AGV vehicle differential drive model

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Abstract. Based on the establishment and analysis of the position model and forward kinematics model of the AGV vehicle, this paper derives the motion trajectory and related motion parameters of the fixed standard wheel and the manipulated standard wheel, and analyzes the motion of the AGV vehicle based on the specific motion parameters, to obtain the constraint sum of the motion and rolling and sliding of the differential AGV vehicle, and the relationship between the wheel speed of the AGV vehicle.

1. Introduction
Deriving a model for the entire AGV vehicle movement is a bottom-up process. Each wheel contributes to the motion of the AGV vehicle, and at the same time imposes constraints on the motion of the AGV vehicle [1-2]. According to the geometric characteristics of the chassis of the AGV vehicle, multiple wheels are connected together. Therefore, their constraints are combined to form a constraint on the entire moving part of the chassis of the AGV vehicle. Due to its independent and mobile nature in mobile AGV vehicle science, it is necessary to establish a clear and consistent mapping between the global and local reference frames to express the forces and constraints of each wheel.

2. Location model of AGV vehicles
In order to determine the position of the AGV vehicle on the plane, as shown in Figure 1, the relationship between the plane global frame and the local frame of the AGV vehicle is established. Axis XI and YI define any inertial basis on the plane as a global reference frame starting from the origin O: {XI, YI}. In order to determine the position of the AGV vehicle, a point P on the chassis of the AGV vehicle is selected as its position reference point. Based on {XR, YR}, the two axes on the chassis of the AGV vehicle relative to point P are defined, thereby defining the local reference frame of the AGV vehicle. In the local frame of reference, P's position is determined by the coordinates x
and y, and the angular difference between the global reference frame and the local reference frame is given by \( \theta \).

**Figure 1.** Meshing diagram of front end face.

Describe the attitude of the AGV vehicle as a vector with these 3 elements. Use the subscript I to clarify that the basis of the attitude is the global reference frame:

\[
\begin{bmatrix}
  x_I \\
  y_I \\
  \theta_I
\end{bmatrix}
\]

(1)

In order to describe the movement of the AGV vehicle based on the movement of the components, it is necessary to map the movement along the global frame to the movement along the local reference frame of the AGV vehicle [3]. The mapping is the current position function of the AGV vehicle. The mapping is described by an orthogonal rotation matrix:

\[
R(\theta) = \begin{bmatrix}
  \cos \theta & \sin \theta & 0 \\
  -\sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]

(2)

Therefore, the motion in the local reference frame can be expressed as:

\[
\dot{\xi}_R = R(\theta) \dot{\xi}_I
\]

(3)
Given a certain speed in the global frame of reference, you can calculate the motion component along the AGV vehicle's local axis using the following formula:

\[
\begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x}_R \\
\dot{y}_R \\
\dot{\theta}_R
\end{bmatrix}
\]

From the formula (3):

\[
\begin{bmatrix}
\dot{x}_R \\
\dot{y}_R \\
\dot{\theta}_R
\end{bmatrix}
= \begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix}
= \begin{bmatrix}
x_I \cos \theta + y_I \sin \theta \\
x_I \sin \theta - y_I \cos \theta \\
0
\end{bmatrix}
\]

(4)

For example: when AGV vehicles are side by side with global coordinates \( \theta = \frac{\pi}{2} \), as shown in Figure 2. The instantaneous rotation matrix \( R(\frac{\pi}{2}) \) can be calculated by formula 2:

\[
R(\frac{\pi}{2}) = \begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(5)

From equation (4), when \( \theta = \frac{\pi}{2} \), the instantaneous speed of the global reference frame is:

\[
\begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix}
= \begin{bmatrix}
x_R \\
y_R \\
\dot{\theta}_R
\end{bmatrix}
\]

Local instantaneous speed of the AGV vehicle at time.
It can be seen from the calculation result that when \( \frac{\pi}{2} \), the moving speed along is equal to \( y_I \), and the moving speed along is \( -x_I \).

