A joint economic lot size model with price and environmentally sensitive demand

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This paper presents a joint economic lot size (JELS) model for coordinated inventory replenishment decisions considering price and environmentally sensitive demand. It assumes a single product that flows along a two-level supply chain (vendor–buyer). The buyer’s demand is linear and sensitive to the product’s price and its environmental performance. A capital investment is considered necessary to improve the production process resulting in an indirect improvement of the product’s environmental quality. A mathematical model is developed to represent this situation and solved to maximize the total profit of the supply chain for: (1) the vendor’s production lot size quantity and the number of shipments to the buyer, and (2) the selling price and the amount invested to improve the production process. Numerical examples are provided with their results discussed.

Keywords: JELS; price sensitive demand; environmental quality

1. Introduction

Supply chain management integrates business processes to provide products, services and information with an added value to customers and stakeholders (Lambert, 2008). As global markets became more competitive, supply chain coordination became a key component for enhancing a supply chain’s profitability and responsiveness. When there is no coordination, the members of a supply chain independently maximize their own profits. This does not ensure that the all chain members reach optimal economic and environmental performance (Sajadieh & Akbari Jokar, 2009). When there is coordination, where the total supply chain profit (cost) is maximized (minimized), the savings from coordination shift to the side of the vendor as the buyer is the one who will operate off its optimal policy. The losing party is usually compensated (e.g. Jaber & Zolfaghari, 2008, Chapter 17). In this regard, this paper investigates the inventory policy for a two-echelon (vendor–buyer) supply chain when the players have economic and environmental objectives to attain.

Achieving an effective coordination between a vendor and its buyer(s) is a managerial concern and a challenging research issue (Qin, Tang, & Guo, 2007). The vendor–buyer coordination problem has been receiving increasing attention by practitioners and academicians (Ertogral, Darwish, & Ben-Daya, 2007; Jaber & Zolfaghari, 2008, Chapter 17; Toptal, Çetinkaya, & Lee, 2003), where the economic order/production quantity

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(EOQ/EPQ) models have been the foundation for the two-echelon supply chain models (e.g. Andriolo, Battini, Persona, Sgarbossa, & Grubbström, in press). The stream of research dealing with vendor–buyer coordination problem is referred to in the literature as the joint economic lot sizing (JELS) problem, the most recent review is found in Glock (2012).

Traditionally, inventory and shipment policies for a vendor and a buyer in a two-echelon supply chain are managed independently. As a result, the optimal lot size policy for the buyer may not be optimal for the vendor, and vice versa. Goyal (1977) is believed to be the first to introduce the idea of a joint total cost for a vendor–buyer system, where he assumed an instantaneous production rate for a vendor with a lot-for-lot (LFL) shipment policy. The work of Goyal (1977) was extended by Banerjee (1986), who assumed a finite rather than an instantaneous production rate, and later by Goyal (1988), who assumed that the vendor’s inventory is accumulated (not LFL) and is delivered to the buyer in shipments of equal sizes. In these models, the optimal inventory and shipment policies for the two-echelon system were determined by the joint optimization of the cost functions of the vendor and the buyer.

Following the works of Goyal and Banerjee, the basic JELS model has been extended in many different directions. Broadly speaking, the existing literature on the extended JELS may be divided into different categories, such as ‘quality’ (e.g. El Saadany, Jaber, & Bonney, 2011; Glock, Jaber, & Searcy, 2012), ‘controllable lead times’ (e.g. Hoque & Goyal, 2006), ‘transportation’ (e.g. Ertrogral et al., 2007; Toptal et al., 2003) and many others (Jaber & Zolfaghari, 2008, Chapter 17; Sajadieh & Akbari Jokar, 2009). ‘Quality’ may be defined in many ways. For example, Reeves and Bednar (1994) defined quality as the value, excellence, conformance to specifications and meeting or exceeding customers’ expectations. Other works have treated quality in a supply chain as an aggregated measure, including the environmental performance of the processes involved in producing and delivering a product, and considered demand to be price and quality driven (El Saadany et al., 2011). A common measure of environmental performance has been the amount of greenhouse gases (GHG) emissions (e.g. CO₂).

Accounting for CO₂ emissions when modelling supply chains started to receive attention very recently. For example, Wahab, Mamun, and Ongkunaruk (2011) proposed an EOQ model for a coordinated two-echelon international supply chain considering imperfect items with carbon emissions costs. Jaber, Glock, and El Saadany (2013) proposed a two-echelon supply chain model where emissions are generated from the vendor’s production process. They considered tax and emissions penalty according to the European Union Emissions Trading System, and presented several numerical examples to illustrate the behaviour of the model for different scenarios of emissions tax and penalties. Zanoni, Mazzoldi, and Jaber (2014) showed, by considering the same setting as in Jaber et al. (2013), how consignment stock coordination mechanism can reduce cost and GHG emissions.

