Gaugino and Scalar Masses in the Landscape

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Abstract: In this letter we demonstrate the genericity of suppressed gaugino masses $M_a \sim \frac{m_{3/2}}{\ln(M_{\text{Planck}}/m_{3/2})}$ in the IIB string landscape, by showing that this relation holds for D7-brane gauginos whenever the associated modulus is stabilised by nonperturbative effects. Although $m_{3/2}$ and $M_a$ take many different values across the landscape, the above small mass hierarchy is maintained. We show that it is valid for models with an arbitrary number of moduli and applies to both the KKLT and exponentially large volume approaches to Kähler moduli stabilisation. In the latter case we explicitly calculate gaugino and moduli masses for compactifications on the two-modulus Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$. In the large-volume scenario we also show that soft scalar masses are approximately universal with $m_i^2 \sim m_{3/2}^2 (1 + \epsilon_i)$, with the non-universality parametrised by $\epsilon_i \sim \frac{1}{\ln(M_P/m_{3/2})^2} \sim \frac{1}{1000}$.

We briefly discuss possible phenomenological implications of our results.
1. Introduction

One of the most important problems in string theory is to make contact with low-energy phenomenology. In this regard, the recent progress made in moduli stabilisation represents a substantial advance [1, 2]. Once the moduli potential has been calculated, it becomes possible to study in a top-down fashion supersymmetry breaking and, with suitable assumptions about the loci of matter fields, the structure of soft terms such as gaugino or scalar masses.

On the other hand, the same advances in moduli stabilisation have led to a concern that the large discrete degeneracies present destroy any possibility of low-energy predictivity. There seem to be many consistent discrete choices for the fluxes that must be present (a standard estimate is $10^{500}$). The worry is that the huge number of possible choices will wash out any low-energy signals of high-energy physics. This set of ideas and problems is encapsulated in the word ‘landscape’.

On a technical level, we use ‘landscape’ to refer to IIB compactifications with moduli stabilised by fluxes and non-perturbative superpotential effects. We shall work in this framework and use the formulae appropriate to it. The dilaton and complex structure moduli are stabilised by 3-form fluxes, while the Kähler moduli are stabilised by nonperturbative superpotential effects. This is true in both the KKLT case [2] and the exponentially large volume scenario developed in [3]. In principle, purely perturbative stabilisation may be possible. However we shall not consider this as no fully explicit examples exist even in a one modulus case.

The first aim of this paper is to show that in the IIB landscape, whenever a modulus $T_a$ is stabilised by nonperturbative effects there is a small hierarchy between the masses of the gravitino $m_{3/2}$ and the associated D7 gaugino $M_a$,

$$M_a^2 \sim \frac{m_{3/2}^2}{\ln(m_{3/2})^2}. \quad (1.1)$$

We use natural units with $M_{\text{Planck}} = 2.4 \times 10^{18}\text{GeV} \equiv 1$. This small hierarchy was first identified in single-modulus KKLT models [4] (with a two-modulus example also studied in [5]). For the KKLT case this suppression of $M_a$ leads to mixed modulus-anomaly mediation and the phenomenology of this scenario has been analysed in [6–16].\footnote{In these references this hierarchy is present for all soft supersymmetry breaking terms. We can establish it in generality only for gaugino masses. As we discuss in section 4 for the exponentially large volume scenario scalar masses are generically $O(m_{3/2})$.} Here we show this relation to be a truly generic feature of the landscape, by showing it to hold for arbitrary multi-modulus models and to be independent of the precise details of the scenario used to stabilise the moduli. The fact that (1.1) relates $M_a$ to $m_{3/2}$ and $M_P$ emphasises that it is a relation only among observable quantities and is independent of the values of the fluxes that determine the structure of the landscape.

In section 4 we also demonstrate soft scalar mass universality for the exponentially large volume scenario and estimate the fractional non-universality to be $1/\ln(m_{3/2})^2 \sim 1/1000$. This is an interesting result as flavour non-universality is usually a serious problem
for gravity-mediated models. We briefly discuss the phenomenology but defer a detailed
discussion to [17].

2. Gaugino Masses

For IIB flux compactifications the appropriate Kähler potential and superpotential are
given by

\[
K = -2 \ln \left( \mathcal{V} + \frac{\hat{\xi}}{2} \right),
\]
\[
W = W_0 + \sum A_i e^{-a_i T_i}.
\] (2.1)

Here \( \mathcal{V} = \frac{1}{6} k_{ijk} t^i t^j t^k \) is the Calabi-Yau volume, \( \hat{\xi} = -\chi(M) \zeta(3) \) and \( T_i = \tau_i + i b_i \) are the
Kähler moduli, corresponding to the volume \( \tau_i = \frac{\partial \mathcal{V}}{\partial t_i} \) of a 4-cycle \( \Sigma_i \), complexified by the
axion \( b_i = \int_{\Sigma_i} C_4 \). \( W_0 = \langle \int G_3 \wedge \Omega \rangle \) is the flux-induced superpotential that is constant
after integrating out dilaton and complex structure moduli. In general there does not exist
an explicit expression for \( \mathcal{V} \) in terms of the \( T_i \). We have included the perturbative Kähler
corrections of [18]. These are crucial in the exponentially large volume scenario but are not
important for KKLT. We use single exponents in the superpotential and do not consider
racetrack scenarios.