3. Forward kinematics model of AGV vehicle

The purpose of studying the forward kinematics model of AGV vehicle is to calculate how the AGV vehicle moves given the geometric characteristics of the AGV vehicle and the speed of its wheels [4]. Two-wheel differential speed model, the radius of the wheels is, the length of each wheel from the center point P of the two-wheel spacing is, given, \( r \), and the speed of each wheel \( \phi_1 \) and \( \phi_2 \), the forward kinematics model will predict the total AGV vehicle in the global reference frame speed:

\[
\begin{bmatrix}
\dot{x}_R \\
\dot{y}_R \\
\dot{\theta}_R
\end{bmatrix}
= R\left(\frac{\pi}{2}\right) \begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix}
\]

From equation (3), it can be seen from the motion of the AGV vehicle in the local reference frame to calculate its motion in the global reference frame:

\[
\begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix}
= f(l, r, \theta, \phi_1, \phi_2)
\]

(6)
First calculate the contribution of each round in the local framework: $\ddot{\xi}_R$.

As shown in Figure 1, it is assumed that the local reference frame of the AGV vehicle is arranged so that the AGV vehicle moves forward. First consider the contribution of the rotation speed of each wheel in the direction to the translational speed of point $P$. If one wheel rotates and the other wheel does not contribute and does not move, then because $P$ is in the middle of the two wheels, it will instantly move at half speed: $\dot{x}_{r1} = \frac{1}{2}r_1\dot{\phi}_1$ and $\dot{x}_{r2} = \frac{1}{2}r_2\dot{\phi}_2$. In a differentially driven AGV vehicle, these two contributions can be added together to calculate $\ddot{\xi}_R$ the component of $X_R$. For example: In a differential AGV vehicle, where the wheels rotate at a constant speed but in opposite directions, the result is to rotate along the ground, in this case $X_R$ will be zero. Because the AGV vehicle does not have a wheel to provide lateral movement in the local reference frame, $Y_R$ is always zero. Study the right wheel (wheel 1), which rotates forward and rotates counterclockwise at $P$. If wheel 1 rotates alone, the pivot of the AGV vehicle revolves around wheel 2, and the rotation speed can be calculated at point $P$, because the wheel instantaneously moves along an arc of radius $2l$:

$$\omega_1 = \frac{r_1\dot{\phi}_1}{2l}$$  

The same calculation is applied to the left wheel, except that the forward rotation produces a clockwise rotation at point $P$,

$$\omega_2 = \frac{r_2\dot{\phi}_2}{2l}$$  

Combine equations (8) and (9) to obtain the motion model of the AGV vehicle for the differential drive example:

$$\dot{\xi}_I = R(\theta)^{-1} \ddot{\xi}_R$$  

$$R(\theta)^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Substituting $[\cos(\alpha + \beta) \sin(\alpha + \beta) l \sin \beta] R(\theta) \dot{\xi}_1 = 0$ into (10), we get: (11)
For example: when the AGV vehicle is located at \( \theta = \frac{\pi}{2} \), \( r = 1 \) and \( l = 1 \). If the AGV vehicle engages its wheels unbalanced, speed \( \varphi_1 = 4 \), \( \varphi_1 = 2 \), it can calculate its speed in the global reference frame:

\[
\begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I \\
\end{bmatrix} =
\begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_I \\
y_I \\
\theta_I \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
3 \\
1 \\
\end{bmatrix}
\]

(12)

Under this condition, the AGV vehicle will move along the y-axis of the global frame of reference, rotating at speed 1 and moving instantaneously at speed 3.

4. Wheel movement constraints

The steps to establish the motion model of the AGV vehicle are:

- First express the constraints imposed on individual wheels;
- Combine the movements of the individual wheels to get the movement model of the entire AGV vehicle.

