"Super-radiance" and the width of exotic baryons

N. Auerbach
School of Physics and Astronomy, Tel Aviv University, Tel Aviv, 69978, Israel

V. Zelevinsky
NSCL, Michigan State University, East Lansing, MI 48824-1321, USA

A. Volya
Department of Physics, Florida State University, Tallahassee, FL 32306-4350, USA

Abstract

It is suggested that the narrow width of the recently observed resonance \(\Theta^+ (1540)\) with strangeness \(S = +1\) could be a result of the super-radiance mechanism of the redistribution of the widths of overlapping resonances due to their coupling through common decay channels.

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Fifty years ago, Dicke [1] introduced the notion of “super-radiance” when discussing the formation of a short-lived state in a radiant gas of \(N\) identical two-level atoms. When the gas is confined to a volume with a linear size smaller than the wave length of the radiation, the atoms are coherently coupled through the common radiation field. This leads to a formation of the many-body state that has a width \(\sim N\Gamma_a\), where \(\Gamma_a\) is the radiation width of an isolated individual atom. This is the “super-radiant” (SR) state observed in the transmission of the laser pulse through the medium. The SR state gets its width at the expense of other states of the system, which are “robbed” of their decay widths and become very narrow.

The phenomenon of the SR is in fact of a more general nature [2–6]. The formalism can be derived with the use of the Feshbach projection method.
[7,8], and in some situations the SR state can be interpreted as a doorway for the continuum coupling although we need to stress that the conventional notion of the doorway state refers to the collectivization of strengths in intrinsic dynamics that may or may not coincide with the width collectivization and segregation of direct and compound processes in the SR dynamics. From the late eighties, the ideas related to this concept have been applied to many fields, including atomic physics [9], molecular physics [10], and condensed matter physics [11]. In low-energy nuclear physics [12–19] the main interest was associated with resonances embedded in the continuum, collective dynamics of giant resonances and description of loosely bound nuclei. Applications to intermediate-energy nuclear physics [20] include meson resonances [15,16,20], nucleon-antinucleon states [20] and, more recently, Δ-isobar resonances in nuclei [21].

In many examples, including the last two cases, the emphasis was placed not only on the wide SR state, but also on the fact that the mechanism of the SR theory creates narrow resonances superimposed on a background formed by the broad SR state leading in this way to the separation of fast (direct) processes from the long-lived structures. The consequences of this were examined and it was suggested that this mechanism can explain the existence of narrow autoionizing states in atoms and narrow resonances in a number of strongly interacting hadronic systems, such as dibaryons, hypernuclei etc.

The case of Δ-resonances [21] is especially relevant for our discussion. In a $^{12}\text{C}(e, e'p\pi^-)^{11}\text{C}$ experiment [22] at the Mainz microtron, at the energy near the Δ-isobar excitation narrow resonances (several MeV wide) in the $\Delta-^{11}\text{C}$ system were observed superimposed on a broad peak with the width $\sim 100$ MeV, see Fig. 1(a). In Ref. [21] the broad peak was interpreted as the SR state coherently formed out of the $\Delta N^{-1}$ ($\Delta$ particle - nucleon hole in $^{11}\text{C}$) configurations, while the narrow states corresponded to the remaining, long-lived combinations of $\Delta N^{-1}$ excitations.

Before proceeding with another application, we present a brief and simplified account of the SR state theory. Using a standard projection technique, we consider a set of states $|q\rangle$ (termed “internal”) and a set of few “external” decay channels $|c\rangle$ [7,8]. We refer to the spaces containing these states as $\{q\}$ and $\{c\}$, respectively, whereas we denote the projection operators onto these spaces as $Q$ and $P$. The internal states $q$ with the same quantum numbers, such as spin, isospin, parity, strangeness, may couple to each other via a Hermitian interaction matrix $V_{qq'}$. Apart from that, in the presence of decay channels, the intrinsic states can also interact indirectly through the coupling $\langle q|V|c\rangle\langle c|V|q'\rangle$ via the channel states $c$, the analog of the Dicke interaction through the common radiation field. The effective Hamiltonian $\mathcal{H}$ at given energy $E$, which belongs to the continuum, can be put in the symbolic form
with obvious notations,
\[ \mathcal{H}_{qq'} = H_{qq'} + H_{QP} G^{(+)}(E) H_{Pq'}, \] (1)

where \( H \) includes bare energies of the internal states and their direct Hermitian interaction; the propagator through the intermediate states \( |c; E⟩ \),
\[ G^{(+)}(E) = \frac{1}{E^{(+)} - H_{Pp}}, \] (2)

has to be taken at \( E^{(+)} = E + i0 \).

