Hilltop Quintessence

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We examine hilltop quintessence models, in which the scalar field is rolling near a local maximum in the potential, and \( w \approx -1 \). We first derive a general equation for the evolution of \( \phi \) in the limit where \( w \approx -1 \). We solve this equation for the case of hilltop quintessence to derive \( w \) as a function of the scale factor; these solutions depend on the curvature of the potential near its maximum. Our general result is in excellent agreement (\( \delta w \lesssim 0.5\% \)) with all of the particular cases examined. It works particularly well (\( \delta w \lesssim 0.1\% \)) for the pseudo-Nambu-Goldstone Boson potential. Our expression for \( w(a) \) reduces to the previously-derived slow-roll result of Sen and Scherrer in the limit where the curvature goes to zero. Except for this limiting case, \( w(a) \) is poorly fit by linear evolution in \( a \).

I. INTRODUCTION

Observational evidence [1, 2] suggests that approximately 70% of the energy density in the universe is in the form of an exotic, negative-pressure component, dubbed dark energy. (For a recent review, see, e.g., Ref. [3].) The observational bounds on the properties of the dark energy have continued to tighten. Taking \( w \) to be the ratio of pressure to density for the dark energy:

\[
 w = \frac{p_{DE}}{\rho_{DE}},
\]

recent observational constraints are typically \(-1.1 \lesssim w \lesssim -0.9\) when \( w \) is assumed constant (see, e.g., [4, 5] and references therein).

One possibility is that the dark energy is, in fact, merely a cosmological constant, with \( w \) exactly equal to \(-1\). This is the standard \( \Lambda \)CDM model, in which the universe today is roughly 30% matter, and 70% vacuum energy, with the latter having a constant density. In this case, we expect future observations to continue to converge on this value of \( w \). A second possibility is that \( w \) is close to \(-1\), but not exactly equal to it. In this case, an alternative explanation for the dark energy is required.

One possible model, dubbed quintessence, assumes that the dark energy arises from a minimally coupled scalar field; these models have been extensively explored. (See, for instance, Refs. [4, 7, 8, 9, 10] for some of the earliest discussions.) The equation of motion for the scalar field simplifies considerably in the limit where the background expansion is well-described by \( \Lambda \)CDM, which is the case whenever \( w \) for the quintessence field is always close to \(-1\). This fact was exploited in Ref. [11], which considered quintessence models in a potential satisfying the slow-roll conditions, i.e.,

\[
 \left( \frac{1}{V} \frac{dV}{d\phi} \right)^2 \ll 1,
\]

and

\[
 \frac{1}{V} \frac{d^2V}{d\phi^2} \ll 1.
\]

It was shown in this paper that the evolution of \( w(a) \) in these models, which we will call “slow-roll quintessence,” approaches a single functional form, given by the present-day values of \( w \) and \( \Omega_\phi \). (Note that the term “slow-roll quintessence” refers to the fact that the potential satisfies the slow-roll conditions; the well-known slow-roll approximation for inflation cannot be applied [11]. For alternative approaches to this problem, see Refs. [12, 13, 14].)

However, the slow-roll conditions given by Eqs. 2 and 3, while sufficient to give \( w \) near \(-1\) today, are not necessary. In this paper, we consider a second possibility: a scalar field rolling down near a local maximum in the potential. We assume that we are sufficiently close to the maximum that Eq. 2 still applies, but we relax our assumption in Eq. 3. In analogy with recent discussions of similar models in inflation [15, 16, 17], we call these models “hilltop quintessence.” A particularly important model that is well-described by this methodology is the pseudo-Nambu-Goldstone Boson (PNGB) model [18].

In the next section, we derive the version of the scalar field equation of motion that applies in the limit where the background expansion is well-approximated by a \( \Lambda \)CDM model. In Sec. III, we expand the potential for hilltop quintessence as a quadratic function near its maximum and solve this equation to derive the evolution of the scalar field and thus, the evolution of its equation of state, for generic hilltop quintessence models. We find that all of these models can be characterized by a single set of functional forms for \( w(a) \) that depends on the present-day values of \( \Omega_\phi \) and \( w \) for the quintessence, along with a parameter \( (K) \) that depends on the curvature of the scalar field potential at its maximum. This result is given in Eq. 31. Our results are discussed in Sec. IV.

