The $K^{-}p \to f_{1}(1285)\Lambda$ reaction within an effective Lagrangian approach

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The production of the $f_{1}(1285)$ resonance in the reaction of $K^{-}p \to f_{1}(1285)\Lambda$ is studied within an effective Lagrangian approach. The production process is described by the $t$ channel $K^{*+}$ meson exchange. My theoretical approach is based on the results of chiral unitary theory where the $f_{1}(1285)$ resonance is dynamically generated from the single channel $KK^{*} - c.c.$ interaction. The total and differential cross sections of the $K^{-}p \to f_{1}(1285)\Lambda$ reaction are evaluated. Within the coupling constant of the $f_{1}(1285)$ to $KK^{*}$ channel obtained from the chiral unitary approach and a cut off parameter $\Lambda_{c} \sim 1.5$ GeV in the form factors, the experimental measurement can be reproduced. This production process would provide further evidence for the $KK^{*} - c.c.$ nature of the $f_{1}(1285)$ state.

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I. INTRODUCTION

The $f_{1}(1285)$ meson was discovered in the 1960s as a bump in the $\pi KK$ mass distribution in $p\bar{p}$ annihilations and in the reaction of $\pi^{-}p \to n\pi KK$. Its quantum numbers have been quite well established, thus, it is now an axial-vector state with $[P G J^{PC}(J^{PCC}) = 0^{+}(1^{++})]$ and mass $M_{f_{1}} = 1281.9 \pm 0.5$ MeV and total decay width $\Gamma_{f_{1}} = 24.2 \pm 1.1$ MeV. Within quark models, it is described as a $q\bar{q}$ state, while the recent studies of Refs. have shown that in addition to the well-established quark model picture, the $f_{1}(1285)$ resonance can also be understood as a dynamically generated state made from the single channel $KK^{*} - c.c.$ interaction using the chiral unitary approach. The $f_{1}(1285)$ resonance cannot couple to other pseudoscalar-vector channels because it has positive $G$ parity, and it cannot decay into two pseudoscalar mesons (in principle $KK$ in this case) for parity and angular momentum conservation reasons. Furthermore, the $f_{1}(1285)$ resonance is located below the $KK^{*}$ threshold, hence its observation in two-body decays is very difficult. Indeed, the main decay channels of the $f_{1}(1285)$ resonance are $4\pi$ (branching ratio $= 33\%$), $\eta\pi\pi$ (52%), and $\pi KK$ (9%).

In Ref. the work of Ref. on the pseudoscalar-vector interaction was extended to include the higher order terms in the Lagrangian, and it was shown that the effect of the higher order terms is negligible. The inclusion of the higher-order kernel does not change the results obtained in Ref. in any significant way, and thus, it lends more confidence to the molecular picture of the $f_{1}(1285)$ state. Using the dynamical picture, predictions for lattice simulations in finite volume have been done in Ref. On the other hand, the three-body decays of $f_{1}(1285) \to \eta\pi\pi$ and $f_{1}(1285) \to \pi KK$ were studied using the picture that the $f_{1}(1285)$ is dynamically generated from the single channel $KK^{*} - c.c.$ interaction in Refs. , where the theoretical calculations are compatible with the experimental measurements. In Ref. , the role of the $f_{1}(1285)$ resonance in the decays of $J/\psi \to \phi KK^{*} + c.c.$ and $J/\psi \to \phi f_{1}(1285)$ was investigated.

On the experimental side, the production and decay of the $f_{1}(1285)$ resonance have been studied in the reaction of $K^{-}p \to f_{1}(1285)\Lambda$ at an incident $K^{-}$ momentum of $4.2$ GeV . The total cross section for this reaction is $11 \pm 3 \mu b$ at $p_{K^{-}} = 4.2$ GeV. Later, in Refs. , the production of $f_{1}(1285)$ resonance in the decays of $J/\psi \to \phi f_{1}(1285) \to \phi 2(\pi^{+}\pi^{-})$ and $J/\psi \to \phi f_{1}(1285) \to \phi \eta\pi^{+}\pi^{-}$ were studied by the DM2 Collaboration .

