Biham-Middleton-Levine Traffic Model With Origin-Destination Trips

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Abstract

We extended the Biham-Middleton-Levine model to incorporate the origin and destination effect of drivers trips on the traffic in cities. The destination sites are randomly chosen from some origin-destination distances probability distribution "ODDPD". We use three different distributions: exponential, uniform and power-law. We consider two variants of the model. In conserved particles model (Model A), drivers continue their travelling even if they reached their destinations. In non-conserved particles model (Model B), a driver which reaches its destination disappears with rate $\beta$. It is found that the traffic dynamics in model A and the evacuation processes in model B are greatly influenced by the ODDPD. On one hand, we found that we can adjust the ODDPD to enhance the road capacity of the city and to minimize the arrival times of drivers in particles conserved system and to optimize the evacuation time of drivers in non-conserved case. On the other hand, we find that, independently on the ODDPD, the evacuation time $T$ of drivers diverges in the form of a power law $T \propto \beta^{-\nu}$, with $\nu = 1$.

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1 Introduction

Ever since O. Biham, A.A. Middleton and D. Levine elaborated their traffic model (BML) \cite{1}, the two-dimensional cellular automata (CA) traffic models are used enormously in order to understand the complex dynamic behavior of the traffic in cities (see the reviews \cite{2-4}). In CA, time and space are discrete. The space is represented as a uniform lattice of cells with finite number of states, subject to a uniform set of rules, which drives the behavior of the system. These rules compute the state of a particular cell as a function of its previous state and the state of the neighboring cells.

The BML model is a three-state CA with periodic boundary conditions where at a given time step each site can be occupied by a car moving upward, a car moving rightward, or the site is empty. There are equal number of cars in each direction and the dynamics is governed by synchronous traffic lights that allow alternatively vertical or horizontal movements. At their turn each car jumps to its next place whenever it is empty. Thus, an essential ingredient of the BML model is the excluded volume at crossings which determines the sharp transition between two phases. A first order phase transition from the free phase to the completely jamming phase occurs as the car density of the system increases. Recently, many generalizations and extensions of the BML model are elaborated to take into account several realistic features of traffic in cities. For example, the effect of asymmetry distributions of vehicles among the east-bound and north-bound streets are investigated in Ref. \cite{5}. Cuesta et al. \cite{6} introduced the turn capability of urban car movement in cities and they found a first order phase transition from a freely moving regime to a jammed state. Tadaki and Kikuchi \cite{7} found two types of jam phases in the BML model, the self-organized jam at relatively low density and the random jam at high density. The anisotropy effect of directions of move on the BML model, in the periodic and open boundaries conditions are investigated in Ref \cite{8}. Chopard et al. \cite{9} have developed a more realistic CA model of city traffic where the stretches of the streets in between successive crossings appear explicitly. The rule for implementing the motion of the vehicles at the crossing is formulated assuming a rotary to be located at each crossing. The BML model has been extended also to incorporate the effects of the static hindrances or road blocks (e.g., vehicles crashed in traffic accident) \cite{10,11}. It was shown that the dynamical jamming transition occurs at lower density of cars with increasing delay time of a car passing over the position of the traffic accident. J. Freund and T. Pöschel
studied another generalization of the BML model. Each site of this
generalized model represents a crossing where a finite number of cars can
wait approaching the crossing from each of the four directions. The mean
velocity of cars, as a function of the global traffic density, is determined
numerically and derived analytically applying combinatorics and statistical
methods. The spatial extension of the streets between two intersections is
completely neglected in the original model. To incorporate this feature, D.
Chowdhury and A. Schadschneider [13] proposed a new cellular automata
model for vehicular traffic in cities by combining ideas borrowed from the
BML model of city traffic and the Nagel-Schreckenberg model [14] of highway
traffic. It was found a phase transition from the free-flowing dynamical phase
to the completely ”jammed” phase at a vehicle density which depends on the
time periods of the synchronized signals and the separation between them
[15, 16].

In Sec. 2, we describe our models for traffic of cars in cities with origins
and destinations. In Sec. 3, we present our numerical results where we give
the mean velocity diagrams of the model A. A detailed description of the
distributions of the arrival times of drivers are also presented. For model B,
we present results concerning the evacuation processes of drivers inside the
lattice. Finally, we conclude with some conclusions in Sec. 4.