For a D7-brane wrapped on a cycle \( \Sigma_i \), the gauge kinetic function is given by

\[
f_i = \frac{T_i}{2\pi}.
\] (2.2)

Given the minimum of the moduli potential, the gaugino masses can be computed through
the general expression

\[
M_a = \frac{1}{2} \text{Re} f_a \sum F_{\alpha} \partial_{\alpha} f_a,
\] (2.3)

where \( a \) runs over gauge group factors and \( \alpha \) over the moduli fields. The F-term \( F_{\alpha} \) is
defined by

\[
F_{\alpha} = e^{\mathcal{K}/2} \sum_{\beta} \mathcal{K}_{\alpha \beta} D_\beta \mathcal{W}
\]
\[
= e^{\mathcal{K}/2} \sum_{\beta} \mathcal{K}_{\alpha \beta} \left( \partial_\beta \mathcal{W} + (\partial_\beta \mathcal{K}) \mathcal{W} \right),
\] (2.4)

where we have expanded the covariant derivative \( D_\beta \mathcal{W} = \partial_\beta \mathcal{W} + (\partial_\beta \mathcal{K}) \mathcal{W} \).

Thus for a brane wrapping cycle \( k \) we have

\[
M_k = \frac{1}{2} \frac{F^k}{\tau_k}.
\] (2.5)

It is a property of the Kähler potential \( \mathcal{K} = -2 \ln(\mathcal{V} + \hat{\xi}) \) that

\[
\sum_j \mathcal{K}^{kj} \partial_j \mathcal{K} = -2 \tau_k \left( 1 + \frac{\hat{\xi}}{4\mathcal{V}} \right) \equiv -2 \tilde{\tau}_k.
\] (2.6)
We therefore obtain

\[ F^k = e^{K/2} \left( \sum_j K^{kj} \partial_j \tilde{W} - 2\tilde{\tau}_k \tilde{W} \right). \tag{2.7} \]

From the superpotential (2.1), we see that \( \partial_j \tilde{W} = -a_j \tilde{A}_j e^{-a_j \tilde{T}_j} \), and so

\[ F^k = e^{K/2} \left( \sum_j -K^{kj} a_j \tilde{A}_j e^{-a_j \tilde{T}_j} - 2\tilde{\tau}_k \tilde{W} \right). \tag{2.8} \]

We now show that if the modulus \( T_k \) is stabilised by nonperturbative effects, the two terms in equation (2.8) cancel to leading order. To see this, we start with the F-term supergravity potential,

\[ V_F = e^K K^{ij} \partial_i W \partial_j \tilde{W} + e^K K^{ij} \left((\partial_i K) W \partial_j \tilde{W} + (\partial_j K) \tilde{W} \partial_i W\right) + e^K (K^{ij} K_i \xi - 3) |W|^2. \tag{2.9} \]

Using (2.1) and (2.8), this becomes

\[ V = \sum_{ij} K^{ij} (a_i A_j)(a_j \tilde{A}_j) e^{-a_i T_i - a_j \tilde{T}_j} V^2 + \sum_j 2\tilde{\tau}_j \left( a_j \tilde{A}_j e^{-a_j \tilde{T}_j} W + a_j A_j \tilde{W} e^{-a_j \tilde{T}_j} \right) + 3\tilde{\xi} |W|^2 V^2. \tag{2.10} \]

The perturbative Kähler corrections of (2.1) break no-scale, giving the third term of (2.10). We have only displayed the leading large-volume behaviour of these corrections. This is reasonable as in KKLT such corrections are not important, while in the exponentially large volume scenario \( V \gg 1 \) and the higher volume-suppressed terms are negligible. We shall also assume throughout that \( m_{3/2} \ll M_P \). This is motivated by phenomenological applications and is any case necessary to make sense of a small hierarchy governed by \( \ln(M_P/m_{3/2}) \).

We assume the modulus \( T_k \) is stabilised by effects non-perturbative in \( T_k \) and locate the stationary locus by extremising with respect to its real and imaginary parts. We first perform the calculation keeping the dominant terms to demonstrate the cancellation in (2.8). We subsequently show that the subleading terms are indeed subleading and estimate their magnitude.

### 2.1 Leading Terms

We first solve for the axionic component, \( \partial V/\partial b_k = 0 \). The axion only appears in the superpotential and we have

\[ \frac{\partial V}{\partial b_k} = \frac{i}{V^2} \left[ \sum_{ij} K^{ij} (a_i A_j)(a_j \tilde{A}_j) \left[-a_i \delta_{ik} + a_j \delta_{jk}\right] e^{-(a_i T_i + a_j \tilde{T}_j)} + \right. \]

\[ + \sum_j 2a_j \tilde{\tau}_j \left( A_j W a_j A_j e^{-a_j \tilde{T}_j} - A_j \tilde{W} a_j \tilde{A}_j e^{-a_j \tilde{T}_j} \right) \left] + \sum_j 2\tilde{\tau}_j \left( a_j \tilde{A}_j e^{-a_j \tilde{T}_j} (\partial_b W) + a_j A_j (\partial_b \tilde{W}) e^{-a_j \tilde{T}_j} \right) \right. \]

\[ + \frac{3\tilde{\xi}}{4V^3} \left( (\partial_b W) \tilde{W} + W (\partial_b \tilde{W}) \right) \tag{2.11} \]
\[ \frac{i}{\sqrt{2}} \left[ -\sum_j \mathcal{K}^{k\bar{j}}(a_k^2 A_k)(a_j \bar{A}_j) e^{-(a_k T_k + a_j T_j)} + \sum_i \mathcal{K}^{i\bar{k}}(a_i A_i)(a_k^2 \bar{A}_k) e^{-(a_i T_i + a_k T_k)} + 2a_k^2 \tilde{\tau}_k \left( \bar{A}_k W e^{-a_k T_k} - A_k \bar{W} e^{-a_k T_k} \right) \right]. \]  

(2.12)

In going from (2.11) to (2.12) we have dropped the third line of (2.11) as subleading. We will estimate the magnitude of these subleading terms in section 2.2.

