CONFORMAL PHASE TRANSITION IN QCD LIKE THEORIES
AND BEYOND

V. A. Miransky∗

Department of Applied Mathematics, University of Western Ontario,
London, Ontario N6A 5B7, Canada
∗E-mail: vmiransk@uwo.ca

The dynamics with an infrared stable fixed point in the conformal window in QCD like theories with a relatively large number of fermion flavors is reviewed. The emphasis is on the description of a clear signature for the conformal window, which in particular can be useful for lattice computer simulations of these gauge theories.

Keywords: conformal phase transition; infrared fixed point; scaling law for hadron masses; light glueballs.

1. Introduction

The Landau, or σ-model-like, phase transition is characterized by the following basic feature. Around the critical point \( z = z_c \) (where \( z \) is a generic notation for parameters of a theory, as the coupling constant \( \alpha \), number of particle flavors \( N_f \), etc.), an order parameter \( X \) is

\[
X = \Lambda f(z),
\]

where \( \Lambda \) is an ultraviolet cutoff and the function \( f(z) \) has such a non-essential singularity at \( z = z_c \) that \( \lim_{z \to z_c} f(z) = 0 \) as \( z \) goes to \( z_c \) both in symmetric and non-symmetric phases. The standard form for \( f(z) \) is \( f(z) \sim (z - z_c)^\nu \), \( \nu > 0 \), around \( z = z_c \) [for convenience, we assume that \( z > z_c \) (\( z < z_c \)) in the nonsymmetric (symmetric) phase\]. The conformal phase transition (CPhT), whose conception was introduced in Ref. is a very different continuous phase transition. It is defined as a phase transition in which an order parameter \( X \) is given by Eq. (1) where \( f(z) \) has such an essential singularity at \( z = z_c \) that while

\[
\lim_{z \to z_c} f(z) = 0
\]
as \( z \) goes to \( z_c \) from the side of the non-symmetric phase, \( \lim f(z) \neq 0 \) as \( z \to z_c \) from the side of the symmetric phase (where \( X \equiv 0 \)). Notice that since the relation

∗ Strictly speaking, Landau considered the mean-field phase transition. By the Landau phase transition, we understand a more general class, when fields may have anomalous dimensions.
(1) In the case of the LPhT, masses of light excitations are continuous functions of the parameters $z$ around the critical point $z = z_c$ (though they are non-analytic at $z = z_c$). In the case of the CPhT, the situation is different: there is an abrupt change of the spectrum of light excitations, as the critical point $z = z_c$ is crossed. This implies that the effective actions describing low energy dynamics in the phases with $z < z_c$ and $z > z_c$ are different in a system with CPhT.

(2) Unlike the LPhT, the parameter $z$ governing the CPhT is connected with a marginal operator [in the LPhT phase transition, such a parameter is connected with a relevant operator; it is usually a mass term].

(3) The fact that the parameter $z$ is connected with a marginal operator in the CPhT implies that in the continuum limit, when $z \to z_c + 0$, the conformal symmetry is broken by a marginal operator in nonsymmetric phase, i.e., there is a conformal anomaly.

(4) Unlike the LPhT, in the case of CPhT, the structures of renormalizations (i.e., the renormalization group at high momenta) are different in symmetric phase and nonsymmetric one.

In relativistic field theory, the CPhT is realized in the two dimensional Gross-Neveu (GN) model at the critical coupling constant $g_c = 0$, reduced (or defect) QED and quenched QED. It was suggested that the chiral phase transition with respect to the number of fermion flavors $N_f$ in QCD is a CPhT one. In condensed matter physics, a CPhT like phase transition is realized in the Berezinskii-Kosterlitz-Thouless (BKT) mode and, possibly, graphene.

Recently, the interest to the dynamics with the CPhT phase transition has essentially increased. It is in particular connected with a progress in numerical lattice studies of gauge theories with a varied number of fermion flavors (for a recent review, see Ref. [15]), the revival of the interest to the electroweak symmetry breaking based on the walking technicolor like dynamics (for a recent review, see Ref. [18]), and intensive studies of graphene, a single atomic layer of graphite (for a review, see Ref. [19]).