4.1. Fixed standard wheel

The fixed standard wheel does not use a steerable vertical rotation axis. Therefore, it is relatively fixed at the angle of the chassis, thus restricting back and forth movement along the wheel plane and rotation around contact with the ground. Figure 3 depicts the fixed wheel A, and illustrates its position and posture relative to the local reference frame \( \{X_R, Y_R\} \) of the AGV vehicle. The position of A is represented by the length \( l \) and the angle \( \alpha \) in polar coordinates. The angle of the wheel plane relative to the chassis is represented by \( \beta \), because the fixed standard wheel is not steerable, so \( \beta \) is fixed. A wheel with a radius of \( r = 1 \) can rotate with it, so the position at which it rotates around its horizontal axis is a function of time \( \varphi(t) \).:
The rolling constraint of the wheel forces all the movement of the wheel in the plane direction to be accompanied by an appropriate amount of wheel rolling, so that there is pure rolling at the contact point:

\[ \sin(\alpha + \beta) - \cos(\alpha + \beta)(-l) \cos \beta R(\theta) \dot{\xi} - r \dot{\varphi} = 0 \]  

(13)

The first term of the sum represents the total movement along the plane of the wheel. For movement along the wheel plane, the third element of the left vector represents the transformation from each \( x, y, \theta \) to their contribution. Note that as shown in the example equation (5), we use the term \( R(\theta) \dot{\xi} \) to transform the reference \( \dot{\xi}_R \) in the global reference frame \( \dot{\xi}_I \) to the motion reference \( \{X_I, Y_I\} \) in the local reference frame \( \{X_R, Y_R\} \). This is necessary because all other references in the equation \( \alpha, \beta, l \) are based on the local reference frame of the AGV vehicle. According to this constraint, the movement along the wheel plane \( \dot{\xi}_R \).

The sliding of the wheel is also restricted, forcing the wheel motion component orthogonal to the wheel plane to be zero:

\[ \cos(\alpha + \beta) \sin(\alpha + \beta) l \sin \beta |R(\theta)| \dot{\xi}_1 = 0 \]  

(14)

For example, suppose wheel A is in a position such as \( \{\alpha = 0, \beta = 0\} \), which will place the contact point on \( X_I \) in the wheel plane facing parallel to \( Y_I \). If \( \theta = 0 \), the sliding constraint can be expressed as:

---

**Figure 3.** Fixed standard wheel and its parameters.
\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{\theta} \end{bmatrix} = [1 \ 0 \ 0] \begin{bmatrix} x \\ y \\ \dot{\theta} \end{bmatrix} = 0 \]

4.2. Steered standard wheel
The standard difference between the steered standard wheel and the fixed wheel is that the former adds an additional degree of freedom: the wheel passes through the center of the wheel and the ground contact and rotates around vertically. The position equation of the steered standard wheel (Figure 4), unlike the fixed wheel, the direction of the standard wheel set AGV vehicle is no longer fixed to \( \beta \) alone, but a function of time change \( \beta(t) \).

Figure 4. The controlled standard wheel and its parameters.

The rolling and sliding constraints are:
\[
\begin{align*}
\{ \sin[\alpha + \beta(t)] & - \cos[\alpha + \beta(t)] \} ( -l ) \cos \beta(t) R(\theta) \ddot{z} - \dot{r} & = 0 \quad (15) \\
\{ \cos[\alpha + \beta(t)] & \sin[\alpha + \beta(t)] \} l \sin \beta(t) R(\theta) \ddot{z} & = 0 \quad (16)
\end{align*}
\]

5. AGV vehicle motion constraints
Given an AGV vehicle with M wheels, the kinematic constraints of the chassis of the AGV vehicle can be calculated. The main idea is that each wheel adds zero or more constraints to the motion of the AGV vehicle, so the process is just to properly link all the kinematic constraints caused by all wheels according to the configuration of those wheels on the AGV vehicle ground.

