Revisiting the phenomenology on the QCD color dipole picture

A.I. Lengyel\textsuperscript{a} and M.V.T. Machado \textsuperscript{b}

\textsuperscript{a}Institute of Electron Physics, National Academy of Sciences of Ukraine, Universitetskaya 21, UA-88016 Uzhgorod, Ukraine

\textsuperscript{b}High Energy Physics Phenomenology Group, GPPAE IF-UFRGS, Caixa Postal 15051, CEP 91501-970, Porto Alegre, RS, Brazil

Using the QCD dipole picture of the hard BFKL Pomeron, we perform a 3 parameter fit analysis of the recent inclusive structure function experimental measurements at small-$x$ and intermediate $Q^2$. As a byproduct, the longitudinal structure function and the gluon distribution are predicted without further adjustments. The data description is quite reasonable, being timely a further study using resummed NLO BFKL kernels along the lines of recent theoretical developments.

1. Introduction

Deep inelastic electron-proton scattering (DIS) experiments at HERA have provided measurements of the inclusive structure function $F_2(x, Q^2)$ in very small values of the Bjorken variable $x \gtrsim 10^{-5}$. In these processes the proton target is analyzed by a hard probe with virtuality $Q^2 = -q^2$, where $x \sim Q^2/2pq$ and $p, q$ are the four-momenta of the incoming proton and the virtual photon probe. In this domain, the gluon is the leading parton driving the small-$x$ behavior of the deep inelastic observables.

A sound approach encoding all order $\alpha_s \ln(1/x)$ resummation is the QCD dipole picture \cite{1}. It was proven that such approach reproduces the BFKL evolution \cite{2}. The main process is the onium-onium scattering, that is the reaction between two heavy quark-antiquark states (onia). This process is basically perturbative due to the onium radius being the natural hard scale at which the strong coupling is computed. In the large $N_c$ limit, the heavy pair and the soft gluons are represented as a collection of color dipoles. The cross section is written as a convolution between the number of the dipoles in each onium state and the basic cross section for dipole-dipole scattering due to two-gluon exchange. The QCD dipole model can be applied to DIS process, assuming that the virtual photon at high $Q^2$ can be described by an onium. Furthermore, the proton is described by a collection of onia with an average onium radius to be determined from phenomenology. This model has produced a successful description of the old structure function data \cite{3}. These achievements are our main motivation in revisiting the QCD color dipole picture and in applying it for description of the currently more accurate $F_2(x, Q^2)$ experimental results.

The approach also allows a systematic framework for testing the resummed next-to-leading order BFKL evolution kernels, producing predictions for the proton structure function. A method for doing this has been proposed in Ref. \cite{4}, where the resummation schemes can tested through the Mellin transformed $j$-moments of $F_2$. Moreover, it has been shown that a geometric scaling for the photon-proton cross section and the symmetry between low and high $Q^2$ regions are associated to the symmetry of the two-gluon dipole-dipole cross section \cite{5}. Furthermore, within the approach, a simple analytic expression for the dipole-proton scattering amplitude has been computed taking the scattering amplitude as a solution of the BFKL equation in the vicinity of the saturation line for dipole sizes $r$ (in the photon wavefunction) obeying $r \lesssim 1/Q_{\text{sat}}$ \cite{6}, where $Q_s^2(x) \propto e^{\lambda \log(1/x)}$ is the saturation scale. Fi-
nally, the approach has recently been used to describe hard processes initiated by virtual-gluon probes [2].

In this contribution we summarize our fit analysis using the QCD dipole phenomenology applied to DIS process [5]. In what follows, the main expressions are presented and the fitting results are shown and discussed in conclusion.

2. The QCD dipole picture applied to DIS

The starting point in the QCD color dipole picture is the onium-onium scattering. An onium is a heavy quark-antiquark state, turning out the scattering process perturbative once the onium radius provides the hard scale entering into the calculation of the dipoles in each onium state and the basic cross section is written as a convolution between the number of onia with an unknown average onium radius. Then, the DIS cross section is written as a convolution between the number of onia and the photon-onium cross section. Requiring onia, the approach has recently been used to describe hard processes initiated by virtual-gluon probes [2].

On the other hand, in the DIS process one has a two-scale problem where the hard scale is given by the photon virtuality and the soft one is associated to the proton typical size. Hence, the proton is approximately described by a collection of onia with an unknown average onium radius. Then, the DIS cross section is written as a convolution of the probability of finding an onium in the proton and the photon-onium cross section. Relying on renormalization group properties, a suitable ansatz for the former quantity was proposed [5]. It depends on the average number of primary dipoles in the proton and on their average transverse diameter $r_0 \equiv 2/Q_0$. Under these assumptions and the convolution integral approximated by a steepest-descent method (using the expansion of the BFKL kernel near $\gamma = 1/2$), the structure functions take a simple form [5].