II. SCALAR FIELD EVOLUTION IN THE \( \Lambda \)CDM LIMIT

Consider a minimally-coupled scalar field, \( \phi \), in a potential \( V(\phi) \). The density and pressure of the scalar field are given by

\[
 \rho = \frac{\dot{\phi}^2}{2} + V(\phi),
\]

\[
 p = \frac{\dot{\phi}^2}{2} - V(\phi).
\]
and
\[ p = \frac{\dot{\phi}^2}{2} - V(\phi), \]  
respectively, and the equation of state parameter, \( w \), is given by equation (1). The equation of motion for this field in an expanding background is
\[ \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \]
where the Hubble parameter \( H \) is given by
\[ H = \left( \frac{\dot{a}}{a} \right) = \sqrt{\frac{\rho_T}{3}}, \]
and we have assumed a flat universe. Here \( a \) is the scale factor, \( \rho_T \) is the total density, and we work in units for which \( 8\pi G = 1 \). The evolution of the scale factor is further described by
\[ \ddot{a} = -\frac{1}{3} \rho_T + 3\rho_T, \]
where \( \rho_T \) is the total pressure.

We first eliminate the \( \phi \) term in Eq. (11) by making the change of variables
\[ \phi(t) = u(t)/a(t)^{3/2}, \]
which gives
\[ \ddot{u} - \frac{3}{2} \left( \frac{\dot{a}}{a} + \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \right)u + a^{3/2}V'(u/a^{3/2}) = 0. \]  
Applying the expressions for \( \dot{a}/a \) and \( \ddot{a}/a \) from Eqs. (7) and (5) gives
\[ \ddot{u} + \frac{3}{4} p_T u + a^{3/2}V'(u/a^{3/2}). \]  
This equation is as yet exact.

Eq. (11) takes a particularly simple form for two special cases: a matter-dominated universe (for which \( p_T = 0 \)) and an LCDM universe (for which \( p_T \) is constant). In the former case, we obtain
\[ \ddot{u} + TV'(u/t) = 0, \]  
and it is easy to use this equation to derive, in a straightforward way, the results of Refs. 2 and 3. However, the subject of this paper is the second case.

We assume a universe containing matter and quintessence, but such that the quintessence always has \( w \) near \(-1\). In this limit, \( p_T \) is well-approximated by a constant: \( p_T \approx -\rho_{\phi 0} \), where \( \rho_{\phi 0} \) is the nearly constant density contributed by the quintessence in this limit. Then Eq. (11) becomes
\[ \ddot{u} - \frac{3}{4} \rho_{\phi 0} u + a^{3/2}V'(u/a^{3/2}) = 0. \]

We expect Eq. (13) to provide a good approximation to the evolution of any scalar field in the limit where the Hubble parameter in Eq. (3) is well-approximated by LCDM, i.e., any model in which the kinetic term in Eqs. (4) and (5) is dominated by the potential term, so that \( w \) for the scalar field never evolves very far from \(-1\). Eq. (13) applies, for example, to the limiting model examined in Ref. 11. We now apply it to a different class of models.

### III. Hilltop Scalar Field Evolution

We are interested in models in which the scalar field evolves near a local maximum in the potential. The most important model of this type is the PNGB model 18, for which the potential is given by
\[ V(\phi) = M^4 [\cos(\phi/f) + 1], \]
where \( M \) and \( f \) are constants. (For recent discussions of the PNGB model in the context of dark energy, see, e.g., Refs. 19, 20, 21 and references therein). Other, less-well-motivated models with a local maximum in the potential include the Gaussian potential,
\[ V(\phi) = M^4 e^{-\phi^2/\sigma^2}, \]
and the quadratic potential
\[ V(\phi) = V_0 - V_2 \phi^2. \]

Our purpose in this paper, however, is not to examine an exhaustive list of such models, but to show that they all converge to a common evolution under certain conditions.