We now change the dummy index in (2.12) from \( j \) to \( i \), and use \( \mathcal{K}^{k\bar{i}} = \mathcal{K}^{i\bar{k}} \) together with \( \frac{\partial V}{\partial b_k} = 0 \) to obtain

\[ 2\tilde{\tau}_k (\bar{A}_k W e^{-a_k T_k} - A_k \bar{W} e^{-a_k T_k}) = \sum_i \mathcal{K}^{k\bar{i}} \left( (a_i \bar{A}_i) A_k e^{-(a_k T_k + a_i T_i)} - (a_i A_i) \bar{A}_k e^{-(a_k T_k + a_i T_i)} \right) + \text{(subleading terms)}. \]  

(2.13)

As the axion does not appear (at least in perturbation theory) in the Kähler potential, its stabilisation is always entirely due to nonperturbative superpotential effects.

We next consider the stabilisation of \( \tau_k = \text{Re}(T_k) \). As stated above, our main assumption is that \( T_k \) is stabilised by superpotential effects nonperturbative in \( T_k \). Another way of stating this is to say that, when computing \( \frac{\partial V}{\partial T_k} \), the dominant contribution must arise from the superpotential term \( A_k e^{-a_k T_k} \); if this were not the case, our assumption about how \( T_k \) is stabilised is invalid. In evaluating \( \frac{\partial V}{\partial T_k} \), we therefore focus on such terms as dominant and neglect terms arising from e.g. \( \frac{\partial}{\partial \tau_k} \left( \frac{\mathcal{K}_{k\bar{j}}}{V^2} \right) \) as subdominant. We show in section 2.2 that the magnitude of the subdominant terms is suppressed by factors of \( \ln m_{3/2} \).

If we only consider superpotential terms, the calculation of \( \frac{\partial V}{\partial \tau_k} \) exactly parallels that of \( \frac{\partial V}{\partial b_k} \) above. The only differences lie in the signs, as

\[ \frac{\partial T_k}{\partial \tau_k} = \frac{\partial \bar{T}_k}{\partial \tau_k} = 1, \quad \text{whereas} \quad \frac{\partial T_k}{\partial b_k} = -\frac{\partial \bar{T}_k}{\partial b_k} = i. \]

In a similar fashion to (2.12) we therefore obtain

\[ \frac{\partial V}{\partial \tau_k} = \frac{a_k^2}{\sqrt{2}} \left[ \sum_i \mathcal{K}^{k\bar{i}} \left( -(a_i \bar{A}_i) A_k e^{-(a_k T_k + a_i T_i)} - (a_i A_i) \bar{A}_k e^{-(a_k T_k + a_i T_i)} \right) - 2\tilde{\tau}_k \left( \bar{A}_k W e^{-a_k T_k} + A_k \bar{W} e^{-a_k T_k} \right) \right] + \text{(subleading terms)}. \]  

(2.14)

Setting \( \frac{\partial V}{\partial \tau_k} = 0 \) then implies

\[ 2\tilde{\tau}_k (\bar{A}_k W e^{-a_k T_k} + A_k \bar{W} e^{-a_k T_k}) = -\sum_i \mathcal{K}^{k\bar{i}} \left( a_i \bar{A}_i A_k e^{-(a_k T_k + a_i T_i)} + a_i A_i \bar{A}_k e^{-(a_k T_k + a_i T_i)} \right) + \text{(subleading terms)}. \]  

(2.15)

We now sum (2.13) and (2.14) to obtain

\[ 4\tilde{\tau}_k \bar{A}_k W e^{-a_k T_k} = -2 \sum_i \mathcal{K}^{i\bar{k}} a_i A_i \bar{A}_k e^{-(a_k T_k + a_i T_i)} \]

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\[ -2\hat{\tau}_k W = \sum_i \mathcal{K}^{ki} a_i A_i e^{-a_i T_i} \]
\[ \Rightarrow -2\hat{\tau}_k \bar{W} = \sum_i \mathcal{K}^{ki} a_i \bar{A}_i e^{-a_i \bar{T}_i}. \] (2.16)

Comparison with equations (2.5) and (2.8) shows that there exists a leading-order cancellation in the computation of the gaugino mass. This cancellation has followed purely from the assumption that the modulus \( \tau_k \) was stabilised by non-perturbative effects: we have only required \( \frac{\partial V}{\partial \tau_k} = 0 \) and not \( D T_k W = 0 \).

In deriving equations (2.12) and (2.14) we dropped subleading terms suppressed by \( \ln \left( \frac{M_p}{m_{3/2}} \right) \). We then expect the cancellation from (2.16) and (2.8) to fail at this order, giving
\[ F^k \sim -2\hat{\tau}_k e^{K/2} \bar{W} \ln(m_{3/2}). \] (2.17)

Equation (2.3) then gives
\[ M_k \sim \frac{e^{K/2} \bar{W}}{\ln(m_{3/2})} = \frac{m_{3/2}}{\ln(m_{3/2})}, \] (2.18)
the relation we sought.

The above argument is general and model-independent. We have used the Kähler potential appropriate to an arbitrary compactification, making no assumptions about the number of moduli. Furthermore, as the result comes from directly extremising the scalar potential it is independent of whether the moduli stabilisation is approximately supersymmetric or not. Indeed, the argument above has not depended on finding a global minimum of the potential, or even on extremising the potential with respect to any of the moduli except \( T_k \). This result shows that the small hierarchy of (1.1) will exist in both KKLT and exponentially large volume approaches to moduli stabilisation. In the latter case this is possibly surprising\(^2\), as the minimum is in no sense approximately susy: each contribution to the sum in (2.8) individually gives \( M_a \sim m_{3/2} \): it is only when summed the mass suppression is obtained.

