ADJUSTABLE ROBUST OPTIMIZATION IN ENABLING OPTIMAL DAY-AHEAD ECONOMIC DISPATCH OF CCHP-MG CONSIDERING UNCERTAINTIES OF WIND-SOLAR POWER AND ELECTRIC VEHICLE

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ABSTRACT. At present, electric vehicles (EVs), small-scale wind power, and solar power have been increasingly integrated into modern power system via the combined cooling heating and power based microgrid (CCHP-MG). However, inside the microgrid the uncertainties of EVs charging, wind power, and solar power significantly impact the economy of CCHP-MG operation. Therefore to improve the economy deteriorated by the uncertainties, this paper presents a two-stage adjustable robust optimization to achieve the minimal operational cost for CCHP-MG. Before the realizations of the uncertainties, the day-ahead stage as the first stage decides an operational strategy that can withstand the worst-case uncertainties. As long as the uncertainties are observed, the real-time stage as the second stage adjusts the operational units to compensate the errors caused by the day-ahead operational strategy. Due to the difficulties of the model solution, this paper further adopts the duality theory, Big-M method, and column-and-constraint generation (C&CG) decomposition to convert the model into two tractable mixed integer linear programming (MILP) problems. Further, C&CG iteration algorithm is also employed to solve the MILPs, which can ultimately provide an optimal economic day-ahead dispatch strategy capable of handling uncertainties. The experimental results demonstrate the effectiveness of the presented approach.

1. Introduction. At present, renewable energy resources (RESs) and electric vehicles (EVs) have been incrementally developed due to their environmental friendly features [46, 31]. A number of researches [15, 37, 28, 11] pointed out that microgrid (MG) is one of the most effective ways to integrate EVs and RESs for example wind power and solar power to the power system. Therefore, MG with RESs and EVs has been widely studied [26, 47, 31, 15, 37]. However, along with the developments of energy technologies and load demands [14, 7, 10], traditional MG needs to be upgraded to integrated energy system based MG (IES-MG), so that multiple types of energies for example cooling, heating, and electricity can be supplied simultaneously. Among the various types of IES-MGs, combined cooling, heating, and power based MG (CCHP-MG) shows great significance to meet the various load demands due to its energy integration characteristics [14, 7, 10, 8, 21, 25]. For example, research [14] presented a CCHP consisting heat storage system (HSS), absorption chiller (AC),

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**Nomenclature**

| Parameters | Description |
|------------|-------------|
| **T**      | Dispatch horizon, $T = 24$ hours. |
| **t**      | Time steps, $t = 1 \ldots T$. |
| $\Delta t$ | Time duration of $T$, $\Delta t = 1$ hour. |
| $a_{MT}/a_{FC}/b_{MT}/b_{FC}$ | Coefficient of electricity generation. |
| $\lambda_{Up}/\lambda_{Up}/\lambda_{Dn}/\lambda_{Dn}/\lambda_{MT}/\lambda_{FC}/\lambda_{EB}$ | Coefficient of up/down-regulation. |
| $\lambda_{Buy}/\lambda_{Buy}/\lambda_{Sell}/\lambda_{Sell}$ | Coefficient of day-ahead transaction. |
| $\lambda_{RT}/\lambda_{RT}$ | Coefficient of real-time transaction. |
| $\lambda_{Wind}/\lambda_{Solar}/\lambda_{Load}$ | Coefficient of WPG curtailment/SPG curtailment/load shedding. |
| $\eta_{EH}$ | Conversion coefficient of MT/EB. (Electricity to heating) |
| $\eta_{HC}$ | Conversion coefficient of MT/EB/HSS. (Heating to cooling) |
| $\eta_{Cha}$ | Charging coefficient of ESS/HSS. |
| $\eta_{Dis}$ | Discharging coefficient of ESS/HSS. |
| $\delta_{ESS}/\delta_{HSS}$ | Energy loss coefficient of ESS/HSS. |
| $P_{min}^{MT}/P_{min}^{FC}/P_{min}^{EB}$ | Minimal generation of MT/FC/EB. |
| $P_{max}^{MT}/P_{max}^{FC}/P_{max}^{EB}$ | Maximal generation of MT/FC/EB. |
| $R_{Up}/R_{Up}/R_{Dn}/R_{Dn}$ | Ramp-up limitation of MT/FC/EB. |
| $R_{Dn}/R_{Dn}$ | Ramp-down limitation of MT/FC/EB. |
| $P_{min}^{Cha}/Q_{min}^{Cha}$ | Minimal charging power of ESS/HSS. |
| $P_{max}^{Cha}/Q_{max}^{Cha}$ | Maximal charging power of ESS/HSS. |
| $P_{min}^{Dis}/Q_{min}^{Dis}$ | Minimal discharging power of ESS/HSS. |
| $P_{max}^{Dis}/Q_{max}^{Dis}$ | Maximal discharging power of ESS/HSS. |
| $P_{Grid}$ | Maximal trading electricity. |
| $P_{Load}$ | Residential electricity load. |
| $P_{min}^{Wind}(t)/P_{min}^{Solar}(t)/P_{min}^{EV}(t)$ | Minimum of WPG/SPG/EVs charging. |
| $P_{max}^{Wind}(t)/P_{max}^{Solar}(t)/P_{max}^{EV}(t)$ | Maximum of WPG/SPG/EVs charging. |
| $P_{pre}^{Wind}(t)/P_{pre}^{Solar}(t)/P_{pre}^{EV}(t)$ | Prediction of WPG/SPG/EVs charging. |
| $P_{Wind}^{+}(t)/P_{Solar}^{+}(t)/P_{EV}^{+}(t)$ | Increase range based on WPG/SPG/EVs charging prediction. |
| $P_{Wind}^{-}(t)/P_{Solar}^{-}(t)/P_{EV}^{-}(t)$ | Decrease range based on WPG/SPG/EVs charging prediction. |
| $Q_{Load}$ | Residential heating load. |
| $O_{Load}$ | Residential cooling load. |
| $E_{min}^{ESS}/E_{min}^{HSS}$ | Minimal capacity of ESS/HSS. |
| $E_{max}^{ESS}/E_{max}^{HSS}$ | Maximal capacity of ESS/HSS. |
| $c/d/e$ | Matrix of objective. |
| $A/C/D/g/i$ | Matrix of equality constraint. |
| $B/E/F/G/h/j$ | Matrix of inequality constraint. |
| $\Gamma$ | Budget parameter. |
### Nomenclature

#### Day-ahead stage variable
- \( x \) \( P_{MT}(t)/P_{FC}(t)/P_{EB}(t) \): Day-ahead dispatch decision vector.
- \( P_{Cha}(t)/P_{Dis}(t) \): Base generation of MT/FC/EB.
- \( P_{Buy}(t)/P_{Sell}(t) \): Day-ahead charging/discharging power of ESS.
- \( Q_{MT}(t)/Q_{EB}(t) \): Electricity bought/sold in day-ahead market.
- \( Q^H_{MT}(t)/Q^H_{EB}(t)/Q^H_{Dis}(t) \): Day-ahead heating power from MT/EB/HSS to heating load.
- \( Q^C_{MT}(t)/Q^C_{EB}(t)/Q^C_{Dis}(t) \): Day-ahead heating power from MT/EB/HSS to AC.
- \( Q_{Cha}(t)/Q_{Dis}(t) \): Day-ahead discharging state of ESS/HSS in hour.
- \( O_{MT}(t)/O_{EB}(t)/O_{HSS}(t) \): Day-ahead discharging state of ESS/HSS in hour.
- \( E_{ESS}(t)/E_{HSS}(t) \): Day-ahead cooling power from MT/EB/HSS to heating load.
- \( S_{MT}(t)/S_{FC}(t)/S_{EB}(t) \): Day-ahead operational state of MT/FC/EB in hour. \( S = 1 \) if it is on; \( S = 0 \) otherwise.
- \( S_{Cha}(t)/S_{HSS}(t) \): Day-ahead charging state of ESS/HSS in hour. \( S = 1 \) if it charges; \( S = 0 \) otherwise.
- \( S_{Dis}(t)/S_{HSS}(t) \): Day-ahead discharging state of ESS/HSS in hour. \( S = 1 \) if it discharges; \( S = 0 \) otherwise.
- \( S_{Buy}(t)/S_{Sell}(t) \): Day-ahead buy/sell state in hour. \( S = 1 \) if it buys/sells; \( S = 0 \) otherwise.

