A Remark on Non-conformal Non-supersymmetric Theories with Vanishing Vacuum Energy Density

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Abstract

We discuss non-conformal non-supersymmetric large \( N \) gauge theories with vanishing vacuum energy density to all orders in perturbation theory. These gauge theories can be obtained via a field theory limit of Type IIB D3-branes embedded in orbifolded space-times. We also discuss gravity in this setup.
I. INTRODUCTION

The cosmological constant problem is probably the most severe problem of the “naturalness and hierarchy” type. Its smallness compared with the energy scales believed to be “more” fundamental is puzzling. However, perhaps even more puzzling is the fact that generically in known non-supersymmetric field theories loop corrections to the vacuum energy density are expected to be as large as $\sim M_\text{SUSY}^4$, where $M_\text{SUSY}$ is the supersymmetry breaking scale.

In principle, when discussing the cosmological constant problem, one must include gravity. This makes the problem even more complicated, in particular, to treat gravity quantum mechanically at present we must appeal to string theory, where supersymmetry breaking is still poorly understood. However, even if we treat gravity as non-dynamical, the problem is still non-trivial. Thus, imagine that we have an interacting renormalizable four-dimensional field theory with supersymmetry broken at some scale $M_\text{SUSY}$, which can be consistently coupled to gravity. Suppose now we treat gravity purely classically (that is, we choose an appropriate gravitational background, and ignore fluctuations around it), and wish to compute loop contributions to the vacuum energy density coming from diagrams where only the field theory states run in the loops. Generically, that is, without fine-tuning, we still expect these loop contributions\(^1\) to be as large as $\sim M_\text{SUSY}^4$, with the possible exception of conformal field theories, where such contributions could vanish.

The purpose of this note is to present examples of non-conformal non-supersymmetric gauge theories with vanishing vacuum energy density to all orders in perturbation theory. The theories we discuss here are not realistic as they are large $N$ gauge theories. Nonetheless, at least to the best of our knowledge, they are the first examples of the aforementioned type. The key fact that enables one to show that in these theories the vacuum energy density vanishes to all orders in perturbation theory is that these gauge theories can be obtained via a field theory limit of Type IIB D3-branes embedded in orbifolded space-times. One then can use the power of string perturbation techniques to prove a non-renormalization theorem for the vacuum energy density.

This is precisely where ’t Hooft’s large $N$ limit\(^2\) becomes crucial. In this limit the gauge theory diagrams are organized in terms of Riemann surfaces, where each extra handle on the surface suppresses the corresponding diagram by $1/N^2$. The large $N$ expansion, therefore, resembles perturbative expansion in string theory. In the case of four-dimensional gauge theories this connection can be made precise in the context of type IIB string theory in the presence of a large number $N$ of D3-branes\(^3\). Thus, we consider a limit where $\alpha' \to 0$, $g_s \to 0$ and $N \to \infty$, while keeping $\lambda \equiv N g_s$ fixed, where $g_s$ is the type IIB string coupling. Note that in this context a world-sheet with $g$ handles and $b$ boundaries is weighted with

\[ (N g_s)^b g_s^{2g-2} = \lambda^{2g-2+b} N^{-2g+2} . \] (1)

\(^1\)Note that if there is a condensate in a field theory such as the Higgs condensate in the Standard Model, we will have additional contributions to the vacuum energy density $\sim M_*^4$, where $M_*$ is the scale of the condensate.

\(^2\)The generalization of this setup in the presence of orientifold planes was discussed in\(^3\).
Once we identify $g_s = g_{YM}^2$, this is the same as the large $N$ expansion considered by 't Hooft. Note that for this expansion to make sense we must keep $\lambda$ at a small value $\lambda < 1$. In this regime we can map the string diagrams directly to (various sums of) large $N$ Feynman diagrams. Note, in particular, that the genus $g = 0$ planar diagrams dominate in the large $N$ limit.

If the space transverse to the D3-branes in the setup of [3] is $\mathbb{R}^6$, then we obtain the $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory on the D3-branes, which is conformal. On the other hand, we can also consider orbifolds of $\mathbb{R}^6$, which leads to gauge theories with reduced supersymmetry. As was shown in [2], if we cancel all twisted tadpoles in such models, in the large $N$ limit the corresponding $\mathcal{N} = 0, 1, 2$ gauge theories are conformal. Moreover, in the planar limit the (on-shell) correlation functions in such theories are the same as in the parent $\mathcal{N} = 4$ gauge theory [2].

Recently, a generalization of the setup of [2] was discussed in [8], which allows one to obtain non-conformal gauge theories by allowing some twisted tadpoles to be non-vanishing. In particular, we can have consistent embeddings of non-conformal gauge theories if we allow logarithmic tadpoles, which correspond to the twisted sectors with fixed point loci of real dimension two. Thus, even though the corresponding string backgrounds are not finite (in the sense that we have logarithmic ultra-violet divergences), they are still consistent as far as the gauge theories are concerned, and the divergences correspond to the running in the four-dimensional gauge theories on the D3-branes.

