Gravitationally Induced Neutrino–Oscillation Phases

D. V. Ahluwalia

Mail Stop H–846, Physics (P–25) and Theory (T–5) Division
Los Alamos National Laboratory, Los Alamos, NM 87545, USA

C. Burgard

Universität Hamburg/DESY, II. Institut für Experimentalphysik
Notkestr.85, D-22607 Hamburg, Germany

Abstract

In this essay, we introduce a new effect of gravitationally induced quantum mechanical phases in neutrino oscillations. These phases arise from an hitherto unexplored interplay of gravitation and the principle of the linear superposition of quantum mechanics. In the neighborhood of a 1.4 solar–mass neutron star, gravitationally induced quantum mechanical phases are roughly 20% of their kinematical counterparts. When this information is coupled with the mass square differences implied by the existing neutrino–oscillation data we find that the new effect may have profound consequences for type-II supernova evolution.

Neutrinos [1,2] play a profound role on almost all length scales. These scales range from the evolution of the nuclei to the evolution of biological structures [3], to the evolution of the stars, the galaxies, and the universe [4,5]. The detection [6] of an ancient neutrino burst on February 23, 1987 provided a dramatic confirmation of the essential elements of the physics of type-II supernovae [7] and placed an upper limit of about 10 eV on the mass of the electron neutrino [8]. The detection of solar neutrinos in terrestrial detectors provides a similarly convincing confirmation of our understanding of the solar evolution [9]. These important observational triumphs mark the beginning of a new era: The era of neutrino astronomy. The observations on the solar neutrinos have already given birth to the more controlled accelerator– and reactor–based neutrino oscillation experiments.

1 E-mail address: ahluwalia@nus.lampf.lanl.gov.
2 E-mail address: christoph.burgard@cern.ch.
For decades the solar neutrino anomaly [10] has indicated that the neutrino flavor eigenstates may be a linear superposition of mass eigenstates [11]. This view has been further strengthened by the data on atmospheric neutrinos [12], and most recently by the observation of excess $\nu_e$ observed at the Liquid Scintillator Neutrino Detector (LSND) Neutrino Oscillation Experiment (NOE) at LAMPF [13]. The excess events observed at LSND NOE have been tentatively interpreted as $\nu_\mu \rightarrow \nu_e$ oscillations [13].

At the present time, (a) the question of neutrino masses [14], (b) the relation of the neutrinos to the spacetime symmetries as manifested in Dirac [15], or Majorana [16–18], or other fundamental constructs appropriate to the neutral particles [19], (c) the question of CP violation in the leptonic sector [21], and (d) other related issues that extend the physics beyond the standard model [22], are being probed with intense theoretical and experimental vigor.

Gravitation plays no direct role on neutrino oscillations in the existing literature. In this essay we shall introduce the notion of gravitationally induced quantum mechanical neutrino–oscillation phases and discuss the possibility of their observation for type-II supernova.

A bit of history is necessary at this point. In one of the classic experiments of physics, Colella, Overhauser, and Werner (COW) established that quantum mechanics and gravitation, despite the well–known conceptual problems, behave in a manner expected for any other interaction [26]. Given the fact that the experiment involved thermal neutrons (non-relativistic quantum realm) and the Earth’s gravitational field (weak gravity), this may not be too unexpected. Nevertheless, theoretical investigation of this elegant experiment provides a deep understanding of gravitation in the context of quantum mechanics [27]. From a formal point of view the COW experiment studies the effect of gravitation on the quantum mechanical evolution of a single–mass eigenstate. For neutrinos, when we extend the COW–like considerations to the linear superposition of mass eigenstates, a new gravitationally induced quantum mechanical effect emerges. Despite the similarities between the COW effect and the new effect introduced here, there are important conceptual differences between the two effects. These differences shall also be enumerated at the appropriate place in this essay. In a certain sense (to become precise below), the two effects shall be seen to be complementary.

Let us assume that in the “creation region,” $\mathcal{R}_c$, located at $\vec{r}_c$, a weak

---

3 However, see (a) Ref. [23], where Violation of Equivalence Principle (VEP) in the context of neutrino oscillation experiments is considered; (b) the work of Goldman et al. on “Kaons, Quantum Mechanics, and Gravity” in Ref. [24] (Ref. [24] also contains references to many classic works on the subject); and (c) the work of J. Anandan on the gravitational phase operator [25].