2 The BML Traffic Model With Origin-Destination
Trips

In the original BML Traffic Model, cars move, whenever the traffic lights
allow it, all along the lanes without any destinations. In this paper, we
extend the model to describe cars movements with origin-destination trips.
Each car is associated with some given origin-destination sites. The distance
$\delta$ between origin and destination sites is chosen from certain probability
distributions ”ODDPD”. We shall consider here three types of distributions:
1) The exponential distribution,

$$f^e(\delta) = \frac{\mu}{e^{-\mu \delta_m} - e^{-\mu \delta_M}} e^{-\mu \delta}$$

Here we focus the case where $\mu = 0.1$ and we suppose that $f(\delta)$ has a support
on some interval $[\delta_m, \delta_M]$. The numerical values of these parameters will be
2) The power-law distribution,

\[ f^p(\delta) = \frac{n + 1}{(\delta_M - \delta_m)^{n+1}}(\delta - \delta_m)^n \] (2)

Here we focus the case of \( n = 2 \) and we suppose that \( f(\delta) \) has a support on the interval \([\delta_m, \delta_M]\).

3) The uniform distribution,

\[ f^u(\delta) = \frac{1}{\delta_M - \delta_m} \] (3)

Here \( f^u(\delta) \) has a support on the interval \([\delta_m, \delta_M]\).

The distance \( \delta \) between origin and destination sites, used all along this paper is defined as follows. Let \((x_o, y_o)\) and \((x_d, y_d)\) be the coordinates of the origin and destination sites respectively. We define also the variables \( \delta_x \) and \( \delta_y \) as:

\[
\begin{cases} 
\delta_x = x_d - x_0 & \text{if } (x_d - x_0) \geq 0; \\
\delta_x = L + (x_d - x_0) & \text{if } (x_d - x_0) < 0.
\end{cases}
\] (4)

Similar equations can lead to defining \( \delta_y \) with replacing \( x \) by \( y \). The distance \( \delta \) is then given by,

\[ \delta = \delta_x + \delta_y \] (5)

Initially, all the origins sites of drivers are chosen randomly from the lattice points. According to traffic lights that permit horizontal motion at even time steps and vertical motion at odd time steps, cars will move towards their destinations by selecting some appropriate paths. Suppose that a car is initially moving rightward (upward). If the origin and destination sites are in the same lane (column), then the car will move horizontally (vertically) until it reaches its destination. Otherwise, the car will move horizontally (vertically) until it reaches the site belonging to the column (lane) of the destination site where the car turns and moves upward (rightward) until it reaches its destination (see figure 1). In this paper, we shall consider two simplest versions of the model, which we name model A and model B. In conserved particles model (model A), cars travel from the origin sites towards the destination sites. A car which reaches its destination site continues to travel from that site to a new destination site, chosen according to the ODDPD, and so on. In non-conserved particles model (model B), cars which completes its journey disappears with certain probability \( \beta \), or continues to travel from that site to a new destination site (as in model A) with probability \( 1 - \beta \).
3 Simulation experiments and results

We carry out our computer simulations of the model by considering a square lattice of size $L$. Initially, we put randomly a number $N$ of cars into the lattice with half arrows up and half right. The density of cars is denoted as $\rho = N/L^2$. We use a parallel updates scheme and a periodic boundary conditions. The average velocity $<v>$ of cars is defined as the rate of moving cars to the total number of cars allowed to move by the traffic lights. The velocity of each car can be either 1 or 0. We perform numerical simulations of the CA model starting with a set of random initial conditions for the system size $L = 40 - 200$. After a transient period that depends on the system size, on the random initial configuration and on the density of cars, the system reaches its asymptotic state. The duration of each simulation run is 50,000 time steps with the first 20,000 time steps to initiate the simulation and the latter 30,000 used to generate performance statistics. Finally, we define the parameter $\delta_m$ (resp. $\delta_M$) of the support interval of the ODDPD as the minimal (resp. maximal) distance between the origin and the destination sites, of the travelling paths of drivers. In our numerical simulations, we set $\delta_m = 20$ cells and $\delta_M = 2 \times (L - 1)$ cells. The first numerical value is chosen without any specification while the second rises from the nature of the path selection procedure followed in this paper (see for example the gray car and its gray destination in figure 1).

