Slow light with fractional optical angular momentum in a photonic crystal ring

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Whispering gallery modes (WGMs) in circularly symmetric optical microresonators exhibit integer quantized angular momentum numbers due to the boundary condition imposed by the geometry. Here, we show that incorporating a photonic crystal pattern in an integrated microring can result in WGMs with fractional optical angular momentum. By choosing the photonic crystal periodicity to open a photonic bandgap with a band-edge momentum lying between that of two WGMs of the unperturbed ring, we observe hybridized WGMs with half-integer quantized angular momentum numbers ($m \in \mathbb{Z} + 1/2$). Moreover, we show that these modes with fractional angular momenta exhibit high optical quality factors with good cavity-waveguide coupling, an order of magnitude reduced group velocity, and also can co-exist with more localized states when a defect is introduced in the photonic crystal lattice. Our work unveils WGMs with fractional angular momentum, which can be used in sensing/metrology, nonlinear optics, and cavity quantum electrodynamics.

Spin angular momentum and orbital angular momentum of light, defining the electromagnetic wave oscillation direction (i.e., polarization) and the spatial distribution of field phase, are two quantization numbers describing light’s total angular momentum. In analogy to fractional spin particles in two-dimensional systems,[10] recent works show that light beams can carry fractional orbital angular momentum.[13,14] A fractional $2\pi$-phase shift along the azimuth of light’s propagation direction is the signature feature of these works. This unusual quantization can be realized by symmetry breaking via various different mechanisms, such as spiral phase modulation,[15] biaxial crystals,[16] astigmatic elements,[17] differential operators acting on a Gaussian beam,[18] and superposition of light modes of different integer angular quantization.[19] However, all these experiments are done on optical beams in free-space.

Optical microresonators supporting whispering gallery modes (WGMs) have long been studied due to their ability to simultaneously support long cavity photon lifetime (high-$Q$) and a strong degree of spatial mode localization (small mode volume $V$). WGMs have rotational symmetry in a circular round trip and, within a given wavelength band, have almost constant free spectral range (FSR). The circular geometry dictates the behavior of the electromagnetic field in the azimuthal ($\phi$) direction, as matching the field phase across one round trip (i.e., a $\phi=2\pi$ rotation) results in integer azimuthal mode numbers. By disturbing the geometry from the circular shape, geometric ray optics predicts more generalized whispering gallery modes with fractional angular momentum numbers of $m Q \in \mathbb{Z}$,[11,13] that is, the optical field exhibits $m \times 2\pi$ phase oscillations across $n$ round trips. Recently, such unconventional WGMs have been demonstrated in a quadrupolar microdisk whose $Q$ was largely deteriorated due to the deformed geometry.[19] Apart from the degraded $Q$, they are mostly observed in microspheres/microdisks instead of microrings, as such modes are not always propagating along the device perimeter. As a result, they inherently exhibit limited coupling to other degrees of freedom, including conventional evanescent wave excitation through a bus waveguide. These factors hinder their application in proven successful application areas of WGMs, such as optomechanics,[20] frequency combs,[21] and lasing.[22] Optical modes with fractional orbital angular momentum (fractional-$m$) and properties like conventional WGMs (e.g., high-$Q$ with the same radial profile) have not been demonstrated yet.

In this work, we report fractional angular momentum WGMs in a ‘microgear’ photonic crystal ring (MPhCR).[23] By creating a band gap much larger than the FSR, we successfully hybridize two sets of WGMs with different integer angular momentum. This mode superposition leads to WGMs of fractional angular momentum, while preserving the high $Q$, good coupling, and intuitive design common to conventional WGM resonators. Beyond that, we demonstrate that the fractional-$m$ WGMs exhibit reduced group velocity (slow light) and defect localization. Our work provides an exciting platform to study the physics of, and find uses for, fractional angular momentum modes within the broad application space in which conventional WGM resonators are being used.

To generate WGMs with fractional angular momentum, the methodology we follow is to hybridize WGMs of odd and even angular momentum. Once there is coupling/interaction between such WGMs, new hybrid modes are formed. For example, the clockwise (CW) and counter-clockwise (CCW) traveling WGMs can be hybridized into a set of standing WGMs through backscattering, which can be realized through fabrication imperfections (in a non-selective way) or intentionally through a modified resonator geometry.

