Absence of two-body delocalization transitions in the two-dimensional Anderson-Hubbard model

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We investigate Anderson localization of two particles moving in a two-dimensional (2D) disordered lattice and coupled by contact interactions. Based on transmission-amplitude calculations for relatively large strip-shaped grids, we find that all pair states are localized in lattices of infinite size. In particular, we show that previous claims of an interaction-induced mobility edge are biased by severe finite-size effects. The localization length of a pair with zero total energy exhibits a nonmonotonic behavior as a function of the interaction strength, characterized by an exponential enhancement in the weakly interacting regime. Our findings also suggest that the many-body mobility edge of the 2D Anderson-Hubbard model disappears in the zero-density limit, irrespective of the (bosonic or fermionic) quantum statistics of the particles.

I. INTRODUCTION

It is well known that in certain disordered media wave propagation can be completely halted due to the backscattering of the randomly distributed impurities. This phenomenon, known as Anderson localization, has been reported for different kinds of waves, such as light waves in diffusive media or in disordered photonic crystals, ultrasound, microwaves, and atomic matter waves. Its occurrence is ruled by the spatial dimension of the system and by the symmetries of the model, which determine its universality class. When both spin-rotational and time-reversal symmetries are preserved, notably in the absence of magnetic fields and spin-orbit couplings, all wave-functions are exponentially localized in one and two dimensions. In three and higher dimensions the energy spectrum contains both localized and extended states, separated by a critical point, dubbed the mobility edge, where the system undergoes a metal-insulator transition. Anderson transitions have recently been detected using noninteracting atomic quantum gases exposed to three-dimensional (3D) speckle potentials. Theoretical predictions for the mobility edge of atoms have also been reported and compared with the experimental data.

Interactions can nevertheless significantly perturb the single-particle picture of Anderson localization. Puzzling metal-insulator transition, discovered in high-mobility 2D electron systems in silicon, were later interpreted theoretically in terms of a two-parameter scaling theory which combines disorder and strong electron-electron interaction. In more recent years a great interest has emerged around the concept of many-body localization (MBL), namely the generalization of Anderson localization to disordered interacting quantum systems at finite particle density (for recent reviews see ). In analogy with the single-particle problem, MBL phases are separated from (ergodic) thermal phases by critical points located at finite energy density, known as many-body mobility edges.

While MBL has been largely explored in one-dimensional disordered systems with short range interactions, both experimentally and theoretically, its very existence in systems with higher dimensions remains somewhat unclear. In particular it has been suggested that the MBL is inherently unstable against thermalization in large enough samples. This prediction contrasts with subsequent experimental and numerical studies of 2D systems of moderate sizes, showing evidence of a many-body mobility edge. It must be emphasized that thorough numerical investigations, including a finite-size scaling analysis, are computationally challenging beyond one spatial dimension.

In the light of the above difficulties, it is interesting to consider the zero density limit of the many-body problem, focusing on the localization properties of few interacting particles in large (ideally infinite) disordered lattices. Although these systems may represent overly simplified examples of MBL states, they can show similar effects, including interaction-induced delocalization transitions with genuine mobility edges.

In a seminal paper, Shpeleyansky showed that two particles moving in a one-dimensional lattice and coupled by contact interactions can travel over a distance much larger than the single-particle localization length, before being localized by the disorder. This intriguing effect was confirmed by several numerical studies, trying to identify the explicit dependence of the pair localization length on the interaction strength. Quantum walk dynamics of two interacting particles moving in a disordered one-dimensional lattice has also been explored, revealing subtle correlation effects. Interacting few-body systems with more than two particles have also been studied numerically in one dimension, confirming the stability of the MBL phase. For instance Ref. investigated a model of up to three bosonic atoms with mutual contact interactions and subject to a spatially correlated disorder generated by laser speckles, while Ref. addressed the localization in the few-particle regime of the XXZ spin-chain with a random magnetic field.

The few-body problem in higher dimensions has been much less explored. Based on analytical arguments, it
has been suggested\cite{99,100} that all two-particle states are localized by the disorder in two dimensions, whereas in three dimensions a delocalization transition for the pair could occur even if all single-particle states are localized. In contrast, subsequent numerical investigations\cite{111,112} of the two-particle system in two dimensions reported evidence of an Anderson transition of the pair, providing explicit results for the position of the mobility edge and the value of the critical exponent.