Using the setup of [8], we can construct non-conformal non-supersymmetric large $N$ gauge theories with vanishing vacuum energy density to all orders in perturbation theory. In fact, in the planar limit the (on-shell) correlation functions in these theories are the same as in the parent $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetric non-conformal gauge theories. In the former case these gauge theories are not renormalized beyond one loop.

The remainder of this paper is organized as follows. In section II we review the setup of [8]. In section III we discuss non-conformal non-supersymmetric large $N$ gauge theories which can be constructed within this setup, and prove the non-renormalization theorem for the vacuum energy density. Section IV contains some concluding remarks, in particular, we discuss gravity in this setup.

II. SETUP

In this section we review the setup of [8]. Thus, consider type IIB string theory in the presence of $N$ coincident D3-branes with the space transverse to the D-branes $\mathcal{M} = \mathbb{R}^6/\Gamma$. The orbifold group $\Gamma = \{g_a | a = 1, \ldots, |\Gamma|\}$ ($g_1 = 1$) must be a finite discrete subgroup of $\Gamma$.

3Note that if $\lambda > 1$, then no matter how large $N$ is, for sufficiently many boundaries the higher genus terms become relevant, and we lose the genus expansion. In fact, in this regime one expects an effective supergravity description to take over as discussed in [4–6].

4The $\lambda > 1$ versions of these orbifold theories via the compactifications of type IIB on AdS$_5 \times (S^5/\Gamma)$ (where $\Gamma$ is the orbifold group) were originally discussed in [7].
Spin(6). If $\Gamma \subset SU(3)(SU(2))$, we have $\mathcal{N} = 1$ ($\mathcal{N} = 2$) unbroken supersymmetry, and $\mathcal{N} = 0$, otherwise.

We will confine our attention to the cases where type IIB on $\mathcal{M}$ is a modular invariant theory\textsuperscript{5}. The action of the orbifold on the coordinates $X_i$ ($i = 1, \ldots, 6$) on $\mathcal{M}$ can be described in terms of $SO(6)$ matrices: $g_a : X_i \rightarrow (g_a)_{ij}X_j$. The world-sheet fermionic superpartners of $X_i$ transform in the same way. We also need to specify the action of the orbifold group on the Chan-Paton charges carried by the D3-branes. It is described by $N \times N$ matrices $\gamma_a$ that form a representation of $\Gamma$. Note that $\gamma_1$ is an identity matrix, and $\text{Tr}(\gamma_1) = N$.

The D-brane sector of the theory is described by an oriented open string theory. In particular, the world-sheet expansion corresponds to summing over oriented Riemann surfaces with arbitrary genus $g$ and arbitrary number of boundaries $b$, where the boundaries of the world-sheet are mapped to the D3-brane world-volume. Moreover, we must consider various “twists” around the cycles of the Riemann surface. The choice of these “twists” corresponds to a choice of homomorphism of the fundamental group of the Riemann surface with boundaries to $\Gamma$.

For example, consider the one-loop vacuum amplitude ($g = 0, b = 2$). The corresponding graph is an annulus whose boundaries lie on D3-branes. The annulus amplitude is given by

$$C = \int_0^{\infty} \frac{dt}{t} \mathcal{Z}.$$  \hspace{1cm} (2)

The one-loop partition function $\mathcal{Z}$ in the light-cone gauge is given by

$$\mathcal{Z} = \frac{1}{|\Gamma|} \sum_a \text{Tr} \left( g_a \frac{1 - (-1)^F}{2} e^{-2\pi tL_0} \right),$$  \hspace{1cm} (3)

where $L_0$ is the light-cone Hamiltonian, $F$ is the fermion number operator, $t$ is the real modular parameter on the cylinder, and the trace includes sum over the Chan-Paton factors. The states in the Neveu-Schwarz (NS) sector are space-time bosons and enter the partition function with weight +1, whereas the states in the Ramond (R) sector are space-time fermions and contribute with weight −1.

The elements $g_a$ acting in the Hilbert space of open strings act both on the left end and the right end of the open string. This action corresponds to $\gamma_a \otimes \gamma_a$ acting on the Chan-Paton indices. The partition function \textsuperscript{3}, therefore, has the following form:

$$\mathcal{Z} = \frac{1}{|\Gamma|} \sum_a (\text{Tr}(\gamma_a))^2 \mathcal{Z}_a,$$  \hspace{1cm} (4)

where $\mathcal{Z}_a$ are characters corresponding to the world-sheet degrees of freedom. The “untwisted” character $\mathcal{Z}_1$ is the same as in the $\mathcal{N} = 4$ theory for which $\Gamma = \{1\}$. The information about the fact that the orbifold theory has reduced supersymmetry is encoded in the “twisted” characters $\mathcal{Z}_a$, $a \neq 1$.

