Chiral Phonon Transport Induced by Topological Magnons

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The plethora of recent discoveries in the field of topological electronic insulators has inspired a search for boson systems with similar properties. There are predictions that ferromagnets on a two-dimensional honeycomb lattice may host chiral edge magnons. In such systems, we theoretically study how magnons and phonons couple. We find topological magneto-polarons around the avoided crossings between phonons and topological magnons. Exploiting this feature along with our finding of Rayleigh edge phonons in armchair ribbons, we demonstrate the existence of chiral edge modes with a phononic character. We predict that these modes mediate a chirality in the coherent phonon response and suggest to measure this effect via elastic transducers. These findings reveal a possible approach towards heat management in future devices.

Introduction.— Topological electronic insulators [1–5] are characterized by an insulating bulk with conducting ‘chiral’ edge states. The unidirectional propagation of these chiral modes is ‘topologically protected’ against defects at low temperatures when we can disregard inelastic scattering from phonons [6]. This has led to the development of a wide range of essential concepts, including Majorana modes [6–8], topological quantum computation [10–11], and chiral transport. Inspired by these findings, there has been an upsurge of efforts towards finding similar states in other systems [12] with an emphasis on bosonic excitations [13–17]. There are predictions of topological magnons [15, 17] in honeycomb ferromagnets with an engineered Dzyaloshinskii-Moriya interaction [15, 19] that induces the necessary band gap. In contrast to fermionic systems with Fermi energy within this band gap, the bulk is not necessarily insulating in bosonic systems [20].

The field of magnonics [21–24] focuses on pure spin transport mediated by magnons [25]. It is possible to exploit the low-dissipation and wave-like nature of these excitations in information processing [26, 27]. The coherent pumping of chiral surface spin wave (Damon-Eshbach) modes induces cooling via incoherent magnon-phonon scattering [28]. Besides application oriented properties, the bosonic nature of magnons, combined with spintronic manipulation techniques [22, 29], allows for exciting physics [30–33]. The coupling [34] between magnons and phonons fundamentally differs from the electron-phonon interaction and results in a coherent hybridization of the modes [35], in addition to the temperature dependent incoherent effects [28, 36] discussed above. The direct influence of the hybridization between magnons and phonons, known as magneto-polarons [37, 38], has been observed in spin and energy transport in magnetic systems [39–41].

In this Letter, we address the robustness of the topological magnons [15, 17] in a honeycomb ferromagnet against their coupling with the lattice vibrations. In contrast to the case of electron-phonon coupling, where phonons can be disregarded at low temperatures, the magnon dispersion may undergo significant changes with new states emerging in the band gap [33, 44]. To explore this, we evaluate and analyze the coupled spin and out-of-plane phonon modes for an infinitely large plane as well as for a finite ribbon geometry. We quantify the effect of the magneto-elastic coupling on the magnon Hall conductivity and find a non-monotonic dependence on the coupling strength. Our numerical analysis of the finite ribbons demonstrates that topological magnons hybridize with bulk phonons around the avoided crossings in their coupled dispersion, forming magneto-polarons with topological chiral properties. Hence, while their edge localization is weakened, the magneto-elastic coupling does not completely remove the topological magnons. Furthermore, we find that armchair ribbon edges support Rayleigh edge phonon modes in contrast to the zigzag edges. When topological magnons hybridize with these edge phonons, edge magneto-polarons with almost undiminished chirality are formed. We suggest a setup which utilizes this induced chirality in coherent phonon transport. Such systems enable the observation of the topological physics and serve as a prototype for a unidirectional heat pump.

Model.—We consider a ferromagnetic material with localized spins on a two-dimensional honeycomb lattice, allow for out-of-plane vibrations of the lattice sites, and assume there is magneto-elastic coupling. This system can be modelled by a Hamiltonian of the form

\[ H = H_m + H_{ph} + H_{me}, \]

where \( H_m \) is the magnetic Hamiltonian, \( H_{ph} \) describes the phonons, and \( H_{me} \) represents the magneto-elastic coupling.

