Design and Development of an Indian Harmonium with Automatic Bellow Mechanism

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Abstract. Harmonium is a reed instrument widely used in Indian classical music, utilizing a hand-driven bellow to pump air into the soundbox. The pumped air is released through the metallic reeds producing musical notes. The hand-driven bellow action can be challenging and tiresome for beginners as well as professionals during long practice sessions. This problem can be resolved by automating the bellow. The design process is twofold – 1st is the synthesis of a motor-driven linkage capable of generating the rocker motion of the bellow, and 2nd is to obtain the required specification of the motor that can drive the mechanism. A 4-bar crank-rocker linkage has been synthesized using a modified Freudenstein equation, which allows the bellow to compress and expand in one complete rotation of the motor shaft. The novelty of this work is that the nonlinear dynamics of the harmonium bellow has been modeled using a spring-mass equivalence technique. This helps in accurately predicting the operating force for the bellow, the force transmission through the linkage, and estimating the required motor torque. The final design is simple and ergonomic. Based on the design, a working prototype has been developed at the Machine Elements Laboratory of Jadavpur University.

1. Introduction

In the late 18th century, Dwarkanath Ghose made the first Indian harmonium inspired by the pump organ of Europe [1]. The instrument comprises 3 octaves and uses a hand-operated bellow placed at the rear end. Being a compact and portable instrument, it became an indispensable part of Hindustani classical music. The harmonium utilizes a bellow that pumps air into the soundbox, and this air is released through the metallic reeds when a key is pressed to produce a musical note [2]. Singers and musicians must bellow the harmonium continuously, which can become very tedious when played for long practice sessions. People who are new to playing the instrument may find it quite difficult to synchronize between playing the melody on one hand and bellowing on the other.

Modern-day music is transitioning from analog to digital technology. Musical instruments are now available digitally in the form of VST’s (Virtual Studio Technology) plugin software, such that anyone can make music without purchasing the actual instrument or mastering the skill to play it. VST’s can be developed in two ways – using wavetables or using audio samples [3]. Wavetables of an instrument are generated by analyzing the sound profile of the original instrument to obtain the complex mathematical waveforms and using a sound oscillator with different filters to generate those waveforms. An alternate technique is triggering sound samples directly, which are recorded by playing the hardware instrument. However, a virtual instrument needs to incorporate a physical modeling technique to replicate the original instrument's nuances [4]. Different control parameters must be modelled for developing an expressive VST plugin of an instrument [5]. For the harmonium, one such critical control parameter is the bellow speed which not only controls the airflow rate but also controls the length and decay of each note. Robotic orchestra [6,7] is a challenging aspect of mechatronics, which requires an unmanned sophisticated control of the instruments to make it sound authentic as if a professional musician were...
playing it. One such attempt was the Kritaanjli – a robotic harmonium comprising of an automatic bellow and a DC solenoid array to play the keys [8,9]. Identifying the different parts and understanding the working principle is the foundation for designing an instrument's automation system [10]. Automation paves the way for a modern approach towards learning instruments such as VAR (Virtual-Augmented-Reality) platform, where students could learn and teachers could teach how to play an instrument from the comfort of their home with visual and haptic feedback [11].

The current work aims to develop an automatic bellow mechanism of the harmonium, which would help reduce the playing fatigue and ease the learning process for novice players. A speed-controlled; motorized bellow would help sound designers to accurately model the bellow-speed control parameter for developing improved VST plugins. Unlike the Kritaanjli [8] which was designed specifically for robotic orchestras, the present work focuses on an ergonomic design and flexible automation, so that the instrument becomes more comfortable to play.