#### Real-time stage variable
- \( y \) \( P^{Up}_{MT}(t)/P^{Up}_{FC}(t)/P^{Up}_{EB}(t) \): Real-time dispatch decision vector.
- \( P^{Up}_{MT}(t)/P^{Up}_{FC}(t)/P^{Up}_{EB}(t) \): Up-regulation of MT/FC/EB.
- \( P^{Up}_{MT}(t)/P^{Up}_{FC}(t)/P^{Up}_{EB}(t) \): Down-regulation of MT/FC/EB.
- \( P^{Up}_{Cha}(t)/P^{Up}_{Dis}(t)/P^{Up}_{Dis}(t) \): Up/down-regulation of ESS charging.
- \( P^{Up}_{Dis}(t)/P^{Up}_{Dis}(t) \): Up/down-regulation of ESS discharging.
- \( P^{RT}_{Buy}(t)/P^{RT}_{Sell}(t) \): Energy bought/sold in real-time market.
- \( P^{RT}_{Wind}(t)/P^{RT}_{Solar}(t)/P^{RT}_{Load}(t) \): Real-time injected WPG/SPG/load.
- \( Q^{RT}_{MT}(t)/Q^{RT}_{EB}(t) \): Real-time heating power from MT/EB to heating load.
- \( Q^{RT}_{MT}(t)/Q^{RT}_{EB}(t) \): Real-time heating power from MT/EB to AC.
- \( Q^{Up}_{Cha}(t)/Q^{Up}_{Dis}(t)/Q^{Up}_{Dis}(t) \): Up/down-regulation of HSS charging.
- \( Q^{Up}_{Dis}(t)/Q^{Up}_{Dis}(t) \): Up/down-regulation of HSS discharging.
- \( Q^{RT}_{Dis}(t)/Q^{RT}_{Dis}(t)/Q^{RT}_{Dis}(t) \): Real-time discharging power of HSS for heating/cooling load.
- \( Q^{RT}_{Dis}(t)/Q^{RT}_{Dis}(t)/Q^{RT}_{Dis}(t) \): Real-time discharging power of HSS for heating/cooling load.
- \( O^{RT}_{MT}(t)/O^{RT}_{EB}(t)/O^{RT}_{HSS}(t) \): Real-time cooling power from MT/EB/HSS.
- \( E^{RT}_{ESS}(t)/E^{RT}_{HSS}(t) \): Real-time capacity of ESS/HSS.
- \( S_{CG}(t) \): Up-regulation state of CG in hour. \( S = 1 \) if it ramps up; \( S = 0 \) otherwise.
- \( S_{CG}(t) \): Down-regulation state of CG in hour. \( S = 1 \) if it ramps down; \( S = 0 \) otherwise.
electric boiler (EB), and combined heating and power system. The authors revealed the high-efficiency of the presented system in terms of energy utilization. Research [7] developed a CCHP system with an extra heat recovery system (HRS). Based on the experimental results, the authors claimed that the improvements of the energy efficiency can be achieved based on their CCHP system. In researches [10, 8], the authors also proved that CCHP-MG shows great potential to efficiently supply the various types of loads. However, researches [21, 25] pointed out that the economy of CCHP-MG operations extremely suffers from the uncertainties caused by the uncertain behaviors of EVs charging, wind power generation (WPG) fluctuations, and solar power generation (SPG) fluctuations. Therefore, the day-ahead economic dispatch (DED) [21, 25, 22] is developed to improve the economy of CCHP-MG involving multiple uncertainties.

In this regard, a number of optimizations have been presented to achieve the optimal DED strategies. Researches [33, 32, 3] employed the heuristic algorithms to implement the economic dispatch (ED) in the MG system. However, researches [21, 25] revealed that the locally optimal and computationally intensive issues prevent these algorithms from handling the optimization effectively. Consequently, mathematical programming approaches especially deterministic optimization (DO), stochastic optimization (SO), and robust optimization (RO) have been developed to carry out the DED optimizations. Li et al. [22] presented a bi-level deterministic DED model for an IES-MG. The experimental results illustrated the economy of the system has been significantly improved. Similarly, research [4] employed DO in enabling economic operation for the IES-MG systems. The authors claimed that the presented optimization is highly tractable for dealing with the economic operation tasks. However, researches [30, 34, 24, 16, 29, 44] pointed out that the

| Nomenclature |
|---------------|
| **Function** |
| \( C_{DA}/C_{RT} \) | Cost of day-ahead/real-time stage. |
| \( C_{DA}^{Grid} \) | Cost of day-ahead electricity transaction. |
| \( C_{MT}/C_{FC} \) | Cost of MT/FC day-ahead generation. |
| \( C_{MT}^{UP}/C_{FC}^{UP}/C_{EB}^{UP} \) | Cost of MT/FC/EB up-regulation. |
| \( C_{MT}^{DN}/C_{FC}^{DN}/C_{EB}^{DN} \) | Cost of MT/FC/EB down-regulation. |
| \( C_{RT}^{Grid} \) | Cost of real-time electricity transaction. |
| \( C_{Wind}/C_{Solar}/C_{Load} \) | Cost of WPG curtailment/SPG curtailment/load shedding. |

| **Stochastic variable and auxiliary variable** |
| \( \hat{P}_{\text{Wind}}(t)/\hat{P}_{\text{Solar}}(t)/\hat{P}_{\text{EV}}(t) \) | Stochastic variable of WPG/SPG/EVs charging. |
| \( u^{up}/u^{down}/u^{pre} \) | Maximal/minimal/predicted value of \( u \). |
| \( \xi^{+}/\xi^{-} \) | Positive/negative value of \( \xi \). |
| \( \alpha/\beta/\gamma \) | Duality variable. |
| \( \rho^{+}(t)/\rho^{-}(t) \) | Auxiliary 0-1 variable introduced by Big-M. |
| \( \mu^{+}/\mu^{-} \) | 0-1 variable introduced by Big-M method. |
DO-based optimal economic dispatch strategies encounter locally optimal issue due to the ignorance of uncertainties. Consequently, because of the uncertainties modeling ability using a large-size of stochastic scenarios, SO has been widely employed to solve the power system optimization tasks [30, 34, 24, 16, 29, 44]. Although the authors claimed that SO can significantly reduce the expected operational cost than DO-based methods do, research [35] pointed out that SO-based decisions are over-optimistic as the worst-case scenarios are almost ignored. Additionally, two flaws of SO still need to be further addressed: massive scenarios result in computationally intensive issue [43, 17]; it is difficult to obtain precise probability distribution function (PDF) of the uncertainties in practice [18]. Therefore to overcome the issues existing in SO, RO [2] has been developed. Modeling uncertainties using interval set instead of PDF, RO focuses on handling the worst-case scenarios to improve the computational efficiency and robustness. For example, researches [38, 40] successfully applied RO to optimize the dispatch tasks. Their results show the security and efficiency of RO. Nevertheless, researches [41, 42, 5, 6] stated that the traditional RO is over-conservative due to the pessimistic optimization mechanism. In this regard, Ben-Tal et al. [1] significantly extended RO to adjustable RO (ARO) by introducing a budget parameter. The parameter can flexibly adjust the pessimistic degree of the worst-case scenario according to an acceptable operation budget, which can effectively balance the economy and robustness of the optimization.

As a result, motivated by the previous researches, this paper presents an ARO-based approach in enabling the optimal DED for CCHP-MG incorporating uncertain WPG, SPG, and EVs charging. First, a two-stage ARO DED model (A-DED) is constructed for CCHP-MG. Before observing the uncertainties, the day-ahead stage decides the UC strategy and the base generation of the operational units considering the worst-case scenario. Once the uncertainties are realized, the real-time stage adjusts the output of controllable generators (CGs) to correct the day-ahead operational strategy. And then, to facilitate the solution, the model is decomposed into a mixed integer linear programming (MILP) master problem (MP) and sub problem (SP) using column-and-constraint generation (C&CG) method [45]. Following, SP is derived into MILP using Big-M method [35] and duality theory [17, 19]. At last, C&CG iteration algorithm is adopted to solve MP and SP, which finally provides the optimal DED strategy.

The rest of the paper is organized as: section 2 presents the mathematical modeling of A-DED; the solution of A-DED is given in section 3; section 4 shows and discusses the experimental results; section 5 concludes the paper.

2. A-DED modeling for CCHP-MG. Figure 1 shows the structure of the CCHP-MG system. WPG, SPG, fuel cell (FC), and micro turbine (MT) provide electricity power for the system. MT generates waste heat in the process of electricity generation. EB consumes the electricity to generate heat. HRS collects all the heating power of MT, EB, HSS discharging. And then, HRS allocates a part of the heating power to meet the heating load directly and distributes the remained heating power to AC. AC converts the heating power into the cooling power for meeting the cooling load. CCHP-MG sells the abundant electricity for the profit and purchases electricity to handle the electricity shortage. With the premise of meeting multiple loads, DED aims at achieving the lowest operational cost.
2.1. Basic model of A-DED. The model of A-DED is represented by (1).

\[
\begin{align*}
\min_{x} C_{DA}(x) \\
\text{s.t. } H_{DA}(x) &= 0 \\
G_{DA}(x) &\leq 0
\end{align*}
\]  

Let \( x \) represent day-ahead dispatch decision vector, including states and base generation of CGs, states and power of ESS and HSS, and day-ahead electricity transaction plan; \( H_{DA}(x) = 0 \) and \( G_{DA}(x) \leq 0 \) denote all the equality constraints and the inequality constraints. Before the realizations of the uncertainties, \( x \) is decided based on the prediction of EVs charging, WPG, and SPG [35]. However, the day-ahead prediction errors due to the uncertainties are inevitable [27]. Therefore, the real-time economic dispatch (RED) is employed to handle the power imbalance caused by the errors. This paper introduces a second stage to simulate the RED process. As a result, (1) is extended to a two-stage DED model shown by (2).