The effective intrinsic Hamiltonian (1) is non-Hermitian since the presence of decay thresholds \( E_c^r \) creates an imaginary part at energy above threshold. The real part of the propagator in eq. (2) (principal value) describes the renormalization of the Hermitian interaction by the coupling through all (open and closed) channels, while the imaginary part of \( \mathcal{H} \), \(-i/2) W_{qq'}\) with
\[ W = 2\pi \sum_{c(\text{open})} V |c⟩⟨c|V, \] (3)

comes from the \( \delta \)-functions in the propagator corresponding to the on-shell decay into the channels open at given energy.

The main features of the SR mechanism can be illustrated by the simplest example of a single open channel \( c_0 \) and a diagonal and degenerate real part of the effective Hamiltonian \( \mathcal{H} \), when the matrix elements of \( \mathcal{H} \) have the form
\[ \mathcal{H}_{qq'} = \epsilon \delta_{qq'} - \frac{i}{2} A_q A_{q'}^*, \] (4)

where \( A_q = \sqrt{2\pi} \langle q | V | c_0⟩ \) are the decay amplitudes. The anti-Hermitian part here has a separable form, similar to the classic model [23] for giant resonances, with the only difference that the collectivization now occurs along the imaginary axis via coupling through the continuum rather than along real energy axis through multipole-multipole interaction [13]. With the rank of the matrix (4) equal to 1, all its eigenvalues are zero except for one (denoted by \( r \)) that is equal to the imaginary part of the trace \( \mathcal{H} \) and accumulates the total summed width of all original states. For the dimension \( N \) of the internal space, the \( N \) complex eigenvalues of \( \mathcal{H} \) are
\[ \mathcal{E}_n = E_n - \frac{i}{2} \Gamma_n = \epsilon - i\pi \sum_q |⟨q|V|c_0⟩|^2 \delta_{nr \equiv \epsilon - \frac{i}{2} \sum_q \Gamma_q \delta_{nr}}. \] (5)
We see in this schematic model that the coupling of the states to a single
decay channel leads to the formation of a special state that has a width equal
to the sum of widths of all $N$ original states, while the remaining $N-1$ states
are stripped of their decay widths and become stable. The broad state is just
the SR one. The separable form of the continuum interaction is not arbitrary
being dictated by the unitarity of the scattering matrix.

In a more realistic situation, when the intrinsic energies $\epsilon_q$ are not degenerate
but their spacings are small compared to widths, $\Delta \epsilon = |\epsilon_q - \epsilon_q'| < \Gamma_q$ (over-
lapping resonances), a wide SR state still appears but the rest of the states
acquire their own widths, which are much smaller than the width of the SR
state (for more details see Refs. [2,4,20,21]). It was shown that the sum of the
widths, $\tilde{\Gamma}$, of all long-lived (“trapped”) states is given by

$$\tilde{\Gamma} \approx \frac{4(\Delta E)^2}{\Gamma},$$

where $\Delta E$ is the energy spread of the $N$ internal states, and $\Gamma = \sum_q \Gamma_q$.

As the number $k$ of open channels increases, so does the number of wide states.
However, the width segregation becomes less pronounced because of more or
less random phase interference of the amplitudes for different channels in the
imaginary part of the Hamiltonian, eq. (3), that reduces the effects of coupling
through the continuum.

The narrow resonances observed in the above mentioned $^{12}\text{C}(e,e'p\pi)^{11}\text{C}$ ex-
periment [22] appear in the SR theory in a natural way because the dynamics
of the $\Delta_{33}$-isobar in nuclei satisfies the validity conditions of such theory. The
wide peak around the $\Delta_{33}$ mass can be interpreted [21] as the SR state formed
of a set of $\Delta N^{-1}$ configurations, and the narrow states are the remaining con-
figurations of the same class with the same quantum numbers but with decay
probabilities strongly reduced by the SR mechanism.

