Magnetic moments of the spin-$\frac{3}{2}$ singly heavy baryons

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We calculate the magnetic moments of spin-$\frac{3}{2}$ singly charmed baryons in the heavy baryon chiral perturbation theory (HBChPT). The analytical expressions are given up to $O(p^3)$. The heavy quark symmetry is used to reduce the number of low energy constants (LECs). With the Lattice QCD simulation data as the magnetic moments of the charmed baryons, the numerical results are given up to $O(p^3)$ in three scenarios. In the first scenario, we use the results in the quark model as the leading order input. In the second scenario, we use the heavy quark symmetry and neglect the contribution of heavy quark. In the third scenario, the heavy quark contribution is considered on the basis of the scenario II and the magnetic moments of singly bottom baryons are given as a by-product.

I. INTRODUCTION

The singly heavy baryon contains a heavy quark and two light quarks. The two light quarks form the $\bar{3}_f$ and the $6_f$ representation in the SU(3) flavor symmetry. With the constraint of Fermi-Dirac statistics, the spin of the $\bar{3}_f$ and the $6_f$ diquarks are 0 and 1, respectively. Thus, the total spin of the $\bar{3}_f$ heavy baryon is $\frac{1}{2}$ while that of the $6_f$ heavy baryon is either $\frac{1}{2}$ or $\frac{3}{2}$.

The electromagnetic form factors are important properties of the hadrons, which can reveal their inner structures. The magnetic moments of hadrons especially attract much attention from the theorists and experimentalists [1–7]. The magnetic moments of the singly charmed baryons were investigated in naive quark model in Ref. [8]. In Ref. [9], the relativistic effect was considered. The magnetic moments and charge radii of the charmed baryons are calculated [9]. The SU(4) chiral constituent quark model was also adopted to calculated the (transition) magnetic moments of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ charmed baryons [10]. The magnetic moments of heavy flavor baryons were calculated in hypercentral model in Ref. [11]. The magnetic moments of spin-$\frac{3}{2}$ heavy baryons were obtained using the effective mass and screened charge scheme [12]. Besides the above quark models, the MIT bag model was employed to get the magnetic moments of heavy baryons [13], which were reexamined in Ref. [14]. The magnetic moments of charmed baryons were calculated in the skyrmion description [15]. The magnetic moments of heavy flavored baryons were calculated in the QCD sum rules [16–18]. The magnetic moments of the lowest-lying singly heavy baryons were investigated in the chiral quark-soliton model [19]. The (transition) magnetic moments and charge radii of charmed baryons were simulated with the Lattice QCD recently [20–23].

The chiral perturbation theory (ChPT) is a model-independent method to study the hadron properties [24–26]. When one performs the ChPT in the baryon sector, the nonvanishing baryon mass in chiral limit will mess up the power counting used in the pure meson sector. The heavy baryon chiral perturbation theory (HBChPT) was introduced to solve the problem [27,28]. The HBChPT is expanded by the momenta of pseudoscalar mesons and the residual momenta of heavy baryons. The HBChPT was widely performed to calculated the electromagnetic properties of baryons. The magnetic moments of octet and decuplet baryons were calculated in HBChPT scheme [29–34]. The (transition) magnetic moments of doubly heavy baryons were investigated in Refs. [35–37]. The magnetic moments of singly charmed baryons were calculated up to the next-to-next-to-leading order in HBChPT [38,39]. In our recent work, we calculated the magnetic moments of spin-$\frac{1}{2}$ singly charmed baryons up to the $O(p^3)$ [40].

The dynamics of singly heavy hadron is constrained by both the chiral symmetry in light quark sector and heavy quark symmetry in heavy quark sector. The heavy quark symmetry and the chiral symmetry were often combined to investigate the singly heavy hadrons. In Ref. [41], the authors constructed the chiral Lagrangians of heavy mesons...
(Qq) and heavy baryons (Qqq) and calculated their strong and semileptonic weak decays incorporating with heavy quark symmetry. The decay properties of singly heavy hadrons were calculated in a formalism which combines the chiral symmetry and the heavy quark symmetry. The electromagnetic decays of \( D_{s0}(2317) \) and \( D_{s1}(2460) \) are investigated in the heavy-hadron chiral perturbation theory with the heavy quark symmetry.

In this work, we calculate the magnetic moments of the spin-\( \frac{3}{2} \) singly heavy baryons in the HBChPT scheme. In Section II, we perform the multiple expansion of the electromagnetic current matrix element for spin-\( \frac{3}{2} \) baryons. In Section III, we construct the Lagrangians used in calculating the magnetic moments. In Section IV, we calculate the analytical expressions of the magnetic moments order by order up to \( O(p^3) \). In Section V, we reduce the numbers of independent LECs in our analytical results with the heavy quark spin symmetry. We give the numerical results in three scenarios in Section VI. Some discussions and a brief conclusion are given in the Section VII. The integrals used in this work and some by-products are listed in the Appendix.

## II. ELECTROMAGNETIC FORM FACTORS OF THE SPIN-\( \frac{3}{2} \) BARYONS

Constrained by the time reversal (T), the parity (P), charge conjugate (C) and the gauge invariance, the matrix element of the electromagnetic current for spin-\( \frac{3}{2} \) particles takes the following form:

\[
\langle T(p')|J_\mu|T(p)\rangle = \bar{u}^\rho(p')O_{\rho\mu\sigma}(p', p)u^\sigma(p),
\]

with

\[
O_{\rho\mu\sigma}(p', p) = -g_{\rho\sigma} \left[ \gamma_\mu F_1(q^2) + \frac{i\sigma_{\rho\sigma} q^\alpha}{2M_T} F_2(q^2) \right] - \frac{q_\rho q_\sigma}{4M_T^2} \left[ \gamma_\mu F_3(q^2) + \frac{i\sigma_{\rho\sigma} q^\alpha}{2M_T} F_4(q^2) \right],
\]

where \( p \) and \( p' \) are the momenta of the spin-\( \frac{3}{2} \) baryons. \( P = p + p', q = p' - p \). \( M_T \) is the baryon mass and \( u_\sigma \) is the Rarita-Schwinger spinor.\[51\]

The charge (E0), electro-quadrupole (E2), magnetic-dipole (M1), and magnetic octupole (M3) form factors read

\[
\begin{align*}
G_{E0}(q^2) &= F_1 - \tau F_2 + \frac{3}{2} \tau G_{E2}, \\
G_{E2}(q^2) &= F_1 - \tau F_2 - \frac{3}{2}(1 + \tau)(F_3 - \tau F_4), \\
G_{M1}(q^2) &= F_1 + F_2 + \frac{3}{2} \tau G_{M3}, \\
G_{M3}(q^2) &= F_1 + F_2 - \frac{1}{2}(1 + \tau)(F_3 + F_4).
\end{align*}
\]

where \( \tau = -\frac{q^2}{2M_T^2} \). On the right-hand side, we omit the variable \( q^2 \) of \( F_i \) for convenience. The magnetic-dipole form factor is related to the magnetic moment as

\[
\mu_T = G_{M1}(0) \frac{e}{2M_T}.
\]

In HBChPT scheme, The baryon momentum \( p^\mu \) is decomposed into the \( M_T v^\mu \) and a residual momentum \( k^\mu \), where \( v_\mu \) is the velocity of the baryon and \( v^2 = 1 \). The baryon field \( T \) is decomposed into a “light” field \( \mathcal{T}(x) \) and a “heavy” field \( \mathcal{N}(x) \),

\[
\begin{align*}
\mathcal{T}(x) &= e^{iM_T v \cdot x} \frac{1 + \frac{\tau}{2}}{1 - \frac{\tau}{2}} T(x), \\
\mathcal{N}(x) &= e^{iM_T v \cdot x} \frac{1 - \frac{\tau}{2}}{1 + \frac{\tau}{2}} T(x).
\end{align*}
\]

After integrating out the heavy degrees of freedom, one gets the nonrelativistic Lagrangians. In the HBChPT scheme, the theory is expanded by either the momenta of the pseudoscalar mesons or the residual momenta of the baryons.

