LTI system order reduction approach based on asymptotical equivalence and the Co-operation of biology-related algorithms

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Abstract. A novel order reduction method for linear time invariant systems is described. The method is based on reducing the initial problem to an optimization one, using the proposed model representation, and solving the problem with an efficient optimization algorithm. The proposed method of determining the model allows all the parameters of the model with lower order to be identified and by definition, provides the model with the required steady-state. As a powerful optimization tool, the meta-heuristic Co-Operation of Biology-Related Algorithms was used. Experimental results proved that the proposed approach outperforms other approaches and that the reduced order model achieves a high level of accuracy.

1. Introduction
In this study optimization problems were solved by a meta-heuristic called Co-Operation of Biology-Related Algorithms (COBRA) [1], which is based on the cooperation of five similar nature-inspired algorithms. The workability of the COBRA approach and its usefulness were shown in [1] on a set of benchmark functions.

The proposed algorithm was applied to the order reduction of linear time-invariant (LTI) system models. The described problem is important in system theory and can be applied to single input single output (SISO) systems and for multiple input multiple output (MIMO) systems. Various approaches to solving the order reduction problem exist, and we suggest a novel one that in some cases outperforms other approaches. Many of these approaches are based on combining the stability equation [2] or a similar technique and a reduced extremum problem, which normally involves the minimization of the cumulative error. The minimization problem can be efficiently solved using evolutionary optimization algorithms, nature inspired optimization algorithms or other mimetic algorithms, since these algorithms are a powerful tool for solving complex black-box optimization problems. Since applying these algorithms, the described problem solving methods have started to be developed again.

In the following studies, the same order reduction problem was considered. The current investigation is related to the research below and solves the same problems, so the results can be compared. In the studies [3] and [4] a genetic algorithm is used for solving the extremum problem, in the study [5] the big bang big crunch algorithm is used and in [6] the cuckoo search algorithm is used.
The last three approaches use the stability equation method. Thus the parameters of the denominator can be calculated and these parameters provide the stability of the LTI model.

In this paper all the parameters of the denominator are to be identified. The stability of the LTI system is ensured by using a criterion that is based on the transient matrix eigenvalues and their implementation into the objective function. The equality of model steady-states is carried out by the nominator coefficient determination, which is based on the asymptote value and requires the denominator coefficients.

The approach performance is estimated on the basis of specific criteria values and comparing them with criteria values for different approaches.

This paper contains three main sections, as well as an introduction and a conclusion. The first main section states the main principles of the meta-heuristics we applied to reduced optimization problem solving. The method of forming the objective function and of determining the solution is described in the second section. In the third section the experimental results are described: we compare the result with the initial model of higher order and with the results found in other work.

2. Co-Operation of Biology-Related Algorithms

The meta-heuristic approach called Co-Operation of Biology-Related Algorithms (COBRA) was originally developed for solving real-valued optimization problems [1]. The approach outlined above is based on the collective work of five well-known nature-inspired algorithms, namely Particle Swarm Optimization (PSO) [7], Wolf Pack Search (WPS) [8], the Firefly Algorithm (FFA) [9], the Cuckoo Search Algorithm (CSA) [10] and the Bat Algorithm (BA) [11].

The algorithm COBRA involves generating five populations which are then executed in parallel cooperating with each other, so each algorithm corresponds to its own population. The key advantage of the optimization method COBRA consists of its self-tuning work. Thus there is no need to choose the population size for components of COBRA. The approach settings were taken as they were recommended by the authors of the papers mentioned above.

The number of individuals in the population of each component-algorithm can increase or decrease depending on whether the fitness value improves: if the fitness value does not improve over a given number of generations, then the size of all populations increases and vice versa. Besides, on every generation the component algorithms compete with each other, so a “winner algorithm” can be determined. Namely the algorithm with the best population’s average fitness value is considered as a “winner”. The population of the “winner” algorithm “grows” by accepting individuals removed from other populations. The migration operator of the given approach consists in replacement of the worst individuals of each population by the best individuals of others.

