Electron Impact Excitation of Singly Charged Indium Ion

Swati Bharti and Lalita Sharma* 

Indian Institute of Technology Roorkee, Roorkee, Uttarakhand 247667, India

*Corresponding author: lalita.sharma@ph.iitr.ac.in

Abstract. We study electron impact excitation of In$^+$ using relativistic distorted wave method and report cross sections for 37 transitions among the fine-structure levels of 5s$^2$, 5s5p, 5s5d, 5s6s and 5s6p configurations at the scattered electron energies upto 200 eV. The bound state wavefunctions of the target ion are obtained within multi-configuration Dirac-Fock approach. Our calculated oscillator strengths are compared with the results from the NIST database and other theoretical calculations. Cross sections are fitted with an analytical formula and the fitting parameters are made available for their possible applications in plasma modeling.

Keywords. Relativistic distorted waves; Multiconfiguration Dirac-Fock method; Dipole allowed transition; Cross sections; Oscillator strength; Fitting parameters

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1. Introduction

Singly charged indium ion has been the subject of various experimental and theoretical studies due to its significant presence in interstellar medium and application in solid state lasers [1-3]. It is also a good candidate for optical frequency precision [4]. Most of the previous work on In$^+$ is focused on its spectroscopic properties, while inelastic collisions of electrons with In$^+$ have been subjected to rare investigations. Studies on electron impact excitation of In$^+$ are required not only for better understanding of its atomic structure, but also to model astrophysical and laboratory plasmas. Theoretical calculations are major source of cross section results since measurements can be very complicated for metal ions due to various technical reasons. Thus, in the present paper we focus on the study of electron impact excitation of In$^+$ using the RDW method.
Gomonai and co-workers did few experimental studies related to electron collisions with indium ions [5-9]. The most relevant to our present work is their experiment in which Gomonai et al [5] reported the effective electron excitation cross-section of In$^+$ for the resonance transition $5s^2 \, 1S_0 - 5s5p \, ^1P_1^o$ in the energy range 7–300 eV. The absolute values of these cross-sections were determined by normalizing their measurement at 300 eV energy with Van-Regemorter formula. Smirnov measured excitation cross sections for 123 spectral lines of In$^+$ at a single electron energy of 30 eV using the extended crossed beam technique [10]. Apart from these measurements, no other theoretical or experimental work has been reported for excitation cross sections of In$^+$. However, there are several extensive theoretical as well as experimental studies related to spectral properties of In$^+$. Karlsson and Litzén [11] determined improved wavelengths and energy levels for 54 lines of In$^+$ using a high-resolution Fourier transform spectrometer. Energy level and hyperfine structures constants of $5s5p \, ^3P_0, 1, 2$ and $5s6s \, ^3S_1$ have been precisely measured by Larkin and Hannaford [4]. They reported transition energy of the forbidden transition $5s^2 \, 1S_0 - 5s5p \, ^3P_0^o$ at 42275.986(7) cm$^{-1}$ which has been found to be a strong candidate for the optical frequency standard. Peik et al performed the laser spectroscopy to measure the hyperfine splitting and isotope shifting for $5s^2 \, 1S_0 - 5s5p \, ^3P_1^o$ transition at 230.6 nm in In$^+$ ion [12] for the optical applications. Among theoretical studies Biémont and Zeippen [13] calculated the life time and transition probabilities of singly charged indium ion using relativistic Hartee-Fock approach. Jönsson and Andersson [14] reported extensive data for oscillator strengths and hyperfine structures for a large number of electric dipole transitions of this ion by employing relativistic multiconfiguration Dirac-Hartree-Fock method. Recently, Kramida [15] systemized a detailed list of the best measured wavelengths in the spectra of In$^+$. In addition, the references to previous experimental and theoretical investigations on spectroscopic properties of In$^+$ can be found in [15].