The present paper contributes to this emerging trend of research by investigating a two-echelon supply chain model where demand is sensitive to the product’s price and its environmental performance. The profit function for the supply chain, which is the sum of the profit functions of the vendor and the buyer, is jointly optimized to determine the production and shipping policies, and ordering and pricing policies, respectively. Here, the product’s environmental performance is assumed to be proportional to an investment amount, where more investment leads to greener operations.

The paper is organized as follows: in Section 2, the problem is defined and the notations and assumptions are presented. Section 3 is for mathematical modelling. Section 4 discusses the independent policies of the vendor and the buyer. Section 5 discusses the
integrated supply chain policy. Section 6 presents and discusses some numerical results, while in Section 7 a sensitivity analysis on some problem parameters is presented. Finally, the paper summarizes and concludes in Section 8, where some future research directions are also presented.

2. Problem definition and notation

Consider a two-echelon (vendor–buyer) supply chain for a single product. The vendor produces a lot of size $nQ$ and delivers it to the buyer in $n$ batches of size $Q$ each. The objective is to determine the optimal values of $Q$, $n$, $\delta$ (selling price) and the investment amount $I$ necessary to improve the production process, thus resulting in an improvement of the environmental quality of the product (which directly influences the production costs of the considered product and its market demand) by jointly maximizing the total profit functions of the vendor and the buyer. Next, the assumptions and notations are presented.

Assumptions:

1. Demand, $D = D(\delta, G)$, at the buyer’s side, which is a linear function of the selling price $\delta$ and the product’s environmental performance measure $G$ is deterministic, where $G$ is a function the investment amount $I$.
2. Vendor’s production rate $P$ is finite, with $P > D(\delta, G)$ for every $\delta$ and $G$.
3. The inventory is continuously reviewed, where the buyer orders $Q$ units when its on-hand inventory reaches the reorder point and its inventory is replenished instantaneously upon the depletion of the last item of inventory.
4. The inventory holding cost for the buyer is higher than that for the vendor, i.e. $h_B > h_V$.
5. Backorders and shortages are not allowed.
6. The time horizon is infinite.

Notations:

$P$ production rate of the vendor
$D$ demand rate (as function of buyer’s unit selling price and of product environmental performance); $P > D$
$Q$ buyer’s order quantity
$A_V$ vendor’s set-up cost
$A_B$ buyer’s ordering cost
$c$ buyer’s unit cost (vendor’s unit selling price)
$c_p$ vendor’s unit production cost
$\delta$ buyer’s unit selling price
$G$ product environmental performance measure
$a$ intercept of the demand function
$b$ demand sensitivity coefficient to economic aspects related to the product
$e$ demand sensitivity coefficient to environmental aspects related to the product
$h_V$ inventory holding cost for an item stored at the vendor’s side
$h_B$ inventory holding cost for an item stored at the buyer’s side
$n$ number of shipments
$I$ investment necessary to improve the product’s environmental performance
Figure 1 illustrates the behaviour of the inventory level at the sides of the vendor and the buyer. The vendor produces $nQ$ units in $nQ/P$ units of time and depletes them over $nQ/D$ units of time, where $Q/D$ is the buyer’s cycle time. The vendor stops production once $nQ$ units have been produced. During the vendor’s cycle time, $n$ shipments are delivered to the buyer.

3. Model’s formulation

The annual demand function can be written as:

$$D(\delta, G) = a - b\delta + eG \quad (a \geq b - e > 0, G > 0),$$

where a higher value of $G$ represents a higher environmental performance of the product sold.

The relationship between demand and price is well documented in the literature; i.e. demand increases as price decreases. Also, increasing product quality, say environmental, and increase of demand has been of a recent and growing interest to researchers (El Saadany et al., 2011; Glock et al., 2012). A recent survey (European Commission, 2013) found that four out of ten (40.7%) consumers claimed that the environmental performance of goods or services influenced their purchasing decisions.

