Influence of Roughness Parameter in Hydrodynamic Lubrication: A Special Case of Thrust Bearing

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Abstract. Basic hydrodynamic lubrication theory has been considered in the present work. In this project, Reynolds equation in polar coordinates (r, θ) was derived from the Navier-Stokes and then it has been augmented with appropriate boundary condition and solved using a notable numerical technique namely finite difference method with a fast solver Gauss-Seidel (with relaxation factor 1.0). In the present investigation, hydrodynamic pressure generation in an oil film has been computed without the embodiment of roughness factor first and then compared with the incorporation of roughness factor between the sliding surfaces. It was behold from this study that there is a significant change in the pressure generation. So its conspicuous from the present investigation, the lubricant performance also relies on roughness factor at large. Therefore, it is the fruitful information for the designer while designing the hydrodynamic bearings irrespective of their type it is imperative to consider roughness factor suitably.

1. Introduction

Bearings can be generally divided into four basic types: Rolling contact bearing were balls or rollers support the load. Hydrostatic were high pressure fluid is used to balance the load. Hydrodynamic were velocity of the fluid is used to develop a lubricant film which further is used to support the load and Magnetic bearings in which magnetic field generates the load supporting capacity[1]. Thrust bearings are essentially used to support axial load, in thin film lubrication and are preferred in large industries such as hydro-power plants. A tilting pad thrust bearing consists of number of annular segments, tilting independently on pivots [2]. Thrust bearings have the ability to operate at optimum film geometry under given set of input conditions. The top surface of the tilting pad is mainly made up of tin-based Babbitt lining, which is cast on the steel base metal. The pads are usually supported on the pivots or spring of mattresses [3]. Point pivots are mostly used for small loads while others (such as line pivots) are used for heavy duty applications. Operation of thrust bearing depends on the theory of hydrodynamic lubrication [4]. When a convergent space is formed between two surfaces in relative motion the lubricating oil (due to wedge action), squeezes oil out from the trailing edge and the side edges. The pressure at outer edges (around the boundary) is atmospheric, while the pressure in the interior of oil film rises (to a higher level) resulting in pressure field which helps in raising the runner while the load applied pushes it down. The runner and tilting pads achieve stability at a typical height or film thickness (at which these two forces are equal). Viscous shearing of lubricating oil results in generation of heat.
This raises the temperature of oil, which causes lower the value of its viscosity. The change in the viscosity significantly reduces the pressure generation in the oil film. Various Studies have been done on thrust bearing. Performance of thrust bearing is varying with the curvature of thrust pad [6]. Also, Film thickness and film thickness play an important role in bearing performance [7]. The shape of circumferential edge also influences the pressure. A converging wedge influencing the performance of the thrust bearing has been reported [8]. Numerical investigation of thrust bearing at constant film thickness and viscosity has been performed [9]. Figure 1 shows various forms of hydrodynamic bearings.

2. Mathematical Modeling

Reynolds equation is employed to study the pressure distribution in an oil film when viscosity is kept constant. It can be developed from hydrodynamic lubrication theory, when a fluid element is taken into consideration. Earlier, Reynolds equation have been reported for sector shaped thrust bearing [10]. In present work the lubricant inside the thrust bearing is considered to be Newtonian and incompressible. The flow is considered to be steady state with changes on viscosity to be negligible. The Osborne Reynold developed the theoretical model for hydrodynamic lubrication which was further used to develop the mathematical equation. Certain assumptions were made by the Reynold:

The oil or lubricant is assumed to be Newtonian, Laminar and the shear stress between the flow layers is proportional to the velocity gradient in the direction perpendicular to the flow (Newton’s law of viscosity):

$$\tau = \mu \frac{\partial u}{\partial x}$$

The fluid is considered to be incompressible, inertial forces acting on the lubricant under acceleration are neglected. Change in fluid pressure along the direction perpendicular to the laminar flow is zero and the lubricating film is also thin: $dp(dy) = 0$. The viscosity is assumed to be constant for the lubrication film. Hydrodynamic pressure profile is shown in figure 2[4].
Let us consider the unit volume of lubricant film under the equilibrium conditions. Since the flow is considered to be laminar, the shear forces act along the upper and lower surfaces of the unit volume taken under consideration. These shear forces are due to the relative motion of the laminar layers. Assuming, The flow is only along the x and y direction, then the equilibrium condition can be from the fig (3):

\[ p dydz - (p + dp) dydz - \tau dx dz + (\tau + d\tau) dy dz = 0 \]
\[ dp dy - d\tau dx = 0 \]
\[ \frac{dp}{dx} = \frac{d\tau}{dy} \]

