Spin–orbit interaction and weak localization in heterostructures

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Abstract

Theory of weak localization in two-dimensional high-mobility semiconductor systems is developed with allowance for the spin–orbit interaction. The obtained expressions for anomalous magnetoresistance are valid in the whole range of classically weak magnetic fields and for arbitrary strengths of bulk and structural inversion asymmetry contributions to the spin splitting. The theory serves for both diffusive and ballistic regimes of electron propagation taking into account coherent backscattering and nonbackscattering processes. The transition between weak localization and antilocalization regimes is analyzed. The manifestation of the mutual compensation of Rashba and Dresselhaus spin splittings in magnetoresistance is discussed. A perfect description of experimental data on anomalous magnetoresistance in high-mobility heterostructures is demonstrated. The in-plane magnetic-field dependence of the conductivity caused by an interplay of the spin–orbit splittings and Zeeman effect is described theoretically.

1. Introduction

The electron, being a quantum object, manifests both particle and wave properties while propagating in a solid. In the state-of-the-art semiconductor heterostructures where the electron mean free path $l$ caused by scattering from remote impurities, phonons and interface imperfections exceeds by far the electron wavelength $\frac{2\pi}{k_F}$ ($k_F$ is a Fermi wave vector) the electron transport is known to be primarily classical and described by Drude theory. However, a quantum nature of an electron is clearly demonstrated by a variation of the conductivity at low temperatures with a small magnetic field perpendicular to the system plane. Low-field magnetoresistance is caused by the weak localization of electron waves: a particle can propagate via different paths, and among them there is a number of self-crossing paths with loops. An electron can pass a loop by two trajectories: clockwise and counter-clockwise, which leads to a constructive interference resulting in the increase of the return probability and the decrease of the conductivity. Due to unusual field dependence this phenomenon is known as anomalous negative magnetoresistance or positive magnetoconductivity [1].

Crucial effect on the above picture makes a spin–orbit interaction. If it is sufficiently strong, the electronic waves also interfere after passage the loops in two opposite directions, but this interference is destructive. As a result, the probability of return is smaller than the classical value, so the conductivity correction is positive. A perpendicular magnetic field destroys this interference as well but in this case it leads to a decrease of conductivity, i.e. to positive magnetoresistance. Since the situation is totally opposite to the spinless case, the interference effect in the presence of spin–orbit interaction is called weak antilocalization.

In semiconductor heterostructures the spin–orbit interaction is described by the following Hamiltonian [2]

$$H(k) = \hbar \mathbf{\sigma} \cdot \mathbf{\Omega}(k),$$

(1)

where $k$ is the electron wave vector, $\mathbf{\sigma}$ is the vector of Pauli matrices, and $\mathbf{\Omega}(k)$ is an odd function of $k$. The spin splitting due to the spin–orbit interaction (1) equals to $2\hbar \mathbf{\Omega}(k)$.

Theory of weak-antilocalization-induced alternating magnetoresistance has been developed by Pikus et al in the middle of 1990s [3, 4]. It had successfully described weak-antilocalization experiments on available in that time low-mobility heterostructures [5]. However the obtained
expressions are valid only for weak spin–orbit interaction and very low magnetic fields. The former assumption means that \( \Omega \tau \ll 1 \), where \( \tau \) is the scattering time, and the latter condition reads as \( l_B \gg l \), where \( l_B = \sqrt{\hbar/ceB} \) is the magnetic length. This so-called ‘diffusive’ regime takes place in fields \( B \ll B_t \), where

\[
B_t = \frac{\hbar}{2eI^2}
\]

is the ‘transport’ field at which the magnetic length \( l_B \) equals the mean free path \( l \). In the theory [3, 4] \( B_t \) is assumed to be infinitely large which is a good approximation for low-mobility samples.

However, starting from the early 2000s, anomalous magnetoresistance measurements are being performed on high-mobility samples in different laboratories around the world, see e.g. [6–9]. The motion of the particle on the trajectories relevant for the interference in such systems is ballistic rather than diffusive. The field \( B_t \) is small in these structures being less than 1 mT. The characteristic magnetoresistance maximum occurs at \( B > B_t \), i.e. out of the range of applicability of the theory existed that time. Moreover the systems started to appear having large spin–orbit splitting (1) and long scattering times so the product \( \Omega \tau \) is even larger than unity. The question to theory was sharply raised after publication of the paper by Studenikin et al [7]: it has been demonstrated that both low-field and high-field parts of the magnetoresistance curve can be fitted by the theory [3] but with absolutely different sets of fitting parameters. After paper [7] it became especially clear that a new theory of weak localization is required.

