Analytical model of strange star in Durgapal spacetime

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Abstract A new strange star model based on Durgapal IV metric (Durgapal in J. Phys. A 15:2637, 1982) is presented here. Here we have applied a specific method to study the inner physical properties of the compact objects 4U 1702-429, 2A 1822-371, PSR J1756-2251, PSR J1802-2124 and PSR J1713+0747. The main objective of our study is to determine central density ($\rho_0$), surface density ($\rho_b$), central pressure ($p_0$), surface redshift ($Z_s$), compactness and radius. Further we perform different tests to study the stability of our model and finally we are able to give an equation based on pressure and density i.e. probable equation of state (EoS) which has an important significances in the field of Astrophysics.

Keywords Stability · Mass-radius relation · Compactness · Surface red-shift · Equation of state

1 Introduction

Now a days people are very much interested to the study of dense objects (compact objects) in relativistic binary system. Neutron Stars, Pulsars, Strange Stars are considered as compact objects. Neutron stars are made by mostly neutron particles, where as for strange stars, its strange quark particles. Actually in strange stars, up-down quarks are transformed to strange quarks which plays a crucial role at the core of the strange stars (Drago et al. 2014; Haensel et al. 1986). It is found that gravitational force of attraction is responsible for the stability of neutron star where as, strange stars are stabilized not only by gravitational force but also by strong nuclear force. Therefore one can demand neutron star have lower force of attraction in comparison to strange star. Consequently, we may say that the neutron star become larger in size over strange star of same mass. It has been found that for neutron star surface matter density is almost zero, while for strange star it is not, at the boundary matter density will exist (Haensel et al. 1986; Alcock et al. 1986; Farhi and Jaffe 1984; Dey et al. 1998). After the formation of neutron star, its temperature has reduced to below the Fermi energy, hence the mass and radius of the neutron stars depends only on central density for a given equation of state. Also, it is very difficult to enumerate the mass and radius of that particular neutron star. For better conception one can go through the review work of Lattimer and Prakash (2007). It is to be mentioned here that for spherically symmetric static compact stars, theoretical calculation on mass and radius are the result of analytical solution of Tolman-Oppenheimer-Volkov i.e., TOV equations. In order to study the astrophysical objects people used various method as computational, observational or theoretical analysis. Mass and radius of a compact star (neutron stars/strange stars) can be measured by pulsar timing, thermal emission from cooling stars, surface explosions and gravity wave emissions, which are basically in observational point of view. Main objective is to obtain the proper equation of state in order to describe the interior features of the neutron star (Lattimer and Prakash 2007; Özel 2006; Özel et al. 2009a; Özel and Psaltis 2009b; Güver et al. 2010a, 2010b). Although a very few number of compact star masses have been calculated accurately (to some extend) in binaries (Heap and Corcoran 1992; Lattimer and Prakash 2007; Özel 2006; Özel et al. 2009a; Özel and Psaltis 2009b; Güver et al. 2010a, 2010b).
2005; Stickland et al. 1997; Orosz and Kuulkers 1999; Van Kerkwijk et al. 1995) but there is no information about the radius of that compact star. Therefore, people has realized that theoretical study is one of the important way for the study of stellar structure of newly observed masses and radius. Here, we want to mention some theoretical study on compact stars (Rahaman et al. 2012a, 2012b; Kalam et al. 2012, 2013a, 2013b, 2014a, 2014b, 2016, 2017, 2018; Hossein et al. 2012; Jafry et al. 2017; Lobo 2006; Bronnikov and Fabris 2006; Maurya et al. 2016a, 2016b; Dayanandan et al. 2016; Ngubelanga and Maharaj 2015; Maharaj et al. 2014; Paul et al. 2015; Pradhan and Pant 2014; Sharma et al. 2015).

