Black hole fluctuations and dynamics from back-reaction of Hawking radiation: Current work and further studies based on stochastic gravity

B. L. Hu\textsuperscript{1}\textsuperscript{*} and Albert Roura\textsuperscript{1,2}\textsuperscript{†}

\textsuperscript{1}Department of Physics, University of Maryland, College Park, Maryland 20742-4111
\textsuperscript{2}Theoretical Division, T-8, Los Alamos National Laboratory, M.S. B285, Los Alamos, NM 87545

We give a progress report of our research on spacetime fluctuations induced by quantum fields in an evaporating black hole and a black hole in quasi-equilibrium with its Hawking radiation. We note the main issues involved in these two classes of problems and outline the key steps for a systematic quantitative investigation. This report contains unpublished new ideas for further studies.

\section{I. BLACK HOLE BACK-REACTION AND FLUCTUATIONS: OUR CURRENT WORK}

Back-reaction here refers to the influence of particle creation as in the Hawking radiation and other quantum effects (such as trace anomaly) on the structure and dynamics of the background spacetime. It is believed to grow in importance when the energy reaches the Planck scale, as in the very early universe and at the final stages of black hole evolution. In semiclassical black hole physics this is perhaps one of the more difficult but important unsolved problems. It is important because its solution is necessary for us to understand better the black hole end-state and information loss puzzles. It also provides a check on the range of validity for Hawking’s derivation of black hole radiance in the framework of semiclassical gravity. As we found out from the stochastic gravity perspective\cite{A1, A2}, the back-reaction problem is tied to dissipation and fluctuation phenomena. Thus it provides a natural generalization to non-equilibrium black hole thermodynamics and can reveal deeper connections between gravity, quantum field theory and statistical mechanics.

Difficulties in the black hole back-reaction problems start with finding a renormalized energy momentum tensor\textsuperscript{1}. The stochastic gravity program\cite{A2} introduces fresh insight and new methodology into the back-reaction problem by a) stressing the importance of an in-in (closed-time-path, or CTP) formulation\cite{A3, A4} which gives real and causal equations of motion, b) imparting a statistical mechanical meaning to the back-reaction effects via quantum open system concepts and techniques\cite{A3, A4}, c) noting the necessity of including fluctuations in conjunction with dissipation in the system dynamics and showing how noise arises with the help of the influence functional formalism\cite{A7, A8, A9}. The quantities of importance are the dissipation kernel, which enters into the Einstein-Langevin (E-L) equation\cite{A10}, and the noise kernel, which characterizes the correlations of its stochastic source.

The back-reaction problems of interest to us fall into two classes: 1) A black hole in a box in quasi-equilibrium with its Hawking radiation. 2) An evaporating black hole emitting Hawking radiation under fully non-equilibrium conditions. These problems have been treated before in varying degrees of completeness. The former is easier to understand but has some remaining technical challenges. We approach this problem at two levels: i) At the quantum field theory level is the derivation of the influence action, which demands a computation of the noise kernel for quantum fields near the Schwarzschild horizon. ii) At the statistical thermodynamic level we approach this problem by viewing the back-reaction as an embodiment of the fluctuation-dissipation relation (FDR). We have demonstrated the usefulness of this way of thinking in cosmological back-reaction problems\cite{A7}. The problems for evaporating black holes have only been treated sparsely and qualitatively, even with contradictory claims. Our current effort has focused on clarifying some existing conceptual confusion and building a unifying framework capable of producing more quantitative results.

\textsuperscript{*} Email address: hub@physics.umd.edu
\textsuperscript{†} Email address: roura@lanl.gov

\textsuperscript{1} For a list of papers dedicated to this task, see footnote 1 of \textsuperscript{D32}. Some notable work under the framework of semiclassical gravity (in addition to those few discussed in more detail below) includes Anderson et al.\cite{D32} for a quantum scalar field in a spherically symmetric spacetime, the back-reaction in the interior of a black hole\cite{D32} (easier technically since it is a cosmological Kantowski-Sachs spacetime) and for two-dimensional dilaton gravity theory\cite{D34}.
II. QUASI-EQUILIBRIUM CONDITIONS: BLACK HOLE IN A BOX