Substituting the value of \( \tau \) from Newton’s law of viscosity and we get:

\[ \frac{dp}{dx} = \mu \frac{\delta^2 y}{\delta x^2} \]

Similarly,
\[
\frac{dp}{dz} = \mu \frac{\partial^2 w}{\partial y^2}
\]

Integrating with respect to \( Y \), we get,
\[
\frac{dp}{dx} \cdot y = \mu \left[ \frac{du}{dy} + Ay \right]
\]

Integrate again with respect to \( Y \), we have,
\[
\frac{dp}{dx} \cdot y^2 = \mu \left[ u + Ay^2 + B \right]
\] (1)

Apply boundary conditions we get,

When, \( y = 0, u = U_1 \) and \( w = W_1 \)

\( B = \mu U_1 \)

When, \( y = h, u = U_2 \) and \( w = W_2 \)

\( A = \mu [U_2 - U_1] - \frac{dp}{dx} \frac{h^2}{2} \) (2)

Using the Value of \( A \) and \( B \) in Equation (1), We get

\[
U = -\frac{1}{2\mu} \frac{dp}{dx} \cdot y(h - y) + \left[ (1 - \frac{y}{h})U_1 + \frac{y}{h}U_2 \right]
\] (3)

\[
W = -\frac{1}{2\mu} \frac{dp}{dx} \cdot y(h - y) + \left[ (1 - \frac{y}{h})W_1 + \frac{y}{h}W_2 \right]
\] (4)

In the calculations, it is assumed that the pressure \( p \) is constant in \( y \) direction. The latter half of RHS of the equation (6.5) shows the flowed velocity of the particles in \( x \) direction. It changes linearly and it is assumed that \( U_2 = 0 \), this is called shear flow or coquette flow. The other half of the RHS shows pressure gradient flow velocity. It is infinite pressure and changes parabolically across the film thickness. This is called pressure flow or poiseuille flow [11].

From continuity equation, we have

\[ \nabla \cdot u = 0 \]

Integrate along \( y \) axis within the limits from \( y = 0 \) to \( y = h \), we have,
\[
\int_0^h \frac{\partial u}{\partial x} \, dy + \int_0^h \frac{\partial v}{\partial y} \, dy + \int_0^h \frac{\partial w}{\partial z} \, dy = 0
\] (5)

Using Leibnitz rule we have,
\[
\frac{d}{dx} \int_0^v f(x, t) \, dt = f(x, v) \frac{dv}{dx} - f(x, u) \frac{dv}{dx} + \int_0^v f(x, t) \, dt
\]
Therefore equation (5) becomes,
\[
\frac{\partial}{\partial x} \int_0^h U dy - U_2 \frac{dh}{dx} + \frac{\partial}{\partial z} \int_0^h W dy - U_1 \frac{dh}{dz} + V_2 - V_1 = 0 \quad (6)
\]

We know that,
\[
q_{xss} = \frac{\partial}{\partial x} \int_0^h U dy = - \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} + \frac{h}{2} (U_1 + U_2) \quad (7)
\]
\[
q_z = \frac{\partial}{\partial z} \int_0^h W dy = - \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} + \frac{h}{2} (W_1 + W_2) \quad (8)
\]

Using the values of equations (7) and (8) in equation (6), we get,
\[
- \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} + \frac{h}{2} (U_1 - U_2) - U_2 \frac{dh}{dx} - \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} + \frac{h}{2} (W_1 - W_2) - U_1 \frac{dy}{dx} + (V_2 - V_1) \quad (9)
\]

