Cosmological extrapolation of MOND

V.V.Kiselev\textsuperscript{1,2} and S.A.Timofeev\textsuperscript{1,2}

\textsuperscript{1}Russian State Research Center Institute for High Energy Physics, Pobeda 1, Protvino, Moscow Region, 142281, Russia
\textsuperscript{2}Department of Theoretical Physics, Moscow Institute of Physics and Technology (State University), Institutsky 9, Dolgoprudny, Moscow Region, 141701, Russia

E-mail: Valery.Kiselev@ihep.ru, serg_timofeev@list.ru

Abstract. Regime of MOND, which is used in astronomy to describe the gravitating systems of island type without the need to postulate the existence of a hypothetical dark matter, is generalized to the case of homogeneous distribution of usual matter by introducing a linear dependence of the critical acceleration on the size of region under consideration. We show that such the extrapolation of MOND in cosmology is consistent with both the observed dependence of brightness on the redshift for type Ia supernovae and the parameters of large-scale structure of Universe in the evolution, that is determined by the presence of a cosmological constant, the ordinary matter of baryons and electrons as well as the photon and neutrino radiation without any dark matter.

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1. Introduction

In the framework of modified Newtonian dynamics (MOND) we get quite the reasonable theoretically and successful phenomenologically explanation for the empirical Tully–Fisher law, that regularly relates the visible masses of spiral galaxies to asymptotically constant rotation-velocities of stars in the region of dominating “dark matter halo” (see figure 1) as well as for profiles of rotational velocities of stars in galaxies, wherein the regime of flattening the rotation curves does not take place. Moreover, the MOND is able to explain many other phenomena caused by the inhomogeneity of baryonic matter-distribution in the Universe at the scales of gravitationally coupled systems (see reviews in [2, 3, 4]). For instance, in disk galaxies, wherein a dominant contribution to the baryonic mass is given by the interstellar gas, the matter density is determined with a high accuracy, since an uncertainty, caused by the extraction of stellar masses from their visible magnitudes, is significantly reduced. The MOND gives a precise agreement of its predictions with the experimental data on the rotation speeds of matter around the centers of galaxies without any free parameters, while the uncertainties of results are caused by errors of measurements, only [5].

By construction, the basic conclusion of MOND is the statement that the observational data can be confidently described without any introduction of non-baryonic dark matter at the galactic scales of matter-inhomogeneity as well as at the scales of gravitationally coupled galactic clusters due to the specially fitted universal modification of the gravity law if the acceleration of free falling \( g \) is less than the critical

\[
\frac{v^4}{GM_\odot} = \frac{G}{M_\odot} \nu_0^2
\]

Figure 1. The correlation of visible baryonic masses in the disk galaxies with the rotation velocities of stars in the region of flat rotation curves, as shown in comparison with the MOND prediction: \( v^4 = GM_\odot \nu_0 \). The mass is given in units of solar mass \( M_\odot \). The figure is taken from [2].
value $\tilde{g}_0 \approx 1.2 \cdot 10^{-10} \text{m/s}^2$, so that $g^2 = GM\tilde{g}_0/r^2$, where $M$ is the matter mass, while $r$ is a distance to a reference point.

However, such the conclusion conflicts with cosmological modeling the Universe evolution and properties of its large scale structure, that is characterized by a high degree of homogeneity for the spatial distribution of matter ($\delta \rho/\rho \sim \delta T/T \sim 10^{-5}$ for fluctuations of energy density $\rho$ and temperature of cosmic microwave background radiation (CMBR), $T$), because such calculations lead to the necessary introduction of dark matter with the density being approximately 5 times greater than the baryonic density [6], if we follow the equations of general relativity (GR).

In cosmology of homogeneous and isotropic Universe, the points in the space with coordinates $r$ move as given by a dependence of scale factor on the time, $a = a(t)$, so that velocity $v$ and acceleration $g$ of material point are determined by

$$x = a(t)r, \quad v = \dot{x} = \dot{a}r, \quad g = \ddot{x} = \ddot{a}r,$$

(1)

where the dot over a symbol denotes the derivative with respect to time. Due to the homogeneity and isotropy, the evolution equations do not contain the co-moving coordinate $r$. Therefore, the evolution law is universal, and it can be considered at small $r$, i.e. in the region of applicability of the non-relativistic mechanics ($v \to 0$) and gravity law by Newton, namely, the acceleration of free falling along the radius vector $g = \ddot{a}r$ reads off

$$g = -G \mathcal{M}_{KT}/|x|^2,$$

(2)

where we introduce the Komar–Tolman gravitational mass for a spherically symmetric distribution of the matter with the energy density $\rho$ and pressure $p$ inside the sphere with radius $|x|$: $\mathcal{M}_{KT} = \frac{4\pi}{3} (\rho + 3p) |x|^3$, (3) so that

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p),$$

(4)

that naturally does not include the co-moving coordinate $r$.