A black hole can be in quasi-equilibrium with its Hawking radiation if placed inside a box of the right size \(^2\), or in an anti-de Sitter universe \(^3\). We divide our consideration into the far-field case and the near-horizon case. The far field case has been studied before \(^4\). In \(^5\) we consider the near-horizon case. Using the model of a black hole described by a radially-perturbed quasi-static metric and Hawking radiation by a conformally coupled massless quantum scalar field, we showed that the closed-time-path (CTP) effective action yields a non-local dissipation term as well as a stochastic noise term in the equation of motion. We have presented the overall structure of the theory and the strategy of our approach in \(^6\), but due to the lack of an analytic form of the Green function for a scalar field in the strong field region of the Schwarzschild metric, numerical calculations may be the only way to go. Being based on a quasilocal expansion, Page’s approximation \(^7\), though reasonably accurate for the stress tensor, is insufficient for the noise kernel since one needs to consider arbitrarily separated points in that case. In \(^8\) we also presented an alternative derivation of the CTP effective action in terms of the Bogoliubov coefficients, thus connecting with the interpretation of the noise term as measuring the difference in particle production in alternative histories. [This will be useful for a grand-canonical ensemble description of black hole fluctuations. (See below.)]

A. Connecting different approaches

On the black-hole-in-a-box back-reaction problem we take a two-prong approach: via quantum field theory and statistical thermodynamics. We are currently performing a calculation of the noise kernel near the Schwarzschild horizon, making use of the results of \(^9\) but with points kept separate \(^10\). We will use the result of this calculation to show the existence of a fluctuation-dissipation relation (described below). In addition we want to integrate the master equation approach of Zurek \(^11\) and the transition probability approach of Bekenstein, Meisel \(^12\) and Schiffer \(^13\) (see also \(^14\)) under the stochastic gravity framework. Within the thermodynamic descriptions we want to compare the results from the canonical with the microcanonical \(^15\) and the grand canonical ensembles \(^16\). For the last task we will invoke results obtained earlier \(^17\) for the number fluctuations in particle creation and the relation we obtained recently in \(^18\) expressing the CTP effective action in terms of the Bogoliubov coefficients for the black hole particle creation, to derive the susceptibility and isothermal compressibility functions of the black hole. This would move us a step closer to establishing a linear response theory (LRT) of non-equilibrium black hole thermodynamics.

B. Back-reaction manifested through a fluctuation-dissipation relation

Historically Candelas and Sciama \(^19\) first suggested a fluctuation-dissipation relation for the depiction of dynamic black hole evolution. They proposed a classical formula relating the dissipation in area linearly to the squared absolute value of the shear amplitude. The quantized gravitational perturbations (they choose the quantum state to be the Unruh vacuum) approximate a flux of radiation emanating from the hole at large radii. Thus they claim that the dissipation in area due to the Hawking flux of gravitational radiation is related to the quantum fluctuations of the metric perturbations. However, as was pointed out in \(^20\), it is not clear that their relation corresponds to a FDR in the correct statistical mechanical sense and does not include the effect of matter fields. The FDR for the contribution of the matter fields should involve the fluctuations of the stress tensor (the “generalized force” acting on the spacetime), which are characterized

\(^2\) Note that one needs to introduce the appropriate redshift factors that account for the finite size of the box. Moreover, for a sufficiently small box, as required in general to have (meta)stable equilibrium, curvature corrections, which were not included in Ref. \(^21\) may not be negligible.

\(^3\) Note, however, that for massless particles with vanishing chemical potential the canonical and grand canonical ensembles coincide.
by the noise kernel. With an explicit calculation of the noise kernel in a similar context one could obtain the correct FDR.

For the quasi-static case Mottola \cite{D29} introduced a FDR based on linear response theory for the description of a black hole in quasi-equilibrium with its Hawking radiation. He showed that in some generalized Hartle-Hawking state a FDR exists between the expectation values of the commutator and anti-commutator of the energy-momentum tensor. This is the standard form of FDR. However, in his case the dynamical equation for the linear metric perturbations describes just their mean evolution, which corresponds to taking the stochastic average of the linearized E-L equation. The E-L equation from stochastic gravity does more in providing the dynamics of the fluctuations.

In a recent paper \cite{D36} we laid out the road map for treating the quasi-static case in the stochastic gravity framework and point out that a non-local dissipation and a fluctuation term will arise which should match with the analytic results in the far field limit derived earlier \cite{D33}. These terms are absent in York’s work \cite{D30} because the approximate form he uses for the semiclassical Einstein equation corresponds to the variation of terms in the CTP effective action which are linear in the metric perturbations around the Schwarzschild background geometry, whereas the non-local dissipation and noise kernels appear at quadratic order. The noise kernel, which is connected to the fluctuations of the stress tensor, would give no contribution to his equation for the mean evolution, even if higher order corrections were included.