2. Dynamics in the conformal window in QCD-like theories

2.1. General description

In this section, we will consider the problem of the existence of a nontrivial conformal dynamics in 3+1 dimensional non-supersymmetric vector like gauge theories, with a relatively large number of fermion flavors $N_f$. We will discuss their phase diagram in the $(\alpha(0), N_f)$ plane, where $\alpha(0)$ is the bare coupling constant. We also discuss a
clear signature for the conformal window in lattice computer simulations of these theories suggested quite time ago in Ref. [20].

The roots of this problem go back to a work of Banks and Zaks [21] who were first to discuss the consequences of the existence of an infrared-stable fixed point $\alpha = \alpha^*$ for $N_f > N_f^\ast$ in vector-like gauge theories. The value $N_f^\ast$ depends on the gauge group: in the case of SU(3) gauge group, $N_f^\ast = 8$ in the two-loop approximation. In Nineties, a new insight in this problem [3,12] was, on the one hand, connected with using the results of the analysis of the Schwinger-Dyson (SD) equations describing chiral symmetry breaking in quenched QED [8–11] and, on the other hand, with the discovery of the conformal window in $N = 1$ supersymmetric QCD [23].

In particular, Appelquist, Terning, and Wijewardhana [12] suggested that, in the case of the gauge group SU($N_c$), the critical value $N_f^\ast \simeq 4N_c$ separates a phase with no confinement and chiral symmetry breaking ($N_f > N_f^\ast$) and a phase with confinement and with chiral symmetry breaking ($N_f < N_f^\ast$). The basic point for this suggestion was the observation that at $N_f > N_f^\ast$ the value of the infrared fixed point $\alpha^*$ is smaller than a critical value $\alpha_{cr} \simeq \frac{4N_c}{N_c^2 - 1} \frac{\pi}{3}$, presumably needed to generate the chiral condensate [8–11].

The authors of Ref. [12] considered only the case when the running coupling constant $\alpha(\mu)$ is less than the fixed point $\alpha^*$. In this case the dynamics is asymptotically free (at short distances) both at $N_f < N_f^\ast$ and $N_f^\ast < N_f < N_f^{**} \equiv \frac{11N_c}{2}$. Yamawaki and the author [13] analyzed the dynamics in the whole $(\alpha(0), N_f)$ plane and suggested the $(\alpha(0), N_f)$-phase diagram of the SU($N_c$) theory, where $\alpha(0)$ is the bare coupling constant (see Fig 1 below) [14]. In particular, it was pointed out that one can get an interesting non-asymptotically free dynamics when the bare coupling constant $\alpha(0)$ is larger than $\alpha^*$, though not very large.

The dynamics with $\alpha(0) > \alpha^*$ admits a continuum limit and is interesting in itself. Also, its better understanding can be important for establishing the conformal window in lattice computer simulations of the SU($N_c$) theory with such large values of $N_f$. In order to illustrate this, let us consider the following example. For $N_c = 3$ and $N_f = 16$, the value of the infrared fixed point $\alpha^*$ calculated in the two-loop approximation is small: $\alpha^* \approx 0.04$. To reach the asymptotically free phase, one needs to take the bare coupling $\alpha(0)$ less than this value of $\alpha^*$. However, because of large finite size effects, the lattice computer simulations of the SU(3) theory with such a small $\alpha(0)$ would be unreliable. Therefore, in this case, it is necessary to consider the dynamics with $\alpha(\mu) > \alpha^*$.

In Ref. [20] this author suggested a clear signature of the existence of the infrared fixed point $\alpha^*$, which in particular can be useful for lattice computer simulations. The signature is based on two characteristic features of the spectrum of low energy excitations in the presence of a bare fermion mass in the conformal window:

b) a strong (and simple) dependence of the masses of all the colorless bound states

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[1] This phase diagram is different from the original Banks-Zaks diagram [21].
(including glueballs) on the bare fermion mass, and b) unlike QCD with a small \( N_f \) (\( N_f = 2 \) or 3), glueballs are lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large.

