Pattern of Light Scalar Mesons

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Combining the recent lattice calculation of $a_0(1450)$ and $\sigma(600)$ mesons with the overlap fermion in the chiral regime with the pion mass less than 300 MeV, the quenched lattice calculation of the scalar glueball, and the phenomenological study of the mixing of isoscalar scalar mesons $f_0(1710)$, $f_0(1500)$, $f_0(1370)$ through their decays, a simple pattern for the light scalar mesons begins to emerge. Below 1 GeV, the scalar mesons form a nonet of tetraquark mesoniums. Above 1 GeV, the nonet $q\bar{q}$ mesons are made of an octet with largely unbroken $SU(3)$ symmetry and a fairly good singlet which is $f_0(1370)$. $f_0(1710)$ is identified as an almost pure scalar glueball with a $\sim 10\%$ mixture of $q\bar{q}$.

§1. Introduction

In light meson spectroscopy, the pseudoscalar, vector, axial, and tensor sectors are reasonably well known in terms of their $SU(3)$ classification and quark contents. The scalar sector, on the other hand, is poorly understood in this regard. First of all, there are too many of them. There are 19 states which are more than twice the usual $q\bar{q}$ nonet as in other sectors. We show in Fig. 1 the known scalar mesons which include $\sigma(600)$, $\kappa(800)$, and $f_0(1710)$ which are better established experimentally nowadays. There are several puzzling characteristics which have been observed over the years. The first question one might raise is the whereabouts of the $q\bar{q}$ $a_0$, the $^3P_0$ partner of $a_1(1260)$ ($^3P_1$) and $a_2(1320)$ ($^3P_2$) according to the quark model classification. From the order of spin-orbit splitting of the P-wave $q\bar{q}$ spectrum, it seems natural to identify it with $a_0(980)$. However, there are a host of difficulties in such an assignment:

- In this case, the member of the octet $K_0^*$ (e.g. $s\bar{u}$ with one strange quark) is expected to lie $\sim 100$ MeV above, which would place it around 1100 MeV. But there is no state there, it would be $\sim 300$ MeV below $K_0^*(1430)$ and $\sim 300$ MeV above $\kappa(800)$.
- The widths of $a_0(980)$ and $f_0(980)$ are substantially smaller than those of $a_0(1450)$ and $f_0(1370)$. In particular, they are much smaller than that of $\kappa(800)$ which should be a nonet partner of $a_0(980)$ and $f_0(980)$.
- The $\gamma\gamma$ widths of $a_0(980)$ and $f_0(980)$ are much smaller than expected of a $q\bar{q}$ state.
- It is hard to understand why $a_0(980)$ and $f_0(980)$ are practically degenerate. The experimental data on $D_s^+ \rightarrow f_0(980)\pi^+$ and $\phi \rightarrow f_0(980)\gamma$ imply copious $f_0(980)$ production via its $s\bar{s}$ component. Yet, there cannot be $s\bar{s}$ in $a_0(980)$ since it is an $I = 1$ state.

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• The radiative decay $\phi \to a_0(980)\gamma$, which cannot proceed if $a_0(980)$ is a $q\bar{q}$ state, can be nicely described in the kaon loop mechanism.\textsuperscript{4} This suggests a considerable admixture of the $K\bar{K}$ component which is in contradiction with assigning $a_0(980)$ as the $q\bar{q}$ meson.

![Figure 1](image.png)

Fig. 1. Spectrum of scalar mesons together with $\pi$, $\rho$, $a_1$ and $a_2$ mesons.