In addition, the adiabatic law of energy conservation for the substance with the energy density $\rho$ and pressure $p$ reads off

$$dE = -p dV$$

(5)

so under the substitutions of $E = \rho V(t)$, $V(t) = a^3 V_0$, we get

$$\dot{\rho} + 3H (\rho + p) = 0,$$

(6)

† The source of gravitational field in Einstein’s equations is the tensor of $2(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$, where the symbol $T$ corresponds to the tensor of energy-momentum and its trace, and $g_{\mu\nu}$ denotes the metric. Therefore, the static gravitational potential is determined by not the energy density alone, but by the temporal component of the source. The integration of temporal component over the volume gives the Komar–Tolman mass, exactly.
Then, after multiplying (4) by $2\ddot{a}a$ we can easily integrate and get the Friedmann equation
\[ H^2 = \frac{8\pi G}{3} \left( \rho + \frac{\rho_E}{a^2} \right), \] (7)
where we have introduced the Hubble constant $H = \dot{a}/a$, while in GR the constant of integration $\rho_E$ is presented by the sum of contributions given by a constant spatial curvature and a matter with equation of state $p = -\frac{1}{3}\rho$. These terms do not contribute to the Komar–Tolman mass, i.e. they do not produce the gravitational force.

Thus, we get the equations of GR for the evolution of homogeneous and isotropic Universe (6) and (7). Then, the description of such evolution is reduced to the non-relativistic dynamics at $r \to 0$.

The straightforward application of MOND paradigm at $r \to 0$ implies the transition to the regime of modified law of gravity, when
\[ g \mapsto g \left| \frac{g}{g_0} \right| , \quad \text{at} \quad |g| \ll g_0, \] hence,
\[ \frac{\ddot{a}}{a} |\ddot{a}| = -\frac{4\pi G}{3} (\rho + 3p) \frac{g_0'}{r}. \] (8)
Notice that a direct observation of the scale factor evolution by the registration of type Ia supernovae \[7, 8, 9, 10, 11\] reliably shows that the accelerated Universe expansion ($\ddot{a} > 0$) at present has changed the decelerated expansion ($\ddot{a} < 0$), which took place at the redshift $z > z_t$, where $z_t \approx 0.4 - 1.0$ is extracted in the procedure of fitting the deceleration parameter $q(z) = -\ddot{a}/(aH^2)$ by the linear function $q(z) \mapsto q_0 + z q_0'$. Consequently, in vicinity of transition point $z_t$, where $\ddot{a} = 0$, the validity of MOND in cosmology is quite justified.

However, eq. (8) at $g_0 = \text{const}$ is inconsistent with the condition of homogeneity and isotropy of matter, since the actor $\frac{1}{r}$ explicitly depends on the co-moving coordinate $r$, hence, such the modification of gravity law is inherently related to the inhomogeneity of matter distribution. In particular, eq. (8) results in the vacuum catastrophe: the vacuum with $\rho_\Lambda = -p_\Lambda = \text{const}$ is to be unstable, namely, due to eq. (8) the inhomogeneous distribution of matter is initiated. Thus, the straightforward transfer of MOND to the cosmology is theoretically inconsistent. Nevertheless, keeping in mind that MOND is related to the spatial inhomogeneity of matter distribution, we can easily suggest its cosmological extrapolation for the homogeneous and isotropic Universe, indeed: by setting
\[ g_0 \mapsto g_0 = g_0' |x|, \] (9)
we arrive to the evolution equation at $|\ddot{a}/a| \ll g_0'$
\[ \frac{\ddot{a}}{a} |\ddot{a}| = -\frac{4\pi G}{3} (\rho + 3p) g_0'. \] (10)

\[ \S \] The redshift is related to the scale factor by $z = \frac{1}{a(t)} - 1.$
which is consistent with the initial conditions imposed on the matter as well as with the vacuum stability.

On the other hand, the visible large scale structure (LSS) is seen at the angle distances of \( \delta \theta_{\text{LSS}} \approx 1.5 - 2^\circ \sim \frac{1}{30} - \frac{1}{30} \) radians, that corresponds to \( |x|_{\text{LSS}} \sim \delta \theta_{\text{LSS}} \cdot x_H \), where the Hubble horizon \( x_H \approx \frac{1}{H_0} \) at the current value \[ H_0 \approx (69 - 72) \frac{\text{km}}{\text{s} \cdot \text{Mpc}}. \]

Empirically in MOND \[ \tilde{g}_0 \approx \frac{H_0}{2\pi}, \] so that if we put \( \tilde{g}_0 \approx g'_0 |x|_{\text{LSS}} \), then we expect

\[ g'_0 \approx \frac{1}{2\pi \delta \theta_{\text{LSS}}} H_0^2. \]

It is convenient to parameterize the quantity \( g'_0 \) in the form

\[ g'_0 = K_0 H_0^2, \]

whereas by the order of magnitude

\[ K_0 \sim \frac{1}{2\pi \delta \theta_{\text{LSS}}} \sim 10. \]

Thus, we can assume that the change of regime in the cosmological extrapolation of MOND as given by (9) is caused by the presence of spatial inhomogeneity of matter, that allows us theoretically to conform the existence of two regimes for the critical acceleration of gravity.

