I. INTRODUCTION

The main purpose of the second part of the paper is to properly analyze other two generation models of high frequency gravitational waves (HFGW) in the interaction of high power laser pulse with a medium, the ablation (rarefaction) and piston models. These models were suggested in [3, 4]. The theory and the basic information about the models was reviewed in the part I where we investigated the shock wave model in detail. Therefore we will move faster in this second part and will concentrate on new results for the ablation and piston models.

The paper is organized as follows. In Section II we derive and analyze the analytical formulae for the perturbations and the luminosity of the gravitational radiation. We present the estimations for the experiment and measurement for the specific data for ablation model.

In Section III we concentrate on the piston model and provide the analytical formulae for perturbation, the luminosity and estimations for an experiment.

In Section IV we derive and analyze the polarization properties of the gravitational radiation and the different radiative properties with dependence on the orientation of the wave vector in the assumed ablation and piston model.

The Section V we concentrate on derivation and analysis of the behaviour of the test particles in the field of passing gravitational waves in both models, ablation and piston one.

The main results are summarized in the concluding Section VI.

II. THE DERIVATION OF GRAVITATIONAL WAVE CHARACTERISTICS FOR ABLATION MODEL

The calculations are made in linear approximation to full gravity theory up to quadrupole moment in the multipole expansion, for details in theory see [5].

In the configuration pictured in Fig. 1, the laser is interacting with a planar thick foil with more than 100 µm thickness. The material is accelerated in the ablation zone and in the shock front. The points on the axis indicate the areas where the gravitational waves start to be generated. These two possibilities are divided into two separate models, the shock wave model and the ablation zone generation model. In the experiment, the two models are put together since each model represents one faze of the same experiment and therefore the radiation could be measured simultaneously.

In the following text we are going to investigate the ablation model in detail.

A. The ablation zone generation model

In this case the gravitational radiation is produced in the ablation zone with starting point \(z_r\). The density profile for this model is visible in the Fig. 1. The expressions will be very similar to ones for the shock wave model therefore we will proceed in a shorter way.

1. The limitations of the theory

Let see whether the low velocity limit Eq. (7) is satisfied for ablation model. The linear size (diameter) of the source (the focus size) is \(d = 1\text{mm} = 10^{-3}\text{m}\) and the reduced generated wavelength is \(\lambda = 4.7746 \times 10^{-2}\text{m}\) for the gravitational wave length \(\lambda_g = 0.3\text{m}\), which is the same as for shock wave model [3]. The comparison
Eq. (7) in [3] results into
\[0.021 \ll 1,\] (1)
The low velocity condition is still satisfied for the ablation wave experiment, while we have obtained the condition for the size of the target to satisfy the low velocity condition. We can generalize the estimation with the fact that
\[\lambda = \frac{1}{2\pi}\tau c,\] (2)
where \(\tau\) is the duration of the pulse and \(c\) is the speed of light, then we can rewrite this condition as
\[d \ll \frac{1}{2\pi}\tau c,\] (3)
which could be useful in general setup of the experiment according to the duration of the pulse.

2. Set up of the experiment

This section is devoted to the derivation of fully analytical formulae of the luminosity \(\mathcal{L}_{GW}\) and the perturbation of the metric \(h_{GW}\) for the shock wave model in Section III using the linearized gravity theory from Section II. The results are new, as well as the results in the following sections about polarization and behaviour of test particles in the gravitational field of gravitational wave.

![FIG. 1. The representation of density profile for the ablation zone model.](image)

The set up of the geometry of the experiment is similar to shock wave model. We assume the rectangular shape of the foil with parameters, \(a, b, l\), and we choose the orthogonal coordinate system \(x, y, z\). The parameter \(l\) is the thickness of the foil in the \(z\) direction. The distance of the laser and the detection desk/point is \(z_L\), for full set up see Fig. 3 in [5]. We assume the whole process happens in the box of rectangular shape with parameters \(a, b, z_L\) for simplicity. The start of the coordinate system corresponds with the position where the detector would be possibly positioned. The moving point where the density of the beam changes will be denoted as \(z_r\) with a form
\[z_r(t) = -v_r t + d,\] (4)
where the velocity is defined as
\[v_r(t) \approx c_r = \sqrt{\frac{P_r}{4\rho_0}},\] (5)
where \(P_r\) is ablation pressure and \(\rho_0\) is material density. We assume that for \(t = 0, z_r(0) = d\), therefore the constant in (4) is \(d = f_2\) according the the Fig. II.

In the following, we will calculate everything with general function \(z_r(t)\) and then we will substitute the explicit function \(f_2\) at convenient places. General expressions might be useful for other forms of \(z_r(t)\). At this point in time, we are not aware of better ansatz for this function.

The basic input for the calculation is the density profile from Fig. I. The step function for the density profile can be written as
\[\rho(t, x) = \begin{cases} 4\rho_0 & \text{if } z < z_r, \\ 4\rho_0e^{-m(z,t)} & \text{if } z > z_r, \end{cases}\] (6)
where we denote \(m(z,t)\) as
\[m(z,t) = -\frac{z - z_r}{z_r},\] (7)
The density does not satisfy the mass conservation law because we integrate the mass moment to the finite value \(z_L\) instead of the \(\infty\) value. This property of the ablation model has its consequences in obtaining artificial gravitational waves in the direction of the laser propagation, which will be discussed later in the paper. Such a property of a model was also observed in [6].

The first step in the calculation is the mass moment derivation.

3. The mass moment

The values for integration of the density \([6]\) in Eq. (11) in [5] are \(x \in (0, a)\), \(y \in (0, b)\) and \(z \in (0, z_L)\) which splits into \((0, z_r)\) and \((z_r, z_L)\). In other words, we integrate over the box in the Fig. 3 in [5].

We denote
\[a_f \equiv m(z_r, t) = 0, \quad b_f \equiv m(z_L,t) = -\frac{z_L - z_r}{z_r},\] (8)
and when \(z = 0\) the function \(m(0,t) = 1\) for every \(t\).

The mass moment Eq. (11) in [5] is listed in Appendix B1 and B2 where we used (8), then the diagonal com-
ponents then read

\[ M_{xx} = \frac{4}{3} S a^2 \rho_0 \alpha t, \quad M_{yy} = \frac{4}{3} S b^2 \rho_0 \alpha t, \]
\[ M_{zz} = 4S \rho_0 \alpha^3 t \left( -\frac{2}{3} + (b_2^2 + 1)e^{-bt} \right), \quad (9) \]
and non–diagonal components \( M_{xy}, M_{yz}, M_{xz} \),

\[ M_{xy} = S^2 \rho_0 \alpha t, \quad M_{yz} = 2Sb \rho_0 \alpha t \left( \frac{1}{2} + b tf e^{-bt} \right), \]
\[ M_{xz} = 2S \rho_0 \alpha t \left( \frac{1}{2} + b tf e^{-bt} \right). \quad (10) \]

4. The quadrupole moment

The next step is the calculation of the quadrupole moment Eq. (10) in [5]. The non–diagonal components \( I_{xy}, I_{yz}, I_{xz} \) are

\[ I_{xy} = M_{xy}, \quad I_{yz} = M_{yz}, \quad I_{xz} = M_{xz}. \quad (11) \]

The diagonal components \( I_{ti} = M_{ti} - \frac{1}{2} TrM \) read

\[ I_{xx} = \frac{4S \rho_0 \alpha t}{3} \left( \frac{2}{3} - (b_2^2 + 1)e^{-bt} \right) + \left( \frac{2a^2 - b_2^2}{3} \right) e^{-bt}, \]
\[ I_{yy} = \frac{4S \rho_0 \alpha t}{3} \left( \frac{2}{3} - (b_2^2 + 1)e^{-bt} \right) + \left( \frac{2b_2^2 - a^2}{3} \right) e^{-bt}, \]
\[ I_{zz} = \frac{4S \rho_0 \alpha t}{3} \left( \frac{2}{3} - (b_2^2 + 1)e^{-bt} \right) - \left( \frac{a^2 + b_2^2}{3} \right) e^{-bt}. \quad (12) \]

Similarly to the shock wave model, the diagonal components of quadrupole moment show cubic dependence on the function \( \alpha t \), and are missing quadratic term. The non–diagonal components \( I_{yz} \) and \( I_{xz} \) are missing the linear dependence on \( \alpha t \). The trace \( TrM_{ii} \) reads

\[ TrM_{ii} = \frac{4}{3} S \rho_0 \alpha t \left( (a^2 + b_2^2)e^{-bt} + 3z^2 \left( -\frac{2}{3} + (1 + b_2^2)e^{-bt} \right) \right). \quad (13) \]

When we substitute the function \( z(t) \) into \( I_{zz} \) component we will get the time dependency

\[ I_{zz} = \frac{4S \rho_0 \alpha t}{3} \left( 4(-v_0^3 t^3 + 3v_0^2 t^2 f_2 - 3v_0 t f_2^2 + f_2^4) \times \right. \]
\[ \left( -\frac{2}{3} + (2z_0 \frac{2}{2} - 1) e^{-bt} \right) - (-v_0 t + f_2 \left( \frac{a^2 + b_2^2}{3} \right) e^{-bt} \right). \quad (14) \]

The quadrupole moment in the \( zz \) direction is given by a cubic polynomial in \( t \) variable as in the shock model [5]. The most dominant term is then the cubic term with a new term \( e^{-bt} \) which behaves as \( e^{-1} \) when \( t \to 0 \) and creates dumping as time progresses. The other terms are new, the quadratic, linear and constant terms. The geometry of the setup influences the quadrupole moment from the quadratic term and lower.

5. The analytical form of perturbation and luminosity

Now, we calculate the components of the perturbation tensor according to Eq. (9) in [5] without projector \( \Lambda_{ij,kl} \).

In other words, we got the components of the perturbation tensor in general form, the components read

\[ h_{xx} = \frac{8G}{3rc^4} S \rho_0 \left( \frac{2a^2 - b_2^2}{3} z^2 - z_0 \right) e^{-bt} - (z_r^2 D) \],
\[ h_{yy} = \frac{8G}{3rc^4} S \rho_0 \left( \frac{2b_2^2 - a^2}{3} z^2 - z_0 \right) e^{-bt} - (z_r^2 D) \],
\[ h_{zz} = \frac{8G}{3rc^4} S \rho_0 \left( -\frac{a^2 + b_2^2}{3} z^2 - 4(z_r^2 D) \right), \quad (15) \]
and the non-diagonal terms are

\[ h_{xy} = -\frac{2G}{r c^4} S \rho_0 \left( \frac{2}{3} + b_2 e^{-bt} \right), \]
\[ h_{yz} = \frac{4G}{rc^4} S \rho_0 \left( \frac{2}{3} + b_2 e^{-bt} \right) + e^{-bt} \frac{2}{3} (2z_0^2 - (z_r + z_L)), \quad (16) \]
\[ h_{xz} = \frac{4G}{rc^4} S \rho_0 \left( \frac{2}{3} + b_2 e^{-bt} \right) + e^{-bt} \frac{2}{3} (2z_0^2 - (z_r + z_L)), \]
where we have used

\[ D = -\frac{2}{3} + e^{-bt} (b_2^2 + 1), \quad (17) \]
and conveniently \( \tilde{z}_r = 0 \) for substitution [4] to simplify the expressions. We are not going to list all the derivatives in Appendix for this model because of the complexity of expressions.

Contrary to the shock wave model calculations, all components of \( h_{ij} \) are time dependent components of the tensor thanks to functions \( a_1 \) and \( b_1 \). Just in the diagonal components the first term vanishes for

\[ f_2 = v, t, \quad (18) \]
which is the position of the detector.