Artificial scatterers with designated size and position can efficiently tune intermodal coupling and loss.[24,25] However, the introduced scattering loss can deteriorate the resonator $Q$.[26] Instead, a well-designed photonic crystal where periodic scatterers are patterned to generate a frequency bandgap at a designated wavevector can realize coherent scattering between coupled modes and avoid coupling to radiation loss channels. Recently, a ‘microgear’ photonic crystal micror-
FIG. 1: Fractional angular momentum whispering gallery modes enabled by mode superposition near a bandgap. a, The combination of rotational symmetry ($\mathcal{R}$) and period translation symmetry ($\mathcal{T}$) in a photonic crystal ring hybridizes the modes $|m_1 + m_2| = N$, forming WGMs with fractional angular momentum when $N$ is odd. For visualization, we choose a small number of unit cells $N = 11$. The first order mode is formed by $|m_1| = 5$ and $|m_2| = 6$, i.e. the circles on the unperturbed dispersion line. Counterclockwise (CCW) modes ($m < 0$) are mirrored ($\mathcal{M}$) about the $y$-axis of $m = 0$. b, Band diagram of a photonic crystal ring with an odd number of unit cells. The air band is pushed towards the light cone (grey) while the dielectric band is well preserved. In a real experiment, the degenerate dielectric (air) modes split due to geometric imperfections, forming eigenmodes with higher and lower frequencies. We label them with $s(a)m^+$ and $s(a)m^-$, respectively. c–d, Qualitative illustrations of the four WGMs before (c) and after (d) hybridization. They correspond to the four purple circles framed by the dashed lines in (a) and (b), respectively. e, An example of the modes with Möbius-type profile for both angular momentum phase (upper panel) and envelope modulation, respectively, of the mixed WGMs. Notably, the complete basis of four mixed modes can be written as $E_{m_1} (r,z) \left( e^{i|m_1|\phi} + e^{-i|m_1|\phi} \right) + E_{m_2} (r,z) \left( e^{i|m_2|\phi} + e^{-i|m_2|\phi} \right)$.

For a microring with continuous rotational symmetry, the dominant traveling electric field of WGMs can be written as $E(r,z,\phi) = E_{m}(r,z) \ e^{i\phi}$, where $r$, $z$, $\phi$ are the radial, vertical, and azimuthal angular coordinate. $m$ represents the azimuthal angular momentum number of the mode, and as previously mentioned, matching the field phase across one round trip necessitates $m$ being an integer, with positive and negative integers corresponding to the frequency degenerate CW and CCW WGMs. Figure 1(a) shows a schematic of the dispersion relation (i.e., frequency-wavevector map) in a conventional microring. Over a certain range of frequencies (and if material dispersion is not significant), the WGMs have nearly equal FSR and symmetric CCW/CW modes due to the rotational symmetry ($\mathcal{R}$) and mirror symmetry ($\mathcal{M}$) of the microring.

When $N$ azimuthally periodic modulations are introduced on the microring, it possesses translation symmetry ($\mathcal{T}$) (in the azimuthal direction) with a period of $2\pi/N$ and becomes a PhCR. The translation symmetry of the PhCR hybridizes four modes that exist near the bandgap, with angular momentum $|m_1 + m_2| = N$ generating dielectric and air bands, as shown in Fig. 1(b). To facilitate understanding, we consider that the four constituent traveling wave modes $\{|m_1|, -|m_1|, |m_2|, -|m_2|\}$ prior to mixing via the PhC are in-phase:

$$E_{m_1} (r,z) \left( e^{i|m_1|\phi} + e^{-i|m_1|\phi} \right) + E_{m_2} (r,z) \left( e^{i|m_2|\phi} + e^{-i|m_2|\phi} \right) \tag{1}$$