\[
\begin{align*}
\min_{x} \{C_{DA}(x) + & \min_{y} C_{RT}(u, y)\} \\
\text{s.t. } H_{DA}(x) &= 0, \ H_{RT}(x, u, y) = 0, \\
G_{DA}(x) \leq 0, G_{RT}(x, u, y) \leq 0
\end{align*}
\]  

Let \( u \) indicate the stochastic variable including WPG, SPG, and EVs charging; \( y \) represent the real-time adjustment decision vector involving regulations of CGs generation, storage systems power, and electricity transaction; \( H_{DA}(x, u, y) = 0 \) and \( G_{RT}(x, u, y) \leq 0 \) represent all the real-time equality constraints and the inequality constraints. Ultimately, (2) is extended to a two-stage A-DED model shown by (3).
\[
\begin{align*}
\min_{x} & \left\{ C_{DA}(x) + \max_{u} R(x, u) \right\} \\
\text{s.t.} & \quad H_{DA}(x) = 0, G_{DA}(x) \leq 0. \\
& \text{where } R(x, u) = \min_{y} C_{RT}(u, y) \\
\text{s.t.} & \quad H_{RT}(x, u, y, \Gamma) = 0, G_{RT}(x, u, y, \Gamma) \leq 0.
\end{align*}
\] (3)

As a two-stage adjustable robust optimization model, equation (3) is min-max-min structure with a budget parameter \( \Gamma \) \cite{38}. The outer min function aims at minimizing the total operational cost by optimizing \( x \) \cite{2}. In the inner max-min function, the max function is to search for the worst-case scenarios of stochastic variable \( u \), and then the worst-case scenario is input to the inner min function. As a result, considering \( x \) and worst-case realization of \( u \), the inner min function minimizes the RED cost by optimizing \( y \) \cite{45}.

2.2. Uncertainty description. The uncertainties of WPG, SPG, and EVs charging are characterized using uncertainty set theory \cite{27, 35}, which are expressed by (4)–(6).

\( \tilde{P}_{Wind}(t) \in [P_{min}^{Wind}(t), P_{max}^{Wind}(t)] \) (4)
\( \tilde{P}_{Solar}(t) \in [P_{min}^{Solar}(t), P_{max}^{Solar}(t)] \) (5)
\( \tilde{P}_{EV}(t) \in [P_{min}^{EV}(t), P_{max}^{EV}(t)] \) (6)

According to researches \cite{41, 6}, the uncertainties hardly remain the worst-case realizations for 24 hours. Therefore to prevent the strategy from being over-conservative, (7)–(9) are introduced to restrict the number of the worst-case realizations.

\[
\sum_{t=1}^{T} \left[ \frac{\tilde{P}_{Wind}(t) - P_{pre}^{Wind}(t)}{P_{Wind}^{+}(t)}, \rho_{Wind}^{+}(t) + \frac{P_{pre}^{Wind}(t) - \tilde{P}_{Wind}(t)}{P_{Wind}^{-}(t)}, \rho_{Wind}^{-}(t) \right] \leq \Gamma_{Wind}
\] (7)

\[
\sum_{t=1}^{T} \left[ \frac{\tilde{P}_{Solar}(t) - P_{pre}^{Solar}(t)}{P_{Solar}^{+}(t)}, \rho_{Solar}^{+}(t) + \frac{P_{pre}^{Solar}(t) - \tilde{P}_{Solar}(t)}{P_{Solar}^{-}(t)}, \rho_{Solar}^{-}(t) \right] \leq \Gamma_{Solar}
\] (8)

\[
\sum_{t=1}^{T} \left[ \frac{\tilde{P}_{EV}(t) - P_{pre}^{EV}(t)}{P_{EV}^{+}(t)}, \rho_{EV}^{+}(t) + \frac{P_{pre}^{EV}(t) - \tilde{P}_{EV}(t)}{P_{EV}^{-}(t)}, \rho_{EV}^{-}(t) \right] \leq \Gamma_{EV}
\] (9)

Let \( \tilde{P}(t) \) represent the stochastic realizations of the uncertainties; \( P_{pre}(t) \) denote the predicted values of the uncertainties. If \( P_{pre}(t) \leq \tilde{P}(t) \), \( \rho^{+}(t) = 1, \rho^{-}(t) = 0 \); otherwise, \( \rho^{+}(t) = 0, \rho^{-}(t) = 1 \). \( \Gamma \) is equal to the times that the uncertainties reach the boundary values. Therefore, the conservativeness of the strategy can be adjusted by tuning the value of \( \Gamma \). Ultimately, A-DED of CCHP-MG is modeled by (3)–(9).
2.3. Objective and constraints of A-DED.

2.3.1. Objective. The total operational costs are the sum of $C_{DA}(x)$ and $C_{RT}(u, y)$. $C_{DA}(x)$ includes the costs of CGs base generation (11)–(12) and the day-ahead electricity transaction (13). $C_{RT}(u, y)$ comprises the real-time costs of CGs regulation (15)–(17), RES curtailment (18)–(19), load shedding (20), and real-time electricity transaction (21).

\[ C_{DA}(x) = C_{MT} + C_{FC} + C_{DA}^{Grid} \] (10)

\[ C_{MT} = \sum_{t=1}^{T} [a_{MT} \cdot P_{MT}(t) + b_{MT}] \cdot \Delta t \] (11)

\[ C_{FC} = \sum_{t=1}^{T} [a_{FC} \cdot P_{FC}(t) + b_{FC}] \cdot \Delta t \] (12)

\[ C_{DA}^{Grid} = \sum_{t=1}^{T} [\lambda_{Buy}^{DA} \cdot P_{Buy}(t) - \lambda_{Sell}^{DA} \cdot P_{Sell}(t)] \cdot \Delta t \] (13)

\[ C_{RT}(u, y) = C_{MT}^{Up} + C_{MT}^{Dn} + C_{FC}^{Up} + C_{FC}^{Dn} + C_{EB}^{Up} + C_{EB}^{Dn} + C_{Wind} + C_{Solar} + C_{Load} + C_{Grid}^{RT} \] (14)

\[ C_{MT}^{Up} + C_{MT}^{Dn} = \sum_{t=1}^{T} [\lambda_{MT}^{Up} \cdot P_{MT}^{Up}(t) + \lambda_{MT}^{Dn} \cdot P_{MT}^{Dn}(t)] \cdot \Delta t \] (15)

\[ C_{FC}^{Up} + C_{FC}^{Dn} = \sum_{t=1}^{T} [\lambda_{FC}^{Up} \cdot P_{FC}^{Up}(t) + \lambda_{FC}^{Dn} \cdot P_{FC}^{Dn}(t)] \cdot \Delta t \] (16)

\[ C_{EB}^{Up} + C_{EB}^{Dn} = \sum_{t=1}^{T} [\lambda_{EB}^{Up} \cdot P_{EB}^{Up}(t) + \lambda_{EB}^{Dn} \cdot P_{EB}^{Dn}(t)] \cdot \Delta t \] (17)

\[ C_{Wind} = \sum_{t=1}^{T} \lambda_{Wind} \cdot [\hat{P}_{Wind}(t) - P_{Wind}^{RT}(t)] \cdot \Delta t \] (18)

\[ C_{Solar} = \sum_{t=1}^{T} \lambda_{Solar} \cdot [\hat{P}_{Solar}(t) - P_{Solar}^{RT}(t)] \cdot \Delta t \] (19)

\[ C_{Load} = \sum_{t=1}^{T} \lambda_{Load} \cdot [\hat{P}_{Load}(t) - P_{Load}^{RT}(t)] \cdot \Delta t \] (20)

\[ C_{Grid}^{RT} = \sum_{t=1}^{T} [\lambda_{Buy}^{RT} \cdot P_{Buy}^{RT}(t) - \lambda_{Sell}^{RT} \cdot P_{Sell}^{RT}(t)] \cdot \Delta t \] (21)
2.3.2. Day-ahead power balance. Equations (22)–(24) indicate the balance constraints for the different types of powers; (25) implies the efficiency of waste heat generation of MT; (26) presents the relation of the electrothermal conversion of EB; (27)–(29) represent that HRS decides the optimal allocation of the heating power from MT, EB, and HSS discharging [14, 7]; (30)–(32) suggest that AC converts the heating power allocated by HRS to cooling power in a certain proportion.

\[
P_{MT}(t) + P_{FC}(t) + P_{Dis}(t) + P_{pre}^{\text{Wind}}(t) + P_{pre}^{\text{Solar}}(t) \\
+ P_{Buy}(t) = P_{Load}(t) + P_{Sell}(t) + P_{Cha}(t) + P_{EB}(t) \tag{22}
\]

\[
Q_{MT}^H(t) + Q_{EB}^H(t) + Q_{Dis}^H(t) = Q_{Load}(t) + Q_{Cha}(t) \tag{23}
\]

\[
O_{Load}(t) = O_{MT}(t) + O_{EB}(t) + O_{HSS}(t) \tag{24}
\]

\[
Q_{MT}(t) = \eta_{MT}^E \cdot P_{MT}(t) \tag{25}
\]

\[
Q_{EB}(t) = \eta_{EB}^E \cdot P_{EB}(t) \tag{26}
\]

\[
Q_{MT}(t) = Q_{MT}^H(t) + Q_{MT}^C(t) \tag{27}
\]

\[
Q_{EB}(t) = Q_{EB}^H(t) + Q_{EB}^C(t) \tag{28}
\]

\[
Q_{Dis}(t) = Q_{Dis}^H(t) + Q_{Dis}^C(t) \tag{29}
\]

\[
O_{MT}(t) = \eta_{MT}^H \cdot Q_{MT}^C(t) \tag{30}
\]

\[
O_{EB}(t) = \eta_{EB}^H \cdot Q_{EB}^C(t) \tag{31}
\]

\[
O_{HSS}(t) = \eta_{HSS}^H \cdot Q_{Dis}^C(t) \tag{32}
\]

