General scalar interaction in the supersymmetric FRW model

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Abstract

In this work we have constructed the most general action for a set of complex homogeneous scalar supermultiplets interacting with the scale factor in the supersymmetric FRW model. It is shown, that local conformal time supersymmetry leads to the scalar fields potential, which is defined in the same combination: Kähler potential and superpotential as in supergravity (or effective superstring) theories. This scalar fields potential depends on arbitrary parameter $\alpha$, which is not fixed by conformal time supersymmetry.

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The study of supersymmetric minisuperspace models have lead to important and interesting results. To find the physical states it is sufficient to solve the Lorentz and supersymmetric constraints\(^1\)\(^-\)\(^3\). Some of this results have already been presented in two comprehensive and organized works: a book\(^4\) and an extended review\(^5\).

In the previous works\(^6\)\(^,\)\(^7\) we have proposed a new approach to the study of supersymmetric quantum cosmology. The main idea is to extend the group of local time reparametrization of the cosmological models to the \(n = 2\) local conformal time supersymmetry. For this purpose the odd “time” parameters \(\eta, \bar{\eta}\) were introduced (where \(\bar{\eta}\) is the complex conjugate to \(\eta\)), which are the superpartners of the usual time parameter \(t\). After we have introduced the new “time” parameters our new functions, previously only time \(t\) functions, become superfunctions depending on \((t, \eta, \bar{\eta})\), which are called superfields. Following the superfield procedure we have constructed the superfield action for the cosmological models, which is invariant under the \(n = 2\) local conformal time supersymmetry. The fermionic superpartners of the scale factor and of the homogeneous scalar fields in the quantum level are elements of the Clifford algebra.

In the present work we have studied the supersymmetric FRW model interacting with a set of \(n\) complex homogeneous scalar supermatter fields. We have shown, that in this case the potential of scalar matter fields is a function of the Kähler potential and one arbitrary parameter \(\alpha\). Furthermore, when \(\alpha = 1\) the scalar fields potential becomes energy vacuum of the scalar fields interacting with chiral matter multiplets in the case of \(N = 1\) supergravity theory\(^8\).

Let’s begin by considering the FRW action

\[
S_{\text{grav}} = \frac{6}{8\pi G_N} \int \left( -\frac{\dot{R}^2}{2N} + \frac{1}{2} k N R + \frac{d}{dt} \left( \frac{R^2 \dot{R}}{2N} \right) \right) dt, \tag{1}
\]

for \(k = 1, 0, -1\) stands for a spherical, plane and hyperspherical three-space respectively, \(\dot{R} = \frac{dR}{dt}\), \(G_N\) is the Newtonian gravitational constant, \(N(t)\) is the lapse function and \(R(t)\) is the scale factor depending only on \(t\). In this work we will apply the system units
\[ c = \hbar = 1. \]

It is well known, that the action (1) preserves the invariance under time reparametrization,

\[ t' \to t + a(t), \quad (2) \]

if \( R(t) \) and \( N(t) \) are transformed as

\[ \delta R = a \dot{R}, \quad \delta N = (aN'). \quad (3) \]

In order to obtain the superfield formulation of the action (1) in ref. [6] the transformations of the time reparametrization (2) were extended to the \( n = 2 \) local conformal time supersymmetry \( (t, \eta, \bar{\eta}) \). These transformations can be written as

\[ \delta t = IL(t, \eta, \bar{\eta}) + \frac{1}{2} \bar{\eta} D_{\eta} IL(t, \eta, \bar{\eta}) - \frac{1}{2} \eta D_{\bar{\eta}} IL(t, \eta, \bar{\eta}), \]
\[ \delta \eta = \frac{i}{2} D_{\eta} IL(t, \eta, \bar{\eta}), \]
\[ \delta \bar{\eta} = -\frac{i}{2} D_{\bar{\eta}} IL(t, \eta, \bar{\eta}) \quad (4) \]

with the superfunction \( IL(t, \eta, \bar{\eta}) \) written as

\[ IL(t, \eta, \bar{\eta}) = a(t) + i\eta \bar{\beta}'(t) + i\bar{\eta} \beta'(t) + b(t) \eta \bar{\eta}, \quad (5) \]

