Purcell effect in complex photonic structures based on optical Fibonacci lattices

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Abstract. The modification of probability of spontaneous emission for a dipole emitter that placed in Fibonacci lattice-based palindrome structures was examined. Localization properties of eigenstates, that appears in photonic band gaps of palindrome structure was investigated. It was shown, that for a dipole placed in centre of palindrome structure an increase in the spontaneous emission rate will be observed and, in addition, calculated value of Purcell factor is significantly greater than for critical localized states of usual Fibonacci lattices, and close to microcavity eigenmode case.

1. Introduction

Photonic quasicrystals (PQC) have some interesting properties like fractal density of states and connected with that property light localization [1,2]. It is well known, that environment (media with spatially inhomogeneous dielectric constant) modify spontaneous emission rate of a point dipole [3]. Environment of Fibonacci-type PQC gives increasing of spontaneous emission rate if frequency and direction of a dipole emission correspond to critical localized eigenstate of structure, but effect is less expressed than in planar microcavities [4].

Photonic structures with complex geometrical properties that formed by stacking the various types of PQC can bring new aspects in light behavior in dielectric media. In the review [5] was reported about results of analysis of optical properties of complex structures, based on PQC structures (Fibonacci) and some aperiodical types structures (Thue–Morse, Rudin–Shapiro, period-doubling sequences). Some eigenstates, that appears in photonic band gap (PBG) of hybrid photonic multilayer structures may correspond to perfect transmission resonances. That fact may be useful in design of non-reciprocal optical devices [6,7].

The localization behavior of these, symmetry-induced eigenstates can be different from critical localized states, and has nature close to microcavity eigenmode. The aim of present paper is to investigate, how environment of Fibonacci palindrome structures make change spontaneous emission rate for a dipole, and, particularly contribution of PBG states to effect by using S-quantization formalism.
2. Results and discussion
Optical Fibonacci lattice is a sequence of layers of two types: a large one with thickness $L$, and a small one with thickness $S$, and with refractive indexes $n_L$ and $n_S$ respectively. Layers parameters are satisfied to relation ($\tau$ – the golden ratio)

$$\frac{L}{S} = \frac{n_S}{n_L} = \tau \approx 1.618.$$ (1)

Fibonacci structure of order $F_i$ is constructed using the recurrent rule $F_{i+1} = \{F_{i-1}, F_i\}$ with initial values $F_0 = L$ and $F_1 = S$. Spatial scale of structure ($L$ and $S$) sets characteristic frequency

$$\omega_0 = \frac{\pi c}{(n_S S + L)}. \quad \text{(2)}$$

![Figure 1](image1.png)

**Figure 1.** (color online) Transmission spectrum of $F_{p6}$ palindrome structure (solid curve) and $F_{p8}$ structure (dotted curve). Arrows indicate two eigenmodes in PBG with frequencies $\omega_1=0.743\omega_0$ and $\omega_2=1.261\omega_0$.

Now consider the $F_{p6}$ palindrome structure based on $F_6$ lattice. The structure was constructed by stacking of two mirror-mapped ($\overline{F_6}$ and $\overline{F_6}$) optical lattices as shown in figure 2 (a). As you can see, in the center of $F_{p6}$ palindrome structure appears $\lambda/2$ defect, which gives enhancing of EM field localization (like in microcavities) in structure. The transmission spectrum of $F_{p6}$ and $F_{p8}$ structures, obtained by transfer matrix method shown in figure 1. Spectrum looks like the case of regular Fibonacci structure, except of presence of two peaks in PBG’s, that corresponds to states with frequencies $\omega_1=0.743\omega_0$ and $\omega_2=1.261\omega_0$. These peaks are having place in $F_{p8}$ spectrum too, but expressed less. This fact indicates the presence of that states in structures of higher order.
The states in PBG are appears only when parts of structure are composing in mirror-mapping style (for example, if compose the $F^p_6$ lattice with itself, as a result will be only modified spectrum of states of the higher order lattice). In case of stacking lattices of odd order, due to edge layers of lattice has $n_L$ refractive index there is no way to get $\lambda/2$ cavity in the centre, so there are no states in PBG.

S-quantization formalism - rigorous self-consistent procedure, that equating eigenvalues of scattering matrix of the system to unity to calculate the directional dependence of spontaneous emission rate [8,9]. Procedure of S-quantization method is based on analysis of changing of spatial eigenmode profile in layered structure $\tilde{\varepsilon}^{(1,2)}$ in relation to uniform media $\varepsilon^{(1,2)}$. For TE polarization eigenmodes of structure $\tilde{\varepsilon}^{(1)}$ (symmetric) and $\tilde{\varepsilon}^{(2)}$ (antisymmetric) relate to modes in uniform media by coefficients $Y^{(1,2)}$

$$\tilde{\varepsilon}^{(1,2)} = \begin{pmatrix} 0 & \tilde{\varepsilon}^{(1,2)}_y & 0 \\ 0 & Y^{(1,2)} \varepsilon^{(1,2)}_y & 0 \end{pmatrix}.$$  \hspace{1cm} (3)

