Adaptive binarization based on fuzzy integrals

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Abstract—Adaptive binarization methodologies threshold the intensity of the pixels with respect to adjacent pixels exploiting the integral images. In turn, the integral images are generally computed optimally using the summed-area-table algorithm (SAT). This document presents a novel adaptive binarization technique based on fuzzy integral images through an efficient design of a modified SAT for fuzzy integrals. We define this new methodology as FLAT (Fuzzy Local Adaptive Thresholding). The experimental results show that the proposed methodology has produced an image quality thresholding often better than traditional algorithms and saliency neural networks. We propose a new generalization of the Sugeno and CF, I, 2 integrals to improve existing results with an efficient integral image computation. Therefore, these new generalized fuzzy integrals can be used as a tool for grayscale processing in real-time and deep-learning applications.

Index Terms—Image Thresholding, Image Processing, Fuzzy Integrals, Aggregation Functions

I. INTRODUCTION

M ost of the binary segmentation algorithms based both on deep learning (DL) or traditional models are built on taking advantage of the foreground/background recognition [1]. Despite the multi-class semantic segmentation problems, the binary segmentation is specifically demanded in those applications where real-time performance is required and a simple but accurate structural and semantic representation is mandatory. In the literature, several image binarization algorithms based on both traditional and neural networks models are proposed for different applicative problems. For example, Cheremkhin et al. [2] provide an extended review of traditional methodologies based on global and local binarization methods for hologram compression; Kalaiselvi et al. [3] present a comparison between thresholding methodologies for real-world and brain MRI image segmentation. Furthermore, Roy et al. [4] provide a comparative study for the most common adaptive techniques. Recently, models based on convolutional networks are adopted for binarization and beyond. In particular, one of the natural evolutions of binarization approaches relies on the study of visual perception, better defined as visual saliency, and in the ability to distinguish and keep imprinted an object, a person or more generally a group of pixels on which the human attention is focused, both in the retina and in the post-processing phase, including the memorization step [5]. At several level, the traditional approaches are embedded in DL models showing a fair balance between accuracy, generalization power and computational time costs [6]–[9]. After all, the images are a matrix of values, thus enabling researchers to use binarization in complex networks [10], Bayesian networks [11] and biological networks/pathways [12]. Even if there are several binarization techniques in literature, none is the gold standard. The traditional global thresholding algorithms are generally worse than the local ones. Moreover, combined models of local and global techniques process the same image several times showing a lack of performance over time [13]. [14]. Furthermore, the perturbations that could affect an image are heterogeneous (illumination changes, experimental noise, variable contrast, etc.) and depends on the represented subjects. At several digital processing levels, such for example in the compressive sampling and lossy compression, the different types and degrees of digital degradation could influence binarization accuracy. Also for this kind of problems, Information Theory provides quantization strategies, but at the cost of much greater estimation complexity [15]. In genomic and proteomic analyses, the adaptive thresholding is exploited for the study of differential microarray spot intensities [16], [17]. The objects analyzed in the images can be static or in motion, multiple or single. Traditional or neural-network-based binarization of real-world [18], as well as, of micro-world [19] could be used to establish relations between frames [20]. In this work, we focus our attention on local adaptive thresholding methods. In general, the latter are more accurate than the global ones and could be fine-tuned in an automatic way [19], [21]. The idea behind the local adaptive thresholding relies on considering a threshold value for every pixel intensity or region of pixel intensities basing the analysis on its neighboring pixels on a fixed or variable local window. After all, the notion of adaptation has its roots in the concept of multi-scale analysis, structural variational analysis and representation of differential intensity values. As described by Bradley and Roth [14], one of the most efficient local adaptive thresholding method comes from an extension of the Wellner’s method [22] and it is a generalized form of the Niblack algorithm [23]. In particular, Bradley and Roth adaptive thresholding method (known as Bradley algorithm) exploits the representation power of integral images. Nevertheless, as proved in Debayle and Pinoli [24], the fuzzy integrals in the context of local adaptiveness show to outperform methodologies based on simple integral images. On the other hand, in the literature,
there have already been attempts to modify the Bradley algorithm. In particular, in these cases, a modification in the computation of the average neighbouring pixel intensities is used, for example considering a weighted integral image [25]. On the same line with the precedent authors, in this work, we propose a novel LAT, and we define it as FLAT, which is the acronym of Fuzzy Local Adaptive Thresholding algorithm. FLAT is based on the logic of the Bradley algorithm [14]. In particular, FLAT improves the thresholding accuracy leveraging a generalized form of the fuzzy integral images; for what is our knowledge, the latter approach has never been applied. The fuzzy integral images are computed from the integral images with a new efficient algorithm based on a modification of the summed-area-table algorithm (SAT) [26] showing real-time performances (1100 fps over 200 × 200 pixels). The document is organized as follows: the theoretical aspects of the three FLAT variants based on the generalizations of Sugeno and $CF_{1, 2}$ are explained in Section II. Instead, in Section III the new algorithms and changes to the SAT algorithm are introduced. In Section IV the results produced by our algorithms are compared to traditional and CNN-based adaptive approaches, both in terms of quality of the output and of performance. In particular, in the first subsection IV-A the goodness of our algorithms is evaluated on a toy data set with controlled perturbations. Next, in subsection IV-B a larger data set of real world images portraying single/multiple objects (≈ 2500 samples) is analyzed and the binarizations are compared. Finally, in sub-section IV-C our models, Bradley algorithm and a state of the art CNN, in their optimal configurations, are compared on a dataset of ≈ 300 images hard to binarize. In conclusion, our 3 FLAT algorithms show very accurate results and performances. Moreover, they appear to have better binarization capability than some state-of-the-art algorithms trained for convolutional networks. The implementation of 3 different variants of FLAT, the pipeline and novel challenging datasets, are available at: https://github.com/lodeguns/FuzzyAdaptiveBinarization.

