Collision of gravitational shock waves in AdS and holography

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Abstract. We study numerically the formation of marginally closed trapped surfaces as the result of the collision of two shock gravitational waves in AdS in various dimensions ($D = 4, 5, 6, 7$ and 8). In all cases a critical value of the impact parameter is found above which no trapped surface of the type sought exists. We obtain a very simple scaling between the critical impact parameter and the energy of the incoming waves. The holographic implications of our results are discussed in the context of the AdS/CFT correspondence.

1. Introduction
The observed phenomenology at RHIC, and more recently at LHC, hints at strongly coupled quark-gluon plasma being produced in heavy ion collisions. This has motivated the use of AdS/CFT duality as a non-perturbative framework to describe the relevant physics involved. In this context it has been proposed that the collision of two gravitational waves propagating in AdS5 could provide a good gravitational dual to model the formation of a quark-gluon plasma as the result of the collision of two energy lumps in the strongly-coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) on the boundary [1]. In spite of the difference between real QCD and $\mathcal{N} = 4$ SYM, the AdS/CFT calculations of some relevant observables show a good agreement with the experiments.

Here we study the collision of two gravitational shock waves in AdS$_D$. Before the collision takes place (i.e., $u < 0$ or $v < 0$), the bulk geometry in Poincare coordinates is given by

$$ds^2 = \frac{L^2}{z^2} \left[ dz^2 - du dv + \frac{z}{L} \Phi_+(z, \vec{x}) \delta(u) du^2 + \frac{z}{L} \Phi_-(z, \vec{x}) \delta(v) dv^2 \right],$$

where $L$ is the AdS$_D$ scale and $\Phi_\pm(z, \vec{x})$ are the wave profiles. For Aichelburg-Sexl waves the profiles satisfy the Poisson-like equation

$$\left( \Box_{D-2} - \frac{D-2}{L^2} \right) \Phi_\pm(z, \vec{x}) = -16\pi G_N \mu_\pm \left( \frac{z_\pm}{L} \right) \delta(z - z_\pm) \delta^{(D-3)}(\vec{x} - \vec{b}_\pm),$$

with $\mu_\pm$ the energy of the wave, and $\Box_{D-2}$ the hyperbolic space transverse to the collision. Finally, $z_\pm$ and $b_\pm$ are the impact parameters along the holographic and “field theory” coordinates respectively.

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The production of an apparent horizon as a result of the shock-wave collision can be interpreted as signaling the thermalization of the dual SYM plasma. A way to detect the formation of this apparent horizon while bypassing a full solution of the Einstein equations in the interaction region $u > 0$, $v > 0$ is to look for marginally outer closed surfaces lying on the region $\{u \leq 0, v = 0\} \cup \{u = 0, v \leq 0\}$ [2]. Notice that this region knows about the interaction between the waves because it includes the point $u = 0$, $v = 0$ common to both wavefronts.

Choosing appropriate coordinates $(U, V, Z, \vec{X})$ in which the null geodesics normal to the wavefronts are continuous, the closed trapped surface candidate $S = S_+ \cup S_-$ is parametrized by two functions $\Psi(z, \vec{x})$ as

$$
S_+ = \{(U, V, Z, \vec{X}) : U = 0, V = -\psi_+(Z, \vec{X}), \psi_+(Z, \vec{X}) \geq 0\},
$$

$$
S_- = \{(U, V, Z, \vec{X}) : U = -\psi_-(Z, \vec{X}), V = 0, \psi_-(Z, \vec{X}) \geq 0\}.
$$

(3)

Imposing the marginally trapped surface condition to the shape functions $\Psi(z, \vec{x}) \equiv \tilde{\psi}_\pm(z, \vec{x})$ we arrive at the boundary problem

$$
\left(\Box_H - D - 2 \frac{L^2}{L^2}\right) (\Phi_\pm - \Psi_\pm) = 0, \quad \Psi_\pm(z, \vec{x})|_C = 0, \quad g^{ab} \partial_a \Psi_+ \partial_b \Psi_-|_C = 4.
$$

(4)

where $C = S_+ \cap S_-$. The first equation comes from the zero-convergence condition over the congruence of outer null geodesics normal to $S_\pm$. The second one follows from the definition of $S_\pm$ in (3). The last condition comes from the continuity of the congruence of null geodesics at the intersection $C$ of the two branches.

