Optimization of energy consumption at thermal vacuum liquid evaporation in the closed volume

V I Trushlyakov¹, A V Panichkin²

¹Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
²Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail: VATrushlyakov@yandex.ru

Abstract. The mathematical model (MM) of the process describing the evaporation of model liquid with a free "mirror"-type surface, changes of liquid and vapour-gas mixture (VGM) temperature in the closed volume with application of power consumption optimization at thermal vacuum influence is developed. For MM optimization of power consumption on the basis of the Pontryagin maximum principle with application to process of liquid evaporation from a free surface at joint thermal and vacuum influence was considered. For modeling heat and mass transfer processes with evaporation in the closed volume at thermal vacuum influence, the system of the ordinary differential equations within some assumptions with simplification of a problem definition was used. The MM at some deviations of parameters allows to perform optimization of the cyclorama of pumping out and heating systems works and to define a tendency of power consumption reduction at consecutive inclusions and switching off VGM pumping out and liquid heating. As an optimum condition criterion are accepted power costs of liquid conductive heating and VGM pumping from the closed volume. Based on numerical modeling intervals of liquid heating and VGM pumping time providing minimization of power consumption for evaporation of the set mass of liquid are defined. The test calculations without optimization of power consumption showing satisfactory coordination with experimental data are carried out.

1. Introduction
There is a significant amount of technologies for drying materials which are widely applied in various fields of activity, for example, for drying agricultural products and timber as well as in aviation industry and the missile and space equipment (fuel tanks and highways) after a number of the technological operations connected with their filling with technological liquid (cleaning, calibrating, durability tests, etc.)

Now the task of increasing the efficiency of heat and a mass exchange processes, occurring in various technical drying systems is actual. In this paper, the system of thermal vacuum drying in the closed volume is considered as one of examples of such technical systems.

Numerous theoretical and experimental studies including development of the mathematical models (MM) on materials drying when using various feeders of heat, acoustic influence, change of pressure, etc., which can be divided conditionally into groups, are known, including the following:

- According to various boundary conditions of liquid arrangement on a firm surface ("drop", "mirror"), and also various types of surfaces on which liquid (micro and nanocoverings) is located [1-6].
- According to types of impact on liquid (convective and conductive heating, thermo-capillary, vacuum) [7-17].
- According to optimization of processes and designs of drying systems [18-20].

In this work, the increase in heat and mass exchange efficiency is considered at liquid evaporation from a free surface at joint thermal and vacuum influences with application of the Pontryagin
maximum principle [21]. The application specificity of the principle which is widely used for problems of optimum control is its applicability for the objects described by systems of the ordinary differential equations, for example, [22].

Thus, to apply the optimization of power consumption on the basis of the Pontryagin maximum principle, it is necessary to develop a simplified MM, describing liquid evaporation process from a free surface at joint thermal and vacuum influence on the basis of the ordinary differential equations that, naturally, will lead to methodical errors, but at the same time will allow to estimate a tendency of power consumption reduction due to the optimization of the pumping out and heating systems operation cyclograma.

2. Definition of liquid evaporation at thermal vacuum influence problem

Based on the literature review carried out above it is possible to formulate the following directions of researches:

- Development of MM of liquid evaporation at thermal vacuum influence based on the ordinary differential equations.
- Development of thermal vacuum process optimum control based on the Pontryagin maximum principle.

For liquid evaporation from the model volume (MV) in the vacuum camera (VC) with pumping of steam-gas mix (VGM) and conductive heating of liquid is considered as criterion of power consumption optimization.

Assumptions:

- The case of limited time (till 10 minutes) interactions of VGM with MV, liquid and the VC metal case when it is possible to present mutual heat exchange between them in the simplified form or to neglect it, allowing some deviations of mathematical modeling results from experimental data is considered;
- The case when in VC there is a constant volume of the air mix \( V_{V0} \), which directly interacts with liquid and in an initial time point is a part of atmospheric volume with the preset parameters is considered.

3. Development of MM of liquid evaporation at thermal vacuum influence

Let us obtain the equations with consideration of VGM pumping from the constant volume of \( V_{V0} \) taking into account pumping out with a volume speed of \( U_v \) when in the initial equations the current pressure \( P(t) \) and temperature of \( T_v(t) \) of VGM will be defined from adiabatic expansion with constant \( k \).

The following sizes for the processes will be considered:

\[ V_i, P_i, T_i, m_i, Cp_i \] are the volumes, pressure, temperatures, masses and heat capacities for a bathtub, liquids, VGM, dry air and steam of liquid at \( i = s, w, v, Ai, p \), respectively.

Based on the considered assumptions we will further give a MM, describing the process of heat and mass exchange between elements in VC, in the form of the ordinary differential equations.

For MM simplification, describing the process of heat and mass exchange at thermal vacuum influence, the following notations are used [23]:

\[ W_w(t,T,P) = K_0 (P_w(T_w) - P_v(m_p,T_v)) \left( \frac{P_v}{P_w} \right)^\delta \]

is the speed of liquid evaporation from the unit of area, \( F_s(t,m_s) = U_v \frac{m_s}{V_{V0}} \) is the function of the rate of dry air decay from VC, \( F_p(t,m_p) = U_v \frac{m_p}{V_{V0}} \)

is the function of the rate of liquid vapors decay from VC, \( F_{ww}(t,T) = \frac{(L_w - Cp_w T_w)}{m_w Cp_s + m_w Cp_w} \) is the function of change of liquid mass-averaged temperature from its evaporation,
\[ F_t(t) = \frac{Q_t}{m_A C_p + m_w C_p} \] is the function of change of liquid mass-averaged temperature from the power of thermal influence, \( F_t(t) = \frac{1}{k-1} \frac{T_v}{V_0} \) is the speed of change of VGM temperature at adiabatic expansion, \( F_{av}(t, T) = \frac{(L_w - C_p T_w)}{m_a C_p + m_p C_p} \) is the function of change of VGM mass-averaged temperature from liquid evaporation, \( F_{m}(t, m, \mu) = \left(\frac{m_a(t)}{\mu_A} + \frac{m_p(t)}{\mu_w}\right) \) is the a molar ratio of air mixture mass.