Nättilä et al. (2017) has used the Rossi X-ray Timing Explorer Observations of five hard-state X-ray bursts to evaluate the mass of compact star in 4U 1702-429 and was found to be 1.9 ± 0.3M⊙. Jonker et al. (2003) gave another approach for the mass limit of neutron star 2A 1822-371 on the basis of phase resolved spectroscopic observations and pulse timing analysis and it was measured as 0.97 ± 0.24M⊙. On the other hand, Ferdman et al. (2014) has found the timing analysis and it was measured as 0 from the Parkes and Nancay observatories. Ferdman et al. (2010) has measured (data basis of phase resolved spectroscopic observations and pulse timing analysis and it was measured as 0 from the Parkes and Nancay observatories).

According to Durgapal (1982),

\[ e^{-\nu} = \left( \frac{7 - 10Cr^2 - C^2r^4}{7(1 + Cr^2)^2} + \frac{KC^2}{(1 + Cr^2)^2(1 + 5Cr^2)^{2/5}} \right) \]

\[ \nu = \ln(A(1 + Cr^2)^4) \]

where \( A \) (dimensionless), \( K \) (dimensionless) and \( C \) (length⁻²) are constants. Solving the above equations, we get density (\( \rho \)), central density (\( \rho_0 \)), surface density (\( \rho_b \)), pressure (\( p \)), central pressure (\( p_0 \)) as follows

\[ \rho = \frac{7CK(9C^2r^4 - 10Cr^2 - 3)}{56\pi(1 + Cr^2)^3(1 + 5Cr^2)^{7/5}} + \frac{8C(1 + 5Cr^2)^{2/5}(9 + 47Cr^2 + 11C^2r^4 + 45C^3r^6)}{56\pi(1 + Cr^2)^3(1 + 5Cr^2)^{7/5}} \]

\[ \rho_0 = \frac{C(72 - 21K)}{56\pi} \]

\[ \rho_b = \frac{7CK(9C^2b^4 - 10Cb^2 - 3)}{56\pi(1 + Cb^2)^3(1 + 5Cb^2)^{7/5}} + \frac{8C(1 + 5Cb^2)^{2/5}(9 + 47Cb^2 + 11C^2b^4 + 45C^3b^6)}{56\pi(1 + Cb^2)^3(1 + 5Cb^2)^{7/5}} \]

where \( b \) = radius of the star.

\[ p = \frac{7CK(1 + 9Cr^2)}{56\pi(1 + Cr^2)^3(1 + 5Cr^2)^{2/5}} - \frac{16C(1 + 5Cr^2)^{2/5}(C^2r^4 + 7Cr^2 - 2)}{56\pi(1 + Cr^2)^3(1 + 5Cr^2)^{2/5}} \]

\[ p_0 = \frac{C(32 + 7K)}{56\pi} \]

3 Exploration of physical properties

In this section we will try to find out the following nature of the compact object:
3.1 Density and pressure behavior of the compact object

From Fig. 1 and Fig. 2 it is clear that, at the centre the density and pressure of the star is maximum and it decreases radially outward. Thus, the energy density and pressure are well behaved in the interior of the stellar structure. Interestingly, pressure drops to zero at the boundary, though density does not. Therefore, it may be justified to take these compact stars as a strange stars where the surface density remains finite rather than the neutron stars for which the surface density vanishes at the boundary (Haensel et al. 1986; Alcock et al. 1986; Farhi and Jaffe 1984; Dey et al. 1998).

It is to be mentioned here that, we set the values of the constants \( K = 0.00053 \) and \( C = 0.00123 \) km\(^{-2} \), such that the pressure drops from its maximum value (at centre) to zero at the boundary.

3.2 Energy conditions

In our study, we have tested all the energy condition like null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC) at the centre of the compact stars. Therefore from Fig. 1, Fig. 2 and Table 1 one can obtained the following energy conditions:

(i) NEC: \( \rho_0 + p_0 \geq 0 \),
(ii) WEC: \( \rho_0 + p_0 \geq 0, \rho_0 \geq 0 \),
(iii) SEC: \( \rho_0 + p_0 \geq 0, 3\rho_0 + p_0 \geq 0 \),
(iv) DEC: \( \rho_0 > |p_0| \).