In the HBChPT scheme, the matrix element of the electromagnetic current \( J_\mu \) is reduced as

\[
\langle T(p')|J_\mu|T(p)\rangle = \bar{u}^\rho(p')O_{\rho\mu\sigma}(p', p)u^\sigma(p),
\]

with

\[
O_{\rho\mu\sigma}(p', p) = -g_{\rho\sigma} \left[ v_\mu \left( F_1 - \tau F_2 \right) + \frac{[S_\mu, S_\sigma]}{M_T} q^\alpha \left( F_1 + F_2 \right) \right] - \frac{q_\rho q_\sigma}{4M_T^2} \left[ v_\mu \left( F_3 - \tau F_4 \right) + \frac{[S_\mu, S_\sigma]}{M_T} q^\alpha \left( F_3 + F_4 \right) \right],
\]

where \( S_\mu = \frac{i}{2} \gamma_5 \sigma_{\mu\nu} v^\nu \) is the covariant spin-operator.
The tree and loop Feynman diagrams contributing to the magnetic moments are shown in Figs. 1 and 2, respectively. According to the standard power counting \[30, 52\], the chiral order \(D_X\) of a Feynman diagram is

\[
D_X = 2L + 1 + \sum_d (d - 2) N_d^\phi + \sum_d (d - 1) N_d^{\phi B}
\]

where \(L\), \(N_d^\phi\) and \(N_d^{\phi B}\) are the numbers of loops, pure meson vertices and meson-baryon vertices, respectively. \(d\) is the chiral dimension. The chiral order of the magnetic moment \(\mu_T\) is counted as \((D_X - 1)\).

### III. CHIRAL LAGRANGIANS

#### A. The leading order chiral Lagrangians

We choose the nonlinear realization of the chiral symmetry,

\[
U = u^2 = \exp(i\phi / F_0),
\]

where \(\phi\) is the matrix for octet Goldstones,

\[
\phi = \left( \begin{array}{ccc} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}}\eta \end{array} \right),
\]

\(F_0\) is the decay constant of the pseudoscalar meson in chiral limit. We adopt \(F_\pi = 92.4\) MeV, \(F_K = 113\) MeV and \(F_\eta = 116\) MeV in this work. Under the \(SU(3)_L \times SU(3)_R\) chiral transformation, the \(U\) and \(u\) respond according to

\[
U \to RUL^\dagger, \quad u \to RuK^\dagger = K u L^\dagger,
\]

where \(R\) and \(L\) are \(SU(3)_R\) and \(SU(3)_L\) transformation matrices, respectively. \(K = K(R, L, \phi)\) is a unitary transformation.

We use the notations \(B_3\), \(B_6\) and \(B_6^*\) to denote the antitriplet, spin-\(\frac{1}{2}\) sextet and spin-\(\frac{3}{2}\) sextet, respectively. These baryon fields are realized as \[41\]:

\[
B_3 = \left( \begin{array}{ccc} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^- & 0 & \Xi_c^0 \\ -\Xi_c^- & -\Xi_c^0 & 0 \end{array} \right), \quad B_6 = \left( \begin{array}{ccc} \Sigma_c^{++} & \Sigma_c^+ & \Xi_c^+ \\ \frac{\Sigma_c^0}{\sqrt{2}} & \Sigma_c^0 & \Xi_c^0 \\ \frac{\Xi_c^0}{\sqrt{2}} & \Xi_c^0 & \Omega_c^0 \end{array} \right), \quad B_6^* = \left( \begin{array}{ccc} \Sigma_c^{++} & \Sigma_c^+ & \Xi_c^+ \\ \frac{\Sigma_c^0}{\sqrt{2}} & \Sigma_c^0 & \Xi_c^0 \\ \frac{\Xi_c^0}{\sqrt{2}} & \Xi_c^0 & \Omega_c^0 \end{array} \right)^\dagger
\]

The chiral transformation can be established:

\[
B \to KBK^T
\]

where \(B\) represents the \(B_3\), the \(B_6\) or the \(B_6^*\) field.

We introduce the left-handed and the right-handed external fields as the electromagnetic fields:

\[
r_\mu = l_\mu = -eQ_{m(c)}A_\mu,
\]

where \(A_\mu\) is the electromagnetic field and \(Q_{m(c)}\) represents the meson (singly charmed baryon) charge matrix. In this work, \(Q_m = \text{diag}(2/3, -1/3, -1/3)\) and \(Q_c = \text{diag}(1, 0, 0)\).

We define some “building blocks” before constructing Lagrangians. The chiral connection and axial vector field are defined as \[31\], \[52\],

\[
\Gamma_\mu = \frac{1}{2} \left[ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right],
\]

\[
\Gamma_\mu = \frac{1}{2} \left[ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right],
\]
\[ u_\mu = \frac{i}{2} \left[ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \right], \] (18)

The chiral covariant QED field strength tensors \( F_{\mu\nu}^\pm \) are defined as
\[
\begin{align*}
F_{\mu\nu}^+ &= u^\dagger F_{\mu\nu}^R u \pm u F_{\mu\nu}^L u^\dagger, \\
F_{\mu\nu}^- &= \partial_\mu r_\nu - \partial_\nu r_\mu - i [r_\mu, r_\nu], \\
F_{\mu\nu}^L &= \partial_\mu l_\nu - \partial_\nu l_\mu - i [l_\mu, l_\nu].
\end{align*}
\] (19-21)

In order to introduce the chiral symmetry breaking effect, we define \( \chi_\pm \),
\[
\begin{align*}
\chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\
\chi &= 2B_0 \text{ diag}(m_u, m_d, m_s)
\end{align*}
\] (22)

where \( B_0 \) is a parameter related to the quark condensate and \( m_{u,d,s} \) is the current quark mass.

The leading order \( \langle O(p^2) \rangle \) pure-meson Lagrangian is
\[
\mathcal{L}_{\phi\phi}^{(2)} = \frac{F_0^2}{4} \langle \nabla_\mu U (\nabla^\mu U)^\dagger \rangle,
\] (23)

where the superscript denotes the chiral order. The \( \langle X \rangle \) means the trace of field \( X \). The covariant derivative of Goldstone fields is defined as
\[
\nabla_\mu U = \partial_\mu U - i r_\mu U + iU l_\mu.
\] (24)

The leading order Lagrangians for singly heavy baryons read
\[
\begin{align*}
\mathcal{L}_{B_6}^{(1)} &= \frac{1}{2} \langle \bar{B}_6 (i \slashed{D} - M_3) B_3 \rangle + \langle \bar{B}_6 (i \slashed{D} - M_6) B_6 \rangle \\
&\quad + \langle \bar{B}_6 \gamma^\mu [-g_{\mu\nu} (i \slashed{D} - M_6) + i (\gamma_\mu D_\nu + \gamma_\nu D_\mu) - \gamma_\mu (i \slashed{D} + M_6) \gamma_\nu] B_6^{\ast \mu} \rangle \\
&\quad + g_1 \langle \bar{B}_6 \gamma_5 \gamma_5 u^\mu B_3 \rangle + g_2 \langle \bar{B}_6 \gamma_5 \gamma_5 u^\mu B_3 + \text{H.c.} \rangle + g_3 \langle \bar{B}_6 \gamma_5 \gamma_5 B_6 + \text{H.c.} \rangle \\
&\quad + g_4 \langle \bar{B}_6 \gamma_5 u^\mu B_3 + \text{H.c.} \rangle + g_5 \langle \bar{B}_6 \gamma_5 \gamma_5 u^\mu B_6^{\ast \mu} \rangle + g_6 \langle \bar{B}_6 \gamma_5 \gamma_5 u^\mu B_3 \rangle,
\end{align*}
\] (25)

where \( g_i \) is the axial charge. In this work, we ignore the mass splitting among the particles in the same multiplet. \( M_3, M_6 \) and \( M_{6^*} \) are the average baryon masses for the antitriplet, spin-\( \frac{1}{2} \) sextet and spin-\( \frac{3}{2} \) sextet, respectively.

In the framework of HBChPT, the leading order nonrelativistic Lagrangians read
\[
\begin{align*}
\mathcal{L}_{B_6}^{(1)} &= \frac{1}{2} \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle + \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_6 \rangle - \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle \\
&\quad + 2g_1 \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle + g_2 \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 + \text{H.c.} \rangle + g_3 \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle + g_4 \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle + g_5 \langle \bar{B}_6 \gamma_5 \gamma_5 \gamma_5 \gamma_5 u^\mu B_3 \rangle,
\end{align*}
\] (26)

where we ignore the terms suppressed by \( \frac{1}{M_6^2} \). \( \delta_{1,2,3} \) are the mass differences between different multiplets,
\[
\begin{align*}
\delta_1 &= M_{6^*} - M_6, \quad \delta_2 = M_6 - M_3, \quad \delta_3 = M_{6^*} - M_3.
\end{align*}
\] (27)

