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Evaluation of reference temperature in the pseudo density model for solving internal natural convection of gases

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Abstract. This paper attempts to investigate a numerical solution for internal natural convection in gases. There are two kinds of models used to handle buoyancy force in the internal natural convection problems, the pseudo density difference model and the direct model. It was shown that pseudo-density-difference model is originally a solution for external natural convection. However, when this model is adopted to solve the internal natural convection problem the reference temperature must be treated properly. The advantage of using pseudo-density-difference model is less memory in the computational process issued. The direct model was used to evaluate the appropriate reference temperature for pseudo density difference model. Calculations for air in a two dimensions square cavity, which is bounded by adiabatic horizontal planes and isothermal vertical planes, with temperature difference $\Delta T$ and Rayleigh numbers $10^3 < Ra < 10^5$ were carried out. The conclusion of this research is the pseudo density difference model reveals better results when the average temperature is chosen as the reference temperature.

1. Introduction

Internal natural convection of gases is a common phenomenon in engineering applications. Noticeable examples include solar-collectors [1 – 21], double-wall insulations, and air circulations through the room in a building. Even the natural circulation in the atmosphere can be explained clearly using the internal natural convection. Based on those facts the internal natural convection has been an interesting topic to be investigated. In the internal natural convection fluid flow and transfer of heat are induced by buoyancy force. From the study of literatures, we found that there are two kinds of models that have been reported in order to treat the buoyancy force, pseudo density difference model and direct model.

Investigation of analytical approach to natural convection started with pioneering work of Schmidt and Bechmann [22]. They investigated natural convection around a vertical heated plate and proposed reduced-pressure to produce density difference as a buoyancy force. The occurrence of the density difference is based on the logical representation of the vertical pressure gradient in the quiescent environment outside the boundary layer. They employed Boussinesq approximation to transform the density difference of the gas temperature difference. The aforementioned treatment of the buoyancy force will be repeated in the following paragraph.
Gravitational force, as a body force, generally will be present in the $y$-direction of momentum equation ($y$-being positive upwards). In steady state form and no others source term, the $y$-direction of momentum equation can be written as

$$u \frac{\partial (\rho v)}{\partial x} + v \frac{\partial (\rho v)}{\partial y} = - \frac{\partial P}{\partial y} + \mu \nabla^2 \cdot v - \rho g$$

(1)

The reduced pressure ($P'$) is defined as local pressure minus hydrostatic pressure of gas outside of the boundary layer.

$$P' = P - P_s$$

(2)

Where

$$P_s = -\rho g y$$

(3)

Hydrostatic pressure is calculated by using the density at the reference temperature. When equation (3) is substituted into equation (2) and differentiated, the result is

$$-\frac{\partial P}{\partial y} = -\frac{\partial P'}{\partial y} + \rho_s g$$

(4)

If equation (4) is substituted into equation (1), the density difference will appear as

$$u \frac{\partial (\rho v)}{\partial x} + v \frac{\partial (\rho v)}{\partial y} = - \frac{\partial P}{\partial y} + \mu \nabla^2 \cdot v - (\rho_s - \rho) g$$

(5)

The density difference in equation (5) is known as the pseudo-density difference [22].

The Boussinesq approximation was employed to transform the pseudo-density difference into temperature difference. If the fluid behaves according to the ideal gas model [24] at any level of density can be written as

$$\rho = \frac{P}{RT} \quad \text{and} \quad \rho_s = \frac{P_s}{RT_s}$$

(6)

Algebra manipulation of equation (6) yields

$$\frac{\rho_s - \rho}{\rho_s} \left(1 - \frac{\rho_s - \rho}{\rho_s} \right)^{-1} = \frac{T - T_s}{T_s}$$

(7)

Which in the limit

$$(T - T_s) \ll T_s$$

(8)

Equation (7) yields to

$$\rho = \rho_s \left[1 - \frac{1}{T_s} (T - T_s) + ...\right]$$

(9)

Defining $\beta$ as the volume expansion coefficient at constant pressure, equation (9) can be written as

$$\rho = \rho_s \left[1 - \beta (T - T_s) + ...\right]$$

(10)