\textsuperscript{5}This is always the case if there are some unbroken supersymmetries. If all supersymmetries are broken, this is also true if $\not\exists \mathbb{Z}_2 \subset \Gamma$. If $\exists \mathbb{Z}_2 \subset \Gamma$, then modular invariance requires that the set of points in $\mathbb{R}^6$ fixed under the $\mathbb{Z}_2$ twist has real dimension 2.
A. Tadpole Cancellation

In this subsection we discuss one-loop tadpoles arising in the above setup. As was pointed out in [2], if all tadpoles are canceled, then the resulting theory is finite in the large $N$ limit. However, not all tadpoles need to be canceled to have a consistent four-dimensional gauge theory. In fact, we can obtain non-conformal gauge theories if we allow such tadpoles. The characters $Z_a$ in (4) are given by

$$Z_a = \frac{1}{(8\pi^2\alpha' t)^2} \left[ \frac{1}{\eta(e^{-2\pi t})} \right]^{2+d_a} \left[ X_a(e^{-2\pi t}) - Y_a(e^{-2\pi t}) \right],$$

(5)

where $d_a$ is the real dimension of the set of points in $\mathbb{R}^6$ fixed under the twist $g_a$. The factor of $(8\pi^2\alpha' t)^2$ in the denominator comes from the bosonic zero modes corresponding to four directions along the D3-brane world-volume. Two of the $\eta$-functions come from the oscillators corresponding to two spatial directions along the D3-brane world-volume (the time-like and longitudinal contributions are absent as we are working in the light-cone gauge). The other $d_a \eta$-functions come from the oscillators corresponding to the directions transverse to the D-branes untouched by the orbifold twist $g_a$. Finally, the characters $X_a, Y_a$ correspond to the contributions of the world-sheet fermions, as well as the world-sheet bosons with $g_a$ acting non-trivially on them (for $a \neq 1$):

$$X_a = \frac{1}{2} \text{Tr}' \left[ g_a e^{-2\pi t L_0} \right],$$

(6)

$$Y_a = \frac{1}{2} \text{Tr}' \left[ g_a (-1)^F e^{-2\pi t L_0} \right],$$

(7)

where prime in $\text{Tr}'$ indicates that the trace is restricted as described above.

For the annulus amplitude we therefore have

$$C = \frac{1}{(8\pi^2\alpha' t)^2} \left[ \sum_a \left[ A_a - B_a \right] \right],$$

(8)

where

$$A_a = (\text{Tr} (\gamma_a))^2 \int_0^\infty dt t^3 \left[ \frac{1}{\eta(e^{-2\pi t})} \right]^{2+d_a} X_a(e^{-2\pi t}),$$

(9)

$$B_a = (\text{Tr} (\gamma_a))^2 \int_0^\infty dt t^3 \left[ \frac{1}{\eta(e^{-2\pi t})} \right]^{2+d_a} Y_a(e^{-2\pi t}).$$

(10)

These integrals are generically divergent as $t \to 0$ reflecting the presence of tadpoles. To extract these divergences we can change variables $t = 1/\ell$ so that the divergences correspond to $\ell \to \infty$.

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6For space-time supersymmetric theories the total tadpoles vanish: $A_a - B_a = 0$. (The entire partition function vanishes as the numbers of space-time bosons and fermions are equal.) For consistency, however, we must extract tadpoles from individual contributions $A_a$ and $B_a$. Thus, for instance, cancellation of certain tadpoles coming from $B_a$ is required for consistency of the equations of motion for the twisted R-R four-form which couples to D3-branes (see below).
\[ A_a = (\text{Tr}(\gamma_0))^2 \int_0^\infty \frac{d\ell}{\ell^{d_a/2}} \sum_{\sigma_a} N_{\sigma_a} e^{-2\pi \ell \sigma_a}, \]
\[ B_a = (\text{Tr}(\gamma_0))^2 \int_0^\infty \frac{d\ell}{\ell^{d_a/2}} \sum_{\rho_a} N_{\rho_a} e^{-2\pi \ell \rho_a}. \]

The closed string states contributing to \( A_a \) (or \( B_a \)) in the transverse channel are the NS-NS (R-R) states with \( L_0 = T_0 = \sigma_a(\rho_a) \) (and \( N_{\sigma_a}(N_{\rho_a}) > 0 \) is the number of such states). The massive states with \( \sigma_a(\rho_a) > 0 \) do not lead to divergences as \( \ell \to \infty \). On the other hand, the divergence property of the above integrals in the \( \ell \to \infty \) limit is determined by the value of \( d_a \). Given the orientability of \( \Gamma \) the allowed values of \( d_a \) are 0, 2, 4, 6. For \( d_1 = 6 \) there is no divergence in \( B_1 \), so we have no restriction for \( \text{Tr}(\gamma_1) = N \). For \( d_a = 4 \) the corresponding twisted NS-NS closed string sector contains tachyons. This leads to a tachyonic divergence in \( A_a \) unless \( \text{Tr}(\gamma_0) = 0 \) for the corresponding \( g_a \) twisted sector. Next, if a twist \( g_a \) with \( d_a = 2 \) (\( d_a = 0 \)) is non-supersymmetric, that is, if \( g_a \in \text{Spin}(6) \) but \( g_a \not\in \text{SU}(2) \) (\( g_a \not\in \text{SU}(3) \)), then we have tachyons in the corresponding NS-NS twisted sector, so we must require \( \text{Tr}(\gamma_0) = 0 \) to avoid a tachyonic divergence. Finally, if a twist \( g_a \) with \( d_a = 2 \) (\( d_a = 0 \)) is supersymmetric, that is, if \( g_a \in \text{SU}(2) \) (\( g_a \in \text{SU}(3) \)), then we have massless states in the corresponding R-R as well as NS-NS twisted sectors, so we get divergences due to massless R-R states in the integral in \( B_a \) and due to massless NS-NS states in the integral in \( A_a \) for large \( \ell \) for such twists with \( d_a = 0, 2 \).