2. Planning
Automation of the bellow motion requires some planning before the design process. The design should be ergonomic, and convenient to assemble and dismantle. The wooden plank attached to the bellow moves in a circular arc, hinged at one end. A motorized 4 bar linkage can be designed with a complete 360° rotation of the crank and the bellow undergoing a rocker motion. The bellow would then expand (open) and compress (close) during crank rotations from (0° to 180°) and (180° to 360°) respectively. To simplify the control and reduce cost, a continuous rotation DC motor is preferred with speed adjustment using a potentiometer. Harmoniums with two types of bellow configurations are available – side opening and top opening. For side opening configurations, the mechanism and has to be installed on the top surface which would obstruct the air exhaust from the soundbox causing a fluttering sound, as shown in Figure 1.a. The top opening configuration is ideal, as the mechanism can be installed on the side causing no hindrance, as shown in Figure 1.b. So, for design and prototype development, the top opening model has been selected.

3. Design of a 4 bar Crank-Rocker Mechanism
Now that the configuration and orientation of the mechanism have been planned, the 4-bar linkage needs to be designed. This is done using the Freudenstein method [12], by predefining any three positions of input and output links with the link length of any one link. For the current problem, the fixed link, output link and, output angles are identified and a reference frame is assigned, as shown in Figure 2. The orientation angles of each link are measured w.r.t the vertical x-axis. The wooden plank attached to the bellow forms the output link, of length $l_o$ and inclined at an angle $\phi$. The line joining the reference frame and bellow pivot is the fixed link, of length $l_f$ and inclined at an angle $\xi$. Let the crank length be

![Figure 1. Possible configurations of the 4-bar mechanism based on bellow type](image-url)
making an angle $\theta$. The coupler link joining the crank and output link, has a length $l_{cc}$ making an angle $\beta$ with the horizontal $y$-axis. The DC motor is attached at the reference frame to rotate the crank. The limits of the input (crank) and output (bellow) are obtained as: $-270^\circ \leq \theta \leq +90^\circ$ and $0^\circ \leq \phi \leq 20^\circ$, considering anti-clockwise rotation to be positive. The Freudenstein equation for the given configuration is obtained by resolving the link lengths along $x$ and $y$ directions.

Along $x$- direction:
$$l_{cc}\sin \beta = l_{cr}\cos \theta - l_b \cos \phi - l_f \cos \xi$$

Along $y$- direction:
$$l_{cc}\cos \beta = -l_{cc}\sin \theta + l_b \sin \phi + l_f \sin \xi$$

Simplifying we get:
$$\cos(\theta - \phi) = k_1 \cos(\theta - \xi) + k_2 \cos(\phi - \xi) + k_3$$

Eq. (1)

Where,
$$k_1 = -l_f/l_b; \quad k_2 = l_f/l_{cr}; \quad k_3 = \left(l_{cr}^2 - l_{cc}^2 + l_b^2 + l_f^2\right)/2l_{cr}l_b;$$

Eq. 1. is the modified Freudenstein equation, from which the link lengths can be obtained.

3.1. Bellow Kinematics

The relation between the input crank angle and the output bellow angle can be computed from Eq. 1, which is also the displacement equation. For a given value of $\theta$, $\phi$ is obtainable using any root-search optimization algorithm, such as the Golden-section method.

Let, the angular velocity of the crank and bellow be $\omega_c$ and $\omega_b$ respectively. Considering the DC motor to rotate at a constant rpm $N_{motor}$, then $\omega_c$ becomes a constant. The expression for $\omega_b$ is obtained by differentiating Eq. 1 w.r.t time $t$.

$$\omega_b = \omega_c(\sin(\theta - \phi) - k_1 \sin(\theta - \xi))/(\sin(\theta - \phi) + k_2 \sin(\phi - \xi))$$

Eq. (2)

