Hybrid Maximum Likelihood Based Linear Modulation Classification with Multiple Sensors via Generalized EM Algorithm

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Abstract—In this paper, we consider the problem of automatic modulation classification with multiple sensors in the presence of unknown time offset, phase offset and received signal amplitude. We develop a novel hybrid maximum likelihood (HML) classification scheme based on a generalized expectation maximization (GEM) algorithm. GEM is capable of finding ML estimates numerically that are extremely hard to obtain otherwise. Assuming a good initialization technique is available for GEM, we show that the classification performance can be greatly improved with multiple sensors compared to that with a single sensor, especially when the signal-to-noise ratio (SNR) is low. We further demonstrate the superior performance of our approach when simulated annealing (SA) with uniform as well as nonuniform grids is employed for the initialization of GEM in low SNR regions. The proposed GEM based approach employs only a small number of samples (in the order of hundreds) at a given sensor node to perform both time and phase synchronization, signal power estimation, followed by modulation classification. We provide simulation results to show the efficiency and effectiveness of the proposed algorithm.

Index Terms—Modulation classification, hybrid maximum likelihood, generalized expectation maximization algorithm, data fusion, multiple sensors

I. INTRODUCTION

The problem of automatic modulation classification (AMC) is concerned with determining the modulation type of a received noisy signal. With the recent advances in software defined radios, AMC is becoming an integral part of various cognitive radio applications that use adaptive modulation techniques, e.g., adaptive cognitive radios for space communications [1]. Widely used AMC methods can be divided into two general classes: i) likelihood-based (LB) and ii) feature-based (FB) methods. An extensive overview of these methods is given in [2]. The LB method is based on the likelihood function of the received signal, where the decision is made using a Bayesian hypothesis testing framework. A classifier obtained by the LB method is optimal in the Bayesian sense, i.e., it minimizes the probability of classification error. Computation of the likelihood function is challenging when there are unknown parameters. Various LB based AMC techniques have been proposed in the literature depending on how the unknown parameters are treated. These techniques are known as generalized likelihood ratio test (GLRT), average likelihood ratio test (ALRT) and hybrid likelihood ratio test (HLRT) [3]. Despite its computational appeal/ lower computational complexity, the traditional GLRT [4] has been shown to provide poor performance in classifying nested constellation schemes such as QAM and PSK [5]. In ALRT [6], which is a fully Bayesian approach, the conditional likelihood function (LF) is averaged over the unknown signal parameters by assuming certain prior distributions, thereby converting the problem into a simple hypothesis testing problem. In the HLRT approach [3], the LF is marginalized over the unknown constellation symbols and then the resulting average LF is maximized over the remaining parameters which are treated as deterministic unknowns. A variant of HLRT is quasi HLRT (qHLRT) [6], [7], where the unknown signal parameters are replaced by their moments based estimates.

A large number of AMC techniques developed so far make the common assumption that perfect timing information is available at the receiver. This assumption is unrealistic for practical AMC scenarios due to a number of reasons. First, AMC usually needs to be performed in a noncooperative environment. Therefore, there is no training sequence available at the receiver for accurate time synchronization. The receiver has to employ blind time synchronization techniques which result in residual errors, namely time offsets in the received signal [8]. Second, AMC is generally based on batch techniques where only a finite number of samples are available for classification. Both blind synchronization and modulation classification need to be carried out using these limited number of samples. In other words, the receiver does not have the luxury to obtain massive amounts of data for perfect time synchronization in practice. This fact should be taken into account in the design of an AMC algorithm. Although time offsets are unavoidable in most AMC scenarios, there have been only a few research works that have addressed this issue [9]–[13]. Among these works, MFSK modulations are considered in [9],[10], whereas PSK and QAM modulations are the focus in this paper. In one of the earlier papers on likelihood based AMC [11], the authors consider only PSK modulations and derive approximate forms of the likelihood functions that are obtained by marginalizing out both phase and time offsets in addition to unknown constellation symbols.
(i.e. ALRT) under low signal-to-noise ratio (SNR) assumptions. It has been shown in [11] that the general ALRT does not have a closed-form analytical expression. In [12], the authors consider only QAM modulations. They adopt the square timing recovery technique for blind synchronization followed by a cumulant based hierarchical modulation classifier. Aside from its limitation to QAM modulations, the method proposed in [12] requires a large number of samples for accurate blind synchronization and it can only classify a limited number of QAM modulations for which appropriate cumulants need to be selected. More recently, the authors in [13] consider linear modulations (amplitude-phase modulations). They propose moment based estimators to estimate the unknown time offset along with unknown phase offset and SNR in their qHLRT approach. They collect a number of samples for estimation and then an additional number of samples for modulation classification. The shortcomings of this approach are two-fold. First, moment based estimators do not always provide meaningful estimates, especially when the number of samples for estimation is small. For example in [13], 10000 samples are used to obtain acceptable classification performance. Second, moment based estimators do not necessarily maximize the likelihood function which results in unavoidable sub-optimality in the classification step. The reason behind the use of moment based estimators for AMC is the computational complexity associated with maximum likelihood (ML) estimators as pointed out in [13].

