Formulation of lagrangian equation and dynamic analysis of 2-DoF Industrial robot using Roboanalyzer

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Abstract. Industrial robot is extensively used in all major manufacturing and assembly line throughout the world. In order to properly actuate the robot links, it is necessary to calculate the torque required at link joints. In this paper we consider a two degrees of freedom robot manipulator having two rotary joints J₁ and J₂. This is a classic example of inverse dynamics problem, inverse dynamics in robotics used to calculate torque or force required at joints from known values of link mass, velocity, and acceleration and we choose lagrangian formulation to solve equation of motion. In addition, Robo analyzer software we plot the torque values for predefined value of centre of gravity and link mass

Keywords: acceleration, calculus of variation, degrees of freedom, dynamic analysis Industrial robot, lagrangian, torque

1. Introduction

The usage of the robotic arms are everywhere from medium scale sector to large scale sector in order to have 24/7 production the arms are widely used. Industrial robots have been extensively used in manufacturing industries after the uprising in computer science technology, faster calculation, precise movement and the support from academia helped the rise of industrial robot. A range of industrial robot was designed to suit the application of the customer such as serial manipulator and parallel manipulator to name the few. Design had been always a major constraint in developing any product, industrial robot designers faced the same concern as different customer requires dissimilar application robots to gratify their needs [2][3][5][6].

2. Literature survey

Robot manufacturer started building a variety of degrees of freedom robots to suit various application. For example a pick and place operation need only 3 dof robot but a painting job robot requires 6 dof robot[1][4]. Degrees of freedom (dof) is the primary design parameter for industrial robot and is defined as number of independent motion a particular system can make or number of variables required to define the position and orientation of robot end effector .The industrial robot plays an essential role in reducing expenditure of production and haste delivery of products to its customers. By accepting the current situation even medium scale industries rely on industrial robots. The robot manufacturer had faced a lot of issue regarding selecting proper motor for the applications.
To decide on proper motors we should be acquainted with the payload capacity and other parameters such as acceleration and mass of the robot manipulator. In order to discover the solution, we need to solve the inverse dynamics problem to find the equation of motion of the system. Inverse dynamics is defined as finding the required torque (rotary joint) and force (prismatic joint) of an industrial robot [9]. There are a variety of methods in classical mechanics to solve the system such as Newton - Euler, Lagrangian, Hamiltonian and Featherstone formulation. Each method have its own advantages and disadvantages. When we gaze into Newton Euler method [10], it is incredibly simple to solve the equation of motion for simple system such as single degrees of freedom but as the system becomes complex solving the equation of motion is very time consuming by using Newton - Euler. At this point we call for lagrangian and Hamiltonian formulation. Both these method uses the kinetic and potential energy of the individual links of the robot to solve the inverse dynamics, former uses the difference of kinetic and potential energy and later uses addition of kinetic and potential energy[7][8].

3. Calculus of Variation

It is concerned with maximizing and minimizing quantities called functional, which depend not on one or more independent variables, but on one or more functions. These quantities are typically definite integrals involving some function in the integrand which must be chosen to extremize the integral.

For every continuously differentiable function η such that η(x1) = η(x2) = 0 it is possible to represent any member of {Y} by an expression of the form

\[ Y(x) = y(x) + \psi \eta(x) \quad x_1 \leq x \leq x_2 \]

Whose derivative are given by

\[ Y'(x) = y'(x) + \psi' \eta'(x) \quad x_1 \leq x \leq x_2 \]

replacing y and y’ by Y and Y’ respectively we obtain the integral

\[ I(\psi) = \int_{x_1}^{x_2} f(x, Y, Y') \, dx \]

\[ I'(0) = 0 \]

\[ \frac{dI}{d\psi} = I'(\psi) = \int_{x_1}^{x_2} \left[ \frac{\partial f}{\partial y} \frac{\partial \eta}{\partial \psi} + \frac{\partial f}{\partial y'} \frac{\partial \eta'}{\partial \psi} \right] \, dx = \int_{x_1}^{x_2} \left[ \frac{\partial f}{\partial y} \eta + \frac{\partial f}{\partial y'} \eta' \right] \, dx \]