4. Mathematical Model of the Bellow Mechanism

After the link lengths are obtained, a commercially available DC motor must be selected that can drive the mechanism. Hence, the force required to operate the bellow must be calculated. A mathematical model of the bellow can be developed to estimate the required bellow force. The harmonium bellow pumps air into a soundbox that stores the compressed air. When the keys are pressed, the air escapes through the metallic reeds producing a musical note. The bellow process can be converted into an equivalent spring-mass model [13,14], as shown in Figure 3. The bellow force is contributed by 3 factors – air compression/expansion, bellow weight, and the material stiffness. The bellow is made of laminated cardboard material with folds that have negligible stiffness to resist deformation. Hence, the effect of material stiffness is not considered in the present model.
4.1. Bellow Force due to Air Compression and Expansion

The harmonium bellow can be compared to an air-spring [15], although the former being highly nonlinear. The work done due to air compression and expansion can be equated to the energy stored in a virtual spring of stiffness \( K_{Th} \). To accommodate the system nonlinearity, the stiffness is considered to be a function of the spring displacement \( x(t) \). The bellow has an air intake valve in the form of a plastic flap that allows air to enter the system, but does not allow the compressed air to escape, as shown in Figure 4. So, the harmonium bellow process can be assumed to be a thermodynamic cycle with the following processes:

- Process 1-2: adiabatic compression of air as the bellow closes,
- Process 2-3: isochoric air release through the reeds caused during key press,
- Process 3-1: isobaric expansion of air as the bellow opens.

Let the volume of the soundbox and bellow be \( V_s \) and \( V_b \) respectively. Let \( l_p \) be the length of the harmonium measured along the keys. From the CAD model, \( V_s = 0.018 \, m^3 \) and \( V_b = 0.002752 \, m^3 \).

The ambient pressure is \( p_{atm} = 1.013 \, bar \) and \( \gamma = 1.4 \).

During the air-compression process 1-2: \( p_1 = p_{atm} \); \( v_1 = v_s + v_b \); \( v_2 = v_s \);

Compression time: from \( t = 0 \) to \( t = 30/N_{motor} \) i.e., for 180° rotation of the crank.

The virtual spring-displacement for compression \( x_c(t) \) is obtained from Figure 4.a. as:

\[
x_c(t) = l_p \sin(\phi_{max} - \phi(t)) \tag{3}
\]

Where \( \phi \) is measured in degrees.
Volume at any instant of time $t$, for $t \in [0 \text{ to } 30\,/N_{motor}]$ is given by:

$$v(t) = v_2 + C_f \left(\pi l^2 b \phi(t)/360\right)$$

Eq. (4)

Where, $C_f$ is a volume correction factor for the bellow, defined by the ratio of the actual bellow volume obtained from the 3D-CAD model to the mathematically calculated volume. This is done to incorporate the volume reduction caused due to the bellow folds.

The pressure at any instant $t$ is given by the relation:

$$p = p_1 v_1^2 / v(t)^2$$

Eq. (5)

Work-done due to air compression at any instant of time $t$ is given by:

$$W_{\text{compression}}(t) = \int p \, dv = \frac{\pi}{360} C_f l^2 b p_1 v_1^2 \int_{t=0}^{t} \frac{\omega_b(t)}{v_2 + (\pi C_f l^2 b \phi(t)/360)} \, dt$$

Eq. (6)

Work-done due to virtual spring compression:

$$W_{\text{spring-comp}}(t) = -\int k x_e \, dx_e = \frac{k}{2} \int_{t=0}^{t} K_{Th}(t) \omega_b(t) \sin(2\phi_{\text{max}} - 2\phi(t)) \, dt$$

Eq. (7)

$K_{Th}(t) |_{\text{comp}}$ is obtained by equating Eq. 6 and Eq. 7 using any numerical integration technique. The integration is carried out in small time steps of $\Delta t$. At each time step, the corresponding $K_{Th}(t) |_{\text{comp}}$ is obtained.

During the air-release process 2-3: $p_2 = p_1 v_1^2 / v_2^2$; $p_3 = p_{\text{atm}}$; $v_2 = v_3$;

Being an isochoric process, work done is zero and it does not contribute to the virtual spring stiffness.

