Wobbling excitation of triaxial nuclear molecule \( ^{28}\text{Si} - ^{28}\text{Si} \)

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Abstract. High-spin resonances observed in the \( ^{28}\text{Si} + ^{28}\text{Si} \) collisions are investigated with a molecular model. At high spins, a stable dinuclear configuration is found to be an equator-equator touching one. Since the E-E configuration is slightly triaxial, rotations of the total system induce mixing of \( K \) quantum numbers, called wobbling motion, which clearly explains the particle-\( \gamma \) angular correlations observed as well as the disalignments in a simple and natural way. Furthermore, predictions are given for the angular correlations of the wobbling excited states. The first excited state of wobbling shows strong alignments, which is quite different from the molecular ground state.

1. Introduction

High-spin resonances well above the Coulomb barrier in the \( ^{28}\text{Si} + ^{28}\text{Si} \) system exhibit a number of sharp peaks correlated among the elastic and inelastic channels. In their angle-averaged excitation functions, bumps are seen corresponding to the grazing angular momenta, and several sharp peaks are found on each bump [1]. The high level density of the resonance states indicates activation of some internal degrees of freedom other than the relative motion. Hence the rotational or the orientation degrees of freedom of the constituent deformed nuclei have to be taken into account. As a first step for that, we solved normal modes of motion in multidimensional space around the equilibrium configurations of interacting two oblate nuclei [2].

Recently new experimental data were obtained for \( ^{28}\text{Si} + ^{28}\text{Si} \) at \( E_{\text{cm}} = 55.8 \) MeV [3]; the angular distributions exhibit disalignments between the orbital angular momentum and the fragment spins, and the particle-\( \gamma \) angular correlation measurements show characteristic features. We analyzed the correlations by using the normal modes of the equilibrium configuration, and found that the molecular ground state reproduces those characteristic features of the angular correlations very well [4]. Furthermore, we expect that some of those sharp peaks on the bump correspond to wobbling excited states, and thus we have analyzed their angular correlations. The results for a couple of the excited states are compared between them, which show quite different characteristics, respectively.

2. Dinuclear molecular structure of \( ^{28}\text{Si} - ^{28}\text{Si} \)

2.1. Normal mode analyses

Assuming a constant deformation and axial symmetry of the constituent nuclei for simplicity, we have seven degrees of freedom \( (q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2) \), as illustrated in fig. 1, where
\( (R, \theta_2, \theta_1) \) is the relative vector of two \(^{28}\text{Si}\) nuclei. The orientations of the symmetry axes of the \(^{28}\text{Si}\) nuclei are described with Euler angles \((\alpha_1, \beta_1)\) referring to the molecular axes, where \(\alpha_1\) and \(\alpha_2\) are combined into \(\theta_3 = (\alpha_1 + \alpha_2)/2\) and \(\alpha = (\alpha_1 - \alpha_2)/2\). Consistently with the coordinate system, at first, we introduce a rotation-vibration type wave function \(D_J^M(K, \theta_i, \chi_K(R, \alpha, \beta_1, \beta_2))\), as basis one, where \(\chi_K\) describes internal motions. Details are given in ref. [5].

By the molecular model the authors investigated an equilibrium configuration of two interacting oblate nuclei, and showed it to be the equator-equator (E-E) one, illustrated in fig. 2. Couplings among various molecular configurations are taken into account by the method of normal mode around the equilibrium, which gives rise to the molecular modes of excitation, such as butterfly with \(\beta_+ = (\Delta \beta_1 + \Delta \beta_2)/\sqrt{2}\) and anti-butterfly with \(\beta_- = (\Delta \beta_1 - \Delta \beta_2)/\sqrt{2}\) around \(\alpha = \pi/2\), respectively [2, 5]. In fig. 3(a) molecular normal modes of \(^{28}\text{Si} + ^{28}\text{Si}\) with spin 38 are displayed, classified with quantum numbers \(K\), for \(K = \text{even}\).