Environmental performance of a product can be improved in several ways, including, but not limited to, the use of less virgin and more recycled material, greener transportation modes, less energy, less material for packaging, etc. (Bonney & Jaber, 2011; El Saadany et al., 2011). This paper contributes along the same lines by studying the effect of the production process on the environmental performance of a product. This is quite strategic and important point since the reduction of pollution that a production site produces (vendor’s plant) requires huge investments that make production costly (e.g. air treatment equipment and process water recycling). However, it also takes affect at the very source of the ‘green’ value of the product, including the sustainability of the plant in relation to its location. Therefore, the ‘greening’ investment modelled here is assumed to directly affect the vendor’s marginal costs.

The assumption of linearity with respect to price and non-price variables, such as quality or service, has been made by several studies in the operations management and marketing literature (Banker, Khosla, & Sinha, 1998; Choi, 1991; Tsay & Agrawal, 2000).
Figure 2 illustrates the behaviour of the demand function over the product’s unit selling price and environmental performance. The parameter $e$ measures the sensitivity of the demand function to changes in the level of environmental quality. When $b > e$, customers tend to purchase the product because it is economical. It is also assumed that a low price would dominate the purchasing behaviour of customers, especially when $b \gg e$, whereas the product’s environmental performance or quality dominates customers demand, especially when $e \gg b$.

Considering the link between the abatement cost (here intended as the investment) and pollution abatement in a production process (here intended as the product environmental performance), the majority of studies expressed that investment is necessary and it is of an exponential function of the pollution abatement level, i.e. by considering the most cost-effective method of the application of measures. For example, Iijima et al. (1996) showed the relationship between the annual average sulphur dioxide concentration and the total environmental costs (cost of pollution preventing equipment + cost of compensation to certified victims). Dellink, Hofkes, van Ierland, and Verbruggen (2004) used a logarithmic-type function to describe the relationship between pollutants reduction ($\text{CO}_2$ equivalent and acid-equivalents) and the related expenditure required to achieve such results. Beaumont and Tinch (2004) observed that reducing industrial waste can, in some circumstances, result in a ‘win–win’ scenario for the industry and the environment. They expressed the pollutants abatement quantities and their related costs as a logarithmic-type function. A marginal abatement cost function of a similar form to those described above can also be obtained from Vijay, DeCarolis, and Srivastava (2010).

Therefore, in this paper, the product’s environmental performance, $G$, is expressed as a function of the investment amount, $I$, as:

$$ G = \gamma \ln(I), \quad (2) $$

where $\gamma \in (0,1)$ is a function parameter. The form of Equation (2) can be derived from the abatement cost curves, which are technology related and of the same behaviour; an example of how one can derive abatement cost curves can be found in Beaumont and Tinch (2004). Figure 3 shows the general relationship between $I$ and $G$.

Therefore, the annual demand rate function can be rewritten as follows:

$$ D(\delta, I) = a - b\delta + e\gamma \ln(I). \quad (3) $$

Moreover, it is considered that the vendor’s production cost is influenced by the investment amount ($I$) needed to improve the production process, i.e. the environmental

![Figure 2. Demand sensitivity with respect to price and environmental quality.](image)
performance of the product (e.g. an investment in more sophisticated equipment so as to reduce the emissions or wastes generated by the vendor’s processes). Then, the production cost can be expressed as $c_p = c^0_p + I/D$, where $c^0_p$ is the intercept representing the initial production cost when $I=0$. Thus, for larger demand it is possible to allocate the investment cost to improve environmental quality of a process (i.e. $I$) producing several products and therefore the specific production cost decreases.

Moreover, since the production rate is larger than the demand rate, $P > D$, then the following inequality must be satisfied:

$$P \geq D(\delta, I) = a - b\delta + e\gamma \ln(I),$$

where the relationship between the coefficients of $D$, $P$ and $G$ are:

$$P \geq a - b\delta + eG,$$

and

$$G \leq \frac{P - a + b\delta}{e}.$$

4. The Independent policy

In this section, we derive the inventory policies for the buyer and the vendor assuming that each player optimizes its profit function independently. In this case, the buyer is free to choose its own marketing and ordering policies ($Q$, $\delta$), and the vendor is free to choose its number of shipment $n$ (i.e. its production batch), together with an investment $I$ to improve environmental quality.

4.1. The Buyer total profit function

The buyer’s annual profit function, $TP^E_B(Q, \delta)$, is given as

$$TP^E_B(Q, \delta) = [a - b\delta + e\gamma \ln(I)](\delta - c) - \frac{[a - b\delta + e\gamma \ln(I)]A_B}{Q} - \frac{h_BQ}{2}.$$
Consequently, the vendor investment amount to improve its environmental performance, i.e. \( \delta \), have information on the amount of invested \( I \), which is decided upon by the vendor. However, the vendor needs to have information on the buyer’s order quantity and unit selling price \( (Q^* , \delta^* ) \) in order to determine its number of shipments to the buyer, and investment amount to improve its environmental performance, i.e. \( (n^* , I^* ) \), which are determined by solving Equation (8) below; Equation (8) shows that the buyer’s and the vendor’s policies are connected even though they are independent.

