Probing the density dependence of the symmetry potential with peripheral heavy-ion collisions

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Abstract

The peripheral heavy-ion collisions of $^{112,124}Sn + ^{86}Kr$ at $E_0 = 25 AMeV$ are studied by means of the Improved Quantum Molecular Dynamics Model(ImQMD). It is shown that the slope of the average N/Z ratio of emitted nucleons vs impact parameters for these reactions is very sensitive to the density dependence of the symmetry energy. Our study also shows that the yields of $^3H$ and $^3He$ decrease with impact parameters and slope of the yield of $^3H$ vs impact parameters as well as the ratio of $Y(^3H)/Y(^3He)$ depend on the symmetry potential strongly for peripheral heavy-ion collisions.

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Nowadays, the nuclear equation of state (EOS) for asymmetric nuclear systems has attracted a lot of attention. The equation of state for asymmetric nuclear matter can be described approximately by the parabolic law

\[ e(\rho, \delta) = e_0(\rho, 0) + e_{sym}(\rho)\delta^2, \]

where \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) is the isospin asymmetry. \( e_0 \) is the energy per nucleon for symmetric nuclear matter and \( e_{sym}(\rho) \) is the bulk symmetry energy. The nuclear symmetry energy term \( e_{sym}(\rho) \) is very important for understanding many interesting astrophysical phenomena, but it is also a subject with large uncertainties, especially, its density dependence. Therefore, to acquire more accurate knowledge of the symmetry energy term becomes one of the main goals in nuclear physics at present, and has driven a lot of theoretical and experimental efforts. The facilities of rare isotope beams provide opportunities to prepare and study the dynamical evolution of nuclear systems with a range of isospin asymmetries. This increases the domain over which a spatially uniform local isospin asymmetry \( \delta(r) \) may be achieved. The isospin effect on the multifragmentation in central neutron-rich heavy ion collisions at intermediate energies has been widely studied both theoretically and experimentally and a amount of the information for isospin dependence part, \( e_{sym}(\rho) \), of nuclear EOS has been obtained. Recently, a large enhanced production of neutron rich rare isotopes in peripheral collisions of Fermi energy for neutron rich system was observed. For peripheral collisions, the neck emission is the most important source. Under the influence of the symmetry potential, the motion of neutrons and protons towards to neck region will be different and thus the ratio between the numbers of neutrons and protons in neck area depends on the the density dependence of the symmetry potential sensitively. Whether the neck emission in peripheral reactions carries the information of the symmetry energy term? In this work we investigate various observables in peripheral heavy ion reactions at Fermi energies to seek those which are sensitive to the symmetry potential. The calculations are performed for peripheral reactions of \( ^{124}Sn + ^{86}Kr \) and \( ^{112}Sn + ^{86}Kr \) at incident energy of 25AMeV.

For the theoretical description of heavy ion collisions, the improved quantum molecular dynamic model (ImQMD) is adopted in this work. The effective interaction potential energy includes the nuclear local interaction potential energy and Coulomb interaction
potential energy,
\[ U = U_{\text{loc}} + U_{\text{Coul}}. \]  

$U_{\text{loc}}$ is obtained from the integration of the nuclear local interaction potential energy density functional. The nuclear local interaction potential energy density functional $V_{\text{loc}}(\rho(r))$ in the ImQMD model reads
\[ V_{\text{loc}} = \alpha \frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma + 1} \frac{\rho^{\gamma+1}}{\rho_0} + \frac{g_{\text{sur}}}{2\rho_0} (\nabla \rho)^2 + \frac{C_s}{2\rho_0} (\rho^2 - \kappa_s (\nabla \rho)^2) \delta^2. \]  

The first three terms in above expression can be obtained from the potential energy functional of Skyrme force directly. The fourth term is the symmetry potential energy where both the bulk and the surface symmetry potential energy are included. The surface symmetry potential energy term modifies the symmetry potential at the surface region and it is important for having a correct neutron skin and neck dynamics in heavy ion collisions. The Coulomb energy includes both the direct and exchange contributions. In the collision term, isospin dependent nucleon-nucleon scattering cross sections are used\cite{16} and the Pauli blocking effect is treated as in\cite{7}. We have applied this model to study heavy ion reactions at intermediate energies. The relevant observable to present study is the charge distribution in heavy ion collisions at intermediate energies. As an example, in Fig.1 we show the charge distribution of products in the central collisions of $^{120}Xe + ^{124}Sn$ at 50 AMeV and