III. EVAPORATING BLACK HOLE: NON-EQUILIBRIUM CONDITIONS

This time-dependent problem has a very different physics from the quasi-equilibrium case, and is in general more difficult to treat. Starting in the early 80’s, it has been approached by Bardeen \cite{E37}, Israel \cite{E38} and Massar \cite{E40}, who considered the mean evolution. The role of fluctuations was initially studied by Bekenstein \cite{F41}, and has received further attention in recent years by Ford \cite{F46, F47}, Frolov \cite{F49}, Sorkin \cite{F43}, Marolf \cite{F44}, and their collaborators (see also Ref. \cite{F42}), largely based on qualitative arguments. On some issues, such as the size of black hole horizon fluctuations, there are contradictory claims.

To make progress one needs to introduce a theoretical framework where all prior claims can be scrutinized and compared. Because of its non-equilibrium nature we expect the stochastic gravity program \cite{A1} to provide some useful quantitative results. In \cite{F47} we wrote down the Einstein-Langevin equation for the fluctuations in the mass of an evaporating black hole and found that the fluctuations compared to the mean is small unless the mean solution is unstable with respect to small perturbations. Our 1998 paper emphasized the first part of this statement, which is in apparent contradiction to what Bekenstein claimed in his 1984 paper \cite{F41}. Recently we revisited this question with a closer analysis \cite{F51}. Since for an evaporating black hole there exist unstable perturbations around the solution of the semiclassical Einstein equation for the mean evolution, the second part of the statement applies.

A key assumption in these studies is that the fluctuations of the incoming energy flux near the horizon are directly related to those of the outgoing one (a condition that does hold for the mean flux). If this condition were true, one can indeed show that the fluctuations of a black hole horizon become important (growing slowly, but accumulating over long times), as Bekenstein claimed. However, we have serious reservations on its validity for energy flux fluctuations (see below). Using the E-L equation we also point out how the different claims of Bekenstein and Ford-Wu can be reconciled by recognizing the different physical assumptions they used in their arguments. The stochastic gravity theory which our present work is based on should provide a platform for further investigations into this important issue.

A. Bardeen’s evaporating black hole and Bekenstein’s fluctuation theory

In 1981 Bardeen \cite{E37} considered the back-reaction of Hawking radiation on an evaporating black hole by invoking a Vaidya-type metric. (This model was later used by Hiscock \cite{E39} for similar inquiries.) Bardeen’s calculation with this model affirmed the validity of the semiclassical picture assumed in Hawking’s original derivation of thermal radiance. His results including back-reaction indicate that the black hole follows an evolution which is largely determined by the semiclassical Einstein equations down to where the black hole mass drops to near the Planck mass ($\sim 10^{-5}$ g), the point where most practitioners of semiclassical gravity would agree that the theory will break down. Bardeen’s result was developed further by Massar \cite{E40}.
For black hole mass fluctuations, in 1984 Bekenstein [F41] considered the mass fluctuations of an isolated black hole due to the energy fluctuations in the radiation emitted by the hole, and asks when such fluctuations become large. According to his calculation, depending on the initial mass, mass fluctuations can grow large well before the mass of the black hole reaches the Planck scale. On the other hand, of the few studies in this setting, the result of Wu and Ford [F47] supports a scenario in which fluctuations do not become important before reaching the Planckian regime. In view of such contradictory claims in the literature, it is highly desirable to have a more solid and complete theoretical framework where all prior claims can be scrutinized and compared. We expect the stochastic gravity program to be useful for this purpose. At the most rudimentary level, Bekenstein’s approach shares similar conceptual emphasis as does the stochastic gravity program in that both attribute importance to the fluctuations of stress tensor and the black hole mass, characterized by their correlation functions.

B. Non-equilibrium conditions

Investigations in this case may assist in answering two important questions: 1) Are the fluctuations near the horizon large or small? 2) How reliable are earlier results from test quantum fields in fixed curved spacetime?