From the above discussion it is clear that studies related to electron collisions with In$^+$ are sparse and will be worthwhile because of their practical applications in various fields. The ground state electronic configurations for In$^+$ is $[1s^22s^22p^63s^23p^63d^{10}4s^24p^64d^{10}]5s^2$. From here onwards, we will exclude the part of configuration inside square brackets for the sake of brevity. In the present work, we have considered electron impact excitation of the dipole allowed (E1) transitions among fine-structure levels of configurations $5s^2$, $5s5p$, $5s6s$, $5s6p$ and $5s5d$. The wavefunctions of the incoming and outgoing electrons are calculated by solving coupled Dirac equations using RDW approximation [16]. The required bound state wavefunctions of In$^+$ are obtained within the framework of multi-configuration Dirac-Fock method using GRASP2K [17]. Since the accuracy of the target bound states play major role in determining the collision parameters, we ensure the quality of the ionic wavefunctions by comparing our calculated oscillator strengths with the previously reported calculations [13,14] and values from the NIST database [18]. Finally, the excitation cross sections for all the 37 transitions are obtained in the range of scattered electron energy from 15 to 200 eV.

In Section 2 we describe RDW method as well as determination of bound state wavefunctions briefly. Results are presented in Section 3 followed by concluding remarks in Sections 4.
2. Theoretical Method

The RDW T-matrix for electron impact excitation of an ion having N electrons and nuclear charge Z from an initial state with total angular momentum \(J_i\) to a higher lying state with total angular momentum \(J_f\) can be expressed as (atomic units are used throughout)

\[
T_{J_i \rightarrow J_f}^{DW} = \langle \chi_f^\pm(1,2,\ldots,N+1)|V - U_f(N+1)|\mathcal{A}\chi_i^\pm(1,2,\ldots,N+1) \rangle.
\]

(2.1)

Here \(V\) is the interaction of projectile electron with target ion, given by

\[
V = -\frac{Ze}{r_{N+1}} + \sum_{i=1}^{N} \frac{1}{r_i - r_{N+1}}.
\]

(2.2)

In above equation \(r_i\) \((i = 1 \ldots N)\) and \(r_{N+1}\) represent the position coordinates of the target and projectile electrons, respectively, with nucleus considered as origin. \(U_f(N+1)\) refers to the spherically symmetric static potential of the target in its final states. \(\chi_{i/f}\) denote the product of atomic wavefunction \(\Phi_{i/f}\) and projectile electron distorted wavefunction \(F_{i/f}^{DW\pm}\) in initial/final states, i.e.,

\[
\chi_{i/f}^\pm(1,2,\ldots,N+1) = \Phi_{i/f}(1,2,\ldots,N)F_{i/f,\mu_{i/f}}^{DW\pm}(k_{i/f},N+1).
\]

(2.3)

Here +/- sign refer to the incoming/outgoing waves with their respective wavevectors, \(k_{i/f}\). \(\mathcal{A}\) is the antisymmetrization operator which includes probability of exchange of projectile electron with the target electrons. The spin projections of the incident and scattered electrons are denoted by \(\mu_{i/f}\). The procedure of obtaining distorted wavefunctions \(F_{i/f,\mu_{i/f}}^{DW\pm}\) can be followed from our previous work [19].

The wavefunctions for initial/final states of \(\text{In}^+\) are obtained within multiconfigurational Dirac-Fock (MCDF) approach using GRASP2K code [17]. In this method atomic state functions (ASF) are expressed as linear combinations of configuration state functions (CSFs) with same parity and angular momentum quantum number \(J\). The single particle orbital radial functions and mixing coefficients of CSFs are obtained by solving the Dirac-Coulomb Hamiltonian using self-consistent field method.