where \( D_{\eta} = \frac{\partial}{\partial \eta} + i\bar{\eta} \frac{\partial}{\partial \bar{\eta}} \) and \( D_{\bar{\eta}} = -\frac{\partial}{\partial \bar{\eta}} - i\eta \frac{\partial}{\partial \eta} \) are the supercovariant derivatives of the global conformal supersymmetry, which have dimension \( [D_{\eta}] = t^{-1/2} \), \( a(t) \) is a local time reparametrization parameter, \( \beta'(t) = N^{-1/2} \beta(t) \) is the Grassmann complex parameter of the local conformal susy transformations (1) and \( b(t) \) is the parameter of local \( U(1) \) rotations on the complex \( \eta \).
The superfield generalization of the action (1), which is invariant under the transformations (4), was found in our previous work and has the form

\[ S_{grav} = \frac{6}{\kappa^2} \int \left\{ -\frac{N}{2} R D_\eta R D_\eta R + \frac{\sqrt{\kappa}}{2} R^2 + \frac{1}{4} D_\eta \left( N^{-1} R^2 D_\eta R \right) \right\} d\eta d\bar{\eta} dt, \]

where we introduce the parameter \( \kappa^2 = 8\pi G_N \). We can see, that this action is hermitian for \( k = 0, 1 \). The last two terms in (6) are a total derivative, which are necessary when we consider interaction and \( N(t, \eta, \bar{\eta}) \) is a real one-dimensional gravity superfield, which has the form

\[ N(t, \eta, \bar{\eta}) = \bar{N}(t) + i\bar{\eta}\bar{\psi}'(t) + i\eta\psi'(t) + \eta\bar{\eta}V'(t), \]

where \( \psi'(t) = N^{1/2}\psi(t) \), \( \bar{\psi}'(t) = N^{1/2}\bar{\psi}(t) \) and \( V'(t) = NV + \bar{\psi}\psi \). This superfield transforms as

\[ \delta N = (\bar{L}L)\dot{N} + \frac{i}{2} D_\eta \bar{L}D_\eta N + \frac{i}{2} D_\eta L D_\eta N. \]

The components of the superfield \( N(t, \eta, \bar{\eta}) \) in (7) are gauge fields of the one-dimensional \( n = 2 \) extended supergravity.

The superfield \( R(t, \eta, \bar{\eta}) \) may be written as

\[ R(t, \eta, \bar{\eta}) = R(t) + i\eta\bar{\lambda}'(t) + i\bar{\eta}\lambda'(t) + \eta\bar{\eta}B'(t), \]

where \( \lambda'(t) = \frac{\kappa N^{1/2}}{\sqrt{R}}\lambda(t) \), \( \bar{\lambda}'(t) = \frac{\kappa N^{1/2}}{\sqrt{R}}\bar{\lambda}(t) \) and \( B'(t) = \kappa NB - \frac{\kappa}{6\sqrt{R}}(\bar{\psi}\lambda - \psi\bar{\lambda}) \). The transformation rule for the real scalar superfield \( R(t, \eta, \bar{\eta}) \) is

\[ \delta R = (\bar{L}L)\dot{R} + \frac{i}{2} D_\eta \bar{L}D_\eta R + \frac{i}{2} D_\eta L D_\eta R. \]

The component \( B(t) \) in (9) is an auxiliary degree of freedom; \( \lambda(t) \) and \( \bar{\lambda}(t) \) are the fermionic superpartners of the scale factor \( R(t) \). The superfield transformations (8,9,10) are the generalization of the transformations for \( N(t) \) and \( R(t) \) in (3).
The complex matter supermultiplets $Z^A(t, \eta, \bar{\eta})$ and $\bar{Z}^\dagger(t, \eta, \bar{\eta}) = (Z^A)^\dagger$ consist of a set of spatially homogeneous matter fields $z^A(t)$ and $\bar{z}^\dagger(t)$ ($A=1,2,\ldots,n$), four fermionic degrees of freedom $\chi^A(t)$, $\bar{\chi}^\dagger(t)$, $\phi^A(t)$ and $\bar{\phi}^\dagger(t)$, as well as bosonic auxiliary fields $F^A(t)$ and $\bar{F}^A(t)$.