2.3.3. Day-ahead CGs operational constraints. CGs include MT, FC, and EB. Equations (33)–(35) represent the generation limitations; (36)–(38) indicate the ramping capacity in successive hours.

\[
S_{MT}(t) \cdot P_{MT}^{\text{min}} \leq P_{MT}(t) \leq S_{MT}(t) \cdot P_{MT}^{\text{max}} \tag{33}
\]

\[
S_{FC}(t) \cdot P_{FC}^{\text{min}} \leq P_{FC}(t) \leq S_{FC}(t) \cdot P_{FC}^{\text{max}} \tag{34}
\]

\[
S_{EB}(t) \cdot P_{EB}^{\text{min}} \leq P_{EB}(t) \leq S_{EB}(t) \cdot P_{EB}^{\text{max}} \tag{35}
\]

\[
- R_{MT}^{\text{Dn}} \leq P_{MT}(t) - P_{MT}(t - 1) \leq R_{MT}^{\text{Up}} \tag{36}
\]

\[
- R_{FC}^{\text{Dn}} \leq P_{FC}(t) - P_{FC}(t - 1) \leq R_{FC}^{\text{Up}} \tag{37}
\]

\[
- R_{EB}^{\text{Dn}} \leq P_{EB}(t) - P_{EB}(t - 1) \leq R_{EB}^{\text{Up}} \tag{38}
\]
2.3.4. Day-ahead storage system constraints. Equations (39)–(40) indicate the mutual exclusion between the charging state and the discharging state; (41)–(44) constrain the charging/discharging power; (45)–(46) suggest the state of the storage capacity (SOC) for each hour; (47)–(48) stipulate the SOC within an interval; (49)–(50) represent that SOC of the 1st hour is equal to that of the 24th hour.

\[
S_{\text{ESS}}^{\text{Cha}}(t) + S_{\text{ESS}}^{\text{Dis}}(t) \leq 1 \tag{39}
\]

\[
S_{\text{HSS}}^{\text{Cha}}(t) + S_{\text{HSS}}^{\text{Dis}}(t) \leq 1 \tag{40}
\]

\[
S_{\text{ESS}}^{\text{Cha}}(t) \cdot P_{\text{min}}^{\text{Cha}} \leq P_{\text{Cha}}(t) \leq S_{\text{ESS}}^{\text{Cha}}(t) \cdot P_{\text{max}}^{\text{Cha}} \tag{41}
\]

\[
S_{\text{ESS}}^{\text{Dis}}(t) \cdot P_{\text{min}}^{\text{Dis}} \leq P_{\text{Dis}}(t) \leq S_{\text{ESS}}^{\text{Dis}}(t) \cdot P_{\text{max}}^{\text{Dis}} \tag{42}
\]

\[
S_{\text{HSS}}^{\text{Cha}}(t) \cdot Q_{\text{min}}^{\text{Cha}} \leq Q_{\text{Cha}}(t) \leq S_{\text{HSS}}^{\text{Cha}}(t) \cdot Q_{\text{max}}^{\text{Cha}} \tag{43}
\]

\[
S_{\text{HSS}}^{\text{Dis}}(t) \cdot Q_{\text{min}}^{\text{Dis}} \leq Q_{\text{Dis}}(t) \leq S_{\text{HSS}}^{\text{Dis}}(t) \cdot Q_{\text{max}}^{\text{Dis}} \tag{44}
\]

\[
E_{\text{ESS}}(t) = (1 - \delta_{\text{ESS}}) \cdot E_{\text{ESS}}(t-1) + \left[\eta_{\text{ESS}}^{\text{Cha}} \cdot P_{\text{Cha}}(t-1) - \frac{P_{\text{Dis}}(t-1)}{\eta_{\text{ESS}}^{\text{Dis}}}\right] \cdot \Delta t \tag{45}
\]

\[
E_{\text{HSS}}(t) = (1 - \delta_{\text{HSS}}) \cdot E_{\text{HSS}}(t-1) + \left[\eta_{\text{HSS}}^{\text{Cha}} \cdot Q_{\text{Cha}}(t-1) - \frac{Q_{\text{Dis}}(t-1)}{\eta_{\text{HSS}}^{\text{Dis}}}\right] \cdot \Delta t \tag{46}
\]

\[
E_{\text{min}}^{\text{ESS}} \leq E_{\text{ESS}}(t) \leq E_{\text{max}}^{\text{ESS}} \tag{47}
\]

\[
E_{\text{min}}^{\text{HSS}} \leq E_{\text{HSS}}(t) \leq E_{\text{max}}^{\text{HSS}} \tag{48}
\]

\[
E_{\text{ESS}}(T) = E_{\text{ESS}}(0) \tag{49}
\]

\[
E_{\text{HSS}}(T) = E_{\text{HSS}}(0) \tag{50}
\]

2.3.5. Day-ahead electricity transaction constraints. Equations (51)–(53) constrain the amount of the day-ahead electricity transaction.

\[
0 \leq P_{\text{Buy}}(t) \leq S_{\text{Buy}}(t) \cdot P_{\text{max}}^{\text{Grid}} \tag{51}
\]

\[
0 \leq P_{\text{Sell}}(t) \leq S_{\text{Sell}}(t) \cdot P_{\text{max}}^{\text{Grid}} \tag{52}
\]

\[
S_{\text{Buy}}(t) + S_{\text{Sell}}(t) \leq 1 \tag{53}
\]
2.3.6. Real-time power balance constraints. To accommodate the uncertainty realization, the day-ahead dispatch plan has to be corrected in the real-time stage. Equations (54)–(56) indicate the real-time power balance constraints; (57) represents the real-time generation of the waste heat of MT; (58) defines the electrothermal conversion relationship of EB; (59)–(61) suggest the optimal allocation of the heating power; (62)–(64) present that AC converts the heating power of MT, EB, and HSS to cooling power in a certain proportion.

\[
P_{MT}(t) + P_{MT}^{Up}(t) - P_{MT}^{Dn}(t) + P_{FC}(t) + P_{FC}^{Up}(t) - P_{FC}^{Dn}(t) - P_{Dis}(t) + P_{Dis}^{Dn}(t) + P_{Wind}(t) + P_{Solar}(t) + P_{Buy} + P_{Buy}^{RT}(t) = P_{Load}(t) + P_{Sell} + P_{Sell}^{RT}(t) + P_{EB}(t) + P_{EB}^{Up}(t) - P_{EB}^{Dn}(t) + P_{Cha}(t) + P_{Cha}^{Up}(t) - P_{Cha}^{Dn}(t)
\]

\[
Q_{MT}^{RTH}(t) + Q_{EB}^{RTH}(t) + Q_{Dis}^{RTH}(t) = Q_{Load}(t) + Q_{Cha}(t) + Q_{Cha}^{Up}(t) - Q_{Cha}^{Dn}(t)
\]

\[
O_{Load}(t) = O_{MT}^{RT}(t) + O_{EB}^{RT}(t) + O_{HSS}^{RT}(t)
\]

\[
Q_{MT}^{RT}(t) = \eta_{MT}^{EH}(P_{MT}(t) + P_{MT}^{Up}(t) - P_{MT}^{Dn}(t))
\]

\[
Q_{EB}^{RT}(t) = \eta_{EB}^{EH}(P_{EB}(t) + P_{EB}^{Up}(t) - P_{EB}^{Dn}(t))
\]

\[
Q_{MT}^{RTC}(t) = Q_{MT}^{RTH}(t) + Q_{MT}^{RTC}(t)
\]

\[
Q_{EB}^{RTC}(t) = Q_{EB}^{RTH}(t) + Q_{EB}^{RTC}(t)
\]

\[
Q_{Dis}(t) + Q_{Dis}^{RT}(t) - Q_{Dis}^{Dn}(t) = Q_{Dis}^{RTH}(t) + Q_{Dis}^{RTC}(t)
\]

\[
O_{MT}^{RT}(t) = \eta_{MT}^{HC} \cdot Q_{MT}^{RTC}(t)
\]

\[
O_{EB}^{RT}(t) = \eta_{EB}^{HC} \cdot Q_{EB}^{RTC}(t)
\]

\[
O_{HSS}^{RT}(t) = \eta_{HSS}^{HC} \cdot Q_{Dis}^{RTC}(t)
\]

2.3.7. Real-time RES utilization and injected load constraints. If too much power is generated in the day-ahead stage, WPG and SPG will be curtailed in the real-time stage. In the case of insufficient power supply, electricity load will be shed to guarantee the power balance. Both RESs curtailment and load shedding are penalized. Equations (65)–(67) set the bounds for the amount of RESs utilization and the injected load; (68) indicates that the total electricity load is the sum of the constant residential load and the uncertain EVs charging power.

