Jetting formation of the explosively loaded powders

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Abstract. The formation of jet-like structures is widely reported in the explosive dispersal of powders surrounding high explosive charges. The jetting of powder beds initiates upon the shock wave reaches the outer edge of the charge. Opposed to the interface instability theory, a hollow sphere based bulk fragmentation model is established to account for the jetting of powders. A two-phase process, namely the nucleation and free expansion of hollow spheres, corresponds to the unloading process of the powder compact caused by the rarefaction waves which governs the fragmentation of the powders. The separation between adjacent hollow spheres dictates the size of the particle clusters, which would evolve into particle jets in later times. The predicted breakup time and the size of particle jets agree well with the experimental results. The increased moisture content in powders results in an increased number of particle jets. This moisture effect can be understood in light of the varied energy distribution due to the incompressibility of the interstitial liquids trapped inside the inter-grain pores. The portion of shock energy which is not consumed in the shock compaction of the wet powders would be dissipated through the viscous shear flows during the unloading of the wet powder compact. The excessive viscous energy requires to activate more localized shear flows, accordingly leading to an increased number of particle jets.

1. Introduction
When a layer of solid particles is dispersed by a blast or shock wave, the expanding particle cloud typically forms a non-uniform structure comprised of coherent jets [1-7]. A growing body of photographic and radiographic evidence suggests that the jets form early during the dispersal process, on the timescale of the propagation of the shock wave through the particle layer [2, 3]. The observed instability timescale is much shorter than that predicted by the hydrodynamic instability theories, such Rayleigh-Tailor, and Richtmyer-Meshkov (RM) instability [2, 3]. Besides, the length scale of the instabilities assumed by the RM instability model is on the order of the particle scale, which obviously contradicts with the macroscopic...
jets consisting of a large number of constituent grains [1-8]. Thus investigators increasingly focus their attentions to the dynamic fragmentation of the shock compressed particle bed which is regarded as the dominant mechanism pertaining to the particle jetting.

In this work, we establish a bulk fragmentation model based on the expansion of hollow spheres to describe the decomposition process of the shocked powder rings caused by the rarefaction waves. The matrix of the hollow sphere represents the dense particle flows whose behavior satisfies the Bingham constitutive relations. Via keeping the thermodynamic variables derived from the expansion of the microscopic hollow spheres consistent with the those obtained from the macroscopic continuum deformation, we can determine the separation of the neighboring hollow spheres which corresponds to the jet size. The consistency between the theoretical jet size and the experimental results validates our model.

Both experiments and our theoretical model have found that increasing moisture content leads to an increased number of jets. This moisture effect is found to be correlated with the viscous energy dissipated through the localized shear flows. Wet powders endure much higher overpressure during the shock compaction due to the reduced compressibility. The ensuing viscous energy dissipation which depends on the overpressure becomes much intensified in the wet powders. Accordingly more particle clusters, equally particle jets, as the carriers of shear flows emerge in wet powders.

2. Theoretical model based on the expansion of the hollow sphere

Our previous hydrodynamic simulations of the shock interaction with the dry and saturated sand shells revealed that the decomposition of the shocked powder compact coincides with the propagation of the rarefaction waves [9]. What is worth noting is that the overpressure in the saturated sand is higher than that in the dry sand by one order of magnitude.

![Figure 1](image)

**Figure 1.** Illustration of the hollow sphere model applied on the spallation. (a) the unloaded sub-volume element in the wake of the rarefaction wave. (b) sub-volume element consisting of a large number of hollow spheres; (c) the expansion of the hollow sphere.

The illustration of the hollow sphere model is shown in figure 1. This model assumes that the sub-volume element $V'$ in the relaxed body is in the thermodynamic uniform state, and the growth of the voids inside can be approximated by the expansion of the hollow spheres. The inner and outer radii of the hollow sphere are $a(t)$ and $b(t)$, respectively, where the $b(t)$ represents the separation between two neighboring nucleate sites of voids. The porosity can be calculated by $f = \frac{a^3}{b^3}$. In the frame of the hollow sphere model, the matrix outside the hollow is treated as the continuum consisting of the dense particle flows. The hollow relates to the porosity rather than the actual voids.
The expansion of the hollow sphere consists of two sequent phases: 1) $0 < t < \tau$, the hollow sphere expands while being subjected to the rarefaction, the expansion rate kept identical with the total volume change rate; 2) $t > \tau$, the hollow sphere undergoes free expansion through the momentum obtained in the first phase. The loading conditions applied on the hollow spheres is prescribed by two parameters, the expansion rate $D$ and the duration of rarefaction waves $\tau$ derived from the hydrodynamic simulations. Table 1 presents the parameters pertaining the rarefaction waves in the dry and saturated sands.

