On the Role of Postconditions in Dynamic First-Order Epistemic Logic

Côme Neyrand¹ Sophie Pinchinat²

May 3, 2022

Abstract

Dynamic Epistemic Logic (DEL) is a logic that models information change in a multi-agent setting through the use of action models with pre- and post-conditions.

In a recent work, DEL has been extended to first-order epistemic logic (DFOEL), with a proof that the resulting Epistemic Planning Problem is decidable, as long as action models pre- and post-conditions are non-modal and the first-order domain is finite. Our contribution highlights the role post-conditions have in DFOEL. We show that the Epistemic Planning Problem with possibly infinite first-order domains is undecidable if the non-modal event post-conditions may contain first-order quantifiers, while, on the contrary, the problem becomes decidable when event post-conditions are quantifier-free. The latter result is non-trivial and makes an extensive use of automatic structures.

1 Introduction

First-order modal logic has been around since the first introduction of modal logic. Modal logic (see [BVBW06]) was introduced as an extension of propositional logic, and its semantics relies on Kripke models i.e., with different valuations in modal relations to one another. These modal relations can be of different sorts creating a wide range of logics that are direct applications of modal logic. For example, temporal logic and epistemic logic are modal logics in which modal relations are defined as relations of time and knowledge respectively. First-order modal logic is an extension of modal logic in which the Kripke model valuations are replaced by first-order structures, and every modal logic can be extended to its first-order counterpart (see [BVBW06, Chapter 9]).

As a remarkable application of modal logic, Dynamic Epistemic Logic (DEL) [VDxDHK07, BR16], provides a logic that describes changes on an epistemic model – a particular Kripke model – through the use of action models and updates. The links between DEL and Epistemic Temporal Logic (ETL) have been studied in [Mau14]. Under some hypothesis on DEL action models, the underlying ETL model is regular, allowing to decide central problems in artificial intelligence. A problem of particular interest is Epistemic Planning, an automated planning problem in the DEL setting introduced by [BA11]. Epistemic Planning has been thoroughly studied (see [BCPS20]...
for a survey). Recently, an extension of Epistemic Planning in the setting of Dynamic First-order Epistemic Logic (DFOEL) has been introduced in [OAR20] – following the pioneer approach of [Koo08] – where first-order logic is used as a means to compactly specify an essentially propositional input.

In [OLR20], the authors prove that so-called non-modal Epistemic Planning in the DFOEL setting remains decidable provided that the first-order domain is finite. Even though one can foresee a full propositional encoding of the entire problem hence its decidability, the authors of [OLR20] provide an elegant proof based on bisimulations. However, this bisimulation approach does not apply when we relax the finiteness assumption on the first-order domain.

In this paper we investigate this Epistemic Planning Problem where infinite domain are allowed. Discarding the hopeless case of action models with modal (pre- and post-) conditions, we focus on pure first-order ones. Surprisingly, we establish that even though pre-conditions are non-modal, the decidability of the Epistemic Planning Problem is sensitive to the nature of post-conditions. To finitely represent our input of the problem, we require that the first-order structures are automatic, and regarding the action model effects, we allow arbitrary first-order interpretations, thus strictly extending the framework of [OAR20]. In this setting, we prove that Epistemic Planning with arbitrary pure first-order post-conditions (FOEPPnm) is undecidable, while Epistemic Planning with quantifier-free post-conditions (FOEPPnmqf) is decidable. For the latter case, our proof is involved since the infinite-domain structures generated along histories are infinitely many, in general.

The paper is organized as follows. In Section 2, we define the full framework of DFOEL. In Section 3, we introduce the First-order Epistemic Planning problem arising in this framework, and we establish our results: the undecidability of FOEPPnm in Section 3.3 and the decidability of FOEPPnmqf in Section 3.5. We conclude on these achievements in Section 4.

2 Dynamic First-Order Epistemic Logic

In this section we describe our proposal for DFOEL, inspired from [OAR20] but that offers a wider expressiveness in both the epistemic and the actions models: first, we allow one to consider infinite (but still finitely presentable) first-order structures, and second, we relax the action model post-conditions, i.e., the predicate updates, as arbitrary first-order interpretations (in the sense of model theory, see [H+97]) but where, for semantical reasons, the domain remains unchanged.

We will see that this extra expressiveness yields an undecidable epistemic planning problem. We will then better control the expressiveness to retrieve decidability, still in a setting that strictly extends the results from [OAR20] for allowing quantifier-free predicate updates, that in general may involve an infinite set of tuples.

For the rest of this paper, we let $P$ be a first-order signature. For a predicate $p \in P$ of arity $k$, we take the convention to write $p(k) \in P$. Also, we let $V$ be a countably infinite set of variables, whose typical elements are $x, y, z, x_1, \ldots$. Finally, $ag$ is a finite set of agents.
2.1 Preliminaries on First-Order Epistemic Logic

We restrict our definition to pure relational first-order structures (i.e., no functions in the signature)\(^1\).

**Definition 1.** The language of First-Order Epistemic Logic (FOEL) FOEL extends first-order logic with epistemic modalities. It is given by the following syntax:

\[
\text{FOEL} \ni \varphi, \psi ::= p(x_1, \ldots, x_k) \mid \neg \varphi \mid \varphi \land \psi \mid \forall x \varphi \mid K_a \varphi
\]

where \(x, x_1, \ldots, x_k \in V\), \(a \in \text{agt}\) and \(p(k) \in P\). An atomic formula is a formula of the form \(p(x_1, \ldots, x_k)\). We denote by FO the pure first-order logic, that is the \(K_a\)-free fragment of FOEL, and by \(\text{FO}^{qf}\) the sub-set of FO with only quantifier-free formulas.

Clearly \(\text{FO}^{qf} \subset \text{FO} \subset \text{FOEL}\).

In an epistemic logic setting, models are based on Kripke models, with possible worlds related through epistemic relations, one for each agent. Such a relation, say for agent \(a\), specifies which worlds of the Kripke models, agent \(a\) cannot distinguish. In the richer setting of DFOEL, we consider first-order epistemic models where each possible world is assigned an entire first-order structure.

**Definition 2.** A first-order epistemic model (or simply epistemic model) over domain \(D\) is a structure \(M = (W, (R_a)_{a \in \text{agt}}, (I_w)_{w \in W})\) where:

1. \(W\) is a non-empty set of worlds;
2. For each \(a \in \text{agt}\), \(R_a \subseteq W \times W\) is an accessibility relation. Denote by \(R_a(w) := \{w' \in W \mid (w, w') \in R_a\}\)
3. For every \(w \in W\), \(I_w = (D, (p^w)_p \in P)\) is a \(P\)-structure associated to \(w\).

Remark that in Definition 2, the structures \(I_w\) all share the same domain \(D\), but that their predicate interpretations may differ.

