Performance Analysis of Irregular Repetition Slotted Aloha with Multi-Cell Interference

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Abstract—Irregular repetition slotted aloha (IRSA) is a massive random access protocol in which users transmit several replicas of their packet over a frame to a base station. Existing studies have analyzed IRSA in the single-cell (SC) setup, which does not extend to the more practically relevant multi-cell (MC) setup due to the inter-cell interference. In this work, we analyze MC IRSA, accounting for pilot contamination and multiuser interference. Via numerical simulations, we illustrate that, in practical settings, MC IRSA can have a drastic loss of throughput, up to 70%, compared to SC IRSA. Further, MC IRSA requires a significantly higher training length (about 4-5x compared to SC IRSA), in order to support the same user density and achieve the same throughput. We also provide insights into the impact of the pilot length, number of antennas, and signal to noise ratio on the performance of MC IRSA.

Index Terms—Irregular repetition slotted aloha, pilot contamination, multi-cell interference, massive random access

I. INTRODUCTION

Massive machine-type communications (mMTC) require random access protocols that serve large numbers of users [1], [2]. One such protocol is irregular repetition slotted aloha (IRSA), a successive interference cancellation (SIC) aided protocol in which users transmit multiple packet replicas in different resource blocks (RBs) [3]. Channel estimation in IRSA is accomplished using training or pilot sequences transmitted by the users at the start of their packets. Assigning mutually orthogonal pilots to users avoids pilot contamination, but is prohibitive in mMTC, since the pilot overhead would be proportional to the total number of users [4]. Thus, pilot contamination (PC), which reduces the accuracy of channel estimation and makes the estimates correlated [5], is unavoidable in mMTC, and significantly degrades the throughput of IRSA. PC is caused by both within-cell and out-of-cell users, termed intra-cell PC and inter-cell PC, respectively. The goal of this paper is to analyze the performance of IRSA, accounting for both intra-cell PC and inter-cell PC.

Initial studies on IRSA with focused on MAC [3] and path loss channels [6]. IRSA has been analyzed in a single-cell (SC) setup, accounting for intra-cell PC, estimation errors, path loss, and MIMO fading [7], [8]. Multi-user interference from users within the same cell is termed intra-cell interference and from users across cells is termed inter-cell interference. In the SC setup, only intra-cell interference affects the decoding of users since users do not face inter-cell interference. In practice, multiple base stations (BSs) are deployed to cover a large region, and thus inter-cell interference is inevitable [9]. Furthermore, MC processing (e.g., MC MMSE combining of signals) schemes can achieve better performance compared to SC processing, since it accounts for inter-cell interference [10].

Our main contributions in this paper are as follows:

1) We derive the channel estimates in MC IRSA accounting for path loss, MIMO fading, intra-cell PC, and inter-cell PC.
2) We analyze the SINR achieved in MC IRSA, accounting for PC, channel estimation errors, intra-cell interference, and inter-cell interference.
3) We provide insights into the effect of system parameters such as number of antennas, pilot length, and SNR on the throughput performance of MC IRSA.

To the best of our knowledge, no existing work has analyzed the effect of MC interference on IRSA. Through numerical simulations, we show that inter-cell PC and inter-cell interference result in up to 70% loss in throughput compared to the SC setup. This loss can be overcome by using about 4–5x longer pilot sequences. Thus, it is vital to account for the effects of MC interference, in order to obtain realistic insights into the performance of IRSA.

Notation: The symbols $a$, $A$, $|a|$, $|A|$, $0_N$, $1_N$, and $I_N$ denote a scalar, a vector, a matrix, the $i$th row of $A$, the $j$th column of $A$, all-zero vector of length $N$, all ones vector of length $N$, and an identity matrix of size $N \times N$, respectively. $[a]_S$ and $[A]_S$ denote the columns of $A$ indexed by the set $S$, respectively. $\text{diag}(a)$ is a diagonal matrix with diagonal entries given by $a$. $[N]$ denotes the set $\{1, 2, \ldots, N\}$. $\| \cdot \|$ is the Euclidean norm, and $\| \cdot \|_F$ is the Frobenius norm, and $\text{diag}(a)$ is the diagonal matrix with diagonal entries given by $a$. $\ell_2$ norm, and Hermite operators.

