Supersymmetry breaking and dilaton stabilization in string gas cosmology

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Received April 28, 2012
Accepted August 14, 2012
Published September 12, 2012

Abstract. In this Note we study supersymmetry breaking via gaugino condensation in string gas cosmology. We show that the same gaugino condensate which is introduced to stabilize the dilaton breaks supersymmetry. We study the constraints on the scale of supersymmetry breaking which this mechanism leads to.

Keywords: string theory and cosmology, alternatives to inflation, physics of the early universe, supersymmetry and cosmology

ArXiv ePrint: 1103.1389
1 Introduction

String Gas Cosmology [1] (see also [2] for an original reference, [3, 4] for reviews, and [5] for a selection of other earlier papers) is an approach to superstring cosmology based on making use of degrees of freedom and symmetries which are particular to string theory and studying their effects in the very early universe. More specifically, string gas cosmology rests on coupling the energy-momentum tensor of a thermal gas of perturbative string states including modes with momentum and winding about the extra spatial dimension s to a background space-time and studying the resulting dynamics.

One of the first results which emerges [1] is that the temperature singularity of Standard (and also Inflationary) Cosmology is resolved: the temperature $T$ of the string gas can never exceed a maximal temperature $T_H$ called the Hagedorn temperature [6]. In fact, if space is compact and isotropic, described by a radius $R$, the T-duality symmetry of string theory implies

$$T(R) = T(1/R),$$

where in this formula the radius is expressed in units of the string length. Thus, the evolution of the very early universe is clearly going to be very different from what is expected based on intuition gained from Standard and Inflationary cosmology. In fact, it is possible that in string theory the universe begins with a long quasi-stationary phase with a temperature close to $T_H$. This conjectured phase is called the “Hagedorn phase”. The Hagedorn phase has a smooth transition to the expanding phase of Standard Cosmology which is given by the decay of string winding modes into string loops.

As was recently realized [10] (see also [11]), thermal fluctuations in the Hagedorn phase of string gas cosmology evolve into a scale-invariant spectrum of cosmological perturbations.  

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1 This phase cannot be described by either General Relativity or Dilaton Gravity (see [7, 8] for discussions of this point), and thus the background for string gas cosmology is not yet under good control. For an attempt to obtain a Hagedorn phase in the context of an effective field theory see e.g. [9].
at late times.\footnote{This analysis assumes the existence of a quasi-static Hagedorn phase during which also the dilaton is frozen.} Thus, string gas cosmology provides an alternative to inflationary cosmology in providing an origin for the observed fluctuations in the distribution of matter and microwave radiation in the universe. A specific prediction of string gas cosmology with which the scenario can be distinguished from any inflationary model\footnote{Any model based on General Relativity as the theory of space and time and matter obeying the usual energy conditions.} is a slightly blue spectrum of gravitational waves\cite{12}.

In order for string gas cosmology to be consistent with late time cosmology, the moduli describing the size and shape of the extra dimensions of space must be stabilized. In Heterotic string theory this is achieved naturally by means of string states containing both winding and momentum about the extra dimensions which become massless at the self-dual radius (of the extra spatial dimensions). These states fix both the size moduli\cite{13} (see also\cite{14–16} for earlier work) and shape moduli\cite{17}.\footnote{See also\cite{19} for a selection of other papers on moduli stabilization in string gas cosmology.} Note that at this stage no new extra ingredients had to be introduced.

In order to be phenomenologically viable, the dilaton must also be fixed at late times. This has recently been achieved by making use of a nonperturbative mechanism used frequently in different contexts: gaugino condensation. In\cite{18} it was shown that gaugino condensation fixes the dilaton without destroying the stabilization mechanism of the radion.\footnote{See\cite{19} for other works on moduli stabilization in string gas cosmology.}

In this Note we wish to point out the the same gaugino condensation mechanism used to fix the dilaton automatically breaks supersymmetry. We study the breaking of supersymmetry in detail and show that supersymmetry breaking at both low and high scales is consistent with dilaton and radion stabilization. Thus, it appears that string gas cosmology is consistent with both late time cosmology and particle phenomenology.

In the following section we briefly review string gas cosmology and the stringy mechanism which stabilizes both the size (radion) and shape moduli associated with the extra spatial dimensions. In section 3 we review the work of\cite{18} which shows how assuming gaugino condensation leads to a mechanism to stabilize the dilaton without disrupting the stringy stabilization of the radion. We then proceed to demonstrate that the same gaugino condensation mechanism breaks supersymmetry. Finally, we compute the dilaton mass, the gravitino mass and the supersymmetry breaking scale in terms of the constants which appear in the superpotential of gaugino condensation and discuss phenomenological constraints.

\section{Review of string gas cosmology}

In string gas cosmology\cite{1}, matter is taken to be a thermal gas of perturbative string states. These include modes which contain both momentum and winding about the extra spatial dimensions. We assume that the topology of the internal spatial manifold is such that long-lived winding modes exist (see e.g.\cite{20} for a discussion of which orbifold compactifications admit long-lived winding modes).

Let us now follow the evolution of this thermal string system as we go backwards in time when space decreases in size and the energy density rises. Initially, the energy density of the string gas is in the states which are light at large radii, namely the string momentum modes. However, as we go backwards in time, eventually the thermal energy will be so large that it becomes possible to excite the string oscillatory modes. Since the number of
string states rises exponentially with energy [6], there is a maximal temperature which the thermal string gas can reach, the so-called Hagedorn temperature $T_H$. Instead of increasing the energy of the modes which are already excited, the extra energy density which is obtained if the volume of space decreases goes to exciting new modes. Eventually, even string winding modes are excited. The string gas of the very early universe is hence expected to contain all perturbative string states, in particular modes which wind the internal spatial dimensions.

