Nonlinear robust compensator design for postural control of a biomechanical model with delayed feedback

Nadia Sultan, Muhammad Najam ul Islam and Asif Mahmood Mughal

Abstract
Postural stability and balance regulation is an intricate neurophysiological task which entails coordination of movements for successful execution. This task is proficiently regulated by central nervous system. The sensory feedback through muscles via proprioceptors has neural transmission delays which make the movement coordination and computations by central nervous system a complex problem to deal with. This paper addresses a nonlinear robust technique based on feedback linearization for postural stabilization of a single-link biomechanical model in the presence of physiological latencies. We included neural transmission delays in sensory feedback from proprioceptors. We developed $H_2$ optimal controller and integrated it with feedback linearization to calculate the joint torque for the biomechanical task. This modeling scheme is simulated in MATLAB/SimMechanics, and the simulation results for the nonlinear biomechanical model are developed. The joint torque compensates for the delays and settles the motion profiles within anatomical constraints. The position profile shows a bit higher overshoot (0.02, 0.03 rad) in case of delays; however, the settling time is same for the profiles with and without delay. The extensor torque is same for all profiles; however, the flexion torque increases for the delayed case. The simulation results show the applicability of this scheme for further analysis of the biomechanical task.

Keywords
Biomechanical model, delayed feedback, feedback linearization, $H_2$ compensator, nonlinear control

Introduction
Maintenance of balance in an upright posture is basic and perhaps the most essential requirement in daily life. In the context of the subject research, human postural movements are of interest for a variety of purposes, for example, designing and control of prosthetics, diagnosis and rehabilitation processes, balance recovery, stability during motion, fall prevention in elderly ages and analysis of neuro-muscular interaction. A complex neuromusculoskeletal system maintains postural stability and balance in humans. Postural stabilization entails proprioceptive and somatosensory as well as visual and vestibular inputs to control posture managing muscles in the entire body, particularly in the lower leg and trunk. The central nervous system (CNS) therefore simultaneously regulates numerous muscles, depending on the respective multisensory outputs. Joint torques are low in elderly people because of slower postural movements since they have muscular deficiencies. Muscle feedback signals through proprioceptors have neural transmission delays. These delays make the movement coordination and calculations by CNS an exceptionally intricate problem to deal with. The mechanism by which humans control balance in upright stance is not yet fully understood due to complex neural control processes and is an area of active research in biomechanics.

Different strategies are adopted by humans for balance recovery after perturbations. The ankle strategy is adopted for small perturbations. It involves the stabilizing movement about the ankle joint moving whole body as a single-link inverted pendulum. For larger perturbations, hip and ankle strategies are adopted, and for still larger perturbations, motion at the knee joint is allowed along with the ankle and hip joint motion. To maintain balance, the CNS needs data about the overall position of body segments as well as the magnitude of

Department of Electrical Engineering, Bahria University, Islamabad, Pakistan

Corresponding author:
Nadia Sultan, Department of Electrical Engineering, Bahria University, Islamabad 44000, Pakistan.
Email: nadiaimran.buic@bahria.edu.pk
forces acting on the body. Three different classes of sensors are used for this purpose: visual, vestibular and somatosensory. These sensory receptors may act differently for each disorder in the human body, for example, injury, disease, certain drugs or aging process. The minimum and essential task of postural control framework in a static position is to keep the vertical projection of center of mass (COM) of an individual within the base of support (BOS) defined by the length of stationary foot. The stability limits rely upon the COM height, weight of body and the area of BOS. The COM position and velocity also influence stability limits. The stiffness of the musculoskeletal system plays a vital role in balance recovery.

The active or feedforward and stiffness of the musculoskeletal system plays a vital role in position and velocity also influence stability limits. The height, weight of body and the area of BOS. The COM position and velocity also influence stability limits. The stiffness of the musculoskeletal system plays a vital role in balance recovery. The active or feedforward and the passive or feedback mechanisms are involved in postural and voluntary movements stabilization.

