Optimization of the force impulse parameters during collision of the vehicles using the evolutionary algorithm

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Abstract. The paper presents the use of the evolutionary algorithm in the reconstruction of vehicle collisions to look for the point of the force impulse – Point of Impact (POI) and the Angle of the Tangent Plane (ATP) during the collision of two cars. The location of this point and the angle of the tangent plane of the collision are often decisive for the simulation of the vehicles’ motion and the calculated pre-crash speeds. The simulation calculations have shown that the evolutionary algorithm allows the location of POI in the area of the highest energy consumption of a given collision. This is the area with the highest stiffness of the car's structure. The search for ATP has also been successfully completed. This allowed for a reliable calculation of vehicle pre-crash speeds. Calculations were validated based on the actual crash test. The presented quality and limitation functions (vehicle stopping area and track run) are based on checking whether the vehicle in the simulation movement has stopped at the place determined by the surface with given dimensions, and the trajectories of the wheels tracks coincide with the set corridors. The uncertainty of calculations is indicated.

1. Introduction

The article presents the use of the evolutionary algorithm [1, 2, 10] with the quality function to look for the point of application of the impact force impulse (POI) of two vehicles and the angle of the tangent plane (ATP), Fig. 1. The location of this point and the tangent plane of the collision is often of decisive importance for simulation of the vehicle motion and calculated pre-crash speeds. The optimization calculations were verified based on the actual crash test. The presented function of quality and limitations is based on the study of whether the vehicle in the simulation movement has stopped at the place defined by the rectangle of error, and the trajectories of the wheels coincide with the real ones. The uncertainty of the calculations was indicated.

In the presented algorithm, the individual means the parameters sought i.e. the coordinates of the point of impact (POI), angle of the tangent plane (ATP) and the speeds of both vehicles. Each individual will therefore be represented by 4 numbers. Next generations of individuals are created on evolutionary principles, i.e. through reproduction (crossing, mutation), succession (total or partial substitution), selection (e.g. dying out).
2. Dynamic model of vehicle and model of impact
Simulation tests used a vehicle model with six degrees of freedom [14]. A linear tire model was adopted. The Kudlich-Slibar impact model was used [4]. These are commonly used models in the reconstruction of road accidents. The Kudlich-Slibar impact model has the advantage, that its input parameters can usually be measured and can be sufficiently defined numerically. The use of other more advanced models does not exclude the use of the optimization algorithm.

3. Quality function and constrains
There are many criteria for correctness of the solution in the optimization of collisions and vehicle motion after collision. In the model of the vehicle motion dynamics after collision, the criteria are the compatibility of the linear and angular position of the vehicles in the simulation with the positions revealed during the visual inspection, deformation of the car body, etc. [11, 14]. The quality function, which is subject to minimization in the optimization process, can take the form:

$$ Q = \frac{\sum_i (w_i \cdot x_i)^2}{\sum_i w_i} \cdot 100\% $$

where:
- $w_i$ - weight factor for criterion $x_i$
- $x_i$ - criterion: e.g. error of vehicle position after collision (after stopping), intermediate position error (during movement), direction / rotation angle error (after stopping), direction / rotation angle error (intermediate position), deformation error (EES).

In the case of collision analysis, the application of the popular formula (1) of the sum of weighted criteria, i.e. the weight combination of normalized angular values with values expressed in linear terms in meters, carries a risk of error. For example, if the vehicle's path in the post-crash motion is, for example, 100 m and the angle of rotation 360º, then 100% error of the location of the vehicle centre of mass is not energetically identical with 100% error of the angular position. It takes incomparably less energy to rotate the vehicle 360º than it would to travel 100 m with the lateral slip or braking of the vehicle. It means that the quality function according to (1) often does not take proportional values to match the simulation movement to the real one. This function can take a small value while the energy consumed in simulated motion is significantly different from reality. This can lead to incorrect calculation results. The quality function presented by dependence (2) was therefore constructed as:
\[ Q_{Pi} = \frac{w_{1Pi} \cdot Q_{1Pi} + w_{2Pi} \cdot Q_{2Pi}}{2} \]  \hspace{1cm} (2)

where:

\( Q_{1Pi} \) – a component of the quality function describing the adjustment of the position of the wheel for vehicle No. 1 in the final position in the simulation to the real position in [m];

\( Q_{2Pi} \) – a component of the quality function describing the adjustment of the location of the centre of mass of the vehicle No. 1 in the final position in [m];

\( w_{1Pi} \) – weight of the component of the quality function \( Q_{1Pi} \) for vehicle number 1;

\( w_{2Pi} \) – weight of the component of the quality function \( Q_{2Pi} \) for vehicle number 1;

The quality function for both vehicles takes the form:

\[ Q = \frac{Q_{1Pi} + Q_{2Pi}}{2} \]  \hspace{1cm} (3)

This function can be normalized to obtain its value in %, but this is not a necessary procedure for the operation of the optimizer.