In this paper, we focus on the problem of classifying linear modulations in the presence of unknown time offset, phase offset and signal amplitude with multiple sensors. This work is based on our initial work for asynchronous AMC with a single sensor [14]. In [14], we developed a hybrid maximum likelihood (HML) based approach to AMC in the presence of unknown time offset, phase offset and signal amplitude with a single sensor. To find unknown parameters via maximum likelihood estimation, a computationally efficient numerical algorithm was proposed based on generalized expectation maximization (GEM). The GEM algorithm provides a tractable procedure to obtain ML estimates which are extremely hard to obtain otherwise. In the current work, we aim to improve the performance of HML based AMC in the low SNR region by employing multiple sensors. We assume that the observations collected at multiple sensors are available at a fusion center to perform classification. With multiple sensors, we observe that the performance of the GEM based approach for joint classification is more susceptible to parameter initialization of the GEM algorithm compared to that with the single sensor case. With a reasonable initialization technique, it is shown that the GEM algorithm with multiple sensors provide promising results irrespective of the nature of the other relevant parameters. We first evaluate the performance of the GEM algorithm with multiple sensors assuming that an initialization technique is given. For any initialization technique, we can express the initial values as true values of the unknown parameters plus some error. The error term determines how good the initialization technique is. Under this assumption, we provide simulation results to show the performance gain achievable with multiple sensors in the presence of unknown time offset, phase offset and channel gain, and also illustrate the impact of the GEM initialization on the overall performance. It is seen that, when the initial values are not significantly far away from the true values, the proposed GEM algorithm provides comparable performance to the Clairvoyant classifier (which assumes that the unknown parameters are exactly known) with multiple sensors. Next, we consider simulated annealing (SA) based stochastic initialization technique for GEM initialization which is shown to provide good results in the low SNR region with multiple sensors.

Our approach is applicable to all QAM and PSK modulations especially in the low SNR region. Moreover, the proposed scheme employs only a small number of samples (in the order of hundreds) at a given node, as opposed to thousands as in [13], to perform both time synchronization and modulation classification. More importantly, since the proposed approach maximizes the original likelihood function, it is expected to perform better than the qHLRT approach. The proposed approach also enables maximum a posteriori (MAP) decoding of the unknown constellation symbol sequence as a by-product of the GEM algorithm. Our simulation results show that the proposed approach provides excellent classification performance with, for example, only $N = 100$ samples per node.

The rest of the paper is organized as follows. In Section II we introduce the system model and formulate the HML based modulation classification problem with multiple sensors. The details of our proposed GEM based classifier are presented in Section III and subsections therein. We provide numerical results to depict the performance of the proposed approach in Section IV. Finally, concluding remarks along with avenues for future work are provided in Section V.

### II. Problem Formulation

We consider $L$ radio receivers observing a linearly modulated communication signal that undergoes block fading. The received baseband signal at the $l$-th radio can be expressed as

$$y_l(t) = a_l e^{j\theta_l} \sum_n I_n g(t - nT - \varepsilon_l T) + w_l(t), \quad 0 \leq t \leq T_0$$

for $l = 1, \cdots, L$, where $T_0$ is the observation interval, $T$ is the symbol duration, $g(t)$ is the transmitted pulse, $I_n$ is the $n^{th}$ complex constellation of the transmitted symbol, $w_l(t)$ is the additive complex zero-mean white Gaussian noise process with two-sided power spectral density (PSD) $N_0/2$, $a_l > 0$ is the channel gain, $\theta_l \in [-\pi, \pi]$ is the channel phase, and $\varepsilon_l T$ is the residual time offset at the receiver. We assume that the estimation of the pulse shape $g(\cdot)$, the symbol duration $T$ and the carrier frequency has been accomplished at each receiver. These are commonly made assumptions in the modulation classification literature [2], [6]. [7], [12], [13], [15], [16], and these estimates can be obtained using the techniques

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1HML is also referred to as the hybrid likelihood ratio test (HLRT) in some modulation classification literature [2], [6]. [7]. [13]. [15]. [16].

2The phase term $\theta_l$ subsumes both the channel phase and the residual phase offset at the receiver.
Let $H_i$ denote the hypothesis associated with the modulation $i$. In a Bayesian setting, the optimal classifier which minimizes the probability of classification error is the MAP classifier. We assume that each modulation is equally likely, i.e., each has the same prior probability. In this case, the optimal classifier takes the form of a ML classifier. As mentioned in Section I, we focus on the HML approach [2], where the LF is marginalized over the unknown random constellation symbols $I_n$ and then maximized over the remaining unknown (nuisance) parameters. Let $u \triangleq [a_1, \ldots, a_L, \theta_1, \ldots, \theta_L, \varepsilon_1, \ldots, \varepsilon_L]$ denote the deterministic unknown parameter vector. We define $s_l(t)$ as

$$s_l(t) \triangleq a_le^{\theta_l} \sum_n I_n g(t - nT - \varepsilon_l T), \quad 0 \leq t \leq T_0$$

for $l = 1, \ldots, L$. Let $y(t)$ denote a vector representation of $y_i(t)$. We also define $I \triangleq [I_0, \ldots, I_{N-1}]^T$ and $y \triangleq [y^{(1)}, \ldots, y^{(N)}]^T$ where $(\cdot)^T$ denotes vector/matrix transpose. It should be clear from the context when $T$ represents symbol duration or transpose. It can be shown that the conditional LF of the noisy received signals is given by [4] as

$$p_i(y|u, I) \propto \exp \left\{ \frac{2}{N_0} \sum_{l=1}^L \int_0^{T_0} \Re \{y_i(t)s^*_l(t)\} dt - \frac{1}{N_0} \sum_{l=1}^L \int_0^{T_0} |s_l(t)|^2 dt \right\}$$

