NATURAL R PARITY CONSERVATION
WITH HORIZONTAL SYMMETRIES.
A FOUR GENERATION MODEL

ZURAB BEREZHIANI\textsuperscript{a,b} and ENRICO NARDI\textsuperscript{c}

\textsuperscript{a}INFN, Sezione di Ferrara, 44100 Ferrara, Italy
\textsuperscript{a}Institute of Physics, Georgian Academy of Sciences, 380077 Tbilisi, Georgia
\textsuperscript{c}Department of Particle Physics, Weizmann Institute of Science, P.O.B. 26, Rehovot, 76100 Israel

Abstract
In most supersymmetric models the stability of the proton is ensured by invoking R-parity. A necessary ingredient to enforce R-parity is the possibility of distinguishing the lepton superfields from the Higgs ones. This is generally achieved either by assuming different charges under some matter parity, or by assigning the superfields to different representations of a unified gauge group. We want to put forward the idea that the replica of the fermion generations, which constitute an intrinsic difference between the fermions and the Higgs superfields, can give a clue to understand R-parity as an accidental symmetry. More ambitiously, we suggest a possible relation between proton stability and the actual number of fermion generations. We carry out our investigation in the framework of non-Abelian horizontal gauge symmetries. We identify $SU(4)$\textsubscript{H} as the only acceptable horizontal gauge group which can naturally ensure the absence of R-parity violating operators, without conflicting with other theoretical and phenomenological constraints. We analyze a version of the supersymmetric standard model equipped with a gauged horizontal $SU(4)$\textsubscript{H}, in which R-parity is accidental. The model predicts four families of fermions, it allows for the dynamical generation of a realistic hierarchy of fermion masses without any ad hoc choice of small Yukawa couplings, it ensures in a natural way the heaviness of all the fourth family fermions (including the neutrino) and it predicts a lower limit for the $\tau$-neutrino mass of a few eV. The scale of the breaking of the horizontal symmetry can be constrained rather precisely in a narrow window around $\sim 10^{11}$ GeV. Some interesting astrophysical and cosmological implications of the model are addressed as well.

Electronic mail: berezhiani@fe.infn.it, ftnard@wiswic.weizmann.ac.il
1 Introduction

In the Standard Model (SM), Baryon (B) and Lepton (L) numbers are conserved as a result of accidental global $U(1)_B$ and $U(1)_L$ symmetries that follow from the requirement of gauge invariance and renormalizability. These symmetries are violated only by higher order non-renormalizable operators, cutoff at the Planck scale, which can arise from non-perturbative quantum gravity or string effects [1]. In the Supersymmetric (SUSY) version of the Standard Model (SSM) this is not true anymore. Consider in fact the quark and lepton left-handed chiral superfields, which transform under SU(3) $\times$ SU(2) $\times$ U(1) as follows:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1/6), \quad u^c \sim (\bar{3}, 1, -2/3), \quad d^c \sim (\bar{3}, 1, 1/3),$$
$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1, 2, -1/2), \quad e^c \sim (1, 1, 1).$$

The two Higgs superfields $\Phi_{1,2}$ transform as $(1, 2, \mp 1/2)$ respectively. As the scalar components of $\Phi_{1,2}$ acquire nonzero vacuum expectation values (VEVs), the fermions acquire a mass through the superpotential terms $\Phi_1 q d^c$, $\Phi_2 q u^c$ and $\Phi_1 l e^c$. As for the Higgsinos, their mass is provided by the SUSY invariant term $\mu \Phi_1 \Phi_2$. Since $l$ and $\Phi_1$ have the same transformation properties under the gauge group, the L and B violating terms obtained by substituting $\Phi_1 \rightarrow l$ are also allowed by the gauge symmetry, as well as an additional term involving three quark superfields. These terms read (family and gauge indices are suppressed)

$$\mu' l \Phi_2, \quad \lambda' l e^c, \quad \lambda' l q d^c, \quad \lambda'' u^c d^c d^c.$$  

The simultaneous presence of all the terms in (2) is phenomenologically unacceptable. In particular, if both the third and fourth of these terms are present, their combination would lead to catastrophically fast proton decay mediated by $d^c$-type squark exchange, unless the relevant couplings are fine-tuned to extremely small values $\lambda' \cdot \lambda'' \lesssim 10^{-26} (\frac{m_{\tilde{d}^c}}{\text{TeV}})^2$. As for the $\mu'$ term, in the SSM it is possible to eliminate it through a suitable rotation among the $l$ and $\Phi_1$ superfields [2]. After such a redefinition, the mass Yukawa terms $\Phi_1 q d^c$ and $\Phi_1 l e^c$ give rise to (corrections to) the $\lambda$ and $\lambda'$ terms in (2). However, in some extensions of the SSM $l$ and $\Phi_1$ will not have the same quantum numbers, thus this rotation is not always possible so that we will keep explicitly the $\mu'$ term.

The relevant symmetry that ensures the B and L conservation in the SSM is called R parity, which is defined as $R \equiv (-1)^{2J+3B+L}$, where $J$ is the spin of the particle and $B(L)$ its baryon (lepton) number [3]. R parity is an automatic consequence of a $Z_2$ matter parity under which the fermion superfields change the sign while the ‘Higgs’ ones $\Phi_{1,2}$ remain invariant. R parity is trivially related to $Z_2$ matter parity by a factor of -1 for fermions, and hence does not commute with supersymmetry. It is a well known fact that

---

1 For example, the dimension 5 lepton number violating term $(1/M_P) l \Phi \Phi$ provides a neutrino Majorana mass of about $10^{-5}$ eV, which could be relevant for the solar neutrino oscillations. However, analogous terms violating baryon number are dimension 6 or higher, and hence too small to cause any observable effect.

2 This term, together with an analogous term in the scalar potential which leads to non-vanishing sneutrino VEVs, induces a mixing between the neutrinos and the neutralinos, and can easily generate a too large value for the $\nu$ mass.
an unsatisfactory feature of the SSM is that the $Z_2$ (or equivalently R) parity conservation has to be imposed by hand.

In the context of Grand Unification Theories (GUT) based on the gauge group $SU(5)$, the fermion superfields are assigned to the $10 + 5\bar{5}$ representation of $SU(5)$, while $\Phi_1$ and $\Phi_2$ belong respectively to the $5\Phi_1$ and $\bar{5}\Phi_2$. The down-quark and lepton masses are generated through the coupling $10 \bar{5} 5\Phi_1$, and the gauge invariant terms obtained by $\bar{5}\Phi_1 \rightarrow \bar{5}$, (namely $10 \bar{5} 5\Phi_1$ and $5 \bar{5} 5\Phi_2$) lead again to the set of B and L violating couplings in (2). Thus, with respect to automatic R-parity conservation the supersymmetric $SU(5)$ model does not differ much from the SSM. In addition to this, in SUSY $SU(5)$ the effectiveness of imposing R parity as an additional global discrete symmetry is also questionable. In fact, even if at the renormalizable level the $10\bar{5}5\bar{5}$ term is forbidden, a problem can appear due to non-perturbative quantum gravity effects (virtual black holes or wormholes) which in general do not respect the global charges. These effects can induce in the superpotential higher order terms as $(1/M_{Pl})\bar{5}5\bar{5}1024$, where the 24 is the adjoint Higgs representation which breaks the $SU(5)$ symmetry. After substituting its VEV $(24) \sim 10^{16}$ GeV), this operator reduces to the terms given in (2) with a Yukawa coupling $(24)/M_{Pl} \sim 10^{-3}$, which would be catastrophic for proton decay. This argument supports the idea that it is highly desirable to achieve R parity conservation in an automatic way as a natural consequence of gauge invariance, which could then protect the symmetry from effects of this kind.

As is well known, this is the case for $SO(10)$ models. Indeed $SO(10)$ offers an elegant solution to this problem, since the fermion superfields are in the spinor representation 16 whereas the Higgs ones are generally assigned to vector representations as $10, 45, 54, 126$ etc.. The masses of the fermions, including the neutrinos, can be generated through the gauge invariant couplings $161610\Phi$ and $1616126\Phi$ [4], while the terms $(16)^3$ and $1610\Phi$ are forbidden since they are not $SO(10)$ invariant. In other words, as long as all the $SO(10)$ invariant couplings allow for only pairs of 16-plets, the theory has an automatic $Z_2$ matter parity under which 16-plets change the sign whereas the superfields in vector representations remain invariant. This is not true anymore for the $SO(10)$ models in which the symmetry breaking is triggered also by the scalar components of superfields belonging to the $16\Phi + \bar{16}\Phi$. (Examples of SUSY models in which these representations play the role of the standard $126 + \bar{126}$ can be found in ref. [3]). In fact, after substituting the VEV $\langle 16\Phi \rangle$, the couplings $\frac{1}{M}16^316\Phi$ which are allowed by the gauge symmetry lead again to R-parity violating terms. Since in these models the right handed neutrino masses are generated by operators $\sim \frac{1}{M}16^316^2\Phi$ which have the same structure, the ratio $\langle 16\Phi \rangle / M$ cannot be very small, implying that the magnitude of the resulting R-parity violating terms is again in conflict with the limits on the proton lifetime. In addition to this, a direct term $m^21616\Phi$ in the scalar potential will induce a nonvanishing VEV for some scalar field carrying lepton number, leading also to spontaneous R-parity violation trough sneutrino VEVs. We conclude that R-parity conservation is not automatic anymore for the $SO(10)$ models with Higgs fields belonging to the $16\Phi$. In this case some additional discrete symmetry has to be imposed by hand in order to distinguish the fermion 16-plets from the Higgs ones.

