Petri net modelling of bridge asset management using maintenance-related state conditions

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Bridge structures are an important part of the UK transportation network. They are also experiencing increasing rates of deterioration due to the increasing traffic volume and load intensity. Available bridge models have many restrictions due to the assumptions of the analytical method used and the means by which the model states are defined to represent the condition of the structure. These models also lack the complexity to allow detailed maintenance and renewal options to be explored. This paper presents a bridge model developed based on the Petri net (PN) approach. The method allows for detailed modelling of the individual components in the structure whilst maintaining the size of the analytical problem to a manageable size and resulting in an efficient analysis. The bridge model is formed from sub-models of each of the bridge components and takes into consideration the component deterioration process, the interaction and dependency between different component deterioration processes, along with the inspection and maintenance processes. The model states are defined based on actual degraded component conditions which they experience. It is therefore easy to relate these to the appropriate maintenance options. This gives a considerable advantage over those models based on the condition scores or ratings. The state residence times between changes in state resulting from deterioration and maintenance are governed by appropriate Weibull distributions. Thus, avoiding the restriction of constant failure rates used in Markov approaches which are rarely appropriate to model deteriorating asset conditions. The application of the model is demonstrated on a typical bridge structure where the PN model is solved using Monte Carlo simulation, the model results are also presented and discussed.

Keywords: bridge; asset management; deterioration; maintenance modelling; life-cycle cost; Petri net

1. Introduction
The issues in managing civil infrastructure such as bridges are well known and experienced worldwide over the last few decades. There are several bridge models which have been developed to support the asset management decision-making and they can be classified into Markov, semi-Markov and lifetime analysis-based models. All of these models are stochastic and they predict the future asset condition in terms of the probability of being in each potential state. Markov models (Cesare, Santamarina, Turkstra, & Vanmarcke, 1991; Chase & Gáspár, 2000; Jiang & Sinha, 1989; Morcous, 2006; Ortiz-García, Costello, & Snaith, 2006; Robelin & Madanat, 2007) are the most popular since they are state-based, relatively simple to construct and allow a fast solutions to be attained. These models account for the present condition and predict the future condition, most of them were developed for bridge components and aggregate the proposed maintenance actions to the bridge level to estimate the work programme (Fernando, Adey, & Walbridge, 2013).

The limitations associated with such models are the assumption of constant deterioration rates and that the detailed effects of maintenance are not captured (Agrawal, Kawaguchi, & Chen, 2010). Semi-Markov models (Kleiner, 2001; Mishalani & Madanat, 2002; Ng & Moses, 1996; Sobanjo, Mtenga, & Rambo-Roddenberry, 2010; Yang, Pam, & Kumaraswamy, 2009) often uses a Weibull distribution to model the time residing in different states, thus it is capable of modelling the commonly experienced increasing deterioration rates. Though, it still has the same limitation as in the traditional Markov model experiencing an exponentially increasing number of model states as the model complexity increases (Andrews, 2013). In lifetime analysis-based models (DeStefano & Grivas, 1998; Frangopol, Kong, & Gharaibeh, 2001; Noortwijk & Klatter, 2004; Sobanjo et al., 2010), the degradation process of bridges or bridge components is modelled based on the lifetime analysis techniques. It was demonstrated that the Weibull distribution provides a good fit (Agrawal et al., 2010) to the lifetime data of bridge components and indicates that the bridge components follow a non-constant deterioration process. Although this method is robust to model the degradation process between different states, a complete bridge model comprising of individuals components and their condition states has not been developed.

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Whilst models developed using each of the alternative approaches apply different methods to predict the degradation process, the majority of them define the model states based on condition scores/ratings. For example, bridge assets in the US are often rated in discrete values between 0 and 9 (American Association of State Highway Transportation Officials. Subcommittee on Bridges Structures, 2011; Federal Highway Administration, 2006), and in the UK, railway bridge condition is described by values from 0 to 100 (Network Rail, 2004). Thus, the resulting bridge models would have states that correspond to either a single score value or a range of these condition scores. States in the model based on these scores rarely indicate the maintenance activity required to rectify the condition. This is especially true where scores or indices defined for different degradation processes (such as concrete pitting and corrosion of the metal reinforcements) are combined to define the asset state in the model. Similar limitations apply when one model is used to represent the whole structure, and its condition is defined by combinations of the condition scores for its individual components. Model states defined on the condition ratings are inadequate for accurate results, but these are commonly used as there is little other data available to enable alternative model formulations. The score/rating is also based on subjective evaluations by bridge inspectors with the reliability of the ratings dependent on the experience of the inspectors (Office of Rail Regulation, 2007). The effects of maintenance are not captured in this data (Madanat, Mishalani, & Ibrahim, 1995).

To avoid the restrictions of alternative analytical methods reported in the literature, this paper proposes a bridge model based on the Petri net (PN) modelling method. The bridge model uses states, for each individual component, based on the maintenance options that relate to the defined condition. Where required, the models states account for different degradation processes experienced by the same component. For each degradation process, the level of deterioration is also considered and is expressed in terms of percentage deteriorated area exhibited on a bridge component. Sub-models are developed for the bridge major components (deck, girders, and abutments) considered and these are combined to form a complete bridge model. The methodology presented in this paper models the dynamic states of the bridge components using Weibull distributions for the state residence times. The parameters for these distributions are obtained from the historical records of the maintenance actions done on structures, which can be regarded as similar and from a homogeneous sample. Data extracted from these records are used to reconstruct the state transitions experienced throughout their lifetime.

The PN method offers a flexible and efficient method to model the deterioration and maintenance process of bridge assets, it offers a substantial improvement in the models developed to date in its capacities to model the reality. The example structure model presented in this paper intended to show the features and flexibility of the method and how the limitations of the model states defined based on condition ratings can be avoided. The development of the model states is based on the main degradation processes of the structure and the maintenance that is required to address each of these condition states to an acceptable, improved condition. The adopted approach was formulated based on the current observation schemes by which several bridge authorities in the UK (Network Rail, 2004) identify the condition of the structure in terms of the degradation mode (e.g., paint flaking, corrosion for metal girders) and its severity (% coverage). Where appropriate, for example with structures made of different materials or operating in different environments, the flexibility of the method allows for alternative degradation processes to be easily incorporated. The method also has the ability to model many processes (dependent deterioration process, inspection, maintenance process, etc.) whilst maintaining a manageable analytical problem. The explosion of the model size and states is a key issue in other asset models that is preventing them from modelling increasingly complex systems. More importantly, dynamic processes considered could be modelled to follow any appropriate distributions so that time-dependent processes such as the non-constant deterioration rates of bridge components can be captured.