II. Background

A. Fuzzy measures and fuzzy integrals

Let $n \in \mathbb{N}$, $[n] = \{1, \ldots, n\}$. A set function $\mu : 2^{[n]} \to [0, 1]$ is a fuzzy measure, if the following conditions are satisfied:

- $\mu(\emptyset) = 0$, $\mu([n]) = 1$.

A fuzzy measure $\mu$ is symmetric, if for any $A, B \subseteq [n]$, $|A| = |B|$ implies $\mu(A) = \mu(B)$ (here $|E|$ stands for the cardinality of the set $E$). For example, the uniform fuzzy measure $\mu_{uni}$ given by

$$\mu_{uni}(E) = \frac{|E|}{n},$$

for $E \subseteq [n]$, is symmetric.

A function $A : [0, \infty]^n \to [0, \infty]$ is an aggregation function, if $A$ is nondecreasing and $\inf_{x \in [0, \infty]^n} A(x) = 0$, $\sup_{x \in [0, \infty]^n} A(x) = \infty$.

An aggregation function $A : [0, \infty]^n \to [0, \infty]$ is

- internal, if $\bigwedge_{i=1}^n x_i \leq A(x_1, \ldots, x_n) \leq \bigvee_{i=1}^n x_i$, for each $(x_1, \ldots, x_n) \in [0, \infty]^n$.
- translation invariant, if $A(x_1 + c, \ldots, x_n + c) = A(x_1, \ldots, x_n) + c$, for all $c \in [0, \infty]$ and $(x_1, \ldots, x_n) \in [0, \infty]^n$.
- idempotent, if $A(x, \ldots, x) = x$, for each $x \in [0, \infty]$.
- positively homogeneous, if $A(cx) = c A(x)$, for each $x \in [0, \infty]^n$ and $c > 0$.
- comonotone additive, if $A(x + y) = A(x) + A(y)$, for all comonotone vectors $x, y \in [0, \infty]^n$ (vectors $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n)$ are comonotone, if $(x_i - y_i)(y_i - y_j) \geq 0$ for all $i, j \in \{1, \ldots, n\}$).
- comonotone maxitive (comonotone minitive), if $A(x \vee y) = A(x) \vee A(y)$ (and $A(x \wedge y) = A(x) \wedge A(y)$), for all comonotone vectors $x, y \in [0, \infty]^n$.

Let $\mu : 2^{[n]} \to [0, 1]$ be a fuzzy measure. The discrete Choquet integral with respect to the fuzzy measure $\mu$ is given by

$$Ch_{\mu}(x) = \sum_{i=1}^n (x(i) - x(i-1)) \cdot \mu(E(i)),$$

for any $x = (x_1, \ldots, x_n) \in [0, \infty]^n$, where $(\cdot)$ is a permutation on $[n]$ such that $x(1) \leq \cdots \leq x(n)$, with the convention $x(0) = 0$ and $E(i) = \{(i), \ldots, (n)\}$ for $i = 1, \ldots, n$.

The Sugeno integral with respect to the fuzzy measure $\mu$ is given by

$$Su_{\mu}(x) = \bigvee_{i=1}^n (x(i) \wedge \mu(E(i))),$$

for $x = (x_1, \ldots, x_n) \in [0, \infty]^n$, with the same meaning of $x(i)$ and $E(i)$, $i = 1, \ldots, n$, as above.

The Choquet integral is an internal function, which is idempotent and positively homogeneous and gives back the considered fuzzy measure, i.e., $Ch_{\mu}(1_E) = \mu(1)E$ for each $E \subseteq [n]$, where $1_E$ stands for the indicator of the set $E$.

The Sugeno integrals is not bounded by the minimum from below, but it is bounded by the maximum from above. It is neither idempotent nor positively homogeneous (however, the Sugeno integral is an idempotent, internal, positively homogenous function on the interval $[0, 1]$). It gives back the considered fuzzy measure, i.e., $Su_{\mu}(1_E) = \mu(E)$, for each $E \subseteq [n]$.

Moreover, the Choquet integral is comonotone additive and translation invariant, while the Sugeno integral is comonotone maxitive and comonotone minitive (for more details see, e.g., [27]).

B. Generalized Sugeno integral

We modify formula (3) defining Sugeno integral by replacing maximum and minimum operators by some more general functions. The obtained functional can be regarded as a generalization of the Sugeno integral.