On rearranging the equation (9) we get,
\[
\frac{\partial}{\partial x} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right] = \frac{1}{2} \frac{\partial}{\partial x} \{(U_2 - U_1)h\} + \frac{1}{2} \frac{\partial}{\partial z} \{(W_2 - W_1)h\} \quad (10)
\]

In many practical cases, the x axis can be taken as the direction of relative motion of the two surfaces in which case we have \(V_1 = W_1 = W_2 = 0\), the above equation becomes
\[
\frac{\partial}{\partial x} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right] = \frac{1}{2} \frac{\partial}{\partial x} (U_2 - U_1) \frac{dh}{dx} + \frac{1}{2} \frac{\partial}{\partial z} (U_1 - U_2) + V_2 \quad (11)
\]

Furthermore, if two surfaces are rigid, density, expansion and contraction becomes zero, then
\[
\frac{\partial}{\partial x} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right] = \frac{1}{2} \frac{\partial}{\partial x} (U_2 - U_1) \frac{dh}{dx} \quad (12)
\]

This is the pressure equation derived based on Reynold’s assumptions known as Reynold’s equation. Equation (12) is the generalized Reynold’s equation because they were derived within the rigid body assumptions and in viscous assumptions along x and z directions respectively. If the flow is one dimensional, then \(U_1 = V_2 = 0\)
\[
\frac{\partial}{\partial x} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right] = \frac{1}{2} \frac{\partial}{\partial x} (U_2 - U_1) \frac{dh}{dx} \quad (13)
\]

In cylindrical coordinates, equation (14) can be written as
\[
\frac{\partial}{\partial \theta} \left[ \frac{r h^3}{\mu} \frac{\partial p}{\partial \theta} \right] + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \frac{h^3}{12 \mu} \frac{\partial p}{\partial x} \right] = 6 \rho \omega \frac{\partial h}{\partial \theta} \quad (14)
\]

This is Reynolds equation in cylindrical form.

The obtained differential equation for Reynolds Equation is discretised using proper tools of FDM on the surface of pad on thrust bearing to find the pressure distribution in lubricant oil film.

This Reynolds equation can be converted into non dimensional form by the following substitution[9],
\[
R^* = \frac{r}{R_0}; \theta^* = \theta; H^* = \frac{h}{h_0}; \mu^* = \frac{\mu}{\mu_1}; P^* = \frac{P h_0^2}{12 \pi h_0^2 \mu_1 R_0^2};
\]

The non-dimensional form of Reynolds equation is shown below:
\[
\frac{\partial}{\partial R^*} \left( \frac{H^3}{\mu^*} R^* \frac{\partial P^*}{\partial R^*} \right) + \frac{1}{R^*} \frac{\partial}{\partial \theta^*} \left( \frac{H^3}{\mu^*} \frac{\partial P^*}{\partial \theta^*} \right) = R^* \frac{\partial H^*}{\partial \theta^*}
\]

2.1 Equation for film thickness

For discretization of the non-dimensional form of Reynolds Equation we need to introduce film thickness and Load carrying capacity. This study involves sector shaped pad. The film thickness in \((r - \theta)\) is expressed as[8]:

\[
H = H_0 + H_s \left[ 1 - \frac{\theta}{\theta_t} \right]
\]  \tag{15}

Converting above equation into non dimensional form by dividing the above equation by \(H_0\), we get

\[
H^* = 1 + \frac{H_s}{H_0} \left[ 1 - \frac{\theta}{\theta_t} \right]
\]  \tag{16}

Where, \(H_0\) = Minimum oil film thickness (\(\mu m\))

\(H_s\) = Amount of taper

\(\theta_t\) = Angular extent of pad in degrees

\(H^*\) = Non dimensional oil film thickness

In non- dimensional form the load carrying capacity is given by [12]

\[
LCC = \frac{W}{K \rho_0} R_0^{\theta_t} \int_0^{\theta_t} \left( \frac{P R}{\mu^*} \right) d\theta dR
\]  \tag{17}

