Mechanism synthesis for knee rehabilitation of patients with different heights taking the ankle as reference point

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Abstract. This article analyzes different trajectories that different patients take when performing knee rehabilitation movements, which are flexion-extension movements in the supine position, analyzing the trajectory taking the patient's ankle as a reference point, aiming to design a mechanism that can offer a solution to the following more than one trajectory issue, which will allow it to be used for more than one movement, in more than one patient.

1. Introduction
The previously designed mechanisms were based on design, the trajectories with a reference point in the knee, taking the trajectory of a specific patient when using these mechanisms in other patients with different measures, a limitation was observed since they did not work, it can also be since the trajectory in the same patient is not completely followed as observed in [1,2]. When observing the two mechanisms previously designed to provide a solution to the tracking of trajectories due to rehabilitation problems, it is observed that these were designed from the movements made by the knee, and this turns out to be a limitation to find the necessary measures that allow the design and calculation of a mechanism that can better perform rehabilitation exercises with different patients according to [3,4].

From this analysis, it was possible to determine that the trajectory that must be taken into account in order to create a more effective design that can meet the necessary requirements seen in [5], is the one where the ankle is a reference point since it is the point that the physical therapist uses to grasp the patient's foot to help perform the relevant. This deduction allows us to appreciate that the best perspective of the mechanism is the one that involves the ankle being observed [2,6-11].

This article will present the analysis made of the trajectory of a patient 0 with measurements taken corresponding to the distance between the hip and the knee, the distance measured between the knee and the ankle. Obtaining the anthropometric measurements of different patients are taken, and by means of an algorithm that uses the angles found in patient 0. Subsequently, applying a methodology for the synthesis of mechanisms considering the mobility equations and based on the graphs obtained from this analysis, a mechanism with probabilities is obtained to fulfil the required task.

2. Stability model
The trajectory that represents the flexion-extension movement is found, since it is desired that the mechanism works on more than one patient, and taking the data of each person is impractical when carrying out the rehabilitation, an analysis of the angles presented by the extremities of the patient from whom the data was taken is performed as it can see in Figure 1, including the arch of the mobility of the knee to make a comparison between the X, Y values of the function that describes the
trajectory from the patient from which the data is obtained, the measurements of angles to be reached at each position on the trajectory as a basis, the calculations are analyzed as follows.

Figure 1. Patient 0 paths analyzed for motion calculations needed to find other trajectories.

Where, the final position (x and y) is known, $L_1$ is the distance from hip to knee, and $L_2$ is the distance from knee to ankle. $C$ is the distance between the hip to the ankle along the $Xt$ axis at each of the points of the trajectory, when the patient's leg is fully stretched, this variable will have a value of $L_1 + L_2$ as shown in Equation (1). The following consideration is taken when the leg is fully stretched when, $\alpha=180^\circ$, $\beta = 0$, and $\theta = 0$.

$$C = L_1 + L_2 - Xt,$$

(1)

when following the trajectory obtained, the variable $J$ is found, which is the distance from the hip to the ankle, Equation (2), the angle between $C$ and $J$ Equation (3), is found, since, this angle is needed to find the total angle between the horizontal with $L_1$.

$$J = \sqrt{C^2 + Yt^2},$$

(2)

$$\theta = \tan \left(\frac{Yt}{C}\right).$$

(3)

Finding the angle $\beta$ using the law of cosines as shown in Equation (4) to Equation (6), in this case the ELBOW = 1 is considered so that $\beta$ is positive with respect to the horizontal. The angle $\alpha$ is found as can be seen in Equation (7) to Equation (9), in this case, the ELBOW = 1 is considered so that $\alpha$ is positive with respect to the horizontal.

$$\cos \beta = \frac{L_1^2 + L_2^2 - L_2^2}{2 \cdot L_1 \cdot |J|},$$

(4)

$$\sin \beta = \text{ELBOW} \cdot \sqrt{1 - (\cos \alpha)^2},$$

(5)

$$\beta = \tan (\sin \beta / \cos \beta),$$

(6)

$$\cos \alpha = \frac{L_1^2 + L_2^2 - L_2^2}{2 \cdot L_1 \cdot L_2},$$

(7)

$$\sin \alpha = \text{ELBOW} \cdot \sqrt{1 - (\cos \alpha)^2},$$

(8)

$$\alpha = \tan (\sin \alpha / \cos \alpha),$$

(9)

where $\alpha$ and $\beta$ are known, $L_1$ is the measurement from hip to knee, and $L_2$ is the measurement from ankle to knee. The angles from the measured patient are observed, and using this, the position in X, Y
that the leg of different patients must reach is found as you can see in Figure 2; α it can be associated with the angle of the arc of motion in the patient as follows in Equation (10).