2. Bridge components and their deterioration processes that trigger maintenance

The bridge type considered to demonstrate that the asset condition modelling process is of a particular type. They have reinforced concrete decks, metal girders and masonry abutments (as shown in Figure 1). The bridge deck is the part of the bridge that directly supports the running surface and traffic. The girder is a linear structural member that spans from one support to another. The abutments support the extreme ends of the structure, and transfer the loads to the foundations or ground. Waterproofing is a protective coating applied to the bridge deck and abutments to protect them from the ingress of harmful agents, e.g., chloride-contaminated water and the growth of vegetation. Waterproofing, whilst not a major bridge component, is an important component to consider as it directly affects degradation processes of the bridge deck and abutments, it comes under the category of ‘servicing’ and is included in the generated bridge model.

Over time, the bridge component deteriorates and structural defects start to appear. These defects are revealed and measured following an inspection. Based on the current observation scheme employed by the majority of bridge authorities, the severity and extent of these
defects can be used to describe the condition of the bridge component. Adopting this approach, this paper identifies the condition of the structure in terms of the main degradation processes and their extent (% coverage). Table 1 gives the deterioration processes for each major bridge component. There can be several deterioration processes for a single component and they can be independent or dependent on the others, some deterioration processes represent the progression from another less severe condition. For example, the deterioration of the bridge deck starts with the concrete surface spalling, this then enables deep spalling to develop which exposes the metal reinforcements and eventually, the exposed reinforced steel bars experience corrosion. This degradation progression is considered in this paper as they are observed in the current structure monitoring scheme employed.

It is important to note that the deterioration processes of different bridge components can also be dependent on each other, e.g., the degradation of the waterproofing promotes the degradation process of both the bridge deck and the abutments. The levels of degradation are used to define the condition state of a component and at these states, the appropriate type of maintenance actions is necessary (Table 1). Generally, different degradation processes require different repair processes which require different scheduling and repair times. The repair time also differs according to the percentage coverage of the defects on the component.

3. PN method

The PN method (Petri, 1963) is increasingly being used to model dynamic systems in many engineering, science and business fields due to its flexibility (Girault & Valk, 2002). The concept of a PN (Schneeweiss, 1999) is a directed graph with two types of nodes called places and transitions, they are linked by arcs. A place represents a

| Bridge component | Material       | Degradation process | Levels of degradation (% coverage of defects) | Maintenance actions                                      |
|------------------|----------------|---------------------|---------------------------------------------|--------------------------------------------------------|
| Abutment         | Masonry        | Vegetation infestation | 5%, 10%, 15%, 20%, ≥25%  | Vegetation remove                                      |
|                  |                | Pointing degradation | 5%, 10%, 15%, 20%, ≥25%  | Re-pointing work                                      |
|                  |                | Brick erosion        | 5%, 10%, 15%, 20%, 25%, ≥30% | Brickwork repair                                      |
| Girder           | Metal          | Paint flaking        | 5%, 10%, 15%, 20%, 25%, ≥30% | Patch painting/full painting                           |
|                  |                | Minor corrosion      | 5%, 10%, 15%, 20%, 25%, ≥30% | Minor corrosion repair                                |
|                  |                | Major corrosion      | 5%, 10%, ≥20%                  | Major corrosion repair (corrosion clean and re-paint)  |
| Deck             | Reinforced     | Surface spalling     | 5%, 10%, 20%, 30%, 40%, ≥50% | Concrete patching                                     |
|                  | concrete       | Deep spalling        | 5%, 10%, 20%, ≥30%            | Filling/concrete patching                              |
|                  |                | Steel corrosion      | 5%, 10%, 20%, ≥30%            | Structure repair                                       |

Figure 1. Typical bridge structure.

Table 1. Degradation levels and corresponding maintenance actions.
particular state or condition of the system. Tokens are located in the places to indicate that current state of the system. A transition enables the token to move from one place to another, this models the changing state of the system, thus the dynamic behaviour of the system is modelled. The PN models formulated in this paper are an extension to the traditional PN method. Additions to the traditional features of the net are introduced to suit the problem of bridge asset management and to make the model more concise and efficient. The key features and extensions of the PN that are implemented in this paper are discussed below.

A simple PN is shown in Figure 2. The places are circles and the transition is denoted by the rectangular box. Places and transition are linked by arrows called arcs, indicating the input and output places for a transition. Arcs can have associated multiplicity which is represented by the number on the arc; arcs with no number have the default multiplicity of one. Tokens are represented by dots of different ‘colours’, i.e., black and white dots. Each of the type of these dots is unique and independent from each other, this feature is used for modelling different bridge components of the same type so that each token represents an individual bridge component. Connecting place P4 to transition T1 is an arc with a circle end, this is called an inhibitor arc. The feature of this arc indicates that place P4 is an inhibitor place and whenever this place is marked with the correct number of tokens, depending on the arc multiplicity, the transition connected to the inhibiting place is prohibited from firing.

The transition governs the firing process during which tokens are consumed from the input places and deposited in the output places according to the arc multiplicities. The firing changes the marking of the places and thus the status of the system. For the firing process to happen, the transition must first be enabled. This requires, for the same type of token, at least the arc multiplicities of tokens in each input place. The transition T1 is enabled for both the black and the white tokens. Even though the transition is enabled for the black token, the presence of a black token in place P4 inhibits the firing process. Thus only the white tokens will be fired through transition T1, and as shown in Figure 2 after the firing, the white tokens are removed from the input places and one white token is deposited into place P5. The firing happens at time $t$ after the enabling, $t$ is derived from a random sample taken from a Weibull distribution with parameters $(\beta, \eta)$. It was found that in order to produce an efficient model, new types of transitions are needed to accommodate certain tasks. They are labelled as shown in Figure 3.

1. Reset transition
   A reset transition (Andrews, 2013; Prescott, 2012), when it fires, resets the marking of specified places in the PN to the desired number of tokens. This transition has an associated list of places and number of tokens that they will contain after reset. A reset action on a network can be carried out using conventional PN features but would require a large number of transitions and places to be added which would increase the size and complexity and destroy the ‘readability’ of the model.