In the case of TE polarization, for the dipole oriented along $y$-axis (emitter is co-directed with modes electric field), directional Purcell factor (Purcell factor) is

$$F^{(TE)}_\theta = \sum_{i=1,2} |Y^{(i)}|^2.$$  \hspace{1cm} (4)

Figure 2 (b) shows distribution of electric field amplitude in $Fp_6$ corresponds to symmetric modes $\tilde{\varepsilon}^{(1)}$ with frequencies $\omega_1 = 0.743\omega_0$ and $\omega_2 = 1.261\omega_0$. As it might be seen, amplitude of field grows exponential to the centre of structure. That type of field distribution usually inherent in microcavities. Field is localized in center cavity of media, which means for a dipole placed in center layer should be observed the enhancing of spontaneous emission rate.

Figure 3 shows dependency of the directional Purcell factor $F^{(TE)}_\theta$ on the emission angle and frequency for $Fp_6$ palindrome structure, when emitter is positioned at the centre of the structure. It can be seen that when frequency and angle corresponds to the considering modes, there is maximum value of Purcell factor (396). Contribution of PBG modes in picture of effect is higher than contribution of critical localized states, due to higher localization of electric field.
Figure 3. (color online) Dependency of the directional Purcell factor $F_{\theta}^{(TE)}$ on the emission angle $\theta$ and frequency for $F_{p6}$ palindrome structure; emitter is positioned at the centre of the structure.

Figure 4. (color online) Dependency of the directional Purcell factor $F_{\theta}^{(TE)}$ on the emission angle $\theta$ and frequency for $F_{p8}$ palindrome structure; emitter is positioned at the centre of the structure.

When the order of palindrome structure is increasing, in spectrum of eigenmodes appears more and more critical localized states (figure 1), but frequency of considered states almost the same. These facts correlate with results of calculation of directional Purcell factor $F_{\theta}^{(TE)}$ frequency for $F_{p6}$ palindrome structure, when emitter is positioned at the centre of the structure (figure 4). As you can see, dispersion of considered PBG states is similar to case of $F_{p6}$ palindrome structure, and maximum value of Purcell factor again falls on frequencies and angles, that corresponds to PBG states and reaches to $4.6 \times 10^5$. This value is significantly greater than values that can be achieved in usual Fibonacci lattices. For clarity, maximum of Purcell factor value $F_{max}^{(TE)}$ was built as a function of length for a set of planar photonic structures eigenstates (figure 5). Due to strong (exponential) localization of PBG modes, behavior of dependency $F_{max}^{(TE)} (D/S)$ in palindrome structures is closer to planar microcavities eigenmode scenario, and approximately like exponent function of normalized length. When the case of usual Fibonacci lattices dependency is closer to behavior of Bragg reflectors edge state (as power law of normalized length), due to less marked field localization. The considered symmetry-induced states, that appears in PBG’s of palindrome structures are characterized by high localization, and their maximum of electric field are isolated from field other critical localized states, that would be useful in design of some new optical devices.
3. Conclusions
Calculation of spontaneous emission rate for emitter placed in Fibonacci lattice-based palindrome structures using $S$-quantization formalism was carried out. Was studied how directional Purcell factor for TE polarization depend on palindrome structures order when the dipole is co-directed with electric field of eigenmode. It was shown, that couple of states, that appears in palindrome structures, behavior is close to microcavity eigenstate, namely described by strong localization and gives great contribution in Purcell effect in photonic structure.

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References
[1] W. Gellermann, M. Kohmoto, B. Sutherland, P.C. Taylor, Phys. Rev. Lett. 72 633 (1994).
[2] M.A. Kaliteevski, R.A. Abram, S. Brand, V.V. Nikolaev, Opt. Spect. 91 109-118 (2001).
[3] E. M. Purcell, Phys. Rev. 69, 681 (1946).
[4] K.M. Morozov, K.A. Ivanov, A.R. Gubaidullin, M.A. Kaliteevski, Opt. Spect. 122, No. 2, 235–242 (2017).
[5] Enrique Macià, Rep. Prog. Phys. 75 036502 (2012).
[6] S V Zhukovsky, Phys. Rev. A, 81, 053808 (2010).
[7] R. W. Peng, X. Q. Huang, F. Qiu, Mu Wang, A. Hu, S. S. Jiang & M. Mazzer, Appl. Phys. Lett., 80, 3063-3065 (2002).
[8] M.A. Kaliteevski, V.A. Mazlin, K.A. Ivanov, A.R. Gubaidullin, Opt. Spect. 119, 832-837 (2015).
[9] M.A. Kaliteevski, A.R. Gubaidullin, K.A. Ivanov, V.A. Mazlin, Opt. Spect. 121, 71-81 (2016).