2. Off-center collisions

The boundary problem (4) can be solved analytically in the case of head-on collisions [1]. Here, however, we will be interested in the situation where the two waves collide with a nonvanishing impact parameter, in which case the problem has to be tackled numerically. In particular, we consider off-center collisions where the impact parameter is along the “field theory coordinates”, i.e. we take $z_+ = z_- = L$ in eq. (2).

The problem at hand has an $O(D - 3)$ symmetry corresponding to rotations in $H_{D-2}$ around the axis joining the two waves sources. For convenience we use radial coordinates where the metric of $H_{D-2}$ reads

$$
ds_{D-2}^2 = \frac{dr^2}{1 + r^2/L^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\Omega_{D-4}^2.
$$

(5)

Without loss of generality we take the position of the wave sources in $H_{D-2}$ at $r = \frac{b}{2}$, $\theta = 0$ and $r = b$, $\theta = \pi$. The intersection $C$ is a $(D - 3)$-dimensional hypersurface that in these coordinates is defined by a function $r = LG(\theta)$, while the symmetry of the problem implies that $\Psi(z, \vec{x}) = \Psi(r, \theta)$. This reduces the boundary problem (4) to a two-dimensional one. In addition, for the collision of two identical shock waves we have that $\mu_+ = \mu_-$, $\Phi_+(r, \theta) = \Phi_-(r, \theta) \equiv \Phi(r, \theta)$, and therefore by symmetry $\Psi_+(r, \theta) = \Psi_-(r, \theta) \equiv \Psi(r, \theta)$.

Moreover $G(\theta) = G(\pi - \theta)$ and $\Psi(r, \theta) = \Psi(r, \pi - \theta)$.

Defining $H(r, \theta) \equiv \Phi(r, \theta) - \Psi(r, \theta)$, the boundary problem (4) reduces to

$$
\left[\left(1 + \frac{r^2}{L^2}\right) \partial_r^2 + \frac{(D - 3)L^2 + (D - 2)r^2}{rL^2} \partial_r + \frac{D - 4}{r^2 \tan \theta} \partial_\theta - \frac{D - 2}{L^2}\right] H = 0,
$$

$$
\left[\left(1 + \frac{r^2}{L^2}\right) \partial_r \Psi^2 + \frac{1}{r^2} (\partial_\theta \Psi)^2\right]_{r = LG(\theta)} = 4, \quad H_{r = LG(\theta)} = \Phi_+(LG(\theta), \theta).
$$

(6)
To solve it we use a finite difference method (with a 50×100 grid) combined with a trial-and-correction loop. First we give a trial shape for $C$, $r = r_0$, and solve the partial differential equation with the second boundary condition in (6). Next, we use the solution to compute $T(\theta) = g^{ab} \partial_a \Psi \partial_b \Psi - 4$ on $C$, and redefine the shape of $C$ according to $r = r_0 + \epsilon T(\theta)$ with $\epsilon \ll 1$. The routine is repeated until $T(\theta) \ll 1$ and convergence is found. We have carried out the numerical analysis for $D = 4, 5, 6, 7$ and 8 with various values of the incoming energy. The details can be found in ref. [3]. For $D = 5$ we reproduce the results of [4].

In fig. 1 the sections of the closed trapped surfaces are shown for various values of the impact parameter in $D = 7$. Our results show that for all explored dimensions a critical impact parameter is always found. That is, for each energy there exists a value $b_c$ such that no trapped surface is formed whenever $b > b_c$, whereas for $b = b_c$ a critical surface forms with finite size. In fig. 2 the values of $b_c$ as a function of $\mu$ are plotted for various dimensions. The results are very well fit by the following scaling of the critical impact parameter with the energy

$$\frac{b_c}{L} \sim \left( \frac{G_N \mu}{L^{D-3}} \right)^\frac{1}{D-2},$$

where the proportionality constant is of order one. For large values of the dimension this scaling implies the independence of the critical impact parameter with the shock wave energy.