$^{40}Ca + ^{40}Ca$ at 35 AMeV. The experimental data from\cite{17,18} are also given in the figure. Clusters are constructed by means of the coalescence model widely used in QMD calculations in which particles with relative momenta smaller than $P_0$ and relative distances smaller than $R_0$ are coalesced into one cluster (here $R_0 = 3.0f m$ and $P_0 = 250MeV/c$) is adopted\cite{16}. Concerning light charged particles of $Z \leq 2$ emission, we further introduce the mechanism of the coalescence of free nucleons adopted by Neubert and Botvina(meant of N.B.) in ref.\cite{19}, where this mechanism was shown to be important for light charged particles. We find that the charge distribution at $Z \leq 2$ region is better described by introducing the production of light charged particles by the coalescence of free nucleons adopted in ref.\cite{19}. One can see that the calculation results are in good agreement with experiments. Now we apply this model to seek the sensitive observable for testing the density dependence of the symmetry potential in heavy ion collisions at energies around Fermi energy. In Eq.3 a linear form of the density dependence of symmetry potential is used. In order to study the dynamical effect of the density dependence of the symmetry potential on heavy ion collisions by means of the
transport theory, we verify the form of the density dependence of the bulk symmetry potential energy in the potential energy density functional. For simplicity, the bulk symmetry potential energy density is taken to be a form of

$$w_{\text{sym}}(\rho) = \frac{C_s}{2} u \gamma \rho \delta^2,$$

(4)

where $u = \rho/\rho_0$. In the following, we refer $\gamma = 0.5$ to soft symmetry potential ($soft\ -\ sym$) case and $\gamma = 1.0$ to the stiff symmetry potential ($stiff\ -\ sym$) case. The single particle symmetry potential can be derived from Eq.(4) and reads

$$v_{\text{sym}}^q = \frac{\partial w_{\text{asy}}}{\partial \rho_q} = \frac{C_s}{2} [(\gamma - 1)u \gamma \delta^2 \pm 2u \gamma \delta]$$

(5)

where $q$ denotes neutron and proton and symbols ”+” and ”-” are for neutron and proton, respectively. The parameters used in this work are listed in the Table.1.

| Parameter | Value |
|-----------|-------|
| $\alpha$ (MeV) | -356 |
| $\beta$ (MeV) | 303 |
| $\sigma$ | 7/6 |
| $g_{\text{sur}}$ (MeV fm$^2$) | 22 |
| $\kappa_s$ (fm$^2$) | 0.08 |
| $C_s$ (MeV) | 32 |
| $\gamma$ | 0.5 or 1.0 |

Fig.2 shows (a) the symmetry energy term $e_{\text{sym}}(\rho)$ in the asymmetry nuclear matter for the $soft\ -\ sym$ case and the $stiff\ -\ sym$ case, (b) the single particle symmetry potential $v_{\text{sym}}^{n(p)}$, and (c) the chemical potential for neutrons and protons. The figure shows that the single particle symmetry potential for neutron and proton as well as the chemical potential of neutron and proton depend on the form of the density dependence of the symmetry potential strongly, which will influence the motion of neutrons and protons, especially, the motion towards to neck. We will see this effect on the N/Z ratio at neck region later. The difference between the neutron and proton chemical potentials is given by $\mu_n - \mu_p = 4\left(\frac{C_s}{2}u \gamma + \frac{1}{3}c^0 F u^2/3\right) = 4e_{\text{sym}}(\rho)\delta$, which is directly related with the emission rate of neutrons and protons.