2. Place conditional transition
   A place conditional transition (Andrews, 2013; Prescott, 2012) is a transition in which the delay time is sampled from different distributions depending on the number of tokens residing in a specific place in the network to which they are linked by a dashed arc. Note that, the dashed arc only indicates the link between the conditional places to the place conditional transition. It does not act like a normal arc so that when the place conditional transition fires, tokens in conditional places are not consumed by the transition.

3. Opportunistic transition
   The key property of this transition is that it can be enabled with tokens of different types as long as the firing rule of the PN is satisfied. All other types of transitions discussed previously require the presence of the same type of tokens in all of the input places for it to become enabled. In contrast, the opportunistic transition (OPP) requires any type of

Figure 3. Representation of the new types of transitions.
tokens to be in all the input places to be enabled. This type of transition also warrants the correct deposition of tokens after firing. This transition is used in the modellling of opportunistic maintenance action on bridge components and each component is modelled by different types/colours of token.

The complete bridge model is constructed in modules (sub-models) which correspond to each individual bridge component. Each of the sub-models accounts for the several dependent deterioration processes. The condition of the bridge component is revealed through an inspection after which the appropriate repair process can be activated. The PN modelling technique used is capable of accounting for all of these features. This complex process for bridge components is modelled by dynamic changes in the state of the net. The development of the nets for the bridge components is described in the next sections.

4. Development of PN for bridge deck

4.1 Deterioration process

The section of the net modelling the degradation process of a concrete bridge deck is shown in Figure 4. The degradation process exhibits three progressive processes: surface spalling, deep spalling and steel corrosion (Table 1) represented by places P1–7, P8–12 and P13–17 respectively. The levels of degradations are described in terms of the percentage areas of the defects over the structure. Place P1 is the ‘as new’ state when there is no defects present on the bridge deck. Places P2 to P7 represent degraded conditions when there is 5%, 10% ... > 50% surface spalling. Similarly, the levels of deterioration for deep spalling and steel corrosion can be seen in Figure 4.

Between these degraded states are transitions T1–14, which specify the transition times of moving between different levels of deterioration of the bridge deck. The transition times are sampled from the appropriate distributions that can be obtained by studying the deterioration characteristics of the bridge deck or estimated by expert opinion. In this paper, they are assumed to follow Weibull distributions with parameters \( (\beta_1, \eta_1, \beta_2, \eta_2, \ldots, \beta_{14}, \eta_{14}) \). It can also be seen that these transitions are place conditional transitions (marked by P.C), thus the transition times are dependent upon the number of tokens in places P(WP.1), P(WP.2) and P(WP.3).

This is shown in Figure 4 by dotted arrows from places P(WP.1–3) to transitions T1–14. Places P(WP.1–3) represent the three conditions of the waterproofing: good, adequate and poor. A token present in place P(WP.3) indicates poor waterproofing, and the transition times governed by transitions T1–14 are sampled from the appropriate distributions so that it reflects the degradation process of the bridge deck when the waterproofing is in a poor condition. With worse waterproofing conditions, the degradation rate of the bridge decks increases slightly and the mean time to move to a more degraded states decreases. It is assumed that for each worse condition of the waterproofing there is a 10% increment and 10% decrement in the value of the shape and scale parameters of the Weibull distributions that govern the deterioration processes.

Note that the arbitrary (10%) adjustments to the Weibull distribution parameters are assumed to illustrate the capability of the model: they are estimated by expert judgment of the bridge engineers and reflect the increasing...
deterioration rate of the bridge deck when the waterproofing degrades. It is worth noting that the degradation characteristic of the bridge deck does not simply relate only to the waterproofing conditions; however, these chosen processes demonstrate the model capability of modelling-dependent deterioration processes for complex systems. The appropriate distributions can be obtained by studying the degradation characteristic of the bridge deck given different waterproofing conditions.

It is important to note that, there are dependencies between the three degradation processes. Deep spalling cannot start until the concrete surface is in the spalled condition, and the reinforcing steel corrosion can only start when deep spalling has occurred. This relationship is modelled by introducing inhibitor arcs (Figure 4) connecting place P2 and transition T8, P3 and T9, P9 and T12, etc. A token in place P1 represents that the bridge deck is in the ‘as new’ state. When transition T1 fires, tokens are placed in P2 and P8, which means surface spalling has now appeared on the deck structure and the deep spalling process can now start. The token in place P2 inhibits the transition T8, thus transition T8 can only fire when the transition T2 has fired and the token is removed from place P2. Sequentially, transition T12 can only fire when there is no token in place P9. This means that at least this degree of surface spalling must happen for the deep spalling and then steel corrosion to occur.

### 4.2 Dependency on waterproofing conditions

The deterioration process of the bridge deck depends on the condition of the waterproofing. Figure 5 shows the part of the net that models the life of the waterproofing. There are three conditions of the waterproofing, and the transition time for T(WP.1) is the time that the waterproofing reaches the adequate condition from the good condition. Similarly, T(WP.2) governs the deterioration time from the adequate to the poor condition. It is assumed that the waterproofing is renewed after a predetermined lifetime, which is every 15 years. This assumption is based on the current maintenance regimes of several bridge authorities in the UK.

![Figure 5. PN for modelling the life of waterproofing.](image)

When the waterproofing is in a good condition, the place P(WP.1) is marked. Transition T(WP.1) and T(WP.3) are both enabled, T(WP.3) fires immediately and tokens are placed in output places P(WP.1) and P(WP.4). Transition T(WP.3) is now inhibited and the token in place P(WP.1) starts the degradation process of the waterproofing. A token in place P(WP.4) indicates that the waterproofing is within the planned lifetime of 15 years. After this lifetime, transition T(WP.4) will fire and move the token to place P(WP.5), which indicates that the waterproofing is now being renewed. T(WP.5) controls the time for waterproofing renewal. It is also a reset transition so that when it fires, it deposits a token into P(WP.1) and resets the tokens in other places in this net, hence indicating that the waterproofing has started a new life. It should also be noted that the places and transitions include the WP (waterproofing) notation to avoid confusion with the labels on other sections of the PN.

