Vacuum Cherenkov effect in logarithmic nonlinear quantum theory

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We describe the radiation phenomena which can take place in the physical vacuum such as Cherenkov-type shock waves. Their macroscopical characteristics - cone angle, flash duration, radiation yield and spectral distribution - are computed. It turns out that the radiation yield is proportional to the square of the proper energy scale of the vacuum which serves also as the vacuum instability threshold and the natural ultraviolet cutoff. While the analysis is mainly based on the theory engaging the logarithmic nonlinear quantum wave equation, some of the obtained results must be valid for any Lorentz-invariance-violating theory describing the vacuum by (effectively) continuous medium in the long-wavelength approximation.

PACS numbers: 04.60.Bc, 41.60.Bq, 04.70.Dy, 98.70.Sa

1. INTRODUCTION

Current observational data in astrophysics seem to indicate the existence of the deviations from the classical theory of relativity. In absence of a fully satisfactory axiomatic theory explaining them, there appeared numerous non-axiomatic theories and effective approaches broadly referred as the effective quantum gravity theories. One of the candidate theories has been proposed in based on the nonlinear logarithmic quantum mechanics which is still a subject of intensive study nowadays, in particular, in the connection with the quantum locality issues. The idea was alternatively formulated on the field-theoretical language in the subsequent paper. There the necessity of introducing the universal nonlinearity in the quantum wave equation was explained by the arguments that the physical vacuum is a kind of the non-removable background Bose liquid or Bose-Einstein condensate (BEC) located in a fictitious Euclidean space. According to the superfluid vacuum approach, Lorentz symmetry is an emergent low-energy phenomenon (on a vacuum energy scale), and the Standard-Model particles and gravity can be treated as the small fluctuations of the non-relativistic background superfluid.

As long as we take the point of view that the physical vacuum can be described as the nontrivial medium which is continuous in the long-wavelength approximation, one of the predictions of the phenomenological approach comes by analogy with the (Vavilov-)Cherenkov effect. The latter is known to happen in the conventional materials because the phase velocity of light in those media is less than the fundamental velocity, and particles of non-zero rest mass can propagate faster than photons. Unlike other associated radiation phenomena, such as Bremsstrahlung, Cherenkov radiation is the collective response of the whole medium which is essentially universal (in particular, material-independent), polarized and directed along the beam, also its spectrum is continuous with maximum of intensity shifted to the higher-frequency (“ultraviolet”) side.

2. DEFORMED DISPERSION RELATIONS

The classical theory of relativity clearly forbids any superluminal phenomena: a particle with non-zero rest mass can reach speed of light only at infinite energy (besides, the nontrivial vacuum itself would create a preferred frame of reference, in violation of one of the relativistic postulates). On the contrary, in the Lorentz-invariance violating (LIV) theories the dispersion relations alter hence the vacuum Cherenkov radiation is not excluded indeed. For instance, in the theory the (velocity) dispersion relation for the particles approaching the speed-of-light barrier is derived as

\[ \frac{v}{c} = \left[ 1 + \mu \left( 1 - \frac{E}{E_0} \right)^2 \right]^{-1/2}, \]

where \( \mu \ll 1 \) is the emerging effective parameter which by construction does not depend on energy of a particle but may vary for different species of particles \( E \) is the energy of a particle, and \( E_0 \) is the proper energy of vacuum (we remind that it is not necessary positive). Thus, below we shall describe a formal experiment where assume that physical particles (including photons) obey dispersion relations of the type, differing only by the values of the parameters \( \mu \).

Before proceeding any further, we define what is meant by the speed of light in our case. The old definition, the fundamental constant \( c = 299792458 \text{ m/s} \), becomes now

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1 In literature the distinct transliteration, Čerenkov, is often used. It is actually a misnomer because checked chars exist neither in Russian nor in Latin alphabets.

2 We emphasize that this dispersion relation is valid in the vicinity \( v \approx c \) and for non-zero \( \mu \) only. In the classical limit the vacuum becomes trivial, \( \mu \) must be set to zero (since its origin is essentially quantum), and the relation turns into an identity.
a unit conversion number which refers to the (approximate) speed of the photon whose energy is very small comparing to \(|E_0|\) - because this is how this speed was measured in past. The genuine physical velocity of photons \(c_n\) is energy-dependent and determined by Eq. (1),

\[
c_n/c = \left[1 + \mu_\gamma \left(1 - \frac{E_n}{E_0}\right)^2\right]^{-1/2},
\]

