda}{4M}(e^{4M/\lambda} - 1)\right), \quad (xs)\]

where the apparent horizon and singularity meet. For \(x^- > x_s^-\), the solution is joined onto the shifted linear dilaton vacuum

\[\Omega = -\lambda^2 x^+(x^- + \frac{M}{\lambda^3}) - \frac{1}{4} \ln[-4\lambda^2 x^+(x^- + \frac{M}{\lambda^3})]. \quad (glp)\]

\(\Omega\) is continuous along the thunderpop at \(x^- = x_s^-\). \((glp)\) and \((gsolp)\) obey the constraints \((cstr)\) with

\[
t_- = -\frac{1}{4x^-} \quad x^- < x_s^-, \quad (zzz)
\]

\[
t_- = \frac{\lambda^3 e^{4M/\lambda}}{4M}(1-e^{-4M/\lambda})^2 \delta(x^- - x_s^-) \quad x^- = x_s^-, \quad (zzz)
\]

\[
t_- = -\frac{1}{4(x^- + \frac{M}{\lambda^3})} \quad x^- > x_s^- . \quad (zzz)
\]

The metric corresponding to \((glp)\) and \((gsolp)\) is asymptotically flat on \(I^+\). Quantum states on \(I^+\) are best described in terms of creation and annihilation operators defined with respect to the natural flat coordinates. Defining

\[
y^- = -\ln \left[\frac{\lambda^3 x^- + M}{\lambda^3 x_s^- + M}\right], \quad (xxyy)
\]

\[
y^+ = \ln(\lambda x^+), \quad (sy)
\]

one finds that asymptotically on \(I^+\)

\[ds^2 \rightarrow -dy^+ dy^- . \quad (sy)
\]

In these coordinates, the radiation flux on \(I^+\) is given by \(t_-\). To determine \(t_-\), note that the constraints in a general gauge are

\[2(e^{-2\phi} + \frac{1}{4}) \nabla^-_2 \phi = \frac{1}{2} \sum_{i=1}^N \partial_- f_i \partial_- f_i + \frac{N}{12}(t_- - \partial_- \rho \partial_- \rho + \partial_-^2 \rho), \quad (gcst)
\]

where \(\nabla^-\) is the covariant derivative. Both sides of this equation must transform covariantly under a conformal coordinate transformation \(\bar{x}^- = \bar{x}^-(x^-)\). This requires that

\[(\partial_- \bar{x}^-)^2 t_- = t_- - (\partial_- \bar{x}^-)^{1/2} \partial^-_-(\partial_- \bar{x}^-)^{-1/2}. \quad (ttr)\]
One then finds that in $y$ coordinates
\[
    t_- = \frac{2e y^- (e^{4M/\lambda} - 1) + e^2 y^- (e^{4M/\lambda} - 1)^2}{4(1 + e^{y^-} (e^{4M/\lambda} - 1))^2} \quad y^- < 0, \\
    t_- = -\frac{1}{4} (1 - e^{-4M/\lambda}) \delta(y^-) \quad y^- = 0, \\
    t_- = 0 \quad y^- > 0.
\]

A thunderpop state $|T\rangle$ clearly can not be described as a coherent state in the Fock basis associated to $y$ coordinates because the energy density of such states is non-zero only when $\langle f_i \rangle$ is non-zero. To represent $|T\rangle$ as a coherent state, we must find a coordinate system in which $t_-$ vanishes. This is accomplished by solving (ttr) with the left hand side set to zero and $t_-$ given by (yyy). One finds that $t_-$ vanishes in $z$ coordinates
\[
    z^+ = y^+, \\
    z^- = -\ln \left[ e^{-y^- -4M/\lambda} + 1 - e^{-4M/\lambda} \right] \quad y^- < 0, \\
    z^- = \frac{4y^- e^{-4M/\lambda}}{4 + (1 - e^{-4M/\lambda}) y^-} \quad y^- > 0.
\]

A thunderpop may then be represented as the $z$ vacuum
\[
    |T\rangle = |0_z\rangle,
\]
where $|0_z\rangle$ is annihilated by modes of the $f$ field which are negative frequency with respect to $z^-$. To represent this state in the $y$ Fock basis - which corresponds to particle states detected by inertial asymptotic observers - one must perform the Bogoliubov transformation corresponding to (zzyy). A nice discussion of these transformations in two dimensions can be found in [gine].

Note that an isolated thunderpop - unaccompanied by nearby positive energy density - can not be represented by a quantum state as in (tpop). The analog of the $z$ coordinates in which $t_-$ vanishes would not cover all of $\mathcal{I}^+$. Thus the global existence of $z$ coordinates is an important consistency check for the RST model.

Finally, we would like to remind the reader that the state (tpop) is not the full outcome of shock wave collapse. The state on $\mathcal{I}^+$ will be a mixed state, or density matrix. To find that density matrix one must first compute the Bogoliubov transformation from the incoming pure state on $\mathcal{I}^-$ to the outgoing pure state on $\mathcal{H}^+ \cup \mathcal{I}^+$, and then trace over states on $\mathcal{H}^+$. (tpop) is presumably one of the pure states which appears on $\mathcal{I}^+$ with some probability.
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FIGURE 1. A shock wave collapses to form a black hole. The final mixed state consists of outgoing Hawking radiation and a thunderpop. The region of future trapped points is shaded.
FIGURE 2. The CPT reverse of black hole formation/evaporation depicted in figure 1 is a white hole which is nucleated by an incoming thunderpop and subsequently evaporates.
FIGURE 3. A doubled spacetime is formed by sewing the future horizon of figure 1 to the past horizon of figure 2. Superscattering matrix elements are obtained from S–matrix elements on this doubled spacetime. The fat arrows denote the direction of time flow.
FIGURE 4. The doubled spacetime appropriate to superscattering processes with one white and one black hole. The two horizons connected by the fat arrow are identified.