TIMING EXCESS RETURNS
A CROSS-UNIVERSE APPROACH TO ALPHA

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Abstract. We present a simple model that uses time series momentum in
order to construct strategies that systematically outperform their benchmark.
The simplicity of our model is elegant. We only require a benchmark time
series and several related investable indices, not requiring regression or other
models to estimate our parameters.

We find that our one size fits all approach delivers significant outperformance
in both equity and bond markets while meeting the ex-ante risk requirements,
nearly doubling yearly returns vs. the MSCI World and Bloomberg Barclays
Euro Aggregate Corporate Bond benchmarks in a long-only backtest. We then
combine both approaches into an absolute return strategy by benchmarking
vs. the Eonia Total Return Index and find significant outperformance at a
sharpe ratio of 1.8.

Furthermore, we demonstrate that our model delivers a benefit versus a static
portfolio with fixed mean weights, showing that timing of excess return mo-
mentum has a sizeable benefit vs. static allocations. This also applies to the
passively investable equity factors, where we outperform a static factor expo-
sure portfolio with statistical significance.

Also, we show that our model delivers an alpha after deducting transaction
costs.

We wish to thank Frank Leyers, without whom this paper would not have been written.
CONTENTS

1. Introduction .................................................. 3
2. Methodology .................................................. 4
3. Universes ................................................... 5
4. Results ..................................................... 9
5. Conclusion .................................................. 19
6. Appendix ................................................... 20
References ..................................................... 23
1. Introduction

1.1. State of the research. Time series momentum is a long studied effect finding time stability of excess returns across a wide range of asset classes [15]. The most common application is to equity factors, relying on the timing of excess returns associated with the latter in order to construct an optimal portfolio. There is a wide range of models and research on factor momentum, most show a significant excess return with respect to the benchmark. Some models rely on time series information only [6, 10, 14], while others include macro data [11, 13]. There has also been research linking excess factor returns to industry excess returns [1].

1.2. What we do differently. There is a fundamental aspect separating our approach from the others: Simplicity. We do not rely on anything but the time series of the indices across a look-back period $T$, a certain rebalancing frequency and a target tracking error for our portfolio, denoted by $\sigma$ in the course of this paper. The simplicity has a profound advantage: Without having to delve into the specifics of the asset class and without having to use any assumption of return and volatility models, we can compute an optimal allocation solely based on time series data. Furthermore, we demonstrate that our risk targets are well met.
2. Methodology

In order to construct our portfolios, we use the following simple approach: Take several indices $X_i(t), 1 \leq i \leq n$ and a benchmark $X_0(t)$. Consider the historic excess total return of the indices:

$$R_i := \frac{X_i}{X_0}$$

and the associated return:

$$\alpha_i := \frac{dR_i}{dt}$$

Next, take an arbitrary time period $T$ and define the average excess return:

$$\Delta^T_i(t) := \frac{1}{T} \int_{t-T}^{t} \alpha_i(s) ds$$

and the covariance coefficients:

$$\Omega^T_{ij}(t) := \frac{1}{T} \int_{t-T}^{t} \alpha_i(s) \alpha_j(s) ds - \Delta^T_i(t) \Delta^T_j(t)$$

For simplicity, define:

$$\Omega^T_{0i} = \Omega^T_{i0} = 0$$

$$\Delta^T_0 = 0$$

Remark 1. Consider a portfolio of weights $P^T = (x_0^T, ..., x_n^T)$. For given boundaries $M^u_i \geq 0, M^l_i \leq 0, i \geq 1$ denote the set of admissible weights $S_M$ by

$$S_{M, \sigma} := \left\{ (x_i) \mid \sum_{i=0}^{n} x_i = 1, M^l_i \leq x_i \leq M^u_i \forall i \geq 1, \sum_{i,j} x_i \Omega^T_{ij} x_j \leq \sigma^2 \right\}$$

