Maximum Mass of the Hot Neutron Star with the Quark Core

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Abstract

We have considered a hot neutron star with a quark core, a mixed phase of quark-hadron matter, and a hadronic matter crust and have determined the equation of state of the hadronic phase and the quark phase, we have then found the equation of state of the mixed phase under the Gibbs conditions. Finally, we have computed the structure of hot neutron star with the quark core and compared our results with those of the neutron star without the quark core. For the quark matter calculations, we have used the MIT bag model in which the total energy of the system is considered as the kinetic energy of the particles plus a bag constant. For the hadronic matter calculations, we have used the lowest order constrained variational (LOCV) formalism. Our calculations show that the results for the maximum gravitational mass of the hot neutron star with the quark core are substantially different from those of without the quark core.
I. INTRODUCTION

A hot neutron star is born following the gravitational collapse of the core of a massive star just after the supernova explosion. The interior temperature of a neutron star at its birth is of order $20 - 50$ MeV [1]. Therefore, the high temperature of these stages cannot be neglected with respect to the Fermi temperature throughout the calculation of its structure. This shows that the equation of state of the hot dense matter is very important for investigating the structure of a newborn neutron star. Depending on the total number of nucleons, a newborn neutron star evolves either to a black hole or to a stable neutron star [2]. Hence, calculation of the maximum mass of hot neutron star is of special interest in astrophysics.

As we go from surface to the center of a neutron star, at sufficiently high densities, the matter is expected to undergo a transition from hadronic matter, where the quarks are confined inside the hadrons, to a state of deconfined quarks. Finally, there are up, down and strange quarks in the quark matter. Other quarks have high masses and do not appear in this state. Glendenning has shown that a proper construction of the hadron-quark phase transition inside the neutron stars implies the coexistence of nucleonic matter and quark matter over a finite range of the pressure. Therefore, a mixed hadron-quark phase exists in the neutron star and its energy is lower than those of the quark matter and nucleonic matter [3]. These show that we can consider a neutron star as composed of a hadronic matter layer, a mixed phase of quarks and hadrons and, in core, a quark matter. Recent Chandra observations also imply that the objects RX J185635-3754 and 3C58 could be neutron stars with the quark core [4].

Burgio et al. have investigated the structure of neutron stars with the quark core at zero [5] and finite temperature [6] with the Brueckner-Bethe-Goldstone formalism to determine the equation of state of the hadronic matter, they have used. We have calculated the structure properties of the cold neutron star by considering a quark phase at its core [7] and compared the results with our previous calculations for the neutron star without the quark core [8]. In these works, we have employed the lowest order constrained variational (LOCV) method for the hadronic matter calculations. In the present paper, we intend to extend these calculations for the hot neutron star with the quark core.
II. EQUATION OF STATE

As it was mentioned in the previous section, we consider a neutron star composed of a hadronic matter (hadron phase), a mixed phase of quarks and hadrons, and a quark core (quark phase). Therefore, we should calculate the equation of state of these phases separately as follows.

A. Hadron Phase

For this phase of the neutron star matter, we consider the total energy per nucleon as the sum of contributions from the leptons and nucleons,

\[ E = E_{lep} + E_{nuc}, \]

(1)

The contribution from the energy of leptons (electrons and muons) is

\[ E_{lep} = E_e + E_\mu, \]

(2)

where \( E_e \) and \( E_\mu \) are the energies of electrons and muons, respectively,

\[ E_i = \frac{m_i^4 c^5}{\pi^2 n \hbar^3} \int_0^\infty \frac{\sqrt{1 + x^2}}{1 + \exp\{\beta[m_i c \sqrt{1 + x^2} - \mu_i]\}} x^2 dx. \]

(3)

Here \( \mu_i \) and \( m_i \) are the chemical potential and mass of particle \( i \), \( \beta = \frac{1}{k_B T} \) (\( T \) is the temperature), \( n \) is the total number density of nucleons (\( n = n_p + n_n \)), \( c \) is speed of light and \( x \) is as follows,

\[ x = \frac{\hbar k}{m_i c}. \]

