Six-dimensional Myers-Perry rotating black hole cannot be overspun

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Though under non-linear accretion, all black holes in four and higher dimensions obey the weak Cosmic Censorship Conjecture, however they generally violate it for the linear test particle accretion with the exception of five dimensional rotating black hole with a single rotation. In dimension greater than five, there exists no extremal condition for black hole with single rotation and hence it can never be overspun. However it does exist for two rotations of five dimensional rotating black hole and then black hole could indeed be overspun under linear accretion. In this paper we study the case of six dimensional rotating black hole with two rotations and show that unlike the five dimensional black hole it cannot be overspun under linear accretion. We would conjecture that so should also be the case in all dimensions greater than six. Thus CCC may always be obeyed for rotating black hole in all dimensions greater than five.

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I. INTRODUCTION

Since Cosmic Censorship Conjecture [1] has remained unproven yet, testing its validity in various circumstances and consideration of physical processes that may lead to destruction of event horizon laying bare singularity have attracted attention of many works. For the latter case, a gedanken experiment is envisioned in which test particles of suitable parameters implored on black hole so as to overspin or overcharge a black hole, and thereby destroying its horizon and creating a naked singularity. Naked singularity is however one of the most important unanswered questions in general relativity. Its theoretical existence is important because it would mean that it is possible to observe collapse of an object to infinite density. The formation of naked singularity in gravitational collapse has long history beginning with Chrtodoulou (1986) [2], and several others, for instance [3–11]. Black holes have taken the center stage after the LIGO-VIRGO detection of gravitational waves produced by stellar mass black hole mergers [12, 13]. Gravitational wave is new and very potent tool to explore black hole properties which have so far remained unexplored.

The question, could a non-extremal black hole be converted into an extremal one by throwing in test particles of suitable parameters was first addressed by Wald [14] and it was shown that it cannot be done. What happens is that as extremality is approached the parameter window required for particle to hit the horizon pinches off, and so non-extremal black hole can never be converted into an extremal one. Then a new and novel setting was envisioned that though extremality cannot be attained but it may however be jumped over in a discontinuous manner. That is transition to over extremality is not through extremality but through discontinuous jump over extremality. This was initiated by Hubeny [15] in which a charged black hole was shown to be overcharged. Following that Jacobson and Sotirio (JS) [16] had shown that a rotating black hole could be overspun by impinging black hole with test particles of suitable parameters. This opened up a new vista of investigations to study the phenomenon of destroying black hole horizon by over-spinning/charging. In this thought experiment one begins with a near extremal black hole, and then let test particles of suitable parameters fall into the black hole, and thereby over-extremalizing it. Of course this is a linear order process in which back reaction and self force are ignored. There is an extensive body of work considering various situations, here we give some representative references [see, e.g. 15–27].

Recently, the mechanism of destroying a Kerr-Newman-AdS black hole has been considered by neglecting radiative and self-force effects [28]. In all these works, it was assumed that test particle follows a geodesic or Lorentz force trajectory and backreaction effects were neglected. It has been argued that if self force and back reaction effects are taken into account, particle that could over-extremalize black hole may not indeed be able to reach the horizon [29–34]. The question of inclusion of back reaction effects has also been studied for a regular black hole [23] and magnetized Reissner-Nordström black hole as well [35]. Further an extensive analysis involving complex scalar test fields around a black hole has also been considered in testing CCC [see,e.g. 36–40].

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Recently, CCC has also been considered for a rotating anti-de Sitter black hole [41, 42]. Non-extremalization of black hole was also considered [43–45] in the context of black hole dynamics [46]. It is worth noting that the gedanken experiment [14] was also extended to an extremal magnetized black hole case [47]. The weak CCC was considered in the case of BTZ black holes by throwing in test particles and fields [48]. Chakraborty et al. [49] approached the issue by considering spin precession in the vicinity of black hole and naked singularity, showing a clear distinction between the two.

This was all in the linear test particle accretion, however Wald and Sorce [50, 51] have studied the gedanken experiment for non-linear flow accretion, and have shown that black hole cannot be overextremalized and CCC is always obeyed. Following that An et al. [52] have shown how the situation gets miraculously reversed when second order effects are taken into account. That is, black hole could be overextremalized at linear order accretion but not when second order effects are included. Similar conclusions were also obtained for non-linear order effects [53–55].

