Cosmological constant and noncommutativity: A Newtonian point of view.

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Abstract

We study a Newtonian cosmological model in the context of a non-commutative space. It is shown that the trajectories of a test particle undergo modifications such that it no longer satisfies the cosmological principle. For the case of a positive cosmological constant, spiral trajectories are obtained and corrections to the Hubble constant appear. It is also shown that, in the limit of a strong noncommutative parameter, the model is closely related to a particle in a Gödel-type metric.

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1 Introduction

It is well known that Newtonian gravity is no-relativistic, however, from it one can get to conclusions which fully agree with general relativity. For instance, J. Michell (1784) [1] and P. S. Laplace (1796) [2] have shown that the idea of black hole can be obtained from pure Newtonian gravity; though black holes do require general relativity to be completely justified. On the other hand, in a cosmological level E. A. Milne (1934) [3] did show that, from fluid equations and the cosmological principle, the Friedmann equation for the case of pressureless matter can be deduced. This approach is still valid when pressure is much less than the energy density; otherwise general relativity must be used [4]. Extensions of this approach can be found in Ref. [5]. Moreover, the same equation can be obtained even from Newton’s second law [6]. For a recent discussion of these topics see e.g. Refs. [7, 8].

A great advantage of Newtonian gravity is its simplicity as it is just an external potential in a flat space. This simplicity allows one to study concepts otherwise difficult to understand from general relativity.

A difficult concept to introduce in general relativity is that of a noncommutative (NC) space, however it can be incorporated in other field theories where it has proved useful for exploring possible modifications to the physics of small scales. The seminal idea of this proposal has been attributed to Heisenberg [9]. NC spaces appear in a natural fashion in the context of string theory under some backgrounds [10]. Moreover, one can also construct in an independent manner a field theory in a NC space [11]. The way to build this field theory starts by changing in the action the usual product of functions by a deformed product of the form

\[(f \ast g)(x) = f(x)g(x) + \theta \frac{i}{2} \{f(x), g(x)\}_{PS} + \mathcal{O}(\theta^2),\]  

(1)

where \(\{., .\}_{PS}\) denotes a Poisson structure and \(\theta\) is a parameter controlling deformation such that when it is negligible the commutative limit is regained [12]. By considering the usual Poisson structure and making \(\theta = \hbar\), the usual field theories are obtained. However, if one considers alternative Poisson structures one gets to new theories. For instance, taking

\[\{x_i, x_j\} = \Theta_{ij},\]  

(2)
with $\Theta_{ij}$ a constant, real and antisymmetric tensor; and requesting associativity of the product, then the Moyal product

$$f(x) \star g(x) = e^{i\hbar \Theta_{ij} \partial_i \partial_j} f(x)g(y)|_{x=y},$$  \hspace{1cm} (3)

is obtained. In particular, the noncommutativity relation

$$[x_i, x_j]_\star = x_i \star x_j - x_j \star x_i = i\hbar \Theta_{ij},$$  \hspace{1cm} (4)

holds. An interesting property of field theories in these spaces is the mixing of infrared and ultraviolet (UV/IR) divergences \[13\]. This divergence mixing implies that the physics at large distances is not disconnected from the physics at short scales. Several interactions have been investigated in this context, for example, for the case of the Standard Model see Ref. \[14\]. Nevertheless, despite considerable progress, the task of accomplishing a final version of NC general relativity has not been completed yet; though versions of topological models with torsion already exist. For a detailed description see Ref. \[15\] and references therein.