### 4.2. The vendor total profit function

The average inventory level for the vendor is computed by subtracting the average buyer’s inventory level from the average inventory level of the system (see Appendix 1). From Figure 1, the expression for the average inventory of the vendor is given as:

\[
\text{Inv}_V = \frac{Q^*}{2} \left( n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right). \tag{6}
\]

Substituting \( D \) in Equation (6) with its expression \( [a - b\delta + cI\ln(I)] \), Equation (6) becomes:

\[
\text{Inv}_V = \frac{Q^*}{2} \left( n \left( 1 - \frac{[a - b\delta + cI\ln(I)]}{P} \right) - 1 + \frac{2[a - b\delta + cI\ln(I)]}{P} \right). \tag{7}
\]

Consequently, the vendor’s annual profit function can be written as:

\[
TP^E_V(n, I) = (c - c_p)[a - b\delta + cI\ln(I)] - \frac{[a - b\delta + cI\ln(I)]A_V}{nQ^*} + \frac{h_V}{2} \left[ n \left( 1 - \frac{[a - b\delta + cI\ln(I)]}{P} \right) - 1 + \frac{2[a - b\delta + cI\ln(I)]}{P} \right]. \tag{8}
\]

To show that \( TP^E_V(n, I) \) is strictly concave with respect to \( n \) and \( I \), its hessian matrix is determined as:

\[
H_{TP^E_V} = \begin{pmatrix}
\frac{\partial^2 TP^E_V(Q, \delta)}{\partial^2 n} & \frac{\partial^2 TP^E_V(Q, \delta)}{\partial I \partial n} \\
\frac{\partial^2 TP^E_V(Q, \delta)}{\partial n \partial I} & \frac{\partial^2 TP^E_V(Q, \delta)}{\partial^2 I}
\end{pmatrix}
= \begin{pmatrix}
-2A_V[a - b\delta + cI\ln(I)] & \frac{A_Ve^I}{IQ^*n^2} + \frac{Qe^Ih_V}{2IP} \\
\frac{A_Ve^I}{IQ^*n^2} + \frac{Qe^Ih_V}{2IP} & \left( \frac{e^I}{\sqrt{T}} \right)^2 \left[ \frac{Qh_V}{P} \left( 1 - \frac{n}{2} \right) - (c - c_p) + A_V \right]
\end{pmatrix}. \tag{9}
\]
Equation (8) is concave with respect to \( n \) and \( I \), if the determinant of \( H_{TP_f} > 0 \), where
\[
-2A_f \frac{Qh}{Q^n} \left( \frac{1 - n}{2} - (c - c_p) + \frac{A_f}{Qn} \right) > \left[ e_f \left( \frac{A_f}{Qn^2} + \frac{Qh}{2P} \right) \right]^2. \tag{10}
\]
The first term of \( H_{TP_f} = -2A_f[a - b\delta + e_f\ln(I)] \) \( Q^n \) is critical; i.e. Equation (10) must be checked for each set of \((n^*, I^*)\). If \( TP_f([n^*], I) < TP_f([n^*], I) \), then the optimal number of shipments to be considered is \([n^*] \), otherwise it is \([n^*] \).

5. The integrated policy

In this section, we present the solution for the models developed in Sections 4 and 5.

\[
TP_f(Q, \delta, n, I) = (\delta - c_p)[a - b\delta + e_f\ln(I)] - \frac{[a - b\delta + e_f\ln(I)](A_f + nA_B)}{nQ} + \frac{h_BQ}{2} - \frac{h_vQ}{2} \left[ n \left( 1 - \frac{[a - b\delta + e_f\ln(I)]}{P} \right) - 1 + 2[a - b\delta + e_f\ln(I)] \right].
\]

Equation (11) is concave with respect to \( Q, \delta, n \) and \( I \), if its hessian matrix \( H_{TP_f} \) is negative definite, where its eigenvalues are < 0, or alternatively Equation (13) below is valid. \( H_{TP_f} \) and the condition for concavity are given respectively as:
\[
H_{TP_f} = \begin{pmatrix}
\frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q^2} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial \delta} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial n} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial I} \\
\frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial \delta} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial \delta^2} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial \delta \partial n} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial \delta \partial I} \\
\frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial n} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial \delta \partial n} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial n^2} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial n \partial I} \\
\frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial Q \partial I} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial \delta \partial I} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial n \partial I} & \frac{\partial^2 TP_f(Q, \delta, n, I)}{\partial I^2}
\end{pmatrix},
\]

and
\[
x^T \cdot H_{TP_f} \cdot x < 0, \tag{13}
\]
where \( x \) (and its transpose \( x^T \)) is a non-zero column vector of \( n \) real numbers; if Equation (13) is valid for all values of \( x \), then \( TP_f \) can be said to be negative definite.