**B. The next-to-leading order chiral Lagrangians**

The \( O(p^2) \) baryon-photon Lagrangians contributing to the magnetic moments read:
\[
\begin{align*}
\mathcal{L}_{B_γ}^{(2)} &= \frac{d_2}{8M_N} \langle \bar{B}_3 \sigma^{\mu\nu} \gamma^5 \gamma^5 F_{\mu\nu}^+ B_3 \rangle + \frac{d_3}{8M_N} \langle \bar{B}_3 \sigma^{\mu\nu} F_{\mu\nu}^+ B_3 \rangle + \frac{d_4}{8M_N} \langle \bar{B}_6 \sigma^{\mu\nu} \gamma^5 \gamma^5 B_6 \rangle + \frac{d_5}{8M_N} \langle \bar{B}_6 \sigma^{\mu\nu} \gamma^5 \gamma^5 B_6 \rangle \\
&\quad + \frac{f_2}{8M_N} \langle \bar{B}_3 \sigma^{\mu\nu} \gamma^5 \gamma^5 B_6 \rangle + \text{H.c.} + \frac{f_4}{8M_N} \langle \bar{B}_3 \sigma^{\mu\nu} \gamma^5 \gamma^5 B_6 \rangle + \text{H.c.} + \frac{f_6}{8M_N} \langle \bar{B}_6 \sigma^{\mu\nu} \gamma^5 \gamma^5 B_6 \rangle + \text{H.c.} \\
&\quad + \frac{if_7}{8M_N} \langle \bar{B}_6 \gamma^5 \gamma^5 B_6 \rangle \langle F_{\mu\nu}^+ \rangle + \text{H.c.} + \frac{if_9}{4M_N} \langle \bar{B}_6 \gamma^5 \gamma^5 B_6 \rangle \langle F_{\mu\nu}^+ \rangle + \frac{if_{10}}{4M_N} \langle \bar{B}_6 \gamma^5 \gamma^5 B_6 \rangle \langle F_{\mu\nu}^+ \rangle,
\end{align*}
\] (28)
There are two unknown LECs \( f_9 \) and \( f_{10} \) contribute to the leading order magnetic moments of spin-\( \frac{3}{2} \) heavy baryons in the tree diagrams. Other terms will contribute to the higher order magnetic moments in the loop diagrams. The nonrelativistic form of Eq. (28) reads

\[
L^{(2)}_{\gamma} = - \frac{id_2}{4M_N} \langle \delta_3[S^\mu, S^\nu] \delta_3^+ \rangle (F_{\mu\nu}) - \frac{id_3}{4M_N} \langle \delta_3[S^\mu, S^\nu] \delta_3^+ \rangle (F_{\mu\nu}) - \frac{id_5}{4M_N} \langle \delta_6[S^\mu, S^\nu] \delta_6^+ \rangle (F_{\mu\nu})
\]

where \( d_i \) and \( f_i \) are the coupling constants. \( \hat{X} = X - \frac{1}{3}(X) \) is the traceless part of the field \( X \). The \( f_9 \) and \( f_{10} \) terms contribute to the leading order magnetic moments of spin-\( \frac{3}{2} \) heavy baryons. The solid dot and black square represent \( O(p^2) \) and \( O(p^4) \) vertices, respectively.

We also construct the \( O(p^2) \) meson-meson-baryon interaction Lagrangian as followings, which contributes to the \( O(p^3) \) magnetic moments through the loop (j) in Fig. 2.

\[
L^{(2)}_{\phi} = - \frac{f_8}{2M_N} \langle \delta_6^{* \mu}[u_\mu, u_\nu] \delta_6^{* \nu} \rangle.
\]

C. The higher order chiral Lagrangians

According to the group representation theory, there are seven interaction terms in \( O(p^4) \) Lagrangians which contribute to the \( O(p^3) \) magnetic moments in the tree diagrams. The \( \chi^+ = 4B_0 \text{ diag}(0, 0, m_s) = 4B_0m_s \chi^+ \) at the leading order. We use the \( \chi^+ \) as the building block and the \( B_0 \) and \( m_s \) are absorbed into the LECs. There are only two independent nonvanishing terms

\[
L^{(4)}_{\phi} = \frac{i\hbar_2}{4M_N} \langle \delta_6^{* \mu}(F_{\mu\nu}^+) \delta_6^{* \nu} \rangle + \frac{i\hbar_4}{M_N} \langle \delta_6^{* \mu} F_{\mu\nu}^+ \delta_6^{* \nu} \delta_6^T \rangle.
\]

IV. ANALYTICAL EXPRESSIONS

The leading order magnetic moments are at \( O(p) \), which stem from \( O(p^2) \) vertices in Eq. (29):

\[
\mu^{(1)}_{\frac{3}{2}++} = - \left( \frac{2}{3}f_9 + f_{10} \right) \mu_N, \quad \mu^{(1)}_{\frac{3}{2}++} = - \left( \frac{1}{6}f_9 + f_{10} \right) \mu_N, \quad \mu^{(1)}_{\frac{3}{2}++} = \mu^{(1)}_{\frac{3}{2}++} = \mu^{(1)}_{\frac{3}{2}++} = \frac{1}{3} \left( \frac{2}{3}f_9 + f_{10} \right) \mu_N.
\]

There are two unknown LECs \( f_9 \) and \( f_{10} \) at this order.

Four loop diagrams (a)-(d) in Fig. 2 contribute to the \( O(p^2) \) magnetic moments. The meson-photon vertex arises from the \( L^{(2)}_{\phi} \), while the meson-baryon vertex is from the \( L^{(1)}_{\phi} \). The diagrams (c) and (d) vanish for the structure \( u_\mu u^\mu \) in the amplitude. The corrections from the loops (a)-(d) read

\[
\mu^{(2,a)} = \beta_0 \frac{g_\phi^2 M_N 3 - d}{2F_\phi^2} \frac{d - 1}{d - 1} n^T_1 (0, m_\phi) \mu_N.
\]
The loop diagrams contribute to the magnetic moments of the spin-$\frac{3}{2}$ heavy baryons. The single and double lines represent the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ heavy baryons, respectively. The solid dots denote the next-leading order vertices, while the other vertices are at the leading order. The diagrams (a)-(d) contribute to the $\mathcal{O}(p^2)$ magnetic moments, while the (e)-(l) diagrams contribute to the $\mathcal{O}(p^3)$ magnetic moments.

$$\mu^{(2,b)} = -\beta^g \frac{g_3^2 M_N}{4 F_\phi^2} n^I_1 (\delta_1, m_\phi) \mu_N - 2\beta^g \frac{g_2^2 M_N}{4 F_\phi^2} n^I_1 (\delta_3, m_\phi) \mu_N,$$

where the $n^I_1(\omega, m_\phi)$ is the loop integral given in Appendix A. The $\beta^g$ is the coefficient in Table I. There exist three LECs $g_{3,4,5}$ to be determined at this order.

The $\mathcal{O}(p^3)$ magnetic moments come from both the tree diagrams and the loop diagrams. The vertices of tree diagrams are from the interaction in Eq. (31). The results of the tree diagram read,

$$\mu^{(3,tree)}_{\Sigma^+} = \frac{8}{9} h_4 \mu_N, \quad \mu^{(3,tree)}_{\Sigma^0} = \frac{2}{9} h_4 \mu_N, \quad \mu^{(3,tree)}_{\Sigma^0} = -\frac{4}{9} h_4 \mu_N,$$

$$\mu^{(3,tree)}_{\Xi^+} = -\left(\frac{1}{2} h_2 + \frac{10}{9} h_4\right) \mu_N, \quad \mu^{(3,tree)}_{\Xi^0} = -\left(\frac{1}{2} h_2 - \frac{2}{9} h_4\right) \mu_N, \quad \mu^{(3,tree)}_{\Omega^0} = -\left(h_2 - \frac{8}{9} h_4\right) \mu_N. \quad (35)$$

The loop diagrams (e)-(l) in Fig. 2 contribute to the $\mathcal{O}(p^3)$ magnetic moments. The baryon-photon vertices in loop diagrams (e)-(h) come from the $\mathcal{L}_{2B\gamma}^{(2)}$ in Eq. (29). The baryon-meson vertices are from the axial coupling Lagrangian in Eq. (26). The vertex in the loop diagram comes from the $f_9$ term in Eq. (29). The meson-meson-baryon vertex in the loop diagram (j) comes from the interaction (30). Diagrams (k) and (l) are the renormalization of the spin-$\frac{3}{2}$ baryon fields. The $\mathcal{O}(p^3)$ corrections from the above loop diagrams read,

$$\mu^{(3,i)} = 2\beta^g \frac{f_9 m_\phi^2}{128 \pi^2 F_\phi^2} \ln \frac{m_\phi^2}{\Lambda^2} \mu_N,$$

$$\mu^{(3,j)} = -4\beta^g \frac{f_9 m_\phi^2}{256 \pi^2 F_\phi^2} \ln \frac{m_\phi^2}{\Lambda^2} \mu_N,$$

$$\mu^{(3,e)} = \gamma^g \frac{g_2^2}{2 F_\phi} \left(\frac{1 - d}{2} + \frac{4}{d - 1} - \frac{4}{d - 1} \right) J_0^2(0) \mu_N,$$

where $\gamma^g$ is the coefficient in Table I. There exist three LECs to be determined at this order.
There are eighteen unknown LECs in the analytical expressions in Eqs. (32)-(42), including five axial charges \((p_2^a, p_2^b, p_2^c, \phi_2^a, \phi_2^b)\), five baryon-photon coupling constants \(d_2, d_3, d_4, d_5, d_6\), five chiral symmetry breaking coupling constants \(\rho_2^a, \rho_2^b, \rho_2^c, \phi_2^a, \phi_2^b\), and three meson-meson-baryon coupling constants \(f_2, f_3, f_4\). Since the number of the LECs is larger than that of the ground heavy baryons, we use the heavy quark symmetry to reduce the number of independent LECs.

