Longitudinal Plasmons in a Thin Flat Conductive Film

A A Zotov and N V Zverev
Moscow Region State University,
Very Voloshinoi str. 24, 141014 Mytishchi, Moscow Region, Russian Federation
E-mail: aleksandr.zotov.99@mail.ru, zverev_nv@mail.ru

Abstract. The longitudinal plasmons in the plasma of conductivity electrons between the surfaces of a thin flat conductive film are investigated. It is shown that these plasmons lead to a resonant behaviour of the optical power coefficient of E-wave interaction with this film. The conditions for appearance of these plasmon resonances are found, and peculiarities of the dependence of the resonant frequencies on the film characteristics are revealed.

1. Introduction
A modern development of the nanotechnologies raise a question of a measurement of linear sizes of thin objects from 1 nanometer up to 1 micrometer with large enough precision. A perspective method for such measurements is a method of the plasmon resonances [1, 2].

This method is based on the longitudinal plasmon resonances appearing in thin layers of conductive matter like metals, semimetals and semiconductors, at the interaction of electromagnetic radiation with these layers.

This paper is devoted to investigation of the longitudinal plasmons in thin flat conductive film and an influence of these plasmons to the interaction of electromagnetic radiation with the film.

2. Longitudinal plasmons
Let us consider in more details the mechanism of appearance of the longitudinal plasmons [3]. These waves are the electromagnetic waves in the plasma matter when the electric field strength is co-directed with their propagation direction i.e. $E \parallel k$ where $k$ is the wave vector. Electric field strength and electric displacement vectors of the longitudinal plasmon look as

$$E(r, t) = E(\omega, k) e^{i(kr - \omega t)} , \quad D(r, t) = D(\omega, k) e^{i(kr - \omega t)} ,$$

where $\omega$ is the cyclic frequency of the wave, and $E(\omega, k)$ and $D(\omega, k)$ are amplitudes of the longitudinal plasmons. Their components are related by the equation

$$D_a(\omega, k) = \varepsilon_0 \sum_b \varepsilon_{ab}(\omega, k) E_b(\omega, k) \quad (a, b = 1, 2, 3).$$
Here $\varepsilon_0$ is the electric constant in SI units and $\varepsilon_{ab}(\omega, k)$ is the plasma dielectric permittivity tensor. In case of uniform isotropic plasma, the tensor looks as

$$
\varepsilon_{ab}(\omega, k) = \varepsilon_l(\omega, k) \frac{k_a k_b}{k^2} + \varepsilon_{tr}(\omega, k) \left( \delta_{ab} - \frac{k_a k_b}{k^2} \right),
$$

where $\varepsilon_l(\omega, k)$ and $\varepsilon_{tr}(\omega, k)$ are respectively the longitudinal and the transverse plasma dielectric permittivity, $k = |k|$ is the wave number, $k_a$ are components of the $k$ vector and $\delta_{ab}$ is the Kronecker symbol.

Substituting the right equation in (1) in the Maxwell equation

$$
\nabla \cdot \mathbf{D}(r, t) = 0
$$

and using the equations (2) and (3), one gets the relation

$$
\varepsilon_l(\omega, k) k \mathbf{E}(\omega, k) = 0.
$$

Since $\mathbf{E} \parallel k$ one arrives at the longitudinal plasmon dispersion law [3]:

$$
\varepsilon_l(\omega, k) = 0.
$$

It worth to note that the longitudinal plasmons arise in plasma owing to the dependence of the longitudinal permittivity $\varepsilon_l$ not only on the frequency $\omega$ but also on the wave number $k$.

However, the longitudinal and transverse permittivities $\varepsilon_l(\omega, k)$ and $\varepsilon_{tr}$ are the complex-valued functions [3]. This is caused by the interaction of plasma particles between each other and also with the radiation and with external particles. Such a feature of the $\varepsilon_l$ leads from (4) to the complex-valued dependence $\omega = \omega(k)$ which shows that the longitudinal plasmons are the damped waves.

At the same time, these waves contribute to interaction of electromagnetic radiation with thin film of conductive matter. The longitudinal plasmons arise between borders of the film and hence for their appearance, the electric field strength $\mathbf{E}$ of the external electromagnetic wave has to possess a constituent which is orthogonal to the borders of the film. Therefore the longitudinal plasmons can arise only in the E-wave for which the electric field strength $\mathbf{E}$ lies in the incidence plane and which falls on the film surface under nonzero incidence angle.