II. SYSTEM MODEL

We consider an uplink MC system with $Q$ cells, where each cell has an $N$-antenna BS located at its center. We refer to the BS at the center of the $q$th cell as the $q$th BS. Every cell has $M$ single antenna users arbitrarily deployed within the cell who wish to communicate with their own BS. The time-frequency resource is divided into $T$ RBs. These $T$ RBs are common to all the cells, and thus, a total of $QM$ users contend over the $T$ RBs. Each user randomly accesses a subset of the available RBs according to the IRSA protocol, and transmit packet replicas in the chosen RBs. Each replica comprises of a header containing pilot symbols for channel estimation, and a payload containing data and error correction symbols.
The access of the RBs by the users can be represented by an access pattern matrix $G = [G_1, G_2, \ldots, G_Q] \in \{0, 1\}^{T \times Q}$. Here $G_j \in \{0, 1\}^{T \times M}$ represents the access pattern matrix of the users in the $j$th cell, and $g_{tji} = |G_j|_{t}$ is the access coefficient such that $g_{tji} = 1$ if the $i$th user in the $j$th cell transmits in the $t$th RB, and $g_{tji} = 0$ otherwise. The $i$th user in the $j$th cell samples its repetition factor $d_{tji}$ from a preset probability distribution. It then chooses $d_{tji}$ RBs from the $T$ RBs uniformly at random for transmission. The access pattern matrix is known at the BS, which is made possible by using pseudo-random matrices generated from a seed that is available at the BS and the users [8]. This can be done in an offline fashion.

The received signal at any BS in the $t$th RB is a superposition of the packets transmitted by the users who choose to transmit in the $t$th RB, across all cells. In the pilot phase, the $i$th user in the $j$th cell transmits a pilot $p_{tji} \in C^r$ in all the RBs that it has chosen to transmit in, where $r$ denotes the length of the pilot sequence. The received pilot signal at the $q$th BS in the $t$th RB, denoted by $Y_{tq} \in C^{N \times r}$, is

$$Y_{tq} = \sum_{j=1}^{Q} \sum_{i=1}^{M} g_{tji} h_{tji} p_{tji} + N_{tq},$$

where $N_{tq} \in C^{N \times r}$ is the additive complex white Gaussian noise at the $q$th BS with $[N_{tq}]_{nr} \sim i.i.d. \mathcal{CN}(0, N_0)$, $r \in [N]$, and $t \in [T]$, and $N_0$ is the noise variance. Here, $h_{tji} \in C^N$ is the uplink channel vector between the $i$th user in the $j$th cell and the $q$th BS on the $t$th RB. The fading is modeled as block-fading, quasi-static and Rayleigh distributed. The uplink channel is distributed as $h_{tji} \sim i.i.d. \mathcal{CN}(0, \sigma^2_j I_N)$, $\forall t \in [T]$, $i \in [M]$, and $j \in [Q]$, where $\sigma^2_j$ is the fading variance, and $\beta^2_{tji}$ is the path loss coefficient between the $i$th user in the $j$th cell and the $q$th BS.

In the data phase, the received data signal at the $q$th BS in the $t$th RB is denoted by $y_{tq} \in C^N$, and is given by

$$y_{tq} = \sum_{j=1}^{Q} \sum_{i=1}^{M} g_{tji} h_{tji} x_{tji} + n_{tq},$$

where $x_{tji}$ is a data symbol with $E[x_{tji}] = 0$ and $E[|x_{tji}|^2] = p_{tji}$, i.e., with transmit power $p_{tji}$, and $n_{tq} \in C^N$ is the complex additive white Gaussian noise at the BS, with $[n_{tq}]_n \sim \mathcal{CN}(0, N_0)$, $\forall n \in [N]$ and $t \in [T]$.