3.1 Conserved particles model (Model A)

3.1.1 Velocity diagrams

In model A, cars travel from origins to destinations. After the arrival, the destination sites will be considered as new origins, from where the travelling will continue towards some new chosen destination sites, and so on. In figure 2, we carried out the plots of the mean velocity of cars as a function of the density, for the three ODDPD. This diagram shows the presence of the two known states, namely the moving phase and the completely jammed state where all cars are blocked. In the freely moving state, interaction between cars is weak and the propagation is important inside the lattice. In contrast, for large density, the interaction becomes strong and jamming takes place where car movements become rare. In contrast with the original BML model, where a sharp transition separates the two phases, our model shows that the
mean velocity goes down to zero gradually with increasing the density and vanishes above the transition point. On one hand, the power-law and the uniform ODDPD lead to almost the same curves of the plot of $\langle v \rangle$. On the other hand, the exponential ODDPD leads to some differently curve. So, for low density the mean velocity of cars of the exponential ODDPD is smaller than that corresponding to power-law (or uniform) and higher for high density. Now, let us study the transition point $\rho_t$ separating the moving phase and the completely jammed state where the average velocity vanishes. From figure 2, we see clearly that $\rho_t$ resulting from the exponential ODDPD is higher than that resulting from the power-law or the uniform ones. Hence, the capacity of traffic in cities can be improved if the travelling paths of drivers are likely shorts.

To study the finite size effect on the velocity diagrams of the system, we compute the mean velocity for different lattice sizes. That is in contrast with the original BML traffic model, where the transition point decreases when increasing the system size $L$ ($\rho_t \propto L^{-0.14}$) [17], the velocity diagrams in our models are unchanged with respect to large lattice sizes (Figure 3a,b). The exponential ODDPD exhibits an interesting dependence of the velocity diagram on the lattice size $L$ (Fig.3a). At low densities, $\langle v \rangle$ remains unchanged with respect to the variation of $L$. However, in the high density region, one may distinguish some particular density where the mean velocity does not depend once again on $L$. Below (resp. above) this density, $\langle v \rangle$ decreases (resp. increases) with $L$ and reaches a stationary value at larger sizes ($L > 120$). One sees from figure 3b that a similar behaviour may occurred for the power-law ODDPD with the only difference is that the dependence on $L$ is inverted.

### 3.1.2 Arrival time distributions

When dealing with a realistic traffic in cities, it is important to know the time necessary to travel from a given origin to a given destination, which is called ”arrival time”. This time is interesting to the drivers because it determines when they must leave their house in order to be on time at their work.

In figure 4, we give the arrival time probability distribution of drivers where the chosen ODDPD is exponential. When the cars density is low, times shorter than $\tau_1 \sim 2*\delta_m$ are strongly suppressed while beyond the probability decays exponentially and exhibits a single peak at $\tau_1$. Since the traffic light
turns periodically and the minimal distance taken by each driver is supposed equal to $\delta_m = 20$ cells, the shortest arrival time will be closely equal to 40 time steps. In addition, since cars interact very weakly at low densities, the arrival time of any driver is almost equal to $2 \times \delta$ where $\delta$ is the origin-destination distance chosen by a driver. This justifies the fact that the arrival time distribution keeps the same shape (exponential) as the ODDPD. For higher densities, the probability distribution of arrival times presents a single peak at time approximately equal to $\tau_1$; reflecting the short routes chosen by almost all drivers. Beyond $\tau_1$, the arrival time distribution decreases exponentially. Consequently, with exponential ODDPD, arrival times of drivers will be likely shorts even if the density is relatively high. In another hand, there are no significant changes of the probability distribution of arrival times with respect to relatively large lattice sizes and relatively low densities. This result can be clarified from two points. First, only small distances from origins to destinations can be drawn from the exponential distribution of equation (1). Second, all the origins points of drivers are chosen randomly from the lattice points. Hence, standing times lost by a driver are usually shorts since the interaction with others cars is weak. Nevertheless, when the cars density is close to $\rho_t$, cars self-organized into a large cluster (large jam) and almost all vehicles do not move. Thus, drivers situated in the interior of this large jam do not move and rest inside for an infinite time. The only moving cars are situated at the boundaries of the large jam which can reach their destinations at finite times.