Substituting equation (10) into equation (5) yields

$$u \frac{\partial (\rho v)}{\partial x} + v \frac{\partial (\rho v)}{\partial y} = - \frac{\partial P}{\partial y} + \frac{\partial P'}{\partial y} + \mu \nabla^2 \cdot v + \frac{\beta g \rho_s (\rho_s - \rho)}{\rho_s}$$

(11)

Assume $\rho$ is equal in the whole equation.

$$\rho_s \approx \rho$$

(12)

Finally, equation (11) can be written in a simpler form:

$$u \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial P'}{\partial y} + \frac{\mu}{\rho} \nabla^2 \cdot v + \frac{\beta g \rho_s (T - T_s)}{\rho_s}$$

(13)

The temperature of fluid in quiescent environment outside the boundary layer is stated as reference temperature. The advantage of employing equation (13) is that all of the transport properties, included
density, are constant and evaluated at the reference temperature in the entire equation. This results in using less memory in the computational process.

The analytical solution of Schimedt and Bechmann [22], originally, is a solution for the external natural convection problem, but many researchers [23-31] have adopted this original model to solve natural convection problems for gas in enclosure. However, in the internal natural convection problems since there is no quiescent environment outside the boundary layer the reference temperature must be stated carefully.

Some researchers [23-29] kept employing equation (13) and stated the colder temperature in computational domain as a reference temperature. The other researches [30, 31] adopted the pseudo density difference but did not employ the Boussinesq approximation. They used equations (5) and employed the ideal gas law, instead of Boussinesq approximation, to transform pseudo density difference into temperature difference. The reference-density is assumed to be constant in [30] and an exponentially varying function of the vertical coordinate in [31]. These numerical solutions took more computational effort because density must be calculated at each iteration process. The paper [31] reported that their calculated Nusselt numbers is different from the previous work [23-25]. They also investigated the pressure effect and reported that the pressure affects the calculated Nusselt number strongly.

The paper [34] proposed a new solution, without using the pseudo density difference model, to solve internal natural convection problem. They employed momentum equation directly, equation (1), and used ideal gas law to calculate density variation in whole governing equations. Thermo-physical properties \( k \) and \( \mu \), respectively the thermal conductivity and the viscosity, were constant and evaluated at the average temperature. This model of solution was known as the direct model because no algebra manipulation was employed.

There is no doubt that employing the direct model to solve internal natural convection provides the best numerical solution, however this model since density is variable, it increases the computational effort considerably. From the computational effort point of view, there is no value to using the pseudo density difference with the ideal gas law, instead of using the Boussinesq approximation. The advantage of employing equation (13) is to save computational effort, but the problem is determination of the reference temperature. There are two strict conditions which must be satisfied when the reference temperature is imposed. The reference temperature is supposed to be close to all local temperatures, which then satisfies equation (8), and the assumption in equation (12).

Imagine the natural convection problem of gas in a two-dimensional cavity which is bounded adiabatic horizontal surfaces and isothermal vertical surfaces. One of the vertical surfaces is fixed at a cold temperature and the other is fixed at a hot temperature. The fluids in entire computational domain can be divided into two parts, the fluids with temperature close to the cold temperature and the fluids close to the hot temperature. When the cold temperature is stated as reference temperature the fluids with temperature close to the cold temperature will agree well with equation (8) but not the fluids with temperature close to the hot temperature. Consequently equation (14) will not be satisfied and the numerical result will lead to a big error. It is true that if the difference of the cold temperature and hot temperature is small enough this numerical error can be reduced but this kind of reference temperature should not be employed.

The best reference temperature is the average temperature of all computational domains. When the average temperature is imposed as the reference temperature, it does mean the temperatures of fluid in the entire computational domains are relatively close to the reference temperature. This is true even when the difference of the temperature of the vertical walls is quite large, therefore equation (8) will be satisfied and consequently equation (12) is also satisfied. The objective of the present study is to numerically investigate the effect of employing the different reference temperature in the pseudo density difference model. The result is expected to supply a necessary consideration of solving numerically internal natural convection if pseudo density difference model is employed. To acquire this objective four of models of numerical solution were carried out. The models are explained as:
a. Model 1, the standard pseudo-density difference which is coupled with Boussinesq approximation is employed. The colder temperature was used as the reference temperature. All of the thermal properties are constant and evaluated at the reference temperature.