We consider a magnetic Hamiltonian directly inspired by the Haldane model [4] that can be written in the form [15, 20]

\[ H_m = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_{\langle i,j \rangle} \nu_{ij} \hat{z} \cdot \mathbf{S}_i \times \mathbf{S}_j - B \sum_i S_i^z, \quad (1) \]

where the first term describes the ferromagnetic exchange coupling between nearest neighbour sites, while the second accounts for the Dzyaloshinskii-Moriya interaction...
between next-to-nearest neighbours \[18\] [19]. The Hal- danne sign \(\nu_{ij} = \pm 1\) depends on the relative orientation of the next-to-nearest neighbours as shown in Fig. 1(a), and is the crucial factor causing non-trivial topological properties. We let the nearest neighbour distance be \(d\) and the next-to-nearest neighbour distance be \(a\). Ref. \[15\] discusses the dispersion relation and Berry curvature of this spin model.

For the phonon Hamiltonian, we consider only the out-of-plane degree of freedom since only these modes couple to the spin to lowest order in the linear spin wave expansion. We assume nearest neighbour interactions with elastic constant \(C\), and let the mass associated with the spins on the lattice sites be \(m\). Introducing \(S_k = \sum_\beta \cos(\mathbf{k} \cdot \mathbf{\beta})\), where the sum is over the three next-to-nearest neighbour vectors \(\mathbf{\beta}\) of Fig. 1(a), we obtain the dispersion relation

\[
\omega_{ph}^k(k) = \sqrt{\frac{C}{m} \sqrt{3 \pm \sqrt{3 + 2S_k}}} \tag{2}
\]

for the free phonon modes.

Motivated by the continuum limit description \[34\] [35], we write down the lattice magneto-elastic coupling to linear order in the magnon amplitude, obtaining

\[
H_{me} = \kappa \sum_D \sum_{i_D,\alpha_D} \left(\left(\hat{x} \cdot \mathbf{\alpha}_D\right)S^x_{i_D} + \left(\hat{y} \cdot \mathbf{\alpha}_D\right)S^y_{i_D}\right)(u^{+\alpha}_{i_D} - u_{i_D + \alpha_D}^\dagger),
\tag{3}
\]

where \(\kappa\) parametrizes the strength of the magnon-phonon coupling, \(\sum_D\) denotes the sum over sublattices, \(\sum_{i_D}\) is the sum over the lattice sites on the \(D\) sublattice, and \(\mathbf{\alpha}_D\) are the corresponding nearest neighbour vectors. The out-of-plane deviation for lattice site \(i_D\) is denoted by \(u_{i_D}^\dagger\).

Bulk spectrum.—We introduce the Holstein-Primakoff representation of spins and use linear spin wave theory in the spin- and magneto-elastic terms \[25\]. Within the rotating wave approximation \[45\], the resulting Hamiltonian describing the phonon and magnon modes of the system is obtained as \(H = \sum_k \psi_k^\dagger M_k \psi_k\), where \(\psi_k^\dagger = (a_k^\dagger, b_k^\dagger, c_k^\dagger, c_k^\dagger)\). Here, \(a_k\) and \(b_k\) are annihilation operators for the sublattice magnon modes on the \(A\) and \(B\) sublattices, while \(c_k\) are the annihilation operators for the phonon modes. The matrix \(M_k\) takes the form

\[
M_k = \begin{pmatrix}
A + h^z & h^y & g_{A-} & g_{A+} \\
A - h^z & -h^y & g_{B-} & g_{B+} \\
g_{A-} & g_{B-} & -\omega_k^\text{ph} & 0 \\
g_{A+} & g_{B+} & 0 & \omega_k^\text{ph}
\end{pmatrix},
\tag{4}
\]

where \(A = 3JS + B\), \(h^z(k) = 2DS\sum_\beta \sin(\mathbf{k} \cdot \mathbf{\beta})\), \(h^y(k) = -JS\sum_\alpha \exp(-i\mathbf{k} \cdot \mathbf{\alpha})\), \(h^+ = (h^\ast)^\dagger\). The coupling between the \(D\)-sublattice magnons and the phonon branch \(±\) is captured by \(g_{Dz\pm}\), which is proportional to the dimensionless coupling strength \(\tilde{\kappa} = (\kappa d/JS) \sqrt{\hbar^2 S^2/16m^2(C/m)}\).

The spectrum obtained by diagonalizing this matrix is plotted in Fig. 1(c) along the paths displayed in Fig. 1(b).