Experimental results showed that the meta-heuristic approach COBRA is reliable and works efficiently. Moreover, it was established that the meta-heuristic COBRA outperforms its component-algorithms and can be used instead of the described bio-inspired optimization techniques.

While solving the optimization problems described in this study, populations were initialised uniformly on the closed interval [-10; 10]. The minimal population size for each component was equal to 6 individuals and calculations were terminated when the number of function evaluations was equal to 2500. This number for the criterion evaluation was taken from paper [6].

In this paper, the decision was taken to use the transformation of the main criterion to a fitness function as follows:

\[
\text{fitness}(x) = 1 + C(x) \quad (1)
\]

where \( x \in \mathbb{R}^x \) is an alternative, \( \mathbb{R}^x \) is the search space and \( C(\cdot): \mathbb{R}^x \rightarrow [0, +\infty) \) is a criterion. According to this definition, \( \text{fitness}(\cdot): \mathbb{R}^x \rightarrow (0, 1] \).
3. The order reduction problem and a proposed approach
Let the SISO LTI system model be determined by the following differential equation

\[ \sum_{i=0}^{n} a_i \cdot x^{(i)}(t) = \sum_{i=0}^{m} b_i \cdot u^{(i)}(t) \]  

(2)

where \( a_i \in R, i = 1, \ldots, n \) and \( b_i \in R, i = 1, \ldots, m \) are the coefficients, \( n : n > m \) is the order, \( t \in R^+ \cup \{0\} \) is the time variable, \( x^{(i)}(t) \) is the \( i \)th derivative of the output, \( u^{(i)} \) is the \( i \)th derivative of the control input and \( x^{(0)}(t) = x(t) \).

For the initial value \( x^{(i)}(0) = 0 \) equation (2) after using the Laplace transformation can be represented with a transfer function

\[ G(s) = \sum_{j=0}^{m} b_j \cdot s^j \cdot \left( \sum_{i=0}^{n} a_i \cdot s^i \right)^{-1} \]  

(3)

The MIMO LTI system can be represented similarly,

\[
\begin{align*}
\frac{d}{dt} X(t) &= A \cdot X(t) + B \cdot U(t), \\
Y(t) &= C \cdot X(t) + D \cdot U(t),
\end{align*}
\]  

(4)

where \( Y(t) : R^+ \cup \{0\} \to R^{N_y} \) is the output function, \( N_y \) is the number of outputs, \( U(t) : R^+ \cup \{0\} \to R^{N_u} \) is the input function, \( N_u \) is the number of inputs, \( X(t) : R^+ \cup \{0\} \to R^{N_x} \) is the space variable, the system matrix is \( A \in R^{N_x \times N_x} \), the control matrix is \( B \in R^{N_x \times N_u} \), the output matrix is \( C \in R^{N_y \times N_x} \) and the feed-forward matrix is \( D \in R^{N_y \times N_u} \). The transfer function for the MIMO system is the following:

\[ W_s(s) = C \cdot (s \cdot I - A)^{-1} \cdot B + D. \]  

(5)

In this paper we consider the MIMO system with two inputs and two outputs, thus its transient function (5) can be determined with the expression

\[
W_s(s) = \begin{bmatrix}
W^{1,1}(s) & W^{1,2}(s) \\
W^{2,1}(s) & W^{2,2}(s)
\end{bmatrix}
\]  

(6)

where \( W^{i,j}(s) = \frac{D^{i,j}(s)}{N^{i,j}(s)} \), \( D^{i,j}, N^{i,j}, i, j = 1,2 \), are the denominator and nominator, respectively. Factoring out the denominator gives
\[ W_i(s) = \frac{1}{D_i(s)} \begin{pmatrix} N_{s,i}^1(s) & N_{s,i}^{1,2}(s) \\ N_{s,i}^{2,1}(s) & N_{s,i}^{2,2}(s) \end{pmatrix}, \]

\[ D_i(s) = \prod_{j} D_{i,j}^{(i)}(s), \quad N_{p,q}^{i,j}(s) = N_{p,q}^{i,j}(s) \prod_{(i',j')} D_{i',j'}^{(i')}(s). \]

(7)

It is known that linear system (1) is in a steady-state in the case of a stable system and input \( u(t) = H(t) \), \( H(t) \), the unit-step function, is a constant

\[ a' = \lim_{t \to +\infty} x(t) = \frac{a_0}{b_0} \]

(8)

the coefficients \( a_0 \) and \( b_0 \) are given in (3). This means that if the model output asymptote is \( a' \), the parameters of the reduced model are to be determined by fraction (8).