Where, \(P\) = hydrodynamic pressure \((\frac{N}{m^2})\)

\(R_0\) = Outer Radius of pad (m)

\(R_t\) = Inner Radius of pad (m)

\(W\) = Load on bearing (KN)

\(K\) = Convergence ratio

Convergence ratio can be written as;

\[
K = \frac{H_s + H_0}{H_0}
\]

Finite difference method is used as treatment of dimensionless Reynolds equation for discretization of sector shaped bearing pad on different grid sizes (M x N) and convergence ratios. Taylor series expansion of dimensionless Reynolds equation gives the finite difference linear algebraic equation. The system of finite difference equations arising from the five-point second-order central difference approximation of the Laplace equation is always diagonally dominant. Therefore, convergence is assured when the iterative methods are applied on the finite difference approach to the solutions of PDEs. The solution is obtained by iterating with the relevant boundary conditions and hence Nodal
pressure is computed. In this way the algebraic linear finite difference is obtained therefore we get a discretised non-dimensionalised Reynolds Equation, which is as follows:

\[
P_{i+1}^* \left[ \frac{3H_i^2}{\mu_i^*R_{i,j}^*} \left( \frac{H_{i+1,j}^2 - H_{i-1,j}^2}{4\Delta\theta^2} \right) \right] + P_{i-1,j}^* \left[ \frac{H_{i,j}^2}{\mu_i^*R_{i,j}^*} \left( \frac{\mu_{i+1,j}^* - \mu_{i-1,j}^*}{4\Delta\theta^2} \right) \right] + P_{i,j+1}^* \left[ \frac{H_{i,j}^2}{\Delta R^2 \mu_i^*} \left( \frac{\mu_{i,j+1}^* - \mu_{i,j-1}^*}{4\Delta R^2} \right) \right] + P_{i,j-1}^* \left[ \frac{H_{i,j}^2}{\Delta R^2 \mu_i^*} \left( \frac{\mu_{i,j+1}^* - \mu_{i,j-1}^*}{2\Delta R^*} \right) \right] = 0
\]

Now we incorporate the roughness parameter (which is a dimensionless factor) into the Non dimensional Reynolds equation in circumferential coordinate system. The pressure profiles with radius and theta are obtained. The study is done on various grids and hence, Change is obtained. The purpose of this study is to develop a mathematical model for Steady state Reynolds equation when roughness parameter is included. The Mathematical Model is obtained by following steps:

\[
\frac{\partial}{\partial R^*} \left( \frac{H^3}{\mu^*} R^* \frac{\partial p^*}{\partial R^*} \right) + \frac{1}{R^*} \frac{\partial}{\partial \theta} \left( \frac{H^3}{\mu^*} \frac{\partial p^*}{\partial \theta} \right) = R^* \frac{\partial H^*}{\partial R^*}
\]

(19)

Or

\[
\frac{\partial}{\partial R^*} \left( \frac{H^3}{\mu^*} \frac{\partial p^*}{\partial R^*} \right) + \frac{1}{R^*} \frac{\partial}{\partial \theta} \left( \frac{H^3}{\mu^*} \frac{\partial p^*}{\partial \theta} \right) = 6 \frac{\partial H^*}{\partial \theta}
\]

(20)

The new factors[13,14] \( \alpha \) and \( \beta \) are introduced were

\[
\alpha = H^3 - \frac{2}{3} H^* C^2
\]

(21)

\[
\beta = H^3 + \frac{1}{3} H^* C^2
\]

(22)

Taking derivatives of (21) and (22),

\[
\frac{\partial \alpha}{\partial R^*} = 3H^2 - \frac{2}{3} C^2
\]

(23)

\[
\frac{\partial \beta}{\partial R^*} = 3H^2 + \frac{1}{3} C^2
\]

(24)

Using these derivates in above equations we get,

\[
\frac{\partial^2 p^*}{\partial R^*^2} + \frac{\partial p^*}{\partial R^*} \frac{\partial \alpha}{\partial R^*} + \frac{1}{R^*} \frac{\partial}{\partial \theta} \left( \frac{\partial^2 p^*}{\partial \theta^2} \right) + \frac{1}{R^*} \frac{\partial^2 p^*}{\partial \theta^2} \frac{\partial \beta}{\partial \theta} = 6 \frac{\partial H^*}{\partial \theta}
\]