Such theory has recently been developed in [10, 11]. The obtained expressions for the magnetoconductivity are valid in the whole range of classically weak magnetic fields and for any values of the spin splittings, i.e. for arbitrary values of \( B/B_t \) and \( \Omega \tau \). The theory takes into account low symmetry of [001] grown heterostructures where both bulk and structure inversion asymmetry contributions to \( \Omega(k) \) in equation (1) with linear and cubic in the wave vector terms coexist. This theory opened a possibility to describe anomalous magnetoresistance experiments and to extract adequately spin-splitting and kinetic parameters of high-mobility two-dimensional (2D) semiconductor systems.

In the magnetic field normal to the heterostructure plane only orbital effects are important in the anomalous magnetoresistance while the Zeeman splitting plays no role. Another interesting possibility opens up in the case of the magnetic field applied in the plane of the heterostructure. In such a case the orbital effect of the magnetic field is relatively unimportant while the Zeeman splitting dramatically affects weak localization. In strong in-plane magnetic fields the Zeeman effect completely overcomes spin–orbit effect and restores the spin–orbit-less value of the quantum conductivity correction. However in the intermediate regime the magnetoresistance is formed as a result of interplay of the Zeeman and spin–orbit splittings. The in-plane magnetoresistance is not sufficiently studied at present.

In this paper we describe the weak-antilocalization theory for high-mobility 2D semiconductor systems and present the expressions for anomalous magnetoconductivity valid in the whole range of classically weak fields and for arbitrary large spin–orbit splitting. We demonstrate that this theory perfectly describes the experimental data on anomalous magnetoresistance of high-mobility heterostructures. We also put forward a theory describing quantum corrections to the conductivity due to an interplay of spin–orbit effects and Zeeman effect of arbitrary strengths caused by an in-plane magnetic field.

2. Theory

There are two contributions of different nature to the spin–orbit interaction Hamiltonian (1) in 2D semiconductor systems: the Rashba term \( \Omega_R \) and the Dresselhaus term \( \Omega_D \). In heterostructures grown along the direction \( z \parallel [001] \) both vectors \( \Omega_R \) and \( \Omega_D \) lie in the 2D plane. The Rashba term contains only first angular harmonics of the wave vector, while the Dresselhaus term contains both first and third harmonics. They have the following form

\[
\begin{align*}
\Omega(k) &= \Omega^{(1)}(k) + \Omega^{(3)}(k), \\
\Omega^{(1)}_R(k) &= \Omega_R(\sin \varphi, -\cos \varphi), \\
\Omega^{(1)}_D(k) &= \Omega_D(\cos \varphi, -\sin \varphi), \\
\Omega^{(3)}(k) &= \Omega_D(3\cos \varphi, \sin 3\varphi),
\end{align*}
\]

where \( \varphi \) is an angle between \( k \) and the axis \( x \parallel [100] \).

We consider low magnetic fields

\[
\omega_c \ll \Omega \sim \tau^{-1} \ll E_F/\hbar,
\]

where \( \omega_c \) is the cyclotron frequency, and \( E_F \) is the Fermi energy, i.e. those fields where the cyclotron motion of electrons is unimportant. The conductivity correction due to weak localization is given by a sum of two terms

\[
\sigma(B) = \sigma_a + \sigma_b,
\]

where \( \sigma_a \) and \( \sigma_b \) can be interpreted as backscattering and nonbackscattering interference corrections to conductivity. In the case of isotropic scattering they are given by [10, 11]

\[
\begin{align*}
\sigma_a &= -\frac{e^2}{2\pi^2\hbar} \left( \frac{l_B}{l} \right)^2 \left\{ \text{Tr}[A^2(I - A)^{-1}] - \sum_{N=0}^{\infty} \frac{P_N}{1 - P_N} \right\}, \\
\sigma_b &= \frac{e^2}{\pi^2\hbar} \left( \frac{l_B}{l} \right)^2 \left\{ \text{Tr}[(k^2 A(I - A)^{-1})^2] - \frac{1}{4} \sum_{N=0}^{\infty} Q_N^2 \frac{P_N}{1 - P_N} + \frac{P_{N+1}}{1 - P_{N+1}} \right\},
\end{align*}
\]

