New Global Fit to the Total Photon-Proton Cross-Section $\sigma_{L+T}$ and to the Structure Function $F_2$

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A fit to world data on the photon-proton cross section $\sigma_{L+T}$ and the unpolarised structure function $F_2$ is presented. The 23-parameter ALLM model based on Reggeon and Pomeron exchange is used. Cross section data were reconstructed to avoid inconsistencies with respect to $R$ of the published $F_2$ data base. Parameter uncertainties and correlations are obtained.

1 Introduction

Deep-inelastic scattering on protons has been studied precisely in the last decades at various energies covering a large kinematic region provided by collider and fixed target experiments, thus providing us with our modern understanding of the proton structure.

The inclusive DIS cross section in the one-photon-exchange approximation is related to the unpolarized structure function $F_2(x, Q^2)$ and the ratio $R(x, Q^2)$ of longitudinal and transverse photo-absorption cross section:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2_{em}}{Q^4} \frac{F_2(x, Q^2)}{x} \left\{ 1 - y \frac{Q^2}{4E^2} + \left( 1 - \frac{2M^2}{Q^2} \right) \frac{y^2 + Q^2/E^2}{2[1+R(x, Q^2)]} \right\}. \quad (1)$$

Here, $Q^2$ is the square of the photon 4-momentum and $x = Q^2/2M\nu$ with the proton mass $M$ and the photon energy $\nu$ in the proton rest frame.

From Eq. (1) it follows that a measurement of the cross section alone is not sufficient to extract both, $F_2$ and $R$, and that only a variation of the beam energy $E$ in the proton rest frame for fixed kinematic conditions can give access to both quantities. Alternatively, $F_2$ can be extracted using parameterizations of world data on $R$: two common examples are $R_{1990}$\textsuperscript{1} and $R_{1998}$\textsuperscript{2}, whose differences reflect the states of world knowledge at the time they were obtained. The sensitivity of the cross section to $R$ increases with $y$ as it can be seen in Eq. (1). The discrepancy in the extracted values of $F_2$ using the two parameterizations can exceed 4% in the regions of maximum $y$.

The structure function $F_2$ is related to the photon-proton cross section $\sigma_{L+T}$ by the expression:

$$\sigma_{L+T} = \frac{4\pi^2\alpha_{em}}{Q^4} \frac{Q^2 + 4M^2x^2}{1-x} F_2. \quad (2)$$

For virtual photons this relation employs the Hand convention for the virtual photon flux. It was used for technical convenience of consistency between real and virtual photon processes.

This paper reports on a new fit of the photon-proton cross section $\sigma_{L+T}$ which reflects the recent world knowledge on the cross section and is self-consistent with respect to the use of $R$, since the cross sections were reconstructed in each case using the value of $R$ that had been used to extract the published values of $F_2$. A result of the fit is a facility to calculate values of $F_2$ based on a single parameterization of $R = R_{1998}$.
2 The fit

The fit includes 2740 data points: 574 from the SLAC experiments E49a, E49b, E61, E87, E89a, E89b [3]; 292 from NMC [4]; 787 from H1 [5]; 570 from ZEUS [6]; 91 from E665 [7]; 229 points from BCDMS [8]. Real photon data comprise 196 points from Ref. [9] and 1 from ZEUS [10].

The ALLM functional form is a 23-parameter model of \( \sigma_{L+T} \) where \( F_2 \) is described by Reggeon and Pomeron exchange, valid for \( W^2 > 4 \text{GeV}^2 \), i.e., above the resonance region, and any \( Q^2 \) including the real \( \gamma \) process. Here, \( W^2 \) is the invariant squared mass of the photon-proton system. For details on the parameterization we refer to the original papers [11, 12]. The new fit was performed by minimizing the \( \chi^2 \) defined in Eq. (3) where \( D_{i,k} \pm \sigma_{i,k}^{\text{stat}} \pm \sigma_{i,k}^{\text{syst}} \) are the values of \( \sigma_{L+T} \) for data point \( i \) within the data set \( k \), \( \delta_k \) is the normalization uncertainty in data set \( k \) quoted by the experiment, \( \nu_k \) is a parameter for the normalization of each data set in units of the normalization uncertainty, \( T(p, W^2, Q^2) \) is the functional form of the 23-parameter ALLM parameterization.