We provide here our running example, borrowed from formal language theory, that we will incrementally enrich as we progress along the paper.

**Example 1.** Given regular languages \(L, L_0, \ldots, L_m\) over an alphabet \(\Sigma\), we first introduce the signature \(P = (L(1), L_0(1), \ldots, L_m(1), \cdot(3), X(1))\), and the single-agent epistemic model \(M_{ex} = (\{\emptyset\}, R, I_\emptyset)^2\) where:

1. \(R := \{(\emptyset, \emptyset)\}\);\(^2\)
2. \(I_\emptyset\) has domain \(D = \Sigma^*\) and the predicates interpretations are \(L^0 := L\), and \(L_i^0 := L_i\) for every \(0 \leq i \leq m\);
3. \(X^0 := \{(u, u', u'') \in (\Sigma^*)^3 \mid u = u'u''\}\);
4. \(X^0 := \emptyset\), whose role is to iteratively compute a language.

---

\(^1\)This is no loss of expressiveness since functions and constants can be modeled by predicates through their graph.

\(^2\)Notation \(\emptyset\) for this single world relates to a starting computation of the language of interest.
Definition 3. Let $\mathcal{M}$ be an epistemic model over domain $D$. An assignment is a mapping $\nu : V \rightarrow D$, and an $x$-variant $\nu_x$ of $\nu$ is an assignment s.t. $\nu_x(y) = \nu(y)$ for all $y \in V \setminus \{x\}$.

Definition 4. Let $\mathcal{M}$ be a model and $\nu$ be an assignment of the variables in $V$. The satisfaction relation between an epistemic model $\mathcal{M}$ and a formula of FOEL is given inductively by:

- $\mathcal{M}, w \models \nu p(x_1, \ldots, x_k)$ iff $(\nu(x_1), \ldots, \nu(x_k)) \in p^w$ for all $p(k) \in P$.
- $\mathcal{M}, w \models \nu \neg \varphi$ iff not $\mathcal{M}, w \models \nu \varphi$.
- $\mathcal{M}, w \models \nu \varphi \land \psi$ iff $\mathcal{M}, w \models \nu \varphi$ and $\mathcal{M}, w \models \nu \psi$.
- $\mathcal{M}, w \models \nu \forall x \varphi$ iff $\mathcal{M}, w \models \nu_x \varphi$ for every $x$-variant $\nu_x$ of $\nu$.
- $\mathcal{M}, w \models \nu K_a \varphi$ iff $\mathcal{M}, w' \models \nu \varphi$ for all $w' \in R_a(w)$.

Note that if $\varphi \in \text{FO}$ i.e., $\varphi$ is modal-free, then $\mathcal{M}, w \models \nu \varphi$ iff $\mathcal{I}_{w'} \models \nu \varphi$. We write $\mathcal{M}, w \models \varphi$ whenever for any $\nu$, $\mathcal{M}, w \models \nu \varphi$; in particular, if $\varphi$ has no free variables.

It is clear that classical propositional epistemic logic is a fragment of FOEL where predicate symbols have all arity 0.

2.2 Dynamic First-Order Epistemic Logic

We now enrich the setting of FOEL with action models that provide the dynamics, via the update product, similarly to the approach in DEL. An action is a Kripke model whose elements are events with their respective precondition and their postconditions.

Definition 5. An action model is a tuple $\mathcal{E} = (E, (Q_a)_{a \in \text{agt}}, \text{pre}, \text{post})$ where:

1. $E$ is a non-empty finite set of events.
2. For $a \in \text{agt}$, $Q_a \subseteq E \times E$ is an accessibility relation.
3. $\text{pre} : E \rightarrow \text{FOEL}$ assigns to each $e \in E$ a precondition, that is a closed$^3$ FO-formula.
4. $\text{post} : E \rightarrow (P \rightarrow \text{FOEL})$ assigns to each $e \in E$ a postcondition $\text{post}(e)(p)(x_1, \ldots, x_k)$ for each $p(k) \in P$.

We distinguish particular action models.

Definition 6. $\mathcal{E} = (E, (Q_a)_{a \in \text{agt}}, \text{pre}, \text{post})$ is non-modal whenever:

- $\text{pre} : E \rightarrow \text{FO}$, and
- $\text{post} : E \rightarrow (P \rightarrow \text{FO})$.

$^3$meaning without free-variables.
Example 2. We define the action model, $E$ that represents the event of the application of complement, and union and concatenation with languages among $L_0, \ldots, L_m$. Formally, we let $E := (E_{ex}, Q, \text{pre}, \text{post})$ with

- $E_{ex} := \{\nu\} \cup \bigcup_{1 \leq i \leq m} \{\cup_i\} \cup \bigcup_{1 \leq i \leq m} \{\cdot_i\}$
- $Q := \{(e, e) \mid e \in E\}$
- for every $e \in E$,
  - $\text{pre}(e) := \text{true}$;
  - for every predicate $p$ among $L, L_0, \ldots, L_m, \cdot$, $\text{post}(e)(p) = p$
  - regarding predicate $X$, for every $0 \leq i \leq m$:
    \[
    \begin{align*}
    \text{post}(\cup_i)(X)(x) & := X(x) \lor L_i(x) \\
    \text{post}(\cdot_i)(X)(x) & := \exists y \exists z X(y) \land L_i(z) \land \cdot(x, y, z)
    \end{align*}
    \]

The update of an epistemic state with an action model filters the worlds that verify the preconditions and updates their structure through the postconditions: the domain is unchanged, but predicates might be updated.

Definition 7. Let $M = (W, (R_a)_{a \in \text{agt}}, (I_w)_{w \in W})$ and $E = (E, (Q_a)_{a \in \text{agt}}, \text{pre}, \text{post})$ be given. The product update of $M$ and $E$, is the epistemic model defined by

$$
M \otimes E = (W', (R'_a)_{a \in \text{agt}}, (I_w)_{w \in W'}),
$$

where:

- $W' = \{(w, e) \in W \times E : M, w \models \text{pre}(e)\};$ we will simply write $we$ instead $(w, e)$.
- for an agent $a \in \text{agt}$, we $R'_a we'$ iff $w R_a w$ and $e Q_a e'$.
- for $we \in W'$, $I_{we} = (D, (p^{we})_{p \in P})$ where for each $p(k) \in P$, $p^{we}$ is the set of tuples $(d_1, \ldots, d_k)$ such that
  \[
  M, w \models_{[x_1 \mapsto d_1, \ldots, x_k \mapsto d_k]} \text{post}(e)(p)(x_1, \ldots, x_k)
  \]

A post-condition can be for example $\text{post}(e)(p)(x_1, x_2) = p(x_1, x_2) \lor \exists y (p(x_1, y) \land p(y, x_2))$ which approximates the transitive closure of the binary predicate $p$ as iterated updates progress.