6. Numerical study

In this section, we present the solution for the models developed in Sections 4 and 5, and the results for the independent and the integrated inventory lot sizing policies for a two-echelon (vendor–buyer) supply chain.
We consider the input parameters for the numerical example from Sajadieh and Akbari Jokar (2009), where: \( P = 3200 \text{/year}, A_p = 400\$/set-up, A_B = 25\$/order, h_V = 4\$/unit/year, h_B = 5\$/unit/year, a = 1500, b = 10 \) and \( c = 5\$/unit. The remaining input parameters were assumed as follows: \( c_p^0 = 1, e/b = .3 \) and \( = .2 \). Table 1 summarizes the results for the independent policy, whose total cost is the sum of Equations (4) and (8), and for the integrated policy, whose total cost is given by Equation (11). The results show that the independent policy is less profitable for the buyer (52,165.56\$ < 52,261.04\$) and the vendor (1647.27 < 1688.97). A joint vendor–buyer policy provides the supply chain with a competitive position where the buyer can offer a lower price (from 77.67\$ (independent) to 76.11\$ (integrated)), and the product’s environmental performance is improved (from .13 to .76). Further, an integrated policy allows ordering in larger batch sizes (110.61 instead of 85.07) and less frequent shipments (4 instead of 5), which implies less greenhouse gases emissions from transportation activities.

For the independent policy, \( TP_{E_{J}} \) must be intended as the sum of \( TP_{E_B} \) and \( TP_{E_V} \), while, for the integrated policy, \( TP_{E_B} \) and \( TP_{E_V} \) are the profit components of \( TP_{E_J} \) relating to the buyer and the vendor, respectively.

In order to test the concavity of \( TP_{E_B} \), \( TP_{E_V} \) and \( TP_{E_J} \) functions, the hessian matrix determinant test have been performed for the points \((Q^*, \delta^*, n^*, I^*)\) for \( TP_{E_B} \) and \( TP_{E_V} \), respectively, for the independent policy, while the negative definite matrix test has been performed for the point \((Q^*, \delta^*, n^* \text{ and } I^*)\) on the \( TP_{E_J} \) function, for the integrated policy.

In Table 2, \( Det(H_X) \) represents the determinant of the hessian matrix related to the function \( X \) (\( TP_{E_B} \) for the buyer and \( TP_{E_V} \) for the vendor), while \( \bar{v} \) is the vector representing the optimization variables considered in the hessian matrix test (\( Q^* \) and \( \delta^* \) for \( TP_{E_B} \), \( n^* \) and \( I^* \) for \( TP_{E_V} \)) and \( v_1 \) represents the first-row, first-column element of the considered hessian matrix.

In Table 3, \( \bar{w} \) represents a generic vector of non-zero elements (in \( \mathbb{R}^4 \)) involved in the concavity test for \( TP_{E_J} \): if the scalar resulting from \( \bar{w}^T \cdot H_{TP_{E_J}} \cdot \bar{w} \) is negative, the hessian matrix related to \( TP_{E_J}(Q^*, \delta^*, n^*, I^*) \) is negative definite and so the function \( TP_{E_J} \) is concave in \((Q^*, \delta^*, n^*, I^*)\).

The results of the two policies, the independent and the integrated, for different values of \( b \) are summarized in Tables 4 and 5, respectively. For both cases, by increasing \( b \),

**Table 1. Decision variables under independent optimization vs. joint optimization.**

| Policy   | \( D \)  | \( Q^* \)  | \( \delta^* \) | \( n^* \) | \( I^* \) | \( G \) | \( TP_{E_B} \) | \( TP_{E_V} \) | \( TP_{E_J} \) |
|----------|---------|------------|---------------|---------|---------|-------|------------|------------|------------|
| Independent | 723.73 | 85.07 | 77.67 | 5 | 1.93 | .13 | 52,165.56 | 1647.27 | 53,812.83 |
| Integrated | 741.18 | 110.61 | 76.11 | 4 | 44.47 | .76 | 52,261.04 | 1688.97 | 53,950.02 |