The equations which govern the dynamics of the early phase of string gas cosmology when the temperature hovers close to (but below) $T_H$ are not known. They cannot be those of General Relativity nor those of Dilaton Gravity, since neither set of equations are consistent with the full set of symmetries of string theory.\textsuperscript{6} However, at late times when the temperature is significantly smaller than $T_H$, the equations of motion which describe the background space-time must reduce to those of Dilaton Gravity coupled to a thermal gas of strings as the matter content.

Thus, the action which describes string gas cosmology in ten space-time dimensions at late times is

$$S = \frac{1}{k}(S_g + S_\phi) + S_{SG},$$

where $S_g$ is the string frame gravitational action (which has a dilaton dependence), $S_\phi$ is the dilaton action and $S_{SG}$ is action for the string gas. Also,

$$k = 16\pi G = \frac{16\pi}{M_{10}^8},$$

where $M_{10}$ is the 10D Planck mass.

The dilaton action is given by

$$S_\phi = -\int d^{10}x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + kV(\phi) \right],$$

where $\phi$ is the ten-dimensional dilaton. In the absence of non-perturbative effects, the dilaton potential $V(\phi)$ vanishes. It is known [7] that without a potential the dilaton cannot be stabilized. We will follow [18] and consider the non-perturbative gaugino condensation mechanism which leads to a non-vanishing dilaton potential.

The string gas is treated as ideal gas. Hence, the action can be written as (see e.g. [13, 16] for extensive discussions)

$$S_{SG} = -\int d^{10}x \sqrt{-g} \sum_\alpha \mu_\alpha \epsilon_\alpha,$$

where the index $\alpha$ runs over all string states, $\mu_\alpha(x, t)$ is number density of strings in the state $\alpha$, and $\epsilon_\alpha(t)$ is the energy of this string state. Factoring out the expansion of the universe we have

$$\mu_\alpha = \sqrt{-g_{ss}}^{-1}\mu_{0,\alpha}(t)$$

where $\mu_{0,\alpha}$ is the comoving number density and $g_{ss}$ is the determinant of the spatial part of the metric:

$$\sqrt{-g} = \sqrt{-g_{ss}} \sqrt{-g_{00}}.$$

\textsuperscript{6}Dilaton gravity is consistent with the T-duality symmetry of string theory, but for large values of the dilaton the string states are no longer the lightest ones and instead D-brane states become dominant.
With these substitutions, the string gas action becomes

\[ S_{SG} = - \int d^{10}x \sqrt{-g_{00}} \sum_{\alpha} \mu_{0,\alpha}(t) \epsilon_{\alpha}. \]  

(2.7)

Since it is important to understand how size moduli stabilization in string gas cosmology comes about, we list below the expressions for the energy density and the pressures of the string gas in the various spatial directions.

We will write down the expressions for the energy density and the various pressures in the case of an anisotropic but homogeneous backgroundmetric given by

\[ ds^2 = -dt^2 + a(t)^2 dx^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2, \]  

(2.8)

where \( a(t) \) is the scale factor of our three-dimensional space and \( b_a(t) \) is the scale factor of the \( a \)'th direction of the internal space (here, for simplicity, we have an internal torus in mind).

By varying the above string gas action with respect to the metric, the expressions for the non-vanishing components of the energy momentum-tensor can be derived. The contribution of the string state \( \alpha \) to the energy density is given by

\[ \rho_{\alpha} = \frac{\mu_{0,\alpha}}{\epsilon_{\alpha} \sqrt{-g}} \epsilon_{\alpha}^2, \]  

(2.9)

and that to the three (large) dimensional pressure by,

\[ P_{d\alpha} = \frac{\mu_{0,\alpha}}{\epsilon_{\alpha} \sqrt{-g}} \frac{P_d^2}{3} \]  

(2.10)

where \( P_d \) is the momentum in the \( d = 3 \) large dimensions. The contribution of the string state \( \alpha \) to the pressure in the \( a \)'th direction of the internal compact space is given by

\[ P_{a\alpha} = \frac{\mu_{0,\alpha}}{\epsilon_{\alpha} \sqrt{-g}} \left( \frac{n_a^2}{b_a^2} - \frac{w_a^2}{b_a^2} \right) \]  

(2.11)

where \( n_a \) and \( w_a \) are the momentum and winding numbers, respectively, of the string state in the \( a \)'th direction (we are omitting the label \( \alpha \) on \( n, w \) and \( N \) to keep the notation simpler).

For the sake of completeness we also give the expression for the energy \( \epsilon_{\alpha} \) of the state \( \alpha \):

\[ \epsilon_{\alpha} = \frac{1}{\sqrt{\alpha}} \left[ \alpha' P_d^2 + b^{-2}(n, n) + b^2(w, w) + 2(n, w) + 4(N - 1) \right]^{1/2} \]  

(2.12)

where \( n \) and \( w \) are the momentum and winding number vectors in the internal space, and \( N \) is the oscillator level. The parentheses in \( (n, n), (w, w) \) and \( (n, w) \) indicate the scalar product of the momentum and winding number vectors in the internal dimensions.

