Updated NNLO QCD predictions for the weak radiative $B$-meson decays

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I. INTRODUCTION

The inclusive decays $B \to X_s \gamma$ and $\bar B \to X_d \gamma$ are considered among the most interesting flavor changing neutral current processes. They contribute in a significant manner to current bounds on masses and interactions of possible additional Higgs bosons and/or supersymmetric particles. Measurements of the CP- and isospin-averaged $\bar B \to X_s \gamma$ branching ratio by CLEO$^4$, Belle$^2,3$, and BABAR$^4,5$ lead to the combined result$^6$

$$B_{\gamma s}^\text{exp} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4},$$

(1)

for the photon energy $E_\gamma > E_0 = 1.6$ GeV in the decaying meson rest frame. The combination involves an extrapolation from measurements performed at $E_0 \in [1.7, 2.0]$ GeV. Applying the same extrapolation method to the available $\bar B \to X_d \gamma$ measurement$^4$, one finds

$$B_{\gamma d}^\text{exp} = (1.41 \pm 0.57) \times 10^{-5},$$

(2)

at $E_0 = 1.6$ GeV$^10$. More precise determinations of $B_{\gamma q}^\text{exp}$ for $q = s, d$ are expected from Belle II$^{11}$.

Theoretical calculations of $B_{\gamma q}$ have a chance to match the experimental precision only in a certain range of $E_0$ where the non-perturbative contribution $\delta \Gamma_{\text{nonp}}$ in the relation

$$\Gamma(\bar B \to X_q \gamma) = \Gamma(b \to X_q^\gamma) + \delta \Gamma_{\text{nonp}}$$

remains under control. Here, $\Gamma(b \to X_q^\gamma)$ denotes the perturbatively calculable rate of the radiative $b$-quark decay involving only charmless partons in the final state. Their overall strangeness vanishes for $X_q^\gamma$ and equals $-1$ for $X_q^\gamma$. The analysis of Ref. $^{12}$ implies that unknown contributions to $\delta \Gamma_{\text{nonp}}$ are potentially larger than the so-far determined ones, and induce around $\pm 5\%$ uncertainty in $B_{\gamma q}$ at $E_0 = 1.6$ GeV. Non-perturbative uncertainties in $B_{\gamma q}$ receive additional sizeable contributions$^{12}$ due to collinear photon emission in the $b \to du \gamma$ process whose Cabibbo-Kobayashi-Maskawa (CKM) factor is only a few times smaller than the one in the leading term.

Apart from possible future progress in analyzing non-perturbative effects, one needs to determine $\Gamma(b \to X_q^\gamma)$ to a few percent accuracy. It requires evaluating next-to-next-to-leading order (NNLO) QCD corrections that involve Feynman diagrams up to four loops. The first standard model (SM) estimate of the $\bar B \to X_s \gamma$ branching ratio at this level was presented in Ref. $^{14}$ where all the corrections calculated up to 2006 were taken into account. A part of the $O(a_s^2)$ contribution was obtained via interpolation$^{13}$ in the charm quark mass between the large-$m_c$ asymptotic expression$^{13}$ and the $m_c = 0$ boundary condition that was estimated using the Brodsky-Lepage-Mackenzie (BLM) approximation$^{17}$.

In the present paper, we provide an updated prediction for $B_{\gamma q}$, including all the contributions and estimates
worked out after 2006. They are listed in Sec. III where the
necessary definitions are introduced. The interpolation
in $m_c$ is still being applied. However, the $m_c = 0$
boundary condition is no longer a BLM-based estimate
but rather comes from an explicit calculation [18].

The paper is organized as follows. After discussing
$B_{s\gamma}$ in Sec. III our NNLO analysis is extended to
$B_{d\gamma}$ in Sec. IIII Next, in Sec. IV we consider $R_u =
(B_{s\gamma} + B_{d\gamma})/B_{c\tau
v}$ which may sometimes be more conve-
"nent than $B_{s\gamma}$ for deriving constraints on new physics.
Sec. VII is devoted to presenting a generic expression for
beyond-SM contributions, as well as an updated bound for
the charged Higgs boson mass in the two-Higgs-
doublet-model II (THDM II). We conclude in Sec. VII

II. $B_{s\gamma}$ IN THE SM

Radiative $B$-meson decays are most conveniently
described in the framework of an effective theory that arises
after decoupling of the $W$ boson and heavier particles.