After obtaining the bound and continuum state wavefunctions, we can compute T-matrix and finally get the excitation cross section \(\sigma\) using the expression as given below,

\[
\sigma(J_i \rightarrow J_f) = \frac{4\pi^2}{2(2J_i + 1)k_f} \sum_{M_f,\mu_f} |T_{i/f}^{DW\pm}|^2 d\Omega.
\]

(2.4)

3. Results and Discussion

The first step of our calculations involves performing MCDF calculations using GRASP2K program to find wavefunctions of the bound states of \(\text{In}^+\). In the present work, we have considered 5s\(^2\), 5s6s, 5s5d, 5s7s, 5s6d, 5s8s, 5p\(^2\) as even parity configurations and 5s5p, 5s6p, 5s4f, 5s7p, 5s5f, 5p5d, 5s7p as odd parity configurations in non-relativistic scheme. These wavefunctions are optimized on the basis of the comparison, as given in Table 1 of our calculated oscillator strengths with the corresponding values from the NIST database [18] as well as other theoretical and experimental studies. Our results are, generally, in good agreement with the the NIST data and lie within the quoted errors, excluding the transitions which have values of oscillator strength less than \(10^{-2}\).
Table 1. Comparison of our calculated oscillator strengths with the NIST values \[18\] and other theoretical results \[13,14\]. The integers inside parenthesis must be raised to power of 10.