Using large-scale numerics, we recently investigated\cite{100,101} Anderson transitions for a system of two interacting particles (either bosons or fermions with opposite spins), obeying the three-dimensional Anderson-Hubbard model. We showed that the phase diagram in the energy-interaction-disorder space contains multiple metallic and insulating regions, separated by two-body mobility edges. In particular we did find metallic pair states in regions where all single-particle states were localized, which can be thought of as a proxy for interaction-induced many-body delocalization. Importantly, our numerical data for the metal-insulator transition were found to be consistent with the (orthogonal) universality class of the noninteracting model (single-particle excitations in a disordered electronic system with Coulomb interaction also undergo a metal-insulator transition which belongs to the noninteracting universality class\cite{102}).

In this work we revisit the Shpelyansky problem in two dimensions and shed light on the controversy. We find that no mobility edge exists for a single pair in an infinite lattice, although interactions can dramatically enhance the pair localization length. In particular we show that previous claims\cite{103,104} of 2D interaction-driven Anderson transitions were plagued by strong finite-size effects.

The paper is organized as follows. In Sec. II we revisit the theoretical approach based on the exact mapping of the two-body Schrödinger equation onto an effective single-particle problem for the center-of-mass motion. The effective model allows to recover the entire energy spectrum of orbitally symmetric pair states and is therefore equivalent to the exact diagonalization of the full Hamiltonian in the same subspace; an explicit proof for a toy Hamiltonian is given in Sec. III. In Sec. IV we present the finite-size scaling analysis used to discard the existence of the 2D Anderson transition for the pair, while in Sec. V we discuss the dependence of the two-body localization length on the interaction strength. Finally in Sec. VI we provide a summary and an outlook.

II. EFFECTIVE SINGLE-PARTICLE MODEL FOR THE PAIR

The Hamiltonian of the two-body system can be written as $\hat{H} = \hat{H}_0 + \hat{U}$, whose noninteracting part $\hat{H}_0$ can be decomposed as $\hat{H}_0 = \hat{H}^p \otimes \hat{1} + \hat{1} \otimes \hat{H}^p$. Here $\hat{1}$ refers to the one-particle identity operator, while $\hat{H}^p$ denotes the single-particle Anderson Hamiltonian:

$$\hat{H}^p = -J \sum_{\langle \mathbf{n}, \mathbf{m} \rangle} \langle \mathbf{n} | \mathbf{m} \rangle \langle \mathbf{m} | \mathbf{n} \rangle + \sum_{\mathbf{n}} V_n \langle \mathbf{n} | \mathbf{n} \rangle,$$

where $J$ is the tunnelling amplitude between nearest neighbor sites $\mathbf{m}$ and $\mathbf{n}$, whereas $V_n$ represents the value of the random potential at site $\mathbf{n}$. In the following we consider a random potential which is spatially uncorrelated $\langle V_n V_{n'} \rangle = \langle V_n^2 \rangle \delta_{nn'}$ and obeys a uniform on-site distribution, as in Anderson’s original work.

$$P(V) = \frac{1}{W} \Theta(W/2 - |V|),$$

where $\Theta(x)$ is the Heaviside (unit-step) function and $W$ is the disorder strength. The two particles are coupled together by contact (Hubbard) interactions described by

$$\hat{U} = U \sum_{\mathbf{m}} |\mathbf{m}, \mathbf{m}\rangle \langle \mathbf{m}, \mathbf{m}|,$$

where $U$ represents the corresponding strength. We start by writing the two-particle Schrödinger equation as $(E - \hat{H}_0) |\psi\rangle = \hat{U} |\psi\rangle$, where $E$ is the total energy of the pair. If $U |\psi\rangle = 0$, then $E$ must belong to the energy spectrum of the noninteracting Hamiltonian $\hat{H}_0$. This occurs for instance if the two-particles correspond to fermions in the spin-triplet state, as in this case the orbital part of the wave-function is antisymmetric and therefore $\langle \mathbf{m}, \mathbf{m} | \psi \rangle = 0$.