We must therefore consider massless tadpoles arising for supersymmetric twists with \( d_a = 0, 2 \). For \( d_a = 0 \) the corresponding integrals are linearly divergent with \( \ell \) as \( \ell \to \infty \). To cancel such a tadpole we must require that \( \text{Tr}(\gamma_0) = 0 \) for the corresponding \( g_a \) twisted sector. On the other hand, if such a tadpole is not canceled, in the four-dimensional field theory language this would correspond to having a quadratic (in the momentum) divergence at the one-loop order. This would imply that the corresponding four-dimensional background is actually inconsistent, in particular, the equation of motion for the corresponding R-R twisted four-form (which couples to the D3-brane world-volume) is inconsistent. Thus, we must require that for such twists \( \text{Tr}(\gamma_0) = 0 \).

Finally, let us discuss supersymmetric twists with \( d_a = 2 \). For such twists the corresponding integrals are only logarithmically divergent as \( \ell \to \infty \). If such a tadpole is not canceled, that is, if the corresponding \( \text{Tr}(\gamma_0) \neq 0 \), in the four-dimensional field theory language this corresponds to having a logarithmic divergence (in the momentum) at the one-loop order. As was pointed out in \cite{8}, these logarithmic divergences are precisely related to the running in the corresponding gauge theories, which are not conformal (even in the large \( N \) limit). Note that the corresponding twisted closed string states propagate in two extra dimensions transverse to the D3-branes (which correspond to the locus of the points fixed under the twist \( g_a \)), so that the four-dimensional backgrounds are perfectly consistent - the tadpoles for these fields simply imply that these fields have non-trivial (logarithmic) profiles in these two extra dimensions (while the four dimensions along the D-brane world-volume are flat).\footnote{In fact, as was shown in \cite{8} for general \( \Gamma \), the presence of such tadpoles does not introduce any anomalies. This was shown for \( \Gamma \approx \mathbb{Z}_m \otimes \mathbb{Z}_n \) in \cite{8} in a somewhat more complicated way.}

Thus, we conclude that, to obtain consistent four dimensional gauge theories in the D3-
brane world-volume, we must require that all twisted $\text{Tr}(\gamma_a) = 0$ except for supersymmetric twists with $d_a = 2$. If $\text{Tr}(\gamma_a) = 0$ for all such twists as well, then in the large $N$ limit we get conformal theories \[4\]. On the other hand, if any of such twisted $\gamma_a$ is not traceless, the corresponding theories are not conformal even in the large $N$ limit \[8\].

III. NON-SUPERSYMMETRIC THEORIES

In this section we discuss non-supersymmetric large $N$ gauge theories arising in the above setup with some supersymmetric twists $g_a$ with $d_a = 2$ such that $\text{Tr}(\gamma_a) \neq 0$. As we have already mentioned, such theories are not conformal. In these theories the gauge group is a product of $U(N_k)$ factors, and we also have matter, which can be obtained using the corresponding quiver diagrams (see \[10,11\]). There is always an overall center-of-mass $U(1)$, which is free. Other $U(1)$ factors, however, run as the matter is charged under them.

In the large $N$ limit, however, these $U(1)$’s decouple in the infra-red, and can therefore be ignored. More precisely, in some cases we have anomalous $U(1)$’s. Thus, in cases where we have twists with $d_a = 0$ some of the $U(1)$ factors are actually anomalous (in particular, we have mixed $U(1)_kSU(N_l)^2$ anomalies), and are broken at the tree-level via the generalized Green-Schwarz mechanism \[13,14\]. In other cases, where we have non-anomalous $U(1)$’s, the latter decouple in the infra-red in the large $N$ limit. At any rate, all of the $U(1)$ factors can be ignored in the large $N$ limit, so that we can focus on the non-Abelian part of the gauge group.

To obtain such models, consider an orbifold group $\Gamma$, which is a subgroup of $\text{Spin}(6)$ but is not a subgroup of $SU(3)$. Let $\tilde{\Gamma}$ be a non-trivial subgroup of $\Gamma$ such that $\tilde{\Gamma} \subset SU(2)$. We will allow the Chan-Paton matrices $\gamma_a$ corresponding to the non-trivial twists $g_a \in \tilde{\Gamma}$ (which have $d_a = 2$) not to be traceless, so that the corresponding $\mathcal{N} = 2$ model is not conformal. However, we will require that the other Chan-Paton matrices $\gamma_a$ for the twists $g_a \notin \tilde{\Gamma}$ be traceless. The resulting $\mathcal{N} = 0$ model is not conformal. However, as was shown in \[8\], in the planar limit the on-shell correlation functions in the $\mathcal{N} = 0$ gauge theory are the same as in the parent $\mathcal{N} = 2$ gauge theory corresponding to the orbifold group $\tilde{\Gamma}$. That is, in the large $N$ limit the perturbative $\mathcal{N} = 0$ gauge theory amplitudes are not renormalized beyond one loop (as usual, various running $U(1)$’s decouple in the IR in this limit).