Then there exists a unique portfolio $P^T$ satisfying:

$$P^T \cdot \Delta^T = \max_{x \in S_{M, \sigma}} \{ x \cdot \Delta^T \}$$

$$=: m_T$$

$$(P^T)^T \cdot \Omega^T \cdot P^T = \min_{x \in S_{M, \sigma}} \{ x | x \cdot \Delta^T = m_T \}$$

This portfolio maximises the historic return over past time period $T$ while having minimal tracking error below $\sigma$ and satisfying the allocation restrictions $M$. 
3. Universes

As our model is quite general and only requires the notion of a benchmark and several related indizes, we show that it can be applied to two very different asset classes to generate excess returns in comparison to the benchmark, irrespective of their different nature: Equity and bonds. We implement above algorithm in Python using the cvxpy library and consider weights $M = \{(M^t_l = 0, M^t_u = 1)\}$ and $T = 91$ days for all backtests.

3.1. **Equity.** The first application of our timing model, equity, has a natural set of indizes related to the standard market benchmark: Factor indizes. As factors intrinsically carry an excess return, one can ask themselves whether timing their allocation provides a benefit with regards to the benchmark.

For these backtests, we consider the MSCI Benchmarks Europe, World and USA and add to each benchmark five factor indizes: Minimum Volatility, Momentum, Size, Value and Quality. As we are interested in the effect of rebalancing on the portfolio, we conduct two backtests for each benchmark, one with a rebalancing frequency of seven days, and one with 28 days between rebalancing dates. As we base our backtest on daily data, the backtest period starts the first year all indizes have daily data available. We fix the tracking error $\sigma = 4\%$, which is a tracking error typically seen in actively managed portfolios and define our sets of benchmarks and indizes as follows.

3.1.1. **World.** The World subuniverse consists of the MSCI World Benchmark and the following indizes (we also list their Bloomberg tickers):

1. Benchmark: MSCI World / NDDUWI Index
2. Min. Vol.: MSCI World Minimum Volatility Index / M00IWO$O$ Index
3. Momentum: MSCI World Momentum Index / M1WOMOM Index
4. Size: MSCI World Size Tilt Index / M1WOMEQ Index
5. Value: MSCI World Enhanced Value Index / M1WOEV Index
6. Quality: MSCI World Sector Neutral Quality Index / M1WONQ Index

The World universe has the same backtest period as the US universe.

3.1.2. **US.** The US subuniverse consists of the MSCI USA Benchmark and the following indizes:

1. Benchmark: MSCI USA / NDDUUS Index
2. Min. Vol.: MSCI USA Minimum Volatility Index / M1USMVOL Index
3. Momentum: MSCI USA Momentum Index / M1US000$S$ Index
4. Size: MSCI USA Size Tilt Index / M1CXBRG Index
5. Value: MSCI USA Enhanced Value Index / M1USEV Index
6. Quality: MSCI USA Sector Neutral Quality Index / M1USSNQ Index

The backtest in the US universe starts on 2000-01-07 and ends on 2020-01-14 due to the daily data for all factors being available from 1999 onwards.

3.1.3. **Europe.** The Europe subuniverse consists of the MSCI Europe Benchmark and the following indizes:

1. Benchmark: MSCI Europe / MSDEE15N Index
2. Min. Vol.: MSCI Europe Minimum Volatility Index / MAEUVOE Index
3. Momentum: MSCI Europe Momentum Index / MAEUMMT Index
(4) Size: MSCI Europe Size Tilt Index / M7EUIMEW Index
(5) Value: MSCI Europe Enhanced Value Index / M7EUEV Index
(6) Quality: MSCI Europe Sector Neutral Quality Index / M7ESNQ Index

The Europe universe has a backtest running from 2003-01-07 till 2020-01-14, as unlike the other two universes the daily data is available from 2002 onwards.
3.2. **Bonds.** The second universe of assets we look at is bonds. Unlike equity, there is no readily investable factor universe for bonds, albeit there having been recent research about bond factors [12][4]. For timing to work, though, in theory, we only need indices that differ predictively from the benchmark. In order to provide access to systematic performance deviation from the benchmark, we select from a pool of systematic indices that have a broad range of performance drivers: inflation, securitization, credit risk and interest rate risk. We choose a rebalancing frequency of 28 days and a tracking error of 2%.