Referring to the universality of the SR mechanism, we want to point out in this
paper that in the recent experiments [24–29], in which evidence was presented
for a narrow strange ($S = +1$) baryon resonance around $E = 1540$ MeV,
similar conditions may occur as in the $^{12}\text{C}(e,e'p\pi)^{11}\text{C}$ experiment in the $\Delta_{33}$
region.

In various experiments one observed the width of this resonance, coined as $\Theta^+$,
or $Z^+$, smaller than 25 MeV; the resonance is situated on top of a very broad
(over 100 MeV) background peak, Fig. 1(b). In order to avoid a contradiction
with available data on kaon-nucleon scattering, the actual width of the narrow
resonance should be of the order of MeV [30,31]. It is difficult to understand
the existence of this very narrow resonance structure in the framework of
the usual mechanism for decay of baryonic resonances, whether non-exotic or exotic. The narrow width of the $S = +1$ exotic baryon indicates that the decay into the $K\bar{N}$ continuum is quenched either by selection rules, for example if the $\Theta^+$ is an isotensor [32], or by some special dynamical features.

Considering the Osaka experiment [24] with $^{12}$C as a target and the ITEP experiment [25] with the Xe nucleus as a target, the immediate thought would be that a mechanism similar to that in the $^{12}$C($e,e'p\pi$)$^{11}$C reaction is at work here and the narrow width of the resonance is a result of a many-body nuclear effect. The $\Delta N^{-1}$ configurations would be replaced by the $\Theta^+ N^{-1}$ ones and the $\pi N$ continuum channel by the $K^+n$, or $K^0p$ in the ITEP experiment, channel. The energy spacings between the various $\Theta^+ N^{-1}$ configurations with the same quantum numbers coupled to the $K^+n$ or $K^0p$ channel are of the order of $\hbar\omega_N$ or $\hbar\omega_\Theta$, which are the spacings between the energies of major orbits of the nucleon or $\Theta^+$-particle in the field of the nuclear core. These spacings are typically of the order of 10-15 MeV (and can be reduced by the larger mass of $\Theta^+$), i.e. much smaller than the expected decay width into the kaon-nucleon continuum.

The CLAS experiment [26,28], in which the deuteron is used as a target, and the SAPHIR experiment [27] with a hydrogen target seemingly do not support the idea that the $\Theta^+ N^{-1}$ configurations are the cause for the quenched decay of the resonance. However, as we explain, the SR state mechanism may still be the cause for the quenching, and there is not much difference in these cases compared to the experiments on a complex nucleus as C or Xe.

The simplest scenario would be to consider the $K^+n$ system ($\bar{s}u + ddu$ quarks) to be nonrelativistic and to form a quasimolecule. Molecular-like structures
for non-strange pentaquarks were discussed by Iachello [33]. The two particles interact via an attractive potential of a typical range of 1 fm producing a $p$-wave resonance at energy 100 MeV above threshold (an $f$-wave resonance was considered in [34]). In Ref. [35] the authors estimate that the width of the resonance in such a potential is more than 175 MeV, the value typical for strong decays of baryons in this mass region. A direct calculation gives the $p$-resonance width 190 MeV for the well of radius $a = 1$ fm obtained at the needed energy, this required the depth of the well to be $V = 333$ MeV; the width of 327 MeV is obtained for $a = 2$ fm and $V = 14$ MeV. These exact results can be compared with the approximate formula for the $l$-wave resonance width

$$
\Gamma_l = \frac{2}{\mu a^2} \frac{(ka)^{2l+1}}{[(2l-1)!]^2} \left(\frac{2l-1}{2l+1}\right), \quad l \neq 0. \tag{7}
$$

Although the fixed resonance energy $k^2/2\mu$ puts the relative momentum of $K^+n$ system at $k = 1.35$ fm$^{-1}$ that, with our choices of the well size, does not satisfy the formal condition of validity of Eq. (7), $(ka)^2 < l(l+1)$, this formula still turns out to give a good estimate and describe reasonably well the scaling behavior of the $p$-wave width as a function of $a$. An exact calculation for the $f$-wave resonance suppressed by the centrifugal barrier gives the width of 4 MeV for $a = 1$ fm that required $V = 1829$ MeV, and 49 MeV for $a = 2$ fm with $V = 316$ MeV.