b. Model 2, the standard pseudo-density difference which is coupled with Boussinesq approximation is employed. Equation (11) is employed (not equation (13)). The buoyancy force used colder temperature as a reference temperature but other thermal properties, $\rho$, $\mu$, and $c_p$ in the governing equations are evaluated at an average temperature.

c. Model 3, the standard pseudo-density difference which is coupled with Boussinesq approximation is employed. The average temperature was used as the reference temperature. All of the thermal properties are constant and evaluated at the reference temperature.

d. Model 4, directly using governing equation such as equation (1) and the appearance of buoyancy force is the gravitational force. All of the thermal properties are variable. Density variation is a function of local pressure and temperature and is calculated by using ideal gas law. The other thermo-physical properties $k$, $\mu$, and $c_p$ now depend on temperature and are respectively calculated by using special equations which are obtained by curve-fitting from the air table of properties.

2. Mathematical Formulation

Since the purpose of this study is to enhance the understanding of choosing the reference temperature, it is helpful to select the simplest geometry problem that still contains the relevant physics. The physical situation of the problem is depicted in Figure 1. The square cavity with dimension $L$ is bounded by two adiabatic horizontal walls and two isothermal vertical walls. The isothermal walls are maintained at a uniform colder temperature $T_c$ and a hotter temperature $T_h$. Fluid inside the cavity is dry air.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(14)
\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{1}{\rho} \frac{\partial p'}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= \frac{1}{\rho} \frac{\partial p'}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\rho g}{\rho} g (T - T_c) \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\end{align*}
\]

Since all of the thermal properties are variable, the governing equations for model 4 are as follows.

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]

\[
\frac{\partial (\rho u)}{\partial x} + v \frac{\partial (\rho u)}{\partial y} = - \frac{\partial p'}{\partial y} + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial x} \right) + \frac{\partial u}{\partial y} \left( \frac{\partial u}{\partial y} \right) \right) + S_u
\]

\[
\frac{\partial (\rho v)}{\partial x} + v \frac{\partial (\rho v)}{\partial y} = - \frac{\partial p'}{\partial y} + \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \left( \frac{\partial v}{\partial x} \right) + \frac{\partial v}{\partial y} \left( \frac{\partial v}{\partial y} \right) \right) - \rho g + S_v
\]

\[
\frac{\partial (\rho c_p u T)}{\partial x} + v \frac{\partial (\rho c_p v T)}{\partial y} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right)
\]

Where \( S_u \) and \( S_v \) are source terms according to transport properties different.

\[
S_u = \frac{1}{3} \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} - 2 \frac{\partial v}{\partial y} - \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) \right)
\]

\[
S_v = \frac{1}{3} \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial x} - 2 \frac{\partial u}{\partial x} - \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \right) \right)
\]

In the present study all of the governing equations did not change into non-dimensional forms. Thermophysical properties \( k, \mu, \) and \( c_p \) depend on temperature (K) were calculated by using the following equations:

\[
\mu = \frac{1.458 \times 10^{-7} T^{1.5}}{110.4 + T}
\]

\[
\mu = \frac{2.6482 \times 10^{-5} T^{0.5}}{1 + \frac{245.4 \times 10^{22/7}}{T}}
\]

\[
c_p = 9 \times 10^{-7} (T - 273)^3 - 7 \times 10^{-4} (T - 273)^2 + 2.134(T - 273) + 977.51
\]

Since air behaves according to the ideal gas law the density at any condition, depends on temperature and pressure. This was evaluated based on density \( \rho_o \) at initial condition 273K and atmospheric pressure \( p_o \).

\[
\rho = \frac{P}{\rho_o \times \frac{273}{T}}
\]

In order to compare the results of model 1, 2, and 3 against model 4 the average heat transfer coefficient, average Nusselt numbers, and heat lines are used. To estimate the heat transfer rate at the isothermal walls, average heat transfer coefficient and average Nusselt numbers are used.

\[
\bar{h} = \frac{1}{L_o} \int_{h \times dy}
\]
\[
\bar{N}_u = \frac{\bar{h}L}{k}
\]