3.2.1. **European Bonds.** The Europe universe consists of the Bloomberg Barclays Euro Aggregate Corporate Bond Index Benchmark and the following indices:

1. **Benchmark:** Bloomberg Barclays Euro Aggregate Corporate Bond Index / LECPTREU Index
2. **Covered Bonds:** Bloomberg Barclays Securitized - Covered Bond Index / LSC1TREU Index
3. **Government Bonds:** Barclays EuroAgg Treasury Index / LEATTREU Index
4. **Inflation Linked Bonds:** Bloomberg Barclays Euro Govt Inflation-Linked Bond All Maturities Index / BEIG1T Index
5. **Long Duration:** Bloomberg Barclays Euro Government 30 Year Term Index / BCEX1T Index
6. **Short Duration:** Bloomberg Barclays Euro-Aggregate Government 1-3 Year Index / LEG1TREU Index
7. **High Yield:** Bloomberg Barclays Pan-European High Yield / LP01TREU Index

As with the equity backtests, our backtests starts Jan. 7th on the first year having daily data for all indices, which is 2005-01-07.
3.3. **Absolute Return.** As a fun exercise demonstrating the simplicity of our model, we combine above indizes into an absolute return strategy: We again use $T = 91$ days, rebalance monthly and set our target tracking error to $\sigma = 2\%$ to the Eonia TR benchmark, use a both long only approach as well as a backtest with $M = \{(M^u_0 = 0, M^u_i = 1), (M^l_i = -1, M^u_i = 1)|i > 0\}$, i.e. a long-short approach, and define our universe as follows:

3.3.1. **Absolute Return.** The Europe universe consists of the Bloomberg Barclays Euro Aggregate Corporate Bond Index Benchmark and the following indizes:

1. Benchmark: Eonia Total Return Index / DBDCONIA Index
2. World: MSCI World / NDDUWI Index
3. Min. Vol.: MSCI World Minimum Volatility Index / M00IWO$O$ Index
4. Momentum: MSCI World Momentum Index / M1WOMOM Index
5. Size: MSCI World Size Tilt Index / M1WOMEQ Index
6. Value: MSCI World Enhanced Value Index / M1WOEV Index
7. Quality: MSCI World Sector Neutral Quality Index / M1WONQ Index
8. Corp. Bonds: Bloomberg Barclays Euro Aggregate Corporate Bond Index / LECPTREU Index
9. Covered Bonds: Bloomberg Barclays Securitized - Covered Bond Index / LSC1TREU Index
10. Government Bonds: Barclays EuroAgg Treasury Index / LEATTREU Index
11. Inflation Linked Bonds: Bloomberg Barclays Euro Govt Inflation-Linked Bond All Maturities Index / BEIG1T Index
12. Long Duration: Bloomberg Barclays Euro Government 30 Year Term Index / BCEX1T Index
13. Short Duration: Bloomberg Barclays Euro-Aggregate Government 1-3 Year Index / LEG1TREU Index
14. High Yield: Bloomberg Barclays Pan-European High Yield / LP01TREU Index

The start date for the backtest is the same as for the bonds one due to sharing all indizes, i.e. 2005-01-07.
4. Results

We report our data based on daily arithmetic returns and annualize the figures. We compute the following statistics:

- Volatility (VOL) (mean daily volatility \( \cdot \sqrt{365.25} \))
- Return, annualized (i.e. mean daily return \( \cdot 365.25 \))
- Sharpe Ratio (SR), the ratio of return to volatility
- Alpha (mean daily difference between portfolio and benchmark, annualized)
- Tracking Error (TE), the standard deviation of alpha
- Information Ratio (IR): alpha divided by tracking error
- Maximum relative drawdown (MRDD): The maximum relative drawdown of the portfolio with respect to the benchmark. If the MRDD is -15%, the portfolio has a negative alpha of -15% from the highest point to the lowest relative to the benchmark.
- TER: Our total expense ratio per annum. It is an estimate based on an index bid/ask spread of 5 bp.

