Sheath losses correction factor for cross-bonded cable systems with unknown minor section lengths: Analytical expressions

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Abstract
The IEC Standard 60287-1-1 contains the formulae for the calculation of the sheath and armour loss factors of a three-phase cable circuit where sheath circulating current losses are computed for single-, multiple- or cross-bonded systems. For cross-bonded systems, the standard gives a formula allowing calculation of the circulating current losses when the lengths of the minor sections are known. When the lengths of the minor sections are not known, the standard suggests using a correction factor of 0.004, which multiplies the circulating current loss factor computed for a multi-point bonded system. Previous studies by the authors have shown that this assumption may lead to a significant error in the evaluation of the sheath loss factor for some combinations of the minor section lengths. This paper provides analytical formulae for the correction factor in the cross-bonded system with unknown section lengths with the aim of improving the accuracy of the IEC 60287-1-1. Several numerical examples illustrate the applicability and accuracy of the proposed approach.

1 INTRODUCTION

In cable rating calculations, in addition to the dielectric losses in the insulation and joule losses from conductors, the losses in the metallic cable screens and armouring are of significant importance. These losses are caused by the induced magnetic field generated by the currents flowing in the conductors, which in turn generates ohmic and potentially hysteresis losses in other metallic components of the cable.

Sheath losses are current dependent and can be divided into two categories. The first category is the circulating current losses, which flow in the metallic screens of single- or three-core SL-type cables if the sheaths are bonded together at multiple points. The second category is the eddy current losses, which circulate radially (skin effect) and azimuthally (proximity effect). Eddy current losses occur in both three- and single-core cables, irrespective of the method of bonding.

Current-dependent losses are the major source of heat and can be calculated in both conductor and metallic sheath of the power cables [1–3]. The subject of loss calculation has a long history, almost as long as the cable industry itself. Reference [3] provides a brief review of the development of the basic equations for calculating sheath circulating current losses.

Sheaths of the high-voltage power cables are, in the majority of cases, cross-bonded to reduce the losses caused by circulating currents present in the multi-point bonded systems. This paper concentrates on the cross-bonded systems with unequal minor section lengths. Such systems are commonly found in practice since the location of the joint pits, where the cross-bonding takes place, is dictated by the urban laying conditions seldom allowing equal distances between them.

When the length of the minor sections is known, calculation of the circulating current losses is well documented [1–3]. In many practical cases, the minor section lengths are unknown and then the only analytical approach used in rating calculations is to compute the circulating current loss factor as if the cables were multi-point bonded and multiply it by 0.004. The origin of this constant is explained in the next section together with a brief review of the cross-bonded cable systems. This is followed by the development of the analytical formulae for cross-bonded...
Cable systems with unknown minor section lengths illustrated by several numerical examples showing also the effect of using the developed expression in the ampacity calculations.

2 CROSS-BONDING OF CABLE SYSTEMS

Cross-bonding of single-core cable sheaths is a method of avoiding circulating currents and excessive sheath voltages while permitting increased cable spacing and long run lengths. The cross-bonding divides the cable run into three sections, and cross-connects the sheaths in such a manner that the induced voltages cancel each other. One disadvantage of this system is that it is expensive and therefore is applied mostly in high-voltage (HV) installations.

The major section of the cable is divided into three equal lengths, called minor sections, and the sheath continuity is broken at each joint. If the minor sections have equal lengths, the induced sheath voltages in each section of each phase are equal in magnitude and 120° out of phase. When the sheaths are cross-connected, as shown in Figure 1, each sheath circuit contains one section from each phase such that the total voltage in each sheath circuit sums to zero. If the sheaths are then bonded and earthed at the end of the major section, the net voltage in the loop and the circulating currents will be zero and the only sheath losses will be those caused by the eddy currents.

This method of bonding allows the cables to be spaced to take advantage of improved heat dissipation without incurring the penalty of increased circulating current losses. In practice, the lengths and cable spacings in each section may not be identical, and therefore some circulating currents will be present. The length of each section and cable spacings are limited by the voltages, which exist between the sheaths and between the sheaths and earth at each cross-bonding position. For long runs, the route is divided into a number of major sections, each of which is divided into three minor sections. Cross-bonding as described above can be applied to each major section independently.

The cross-bonding scheme described above assumes that the cables are arranged symmetrically; that is, in trefoil. It is usual that single-core cables are laid in a flat configuration. In this case, it is a common practice in long-cable circuits or heavily loaded cable lines to transpose the cables as shown in Figure 1b, so that each phase by rotation occupies each of the three phase positions in a circuit.