where $E_{m_1} (r,z)$ and $E_{m_2} (r,z)$ are considered to be equal to $E_{m_0} (r,z)$ for $m \gg 1$. $m = (|m_1| + |m_2|)/2$ and $\tilde{m} = (|m_1| - |m_2|)/2$ are the numbers describing the angular momentum and envelope modulation, respectively, of the mixed WGMs. Notably, the complete basis of four mixed modes can be written as $E_{m_0} (r,z) \cos(\tilde{m}\phi) \cos(\tilde{m}\phi)$ when the relative phase of the traveling waves are taken into consideration. The first $\pm\pi/4$ corresponds to the dielectric and air bands, and second one represents the two orthogonal envelope distributions. $\phi_0$ describes the phase offset given by the geometry of the photonic crystal modulation, while $\phi_1$ arises from fabrication imperfection induced rotational symmetry breaking. When $N$ is odd, the hybridization of four modes of $|m_1 - m_2| = 1, 3, 5, \ldots$ gives mixed modes with fractional $\tilde{m} = N/2$.
with the fractional angular momentum, phase matching cannot be fulfilled in one round trip of light around the ring. Instead, the mixed WGMs with fractional angular momentum $\hat{m} = N/2$ should undergo two round trips before the phase is matched. As a result, its phase presents a Möbius-type distribution along the propagation path of light, which for simplicity we take as a black circle with radius close to that of the microring’s average radius. Figure 2(e) shows the angular momentum phase and envelope phase topologies of a hybridized mode in the upper and lower panels. For the dots on the edges of the Möbius strip in each panel, their azimuthal angle about the propagation path shows the local phase (i.e., $\phi$) following an additional $\pi$ phase delay. The uncertainty in $\phi$ comes from the fit, and represents a one standard deviation value.

The frequencies (blue) and the slow down ratios (red) of each fractional-$m$ mode near the dielectric band-edge. The blue and red curves are theoretically calculated frequencies and SR ratios, respectively, using a unit period of 452 nm using MIT Photonic Bands method. The uncertainty in SR comes from estimating the free spectral ranges of modes that are frequency split, using the same approach as outlined in Ref. 22. The uncertainty in the eigenfrequency is from the nonlinear fit. Infrared images of the fractional-$m$ modes from a top view of the MPhCR. The dashed lines mark the antinodes (local maxima) of the light intensity distribution.

FIG. 2: Fractional angular momentum in a ‘microgear’ photonic crystal ring (MPhCR). a,b, Scanning electron microscope image of the MPhCR device and zoom-in image of the ‘microgear’ structure. The number of unit cells along the ring perimeter is $N=333$, resulting in a fractional angular momentum value $\hat{m}=166.5$ for the modes discussed below. c, Finite-element method simulated mode profile in one unit cell for the $s0.5^+$ mode at 194.25 THz (top and cross-section views). d, Linear transmission spectrum of the MPhCR, showing a few pairs of fractional-$m$ slow light modes at the dielectric band-edge (labeled $\{s0.5^\pm, s1.5^\pm, s2.5^\pm, s3.5^\pm, s4.5^\pm\}$). The airband modes at the other side of the bandgap (BG) are pushed into the light cone and not covered in the spectrum. These modes results from the hybridization of $m = \{\pm 166, \pm 167, \pm 165, \pm 166, \pm 164, \pm 169, \pm 163, \pm 170, \pm 162, \pm 171\}$. e, Zoom-in of the $s0.5^+$ mode (blue), along with a nonlinear least squares fit to resonance (red). The uncertainty in $Q$, comes from the fit, and represents a one standard deviation value. f, The frequencies (blue) and the slow down ratios (red) of each fractional-$m$ mode near the dielectric band-edge. The blue and red curves are theoretically calculated frequencies and SR ratios, respectively, using a unit period of 452 nm using MIT Photonic Bands method. The uncertainty in SR comes from estimating the free spectral ranges of modes that are frequency split, using the same approach as outlined in Ref. 22. The uncertainty in the eigenfrequency is from the nonlinear fit. g, Infrared images of the fractional-$m$ modes from a top view of the MPhCR. The dashed lines mark the antinodes (local maxima) of the light intensity distribution.

and $\hat{m} = 0.5, 1.5, 2.5, \ldots$, respectively (i.e., the corresponding modes with the same color in Fig. 1(a) and 1(b)). For example, the four WGMs with $|m_1|=5$ and $|m_2|=6$ shown in Fig. 1(c) hybridize in a PhCR of $N=11$, generating four mixed WGMs of $\hat{m}=5.5$ and $\hat{m}=0.5$ shown in Fig. 1(d). Upper (lower) modes correspond to the air (dielectric) band, and left/right are the two orientations of the envelope.