From the boundary field theory point of view, the situation analyzed in this section corresponds to two relativistic energy lumps colliding with the same energy and impact parameter $b$. Assuming the existence of the trapped surface signals the eventual formation of an event horizon in the bulk, the critical behavior observed might be interpreted as indicating the absence of thermalization in the CFT plasma resulting from the collision for large enough values of the impact parameter.
3. Collisions between energy lumps of different size

In the field theory on the boundary the size of the incoming lumps is determined by the value of the holographic coordinate of the source as can be seen from

\[ \frac{\int d^{D-2}x \tilde{x}^2 \langle T_{uu} \rangle_{\text{CFT}}}{\int d^{D-2}x \langle T_{uu} \rangle_{\text{CFT}}} = z_+^2, \]  

(8)

and equivalently for the second wave replacing \( \langle T_{uu} \rangle_{\text{CFT}} \) by \( \langle T_{vv} \rangle_{\text{CFT}} \) and \( z_+ \) by \( z_- \). Thus collisions with \( z_+ \neq z_- \) are the dual descriptions for head-on collisions between energy lumps of different sizes.

The isometries of AdS can be exploited to link the computation of last section to collisions of gravitational waves with \( z_\pm \neq z_- \) [3]. In particular, using these symmetries the collision of two shock waves with \( z_\pm = L(\sqrt{1 + \beta^2} \pm \beta)^{-1} \) and \( b_\pm = 0 \) can be “rotated” into a symmetric one with \( z_\pm'' = L \) and \( b_\pm'' = \mp L\beta \), with \( \beta \) a real parameter. This is done in the following steps:

- An O(2) rotation of angle \( \theta \) in the plane \( XY \) (cf. fig. 1). This moves the sources to

\[ z_\pm' = \frac{L}{\sqrt{1 + \beta^2 \pm \beta \cos(\theta)}}, \quad b_\pm' = \frac{L\beta \sin(\theta)}{\sqrt{1 + \beta^2 \pm \beta \cos(\theta)}}. \]

(9)

For \( \theta = \frac{\pi}{2} \) the two sources have the same value of the holographic coordinate \( z_\pm' = L(1 + \beta^2)^{-\frac{1}{2}} \). Because of the UV/IR connection in AdS/CFT this rotation changes the energy of the shock waves to \( \mu_\pm = \mu_\pm \sqrt{1 + \beta^2 (\sqrt{1 + \beta^2} \pm \beta)}^{-1} \).

- A longitudinal boost \( \bar{u} = \mu_\pm \bar{u}, \bar{v} = \lambda^{-1}v \), with \( \lambda = (\mu_+/\mu_-)(\sqrt{1 + \beta^2} - \beta)(\sqrt{1 + \beta^2} + \beta)^{-1} \). This results in the two shock waves having the same energy \( \bar{\mu}_\pm = \sqrt{\mu_+ \mu_-} \).

- Finally, a coordinates rescaling \( z'' = z' \sqrt{1 + \beta^2} \) and \( \bar{x} = \bar{x}'' \sqrt{1 + \beta^2} \). The point of this last isometry is to eliminate the dependence of the holographic coordinate on the parameter \( \beta \), so we are left with \( z''_\pm = L \) and \( b''_\pm = \mp L\beta \). The energy of the waves is now given by

\[ \mu''_\pm = \sqrt{\mu_+ \mu_-} (1 + \beta^2)^{-\frac{1}{2}}. \]

The analysis of the formation of closed trapped surfaces for the collision of unequal lumps can be now carried out by applying in reverse order this series of AdS isometries to the results of the previous section [3]. This leads to the conclusion that there is a critical value for the size difference \(|z_+ - z_-|\) below which no trapped surface is formed. Physically this can be interpreted as indicating that a too small energy lump does not have enough degrees of freedom to induce thermalization after a collision with a big one. The fact that this depends only on the size difference is a consequence of conformal invariance.

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