3.3 Matching conditions

Interior metric of the strange star will be matched to the Schwarzschild exterior solution at the boundary i.e., at \( r = b \)

\[
d s^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{12}
\]

For the continuity of the metric functions \( g_{tt}, g_{rr} \) and \( \frac{\partial g_{tt}}{\partial r} \) at the boundary, we get

\[
\left(\frac{7 - 10Cb^2 - C^2b^4}{7(1+Cb^2)^2}\right) + \frac{Kcb^2}{(1+Cb^2)^2(1+5Cb^2)^{2/5}} = 1 - \frac{2M}{b}, \tag{13}
\]

\[
A(1+Cb^2)^4 = \left(1 - \frac{2M}{b}\right). \tag{14}
\]

Now from the Eq. (13), we get the compactification factor as

\[
u = M(b) = Cb^2\left(\frac{24 + 8b^2C - \frac{7K}{(1+5b^2C)^{2/5}}}{14(1+b^2C)^2}\right) \tag{15}
\]

3.4 TOV equation

The generalized TOV equation for fluid distribution takes the form as

\[
\frac{dp}{dr} + \frac{1}{2} \nu' (\rho + p) = 0. \tag{16}
\]
The gravitational \((F_g)\) and hydrostatic \((F_h)\) forces at the stellar interior (taking \(K = 0.00053\) and \(C = 0.00123\) km\(^{-2}\)) of the strange star remain at equilibrium position by effective gravitational forces \((F_g)\) and effective hydrostatic \((F_h)\) forces.

\[
F_h + F_g = 0, \quad (17)
\]

where,

\[
F_g = -\frac{1}{2} \nu'(\rho + p) \quad (18)
\]

\[
F_h = -\frac{dp}{dr} \quad (19)
\]

Now Fig. 3 shows the equilibrium of gravitational and hydrostatic forces in the stellar structure.

### 3.5 Adiabatic index

In order to more tuning the model, one should check the infinitesimal radial adiabatic perturbation. Chandrasekhar (1964) had used this concept at first. Later this stability condition was used to various astrophysical cases by Bardeen et al. (1966), Knutsen (1988), Mak and Harko (2013). It is known fact that for star modeling adiabatic index should be \(\gamma = \frac{\partial u}{\partial p} > \frac{4}{3}\). Therefore from Fig. 4, it is clear that \(\gamma > \frac{4}{3}\) every point inside the strange stars.

### 3.6 Mass-radius relation and surface redshift

In this subsection, we will try to investigate the maximum allowable mass-radius ratio of strange stars. Buchdahl (1959) has mentioned the mass-radius ratio limit for spherically perfect fluid sphere should be \(\frac{M}{R} < \frac{4}{5}\). In our study the relation between gravitational mass \((M)\) and energy density \((\rho)\) can be written as

\[
M = 4\pi \int_0^b \rho r^2 dr = C b^3 \left( \frac{24 + 8b^2C - \frac{7K}{(1 + 5b^2C)^{3/5}}}{14(1 + b^2C)^2} \right) \quad (20)
\]

The compactness, \(u\) is given by

\[
u = \frac{M(b)}{b} = C b^2 \left( \frac{24 + 8b^2C - \frac{7K}{(1 + 5b^2C)^{3/5}}}{14(1 + b^2C)^2} \right) \quad (21)
\]

Variation of mass function and compactness of the strange star are shown in Fig. 5 and Fig. 6.
4 Discussion and concluding remarks

Based on present technique, we have proposed a new model of isotropic strange stars corresponding to the exterior Schwarzschild spacetime which is singularity free. Here, we have studied several physical behaviour of the strange star namely 4U 1702-429, 2A 1822-371, PSR J1756-2251, PSR J1802-2124 and PSR J1713+0747 under Durgapal (1982) IV metric spacetime. Conventionally, the mass-radius curve of compact stars are calculated under a given equation of state for various values of central density; by a given value of the central density, the mass and radius of a compact star are fixed. It is to be mentioned here that our strange star model are different and theoretically interesting. According to our model, different strange stars are depends on the same parameter values ($K = 0.00053$, $C = 0.00123$ km$^{-2}$) and consequently the same central density and the same equation of state. Therefore, interestingly, if we starts from the centre with a certain central density, the model of a compact star can be determined by stopping at any radius where pressure becomes zero. We think this model will give new dimension to study of compact stars.