Several publications deal with installations when the minor sections of a cross-bonded system are not equal, but no analytical formula exists when the minor section lengths are unknown. The IEC Standard 60287-1-1, [1] gives formulae for unequal minor section lengths allowing calculation of the circulating current losses when the lengths of the minor sections are known. When the lengths of the minor sections are not known, the standard suggests using a factor of 0.004 (see below for the derivation of this value), which multiplies the circulating current loss factor computed for a multi-point bonded system. Several numerical examples employing standard equations for sheath voltages and currents presented, for example in [2], to obtain sheath loss factors in cable sections of different axial spacing and/or different conductor sizes are offered in [4] and are also compared with the results obtained with the IEC formulation.

An earlier paper by the authors [3] discussed the precision and applicability of the 0.004 factor for the evaluation of sheath circulating losses for the actual cable systems with different minor section lengths. The aim of this paper is to present a development of general formulae applicable to all practical cable constructions and possible minor section lengths.

3 PRESENT METHOD FOR CALCULATING SHEATH LOSSES

Electrical losses within HV cables are most commonly calculated using the methods shown in the IEC Standard 60287-1-1, [1]. All equations for sheath losses given in this standard assume that the phase currents are balanced. The equations also require a knowledge of the temperature of the sheath and armour, which cannot be calculated until the cable rating is known; therefore, an iterative process is required. For the first calculation, these temperatures must be estimated; this estimate can be checked later after the current rating has been calculated. If necessary, the sheath and armour losses, and hence the current rating, must be recalculated with the revised temperatures.

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1 The review presented in this chapter is taken from [2].

2 The review in this chapter is taken from [3].
The power loss in the sheath or screen (\(\lambda_1\)) consists of losses caused by circulating currents \(\lambda'_1\) and eddy currents \(\lambda''_1\). Thus,
\[
\lambda_1 = \lambda'_1 + \lambda''_1.
\]  

In this presentation, only circulating current losses in a cross-bonded system are considered. In order to address this issue, a brief review of the calculation procedure of the sheath circulating current loss factor for solidly bonded screens is required.

### 3.1 Sheath circulating current loss factor

With sheaths bonded at both ends of an electrical section, the losses caused by circulating currents will dominate sheath loss factor calculations. An electrical section is defined as a portion of the route between points at which the sheaths or screens of all cables are solidly bonded. Of course, there are no circulating currents when the sheaths are isolated or bonded at one point only. The complex currents flowing in the conductor, sheath, and armour are denoted by \(I_a, I_b,\) and \(I_{as}\) respectively. Then, the sheath loss factor due to circulating currents is defined as:
\[
\lambda'_1 = \frac{|I_a|^2 R_s}{|I_a|^2 R}
\]  

where \(R (\Omega/m)\) denotes the ac resistance of the conductor at operating temperature and the subscript \(s\) represents the sheath.

The first step in computing the sheath loss factor is to obtain the sheath resistance and reactance. If the sheath is reinforced with a nonmagnetic tape or tapes, a parallel combination of both resistances is required. Once the sheath resistance is obtained, the sheath reactance needs to be computed. The appropriate formulae for the calculation of the sheath reactance are as follows.

For single-conductor and pipe-type cables:
\[
X = 4\pi f \times 10^{-7} \times \ln \frac{2s}{d}
\]  

where \(f (\text{Hz})\) is the system frequency, \(s (\text{mm})\) is the spacing between conductor centres and \(d (\text{mm})\) is the mean diameter of the sheath.

For single-conductor cables in flat formation, regularly transposed, sheaths bonded both ends:
\[
X_1 = 4\pi f \times 10^{-7} \times \ln \left[2 \times \sqrt{2} \left(\frac{s}{d}\right)\right].
\]  

For cables in flat formation with equal distance between the middle and outer cables, the mutual reactance per unit length between the outer cable sheath and the conductor of the other two cables is given by [1]
\[
X_{as} = 4\pi f \ln(2) \times 10^{-7}.
\]

### 3.1.1 Sheaths solidly bonded

For single-conductor cables in a flat configuration with sheaths solidly bonded at both ends, the sheath loss factor depends on the spacing. If it is not possible to maintain the same spacing in the electrical section (i.e. between points at which the sheaths of all cables are bonded), the following allowances should be made:

1. If the spacings are known, the value of \(X\) is computed from
\[
X = \frac{l_a X_a + l_b X_b + \cdots + l_s X_s}{l_a + l_b + \cdots + l_s}
\]  

where
\[
l_a, l_b, \ldots, l_s\]
are lengths with different spacing along an electrical section
\[
X_a, X_b, \ldots, X_s\]
are the reactances per unit length of cable, given by Equations (3) and (4) where appropriate values of spacing \(s_a, s_b, \ldots, s_s\) are used.

