Stopping of protons in pA collisions at SPS and NICA energies in analytical hydrodynamic model and in SMASH event generator

V Ermakova, V Sandul and G Feofilov
Saint Petersburg State University, SPb
E-mail: st056104@student.spbu.ru

Abstract.
Our study is motivated by the first experimental results on pion production and stopping obtained in pA collisions in E910 experiment at BNL. The effect of stopping is the deceleration of an incident high energy proton traversing the target nucleus. It appears due to the energy losses relevant to the production of secondary particles in the inelastic interactions with nucleons.

A hydrodynamic effective model of stopping is proposed that is based on the consideration of the nuclear matter as a liquid and by the introduction of an effective stopping force acting on a fast-moving proton. With this force, we obtain a differential equation that describes the relativistic motion of a proton in a nucleus. Calculations in the framework of this analytical model are performed for the correlation between the mean multiplicity of charged pions and a number of binary collisions.

1. Introduction
The effects of the nucleon stopping due to the energy losses in the inelastic collisions in nuclear matter and related phenomena, such as the production of so-called gray particles, could be important in determining the relevant number of binary-collisions, i.e. the number of times the projectile nucleon is re-scattered in its path through the nucleus. The gray particles are the products of the recoil nucleon shower induced by the incoming nucleon (proton) \[1,2\], thus providing possible numerical estimations of such quantities as the collision centrality.

Our study is motivated by the experimental results on pion production and stopping obtained in pA collisions in E910 experiment at BNL \[1\]. Measurements of the mean multiplicity \(<N_{ch}>)\) of negatively charged pions (\(\pi^-\)) as a function of \(N_{gray}\) \[1\] and the related number of binary collisions \(\nu\) demonstrated linear relationship in case of peripheral collisions (for small values of \(N_{gray}\)) and some tendency for saturation at \(N_{gray} > 3\), see figure \[1\].

One of the theoretical explanations proposed for the effect of stopping of nucleons in AA collision could be found in \[3\]. Results shown in the figure \[2\] were obtained at several nucleon-nucleon collision energies in c.m. under assumptions that nucleons are losing some fixed amount of energy \(\sigma\) when passing along the path \(dz\) inside the nuclear matter. It turned out that in \[1\] In this work we use both designations of number of binary collisions: \(\nu\) as it was introduced in \[1\] and \(N_{bin}\) as it is always used in Glauber-like approaches.
this case, nucleons can completely stop inside the target if the collision happens at rather low energy.

In our study of stopping of fast-moving nucleons (protons) in the nuclear matter, we propose the Effective Hydrodynamic Model (EHM) described below in Section 2. The results of the calculations of correlation of mean yields of pions with the number of binary nucleon-nucleon collisions are compared in Section 3 to the MC event generators and to the existing experimental data on pAu collisions at $p_{lab} = 18$ GeV/c. The non-linear behavior of the correlation observed in the experiment is also discussed.

2. Effective hydrodynamic model (EHM)

The idea of the Effective Hydrodynamic Mode is to treat the target nucleus as a liquid drop with some given internal energy density $\epsilon$, nuclear density $\rho$, mass $M$ and the volume $V$. So, when the projectile-proton gets into the target-nucleus, the stopping force begins to act on it. According to Landau and Lifshitz [4], liquid acts on the body with the force $dF = T^{ik} df_k$, where $T^{ik}$ is components of momentum tensor and $df_k$ is transverse area of the body.

In the local rest system the momentum is

$$T^{\alpha\beta} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}. \quad (1)$$

