MEASURING THE CHAPLYGIN GAS EQUATION OF STATE FROM ANGULAR AND LUMINOSITY DISTANCES

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Received 2004 July 12; accepted 2004 September 9

ABSTRACT

An exotic dark component, called generalized Chaplygin gas (Cg) and parameterized by an equation of state $p = -A/R_C^a$, where $A$ and $a$ are arbitrary constants, is one of the possible candidates for dark energy as well as for a unified scenario of dark matter and dark energy. In this paper we investigate qualitative and quantitative aspects of the angular size—redshift test in cosmological models driven by such a dark component. We discuss the prospects for constraining the Cg equation of state from measurements of the angular size at low- and high-redshift radio sources and also from a joint analysis involving angular size and supernova data. A detailed discussion about the influence of the Cg on the minimal redshift at which the angular size of an extragalactic source takes its minimal value is also presented.

Subject headings: cosmological parameters — dark matter — equation of state

1. INTRODUCTION

The impressive convergence of recent observational facts along with some apparently successful theoretical predictions seem to indicate that the simple picture provided by the standard cold dark matter (SCDM) model is insufficient to describe the present stage of our universe. From these results, the most plausible picture for our world is a spatially flat scenario dominated basically by CDM and an exotic component endowed with large negative pressure, usually named dark energy. Despite the good observational indications for the existence of these two components, their physical properties constitute a completely open question at present, which gives rise to the so-called dark matter and dark energy problems.

Dark matter, for which the leading particle candidates are the axions and the neutralinos, was originally inferred from galactic rotation curves that show a general behavior that is significantly different from the one predicted by Newtonian mechanics. Dark energy or “quintessence,” for which the main candidates are a cosmological constant and a relic scalar field $\phi$, has been inferred from a combination of astronomical observations that includes distance measurements of Type Ia supernovae (SNe Ia) indicating that the universe is speeding up, not slowing down (Perlmutter et al. 1999; Riess et al. 1998, 2004), cosmic microwave background (CMB) data suggesting $\Omega_{\text{total}} \simeq 1$ (de Bernardis et al. 2000; Spergel et al. 2003), and clustering estimates providing $\Omega_m \simeq 0.3$ (Carlborg et al. 1996; Dekel et al. 1997). While the combination of these two latter results implies the existence of a smooth component of energy that contributes with $\simeq 2/3$ of the critical density, the SNe Ia results require this component to have a negative pressure, thereby leading to a repulsive gravity (for reviews, see Peebles & Ratra 2003; Padmanabhan 2003).

By assuming the existence of these two dominant forms of energy in the universe, one finds that the main distinction between them comes from their gravitational effects. CDM agglomerates at small scales, whereas dark energy seems to be a smooth component. In a certain sense, such properties are directly linked to the equation of state of both components (for a short review see Lima 2004). On the other hand, the idea of a unified description for the CDM and dark energy scenarios has received much attention (Matos & Ureña-Lopez 2000, 2001; Davidson et al. 2001; Kasuya 2001; Wetterich 2002; Padmanabhan & Choudhury 2002). For example, Wetterich (2002) suggested that dark matter might consist of quintessence lumps, while Kasuya (2001) showed that quintessence-like scenarios are generally unstable for the formation of $Q$ balls that behave as pressureless matter. More recently, Padmanabhan & Choudhury (2002) investigated such a possibility via a string theory—motivated tachyonic field.

Another interesting attempt of unification was suggested by Kamenshchik et al. (2001) and developed by Bilic’ et al. (2002) and Bento et al. (2002). It refers to an exotic fluid, the so-called Chaplygin gas (Cg), whose equation of state is given by

$$p_{\text{Cg}} = -A/R_C^a,$$  \hspace{1cm} (1)

where $\alpha = 1$ and $A$ is a positive constant related to the present-day Chaplygin adiabatic sound speed, $v_s^2 = \alpha A/\rho_{\text{Cg}0}$, where $\rho_{\text{Cg}0}$ is the current Cg density. In actual fact, the above equation for $\alpha \neq 1$ constitutes a generalization of the original Chaplygin gas equation of state proposed by Bento et al. (2002). The idea of a dark matter–dark energy unification from an equation of state like equation (1) comes from the fact that the Cg or the generalized Chaplygin gas (from now on we use Cg to denote the Chaplygin gas as well as the generalized Chaplygin gas) can naturally interpolate between nonrelativistic matter and negative-pressure dark energy regimes. It can be easily seen by inserting equation (1) into the energy conservation law ($\mu_{\rho}T_{\rho}^{\mu\nu} = 0$). One finds