4.1. Equity universes.

4.1.1. World. Both strategies examined have a significant information ratio with respect to the benchmark, having a p-value of 0.000028. Also, the observed volatility is similar to the benchmark, albeit with a 68% higher annual return. The tracking error metrics are comparable to those of the factors, whereas the MRDD is strikingly lower. The target tracking error of 4% is reasonably attained with 4.5% realized tracking error.

| Benchmark | 28 Day Rebalancing | 7 Day Rebalancing | Min. Vol. | Momentum | Quality | Size | Value |
|-----------|------------------|------------------|----------|----------|---------|------|-------|
| Return    | 5.6%             | 9.5%             | 9.4%     | 7.1%     | 7.7%    | 7.0% | 7.3%  | 8.2%  |
| VOL       | 16.3%            | 15.6%            | 15.6%    | 12.8%    | 17.1%   | 16.5%| 15.2% | 16.4% |
| SR        | 0.34             | 0.61             | 0.60     | 0.56     | 0.45    | 0.43 | 0.48  | 0.50  |

| Alpha     | —                | 4.0%             | 4.0%     | 1.1%     | 1.6%    | 1.1% | 1.3%  | 1.9%  |
| TE        | —                | 4.5%             | 4.4%     | 6.8%     | 7.5%    | 3.1% | 5.3%  | 5.7%  |
| IR        | —                | \textbf{0.90}    | \textbf{0.90} | 0.17 | 0.21   | 0.34 | 0.24  | 0.34  |
| MRDD      | —                | -8.5%            | -7.1%    | -17.6%   | -20.7%  | -10.9%| -14.6%| -25.3%|

Examining the mean allocations per index, one can see that based on the mean allocation, one would expect a mean excess return of around 1.4% based on average allocation statistics, implying the 2.6% additional excess return is an active contribution from timing. We also tested the performance of the portfolio vs. an equal weighted portfolio, not finding any significant difference between the performance
vs. mean or vs. equal weighted portfolios, hence we omit the latter. Further examined is the strategy alpha vs. a static strategy possessing the same mean allocation ("Mean"). The outperformance of the timing strategy vs. the mean allocation is significant with a p-value of 0.011304.

| Index       | 28 Day | 7 Day |
|-------------|--------|-------|
| Mean Weights| Benchmark | 6.5% | 6.4% |
|             | Min. Vol. | 16.0% | 15.3% |
|             | Momentum | 25.9% | 25.4% |
|             | Quality  | 15.6% | 16.0% |
|             | Size     | 10.0% | 10.6% |
|             | Value    | 26.0% | 26.1% |
| Allocation  | 1.4%    | 1.4%  |
| Active      | 2.6%    | 2.6%  |
| TER         | 0.57%   | 1.15% |
| Turnover    | 1147%   | 2299% |
| Alpha vs. Mean | 1.9% | 1.8% |
| TE vs. Mean | 3.7%   | 3.8%  |
| IR vs. Mean | 0.51   | 0.53  |

With a TER of 0.57%, our 28-day strategy is still well viable after trading costs whereas the 7-day strategy would perform significantly worse.
4.1.2. US. Same as in the world universe, the outperformance of the strategy with respect to the benchmark is significant with a p-value of 0.004145. Again, the volatility is similar to that of the benchmark with a 38% higher annual return, the further picture is similar as well: A tracking error that well matches the 4% target and a lower MRDD than the factors themselves.