where $p_i(\cdot) \triangleq p(\cdot|H_i)$ and $\Re(\cdot)$ denotes the real part of a complex number. The observation interval $T_0$ is based on designer’s choice, so we assume that $T_0$ is a multiple of $T$ and define $N \triangleq T_0/T$. The symbol pulse $g(t)$ is a finite-length pulse (e.g., symmetrically truncated root-raised cosine (RRC) pulse) with duration $T_p$. We assume that $T_0 \gg T_p$, i.e., the observation interval is much larger than the duration of the transmit pulse. This assumption is well justified in all practical modulation classification applications as it is typical to observe at least $\sim 100$ symbols before making a decision. Under these assumptions, we get the following two expressions [8]

$$\int_0^{T_0} \Re \{y_i(t)s^*_l(t)\} dt = a_l \Re \left\{ e^{-j\theta_l} \sum_{n=0}^{N-1} I_n^* \int_0^{T_0} y_i(t)g^*(t - nT - \varepsilon_l T) dt \right\},$$

$$\int_0^{T_0} |s_l(t)|^2 dt \approx E_g a_l^2 \sum_{n=0}^{N-1} |I_n|^2,$$

where $E_g$ is the energy of the transmit pulse

$$E_g \triangleq \int_{-\infty}^{\infty} g^2(t) dt.$$

The approximation in [8] is based on the assumption that $T_0 \gg T_p$. In other words, the contribution of the symbols in the beginning and at the end of the observation interval to the total energy of the received signal will be negligible for $T_0 \gg T_p$.

With this approximation, the LF can be written as

$$p_i(y|u, I) \propto \exp \left\{ \frac{2}{N_0} \sum_{l=1}^L \sum_{n=0}^{N-1} a_l \Re \left\{ I_n^*e^{-j\theta_l} \int_0^{T_0} y_i(t)g^*(t - nT - \varepsilon_l T) dt \right\} \right\} \cdot \exp \left\{ -\frac{E_g}{N_0} \sum_{n=0}^{N-1} |I_n|^2 \sum_{l=1}^L a_l \right\}.$$  

Note that (7) now denotes approximate proportionality due to [8]. Now we turn our attention to our original problem where we need to marginalize the distribution over the constellation symbols, i.e., we need to compute

$$p_i(y|u) = \sum_{I^{(i)}} p_i(y|u, I^{(i)}) P(I^{(i)})$$

Finally, the HML modulation classifier is

$$\hat{i} = \arg \max_i \Lambda_i(\hat{u}_i).$$

Due to the marginalization over the constellation symbols, the resulting ML estimation problem in (10) is not tractable. This is because it is a three dimensional non-convex optimization problem and there is no closed-form solution. Therefore, finding theMLE $\hat{u}_i$ from (10) would normally require an exhaustive search which is computationally expensive and is impractical in real AMC applications. In order to solve this problem, we propose an efficient algorithm which is based on the Generalized Expectation Maximization (GEM) algorithm [17].

III. THE EM ALGORITHM

The Expectation Maximization (EM) algorithm is an iterative method which enables the computation of ML estimates. It is especially well suited to problems where ML estimation is intractable due to the presence of hidden (unobserved) data. For the problem addressed in this paper, the actual sequence of transmitted constellation symbols $I$ can be treated as hidden data. We can formally describe the EM algorithm for our problem in [8] as follows [17]. Let us define the so-called
\[ \Lambda_i(u) = \sum_{n=0}^{N-1} \ln \left( \sum_{k=1}^{M} \exp \left\{ \frac{2}{N_0} \sum_{l=1}^{L} a_l \Re \left( I_n^k e^{-j\theta_l} \int_0^{T_0} y(t)g^*(t - nT - \varepsilon_l T)dt \right) \right\} \right) - \frac{E_g}{N_0} \sum_{l=1}^{L} a_l^2 |I_n^k|^2 \]  

(9)

**Completely data** \( x = [y^T, I^T]^T \). Starting from an initial estimate \( \tilde{u}_i^{(0)} \) under the hypothesis \( H_i \), the EM algorithm performs the following two steps: the expectation step (E-step) and the maximization step (M-step).

**E-step:** \( Q(u|\tilde{u}_i^{(r)}) = E \left\{ \ln p_i(x|u, y, \tilde{u}_i^{(r)}) \right\} \), \hspace{1cm} (12)

**M-step:** \( \tilde{u}_i^{(r+1)} = \arg \max_u Q(u|\tilde{u}_i^{(r)}) \). \hspace{1cm} (13)

Given the fact that the unknown parameter vector \( u \) is independent of the transmitted constellation symbols \( I \), the E-step in (12) reduces to

\[ Q(u|\tilde{u}_i^{(r)}) = \sum_{l=1}^{L} \ln p_i(y|I, u)P_l \left\{ I|y, \tilde{u}_i^{(r)} \right\}, \hspace{1cm} (14) \]

where \( \ln p_i(y|I, u) \) is as given in (7). Suppose we have \( \tilde{u}_i^{(r)} \) at the end of the \( r \)-th iteration. We define \( y_{n,l}^{(r)} \) as

\[ y_{n,l}^{(r)} \triangleq y(nT + \varepsilon_l^{(r)} T) - \int_0^{T_0} y(t)g^*(t - nT - \varepsilon_l^{(r)} T)dt. \hspace{1cm} (15) \]