(25)

Applying FDM Scheme:

\[
2P_{i,j}^* \left[ \frac{H^3 - \frac{2}{3} H^* C^2}{(\Delta \theta)^2} \right] + \frac{1}{3} H^3 C^2 \left( \frac{P_{i,j+1}^* + P_{i,j-1}^*}{\Delta R^2} \right) + \left( 3H^2 - \frac{2}{3} C^2 \right) \left( \frac{H_{i+1,j}^* - H_{i-1,j}^*}{2\Delta \theta} \right) \left( \frac{P_{i,j+1}^* - P_{i,j-1}^*}{2\Delta R^*} \right) - 6 \left( \frac{H_{i+1,j}^* - H_{i-1,j}^*}{2\Delta \theta} \right)
\]

(26)
The above equation is the linear finite difference equation for Reynolds Equation when roughness ‘C’ is incorporated. The algorithm works on constant film thickness of 75µm and High tolerance limit. Mathematically Tolerance limit is set as:

\[
\sum_{i=2}^{M-1} \sum_{j=2}^{N-1} \left| \frac{p_{ij}^{new} - p_{ij}^{old}}{p_{ij}^{new}} \right| \leq \varepsilon_r
\]  

(27)

\(\varepsilon_r\) is called the tolerance limit and for efficiency of the algorithm, it is kept less or equal to 0.001

### 3. Results and discussions

The variation of pressure is analysed at various positions along the sector shaped pad of the thrust bearing. Study is performed on six grids and it has been observed that with grid refinement (i.e. increase in grid size) peak pressure is reduced. Keeping the dimensions constant (radius and angle), for a specific roughness factor, the peak pressure is reduced. The schematic representation of variation in peak pressure at the constant radius (inner and outer) and angle is shown in table 1.

| Roughness ('C') | Angle (degrees) | Outer Radius (r_o) (m) | Inner Radius (r_i) (m) | Non-dimensional Peak Pressure |
|-----------------|-----------------|------------------------|------------------------|-------------------------------|
|                 | 0               | .11430                 | .05715                 | Grid (21x21)                  |
|                 | 0.5             | .11430                 | .05715                 | Grid (41x41)                  |
|                 | 1.0             | .11430                 | .05715                 | Grid (61x61)                  |
|                 | 1.5             | .11430                 | .05715                 | Grid (81x81)                  |
|                 |                 |                        |                        | Grid (101x101)                |
|                 |                 |                        |                        | Grid (121x121)                |

The data shown in the table 1 can be shown on pressure plots for comparative analysis. For roughness factor equal to zero and angle 50 degrees at outer radius equal to 0.11430 m and inner radius equal to 0.05715 m, the pressure plot for grid size 21x21 and 121x121 is shown in figure 4 and figure 5.
Figure 4. Non dimensional Pressure distribution on grid size 21x21 with C = 0, \( \theta = 50^\circ \), \( r_i = 0.05715 \) m and \( r_o = 0.11430 \) m.

Figure 5. Non dimensional Pressure distribution on grid size 121x121 with C = 0, \( \theta = 50^\circ \), \( r_i = 0.05715 \) m and \( r_o = 0.11430 \) m.

Since the calculations are non-dimensional so we can represent \( r, \theta \) on any of the two axes (X, Y) while the pressure is represented by Z-axis. From figure 4 and figure 5, it can be observed that the peak pressure is reduced due to the grid refinement. With the increase in grid size, the peak pressure is shifted towards the leading edge. The graphical representation of variation of pressure with change in grid size at C=0 is shown in figure 6.
The peak pressure is also altered by the change in roughness factor. Considering the specific group (61x61), the pressure profiles for roughness factor $C = 0$, $C = 1.5$ are shown in Fig 7 and Fig 8 respectively. Peak pressure has been reduced due to the incorporation and increase in roughness factor while keeping the radius and angle constant.