The $KK^{*}$ channel is bound for the energy of the $f_{1}(1285)$ by about 100 MeV, hence this decay is not observed experimentally . However, in this work, I study the production (rather than decay) of the $f_{1}(1285)$ resonance from the $KK^{*}$ interaction in the $K^{-}p \to f_{1}(1285)\Lambda$ reaction within the effective Lagrangian approach. From the perspective that the $f_{1}(1285)$ is generated from the single channel $KK^{*} - c.c.$ interaction, the $K^{-}p \to f_{1}(1285)\Lambda$ reaction is dominant by the $t$ channel $K^{*+}$ exchange, and the interaction of $K^{-}$ and $K^{*+}$ generating the $f_{1}(1285)$ resonance. This process should be tied to the $KK^{*} - c.c.$ nature of the $f_{1}(1285)$ state.

Before the end of this introduction, it will be helpful to mention that, based on phenomenological Lagrangians, only the tree-level diagram contributions are considered, in which the re-summation of the Born amplitudes are

$^{1}$ It was assumed that the signal observed in the $\eta\pi^{+}\pi^{-}$ invariant mass distribution at 1297 MeV is the $f_{1}(1285)$ resonance.

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not taken into account. However, the model can give a reasonable description of the experimental measurement in the considered energy region. Meanwhile, the present calculation offers some important clues for the mechanisms of the $K^- p \rightarrow \Lambda f_1(1285)$ reaction and makes a first effort to study the production of the $f_1(1285)$ resonance in the $K^- p \rightarrow \Lambda f_1(1285)$ reaction. This production process would provide further evidence for the $\bar{K}K^* - c.c.$ nature of the $f_1(1285)$ state.

This paper is organized as follows. In Sec. II I discuss the formalism and the main ingredients of the model. In Sec. III I present my main results and, finally, a short summary and conclusions are given in Sec. IV.

II. FORMALISM

The basic tree level Feynman diagram for the $K^- p \rightarrow f_1(1285)\Lambda$ reaction is depicted in Fig. 1 where the $t$ channel $K^{*+}$ exchange is considered. In this work, the contributions from $s$ and $u$ channels are ignored, because the information about the $\Lambda\Lambda f_1(1285)$ and $\Lambda^*\Lambda f_1(1285)$ vertices in the $s$ channel and $NN f_1(1285)$ and $N^* N f_1(1285)$ vertices in the $u$ channel is scarce.

\[ \begin{array}{c}
K^- \quad \bullet \quad p_3 \quad f_1(1285) \\
\quad \bullet \quad q \\
p_2 \quad \bullet \quad p_4 \quad \Lambda
\end{array} \]

FIG. 1: Feynman diagram for the $K^- p \rightarrow f_1(1285)\Lambda$ reaction. It consists of $t$ channel $K^{*+}$ exchange. In this diagram, we also show the definition of the kinematical $(p_1, p_2, p_3, p_4)$ variables that we used in the present calculation. In addition, we use $q = p_3 - p_1 = p_2 - p_4$.

To compute the contribution of the diagram shown in Fig. 1 I need the effective interaction of the $\Lambda p K^{*+}$ vertex, which is given in terms of the interaction Lagrangian density by [16, 17]:

\[ \mathcal{L}_{\Lambda NK^*} = -g_{\Lambda NK^*} \Lambda (\gamma^\mu - \frac{\kappa_{K^*}}{m_{\Lambda} + m_p} \sigma^{\mu\nu} \partial_\nu) K^*_\mu N + h.c., \] (1)

which consists of a vector part with $\gamma^\mu$ and a tensor part with $\sigma^{\mu\nu}$. Within SU(3) flavor symmetry, the value of the coupling constant, $g_{\Lambda NK^*} = -6.41$, is obtained from the relationship $g_{\Lambda NK^*} = -g_{NN\Lambda}(1 + \alpha_{BBV})/\sqrt{3}$ with $g_{NN\Lambda} = 3.36$ and $\alpha_{BBV} = 1.15$ [16, 17]. I take $\kappa_{K^*} = 2.77$ which is obtained from $\kappa_{K^*} = 1.5k_{p}/(1 + 2\alpha_{BBV})$ with $k_{p} = 6.1$ [18, 22].