To understand and improve human postural balance and movement mechanism, researchers used mathematical models of the musculoskeletal system. According to Kuo, analysis of human postural balance in upright stance is particularly well suited for modeling and representation in this paradigm is convenient and can be utilized by the researchers to test the hypothesis of motor control. These models are based on inverted pendulum on account of its resemblance to the human body and its inherent instability. The controllers for these models imitate the control function of CNS that issue commands to muscle actuation. Feedback-controlled models of standing stance have been used by researchers to pick up understanding into the neural control by reactivating calm position or subjecting to postural perturbations. Davidson et al. collected the kinematic data from 16 young (ages: 19.4 ± 1.4 years) and 16 older adults (ages: 62.2 ± 5.1 years). They simulated the recorded data on a time-delayed single-link biomechanical model in the sagittal plane. They found that the sensory input and time delay in feedback loops are significantly different in two age groups. Hidler and Rymer conducted a simulation study of reflex loop instability. They reported that a combination of excessive time delays and increased motoneuron sensitivity may lead to instability. Insperger et al. proposed a single-link inverted pendulum model to study human postural control. They represented the sensory dead zones with open-loop periods. The proportional–derivative–acceleration (PDA) provided feedback control during closed-loop periods with appropriate feedback gain to stabilize the inverted pendulum. In this way, it merits referring that the acceleration information can be likewise important for stabilization. Roy and Iqbal studied the postural stabilization problem with delays in position and velocity feedback. In their research, they stabilized the inverted pendulum–based single-link biomechanical model with proportional–integral–derivative (PID) controller as CNS function. The PID controller stabilized the plant, but one can observe oscillations in the transient state which are not desirable. These oscillations are due to linearization of the model at upright stance. Mughal and Iqbal developed the bond graph model of the human musculoskeletal system. They investigated the designed system for human postural equilibrium with $H_2$ and PID control techniques. In their proposed control scheme, the system oscillated during the transient state and the simulations took a long time of 6–10 s to settle down the plant. The biomechanical model with inertial and gravitational components is nonlinear. Linearizing a nonlinear model leads to linearization error due to unmodeled dynamics, resulting in the higher amplitude of joint input torques as well as an undesired deviation in the angular profiles which violates the physiological boundaries. The linear controllers provide control in a small neighborhood of the operating region where linear approximations are valid. These issues can be addressed with the use of nonlinear control techniques.

Motivated by the nonlinear observer-based control, we propose a modeling simulation technique for maintenance of postural stability in the presence of neural delays and white noise disturbance. The simulation results presented in this paper show an improvement upon the previous research where results were not according to expectations. Our proposed technique is based on feedback linearization combined with $H_2$. Proportioceptive feedback delays are included in position and velocity from muscle spindles (MSs). The Padé-approximation technique approximates the delays in states. The task-specific command signals from CNS and force generation in muscles are represented by the torque at ankle joint. The strength of feedback linearization lies in the fact that due to the exact cancelation of nonlinearities, the techniques viable for linear systems are also viable for it. Unlike the previous work on postural stability, our current study has the following features: (1) it proves asymptotic stability not only of estimation error but of closed-loop system in the presence of delays and noise disturbance; (2) it compensates for the delays and regulates the body movement in the presence of perturbations and noisy joint sensors and thus demonstrates the performance recovery; (3) it settles the torque and motion profiles within anatomical constraints, and the profiles are physiologically more relevant. However, this study is limited to restricted knee and hip joint motion. Summarily, this paper presents a nonlinear robust compensator design which represents the CNS function to regulate and control the movement coordination during maintenance of postural stability in the presence of neural transmission delays and noisy joint sensors.

**Biomechanical model formulation**

For small perturbations, the stabilizing movements are produced around the ankle joint. Keeping in view the stabilization problem for small perturbations, the mechanics of human body in performance of postural
movements are modeled as two segments structure with torque actuation at the ankle joint. Head–arm–trunk, thighs and legs are represented by a single segment over stationary feet. This segment can rotate about the ankle joint. The foot length represents the BOS in anterior–posterior directions. This biomechanical model represents symmetrical arrangement of feet along with the body segment in the sagittal plane as shown in Figure 1. The COM for static postural stability and the center of pressure (COP) for dynamic stability must be restricted to the BOS. The mass of segment is represented by $m$, length of segment is represented by $l$ and inertia by $I$. The distance from COM to joint is represented by $k$, and $\theta$ is the ankle angle from horizontal. It represents the segment position in terms of angle. The CNS command signals and force generation in muscles are represented by the torque actuation $\tau$ at the ankle joint. Two forces act on the biomechanical model, that is, the gravitational force $mg$ and the net torque generated by muscle tendon complexes about the ankle joint.

Nonlinear biomechanical model

The nonlinear biomechanical model is represented by the following dynamic equation

$$\dot{\theta} = \frac{\tau + mgk \sin \theta}{I + mk^2}$$

This dynamical equation can be represented in nonlinear state space as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{mgk \sin x_1}{I + mk^2} \end{bmatrix} + \begin{bmatrix} 0 \\ \tau \end{bmatrix}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$.

In the above equation, $\theta$ and $\dot{\theta}$ represent the angular position and angular velocity of the segment, respectively. The inertial component of model establishes the muscle force which then transforms into ankle joint torque. The angular position and angular velocity of segment transform into fascicle-length and fascicle-velocity as a contribution to the muscle-spindle assembly.