### 4.3 Inspection process

The state of the component will only be revealed following an inspection. This is modelled using the inspection transitions (transitions marked with INSP), T15–28 in Figure 6. Assuming the inspection time is set to inspect at every θ time units and the concrete deck reaches 5% surface spalling (represented by place P2 in Figure 4), there are now two transitions possible (T2 and T15). If the transition T2 fires first, the token is transferred to place P3, which means the deck has degraded to a worse state before it was inspected. However, if transition T15 fires first, the token is entered in place P18, which means the deck is now been inspected and its current condition revealed. The transition time T15 is the time between when the token arrives at P2 to the time when the deck is next inspected. Note that when T15 fires, not only is place P18 marked but also places P2 and P32. The re-marking of place P2 ensures that the deterioration process of the bridge deck continues even after the inspection. Places P32–34 ensure that an inspection will not take place again whilst scheduled for repair. The inspection takes place every year and therefore the inspection period set for inspection transitions T15–28 is 1 year.

### 4.4 Maintenance and repair scheduling process

When the deck condition is revealed, then the appropriate type of maintenance work can be applied. The maintenance issues are considered below. These processes are considered upon the consultation with bridge engineers, based on their current maintenance regimes. Alternative processes can also be implemented into the model if necessary.
1. Maintenance actions can have scheduling times or can be repaired immediately.
2. The repair times are different depending on the type of repair and the severity of the defects, i.e., it would take a longer repair time for a larger percentage of degraded area.
3. The component continues to degrade after the inspection and while the component is scheduled for repair, thus the appropriate maintenance actions must be applied depending on the condition of the component at the point when repair begins.

Figure 7 shows part of the net modelling the maintenance and repair delay process. Places P18–31 represent the known condition of the bridge deck following an inspection, as shown in Figure 4, at these states, the repair is scheduled. The maintenance process does not usually happen immediately and often has an associated scheduling delay. Transitions T29–42 govern these scheduling times. Places P35–37 represent the states where the repairs actually start and transitions T43–45 govern the appropriate repair times. Arcs that are connecting transitions T43–45 to place P1 complete the maintenance process which restores the condition of the bridge deck.

It is important to realise that the transitions T43–45 are special conditional transitions with two key properties: place conditional (P.C) and reset (RST) properties. Places P38–40 are conditional places which input to the transitions T43–45 by dash arrows, hence the transition times of T43–45 are generated based on the number of tokens present in these conditional places. In particular, the transition T43 governs the surface spalling repair time. T44 governs total surface spalling and deep spalling repair time. T45 governs total repair time when the work involves surface spalling, deep spalling and reinforcement corrosion repair. The number of tokens to be deposited in conditional places P38–40 depends on which transition of T29–42 has fired. Noticing that transitions T29–34 connect to conditional place P38 by arcs with different multiplicities, this can also be observed with arcs connecting T35–38 to P39 and T39–42 to P40. The reset properties ensure that when the transitions fire, the tokens in conditional places P35–37 are cleared. In addition, places P32–34 are also reset when T43–45 fire respectively; this is to enable the inspection process again after it was inhibited during the maintenance process.
To explain the function of the model in more detail, consider a situation when, following an inspection, the bridge deck is revealed to have 20% surface spalling, 10% deep spalling and 10% reinforcing steel corrosion. The marking shown in Figure 7 represents this situation with tokens present in places P20, 25, 29. Transitions T31, 36 and 40 are now enabled. If the maintenance action is scheduled for two weeks, the transitions will fire after this set scheduling time. Once transition T31 has fired, the token in place P20 will be removed and a token is deposited in place P35, indicating that the repair action for surface spalling will now start. Also three tokens will be deposited in conditional place P38, indicating that the repair action required is to rectify 20% of surface spalling. Similarly, when T36 and T40 fire, places P36, 37, 39, 40 are marked with 1, 1, 2, 2 tokens respectively. At this state, transition T45 is enabled, the transition time is governed by the conditional places P38–40.

Table 2 gives the repair time required for different maintenance actions. It should be noted that these figures are illustrative to demonstrate how different repair times are taken into account with different maintenance actions. These times vary with different management authorities and they can be changed accordingly. With 3, 2, 2 tokens present in conditional places P35–37 respectively, the repair time required for 20% surface spalling repair, 10% deep spalling repair and 10% steel corrosion repair would be 3, 4 and 8 days respectively. Finally, the total repair times would be 15 days and would be the transition time for transition T45.

Inhibitor arcs connecting places P36 to transitions T43 and from P37 to T43–44 impose the rule that all the maintenance actions start at the same time after the delay period. It is also worth noting that if the scheduling times are different for the different maintenance actions, inhibitor arcs must be introduced that connect places P24–27 to transitions T43, P27–31 to T43 and P27–31 to T44. Again, this is to ensure that different maintenance actions, with different scheduling times, all start at the same time.

Table 2. Repair times for different maintenance actions according to different levels of degradation.

| Number of tokens | P35 | P36 | P37 | Level of degradation (%) | Repair times (T43, T44, T45) (days) |
|------------------|-----|-----|-----|--------------------------|-------------------------------------|
|                  | 1   | 1   | 1   | 5                        | Surface spalling: 1, Deep spalling: 2, Steel corrosion: 4 |
|                  | 2   | 2   | 2   | 10                       |                                     |
|                  | 3   | 3   | 3   | 20                       |                                     |
|                  | 4   | 4   | 4   | 30                       |                                     |
|                  | 5   | –   | –   | 40                       |                                     |
|                  | 6   | –   | –   | >50                      |                                     |
4.5 Maintenance actions based on true component condition

When the component is scheduled for a repair, it continues to deteriorate. In the case when the scheduled time is long enough, the component might deteriorate to a worse condition before the maintenance action takes place. Figure 8 shows this case. Place P18 is marked to represent the deck scheduled for a 5% surface spalling repair; however, place P3 is marked, which means that the actual condition of the bridge deck surface has now reached 10% spalling. When transition T29 fires, a token is deposited in place P35, indicating that the repair is now starting and place P38 is marked with 1 token, indicating that 5% surface spalling repair is required. With places P3 and P35 marked, transition T46 is now enabled, which then fires and places a token in place P35 and two tokens in P38.