In addition to the $\Lambda NK^*$ vertex, I need also the interaction of the $\bar{K}K^* f_1(1285)$ vertex. As mentioned before, in the chiral unitary approach of Ref. [7], the $f_1(1285)$ resonance results as dynamically generated from the interaction of $\bar{K}K^* - c.c.$ One can write down the $K^- K^{*+} f_1(1285)$ vertex of fig. 1 as,

\[ v = -\frac{1}{2}g_{f_1} \varepsilon^\mu(K^*) \varepsilon^{\mu\ast}(f_1), \] (2)

where $\varepsilon^\mu(K^*)$ is the polarization vector of the $K^{*+}$ and $\varepsilon^{\mu\ast}(f_1)$ is the polarization vector of the $f_1(1285)$ resonance. The factor $-\frac{1}{2}$ in Eq. 2 is due to the fact that the $f_1(1285)$ resonance couple to the $I = 0$, $C = +1$ and $G = +1$ combination of $\bar{K}$ and $K^*$ mesons, which is represented by the state

\[ \frac{1}{\sqrt{2}}(\bar{K}K^* - \bar{K} \bar{K}^*) = -\frac{1}{2}(K^- K^{*+} + \bar{K}^0 K^0) - K^+ K^{*-} - K^0 \bar{K}^{*0}, \] (3)

where I have taken $C|K^*| = -|K^*|$, which is consistent with the standard chiral Lagrangians.

The coupling $g_{f_1}$ of the $f_1(1285)$ resonance to the $\bar{K}K^*$ channel was obtained in Refs. [7, 11] from the residue in the pole of the scattering amplitude for $\bar{K}K^* \rightarrow \bar{K} \bar{K}^*$ in $I = 0$. In the present calculation, I take $g_{f_1} = 7555$ MeV as used in Ref. [11].

With the effective interaction Lagrangian density for the $\Lambda NK^*$ vertex and the $K^- K^{*+} f_1(1285)$ interaction shown in Eq. 2, one can easily construct the invariant scattering amplitude for the reaction of $K^- p \rightarrow f_1(1285)\Lambda$ as,

\[ \mathcal{M} = \frac{i g_{f_1} g_{\Lambda NK^*}}{2} \bar{u}(p_4, s_{\Lambda}) \left[ \gamma_{\mu} + \frac{\kappa_{K^*}}{m_{\Lambda} + m_p}(q_{\mu} - \bar{q} q_{\mu}) \right] u(p_2, s_p) G_{K^*}^{\mu\nu}(q) \varepsilon_{\nu}(p_3, s_{f_1}) F_1(q) F_2(q), \] (4)

where $s_{\Lambda}$, $s_p$, and $s_{f_1}$ are the spin polarization variables for the $\Lambda$, proton, and $f_1(1285)$ resonance, respectively. The $G_{K^*}^{\mu\nu}$ is the $K^*$ meson propagator with the form as,

\[ G_{K^*}^{\mu\nu}(q) = -i \frac{\eta^{\mu\nu} - q^\mu q^\nu/m_{K^*}^2}{q^2 - m_{K^*}^2}. \] (5)

with $m_{K^*}$ the mass of the exchanged $K^{*+}$ meson. I take $m_{K^*} = 891.66$ MeV in the present calculation.

Finally, because the hadrons are not point-like particles and the exchanged $K^{*+}$ meson is off mass shell, one needs to include the form factors, $F_1(q)$ and $F_2(q)$ in Eq. 4, where $F_1(q)$ is for the $\Lambda NK^*$ vertex and $F_2(q)$ is for the $K^- K^{*+} f_1(1285)$ vertex. To minimize the number of free parameters, one can choose the same form for $F_1(q)$ and $F_2(q)$. I adopt here the common scheme used in many previous works [16, 22].

\[ F_1(q) = F_2(q) = \left( \frac{\Lambda_c^2 - m_{K^*}^2}{\Lambda_c^2 - q^2} \right)^2. \] (6)

This form has advantages and disadvantages because $(\Lambda_c^2 - m_{K^*}^2)^2$ will be small and cut much if $\Lambda_c$ is not
far from the $m_{K^-}$. Actually, the value of $\Lambda_c$ can be directly related to the hadron size. But, the question of hadron size is still very open, one has to adjust $\Lambda_c$ to fit the related experimental data. Empirically the cutoff parameter $\Lambda_c$ should be at least a few hundred MeV larger than the $K^-$ mass, hence, one can constrain it in the range of 1.3 to 1.7 GeV as used in previous works \cite{20} for other reactions.