However, the next excited (vibrational) states with the same quantum numbers, that can be viewed as radial excitations of the relative $K-N$ motion being analogous to the $\Theta^+N^{-1}$ excitations in heavier nuclei, are too high in energy and overlap weaker although their widths are very large.

In addition to quasimolecular states, one should consider many-quark bag dynamics in a system of five or even seven quarks [36] formed after the photon absorption by the original proton. Among the intrinsic states there are groups with the same quantum numbers, including “normal” quark states, quark-gluon states, paired states with singlet or triplet diquark(s), states with pions and so on. If the decay width of each individual state is much greater than the spacings, the unperturbed spectrum will be that of overlapping resonances. Under such conditions, the dynamics will be completely dominated by the coupling to the decay channel, and the anti-Hermitian part of the Hamiltonian will be the crucial factor. As a result, the SR state mechanism will give rise to the observed spectrum with a very broad (several hundred MeV) background peak, the SR state, and one or several very narrow resonances with width of order of few MeV. For example, for two overlapping resonances with $\Delta \epsilon \approx 20$ MeV, and the bare width of $\Gamma_{1,2} \approx 200$ MeV, the estimate above, eq. (6), gives for the narrow resonance $\Gamma_{nar} \approx 4$ MeV. The molecular $KN$-state can play a role of a doorway state [4,20,34] for the coupling to the continuum.
and give the main contribution to the SR state after the width redistribution. Then the direct (SR) processes are spatially separated from the trapped states (compound processes), similarly to what has been found in the open quantum wire model [4].

For a more quantitative discussion we consider a two-resonance model discussed in detail in refs. ([15]) and ([19]) for \( \rho - \omega \) interference, halo nuclei and other applications. As an example one can have in mind a quasimolecular state and a bag state of a tightly bound five-quark system, although any two intrinsic states with the same quantum numbers and a common decay channel would be suitable as well. We assume that the unperturbed energies of these states are \( \epsilon_1 \) and \( \epsilon_2 \), respectively. The states are coupled to a single \( KN \) channel (for definitiveness we assume \( p \)-wave relative motion). The squared amplitudes of these couplings \( |A_1(E)|^2 \) and \( |A_2(E)|^2 \) are greater than \( \Delta \epsilon \) and have correct threshold energy dependence. For the quasimolecular state the amplitude \( A_1(E) \) can be calculated assuming a square well potential; the resulting behavior is well described by Eq. (7), where \( \Gamma_1 = |A_1(E)|^2 \). The continuum coupling of a quark-bag state in the near-threshold region is expected to have a similar energy-dependence \( A(E) \sim (E - E_t)^{3/4} \) being dominated by the \( p \)-wave kinematics. Finally, both internal mixing and the interaction through the continuum produce real coupling \( v \) between these states. Due to a very different nature of the states we expect this matrix element to be small. As a result, we come to the effective non-Hermitian Hamiltonian,

\[
\mathcal{H} = \begin{pmatrix}
\epsilon_1 - \frac{i}{2} \gamma_1 & v - \frac{i}{2} A_1 A_2 \\
v - \frac{i}{2} A_1 A_2 & \epsilon_2 - \frac{i}{2} \gamma_2
\end{pmatrix},
\]

(8)

where \( \gamma_i = |A_i(E)|^2 \) (\( i = 1, 2 \)).