At the considered assumptions in a problem statement, the system of the ordinary differential equations taking into account exchange of masses (liquid evaporation, pumping out of VGM) and heat exchange for liquid and VGM, will have the following form:

\[
\begin{align*}
\frac{\partial m_w}{\partial t} &= -SW_w(t, T, P), \\
\frac{\partial m_p}{\partial t} &= -C_U(t) \frac{\partial V_v}{\partial t} V_0 = -C_U(t) F_A(t, m_A), \\
\frac{\partial T_v}{\partial t} &= -F_v(t) C_U(t) U_V + F_{av}(t, T) SW_w(t, T, P), \\
\frac{\partial T_v}{\partial t} &= -F_v(t) C_U(t) U_V + F_{av}(t, T) SW_w(t, T, P), \\
\end{align*}
\]

where \( P_v(t) = \left(\frac{m_a(t)}{\mu_A} + \frac{m_p(t)}{\mu_w}\right) \frac{RT_v(t)}{V_0(t)} \) is the the current pressure in ME (Pa), \( P_0 \) is the initial atmospheric pressure (Pa), \( m_w \) is the liquid weight (kg), \( V_v \) is the volume of VGM (m³), \( T_w \) is the liquid temperature (K), \( T_v \) is the VGM temperature (K), \( S \) is the area of a free surface liquid (m²), \( W_w \) is the liquid evaporation velocity from a unit of area (kg / (c·m²)), \( L_w \) is the specific heat of evaporation liquid (J/kg), \( W_w \) is the liquid evaporation speed from the unit of area (kg / (c·m²)), \( C_{U}(t) \) is the function of thermal heating on/off control (1 or 0), \( R = 8.314 \) is the universal gas constant (J/(mole ·K)), \( K_0 \) is the coefficient of a condition of the evaporated liquid surface (undistorted is the 1.39·10⁻⁸⁷, wavy is the 4.17·10⁻⁸⁷[24] and the movements of the air medium (at 5 m/s is the 1.67·10⁻⁷kg / (Pa · m³ · c)), \( P_w \) is the partial pressure of evaporated liquid saturation at its current temperature of \( T_w \) (Pa), \( P_P \) is the partial pressure of liquid in air (Pa), \( q \) is the proportionality degree of pressure change for this liquid (0.66+1.18) [24], \( m_{l} \) is the mass of dry air (kg/mole), \( m_{p0} \) is the initial mass of water (liquid) vapors (kg), \( m_p = m_{p0} + m_{w0} - m_w \) is the current mass of liquid vapors (kg), \( \mu_w \) is the liquid molar weight (kg/mole), \( \mu_A \) is the molar mass of dry air (kg/mole), \( U_V \) is the speed of pumping (expansion) of VGM volume (m³/s), \( C_{l}(t) \) is the function of pumping on/off control (1 or 0), \( m_{l} \) and \( m_{p} \) (kg) is the mass of dry air and liquid vapors in a unit of volume of atmospheric \( V_{v1} \) air (m³), \( m_i \) is the mass of MV (kg), \( C_{p3} \) is the MV heat capacity (J / (kg · K)), \( C_{p_{w}} \) is the liquid heat capacity (J / (kg · K)), \( C_{p_{p}} \) is the heat capacity of liquid vapors (J / (kg · K)), \( C_{p_{A}} \) is the heat capacity of dry air (J / (kg · K)).
At temperatures of liquid condition from 273.1 K and up to 303.1 K and above we shall approximately present the partial pressure of the evaporated liquid saturation in the following form, according to tabular data [24]:

\[ P_w(T_w) = 174727.8 - 1312.1 T_w + 2.47 T_w^3 \quad (Pa) \]  

(6)

Based on Clapeyron-Mendeleyev's equation for mixture gases the partial pressure of liquid in air \( P_P \) can be presented in the following form:

\[ P_p(t) = \frac{m_p(t) RT_v(t)}{V_v(t)}. \]  

(7)

For the system of equations (1) - (5), initial and boundary conditions can be set from time of \( m_w, m_A, m_p, T_w, T_V \) in the following form:

\[ t_0 = 0; m_w = m_{w0}, m_A = m_{A0}, m_p = m_{p0}, T_w = T_{w0}, T_V = T_{V0}; t = t_k : m_w = m_{wk} \]  

(8)

where \( t_k \) is the undetermined value.