We note, as an aside, that if all moduli are stabilised by non-perturbative effects then by contracting (2.16) with \( \mathcal{K}_{jk} \) we obtain
\[ -2 \sum_k \mathcal{K}_{jk} \hat{\tau}_k \bar{W} = a_j \bar{A}_j e^{-a_j T_j}. \] (2.19)

Now, \( \mathcal{K}_j \) is homogeneous of degree -1 in \( \tau_k \), so recalling that \( \frac{\partial}{\partial \tau_k} = 2 \frac{\partial}{\partial T_k} \),
\[ \sum_k -2\mathcal{K}_{jk} \hat{\tau}_k = \sum_k -\frac{\partial \mathcal{K}_j}{\partial \tau_k} \tau_k = \mathcal{K}_j, \]
and therefore to leading order
\[ \partial_j \bar{W} + (\partial_j \mathcal{K}) \bar{W} = 0. \] (2.20)

Consequently if all moduli are stabilised by nonperturbative effects then the stabilisation is always approximately supersymmetric.

\(^2\)and was to the authors.
2.2 Subleading terms

We now want to show that the terms neglected in computing $\frac{\partial V}{\partial \tau_k}$ are all suppressed, under the assumption that the modulus is solely stabilised by nonperturbative effects. For concreteness we focus on the term in the potential

$$\sum_{i,j} \left( \frac{\kappa_{ij}}{V^2} \right) (a_i A_j) (a_j \bar{A}_j) e^{-\left( a_i T_i + a_j \bar{T}_j \right)}.$$

(2.21)

The argument used for this term will also apply to the other terms of (2.10). $\kappa_{ij}$ is homogeneous in the $\tau_k$ of degree $-1$. To see this, we note that as by dimensional analysis $V$ is homogeneous in the $\tau_i$ of degree $3/2$, $\kappa_{ij} = \frac{\partial}{\partial T_i} \frac{\partial}{\partial \bar{T}_j} (-2 \ln(V))$ is homogeneous in the $\tau_i$ of degree $-2$, and so $\kappa_{ij}$ is homogeneous in the $\tau_i$ of degree $2$. Therefore, summing over $k$,

$$\sum_k \tau_k \frac{\partial}{\partial \tau_k} \left( \frac{\kappa_{ij}}{V^2} \right) = -\frac{\kappa_{ij}}{V^2},$$

(2.22)

and so

$$\frac{\partial}{\partial \tau_k} \left( \frac{\kappa_{ij}}{V^2} \right) \lesssim \frac{\kappa_{ij}}{\tau_k V^2}.$$

(2.23)

Cosequently, differentiating (2.21) w.r.t $\tau_k$ gives

$$O \left( \frac{1}{\tau_k} \sum_{i,j} \left( \kappa_{ij} (a_i A_j) (a_j \bar{A}_j) e^{-\left( a_i T_i + a_j \bar{T}_j \right)} \right) + a_k \sum_j \left( \kappa^{kj} (a_k A_k) (a_j \bar{A}_j) e^{-\left( a_k T_k + a_j \bar{T}_j \right)} + c.c. \right) \right).$$

The basic assumption we make is that the location of the minimum for $\tau_k$ is dominantly determined by the effects nonperturbative in $\tau_k$. Therefore in the first sum we should only include the terms which depend nonperturbatively on $a_k T_k$. This gives

$$O \left( \frac{1}{\tau_k} \sum_j \left( \kappa^{kj} (a_k A_k) (a_j \bar{A}_j) e^{-\left( a_k T_k + a_j \bar{T}_j \right)} + c.c. \right) + a_k \sum_j \left( \kappa^{kj} (a_k A_k) (a_j \bar{A}_j) e^{-\left( a_k T_k + a_j \bar{T}_j \right)} + c.c. \right) \right).$$

(2.24)

We then see that the first term of (2.24) is suppressed compared to the second by a factor $a_k \tau_k$.

We note that there can exist moduli $\tau_k$ not stabilised by effects nonperturbative in $\tau_k$. This certainly holds for the volume modulus in the exponentially large volume models of [3]. Furthermore, one can argue that that in order to avoid generating a potential for the QCD axion, the modulus $\tau_k$ associated with the QCD cycle should be stabilised without using effects nonperturbative in $\tau_k$. The relationship between moduli stabilisation and the existence of a QCD axion is discussed further in [19] (also see [20]). Our argument above is restricted to the case where the modulus $\tau_k$ is stabilised by effects nonperturbative in $\tau_k$.

An identical analysis applies to the other two terms of equation (2.10). As the Kähler dependent terms are polynomials in $\tau_k$, derivatives of these with respect to $\tau_k$ also give a
suppression factor of $\tau_k$, while the derivatives of superpotential exponents are enhanced by a factor $a_k$. The latter (which we keep) are therefore larger by a factor $a_k\tau_k$ than the terms discarded.

In passing from (2.11) to (2.12) we dropped the last line of (2.11). This is self-consistent so long as $\sum_j A_j e^{-a_j\tau_j}$ is suppressed compared to $W$. In the exponentially large-volume scenario this is trivial as $e^{-a_k\tau_k} \sim \frac{1}{V}$ while $W \sim \mathcal{O}(1)$. In KKLT models, as

$$\partial_T K = -2 \frac{\partial_T V}{V} \lesssim \frac{2}{\tau_i},$$

we can use (2.20) to see that

$$\bar{W} \gtrsim (a_k\tau_k) A_k e^{-a_k\tau_k},$$

and so the third line of (2.11) is suppressed compared to the second by a factor of (at least) $a_k\tau_k$.

The above arguments imply that the terms dropped in our evaluation of $\frac{\partial V}{\partial \tau_k}$ either

1. are suppressed by a factor of $a_k\tau_k \sim \ln(m_{3/2})$ or
2. have no non-perturbative dependence on $\tau_k$.

Consequently the gaugino mass suppression found above will hold at leading order in $\frac{1}{\ln(m_{3/2})}$.