Note that the large $N$ property is crucial here. The reason is that in the cases where the orbifold group $\Gamma \notin SU(3)$, we always have twisted NS-NS closed string sectors with tachyons. Their contributions to the corresponding part of the annulus amplitude \[14\] is then exponentially divergent unless we require that

$$\text{Tr}(\gamma_a) = 0, \quad g_a \notin SU(3).$$

(13)

However, even if this condition is satisfied, we must take the ’t Hooft limit - indeed, otherwise it is unclear, for instance, how to deal with the diagrams with handles, which contain tachyonic divergences. In fact, the same applies to some non-planar diagrams without handles, that is, diagrams where the external lines are attached to more than one boundaries (such diagrams are subleading in the large $N$ limit).

\[8\] A T-dual description of such models can be studied in the context of the brane-box models \[12\].
A. Vanishing of the Vacuum Energy Density

As we already mentioned, in the planar limit the on-shell correlation functions in the $\mathcal{N} = 0$ gauge theories of the aforementioned type are the same as in the corresponding parent $\mathcal{N} = 2$ gauge theories. In particular, this applies to zero-point functions corresponding to the perturbative contributions to the vacuum energy density. That is, even though these gauge theories are non-supersymmetric, the vacuum energy density vanishes to all order in perturbation theory in such models.

The proof of this statement is straightforward. Thus, consider a vacuum amplitude with $b$ boundaries but no handles (such a diagram corresponds to a $(b - 1)$-loop diagram in the field theory language). Next, we need to specify the twists on the boundaries. A convenient choice (consistent with that made for the annulus amplitude (3)) is given by

$$\gamma_{a_1} = \prod_{s=2}^{b} \gamma_{a_s},$$

(14)

where $\gamma_{a_1}$ corresponds to the outer boundary, while $\gamma_{a_s}$, $s = 2, \ldots, b$, correspond to inner boundaries. Then the above vacuum amplitude has the following Chan-Paton group-theoretic dependence:

$$\sum \prod_{s=1}^{b} \text{Tr}(\gamma_{a_s}),$$

(15)

where the sum involves all possible distributions of the $\gamma_{a_s}$ twists that satisfy the condition (14). Note that the diagrams with all twists $g_s \in \tilde{\Gamma}$ are (up to overall numerical coefficients) the same as in the parent $\mathcal{N} = 2$ theory, and therefore vanish. All other diagrams contain at least one twist $g_s \notin \tilde{\Gamma}$. This then implies that all such diagrams vanish as

$$\text{Tr}(\gamma_a) = 0, \quad g_a \notin \tilde{\Gamma}.$$  

(16)

That is, in the large $N$ limit the vacuum energy density vanishes in such theories to all orders in perturbation theory. In fact, as was shown in [8], in these theories on-shell correlation functions are not renormalized beyond one loop. For instance, the non-Abelian gauge couplings do not run in the large $N$ limit beyond one loop\(^9\).

\(^9\)Here some care is needed in the cases where the orbifold group $\Gamma$ is non-Abelian, and we have to choose base points on the world-sheet to define the twists. Our discussion here, however, is unmodified also in this case.

\(^{10}\)More precisely, the higher loop contributions to the gauge coupling running, which come from the diagrams with handles, are subleading in the large $N$ limit compared with the leading one-loop contribution. This is analogous to what happens in theories discussed in [15]. In fact, the techniques used in [15] to prove that the higher loop corrections are subleading are very similar to the ones used in [8].
B. A Generalization

Above we constructed $\mathcal{N} = 0$ non-conformal gauge theories that have parent $\mathcal{N} = 2$ gauge theories. As far as vanishing of the vacuum energy density to all loop orders is concerned, it actually suffices that the parent theories are $\mathcal{N} = 1$ supersymmetric (albeit in such cases the non-renormalization property beyond one loop for higher point functions is lost). Such theories can be obtained as follows. Thus, consider an orbifold group $\Gamma$, which is a subgroup of $\text{Spin}(6)$ but is not a subgroup of $SU(3)$. Let $\tilde{\Gamma}$ be a non-trivial subgroup of $\Gamma$ such that $\tilde{\Gamma} \subset SO(3)$, but $\tilde{\Gamma} \not\subset SU(2)$. Note that all the non-trivial elements $g_a \in \Gamma$ have $d_a = 2$. We will therefore allow the corresponding Chan-Paton matrices $\gamma_a$ not to be traceless, so that the corresponding $\mathcal{N} = 1$ model is not conformal. However, we will require that the other Chan-Paton matrices $\gamma_a$ for the twists $g_a \not\in \tilde{\Gamma}$ be traceless. The resulting $\mathcal{N} = 0$ model is not conformal. In the planar limit the on-shell correlation functions in the $\mathcal{N} = 0$ gauge theory are the same as in the parent $\mathcal{N} = 1$ gauge theory corresponding to the orbifold group $\tilde{\Gamma}$. In particular, the vacuum energy density vanishes to all orders in perturbation theory.