In this paper we investigate the scheme proposed in (9) and (13) for the cosmological extrapolation of MOND with the parametrization in the transition region as derived in the holographic description of gravity as the entropic force \[12, 13\] at low temperatures (the MOND regime) \[14\]. This way essentially differs from the relativistic theory of gravity for the MOND paradigm offered by J.Bekenstein in \[15\], wherein he constructed the action satisfying a set of requirements, which include the appropriate limit to the MOND regime as well as, in principle, a correct description of cosmological effects, too. The modification is very specific due to additional gravitational scalar and vector fields in the action. Additional fields appear in the modified gravity formulated by J.W.Moffat in \[16\], too. Then, the gravitational constant becomes the field satisfying the equations of motion. So, J.W.Moffat has obtained the modification of gravity law \[17\] similar to the MOND, but it is distinguishable from the MOND, while some cosmological effects can be treated in agreement with observations \[18\]. Thus, approaches in \[15\] and \[16\] represent the axiomatic attempts to construct the consistent theory of modified gravity from the primary principles of field theory, and they are able to produce
the framework for complete calculations including the propagation of cosmological perturbations in the relativistic way, of course. We follow a more pragmatic way: without an introduction of any new additional notions, the modification of gravitational dynamics in the form of (9) and (13) can be consistent with the MOND at galactic scales, and it is enough for the investigation of modified cosmological evolution essentially different from the evolution in the framework of general relativity if we consider the ordinary components of matter. Therefore, the extrapolated MOND could exhibit some general features, which are common for meaningful extensions of general relativity as they tend to exclude the dark matter from the cosmology *dynamically*. In this way, we understand that the specific additional gravitational degrees of freedom defined explicitly as in [15] and [16] should be inevitably considered if we try to take into account the modification of theory for the propagation of matter inhomogeneity in cosmology, hence, in the construction of full range description of large scale structure of Universe, that includes the matter power spectrum and anisotropy of cosmic microwave background radiation. In the general relativity, that is the dark matter, which is responsible for the actual description of all features in the large scale structure due to corrections to the matter distributions during the cosmological evolution, but it is commonly known that basic cosmological characteristics such as the accelerated-decelerated regimes of Universe expansion, and the visible angular scale of matter inhomogeneity in the sky are mainly given by the appropriate evolution rate of Universe, and we show that such the evolution can be achieved in the framework of cosmological extrapolation of MOND without any dark matter. This extrapolation does not provide us with the complete theory of inhomogeneity propagation (especially, because such the theory is expected to be nonlinear in contrast to the well-working ordinary linear perturbation theory in the general relativity), hence, we do not suppose to reproduce the full description of angular dependence of cosmic microwave background radiation subject to the given primary spectrum of matter distribution, of course, or the big-bang nucleosynthesis, for instance. Nevertheless, the baryonic matter inhomogeneities themselves can produce the potential gravitational wells for the further concentration of matter, though such the wells would be affected by the sound propagation in the photon-electron-baryon plasma, and this evolution of inhomogeneities will be mainly determined by the strong field regime of MOND, which is consistent with the general relativity, while the nonlinear effects are expected to be subleading. Thus, we can expect the correct reproduction of main features of large scale structure in the framework of cosmological extrapolation of MOND. So, our preliminary studies show that the first acoustic peak in the cosmic microwave background anisotropy is successfully obtained within the cosmological extrapolation of MOND under the very standard settings of primary spectrum of matter power without any dark matter, of course, while the second peak can be also correctly described by an appropriate variation of the matter power spectrum, but the subleading structures need the construction of full theory of cosmic perturbations beyond of purposes of our paper. So, the analysis of anisotropy in the cosmic microwave background radiation would be presented elsewhere, since it
requires a deep and comprehensive investigation in details. The similar note can be addressed to the problem of nucleosynthesis, too. Here, we present the general scheme for the cosmological extrapolation of MOND and its applications to the basic features of Universe evolution.

In this paper, we apply the minimal scheme of \ref{9} and \ref{13} to the description of data on type Ia-supernovae. We show that in the redshift region accessible for observations, such the astronomical data can be reliably described in the proposed model if there are the cosmological constant and baryonic matter, only, without any dark matter, whereas the numerical estimate of \ref{14} is correct. While comparing with the Friedmann cosmology at presence of dark matter, we can evaluate the imitated contribution of dark matter \( \rho_M \) to the critical density of \( \rho_c = 3H_0^2/8\pi G \). The ratio of \( \Omega_M = \rho_M/\rho_c \) to the baryonic contribution \( \Omega_b = \rho_b/\rho_c \) is roughly determined by \( K_0 \) in order of magnitude. In accordance to the standard Friedmann cosmology-model with the cold dark matter and cosmological constant (ΛCDM), the parameters correspond to \( \Omega_M \approx 0.27 \) at \( \Omega_b = 0.045 \).

Further, we analyze the large scale structure of inhomogeneity in the matter distribution of Universe in calculations of baryon acoustic oscillations, measured by observing the luminous matter in the sky versus the redshift \cite{19}, and in estimation of “acoustic scale” in the CMBR anisotropy \cite{6}. The estimates obtained in the framework of cosmological extrapolation of MOND agree with the measured values of LSS without any postulation of hypothetic dark matter: it is enough to adjust the value of baryonic matter in the presence of cosmological constant. Then, the density of baryonic matter is roughly twice its value in ΛCDM of GR.

After modeling the acoustic scale in the CMBR anisotropy we calculate the critical acceleration \( \dot{g}_0 \) in the framework of the cosmological extrapolation of MOND and find preferable values of cosmological parameters of the Universe (the density of baryonic matter). We analyze the evolution into the future, too.

In Conclusion we summarize our results and discuss its meaning.

2. MOND parameters from data on supernovae

At present, measuring the stellar magnitude of type Ia supernovae versus the redshift gives the only direct observation of scale factor versus time in cosmology. Such the observation does not base on any model of Universe expansion, so that even the theory beyond the Friedmann equation can be confronted to the experiment without any additional assumptions. In this respect, such the data are unique and they are suitable for testing the cosmological extrapolation of MOND.