\[
0 \leq P_{Wind}(t) \leq \hat{P}_{Wind}(t)
\]

\[
0 \leq P_{Solar}(t) \leq \hat{P}_{Solar}(t)
\]

\[
0 \leq P_{Load}(t) \leq \hat{P}_{Load}(t)
\]

\[
\hat{P}_{Load}(t) = P_{Rload}(t) + \hat{P}_{EV}(t)
\]
2.3.8. Real-time CGs operational constraints. The real-time generation of CGs can be ramped up/down based on the day-ahead base generation and on/off states. Equation (69) stipulates the up/down-regulation range within the capacity; (70) demonstrates the generation limitations; (71) indicates the ramping capacity.

\[
\begin{align*}
0 \leq P_{CG}^{up}(t) & \leq S_{CG}^{up}(t) \cdot P_{CG}^{max} \\
0 \leq P_{CG}^{dn}(t) & \leq S_{CG}^{dn}(t) \cdot P_{CG}^{max} \\
S_{CG}(t) \cdot P_{CG}^{min} & \leq P_{CG}(t) + P_{CG}^{up}(t) - P_{CG}^{dn}(t) \leq S_{CG}(t) \cdot P_{CG}^{max} \\
R_{CG}^{dn} & \leq P_{CG}(t) + P_{CG}^{up}(t) - P_{CG}^{dn}(t) \\
- [P_{CG}(t-1) + P_{CG}^{up}(t-1) - P_{CG}^{dn}(t-1)] & \leq R_{CG}^{up}
\end{align*}
\] (69)

2.3.9. Real-time storage system constraints. The charging/discharging power of the storage systems can be adjusted based on the day-ahead set points and charging/discharging states. Equation (72)–(79) constrain the adjustment range of power; (80)–(83) indicate the power capacity; (84)–(85) represent the SOC for each hour; (86)–(87) stipulate the SOC within a range; (88)–(89) represent that the initial SOC should be equal to the SOC in the last hour \(T\).

\[
\begin{align*}
S_{ESS}^{cha}(t) \cdot P_{cha}^{min} & \leq P_{cha}(t) \leq S_{ESS}^{cha}(t) \cdot P_{cha}^{max} \\
S_{ESS}^{cha}(t) \cdot P_{cha}^{min} & \leq P_{cha}(t) \leq S_{ESS}^{cha}(t) \cdot P_{cha}^{max} \\
S_{ESS}^{dis}(t) \cdot P_{dis}^{min} & \leq P_{dis}(t) \leq S_{ESS}^{dis}(t) \cdot P_{dis}^{max} \\
S_{ESS}^{dis}(t) \cdot P_{dis}^{min} & \leq P_{dis}(t) \leq S_{ESS}^{dis}(t) \cdot P_{dis}^{max} \\
S_{HSS}^{cha}(t) \cdot Q_{cha}^{min} & \leq Q_{cha}(t) \leq S_{HSS}^{cha}(t) \cdot Q_{cha}^{max} \\
S_{HSS}^{cha}(t) \cdot Q_{cha}^{min} & \leq Q_{cha}(t) \leq S_{HSS}^{cha}(t) \cdot Q_{cha}^{max} \\
S_{HSS}^{dis}(t) \cdot Q_{dis}^{min} & \leq Q_{dis}(t) \leq S_{HSS}^{dis}(t) \cdot Q_{dis}^{max} \\
S_{HSS}^{dis}(t) \cdot Q_{dis}^{min} & \leq Q_{dis}(t) \leq S_{HSS}^{dis}(t) \cdot Q_{dis}^{max} \\
P_{cha}^{min} & \leq P_{cha}(t) + P_{cha}^{up}(t) - P_{cha}^{dn}(t) \leq P_{cha}^{max} \\
P_{dis}^{min} & \leq P_{dis}(t) + P_{dis}^{up}(t) - P_{dis}^{dn}(t) \leq P_{dis}^{max} \\
Q_{cha}^{min} & \leq Q_{cha}(t) + Q_{cha}^{up}(t) - Q_{cha}^{dn}(t) \leq Q_{cha}^{max} \\
Q_{dis}^{min} & \leq Q_{dis}(t) + Q_{dis}^{up}(t) - Q_{dis}^{dn}(t) \leq Q_{dis}^{max}
\end{align*}
\] (72)–(83)
\[ E_{\text{ESS}}^{\text{RT}}(t) = (1 - \delta_{\text{ESS}}) \cdot E_{\text{ESS}}^{\text{RT}}(t - 1) \]
\[ + \left[ \eta_{\text{ESS}} \cdot (P_{\text{Cha}}(t) + P_{\text{Up Cha}}(t) - P_{\text{Dis Cha}}(t)) - \frac{P_{\text{Dis}}(t) + P_{\text{Up Dis}}(t) - P_{\text{Dn Dis}}(t)}{\eta_{\text{ESS}}^{\text{Dis}}} \right] \cdot \Delta t \]
\[ (84) \]

\[ E_{\text{HSS}}^{\text{RT}}(t) = (1 - \delta_{\text{HSS}}) \cdot E_{\text{HSS}}^{\text{RT}}(t - 1) \]
\[ + \left[ \eta_{\text{HSS}} \cdot (Q_{\text{Cha}}(t) + Q_{\text{Up Cha}}(t) - Q_{\text{Dis Cha}}(t)) - \frac{Q_{\text{Dis}}(t) + Q_{\text{Up Dis}}(t) - Q_{\text{Dn Dis}}(t)}{\eta_{\text{HSS}}^{\text{Dis}}} \right] \cdot \Delta t \]
\[ (85) \]

\[ E_{\text{ESS}}^{\text{min}} \leq E_{\text{ESS}}^{\text{RT}}(t) \leq E_{\text{ESS}}^{\text{max}} \]
\[ (86) \]

\[ E_{\text{HSS}}^{\text{min}} \leq E_{\text{HSS}}^{\text{RT}}(t) \leq E_{\text{HSS}}^{\text{max}} \]
\[ (87) \]

\[ E_{\text{ESS}}^{\text{RT}}(T) = E_{\text{ESS}}^{\text{RT}}(0) \]
\[ (88) \]

\[ E_{\text{HSS}}^{\text{RT}}(T) = E_{\text{HSS}}^{\text{RT}}(0) \]
\[ (89) \]

2.3.10. Real-time electricity transaction constraints. Equation (90) limits the range of the real-time exchanged electricity; (91) indicates the limitation of the total exchanged electricity.

\[ \begin{align*}
0 & \leq P_{\text{Buy}}^{\text{RT}}(t) \leq S_{\text{Buy}}(t) \cdot P_{\text{max}}^{\text{Grid}} \\
0 & \leq P_{\text{Sell}}^{\text{RT}}(t) \leq S_{\text{Sell}}(t) \cdot P_{\text{max}}^{\text{Grid}} \\
0 & \leq P_{\text{Buy}}(t) + P_{\text{Buy}}^{\text{RT}}(t) \leq S_{\text{Buy}}(t) \cdot P_{\text{max}}^{\text{Grid}} \\
0 & \leq P_{\text{Sell}}(t) + P_{\text{Sell}}^{\text{RT}}(t) \leq S_{\text{Sell}}(t) \cdot P_{\text{max}}^{\text{Grid}}
\end{align*} \]
\[ (90) \]

2.4. Compact formulation of A-DED. Ultimately, the presented A-DED ((3)–(91)) can be rewritten into a compact formulation (92). The day-ahead operational strategy \( x \) and the real-time adjustment strategy \( y \) are represented by (93)–(94).

\[ \begin{align*}
\min \{ c^\top x + \max \min \{ d^\top y + e^\top u \} \} \\
\text{subject to:} \quad Ax = g, Bx \leq h, \\
Cx + Dy = i, Ex + Fy \leq j, \\
Gy \leq u.
\end{align*} \]
\[ (92) \]

\[ x = \begin{bmatrix}
P_{\text{MT}}(t), P_{\text{FC}}(t), P_{\text{EB}}(t), P_{\text{Cha}}(t), P_{\text{Dis}}(t), P_{\text{Buy}}(t), P_{\text{Sell}}(t), \\
Q_{\text{Cha}}(t), Q_{\text{Dis}}(t), Q_{\text{MT}}^{\text{CH}}(t), Q_{\text{MT}}^{\text{CH}}(t), Q_{\text{EB}}^{\text{CH}}(t), Q_{\text{EB}}^{\text{CH}}(t) \\
S_{\text{ESS}}^{\text{MT}}(t), S_{\text{ESS}}^{\text{MT}}(t), S_{\text{ESS}}^{\text{MT}}(t), S_{\text{ESS}}^{\text{MT}}(t), S_{\text{Buy}}(t), S_{\text{Sell}}(t)
\end{bmatrix} \]
\[ (93) \]
3. Model solution.