| Lower state | Upper state | Oscillator strength |
|-------------|-------------|---------------------|
|             | This work   | NIST                | Ref [13] | Ref [14] |
| $5s^2 \ 1s_0$ | $5s5p \ 3p_1^2$ | 2.282(-3)           | 5.000(-3) | 7.300(-3) | 4.910(-3) |
| $5s5p \ 1p_1^2$ | $5s6p \ 3p_1^2$ | 1.803               | 1.450     | 1.636     | 1.560     |
| $5s5p \ 1p_1^2$ | $5s6p \ 3p_1^2$ | 3.293(-5)           | 1.900(-3) |           |           |
| $5s6p \ 1p_1^2$ |             | 8.338(-2)           | 2.700(-3) | 2.700(-3) | 7.800(-4) |
| $5s5p \ 3p_0^2$ | $5s6s \ 3s_1^0$ | 0.152               | 0.160     | 0.176     | 0.161     |
| $5s5d \ 3d_2^1$ |             | 0.982               | 0.920     | 0.855     | 0.877     |
| $5s5p \ 3p_1^2$ | $5s6s \ 3s_1^0$ | 0.151               | 0.160     | 0.171     | 0.160     |
| $5s6s \ 1s_0^0$ | $5s5d \ 1d_2^1$ | 2.300(-3)           | 8.000(-4) | 3.001(-3) | 8.167(-4) |
| $5s5d \ 1d_2^1$ | $5s5d \ 3d_2^1$ | 5.685(-3)           | 3.100(-3) | 1.818(-3) | 3.103(-3) |
| $5s5d \ 3d_2^1$ |             | 0.159               | 0.179     | 0.208     | 0.179     |
| $5s5d \ 3d_2^1$ |             | 0.614               | 0.620     | 0.629     | 0.620     |
| $5s5p \ 3p_0^2$ | $5s6s \ 3s_1^0$ | 0.169               | 0.162     | 0.163     | 0.162     |
| $5s5d \ 1d_2^1$ |             | 5.526(-3)           | 6.000(-3) | 8.158(-3) | 6.000(-3) |
| $5s5d \ 3d_2^1$ |             | 0.097               | 0.110     | 0.118     | 0.110     |
| $5s5d \ 3d_2^1$ |             | 0.660               | 0.700     | 0.673     | (0.686)   |
| $5s5d \ 3d_2^1$ |             | 6.680(-4)           | 4.500(-3) | 0.008     | 4.520(-3) |
| $5s5p \ 1p_1^2$ | $5s6s \ 3s_1^0$ | 6.340(-4)           | 1.4(-3)   |           | 1.360(-3) |
| $5s6s \ 1s_0^0$ | $5s5d \ 1d_2^1$ | 0.156               | 0.149     | 0.227     | 0.153     |
| $5s5d \ 1d_2^1$ |             | 7.725(-3)           | 4.200(-2) | 1.448     | 3.800(-2) |
| $5s5d \ 1d_2^1$ |             | 4.911(-4)           | 1.2(-3)   | 8.548(-4) | 1.187(-3) |
| $5s5d \ 3d_2^1$ |             | 1.190(-3)           | 2.5(-3)   | 2.482(-3) | 2.520(-3) |
| $5s6s \ 3s_1^0$ | $5s6p \ 3p_0^2$ | 0.159               | 0.149     | 0.155     | 0.147     |
| $5s6p \ 3p_1^2$ |             | 0.473               | 0.440     | 0.456     | 0.430     |
| $5s6p \ 3p_1^2$ |             | 6.261(-3)           | 1.8(-2)   | 1.800(-2) | 1.747(-2) |
| $5s6p \ 3p_2^2$ |             | 0.814               | 0.770     | 0.821     | 0.763     |
| $5s6s \ 1s_0^0$ | $5s6p \ 3p_1^2$ | 1.206(-2)           | 4.0(-2)   | 3.974(-2) | 3.920(-2) |
| $5s6p \ 1p_1^2$ |             | 1.268               | 1.210     | 1.321     | 1.190     |
| $5s5d \ 1d_2^1$ | $5s6p \ 3p_1^2$ | 1.583(-3)           | 5.4(-3)   |           | 5.240(-3) |
| $5s6p \ 1p_1^2$ |             | 0.167               | 0.149     |           | 0.146     |