4 The creation region $\mathcal{R}_c$ is assumed fixed in the global coordinate system.
eigenstate with energy $E_\nu$, denoted by $|\nu_\ell, \mathcal{R}_c\rangle$, is produced with the clock set to $t = t_c$. Each of the three neutrino mass eigenstates shall be represented by $|\nu_i\rangle; i = 1, 2, 3$. So we have the linear superposition:

$$|\nu_\ell, \mathcal{R}_c\rangle = \sum_{i=1,2,3} U_{\ell i} |\nu_i\rangle,$$  \hspace{1cm} (1)

where $\ell = e, \mu, \tau$ represents the weak flavor eigenstates (corresponding to electron, muon, and tau neutrinos respectively).\(^5\) The $|\nu_1\rangle$, $|\nu_2\rangle$, and $|\nu_3\rangle$ correspond to the three mass eigenstates of masses $m_1$, $m_2$, and $m_3$, respectively. Under the already–indicated assumptions the $3 \times 3$ unitary mixing matrix $U_{\ell i}$ may be parameterized by three angles and reads [20, Eq. 6.21 with the CP phase $\delta = 0$]:

$$U(\theta, \beta, \psi) = \begin{pmatrix}
    c_\theta c_\beta & s_\theta c_\beta & s_\beta \\
    -c_\theta s_\beta s_\psi - s_\theta c_\psi & c_\theta c_\psi - s_\theta s_\beta s_\psi & c_\beta s_\psi \\
    -c_\theta s_\beta c_\psi + s_\theta s_\psi & -s_\theta s_\beta c_\psi - c_\theta s_\psi & c_\beta c_\psi
\end{pmatrix},$$ \hspace{1cm} (2)

where $c_\xi = \cos(\xi)$, $s_\xi = \sin(\xi)$, with $\xi = \theta, \beta, \psi$.

At a later time $t = t_d > t_c$, we wish to study the weak flavor eigenstate in the “detector region,” $\mathcal{R}_d$, located at $\vec{r}_d$.\(^6\) The neutrino evolution from $\mathcal{R}_c$ to $\mathcal{R}_d$ is given by the expression:

$$|\nu_\ell, \mathcal{R}_d\rangle = \exp\left(-\frac{i}{\hbar} \int_{t_c}^{t_d} H dt + \frac{i}{\hbar} \int_{\vec{r}_c}^{\vec{r}_d} \vec{P} \cdot d\vec{x}\right) |\nu_\ell, \mathcal{R}_c\rangle.$$ \hspace{1cm} (3)

Here $H$ is the time translation operator (the Hamiltonian) associated with the system; $\vec{P}$ is the operator for spatial translations (the momentum operator), and $[H(t, \vec{x}), \vec{P}(t, \vec{x})] = 0$. Consider that both $\mathcal{R}_c$ and $\mathcal{R}_d$ are located in the Schwarzschild gravitational environment [4] of a spherically symmetric object of mass $M$. The direction of neutrino propagation is along $\vec{L} = \vec{r}_d - \vec{r}_c$.

\(^5\) We shall assume that CP is not violated for the purposes of this analysis. Neutrinos shall be assumed to be of the Dirac type (for a recent analysis of various quantum field theoretic possibilities for the description of neutral particles of spin–1/2 and higher, and their relation with space–time symmetries, see Ref. [19] and references therein). In addition, we shall assume that both $\nu_\ell$ and $\nu_m$ are relativistic in the frame of the experimenter.

\(^6\) Like the creation region, the detection region $\mathcal{R}_d$ too is fixed in the global coordinate system.
We now confine to the weak field limit, and neglect the spin–dependent terms for the present, as we do not wish to study effects of the astrophysical magnetic fields, or the effects of interaction between spin (of neutrino) and angular momentum (of the astrophysical object). Under these conditions, the work of Stodolsky implies [28]:

\[
\exp \left( -\frac{i}{\hbar} \int t_c^d H dt + \frac{i}{\hbar} \int \vec{r}_c^d \cdot d\vec{x} \right) |\nu_\ell\rangle = \exp \left[ -\frac{i}{\hbar} \int_{\mathcal{R}_c}^{\mathcal{R}_d} \left( \eta_{\mu\nu} + \frac{1}{2} h_{\mu\nu} \right) p^\mu \, dx^\nu \right] |\nu_\ell\rangle, \quad (4)
\]