Some of the above difficulties can be reconciled if $a_0(980)$ and $f_0(980)$ are part of the nonet of four-quark (two-quarks and two-antiquarks) mesons which was first proposed by Jaffe based on the MIT bag model calculation.\textsuperscript{5} The ensuing potential model studies of these four-quark mesons are also carried out\textsuperscript{6,7} and it was suggested that $a_0(980)$ and $f_0(980)$ are the $K\bar{K}$ molecular states.\textsuperscript{7} We shall refer them generically as tetraquark mesoniums, not to be concerned with their possible clustering structure. With the four quark content, it is relatively easy to understand the degeneracy of $a_0(980)$ and $f_0(980)$ and their narrow widths. Since they have the quark content $u(d)\bar{u}(\bar{d})s\bar{s}$ and sit at $K\bar{K}$ threshold, they do not have much phase space to decay to $K\bar{K}$ ($a_0(980)$ decay to $\eta\pi$ is suppressed by having to go through the $s\bar{s}$ in $\eta$); whereas, $\sigma(600)$ and $\kappa(800)$ are relatively far above the respective $\pi\pi$ and $\pi K$ thresholds and hence have much larger widths.

Recent experimental finding of $\sigma(600)$ in $D^+ \to \pi^+\pi^-\pi^+\pi^+$ and $J/\psi \to \omega\pi^+\pi^-$\textsuperscript{8} and the dispersion analysis of $\pi\pi$ scattering with the Roy equation\textsuperscript{8} which found a resonance at $441_{-8}^{+16}$ MeV with a width of $544_{-25}^{+18}$ MeV have helped establish the existence of the broad $\sigma$ resonance. Besides the low-lying scalar mesons, other candidates for tetraquark mesoniums include those vector mesons pairs produced in $\gamma\gamma$ reactions\textsuperscript{9} and hadronic productions\textsuperscript{10} and the recently discovered charmed narrow resonances.\textsuperscript{11}

Given that the spectrum below 1 GeV is better understood, many questions about classification of scalar mesons above 1 GeV are still outstanding. For example:

• The $K_0^*(1430)$, which is a $q\bar{q}$ state in all the models,\textsuperscript{1} lies higher than the axial-
vector mesons $K_1(1270)$ and $K_1(1400)$. This is a situation which parallels the
case of non-strange mesons where $a_0(1450)$ is higher than $a_1(1260)$ and $a_2(1320)$
and is contrary to the conventional wisdom in the quark model as far as the
order of spin-orbit splitting is concerned.

- It is not clear why $K_0^*(1430)$, having one strange quark, is almost degenerate
  with $a_0(1450)$, assuming the later is $(u\bar{u} - d\bar{d})/\sqrt{2}$. This is in contrast with all
  the other meson sectors.
- In the $I = 0$ channel, there are three states – $f_0(1370), f_0(1500)$ and $f_0(1710)$
  and they are expected to be $(u\bar{u} + d\bar{d})/\sqrt{2}, s\bar{s}$ and glueball. Which is which? Is
  the mixing more like that of the pseudoscalar sector where there is substantial
  mixing between $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$, or those of the vector and tensor sectors
  where the mixing are nearly ideal between the octet and the singlet?

In the following, we shall use a recent lattice calculation to verify the existence of
$\sigma(600)$ as a tetraquark mesonium to help establish the classification of the low-lying
scalars below 1 GeV. We will also use lattice calculations of $a_0(1450), K_0^*(14300)$ and
 glueball together with the analysis of various decays to discern the mixing among
$f_0(1370), f_0(1500)$ and $f_0(1710)$. Based on these, a simple pattern of scalar mesons is
beginning to surface as will be described in the subsequent sections.