At the constant volume of VC in the system of equations (1)-(5) we suggest that the VGM volume is equal to \( V_V = V_0 \), and at pumping out of air the equation is carried out from VC with change of pressure and temperature on adiabatic process with \( k = 1.4 \) \( (TV^{k-1} = \text{const}, PV^k = \text{const}) \) – taking into account the VGM expansion on a differential ratio in the following form:

\[ \frac{dV_V}{dt} = U_V (t) = C_U (t) U_V. \]  

(9)

4. Criterion of optimization

For formation of optimality criterion for heat and a mass exchange process at thermal vacuum influence we will consider minimization of power costs of a certain liquid mass evaporation with \( m_0 \) to \( m_k \) for time from \( t_0 = 0 \) to \( t_k \) with change of whole energy in the considered system of elements (MV + liquid + VGM).

The change of energy of the whole system of material bodies with liquid and VGM have to be corresponded by expenses of the used energy of pumping with expansion of volume on adiabatic process (with pressure decline and temperatures) and the additional conductive energy supplied for heating that is possible to present as integral from a difference of pressure in the form of [25]:

\[ E_z = \int_0^t C_U S_V (P_0 - P_t) \frac{U_V}{S_V} dt + E_r = \int_0^t C_U (P_0 - P_t) U_V dt + \int_0^t C_T Q_r dt, \]  

(10)

which corresponds to the work on of forces \( S_V (P_0 - P_t) \) overcoming on distances \( U_V / S_V \) during \( t_k \), where \( U_V \) is the speed of volume expansion (or pumping) on the area \( S_v \), \( P_0 \) is the external atmospheric pressure, \( P_V \) is the internal pressure of air mixture in the considered volume (in this case, in the constant volume of \( V_0 \)).

Numerical results of modeling the system of equations (1) – (5) and comparison with experimental studies results [23] are given in paragraph 5

5. Development of optimum control of thermal vacuum process of evaporation based on the principle of a maximum of Pontryagin

Let us consider optimization of power consumption of evaporation of the preset liquid mass by means of the Pontryagin method [21].

According to this theory, in solving the optimization problem with functionality of Hamilton:
\[ H = \lambda_1 \frac{\partial m_w}{\partial t} + \lambda_2 \frac{\partial m_A}{\partial t} + \lambda_3 \frac{\partial m_p}{\partial t} + \lambda_4 \frac{\partial T_w}{\partial t} + \lambda_5 \frac{\partial T_v}{\partial t} - f_H \]  

(11)

It is necessary to introduce the conjugate functions $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$ and the function $f_H$ for power consumption minimization.

The expression for $f_H$ can be written as:

\[ f_H = \frac{\partial E_v}{\partial t} = C_v(t)(P_0 - \frac{m_x(t)}{\mu_A} + \frac{m_p(t)}{\mu_w} RT_v(t) V_{v_0}(t))U_v + C_f Q_r. \]  

(12)

According to equations (1) - (5) and (12) final expression for the function (11) determining the maximizing functional with minimization of expenses of energy by function (12) during time is based $[0, t_k]$:

\[ H = \lambda_1 \frac{\partial m_w}{\partial t} + \lambda_2 \frac{\partial m_A}{\partial t} + \lambda_3 \frac{\partial m_p}{\partial t} + \lambda_4 \frac{\partial T_w}{\partial t} + \lambda_5 \frac{\partial T_v}{\partial t} - 
-C_v(t)(P_0 - \frac{m_x(t)}{\mu_A} + \frac{m_p(t)}{\mu_w} RT_v(t) V_{v_0}(t))U_v - C_f Q_r = 
\]

\[ = \lambda_1[-SW_w(t,T,P)] + \lambda_2[-C_v(t)U_v \frac{m_x}{V_{v_0}}] + 
+ \lambda_3[-C_v(t)U_v \frac{m_p}{V_{v_0}} + SW_w(t,T,P)] + 
+ \lambda_4[-C_v(t)U_v \frac{m_p}{V_{v_0}} + SW_w(t,T,P)] + 
+ \lambda_5[-C_v(t)U_v \frac{m_p}{V_{v_0}} + SW_w(t,T,P)] + 
\]

To develop the conjugate system of the equations for definition of functions $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_5$ separate derivatives of expressions (1) – (5) and (11) are firstly determined by the chosen independent variables $(m_w, m_A, m_p, T_w, T_v)$. After their application in derivative of function $H$ of Hamilton (derivative of $H$, including $f_H$) the conjugate system of the equations to (1) - (5) is developed in the following form [21]:

\[ \dot{\lambda}_1 = -\frac{\partial H}{\partial m_w} = -\lambda_1[C_p(T_w)SW_w(t,T,P)F_{m_w}(t,T) - C_p(T_r)F_r(t)]F_{c_1}(t), \]  

(13)

\[ \dot{\lambda}_2 = -\frac{\partial H}{\partial m_A} = -(\lambda_2 - \lambda_5) L_w F_{c_1}(t) + \lambda_5 F_{m_p}(t,T)[SW_w(t,T,P) \frac{q}{\mu_A F_{m_p}(t,T,m,\mu)}] + 
+ \lambda_2 C_v(t)U_v(t) \frac{1}{V_{v_0}} - \lambda_5 [-C_p(T_m)SW_w(t,T,P)F_{c_2}(t)] + C_v(t)F_{m_1}(t)U_v, \]  