The size moduli \( b_a(t) \) and shape moduli (which we have implicitly set to zero in the above ansatz for the metric) must be stabilized at low energy densities in order that the resulting low energy effective theory be consistent with experimental and observational constraints. The stabilization of the size moduli can be understood by looking at the above formula (2.11) for the pressure in one of the compact directions: there are special states which have both momentum and winding about the extra dimensions for which this pressure vanishes at the self-dual radius. If these states dominate the string gas partition function, then the effective
potential for the dynamics of $b_a(t)$ will have a stable minimum at a value given by the “self-dual radius” which in general is the string scale. As discussed in [13], in heterotic string theory these special states are massless at the self-dual radius. Hence, if the value of $b$ is close to the self-dual radius, these states will dominate the string partition function, and they will lead to radion stabilization. They are enhanced symmetry states which trap the radion at the fixed point. Assuming that the dilaton is fixed, it was shown in [13] that the enhanced symmetry point is a stable fixed point for the radion, and that the fluctuations about this enhanced symmetry point are phenomenologically safe.

To be phenomenologically viable, string gas cosmology must also admit a mechanism which stabilizes the dilaton. In [18], a gaugino condensation mechanism as a means to stabilize the dilaton was explored. By studying the dilaton potential $V$ arising from the superpotential of gaugino condensation, it was shown that the dilaton can be trapped at a stable fixed point. In fact, it was shown that dilaton stabilization by gaugino condensation is consistent with radion stabilization by string gases. This was shown by expanding the equations of motion for both the radion and the dilaton about the fixed point, and showing that the mass matrix for the fluctuations is positive definite.

Gaugino condensation is a well-known mechanism to break local supersymmetry. In the following, we will explore this mechanism in the context of string gas cosmology.

3 Gaugino condensation and supersymmetry breaking

We now study the gaugino condensation mechanism and the resulting breaking of supersymmetry. We are working in the context of perturbative heterotic $E_8 \times E_8$ string theory. Our goal is to find a stable state that breaks supersymmetry but keeps the cosmological constant zero.

We consider a background with space-time $M^4 \times K$, where $M^4$ is our four-dimensional Minkowksi space-time and $K$ is a six-dimensional manifold of SU(3) holonomy (and needs to be chosen such that the enhanced symmetry states which can lead to the size and shape moduli stabilization actually exist — for a specific example see [18]). This compactification breaks $E_8 \times E_8$ to $E_6 \times E_8$ [22]. Subsequently the $E_6$ breaks to the Standard Model gauge group. The second $E_8$ is the hidden sector of low energy supergravity which breaks to a subgroup $Q$.

Let us consider the ten-dimensional supergravity Lagrangian for gauge fields and gravitons which is given by (following the notation in the textbook [23]):

$$
\mathcal{L} = \frac{M_{10}^8}{16\pi} R - \frac{M_{10}^8 e^{-\varphi}}{32\pi} |H_3|^2 - \frac{e^{\varphi/2}}{4g_{10}^2} \text{Tr} F_{AB} F^{AB}
$$

where $g_{10}$ is the ten-dimensional gauge coupling, $H_3$ is the NS-NS field and $F_{AB}$ is the gauge field (the capitalized Latin indices run over all of the dimensions). The above action can be reduced to a four-dimensional gauge action which is given by

$$
S = \int d^4x \frac{1}{4} e^{-2\varphi} b_{\mu}^E (M_{10}^8 g_{10}^2)^{-1} \text{Tr} F_{\mu\nu}^2
$$

A heuristic way to understand radion stabilization is as follows [14]: the winding modes counteract an increase in the value of the radion (see appendix A for the definition of the radion field $\sigma(t)$ in terms of $b(t)$), whereas the momentum modes oppose a contraction, leaving the self-dual point as a stable point of the dynamics.

See [15, 21] for more general discussions of moduli trapping at enhanced symmetry points.

Shape moduli stabilization in the same perturbative superstring context was studied in [17].
where the Einstein frame scale factor $b_E$ of the extra dimensions yields the radion $\sigma$ (see appendix A). Setting $M_p^6 g_{10}^2 = 1$, the four-dimensional gauge coupling can be read off from eq. (3.2) as

$$g_4^2 = e^{2\sigma} b_E^{-6}. \quad (3.3)$$

The low energy subgroup $Q$ becomes strong at a mass scale $\mu$ given by

$$\mu \sim M_{10} b_E^{-4} \exp \left( -1/(2 b_0 e^{2\sigma} b_E^6) \right), \quad (3.4)$$

where $b_0$ is the one-loop $\beta$ function coefficient of $Q$. In order for the ten-dimensional Planck mass to coincide with the four-dimensional Planck mass, the corresponding metrics are related by

$$g^{(10)}_{\mu\nu} = b_E^{-6} g^{(4)}_{\mu\nu}. \quad (3.5)$$

The massive gauge boson mass is of order

$$M_{\mathrm{GUT}} = M_{10} b_E^{-4}. \quad (3.6)$$

Inserting the value of $g_4$ from eq. (3.3) into eq. (3.4), the mass scale $\mu$ depends on $\varphi$ as [24] (see also [25])

$$\mu \sim M_{\mathrm{GUT}} \exp \left( -1/(2 b_0 e^{2\sigma} b_E^6) \right) \quad (3.7)$$

$$\sim M_{\mathrm{GUT}} \exp \left( -1/(2 b_0 e^{2\sigma}) \right), \quad (3.8)$$

where we have used that $b_E$ can be set to 1 if radion stabilization occurs at the string scale (see appendix A). The above equation indicates that for small values of $\varphi$, the characteristic scale of $Q$, namely $\mu$, is much less than $M_p$.