We will investigate the component \( zz \) of perturbation because it is the most complex component in the direction of motion of the experiment, the components \( h_{xx} \) and \( h_{yy} \) has similar terms in their expression and therefore for the purposes of estimation and functional dependence it is enough to investigate just \( zz \) component.

First, we investigate the component of perturbation \( h_{GW}^{zz} \) which can be rewritten as

\[ h_{zz} = \frac{8G}{3rc^4} S \rho_0 \left( 24(z_r)^2 \left[ -z_r \left( \frac{2}{3} + e^{-bt} \right) + \frac{3}{2} z_L e^{-bt} \right] \right. \]
\[ + 24z_L z_r e^{-bt} \left( 1 - \frac{z_L}{z_r} \right) + 4 e^{-bt} (4z_L - 3z_r) \frac{z_r^2}{z_r^2} \left( \frac{a^2 + b_2^2}{3} e^{-bt} \right), \left( \frac{a^2 + b_2^2}{3} e^{-bt} \right), \quad (19) \]
For the purposes of an estimation we will evaluate just the first term of \[\text{(19)}\] which is linear in \(z_t\), and most dominant. The second term behaves as \(O\left(\frac{e^{-b_I}}{z_t}\right)\), the third as \(O\left(\frac{\lambda}{z_t}\right)\) and the fourth as \(O\left(\frac{e^{-b_I}}{z_t^2}\right)\) which in limit \(t \to \infty\) approach zero. According to the fourth term the parameters of the foil then contribute in the small way to the value of perturbation.

The expression \(\text{(19)}\) becomes using \(\text{(4)}, \text{(5)}\),

\[
h_{zz} = \frac{64G}{rc^4} \left( v_t^3 \left( \frac{2}{3} + e^{-b_I} \right) - v_e^2 \left( \frac{2}{3} + e^{-b_I} \right) - \frac{3}{2} z_t e^{-b_I} \right)
\]

(20)

The previous expression can be rewritten even further using \(\text{(5)}\) and \(\text{(24)}\) as

\[
h_{zz} = \frac{64G}{rc^4} \left( \frac{1}{6} \left( \frac{R_t}{\rho_0} \right) \right)^{1/2} E_L \left( \frac{2}{3} + e^{-b_I} \right)
\]

\[- \frac{S I_t^{1/3} I_L^{2/3}}{4} \left( f_2 \left( \frac{2}{3} + e^{-b_I} \right) - \frac{3}{2} z_t e^{-b_I} \right) \right) .
\]

(22)

where we used the pressure and the energy of the laser,

\[
P_L = S I_t, \quad E_L = S I_t t.
\]

(23)

When we compare this final formula with one for shock wave model \(\text{(5)}\) we observe that the perturbation is more general in terms with \(e^{-b_I}\). This is a natural consequence of the more general density ansatz \(\text{(6)}\) when compared with one for shock wave model. Thanks to the ansatz the constant \(z_t\) appears in the final expression. The value of the perturbation decreases with the distance as \(1/r\) and will be zero in the infinity. We have obtained additional time dependent terms which contribute to the first term in the brackets.

We use more general expression for \(P_t\) and \(I_L\) \(\text{(10)}\) which will allow us to have control over more parameters than the formulae suggested in \(\text{(3)}, \text{(4)}\),

\[
P_t = R_t^{1/3} I_L^{2/3},
\]

(24)

and \(R_t\) denotes the target 'density' as \(R_t = \frac{1}{4 \pi} m_p n_c\), and \(n_c\) is the critical density defined as \(n_c = \frac{\epsilon_0 m_p}{\pi c^2 \lambda L^2}\), where \(\epsilon_0\) is vacuum permittivity of vacuum, \(m_p\) is the rest mass of the electron, \(e\) is the charge of electron and \(\lambda L\) is the wavelength of the laser. All of the parameters in \(n_c\) are constants except the laser wavelength \(\lambda L\) which is constant given by the specific experiment.

The luminosity Eq. \(\text{(12)}\) can be rewritten as Eq. \(\text{(27)}\) in \(\text{(3)}\). After substituting the quadrupole moment components into Eq. \(\text{(27)}\) in \(\text{(3)}\), we get general expression as

\[
L_{\text{quad}} = \frac{G}{5c^5} S^2 \rho_0^2 \left\{ \frac{16}{9} \left\{ 18 \left( \frac{z_t^3}{r} \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) \right) \right] ^2 - \frac{10}{3} \left( \frac{z_t e^{-b_I}}{r} \right) \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) \left( a^2 + b^2 \right) \right] ^2 + \frac{1}{9} \left( \frac{z_t e^{-b_I}}{r} \right) ^2 \left( \left( a^2 + b^2 \right) - \left( \frac{2}{3} + \frac{3}{2} \frac{z_t e^{-b_I}}{r} \right) \right) ^2 + \frac{8}{9} \left( \frac{z_t e^{-b_I}}{r} \right) ^2 \left( \frac{1}{2} + b_I e^{-b_I} \right) ^2 \right\} .
\]

(25)

We observe that the expression is in fact generalized luminosity for shock wave model \(\text{(5)}\) with terms with \(b_I\) as in previous results. Contrary to result for shock wave model the result it time dependent. In order to obtain the most dominant contribution we neglect the higher derivatives of such terms the higher the power of \(z_t\) in denominator and lower contribution. Then we obtain

\[
L_{\text{quad}} = \frac{G}{5c^5} S^2 \rho_0^2 \left\{ \frac{16}{9} \left\{ 18 \left( \frac{z_t^3}{r} \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) \right) \right] ^2 - \frac{10}{3} \left( \frac{z_t e^{-b_I}}{r} \right) \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) \left( a^2 + b^2 \right) \right] ^2 + \frac{1}{9} \left( \frac{z_t e^{-b_I}}{r} \right) ^2 \left( \left( a^2 + b^2 \right) - \left( \frac{2}{3} + \frac{3}{2} \frac{z_t e^{-b_I}}{r} \right) \right) ^2 + \frac{8}{9} \left( \frac{z_t e^{-b_I}}{r} \right) ^2 \left( \frac{1}{2} + b_I e^{-b_I} \right) ^2 \right\} .
\]

(26)

which further simplifies to

\[
L_{\text{quad}} = \frac{1152G}{10c^5} S^2 \rho_0^2 \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) ^2 .
\]

(27)

Finally, we will use the explicit expression for the velocity \(v_s\) via \(\text{(5)}\) and \(\text{(24)}\), we will obtain the final expression for luminosity of gravitational radiation,

\[
L_{\text{quad}} = \frac{9G}{10c^5} \frac{R_L^2}{\rho_0^2} \left( \frac{2}{3} + e^{-b_I} \left( b_I^2 + 1 \right) \right) ^2 ,
\]

(28)

where the first term in the brackets is constant, second one is \(O\left(\frac{e^{-b_I}}{z_t^2}\right)\) and third one \(O\left(\frac{e^{-b_I}}{z_t}\right)\). The terms with \(b_I\) are corrections to the most dominant constant term. The luminosity then depends on the power of the laser, the density of the material and the laser wavelength. The result generalizes \(\text{(3)}, \text{(4)}\) in the dependency on the laser wavelength and correction terms with \(b_I\) and constant \(R_t\). The numerical factor in front of the fraction for estimation will be presented in the next subsection.

Interestingly, the quadrupole moment using \(\text{(28)}\),

\[
I_{zz} = \frac{4S \rho_0}{3} \left\{ \left( \frac{R_t^{1/2} E_L}{\rho_0^2} I_L^{1/3} \right) ^2 + \left( \frac{R_t^{1/3} I_L^{2/3} I_L^{1/3}}{\rho_0} \right) \right\} - 6 \frac{R_t^{1/6} I_L^{1/3} f_2}{\rho_0} \left( \frac{2}{3} + \frac{3}{2} \frac{z_t e^{-b_I}}{r} \right) \left( \frac{2}{3} + \frac{3}{2} \frac{z_t e^{-b_I}}{r} \right) + \frac{8}{3} \left( \frac{z_t e^{-b_I}}{r} \right) ^2 \left( \frac{1}{2} + b_I e^{-b_I} \right) ^2 \right\} .
\]

(29)
has similar form as for the shock wave model [8] generalized with terms \(b_I\).

In this subsection, we have derived explicit expressions for perturbation component \(h_{zz}^{GW}\) and \(\mathcal{L}_{\text{quad}}\) which generalize previously published results with additional time dependent terms with function \(b_I\) and constant \(R_I\).

6. The estimations for the \(h_{\nu\nu}\) and \(\mathcal{L}_{\text{quad}}\) for real experiment

We will evaluate the numerical factors in final results for luminosity (28) and the perturbation \(h_{zz}^{GW}\) of the space by the gravitational wave in \(zz\) direction, which will be useful for real experiment.

Now, we arrive to the expression for the luminosity as

\[
\mathcal{L}_{\text{quad}}^{[\text{erg}/s]} = 2.51 \times 10^{-22} \left[ \frac{s^3}{\text{kg m}^2} \right] \frac{R_i}{\rho_0} \left[ \text{g/cm}^3 \right] P_L^{2}\left[ \text{PW} \right] \\
\times \left( \frac{2}{3} + e^{-b_I(b_I^2 + 1)} \right)^2
\]

and we denote the part without the \(b_I\) function as

\[
\mathcal{L}_{\text{quad}}^{1}\left[ \text{erg/s} \right] = 2.51 \times 10^{-22} \left[ \frac{s^3}{\text{kg m}^2} \right] \frac{R_i}{\rho_0} \left[ \text{g/cm}^3 \right] P_L^{2}\left[ \text{PW} \right] \left( \frac{2}{3} \right)^2.
\]

First, we will investigate the first time dependent part of (31), we obtain

\[
h_{zz} = 2.817 \times 10^{-39} \left[ \frac{s^2}{\text{kg m}^2} \right] \frac{1}{r[m]} \left( \frac{R_i}{\rho_0} \left[ \text{g/cm}^3 \right] \right)^{1/2} \\
\times E_L[\text{MJ}] \left( \frac{2}{3} + e^{-b_I} \right),
\]

and the second constant term is a new contribution to the result which depends on the geometry of the setup and the choice of \(f_1\),

\[
h_{zz}^{1/2} = -6.201 \times 10^{-43} \left[ \frac{s^2}{\text{kg m}^2} \right] \frac{S[\text{cm}^2]}{r[m]} \frac{R_{t}^{1/3}}{[\text{g/cm}^3]} \frac{1}{f_{L}^{2/3}} \left[ \text{PW/cm}^2 \right] \\
\times \left[ f_2[m] \left( \frac{2}{3} + e^{-b_I} \right) - \frac{3}{2} \frac{E_L[\text{MJ}]}{e^{-b_I}} \right].
\]

The first expression in the second term has no physical meaning because we can make it zero by choosing different center of coordinate system with start at \(d = f_2 = 0\).

The value of \(R_i\) for Carbon as a material for the target with \(A = 12\), \(Z = 6\) and wavelength \(\lambda_t = 0.35 \times 10^{-4}\) cm, we will obtain \(R_i = 15.144[\text{kg/m}^3]\) from Eq. (24).

For evaluation we will use the experimental values

\[
P_L = 0.5 \text{ PW}, \rho_0 = 30 \text{ mg/cm}^3, E_L = 0.5 \text{ MJ}, \tau = 1 \text{ ns},
\]

and the detection distance is \(R = 10 \text{ m}\) or equivalently \(f_2 = f = 10 \text{ m}, \ z_L = 12 \text{ m}\), parameters \(a, b\) of the target foil are \(a = b = 1 \text{ mm} = 0.1 \text{ cm}\) and therefore \(I_L = 50[\text{PW/cm}^2]\).

The outgoing gravitational radiation has frequency \(v_g = 1 \text{ GHz}\) and wavelength \(\lambda_g = 0.3 \text{ m}\). The velocity \(v_r = 1.14 \times 10^6[\text{m/s}]\), \(b_I = 0.2\) for time \(t = 10^{-9}\) s.