2. If the spacings are not known, the IEC Standard recommends that the value of \(\lambda'_1\) calculated below should be increased by 25%.

The following equations provide sheath loss factors for single-conductor cables for various bonding arrangements and cable configurations [1].

- Sheath bonded both ends—triangular configuration or flat formation with a regular transposition:
\[
\lambda'_1 = \frac{R_s}{R} \cdot \frac{1}{1 + \left(\frac{R_s}{X_1}\right)^2}.
\]  

When the cables in triangular formation are not transposed, the value of \(X_1\) is replaced by \(X\) given in (3).

- Sheath bonded both ends—flat configuration, no transposition. Centre cable equidistant from other cables.
\[
\lambda'_{1s} = \frac{R_s}{R} \left(\frac{\frac{1}{2}\sqrt{2} \left(\frac{s_a}{d}\right)}{R_s^2 + Q^2} + \frac{\frac{1}{2}p^2}{R_s^2 + p^2} - \frac{2R_s Q X_s}{\sqrt{3} (R_s^2 + Q^2) (R_s^2 + p^2)}\right)
\]  

in the leading phase
\[
\lambda'_{1m} = \frac{R_s}{R} \frac{Q^2}{R_s^2 + Q^2}
\]  

in the middle cable
\[
\lambda'_{1o} = \frac{R_s}{R} \left(\frac{\frac{1}{2}\sqrt{2} \left(\frac{d}{s_a}\right)}{R_s^2 + Q^2} + \frac{\frac{1}{2}p^2}{R_s^2 + p^2} + \frac{2R_s Q X_s}{\sqrt{3} (R_s^2 + Q^2) (R_s^2 + p^2)}\right)
\]  

in the lagging phase
where \(P\) and \(Q\) are defined by

\[
P = X_m + XQ = X - X_m/3.
\]

(9)

Derivation of these equations can be found in [2]. Ratings for cables in air should be calculated using \(\lambda_1\).

### 3.1.2 Sheaths cross-bonded

The ideal cross-bonded system will have equal lengths and spacing in each of the three sections. If the section lengths are different, the induced voltages will not sum to zero and circulating currents will be present. These currents are taken into account by calculating the circulating current loss factor \(\lambda'_c\), assuming the cables were not cross-bonded, and multiplying this value by a factor to take into account the length variations. In the IEC Standard [1], this factor \(F_c\) is given by

\[
F_c = \frac{p^2 + q^2 + 1 - p - pq - q}{(p + q + 1)^2}
\]

(10)

where

- \(p\alpha\) = length of the longest section
- \(q\alpha\) = length of the second-longest section
- \(a\) = length of the shortest section.

This formula deals only with differences in the length of minor sections. Any deviations in spacing must also be taken into account. The development of this formula is presented in [3].

Where lengths of the minor sections are not known, the IEC standard [1] recommends that \(p\) should be set to 1 and \(q\) to 1.2, this gives the value of 0.004. The aim of this paper is to develop an analytical formula for the calculation of the factor \(F_c\) in two situations. The first one is when no information is available about the range of values of \(p\) and \(q\). Since, in a general case, the only readily available parameter is the sheath resistance at 20 °C, denoted here by \(R_s\), the goal was to develop an expression:

\[
F_c = f (R_s).
\]

(11)

As will be shown later, relying on the sheath resistance alone may lead in some extreme cases to a substantial error when Equation (11) is used. In such cases, an alternative formulation will be developed in Chapter 5 and its applicability discussed in Chapter 6.

### 4 DEVELOPMENT OF THE ANALYTICAL EXPRESSIONS

Since the aim is to find function \(f\) in (11) applicable to all practical cases, the approach adopted by the authors was to investigate thousands of possible combinations of the sheath resistance and \((p, q)\) pairs. The process is described below.

#### 4.1 Generation of \((p, q)\) pairs

The values of \(p, q\) are selected based on practical observations to cover all possible changes of these parameters in the range of 1–5 in an increment of 0.1. Thus, we have 50 values for “\(p\)" as \(p = \{1, 1.1, 1.2, \ldots, 5\}\), similarly, \(q = \{1, 1.1, 1.2, \ldots, 5\}\). Then, 860 different pairs of \(p\) and \(q\) are generated with all possible \((p, q)\) pairs excluding the cases when \(p = q\).

