Flux Transport Dynamo: From Modelling Irregularities to Making Predictions

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Abstract

The flux transport dynamo, in which the poloidal magnetic field is generated by the Babcock–Leighton mechanism and the meridional circulation plays a crucial role, has emerged as an attractive model for the solar cycle. Based on theoretical calculations done with this model, we argue that the fluctuations in the Babcock–Leighton mechanism and the fluctuations in the meridional circulation are the most likely causes of the irregularities of the solar cycle. With our increased theoretical understanding of how these irregularities arise, it can be possible to predict a future solar cycle by feeding the appropriate observational data in a theoretical dynamo model.

Keywords: dynamo, solar cycle

1. Introduction

The flux transport dynamo model, which started being developed about a quarter century ago (Wang et al., 1991; Choudhuri et al., 1995; Durney, 1995), has emerged as an attractive theoretical model for the solar cycle. There are several modern reviews (Choudhuri, 2011, 2014; Charbonneau, 2014; Karak et al., 2014a) surveying the current status of the field. The present paper is not a comprehensive review, but is based on a talk in a Workshop at the International Space Science Institute (ISSI) highlighting the works done by the author and his coworkers. Readers are assumed to be familiar with the phenomenology of the solar cycle and the basic concepts of MHD (such as flux freezing and magnetic buoyancy). Readers not having

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this background are advised to look at the earlier reviews by the author (Choudhuri, 2011, 2014), which were written for wider readership.

The initial effort in this field of flux transport dynamo was directed towards developing periodic models to explain the various periodic aspects of the solar cycle. Once sufficiently sophisticated periodic models were available, the next question was whether these theoretical models can be used to understand how the irregularities of the solar cycle arise. There is also a related question: if we understand what causes the irregularities of the cycle, then will that enable us to predict future cycles?

We discuss the basic periodic model of the flux transport dynamo in the next Section. Then § 3 discusses the possible causes of the irregularities of the solar cycle. Afterwards in § 4 we address the question whether we are now in a position to predict future cycles. Finally, in § 5 we summarize the limitations of the 2D kinematic dynamo models and the recent efforts of going beyond such simple models.

2. The Basic Periodic Model

One completely non-controversial aspect of solar dynamo models is the generation of the toroidal field from the poloidal field by differential rotation. Since differential rotation has now been mapped by helioseismology, this process can now be included in theoretical dynamo models quite realistically. The toroidal field is primarily produced in the tachocline at the bottom of the convection zone and rises from there due to magnetic buoyancy to create the sunspots. Although some authors have argued that the near-surface shear layer discovered by helioseismology can also be important for the generation of the toroidal field (Brandenburg, 2005), the general view is that magnetic buoyancy would limit the growth of magnetic field in this region of strong super-adiabatic gradient. To this generally accepted view that the toroidal field is primarily produced in the tachocline, the flux transport dynamo model adds the following assumptions.

• The generation of the poloidal field from the toroidal field takes place due to the Babcock–Leighton mechanism.

• The meridional circulation of the Sun plays a crucial role in the dynamo process.

We now comment on these two assumptions.
Bipolar sunspots on the solar surface appear with a tilt statistically increasing with latitude, in accordance with the so-called Joy’s law. This tilt is produced by the Coriolis force acting on the rising flux tubes (D’Silva and Choudhuri, 1993). Babcock (1961) and Leighton (1964) suggested that the poloidal field of the Sun is produced from the decay of such tilted bipolar sunspot pairs. There is now enough evidence from observations of the solar surface that the poloidal field does get built up in this way.

The meridional circulation is observed to be poleward at the solar surface and advects the poloidal field generated there, in conformity with observational data of surface magnetic fields. The material which is advected to the poles has to flow back equatorward through deeper layers within the solar convection zone. Since this circulation is driven by the turbulent stresses in the convection zone, we expect the meridional circulation not to penetrate much below the bottom of the convection zone, although a slight penetration helps in explaining several aspects of observational data (Nandy and Choudhuri, 2002; Chakraborty et al., 2009). The early models of the flux transport dynamo assumed the return flow of the meridional circulation to take place at the bottom of the convection zone, where the toroidal field generated by the differential rotation is advected equatorward with this flow, giving a natural explanation of the butterfly diagram representing the equatorward shift of the sunspot belt (Choudhuri et al., 1995). Such dynamo models have been remarkably successful in explaining many aspects of the observational data (Chatterjee et al., 2004).