(14)
\[ \dot{\lambda}_3 = -\frac{\partial H}{\partial m_p} = -[\dot{\lambda}_1 - \dot{\lambda}_3 + \dot{\lambda}_4 L_w F_{c_1}(t) + \dot{\lambda}_4 F_{m_w}(t, T)] \cdot \]

\[ \{SW_w(t, T, P) \frac{q}{\mu_w F_{m_w}(t, m, \mu)} + SK_0(P_0) F_{kw}(t) F_{p_{pq}}(t) \} - \]

\[ -\lambda_3 \left[ -C_u(t) U_V(t) \right] \frac{1}{V_{V_y}} \left[ C_p F_{m_v}(t, T) SW_w(t, T, P) F_{c_2}(t) \right] + \left[ -C_u(t) F_{kw}(t) U_V \right], \]

\[ \dot{\lambda}_4 = -\frac{\partial H}{\partial T_w} = (\lambda_1 - \lambda_3)[SK_0(P_0)[-1312.1 + 4.94 \cdot Tw] F_{p_{pq}}(t)] - \]

\[ -\lambda_4 \left[ -L_w F_{c_1}(t)[SK_0(P_0)[-1312.1 + 4.94 \cdot Tw] F_{p_{pq}}(t)] \right] - \]

\[ -\lambda_3 \left[ -C_p F_{c_2}(t) SW_w(t, T, P) + F_{m_v}(t, T) \cdot [SK_0(P_0)[-1312.1 + 4.94 \cdot Tw] F_{p_{pq}}(t)] \right], \]

\[ \dot{\lambda}_5 = -\frac{\partial H}{\partial T_v} = \left[ -\lambda_1 - \lambda_3 + \lambda_4 L_w F_{c_1}(t)] [SK_0(P_0) F_{rw}(t) F_{p_{pq}}(t) + SW_w(t, T, P) \frac{q}{T_v(t)} \right] - \]

\[ -\lambda_5 \left[ -1 - \frac{1}{k} \frac{1}{V_{V_y}} \left[ C_u(t) U_V(t) \right] - F_{m_v}(t, T) [SK_0(P_0) F_{rw}(t) F_{p_{pq}}(t) + \right. \]

\[ + SW_w(t, T, P) \frac{q}{T_v(t)} \} - C_u(t) F_{m_w}(t) \frac{R}{V_{V_y}(t)} U_V, \]

where the following designations are used

\[
F_{c_1}(t) = \frac{1}{m_s Cp_s + m_p Cp_w}, F_{c_2}(t) = \frac{1}{m_s Cp_s + m_p Cp_p}, F_{p_{pq}}(t) = \left( \frac{P_0}{P'_0(t)} \right)^g, F_{rw}(t) = \frac{1}{\mu_k} \frac{RT_v(t)}{V_{V_y}(t)}, \]

\[
F_{kw}(t) = \frac{1}{\mu_w} \frac{RT_v(t)}{V_{V_y}(t)}, F_{rw}(t) = \frac{m_p(t) R}{\mu_k V_{V_y}(t)}. \]

For equations (13) - (17) initial data \( \lambda_{10}, \lambda_{20}, \lambda_{30}, \lambda_{40}, \lambda_{50} \) and final data \( \lambda_{1f}, \lambda_{2f}, \lambda_{3f}, \lambda_{4f}, \lambda_{5f} \) are unknown; besides, final values for four functions \( m_{w_b}, m_{p_b}, T_{W_b}, T_{R_b} \) are not set yet, therefore, short circuit of boundary conditions in systems of the equations with the unknown final time \( t_k \) will require five conditions of transversality at \( t = t_k \), which with variables \( y_1 = m_{w_b}, y_2 = m_{p_b}, y_3 = m_{p_k}, y_4 = T_{w_b}, y_5 = T_{R_b} \) the first variation of the optimizing functionality will be:

\[ J = \int_0^t H(y_i, \dot{y}_i, t) dt \]

at the requirement of achieving the extremum has to be equal to zero:

\[ \delta J |_t = \sum_{i=1}^{\lambda} \frac{\partial H}{\partial y_i} \delta y_i(t_k) + H(y_i, \dot{y}_i, t_k) \] \[ = 0. \]

Thus, for achieving the maximum, the second variation has to be negative:

\[ \delta^2 J |_t = \sum_{i=1}^{\lambda} \frac{\partial^2 H}{\partial y_i^2} \delta y_i^2 + \sum_{i=1}^{\lambda} \frac{\partial H}{\partial y_i} \delta y_i + \frac{\partial H}{\partial t} (\delta t_k)^2 < 0. \]

This condition is satisfied from creation of the interfaced system of the equations (13) - (17).

The equation (19), taking into account that already \( \delta y_1(t_k) = 0 \), will be valid at equalities to zero coefficients at other variations on unknown variables on the right border, i.e. at
\[
\frac{\partial H}{\partial y_2} = 0, \quad \frac{\partial H}{\partial y_3} = 0, \quad \frac{\partial H}{\partial y_4} = 0, \quad \frac{\partial H}{\partial y_5} = 0, \quad H(y, t_k) = 0,
\]

or
\[
\left. \frac{\partial H}{\partial m_{\lambda_i}} \right|_{y_i} = 0, \quad \left. \frac{\partial H}{\partial m_p} \right|_{y_i} = 0, \quad \left. \frac{\partial H}{\partial T_w} \right|_{y_i} = 0, \quad \left. \frac{\partial H}{\partial T_v} \right|_{y_i} = 0,
\]
\[
f_H = -m_w \frac{\partial H}{\partial m_w} - m_A \frac{\partial H}{\partial m_A} - m_p \frac{\partial H}{\partial m_p} - T_w \frac{\partial H}{\partial T_w} - T_v \frac{\partial H}{\partial T_v} = -H_{y_i} = 0.
\]