So for this region gravity is a small correction as compared to gauge forces of $Q$. Now considering the possibility of supersymmetry breaking in this sector, there is a possibility of formation of gaugino condensates through which supersymmetry breaking can be triggered. Since $\mu$ is the mass scale for $Q$, gauginos of the gauge superfield $Q$ can condense with

$$\langle \text{Tr} \chi \bar{\chi} \rangle \sim \mu^3 \quad (3.9)$$

from instanton effects. However it has been shown in [26] that these gaugino condensates can not break supersymmetry in pure gauge theories (i.e. in the case of global supersymmetry). The situation is improved [24] by coupling the gauge theory to supergravity that contains an axion field. This axion fields arises while compactifying ten-dimensional supergravity to four dimensions, and it is the Hodge dual of the field $H_3$.

Thus, the gaugino condensation in four dimensions corresponds to

$$\langle \text{Tr} \chi \Gamma_{ijk} \bar{\chi} \rangle = A \epsilon_{ijk} \quad (3.10)$$

in ten dimensions, where $A$ is a complex number.

One way to see that gaugino condensation triggers supersymmetry breaking is to apply a supersymmetry transformation on the supergravity multiplet. Under a supersymmetry transformation, the massless fermion $\chi$ of the supergravity multiplet transforms as

$$\delta \lambda \sim \langle \text{Tr} \chi \rangle \epsilon + \langle \text{Tr} \chi \gamma_5 \chi \rangle \gamma_5 \epsilon \quad (3.11)$$

so that if $\text{Tr} \chi \chi$ or $\text{Tr} \chi \gamma_5 \chi$ is non-zero (refer to eq. (3.9)) then $\delta \lambda \neq 0$, $\chi$ is a Goldstone fermion and supersymmetry is spontaneously broken.
The scale of supersymmetry breaking $M_s$ and the mass of the gravitino $m_{3/2}$ will be used later. Their values in terms of $\mu$ are given in appendix B. In the following we will also need the expression for the dilaton mass. To obtain its expression, we need to study the dilaton potential induced by gaugino condensation.

Gaugino condensation is usually analyzed in terms of the fields appearing in the four-dimensional effective field theory which describes our dimensions. We must thus pause for a moment and relate the fields in the four-dimensional theory to those in the ten-dimensional space. The four-dimensional dilaton $\Phi$ is given by

$$\Phi = 2\varphi - 6 \ln b_E,$$

which leads to the relationship between gauge coupling and dilaton field in four dimensions,

$$g_4^2 = e^{\Phi}.$$  

The first basic potential in a supersymmetric theory is the Kähler potential $K(S,T)$. We take it to be of minimal form, namely

$$K(S,T) = -\ln(S + S^*) - 3\ln(T + T^*),$$

where $S$ is the dilaton-axion multiplet

$$S = e^{-\Phi} + ia,$$

$a$ being the axion field, and $T$ is

$$T = b^2 + i\beta,$$

$b$ corresponding to the radion and $\beta$ describing the flux about the compact directions (which we set to zero in our model).

Compactifications with gaugino condensation lead to a superpotential of the form

$$W = M_4^3(A - A e^{-a_0 S})$$

where $C, a_0$ and $A$ are constants, and $M_4$ is the four dimensional Planck mass. The second term arises from gaugino condensation, the first comes from fluxes and a Chern-Simons term. The flux term must be added in order to obtain vanishing potential energy in the ground state.

The scalar potential in the four-dimensional Einstein frame is given by

$$V_F = \frac{1}{M_4^2} e^K [K^{AB} D_A W D_W W - 3|W|^2]$$

where $A, B$ run over all moduli fields ($\Phi$ and $b$ in our case). The Kähler covariant derivatives are given by

$$D_A W = \partial_A W + (\partial_A K) W.$$ 

Here the superpotential is independent of the $T$ modulus field. Thus, eq. (3.18) reduces to

$$V = \frac{1}{M_4^2} e^K K^{ab} D_a W D_b W,$$
where $a, b$ now runs over $S$. After simplifying the above expression the scalar potential is given by,

$$V = \frac{M^4}{4} b^{-6} e^{-\Phi} \left[ \frac{C^2}{4} e^{2\Phi} - A e^\Phi \left( a_0 + \frac{1}{2} e^\Phi \right) e^{-a_0 e^{-\Phi}} + A^2 \left( a_0 + \frac{1}{2} e^\Phi \right)^2 e^{-2a_0 e^{-\Phi}} \right]. \quad (3.21)$$

Minimising the above potential and expanding around a stable minimum $\Phi = \Phi_0$, the approximate form of the potential is given by

$$V = \frac{M^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} (e^{-\Phi} - e^{-\Phi_0})^2. \quad (3.22)$$

From this expression for the dilaton potential we can compute the dilaton mass obtained if the dilaton sits at the minimum of its potential.

For the sake of completeness we can also write down the potential in terms of ten-dimensional fields. Using the relation (3.12) between the four-dimensional and ten-dimensional dilatons and the following relation between the radions in the string and Einstein frames

$$b_s = e^{\varphi/4} b_E, \quad (3.23)$$

the approximate form of the scalar potential can be written as

$$V(b, \varphi) = \frac{M_{10}^8 \hat{V}}{4} e^{-\Phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} (e^{-\varphi} - e^{-\Phi_0})^2. \quad (3.24)$$

where we have used the following relation between the four-dimensional Planck mass $M_4$ and the ten-dimensional one $M_{10}$:

$$M^2_4 = M_{10}^8 \hat{V}. \quad (3.25)$$

### 4 Phenomenological considerations

It is now important to investigate what values of the supersymmetry breaking scale can arise from string gas cosmology. We must consider a range of constraints. Firstly, string gas cosmology is based on the assumption that the string coupling constant is small, i.e. that

$$\Phi_0 < 0. \quad (4.1)$$

If this condition were not satisfied, then the gauge coupling is not weak and the string states would not be the lightest states. In particular, D-branes would have to be considered. On the other hand, if the string coupling constant is small, then D-branes will decouple early in the cosmological evolution, leaving the perturbative string states to dominate the late-time dynamics [27].

