Sigma terms of octet baryons in the extended chiral constituent quark model

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Abstract

Background: Quantitative insight into the respective roles played by the valence flavors and the sea quark-antiquark pairs in the baryons is crucial in deepening our comprehension of nonperturbative QCD.

Purpose: Study the meson-baryon $\sigma$-terms for the ground-state octet baryons $B \equiv N, \Lambda, \Sigma, \Xi$.

Methods: Within an extended chiral constituent quark model, we investigate contributions from all possible five-quark components to the $\sigma$-terms. The probabilities of the quark-antiquark components in the baryons wave functions are calculated by taking the baryons to be admixtures of three- and five-quark components, with the relevant transitions handled via the $^3P_0$ mechanism.

Results: Predictions are obtained by using input parameters taken from the literature. Numerical results for the meson-nucleon and the dimensionless $\sigma$-terms, $\bar{\sigma}_{Bl}$ and $\bar{\sigma}_{Bs}$, are reported.

Conclusions: Our results turn out to be, in general, consistent with the findings via lattice QCD and chiral perturbation theory. A thorough handling of the probabilities of light and strange quark-antiquark pairs in baryons is crucial.

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**Introduction.** The quark-antiquark $\bar{q}q$ components of the intrinsic five-quark Fock states in the baryons wave-functions, at the origin of non-perturbative phenomena, are of paramount interest in hadron physics. An appropriate entity here is the ratio of the strange component over the light ones

$$y_N = 2\frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}, \quad (1)$$

where $u$, $d$ and $s$ denote up, down and strange quark fields, respectively.

The importance of the $y_N$-parameter in the determination of the meson-baryon $\sigma$-terms is well established and, in recent years, its relevance to dark matter studies has been receiving much interest [1–3] emphasizing the highly significant impact of the sea quarks on the nucleon dark matter cross section.

The pion nucleon sigma term $\sigma_{\pi N}$, which provides critical information on the nature of explicit chiral symmetry breaking in QCD and the decomposition of the mass of the nucleon can also be related to the $\bar{q}q$ probability in the nucleon and hence to $y_N$. Extraction of $\sigma_{\pi N}$ from the $\pi N$ scattering data appears to be challenging [4–6] and there are no experimental constraints on the other meson-baryon $\sigma$-terms. However, intensive theoretical investigations on these quantities are in progress within various approaches, such as lattice QCD [7–11], chiral perturbation theory [1, 12, 13], and chiral quark model [14, 15].

Recently, the traditional constituent quark model was extended to include higher Fock components and successfully applied to describe the properties of the baryons, such as the magnetic form factor of the nucleon [16–18], the $\bar{s}s$ asymmetry of the nucleon and the NuTeV anomaly [19], the decays of baryon resonances [20–25]. In Ref. [26], the sea content of the ground-state octet baryons was investigated by taking the mechanism of transition between three- and five-quark components in the baryons to be the $^3P_0$ quark-antiquark creation. The light antiquarks asymmetry of the nucleon was then well reproduced, and $\bar{u}u$, $\bar{d}d$, and $\bar{s}s$ sea quark content in the ground-state of octet baryons were predicted. In this Brief Report, we follow the predictions of Ref. [26] on the sea quarks probabilities, to study the $\sigma$-terms of the baryons.

**Theoretical frame.** We briefly review the main ingredients of the extended chiral constituent quark model ($E\chi CQM$), which was explicitly presented in Refs. [18, 26]. The wave
function for the baryon $B$ is expressed as

$$|\psi\rangle_B = \frac{1}{\sqrt{N}} \left[ |qqq\rangle + \sum_{i,n_r,l} C_{in,l} |qqq(\bar{q}q), i, n_r, l\rangle \right], \quad (2)$$

where the first term is the conventional wave function for the baryon with three constituent quarks and the second term is a sum over all possible higher Fock components with a $\bar{q}q$ pair. Different possible orbital-flavor-spin-color configurations of the four-quark subsystems in the five-quark system are numbered by $i$; $n_r$ and $l$ denote the inner radial and orbital quantum numbers, respectively. Finally, $C_{in,l}/\sqrt{N}$ represents the probability amplitude for the corresponding five-quark component.

The coefficient $C_{in,l}$ for a given five-quark component can be related to the transition matrix element between the three- and five-quark configurations of the studied baryon. To calculate the corresponding transition matrix element, we use a $^3P_0$ version for the transition coupling operator $\hat{T}$

$$\hat{T} = -\gamma \sum_j \mathcal{F}_{i5}^{00} C_{i5}^{00} C_{OFSC} \sum_m \langle 1, m; 1, -m | 00 \rangle \chi_{i5}^{1,m} Y_{i5}^{j,-m} (\vec{p}_j - \vec{p}_5) b^\dagger (\vec{p}_j) d^\dagger (\vec{p}_5), \quad (3)$$

where $\gamma$ is a dimensionless constant of the model, $\mathcal{F}_{i5}^{00}$ and $C_{i5}^{00}$ denote the flavor and color singlet of the quark-antiquark pair $\bar{q}q_i$ in the five-quark system, and $C_{OFSC}$ is an operator to calculate the orbital-flavor-spin-color overlap between the residual three-quark configuration in the five-quark system and the valence three quark system.