We need to make a model of the 2\(^{nd}\) order which is determined by the following transfer function

\[ G_m(s,p) = \frac{D_s \cdot s + a' \cdot P_1}{s^2 + p_0 \cdot s + P_1}, \]

(9)

for the SISO systems and

\[ G_m(s,p) = \frac{D_m \begin{pmatrix} N_{m,1}^{1,1} & N_{m,1}^{1,2} \\ N_{m,2}^{2,1} & N_{m,2}^{2,2} \end{pmatrix}}{s^2 + p_0 \cdot s + p_1, \quad N_{m}^{i,j}(s,p) = p_{1+2(i-1)+j} \cdot s + a'_{2(i-1)+j} \cdot P_1}, \]

(10)

for the MIMO systems. Hence, the parameters \( p \) are to be identified. As can be seen, the absolute term of the nominator is defined by (8) to ensure the asymptotic equality.

We want the model with reduced order to be a good estimation of the LTI system, so its response is close to the response of the model with higher order on the same control input \( u(t) \). The response is a function on a time domain and for both models it can be found by solving the Cauchy problem for (2) and (4). Since we consider the unit-step function as an input and its first derivative is the Dirac delta function, the invariant Laplace transformation is useful:

\[ x(t,p) = L^{-1} \left( G_m(s,p) \cdot L(u(t)) \right), \quad \dot{x}(t) = L^{-1} \left( G(s) \cdot L(u(t)) \right), \]

(11)

for the SISO LTI systems and

\[ x^{i,j}(t,p) = L^{-1} \left( \frac{N_{m}^{i,j}(s,p) \cdot L(u(t))}{D_m(s,p)} \right), \quad \dot{x}^{i,j}(t) = L^{-1} \left( \frac{N_{s}^{i,j}(s) \cdot L(u(t))}{D_i(s)} \right), \]

(12)

for the MIMO LTI systems. Let \( u(t) = H(t) \), so the parameters are the solution of the extremum problem...
\[ C_{\text{siso}}(p) = \sum_{i=0}^{N} (\hat{x}(t_i) - x(t_i, p))^2, \quad C_{\text{siso}}(p) \rightarrow \min_{p \in R^*}, \] (13)

or

\[ C_{\text{mimo}}(p) = \sum_{i,j} (\hat{x}^{i,j}(t_k) - x^{i,j}(t_k, p))^2 \quad 2 + N_i, \quad C_{\text{mimo}}(p) \rightarrow \min_{p \in R^*}, \] (14)

where \( t_i = \frac{T \cdot i}{N}, i = 1, N \), \( T \) is the final time, \( N \) is the number of points, and all the responses are calculated due to (11) or (12) for the SISO problems and MIMO problems, respectively.

To ensure that the reduced order model is stable, the criteria (13) and (14) were modified: the stability condition based on the eigenvalues was implemented. A LTI system of the 2\(^{nd}\) order can be described with an equation from the denominator of (3), \( a_0^* \cdot s^2 + a_1^* \cdot s + a_2^* \), whose parameters are related to the coefficients of the system matrix, so the system is asymptotically stable when

\[ -\frac{a_2^*}{a_0^*} < 0 \] (15)

because inequality (14) means that all the eigenvalues are less than zero. However, the 2\(^{nd}\) order LTI system can always be determined by the equation (9), so \( a_2^* = 1 \) and thus

\[-a_1^* < 0 \Rightarrow a_1^* > 0 \Rightarrow p_0 > 0 \] (16)