Since the dilaton potential takes the form of a perfect square, the cosmological constant will vanish at the minimum of the potential. The conditions for $\Phi_0$ to be a minimum of the potential are

$$|D_T W|^2 = 3|W|^2 \quad (4.2)$$

and

$$|D_S W|^2 = 0, \quad (4.3)$$
the first of which is automatically satisfied, leaving (4.3) as the condition which is to be verified. For our minimal Kähler potential (3.14) and our superpotential (3.17) this condition becomes

$$A(2\text{Re}(S)a_0 + 1)e^{-a_0S} = C,$$

(4.4)

where $S$ is evaluated at the minimum $\Phi_0$.

The equation (4.4) provides a relation between the three constants $A$, $C$, and $a_0$ in the superpotential of gaugino condensation. Note that it should not be unexpected that the constraint of vanishing potential at the minimum leads to a constraint on the parameters in the underlying theory: in all of our current models of particle physics a fine-tuning of coefficients is required in order to eliminate what would otherwise be a large cosmological constant.

Furthermore, there are phenomenological constraints on the dilaton and the gravitino. They must either be extremely light or quite heavy. If they are very light, their decay time would be cosmological, and they could dominate the energy density of the universe. This leads to the constraint that the mass of a long lived gravitationally coupled particle should be less than about 1 KeV [28]. A much tighter constraint comes from fifth force constraints and from the life-time of white dwarfs and red giants [29]. The bound is

$$m_X < 10^{-33} \text{eV},$$

(4.5)

where $X$ refers to both the gravitino and the dilaton. This window will be uninteresting for us.

If the particles are heavy and decay on time scales shorter than the life-time of the universe there are also cosmological constraints. In particular, the mass should obey [30, 31]

$$m_X > 10^4 \text{GeV},$$

(4.6)

otherwise the particle decay would lead to excessive cosmological entropy production which would destroy the successful predictions of cosmological nucleosynthesis. The constraint yields the following bounds on the scale of supersymmetry breaking scale and on the gaugino condensation scale,

$$M_s > 10^{11} \text{GeV}, \quad \mu > 10^{14} \text{GeV}.$$  

(4.7)

Let us now study these constraints in our model. We first need to find the location of the local minimum of the potential $V(\Phi)$ given in (3.21). The potential tends to zero at large negative values of $\Phi$ and diverges for large positive values. The slope of the potential in the limit $\Phi \to -\infty$ is positive. Hence, the first extremum of the potential is a local maximum, and the second is a local minimum. Under the hypothesis that $\Phi_o$ is negative, then the approximate form of the derivative of the potential is

$$\frac{\partial V}{\partial \Phi} \simeq V_0 \left[ \frac{C^2}{4} e^{2\Phi} - 2a_0^2 A C e^{-\Phi} + 4A^2 a_0^3 e^{-2\Phi} \epsilon^2 \right],$$

(4.8)

with

$$V_0 = \frac{M_4^4}{4} b^{-6}$$

(4.9)

and

$$\epsilon \equiv e^{-a_0 e^{-\Phi}}.$$  

(4.10)
Given the hypothesis that $\Phi_0$ is negative, and given that $a_0$ is positive, $\epsilon$ is indeed a small quantity. This explains the suggestive notation.

The local maximum of $V$ is given by the zero of (4.8) obtained by balancing the first and the second term, the local minimum $\Phi_0$ which we are looking for is given by balancing the second and third terms. This yields

$$e^{-\Phi_0}\epsilon = \frac{C}{2Aa_0}.$$  \hfill(4.11)

Making use once again of the hypothesis that $\Phi_0$ is negative, we can simplify (4.11) to obtain

$$\Phi_0 \simeq - \log \left[\frac{1}{2a_0} \log \left(\frac{2a_0A}{C}\right)\right].$$  \hfill(4.12)

Thus, the self-consistency condition for $\Phi_0$ to be negative is

$$\log \left(\frac{2a_0A}{C}\right) > 2a_0.$$  \hfill(4.13)

Comparing (4.4) with the condition (4.13) required to have small string coupling, we see that it is possible to satisfy both simultaneously, provided that $C$ is a small number. Ways to achieve this were discussed in [32].

The dilaton mass can be obtained from the second derivative of the potential $V(\Phi)$ at the minimum $\Phi = \Phi_0$. The result is

$$m_{\Phi}^2 = \frac{M_P^2}{8} C^2 \frac{1}{2a_0} \log \left(\frac{2a_0A}{C}\right).$$  \hfill(4.14)

Thus, we see that the dilaton mass is suppressed compared to the Planck mass by the constant $C$ which is required to be much smaller than 1 in order to ensure weak string coupling.

From eq. (3.4) the scale of the gaugino condensate is given by

$$\mu \sim M_4 e^{\frac{1}{2a_0S}} \sim M_4 (2a_0 e^{-\Phi_0}\epsilon)^{1/3},$$  \hfill(4.15)

where we have used the definition of $\epsilon$ from (4.10). From (4.11) we then see that $\mu$ is suppressed compared to the Planck scale by powers of $\frac{C}{A}$, more precisely

$$\mu \sim M_4 \left(\frac{C}{A}\right)^{1/3}.$$  \hfill(4.16)

The result depends on (4.11), which requires $2a_0S \ll 1$. If this condition is not met then

$$\mu \sim M_4 (C - Ae^{-a_0S})^{1/3}.$$  \hfill(4.17)

Clearly, however, a high scale of gaugino condensation and hence also of supersymmetry breaking appears more natural than a low scale. Hence, the phenomenological constraints on the gaugino mass are easily satisfied.
5 Conclusions and discussion

We have shown that the gaugino condensation mechanism introduced in [18] to stabilize the dilaton also leads to supersymmetry breaking. The typical scale of supersymmetry breaking is high. String gas cosmology (in the context of perturbative heterotic string theory) thus provides a natural way to stabilize all of the moduli fields and at the same time leads to a non-supersymmetric low energy field theory.