The result of this brief analysis shows that in order to ensure the absence of the operators (2), some ‘label’ to distinguish the Higgs superfields from the fermion ones is required. Such a label can be chosen ad hoc, as in the SSM and in the $SU(5)$ model, by assigning
to the Higgs and fermion superfields different $Z_2$ parities or, as in $SO(10)$, it can arise in a more natural way by assigning the different superfields to different representations of the GUT gauge group.

Our work stems from the observation that there is a natural distinction between the fermion and the Higgs superfields: namely that fermions superfields replicate in different generations, while Higgs superfields do not. In this paper we wish to investigate if, in the framework of SUSY models, any natural link can be found between two striking but apparently unrelated experimental evidences: the stability of the proton (or more generally the conservation of R-parity) and the replica of fermion generations. More precisely, we want to put forward the idea that R (or equivalently $Z_2$) parity could arise as a consequence of a gauged horizontal symmetry, broken at some high scale, which constitutes also a natural framework to account for the replica of fermion generations. Clearly, the horizontal group $G_H$ should act only on the quark-lepton superfields, while the Higgses $\Phi_{1,2}$ responsible for the electroweak symmetry breaking are $G_H$-singlets. In such a picture, independently of the choice of the vertical gauge group and/or of the particular superfield assignments to its representations, the Higgs and fermion superfields can be always distinguished by their different transformation properties under the horizontal gauge group, and this leads to the possibility of allowing the necessary mass terms which are bilinear in the fermion superfields, while forbidding the B and L violating linear and trilinear couplings in (2).

Beyond accounting for the number of generations, models based on gauged horizontal symmetries present several additional interesting features (see for example the models \cite{7, 8} based on $SU(3)_H$ horizontal symmetry). They naturally embed the principle of ‘flavor democracy’ \cite{6}, namely that all fermions with the same gauge quantum numbers should have the same short distance interactions, while the observed mass differences arise from dynamics at some large energy scale. They can also explain qualitatively the observed pattern of fermion masses and mixing without appealing to unnaturally small values for any fundamental parameter \cite{4, 8}. The structure of the fermion mass matrices can in fact be related to the horizontal symmetry breaking pattern. Then the mass hierarchy between families arises dynamically from certain hierarchies in this breaking, while the fermion Yukawa couplings are generation blind, and can be assumed to be all of order unity.

The paper is organised as follows: in Section 2 we will carry out a general analysis of the non-Abelian horizontal symmetries that can forbid the R-parity violating operators (2), and that at the same time satisfy a certain number of theoretical and phenomenological constraints. We will show that $SU(4)_H$, with the fermions assigned to the fundamental $4_H$ representation, is the only viable candidate. Then in our scheme the presence of one additional family of fermions results as an unavoidable prediction. In Section 3 we will explicitly construct a model based on $SU(4)_H$. We will discuss the pattern of the Horizontal symmetry breaking, and the peculiar form of the resulting fermion mass matrices. We will show that the minimal number of horizontal scalars needed to reduce completely the rank of the group, can also ensures that all the fourth family fermions (including the neutrino) acquire naturally large masses, typically of the order of the electroweak scale. An additional interesting feature of our model is that the fourth family fermions are unmixed with the lighter ones. In Section 4 we will address some of the phenomenological consequences of the model. With the aid of cosmological and astrophysical arguments,
the scale at which the horizontal symmetry is completely broken can be constrained to a narrow window around $10^{11}$ GeV. This results in a strict lower limit of a few eV for the mass of the $\tau$ neutrino. We also discuss the possible decay channels and lifetime for the unmixed fourth family quarks. We show how our model leads to the prediction of a possible cosmological signal of diffuse gamma ray flux over background from decays of relic $b'$. The problem of the baryogenesis mechanisms viable in our model is also briefly addressed in this section. Finally, in Section 5 we will collect the main results and draw our conclusions.

2 General Analysis of Horizontal Gauge Groups

Our aim is to find and classify the theories in which the horizontal gauge group $G_H$ naturally forbids the terms in (2) due to gauge principles, or in other words in which $R$ parity (or equivalently $Z_2$ matter parity) appears as an automatic consequence of the horizontal gauge symmetry and of the field content of the model. We demand that the models we are interested in should satisfy the following list of basic requirements:

(i) In order to ensure a straightforward definition for the horizontal gauge symmetry as a symmetry intrinsically related to the number of fermion generations, all the fermion superfields with the same quantum numbers must fill up one irreducible representation of the horizontal group $G_H$. In particular, we forbid $G_H$ singlet families.

(ii) On the other hand, in order to implement consistently our idea, the standard Higgs superfields $\Phi_1, 2$ must be singlets with respect to $G_H$. This requirement is needed also to prevent the proliferation of Higgs doublets with masses at the electroweak scale, that would spoil the natural suppression of Flavor Changing Neutral Currents (FCNC) \cite{9}. Moreover, Higgs in non-singlet horizontal representations would also destroy gauge coupling unification, thus preventing any attempt to embed the model in some vertical GUTs.

(iii) We demand that the couplings in (2) are forbidden as a consequence of conditions (i) and (ii), together with the requirement of horizontal gauge invariance. Clearly, any non-Abelian group satisfying (i) and (ii) will forbid the first term in (4), since it is linear in the fermion superfields, so in the following we will concentrate on the additional conditions needed to forbid the trilinear terms.

Since we are investigating the possibility of relating the absence of R-parity violating operators to the number of generations, we wish to treat the latter one as a free parameter, to be determined by the dimension $N$ of the representation of the non-Abelian horizontal group. However, in order to have phenomenologically realistic theories, $N$ should not be too large. For example additional families would contribute to the radiative corrections to electroweak quantities. Detailed analyses of the precise electroweak data have been recently performed, and they rule out $N \geq 6$ \cite{12}. We will then restrict our analysis to groups with representations of dimension $N = 3, 4, 5$.

To have phenomenologically appealing models, the following additional constraints should be also imposed:

(iv) We require that a realistic pattern of fermion masses and mixings should arise naturally as a result of the dynamics of the horizontal symmetry breaking, and in partic-
ular, since no new state in addition to the three families of fermions have been observed yet, possible new generations should be naturally heavy.

More specifically, the fermion masses should arise from effective operators with the structure

\[ \mathcal{O}_{\text{eff}} \sim \frac{\mathcal{P}^{(n)}(\xi_k)}{M^n} ff^c \Phi_{1,2}, \]  

where \( f, f^c \) are the fermion superfields in eq. (1), \( \mathcal{P}^{(n)} \) represents some \( n \)-order polynomial of the scalars \( \xi_k \) responsible for the breaking of \( G_H \), and \( M \) is some cutoff mass scale. Clearly, in order to ensure \( G_H \) invariance, \( \mathcal{P}^{(n)} \) should transform as the conjugate of the tensor product of \( f \) and \( f^c \).

This ‘naturalness’ condition is a rather strong one, since it rules out at once all the self-conjugate representations \( N_S \), as well as the groups that have only self-conjugate representations. In fact, the horizontal gauge invariant term \( N_S N_S \Phi_{1,2} \) would imply equal masses for the different generations. The mass splitting between different families could then be achieved by means of the additional effective operators (3) only in a very unnatural way, at the price of many fine tunings or ad hoc choices for the relevant parameters. Indeed, generating a hierarchy in this way can be hardly regarded as realistic.

\( (v) \) An additional condition is that R-parity breaking terms should not appear even after \( G_H \) breaking. More precisely, all effective operators of the form

\[ \frac{\mathcal{P}^{(n)}(\xi_k)}{M^n} ff^c, \quad \frac{\mathcal{P}^{(n)}(\xi_k)}{M^n} f^c f^c, \]  

should be forbidden by the \( G_H \) symmetry, since after the horizontal symmetry breaking \( \xi_k \to \langle \xi_k \rangle \) these terms would generate again the R parity violating couplings (2).

We start our analysis with the only simple groups that have three dimensional representations, namely \( SO(3)_H \) and \( SU(3)_H \).

For \( SO(3)_H \), the term \( 3^3 \) (as well as \( 5^3 \)) contains a gauge singlet, and hence does not forbid the couplings (3). Moreover, \( SO(3)_H \) contains only self-conjugate representations, so even if we had to assign the fermions to the \( 4 \), the phenomenological requirement (iv) would not be satisfied.

For \( SU(3)_H \) there are two possibilities. Vectorlike \( SU(3)_H \), with \( q, l \) transforming as \( 3 \) and \( u^c, d^c, e^c \) as \( 3^c \), is an interesting possibility, since it forbids the second and third terms in (4), which is enough to ensure the proton stability. However, the \( SU(3)_H \) invariant terms \( ff^c \Phi_{1,2} \) is allowed, and thus once more condition (iv) is not fulfilled. In addition vectorlike \( SU(3)_H \) would also impede the unification of the fermions (1) within one irreducible GUT multiplet.

Chiral \( SU(3)_H \), with all the fermions assigned to the same representation \( 3 \) (or \( 3^c \)) fails to satisfy condition (iii), since \( 3^3 \) contains a gauge singlet. However, models based on chiral \( SU(3)_H \) \( [7, 8] \) have proven to be quite effective in relating the fermion mass hierarchy to the hierarchy in the \( SU(3)_H \) breaking, that is to the hierarchy among the horizontal VEVs \( \langle \xi_k \rangle \). Since this success in accounting for the pattern of fermion masses and mixings is intimately related to the non self-conjugate nature of the fundamental

---

3 The non-renormalizable couplings (3) with \( M \sim M_{Pl} \) could appear due to quantum gravity effects. Alternatively, these operators with arbitrary \( M \) can be effectively generated through the exchange of some superheavy fields with \( O(M) \) masses (4).
representation, it appears to be a general feature of chiral $SU(N)$ models. This points towards chiral $SU(N)$ as possible interesting candidates, since we immediately notice that in this case the term $N^3$ does not contain gauge singlets, and thus the terms in (3) are automatically forbidden.