Interactions are instead relevant for orbitally symmetric wave-functions, describing either bosons or fermions with opposite spins in the singlet state. In this case from Eq. (3) we find that the wave-function obeys the following self-consistent equation

$$|\psi\rangle = \sum_{\mathbf{m}} U \hat{G}(E) |\mathbf{m}, \mathbf{m}\rangle \langle \mathbf{m}, \mathbf{m}| \psi\rangle,$$

where $\hat{G}(E) = (E \hat{1} - \hat{H}_0)^{-1}$ is the non-interacting two-particle Green’s function. Eq. (4) shows that for contact interactions the wave-function of the pair can be completely determined once its diagonal amplitudes $f_{\mathbf{m}} = \langle \mathbf{m}, \mathbf{m} | \psi \rangle$ are known. By projecting Eq. (4) over the state $|\mathbf{n}, \mathbf{n}\rangle$, we see that these terms obey a closed equation\cite{113,114}:

$$\sum_{\mathbf{m}} K_{\mathbf{nm}} f_{\mathbf{m}} = \frac{1}{U} f_{\mathbf{n}},$$

where $K_{\mathbf{nm}} = \langle \mathbf{n}, \mathbf{n} | \hat{G}(E) |\mathbf{m}, \mathbf{m}\rangle$. Eq. (5) is then interpreted as an effective single-particle problem with Hamiltonian matrix $K$ and pseudo-energy $\lambda = 1/U$, corresponding to the inverse of the interaction strength. In the following we will address the localization properties of this effective model for the pair.

The matrix elements of $K$ are unknown and must be calculated explicitly in terms of the eigen-basis of the
single-particle model, $\hat{H}^{sp}\ket{\phi_r} = \varepsilon_r\ket{\phi_r}$, as

$$K_{nm} = \sum_{r,s=1}^{N} \frac{\phi_{nr}^* \phi_{ms} \phi_{ns} \phi_{mr}}{E - \varepsilon_r - \varepsilon_s}$$  \hspace{1cm} (6)$$

where $N$ is the total number of lattice sites in the grid and $\phi_{nr} = \langle n | \phi_r \rangle$ are the amplitudes of the one-particle wave-functions.

III. EQUIVALENCE WITH EXACT DIAGONALIZATION OF THE FULL MODEL

The effective single-particle model of the pair, Eq. (5), allows to reconstruct the entire energy spectrum of orbitally symmetric states for a given interaction strength $U$. At first sight this is not obvious because the matrix $K$ is $N \times N$, and therefore possesses $N$ eigenvalues, while the dimension of the Hilbert space of orbitally symmetric states is $N(N+1)/2$, which is much larger. The key point is that one needs to compute the matrix $K$ and the associated eigenvalues $\lambda_r = \lambda_r(E)$, with $r = 1, 2 \ldots N$, for different values of the energy $E$. The energy levels for fixed $U$ are then obtained by solving the equations $\lambda_r(E) = 1/U$ using standard root-finding algorithms.

Let us illustrate the above point for a toy model with $N = 2$ lattice sites in the absence of disorder. In this case the Hilbert space of symmetric states is spanned by the three vectors $|1,1\rangle$, $|2,2\rangle$ and $(|1,2\rangle + |2,1\rangle)/\sqrt{2}$. The corresponding energy levels of the pair can be found from the exact diagonalization of the $3 \times 3$ matrix of the projected Hamiltonian:

$$H_{cd} = \begin{pmatrix} U & -\sqrt{2} & 0 \\ -\sqrt{2} & 0 & -\sqrt{2} \\ 0 & -\sqrt{2} & U \end{pmatrix}.$$  \hspace{1cm} (7)$$

An explicit calculation yields $E = U$ and $E = (U \pm \sqrt{U^2 + 16})/2$. Let us now show that we recover exactly the same results using our effective model. The single-particle Hamiltonian is represented by the matrix

$$H^{sp} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$  \hspace{1cm} (8)$$

whose eigenvalues are given by $\varepsilon_1 = -1$ and $\varepsilon_2 = 1$. The associated wavevectors are $\ket{\phi_1} = (|1\rangle + |2\rangle)/2$ and $\ket{\phi_2} = (|1\rangle - |2\rangle)/2$. From Eq. (8) we immediately find