Very recently an interesting feature of five dimensional rotating black hole has been brought out [56] showing that though it could be overspun under linear accretion when both rotations are present but could not be when there is only one rotation. It is interesting that a five dimensional black hole with single rotation defies the general result that CCC could be violated at the linear accretion. That is, a black hole with one rotation behaves radically differently from the one with two rotations. Then an interesting question arises, what happens in the case of a five dimensional charged rotating black hole – an analogue of Kerr-Newman? The question is interesting because overcharging should be possible in this case while no overspinning. Strange may it sound but it is true that there exists no exact solution of Einstein-Maxwell equation describing a true analogue of Kerr-Newman black hole in five dimension. That is, it is not possible to put a charge on a five dimensional rotating black hole [57, 58]. The closest that comes to it is the minimally gauged supergravity charged rotating black hole [59], and in that case it turns out that for single rotation the ultimate outcome for linear accretion depends on which of the particle parameter, charge or rotation is dominant [60]. If angular momentum is larger than charge, black hole cannot be overextremalized while the opposite is the result if the case is other way round. For non-linear accretion, however it cannot be overextremalized and CCC always holds good.

In dimension $> 5$, a black hole with one rotation has no extremal condition and hence the question of its overspinning doesn’t arise. The natural question then arises is, what happens for a six dimensional black hole with two rotations, could it be overspun or not for linear accretion. This is the question we wish to address in this paper and the answer turns out to be no; i.e. a six dimensional rotating black hole cannot be overspun even for linear accretion process. We would like to conjecture that the same should be the case in all dimensions greater than six as well. Thus CCC may always be obeyed for rotating black holes in all dimensions greater than and equal to six.

In Sec. II we describe briefly the six dimensional rotating black hole which is followed by the main concern of the study — whether overspinning is possible or not. We conclude with a discussion in Sec. III.

II. SIX DIMENSIONAL ROTATING MYERS-PERRY BLACK HOLE SPACETIME

Rotating Myers-Perry black hole metric [58] in six dimension in the Boyer-Lindquist coordinates is given by

$$ds^2 = -\frac{\Delta}{\Sigma} (dt - a \sin^2 \theta d\phi - b \cos^2 \theta d\psi)^2 + \frac{\Sigma}{\Delta} \left( (r^2 + a^2) d\phi - b dt \right)^2$$

$$+ \frac{\Sigma}{\Delta} \left( (r^2 + b^2) d\psi - b dt \right)^2$$

$$+ r^2 \left[ \cos^2 \theta + \sin^2 \phi \right] \left( d\psi^2 + \sin^2 \psi d\eta^2 \right),$$

where

$$\Delta = \frac{r^2}{r^2} - \frac{\mu}{r},$$

$$\Sigma = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta.$$ (2)

Here $\mu = \frac{3M}{\pi}$ is related to the black hole mass, while $a = 3J_\phi/2M$ and $b = 3J_\psi/2M$ are two rotation parameters of the black hole. The event horizon is located at the largest real root of $\Delta = 0$, and it is given by

$$r_\pm = \frac{1}{2} \left( \frac{2^{1/3} (a^2 + b^2) 2\pi}{(A + \sqrt{A^2 - 4B^2})^{1/3}} - \frac{2}{3} \left( \frac{a^2 + b^2}{9 \times 2^{1/3} \pi} \right) \right)^{1/2} + \frac{1}{2} \left( \frac{16M}{3\pi} \left( \frac{2^{1/3} (a^2 + b^2) 2\pi}{(A + \sqrt{A^2 - 4B^2})^{1/3}} \right) \right)^{1/2}$$

$$- \frac{2}{3} \left( \frac{a^2 + b^2}{9 \times 2^{1/3} \pi} \right)^{-1/2} - \frac{2^{1/3} (a^2 + b^2) 2\pi}{(A + \sqrt{A^2 - 4B^2})^{1/3}} - \frac{4}{3} \left( \frac{a^2 + b^2}{9 \times 2^{1/3} \pi} \right)^{1/2}.$$ (4)
where
\[ A = 54\pi^3 (a^2 + b^2)^3 - 1944\pi^3 a^2 b^2 (a^2 + b^2) + 5184\pi M^2, \]
(5)
\[ B = 108\pi^2 a^2 b^2 + 9\pi^2 (a^2 + b^2)^2. \]
(6)
The extremality condition then reads \( A^2 - 4B^3 = 0 \), we write
\[ A^2 - 4B^3 = 12\pi^2 [768M^4 + 16\pi^2 M^2 (a^6 - 33a^4b^2 - 33a^2b^4 + b^6) - 9\pi^4 a^2 b^2 (a^2 - b^2)^4]. \] (7)
Clearly when one of rotations is zero, there occurs no extremal limit because the above expression is always positive. In general for \( d > 5 \), there occurs no extremality limit for black hole with only one rotation parameter. For overspinning the above expression has to be negative.