As a first result, our analysis implies that in order to implement our scheme the number of generations must be larger than 3, and also suggests that the $SU(N)$ groups represent a class of interesting candidates.

We now move to groups with 4 dimensional representations. Apart from $SU(4)$, which we will discuss in detail in the following, also for $SO(4)$ and $SO(5)$ the lowest dimensional representation is the 4. In both cases, while the $4^3$ terms are forbidden and thus condition (iii) is fulfilled, the requirement (iv) is not satisfied. In fact, both these groups have only self-conjugate representations, the invariant Yukawa terms $ff^c\Phi_{1,2}$ are again allowed, and a hierarchy in the fermion masses cannot be generated in a natural way.

Apart for $SU(5)$, there are no new groups with five dimensional representations. As we will now show, the last condition (v) restrict the viable $SU(N)$ models to the cases when $N$ is even, thus ruling out $SU(5)$ as a satisfactory horizontal symmetry. Consider in fact $SU(N)$ with the $f$ and $f^c$ fermion superfields assigned to the fundamental $N$ dimensional representation. The mass terms transform as $N \times N$ and thus belong to two-index (symmetric and antisymmetric) representations. In order to construct horizontal gauge invariant mass terms, we can take also the horizontal Higgses $\xi_k$ in two-index representations. Then for $N=4,6,\ldots$ terms of the form $N \times N \times \mathcal{P}^{(n)}$ (that is the $R$-parity violating effective operators (4)) cannot arise, since it is impossible to saturate all the indices and construct horizontal gauge invariants. In contrast, for $SU(N)$ with $N$ odd the totally antisymmetric $\epsilon$ tensor allows to rewrite some combinations of Higgs fields with an even number of free indices as tensors with an odd number of free indices, which are suitable for generating gauge invariants when matched with the $N \times N \times N$ term. Indeed, after the horizontal symmetry breaking ($\xi \rightarrow \langle \xi \rangle$) operators of the form

\[
    f^{(c)}_{\alpha} f^{(c)}_{\beta} f^{(c)}_{\gamma} (\xi_1 \xi_2 \ldots \xi_n)^{\delta \ldots \sigma} M^n \epsilon_{\alpha \beta \gamma \delta \ldots \sigma}
\]

(which can be constructed also when the $\xi$’s belong to the symmetric part of $N \times N$) will again spoil R-parity.

Our analysis suggests that natural conservation of $R$-parity, complemented with the additional phenomenological constraints (iv) and (v), can be achieved in models based on chiral horizontal symmetries $SU(N)$ with $N$ even, under which the quark and lepton superfields transform as fundamental $N$-plets. The unwanted terms transforming as $N^3$ are automatically forbidden by horizontal gauge invariance, and the generation of a realistic pattern of masses and mixings appears viable. Clearly, in order to implement our scheme, the number of families must be extended to $N_f > 3$ and even. As is well known, the possibility of extra families with a light neutrino is ruled out by the results of the Mark II and LEP collaborations [11]. However, these results do not exclude sequential generations with heavy neutrinos ($m_\nu > M_Z/2$). On the other hand, as we have already mentioned, detailed studies [12] of the effects of radiative corrections due to additional families show that precise electroweak data are not incompatible with a fourth family, while six families (which would be our next interesting case) are ruled out [12]. In ad-
dition to this, a dedicated analysis showing the viability of supersymmetric models with four families with respect to gauge coupling unification was presented in ref. \[13\]. These results are relevant for our analysis, since condition (ii) ensures that the field content in our $SU(4)_H$ model is the same than that of the four family SSM of ref. \[13\], up to some large energy scale where the horizontal symmetry breaks down (see Sect. 3).

We can conclude that the stability of the proton in SUSY models can be naturally related to the replica of the fermion generations by assuming a suitable horizontal gauge symmetry. Such a symmetry ensures that R parity arises as an accidental symmetry of the gauge model. Moreover, theoretical and phenomenological constraints allow to single out $SU(4)_H$ as the only satisfactory horizontal gauge group, on which we will concentrate in the rest of the paper.

### 3 A Model with Horizontal Symmetry $SU(4)_H$

Let us now consider the standard $SU(3) \times SU(2) \times U(1)$ vertical gauge group, with local chiral $SU(4)_H$ horizontal symmetry acting on four families of left chiral superfields

\[
\begin{align*}
    f_\alpha &: \quad q_\alpha = \left( \begin{array}{c} u \\ d \end{array} \right)^\alpha \sim (3, 2, 1/6, 4), \\
    f'_\alpha &: \quad u^c_\alpha \sim (\bar{3}, 1, -2/3, 4), \quad d^c_\alpha \sim (3, 2, 1/3, 4), \quad e^c_\alpha \sim (1, 1, 1, 4)
\end{align*}
\]

where each superfield is assigned to the fundamental 4 representation ($\alpha = 1, \ldots, 4$ is the $SU(4)_H$ index). With this field content the horizontal $SU(4)_H$ is anomalous. In order to cancel the horizontal anomaly we introduce the following superfields which are vectorlike with respect to $SU(3) \times SU(2) \times U(1)$ and belong to the $\overline{4}$ of $SU(4)_H$:

\[
\begin{align*}
    F^\alpha &: \quad U^\alpha \sim (3, 1, 2/3, 4), \quad D^\alpha \sim (3, 1, -1/3, 4), \quad E^\alpha \sim (1, 1, -1, 4) \\
    F'^\alpha &: \quad U^c_\alpha \sim (\bar{3}, 1, -2/3, 4), \quad D^c_\alpha \sim (\bar{3}, 1, 1/3, 4), \quad E^c_\alpha \sim (1, 1, 1, 4), \quad N^c_\alpha \sim (1, 1, 0, 4)
\end{align*}
\]

As we will see in short, this same set of superfields turn out to be necessary also for providing masses to the known fermions.

In the Higgs sector, we choose the standard Higgs doublet superfields $\Phi_{1,2}$ to be singlets under $SU(4)_H$. The additional Higgs scalars needed for the breaking of the horizontal symmetry at some large scale cannot couple to the standard $SU(2) \times U(1)$ gauge bosons, and thus must be singlets under the electroweak group. In order to break completely the horizontal symmetry and to generate realistic mass matrices for the fermions, we introduce a set of $SU(3) \times SU(2) \times U(1)$ singlet ‘horizontal’ superfields, transforming either as the symmetric 10 ($\xi_{(\alpha\beta)}$) or as the antisymmetric 6 ($\chi_{[\alpha\beta]}$) representations of $SU(4)_H$. Let us first consider the case with the fields $\xi^k$, $k = 1, 2, \ldots$ in symmetric representation. Additional superfields $\bar{\xi}^k$ transforming as $\overline{10}$ are needed to render the Higgsino sector free from chiral anomalies. However, these additional scalars do not contribute to the fermion masses.\footnote{Let us note that $\bar{\xi}^{(\alpha\beta)}$ in the $\overline{10}$ cannot couple in renormalizable way to the heavy vectorlike ‘matter’ fields in the $\overline{4}$, and being an $SU(2)$ singlet, neither it can couple to quarks and leptons. However, it is still possible to introduce a direct non-renormalizable terms cutoff by the Planck scale $(1/\sqrt{M_{Pl}}) f_{\alpha} f_{\beta} \Phi_{1,2} \bar{\xi}^{\alpha\beta}$.}
What remains now to show, is that within the framework of the gauge horizontal symmetry $SU(4)_H$ one can obtain a realistic mass pattern for three families, ensuring at the same time that all fermions of the fourth family are naturally heavy, say in the 100 GeV range. Although we have started our considerations by a general analysis which included also non-renormalizable operators as in (3) and (4), in building the model we will restrict ourselves to consider only renormalizable interactions.

The most general Yukawa superpotential for the down (up) quark and for the lepton superfields allowed by gauge invariance reads

$$W_F = g_f f^\alpha F_c^{\alpha} \Phi_{1(2)} + \sum_k h^k_F F^\alpha_c F^{\beta}_c \xi^k_{\alpha\beta} + \mu_f F^\alpha f^c_\alpha$$  \hspace{1cm} (8)

with $f$ and $F$ respectively from (3) and (4). The analogous couplings for the neutrinos have the form

$$W_N = g_\nu l^\alpha \nu^\alpha \Phi_2 + \sum_k h^k_N N^\alpha_c N^\beta_c \xi^k_{\alpha\beta}.$$  \hspace{1cm} (9)

Here the $g$'s and $h$'s are Yukawa couplings which we assume to be $O(1)$. The last term in eq. (8) is a gauge invariant bilinear, and the $\mu_f$'s are gauge invariant large mass parameters. As already stated, no terms trilinear in the quark and lepton superfields are allowed by the $SU(4)_H$ gauge symmetry, ensuring naturally the absence of the B and L violating couplings $lqd^c$, $lle^c$ and $u^cd^c$. We are facing here a situation analogous to the $SO(10)$ model, since R-parity does not have to be imposed by hand, but appears as an accidental symmetry that follows from the requirement of horizontal gauge invariance. Indeed, the superpotential is invariant with respect to a $Z_2$ transformation under which the fermion superfields $f, f^c, F$ and $F_c$ (which have an odd number of $SU(4)_H$ indices) change sign, while the Higgs superfields $\Phi_{1,2}$ and $\xi$ (with an even number of $SU(4)_H$ indices) stay invariant. More in general, the superpotential $W_F + W_N$ has an automatic global symmetry $U(1)_H$ under the following transformations:

$$f, f^c \rightarrow e^{i\omega} f, f^c, \quad F, F^c \rightarrow e^{-i\omega} F, F^c, \quad \xi^k \rightarrow e^{2i\omega} \xi^k, \quad \Phi_{1,2} \rightarrow \Phi_{1,2} \quad (\bar{\xi}^k \rightarrow e^{-2i\omega} \bar{\xi}^k)$$  \hspace{1cm} (10)

where a $Z_2$ subgroup ($\omega = \pi$) remains unbroken even when the scalars $\xi$ get non-zero VEVs. This $Z_2$ matter parity ensures R parity conservation and hence proton stability.