While we still do not have unambiguous measurements of the return flow of the meridional circulation, some groups claim to have found evidence for the return flow well above the bottom of the convection zone (Hathaway, 2012; Zhao et al., 2013; Schad et al., 2013). However, Rajaguru and Antia (2015) argue that the available helioseismology data still cannot rule out a one-cell meridional circulation spanning the whole of the convection zone in each hemisphere. Hazra et al. (2014a) showed that, even with a meridional circulation much more complicated than the one-cell pattern assumed in the earlier flux transport dynamo papers, it is still possible to match the relevant observational data as long as there is an equatorward flow at the bottom of the convection zone (see Figure 1).
3. The Origin of the Irregularities of the Solar Cycle

The earliest attempts of explaining irregularities of the solar cycle were by regarding them as nonlinear chaos arising out of the nonlinearities of the dynamo equations ([Weiss et al., 1984]). Charbonneau et al. (2007) argued that the Gnevyshev-Ohl rule in solar cycles (i.e. the tendency of alternate cycles to lie above and below the running mean of cycle amplitudes) arises out of period doubling due to nonlinearities. However, the simplest kinds of nonlinearities expected in dynamo equations tend to make the cycles more stable rather than producing irregularities and it has been suggested that stochastic fluctuations are more likely to be the primary reason behind producing irregularities (Choudhuri, 1992).

The Babcock–Leighton mechanism for the generation of the poloidal field depends on the tilts of bipolar sunspots. While the average tilt is given by Joy’s law, we see considerable scatter around this average tilt, presumably caused by the action of turbulence in the convection zone on the rising flux tubes (Longcope and Choudhuri, 2002). This scatter in the tilt angles is expected to introduce fluctuations in the Babcock–Leighton mechanism (Choudhuri et al., 2007). By including this fluctuation in the dynamo mod-
els, it is possible to explain many aspects of the irregularities of the cycles including the grand minima (Choudhuri and Karak, 2009).

One other source of irregularities is the fluctuations in the meridional circulation. A faster meridional circulation will make the solar cycles shorter and vice versa. While we have actual data of meridional circulation variations for not more than a couple of decades, we have data for durations of solar cycles for more than a century, providing indications that the meridional circulation had fluctuations in the past with correlation times of the order of 30–40 years (Karak and Choudhuri, 2011). When the meridional circulation is slow and the cycles longer, diffusion has more time to act on the magnetic fields, making the cycles weaker. On such ground, we expect longer cycles to be weaker and shorter cycles to be stronger, leading to what is called the Waldmeier effect (Karak and Choudhuri, 2011). Also, when the meridional circulation is sufficiently weak, theoretical dynamo models show that even grand minima can be induced (Karak, 2010). To get these results, the correlation time of the meridional circulation fluctuations was taken to be considerably longer than the cycle period. If the correlation time is taken too short, then one may not get these results (Muñoz-Jaramillo et al., 2010).

We also emphasize that the effect of diffusion in making longer cycles weaker is vital for getting these results. We need to take the value of diffusivity sufficiently high such that the diffusion time scale is shorter than or of the order of the cycle period. This is not the case in the model of Dikpati and Gilman (2006) in which diffusivity is very low. A longer cycle in such a low-diffusivity model tends to be stronger because differential rotation has time to generate more toroidal field during a cycle, giving the opposite of the Waldmeier effect. Differences between high- and low-diffusivity dynamos were studied by Yeates et al. (2008). Clearly the high-diffusivity model yields results more in conformity with observational data.

By analyzing the contents of C-14 in old tree trunks and Be-10 in polar ice cores, it has now been possible to reconstruct the history of solar activity over a few millenia (Usoskin, 2013). It has been estimated that there have been about 27 grand minima in the last 11,000 years (Usoskin et al., 2007). Since grand minima can be caused both by fluctuations in the Babcock-Leighton mechanism and fluctuations in the meridional circulation, a full theoretical model of grand minima should combine both types of fluctuations. If, at the beginning of a cycle, the poloidal field is too weak due to the fluctuations in the Babcock–Leighton mechanism or the meridional circulation is too weak, then the Sun may be forced into a grand minimum. Assuming a Gaussian
distribution for fluctuations in both the Babcock–Leighton mechanism and the meridional circulation, Choudhuri and Karak (2012) developed a comprehensive theory of grand minima that agreed remarkably well with the statistical data of grand minima (see Figure 2 and its caption). However, if there are no sunspots during grand minima, then the Babcock–Leighton mechanism which depends on sunspots may not be operational and how the Sun comes out of the grand minima is still rather poorly understood (Karak and Choudhuri, 2013; Hazra et al., 2014b).