### 2.2. Wobbling motion with triaxial deformation

Since the E-E configuration is triaxial, rotations of the total system induce mixing of quantum numbers \(K\) at high spins, which is known as wobbling. Such a motion is consistent with the measured angular correlations as well as the disalignments. At a given angular momentum \(J\), this configuration rotates in a triaxial way preferentially about the axis corresponding to the largest moment of inertia in the state with the lowest energy. Therefore the whole system rotates about the normal to the plane defined by the two pancake-like nuclei. With such a characteristic rotation, the spins of the \(^{28}\text{Si}\) fragments are preferentially in the reaction plane, because they are perpendicular to the symmetry axes of \(^{28}\text{Si}\). After \(K\)-mixings due to the axial asymmetry, we obtain a new energy spectrum, displayed in fig. 3(b).

In the high-spin limit \((|K|/J \sim 0)\) we obtain analytical wobbling \((K\)-mixing) solutions as \(F_n(K) = H_n(K/b) \exp(-K^2/2b^2)\), with the Hermite polynomial \(H_n\) for the n-th excited state, which provide a good perspective [5, 4]. The width \(b\) is given by the asymmetry of the moments of inertia.

![Figure 1](image1.png)  
**Figure 1.** Dinituclear molecular coordinates; seven degrees of freedom.

![Figure 2](image2.png)  
**Figure 2.** Equilibrium configuration of two interacting oblate nuclei.

![Figure 3](image3.png)  
**Figure 3.** Energy spectra of the \(^{28}\text{Si} + ^{28}\text{Si}\) system for \(J = 38\). (a) Molecular normal modes without \(K\)-mixing. (b) After \(K\)-mixing, with indications of the modes under the levels.
3. Angular correlations of the wobbling excitations

Figure 4 shows \( \gamma \)-ray intensities emitted from the first excited \( 2^+ \) state of \(^{28}\text{Si}\) fragments, obtained for the mutual \( 2^+ \) channel decays in the particle-\( \gamma \) angular correlation measurements [3]. Data on \( 4\pi \) solid angle are classified with three different quantization \( z \) axes: (a) beam direction, (b) normal to the plane and (c) fragment direction taken perpendicular to the axes (a) and (b), respectively. Solid lines show theoretical results of the molecular ground state with \( J = 38 \), which have dominant \( m = 0 \) (disalignment) in (b). It is found that the molecular ground state reproduces the measured correlations very well [3, 4]. Furthermore, we have predicted the first and second excited states of the wobbling motion (for the states denoted ‘Ground wobbling’ in fig. 3(b)). Apparently the excited states exhibit different characteristics from that of the molecular ground state. For the first excited state (dashed lines), with a dominant \( m = 2 \) pattern in (b), strong alignments are indicated. The second excited state (dotted lines) gives dominant \( m = 2 \) pattern in (a). Due to high \( K \)-rotation, the fragment spins are almost parallel to the molecular \( z' \) axis. Those states are possible origin of the resonance peaks observed on the same bump of the 55.8 MeV resonance.

Decay properties have also been investigated; due to the strong \( K \)-mixing, the wobbling excited states show strong suppression in the elastic channel, which is consistent with the data around \( E_{\text{cm}} = 57 \) MeV [1].

\[ \text{(a)} \]
\[ \text{(b)} \]
\[ \text{(c)} \]

\( \theta_\gamma \) (degree)

\( W(\theta_\gamma) \)

Figure 4. Angular correlations for three quantization \( z \) axes (see text for the axes). Solid lines are those of the molecular ground state, dashed lines for the first excited state and dotted lines for the second excited state.

4. Conclusions

The molecular ground state with \( J = 38 \) is a good candidate for the resonance at \( E_{\text{cm}} = 55.8 \) MeV [4, 6]. In addition, we have predicted angular correlations of the wobbling excited states. Contrary to the molecular ground state with the property of disalignments, the first excited state of wobbling motion shows strong alignments. In brief, it has turned out that particle-\( \gamma \) angular correlations provide decisive information on orientations or motions of the constituent nuclei in resonance states. Experiments on the nearby resonances are strongly called for, which unveil the long-standing mystery of the high-spin resonances in heavy-ion collisions.

References

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