where

\[
\begin{align*}
P_N &= \frac{l_B}{l} \int_0^\infty dx \exp \left( -x \frac{l_B}{l} - \frac{x^2}{2} \right) L_N(x^2), \\
Q_N &= \frac{1}{\sqrt{N + 1}} \frac{l_B}{l} \int_0^\infty dx \exp \left( -x \frac{l_B}{l} - \frac{x^2}{2} \right) x L_N^2(x^2),
\end{align*}
\]

with \( l = l/(1 + \tau/\tau_d) \), \( \tau_d \) being the dephasing time, \( L_N^2 \) are the associated Laguerre polynomials, and \( I \) is the unit operator. The matrix elements of the operators \( A \) and \( K \) in the basis
of Landau levels of a particle with a charge $2e$ and in the representation of the total momentum of interacting particles $S$ ($S = 1$, the momentum projection $m = 1, 0, -1$) are given by

$$A(N, m; N', m') = \int d^2 R \frac{\exp(-R/l)}{2\pi R l} F_{NN'}(R) \times \langle m'|[2\pi \sigma S \cdot \omega(R)]|m\rangle.$$

(5)

Here $\omega(R) = \Omega(kn)R/l$, where $n = (\cos \vartheta, \sin \vartheta)$ is a unit vector pointing along $R$, and $t = R/l_B$. The expression for matrix elements $K(N, m; N', m')$ are different from equation (5) by the additional factor $i \cos \vartheta$ in the integrand. In equations (3) and (4) the trace, $Tr$, is a sum of matrix elements with $m = m'$ and $N = N$.

3. Results and discussion

In this section we consider consequently the weak antilocalization caused by isotropic spin–orbit splitting, interplay of Rashba and linear Dresselhaus splittings, cubic in $k$ Dresselhaus splitting, and present comparison of the theory with experimental data.

3.1. Isotropic spin splitting

First we discuss the Rashba or linear Dresselhaus spin–orbit interaction dominance. In both cases the spin splitting $2\hbar\Omega(k)$ is isotropic in $k$-space. This makes possible a partial diagonalization of the operators $A$ and $K$. For Rashba spin–orbit interaction, it takes place in the basis of the states $|N, m\rangle$ with equal $N + m$: $|N + 2, 1\rangle, |N - 1, 0\rangle, |N, -1\rangle$, while for Dresselhaus term this takes place for the states with equal $N - m$. In both cases we have [11]

$$\sigma_{\alpha} = \frac{-e^2}{2\pi^2 \hbar} \left( \frac{l}{l_B} \right)^2 \sum_{N=0}^\infty \left[ Tr \left[ A_N^\dagger (I - A_N)^{-1} \right] - \frac{p_N^3}{1 - p_N} \right],$$

(6)

$$\sigma_{\beta} = \frac{e^2}{4\pi^2 \hbar} \left( \frac{l}{l_B} \right)^2 \sum_{N=0}^\infty \left[ Tr \left[ K_N^T K_N A_N (I - A_N)^{-1} \right] + Tr \left[ K_N^T K_N A_N (I - A_N)^{-1} \right] \right.\nonumber \left. - Q_N^2 \left( \frac{P_N}{1 - P_N} + \frac{P_{N+1}}{1 - P_{N+1}} \right) \right].$$

(7)

Here $I$ is a $3 \times 3$ unit matrix,

$$A_N = \begin{pmatrix} P_{N-2} - S_{N-2}^{(0)} & R_{N-2}^{(1)} \ & S_{N-2}^{(2)} \\ R_{N-2}^{(1)} & P_{N-1} - 2S_{N-1}^{(0)} & R_{N-1}^{(2)} \ & S_{N-1}^{(3)} \\ S_{N-1}^{(2)} & R_{N-1}^{(2)} & P_N - S_N^{(0)} \ \end{pmatrix},$$

(8)

$$K_N = \begin{pmatrix} Q_{N-2} - S_{N-2}^{(1)} & R_{N-2}^{(2)} \ & S_{N-2}^{(3)} \\ -R_{N-2}^{(2)} & Q_{N-1} - 2S_{N-1}^{(1)} & R_{N-1}^{(3)} \ & S_{N-1}^{(4)} \\ S_{N-1}^{(3)} & R_{N-1}^{(3)} & Q_N - S_N^{(1)} \ \end{pmatrix}.$$}

(9)

Note that the values with negative indices appearing in the above equations for $A_N$ and $K_N$ at $N = 0$, 1 should be replaced by zeros.