The final estimations for our expressions of the luminosity (30) and the perturbation (32) are:

\[
\mathcal{L}_{GW} \simeq 3.61 \times 10^{-20}[\text{erg/s}], \quad h_{zz}^{GW} \simeq 4.7 \times 10^{-39}.
\]

The estimations are one lower lower in \(\mathcal{L}_{GW}\) and three orders higher in \(h_{zz}^{GW}\) compared to [3, 4]. Our results contain new time dependent terms with function \(b_I\) which modify the results and provide more precision.

The estimation for the constant term \(\mathcal{L}_{GW}^{1}\) and second term in \(h_{zz}^{1/2}\) are

\[
\mathcal{L}_{GW}^{1} = 4.699 \times 10^{-19}[\text{erg/s}], \quad h_{zz}^{1/2} = -2.45 \times 10^{-39},
\]

which corresponds to the result in [3, 4] but the order of \(\mathcal{L}_{GW}\) is one order lower due to the \(b_I\) terms.

Interestingly, the second term [33] results in the estimation to a number \(h_{zz}^{1/2} = -2.45 \times 10^{-39}\) which has the same order as [33]. The term is partially of coordinate nature therefore we did not include it into final results.

We have derived and investigated generalized formulae for the luminosity (28) and the perturbation tensor \(h_{zz}\) which newly shows non–trivial time dependence and depends on the function \(b_I\) and on the laser wavelength \(\lambda_g\) through \(R_i\).

III. THE DERIVATION OF GRAVITATIONAL WAVE CHARACTERISTICS FOR PISTON MODEL

A. The piston model

![FIG. 2. The structure of the ion density profile of the piston caused by radiation pressure where the frame moves with the piston velocity \(v_p\).](image)

The recent progress in focal intensities of short-pulse lasers allows us to achieve intensities larger than \(10^{20}\)
W/cm² where the radiation pressure becomes the dominant effect in driving the motion of a particle in the material (target). The ponderomotive potential pushes the electrons steadily forward and the charge separation field forms a double layer (electrostatic shock or piston) propagating with \( v_p \) where the ions are then accelerated forward. This strong electrostatic field forms a shocklike structure [2].

The use of circularly polarized laser light improves the efficiency of ponderomotive ion acceleration while avoiding the strong electron overheating. Then we will obtain quasi monoenergetic ion bunch in the homogeneous medium consisting of fast ions accelerated at the bottom of the channel with 20\% efficiency. The depth of penetration depends (in microns) on the laser fluence which should exceed tens of GJ/cm².

The model generates gravitational waves in THz frequency range with the duration of the pulse in picoseconds. The mass is accelerated with radiation pressure with circularly polarized pulse with intensity \( I_L \geq 10^{21} \text{ W/cm}^2 \) which pushes the matter thanks to ponderomotive force. The mass is accelerated to the velocity \( v_p \) which could be \( 10^9 \text{ cm/s} \) and even more.

1. The limitations of the theory

Let see whether the low velocity condition Eq. (21) in [5] is satisfied for ablation model. The linear size of the source (focus size) is more than \( d = 1 \mu m = 10^{-6} \text{ m} \) and the reduced generated wavelength is \( \lambda = 4.778 \times 10^{-5} \text{ m} \) for the gravitational wave length \( \lambda_g = 300 \mu m \). The comparison Eq. (22) in [5] results into

\[ 0.021 \ll 1. \tag{37} \]

The low velocity condition is still satisfied for the piston model experiment, while we have a limit for the size of the target for the piston model.

2. Set up of the experiment

The set up for the experiment is visible in Fig. 2. The target is positioned at the start of the coordinate system \( x, y, z \) and we expect that the depth of hole boring is very small. The detector is positioned in the same distance as in the previous models, in the distance \( z_D = 10 \text{ m} \).

The material is accelerated in the direction of the \( z \) coordinate. The function of the shock position is again taken

\[ z_p(t) = v_p t + d, \tag{38} \]

like in the previous models, see [5] and [4] for comparison.

The velocity of a piston is denoted as

\[ v_p \simeq \sqrt{\frac{I_L}{c \rho_0}}, \tag{39} \]

where \( \rho_0 \) is material density and \( I_L \) is the intensity of the laser in PW/cm². We have denoted the velocity as \( (39) \) and we assume that for \( t = 0, z_s(0) = 0 \), therefore \( d = 0 \) according the the Fig. 2.

The time when the radiation reaches the detector is defined as

\[ t_D = \frac{z_D}{v_p}. \tag{40} \]

Again, we will calculate everything with general function \( z_p(t) \) and then we will substitute the explicit function \( (33) \) at convenient places which might be useful for other forms of \( z_p(t) \).

The basic input for the calculation is the density profile from Fig. 2. The step function for the density profile can be written as

\[ \rho(t, x) = \begin{cases} 2\rho_0 & \text{if } z < z_p, \\ \rho_0 & \text{if } z > z_p. \end{cases} \tag{41} \]

The first step in the calculation is the mass moment derivation.

3. The mass moment

The values for integration of the density \( (11) \) in Eq. (11) in [5] are \( x \in (0, a), y \in (0, b) \) and \( z \in (0, z_D) \) which splits into \( z_s < 0, z_p \) and \( (z_s, z_D) \). The mass moment diagonal components then read

\[ M_{xx} = \frac{S \rho_0}{3} z_p (z + z_D), \quad M_{yy} = \frac{S b^2}{3} \rho_0 (z + z_D), \]

\[ M_{zz} = \frac{S}{3} \rho_0 (z_p^3 + z_D^3), \tag{42} \]

and non–diagonal components \( M_{xy}, M_{yz}, M_{zz}, \)

\[ M_{xy} = \frac{S^2}{4} \rho_0 (z + z_D), \quad M_{yz} = \frac{S b}{4} \rho_0 (z_p^2 + z_D^2), \]

\[ M_{zz} = \frac{S a}{4} \rho_0 (z_p^2 + z_D^2). \tag{43} \]

These semi–results will be usefull for the polarization because it shows that it is sometimes more convenient to use the mass moment for calculations instead of the quadrupole moment.

4. The quadrupole moment

The non–diagonal components \( I_{xy}, I_{yz}, I_{zz} \) are

\[ I_{xy} = M_{xy}, \quad I_{yz} = M_{yz}, \quad I_{zz} = M_{zz}. \tag{44} \]

The diagonal components \( I_{ii} = M_{ii} - \frac{1}{3} Tr M \) read

\[ I_{xx} = \frac{S \rho_0}{9} \left\{ -z_p^3 + (2a^2 - b^2)(z + z_D) - z_D^3 \right\}, \]

\[ I_{yy} = \frac{S \rho_0}{9} \left\{ -z_p^3 + (2b^2 - a^2)(z + z_D) - z_D^3 \right\}, \]

\[ I_{zz} = \frac{S \rho_0}{9} \left\{ 2z_p^3 - (a^2 + b^2)(z + Z_D) + z_D^3 \right\}. \tag{45} \]
The functional dependence is almost the same as in the previous models thanks to the linearity of the function \( z_p(t) \). The component \( I_{zz} \) then becomes explicitly

\[
I_{zz} = \frac{S\rho_0}{9} \left\{ 2v_p^3t^3 - (a^2 + b^2)v_p t + z_D(2z_D^2 - (a^2 + b^2)) \right\}. 
\]  

(46)

The quadrupole moment in the \( zz \) direction is given by a cubic polynomial in \( t \) time variable.

When we compare our result [40] with [3,4] we observe (again) that just the most dominant term was used for their calculations. The other terms are new, linear and constant terms. The geometry of the setup influences the quadrupole moment from the linear term and lower. The derivatives of the quadrupole moment and mass moment are listed in Appendix A, the derivatives with dependence on \( z_p \) in (C1) and with substitution of \( z_p \) in (C2).

5. The analytical form of perturbation and luminosity

Now, we calculate the components of the perturbation tensor according to Eq. (9) in [3] without projector \( \Lambda_{ij,kl}(n) \). In other words, we got the components of the perturbation tensor in general form, the components read

\[
\begin{align*}
    &h_{xx} = \frac{2G}{9c^4}S\rho_0 \left\{ (2a^2 - b^2)z_p^3 - (z_p^3) \right\}, \\
    &h_{yy} = \frac{2G}{9c^4}S\rho_0 \left\{ (2b^2 - a^2)z_p^3 - (z_p^3) \right\}, \\
    &h_{zz} = \frac{2G}{9c^4}S\rho_0 \left\{ (z_p^3) - (a^2 + b^2)z_p^3 \right\},
\end{align*}
\]

(47)

and the non-diagonal terms are

\[
\begin{align*}
    &h_{xy} = \frac{G}{2c^2}S^2\rho_0 z_p^3, \\
    &h_{xz} = \frac{G}{2c^2}S\rho_0 (z_p^3), \\
    &h_{yz} = \frac{G}{2c^2}S\rho_0 (z_p^3),
\end{align*}
\]

(48)

The perturbation tensor with substitution of \( z_p(t) \) reads

\[
\begin{align*}
    h_{xx} &= -\frac{4G}{3c^4}S\rho_0 v_p^3 t, \\
    h_{yy} &= -\frac{4G}{3c^4}S\rho_0 v_p^3 t, \\
    h_{zz} &= \frac{8G}{3c^4}S\rho_0 v_p^3 t,
\end{align*}
\]

(49)

and the non-diagonal terms are

\[
\begin{align*}
    h_{xy} &= 0, \\
    h_{xz} &= \frac{G}{c^2}S\rho_0 v_p^3, \\
    h_{yz} &= \frac{G}{c^2}S\rho_0 v_p^3,
\end{align*}
\]

(50)

where we used the derivatives of \( z_p \) listed in Appendix A.

After substituting the quadrupole moment components into Eq. (10) in [3], we get general expression as

\[
\begin{align*}
    \mathcal{L}_{\text{quad}} &= \frac{G}{405c^6}S^2 \rho_0^2 \left\{ 6\left((z_p^3)\right)^{\cdot \cdot} - 6\dot{z}_p \left((z_p^3)\right)^{\cdot \cdot} (a^2 + b^2) \\
    &+ (\dot{z}_p)^2((a^2 + b^2)) + (2a^2 - b^2)^2 + (2b^2 - a^2)^2 + \frac{81}{16}S^2 \\
    &+ \frac{81}{16}(a^2 + b^2) \left((z_p^3)\right)^{\cdot \cdot} \right\}.
\end{align*}
\]

(51)

The explicit substitution \( z_p \) simplifies the expression Eq. (10) in [3] that just the diagonal components of quadrupole moment contribute to the result, see (C11). The expression (51) further simplifies to

\[
\mathcal{L}_{\text{quad}} = \frac{8G}{15c^6}S^2 \rho_0 v_p^6. 
\]

(52)

After inserting (39) and (24), we will obtain the final expression for luminosity of gravitational radiation,

\[
\mathcal{L}_{\text{quad}} = \frac{8}{15} \frac{G}{c^2} \left( \frac{P_L}{c} \right)^3, 
\]

(53)

where we have used the pressure (24).

The luminosity then depends on the power of the laser, the density of the material and the laser wavelength and the surface of the focal spot \( S \). The numerical factor in front of the fraction for estimation will be presented in the next subsection.

The perturbation component \( h_{zz}^{GW} \) becomes using (39), (24) and (23),

\[
|h_{zz}^{GW}|^2 \leq \frac{8G}{15c^6}S^2 \rho_0 v_p^6 t. 
\]

(54)

This is the final formula for the perturbation of the space by gravitational wave in the \( zz \) direction. The formula has different power of laser power than the previous models. The value of the perturbation decreases with the distance as \( 1/r \) and will be zero in the infinity. The numerical factors will be evaluated in the next subsection for specific values for an experiment.