where \( h_{\mu\nu} = g^W_{\mu\nu} - \eta_{\mu\nu} \); \( g^W_{\mu\nu} \) is the Schwarzschild space–time metric in the weak field limit and \( \eta_{\mu\nu} \) is the flat space–time metric. In addition, \( h_{\mu\nu} = 2 \phi \delta_{\mu\nu} \) with the dimensionless gravitational potential \( \phi = -GM/(c^2 r) \). Further, it shall be noted that \( p^\mu_\ell \) is the four–momentum of special relativity with \( p^\mu \equiv m dx^\mu/ds_0 \) in the notation of Ref. [28]. The limits of applicability of this formalism are further enumerated in the above–cited paper of Stodolsky. To avoid notational confusion we remind the reader that \( p_\ell \equiv |\vec{p}_\ell| \) — the subscript \( \ell \) identifies the mass eigenstate, and does not refer to \( \ell \)th component of the momentum vector.

We now calculate the “neutrino oscillation probability” from a state \( |\nu_\ell, \mathcal{R}_c\rangle \) to another state \( |\nu_\ell', \mathcal{R}_d\rangle \) following closely the standard arguments, appropriately adapted to the present situation [20,29,30]. The oscillation probability is obtained by calculating the projection \( \langle \nu_\ell', \mathcal{R}_d | \nu_\ell, \mathcal{R}_c \rangle \), i.e., the amplitude for \( |\nu_\ell, \mathcal{R}_c\rangle \rightarrow |\nu_\ell', \mathcal{R}_d\rangle \), and then multiplying it by its complex conjugate. An algebraic exercise that (a) exploits the unitarity of the neutrino mixing matrix \( U(\theta, \beta, \psi) \), (b) exploits orthonormality of the mass eigenstates, (c) exploits certain trigonometric identities, and (d) takes care of the fact that now \( dx \) and \( dt \) are related by

\[
dx \simeq \left[ 1 - \left( \frac{2GM}{c^2 r} \right) \right] c dt , \quad (5)
\]

yields:

\[
P[|\nu_\ell, \mathcal{R}_c\rangle \rightarrow |\nu_\ell', \mathcal{R}_d\rangle] = \delta_{\ell\ell'} - 4 U_{\ell1} U_{\ell1} U_{\ell2} U_{\ell2} \sin^2 \left[ \varphi^0_{21} + \varphi^G_{21} \right] - 4 U_{\ell1} U_{\ell1} U_{\ell3} U_{\ell3} \sin^2 \left[ \varphi^0_{31} + \varphi^G_{31} \right] - 4 U_{\ell2} U_{\ell2} U_{\ell3} U_{\ell3} \sin^2 \left[ \varphi^0_{32} + \varphi^G_{32} \right] \quad (6)
\]

The arguments of \( \sin^2(\cdots) \) in the neutrino oscillation probability now contain two types of phases. The usual kinematic phase, denoted here by \( \varphi^0_{ji} \), and
defined as
\[ \phi^0_\mu \equiv \frac{c^3}{4\hbar} \frac{|\vec{r}_d - \vec{r}_c| \Delta m^2_{\mu}}{E} = \frac{c^3 L \Delta m^2_{\mu}}{4\hbar E} ; \] (7)

and the new *gravitationally induced quantum mechanical phase*, denoted here by \( \phi^G_\mu \), and defined as
\[ \phi^G_\mu \equiv \frac{GM c}{4\hbar} \left[ \int_{\vec{r}_c}^{\vec{r}_d} \frac{dL}{r} \right] \frac{\Delta m^2_{\mu}}{E} . \] (8)

It is readily seen that the gravitationally induced phases and the kinematic phases, for neutrino oscillations, are related via the identity
\[ \phi^G_\mu = - \langle \phi \rangle \phi^0_\mu , \] (9)

where \( \langle \phi \rangle \) is the average dimensionless gravitational potential over the semi-classical neutrino path
\[ \langle \phi \rangle \equiv - \frac{1}{L} \int_{\vec{r}_c}^{\vec{r}_d} dL \frac{GM}{c^2 r} . \] (10)

Two immediate questions seem relevant. First, what are the conceptual similarities, and differences, between the neutron interferometer experiment of COW and the phenomenon of neutrino oscillations in the presence of gravity? Second, what are the astronomical, or experimental chances, of detecting the gravitationally induced modification to neutrino oscillations? We shall discuss these two questions in turn, and then proceed to explore the astrophysical consequences of our study.