$$\rho_{\text{Cg}} = \rho_{\text{Cg}0} \left[ A_s + (1 - A_s) \left( \frac{R_0}{R} \right) ^{3(1+\alpha)} \right] ^{1/(1+\alpha)} ,$$ \hspace{1cm} (2)

where the subscript 0 denotes present-day quantities, $R(t)$ is the cosmological scale factor, and $A_s = A/\rho_{\text{Cg}0}^{1-\alpha}$ is a quantity.
related to the present sound speed for the Chaplygin gas \((c_s^2 = \alpha A_3)\). As can be seen from the above equations, the Chaplygin gas interpolates between nonrelativistic matter \([\rho_{\text{Cg}}(R \to 0) \propto R^{-3}]\) and negative-pressure dark component regimes \([\rho_{\text{Cg}}(R \to \infty) \propto \text{constant}]\).

Motivated by these potentialities, there has been growing interest in exploring theoretical (Bordemann & Hoppe 1993; Hoppe 1993; Jackiw 2000; González-Díaz 2003a, 2003b; Kremer 2003; Khalatnikov 2003; Balakin et al. 2003; Bilic et al. 2002) and observational consequences of the Chaplygin gas, not only as a possibility of unification for dark matter and dark energy but also as a new candidate for dark energy only. The viability of such scenarios has been tested by a number of cosmological tests, including SNe Ia data (Fabris et al. 2002; Colistete et al. 2004; Avelino et al. 2003b; Makler et al. 2003; Bertolami et al. 2004), lensing statistics (Dev et al. 2003, 2004; Silva & Bertolami 2003), CMB measurements (Bento et al. 2004), future lensing and SNe Ia experiments (Avelino et al. 2003a, 2003b; Carturan & Finelli 2003; Amendola et al. 2003), the age-redshift test (Alcaniz et al. 2003), measurements of X-ray luminosity of galaxy clusters (Cunha et al. 2004; Zhu 2004), future lensing and SNe Ia experiments (Avelino et al. 2003b; Silva & Bertolami 2003; Sahni et al. 2003), and Statefinder parameters (Sahni et al. 2003), as well as by observations of large-scale structure (Multamäki et al. 2004; Bilic et al. 2003; Beca et al. 2003; Bean and Dore 2003; Avelino et al. 2004). In particular, the latter reference has shown that in the context of unified dark matter and dark energy models, the onset of the nonlinear regime on small cosmological scales may lead to the breakdown of the background solution, even on large cosmological scales (if \(\alpha\) is not equal to zero).

Although carefully investigated in many of its theoretical and observational aspects, the influence of a Cg component in some kinematic tests, such as the angular size-redshift (\(\theta-z\)) relation, still remains to be studied. This is the goal of the present paper. In the next sections we investigate qualitative and quantitative aspects of the angular size–redshift test in cosmological models driven by this dark matter and dark energy component. We first investigate the influence of the Cg on the minimal redshift at which the angular size of an extragalactic source takes its minimal value. Afterward, we consider the \(\theta(z)\) data recently updated and extended by Gurvits et al. (1999) to constrain the equation of state of the Cg component as well as a combination between these \(\theta(z)\) observations and the latest SNe Ia data, as provided by Riess et al. (2004). To do so, we follow Alcaniz et al. (2003) and consider two different cases, namely, a flat scenario in which the generalized Chaplygin gas together with the observed baryonic content are responsible by the dynamics of the present-day universe (unifying dark matter–energy (UDME)) and a flat scenario driven by nonrelativistic matter plus the generalized Chaplygin gas (CgCDM). For UDME scenarios we adopt in our computations \(\Omega_b = 0.04\), in accordance with the estimates of the baryon density at nucleosynthesis (Burles et al. 2001) and the latest measurements of the Hubble parameter (Freedman et al. 2001). For CgCDM models we assume \(\Omega_m = 0.3\), as suggested by dynamical estimates on scales up to about 2 h\(^{-1}\) Mpc (Carlborg et al. 1996; Dekel et al. 1997). For the sake of completeness, an additional analysis for the original Cg model (\(\alpha = 1\)) is also included.