1) SIC-based Decoding: In this work, the decoding of a packet is abstracted into an signal to interference plus noise ratio (SINR) threshold model. Here, if the SINR of a packet in a given RB in any decoding iteration exceeds a threshold $\gamma_{th}$, then the packet can be decoded correctly [6]. [11].

We now describe the performance evaluation of IRSA via the SINR threshold model. In each cell, the BS computes channel estimates and the SINRs of all users in all RBs. If it finds a user with SINR $\geq \gamma_{th}$ in some RB, it marks that user’s packet as decoded, and performs SIC from all RBs in which the same user has transmitted a replica. This process of estimation and decoding is carried out iteratively. Decoding stops when no more users are decoded in two successive iterations. The throughput is calculated as the number of correctly decoded packets divided by the number of RBs.

2) Power Control: To ensure fairness among users within each cell, we implement a power control policy. Each user performs path loss inversion with respect to the BS in its own cell [12]. That is, the $i$th user in the $j$th cell transmits its symbol $x_{tji}$ at a power $p_{tji}$, i.e., $E[|x_{tji}|^2] = p_{tji}$, according to $p_{tji} = P/\beta^2_{tji}$, where $P$ is a design parameter. The same power control policy is used in the pilot phase where the transmit power of the $i$th user in the $j$th cell is $p_{tji}^0 = P/\beta^2_{tji}$, and $P \geq P^0$ is a design parameter, with $\|P_{tji}^0\| = \tau p_{tji}^0$. This ensures a uniform SNR at the BS across all users, with the pilot SNR being $P\sigma^2_j/N_0$ and the data SNR being $P\sigma^2_j/N_0$. This ensures the power disparity between cell edge users and users located near the BS is reduced, thus ensuring fairness [12].

III. CHANNEL ESTIMATION

Channel estimation is performed based on the received pilot signal in each cell. The signals and the channel estimates are indexed by the decoding iteration $k$, since they are recomputed in every iteration. We denote the set of users in the $j$th cell who have not yet been decoded up to the $k$th decoding iteration by $S_{kj}$. For some $m \in S_{kj}$, denote the set of users in the $j$th cell who have not yet been decoded up to the $k$th decoding iteration by $S_{kj}$. For some $m \in S_{kj}$, let $S_{kj} = S_{kj} \setminus \{m\}$, with $S_{kj} = [M]$. Let the set of all cell indices be denoted by $Q = \{1, 2, \ldots, Q\}$, and let $Q^t = Q \setminus \{q\}$. The received pilot signal at the $q$th BS in the $t$th RB in the $k$th decoding iteration is given by

$$y_{tq}^{pk} = \sum_{i \in S_{kj}} g_{tqi} h_{tji} p_{tji}^k + \sum_{j \in Q^t} \sum_{i \in S_{ij}} g_{tji} h_{tji}^k p_{tji}^k + N_{tq}^{pk}$$

where the first term contains signals from users within the $q$th cell who have not yet been decoded up to the $k$th decoding iteration, i.e., $\forall i \in S_{kj}$. The second term contains signals from all users outside the $q$th cell, i.e., from every $i \in S_{kj}, \forall j \in Q^t$. We note that there is no coordination among BSs, and thus, all the users outside the $q$th cell do not get decoded by the $q$th BS, and they permanently interfere with the decoding of users in other cells, across all the decoding iterations.