Let \( \alpha_n^{m,(r)} \triangleq P_l \left\{ I_n = I_n^m|y_{n,1}^{(r)}, \ldots, y_{n,L}^{(r)}, \tilde{u}_i^{(r)} \right\} \), \( m = 1, \ldots, M_i \), denote the \textit{a posteriori} probability of the \( n \)-th unknown constellation symbol which can be calculated as

\[ \alpha_n^{m,(r)} \triangleq \frac{P_l \left\{ I_n = I_n^m|y_{n,1}^{(r)}, \ldots, y_{n,L}^{(r)}, \tilde{u}_i^{(r)} \right\}}{P_l \left\{ y_{n,1}^{(r)}, \ldots, y_{n,L}^{(r)}|\tilde{u}_i^{(r)} \right\}} = \frac{P_l \left\{ I_n = I_n^m|y_{n,1}^{(r)}, \ldots, y_{n,L}^{(r)}, \tilde{u}_i^{(r)} \right\}}{P_l \left\{ y_{n,1}^{(r)}, \ldots, y_{n,L}^{(r)}|\tilde{u}_i^{(r)} \right\}}. \hspace{1cm} (16) \]

In (a), we have used the assumption that each data symbol has the same \textit{a priori} probability, i.e., \( P_l \left\{ I_n = I_n^m|\tilde{u}_i^{(r)} \right\} = 1/M_i \), \( m = 1, \ldots, M_i \). Substituting (7) in (14) along with \( \alpha_n^{m,(r)}, Q(u|\tilde{u}_i^{(r)}) \) reduces to

\[ Q(u|\tilde{u}_i^{(r)}) = \sum_{l=1}^{L} Q_l(u|\tilde{u}_i^{(r)}) \hspace{1cm} (17) \]

where \( u_l \) contains the unknowns in \( u \) that correspond to the \( l \)-th node so that \( u_l = [a_l, \theta_l, \varepsilon_l]^T \) for \( l = 1, \ldots, L \) and

\[ Q_l(u|\tilde{u}_i^{(r)}) \] is given by

\[ Q_l(u|\tilde{u}_i^{(r)}) = \frac{2a_l}{N_0} \sum_{n=0}^{N-1} \sum_{m=1}^{M_i} \alpha_n^{m,(r)} \Re \left\{ I_n^m e^{-j\theta_l} \int_0^{T_0} y(t)g^*(t - nT - \varepsilon_l T)dt \right\} - \frac{E_g}{N_0} \sum_{l=1}^{L} a_l^2 |I_n^k|^2 \]

(18)

where \( \hat{I}_n^{(r)} \) and \( \hat{E}_l^{(r)} \) denote the posterior expectations of the \( n \)-th transmitted symbol and the average normalized signal energy, respectively, which are defined as

\[ \hat{I}_n^{(r)} \triangleq \sum_{m=1}^{M_i} \alpha_n^{m,(r)} I_n^m, \hspace{1cm} \hat{E}_l^{(r)} \triangleq \sum_{n=0}^{N-1} \sum_{m=1}^{M_i} \alpha_n^{m,(r)} |I_n^m|^2. \hspace{1cm} (19) \]

Then the maximization step in (13) at \( r \)-the iteration of the EM algorithm reduces to

\[ \tilde{u}_i^{(r+1)} = \arg \max_{u_l} Q_l(u|\tilde{u}_i^{(r)}) \hspace{1cm} (20) \]

for \( l = 1, \ldots, L \) where \( Q_l(u|\tilde{u}_i^{(r)}) \) is as given in (18).

The maximization step in (20) can be carried out in two steps:

\[ \left( \theta_l^{(r+1)}, \varepsilon_l^{(r+1)} \right) = \arg \max_{\theta_l, \varepsilon_l} \sum_{n=0}^{N-1} \Re \left\{ \hat{I}_n^{(r)} e^{-j\theta_l} \int_0^{T_0} y(t)g^*(t - nT - \varepsilon_l T)dt \right\} \hspace{1cm} (21) \]

\[ \hat{\theta}_l^{(r+1)} = \frac{1}{\hat{E}_l^{(r)}} \sum_{n=0}^{N-1} \Re \left\{ \hat{I}_n^{(r)} e^{-j\theta_l^{(r+1)}} \int_0^{T_0} y(t)g^*(t - nT - \varepsilon_l^{(r+1)} T)dt \right\} \hspace{1cm} (22) \]

for \( l = 1, \ldots, L \). At the \((r+1)\)-th iteration, the maximization step in (21) constitutes a weighted correlation (or matched filtering) based estimation of the unknown phase and time offsets. After these estimates are obtained, the estimate of the signal amplitude is computed in closed-form using (22). Even though the EM algorithm simplifies significantly the ML estimation procedure, the optimization problem in (21) still requires maximization over two dimensions which can
be computationally expensive. Thus, in the following, we consider a computationally efficient approach to overcome this issue.