(4)

In our calculations, the equation of state of hot nucleonic matter is determined using the lowest order constrained variational (LOCV) method as follows \[9, 10\]. We adopt a trail wave function as

\[ \psi = F\phi, \]

(5)

where \( \phi \) is the Slater determinant of the single-particle wave function and \( F \) is the correlation function which is taken to be

\[ F = S \prod_{i>j} f(ij). \]

(6)
\( S \) is a symmetrizing operator. For the energy of nucleonic matter, we consider up to the two-body term in the cluster expansion,

\[ E_{\text{nuct}} = E_1 + E_2. \tag{7} \]

The one body term \( E_1 \) for the hot asymmetrical nucleonic matter that consists of \( Z \) protons and \( N \) neutrons is simply the fermi gas kinetic energy,

\[ E_1 = \sum_{i=1,2} \mathcal{E}_i \tag{8} \]

Label 1 and 2 are used instead of proton and neutron, respectively, and \( \mathcal{E}_i \) is

\[ \mathcal{E}_i = \sum_k \frac{\hbar^2 k^2}{2m_i} f_i(k, T, n_i), \tag{9} \]

where \( f(k, T, n_i) \) is the Fermi-Dirac distribution function \[18\],

\[ f(k, T, n_i) = \frac{1}{e^{\beta \left[ \epsilon_i(k, T, n_i) - \mu_i(T, n_i) \right]} + 1}. \tag{10} \]

In the above equation, \( n_i \) are the number densities and \( \epsilon_i \) are the single particle energies associated with the protons and neutrons,

\[ \epsilon_i(k, T, n_i) = \frac{\hbar^2 k^2}{2m_i^* (T, n_i)}, \tag{11} \]

where \( m_i^* \) are the effective masses.

The two-body energy, \( E_2 \), is

\[ E_2 = \frac{1}{2A} \sum_{ij} <ij|\nu(12)|ij - ji>, \tag{12} \]

where

\[ \nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12). \tag{13} \]

\( f(12) \) and \( V(12) \) are the two-body correlation and inter-nucleonic potential.

We note that the conditions of charge neutrality and beta stability impose the following constraints on the number densities and chemical potentials,

\[ n_p = n_e + n_\mu \tag{14} \]

\[ \mu_n - \mu_p = \mu_e = \mu_\mu. \tag{15} \]

The procedure to calculate the nucleonic matter has been fully discussed in the Refs. \([9, 10]\).
B. Quark Phase

We use the MIT bag model for the quark matter calculations. In this model, the energy density is the kinetic energy of quarks plus a bag constant ($B$) which is interpreted as the difference between the energy densities of non-interacting quarks and interacting ones [17],

$$\mathcal{E}_{\text{tot}} = \mathcal{E}_u + \mathcal{E}_d + \mathcal{E}_s + B,$$

(16)

where $\mathcal{E}_i$ is the kinetic energy per volume of particle $i$,

$$\mathcal{E}_i = \frac{g}{2\pi^2} \int_0^\infty (m_i^2 c^4 + \hbar^2 k^2 c^2)^{1/2} f(k, T, n_i) k^2 dk.$$

(17)

In above equation, $g$ is the degeneracy number of the system and $n_i$ is the number density of particle $i$,

$$n_i = \frac{g}{2\pi^2} \int_0^\infty f(k, T, n_i) k^2 dk.$$

(18)

For the quark phase, the Fermi-Dirac distribution function, $f(k, T, n_i)$, is given by

$$f(k, T, n_i) = \frac{1}{\exp\{\beta((m_i^2 c^4 + \hbar^2 k^2 c^2)^{1/2} - \mu_i)\} + 1}.$$

(19)

We assume that the up and down quarks are massless, the strange quark has a mass equal to 150 $MeV$ and $B = 90$ $MeV fm^{-3}$. Now, by applying the beta stability and charge neutrality conditions, we get the following relations for the chemical potentials and number densities,

$$\mu_d = \mu_u + \mu_l,$$

(20)

$$\mu_s = \mu_u + \mu_l,$$

(21)

$$\Rightarrow \mu_d = \mu_s,$$

(22)

$$\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_l = 0,$$

(23)

$$n_B = \frac{1}{3} (n_u + n_d + n_s),$$

(24)

where $n_l$ and $\mu_l$ are the leptonic number density and chemical potential, and $n_B$ is the baryonic number density.