**Definition 1.** Let $\mu : 2^{[n]} \to [0, 1]$ be a symmetric fuzzy measure, $F : [0, \infty] \times [0, 1] \to [0, \infty]$ be a binary function,
Let \( G: [0, \infty]^n \rightarrow [0, \infty] \) be an \( n \)-ary function. A Sugeno-like \( FG \)-functional is a function \( A: [0, \infty]^n \rightarrow [0, \infty] \) given by

\[
A(x_1, \ldots, x_n) = G\left(F(x_1, \mu(E_1)), \ldots, F(x_n, \mu(E_n))\right),
\]

for \( x = (x_1, \ldots, x_n) \in [0, \infty]^n \), with the same meaning of \( x(i) \) and \( E(i) \), \( i = 1, \ldots, n \), as above.

The correctness of the definition depends on whether the functional \( A \) given by formula (4) gives back the same value if some ties occur in a vector \( x \) and there is more than one permutation ordering this vector nondecreasingly. The symmetry of the fuzzy measure \( \mu \) considered in Definition 1 ensures that functional \( A \) is well-defined. In fact, for particular cases of \( G \), assumptions under which \( A \) is well-defined can be weakened. For example, the case of \( G \) being the maximum operator and \( F \) an arbitrary fusion function was deeply studied in [28], wherein assumptions under which \( A \) is well-defined for an arbitrary fuzzy measure \( \mu \) and a complete characterization of the functional \( A \) and its properties can be found.

The following three instances of Sugeno-like \( FG \)-functionals are of particular interest for us:

(i) Let \( G(x_1, \ldots, x_n) = \bigvee_{i=1}^n x_i \) and \( F(x, y) = x \land y \). Then we get

\[
A_1(x) = \bigwedge_{i=1}^n \left( x(i) \land \mu(E(i)) \right),
\]

so we recover the Sugeno integral, i.e. \( A_1 = S_u \).

(ii) Let \( G(x_1, \ldots, x_n) = \sum_{i=1}^n x_i \) and \( F(x, y) = x \cdot y \). Then we obtain

\[
A_2(x) = \sum_{i=1}^n \left( x(i) \cdot \mu(E(i)) \right).
\]

(iii) Let \( G(x_1, \ldots, x_n) = \sum_{i=1}^n x_i \) and \( F(x, y) = \frac{x y}{x + y} \).

Then we obtain

\[
A_3(x) = \sum_{i=1}^n \frac{x(i) \cdot \mu(E(i))}{x(i) + \mu(E(i)) - x(i) \cdot \mu(E(i))}.
\]

Note, that \( F \) is the Hamacher t-norm corresponding to the parameter \( \lambda = 0 \).

A straightforward computation gives us the following properties of \( A_2 \) and \( A_3 \): Both \( A_2 \) and \( A_3 \) are aggregation functions, since they are nondecreasing and

\[
\inf_{x \in [0, \infty]^n} A_2(x) = \inf_{x \in [0, \infty]^n} A_3(x) = 0,
\]

\[
\sup_{x \in [0, \infty]^n} A_2(x) = \sup_{x \in [0, \infty]^n} A_3(x) = \infty.
\]

Both \( A_2 \) and \( A_3 \) are bounded by the minimum from below, but not bounded by the maximum from above.

\( A_2 \) is positively homogeneous, but \( A_3 \) is not. Neither \( A_2 \) nor \( A_3 \) are idempotent, giving back the capacity, comonotone additive, comonotone maxitive, translation invariant.

Finally, we define \( A_4 = Ch \) in order to keep uniformity of the notation in the following paragraphs.

C. Computation of the integral image \( S \) with SAT

Let \( n, m \in \mathbb{N} \), \( [n] = \{1, \ldots, n\} \), \( [m] = \{1, \ldots, m\} \).

An original image \( I \) consisting of \( n \times m \) pixels arranged in \( n \) rows and \( m \) columns is associated with the matrix \( (p(x, y))_{(x, y) \in [n] \times [m]} \) assigning the intensity \( p(x, y) \) to the each pixel \( (x, y) \in [n] \times [m] \).

In the Bradley algorithm, the binarized pixel values are determined considering the average pixel intensities \( p_a \) of its neighboring pixels. The central role in determining the value of \( p_a \) is played by the so-called integral image. The integral image \( S \) is the matrix \( (S(x, y))_{(x, y) \in [n] \times [m]} \), defined for any pixel \( (x, y) \in [n] \times [m] \) by the following formula [9]

\[
S(x, y) = \sum_{i \leq x} \sum_{j \leq y} p(i, j).
\]

Determination of \( S \) has time complexity of \( O(l \ast (n \ast m)) \), which is derived by the \( l \) number of times that the Equation above is applied. However, leveraging the summed-area table algorithm (SAT) [29], the computation of \( S(x, y) \) can be maintained constant and the SAT time complexity remains fixed to \( O(n \ast m) \). The SAT can be developed efficiently computing for each pixel \( (x, y) \in [n] \times [m] \) the column-wise prefix-sums and the row-wise prefix-sums [26], as it is shown in Equation 10:

\[
S(x, y) = p(x, y) + S(x, y - 1) + S(x - 1, y) - S(x - 1, y - 1),
\]

with convention \( S(0, k) = 0 \), for each \( k = 0, \ldots, m \) and \( S(l, 0) = 0 \), for each \( l = 0, \ldots, n \).