Here $\epsilon = M/V$ and $p = 0$, because static pressure in the core is absent. Taking into consideration the speed of liquid $u^\alpha$, momentum tensor should be rewritten as

$$T^{\alpha\beta} = \epsilon u^\alpha u^\beta. \quad (2)$$

In local rest system $u^0 = 1, u^i = 0$. In any other system, where liquid moves with the speed $\vec{v}$

$$T^{ik} = \frac{\epsilon v^i v^k}{(1 - v^2)}. \quad (3)$$

---

2 As usual, we will use an agreement that $i, k = 1, 2, 3$ and $\alpha, \beta = 0, 1, 2, 3$. 
The liquid moving with the speed \( \vec{v} \) acts on the resting body with the force
\[
dF = T^{ik} df^k.
\]
This force is the same in magnitude for the opposite case when the body moves through the liquid at rest, so that its projection on \( z \)-axis is:
\[
F = -\gamma^2 \epsilon v^2 \cdot S,
\]
where \( v = v(t) \) is a speed of proton, \( S = \pi r_p^2 \) transverse area of proton and \( \gamma = \frac{1}{\sqrt{1-v^2}} \). Then one can write down the second Newton’s law for projectile and obtain a differential equation that describes relativistic motion of a proton in a nucleus:
\[
\frac{dv}{dt} = \text{const} \cdot v^2 \sqrt{1 - v^2},
\]
where \( \text{const} = -\frac{\epsilon S_{m_0}}{m_0} \). Assuming the projectile’s movement to be along the \( z \)-axis in a straight line and setting some final speed value \( (V_0) \), the speed of nucleon after which the relevant binary collision cannot contribute to the multiplicity, one may calculate the length \( l \) of the proton’s path in the nucleus. One may see that in case of rather low momenta of incoming protons this path length cuts out a region \( \Omega \) in the nucleus (see figure 3). (One can lay the path length from the border of the target nucleus for the different impact parameters and get the region \( \Omega \).

![Figure 3. Area \( \Omega \). \( \Omega = S(R, 0) - S(R, 0) \cap S_1(R, l + R) \)](image)

**Table 1.** Quantities used for calculations. The \( p-Au \) collisions with given \( z \)-momentum are considered in the laboratory system. Quantities with (*) could be found in [5, 6]. Radius of nuclei here is \( R = r_p \times (1.12A^{1/3} - 0.86A^{-1/3}) \).

| \( R, \text{ fm} \) | \( a^*, \text{ fm} \) | \( A \) | \( V_0 \) | \( \sigma_{inel}^{NN}, \text{ fm}^2 \) | \( r_p, \text{ fm} \) | \( m_{0p}, \text{ GeV} \) | \( M, \text{ GeV} \) | \( p_{lab}, \text{ GeV}/c \) |
|-----------------|-----------------|------|-------|----------------|---------------|----------------|-----------|-------------|
| 7.64            | 0.538           | 197  | 0.5   | 2.85           | 0.98          | 1              | 197.09    | 18          |

We can calculate number of binary collisions \( (N_{bin}) \) and \( (N_{ch}) \) by using a Glauber-like approach [7].

The Woods-Saxon distribution for nucleus density is taken as
\[
\rho = \rho_0 \frac{1}{1 + e^{r-R/a}},
\]
where \( \rho_0 \) is such that Wood-Saxon distribution normalized to unity, and \( a \) characterizes the surface thickness of the nucleus.

The following quantities are also introduced:
• probability to find baryon with impact parameter $b_A$ in the volume element $d b_A \, d z_A$: $\rho(b_A, z_A) \, d b_A \, d z_A$;

• probability for the projectile (a single proton) with the impact parameter $b$ to collide with any nucleus from the target $T$ in volume element $d b_A \, d z_A$: $\delta(b - b_A) \, \rho(b_A, z_A) \, d b_A \, d z_A$;

• probability for the projectile (single proton) with the impact parameter $b$ to have inelastic collision with any nucleus from target in volume element $d b_A \, d z_A$: $\sigma_{inel}^{N N} \, \delta(b - b_A) \, \rho(b_A, z_A) \, d b_A \, d z_A$.