In this work a penalty function was used to provide the implementation of the stability condition into criteria (13) and (14). Modified criteria are as follows,

\[ \tilde{C}_{\text{siso}}(p) = C_{\text{siso}} + c \cdot P(p_0) \rightarrow \min_{p \in R^*}, \] (17)

\[ \tilde{C}_{\text{mimo}}(p) = C_{\text{mimo}}(p) + c \cdot P(p_0) \rightarrow \min_{p \in R^*}, \] (18)

where \( P(\cdot) : R \rightarrow R^* \cup \{0\} \) is a static penalty function and \( c > 0 \) is a coefficient. The penalty function activates as the parameter becomes less than zero:

\[ P(x) = \begin{cases} 0, \ x > 0 \\ \|s\|, \ x \leq 0 \end{cases} \] (19)

To analyse solution accuracy three more criteria are used. These criteria are involved in comparing the efficiency of approaches. Let \( x(t) = x(t, p^*) \), \( \hat{x}(t) \) be the solutions of (11) or (12), the input is \( u(t) = H(t) \) and \( p^* = \arg \min_{p \in R^*} C_{\text{siso}}(p) \) or \( p^* = \arg \min_{p \in R^*} C_{\text{mimo}}(p) \), depending on the problem. The first criterion we want to calculate is the integral square error

\[ E = \int_{t_0}^{t_f} (x(t) - \hat{x}(t))^2 dt \]
\[ I_1 = \int_{0}^{\infty} \left( x(t) - \hat{x}(t) \right)^2 dt. \] (20)

Its estimation was used to identify the parameters by solving problems (13) or (14). The integral (20) is divergent if \( \lim_{t \to +\infty} x(t) \neq \lim_{t \to +\infty} \hat{x}(t) \), and for this reason expression (8) was implemented and the stability condition is required.

The next criteria are the relative integral square error and the criteria are given in [3] which are proposed to check the accuracy of the model. Both criteria are expressed by the fraction:

\[ I_2 = \frac{\int_{0}^{\infty} \left( x(t) - \hat{x}(t) \right)^2 dt}{\int_{0}^{\infty} \left( x(t) - x(+\infty) \right)^2 dt}, \] (21)

and for \( u(t) = \delta(t) \),

\[ I_3 = \frac{\int_{0}^{\infty} \left( x(t) - \hat{x}(t) \right)^2 dt}{\int_{0}^{\infty} \left( x(t) \right)^2 dt}. \] (22)

The result of the inverse Laplace transformation (11) and (12) was found symbolically for the current problems, where the initial and reduced order models are linear. Due to the decomposition of fraction (3) each LTI 2nd order model belongs to one of the following solution classes:

- \( b_0^2 - 4 \cdot b_1 = 0, b_0 \neq 0; \)
- \( b_0^2 - 4 \cdot b_1 = 0, b_0 = 0; \)
- \( b_0^2 - 4 \cdot b_1 < 0, a_0 = a_1 \cdot b_0 \neq 0; \)
- \( b_0^2 - 4 \cdot b_1 < 0, a_0 = a_1 \cdot b_0 = 0; \)
- \( b_0^2 - 4 \cdot b_1 > 0, b_0 \neq 0; \)
- \( b_0^2 - 4 \cdot b_1 > 0, b_0 = 0, \)

and there is a symbolic expression of the LTI model output, which is the unit-step function \( H(t) \) or impulse function \( \delta(t) \) response, for each solution class. Using the symbolic expression of the response it is possible to calculate criteria (17) and (18) for an alternative and (20), (21) and (22) for the solution found.

4. Performance investigation

The first order reduction problem we consider is related to the SISO LTI system model

\[ G(s) = \frac{s^3 + 7 \cdot s^2 + 24 \cdot s + 24}{s^3 + 10 \cdot s^2 + 35 \cdot s^2 + 50 \cdot s + 24} \] (23)
which is a 4\textsuperscript{th} order linear differential equation given, for example, in [6]. The linear dynamic model of the 2\textsuperscript{nd} order, (9), is to be found as a solution of the extremum problem (17) and $u(t) = H(t)$. The transformation in (1) was used to calculate the fitness of an individual; the settings of COBRA are given above. After 100 algorithm launches, the following solution was found:

\[ G_m^*(s) = \frac{0.761968643 \cdot s + 1.73503772}{s^2 + 2.639490066 \cdot s + 1.73503772} \]  \hspace{1cm} (24)

In Figure 1 the higher order model response and the lower order model response are given. The best solution found has a fitness value of 0.99914635, the worst solution fitness value is 0.99157284, the mean fitness value of the solution found is 0.99856314 and the variance of the fitness function value is 0.00102815. The values of criteria (20), (21) and (22) are given in Table 1, where the proposed approach is compared to [6], [5] and [3].