D. Bradley algorithm based on the integral image \( S \)

Let us denote by \( [(x_1, y_1), (x_2, y_2)] \) the rectangle determined by the upper left corner \( (x_1, y_1) \) and the lower right corner \( (x_2, y_2) \). Once \( S \) is obtained, the sum of the pixel intensities in a rectangle \( [(x_1, y_1), (x_2, y_2)] \) denoted by \( p_s(x_1, y_1, x_2, y_2) \), is given by Equation 11:

\[
p_s(x_1, y_1, x_2, y_2) = S(x_2, y_2) - S(x_2, y_1) - S(x_1, y_2) + S(x_1, y_1),
\]

where \( 1 \leq x_1 \leq x_2 \leq m \), \( 1 \leq y_1 \leq y_2 \leq n \). Thus, the average value of the pixel intensities in the rectangle \( [(x_1, y_1), (x_2, y_2)] \), is given by Equation 12:

\[
p_a(x_1, y_1, x_2, y_2) = \frac{p_s(x_1, y_1, x_2, y_2)}{(x_2 - x_1) \times (y_2 - y_1)}
\]

with \( 1 \leq x_1 \leq x_2 \leq m \), \( 1 \leq y_1 \leq y_2 \leq n \). In the second part of the process, the pixel intensity of the original image \( I \) is compared pixel-by-pixel with the average value of the pixel intensities in the local window around the current pixel. For a given size \( 2s \times 2s \) of the local window, the Bradley algorithm iteratively binarizes the original image and provides the binary values \( I_b(x, y) \) for each pixel \( (x, y) \), as described in the following formula:

\[
I_b(x, y) = \begin{cases} 
1 & \text{if } p(x, y) \leq p_a(x_1, y_1, x_2, y_2) \times t, \\
0 & \text{otherwise,}
\end{cases}
\]
where \((x_1, y_1, x_2, y_2) = (x - s, y - s, x + s, y + s)\). Note that the local window needs to be changed, if it is not within the borders of the original image. Note also that it is considered only a percentage of \(p_a\) controlled by the sensitivity value \(r\), which is defined in the interval \([0, 1]\).

\[
\begin{array}{cccc}
0.5 & 1 & 0 & 0.2 \\
0 & 1 & 1 & 0 \\
0 & 1 & 0.0 & 0.8 \\
1 & 1 & 0.4 & 1 \\
\end{array}
\]

\[
\begin{array}{cccc}
0.5 & 1.5 & 1.5 & 1.7 \\
0.5 & 2.5 & 3.5 & 3.7 \\
1.5 & 4.5 & \text{max} & \end{array}
\]

\[
\begin{array}{cccc}
0.5 & 0.75 & 0.75 & 0.85 \\
0.5 & 1.25 & 2.25 & 2.60 \\
1.0 & 2.25 & \end{array}
\]

\[
\begin{array}{cccc}
0.5 & 0.75 & 0.75 & 0.85 \\
1.0 & 2.25 & \end{array}
\]

**A. Fuzzy integral image computation (\(F_{A_i}\))**:

Fuzzy integrals are used to avoid uncertainty in binarization and beyond, showing various application fields in the most different research areas [30], [31]. The main disadvantage of fuzzy integrals, like the Choquet integrals, relies in allocating further computational effort to the element sorting, in order to respect the monotonicity property (see section II). Despite this last observation and looking closely at the cascade construction of an integral image, in a constant sorting time, it is possible to adopt the procedure applied in the SAT and optimally generate the fuzzy integral image \(F\). As shown in Figure I - Box (a-b), once the integral image \(S\) is computed (see subsection II-C), for each pixel \((x, y)\) in the \(j\)-th operative window, it is possible to compute the fuzzy integral image \(F_{A_i}\), as follows:

\[
F_{A_i}(x, y) = A_i(S(x, y), S(x, y - 1), S(x - 1, y), S(x - 1, y - 1),)
\]

where \(A_i : [0, \infty]^4 \rightarrow [0, \infty]\), for \(i = 1, 2, 3, 4\), is one of the fuzzy integral-based functionals mentioned in the previous section, namely the Sugeno integral \(A_1 = Su\), the Sugeno-like \(FG\)-functionals \(A_2\) and \(A_3\), respectively and the Choquet integral \(A_4 = Ch\). As a fuzzy measure we adopt the uniform fuzzy measure \(\mu_{uni}\) defined by formula (11).

As shown in Algorithm 1 the procedure takes advantage of the natural ordering of the four elements aggregated in (14), obtaining a vector of ordered values \(\vec{o}\) and an associated static vector of fuzzy measures \(\vec{m}\).