Flavor-changing weak interactions that are relevant for
the $b \to X_s^p \gamma$ with $q = s, d$ are given by

$$
\mathcal{L}_{\text{eff}} \sim V_{ts}^* V_{tb} \left[ \sum_{i=1}^{8} C_i Q_i + \kappa_q \sum_{i=1}^{2} C_i (Q_i - Q_i^\ast) \right].
$$

Explicit expressions for the current-current ($Q_{1,2}$), four-
quark penguin ($Q_3,...,6$), photonic dipole ($Q_7$) and
gluonic dipole ($Q_8$) operators can be found, e.g., in
Eq. (2.5) of Ref. [17]. The CKM element ratio $\kappa_q =
(V_{ts}^* V_{tb})/(V_{ts}^* V_{tb})$ is small for $q = s$, and it affects $B_{s\gamma}$ by
less than 0.3%. Barring this effect and the higher-order
electroweak ones, $\Gamma(b \to X_s^p \gamma)$ in the SM is given by
a quadratic polynomial in the real Wilson coefficients $C_i$

$$
\Gamma(b \to X_s^p \gamma) \sim \sum_{i,j=1}^{8} C_i C_j G_{ij}.
$$

A series of contributions to the above expression from
our calculations in Refs. [18, 27] makes the current anal-
ysis significantly improved with respect to the one in
Ref. [14]. In particular, the NNLO Wilson coefficient

is calculated complete after including the four-
loop anomalous dimensions that describe $Q_1,...,6 \to Q_8$
mixing under renormalization [14]. Effects of the charm
and bottom quark masses in loops on the quark lines in
$G_{77}$ [20], $G_{78}$ [21] and $G_{(1,2)7}$ [22], as well as a complete
calculation of $G_{78}$ [23] are now available. Three-
and four-body final-state contributions to $G_{88}$ [24, 27]
and $G_{(1,2)8}$ [25] are included in the BLM approximation.

Four-body final-state contributions involving the penguin
and $Q_{1,2}^{''}$ operators are taken into account at the leading
order (LO) [26] and next-to-leading order (NLO) [27].

Last but not least, the complete NNLO calculation [18]
of $G_{77}$ and $G_{78}$ at $m_c = 0$ is used as a boundary for
interpolating their unknown parts in $m_c$.

Following the algorithm described in detail in Ref. [18],
taking into account new non-perturbative effects [12, 28]
[29], as well as the previously omitted parts of the NNLO
B decays [31], we arrive at the following SM prediction

$$
B^{\text{SM}}_{s\gamma} = (3.36 \pm 0.23) \times 10^{-4} \quad \text{for } E_0 = 1.6 \text{ GeV. (6)}
$$

Individual contributions to the total uncertainty are of
non-perturbative ($\pm 5\%$), higher-order ($\pm 3\%$), interpol-
ation ($\pm 3\%$) and parametric ($\pm 2\%$) origin. They are com-
bined in quadrature. The parametric one gets reduced with
respect to Ref. [14], which becomes possible thanks to
the new semileptonic fits of Ref. [30]. Unfortunately, the
interpolation uncertainty cannot be reduced because the
interpolated parts of the $O(\alpha_s^2)$ non-BLM contribu-
tions to $G_{(1,2)7}$ turn out to be sizeable. Their effect on
$B^{\text{SM}}_{s\gamma}$ grows from 0 to around 5% when $m_c$ changes from
0 up to the measured value.

III. $B_{d\gamma}$ IN THE SM

Extending our NNLO calculation to the $B_{d\gamma}$ case be-
gins with inserting the proper CKM factors in Eq. (4).

Contrary to $\kappa_s$, the ratio $\kappa_d$ is not numerically small.
Using the CKM fits of Ref. [32], one finds

$$
\kappa_d = (0.007_{-0.011}^{+0.015}) + i (-0.404_{-0.014}^{+0.012}).
$$

The small real part implies that the effects of $\kappa_d$ on the
CP-averaged $B_{d\gamma}$ are dominated by those proportional to
$|\kappa_d|^2$. In such terms, perturbative two- and three-body fi-
nal state contributions are only at the NNLO and NLO,
respectively. They vanish in the $m_c = m_b$ limit, which
effectively makes them suppressed by $m_c^2/m_b^2 < 0.1$. In
consequence, the main $\kappa_d$-effect comes from $b \to d\bar{u}\gamma$
at the LO, where phase-space suppression is partially com-
penated by the collinear logarithms.