Equations (6) and (7) yield the weak-antilocalization correction to the conductivity in the whole range of classically weak magnetic fields and for arbitrary values of $\Omega \tau$. In the limit of zero spin splitting,

$$\sigma_{\alpha} = \frac{-e^2}{2\pi^2 \hbar} \left( \frac{l}{l_B} \right)^2 \sum_{N=0}^\infty \frac{p_N^3}{1 - p_N},$$

(10)

$$\sigma_{\beta} = \frac{e^2}{2\pi^2 \hbar} \left( \frac{l}{l_B} \right)^2 \sum_{N=0}^\infty Q_N^2 \left( \frac{P_N}{1 - P_N} + \frac{P_{N+1}}{1 - P_{N+1}} \right).$$

(11)

Equations (10) and (11) were obtained as results of non-diffusive theory developed for $\Omega = 0$ in [12]. In a magnetic field $B \gg (\Omega \tau)^2 B_c$, the conductivity is independent of $\Omega$, and it is also described by equations (10) and (11). The reason is that in a so strong field the dephasing length due to magnetic field $\sim l_B$ is smaller than that due to spin–orbit interaction, $l/\Omega \tau$. As a result, the particle spins keep safe at characteristic trajectories. The conductivity for any finite $\Omega \tau$ has the zero-$\Omega$ asymptotic. For $\Omega \tau < 1$ this dependence is achieved at $B \lesssim B_c$. In high magnetic field $B \gg B_c$, $(\Omega \tau)^2 B_c$, the conductivity correction has the high-field asymptotic [12]

$$\sigma_{\text{id}}(B) = -0.25 \sqrt{\frac{B_c}{B}} \frac{e^2}{\hbar}.$$ (12)

In figure 1 the conductivity correction is plotted for different strengths of spin–orbit interaction. The nonmonotonous dependence $\sigma (B)$ can qualitatively be explained by noting that the spin state of two interfering electrons can be either triplet or singlet [1]. Indeed, the singlet configuration corresponding to the total spin $S = 0$ is unaffected by the spin–orbit interaction while the triplet contribution is suppressed. These singlet states contribute to the conductivity correction with opposite signs: singlet contribution is positive while triplet one is negative. The small magnetic field suppresses the singlet term only, leading to the decrease of the conductivity while in higher fields both singlet and triplet states are suppressed. Therefore, the magnetococonductivity in high fields is positive. As a result the conductivity as a function of the magnetic field is non-monotonic with a minimum at a certain value of the field, $B_{\text{min}}$.

Figure 1 shows that for $\Omega \tau \lesssim 1$, $\sigma (B)$ coincides with the zero-$\Omega$ dependence for $B > B_{\text{min}}$. The asymptotic $\sigma_{\text{id}}(B)$ is
reached at \( B \approx 100B_n \) for all presented values of \( \Omega \tau \). The positions of minima in the curves are shown in the inset. One can see that \( B_{min} \) almost linearly depends on the spin splitting at \( \Omega \tau > 0.8 \). Fitting yields the following approximate law

\[
B_{min} \approx (3.9 \Omega \tau - 2)B_n.
\]

In the limit \( \Omega \tau \rightarrow \infty \) one can see a decrease of conductivity in the whole range of magnetic fields. At \( B > B_n \), the correction tends to zero as 0.035 \( e^2/h \sqrt{B_n/B} \).

### 3.2. Interplay of Rashba and Dresselhaus terms

Here we study the effect of interference of Dresselhaus and Rashba spin–orbit interactions on weak localization. We assume that, in the effective field (2), the first angular harmonics \( \Omega^{(1)}(k) \) prevail over \( \Omega^{(3)}(k) \).

In the presence of both \( k \)-linear Dresselhaus and Rashba spin splittings the system has \( C_{2v} \) point symmetry which does not allow even a block diagonalization of the operators \( A \) and \( K \). In this case the calculations are performed numerically by using equations (3)-(5) (see [11] for details).