In fact, the maximum value \(v_4\) is the element \(S(x, y)\) present in the right lower corner of the \(j\)-th operative window, while, the minimum value \(v_1\) is \(S(x - 1, y - 1)\) element ((see Figure I - Box (b)) - min in violet, max in blue). In order to complete the sorting, we need just to eventually to swap \(s_1 = S(x, y - 1)\) and \(s_2 = S(x - 1, y)\) by the following swap operation \(P_{swap}\) ((see Figure I - Box (b)) - double green arrow):

\[
P_{swap}(s_1, s_2) = \begin{cases} 
    v_2 = s_1, v_3 = s_2, & \text{if } s_1 < s_2 \\
    v_2 = s_2, v_3 = s_1, & \text{otherwise}
\end{cases}
\]

Thus, the final result is a one dimensional array of sorted values: \(\vec{o} = [v_0, v_1, v_2, v_3, v_4]\), where by convention \(v_0 = 0\). Moreover, since \(E_{(i)} = \{(1), \ldots, (4)\}\) is the subset of indices of the \(4 - i + 1\) greatest component of \(\vec{o}\), for the uniform fuzzy measure \(\mu_{uni}\) defined by formula (11), we have

\[
\mu_{uni}(E_{(i)}) = \frac{4 - i + 1}{4}
\]

Hence, we deal always with the same vector of fuzzy measures \(\vec{m} = [\mu_{uni}(E_{(1)}), \mu_{uni}(E_{(2)}), \mu_{uni}(E_{(3)}), \mu_{uni}(E_{(4)})] = [1, 0.75, 0.50, 0.25]\). In Algorithm 1 a bridge function \(f_i(\vec{o}, \vec{m})\) for each pixel \((x, y)\) is defined, in order to map each fuzzy integral-based functional computation (for \(i = 1, 2, 3, 4\) on the two vectors: \(\vec{o}\) and \(\vec{m}\) for the \(j\)-th operative window. As shown in Figure I the fuzzy integral image \(F_{A_i}\) could be computed with different bridge functions \(f_i(\vec{o}, \vec{m})\), varying only the values of \(\vec{o}\) and \(\vec{m}\) and maintaining the algorithmic structure unchanged. Only the 4 operative window corners are considered at a time \((O_{f_i}(4))\) leaving the computational complexity polynomial in time, as it is for the original SAT \(O(n \times m + O_{f_i}(4) + \cdots = O(n \times m))\).
B. Adaptive binarization with the $F_{A_{r}}$:

Algorithm 1 outputs the fuzzy integral image $F_{A_{r}}$. Then, the latter is given in input to Algorithm 2 for binarization. For what is concerning the binarization, $F_{A_{r}}$ will be leveraged as $S$ is exploited in Bradley algorithm (see section II-C). However, in Algorithm 2 a modified version of the Bradley algorithm is presented according to our constraints. In detail, $F_{A_{r}}$ is computed with the different integral generalization presented in sections II-A and II-B. Furthermore, the size and the coordinates of the sliding nearest neighbor’s pixel local window $w_n$ is set with the dimensional parameter $n_a$. The latter is computed through 2 empirical parameters: $a_1$ and $a_2$. As it is described above, the $w_n$ is used to locally binarize the central pixels. Thus, $w_n$ is sized and positioned following the procedure described in Algorithm 2. The area of $w_n$ is equal to $n_a^2$. The dimensional parameter $n_a$ is computed as follows:

$$n_a = \left\lfloor \frac{\min(n, m)}{a_1 \times a_2} \right\rfloor.$$  \hspace{1cm} (16)

and it is based on the $(n, m)$ dimensions of $I$. The parameters $a_1$ and $a_2$, as well as $t$, can be varied iteratively to improve the accuracy until the binarized image at the optimum ($I^*_t$) is found. In particular, the $I^*_t$ indicates the optimum binarization in terms of the best $F_m$ value [32] with respect to the ground truth. The subscript b indicates which method of binarization is applied, such as, for example, if we consider $b$ equal to $F_{A_{r}}$, $I_{F_{A_{r}}}$ is the original image binarized with the Sugeno integral image and $I_{F_{A_{r}}}^*$ is its binarization at the optimum.

C. Dataset

In order to test and compare our algorithms, we leverage a controlled toy dataset and a saliency MSRA-B dataset [33]. These datasets are provided with ground truths (GTs) and have the following characteristics:

**Toy dataset:** The toy dataset is a novel challenging set of 8 images in which are applied several types of perturbations. In particular, images are labeled alphabetically from a to h. These challenging images have a very small size with odd and even dimension (9 x 9 and 8 x 8 pixels). The pixel intensity is normalized in the interval [0, 1] and with a decimal precision of 0.01. In particular, these images have been designed in a methodological way to present increasing levels of difficulty for the binarization. Furthermore, the odd and even sizes are suitable for testing the correct sliding of the local window $w_n$. In particular, the dataset is built with an accurate modification of the pixel intensities with respect to the interplay of 5 specific challenging characteristics. Thus, the design of the images reflects 5 challenges: high-low contrast variations ($\gamma_0$), spatial variations in lighting ($\gamma_1$), additive random noise ($\gamma_2$), motifs of structured noise ($\gamma_3$), smoothed borders ($\gamma_4$). Moreover, in Table[7] the percentage of extension of the applied perturbations and the variability in intensity between the maximum and minimum average perturbation intensity are shown.

**Test set:** The second dataset comes from an accurate selection of 5,000 images collected from the MSRA-B dataset [33]. This dataset is used for saliency analyses and the GTs are suited for testing saliency foreground/background extraction. Thus, 2,413 images are selected from MSRA-B applying a global threshold filtering (Otsu method [34]). In particular, the Otsu predicted masks are compared with only the original images with an $F_1$ measure greater than or equal to 0.7 are selected. This guarantees to make fair comparisons between the binarizations/predictions made by DSS-Net [35] (see also Section ref) and those obtained by traditional algorithms and our fuzzy algorithms.