The biomechanical parameters, that is, the segment mass $m$, length $l$, inertia $I$ and distance from COM to joint $k$ are based on average anatomical proportions defined in Table 1 taken from the study by De Leva. Since we are interested in ankle position posture, so we are taking output from $x_1$ state

$$y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Neural delays

The nonlinear model defined in equation (2) is a second-order model with angle and velocity variables at the ankle joint. The joint angle and velocity are assumed to have physiological transmission delays to CNS. These latencies represent the overall feedback delays and are due to proprioceptors (MSs and Golgi tendon organs (GTOs)). The ideal time delay is expressed as $e^{-Ts}$ in frequency domain. Since $e^{-Ts}$ cannot be comprehend in finite dimensions, this term should be represented in finite-dimension rational-approximation for analysis and synthesis purposes. Researchers have used hyperbolic functions, Bessel functions, Laguerre polynomials and Pade approximations to approximate time delay. Among these techniques, Pade approximation seems appropriate for the current application since it is widely used in control system theory to approximate delays. The delay in position and velocity feedback is represented by the following transfer function

$$e^{-Ts} \approx \frac{T_2 s^2 + T_1 s + 1}{T_2 s^2 + T_1 s + 1}$$

The system delay is represented as

$$m(t) = \begin{bmatrix} \ddot{x}(t - e^{-Ts}) \\ \ddot{x}(t - e^{-Ts}) \end{bmatrix}$$

where $m(t)$ represents the delayed measurements in both states. The delay in ankle position posture and velocity is caused by the proprioceptive feedback. These delays are finite and time-invariant evaluated through second-order Pade approximation given in equation (4). In an
experimental study carried out by Rowland et al., the feedback delays were recorded to be 15–18 ms when both the visual and auditory stimuli were individually effective at eliciting responses, whereas delays were 28–30 ms when only the auditory stimulus was effective. Keeping in view the various ranges of delays, we select the two values (lower and upper bounds) of short delays. We simulate our model for time delays $T = 15$ and $30$ ms, which are approximated and observed through nonlinear compensator design.

**Robust nonlinear compensator design**

**Feedback linearization controller design**

Feedback linearization is a powerful technique based on vector field methods used in controlling nonlinear systems. The central idea of this approach involves coming up with the transformation of nonlinear system dynamics into partially or full linearized system, either by the change of variables or suitable control input. The loop control strategy for the resulting linear system can then be applied. It is entirely different from the Jacobian linearization since the linearization is achieved either by cancelation or by exact state transformation.

In order to define the relative degree of the output in equation (3), we take Lie derivative

$$L_{y} \mu(x) + \partial_y \mu(x) u$$

where $L_{y} \mu(x) = (\partial \mu / \partial x)(x)$.

The coefficient of $u$ in equation (7) is $L_{y} \mu(x) = 0$. Thus, $\dot{y} = L_{y} \mu(x)$ is independent of $u$. Now, we need to calculate the second derivative of $y$

$$y^{(2)} = \partial_y \mu(x) u$$

The input $u$ appears in equation (8) with non-zero coefficient which shows that the output has a vector relative degree of order 2. The superscript $y$ in equation (8) shows its relative degree. It is clear from the above equation that the system is input–output linearizable and the state feedback control law can be defined as

$$u = \frac{1}{L_{y} \mu(x)} \left(-L_{y}^{2} \mu(x) + \omega\right)$$

where $\omega$ defines the linear gains in above equation and $u$ is the controlled input torque $\tau$. The nonlinear feedback gains in equation (9) eliminate the nonlinearities in the system reducing the input/output map to

$$y^{(2)} = \omega$$

Now we need to choose $\omega$ such as to assign the closed-loop eigenvalues in the left half-plane and achieve asymptotic stabilization. We have used $H_2$ compensator to calculate linear gains, discussed in section “$H_2$ compensator.”

**Feedback linearization observer design**

Since the delayed states are not available for measurement, so there is a need to design a state estimator that gives an approximation of position, velocity and delays in these states. In our proposed research, a full-state observer is designed using output feedback linearization. The nonlinear plant defined in equation (2) is fully observable as can be seen from the observability matrix

$$\begin{bmatrix} L_{y}^{0}(Dh) \\ L_{y}^{1}(Dh) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The controlled input torque $u$ in equation (9) stabilizes the origin $x = 0$ of the closed-loop system. To execute the feedback control utilizing estimations of the output $y$, we again consider equations (2) and (3). The state observer can be defined as

$$\begin{align*}
\dot{x}_1 &= \dot{x}_2 + G_1(y - \dot{x}_1) \\
\dot{x}_2 &= \frac{mgk \sin \dot{x}_1}{I + mk^2} + \frac{u}{I + mk^2} + G_2(y - \dot{x}_1)
\end{align*}$$

$G_1$ and $G_2$ represent the linear gains. The $H_2$ control technique is used to calculate the linear gains and estimate the delayed states, which diminish the effects of noise disturbance and compensate for delays as explained in the next section.