The components of the matter superfields $Z^A(t, \eta, \bar{\eta})$ and $\bar{Z}^\dagger(t, \eta, \bar{\eta})$ may be written as

$$Z^A = z^A(t) + i\eta \chi^A(t) + i\bar{\eta} \bar{\phi}^\dagger(t) + F^A(t)\eta\bar{\eta},$$

$$\bar{Z}^\dagger = \bar{z}^\dagger(t) + i\eta \bar{\phi}^\dagger(t) + i\bar{\eta} \chi^A(t) + \bar{F}^A(t)\eta\bar{\eta},$$

where

$$\chi^A(t) = N^{1/2}R^{-3/2} \chi^A(t), \quad \phi^A(t) = N^{1/2}R^{-3/2} \phi^A(t),$$

$$F^A(t) = NF^A - \frac{1}{2\sqrt{R}}(\psi \chi^A - \bar{\psi} \bar{\phi}^\dagger).$$

The transformation rule for the superfields $Z^A(t, \eta, \bar{\eta})$ and $\bar{Z}^\dagger(t, \eta, \bar{\eta})$ may be written as

$$\delta Z^A = \delta \bar{Z}^\dagger = \delta \bar{Z}^\dagger = \lambda \bar{Z}^\dagger + i\frac{D_{\eta}}{2} D_{\eta} Z^A + i\frac{D_{\bar{\eta}}}{2} D_{\bar{\eta}} \bar{Z}^\dagger,$$

The most general interacting action for the set of complex homogeneous scalar supermultiplets $Z^A(t, \eta, \bar{\eta})$ with the scale superfactor $\mathbb{R}(t, \eta, \bar{\eta})$ up to the second-order derivatives has the following form

$$S = \int \left\{ \Phi \left[ N^{-1} \mathbb{R} D_{\eta} \mathbb{R} D_{\bar{\eta}} \mathbb{R} - \sqrt{k} \mathbb{R}^2 \right. ight. - \frac{1}{2} \left. \left. \left( \mathbb{R} \left[ N^{-1} \mathbb{R}^2 \mathbb{R} D_{\eta} \mathbb{R} D_{\bar{\eta}} \mathbb{R} \right] - D_{\eta} \left( N^{-1} \mathbb{R}^2 \mathbb{R} D_{\eta} \mathbb{R} \right) \right) \right] ight.$$

$$+ \frac{1}{2\alpha} N^{-1} \mathbb{R}^3 \left[ D_{\eta} \bar{Z}^\dagger D_{\eta} Z^A + D_{\eta} Z^B D_{\bar{\eta}} \bar{Z}^\dagger \right] \left[ -\Phi^{-1} \frac{\partial \Phi}{\partial \bar{Z}^\dagger} \frac{\partial \Phi}{\partial Z^A} + \frac{\partial^2 \Phi}{\partial \bar{Z}^\dagger \partial Z^A} \right]

$$- \frac{2}{\kappa^3} \left| g(Z) \right|^\alpha + \frac{1}{4} N^{-1} \mathbb{R}^3 \Phi^{-1} D_{\eta} \Phi D_{\eta} \Phi \right\} d\eta d\bar{\eta} dt.$$
This interaction depends on two arbitrary superfunctions \( \Phi(Z^A, \bar{Z}^\bar{A}) \), \( g(Z^A) \) which is dimensionless superpotential and of the arbitrary parameter \( \alpha \). The action (15) is invariant under the local conformal supersymmetric transformations (4). If we make the following Weyl redefinitions

\[
N \to \exp \left( \frac{\alpha K}{6} \right) N, \quad R \to \exp \left( \frac{\alpha K}{6} \right) R,
\]

\[
\Phi \exp \left( \frac{\alpha K}{3} \right) = -\frac{3}{\kappa^2},
\]

then, our superfield action (15) takes the form

\[
S = \int \left\{ -\frac{3}{\kappa^2} N^{-1} R D_\eta R D_\eta R + \frac{3}{\kappa^2} \sqrt{k} R^2 - \frac{2}{\kappa^3} R^3 e^{2G} \\
+ \frac{1}{2\kappa^2} N^{-1} R^3 G_{\bar{A}B} \left[ D_\eta Z^{\bar{A}} D_\eta Z^B + D_\eta Z^B D_\eta Z^{\bar{A}} \right] \right\} d\eta d\bar{\eta} dt.
\]