Example 3. Returning to our running example on languages, the model $M_{ex} \otimes E$ is such that:

- $W' = \{\emptyset\} \times E$;
- $R' = \{((\emptyset e, \emptyset e) \mid e \in E\};$
Thus, from world $\emptyset$, after applying the sequence of events $(\cup L_1)(\cup L_2)(\cdot (\cdot L_3))$, one reaches a first-order structure where $X$ is interpreted as $(L_1 \cup L_2)^c \cdot L_3$; otherwise said:

$$
X^0(\cup L_1)(\cup L_2)(\cdot (\cdot L_3)) = (L_1 \cup L_2)^c \cdot L_3
$$

One can easily enrich the setting, to capture other language operations such as complementing inside $L_i$, etc.

In the following, we aim at capturing the single infinite epistemic model comprised of all the updates, in a way similar to the DEL structure introduced in [AMP14, BCPS20] for proposition DEL.

### 2.3 The infinite epistemic model of histories

Given $M$ an epistemic model and $E$ an action model, we consider the family of updates $(M^E_n)_{n \in \mathbb{N}}$ defined by:

- $M^E_0 = M$, and
- $M^E_{n+1} = M^E_n \otimes E$.

The epistemic $M^E_n$ is called the $n$-th update, and we introduce the following notations for its components:

- $M^E_n = (W_n, (R_a^n)_{a \in \text{agt}}, (I_h)_h \in W_n)$.

We call a history any element $h \in W_n$, for some $n$, which is of the form $we_1...e_n$, where $w \in W$ and $e_1, \ldots, e_n \in E$; we then say that history $h$ starts from $w$. Given $w \in W$, we denote by $H_w$ be the set of histories starting from $w$.

We now gather all possible iterated updates in a single infinite epistemic model.

**Definition 8 (The epistemic model of histories).** The epistemic model of histories is

$$
M^E^* = (H, (R_a)_{a \in \text{agt}}, (I_h)_h \in H)
$$

where

- $H := \bigcup_{n \in \mathbb{N}} W_n$ is the set of histories;
- $R_a := \bigcup_{n \in \mathbb{N}} R_a^n$ is the accessibility relation over all the histories for agent for $a \in \text{agt}$;
- $I_h$ is the first-order structure resulting from the updates induced by the sequence of events along history $h$.

In the next section, we introduce epistemic planning problems in the FOEL framework in a way that exploit the prism provided by the infinite model $M^E^*$. 

6
3 Epistemic Planning in DFOEL

We consider a planning problem over the whole epistemic model $\mathcal{M}E^*$ of histories. Because the model $\mathcal{M}$ is infinite in general, we rely on finite-state automata to represent it, leading to classic automatic structures.

More precisely, we define the class of first-order epistemic models with finitely many worlds but whose first-order interpretations in worlds are automatic structures, and then introduce the first-order epistemic planning problem in this setting.

3.1 Automatic presentations of epistemic models

Automatic structures are first-order structures with no function symbols, such that their domain and predicates are regular, i.e., recognized by finite-state automata. An automatically presentable structure is a first-order structure isomorphic to an automatic structure; the reader may refer to [Rub08] for an exhaustive survey.

Definition 9. An automatic presentation over the set of predicates $P$ is a finite tuple $A = (A_D, (A_p)_{p \in P})$ of finite-state automata, where:

- $A_D$ is an automaton over alphabet $\Sigma$, and
- for every $p(k) \in P$, $A_p$ is an automaton over alphabet $\Sigma \cup \{\Box\}^k$.

An automatic presentation $A = (A_D, (A_p)_{p \in P})$ denotes a first-order structure over signature $P$ defined by

$S_A = \langle L(A_D), L(A_p)_{p \in P} \rangle$

A first-order structure $S = (S, (p^S)_{p \in P})$ is automatic if there exists a bijection $\text{enc} : S \rightarrow L(A_D)$, called an encoding, for some automatic presentation $A = (A_D, (A_p)_{p \in P})$, that is an isomorphism between $S$ and $S_A$.

Theorem 1 ([BG00]). The first-order theory of an automatic structure is decidable.

In our setting, we allow the domain of the (first-order) epistemic models $\mathcal{M}$ to be infinite as long as the first-order structure attached to each world is automatic. Formally,

Definition 10 (Automatic (first-order) epistemic models). An epistemic model $\mathcal{M} = (W, (R_a)_{a \in \text{agt}}, (I_w)_{w \in W})$ is automatic if $W$ is finite and $I_w$ is automatic, for each $w \in W$.

It can be seen that the predicates updates $\text{post}(e)(p)$ for a non-modal action model are mere first-order interpretations which by [BG00, Proposition 5.2] preserve automaticity. As a consequence, we have the following.

Proposition 1. If $\mathcal{M}$ is an automatic and $\mathcal{E}$ is non-modal, then $\mathcal{M} \otimes \mathcal{E}$ is automatic.
3.2 The First-order Epistemic Planning Problem

As originally defined in [BA11], an instance of the epistemic planning problem is composed of a (propositional) epistemic model, a distinguished world in this model, an action model and a first-order epistemic formula called the goal formula. The epistemic planning problem is to decide whether or not there exists an executable sequence of events from the distinguished world so that the goal formula holds.

We restate the epistemic planning problem in order to allow inputs with possibly infinite domains.

Definition 11 (The Epistemic Planning Problem (FOEPP)).

Input: an automatic epistemic model $M$, a distinguished world $w$ in $M$, an action model $E$, and a first-order epistemic formula $\varphi$;

Output: “yes” if there is a history $h$ starting from $w$ such that $M E^*, h \models \varphi$, otherwise “no”.

Example 4. We can rephrase formal language questions as epistemic planning ones, such as: Given a language $L$, can it be built using generators $L_0, \ldots, L_n$ and operations among union, complement and concatenation? Indeed, this question amounts to querying the epistemic planning problem with inputs the epistemic model $M_{ex}$, the action model $E$ and the epistemic formula $\varphi_{ex}$ expressing that languages $X$ and $L$ coincide i.e., $\varphi_{ex} := \forall x(X(x) \leftrightarrow L(x))$.

Epistemic planning problems have been widely investigated in the literature [BCPS20, BA11, CMS16, CPS18, AMP14, OAR20, OLR20]. The first family of contributions [BCPS20, BA11, CMS16, CPS18, AMP14] consider propositional epistemic and action models. This setting is clearly captured by our framework, simply because propositional logic can be embedded into first-order logic. It is well-known that, in the propositional setting, the epistemic planning problem is undecidable as soon as one allows modal event pre-condition formulas ([BCPS20]). As a corollary, we can state the following.

Theorem 2. FOEPP is undecidable.