During air-expansion process 3-1: $p_1 = p_3$; $v_3 = v_2$;

Expansion time from $t = 30\,/N_{motor}$ to $t = 60\,/N_{motor}$; i.e., for next 180° rotation of the crank, the virtual spring-displacement for expansion $x_e(t)$ is obtained from Figure 4.b. as:

$$x_e(t) = l_b \sin(\phi(t))$$

Eq. (8)

Work-done due to expansion at any instant of time $t$ is given by:

$$W_{\text{expansion}}(t) = \int p \, dv = \frac{\pi}{360} C_f l^2 b p_1 \int_{t=0}^{t} \omega_b(t) \, dt$$

Eq. (9)

Work-done due to spring extension:

$$W_{\text{spring-extn}}(t) = \int k x_e \, dx_e = \frac{k}{2} \int_{t=0}^{t} K_{Th}(t) |_{\text{exp}} \omega_b(t) \sin(2\phi(t)) \, dt$$

Eq. (10)

$K_{Th}(t) |_{\text{exp}}$ is obtained by equating Eq. 9 and Eq. 10 using a similar technique as $K_{Th}(t) |_{\text{comp}}$.

Hence, the bellow force required for air compression and expansion is given by:

$$F_{Th}(t) = x(t) K_{Th}(t)$$

Eq. (11)
4.2. Bellow Force due to Weight of the Bellow

The weight of the bellow including the supporting wooden plank also contributes to bellow force. Let \( m_b g \) be combined weight of the bellow, acting at \( l_b / 2 \). The 4-bar linkage mechanism pulls/pushes the bellow with a force \( F_{mg}(t) \) at distance \( l_b \) along y-axis. The push/pull force is obtained by a moment balance about the bellow pivot and, the virtual spring stiffness \( K_{mg}(t) \) as:

For Compression:
\[
F_{mg}(t)_{\text{comp}} = 0.5 (mg) \tan(\phi(t)) \quad \text{Eq. (12)}
\]

For Expansion:
\[
F_{mg}(t)_{\text{exp}} = -0.5 (mg) \tan(\phi(t)) \quad \text{Eq. (13)}
\]

\[
K_{mg}(t)_{\text{comp}} = \frac{F_{mg}(t)_{\text{comp}} / x_c(t)}{\phi(t)} ; \quad 0 \leq t \leq 30 / N_{motor} \quad \text{Eq. (14)}
\]

\[
K_{mg}(t)_{\text{exp}} = \frac{F_{mg}(t)_{\text{exp}} / x_e(t)}{\phi(t)} ; \quad 30 / N_{motor} \leq t \leq 60 / N_{motor}
\]

4.3. Computing the Motor Specifications

From Eq. 11. and Eq. 14. the net force \( F_{\text{net}} \) required to drive the bellow during compression and expansion is obtained. This force is provided by the motor-driven linkage and acts along the y-axis.

\[
F_{\text{net}}(t) = (K_{\text{Th}}(t) + K_{mg}(t)) x(t) = \begin{cases} 
(K_{\text{Th}}(t)_{\text{comp}} + K_{mg}(t)_{\text{comp}}) x_c(t) ; & \text{compression} \\
(K_{\text{Th}}(t)_{\text{exp}} + K_{mg}(t)_{\text{exp}}) x_e(t) ; & \text{expansion} 
\end{cases} \quad \text{Eq. (15)}
\]

The linkage is considered to be lightweight and having negligible inertia compared to the system, hence, the torque required to rotate that crank is obtained by resolving the forces in each link [16]. The links carry a radial force that acts along the link and a tangential force that acts normal to the link. The radial forces acting on the coupler and crank is denoted by \( F_{ccl} \) and \( F_{cr} \), respectively, while the tangential forces are denoted by \( F_{ccl} \) and \( F_{cr} \). The radial force on the crank \( F_{cr} \) becomes the bending load on the motor shaft. The tangential force \( F_{cr} \) on the crank generates a turning moment which is equivalent to the torque generated by the motor. The mathematical relations are derived from the force transmission diagram, as shown in Figure 5.