### 2.3 The uplift term

In order to attain almost vanishing but positive vacuum energy, an uplift term must also be included. In KKLT this is in a sense responsible for the soft masses, as the original AdS minimum is supersymmetric. In the exponentially large volume case the AdS minimum is already non-supersymmetric and the contribution of the uplift to soft terms is less relevant.

We take as uplift

$$V_{uplift} = \frac{\epsilon}{V^\alpha},$$

where the power $4/3 \leq \alpha \leq 2$ depends on the uplift mechanism [2, 21, 22]. Including this phenomenological uplift term, the full potential is

$$V_{full} = V_{SUGRA} + V_{uplift}.$$  

At the minimum $\langle V_{full} \rangle = 0$ and so we must have

$$\langle V_{SUGRA} \rangle = -\langle V_{uplift} \rangle.$$  

$V^{-\alpha}$ is homogeneous of degree $-3\alpha/2$ in the $\tau_i$ and so

$$\sum_k \tau_k \frac{\partial}{\partial \tau_k} V^{-\alpha} = -\frac{3\alpha}{2} V^{-\alpha},$$

implying

$$\frac{\partial}{\partial \tau_k} V^{-\alpha} \lesssim -\frac{3\alpha}{2\tau_k} V^{-\alpha}.$$  

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Thus $\frac{\partial V_{\text{uplift}}}{\partial \tau_k}$ is suppressed compared to $V_{\text{uplift}}$ by a factor of $\tau_k$. In contrast, the derivatives of $V_{\text{SUGRA}}$ involve an enhancement by $a_k$ due to the exponentials. As the two terms of (2.27) are by definition equal at the minimum, we see that

$$\frac{\partial V_{\text{SUGRA}}}{\partial \tau_k} \gtrsim a_k \tau_k \frac{\partial V_{\text{uplift}}}{\partial \tau_k}.$$  

(2.30)

This fits in with our previous analysis: the terms giving rise to the cancellation in (2.8) are the leading ones, with subleading terms suppressed by $a_k \tau_k$. We do not control the subleading terms and they will generically be non-vanishing, giving further contributions to the gaugino masses at $\mathcal{O}\left(\frac{m_3}{2 \ln(m_3/2)}\right)$.

As an aside, we note that for the exponentially large volume models

$$\frac{\partial V_{\text{uplift}}}{\partial \tau_k} \sim \frac{1}{V} V_{\text{uplift}},$$  

(2.31)

and so the presence of the uplift term does not significantly affect the stabilisation of $\tau_k$.

3. Explicit Calculation for $\mathbb{P}^4_{[1,1,1,6,9]}$

The above has established that a suppression of gaugino masses compared to the gravitino mass is generic in the landscape. We now illustrate the above with explicit calculations for compactifications on $\mathbb{P}^4_{[1,1,1,6,9]}$ in the exponentially large volume scenario.

3.1 Gaugino Masses

We calculate the gaugino masses explicitly for exponentially large volume flux compactifications on $\mathbb{P}^4_{[1,1,1,6,9]}$. We briefly recall the relevant properties of the model - a fully detailed analysis can be found in [3, 23]. As the manifold has $h^{1,1} = 2$, there are two moduli, $T_b$ and $T_s$. $T_b$ controls the overall volume and $T_s$ is a blow-up mode. At the minimum $\tau_b = \text{Re}(T_b) \gg \tau_s = \text{Re}(T_s) \sim \ln(\tau_b)$.

The Kähler and superpotential are given by

$$K = -2 \ln \left( V + \frac{\xi}{2} \right) = -2 \ln \left( \tau_b^{3/2} - \tau_s^{3/2} + \frac{\xi}{2} \right),$$  

(3.1)

$$W = W_0 + A_s e^{-a_s T_s} + A_b e^{-a_b T_b}.$$  

(3.2)

The resulting Kähler metric is (dropping terms subleading in powers of $\mathcal{V}$)

$$K_{i\bar{j}} = \begin{pmatrix} K_{bb} & K_{bs} \\ K_{sb} & K_{ss} \end{pmatrix} = \begin{pmatrix} \frac{3}{4V^{2/3}} & \frac{9\tau_b^{2}}{8V^{8/3}} \\ \frac{9\tau_s^{2}}{8V^{8/3}} & \frac{3}{8\sqrt{\tau_s} V} \end{pmatrix},$$  

(3.3)

$^3$The prefixes $b$ and $s$ indicate ‘big’ and ‘small’ moduli. It should be properly understood that this means that $\tau_b$ is an exponentially large modulus determining the overall volume, whereas $\langle \tau_s \rangle$ is smaller than $\langle \tau_b \rangle$ but still larger than the string scale, and so the low-energy 4D effective theory is justified.

$^4$For simplicity we do not include a factor of $\frac{1}{\tau_b^{2}}$ in $\mathcal{V}$: this does not affect the results.
with inverse metric
\[ K^{ij} = \begin{pmatrix} K^{bb} & K^{bs} \\ K^{sb} & K^{ss} \end{pmatrix} = \begin{pmatrix} 4V^{4/3} & 4\tau_s \tau_b \\ 4\tau_s \tau_b & 8\sqrt{s} \end{pmatrix} \tag{3.4} \]

In a limit \( V \equiv \tau_b^{3/2} - \tau_s^{3/2} \gg 1 \) with \( \tau_s \sim O(1) \), direct evaluation of the scalar potential gives (dropping terms subleading in \( V \)),
\[ V = \frac{\lambda a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s \tau_s}}{V} - \frac{\mu a_s A_s \tau_s |W_0| e^{-a_s \tau_s}}{V^2} + \frac{\nu |W_0|^2}{V^3}. \tag{3.5} \]