In the first (rough) approximation, one evaluates the
tree-level $b \to d\bar{u}\gamma$ diagrams retaining a common light-
quark mass $m_q$ inside the collinear logarithms [25], and
varying $m_b/m_q$ between $10 \sim m_B/m_K$ and $50 \sim m_B/m_\pi$
to estimate the uncertainty. The considered effect varies
then from 2% to 11% of $B_{d\gamma}$. A more involved analy-
sis with the help of fragmentation functions gives a very
similar range [13]. Including this contribution in our eval-
uation of the entire $B_{d\gamma}$ from Eq. (4), we find

$$
B^{\text{SM}}_{d\gamma} = (1.73_{-0.22}^{+0.12}) \times 10^{-5} \quad \text{for } E_0 = 1.6 \text{ GeV, (8)}
$$

where the central value corresponds to $m_b/m_q = 50$. Our
result is about 12% larger than the one given in Ref. [10]

where the $b \to d\bar{u}\gamma$ contributions were neglected. The
uncertainty estimate in Eq. (8) improves with respect to
Ref. [10] thanks to including the NNLO QCD corrections and
using the updated CKM fit [32]. Interestingly, the parametric
uncertainty due to the CKM input amounts to $\pm 2.5\%$ only.
The collinear logarithm problem might seem artificial because isolated photons are required in the experimental signal sample. Unfortunately, requiring photon isolation on the perturbative side would necessitate introducing an infrared cutoff on the gluon energies, e.g., in the NLO corrections to the dominant $G_{\gamma\gamma}$ term. Without a dedicated analysis (which is beyond the scope of the present paper), it is hard to verify whether such an approach would enhance or suppress the uncertainty in $B_{d\gamma}$.

Another question concerning the $|\kappa_d|^2$-terms is whether the off-shell light vector meson conversion to photons can be assumed to be included in our overall $\pm5\%$ non-perturbative uncertainty. Much smaller effects found in the vector-meson-dominance analysis of Ref. \cite{33} imply that it is likely to be the case.

IV. THE RATIO $R_\gamma$

In the fully inclusive measurements of radiative $B$-meson decays \cite{1, 2, 3}, the final hadronic state strangeness is not verified. The actually measured quantity is $B_{s\gamma} + B_{d\gamma}$. Next, the result is divided by $(1 + |V_{td}|^2)/(V_{ts}V_{tb})^2$ to obtain $B_{s\gamma}$. To avoid such a complication, we provide here our SM prediction for $B_{s\gamma} + B_{d\gamma}$ with all the correlated uncertainties properly taken into account. Moreover, we normalize it to the CP- and isospin-averaged inclusive semileptonic branching ratio $B_{ctd}$. In the $B_{\gamma}$ case, such a normalization reduces the parametric uncertainty from $\pm2.0\%$ to $\{\pm1.2, -1.4\}\%$. It may also be useful on the experimental side because the inclusive semileptonic events can serve for determining the $B$-meson yield. Proceeding as in the previous sections, we obtain for $E_\gamma = 1.6 \text{GeV}$

$$R_\gamma^{SM} = (B_{s\gamma}^{SM} + B_{d\gamma}^{SM})/B_{ctd} = \left(3.31 \pm 0.22 \right) \times 10^{-3}. \quad (9)$$

The relative uncertainties are identical to those in $B_{s\gamma}$ (as given below Eq. (9)), except for the parametric one which amounts to $\{\pm1.2, -1.7\}\%$ including the effect of $m_b/m_q$. The gain in the overall theory uncertainty is hardly noticeable, but this may change with the future progress in determining the perturbative and non-perturbative corrections.