Which upon setting \( \psi = 0 \) and making use of \( I'(0) = 0 \)

\[ I'(0) = \int_{x_1}^{x_2} \left[ \frac{\partial f}{\partial y} \eta + \frac{\partial f}{\partial y'} \eta' \right] \, dx \]

Applying integration by the parts to the last term in this integral we obtain

\[ I'(0) = \int_{x_1}^{x_2} \left[ \frac{\partial f}{\partial y} \eta + \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \eta' \right) \right] \, dx = \int_{x_1}^{x_2} \left( \frac{d}{dx} \frac{\partial f}{\partial y'} \eta' \right) \, dx = 0 \]

\[ \frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0 \]  

\[ \ldots \ldots \ldots \ldots \ldots (a) \]
Equation ‘a’ is known as Euler- Lagrangian equation

A. Equation of motions for the 2 DoF robot arm. The two links have distributed masses that requires moments of inertia on the calculation of kinetic energy. We calculate the velocity of the centre of masses of link 2 by differentiating its position. Fig 1 shows 2-Dof industrial robot.

\[ x_D = l_1 s_1 + 0.5 l_2 s_{12} \Rightarrow \dot{x}_D = l_1 \dot{s}_1 + 0.5 l_2 s_{12} (\dot{s}_1 + \dot{s}_2) \] \hspace{1cm} (1)

\[ y_D = l_1 \dot{s}_1 + 0.5 \dot{l}_2 s_{12} \Rightarrow \dot{y}_D = l_1 \dot{s}_1 + 0.5 l_2 \dot{s}_{12} (\dot{s}_1 + \dot{s}_2) \] \hspace{1cm} (2)

Therefore, the total velocity of the centre of mass of link 2 is:

\[ \dot{v}_D = \ddot{x}_D + \ddot{y}_D = l_1 \dot{\theta}_1^2 (l_1^2 + 0.25 l_2^2 + l_1 l_2 C_2) + (0.25 l_2^2 + l_1 l_2 C_2) \] \hspace{1cm} (3)

The total kinetic energy of the system is the sum of the kinetic energies of links 1 and 2.

\[ K = K_1 + K_2 = \frac{1}{2} \dot{l}_1 \dot{\theta}_1\dot{\theta}_1 + \frac{1}{2} \dot{l}_2 \dot{\theta}_2\dot{\theta}_2 + \frac{1}{6} m_1 \dot{y}_1 \dot{v}_1 + \frac{1}{3} (\dot{m}_1 l_1 \dot{\theta}_1\dot{\theta}_1\dot{\theta}_1) + \frac{1}{12} (\dot{m}_2 l_2 \dot{\theta}_1\dot{\theta}_1\dot{\theta}_1) + \frac{1}{2} m_2 y_1^2 \] \hspace{1cm} (4)

Substituting equation 3 in 4

\[ K = \theta_1^2 (\frac{1}{6} m_1 l_1^2 + \frac{1}{2} m_1 l_2^2 + \frac{1}{2} C_2 l_2 m_1 \dot{s}_1 + \frac{1}{2} m_1 \dot{s}_2) + \dot{\theta}_1 \dot{\theta}_2 (\frac{1}{6} m_2 l_2^2 + \frac{1}{2} C_2 l_2 m_1 \dot{s}_1) + \dot{\theta}_1 \dot{\theta}_2 (\frac{1}{6} m_1 \dot{s}_1 l_1 + \frac{1}{2} m_1 \dot{s}_2) \] \hspace{1cm} (5)

The potential energy of the system is the sum of potential energies of the two links:

\[ P = m_1 g l_1 S_1 + m_2 g (l_1 S_1 + l_2 S_{12}) \] \hspace{1cm} (6)