6. The estimations for the \( h_{\mu \nu} \) and \( \mathcal{L}_{\text{quad}} \) for real experiment

We will evaluate the numerical factors in final results for luminosity (53) and the perturbation \( h_{zz}^{GW} \) of the space by the gravitational wave in \( zz \) direction, (54), which will be useful for real experiment. Now, we arrive to the expression for the luminosity as

\[
\mathcal{L}_{\text{quad}} \leq \frac{5.572 \times 10^{-30}}{\text{kg m}^3} \frac{5^6}{S[m^2]} \frac{P_L^3[\text{PW}]}{[\text{kg m}^3]}.
\]

(55)

Similarly to the previous case, we obtain

\[
|h_{zz}^{GW}| \leq 2.2267 \times 10^{-35} \frac{kg^{7/2}}{S^2 m^{5/2}} \frac{1}{[\text{m}]} \frac{5^{3/2}}{[\text{PW}]} \frac{[\text{PW}]}{[\text{ps}]} \frac{[\text{PW}]}{[\text{ps}]}.
\]

(56)

When we substitute achievable laser parameters into expressions for luminosity and the perturbation we will get the estimations for the experiment:

\[
P_L = 7 \text{ PW}, \rho_0 = 1 \text{ g/cm}^3, \Phi = 30 \mu \text{m}, \tau = 1 \text{ ps},
\]

(57)
and the detection distance is again \( R = 10 \text{ m} \) and \( S = \Phi^2 \pi/4 \) where \( \Phi \) is diameter of the target. The detection distance is \( R = 10 \text{ m} \) or equivalently \( f_2 = f = 10 \text{ m} \), \( z_L = 12 \text{ m} \), parameters \( a, b \) of the target foil are \( a = b = 1 \mu \text{m} = 1 \times 10^{-6} \text{ m} \) and therefore \( L_z = 7 \times 10^5 \text{ [PW/cm}^2\text{]} \) and the velocity \( v_r = 153008 \text{ [km/s]} \). The wavelength of the gravitational wave is \( \lambda_g = 300 \mu \text{m} \) and the frequency is \( \nu_g = 1 \text{ THz} \).

The final estimations for the luminosity and the per-

The waves are linearly polarized in the direction of the propagation, in the \( z \)-coordinate.

The time dependency is hidden in \( z_r \) [4]. Contrary to the shock wave model [3] and piston model \(\text{(IV B)}\) the amplitudes do not vanish but are quite small \( O(e^{-b t}) \) and vanish as \( t \to \infty \) or \( r \to \infty \). The amplitude \( A_{z_1}^+ = 0 \) because of our choice of square target \( b^2 - a^2 = 0 \). The remaining amplitude \( A_{z_2}^+ \) is pictured in Fig. 3 where we observe that the amplitude approaches zero quickly. Therefore waves do radiate along the \( z \) axis in which the motion occurs but very weakly. It is surprising result because in the linear gravitation such waves do not exist, just the transversal ones. It is the consequence of the non–conservation of mass by the ablation model and the finite integration boundary \( z_L \).

![FIG. 3. The amplitudes \( A_{z_2}^+ \) (60) is pictured with dependence on time \( t\) [s]. The amplitude approaches zero quickly.](image)

The gravitational radiation is strongly non–zero in the other directions, for example in the direction of the \( x \) and \( y \) axes, see the next subsections.

The amplitudes are one order lower than the result in [3, 4].

### IV. THE POLARIZATION OF GRAVITATIONAL WAVES

In this section, we are going to investigate the two polarization modes of the gravitational waves which are generated by ablation and piston models. We derive the amplitudes of the gravitational wave in two independent modes, + and –, and focus on their interpretation which would be useful for real experiment conditions while we will refer to the theory part in the first part of this paper [3].

#### A. The \( x, y \) and \( z \) directions of the wave vector for ablation model

First, we are going to investigate the gravitational perturbations in the direction of the propagation, in the \( z \)-coordinate.

1. The wave propagation in the \( z \)-direction

The \( h_{ij}^{TT} \) Eq. (9) in [3] has then the only non–vanishing components

\[
\begin{align*}
  h_{z_2}^{TT} &= -h_{zy}^{TT} = \text{Re}\{A_+ e^{-i \omega(t+z/c)}\}, \\
  h_{xy}^{TT} &= h_{yx}^{TT} = \text{Re}\{A_x e^{-i \omega(t+z/c)}\},
\end{align*}
\]

for the wave propagation vector in the \( z \)-direction \( n = (0, 0, -1) \).

The waves are linearly polarized in the direction of propagation as in the case of shock wave model [3]. We obtain the amplitudes of the polarization modes for the ablation model in the form, Eq. (49) in [3] then we use the mass moments expressed in terms of derivatives of function \( z \),

\[
\begin{align*}
  A_{z_2}^+ &= \frac{4 G}{3 r c^4} S \rho_0 (b^2 - a^2) z_L e^{-b_1} \frac{z}{z^2} = 0, \\
  A_{x}^+ &= -\frac{2 G}{r c^4} S \rho_0 z_L e^{-b_1} \frac{z}{z^2}.
\end{align*}
\]

The waves are linearly polarized as in the previous case. We obtain the amplitudes of the polarization modes, Eq. (54) in [3] then we use the mass moments expressed in terms of derivatives of function \( z \), the am-
amplitudes read as follows,

\[
A^+_t = \frac{4G}{r c^4} S \rho_0 \left[ -6 \dot{z}_r (\dot{z}_r)^2 \left( -\frac{2}{3} + e^{-b_1} (b_1^2 + 1) \right) + 4 \dot{z}_r e^{-b_1} z_L \frac{\dot{z}_L}{z_r} - 1 \right] 12 (\dot{z}_r)^2 z_L e^{-b_1} - \frac{e^{-b_1}}{z_r} \frac{e^{-b_1}}{3 z_r^2} z_r^2 b_1^2 \right] ,
\]

(62)

\[
A^+_s = \frac{4G}{r c^4} S \rho_0 \left[ 2 b_1 e^{-b_1} \right] .
\]

(63)

We have obtained non-zero amplitudes for both '+' and '×' polarization modes. The amplitudes depend on the focus area \(S\), the density of the material \(\rho_0\), the velocity of the ions \(v_r\) and constant \(z_L\). The amplitudes vanish as the radial distance \(r \to \infty\) and they decrease like \(1/r\).

Importantly, both amplitudes of '+' and '×' polarization are time dependent. The dependency originates from the expression \(b_1\) which was not present in the shock wave model and in fact generalizes the results of the shock wave model [3]. The amplitude for '×' polarization was not time dependent.

We observe that the terms containing \(b_1\) in the numerator contribute less in the limit \(t \to \infty\), such as \(\lim_{t \to \infty} e^{-b_1} = e^{-1}\) and \(\lim_{t \to \infty} b_1 = 1\), the terms as \(\frac{e^{b_1}}{z_r}\), where \(k = 1, 2, 3\), vanish in the limit. The most dominant terms remain the first terms in the expressions for the amplitudes (62) and (63) which have functional similar character, except the terms with \(b_1\), as the shock wave model.

When the radiation reaches the detector at \(t_{det} = f_2/v_r\), the most dominant term in \(A^+_t\) vanishes, the last two diverge since the division by 0. The \(A^+_s\) has just the first term non-divergent.

The amplitudes then reduce to

\[
A^+_t = \frac{8G}{r c^4} S \rho_0 v_r^2 \left[ 3 \dot{z}_r \left( -\frac{2}{3} + e^{-b_1} (b_1^2 + 1) \right) - 2 e^{-b_1} z_L (3 - \frac{1}{2} \dot{z}_r) \right] ,
\]

(64)

\[
A^+_s = \frac{4G}{r c^4} S \rho_0 \left[ 2 b_1 (\frac{1}{2} + b_1 e^{-b_1}) \right] .
\]

(65)

while we have omitted the terms of type \(e^{-b_1}/z_r\) which diverge for our choice of the start of coordinate system and have smaller additional contribution than the remaining terms. The amplitudes are depicted in the Fig. 4 for experimental values specified in estimations part (III A 6).

The amplitude \(A^+_s\) shows jump down at \(t_{det}\) because of the \(zr = 0\) and then grows like the amplitude \(A^+_t\). The amplitude \(A^+_s\) shows open profile function which continues to \(\infty\). Correctly, the function should close down because GW loses its energy. The opened function is again caused by the mass non-conservation in the ablation model. We will investigate the influence of the wave on test particles in Section (VI).

![FIG. 4. The amplitudes \(A^{+}_t\) (64) and \(A^{+}_s\) (65) are pictured in dependence on time \(t[s]\). The amplitudes do not vanish in time due to fact that mass is not conserved by the ablation model.](image)

3. The wave propagation in the \(y\)–direction

The last direction we are going to investigate is the \(y\)-direction transversal to the direction of motion in \(z\)-coordinate. The perturbation tensor Eq. (9) in [3] for the wave vector in the \(y\)-direction \(n = (0, 1, 0)\) reads

\[
\begin{align*}
\tilde{h}_x & = \tilde{h}_y = \Re \left\{ A \times e^{-i \omega (t - y/c)} \right\} , \\
\tilde{h}_z & = h_{zz} = \Re \left\{ A \times e^{-i \omega (t - y/c)} \right\} .
\end{align*}
\]

(66)

Again, the waves are linearly polarized as in the previous cases. The amplitudes of the polarization modes become, Eq. (59) in [3] then we use the mass moments expressed in terms of derivatives of function \(z\),

\[
\begin{align*}
A^+_t & = \frac{4G}{r c^4} S \rho_0 \left[ -6 \dot{z}_r (\dot{z}_r)^2 \left( -\frac{2}{3} + e^{-b_1} (b_1^2 + 1) \right) + 4 \dot{z}_r e^{-b_1} z_L \frac{\dot{z}_L}{z_r} - 1 \right] 12 (\dot{z}_r)^2 z_L e^{-b_1} - \frac{e^{-b_1}}{z_r} \frac{e^{-b_1}}{3 z_r^2} z_r^2 b_1^2 \right] , \\
A^+_s & = \frac{4G}{r c^4} S \rho_0 \left[ 2 b_1 (\frac{1}{2} + b_1 e^{-b_1}) \right] + \frac{z_L^2}{z_r^2} e^{-b_1} (2 (\dot{z}_r)^2 - (z_r + z_L)) .
\end{align*}
\]

(67)

(68)
The resulting amplitudes $A_+^a$ and $A_-^a$ have the form like in the direction $x$ (62) and (63) apart from the sign in $A_+^a$ and parameter $a$ instead $b$. Importantly, the $A_+^a$ and $A_-^a$ amplitudes are dependent on time. The results have the same character as in the previous case. The amplitudes vanish as the radial distance $r \to \infty$ and decrease as $1/r$.

\[
A_+^a = - \frac{8G}{r^2} S\rho_0 v_r^2 \left[ 3z_r \left( -\frac{2}{3} + e^{-b_i} (b_i^2 + 1) \right) \right. \\
-2e^{-b_i} z_L (3 - \frac{1}{z_r})], \quad (69)
\]

\[
A_-^a = - \frac{4G}{r^2} S\rho_0 \left[ 2v_r^2 \left( \frac{1}{2} + b_i e^{-b_i} \right) \right], \quad (70)
\]

while we have omitted the terms of type $e^{-b_i}/z_r$ which diverge for our choice of the start of coordinate system and have smaller additional contribution than the remaining terms. The amplitudes are depicted in Fig. 5 which is just rotated Fig. 4 because of the minus sign in (70).

FIG. 5. The amplitude $A_+^a$ (70) is pictured in dependence on time $t[s]$. The image for $A_-^a$ in Fig. 4 is the same for this case.