With the fractional angular momentum, phase matching cannot be fulfilled in one round trip of light around the ring. Instead, the mixed WGMs with fractional angular momentum $\hat{m} = N/2$ should undergo two round trips before the phase is matched. As a result, its phase presents a Möbius-type distribution along the propagation path of light, which for simplicity we take as a black circle with radius close to that of the microring’s average radius. Figure 2(e) shows the angular momentum phase and envelope phase topologies of a hybridized mode in the upper and lower panels. For the dots on the edges of the Möbius strip in each panel, their azimuthal angle about the propagation path shows the local phase (i.e., $\phi$ and $\tilde{\phi}$). After the first round trip (red dots), it has $\pi$ phase delay. The second round trip (blue dots) fulfills the phase matching with an additional $\pi$ phase delay, so that the requisite $2\pi$ phase
FIG. 3: Fractional-$m$ optical angular momentum in defect MPhCRs. a, Transmission spectrum of the fractional-$m$ defect MPhCR, showing the defect mode (labeled $g$) that is localized from the bandedge slow light mode ($s0.5^+$), and the remaining slow light modes (all of which exhibit half-integer angular momentum). b, Zoom-in of the $g$ mode (blue) and a nonlinear least squares fit to its transmission resonance (red). c, The frequency (blue) and SR (red) of each fractional-$m$ mode near the dielectric band-edge. The uncertainty in $SR$ comes from estimating free spectral ranges with the split modes. The uncertainty in the eigenfrequency is from the nonlinear fit. d, Infrared images of the defect mode and the slow light modes of the MPhCR. The dashed lines mark the antinodes (local maxima) of the light intensity distribution. The stray light off the rings is arising from reflection at the waveguide-ring junctions.

delay is reached. Their projection on the $x - y$ plane shows the local amplitude $\propto \cos(\tilde{m}\phi)$ and $\propto \cos(\tilde{m}\phi)$, respectively. The mode shape of mixed WGMs with $\{\tilde{m}, \tilde{n}\} = \{5.5, 0.5\}$ is shown in the middle panel where $N = 11$. The odd antinode number of the envelope phase and angular momentum phase is the signature of fractional angular momentum.

In a real experiment, $\tilde{m}$ is a number much larger than that in the toy schematic shown above. For example, we use a photonic crystal ring to demonstrate fractional-$m$ with $\tilde{m} = 166.5$. Figure 2(a) shows the scanning electron microscope (SEM) images of the nanofabricated MPhCRs made from a 500-nm-thick stoichiometric silicon nitride layer. The ring has a radius of $\approx 25 \mu m$ and is spatially modulated with a nominal average width $\approx 1500 \text{ nm}$ and modulation amplitude $\approx 1400 \text{ nm}$, as shown in Fig. 2(b). Such a device supports fundamental transverse-electric-like (TE) modes, whose dominant electric field is in the radial direction and can be theoretically modeled by Eq. (1). We carry out finite-element method simulations of these devices over a unit cell. Top and cross-sectional views of the generated field profile are shown in Fig. 2(c), corresponding to $E_{\tilde{m}}(r, z_0)\cos(\tilde{m}\phi)$ and $E_{\tilde{m}}(r, z)$, respectively, in Eq. (1), where $z = z_0$ corresponds to a plane located midway through the thickness of the ring. The dielectric mode is well confined in the silicon nitride core, preserving conventional WGM profiles. We couple light to the device via on-chip waveguides (Fig. 2(a)), and its normalized transmission spectrum is shown in Fig. 2(d). The MPhCR with odd $N = 2 \times 166.5 = 333$ opens a bandgap between the $m = 166$ and $m = 167$ WGMs, hybridizing them and their neighbors, and generating the mixed modes $\{\tilde{m}^+, \tilde{m}^-, \tilde{m}0^+, \tilde{m}0^-\}$ where $\tilde{m} = 0.5, 1.5, 2.5$.... The fabrication imperfections break the degeneracy between the modes, resulting in mode splitting. We label the mode with higher (lower) frequency with + (-) sign. Only dielectric modes $\tilde{m}^\pm$ are observed since the airband is pushed into the light cone, as illustrated in Fig. 1(b).