3.1. Model decomposition. A-DED (92) is a min-max-min optimization problem. In the three-level optimization, the real-time adjustment $y$ for the worst-case scenario $u$ is based on the day-ahead strategy $x$ while $x$ is also affected by $y$. As a result, the intertwine between $x$ and $y$ leads to the difficulty of the model solution. Therefore, CcCG method [35, 45] is employed to decompose the model into MILP MP (95) and SP (96).

\[
\begin{align*}
\min_{x} & \ c^\top x + \Theta \\
\text{s.t.} & \ d^\top y + e^\top u \\
& A x = g, B x \leq h, \\
& C x + D y = i, E x + F y \leq j, \\
& G y \leq u.
\end{align*}
\]  

(95)

\[
\begin{align*}
\max_{u, y} & \ d^\top y + e^\top u \\
\text{s.t.} & \ C x + D y = i, \\
& E x + F y \leq j, \\
& G y \leq u.
\end{align*}
\]  

(96)

3.2. Reformulation of SP and embedding of adjustable parameters. The max-min structure of SP (96) makes it difficult for SP to be solved directly. Therefore, the duality theory [35, 17] is employed to derive the MILP SP to a single-level counterpart (97).

\[
\begin{align*}
\max_{u, \alpha, \beta, \gamma} & \ u^\top \xi + i^\top \alpha - x^\top C^\top \alpha + j^\top \beta - x^\top E^\top \beta \\
\text{s.t.} & \ D^\top \alpha + F^\top \beta + G^\top \gamma = d, \\
& \beta \leq 0, \gamma \leq 0, \xi = e + \gamma.
\end{align*}
\]  

(97)

Where $\alpha$, $\beta$, and $\gamma$ are duality variables; $\xi$ is auxiliary variable. These variables are only introduced to facilitate the duality process rather than to represent specific physical meaning.

Although the SP (97) has been simplified using the duality theory, it is still challenging to be solved due to the bilinear term $u^\top \xi$. Therefore, Big-M method [35] is applied to tackle the bilinear term. Ultimately, SP is equivalently converted to a tractable MILP (98).
to researches [38, 40, 9], commercial solver CPLEX can solve MILPs using branch-cut
Model iteration.

3.3. Model iteration. Based on the above reformulation, the min-max-min model
(92) is finally decomposed into MP (95) and SP (98), which are MILPs. According
to researches [38, 40, 9], commercial solver CPLEX can solve MILPs using branch-and-bound approach while providing globally optimal solution. As a result, based on CPLEX, this paper implement optimization using C&CG iteration algorithm
[45], of which the steps are listed in Table 1.

Table 1. Steps of C&CG iteration algorithm.

| C&CG Iteration Algorithm          |
|-----------------------------------|
| **Step 1 (Initialization):** Set initial scenario \( u_1 \) and convergence gap \( \delta \). Initialize upper bound \( U_0 = +\infty \), lower bound \( L_0 = -\infty \), and iteration number \( k = 1 \). |
| **Step 2 (Solve MP):** Input the scenario set \( u_1 \) into (95) to solve MP. Record optimal solution \( (x_k, y_l) \), optimal value \( \alpha \) of objective, and \( c^T x \). Update the lower bound \( L_k = \alpha \), \( l = 1, 2, \ldots, k \). |
| **Step 3 (Solve SP):** Input \( x_k \) into (98) to solve SP. Record optimal solution \( (u_k^0, y_l^0) \) and optimal value \( \beta \) of objective. Set the worst scenario \( u_{k+1} \) to \( u_k^0 \). Update the upper bound \( U_k = \beta + c^T x \). |
| **Step 4 (Check Convergence):** If \( U_k - L_k \leq \delta \), terminate algorithm and record optimal value \( \nu \) as the expected cost. Otherwise, add constraints (99) and real-time adjustment variables \( y_{k+1} \) related to \( u_{k+1} \); return to **Step 2**, set \( k = k + 1 \). |

\[
\begin{align*}
\max \ u^u \xi^+ + u^d \xi^- + u^{pre}(1 - \xi^+ - \xi^-) \\
+ i^T \alpha - x^TC^T \alpha + j^T \beta - x^T E^T \beta \\
\text{s.t.} \ D^T \alpha + F^T \beta + G^T \gamma = d, \\
\beta \leq 0, \gamma \leq 0, \xi = e + \gamma, \\
-M(1 - \mu^+_i) + \xi_i \leq \xi^+_i \leq -M(1 - \mu^+_i) + \xi_i, \\
-M(1 - \mu^-_i) + \xi_i \leq \xi^-_i \leq -M(1 - \mu^-_i) + \xi_i, \\
-M \mu^+_i \leq \xi^+_i \leq M \mu^+_i, -M \mu^-_i \leq \xi^-_i \leq M \mu^-_i, \\
\mu^+_i + \mu^-_i \leq 1, \sum_i (\mu^+_i + \mu^-_i) \leq \Gamma.
\end{align*}
\]
4. Experimental results. This section evaluates the performances of the presented A-DED. The parameters and details of CGs, penalty prices of RES curtailment and load shedding, electricity prices of day-ahead market and real-time market, storage system parameters, and energy conversion coefficients are listed in Appendix Tables 6, 7, 8, 9, and 10 respectively. Data of load demand, predicted WPG, SPG, and EVs charging are shown as Figure 2 [31, 35].

All the uncertainty sets ((4)–(6)) are constructed using 30% prediction error, which are shown as the curves in Figure 3. As shown in the figure, Monte Carlo Simulation (MSC) generates 500 stochastic scenarios to serve as the real-time testing scenarios for evaluating the optimal day-ahead strategy $x$. All the scenarios are based on Gaussian distribution with 10% root mean square error. Based on a PC with Core i3-7100@3.0GHz CPU and 4 GB memory, all the experiments are carried out using MATLAB with CPLEX.

4.1. Comparison of D-DED, S-DED, R-DED, and A-DED. To evaluate the effectiveness of A-DED, the performance of A-DED with $\Gamma_{WPG} = \Gamma_{SPG} = \Gamma_{EV} = \Gamma = 6$ is compared to that of DO-based DED (D-DED), SO-based DED (S-DED), and RO-based DED (R-DED). Table 2 shows that S-DED consumes more
computing time. The computing times of D-DED, R-DED, and A-DED are quite comparable. In terms of the day-ahead operational cost and expected cost, R-DED is the most pessimistic as it only focuses on the worst-case scenario; D-DED is the most optimistic due to the ignorance of the prediction error; A-DED is more conservative than S-DED. Table 2 also reveals that the differences of the day-ahead costs mainly caused by FC generation plans and transaction strategies. Figure 4 to Figure 7 indicate that compared to D-DED and S-DED, A-DED schedules FC to generate less power. In addition, Table 3 illustrates that all the methods except R-DED benefit from the electricity market. Figure 6 points out the reason is that conservative R-DED purchases too much day-ahead electricity to withstand RES shortage and load surge. Although the day-ahead operational cost of A-DED is more conservative than S-DED and D-DED, Table 2 reveals that the total cost of A-DED is the lowest. The economy improvement can be attributed to suitable FC base generation and reasonable electricity transaction.

Table 2. Comparison of efficiencies and costs of different methods.

| Method | Time (s) | Day-ahead Cost ($) | Expected Cost ($) | Actual ($) |
|--------|---------|--------------------|-------------------|------------|
|        |         | $C_{MT}$ | $C_{FC}$ | $C_{DA}$ | RT | SUM |
| D-DED  | 6.52    | 5422.48 | 2251.42 | 5972.69 | 613.25 | 6585.95 |
| S-DED  | 1695.69 | 5336.95 | 2099.66 | 6011.19 | 566.70 | 6577.90 |
| R-DED  | 6.92    | 5425.33 | 1146.89 | 6503.61 | 10916.15 | 457.79 | 6961.91 |
| A-DED  | 8.37    | 5422.83 | 1796.67 | 6203.84 | 8305.34 | 367.47 | 6571.81 |

Table 3. Comparison of electricity transactions of different methods.

| Method | Day-ahead Transaction | Actual Transaction |
|--------|-----------------------|--------------------|
|        | Revenue ($) | Loss ($) | State | Revenue ($) | Loss ($) | State |
| D-DED  | 1701.19    | –        | Profit | 1303.22    | –        | Profit |
| S-DED  | 1425.36    | –        | Profit | 1051.32    | –        | Profit |
| R-DED  | –          | 68.61    | Loss  | –          | 275.65   | Loss  |
| A-DED  | 1015.67    | –        | Profit | 671.63     | –        | Profit |

Figure 4. Day-ahead dispatch decision of D-DED.
4.2. **Comparison of A-DED with different budgets.** The impact of the budget parameter is investigated in this section. $\Gamma_{WPG}$, $\Gamma_{SPG}$, and $\Gamma_{EV}$ are equal. The value is configured as 4, 8, 12, 16, and 20 respectively. Table 4 illustrates that larger budgets result in higher computational time. The reason is that A-DED with larger budgets requires more iterations to achieve the convergence. Additionally, the costs of the day-ahead strategies and the expectation increase with the increasing budgets. This occurs as the conservative strategies purchase more electricity from the day-ahead market to ensure the operational security. In terms of total cost, it reveals...
that the strategies with smaller budgets have better ability to improve the economy. Figure 8 further reveals that with the increasing budgets, both RESs penalty cost and RESs curtailment probability become lower. This point reveals that although the larger budget leads to more conservative strategy, the improvement of the RESs utilization can be achieved.