A. Generalized EM

The EM algorithm is theoretically guaranteed to converge to a stationary point as long as the \( Q(u_i, \hat{u}_i^{(r)}) \) function increases at every iteration [18]. In other words, the maximization step in (13) can be replaced with an improvement step, which does not impact the convergence property of the EM algorithm. These variants of the EM algorithms are referred to as Generalized EM (GEM) algorithms [17]. Due to this theoretical result, we can replace the maximization step in (21) with the following ‘block coordinate ascent’ type procedure

\[
\hat{e}_i^{(r+1)} = \arg\max_{e_i} \sum_{n=0}^{N-1} \Re \left\{ \int_0^{T_0} y(t) e^{-j \theta_i^{(r)}} \int_0^{T_0} y(t) g^*(t - nT - e_i T) dt \right\},
\]

for \( l = 1, \ldots, L \) where \( \Re(\cdot) \) denotes the imaginary part of a complex number, \( \hat{f}_i^{(r)} \triangleq \left[ \hat{f}_0^{(r)}, \ldots, \hat{f}_{N-1}^{(r)} \right]^T \), and \( y_i^{(r+1)} \triangleq \left[ y_0^{(r+1)}, \ldots, y_{N-1}^{(r+1)} \right]^T \), in which \( y_{n,l}^{(r+1)} \) is obtained from (15), i.e., \( y_{n,l}^{(r+1)} \triangleq y_l(nT + \hat{e}_l^{(r)} T) \). Note that the above two steps are much simpler to implement than (21), since (23) requires a line search which can be carried out by methods such as the Newton-Raphson method and (24) is a closed-form expression. Even though the \( (\hat{\theta}_l^{(r+1)}, \hat{e}_l^{(r+1)}) \) pair obtained by (23)–(24) does not necessarily maximize \( Q(u_i, \hat{u}_i^{(r)}) \), the EM algorithm is guaranteed to converge to a stationary point of the original likelihood function after a sufficient number of iterations.

We should also mention that if the time offset \( e_i \) is perfectly known, the EM algorithm simplifies significantly. Under this scenario, the EM algorithm would iterate over (19), (23) and (24) which are all closed-form expressions. Numerical results for such a scenario are provided in Section IV.

B. Initialization of unknown parameters

The initialization of the EM algorithm, namely obtaining the initial estimate \( \hat{u}_i^{(0)} \), has a large impact on the stationary point the EM will converge to. A good initial point increases the likelihood that the algorithm will converge to the global maximum rather than to some local maxima. Our GEM based approach for AMC with multiple sensors is seen susceptible to convergence to a local maxima as the number of sensors increases when a good initialization technique is not available. There are a number of methods that can be used to initialize the EM algorithm. The initial estimates obtained with any initialization technique can be expressed as the true value of the parameter plus some error. Obviously, larger the error, the worse the initialization technique is. More specifically, we represent the initial values in the form of \( \hat{u}_i^{(0)} = u + \epsilon \) where \( \epsilon \) is a 3L \times 1 vector which denotes the deviation of the initialization points of unknown parameters from their true values. We numerically show the performance gain achievable by GEM with multiple sensors as \( \epsilon \) varies. This approach provides insights into how much performance gain is achievable with proposed GEM approach for AMC with multiple sensors. In the following, we consider a practical scheme for EM initialization, which provides us with good initial estimates in low SNR regions (which is the most interesting scenario).

1) EM Initialization with simulated annealing (SA): We adopt a modified SA method which is implemented over a coarse grid and over a predefined finite number of iterations. Specifically, we construct the following grid sets \( \Theta \triangleq \{-\pi, -\pi + \Delta \theta_1, -\pi + 2\Delta \theta_1, \ldots, -\pi - \Delta \theta_i\}, \xi \triangleq \{0, \Delta \xi_1, \ldots, 1 - \Delta \xi_j\}, \) and \( \Lambda \triangleq \{\Delta a_1, 2\Delta a_1, \ldots, a_i\} \), where \( a_i \) is some upper bound which is based on designer’s choice and can be selected depending on the channel characteristics for a given scenario. The increments (denoted by \( \Delta s \)) determine the resolutions of the grid sets, i.e., how coarse the grids are. Let us define \( \Omega \triangleq \Theta \times \xi \times A \). Let \( K \) denote the maximum number of iterations and \( d \) denote the predefined SA parameter. The parameters \( K \) and \( d \) are adjusted by the user. The SA algorithm is summarized in Algorithm 1. Note that, instead of iterating until convergence, a maximum number of iterations is employed for the SA algorithm. This is in part to keep the overall computational complexity low and in part due to the fact that the GEM algorithm takes care of the fine maximization step. The overall goal is to find a ‘good’ initial point for the GEM algorithm. Other methods such as moment based estimators can also be employed for initialization as long as they have low computational complexity and provide ‘good’ initial points.

**Algorithm 1** SA for GEM Initialization under \( \mathcal{H}_i \)

1. Randomly select \( \omega_1 \in \Omega \). Initialize \( \omega_F = \omega_1 \).
2. FOR \( k = 2, \ldots, K \)
3. \( T = d/\log(k) \).
4. Randomly select a neighbor of \( \omega_k \), denoted by \( \omega_N \).
5. If \( \Lambda_i(\omega_k) \leq \Lambda_i(\omega_N) \), set \( \omega_{k+1} = \omega_N \).
6. Else, \( \omega_{k+1} = \omega_N \) with probability
   \[
   \exp \left( \frac{\Lambda_i(\omega_N) - \Lambda_i(\omega_k)}{T} \right),
   \]
   \( \omega_{k+1} = \omega_k \) otherwise.
7. If \( \Lambda_i(\omega_F) \leq \Lambda_i(\omega_{k+1}) \), set \( \omega_F = \omega_{k+1} \) and continue.
8. ENDFOR
9. Set \( \hat{u}_i^{(0)} = \omega_F \).