Figure 2 shows the dependence of the weak localization correction to the conductivity on magnetic field assuming that the magnitude of the first harmonics of the Dresselhaus term is constant, \( \Omega_D \tau = 1 \). Different curves refer to different ratios between Rashba and Dresselhaus contributions (\( \Omega_R/\Omega_D = 0, 0.5, 0.7, 0.85 \) and 1.0).

We begin analyzing the results with the case of \( \Omega_R = \Omega_D \). In this limit, the energy spectrum splits into two independent paraboloids. Each of them provides the universal contribution to the magnetoresistance, so the total correction is the same as in the absence of spin–orbit interaction, which is described by equations (10) and (11). The effective field \( \Omega(k) \) in this situation points along the fixed axis; therefore the spin rotation angle on the closed loop is zero. The absolute value of the quantum correction to the conductivity steadily increases. In high fields \( B/B_n > 1 \), the correction is described by the asymptotic expression (12).

With unequal Rashba and Dresselhaus terms, the rotation angle of the spin at closed paths is no longer zero. This yields (i) a smaller magnitude of the correction and (ii) an alternative magnetoresistance. Since, for a given path, the rotation angle of the spin is larger the larger is the quantity \( \Omega_R^2/\Omega_D^2 \), a decrease in the Rashba term (at a fixed Dresselhaus term) manifests itself as an enhancement of the spin–orbit coupling. In fact, as is evident from figure 2, the depth of the minimum increases with decreasing ratio \( \Omega_R/\Omega_D \), and the minimum itself shifts to higher magnetic fields. These results are in qualitative agreement with the diffusion theory [4]. In contrast to [4], the theory developed here provides a correct asymptotic behavior of the quantum correction to the conductivity at \( B > B_n \): irrespective of the quantity \( \Omega_R/\Omega_D \), all curves approach the same dependence (12).

### 3.3. Cubic in momentum splitting

Here we discuss the situation where the third harmonic \( \Omega_D^{(3)}(k) \) is the only one in the energy spectrum. This situation is relevant to \( p \)-type heterostructures and, to a large extent, to the case of the equal first harmonics of Dresselhaus contribution and Rashba contribution (\( \Omega_R = \Omega_D \)).

If \( \Omega_D^{(3)}(k) \) is dominant then the spin splitting is isotropic in \( k \)-space. Therefore, the operators \( A \) and \( K \) are again separated into blocks of sizes 3 \( \times \) 3. The third harmonic of Dresselhaus contribution mixes the states with equal values of \( N + 3m \). The corresponding expressions for the quantum correction to the conductivity are given in [11].

Figure 3 shows the dependences of the conductivity correction on the magnetic field, calculated for different
3.4. Comparison with experiment

The theory developed in [10, 11] has been successfully applied for description of experimental data on anomalous magnetoresistance in various high-mobility heterostructures. For the first time it has been done in the work by Guzenko et al [13] where the parameter $\Omega$ as well as the temperature dependence of the dephasing time $\tau_0$ have been determined from the fitting of the magnetoconductivity curves. Note that the experimental data were fitted in a temperature range 0.8–4 K by the same parameter $\Omega \approx 0.5$ for magnetic fields $B \leq 8 B_n$ while the minimum took place at $B \approx 2 B_n$. The extracted spin–orbit splitting $\approx 1$ meV is a reasonable value for the Rashba splitting for the studied 2D electrons in InGaSb heterostructures. Note that this splitting could not be determined e.g. by beatings in the Shubnikov–de Haas oscillations because it has an order of $\hbar/\tau$. At the same time the anomalous magnetoconductivity had a pronounced minimum allowed extraction of the Rashba spin splitting.