**$H_2$ compensator**

Deterministic robust compensator has gained eminence considering its stability in the presence of both disturbances and parametric uncertainties. Due to this quality, the robust compensators are utilized for stabilization of biomechanical models in different applications, for example, active prosthetics. The lack...
of robustness and excessive bandwidth requirement of controller prompted the formulation of $H_2$ control problems.\textsuperscript{38} $H_2$ control is an efficient, robust and optimal control method which minimizes the effects of disturbances, noises and bound the power gain of the system. The linear gains of state feedback law defined in equation (9) and the observer in equation (11) are calculated using $H_2$ control technique. The model defined in equation (1) is linearized at the upright standing posture $\theta = 90^\circ$ with ground as a frame of reference. The equilibrium points in the static upright standing position. This increases the order of angle and velocity states after linearizing the model at present at the plant input and can be part of either process or sensor noise. We modeled input noise with signal power $S_{\phi}(\tau) = 1 \mu W$ as bandlimited white noise. The measurement noise disturbance defined in equation (15) of signal power $S_{\varphi}(\tau) = 4nW$ has been added at the output and provides for any noise that can be present at the output of plant. The neural delays defined in equation (5) are approximated with second-order Pade approximation as defined in equation (4). In order to compensate for these delays, we propose a fourth-order compensator. The overall system is

$$
P = \begin{bmatrix} A & B_w & B_u \\
C_y & D_{yw} & D_{yu} \\
C_m & D_{mw} & D_{mu} \end{bmatrix}
$$

We incorporate second-order Pade approximation in angle and velocity states after linearizing the model at standing position. This increases the order of $A$ matrix to fourth order. $B_u$ is $4 \times 2$ matrix and represents the measurement and process noise in the system. $B_w$ is $4 \times 1$ matrix and represents the input torque at ankle joint. $C_y$ is $2 \times 4$ matrix and represents the state weighting function for controller and estimator design. $C_m$ is $1 \times 4$ matrix and represents the measurement output angle. $D_{yu}$ and $D_{mu}$ are $2 \times 2$ and $1 \times 1$ matrices and are taken to be zero. This is the requirement of $H_2$ control in order to obtain finite $H_2$ norm. $D_{yu} = 0$ results in zero direct feedthrough from $w$ into objective function. $D_{mu} = 0$ results in zero direct feedthrough from $u$ into measurement function. $D_{yu}$ represents input weighting function and is a $2 \times 1$ matrix. $D_{mu}$ represents the “disturbance to plant output” weighting function. It is a $1 \times 2$ matrix and is kept very small. In this designed problem, two states are measurable and two states are estimated. These two states are introduced by Pade approximation delays. The values of state and input weighting functions represented by $C_y$ and $D_{yu}$ are scaled for states and input optimization. Without scaling the delayed states will get equivalent priority in optimization. The whole system defined in equation (16) must satisfy the conditions given in the study by Doyle et al.\textsuperscript{38} for stable operation of the system. The assumptions on $P$ matrix are given below:

1. The pair $(A, B_u)$ and $(A, B_w)$ are stabilizable. $(A, B_u)$ has no uncontrollable modes on the imaginary axis;
2. $(C_m, A)$ and $(C_y, A)$ are detectable. $(C_y, A)$ has no unobservable mode on the imaginary axis;
3. $D_{yw}^T C_y = 0$ and $B_w D_{mu}^T = 0$;
4. $D_{yu}^T D_{yu} = I$ (invertible);
5. $D_{mu}^T D_{mu} = I$ (invertible).

These conditions are necessary for stabilizing controller and state estimator. The measured and controlled outputs are given as

$$
\begin{bmatrix} m(t) \\
y(t) \end{bmatrix} = \begin{bmatrix} C_m & C_y \\
D_{yu} & D_{mu} \end{bmatrix} x(t) + \begin{bmatrix} D_{nu} & D_{mu} \end{bmatrix} u(t) + \begin{bmatrix} D_{nu} & D_{wu} \end{bmatrix} w(t)
$$