The stellar magnitude \( \mu \) depends on the redshift according to the formula

\[ \mu = \mu_{\text{abs}} + 5 \log_{10} d_L(z) + 25, \]

where \( \mu_{\text{abs}} \) is the absolute stellar magnitude, i.e. the stellar magnitude of light source at the distance of 10 pc. The photometric distance \( d_L \) measured in Mpc, is determined
by the Hubble constant evolution
\[ d_L(z) = (1 + z) \int_0^z \frac{c \, dz}{H(z)}, \] (16)
where \( c \) is the speed of light. In [7] the following dependence of the deceleration parameter on the redshift has been used as the working hypothesis:
\[ q(z) \mapsto q_{\text{lin.}} = q_0 + zq'_0, \] (17)
that allows it to describe the supernovae data at values
\[ q_0 = -0.85 \pm 0.35, \quad q'_0 = 1.8 \pm 1, \] (18)
essentially different from zeros, whereas the parameters significantly correlate (see details in the original paper [7]).

According to (17), the Hubble constant
\[ H_{\text{lin.}} = H_0(1 + z)^{1+q_0-q'_0} \exp\{q'_0z\}, \] (19)
that should be reasonably compared with the case of zero deceleration parameter, when
\[ H_{\text{nil}} = H_0(1 + z). \] (20)

From eqs. (15)–(20) we see that normalizing the data at low redshifts, i.e. measuring the value of \( H_0 \), allows us to plot a fiducial dependence of supernovae stellar magnitude \( \mu_0 \) with the deceleration parameter equal to zero, and further, systematically to study the deviations from this dependence versus the increasing redshift.

Data from [7] are shown in figures 2 and 3. They confidently signalize on the accelerated expansion of Universe at present as well as on the existence of transition from the decelerated expansion to the accelerated one at \( z = z_t \sim 0.5 \). The best fit of data takes place at parameter values listen in (18), as it shown in figure 3 by the dashed curve.

In the cosmological extrapolation of MOND we use the following equation for the acceleration at the presence of both the cosmological term with the energy density \( \rho_\Lambda \) and baryonic matter with the density \( \rho_b \):
\[ \frac{\ddot{a}}{a} \mathcal{D} \left( \frac{\pi^2}{6} \frac{ag'_0}{|\dot{a}|} \right) = \frac{4\pi G}{3} (2\rho_\Lambda - \rho_b), \] (21)
where the region of transition from the Friedmann regime to the MOND is described by the 1-dimensional Debye function as accepted in [14], hence,
\[ \mathcal{D}(x) = \frac{1}{x} \int_0^x \frac{y \, dy}{\exp\{y\} - 1}, \] (22)
so that at the acceleration greater than critical one, i.e. in the limit of standard cosmology at \( |\dot{a}|/a \gg g'_0 \), we get
\[ \mathcal{D} \left( \frac{\pi^2}{6} \frac{ag'_0}{|\dot{a}|} \right) \to 1, \]
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Figure 2. The stellar magnitude $\mu$ of type Ia supernovae versus the redshift $z$: the bottom dotted curve presents the case of zero deceleration parameter, the solid curve shows the result of cosmological model of MOND, the top dotted curve depicts the fit in ΛCDM. The supernovae discovered by the ground based telescopes are marked by rhombuses, the circles show the supernovae discovered by the Hubble Space Telescope (data from [7]).

while in the deep regime of MOND, i.e. at $|\ddot{a}|/a \ll g'_0$, asymptotically we find

$$D \left( \frac{\pi^2}{6} \frac{a g'_0}{|\ddot{a}|} \right) \rightarrow \frac{|\dddot{a}|}{a g'_0},$$

and we arrive to eq. (10).

The critical density defined by

$$\rho_c = \frac{3}{8 \pi G} H_0^2,$$

allows us to write down the baryonic density as

$$\rho_b = \rho_c \frac{\Omega_b}{a^3},$$

while the requirement of vanishing the acceleration at redshift $z_t$ leads to the following expression for the cosmological contribution:

$$\rho_\Lambda = \frac{1}{2} \rho_c \Omega_b (1 + z_t)^3.$$
Figure 3. The difference of stellar magnitudes $\Delta(\mu - \mu_0)$ for the type Ia supernovae versus the redshift $z$ after subtracting the fiducial magnitude at zero deceleration parameter. The solid line refers to the cosmological MOND, the dashed line shows the best fit at the deceleration parameter $q(z) = q_0 + z q'_0$ \cite{7}. The bottom panel serves for the systematical illustration of supernovae data as averaged for the similar redshifts. The dotted and dashed-dotted lines represent the evolution with appropriate constant positive and negative deceleration parameter, respectively.

Thus, the model for the cosmological extrapolation of MOND is described by equation

$$\frac{\ddot{a}}{a} D \left( \frac{\pi^2}{6} \frac{a}{|\dot{a}|} K_0 H_0^2 \right) = \frac{1}{2} H_0^2 \Omega_\delta \left( (1 + z_t)^3 - \frac{1}{a^3} \right),$$

combined with initial data

$$a_0 = 1, \quad \dot{a}_0 = H_0.$$  \hspace{1cm} (23)

Figures 2 and 3 illustrate the results of cosmological MOND at the typical value of $\Omega_\delta = 0.045$ as in $\Lambda$CDM, while

$$q_0 = -0.775, \quad z_t = 0.375, \quad \Rightarrow K_0 = 16.7,$$  \hspace{1cm} (24)

so that in the region of available data, the calculated profile is practically indistinguishable from the fit obtained at the linear dependence of deceleration parameter on the redshift as performed in \cite{7}. Note, that the difference of stellar magnitudes presented in figure 3 is independent of the normalization of Hubble constant $H_0$, while data shown in figure 2 allows us to fix both the value of $H_0$ at small $z$ and the fiducial curve at $\ddot{a} \equiv 0$.