The Yukawa couplings (3) lead to the so called ”universal seesaw” mechanism [14] for the fermion mass generation, which for the case of neutrinos reduces to the ordinary seesaw mechanism [15]. Indeed, after the horizontal scalars $\xi_k$ develop non-zero VEVs, the extra fermions $F$ and $F_c$ of eq. (7) acquire large masses through the second term in eq. (8). Then the first and third terms cause a “seesaw” mixing of the ordinary quarks

In this case, in order to reproduce the observed values of the fermion masses, the $SU(4)_H$ symmetry should be broken at a scale very close to $M_{Pl}$. On the other hand, as we will see in Sect. 4, the phenomenology of the model requires a horizontal symmetry breaking scale substantially smaller than $M_{Pl}$. As a consequence, these non-renormalizable terms would provide only negligible contribution to the fermion masses.
and leptons \( f, f^c \) with the heavy ones. As a result, in the base \((f, F)(f^c, F^c)\), the \( 8 \times 8 \) mass matrix for the down (up) type charged fermions \( f = e, d, (u) \) reads

\[
\mathcal{M}_f = \begin{pmatrix}
0 & \mu_f \\
\mu_f & M_f
\end{pmatrix}, \quad \mathcal{M}_F = \sum_k h_F^k \langle \xi_k \rangle
\]

(11)

where \( v_{1,2} = \langle \tilde{\Phi}_{1,2} \rangle \) are the VEVs of the two electroweak Higgs doublets. As for the neutrinos, in the base \((\nu, N_c)\) the \( 8 \times 8 \) Majorana mass matrix has the form

\[
\mathcal{M}_\nu = \begin{pmatrix}
0 & g_\nu v_2 \\
g_\nu v_2 & M_N\end{pmatrix}, \quad \mathcal{M}_N = \sum_k h_N^k \langle \xi_k \rangle.
\]

(12)

The universal seesaw picture provides a natural possibility to obtain three light families, while the fourth one is heavy, say with masses of the order of the electroweak scale. Indeed, let us assume that the \( 4 \times 4 \) mass matrices \( \hat{M}_{F(N)} \) for the heavy fermions are rank-3 matrices of the following form

\[
\hat{M}_F = \begin{pmatrix}
M_F^{(3)} & 0 \\
0 & 0
\end{pmatrix}, \quad F = U, D, E, N
\]

(13)

where the \( 3 \times 3 \) blocks \( M_F^{(3)} \) contain non-zero entries. In other words, we assume that all the VEVs of the type \( \langle \xi_{\alpha_4} \rangle \) are vanishing, so that a diagonal \( \tilde{U}(1) \) subgroup of \( SU(4)_H \times U(1)_H \), given by the generator \( \tilde{T} = \text{diag}(0, 0, 0, 1) \), is left unbroken. In this case there is no seesaw mechanism for the fermions of the fourth family: the right-handed components of the fields \( f_4 = b', t', \tau', \nu' \) are actually the \( F_4^c \) states, whereas the \( f_4^c \) form with the \( F_4 \) superheavy particles of mass \( \mu_f \).

From eqs. (11) and (12) we obtain for the fourth family fermions

\[
m_{b'} = g_d v \cos \beta \quad m_{t'} = g_u v \sin \beta \quad m_{\tau'} = g_e v \cos \beta \quad m_{\nu'} = g_\nu v \sin \beta
\]

(14)

where \( v = 174 \text{ GeV} \) is the electroweak breaking scale and \( \tan \beta = v_2/v_1 \). Since all the Yukawa couplings are assumed to be \( O(1) \), for moderate values of \( \tan \beta \) all the masses in (14) are of the order \( \sim 100 \text{ GeV} \). On the experimental side, the firmest constraints on the masses of any new sequential fermion, quark or lepton, have been set at LEP: \( m_f \gtrsim M_Z/2 \). This indeed represent the best constraint on \( m_{\tau'} \) and \( m_{\nu'} \). Searches for new quarks at the TEVATRON collider could in principle give much better bounds for \( m_{t'} \) and \( m_{b'} \).

However, let us note that the structure of the heavy mass matrix (13) implies that the fourth family is unmixed with the three lighter ones. Hence the usual signatures, as for example \( b' \to c, u \), that have been used to set the limits on new sequential fermion, quark or lepton, have been set at LEP: \( m_f \gtrsim M_Z/2 \). This indeed represent the best constraint on \( m_{\tau'} \) and \( m_{\nu'} \). Searches for new quarks at the TEVATRON collider could in principle give much better bounds for \( m_{t'} \) and \( m_{b'} \).

However, let us note that the structure of the heavy mass matrix (13) implies that the fourth family is unmixed with the three lighter ones. Hence the usual signatures, as for example \( b' \to c, u \), that have been used to set the limits on new sequential quarks do not occur in our case. In the absence of a detailed experimental analysis of the unmixed case, the only reliable limit is again the LEP one also for the new quarks. Hence we can safely conclude that the predictions in (14) are by no means in conflict with the existing experimental limits. However, it is clear that for the masses of the fourth family fermions

\[\text{The scheme considered here is a direct } SU(4)_H \text{ extension of the model } [16] \text{ based on the horizontal symmetry } SU(3)_H. \text{ In the case when the vertical gauge symmetry is extended to the left-right symmetric model } SU(2)_L \times SU(2)_R \times U(1)_{B-L}, \text{ the values of } \mu \text{’s are given by the scale of the } SU(2)_R \text{ breaking } [16].\]
not much room is left. The allowed parameter space is in fact strongly constrained by the CDF measurement of the top mass, \(m_t = 174 \pm 10 \pm 13\) GeV \[17\], by the precision tests of the SM which do not leave much space for additional sizeable radiative corrections as would be induced by a too large \(m_{\nu}-m_{\nu'}\) splitting, and by renormalization group (RG) analysis of the Yukawa couplings, much in the spirit of ref. \[13\].

In particular, while the general analysis in \[13\] does allow for the possibilities \(m_{\nu'} < m_t\) or \(m_{\nu} < m_{\nu'}\), in our model these pattern of masses are not allowed. In fact the universal seesaw mechanism outlined before implies \(m_{\nu'} \geq m_t\), and most likely \(m_{\nu'} \geq m_{\nu} \). Then, according to \[13\], for \(m_{\nu'} \geq m_t > 150\) GeV the consistency of the model implies not too large values for the masses of the other fermions in fourth family. Namely, for the low values of \(\tan \beta\) we are interested in (e.g. \(\tan \beta \sim 2\)), the maximal values allowed are about \(m_{\nu} \sim 100\) GeV and \(m_{\nu', \nu'} \sim 50\) GeV, that is within the reach of LEP II.

Let us now consider the mass matrices for the first three families. In this case the seesaw mechanism is effective for suppressing the fermion masses from the electroweak scale down to the observed values. By assuming \(M_F^{(3)} > \mu_f\), it is apparent from (11) that the fermions of the first three families will acquire their masses through a mixing with the superheavy \(F\) fermions. Namely, after decoupling the heavy states, the \(3\times3\) mass matrices of the light charged down (up) type fermions are

\[
m_f^{(3)} = g_f \mu_f (M_F^{(3)})^{-1} v_1(2), \quad f = d, e, (u)
\]

while the \(3\times3\) Majorana mass matrix for the light neutrinos obtained from (12) reads

\[
m_{\nu}^{(3)} = (M_N^{(3)})^{-1} (g_\nu v_2)^2.
\]

In contrast to the SM and to most GUT models, in our picture the fermion mass hierarchy is not generated by an \(ad\ hoc\) choice of the Yukawa coupling constants. In fact, in our scheme all the Yukawas are assumed to be of the same order of magnitude, for example \(O(1)\) or close to the size of the gauge couplings. As long as the off-diagonal blocks in eqs. (11) and (12) are flavour blind (unit) matrices, all the informations on the fermion mass and mixing pattern is contained in the heavy fermion mass matrices \(M_F^{(3)}\). Since the structure of the latter is determined by the different VEVs \((\xi^k)\) (modulo differences in the Yukawa constants \(h^F_k\)), the observed hierarchy of the light fermion masses is ultimately determined by the hierarchy in the VEVs which break the horizontal symmetry. In other words, the VEV pattern should provide the step-by-step breaking of the chiral horizontal symmetry

\[
SU(4)_H \times U(1)_H \xrightarrow{V_1} SU(3)_H \times U(1)'_H \xrightarrow{V_2} SU(2)_H \times U(1)''_H \xrightarrow{V_3} \tilde{U}(1)
\]

so that the first breaking (at the scale \(V_1 \sim \langle \xi_{11}\rangle\)) defines the mass terms for the first heavy family \(F_1\), the second breaking (at \(V_2 \sim \langle \xi_{12}, \xi_{22}\rangle\)) for the second family \(F_2\) etc. Through the seesaw mechanism this horizontal VEV hierarchy is reflected in the observed pattern of fermion masses. Namely, from (13) and (16) it is clear that the hierarchy

\[
6 \text{ After decoupling the heavy states at the horizontal symmetry scale } V_H, \text{ our model simply reduces to the SSM with four families. In fact, eqs. (14), (15) and (16) define the fermion running masses at } \mu = V_H. \text{ In order to deduce the fermion physical masses the RG running has to be taken into account.}
\]
down to the vacuum state would have a continuous degeneration – there will be vacuum valleys. The consideration, according to our ‘renormalizability’ paradigm, the general superpotential

$$W_H = M_k \xi k \bar{\xi}_k + \lambda_{aik} S_a \xi k \bar{\xi}_l + \lambda'_{aik} \xi k \Sigma_a \bar{\xi}_l + P(S, \Sigma)$$  \hfill (18)$$

where $P(S, \Sigma)$ is a general $3^{rd}$ order polynomial of the $S_a$ and $\Sigma_a$ fields (containing linear, bilinear and trilinear terms). Notice that this superpotential automatically respects the $U(1)_H$ invariance \[^{10}\], but has no additional accidental global symmetries.