### Table 4. Comparison of A-DED with different budgets.

| Γ | Iteration Number | Time (s) | Day-ahead Cost ($C_M$) | $C_{FC}$ | $C_{DA}^{Grid}$ | $C_{DA}$ | Expected Cost ($) | Actual Cost ($) | SUM ($) |
|---|------------------|---------|------------------------|---------|----------------|---------|------------------|----------------|---------|
| 4 | 1                | 17.5    | 5422.8                | 1945.8  | -1249.8        | 6118.6  | 7580.2           | 6568.5         |
| 8 | 2                | 28.7    | 5422.8                | 1596.3  | -715.1         | 6304.1  | 8975.5           | 6753.4         |
| 12| 3                | 36.2    | 5422.8                | 1332.3  | -323.9         | 6431.6  | 9981.2           | 6880.8         |
| 16| 5                | 37.1    | 5422.8                | 1237.5  | -203.4         | 6457.3  | 10594.5          | 6900.3         |
| 20| 6                | 42.0    | 5422.8                | 1180.7  | -118.4         | 6485.0  | 10809.4          | 6928.3         |

**Figure 8.** RESs utilization results of A-DED with different budgets.

#### 4.3. Comparison of A-DED with different prediction errors.

Based on $\Gamma_{WP} = \Gamma_{SP} = \Gamma_{EV} = \Gamma = 12$, the performances of A-DED with 10%, 20%, 30%, 40%, and 50% errors are compared. As shown in Table 5, although outliers exit, generally larger errors result in longer computing time. Table 5 also indicates that larger prediction errors lead to significant increase of the day-ahead cost, expected cost, real-time cost, and total cost. The reason is that the optimizations with larger errors have to immunize the day-ahead strategies against worse scenarios of the uncertainty set. Therefore, strategies based on larger errors purchase more power in day-ahead market, while reducing base generation of FC to provide more reserve in real-time stage. Ultimately, the economy is reduced to guarantee the robustness in the case of larger prediction errors. Figure 9 further compares the RESs utilization results. As can be seen in the figure, larger prediction errors result in increasing RESs curtailment probability and penalty.
Table 5. Comparison of A-DED with different budgets.

| Error Number | Iteration Time (s) | Day-ahead Cost ($) | Expected Cost ($) | Actual Cost ($) | RT | SUM |
|--------------|--------------------|--------------------|------------------|-----------------|----|-----|
| 10%          | 1                  | 15.3               | 5422.5           | 2098.4          | -1500.2 | 6020.2 | 7152.4 | 103.5 | 6123.7 |
| 20%          | 3                  | 36.3               | 5422.8           | 1715.5          | -914.3 | 6224.8 | 8562.3 | 235.7 | 6460.5 |
| 30%          | 3                  | 29.2               | 5422.8           | 1332.7          | -323.9 | 6431.4 | 9987.2 | 449.4 | 6880.8 |
| 40%          | 4                  | 30.3               | 5409.2           | 1050.6          | 176.8  | 6636.0 | 11425.6 | 479.9 | 7385.9 |
| 50%          | 8                  | 36.4               | 5306.4           | 937.1           | 752.9  | 6996.3 | 13021.7 | 1023.3 | 8019.6 |

Figure 9. RESs utilization results of A-DED with different prediction errors.

5. Conclusion. This paper presents a two-stage adjustable robust optimization in enabling the optimal day-ahead dispatch for CCHP-MG incorporating the uncertainties of WPG, SPG, and EVs charging. Before the realization of the uncertainties, the base generation and states of the operational units are decided in the day-ahead stage. As long as the uncertainties are observed, the real-time dispatch adjusts the units to correct the day-ahead operational strategy. Based on C&C approach, the presented model is decomposed into MILP MP and SP. SP can be further converted to a tractable MILP using duality theory and Big-M method. As a result, the optimization can be implemented using C&C iteration algorithm. Based on the experimental results, the presented two-stage ARO approach is able to provide the optimal day-ahead operational strategy for CCHP-MG considering the worst realization of the uncertainties. Compared to DO-based, SO-based, and traditional RO-based DEDs, ARO-based DED with a suitable budget value achieves reasonable equilibrium in terms of robustness and the economy efficiently. In future work, the uncertainties of the heating load and the cooling load will be integrated into the demand side. In addition, the dynamic nonlinear production processes of CGs will be further considered.
Appendix. The parameters of CGs are proved in Table 6 [39]. The penalty prices of RESs curtailment and load shedding are given in Table 7 [9]. The electricity prices of day-ahead market and real-time market are listed in Table 8 [9]. The storage systems parameters are provided in Table 9 and Table 10 [9]. The energy conversion coefficients are shown in Table 11 [25].

Table 6. CGs parameters.

| CG | \( P_{\text{min}} \) (kW) | \( P_{\text{max}} \) (kW) | \( R_{\text{Up}} \) (kW/min) | \( R_{\text{Dn}} \) (kW/min) | \( \alpha \) | \( \beta \) | \( \lambda_{\text{Up}} \) ($/kWh) | \( \lambda_{\text{Dn}} \) ($/kWh) |
|----|-----------------|-----------------|-----------------|-----------------|-------|-------|-----------------|-----------------|
| MT | 50              | 550             | 6               | 6               | 0.67  | 0     | 2.5             | 1.5             |
| FC | 50              | 240             | 2               | 2               | 0.60  | 0     | 2.5             | 1.5             |
| EB | 20              | 500             | 5               | 4               | –     | –     | 0.5             | 0.5             |

Table 7. Penalty prices.

| \( \lambda_{\text{Wind}} \) ($/kWh) | \( \lambda_{\text{Solar}} \) ($/kWh) | \( \lambda_{\text{Load}} \) ($/kWh) |
|-----------------|-----------------|-----------------|
| 0.536           | 0.536           | 5               |

Table 8. Electricity market prices.

| Hour | Day-ahead stage | Real-time stage |
|------|-----------------|-----------------|
|      | \( \lambda_{\text{Buy}}^{\text{DA}} \) ($/kWh) | \( \lambda_{\text{Sell}}^{\text{DA}} \) ($/kWh) | \( \lambda_{\text{Buy}}^{\text{RT}} \) ($/kWh) | \( \lambda_{\text{Sell}}^{\text{RT}} \) ($/kWh) |
| (00:00-08:00) | 1.35 | 1.04 | 2.70 | 0.11 |
| (08:00-09:00, 12:00-19:00) | 0.90 | 0.69 | 1.80 | 0.07 |
| (09:00-12:00, 19:00-24:00) | 0.50 | 0.39 | 1.00 | 0.04 |

Table 9. Energy storage system parameters.

| \( P_{\text{min}} / P_{\text{max}} \) Cha (kW) | \( P_{\text{min}} / P_{\text{max}} \) Dis (kW) | \( \eta_{\text{ESS}}^{\text{Cha}} / \eta_{\text{ESS}}^{\text{Dis}} \) | \( \delta_{\text{ESS}} \) | \( E_{\text{max}}^{\text{ESS}} \) (kWh) | \( E_{\text{min}}^{\text{ESS}} \) (kWh) | \( E_{\text{ESS}}(0) \) (kWh) |
|-----------------|-----------------|-----------------|----------|-----------------|-----------------|-----------------|
| 0/200           | 0/200           | 0.9/0.9         | 0.001    | 480             | 120             | 120             |
Table 10. Heat storage system parameters.

| Parameter         | Value       | Parameter         | Value       | Parameter         | Value       | Parameter         | Value       |
|-------------------|-------------|-------------------|-------------|-------------------|-------------|-------------------|-------------|
| $Q_{\text{Cha}}^{\min}$ (kW) | 0/200       | $Q_{\text{Cha}}^{\max}$ (kW) | 0/200       | $\eta_{\text{Cha}}^{\text{HSS}}/\eta_{\text{Dis}}^{\text{HSS}}$ | 0.9/0.9   | $\delta_{\text{HSS}}$ | 0.01       |
| $\eta_{\text{Dis}}^{\text{HSS}}$ (kWh) | 600         | $E_{\text{HSS}}^{\max}$ (kWh) | 0          | $E_{\text{HSS}}^{\min}$ (kWh) | 0          | $E_{\text{HSS}}(0)$ (kWh) | 0          |

Table 11. Energy conversion coefficients.

| Parameter         | Value       |
|-------------------|-------------|
| $\eta_{\text{EH}}^{\text{MT}}$ | 0.8         |
| $\eta_{\text{EH}}^{\text{EB}}$ | 0.8         |
| $\eta_{\text{HC}}^{\text{MT}}$ | 0.8         |
| $\eta_{\text{HC}}^{\text{EB}}$ | 0.8         |
| $\eta_{\text{HC}}^{\text{HSS}}$ | 0.8         |