2.2. Phase diagram

The phase diagram in the \( (\alpha^{(0)}, N_f) \)-plane in the \( SU(N_c) \) gauge theory is shown in Fig. 1. The left-hand portion of the curve in this figure coincides with the line of the infrared-stable fixed points \( \alpha^*(N_f) \):\(^{22}\)

\[
\alpha^{(0)} = \alpha^* = -\frac{b}{c},
\]

where

\[
b = \frac{1}{6\pi}(11N_c - 2N_f),
\]

\[
c = \frac{1}{24\pi^2}(34N_c^2 - 10N_cN_f - 3\frac{N_c^2 - 1}{N_c}N_f).
\]

It separates two symmetric phases, \( S_1 \) and \( S_2 \), with \( \alpha^{(0)} < \alpha^* \) and \( \alpha^{(0)} > \alpha^* \), respectively. Its lower end is \( N_f = N_f^\text{cr} \) (with \( N_f^\text{cr} \approx 4N_c \) if \( \alpha_{\text{cr}} \approx \frac{1}{N_c} \)): at \( N_f^* < N_f < N_f^\text{cr} \) the infrared fixed point is washed out by generating a dynamical fermion mass (here \( N_f^* \) is the value of \( N_f \) at which the coefficient \( c \) in Eq. (5) becomes positive and the fixed point disappears).

The horizontal, \( N_f = N_f^\text{cr} \), line describes a phase transition between the symmetric phase \( S_1 \) and the phase with confinement and chiral symmetry breaking. As it was suggested in Ref.12 based on a similarity of this phase transition with that in quenched \( QED_4 \), there is the following scaling law for \( m_{\text{dyn}}^2 \):

\[
m_{\text{dyn}}^2 \sim \Lambda_{\text{cr}}^2 \exp \left( \frac{C}{\frac{\alpha^*(N_f)}{\alpha_{\text{cr}}^2} - 1} \right),
\]

where the constant \( C \) is of order one and \( \Lambda_{\text{cr}} \) is a scale at which the running coupling is of order \( \alpha_{\text{cr}} \). It is a CPhT phase transition with an essential singularity at \( N_f = N_f^\text{cr} \).

At last, the right-hand portion of the curve on the diagram occurs because at large enough values of the bare coupling, spontaneous chiral symmetry breaking takes place for any number \( N_f \) of fermion flavors. This portion describes a phase transition called a bulk phase transition in the literature, and it is presumably a first order phase transition.\(^\text{3}\) The vertical line ends above \( N_f = 0 \) since in pure gluodynamics there is apparently no phase transition between weak-coupling and strong-coupling phases.

\textsuperscript{3} The fact that spontaneous chiral symmetry breaking takes place for any number of fermion flavors, if \( \alpha^{(0)} \) is large enough, is valid at least for lattice theories with Kogut-Susskind fermions. Notice however that since the bulk phase transition is a lattice artifact, the form of this portion of the curve can depend on the type of fermions used in simulations.
2.3. Signature for the conformal window

Up to now we have considered the case of a chiral invariant action. But how will the dynamics change if a bare fermion mass term is added in the action? This question is in particular relevant for lattice computer simulations: for studying a chiral phase transition on a finite lattice, it is necessary to introduce a bare fermion mass. As was pointed out in Ref. [20], adding even an arbitrary small bare fermion mass results in a dramatic changing the dynamics both in the $S_1$ and $S_2$ phases.

Recall that in the case of confinement SU($N_c$) theories, with a small, $N_f < N_f^c$, number of fermion flavors, the role of a bare fermion mass $m(0)$ is minor if $m(0) \ll \Lambda_{QCD}$ (where $\Lambda_{QCD}$ is a confinement scale). The only relevant consequence is that massless Nambu-Goldstone pseudoscalars get a small mass (the PCAC dynamics).

The reason for that is the fact that the scale $\Lambda_{QCD}$, connected with a conformal anomaly, describes the breakdown of the conformal symmetry connected both with perturbative and nonperturbative dynamics: the running coupling and the formation of bound state. Certainly, a small bare mass $m(0) \ll \Lambda_{QCD}$ is irrelevant for the dynamics of those bound states.

Now let us turn to the phases $S_1$ and $S_2$, with $N_f > N_f^c$. There is still the conformal anomaly in these phases: because of the running of the effective coupling
constant, the conformal symmetry is broken. It is restored only if $\alpha(0)$ is equal to the infrared fixed point $\alpha^*$. However, the essential difference with respect to confinement theories is that this conformal anomaly have nothing to do with the dynamics forming bound states: Since at $N_f > N_f^*$ the effective coupling is relatively weak, it is impossible to form bound states from massless fermions and gluons (recall that the $S_1$ and $S_2$ phases are chiral invariant).