V. BEYOND-SM EFFECTS

In most of the new-physics scenarios considered in the literature, beyond-SM effects on $B_{s\gamma}$ are driven by new additive contributions to the Wilson coefficients of the dipole operators at the matching scale $\mu_0$ where the heavy particles ($t$, $W$, $Z$, $H^0$, . . .) are decoupled. Denoting such contributions by $\Delta C_{7,8}$ and setting $\mu_0$ to 160 GeV, we find

$$B_{s\gamma} \times 10^4 = (3.36 \pm 0.23) - 8.22 \Delta C_7 - 1.99 \Delta C_8,$$

$$R_\gamma \times 10^3 = (3.31 \pm 0.22) - 8.05 \Delta C_7 - 1.94 \Delta C_8. \quad (10)$$

The above expressions are linearized, i.e. it is assumed that the quadratic terms in $\Delta C_{7,8}$ are negligible when they enter with $O(1)$ coefficients into the above equations. If they are not, a detailed analysis of QCD corrections in the considered beyond-SM scenario is necessary.

Such an analysis is available in the THDM II \cite{34} for which the NLO \cite{32, 33} and NNLO \cite{35} corrections to $\Delta C_{7,8}$ are known. They are always negative and remain practically independent of the vacuum expectation value ratio $\tan \beta$ when $\tan \beta \gtrsim 2$. Sending $\tan \beta$ to infinity in the expressions for $\Delta C_{7,8}$, we find the following updated bounds from $B_{s\gamma}$ on the charged Higgs boson mass in this model

$$M_{H^\pm} > 480 \text{ GeV at 95\%C.L.},$$

$$M_{H^\pm} > 358 \text{ GeV at 99\%C.L.}. \quad (11)$$

For $\tan \beta \lesssim 2$ the bounds become considerably stronger, but at the same time other observables provide competitive limits \cite{32}. In the supersymmetric case, in which the charged scalar and the neutral pseudoscalar tend to be almost degenerate, the current direct search bounds \cite{40, 41} exceed 500 GeV for $\tan \beta \gtrsim 20$.

VI. SUMMARY

We presented an updated prediction for $B_{s\gamma}$ in the SM taking into account all the perturbative and non-perturbative effects worked out after the 2006 publication \cite{13} of the first NNLO estimate for this quantity. Some of the $O(\alpha_s^2)$ corrections are still interpolated in $m_c$, but the $m_c = 0$ boundary condition now comes from an explicit calculation. Despite this improvement, the interpolation uncertainty cannot be reduced because the interpolated correction is sizeable. Future progress requires extending the calculation of $G_{(1,2)}$ to arbitrary $m_c$, which is considered a difficult but manageable task. In parallel, one should investigate whether non-perturbative uncertainties can be suppressed by combining lattice inputs with measurements of observables like the CP- or isospin asymmetries in $\bar{B} \to X_q\gamma$.

The main outcome of the current update is an upwards shift by around $6.4\%$ in the central value of $B_{s\gamma}^{SM}$. It originates mainly from fixing the $m_c = 0$ boundary (+3\%) and including the complete NNLO BLM corrections to the three- and four-body final state channels (+2\%). Since $B_{s\gamma}^{SM}$ is now closer to $B_{s\gamma}^{exp}$ (but still $B_{s\gamma}^{SM} < B_{s\gamma}^{exp}$), the bound on $M_{H^\pm}$ in the THDM II becomes significantly stronger.