The probability of the sea quarks in baryon $B$ and the normalization factor read, respectively,

$$P_{B}^{\bar{q}q} = \frac{1}{\mathcal{N}} \sum_{i=1}^{17} \left[ \frac{T^q_{iq}}{M_B - E_{iq}^{qq}} \right]^2, \quad (4)$$

$$\mathcal{N} \equiv 1 + \sum_{i=1}^{17} \mathcal{N}_i = 1 + \sum_{i=1}^{17} \sum_{\bar{q}q} \left[ \frac{T^q_{iq}}{M_B - E_{iq}^{qq}} \right]^2. \quad (5)$$

where the first term is due to the valence three-quark state, while the second term comes from the five-quark mixtures; the second sum is over $\bar{q}q \equiv \bar{u}u, \bar{d}d, \bar{s}s$.

The probabilities $P_{B}^{\bar{q}q}$ provide us with all needed matrix elements to extract $y_N$.

**Results.** Our numerical results for the $\bar{q}q$ probabilities in the ground state baryons are shown in Table I. The reported uncertainties come from a model parameter $V = 570 \pm 46$...
# Predictions for the sea content of the octet baryons (in %).

| Baryon | $\bar{u}u + \bar{d}d$ | $\bar{s}s$ | $\bar{u}u + \bar{d}d + \bar{s}s$ |
|--------|----------------------|------------|-------------------------------|
| N      | 31.8±3.2             | 5.8±0.6    | 37.6±3.8                      |
| Λ      | 28.6±3.0             | 5.8±0.6    | 34.4±3.7                      |
| Σ      | 26.6±2.9             | 6.4±0.7    | 33.1±3.6                      |
| Ξ      | 26.9±3.0             | 5.7±0.6    | 32.6±3.6                      |

MeV, which represents a common factor of the matrix elements of the transitions between three- and five-quark components in the studied baryons.

Having obtained the required ingredients to determine the $y_N$-parameter, we focus on the meson-nucleon and the nucleon strangeness $\sigma$-terms formulated as functions of $y_N$,

$$
\sigma_{\pi N} = \frac{\hat{\sigma}}{1 - 2y_N}, \quad (6)
$$
$$
\sigma^{I=0}_{KN} = \frac{1}{4}(1 + \frac{m_s}{m_l})(1 + 2y_N)\sigma_{\pi N}, \quad (7)
$$
$$
\sigma_{\eta N} = \frac{1}{3}(1 + 2\frac{m_s}{m_l}y_N)\sigma_{\pi N}, \quad (8)
$$
$$
\sigma_{N^*} = \frac{m_s}{m_l}y_N\sigma_{\pi N}, \quad (9)
$$

with the nonsinglet contribution $\hat{\sigma} = 33\pm5$ MeV as extracted within the chiral perturbation theory [13] and the PDG masses ratio [27] $m_s/m_l = 27.5 \pm 1.0$, where $m_l \equiv (m_u + m_d)/2$ is the average current mass of the up and down quarks.

# Predictions for the meson-nucleon $\sigma$-terms (in MeV).

| Reference | Approach | $\sigma_{\pi N}$ | $\sigma_{KN}$ | $\sigma_{\eta N}$ |
|-----------|----------|------------------|---------------|-------------------|
| Present work | $E\chi CQM$ | 35±6           | 267±53        | 32±8              |
| Shanahan et al. [1] | $FRR$ | 45±6           | 300±40        |                   |
| Durr et al. [7] | $LQCD$ | 39±4           |               |                   |
| Bali et al. [8] | $LQCD$ | 37±10          |               |                   |

In Table II, we report our results for the meson-nucleon $\sigma$-terms, as well as those from other approaches: the chiral perturbation extrapolations of lattice data for the nucleon mass...
using the finite-range regularization (FRR) \cite{1}, and the lattice QCD calculations \cite{7,8}. Within the reported uncertainties, the predictions from various models produce comparable results for the meson-nucleon $\sigma$-terms.

Finally, the nucleon-strange quark $\sigma$-term in our model, $\sigma_{Ns}=31\pm 8$ MeV, is consistent with those from the FRR approach \cite{1} (21\pm 6 MeV) and LQCD \cite{7} (67\pm 58 MeV). We refer the reader to Ref. \cite{1} for an instructive discussion on $\sigma_{Ns}$.