#### 4.2 Selection of sheath resistances and study cases

In order to cover the majority of cases encountered in practice, 50 cables with different sheath constructions (continuous and concentric neutral wires) and materials (Al, Cu, Lb) were investigated. The analysis also covered different conductor sizes and voltage levels. Each case was studied with 860 different \((p, q)\) pairs generating 43,000 values of the correction factor \(F_c\). After excluding the outliers, 41,699 different values of this parameter were obtained. A summarized description of the studied cable constructions is presented in Table 1.

#### 4.3 Description of the calculation procedure

To obtain the required relationship in (11), the following procedure was adopted:

1. For a given cable construction and \(p, q\) values, circulating current loss for a cross-bonded system was computed.
2. For the same conditions assuming two-points bonding, circulating current loss was computed.
3. By comparing results from 1 and 2, the value of \(F_c\) was obtained.
4. Steps 1–3 were repeated for different \((p, q)\) pairs for a given value of \(R_s\) with selected cable construction.
5. For the 50 cases of different cables and hence different \(R_s\), steps 1 to 4 were repeated. This resulted in 43,000 different values of \(F_c\).
6. From the set obtained in step 5, the outliers were excluded which reduced the number of the \(F_c\) values to 41,699.
7. The 41,699 values obtained in step 6 were divided into four groups, each group covering certain range of the studied cases. This gives the four groups of data in terms of \((F_c, R_s)\) pairs.
8. In each group, \(F_c\) is divided by its corresponding \(R_s\) that gives the four groups of data in terms of \((F_c, F_c/R_s)\) pairs.
9. A trendline with its analytical expression that best fits the computed values of each group is developed and \(R^2\) values are found as a statistical measure of how close the data are to the fitted regression line.

When the required functional relationship was obtained, the results were compared with the IEC values with \(p\) and \(q\) known for reliability and applicability validation. This is followed by a discussion of validity of the multiplier 0.004 in the IEC standard.
| Case | Conductor size | Area (mm²) | Voltage (kV) | Metallic sheath | Jacket | Manufacturer |
|------|----------------|------------|--------------|----------------|--------|--------------|
| 1    | Cu             | 1200       | 132          | Lead alloy     | HDPE   | Silec Cable  |
| 2    | Cu             | 1200       | 132          | Lead alloy     | HDPE   | ABB          |
| 3    | Cu             | 2500       | 400          | Lead alloy+ Concentric metallic wire screen | HDPE | Exsym        |
| 4    | Cu             | 1200       | 220          | Lead alloy+ concentric metallic wire screen | HDPE | Silec Cable  |
| 5    | Cu             | 800        | 66           | Concentric metallic Cu wire screen | HDPE | Elsewedy Cables |
| 6    | Cu             | 1000       | 66           | Corrugated aluminium sheath | PE | Unistar      |
| 7    | Cu             | 800        | 66           | Stainless steel | HDPE | ABB          |
| 8    | Cu             | 800        | 132          | Lead screen    | PE     | ABB          |
| 9    | Cu             | 1200       | 220          | Lead alloy     | HDPE   | Taihan       |
| 10   | Cu             | 1200       | 220          | Lead alloy     | HDPE   | Showa        |
| 11   | Cu             | 630        | 132          | Lead sheath    | PE     | Demirer Kablo |
| 12   | Cu             | 2000       | 400          | Corrugated aluminium sheath | PE | Taihan       |
| 13   | Cu             | 500        | 110          | Corrugated aluminium sheath | PE | Taihan       |
| 14   | Cu             | 500        | 110          | Lead alloy+ concentric metallic wire screen | PE | Taihan       |
| 15   | Cu             | 400        | 66           | Aluminium sheath | PE | LS Cable     |
| 16   | Cu             | 400        | 66           | Lead sheath    | PE     | LS Cable     |
| 17   | Cu             | 400        | 66           | Copper wire    | PE     | LS Cable     |
| 18   | Cu             | 3000       | 400          | Aluminium sheath | PE | LS Cable     |
| 19   | Cu             | 3000       | 400          | Lead sheath    | PE     | LS Cable     |
| 20   | Cu             | 1600       | 132          | Corrugated copper sheath | PE | Phelps Dodge |
| 21   | Cu             | 1400       | 150          | Corrugated copper sheath | PE | Phelps Dodge |
| 22   | Cu             | 2000       | 220          | Lead sheath    | HDPE   | Elsewedy Cables |
| 23   | Cu             | 1000       | 132          | Copper wire    | HDPE   | Elsewedy Cables |
| 24   | Cu             | 500        | 66           | Corrugated aluminium | PE | ILJIN Electric |
| 25   | Cu             | 1200       | 66           | Copper wires   | PE     | ILJIN Electric |
| 26   | Cu             | 1600       | 66           | Lead alloy     | PE     | ILJIN Electric |
| 27   | Cu             | 630        | 66           | Smoothed aluminium | PE | ILJIN Electric |
| 28   | Cu             | 1000       | 110          | Corrugated aluminium sheath | PE | ILJIN Electric |
| 29   | Cu             | 1200       | 110          | Copper wires   | PE     | ILJIN Electric |
| 30   | Cu             | 1600       | 110          | Lead alloy     | PE     | ILJIN Electric |
| 31   | Cu             | 2000       | 110          | Smoothed aluminium | PE | ILJIN Electric |
| 32   | Cu             | 1200       | 132          | Corrugated aluminium | PE | ILJIN Electric |
| 33   | Cu             | 1600       | 132          | Copper wires   | PE     | ILJIN Electric |
| 34   | Cu             | 2000       | 132          | Lead alloy     | PE     | ILJIN Electric |
| 35   | Cu             | 2500       | 132          | Smoothed aluminium | PE | ILJIN Electric |
| 36   | Cu             | 500        | 150          | Corrugated aluminium | PE | ILJIN Electric |
| 37   | Cu             | 630        | 150          | Copper wires   | PE     | ILJIN Electric |
| 38   | Cu             | 800        | 150          | Lead alloy     | PE     | ILJIN Electric |
| 39   | Cu             | 1000       | 150          | Smoothed aluminium | PVC | ILJIN Electric |
| 40   | Cu             | 1200       | 220          | Corrugated aluminium | PVC | Demirer Kablo |
| 41   | Cu             | 2500       | 220          | Lead sheath    | PVC    | Demirer Kablo |
| 42   | Cu             | 1000       | 66           | Lead sheath    | HDPE   | Elsewedy Cables |