A. Two equal rotation \( a = b \)

The black hole horizon for \( a = b \) is given by
\[ r_{\pm} = 6^{1/2} A'^{1/2} + \frac{1}{2} \left( 8\sqrt{\frac{7}{3}} a^2 A'^{-1/2} - \frac{8a^2}{3} \right) \]
\[ \quad - \frac{27/3a^4\pi^2/3}{3B'^{1/3}} \left( \frac{32}{27\pi^2} B'^{2/3} \right)^{1/2}, \] (8)
where
\[ A' = \left( \frac{4}{3} B' \right)^{1/3} - 2a^2 + (16\pi^2)^{1/3} a^4 B'^{-1/3}, \] (9)
\[ B' = 3M^2 - 2\pi a^6 + \sqrt{3M\sqrt{3M^2 - 4\pi^2a^6}}, \] (10)
and extremality is indicated by \( 3M^2 = 4\pi^2 a^6 \), which also follows from Eq. (7) when \( a = b \). However, if the following inequality
\[ 3M^2 < 4\pi^2 a^6, \] (11)
is satisfied the final object turns into a naked singularity. We consider the following three scenarios for test particle accretion: (i) it has two equal rotations, \( \delta J_\phi = \delta J_\psi = \delta J \), (ii) has one rotation, \( \delta J \) having projections on both rotation axes, and (iii) has only one rotation associated with \( \delta J_\psi \).

(i) For equal rotation case, from Eq. (11) the minimum threshold value would be defined by
\[ \sqrt[6]{\frac{3}{4\pi^2} (M + \delta E)}^{1/3} < \frac{3}{2} \left( \frac{J + \delta J}{M + \delta E} \right). \] (12)
That is, particle adds equal amount to both rotations of the black hole, \( \delta J_\phi \) and \( \delta J_\psi \). From the above equation, we write the minimum threshold value as
\[ \delta J_{\min} = \left( \frac{2}{3} \sqrt[6]{\frac{3}{4\pi^2} M^{4/3} - J_\psi} \right) + 2 \sqrt[6]{\frac{3}{4\pi^2} M^{1/3} \delta E} \]
\[ + \frac{2}{9} \sqrt[6]{\frac{3}{4\pi^2} M^{-2/3} \delta E^2}. \] (13)
We should here note that black hole is endowed with two equal rotations and test particle also adds equal amount to both rotations of black hole about two axes. We begin with a near extremal black hole, \( a = b = \sqrt[6]{\frac{3}{4\pi^2} M^{1/3} (1 - \epsilon^2)} \) with \( \epsilon \ll 1 \), and then \( \delta J_{\min} \) for either \( \phi \) or \( \psi \) rotation is given by
\[ \delta J_{\min} = \frac{2}{3} \sqrt[6]{\frac{3}{4\pi^2} M^{4/3} \epsilon^2 + \frac{4}{3} M^{1/3} \delta E} \]
\[ + \frac{2}{9} M^{-2/3} \delta E^2, \] (14)
and adding the two together, we write
\[ (\delta J_\phi + \delta J_\psi)_{\min} = \frac{4}{3} \sqrt[6]{\frac{3}{4\pi^2} M^{4/3} \epsilon^2 + \frac{4}{3} M^{1/3} \delta E} \]
\[ + \frac{2}{9} M^{-2/3} \delta E^2. \] (15)
This is the minimum threshold angular momentum for an impinging particle.

(ii) When infalling particle has only one rotation \( \delta J \) having equal projections on both rotation axes of black hole. We will follow Eq. (7) to find the minimum threshold value of the particle angular momentum. In this case, we assume that a test particle’s angular momentum \( \delta J \) is equally shared between the two axes, \( J_\phi = \delta J/2 \) and \( J_\psi = \delta J/2 \). On accretion of particle the black hole parameters change as \( J_\phi + \delta J/2 \), \( J_\psi + \delta J/2 \), and \( M + \delta E \). From Eq. (7) it then follows:
\[ \frac{1024}{243\pi^2} (M + \delta E)^2 < \left( \frac{2J_\psi + \delta J}{M + \delta E} \right)^6, \] (16)
leading to
\[ \delta J_{\min} = \left( \frac{4}{3} \sqrt[6]{\frac{1024}{243\pi^2} M^{4/3} - 2J_\psi} \right) + \frac{4}{3} \sqrt[6]{\frac{1024}{243\pi^2} M^{1/3} \delta E} \]
\[ + \frac{2}{9} \sqrt[6]{\frac{1024}{243\pi^2} M^{-2/3} \delta E^2}. \] (17)
Now writing \( a = b = \sqrt[6]{\frac{3}{4\pi^2} M^{1/3} (1 - \epsilon^2)} \), we obtain \( \delta J_{\min} \) as given by
\[ \delta J_{\min} = \frac{4}{3} \sqrt[6]{\frac{3}{4\pi^2} M^{4/3} \epsilon^2 + \frac{4}{3} M^{1/3} \delta E} \]
\[ + \frac{2}{9} M^{-2/3} \delta E^2. \] (18)
Note that this is the same as Eq. (14) indicating that the minimum threshold for angular momentum is the same in both the cases.