In this work we introduce noncommutativity in a cosmological Newtonian model and study its consequences. To this end we first construct a NC classical mechanics. Starting from the Poisson structure in Eq. (2), we extend to the phase space as

$$\{x_i, x_j\} = \Theta_{ij}, \quad \{x_i, p_j\} = \delta_{ij}, \quad \{p_i, p_j\} = 0.$$  \hspace{1cm} (5)

Notice that changing to a commutator this Poisson structure, the commutation rules of a NC quantum mechanics are obtained \[16\]. However by using Eq. (5) a NC classical mechanics can also be constructed. For example, with a Hamiltonian of the form

$$H = \frac{p_i p^i}{2m} + V(x),$$  \hspace{1cm} (6)

one gets the equations of motion

$$\dot{x}_i = \frac{p_i}{m} + \Theta_{ij} \frac{\partial V}{\partial x^j},$$  \hspace{1cm} (7)

$$\dot{p}_i = -\frac{\partial V}{\partial x^i},$$  \hspace{1cm} (8)
which yields,

\[ m\ddot{x}_i = -\frac{\partial V}{\partial x_i} + m\Theta_{ij} \frac{\partial^2 V}{\partial x_j \partial x_k} \dot{x}_k. \tag{9} \]

Notice that there is a correction to Newton’s second law that depends on the NC \( \Theta \) parameter and on variations of the external potential \( V(x) \). This correction term can be regarded as a perturbation to the space due to the external potential. Eq. (9) has interesting properties itself. One of them is that the \( t \rightarrow -t \) symmetry is broken unless the NC parameter is also transformed as \( \Theta \rightarrow -\Theta \). A similar phenomenon occurs in field theory \[17\]. Also the rotational symmetry for a central potential is broken. It can be shown that for the Kepler potential Eq. (9) yields a perihelion shift of the planets. In particular, for the case of Mercury this shift imposes a bound to the noncommutativity scale of the order of \( 10^{15}\text{GeV} \) \[18\]. This is a remarkable bound as the lowest one previously found was of the order of \( 10^{17}\text{GeV} \), given by NC-QCD \[19\]. A detailed study of Eq. (9) can be found in Refs. \[18,20\] and some other aspects of a NC classical mechanics in Ref. \[21\].

The main aim of this letter is to study Eq. (9) for a Newtonian cosmological model. The first outcome is that a NC space cannot be consistent with the cosmological principle as there is a privileged direction. Both, the cases with positive and negative cosmological constant \( \Lambda \) are discussed. For a negative \( \Lambda \), the solutions obtained oscillate about the origin with a frequency depending on \( \Theta \). In the positive \( \Lambda \), but otherwise commutative case, the trajectories are straight lines in which the particle departs from the origin exponentially with time. However, in the NC case, the trajectories are spirals in which the particle distance to the origin also grows exponentially with time but now \( \Theta \) introduces small oscillations. The contribution of these oscillations to the distance become important just after every time period \( T \propto 1/\Theta \); and in this period the distance \( r \propto e^T = e^{1/\Theta} \) is traveled. I.e. noncommutativity at small distances (\( \Theta \) small) produces effects at large distances, and therefore it connects short and large distances. This phenomenon is analogous to the mixing of UV/IR divergences in field theory. Also, as it will be shown shortly, the corrections to the distance \( r \) give raise to corrections in the Hubble constant of the model. Finally, in the strong \( \Theta \) limit, it is shown that the model is closely related to a particle in a Gödel-type metric and that the trajectories are circles which at the quantum mechanical level have quantized radius.
This work is organized as follows. In section 2 the Newtonian limit of
general relativity without matter but with cosmological constant is briefly
reviewed. In section 3 the NC case is studied. The strong $\Theta$ limit is consid-
ered in section 4 and finally in section 5 the results are summarized.

2 AdS and dS in the Newtonian limit

Einstein’s equations with cosmological constant, $\Lambda$, in vacuum are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}. \quad (10)$$

Their solutions for maximum symmetry are of the form

$$ds^2 = -(1 - \frac{\Lambda r^2}{3}) c^2 dt^2 + \frac{dr^2}{(1 - \frac{\Lambda r^2}{3})} + r^2 d\Omega^2. \quad (11)$$