**Table 2. Independent policy profit functions concavity: test results.**

| Policy   | \( Q^* \)  | \( \delta^* \) | \( n^* \) | \( I^* \) | \( Det(H_X) \) | \( \frac{\partial TP_{E_X}(\bar{v})}{\partial^2 v_1} \) | Test result |
|----------|------------|---------------|---------|-------|----------------|-----------------|-------------|
| Independent Buyer | 85.07 | 77.67 | – – | 1.17 (>0) | 1.93 >0 | 52,165.56 | TP_{E_B} concave in \((Q^*, \delta^*)\) |
| Vendor    | – – | 5 | 1.93 | 28.16 (>0) | 54.45 (<0) | TP_{E_V} concave in \((n^*, I^*)\) |
the optimum selling price $\delta^*$ and the demand $D$ decrease, thus the profits for the vendor and the buyer decrease. The integrated case performs better for all values of $b$ when considering the system total profit, and the vendor’s profit, but for the case of the joint optimization policy, the buyer’s profit is lower for the integrated policy than for the independent policy for values of $b$ larger than 20. In particular, as $b$ increases from 10 to 100, $TP^E_B$ and $TP^E_V$ decrease by about 96% (from 52,165.56$ to 2233.88$) and 46% (from 1647.27$ to 888.88$) for the independent policy, and by about 96% (from 52,261.04$ to 1980.80$) and 16% (from 1688.97$ to 1417.90$) for the integrated policy, respectively. This suggests that $TP^E_B$ is as sensitive (96%) for changes in the value of $b$ for either policies, while $TP^E_V$ is more sensitive (from 46 to 16%). This suggests that it is advantageous for the vendor to go into an integrated policy as it guarantees a higher profit margin. The sensitivity of $TP^E_V$ is due to changes in demand, where $D$ decreases by about 32% (from 723.73 to 490.82) and 11% (from 741.18 to 657.06) for the

### Table 3. Integrated policy profit function concavity: test results.

| Policy | $Q^*$ | $\delta^*$ | $n^*$ | $I^*$ | $\bar{w}^T \cdot H_{TP} \cdot \bar{w}$ | Test result |
|--------|-------|------------|-------|-------|---------------------------------|-------------|
| Integrated | 110.61 | 76.11 | 4 | 44.47 | $<0, \ \forall \ \bar{w} \in R^4$ | $TP^E_J$ concave in $(Q^*, \delta^*, n^*, I^*)$ |

### Table 4. Results for individual optimization (independent policy) model.

| $b$ | $D$ | $Q^*$ | $\delta^*$ | $n^*$ | $I^*$ | $G$ | $TP^E_B$ | $TP^E_V$ | $TP^E_J$ |
|-----|-----|-------|------------|-------|-------|-----|---------|---------|---------|
| 10  | 723.73 | 85.07 | 77.67 | 5 | 1.93 | .13 | 52,165.56 | 1647.27 | 53,812.83 |
| 20  | 697.81 | 83.54 | 40.19 | 5 | 3.84 | .27 | 24,138.40 | 1560.15 | 25,698.56 |
| 30  | 672.00 | 81.98 | 27.70 | 5 | 5.72 | .35 | 14,847.64 | 1473.94 | 16,321.58 |
| 40  | 646.21 | 80.39 | 21.47 | 5 | 7.57 | .40 | 10,238.71 | 1388.47 | 11,627.18 |
| 50  | 620.43 | 78.77 | 17.73 | 5 | 9.40 | .45 | 7501.64 | 1303.66 | 8805.31 |
| 60  | 594.62 | 77.11 | 15.23 | 5 | 11.19 | .48 | 5700.11 | 1219.48 | 6919.59 |
| 70  | 568.77 | 75.42 | 13.46 | 5 | 12.94 | .51 | 4432.95 | 1135.91 | 5568.86 |
| 80  | 542.87 | 73.68 | 12.13 | 5 | 14.65 | .54 | 3499.66 | 1052.95 | 4552.61 |
| 90  | 516.89 | 71.90 | 11.09 | 5 | 16.32 | .56 | 2788.89 | 970.60 | 3759.50 |
| 100 | 490.82 | 70.06 | 10.27 | 5 | 17.94 | .58 | 2233.88 | 888.88 | 3122.76 |

