Supersymmetric $\Delta A_{CP}$

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There is experimental evidence for a direct CP asymmetry in singly Cabibbo suppressed $D$ decays, $\Delta A_{CP} \sim 0.006$. Naive expectations are that the Standard Model contribution to $\Delta A_{CP}$ is an order of magnitude smaller. We explore the possibility that a major part of the asymmetry comes from supersymmetric contributions. The leading candidates are models where the flavor structure of the trilinear scalar couplings is related to the structure of the Yukawa couplings via approximate flavor symmetries, particularly $U(1)$, $[U(1)]^2$ and $U(2)$. The recent hints for a lightest neutral Higgs boson with mass around 125 GeV support the requisite order one trilinear terms. The typical value of the supersymmetric contribution to the asymmetry is $\Delta A_{CP}^{SUSY} \sim 0.001$, but it could be accidentally enhanced by order one coefficients.

INTRODUCTION

The world average for the direct CP asymmetry in singly Cabibbo suppressed $D$ decays, based on measurements by E687 [1], CLEO [2, 3], E791 [4], FOCUS [5], BaBar [6], Belle [7], CDF [8, 9] and LHCb [10], is now $4.3\sigma$ away from zero [11]:

$$\Delta A_{CP} \equiv A_{CP}(K^+ K^-) - A_{CP}(\pi^+ \pi^-) = -0.00656 \pm 0.00154.$$  

Here,

$$A_{CP}(f) = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}.$$  

In $\Delta A_{CP}$, that is the difference between asymmetries, effects of indirect CP violation largely cancel out [12]. Thus, $\Delta A_{CP}$ is a manifestation of CP violation in decay.

The Standard Model (SM) contribution to the individual asymmetries is suppressed by a CKM factor of order $10^{-3}$, and by a loop factor of order $\alpha_s(m_c)/\pi \sim 0.1$. While one cannot exclude an enhancement factor of order 30 from hadronic physics [13–21], it is interesting to explore the possibility that new physics contributes a major part of $\Delta A_{CP}$.

The size of new physics contributions to $\Delta A_{CP}$ is often constrained by other flavor-related observables, such as $D^0 - \bar{D}^0$ mixing or $\epsilon'/\epsilon$ [22]. Supersymmetric models, via their contribution to the chromomagnetic operator, can generate large enough asymmetry in $D$ decays without conflicting with these observables [12, 22–24]. In this work, we investigate whether this scenario is likely to be realized in supersymmetric models with viable and natural flavor structure.

THE SUPERSYMMETRIC PARAMETERS

The $6 \times 6$ mass-squared matrix for the up- and down-type squarks can be decomposed into $3 \times 3$ blocks, $q = u, d$,

$$\hat{M}^{2q} = \begin{pmatrix} \hat{M}^{2q}_{LL} & \hat{M}^{2q}_{LR} \\ \hat{M}^{2q}_{RL} & \hat{M}^{2q}_{RR} \end{pmatrix} + D, F,$$  

where $L$ and $R$ denote SU(2) doublets and singlets, respectively. We denote the average squark mass by $\tilde{m}$. Then, it is convenient to parameterize the supersymmetric contributions to flavor changing processes in terms of dimensionless parameters,

$$(\delta_{MN}^q)_{ij} = \frac{(\hat{M}^{2q}_{MN})_{ij}}{\tilde{m}^2}.$$  

where $M, N = L, R$. When, to a good approximation, only two squark generations are involved, one can express these parameters in terms of the supersymmetric mixing angles, $(K_M^q)_{ij}$, and the mass-squared splittings between squarks, $\Delta \tilde{m}^2_{ij}$:

$$(\delta_{MN}^q)_{ij} = \frac{\Delta \tilde{m}^2_{MN}}{\tilde{m}^2} (K_M^q)_{ij} (K_M^q)_{ij}.$$  