In order to extend our investigation to strange baryons, we use the $\sigma$-terms for the quark flavor-baryon $B$ defined as

$$\sigma_{Bq} = m_q \langle B | \bar{q}q | B \rangle, \quad \bar{\sigma}_{Bq} = \sigma_{Bq}/M_B,$$

where $m_q$ denotes the current mass of the quark $q$, and $M_B$ the mass of the baryon $B$; $\bar{\sigma}_{Bq}$ are the dimensionless $\sigma$-terms for $B \equiv N, \Lambda, \Sigma, \Xi$.

To separate contributions from the light and strange quark-antiquark pairs, the following expressions are introduced:

$$\sigma_{Bl} = \sigma_{Nl} R_l, \quad \sigma_{Bs} = \sigma_{Ns} R_s,$$

$$R_l = \frac{\langle B | \bar{u}u + \bar{d}d | B \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle}, \quad R_s = \frac{\langle B | \bar{s}s | B \rangle}{\langle N | \bar{s}s | N \rangle}.$$

In Table III our results for the dimensionless $\sigma$-terms are given. Therein, are depicted also results from the above mentioned FRR approach \cite{1} and from the covariant baryon chiral perturbation theory ($\chi PT$) \cite{28} up to $N^3LO$.

|          | Present work | Shanahan et al. \cite{1} | Ren et al. \cite{28} |
|----------|--------------|--------------------------|----------------------|
|          | $E\chi CQM$  | $FRR$                    | $\chi PT$            |
| $B$      | $\bar{\sigma}_{Bl}$ | $\bar{\sigma}_{Bs}$ | $\bar{\sigma}_{Bl}$ | $\bar{\sigma}_{Bs}$ |
| $N$      | 0.038\pm 0.006 | 0.033\pm 0.009 | 0.047\pm 0.008 | 0.022\pm 0.006 | 0.046\pm 0.006 | 0.134\pm 0.063 |
| $\Lambda$ | 0.022\pm 0.004 | 0.270\pm 0.051 | 0.026\pm 0.004 | 0.141\pm 0.008 | 0.017\pm 0.006 | 0.241\pm 0.063 |
| $\Sigma$ | 0.021\pm 0.003 | 0.252\pm 0.047 | 0.020\pm 0.003 | 0.172\pm 0.008 | 0.015\pm 0.005 | 0.248\pm 0.046 |
| $\Xi$    | 0.011\pm 0.002 | 0.428\pm 0.078 | 0.009\pm 0.001 | 0.239\pm 0.008 | 0.003\pm 0.003 | 0.301\pm 0.045 |

Our results for $\bar{\sigma}_{Bl}$ are consistent with the predictions provided by the two other approaches \cite{1,28}. For $\bar{\sigma}_{Bs}$ the case is less homogeneous: for the nucleon, our results are

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compatible with those of FRR, but much smaller than the \( \chi PT \) value, whereas for the three strange baryons, we get results consistent with those of \( \chi PT \), but significantly larger than those obtained within the FRR approach.

Finally, we comment on the uncertainties reported in Tables II and III. For our results the uncertainties are due to \( V = 570 \pm 46 \) MeV, \( \hat{\sigma} = 33 \pm 5 \) MeV, and \( m_s/m_l = 27.5 \pm 1.0 \). For the results from other authors, using statistical and systematic uncertainties reported in their papers, we give \( \delta = \sqrt{\delta_{\text{stat}} + \delta_{\text{sys}}} \).

Conclusions. In the present work, we investigated the \( \sigma \)-terms of the ground-state octet baryons, employing the recently developed extended chiral constituent quark model, within which the baryons are considered as admixtures of three- and five-quark states.

Probabilities of the five-quark components were calculated using the \( ^3P_0 \) transition operator. It is worth noting that out of the 17 five-quark configurations in the nucleon, only 3 of them contribute to both light and \( \bar{s}s \) pairs probabilities, whereas 5 of them have only \( \bar{u}u + \bar{d}d \) components and the remaining 9 configurations are exclusively composed of \( \bar{s}s \) pairs, see Table 1 in Ref. [29]. Accordingly, any truncated configurations set would alter not only the respective probabilities in light and strange sectors, but also will change their relative probabilities and hence \( y_N \). Our complete calculation gives \( y_N = 0.032 \pm 0.003 \).

The obtained predictions for the pseudoscalar meson-nucleon \( \sigma \)-terms and \( \sigma_{Ns} \) turn out to be in agreement with the results from the \( LQCD \) and \( \chi PT \) approaches. With respect to the dimensionless \( \sigma \)-terms, our findings for \( \bar{\sigma}_{Bl} \) are consistent with the quoted \( \chi PT \) predictions. For \( \bar{\sigma}_{Ns} \) our model and the FRR approach [1] lead to consistent results. However, for the strange baryons \( \bar{\sigma}_{Bs} \) our values are in agreement with those from the (\( \chi PT \)) approach [28], but significantly larger than the (FRR) results. This might be due to different probabilities of the \( \bar{q}q \) components of the strange baryons in our model and the FRR approach.

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