| Benchmark        | 28 Day Rebalancing | 7 Day Rebalancing | Min. Vol. | Momentum | Quality | Size | Value |
|------------------|---------------------|-------------------|-----------|-----------|---------|------|-------|
| Return           | 7.0% 9.6%           | 10.1%             | 8.1%      | 9.4%      | 7.7%    | 9.8% | 9.8%  |
| Vol.             | 20.6% 20.2%         | 20.0%             | 17.3%     | 21.0%     | 19.8%   | 22.4%| 21.5% |
| SR               | 0.34 0.47           | 0.51              | 0.47      | 0.45      | 0.39    | 0.44 | 0.46  |
| Alpha            | — 2.5%              | 3.1%              | 0.8%      | 1.7%      | 0.5%    | 2.0% | 2.0%  |
| TE               | — 4.3%              | 4.2%              | 6.4%      | 7.4%      | 3.2%    | 6.4% | 5.0%  |
| IR               | — **0.59**          | **0.75**          | 0.13      | 0.23      | 0.14    | 0.31 | 0.40  |
| MRDD             | — -9.4%             | -7.1%             | -15.2%    | -23.0%    | -10.5%  | -18.9%| -18.2%|

The other characteristics of the US universe match the pattern found in world: An allocation contribution to excess return that is lower than the observes excess return, leaving an active contribution of roughly equal magnitude. However, the outperformance vs. a static portfolio of same mean weight is not statistically significant. Interestingly, after costs, the 28-day and 7-day strategy have approximately the same alpha.
| Mean Weights       | Benchmark | 28 Day | 7 Day |
|--------------------|-----------|--------|-------|
| Min. Vol.          | 15.2%     | 15.3%  |       |
| Momentum           | 22.9%     | 21.1%  |       |
| Quality            | 13.6%     | 14.2%  |       |
| Size               | 20.6%     | 20.4%  |       |
| Value              | 21.0%     | 23.2%  |       |

| Allocation       | 1.4% | 1.4% |
| Active           | 1.1% | 1.7% |

| TER               | 0.59% | 1.26% |
| Turnover          | 1172% | 2501% |

| Alpha vs. Mean   | 0.7% | 1.3% |
| TE vs. Mean      | 3.7% | 3.7% |
| IR vs. Mean      | 0.19 | 0.35 |
4.1.3. Europe. The pattern repeats. We have a significant outperformance (p-value of 0.021692) with respect to the benchmark, at similar volatility. The tracking error is comparable to the factors again, with only Quality having a lower MRDD than the strategy.

|              | Benchmark | 28 Day Rebalancing | 7 Day Rebalancing | Min. Vol. | Momentum | Quality | Size | Value |
|--------------|-----------|---------------------|-------------------|-----------|----------|---------|------|-------|
| Return       | 8.2%      | 10.3%               | 10.0%             | 9.0%      | 11.3%    | 9.9%    | 9.8% | 9.4%  |
| Vol.         | 18.1%     | 17.5%               | 17.2%             | 13.7%     | 17.5%    | 17.4%   | 18.2%| 19.6% |
| SR           | 0.45      | 0.59                | 0.58              | 0.66      | 0.64     | 0.57    | 0.54 | 0.48  |
| Alpha        | —         | 2.4%                | 2.1%              | 0.6%      | 2.2%     | 1.2%    | 1.2% | 0.8%  |
| TE           | —         | 4.3%                | 4.2%              | 6.1%      | 7.3%     | 3.5%    | 5.5% | 4.2%  |
| IR           | —         | 0.55                | 0.49              | 0.1       | 0.31     | 0.34    | 0.21 | 0.2   |
| MRDD         | —         | -9.4%               | -8.3%             | -16.5%    | -21.1%   | -8.0%   | -20.2%| -22.4%|