Where local heat transfer coefficient is calculated by
\[
h = \frac{-k_{\text{wall}} \times \partial T / \partial x}{(T_h - T_c)}
\]

To visualize the flow of fluid in the computational domain the well-known streamlines function was used, but to visualize the energy flow the heat function was used. The aim of the heat function is to satisfy the energy equation. The heat function is defined as \([34]\):
\[
\frac{\partial H}{\partial y} = \rho c_p u (T - T_c) - k \frac{\partial T}{\partial x}
\]
\[
\frac{\partial H}{\partial x} = \rho c_p v (T - T_c) - k \frac{\partial T}{\partial y}
\]

3. Numerical Procedure

A FORTRAN code program to solve all of the governing equations have been developed. The governing equations were solved by using staggered grid system. In this staggered grid system pressure, temperature, density and all thermal properties \((k, \mu, \text{and} c_v)\) are placed on the centre of the grid. When these parameters are wanted at the faces of the grid the harmonic mean interpolation \([33]\) was employed. The velocities component, \(u\) and \(v\), are placed at the faces of the grid. Linear interpolation was used when the velocities are wanted in the centre of the grid.

Methods of deriving the discretization equations based on the control-volume formulation. To handle convective and diffusion terms the power law scheme was employed. The sets of discretization equations were solved by using line-by-line method which is combined with the Thomas algorithm. The well-known SIMPLE algorithm \([36]\) was employed to coupling pressure, velocities field and temperature field. In the SIMPLE algorithm under-relaxation factor is essential and it is better to use minimal values of under-relaxation factor for momentum equations \([32]\). In this present study, the under-relaxation factor for momentum equations is about 0.02, for temperature it is about 0.5, and for thermal properties calculation, only for model 4, is about 0.8. This numerical study employed non-uniform grids, which grid size becomes smaller closer to the wall. To ensure that the results are independent of the grid size, a grid sensitivity analysis was carried out. Calculations with different number of grids \(30 \times 30\), \(40 \times 40\), \(50 \times 50\), and \(60 \times 60\) were carried out and the \(50 \times 50\) numbers of grid were selected.

3.1. Numerical validation

To validate the developed numerical method a preliminary calculation was made and results were compared with the previous work. By using this present method, the problem that has been investigated by Davish \([23]\) and Hortmann \([24]\) was calculated again. Since Davish \([23]\) and Hortmann \([34]\) used \(T_c\) as reference temperature, their models are equivalent to the model 1 of this present study. Dimensions and the temperature of the square cavity are: \(L = 0.045841\text{m}, T_c = 12^\circ\text{C}, T_r = 2^\circ\text{C}\), and the air properties are: \(\rho = 1.19\text{kg/m}^3, \mu = 1.8 \times 10^{-5}\text{kg/s/m}, \text{Pr}=0.71, \text{and} \beta = 0.00341\text{K}^{-1}, g = 9.81\text{m/s}^2\). In this preliminary calculation, all of the parameters are same except the grid dimensions. The results are shown in the following Table 1. Data of Table 1 shows that the results of this developed program, even though it is only the result of model 1, still agree well with the previous work. There is a little bit of difference in those data of Table 1 but those differences are not essential and probably happened because of the different grid size or round-off in the computational process. Based on this successfully validation, these developed numerical methods are applied to solve and analyze the internal natural convection problem.
Table 1. Comparison of the general parameters at Ra=10^5

| Reference (grids) | Davis [23] (80x80) | Hortmann [24] (80x80) | Present Study (50x50) |
|------------------|--------------------|-----------------------|-----------------------|
| \( \bar{N}_v \)  | 4.510              | 4.52556               | 4.515                 |
| \( U_m \)        | 34.81              | 34.7132               | 34.02                 |
| \( Y_m \)        | 0.855              | 0.85665               | 0.8625                |
| \( V_m \)        | 68.22              | 68.5383               | 68.54                 |
| \( X_m \)        | 0.066              | 0.06902               | 0.06875               |

4. Result and Discussion

Numerical calculations for all models were carried out. To make sure that flow is laminar, \( Ra \leq 10^5 \), for the given temperature difference the dimension of the square cavity \( L = 0.05m \) was selected. The colder temperature was fixed at \( T_c = 0^\circ C \). Each calculation was made for the variation of the hotter temperature from 0.1 \( ^\circ C \) until 200 \( ^\circ C \). Because direct model (model 4) employs no manipulation in handling buoyancy force, the result of model 4 should be the closest to the theoretical solution of the problem. Based on this fact the calculated heat transfer coefficient from model 1, 2, and 3 were compared with result of model 4. Rayleigh numbers, average heat transfer coefficient, and the differences of the average heat transfer coefficient from model 4 are showed in Table 2.