Transition T46 is an instant transition with 0 time delay, and has the reset property which initialises the marking of place P38 when it fires. Thus, the token already present in P38 before the firing is removed and two tokens are deposited. At this point, the markings represent the start of the surface spalling repair for 10% of the area. Effectively, transition T46 is introduced to ensure that the appropriate type of repair will be carried out based on the true condition of the component at the end of the scheduling period. Similarly, more instant transitions (T46 to T59) are introduced into the model, to cover other cases when the component deteriorates to worse states during the scheduling period.

If there is deep spalling or corrosion on the component at the point of surface spalling repair starts, the complete repair must also be carried out. Figure 9 shows the PN that
enables the deep spalling and corrosion repair process when the surface spalling repair process begins. When instant transitions $T_{60}–67$ are enabled, the firing will remove the tokens in any places $P_{9}–12$, $P_{14}–17$, and deposit tokens in places $P_{35}$, $36$, $37$, $39$ and $40$. The markings of these places indicate that the repair process is updated and repair actions are based on the present component condition. The rest of the repair follows the process through the net, as shown in Figure 7.

4.6 Renewal process

At some time, the bridge deck will reach a point where it is cheaper to replace it rather than continue to maintain it. There are many possibilities as to how this time can be established. It can be after a predetermined lifetime, after the condition is deemed to be too poor or after a set number of maintenance actions. Any of these strategies can be implemented within the PN. In this section, the renewal of the bridge deck is carried out after a certain number of specific maintenance actions. Figure 10 shows this section of the PN.

When transitions $T_{43}–45$ fire, this means that maintenance actions have been carried out, tokens are deposited into places $P_{41}–43$ which record the number of specific maintenance actions which have happened during the lifetime of the bridge deck. Places $P_{41}–43$ are inputs to transition $T_{68}$ with arc multiplicities $5$, $3$ and $2$, respectively. This represents the renewal strategy that once the number of surface spalling repairs, deep spalling repairs and corrosion repairs reach $5$, $3$ and $2$ times respectively, the deck will be renewed. Place $P_{32}$ is also an input to transition $T_{68}$, this implies that the deck renewal is enabled on the next maintenance action following the condition that the deck renewal point has been reached. Place $P_{44}$ represents the state where the deck is now being renewed, and transition $T_{69}$ returns the token to the state where the deck is in the ‘as new’ condition (place $P_{1}$). Note that transition $T_{69}$ is a reset transition which, when it fires, initialises the tokens in any other places in the net to zero. This action is necessary to model the fact that once the deck is renewed, the deck starts a new life and the number of previous maintenance actions is zero.

4.7 Maintenance policy

Different maintenance policies can be set in the model by placing the appropriate number of tokens into the places $P_{45}–47$ as shown in Figure 11. These places inhibit the inspection transitions $T_{15}–20$, $T_{21}–24$ and $T_{25}–28$, thus disabling a certain type of repair. For example, where the maintenance policy is to trigger the maintenance when the concrete spalling has reached $30\%$ surface occupation, $3$ tokens are placed in $P_{45}$. Because the arcs connecting $P_{45}$ to $T_{15}–17$ have arc multiplicities equal to or smaller than $3$, transitions $T_{15}–17$ are inhibited, thus the component is allowed to deteriorate to place $P_{5}$ or worse ($P_{6}$, $P_{7}$) where an inspection will reveal the true component condition and the corresponding type of repair is triggered.

More strategies can be implemented in the PN by marking the corresponding number of tokens in places $P_{46}$ and $P_{47}$, so that different intervening conditions can be set for deep spalling and steel corrosion. Table 3 gives the number of tokens required in places $P_{45}–47$ for different maintenance policies to be modelled by the net. As an example of a policy when no maintenance actions is applied to the bridge deck at all, the number of tokens in
4.8 Building a complete bridge deck model

The parts of the PN presented are integrated to form a complete PN for the bridge deck by linking the same places present in each of the individual section models. The dynamic process of the token moving around the PN models the associate degradation, inspection, maintenance and renewal process over the bridge deck lifetime.

5. Development of PN for bridge girders

5.1 Deterioration process

The main deterioration process for bridge metal girders is corrosion. In addition, the corrosion process is enabled by the degradation of the protective paint condition. Figure 12 shows the PN section that models the degradation process of a metal bridge girder, taking into account the relationship between the two main processes: paint flaking and metal corrosion. As with the deck, these degradation processes are progressive and dependent. Place P1 is the ‘as new’ state of the paint coating at which there is no flaking. Places P2 to P7 are the degraded conditions when there is 5%, 10%... >30% paint flaking respectively. Transitions T1–6 govern the degradation times between these degraded paint conditions. These times are sampled from the appropriate Weibull distributions that have parameters \((b_1, \eta_1), (b_2, \eta_2)\ldots (b_6, \eta_6)\) as shown in Figure 12.

Transitions T1–6 are place conditional transitions, and the transition times sampled are dependent upon the number of tokens present in the condition place P19. This is represented in the figure by the dotted arrows from P19 to T1–6. It is expected that the time for the coating to

Figure 11. PNs for different maintenance policies.

places P45–47 must be equal or greater than 6, 4 and 4 respectively.

Table 3. Maintenance policies implemented in the PN for bridge deck.

| No of token in P45 | Maintenance action is triggered when surface spalling is | No of token in P46 | Maintenance action is triggered when deep spalling is | No of token in P47 | Maintenance action is triggered when steel corrosion is |
|-------------------|--------------------------------------------------------|-------------------|------------------------------------------------|------------------|--------------------------------------------------------|
| 0                 | ≥5%                                                   | 0                 | 5%                                               | 0                | ≥5%                                                   |
| 1                 | ≥10%                                                  | 1                 | ≥10%                                            | 1                | ≥10%                                                  |
| 2                 | ≥20%                                                  | 2                 | ≥20%                                            | 2                | ≥20%                                                  |
| 3                 | ≥30%                                                  | 3                 | ≥30%                                            | 3                | ≥30%                                                  |
| 4                 | ≥40%                                                  | ≥4                | No action                                       | 4                | No action                                             |
| ≥6                | No action                                             |                    |                                                  |                  |                                                        |
reach a degraded state should be shorter every time patch painting is carried out. The reason for this is that patch painting only repaints the area of the flaked paint, other areas that have already started the degradation process and would have reduced times of reaching the degraded state. This effect is captured and modelled according to the number of previous painting patches that have been performed on the girder, which is recorded in place P19. Again, the arbitrary (10%) adjustments to the Weibull distribution parameters are assumed to model the slight increase in degradation rates and shorter mean times to reach the degraded states each time patch painting is undertaken.