In order to make the question about FOEPP less trivial, we consider its restricted variant, written FOEPP$_{nm}$, where pre-conditions and post-conditions formulas are non-modal, i.e., for each event $e$, the formulas $\text{pre}(e)$ and $\text{post}(e)(p)$ are FO-formulas.

We below show that the problem FOEPP$_{nm}$ is undecidable (Theorem 3). This makes the decidability result of [OAR20] more anecdotal for failing in the setting of infinite first-order domains. Additionally, we establish in Theorem 5 that there is room for a decidable subcase of FOEPP$_{nm}$, written FOEPP$_{nmqf}$, where post-conditions of the input action model are non-modal but also quantifier-free, i.e., all formulas $\text{post}(e)(p)$ belong to FO$^q$. This latter result makes the decidability of Epistemic Planning in the (propositional action models) DEL setting a mere corollary.
3.3 FOEPP is undecidable

Because with a $\text{post}(e)(p) \in \text{FO}$, we can capture the transitive closure of a graph, we exhibit a reduction from the emptiness problem of Turing machines (TME). The problem TME is known to be undecidable [HMU01, Theorem 9.10] and is defined as follows.

**Input:** a Turing machine $T$;

**Output:** do we have $L(T) = \emptyset$?

The reduction from TME to FOEPP relies on the fact that the configuration graph of a Turing machine is automatically presentable [KNRS07, Lemma 5.1], and on the fact that we can design an event that updates the binary successor predicate of this graph in a way that this iterated updates approximates its transitive closure.

We now formalize this reduction. Let $T$ be a Turing machine. We consider the first-order structure $G_T$ over the signature comprised of the binary predicate $p$, and two unary predicates $i$ and $f$ defined by:

$$G_T := (\text{Conf}(T), p^{G_T}, i^{G_T}, f^{G_T})$$

where its domain $\text{Conf}(T)$ is the set of configurations of $T$, $p^{G_T}$ is the binary relation $\rightarrow$ composed of configuration pairs such that $T$ can move from the former configuration to the latter configuration, $i^{G_T}$ is the set of initial configurations of $T$, and $f^{G_T}$ its the set of final configurations.

As for any first-order structure, the structure $G_T$ can be embarked into a single-agent single-world first-order epistemic model as follows: $M_T = (\{w\}, \{(w, w)\}, G_T)$.

**Lemma 1.** $M_T$ is an automatic epistemic model.

**Proof.** By [KNRS07, Lemma 5.1], the structure $(\text{Conf}(T), p^w)$ is automatically presentable and an accurate look at the encoding of the structure used in the proof of lemma shows that this encoding also makes the remaining relations $i^w$ and $f^w$ regular, which concludes. \hfill $\square$

We next design the single-event action model

$$E_T = (\{e\}, \{(e, e)\}, \text{pre, post})$$

where the effect of $e$ is to augment the current interpretation of predicate $p$ between configurations by its self-composition. Formally, we let:

- $\text{pre}(e) := \top$;
- $\text{post}(e)(p)(x_1, x_2) := p(x_1, x_2) \lor \exists y(p(x_1, y) \land p(y, x_2));$
- $\text{post}(e)(i)(x) := i(x)$ and $\text{post}(e)(f)(x) := f(x)$, i.e., the initial and final configurations remain unchanged.
Finally, we define the goal formula
\[ \varphi_T := \exists x \exists y (i(x) \land p(x, y) \land f(y)) \]

stating the existence of an initial and a final configuration related by \( p \).

**Lemma 2.** \( T \) is a positive instance of TME if, and only if, \( \mathcal{M}_T, \mathcal{E}_T, \varphi_T \) is a positive instance of FOEPP

**Proof.** By definition of \( \text{post}(e)(p) \), a sequence \( ee \ldots e \) of updates makes predicate \( p \) incrementally closer to \( \rightarrow^* \), the transitive closure of \( \rightarrow \): indeed, one can show by induction over \( \ell \) that after \( \ell \) triggers of event \( e \), two configurations are related by (the interpretation of) \( p \) if, and only if, there is a path of length at most \( \ell \) between them. Therefore, \( L(T) \neq \emptyset \)

iff some final configuration is reachable from some initial configuration in \( G_T \)

iff there exists a path of some length \( \ell \) from some final configuration to some initial configuration \[ \square \]

iff there exists some \( \ell \) such that \( \mathcal{M}_T \mathcal{E}_T^\ell, we^{\ell} \models \exists x \exists y (i(x) \land p(x, y) \land f(y)) \)

This concludes the proof of Theorem 3, which entails the following.

**Theorem 3.** The Epistemic Planning Problem FOEPP is undecidable.

As a preliminary step towards the decidability proof of FOEPP (Theorem 5), we reduce first-order epistemic planning to an existential first-order formula model checking query in a first-order structure \( \mathbb{H} \) derived from the epistemic model of histories \( \mathcal{M}_E^* \).

### 3.4 The first-order structure of the epistemic model of histories

Fix \( \mathcal{M} = (W, (R_a)_{a \in \text{agt}}, (I_w)_{w \in W}) \) a first-order epistemic model, and an action model \( \mathcal{E} = (E, (Q_a)_{a \in \text{agt}}, \text{pre}, \text{post}) \).

On the basis of \( \mathcal{M}_E^* = (H, (R_a)_{a \in \text{agt}}, (I_h)_{h \in H}) \), we consider the domain, called the universe, defined by \( U = H \cup \bigcup_{h \in H} D_h \)

where for every history \( h \in H \), the set \( D_h \) is a fresh copy of domain \( D \) from \( \mathcal{M} \), meant to denote the elements of the first-order structure \( \mathcal{I}_h \).

We consider the signature \( \tau \), obtained from the \( P \) (the signature of \( \mathcal{M} \)) and three other kinds of predicates, and provide their interpretations to obtain a first-order structure \( \mathbb{H} \). Formally,

\[ \tau = ((R_a)_{a \in \text{agt}}, (p^*)_{p \in P}, (\text{from}_w)_{w \in W}, \text{Dom}) \]

where:
• each $R_a$ has arity 2 and is set to: $R_a^H$ as the pairs of histories related by $R_a$ in $\mathcal{ME}^*$; 

• each $p^*$ is reshaping of predicate $p(k) \in P$ with arity $k + 1$, and set to:

$$p^H := \{(h, d_1, \ldots, d_k) \mid (d_1, \ldots, d_k) \in p^h\}$$

• predicate $\text{from}_w$ has arity 1, with $\text{from}_w^H$ equals to the subset $H_w$ of histories that start from $w$;

• predicate $\text{Dom}$ has arity 2, with $\text{Dom}^H$ the binary relation between every history and each element of its domain $D_h$.