where \(E_n\) is the photon’s energy, and the parameter \(\mu_\gamma\) must be positive for the restriction \(c_n \leq c\) to hold.\(^3\) From these dispersions one can immediately deduce that the behavior of the velocity of a particle in the vicinity of either of speed-of-light points is regular. This essentially means that the “barriers” corresponding to the physical speed of light \(c_n = c/n\) and fundamental velocity \(c\) are not infinite anymore and the velocity of the particle of sufficiently high yet finite energy can reach these values exactly. For the classical particle the required energy scale would be about \(E_0\) which can take any value up to the Planck scale. However, quantum particles can penetrate these barriers at lower energies, and the probability of this grows along with the particles’ energy and velocity. According to the above-mentioned BEC interpretation, \(E_0\) is a critical scale at which the fundamental background becomes unstable against the phase transition where the physical degrees of freedom alter. In practice this instability should most probably cause the quantum-many-body effects, such as the vacuum polarization and creation of pairs, which would drain energy away. In that case we do not have a single-particle problem anymore, also we cannot neglect the particle’s velocity change.

As long as the primary target of our paper is the Cherenkov radiation, in what follows by the speed-of-light barrier we will understand the \(c_n\) one, all particles are assumed to be in the subluminal mode, \(c_n \leq v \leq c\), as defined in Ref. [2]. As for the superluminal particle which crosses the \(c_n\)-barrier while decelerating then the physical picture is not clear yet, as one should deal with the problem in an essentially many-body way [2].

To proceed with derivation of the properties of the Cherenkov radiation we first recall that among all nonlinear extensions of quantum mechanics it is only the logarithmic one which jointly conserves two physically very important features of the conventional (linear) quantum mechanics. The first property is the additivity of energy for uncorrelated systems, and the second one is that the Planck relation, \(E = h\omega\), holds for stationary states [3]. Using the latter and Eq. (2) one can immediately write down an expression for the effective refractive index in the Cauchy form

\[
n^2 = 1 + \mu_\gamma \left[1 + \mathcal{M}(\omega)(\omega/2c)^2\right],
\]

where the parameter \(\mu_\gamma\) can be thus interpreted as indeed the constant of refraction of the physical vacuum, \(\mathcal{M}(\omega) = \frac{4\pi\epsilon_0^2}{\omega^2} (1 + 2\xi\omega)\) is the effective dispersion coefficient, \(\omega = E_\gamma/h\) is the angular frequency of the electromagnetic wave, \(\omega_0 = |E_0|/h\) is the natural frequency of the vacuum, and \(\xi = -\text{sign}(E_0)\). All this means that both the elementary particles and electromagnetic waves propagating through the physical vacuum get affected by it, and once again confirms that the physical vacuum is a medium with non-trivial properties.

As long as the Cherenkov effect is an essentially macroscopic long-wavelength phenomenon (the particles’ Compton wavelengths are much larger than the characteristic size \(\ell_0 = hc/|E_0|\)), its main properties can be easily computed just using the dispersion relations above.

### 3. Cherenkov Cone Angle

We consider the following physical setup: a particle moving with speed \(v\), \(c_n \leq v < c\), momentum \(p\) and energy \(E\) emits at some point the photon with energy \(E_\gamma\), momentum \(p_\gamma\), and velocity \(v_\gamma\). After this event the particle acquires speed \(v'\), momentum \(p'\) and energy \(E'\).

Before and after the moment of emission the particles are assumed to be uncorrelated. Then, due to the above-mentioned energy additivity property we can write the energy conservation law in the standard form, \(E' = E - E_\gamma\), where the photon energy can be expressed as \(E_\gamma = hc/\lambda\) with \(\lambda\) being the photon wavelength. For momenta we can write the standard conservation law as well if we work in the reference frame where the momentum of the background is set to zero, and obtain the standard expression for the conical angle: \(\cos \theta = 1 - \frac{p'^2 - (p - p_\gamma)^2}{2p p_\gamma}\) provided \(p < p' + p_\gamma\).