C. Examples

Let us consider a simple example of such a theory. Let $\Gamma \approx \mathbb{Z}_2 \otimes \mathbb{Z}_3$, where the action of the generators $R$ and $\theta$ of the $\mathbb{Z}_2$ respectively $\mathbb{Z}_3$ subgroups on the complex coordinates $z_\alpha$ on $\mathcal{M} = \mathbb{C}^3/\Gamma$ is as follows: $R : z_1 \rightarrow z_1$, $R : z_{2,3} \rightarrow -z_{2,3}$, $\theta : z_1 \rightarrow \omega z_1$, $\theta : z_{2,3} \rightarrow z_{2,3}$, where $\omega \equiv \exp(2\pi i/3)$. The twisted Chan-Paton matrices are given by: $\gamma_R = I_{3N}$, $\gamma_{\theta} = \text{diag}(I_N, \omega I_N, -\omega^{-1} I_N)$. (Note that in this case $\tilde{\Gamma} \approx \mathbb{Z}_2 \subset SU(2).$) Then the theory is a non-supersymmetric $SU(N) \otimes SU(N) \otimes SU(N)$ gauge theory (we are dropping the $U(1)$ factors for the reasons discussed above) with matter consisting of complex scalars in $(\mathbf{N}, \overline{\mathbf{N}}, 1)$, $(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}})$ and $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$, as well as chiral fermions in the above representations plus their complex conjugates. Note that the numbers of the physical bosonic and fermionic degrees of freedom are the same. In fact, this is the case for all such gauge theories.

Note that in the above example we have a non-chiral non-supersymmetric gauge theory. Moreover, this theory contains massless scalars in the bifundamental representations. We can, however, construct examples of chiral non-supersymmetric gauge theories. Moreover, such theories need not contain scalars. Thus, let $\Gamma \approx \mathbb{Z}_2 \otimes \mathbb{Z}_2 \otimes \mathbb{Z}_3$, where the action of the generators $R_1$, $R_2$ and $\theta$ of the first $\mathbb{Z}_2$, second $\mathbb{Z}_2$ and $\mathbb{Z}_3$ subgroups, respectively, on the complex coordinates $z_\alpha$ on $\mathcal{M} = \mathbb{C}^3/\Gamma$ is as follows: $R_1 : z_1 \rightarrow z_1$, $R_1 : z_{2,3} \rightarrow -z_{2,3}$, $R_2 : z_2 \rightarrow z_2$, $R_2 : z_{1,3} \rightarrow -z_{1,3}$, $\theta : z_1 \rightarrow \omega z_1$, $\theta : z_{2,3} \rightarrow z_{2,3}$, where $\omega \equiv \exp(2\pi i/3)$. The twisted Chan-Paton matrices are given by: $\gamma_{R_1} = \gamma_{R_2} = I_{3N}$, $\gamma_{\theta} = \text{diag}(I_N, \omega I_N, -\omega^{-1} I_N)$. (Note that in this case $\tilde{\Gamma} \approx \mathbb{Z}_2 \otimes \mathbb{Z}_2 \subset SO(3)$, but $\tilde{\Gamma} \not\subset SU(2).$) Then the theory is a non-supersymmetric $SU(N) \otimes SU(N) \otimes SU(N)$ gauge theory (once again, we are dropping the $U(1)$ factors) with matter consisting of chiral fermions $\Psi_1, \Psi_2, \Psi_3$ in $(\mathbf{N}, \overline{\mathbf{N}}, 1)$, $(\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}})$ and $(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$, respectively. Since we have no scalars, even classically there is no moduli space in this theory, so it corresponds to an isolated non-supersymmetric vacuum. Moreover, this theory is chiral. Also note that the operator

$$B \equiv \Psi_1 \Psi_2 \Psi_3$$  (17)
corresponds to a baryon of this theory.

IV. REMARKS

Thus, as we see, we can construct an infinite number of non-supersymmetric non-conformal large $N$ gauge theories with vanishing vacuum energy density to all orders in perturbation theory. In such a gauge theory the gauge group is a product of $SU(N_k)$ factors, and we have charged matter. Here we would like to comment on gravity in this setup.

Note that the Type IIB setup within which we discussed these gauge theories can be thought of as a brane world scenario [16–32]. In particular, in this case we have infinite-volume extra dimensions [30,31]. Since the brane matter is not conformal, we expect the Einstein-Hilbert term to be generated in the D3-brane world-volume already at the one-loop order [30,31]. In fact, the corresponding induced four-dimensional Planck scale

$$M_P^2 \sim N^2 \Lambda^2,$$

where $\Lambda$ is the gauge theory cut-off\footnote{One might wish to identify $\Lambda$ with the string scale $M_s$. Note, however, that the string backgrounds we are considering here are not finite - we need an ultra-violet cut-off to regularize logarithmic divergences (see \S\ for details), so identifying $\Lambda$ with $M_s$ is not necessary.}. The factor $N^2$ comes from the fact that the number of brane matter degrees of freedom propagating in the loops is of order $N^2$.