We wish to emphasize that the stabilization of shape and size moduli of the extra spatial dimensions is completely natural in string gas cosmology. It relies on the effects of string modes which carry both momentum and winding. These modes are not seen in an effective field theory approach to string cosmology. A single non-perturbative mechanism — namely gaugino condensation — is sufficient to provide both dilaton stabilization and supersymmetry breaking.

At this stage of the analysis, it appears that a high scale of supersymmetry breaking is favored. Thus, it does not appear that a natural solution of the hierarchy problem emerges. However, it would be of great interest to extend our analysis to more realistic compactifications. With more realistic toroidal orbifold compactifications it would then also be possible to obtain low energy field theories close to the Standard Model (see e.g. [33] for a selection of references connecting the Standard Model to heterotic string models).

Acknowledgments

We wish to thank Keshav Dasgupta, Andrew Frey, and Bret Underwood for stimulating discussions. One of us (R.B.) wishes to thank Joe Polchinski for raising probing questions which motivated this work. We thank Andrew Frey and Scott Watson for comments on the draft. The research of R.B. is supported in part by an NSERC Discovery Grant at McGill and by funds from the Canada Research Chairs program. The visit of S.M. to McGill University was supported by a Commonwealth Scholarship for which we are grateful. W.X. is supported in part by a Schulich Fellowship. U.Y. thanks the members of the McGill High Energy Theory group for hospitality and financial support during a sabbatical visit.

A Radion

The Einstein frame scale factor $b_E$ of an extra spatial dimension becomes a radion field $\sigma$ in the four-dimensional effective action. This field can be defined by

$$\sigma = \sigma_o \ln \frac{b_E}{b_s},$$

(A.1)

where $\sigma_o$ is a constant with dimension of mass. The constant $b_s$ corresponds to the radius of an extra dimension at the string scale. If moduli stabilization occurs at the string scale (as in string gas cosmology in the absence of a chemical potential for winding number) then $b_E = b_s$ and hence $\sigma = 0$.

B Gravitino mass and supersymmetry breaking

The gravitino mass $m_{3/2}$ induced by the gaugino condensation mechanism is given in terms of the scale $\mu$ by

$$m_{3/2} \sim \frac{\mu^3}{M_4^2}.$$  

(B.1)
And from the superpotential, the mass of the gravitino can be expressed as,

$$m_{3/2} \sim \frac{M_s^2}{M_4} \sim \frac{W}{M_4} \sim M_4 \times (C - A e^{-a_0 S}) \sim 2M_a a_0 e^{-\Phi e}$$

(B.2)

The supersymmetry breaking scale $M_s$ is in turn given by

$$M_s^2 \sim \frac{\mu^3}{M_4}.$$  

(B.3)

Hence, for the gravitino mass to be order of TeV the supersymmetry breaking scale must be about $10^{14}$ GeV. This constrains the value of $e^\tau$ in eq. (3.8).

References

[1] R.H. Brandenberger and C. Vafa, Superstrings in the early universe, Nucl. Phys. B 316 (1989) 391 [inSPIRE].
[2] J. Kripfganz and H. Perlt, Cosmological impact of winding strings, Class. Quant. Grav. 5 (1988) 453 [inSPIRE].
[3] R.H. Brandenberger, String gas cosmology, arXiv:0808.0746 [inSPIRE].
[4] T. Battefeld and S. Watson, String gas cosmology, Rev. Mod. Phys. 78 (2006) 435 [hep-th/0510022] [inSPIRE].
[5] G.B. Cleaver and P.J. Rosenthal, String cosmology and the dimension of space-time, Nucl. Phys. B 457 (1995) 621 [hep-th/9402088] [inSPIRE]; M. Sakellariadou, Numerical experiments on string cosmology, Nucl. Phys. B 468 (1996) 319 [hep-th/9511075] [inSPIRE]; D.A. Easson, Brane gases on K3 and Calabi-Yau manifolds, Int. J. Mod. Phys. A 18 (2003) 4295 [hep-th/0110225] [inSPIRE]; S. Watson and R.H. Brandenberger, Isotropization in brane gas cosmology, Phys. Rev. D 67 (2003) 043510 [hep-th/0207168] [inSPIRE]; T. Boehm and R.H. Brandenberger, On T-duality in brane gas cosmology, JCAP 06 (2003) 008 [hep-th/0208188] [inSPIRE]; R. Easther, B.R. Greene, M.G. Jackson and D.N. Kabat, Brane gas cosmology in M-theory: late time behavior, Phys. Rev. D 67 (2003) 123501 [hep-th/0211124] [inSPIRE]; S.H. Alexander, Brane gas cosmology, M-theory and little string theory, JHEP 10 (2003) 013 [hep-th/0212151] [inSPIRE]; A. Kaya and T. Rador, Wrapped branes and compact extra dimensions in cosmology, Phys. Lett. B 565 (2003) 19 [hep-th/0301031] [inSPIRE]; B.A. Bassett, M. Borunda, M. Serone and S. Tsujikawa, Aspects of string gas cosmology at finite temperature, Phys. Rev. D 67 (2003) 123506 [hep-th/0301180] [inSPIRE]; A. Kaya, On winding branes and cosmological evolution of extra dimensions in string theory, Class. Quant. Grav. 20 (2003) 4533 [hep-th/0302118] [inSPIRE]; A. Campos, Late time dynamics of brane gas cosmology, Phys. Rev. D 68 (2003) 104017 [hep-th/0304216] [inSPIRE]; R.H. Brandenberger, D.A. Easson and A. Mazumdar, Inflation and brane gases, Phys. Rev. D 69 (2004) 083502 [hep-th/0307043] [inSPIRE]; R. Easther, B.R. Greene, M.G. Jackson and D.N. Kabat, Brane gases in the early universe: thermodynamics and cosmology, JCAP 01 (2004) 006 [hep-th/0307233] [inSPIRE]; T. Biswas, Cosmology with branes wrapping curved internal manifolds, JHEP 02 (2004) 039 [hep-th/0311076] [inSPIRE]; A. Campos, Late cosmology of brane gases with a two form field, Phys. Lett. B 586 (2004) 133 [hep-th/0311144] [inSPIRE];
S. Watson and R.H. Brandenberger, Linear perturbations in brane gas cosmology, *JHEP* **03** (2004) 045 [hep-th/0312097] [InSPIRE];
S. Watson, UV perturbations in brane gas cosmology, *Phys. Rev. D* **70** (2004) 023516 [hep-th/0402015] [InSPIRE];
T. Battefeld and S. Watson, Effective field theory approach to string gas cosmology, *JCAP* **06** (2004) 001 [hep-th/0403076] [InSPIRE];
F. Ferrer and S. Rasanen, Dark energy and decompactification in string gas cosmology, *JHEP* **02** (2006) 016 [hep-th/0509225] [InSPIRE];
M. Borunda and L. Boubekeur, The effect of $\alpha'$ corrections in string gas cosmology, *JCAP* **10** (2006) 002 [hep-th/0604086] [InSPIRE].