To obtain the general information of the process of the peripheral reactions, we show the time evolution of contour plots of neutron and proton densities for a typical event of reaction $^{124}Sn + ^{86}Kr$ at $E_b = 25AMeV$, $b = 10fm$ within the time period of $25fm/c$ to $200fm/c$ in Fig.3. Calculations are performed by using $stiff\ -\ sym$. From Fig.3 one can see that the neutrons reach the neck region earlier than the protons. After the touching of projectile and target at about $50fm/c$, there is a very gentle compression in the neck region. At this
stage the density at the neck region reaches a value greater than the normal density. And after then, this region expands and finally ruptures during 125fm/c – 200fm/c. While, the density of $PLF^*/TLF^*$ is always around the normal density and it means that there is no compression happening for $PLF^*/TLF^*$. Thus, nucleons, light charged particles and very few intermediate mass fragments, are emitted mainly in the neck region. In Fig.4, we show the time evolution of the average $N/Z$ ratio of the nucleons in a cube with a side length of 8fm centered at the point where it has an equal distance from the surface of projectile and target (see the small inserted figure in Fig.4(a)) for reactions of $^{112,124}Sn + ^{86}Kr$. The left panel is the results with stiff-symmetry potential and the right one is those with soft-symmetry potential. The general behavior of the time evolution of the $N/Z$ ratio of nucleons at neck region is: At about 25 fm/c it reaches a very high value because neutrons move faster than protons at early time, then it reduces till it reaches a minimum at about 100fm/c-110fm/c depending on the symmetry potential being stiff or soft, and then remains at this state for a while which form a plateau. The $N/Z$ value of nucleons at the plateau is higher for the soft-sym case than for the stiff-sym case for the same reactions. After the plateau, it gradually increases. The increasing slope depends on the impact parameters, isospin asymmetry of the system and the symmetry potential. The larger the asymmetry of the system and the stiffer of the symmetry potential is, the larger the increasing slope is at the fixed impact parameters. The behavior of the time evolution of the $N/Z$ ratio at neck area shown in Fig.4 clearly reveals the effects from both the neutron skin and the symmetry potential. As is shown in Fig.2 that the neutron chemical potential from high density to the minimum for the stiff-sym case decreases more steeply than that for the soft-sym case and vice versa for proton chemical potential, which leads to the enhancement of both the $N/Z$ ratio and the increasing slope of the $N/Z$ ratio for the stiff-sym case compared with that for the soft-sym case. While the increase of the $N/Z$ ratio with impact parameters reveals the neutron skin effect in peripheral heavy ion collisions. The comparison between reactions of $^{124}Sn + ^{86}Kr$ and $^{112}Sn + ^{86}Kr$ indicates much stronger impact dependence for the former reaction and it can be attributed to the stronger neutron skin effect for the former case. The different $N/Z$ ratio of nucleons in the neck area with the soft and stiff symmetry potential will lead to different ratios between the neutron to proton emission rate and the isobaric ratio of yields of $^3H$ and $^3He$, and other particles, which emit from the neck area.

How does the emission rates of neutrons and protons from the neck area depend on the
density dependence of the symmetry potential? Fig.5 shows the average $N/Z$ ratio of emitted free nucleons and heavy residues ($Z \geq 30$) versus impact parameters. The left panel shows the results for $^{112}Sn + ^{86}Kr$ and the right panel for $^{124}Sn + ^{86}Kr$. From the figure one sees that the $N/Z$ ratio of emitted nucleons is higher with soft symmetry potential than with stiff symmetry potential. This behavior has been explored in [2]. As far as the impact parameter dependence is concerned, the $N/Z$ ratio is rather flat for central collisions (at $b = 2, 4 \text{ fm}$), while it increases with the impact parameters for peripheral collisions ($b = 7 - 10 \text{ fm}$). The increasing slope of the $N/Z$ ratio of emitted nucleons with impact parameters is sensitive to the symmetry potential. For reactions of $^{112}Sn + ^{86}Kr$ and $^{124}Sn + ^{86}Kr$, the increasing slope of the $N/Z$ ratio of emitted free nucleons versus impact parameters calculated with $stiff - sym$ is about 2-3 times larger than that with $soft - sym$ in the range of $b = 7 \text{ fm} - 10 \text{ fm}$. And the slope for $^{124}Sn + ^{86}Kr$ is larger than that for $^{112}Sn + ^{86}Kr$. The behavior of the dependence of the $N/Z$ ratio of emitted nucleons on impact parameters, the isospin asymmetry of systems and symmetry potentials shown in Fig.5 is closely related with the different behaviors of the time evolution of $N/Z$ ratio of nucleons in neck area at corresponding conditions shown in Fig.4. Correspondingly, the $N/Z$ ratio of heavy residues is larger for $stiff - sym$ case than for $soft - sym$ case due to the balance of emitted protons and neutrons and those in heavy residue. The detailed analysis for heavy residues is in progress. For the same reason, the slope of the $N/Z$ ratio of heavy residues vs impact parameters also depends on the symmetry potential. We find that introducing the production of light charged particles by the coalescence of free nucleons does not change the feature of the influence of the different forms of the symmetry potential on the $N/Z$ ratio of emitted nucleons vs impact parameters.