We supplemented our analysis with a prediction for $B_{d\gamma}$, as well as the ratio $R_\gamma = (B_{s\gamma} + B_{d\gamma})/B_{ctd}$ where correlated uncertainties are treated in a consistent manner. The ratio $R_\gamma$ may serve in the future as a more convenient observable for testing beyond-SM theories with minimal flavor violation.
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[1] S. Chen et al. (CLEO Collaboration), Phys. Rev. Lett. 87, 251807 (2001) [hep-ex/0108032].
[2] K. Abe et al. (Belle Collaboration), Phys. Lett. B 511, 151 (2001) [hep-ex/0103042]. This measurement has recently been superseded by a new one in Ref. [12], which is not yet taken into account in the world average of Ref. [5].
[3] A. Limosani et al. (Belle Collaboration), Phys. Rev. Lett. 103, 241801 (2009) [arXiv:0907.1384].
[4] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. Lett. 109, 191801 (2012) [arXiv:1207.2690].
[5] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 112008 (2012) [arXiv:1207.5772].
[6] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 052012 (2012) [arXiv:1207.2520].
[7] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 77, 051103 (2008) [arXiv:0711.3889].
[8] Y. Amhis et al. (Heavy Flavor Averaging Group), arXiv:1412.7515.
[9] P. del Amo Sanchez et al. (BABAR Collaboration), Phys. Rev. D 82, 051101 (2010) [arXiv:1005.4087].
[10] A. Crivellin and L. Mercolli, Phys. Rev. D 84, 114005 (2011) [arXiv:1105.5499].
[11] T. Aushev et al., arXiv:1002.5012.
[12] M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008, 099 (2010) [arXiv:1006.5012].
[13] H. M. Asatrian and C. Greub, Phys. Rev. D 88, 074014 (2013) [arXiv:1305.0464].
[14] M. Misiak et al., Phys. Rev. Lett. 98, 022002 (2007) [hep-ph/0609232].
[15] M. Misiak and M. Steinhauser, Nucl. Phys. B 764, 62 (2007) [hep-ph/0609241].
[16] M. Misiak and M. Steinhauser, Nucl. Phys. B 840, 271 (2010) [arXiv:1005.1178].
[17] S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D 28, 228 (1983).
[18] M. Czakon, P. Fiedler, T. Huber, M. Misiak, T. Schutzmeier and M. Steinhauser, to be published.
[19] M. Czakon, U. Haisch and M. Misiak, JHEP 0703, 008 (2007) [hep-ph/0612329].
[20] H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647, 173 (2007) [hep-ph/0611123].
[21] T. Ewerth, Phys. Lett. B 669, 167 (2008) [arXiv:0805.3911].
[22] R. Bougezal, M. Czakon and T. Schutzmeier, JHEP 0709, 072 (2007) [arXiv:0707.3090].
[23] H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, Phys. Rev. D 82, 074006 (2010) [arXiv:1005.5587].
[24] A. Ferroglia and U. Haisch, Phys. Rev. D 82, 094012 (2010) [arXiv:1009.2144].
[25] M. Misiak and M. Poradziński, Phys. Rev. D 83, 014024 (2011) [arXiv:1009.5685].
[26] M. Kamiński, M. Misiak and M. Poradziński, Phys. Rev. D 86, 094004 (2012) [arXiv:1209.0965].
[27] T. Huber, M. Poradziński and J. Virto, JHEP 1501, 115 (2015) [arXiv:1411.7677].
[28] T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830, 278 (2010) [arXiv:0911.2175].
[29] A. Alberti, P. Gambino and S. Nandi, JHEP 1401, 147 (2014) [arXiv:1311.7381].
[30] A. Alberti, P. Gambino, K. J. Healey and S. Nandi, Phys. Rev. Lett. 114, 061802 (2015) [arXiv:1411.0560].
[31] Z. Ligeti, M.E. Luke, A.V. Manohar and M.B. Wise, Phys. Rev. D 60, 034019 (1999) [hep-ph/9903305].
[32] J. Charles et al. (CKMfitter Group Collaboration), arXiv:1301.0503.
[33] G. Ricciardi, Phys. Lett. B 355, 313 (1995) [hep-ph/9502286].
[34] L. P. Abbott, P. Sikivie and M. B. Wise, Phys. Rev. D 21, 1393 (1980).
[35] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B 527, 21 (1998) [hep-ph/9710335].
[36] F. Borzumati and C. Greub, Phys. Rev. D 58, 074004 (1998) [hep-ph/9802391].
[37] F. Borzumati and C. Greub, Phys. Rev. D 59, 057501 (1999) [hep-ph/9809438].
[38] T. Hermann, M. Misiak and M. Steinhauser, JHEP 1211, 036 (2012) [arXiv:1208.2788].
[39] O. Eberhardt, U. Nierste and M. Wiebusch, JHEP 1307, 114 (2013) [arXiv:1305.1649].
[40] V. Khachatryan et al. (CMS Collaboration), JHEP 1410, 160 (2014) [arXiv:1408.3316].
[41] G. Aad et al. (ATLAS Collaboration), JHEP 1411, 056 (2014) [arXiv:1409.6064].
[42] T. Saito et al. (Belle Collaboration), arXiv:1411.7198.