The \( \chi^2 \) takes into account uncorrelated point-by-point statistical and systematic uncertainties and overall normalization uncertainties. The normalization parameters \( \nu_k \) determine the size of the shifts in units of the normalization uncertainties \( \delta_k \).

\[
\chi^2(p, \nu) = \sum_{i,k} \frac{|D_{i,k}(W^2, Q^2) \cdot (1 + \delta_k \nu_k) - T(p, W^2, Q^2)|^2}{(\sigma_{i,k}^{\text{stat}} + \sigma_{i,k}^{\text{syst}}) \cdot (1 + \delta_k \nu_k)^2} + \sum_k \nu_k^2
\]

\[
\approx \sum_{i,k} \frac{|D_{i,k}(W^2, Q^2) - T(p, W^2, Q^2) \cdot (1 - \delta_k \nu_k)|^2}{\sigma_{i,k}^{\text{stat}} + \sigma_{i,k}^{\text{syst}}} + \sum_k \nu_k^2, \tag{3}
\]

In order to keep the number of free parameters as small as possible, the normalization

| Parameter | ALLM97 | this fit | uncertainty |
|-----------|--------|---------|-------------|
| \( m_0^2(\text{GeV}^2) \) | 0.31985 | 0.454 | 0.137 |
| \( m_0^2(\text{GeV}^2) \) | 49.457 | 30.7 | 13.4 |
| \( m_R^2(\text{GeV}^2) \) | 0.15052 | 0.118 | 0.224 |
| \( Q_0^2(\text{GeV}^2) \) | 0.52544 | 1.13 | 1.47 |
| \( \Lambda_0^2(\text{GeV}^2) \) | 0.06527 | 0.06527 | - |
| \( a_{p1} \) | -0.0808 | -0.105 | 0.024 |
| \( a_{p2} \) | 0.44812 | -0.496 | 0.154 |
| \( a_{p3} \) | 1.1709 | 1.31 | 1.04 |
| \( b_{p4} \) | 0.36292 | -1.43 | 2.31 |
| \( b_{p5} \) | 1.8917 | 4.50 | 2.46 |
| \( b_{p6} \) | 1.8439 | 0.554 | 0.531 |
| \( c_{p7} \) | 0.28067 | 0.339 | 0.093 |
| \( c_{p8} \) | 0.22291 | 0.128 | 0.104 |
| \( c_{p9} \) | 2.1979 | 1.17 | 1.14 |
| \( a_{R1} \) | 0.584 | 0.373 | 0.150 |
| \( a_{R2} \) | 0.37888 | 0.994 | 0.443 |
| \( a_{R3} \) | 2.6063 | 0.781 | 0.524 |
| \( b_{R4} \) | 0.01147 | 2.70 | 1.84 |
| \( b_{R5} \) | 3.7582 | 1.83 | 2.39 |
| \( b_{R6} \) | 0.49338 | 1.26 | 1.33 |
| \( c_{R7} \) | 0.80107 | 0.837 | 0.500 |
| \( c_{R8} \) | 0.97307 | 2.34 | 2.34 |
| \( c_{R9} \) | 3.4942 | 1.79 | 0.93 |

Table 1: Parameters of the functional form used in the ALLM parameterization [11]. Results of the ALLM97 fit [12] without uncertainties in comparison to the results discussed in this paper with uncertainties. These uncertainties correspond only to the diagonal elements of the full covariance matrix which must be used to calculate uncertainties in \( F_2 \) or cross sections. The parameter \( \Lambda_0^2 \) has no uncertainty as it was fixed in the fit.
parameters are determined analytically in each minimization step using the relation

\[ \nu_k = \frac{\sum_i \delta_k T_{i,k} (T_{i,k} - D_{i,k}) / \sigma_{i,k}^2}{\sum_i T_{i,k}^2 \delta_k^2 / \sigma_{i,k}^2 + 1}, \]  

(4)

obtained by requiring \( \partial \chi^2 / \partial \nu_k = 0 \) in the context of the approximation for \( \chi^2 \) in the second line of Eq. (3). Here \( \sigma_{i,k}^2 = \sigma_{i,k}^{\text{stat}}^2 + \sigma_{i,k}^{\text{syst}}^2 \). This separate extraction is possible since the normalization parameters are not correlated and depend only on the involved data points and the functional parameters. The resulting fit has a reduced \( \chi^2 \) equal to 0.94; the contributions from each data set, together with the normalization parameters can be found in Ref. [13]. Table 1 shows the final parameters from this fit with the corresponding uncertainties and, for comparison, the parameters from the ALLM97 fit. Figure 3 shows the new fit in comparison with world data and with the ALLM97 fit. A full comparison between the two fits is not possible as in the ALLM97 fit parameter uncertainties were not provided. Presumably, these uncertainties are larger than those of the new fit, since the size of the current data set is nearly twice as large. The uncertainties in the cross sections calculated from the fit as represented by the error bands in the figure are much smaller than individual error bars on the original data points because of the smoothness constraint inherent in the fitted model. The fit evaluated at any kinematic point is effectively an average of a number of data points.

In conclusion, a new fit of world data on \( \sigma_{L+T} \) and \( F_2 \) is presented. Such a fit is consistent in the choice of the \( R \) parameterization \( R_{1998} \). Also, for the first time, parameter and fit uncertainties are calculated. A subroutine that allows the calculation of \( \sigma_{L+T} \) and \( F_2 \) with their fit uncertainties is available upon request from the authors.

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