![Figure 1](image-url)

**Figure 1.** The unit-step function response of the reduced order model (24) and the initial model (23).

| Approach          | $I_1$, (19) | $I_2$, (20) | $I_3$, (21) |
|-------------------|------------|------------|------------|
| Proposed          | 7.408 \cdot 10^{-3} | 7.458 \cdot 10^{-3} | 6.09 \cdot 10^{-3} |
| Narval et al.     | 9.98 \cdot 10^{-3} | 1.6 \cdot 10^{-4} | 8.52 \cdot 10^{-4} |
| Desai et al.      | 2.178 \cdot 10^{-4} | 2.01 \cdot 10^{-4} | 4.3 \cdot 10^{-5} |
| Parmar et al.     | 4.192 \cdot 10^{-4} | 4.19 \cdot 10^{-4} | 1.85 \cdot 10^{-2} |

As can be seen, the proposed approach outperforms the approaches presented in Table 1, but not in terms of criterion $I_3$ values.

The bode diagram shows the frequency response of the initial and the reduced model and is given in Figure 2 and Figure 3 for the amplitude and the phase, respectively.
The unit-step function response magnitude of the reduced order model (24) and the initial model (23).

The unit-step function phase response of the reduced order model (24) and the initial model (23).

The same investigation was performed for the MIMO LTI system model, which is determined by the following equation:

\[
H(s) = \begin{pmatrix}
\frac{2 \cdot (s + 5)}{(s + 1) \cdot (s + 10)} & \frac{s + 4}{(s + 2) \cdot (s + 5)} \\
\frac{s + 10}{(s + 1) \cdot (s + 20)} & \frac{s + 6}{(s + 2) \cdot (s + 3)}
\end{pmatrix}.
\]

(25)

It must be emphasized that there is a limited number of decimal places, but the accuracy can be increased by calculating the 2nd element in total for each nominator using formula (8) and knowing that \(a_1 = 1, a_1' = 0.4, a_2 = 0.5\) and \(a_2' = 1\).

As can be seen, the form of its representation in (7) would give us a denominator of the 6th order and nomina tors of the 5th order. The second order model, (10), is to be identified. Extremum problem (18) for \(u(t) = H(t)\) was solved in a similar way 100 times. The responses are given in Figure 4, Figure 5, Figure 6 and Figure 7 for (1,1) matrix (6) component, (1,2), (2,1) and (2,2), respectively. The best solution found has a fitness value of 0.98445642, the worst solution fitness value is 0.98052264, the mean fitness value of the solution found is 0.98427018 and the variance of the fitness function value is 0.00045411. The values of criterion (20) for the proposed approach and the other approaches are given in Table 2 for different equations matrix components. In Table 2 the proposed approach was compared with [6], [5], [12] and [13]. The same was performed for criteria (21) and (22), Table 3 and Table 4, respectively. The following solution in the form of (10) was found:

\[
G^*_m(s) = \frac{1}{D^*_m} \begin{pmatrix}
N^{*1,1}_m & N^{*1,2}_m \\
N^{*2,1}_m & N^{*2,2}_m
\end{pmatrix},
\]

\[
D^*_m(s) = s^2 + 2.3579688 \cdot s + 1.2967903,
\]

\[
N^{*1,1}_m(s) = 1.1830408 \cdot s + 1.2967903,
\]

\[
N^{*1,2}_m(s) = 0.7282982 \cdot s + 0.51871609,
\]

\[
N^{*2,1}_m(s) = 0.5561906 \cdot s + 0.6483951,
\]

\[
N^{*2,2}_m(s) = 1.4883673 \cdot s + 1.2967903.
\]