### Table 5. Results for joint optimization (integrated policy) model.

| $b$ | $D$ | $Q^*$ | $\delta^*$ | $n^*$ | $I^*$ | $G$ | $TP^E_B$ | $TP^E_V$ | $TP^E_J$ |
|-----|-----|-------|------------|-------|-------|-----|---------|---------|---------|
| 10  | 741.18 | 110.61 | 76.11 | 4 | 44.47 | .76 | 52,261.04 | 1688.97 | 53,950.02 |
| 20  | 732.26 | 109.86 | 78.61 | 4 | 43.95 | .76 | 24,172.88 | 1659.96 | 25,832.84 |
| 30  | 723.25 | 109.10 | 76.12 | 4 | 43.41 | .75 | 14,835.04 | 1630.71 | 16,465.75 |
| 40  | 714.14 | 108.33 | 74.87 | 4 | 42.86 | .75 | 10,185.05 | 1601.19 | 11,786.24 |
| 50  | 704.92 | 107.54 | 73.13 | 4 | 42.31 | .75 | 7410.42 | 1571.40 | 8981.82 |
| 60  | 695.59 | 106.75 | 71.63 | 4 | 41.76 | .75 | 5573.68 | 1541.32 | 7115.00 |
| 70  | 686.15 | 105.94 | 69.85 | 4 | 41.20 | .74 | 4273.04 | 1510.94 | 5783.98 |
| 80  | 676.58 | 105.11 | 67.52 | 4 | 40.63 | .74 | 3307.65 | 1480.24 | 4787.89 |
| 90  | 666.89 | 104.28 | 65.08 | 4 | 40.06 | .74 | 2565.88 | 1449.23 | 4015.12 |
| 100 | 657.06 | 103.42 | 8.65 | 4 | 39.42 | .73 | 1980.80 | 1417.90 | 3398.70 |

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independent and integrated policies, respectively. The reason why $D$ is less sensitive to changes in $b$ for the integrated policy is related to the level of investment in environmental quality being kept at about the same level (reduced by about 11%, from 44.47$ to 39.42$). The results in Tables 4 and 5 show that the supply chain model considered becomes more competitive (lower $\delta^*$ values) for high values of $b$ when an integrated policy is adopted.

7. Sensitivity analysis

In this section, we perform a sensitivity analysis to study the changes in the values of the parameter $\gamma$ as it impacts the level of investment $I$, which affects the product’s environmental quality and subsequently the demand rate (the $b$ value affects the preference of customers when buying the product). Figure 4 shows the behaviour of the profits for the buyer, the vendor and the system (independent policy vs. integrated one) for different values of $b$, when $\gamma = .2$, where the percentage values representing the change ($-/+$) in the profits for the buyer, the vendor and the system with the independent policy being the reference point. Figure 5 shows the behaviour of the system profit (integrated policy.
vs. independent one) for different values of $b$, for $\gamma = .2, .3$ and .5. The results in Figure 5 show that the system’s total cost is not that sensitive to changing values of $\gamma$.

Figure 6 shows that the system total profit variations (independent policy vs. integrated one) for different values of $b$ and $e/b$, when $\gamma = .2$. The results show that the percentage difference in the system’s total profits for the independent and the integrated policies is more sensitive to lower values of the ratio $e/b$.

8. Conclusions

This paper developed an integrated production-inventory-marketing model for a two-echelon (vendor–buyer) supply chain where an investment is needed to elevate the environmental burdens of a production process. The contribution of the paper to the JELS literature is that it considers a demand function that is sensitive to the product’s unit price and its environmental performance (quality). The proposed demand function is used to develop a new JELS model. Four decision variables were considered in the model: the size of a lot, the number of shipments, the unit price and the amount of investment needed to improve the product’s environmental performance.

The results confirmed some findings in the literature, i.e. that investing to enhance environmental performance may lead to better profit levels (Ambec & Lanoie, 2008; Klassen & McLaughlin, 1996; Rao & Holt, 2005). The results also showed that an integrated supply chain policy is more competitive as it can deliver a product to the final customer at a lower price, while maintaining a good environmental performance. This suggests that adopting an integrated policy would lead to eco-efficient solutions. The results further showed that more emphasis should be put on improving the environmental performance of a product when the profit margin is high, rather than when it is low. Moreover, the vendor and the buyer can improve the coordination of their production and inventory activities by appropriately agreeing on a suitable profit-sharing mechanism that properly (or fairly) distributes the net savings between the vendor and the buyer: one option could be to adopt a model from the literature (Jaber & Osman, 2006; Jaber, Osman, & Guiffrida, 2006; Ouyang, Wu, & Ho, 2004). With such an arrangement, the vendor and the buyer preserve their relative shares of the total profits. However, there are also several other possible arrangements, for example based on game-theory approaches.
This work can be extended by considering transport costs and waste generated by a production process or remanufacturing activities (Hasanov, Jaber, Zanoni, & Zavanella, 2013; Jaber, Zanoni, & Zavanella, 2014). Another extension is to consider the environmental performance of transportation activities as a product quality indicator, thus verifying whether coordination between the vendor and the buyer can reduce environmental impact of transportation activities. Moreover, it would be interesting to link investment in ‘greener’ logistics processes to product performance.