$$K = \begin{pmatrix} A & B \\ B & A \end{pmatrix},$$  \hspace{1cm} (9)$$

where $A = (E/(E^2 - 4) + 1/E)/2$ and $B = (E/(E^2 - 4) - 1/E)/2$. The corresponding eigenvalues of $K$ are given by $\lambda_1(E) = A - B = 1/E$ and $\lambda_2(E) = A + B = E/(E^2 - 4)$. The condition $\lambda_1 = 1/U$ yields $E = U$, while $\lambda_2 = 1/U$ admits two solutions, $E = (U \pm \sqrt{U^2 + 16})/2$, allowing to recover the exact-diagonalization energy spectrum. In Fig. 1 we plot the energy dependence of the two eigenvalues of $K$ for our toy model. Intersecting the curves with the horizontal line $\lambda = 1/U$ (dashed red line) yields visually the three sought energy levels for the orbitally symmetric states.

We stress that extracting the full energy spectrum of the pair based on the effective model, for a fixed value of the interaction strength $U$, is computationally demanding as $N$ becomes large. The effective model is instead very efficient, as compared to the exact diagonalization, when we look at the properties of the pair as a function of the interaction strength $U$, for a fixed value of the total energy $E$. This is exactly the situation that we will be interested in below.

IV. ABSENCE OF 2D DELOCALIZATION TRANSITIONS FOR THE PAIR

Numerical evidence of 2D Anderson transition for two particles obeying the Anderson-Hubbard model in two dimensions was first reported on the basis of transmission-amplitude calculations performed on rectangular strips of length $L = 62$ and variable width up to $M = 10$. For a pair with zero total energy and for interaction strength $U = 1$, the delocalization transition was found to occur for $W = 9.3 \pm 0.5$. The result was also confirmed from the analysis of the energy-level statistics, although with slightly different numbers.

The existence of a 2D mobility edge for the pair was also reported in Ref. 73, where a decimation method was employed to compute the critical disorder strength as a
function of the interaction strength $U$, based on lattices of similar sizes. For $U = 1.59$, a pair with zero total energy was shown to undergo an Anderson transition at $W = 9 \pm 0.13$.

Below we verify the existence of the 2D delocalization transition of the pair based on the transmission-amplitude computation\cite{40}, for the effective model $\mathcal{H}$, following the procedure developed in Ref.\cite{41}. In order to compare with the previous theoretical predictions, we set $E = 0$ and $W = 9$. We consider a rectangular strip of dimensions $L, M$, with $L \gg M$, containing $N = ML$ lattice sites. In order to minimize finite-size effects, the boundary conditions on the single-particle Hamiltonian $H^{\text{op}}$ are chosen periodic in the orthogonal direction $(y)$ and open along the transmission axis $(x)$. We rewrite the rhs of Eq. (6) as

$$K_{nm} = \sum_{r=1} G^{\text{op}}(E - \varepsilon_r) |m\rangle \langle n|, \quad (10)$$

where $G^{\text{op}}(E) = (\varepsilon I - H^{\text{op}})^{-1}$ is the Green’s function (e.g. the resolvent) of the single-particle Anderson Hamiltonian $H^{\text{op}}$, $I$ being the identity matrix. Due to the open boundary conditions along the longitudinal direction, the Anderson Hamiltonian possesses a block tridiagonal structure, each block corresponding to a transverse section of the grid. This structure can be exploited to efficiently compute the Green’s function $G^{\text{op}}(E)$ in Eq. (10) via matrix inversion. In this way the number of elementary operations needed to compute the matrix $K$ scales as $M^4 L^3$, instead of $M^4 L^4$, as naively expected from the rhs of Eq. (6).

Once computed the matrix $K$ of the effective model, we use it to evaluate the logarithm of the transmission amplitude between two transverse sections of the strip as a function of their relative distance $n_x$:

$$F(n_x) = \ln \sum_{m_y, n_y} |\langle 1, m_y | G^{\text{op}}(\lambda) | n_x, n_y \rangle|^2. \quad (11)$$

In Eq. (11) $G^{\text{op}}(\lambda) = (\lambda I - K)^{-1}$ is the Green’s function associated to $K$ with $\lambda = 1/U$ and the sum is taken over the sites $m_y, n_y$ of the two transverse sections.