The parameters that are most relevant to $\Delta A_{CP}$ are $\delta_{LL} \equiv (\delta_{LL}^q)_{12}$ and $\delta_{LR} \equiv (\delta_{LR}^q)_{12}$, which generate the chromomagnetic operator with Wilson coefficient given by

$$C_{8q} = F(x) \delta_{LL} + G(x) \frac{\tilde{m}}{m_c} \delta_{LR},$$  

where $x = (m_q^2/\tilde{m}^2)$, and the functions $F$ and $G$ can be found, for example, in Ref. [12]. Given that $G(x)$ is larger than $F(x)$ by a factor of a few, and the enhancement factor of $m_q/m_c$, the dominant contribution in the models
that we consider comes from $\delta_{LR}$. It can be estimated as follows \[23\]:

$$\Delta A_{CP}^{SUSY} \sim 0.006 \frac{\mathcal{I}m(\delta_{LR})}{0.001} \frac{1 \text{ TeV}}{\tilde{m}}.$$ \hspace{1cm} (7)

In the following sections, we investigate whether

$$\mathcal{I}m(\delta_{LR}) \sim 0.001$$ \hspace{1cm} (8)

can plausibly arise in supersymmetric flavor models.

**SUPERSYMMETRIC FLAVOR MODELS**

If the soft supersymmetry breaking terms had a generic flavor structure (“anarchy”), the supersymmetric contributions to flavor changing neutral current processes would exceed experimental constraints by orders of magnitude. Thus, these terms must have a special structure.

The most extreme solution to this “supersymmetric flavor puzzle” is a constrained version of minimal flavor violation (MFV): at the supersymmetry breaking mediation scale, squark masses are universal and the trilinear flavor asymmetry scale values of the $A$-terms of the Standard Model, but is related to that of the $\delta_{LR}$.

Given that the $U(1)$ models are only viable if $m_{1/2} \gtrsim 7m_0$, the optimal enhancement occurs for $a_0 > m_{1/2} \gg m_0$ which might, however, lead to negative squark masses-squared.

The single $U(1)$ models lead to the following simple parametric relation between the up and down sectors:

$$(\delta_{LR})_{12} \sim \frac{m_c}{m_s}.$$ \hspace{1cm} (15)

The $U(1)_{LR}$ parameter is constrained, however, by $\epsilon'/\epsilon$: $(\delta_{LR})_{12} \lesssim 4 \times 10^{-5}$ (m/TeV) (see \[30\], \[31\] and references therein). We thus obtain an upper bound that is independent of the flavor-diagonal scales,

$$\Delta \tilde{m}_{12} \lesssim 5 \times 10^{-4} \frac{\tilde{m}}{\text{TeV}}.$$ \hspace{1cm} (16)

Moreover, the approximate symmetry relates $(\delta_{LR})_{12}$ to flavor diagonal parameters,

$$\frac{\mathcal{I}m(\delta_{LR})_{12}}{\mathcal{I}m(\delta_{LR})_{11}} \sim \frac{m_c |V_{us}|}{m_q}, \quad (q = u, d).$$ \hspace{1cm} (17)
Assuming phases of order one (which we must do to explain $\Delta A_{CP}$), these flavor diagonal parameters are bounded by electric dipole moment (EDM) constraints (see Ref. [30] and references therein), $\delta_{LR}^{u,11} \lesssim 3 \times 10^{-6}$ (m/TeV) and $(\delta_{LR}^{d,11}) \lesssim 2 \times 10^{-6}$ (m/TeV). The resulting bounds are

\[(\delta_{LR}^{u,12}) \lesssim 3 \times 10^{-4} \frac{\bar{m}}{\text{TeV}} \quad \text{from (}\delta_{LR}^{u,11}\text{),} \quad (18)\]
\[(\delta_{LR}^{d,12}) \lesssim 8 \times 10^{-5} \frac{\bar{m}}{\text{TeV}} \quad \text{from (}\delta_{LR}^{d,11}\text{).} \quad (19)\]

We conclude that FN models with a single $U(1)$ are unlikely to account for $\Delta A_{CP} \gg 0.001$. Of course, since the FN mechanism only dictates the parametric suppression, it is impossible to exclude an accidental enhancement of $(\delta_{LR}^{u,12})$ by the order-one coefficient.