The fractional-$m$ WGMs preserve all the main features of conventional WGMs, including high-$Q$, ease in design and fabrication, and strong resonator-waveguide coupling. As shown in Fig. 2(e), the total optical $Q$ ($Q_t$) of the mode $s0.5^+$ is measured to be $Q_t = (5.4 \pm 0.4) \times 10^3$, comparable to conventional WGMs in microrings. The large bandgap pushes away the dielectric modes near the photonic band-edge, similar to waveguide-type photonic crystal. The mode spectrum is compressed near the band-edge (with smaller FSRs), distinct from that of an unmodulated ring (with nearly constant FSRs). Particularly, the reduced FSR indicates a decrease in group velocity. The factor by which the group velocity is reduced, called the slowdown ratio (SR), is used as a
figure of merit following previous work. The FSR between s0.5 and s1.5 is measured to be (122 ± 33) GHz, corresponding to $SR = 8.0 ± 2.3$ where the uncertainty is arising from the mode splitting of s0.5 and s1.5. We collect the frequencies of the first five pairs of modes in Fig. 2(d), and calculate their SRs, as shown in Fig. 2(f). The simulated frequencies ([blue curve] and SR ratios [red curve] with MIT Photonic Bands method show good agreement with experimental data. Additionally, in the dielectric band, the mode splitting of degenerate modes near the band-edge is much larger than for other modes (Fig 2(d)), indicating stronger coupling near the band-edge. It can be attributed to the slow light enhanced coupling, i.e. higher group index leads to a larger backscattering ratio, although its interplay with fabrication imperfection requires further investigation.

Infrared images of scattered light from the hybridized dielectric band-edge modes are illustrated in Fig. 2(g). The dashed lines in the images mark the antinodes (local maxima) of the intensity distribution. The images in Fig. 2(g) clearly display the number of envelope antinodes to be odd for each displayed mode, and equal to $2\pi n$. Remarkably, different from integer-m WGMs, the fundamental dielectric mode s0.5+ and s0.5- are localized orthogonally at opposite sides of the ring. This mode localization can be beneficial for increasing coupling to other degrees of freedom, such as mechanical motion or introduced quantum emitters. However, the orientation ($\phi_1$) of these two modes is currently dictated by random fabrication imperfections, limiting a priori knowledge of where localization along the ring will occur.

We next introduce defect localization to the half-m MPPhCR. We construct a defect region by varying the PhC modulation amplitude quadratically across 24 cells, with a maximum modulation depth deviation of 10 %, as shown in the inset of Figure 3(a). Figure 3(a) shows the normalized transmission spectrum of the defect MPPhCR, where most of modes stay unperturbed relative to non-defect MPPhCR, and retain comparable $Q$ (e.g. s1.5+ has $Q_r = (2.1 ± 0.3) \times 10^5$). Only one of the fundamental dielectric modes is pulled into the bandgap, becoming a defect mode (g), with $Q_\text{g} = (4.0 ± 0.2) \times 10^5$, as shown in Fig. 3(b). Compared to Fig. 2(e), the relatively low total quality factor $Q_r$ is attributed to the much stronger coupling loss to the waveguide adjacent to the defect. Given that this is a standing wave resonance in which strong over-coupling corresponds to a transmission level near zero, it is difficult to accurately extract the intrinsic quality factor from this data. The mechanism of mode selection in localization is still under investigation, though we note that in this case, it was the mode closest to the band-edge (s0.5+) from which g is localized. Figure 3(c) displays the dielectric band-edge mode frequencies ([blue curve] and $SR$ (red) for the modes shown in Fig. 3(a). The SR of the modes near band-edge is measured to be as large as 9.1±3.0, indicating that the slow light property of fractional-m WGMs does not degrade by the introduced defects. Infrared images of the defect mode and fractional-m WGMs are shown in Fig. 3(d). Notably, the g mode of fractional-m WGMs is highly localized at the position of the defect at the bottom adjacent to the waveguide.

There are many interesting questions that rise from this work on half-integer angular momentum modes in the WGM platform. An immediate question is whether this work can be extended to arbitrary fractional angular momentum ($m/n$, with both m and n as integers) as has been demonstrated in deformed microcavities that subscribe to a ray-optics description. One possible path towards this end is to use a photonic crystal ring with multiple coherent mode couplings simultaneously. Another interesting topic is the precise nature of both half-m localization (i.e., how to control the position of the envelope antinodes) and multiple defects localization in a photonic crystal ring (i.e., confirming that it is the closest slow light modes to the band-edge that become localized).

In summary, we have presented optical modes with fractional (half-integer) angular momentum in a ‘microgear’ photonic crystal ring resonator. The photonic crystal patterning enables controlled hybridization of whispering gallery modes into half-integer angular momentum states that retain high-Q while exhibiting reduced group velocity. Furthermore, these states can co-exist with highly localized defect states and conventional whispering gallery modes. Apart from further studies of the physics and engineering of mode hybridization in this platform, we anticipate that our platform may broaden the usage of fractional-m light in applications including nonlinear photonics, quantum photonics and cavity optomechanics.

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