$$F_{T,L} = H_{T,L} \frac{\bar{c}_s \pi^2 e_f^2 n_{\text{eff}}}{96} \left( \frac{x_0}{x} \right) \frac{\omega P}{Q_0} \times \sqrt{2\kappa(x)/\pi} \exp \left[ -\frac{\kappa(x)}{2} \ln^2 \frac{Q}{Q_0} \right],$$ (1)

where $H_T = 9/2$ and $H_L = 1$. The hard Pomeron intercept is given by $\alpha_P = 1 + \omega P$, with $\omega P = 4\bar{c}_s \ln 2$ and $\bar{c}_s = c_s N_c/\pi$. The BFKL diffusion coefficient at rapidity $Y = \ln(x_0/x)$ is written as $\kappa(x) = [\bar{c}_s 7 \zeta(3) \ln \frac{x_0}{x}]^{-1}$. The conditions to obtain Eq. (1) from the saddle-point method constrain its region of applicability. Namely, the relation $\kappa(x) \ln(Q/Q_0) \ll 1$ should be obeyed. This is realized for the region of moderate $Q/Q_0$ when compared to the range on $x_0/x$.

3. Results and Conclusions

Let us present the fitting procedure using the recent HERA experimental data on the proton structure function [9,10] and taking Eq. (1), where $F_2 = F_T + F_L$. We defined the overall normalization for $F_2$, $N_p = (H_T + H_L) \bar{c}_s \pi^2 e_f^2 n_{\text{eff}}/96$. For the fit procedure we have considered only the small $x \leq 10^{-2}$ data, covering the range of virtualities $1.5 \leq Q^2 \leq 150$ GeV$^2$. We have also fixed $x_0 = 1$, since its value is reasonably stable for different data sets. Therefore, we are left with a reduced number of parameters ($N_p, \alpha_P, Q_0$). The resulting parameters for H1 and ZEUS experimental data sets are presented in Table 1.

| PARAMETER | ZEUS data set | H1 data set |
|-----------|---------------|-------------|
| $N_p$     | 0.0977        | 0.0995      |
| $Q_0$     | 0.571         | 0.587       |
| $\alpha_P$| 1.24          | 1.23        |
| $\chi^2$/d.o.f. | 1.08        | 1.02        |

The quality of fit is quite good for both H1 and ZEUS data sets. We have performed also an extrapolation of the fit using in all range on $x$...
(adding E665 the EMC data) and $Q^2$. In order to do this, a non-singlet contribution was added and large-$x$ threshold factors were considered. We quote Ref. [8] for further details. The procedure presented here is similar to previous analysis on Refs. [3], with an even lower effective power ($\alpha_F \approx 1.282$ in [3]). The low value for the fixed coupling constant $\alpha_s \approx 0.1$ reveals the well known necessity of sizeable higher order corrections to the approach. Accurate analysis in this lines, considering resummed NLO BFKL kernels, has been proposed recently [4] producing a reasonable $\alpha_s \sim 0.2$ for the typical $Q^2$ range considered in phenomenology for structure functions.

Within the approach above, it is possible to determine longitudinal structure function $F_L$ without further adjustments. From Eq. (1) and the definition of the overall normalization one has $F_L = (2/11) F_2$, since $H_T + H_L = 11/2$ and $H_L = 1$. The results are in good agreement with the recent data (see Fig. (3) in [8]). Moreover, in the QCD dipole approach the gluon distribution function can be calculated in a straightforward way. The result is independent of the overall normalization, which contains part of the non-perturbative inputs of the model. The gluon distribution function is given by [3],

$$x G(x, Q^2) = \frac{F_2(x, Q^2)}{h_T(\gamma = \gamma_s) + h_L(\gamma = \gamma_s)},$$

where $h_{T,L}(\gamma)$ are the LO photon impact factors, corresponding to the perturbative coupling to the photon [3]. The quantity $\gamma_s = 1/2 \left[ 1 - \kappa(x) \frac{Q}{Q_0} \right]$ comes from the integration over $\gamma$ in the convolution of onia density inside the proton and photon-onium cross section. It is obtained via steepest descent method and taking the expansion of the LO BFKL kernel, $\chi(\gamma)$, near $\gamma = 1/2$, as referred before.

We have compared expressions Eq. (2) with NLO DGLAP resummed kernels. A sizeable deviation between the recent H1 and ZEUS fits for that parton density function and the QCD dipole result is found [8], whereas it is consistent with previous analysis. However, it should be stressed that the gluon distribution is an indirect observable, with its determination being model-dependent, and a direct comparison is not straightforward. We quote Ref. [8] for more details on this discussion.

As a final remark, very recently a pioneering phenomenological analysis using resummed NLO BFKL kernels in the saddle-point approximation has been done in Ref. [11]. The LO fit provides parameters similar to ours, with a slightly higher $\chi^2$/d.o.f.. The NLO fits give a qualitatively satisfactory account of the running $\alpha_s$ effect but quantitatively the quality of fit remains sizeably higher than the LO fit. This feature suggests the investigation of other proposed theoretical resummation schemes and/or to improve those ones considered in the first analysis presented in Ref. [11].

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