Towards Radiative Transitions in Charmonium

Cian O’Hara

Trinity College Dublin

Supervised by Prof. Sinead Ryan (TCD), Dr Christopher E. Thomas (University of Cambridge, DAMTP), Dr Graham Moir (University of Cambridge, DAMTP).

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Overview

1. Introduction

2. General Framework and Method

3. Current Work

4. Summary and Future Work
Why do we study the charmonium spectrum?

- Charmonium is the "hydrogen atom" of meson spectroscopy.
- Studies of the charm spectrum allow us to bridge the gap between theory and experiment, and to probe QCD.
- It is non-relativistic enough to be fairly well described by potential models, yet there are many states not accounted for.
- The discovery of a plethora of unexpected charmonium-like states has highlighted the need for a more complete theoretical understanding of the hadronic spectrum.
From a presentation by R. Mitchell
Why do we study radiative transitions?

- Transition from initial state to final state via photon emission.
- Below the $D\bar{D}$ threshold, states have relatively small widths and so the radiative transition rates constitute significant branching fractions.
- The computation of photocouplings is of interest for experiments such as LHCb and Panda.
- Allows us to probe the underlying quark current and charge distributions within hadrons.
- Want to extract form factors $F(Q^2)$, where $Q^2$ is the virtuality of the photon.
Radiative transitions in Charmonium

Photon has $C = -1$ so only some transitions are allowed
Lattice details

Current study - Dynamical $2 + 1$ anisotropic gauge configurations generated by the Hadron Spectrum Collaboration

| Volume   | $N_{\text{cfgs}}$ | $N_{\text{tsrcs}}$ | $N_{\text{vecs}}$ | $M_\pi$   |
|----------|------------------|-------------------|------------------|-----------|
| $20^3 \times 128$ | 603(50)          | 4(1)              | 128              | $\sim 400$ MeV |

Previous studies

- **Exotic and excited-state radiative transitions in charmonium from lattice QCD** Jozef J. Dudek, Robert G. Edwards and Christopher E Thomas, Phys. Rev. D 79, 094504, (2009)

- **Excited meson radiative transitions from lattice QCD using variationally optimised operators** Christian J. Shultz, Jozef J. Dudek and Robert G. Edwards, Phys. Rev. D 91, 114501, (2015)
\( \eta_c \) dispersion relation on the lattice

\[
(a_t E)^2 = (a_t m)^2 + \left( \frac{2\pi}{\xi (L/a_s)} \right)^2 |\vec{n}_{\vec{p}}|^2
\]

Squared energies as a function of \( |\vec{n}_{\vec{p}}|^2 \) for the pseudoscalar state. The anisotropy was found to be \( \xi = 3.488(2) \).
Three point correlation functions

- The central object of interest is the three point correlation function, with a vector current insertion $j^\mu$

$$C_{ij}^{\mu}(\Delta t) = \langle 0 | O_i(\Delta t) j^\mu(t) O_j^\dagger(0) | 0 \rangle.$$

- Within this we have three classes of diagrams
We can expand the correlation function as

$$C_{ij}^{\mu}(\Delta t) = \sum_{m,n} \frac{1}{2E_m} \frac{1}{2E_n} e^{-E_m(\Delta t-t)} e^{-E_nt} \langle 0 | O_i(0) | m \rangle \langle m | j^{\mu}(0) | n \rangle \langle n | O_j^\dagger(0) | 0 \rangle$$

Large sum containing contamination from many excited states.

Could reduce pollution by separating the operators by large times - leads to increase in noise, also want to look at excited state transitions.

Need to use appropriate interpolators with large overlap onto states of interest.
Calculation framework

We use the framework devised by the Hadron Spectrum Collaboration to accurately compute these correlators using optimised operators.

- Compute the spectrum from a matrix of two point functions, $C_{ij}$, via the generalized eigenvalue problem method.

$$C_{ij}(t) = \langle 0 | O_i(t) O_j^\dagger(0) | 0 \rangle.$$  

- We then extract the optimal linear combination of interpolators that overlap most strongly with the individual states in the spectrum.

$$\Omega_n^\dagger \sim \sum_i \nu_i^{(n)} O_i^\dagger$$
Three point correlation functions

- Using these optimised operators, the three point functions can be written as

\[
\langle 0 | \Omega_{n_f}(\Delta t) j\mu(t) \Omega_{n_i}^\dagger(0) | 0 \rangle = e^{-E_{n_f}(\Delta t-t)} e^{-E_{n_i}t} \langle n_f | j\mu | n_i \rangle + \ldots
\]

- After removing the leading time dependence we can in general expand the desired matrix element as

\[
\langle n_f | j\mu | n_i \rangle = \sum_i K_i\mu F_i(Q^2) + \ldots
\]

- For example, the pseudoscalar meson form factor is gotten from

\[
\langle \eta_c(p') | j\mu | \eta_c(p) \rangle = (p' + p)^\mu F_{\eta_c}(Q^2)
\]
Two point correlation functions \((2E_n)^{-1} e^{E_n t} \langle \Omega_n(t) \Omega_n^\dagger(0) \rangle\) for the pseudoscalar ground state and first excited state projected operators in the A2 irrep.
2m_\pi e^{m_\pi t} \langle 0 | O(t) O^\dagger(0) | 0 \rangle / | \langle 0 | O | \pi \rangle |^2 \) plotted for the rest frame pion using the optimised pion-like operator vs the standard \( \gamma_5 \) operator.
Renormalization of the vector current

- We use the local vector current $\bar{\psi} \gamma^\mu \psi$ - Not conserved on the lattice!
- Need to multiplicatively renormalize by a factor $Z_V$, which can be different for the spatial and temporal currents due to the anisotropic lattice.
- Can extract $Z_V$ from the pseudoscalar charge form-factor at zero momentum transfer

$$Z_V = \frac{F^{cont.}(0)}{F^{lat.}(0)} = \frac{1}{F^{lat.}(0)}$$
Three-point correlators

Three-point correlation functions with a $\gamma_0$ insertion for momentum up to $|\vec{n}_\beta|^2 = 4$. 
Ground state $\rho$ to ground state $\pi$ transition form factor. Curves in grey show fits used to interpolate between spacelike and timelike regions to determine the photocoupling, $F_{\rho\pi}(0)$. Experimental decay widths are converted to photocouplings shown for orientation.
An analysis of radiative transitions in the charmonium spectrum is both interesting and timely.

We have seen that using variationally optimised interpolators allows us to extract three point correlation functions for various different momenta.

The next step is to extract the various form factors for desired transitions.
Thank you for listening.