| materials      | Expansion rate $D$ (s$^{-1}$) | Duration $\tau$ ($\mu$s) |
|----------------|-------------------------------|-------------------------|
| Dry sand       | $7.84 \times 10^3$            | 20                      |
| Saturated sand | $1.0 - 1.46 \times 10^3$      | 2                       |

The strain rate tensor of the radially expanding hollow sphere in the spherical coordinates can be expressed as equation (1)

$$d_{\nu} = \frac{2\phi}{3V}, \quad d_{m0} = \frac{\phi}{3V},$$

where $x_r$ is the radial coordinate of the particle, $\omega(t)$ represents the volume expansion of the hollow, $\omega(t) = x_r^3 - a^3$. The behavior of the matrix of the hollow sphere is described by the constitutive relation of the dense particle flows [10]. The stress tensor is composed by the hydrostatic pressure $P$ and the shear stress $\tau_{ij}$:

$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij},$$

$$\tau_{ij} = \eta_{eff} d_{ij}, \quad \eta_{eff} = \frac{\mu(I, \beta) P}{|d|}$$

where $|d|$ is the second order invariant of the shear rate tensor, $|d| = \sqrt{2d_{ij}d_{ij}}$, $\eta_{eff}$ is the effective viscosity. The effective fictional coefficient $\mu$ is the function of the inertial number $I$ and the cohesion number $\beta$ as shown in equation (4)

$$\mu(I, \beta) = \mu_{min}(\beta) + b(\beta) I,$$

$$I = |d| \sqrt{\frac{m}{P}}, \quad \beta = \frac{N_c^p}{Pd_p^2}$$

where $m$ is the mass of the particle, $d_p$ is the diameter of particle; $N_c^p$ is the largest cohesion between particles, for dry sand, $N_c^p (\sim 10^4 \text{ N-m})$ is the van der Waals force, for saturated sand, $N_c^p (\sim 10^4 \text{ N-m})$ is the cohesion induced by the surface tension of the liquid. For $P \sim 10^7 - 10^8 \text{ Pa}$, $d_p \sim 10^{-3} \text{ m}$, $m\sim 7 \times 10^{-7} \text{ kg}$, shear rate $|d| \sim 10^3 \text{ s}^{-1}$, we can obtain

$$I \sim 10^{-3}, \beta \sim 10^{-3}, \mu \sim \mu_{min}(0) \sim 0.25$$

Substituting equations (3-5) into equation (2), we have
\[
[\sigma] = -P \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}+ \frac{\sqrt{3}\mu(t)P}{3} \begin{pmatrix}
-2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]  

(6)

The Euler form of the momentum equation under the spherical symmetric condition is

\[
\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (2\sigma_{rr} - 2\sigma_{tt}) = \rho \gamma .
\]  

(7)

For dry sand, there is no traction on the inner surface of the hollow. For the saturated sand,

\[
\sigma_{rr}(r = a, t) = 2\gamma / a .
\]  

(8)

Integrating equation (7) from \(a\) to \(b\), for dry sand it becomes

\[
\left(1 + \frac{2\sqrt{3}}{3} \mu(t)P \right) P + 2\sqrt{3}\mu(t)P \ln \frac{b}{a} = \rho \left[ \frac{\partial}{\partial r} \left( \frac{1}{b^2/a^2} - 1 \right) + \frac{\partial^2}{\partial a^2} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \right]
\]  

(9)

Substituting (8) into (7), for the saturated sand, integrating the momentum equation leads to

\[
\left(1 + \frac{2\sqrt{3}}{3} \mu(t)P \right) P + 2\gamma / a + 2\sqrt{3}\mu(t)P \ln \frac{b}{a} = \rho \left[ \frac{\partial}{\partial r} \left( \frac{1}{b^2/a^2} - 1 \right) + \frac{\partial^2}{\partial a^2} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) \right]
\]  

(10)

Keeping the expansion rate of the hollow sphere identical with the macroscale volume change rate:

\[
D = \frac{\dot{b}}{b} = \frac{\dot{a}}{b_0 + \omega}
\]  

(11)

The initial condition is \(\omega(0) = 0\), \(\dot{\omega}(0) = D b_0^3\).

**Figure 2.** (a) Temporal variations in the microscopic pressure with different \(b_0\) and the microscopic pressure in the layer with \(r = 56.8\) mm. The theoretically estimated breakup radius and the fragment size of layers with increasing radius in dry (b) and saturated sand (c).