The obtained structure $H$ is called the first-order structure of $\mathcal{ME}^*$, and boils down to being the following $\tau$-structure:

$$H = (\{1\}, (R_a)_{a \in \text{agt}}, H, \{h\} \times p^h)_{p \in P}, \{\text{from}_w^H\}_{w \in W}, \text{Dom}^H_{\{\{h\} \times D_h\}}$$

The structure $H$, equipped with first-order logic over signature $\tau = ((R_a)_{a \in \text{agt}}, (p^*)_{p \in P}, (\text{from}_w)_{w \in W}, \text{Dom})$, is at least as expressive as the epistemic model of histories $\mathcal{ME}^*$ (see Proposition 2), via the standard translation of FOEL into first-order logic inspired from [BVBW06], defined as follows.

**Definition 12.** The standard translation $ST_y$ of FOEL into the first-order logic over signature $\tau$ is inductively defined by:

- $ST_y(p(x_1, \ldots, x_k)) := \{p^*(y, x_1, \ldots, x_k) \land \bigwedge_{i=1}^k \text{Dom}(y, x_i)\}$
- $ST_y(\neg \varphi) := \neg ST_y(\varphi)$
- $ST_y(\varphi \land \psi) := ST_y(\varphi) \land ST_y(\psi)$
- $ST_y(\forall x \varphi(x)) := \forall x (\text{Dom}(y, x) \rightarrow ST_y(\varphi(x)))$
- $ST_y(K_a \varphi) := \forall y(K_a(y, y') \rightarrow ST_y(\varphi))$

**Proposition 2.** For any assignment $\nu : V \rightarrow D$ and any history $h \in H$, we let $\nu_h : V \rightarrow D_h$ be defined by $\nu_h(x) := \text{copy}_h(\nu(x))$. Then, for any formula $\varphi \in \text{FOEL}$,

$$\mathcal{ME}^*, h \models^y \varphi \iff H \models_{\nu_h[y \rightarrow h]} ST_y(\varphi),$$

**Proof.** We proceed by induction over $\varphi$:

- $\mathcal{ME}^*, h \models^y p(x_1, \ldots, x_k)$ iff $\{\nu(x_1), \ldots, \nu(x_k)\} \in p^h$
- iff (by definition of $p^H$) $(h, \nu_h(x_1), \ldots, \nu_h(x_k)) \in p^H$
- iff $H \models_{\nu_h[y \rightarrow h]} p^*(y, x_1, \ldots, x_k)$
- iff $H \models_{\nu_h[y \rightarrow h]} p^*(y, x_1, \ldots, x_k) \land \bigwedge_{i=1}^k \text{Dom}(y, x_i)$, and this latter formula is precisely $ST_y(p(x_1, \ldots, x_k))$;
• the cases for formulas of the form ¬ϕ and ϕ ∧ ψ is smooth;

• MME*, h □ν ∀xϕ
if M, h □νx ϕ for every x-variant ν of ν
if (by induction) H □ν (νx[y → h]) STy(ϕ) for every x-variant (νx) of νh;

• MME*, h □ν Kaϕ iff for all νx[h → h′] E νx[νx[h → h′]] STy(ϕ) (by induction)
if H □ν (νx[h → h′]) STy(ϕ) (by definition of STy).

\[ \square \]

An immediate corollary of Proposition 2 is a reduction of the entire FOEPP problem into the model-checking problem over H against a first-order logic:

**Proposition 3.** There exists h starting from w such that MME*, h □ ϕ if, and only if, H □ \exists y(ϕ ∧ χomw(y)).

Notice that by, Proposition 3 above and because we proved that FOEPPnm is undecidable, there must exist a structure H arising from action models with some non-quantifier-free event post-condition that is not automatic.

We now have gathered all the material to prove our last result.

### 3.5 FOEPPnmqf is decidable

We take inspiration from the methodology of [DPS18, BCPS20]: In order to prove that FOEPPnmqf is decidable, it is sufficient to show that the resulting structure H is automatic (Theorem 4). This is because, by Proposition 3, the epistemic planning problem then reduces to the decidable model-checking against first-order formula over the automatic structure H.

The section is therefore dedicated to the proof of the following Theorem 4, and ends with the statement that FOEPPnmqf is decidable (Theorem 5) as a corollary.

**Theorem 4.** Let M be an automatic epistemic model and E be an action model where all pre- and post-conditions are non-modal, and post-conditions are quantifier-free.

Then the derived first-order structure H of the epistemic model of histories (see Section 3.4) is automatic.

As pre- and post-conditions are non-modal, the update by an event e of the predicate interpretations at some history h ∈ H only depends on Iy and e. We can thus keep track of the interpretation along we1...en after the trigger of each event ei by remembering the current interpretation.

We show that, in the case where all post-conditions in E are quantifier-free, there are only finitely many different interpretations Iy. Otherwise said, the natural equivalence relation ∼ between histories induced “having isomorphic interpretation” (Definition 13) yields a finite partition of H (Proposition 4).
Definition 13. Define the relation $\sim \subseteq H \times H$ by: for all histories $h, h' \in H$,

$$h \sim h' \iff p^h = p^{h'}, \text{ for every } p \in P.$$ 

We will denote by $[h] = \{h' \mid h \sim h'\}$ the $\sim$-equivalence class of history $h$ and by $H/\sim$ the set of all the equivalence classes, with typical element $\alpha$.

By Definition 13 of $\sim$, it is clear that $I_{h'} = I_h^4$, for every $h' \in [h]$. This allows us to consistently set for a $\sim$-class $\alpha \in H/\sim$, $[\alpha] := I_h$, for some $h \in \alpha$, and to define $I_\alpha := I_h$.

We introduce some technical material that will be usefull in the proof that the set $[h]$ is regular (Proposition 5). Let $I$ be a $P$-interpretation with domain $D$, and $e \in E$, if $I \models \text{pre}(e)$, we define the $P$-interpretation $I \otimes e := (D, (p_{I \otimes e})_{p \in P})$ where $p_{I \otimes e}$ is the set of $(d_1, \ldots, d_k)$ such that:

$$I \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] \text{ post}(e)(p)(x_1, \ldots, x_k).$$

Lemma 3. For every $he \in H$, we have:

$$[he] = [h] \otimes e.$$

Proof. As $\text{post}(e)(p)$ is non-modal,

$$I_{he} \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] \text{ post}(e)(p)(x_1, \ldots, x_k)$$

if, and only if,

$$\mathcal{M}\mathcal{E}^*, he \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] \text{ post}(e)(p)(x_1, \ldots, x_k)$$

if, and only if,

$$\mathcal{M}\mathcal{E}^*, h \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] \text{ post}(e)(p)(x_1, \ldots, x_k)$$

if, and only if,

$$I_h \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] \text{ post}(e)(p)(x_1, \ldots, x_k)$$

if, and only if,

$$I_h \models [x_1 \mapsto d_1, \ldots, x_k \mapsto d_k] p_{I \otimes e}(x_1, \ldots, x_k).$$

Recall that in this section, $\mathcal{M}$ is automatic and $\mathcal{E}$ is an action model where all pre- and post-conditions are non-modal, and post-conditions are quantifier-free.