This paper is organized as follows. In § 2 we present the basic equations and distance formulae necessary for our analysis. Following the method developed in Lima & Alcaniz (2000a, 2000b), the influence of a Cg-like component on the minimal redshift \(z_{\text{min}}\) is investigated. In § 3 we analyze the constraints on the equation of state of the Cg component from measure-
ments of the angular size of compact radio sources and SNe Ia data and compare them with other independent limits. We end the paper by summarizing the main results in § 4.

2. DISTANCE FORMULAE AND THE MINIMAL REDSHIFT

2.1. Distance Formulae

Let us now consider the flat FRW line element (\(c = 1\))

\[
\text{d}s^2 = dt^2 - R^2(t)[d\chi^2 + \chi^2(d\theta^2 + \sin^2\phi d\phi^2)],
\]

where \(\chi, \theta, \phi\) are dimensionless, comoving coordinates. In this background, the angular size–redshift relation for a rod of intrinsic length \(\ell\) is easily obtained by integrating the spatial part of the above expression for fixed \(\chi\) and \(\phi\), i.e.,

\[
\theta(z) = \frac{\ell}{d_A}.
\]

The angular diameter distance \(d_A\) is given by

\[
d_A = \frac{R_0 \chi}{(1 + z)} = \frac{H_0^{-1}}{(1 + z)} \int_{(1+z)^{-1}}^{1} \frac{x^{-2} dx}{E(\Omega_j, A_j, \alpha, x)},
\]

where \(x = R(t)/R_0 = (1 + z)^{-1}\) is a convenient integration variable. For the kind of models considered here, the dimensionless function \(E(\Omega_j, A_j, \alpha, x)\) takes the following form:

\[
E = \left[\frac{\Omega_j}{x^3} + (1 - \Omega_j) \left[A_j + \frac{(1 - A_j)}{x^{3(1+\alpha)}}\right]^{1/(1+\alpha)}\right]^{1/2},
\]

where \(\Omega_j\) stands for the baryonic matter density parameter \((j = b)\) in UDME scenarios and the baryonic + dark matter density parameter \((j = m)\) in CgCDM models. Note that, from the above equation, UDME models reduce to the LCDM case for \(\alpha = 0\), whereas CgCDM models get the same limit, regardless of the value of \(\alpha\), when \(A_j = 1\). In both cases, the standard Einstein–de Sitter behavior is fully recovered for \(A_j = 0\) (Avelino et al. 2003a; Fabris et al. 2004).

2.2. Minimal Redshift

As is widely known, the existence of a minimal redshift \(z_{\text{min}}\) on the angular size–redshift relation may qualitatively be understood in terms of an expanding space; the light observed today from a source at high \(z\) was emitted when the object was closer (for a pedagogical review on this topic, see Janis 1986). The relevant aspect here is to demonstrate how this effect may be quantified in terms of the Chaplygin parameters \(\alpha\) and \(A_j\). To analyze the sensitivity of the critical redshift to this dark component, we adopt here the approach originally presented by Lima & Alcaniz (2000a, 2000b). The numerical results of this method have been confirmed by Lewis & Ibata (2002) through a Monte Carlo analysis.

The redshift \(z_{\text{min}}\) at which the angular size takes its minimal value is the one canceling out the derivative of \(\theta\) with respect to \(z\). Hence, from equations (4)–(6) we have the following condition:

\[
\chi_{\text{min}} = (1 + z_{\text{min}}) \chi_{\ell z=z_{\text{min}}}',
\]
where a prime denotes differentiation with respect to $z$ and by definition $\chi_{\text{min}} = \chi(z_{\text{min}})$. Note that equation (5) can readily be differentiated, yielding

$$(1 + z_{\text{min}}) \chi'(z_{\text{min}}) = \frac{1}{R_{0}H_{0}} F(\Omega_{j}, A_{s}, \alpha, z_{\text{min}}),$$

where

$$F = \left\{ \frac{\Omega_{j}}{(1 + z_{\text{min}})^{3}} \right\} \left[ 1 - \frac{A_{s}}{(1 + z_{\text{min}})^{\alpha}} \right]^{1/(\alpha+1)}.$$