These conditions are simplified to the following form:
\[
\lambda_2 = 0, \quad \lambda_3 = 0, \quad \lambda_4 = 0, \quad \lambda_5 = 0, \quad f_H - \lambda_i m_w = 0
\]
\[
C_U(t)\left( P_0 - \left( \frac{m_A(t)}{\mu_A} + \frac{m_p(t)}{\mu_p} \right) \frac{RT_v(t)}{V_{v0}(t)} \right) U_V + C_T Q_t +
\]
\[
+ \lambda_2 [S K_0(P_0)[174727.8 - 1312.1 \cdot Tw + 2.47 \cdot Tw^2] -
\]
\[
- \left( \frac{m_p(t)}{\mu_p} \right) \frac{RT_v(t)}{V_{v0}(t)} \left[ \left( \frac{m_A(t)}{\mu_A} + \frac{m_p(t)}{\mu_p} \right) \frac{RT_v(t)}{V_{v0}(t)} \right]^y = 0.
\]

For obtaining the maximum value of functionality from \( H(t) \) on an interval \((0, t_k)\) on/off control of VGM pumping of on the \( C_U(t) \) function will be carried out from the following conditions:
\[
U_V \frac{\partial H}{\partial C_U} \geq 0 \text{ at } C_U(t) = 1,
\]
\[
U_V \frac{\partial H}{\partial C_U} < 0 \text{ at } C_U(t) = 0.
\]

From here, VGM pumping switch on will be under the condition
\[
U_U = -\lambda_2 U_V \left( \frac{m_A}{V_{v0}} - \lambda_3 U_V \left( \frac{m_p}{V_{v0}} - \lambda_5 \left[ \frac{1}{k - 1} \frac{T_v}{V_{v0}} \right] U_V \right) -
\]
\[
- (P_0 - \left( \frac{m_A(t)}{\mu_A} + \frac{m_p(t)}{\mu_p} \right) \frac{RT_v(t)}{V_{v0}(t)}) U_V \geq 0.
\]

For \( C_T \) condition of thermal influence switching and off is defined similarly (23):
\[
U_T \frac{\partial H}{\partial C_T} \geq 0 \text{ at } C_T(t) = 1,
\]
\[
U_T \frac{\partial H}{\partial C_T} < 0 \text{ at } C_T(t) = 0.
\]

The requirement of positivity of control functions \( U_U, U_T \) indicate an increase of coefficients \( C_U, C_T \) (here only with 0 to 1), in order to achieve maximum values for functional \( J \).

On the basis of (11), function of inclusion \( U_T \) in the following form is obtained:
\[ U_r = \lambda_1 \frac{Q_r}{m_S C_{p_S} + m_W C_{p_W}} - Q_r. \]  

6. Numerical modeling of thermal vacuum evaporation process with power consumption optimization

Numerical modeling of system (1) – (5), with an initial mass of liquid \( m_{w0} = 10 \) g to final \( m_{w1} = 7.1 \) g from volume with mass \( m_s = 22 \) g and area \( S = 0.01275 \) m\(^2\) with minimization of energy expenses when using heating with power \( Q_r = 3.57 \) W and pumping out with speed \( U_V = 0.0065 \) m/s of VGM from VC volume \( V_K = 0.463 \) m\(^3\) was carried out with an initial pressure of VGM \( P_0 = 105.6 \) kPa and temperature \( T_0 = 293.1^\circ \)K for air, liquid, the camera and the tray from iron with mass \( m_s = 22 \) g.

In an initial time point in VC air with humidity of liquid vapors of 50 - 100% that demands, as for the most intense cases, pumping out of VGM from VC is considered.

For the numerical solution of system of the differential equations (1) – (5) and the associated system (13) – (17), the Runge-Kutta method of the 4th order of accuracy on time was used. Calculations were carried out before evaporation of 2.9 g of liquid, and when using optimization conditions, the updating of initial data for variables of the associated system was carried out (at the beginning: \( \lambda_{10} = 1, \lambda_{20} = 1, \lambda_{30} = -1, \lambda_{40} = -1, \lambda_{50} = -1 \)). In optimization calculations, conditions (29) – (32) were used.

By analogy with experimental data given in [23], the following physical quantities were taken: \( L_w = 2256.0 \) kJ/kg, \( R = 8.314 \) J/(mol · K), \( K_0 = 1.39 \times 10^{-4} \) kg/(Pas · m\(^2\) · c), \( q = 1.18 \), \( \mu_s = 0.0182 \) kg/mol, \( \mu_A = 0.02897 \) kg/mol, \( C_{p_S} = 460 \) J/(kg · K), \( C_{p_w} = 4180 \) J/(kg · K), \( C_{p_a} = 2020 \) J/(kg · K), \( C_{p_i} = 1005 \) J/(kg · K).

For the accounting of heat exchange between VGM, trays with liquid and the VC walls, the following sizes were considered: the tray sizes – 0.150m x 0.085m, the VC walls sizes – 0.755m x 0.785m x 0.78m with thickness 0.018m, a heat capacity of walls of the VC - \( C_{p_K} = 460 \) J/(kg · K), liquid heat conductivity coefficients - \( \lambda_w = 0.63 \) W/(m · K), VGS – \( \lambda_s = 0.022 \cdot (P_v/P_0) \) W/(m · K), VC tray and body - \( \lambda_s = 92.0 \) W/(m · J) modeling of heat exchange was carried out according to the methods presented in [23].

The case of joint continuous influence of the heater with VGM pumping out and with control for achievement of total power consumption of \( E_s \) minimization according to expression (10) was considered.