Hall conductivity.—The topological nature of the spin model is manifested in the magnon Hall conductivity that arises because of the time-reversal symmetry breaking caused by the Dzyaloshinskii-Moriya interaction.

The spin current operator in linear spin wave theory can be derived from the magnetic Hamiltonian by a continuity equation or magnon group velocity approach \[46\], both resulting in a current

\[
J_\gamma = \sum_k \left( a_k^\dagger b_k^\dagger \left( \frac{\partial H_m(k)}{\partial \theta_\gamma} \right) \right) \left( a_k b_k \right)
\tag{5}
\]

along the Cartesian direction \(\gamma\). Here, \(H_m(k)\) is the matrix representation of the magnon Hamiltonian. Assuming we apply a weak in-plane magnetic field gradient \(\nabla B\), we are interested in the current \(\mathbf{j} = \sigma \nabla B\), which is determined by the conductivity tensor \(\sigma\) \[46\]. The Hall conductivity can be calculated using the Kubo formula, giving
\[ \sigma_{xy} = \sum_k \sum_{\alpha, \beta \neq \alpha} n_B(E_{k\alpha}) C_{\alpha\beta}(k), \] 

where \( E_{k\alpha} \) is the energy eigenvalue of band \( \alpha \) and \( n_B(E_{k\alpha}) \) is the corresponding Bose factor. The curvature-tensor \( C_{\alpha\beta} \) is given by

\[ C_{\alpha\beta}(k) = i J_{y}^{\alpha\beta}(k) J_{x}^{\alpha\beta}(k) - J_{x}^{\alpha\beta}(k) J_{y}^{\alpha\beta}(k) \frac{(E_{k\alpha} - E_{k\beta})^2}{(E_{k\alpha} - E_{k\beta})^2}, \]

where \((\alpha, \beta)\) are band-indices, and \( J_{\gamma}^{\alpha\beta}(k) \) are the energy eigenstate matrix elements of the current operator at quasimomentum \( k \). Disregarding the magneto-elastic coupling, the band-curvature \( C_{\alpha\beta} = \sum_{\beta \neq \alpha} C_{\alpha\beta} \) can be identified as the Berry curvature.

Expressing the sublattice magnon operators in terms of the eigenmode operators, one may identify the current matrix elements \( J_{\gamma}^{\alpha\beta}(k) \) and integrate the curvature over the Brillouin zone to obtain the Hall conductivity. We are particularly interested in the effect of the magneto-elastic coupling, and therefore present the dependence of the Hall conductivity on the dimensionless coupling \( \kappa \) in Fig. 2.

To understand this dependence, we consider the curvature-tensor \( C_{\alpha\beta} \). When the bands \( \alpha \) and \( \beta \) both have a predominant magnon content, the topological nature of the underlying magnons gives a finite curvature. This magnon curvature is largest close to the Dirac points \[ \pm \frac{\pi}{a} \]. Close to an avoided crossing, the magneto-elastic coupling changes the spectrum and causes transfer of band-curvature between the relevant bands \( \alpha \) and \( \beta \). The latter can be seen by plotting the curvature-tensor element \( C_{\alpha\beta} \) for the band-pairs with avoided crossings, as shown in the insets of Fig. 2. The resulting change in Hall conductivity is given by these curvature-tensor elements weighted with the difference between the Bose factors of the relevant bands. This follows from the anti-symmetry property of the curvature-tensor. The two band-pairs in the insets contribute oppositely to the Hall conductivity, and the competition between their curvature transfer explains the non-monotonic behaviour of the Hall conductivity.

**Ribbons geometry and coherent transport.**—Due to the topological nature of the magnon model under consideration and the bulk-boundary correspondence, there are gapless magnon edge states in a finite sample \[ \Box \ Box \]. Considering a ribbon geometry with a finite number of hexagon layers, the one-dimensional projection of the energy spectrum is plotted in Fig. 3 for zigzag and armchair edges. Magnon and phonon modes hybridize in regions with an avoided crossing. When the upper phonon band lies within the bandgap of the pure magnon spectrum, there are modes with a mixed content of chiral magnon edge states and phonons. Although the spectra look qualitatively similar, there is a crucial distinction between the two cases. Namely, for the zigzag edge configuration, all the phonon modes are delocalized throughout the sample, while the armchair edges host Rayleigh modes localized near the edges with a momentum dependent localization length. The latter modes are the phonon modes where the half-hexagon protrusions of the armchair edge pivot around the bonds parallel to the edges connecting these protrusions, see Fig. 3(b). No such parallel bonds exist for the zigzag edge, see Fig. 3(a).