The pressure of the system is calculated from free energy using the following equation,

$$P = \sum_i n_i \frac{\partial \mathcal{F}_i}{\partial n_i} - \mathcal{F}_i,$$

(25)
where the Helmholtz free energy per volume ($F$) is given by

$$ F = \mathcal{E}_{\text{tot}} - T S_{\text{tot}}. $$  

The entropy of quark matter ($S_{\text{tot}}$) can be written as follows,

$$ S_{\text{tot}} = S_u + S_d + S_s, $$

where $S_i$ is the entropy of particle $i$,

$$ S_i(n_i, T) = -\frac{3}{\pi^2} k_B \int_0^\infty \left[ f(k, T, n_i) \ln(f(k, T, n_i)) + (1 - f(k, T, n_i)) \ln(1 - f(k, T, n_i)) \right] k^2 dk. $$

### C. Mixed phase

For the mixed phase, where the fraction of space occupied by quark matter smoothly increases from zero to unity, we have a mixture of hadrons, quarks and electrons. In the mixed phase, according the Gibss equilibrium condition, the temperatures, pressures and chemical potentials of the hadron phase (H) and quark phase (Q) are equal \[3\]. Here, for each temperature we let the pressure to be an independent variable.

The Gibss conditions implies that

$$ \mu_Q^N = \mu_H^N, $$

$$ \mu_Q^P = \mu_H^P, $$

where $\mu_H^N$ and $\mu_Q^N$ ($\mu_H^P$ and $\mu_Q^P$) are the neutron (proton) chemical potentials in the hadron phase and the quark phase, respectively,

$$ \mu_n = \frac{\partial \mathcal{E}}{\partial n_n}, $$

$$ \mu_p = \frac{\partial \mathcal{E}}{\partial n_p}. $$

In above equations, $\mathcal{E}$ is the energy density of the system,

$$ \mathcal{E} = n(E + mc^2). $$

To obtain $\mu_H^N$ and $\mu_H^P$ for the hadronic matter in mixed phase, we use the semiempirical mass formula \[19-21\],

$$ E = T(n, x) + V_0(n) + (1 - 2x)^2 V_2(n), $$

$$ E = T(n, x) + V_0(n) + (1 - 2x)^2 V_2(n), $$
where $x = \frac{2n}{n}$ is the proton fraction. $T(n, x)$ is kinetic energy contribution and the functions $V_0$ and $V_2$ represent the interaction energy contributions which are determined from the energies of the symmetric nuclear matter ($x = \frac{1}{2}$) and pure neutron matter ($x = 0$). We calculate $V_0$ and $V_2$ using our results for the LOCV calculation of nucleonic matter with the $UV_{14} + TNI$ nuclear potential which is discussed in section II A. Now, we can obtain the chemical potentials of neutrons and protons from Eqs. (31)-(34) as follows,

$$
\mu^H_p = T(n, x) + n \frac{\partial T(n, x)}{\partial n} + \frac{\partial T(n, x)}{\partial x} + V_0(n) + nV'_0(n) + \frac{(-3 + 8x - 4x^2)V_2(n) + (1 - 2x)^2nV'_2(n) + mc^2}{2},
$$

$$
\mu^H_n = T(n, x) + n \frac{\partial T(n, x)}{\partial n} - \frac{\partial T(n, x)}{\partial x} + V_0(n) + nV'_0(n) + (1 - 4x^2)V_2(n) + (1 - 2x)^2nV'_2(n) + mc^2.
$$

For the quark matter in mixed phase, we have

$$
\mu^Q_p = 2\mu_u + \mu_d,
$$

$$
\mu^Q_n = \mu_u + 2\mu_d.
$$

At a certain pressure, we calculate $\mu_u$ for different $\mu_d$ under the condition that the densities yield this certain pressure. By calculating $\mu_u$ and $\mu_d$, we obtain $\mu^Q_p$ and $\mu^Q_n$.