§2. Lattice Calculation

Although there are MIT bag model\(^5\) and potential model calculations\(^6,7\) of
tetraquark mesoniums, lattice QCD is perhaps the most desirable theoretical tool
to adjudicate whether these four-quark states exist and if the low-lying scalars are
indeed the predicted tetraquark mesoniums. To begin with, we note that a resonance
can be viewed as a mixture of a bound state and the continuum of scattering states.
To establish the existence of a resonance on the Euclidean lattice, one can utilize
the volume effect of a finite box where all the eigenstates are discrete (e.g. with a
periodic boundary condition, the available momenta are $p_L = n\frac{2\pi}{a_L}, n = 0, \pm 1, \pm 2...$)
and check if there exists a bound state which is separated from the discrete scattering
states. In the context of the existence of $\sigma(600)$, one needs to first work in the
chiral region where $m_\pi < 300$ MeV in recognition of the fact that the occurrence
of $\sigma$ is on the basis of ‘current algebra, spontaneous symmetry breakdown, and
unitarity’.\(^8\) Secondly, one needs to identify both the tetraquark mesonium and the
collateral $\pi\pi$ scattering states. Thirdly, it is necessary to work on a lattice where
the scattering states and the bound state are well separated (e.g. further apart than
half of the ‘would be’ resonance width) in order to discern the nature of these states
separately to make sure that $\sigma$ is indeed a one-particle state and not a two-particle
scattering state. To this end, a recent lattice QCD calculation was carried out on
$12^3 \times 32$ and $16^3 \times 32$ lattices with $a = 0.2$ fm and 300 configurations to examine the
spectrum with the $\bar{\Psi}\gamma_5\Psi\bar{\Psi}\gamma_5\Psi$ type of four-quark interpolation operators. Although
a quenched calculation, it incorporates the chiral fermion (overlap fermion) in the
chiral region with the pion mass as low as 182 MeV. Results on the $12^3 \times 28$ lattice
are presented in Fig. 2 as a function of $m_\pi^2$ for the pion mass range from 182 MeV
to 250 MeV. The lowest state is about 100(30) MeV below the $\pi\pi$ threshold which is indicated by the solid line. This is the lowest interacting state of two pions which is attractive in the $I = 0$ channel and is reasonably well described by the quenched chiral perturbation theory of $\pi\pi$ scattering.\textsuperscript{13) The third state is above the non-interacting $\pi\pi$ scattering state with each pion having one unit of lattice momentum (i.e. $p_1 = 2\pi/La$) and is supposed to include the higher excited states which are not fitted. The interesting thing is that there is an extra state at $\sim 550$ MeV which falls in between the two states. To discern the nature of this state, we studied the volume dependence of the spectral weight $W_i$ from the fitting function $\sum_i W_i e^{-E_i t}$ of the tetraquark correlator. The details are given in Ref.\textsuperscript{12) To summarize the results, we found that, by examining the characteristic 3-volume dependence of the spectral weight, the state at $\sim 550$ MeV is a one-particle state, while the lowest state is a two-particle state which is consistent with the quenched chiral perturbation prediction of the interacting $I = 0 \pi\pi$ scattering state.\textsuperscript{13)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{four_quark_spectrum.png}
\caption{The lowest three states from the scalar tetraquark correlator as a function of $m_\pi^2$ for $m_\pi$ from 182 MeV to 250 MeV on the $12^3 \times 28$ lattice. The solid lines indicate the energies of the two lowest non-interacting pions in S-wave with lattice momenta $p_0 = 0$ and $p_1 = 2\pi/La$.}
\end{figure}

This verifies that the tetraquark mesonium exists and the lattice calculation, which gives a mass of $540 \pm 170$ MeV at the chiral limit, suggests that $\sigma(600)$ is such a state. However, one important question remains. Experimentally, $\sigma$ is a very broad resonance with a width of 544 MeV.\textsuperscript{8) How does one find its width on the lattice? After finding $\sigma(600)$ which is separated from the $\pi\pi$ scattering states on the present lattice, one can increase the box which will lower the energies of the scattering state above it. When it is lowered to within the range of the ”width”, it mixes with the bound state and avoids level crossing. From the energy of the mixed state one can deduce the scattering phase shift from Lüscher’s formula.\textsuperscript{14) This is valid for elastic scattering irrespective how broad the resonance is. This is studied in detail in a spin...
model\textsuperscript{15}) which illustrates how the scattering state mixes with the bound state and gives rise to the phase shift as the volume is increased. In a sense, by varying the lattice volume, hence the momentum, one can use a scattering state to mix with the bound state and scan the energy range to obtain the phase shift and therefore the width of the resonance. The information of the width can also be obtained by determining how far apart in energy the scattering and bound state start to avoid the level crossing.

