Exotic Quarkonia from Anisotropic Lattices

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We study in detail the spectrum of heavy quarkonia with different orbital angular momentum along with their radial and gluonic excitations. Using an anisotropic formulation of Lattice QCD we achieved an unprecedented control over statistical errors and were able to study systematic errors such as lattice spacing artefacts, finite volume effects and relativistic corrections. First results on the spin structure in heavy hybrids are also presented.

1. INTRODUCTION

Heavy quarkonia are an interesting testing ground for our theoretical understanding of QCD. Many models have been developed to describe the vast amount of experimental data, but more accurate predictions from first principles will be needed for future experiments with heavy quark systems. However, the conventional lattice approach becomes overly expensive if one tries to accommodate all the non-relativistic energy scales on a single grid: $M v^2 \ll M v \ll M$. Here $v$ is the small velocity of the heavy quark with mass $M$.

The non-relativistic nature of such problems has frequently been employed to formulate effective theories in which spatial and temporal directions are treated differently at the level of the quark action\cite{1,2}. In particular the NRQCD approach has lead to very precise calculations of the low lying spectrum in heavy quarkonia\cite{3}.

Additional problems arise if high energetic excitations are to be resolved on isotropic lattices. Very often the temporal discretisation is too coarse and the correlator of such heavy states cannot be measured accurately for long times. Glueballs and hybrid states are among the prominent examples of such high-lying excitations and they receive much theoretical and experimental attention as they are non-perturbative revelations of the gluon degrees of freedom in QCD. Early lattice studies have predicted such states starting at around 1.4 GeV, but those results were spoilt by large statistical uncertainties\cite{4-7}. This suggests that the inverse lattice spacing should at least be 3 GeV or higher. More recently anisotropic lattices have been used to circumvent this problem in glueball calculations by giving the lattice a fine temporal resolution whilst maintaining a coarse discretisation in the spatial direction\cite{8}. The success of this approach has triggered new efforts to measure hybrid potentials in an anisotropic gluon background and a comparative NRQCD analysis has demonstrated the validity of this adiabatic approximation for $b\bar{b}$ hybrids. These attempts were reviewed in\cite{9}.

In a previous study we reported on first quantitative results for charmonium and bottomonium hybrid states from anisotropic lattices\cite{10}. Here we extend those methods to study also other excitations and the spin structure in heavy quarkonia more carefully.

In Section 2 we present the details of our calculation and results for the spin-averaged spectrum. In Section 3 we investigate the spin structure in heavy quarkonia and report on novel results for the splittings in heavy hybrids and D-states.

2. SPIN-AVERAGED SPECTRUM

In order to study excited states with small statistical errors it is mandatory to have a fine resolution in the temporal lattice direction, along which we measure the multi-exponential decay of meson correlators. To this end we employ an anisotropic and spatially coarse gluon action:
\[ S = -\beta \sum_{x,i>j} \xi^{-1} \left\{ \frac{5}{3} P_{ij} - \frac{1}{12} (R_{ij} + R_{ji}) \right\} \]
\[ -\beta \sum_{x,i} \xi \left\{ \frac{4}{3} P_{it} - \frac{1}{12} R_{it} \right\} . \quad (1) \]

Here \((\beta, \xi)\) are two parameters, which determine the gauge coupling and the anisotropy of the lattice. Action (1) is Symanzik-improved and involves plaquette terms, \(P_{\mu \nu}\), as well as rectangles, \(R_{\mu \nu}\). It is designed to be accurate up to \(O(a_s^4, a_t^2)\), classically. To reduce the radiative corrections we invoked mean-field improvement and divided all spatial and temporal links by a ’tadpole’ coefficient \(u_0s\) and \(u_0t\), respectively. We determined those coefficients self-consistently by measuring spatial and temporal plaquettes: \(u_0s = (\text{Tr} \, P_{ij})^{1/4}\) and \(u_0t = (\text{Tr} \, P_{it})^{1/4}\). With this prescription we expect only small deviations of \(\xi\) from its tree-level value \(a_s/a_t\).

To describe the forward propagation of heavy quarks in the gluon background \((A_\mu)\) we used the NRQCD approach introduced in [1]:

\[ G_{t+a_t} = \exp \left[ -a_t \left( H(g, M) + igA_t \right) \right] G_t . \quad (2) \]

Here the NRQCD Hamiltonian, \(H\), is designed to account for relativistic corrections and includes spin-dependent operators up to \(O(mv^6)\). The implementation details of Equation (2) on anisotropic lattices are given in [11]. Since we are working with spatially coarse lattices it is crucial to improve all lattices derivatives and colour-electromagnetic fields in Equation (2). Following the prescription of [1] we also achieved an accuracy of \(O(a_s^4, a_t^2)\) in the quark sector.

At each value of the coupling, \(\beta\), we carefully tuned the heavy quark mass, \(M\), so as to reproduce the experimental ratios \(M_{\text{kin}}/(1P-1S)\) very accurately. Fortunately, the spin-independent quantities, such as \(1P - 1S\), are not very sensitive to the actual value of \(M\), but the spin structure will show a strong dependence. For our calculation we chose several different values of the coupling, \(\beta\), which correspond to spatial lattice spacings between 0.15 fm and 0.47 fm. On even coarser lattices we cannot expect to control discretisation errors with our simple-minded approach, while much finer lattices will violate the validity of NRQCD which requires \(a_sM > 1\).