The calculations results showing the advantage of applying the operating impacts of thermal vacuum drying process (VGM heating and pumping) in comparison with influences on time are given in tables 1 – 4. Thus, the factor of smaller evaporation time is not less important.

Calculations results with initial humidity \( C_{w0} = 0 \% \), 50% and 100% without heating are given in table 1 when the given mass of liquid in view of the VC limited volume (line 1-3) can not evaporate. In the same place results with heating at the constant power of \( Q_r = 3.57 \) W are given when long heating resulted in considerable liquid overheating’s (actually it had to be evaporated) which did not correspond to an assumption about small time impact of various factors on process of evaporation (line 4-6).

\[ \textbf{Table 1. Energy consumption for liquid evaporation without VGM pumping and without optimization} \]

| \(N\) | Time \(t\), s | \(C_{w0}\) | \(Q_r\), W | \(U_V\), m/s | \(\lambda_{10}\) | \(\lambda_{20}\) | \(\lambda_{30}\) | \(\lambda_{40}\) | \(\lambda_{50}\) | \(P_s\), kPa | \(T_{w0},^\circ\) | \(T_{w1},^\circ\) | \(\Delta m\), g | \(E_s\), kJ |
|-------|--------------|-----------|-----------|-------------|----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|----------|
| 1     | \(t = 44187.7\) | 0%        | --        | --         | --       | --       | --       | --       | --       | 108.611   | 198.4     | 299.1     | <2.9      | 0.0       |
| 2     | \(t = 43050.0\) | 50%       | --        | --         | --       | --       | --       | --       | --       | 107.968   | 223.7     | 297.5     | <2.2      | 0.0       |
Results of modeling of joint turning on of the thermal heater and VGM pumping out are given in table 2. Parameters for evaporation of 2.9 g of liquid at continuous impact of VGM pumping out and liquid heating and without optimization are specified in the first line ($\lambda_i = 0$).

**Table 2.** Comparison of energy consumption for liquid evaporation with pumping and heating optimization

| №  | Time t, s | $Q_r$, W | $U_V$ | $U_Dm^3/s$ | $\lambda_{10}$ | $\lambda_{20}$ | $\lambda_{30}$ | $\lambda_{40}$ | $\lambda_{50}$ | $E_\Sigma$ kJ |
|----|-----------|----------|-------|-------------|----------------|----------------|----------------|---------------|---------------|--------------|
| 1  | $t = 143.2$ | 3.57     | 6.5   | –           | –              | –              | –              | –             | –             | 84.7426      |
| 2  | $t = 629.7$ | 3.57     | 6.5   | 1182.3      | 2255.40        | 5427.80        | –216.3         | –68.14        | 35.7568       |
|    | $t = 629.7$ | 3.57     | 6.5   | 1159.7      | 2255.10        | 5428.0         | –218.1         | –66.85        | 34.1021       |
| 3  | $t = 727.7$ | 3.57     | 6.5   | 1089.0      | 2250.2         | 5429.2         | –226.4         | –58.24        | 26.0731       |
| 4  | $t = 3191.9$ | 3.57    | 6.5   | 1182.3      | 2255.40        | 5427.80        | –216.3         | –68.14        | 35.7568       |

Table 2 (line 2-4) shows that by means of controlling the evaporation of the given mass of liquid, power consumption are reduced by three times.

In figures 1-2 cyclogramas of turns-on and turns-off of pumping out and heating to lines 3-4 of table 2 are shown. Besides, the graphs of liquid mass evaporation on time are shown there. As it appears from the graphs, short-term impacts of pumping out and heating lead to liquid evaporation acceleration.
Figure 1. Graph of liquid evaporation with the cyclogram of turns-on - calculation 3, table 2.

Figure 2. Graph of Liquid Evaporation with the cyclograma of turns-on - calculation 4, table 2.

In table 3 under numbers 2-5 test calculations for liquid evaporation at thermal influence with heat exchange of VGM with liquid in a tray and case walls without optimization are given (in line 2 evaporation at various initial humidity of Cw0bez of heating is considered). All these results are given as an example since they are in a contradiction with an assumption about a time smallness of thermal vacuum influence (till 10 min.).
Table 3. Energy consumption at liquid evaporation with heat exchange between VGM and liquid in a tray and case walls without optimization and comparison with the experimental results

| №   | Time, s | C_w0 | Q_T, W | U_V, Dm³/s | λ_10 | λ_20 | λ_30 | λ_40 | P_v, kPa | T_w, °K | T_v, °K | Δm, g | E_Σ, KJ |
|-----|--------|------|--------|------------|------|------|------|------|----------|--------|--------|-------|---------|
| 1   | t=41865.7 | 50%  | –      | –         | –    | –    | –    | –    | 106.447  | 293.0  | 293.1  | 2.9   | 0.0     |
| 2   | t=23613.9 | 50%  | 3.57   | –         | –    | –    | –    | –    | 106.447  | 296.3  | 293.1  | 2.9   | 84.301  |
| 3   | t=41557.1 | 70%  | 3.57   | –         | –    | –    | –    | –    | 106.447  | 296.5  | 293.1  | 2.9   | 148.358 |
| 4   | t=74901.8 | 80%  | 3.57   | –         | –    | –    | –    | –    | 106.447  | 296.7  | 293.1  | 2.9   | 267.391 |
| 5   | t=77550.5 | 90%  | 3.57   | –         | –    | –    | –    | –    | 106.447  | 296.9  | 293.1  | 2.2   | 276.889 |
| 6   | t=442.8   | 80%  | 3.57   | 6.5       | –    | –    | –    | –    | 0.464    | 266.9  | 290.3  | 2.9   | 255.726 |
| 7   | t=406.1   | 80%  | –      | 6.5       | –    | –    | –    | –    | 0.618    | 272.6  | 289.6  | 2.2   | 232.099 |