C. GEM Summary

For clarity, we summarize the proposed GEM based asynchronous modulation classifier (MC) in Algorithm 2. After a classification decision has been made, the MAP decoding of the received symbol sequence can be easily obtained using the final \textit{a posteriori} probabilities \( a_{n,m}^{(r)} \) which have already been calculated in the GEM algorithm for modulation \( i \).
Algorithm 2 GEM Based Asynchronous MC

1: Set stopping criterion $\delta$.
2: FOR $i = 1, \ldots, S$
3: Set $r = 0$. Initialize $\hat{u}_i(0)$
4: For $n = 0, \ldots, N - 1$; $m = 1, \ldots, M_i$; compute $\alpha^{m,(r)}_{n}$ from (16).
5: For $n = 0, \ldots, N - 1$; compute $I_{n}^{(r)}$ from (19).
6: Compute $E_{n}^{(r)}$ from (19).
7: Set $r = r + 1$
8: Compute $\hat{\theta}_l^{(r+1)}$ using (22).
9: Compute $\hat{\theta}_l^{(r+1)}$ using (24).
10: Compute $\hat{\theta}_l^{(r+1)}$ using (22).
11: If $\Lambda_i(\hat{u}_i^{(r+1)}) - \Lambda_i(\hat{u}_i^{(r)}) > \delta$, go to Step 4, else set $\hat{u}_i^{(r+1)}$ and continue.
12: ENDFOR
13: Final decision $\hat{i} = \arg\max_i \Lambda_i(\hat{u}_i)$.

IV. Numerical Results

In this section, we provide numerical results to illustrate the performance gain achievable with multiple sensors using the proposed GEM based modulation classification scheme compared to that with a single sensor. We consider a scenario where $g(t)$ is a symmetrically truncated RRC pulse, i.e., $g(t) = g(-t)$, with a roll-off factor $\alpha = 0.3$ and duration 8T. Without loss of generality, we assume that $\mathbb{E}\{|I_n|^2\} = 1$ and $N_0 = 1$. The channels between the transmitter and each sensor are modeled as Rayleigh fading channels, i.e., $a_l$ is a Rayleigh distributed random variable with scale parameter $\sigma$ for $l = 1, \ldots, L$. With these assumptions, the channel signal-to-noise ratio (SNR) is $\mathbb{E}\{|a_l^2|I_n|^2\}/N_0 = 2\sigma^2$. We assume $T = 1$, $\theta_l \sim U[-\pi, \pi]$ and $\varepsilon_l \sim U[0, 1]$, for $l = 1, \ldots, L$ where $U[a, b]$ denotes uniform distribution with support $[a, b)$. The observation interval is set as $T_p = NT$. We consider a quaternary classification scenario. The modulations to be classified are 8-PSK, 8-QAM, 16-PSK, and 16-QAM. In the following figures, we assess the classification performance with respect to different aspects including the channel SNR, the true modulation format, the initial values of the unknowns used for the GEM algorithm, the number of sensors, the number of samples per node, and the impact of ignoring the time offset on the classification performance.

A. Impact of the initial values of deterministic unknowns on the GEM performance

It is known that the performance of the GEM algorithm greatly depends on the selection of initial values of the unknown parameters. When the initial values deviate significantly from the true values, there is a high probability that the estimates of GEM are trapped in local maxima especially when there is a large number of unknowns. To demonstrate the impact of the initial values of unknowns on the performance of the GEM algorithm, we take initialization points of unknown parameters as the true value of unknowns plus some error. More specifically, we consider the initial values for the unknown parameters $a_l$, $\theta_l$ and $\varepsilon_l$ can take any values uniformly in the regions $[0, a_l + \delta_a]$, $[\theta_l - \delta_\theta, \theta_l + \delta_\theta]$, and $[\varepsilon_l - \delta_\varepsilon, \varepsilon_l + \delta_\varepsilon]$, respectively, for $l = 1, \ldots, L$ where $\delta_a, \delta_\theta, \delta_\varepsilon > 0$ are the maximum errors for each unknown. These error bounds determine how close the initialization points to the true values are.

In Fig. 1, we present the probability of correct classification vs channel SNR. Given the $i$-th modulation format, the probability of correct classification is denoted by $P(H_i|H_i)$ which means that the classifier decides $H_i$ when the true hypothesis is $H_i$. We let $L = 5$, $N = 100$. Three sets of initial values are considered taking $\{\delta_a = 1, \delta_\theta = \pi/20, \delta_\varepsilon = 0.05\}$ and $\{\delta_a = 5, \delta_\theta = \pi/10, \delta_\varepsilon = 0.1\}$ and $\{\delta_a = 10, \delta_\theta = \pi/5, \delta_\varepsilon = 0.1\}$. Four subplots in Fig. 1 correspond to 4 different modulation formats as the true format. Each result is based on 500 Monte-Carlo runs. We also plot the probability of correct classification with Clairvoyant classifier which assumes that the unknown parameter vector $\mathbf{u}$ is known and the classification is done based on the marginalized LF over the distribution of constellations.

It can be seen from Fig. 1 that the GEM algorithm is capable of providing comparable performance compared to the Clairvoyant classifier when the initial values of unknowns are not considerably away from the true values irrespective of the true modulation format. It is noted that with the third set of values for error, the initial values can be considerably away from the true values of unknowns. Based on Fig. 1, when the true modulation format is either 8-PSK, or 16-PSK the GEM algorithm does not seem to depend much on the initial values even though they (initial values) considerably deviate from the actual values. However, when 8-QAM or 16-QAM is the true format, it is observed that the GEM performance seems to degrade as the initial values significantly deviate from the true values. With multiple sensors, (the number of unknowns is proportional to the number of sensors) the likelihood function shows a large number of local maxima. Thus, when the initial values are significantly far away from the actual values, the GEM estimates for unknowns can be easily trapped at local maxima leading to poor performance. From Fig. 1, we can conclude that, if a reasonably good initialization method is available, the proposed GEM algorithm for AMC is a promising technique with performance that is comparable to Clairvoyant classifier.