For negative $\Lambda$ this is the anti de Sitter space and for positive $\Lambda$ is the de
Sitter space. In the Newtonian limit, $c \to \infty$ and $\Lambda \to 0$ but $c\Lambda$ finite, the
metric in Eq. (11) generates the potential

$$V = -\frac{\Lambda mc^2}{6} r^2. \quad (12)$$

The Hamiltonian of the system is

$$H = \frac{p_i p_i}{2m} - \frac{\Lambda mc^2}{6} r^2. \quad (13)$$

Assuming the usual Poisson structure the equations of motion

$$m \frac{d^2 \vec{r}}{dt^2} = m \frac{\Lambda c^2}{3} \vec{r}, \quad (14)$$

are obtained. For negative $\Lambda$ there is an attractive force and the particle
oscillates about the origin with a distance (to the origin) given by $r_{\Lambda<0}(t) = A|\sin \sqrt{\Lambda}t|;$ but for positive $\Lambda$ the force is repulsive and the particle departs
exponentially fast from the origin, i.e. $r_{\Lambda>0}(t) = Be^{\sqrt{\Lambda}t}.$

Hubble’s constant $H$ can be defined by the formula

$$\dot{r} = H r, \quad (15)$$

which for $\Lambda > 0$ yields $H = \sqrt{\Lambda}$. For a study of the symmetries of Eq. (13)
see Ref. [8].
3 Noncommutative case

Let us now study the same system as in the previous section, but now introducing the Poisson structure from Eq. (5). Firstly, we shall see that a NC space is not compatible with the cosmological principle. For that, as $\Theta_{ij}$ is a $3 \times 3$ antisymmetric matrix, it can be written as $\Theta_{ij} = \epsilon_{ijk}\Theta_k$. Therefore, $\Theta$ defines a privileged direction. For instance, if this vector is taken along the $z$ direction (i.e. $\Theta_k = \delta_{k3}\Theta$) and Poisson brackets from Eq. (5) are assumed, we obtain the relationships

$$\{x_1, x_2\} = \Theta, \quad \{x_2, x_1\} = -\Theta, \quad \{x_3, x_i\} = 0.$$  \hspace{1cm} (16)

That is, noncommutativity only affects the plane perpendicular to the $z$ direction. This defines a privileged direction and in such a space the cosmological principle is not possible.

Let us now turn to the Hamiltonian in Eq. (13) with the Poisson structure from Eq. (5). By taking $\Theta_{ij} = \epsilon_{ij3}\Theta$ one gets to the equations of motion

$$m\ddot{x} = \frac{m\Lambda c^2}{3}x - \frac{\Lambda c^2}{3}m^2\Theta \dot{y},$$ \hspace{1cm} (17)

$$m\ddot{y} = \frac{m\Lambda c^2}{3}y + \frac{\Lambda c^2}{3}m^2\Theta \dot{x},$$ \hspace{1cm} (18)

$$m\ddot{z} = \frac{m\Lambda c^2}{3}z.$$ \hspace{1cm} (19)

These equations are essentially identical to those for a harmonic oscillator in a constant magnetic field along the $z$ direction. As expected, movement along the $z$ direction is not affected by noncommutativity, but it is in the plane perpendicular to $z$. This is the reason behind being unable to take solutions of the form $\vec{r} = R(t)\vec{r}(t_0)$, which are consistent with the cosmological principle.

As the $\Theta$ parameter affects only the $x$-$y$ plane, for simplicity (unless stated otherwise), we will assume the solution $z(t) = 0$ along the $z$ direction. Now, looking for solutions of the form $x = x_0 e^{\omega t}$ and $y = y_0 e^{\omega t}$, we find that

$$\omega = \sqrt{\alpha} \left[ \left( 1 - \frac{\beta^2}{2\alpha} \right) \pm \sqrt{\left( 1 - \frac{\beta^2}{2\alpha} \right)^2 - 1} \right]^{1/2}.$$ \hspace{1cm} (20)
\[ y_0 = x_0 \frac{\beta \omega}{\alpha - \omega^2}, \]  
with \( \alpha = \Lambda c^2/3 \) and \( \beta = m\Theta \alpha \). To second order in \( \Theta \) this yields
\[ \omega = \sqrt{\alpha} \left( 1 - \frac{\beta^2}{4\alpha} \pm \frac{\beta}{2\alpha} \sqrt{1 - \frac{\beta^2}{4\alpha}} \right). \]