The COW experiment studies the effect of gravitation on a *single mass eigenstate* (i.e., neutron of mass \( m_n \approx 940 \text{ MeV} \)). The observable physical effect arises because *spatially distinct* parts (spatial spread \( \simeq \) a few cm) of the wave function pick up different gravitationally induced quantum mechanical phases. In the case of neutrino oscillations the gravitationally induced modification to the \( |\nu_\ell \rangle \rightarrow |\nu_\ell' \rangle \) oscillations arises from the difference in phase that *each of the different mass eigenstates* picks in a given gravitational environment. *No spatial spread* of the wave function is required. Thus, the formal difference between the effect induced by gravity in the COW experiment, and the one induced in neutrino oscillations, is that in the former the different strengths of the gravitational field at different locations (and interacting with the different superimposed amplitudes of the wave function associated with a neutron
mass eigenstate) manifest into a physically observable result; whereas in the latter the different gravitational interaction energies–momenta of the respective mass eigenstates leave their trace in the gravitationally induced quantum mechanical phases. The magnitude of the two effects is also affected by the non-relativistic nature of the neutrons in the COW experiment, and the extreme relativistic nature of the neutrinos.

An important question in the context of our present discussion is related to the observability of the new effect. Let us again note that in the COW experiment the relevant quantity, apart from Earth’s gravitational field, is the product of mass of neutron and the (vertical) spatial spread of the wave function. For neutrino mass eigenstate of the order of an eV (nine orders of magnitude below the neutron mass) one may expect that if the flavor eigenstate of a neutrino is allowed to travel a (vertical or horizontal!) distance of the order of $10^9$ cm (actually ten to a hundred times less than this will do because of the fact that the COW experiment saw a shift in fringes by a rather large number) one may see gravitationally induced effect on neutrino oscillations in terrestrial experiments (such as those involving atmospheric neutrinos [12]). However, this is not so. The reason is that the COW experiment used thermal neutrons (i.e., non-relativistic particles) and hence the effect was proportional to the mass of the neutron. For the MeV–GeV neutrinos (i.e., relativistic particles), as is the case for the accelerator and the atmospheric neutrinos, the effect is proportional to mass squared differences. As a result, the effect may only be seen in astrophysical environments.

The above considerations complete the formal aspects of this paper. We now turn to what is essentially a back–of–the–envelope exploration of the astrophysical consequences.

In the context of supernova explosions, and the problem of obtaining successful explosions, we follow Colgate et al. [7] and assume that the matter next to the neutron star is heated by neutrinos from the cooling neutron star. Colgate et al. note that in some models “this results in strong, large scale convective flows in the gravitational field of the neutron star that can drive successful, albeit weak, explosions.” Now we recall that the energy flux in each of the electron neutrinos and antineutrinos is about $L_{\nu_e} \approx L_{\bar{\nu}_e} \approx \text{few } \times 10^{52}$ ergs s$^{-1}$, with comparable fluxes of $\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$ [31]. A particularly relevant fact about these fluxes is that while average energy of $\nu_e$ is about 10 MeV, the average energy of other neutrinos may be higher by a factor of about 2 to 3. Because of the extremely large fluxes, and different energies (and hence different cross sections) associated with the neutrinos, even a small variation in the neutrino oscillation probabilities may profoundly affect the success of the supernova explosion.
explosion provided at least one of the $\lambda_{\text{osc}}$ has appropriate length scale.\footnote{In neutrino oscillation literature “oscillation length” is defined as $\lambda_{\text{osc}} = \frac{2\pi}{\eta \frac{E}{\Delta m_{\text{osc}}^2}}$, where $\Delta m_{\text{osc}}^2$ are measured in eV$^2$, $E$ in MeV, and $\eta = 1.27$.}

The existing experiments imply that the two independent mass square differences [32] that define $\varphi_{\mu}^0$ are as follows: $\Delta m_{21}^2 \approx 10^{-2}$ eV$^2$ if the zenith–angle dependence of the atmospheric neutrino anomaly is explained by neutrino oscillations (or [33], $\Delta m_{21}^2 \approx 10^{-5}$ eV$^2$ if the energy dependence of the solar neutrino deficit is explained by invoking the MSW phenomenon [34]) and $\Delta m_{32}^2 \approx 0.5$ eV$^2$ (and opposite these values if the inverted mass hierarchy is considered [35]). Taking the typical energy of a supernova neutrino to be 10 MeV, these mass square differences yield oscillation lengths $\lambda_{\text{osc}} \approx 5$ km (or, $\lambda_{\text{osc}} \approx 5 \times 10^3$ km) and $\lambda_{\text{osc}} \approx 100$ m. These length scales are certainly relevant to the supernova–evolution processes of neutrino diffusion, neutrino trapping, and neutrino heating.\footnote{For details on neutrino heating in the context of supernova explosion we refer the interested reader to Ref. [7], and on supernova–evolution processes of neutrino diffusion and neutrino trapping the reader may find Ref. [36] very valuable.}