Models with an FN symmetry $[U(1)]^2$ allow one to take advantage of the holomorphy of the superpotential to obtain vanishing entries in the Yukawa and $A$ matrices and to strongly suppress entries in the squark mass-squared matrices (compared to the single $U(1)$ case). This feature was first employed in Refs. [27, 28] to align in a very precise way the squark and quark mass matrices in the down sector.

The flavor structure of the Yukawa and $A$ terms in these models can be written as follows:

\[Y^d \sim \frac{M_{2d}^{LR}}{\bar{a}v_d} \sim \begin{pmatrix} y_d & 0 & 0 & |V_{ub}| \\ 0 & y_u & y_b & |V_{tb}| \\ 0 & 0 & y_b \end{pmatrix}, \quad (20)\]
\[Y^u \sim \frac{M_{2u}^{LR}}{\bar{a}v_u} \sim \begin{pmatrix} y_u & y_v |V_{us}| & y_t |V_{ub}| \\ Y_{21}^{u} & y_c & y_t |V_{tb}| \\ Y_{31}^{u} & y_c |V_{cb}| & y_t \end{pmatrix}. \quad (21)\]

The four holomorphic zeros in the down sector are essential to obtain an effective alignment [29]. The (21) and (31) entries in the up sector either (i) get their naive parametric suppression (of order $y_u/|V_{us}|$ and $y_a/|V_{ab}|$, respectively) or (ii) vanish.

In both cases, the contribution to $\epsilon_K$ from $(\delta_{LR}^{u,12})$ is too small, unless RGE generates degeneracy, $r_3 \gtrsim 18$ (TeV/\bar{m}). (For moderate suppression of phases, $D^{0} - \bar{D}^{0}$ mixing provides the strongest constraint, requiring a milder degeneracy [32].) In case (i), the estimates [10] and [11] hold, and the contribution from $(\delta_{LR}^{u,12})$ is too large, unless RGE generates degeneracy, $r_3 \gtrsim 7$ (TeV/\bar{m}) [30].

In case (ii), $(\delta_{LR}^{u,12})$ is suppressed compared to Eq. (11), and the contribution from $[(\delta_{LR}^{u,12})^{2}]$ only requires a very mild degeneracy.

In either case, the parametric suppression of $(\delta_{LR}^{u,12})$ is as in Eq. (12), and the numerical estimate is as in Eq. (13). The parametric relation between the up and down sectors of Eq. (15) does not hold in the $[U(1)]^2$ models since $(\delta_{LR}^{u,12})$ is further suppressed, and so the constraint of Eq. (16) does not hold. The constraints of Eqs. (18) and (19) hold, leading to the same conclusion as in the single $U(1)$ case: The parametric suppression is such that the contribution to $\Delta A_{CP}$ falls an order of magnitude short compared to the benchmark value of Eq. (5). However, one cannot exclude the possibility that: (i) the large hadronic uncertainties in the EDM calculation are such that the bounds are weaker by an order of magnitude; or (ii) the order one uncertainty from the unknown coefficients provides further enhancement; or a combination of the above.

Non-Abelian Symmetries

In $U(2)$ models, the first two generations are in a doublet and the third generation in a singlet of the $U(2)$ symmetry [33-36]. With a two-stage symmetry breaking, the structure of the Yukawa, $A$ and $M^2$ matrices is as follows (see, for example, [30]):

\[Y^d \sim \frac{M_{2d}^{LR}}{\bar{a}v_d} \sim \begin{pmatrix} 0 & \epsilon_1 & 0 \\ -\epsilon_1 & \epsilon_2 & \epsilon_2 \\ 0 & \epsilon_2 & 1 \end{pmatrix}, \quad (22)\]

As in the Abelian case, all non-vanishing entries have unknown coefficients of order one, but $Y_{12}^{d} = -Y_{21}^{d}$ and there are relations for the $M_{NN}^{2}$ matrices that follow from hermiticity.