Places P8–14 and P15–18 represent the degradation levels of minor and major corrosion respectively. The distinction between minor and major corrosion is the maintenance required to rectify the condition. For minor corrosion, the area is cleaned and repainted. For major corrosion, the metal loss has to be replaced which requires plating/welding to be undertaken. Transitions T7–12 govern the transition times between different states of minor corrosion. They are place conditional transitions that depend upon the presence of the tokens in places P2–7. These conditional transitions result due to the rate of metal corrosion increasing with the worsening paint condition. There is also a dependency between the degradation of the coating and the metal component, such that the corrosion happens as a consequence of the paint flaking. This dependency is modelled by introducing inhibitor arcs from places P2 to transition T8, from P3 to T9, from P4 to T10, etc.

Once minor corrosion has occurred on the structure, major corrosion can start. Transitions T13–30 govern the transition times between levels of major corrosion given the current level of minor corrosion. These transitions are also place conditional transitions with the firing times dependent on the number of tokens present in place P20 which records the number of previous major corrosion repairs. This feature of the net models the effects of previous major repairs on the degradation process of the girder, and this is similar to how the number of patch painting previously performed effects the degradation process of the paint.

5.2 Inspection process

The condition of the girder is revealed following an inspection. The section of the PN which connects degraded states in each condition to the revealed degraded states follows the same structure as show in Figure 6 for the bridge deck.

5.3 Maintenance and renewal process

The following maintenance actions are considered:

1. Flaked paint is patch painted; however patch painting is only allowed up to 5 times, after this, the whole girder is fully repainted.
2. When repairing corrosion, the corroded area will also be repainted.
3. Maintenance actions can have scheduled times or can be repaired immediately.
4. The repair times are different depending on the type of repair and the severity of the defects, i.e., it would take longer to repair larger percentages of degraded area.

5. The component continues to degrade after the inspection and while the component is scheduled for repair.

6. The girder is replaced once major corrosion covers more than 20%.

7. Opportunistic maintenance is considered on adjacent girders if their condition, while not currently triggering maintenance, is expected to do so in the near future.

Figure 13 shows part of the net modelling the maintenance and repair delay process. Places P21–36 represent the known condition of the bridge girder following an inspection, from these conditions, the repair is scheduled. The scheduling time for repainting work, minor corrosion repair, major corrosion repair and girder replacement is governed by transitions T47–52, T53–58, T59–61 and T62 respectively. Places P40–44 represent the states where the repairs begin and transitions T64–70 govern the appropriate repair times. Finally, arcs connect these transitions to place P1 that represents the restored condition of the girder after maintenance. The number of times that patch painting and major corrosion repairs are performed also is recorded by tokens placed in P19 and P20, which link to the net shown in Figure 12[!–insert–], and this completes the maintenance process.

Transitions T64, T66–69 govern the repair time, and as with the repair process of the bridge deck, these transitions contains conditional and reset properties so that the appropriate repair time is generated based on the level of deteriorations. The arc from place P19 to transition T63 has multiplicity of 5, indicating that when P19 contains 5 tokens and P40 is marked, T63 is enabled and a token is instantly deposited in place P41. This feature of the net triggers a full-girder repaint when patch painting has been carried out for more than 5 times (represented by P41). Transition T65 fires when a full repaint has been completed and it zeros the tokens in place P45.

When major corrosion reaches 20% and the condition is revealed, place P44 is marked, indicating the girder is to be replaced. When transition T70 fires, the token is transferred to place P1, which represents the new girder condition, this transition is a reset transition which resets the tokens in all the parts of the net indicating the new life of the girder.

Figure 13. PN for the maintenance, scheduling and renewal process.
5.4 Maintenance based on true component condition

The feature of the PN modelling the inspection and deterioration process allows the girder to continue to deteriorate after the inspection and while awaiting maintenance. Figure 14 shows the net that updates the corresponding maintenance actions based on the true condition of the girder at the point where the repair process actually starts. It is worth noting that when the true condition of the girder reaches more than 20% major corrosion (P18), transition T86 is enabled, which means the major corrosion repair process is no longer appropriate and the girder replacement process will begin.

At inspection, if only the degradation processes relating to paint flaking and minor corrosion are detected, however when the repainting or minor corrosion repair processes start, any major corrosion will also be revealed and dealt with. Figure 15 shows this aspect of the model.

T87–90 are immediate transitions which are enabled when maintenance is to be carried out (marking P40 and P42), and the girder true condition has established major corrosion (the marking in any of places P15–18). The firing of these transitions will trigger the appropriate major corrosion repair or girder replacement depending on the level of major corrosion developed (by marking places P43, 47 or P44).

5.5 Complete PN model for all girders in a bridge structure

By connecting different parts of the net developed previously, a complete PN for the bridge girders is formed. A bridge structure contains many major and minor girders, the same net structure can be used to model all of these girders. The way it is modelled is by adding more

Figure 14. PN net for updating the correct maintenance action required based on the true condition of the component at the point the repair starts.

Figure 15. PN net for enabling major corrosion repair process when the repair processes for repainting and minor corrosion repair begin.
Figure 16. PNs for opportunistic maintenance of different girders modelled using different coloured tokens. (black, green, red, white, etc.). The transition times are sampled separately with each token.

5.6 Maintenance policy

Different maintenance strategies can be implemented for each girder. In particular, different levels of degradation to trigger maintenance can be set and the PN net to implement this is similar to Figure 11.

5.7 Opportunistic maintenance

Since there is a group of girders on the bridge, it is possible to implement opportunistic maintenance. Opportunistic maintenance considers carrying out maintenance on components which have a deteriorated condition but would not normally instigate maintenance. Transitions T91–96 (denoted OPP in Figure 16) are introduced to model this for paint flaking, the firing rules for these transitions are based of different colours of the tokens. These transitions are instantaneous to ensure that opportunistic maintenance is implemented when the maintenance process starts. In Figure 16, the white token models the girder is being repaired, the firing of transitions T91 transfers the black token to place P40. This process indicates that opportunistic maintenance is performed on the girder represented by the black token. The PNs for opportunistic maintenance for minor, major corrosion repair and replacement have a similar structure.