| $5s5d \ 3d_1^1$ | $5s6p \ 3p_0^2$ | 7.894(-2)           | 8.0(-2)   |           | 7.767(-2) |
| $5s6p \ 3p_1^2$ |             | 6.137(-2)           | 6.000(-2) |           | 5.800(-2) |
| $5s6p \ 1p_1^2$ |             | 9.825(-4)           | -         | 2.730(-3) |           |
| $5s6p \ 3p_2^2$ |             | 3.814(-3)           | -         |           | 4.200(-3) |
| $5s5d \ 3d_2^1$ | $5s6p \ 3p_1^2$ | 0.111               | 0.107     |           | 0.104     |
| $5s6p \ 1p_1^2$ |             | 1.332(-3)           | 4.600(-3) |           | 4.480(-3) |
| $5s6p \ 3p_2^2$ |             | 3.976(-2)           | 4.0(-2)   |           | 3.860(-2) |
The present oscillator strengths also match fairly well with those obtained by Biémont and Zeippen [13] using relativistic Hartree Fock by considering the effect of intravalence configuration interactions on transition probabilities. It can be observed from Table 1 that MCDF calculations of Jönsson and Andersson [14] show better agreement with the NIST results in comparison to ours. This is due to the fact that in ref. [14] the configuration state functions resulting from single excitations of all the core-shells have been considered in the expansion of an atomic state function. The maximum number of relativistic orbitals and CSFs taken are, respectively, 49 and 157000 for even parity states while for odd parity states their respective numbers are 52 and 235000. Thus the spin-polarization, in addition to the core polarization effects via configuration interaction are included more rigorously by Jönsson and Andersson [14]. On the other hand, we have limitations of comprising this kind of large number of CSFs in our RDW method as our main aim is to perform RDW calculations to obtain excitation cross sections using a reasonably accurate set of wavefunctions, which has been finalized on the basis of comparison as shown in Table 1. Since the RDW approximation is a first order perturbation theory, the scattering amplitudes for two electron transitions vanish and hence, the CSFs involving two electron transitions will not contribute. Furthermore, we have ensured in our calculations to include all those CSFs having value of mixing coefficient of the order of $10^{-3}$.

Thus, the present optimized wavefunction can be counted on to provide accurate cross sections.

![Figure 1](image_url)

**Figure 1.** Cross sections for electron impact excitation of the ground $5s^2 1S_0$ state to the excited (a) $5s5p^1P_1^o$ state compared with the measurements of Gomonai et al [5] (b) $5s5p^3P_1^o$ and $5s6p^1P_1^o$ state.
respect to the scattered electron energy. This feature is the consequence of the dipole allowed nature of these transitions. Further, it can be seen that the cross sections for same spin transitions are nearly two order of magnitude more than those for spin-change transitions.