Now, in the large $N$ limit $M_P$ goes to infinity, so, not surprisingly, we have no gravity on the branes. In the case of non-supersymmetric gauge theories we are essentially forced to consider the large $N$ limit to avoid problems with bulk tachyons. That is, in this context we do not get four-dimensional gravity unless we are able to consider finite $N$ gauge theories. Since the corresponding non-supersymmetric gauge theories are perfectly consistent\footnote{Note that the non-Abelian gauge theories we are discussing here are actually asymptotically free if we choose the Chan-Paton matrices $\gamma_a$ corresponding to the elements $g_a$ of the orbifold subgroup $\tilde{\Gamma}$ of the parent theory such that if $\mathrm{Tr}(\gamma_a) \neq 0$ then $\gamma_a$ is an identity matrix.}, one might wish to argue that the tachyon problem in their embedding in the Type IIB string theory context might be an artifact of sorts. If so, then one might hope to make sense of such embeddings for finite $N$ (perhaps a way to make this precise is to consider $\alpha' \rightarrow i\epsilon$ \cite{2}). Note, however, that for finite $N$ we would lose any control over the vacuum energy density as the arguments of the previous section crucially depend on the large $N$ property.

One tempting possibility around this difficulty is to start with a supersymmetric gauge theory. In this case we can consider finite $N$ gauge theories. A rosy scenario then goes as follows. Suppose the gauge theory on the branes is actually $\mathcal{N} = 1$ supersymmetric. Moreover, suppose supersymmetry is dynamically broken on the branes via non-perturbative gauge dynamics. Then, since the volume of the extra dimensions is infinite, bulk supersymmetry is intact even if brane supersymmetry is completely broken [33,34]. Then in some cases un-
broken bulk supersymmetry might protect the brane cosmological constant \[33–36\]. This way we might hope to obtain a scenario where the brane cosmological constant vanishes even though the brane supersymmetry is completely broken. Moreover, the induced four-dimensional Planck scale on the branes in this case would be finite as we are dealing with a finite \( N \) gauge theory.

However, the above scenario seems to run into the usual problem of runaway moduli. Indeed, for dynamical supersymmetry breaking to take place we need quantum modification of the moduli space. The latter does occur in some non-conformal \( \mathcal{N} = 1 \) supersymmetric models of the type we are discussing here. In fact, a general model of this type can be constructed as follows. Let the orbifold group \( \Gamma \) be a subgroup of \( SU(3) \) but not a subgroup of \( SU(2) \). Let \( \tilde{\Gamma} \) be a non-trivial subgroup of \( \Gamma \) such that \( \tilde{\Gamma} \subset SO(3) \). Note that all the non-trivial elements \( g_a \in \tilde{\Gamma} \) have \( d_a = 2 \). We can therefore have \( \text{Tr}(\gamma_a) \neq 0 \) for such elements \( g_a \in \tilde{\Gamma} \). For all the elements \( g_a \in \Gamma \) such that \( g_a \notin \tilde{\Gamma} \) we will, however, require \( \text{Tr}(\gamma_a) = 0 \). The resulting \( \mathcal{N} = 1 \) gauge theory is then non-conformal. Note that if \( \tilde{\Gamma} \not\subset SU(2) \), then we do not have a parent \( \mathcal{N} = 2 \) theory, and the \( \mathcal{N} = 1 \) theory is “truly” \( \mathcal{N} = 1 \) supersymmetric in the sense that even in the large \( N \) limit the on-shell correlation functions are not the same as in an \( \mathcal{N} = 2 \) supersymmetric model. On the other hand, if \( \tilde{\Gamma} \subset SU(2) \), then we have a parent \( \mathcal{N} = 2 \) theory, so that in the large \( N \) limit the on-shell correlation functions are the same as in the parent \( \mathcal{N} = 2 \) theory. As was pointed out in \[8\], in some \( \mathcal{N} = 1 \) theories of the latter type we have quantum modification of the moduli space (such a modification is also present in some of the theories of the former type).