V. INDEPENDENT LECs IN THE HEAVY QUARK LIMIT

| Loop | \(\Sigma_c^{++}\) | \(\Sigma_c^{+}\) | \(\Sigma_c^0\) | \(\Xi_c^+\) | \(\Xi_c^0\) | \(\Omega_c^0\) |
|------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \((a),(b), \beta^n\) | 2 | –2 | 1 | –1 | \(\frac{1}{2}\) | \(\frac{1}{2}\) |
| \((i),(j), \beta^K\) | 2 | 1 | –1 | –2 | \(\frac{1}{2}\) | \(\frac{1}{2}\) |

\[ \gamma_1^n = \frac{g_2}{4F^2_\phi} J'_2(\delta_1) \mu_N + \rho^K \frac{g_2^2}{4F^2_\phi} J'_2(\delta_3) \mu_N + 2\delta^K \frac{g_2g_4}{4F^2_\phi} J_2(\delta_1) - J_2(\delta_3) \mu_N, \]

\[ \mu^{(3,f)} = \gamma_2^n \frac{g_2^2}{4F^2_\phi} J'_2(\delta_1) \mu_N + \rho^K \frac{g_2^2}{4F^2_\phi} J'_2(\delta_3) \mu_N + 2\delta^K \frac{g_2g_4}{4F^2_\phi} J_2(\delta_1) - J_2(\delta_3) \mu_N, \]

where \(\beta^n, \gamma_1^n, \rho^K, \delta^K, \xi^n, \eta^n, \xi^K, \eta^K, \xi^\phi, \eta^\phi\) and \(r^n\) are the coefficients of loops, which are given in Table I. There are thirteen new LECs introduced at this order.
The spin-$1$ and the spin-$\frac{3}{2}$ sextet are degenerate states in the heavy quark limit. The heavy quark symmetry can relate some LECs to others. We define a superfield $\mathcal{H}_\mu$ to denote $\mathcal{B}_6$ and $\mathcal{B}_{6\mu}$ \cite{44,47},

\[
\begin{align*}
\mathcal{H}_\mu &= \mathcal{B}_{6\mu}^* - \sqrt{\frac{2}{3}}(\gamma_\mu + v_\mu)\gamma^5 \mathcal{B}_6, \\
\mathcal{H}_\mu &= \mathcal{B}_{6\mu}^* + \sqrt{\frac{2}{3}}\mathcal{B}_6\gamma^5(\gamma_\mu + v_\mu), 
\end{align*}
\]

where $\bar{\mathcal{H}}_\mu$ is the conjugate field of $\mathcal{H}_\mu$. $v_\mu$ is the velocity of heavy quark. In the heavy quark limit, the $v_\mu$ also corresponds to the velocity of the heavy baryon. This field $\mathcal{H}_\mu$ is constrained by

\[
v \cdot \mathcal{H} = 0, \quad \gamma \mathcal{H} = \mathcal{H}.
\]

The $\mathcal{H}_\mu$ follows the same chiral transformation in Eq. \cite{15}.

In Refs. \cite{44,47}, the authors constructed the axial coupling Lagrangian of the sextet baryons in heavy quark symmetry,

\[
\mathcal{L}_{\mathcal{B}_6}^{(1)} = i g_a \epsilon_{\mu\nu\rho}(\bar{\mathcal{H}}^\mu \varphi^\nu \varphi^\rho) + g_b(\bar{\mathcal{H}}^\mu u_\mu B_3 + \text{H.c.}).
\]

The LECs in $\mathcal{L}_{\mathcal{B}_6}^{(1)}$ are reduced to two independent LECs, $g_a$ and $g_b$:

\[
g_5 = g_a, \quad g_1 = -\frac{2}{3}g_a, \quad g_3 = -\sqrt{\frac{1}{3}}g_a, \quad g_4 = g_b, \quad g_2 = -\sqrt{\frac{1}{3}}g_b, \quad g_6 = 0.
\]

$g_b$ is the coupling constant between pseudoscalar mesons and antitriplet heavy baryons. The light spin $S_L = 0$ for the antitriplets. The pseudoscalar mesons only interact with the light degree in the heavy baryon. Thus, the parity and angular momentum conservation forbid the $g_b$ vertex.

The interaction in $\mathcal{L}_{\mathcal{B}_T}^{(2)}$ in heavy quark symmetry corresponds to

\[
\mathcal{L}_{\mathcal{B}_T}^{(2)} = i \frac{g_c}{4 M_N} (\bar{\mathcal{H}}^\mu \tilde{F}^\mu_+ \varphi^\nu) + i \frac{g_d}{4 M_N} (\bar{\mathcal{H}}^\mu \varphi^\nu \varphi^\rho) - \frac{g_e}{4 M_N} \epsilon^{\sigma\mu\nu\rho} (\bar{\mathcal{H}}_\sigma \tilde{F}^\mu_+ \varphi^\nu B_3) + \text{H.c.}
\]

The eight LECs $d_5$, $d_6$, $f_2$, $f_4$, $f_6$, $f_7$, $f_9$ and $f_{10}$ in $\mathcal{L}_{\mathcal{B}_T}^{(2)}$ are reduced to three LECs $g_c$, $g_d$ and $g_e$:

\[
\begin{align*}
f_9 &= g_c, \quad f_6 = \frac{2}{\sqrt{3}}g_c, \quad d_5 = -\frac{2}{3}g_c, \\
f_{10} &= g_d, \quad f_7 = \frac{2}{\sqrt{3}}g_d, \quad d_6 = -\frac{2}{3}g_d, \\
f_2 &= -\frac{2}{\sqrt{3}}g_c, \quad f_4 = -4g_c.
\end{align*}
\]

For spin-$\frac{1}{2}$ singly heavy baryons, we construct the $\mathcal{O}(p^2)$ meson-baryon interaction and $\mathcal{O}(p^4)$ photon-baryon interaction \cite{41}:

\[
\begin{align*}
\mathcal{L}_{\mathcal{B}_T}^{(2)} &= \frac{d_4}{2 \sqrt{2} M_N} (\bar{B}_6 \sigma^{\mu\nu}[u_\mu, u_\nu] B_6), \\
\mathcal{L}_{\mathcal{B}_T}^{(4)} &= \frac{g_s}{8 \sqrt{2} M_N} (\bar{B}_6 \sigma^{\mu\nu}(F^\mu_+ \chi_B) + \frac{1}{2} \bar{B}_6 \sigma^{\mu\nu} F^\mu_+ B_6 \chi^T).
\end{align*}
\]

In the heavy quark symmetry, the Lagrangians in Eqs. \cite{50} and \cite{51} can be related to those in Eqs. \cite{50} and \cite{51}. The Lagrangians in the heavy quark limit read

\[
\begin{align*}
\mathcal{L}_{\mathcal{H}_\mu}^{(2)} &= \frac{g_a}{2 \sqrt{2} M_N} (\bar{\mathcal{H}}^\mu[u_\mu, u_\nu] \varphi^\nu) \\
\mathcal{L}_{\mathcal{H}_\mu}^{(4)} &= \frac{g_a}{2 \sqrt{2} M_N} (\bar{\mathcal{H}}^\mu F^\mu_+ \chi_B) + \frac{1}{2} \bar{\mathcal{H}}^\mu \tilde{F}^\mu_+ \varphi^\nu \chi^T
\end{align*}
\]

The LECs are related as

\[
\begin{align*}
d_4 &= \frac{1}{6}g_f, \quad f_8 = -\frac{1}{2}g_f, \quad s_2 = -\frac{2}{3}g_f, \quad h_2 = g_g; \quad s_4 = \frac{1}{6}g_h, \quad h_4 = \frac{g_h}{4}.
\end{align*}
\]