The probability of a single baryon-baryon collision, when the projectile and target nucleons collide, is

$$\sigma_{inel}^{N N} T(b) = \int_{\Omega} \sigma_{inel}^{N N} \rho(z, b_A) \delta(b - b_A) \, \rho(b_A) \, d b_A \, d z.$$  \hspace{1cm} (8)

This equation is simplified by omitting index and replacing $z_A$ by $z$. $\sigma_{inel}^{pp}$ is a constant value and $T(b)$ is usually called thickness function. Then the probability to have $n$ baryon-baryon inelastic collision when proton and target situated at an impact parameter $b$ relative to each other

$$P(n, b) = \frac{A!}{n!(A - n)!} \left( T(b) \sigma_{inel}^{N N} \right)^n (1 - T(b) \sigma_{inel}^{N N})^{A - n}.  \hspace{1cm} (9)$$

The average number on inelastic baryon-baryon collisions at the impact parameter $b$ can be calculated as:

$$N_{bin}(b) = \sum_{n=1}^{A} n \, P(n, b). \hspace{1cm} (10)$$

The value of the mean multiplicity may be obtained as:

$$N_{ch}(b) = N_{ch}^{pp} \cdot N_{bin}(b), \hspace{1cm} (11)$$

where $N_{ch}^{pp}$ is the value of the mean multiplicity of charged particles in a binary collision. As the first approximation, this value is considered as the function of speed (or energy) because the projectile slows down in the target. It means that we also need $N_{bin}(b)$ to be the function of speed as well: $N_{bin}(v|b)$. Therefore, all previous calculations are repeated for different $V_0$ by changing $V_0$ from the initial speed of the projectile to the final value of 0.5 (the last one is in terms of $v/c$ and is supposed to be as a limiting speed of nucleon in charged-particles production $[2]$). So, finally, the expression for multiplicity could be written down like this:

$$N_{ch}(b) = \sum_{i=1}^{N} N_{ch}^{pp}(v_i) \cdot \Delta N_{bin}(v_i|b), \hspace{1cm} (12)$$

where $v_i$ is the speed of the projectile relevant to the $i$-th interval of $[v_i, V_0]$ divided into $N$ parts, $\Delta N_{bin}(v_i|b)$ is an increment of the number of binary collisions when speed changes from $v_i$ to $v_{i+1}$ (this basically means that projectile passes small part of $l$) with a given impact parameter. Multiplicity $N_{ch}^{pp}(v_i)$ could be obtained from $[8]$:

$$N_{ch}^{pp}(s_{NN}) = a + b \cdot \ln(s_{NN}) + c \cdot (\ln(s_{NN}))^2, \hspace{1cm} (13)$$

where $a = 0.88 \pm 0.1$, $b = 0.44 \pm 0.05$, $c = 0.118 \pm 0.006$.

Note, that equation $[12]$ could be written as integral if $N \rightarrow \infty$ and $N_{bin}(v|b)$ is a continuous function.

3 $b$ is two-dimensional vector in $xy$-plane, that is transverse to the direction of projectile movement.

4 Note that in Glauber approach there is also function $t(b)$ that shows probability to have any collision at all with given impact parameter, because it is not always equivalent to unity. But in our Glauber-like approach let us consider $t(b) = 1$.

5 Speed of projectile $v$ could be easily converted into energy per nucleon-nucleon pair $\sqrt{s_{NN}}$. 

3. Comparing with experiment and MC event generators

The results of our analytic calculations are compared with the MC event generators and to the available experimental data obtained in p-Au collisions at the BNL [1]. It was assumed in the comparison that the mean number of negatively charged pions produced in pA collision is equal to one half of the total mean multiplicity:

$$\langle N_{\pi^-} \rangle = \langle N_{\text{ch}} \rangle / 2$$ (14)

Results of our model obtained in the full acceptance are presented in the figure 3 and compared to SMASH [9] MC data and to the RQMD event generator results taken from [1].

Each point on our model graph corresponds to some impact parameter (remember, that it is possible to calculate $N_{\text{bin}}(b)$, see equation (10) and $N_{ch}(b)$, equation (12)). One can see that the model gives a linear relationship between the average multiplicity of pions and the number of binary collisions. At the same time, the RQMD and SMASH event generator results show some level of saturation.

This deviation from the linearity is found to be quite dramatic for the case of the real experimental p-Au collisions data shown in the figure 1 and obtained in the limited acceptance (see details in [1]). Therefore, before discussing some possible physics reasons for the observed “saturation” effects in the correlations of pion multiplicity with the number of binary collisions, one has to study the influence of trivial factors. The limited acceptance might decrease the probability to register the pions produced in the collision with a given number of $N_{\text{grey}}$ (or with the relevant number $N_{\text{bin}}$).