Let $G_{tq} = \{t \in S_{1q}, g_{tq} = 1\}$ denote the set of users within the $q$th cell who have transmitted in the $t$th RB, with $M_{tq} = |G_{tq}|$. We denote the set of users in the $q$th cell who have transmitted on the $t$th RB but have not yet been decoded up to the $k$th decoding iteration by $M_{tq}^{sk} = G_{tq} \cap S_{kq}$, with $M_{tq}^{sk} = \{t \in S_{1q}, g_{tq} = 1, \forall i \in S_{kj} \}$
coefficients between the users within the $j$th cell and the $q$th BS, with $\hat{H}_{tq}^q \triangleq [\hat{H}_{tq}^q]_{M_q^k \times 1}$ and $\hat{H}_{tq}^{qk} \triangleq [\hat{H}_{tq}^{qk}]_{1 \times 1}, \forall j \in Q^q$. Let $P_j^q \triangleq [P_{j1}^q, P_{j2}^q, \ldots, P_{jM_q^k}^q]$ contain the pilots of all users within the $j$th cell, with $P_{tq}^{qk} \triangleq [P_{tq}^{qk}]_{M_q^k \times 1}$ and $P_{tj}^q \triangleq [P_{tj}^q]_{1 \times M_q^k}, \forall j \in Q^q$. Let $B_{tq}^{qk} \triangleq \sigma_q^2 \text{diag}(\hat{q}_{11}^q, \hat{q}_{22}^q, \ldots, \hat{q}_{M_q^k, M_q^k}^q)$ contain the path loss coefficients between the users within the $j$th cell and the $q$th BS, with $B_{tq}^q \triangleq [B_{tq}^q]_{M_q^k \times 1}$ and $B_{tj}^q \triangleq [B_{tj}^q]_{1 \times M_q^k}, \forall j \in Q^q$. Let $\hat{B}_{tq}^{qk} \triangleq [\hat{B}_{tq}^{qk}]_{M_q^k \times M_q^k} \text{ and } \hat{B}_{tj}^q \triangleq [\hat{B}_{tj}^q]_{1 \times M_q^k}, \forall j \in Q^q$. Let $\Phi = [\Phi_{j1}, \Phi_{j2}, \ldots, \Phi_{jM_q^k}]$ denote the pilots all users within the $j$th cell, with $\hat{\Phi}_{tq}^{qk} \triangleq [\hat{\Phi}_{tq}^{qk}]_{M_q^k \times 1}$ and $\hat{\Phi}_{tj}^q \triangleq [\hat{\Phi}_{tj}^q]_{1 \times M_q^k}, \forall j \in Q^q$.

Here, the received pilot signal from (3) can be written as

$$Y_{tq}^{qk} = \hat{H}_{tq}^{qk} P_{tq} H^* + \sum \hat{H}_{tq}^{qk} B_{tq}^{qk} + N_{tq} = \hat{H}_{tq}^{qk} P_{tq} H^* + N_{tq},$$

where $\hat{H}_{tq}^{qk} \triangleq [\hat{H}_{tq}^{qk}, \hat{H}_{tq}^{q1}, \hat{H}_{tq}^{q2}, \hat{H}_{tq}^{q3}, \ldots, \hat{H}_{tq}^{qM_q^k}] \in \mathbb{C}^{N \times M_q^k},$ with $\hat{M}_q^k \triangleq M_q^k + \sum_{j \in Q^q} M_j, \text{ and } P_{tq}^{qk} \triangleq [P_{tq}^{qk}, P_{tq}^{q1}, P_{tq}^{q2}, \ldots, P_{tq}^{qM_q^k}] \in \mathbb{C}^{M_q^k \times M_q^k}.$

**Theorem 1.** The minimum mean squared error (MMSE) channel estimate $\hat{H}_{tq}^{qk}$ of $H_{tq}^{qk}$ in the $q$th BS in the $k$th decoding iteration can be calculated as

$$\hat{H}_{tq}^{qk} = Y_{tq}^{qk} P_{tq}^{qk} \bar{P}_{tq}^{qk} (\hat{B}_{tq}^{qk} H_{tq}^{qk} + N_{tq} \bar{P}_{tq}^{qk})^{-1}.$$