In figure 5, we plot the arrival time probability distribution of drivers where the ODDPD is power-law. As before, when the cars density is low, the form of the arrival time distribution is similar to that of the ODDPD because the system is in free regime. However, in contrast to the exponential ODDPD, the power-low one exhibits a strong dependence on the lattice size. In deed, the distances drawn from the power-law ODDPD are usually large; leading therefore to an enhancement of the arrival times of drivers if the lattice size is increased. If the car density increases, one finds evidently a broad distribution of arrival times, because cars are moving for quite long times before reaching their destinations. The higher is $\rho$, the higher is the arrival time. Yet, the arrival time distribution exhibits two peaks. The arrival time at the first peak which is located at $\tau_2 \sim 2 \times \delta_M$ changes little with increasing the density. Beyond $\tau_2$, the distribution spreads to the higher value of arrival times. On the other hand, times shorter than $\tau_2$ represent the arrival times of cars which terminate their journey without that they are stopped. However,
times longer than $\tau_2$ correspond to cars which stopped for a while during their routes. In the long times region, the probability distribution increases, reaches a maximum and then decreases asymptotically towards zero. This peak is considered as the most probable arrival time of a driver who travels in congested region of the lattice. This behaviour which occurs also for the time-headway in highways [5], rises from the strong interactions between cars at high densities. Furthermore, long arrival times should be taken by drivers in large lattice sizes, because larger distances could be drawn from the probability distribution of Eq.(2).

If the uniform ODDPD is used and the cars density is low, one sees that the corresponding arrival times distribution is uniform (Fig. 6). For higher densities, we find that the arrival times distribution behaves as the power-law case, especially in long arrival times region. The main difference is that, in the power-law case, the distribution presents two peaks while, in the uniform case, it presents three peaks. The first peak is the higher corresponding to the arrival time $\tau_1$ while the second peak is shifted a little bit towards the higher values with respect to $\tau_2$. Drivers with short trips can avoid congested regions and move towards their destinations with a minimal arrival times. However, those which travel far away, cannot avoid congested regions and their arrival times will be larger than the distances of their trips. The third peak gives the most probable arrival time of such drivers.

### 3.2 Non-Conserved particles model (Model B)

In model B, cars move from origins to destinations. In contrast to model A, cars which completes its journey disappears with certain probability $\beta$, or continues to move from that site to a new destination site with probability $1 - \beta$. In model B, the number of cars present in the lattice, $N(t)$, is not conserved but decreases with time until becomes zero above certain time $T$, called hereafter as the evacuation time, i.e., the time it takes for all cars to leave the lattice. Hereafter, we shall investigate the effect of the ODDPD on $N(t)$.

If $\beta = 1$, the number of cars remains constant up to certain time $\tau_1 \sim 40$ time steps which is the same for the three different ODDPD. This is the time needed to reach a destination for a driver which have an origin-destination distance equal to $\delta_m = 20$ cells. Above $\tau_1$, the number of cars $N(t)$ shows a polynomial or a linear or an exponential decreases if we used respectively a power-law, or a uniform or an exponential ODDPD. Moreover, as figure 7a
shows, the evacuation time is much shorter for the exponential distribution than for the two others distributions. As β diminishes, \( N(t) \) decreases slowly with time; leading therefore to an increasing of the evacuation time. Most interestingly, if β becomes very small, \( N(t) \) will decrease almost exponentially for the three different ODDPD. In addition, the evacuation times are found to be very long for the power-law ODDPD (see Fig. 7b).

It is found that, when the time \( t \) exceeds \( \tau_1 \), the exponential ODDPD produces an exponential decrease of the number \( N(t) \) whatever the values of the system parameters. Thus, \( N(t) \) follows the equation,

\[
N(t) = N_0 e^{-\lambda t}
\]

where \( N_0 \) is the initial number of drivers present in the lattice. Our results depicted in figures 8a and 8b show that the coefficient \( \lambda \) is independent on both the initial density \( \rho_0 \) and the lattice size \( L \). However, it is found that \( \lambda \) depends strongly on \( \beta \) and \( \lambda = f(\beta) \) is an increasing function which shows a strong increase as \( \beta \) approaches the maximal value (\( \beta = 1 \)). This function is illustrated in figure 8c.