The amplitudes of radiation and the radiative characteristics of the radiation are one of the main results of this paper.

B. The $x$, $y$ and $z$ directions of the wave vector for piston model

First, we are going to investigate the gravitational perturbations in the direction of the propagation, in the $z$–coordinate.

1. The wave propagation in the $z$–direction

The $h_{ij}^{TT}$ Eq. (9) in \[ used \] has then the only non–vanishing components \[ for \] the wave propagation vector in the $z$–direction $n = (0,0,1)$. The amplitudes are given by Eq. (49) in \[ used \], after substituting the $z_p$ \[ read as \] follows,

\[
A_+^p = \frac{1}{3r} S\rho_0 \left( a^2 - b^2 \right) = 0, \quad A_-^p = \frac{1}{2r} S\rho_0 \left( a^2 - b^2 \right) = 0.
\]

Therefore the radiation $h_{ij}^{TT}$ is vanishing for the orientation of the wave vector into the direction of motion of the experiment. The waves do not radiate along the $z$ axis.

2. The wave propagation in the $x$–direction

The $h_{ij}^{TT}$ Eq. (9) in \[ used \] has the only non–vanishing components for the wave vector in the $x$–direction $n = (1,0,0)$ \[ used \] where the amplitude are given by Eq. (54) in \[ used \] and after substitution to $z_p$ we get,

\[
A_+^x = \frac{1}{3r} S\rho_0 \left( b^2 z_p^2 - \left( z_p^3 \right) \right) = - \frac{2G}{r^2} S\rho_0 v_p^3 t, \quad (72)
\]

\[
A_-^x = \frac{1}{2r} S\rho_0 \left( b^2 z_p^2 - \left( z_p^3 \right) \right) = - \frac{1G}{r^2} S\rho_0 v_p^3 t. \quad (73)
\]

3. The wave propagation in the $y$–direction

The perturbation tensor on $TT$ calibration Eq. (9) in \[ used \] for the wave vector in the $y$–direction $n = (0,1,0)$ are \[ used \]. The amplitudes are given by Eq. (59) in \[ used \] and after substitution for $z_r$ we get,

\[
A_+^y = \frac{1}{3r} S\rho_0 \left( a^2 z_r^2 - \left( z_r^3 \right) \right) = - \frac{2G}{r^2} S\rho_0 v_p^3 t, \quad (74)
\]

\[
A_-^y = - \frac{1G}{r^2} S\rho_0 \left( a^2 z_r^2 - \left( z_r^3 \right) \right) = - \frac{1G}{r^2} S\rho_0 v_p^3 t. \quad (75)
\]

The resulting amplitudes $A_+^p$ and $A_-^p$ have the form as in the direction $x$ \[ used \] and \[ used \] apart from the sign in $A_+^p$. Importantly, the $A_-^p$ amplitude depends linearly on time and again the other one $A_+^p$ is constant in time. The results have the same character as in the previous case and correspond to results for shock wave model \[ used \]. The amplitudes vanish as the radial distance $r \to \infty$ and decrease as $1/r$.

The GW amplitudes are the main result of the paper.

C. The general direction of the wave vector

Finally, we are going to investigate the amplitudes with the general wave vector of propagation. The general direction of the wave propagation can be expressed in the spherical coordinates as $n = (\sin \theta \sin \phi, \sin \theta \cos \phi, \cos \theta)$, and the perturbation tensor can be obtained via Eq. (9) in \[ used \] and the projector $A_{ij,kl}$. 
1. The case of ablation model

The general expressions for the two modes of polarizations are Eq. (62-63) in [2], ([3]), Afterwards we use the mass moments expressed in terms of derivatives of function $z$, the amplitudes read as follows,

$$A_+^a(t; \theta, \phi) = \frac{1}{r^3} S \rho_0 \frac{4G}{v_r^2} \left[ a^2 (\cos^2 \phi - \sin^2 \phi \cos^2 \theta) + b^2 (\sin^2 \phi - \cos^2 \phi \cos^2 \theta) \right] \sin \theta (a \sin \phi + b \cos \phi)$$

$$-3 \sin^2 \theta \left[ 3(-v_r t + f_2) (-\frac{2}{3} + e^{-b_1} (b_1^2 + 1)) - 2z_L e^{-b_1} (3 - \frac{1}{z_r}) \right]$$

$$A_\times^a(t; \theta, \phi) = \frac{1}{r^3} S \rho_0 \frac{4G}{v_r^2} \left[ a^2 (\cos^2 \phi - \sin^2 \phi \cos^2 \theta) + b^2 (\sin^2 \phi - \cos^2 \phi \cos^2 \theta) \right] \sin \theta (a \cos \phi - b \sin \phi)$$

To visualize the amplitudes it is convenient to rewrite them as

$$A_+^a(t; \theta, \phi) = \frac{4G}{v_r^2} S \rho_0 v_r^2 P_{A_+^a}(\theta),$$

$$A_\times^a(t; \theta, \phi) = -\frac{4G}{v_r^2} S \rho_0 v_r^2 P_{A_\times^a}(\theta),$$

where the angular dependence is denoted as

$$P_{A_+^a}(\theta, r) = \frac{1}{r} \left\{ \frac{1}{2} + b_1 e^{-b_1} \right\} \sin \theta (a \sin \phi + b \cos \phi)$$

$$-3 \sin^2 \theta \left[ 3(-v_r t + f_2) (-\frac{2}{3} + e^{-b_1} (b_1^2 + 1)) - 2z_L e^{-b_1} (3 - \frac{1}{z_r}) \right]$$

We have included the $r$ dependence in the angular parts of the amplitudes in order to investigate the dependence. Let us note that the time when the radiation reaches the detector is

$$t_{det} = f_2/v_r,$$ 

then the geometrical structure of $P_{A_\times^a}(\theta, r)$ changes because of $f_2 - v_r t_{det} = 0$. The choice of coordinates enables us to choose $f_2$, this change of structure is then just of coordinate nature and has no physical meaning. We have plotted the amplitude $A_+^a$ in the following graphs Fig. 6 and Fig. 7. The graphs were made for values $a = b = 1$ mm $= 0.1$ cm, $I_L = 50$ [PW/cm$^2$] and $R_t = 15.144$ [kg/m$^3$] for Carbon. The velocity $v_r = 1.14 \times 10^8$ [m/s] and $b_1 = 0.2$ starts at this value as is growing in time. The amplitude $A_\times^a = 4.34 \times 10^{-41}$ and $A_+^a = -4.34 \times 10^{-41}$.

The angular shape of $A_\times^a(\theta, t)$ of the ablation wave at start $t = 0$ is depicted in Fig. 6. The angular dependence has a symmetric shape of toroid with the center at $z = 0$ ($\theta = \phi = 0$). The surfaces inside the toroid represent angular structure for larger $r$ and we observe that the magnitude of the toroid becomes smaller as expected as $1/r$. Before the radiation reaches the detector $t < t_{det}$, the amplitude is smaller than Fig. 6. The image of the amplitude $A_\times^a$ is depicted in the Fig. 8 where the first image is for $t = 0$ and the second for $t = 8 \mu s$. The amplitude is slightly decreasing in time as the previous $A_+^a$ amplitude.

The orientation of the both amplitudes on left toward each other are very similar to ones for the shock wave model, see Fig. 8 in [3], therefore we will not present them again.

The difference in the time dependency of the two independent polarization modes might be very important for the experimental detection, because it would be possible to distinguish the two modes of polarization.
FIG. 6. The angular part of amplitude $P_{A_0^+}(\theta, r)$ pictured in dependence on $\theta$ angle and additional $\phi$ angle in radians at the time $t = 0$ [s] in 3D and 2D figures. The amplitude has a shape of toroid with symmetry around axes $z = 0$. The dependence on $1/r$ is depicted in smaller surfaces in the figure, the biggest surface is $r = 1$ m, then $r = 1.5$ m and $1.8$ m. The surface is getting smaller as $r \to 10$ m (at the distance of the detector) and approaches 0 as $r \to \infty$. The toroid was cut on purpose to see the inner surfaces of lower $r$. The polar 2D diagram was plotted for fixed angle $\phi = \pi/2$.

FIG. 7. The angular part of amplitude $P_{A_0^+}(\theta, r)$ pictured in dependence on $\theta$ angle and additional $\phi$ angle in radians at the time $t = 8\mu s$ in 3D and 2D figures. The amplitude has a shape of toroid with symmetry around axes $z = 0$. The dependence on $1/r$ is depicted in smaller surfaces in the figure, the biggest surface is $r = 1$ m, then $r = 1.5$ m and $1.8$ m. The toroid was cut on purpose to see the inner surfaces of lower $r$.

2. The case of piston model

Afterwards we use the mass moments expressed in terms of derivatives of function $z$, the amplitudes read
FIG. 8. The angular part of amplitude $P_{A^p}(\theta, r)$ pictured in dependence on $\theta$ angle and additional $\phi$ angle in radians at the time $t = 0$ in 3D. The amplitude has a shape of a ball with start at $z = 0$.

as follows,

$$A^p_\theta(t; \theta, \phi) = \frac{1}{r} G \frac{c^4}{r} S \rho_0 \left[ -\frac{1}{3} z_p^2 \sin^2 \theta 
+ \frac{1}{4} (z_p^2) \sin (2 \theta) (a \sin \phi + b \cos \phi) 
+ \frac{1}{3} z_p a^2 (\cos^2 \phi - \sin^2 \phi \cos^2 \theta) 
+ \frac{1}{3} z_p b^2 (\sin^2 \phi - \cos^2 \phi \cos^2 \theta) 
- \frac{3}{4} S z_p \sin (2 \phi) (1 + \cos^2 \theta) \right], \quad (85)$$

$$A^p_\phi(t; \theta, \phi) = \frac{1}{r} G \frac{c^4}{r} S \rho_0 \left[ -\frac{1}{2} z_p^2 \sin \theta (a \cos \phi - b \sin \phi) 
+ \frac{1}{3} (a^2 - b^2) \sin 2 \phi + \frac{1}{2} S \cos 2 \phi \right]. \quad (86)$$

After we use the ansatz for the $z_p$, we get

$$A^p_\theta(t; \theta, \phi) = \frac{1}{2 r} G \frac{c^4}{r} S \rho_0 v_p^2 \left[ -4 v_p t \sin^2 \theta + \sin 2 \theta (a \sin \phi + b \cos \phi) \right], \quad (87)$$

$$A^p_\phi(t; \theta, \phi) = -\frac{1}{r} G \frac{c^4}{r} S \rho_0 v_p^2 \sin \theta (a \cos \phi - b \sin \phi). \quad (88)$$

The final expressions (87) and (88) are very similar to results in Eq. (66–67) [5]. The difference is in the positive sign of the second term in (87) and minus sign in the whole expression (88). We will rewrite the amplitudes into

$$A^p_\theta(t; \theta, \phi) = \frac{1}{2} G \frac{c^4}{r} S \rho_0 v_p^2 P_{A^p_\theta}, \quad (89)$$

$$A^p_\phi(t; \theta, \phi) = -\frac{1}{r} G \frac{c^4}{r} S \rho_0 v_p^2 P_{A^p_\phi}, \quad (90)$$

where we denote the angular part of the amplitude

$$P_{A^p_\theta}(\theta, r) = \frac{1}{r} \left[ -4 v_p t \sin^2 \theta + \sin 2 \theta (a \sin \phi + b \cos \phi) \right], \quad (91)$$

$$P_{A^p_\phi}(\theta, r) = \frac{1}{r} \sin \theta (a \cos \phi - b \sin \phi). \quad (92)$$

The graphs were made with the parameters, $r = 10$ m, parameters $a, b$ of the target foil are $a = b = 1 \mu m = 1 \times 10^{-6}$ m and therefore $I_L = 7 \times 10^8$ [PW/cm$^2$] and the velocity $v_p = 153008$ [km/s].