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REFERENCES

[1] A. Ben-Tal, A. Goryashko, E. Guslitzer and A. Nemirovski, Adjustable robust solutions of uncertain linear programs, Math. Program., 99 (2004), 351–376.
[2] A. Ben-Tal and A. Nemirovski, Robust convex optimization, Math. Oper. Res., 23 (1998), 769–1024.
[3] C. Chen, Simulated annealing-based optimal wind-thermal coordination scheduling, IET Generation, Transmission & Distribution, 1 (2007), 447–455.
[4] C. M. Correa-Posada and P. Sánchez-Martín, Integrated power and natural gas model for energy adequacy in short-term operation, IEEE Transactions on Power Systems, 30 (2015), 3347–3355.
[5] C. Duan, L. Jiang, W. Fang and J. Liu, Data-driven affinely adjustable distributionally robust unit commitment, IEEE Transactions on Power Systems, 33 (2018), 1385–1398.
[6] C. Duan, L. Jiang, W. Fang, J. Liu and S. Liu, Data-driven distributionally robust energy-reserve-storage dispatch, IEEE Transactions on Industrial Informatics, 14 (2018), 2826–2836.
[7] F. Fang, Q. H. Wang and Y. Shi, A novel optimal operational strategy for the CCHP system based on two operating modes, IEEE Transactions on Power Systems, 27 (2012), 1032–1041.
[8] F. Farmani, M. Parvizimosaeid, H. Monsef and A. Rahimi-Kian, A conceptual model of a smart energy management system for a residential building equipped with CCHP system, Internat. J. Electrical Power Energy Systems, 95 (2018), 523–536.
[9] H. Gao, J. Liu, L. Wang and Z. Wei, Decentralized energy management for networked microgrids in future distribution systems, IEEE Transactions on Power Systems, 33 (2018), 3599–3610.
[10] W. Gu, S. Lu, Z. Wu, X. Zhang, J. Zhou, B. Zhao and J. Wang, Residential CCHP microgrid with load aggregator: Operation mode, pricing strategy, and optimal dispatch, Appl. Energy, 205 (2017), 173–186.
[11] Y. Guo, J. Xiong, S. Xu and W. Su, Two-stage economic operation of microgrid-like electric vehicle parking deck, IEEE Transactions on Smart Grid, 7 (2016), 1703–1712.
[12] Z. Guo and X. Xiao, Wind power assessment based on a WRF wind simulation with developed power curve modeling methods, Abstract Appl. Anal., 2014 (2014), 1–15.
[13] N. Haouas and P. R. Bertrand, Wind farm power forecasting, Math. Probl. Eng., 2013 (2013), 5pp.
[14] R. Hashemi, A developed offline model for optimal operation of combined heating and cooling and power systems, IEEE Transactions on Energy Conversion, 24 (2009), 222–229.
[15] S. Jin, Z. Mao, H. Li and W. Qi, Dynamic operation management of a renewable microgrid including battery energy storage, Math. Probl. Eng., 2018 (2018), 19pp.
[16] Y. Lee and R. Baldick, A frequency-constrained stochastic economic dispatch model, IEEE Transactions on Power Systems, 28 (2013), 2301–2312.
[17] B. Li, X. Qian, J. Sun, K. L. Teo and C. Yue, A model of distributionally robust two-stage stochastic convex programming with linear recourse, Appl. Math. Model., 58 (2018), 86–97.
[18] B. Li, J. Sun and K. L. Teo, A distributionally robust approach to a class of three-stage stochastic linear programs, Pac. J. Optim., 15 (2019), 219–236.
[19] B. Li, J. Sun, H. Xu and M Zhang, A class of two-stage distributionally robust games, *J. Ind. Manag. Optim.*, **15** (2019), 387–400.

[20] G. Li, G. Li and M. Zhou, Model and application of renewable energy accommodation capacity calculation considering utilization level of inter-provincial tie-line, *Protection and Control of Modern Power Systems*, **4** (2019), 1–1.

[21] G. Li, R. Zhang, T. Jiang, H. Chen, L Bai, H. Cui and X. Li, Optimal dispatch strategy for integrated energy systems with cchp and wind power, *Appl. Energy*, **192** (2017), 408–419.

[22] G. Li, R. Zhang, T. Jiang, H. Chen, L Bai and X. Li, Security-constrained bi-level economic dispatch model for integrated natural gas and electricity systems considering wind power and power-to-gas process, *Appl. Energy*, **194** (2017), 696–704.

[23] Y. Liu, Y. Liu, J. Liu, M. Li, T. Liu, G. Taylor and K. Zuo, A MapReduce based high performance neural network in enabling fast stability assessment of power systems, *Math. Probl. Eng.*, **2017** (2017), 1–12.

[24] Y. Liu and N. C. Nair, A two-stage stochastic dynamic economic dispatch model considering wind uncertainty, *IEEE Transactions on Sustainable Energy*, **7** (2016), 819–829.

[25] C. Marino, M. Marufuzzaman, M. Hu and M. D. Sarder, Developing a CCHP-microgrid operation decision model under uncertainty, *Comput. Industrial Eng.*, **115** (2018), 354–367.

[26] M. H. Sarparandeh and M. Ehsan, Pricing of vehicle-to-grid services in a microgrid by Nash bargaining theory, *Math. Probl. Eng.*, **2017** (2017).

[27] X. Shen, Y. Liu and Y. Liu, A multistage solution approach for dynamic reactive power optimization based on interval uncertainty, *Math. Probl. Eng.*, **2018** (2018), 10pp.

[28] R. Shi, C. Sun, Z. Zhou, L Zhang, and Z. Liang, A robust economic dispatch of residential microgrid with wind power and electric vehicle integration, Chinese Control and Decision Conference (CCDC), 2016, 3672–3676.

[29] J. Soares, B. Canizes, M. A. F. Ghazvini, Z. Vale and G. K. Venayagamoorthy, Two-stage stochastic model using benders’ decomposition for large-scale energy resource management in smart grids, *IEEE Transactions on Industry Appl.*, **53** (2017), 5905–5914.

[30] Y. Tan, Y. Cao, C. Li, Y. Li, J. Zhou and Y. Song, A two-stage stochastic programming approach considering risk level for distribution networks operation with wind power, *IEEE Systems Journal*, **10** (2016), 117–126.

[31] L. Tian, S. Shi and Z. Jia, A statistical model for charging power demand of electric vehicles, *Power System Technology*, **11** (2010), 126–130.

[32] T. A. Victoire and A. Jeyakumar, Hybrid PSO–CSQP for economic dispatch with valve-point effect, *Electric Power Systems Research*, **71** (2004), 51–59.

[33] D. C. Walters and G. B. Sheble, Genetic algorithm solution of economic dispatch with valve point loading, *IEEE Transactions on Power Systems*, **8** (1993), 1325–1332.

[34] J. Wang, J. Wang, C. Liu and, J. Ruiz, Stochastic unit commitment with sub-hourly dispatch constraints, *Appl. Energy*, **105** (2013), 418–422.

[35] P. Wei and Y. Liu, The integration of wind-solar-hydropower generation in enabling economic robust dispatch, *Math. Probl. Eng.*, **2019** (2019), 12pp.

[36] H. Wu, X. Hou, B. Zhao and C. Zhu, Economical dispatch of microgrid considering plug-in electric vehicles, *Automation of Electric Power Systems*, **38** (2014), 77–84.

[37] T. Wu, Q. Yang, Z. Bao and W. Yan, Coordinated energy dispatching in microgrid with wind power generation and plug-in electric vehicles, *IEEE Transactions on Smart Grid*, **4** (2013), 1453–1463.

[38] W. Wu, J. Chen, B. Zhang and H. Sun, A robust wind power optimization method for look-ahead power dispatch, *IEEE Transactions on Sustainable Energy*, **5** (2014), 507–515.

[39] Y. Xiang, J. Liu and Y. Liu, Robust energy management of microgrid with uncertain renewable generation and load, *IEEE Transactions on Smart Grid*, **7** (2016), 1034–1043.

[40] L. Xie, Y. Gu, X. Zhu and M. G. Genton, Short-term spatio-temporal wind power forecast in robust look-ahead power system dispatch, *IEEE Transactions on Smart Grid*, **5** (2014), 511–520.

[41] P. Xiong, P. Jirutitijaroen and C. Singh, A distributionally robust optimization model for unit commitment considering uncertain wind power generation, *IEEE Transactions on Power Systems*, **32** (2017), 39–49.

[42] P. Xiong and C. Singh, Distributionally robust optimization for energy and reserve toward a low-carbon electricity market, *Electric Power Systems Res.*, **149** (2017), 137–145.

[43] Y. Yang, Practical robust optimization method for unit commitment of a system with integrated wind resource, *Math. Probl. Eng.*, **2017** (2017), 13pp.
[44] J. Yu, Q. Feng, Y. Li and J. Cao, Stochastic optimal dispatch of virtual power plant considering correlation of distributed generations, *Math. Probl. Eng.*, **2015** (2015).

[45] B. Zeng and L. Zhao, Solving two-stage robust optimization problems using a column-and-constraint generation method, *Oper. Res. Lett.*, **41** (2013), 457–461.

[46] Y. Zhang, J. Meng, B. Guo and T. Zhang, An improved dispatch strategy of a grid-connected hybrid energy system with high penetration level of renewable energy, *Math. Probl. Eng.*, **2014** (2014), 18pp.

[47] Y. Zhao, C. Li, M. Zhao, S. Xu, H. Gao and L. Song, Model design on emergency power supply of electric vehicle, *Math. Probl. Eng.*, **2017** (2017), 6pp.

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