Further, for future it is convenient to introduce the following dimensionless quantities \(M = v/c\), \(M' = v'/c\), \(M_\gamma = c_n/c = 1/n\), \(\epsilon = E/E_0\), \(\epsilon' = E'/E_0\), \(\epsilon_\gamma = E_\gamma/E_0 = (hc/\lambda)/E_0\), \(\sigma = \Lambda/\lambda\), and their combinations such as the inverse Lorentz factors \(\Gamma = \sqrt{1 - M^2}/M\), \(\Gamma'\), \(\Gamma_\gamma\), etc. Then the velocity dispersions for our setup can be written as \(M = 1/\sqrt{1 + \mu (1 - \epsilon)^2}\), \(M' = 1/\sqrt{1 + \mu' (1 - \epsilon')^2}\) and \(n = \sqrt{1 + \mu_\gamma (1 - \epsilon_\gamma)^2}\). Further, as long as \(M\) and \(M'\) refer to the same particle in similar physical condition we must impose \(\mu' = \mu\). From which we obtain the velocity transformation formula

\[
M' = \left[1 + \Gamma^2 \left(\frac{1 - \epsilon'}{1 - \epsilon}\right)^2\right]^{-1/2},
\]

which can be used to eliminate \(M'\) where necessary. Then, using the momentum dispersion relations one can...
write the cone angle in the form
\[ \cos \theta = 1 - \frac{P^2(x, y)}{2pP(x, y)} - \frac{p - P(\epsilon, \Gamma)}{2pP(\epsilon, \Gamma)}, \]
where we defined the function \( P(x, y) = E(x, y) \cdot \frac{\tau(x, y)}{\tau(y)} \) assuming \( \tau(x) = x \sqrt{1 + x^2 + \arcsinh x} \).

Using the formulae written above, it is straightforward to write down the exact expression for the cone angle as a function of \( M, n, \sigma \) and \( \epsilon_c \). In general, this expression is quite bulky (in particular, it involves the solving of the transcendental equation for \( \epsilon(p) \) if one wants to obtain an expression in terms of de Broglie’s wavelengths) and thus it is suitable more for a numerical analysis. For analytical purposes, one can write it in a perturbative form, using the smallness of the parameters \( \eta_1 = 1 - M^2 \) and \( \eta_2 = 1 - M^2 = (n^2 - 1)/n^2 \). This approximation is valid as long as all the velocities in the problem are close to \( c \). We obtain
\[ \cos \theta = \frac{1}{Mn} + \Theta_\eta, \]
where \( n \) is given by Eq. (3), and by \( \Theta_\eta \) we denote the correction term
\[ \Theta_\eta = \frac{1}{2} \eta_\gamma \sigma + \frac{1}{3} \eta_\gamma \epsilon_c \left( \epsilon_c - 3/2(1 - \sigma) \right) + \frac{1}{2} \frac{\sigma^3 - (\sigma^3 - 3\sigma^2 + 6\sigma - 2)\epsilon_c + \frac{1}{3} (\sigma^2 - 4\sigma + 6) \epsilon_c^2}{(\sigma - \epsilon_c)^2} + O(\eta_\gamma^2, \eta_\sigma, \eta_\gamma^2), \]
and it is implied that \( \Theta_\eta \ll 1/(Mn) \), of course. The latter condition obviously fails when \( \sigma \) approaches \( \epsilon_c \).

The relations \( \sigma = \epsilon_c \) and \( \epsilon_c = 1 \) are the horizon-type resonance conditions which can be fulfilled only when the particles’ Compton wavelengths become comparable to the wavelength of the physical vacuum do, see also the Footnote 2.

4. FLASH DURATION

In the non-dispersive medium the wavefront of the Cherenkov shock is infinitely thin, therefore, the light pulse an observer sees when the wave hits a detector has an infinitely short duration. However, as long as our vacuum is a dispersive medium, the cone angle is different for different wavelengths. Therefore, an observer tuned to the frequency band \( [\omega_1, \omega_2] \) will see the light flash with a finite duration \( \Delta t = \frac{\rho}{c} (\tan \theta(\omega_2) - \tan \theta(\omega_1)) \), where \( \rho \) is a distance from the axis of particle’s trajectory. Using the expression for the cone angle derived above, we obtain
\[ \Delta t \approx \frac{\rho}{c} \sqrt{\frac{\omega_0}{\omega_1 - \sigma}} \left( \frac{1}{\sqrt{\omega_1}} - \frac{1}{\sqrt{\omega_2}} \right) \left( 1 - \frac{\mu \gamma}{4} \right), \]
where we neglected terms of the order \( O(\mu_3/2, \eta_3/2, \eta_3/2, \sqrt{\omega_1/\omega_0}, \sqrt{\omega_2/\omega_0}) \). The last term again consists of two contributions - due to not only the Cherenkov wave but also the emitting particle are affected by the vacuum.