Later on, the theory has been applied independently to different experimental groups allowing characterization of electron spin properties in various 2D semiconductor heterostructures (see, e.g. [14–18]). In the work by Yu et al [18] the structures with an electric gate were investigated with allowance for a change of concentration and mobility of 2D electrons. The results of this study are summarized in figure 4. The left panel of figure 4 shows the conductivity correction as a function of magnetic field plotted for different temperatures at a fixed gate voltage. All curves approach the same zero-$\Omega$ dependence (12). With increasing $\Omega$, the minimum of the conductivity shifts to higher fields. The depth of the minimum behaves nonmonotonically: with increasing spin splitting, it increases at small $\Omega$ and decreases at large $\Omega$.

magnitudes of the third harmonic of Dresselhaus term, with the first harmonic $\Omega_1(k)$ equal to zero. Qualitatively, the form of the dependences is consistent with the results for the spin splitting described by the first harmonic, figure 1, but the range of variation of $\sigma(B)$ is about an order of magnitude larger. In the magnetic fields $B \gg \max[(\Omega_1)^2, 1]B_n$, all curves approach the same zero-$\Omega$ dependence (12). With increasing $\Omega$, the minimum of the conductivity shifts to higher fields. The depth of the minimum behaves nonmonotonically: with increasing spin splitting, it increases at small $\Omega$ and decreases at large $\Omega$.

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4. Effect of an in-plane magnetic field

Magnetic field applied in the plane of the quantum well affects an interference of electrons in a two-fold way. First, inevitable fluctuations of the quantum well width caused by the imperfections of interfaces lead to the fluctuations of the magnetic flux through the cross-section of electron wavefunction. This leads to dephasing similar to the case of the perpendicular field [19]. This effect is purely orbital, and it acts in the same manner on all spin states of interfering electrons. The ‘microroughness’ effect can be taken into account by a proper renormalization of dephasing time, $\tau_\phi$.

The other possibility for an in-plane magnetic field to affect weak localization or antilocalization of electrons is the Zeeman effect. The corresponding term in the electron Hamiltonian is

$$H_Z = \frac{\hbar}{2} \bar{\sigma} \cdot \Delta,$$

where $\Delta = \tilde{g} \mu_B B_1 / \hbar$, $B_1$ is the in-plane magnetic field, $\mu_B$ is the Bohr magneton, and $\tilde{g}$ is the in-plane electron Landé-factor tensor. For simplicity we consider here the situation where only isotropic spin splitting is present ($\Omega_k \neq 0$, $\Omega_D = 0$).

An in-plane magnetic field admixes triplet states of the interfering electron pair to the singlet one, thus causing a dephasing of the singlet state [19, 20]. Theory [19] describes anomalous magnetoresistance in the presence of in-plane field [21]. However, the expressions obtained in [19, 20] cover only the case of small spin splittings and relatively weak magnetic fields: $\Omega \tau \ll 1$, $\Delta(\tau_\phi)^{1/2} \ll 1$. Below we present a general theory describing an in-plane magnetoresistance for the arbitrary values of the spin–orbit and Zeeman splittings.

The conductivity corrections in this case are given by

$$\sigma_\alpha(B_1) = - \frac{e^2}{2\pi \hbar^2} \int \frac{d^2q}{(2\pi)^2} [\mathcal{P}(I - \mathcal{P})^{-1}]_{\alpha\beta\mu\nu},$$

$$\sigma_\beta(B_1) = \frac{e^2}{\pi \hbar^2} \int \frac{d^2q}{(2\pi)^2} \mathcal{Q}_{\mu\nu\gamma\delta} [\mathcal{P}(I - \mathcal{P})^{-1}]_{\gamma\delta\nu\alpha}.$$  \hspace{1cm} (15)

Here

$$\mathcal{P}(q) = \int d^2R \frac{\exp(-R/l)}{2\pi RI}$$

$$\times \exp [i(q \cdot R - 2i\mathbf{S} \cdot \omega(R) - i\mathbf{L} \cdot \Delta)],$$

and the operator $\mathcal{Q}(q)$ differs by an additional factor $iX/R$ in the integrand (16). $L$ is the operator of spin difference of interfering particles. Greek subscripts $\alpha \ldots \nu = \pm 1/2$ enumerate spin states of the interfering particles, the summation over repeated subscripts is assumed. It is worth noting that the Zeeman effect in interference is determined by difference of spins since Zeeman splitting is an even function of the wave vector, similar to the effect of longitudinal-transverse splitting of exciton polaritons [22]. Therefore singlet and triplet contributions are mixed by the in-plane magnetic field. This is a manifestation of $C_6$ point symmetry of the system in the presence of both $B_1$ and Rashba spin–orbit interaction.