We decompose the $F^\mu_+$ into the trace part $(F^\mu_+)$ and traceless part $\hat{F}^\mu_+$, which correspond to the contributions from the light quarks and the heavy quark, respectively. The contribution from the heavy quark to the magnetic moments is order of $\frac{1}{M_N}$. Thus, heavy quark contribution and the LECs, $s_4, h_2$ vanish in the heavy quark limit. Thus, in this limit, there are seven nonvanishing independent LECs, $d_2$ and $g_{a,b,c,e,f,h}$, contributing to the magnetic moments of the sextet baryons.
TABLE II: The Lattice QCD simulation results [20, 21, 23]. “√” represents the results used as input.

|       | Ξ_0^+ | Ξ_0^0 | Ξ_0^+ | Ξ_0^0 | Ω_0^+ | Ω_0^0 |
|-------|-------|-------|-------|-------|-------|-------|
| LQCD  | 0.235(25) | 0.192(17) | 1.499(202) | -0.875(103) | 0.315(141) | -0.599(71) | -0.667(97) | -0.730(23) |
| SI Input | √ | √ | √ | √ | √ | √ |
| SII Input | √ | √ | √ | √ | √ | √ |
| SIII Input | √ | √ | √ | √ | √ | √ |

TABLE III: The (transition) magnetic moments \( \mu_{B_3}, \mu_{B_0 \rightarrow B_3}, \) and \( \mu_{B_0^\ast \rightarrow B_3} \) from the quark model and the leading order results in HBChPT.

|       |          |          |          |          |       |
|-------|----------|----------|----------|----------|-------|
| \( \mu_{B_3} \) | \( \Lambda_0^\pm \) | \( \Xi_0^\pm \) | \( \Xi_0^0 \) |
| QM    | \( \mu_c \) | \( \mu_c \) | \( \mu_c \) |
| \( \mathcal{O}(p^3) \) | \( \frac{1}{2}g_2 + 2d_3 \) | \( \frac{1}{2}g_2 + 2d_3 \) | \( -\frac{3}{2}d_2 + 2d_3 \) |
| \( \mu_{B_0 \rightarrow B_3} \) | \( \Sigma_c^+ \rightarrow \Lambda_0^+ \gamma \) | \( \Xi_0^+ \rightarrow \Xi_0^0 \gamma \) | \( \Xi_0^0 \rightarrow \Xi_0^0 \gamma \) |
| QM    | \( \sqrt{\frac{1}{2}}(\mu_d - \mu_u) \) | \( \sqrt{\frac{1}{2}}(\mu_d - \mu_u) \) | \( \sqrt{\frac{1}{2}}(\mu_d - \mu_d) \) |
| \( \mathcal{O}(p^3) \) | \( \sqrt{\frac{1}{2}}f_2 \) | \( \sqrt{\frac{1}{2}}f_2 \) | 0 |
| \( \mu_{B_0^\ast \rightarrow B_3} \) | \( \Sigma_c^+ \rightarrow \Lambda_0^+ \gamma \) | \( \Xi_0^+ \rightarrow \Xi_0^0 \gamma \) | \( \Xi_0^0 \rightarrow \Xi_0^0 \gamma \) |
| QM    | \( \frac{3}{\sqrt{2}}(\mu_d - \mu_u) \) | \( \frac{3}{\sqrt{2}}(\mu_d - \mu_u) \) | \( \frac{3}{\sqrt{2}}(\mu_d - \mu_u) \) |
| \( \mathcal{O}(p^3) \) | \( -\sqrt{\frac{1}{2}}f_4 \) | \( -\sqrt{\frac{1}{2}}f_4 \) | 0 |

VI. NUMERICAL RESULTS

In the present work, we perform the numerical analysis with three scenarios. In the first scenario, we use the LECs determined by our previous work [40]. Three new LECs, \( f_8, h_2 \) and \( h_4 \) are related to \( d_4, s_2 \) and \( s_4 \) through the heavy quark spin symmetry. In the second scenario, we reduce the number of the LECs in the heavy quark limit and adopt the Lattice QCD simulation results as input. In the third scenario, we include the heavy quark contribution on the basis of the scenario II. As a by-product, we also give the magnetic moments of singly bottom baryon.

In the three scenarios, we all use the same axial coupling values. The axial coupling constants \( g_2 \) and \( g_4 \) in Eq. (26) are estimated through the decay widths of \( \Sigma_c \) and \( \Sigma_c^* \), respectively [53, 54]. The other \( g_i \) are related to \( g_2 \) and \( g_4 \) with the help of quark model. Their values are

\[
\begin{align*}
g_2 &= -0.60, & g_4 &= -\sqrt{3}g_2 = 1.04, & g_1 &= -\sqrt{\frac{3}{3}}g_2 = 0.98, \\
g_3 &= \frac{\sqrt{3}}{2}g_1 = 0.85, & g_5 &= -\frac{3}{2}g_1 = -1.47, & g_6 &= 0.
\end{align*}
\]

A. Scenario I

In our previous work [40], we calculated the magnetic moments of the spin-\( \frac{1}{2} \) singly heavy baryons. All the LECs in Eq. (29) have been evaluated through the quark model. Here, we review the idea in brief. The vertices in Eq. (29) contribute to the leading order (transition) magnetic moments in the HBChPT scheme. We assume that their values are approximate to those estimated by the naive quark model. Then, we can extract these LECs. The (transition) magnetic moments from the quark model and the leading order results in HBChPT are given in Table III and IV.

Apart from the axial coupling constants and the \( \mathcal{O}(p^3) \) baryon-photon coupling constants, we have three new LECs in the present work. In our previous work, \( d_4, s_2 \) and \( s_4 \) have been determined by fitting three Lattice QCD results,
In this scenario, we make use of six Lattice QCD results in Table II to determine them. In the heavy quark limit, the mass splitting between the spin-$\frac{1}{2}$ sextet and the spin-$\frac{3}{2}$ sextet.

\[ \delta_1 = M_{\Sigma^*_6} - M_6 = 67 \text{ MeV}, \]
\[ \delta_2 = M_6 - M_3 = 127 \text{ MeV}, \]
\[ \delta_3 = M_{\Sigma^*_6} - M_3 = 194 \text{ MeV.} \] (55)

We have determined all the LECs in the analytical expressions up to $\mathcal{O}(p^3)$. We give the numerical results in two schemes. In the first scheme, we include the spin-$\frac{3}{2}$ antitriplet, the spin-$\frac{1}{2}$, and the spin-$\frac{3}{2}$ sextet as the intermediate states in the loops. The numerical results are listed in the left panel of Table IV. The chiral convergence is not good enough. The $\mathcal{O}(p^3)$ contribution is larger than that at $\mathcal{O}(p^2)$ for $\Sigma^{*+}$ and $\Sigma^{*0}$. In the second scheme, we only take the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet as the intermediate states. The results are given in the right panel of Table IV. The chiral convergence becomes much better.

The mass splittings $\delta_{1,2,3}$ do not vanish in the chiral limit, which will worsen the chiral convergence. Due to the large mass splitting $\delta_3$, about 194 MeV, including the spin-$\frac{3}{2}$ antitriplet will destroy the chiral convergence. As for the spin-$\frac{1}{2}$ sextet, the mass splitting $\delta_1$ is small. Taking spin-$\frac{1}{2}$ sextet as intermediate states has almost no negative impact on the convergence. Meanwhile, the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet form doublet in the heavy quark limit. To calculate the magnetic moments of the spin-$\frac{3}{2}$ sextet, the contribution from the chiral fluctuation around the spin-$\frac{1}{2}$ sextet is important. Thus, we choose the results from the second scheme, in the right panel of Table IV as our final results.

### B. Scenario II

According to Section IV, we reduce the LECs to five unknown independent ones with the heavy quark symmetry. In this scenario, we make use of six Lattice QCD results in Table II to determine them. In the heavy quark limit, the spin-$\frac{1}{2}$ and $\frac{3}{2}$ are degenerate states. The mass splittings are

\[ \delta_1 = M_{\Lambda_{6c}} - M_6 = 0 \text{ MeV}, \]
\[ \delta_2 = \delta_3 = M_{\Lambda(6c)} - M_3 = 161 \text{ MeV.} \] (56)

In this scenario, we also apply two schemes to estimate the LECs. In the first scheme, we consider all the singly charmed baryons as the intermediate states. The results are given in the left panel of Table V. In the second scheme, we set $g_c = 0$ and decouple the $3f$ and $6f$ singly charmed baryon in the loop diagrams. The results are given in the right panel of Table V.