$m(t)$ represents the delayed measurements as defined in equation (5). The algebraic Riccati equations for controller and state estimator are given in equations (18) and (19) as defined in the study by Doyle et al.\textsuperscript{38} and Zhou et al.\textsuperscript{39} A steady state solution is obtained after solving these two equations if a positive semi-definite solution exists for the designed problem

$$
S A + A^T S - S (B_u B_u^T) S + C_y^T C_y = 0
$$

$$
Q A + A^T Q - Q (C_m^T C_m) Q + B_u B_u^T = 0
$$

The fourth-order compensator equation is given as

$$
\dot{x}_c = A_c x_c(t) + B_c m(t)
$$

where $A_c$ is fourth-order matrix with ankle position, velocity and delays in these states

$$
A_c = A - B_u B_u^T S - Q C_m C_m^T
$$

$B_c$ is $4 \times 1$ order matrix for measurement and represents the linear estimator gain matrix

$$
P = \begin{bmatrix} A & B_w & B_u \\
C_y & D_{yw} & D_{yu} \\
C_m & D_{mw} & D_{mu} \end{bmatrix}
$$
The state feedback law defined in equation (9) consists of two parts. One part contains the nonlinear feedback gains which control the subsequent linear system. We designed a state estimator defined in equation (11) which gives the estimation of states. In control law implementation defined in equation (25), the state estimations are used rather than the states themselves. The state observer compensates for the delays in angle and velocity by measuring these states asymptotically from output measurement. The nonlinear observer equation defined in equation (11) is combined with $H_2$ given as

$$
\dot{x} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} x + \begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
\end{bmatrix} (y - \hat{y}) \\
+ \begin{bmatrix}
0 \\
mg \sin \hat{\theta} \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
I + mk^2 \\
0 \\
\end{bmatrix} u
$$

(24)

The initial conditions of state observer are set to zero. The state observer in equation (24) imitates the position and velocity profiles created by CNS for stable body movement. It gives the estimation of states and compensates for delays in angle and velocity.

The linear feedback gains are calculated using $H_2$ control technique defined in equation (23). Thus, our robust nonlinear controller equation is defined as

$$
u = \frac{1}{L_e L_j \mu(x)} \left( -L_j \mu(x) + (-B_u^T S) \right)
$$

(25)

The controlled input torque defined in equation (26) is utilized as input to the nonlinear model which controls the body movement in the presence of perturbations. The weighting matrices $C_p$ and $D_{in}$ play a vital role in optimal control design.

**Simulation results**

The nonlinear model in equation (1) and compensator defined in equations (24) and (25) act as a closed-loop system in the presence of physiological delays, input and measurement noise as shown in Figure 2. We implement the nonlinear model in SimMechanics to test the system-level performance as well as to ascertain the viability of the designed scheme. In Figure 2, machine environment describes the simulation environment. Weld block represents the foot in halt position. The body segment is defined in block parameter such that the model is defined in ground frame and standing position. The ankle joint is defined as revolute with $z$ as an axis of rotation. To measure the posture position $\theta$, the ankle joint is equipped with a sensor ($j_2$). The sensor ($j_2$) measures the ground reaction forces. $n(t)$ is the input noise disturbance present at the plant input, and $\rho(t)$ is the measurement noise present at the plant output. Postural regulations are initiated as a result of internal or external perturbations to BOS. We initially placed COM behind heel which represents perturbation of 1.47 rad at the ankle joint. The COM might have a nonzero anterior velocity at movement inception which then converts into joint angular velocity. However, we choose the initial COM velocity to be zero to include a static position in the controller performance evaluation. We introduced proprioceptive feedback delays of 15 and 30 ms at the ankle joint. The COM is located 35% behind heel at the movement initiation. The intended position of the body after perturbation is the equilibrium standing position making an angle of $\pi/2$ radians with the BOS. The ankle joint is driven by an actuator ($T$) which is regulated by the controlled input torque ($u$). We synthesize a full-order compensator based on feedback linearization in combination with $H_2$ control technique.

Figure 3 shows the simulation results for joint profiles with and without delay in the presence of noisy measurements. These profiles are plotted in radians with ground as a frame of reference. The joint profiles show a smooth and stable response. The nonlinear compensator compensates for the delays and noises. The desired position is the equilibrium standing position, that is, $\pi/2$ radians with ground as a frame of reference. All joint profiles reach the equilibrium standing position in 2 s. In previous studies, one can observe initial ankle movement in the opposite direction. This is due to linearization of the model. The position profile shows an increase in overshoot in case of delays. This overshoot is found to be 0.018 rad for no delay, 0.02 and 0.03 rad for 15- and 30-ms delay. However, the compensator settles all the profiles concurrently.

Figure 4 displays the simulation results for angular velocity. We assume initial angular velocity to be zero. This posture characterizes a person leaning with a wall statically. Including nonzero initial angular velocity means including reaction forces in simulations. The angular velocity profiles show an increase in overshoot.
Figure 2. Postural control simulations developed in SimMechanics.