[6] R. Hagedorn, Statistical thermodynamics of strong interactions at high-energies, *Nuovo Cim. Suppl.* **3** (1965) 147 [InSPIRE].

[7] R.H. Brandenberger et al., More on the spectrum of perturbations in string gas cosmology, *JCAP* **11** (2006) 009 [hep-th/0608186] [InSPIRE].

[8] N. Kaloper, L. Kofman, A.D. Linde and V. Mukhanov, On the new string theory inspired mechanism of generation of cosmological perturbations, *JCAP* **10** (2006) 006 [hep-th/0608200] [InSPIRE];
N. Kaloper and S. Watson, Geometric precipices in string cosmology, *Phys. Rev. D* **77** (2008) 066002 [arXiv:0712.1820] [InSPIRE].

[9] R.H. Brandenberger, A.R. Frey and S. Kanno, Towards a nonsingular tachyonic big crunch, *Phys. Rev. D* **76** (2007) 063502 [arXiv:0705.3265] [InSPIRE].

[10] A. Nayeri, R.H. Brandenberger and C. Vafa, Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology, *Phys. Rev. Lett.* **97** (2006) 021302 [hep-th/0511140] [InSPIRE].

[11] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, String gas cosmology and structure formation, *Int. J. Mod. Phys.* **A** **22** (2007) 3621 [hep-th/0608121] [InSPIRE].

[12] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, Tensor modes from a primordial Hagedorn phase of string cosmology, *Phys. Rev. Lett.* **98** (2007) 231302 [hep-th/0604126] [InSPIRE].

[13] S.P. Patil and R.H. Brandenberger, The cosmology of massless string modes, *JCAP* **01** (2006) 005 [hep-th/0502069] [InSPIRE].

[14] S. Watson and R.H. Brandenberger, Stabilization of extra dimensions at tree level, *JCAP* **11** (2003) 008 [hep-th/0307044] [InSPIRE].

[15] S. Watson, Moduli stabilization with the string Higgs effect, *Phys. Rev. D* **70** (2004) 066005 [hep-th/0404177] [InSPIRE].

[16] S.P. Patil and R.H. Brandenberger, Radion stabilization by stringy effects in general relativity, *Phys. Rev. D* **71** (2005) 103522 [hep-th/0401037] [InSPIRE].

[17] R.H. Brandenberger, Y.-K. Cheung and S. Watson, Moduli stabilization with string gases and fluxes, *JHEP* **05** (2006) 025 [hep-th/0501032] [InSPIRE].

[18] R.J. Danos, A.R. Frey and R.H. Brandenberger, Stabilizing moduli with thermal matter and nonperturbative effects, *Phys. Rev. D* **77** (2008) 126009 [arXiv:0802.1557] [InSPIRE].