Comparison with experiment

| №   | Time, s | C_w0 | Q_T, W | U_V, Dm³/s | λ_10 | λ_20 | λ_30 | λ_40 | P_v, kPa | T_w, °K | T_v, °K | Δm, g | E_Σ, KJ |
|-----|--------|------|--------|------------|------|------|------|------|----------|--------|--------|-------|---------|
| 8   | t=435.0 | 80%  | 3.57   | 6.5        | –    | –    | –    | –    | 0.573    | 273.1  | 282.1  | 2.9   | 252.621 |
| 9   | t=400.0 | 80%  | –      | 6.5        | –    | –    | –    | –    | 0.613    | 277.9  | 279.3  | 2.2   | 230.868 |

Different entry conditions on VGM humidity which showed essential increase of both time for evaporation of 2.9 g of liquid and power consumption at high initial humidity (80% - 90%) were considered.

At humidity of C_w0 = 90% the evaporation process stopped at 2.2 g with continuation of power consumption when heating liquid that is characteristic for the closed camera with heat exchange.

With introduction of VGM pumping out (calculations 6 and 7) at humidity of 80% the essential gain in power consumption for evaporation of 2.9 g and 2.2 g of liquid, is respectively observed. These calculations will well be coordinated with the experimental data [23] given in the table in lines 8 and 9.

In table 4 comparison of energy consumption on liquid evaporation with optimization at the accounting of heat exchange of VGM with liquid is given in a tray and VC case walls.

Table 4. Comparison of energy consumption on evaporation of liquid with optimization and heat exchange of VGM with liquid in a tray and VC case walls.

| №   | Time, s | Q_T, W | U_V, Dm³/s | λ_10 | λ_20 | λ_30 | λ_40 | λ_50 | E_Σ, KJ |
|-----|--------|--------|------------|------|------|------|------|------|---------|
| 1   | t=442.8 | 3.57   | 6.5        | –    | –    | –    | –    | –    | 255.726 |
| 2   | t=435.0 | 3.57   | 6.5        | –    | –    | –    | –    | –    | 252.621 |
| 3   | t=409.5 | 3.57   | 6.5        | 2632.1| 13412| 8083 | –1.918| –70.61| 234.035 |
|     | t=t_0=105.7 | t_v=0 |       |     |     |     |     |     |         |
|     | t=t_0=409.5 | t_v=409.5 |       |     |     |     |     |     |         |
| 4   | t=403.7 | 3.57   | 6.5        | 2487.2| 11380| 8903 | 130.2 | –85.01| 230.457 |
|     | t=t_0=95.3 | t_v=0 |       |     |     |     |     |     |         |
In line 1 calculation with evaporation of 2.9 g of liquid without control at constant heating and continuous pumping out is given and corresponds to energy consumption to the experimental data given as No. 2. In lines 3 and 4 two calculations at the same input parameters with optimization which at consecutive approximations show reduction of energy consumption by evaporation of 2.9 g of liquid due to later inclusion of liquid heating are given. Using control parameters with specification of initial data $\lambda_{10} \sim \lambda_{50}$ provides the decrease of energy consumption by 10% and even more.

### Figure 3. Graph of Liquid Evaporation with the cyclogram of inclusions—calculation 4, table 4.

In figure 3 shown cyclograms of pumping out and heating to line 4 of table 4 and the graph of evaporation of liquid mass on time are shown.

#### 7. Discussion of the results

With optimization and without optimization of energy consumption (table 2) follows from results of modeling of a thermal vacuum drying (VGM heating and pumping) that there appears the gain in optimization due to reduction of an initial interval of time to VGM pumping out and later beginning of liquid heating-up (line 4). Thus, with reduction of energy consumption more optimum control leads to lengthening the time of evaporation of the set mass of liquid. In line 4 more optimum case with heating turn-off with an incomplete mass of evaporation with reduced power consumption is given, but the general time for evaporation of the set weight due to evaporation of liquid in no saturated VGM considerably increases.

The results of modeling of liquid evaporation at thermal influence with heat exchange of VGM with liquid in a tray and case walls at initial humidity > 70% showed the requirement in considerable energy consumption and long time (table 3), at humidity of 90% because of limitation of volume with saturation of VGM evaporation stops (in the considered case at value 2.2 g).
Application of pumping out leads to considerable reduction of time of evaporation of the set mass of liquid and a power prize. Thus comparison with experimental data when the heating and pumping of VGM are constantly involved, gives deviations within 1-3% in different parameters. With application of optimization (table 4) when there is later inclusion of heating, the power prize > 10% can be received.

The further direction of researches provides an assessment of the methodical errors caused by replacement of the differential equations in private derivatives on the ordinary differential equations, and also experimental check of the received results of optimum control of process of thermal vacuum evaporation of liquids with other parameters of system (volumes, masses, speeds of pumping, heat feeders in volume).

8. Conclusion

1. The mathematical model of liquid evaporation at thermal vacuum influence based on the ordinary differential equations is developed.