As far as the first question is concerned, one should distinguish between fluctuations with short and long characteristic time scales. For time scales comparable to the evaporation time, one expects spherically-symmetric modes ($l = 0$) to be dominant. Moreover, if one assumes a direct correlation between the energy flux fluctuations on the horizon and those far from it, as done in earlier work, an explicit result supporting Bekenstein’s conclusion can be obtained and the origin of the discrepancy with Ford and Wu’s result can be understood. However, a more careful analysis reveals that such an assumption is not correct (see below) and it is necessary to compute the noise kernel near the horizon to get an accurate answer. One can present arguments to the effect that this kind of fluctuations may not modify in a drastic way the result obtained by Hawking for test fields evolving on a fixed black hole spacetime, and later extended by Bardeen and Massar to include their back-reaction effect on the mean evolution of the spacetime geometry. On the other hand, in principle, fluctuations with much shorter correlation times, which also involve higher multipoles ($l ≠ 0$), could alter substantially Hawking’s result.

Detailed calculations within the framework of stochastic gravity can address those issues in a natural way, at least for fluctuations with typical scales much larger than the Planck length. Nevertheless, one needs to pay attention to the subtleties. Preliminary results, briefly described below, signal a possible breakdown of the geometric optics approximation for the propagation of test fields when fluctuations are included. This would require finding alternative ways of probing satisfactorily those metric fluctuations and extracting physically meaningful information.

1. Spherically symmetric sector ($l = 0$)

Consideration of this problem with restriction to spherically symmetric modes was done in [C27, F41, F47, F48, F51] as well as two-dimensional dilaton-gravity models [D34]. All those studies restricted from the outset the contribution to the classical action to s-wave modes for both the metric and the matter fields, which only allows $l = 0$ modes for each matter field to contribute to the noise kernel. Our approach goes well beyond that approximation since modes with all possible values of $l$ for the matter fields can contribute to the $l = 0$ sector of the noise kernel.

We focus on the physics in the adiabatic regime, which is the time when the black hole mass $M$ remains much larger than the Planck mass $m_p$. This allows one to use $\langle T_{ab} \rangle_{\text{ren}}$ in the Schwarzschild spacetime for each instant of time but with a mass slowly evolving in time. Technically this saves one the trouble of solving the integro-differential (semiclassical Einstein) equation to obtain the mean evolution.

In the adiabatic regime, one can show from energy-momentum conservation that the outgoing (for $r \gg 2M$) and ingoing (for $r ≈ 2M$) mean fluxes are related. If a similar argument is employed to relate the outgoing and ingoing energy fluxes for the stochastic source characterizing the stress tensor fluctuations, one can proceed in the same way as was done for the mean fluxes to provide a justification for Bekenstein’s approach.

This energy conservation argument has been assumed to be valid for energy flux fluctuations by Bekenstein [F41] as well as Wu and Ford [F47] for an evaporating black hole. With this assumption one can also clarify
the apparent discrepancy with Wu and Ford as follows. The growth in time of the fluctuations can be understood in terms of the following “instability” exhibited by the mean evolution: if the initial mass of a macroscopic black hole with $M \gg m_p$ is slightly perturbed by a small amount of the order of the Planck mass, the difference between the masses of the perturbed and unperturbed black holes becomes of the same order as the mass of the unperturbed black hole when the latter becomes of order $(m_p^2M)^{1/3}$, i.e., still much larger than the Planck mass. This growth of the fluctuations, first found by Bekenstein, is a consequence of the secular effect of the renormalized stress energy tensor of the perturbations, whose effect builds up in time and gives a contribution to the mass evolution of the same order as the mean evolution for times of the order of the black hole evaporation time even when the mass of the black hole is still much larger than the Planck mass. This term was not taken into account by Wu and Ford, which explains why they found much smaller fluctuations than Bekenstein for times of the order of the evaporation time.\footnote{One expects that a small region near the event horizon for a very large black hole should be very similar to a Rindler horizon. The arguments of Casher et al. would lead to large fluctuations (actually infinite in the limit of infinite radius), but one knows that the fluctuations in Minkowski spacetime are small.}

Moreover, due to the nonlinear dependence of the flux of radiated energy on the mass of the black hole, terms of higher order in the perturbations become relevant when the fluctuations become of the same order as the mean value of the mass. As pointed out by Bekenstein, this implies a deviation from the usual semiclassical Einstein equation for the evolution of the ensemble/stochastic average of the mass.\footnote{An interesting related question is whether decoherence effects render those fluctuations effectively classical, so that they can be regarded as fluctuations within an incoherent statistical ensemble rather than coherent quantum fluctuations.}