In the following figures, we will observe the effect of time dependence of the $A^p_\theta$ amplitude. The angular shape of $P_{A^p_\theta}(\theta, t)$ of the piston at start $t = 0$ is depicted in Fig. 9. The angular dependence has a symmetric shape of cloverleaf with the center at $z = 0$ ($\theta = \phi = 0$), because the first term in (81) vanishes. The surfaces inside the cloverleaf represent angular structure for larger $r$ and we observe that the magnitude of the cloverleaf becomes smaller as expected as $1/r$. For shock wave model, we
got this geometry structure for the detection time and the shape was of coordinate nature – choice of the start of coordinates. The reason we obtain the geometry here is because we have chosen the start of coordinates in the opposite way than the shock wave model set up, therefore we get the structure at the start of the experiment.

At the time shortly before the detector \( t < t_{\text{det}} \), the angular dependence is larger in Fig. 10 than the one at \( t = 0 \) in the previous Fig. 9 and the geometry changes to the toroid geometry as in the shock wave model. Then the time when radiation reaches detector is \( t_{\text{det}} = 1.3 \times 10^{-6} \text{s} \) and the amplitudes \( A_{A^+} = 9.7 \times 10^{-38} \) and \( A_{A^\times} = 1.95 \times 10^{-37} \).

At the moment \( t_{\text{det}} \) when the radiation reaches the detector, the geometry does not change in Fig. 11, we can see the structure of toroid again. We observe that the amplitude of the angular dependence is much larger than the two previously pictured.

The amplitude for polarization mode \( \times \) is the almost identical to Fig. 6 in \( \mathbb{E} \) up to amplitude \( \mathbb{E} \) which has opposite sign. Also the orientation of both amplitudes \( x \) and \( \times \) is similar to Fig. 8 on the left in \( \mathbb{E} \). The toroidal amplitudes are rotated for 180° in \( \theta \) compared to the images for shock wave model, the Figs. 10 and 11 are rotated accordingly to show off the inside layers.

The difference in the time dependency of the two independent polarization modes might be very important for the experimental detection in both shock wave and piston models in the quadrupole approximation of linear gravity.
Then on piston model.

along the expressions in subsection (4.3) in chapter IV.

where

\begin{align}
S_n^a &= \frac{16}{9} \frac{c^3}{G \pi r^2 S^2 \rho_0^2 v_r^6} \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2, \\
S_n^a &= \frac{16}{3} \frac{c^3}{G \pi r^2} \frac{S^2 \rho_0^2 v_r^6}{v_s} \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2,
\end{align}

and the expression for the general wave vector Eq. (77) in \[5\] results in
\begin{align}
S_n^a &= \frac{16}{9} \frac{S^2 c^3 \rho_0^2 v_r^6}{G \pi r^2} \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2 \times 
\left[ 9 - (\sin^2 \theta + 16 \cos^2 \theta) + (2 \cos^2 \theta + \sin^2 \theta)^2 \right].
\end{align}

\(P_{\text{GW}}\) depends only on the \(\theta\) angle which is a consequence of the axis symmetry of the problem. The characteristics behave as \(1/r^3\) as \(r \to \infty\) contrary to \(1/r\) decay of amplitudes. To visualize the characteristic, it is useful to separate the angular part from its amplitude as
\begin{align}
S_n^a &= \frac{16}{9} \frac{S^2 c^3 \rho_0^2 v_r^6}{G \pi} \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2 \left( \frac{9}{r^2} \right)^{1/2} P_{\text{GW}}(\theta) \quad (96)
\end{align}

where the angular dependence
\begin{align}
P_{\text{GW}}(\theta) &= \frac{1}{r^2} \left[ 9 - (\sin^2 \theta + 16 \cos^2 \theta) + (2 \cos^2 \theta + \sin^2 \theta)^2 \right].
\end{align}

In the calculations we have neglected the terms of type \(e^{-b_l} r\) as in previous calculations.

The radiation structure is pictured in Fig. 12. The amplitude \(A_{\text{GW}}\) has a specific value \(A_{\text{GW}} = 8.94 \times 10^{44}\), for values \(a = b = 1\) mm, \(I_L = 50\) [PW/cm²], where the velocity \(v_s = 1.14 \times 10^6\) [m/s]. The dependence on \(\theta\) and \(r\) is plotted in Fig. 12, and the polar dependence on \(\theta, \phi\) and \(r\) is plotted in Fig. 13 (2D and 3D).

This directional characteristic would help with the experimental set up and positions of the detectors. The directional structure [12] has similar toroidal shape as the structure for shock wave model Fig. 9 in [5] and piston model [15] but with the additional radiative part in the \(z = 0\) direction of shape of a dumbbell. It suggests existence of longitudinal GW radiation in the direction of the laser propagation, which should not occur in linear gravity, and is the consequence of the broken mass conservation law as mentioned earlier.

2. The case of piston model

Again, the \(A_x\) amplitude is time-independent, therefore just the \(A_{\theta}^p\) contributes to the effective tensor,
\begin{align}
t_{00}^{\text{GW}} &= \frac{c^4}{32 \pi G} \left( A_{\theta}^p \right)^2 = \frac{1}{8 \pi r^2 c^4} \frac{S^2 \rho_0^2 v_r^6}{v_s} \sin^4 \theta,
\end{align}

D. The radiative characteristics for generated gravitational waves

In this part, we will calculate radiative characteristics along the expressions in subsection (4.3) in chapter IV [3]. First, we will concentrate on ablation model and then on piston model.

1. The case of ablation model

In our case, the amplitudes are time-independent for ablation model, then the invariant density Eq. (71) in [3] reads,
\begin{align}
t_{00}^{\text{GW}} &= \frac{81}{c^4} \frac{G}{\pi r^2 c^4} S^2 \rho_0^2 v_r^6 \sin^4 \theta \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2,
\end{align}

which functionally depends on \(r\) and \(\theta\) angle. The energy goes to zero as \(r\) approaches infinity, where we have neglected terms of type \(e^{-b_l} r\).

The energy spectrum is then trivial
\begin{align}
\frac{dE}{dr} &= \frac{c^3}{16 \pi G} \rho_0 v_r^6 \sin^2 \theta \int_0^\tau \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right) \right) d\Omega.
\end{align}

Now, we are able to substitute the ansatz for the \(z_s\) into into Eqs. (74–76) in [3]. Then the expressions for the wave vector in directions \(n = x, y, z\) read,
\begin{align}
S_n^a &= S_n^a = \frac{11}{12} \frac{c^3}{G \pi r^2} S^2 \rho_0^2 v_r^6 \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2,
\end{align}

\begin{align}
S_n^a &= \frac{16}{3} \frac{c^3}{G \pi r^2} S^2 \rho_0^2 v_r^6 \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2,
\end{align}

and the expression for the general wave vector Eq. (77) in [3] results in
\begin{align}
S_n^a &= \frac{16}{9} \frac{S^2 c^3 \rho_0^2 v_r^6}{G \pi r^2} \left( \frac{2}{3} + e^{-b_l (b_l^2 + 1)} \right)^2 \times 
\left[ 9 - (\sin^2 \theta + 16 \cos^2 \theta) + (2 \cos^2 \theta + \sin^2 \theta)^2 \right].
\end{align}

FIG. 11. The angular part of amplitude \(P_{\text{GW}}^a(\theta, r)\) pictured in dependence on \(\theta\) angle and additional \(\phi\) angle in radians at the detector in 3D and 2D figures. The magnitude of the angular part of amplitude is much smaller than previous ones. The polar 2D diagram was plotted for fixed angle \(\phi = \pi/2\).
The radiation characteristics $S_{n}(96)$ pictured in dependence on $\theta$ angle and $r$. We have plotted just the angular dependence $P_{S_{n}}(\theta)$ (97), $S_{n} = A_{S_{n}}P_{S_{n}}(\theta)$, (98). The surface is approaching 0 at the distance of the detector $r = 10$ m, also while $r \to \infty$ the surface approaches zero.

which functionally depends on $r$ and $\theta$ angle. The energy goes to zero as $r \to \infty$. The energy spectrum is then trivial $\frac{dE}{dA} = c\frac{3}{16\pi G}$.

Now, we are able to substitute the ansatz for the $z_{s}$ into Eqs. (74–76) in [5]. Then the expressions for the wave vector in directions $n = x, y, z$ read,

$$S_{n}^{\alpha} = S_{n}^{\beta} = \frac{c^{3}}{36G\pi r^{2}} S_{n}^{\alpha \beta \gamma \delta} \rho_{0}^{2} v_{p}^{6}, S_{n \alpha} = 0,$$

and the expression for the general wave vector Eq. (120) in [5] results in

$$S_{n}^{\alpha} = \frac{S_{n}^{\alpha \beta \gamma \delta}}{324G\pi r^{2}} [12 - 4(\sin^{2}\theta + 4 \cos^{2}\theta) + (2 \cos^{2}\theta - \sin^{2}\theta)^{2}]^{\frac{1}{2}}$$

(99)

The radiative characteristics (99) depends only on the $\theta$ angle which is a consequence of the axis symmetry of the problem. The characteristics behave as $1/r^{2}$ as $r \to \infty$ contrary to $1/r$ decay of amplitudes. To visualize the characteristic, it is useful to separate the angular part from its amplitude as

$$S_{n}^{\alpha} = 324G\pi r^{2} P_{S_{n}}(\theta)$$

(100)

where the angular dependence

$$P_{S_{n}}(\theta) = \frac{1}{r^{2}} [12 - 4(\sin^{2}\theta + 4 \cos^{2}\theta) + (2 \cos^{2}\theta - \sin^{2}\theta)^{2}]^{\frac{1}{2}}.$$  

(101)

The radiation structure is pictured in Fig. 13 which is the same as for the shock wave model thanks to the same resulting formulae (101). The amplitude $A_{S_{n}}$ (101) has a specific value $A_{S_{n}} = 4.54 \times 10^{69}$ for $a = b = 10^{-6}$ m, $I_{L} = 7 \times 10^{8}$ PW/m$^{2}$ and $v_{s} = 153008$ m/s. The polar dependence on $\theta, \phi$ and $r$ in Fig. 14 and the dependence on $\theta$ and $r$ is plotted in Fig. 15.

The directional structure of radiation is the same for the shock wave model and for the piston model in the approximation we use in the paper, etc. the gravity in linear approximation up to quadrupole moment in the moment expansion. The differences might appear in higher orders of the expansion.
E. The angular momentum

The angular momentum carried away per unit time by the gravitational waves is given by Eq. (81) in [5], we obtain for ablation and piston model (using derivatives in (C 2))

\[
\left( \frac{dJ^i_{\text{ablation}}}{dt} \right)_{\text{quad}} = 0 \rightarrow J^i_{\text{ablation}} = \text{const},
\]

and the angular momentum of the radiation in the shock wave model stays constant in time due to the single dimension of the experiment. In case of ablation model, we have neglected the terms of type \( e^{-b r}/z_r \) to obtain the result.

V. THE BEHAVIOUR OF TEST PARTICLES IN THE PRESENCE OF GRAVITATIONAL WAVE

We will analyse the test particles for the ablation and piston models in the same way as section V in [5].

A. The predictions for detector

According to Eq. (83) in [5], we can estimate the linear size \( L \) of the possible detector

\[
L_{\text{ablation}} \ll 4.7746 \times 10^{-2} \text{ m}, \quad L_{\text{piston}} \ll 4.778 \times 10^{-5} \text{ m},
\]

which might serve as useful estimation for validity of the future experiment and the detector. We have used the numerical values mentioned in the evaluation of the low limit condition [II A 1] and [II A 1]

We can rewrite the condition in general way using (2) as

\[
L_{\text{experiment}} \ll \frac{1}{2\pi} \tau c,
\]

which connects the linear size of the detector with duration of the pulse in the experiment.
B. Movement of particles

Again, we will investigate the behaviour of test particles in $x$ direction in the mode $+$ and $\times$ for both models, ablation model and piston model. We will use the geodesic equation Eq. (82), which can be rewritten in a form of ellipse.