Lagrangian for two link robot will be

\[ L = K - P = \theta_1^2 (\frac{1}{6} m_1 l_1^2 + \frac{1}{2} m_1 l_2^2 + \frac{1}{2} C_2 l_2 m_1 \dot{s}_1 + \frac{1}{2} m_1 \dot{s}_2) + \dot{\theta}_1 \dot{\theta}_2 (\frac{1}{6} m_2 l_2^2 + \frac{1}{2} C_2 l_2 m_1 \dot{s}_1) - m_1 g l_1 S_1 - m_2 g l_1 S_1 + l_2 S_{12} \] \hspace{1cm} (7)

Taking the derivatives lagrangian we get two equation of motions:

\[ T_1 = (\frac{1}{3} m_1 l_1^2 + \frac{1}{2} m_2 l_2^2 + C_2 l_2 m_1) \dot{\theta}_1 + (\frac{1}{3} m_1 l_1^2 + \frac{1}{2} C_2 l_2 m_1) \dot{\theta}_2 - (m_1 l_1 l_2 S_2) \dot{\theta}_1 \dot{\theta}_2 - (\frac{1}{2} m_1 l_1 l_2 S_2) \dot{\theta}_1^2 \] \hspace{1cm} (8)

\[ T_2 = (\frac{1}{3} m_1 l_1^2 + \frac{1}{2} C_2 l_2 m_1) \dot{\theta}_1 + (\frac{1}{3} m_1 l_1^2) \dot{\theta}_2 + (\frac{1}{2} m_1 l_1 l_2 S_2) \dot{\theta}_1 \dot{\theta}_2 - \frac{1}{2} m_2 g l_1 C_{12} \] \hspace{1cm} (9)
Analysis of Inverse dynamics is shown in table 1

| Link no: | Joint offset(m) | Joint angle (deg) | Link length(m) | Twist angle(deg) | Centre of gravity (m) | Mass (kg) |
|---------|----------------|-------------------|----------------|------------------|-----------------------|-----------|
| 1       | 0              | 0-180             | 0.2            | 0                | 0.1                   | 1         |
| 2       | 0              | 0-90              | 0.2            | 0                | 0.1                   | 1         |

**Table 1**: Analysis of Inverse dynamics

The below figure 2-3 shows the inverse dynamic analysis for link 1 and 2 (Torque) and the inverse dynamic analysis for link 1 and 2 (Acceleration).

Fig 2 shows inverse dynamic analysis for link 1 and 2 (Torque)

Fig 3 shows inverse dynamic analysis for link 1 and 2 (Acceleration)

4. Results and Discussion

Fig 2 shows the value of torque required for link 1 and link 2. It is clear that link 1 need more positive torque ~6 Nm compare with link 2 which in turn require less positive torque of 2 Nm

\[ T_1 = (\frac{1}{3}m_1l_1^2 + m_2l_2^2 + \frac{1}{3}m_3l_3^2 + C_1l_1m_2l_1)\ddot{\theta}_1 + (\frac{1}{3}m_3l_3^2 + \frac{1}{2}C_1l_1m_2l_1)\ddot{\theta}_2 - (m_2l_1l_2S_2)\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2} \]
m_1 \dot{l}_1 \dot{s}_1 + \text{(10)}

T_2 - \frac{1}{2} m_2 l_2 \dot{\theta}_1 + \text{(11)}

Table 2: shows comparison of torque values for link1 and link2 using lagrangian formulation and simulation software Robo analyzer

| Link | Torque (simulated), Nm | Torque values (Theoretical), Nm |
|------|------------------------|---------------------------------|
| 1    | 6                      | 4.112                           |
| 2    | 2                      | 1.046                           |

5. Conclusion

It is not essential to apply lagrangian formulation for solving inverse dynamics problem, there are other methods such as Newton- Euler, Hamiltonian and Featherstone method to obtain the equation of motion. The similar method shall be used for higher degrees of freedom system however the complexity of solving it increases theoretically. It is far better to use simulated software to gain the result provided if the simulation does consider centrifugal and Coriolis Effect. In future studies, researchers can check the theoretical results of a range of configured industrial robot using lagrangian and hamiltonian formulation

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