Further, the estimation error $\hat{h}_{tji}^q = \hat{h}_{tji}^q - h_{tji}^q$ is distributed as $h_{tji}^q \sim \mathcal{CN}(0, \delta_{tji}^q I_N),$ where $\delta_{tji}^q$ is calculated as

$$\delta_{tji}^q = \frac{\tau P_{tj} \sigma_q^2 \sigma_h^2}{\left( N_0 + \sum_{n \in S_k} \tau P_{tj} \gamma_n^q \sigma_h^2 \right) + \sum_{n \in S_q} \tau P_{tj} \sigma_q^2 \sigma_h^2}.$$
that arises due to estimation errors, intra-cell interference, and
Remark 3: Proof.
that does not scale with the number of antennas
Thus it is vital to account for both while analyzing the
the throughput of MC IRSA via Monte Carlo simulations and
We now present simple and interpretable expressions for the
Theorem 3. As the number of antennas $N$ gets large, the SINR
where $\Sigma_{S_{tqm}}$ is the desired signal, $\text{IntC}_{tqm}$ represents
and IntC$^k_{tqm}$ represents the coherent interference. These can be evaluated as

\[ \epsilon_{tqm}^k = \left( N_0 ||c_{tqm}^k||^2 + \sum_{n \in S_{tqm}} |p_n^H c_{tqm}^k|^2 g_{tqm}\beta_n^q \sigma_n^2 \right) \]

\[ \text{Sig}_{tqm}^k = N p_{qmn} g_{tqm}(\epsilon_{tqm}^k)^2 \]

\[ \text{IntC}_{tqm}^k = N \left( \sum_{n \in S_{tqm}} |p_n^H c_{tqm}^k|^2 p_{qmn} g_{tqm}\beta_n^q \sigma_n^2 \right) \]

\[ \text{IntC}_{tqm}^k = N \left( \sum_{n \in S_{tqm}} |p_n^H c_{tqm}^k|^2 |p_n^H g_{tqm}\beta_n^q \sigma_n^2| \right) \]

Proof. See Appendix [3].

Remark 3: IntC$^k_{tqm}$ represents the non-coherent interference that arises due to estimation errors, intra-cell interference, and inter-cell interference. IntC$^k_{tqm}$ is the coherent interference that arises due to intra-cell PC and inter-cell PC. The former does not scale with the number of antennas $N$, whereas the latter scales linearly with $N$. Both inter-cell PC and inter-cell interference degrade the performance of the system [9], and thus it is vital to account for both while analyzing the performance of IRSA.

V. NUMERICAL RESULTS

In this section, the derived SINR analysis is used to evaluate the throughput of MC IRSA via Monte Carlo simulations and provide insights into the impact of various system parameters on the performance of the system. In each simulation, we generate independent realizations of the user locations, the access pattern matrix, and the channels. The throughput in each run is calculated as described in Sec. [11], and the effective system throughput is calculated by averaging over the runs. We consider a set of $Q = 9$ square cells, stacked in a $3 \times 3$ grid, and report the performance of the center cell [5].

Each cell has $M$ users spread uniformly at random across an area of $250 \times 250$ m$^2$, with the BS at the center [10].

The results in this section are for $T = 50$ RBs, $N_s = 10^3$, Monte Carlo runs, $\sigma_n^2 = 1$, SINR threshold $\gamma_{th} = 10$. The number of users contending for the $T$ RBs in each cell is computed based on the load $L$ as $M = [LT]$. The path loss is calculated as $\beta_j$ (dB) = $-37.6 \log_{10} (d_j^{10}/10m)$, where $d_j$ is the distance of the $i$th user in the $j$th cell from the $q$th BS [10]. The pilot sequences are chosen as the columns of the $\tau \times \tau$ discrete Fourier transform matrix normalized to have column norm $\sqrt{\tau P_r}$. The soliton distribution [7] with $d_{\max} = 8$ maximum repetitions is used to generate the repetition factor $d_j$, for the $i$th user in the $j$th cell, whose access vector is formed by uniformly randomly choosing $d_j$ RBs from $T$ RBs without replacement [3]. The access pattern matrix is formed by stacking the access vectors of all the users. The power level is set to $P = P_r = 10$ dBm [10] and $N_0$ is chosen such that

1 Due to path loss inversion, the area of the cell does not significantly affect the throughput, but affects the area spectral efficiency, which we do not analyze here.