(26)
The results in Table 2 show that the proposed approach is an efficient tool for solving the reduced order model parameter identification problem. The sum of the error values, which is the sum of the values in a row, for the proposed approach is less than the sum of the values for any other approach in Table 2.
Table 3. Criterion (21) values for different approaches.

| System component | Approach     | (1,1)    | (1,2)    | (2,1)    | (2,2)    |
|------------------|-------------|----------|----------|----------|----------|
| Proposed         | 2.082 · 10^{-3} | 2.1 · 10^{-2} | 5.537 · 10^{-2} | 4.136 · 10^{-3} |
| Narval et al.    | 1.08 · 10^{-2} | 3.68 · 10^{-2} | 8.7 · 10^{-2} | 1.65 · 10^{-2} |
| Desai et al.     | 3.4 · 10^{-2} | 2.17 · 10^{-2} | 9.5 · 10^{-3} | 2.47 · 10^{-2} |

The results in Table 3 confirm the efficiency of the proposed approach and that it outperforms the other approaches, since it gives more accurate models; the criterion (21) values provide both higher and lower order model response closure for all the MIMO LTI model components.

Table 4. Criterion (22) values for different approaches.

| System component | Approach     | (1,1)    | (1,2)    | (2,1)    | (2,2)    |
|------------------|-------------|----------|----------|----------|----------|
| Proposed         | 4.3 · 10^{-2} | 3.4 · 10^{-2} | 3.1 · 10^{-2} | 2.9 · 10^{-2} |
| Narval et al.    | 1.72 · 10^{-2} | 3.62 · 10^{-2} | 1.62 · 10^{-2} | 2.3 · 10^{-3} |
| Desai et al.     | 2.49 · 10^{-2} | 4.87 · 10^{-2} | 2.43 · 10^{-2} | 3.7 · 10^{-3} |

The proposed approach improves the solution properties: integral square error, (20), and relative integral square error, (21), but it does not outperform the other approaches in terms of criterion (22) values. The reason for this is the problem statement, which provides the following aim: to receive the model that has a response on the unit-step function that is maximally close to the initial model response on that function. In reality, a different input function corresponds to a different extremum problem with its extremum points. Nevertheless, the proposed approach criterion (22) values are close to the values for the other approaches.

The bode diagram shows the difference between the initial and the reduced model in the frequency domain and is given in Figures 8-9 for the (1,1) MIMO system component, Figures 10-11 for the (1,2) component, Figures 12-13 for the (2,1) component and Figures 14-15 for the (2,2) component.

![Figure 8](image1.png)  
**Figure 8.** The unit-step function response magnitude of the reduced order model (26) and the initial model (25), (1,1)-component.

![Figure 9](image2.png)  
**Figure 9.** The unit-step function phase response of the reduced order model (26) and the initial model (25), (1,1)-component.
5. Conclusion
The order reduction problem for the LTI, SISO and MIMO systems was successfully solved in this study by applying COBRA to the extremum problem, based on asymptotic stability and its equivalence to the higher order LTI system model. The proposed approach significantly outperformed other approaches in terms of the relative integral square error value for the unit-step function response. Also the integral square error was the least for the first problem and the sum of the integral square
errors for the second problem was the least, compared to other approaches. By this we can conclude that COBRA and the reduced order model determination improved the solutions found earlier. The determination rule allows a stable LTI system to be achieved, whose asymptote is equal to the desired value.

The COBRA statistics prove that it is a reliable technique, since the estimation of the integral square error is sufficiently low, which means that the expected value for the fitness function is close to its maximum value, so also to its worst value. Moreover, it can be seen that fitness function variance is low for both problems.

Since the 2nd order LTI system model response can be transformed into a symbolic form, the future work is related to implementing integral square error directly to the fitness function. It would solve a problem by estimating the main criteria with its numerical analogue: different estimations often give different results.

Another important task is to use as a main criterion the distance between responses of the higher and lower orders systems on the impulse function. This is necessary to compare the solutions and their properties in the case of a different main criterion.

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