The superpotential (18) in itself does not break SUSY. Moreover, in the exact SUSY limit the vacuum state is highly degenerated – there are several zero-energy vacua with different configurations of horizontal VEVs. It would be a difficult task to provide an exhaustive analysis of all the possible vacua in the general case, that is to decide which configuration of VEVs is chosen as the true vacuum once the soft SUSY breaking terms are included. However, taking into account that after SUSY breaking the potential of the horizontal scalars has to a large extent the general structure of usual (non-SUSY) Higgs polynomial, one can argue that for a certain choice of parameters it is possible to obtain the needed pattern of VEVs (see for example the analysis in refs. \[^{8, 20}\] for the case of $SU(3)_H$ symmetry).

Indeed, let us consider a first case with only one pair of $\xi_1 + \bar{\xi}_1$ superfields, the ones which have the largest VEV ($V_1$) in the exact SUSY limit. The constraint from the $D$-term tells us that in this case $\langle \xi_1 \rangle = \langle \xi_1 \rangle$. Then it is easy to show that after SUSY breaking, for a proper choice of the range of values for the relevant parameters, the true vacuum can have the configuration $\langle \xi_1 \rangle = V_1 \text{diag}(1, 0, 0, 0)$ which breaks $SU(4)_H \times U(1)_H'$ down to $SU(3)_H \times U(1)'_H$. Therefore, at this stage only the first family of $F$ fermions gets a mass through the couplings \[^{8}\] while the others, being protected by the residual chiral symmetry $SU(3)_H \times U(1)'_H$, remain massless. \[^{7}\]

In analysing the scalar potential of the fields $\xi_2 + \bar{\xi}_2$ with next largest VEV ($V_2$), we have to take into the account that after ‘decoupling’ $\xi_1$ (i.e. substituting $\xi_1 \rightarrow \langle \xi_1 \rangle$), the symmetry group is reduced to $SU(3)_H \times U(1)'_H$, under which $\xi_2$ branches as $10 = 6 + 3 + 1$. The VEVs which give masses to the second heavy generation belong to the 6 and 3, and can be chosen in the form $\langle \xi_{21} \rangle$ and $\langle \xi_{12} \rangle$ respectively. These break $SU(3)_H \times U(1)'_H$ down to $SU(2)_H \times U(1)''_H$, thus respecting a residual chiral symmetry for the third and fourth family of heavy fermions which at this stage remain massless. Finally, yet another pair $\xi_3 + \bar{\xi}_3$ can develop VEVs in the $\alpha 3$ directions ($\alpha = 1, 2, 3$) thus breaking $SU(2)_H \times U(1)''_H$ down to $\tilde{U}(1)$, which acts only on the fourth family.

One could try to avoid introducing the adjoint representations $\Sigma_a$ and keep in the superpotential (18) only the singlet fields $S_a$. Then, in the exact SUSY limit the vacuum state would have a continuous degeneration – there will be vacuum valleys. The

---

\[^{7}\] Alternatively, for the complementary choice of the parameter range, one would have the vacuum $\langle \xi_1 \rangle \propto \text{diag}(1, 1, 1, 1)$ which breaks $SU(4)_H \times U(1)'_H$ down to $SO(4)_H$. This pattern, however, does not maintain chirality and leads to degenerate fermion masses.
reason for this is that in this case the superpotential would have an accidental global symmetry $SU(10)$ larger than local $SU(4)_H$. In the SUSY limit these valleys correspond to massless Goldstone modes given by certain components of the horizontal superfields. When SUSY breaking is taken into account, the radiative corrections explicitly break the extra global symmetry, lifting the vacuum degeneracy and providing $\sim 100$ GeV masses to the horizontal Goldstone modes, which would then become pseudo-Goldstone, massive familon-like scalars. In general these states would have diagonal as well as non-diagonal Yukawa couplings with the fermions, and in particular the strength of the couplings to the light fermions would be suppressed by a factor $\sim v/V_H$.

It is a very difficult task to provide a full analysis of the VEV pattern in this case and deduce which configuration of VEVs is fixed as the true vacuum state after SUSY breaking. Namely, already for two pairs of $\xi + \bar{\xi}$ the general VEV structure of vacuum valleys completely breaks the $SU(4)_H$ symmetry, not maintaining the $U(1)$ subgroup. One can still argue that for a proper choice of the relevant parameters the needed pattern of VEVs can be obtained. Though this possibility can be interesting from the phenomenological point of view, it deserves a special investigation. Therefore, in the following we will assume that there are no light familon-like scalars and that all the horizontal fields have $O(V_H)$ masses.

As we have mentioned at the beginning of this section, in order to generate masses for the heavy states $F$ and $F_c$ it is also possible to introduce horizontal Higgs fields $\chi_{[\alpha\beta]}$ in the antisymmetric 6 representations of $SU(4)_H$. The $\langle \chi_k \rangle$ VEVs would then contribute to the mass matrices of the heavy charged fermions through the term $\sum_k h_{F}^{k} F_{\alpha}^{\beta} F_{c}^{\beta} \chi_{k}^{[\alpha\beta]}$, while the corresponding term for the Majorana mass matrix of the heavy neutral states $N^c$ is forbidden due to the antisymmetry of the representation. However, in this case the appearance of the terms like $M_{F}^{\prime} \chi_k \chi_k$ in the superpotential for the horizontal fields would break explicitly the global $U(1)_H$ in eq. (10) and hence the residual $U(1)$ invariance, thus rendering unnatural the degenerate structure of the heavy matrices $\hat{M}_F$ in eq. (13). Although for $\langle \chi_k \rangle \leq \mu_f$ the heaviness of the fourth family charged fermions would still be guaranteed, we would lose a natural explanation for a heavy $\nu'$. In fact, through additional terms like $\xi \sum \chi$ sizable VEVs in the $\alpha 4$ directions would be induced also for the $\xi$ fields. If, as it seems natural to occur, the induced VEVs are larger than the electroweak scale, the fourth family neutrino mass will also result from a seesaw giving $m_{\nu'} \sim (g_{\nu} v_2)^2 / \langle \xi \rangle_{\text{induced}}$. That is, the $\nu'$ will also be light, thus rendering the model phenomenologically unacceptable.

4 Phenomenological consequences of the model

As was discussed above, the quark and charged lepton masses at the scale $V_H$ are given by eqs. (14) and (15). By assumption, the heavy fermion mass matrices $M_{F}^{(3)}$ are non-degenerate, and thereby have three massive eigenstates, with mass hierarchy reflecting the $SU(4)_H$ symmetry breaking pattern. The weak mixing angles are determined by the structure of these matrices, whereas the quark and lepton masses are inversely propor-
tional to the masses of their heavy partners. For the down quark and charged lepton masses we have

\[ m_{d,s,(b)} \simeq \frac{\mu_d}{M_{D,S,B}} m_{b'} \quad \text{and} \quad m_{e,\mu,(\tau)} \simeq \frac{\mu_e}{M_{E,M,T}} < m_{b'} \]

where \( D, S, B \) are the mass eigenstates of \( M^{(3)}_{D(S)} \), and the factors \( \eta \) account for the differences in the RG running of masses from the horizontal scale to lower energies.

The fact that the \( b \) and \( \tau \) masses are of order a few GeV, implies that the masses of the corresponding heavy states \( B \) and \( T \) are not much larger (say, within one or two orders of magnitude) than the mass scale \( \mu_{d,e} \). As for the top quark, the value of its mass \( m_t \gtrsim 150 \text{ GeV} \) requires \( M_T \sim \mu_u \). In this case corrections to the seesaw formula (15) should be taken into account in relating \( m_t \) to the heavy scales (see e.g. ref. [19]).

As a result of the seesaw mechanism for the fermion masses generation, the light charged states correspond to some superposition of the \( (f, f^c) \) and \( (F, F^c) \) states. It is well known that a mixing between the light \( SU(2) \) doublet states \( f \) and the heavy \( SU(2) \) singlets \( F \) could induce FCNC in the electroweak interactions and will also alter the flavor diagonal couplings of the light states [21]. However, in our case such a mixing is suppressed as the ratio \( v_1(2)/\hat{M}_F \) and thus negligibly small when compared with the present experimental bounds [22, 23]. On the other hand, the mixing between the \( SU(2) \) singlets \( f^c \) and \( F^c \) can be large, since it is controlled by the ratio of the two mass scales \( \mu_f \) and \( \hat{M}_F \) which, as we have seen, can be as large as \( \sim 10^{-1} \) or even close to unity in the case of the \( t \) quark. However, this kind of mixing between states transforming in the same way under \( SU(2) \times U(1) \) cannot affect the electroweak quantities, and is essentially unobservable.