Table 2 shows that the difference of the average heat transfer coefficient for all models, model 1, model 2, and model 3, are greater if the hotter temperature is greater. However, for small temperature difference the differences of the average heat transfer coefficients are small. When the discrepancies are ordered, model 2 is the biggest, followed by model 1, and model 3 respectively. These results are already expected. Since thermal properties and buoyancy force are evaluated at different reference temperature the result of model 2 provided the biggest deviation. Model 1 also provided a significant deviation, as expected, because colder temperature was chosen as the reference temperature. Since this reference temperature is not close to all temperatures in the entire computational domain, the limitation and assumption in equation (8) and equation (12) will not be satisfied and the results lead to a larger numerical error.

Table 2. Comparison of the average Heat Transfer Coefficient

| \( T_h \) (°C) | Ra      | Model-1 | Diff.(%) | Model-2 | Diff.(%) | Model-3 | Diff.(%) | Model-4 | Diff.(%) |
|--------------|---------|---------|----------|---------|----------|---------|----------|---------|----------|
| 0.1          | 1.82E+03 | 0.621   | 0.0      | 0.621   | 0.0      | 0.621   | 0.0      | 0.621   | 0.0      |
| 0.5          | 9.08E+03 | 1.053   | 0.0      | 1.053   | 0.0      | 1.053   | 0.0      | 1.053   | 0.0      |
| 1            | 1.81E+04 | 1.306   | 0.077   | 1.306   | 0.077   | 1.305   | 0.0      | 1.305   | 0.0      |
| 4            | 7.05E+04 | 1.983   | 0.303   | 1.985   | 0.405   | 1.976   | 0.051   | 1.977   | 0.0      |
| 8            | 1.36E+05 | 2.440   | 0.702   | 2.444   | 0.867   | 2.423   | 0.000   | 2.423   | 0.0      |
| 14           | 2.27E+05 | 2.878   | 1.195   | 2.886   | 1.477   | 2.843   | 0.035   | 2.844   | 0.0      |
| 20           | 3.08E+05 | 3.193   | 1.656   | 3.206   | 2.069   | 3.140   | 0.032   | 3.141   | 0.0      |
| 50           | 6.06E+05 | 4.147   | 3.805   | 4.196   | 5.031   | 3.993   | 0.050   | 3.995   | 0.0      |
| 80           | 7.72E+05 | 4.730   | 5.817   | 4.823   | 7.897   | 4.466   | 0.089   | 4.470   | 0.0      |
| 100          | 8.34E+05 | 5.030   | 7.090   | 5.157   | 9.793   | 4.693   | 0.085   | 4.697   | 0.0      |
| 120          | 8.70E+05 | 5.289   | 8.359   | 5.449   | 11.637  | 4.876   | 0.102   | 4.881   | 0.0      |
| 150          | 8.89E+05 | 5.621   | 10.172  | 5.835   | 14.367  | 5.095   | 0.137   | 5.102   | 0.0      |
| 175          | 8.82E+05 | 5.862   | 11.657  | 6.120   | 16.571  | 5.241   | 0.171   | 5.250   | 0.0      |
| 200          | 8.64E+05 | 6.078   | 13.121  | 6.381   | 18.760  | 5.362   | 0.205   | 5.373   | 0.0      |