### 3. Model of E-wave interaction with conductive film

Now we consider thin flat conductive film of the thickness $d$ localized between two transparent isotropic dielectric media having positive dielectric constants $\varepsilon_1$ and $\varepsilon_2$. Let the plane E-wave falls on the surface of the film from the $\varepsilon_1$ medium under the incidence angle $\theta$. Then the reflection and transmission optical power coefficients, $R$ and $T_r$, for the wave look as [4]

$$
R = \left| \frac{U^{(1)} + U^{(2)}}{V^{(1)} + V^{(2)}} \right|^2, \quad T_r = \text{Re} \left( \frac{\cos \theta'}{\cos \theta} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \right) \left| \frac{U^{(1)} V^{(2)} - U^{(2)} V^{(1)}}{V^{(1)} + V^{(2)}} \right|^2,
$$

where the values $U^{(j)}$ and $V^{(j)}$ ($j = 1, 2$) are equal to

$$
U^{(j)} = \frac{\cos \theta - Z^{(j)} \sqrt{\varepsilon_j}}{\cos \theta' + Z^{(j)} \sqrt{\varepsilon_j}}, \quad U^{(j)} = \frac{\cos \theta + Z^{(j)} \sqrt{\varepsilon_j}}{\cos \theta' + Z^{(j)} \sqrt{\varepsilon_j}}.
$$

In the equations (5) and (6), $\theta'$ is the refraction angle satisfying to the law

$$
\sqrt{\varepsilon_1} \sin \theta = \sqrt{\varepsilon_2} \sin \theta',
$$

where

$$
\sqrt{\varepsilon_1} \sin \theta = \sqrt{\varepsilon_2} \sin \theta',
$$

and

$$
\sqrt{\varepsilon_1} \sin \theta = \sqrt{\varepsilon_2} \sin \theta'.
$$
and \( Z^{(j)} \) is the surface impedance of the conductive film for the E-wave. When one supposes the mirror reflection of the conductivity electrons from the borders of the film, the surface impedance is evaluated by the equation [5]

\[
Z^{(j)} = \frac{2i\text{c}\omega}{d} \sum_n \frac{1}{k_n^2} \left( \frac{k_n^2}{\omega^2 \varepsilon_I(\omega, k_n)} + \frac{(\pi n/d)^2}{\omega^2 \varepsilon_{tr}(\omega, k_n) - (ck_n)^2} \right).
\]  

(7)

Here \( \omega \) is the E-wave frequency, \( \text{c} \) stands for the speed of light, the \( k_n \) and \( k_x \) values are equal to

\[
k_n = \sqrt{(\pi n/d)^2 + k_x^2}, \quad k_x = \frac{\omega}{c} \sqrt{\varepsilon_1 \sin \theta},
\]

(8)

and the summation is performed over all odd \( n = \pm 1, \pm 3, \pm 5, \ldots \) at \( j = 1 \) and over all even \( n = 0, \pm 2, \pm 4, \ldots \) at \( j = 2 \).

The permittivities \( \varepsilon_I(\omega, k) \) and \( \varepsilon_{tr}(\omega, k) \) are known for the electron plasma with taking into account the wave properties of electrons [6, 7]. In case of the degenerate electron plasma in metals, the permittivities are evaluated by the equations

\[
\varepsilon_I(\omega, k) = 1 + \frac{3}{4Q^2} \left( \frac{\Omega F(\Omega + i\gamma, Q)F(0, Q)}{\Omega F(0, Q) + i\gamma F(\Omega + i\gamma, Q)} \right),
\]

(9)

\[
\varepsilon_{tr}(\omega, k) = 1 - \frac{1}{\Omega^2} \left( 1 + \frac{\Omega G(\Omega + i\gamma, Q) + i\gamma G(0, Q)}{\Omega + i\gamma} \right),
\]

(10)

where

\[
F(\Omega + i\gamma, Q) = \frac{1}{r} \left[ B_1(\Omega_+ + i\gamma, Q) - B_1(\Omega_- + i\gamma, Q) \right] + 2,
\]

(11)

\[
G(\Omega + i\gamma, Q) = \frac{3}{16r} \left[ B_2(\Omega_+ + i\gamma, Q) - B_2(\Omega_- + i\gamma, Q) \right] + \frac{9}{8} \left( \frac{\Omega + i\gamma}{Q} \right)^2 + \frac{3}{32} Q^2 \gamma^2 - \frac{5}{8},
\]

(12)

\[
B_n(\Omega + i\gamma, Q) = \frac{1}{Q^{2n+1}} \left[ (\Omega + i\gamma)^2 - Q^2 \right]^n \ln \frac{\Omega + i\gamma - Q}{\Omega + i\gamma + Q}.
\]

(13)

In case of the non-degenerate electron plasma in semimetals or semiconductors, these permittivities look as

\[
\varepsilon_I(\omega, k) = 1 - \frac{2}{Q^2} \left( \frac{\Omega H(\Omega + i\gamma, \tilde{Q})H(0, \tilde{Q})}{\Omega H(0, Q) + i\gamma H(\Omega + i\gamma, Q)} \right),
\]