In the present model, the final state interaction (FSI) is not considered because it is difficult to treat the FSI unambiguously due to the lack of the accurate $f_1(1285)\Lambda$ interaction. This FSI effect would give the near threshold enhancement in the total cross section. In this work, I do not take this FSI in account since this estimation is rough thus it would cause uncertainty in the model.

Then the calculation of the invariant scattering amplitude square $|\mathcal{M}|^2$ and the cross section $\sigma(K^-p \rightarrow f_1(1285)\Lambda)$ is straightforward. The differential cross section for $K^-p \rightarrow f_1(1285)\Lambda$ in center of mass (c.m.) frame can be expressed as

$$\frac{d\sigma}{d\cos(\theta)} = \frac{1}{32\pi W^2} \left| \frac{\vec{p}_1}{\vec{p}_3} \right|^2 \left( \frac{1}{2} \sum_{s_p} \sum_{s_{f_1}} |\mathcal{M}|^2 \right),$$

where $W$ is the invariant mass of the $K^-p$ system, whereas, $\theta$ denotes the scattering angle of the outgoing $f_1(1285)$ resonance relative to beam direction in the c.m. frame. In the above equation, $\vec{p}_1$ and $\vec{p}_3$ are the 3-momenta of the initial $K^-$ meson and the final $f_1(1285)$ mesons,

$$|\vec{p}_1| = \frac{\lambda^{1/2}(W^2, m^2_{K^-}, m^2_p)}{2W},$$

$$|\vec{p}_3| = \frac{\lambda^{1/2}(W^2, M^2_{f_1}, m^2_{\Lambda})}{2W},$$

where $\lambda(x, y, z)$ is the Kählen or triangle function. The $m_{K^-}$, $m_p$, and $m_\Lambda$ are the masses of the $K^-$ meson, proton, and $\Lambda$, respectively. I take $m_{K^-} = 493.68$ MeV, $m_p = 938.27$ MeV, and $m_\Lambda = 1115.68$ MeV.

In the effective Lagrangian approach, the sum over polarizations and the Dirac spinors can be easily done thanks to

$$\sum_{s_{f_1}} \varepsilon^\mu(p_3, s_{f_1}) \varepsilon^{\nu*}(p_3, s_{f_1}) = -g^{\mu\nu} + \frac{p_{3}^\mu p_{3}^\nu}{M_{f_1}^2},$$

$$\sum_{s_p} \bar{u}(p_2, s_p) u(p_2, s_p) = \frac{p_2 + m_p}{2m_p},$$

$$\sum_{s_\Lambda} \bar{u}(p_4, s_\Lambda) u(p_4, s_\Lambda) = \frac{p_4 + m_\Lambda}{2m_\Lambda}.$$  

Finally, I get

$$\frac{1}{2} \sum_{s_p} \sum_{s_{f_1}} |\mathcal{M}|^2 = \frac{g_{NNK}^2 g_1^2 F_1^2(q) F_2^2(q)}{8m_p m_\Lambda (q^2 - m^2_{K^-})^2} \times \text{Tr} \left( (\not \! p_4 + m_\Lambda) \Gamma_\mu (\not \! p_2 + m_p) \Gamma_\nu (\not \! p_1 + m_p) \right),$$

with

$$\Gamma_\mu = \left( \gamma^\rho + \frac{\kappa_{K^-}}{m_\Lambda + m_p} (q^\rho - g^\rho_\mu) \right) (g_\mu^\rho - \frac{q_\mu q_\rho}{m^2_{K^-}}) = \gamma^\mu + \frac{\kappa_{K^-}}{m_\Lambda + m_p} q_\mu - \frac{\kappa_{K^-}}{m_\Lambda + m_p} g^\rho_\mu - \frac{q_\mu q_\rho}{m^2_{K^-}}.$$  