For each disorder realization, we evaluate $F(n_x)$ at regular intervals along the bar and apply a linear fit to the data, $f_{fit}(n_x) = p n_x + q$. For a given value of the interaction strength, we evaluate the (disorder-averaged) Lyapunov exponent $\gamma = \gamma(M, U)$ as $\gamma = -\overline{p}/2$, where $\overline{p}$ is the average of the slope. We then infer the localization properties of the system from the behavior of the reduced localization length, which is defined as $\Lambda = (M \gamma)^{-1}$. In the metallic phase $\Lambda$ increases as $M$ increases, whereas in the insulating phase the opposite trend is seen. At the critical point, $\Lambda$ becomes constant for values of $M$ sufficiently large. Hence the critical point $U = U_c$ of the Anderson transition can be identified by plotting the reduced localization length versus $U$ for different values of the transverse size $M$ and looking at their common crossing points.

In Fig. 2 we show the reduced localization length $\Lambda$ as a function of the interaction strength for increasing values of the strip width, ranging from $M = 8$ to $M = 20$. The length of the grid is fixed to $L = 400$. Notice that, since $E = 0$, the reduced localization length is an even function of the interaction strength, $\Lambda(-U) = \Lambda(U)$. We see that $\Lambda$ exhibits a nonmonotonic dependence on $U$, as previously found in the one-dimensional\cite{42} and in the three-dimensional\cite{75} versions of the Anderson-Hubbard model. In particular, interactions favor the delocalization of the pair, the effect being more pronounced near $U = 6$. We also notice from Fig. 2 that the curves corresponding to different values of $M$ intersect each others around $U = 1$, suggesting indeed a possible phase transition, as previously reported in Refs.\cite{76,77}. A closer inspection of the data, however, reveals that the crossing points are spread out in the interval $0.73 < U < 1.1$; in particular, they drift to stronger interactions as the system size increases, in analogy with the three-dimensional case studied earlier\cite{78}.

A key question is whether a further increase of the strip’s width $M$ will only cause a (possibly large) shift of the critical point, or rather, the localized phase will ultimately take over for any value of the interaction strength. To answer this question, we have performed additional calculations using larger grids, corresponding to $M = 30, 40, 50$. In order to guarantee a sufficiently large aspect ratio, the length of the bar was fixed to $L = 500$. The obtained results are displayed in Fig. 3. We notice that the crossing points have completely disappeared and...
the pair is ultimately localized by the disorder, irrespective of the value of the interaction strength. This leads us to conclude that the results of Refs. 71, 73 were plagued by severe finite-size effects and no Anderson transition can actually take place for a pair in a disordered lattice of infinite size.

V. PAIR LOCALIZATION LENGTH

Although the pair cannot fully delocalize in two dimensions, interactions can lead to a drastic enhancement of the two-particle localization length. This quantity can be estimated using the one-parameter scaling ansatz $\Lambda = f(\xi / M)$, stating that the reduced localization length depends solely on the ratio between two quantities: the width $M$ of the strip and a characteristic length $\xi = \xi(U, W, E)$, which instead depends on the model parameters and on the total energy of the pair (but not on the system sizes $L, M$). This latter quantity coincides, up to a multiplicative numerical constant $a$, with the sought pair localization length, $\xi = a\tilde{\xi}$.

We test the scaling ansatz for our effective model 5 using the numerical data for $M = 30, 40, 50$ displayed in Fig. 3 corresponding to the largest system sizes. Let $U_j$, with $j = 1, \ldots, N_U$, be the values of the interaction strength used to compute the reduced localization length (in our case $N_U = 44$). We then determine the corresponding unknown parameters $\xi(U = U_j)$ through a least squares procedure, following the procedure developed in Ref. 22. Plotting our data in the form $\ln \Lambda(M, U)$ vs $\ln M$ results in multiple data curves, each of them containing three data points connected by straight lines (corresponding to linear interpolation). Let $\Lambda_i$ be one of the $(3N_U)$ numerical values available for the reduced localization length. The horizontal line $\ln \Lambda = \ln \Lambda_i$ will generally intersect some of these curves. We find convenient to introduce a matrix $\eta$ which keeps track of such events: if the curve $U = U_j$ is crossed, we set $\eta_{ij} = 1$ and call $\ln M_{ij}$ the corresponding point; otherwise we set $\eta_{ij} = 0$.