(Continues)
TABLE 1 (Continued)

| Case | Type | Area (mm²) | Voltage (kV) | Metallic sheath | Jacket | Manufacturer          |
|------|------|------------|--------------|----------------|--------|------------------------|
| 43   | Cu   | 2500       | 220          | Copper wires   | HDPE   | Elsewedy Cables        |
| 44   | Cu   | 1000       | 110          | Laminated aluminium tape | PE    | Riyadh Cables         |
| 45   | Cu   | 1600       | 132          | Laminated aluminium tape | PE    | Riyadh Cables         |
| 46   | Cu   | 2000       | 220          | Laminated aluminium tape | PE    | Riyadh Cables         |
| 47   | Cu   | 800        | 132          | Lead alloy     | HDPE   | Brugg Cables          |
| 48   | Cu   | 1200       | 132          | Lead alloy     | HDPE   | Brugg Cables          |
| 49   | Al   | 1600       | 150          | Lead sheath    | PE     | Nexans                |
| 50   | Al   | 2000       | 150          | Lead sheath    | PE     | Nexans                |

TABLE 2 Constants in Equation (12)

| Group | Number of \((F_c, F_c/R_s)\) pairs | \(R_s\) (Ω/km) at 20 °C | From | To | Analytical expression | \(R^2\) |
|-------|----------------------------------|--------------------------|------|---|------------------------|---------|
| 1     | 2574                             | 0.01946                  | 0.03752 | \(F_c = \frac{0.0889}{R_s - 39.055}\) | 0.7285  |
| 2     | 11113                            | 0.03753                  | 0.07857 | \(F_c = \frac{0.0757}{18.749 - \frac{1}{R_s}}\) | 0.8354  |
| 3     | 13871                            | 0.07858                  | 0.20076 | \(F_c = \frac{0.032}{8.227 - \frac{1}{R_s}}\) | 0.7030  |
| 4     | 14111                            | 0.20077                  | 0.53172 | \(F_c = \frac{0.0049}{\frac{1}{R_s} - 3.5889}\) | 0.7332  |