As concerns the $\delta_{LR}^{u,12}$ parameters, their parametric suppression is similar to the $U(1)$ model. Hence, Eqs. (12), (16), (18) and (19) all hold.

The main phenomenological difference of the $U(2)$ model with respect to the $U(1)$ model is that the first two quark generations are quasi-degenerate already at the mediation scale, with a mass splitting $\Delta \tilde{m}_{12}^{2}/\bar{m}^{2} \sim \epsilon_2^{2} \sim 10^{-3}$. Hence, the model is viable even without invoking flavor-universal RGE effects.

A flavor $[U(2)]^{3}$ symmetry [37] is motivated by the tension between the measured value of the CP asymmetry $S_{\psi K}$, and its theoretical value in the Standard Model derived from a global CKM fit. To alleviate this tension, a new physics contribution to $B^{0} - \bar{B}^{0}$ mixing of order 10 percent of the total amplitude is required. In a $U(2)$ model, such a contribution entails a contribution to $\epsilon_K$ of order 100 percent, which is unacceptable.

A $U(2)_{Q} \times U(2)_{U} \times U(2)_{D}$ model, with minimal spurion content — $V(2,1,1)$, $\Delta Y_{u}(2,2,1)$ and $\Delta Y_{d}(2,1,2)$ — allows one to suppress the contribution to $\epsilon_K$. 

The structure of the Yukawa and $A$ matrices is as follows [37]:

$$Y^q \sim \frac{\tilde{M}^2 q}{\tilde{m}^2_{u_d}} \sim y_{q3} \left( \begin{array}{cc} \Delta Y_q & x_qV \\ 0 & 1 \end{array} \right),$$

where $y_{q3} = y_q(y_b)$ for $q = u(d)$, $x_q$ is a complex free parameter of order one, $\Delta Y_q$ is a $2 \times 2$ matrix, and $V$ is a $2 \times 1$ vector.

This structure is quite unique in that one and the same spurion, $\Delta Y_q$, determines the structure of the $2 \times 2$ upper left block of both $Y^q$ and $\tilde{M}^2_{LR}$. Consequently, to leading order in the breaking parameters, $(\delta^q_{LR})_{12} = 0$, and the supersymmetric contribution to $\Delta A_{CP}$ vanishes. Corrections to $\delta_{LR}$ arise at the order $y_{LQV}^*V_{ab}$ and are negligible.

**Hybrid Mediation**

In models of hybrid mediation, the dominant source of supersymmetry breaking is MFV, but there are non-negligible contributions from Planck scale physics that do not obey the MFV principle. Examples include high-scale gravity and MFV physics is given by

$$\sim \frac{\tilde{m}^2_{grav}}{\sqrt{r}} \sim \frac{\tilde{m}^2_{MFV}}{\sqrt{r}} \sim \frac{\tilde{m}^2_{grav}}{\sqrt{r}},$$

and the expressions for $\delta_{LR}$ remain as in the pure gravity case, Eqs. (12) and (13).

The EDM constraints of Eqs. (18) and (19) hold. One can now ask what further constraints arise when linking the trilinear terms and the soft masses as is characteristic in hybrid models. If all gravity soft terms are dictated by a single scale, then $a_0 \sim \sqrt{r} \tilde{m}_{MFV}$, where $\tilde{m}_{MFV}$ is the typical messenger scale MFV soft mass. The relevant combination entering $\delta_{LR}$ is then $\tilde{a}/\tilde{m} \lesssim 3\sqrt{r}/r_3$, where the numerical factor stems from RGE and is largest for high scale mediation. In the following we analyze this single gravity scale scenario in the FN context.

In models with a single $U(1)$ and order one phases kaon mixing requires $r/r_3 \lesssim 0.002 (\tilde{m}/\text{TeV})$. Therefore,

$$\Im(\delta_{LR}) \lesssim 0.2 \times 10^{-4} \sqrt{\text{TeV}/\tilde{m}},$$

a stronger constraint than the EDM bound of Eq. (19).