Now, we plot $\mu_p$ versus $\mu_n$ for both hadron and quark phases, the cross point of the two curves satisfies the Gibbs conditions. In the mixed phase, as the chemical potentials determine the densities, the volume fraction occupied by quark matter, $\chi$, can be obtained by the requirement of global charge neutrality,

$$
\chi\left(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s\right) + (1 - \chi)n_p - n_e = 0.
$$

Finally, we can calculate the baryonic density of the mixed phase (M),

$$
n_B = \chi n_Q + (1 - \chi)n_H,
$$

and then the total energy density of mixed phase is found,

$$
\mathcal{E}_M = \chi \mathcal{E}_Q + (1 - \chi)\mathcal{E}_H.
$$
D. Results

We have shown our results for the energy densities of hadron phase, quark phase and mixed phase in Figs. 1 and 2 at two different temperatures. Figs. 1 and 2 show that at low densities the energy density of the hadronic matter is lower than those of other phases. However, as the density increases, at first the energy of mixed phase and finally the energy of quark phase is lower than those of other phases. We also see that there is a mixed phase for a range of densities. Below (beyond) this range, we have the pure hadron (quark) phase. By comparing Figs. 1 and 2 we see that for a given value of the density, the energies of all phases increases by increasing the temperature.

Using the above calculated energy density, we can determine the equation of state and finally the structure of the hot neutron star with the quark core which is discussed in the next section.

III. STRUCTURE OF THE HOT NEUTRON STAR WITH THE QUARK CORE

The structure of neutron star is determined by numerically integrating the Tolman-Oppenheimer-Volkoff equation (TOV) [22–25],

\[
\frac{dP}{dr} = - \frac{G[\mathcal{E}(r) + \frac{P(r)}{c^2}][m(r) + \frac{4\pi r^3 P(r)}{c^2}]}{r^2[1 - \frac{2Gm(r)}{rc^2}]},
\]

\[
\frac{dm}{dr} = 4\pi r^2 \mathcal{E}(r),
\]

where \( P \) is the pressure and \( \mathcal{E} \) is the total energy density. For a given equation of state in the form \( P(\mathcal{E}) \), the TOV equation yields the mass and radius of star as a function of the central mass density.

In our calculations for the structure of hot neutron star with the quark core, we use the following equations of state: (i) Below the density of 0.05 \( fm^3 \), we use the equation of state calculated by Baym [26]. (ii) From the density of 0.05 \( fm^3 \) up to the density where the mixed phase starts, we use the equation of state of pure hadron phase calculated in section II A. (iii) In the range of densities in which there is the mixed phase, we use the equation of state calculated in section II C. (iv) Beyond the density of end point of the mixed phase, we use the equation of state of pure quark phase calculated in section II B. All calculations
are done for $B = 90 \, MeV/fm^{-3}$ at two different temperatures $T = 10$ and $20 \, MeV$. Our results are as follows.

The gravitational mass as a function of the central mass density for the hot neutron star with the quark core at two different temperatures has been presented in Figs. 3 and 4. It is seen that for both relevant temperatures, the gravitational mass increases by increasing the central mass density and finally reaches a limiting value (maximum mass). In Figs. 3 and 4, our results for the case of neutron star without the quark core have been also given for comparison. We see that by including the quark core for the neutron star, our results for the gravitational mass are substantially affected. For the neutron star with the quark core, our results for the gravitational mass at three different temperatures ($T = 0, 10$ and $20 \, MeV$) have been compared in Fig. 5. It is seen that the gravitational mass increases by increasing the temperature.