(17)

In this action the total derivative no appear because is not necessary in our study. The action (17) is determined only by terms of one arbitrary Kähler superfunction \( G(Z^A, \bar{Z}^{\bar{A}}) \), and it is a special combination of \( IK(Z^A, \bar{Z}^{\bar{A}}) \) and \( g(Z^A) \), where

\[
G(Z, \bar{Z}) = IK(Z, \bar{Z}) + \log|g(Z)|^2
\]

is invariant under the transformations

\[
g(Z) \to g \exp f(Z), \\
IK(Z, \bar{Z}) \to IK(Z, \bar{Z}) - f(Z) - \bar{f}(\bar{Z})
\]

(19)

with the Kähler potential \( IK(Z, \bar{Z}) \) defined on the complex superfield \( Z^A \) related to \( \Phi(Z, \bar{Z}) \) (16). The superfunction \( G(Z, \bar{Z}) \) and their transformations are the generalizations of the Kähler function \( G(z, \bar{z}) = K(z, \bar{z}) + \log|g(z)|^2 \) defined on the complex manifold and has the Taylor expansion respect to \( \eta, \bar{\eta} \)

\[
G(Z, \bar{Z}) = G(z, \bar{z}) + G(z, \bar{z})_A(Z^A - z^A) + G(z, \bar{z})_{\bar{A}}(\bar{Z}^{\bar{A}} - \bar{z}^{\bar{A}}) \\
+ \frac{1}{2} G(z, \bar{z})_{AB}(Z^A - z^A)(Z^B - z^B) + \frac{1}{2} G(z, \bar{z})_{\bar{A}B}(\bar{Z}^{\bar{A}} - \bar{z}^{\bar{A}})(\bar{Z}^{\bar{B}} - \bar{z}^{\bar{B}}) \\
+ G(z, \bar{z})_{AB}(\bar{Z}^{\bar{A}} - \bar{z}^{\bar{A}})(Z^B - z^B),
\]

(20)
where the first term in the expansion is the Kähler function. Derivatives of the Kähler function are denoted by \( \frac{\partial G}{\partial z} = G_A \equiv G_A, \frac{\partial G}{\partial \bar{z}} = G_{\bar{A}} \equiv G_\bar{A}, \frac{\partial^n G}{\partial z^m \partial \bar{z}^n} = G_{ABC...D} \equiv G_{ABC...D} \) and the Kähler metric is \( G_{AB} = G_{BA} = K_{AB} \), the inverse Kähler metric \( G^{AB} \), such as \( G^{AB} G_{BD} = \delta^A_B \), can be used to define \( G^A \equiv G^{AB} G_B \) and \( G^B \equiv G_A G^A \).

Perhaps, it is important to mention, that in supergravity it is also possible to introduce Weyl transformations \( \phi(z, \bar{z}) = -\frac{2}{\kappa^2} \exp(-\frac{2}{3} K(z, \bar{z})) \) and the vierbein \( e_\mu^a \rightarrow \exp \frac{2}{3} K(z, \bar{z}) \) with an arbitrary parameter \( \alpha \) (see eq. (16)). However, the terms in the supergravity action can not be represented by the Kähler function \( G(z, \bar{z}) \), which is due to the scalar curvature term, the kinetic term in the complex fields and auxiliary fields \( A_\mu \) in the supergravity multiplet are eliminate only if \( \alpha = 1 \). The action (17) is invariant under the local conformal transformations (9) if the superfields are transformed as (8, 10, 13, 14). After the integration over the Grassmann variables \( \eta, \bar{\eta} \) the action (17) becomes a component action with the auxiliary fields \( B(t), F^A(t) \) and \( \bar{F}^\bar{A}(t) \). These ones may be determined from the component action by taking the variation with respect to them. The equations for these fields are algebraical and have the solutions

\[
B = -\frac{\kappa}{18 R^2} \bar{\lambda} \lambda + \sqrt{\frac{\kappa}{R^2}} + \frac{1}{4 \kappa R^2} G_{AB} (\bar{\chi} \chi^B + \phi^B \bar{\phi}^A) - \frac{R}{\kappa^2} e^{\alpha G} \sqrt{\frac{\kappa}{R^2}} \lambda \lambda - \frac{1}{\kappa} e^{\alpha G} \bar{\lambda} \lambda, \quad (21)
\]

\[
F^D = -\frac{\kappa}{2 R^3} (\bar{\lambda} \phi^D - \lambda \chi^D) - \frac{1}{R^3} G^{DA} G_{ABC} \chi^C \chi^B + \frac{2}{\kappa} G^{D \bar{A}} (\bar{e}^{\alpha G} \sqrt{\frac{\kappa}{R^2}} \bar{\lambda} \lambda) \quad (22)
\]