2 The soliton distribution has been shown to achieve 96% of the throughput that can be achieved with the optimal repetition distribution [6].
In performance.

The threshold, and consequently higher $\gamma_{\text{true}}$ with MMSE combining reduces as we decrease $N$. This holds true with $\gamma_{\text{th}} = 6$ also, which corresponds to a lower SINR threshold, and consequently higher $T_C$. To summarize, at high $L$, there is a high degree of inter-cell interference which SC processing does not account for, resulting in a substantial drop in performance.

Fig. 3 studies the impact of the pilot length $\tau$. The performance of SC IRSA at all $L$ is optimal (note that the throughput is upper bounded by $L$) for $\tau > 10$. In MC IRSA, nearly optimal throughputs are achieved for $L = 1, 2, 3$ at $\tau = 10, 30, 40$, respectively. The throughput for $L = 4$ does not improve much with $\tau$. At high $L$, the impact of inter-cell interference is severe, as expected. Increasing $\tau$ implies that each cell has a higher number of orthogonal pilots, and hence can help in reducing intra-cell PC, but the system is still impacted by inter-cell PC and inter-cell interference. Thus, we see that MC IRSA requires significantly higher $\tau$ (at least $4-5\times$) to overcome inter-cell PC and inter-cell interference to achieve the same performance as that of SC IRSA.

In Fig. 4, we study the effect of $N$ for $L = 1, 2, 3, 4$, with SNR $= 10, -5$ dB. Nearly optimal throughputs can be achieved with $N = 8, 16, 32, 64$ for SNR $= 10$ dB, and with $N = 64$ for SNR $= -5$ dB. The system performance improves because of the array gain and higher interference suppression ability at high $N$. This aids in reducing not only intra-cell interference, but also inter-cell interference. However, as discussed in Remark 3, the SINRs of the users have a coherent interference component that scales with $N$. Thus, while an increase in $N$ helps reducing intra-cell interference and inter-cell interference, and improves the system performance, it does not reduce intra-cell PC and inter-cell PC. Similar observations about $N$ can be made where we study the impact of SNR in Fig. 5. At very low SNR, the system is noise limited, and increasing $N$ does not help increase the throughput, which is at zero. For $N = 16$, the throughput is always zero and nearly zero for $L = 4$ and $L = 3$, respectively. Optimal throughputs are obtained at higher SNRs for $N = 32$ and 64. Since boosting transmit powers of the users scales both the signal and interference components equally, the SINR does not increase, and therefore the system performance saturates with SNR. To summarize, increasing $\tau$, $N$, and the SNR can judiciously help reduce the impact of intra-cell PC and inter-cell PC, as well as intra-cell interference and inter-cell interference.

VI. CONCLUSIONS

This paper studied the effect of MC interference, namely inter-cell PC and inter-cell interference, on the performance of IRSA. The users across cells perform path loss inversion with respect to their own BSs and employ a $\tau$-length pilot codebook for channel estimation. Firstly, the channel estimates were derived, accounting for path loss, MIMO fading, intra-cell PC, and intra-cell interference. The corresponding SINR of all the users were derived accounting for channel estimation errors, inter-cell PC, and inter-cell interference. It was seen that MC IRSA had a significant degradation in performance compared to SC IRSA, even resulting in up to $70\%$ loss of throughput in certain regimes. Recuperating this loss requires at least $4-5\times$ larger pilot length in MC IRSA to yield the same performance as that of SC IRSA. Increasing $\tau$, $N$, and SNR helped improve the performance of MC IRSA. These results underscore the importance of accounting for multiuser interference in analyzing IRSA in multi-cell settings. Future
work could include design of optimal pilot sequences to reduce PC and density evolution \[3\] to obtain the asymptotic throughput.