Fig.6 shows the yields of $^3H$ and $^3He$ as a function of impact parameters for $^{124}Sn + ^{86}Kr$ (left panel) and $^{112}Sn + ^{86}Kr$ (right panel) at 25AMeV calculated with $stiff - sym$ and $soft - sym$. The yield of $^3H$ is greater than that of $^3He$ for the same reaction system calculated with the same symmetry potential for all impact parameters. For central collisions (at $b = 2, 4 \text{ fm}$), the ratio between the yields of $^3H$ and $^3He$ for $^{124}Sn + ^{86}Kr$ is about 2.5 with $soft - sym$ and 1.9 with $stiff - sym$. For reactions of $^{112}Sn + ^{86}Kr$, it is about 1.98 with $soft - sym$ and about 1.54 with $stiff - sym$. Introducing the production of light charged particles by coalescence of free nucleons adopted in [19] largely increases the yields of $^3H$ and $^3He$ but does not change the behavior of the dependence on impact parameters.
parameters. The ratio between yields of $^3H$ and $^3He$ at central collisions changes slightly. They are about 2.33 with soft $-$ sym and 1.86 with stiff $-$ sym for reactions of $^{124}Sn + ^{86}Kr$ and about 1.85 with soft $-$ sym and about 1.44 with stiff $-$ sym for $^{112}Sn + ^{86}Kr$. These values are roughly coincident with the experiment results of ref. [20] where the yields of $^3H$ and $^3He$ for central collisions of $^{36}Ar + ^{58}Ni$ at $Eb = 74 AMeV$ were measured and the ratio between them was about 1.4. For peripheral collisions ($b = 7 - 10 fm$), the yields of $^3H$ and $^3He$ decrease with impact parameters owing to the decreasing of the size of the neck area with impact parameters. While the decreasing slope depends on the symmetry potential adopted, especially for the yield of $^3H$. The average decreasing slope of the yield of $^3H$ vs impact parameters for reactions $^{112}Sn + ^{86}Kr$ and $^{124}Sn + ^{86}Kr$ calculated with soft $-$ sym is about 1.47 and 1.59 times larger than those with stiff $-$ sym, respectively. The average decreasing slope of the yield of $^3He$ vs impact parameters is less sensitive to the symmetry potential.

In summary, we have investigated the dynamical effects of the symmetry energy on the peripheral heavy ion collisions for $^{124,112}Sn + ^{86}Kr$ at 25 AMeV by means of the ImQMD model. Our investigation shows that the neck emission is the main emission source of nucleons and light charged particles for peripheral reactions. The N/Z ratio at neck area strongly depends on the impact parameter, the neutron excess of the system and the the stiffness of the symmetry potential which leads to the N/Z ratio of emitted nucleons and the light charged particles changing with impact parameters, systems and the stiffness of the symmetry potentials for peripheral reactions. Our results show that the average N/Z ratio of emitted nucleons calculated with soft symmetry potential is higher than that with stiff symmetry potential. Furthermore, we have shown that the N/Z ratio increases with impact parameters and the increasing slope with stiff symmetry potential is about twice as large as those with soft symmetry potential. The average N/Z ratio of heavy residues shows the opposite trend but much weaker. Our results show that the yields of $^3H$ and $^3He$ decrease with impact parameters for peripheral reactions and the ratio between the yields of $^3H$ and $^3He$ depends on the symmetry potential. And the reducing slope of the yield of $^3H$ with impact parameters also depends on the symmetry potential strongly. Our calculations also show that introducing the production of light charged particles by the coalescence of free nucleons increases the yields of $^3H$ and $^3He$ but does not change the feature of the influence of different stiffness of the symmetry potential on the N/Z ratio of free nucleons,
the yield of $^3H$ and the ratio between the yields of $^3H$ and $^3He$ and their dependence on impact parameters. From our study we can conclude that for peripheral neutron-rich heavy ion collisions, the slope of average $N/Z$ ratio of emitted nucleons, the yield of $^3H$ and the ratio between the yields of $^3H$ and $^3He$ with respect to impact parameters are sensitive to the density dependence of the symmetry potential and can be used to prob the density dependence of the symmetry potential.