Figure 2 presents cross sections for electron impact excitation from excited 5s5p $^3P_0$, and $^1P_1$ levels to the 6 fine structure levels of 5s6s and 5s5d configurations. All these transitions show the characteristic feature of the dipole allowed transitions, i.e., slower rate of decrease in cross sections with increasing electron energy. As observed in Figure 1(b), the same spin transitions have greater value of cross sections as compared to spin flip transitions. The relative magnitude of cross sections can further be discerned from their oscillator strengths. We find that more the value of oscillator strength for a particular transition, greater is the value of cross section at a given energy. For example, transition 5s5p $^3P_0 \rightarrow 5s5d \, ^3D_1$ has the largest cross sections among all the 17 transition displayed in Figure 2, owing to maximum value of its oscillator strength.

![Cross sections for electron impact excitation of the excited states 5s5p $^3P_0$, and $^1P_1$ to the higher lying fine-structure states 5s6s $^3S_1$ and $^1S_0$ as well as 5s5d $^3D_{1,2,3}$ and $^1D_2$](image)

**Figure 2.** Cross sections for electron impact excitation of the excited states 5s5p $^3P_0$, and $^1P_1$ to the higher lying fine-structure states 5s6s $^3S_1$ and $^1S_0$ as well as 5s5d $^3D_{1,2,3}$ and $^1D_2$.

Excitation cross section for transitions from 5s6s $^3S_1$ and $^1S_0$ states to 5s6p $^3P_0$ and $^1P_1$ states are depicted in Figure 3. As observed earlier, the increasing order of cross sections can be traced back to oscillator strength for a particular transition as well as to the fact whether or
not the spin change of the state is involved during the transition. Finally, we have shown cross 
section results for electron impact excitation of 5s5d 3D1,2,3 and 1D2 states to the higher lying 
fine-structure states 5s6p 3P0,1,2 and 1P1 in Figure 4. Here again, the behavior of the cross 
section curves is similar as noticed for other dipole allowed transitions in Figures 1-3. We would 
like to mention here that in spite of small values of oscillator strengths of these transitions, 
magnitude of the cross sections is relatively higher in comparison to the cross sections for 
excitation from the ground state to the farther lying states. This is clearly due to small value of 
excitation threshold, less than 2 eV, for these transitions.

![Figure 3. Cross sections for electron impact excitation of the excited states 5s6s 3S1 and 1S0 to the higher lying fine-structure states 5s6p 3P0 0,1,2 and 1P1.](image)

In order to make our results available for plasma modeling, we have fitted our calculated 
cross sections for all the fine-structure transitions using the two analytical expressions. For low 
to intermediate energy range, the following analytical formula [20] has been employed to fit the 
cross sections,

$$\sigma(J_i \rightarrow J_f) = \sum_{i=0}^{n} b_i E^i$$

Here, incident electron energy $E$ and cross section ($\sigma$) are in atomic units. In the above equation 
$b_i$, $c_0$, $c_1$ and $c_2$ are the fitting coefficients which are listed in Table 2 for the 37 transitions 
considered in the present work. Cross section for a given transition can be directly calculated 
by inserting the fitting coefficients in above expressions. The uncertainty in the fitted cross 
sections is found to be less than 4%. Further, the cross sections for dipole allowed transitions 
fall-off as $\ln E/E$ in high energy range of the projectile electron. Therefore, the Bethe-Born form, 
as given below, is used to fit the cross sections at high energies,

$$\sigma(J_i \rightarrow J_f) = \frac{1}{E}(d_0 + d_1 \ln(E)),$$

where scattered electron energy $E$ and cross section ($\sigma$) are in atomic units. The fitting 
coefficients $d_0$ and $d_1$ are provided in Table 2.
Table 2. The fitting parameters for cross sections of the transitions considered in In⁺ ion. The integers inside the parenthesis must be raised to power of 10