We use this assumption to establish the following, essentially stating that the set $(I_h)_{h \in H}$ is a finite family.

Proposition 4. $H/\sim$ is finite.

In order to establish Proposition 4, it is sufficient to show that every interpretation $p^h$ belongs to the Boolean algebra finitely generated by the interpretations $q^w$, where $q \in P$ and $w \in W$. Note that since this algebra of sets is finitely generated, it is itself finite.

We first introduce some notations.

\footnote{Up to the isomorphism arising from the natural one-to-one mapping between $D^h$ and $D^{h'}$.}
**Definition 14.** • For \( n \in \mathbb{N} \), let \([n] := \{1, \ldots, n\}\).

• For \( l, k \in \mathbb{N} \), \( B \subseteq D^l \), and \( \sigma : [l] \to [k] \), we let:
  \[ B\sigma := \{(d_1, \ldots, d_k) \mid (d_{\sigma(1)}, \ldots, d_{\sigma(l)}) \in B\} \]

• For \( G = \{B_1, \ldots, B_m\} \) comprised of subsets of a given superset, denote by \( \mathbb{B}_G \) the boolean algebra generated by \( G \).

We now show that the interpretation of a predicate after an update is a combination of atoms from the precedent interpretations.

**Lemma 4.** Let \( h \in H \), and \( p(k) \in P \).

If \( \text{post}(e)(p) \in \text{FO}^d \), then

\[ p^h \in \mathbb{B}_0\{q^h \sigma \mid q(l) \in P, \sigma : [l] \to [k]\} \]

**Proof.** We reason by induction over formula \( \text{post}(e)(p) \in \text{FO}^d \). If \( \text{post}(e)(p) \) is an atomic formula of \( \text{FO}^d \), say of the form \( q(x_{\sigma(1)}, \ldots, x_{\sigma(l)}) \), we have:

\( (d_1, \ldots, d_k) \in p^h \)

iff \( I_h \models_{[x_1 \mapsto d_1, \ldots, x_k \mapsto d_k]} q(x_{\sigma(1)}, \ldots, x_{\sigma(l)}) \)

iff \( (d_{\sigma(1)}, \ldots, d_{\sigma(l)}) \in q^h \)

iff \( (d_1, \ldots, d_k) \in q^h \sigma \).

Otherwise \( \text{post}(e)(p) \) is Boolean combination of atomic formulas of \( \text{FO}^d \), and as a result:

\[ p^h \in \mathbb{B}_0\{q^h \sigma \mid q(l) \in P, \sigma : [l] \to [k]\} \]

\( \square \)

We now establish that the interpretation of each predicate \( p^h \) for a history \( h \) starting from \( w \) belongs to the Boolean algebra generated by the initial interpretations \( \mathcal{I}_w(q \in P) \) of the predicates in \( \mathcal{I}_w \).

**Lemma 5.** For every \( h \in H \), for every \( p \in P \),

\[ p^h \in \mathbb{B}_0\{q^w \sigma \mid q(l) \in P, \sigma : [l] \to [k]\} \]

**Proof.** First, observe that given \( k, l, m \in \mathbb{N}, \sigma : [l] \to [k], \nu : [m] \to [l], \) and \( B \subseteq D^m \), we have:

\[(B\nu)\sigma = B(\sigma \circ \nu) \tag{1}\]

Indeed, \((d_1, \ldots, d_k) \in (B\nu)\sigma\) iff \((d_{\sigma(1)}, \ldots, d_{\sigma(l)}) \in B\nu\) iff \((d_{\sigma(1)}, \ldots, d_{\sigma(l)}) \in B\) iff \((d_1, \ldots, d_k) \in B(\sigma \circ \nu)\), which achieves the argument.

We prove Lemma 5 by induction on \( h \).

If \( h = w \), then for every \( p(k) \in P \), we clearly have \( p^w \in \mathbb{B}_0\{q^w \sigma \mid q(l) \in P, \sigma : [l] \to [k]\} \).

Otherwise, for a history of the form \( he \), by Lemma 4, we have that \( p^h \in \mathbb{B}_0\{q^h \sigma \mid q(l) \in P, \sigma : [l] \to [k]\} \). By induction hypothesis, \( q^h \in \mathbb{B}_0\{r^w \nu \mid r(m) \in P, \nu : \).

14
Thus \( q^h \sigma \in \mathbb{B} \{ (r^w \nu) \sigma \mid r \in P, \, \nu : [m] \to [l] \} \), Therefore by Equation (1),

\[
q^h \sigma \in \mathbb{B} \{ (r^w \circ \nu) \sigma \mid r(m) \in P, \, \nu : [m] \to [l] \},
\]
i.e., \( q^h \sigma \in \mathbb{B} \{ r^w \nu \mid r(m) \in P, \, \nu : [m] \to [k] \} \), which entails

\[
p^{he} \in \mathbb{B} \{ q^w \sigma \mid q(l) \in P, \, \sigma : [l] \to [k] \}.
\]

\[\square\]

Since \( \{ q^w \sigma \mid q[l] \in P, \, \sigma : [l] \to [k] \} \) is finite, so is \( \mathbb{B} \{ q^w \sigma \mid q(l) \in P, \, \sigma : [l] \to [k] \} \). Therefore there is a finite number of different \( p^{he} \), which achieves the proof of Proposition 4.

This finiteness property paves the way to defining an automaton that while reading a history \( h \) can determine \( \mathcal{I}_h \) (Proposition 5).

**Proposition 5.** For each \( h \in H, [h] \) is regular.

**Proof.** We establish the regularity of \( [h] \) by constructing a finite-state automaton \( A_{[h]} \) for it.

This automaton (over alphabet \( W \cup E \)) has states ranging over \( \{ s_0 \} \cup H/\sim \) (where \( s_0 \) is a fresh state), initial state \( \{ s_0 \} \), and final states ranging over \( \{ [h] \} \), and the transition relation \( \delta \) defined by:

- Regarding \( s_0 \):
  - \( \delta(s_0, w) = [w] \), for every \( w \in W \);
  - \( \delta(s_0, e) = \emptyset \), for \( e \in E \);

- Regarding every \( \alpha \in H/\sim \):
  - \( \delta(\alpha, w) = \emptyset \), for every \( w \in W \);
  - \( \delta(\alpha, e) = \begin{cases} \alpha \otimes e & \text{if } I_\alpha \models \text{pre}(e), \\ \emptyset & \text{otherwise.} \end{cases} \)

We now show that \( [h] = L(A_{[h]}) \):

\[
we_1...e_n \in [h] \quad \iff \quad \begin{array}{l}
we_1...e_n = [h] \\
we_1...e_n \text{ is final} \\
[w] \otimes e_1 \otimes ... \otimes e_n \text{ is final} \quad (\text{Lemma 3}) \\
\delta^*(s_0, we_1...e_n) \text{ is final} \\
we_1...e_n \in L(A_{[h]})
\end{array}
\]

We have here gathered all the material to show that the structure \( \mathbb{H} \) is automatic.