In $[U(1)]^2$ models the kaon system constrains $r/r_3 \lesssim 0.06 (\tilde{m}/\text{TeV})$, and so

$$\Im(\delta_{LR}) \lesssim 1 \times 10^{-4} \sqrt{\text{TeV}/\tilde{m}},$$

close to the upper bound from EDMs, Eq. (19). We note that it is possible in specific $U(1)^2$ models to further suppress the contribution to the kaon system, such that the strongest bound comes from $D^0 - \bar{D}^0$ mixing and gives $r/r_3 \lesssim 0.8 (\tilde{m}/\text{TeV})$, relaxing the constraint (27) by a factor of 4.

Both $U(2)$ and $[U(1)]^2$ models do not require further flavor suppression, and are viable for $r \lesssim 1$, with predictions as in the non-hybrid models.

We conclude that hybrid models with a $[U(1)]^2$, $U(2)$ or $U(2)^3$ symmetry generate $\Delta A_{CP}$ of the same size as non-hybrid models. The size of $\delta_{LR}$ allowed by hybrid models with a single $U(1)$ is somewhat smaller.

### A Terms and the Lightest Higgs Mass

In supersymmetry $\Delta A_{CP}$ can be interpreted via the left-right mixing $\delta_{LR}$ which requires unsuppressed trilinear couplings with respect to the squark masses, $\tilde{a}/\tilde{m} \sim O(1)$, see Eq. (13). At the same time the recent hints from ATLAS and CMS of a neutral Higgs boson with mass near 125 GeV implies that the stops – if not decoupled – are largely mixed as well [41, 42]:

$$|A_t/y_t - \mu/\tan\beta| \gtrsim M_S,$$  

where $M_S$ denotes the geometric mean of the stop masses. In the FN models, where the flavor structure of the $A$ terms is parametrically similar to that of the Yukawas, $A \sim Y$, the stop $A$ terms at the weak scale can be written as [43]

$$A_t/y_t \simeq \tilde{a} - \Delta a, \quad \Delta a = y_t^2 b_{ab} + m_{1/2} c,$$

with positive RGE-induced coefficients $b, c$ of order one.

For positive $\mu$ or sufficient $\tan\beta$ suppression one obtains from Eq. (28) a lower bound that supports a sizeable supersymmetric $\Delta A_{CP}$,

$$\tilde{a}/\tilde{m} \gtrsim M_S/\tilde{m}$$

for $A_t > 0$ and unsplit spectrum where the stops are not too far away from the other squarks. Negative $A_t < 0$ can arise in scenarios with tiny or vanishing $a_0$ such as gauge mediation, which lead to acceptable phenomenology only for sufficiently large gluino masses.

While the Higgs signal needs to be consolidated, it is interesting that if confirmed, the current mass of $\sim 125$ GeV points to a similar region in supersymmetric parameter space as the interpretation of $\Delta A_{CP}$.

### Conclusions

Supersymmetric models can contribute to direct CP violation in singly Cabibbo suppressed $D$ decays at the
level observed by experiments, $\Delta A_{CP}^{\text{SUSY}} \sim 0.006$, without conflicting with phenomenological constraints from $D^0 - \bar{D}^0$ mixing or $\epsilon'/\epsilon$. This is naturally the case if the flavor changing parameter $|\delta_{LR}^{12}|$, generated by trilinear scalar couplings, is of order $10^{-3}$.

In minimally flavor violating supersymmetric models, such as those of gauge mediation and anomaly mediation, $|\delta_{LR}^{12}|$ is orders of magnitude too small. Thus, to account for $\Delta A_{CP}$, one has to go beyond minimal flavor violation. We examined models where the flavor structure of the soft breaking terms is dictated by an approximate flavor symmetry.