The results in the left panel suffer from the bad convergence, which are even worse than those in the first scheme in the scenario I. The quark model predictions are comparable with the Lattice QCD results. Taking the quark model

| TABLE IV: The (transition) magnetic moments $\mu_{\Sigma^*_6}$, $\mu_{\Sigma^*_{6c}}$, and $\mu_{\Lambda_{6c}}$ from the quark model and the leading order results in HBChPT. |
| $\mu_{\Sigma^*_6}$ | $\Sigma^{*+}_{6c}$ | $\Sigma^{*0}_{6c}$ | $\Sigma^{*+}_{6c}$ | $\Sigma^{*0}_{6c}$ | $\Omega^0_{6c}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Sigma^{*+}_{6c}$ | $\Sigma^{*+}_{6c}$ | $\Sigma^{*0}_{6c}$ | $\Sigma^{*+}_{6c}$ | $\Sigma^{*0}_{6c}$ | $\Omega^0_{6c}$ |
| $\mu_{\Lambda_{6c}}$ | $\Lambda^{*+}_{6c}$ | $\Lambda^{*0}_{6c}$ | $\Lambda^{*+}_{6c}$ | $\Lambda^{*0}_{6c}$ | $\Omega^0_{6c}$ |

\[ \mu_{\Sigma^{*+}_{6c}}, \mu_{\Lambda^{*0}_{6c}}, \text{and } \mu_{\Lambda^{*0}_{6c}} \text{ in Table [V]} \] With the relations in Eq. (55), we obtain the values of $f_8$, $h_2$ and $h_4$. In this scenario, we keep the mass splitting between the spin-$\frac{1}{2}$ sextet and the spin-$\frac{3}{2}$ sextet. The mass splitting reads
TABLE V: The numerical results of spin-$\frac{3}{2}$ singly charmed baryon magnetic moments in the scenario I (in unit of $\mu_N$). We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only consider the $B_6$ and $B_6^*$ in the right panel.

| SI | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|----|-------------------------------|------------------------|
|    | $O(p^1)$ $O(p^2)$ $O(p^3)$ Total | $O(p^1)$ $O(p^2)$ $O(p^3)$ Total |
| $\Sigma^{++}_c$ | 4.10 -1.16 -0.02 2.92 | 4.10 -1.03 -0.16 2.91 |
| $\Sigma^+_c$ | 1.48 -0.72 -0.19 0.57 | 1.48 -0.39 -0.11 0.99 |
| $\Sigma^0_c$ | -1.13 -0.29 -0.35 -1.77 | -1.13 0.26 -0.06 -0.94 |
| $\Xi^+_c$ | 1.48 0.14 -0.50 1.13 | 1.48 -0.13 -0.07 1.28 |
| $\Xi^0_c$ | -1.13 0.58 -0.22 -0.77 | -1.13 0.52 0.01 -0.60 |
| $\Omega^0_c$ | -1.13 1.45 -0.24 0.08 | -1.13 0.78 0.12 -0.24 |

TABLE VI: The numerical results of spin-$\frac{3}{2}$ singly charmed baryon magnetic moments in the scenario II (in unit of $\mu_N$). We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only consider the $B_6$ and $B_6^*$ in the right panel.

| SII | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|-----|-------------------------------|------------------------|
|     | $O(p^1)$ $O(p^2)$ $O(p^3)$ Total | $O(p^1)$ $O(p^2)$ $O(p^3)$ Total |
| $\Sigma^{++}_c$ | 0.82 -1.28 2.42 1.95 | 2.59 -1.11 0.64 2.12 |
| $\Sigma^+_c$ | 0.20 -0.74 0.82 0.28 | 0.65 -0.39 0.18 0.44 |
| $\Sigma^0_c$ | -0.41 -0.20 -0.77 -1.38 | -1.29 0.32 -0.27 -1.24 |
| $\Xi^+_c$ | 0.20 0.10 -0.02 0.29 | 0.65 -0.16 0.04 0.52 |
| $\Xi^0_c$ | -0.41 0.64 -1.26 -1.03 | -1.29 0.55 -0.29 -1.03 |
| $\Omega^0_c$ | -0.41 1.49 -1.81 -0.73 | -1.29 0.78 -0.27 -0.78 |

results as the leading order input at least ensure a dominant $O(p^1)$ contribution in scenario I. Comparing results in the two panels of Table VII although the total values are similar, the chiral convergence in the second scheme improve significantly. Including the antitriplet as the intermediate states break the chiral convergence. Thus, we also choose the results from the second scheme as our final results in this scenario.

In the second scheme, the magnetic moments of sextet baryons do not depend on the antitriplet. In the sextet sector, we determine three unknown LECs and obtain twelve magnetic moments. Thus, this scenario has powerful predictions.

C. Scenario III

In the Lattice QCD simulation [20, 21, 23], the contribution of heavy quark and light quarks to the magnetic moments are given separately. The heavy quark contribution for $\Xi^+_c$, $\Sigma^{++}$, $\Xi^+_c$, $\Omega^0_c$ and $\Omega^0_c$ read

$$\mu^c_{\Xi^+_c} = 0.226\mu_N, \quad \mu^c_{\Sigma^{++}} = -0.066\mu_N, \quad \mu^c_{\Xi^+_c} = -0.059\mu_N, \quad \mu^c_{\Omega^0_c} = -0.061\mu_N, \quad \mu^c_{\Omega^0_c} = 0.239\mu_N,$$

where the superscript “c” denotes the contribution from the charm quark. According to the quark model in Tables III and IV the heavy quark contribution is $\mu_c$, $-\frac{1}{3}\mu_c$ and $\mu_c$ for the antitriplet, spin-$\frac{3}{2}$ sextet and spin-$\frac{1}{2}$ sextet, respectively. Using the Lattice QCD results in Eqs. (57), we get the average $\mu_c = 0.205\mu_N$. In this scenario, the heavy quark contribution is estimated by using the average $\mu_c$ while the light quark contribution is derived through fitting the remaining part of Lattice QCD results. The results are given in Table VII. The right panel of this table is our final results of this scenario.
TABLE VII: The numerical results of spin-$\frac{3}{2}$ singly charmed baryon magnetic moments from the scenario III (in unit of $\mu_N$). We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only take the $B_6$ and $B_6^*$ in the right panel. The “HQ” represents the heavy quark contribution.

|       | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|-------|-----------------------------|-------------------------|
|       | HQ $O(p^1)$ $O(p^2)$ $O(p^3)$ Total | HQ $O(p^1)$ $O(p^2)$ $O(p^3)$ Total |
| $\Sigma^{++}_c$ | 0.21 0.89 -1.28 2.76 2.57 | 0.21 2.68 -1.11 0.64 2.41 |
| $\Sigma^+_c$ | 0.21 0.22 -0.74 1.00 0.68 | 0.21 0.67 -0.39 0.19 0.67 |
| $\Sigma^0_c$ | 0.21 -0.45 -0.20 -0.76 -1.21 | 0.21 -1.34 0.32 -0.27 -1.08 |
| $\Xi^+_c$ | 0.21 0.22 0.10 0.31 0.84 | 0.21 0.67 -0.16 0.10 0.81 |
| $\Xi^0_c$ | 0.21 -0.45 0.64 -1.35 -0.94 | 0.21 -1.34 0.55 -0.32 -0.90 |
| $\Omega^+_c$ | 0.21 -0.45 1.49 -1.98 -0.73 | 0.21 -1.34 0.78 -0.34 -0.69 |

TABLE VIII: The magnetic moments of singly bottom baryon sextet (in unit of $\mu_N$). The “HQ” represents the heavy quark contribution. The light quark contribution is the same as that for singly charmed baryon in Table XIII.

|       | spin-$\frac{1}{2}$ HQ Total | spin-$\frac{3}{2}$ HQ Total |
|-------|----------------------------|-----------------------------|
| $\Sigma^+_b$ | 0.02 1.59 | $\Sigma^{++}_b$ | -0.06 2.14 |
| $\Sigma^0_b$ | 0.02 0.39 | $\Sigma^{+0}_b$ | -0.06 0.40 |
| $\Sigma^-_b$ | 0.02 -0.81 | $\Sigma^{--}_b$ | -0.06 -1.35 |
| $\Xi^+_b$ | 0.02 0.40 | $\Xi^{+0}_b$ | -0.06 0.54 |
| $\Xi^-_b$ | 0.02 -0.73 | $\Xi^{--}_b$ | -0.06 -1.17 |
| $\Omega^-_b$ | 0.02 -0.64 | $\Omega^{--}_b$ | -0.06 -0.96 |

In scenario III, we can easily extend our formalism to calculate the magnetic moments of singly bottom baryons. In the heavy quark limit, the light contribution for a bottom baryon is the same as that for the charmed baryon. The heavy quark part is estimated using the quark model. We adopt the constituent mass $m_b = 4700$ MeV. The magnetic moments of singly bottom baryons are given in Table VIII.