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APPENDIX A: PROOF OF THEOREM 1

The minimum mean squared error (MMSE) channel estimate $\hat{H}_{tq}^k$ of the channel matrix $H_{tq}^k$ in the $t$th RB in the $q$th decoding iteration at the $q$th BS can be calculated as

$$\hat{H}_{tq}^k = Y_{tq}^{pk} P_{tq}^{kH} \tilde{B}_{tq}^{kH} P_{tq}^{kH} \tilde{B}_{tq}^{kH} (\tilde{B}_{tq}^{kH} P_{tq}^{kH} \tilde{B}_{tq}^{kH} + N_0 I_{M_{eq}})^{-1}. \tag{9}$$

1) Channel estimation: The received signal is first vectorized as

$$y_{tq} = \text{vec}(Y_{tq}) = (P_{tq}^H \otimes I_N) h_{tq}^k + n_{tq}, \tag{10}$$

where $h_{tq}^k \triangleq \text{vec}(H_{tq}^k)$, $n_{tq} \triangleq \text{vec}(N_{tq})$, and $\otimes$ is the Kronecker product. The MMSE estimate is $\hat{h}_{tq}^k \triangleq \mathbb{E}_z[h_{tq}^k]$, where $z = y_{tq}$. The estimation error $\hat{h}_{tq}^k - h_{tq}^k$ is uncorrelated with the estimate and with $z$. The conditional statistics of a Gaussian random vector $x$ are

$$\mathbb{E}_z [x] = \mathbb{E}[x] + K_{xx}|z| (z - \mathbb{E}[x]), \tag{11}$$

$$K_{xx}|z| = K_{xx} - K_{xz} K_{zz}^{-1} K_{zx}. \tag{12}$$

Here, $K_{xx}$, $K_{xz}|z|$, and $K_{zx}$ are the unconditional covariance of $x$, the conditional covariance of $x$ conditioned on $z$, and the cross-covariance of $x$ and $z$ respectively. From (11), the MMSE estimate $\hat{h}_{tq}^k$ of the channel can be evaluated as

$$\hat{h}_{tq}^k = E[h_{tq}^k] + E[h_{tq}^k y_{tq}^H] E[y_{tq}^H y_{tq}]^{-1} (y_{tq} - E[y_{tq}]).$$

The terms in the above expression can be calculated as

$$E[h_{tq}^k y_{tq}^H] = B_{tq}^{kH} P_{tq}^{kH} \otimes I_N,$$

$$E[y_{tq}^H y_{tq}] = (P_{tq}^H B_{tq}^{kH} \otimes I_N) \otimes I_N,$$

$$\hat{h}_{tq}^k = (B_{tq}^{kH} P_{tq}^{kH} \otimes I_N) \otimes I_N,$$

and thus, the MMSE estimate $\hat{h}_{tq}^k$ of $H_{tq}^k$ is

$$\hat{h}_{tq}^k = P_{tq}^{kH} B_{tq}^{kH} + N_0 I_{M_{eq}} \otimes I_N, \tag{13}$$

$$\tilde{y}_{tq}^k = (a) P_{tq}^{kH} B_{tq}^{kH} + N_0 I_{M_{eq}} \otimes I_N, \tag{14}$$

where (a) follows from $(AB + I)^{-1}A = A(AB + I)^{-1}$. 2) Error variance: The conditional covariance of $h_{tq}^k$ is calculated conditioned on the knowledge of $z = \tilde{h}_{tq}^k$. Let $C_t = B_{tq}^{kH} P_{tq}^{kH} \otimes I_N$, and $\sigma_k^2 \triangleq [C_1^T, C_2^T, \ldots, C_T^T]$. Thus, we can evaluate

$$K_{h_{tq}^k h_{tq}^k} = E[h_{tq}^k h_{tq}^k] = E[\tilde{h}_{tq}^k \tilde{h}_{tq}^k] = \mathbb{E}_z[\tilde{h}_{tq}^k \tilde{h}_{tq}^k]. \tag{15}$$