We note that any model with a local maximum in the potential at \( \phi = \phi_* \) can be expanded, near this maximum, in the form
\[ V(\phi) = V(\phi_*) + (1/2)V''(\phi_*)\phi^2 + O(\phi^3) + \ldots. \]

This expansion will be a good approximation for the PNGB model when \( \phi \ll f \) and for the Gaussian model as long as \( \phi \ll \sigma \). It is exact for the quadratic potential. The evolution of \( \phi \) for this expansion (or, equivalently, for the quadratic potential of Eq. 16) was previously examined in Refs. 22, 23, 24, 25. The motivation in Ref. 22 was similar to our own investigation, while Refs. 23, 24, 25 were concerned with the future fate of the universe in such a model. The exact solution to the scalar field equation of motion for this potential is given in Ref. 22 for the matter-dominated case, while Refs. 23, 24, 25 gave the evolution for the scalar-field-dominated case (appropriate for the far future of the universe). Here we solve for the general case in which both matter and the scalar field contribute to the evolution, since this is the relevant case at low redshift.

Substituting the expansion of Eq. (17) into Eq. (13) (and taking \( \rho_{\phi 0} = V(\phi_*) \)) gives
\[ \ddot{u} + [V''(\phi_*) - (3/4)V(\phi_*)]u = 0. \]
If we define the constant $k$ to be given by

$$ k \equiv \sqrt{(3/4)V(\phi_*) - V''(\phi_*)}, $$

(19)

then the general solution to Eq. (18) is

$$ u = A \sinh(kt) + B \cosh(kt). $$

(20)

Now we assume that the scale factor is well-approximated by its value in the ΛCDM model, which is again a consequence of the assumption that $w$ is always close to $-1$:

$$ a(t) = \left[ \frac{1 - \Omega_{\phi 0}}{\Omega_{\phi 0}} \right]^{1/3} \sinh^{2/3}(t/t_\Lambda), $$

(21)

where $\Omega_{\phi 0}$ is the present-day value of $\Omega_{\phi}$, and $a = 1$ at present. The time $t_\Lambda$ is defined to be

$$ t_\Lambda = 2/\sqrt{3\rho_{\phi 0}} = 2/\sqrt{3V(\phi_*)}. $$

(22)

With this expression for $a(t)$, the general solution for $\phi(t)$ is given by

$$ \phi(t) = \left[ \frac{\Omega_{\phi 0}}{1 - \Omega_{\phi 0}} \right]^{1/2} A \sinh(kt) + \frac{B \cosh(kt)}{\sinh(t/t_\Lambda)}. $$

(23)

We require that at $t = 0$, $\phi$ is equal to a fixed initial value, $\phi_i$. This forces $B = 0$ and gives us the value of $A$, so that

$$ \phi = \frac{\phi_i}{kt_\Lambda \sinh(t/t_\Lambda)} \sinh(kt), $$

(24)

In the limit where $t \ll t_\Lambda$, our solution reduces to the matter-dominated solution in Ref. 22, while for $t \gg t_\Lambda$ we regain the scalar-field-dominated solution of Refs. 23, 24, 25.

The equation of state parameter for quintessence is given by

$$ 1 + w = \frac{\dot{\phi}^2}{\rho_\phi}. $$

(25)

Taking $\rho_\phi \approx \rho_{\phi 0} \approx V(\phi_*)$, Eqs. (21) and (25) yield

$$ 1 + w = \frac{3 \phi_i^2}{4k^2} \left[ \frac{k \cosh(kt) \sinh(t/t_\Lambda) - (1/t_\Lambda) \sinh(kt) \cosh(t/t_\Lambda)}{\sinh^2(t/t_\Lambda)} \right]^2. $$

(26)

We can normalize this expression to the present-day value of $w$, which we denote $w_0$, and we can use Eq. (21) to express $w$ as a function of the scale factor (or the redshift) rather than $t$. We obtain:

$$ 1 + w(a) = (1 + w_0)a^{-3} \sqrt{\Omega_{\phi 0} k t_\Lambda \cosh (kt(a)) \sinh (kt(a))}
- \sqrt{(1 - \Omega_{\phi 0})a^{-3} + \Omega_{\phi 0} \sinh (kt(a))}],
\frac{\sqrt{\Omega_{\phi 0} k t_\Lambda \cosh (kt_0) - \sinh (kt_0)]]}{\sqrt{\Omega_{\phi 0} k t_\Lambda \cosh (kt_0) - \sinh (kt_0)]]}}. $$