Figure 3. Profiles of angular position with and without delay. The profiles show an initial perturbation of 1.47 rad with ground. The boundary value for position profiles is 1.57 rad (equilibrium standing position) at the movement termination.
in case of delays. The desired value for joint angular velocity at movement termination is 0 rad/s. It is obvious from Figure 4 that all the profiles settle to zero in 1.8 s.

Figure 5 displays the phase portraits of the controlled plant. The angular velocity and position trajectories are plotted for initial perturbation of 0.1 rad with and without delay. The angular velocity trajectories are attracted to zero whereas these trajectories settle to 1.57 rad on \(x\)-axis which show the stable upright position at the movement termination.

Figure 6 displays the ankle joint torque profiles for the controlled plant with and without delay. This torque represents the passive viscoelasticity at the ankle joint. The joint profiles show a maximum torque of 420 N m for all three cases which is within the physiological limits. The ankle joint torque starts as an extensor torque followed by flexion torque which aids in movement termination. The extensor torque is same for the controlled plant with and without delay. However, the flexion torque increases a bit for 15 ms and further for 30-ms delay. The maximum flexion torque for no delay, 15- and 30-ms delay is found to be 80, 90 and 100 N m. The nonlinear observer compensates for the delays and settles all the three profiles in 0.9 s.

Figure 7 shows the ground reaction forces \(GRF_x\) and \(GRF_y\). The X-component of GRF settles to zero whereas the Y-component settles at body weight 755 N (75.5 kg). The \(GRF_y\) settles at the body weight in 0.7 s; however, the \(GRF_x\) profile takes a bit longer time in settling because of balancing after motion. The \(GRF_y\) profile is closer to the experimental data analysis\(^{40,41}\) of the GRF profile in \(y\)-direction. These profiles confirm that the simulated postural movement is physiologically relevant to the experimental data.

These results show that our modeling simulation scheme provides a robust and optimal framework for postural control of a single-link biomechanical model in the presence of physiological delays and noisy sensors.

**Discussion and conclusion**

This study investigates the robust nonlinear compensator design for the postural control of a single-link biomechanical model in the presence of proprioceptive feedback delays and noisy sensor data from joints. The torque actuator positioned at the ankle joint imitates the function of biological muscle actuator in positioning the body being perturbed. The feedback paths form MS, GTO or reflex loops are not modeled explicitly. All the feedback signals from proprioceptors are assumed to be lumped into a single control signal. The proprioceptive feedback delays are introduced in ankle position and velocity. The second-order Pade gives an approximation for the delayed states thus introducing two more states which increase the system to fourth order. The input and measurement noise are considered at the joint sensors. The feedback linearization in augmentation with \(H_2\) control technique results in an optimal and robust framework for postural stabilization in the presence of delays and noisy joint sensors. The feedback linearization calculates the nonlinear feedback gains and eliminates the nonlinearities in states and
output by cancelation. The $H_2$ compensator calculates the linear gains for the nonlinear controller and observer. Since the delayed states are approximated by Pade, so we assume no nonlinearity in these states. The fourth-order $H_2$ in combination with feedback linearization optimizes the strength of compensator by penalizing the inputs. Our simulation results show a smooth and stable response. The passive torque is zero at the movement initiation, and it starts building up with the movement progression. It reaches a peak value followed by a recession and finally reaches zero at the movement termination. The total passive torque consists of two parts: the extension and flexion torque. Initially, the angle between the BOS and the body

Figure 5. Phase portraits of controlled plant.

Figure 6. Controlled input torque at the ankle joint with and without delay.
segment extends due to extensor torque reaching a maximum value of 1.588, 1.59 and 1.6 rad in case of no delay, 15- and 30-ms delay, respectively. The extensor torque is same for all three cases. The flexion torque starts building up at this point to bring the body back to equilibrium standing position, that is, all the profiles settle at 1.57 rad. However, more flexion torque is needed for 30- and 15-ms delay as compared to position profile without delay. The settling time is same for all three cases. In previous studies, the angular deviation of body segment was assumed to be very small such that the biomechanical model was simplified by linearizing the model at standing position. The authors constructed PID controller–based stabilization for the model with delays in position and velocity feedback. However, one can observe the oscillations in angular profiles before reaching a steady state which is physiologically not relevant. Mughal used bond graph technique to model the MS and GTO. The PID controller stabilized the GTO signal, but the response showed a higher overshoot and took a long time of 6s to settle the plant.