These length scales, and hence the associated supernova processes (and very importantly the compatibility arguments between terrestrial neutrino oscillations and supernova evolution [33]), will be altered if we find $\varphi_{\mu}^G$ equals a few percent of $\varphi_{\mu}^0$.

To see if the neutrino oscillation phases can be altered at a level of a few percent in the neighborhood of the neutron star we consider the radially outward motion of a neutrino, and set $\vec{r}_d = \alpha \vec{r}_c$, $1 < \alpha \leq \infty$, then we find

$$\varphi_{\mu}^G \parallel = \frac{GM_c \Delta m_{\mu}^2}{4\hbar} \frac{\ln(\alpha)}{E} \left[ \frac{GM}{c^2 r_c} \frac{\ln \alpha}{\alpha - 1} \right] \varphi_{\mu}^0. \quad (11)$$

For motion transverse to the radial direction (and in the vicinity of $\vec{r}_c$), the corresponding expression is

$$\varphi_{\mu}^G \perp = \frac{GM_c \Delta m_{\mu}^2}{4\hbar} \frac{|\vec{r}_d - \vec{r}_c|}{r_c} = \left[ \frac{GM}{c^2 r_c} \right] \varphi_{\mu}^0. \quad (12)$$

Taking $M$ to be 1.4 solar mass, $1.4 M_\odot$, and $r_c = 10$ km, we have

$$\varphi_{\mu}^G \parallel = 0.21 \left( \frac{\ln(\alpha)}{\alpha - 1} \right) \varphi_{\mu}^0, \quad \varphi_{\mu}^G \perp = 0.21 \varphi_{\mu}^0. \quad (13)$$
We thus find that for a 1.4 solar mass neutron star, with a radius of ten kilometers, $\varphi^C_G\approx G\approx 20\%$ of the $\varphi^0_G$. In astrophysical situations matter effects, and the presence of magnetic fields (if neutrinos have non-zero magnetic moments), will further alter neutrino oscillations [34,37,38]. The gravitationally induced neutrino–oscillation phases arise from an hitherto unexplored interplay of gravitation and the principle of the linear superposition of quantum mechanics and cannot be ignored in many astrophysical environments.

Acknowledgments We wish to gratefully acknowledge useful comments of Yuval Grossman, and the comments of Harry Lipkin forwarded by him to us, on the first draft of this manuscript. These comments led to a correction in the sign of the effect introduced here. We also thank them for providing us with a rough draft of their notes where, after reading our work, they reproduced our results in a covariant analysis. In addition, one of us (DVA) wishes to record the following acknowledgements. I wish to extend my zimpoic thanks to Terry Goldman, Bill Louis, John McClelland, Hywel White, and Nu Xu for our continuing conversations on neutrino oscillations and other matters of physics. With Nu I also realised that a dark photon night is a bright neutrino day, for this too I thank him warmly. Warm thanks are extended to Jeanne Bowles for many stylistic suggestions. Affectionate thanks are due to Marianne Raish for her good cheer and help in taking care of non-academic matters, thus allowing me to devote myself to these studies.

This work was done, in part, under the auspices of the U. S. Department of Energy; and supported by facilities and financial support of the groups P-25 and T-5 of the Los Alamos National Laboratory.

References

[1] Pauli, W. (1930). Letter to Meitner, L. et al., reproduced in Collected Scientific Papers by Wolfgang Pauli (Wiley Interscience, USA), Vol. 2, Kronig, R. and Weisskopf, V. F., eds., p. 1316. English translation: In Inward Bound (Oxford University Press, UK), by Pais A., p. 315.

[2] Reines, F., and Cowan Jr., C. L. (1953). Phys. Rev. 92, 830.

[3] Collar, J. I. (1996). Phys. Rev. Lett. 76, 999.

[4] Weinberg, S. (1972). Gravitation and Cosmology (John Wiley & Sons, USA).