IV. RESULTS AND DISCUSSION

This Section is organized as follows: (i) In Section IV-A several analyses on the toy dataset are performed with comparisons between Bradley algorithm and our algorithms ($A_2$($CF_1$), $A_4$($Choquet$) and $A_3$(Hamacher)). (ii) Instead, in Section IV-B a whole comparison on the test set with a fixed parametrization between traditional adaptive algorithms
Algorithm 2: Binarization based on $F_A$ (FLAT - Step 2)

Require: Gray-scale image $I$ with intensities in $[0,1]$.
Require: The fuzzy integral image $F_A$.
Require: The parameters $a_1$ and $a_2$.
Require: The sensitivity parameter $t$.

function FLAT- $I_b(I, F_A, a_1, a_2, t)$
\[ n, m \leftarrow \text{dim}(I) \]  \Comment{Dimension of $I$}
\[ I_b \leftarrow \text{allocate a zero-matrix with size $(n, m)$} \]
\[ n_a \leftarrow \text{Defined in Formula [16] with $a_1$ and $a_2$} \]
\[ r \leftarrow 1 \text{ to } n \]
\[ \text{for } c \leftarrow 1 \text{ to } m \text{ do} \]
\[ y_0 \leftarrow \text{max}(r - n_a, 0) \]  \Comment{$w_n$}
\[ y_1 \leftarrow \text{min}(r + n_a, r) \]
\[ x_0 \leftarrow \text{max}(c - n_a, 0) \]
\[ x_1 \leftarrow \text{max}(c + n_a, c) \]
\[ p_{\text{area}} \leftarrow (y_1 - y_0) \ast (x_1 - x_0) \]
\[ p_s \leftarrow F_A[y_1, x_1] - F_A[y_0, x_1] - F_A[y_1, x_0] + F_A[y_0, x_0] \]
\[ p_a \leftarrow \frac{p_s}{p_{\text{area}}} \]
\[ \text{if } I[r, c] \leq p_a \times (1 - t) \text{ then} \]
\[ I_b[r, c] \leftarrow 1 \]
\[ \text{else} \]
\[ I_b[r, c] \leftarrow 0 \]
\[ \text{end if} \]
\[ \text{end for} \]
\[ \text{end for} \]
return $I_b$
end function

TABLE I

| Image | Perturbation | Coverage | Intensity Variability |
|-------|-------------|----------|----------------------|
| a     | $\gamma_0$ | $\approx 87\%$ | 0.20                 |
|       | $\gamma_3$ | $\approx 70\%$ | 0.30                 |
| b     | $\gamma_0$ | $\approx 89\%$ | 0.15                 |
|       | $\gamma_3$ | $\approx 50\%$ | 0.20                 |
| c     | $\gamma_0$ | $\approx 10\%$ | 0.35                 |
| d     | $\gamma_1$ | $\approx 90\%$ | 0.03                 |
|       | $\gamma_4$ | $\approx 23\%$ | 0.10                 |
| e     | $\gamma_0$ | $\approx 18\%$ | 0.20                 |
|       | $\gamma_3$ | $\approx 88\%$ | 0.20                 |
| f     | $\gamma_0$ | $\approx 7\%$  | 0.05                 |
|       | $\gamma_1$ | $\approx 62\%$ | 0.01                 |
| g     | $\gamma_0$ | $\approx 64\%$ | 0.03                 |
|       | $\gamma_3$ | $\approx 28\%$ | 0.03                 |
| h     | $\gamma_0$ | $\approx 18\%$ | 0.05                 |
|       | $\gamma_1$ | $\approx 88\%$ | 0.20                 |

and our novel algorithms is described. (iii) Finally, on the same test set, a comparison from the optimal predictions from DSS-Net and our novel algorithms is shown in Section [V-C]. For all the setups, the binarized images are tested on the GTs with 7 metrics: Structural Similarity Index (SSIM), Mean Square Error (MSE), accuracy (Acc), precision (P), recall (R) and the $F_1$ measure ($F_1$) [32]. In addition, the Matthews Correlation Coefficient metric (MCC) [36] is evaluated for the MSRA-B binarizations, to deal with the class imbalance problem in real-world images. The metrics are normalized in the range $[0,1]$, except for MCC and SSIM which are defined in the range $[-1,1]$.