Finally, by combining equations (7)–(9), we find

$$\int_{1/(1+z_{\text{min}})}^{1} \frac{dx}{x^{2}E(\Omega_{j}, A_{s}, \alpha, z_{\text{min}})} = F(\Omega_{j}, A_{s}, \alpha, z_{\text{min}}).$$

In Figure 1 we show the results of the above expression. Figures 1a and 1c show the minimal redshift $z_{\text{min}}$ as a function of the parameter $A_{s}$ for selected values of $\alpha$ in the context of CgCDM ($\Omega_{m} = 0.3$) and UDME ($\Omega_{b} = 0.04$) models, respectively, whereas Figures 1b and 1d display the $\alpha$-$z_{\text{min}}$ plane for some values of $A_{s}$, also for CgCDM and UDME scenarios (note that the scale of $z_{\text{min}}$ in each panel is different). The minimal redshift is a much more sensitive function to the parameter $A_{s}$ than to the index $\alpha$. As can be seen in the panels (and also expected from eq. [5]), regardless of the value of $\alpha$, models with $A_{s} = 0$ reduce to the Einstein–de Sitter case so that the standard result $z_{\text{min}} = 1.25$ is fully recovered. As physically expected (see, e.g., Lima & Alcaniz 2000a, 2000b), the smaller the contribution of the material component (baryonic and/or dark) the higher the minimal redshift $z_{\text{min}}$. From this qualitative argument UDME models would be in a better agreement with the observational data than CgCDM scenarios, since the current data for milliarcsecond radio sources do not show clear evidence for a minimal angular size ($\theta_{\text{min}}$) close to $z = 1.25$ (Gurvits et al. 1999). For the best-fit CgCDM model obtained from an analysis involving galaxy cluster X-ray and supernova data, i.e., $A_{s} = 0.98$ and $\alpha = 0.93$, we find $z_{\text{min}} \approx 1.6$, a value that is very similar to the one predicted by the current concordance model, namely, a flat $\Lambda$CDM scenario with $\Omega_{m} = 0.3$, and also by the standard FRW model with $\Omega_{m} = 0.5$ (see Table 1 of Lima & Alcaniz 2000a). It is worth mentioning that if high-redshift sources present cosmological evolution, such an effect would move the position of the $\theta_{\text{min}}$ to extremely high redshift.

3. CONSTRAINTS FROM ANGULAR SIZE MEASUREMENTS

In this section we study the constraints from angular size measurements of high-$z$ radio sources on the free parameters of...
the Cg model. In order to constrain the parameters $A_s$ and $\alpha$ we use the angular size data for milliarcsecond radio sources recently compiled by Gurvits et al. (1999). This data set is composed by 145 sources at low and high redshifts ($0 \, z / 20 = 0.011 \leq z \leq 4.72$) distributed into 12 bins with 12–13 sources per bin. Since the main difference between the analysis performed in this section and the previous ones that have used these angular size data to constrain cosmological parameters is the background cosmology, we refer the reader to previous works for a more complete analysis and detailed formulae (see, for instance, Jackson & Dodgson 1996, 1997; Jackson 2003; Vishwakarma 2001; Lima & Alcaniz 2002; Alcaniz 2002; Zhu & Fujimoto 2002; Jain et al. 2003; Chen & Ratra 2003).

Following a procedure similar to that described in the quoted reference, we determine the cosmological parameters $A_s$ and $\alpha$ through a $\chi^2$ minimization for a range of $A_s$ and $\alpha$ spanning the interval $[0, 1]$ in steps of 0.02, i.e.,

$$
\chi^2(\ell, \Omega_j, A_s, \alpha) = \sum_{i=1}^{12} \frac{[\theta(z_i, \ell, \Omega_j, A_s, \alpha) - \theta_{\text{obs}}]}{\sigma_{\text{obs}}^2}^2,
$$

where $\theta(z_i, \ell, \Omega_j, A_s, \alpha)$ is given by equations (4)–(6) and $\theta_{\text{obs}}$ is the observed values of the angular size with errors $\sigma_{\text{obs}}$ of the ith bin in the sample. The conventional two-parameter $\chi^2$ levels 2.30 and 6.17 define the 68% and 95% confidence regions, respectively.