It can be seen from Eq. (22) that if \( \Lambda < 0 \) the movement is completely oscillatory as in the commutative case. However, the oscillations now get corrected by the NC parameter. That is, the cosmological constant gets a correction from \( \Theta \). On the other hand, if \( \Lambda > 0 \) the frequencies are no longer real as they get an imaginary correction from \( \Theta \) of the form
\[ \omega = \omega_R \pm \omega_I = \sqrt{\alpha} \left( 1 - \frac{\beta^2}{4\alpha} \right) \pm \frac{\beta}{2}, \]
so that a test particle now departs from the origin and it also oscillates in the \( x-y \) plane, in contrast to the commutative case. To this order, linearly independent trajectories in the \( x-y \) plane are
\[ (x, y) = e^{\omega_R t} \left( \cos(\omega_I t), \left( 1 - \frac{\beta^2}{8\alpha} \right) \sin(\omega_I t) \right), \]
\[ (x, y) = e^{\omega_R t} \left( \sin(\omega_I t), -\left( 1 - \frac{\beta^2}{8\alpha} \right) \cos(\omega_I t) \right). \]

Therefore, in this case the trajectories are no straight lines, but spirals. This trajectories are schematically shown in Figure 1. From these solutions, Eqs. (24) and (25), it can be seen that the cosmological principle is broken in the NC plane.

On the other hand the particle distance to the origin at time \( t \) is
\[ r(t) = A e^{\sqrt{\omega_R} t} \sqrt{1 - \frac{\beta^2}{4\alpha} \cos^2 \omega_I t}. \]
To first order in \( \Theta \) this grows exponentially. However, to second order small oscillations appear. The behaviour of \( r(t) \) is shown schematically in Figure 2. The oscillations in \( r(t) \) are analogous to the scale factor oscillations of a cosmological model of a quasi-steady-state type \[22, 23\].
Figure 1: Trajectories of a particle in a space with positive cosmological constant. Left figure represents the commutative case. Right figure is for the noncommutative one.

From Eq. (26) it can be seen that the effect of the noncommutativity in \( r(t) \) is essentially suppressed, showing up only when \( \omega_I t = n \pi \). That is, when

\[
    t = \frac{n \pi}{\omega_I} = \frac{3 n \pi}{m \Theta \Lambda c^2}.
\]  

The distance traveled by the test particle during this time is

\[
    r(t = n \pi / \omega_I) \propto e^{\left( \frac{4 \pi n}{m \sqrt{\Lambda c^2}} \right)^{\frac{1}{3}}}.
\]

Now, taking the limit \( \Theta \to 0 \), then \( r \to \infty \). Thus, for small \( \Theta \) the first oscillation will appear at a very long distance and this can be interpreted as that noncommutativity at short distances has effects at very long ones; i.e. short and very long distances are no disconnected. This phenomenon is analogous to the mixing of the UV/IR divergences appearing in a field theory in a NC space. Notice that for this phenomenon to be observed, corrections to second order in \( \Theta \) have to be considered. Also note that the time defined in Eq. (27) is proportional to the time the particle requires to turn \( n \) times about the origin.

Now, by defining Hubble’s constant from \( \dot{r} = H r \), for the \( \Lambda > 0 \) case we find

\[
    H = \sqrt{\omega_R} = \sqrt{\alpha} \left( 1 - \frac{\beta^2}{4 \alpha} \right).
\]
Figure 2: Distance to the origin for the particle position as a function of time. Order zero and first order are represented by the line on the left. Second order by the line on the right.

Therefore, to this order a constant correction to $H$ appears. To higher orders Hubble’s constant gets corrections that depend on time.