The constraint function (4) examines whether the simulation of the trace of the car wheel does not move away from the real one beyond the limited area in the successive generations (iterations of the optimization algorithm), Fig. 2. This area is restricted by circles with a radius \( r_{wodz} \) in subsequent points of the trace. Through the optimization calculations, the trace is simulated "attracted" to the real track, and strictly located in the area dependent on the value of the radius \( r_{wodz} \). In subsequent generations of optimization, the radius \( r_{wodz} \) is reduced [5].

\[ O = \sum_{i=1}^{n} \left[ (x_{islry} - x_{islrx})^2 + (y_{islry} - y_{islrx})^2 \right] \leq r_{wodz}^2 \]  \hspace{1cm} (4)

where:

\( n \) - number of real track points,

\( x, y \) - coordinates of the trace points in the global coordinate system,

\( slr, sls \) - respectively: simulated and real trace index,

\( r_{wodz} \) - radius of the circle

The visualization of the operation of the evolutionary algorithm for two colliding vehicles is presented in Fig. 2. The algorithm uses the quality function according to (2).

**Figure 2.** Visualization of the function of quality and constraints with marked circles of radius \( r_{wodz} \).
4. Optimization of the point of impact and the angle of the tangent plane

Performing calculations of vehicle collisions, in the Kudlich-Slibar impact model, requires the selection of the point of impact (POI). The POI is located in the place where the greatest collision forces were concentrated. The location of POI in relation to vehicle center of mass affects very significantly the directions of movement of these vehicles shortly after the impact. The POI position affects the length of the arms of the impulse of force relative to the vehicle's center of mass.

The angle of location of the tangential plane (ATP) also affects the directions of vehicle velocity vectors shortly after the collision. This effect is important when the impulse reaches the surface of the ATP-dependent friction cone. "Manual" changes to the above parameters during simulation calculations are very time-consuming. Therefore, it is recommended to optimize them, which also allows you to search the full area of acceptable solutions. The optimization program enables defining a convex polygon in which POI will be sought, fig. 5. This polygon can be arbitrarily oriented and can overlap with the area of penetration of vehicle bodies. Points from outside the polygon area are discarded. The decision about the shape and size of the polygon is made by the person conducting the calculation based on the assessment of the extent of damage to the vehicles. The search for POI consists in drawing the coordinates of the point from the area of the rectangle described on the previously adopted polygon. The sides of this rectangle are parallel to the axis of the global coordinate system. The evolutionary algorithm looks for POI coordinate in the next populations and calculates, respectively, the arms of the impulse of force relative to the centers of the vehicles masses. Then, the signs of individual arms of the force pulse are examined, and by means of appropriate logical conditions the influence of the angle of the tangent plane on the change of these characters is taken into account when the tangent plane is rotated in the range from 0 to 360°. The originally defined POI area is gradually narrowed. The whole operation is based on the principle of the evolutionary algorithm with the presented function of quality and limitations. POI will never go beyond the area defined by a polygon because the condition of belonging to a polygon is still being examined. In subsequent populations, the area from which the POI is taken drawn decreases as the criterion of the positional placement of the centres of the vehicles mass and the radius criterion is more and more stringent. An actual collision of cars (Ford - Seat) was selected as an example [5]. The program was set to search for speeds and POI coordinates so that the optimization criteria were met in the last population: the position of the vehicle in relation to the real location and course of the traces. The final criterion of the radius for the location of the simulated traces was applied at 0.5 m, and the location of the centre of mass at 1.0 m to allow for a wider range of calculated speeds. The tangent plane has been set at a fixed angle of 268.63°. The results of calculations carried out in the "manual" mode (PC-Crash) are shown in Figure 3.
The speed of the Ford (blue) calculated (without optimization) in the PC-Crash was 52 km/h, and the Seat 0 km/h (it was known that the Seat car was standing). Real values in the experiment: Ford – 50 km/h, Seat - 0 km/h.