Figures 2a and 2b show the binned data of the median angular size plotted as a function of redshift for selected values of $A_s$ and $\alpha$ for UDME and CgCDM models. The characteristic length $D$ has been fixed at (a) $29.58 h^{-1}$ pc and at (b) $24.10 h^{-1}$ pc, which corresponds to the best-fit values obtained from a minimization of eq. (11) relative to the parameters $A_s$, $\alpha$, and $D$. From this analysis we obtain the following best-fit values: $A_s = 0.96$, $\alpha = 0.84$, and $D = 24.1 h^{-1}$ pc ($\chi_{\text{min}}^2 = 4.44$) for UDME scenarios and $A_s = 1.0$ and $D = 29.58 h^{-1}$ pc ($\chi_{\text{min}}^2 = 4.57$) for CgCDM models. Both cases represent accelerating universes with deceleration parameter and the total age of the universe given by $q_0 = -0.88$ and $t_0 \approx 11 h^{-1}$ Gyr (UDME) and $q_0 = -0.55$ and $t_0 \approx 9.4 h^{-1}$ Gyr (CgCDM). Note that, as happens in the calculation of the minimal redshift $z_{\text{min}}$ as well as in other cosmological tests (see, e.g., Cunha et al. 2004), the current angular size data constrain more strongly the parameter $A_s$ than the index $\alpha$. From Figure 3 it is perceptible that while the parameter $A_s$ is constrained to be $>0.68$ (UDME) and $>0.54$ (CgCDM) at 2 $\sigma$, the entire interval of $\alpha$ is allowed.

### 3.1. $\theta(z)$ + SNe Ia

By combining the angular and luminosity distances, interesting constraints on the Cg parameters are obtained. To perform...
such analysis, we follow the conventional magnitude-redshift test (see, e.g., Goliath et al. 2001; Dicus & Repko 2003; Alcaniz & Pires 2004; Padmanabhan & Choudhury 2003; Zhu & Fujimoto 2003; Zhu et al. 2004) and use the latest SNe Ia data set that corresponds to the gold sample (157 events including 9 SNe at \( z > 1 \)) of Riess et al. (2004). In this analysis, both the characteristic angular size \( D \) and the Hubble parameter \( H_0 \) are considered “nuisance” parameters so that we marginalize over them. In the case of the \( \theta(z) \) test, the marginalization over the characteristic length can be easily done by defining a modified \( \chi^2 \) statistics as

\[
\tilde{\chi}^2 = -2 \ln \left[ \int_0^\infty \exp \left( -\frac{1}{2} \chi^2 \right) d\ell \right] = A - \frac{B^2}{C} + \ln \left( \frac{2C}{\pi} \right),
\]

where

\[
A = \theta_{\text{obs}}^2 \sum_{i=1}^n \frac{1}{\sigma_i^2},
\]

\[
B = -\frac{\theta_{\text{obs}}}{d_A} \sum_{i=1}^n \frac{1}{\sigma_i^2},
\]

\[
C = \frac{1}{d_A} \sum_{i=1}^n \frac{1}{\sigma_i^2}.
\]

For a similar procedure concerning the magnitude-redshift test, we refer the reader to Goliath et al. (2001) and Silva & Bertolami (2003).