B. Performance of GEM with SA based initialization

Next, we investigate the performance of the GEM algorithm considering SA, stated in Algorithm 1 as the initialization technique. Using SA, we first get a rough estimate for the initial values of the unknowns using a coarse grid. The accuracy depends on the grid size. As the grid becomes finer, the complexity of the algorithm increases. For SA based initialization as stated in Algorithm 1, we set $a_{l}^{(0)} = F^{-1}_a(0.99; \sigma)$ for $l = 1, \ldots, L$, where $F^{-1}_a(\cdot; \sigma)$ is the inverse cumulative distribution function of Rayleigh distribution parameterized by $\sigma$. The grid increments for initialization ($\Delta$s) are selected such that we have 10 grid points for each unknown, i.e., $\Omega$ consists of 1000 points. We set $K = 200$ and $d = 1.6$ for the SA algorithm. Note that our SA algorithm requires only
Fig. 1. Impact of the initial values of unknowns on the GEM performance; the performance of the GEM with SA based initialization is for GEM are estimated via simulated annealing as considered in Fig. 2. different curves are for different $L$, different modulation formats and these results are not included for brevity. Thus, it appears that GEM with SA based initialization scheme is a promising technique for AMC with any given number of sensors.

While it is expected that the performance of the GEM algorithm with SA based initialization could be further improved by increasing the number of grid points, it is not desirable due to higher computational complexity at the initialization

Fig. 2. Performance of the GEM algorithm as the number of sensors varies; true format is 8-PSK, initial estimates for unknown parameters for GEM are estimated via simulated annealing as considered in Algorithm 1 $N = 100$

200 evaluations of the likelihood function for initialization as opposed to 1000 that would be required by an exhaustive grid search.

In Fig. 2 we plot the performance of the GEM classifier when the initial values are selected based on the SA algorithm assuming that the true modulation format is 8-PSK. In Fig. 3 different curves are for different $L$. It is observed that the performance of the GEM with SA based initialization is monotonically increasing in the low-to-moderate SNR region considered for all $L$. Similar behavior is observed for the other modulation formats and these results are not included for brevity. Thus, it appears that GEM with SA based initialization scheme is a promising technique for AMC with any given number of sensors.

Fig. 3. Performance of the GEM algorithm: SA based initialization with different grid sizes: $N = 100$, $L = 5$. 

While it is expected that the performance of the GEM algorithm with SA based initialization could be further improved by increasing the number of grid points, it is not desirable due to higher computational complexity at the initialization
To solve this problem to a certain extent, we created a nonuniform grid for SA based initialization, where the number of grid points along \( \theta_l \) are increased while those along \( a_l \) are reduced, so that the total number of total number of grid points in \( \Omega \) are kept the same compared to that in the uniform grid considered above. The motivation behind the use of a nonuniform grid is the observation that it is the channel phase that incorrectly estimated most of the time with a uniform grid. In Fig. 3, we plot the probability of correct classification for \( L = 5 \) when the true format is either 8-PSK or 16-PSK. For the nonuniform grid, we take 5, 20 and 10 grid points for \( a_l \), \( \theta_l \) and \( \epsilon_l \), respectively. From Fig. 3, we observe an improved performance of GEM with SA based initialization with nonuniform grid compared to a uniform grid with 16-PSK. With 8-PSK, the performance with nonuniform grid is better than that with the uniform grid only when the SNR is larger. Further, while curves are not included, the performance when the true format is either 8-QAM or 16-QAM does not seem to vary significantly with a nonuniform grid compared to a uniform grid.

Next, we investigate the effectiveness of the SA based initialization scheme for the GEM algorithm with multiple sensors by varying the number of unknowns. It is noted that, we consider three unknown parameters for each node (i.e. \( a_l \), \( \theta_l \) and \( \epsilon_l \) at the \( l \)-th node for \( l = 1, \ldots, L \)). In Figs. 4 and 5, we plot the performance as the number of unknowns varies, assuming that the true modulation formats are 16-PSK, and 16-QAM, respectively. It can be observed that, if the channel gain \( a_l \) and channel phase \( \theta_l \) at the \( l \)-th sensor are assumed to be known, (in other words, if we consider that only the time offset \( \epsilon_l \) is the unknown parameter for \( l = 1, \ldots, L \) then the GEM algorithm with SA based initialization (with a uniform grid) provides comparable performance to that with Clairvoyant classifier. In particular, if less number of parameters per node have to be estimated via ML estimation while performing classification, then GEM with SA as the initialization technique with a coarse grid provides comparable performance to the Clairvoyant classifier.

From Figs. 4 and 5, we conclude the following; (i). Given a relatively good initialization technique, GEM for HML based AMC for linear modulation classification is capable of providing promising performance as the number of sensors increases. In depth investigation of initialization schemes for GEM is beyond the scope of this paper. (ii). When SA with a coarse grid is chosen as the initialization scheme, GEM provides good performance in the low-to moderate SNR region considered in this paper. When the number of unknowns per node to be estimated during the classification process is small, GEM with SA based initialization provides performance that is closer to the Clairvoyant classifier.

In the following, we further investigate the performance of the GEM algorithm for AMC with multiple sensors with respect to several other parameters.