Notice that at present, the regime of deep MOND takes place in cosmology, so that
from eq. (23) we find
\[ q_0^2 = \frac{1}{2} K_0 \Omega_b \left( (1 + z_t)^3 - 1 \right). \] (25)

Hence, we can see that the value of baryonic matter density, i.e. the parameter of \( \Omega_b \), can get quite arbitrary variations in this test on the supernovae data, because the region of MOND applicability near \( \ddot{a} = 0 \) is completely described by the factor of \( K_0 \Omega_b \), entering in eq. (23), while the Hubble constant at low redshifts is mainly determined by initial data, namely, by \( H_0 \) and \( q_0 \), so that the influence of \( \Omega_b \) value on the quality of data description is quite weak. For instance, the twice increase of baryonic fraction is almost invisible on the curve derived from the cosmological MOND as shown in figure \( \ddagger \) i.e. the region of low redshifts is weakly sensitive to \( \Omega_b \).

In the framework of \( \Lambda \)CDM the deceleration parameter and redshift of transition from the deceleration to the acceleration are given by the fractions of densities for the vacuum \( \Omega_\Lambda \) and matter \( \Omega_M \),
\[ q_0 = \frac{1}{2} \Omega_M - \Omega_\Lambda, \quad (1 + z_t)^3 = 2 \frac{\Omega_\Lambda}{\Omega_M}, \] (26)
so that in the case of spatially flat Universe, when \( \Omega_\Lambda + \Omega_M = 1 \), we get
\[ q_0 = \frac{3}{2} \Omega_M - 1, \quad (1 + z_t)^3 = 2 \frac{1 - \Omega_M}{\Omega_M}. \] (27)

The quality of fitting the supernovae data for the flat Universe in \( \Lambda \)CDM at \( \Omega_\Lambda \approx 0.73, \Omega_M \approx 0.27 \) is slightly worse than in the case of the linear approximation \( \ddagger \) for the deceleration parameter, but it has got a little bit of better quality at \( \Omega_\Lambda \mapsto \tilde{\Omega}_\Lambda \approx 0.95, \Omega_M \mapsto \tilde{\Omega}_M \approx 0.45 \) \( \ddagger \).

The parameters brought from \( \Lambda \)CDM into MOND give
\[ \frac{\Omega_M}{\Omega_b} = - \frac{K_0}{q_0} = \frac{K_0}{\Omega_\Lambda - \frac{1}{2} \Omega_M}. \] (28)
At \( |q_0| \approx 1 \) we find \( \Omega_M/\Omega_b \sim K_0 \). However, by taking into account the fact that the point of maximal confidence level of data fit in \( \Lambda \)CDM is shifted from the standard values \( \Omega_\Lambda \approx 0.73, \Omega_M \approx 0.27 \) to \( \Omega_\Lambda \mapsto \tilde{\Omega}_\Lambda \approx 0.95, \Omega_M \mapsto \tilde{\Omega}_M \approx 0.45 \), hence, numerically \( \tilde{\Omega}_M/\Omega_b \approx 10 \), we deduce \( K_0 \approx 7.5 \), that is in agreement with our estimates obtained in consideration of changing the regime of homogeneous matter distribution to the limit of inhomogeneity in MOND.

Thus, in the framework of cosmological extrapolation of MOND we can reliably describe the type Ia supernovae data by making use of baryonic matter and cosmological constant, only, without any postulating the dark matter. In this way, the parameters of extrapolation is naturally conformed with the Milgrom’s critical acceleration in MOND, applicable at scales, when the spatial inhomogeneity of baryonic matter distribution becomes significant. The cosmological extrapolation of MOND allows us, in fact, to calculate the present ratio of density of hypothetical dark matter to the baryonic density \( \Omega_M/\Omega_b \) in terms of the slope of critical acceleration in its dependence on the distance, i.e. the parameter of \( g'_0 \), expressed in units of Hubble constant at present.
Table 1. The comparison of observed parameters for the large scale structure of Universe with estimates in the framework of the cosmological extrapolation of MOND (the parameters are explained in the text).

| quantity | \( \frac{r_s(z_d)}{D_V(0.2)} \) | \( \frac{r_s(z_d)}{D_V(0.35)} \) | \( l_A \) |
|----------|-------------------------------|-------------------------------|---------|
| exp.[6][19] | 0.1905 ± 0.0061 | 0.1097 ± 0.0036 | 302.69 ± 0.76 |
| MOND | 0.1924 | 0.1149 | 302.5 |

Therefore, the supernovae data lead to the necessary introduction of dark matter if we keep the Friedmann evolution, i.e. if the evolution is determined by the Newton law of gravity, indeed, while the modification of gravity law at low accelerations in the case of inhomogeneous distribution of matter in accordance with MOND as well as for the homogeneous distribution of matter in cosmological MOND, gives the confident description of data without any dark matter.

3. The large scale structure of Universe

The Universe evolution essentially transforms the spatial distribution of matter, that is observed as baryonic acoustic oscillations (BAO) [19] and CMBR anisotropy [6].