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Fig. 1 The charge distribution of products in the central collisions for reactions $^{129}$Xe+$^{124}$Sn at 50 AMeV and $^{40}$Ca+$^{40}$Ca at 35 AMeV. The open circles and squares are the results with and without the productions of light charges particle by the coalescence of free nucleons adopted by [19]. The triangles are the experimental data taken from [17] for $^{129}$Xe+$^{124}$Sn and [18] for $^{40}$Ca+$^{40}$Ca.

Fig. 2 (a) The symmetry energy, (b) the single particle symmetry potential and (c) the chemical potential for proton and neutron in the nuclear matter at $\delta = 0.1, 0.3$ calculated with the soft symmetry potential and stiff symmetry potential, respectively.

Fig. 3 The time evolution of contour plots of proton and neutron densities for reactions of $^{124}$Sn+$^{86}$Kr at $Eb = 25$AMeV and $b = 10\, fm$, calculated with the stiff symmetry potential.

Fig. 4 The time evolution of the average $N/Z$ ratio for nucleons in the neck region (see the text) for reactions of $^{124}$Sn+$^{86}$Kr and $^{112}$Sn+$^{86}$Kr at impact parameters of $b = 7−10$. The calculations are performed with stiff − sym and soft − sym, respectively.

Fig. 5 The average $N/Z$ ratios of the emitted free nucleons (circles) and heavy residues with $Z \geq 30$ (squares) vs impact parameters with soft − sym (open symbols) and stiff − sym case (solid symbols), respectively. The Left panel is for $^{112}$Sn+$^{86}$Kr and the right panel for $^{124}$Sn+$^{86}$Kr.

Fig. 6 The yields of $^3$H (squares) and $^3$He (circles) vs impact parameters for reactions of $^{124}$Sn+$^{86}$Kr (right) and $^{112}$Sn+$^{86}$Kr (left) with soft − sym (open symbols) and stiff − sym (solid symbols), respectively. The solid and dashed lines are the results with and without the production of light charged particles by the coalescence of free nucleons, respectively.
$^{129}\text{Xe} + ^{124}\text{Sn}, \ E_b=50 \text{AMeV}$

- without coale. Mec. of NB
- with
- Exp

$^{40}\text{Ca} + ^{40}\text{Ca}, \ E_b=35 \text{AMeV}$

- without coale. Mec. of NB
- with
- Exp
\( \mu (\text{MeV}) \)

\( \delta = 0.3 \)

\( \delta = 0.1 \)

\( \gamma = 0.5 \)

\( \gamma = 1.0 \)

\( V_{asy} \) (MeV)

\( \rho/\rho_0 \)

\( e_{sym} (\rho) \) (MeV)

\( \delta = 0.3 \)

\( \delta = 0.1 \)

\( \gamma = 0.5 \)

\( \gamma = 1.0 \)

\( \rho/\rho_0 \)

\( n \)

\( p \)

\( \text{neutron} \)

\( \text{proton} \)

\( (a) \)

\( (b) \)

\( (c) \)
\( ^{124}\text{Sn} + ^{86}\text{Kr}, b=10\text{fm}, \gamma=1.0 \)
$^{112}_{\text{Sn}} + ^{86}_{\text{Kr}}, E_b = 25\text{AMeV}$

$^{124}_{\text{Sn}} + ^{86}_{\text{Kr}}, E_b = 25\text{AMeV}$

- Nucleons
- $Z \geq 30$
- $\gamma = 1.0$
- $\gamma = 0.5$
The graph shows the yield (arb. units) against the impact parameter (b in fm) for different conditions.

- The solid line represents the yield without the coalescence mechanism of NB.
- The dashed line represents the yield with the coalescence mechanism of NB.

Two isotopes are considered:
- $^3\text{H}$ represented by open squares.
- $^3\text{He}$ represented by solid squares.

Two values of $\gamma$ are shown:
- $\gamma = 1.0$ represented by solid squares.
- $\gamma = 0.5$ represented by dashed squares.

The yield decreases as the impact parameter increases, and the effect of coalescence and $\gamma$ value is evident.