The postural stabilization is a complex human voluntary movement and, in this study, we explore the role of CNS in controlling this task in the presence of proprioceptive feedback delays. These physiological delays have a significant effect on postural stability, especially in elderly or neuro-deficient patients. Iqbal and Roy reported the ankle angle and velocity delay to be 30 ms keeping in view the postural stabilization problem. In an experimental study performed by Sinkjaer et al., the ankle flexor/extensor pathway delay in humans was found to be of the order of 50 ms. Brooks reported 30–80 ms for small and medium loop delay. Our simulation results are supported by the physiological process involved in postural stabilization. Our angular and GRF profiles confirm that the simulated postural movement is physiologically relevant to the experimental results of previous studies. Mughal and Iqbal generated the angular profiles for sit-to-stand (STS) manure using the experimental data. Mughal and Iqbal synthesized the angular profiles for postural movements using the experimental data for STS manure as a reference. Our profiles match comparing GRF in y-direction in all respect of maximum values and shape of profiles. Our simulation results are physiologically relevant to previous studies and better in terms of transient responses and settling time.

This study can be extended for analysis of larger perturbations and different voluntary movements, for example, STS movement by adding more degrees-of-freedom in the model. This study is helpful in improving implants, exoskeleton and prosthetic designs. In future, applications such as rehabilitation robotics will use modeling simulation frameworks for prosthesis design which are both cognitive controlled and optimal in nature to establish natural movements. We will extend our study to three-link biomechanical model to explore the role of knee and hip joints in movement coordination of postural stabilization and STS movement in the presence of neural delays. We will optimize the controller performance with physiological variables

![Figure 7. Ground reaction forces (GRFx, GRFy) with and without delay.](image-url)
COM and GRF to compute better cognitive control for human voluntary motion or biologically inspired robotic applications.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship and/or publication of this article.