By optimizing this collision, the speed range of both vehicles was initially set at 0 – 200 km/h. The result of calculations with the evolutionary algorithm is presented in Fig. 4.

Figure 5 shows the area of POI sampling and the ATP parameter.
The selection of the POI location can not be arbitrary. This point can not lie outside the area of permeation of vehicle bodies. However, it is not always correct to take it anywhere in the penetration area (e.g. at the extremes) just because the simulated movement of the vehicles is the closest to the real one. Carrying out the calculations, taking into account the actual damage to vehicles, must be taken the decision whether the selected POI has a physical sense, e.g. whether it is near deformed stringers, damaged wheel suspension, or in the event of a collision with a non-deformable obstacle the POI does not penetrate inside these obstacle. POI should be close to the place that consumed the greatest collision energy.

This collision proved to be very sensitive to a change in the location of the POI. All matching pulse points are located in a very small area in relation to the polygon of searches, Fig. 5. Even a slight displacement of POI out of this small area caused a much different vehicle movement in the simulation and in the real movement. The important thing is that in the simulation, the permissible POI position is located in the area of the left front wheel of the Seat car (red silhouette), which proves the correctness of the result, because it is compatible with the highest stiffness and energy consumption of the impact area. The algorithm allowed to move the POI in the range of the deformation depth (in the direction similar to the direction of the Ford car), but the shift to it perpendicular was much smaller. This is due to the fact that the shift of POI in the direction of Ford's movement did not significantly change the torques acting on the vehicle centres of masses and on the directions in motion after the collision. On the other hand, the lateral displacement changed these moments and caused incompatibility of the simulated movement with the real motion.

In the case of both vehicles, it is difficult to find a clear tendency for the calculated speed to increase or decrease depending on the POI shifting towards the increase or decrease of the \(x\) or \(y\) coordinate value of this point. This was shown in Figure 6, where the distribution of points representing vehicle speeds is quite chaotic.
The angle of the tangent plane of the collision (ATP) also affects the result of calculations. In a situation where the impulse of the POI collision does not enter the friction cone, the rotation of the tangent plane does not produce any effect, as there is rough friction at the point of impact. If, however, the collision impulse is on the side of friction cone the direction of the traffic of the vehicles changes due to the slip friction. In order to investigate how the angle of the tangent plane affects the pre-crash speeds of vehicles, based on principles analogous to those for POI optimization, the ATP optimization model was built, followed by the sub-program implementing it.

The initial range of ATP optimization was set to the range 0-360°. Fig. 7 shows the accepted (one of the matched) POIs. In optimization calculations, these are many points (corresponding to the number of individuals in the population) imposed on each other, because they have identical coordinates.

The optimization calculations carried out, in this particular case, did not show any significant impact of ATP on pre-crash speed of vehicles, as shown in Figure 8.
Speed of the vehicle 1 (after ATP optimization) ranged from 51.56 to 52.15 km/h, and vehicle 2 from 0.02 to 0.34 km/h.

The result of these calculations was predictable and is consistent with the result of calculations carried out in the "manual" mode. Rotation of the tangent plane in the range of about 252-293° did not affect the directions of the vehicles' motion. However, as soon as the impulse of the collision was on the side of the friction cone, there was a sudden change of the motion direction, especially the vehicle No. 1. The evolutionary algorithm, striving for the best match of the simulated motion to the real motion of the vehicle, correctly eliminated the traits of the tangent plane (ATP) from the population out of the matched range.

Fig. 9 presents the values of the standardized quality function for both vehicles, for the last population of individuals, according to the dependence (3) including dependencies (2) and (4). The value did not exceed the number 1, which means that none of the individuals didn't exceed the predefined final value of the radius $r_{\text{wod}}$ and predefined final position of the mass centres. Normalization consisted in dividing the received value of the quality function by the maximum allowed deviation of the position of vehicles in the last generation. The percentage value of the quality function could not exceed the value of 1 (100%), because the individuals who violated this rule were eliminated from the population.
5. Optimization all parameters
While optimizing both POI and ATP, one should expect to increase the uncertainty of the calculated pre-crash speed. Speeds as well as POI and ATP were optimized in the same ranges as in separate analyzes.