Let us discuss now the neutrino masses. As we have already stated, three neutrinos \( \nu_e, \nu_\mu, \nu_\tau \) are light Majorana particles. Their running masses at \( \mu = V_H \) are determined by the heavy ’right-handed’ neutrino eigenstates \( N_e, N_\mu, N_\tau \) at their decoupling, according to the seesaw formula (16). As for the fourth neutrino \( \nu' \), it appears to be a heavy Dirac particle with mass \( \sim 100 \text{ GeV} \). Then for the neutrino physical masses we have

\[ m_{\nu_e,\nu_\mu,\nu_\tau} = \frac{(m_{\nu'})^2}{M_{N_e,N_\mu,N_\tau}} \quad \text{and} \quad m_{\nu'} \geq \frac{M_Z}{2} \]

where the factor \( \eta_{\nu} \) accounts for the different RG running of Majorana and Dirac masses from the \( SU(4)_H \) breaking scale to lower energies (for the RG running of Majorana neutrino masses see e.g. [21]). Therefore, modulo the different Yukawa couplings \( h_F \), the neutrino mass hierarchy is expected to be qualitatively the same as the hierarchy between the quarks or the charged leptons:

\[ m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \sim m_u : m_c : m_t \quad \text{or} \quad m_e : m_\mu : m_\tau \]

Below the scale \( V_H \) our theory is just the SSM, and all FCNC phenomena related with the horizontal symmetry are strongly suppressed. Therefore, all neutrinos are effectively stable on a cosmological scale. In order to respect the cosmological upper bound [25] on the light stable neutrino masses \( m_{\nu_e} \leq 92 \Omega h^2 \text{ eV} \) (here \( \Omega = \rho/\rho_c \sim 1 \) is the ratio of the energy density of the Universe to the critical density, and \( h = 0.4 - 1 \) is the Hubble
parameter in units of \(H_0 = 100 \text{ Km s}^{-1} \text{ Mpc}^{-1}\) the following lower bound on the mass of \(N_\tau\) must be respected:

\[
M_{N_\tau} = h_N V_H \geq \frac{\eta_\nu}{4\Omega h^2} \left( \frac{m_Z^2}{92 \text{eV}} \right) \approx 10^{11} \text{ GeV.} \tag{22}
\]

In deriving this limit we have taken into account that the present age of Universe \(t_0 \approx 12 \cdot 10^9 \text{ yr}\) requires, for \(\Omega = 1\), the Hubble parameter \(h \approx 0.5\), and we have assumed that \(\eta_\nu \approx 1\). As long as the Yukawa constants \(h_N\) are \(O(1)\), this bound translates into a lower bound on the smallest scale \(V_H = V_3\) in the \(SU(4)_H\) symmetry breaking chain (17).

Let us now address some phenomenological issues regarding the fourth family fermions. The structure of the heavy mass matrix (13) which leaves unbroken the diagonal \(\hat{U}(1)\) subgroup of \(SU(4)_H \times U(1)_H\), also ensures that the fourth family is unmixed with the three lighter ones. We assume that the lightest member of the fourth generation is the neutral one \(\nu'\), as is also suggested by the analysis of ref. [13]. For simplicity we also assume that \(m_{\nu'} > m_{b'}\). Then \(b'\) and \(\nu'\) are stable with respect to electroweak interactions.

The presence of stable neutrinos \(\nu'\) with mass in the 100 GeV range is phenomenologically and in particular cosmologically acceptable, since their contribution to the cosmological energy density is vanishingly small. Only in the presence of a sizeable \(\nu'\)-\(\bar{\nu}'\) primordial asymmetry the stable relics \(\nu'\) would contribute to the present cosmological density, and this contribution would still be acceptable as long as their present number density \(n_{\nu'}\) does not exceed the baryon number density \(n_B\). However, as we will argue in the following, in our model no sizeable asymmetry has to be expected for the fourth family fermions.

In contrast, the existence of stable heavy quarks carrying colour and electric charge would constitute a potential problem for the model, since it will conflict with the constraints arising from superheavy element searches, as well as with other cosmological and astrophysical constraints [26, 27]. Indeed, the stable \(b'\) would behave essentially as \(d\) quarks, hadronising into heavy ‘protons’ and giving rise to heavy hydrogen-like ‘isotopes’ with masses \(\sim 100 \text{ GeV}\). The existing experimental limits on this kind of isotopes are extremely tight. For example for masses \(m_{b'} < 1 \text{ TeV}\) the limit on their abundance relative to normal hydrogen is \(n_{b'}/n_B < 10^{-28}\) [28].

However, the exchange of the \(SU(4)_H\) gauge bosons \(Z_H\) would allow the heavy quark to decay, dominantly through the channel \(b' \rightarrow b\bar{\nu}_\tau\nu',\) with a lifetime \(\tau_{\nu'} \sim \left( \frac{V_H}{v} \right)^4 \left( \frac{m_{\mu}}{m_{b'}} \right)^5 \tau_\mu\), where \(V_H = V_3\) is the lowest scale in the horizontal symmetry breaking (see eq. (17)), \(v\) is the electroweak scale and \(\tau_\mu = \tau(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \approx 2.2 \times 10^{-6} \text{s}\). is the muon lifetime. We can use cosmological arguments, together with the experimental limits on searches of heavy isotopes, to put an upper bound on \(\tau_{\nu'}\), which in turn will translate in an upper limit on \(V_H\). Indeed, taking into account the finite lifetime of the heavy quarks, their present number

\[\text{where } V_H = V_3 \text{ is the lowest scale in the horizontal symmetry breaking (see eq. (17)), } v \text{ is the electroweak scale and } \tau_\mu = \tau(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \approx 2.2 \times 10^{-6} \text{s. is the muon lifetime. We can use cosmological arguments, together with the experimental limits on searches of heavy isotopes, to put an upper bound on } \tau_{\nu'}, \text{ which in turn will translate in an upper limit on } V_H. \text{ Indeed, taking into account the finite lifetime of the heavy quarks, their present number}\]

\[\text{where } V_H = V_3 \text{ is the lowest scale in the horizontal symmetry breaking (see eq. (17)), } v \text{ is the electroweak scale and } \tau_\mu = \tau(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \approx 2.2 \times 10^{-6} \text{s. is the muon lifetime. We can use cosmological arguments, together with the experimental limits on searches of heavy isotopes, to put an upper bound on } \tau_{\nu'}, \text{ which in turn will translate in an upper limit on } V_H. \text{ Indeed, taking into account the finite lifetime of the heavy quarks, their present number}\]
abundance relative to baryons is \( n_{b'}/n_B = r_0 \exp(-t_0/\tau_{b'}) \leq 10^{-28} \), where \( r_0 = (n_{b'}/n_B)_0 \) represents the relic abundance for stable \( b' \). From this equation we get the upper limit on the \( b' \) lifetime

\[
\tau_{b'} \leq 3 \cdot 10^{15} h^{-1}(1 + 0.036 \lg r_0)^{-1} \text{ s.} \tag{24}
\]

One cannot say definitely what is the value of \( r_0 \), due to many theoretical uncertainties related to the actual annihilation cross section for the \( b' \). However, an estimate of the relic abundance of heavy stable \( d' \)-type quarks has been given in \([26]\). Under the assumption that there is no cosmological baryon asymmetry between the \( b' \) and \( b \), it was found that for \( m_{b'} \sim 150 \text{ GeV} \) the energy density of these relics, relative to critical density (namely \( \Omega_{b'} h^2 \)) could range from \( 10^{-9} \) to \( 10^{-4} \) (smaller values are obtained for lighter \( b' \) masses). The lower limit corresponds to the case when the relic density is determined by the annihilation after the QCD phase transition, and it was obtained by taking as an upper bound on the annihilation cross section the geometrical cross section \( (\sigma_0 \sim 100 \text{ mb}) \). The upper limit was obtained under the opposite assumption, namely that annihilation after confinement is negligible, and that the relic density is essentially determined by the QCD annihilation cross section for free quarks. Then the ratio of the \( b' \) to baryon number densities \( r_0 = (n_{b'}/n_B)_0 = \Omega_{b'}/\Omega_B \cdot m_B/m_{b'} \) lies in the range \( 3 \cdot 10^{-10} < r_0 < 3 \cdot 10^{-5} \), where we have taken \( \Omega_B \sim 0.02 \) as suggested by nucleosynthesis estimates. As is discussed in \([26]\), the most reasonable assumption is that the relevant annihilation process happens after confinement, however with a cross section much smaller than the geometrical one, giving \( r_0 \sim 10^{-7} - 10^{-8} \). Clearly, in the presence of a sizeable baryon asymmetry in the fourth family sector, the relic abundance of the heavy \( b' \) quarks would be some orders of magnitude larger than the quoted estimates.

As we see, the bound \( (24) \) very weakly depends on the initial \( b' \) abundance. Even if we allow for a large primordial asymmetry for the \( b' \), and let \( r_0 \) range between \( 1 - 10^{-10} \), by taking \( h = 0.5 \) we obtain \( \tau_{b'} \leq 6 \cdot 10^{15} - 10^{16} \text{ s.} \) On the other hand, according to eq. \( (23) \), this bound translates into an extremely strong upper limit

\[
V_H \leq \frac{(m_{b'}/150 \text{ GeV})^{5/4}}{h^{1/4}(1 + 0.036 \lg r_0)^{1/4}} \cdot 3 \cdot 10^{11} \text{ GeV} \leq 4 \cdot 10^{11} \text{ GeV} \tag{25}
\]

This bound corresponds to \( m_{b'} \leq 150 \text{ GeV} \), and the scale \( V_H \) cannot much exceed this limit unless \( m_{b'} \gg 200 \text{ GeV} \). For the more realistic values \( m_{b'} \simeq 100 \text{ GeV} \) suggested by the analysis in \([13]\) we get \( V_H \leq 2.4 \cdot 10^{11} \text{ GeV} \).