Explicitly, \( \lambda = \frac{8}{3} \) and \( \mu = 4 \). The minus sign in (3.5) comes from minimising the potential with respect to the axion \( b_s \). As \( \tau_b \gg 1 \) all terms nonperturbative in \( \tau_b \) vanish. Noting that \( \frac{\partial}{\partial \tau_s} (V^{-1}) \sim O\left(\frac{1}{V}\right) \), we obtain
\[ \frac{\partial V}{\partial \tau_s} = 0 \text{ and rearranging } (3.6) \text{ gives} \]
\[ e^{-a_s \tau_s} = \left( \frac{\mu}{2\lambda} \right) \frac{|W_0|}{\sqrt{\tau_s}} \left( 1 - \frac{3}{4a_s \tau_s} \right) + O\left(\frac{1}{(a_s \tau_s)^2}\right). \tag{3.7} \]

If we also solve \( \frac{\partial V}{\partial \nu} = 0 \), we obtain [3]
\[ V \sim \left| \frac{W_0}{A_s} \right| e^{a_s \tau_s} \quad \text{with } \tau_s \sim \xi^2. \]

\( V \) is exponentially sensitive to \( \xi \) (which includes \( g_s \)) and \( a_4 \) and so can take on essentially any value. A TeV scale gravitino mass requires \( V \sim 10^{14} \), which we assume.

There are gaugini associated with the moduli \( \tau_b \) and \( \tau_s \). From (2.3) these have masses \( F_b \) and \( F_s \) respectively. We calculate both masses: however we note that the small cycle is the only cycle appropriate for Standard Model matter, as a brane wrapped on the large cycle would have too small a gauge coupling. First,
\[ F^b = e^{K/2} \left( K^{bb} D_b \tilde{W} + K^{bs} D_s \tilde{W} \right) = e^{K/2} \left( -2\tau_b \tilde{W} + K^{bb} \partial_b \tilde{W} + K^{bs} \partial_s \tilde{W} \right), \tag{3.8} \]
where we have used \( K^{bb} \partial_b K + K^{bs} \partial_s K = -2\tau_b \) from (2.6). However, as \( \tau_b \sim V^{2/3} \gg 1 \), \( \partial_b \tilde{W} \sim \exp(-a_b \tau_b) \sim 0 \). Furthermore,
\[ K^{bs} \partial_s \tilde{W} \sim (4\tau_b \tau_s) \times (a_s A_s \exp(-a_s \tau_s)) \sim V^{-1/3}, \]
and so
\[ F^b = \frac{1}{V} \left( -2\tau_b W_0 + O\left(V^{-1/3}\right) \right), \tag{3.10} \]

implying
\[ |M_b| = \left| \frac{F^b}{2\tau_b} \right| = m_{3/2} + O(V^{-1/3}). \tag{3.11} \]
This is an identical relation to that of the fluxed MSSM [24].

We now calculate $M_s$. Using $K_{ij}\bar{K}_{ji} = -2\tau_i$ and $K^{\bar{s}b}\partial_b W \sim 0$, we get

$$F_s = e^{K/2} \left( K^{s\bar{s}} \partial_{\bar{s}} \bar{W} - 2\tau_s \bar{W} \right)$$

$$= e^{K/2} \left( K^{s\bar{s}} (-a_sA_s e^{-a_s\tau_s} - 2\tau_s \bar{W}) \right).$$

(3.12)

(3.13)

From (3.4), $K^{s\bar{s}} = \frac{8\sqrt{\tau_s}}{3}$. Using (3.7) we then have

$$F_s = 2\tau_s \bar{W} V \left( 1 - \frac{3}{4a_s\tau_s} \right).$$

(3.14)

We therefore obtain

$$|M_s| = 3 \frac{m_{3/2}}{4a_s\tau_s} \left( 1 + \mathcal{O} \left( \frac{1}{a_4\tau_4} \right) \right) = \frac{3m_{3/2}}{4\ln(m_{3/2})} \left( 1 + \mathcal{O} \left( \frac{1}{\ln(m_{3/2})} \right) \right),$$

(3.15)

with the expected small hierarchy. We therefore conclude that the gaugino mass associated to the exponentially large modulus $\tau_b$ is equal to the gravitino mass, whereas the gaugino mass associated to the small modulus $\tau_s$ is suppressed by $\ln(m_{3/2})$. While in the $\mathbb{P}^4_{[1,1,1,6,9]}$ case there is only one small modulus, this result will extend to more realistic multi-modulus examples. Notice that it is on the smaller cycles that the Standard Model should be accommodated since on the larger ones the gauge coupling is exponentially small and unrealistic.

### 3.2 Moduli Masses

We also here prove the existence of an enhancement by $\ln(m_{3/2})$ in the mass of the small modulus $\tau_s$ compared to $m_{3/2}$. This is similar behaviour as found in KKLT solutions [6]. This extends the simple volume scaling arguments of [23] which gave $m_s \sim \frac{M_P}{V^{3/2}}$ and $m_b \sim \frac{M_P}{V^{1/2}}$. Focusing purely on the field $\tau_s$, its Lagrangian is

$$\int d^4x K_{s\bar{s}} \partial_{\mu} \tau_s \partial^{\mu} \tau_s + V(\tau_s).$$

(3.16)

As $\tau_s$ is the heavier of the two moduli, a lower bound on its mass is given by

$$m_s^2 \gtrsim \frac{1}{2K_{s\bar{s}}} \left( \frac{\partial^2 V}{\partial \tau_s^2} \right).$$

(3.17)