ORCID iD
Nadia Sultan https://orcid.org/0000-0003-2816-1116

References
1. Wallmann HW. The basics of balance and falls. Home Health Care Manag Pract 2009; 21(6): 436–439.
2. Chiba R, Takakusaki K, Ota J, et al. Human upright posture control models based on multisensory inputs; in fast and slow dynamics. Neurosci Res 2016; 104: 96–104.
3. Haenen WP and Rozendaal LA. Stability of bipedal stance: the contribution of cocontraction and spindle feedback. Biol Cybern 2003; 88(4): 293–301.
4. Peterka RJ. Postural control model interpretation of stabilogram diffusion analysis. Biol Cybern 2000; 82(4): 335–343.
5. Peterka R. Sensorimotor integration in human postural control. J Neurophysiol 2002; 88(3): 1097–1118.
6. Iqbal K and Pai Y-C. Predicted region of stability for balance recovery: motion at the knee joint can improve termination of forward movement. J Biomech 2000; 33(12): 1619–1627.
7. Barin K. Evaluation of a generalized model of human postural dynamics and control in the sagittal plane. Biol Cybern 1989; 61(1): 37–50.
8. Coelho DB and Duarte M. Human postural control during standing posture with a muscle-tendon actuator. Int J Exp Comput Biomech 2014; 2(4): 343–358.
9. Iqbal K. Mechanisms and models of postural stability and control. In: 2011 annual international conference of the IEEE engineering in medicine and biology society, Boston, MA, 30 August–3 September 2011, pp. 7837–7840. New York: IEEE.
10. Welch TD and Ting LH. A feedback model explains the differential scaling of human postural responses to perturbation acceleration and velocity. J Neurophysiol 2009; 101(6): 3294–3309.
11. Kuo AD. An optimal control model for analyzing human postural balance. IEEE T Biomed Eng 1995; 42(1): 87–101.
12. Maurer C, Mergner T and Peterka R. Multisensory control of human upright stance. Exp Brain Res 2006; 171(2): 231–250.
13. Baloh RW, Fife TD, Zwerling L, et al. Comparison of static and dynamic posturography in young and older normal people. J Am Geriatr Soc 1994; 42(4): 405–412.
14. Ishida A, Imai S and Fukuoka Y. Analysis of the posture control system under fixed and sway-referenced support conditions. IEEE T Biomed Eng 1997; 44(5): 331–336.
15. Kuo AD. An optimal state estimation model of sensory integration in human postural balance. J Neural Eng 2005; 2(3): S235.
16. Maurer C and Peterka RJ. A new interpretation of spontaneous sway measures based on a simple model of human postural control. J Neurophysiol 2005; 93(1): 189–200.
17. Qu X, Nussbaum MA and Madigan ML. A balance control model of quiet upright stance based on an optimal control strategy. J Biomech 2007; 40(16): 3590–3597.
18. Alexandrov AV, Frolov AA, Horak F, et al. Feedback equilibrium control during human standing. Biol Cybern 2005; 93(5): 309–322.
19. Jo S and Massaquoi SG. A model of cerebellum stabilized and scheduled hybrid long-loop control of upright balance. Biol Cybern 2004; 91(3): 188–202.
20. Park S, Horak FB and Kuo AD. Postural feedback responses scale with biomechanical constraints in human standing. Exp Brain Res 2004; 154(4): 417–427.
21. Davidson BS, Madigan ML, Southward SC, et al. Neural control of posture during small magnitude perturbations: effects of aging and localized muscle fatigue. IEEE T Biomed Eng 2011; 58(6): 1546–1554.
22. Hidler JM and Rymer WZ. A simulation study of reflex instability in spasticity: origins of clonus. IEEE T Rehabil Eng 1999; 7(3): 327–340.
23. Insperger T, Milton J and Stépán G. Acceleration feedback improves balancing against reflex delay. J Roy Soc Interface 2013; 10(79): 20120763.
24. Roy A and Iqbal K. PID controller stabilization of a single-link biomechanical model with multiple delayed feedbacks. In: SMC’03 conference proceedings. 2003 IEEE international conference on systems, man and cybernetics. Conference theme-system security and assurance, Washington, DC, 8 October 2003, pp. 642–647. New York: IEEE.
25. Iqbal K and Roy A. Stabilizing PID controllers for a single-link biomechanical model with position, velocity, and force feedback. J Biomech Eng 2004; 126(6): 838–843.
26. Mughal AM and Iqbal K. Bond graph modeling of physiological motor control with H∞ controller design. In: IEEE international conference of control applications, Munich, 4–6 October 2006, pp. 947–952. New York: IEEE.
27. Mughal AM. Real time modeling, simulation and control of dynamical systems. Cham: Springer, 2016, chapter 9, pp. 167–172.
28. Mughal AM and Iqbal K. Optimization of biomechanical STS movement with linear matrix inequalities. Mech Syst Control 2019; 47(1): 1–11.
29. Imran N, Mughal A and ul Islam MN. Control synthesis of single link biomechanical model using feedback linearization. In: 2018 international conference on computing, mathematics and engineering technologies (iCoMET), Sukkur, Pakistan, 3–4 March 2018, pp. 1–6. New York: IEEE.
30. Lei J and Khalil HK. Feedback linearization for nonlinear systems with time-varying input and output delays by using high-gain predictors. IEEE T Automat Contr 2015; 61(8): 2262–2268.
31. De Leva P. Adjustments to Zatsiorsky-Seluyanov's segment inertia parameters. J Biomech 1996; 29(9): 1223–1230.
32. Piche R. Low-order rational all-pass approximations to e-s. *IFAC Proc Vol* 1990; 23(8): 183–186.
33. Martinez J. Transfer functions of generalized Bessel polynomials. *IEEE T Circuits Syst* 1977; 24(6): 325–328.
34. Glader C, Högnäs G, Mäkilä P, et al. Approximation of delay systems—A case study. *Int J Control* 1991; 53(2): 369–390.
35. Rasool G, Mughal AM and Iqbal K. Fuzzy biomechanical sit-to-stand movement with physiological feedback latencies. In: *2010 IEEE international conference on systems, man and cybernetics*, Istanbul, 10–13 October 2010, pp. 316–321. New York: IEEE.
36. Rowland BA, Quessy S, Stanford TR, et al. Multisensory integration shortens physiological response latencies. *Soc Neurosci* 2007; 27: 5879–5884.
37. Zak SH and Zak SH. *Systems and control*. New York: Oxford University Press, 2002, chapter 7, pp. 368–372.
38. Doyle JC, Glover K, Khargonekar PP, et al. State-space solutions to standard H/sub 2/ and H/sub infinity/ control problems. *IEEE T Automat Contr* 1989; 34(8): 831–847.
39. Zhou K, Doyle JC and Glover K. *Robust and optimal control*, vol. 40. Englewood Cliffs, NJ: Prentice Hall, 1996, chapter 14, pp. 365–390.
40. Mughal AM and Iqbal K. Physiological LQR design for postural control coordination of sit-to-stand movement. *Cogn Comput* 2012; 4(4): 549–562.
41. Mughal AM and Iqbal K. Synthesis of angular profiles for bipedal sit-to-stand movement. In: *2008 40th south-eastern symposium on system theory (SSST)*, New Orleans, LA, 16–18 March 2008, pp. 293–297. New York: IEEE.
42. Sinkjaer T, Toft E, Andreassen S, et al. Muscle stiffness in human ankle dorsiflexors: intrinsic and reflex components. *J Neurophysiol* 1988; 60(3): 1110–1121.
43. Brooks VB. *The neural basis of motor control* (Posture and Locomotion). New York: Oxford University Press, 1986, pp. 160–180.