TABLE IX: The numerical results of LECs for the three scenarios.

|       | d2 | d3 | f9 | f6 | d5 | f10 | f7 | d6 | f4 | f2 | d4 | f8 | s2 | h2 | s4 | h4 |
|-------|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|----|----|
| SI    | 0.04 | 0.11 | -5.23 | -6.00 | 3.49 | -0.61 | 0.60 | 0.03 | -7.26 | -2.14 | 3.61 | -10.83 | -0.23 | 0.35 | -0.04 | 0.06 |
| SII   | -0.09 | 0 | -3.88 | 0 | 0 | 0.91 | 0 | 0.33 |
| SIII  | 0.03 | 0 | -4.02 | 0 | 0 | 0.66 | 0 | 0.14 |

In scenario III, we can easily extend our formalism to calculate the magnetic moments of singly bottom baryons. In the heavy quark limit, the light contribution for a bottom baryon is the same as that for the charmed baryon. The heavy quark part is estimated using the quark model. We adopt the constituent mass $m_b = 4700$ MeV. The magnetic moments of singly bottom baryons are given in Table VIII.
TABLE X: Comparision of the spin-$\frac{3}{2}$ singly charmed baryon magnetic in the literature, including the Lattice QCD (LQCD) [21], the hyper central model (HCM) [11], effective mass (EM) and screened charge scheme (SC) [12], chiral constituent quark model (χCQM) [13], light-cone QCD sum rules (LCQSR) [14], MIT bag model [15 14], Skyrmion [15] scheme and chiral quark-soliton model (χQSM) [16] (in unit of $\mu_N$).

|      | SI  | SII | SIII | LQCD | HCM | EM  | SC  | χCQM | LCQSR | Bag I | Bag II | Skyrmion | χQSM |
|------|-----|-----|------|------|-----|-----|-----|------|-------|-------|--------|----------|-------|
| Σ⁺⁺⁺ | 2.91| 2.12| 2.41 | -    | -   | -   | -   | 3.68 | 3.56  | 3.63  | 3.92   | 4.81    | 3.91   | 3.13   | 4.52 | 4.58  | 3.22 ± 0.15 |
| Σ⁺⁺⁺ | 0.99| 0.44| 0.67 | -    | -   | -   | -   | 1.20 | 1.17  | 1.18  | 0.97   | 2.00    | 1.34   | 1.09   | 1.12 | 1.31  | 0.68 ± 0.04  |
| Σ⁺⁺⁻ | -0.94| -1.24| -1.08| -    | -   | -   | -   | -0.83| -0.85 | -1.23 | -1.18 | -1.99  | -0.81   | -1.20  | -0.96 | -2.29 | -1.92 | -1.86 ± 0.07 |
| Ξ⁺⁺⁻ | 1.28| 0.52| 0.81 | -    | -   | -   | -   | 1.45 | 1.43  | 1.39  | 1.59   | 1.68    | 1.54   | 1.27   | 2.26 | 2.07  | 0.90 ± 0.04  |
| Ξ⁺⁺⁻ | -0.60| -1.03| -0.90| -    | -   | -   | -   | -0.67| -0.69 | -1.00 | -1.02 | -1.43  | -0.68   | -1.01  | -0.75 | -2.01 | -1.98 | -1.57 ± 0.06  |
| Ω⁺⁺⁻ | -0.24| -0.78| -0.69| -0.73| -0.83| -0.87| -0.77| -0.84| -0.86| -0.62 | -0.78 | -0.55   | -0.87   | -1.23  | -1.28 | -0.08  |       |

VII. DISCUSSION AND CONCLUSION

We calculate the magnetic moments of spin-$\frac{3}{2}$ singly charmed baryons. The analytical expressions are derived up to $O(p^3)$. There are eighteen unknown LECs involved. We reduce them into seven vanishing independent LECs with the heavy quark symmetry. Our numerical results are given up to $O(p^3)$ in three scenarios. In the first scenario, we keep the mass difference between spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ sextets. The quark model results are regarded as the leading order magnetic moments. Five Lattice QCD results are used to determine the LECs. The heavy quark symmetry is used to relate the $O(p^3)$ $B\phi\bar{\phi}$ and $O(p^3)$ $B\gamma$ vertices to those for the spin-$\frac{1}{2}$ heavy baryons. In the second scenario, we adopt the heavy quark symmetry globally. The spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextets belong to the same doublet. The heavy quark contribution explicitly on the basis of scenario II. In this scenario, we also evaluate the magnetic moments of singly bottom baryons as a by-product. Including the spin-$\frac{1}{2}$ antitriplet intermediate states will worsen the chiral divergence, due to its large mass difference with the sextet. We list both the results with all intermediate states and only sextet intermediate states. We take the latter ones as the final results.

We give our final results and those from other schemes in Table X. Compared with the scenario II, the scenario III includes the heavy quark contribution. The results in scenario III tend to be closer to those from other schemes. Thus, the $\frac{1}{m_Q}$ effect may be not negligible. While the bottom quark is much heavier, its contribution in the singly bottom baryons can be neglected. In the scenario I, no Lattice QCD results for spin-$\frac{3}{2}$ heavy baryon is used as input. The value of $\mu_{O^{10}}$ in scenario I may become larger if we use Lattice QCD simulation value as input. In the scenario III, we determined three unknown LECs and $\mu_c$ to give twelve predictions. The scenario III has powerful predictions with twelve predictions. Scenario I and III are quite different methods. The numerical results for the scenario I and III are similar and corroborate each other.

The other schemes in Table X include the Lattice QCD [21], the hyper central model [11], effective mass and screened charge scheme [12], chiral constituent quark model [13], light-cone QCD sum rules [14], MIT bag model [15, 14], Skyrmion scheme [15] and chiral quark-soliton model [19]. Our results from all scenarios are less than those from other schemes in general. Same tendency also appeared in the magnetic moments of spin-$\frac{1}{2}$ charm baryon [40]. In fact the Lattice QCD results which we used as input are also less than other schemes. In the Lattice QCD simulation, in order to extract the results with physical pion mass, the rough linear or quadratic extrapolation was used in Ref. [20].

We have calculated the magnetic moments of spin-$\frac{3}{2}$ singly heavy baryons analytically to $O(p^3)$. The convergence of the chiral expansion is good in our numerical results. For the lack of experimental data, we have to adopt heavy quark symmetry and the quark model to reduce and estimate our LECs. Our numerical results can be improved with the new experimental results and the new Lattice QCD simulation results in the future. Meanwhile, our analytical expressions can help the chiral extrapolation in Lattice QCD simulation. The LECs determined in this work can also be used to study other physical properties, for instance, the electromagnetic decay of singly heavy baryon.
ACKNOWLEDGMENTS

L. Meng is very grateful to H. S. Li, X. L. Chen and W. Z. Deng for very helpful discussions. This project is supported by the National Natural Science Foundation of China under Grants 11575008, 11621131001 and 973 program. This work is also supported by the Fundamental Research Funds for the Central Universities of Lanzhou University under Grants 223000–862637.

Appendix A: Integrals

We give some integrals with the conditions $v \cdot q = 0$ and $q^2 = 0$. All the results are given in the dimension $d = 4$.

- Integrals with one meson propagator and one baryon propagator

$$\int \frac{d^d \lambda^{4-d}}{(2\pi)^d} \frac{l_\alpha l_\beta}{(l^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)} = g_{\alpha \beta} J_2(\omega) + v_\alpha v_\beta J_3(\omega) \quad (A1)$$

$$J_2(\omega) = \begin{cases} 
2\omega(\omega^2 - m^2 + \omega(3m^2 - 2\omega^2)(\ln \frac{\omega^2 + 32\pi^2 L(\omega)}{2} - 4(\omega^2 - m^2)^{3/2}(\cosh^{-1}(\frac{\omega}{m}) - i\pi) \quad (\omega > m) \\
\omega(32\pi^2 L(\lambda) + \ln \frac{m^2 - \omega^2}{2}) + 2\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) \quad (\omega < m) \\
\omega(32\pi^2 L(\lambda) + \ln \frac{m^2 - \omega^2}{2}) - 2\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) \quad (\omega < m) 
\end{cases}$$

where $L(\lambda)$ is the infinite term:

$$L(\lambda) = \frac{\lambda^{d-4}}{16\pi^2} \left[ \frac{1}{d-4} - \frac{1}{2} \left( \ln 4\pi + 1 + \Gamma'(1) \right) \right] \quad (A3)$$