After substituting them again into the component action we get the following action

\[
S = \int \left\{ -\frac{3}{\kappa^2} \frac{R(DR)^2}{N} - N R^3 U(R, z, \bar{z}) + \frac{2i}{3} \lambda D \lambda + \frac{\sqrt{\kappa}}{3 R} \lambda \lambda - \frac{1}{\kappa} e^{\alpha G} \bar{\lambda} \lambda \right. \\
+ \frac{\sqrt{\kappa}}{\kappa} \sqrt{R} (\bar{\psi} \lambda - \psi \bar{\lambda}) + \frac{R^3}{N \kappa^2} G_{AB} D \bar{z} \chi^A D z^B + \frac{i}{2 \kappa} D z^B \left( \bar{\lambda} G_{\bar{A}} \chi^A + \lambda G_{AB} \bar{\phi}^A \right) \\
+ \frac{i}{2 \kappa} D \bar{z} \bar{\chi}^\bar{A} \left( \bar{\lambda} G_{\bar{A}} \chi^B + \lambda G_{AB} \bar{\phi}^B \right) - \frac{i}{\kappa^2} G_{AB} \left( \bar{\chi} \chi^B + \phi^B \bar{\phi}^A \right) \\
- \frac{N}{\kappa^2 R^3} R_{ABC} D \bar{z} \chi^C \bar{\phi}^D - \frac{i}{4 \kappa \sqrt{R^3}} (\bar{\psi} \bar{\lambda} - \psi \lambda) G_{AB} \left( \bar{\chi} \chi^B + \phi^B \bar{\phi}^A \right) \\
+ \frac{3 N}{16 \kappa^2 R^3} \left[ G_{AB} \left( \bar{\chi} \chi^B + \phi^B \bar{\phi}^A \right) \right]^2 + \frac{3 \sqrt{\kappa}}{2 \kappa^2 R} G_{AB} \left( \bar{\chi} \chi^B + \phi^B \bar{\phi}^A \right) \right\}
\]
where the effective potential of the scalar matter fields is

\[
- \frac{3N}{2k^3} e^{\frac{\alpha}{2}} G_{AB} \left( \chi^A \chi^B + \phi^B \phi^A \right) - \frac{2N}{k^3} \left( e^{\frac{\alpha}{2}} \right)_{;AB} \chi^A \phi^B - \frac{2}{k^3} N(e^{\frac{\alpha}{2}})_{;AB} \phi^A \chi^B \\
- \frac{2}{k^3} N(e^{\frac{\alpha}{2}})_{;AB} \left( \chi^A \chi^B + \phi^B \phi^A \right) - \frac{N}{k^3} \chi \left( e^{\frac{\alpha}{2}} \right)_{;A} \phi^A + (e^{\frac{\alpha}{2}})_{;A} \chi^A \\
+ \frac{N}{k^3} \left[ \left( e^{\frac{\alpha}{2}} \right)_{;A} \chi^A + (e^{\frac{\alpha}{2}})_{;A} \phi^A \right] - \sqrt{R^3} \left( \bar{\psi} \lambda - \bar{\psi} \lambda \right) e^{\frac{\alpha}{2}} \\
+ \sqrt{R^3} \left( e^{\frac{\alpha}{2}} \right)_{;A} \left( \bar{\psi} \chi^A - \bar{\psi} \phi^A \right) + \sqrt{R^3} \left( e^{\frac{\alpha}{2}} \right)_{;A} \left( \bar{\psi} \phi^A - \bar{\psi} \chi^A \right) \right) \, dt, \quad (23)
\]