We found that quite generically in such models, $|\delta_{LR}^{12}|$ is flavor-suppressed by $(m_u |V_{us}|)/\bar{m}$, which is of order a few times $10^{-4}$. There is however additional dependence on the ratio between flavor-diagonal parameters, $\bar{a}/\bar{m}$, and on unknown coefficients of order one, that can provide enhancement by a factor of a few.

In most such models, however, the selection rules that set the flavor structure of the soft breaking terms, relate $|\delta_{LR}^{12}|$ to $|\delta_{LR}^{12}|$ and to $|\delta_{LR}^{12}|$, which are bounded from above by, respectively, $\epsilon'/\epsilon$ and EDM constraints. Since both $\epsilon'/\epsilon$ and EDMs suffer from hadronic uncertainties, small enhancement due to the flavor-diagonal supersymmetric parameters cannot be ruled out. Additionally, it is still possible that $|\delta_{LR}^{12}|$ is accidentally enhanced by the order one coefficient.

Chirality-flipping couplings between the first and second generation up squarks can effectively arise also via chirality-flipping in the third generation [25]. In all flavor models considered here, the effective $\delta_{LR}$ generated is at most parametrically of the same order as the direct contribution, accompanied by an additional $1/r_3^2$. For $U(1)$ and $[U(1)]^2$ models this provides extra suppression, while for $U(2)$ models the effective contribution to $\delta_{LR}$ is flavor-suppressed with respect to the direct one. In any event, the constraints remain the same and the analysis stands.

We conclude that it is possible to accommodate $\Delta A_{CP} \sim 0.006$ in supersymmetric models that are nonminimally flavor violating, but -- barring hadronic enhancements in charm decays -- it takes a fortuitous accident to lift the supersymmetric contribution above the permil level.

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[29] Y. Nir and G. Raz, Phys. Rev. D 66, 035007 (2002) [hep-ph/0206064].
[30] G. Hiller, Y. Hochberg and Y. Nir, JHEP 0903, 115 (2009) [arXiv:0812.0511 [hep-ph]]; JHEP 1003, 079 (2010) [arXiv:1001.1513 [hep-ph]].
[31] G. Isidori, Y. Nir and G. Perez, Ann. Rev. Nucl. Part. Sci. 60, 355 (2010) [arXiv:1002.0900 [hep-ph]].
[32] O. Gedalia, J. F. Kamenik, Z. Ligeti and G. Perez, arXiv:1202.5038 [hep-ph].
[33] M. Dine, R. G. Leigh and A. Kagan, Phys. Rev. D 48, 4269 (1993) [hep-ph/9304299].
[34] A. Pomarol and D. Tommasini, Nucl. Phys. B 466, 3 (1996) [hep-ph/9507462].
[35] R. Barbieri, G. R. Dvali and L. J. Hall, Phys. Lett. B 377, 76 (1996) [hep-ph/9512388].
[36] R. Barbieri, L. J. Hall and A. Romanino, Phys. Lett. B 401, 47 (1997) [hep-ph/9702315].
[37] R. Barbieri, G. Isidori, J. Jones-Perez, P. Lodone and D. M. Straub, Eur. Phys. J. C 71, 1725 (2011) [arXiv:1105.2296 [hep-ph]].
[38] J. L. Feng, C. G. Lester, Y. Nir and Y. Shadmi, Phys. Rev. D 77, 076002 (2008) [arXiv:0712.0674 [hep-ph]].
[39] C. Gross and G. Hiller, Phys. Rev. D 83, 095015 (2011) [arXiv:1101.5352 [hep-ph]].
[40] [ATLAS Collaboration], arXiv:1202.1408 [hep-ex]. S. Chatrchyan et al. [CMS Collaboration], arXiv:1202.1488 [hep-ex].
[41] S. Heinemeyer, O. Stal and G. Weiglein, arXiv:1112.3026 [hep-ph].
[42] P. Draper, P. Meade, M. Reece and D. Shih, arXiv:1112.3068 [hep-ph].
[43] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994) [Erratum-ibid. D 78, 039903 (2008)] [hep-ph/9311340].