[19] A. Kaya, Volume stabilization and acceleration in brane gas cosmology, *JCAP* **08** (2004) 014 [hep-th/0405099] [InSPIRE];
A.J. Berndsen and J.M. Cline, Dilaton stabilization in brane gas cosmology, *Int. J. Mod. Phys.* **A** **19** (2004) 5311 [hep-th/0408185] [InSPIRE];
S. Arapoglu and A. Kaya, D-brane gases and stabilization of extra dimensions in dilaton gravity, *Phys. Lett. B* **603** (2004) 107 [hep-th/0409094] [InSPIRE].
T. Rador, Intersection democracy for winding branes and stabilization of extra dimensions, 
*Phys. Lett. B* **621** (2005) 176 [hep-th/0501249] [SPIRE]; 
Vibrating winding branes, wrapping democracy and stabilization of extra dimensions in dilaton gravity, 
*JHEP* **06** (2005) 001 [hep-th/0502039] [SPIRE]; 
Stabilization of extra dimensions and the dimensionality of the observed space, 
*Eur. Phys. J. C* **49** (2007) 1083 [hep-th/0504047] [SPIRE]; 
A. Kaya, Brane gases and stabilization of shape moduli with momentum and winding stress, 
*Phys. Rev. D* **72** (2005) 066006 [hep-th/0504208] [SPIRE]; 
D.A. Easson and M. Trodden, Moduli stabilization and inflation using wrapped branes, 
*Phys. Rev. D* **72** (2005) 026002 [hep-th/0505096] [SPIRE]; 
A. Berndsen, T. Biswas and J.M. Cline, Moduli stabilization in brane gas cosmology with superpotentials, 
*JCAP* **08** (2005) 012 [hep-th/0505151] [SPIRE]; 
S. Kanno and J. Soda, Moduli stabilization in string gas compactification, 
*Phys. Rev. D* **72** (2005) 104023 [hep-th/0509074] [SPIRE]; 
S. Cremonini and S. Watson, Dilaton dynamics from production of tensionless membranes, 
*Phys. Rev. D* **73** (2006) 066007 [hep-th/0601082] [SPIRE]; 
A. Chathrabhuti, Target space duality and moduli stabilization in string gas cosmology, 
*Int. J. Mod. Phys. A* **22** (2007) 165 [hep-th/0602031] [SPIRE]; 
J.Y. Kim, Stabilization of the extra dimensions in brane gas cosmology with bulk flux, 
*Phys. Lett. B* **652** (2007) 43 [hep-th/0608131] [SPIRE]; 
M. Sano and H. Suzuki, Moduli fixing and T-duality in type II brane gas models, 
*Phys. Rev. D* **78** (2008) 064045 [arXiv:0804.0176] [SPIRE]; 
J.Y. Kim, Stabilizing radion and dilaton with brane gas and flux, arXiv:0908.4314 [SPIRE].

[20] R. Easther, B.R. Greene and M.G. Jackson, Cosmological string gas on orbifolds, 
*Phys. Rev. D* **66** (2002) 023502 [hep-th/0204099] [SPIRE].

[21] L. Kofman et al., Beauty is attractive: moduli trapping at enhanced symmetry points, 
*JHEP* **05** (2004) 030 [hep-th/0403001] [SPIRE].

[22] D.J. Gross, J.A. Harvey, E.J. Martinec and R. Rohm, The heterotic string, 
*Phys. Rev. Lett.* **54** (1985) 502 [SPIRE]; 
Heterotic string theory. 1. The free heterotic string, 
*Nucl. Phys. B* **256** (1985) 253 [SPIRE].

[23] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string*, Cambridge University Press, Cambridge U.K. (1998) [SPIRE]; 
String theory. Vol. 2: Superstring theory and beyond, Cambridge University Press, Cambridge U.K. (1998) [SPIRE].

[24] M. Dine, R. Rohm, N. Seiberg and E. Witten, Gluino condensation in superstring models, 
*Phys. Lett. B* **156** (1985) 55 [SPIRE].

[25] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, On the low-energy $D = 4, N = 1$ supergravity theory extracted from the $D = 10, N = 1$ superstring, 
*Phys. Lett. B* **155** (1985) 65 [SPIRE].

[26] G. Veneziano and S. Yankielowicz, An effective Lagrangian for the pure $N = 1$ supersymmetric Yang-Mills theory, 
*Phys. Lett. B* **113** (1982) 231 [SPIRE].

[27] S. Alexander, R.H. Brandenberger and D.A. Easson, Brane gases in the early universe, 
*Phys. Rev. D* **62** (2000) 103509 [hep-th/0005212] [SPIRE].

[28] S. Weinberg, Cosmological constraints on the scale of supersymmetry breaking, 
*Phys. Rev. Lett.* **48** (1982) 1303 [SPIRE].

[29] J.R. Ellis, N.C. Tsamis and M.B. Voloshin, Could a dilaton solve the cosmological constant problem?, 
*Phys. Lett. B* **194** (1987) 291 [SPIRE].

[30] B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings, 
*Phys. Lett. B* **318** (1993) 447 [hep-ph/9308325] [SPIRE].
[31] J.R. Ellis, D.V. Nanopoulos and M. Quirós, *On the axion, dilaton, Polonyi, gravitino and shadow matter problems in supergravity and superstring models*, *Phys. Lett.* B **174** (1986) 176 [arXiv:hep-ph/0310159 [INSPIRE]].

[32] S. Gukov, S. Kachru, X. Liu and L. McAllister, *Heterotic moduli stabilization with fractional Chern-Simons invariants*, *Phys. Rev.* D **69** (2004) 086008 [hep-th/0310159] [INSPIRE].

[33] R. Donagi, B.A. Ovrut, T. Pantev and D. Waldram, *Standard models from heterotic M-theory*, *Adv. Theor. Math. Phys.* **5** (2002) 93 [hep-th/9912208] [INSPIRE];
J.T. Giedt, *Heterotic orbifolds*, hep-ph/0204315 [INSPIRE];
J.E. Kim, *Z_{12-1} orbifold compactification toward SUSY standard model*, *Int. J. Mod. Phys.* A **22** (2007) 5609 [arXiv:0706.3498] [INSPIRE];
H.P. Nilles, S. Ramos-Sanchez, M. Ratz and P.K.S. Vaudrevange, *From strings to the MSSM*, *Eur. Phys. J.* C **59** (2009) 249 [arXiv:0806.3905] [INSPIRE].