(27)

where $t(a)$ and $t_0$ can be derived from Eq. (21):

$$ t(a) = t_\Lambda \sinh^{-1} \left( \frac{\Omega_{\phi 0} a^3}{1 - \Omega_{\phi 0}} \right), $$

(28)

and

$$ t_0 = t_\Lambda \tanh^{-1} \left( \sqrt{\Omega_{\phi 0}} \right). $$

(29)

We now define the constant $K \equiv k t_\Lambda$. In terms of the quintessence potential, $K$ is just

$$ K = \sqrt{1 - (4/3)V''(\phi_*)/V(\phi_*)}. $$

(30)

Thus, $K$ depends only on the value of the potential and its second derivative at its maximum. Note that $V''(\phi_*) < 0$, so $K > 1$. Then Eq. (27) can be written as

$$ 1 + w(a) = (1 + w_0)a^{3(K - 1)} \frac{[(F(a) + 1)^K (K - F(a)) + (F(a) - 1)^K (K + F(a))]^2}{[(\Omega_{\phi 0}^{-1/2} + 1)^K (K - \Omega_{\phi 0}^{-1/2}) + (\Omega_{\phi 0}^{-1/2} - 1)^K (K + \Omega_{\phi 0}^{-1/2})]^2}, $$

(31)
potential, a set of models parametrized by the value of \( \Omega \).

For the case of slow-roll quintessence models, a similar analysis yields only a single form for \( w(a) \), once \( w_0 \) and \( \Omega_{s0} \) are fixed [11]. Here we have more complex behavior, since \( w(a) \) also varies with \( K \). This behavior is illustrated in Fig. 1, where we fix \( w_0 = -0.9 \) and \( \Omega_{s0} = 0.7 \) and then plot \( w(a) \) from Eq. (31) as a function of \( a \). When \( V''(\phi_0)/V(\phi_0) \gg 1 \), we see that the field evolution diverges significantly from slow-roll quintessence. In particular, \( w \) increases more slowly at high redshift, but much more rapidly at low redshift.

This is easily understood from the nature of these potentials: \( dV/d\phi \) increases as \( \phi \) rolls down the potential. In the limit where \( V''(\phi_0)/V(\phi_0) \to 0 \), corresponding to \( K \to 1 \), both slow-roll conditions (Eqs. 2 and 3) apply, and we expect the evolution to converge to the form derived in Ref. 11, namely

\[
1 + w = p \left[ F(a) - \left[ F(a) - 1 \right] \tanh^{-1} \left[ F(a) - 1 \right] \right]^2 \]  

where \( F(a) \) is given by

\[
F(a) = \sqrt{1 + (\Omega_{s0}^{-1} - 1)a^{-3}}. \tag{32}
\]

(Note that \( F(a) = 1/\sqrt{\Omega_\phi(a)} \), where \( \Omega_\phi(a) \) is the value of \( \Omega_\phi \) as a function of redshift, so that \( F(a = 1) = \Omega_\phi^{-1/2} \).

Eq. (31) is our main result. It shows that, in the limit where \( w \) is close to \(-1\) (i.e., the scalar field potential energy dominates the kinetic energy), all of the hilltop quintessence models with a given value of \( w_0 \) form a single set of models parametrized by the value of \( K \), which depends only on \( V''(\phi_\pm)/V(\phi_\pm) \). For the case of slow-roll quintessence models, a similar analysis yields only a single form for \( w(a) \), once \( w_0 \) and \( \Omega_{s0} \) are fixed [11]. Here we have more complex behavior, since \( w(a) \) also varies with \( K \). This behavior is illustrated in Fig. 1, where we fix \( w_0 = -0.9 \) and \( \Omega_{s0} = 0.7 \) and then plot \( w(a) \) from Eq. (31) as a function of \( a \). When \( V''(\phi_0)/V(\phi_0) \gg 1 \), we see that the field evolution diverges significantly from slow-roll quintessence. In particular, \( w \) increases more slowly at high redshift, but much more rapidly at low redshift.