**Figure 6.** Variation of Pressure with change in grid size when $C = 0$, $	heta = 50^\circ$, $r_i = 0.05715$ m and $r_o = 0.11430$ m.

**Figure 7.** Non dimensional Pressure distribution on grid size 61x61 with $C = 0$, $	heta = 50^\circ$, $r_i = 0.05715$ m and $r_o = 0.11430$ m.
Figure 8. Non dimensional Pressure distribution on grid size 61x61 with $C = 1.5$, $\theta = 50^\circ$, $r_i = 0.05715$ m and $r_o = 0.11430$ m.

From figure 7 and 8 it is clear that peak pressure of a certain grid is reduced due to increase in Roughness factor. The graphical representation of variation of pressure with change in grid size at $C=1.5$ is shown in figure 9.

Figure 9 and Figure 6 shows that roughness factor has certainly reduced the pressure in all the grids. The Peak Pressure can also be influenced by the geometry of thrust bearing. The study is carried in two steps. First study is carried with the change in angle with constant roughness and radius. The schematic representation of pressure variation with change angle at constant radius and constant roughness factors are shown in given in tables below:
Table 2. Non-dimensional Peak Pressure at Various Grids with change in angle at constant radius, C

| Roughness 'C' | Angle(θ) (degrees) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|---------------|-----------------|-----------------|-----------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 0             | 50              | .11430          | .05715          | 0.0047       | 0.0042       | 0.0036       | 0.0030       | 0.0025         | 0.0021         |
| 0             | 55              | .11430          | .05715          | 0.0046       | 0.0041       | 0.0035       | 0.0029       | 0.0024         | 0.0020         |
| 0             | 60              | .11430          | .05715          | 0.0045       | 0.0041       | 0.0035       | 0.0029       | 0.0024         | 0.0020         |
| 0             | 65              | .11430          | .05715          | 0.0044       | 0.0040       | 0.0034       | 0.0028       | 0.0024         | 0.0021         |

Table 3. Non-dimensional Peak Pressure at Various Grids with change in angle at constant radius, C

| Roughness 'C' | Angle(θ) (degrees) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|---------------|-----------------|-----------------|-----------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 0.5           | 50              | .11430          | .05715          | 0.0046       | 0.0041       | 0.0035       | 0.0029       | 0.0024         | 0.0020         |
| 0.5           | 55              | .11430          | .05715          | 0.0045       | 0.0041       | 0.0035       | 0.0029       | 0.0024         | 0.0020         |
| 0.5           | 60              | .11430          | .05715          | 0.0044       | 0.0040       | 0.0034       | 0.0028       | 0.0024         | 0.0020         |
| 0.5           | 65              | .11430          | .05715          | 0.0043       | 0.0039       | 0.0034       | 0.0028       | 0.0023         | 0.0019         |

Table 4. Non-dimensional Peak Pressure at Various Grids with change in angle at constant radius, C

| Roughness 'C' | Angle(θ) (degrees) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|---------------|-----------------|-----------------|-----------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 1.0           | 50              | .11430          | .05715          | 0.0044       | 0.0040       | 0.0034       | 0.0028       | 0.0023         | 0.0019         |
| 1.0           | 55              | .11430          | .05715          | 0.0043       | 0.0039       | 0.0033       | 0.0028       | 0.0023         | 0.0019         |
Table 5. Non-dimensional Peak Pressure at Various Grids with change in angle at constant radius, C

| Roughness ‘C’ | Angle(°) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|---------------|----------|-----------------------|-----------------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 1.5           | 50       | 0.11430               | 0.05715               | 0.0041       | 0.0037       | 0.0032       | 0.0026       | 0.0022         | 0.0018         |
| 1.5           | 55       | 0.11430               | 0.05715               | 0.0041       | 0.0036       | 0.0031       | 0.0026       | 0.0021         | 0.0018         |
| 1.5           | 60       | 0.11430               | 0.05715               | 0.0040       | 0.0036       | 0.0030       | 0.0025       | 0.0021         | 0.0017         |
| 1.5           | 65       | 0.11430               | 0.05715               | 0.0039       | 0.0035       | 0.0030       | 0.0025       | 0.0020         | 0.0017         |