6. Development of PN for bridge abutments

The section of the net modelling the degradation process of a masonry abutment is shown in Figure 17. There are three separate processes that drive the degradation: vegetation infestation, pointing degradation and brick degradation represented by places P1–6, P7–12 and P13–18 respectively. Place conditional (P.C) transitions T1–15 control the deterioration times between levels of degradation. Similar to the degradation process of bridge deck, the degradation process of the bridge abutments are also dependent on the condition of the waterproofing given by places P(WP.1–3) and as shown in Figure 5. It is also assumed that deterioration of the waterproofing condition accelerates the degradation process of bridge abutments. The abutments are inspected annually during inspection, this is modelled using inspection (marked as INSP) transitions. The maintenance of the abutments follows a similar process and has a similar PN structure as for the deck. The only addition to the net is that, reset transitions require few extra appropriate places to be reset since following any repointing and brickwork repair any vegetation would be cleared. Also, the renewal of the abutments is not presented in the net since repair rather than renewal is always performed. The complete PN model for the abutment is formed by connecting different parts of the nets for the different processes. Two bridge abutments are then modelled by adding two unique coloured tokens in the net. Opportunistic maintenance is again introduced as if work is done on one abutment it would also be done on the other.

7. Developing a complete PN bridge model

By combining the deck, girder and abutment PNs, the complete bridge model is formed. Sub-models are

Figure 17. PN of the degradation process of the bridge masonry abutment.
connected by the common places, in particular, the nets for the bridge deck and abutments are connected to the net that models the waterproofing condition. The same inspection times are set for all the inspection transitions as a single inspection is carried out for the whole structure at one time. The state of the bridge is represented by the combined marking of the places in the modelled components. A complete bridge established, for the example presented, consists of 1 deck, 24 girders, 2 abutments, if all components are in the ‘as new’ condition, this will require 1 token to be added to place P1 in the bridge deck net, 24 unique tokens adding to place P1 in girder model and 2 tokens adding to place P1 in the abutment PN. Note that when combining sub-nets, the numbering system for places and transitions would be changed to ensure that they are unique. The model can be complexity can be increased by adding more rules and relationships if required. For example, opportunistic maintenance can be considered for the components of different types, or PN nets can be developed to include more bridge components such as bearings. These possibilities can easily be accommodated within the approach presented.

8. Model solution and results

8.1 Model construction and inputs

A computer programme was written to generate the solution of the PN model. There are two main types of model inputs: information regarding the degradation characteristics of bridge components, and parameters that are used to specify a selected maintenance and renewal strategy. In particular, for the degradation characteristics, the model requires:

1. The distributions of times that a component degrades to different condition states. This kind of information can be obtained by studying historical data and analysing the degradation processes of the bridge components.
2. The relationship between dependable components, e.g. the effects on the degradation rate of the bridge deck and abutment provided by the condition of waterproofing.

The inputs that specify maintenance strategy, which can change for different runs of the model, are:

1. The level of degradation at which maintenance action is triggered.
2. The period between inspections for the bridge components.
3. The scheduling time of a maintenance action, which determines the delay time between when the component condition is revealed through inspection and when the maintenance actions begin.
4. The criteria for component replacement/full repaint would need to be entered, which determines the number of maintenance actions/patch paintings before renewal.

8.2 Monte Carlo sampling

Sampling from distributions is required for all stochastic transitions. Except for transitions whose transition times are constant, all of the stochastic transitions used in this paper are governed by Weibull distributions, $\text{Weibull}(\beta, \eta)$ with the cumulative density function:

$$F(t) = 1 - \exp \left( -\left( \frac{t}{\eta} \right)^\beta \right) = X.$$  \hspace{1cm} (1)

The transition times are derived by generating a random number $X$ between 0 and 1, which has the same properties as the cumulative probability, $F(t)$. Equating these two and rearranging Equation (1) gives the sampled transition time:

$$t = \eta [-\ln(X)]^{1/\beta}.$$ \hspace{1cm} (2)

The lifetime of the bridge is simulated using the PN for a number of simulations and the performance statistics are collected. When the results have converged following a large number of simulations the following statistics are collected:

1. The number of maintenance actions implemented on any bridge component over the simulated lifetime. Combining these with the associating cost for each maintenance action, the life-cycle costs can be calculated.
2. The average probability of a component being in each of the possible conditions.
3. The future predicted condition profile can be plotted to illustrate the effects of different maintenance strategies on the component condition.
4. The distribution of times of a component residing in any condition can also be investigated.

8.3 Model results and discussion

An example analysis of all the component models is presented in this section. All components are assumed to have the initial condition of as good as new and the repair strategy is to repair as soon as any defect is revealed. Each simulation was performed over a lifetime period of 60 years. With random sampling, the results must be checked for convergence, and this was found to be achieved in all analyses after 200 simulations. To perform 200 simulations, the solution time for the model is under 10 minutes.
Plots of the average number of maintenance actions performed on the abutment over its lifetime are show in Figure 18. Plots 18(a)–(c) depict the number of maintenance actions for vegetation clearing, repointing and brickwork repair respectively. Furthermore, each of these plots shows the expected number of specific actions, for example, the third plot shows that there are 6.5 repairs when there is 5%- brick defects; about 3 brickwork
repairs at the point where 10% of the brickwork has degraded, about 1 repair at the 15% degraded brick condition and almost no repairs at the more degraded conditions.

For other bridge components, this information can also be easily obtained. Tables 4 and 5 show the statistics obtained for a single-bridge girder. It can be seen that about 12 times patch painting is required and on average on 1.68 times the girder will be fully repainted. The number of maintenance actions required for corrosion is generally low being under 2 times over the 60-year lifetime prediction period and there is no girder replacement expected.

Figure 19 presents the distribution of times that the bridge deck resides in the 'as new' state. This means that there is higher probability of the bridge deck being in a state with a degraded surface than being in state which deep spalling presents. Figure 20(a) and (b) show results of the bridge deck residing in states with surface spalling or deep spalling respectively for different degrees of coverage over predicted timeline. Figure 21 displays the distributions of times residing in different conditions for the waterproofing. The distributions clearly show that over the pre-determined lifetime of 15 years until the waterproofing is reapplied it spends more time in the poor condition than in the good and adequate conditions.