We first start with the encoding \( \text{enc} \) (see (Definition 15) of the elements of the universe \( U = H \cup \bigcup_{h \in H} D_h \)), then we prove that encoding \( \text{enc} \) provides an automatic presentation of the first-order structure \( \mathbb{H} \) (Lemma 7).
Recall that because event updates are particular cases of first-order interpretations, and, according to Proposition 1, we know that each $I_h$ is automatic. As a consequence, each $I_h$ is automatic, say with encoding $\text{enc}[h]$ over some alphabet $\Sigma_h$.

Now, the overall encoding function $\text{enc}$ of the universe $U$ is as follows.

**Definition 15.** The encoding function $\text{enc}$ of the universe $U = H \cup \bigcup_{h \in H} D_h$

$$\text{enc} : U \rightarrow (W \cup E \cup \bigcup_{[h] \in H/\sim} \Sigma_h)^*$$

is defined by:

- for $w_1 \ldots e_m \in H$, $\text{enc}(w_1 \ldots e_m) := w_1 \ldots e_m$
- for $d \in D_h$, $\text{enc}(d) := h \cdot \text{enc}[h](\text{copy}_h^{-1}(d))$; we recall that $\text{copy}_h$ is the bijection between $D$ and $D_h$.

We recall that

$$H = (U, (R^H_a)_{a \in \text{agt}}, (p^H \cdot \text{from}^H_w)_{w \in W}, \text{Dom}^H)$$

where

- $U = H \cup \bigcup_{h \in H} D_h$;
- $R^H_a = R_a$, for $a \in \text{agt}$;
- $p^H = \bigcup_{h \in H} (\{h\} \times p^h)$, for $p \in P$;
- $\text{from}^H_w = H_w$, for $w \in W$; and
- $\text{Dom}^H = \bigcup_{h \in H} (\{h\} \times D_h)$.

**Lemma 6.** $\text{enc}$ is injective and $\text{enc}(U)$ is regular.

**Proof.** Mapping $\text{enc}$ is injective since any element of $H$ is the element of domain $D_h$, for some $h$, and is therefore uniquely identified by the prefix $h$ in its encoding.

Regarding the regularity of $\text{enc}(U)$, we have:

$$\text{enc}(U) = \text{enc}(H \cup \bigcup_{h \in H} D_h) = \text{enc}(H) \cup \bigcup_{h \in H} \text{enc}(D_h)$$

(by definition of $\text{enc}$)

$$= H \cup \bigcup_{h \in H} h \cdot \text{enc}[h](\text{copy}_h^{-1}(D_h))$$

(because $H = \bigcup_{\alpha \in H/\sim} \alpha$)

In the expression above, $\alpha$ is a regular language by Proposition 5, and so is $\text{enc}_\alpha(D)$
It now remains to prove that \( enc \) is a good candidates for the relations of the structure \( \mathbb{H} \).

**Lemma 7.** Relations \( enc(R^H_a) \), \( enc(p^*H) \), \( enc(from_w) \) and \( enc(Dom) \) are regular.

**Proof.** By the definition of \( \mathbb{H} \) from Section 3.4, we have:

- \( enc(R^H_a) = enc(R_a) = R_a = (H \times H) \cap R^0_a \cdot Q^*_a \), where we recall that \( R^0_a \) is the epistemic relation in \( M \), and \( Q^*_a \) is the set of pointwise concatenation of pairs in the binary relation \( Q_a \) in \( E \).

Now, because \( W \) is finite, \( R^0_a \subseteq W \times W \) is regular. Also, since \( E \) is finite, \( Q_a \subseteq E \times E \) is regular and so is \( Q^*_a \). Obviously \( H \times H \) is regular. Since regular languages are closed under intersection, \( enc(R^H_a) \) is a regular language.

- If \( p(k) \in P \), \( enc(p^*H) = enc(\bigcup_{h \in H} \{h\} \times p^h) = enc(\bigcup_{\alpha \in H/\sim} \alpha \times p^\alpha) = \bigcup_{\alpha \in H/\sim} enc(\alpha) \times enc(p^\alpha) \).
  Now because by definition of \( enc \),
  \[
  enc(p^\alpha) = \left( \prod_{i=1}^{k} enc(\alpha) \right) \cdot enc_\alpha(p^\alpha),
  \]
  this set is also equal to \( \bigcup_{\alpha \in H/\sim} enc(\alpha) \times \left( \prod_{i=1}^{k} enc_\alpha(p^\alpha) \right) \). Since regular languages are closed under cartesian product and union, this last expression describes a regular language.

- \( enc(from_w) = enc(H_w) = H_w = H \cap w \cdot E^* \), which clearly is a regular language.

- Finally, \( enc(Dom^H) = enc(\bigcup_{h \in H} \{h\} \times D_h) = \bigcup_{\alpha \in H/\sim} \alpha \times enc_\alpha(D) \).
  As a finite union of regular languages, \( enc(Dom^H) \) is a regular language.

Lemmas 6 and 7 conclude the proof of Theorem 4, and as a consequence, we state the following.

**Theorem 5.** \( FOEPP_{nmqf} \) is decidable.

**Example 5.** Because an event whose effect is the operation of self-concatenation – that we did not consider in Example 2 – would require some existential quantifier in its post-condition, the query whether a given language \( L \) can be obtained by union, complement and concatenation from a finite set of generators, or not, (see Example 4) may not be decidable because of Theorem 3.

On the contrary, fixing a bound \( k \) to the number of occurrences of the concatenation operation in the expression for \( L \), allows us to reduce the problem to the decidable epistemic planning problem \( FOEPP_{nmqf} \): we can control by quantifier-free post-conditions and enough unary predicates \( X_0, \ldots, X_k \) in the signature that the concatenation operator is applied at most \( k \) times.
Remark that our running example on formal languages does not fully exploit the epistemic setting, as all events are distinguishable. To make our formal language problem more exciting on this aspect, one can expand it by introducing “errors” in the action model: the agent may not know whether the applied event is, say, \( \cup L_i \) (the union with languages \( L_i \)) or some other event corresponding to the union with an other language \( L'_i \), say because of computation errors during the process. We then would be able to decide whether or not the agent can still decide if the language \( L \) is obtainable through operations on \( L_0, \ldots, L_n \).

Still, we believe that our running example on formal languages is an interesting one for being reminiscent of the many challenging problems in formal language theory as addressed in [Brz80]. In particular, the case of language concatenation, as considered in Example 5, is tightly connected with the dot-depth hierarchy of [CB71], where many conjectures remain [Pin17].