\section*{2. No correlation between the fluctuations of the energy flux crossing the horizon and far from it}

At the time of this meeting (Nov. 2005) we had serious doubts that the same argument that connects the outgoing flux and the flux crossing the horizon, valid for the expectation value of the stress tensor, could also hold for the stochastic source accounting for the stress tensor fluctuations. The reason we gave was the following: while the time derivative of the expectation value, being of higher order in $(m_p/M)$, is negligible in the adiabatic regime, that is not always the case for the stochastic source. Therefore, when integrating the conservation equation and computing the correlation function for the flux crossing the horizon, one gets additional terms besides the correlation function for the outgoing flux. Soon after we came up with a proof that this relation does not hold. This can be found in \cite{F52}, where the assumption of a simple correlation between the fluctuations of the energy flux crossing the horizon and far from it, which was made in earlier work on spherically-symmetric induced fluctuations, was carefully analyzed and found to be invalid (see also Ref. \cite{F50} for a related result in an effectively two-dimensional model). This recent finding would invalidate the working assumption of prior results on black hole event horizon fluctuations based on semiclassical gravity, and points to the necessity of doing the calculation of the noise kernel near the horizon in all seriousness, an effort barely gotten started a few years ago \cite{B13, B14}.

\section*{3. Non-spherically symmetric sector ($l \neq 0$)}

Sorkin \cite{F43}, Casher \textit{et al.} \cite{F42} and Marolf \cite{F44} have provided qualitative arguments for the existence of large quantum fluctuations of the event horizon involving time scales much shorter than the evaporation time\footnote{This effect can be interpreted as follows: due to the growth of the mass fluctuations, the higher order radiative corrections to the semiclassical equation, which involve Feynman diagrams with internal lines corresponding to correlation functions of the metric perturbations (mass perturbations in this case), can no longer be neglected.} which would give rise to an effective width of order $(R_{S_p}^2)_{M}^{1/3}$, much larger than the Planck length (in all cases induced rather than intrinsic fluctuations are implicitly considered). However, Ford and Svaiter \cite{F46} pointed out that Casher \textit{et al.}’s result was probably an artifact from invoking a wrong vacuum to evaluate the fluctuations.\footnote{In all cases a Schwarzschild spacetime rather than that of an evaporating black hole was considered. However, this is a good approximation if one is interested in analyzing fluctuations with correlation times much shorter than the evaporation time. Moreover, one expects for similar reasons that if those large fluctuations did actually exist, they would also be present in the equilibrium case.} Sorkin’s result is based on Newtonian gravity, but Marolf’s work is intended
to be a generalization of Sorkin’s to the general relativistic case. We intend to clarify these apparently
contradictory claims and treat it with the E-L equation and the noise kernel calculations for this case.
Additional insight into this problem can be gained by studying induced metric fluctuations in de Sitter
spacetime. A static observer in de Sitter spacetime perceives the quantum fluctuations of the Bunch-
Davies vacuum as a thermal equilibrium distribution in the same way a static observer outside a black
hole event horizon would perceive the quantum fluctuations of the Hartle-Hawking vacuum. The high degree
of symmetry of de Sitter spacetime makes it easier to obtain exact analytical results. We can check the
validity of claims of large black hole horizon fluctuations by studying the corresponding problems in the de
Sitter spacetime.

IV. PROBING METRIC FLUCTUATIONS NEAR THE HORIZON

Our recent finding that there exists no simple connection between the outgoing flux and the flux crossing
the horizon implies that one needs to compute the noise kernel near the horizon. Ideally one would compute
the noise kernel everywhere in a Schwarzschild background, but the difficulties mentioned above make it very
hard to obtain an analytical result. On the other hand, having a good approximation for the noise kernel
near the horizon might be enough to get the main features because the emission of Hawking radiation is
mostly dominated by what is happening near the horizon.8

A. Computing the noise kernel near the horizon

The key ingredient to compute the noise kernel in a given background spacetime and for a given (vacuum)
state of the quantum matter fields is the Wightman function $G^+(x, y)$. Page developed an approximation to
obtain two-point Green functions for spacetimes which are a vacuum solution of the Einstein equation and
are conformally related to an ultrastatic spacetime. Thus, it can be applied in particular to the Schwarzschild
spacetime. Page’s approximation involves (among other things) an expansion in terms of the geodetic interval
$\sigma$ between the two points in the Green function starting at order $1/\sigma$ and valid up to order $\sigma^2$. Page used
it to obtain the renormalized expectation value of the stress tensor operator. The expansion up to order $\sigma^2$
was enough for his purpose because, after applying the appropriate differential operators and subtracting
the divergent terms in the renormalization process, the contribution from terms of order $\sigma^2$ or higher in the
expansion of the Green function vanishes when the coincidence limit is finally taken.