**Figure 10(a).** Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 0 when r_i = 0.05715 m, r_o = 0.11430 m and θ = 50°

**Figure 10(b).** Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 0 when r_i = 0.05715 m, r_o = 0.11430 m and θ = 65°
Figure 10(c). Non-dimensional Pressure distribution Contour for Grid size 21x21 at $C = 1.5$ when $r_i = 0.05715$ m, $r_o = 0.11430$ m and $\theta = 50^\circ$.

Figure 10(d). Non-dimensional Pressure distribution Contour for Grid size 21x21 at $C = 1.5$ when $r_i = 0.05715$ m, $r_o = 0.11430$ m and $\theta = 65^\circ$.

Figure 10 clearly shows that Peak pressure is reduced with the increase in angle of the thrust bearing. Above study is carried on grid 21x21 with constant radius. For different roughness factors and grid sizes as shown in Table 3,4,5 and figure 10(c), 10(d), same trend is observed. It must be noted that all the observations are carried on constant film thickness. Both the change in angle and roughness factor influence the pressure profile and maximum pressure in the thrust bearing. The location of peak pressure and area under the peak pressure is also varied. Further investigation is carried on the effect of radius change on Peak Pressure with constant angle at different roughness factors. Grid study is also incorporated. The below tables represent the variation of peak pressure at constant angle different C’s.

Table 6. Non-dimensional Peak Pressure at Various Grids with change in radius at constant angle, C

| Roughness Factor | Outer Radius ($r_o$) (m) | Inner Radius ($r_i$) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|------------------|--------------------------|--------------------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 0                | 50                       | .11430 (.05715)          | 0.0047       | 0.0042       | 0.0036       | 0.0030       | 0.0025         | 0.0021         |
| 0                | 50                       | .13430 (.07715)          | 0.0040       | 0.0036       | 0.0031       | 0.0026       | 0.0021         | 0.0018         |
| 0                | 50                       | .15430 (.09715)          | 0.0034       | 0.0031       | 0.0026       | 0.0022       | 0.0018         | 0.0015         |
| 0                | 50                       | .17430 (.11715)          | 0.0029       | 0.0026       | 0.0023       | 0.0019       | 0.0016         | 0.0013         |
Table 7. Non-dimensional Peak Pressure at Various Grids with change in radius at constant angle, C

| Roughness ‘C’ | Angle(θ) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|--------------|----------|----------------------|----------------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 0.5          | 50       | .11430               | .05715               | 0.0046       | 0.0041       | 0.0035       | 0.0029       | 0.0024         | 0.0020         |
| 0.5          | 50       | .13430               | .07715               | 0.0039       | 0.0035       | 0.0030       | 0.0025       | 0.0021         | 0.0017         |
| 0.5          | 50       | .15430               | .09715               | 0.0033       | 0.0030       | 0.0026       | 0.0020       | 0.0018         | 0.0015         |
| 0.5          | 50       | .17430               | .11715               | 0.0028       | 0.0026       | 0.0022       | 0.0019       | 0.0015         | 0.0013         |

Table 8. Non-dimensional Peak Pressure at Various Grids with change in radius at constant angle, C

| Roughness ‘C’ | Angle(θ) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|--------------|----------|----------------------|----------------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 1.0          | 50       | .11430               | .05715               | 0.0041       | 0.0037       | 0.0032       | 0.0026       | 0.0022         | 0.0018         |
| 1.0          | 50       | .13430               | .07715               | 0.0035       | 0.0032       | 0.0027       | 0.0022       | 0.0018         | 0.0015         |
| 1.0          | 50       | .15430               | .09715               | 0.0030       | 0.0027       | 0.0023       | 0.0019       | 0.0016         | 0.0013         |
| 1.0          | 50       | .17430               | .11715               | 0.0025       | 0.0023       | 0.0019       | 0.0016       | 0.0013         | 0.0011         |