The conditional autocorrelation follows as

$$E_z[h_{tq}^k h_{tq}^k] = K_{h_{tq}^k h_{tq}^k} (z - E[z]) + E_z[h_{tq}^k h_{tq}^k] \mathbb{E}_z[h_{tq}^k]^H = \mathbb{E}_z[\tilde{h}_{tq}^k \tilde{h}_{tq}^k] + \mathbb{E}_z[h_{tq}^k h_{tq}^k] \mathbb{E}_z[h_{tq}^k]^H.$$

The unconditional and conditional means of the estimation error are $E[\tilde{h}_{tq}^k] = E[h_{tq}^k] - h_{tq}^k = 0$ and $E_z[\tilde{h}_{tq}^k] = E_z[h_{tq}^k]$. The MMSE estimate of the error therefore simplifies as

$$K_{\tilde{h}_{tq}^k \tilde{h}_{tq}^k} = K_{h_{tq}^k h_{tq}^k} \mathbb{E}_z[h_{tq}^k h_{tq}^k] = E_z[h_{tq}^k h_{tq}^k] - h_{tq}^k h_{tq}^k = \delta_{tq}^k I_N,$$

and thus, $\delta_{tq}^k$ is also the variance of the estimation error.

APPENDIX B: PROOF OF THEOREM 2

In order to calculate the SINR, we first evaluate the power of the received signal, which is calculated conditioned on the knowledge of the channel estimates $z \triangleq \text{vec}(H_{tq}^k)$ as

$$E_z[\tilde{g}^2_{tq} \tilde{y}_{tq}^2] = E_z[\sum_{i=1}^5 T_i^2].$$

Since noise is uncorrelated with data, $E_z[T_i T_i^H] = E_z[T_i] E_z[T_i^H] = E_z[T_i T_i^H] = E_z[T_i].$
The signal powers of other users who have also transmitted in the same RB (from both in-cell and out-of-cell users). SINR can thus be evaluated as in (7) for all users. The SINR can be calculated by plugging in the channel estimates as detailed in Theorem 2.

APPENDIX C: PROOF OF THEOREM 3

As the number of antennas gets large, both $\|h_{tkm}^q\|^2$ and $[h_{tkm}^qH^c_{tkm}]^2$ converge almost surely (a.s.) to their deterministic equivalents [13]. Evaluating the deterministic equivalents as in [13] and plugging into the SINR expression in place of the original terms, we can find an approximation to the SINR in the high antenna regime. As $N$ gets large, the SINR with maximal ratio combining converges almost surely $(\rho_{tkm} \overset{a.s.}{\rightarrow} \rho_{tkm})$ to

$$\rho_{tkm}^c = \frac{\text{Sig}_{tkm}^c}{\epsilon_{tkm}^c + \text{IntNC}_{tkm}^c},$$

where $\text{Sig}_{tkm}^c$ is the desired signal, $\text{IntNC}_{tkm}^c$ represents the non-coherent interference, and $\text{Int}_{tkm}^c$ represents the coherent interference. These can be evaluated as

$$\epsilon_{tkm}^c = \left(\frac{N_0}{\rho_{tkm}^c} + \text{IntNC}_{tkm}^c\right)^2 + \text{Int}_{tkm}^c,$$

$$\text{IntNC}_{tkm}^c = \frac{\text{Int}_{tkm}^c}{\epsilon_{tkm}^c},$$

$$\text{Int}_{tkm}^c = \frac{\text{Int}_{tkm}^c}{\epsilon_{tkm}^c},$$

Here, $\epsilon_{tkm}^c$ and $\text{IntNC}_{tkm}^c$ are obtained from Theorems 1 and 2, respectively, for the three estimation schemes. The above expressions are obtained by setting $\hat{a}_{tkm}^q = h_{tkm}^q$ [10] and replacing each of the terms involving $h_{tkm}^q$ in (7) with their respective deterministic equivalents.