References
Ambec, A., & Lanoie, P. (2008). Does it pay to be green? A systematic overview. Academy of Management Perspective, 22, 45–62.
Andriolo, A., Battini, D., Persona, A., Sgarbossa, F., & Grubbström, R. W. (In press). A century of evolution from Harris’s basic lot size model: Survey and research agenda. International Journal of Production Economics. doi:10.1016/j.ijpe.2014.01.013
Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. Decision Sciences, 17, 292–311.
Banker, R. D., Khosla, I., & Sinha, K. K. (1998). Quality and competition. Management Science, 44, 1179–1192.
Beaumont, N. J., & Tinch, R. (2004). Abatement cost curves: A viable management tool for enabling the achievement of win–win waste reduction strategies? Journal of Environmental Management, 71, 207–215.
Bonney, M., & Jaber, M. Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-classical era. International Journal of Production Economics, 133, 43–53.
Choi, S. C. (1991). Price competition in a channel structure with a common retailer. Marketing Science, 10, 271–296.
Dellink, R., Hofkes, M., van Ierland, E., & Verbruggen, H. (2004). Dynamic modelling of pollution abatement in a CGE framework. Economic Modelling, 21, 965–989.
El Saadany, A. M. A., Jaber, M. Y., & Bonney, M. (2011). Environmental performance measures for supply chains. Management Research Review, 34, 1202–1221.
Ertogral, K., Darwish, M., & Ben-Daya, M. (2007). Production and shipment lot sizing in a vendor–buyer supply chain with transportation cost. European Journal of Operational Research, 176, 1592–1606.
European Commission. (2013). The consumer conditions scoreboard – Consumers at home in the single market – SWD(2013) 291.
Glock, C. H. (2012). The joint economic lot size problem: A review. International Journal of Production Economics, 133, 671–686.
Glock, C. H., Jaber, M. Y., & Searcy, C. (2012). Sustainability strategies in an EPQ model with price- and quality-sensitive demand. International Journal of Logistics Management, 23, 340–359.
Goyal, S. K. (1977). An integrated inventory model for a single supplier single customer problem. International Journal of Production Research, 15, 107–111.
Goyal, S. K. (1988). A joint economic-lot-size model for purchaser and vendor: A comment. Decision Sciences, 19, 236–241.
Hasanov, P., Jaber, M. Y., Zanoni, S., & Zavanella, L. E. (2013). Closed-loop supply chain system with energy, transportation and waste disposal costs. International Journal of Sustainable Engineering, 6, 352–358.
Hill, R. M., & Omar, M. (2006). Another look at the single-vendor single-buyer integrated production-inventory problem. International Journal of Production Research, 44, 791–800.
Hoque, M., & Goyal, S. K. (2006). A heuristic solution procedure for an integrated inventory system under controllable lead-time with equal or unequal sized batch shipments between a vendor and a buyer. International Journal of Production Economics, 102, 217–225.
Iijima, M., Takemoto, Y., Oka, Y., Kito, H., Nishigaki, Y., Kataoka, K., & Asahi, S. (1996). Economic aspects of environmental investment in plant facilities. Computers & Industrial Engineering, 31, 713–717.
Jaber, M. Y., Glock, C. H., & El Saadany, A. M. A. (2013). Supply chain coordination with emissions reduction incentives. International Journal of Production Research, 51, 69–82.
Appendix 1

The system’s inventory function, according to Hill and Omar (2006) can be written as:

\[ \text{Inv}_S = \frac{(P - D)nQ^*}{2P} + \frac{DQ^*}{P}. \quad (A1) \]

The buyer’s inventory function is:

\[ \text{Inv}_B = \frac{Q^*}{2}. \quad (A2) \]

The vendor’s inventory function is obtained as the difference between system’s and buyer’s inventory functions:

\[ \text{Inv}_V = \frac{Q^*}{2} \left[ n \left( 1 - \frac{D}{P} \right) + \frac{2D}{P} - 1 \right]. \quad (A3) \]