So, the results of survey of the matter distribution on the celestial sphere versus the redshift [19] are particularly reduced to the ratio of sound horizon \( r_s \) for the waves in the baryon-electron-photon substance as written in co-moving coordinates

\[
r_s(z) = \int_0^{t(z)} c_s \, dt
\]

(29)
to the effective distance defined by

\[
D_V(z) = \left\{ (1 + z)^2 D_A^2(z) \frac{cz}{H(z)} \right\}^{\frac{1}{2}},
\]

(30)

where \( c_s \) is the sound speed in the medium, while the angular distance is given by

\[
D_A(z) = \frac{c}{1 + z} \int_0^z \frac{dz}{H(z)}.
\]

(31)

The measurements give the value of \( \frac{r_s(z_d)}{D_V(z)} \) at \( z = 0.2 \) and \( z = 0.35 \), which are shown in table [1]. Here \( z_d \) is the redshift of epoch, when the interaction of baryonic and photonic components of substance via the Compton scattering of electrons off photons and Coulomb attraction of electrons to protons becomes negligible, i.e. when baryons stop to drag photons (the finish of drag epoch).

|| We consider the flat space.
The speed of sound is given by the following expression:

\[
c_s = \frac{c}{\sqrt{3}} \frac{1}{\sqrt{1 + R}},
\]

where \( R \) is the baryon-photon ratio depending on the redshift

\[
R = \frac{3 \rho_b}{4 \rho_{\gamma}} = \frac{3 \Omega_b}{4 (1 + z) \Omega_{\gamma}}.
\]

Introducing the redshift \( z_{eq} \) for the moment, when the density of non-relativistic matter \( \rho_M \) equals to the density of radiation (photons and neutrinos), allows us to write down the formula for the sound horizon as

\[
r_s(z) = \frac{2}{k_{eq}} \sqrt{\frac{6}{R_{eq}}} \ln \frac{\sqrt{1 + R + \sqrt{R + R_{eq}}}}{1 + \sqrt{R_{eq}}},
\]

where

\[
k_{eq}^2 = 2\Omega_M H_0^2 z_{eq}.
\]

Analytical parameterizations of numerical calculations for \( z_{eq} \), \( z_d \) and \( R \) are given in [20], wherein the dependencies on the CMBR temperature \( T = \Theta \cdot 2.7 \) K and parameters \( \Omega_b h^2 \), \( \Omega_M h^2 \) with \( h \) being the coefficient for the Hubble constant written down as \( H_0 = h \cdot 100 \) km \( \cdot \) s\(^{-1} \)/Mpc, are explicitly presented. Thus, the Universe evolution at low redshifts determines the angular distance \( D_A \) in eq. (31), which depends on the initial data on \( H_0 \) and \( q_0 \), while the sound horizon \( r_s \) and redshift \( z_d \) are determined by fractions of baryonic and dark matters in the energy budget of Universe. Since the evolution at low redshifts in the cosmological extrapolation of MOND conforms with the supernovae data in the same way as it was done in \( \Lambda \)CDM, the angular distances are almost coincident in both these models, hence, the description of BAO data is determined by fitting the sound horizon \( r_s \). In the case of the cosmological extrapolation of MOND all of non-relativistic matter is identified to baryons, i.e. \( \Omega_M = \Omega_b \). Hence, we find \( \Omega_b h^2 \approx 0.057 \), i.e. it is twice and a half greater than the baryonic fraction in \( \Lambda \)CDM. At such the value of baryonic density we get the agreement with the BAO data (see table [1]). In numerical estimations we have put \( h = 0.71 \), \( T = 2.725 \) K and \( q_0 = -0.775 \), \( z_t = 0.375 \) as it was in the previous section. Then, \( \Omega_b = 0.113 \). Such the doubling can point to necessity of accounting for the baryonic matter outside luminous stars and visible gas, that is also numerically consistent with the lack of visible baryonic matter in some galactic clusters, which dynamics is described in MOND without any dark matter (see [2, 3]).

At the same parameters we estimate the “acoustic scale”

\[
l_A = (1 + z_s) \frac{\pi D_A(z_s)}{r_s(z_s)},
\]

which is measured in the spectra of CMBR temperature anisotropy by locations of peaks of temperature fluctuations versus the multipole number in WMAP data [6]. Here \( z_s \) is the redshift of decoupling, when due to the recombination of electrons with protons the medium becomes transparent for photons (see analytical approximations for \( z_s \) in terms of baryonic density, matter density and Hubble constant in [21]). Calculating the
value of \( D_A(z_\ast) \), we take into account for the contribution of relativistic particles into the Hubble constant, since
\[
\rho_\gamma + \rho_\nu = \rho_b \frac{1 + z}{1 + z_{eq}}.
\]

As we can see from table I, the cosmological extrapolation of MOND is safely able to describe the large scale structure data given by the measurements of both the baryon acoustic oscillations of visible matter and the anisotropy of CMBR.

It is interesting that our argumentation about changing the MOND regime at distances of the order of the scale for the spatial inhomogeneity of matter distribution, in fact, is confirmed by the calculation of acoustic scale \( l_A \), which points to the angular size of inhomogeneity about \( 360^\circ/l_A \approx 1.2^\circ \) in consistency with our initial assumptions.

4. The Milgrom’s acceleration

Since the critical Milgrom’s acceleration in MOND is experimentally measured with the high accuracy in the gas-reach galaxies \[5\]
\[
\tilde{g}_0 = (1.21 \pm 0.14) \cdot 10^{-10} \text{m/s}^2,
\]
(36) it makes sense to use this information in order to refine the parameters in the cosmological extrapolation of MOND.