Albeit we can have quantum modification of the moduli space, supersymmetry in these models is actually not broken as the twisted moduli that control the corresponding gauge couplings have the usual runaway behavior. Moreover, tree-level couplings required for stabilization of these moduli via the mechanism of \[38\] are also absent as the corresponding singlets would have to come from the closed string sector which is \( \mathcal{N} = 2 \) supersymmetric.\[14\]

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\[13\] Here we note that, since the volume of the extra dimensions is infinite, the issues discussed in \[37\], which arise in scenarios with finite-volume non-compact extra dimensions, need not concern us here.

\[14\] In principle, we can generalize the above construction to include orientifold planes, in which case the closed string sector is \( \mathcal{N} = 1 \) supersymmetric. However, in the presence of orientifold planes some caution is needed due to various issues discussed in \[39–41\]. At any rate, the required tree-level couplings would still be absent in this case unless the corresponding singlets come from twisted open string sectors that arise in the context of non-perturbative orientifolds \[11\].
REFERENCES

[1] G. ’t Hooft, Nucl. Phys. B72 (1974) 461.
[2] M. Bershadsky, Z. Kakushadze and C. Vafa, Nucl. Phys. B523 (1998) 59.
[3] Z. Kakushadze, Nucl. Phys. B529 (1998) 157; Phys. Rev. D58 (1998) 106003; Phys. Rev. D59 (1999) 045007; Nucl. Phys. B544 (1999) 265.
[4] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.
[5] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428 (1998) 105.
[6] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
[7] S. Kachru and E. Silverstein, Phys. Rev. Lett. 80 (1998) 4855.
[8] Z. Kakushadze and R. Roiban, JHEP 0103 (2001) 043.
[9] R.G. Leigh, M. Rozali, Phys. Rev. D59 (1999) 026004.
[10] M. Douglas and G. Moore, hep-th/9603167.
[11] A. Lawrence, N. Nekrasov and C. Vafa, Nucl. Phys. B533 (1998) 199.
[12] A. Hanany, M.J. Strassler and A.M. Uranga, JHEP 9806 (1998) 011.
[13] L.E. Ibanez, R. Rabanad and A.M. Uranga, Nucl. Phys. B542 (1999) 112.
[14] E. Poppitz, Nucl. Phys. B542 (1999) 31.
[15] Z. Kakushadze and T.R. Taylor, Nucl. Phys. B562 (1999) 78.
[16] V. Rubakov and M. Shaposhnikov, Phys. Lett. B125 (1983) 136.
[17] A. Barnaveli and O. Kancheli, Sov. J. Nucl. Phys. 52 (1990) 576.
[18] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.
[19] P. Hořava and E. Witten, Nucl. Phys. B460 (1996) 506; Nucl. Phys. B475 (1996) 94; E. Witten, Nucl. Phys. B471 (1996) 135.
[20] I. Antoniadis, Phys. Lett. B246 (1990) 377; J. Lykken, Phys. Rev. D54 (1996) 3693.
[21] G. Dvali and M. Shifman, Nucl. Phys. B504 (1997) 127; Phys. Lett. B396 (1997) 64.
[22] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004.
[23] K.R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436 (1998) 55; Nucl. Phys. B537 (1999) 47; hep-ph/9807522; Z. Kakushadze, Nucl. Phys. B548 (1999) 205; Nucl. Phys. B552 (1999) 3; Nucl. Phys. B551 (1999) 549.
[24] Z. Kakushadze and T.R. Taylor, Nucl. Phys. B562 (1999) 78.
[25] Z. Kakushadze, Phys. Lett. B434 (1998) 269; Nucl. Phys. B535 (1998) 311; Phys. Rev. D58 (1998) 101901.
[26] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.
[27] G. Shiu and S.-H.H. Tye, Phys. Rev. D58 (1998) 106007.
[28] Z. Kakushadze and S.-H.H. Tye, Nucl. Phys. B548 (1999) 180; Phys. Rev. D58 (1998) 126001.
[29] M. Gogberashvili, hep-ph/9812296; Europhys. Lett. 49 (2000) 396.
[30] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370; Phys. Rev. Lett. 83 (1999) 4690.
[31] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B485 (2000) 208.
[32] G. Dvali and G. Gabadadze, Phys. Rev. D63 (2001) 065007.
[33] A. Iglesias and Z. Kakushadze, hep-th/0011111; hep-th/0012049.
[33] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B484 (2000) 112; Phys. Lett. B484 (2000) 129.
[34] E. Witten, hep-ph/0002297.
[35] Z. Kakushadze, Phys. Lett. B488 (2000) 402; Phys. Lett. B489 (2000) 207; Phys. Lett. B491 (2000) 317; Mod. Phys. Lett. A15 (2000) 1879.
[36] G. Dvali, hep-ph/0004057.
[37] Z. Kakushadze, Nucl. Phys. B589 (2000) 75; Phys. Lett. B497 (2001) 125; O. Corradini and Z. Kakushadze, Phys. Lett. B494 (2000) 302; Phys. Lett. B506 (2001) 167; Z. Kakushadze and P. Langfelder, Mod. Phys. Lett. A15 (2000) 2265.
[38] G. Dvali and Z. Kakushadze, Phys. Lett. B417 (1998) 50.
[39] Z. Kakushadze, Nucl. Phys. B512 (1998) 221; Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686; Nucl. Phys. B520 (1998) 75.
[40] Z. Kakushadze, G. Shiu and S.-H.H. Tye, Nucl. Phys. B533 (1998) 25; Phys. Rev. D58 (1998) 086001.
[41] Z. Kakushadze, Phys. Lett. B455 (1999) 120; Int. J. Mod. Phys. A15 (2000) 3461; Phys. Lett. B459 (1999) 497; Int. J. Mod. Phys. A15 (2000) 3113.