The unknown parameters are then obtained by solving the following set of equations:

$$\sum_j \left\{ \sum_i \eta_{ij} \left( \frac{1}{N_i} - \frac{\delta_{ij}}{N_i} \right) \right\} \ln \tilde{\xi}(U_j) =$$

$$= \sum_j \left\{ \sum_i \eta_{ij} \left( \frac{1}{N_i^2} - \frac{\delta_{ij}}{N_i} \right) \right\} \ln M_{ij}, \quad (12)$$

where $N_i = \sum_j \eta_{ij}$ is the total number of crossing points obtained for each $\Lambda_i$ value. Eq. (12) is of the form $AX = B$ and can easily be solved. Notice however that the solution is not unique because the matrix $A$ is singular. Indeed the correlation length $\xi(U)$ is defined up to a multiplicative constant, $\xi \rightarrow a\tilde{\xi}$, implying that $\ln \tilde{\xi}$ is defined up to an additive constant, $\ln \xi \rightarrow \ln \xi + \ln a$.

In Fig. 4 we verify the correctness of the scaling ansatz, by plotting the reduced localization length as a function of the ratio $\xi / M$, where $\xi$ is obtained from the solution of Eq. (12). We see that our numerical data for different values of the interaction strength and system size do collapse on a single curve, thus confirming the scaling hypothesis.

In the main panel of Fig. 5 we plot the unnormalized localization length of the pair as a function of the in-
The localization length of the Anderson model and $U \xi$ in the weakly interacting regime should obey the relation

$$\Lambda = \xi + c \left( \frac{\xi}{M} \right)^2,$$  \hspace{1cm} (13)

where $c$ is a number. In our case the most localized states are those at weak interactions, where the reduced localization length takes its minimum value. For each values $U = U_j$ falling in this region, we fit our numerical data according Eq. (13), yielding $\xi = \xi(U)$. The estimate of the multiplicative constant, which is defined as $a_{\text{est}} = \xi(U)/\xi(U)$, is displayed in the inset of Fig. 5. Since the estimate of $a$ does not saturates for small $U$, we conclude that, even for the weakest interactions and the largest system sizes considered, the pair has not yet entered the strongly localized regime underlying Eq. (13). The latter is typically achieved for $\Lambda \lesssim 0.1$, whereas our smallest value of the reduced localization length is $\Lambda(M = 50, U = 0.5) \approx 0.2929$. From the inset of Fig. 5 we also see that $a_{\text{est}}$ increases as $U$ diminishes, suggesting that the result obtained for $U = 0.5$ actually provides a lower bound for the multiplicative constant. This allows us to conclude that $a \geq 18.2$.

VI. CONCLUSION AND OUTLOOK

Based on an efficient mapping of the two-body Schrödinger equation, we have addressed the localization properties of two bosons or two fermions in a spin-singlet state obeying the 2D Anderson-Hubbard model. We have found that no interaction-induced Anderson transition occurs for disordered lattices of infinite size in contrast with previous numerical works, which we have shown to be biased by finite-size effects. In this way we reconcile the numerics with the one-parameter scaling theory of localization, predicting the absence of one-particle Anderson transition in two dimensions, in the presence of both time reversal and spin rotational symmetries. The localization length exhibits a non nonmonotonic behavior as a function of $U$, characterized by an exponential growth for weak interactions.

We point out that the absence of the 2D mobility edge for the two-particle system has been proven for the case of contact interactions; similar conclusions should apply also for short but finite-range interactions. The case of true long-range (e.g Coulomb) interactions is conceptually different and can lead to opposite conclusions.

A compelling problem is to investigate the implications of our results for a 2D system at finite density of particles, where many-body delocalization transitions have instead been observed both numerically and experimentally in the strongly interacting regime. We expect that, in the zero density limit, the many-body mobility edge disappears, irrespective of the bosonic or fermionic statistics of the two particles.

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FIG. 5. Unnormalized localization length $\xi$ of the pair plotted as a function of the interaction strength. Notice the logarithmic scale in the $y$ axis, showing that interactions can enhance the 2D localization length of the pair by more than three orders of magnitude. The inset displays the estimate of the multiplicative constant $a$, fixing the absolute scale of the localization length, plotted as a function of the interaction strength. The estimate is obtained by fitting the numerical data in Fig. 3 corresponding to weak interactions using Eq. (13), from which we extract $a_{\text{est}} = \xi/\xi$. This quantity keeps increasing as $U$ diminishes, signaling that the strongly localized regime is not fully reached in our simulations.
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