5. ENERGY AND SPECTRAL DISTRIBUTION

As long as the energy of the Cherenkov photon is small compared to the natural vacuum energy scale one can treat the problem in a linearized way where the vacuum effects are taken into account via the nontrivial refraction index. By doing that we are neglecting the microscopical structure of the vacuum which makes sense as long as the frequency of the electromagnetic wave is smaller than the frequency of the vacuum oscillations \( \omega_0 \). In other words, as a leading approximation we consider the Frank-Tamm approach with \( n \) given by Eq. (3). Following the method, we assume that the \( \omega \)-Fourier images of the vector and scalar potential of the electromagnetic wave emitted by a charge \( Q \) moving at the speed \( v = \text{const} \) along \( z \)-axis are obeying the macroscopic Maxwell equations in the medium
\[ \left( \nabla^2 + \frac{\omega^2 \rho^2}{c^2} \right) A_\omega = -\frac{2Q}{c} e^{-i\omega z/v} \delta(x) \phi(y) e_y, \]
\[ \left( \nabla^2 + \frac{\omega^2 \rho^2}{c^2} \right) \phi_\omega = \frac{4\pi}{\hbar^2} \rho, \]
\[ \nabla \times A_\omega = -(1/c) \partial_t A_\omega - \nabla \phi_\omega. \]

Introducing the cylindrical coordinates \( \rho, \phi \) and \( z \), we assume the (Fourier image of) vector potential in the form \( A_\rho = A_z = 0 \) and \( A_z = u(\rho)e^{-i(\omega t - \omega z/\nu)} \) where \( u(\rho) \) obeys the differential equation
\[ u'' + 2i\nu u' - \kappa u = \frac{Q_\rho}{\pi \nu c} \delta(\rho), \]
where \( \kappa = (\omega/v)^2 (1 - M^2 n^2) \).

This equation can be replaced by the homogeneous one if we impose the singular boundary condition in the origin: \[ \lim_{\rho \to 0} \frac{d^2}{d\rho^2} u(\rho) = -Q/(\pi c). \]

Now, if a charge moves slower than light then \( \kappa \) is positive and the solution exponentially decreases with \( \rho \). Otherwise the solution has an oscillating behavior at large \( \rho \), \( A_\rho \propto -\frac{Q}{2\pi \nu c} \exp[\{i\omega(t - z/\nu) + i(\kappa + 3\pi/4)] \), which indicates the Cherenkov wave’s existence.

The total energy radiated by the charge through the cylindrical surface of length \( l \) whose axis coincides with the charge’s trajectory is given by \[ W = \frac{1}{2} \varepsilon c l_0 \int_{-\infty}^{+\infty} dt \int_{M \geq 1} d\omega d\omega' e^{i(\omega + \omega')t} \left| \mathbf{E}_\omega \times \mathbf{H}_\omega' \right|, \]
where the integration over the frequencies must be performed only for those values at which the charge’s velocity is larger than \( c_n \) but smaller than \( c \). According to dispersion relations, the latter bound imposes the natural ultraviolet cut-off \( E_0/h = -\omega_0 \) such that one does not need to postulate it separately, in contrast to the conventional materials.

By introducing the variable \( x = -\xi \omega/\omega_0 \) we can do both cases \( \xi = \pm 1 \) in a uniform way. The radiation energy per unit path is given by the Frank-Tamm spectral distribution
\[ \frac{dW}{dl} = \frac{\mu \gamma^2 Q_\rho^2}{c^2} \int_0^1 dx \frac{x(x - 1)^2}{\mu \gamma(x - 1)^2 + 1} + O(\eta, \hbar), \quad (11) \]
with the integrand having a local maximum at \( x_{\text{peak}} = 1/3 + O(\mu \gamma) \). Thus, the radiation yield produced by the moving charge \( Q \) amounts to
\[ \frac{dW}{dl} = \frac{\mu \gamma}{3} \left( \frac{c N E_0}{2 \hbar} \right)^2 + O(\eta, \hbar, \mu \gamma), \]
where \( N = Q/e \), \( e \) being the elementary charge. Therefore, in the leading-order approximation we obtain
\[ \frac{dW}{dl} = 3 \times 10^{10} \mu \gamma N^2 E_0^2 \text{GeV}^{-1}\text{cm}^{-1}. \quad (12) \]

Looking at these equations we can immediately notice that the value of the vacuum energy enters the picture in a crucial way. As a matter of fact, the main contributing prefactor, \( \omega_0^2 \sim E_0^2 \), is inherent in the theory of the Cherenkov radiation in effectively continuous media. Moreover, its appearance weakly depends on a specific form of the refractive index - as long as the latter contains the ultraviolet cut-off frequency. Therefore, this factor should necessarily appear in a very large class of theories with the ultraviolet cutoff being determined by the energy of the vacuum. The non-local nature of the superfluid vacuum as an extended object (i.e., non-point-like and possessing internal structure) makes the quantitative properties of the Cherenkov effect in theories with the BEC-type vacuum being different from those in some other LIV theories. Our results are more close to the predictions based on the general arguments about the existence of a preferred frame of reference - which is not surprising though.