More stringent limit on \( r_0 \) and \( \tau_{b'} \) can be derived by considering that the late decay of the \( b' \) can cause a significant contribution to observed cosmic ray fluxes, in particular to the isotropic diffuse gamma-ray background \([24]\). Indeed, at the moment of decay, the \( b' \) quarks are bounded within colorless hadrons like \( b'ud \) or \( \bar{b}u \) \([26]\). Then in the decay \( b' \to b\pi^0 \nu' \) an unstable hadronic state emerges with the excitation energy \( E_0 \simeq \frac{1}{3} m_{b'} \). This will essentially appear as a hadronic jet with the \( b \) quark being the leading particle. The fragmentation of this jet produces \( \pi^0, \eta \) etc., with the subsequent radiative decay resulting in a specific photon spectrum. Obviously, the amount of produced photons is directly proportional to \( r_0 \). In order to estimate their flux in the present Universe, the redshift of their energies has to be taken into account as well. As long as the decay happens at the matter dominated epoch, and the small amount of relativistic decay products does not affect sensibly the Universe expansion rate, we have \( 1 + z = (t_0/\tau_{b'})^{2/3} \sim 10 - 20 \) for
the values of $\tau_{b'}$ estimated above. We also need to know what fraction of the jet energy $E_0$ is taken by the photons and what is the energy spectrum. These issues were studied in ref. [30], where the photon spectra produced at jet hadronization were computed for different leading particles using a Monte Carlo simulation program [31]. It was shown that these spectra exhibit a remarkable scaling property in terms of the variable $x = E_\gamma/E_0$, and in the case of leading particle being a $b$ quark, the photons carry away about 25 percent of the initial jet energy. Using the results of ref. [30] we have computed the value of the isotropic cosmological gamma-flux $d\Phi_\gamma/dE_\gamma$ and we have compared it with the existing observational limits (see [32] and references therein). For example, for $E_\gamma = 100$ MeV the experimental upper bound on the $\gamma$-flux is of about $10^{-7}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ MeV$^{-1}$. We have obtained that the cosmic gamma-flux produced due to $b'$ decay at $z = 10 - 20$, saturates the above bound for $r_0 \sim 10^{-7}$ which is close to, but still not in conflict with our estimate of the $b'$ relic abundance in the $b'$-$\overline{b'}$ symmetric case. Substantially larger $r_0$ would require much larger redshift, and hence much smaller $\tau_{b'}$. On the other hand, the lower bound (22) on the horizontal symmetry breaking scale $V_H$ already excludes much smaller lifetimes.

This analysis implies that $r_0$ should be rather small, so that any sizeable cosmological baryon asymmetry between $b'$ and $\overline{b'}$ is excluded. This severely constrains the possible baryogenesis mechanisms applicable to our model. The appearance of baryon asymmetry in the fourth family in itself is hardly expected, since it is unmixed with the other three families and hence it has no source of CP violation. However, the sphaleron effects [33, 34] would immediately redistribute the baryon asymmetry produced within the first three families to the fourth family fermions. Therefore, no mechanism is acceptable which generates the baryon asymmetry before the sphaleron effects are switched off, that is before the electroweak phase transition. In the context of our model the most appealing possibility is to assume that no baryon asymmetry is produced before the electroweak epoch, and baryogenesis takes place at the electroweak (first order) phase transition. Such a baryogenesis mechanism is associated with the walls of the expanding bubbles of the broken phase [37]. Outside of the bubbles electroweak symmetry is unbroken, quarks are massless and the rate of the fermion number violation due to sphaleron transitions greatly exceeds the Universe expansion rate. Inside the bubbles the quarks are massive due to non-zero VEVs of the Higgs fields, while the sphaleron processes are strongly suppressed and fermion number is effectively conserved. Then baryon asymmetry inside the bubbles could be produced (and maintained) due to CP violating effects, as a difference between the quark and anti-quark fluxes penetrating the walls from the unbroken phase to the broken one. Obviously, this concerns only the first three family fermions. Since the fourth

\[9\]In principle, in our model the baryogenesis with non-zero $B-L$ could occur due to CP violation effects in out-of-equilibrium decays $N^c \rightarrow l + \Phi$ of the heavy right-handed neutrino $N^c$ (for the viability of this mechanism in the SUSY case see ref. [38]), or in the decays of $SU(4)$ gauge or scalar bosons. Then sphaleron effects would immediately transfer the produced net lepton number into a baryon asymmetry also in the fourth family sector. Fortunately, our model naturally avoids the possibility of such a lepto-baryogenesis. As it was shown in ref. [39], the large scale density fluctuations hinted by the COBE measurements require rather low inflationary reheat temperature ($T_R \sim 10^8$ GeV) and correspondingly low inflaton mass ($m_\eta \sim 10^{11}$ GeV). On the other hand, the lower bound (22) on $M_N$ (and respectively on $V_H$) tells us that masses of the right-handed neutrinos and horizontal bosons should exceed $10^{11}$ GeV, and therefore they are not produced after inflation.
family is unmixed, has no CP violation, and moreover all the fermions are very heavy, no baryon excess is expected in this sector. Although the viability of such a baryogenesis in the SM is still disputed in the literature [38], in the context of SSM it could be more effective and sufficient for providing the observed baryon asymmetry. Clearly this topic deserves additional special considerations.

According to eq. (20), the upper limit on the horizontal symmetry breaking scale $V_H \lesssim 4 \cdot 10^{11}$ GeV together with the experimental limit $m_{\nu} \geq M_Z/2$ translates into a lower bound on the $\tau$-neutrino mass:

$$m_{\nu_{\tau}} \geq \frac{\eta_{\nu} m_Z^2}{4V_H} \gtrsim (1 - 10) \text{ eV} \quad (26)$$

where in the numerical estimate we have taken into account the $O(1)$ uncertainties in the relative renormalization factor $\eta_{\nu}$ and in the Yukawa coupling $h_N$ (for perturbativity we have to assume $h_N < 3$ at $\mu = V_H$). A $\nu_{\tau}$ with mass in the range $1 - 10$ eV will give a sizeable contribution to the cosmological energy density as a hot dark matter (HDM) component, while according to (21) $\nu_\mu$ and $\nu_e$ are expected to have much smaller masses. We remind here that the COBE measurements of the cosmic microwave background anisotropy, together with other data on the density distribution of the Universe at all distance scales (galaxy-galaxy angular correlations, correlations of galactic clusters, etc.) can all be fit by assuming some HDM admixture to the dominant CDM component [39]. The best fits hint to a neutrino mass $m_{\nu_{\tau}} \sim 5 - 7$ eV [40] which does appear naturally in our model. As for the CDM itself, in our R parity conserving SUSY model it is naturally provided by the lightest supersymmetric particle (LSP), presumably a neutralino.

As we commented earlier, the neutrino mass hierarchy should be qualitatively the same as that for the charged quarks and leptons. However, the spread in the Yukawa coupling constants $h_F$ does not allow to put severe limits on the other neutrino masses. For example, by taking $m_{\nu_{\mu}}/m_{\nu_{\tau}} \sim m_e/m_{\tau}$, as is suggested by the first estimate in eq. (21), one obtains $m_{\nu_{\mu}} \sim (2 - 5) \cdot 10^{-3}$. This range corresponds to the Mikheyev-Smirnov-Wolfenstein (MSW) solution of the solar neutrino problem [11] via $\nu_e \rightarrow \nu_\mu$ oscillations. Alternatively, if we had to attempt an explanation of the deficit of the atmospheric $\nu_\mu$ via $\nu_\mu \rightarrow \nu_e$ oscillations, then we would need $m_{\nu_{\mu}} \sim 0.1$ eV [12] which is compatible with the second estimate in eq. (21). Obviously the MSW explanation to the solar neutrino deficit would not be viable in this latter case.

5 Conclusions

In this paper we have put forward the idea that natural conservation of R parity in SUSY models can be guaranteed in the presence of some suitable horizontal gauge symmetries. We have shown how these symmetries can indeed forbid all the dangerous terms in the superpotential, which are trilinear in the fermion superfields, and how an accidental $Z_2$ matter parity (equivalent to R parity) then follows in a quite satisfactory way only due to gauge invariance and to the field content of the model. On theoretical and phenomenological grounds, we have uniquely identified $SU(4)_H$ as the only viable horizontal gauge group. As a consequence, our scheme requires a fourth generation of superfields in addition to the three known families. Hence we have focused our analysis on a four generation
SUSY model based on the SM vertical gauge group $SU(3) \times SU(2) \times U(1)$ and equipped with an $SU(4)_H$ anomaly free horizontal gauge symmetry. We have discussed in some details the structure of the fermion mass matrices arising in the model, as well as some possible patterns for the breaking of the horizontal symmetry. We have shown that the simplest symmetry breaking scheme which ensures that all the horizontal modes acquire large masses, can also lead to a particular form for the fermion mass matrices which ensures that the masses for the fourth generation fermions are naturally close to the electroweak scale. A recent RG analysis of SUSY models with four generations [13] does apply straightforwardly to our case, and suggests that if the hypothesis of unification of the vertical gauge group is correct, then at least the new leptons should be well in the reach of LEP II. As regards the masses of the first three families, our model leads to a seesaw suppression of their magnitude from the electroweak scale down to the observed values. In particular, this is achieved without the need of any tuning for the Yukawa couplings, which can be assumed to be all $O(1)$ or close to the typical values of the gauge couplings. By means of cosmological and astrophysical arguments, we have managed to constrain rather precisely the scale $V_H$ at which the horizontal gauge symmetry is completely broken, obtaining a very narrow window around $10^{11}$ GeV. Below this scale, our model is essentially the SSM with four generations. In turn, the upper bound on the scale $V_H$ feeds back into the neutrino mass matrix, implying a mass for the $\tau$-neutrino not much lighter than a few eV. A neutrino mass in this range will then give a sizeable contribution to the present energy density of the Universe. Thus, our model naturally provides cosmological HDM in the form of $\nu_\tau$'s and, due to R parity conservation, also CDM in the form of stable LSPs. Since in our scheme conservation of R-parity is ensured by the horizontal gauge symmetry independently of the particular choice for the vertical gauge group, it would be interesting to extend the present analysis to phenomenologically appealing GUT models, such as $SU(5)$ or $E_6$, for which R-parity conservation is not automatic.