2. The criterion of optimization of pumping and temperature heating control at the fixed values of the heater power and pumping productivity from a condition of minimization of energy for evaporation of the fixed mass of liquid is offered.

3. The procedure of the theory of optimum control application based on the Pontryagin principle of maximum is developed for optimization of thermal vacuum evaporation process.

4. Efficiency of optimum control theory application for the considered problem definition is shown.

Acknowledgements

The research is conducted with financial support of the Ministry of Education and Science of the Russian Federation within the state task to the subordinated educational organizations, the "Increase of Ecological Safety and Economic Efficiency of Carrier Rockets with Mid-flight Liquid Rocket Engines" project task No. 9.1023.2017/PCh.

References

[1] Gatapova E, Filipenkor A, Lyulin Yu, Grauri A, Marchuk I, Kabov O 2015 Experimental research of a temperature field in two-layer system liquid-gas Thermophysics and aeromechanics 6 pp 729–734

[2] Gatapova E, Kabov O 2010 Shear-driven flows of locally heated liquid films Intern. J. Heat and Mass Transfer 53 pp 2795–2807

[3] Goncharova O, Rezanov E, Tarasov Ya 2013 Mathematical modeling of thermo-capillary currents in a thin layer of liquid taking into account evaporation The Mathematician and the Mechani, 3 pp 47–52

[4] Smiths G, Feoktistov D, Orlova E 2016 Evaporation of drops of liquids from a surface of the anodized aluminum Thermophysics and aeromechanics 1 pp 17–22

[5] Semenov A, Feoktistov D, Zaytsev D, Kuznetsov G, Kabov O 2015 A pilot study of evaporation of a drop of liquid on the heated firm surface Thermophysics and aeromechanics vol 22 pp 801–804

[6] Tsygankov A, Alyoshin A 2016 Modeling of processes of condensation and evaporation in the channel of the regenerative heatutilizer Messenger of the International academy of cold 1 pp 82–85

[7] Zudin Yu 2017 Poluempiricheskaya model of intensive evaporation Thermophysics and aeromechanics 4 pp 539–552

[8] Trushlyakov V, Lavruk S 2015 Theoretical and experimental investigations of interaction of hot gases with liquid in closed volume Acta Astronautica 109 pp 241–247
[9] Zudin Yu 2016 Linear kinetic analysis of evaporation and condensation Thermophysics and aeromechanics 3 pp 437–450
[10] Kuznetsov V 2000 Dynamics of locally heated liquid films Russ. J. Eng. Thermophys 10 pp 107–120
[11] Kabov O, Kabova Y, Kuznetsov V 2012 Evaporation of not isothermal film of liquid in the microchannel at a cocurrent stream of gas it is GIVEN 446 p 5
[12] Kuznetsov V, Andreyev V 2013 The movement of a liquid film and gas stream in the microchannel with evaporation Thermophysics and aeromechanics 1 pp 17–28
[13] Iorio G, Goneharova O, Kabov O 2009 Study of evaporative convection in an open cavity under shear stress flow Microgravity Sci. TechnoL 21 p 42
[14] Gatapova E, Filipenko R, Lyulin Yu, Graur I, Marchuk I, Kabov O 2015 A pilot study of a temperature field in two-layer system liquid-gas Thermophysics and aeromechanics 22 pp 729–734
[15] Kuznetsov V 2011 Heat-mass exchange on an interface liquid-steam News of the Russian Academy of Sciences Mechanics of liquid and gas 5 pp 97–107
[16] Trushlyakov V, Lesnyak I, Galfetti L 2017 Pilot studies of process of convective heat exchange at evaporation of kerosene and water in the closed volume Thermophysics and aeromechanics, 5 pp 771–781
[17] Semenov G, Budantsev E, Melamed L, Tropkina A 2011 Mathematical modeling and a pilot study of the combined cycles of vacuum drying of thermolabile materials The bulletin of the international academy of cold 4 pp 5–11
[18] Kabov O, Kabova Yu 2015 Influence of the sizes of the heater on evaporation of a film of the liquid which is carried away by a gas stream in the microchannel at local heating Thermophysics and aeromechanics 22 pp 539–542
[19] Nekrasov S, Volkov V 2017 Computer modeling and optimization of process of thawing of soil by means of energy of the microwave oven The Engineering and physical journal 90 pp 55–63
[20] Diligenskaya A, Rapoport E 2016 Metod of minimax optimization in a coefficient return problem of heat conductivity The engineering and physical journal 89 pp 1007–1012
[21] Pontryagin L. Boltyansky V, Gamkrelidze R, Mishchenko E 1983 Mathematic theory of optimum processes (Moscow: Science) p 393
[22] Gusev E, Bakulin V 2014 Optimum design of structural and non-uniform materials and designs with the demanded properties The Engineering and physical journal 87 pp 250–255
[23] Trushlyakov V, Panichkin A, Prusova O, Zharkov K, Dron’ M 2017 Theoretical and pilot studies of process of evaporation of liquid by vacuum drying Problem of development, production and operation of the missile and space equipment and preparation of engineering shots for aerospace branch: materials XI of the All-Russian scientific conference devoted to memory of the chief designer: "Polyot" A.S. Klinyshkov pp 100–107
[24] Volkov A, Zharsky I 2005 The big chemical reference book (Munich: Modern school) p 608
[25] Lykov A, Mikhaylov Yu 1963 Theory of heat and mass transfer (Moscow-Leningrad: Gosenergoizdat) p 536