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Computing free non-commutative Gröbner bases over \( \mathbb{Z} \) with SINGULAR:LETTERPLACE. (English) [Zbl 07589746]
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Summary: With this paper we present an extension of our recent ISSAC paper about computations of Gröbner(-Shirshov) bases over free associative algebras \( \mathbb{Z}\langle X \rangle \). We present all the needed proofs in details, add a part on the direct treatment of the ring \( \mathbb{Z}/m\mathbb{Z} \) as well as new examples and applications to e.g. Iwahori-Hecke algebras. The extension of Gröbner bases concept from polynomial algebras over fields to polynomial rings over rings allows to tackle numerous applications, both of theoretical and of practical importance. Gröbner and Gröbner-Shirshov bases can be defined for various non-commutative and even non-associative algebraic structures. We study the case of associative rings and aim at free algebras over principal ideal rings. We concentrate on the case of commutative coefficient rings without zero divisors (i.e. a domain). Even working over \( \mathbb{Z} \) allows one to do computations, which can be treated as universal for fields of arbitrary characteristic. By using the systematic approach, we revisit the theory and present the algorithms in the implementable form. We show drastic differences in the behaviour of Gröbner bases between free algebras and algebras, which are close to commutative. Even the process of the formation of critical pairs has to be reengineered, together with implementing the criteria for their quick discarding. We present an implementation of algorithms in the SINGULAR subsystem called LETTERPLACE, which internally uses Letterplace techniques (and Letterplace Gröbner bases), due to La Scala and Levandovskyy. Interesting examples and applications accompany our presentation.

MSC:
16Zxx Computational aspects of associative rings
13Pxx Computational aspects and applications of commutative rings
68Wxx Algorithms in computer science

Keywords:
non-commutative algebra; coefficients in rings; Gröbner bases; algorithms

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