- Integrals with two meson propagators and one baryon propagator

$$\int \frac{d^d \lambda^{4-d}}{(2\pi)^d} \frac{l_\alpha l_\beta}{(l^2 - m^2 + i\epsilon)((l + q)^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)} = n_1^\mu g_{\alpha \beta} + n_2^\mu q_{\alpha \beta} + n_3^\mu v_{\alpha \beta} + n_4^\mu (v_{\alpha q_{\beta} + q_{\alpha} v_{\beta}) \quad (A4)$$

$$n_1^\mu(\omega) = \begin{cases} 
\omega(32\pi^2 L(\lambda) + \ln \frac{m^2 - \omega^2}{2}) + 2\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) \quad (\omega > m) \\
\omega(32\pi^2 L(\lambda) + \ln \frac{m^2 - \omega^2}{2}) + 2\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) \quad (\omega < m) \\
\omega(32\pi^2 L(\lambda) + \ln \frac{m^2 - \omega^2}{2}) - 2\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) \quad (\omega < m) 
\end{cases}$$

- Other integrals

The infinite terms are absorbed by the renormalization of the coefficients and the $L(\lambda)$ term is omitted in the following expression.

$$\frac{3-d}{d-1} n_{1}^\mu(\omega, m) = \begin{cases} 
-\frac{\omega(3\ln \frac{m^2}{\omega^2} + 1) + 6\sqrt{\omega^2 - m^2}\cosh^{-1}(\frac{\omega}{m}) - i\pi}{144\pi^2} \quad (\omega > m) \\
-\frac{\omega(3\ln \frac{m^2}{\omega^2} + 1) - 6\sqrt{m^2 - \omega^2}\cosh^{-1}(\frac{\omega}{m})}{144\pi^2} \quad (\omega < m) \\
-\frac{\omega(3\ln \frac{m^2}{\omega^2} - 5) - 6\sqrt{m^2 - \omega^2}\cosh^{-1}(\frac{\omega}{m})}{36\pi^2} \quad (\omega < m) 
\end{cases}$$

$$\frac{4-d}{d-1} n_{1}^\mu(\omega, m) = \begin{cases} 
\frac{\omega(3\ln \frac{m^2}{\omega^2} - 5) + 6\sqrt{m^2 - \omega^2}\cosh^{-1}(\frac{\omega}{m}) - i\pi}{288\pi^2} \quad (\omega > m) \\
\frac{\omega(3\ln \frac{m^2}{\omega^2} - 5) + 6\sqrt{m^2 - \omega^2}\cosh^{-1}(\frac{\omega}{m})}{36\pi^2} \quad (\omega < m) \\
\frac{\omega(3\ln \frac{m^2}{\omega^2} - 5) - 6\sqrt{m^2 - \omega^2}\cosh^{-1}(\frac{\omega}{m})}{36\pi^2} \quad (\omega < m) 
\end{cases}$$
\[ J'_2(\omega) = \begin{cases} 
\frac{(m^2 - 2\omega^2)}{4\sqrt{\omega^2 - m^2}} \ln \left( \frac{m^2}{\omega^2} \right) - 4\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( \frac{\omega}{m} \right) - i\pi \right] + 2\omega^2 & (\omega > m) \\
\frac{16\pi^2}{10\pi^2} \frac{(m^2 - 2\omega^2)}{10\pi^2} \ln \left( \frac{m^2}{\omega^2} \right) - 4\omega \sqrt{\omega^2 - m^2} \cos^{-1} \left( -\frac{\omega}{m} \right) + 2\omega^2 & (\omega^2 < m^2) \\
\frac{16\pi^2}{10\pi^2} \frac{(m^2 - 2\omega^2)}{10\pi^2} \ln \left( \frac{m^2}{\omega^2} \right) + 4\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( -\frac{\omega}{m} \right) + 2\omega^2 & (\omega < -m) 
\end{cases} \] (A8)

\[ (1 - \frac{d}{4} + \frac{1}{d - 1}) J'_2(\omega) = \begin{cases} 
-15(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} + 60\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( \frac{\omega}{m} \right) - 26m^2 + 22\omega^2 & (\omega > m) \\
-15(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} + 60\omega \sqrt{\omega^2 - m^2} \cos^{-1} \left( -\frac{\omega}{m} \right) - 26m^2 + 22\omega^2 & (\omega^2 < m^2) \\
-15(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} - 60\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( -\frac{\omega}{m} \right) - 26m^2 + 22\omega^2 & (\omega < -m) 
\end{cases} \] (A9)

\[ (1 - \frac{d}{2} + \frac{4}{d - 1} - \frac{4}{(d - 1)^2}) J'_2(\omega) = \begin{cases} 
-33(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} + 132\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( \frac{\omega}{m} \right) - 70m^2 + 74\omega^2 & (\omega > m) \\
-33(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} + 132\omega \sqrt{\omega^2 - m^2} \cos^{-1} \left( -\frac{\omega}{m} \right) - 70m^2 + 74\omega^2 & (\omega^2 < m^2) \\
-33(m^2 - 2\omega^2) \ln \frac{m^2}{\omega^2} - 132\omega \sqrt{\omega^2 - m^2} \cosh^{-1} \left( -\frac{\omega}{m} \right) - 70m^2 + 74\omega^2 & (\omega < -m) 
\end{cases} \] (A10)
Appendix B: Magnetic moments of spin-$\frac{1}{2}$ singly charmed baryon

TABLE XI: The numerical magnetic moments of spin-$\frac{1}{2}$ sextet from the scenario I (in unit of $\mu_N$) [40]. We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only take the $B_6$ and $B_6^*$ in the right panel.

|       | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|-------|-------------------------------|------------------------|
|       | $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total | $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total |
| $\Sigma_{++}$ | 2.36 -1.01 0.15 1.50 | 2.36 -0.70 -0.16 1.50 |
| $\Sigma^+_c$ | 0.61 -0.50 -0.01 0.11 | 0.61 -0.26 -0.10 0.23 |
| $\Sigma^0_c$ | -1.13 0.01 -0.16 -1.29 | -1.13 0.18 -0.03 -0.98 |
| $\Xi^+_c$ | 0.61 -0.003 -0.29 0.32 | 0.61 -0.09 -0.21 0.32 |
| $\Xi^0_c$ | -1.13 0.50 -0.32 -0.95 | -1.13 0.35 -0.05 -0.84 |
| $\Omega^0_c$ | -1.13 1.00 -0.54 -0.67 | -1.13 0.52 -0.05 -0.67 |

TABLE XII: The numerical magnetic moments of spin-$\frac{1}{2}$ sextet from the scenario II (in unit of $\mu_N$). We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only take the $B_6$ and $B_6^*$ in the right panel.

|       | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|-------|-------------------------------|------------------------|
|       | $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total | $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total |
| $\Sigma_{++}$ | 0.55 -0.85 1.81 1.50 | 1.72 -0.74 0.52 1.50 |
| $\Sigma^+_c$ | 0.14 -0.49 0.66 0.31 | 0.43 -0.26 0.18 0.35 |
| $\Sigma^0_c$ | -0.27 -0.14 -0.48 -0.89 | -0.86 0.22 -0.16 -0.81 |
| $\Xi^+_c$ | 0.14 0.07 0.11 0.32 | 0.43 -0.11 -0.001 0.32 |
| $\Xi^0_c$ | -0.27 0.43 -0.92 -0.76 | -0.86 0.37 -0.21 -0.70 |
| $\Omega^0_c$ | -0.27 0.99 -1.37 -0.67 | -0.86 0.52 -0.24 -0.58 |
TABLE XIII: The numerical magnetic moments of the spin-$\frac{1}{2}$ sextet in the scenario III (in unit of $\mu_N$). We take $B_3$, $B_6$ and $B_6^*$ as the intermediate states in the left panel, while we only take the $B_6$ and $B_6^*$ in the right panel. The “HQ” represents the heavy quark contribution.

|             | with $B_3$, $B_6$ and $B_6^*$ | with $B_6$ and $B_6^*$ |
|-------------|-------------------------------|--------------------------|
|             | HQ $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total | HQ $\mathcal{O}(p^1)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p^3)$ Total |
| $\Sigma^+_c$ | -0.07 0.59 -0.85 1.83 1.50 | -0.07 1.79 -0.74 0.51 1.50 |
| $\Sigma^+_c$ | -0.07 0.15 -0.49 0.67 0.26 | -0.07 0.45 -0.26 0.18 0.30 |
| $\Sigma^0_c$ | -0.07 -0.30 -0.14 -0.49 -0.99 | -0.07 -0.89 0.22 -0.15 -0.90 |
| $\Xi^+_c$   | -0.07 0.15 0.07 0.17 0.32 | -0.07 0.45 -0.11 0.04 0.31 |
| $\Xi^0_c$   | -0.07 -0.30 0.43 -0.88 -0.81 | -0.07 -0.89 0.37 -0.23 -0.82 |
| $\Omega^0_c$| -0.07 -0.30 0.99 -1.29 -0.67 | -0.07 -0.89 0.52 -0.29 -0.73 |

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