Indeed, within our approach, the Milgrom’s acceleration is determined by the angular scale of inhomogeneity in the matter distribution \( \delta \theta \), so that in Hubble units
\[
\frac{\tilde{g}_0}{cH_0} = K_0 \delta \theta.
\]
(37) However, the acoustic scale gives
\[
\delta \theta = \frac{2\pi}{l_A}.
\]
(38) Then, by expressing \( K_0 \) in term of deceleration parameter \( q_0 \) and redshift \( z_t \) for the transition from the Universe deceleration to its acceleration in accordance with (25), we find
\[
\frac{\tilde{g}_0}{cH_0} = \frac{4\pi}{l_A\Omega_b} \frac{q_0^2}{(1 + z_t)^2 - 1},
\]
(39)

\[\footnote{We have not considered the “shift parameter” defined in \[22\], \( \mathcal{R} = \sqrt{\Omega_M H_0^2(1+z_\ast)D_A(z_\ast)/c} \), since, as authors of \[22\] have pointed out, this quantity can not be directly extracted from the angular spectrum of CMBR anisotropy because of a poor accuracy of measurements at low values of the multipole number, i.e. at large angles of correlations, wherein the shift parameter makes a significant influence on the amplitude of spectrum, while the high multipoles are not sensitive to \( \mathcal{R} \). It means that the choice of model for fitting the spectrum in the region of acoustic oscillations at \( l \sim l_A \) determines the extrapolation to low values of multipoles, but it is not critical for the data at low multipoles because of poor accuracy of data at low \( l \ll l_A \). Consequently, the value of shift parameter given in \[6\] is actually the model dependent-extrapolation, indeed.} \]
that allows us to plot the dependence of Milgrom’s acceleration on the baryon density at fixed values of \( q_0 \) and \( z_t \), which are determined by the supernovae data. Further, we can compare the result with the empirical value of

\[
\frac{\tilde{g}_0}{cH_0} = 0.180 \pm 0.022,
\]

known with the 12%-accuracy after taking into account for the uncertainty of the Hubble constant.

The comparison is shown in figure 4. It turns out that the agreement of data on the supernovae (SN), baryonic acoustic oscillations (BAO), acoustic scale (WMAP) and Hubble constant \( (H_0) \) with the Milgrom’s acceleration is reached at the deceleration parameter \( q_0 \approx -0.853 \) and accessible 10%-variation of the transition redshift \( z_t \approx 0.375 \). In this way, the account for the small error in the value of acoustic scale leads to

\[
\begin{align*}
\Omega_b & = 0.115 \pm 0.012, \\
\tilde{g}_0 & = (1.10 \pm 0.03) \times 10^{-10} \text{m/s}^2, \\
\frac{r_s(z_d)}{D_V(0.2)} & = 0.185 \pm 0.006, \\
\frac{r_s(z_d)}{D_V(0.35)} & = 0.112 \pm 0.004.
\end{align*}
\]

Thus, we successfully conform the logical consistency of two MOND regimes: the cosmological limit and the locally inhomogeneous case; and we extract the confident intervals for the Universe parameters.
5. The evolution into the future

In the future $a(t) \gg 1$, and the equation of cosmological MOND is reduced to

$$\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} = \frac{1}{2} H_0^4 K_0 \Omega_b (1 + z_t)^3,$$

hence, according to (25)

$$\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} = H_0^4 q_0^2 \frac{(1 + z_t)^3}{(1 + z_t)^3 - 1},$$

so that the limit is the de Sitter Universe with $a(t) = a_\star \exp[\bar{H}_\Lambda(t - t_\star)]$, where the cosmological Hubble constant

$$\bar{H}_\Lambda^4 = H_0^4 q_0^2 \frac{(1 + z_t)^3}{(1 + z_t)^3 - 1}. $$

Since $z_t > 0$ and $\bar{H}_\Lambda \gg H_0$,

$$q_0^2 < 1, \quad z_t \leq \frac{1}{\sqrt[3]{1 - q_0^2}} - 1.$$  \hfill (45)

Therefore, the future epochs at $\bar{H}_\Lambda \gg H_0$ are constrained by condition (45) as illustrated in figure 5.

![Figure 5](image)

**Figure 5.** Delimiting the future epochs versus the deceleration parameter $q_0$ and the redshift of zero acceleration $z_t$. The preferable fit of WMAP+BAO+SN+$H_0+\tilde{g}_0$ data is depicted by the crossing of the shaded bands.

Substituting of $q_0 = -0.853$ as obtained by fitting the data, into eq. (45) gives

$$z_t \leq 0.54,$$

so that according to our estimates for $z_t$ in the previous section, it means that to the end of evolution $\bar{H}_\Lambda > H_0$ (see figure 5).

It is interesting to compare the cosmological term evaluated in MOND in terms of $\bar{H}_\Lambda$ with its value in GR:

$$H_\Lambda^2 = \frac{8\pi G}{3} \rho_\Lambda = \frac{1}{2} \bar{\Omega}_b H_0^2 (1 + z_t)^3,$$

\hfill (46)
so that the equality of $\bar{H}_\Lambda = H_\Lambda$ takes place at

$$\bar{\Omega}_b^2 = \frac{4q_0^2}{(1 + z^*)^3} \frac{1}{(1 + z^*)^3 - 1}.$$ 

Numerically

$$\Omega_b \ll \bar{\Omega}_b.$$ 

Therefore, the limit of de Sitter Universe and, hence, the problem of small-scale cosmological constant cannot get an adequate description in GR, while in order to consider the vacuum we need essentially to modify the equation for the connection between the gravity and matter.