The procedure described above will generate the relation (11), which is dependent on the sheath resistance at 20 °C. This approach has, however, one important flaw. It will give the same circulating current loss factor for a given cable construction independent of the actual minor section lengths. Therefore, an additional study was performed to see whether the factor \(F_c\) can be linked to both \((p, q)\) pairs and the \(R_s\) values. This procedure is described below:

1. Steps 1 to 5 above remain with no change.
2. The 43,000 different values of \(F_c\) obtained in step 5 were divided into four groups, each group covering certain range of the studied cases. This gives the four groups of data in terms of \((F_c, R_s)\) pairs.
3. In each group, \(F_c\) and \(R_s\) were multiplied by its corresponding parameter \(P\), where \(P\) is the product of \(p\) times \(q\) that gives the four groups of data in terms of \((PF_c, PR_s)\) pairs.
4. A trendline with its analytical expression that best fits the computed values of each group is developed and \(R^2\) values are found as a statistical measure of how close the data are to the fitted regression line.

When the required functional relationship was obtained, the results were compared with the IEC values with \(p\) and \(q\) known for reliability and applicability validation.

5 | RESULTS

5.1 | Equation (11) without \((p, q)\) values

The analysis described in Chapter 4 resulted in the following general expression:

\[
F_c = \frac{a}{b R_s - c}.
\]  

The values of the constants are different in each of the four groups of the \(R_s\) range and are summarized in Table 2. Figures 2–5 display the \((F_c, F_c/R_s)\) graphs for each group.

5.2 | Equation (11) with \((p, q)\) values

The following general expression was obtained in this case:

\[
F_c = h R_s^P \exp(-1)
\]

where \(P = p \cdot q\) and the constants are given in Table 3.

The applicability of formulae (12) and (13) is discussed in the next section.
TABLE 3 Values of the constants in Equation (13)

Summary of analytical expressions

| Group | Number of pairs | Rs (20°C) (Ohm/km) | Analytical expression | R-squared |
|-------|----------------|--------------------|-----------------------|-----------|
| 1     | 9460           | 0.019460 to 0.0583207 | $F_\gamma = 3.751(R_s^{1.2022})(P_0^{0.2022})$ | 0.5994 |
| 2     | 12900          | 0.0638087 to 0.1533471 | $F_\gamma = 1.6314(R_s^{1.3927})(P_0^{0.3927})$ | 0.6736 |
| 3     | 10320          | 0.1778360 to 0.21094486 | $F_\gamma = 0.4711(R_s^{1.4914})(P_0^{0.4914})$ | 0.6524 |
| 4     | 10320          | 0.2189243 to 0.53171739 | $F_\gamma = 0.2416(R_s^{1.3784})(P_0^{0.3784})$ | 0.6363 |

FIGURE 2 $F_\gamma - F_c/R_s$ graph of group 1

FIGURE 3 $F_\gamma - F_c/R_s$ graph of group 2

FIGURE 4 $F_\gamma - F_c/R_s$ graph of group 3

FIGURE 5 $F_\gamma - F_c/R_s$ graph of group 4

6 | VALIDATION OF DEVELOPED EXPRESSIONS

6.1 | Validation of Equation (12)

In this section, the verification of the proposed expressions is illustrated through numerical application to eight cases of different cable constructions and/or voltage levels. Lengths of the minor and major sections are unknown and, hence, $(p, q)$ pairs are assumed to be in their extreme combinations. The correction factors computed from the proposed expressions are compared to those using the IEC formula with $p$ and $q$ known. A brief description of the installation parameters and the cable systems is given below:

6.1.1 | Installation parameters

The installation parameters are kept the same for all systems as described below:

- Laying method: Direct buried
- Formation: Flat (transposed)
- Sheath bonding: Cross-bonded
- Depth to cable centre: 1046 mm
- Phase spacing: 200 mm
6.1.2  Description of cable systems used for reliability verification

A brief description of the cable systems is given below:

- **System 1**: Copper conductor, 132 kV, 1200 mm², XLPE insulated, lead alloy sheath and HDPE outer sheath. The metallic sheath is 3.5 mm thick and 92.4 mm average outer diameter ($R_s = 0.218924363 \text{ Ohm/km}$).

- **System 2**: Copper conductor, 132 kV, 1200 mm², XLPE insulated, lead alloy sheath and HDPE outer sheath. The metallic sheath is 3.8 mm thick and 97.3 mm average outer diameter ($R_s = 0.19172 \text{ Ohm/km}$).