Finally, the dependence of the evacuation time \( T \) on the parameter \( \beta \) is depicted in figure 9a for the three ODDPD at low and high densities. We find that, the evacuation time diverges as \( \beta \) approaches the vanishing value.

From figure 9b we see that the evacuation time \( T \) follows a power law behavior of the form,

\[
T \propto \beta^{-\nu}
\]

Except for some minor fluctuations, the dynamic exponent \( \nu \) remains unchanged when varying the density or when changing the type of ODDPD. For examples, if the exponential ODDPD is used, one finds, \( \nu \approx 1.08 \pm 0.001 \) for \( \rho_0 = 0.1 \) and \( \nu \approx 1.09 \pm 0.007 \) for \( \rho_0 = 0.3 \), while for the power-law case one finds, \( \nu \approx 1.10 \pm 0.009 \) for \( \rho_0 = 0.1 \) and \( \nu \approx 1.11 \pm 0.006 \) for \( \rho = 0.3 \).

Assuming that the parameter \( \beta \) is rate of transition for the dynamics of the model B, and as it was demonstrated in Ref. [19] for a one-dimensional traffic model, the exponent \( \nu \) is expected to be theoretically equal to one.

## 4 Conclusions

In summary, we have extended the BML model to incorporate the origin and destination effect of drivers trips on the traffic in cities. The origin-destination distances are drawn from certain probability distribution that is
used for all drivers.
In model A, all drivers continue their travelling even if they reached their destinations. The mean velocity of cars is greatly influenced by the ODDPD. Yet, the transition density that separates the moving phase and the completely jammed state, is higher when we used the exponential ODDPD than if we used the power-law or the uniform ODDPD. The arrival times of drivers depend on the density and on the type of ODDPD. At low density, cars interact very weakly and the arrival time is almost equal to 2\*δ where δ is the origin-destination distance. However, at high density, cars strongly interact and jamming phase occurs. This increases the arrival time of some drivers which stopped for a while during their journeys. Our results indicate that we can adjust the ODDPD to enhance the road capacity of the city and to minimize the arrival time of drivers. The effect of lattice size \( L \) is also studied. Hence, in contrast to the original BML model, the model A shows a non dependence of the mean velocity on the lattice size beyond some large limit. However, the distribution of arrival times becomes broad as we increase the size \( L \), especially for the power-law and the uniform distance distributions.

The other version studied in this paper is the model B where a driver which reaches its destination disappears with rate \( \beta \). We found that the evacuation processes is greatly influenced by the ODDPD. For the three ODDPD and with respect to varying the rate \( \beta \), the evacuation time is found to exhibit a power law behaviour. This evacuation processes is characterized by a dynamical exponent \( \nu \) (\( \tau \propto \beta^{-\nu}, \nu = 1 \)).
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Figures captions

**Figure 1.** Illustration of traffic in a square lattice (with periodic boundary conditions) where each car moves from origins to destinations. Cars (rectangles) located at the origin sites will move towards their destination sites (circles).

**Figure 2.** Mean velocity diagrams of cars versus density for the three different ODDPD.

**Figure 3.** Mean velocity diagrams of cars versus density for different values of lattice size $L$. (a) Exponential ODDPD; (b) Power-law ODDPD.

**Figure 4.** The arrival time probability distribution of drivers for the exponential ODDPD.

**Figure 5.** The arrival time probability distribution of drivers for the power-law ODDPD.

**Figure 6.** The arrival time probability distribution of drivers for the uniform ODDPD.

**Figure 7.** Time evolution of the number of drivers present in the lattice for the three ODDPD and for various values of the rate $\beta$; (a) $\beta = 1.0$ and $\beta = 0.5$; (b) $\beta = 0.1$.

**Figure 8.** Time evolution of the number of drivers present in the lattice for the exponential ODDPD; (a) for various values of the rate $\beta$ and density $\rho$; (b) for various lattice size $L$.

**Figure 9.** The dependence of the evacuation time $T$ on the parameter $\beta$ for the three ODDPD at low and high densities; (a) Linear plots (b) Log-Log plots.
Low density

High density

\( L = 120 \)

Probability vs. Arrival Time

Graph showing probability over arrival time for low and high density cases, with a specific point at \( L = 120 \).