Arch of mobility = 180 − α.  \hspace{1cm} (10)

With the previous angles found from the trajectory of the analyzed patient, the calculations are made to find the trajectory to follow for each of the patients to be treated. The Table 1. Show the anthropometric measurements of lower extremities.

| Patient | Distance from hip to knee (cm) | Distance from knee to ankle (cm) |
|---------|--------------------------------|---------------------------------|
| 1       | 40                             | 40                              |
| 2       | 45                             | 45                              |
| 3       | 43.5                           | 44                              |
| 4       | 50                             | 48                              |

Figure 2. Trajectories found from the anthropometric measurements of Table 1.

3. Mechanism synthesis

This begins by obtaining the structural requirements of the mechanism such as mobility, the workspace where it will be developed, and the number of circuits that will be used to fulfill its objective, these characteristics can be evidenced in Equation (11) and Equation (12). Then the kinematic analysis of the mechanism found is done to obtain dimensions to follow each trajectory.

3.1. Number synthesis method

From of mobility, Equation (11), and the circuit, Equation (12) [12].

\[ M = j - \lambda, \]  \hspace{1cm} (11)
\[ J = j + 1 - n, \]  \hspace{1cm} (12)

where \( M = 2 \) is mechanism mobility, \( \lambda = 3 \) is the workspace, and \( J \) is the number of mechanism circuits. From Equation (11) the number of joins is five (\( f = 5 \)), and from circuit Equation (12), the number of links is five (\( n = 5 \)). The Figure 3 show the link and joint analysis for type synthesis.
3.2. Type synthesis method
Here, the different possible mechanisms derived from number synthesis are found, anchoring any link to the ground would result in the same mechanism as you can see in Figure 3.

Figure 3. Link and joint analysis for type synthesis.

3.3. Kinematic analysis of the 5-bar mechanism
Both the direct kinematics and the inverse kinematics of the 5-bar mechanism are analyzed, since it is necessary to obtain the joint coordinates that the actuators must have to reach each point of the desired trajectory from the points found on the trajectory [13]. This analysis can be seen in Figure 4 and Figure 5. The Figure 4(a) show the link and joint analysis of the 5-bar mechanism and the Figure 4(b) show the inverse Geometric analysis of the 5-bar mechanism.

The distances between points A, C, D, and C, are found, these points are taken as vectors. The values of ELBOW1 = 1 and ELBOW2 = 1 are also assigned to determine the direction of the angles to be found so that the mechanism always tries to have an appearance like that of Figure 4, this can be evidenced in Equation (13) and Equation (14).

\[
a = \text{norm}(C - A) \quad \text{ELBOW1} = 1, \quad (13) \\
b = \text{norm}(C - D) \quad \text{ELBOW2} = 1. \quad (14)
\]

The inverse analysis of the mechanism begins by finding the angle of the horizontal with respect to the magnitudes a and b using the inverse of the tangent, these angles are \(\alpha\) and \(\omega\) as shown in Equation (15) and Equation (16).

\[
\alpha = \arctan \left( \frac{C_y}{C_x} \right), \quad (15) \\
\omega = \arctan \left( \frac{C_y}{C_x - L} \right). \quad (16)
\]

The law of cosines is used to obtain \(\varphi\) and \(\Psi\) as can be seen in Equation (17) to Equation (22).

\[
\cos \varphi = \frac{(L_1^2 + L_2^2 - a^2)}{(2 \cdot L_1 \cdot L_2)}, \quad (17)
\]
\[
\sin \varphi = \text{ELBOW1} \cdot \sqrt{1 - (\cos \varphi)^2},
\]
\[
\varphi = \text{atan} \left( \frac{\sin \varphi}{\cos \varphi} \right),
\]
\[
\cos \Psi = \frac{(L_3^2 + L_4^2 - b^2)}{(2L_3L_4)},
\]
\[
\sin \Psi = \text{ELBOW2} \cdot \sqrt{1 - (\cos \Psi)^2},
\]
\[
\Psi = \text{atan} \left( \frac{\sin \Psi}{\cos \Psi} \right).
\]

Taking auxiliary lines from point C as shown in Figure 4(b) with blue lines, the angles \(\beta\) and \(\sigma\) are found as seen in Equation (23) and Equation (24).

\[
\beta = \text{atan} \left( \frac{L_4 \cdot \text{sen}(\pi - \varphi)}{L_3 + L_4 \cdot \text{cos}(\pi - \varphi)} \right),
\]
\[
\sigma = \text{atan} \left( \frac{L_4 \cdot \text{sen}(\pi - \Psi)}{L_3 + L_4 \cdot \text{cos}(\pi - \Psi)} \right).
\]

The angles \(\theta_1\) and \(\theta_2\) are found with Equation (25) and Equation (26).

\[
\theta_1 = \alpha + \beta,
\]
\[
\theta_2 = w - \sigma.
\]

**Figure 5.** Direct geometric analysis of the 5-bar mechanism.

The points of the mechanism are found in the x and y coordinates according to Equation (27) to Equation (31).