On the other hand, when computing the noise kernel (which involves a product of two Wightman functions)
using Page’s approximation for the Wightman function one obtains an expansion in powers of $\sigma$ starting
at order $1/\sigma^4$, which coincides with the flat space result, and valid through order $1/\sigma$. Furthermore, since
the only additional scale that appears in the problem is the Schwarzschild radius of the black hole ($= 2M$, the
mass in geometrical/Planckian units), one can conclude by dimensional analysis that the dimensionless
expansion parameter is proportional to $\sigma/M$. In contrast to the expectation value of the stress tensor, one
does not need to take the coincidence limit $\sigma \to 0$ when computing the noise kernel. In fact, when projecting
onto the subset of spherically symmetric multipoles, one needs to integrate over the whole solid angle for
the two points appearing in the noise kernel. Unfortunately, that involves considering pairs of points with
$\sigma \sim M$, for which Page’s expansion in terms of $\sigma/M$ would break down. However, since the first few terms
contain inverse powers of $\sigma/M$, the integral of the noise kernel over the whole solid angle is dominated by
the contribution from small angle separations, i.e., pairs of points with $\sigma/M \ll 1$. Therefore, one expects
that using Page’s approximation for the Wightman function when computing the integral of the noise kernel
over the whole solid angle would provide a fairly good approximation to the actual result. 9

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8 The effect from the fluctuations of the potential barrier for the equation of motion of the radial component could also be
important, but it is not taken into account here.
9 Note, however, that one needs to choose the appropriate analytical continuation to go from the Euclidean Green function
obtained by Page to the Lorentzian Wightman function. Page’s scheme provides the Green function for Hartle-Hawking
vacuum, but the Unruh vacuum should be considered for an evaporating black hole.
B. Probing metric fluctuations

Actually the contribution from small separation angles to the integral discussed in the previous paragraph not only dominates the integral but also gives a divergent result. This implies that the event horizon is no longer well defined as a three-dimensional null hypersurface when the effect of fluctuations is included since the amplitude of its fluctuations is infinite. In order to get a finite result, it is necessary to introduce some additional smearing along the transverse null direction. The final result will then depend on the characteristic size of the smearing functions employed. This means that in addition to seeking workable prescriptions one should also understand the physical meaning of the smearing introduced for this task. This can be achieved by analyzing how the propagation of test fields probes the metric fluctuations of the underlying geometry.

For this purpose a natural first step is to study the effect of those metric fluctuations on a bundle of null geodesics near the horizon. This is expected to provide a good characterization of the propagation of a test field whenever the geometric optics approximation is valid \[F49\], which is certainly the case when studying Hawking radiation in the absence of fluctuations. In Ref. \[F49\], where the particular form of the metric fluctuations was simply assumed rather than derived from first principles, the authors found no dramatic effect due to the fluctuations on the Hawking radiation associated with the test field propagating in the black hole spacetime. In contrast, following the stochastic gravity program, an estimate of the metric fluctuations exhibits a much more singular correlation function. Our preliminary analysis suggests a larger effect on the propagation of a bundle of null geodesics near the horizon, which would imply a substantial modification of the Hawking effect derived under a test field approximation.

However, it is likely that the large fluctuations found this way for a bundle of null geodesics sufficiently close to the horizon is signaling a breakdown of the geometric optics approximation rather than an actual drastic modification of Hawking’s result. This assessment stems from a similar situation in flat space: Near a Rindler horizon large fluctuations arise from using the geometric optics approximation, but they are artifacts when examined in a more detailed quantum field theory calculation. If confirmed, this would mean that the propagation of a bundle of geodesics does not constitute an adequate probe of spacetime fluctuations in this context. Although computationally more involved, a more reliable way to extract physically meaningful information about the effect of metric fluctuations on the Hawking radiation is to consider the Wightman two-point function for the test field, which characterizes the response of a particle detector for that field, and analyze the radiative corrections when including the interaction with the quantized metric perturbations.

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Stochastic gravity

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