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The agreement, in all three cases, between Eq. (31) and the exact numerical evolution is excellent, with errors \( \delta w \lesssim 0.5\% \) for the quadratic potential, \( \delta w \lesssim 0.1\% \) for the PNGB potential, and \( \delta w \lesssim 0.3\% \) for the Gaussian potential. Rather surprisingly, the errors for the quadratic potential (which is the basis of our approximation) are actually larger than for the other two potentials. However, this is due to the fact that we have made several approximations in our calculation; these tend to cancel for the PNGB and Gaussian potentials.

The agreement between Eq. (31) and the exact evolution is particularly striking for the case of the PNGB potential, which also happens to be the most interesting and well-motivated of the hilltop potentials. We have therefore examined this potential in more detail. In Fig.
our approximation for \(w(a)\) for hilltop quintessence (Eq. 31) with \(w_0 = -0.9\) and \(\Omega_{\phi 0} = 0.7\) and the exact (numerically-integrated) evolution for \(w(a)\) for the quadratic potential \(V = V_0 - V_2 \phi^2\). Red (dashed) curves give our approximation, and solid (blue) curves give exact evolution, for (left to right), \(K = 1.01, 2, 3, 4\), where \(K\) is defined by Eq. (30).

FIG. 3: As Fig. 2, for the PNGB potential, \(V = M^4 \cos(\phi/f) + 1\).

5, we extend our results to larger values of \(w_0\). It is interesting to see that the agreement remains excellent for \(w_0\) as large as \(-0.7\). Thus, Eq. (31) represents a nearly exact solution for \(w(a)\) for the PNGB model within a wide range for \(w_0\).

Finally, in Figs. 6-9, we use Eq. (31) to construct a \(\chi^2\) likelihood plot for \(w_0\) and \(\Omega_{\phi 0}\) with \(K = 1.01, 2, 3, 4\), using the recent Type Ia Supernovae standard candle data (ESSENCE+SNLS+HST from [2]). While none of these models is ruled out by current supernova data, it is interesting to note that the hilltop quintessence models \((K = 2, 3, 4)\) produce a larger allowed region than the slow-roll quintessence model \((K = 1.01)\), and the allowed region increases with increasing \(K\).

FIG. 4: As Fig. 2, for the Gaussian potential, \(V = M^4 e^{-\phi^2/\sigma^2}\).

FIG. 5: As Fig. 3, for the PNGB potential, extended to larger values of \(w_0\).

IV. DISCUSSION

Our results indicate that hilltop quintessence models with \(w\) near \(-1\) all produce a similar evolution for \(w(a)\), given by Eq. (31). The importance of this result lies in the fact that a very general set of models can be mapped onto a fairly constrained set of behaviors for \(w(a)\). Note that in general, the evolution given by Eq. (31) and shown in Fig. 1 is not well-described by the popular linear parametrization, \(w(a) = w_0 + w_a(1 - a)\) [26]. The one exception is the limiting case \(K \to 1\), where we regain the results of Ref. [11]: in this limit the evolution is roughly linear for \(a > 0.5\) \((z < 1)\), with \(w_a \approx -1.5(1 + w_0)\).

This investigation, along with Ref. [11], can also be thought of as a kind of Taylor expansion of the potential about the initial value of \(\phi\). The results of Ref. [11] apply when the linear term dominates, while the results presented here assume a quadratic expansion. The results of Ref. [11] are then a special case (albeit a very important special case) of the results presented here.

Of course, our results do not apply to all thawing
FIG. 6: Likelihood plot from SNIa data for the parameters $w_0$ and $\Omega_{\phi,0}$, for hilltop quintessence models with generic behavior described by Eq. (31), with $K = 1.01$, where $K$ is the function of the curvature of the potential at its maximum given in Eq. (29). The yellow (light) region is excluded at the 2σ level, and the orange (darker) region is excluded at the 1σ level. Red (darkest) region is not excluded at either confidence level.

Quintessence models with $w$ near $-1$, but only those satisfying Eq. (2). A different approach was taken recently by Cahn, et al. [14]. They looked at the evolution of the scalar field at early times, when $\rho_T \gg \rho_\phi$, while assuming nothing about the detailed nature of the potential. This allows the derivation of a generic result for the evolution of the scalar field before it dominates the expansion (see also Ref. [27] for the special case of tracker fields). Our results for the particular class of potentials considered here agree with those given in Ref. [14] (as they should) in the limit where $\Omega_\phi \ll 1$.

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