4 Conclusion and Future work

We introduced the problem Epistemic Planning Problem FOEPP as a natural problem stemming from the DFOEL setting and that allows for infinite-domain first-order structures. Our framework thus strictly extends the one proposed in [OAR20].

Additionally, in order to have finitely presentable inputs to FOEPP, we proposed to consider so-called automatic epistemic models.

Regarding the problem FOEPP, and inheriting from known results on epistemic planning in the DEL setting (no first-order logic), the former problem, that allows for modal action models, is de facto undecidable. However, as shown here, restricting to non-modal action models is not sufficient in the wider first-order setting, since the resulting subproblem FOEPP_{nm} remains undecidable (Theorem 3).

Theorem 3 and its companion Theorem 5 for the decidability of FOEPP_{nmqf} make evidence that the nature of post-conditions, whether quantifier-free or not, draws a line in the decidability landscape. To our knowledge this phenomenon has never been observed in the literature, likely because infinite first-order domains have too eagerly been discarded. And indeed, dealing with infinite domain requires powerful mechanisms: our proof for that FOEPP_{nmqf} is decidable (Theorem 5) involves a non-trivial machinery, heavily based on automatic structures.

Even though one might wish a simpler proof for the FOEPP_{nmqf} decidability, the chosen approach has some benefits by offering automata constructions that entail a wide range of decidable problems related to epistemic planning (in the same line as [BCPS20]). For instance, with the proof of automaticity, we get an automaton that represents the set of all plan solutions which enables one to address all sorts of queries, e.g., whether they are infinitely many, whether there exists a solution that never resorts to some particular event, etc.

On a longer term perspective, we may investigate event schemes in action models, as done in [OAR20], that, for infinite first-order domains, would result in infinitely many events, hence a powerful but challenging setting. Under some hypothesis of automaticity, we believe that our decidability proof for FOEPP_{nmqf} can be extended.
Finally, another interesting track of research would be to allow different domains in different words, as first-order interpretations do.

References

[AMP14] Guillaume Aucher, Bastien Maubert, and Sophie Pinchinat. Automata techniques for epistemic protocol synthesis. In Fabio Mogavero, Aniello Murano, and Moshe Y. Vardi, editors, *Proceedings 2nd International Workshop on Strategic Reasoning, SR 2014, Grenoble, France, April 5-6, 2014*, volume 146 of *EPTCS*, pages 97–103, 2014.

[BA11] Thomas Bolander and Mikkel Birkegaard Andersen. Epistemic planning for single and multi-agent systems. *J. Appl. Non Class. Logics*, 21(1):9–34, 2011.

[BCPS20] Thomas Bolander, Tristan Charrier, Sophie Pinchinat, and François Schwarzentruber. Del-based epistemic planning: Decidability and complexity. *Artificial Intelligence*, 287:103304, 2020.

[BG00] Achim Blumensath and Erich Grädel. Automatic structures. In *Proceedings of the 15th Annual IEEE Symposium on Logic in Computer Science*, LICS ’00, page 51, USA, 2000. IEEE Computer Society.

[BR16] Alexandru Baltag and Bryan Renne. Dynamic Epistemic Logic. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Winter 2016 edition, 2016.

[Brz80] Janusz Brzozowski. Open problems about regular languages. In *Formal Language Theory*, pages 23–47. Elsevier, 1980.

[BVBW06] Patrick Blackburn, Johan Van Benthem, and Frank Wolter. *Handbook of Modal Logic*. Springer, 2006.

[CB71] Rina S Cohen and Janusz A Brzozowski. Dot-depth of star-free events. *Journal of Computer and System Sciences*, 5(1):1–16, 1971.

[CMS16] Tristan Charrier, Bastien Maubert, and François Schwarzentruber. On the Impact of Modal Depth in Epistemic Planning (Extended Version). Research report, IRISA, équipe LogicA, May 2016.

[CPS18] Sébastien Lê Cong, Sophie Pinchinat, and François Schwarzentruber. Small undecidable problems in epistemic planning. In Jérôme Lang, editor, *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018, July 13-19, 2018, Stockholm, Sweden*, pages 4780–4786. ijcai.org, 2018.

[DPS18] Gaëtan Douéneau-Tabot, Sophie Pinchinat, and François Schwarzentruber. Chain-monadic second order logic over regular automatic trees
and epistemic planning synthesis. In Guram Bezhanishvili, Giovanna D’Agostino, George Metcalfe, and Thomas Studer, editors, Advances in Modal Logic 12, proceedings of the 12th conference on “Advances in Modal Logic,” held in Bern, Switzerland, August 27-31, 2018, pages 237–256. College Publications, 2018.

[H+97] Wilfrid Hodges et al. A shorter model theory. Cambridge university press, 1997.

[HMU01] John E Hopcroft, Rajeev Motwani, and Jeffrey D Ullman. Introduction to automata theory, languages, and computation. Acm Sigact News, 32(1):60–65, 2001.

[KNRS07] Bakhadyr Khoussainov, André Nies, Sasha Rubin, and Frank Stephan. Automatic structures: Richness and limitations. Log. Methods Comput. Sci., 3(2), 2007.

[Koo08] Barteld Kooi. Dynamic term-modal logic. Van Benthem, J., Ju, S. and Veltman, F. (eds.). A meeting of the minds. Proceedings of the workshop on Logic, Rationality and Interaction, Beijing , 2007 Texts in Computing Computer Science 8. pp. 173-185 College publications , London, 8:173–185, 2008.

[Mau14] Bastien Maubert. Logical foundations of games with imperfect information : uniform strategies. Theses, Université Rennes 1, January 2014.

[OAR20] Andrés Occhipinti Liberman, Andreas Achen, and Rasmus Křemmer Rendsvig. Dynamic term-modal logics for first-order epistemic planning. Artificial Intelligence, 286:103305, 2020.

[OLR20] Andrés Occhipinti Liberman and Rasmus Křemmer Rendsvig. Decidability results in first-order epistemic planning. In Christian Bessiere, editor, Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence, IJCAI-20, pages 4161–4167. International Joint Conferences on Artificial Intelligence Organization, 7 2020. Main track.

[Pin17] Jean-Éric Pin. The dot-depth hierarchy, 45 years later. In THE ROLE OF THEORY IN COMPUTER SCIENCE: Essays Dedicated to Janusz Brzozowski, pages 177–201. World Scientific, 2017.

[Rub08] Sasha Rubin. Automata presenting structures: A survey of the finite string case. The Bulletin of Symbolic Logic, 14(2):169–209, 2008.

[VDvDHK07] Hans Van Ditmarsch, Wiebe van Der Hoek, and Barteld Kooi. Dynamic epistemic logic, volume 337. Springer Science & Business Media, 2007.