The Hall conductivity is a hallmark of topological electronic properties and motivates a similar role for the Hall conductivity mediated by topological magnons. However, in contrast to electrons, the bosonic nature of the magnons results in the lack of a general proportionality between the magnon Hall conductivity and the Chern number \[ \Gamma \]. Furthermore, the observation of a magnon planar Hall effect \[ \Gamma \] in a cubic, non-topological magnet suggests that this Hall conductivity may not be regarded as a smoking-gun signature for topological properties. Thus, we suggest a complementary approach to observe the topological nature of the underlying magnons by elastically probing the chirality of the magneto-elastic hybrid modes.

We suggest to observe coherent chiral phonon propagation in the experimental setup of Fig. 4. Elastic energy is injected into the sample middle at the upper edge using a nano-scale variant of the interdigital transducer design \[ \Box \ Box \ Box \]. For a given transducer, modes are excited with fixed wavevectors \( \pm k_x \) and a tunable frequency. Identical transducers can be used to detect an elastic response \( p_{L/R} \) at the left (L) and right (R) edges of the...
sample, see Fig. 4. Here, $p_{L/R}$ is the power absorbed at the transducers.

Choosing the right frequency and wavevector, one may inject energy in a region with an avoided crossing of a phonon and chiral magnon mode. Due to magneto-elastic coupling, the elastic excitations are converted into hybridized phonon and edge magnon excitations. With delocalized phonon modes, the phonon content eventually extends across the entire sample for both the $+k_x$ and $-k_x$ modes. Since this allows hybridization with the chiral magnon modes at both the upper and lower edges, the phononic and magnetic responses are the same at the left and right detectors, and there is no chiral transport.

Since the zigzag edge only hosts delocalized phonon modes, no chiral transport is possible on the zigzag edge within this setup. On the armchair edge, in contrast, one may excite modes in the region of the avoided crossing of the chiral magnon mode and the Rayleigh phonon mode. Since we inject elastic energy at only one of the edges and both modes are localized, the mode at $-k_x$ may hybridize with the edge magnon mode, while the mode at $+k_x$ remains a pure phononic mode. This yields a finite chirality $P = (p_R - p_L)/(p_R + p_L)$ in the observed phonon transport.

The location of the avoided crossing can be tuned with a magnetic field. Performing a frequency integrated measurement over an energy range of the same order as the magneto-elastic coupling, one obtains a peaked chirality when the wavevector of the avoided crossing coincides with the fixed wavevector of the transducer grating. As shown in the inset of Fig. 3(b), the pure magnon and pure phonon modes have group velocities in opposite directions at the avoided crossing. Turning on the magneto-elastic coupling may therefore even flip the direction of propagation for one of the modes with phononic content, ideally resulting in perfect chiral transport.

Summary.—We have examined the robustness of topological magnons in a honeycomb ferromagnet against their interaction with phonons, finding that the topological properties, albeit weakened, survive the magneto-elastic coupling. The magnon Hall conductivity of the system is found to depend on the magneto-elastic coupling strength in a non-monotonic, temperature-sensitive manner. Exploiting the Rayleigh edge phonons in armchair ribbons, we predict the existence of topological magneto-polarons confined to the boundary. We have suggested an experimental setup capable of probing the chiral nature of the topological magneto-polarons by elastic means, which thus serves as a platform for chiral coherent phononic transport.

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FIG. 4. Proposed experimental setup for detecting coherent chiral transport through excitation of phononic modes. Elastic energy is injected in the sample middle at the upper edge and detected at the left (L) and right (R) edges using elastic transducers (purple). Considering propagation along the armchair edge, one may excite modes at wavevectors and energies close to the avoided crossing of the phonon Rayleigh modes and chiral magnons. The elastic excitations (purple arrows) are converted into hybridized modes (green arrow) at only one of the two quasimomenta ±kz. This gives a chiral response. The location of the avoided crossing can be tuned with a magnetic field, and the chirality is peaked when the wavevector of the avoided crossing coincides with the fixed wavevector of the transducer grating. One may also detect chiral spin transport with magnetic receivers (yellow).

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