[5] Kolb, E. W., and Peccei, R. D., eds. (1995). Proceedings of the 1994 Snowmass Summer Study: Particle and Nuclear Astrophysics and Cosmology in the Next
[6] Hirata, K. et al. (1987). *Phys. Rev. Lett.*, **58**, 1490.
Bionta, R. M. (1987). *Phys. Rev. Lett.*, **58**, 1494.
Aglietta, M. et al. (1987). In *The Standard Model, The Supernova 1987a*, J. Tran Thanh Van, ed., Proceedings of the Leptonic Session of the Twenty-Second Rencontre de Moriond (Editions Frontières, France).
Alexeyev, E. N. et al. (1987). In *The Standard Model, The Supernova 1987a*, J. Tran Thanh Van, ed., Proceedings of the Leptonic Session of the Twenty-Second Rencontre de Moriond (Editions Frontières, France).

[7] Colgate, S. A., Herant, M., and Benz, W. (1993). *Phys. Rep.*, **227**, 157.

[8] Roos, M (1987). In *The Standard Model, The Supernova 1987a*, J. Tran Thanh Van ed., Proceedings of the Leptonic Session of the Twenty-Second Rencontre de Moriond (Editions Frontières, France).
Bahcall, J. N., and Glashow, S. L. (1987). *Nature* **326**, 476.

[9] Bahcall, J. (1994). *Neutrino Astrophysics* (Cambridge University Press, U.K.).

[10] Results from SAGE: Abdurashitov, J. N. et al. (1994). *Phys. Lett. B* **328**, 234.
Results from GALLEX: Anselmann, P. (1994). *Phys. Lett. B* **327**, 377.
Results on $^8$B Solar Neutrinos from Kamiokande II: Hirata, K. S. et al. (1991). *Phys. Rev. D* **44**, 2241.
Review of the Homestake Solar Neutrino Experiment ($^37$Cl Experiment): Davis, R. (1994). *Prog. Part. Nucl. Phys.* **32**, 13.

[11] Pontecorve, B. (1957). *Zh. Eksp. Theor. Fiz.* **33**, 549 [English translation: (1958) *Sov. Phys. JETP* **6**, 429].

[12] Fukuda, Y. et al. (1994). *Phys. Lett. B* **335**, 237.

[13] Athanassopoulos, C. et al. (1995). *Phys. Rev. Lett.* **75**, 2650.
For an alternate analysis (based on the partial data from LSND), see Hill, J.E. (1995). *Phys. Rev. Lett.* **75**, 2654.

[14] Rosen, S. P. (1995). In *Symmetries and Fundamental Interactions in Nuclei*, Haxton, W. C., and Henley, E. M., eds. (World Scientific, Singapore), pp. 251–302.

[15] Dirac, P. A. M. (1928). *Proc. Roy. Soc. (London) A* **117**, 610; ibid., **A118**, 351.

[16] Majorana, E. (1937). *Nuovo Cimento* **14**, 171. For an English translation of this classic work, the reader should refer to technical translation TT-542, National Research Council of Canada. The reader may also contact one of
the authors to obtain a copy of the English translation of this and two other Italian language works (Racah’s paper [17], and a 1932 paper of Majorana [18]) on the subject.

[17] Racah, G. (1937). *Nuovo Cimento* 14, 322. Also see, Yang, C. N., and Tiomno, J. (1950). *Phys. Rev.* 79, 495.

[18] Majorana, E. (1932). *Nuovo Cimento* 9, 355. Account in English: Fradkin, D.M. (1966). *Am. J. Phys.* 34, 314. English translation: Orzalesi, C. A. (1968). University of Maryland (Department of Physics and Astronomy) Technical Report number 792.

[19] Ahluwalia, D. V. (1996). *Int. J. Mod. Phys. A* 11, 1855. Dvoeglazov, V. V. (1996). *Int. J. Theo. Phys.* 34, 2467.

[20] Kim, C. W. and Pevsner, A. (1993). *Neutrinos in Physics and Astrophysics* (Harwood Academic Publishers, USA).

[21] Ghuza, J., and Zraelek, M. (1995). *LANL Preprint Archive*: hep-ph/9502284. del Aguila, F., and Zraelek, M. (1995). *Nucl. Phys. B* 447, 211.

[22] Mohapatra, R. N., and Pal, P. B. (1991). *Massive Neutrinos in Physics and Astrophysics* (World Scientific, Singapore).