A. Comparisons on the Toy dataset

1) Exhaustive analyses: A grid search was carried out on all the possible algorithm parameter configurations to find the optimal fuzzy thresholding for the toy dataset. A voting schema is suited for comparisons. For what is concerning $w_n$, we tested all the possible windows $n_a \times n_a$ constrained by Eq. [16] and $1 \leq n_a \leq \min(n, m)$, where $n, m$ are the dimensions of $I$. Instead, for what is concerning the threshold $T_h$, we analyzed all the possible configurations with respect to $w_n$ changing $T_h$ increasingly from 0.01 to 1 with steps of 0.01. For all the possible parameter configurations of $T_h, a_1$ and $a_2$, the binarizations obtained with our algorithms and with the Bradley algorithms are divided into three subsets with respect to three specific sensitivity values. Thus, the obtained binarizations are regrouped with respect to SSIM values that are greater or equal to $\theta = [0.90, 0.55, 0.00]$, respectively. Under the same parameter configurations, the variable $g^*$ indicates the overall number of times when our strategies binarize better than the Bradley algorithm and vice versa. In particular, in order to obtain a strong pairwise comparison, a counting is made of the times in which the SSIM of the best
algorithm is greater then $\theta$ and the algorithm is better than the other. Formally, for each $j = [2, 3, 4]$, the $i$-th image and the $n$-th parameter configuration, $g_{F_{A_j}}^i$ represents the number of times in which $SSIM_{F_{A_j}} \geq \theta \wedge SSIM_{F_{A_j}} > SSIM_{Bradley}$ are satisfied. While, $g_{Bradley}^i$ represents the number of images in which $SSIM_{Bradley} \geq \theta \wedge SSIM_{F_{A_j}} < SSIM_{Bradley}$ are satisfied. These results are shown in Table III. For example, in the subset of binarizations with $SSIM \geq 0.9$, the $SSIM$ of $F_A$ is $g_{F_{A}}^i = 850$ times. As shown in Table III, $F_{A_j}$'s computed with $A_2$ and $A_3$ are the ones that better binarize the images. Moreover, between the two definitions of fuzzy integrals, our $A_2$ approach is both the best-performing one and that with lower computational complexity than others.

2) Robustness and sensitivity analyses: An exhaustive analysis on our toy dataset with 4 increasing percentages of random additive noise (+20%, +30%, +40%, +50% of $\gamma_2$) is provided on the online repository. In particular, the random noise is added twice, on both the images including the other perturbations ($\gamma_1 + \gamma_2$, $i \neq 2$) and on the ground truth images $(GT + \gamma_2)$. As it is shown on the online repository, the approach based on $A_2$ is very stable and binarizes with an average $F_1 \geq 0.95$ in the 75% of the cases, and with $0.87 \leq F_1 \leq 0.93$ in the 25% of the cases with the 20% of additive random noise. Instead, considering the 40% of coverage by using $\gamma_2$ perturbation, the $A_2$ methodology binarizes with an average $F_1 \geq 0.95$ in the 62% of the cases, and with $0.75 \leq F_1 \leq 0.93$ in the 38% of the cases. While, considering the $A_3$-based methodology with the 40% of $\gamma_2$ coverage, the binarization is maintained up to an average $F_m \geq 0.89$ in the 56% of the cases, and with $0.53 \leq F_1 \leq 0.78$ in the rest of the cases.

3) Qualitative and comparative analyses: In Table S-1 on the online repository, our algorithm binarizations (respectively $I_b$, for $b = A_2, A_3, A_4$) and the Bradley algorithm binarizations ($I_B$) are compared with the same parameter configurations ($n_a$ and $t$). In Table S-1 the best values are indicated with asterisks and in bold. The toy images present different types of perturbations, that differently affects an accurate binarization. In Table I the percentages of applied perturbations are indicated. As it is shown in Table S-1 the comparative analysis indicates that the Bradley algorithm seems to be stable only with high-low contrast variations ($\gamma_0$) and spatial variation in lightning ($\gamma_1$) (see also Figure 2 - Image c). While, with our fuzzy algorithms, the binarization is more stable with all the types of perturbations considered. This is proved also with a visual example in Figure 2 where the binarization of Image a is shown. In fact, in this case, Image a presents a high percentage of $\gamma_0$ and $\gamma_3$ (see Table I). The latter perturbation represents motifs (recurrent patterns) of structured noise which are very difficult to threshold. For other visual comparisons please visit our online repository. The FLAT methodology based on $A_1$ does never reach good results, however, its analyses are shown in the online repository.

B. Comparisons with traditional algorithms

Our fuzzy algorithms based on $A_2$, $A_3$ and $A_4$ functions were tested on the test set of 2413 images (see also subsection III-C) with respect to adaptive methods of Sauvola [13], Niblack [23] and Bradley and Roth. As it is shown in Table I (a), for what concerns our algorithms and the algorithm of Bradley et al. [14], three different threshold levels ($Th = [0.25, 0.45, 0.75]$) and two fixed window parameters $a_1 = 3$, $a_2 = 1$ were chosen. On the other hand, for Niblack and Sauvola (Table I (b)) the results are computed considering only the same fixed window parameters. In this case, it is impossible to fix a threshold because these algorithms compute their adaptive threshold value basing their binarizations on the mean and standard deviation of the window centered on the pixel to binarize. Furthermore, it is important to underline that two other parameters have been fixed, in order to make the comparisons as balanced as possible. In particular, the parameter $K$ for Niblack is set to 0, because in such a way, exhibits a generalized behavior like the Bradley algorithm. While, as suggested by Sauvola et al. [13], the values of $K$ and $R$ are set to 0.2 and 128, respectively. As it is shown in Table I our algorithms outperform the binarizations obtained by Sauvola et al. [13].
with *niblack* and Sauvola, and, in particular, our methodology based on $A_2(\text{CF}_{1.2})$ turns out to be the best performing one, with a threshold fixed to 0.65. In Figure 5, a visual comparison of binary maps produced by the proposed algorithms is shown. By looking at the obtained binarizations, also with the DSS-Net predictions (see also sub-section IV-C), our proposed algorithms turn out to be more reliable in printed documents, and on images with shadows. From a first analysis, even if $A_2(\text{CF}_{1.2})$ seems to be the one that performs better, there was no big difference for the threshold values at 0.25 and 0.45 between our algorithms $A_1(\text{Choquet})$, $A_3(\text{Hamacher})$ and Bradley’s. Moreover, in this case, the thresholds were chosen empirically; on the other hand, as we will show in the next paragraph, at the *optimum*, the quality of our binarizations is better than those obtained with the Bradley algorithm.