The results of model 3 provided the smallest discrepancy. This can be happened, because this model employs average temperature as the reference temperature, as stated earlier, consequently reference
temperature is close enough to all local temperatures in the entire computational domain. Even for large temperature differences, about 200°C, model-3 gave the difference of only 0.2% from the direct model. Figure 2 reveals the average heat transfer coefficient for all models as a function of temperature difference. From the graphic it can be seen that model 3 agrees well with model 4. Some researchers [30, 31] kept employing pseudo-density difference approximation but replaced Boussinesq approximation with the ideal gas law. The reason of doing this is the incredulity that at a large temperature difference, the Boussinesq approximation will lead to a large deviation. However this effort will increase the computational effort. This incredulity is correct when the colder temperature is chosen as the reference temperature. Figure 2 shows that if the colder temperature is chosen as the reference temperature, as in model 1, the result agree well with direct model only for temperature difference of less than 20K. For the temperature difference more than 20K discrepancy is large and of course cannot be eliminated. When the reference temperature is treated properly, the average temperature as the reference temperature (as in model 3), results agree well with the direct model. Even for a large temperature difference up to 200K, the difference of the average heat transfer coefficient is still less than 0.25% compared to the direct model. A further discussion of the effect of the different reference temperature in pseudo density difference model is provided by an examination of the temperature fields inside the cavity. From aforementioned discussion, the average heat transfer coefficient of model 2 provided a largest deviation, therefore only the temperature field of model 1, model 3, and model 4 are presented in Figure 3. There are many results of calculated temperature fields but for the present purpose only one of these calculation results are used, that is the case at the hotter temperature 50°C. The isotherm contour temperature of model 1, model 3, and model 4 are expressed as red line, blue line, and black line respectively.

At the first glance, all models provide the same pattern but when carefully observed the difference will be found out. The isotherm contour line of model 3 and model 4 almost coalesce in entire computational domain. It can be seen the isotherm contour line of model 1 differ with model 3 and model 4. This difference can be seen on the top and bottom of the cavity, in Figure 3 represented by circle.

![Figure 2. Average Heat Transfer Coefficient](image)
The next discussion is the calculated streamlines at the same condition $T_c = 0^\circ C$ and $T_h = 50^\circ C$. Streamlines of model 1, model 3, and model 4 are expressed as red line, blue line, and black line respectively and presented in Figure 4. Observation of the figure reveals virtually no significant difference in the flow pattern of model-3 and model-4. The streamlines next to the walls are almost the same for model 3 and model 4. The streamlines of model 1, again, are different than model 3 and model 4. The difference can be seen clearly around the center of the cavity. These differences represented by circle in Figure 4.

Heatlines results of model 1, model 3, and model 4 again are expressed as red line, blue line, and black line respectively and depicted in Figure 5. This figure reveals the heatlines pattern of model-3 and model 4 again is almost same. The equality of heatlines pattern of model-3 and model-4 can be seen.
clearly on the area close to the wall. The heatlines pattern of model 1 is different than model 3 and model 4 and the difference is revealed in Figure 5 by circle.

Figure 5. Heatlines at \( T_c = 0^\circ C \) and \( \Delta T = 50 K \)

A numerical calculation by using model-4 with the different pressure inside the cavity, to investigate the influence of the pressure was carried out. In this calculation, the equation to evaluate thermal properties, respectively \( k, \mu \), and \( c_p \), are the same as the previous calculation because the reference [34] said that the values of \( k, \mu \), and \( c_p \) are not strongly pressure-dependent and may be used over a fairly wide range of pressure. After the calculations, there is no influence of the inside pressure on the heat transfer coefficient, the streamlines, the heatlines pattern, and the temperature distribution.

5. Conclusions
The numerical prediction to internal natural convection of gas by using well known pseudo density difference which is coupled with Boussinesq approximation has been investigated. It was shown that the pseudo density difference model originally is a solution to the external natural convection. Adoption at the pseudo density difference model to analyse internal natural convection will give the reasonable result as long as the reference temperature is treated properly. A direct model has been used to evaluate the appropriate reference temperature for the pseudo density difference model. When the average of the temperatures of the walls which bound the cavity is chosen as a reference temperature, the result of the pseudo density difference model and the result of the direct model will agree well. Even for the wide temperature difference of the walls the pseudo density difference model will provide the satisfactory result. The effect of the pressure inside the cavity to heat transfer rate was investigated also. Pressure inside the cavity does not affect the heat transfer coefficient and fluid flow.

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