C. The amplitudes for ablation model

First, we will look at the mode $+$ for the wave vector in $x$ direction, which is given by relations Eq. (85) of the geodesic equations Eq. (82) in [5].

For convenience, we will shift the start of the coordinates to $z = f_2$, then the coordinates of TT will be $x = y = 0$ and $z = 0$ and $t = \tau + O(h)$. Without losing any information, we perform a phase shift, $+\pi/2$, and get $h_{zz}(\tau) = A_2^x \sin \omega \tau$. When $\tau = 0$ then $h_{zz} = h_{zz}(\tau) \neq 0$ and in fact the function $b_I$ diverge, therefore we will investigate the behavior in small area are around zero $0 < \tau < \epsilon$ where $\epsilon$ is small number. Generally the amplitudes are non–zero for $0 < \tau < \epsilon$, because of the correction terms with $b_I$.

The semi-minor axes are

$$a[1 \pm A \sin \omega \tau], \quad (106)$$

where $A = \frac{1}{2} A_2^y$ and

$$A \equiv A|_{x^1=0} = -\frac{4G}{r c^4} \rho_0 v_r^2 \tau$$

$$\times \left(3(-v_r \tau)(-\frac{2}{3} + e^{-b_I}(b_I^2 + 1)) - 2e^{-b_I}z_L(3 + 1/v_r)\right), \quad (107)$$

the explicit form of $\frac{1}{2} h_{zz}^{TT}(\tau)|_{x^1=0}$ then is

$$\frac{1}{2} h_{zz}^{TT}(\tau)|_{x^1=0} = \frac{4G}{r c^4 \omega} \rho_0 v_r^2 \sin \omega \tau$$

$$\times \left(3(-v_r \omega \tau)(-\frac{2}{3} + e^{-b_I}(b_I^2 + 1)) - 2e^{-b_I}z_L(3 + 1/v_r)\right) \quad (108)$$

where the function $b_I$ becomes

$$b_I = \frac{z_L + v_r \tau}{v_r \tau}. \quad (109)$$

The negativity of the amplitude just means that the change will happen in the transversal direction to the positive one.

For specific values, $a = b = 1 \text{ mm} = 0.1 \text{ cm}$, $I_L = 50 \text{ [PW/cm}^2\text{]}$ and $R_i = 15.144 \text{ [kg/m}^3\text{]}$ for Carbon. The velocity $v_r = 4.884 \times 10^5 \text{ [m/s]}$ and $b_I = 0.2$ for time $t = 10^{-9} \text{ s}$. And $\omega = 2\pi c/\lambda = 6.26 \times 10^9$ where $\lambda = 0.3 \text{ m}$. Then we get the amplitude $A = -\frac{4G}{r c^4} \rho_0 v_r^2 = -1.483 \times 10^{-34}$. The effect of the GW on test particles does not produce ellipses but circles which grow with in time with distance between each circles for $\tau = \pi/2\omega$ from $\tau = 0$, then back to one circle at $\tau = \pi/2\omega$. Then the circles grow equi–distantly with time for $\tau = 3\pi/2\omega$. This effect of expansion of the test particles is definitely connected to the mass non conservation in the ablation model.

In the mode $\times$ we will get deformation of a circle with the only non–zero component $h_{yz}^{TT}$. The equations of motion have form Eq. (87) in [5], where the images for + mode will be rotated for $45^\circ$. Again, we perform a phase shift, $\pi/2$, and get $h_{yz}(\tau) = A_2^x \sin \omega \tau$, then for $\tau = 0$ we get $h_{yz \neq 0}$, the explicit form of $h_{yz}$ then become

$$\frac{1}{2} h_{yz}^{TT}(\tau)|_{x^1=0} = \frac{2G}{r c^4} S \rho_0 [2v_r^2 (\frac{1}{2} + b_I e^{-b_I})] \sin \omega \tau. \quad (110)$$

The circle of test particles under influence of GW in mode $\times$ changes to the shape ellipse of the same magnitude as the original circle at $\tau = 0.01$. When we compare the images for this mode with shock wave model, we observe that the main difference is the much sharper shape of the ellipse.

D. The amplitudes for piston model

In the mode $+$, is described by the Eq. (85) in [5] and the semi-minor axes and the $A = \frac{1}{2} A_2^y$ are for piston model

$$A \equiv A|_{x_2^1=0} = -\frac{1}{r c^4} S \rho_0 v_r^2 \tau, \quad (111)$$

and the explicit form of $\frac{1}{2} h_{zz}^{TT}(\tau)|_{x_2^1=0}$ then is

$$\frac{1}{2} h_{zz}^{TT}(\tau)|_{x_2^1=0} = -\frac{1}{r c^4 \omega} S \rho_0 v_r^2 (v_r \omega \tau) \sin \omega \tau. \quad (112)$$

The negativity of the amplitude just means that the change will happen in the transversal direction to the positive one. 

In the mode $\times$ we will get also deformation of a circle with the only non–zero component $h_{zy}^{TT}$, that is zero for $\tau = 0$, according to Eq. (87) in [5]. The component $h_{zy}^{TT}$ then becomes

$$\frac{1}{2} h_{zy}^{TT}(\tau)|_{x_2^1=0} = \frac{1}{2r c^4} S \rho_0 b v_r^2 \sin \omega \tau. \quad (113)$$

The component $h_{zy}^{TT}$ has constant amplitude therefore the ellipses do not change shape when time grows and the images for piston model will appear the same as for the shock wave model Fig. (11) and (12) in [5].

In this section, we have investigated behaviour of test particles in the presence of GW with two modes of polarization for ablation and piston models.

The main result of this section is that the time dependent amplitudes of polarization $+$ and $\times$ of ablation model influence the circle of particles to change the shape.
FIG. 16. The diagrams depict the position of test particles in time evolution under influence of GW wave with + polarization.

(a) The test particles at \( \tau = 0.1 \).

(b) The test particles at \( \tau = \pi/\omega; 5\pi/\omega; 9\pi/\omega; 13\pi/\omega \) and more.

(c) The test particles at \( \tau = \pi/\omega; 3\pi/\omega; 5\pi/\omega \) and more.

(d) The test particles at \( \tau = 3\pi/\omega; 7\pi/\omega; 11\pi/\omega; 15\pi/\omega \) and more.

FIG. 17. The diagrams depict the position of test particles in time evolution under influence of GW wave with \( \times \) polarization.

(a) The test particles at \( \tau = 0.01, \tau = 2\pi/\omega \) and more.

(b) The test particles at \( \tau = \pi/\omega; 5\pi/\omega; 9\pi/\omega; 13\pi/\omega \) and more.

(c) The test particles at \( \tau = \pi/\omega; 3\pi/\omega; 5\pi/\omega \) and more.

(d) The test particles at \( \tau = 3\pi/\omega; 7\pi/\omega; 11\pi/\omega; 15\pi/\omega \) and more.
to larger circles in magnitude, at $\tau = \pi/2\omega$, and equidistant circles at $\tau = 3\pi/2\omega$. In piston model, just the $+$ amplitude is time dependent and shapes the circle contrary to the $\times$ polarization which does not change the circle of test particles and the shape stays constant in time just for the piston model. This might serve as a measurable quality in the future experiments.

VI. THE CONCLUSION

In the second part of the paper, we have investigated the ablation and piston models for generation of gravitational waves for the possible experiments.

The ablation and piston models were investigated in linearized gravity in quadrupole approximation which proved to be valid for the low velocity condition of the suggested experiments.

We have calculated and analyzed the perturbation tensor $h_{ij}^{GW}$ and the luminosity of gravitational radiation $L_{GW}$ in linear gravity in low (non-relativistic) velocity approximation far away from the source. We have generalized the results presented in [5] where we included the dependence on the laser wavelength and material of the foil for the ablation model. The calculations are presented in detail and estimations for real experimental values are included. The ablation model has estimations for luminosity $L = 3.61 \times 10^{-20}$ [erg/s] and perturbation $h_{zz}^{GW} = 4.7 \times 10^{-39}$ for intensity $I_{L} = 50$ [PW/cm$^2$] and duration of pulse 1ns. The piston model has luminosity $L = 2.7 \times 10^{-18}$ [erg/s] and perturbation $h_{zz}^{GW} = 3 \times 10^{-43}$ for intensity $I_{L} = 7 \times 10^{8}$ [PW/cm$^2$] and duration of pulse 1ps. Let us repeat that the luminosity for $L = 1.69 \times 10^{-23}$ [erg/s] and perturbation $h_{zz}^{GW} = 2.37 \times 10^{-39}$ for intensity $I_{L} = 0.5 \times 10^{8}$ [PW/cm$^2$] and duration of pulse 1ns, [5].

The ablation model shows to have the highest luminosity of all the models and the perturbation of the same order as the shock wave model. Therefore the model might be the most suitable for the real experiment. In reality, it would depend on the technical realization of the possible model and the expenses.

Furthermore, we have investigated the two independent polarization modes of the gravitational radiation in the ablation and piston model. We have derived the amplitudes of the radiation in the three main directions of wave propagation, $x, y, z$. The radiation vanishes in the direction of motion in the $z$ direction for the piston model, the radiation in $\times$ mode appears for ablation model due to the fact that the model does not satisfy the mass conservation law and the existing radiation is an artefact which vanishes in time and distance.

The radiation is non-vanishing in other directions as $x$ and $y$ directions, the amplitude for mode $+$ of the polarization occurs is time dependent and the other amplitude for mode $\times$ is time–independent for piston model. For ablation model, the amplitudes are both time–dependent. This fact might be measured in the real experiment.

We have also investigated the amplitudes in the general wave direction given by angles $\theta$ and $\phi$. Again the amplitudes are for both modes time–dependent in case of ablation model and for piston model the mode $+$ is time dependent and the mode $\times$ is time–independent in the general case. The result might be used for convenient positioning of detectors in real experiment. The $+$ amplitude have toroidal symmetry around $z = 0$ axes for both ablation, piston and shock wave models. For ablation model, the $+$ amplitude is decreasing in magnitude with the distance as in shock wave model, while for piston model, the amplitude is slowly increasing in the magnitude with the distance from the source. The $\times$ amplitude has a shape of a ball which has one point attached to the $z = 0$ axes and remains constant in time and has much smaller amplitude than the $+$ amplitude.

The general directional structure of the radiation produced by the models has toroidal shape with symmetry around $z$ axes for both models, the structure of ablation model has additional radiation along the $z$ axes which is caused by the model does not satisfy the mass conservation and non–zero radiation appears as its consequence. The radiation vanishes as the distance approaches infinity. The angular momentum for all models is vanishing due to the one dimensional character of the models.

Moreover, we have analyzed the influence of gravitational waves on test particles thanks to the geodesics equation. The effects of GW on test particles for piston models are similar to shock wave model [3] where the time–dependent amplitudes changes shape of the ellipse in time contrary to the constant amplitude $\times$ which does not change the shape of the ellipse. In ablation model, both amplitudes $+$ and $\times$ are time–dependent and the $+$ mode amplitudes shape changes just in magnitude as the time progresses, the change to larger circles is growing for $\tau = \pi/2\omega$ which change back to circle for $\tau = \pi/2$ and then they change to circles at higher magnitude which are equidistant for $\tau = 3\pi/2\omega$ and its higher periods. The $\times$ mode changes the circle to sharp ellipse and back to circle as the shock wave model, but the ellipse for ablation model is much sharper. All of the analyzed aspects of the GW radiation might be used to set up the possible experiment in the future.