Table 9. Non-dimensional Peak Pressure at Various Grids with change in radius at constant angle, C

| Roughness ‘C’ | Angle(θ) | Outer Radius(r_o) (m) | Inner Radius(r_i) (m) | Grid (21x21) | Grid (41x41) | Grid (61x61) | Grid (81x81) | Grid (101x101) | Grid (121x121) |
|--------------|----------|----------------------|----------------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 1.5          | 50       | .11430               | .05715               | 0.0041       | 0.0037       | 0.0032       | 0.0026       | 0.0021         | 0.0018         |
| 1.5          | 50       | .13430               | .07715               | 0.0035       | 0.0032       | 0.0027       | 0.0022       | 0.0018         | 0.0015         |
| 1.5          | 50       | .15430               | .09715               | 0.0030       | 0.0027       | 0.0023       | 0.0019       | 0.0016         | 0.0013         |
| 1.5          | 50       | .17430               | .11715               | 0.0025       | 0.0023       | 0.0019       | 0.0016       | 0.0013         | 0.0011         |
Figure 11(a). Non dimensional Pressure distribution Profile for $C = 0$, $\theta = 50^\circ$, $r_o = .11430$, $r_i = .07715$ and grid size (21x21)

Figure 11(b). Non dimensional Pressure distribution Profile for $C = 0$, $\theta = 50^\circ$, $r_o = .17430$, $r_i = .11715$ and grid size (21x21)

Figure 11(c). Non dimensional Pressure distribution Profile for $C = 1.5$, $\theta = 50^\circ$, $r_o = .11430$, $r_i = .13430$ and grid size (21x21)

Figure 11(d). Non dimensional Pressure distribution Profile for $C = 1.5$, $\theta = 50^\circ$, $r_o = .17430$, $r_i = .11715$ and grid size (21x21)
Figure 12(a). Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 0 when ri = 0.07715m, ro = 0.11430 m and $\theta = 50^\circ$

Figure 12(b). Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 0 when ri = .11715m, ro = .17430m and $\theta = 50^\circ$

Figure 12(c). Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 1.5 when ri = 0.07715m, ro = 0.11430 m and $\theta = 50^\circ$

Figure 12(d). Non-dimensional Pressure distribution Contour for Grid size 21x21 at C = 1.5 when ri = .11715m, ro = .17430m and $\theta = 50^\circ$

The similar trend of pressure variation is observed in figure 12 as in figures 11, 5, 6, 7. After reaching the maximum pressure, the pressure drops to zero. Both entering and leaving points have the zero pressure. Also, with the increase of inner and outer radius in the calibrated way, Pressure drop has been Reported. After the incorporation of roughness, the peak pressure area is reduced to the large extent. Hence roughness, angle and radius are the three important parameters in hydrodynamic lubrication.
4. Conclusion
Hydrodynamic lubrication study is performed on thrust bearing with full scale pressure generation. Basic fluid element is taken into consideration to develop the governing equation of fluid flow inside the thrust bearing. The equation is further tailored and a dimensionless factor ‘C’ known as roughness factor is introduced. FDM is employed to break the second order partial differential equation into the finite linear equation using proper and stable boundary conditions. It has been seen that with the increase in the mesh size, the accuracy of the results is improved. The Non dimensional peak pressure is reduced due to grid refinement. This has been observed in both the cases, with or without the roughness factor ‘C’. The pressure reduces gradually when the roughness is increased from 0 to 1.5 when radius and angle are kept constant. It has also been seen that pressure gets reduced with the increase in angle ‘θ’ from 50 degrees to 65 in constant manner. Peak pressure areas have also been reduced due to increase in angle and roughness factor. Inner and outer radius of the sector shaped are also the important parameters of pressure distribution. It has been seen that with the increase in inner and outer radius, the pressure gets decreased. It is can be seen from the tables and plots that pressure is most effected by the combination of roughness and increase in radius rather than the combination of roughness and increase in the pad angle. In the radius varying study, ‘C’ can put the major effect on pressure up to certain value. Here major effect can be seen up to C =1 beyond that the change very minute or negligible.

5. References
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