\[
B_x = L_1 \cdot \cos(\theta_1),
\]
\[
B_y = L_1 \cdot \text{sen}(\theta_1),
\]
\[
D_x = L,
\]
\[
E_x = L_3 \cdot \cos(\theta_2) + L,
\]
\[
E_y = L_3 \cdot \text{sen}(\theta_2).
\]

To determine the coordinates of point C, a line is traced between points E and B as shown in Figure 5, finding the distance R and in this way \(C_x\) and \(C_y\) are found, as shown by Equation (32) to Equation (36).
\[ R = \sqrt{L - L_1 \cos(\theta_1) + L_3 \cos(\theta_2)}^2 + (L_1 \sin(\theta_1) - (L_3 \sin(\theta_2))^2, \] (32)

\[ \theta R = \tan^{-1}(L_3 \sin(\theta_2) - L_1 \sin(\theta_1), L_1 - L_4 \cos(\theta_1) + L_3 \cos(\theta_2)), \] (33)

\[ \theta_{12} = \theta R + \cos(\frac{R^2 + L_3^2 - L_4^2}{2L_1 + L_2^2}), \] (34)

\[ C_x = B_x + L_2 \cos(\theta_{12}), \] (35)

\[ C_y = B_y + L_2 \sin(\theta_{12}). \] (36)

### 4. Results

The dimensions obtained by the 5-bar mechanism were the following: Distance between A and D was 10 cm, value of \( L_1 = L_3 = 17.3687 \) cm and value of \( L_2 = L_4 = 25.4656 \) cm. Looking at Figure 4, the end effector that will follow the path described by the ankle of each patient will be the joint shown at point C of the mechanism. Thanks to the calculations obtained, the joint coordinates that the actuators must follow to perform the trajectory were determined, as shown in Table 2. The Table 2 shows the results obtained considering the first 12 points of the trajectory, for this was taken from patient 0 and patient 1, thus calculating the articular coordinates and position the end effector found.

#### Table 2. Comparison of trajectories and joint coordinates between patient 0 and patient 1.

| X (cm) | Y(cm) | \( \theta_1 \) (rad) | \( \theta_2 \) (rad) |
|--------|-------|----------------------|----------------------|
| Patient 0 with \( L_1 = L_2 = 36 \) cm |
| 0.00   | -1.00 | 0.52                 | -1.00                |
| -2.00  | -2.00 | 0.67                 | -0.90                |
| -3.00  | -3.00 | 0.71                 | -0.84                |
| -4.00  | -4.00 | 0.86                 | -0.81                |
| -5.00  | -5.00 | 0.94                 | -0.78                |
| -6.00  | -6.00 | 1.01                 | -0.77                |
| -7.00  | -7.00 | 1.08                 | -0.77                |
| -8.00  | -8.00 | 1.14                 | -0.77                |
| -9.00  | -9.00 | 1.20                 | -0.77                |
| -10.00 | -10.00| 1.26                 | -0.77                |
| -11.00 | -11.00| 1.32                 | -0.77                |
| Patient 1 with \( L_1 = L_2 = 40 \) cm |
| 0.00   | -1.11 | 0.52                 | -1.00                |
| -2.22  | -2.22 | 0.69                 | -0.89                |
| -3.33  | -3.33 | 0.80                 | -0.82                |
| -4.44  | -4.44 | 0.89                 | -0.78                |
| -5.56  | -5.56 | 0.98                 | -0.75                |
| -6.67  | -6.67 | 1.05                 | -0.73                |
| -7.78  | -7.78 | 1.13                 | -0.72                |
| -8.89  | -8.89 | 1.20                 | -0.72                |
| -10.00 | -10.00| 1.26                 | -0.71                |
| -11.1  | -11.1 | 1.33                 | -0.71                |
| -12.2  | -12.2 | 1.40                 | -0.70                |

When comparing the two trajectories in Figure 6, specifically in the Figure 6(a) show the Patient 0 and Figure 6(b) show the Patient 1. This Figures it can be observed that although the difference in measurements between the patients is not great, there is a notable difference in the trajectories, suggesting the importance of the result achieved when finding a mechanism with adequate dimensions to achieve the different trajectories that have large variations before small changes in the anthropometric measurements of the patients as can be seen in Figure 2.

![Figure 6](image-url)

**Figure 6.** Trajectory of Ankle patients and the 5-bar mechanism: (a) patient 0; (b) patient 1.
5. Conclusion

Analyzing the results obtained in this article, it can be deduced that the trajectories described by each patient have notable changes in the face of small changes in the anthropometric measurements of each person to whom the flexion-extension rehabilitation exercises will be applied. Knowing these trajectories, physiotherapists can get an idea of the implication that this entails when rehabilitating a patient. With the help of this mechanism, a repetitive trajectory can be carried out for any of the patients who are in this range of measurements, which is positive, for the rehabilitation.

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