[23] Mann, R. B., and Sarkar, U. (1996). *Phys. Rev. Lett.* 76, 865. Gasperini, M. (1988). *Phys. Rev. D* 38, 2635.; (1989) 39, 3606. Halprin, A., Leung, C. N., and Pantaleone, J. (1995). *LANL Preprint Archive*: hep-ph/9512220.

[24] Goldman, T., Nieto, M. M., and Sandberg, V. D. (1992). *Mod. Phys. Lett. A* 7, 3455.

[25] Anandan, J. (1996). *Phys. Rev. D* 53, 779. Anandan, J. (1980). In *Quantum Theory and Gravitation*, A. R. Marlow, ed., (Academic Press, USA), p. 157. Also see, Corichi, A. and Pierri, M. (1995). *Phys. Rev. D* 51, 5870.

[26] Colella, R., Overhauser, A. W., and Werner S. A. (1975). *Phys. Rev. Lett.* 34, 1472.

[27] Greenberger, D. M., and Overhauser, A. W. (1979). *Rev. Mod. Phys.* 51, 43. For an elementary, but insightful, textbook treatment, see: Sakurai, J. J. (1985). *Modern Quantum Mechanics* (The Benjamin/Cummings Publishing Co., USA) pp. 126–129.

[28] Stodolsky, L. (1979). *Gen. Rel. and Grav.* 11, 391.

[29] Lipkin, H. (1995). *Phys. Lett. B* 348, 604.

[30] Goldman, T (1996). *LANL archive preprint*: hep-ph/9604357.

[31] Bethe, H. A., and Wilson, J. R. (1985). *Astrophs. J.* 295, 14.
Ahluwalia, D. V. (1995). Talk, at *Santa Fe Workshop on Massive Neutrinos and their Implications*, Santa Fe, July 24 – August 11, 1995. Ahluwalia, D.V., and Burgard, C. (1996). Forthcoming.

Cardall, C. Y., and Fuller, G. M. (1996). *Phys. Rev. D* (in press).
Qian, Y-Z et al. (1993), *Phys. Rev. Lett.* 71, 1965; *LANL Preprint Archive: astro-ph/9602104.*
Jegerlehner, B., Neubig, F., and Raffelt, G. (1996). *LANL Preprint Archive: astro-ph/9601111.*

Mikheyev, S. P., and Smirnov, A. Yu. (1985). *Sov. J. Nucl. Phys.* 42, 913.
Wolfenstein, L. (1978). *Phys. Rev. D* 17, 2369.

Fuller, G. M. (1995). Talk, at *Santa Fe Workshop on Massive Neutrinos and their Implications*, Santa Fe, July 24 – August 11, 1995.
Qian, Y.-Z., and Fuller, G. M. (1995). *Phys. Rev. D* 52, 656.
Fuller, G. M., Primack, J. R., and Qian, Y.-Z. (1995). *Phys. Rev. D* 52, 1288.
Caldwell, D. O., and Mohapatra, R. N. (1995). *Phys. Lett. B* 334, 371.

Bethe, H. A., Brown, G. E., Applegate, J., and Lattimer, J. M. (1979). *Nucl. Phys. A* 324, 487.

Wolfenstein, L. (1978)., *Phys. Rev. D* 18, 958.

Harrison, P. F., Perkins, D. H., and Scott, W. G. (1996). *LANL archive preprint: hep-ph/9601346.*
Bilenky, S. M. and Giunti, C. (1996). *LANL archive preprint: hep-ph/9601389.*

Kayser, B. (1981). *Phys. Rev. D* 24, 110.
Appendix A: Erratum

The following Erratum was published in *Gen. Rel. Grav.* 29, 681 (1997):

The $2\hbar$ in eqs. (7), (8), (11), and (12) should be replaced by $4\hbar$.

In retrospect, this paper shows that neutrino oscillations provide a flavor-oscillation clock *and* this flavor-oscillation clock redshifts as required by the theory of general relativity.
Appendix B: Neutrino oscillations and supernovae

The following is an unedited text of Section 1 of “JRO Fellowship Research Proposal by D. V. Ahluwalia,” which was submitted by Mikkel B. Johnson in July 1996 in his Nomination of Dharam Ahluwalia for Oppenheimer Fellowship. The appendix title coincides with the title of Section 1 of the proposal.

Neutrinos were introduced in physics by Pauli to save conservation of energy and momentum in the $\beta$-decay: Neutron $\rightarrow$ Proton + Electron + Anti-electron Neutrino. All the planets and galaxies are embedded in a sea of neutrinos with a number density of roughly 100 neutrinos/cm$^3$. Our own Sun shines via thermonuclear processes that emit neutrinos in enormous number. Because of their weak interactions, neutrinos, unlike photons, can pass through extremely dense matter very efficiently. This fact makes neutrinos primary agents for energy transport in the dense matter associated with supernova and neutron stars.