4 Strong $\Theta$ limit

The strong $\Theta$ limit in Eqs. (17)-(19) can be obtained as $(m\Lambda c^2/3) \to 0$ and $\Theta \to \infty$, but $(m\Lambda c^2/3)m\Theta$ finite, so that

$$m\ddot{x} = -\frac{\Lambda c^2}{3} m^2 \Theta \dot{y}, \quad (29)$$

$$m\ddot{y} = \frac{\Lambda c^2}{3} m^2 \Theta \dot{x}, \quad (30)$$

$$m\ddot{z} = 0. \quad (31)$$

Eqs. (29)-(31) can also be regarded as the equations of motion of a charged particle in a magnetic field along the $z$ direction. In such a case, $\Lambda$ plays the role of the charge and $\Theta$ the magnetic field. If the particle speed along the $z$ direction is zero, then it describes a circle centred in $(x_0, y_0)$. Whether the movement is clock or anticlockwise depends on the sign of $\Lambda$. Moreover, as
in the quantum formalism the commutation relations,

\[ [\hat{x}, \hat{y}] = i\hbar \Theta, \quad (32) \]

hold and therefore the radius \( \hat{r} \) of the circles is quantized,

\[ \hat{r}^2 = 2\hbar \Theta \left( n + \frac{1}{2} \right); \quad (33) \]

just as it happens in quantum mechanics to a charged particle in a uniform magnetic field \([27]\).

As explained in Sec. [1] the commutative version of this model yields results in agreement with general relativity. Assuming that analogously happens for the noncommutative case, we could say that if at the beginning of the universe matter was dominated by a cosmological constant (i.e. vacuum) and the space is NC with \( \Theta \) strong, then at the quantum level a test particle would describe helices of quantized radius; but as \( \Theta \) becomes weak, the circles get deformed.

Now, by considering the metric

\[ ds^2 = - \left( dt + \frac{\Omega \sinh^2 \rho \rho}{l^2} d\phi \right)^2 + \frac{\sinh^2 2\rho \rho}{4l^2} d\phi^2 + d\rho^2 + dz^2, \quad (34) \]

taking the limit \( l \to 0 \), and expressing it in terms of the coordinates

\[ x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad (35) \]

then the geodesics of this metric are of the same form as the equations of motion \([20] - [31]\); where \( \Omega \) plays the role of the \( \Theta (\Lambda c^2 / 3) \) parameter. Since for the \( l^2 = 2\Omega \) case one has the Gödel metric \([24]\), the metric in Eq. \((34)\) is called a Gödel-type metric and is the solution to Einstein’s equations with cosmological constant and nonzero energy momentum tensor. For more properties of this metric see e.g. Ref. \([25]\). An interesting relationship between this space and Landau’s problem can be found in Ref. \([26]\).

Another way of getting noncommutativity is through a Matrix model. In this formalism it is also possible to propose a Newtonian cosmological model \([28]\). It’s worth mentioning that recently new observations have been obtained suggesting the existence of a positive cosmological constant \([29]\).
5 Summary

In this letter a Newtonian cosmological model in a NC classical mechanics is studied. As in a NC space there are privileged directions, the trajectories of a test particle are not compatible with the cosmological principle. If the cosmological constant is negative, the trajectories are oscillatory, with oscillation frequency depending on the NC parameter. On the contrary, if the cosmological constant is positive, the trajectories on the NC plane are spirals and to first order in $\Theta$ the distance from the origin to the particle, $r(t)$, grows exponentially fast. To this order Hubble’s constant is the same as for the commutative case. To second order, however, $r(t)$ also grows exponentially but develops small oscillations. In this approximation Hubble’s constant gets corrections. The oscillation period in $r(t)$, $T \propto 1/\Theta$, so for $\Theta$ small this period is very long. On the other hand, because $r(t)$ grows exponentially with time, the distances at which perturbations can be observed are very long. Therefore, considering noncommutativity at short distances has implications at large distances. Finally, it is shown that in the strong $\Theta$ limit the trajectories are circles, whose radius are discretized at the quantum level. In this limit there is a relation with a Gödel-type metric.

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