### C. Number of samples per node

Next, we illustrate the impact of the number of samples per node on the classification performance as the number of sensors varies. In Fig. 6, the probability of correct classification vs the number of samples is plotted as the number of samples per node, \( N_s \), varies when the true modulation format is 16-PSK and channel SNR = 5dB. In Fig. 6, GEM is performed with a given initialization scheme as considered in Section V-A with \( \delta_u = 5 \), \( \delta_\theta = \pi/10 \), and \( \delta_x = 0.1 \). Results in Fig. 6 validate the claim that a considerably small number of samples at each node is capable of providing a comparable performance based on the proposed approach as the number of sensors increases even if the SNR is low.

### D. Number of sensors

In Fig. 7, we plot the probability of correct classification vs channel SNR as the number of sensors varies. We consider that the true format is 8-PSK in Fig. 7(a) and 16-PSK in Fig. 7(b). The results are based on the GEM algorithm with initial values as considered in the first paragraph in Subsection V-A with \( \delta_u = 5 \), \( \delta_\theta = \pi/10 \), and \( \delta_x = 0.1 \). It was observed from Fig. 1 that with this set of initial values, the GEM algorithm provides comparable performance to the Clairvoyant classifier for all the modulation formats. In Fig. 7, it can be seen that
It is also interesting to see that the clairvoyant EM classifier upper bound on the performance of all modulation classifiers. With a single sensor, the clairvoyant classifier [19] serves as an initial values are selected as true value plus some error as considered in Section IV-A where \( \delta_a = 5, \delta_\theta = \pi/10, \) and \( \delta_\epsilon = 0.1 \)

does not know \( \epsilon_l \) or \( \theta_l \) for \( l = 1, \cdots, L \) and 3) EM classifier which ignores time offsets, i.e., which assumes that time offsets are zero. We use average probability of correct classification \( P_{cc} \) as the performance criterion which is defined as 

\[
P_{cc} \triangleq \frac{1}{S} \sum_{i=1}^{S} P(H_i|H_i).
\]

In Fig. 8 (a), we let \( L = 1, N = 100 \) while in Fig. 8 (b), we let \( L = 5 \) and \( N = 100 \). We consider two initialization schemes for GEM in Fig. 8. The dashed curve plots the average probability of classification when the initial values are selected as true value plus some error as considered in Section IV-A where \( \delta_a = 5, \delta_\theta = \pi/10, \delta_\epsilon = 0.1 \). The dotted curve with circle marks is for GEM with SA based initialization. It is clear from Figs. 8 (a) and 8 (b), that when time offsets are ignored, the performance of the resulting classifier is extremely poor. This result indicates the fact that residual time offsets need to be taken into account in a modulation classification application. We can see from Fig. 8 (b) that the performance of the proposed GEM based modulation classifier with a relatively good initialization scheme is almost identical to the clairvoyant EM classifier that has perfect information on \( \epsilon_l \) for \( l = 1, \cdots, L \). Further, Figs. 8 (a) and 8 (b) again verify that, with a good initialization scheme for GEM, a significant performance gain can be obtained by increasing the number of sensors compared to that with a single sensor. With SA based initialization, GEM works better with a single sensor. The clairvoyant classifier [19] serves as an upper bound on the performance of all modulation classifiers. It is also interesting to see that the clairvoyant EM classifier performs very close to this upper bound even though it knows neither the channel phase nor the channel gain. Further, as seen in Figs. 8 (a) and 8 (b), the proposed GEM algorithm (without the knowledge of channel gain, channel phase and the time offset) with a good initialization scheme also provides performance that is very close to the Clairvoyant classifier.

**E. Performance with other comparable classifiers**

In Fig. 8 we compare the proposed GEM based classifier with three other classifiers: 1) clairvoyant classifier of [19] which has perfect information on \( u \), 2) clairvoyant EM classifier which has perfect information on \( \epsilon_l \), but does not know \( \theta_l \), for \( l = 1, \cdots, L \) and 3) EM classifier which ignores time offsets, i.e., which assumes that time offsets are zero. We use average probability of correct classification \( P_{cc} \) as the performance criterion which is defined as 

\[
P_{cc} \triangleq \frac{1}{S} \sum_{i=1}^{S} P(H_i|H_i)
\]

In Fig. 6, we compare the proposed GEM based classification with multiple sensors in the presence of unknown time offset in addition to unknown phase offset and received signal amplitude. We considered a centralized fusion scheme, where multiple sensors transmit their observations to a central fusion center to perform classification. We have proposed a novel hybrid maximum likelihood (HML) approach where the unknowns are estimated using a tractable GEM algorithm. We have shown that the performance of AMC can be significantly improved as the number of sensors increases with a good initialization technique for GEM. Our proposed approach employs only a small number of samples to perform both

**V. Conclusion**

We have addressed the problem of linear modulation classification with multiple sensors in the presence of unknown time offset in addition to unknown phase offset and received signal amplitude. We considered a centralized fusion scheme, where multiple sensors transmit their observations to a central fusion center to perform classification. We have proposed a novel hybrid maximum likelihood (HML) approach where the unknowns are estimated using a tractable GEM algorithm. We have shown that the performance of AMC can be significantly improved as the number of sensors increases with a good initialization technique for GEM. Our proposed approach employs only a small number of samples to perform both
time/phase synchronization and modulation classification. The simulation results show that the proposed approach provides excellent classification performance with only a small number of samples.

In this paper, we assumed that the sensors transmit their raw observations to a fusion center to perform classification. An interesting future avenue is to consider the AMC problem when the sensors transmit only a summary of the observations to a fusion center.

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