Acknowledgements

It is a pleasure to thank Venya Berezinsky, Sasha Dolgov and Misha Shaposhnikov for illuminating discussions and useful comments. One of us (E.N.) would like to thank I. Rothstein, R. Akhoury, M. Einhorn, G. Kane, R. Stuart, Y. Tomozawa, M. Veltman, D. Williams, E. Yao, V. Zhakarov and all the post-docs and students of the Particle Theory Group of the University of Michigan, for several useful discussions and for the pleasant working atmosphere during his stay at the University of Michigan.
References

[1] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566; R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. B 90 (1980) 249; E. Akhmedov, Z. Berezhiani and G. Senjanović, Phys. Rev. Lett. 69 (1992) 3013.

[2] L.J. Hall and M. Suzuki, Nucl. Phys. B 231 (1984) 419.

[3] P. Fayet, Phys. Lett. B 69 (1977) 489; G. Farrar and P. Fayet, Phys. Lett. B 76 (1978) 575; Among the first references on the R-parity see for example S. Weinberg, Phys. Rev. D 27 (1982) 2732; L.J. Hall and M. Suzuki, in ref. [4]; S. Dimopoulos and L.J. Hall, Phys. Lett. B 207 (1987) 210.

[4] See e.g. K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845; D.G. Lee and R.N. Mohapatra, UMD-PP-94-166 (June 1994) unpublished.

[5] Z.G. Berezhiani, Yad. Fiz. 42 (1985) 1309 [Sov. J. Nucl. Phys. 42 (1985) 825]; K.S. Babu and S.M. Barr, Phys. Rev. D 48 (1993) 5354.

[6] H. Georgi, A.V. Manohar and A.E. Nelson, Phys. Lett. B 126 (1983) 169.

[7] J.L. Chkareuli, Pis’ma ZhETF 32 (1980) 684 [JETP Lett. 32 (1980) 671]; Z.G. Berezhiani and J.L. Chkareuli, Pis’ma ZhETF 35 (1982) 494 [JETP Lett. 35 (1982) 612]; Yad. Fiz. 37 (1983) 1043 [Sov. J. Nucl. Phys. 37 (1983) 618].

[8] Z.G. Berezhiani and M.Yu. Khlopov, Yad. Fiz. 51 (1990) 1157, 1479 [Sov. J. Nucl. Phys. 51 (1990) 739, 935].

[9] S.L. Glashow and S. Weinberg, Phys. Rev. D 15 (1977) 1958; E.A. Paschos, Phys. Rev. D 15 (1977) 1964.

[10] C.D. Frogatt and H.B. Nielsen, Nucl. Phys. B 417 (1979) 277; Z.G. Berezhiani, Phys. Lett. B 129 (1983) 99; Phys. Lett. B 150 (1985) 177; S. Dimopoulos, Phys. Lett. B 129 (1983) 417.

[11] Mark II Collaboration, G.S. Abrams et. al., Phys. Rev. Lett. 63 (1989) 2173; L3 Collaboration, B. Adeva et. al., Phys. Lett. B 231 (1989) 509; OPAL Collaboration, I. Decamp et. al., ibid., B 231 (1989) 519; DELPHI Collaboration, M.Z. Akrawy et. al., ibid., B 231 (1989) 539.

[12] N. Evans, report SWAT/40 (August 1994) unpublished; P. Bamert and C.P. Burgess, report McGill-94/27, NEIP-94-005 (June 1994) unpublished.

[13] J.F. Gunion, D.W. McKay and H. Pois, Phys. Lett. B 334 (1994) 339.

[14] Z.G. Berezhiani, Phys. Lett. B 129 (1983) 99; D. Chang and R.N. Mohapatra, Phys. Rev. Lett. 58 (1987) 1600; S. Rajpoot, Phys. Lett. B 191 (1987) 122.
[15] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, eds. F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979) p. 315; T. Yanagida, *Proc. of the Workshop on Unified Theory and Baryon Number of the Universe*, KEK, Japan, 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.

[16] Z.G. Berezhiani, ref. [4]; Z.G. Berezhiani and M.Yu. Khlopov, Z. Phys. C 49 (1991) 73.

[17] F. Abe et al., Phys. Rev. Lett. 72 (1994) 2138; Phys. Rev. D 50 (1994) 2966.

[18] F. Abe et al., Phys. Rev. Lett. 64 (1990) 147; Phys. Rev. Lett. 68 (1992) 447.

[19] Z.G. Berezhiani and R. Rattazzi, Phys. Lett. B 279 (1992) 124; Nucl. Phys. B 407 (1993) 249; Z. Berezhiani, *Proc. Int. Workshop 'SUSY 94*', Ann Arbor, May 1994 (hep-ph/9407264).

[20] Z. Berezhiani, J. Chkareuli, G. Dvali and M. Jibuti, *Proc. Int. Seminar 'Quarks-86*', eds. A. Tavkhelidze et al., (Moscow, INR, 1987) p. 209.

[21] P. Langacker and D. London, Phys. Rev. D 38 (1988) 886; E. Nardi, E. Roulet and D. Tommasini, Nucl. Phys. B 386 (1992) 239.

[22] D. London, in *Precision Tests of the Standard Model*, ed. P. Langacker (World Scientific, 1993); E. Nardi, in *Proc. 2nd Int. Workshop on Physics and Experiments at Linear e⁺e⁻ Colliders*, eds. F. Harris et al., (World Scientific, Singapore, 1993) Vol. II, p. 496;

[23] E. Nardi, E. Roulet and D. Tommasini, Phys. Lett. B 327 (1994) 319; CERN-TH.7443/94 (September 1994), unpublished.

[24] P. Chankowski and Z. Pluciennik, Phys. Lett. B 316 (1993) 312; K.S. Babu, C.N. Leung and J. Pantaleone, *ibid.* 191; F. Vissani and A.Yu. Smirnov, IC/94/102 (May 1994) unpublished; A. Brignole, H. Murayama and R. Rattazzi, Phys. Lett. B 335 (1994) 345.

[25] S.S. Gershtein and Ya.B. Zeldovich, Pis’ma ZhETF 4 (1966) 174; R. Cowsik and J. McClelland, Phys. Rev. Lett. 29 (1972) 669. See also in E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley, 1990.

[26] E. Nardi and E. Roulet, Phys. Lett. B 245 (1990) 105.

[27] A. de Rújula, S. Glashow and U. Sarid, Nucl. Phys. B 333 (1990) 173.

[28] P.F. Smith *et al.*, Nucl. Phys. B 206 (1982) 333.

[29] V.S. Berezinsky, Nucl. Phys. B 380 (1992) 478.

[30] H.U. Bengtsson, P. Salati and J. Silk, Nucl. Phys. B 346 (1990) 129.

[31] T. Sjostrand and M. Bengtsson, Computer Phys. Comm. 43 (1987) 367.
[32] V.S. Berezinsky et al., *Astrophysics of Cosmic rays*, North-Holland, 1990; E.W. Kolb et al., *Inner Space/Outer Space*, Univ. of Chicago Press, 1986.

[33] V. Kuzmin, V. Rubakov and M. Shaposhnikov, Phys. Lett. B 155 (1985) 36.

[34] P. Arnold and L. McLerran, Phys. Rev. D 36 (1987) 581; D 37 (1988) 1020.

[35] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45; Phys. Rev. D 42 (1990) 1285.

[36] B. Campbell, S. Davidson and K.A. Olive, Nucl. Phys. B 399 (1993) 111.

[37] G. Farrar and M. Shaposhnikov, Phys. Rev. Lett. 70 (1993) 2833; CERN-TH-6732-93 (May 1993) unpublished.

[38] M.B. Gavela et al., CERN-TH-7262,7263-94 (June 1994) unpublished; P. Huet, SLAC-PUB-6631 (August 1994) unpublished; G. Farrar and M. Shaposhnikov, RU-94-40 (June 1994) unpublished.

[39] Q. Shafi and F. Stecker, Phys. Rev. Lett. 53, 1292, 1984.

[40] E.L. Wright et al., Astrophys. J. 396 (1992) L13; M. Davis, F.J. Summers and D. Schagel, Nature 359 (1992) 393; A.N. Taylor and M. Rowan-Robinson, *ibid.*, 359 (1992) 396; R.K. Schaefer and Q. Shafi, Report No. BA-92-28, 1992 (unpublished) and Nature 359 (1992) 199; J.A. Holtzman and J.R. Primack, Astrophys. J. 405 (1993) 428; A. Klypin et al., *ibid.*, 416 (1993) 1.

[41] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42 (1985) 1441; Nuo. Cim. 9 C (1986) 17; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369; D 20 (1979) 2634.

[42] For an analysis of the most recent data, see Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B 335 (1994) 237.