Now, $K_{s\bar{s}} = \frac{3}{8\sqrt{\tau_s}V}$. If we evaluate $\frac{\partial^2 V}{\partial \tau_s^2}$, we obtain

$$\frac{\partial^2 V}{\partial \tau_s^2} = 4\lambda^4 a_s^4 \sqrt{\tau_s} e^{-2a_s\tau_s} \frac{\tau_s}{V} - \frac{\mu a_s^3 \tau_s e^{-a_s\tau_s} |W_0|}{\sqrt{2}} + \left( \text{terms suppressed by} \frac{1}{a_s\tau_s} \right).$$

(3.18)

Substituting in our evaluation of $\langle e^{-a_s\tau_s} \rangle$ from (3.7), we obtain

$$\frac{\partial^2 V}{\partial \tau_s^2} = -\frac{\mu^2}{2\lambda} \frac{a_s^3\tau_s^{3/2} |W_0|^2}{\sqrt{2}} \left( 1 + \mathcal{O} \left( \frac{1}{a_s\tau_s} \right) \right)$$

(3.19)

The large vev modulus $\tau_b$ has a mass further suppressed by a factor of $V^{1/2}$ and is therefore much lighter than $\tau_s$ [23].
Consequently
\[ m_{\tau_s}^2 \gtrsim \left( \frac{4\sqrt{\tau_s} V}{3} \right) \left( \frac{\mu^2}{2\lambda} \right) \frac{a_{2\tau_s}^{3/2}|W_0|^2}{V^3} \]
\[ = \left( \frac{2\mu^2}{3\lambda} \right) \frac{a_{2\tau_s}^{3/2}|W_0|^2}{V^2}. \] (3.20)

This gives
\[ m_{\tau_s} \gtrsim 2 \ln \left( \frac{M_P}{m_{3/2}} \right) m_{3/2} \left( 1 + O \left( \frac{1}{\ln(m_{3/2})} \right) \right). \] (3.21)

While technically a lower bound, this is actually a very good estimate of \( m_{\tau_s} \), as the canonically normalised heavy modulus has only a very small admixture of \( \tau_s \). This is confirmed by explicit numerical evaluation, which shows the formulae (3.15) and (3.21) to be accurate to within a couple of per cent.

4. Scalar Masses

The suppressed values for the gaugino masses are a direct consequence of a cancellation in the calculation for the F-terms for the ‘small’ moduli. Having suppressed F-terms could naively lead to the conclusion that the other soft terms must also be suppressed. This is not necessarily the case. For example, the general expression for scalar masses depends explicitly on the form of the Kähler potential for matter fields \( \phi \). Suppose we write

\[ K(\phi, \bar{\phi}, T_m, \bar{T}_n) = K_0(T, \bar{T}) + \tilde{K}(T_m, \bar{T}_n)\phi\bar{\phi} + \ldots, \]

where the index labelling different scalar fields has been omitted. This leads to the well known expression for scalar masses [25]:

\[ m_{\bar{\phi}}^2 = m_{3/2}^2 + V_0 - F^m F^\dagger_m \partial_m (\ln \tilde{K}). \] (4.2)

In order to get suppressed values for \( m_{\bar{\phi}}^2 \) there must be a contribution cancelling the leading \( m_{3/2}^2 \) contribution (assuming a negligible vacuum energy \( V_0 \)). In the KKLT models, this is provided by the anti-D3 brane [6]. However, this cancellation is not generic.

For the large-volume models the uplift term is subdominant in susy breaking and so has no significant effect on (4.2). As the F-terms are suppressed by \( \ln(m_{3/2}) \) for all small moduli, the only F-term that can cancel the gravitino mass contribution is that associated to the large volume modulus. The dependence of \( \tilde{K} \) on \( \mathcal{V} \) varies depending on the type of scalar field considered. To leading order we can write [26]:

\[ \tilde{K} = h(\tau_s)\mathcal{V}^{-a} \] (4.3)

with the exponent \( a \geq 0 \) taking different values for the different kinds of matter fields in the model. Here \( h \) is a flavour dependent function of the smaller moduli that in general will be very hard to compute.

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6This form assumes the matter metric is diagonal: the results below are unaffected if we use the fully general expressions [17].
We have found that the F-term contribution only cancels the leading $m_{3/2}^2$, giving scalars suppressed by $\ln(m_{3/2})$, if $a = 2/3$. This applies to D3 brane adjoint scalars and D7 Wilson lines. For adjoint D7 matter, $a = 0$ and the scalar masses are comparable to $m_{3/2}$. Of course, Standard Model matter fields are in bifundamental representations and should correspond to D3-D7 or D7-D7 matter. In this case the only calculations for $\mathcal{K}$ are in the context of toroidal orbifolds, where $0 \leq a < 2/3$. In this case there is no cancellation in (4.2) and the scalars are comparable to the gravitino mass, with positive mass squared, and heavier than the gauginos by the small hierarchy $\ln(m_{3/2})$.

This also allows us to say something about flavour universality. In the exponentially large volume scenario, the physical picture is that Standard Model matter is supported on almost-vanishing small cycles within a very large internal space ($V \sim 10^{14}$). The physics of flavour is essentially local physics which is determined by the geometry of the small cycles and their intersections. Consequently, all flavours should see the large bulk in the same way, as the distinctive flavour physics is local not global. Therefore the power of $a$ in (4.3) should be flavour-universal. The function $h(\tau_s)$, in contrast, is sensitive to the local geometry and so should not be flavour-universal.