While discussing irregularities of the solar cycle, it may be mentioned that these irregularities are correlated reasonably well in the two hemispheres of the Sun. Strong cycles are usually strong in both the hemispheres and weak cycles are weak in both. This requires a coupling between the two hemispheres, implying that the turbulent diffusion time over the convection zone could not be more than a few years (Chatterjee and Choudhuri, 2006; Goel and Choudhuri, 2009).

Figure 2: According to the calculations of Choudhuri and Karak (2012), the poloidal field strength (γ is the value of the poloidal field compared to its average value) and the amplitude of the meridional circulation at the surface have to lie in the shaded region at the beginning of a cycle in order to force the dynamo into a grand minimum. They estimated the probability of this to be about 1.3%, corresponding to about 13 grand minima in 11,000 years.
4. Predicting solar cycles

The first attempts of predicting solar cycles were based on using observational precursors of solar cycles. There is considerable evidence that the polar field at the end of a solar cycle is correlated with the next cycle. Since the polar field at the end of cycle 23 was rather weak, several authors (Svalgaard et al., 2005; Schatten, 2005) predicted that the next cycle 24 would be weak.

The sunspot minimum between the cycles 23 and 24 (around 2005–2008) was the first sunspot minimum when sufficiently sophisticated models of the flux transport dynamo were available. Whether these models could be used to predict the next cycle became an important question. When a kinematic mean field dynamo code is run without introducing any fluctuations, one finds that the code settles down to a periodic solution if the various dynamo parameters are in the correct range. In order to model actual solar cycles, one has to feed some observational data to the theoretical model in an appropriate manner and then run the code for a future cycle to generate a prediction. The crucial issue here is to figure out what kind of observational data to feed into the theoretical model and how. An understanding of what causes the irregularities of the solar cycle is of utmost interest in deciding this. An attempt by Dikpati and Gilman (2006) produced the prediction that the cycle 24 would be very strong, in contradiction to what was predicted on the basis of the weak polar field at the end of the cycle 23.

Assuming that the fluctuation in the Babcock–Leighton mechanism is the main cause of irregularities in the solar cycle, Choudhuri et al. (2007) devised a methodology of feeding observational data of the polar magnetic field into the theoretical model to account for the random kick received by the dynamo due to fluctuations in the Babcock–Leighton mechanism. The dynamo model of Choudhuri et al. (2007) predicted that the cycle 24 would be weak, in conformity with the weakness of the polar field at the end of cycle 23. Jiang et al. (2007) explained the physical basis of what causes the correlation between the polar field at the end of a cycle and the strength of the next cycle. Suppose the fluctuations in the Babcock–Leighton mechanism produced a poloidal field stronger than the usual. This strong poloidal field will be advected to the poles to produce a strong polar field at the end of the cycle and, if the turbulent diffusion time across the convection zone is not more than a few years, this poloidal field will also diffuse to the bottom of the convection zone to provide a strong seed for the next cycle, making
Figure 3: The sunspot number in the last few years. The upper star indicates the predicted amplitude of cycle 24 according to Dikpati and Gilman (2006), while the lower star indicates the predicted amplitude according to Choudhuri et al. (2007). The circle on the horizontal axis indicates the time when these predictions were made.

the next cycle strong. On the other hand, if the poloidal field produced in a cycle is weaker than the average, then we shall get a weak polar field at the end of the cycle and a weak subsequent cycle. This will give rise to a correlation between the polar field at the end of a cycle and the strength of the next cycle. If the diffusion is assumed to be weak—as in the model of Dikpati and Gilman (2006)—then different regions of the convection zone may not be able to communicate through diffusion in a few years and we shall not get this correlation. The prediction of Choudhuri et al. (2007) that the cycle 24 would be weak was a robust prediction in their model because the polar field at the end of a cycle is correlated to the next cycle in their model and they had fed the data of the weak polar field at the end of cycle 23 into their theoretical model in order to generate their prediction. As can be seen in Figure 3, the actual amplitude of cycle 24 turned out to be very close to what was predicted by Choudhuri et al. (2007), making this to be the first successful prediction of a solar cycle from a theoretical dynamo model in the history of our subject.