(14)

\[
\varepsilon_{tr}(\omega, k) = 1 - \frac{1}{\Omega^2} \left( 1 + \frac{\Omega H(\Omega + i\gamma, \tilde{Q}) + i\gamma H(0, \tilde{Q})}{\Omega + i\gamma} \right),
\]

(15)

where

\[
H(\Omega + i\gamma, \tilde{Q}) = \frac{\tilde{Q}^2}{\sqrt{\pi}} \int_0^{+\infty} \frac{[(\Omega_+ + i\gamma)(\Omega_- + i\gamma) + (\tilde{Q}\xi)^2] \exp(-\xi^2)}{[(\Omega_+ + i\gamma)^2 - (\tilde{Q}\xi)^2][(\Omega_- + i\gamma)^2 - (\tilde{Q}\xi)^2]} \, d\xi.
\]

(16)

In the equations (9) – (16), the following dimensionless values are introduced:

\[
\Omega = \frac{\omega}{\omega_p}, \quad \gamma = \frac{\nu}{\omega_p}, \quad \Omega_\pm = \Omega \pm \frac{\hbar k^2}{2m_\nu \omega_p}, \quad Q = \frac{v_F k}{\omega_p}, \quad r = \frac{\hbar \omega_p}{m_\nu v_F}, \quad \tilde{Q} = \frac{v_F k}{\omega_p}.
\]

(17)
Here $\omega_p$, $\nu$ and $m_e$ are respectively the plasma frequency, the collision frequency and the effective mass of the conductivity electrons, $\hbar$ is the Planck constant, $v_F$ is the Fermi velocity of electrons in the degenerate electron plasma at zero absolute temperature $T = 0$ K, and $v_T$ is the thermal velocity of electrons in non-degenerate electron plasma:

$$v_T = \sqrt{\frac{2k_B T}{m_e}},$$

where $k_B$ is the Boltzmann constant.

4. Results and discussion

By use of the equations (5) – (18) we study the reflection and transmission power coefficients $R$ and $T_r$. For numerical simulations, we took the aluminium and graphite materials. The aluminium metal has the parameters $[8]$: $\omega_p = 1.93 \cdot 10^{16}$ s$^{-1}$, $\nu = 10^{-3}\omega_p$, $v_F = 1.34 \cdot 10^6$ m/s, $m_e = 1.35 \cdot 10^{-30}$ kg. For graphite matter being a weak conductor we took the following parameters $[9]$: $\omega_p = 2.54 \cdot 10^{14}$ s$^{-1}$, $\nu = 2.273 \cdot 10^{11}$ Hz, $m_e = 9 \cdot 10^{-31}$ kg.

Comparing the equations (4) and (7), one sees that the longitudinal plasmons between film borders can contribute to the power coefficients (5) when the condition

$$\text{Re} \varepsilon_l(\omega_{res}, k_n) = 0$$

is fulfilled. Here $\omega_{res}$ are the resonant frequencies for which the power coefficients $R$ and $T_r$ have a critical behaviour. Note that it is obvious from (7) that the critical behaviour of power coefficients occurs then the incidence angle $\theta \neq 0$.

It was found that the resonant frequencies are compared with the plasma frequency according to the relations

$$\omega_{res} > \omega_p, \quad \omega_{res} \sim \omega_p.$$

This is turn out that the $\omega_{res}$ values depend mostly on the $\omega_p$, $d$, $v_F$ and $v_T$ parameters. This is obvious from the equations (9), (11), (13), (14), (16) and (17). They are almost independent of $\theta$, $\nu$, $\varepsilon_1$, $\varepsilon_2$ values when the conditions

$$d \ll \frac{c}{\omega_p}, \quad d \gg \frac{\hbar}{m_e v}$$

are valid. Here $v = v_F$ for metals and $v = v_T$ for semimetals and semiconductors.

The left condition in (20) follows from (8) when one requires a negligibility of $k_x$ in comparison with $\pi/d$ at $\omega \sim \omega_p$. One explains this condition as the plasmon resonances in the film are observable when the film width is much less than the skin-depth length $c/\omega_p$. But the right condition in (20) means in framework of classical physics that the measurable film thickness must be much large than the de Broglie wavelength. Then the relation

$$\frac{\hbar}{2m_e \omega_p} \left( \frac{\pi}{d} \right)^2 \ll 1$$

occurs since the $\hbar \omega_p \sim m_e v^2$ for metals and for semimetals and semiconductors at usual temperatures $T$. Independence of the $\omega_{res}$ on $\nu$ follows from the relation $\nu \ll \omega_p$.

In numerical studies, the $d$ values satisfy to the (20) conditions.