In these circumstances we can both show universality for the soft scalar masses and also estimate the fractional level of non-universality. In the sum (4.2) the leading $m_{3/2}^2$ term and the terms involving $F^b$ are flavour-universal and give a universal contribution of $O(m_{3/2}^2)$. Universality fails due to the F-terms associated with the small moduli. We can then rewrite (4.2) as

$$m_i^2 \sim \left( m_{3/2}^2 + F^b \bar{F}^b \partial_b \partial_{\bar{b}} \ln \tilde{K} \right) + \left( \sum_s F^s \bar{F}^s \partial_s \partial_{\bar{s}} \ln \tilde{K} \right).$$

(4.4)

The $F^b F^s + F^b \bar{F}^s$ cross-terms vanish. As $F^s \sim m_{3/2}^2$ we obtain

$$m_i^2 \sim m_{3/2}^2 (1 + \epsilon_i),$$

$$\Rightarrow m_i \sim m_{3/2} \left( 1 + \frac{\epsilon_i}{2} \right),$$

(4.5)

where non-universality is encoded in $\epsilon_i \sim \frac{1}{\ln(m_{3/2})^2}$. As we require $m_{3/2} \sim 1\text{TeV}$, we estimate the fractional non-universality for soft masses as $\sim 1/(\ln(10^{18}/10^3))^2 \sim 1/1000$.

It is remarkable that these general results can be extracted despite our ignorance of the precise dependence of the Kähler potential on the matter fields. This is possible because in the above scenario flavour physics is local while supersymmetry breaking is global, and there exists a controlled expansion in $1/V$.

We also note that the large-volume scenario naturally addresses the $\mu$ problem. This is because the natural scale for any mass term, susy or non-susy, is $\mu \sim \frac{M_P}{V} \sim 1\text{TeV}$. Indeed the dilaton and complex structure moduli, which are stabilised supersymmetrically by the fluxes, do acquire masses of this order. Essentially this arises because the scalar potential has a prefactor $e^K \sim \frac{1}{V}$, which sets the natural scale for any mass term $\mu^2$. 

The phenomenology of flux compactifications has been much studied recently. The above discussions suggest new scenarios beyond those previously considered in [23,27] and the KKLT case [6–13,15,16]. The most obvious case is that of an intermediate string scale and thus a TeV scale gravitino mass, with squarks and sleptons heavy and comparable to the gravitino mass, while gauginos are suppressed by a $(\ln m_{3/2})$ factor. As the gaugino masses are suppressed, it is necessary to include anomaly mediated contributions in addition to the gravity-mediation expressions above. However as the scalar masses are heavy and comparable to the gravitino mass the contribution of anomaly mediation is in that case negligible - this is just as well given the notorious problem of tachyonic sleptons for pure anomaly mediation. It will be also interesting to analyse the phenomenology of the non-universality predicted in equation (4.5). A detailed investigation of these and related scenarios is in progress [17].

5. Conclusions and Outlook

This note has focused on the two complementary topics of gaugino and scalar masses in the IIB string landscape. We first showed that suppressed gaugino masses $M_a \sim \frac{m_{3/2}}{\ln(m_{3/2})}$ are a generic feature of the landscape and occur whenever the stationary locus of a modulus $T$ is purely determined by nonperturbative superpotential effects $e^{-aT}$. This small hierarchy has previously been identified in simple KKLT models. Our results show that this extends to models with arbitrary numbers of Kähler moduli and also to the large volume non-susy models of [3,23] in which the AdS minimum is already non-supersymmetric and the gaugino mass hierarchy is the consequence of a subtle cancellation.

For the large volume non-susy models we have verified this explicitly by performing an exact calculation of the gaugino and moduli masses for compactifications on $\mathbb{P}^4_{[1,1,1,6,9]}$, obtaining the expected suppression as well as the numerical prefactor.

We also studied soft scalar masses in the exponentially large volume scenario. An important deviation from the KKLT scenario is that in this case scalar masses are generically not suppressed compared to the gravitino mass. In this scenario there is thus a small hierarchy between the scalar and gaugino masses,

$$m_i \sim m_{3/2} \sim \ln(m_{3/2}) M_a.$$  \hspace{1cm} (5.1)

For gaugino masses, anomaly mediation and gravity mediation are therefore equally important, but for scalar masses gravity mediation dominates and anomaly mediation is suppressed by the ordinary $1/(16\pi^2)$ factor. This is interesting as it bypasses the standard problems of tachyonic anomaly-mediated scalar masses. A further difference from the KKLT scenario is that in the large-volume models the relevant high-scale for an RG analysis is the intermediate rather than GUT scale. This is because $m_{3/2} \sim \frac{M_p W_0}{V}$ while $m_s \sim \frac{M_p}{\sqrt{V}}$ and for $W_0 \sim 1$ weak-scale soft terms require $V \sim 10^{14}$, giving an intermediate string scale. In this respect it will be interesting to extend the results of [27] to include this small scalar-gaugino mass hierarchy.

Possibly the most interesting result of this paper is that the exponentially large volume scenario naturally gives approximate flavour universality. This has been one of the
main problems for gravity mediated scenarios, but in our scenario it comes out naturally. Roughly, this arises because the physics of flavour is local and hence insensitive to the dominant F-term which is that associated to the overall volume. Flavour can however see the suppressed F-terms associated with the small cycles, and the same factor of $\ln(m_{3/2})$ responsible for the F-term suppression also determines the magnitude of non-universal contributions to soft masses. As this scenario can also solve the hierarchy problem through the dynamical volume stabilisation at exponentially large volumes, we find it phenomenologically appealing. A more detailed further study of the results of this paper will appear in [17].

There is one final note of caution. In attempting to build realistic models, there are sound reasons to suppose that not all Kähler moduli are stabilised by nonperturbative effects. In particular, if all moduli were stabilised by nonperturbative effects then their axionic parts would all also be heavy and a QCD axion capable of solving the strong CP problem would not exist. If the Kähler modulus corresponding to the QCD cycle is partially stabilised through perturbative effects, it is possible that gluinos may be heavier than the remaining gauginos. It is therefore also interesting to analyse the phenomenology of a mixed scenario with an ordering $m_{3/2} \sim m_i \gtrsim m_3 > m_W \sim \frac{m_{3/2}}{\ln(m_{3/2})}$.

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