The non-Hermitian, but symmetric eigenvalue problem (8) leads to the secular equations for the real and imaginary part of resonance energy \( E - (i/2) \Gamma \),

\[
E^2 - E(\epsilon_1 + \epsilon_2) - \frac{\Gamma}{4}(\Gamma - \gamma_1 - \gamma_2) + \epsilon_1 \epsilon_2 - v^2 = 0,
\]

(9)

\[
\Gamma = \frac{E(\gamma_1 + \gamma_2) - \gamma_1 \epsilon_2 - \gamma_2 \epsilon_1 + 2v A_1 A_2}{2E - \epsilon_1 - \epsilon_2}.
\]

(10)

To avoid false solutions emerging because of the energy dependence of the parameters, the two physical roots \( \mathcal{E}_\pm = E_\pm - (i/2) \Gamma_\pm \) are to be “genetically” traced to the original independently decaying states. This model provides a transparent example of superradiance. As follows from Eqs. (9) and (10) at energy above threshold, when both decay amplitudes are non-vanishing and
large, it is still possible that an entire imaginary part is absorbed by one of the two states. The condition for $\Gamma_- = 0$ is

$$v(\gamma_1 - \gamma_2) = A_1 A_2 (\epsilon_1 - \epsilon_2). \quad (11)$$

The parameters of pentaquark physics turn out to be close to satisfying this condition.

For a qualitative comparison we consider what should be seen in $K\bar{N}$ scattering. In the two-resonance model with a single channel, the resonance part of the scattering amplitude,

$$T(E) = \sum_{1,2} A_i^* \left( \frac{1}{E - \mathcal{H}} \right)_{12} A_2, \quad (12)$$

where the denominator contains the full effective Hamiltonian (1), is given by

$$T(E) = \frac{E(\gamma_1 + \gamma_2) - \gamma_1 \epsilon_2 - \gamma_2 \epsilon_1 - 2vA_1 A_2}{(E - \mathcal{E}_+)(E - \mathcal{E}_-)}. \quad (13)$$

Ignoring a non-resonant background, we obtain the cross section $\sigma(E) = (\pi/k^2)|T(E)|^2$. In Fig. 2 the calculated cross section is compared with the experimental graph. The parameters selected for this example are $\epsilon_1 = 1535$ MeV, $\epsilon_2 = 1560$ MeV, and $v = 1$ MeV. For the solid curve, the amplitudes of the continuum coupling for both states were assumed to scale according to Eq. (7), where the threshold energy $E_t = M_K + M_N = 1432.3$ MeV.

The simple power-law scaling of decay amplitudes, Eq. (7), reflects the threshold behavior and breaks down as the parameter $ka$ increases. Being not accounted for in the calculations represented by the solid line, this leads to a large cross section on a high energy side. To correct this we introduce a cutoff energy $\Lambda$ that limits the validity of Eq. (7). For a quasimolecular state, the value of $\Lambda$ is about 200 MeV; for a small quark bag, $\Lambda$ should be higher. The dotted and dashed curves in Fig 2 show the results of the calculation with the same cutoff for both amplitudes. The energy dependence was parameterized as $A(E) \sim \left[ E^3/(1 + (E/\Lambda)^3) \right]^{1/4}$; at $E = 1550$ MeV the bare widths of the decoupled resonances are $\gamma_1 = 120$ MeV and $\gamma_2 = 60$ MeV. The precise form of the energy dependence is not important (here for simplicity of notations we put the threshold at $E = 0$). The solution of the model defines the narrow resonance at required energy 109 MeV above threshold with the width of 2.5 MeV and the broad bump with the centroid at 120 MeV and width 178 MeV.

Certainly, the two-state model still should be considered merely as an illustration. The parameters were not fit to the experiment, being chosen just
following the guiding physical principles so that the set of our parameters is not unique. Additional factors, such as possible isospin or/and flavor SU(3) violation, may also play a role in the dynamics (according to the QCD sum rules [37], the masses of the pentaquarks with strangeness +1 and isospins 0,1 and 2 may be close that enhances their mixing). We conclude that, due to the overlap of unstable intrinsic states, the “super-radiant” mechanism may produce the narrow peak(s) on the broad background in exotic baryon systems with strangeness +1. The dynamics leading to the redistribution of widths and formation of the SR state along with long-lived trapped states are universal in a sense that they are compatible with any intrinsic dynamics being constrained by unitarity only. In various situations, as it was discussed in [13] and [20,21], the nature of trapped intrinsic states can be very different. The observed resonance widths can also be slightly different in different nuclear environments.

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