The fixed standard wheel and the operable standard wheel have an influence on the kinematics of the AGV vehicle chassis. Therefore, when calculating the constraints of AGV vehicle kinematics, consideration needs to be given. Assuming that the AGV vehicle has N standard wheels, it is composed of \( N_f \) fixed standard wheels and \( N_s \) steerable standard wheels. Use \( \beta_i(t) \) to represent the
variable steering angle of the $N_s$ steerable standard wheels; on the contrary, as shown in Figure 3, $\beta_f$ is regarded as the direction of the $N_f$ fixed standard wheels. In the case of wheel rotation, both fixed and steerable wheels have a rotational position around a horizontal axis, which changes as a function of time. The fixed and controllable conditions are expressed as $\varphi_f(t)$ and $\varphi_s(t)$ respectively, and the coalescence matrix of the two is expressed by $\varphi(t)$.

$$\varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix}$$

The rolling constraints of all wheels can be combined with a single expression:

$$J_1(\beta_s)R(\theta) \dot{\xi}_1 - J_2 \varphi = 0$$

This expression is similar to the rolling constraints of individual wheels. However, the matrix replaces a single value, so all wheels are taken into account. $J_2$ is a constant diagonal matrix, and its entity is the radius $r$ of all standard wheels. For all wheels, the $J_1(\beta_s)$ table designs a matrix with a projection, which is projected on the motion along the plane of each wheel.

$$J_1(\beta_s) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \end{bmatrix}$$

Put all the sliding constraints of the standard wheels into a single representation, which is the same as the equation

$$\{ \cos(\alpha + \beta) \sin(\alpha + \beta) \ \sin(\beta) \ \sin(\beta) \} R(\theta) \dot{\xi}_1 = 0$$

$$\{ \cos(\alpha + \beta(t)) \sin(\alpha + \beta(t)) \ \sin(\beta(t)) \ \sin(\beta(t)) \} R(\theta) \dot{\xi}_1 = 0$$

have the same organization:

$$C_1(\beta_s)R(\theta) \dot{\xi}_1 = 0$$

$$C_1(\beta_s) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$$

For all fixed standard wheels and steerable standard wheels, their rows are the three items in the 3-matrix in equations (14) and (16). Therefore, equation (20) is constant for all standard wheels, that is, the motion component of the wheel plane orthogonal to them must be zero. This sliding constraint on all standard wheels has the most significant impact on determining the overall mobility of the AGV vehicle chassis.

6. Conclusions

Combining $J_1(\beta_s)R(\theta) \dot{\xi}_1 - J_2 \varphi = 0$ (18) and $C_1(\beta_s)R(\theta) \dot{\xi}_1 = 0$ (20) can obtain the motion and rolling and sliding constraints of differential AGV vehicles $J_1(\beta_s)$ and $C_1(\beta_s)$, as well as the relationship between AGV vehicle wheel speeds.

Combining these two equations can get the expression:

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta) \dot{\xi}_1 = \begin{bmatrix} J_2 \varphi \\ 0 \end{bmatrix}$$

(22)
Note that for the value of the right wheel $\beta^r$, it must be ensured that forward transmission causes movement in the $+X$ direction (Figure 3). Now we can use the matrix terms of equations (13) and (14) to calculate $J_{1f}$ and $C_{1f}$. Because the two fixed standard wheels are parallel, equation (14) only produces one independent equation (22) giving

$$
\begin{bmatrix}
1 & 0 & l \\
1 & 0 & -l \\
0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
2 & 0 \\
2 & 0 \\
\end{bmatrix}
R(\theta)\dot{\xi}_l =
\begin{bmatrix}
J_{1f} \\
C_{1f} \\
0 \\
\end{bmatrix}
$$

(23)

The inverse matrix of equation (23) is used to obtain the kinematics equation specific to the differential drive AGV vehicle:

$$
\dot{\xi} = R(\theta)^{-1}
\begin{bmatrix}
1 & 0 & l \\
1 & 0 & -l \\
0 & 1 & 0 \\
\end{bmatrix}
^{-1}
\begin{bmatrix}
1 & 0 \\
2 & 0 \\
2 & 0 \\
\end{bmatrix}
R(\theta)^{-1}
\begin{bmatrix}
1 & 0 \\
2 & 0 \\
2 & 0 \\
\end{bmatrix}
$$

(24)

Jointly description of the movement behavior of wheels rolling and sliding.

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