Furthermore, investigations of $R$ and $T_r$ show that the $\omega_{res}$ values evaluated from (19), lead to critical behaviour of the power coefficients only in case of the odd numbers $n = 1, 3, 5, \ldots$ as it is demonstrated at the figures 1 and 2 (see also $[1, 5]$). Such a result can be explained as follows. In case of the even values $n$, the $n = 0$ term in (7) strongly prevails over all remaining
Figure 1. The real part of longitudinal permittivity \( \text{Re} \varepsilon_l(\omega, k_n) \) (left plot, various \( n \)) and transmission power coefficient \( T_r \) (right plot) as functions of the frequency \( \omega \) for aluminium film. Values \( d = 3.5 \) nm, \( \theta = 60^\circ \), \( \omega_p = 1.93 \cdot 10^{16} \) s\(^{-1} \), \( \varepsilon_1 = 1 \) (air), \( \varepsilon_2 = 2 \) (quartz).

Figure 2. The real part of longitudinal permittivity \( \text{Re} \varepsilon_l(\omega, k_n) \) (left plot, various \( n \)) and reflection power coefficient \( R \) (right plot) as functions of the frequency \( \omega \) for graphite film. Values \( d = 20 \) nm, \( \theta = 60^\circ \), \( T = 294 \) K, \( \omega_p = 2.54 \cdot 10^{14} \) s\(^{-1} \), \( \varepsilon_1 = 1 \) (air), \( \varepsilon_2 = 2 \) (quartz).

terms and critical behaviour of \( Z^{(2)} \) is smeared. But in case of odd numbers \( n \), the first positive and negative \( n \) terms in (7) have the same orders and the critical behaviour of \( Z^{(1)} \) is visible for different \( n \) values.

The above analysis of \( \omega_{\text{res}} \) leads to a conclusion that these resonant frequencies delivering a visible critical behaviour of the power coefficient, satisfy to the equation

\[
\omega_{\text{res}} = \omega_p f \left( \frac{\pi n v}{\omega_p d} \right). \tag{21}
\]

Here \( n = 1, 3, 5, \ldots \), \( v = v_F \) for metals and \( v = v_T \) for semimetals and semiconductors, and \( f(Q) \) is a dimensionless function of dimensionless variable \( Q \) which is evaluated from (19) and from (9) for metals or from (14) for weak conductors. Of course, the \( d \) values should satisfy to the (20) conditions.

The dependence (21) is confirmed qualitatively by the numerical simulations of the coefficients (5) at various \( d \) and for weak conductor, at different \( T \) values (figures 3 and 4). These results show that since the resonant frequencies \( \omega_{\text{res}} \) are measured with large enough precision, the relative errors of \( d \) values are less than 1\% [1]. Hence the method of plasmon resonances is suitable for measurements of the thickness \( d \) of conductive film with a large precision.
Figure 3. The reflection and transmission power coefficients $R$ (left plot) and $T_r$ (right plot) as functions of the frequency $\omega$ for aluminium film: $1 - d = 1.75$ nm (solid line), $2 - d = 3.5$ nm (dotted line). Values $\theta = 60^\circ$, $\omega_p = 1.93 \cdot 10^{16}$ s$^{-1}$, $\varepsilon_1 = 1$ (air), $\varepsilon_2 = 2$ (quartz).

Figure 4. The reflection and transmission power coefficients $R$ (left plot) and $T_r$ (right plot) as functions of the frequency $\omega$ for graphite film: $1 - T = 283$ K (dotted line), $2 - T = 305$ K (solid line). Values $d = 40$ nm, $\theta = 60^\circ$, $\omega_p = 2.54 \cdot 10^{14}$ s$^{-1}$, $\varepsilon_1 = 1$ (air), $\varepsilon_2 = 2$ (quartz).

Acknowledgements
The work is supported by the RBRF Grant No 19-07-00537 and by the Moscow Region Governor Grant No 18 on July 24, 2020.

References
[1] Latyshev A V and Yushkanov A A 2015 Quantum Electronics 45(3) 270–74
[2] Valyansky S I, Vinogradov S V, Kononov M A et al 2017 Priljadnaya Fizika (Applied Physics) No 6 103–08
[3] Lifshitz E M and Pitaevskii L P 1981 Physical Kinetics. Vol 10 (1st ed.) (Pergamon Press) p. 462
[4] Yushkanov A A and Zverev N V 2017 Phys. Lett. A 381 679–84
[5] Jones W E, Kliwev K L and Fuchs R 1969 Phys. Rev. 178(3) 1201–03
[6] Latyshev A V and Yushkanov A A 2013 Theor. and Math. Phys. 175(1) 559–69
[7] Latyshev A V and Yushkanov A A 2014 Theor. and Math. Phys. 178(1) 130–41
[8] Fuchs R and Kliewer K L 1969 Phys. Rev. 185(3) 905–13
[9] Yushkanov A A and Zverev N V 2019 J. Phys.: Conf. Ser. 1309 012011