Figures 4 and 5 show the results of our analysis. In Figures 4a and 4b we display contours of the combined likelihood analysis for the parametric space \( \alpha - A_\gamma \) in the context of CgCDM and UDME scenarios, respectively. For UDME models we see that the available parameter space is considerably modified when compared with Figure 3b, with the best-fit value for \( A_\gamma \) provided by the \( \theta(z) \) analysis, i.e., \( A_\gamma = 0.96 \) being off by \( \sim 3 \sigma \) relative to the joint analysis. This analysis also yields \( 0.65 < A_\gamma < 0.90 \) (95% c.l.) for UDME models and \( A_\gamma > 0.85 \) (95% c.l.) for CgCDM scenarios. These particular limits on \( A_\gamma \) are in good agreement with the ones obtained from quasar lensing statistics \( (A_\gamma = 0.72; \text{Dev et al. 2004}) \), the old SNe Ia data \( (A_\gamma = 0.87^{+0.15}_{-0.18}; \text{Fabris et al. 2002}; \text{Avelino et al. 2003b}) \), as well as from the location of the acoustic peaks of CMB as given by BOOMERANG and Archeops \( (0.57 \leq A_\gamma \leq 0.91 \text{ for } \alpha \leq 1; \text{Bento et al. 2003a, 2003b, 2003c}) \). However, they are only marginally compatible with the tight constraint obtained from the expected number of lensed radio source for the Cosmic Lens All-Sky Survey (CLASS) statistical data \( (\text{Dev et al. 2004}) \). Other interesting limits from this analysis are also obtained on the original version of Cg, i.e., by fixing \( \alpha = 1 \). In this case, the plane \( \Omega_m - A_\gamma \) (Fig. 5a) is reasonably constrained with the best-fit values located at \( A_\gamma = 0.81 \) and \( \Omega_m = 0.0 \) with \( \chi^2_{\text{min}}/\nu \approx 1.09 \).
This particular value of the matter density parameter agrees with the one found by Fabris et al. (2002) by using the old sample of SNe Ia from the High-z Supernova Project. Fabris et al. (2002) interpret such a result as a possible backup to the idea of dark matter–energy unification (UDME models). However, in the light of recent CMB data, unification from the original Cg (α = 1) seems to be quite unrealistic, since the location of the acoustic peaks as given by the Wilkinson Microwave Anisotropy Probe (WMAP) and BOOMERANG is in conflict with the predictions of this particular scenario (see, e.g., Carturan & Finelli 2003). Figure 5b shows the same analysis of Figure 5a by assuming the Gaussian prior on the matter density parameter, i.e., Ω_m = 0.27 ± 0.04 (Spergel et al. 2003). As can be seen, the parameter space is tightly constrained with the best-fit scenario located at Ω_m = 0.26, A_2 = 0.97, and χ_2/ν ≈ 1.1. In Table 1 we present the best-fit values for the Cg parameters obtained from the two main analyses performed in this paper.

4. CONCLUSION

Based on a large body of observational evidence, a consensus is beginning to emerge that we live in a flat, accelerated universe composed of ~1/3 of matter (baryonic + dark) and ~2/3 of a negative-pressure dark component. However, since the nature of these dark components (matter and energy) is not well understood, an important task nowadays in cosmology is to investigate the existing possibilities in light of the current observational data. In this paper we have focused our attention on some observational aspects of cosmologies driven by an exotic dark energy component called generalized Chaplygin gas (Cg). These models also constitute an interesting possibility of unification for dark matter and dark energy (where these two dark components are seen as different manifestations of a single fluid). Initially, we have investigated the influence of the Cg on the minimal redshift (z_min) at which the angular sizes of extragalactic sources takes their minimal values. From this analysis it was showed that the location of the minimal redshift is a much more sensitive function to the parameter A_1 than to the index α. By using a large sample of milliarcsecond radio sources recently updated and extended by Gurvits et al. (1999) along with the latest SNe Ia data as given by Riess et al. (2004), we obtained, as the best fit for these data, A_2 = 0.84 and α = 1.0 (UDME) and A_2 = 0.99 and α = 1.0 (CgCDM). Such values are in full disagreement with the CMB analysis by Amendola et al. (2003), which showed that Cg scenarios with α > 0.2 are ruled out by the current WMAP data. Finally, it should be remarked that the background results presented here may be somewhat modified if one takes into account the onset of the nonlinear regime, as recently discussed by Avelino et al. (2004).

The authors are very grateful to L. I. Gurvits for sending his compilation of the data. This work is supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil) and CNPq (62.0053/01-1-PADCT III/Milenio).

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References cited in the text are available from the authors on request.

TABLE 1

| Test    | UDME | CgCDM |
|---------|------|-------|
| θ(z)    | A_2  | α     |
| SnIa    | 0.84 | 1.0   |
| SnIa + θ | 0.84 | 1.0   |

The authors are very grateful to L. I. Gurvits for sending his compilation of the data. This work is supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brazil) and CNPq (62.0053/01-1-PADCT III/Milenio).