- **System 3**: Copper conductor, 400 kV, 2500 mm², XLPE insulated, combination of copper wires and lead alloy sheath and HDPE outer sheath. The lead sheath is 2.8 mm thick and 139.6 mm average outer diameter in addition to 50 concentric metallic wire screen, each of 1.54 mm² cross-sectional area ($R_s = 0.177836037 \text{ Ohm/km}$).

- **System 4**: Copper conductor, 220 kV, 1200 mm², XLPE insulated, combination of copper wires and lead alloy sheath and HDPE outer sheath. The lead sheath is 2.5 mm thick and 107.7 mm average outer diameter in addition to 105 concentric metallic wire screen, each of 1.43 mm² cross-sectional area ($R_s = 0.259005002 \text{ Ohm/km}$).

- **System 5**: Copper conductor, 66 kV, 800 mm², XLPE insulated, copper wire screen and HDPE outer sheath. The metallic sheath is made of 50 concentric metallic wire screen wires, each of 1.43 mm² cross-sectional area ($R_s = 0.046099084 \text{ Ohm/km}$).

- **System 6**: Copper conductor, 66 kV, 1000 mm², XLPE insulated, corrugated aluminium sheath and HDPE outer sheath. The metallic sheath is 1.7 mm thick and 68.2 mm average outer diameter ($R_s = 0.079580 \text{ Ohm/km}$).

- **System 7**: Copper conductor, 66 kV, 800 mm², XLPE insulated, stainless steel sheath and HDPE outer sheath. The metallic sheath is made of 5 concentric metallic wire screen wires, each of 1.43 mm² cross-sectional area ($R_s = 0.370807338 \text{ Ohm/km}$).

- **System 8**: Copper conductor, 400 kV, 2000 mm², XLPE insulated, aluminium sheath and HDPE outer sheath. The metallic sheath is 3 mm thick and 125.8 mm average outer diameter ($R_s = 0.024146615 \text{ Ohm/km}$).

Table 4 displays the selected extreme ($p, q$) combinations and also shows the metallic sheath resistance at 20 °C for each case. In this table, for all systems two extreme values of this pair were selected: the lower extreme $p = 1$ and $q = 1$ and the upper extreme $p = 5$ and $q = 5$.

Table 5 compares the values of the correction factors $F_i$ obtained from the proposed expressions to those computed using the IEC formula for the cases studied. We can observe that the computed values of the $F_i$ fall within the IEC range.

6.1.3  Real system studies

The next study involves the application of the developed expressions to the real cable systems. The minor section lengths are known but assumed to be unknown for this study. The correction factors computed from the proposed expressions are compared to those obtained using the IEC formula with known and unknown $p$ and $q$ values.

1. Installation parameters

The installation parameters are kept the same for all systems as described below:

- Laying method: Direct buried
- Formation: Flat (transposed)
- Sheath bonding: Cross-bonded
- Depth to cable centre: 1046 mm
- Phase spacing: 200 mm for 132 and 220 kV systems and 450 mm for the 400 kV system.

2. Description of the cable systems

- Seven different real cable systems of different voltage levels and/or construction are studied; the seventh system
TABLE 6  Factor $F_c$ computed from (12) and from the IEC formula at actual $(p, q)$ combinations

| Cable system | Actual $p$, $q$ values | $F_c$ | IEC formula [known $(p, q)$] | from (12) | IEC formula [unknown $(p, q)$] |
|--------------|------------------------|-------|-------------------------------|-----------|-------------------------------|
|              | $p$ | $q$ |                              |           |                               |
| 1            | 1.0000 | 1.0368 | 0.0000                       | 0.0045    | 0.004                         |
| 2            | 1.0477 | 1.0955 | 0.0006                       | 0.0045    | 0.004                         |
| 3            | 1.0312 | 1.0312 | 0.0001                       | 0.0045    | 0.004                         |
| 4            | 1.0000 | 2.1228 | 0.0741                       | 0.0038    | 0.004                         |
| 5            | 1.0269 | 1.0371 | 0.0001                       | 0.0086    | 0.004                         |
| 6            | 1.0656 | 1.0656 | 0.0004                       | 0.0102    | 0.004                         |
| 7            | 3.6852 | 6.7222 | 0.1890                       | 0.0086    | 0.004                         |

has highly irregular section lengths. Installation parameters and cable details are listed below:

- System 1: Copper conductor, 132 kV, 1200 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.214 Ohm/km – $p$ and $q$ values are 1.0000 and 1.0368, respectively.
- System 2: Copper conductor, 132 kV, 1200 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.214 Ohm/km – $p$ and $q$ values are 1.0477 and 1.0955, respectively.
- System 3: Copper conductor, 132 kV, 1200 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.214 Ohm/km – $p$ and $q$ values are 1.0312 and 1.0312, respectively.
- System 4: Copper conductor, 132 kV, 800 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.205 Ohm/km – $p$ and $q$ values are 1.0000 and 2.1228, respectively.
- System 5: Copper conductor, 220 kV, 1200 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.0839 Ohm/km – $p$ and $q$ values are 1.0269 and 1.0371, respectively.
- System 6: Copper conductor, 400 kV, 2500 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.196 Ohm/km – $p$ and $q$ values are 1.0656 and 1.0656, respectively.
- System 7: Copper conductor, 220 kV, 1200 mm², XLPE insulated. The metallic sheath resistance at 20 °C is 0.0839 Ohm/km – $p$ and $q$ values are 6.7222 and 3.6852, respectively.

System 7, even though has unrealistic section lengths, was selected to test the extreme laying situations. Table 6 and Figure 6 show the correction factors $F_c$ computed from the proposed expression in comparison with the IEC values with known and unknown sections lengths.

We can see that for the cases of highly irregular section lengths, the difference between the IEC with known section length and the formula (12) does not fare well. This motivated development of Equation (13) and the comparative results are shown below.

Table 7 compares the values of $F_c$ obtained with (13) for four cases of high irregularity in minor sections to the IEC values.

Figure 7 and Table 8 show the improved correction factors $F_c$ computed from expression (13) and compares them with the IEC values with known and unknown sections lengths for the two real cases. Expression (13) was applied to cases 4 and 7 while expression (12) is still applied to remaining cases.

6.2 Selection of the appropriate formula

The two expressions presented above could be used depending on the available information about the minor section lengths. Application of expression (13), the results for the cases if high
irregular minor sections were greatly improved as shown in Figure 7 for real cases 4 and 7. Based on the results obtained above, the following recommendations can be made regarding the use of either Equation (12) or (13).

- For minor section of small or ignorable difference or when they are completely unknown, apply Equation (12).
- For highly irregular minor sections, we can apply Equation (13) where we need to put the values of \( p \) as per best of our knowledge (they are unknown but we may use the values that we think are the closest to the actual ones).

### 7 | AMPACITY CALCULATIONS

In order to compute ampacities of the seven real cases, the loss factors were computed first and compared with the IEC expression with known section lengths. The results are summarized in Table 9.

Next, the ampacities were computed applying the formulas from the IEC 60287-1-1 with known and unknown minor section lengths. The comparison was made with the values calculated based on proposed expressions. Table 10 summarizes the results of these calculations.

### 8 | DISCUSSION AND CONCLUSIONS

The newly developed expressions for the analysis of the losses in cross-bonded cable systems with unknown section lengths are very simple and cover a very wide range of sheath resistances from 0.019460 to 0.531717 Ohm/km. The \( R^2 \) values with a minimum indicate that the selected models well explain all the variability of the data around its mean [6].

Table 5 compares the values of the correction factors \( F_r \) obtained from the proposed expressions to those computed using the IEC formula for the cases studied. The same table also provides the evidence that the correction factors obtained
by applying the proposed formulas are reliable since all values lie within the range between the minimum and maximum of the correction factors computed from IEC formula with known \((p, q)\) values.

For the first three cases presented in Table 6, the correction factors computed by the proposed formulas are the same even with slightly different minor section lengths, because the sheath resistance is the same. The values are very close to those of the IEC with unknown minor section lengths. In case 4, the correction factor computed by the proposed formula (12) is also very close to that of the IEC with the \((p, q)\) unknown. In cases 5 and 7 that have the same sheath resistance, even though the minor section lengths are slightly different in case 5 and highly irregular in case 7, the correction factors computed by the proposed formulas are equal and almost double that of IEC with \((p, q)\) unknown. In case 6, the correction factors computed by the proposed formulae is 2.55 times that of IEC with \((p, q)\) unknown. This motivated to develop Equation (13), which is to be applied for cases with substantially different minor section lengths or for the cases when imprecise but some information is available about these lengths.

Figure 7 suggests that the correction factors obtained by applying the proposed formulas are very close to those provided by the IEC with unknown minor section lengths.

Table 10 indicates that the change in the correction factors has a small and sometimes negligible effect on the continuous current carrying capacity for cross-bonded systems.

Nevertheless, it is important that the best possible approximations are used in the case when no information is available on the minor section lengths.

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