Finally, we have got the variation of density and pressure at the interior of strange star in a systematic way. We observed that density and pressure are maximum at the centre and gradually decreases as we move from centre to surface. Incorporating the value of $G$ and $c$ in the expression, we have calculated the central density ($\rho_0$) as $50.33 \times 10^{-5}$ km$^{-2}$ ($6.79 \times 10^{14}$ g cm$^{-3}$) and central pressure ($p_0$) $22.37 \times 10^{-5}$ km$^{-2}$ ($5.58 \times 10^{35}$ dyne/cm$^2$) (pl. see Table 1). We have verified all the energy conditions, stellar equation (TOV) and stability conditions ($\gamma = \frac{\alpha \rho}{p^2} \frac{dp}{d \rho} > \frac{1}{2}$). We have also matched our interior solution to the exterior Schwarzschild line element at the boundary. From the mass function (Eq. (20)), the desired interior features of a strange star can be evaluated which satisfies Buchdahl (1959) maximum mass-radius ratio. The surface redshift of the strange stars are found within standard value ($Z_s \leq 0.85$) which is satisfactory (Haensel et al. 2000). We have also obtained several physical parameters with numerical values as radius, compactness ($u$) and surface redshift ($Z_s$) (pl. see Table 2) of the above mentioned strange stars. We have estimated the EoS and that is of the form $p = \alpha \rho + \beta$ where $\alpha$ (dimensionless) and $\beta$ (km$^{-2}$) are constants. According to our model, the estimated EoS (Fig. 9) should be a soft equation of state. Therefore, we see that our analytical study of the isotropic strange stars: 4U 1702-429, 2A 1822-371, PSR J1756-2251, PSR J1802-2124 and PSR J1713+0747 under Durgapal (1982) IV metric satisfies all physical requirements of a stable strange star. As a consequence of this, we may conclude that one can find useful relativistic model of strange stars under Durgapal IV metric by using the suitable choice of parameters $K$ and $C$.
Fig. 8 Probable radii of 4U 1702-429, 2A 1822-371, PSR J1756-2251, PSR J1802-2124 and PSR J1713+0747 (taking $K = 0.00053$ and $C = 0.00123$ km$^{-2}$)
Table 2  Radius, compactness and red-shift of strange stars are mentioned below (taking $K = 0.00053$ and $C = 0.00123 \text{ km}^{-2}$)

| Star               | Observed mass ($M_\odot$) | Radius from model (in km) | Compactness from model | Redshift from model |
|--------------------|---------------------------|---------------------------|------------------------|---------------------|
| 4U 1702-429        | $1.9 \pm 0.3$             | $12.31 \pm 0.84$          | $0.240 \pm 0.024$      | $0.394 \pm 0.066$   |
| 2A 1822-371        | $0.97 \pm 0.24$           | $9.33 \pm 0.93$           | $0.155 \pm 0.026$      | $0.206 \pm 0.045$   |
| PSR J1756-2251     | $1.341 \pm 0.007$         | $10.67 \pm 0.04$          | $0.193 \pm 0.001$      | $0.276 \pm 0.002$   |
| PSR J1802-2124     | $1.24 \pm 0.11$           | $10.35 \pm 0.42$          | $0.184 \pm 0.012$      | $0.258 \pm 0.024$   |
| PSR J1713+0747     | $1.3 \pm 0.2$             | $10.51 \pm 0.69$          | $0.188 \pm 0.020$      | $0.269 \pm 0.041$   |

Fig. 9 Possible pressure ($p$)–density ($\rho$) relation (EoS) at the stellar interior

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