| Transition          | Energy range (eV) | $b_1$   | $b_2$   | $b_3$   | $c_4$   | $c_5$   | $c_6$   | $d_6$   | $d_7$   |
|---------------------|-------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| $5s^2 \rightarrow 5s^6p$ | 19.566 - 94.566   | 1.99336 | -2.39449 | 1.66368 | -2.25350(-1) | 2.62942 | -7.04577 | 7.62854 | 1.93120(-1) | 1.22700(-2) |
| $5s^2 \rightarrow 5s^5p$ | 23.073 - 208.073  | 3.19393(+1) | 3.22066(+1) | -5.19678 | 2.55760(-1) | -8.39200(-2) | 1.81101 | -9.19100(-2) | 2.84813(+1) | 2.47467(+1) |
| $5s^2 \rightarrow 5s^6d$ | 27.653 - 162.653  | 1.64727(+2) | -2.02931(+2) | 1.43274(+2) | -1.24176(+1) | 1.17932(+4) | -2.52880(+4) | 1.57574(+4) | 1.58000(-2) | 1.62000(-3) |
| $5s^2 \rightarrow 5s^6p$ | 28.0567 - 213.057 | -1.04724 | -7.89800(-2) | 1.91631 | -1.21080(-1) | 8.96390(-1) | -2.88500 | 2.31498 | 1.54872 | 1.03648 |
| $5s^6p \rightarrow 5s^6s$ | 21.574 - 206.574  | -3.60576 | 6.50334 | 0.56631 | -0.03354 | -1.26378 | 1.425 | 1.62909 | 1.66480 | 1.45576 |
| $5s^6p \rightarrow 5s^5d$ | 22.598 - 207.598  | 7.12479 | 2.15727(+1) | -1.45064 | 8.43200(-2) | -1.30800(-1) | 1.67915 | 2.22040(-1) | 1.25636(+1) | 1.15324(+1) |
| $5s^5p \rightarrow 5s^6s$ | 21.422 - 206.422  | -4.00813 | 6.85772 | 7.06570(-1) | -4.30600(-2) | -1.34566 | 1.33322 | 1.84344 | 1.67519 | 1.43226 |
| $5s^5p \rightarrow 5s^5d$ | 22.099 - 87.099   | 5.05000(-2) | -2.07650(-1) | 3.07360(-1) | -3.08400(-2) | 3.04752 | -9.9364 | 9.44487 | 3.55700(-2) | 2.85400(-2) |
| $5s^5p \rightarrow 5s^6d$ | 21.608 - 106.608  | -5.37000(-2) | -9.19100(-2) | 5.76010(-1) | -5.62100(-2) | 2.31275 | -7.43236 | 6.50492 | 1.79340(-1) | 5.87400(-2) |
| $5s^5p \rightarrow 5s^5d$ | 22.446 - 207.446  | 2.36300(-2) | 6.03852 | -0.22225 | 1.54200(-1) | -5.98970 | 2.68015 | 6.63940(-1) | 1.73967 | 1.62741 |
| $5s^5p \rightarrow 5s^5d$ | 22.453 - 207.453  | 3.95753 | 1.48060(+1) | -8.16900(-1) | 4.97000(-1) | -1.55020(-1) | 1.68838 | 2.87370(-1) | 8.09760 | 7.31614 |
| $5s^5p \rightarrow 5s^6s$ | 21.091 - 206.091  | -3.47966 | 5.40955 | 7.21680(-1) | -4.57600(-2) | -1.03375 | 7.66540(-1) | 1.59623 | 1.75943 | 1.43746 |
| $5s^5p \rightarrow 5s^5d$ | 21.278 - 106.278  | -4.31600(-2) | -7.40800(-2) | 5.51560(-1) | -5.33300(-2) | 1.97411 | -6.47262 | 5.93886 | 1.84750(-1) | 6.50000(-2) |
| $5s^5p \rightarrow 5s^5d$ | 22.122 - 207.122  | 1.51514 | 5.07177 | -2.10570(-1) | 1.31300(-2) | -3.22900(-1) | 3.13561 | 6.69390(-1) | 1.51894 | 1.22756 |
| $5s^5p \rightarrow 5s^5d$ | 22.132 - 207.132  | 3.17316 | 1.62246(+1) | -6.86770(-1) | 4.94100(-2) | -1.82870(-1) | 1.49813 | 3.27310(-1) | 9.43521 | 8.21913 |
| $5s^5p \rightarrow 5s^5d$ | 22.115 - 157.115  | 9.19160(-1) | 2.55340(-1) | 1.60800(-1) | -1.60100(-2) | 1.12685 | 2.91659 | 3.13609 | 1.72650(-1) | 2.27300(-2) |
| $5s^5p \rightarrow 5s^6s$ | 17.915 - 202.915  | -4.44019(+1) | 5.11148(+2) | 9.01417(+2) | -6.41894(+1) | 9.71519(+3) | -3.76227(+4) | 4.68744(+4) | 5.27500(-2) | 1.78200(-2) |
| $5s^5p \rightarrow 5s^6s$ | 18.593 - 203.593  | -6.22254 | 1.37097(+1) | -8.34310(-1) | 9.25900(-2) | -4.23820(-1) | 5.00810(-1) | 7.89160(-1) | 7.21992 | 3.19046 |
| $5s^5p \rightarrow 5s^5d$ | 18.102 - 83.102   | 2.03876 | 5.74180(-1) | 1.66656 | 9.37200(-2) | 1.25950 | -2.37359 | 4.21352 | 1.37332 | 4.47630(-1) |
| $5s^5p \rightarrow 5s^5d$ | 18.939 - 203.939  | -2.30156 | 4.39506 | 3.40230(-1) | 2.10000(-3) | 4.19678(+1) | -1.66219(+2) | 1.69641(+2) | 3.06700(-2) | 6.34000(-3) |
| $5s^5p \rightarrow 5s^5d$ | 18.946 - 73.946   | 4.20910(-1) | 1.69200(-2) | - - | -2.43545 | 6.76446 | -4.01810(-1) | 6.97700(-2) | 2.05900(-2) | 6.88411 |