As we have pointed out in the previous Section, the fluctuations of the meridional circulation also can cause irregularities in the solar cycle. This
was not realized when the various predictions for cycle 24 were made during 2005–2007. It is observationally found that there is a correlation between the decay rate of a cycle and the strength of the next cycle (Hazra et al., 2015). Now, a faster meridional circulation, which would make a cycle shorter, surely will make the decay rate faster and also the cycle stronger, as pointed out already (a slower meridional circulation would do the opposite). If the effect of the fluctuating meridional circulation on the decay rate is immediate, but on the cycle strength is delayed by a few years, then we can explain the observed correlation. This is confirmed by theoretical dynamo calculations (Hazra et al., 2015). This shows that it may be possible to use the decay rate at the end of a cycle to predict the effect of the fluctuating meridional circulation on the next cycle. This issue needs to be looked at carefully.

5. Conclusion

We have pointed out that over the years we have acquired an understanding of how the irregularities of the solar cycle arise and that this understanding helps us in predicting future solar activity. Our point of view is that the fluctuations in the Babcock–Leighton mechanism and the fluctuations in the meridional circulation are the two primary sources of irregularities in the solar cycles. These fluctuations have to be modelled realistically and fed into a theoretical dynamo model to generate predictions.

It may be noted that we now have a huge amount of data on the magnetic activity of solar-like stars (Choudhuri, 2017). Some solar-like stars display grand minima and we have evidence for the Waldmeier effect in some of them—see the concluding paragraph of Karak et al. (2014b). This suggests that dynamos similar to the solar dynamo may be operational in the interiors of solar-type stars and the irregularities of their cycles also may be produced the same way as the irregularities of solar cycles. Work on constructing flux transport dynamo models for solar-like stars has just begun (Karak et al., 2014b). Our ability to model stellar dynamos may throw more light on how the solar dynamo works.

All the theoretical results we discussed are based on axisymmetric 2D kinematic dynamo models. One inherent limitation of such models is that the rise of a magnetic loop due to magnetic buoyancy and the Babcock–Leighton process of generating poloidal flux from it are intrinsically 3D processes and can be included in 2D models only through very crude averaging procedures (Nandy and Choudhuri, 2001; Muñoz-Jaramillo et al., 2010).
Figure 4: A study of magnetic field evolution on the solar surface from the 3D kinematic dynamo model of Hazra et al. (2017), showing how the polar field builds up from a single tilted bipolar sunspot pair due to the Babcock–Leighton mechanism. The different panels show the distribution of magnetic field at the following epochs after the emergence of the bipolar sunspots: (a) 0.025 yr, (b) 0.25 yr, (c) 1.02 yr, (d) 2.03 yr, (e) 3.05 yr, (f) 4.06 yr.

As we have discussed, the fluctuations in the Babcock–Leighton process play a crucial role in producing the irregularities of the solar cycle. In order to model these fluctuations realistically, it is essential to treat the Babcock–Leighton process itself more realistically than what is possible in 2D models. The next step should be the construction of 3D kinematic dynamo models for which efforts have begun (Yeates and Muñoz-Jaramillo, 2013; Miesch and Dikpati, 2014; Miesch and Teweldebirhan, 2015; Hazra et al., 2017). Such 3D kinematic dynamo models can treat the Babcock–Leighton mechanism more realistically (see Figure 4) and should provide a better understanding of how fluctuations in the Babcock–Leighton mechanism cause irregularities in the dynamo. The magnetic field presumably exists in the form of flux tubes throughout the convection zone and one limitation of a mean field model is that flux tubes are not handled properly (Choudhuri, 2003). A 3D kinematic model allows one to model flux tubes in a more realistic fashion. A proper inclusion of flux tubes in a dynamo model is essential for explaining such interesting observations as the predominance of negative helicity in the northern hemisphere (Pevtsov et al., 1995), which is presumably caused by the wrapping of the poloidal field around the rising flux tubes (Choudhuri, 2003; Chatterjee et al., 2006; Hotta and Yokoyama, 2012). This process can be modelled in 2D mean field dynamo models only through drastic simplifications (Choudhuri et al., 2004). It should be possible to model this better through 3D kinematic dynamo models. In other
words, after the tremendous advances made by the 2D kinematic flux transport model during the last quarter century, it appears that that 3D kinematic dynamo models are likely to occupy the centre stage in the coming years.

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