Therefore the absence of a mass for fermions and gluons is a key point for not creating bound states in those phases. The situation changes dramatically if a bare fermion mass is introduced: indeed, even weak gauge, Coulomb-like, interactions can easily produce bound states composed of massive constituents, as it happens, for example, in QED, where electron-positron (positronium) bound states are present. To be concrete, let us consider the case when all fermions have the same bare mass $m^{(0)}$. It leads to a mass function $m(q^2) \equiv B(q^2)/A(q^2)$ in the fermion propagator $G(q) = (\hat{q}A(q^2) - B(q^2))^{-1}$. The current fermion mass $m$ is given by the relation

$$m(q^2)|_{q^2=m^2} = m.$$  

For the clearest exposition, let us consider a particular theory with a finite cutoff $\Lambda$ and the bare coupling constant $\alpha^{(0)} = \alpha(q)|_{q=\Lambda}$ being not far away from the fixed point $\alpha^*$. Then, the mass function is changing in the “walking” regime with $\alpha^{(0)}(q)^{-\frac{1}{2}}$.

Eqs. (7) and (8) imply that

$$m \simeq \Lambda \left( \frac{m^{(0)}}{\Lambda} \right)^{\frac{1}{1+\gamma_m}}.$$  

Recall that the anomalous dimension $\gamma_m \geq 0$, and $\gamma_m \lesssim 2$ in the “walking” regime.

There are two main consequences of the presence of the bare mass:

(a) bound states, composed of fermions, occur in the spectrum of the theory. The mass of a $n$-body bound state is $M^{(n)} \simeq nm$. Therefore they satisfy the scaling

$$M^{(n)} \simeq nm \sim n \left( m^{(0)} \right)^{\frac{1}{1+\gamma_m}}.$$  

(b) At low momenta, $q < m$, fermions and their bound states decouple. There is a pure $SU(N_c)$ Yang-Mills theory with confinement. Its spectrum contains glueballs.

To estimate glueball masses, notice that at momenta $q < m$, the running of the coupling is defined by the parameter $\tilde{b}$ of the Yang-Mills theory,

$$\tilde{b} = \frac{11}{6\pi} N_c.$$  

Therefore the glueball masses $M_{gl}$ are of order

$$\Lambda_{YM} \simeq m \exp\left(-\frac{1}{b\alpha^*}\right).$$  \hspace{1cm} (12)\]

For $N_c = 3$, we find from Eqs.(11), (5), and (11) that $\exp(-\frac{1}{b\alpha^*})$ is $6 \times 10^{-7}$, $2 \times 10^{-2}$, $10^{-1}$, and $3 \times 10^{-1}$ for $N_f=16$, 15, 14, and 13, respectively. Therefore at $N_f=16$, 15 and 14, the glueball masses are essentially lighter than the masses of the bound states composed of fermions.

The situation is similar to that in confinement QCD with heavy (nonrelativistic) quarks, $m >> \Lambda_{QCD}$. However, there is now a new important point. In the conformal window, any value of $m^{(0)}$ (and therefore $m$) is "heavy": the fermion mass $m$ sets a new scale in the theory, and the confinement scale $\Lambda_{YM}$ (12) is less, and rather often much less, than this scale $m$. One could say that the latter plays a role of a dynamical ultraviolet cutoff for the pure YM theory.

This leads to a spectacular “experimental” signature of the conformal window in lattice computer simulations: the masses of all colorless bound states, including glueballs, decrease as $(m^{(0)})^{\frac{1}{1+\gamma_m}}$ with the bare fermion mass $m^{(0)}$ for all values of $m^{(0)}$ less than cutoff $\Lambda$. Moreover, one should expect that glueball masses are lighter than the masses of the bound states composed of quarks.

Few comments are in order:

(1) The phases $S_1$ and $S_2$ have essentially the same long distance dynamics. They are distinguished only by their dynamics at short distances: while the dynamics of the phase $S_1$ is asymptotically free, that of the phase $S_2$ is not. Also, while around the infrared fixed point $\alpha^*$ the sign of the beta function is negative in $S_1$, it is positive in $S_2$. When all fermions are massive (with the current mass $m$), the continuum limit $\Lambda \to \infty$ of the $S_2$-theory is a non-asymptotically free confinement theory. Its spectrum includes colorless bound states composed of fermions and gluons. For $q < m$ the running coupling $\alpha(q)$ is the same as in pure SU($N_c$) Yang-Mills theory, and for all $q \gg m \alpha(q)$ is very close to $\alpha^*$ (“walking”, actually, “standing” dynamics). For those values $N_f$ for which $\alpha^*$ is small (as $N_f=16$, 15 and 14 at $N_c=3$), glueballs are much lighter than the bound states composed of fermions. Notice that unlike the case with $m = 0$, corresponding to the unparticle dynamics, there exists a conventional S-matrix in this theory.