For the European universe, there still is an active component to excess return. However, it is not statistically significant. We find that the alpha ex costs of the 7-day strategy is nearly half that of the 28-day strategy.
### Timing Excess Returns: A Cross-Universe Approach to Alpha

| Mean Weights        | Benchmark | 28 Day | 7 Day |
|---------------------|-----------|--------|-------|
| Min. Vol.           | 16.7%     | 17.4%  |       |
| Momentum            | 24.3%     | 24.1%  |       |
| Quality             | 15.7%     | 15.0%  |       |
| Size                | 17.1%     | 16.3%  |       |
| Value               | 21.8%     | 22.9%  |       |
| Allocation          | 1.2%      | 1.2%   |       |
| Active              | 1.2%      | 0.9%   |       |
| TER                 | 0.53%     | 1.19%  |       |
| Turnover            | 1257%     | 2781%  |       |
| Alpha vs. Mean      | 0.5%      | 0.2%   |       |
| TE vs. Mean         | 3.7%      | 3.6%   |       |
| IR vs. Mean         | 0.14      | 0.06   |       |
### 4.2. Bonds.

The result for the bond universe is stunning. The strategy has an alpha of equal magnitude to that of the benchmark (p-value < 0.00001) and a small maximal relative drawdown of 5.3%, the target tracking error of 2.0% is almost reached.

|                      | Benchmark | 28 Day Rebalancing | Covered Bonds | Government Inflation Bonds | Long Duration | Short Duration | High Yield |
|----------------------|-----------|---------------------|---------------|-----------------------------|---------------|----------------|------------|
| **Return**           | 3.7%      | 7.1%                | 3.7%          | 4.1%                        | 2.7%          | 6.9%           | 2.0%       | 7.5%       |
| **Vol.**             | 2.5%      | 3.4%                | 2.1%          | 3.8%                        | 5.0%          | 8.5%           | 1.1%       | 5.0%       |
| **SR**               | 1.47      | 2.09                | 1.74          | 1.09                        | 0.55          | 0.80           | 1.83       | 1.50       |
| **Alpha**            | —         | 3.4%                | -1.0%         | 0.3%                        | -0.7%         | 2.3%           | -1.2%      | 2.8%       |
| **TE**               | —         | 2.4%                | 1.3%          | 2.7%                        | 4.1%          | 7.1%           | 2.2%       | 4.9%       |
| **MRDD**             | —         | 1.96                | -0.76         | 0.11                        | -0.17         | 0.32           | -0.57      | 0.56%      |

If one looks at the mean allocation, there is a 40% High Yield quota. However, the risk profile of the strategy seems nowhere near that of High Yield, its volatility and tracking error are markedly lower. The MRDD is drastically smaller - at only 5.3% compared to the 37.8% drawdown of the High Yield index compared to the benchmark during the Global Financial Crisis. The active component of the return is sizeable and with a cost of 0.48% p.a. the strategy would still yield an alpha of 2.9%. We have a significant outperformance vs. a static portfolio allocation at a p-value of 0.0005.
### Mean Weights

| Benchmark   | 13.4% |
|-------------|-------|
| Covered Bonds | 10.5% |
| Government Bonds | 6.1% |
| Inflation Linked Bonds | 8.0% |
| Long Duration   | 14.3% |
| Short Duration  | 7.7%  |
| High Yield      | 40.0% |

| indicator  | value  |
|------------|--------|
| TER        | 0.48%  |
| Turnover   | 965%   |
| Allocation | 1.2%   |
| Active     | 2.2%   |

| comparison | result |
|------------|--------|
| Alpha vs. Mean | 1.6%   |
| TE vs. Mean   | 2.4%   |
| IR vs. Mean   | 0.51   |
4.3. **Absolute Return.** The strategy works once more. Both long short and long only have a significant information ratio (p-value < 0.00001). The volatility of the long only strategy is much closer to the target volatility of 2%, the difference might be due to a large absolute position exposure of the long short strategy.

|                | Benchmark | Long Only | Long Short |
|----------------|-----------|-----------|------------|
| Return         | 0.9%      | 5.6%      | 6.9%       |
| VOL            | 0.1%      | 2.5%      | 3.2%       |
| SR             | 7.70      | 2.21      | 2.13       |
| Alpha          | —         | 4.7%      | 6.0%       |
| TE             | —         | 2.5%      | 3.2%       |
| IR             | —         | **1.85**  | **1.85**   |
| MRDD           | —         | -8.7%     | -7.2%      |