where \( DR = \dot{R} - \frac{i\kappa}{6\sqrt{R}} \left( \bar{\psi} \lambda + \psi \bar{\lambda} \right) \), \( Dz^A = \dot{z}^A - \frac{i}{2\sqrt{R}} \left( \bar{\psi} \phi^A + \psi \chi^A \right) \), \( D\chi^B = \dot{\chi}^B - \frac{3}{2} \sqrt{\kappa} \chi^B \), \( D\phi^B = \dot{\phi}^B + \frac{i}{2} \sqrt{\kappa} \phi^B \), \( D\lambda = \dot{\lambda} + \frac{i}{2} \sqrt{\kappa} \lambda \), \( \bar{D} \chi^B = \bar{D} \chi^B + \Gamma^B_{CD} \bar{z}^C \chi^D \), \( \bar{D} \phi^B = \bar{D} \phi^B + \Gamma^B_{CD} \bar{z}^C \phi^D \), \( R_{ABCD} \) is the curvature tensor of the Kähler manifold defined by the coordinates \( z^A, \bar{z}^B \) with the metric \( G_{AB} \), and \( \Gamma^B_{CD} = G^{BD} G_{ACD} \) are the Christoffel symbols in the definition of the covariant derivative and their complex conjugate. Besides, the potential term \( U(R, z, \bar{z}) \) read as

\[
U(R, z, \bar{z}) = - \frac{3k}{k^2 R^2} + \frac{6\sqrt{\kappa}}{k^3 R} e^{\frac{\alpha}{2}} + V_{eff}(z, \bar{z}), \quad (24)
\]

where the effective potential of the scalar matter fields is

\[
V_{eff} = \frac{4}{k^4} \left[ \left( e^{\frac{\alpha}{2}} \right)_{;A} G^{AD} (e^{\frac{\alpha}{2}})_{;D} - \frac{3}{4} e^{2\alpha} G \right] = \frac{e^{\alpha G}}{k^4} \left[ \alpha^2 G^A G_A - 3 \right]. \quad (25)
\]

In the action \((23)\) the Kähler function is a function of scalar fields, i.e \( G(z, \bar{z}) \). When the parameter \( \alpha = 1 \) we obtain the effective potential for supergravity coupling with scalar matter fields. From \((24)\) we can see, that when \( k = 0 \), \( U(R, z, \bar{z}) = V_{eff}(z, \bar{z}) \).

In order to discuss the implications of spontaneous supersymmetry breaking we need to display the superpotential \((24)\) in terms of the auxiliary fields

\[
\mathcal{U}(R, z, \bar{z}) = \frac{\tilde{F}^A G_{AB} F^B}{k^2} - \frac{3B^2}{R^2}, \quad (26)
\]

where the bosonic terms \((24,22)\) are read now as

\[
B = \sqrt{\kappa} - \frac{R}{k^2} e^{\frac{\alpha}{2}}, \quad (77)
\]

\[
F^A = \frac{\alpha}{k} e^{\frac{\alpha}{2}} G^A. \quad (28)
\]
The supersymmetry is a spontaneous breaking, if the auxiliary fields (28) of the matter supermultipletes get non-vanishing vacuum expectation values. According to our assumption at the minimum in (26) $U(R, <z^A>, <\bar{z}^{\bar{A}}>) = 0$, but $<F^A> \neq 0$ and, thus, the supersymmetry is broken. The measure of this breakdown is the term $(-\frac{1}{\kappa}e^{\frac{2}{2}\mathcal{G}(z^A, \bar{z}^{\bar{A}})})\lambda\lambda$ in the action (23). Besides, we can identify

$$m_{3/2} = \frac{1}{\kappa}e^{\frac{2}{2}\mathcal{G}(z^A, \bar{z}^{\bar{A}})}$$

(29)

as the gravitino mass in the effective supergravity theory.

Hence, we can see, that in our model the conformal time supersymmetry (4), being subgroup of space-time SUSY, gives a general mechanism of spontaneous breaking of this SUSY. If we consider dilatonic fields in Kähler function, then the parameter $\alpha$ may be viewed as a dilaton coupling constant.

In the future it will be shown, that in the quantum level, the supersymmetric quantum mechanics action (23) describes the vacuum states of supergravity, as well as supersymmetric cosmological FRW model.

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