To calculate $a_0$ on the same lattices, the two-quark interpolation field $\Psi \Psi$ was used. We plot its mass as a function of the corresponding $m_\pi^2$ in Fig.3 together with that of $a_1$ for comparison. We see that above the strange quark mass, $a_1$ lies higher than $a_0$ as expected from the quark model for heavy quarks. However, when the quark mass is smaller than that of the strange, $a_0$ levels off, in contrast to the $a_1$ case and those of other hadrons that have been calculated on the lattice. This confirms the trend that has been observed in earlier lattice calculations with higher quark masses in quenched approximation\textsuperscript{16),17} as well as with dynamical fermions.\textsuperscript{18} The chirally extrapolated mass $a_0 = 1.42 \pm 0.13$ GeV suggests that the meson $a_0(1450)$ is a $q\bar{q}$ state. By virtue of the fact that we do not see $a_0(980)$, its $q\bar{q}$ content is estimated to be two orders of magnitude smaller than that of $a_0(1450)$.\textsuperscript{12} The $K_0^*(1430)$ mass at $1.41 \pm 0.12$ GeV is calculated with the strange mass fixed to reproduce the $\phi$ mass and the $u/d$ extrapolated to the chiral limit and the corresponding $s\bar{s}$ state from the connected insertion (no annihilation) is $1.46 \pm 0.05$ GeV. These lattice results are consistent with the experimental fact that $K_0^*(1430)$ is basically degenerate with $a_0(1450)$ despite having one strange quark. This resolves one of the puzzles outlined in Sec.1 which is hard for quark models to accommodate.

![Graph](image_url)

**Fig. 3.** Masses of $a_0$, $a_1$ and two pions (dashed line) are plotted as a function of $m_\pi^2$.

Latest large scale calculation of glueball masses on anisotropic lattices gives the
scalar glueball mass at 1710(50)(80) MeV in the quenched approximation\textsuperscript{19} which seems to coincide with $f_0(1700)$ discovered in $J/\Psi$ radiative decays, long suggested to be a channel for copious glueball production.

\section*{§3. Mixing and Decays}

To answer such questions as raised in Sec.\textsuperscript{4} there have been a number of studies on the mixing of the isoscalar mesons $f_0(1370), f_0(1500),$ and $f_0(1710)$ to sort out their glueball and flavor content of $q\bar{q}$\textsuperscript{20}. In considering the mixing matrix, the usual premise is to place the unmixed (connected insertion without annihilation) $s\bar{s} \sim 200$ MeV above $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ to reflect the pattern well-known in other meson sectors as well as in baryons. However, this is not appropriate here. It runs counter to the fact that $K_0^*(1430)$ is basically degenerate with $a_0(1450)$. In view of the lattice results discussed in Sec.\textsuperscript{22} where one finds that $a_0(1450), K_0^*(1430)$, and the unmixed $ss$ are nearly degenerate, an apparent conclusion is that the scalar $q\bar{q}$ mesons have, to first order, an unbroken $SU(3)$ octet. As a result, $f_0(1500)$, which is close to $a_0(1450), K_0^*(1430)$, should be a fairly pure $f_{\text{octet}} = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ state. A mixing model, which takes the degeneracy of unmixed $n\bar{n}$ and $s\bar{s}$ and the quenched prediction of scalar glueball mass at $\sim 1700$ MeV into account with slight $SU(3)$ breaking, is quite successful in delineating the decays into pseudoscalar pairs of the isoscalar mesons as well as various decays from $J/\Psi$. The details of the fit and predictions are given in a previous work\textsuperscript{21}. We want to point out several salient and robust features in the resultant mixing and decay patterns.