In contrast, for Zeeman splitting due to the field $B_z$ applied perpendicularly to the 2D plane the conductivity corrections have the simplified form similar to equations (6) and (7) but with matrices of sizes $4 \times 4$. Partial diagonalization of the operators $\mathcal{P}$ and $\mathcal{Q}$ is possible in this case because the presence of the $B_z$ component does not break an in-plane isotropy of the energy spectrum. Formally this can be seen from equation (16) noting that the operator $L_z$ conserves the sum $N + m$.

Before presenting numerical results let us briefly discuss the situation of small spin–orbit splittings of the conduction band and high Zeeman splitting: $\Omega \tau \ll 1, \Delta \tau \gg 1$. In the absence of the spin–orbit interaction, Zeeman effect splits the electron energy spectrum into two independent parabolas. Provided that $\Delta \tau \gg 1$ the difference of their occupations and relaxation times can be neglected, and each parabolid yields the same universal contribution to the quantum conductivity correction. As a result the latter equals the spin–orbit-less value given by

$$\sigma(0) = - \frac{e^2}{2\pi^2\hbar} \ln \left( \frac{\tau_\phi}{2\tau} \right).$$

Small spin–orbit interaction leads to the spin relaxation and mixing of parabolics. However, at $\Delta \tau \gg 1$ and $\Delta \gg \Omega$ Zeeman effect of the magnetic field quenches spin relaxation completely [23]; therefore even in the presence of the spin–orbit interaction the in-plane magnetoconductivity approaches at high magnetic field $B_1$ the spin–orbit-less value (17).

The quantum corrections to the conductivity calculated by equations (14) and (15) are shown in figure 5. Different curves correspond to the different values of spin–orbit splitting. The dependences start from the zero-field values of conductivity correction at a corresponding spin splitting [10] and, at $\Delta \tau \gg 1$, tend to the spin–orbit-less value equation (17). In small in-plane fields, $\Delta \tau \gg 1$, the conductivity correction decreases because the singlet contribution is suppressed due to an admixture of triplet states. This situation is described by the theory developed in [19, 20]. However, in the intermediate area $\Delta \sim \Omega$ the spectrum of interfering states is rearranged. It results in the non-monotonous dependence of the conductivity correction on the magnetic field. First, Zeeman splitting suppresses the singlet thus decreasing the conductivity. With an increase of $\Delta$, triplet states start to be suppressed as well and the conductivity increases. At very high fields the Zeeman splitting completely overcomes the spin–orbit splitting and restores the spin-less situation thus decreasing the conductivity again. This results in the presence of minimum and maximum in the dependence $\sigma(B_1)$ at $\Delta \sim \Omega$, figure 5, absent in the theories [19, 20].
It is noteworthy that contrary to [19, 21] the nonmonotonous behavior of the conductivity correction takes place due to the interplay of spin splittings only. The nonmonotonous behavior can be manifested in the absence of the orbital (‘microroughness’) effects. The minimum and maximum of magnetococonductivity shown in figure 5 are most in smaller fields $B \lesssim 1$ T, when $\Delta \sim (\tau_0 k_0)^{-1/2} \ll \tau^{-1}$ [21].

Recently the diffusion theory has been applied for the calculation of a low-field magnetoconductivity of thin quantum wires [24]. The theory developed here allows one to describe adequately the weak localization effect even in ballistic structures with dimensions comparable with the mean free path.

5. Conclusion

To summarize, we have presented a theory of quantum conductivity corrections valid for the state-of-the-art semiconductor heterostructures. The theory is applicable in the whole range of classically weak magnetic fields allowing for both the diffusive and ballistic carrier propagation. It takes into account all possible contributions to the spin splitting of the electron energy spectrum such as linear in the wave vector Rashba and Dresselhaus terms and cubic in the wave vector Dresselhaus term. We have demonstrated that in the case of equal Rashba and Dresselhaus terms the magnetococonductivity shows monotonic behavior contrary to the general case of unequal Rashba and Dresselhaus contributions or dominant cubic-in wave vector splitting. The theory is shown to be in the excellent agreement with the recent experimental data.

We have also studied the magnetococonductivity in the in-plane field. Although its orbital effects are relatively unimportant, the Zeeman effect of the field enters in the competition with the conduction band spin–orbit splitting. We predict in high-mobility heterostructures an alternating magnetococonductivity due to the interplay of various spin splittings.

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