![Figure 5. Force Transmission diagram showing the forces acting on the links.](image)

**Coupler Forces:**
\[
\begin{align*}
F_{ccl} & = F_{\text{net}} \cos \beta \\
F_{ccl} & = F_{\text{net}} \sin \beta
\end{align*} \quad \text{Eq. (16)}
\]

**Crank Forces:**
\[
\begin{align*}
F_{ccl} & = F_{ccl} \cos(\theta - \beta) + F_{ccl} \sin(\theta - \beta) \\
F_{cr} & = -F_{ccl} \sin(\theta - \beta) + F_{ccl} \cos(\theta - \beta)
\end{align*} \quad \text{Eq. (17)}
\]

**Motor Torque:**
\[
M_{T} = F_{cr} l_{cr} \quad \text{Eq. (18)}
\]

**Motor Power:**
\[
F_{\text{motor}} = M_{T} (2\pi N_{motor} / 60) \quad \text{Eq. (19)}
\]
5. Results and Discussion
Based on the design and mathematical model, the linkage is developed. The equations are solved to obtain the link lengths; accordingly, the links are designed in Solidworks™ CAD software. The complete harmonium modeling helps in determining mass and volume properties. The equations of the bellow kinematics and mechanics are solved by programming in Matlab™ software. The details are enumerated as below.

5.1. Linkage Synthesis and development of CAD model
Considering, 3 positions of \( \theta = \{-90^\circ, 0^\circ, +90^\circ\} \) and \( \phi = \{0^\circ, 10^\circ, 20^\circ\} \) with the linkage constants \( l_b = 158 \text{mm} \) and \( \xi = 166.68^\circ \), the unknown link lengths are obtained from Eq. 1. as:

\[
\begin{align*}
l_a &= 26.04 \text{mm}, \\
l_c &= 61.816 \text{mm}, \\
l_f &= 144.72 \text{mm}.
\end{align*}
\]

The choice of \( \xi \) is subjective and is selected by the trial-and-error method, such that link lengths are within acceptable limits considering the overall size of the harmonium. The CAD model of the 4-bar crank rocker linkage assembled onto the harmonium is shown in Figure 6.

![Figure 6. CAD model showing the assembly of 4-bar linkage onto the harmonium.](image)

5.2. Simulating the Mathematical model and Motor Selection
The motor torque required to drive the mechanism is obtained from the results of the simulation. The simulation is carried out for \( N_{\text{motor}} = 30 \text{ rpm} \), i.e., the bellow compression and expansion take 1s each. The pressure versus volume plots for process 1-2, 2-3, and 3-1 is shown in Figure 7.a. The peak pressure was observed to be 1.23bar, which is the maximum possible pressure rise in the system. In actual practice, the keys are played during the bellow compression/expansion, i.e., process 2 – 3 overlaps with process 1 – 2 and process 3 – 1. Hence, the actual peak pressure is much lower \[17\] than that calculated. The simulation result is valid if processes 1 – 2, 2 – 3 and 3 – 1 occur independently. The variation of crank angle \( \theta \) versus bellow angle \( \phi \) is obtained by solving Eq. 1. and is shown in Figure 7.b. The variation of crank and bellow angular velocities \( \omega_c, \omega_b \) is obtained by solving Eq. 2, shown in Figure 7.c. For each case, the compression and expansion phases are shown separately. The stiffness of the virtual spring representing the air compression/expansion process \( K_{Th}(t) \) is shown in Figure 7.d. An iterative convergence is used where, the time step \( \Delta t \) is reduced at each iteration, such that there is negligible change in \( K_{Th}(t) \) upon further reduction of \( \Delta t \).
The forces generated due to air compression/expansion \((F_{Th})\) at any instant of time is obtained from Eq. 11. The value of inertial force \(F_{ng}\) is obtained from Eq. 12. and Eq. 13. Both \(F_{Th}\) and \(F_{ng}\) are a function of the instantaneous position of the bellow, which is, in turn, is a function of time. Thus, the forces acting on the bellow \((F_{Th}, F_{ng})\) are obtained and the variation of the net force \(F_{net}\) is shown in Figure 8.a. The effect of bellow weight \(F_{ng}\) is negligible compared to forces generated due to air compression/expansion \(F_{Th}\). The negative values of force arising during expansion imply that force acts in the opposite direction to that of compression. The net force is transmitted along the coupler and crank, which are resolved to obtain the radial and tangential forces as shown in Figure 8.b. and Figure 8.c. The peak tangential and the radial forces on the coupler and crank are of comparable magnitude. The variation of motor torque is obtained from Eq. 18. and shown in Figure 8.d. The peak torque during compression and expansion is 2.66 Nm and 1.07 Nm respectively at 30 rpm. Hence, 2.66 Nm becomes the design torque. As stated previously, in actual practice the peak pressure is much lower than that estimated, hence the running torque will also be much lower. The tangential force on the crank and the motor torque shows a sinusoidal variation. The maximum power consumption is estimated to be 7.57 Watts and 3.38 Watts respectively. These results are sufficient to fabricate the links and to purchase a commercially available DC motor which can be used to automate the bellow of the harmonium.