The remaining problem of the models is the detection of the gravitational waves which have the amplitude of the metric perturbation around $10^{-40}$.

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Appendix A: The derivatives of an ansatz for $z_a$

The derivatives of the arbitrary function $z_i(t)$ are

\[
\begin{align*}
(z_i^2) &= 2z_i\dot{z}_i, \\
(z_i^3) &= 2\left((\dot{z}_i)^2 + z_i(z_i)\right), \\
(z_i^4) &= 2\left(3\dot{z}_i\ddot{z}_i + z_i(z_i)\right), \\
(z_i^3) &= 3z_i^2\dot{z}_i, \\
(z_i^4) &= 3z_i(2\dot{z}_i^2 + z_i\dot{z}_i), \\
(z_i^5) &= 3\left(2\dot{z}_i^3 + 6z_i\dot{z}_i\ddot{z}_i + z_i^2(z_i)\right).
\end{align*}
\]  
(A1)

and after the ansatz for the $z_r$ (Ablation) we get

\[
\begin{align*}
(z_r^2) &= 2\dot{z}_r + d, \\
(z_r^3) &= 2(-\dot{v}_r + d)\ddot{z}_r, \\
(z_r^4) &= 6\dot{v}_r(\dot{v}_r + d), \\
(z_r^5) &= -6\dot{v}_r, (z_r^6) &= 0.
\end{align*}
\]  
(A2)

and for $z_p$ (Piston) we get

\[
\begin{align*}
(z_p^2) &= 2v_p^2, \\
(z_p^3) &= 2v_p, (z_p^4) &= 0, \\
(z_p^5) &= 3v_p^3, (z_p^6) &= 0.
\end{align*}
\]  
(A3)

Appendix B: Integrals for the ablation model

The mass moment Eq. (11) in \( \mathbb{F} \) diagonal components then read

\[
\begin{align*}
M_{xx} &= \frac{4}{3} S a^2 \rho_0 \left[ z_r + \int_{z_r}^{z_l} e^{-m(z,t)} \, dz \right], \\
M_{yy} &= \frac{4}{3} S b^2 \rho_0 \left[ z_r + \int_{z_r}^{z_l} e^{-m(z,t)} \, dz \right], \\
M_{zz} &= 4S \rho_0 \left[ z_s^3/2 + \int_{z_r}^{z_l} z^2e^{-m(z,t)} \, dz \right], \\
M_{xy} &= S^2 \rho_0 \left[ z_r + \int_{z_r}^{z_l} e^{-m(z,t)} \, dz \right], \\
M_{yz} &= 2Sb \rho_0 \left[ z_r^2/2 + \int_{z_r}^{z_l} z e^{-m(z,t)} \, dz \right], \\
M_{xz} &= 2Sa \rho_0 \left[ z_r^2/2 + \int_{z_r}^{z_l} e^{-m(z,t)} \, dz \right].
\end{align*}
\]  
(B1)

The integrals evaluate as

\[
\begin{align*}
\int_{z_r}^{z_l} e^{-m(z,t)} \, dz &= z_r(e^{-b_1} - e^{-a_1}), \\
\int_{z_r}^{z_l} z e^{-m(z,t)} \, dz &= z_r^2(b_1 e^{-b_1} - a_1 e^{-a_1}), \\
\int_{z_r}^{z_l} z^2 e^{-m(z,t)} \, dz &= z_r^3[(1 + b_1^2)e^{-b_1} - (1 + a_1^2)e^{-a_1}].
\end{align*}
\]  
(B3)

Appendix C: Derivatives for the piston model

1. The derivatives of mass moment and quadrupole moment with $z_p$ function

For calculation purposes we will present derivatives, first, second and third derivatives with respect to time, of the quadrupole moments here. The first derivatives of non–diagonal components are

\[
\begin{align*}
\dot{I}_{xy} &= \dot{M}_{xy} = \frac{1}{4} S^2 \rho_0 \dot{z}_p, \\
\dot{I}_{yz} &= \dot{M}_{yz} = \frac{1}{4} Sb \rho_0 (z_p^2), \\
\dot{I}_{xz} &= \dot{M}_{xz} = \frac{1}{4} Sa \rho_0 (z_p^2),
\end{align*}
\]  
(C1)

and the second derivatives are

\[
\begin{align*}
\ddot{I}_{xy} &= \ddot{M}_{xy} = \frac{1}{4} S^2 \rho_0 \ddot{z}_p, \\
\ddot{I}_{yz} &= \ddot{M}_{yz} = \frac{1}{4} Sb \rho_0 (z_p^2), \\
\ddot{I}_{xz} &= \ddot{M}_{xz} = \frac{1}{4} Sa \rho_0 (z_p^2)
\end{align*}
\]  
(C2)

and the third derivatives are

\[
\begin{align*}
\dddot{I}_{xy} &= \dddot{M}_{xy} = \frac{1}{4} S^2 \rho_0 \dddot{z}_p, \\
\dddot{I}_{yz} &= \dddot{M}_{yz} = \frac{1}{4} Sb \rho_0 (z_p^2), \\
\dddot{I}_{xz} &= \dddot{M}_{xz} = \frac{1}{4} Sa \rho_0 (z_p^2)
\end{align*}
\]  
(C3)

The derivatives of diagonal components of the mass moments are

\[
\begin{align*}
\dot{M}_{xx} &= \frac{S a^2}{3} \rho_0 \dot{z}_p, \quad 
\dot{M}_{yy} &= \frac{S b^2}{3} \rho_0 \dot{z}_p, \\
\dot{M}_{zz} &= \frac{4S}{3} \rho_0 \dot{z}_p, \\
\ddot{M}_{xx} &= \frac{S a^2}{3} \rho_0 \ddot{z}_p, \\
\ddot{M}_{yy} &= \frac{S b^2}{3} \rho_0 \ddot{z}_p, \\
\ddot{M}_{zz} &= \frac{4S}{3} \rho_0 \ddot{z}_p.
\end{align*}
\]  
(C4)

The derivatives of the trace of the mass moment,

\[
\begin{align*}
(TrM) &= \frac{S \rho_0}{3} \left\{ (a^2 + b^2)\dot{z}_s + (z_p^2) \right\}, \\
(TrM) &= \frac{S \rho_0}{3} \left\{ (a^2 + b^2)\dot{z}_s + (z_p^2) \right\}, \\
(TrM) &= \frac{S \rho_0}{3} \left\{ (a^2 + b^2)\dot{z}_s + (z_p^2) \right\}
\end{align*}
\]  
(C5)

The derivatives of diagonal components of the quadrupole moment are

\[
\begin{align*}
\dot{I}_{xx} &= \frac{1}{5} S \rho_0 \left\{ (2a^2 - b^2)\dot{z}_p - (z_p^3) \right\}, \\
\dot{I}_{yy} &= \frac{1}{9} S \rho_0 \left\{ (2b^2 - a^2)\dot{z}_p - (z_p^3) \right\}, \\
\dot{I}_{zz} &= \frac{1}{9} S \rho_0 \left\{ (2z_p^3) - (a^2 + b^2)\dot{z}_p \right\}
\end{align*}
\]  
(C6)
the second derivatives

\[ \ddot{I}_{xx} = \frac{1}{9} S \rho_0 \left\{ (2a^2 - b^2) \ddot{z}_s - (z_s^3) \right\}, \]
\[ \ddot{I}_{yy} = \frac{1}{9} S \rho_0 \left\{ (2b^2 - a^2) \ddot{z}_s - (z_s^3) \right\}, \]
\[ \ddot{I}_{zz} = \frac{1}{9} S \rho_0 \left\{ 2(z_s^3) \right\} - (a^2 + b^2) \ddot{z}_s \}, \] (C7)

and third

\[ \dddot{I}_{xx} = \frac{1}{9} S \rho_0 \left\{ (2a^2 - b^2) \dddot{z}_s - (z_s^3) \right\}, \]
\[ \dddot{I}_{yy} = \frac{1}{9} S \rho_0 \left\{ (2b^2 - a^2) \dddot{z}_s - (z_s^3) \right\}, \]
\[ \dddot{I}_{zz} = \frac{1}{9} S \rho_0 \left\{ 2(z_s^3) \right\} - (a^2 + b^2) \dddot{z}_s \}, \] (C8)

When using the ansatz for the function \( z_p \) some derivatives simplify significantly. Let us mention that to this point, we did not use the ansatz for \( z_p \) and the every formula was derived for general function of time \( z_p(t) \).

2. The derivatives of the mass moment and quadrupole moment with substitution for \( z_s \)

The derivatives of the non-diagonal components of quadrupole moment read

\[ \ddot{I}_{xy} = \dddot{M}_{xy} = \frac{1}{4} S a \rho_0 v_p, \quad \ddot{I}_{yx} = \dddot{M}_{yx} = 0, \]
\[ \ddot{I}_{yz} = \dddot{M}_{yz} = \frac{1}{2} S b \rho_0 v_p^2, \quad \ddot{I}_{zy} = \dddot{M}_{zy} = \frac{1}{2} S b \rho_0 v_p^2, \] (C9)

\[ \dddot{I}_{xx} = \dddot{M}_{xx} = \frac{1}{2} S a \rho_0 v_p^3 t, \quad \dddot{I}_{yy} = \dddot{M}_{yy} = \frac{1}{2} S a \rho_0 v_p^3 t, \]
\[ \dddot{I}_{zz} = \dddot{M}_{zz} = \frac{1}{2} S a \rho_0 v_p^3 t, \quad \dddot{I}_{xy} = \dddot{M}_{xy} = 0, \quad \dddot{I}_{yx} = \dddot{M}_{yx} = 0, \quad \dddot{I}_{zz} = \dddot{M}_{zz} = 0 \]

and diagonal components of the mass moment

\[ \dddot{M}_{xx} = \frac{2 S a^2}{3} \rho_0 v_p, \quad \dddot{M}_{xx} = \dddot{M}_{xx} = 0, \]
\[ \dddot{M}_{yy} = \frac{2 S b^2}{3} \rho_0 v_p, \quad \dddot{M}_{yy} = \dddot{M}_{yy} = 0, \] (C10)
\[ \dddot{M}_{zz} = \frac{2 S}{3} a \rho_0 v_p^3 t, \quad \dddot{M}_{zz} = \dddot{M}_{zz} = \frac{2 S}{3} a \rho_0 v_p^3 t, \]

The derivatives of diagonal components of the quadrupole moment are

\[ \dddot{I}_{xx} = \frac{1}{9} S \rho_0 v_p \left\{ (2a^2 - b^2) - 3v_p^2 t^2 \right\}, \]
\[ \dddot{I}_{yy} = \frac{1}{9} S \rho_0 v_p \left\{ (2b^2 - a^2) - 3v_p^2 t^2 \right\}, \]
\[ \dddot{I}_{zz} = \frac{1}{9} S \rho_0 v_p \left\{ 6v_p^2 t^2 - (a^2 + b^2) v_p \right\}, \] (C11)

the second derivatives

\[ \dddot{I}_{xx} = \frac{2 S}{3} a \rho_0 v_p^3 t, \quad \dddot{I}_{yy} = \frac{2 S}{3} a \rho_0 v_p^3 t, \quad \dddot{I}_{zz} = \frac{4 S}{3} a \rho_0 v_p^3 t, \] (C12)

and third derivatives

\[ \dddot{I}_{xx} = \frac{2 S}{3} a \rho_0 v_p^3 t, \quad \dddot{I}_{yy} = \frac{2 S}{3} a \rho_0 v_p^3 t, \quad \dddot{I}_{zz} = \frac{4 S}{3} a \rho_0 v_p^3 t. \] (C13)