Therefore, a value $Q$ is introduced in our study, which is the probability to detect the true multiplicity in case of one inelastic binary collision. Then $Q^m$ is the probability to detect the true multiplicity in $m$ inelastic binary collisions in a given event. With this coefficient, the average multiplicity can be calculated as in equation (15). Please note that this is integral presented in the discrete form. With this parameter $Q$, one can take the acceptance into account to compare the model with the experiment.

$$N_{\text{ch}} = \sum_{i=1}^{N} N_{\text{ch}}(v_i) \cdot \Delta N_{\text{bin}}(v_i | b) \cdot \sum_{k=1}^{i} \Delta N_{\text{bin}}(v_k | b)$$ (15)

Results of calculations with the account of the acceptance of the correlation of pion mean multiplicity with the number of binary collisions are shown in figure 3. The experimental data, showing a tendency of saturation, are found to be well described with the parameter $Q$ chosen equal to 0.74. The RQMD results obtained in full acceptance are well described with the $Q$ equal to 0.96, which is close to unity.

4. Summary

A new hydrodynamic model of nucleon stopping that describes the deceleration of a proton in a nucleus and based on hydrodynamics is proposed. No fitting coefficients are required.

The linear dependence of the mean multiplicity of charged particles vs. number of binary collisions was predicted for the pA collisions in case of the full acceptance study. Similar dependence is demonstrated by the RQMD and SMASH event generators.

SMASH is the event generator based on the hadronic transport approach. Particle dynamics is described by the relativistic Boltzmann equation. According to the relativistic Boltzmann equation, one can predict the trajectory of every particle. This equation is being solved after every time step $\Delta t$, so knowing the initial positions of every particle, we can predict their position at the next moment. At this new position, every particle can experience inelastic collision, elastic collision, decay or string excitation, but only once. It can have no interaction during this time step either. After every time step, SMASH updates the list of particles and removes particles, that experienced interaction, replacing them with particles, that were born during the last interactions. Then Boltzmann equation is solved again for each particle from the updated list to predict their next position.
Figure 4. Results of Effective Hydrodynamics Model calculations for the full acceptance of pion multiplicity vs. number of binary collisions obtained for for \( p - Au \) collisions at 18 GeV/c in comparison to our SMASH MC data and to the RQMD results taken from [1].

Figure 5. The same calculations but for the case for the limited acceptance. The effect of the last one is taken into account by a factor \( Q \) (see the text). The experimental data [1] are shown in the right picture.

The non-linear behavior of mean multiplicity vs. number of binary collisions observed in \( p - Au \) collision at \( p_{\text{lab}} = 18 \text{ GeV/c} \) was found to be a result of the limited acceptance of the experimental data.

Results of these studies of nucleon stopping are important for the future analysis of centrality selection in \( pA \) and \( AA \) collisions at NICA experiments.

Acknowledgments
Job is supported by the RFBR grant 18-02-40097. Authors are grateful to V. Kovalenko, V. Vechernin and A. Puchkov for valuable discussions.

References
[1] Chemakin I et al. 1999 Preprint nucl-ex/9902009
[2] Hegab M K, H"ufner J 1982 Nucl. Phys. A 384 353–70
[3] Bialas A, Bzdak A, Koch V 2016 Acta Phys. Pol. B 49 103
[4] Landau L D, Lifshitz E M 1986 *Theoretical Physics vol 6: Hydrodynamics* (Moscow: Nauka)
[5] De Vries H, De Jager C W, De Vries C 1987 At. Data Nucl. Data Tables 36 495-536
[6] Yao W-M et al. (Particle Data Group) 2006 J. Phys. G 33 1
[7] Wong C-Y 1994 *Introduction to High-Energy Heavy-Ion Collisions* (Singapore: World Scientific) p 251–60
[8] Thomé W, Eggert K, Gibini K et al. 1997 Nucl. Phys. B 129 365–89
[9] Weil J, Steinberg V, Staudenmaier J et al. 2017 Phys. Rev. C 94 054905