- $f_0(1500)$ is indeed a fairly pure octet ($f_{\text{octet}}$) with very little mixing with the singlet and the glueball. $f_0(1710)$ and $f_0(1370)$ are dominated by the glueball and the $q\bar{q}$ singlet respectively, with $\sim 10\%$ mixing between the two. This is consistent with the experimental result $\Gamma(J/\psi \rightarrow \gamma f_0(1710)) \sim 5 \Gamma(J/\psi \rightarrow \gamma f_0(1500))$\textsuperscript{2}\ which favors $f_0(1710)$ to have larger glueball component\textsuperscript{21}.

- The ratio $\Gamma(f_0(1500) \rightarrow K\bar{K})/\Gamma(f_0(1500) \rightarrow \pi\pi) = 0.246 \pm 0.026$ is one of the best experimentally determined decay ratios for these mesons\textsuperscript{1}. When the mixing with glueball and $SU(3)$ breaking are neglected, one obtains

$$\frac{\Gamma(f_0(1500) \rightarrow K\bar{K})}{\Gamma(f_0(1500) \rightarrow \pi\pi)} = \frac{1}{3} \left( 1 + \frac{s}{u/d} \right)^2 \frac{p_K}{p_\pi},$$

where $p_h$ is the c.m. momentum of the hadron $h$, $u/d$ and $s$ are the coefficients for the $u\bar{u}/d\bar{d}$ and $s\bar{s}$ components of the $f_0(1500)$ wavefunction. If $f_0(1500)$ is a glueball (i.e. a flavor singlet) or $s\bar{s}$, the ratio will be 0.84 or larger than unity. Either one is much larger than the experimental value. On the other hand, if $f_0(1500)$ is $f_{\text{octet}}$, then the ratio is 0.21 which is already close to the experimental number. This further demonstrates that $f_0(1500)$ is mainly an octet and its decay ratio can be well described with a small $SU(3)$ breaking\textsuperscript{21}.

- Because the $n\bar{n}$ content is more copious than $s\bar{s}$ in $f_0(1710)$ in this mixing scheme, the prediction of $\Gamma(J/\psi \rightarrow \omega f_0(1710))/\Gamma(J/\psi \rightarrow \phi f_0(1710)) = 4.1$ is naturally large and consistent with the observed value of $6.6 \pm 2.7$. This ratio is not easy to accommodate in a picture where the $f_0(1710)$ is dominated by
s\bar{s}$. One may have to rely on a doubly OZI suppressed process to dominate over the singly OZI suppressed process to explain it\cite{20}.

\[
\begin{array}{c}
f_0(1710) \\
K_0^+(1430) & K_0^0(1430) \\
a_0(1450) & a_0(1450) \\
K_0^0(1430) & K_0^+(1430) \\
f_0(1500) & f_0(1370) \\
\end{array}
\]

\[
\begin{array}{c}
\kappa(800) & \kappa(800) \\
a_0(980) & a_0(980) \\
\kappa(800) & \kappa(800) \\
\end{array}
\]

Fig. 4. Pattern of light scalar mesons – a tetraquark mesonium nonet below 1 GeV, an almost pure $SU(3)$ $q\bar{q}$ nonet and a nearly pure glueball above 1 GeV.

§4. Conclusion

Notwithstanding many detailed questions remain unanswered satisfactorily, lattice QCD calculations of scalar meson and glueball masses and a phenomenological study of meson decays and their mixing have suggested that a pattern for light scalar
mesons is starting to arise – a tetraquark mesonium nonet below 1 GeV, and an almost pure $SU(3)$ $q\bar{q}$ nonet and a fairly pure glueball above 1 GeV. It should be scrutinized by experiments in the future, such as with high statistics $J/\Psi$ and $D$ decays and $p\bar{p}$ annihilation. Lattice calculations with light dynamical fermions are needed to check the pattern and determine the strong decay widths of these mesons.

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