Fig. 10 presents the distribution of optimized POI. Comparing the location of these points with the location in Figure 5 (optimization only POI), it can be seen that the width of the area covered with points increased in the direction similar to the direction of the speed vector of the car No. 1 (Ford). However, there was no noticeable change in the direction of it perpendicular to it, which should be explained as in the example from Fig. 5. There could not be a significant change in the moments acting on the centers of masses. Recognition by the evolutionary algorithm that the "additional" POIs are optimal should be explained by the optimization of the ATP simultaneously. New POIs were created, and their unfavorable features compensated ATP features, which also slightly expanded, by about 2 degrees (Figure 10). Two grades seem to be unimportant. However, when the impulse on the surface of the friction cone is positioned, even the minimum value of the ATP significantly influences the directions of vehicle movement after the impact.

Figure 10. An enlarged view of optimized POI points along with an optimized ATP search range about 42°.

Figure 11 shows the speed range of the vehicles 1 and 2 as a function of POI coordinates for POI and ATP optimization.
As in the optimization of each of the parameters separately, in this particular collision you cannot see a tendency indicating that the change in the POI coordinate influences the increase of the calculated speed. It can only be said that the range of both calculated speeds of both vehicles increased as well as the range of both coordinates – individual traits.

In the case of a standing vehicle, this is an increase of approx. 1 km/h, however, this is a negligible value from the point of view of reconstruction of road accidents.

Fig. 12 presents the speeds of both vehicles as a function of ATP for POI and ATP optimization.

It is not noticeable that the angle of the tangent plane's location influences the calculated velocities. The distribution of individuals is unordered, and the dispersion of results is not significant in terms of reconstruction of the collision.

The range of the standardized quality function has been extended according to the relationship (2), (3) and (4), which can be explained by the "greater" number of matching possibilities resulting from the search for a larger number of parameters (Fig. 13).
However, the algorithm didn’t go beyond the imposed criterion of final positions and the traces. The program realized 52,113 iterations. The time of simultaneous optimization of both parameters increased by 43% compared to the optimization of only POI and by 400% in relation to the optimization of ATP.

Summing up the conducted analyzes, it can be pointed out that for this particular collision, the speed uncertainties for POI optimization, ATP and total for these parameters were respectively:

- only POI: from 51.7 to 53.8 km/h (Ford) – real speed 52 km/h, from 0.05 to 1.48 km/h (Seat) – real speed 0 km/h;
- only ATP: from 51.56 to 52.15 km/h (Ford), from 0.02 to 0.34 km/h (Seat);
- both parameters together: from 51.7 to 53.8 km/h (Ford), from 0.00 to 1.59 km/h (Seat).

Figure 14 shows how individual POIs were distributed in a given polygon during the formation of subsequent generations while optimizing POI and ATP simultaneously.
In the first population with the least severe vehicle location criteria (Figure 14a), POIs are evenly distributed over the entire area. In the second population (b), the distribution is similar and you can see points located identically with (a). This is understandable because the next generation's evolutionary program moved 18 individuals (from 40) from the first to the second generation, because they met the next more stringent criteria for this generation. A visually significant difference appears between Fig. 14 (b) and (c). During the creation of the third generation only 1 individual from generation 2 to generation 3 was transferred. Fig. 14 (c) and (d) are almost identical, from 3 to 4 generation passed as many as 34 individuals. Generation No. 5, Fig. 14 (e) differs significantly from the others - none of the
individuals of the fourth generations have passed into fifth generation. There is no rule to which
generation will be transferred many individuals, and when none. All this depends on the sensitivity of
a given collision, the shape of post-collision motion and the settings of the optimizer. It would be
desirable to narrow down the characteristics of the population in a linear way. Sometimes this
happens, but there are so many parameters that determine the course of optimization, that at that
moment you cannot answer how to set the algorithm to always do this.

6. Conclusions
The simulation calculations have shown that the evolutionary algorithm allows placing the point of the
force impulse (POI) during the collision of two vehicles in a credible area of the highest energy
consumption of a given collision. At the same time, the accuracy criteria for the location of vehicles
after the collision assumed in the simulation are met.
The search for the angle of the tangential plane (ATP) has also been successfully completed.
The use of the evolutionary algorithm to optimize the point of impact (POI) and the position of the
tangential plane (ATP) allowed reliable calculate the pre-crash speeds of the vehicles.

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