### III. NUMERICAL RESULTS AND DISCUSSION

With the formalism and ingredients given above, the total cross section versus the invariant mass ($W$) \(^2\) of the $K^-p$ system for the $K^-p \rightarrow f_1(1285)\Lambda$ reaction is evaluated. The results for $W$ from the reaction threshold to 5.0 GeV are shown in Fig. 2 together with the experimental data \cite{13} for comparison. In Fig. 2 the dashed and solid curves represent the theoretical results obtained with $\Lambda_c = 1.3$ and 1.7 GeV, respectively. One can see that the experimental data can be reproduced with a reasonable value of the cut off parameter $\Lambda_c$. Besides, the calculated cross section is sensitive to the value of cutoff parameter used in the form factors. The obtained results for the total cross section $\sigma$ at $W = 3.01$ GeV are 2.1 $\mu b$ and 50.5 $\mu b$ with $\Lambda_c = 1.3$ and 1.7 GeV, respectively. To have a reliable prediction for the cross section for the reaction $K^-p \rightarrow f_1(1285)\Lambda$ thus requires a good knowledge of the form factors. More and accurate experimental data can be used to constraint the value of the cut off parameter.

![FIG. 2: Total cross section for the $K^-p \rightarrow f_1(1285)\Lambda$ reaction as a function of the invariant mass $W$. The experimental data are taken from Ref. \cite{13}, which can be well reproduced if I take $\Lambda_c = 1.46$ GeV.](Image)
On the other hand, since there is only one available experimental datum, one will always reproduce the experimental data, \( \sigma = 11 \pm 3 \, \mu b \) at \( W = 3.01 \, \text{GeV} \), by adjusting the cut off parameter \( \Lambda_c \) with a fixed coupling \( g_f = 7555 \, \text{MeV} \). Indeed, if one chooses \( \Lambda_c = 1.46 \, \text{GeV} \), \( \sigma = 10.9 \, \mu b \) can be obtained at \( W = 3.01 \, \text{GeV} \).

In addition to the total cross section, I calculate also the differential cross section for the \( K^- p \rightarrow f_1(1285) \Lambda \) reaction as a function of \( \cos(\theta) \) at different energies. The theoretical results are shown in Fig. 3. Since I have considered the contributions from only the \( t \) channel \( K^{*+} \) exchange which will give dominant contributions at the forward angle, the differential cross sections have strong diffractive behavior, and it is stronger when the energies are increased. These distributions can be measured by experiment at corresponding energies. It should be pointed out that, if there are contributions from \( s \) and \( u \) channels, they will cause nondiffractive effects at off-forward angles which can be measured directly.

In brief, either a detailed scan of the total cross section or the angular distributions for the reaction of \( K^- p \rightarrow f_1(1285) \Lambda \) will test the model calculation, and will give more valuable information about the mechanism of this reaction. Note that if there were contributions from the \( s \) channel terms, there will be a clear bump (or peak) in the total cross section. Indeed, in Ref. [12] it was claimed that they found also some backward contributions to the \( K^- p \rightarrow f_1(1285) \Lambda \) reaction. However, the limited experimental data of Ref. [12] were obtained in the 1970s and only a few signal events were observed. The future experimental observation of the total and differential cross sections would provide very valuable information on the reaction mechanism of \( K^- p \rightarrow f_1(1285) \Lambda \).

### IV. SUMMARY

In summary, the production of the \( f_1(1285) \) resonance in the \( K^- p \rightarrow f_1(1285) \Lambda \) reaction is studied within an effective Lagrangian approach. The production process is described by the \( t \) channel \( K^{*+} \) meson exchange, while the coupling constant of \( f_1(1285) \) to \( K^- K^{*+} \) is adopted from the results of chiral unitary theory where the \( f_1(1285) \) resonance is dynamically generated from the single channel \( KK^* - c.c. \) interaction. The total and differential cross sections are calculated which can be tested by future experiments. Note that the theoretical results have a strong dependence on the cutoff parameter \( \Lambda_c \) in the form factors. To have a reliable prediction for the cross section we need a good knowledge of the form factors.

Finally, I would like to stress that, thanks to the important role played by the \( t \) channel \( K^{*+} \) exchange in the \( K^- p \rightarrow f_1(1285) \Lambda \) reaction, one can reproduce the available experimental data with a reasonable value of the cut off parameter in the form factors. More and accurate data for this reaction will provide more valuable information on the reaction mechanism of \( K^- p \rightarrow f_1(1285) \Lambda \) reaction and can be used to test the model calculation which should be tied to the \( KK^* - c.c. \) nature of the \( f_1(1285) \) state. This work constitutes a first step in this direction.

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