The mean weights are interesting with a mean negative equity exposure for the long short strategy and a significant high yield allocation in both strategies. Despite such a significant high yield exposure, our maximal drawdown, which peaks during the global financial crisis, is well under control for an absolute return strategy. However, the high TER and poorer tracking error render the long short strategy inattractive in comparison to the long only approach. The outperformance vs. a static mean weight allocation is significant in the long short approach (p-value < 0.00001), whereas it is not significant for the long only one.
## Mean Weights

| Index                      | Long Only | Long Short |
|----------------------------|-----------|------------|
| Benchmark                  | 17.7%     | 60.6%      |
| Corp. Bonds                | 8.2%      | 2.2%       |
| Covered Bonds              | 10.9%     | 14.1%      |
| Government Bonds           | 3.6%      | 9.3%       |
| Inflation Linked Bonds     | 3.4%      | -2.2%      |
| Long Duration              | 3.9%      | -3.0%      |
| Short Duration             | 11.0%     | -14.9%     |
| High Yield                 | 32.0%     | 32.2%      |
| World                      | 0.1%      | -21.1%     |
| Min. Vol.                  | 3.7%      | 1.6%       |
| Momentum                   | 2.1%      | 9.0%       |
| Quality                    | 0.4%      | 8.5%       |
| Size                       | 1.0%      | -0.5%      |
| Value                      | 2.2%      | 4.4%       |

## Other Metrics

| Metric          | Long Only | Long Short |
|-----------------|-----------|------------|
| TER             | 0.52%     | 2.72%      |
| Turnover        | 1047%     | 5338%      |
| Allocation      | 1.7%      | 3.9%       |
| Active          | 3.0%      | 2.1%       |
| Alpha vs. Mean  | 0.8%      | 6.0%       |
| TE vs. Mean     | 2.2%      | 3.2%       |
| IR vs. Mean     | 0.36      | **1.88**   |
5. Conclusion

To summarize, this paper contributes to the debate whether timing in Portfolio Management decisions are possible or not. In contrast to most of the existing studies we use a simple model that uses time series momentum in order to construct strategies that systematically outperform their benchmark. The one size fits all model works in both the equity and bond markets where it was possible to achieve statistically significant alpha in some cases, and an alpha in all. If you combine stocks and bonds and measure them against a cash benchmark (Absolute Return), the results are even better. The approach presented in this paper has not been discussed in this form until now. It turns out that price momentum alone can lead to significant results.

In contrast to other researchers, we rely only on time series of the indices of equities and bonds across a look-back period, a certain rebalancing frequency and a target tracking error for our portfolio. We were able to demonstrate that the risk targets were met, and, at the same time, significant results could be achieved compared to the selected benchmarks. This fact is due in particular to the rotation model presented which natively includes correlation effects. Although this approach is burdened by high transaction costs, there are still results that can largely be classified as significant. This applies in particular to the strategies with global stocks, bonds and absolute return.

As to our stock results, one might expect that the idiosyncratic risk of smaller universes contributes to noise and hence lower returns due to more volatility. The signal for the benchmark MSCI World was able to achieve better results than would have been the case in isolation for the USA and Europe, which would be in agreement with that hypothesis.

An impressive result was achieved in the area of bonds, which can be classified as statistically highly significant. This is because of a high High Yield quota. However, the risk profile of the strategy seems nowhere near that of High Yield, its volatility and tracking error are markedly lower. The same is true for Absolute Return. High Yield Bonds are a core investment in the long only as well as in the long short strategy.