From the quark propagator, \(G_t\), we construct meson correlators for bound states with spin \(S = (0, 1)\) and orbital angular momentum \(L = (0, 1, 2)\). Magnetic hybrid states are constructed from the colour-magnetic field coupled to a \(Q\bar{Q}\)-pair \((B_t = [\Delta_j , \Delta_k])\). For example, the spin-singlet operators read

\[ \bar{Q}^i Q , \bar{Q}^i \Delta_j Q , \bar{Q}^i \Delta_j \Delta_k Q \text{ and } \bar{Q}^i B_t Q . \quad (3) \]

Those simple operators can be further improved upon in order to optimise their overlap with the states of interest. Here we extract the excitation energies from multi-exponential fits to several different correlators.

Within the NRQCD approach one cannot extrapolate to the continuum limit and it is paramount to establish a scaling region for physical quantities already at finite lattice spacing. In a previous study we found such scaling windows for the spin-averaged gluon excitations in both charmonium and bottomonium [10]. These results are particularly encouraging as they are in excellent agreement with calculations on isotropic lattices [12], but with much smaller errors. In addition we were able to check our predictions against possible systematic errors such as finite volume effects. This is a natural concern since hybrids are expected to be rather large owing to the flat potentials they are living in. In Figure 3 we show our results from lattices all larger than 1.2 fm in extent, beyond which we could not resolve any volume dependence. For the Bottomonium hybrid we also have consistent results from two different anisotropies \((\xi = 3, 5)\) which confirms our initial assumption of small temporal lattice spacing artefacts. In fact, on our lattices the tadpole coefficient \(u_0t\) deviates from its continuum value by only about 5% or less. It is also interesting to notice the presence of scaling violations which are clearly visible if the lattices are too coarse for the physical system.

We have now extended our analysis to study also higher radial excitations and D-states with \(L = 2\). The spin-independent results are sum-
Figure 1. Scaling analysis for spin-averaged hybrids. As we only measure excitation energies relative to the ground state it is natural to present our results as the ratio $R_B = (1B - 1S)/(1P - 1S)$, which gives the normalized splitting of the magnetic hybrid excitation above the 1S.

summarised in Figure 2. The possibility to resolve all these excitations reliably should be considered the main success of anisotropic lattices. We are presently performing a finite volume analysis of the higher radial excitations. However, with the newly achieved accuracy we can also study spin-splittings in more detail.

3. SPIN STRUCTURE

Our inclusion of relativistic corrections to Equation (2) is a significant improvement over previous NRQCD calculations of hybrid states, which were restricted to only leading order in the velocity expansion: $O(mv^2)$. At this level there are no spin-dependent operators and we have a strict degeneracy of all singlet and triplet states. Within the NRQCD framework, the spin-dependent operators appear first as higher order corrections to the Hamiltonian: $O(mv^4)$. This is in accordance with the experimental observation that spin-splittings in quarkonia are suppressed by $v^2$ compared to the spin-independent structure discussed in the previous section. Here we also include spin correction terms up to $O(mv^6)$, and study the breaking of the degeneracy. In particular, we could directly observe the exotic hybrid, $1^{-+}$, which is the state of greatest phenomenological interest.

Our results for fine structure and hyperfine splittings are shown in Figure 3 and 4, respectively. There are several interesting observations one can make. First of all, we find a noticeable reduction of the fine structure in D-states when compared to that of P-states. Similarly, the hyperfine splitting between spin-singlet and spin-triplet states is equally suppressed as the orbital angular momentum is increased. This is in accordance with potential models which predict a hyperfine splitting for only (L=0)-states since all other wavefunctions vanish at the origin. Our data also indicates that the fine structure in hybrid states is enlarged compared to the splittings in P-states, but we could not yet resolve any splitting between the spin-triplet state $(^3B = 5^3B_2 + 3^3B_1 + 3^3B_0)$ and the singlet $^1B_1$.

Finally one should also notice that the scaling behaviour of the spin structure is more involved than that of the spin-independent spectrum. This is not surprising since we have adopted a very simplistic approach to determine all the coeffi-
Figure 3. Fine Structure in Charmonium and Bottomonium on three different lattices. All results come from NRQCD calculations with accuracy $O(a^6, a^4, a^2 t)$. The conversion to dimensionful units is done by setting the $1P - 1S$ splitting to its experimental value.

The coefficients in NRQCD with a single prescription (tree-level tadpole improvement). Namely the hyperfine splitting $3S_1 - 1S_0$ does not scale on the lattices considered here. It is apparent that one needs a better improvement description to account for lattice spacing artefacts in such UV-sensitive quantities.

In conclusion, we find that coarse and anisotropic lattices are extremely useful for precision measurements of higher excited states. This is due to an improved resolution in the temporal direction and the possibility to generate large ensembles of gauge field configurations at small computational cost. It has lead to an unprecedented control over statistical and systematic errors in lattice studies of heavy quarkonia. After the inclusion of relativistic corrections we are also sensitive to spin-spin and spin-orbit interactions and could observe a clear hierarchy in the spin structure, depending on the orbital angular momentum. Spin splittings in hybrid states were found to be larger as a result of the gluon angular momentum to which the spin can couple. The remaining and dominating systematic error for all our predictions is an uncertainty in the scale as the result of the quenched approximation. This is not yet controlled and we find a variation of 10-20%, depending on which experimental quantity is used to set the scale.

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