(iii) In the case of particle having a single rotation about one of the black hole axes, the minimum threshold
for angular momentum again turns out to be the same as given in the above equation. That is, in all three accretion scenarios, $\delta J_{\min}$ is given by the same relation.

The maximum threshold value of the angular momentum allowed for particle to reach black hole horizon is defined by

$$\delta E \geq \Omega_{+}^{(\phi)} \delta J_{\phi} + \Omega_{+}^{(\psi)} \delta J_{\psi},$$

where $\Omega_{+}^{(\phi)}$ and $\Omega_{+}^{(\psi)}$ are angular velocities for two $\phi$ and $\psi$ axes, respectively. Thus, we have maximum threshold as

$$\delta J_{\max} = \delta J_{\phi} + \delta J_{\psi} = \frac{r_{\phi}^{2} + a_{\phi}^{2}}{b} \delta E,$$

which in view of Eq. (19), we write

$$\delta J_{\max} = \left(1 + \epsilon + \frac{2}{3} \epsilon^{2}\right) \frac{4}{3} \sqrt{\frac{3}{4\pi^{2}}} M^{1/3} \delta E.$$

Hence, the difference between $\delta J_{\max}$ and $\delta J_{\min}$ for all cases (i-iii) takes following form

$$\Delta J = - \left[\left(1 + \epsilon - \frac{2}{3} \epsilon^{2}\right) M^{1/3} \delta E + M^{1/3} \epsilon^{2} \frac{2}{9} \left(M^{2/3} \delta E^{2}\right) \frac{4 \epsilon}{3} \sqrt{\frac{3}{4\pi^{2}}} \right].$$

which is clearly negative. Since $\Delta J < 0$ always, that indicates absence of parameter space available for overspinning and so horizon cannot be destroyed. Thus, the CCC is always respected for Myers-Perry black hole in six dimension.

**B. A single rotation $b = 0$**

Here the point to be noted is that (2) and (3) reduce to

$$\Delta = r^{2} + a^{2} - \frac{8M}{3\pi r},$$

$$\Sigma = r^{2} + a^{2} \sin^{2} \theta.$$  

It is certain that $\Delta$ has at least one real positive root as

$$r_{+} = \left(12M + \sqrt{48M^{2} + \pi^{2}a^{2}}\right)^{1/3}$$

$$- \left(\frac{1}{3} \right)^{1/3} a^{2} \frac{3^{2/3} \pi^{1/3}}{(\pi^{1/3} \pi^{1/3})},$$

which will always exist.

This clearly shows that there exists no extremality condition and hence the question of its overspinning does not arise. This is however known that in dimensions greater than five a rotating black hole with a single rotation has no extremality condition. Though five dimensional black hole with single rotation has extremality condition yet it cannot be overspun [56]. Thus a black hole with single rotation cannot be overspun in all higher dimensions greater than four, and what we have shown here is that six dimensional black hole with two rotations also cannot be overspun.

**III. CONCLUSIONS**

For linear accretion process, it was possible to overspin black hole in four dimension. A subtler behavior ensues when one goes to five dimension where overspinning is not possible when black hole has only one rotation but it is possible when it has two rotations [56]. A black hole with one rotation defies what is true for four dimensional rotating black hole and behaves radically differently from the one with two rotations.

Another interesting aspect of rotating higher dimensional black hole is that it has no extremal limit defined for a black hole with one rotation in dimension $> 5$, and hence a rotating black hole with one rotation in higher dimension $> 4$ can never be overspun. A higher dimensional black hole with one rotation always obeys CCC.

Next question is, what happens in six dimension for a black hole with two rotations? This is the question we have addressed here and have shown that it could not be overspun under linear accretion process. Thus, six dimensional rotating black hole always obeys CCC.

What happens in the next higher dimension seven where a black hole can have three rotations. We would like to conjecture that the two rotations case here should be similar to one rotation case in five dimension; i.e., it cannot be overspun and three rotations case should be similar to two rotations case in six dimension, and again there should be no overspinning. Thus a rotating black hole always obeys CCC even at linear accretion in all dimensions greater than five.

What has been alluded above as a conjecture, it would be interesting to further analyse subtler behavior of rotating black hole in higher dimensions. That is what we intend to take up next.

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