Figs. 6 and 7 show the gravitational mass versus the radius for both cases of the neutron star with and without the quark core at two different temperatures. At each temperature, it is seen that there is a reasonable difference between the mass-radius relations of these two cases of the neutron star. However, for both cases, we see that the radius decreases as the mass increases. By comparing Figs. 6 and 7, we can see that the decreasing rate of the radius versus the mass is substantially different for different temperatures.

Our results for the maximum gravitational mass of the hot neutron star with the quark core and the corresponding values of radius and central mass density have been given in Tables I and II for two different temperatures. Our results for the case of hot neutron star without quark core have been also presented for comparison. For different temperatures, it is seen that the inclusion of the quark core considerably reduces the maximum mass of the hot neutron star. This is due to the fact that by including the quark core for the neutron star, the equation of state becomes softer than that without the quark core. However, we do not see any substantial changes for the radius and central mass density of these two cases of the hot neutron star.

IV. SUMMARY AND CONCLUSION

For the hot neutron star, from the surface toward the center, we have considered a pure hadronic matter layer, a mixed phase of quarks and hadrons in a range of densities which
are determined by employing the Gibbs conditions, and a pure quark matter in the core, to calculate its equation of state at finite temperature. For calculating the equation of state of the hot hadronic matter, we have applied the lowest order constrained variational (LOCV) method at finite temperature. The equation of state of the hot quark matter has been computed using the MIT bag model with the bag constant $B = 90 \ MeV \ fm^{-3}$. Using this equation of state, we have solved the TOV equation by numerical method to determine the structure properties of the hot neutron star with the quark core at $T = 10$ and $20 \ MeV$. Then, we have compared the results of these calculations with those for the hot neutron star without the quark core. It is found that our results for the maximum gravitational mass of the neutron star with a quark core are less than those of the neutron star without the quark core.

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TABLE I: Maximum gravitational mass ($M_{\text{max}}$), and the corresponding radius ($R$) and central mass density ($\varepsilon_c$) of the hot neutron star without (NS) and with (NS+Q) the quark core at $T = 10 \ MeV$.

|       | $M_{\text{max}} (M_\odot)$ | $R \ (Km)$ | $\varepsilon_c \ (10^{14} \ gr/cm^3)$ |
|--------|----------------------------|-------------|--------------------------------------|
| NS     | 2.07                       | 10.22       | 26.94                                |
| NS+Q   | 1.76                       | 10.45       | 27.38                                |

TABLE II: As Table I but at $T = 20 \ MeV$.

|       | $M_{\text{max}} (M_\odot)$ | $R \ (Km)$ | $\varepsilon_c \ (10^{14} \ gr/cm^3)$ |
|--------|----------------------------|-------------|--------------------------------------|
| NS     | 2.09                       | 10.64       | 27.01                                |
| NS+Q   | 1.78                       | 11          | 27.37                                |

FIG. 1: Energy density versus the baryonic density at $T = 10 \ MeV$ for the hadron phase (solid curve), quark phase (dotted curve) and mixed phase (dashed curve).
FIG. 2: As Fig. 1 but at $T = 20 \text{ MeV}$.
FIG. 3: Gravitational mass versus the central mass density for the neutron star with (dotted curve) and neutron star without (solid curve) the quark core at $T = 10 \text{ MeV}$. 
FIG. 4: As Fig. K but at $T = 20 \text{ MeV}$.
FIG. 5: Gravitational mass versus the central mass density for the neutron star with the quark core at $T = 0$ (dotted-dashed curve), 10 (solid curve) and 20 $MeV$ (dotted curve).
FIG. 6: Mass-radius relation for the neutron star with (dotted curve) and without (solid curve) the quark core at $T = 10 \text{ MeV}$. 
FIG. 7: As Fig. [4] but at \( T = 20 \text{ MeV} \).