Table 4. Statistics of the number of maintenance actions on a single bridge girder.

| Type of maintenance actions               | 5%   | 10%  | 15%  | 20%  | 25%  | >30% |
|-------------------------------------------|------|------|------|------|------|------|
| Patch painting                            |      |      |      |      |      |      |
| Min                                       | 2    | 0    | 0    | 0    | 0    | 0    |
| Max                                       | 20   | 5    | 1    | 1    | 0    | 0    |
| Avg                                       | 11.30| 1.03 | 0.06 | 0.01 | 0    | 0    |
| S.D.                                      | 2.89 | 1.02 | 0.23 | 0.07 | 0    | 0    |
| Full painting                             |      |      |      |      |      |      |
| Min                                       | 0    |      |      |      |      |      |
| Max                                       | 3    |      |      |      |      |      |
| Avg                                       | 1.68 |      |      |      |      |      |
| S.D.                                      | 0.57 |      |      |      |      |      |
| Minor corrosion repair                     |      |      |      |      |      |      |
| (corrosion clean and repaint)              |      |      |      |      |      |      |
| Min                                       | 0    | 0    | 0    | 0    | 0    | 0    |
| Max                                       | 7    | 1    | 0    | 0    | 0    | 0    |
| Avg                                       | 1.42 | 0.02 | 0    | 0    | 0    | 0    |
| S.D.                                      | 1.34 | 0.14 | 0    | 0    | 0    | 0    |
| Major corrosion repair                     |      |      |      |      |      |      |
| (welding/plating)                         |      |      |      |      |      |      |
| Min                                       | 0    | 0    | 0    | –    | –    | –    |
| Max                                       | 2    | 0    | 0    | –    | –    | –    |
| Avg                                       | 0.10 | 0    | 0    | –    | –    | –    |
| S.D.                                      | 0.32 | 0    | 0    | –    | –    | –    |
| Replacement                               |      |      |      |      |      |      |
| Min                                       | 0    |      |      |      |      |      |
| Max                                       | 0    |      |      |      |      |      |
| Avg                                       | 0    |      |      |      |      |      |
| S.D.                                      | 0    |      |      |      |      |      |

Table 5. Statistics of the mean time (in years) of residing in each degraded condition states.

| Deterioration processes | 0%   | 5%  | 10%  | 15%  | 20%  | 25%  | >30% |
|-------------------------|------|-----|------|------|------|------|------|
| Paint flaking           |      |     |      |      |      |      |      |
| Min                     | 35.30| 0.98| 0    | 0    | 0    | 0    | 0    |
| Max                     | 59.02| 12.56| 2.28 | 0.82 | 0.00 | 0    | 0    |
| Avg                     | 50.20| 6.03 | 0.33 | 0.02 | 0.00 | 0    | 0    |
| S.D.                    | 3.98 | 1.76 | 0.40 | 0.08 | 0.00 | 0    | 0    |
| Minor corrosion         |      |     |      |      |      |      |      |
| Min                     | 39.05| 0    | 0    | 0    | 0    | 0    | 0    |
| Max                     | 60   | 5.37 | 0.66 | 0    | 0    | 0    | 0    |
| Avg                     | 56.04| 0.62 | 0.01 | 0    | 0    | 0    | 0    |
| S.D.                    | 4.13 | 0.90 | 0.05 | 0    | 0    | 0    | 0    |
| Major corrosion         |      |     |      |      |      |      |      |
| Min                     | 0    | 0    | 0    | –    | –    | –    | –    |
| Max                     | 3.87 | 0    | 0    | –    | –    | –    | –    |
| Avg                     | 0.17 | 0    | 0    | –    | –    | –    | –    |
| S.D.                    | 0.65 | 0    | 0    | –    | –    | –    | –    |
Figure 19. Distributions of times a bridge deck residing in the ‘as new’ state represented by place P1.

Figure 20. Probabilities of the bridge deck being in different condition states: (a) states represent level of degradation of surface spalling, (b) states represent level of degradation of deep spalling.
9. Conclusions

This paper presents a new method for modelling the consequences of alternative maintenance strategies adopted to control the condition of the bridge structure. The PN modelling technique has been employed in developing models for each of the bridge components. These models are combined to form a complete bridge model. The flexibility and capability of the technique have been demonstrated with regard to the requirements and complexity of a bridge asset management model. The model is considerably more detailed than others found in the literature, and defines the states of the structure in a way which can be directly related to the maintenance actions needed to rectify its condition. The flexibility of the modelling technique allows many rules to be incorporated into the model to simulate complex opportunistic and dependent maintenance processes whilst keeping the model size within manageable limits.

The model requires the degradation time distributions as input, this information has been obtained by studying the historical lifetime maintenance record data for bridge components. The model is solved using a Monte Carlo simulation with convergence being attained after 200 simulations. The model allows the investigation of many performance parameters such as: the number of maintenance actions required for any components, the life-cycle cost of the asset and the asset or component condition profiles over their lifetime. The model can be used on a bridge with known initial condition, where the conditions of the components is not as good as new and predict the effects of different maintenance strategies based on tactical decisions. The model could provide a tool to be used in practice to aid the maintenance decision-making process by identifying the most appropriate strategy.

In conclusion, the benefits of the developed bridge model based on the PN modelling method are:

Figure 21. Distributions of times of waterproofing residing in different conditions.
Model states are based on actual component deterioration conditions that relate to a required maintenance actions for their restoration.

Bridge components can have several deterioration processes, which can be independent or dependent of each other, which result in different deteriorated conditions.

The model has the capability of modelling bridge components with non-constant deterioration rates.

The model considers inspection process through which the asset condition is revealed and, depending on the maintenance strategy selected, the appropriate maintenance actions are triggered.

The model can account for maintenance scheduling times during which the asset may continue to deteriorate further; the types of maintenance action are adjusted at the point of execution to match the required restoration work.

Using the model, some defects, undetected at inspections, are revealed and acted upon when the maintenance work is executed.

The model records previous maintenance actions performed and their future effectiveness to control the state of the asset is also captured.

The model incorporates the potential to include several routine maintenance, opportunistic maintenance and renewal strategies, each set by placing tokens in controlling places in the PN bridge model.

The same PN bridge model can be used to model components of the same type, thus the model states and size remain the same even if the number of modelled component increases in more complex structure.

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Notes

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2. Lloyd’s Register Foundation supports the advancement of engineering-related education, and funds research and development that enhances safety of life at sea, on land and in the air.
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