The radiation yield of the Cherenkov effect in the usual materials is observed to be relatively small, few keV per cm, but only because the typical ultraviolet cut-off frequency there is tiny small - about an electron-volt per Planck. However, on a scale of the cutoff frequency the energy output is not small at all - it is at least three orders of magnitude larger than the cutoff energy: \( (dW/dl)_{N=1} \sim E_0 \times 10^3 \text{cm}^{-1} \). In vacuum the cutoff energy is higher by many orders of magnitude and also may depend on a physical setup because the background condensate gets affected by geometry of the problem and external fields acting upon which leads to the value of \( E_0 \) can differ for different physical situations (same goes about other parameters such as \( \mu \)'s).

Therefore, in absence of the proper microscopical theory of the physical vacuum, the value \( E_0 \) is difficult to compute theoretically, yet the boundaries can be established already at this stage. The upper boundary for \( |E_0| \) is, of course, the Planck energy, \( 10^{19} \text{GeV} \), the largest energy pertinent to the microworld (debates, however, continue [25]). The lower boundary, \( 10^4 \text{GeV} \), comes from current non-observability data, and thus can be significantly lifted [26]. Using these conservative values, \( 10^4 \lesssim |E_0| \lesssim 10^{19} \text{GeV} \), we give the following estimate
\[ 10^{15} \lesssim \frac{1}{\mu \gamma N^2} \frac{dW}{dl} \lesssim 10^{45} \text{TeV/cm}. \quad (13) \]

Despite the constant of refraction of the vacuum is obviously extremely small, the resulting numbers can be quite substantial - especially considering that \( N \) can be large. In that case the vacuum Cherenkov shocks turn out to be a very efficient, fast and powerful way of draining and releasing energy. This poses the question whether such processes can happen in the astrophysical objects such as super- and hypernovas [27, 28], active galactic nuclei, gamma-ray bursts and ROCSs (radio objects with the continuous optical spectra often having an abnormally strong ultraviolet part [29]).

6. \textbf{BEYOND FRANK-TAMM: "BOOM SHOCK"}

In a conventional theory of the Cherenkov effect the Frank-Tamm formula was derived assuming that the particle’s velocity is constant and any changes of it happen instantaneously. This approximation has been proven to be very robust, yet in reality the speed-of-light barrier is crossed by the particle which is either accelerating or decelerating in a smooth way. The analytical theory of the Cherenkov radiation for such cases is far from being complete, even for the case of conventional materials. There exists, however, a number of heuristic and numerical results which point at the appearance of the separate wave when the velocity of a moving charge exactly
coincides with the speed of light in the medium - the so-called Tyapkin-Zrelov-(Afanasiev) or “luminal boom” wave. In the case of an accelerating charge such wave is indistinguishable from the Cherenkov one as it just closes the cone but for the decelerating motion this wave decouples from the charge when the latter crosses the \( c_n \)-barrier while slowing down. Then this wave continues propagating independently with the velocity \( c_n \).

7. CONCLUSION

In this paper we theoretically outlined basic properties of the Cherenkov-type radiation phenomena in vacuum. It is shown that the macroscopical description of the Cherenkov radiation is based on two parameters, the constant of refraction and the cut-off frequency. From the phenomenological point of view, even in the conventional materials such parameters are quite difficult to determine theoretically but the experimental findings are greatly facilitated by the universal features of the Cherenkov radiation. The same is true for the vacuum case, moreover, when compared to other possible dissipative processes then in terms of released energy the Cherenkov effect in vacuum should play more dominant role than its condensed-matter counterpart (the latter usually accounts for less than one per cent of the energy loss by ionization). This becomes possible because the vacuum itself can be viewed as the (super)fluid with minimum dissipation.

While our study was mainly based on the theory described by the logarithmic nonlinear Schrödinger equation, some of the obtained results must be valid for any Lorentz-invariance violating theory describing the vacuum by (effectively) continuous medium in the long-wavelength approximation.

Acknowledgments

I am grateful to Eugene Tkalya and Ralf Lehner for bringing some references into my attention and fruitful comments. This work was supported under a grant of the National Research Foundation of South Africa.

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