Figure 7. Results of simulation. (a) \(p - v\) diagram for the bellow process. (b) Variation of \(\theta\) vs \(\phi\). (c) Variation of \(\omega_c\) and \(\omega_b\) vs time \(t\), (d) Variation of \(K_{Th}\) vs \(t\).
Prototype Development

Prototype development is essential to test the success of the design and mathematical model. The links are fabricated using 5mm thick acrylic sheets. The CAD design is exported to .dwg format on a 1:1 scale and used directly in the CNC router machine. CNC router uses laser to cut the acrylic sheets, maintaining high dimensional accuracy of the fabricated parts. The links are assembled and screwed onto a Harmonium using fasteners. An aluminum plate is used to fabricate the support link and motor mount which provides rigidity to the linkage. A DC motor with spur-gearbox delivering a running torque of 1.9 Nm at 30 rpm, and a stall torque of 37 Nm is selected to drive the linkage. A 10 kΩ potentiometer is connected across the 12V, 5A DC power supply to regulate the motor rpm. The final working prototype developed in the Machine Elements Laboratory of Jadavpur University has been shown in Figure 9. The prototype can be improved by fabricating all the links with steel or aluminum making it more durable for daily use. Acrylic sheets were used for the current prototype due to their ease of manufacturing and high dimensional accuracy.

Figure 8. Computing forces and torque. (a). Variation of bellow forces $F_{RH}$, $F_{mg}$ and $F_{net}$ vs $t$, (b). Variation of coupler forces $F_{C1} |r$ and $F_{C1} |r$ vs $t$, (c). Variation of crank forces $F_{cr1} |r$ and $F_{cr1} |r$ vs $t$, (d). Variation of Motor torque $M_{F}$ vs $t$.
Conclusion
An automatic bellow mechanism for the harmonium was primarily developed to address the playing fatigue and aid novice players to master this instrument quickly. The key focus was on the design ergonomics with easy installation of the device onto the instrument. The working prototype reveals that design and modeling have been successful. The concepts of thermodynamics, mechanics, design, and mechatronics have been interleaved for the present work. Modeling the nonlinear bellow using an equivalent spring-mass model was effective in estimating the motor torque. This paper’s novelty is this bellow modelling technique which could prove useful in automating complex nonlinear systems such as bellow actuators for use in robotics, automotive suspensions, and medical ventilators. The present work could also be used in developing an accurate physical model for harmonium VST plugins as it provides precise control of bellow speed.

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Figure 9. Prototype developed at the Machine Elements Laboratory of Jadavpur University.