### C. Comparisons with DSS-Net and Bradley at the optimum

At best of our knowledge, Deep Learning models have not been used in image thresholding. Few attempts have been done so far for solving similar tasks as RED-Net (Residual Encoder-Decoder Network - U-Net [37]) for handwritten document binarization and Le-Net5 [38, 39] (a traditional CNN based on the model of [40]) for musical document binarization. In this study we chose DSS-Net [35], that is a CNN trained for saliency on real world images, which we used in our experimental set-up. As far as we know, DSS-Net seems to be the best comparable model concerning our adaptive algorithms, because it retains a strong generalization power, deriving from the use of a very extensive data set on binarizable images. For the comparisons between DSS-Net, Bradley and our fuzzy algorithms, only 280 images with GTs were selected on the test set (see Section III-C). In particular, the images were selected considering an *Otsu* $F_1$ measure greater or equal to 0.8 to ensure a reliable level of thresholdability. Moreover, only the images that are more difficult to be binarized have been selected with a manual control. In fact, they are complex in terms of shading and lighting, relative positions of objects and variable size of objects in the background and foreground. The subset of these thresholdable images with their predicted binary masks is available on the online repository. For each image, the binarization of Bradley and our methodologies are computed at the optimum, selecting only the best results. In particular, the search of the optimum is obtained changing $Th$ increasingly from 0.01 to 1 with steps of 0.01. In Table IV the average results of these comparisons are shown for several metrics. In particular, the table shows that $F_{A_2}$ reaches an $\text{MCC} \approx 0.86 \pm 0.07$ showing the algorithm ability to manage binarizations with a very different ratio between the pixels classified as background and foreground, dealing correctly with true and false positives and negatives. For what is concerning the *precision*, DSS-Net and $F_{A_2}$ show a better ability to recognize false positives. Looking at the recall, Bradley, and our $F_{A_1}$ and $F_{A_3}$ are more accurate in the detection of false negatives. However, the best accurate $F_1$, which is more stable on extreme values, and *accuracy*, according to the $\text{MCC}$, is obtained with $F_{A_2}(F_1 \approx 0.8, \text{accuracy} \approx 0.9)$. The similarity between predictions/binarizations and GTs are evaluated considering, also the presence of noise, with the $\text{SSIM}$. In this case, DSS-Net reaches the same performance of $F_{A_2}$ with $a_1 = 2$ and $a_2 = 1$. For what is concerning $\text{MSE}$, $F_{A_2}$ outperform all the other algorithms with the different $a_2$ configurations. In conclusion, similarly to the experiments on the toy dataset (see section IV-A) and to the experiments on the dataset of 2413 images (see section IV-B), the Choquet ($A_1$) and Hamacher ($A_3$) methodologies, show equal and slightly lower performances than those of $A_2$ and DSS-Net by varying the $a_1$ parameter and fixing $a_2 = 1$. Furthermore, as it is described above, the FLAT methodology based on $\text{CF}_{1.2}$ seems to perform much better than the other fuzzy algorithms and DSS-Net predictions. Moreover, Google Colab [41], a benchmark of 10 images with a fixed size of $200 \times 200$ pixels, was selected for evaluating the binarization time of our algorithms. In detail, our fuzzy algorithms reach $\approx 1100 \text{fps}$. For what is concerning DSS-Net, authors declare a prediction time of $\approx 750 \text{fps}$ on images with a variable size.
Moreover, it is important to underline that the search time of the optimal threshold is a limitation for our algorithms. Therefore, the search time can greatly reduce the number of frames per second in binarization. The search range of the optimum could be restricted, because as it is shown on Table IV, on an extended dataset of images, the average of the optimal threshold values seem to settle on certain values, with a very low standard deviation.

V. Conclusion

Three new models for adaptive binarization, based on the optimized calculation of generalized fuzzy integral images, were introduced. The algorithm optimizations were obtained through a novel modification of the summed area table algorithm which, in particular, is suited for fuzzy integrals. It has been shown that, compared to traditional methods and a state of the art neural network, our adaptive methods have improved the accuracy of binarization without additional computational complexity. In particular, according the the MCC and F1 metrics, one of our proposed algorithms (CF1,2) reaches \( F_1 \approx 0.86 \) with standard deviation of \( \approx 0.07 \) and \( MCC \approx 0.82 \) and a standard deviation of 0.04. Fuzzy thresholding algorithms turn out to be very stable for a correct thresholding of real-world images which are highly perturbed by different lighting conditions, background/foreground size imbalance and several color contrast conditions. Due to the impressive time performances (1100 fps), these new thresholding algorithms could be embedded in deep learning models obtaining a better accuracy and speeding up the network convergence. In conclusion, the results obtained in this paper, both from a theoretical and applied points of view, are really promising. We expect that these novel methodologies will lead to new research opportunities in real time binarization and image processing.

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