Lastly, across all asset universes, we find significant benefits of timing vs. static allocations. In fact, in no backtest, we found negative excess returns of our timing strategy vs. the static allocations. As our model is easily implementable and can be realized using only passively investable products such as ETFs, this enables a new set of “semi-passive” allocations that actively time their exposure with a general model to generate excess returns at a fixed target volatility.
6. Appendix

As a sample implementation, we provide our code in the appendix. The first snippet is an optimiser using cvxpy \[5\] to compute the optimal exposure relative to the benchmark.
Algorithm 1 Excess Return Optimization

```python
import numpy, scipy
import cvxpy as cp
from pandas import Series, import, scipy, import, cvxpy as cp
import Series, import, datetime, import, factor_optimiser
import numpy, import, math

# Sample optimiser using cvxpy. We need our excess return vector and the
covariance matrix of the excess returns as input, as well as the
tracking error and any bounds.
def compute_optimal_portfolio(excess_return_vector, covariance_matrix,
tracking_error, lower_bounds=None, upper_bounds=None,
set_upper_bounds_to_one=True, max_leverage=1, allow_short=False,
no_benchmark=False):
    # We set dim to our number of indices
    dim = len(excess_return_vector)

    # Define a function to compute the historic return
    def historic_return(x):
        return -1 * numpy.dot(excess_return_vector, x).item()

    # And use cvxpy to configure our bounds depending on the arguments
    x = cp.Variable(dim)
    constraints = []

    for i in range(0, dim):
        lower = 0
        upper = 0
        if (not lower_bounds is None) and (len(lower_bounds) == dim):
            lower = lower_bounds[i]
        if (not lower_bounds is None) and (len(upper_bounds) == dim):
            upper = upper_bounds[i]

        elif set_upper_bounds_to_one:
            upper = 1

        if (upper != lower):
            e = [float(j==i) for j in range(0, dim)]
            constraints.append(e @ x >= lower)
            constraints.append(e @ x <= upper)

    es = [1 for j in range(0, dim)]

    # If we do not want benchmark exposure, we set the sum to one.
    if not no_benchmark:
        constraints.append(es @ x == 1)
        constraints.append(cp.quad_form(x, covariance_matrix) <=
                        tracking_error ** 2)
    else:
        constraints.append(es @ x == 1)

    # First optimization calculates the maximally possible return
    opt = cp.Problem(cp.Minimize(-1 * excess_return_vector @ x), constraints)
    opt.solve()
    max_return = -1 * opt.value

    # The second optimization finds the portfolio with minimum
    volatility, if there are several with the same maximum return
    constraints.append(x @ excess_return_vector == max_return)
    opt2 = cp.Problem(cp.Minimize(cp.quad_form(x, covariance_matrix)),
                      constraints)
    opt2.solve()
    return x.value
```

The next snippet then takes that optimal allocation and adds the benchmark exposure.
Algo 2 Optimization With Benchmark

```python
# We use pandas to provide time series data as a Series
from pandas import Series
import datetime
import factor_orthos
import numpy
import math

def compute_return_data(benchmark_time_series, factor_time_series, date, window_length_in_days):
    # Generic function to compute excess return time series and excess return covariance and returns an index for each name
    return alphas, covariance, name_map

# We call the compute_optimal_portfolio function from above code and subtract the sum of exposures from 1 to get an allocation that includes the benchmark

def compute_optimal_portfolio_with_benchmark(benchmark_time_series, factor_time_series, date, window_length_in_days, tracking_error, no_benchmark=False):
    # Get the data
    alphas, covariance, name_map = compute_return_data(benchmark_time_series, factor_time_series, date, window_length_in_days)
    # Optimise
    optimal_portfolio = factor_orthos.compute_optimal_portfolio(alphas, covariance, tracking_error, no_benchmark=no_benchmark)
    allocation = {}
    sum = 0
    for j in factor_time_series:
        allocation[j] = optimal_portfolio[name_map[j]]
        sum += optimal_portfolio[name_map[j]]
    # And compute benchmark exposure
    allocation['Benchmark'] = 1 - sum
    return allocation
```
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