By solving the integration equation (9) or (10), the microscopic pressure \(P_{\text{mic}} = -\sigma_{rr}(b, t)\) and the porosity \(f = \frac{a'(a, t)}{b'(b, t)}\) can be derived from the \(\omega(t)\). Figure 2(a) presents the comparison of \(P_{\text{mic}}\) which is a function of \(b_0\) and the macroscopic pressure, \(P_{\text{mac}}\), obtained by the hydrodynamic simulations. The \(b_0\) is chosen in such way that the deviation between \(P_{\text{mic}}\) and \(P_{\text{mac}}\) is the smallest. The predicted fragment size of the saturated sand, \(b_{0,\text{sat}} \sim 0.6-1\) cm is one order smaller than the predicted one for the dry sand, \(b_{0,\text{dry}} \sim 3-5\) cm (see figure 2 (b) and (c)), indicating the refinement effect of the moisture content on particle jets observed in
experiments. The dependence of the fragment size on the radius of particle layers can be attributed to the reduced strain rate rendered by the declined rarefaction waves. The number of jets around the outer perimeter of the charge is around 100 reported by experiments, consistent with the theoretical estimation on the order of magnitude [2-3].

Applying the fragmentation criteria, $f_c \sim 0.8$, the disintegration of the outer particle layers initiates at $t_c \sim 170-200 \mu s$. $t_c$ is very close to the inception time of the particle jetting recorded by the X-ray radiographic photos. The breakup of the saturated particle layers occurs in the later times, suggesting that the particle jets are formed in the wet particles at delayed times and form more uniform structure [5].

3. Discussion

Grady fragmentation theory argues that the material fragments when the input energy is larger than the energy required by the creation of new surfaces combined by that dissipated during this process [11]. The surface tension is trivial in both dry and saturated particles subjected to the high strain rate loadings. Thus surface energy is negligible during the fragmentation of the particle systems. By contrast, due to the proportionate dependence of viscosity on pressure, the viscous energy dissipation in the shocked particles becomes significant due to the high overpressure.

The energy dissipation rate caused by the inelastic shear flows is

$$\varphi_i = \int_{\alpha}^{\beta} \varepsilon_i d\phi 4\pi x_i^2 dx_i = 4\pi \mu \frac{d^2 \xi}{|d|} dx_i.$$

Throughout the unloading process of the sand compact, the cumulative viscous energy in the unit volume, $\Phi_i$, is

$$\Phi_i = \int_0^\tau \frac{\varphi_i}{1/6 \pi b^2} dt = 48 \frac{1}{\sqrt{3}} \mu \int_0^b P(t) \dot{\phi}(t) \ln \frac{b(t)}{a(t)} dt,$$

where $\tau$ is the duration of the rarefaction waves.

![Figure 3](image)

**Figure 3** (a) the cumulative viscous energy dissipated in the saturated sand under different pressures, where $b_0 = 7.5 \text{ mm}$. The cumulative viscous energy dissipated in the dry (b) and saturated sand (c).

$\Phi_i$ increases with the pressure and $b_0$. Figure 3 demonstrates the pressure dependence of $\Phi_i$. A trivial increase of pressure suffices to result in the significant increase of $\Phi_i$ dissipated in the unloading process of the saturated sand. Although the $b_0$ is smaller in the saturated sand,
the pressure effect dominates $\Phi$, which is one order of magnitude higher in the saturated sand than that in the dry sand. Viscous energy dissipation occurs wherever the shear flows are present. Instead of being prevalent throughout the particle system, the shear flows are localized and intensified within a number of particle clusters which would involve into particle jets in later times. The intensity of shear flows as a function of velocity gradient doesn't change much regardless of the composition of the particle system. Consequently increasing the number of particle clusters where the shear flows occur is the most effective way to dissipate the excessive viscous energy. Increased viscous energy dissipated in the saturated sand demands an increased number of particle clusters, equally more localized shear flows be activated, resulting in an increased number of particle jets.

4. Conclusion

The particle jetting is initiated upon the reflection of the shock wave on the outer edge of the particle ring, and evolves during the unloading process of the compact particles caused by the rarefaction waves. A two-phase hollow sphere model is developed to account for the unloading and decomposition of the particle compact. The predicted breakup time, the breakup radius of the expanding shell as well as the jet size are all consistent with the experimental results as least on the order of magnitude. Increased viscous energy is dissipated in the saturated sand due to its incompressibility, resulting in the intensified shear flows. Accordingly the fragmentation of the saturated sand involves an increased number of particle clusters as the carriers of shear flows, which would evolve into particle jets in later times.

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