(2) In order to get the clearest exposition, we assumed such estimates as $N_f^{cr} \simeq 4N_c$ for $N_f^{cr}$ and $\gamma_m = 1 - \sqrt{1 - \frac{\alpha^*}{\alpha_{cr}}}$ for the anomalous dimension $\gamma_m$. While the latter should be reasonable for $\alpha^* < \alpha_{cr}$ (and especially for $\alpha^* << \alpha_{cr}$), the former is based on the assumption that $\alpha_{cr} \simeq \frac{3N_c}{2N_c - 1}$ which, though seems reasonable, might be crude for some values of $N_c$. It is clear however that the dynamical picture presented above is essentially independent of those assumptions.
2.4. Lattice computer simulations

During last two years, there has been an essential progress in the lattice computer simulations of gauge theories with a varied number of fermion flavors.\(^7\) For a recent review, see Ref. 15 and the papers of Tom Appelquist, George Fleming, Kieran Holland, Julius Kuti, Maria Lombardo, and Donald Sinclair in this volume.

This author is certainly not an expert in lattice computer simulations. Here I would like to discuss this topic only in the connection with the phase diagram and the signature of the conformal window considered in the Secs. 2.2 and 2.3 above.

In Ref. 28, based on the fact that the sign of the beta function changes from negative to positive when the line between the \(S_1\) and \(S_2\) phases is crossed, the existence of the conformal window in QCD with \(N_f = 12\) was studied by using the measurements of the chiral condensate and the mass spectrum. The analysis supports the existence of the conformal window in this theory.

In Ref. 29, the scaling law (10) was rediscovered and applied to the study of the conformal window in the SU(3) lattice gauge theory with two flavors of color sextet fermions (the parameter \(y_m\) in Ref. 29 is connected with the anomalous dimension \(\gamma_m\) as \(y_m = 1 + \gamma_m\)). The main conclusion of that study was that \(y_m \sim 1.5\) \((\gamma_m \sim 0.5)\). This value is smaller than \(\gamma_m \approx 1\) in walking technicolor and at this moment it is unclear whether this theory contains an infrared-stable fixed point.

The authors of Ref. 30 studied the spectrum of mesons and glueballs in the SU(2) lattice gauge theory with adjoint fermions. They found that for light constituent fermions the lightest glueballs are lighter than the lightest mesons. It is tempting to speculate that in accordance with the signature for the conformal window discussed above, this fact indicates on the existence of a infrared fixed point in this theory. However, as the authors point out, a lot of issues should still be clarified in order to reach a solid conclusion.

It is clear that lattice simulations of gauge theories with varied numbers of fermion flavors are crucial for further progress in our understanding of such dynamics. The important point is that CPhT is a long range interactions phenomenon, which is very sensitive to any screening and finite-size effects. The progress made in this area during last two years is certainly encouraging.

3. Final comments

At present the interest to the dynamics with an infrared stable fixed point in the conformal window and related issues, such as the conformal phase transition, is quite high. In such a brief review, it would be impossible to describe all the recent developments in this area. Let me just mention the following ones, intimately related to the issues discussed above:

a) The holographic (gauge/gravity duality) approach to describe the conformal phase transition have been recently considered in Refs. 6,7.

\(^7\)For pioneer papers in this area, see Refs. 25, 27.
b) Nonperturbative approaches to the calculation of the beta function in QCD with different numbers of fermion flavors $N_f$ were developed in Refs. 31,32.

c) QCD with different $N_f$ was studied by using renormalization group flaw equations in Ref. 33.

d) The problem of the existence of the conformal window in QCD on $R^3 \times S^1$ has been analyzed in Ref. 34.

e) Lattice computer simulations of the phase diagram of graphene have been recently realized in Refs. 35,36.

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