Contd. Table 2
| Subshell | Transition | Wavelength (Å) | Energy (eV) | Target State | Target State | Target State | Target State | Target State | Target State | Target State | Target State | Target State | Target State |
|----------|------------|----------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 6s6s 3S → 5p6p 3P | 17.069 - 202.069 | 17.069 - 202.069 | -1.18820 | 4.80634 | -2.30590(-1) | 2.86100(-2) | -1.14052 | 2.27505 | 3.53414 | 6.94360(-1) | 2.26570(-1) |
| 6s6s 3S → 5p6p 3P | 16.729 - 51.729 | 16.729 - 51.729 | 1.77275(+1) | -3.97013 | -1.05171 | - | 2.60500(-2) | 1.25180(-1) | -5.15900(-2) | 1.10974(+2) | 3.44284(+1) |
| 6s6s 3S → 5p6p 3P | 15.987 - 40.987 | 15.987 - 40.987 | 2.40360 | -1.00740(-1) | 3.53414 | 6.94360(-1) | 2.26570(-1) |
| 6s6s 3S → 5p6p 3P | 16.391 - 51.391 | 16.391 - 51.391 | -5.30440 | 4.39856(+1) | 3.68506(+1) | -7.49463 | 3.84000(-3) | -3.51800(-2) | 2.85700(-1) | 2.27408(+2) | 6.20001(+1) |
| 6s6s 3S → 5p6p 3P | 16.478 - 201.478 | 16.478 - 201.478 | 4.45820(-1) | 1.51920 | 3.76300(-2) | -2.24880(-1) | 2.89709 | 3.07780 | 3.34910(-1) | 9.01600(-2) |
| 6s6s 3S → 5p6p 3P | 16.882 - 201.882 | 16.882 - 201.882 | -5.13680(-1) | 2.35127(+1) | 6.95020(-1) | -1.05000(-2) | -6.78600(-2) | 3.90530(-1) | 6.72040(-1) | 2.27381(+1) | 7.83741 |
| 5s5d 3D → 5p6p 3P | 15.617 - 60.617 | 15.617 - 60.617 | -1.07492(+1) | 1.86570(+1) | 3.22486 | -3.86220(-1) | -1.98900(-2) | -1.57200(-1) | 3.67990(-1) | 5.52305(+1) | 1.16315(+1) |
| 5s5d 3D → 5p6p 3P | 15.641 - 60.641 | 15.641 - 60.641 | -7.34721 | 1.30787(+1) | 2.26353 | -2.69520(-1) | -1.44720(-1) | 3.51030(-1) | 3.8535 | 6.00850(-1) |
| 5s5d 3D → 5p6p 3P | 15.705 - 100.705 | 15.705 - 100.705 | -2.96013 | 5.48459 | 4.43170(-1) | -4.36500(-2) | -1.22410(-1) | -5.56750(-1) | 1.53058 | 2.39480(+1) | 5.27641 |
| 5s5d 3D → 5p6p 3P | 16.045 - 201.045 | 16.045 - 201.045 | -4.08150(-1) | 1.94433 | 1.34020(-1) | -6.47000(-3) | -4.59070(-1) | 2.66700(-1) | 5.26069 | 3.19360(-1) | 7.39200(-2) |
| 5s5d 3D → 5p6p 3P | 15.634 - 80.634 | 15.634 - 80.634 | 1.85480(+1) | -2.66365 | 1.10100(-1) | -8.89990(-1) | 4.63800(-2) | 1.95740(-1) | -3.18300(-2) | 7.45894(+1) | 1.58943(+1) |
| 5s5d 3D → 5p6p 3P | 15.698 - 60.698 | 15.698 - 60.698 | -5.67210 | 1.04743(+1) | 1.68231 | -1.93150(-1) | -2.70400(-2) | -1.77710(-1) | 4.61870(-1) | 2.39480(+1) | 5.27641 |
| 5s5d 3D → 5p6p 3P | 16.038 - 201.038 | 16.038 - 201.038 | -7.49130(-1) | 2.61119 | 1.80230(-1) | -8.69000(-3) | 4.51450(-1) | -1.69830(-1) | 5.14794 | 4.41010(-1) | 1.06100(-1) |
| 5s5d 3D → 5p6p 3P | 15.688 - 60.688 | 15.688 - 60.688 | -1.37211(+1) | 2.52997(+1) | 4.4060 | -5.26770(-1) | -1.12410(-1) | 2.87940(-1) | 9.48479(+1) | 2.10234(+1) |
4. Conclusions

We have studied electron impact excitation of singly charged indium ion using RDW method. The cross section calculations are performed for dipole allowed fine-structure transitions among the states arising from the configurations $5s^2$, $5s5p$, $5s5d$, $5s6s$ and $5s6p$ in the scattered electron energy range of 15-200 eV. We have provided a detailed comparison of our calculated oscillator strengths for these transitions with the values listed in the NIST database and other theoretical results and found an overall good agreement. This ensures the quality of bound state wavefunctions employed in our calculations as well as reliability of the cross sections reported here. Cross sections for the transition $5s^2 \, ^1S_0 \rightarrow 5s5p \, ^1P^o_1$ are compared with the measurements of Gomonai et al [5] and a good agreement is found at high energies. Unfortunately, there are no other theoretical and experimental results available to compare with our calculated cross sections for other transitions. Finally, we have provided fitting of our cross sections using two analytical expressions based on low and high energy ranges of the incident electron. We hope our results will be useful for plasma modelers and stimulate more work on electron induced processes in singly charged indium ions.
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Competing Interests
The authors declare that they have no competing interests.

Authors’ Contributions
All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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