6. Discussion and conclusion

In this paper we have conformed the MOND to its cosmological extrapolation, that allows us to eliminate hypothetical dark matter not only at scales of spatial inhomogeneity of baryon matter distribution in galaxies and galactic clusters, but also in the description of Universe evolution up to the redshift of $z \approx 1.8$ in the observation of type Ia supernovae. In this conformation, the angular size of inhomogeneity in the large scale structure of Universe is essential.

Another question, related to the cosmology and modification of gravity, is the necessity to postulate the dark matter in order to describe the CMBR anisotropy in $\Lambda$CDM. To our point of view, this problem is also closely connected to the usage of Friedmann model of expansion at redshifts from zero to $z^*$, when the CMBR inhomogeneity was formed. Clearly, parameters such as the contribution of cosmological term into the energy budget of Universe and the fraction of dark matter can be mostly significant at low redshifts, exactly when the modification of gravitational law becomes remarkable at low accelerations of expansion. However, the forming of CMBR essentially depends on the proportion between the baryons and dark matter. On the other hand, the evolution of the baryonic density versus the redshift is completely determined by the conservation law of matter, hence, the conformation of the baryon densities in both $\Lambda$CDM and MOND allows us to generate similar conditions for the interaction between photons and electrons, which density correlates with baryons. The problem of considering the anisotropy evolution versus the redshift at the whole interval in the framework of cosmological MOND expects a solution, i.e. the question about a detailed investigation of CMBR anisotropy still remains open within the cosmological extrapolation of MOND. In this respect we have to mention the field theory-approaches in [15] and [16], which can, in principle, provide us with complete calculations for the propagation of perturbations, whereas the theory by J.Bekenstein [15] includes the MOND regime for inhomogeneous distribution of matter. However, we follow another motivation based on the minimal extension of MOND that satisfies the reasonable constraint on the transition to the cosmology of homogeneous distribution of matter. In this way, we note that some integral quantities, which values are deduced by the
consideration of CMBR anisotropy, are derived under the assumption of Friedmann evolution and they cannot be straightforwardly transferred to the modified law of gravity. This note is also valid as concerns for the observation of large scale structure, i.e. in studying the baryon acoustic oscillations (BAO) caused by the spatial inhomogeneity. Nevertheless, we have analyzed the “angular scale” in the CMBR anisotropy and parameters of baryon acoustic oscillations in the framework of cosmological MOND and showed that MOND can confidently describe these cosmological phenomena. Then, we can conclude that the cosmological MOND has exhibited its positive potential for the adequate description of Universe evolution at low redshifts as well as up to the redshifts of CMBR forming. We have to note, of course, that the purpose of this paper has been not a complex fitting the type Ia supernovae data, baryon acoustic oscillation and CMBR anisotropy, but the aim has been the demonstration of opportunity to get the successful application of cosmological MOND to those problems.

Finally, we have compared the critical acceleration calculated in the framework of cosmological MOND with its empirical value and extract the intervals of cosmological parameters, wherein there is the good agreement of the model with observations of WMAP+BAO+SN+ $H_0+\tilde{g}_0$. In addition, we have found that the modification of gravity law is essential in studying the problem of cosmological constant.

Thus, the preliminary estimates performed in the framework of cosmological MOND conceptually permit for the confidential description of Universe evolution and point to the necessity to work out a cumbersome analysis of complete base of the data on the supernovae, large scale structure, baryogenesis and abundance of elements in the Universe etc., for the accurate extraction of cosmological parameters, in particularly, the baryonic fraction of energy. Nevertheless, in this paper we have shown that hypothetical dark matter can be completely excluded from the cosmology by the appropriate modification of gravity law at accelerations less than the critical value in analogy to the case, when it was successfully done at the galactic scale in MOND.

Notice, in the framework of ΛCDM the spatial inhomogeneity of both the matter and dark matter are generated, say, by the primordial spectrum of inhomogeneity as given by quantum fluctuations of a scalar filed having caused the Universe inflation. Therefore, the baryonic and dark matter initially has got common locations of compression-decompression regions, that is consistent with the assumption about the joint concentration of baryonic and dark matter. However, the baryon oscillations having the electromagnetic nature, suggest that the inhomogeneities of the photon-electron-baryon medium propagate as sound waves decoupled from the dark matter (except the interaction due to gravitational forces), while the inhomogeneities of dark matter do not propagate at all, since there are no sound in the dark matter. During the evolution after the baryon acoustic oscillations stopped, the gravitational interaction attracts the centers of baryonic and dark matter concentration, that results in a partial nearing the locations of matter lumps. The transfer function of initial perturbations in the energy densities consists of two different parts given by the terms of baryonic and dark matter. Consequently, initially coinciding regions of concentration for the baryons and dark matter become spatially separated, that creates different centers of gravitational contraction of baryons and dark matter in the future evolution. Thus, the existence of baryon acoustic oscillations challenges the dark matter hypothesis by asking for a natural explanation for the coincident locations of baryons and dark matter in galaxies, that rises the additional problem in a dynamical derivation of the Tully–Fisher law in the framework of standard cosmology.
Notice, there are several papers, wherein the influence of modified law of gravity to the Universe evolution at low accelerations was considered \cite{28,29}, however, in those papers the acceleration is evaluated by the temperature of apparent horizon \(T_H = H/2\pi \) giving \( g_H = 2\pi T_H = H \), while the critical acceleration is the constant value independent of distance. In \cite{29} this approach follows the MOND at the linear dependence of factor modifying the acceleration \( \ddot{a}r \) on ratio \( g/g_H \ll 1 \). Hence, it is clear that such the modification of gravity law at the Hubble horizon is extrapolated inside the Hubble sphere, that can be never conformed with our consideration.

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