Since their initial experimental observation by Frederick Reines and C. L. Cowan, neutrinos are now known to exist in three types. These types are called “electron,” “muon,” and “tau” and are generically written as $\nu_e$, $\nu_\mu$, and $\nu_\tau$. A series of empirical anomalies indicates that neutrinos may not have a definite mass but, instead, be in a linear superposition of three different mass eigenstates. The mass differences in the underlying mass eigenstates would cause a neutrino of one type to “oscillate” to a neutrino of another type as may have been seen recently at the LSND neutrino oscillation experiment at LANL. The phenomenon of neutrino oscillations, if experimentally confirmed, will have profound consequences not only for nuclear and particle physics but also for astrophysics and cosmology.

I have already noted the neutrinos to be prime drivers of supernova explosions. The phenomenon of neutrino oscillations will alter the evolution of supernova explosion. The basic problem that still stands unsolved is a robust theory of supernova explosions. In the context of supernova explosions, and the problem of obtaining successful explosions, I now follow Colgate et al. [S. A. Colgate, M. Herant, and W. Benz, Phys. Rep. 227, 157 (1993)] and assume that the matter next to the neutron star is heated by neutrinos from the cooling neutron star. They note that in some models “this result in strong, large scale convective flows in the gravitational field of the neutron star that can drive successful, albeit weak, explosions.” I emphasize that all authors find that without “fine tuning” the explosions are weak and lack about five percent of the energy needed for an explosion. Qualitatively, this missing energy needed for a robust model of explosion may be provided if the length scales over which neutrino oscillations take place are of the same order of magnitude as the spatial extent of a neutron star and neutrino-sphere, because while
the energy flux in each of the electron neutrinos and antineutrinos is about $L_{\nu_e} \approx L_{\bar{\nu}_e} \approx \text{few} \times 10^{52}$ ergs s$^{-1}$, with comparable fluxes of $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$, and $\bar{\nu}_\tau$. 

the average energy of $\nu_e$ is about 10 MeV, the average energy of other neutrinos is higher by a factor of 2 for $\nu_\mu$ and $\bar{\nu}_\mu$, and by a factor of 3 for $\nu_\tau$, and $\bar{\nu}_\tau$.

Any oscillation between neutrinos of different flavors is, therefore, an indirect energy transport mechanism towards the actively interacting $\nu_e$ and $\bar{\nu}_e$. Qualitatively this contributes in the direction of the robustness of the explosion. My resent work, with C. Burgard, on the solution of terrestrial neutrino anomalies provides precisely the neutrino oscillation parameters that yield the oscillation length scales of just the right order of magnitude for supernova physics (and in addition predict the observed solar neutrino deficit).

In order to make these qualitative arguments quantitative two additional physical processes affecting the above indicated vacuum neutrino oscillations must be incorporated: (a) The presence of large electron densities in astrophysical environment makes it necessary that relevant matter induced effects, suggested by Mikheyev, Smirnov and Wolfenstein, be considered, and (b) My work, with C. Burgard, on gravitationally induced neutrino oscillation phases also indicates that strong gravitational fields associated with neutron stars may introduce important modifications to neutrino oscillations, and hence to the suggested energy transport mechanism via neutrino oscillations. As part of my JRO studies I propose to implement the above outlined program quantitatively. My quantitative and qualitative studies so far give reasons to claim that there is every physical reason to believe that the “missing energy” in the non-robust models for supernova explosion, the anomaly in the observed deficit in the solar neutrino flux, the excess $\bar{\nu}_e$ events seen at LSND at Los Alamos, and the anomaly associated with atmospheric neutrinos, all arise from the same underlying new physics —the phenomenon of neutrino oscillations from one type to another. It is of profound physical importance to place these suspected physical connections on firm quantitative foundations.

The above proposal has been widely known informally (without a full access to its text). This appendix fills the gap of its availability.

The above suggestion has been pursued vigorously, though often without being acknowledged. Calculations done by S. Goswami, Pramana 54, 173-184 (2000) [arXiv: hep-ph/0104094] strongly support the the suggestion that neutrino oscillations are indeed a powerful energy transport mechanism and may play a dominant and important role in supernova explosions.