Stable double-pair skyrmion in an antiferromagnetic $F = 1$ Bose–Einstein condensate

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Keywords: double-pair skyrmion, spinor BEC, spin–orbit coupling

Abstract

The recent experimental realization of spin-orbit coupling (SOC) in spin-1 ultracold atoms opens up an interesting avenue for exploring new quantum states and novel quantum phenomena in large-spin systems. Two types of two-dimensional double-pair vortices that are connected by a domain-wall can be accommodated in the antiferromagnetic spin-1 Bose–Einstein condensate (BEC). One type, named a 2D double-pair skyrmion, which differs from the conventional 2D skyrmion in the $F = 1$ polar BEC, hosts a pair of vortices and displays a meron-pair texture with a unit topological charge. The other type has a meron-pair texture but with null topological charge. These two types of double-pair vortices can be naturally generated from a vortex-free Gaussian wave packet by incorporating a non-Abelian gauge field into the spinor condensate.

1. Introduction

Skyrmions have been studied in a wide variety of systems [1–4]. It has been demonstrated that a two-dimensional (2D) skyrmion spontaneously appears as the ground state in the helical magnets MnSi [5] and in the quantum Hall state [6]. The spinor Bose–Einstein condensates (BECs) have provided an ideal pilot to model and explore the various types of skyrmions, such as the 2D skyrmion and the half-skyrmion [7–13]. Experimental interest in the topological defects and textures in spinor BECs is currently accelerating. The traditional method for creating a topological structure is to employ the rotating magnetic-field technique [14, 15]. After much effort, the 2D skyrmion has been experimentally observed in antiferromagnetic spin-1 BECs [14].

Thus far, the 2D skyrmion has been widely investigated in two-component BECs [8, 16]. Motivated by the creation of the 2D skyrmion in the spin-1 polar BEC, theoretical works regarding its dynamics have also been studied [17–19]. It has been shown that skyrmion excitations are energetically unstable and will decay by expanding or shrinking in the antiferromagnetic spin-1 BEC [20]. Therefore it is natural to ask whether some additional stabilizing mechanisms can be adopted to search for stable skyrmions in spinor BECs. In the past few years, spin–orbit coupling (SOC) effects in two-component BECs have attracted significant attention due to the interplay between SOC and the unique properties of dilute atomic gases [21–23]. The recent experimental realization of SOC in the spin-1 BEC [24] may help stabilize the skyrmion and other exotic spin textures [25–27]. The aim of the present paper is to study new spin textures and stabilize them with SOC.

The mean-field OP of the $F = 1$ condensate is described by a wavefunction $\psi(r) = \sqrt{n(r)} \xi(r)$, where $n(r)$ is the density of the condensates and $\xi(r) = (\xi_+^r(r), \xi_0^r(r), \xi_-^r(r))^T$ is a normalized spinor [28]. The topological excitations in the spinor condensate are classified by the homotopy classes of the OP space [29]. This examination can be carried out with the help of the homotopy group theory. For example, the 2D skyrmion is classified by the second homotopy group $\pi_2(M)$, where $M$ is the order parameter (OP) manifold [30]. The OP of a spin-1 polar BEC is $M = (U(1) \times S^2)/Z_2$, which has the nontrivial second homotopy group $\pi_2(M) = Z$. The polar state $|F\rangle = 0$ reflects the $SO(2)$ symmetry around the quantization axis $\hat{d}$, which reads $\xi = \left(\frac{d_+ + id_0}{\sqrt{2}}, \frac{d_0}{\sqrt{2}}, \frac{d_- + id_0}{\sqrt{2}}\right)^T$ [20, 28]. Unlike the skyrmion in the two-component BEC which is displayed by the

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spin vector $\mathbf{S}$, the spin texture in the spin-1 polar BEC has a topological charge defined by

$$q = \frac{i}{4\pi} \mathbf{d} \cdot (\partial_j \mathbf{d} \times \partial_d \mathbf{d})$$

[14].

We consider a spin-1 boson system confined in a harmonic trap with a non-Abelian gauge field. The model Hamiltonian is given by $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}$, with

$$\mathcal{H}_0 = \int \! d\mathbf{r} \left[ \frac{\hbar^2 \mathbf{k}^2}{2m} + V \right] \psi$$

$$\mathcal{H}_{\text{int}} = \int \! d\mathbf{r} \left[ \frac{c_0 n^2}{2} + \frac{c_1}{2} |\mathbf{F}|^2 \right],$$

where $n(r) = \sum_{m=-1}^{1} |\psi_m(r)|^2$ is the total density of the condensate, $V$ is the harmonic trap, $A$ is the non-Abelian gauge field and $F = \sum_{m=-1}^{1} \psi_m^*(r) \tilde{\mathbf{f}}_m \psi_m(r)$ is the spin-polarization vector. The spin operator $\tilde{\mathbf{f}}$ is given by a vector of spin-1 matrices. The interaction term $\mathcal{H}_{\text{int}}$ contains the spin-independent and $|\mathbf{F}|$-dependent interactions. The interaction strengths are given by the $s$-wave scattering lengths $a_s$ in the spin-$f$ channels of colliding spin-1 atoms as $c_0 = (g_s' + 2g_s)/3$ and $c_2 = (g_z - g_y)/3$, with $g_s' = 4\pi \hbar^2 a_s / M$. The system becomes ferromagnetic (FM) when the spin–spin interaction $c_2 < 0$ and polar for $c_2 > 0$. A general FM spinor wavefunction can be constructed from the representative spinor $\zeta = (1, 0, 0)^T$ by imprinting a phase $\phi$ and making a spin rotation for the macroscopic condensate. On the other hand, as the spin–spin interaction $c_2 > 0$, we may construct the general spinor wavefunction of the polar phase by applying a spin rotation and global condensate phase to the representative spinor $\zeta = (0, 1, 0)^T$ [28].

In this paper, we investigate two types of double-pair vortices in the antiferromagnetic spin-1 BEC. To our knowledge, these are new topological structures that have not been studied in spin-1 BEC before. The double-pair vortices (they form a pair of vortices in both $\psi_1$ and $\psi_{-1}$) are connected by a domain wall of the phase difference between the hyperfine states. One type of the double-pair vortices is a double-pair skyrmion. It forms a 1/2 and 1/2 meron-pair, where 1/2 is the topological number of each half plane and the total topological charge is 1. The other type forms a 1/2 and $-1/2$ meron-pair, and the total topological charge is 0. We demonstrate that the two types of double-pair vortices can be naturally generated from a vortex-free Gaussian wave packet by incorporating a non-Abelian gauge field or Rashba-type SOC in the spin-1 BEC. It provides a perspective for studying the topological object and observing the dynamics in the spin-1 BEC.

The paper is organized as follows. In section 2 we introduce two types of double-pair vortices, a 2D double-pair skyrmion with a unit topological charge and a meron-pair with a null topological charge. In section 3, we create a stable 2D double-pair skyrmion from a vortex-free Gaussian wave packet by incorporating a non-Abelian gauge field into the condensates, and we analyze the skyrmion that emerges with the concept of helical modulation. In section 4, another double-pair vortex is created in the spin-1 BEC with Rashba-type SOC. A brief summary is included in section 5.

2. Double-pair vortices in the polar BEC

2.1. Double-pair skyrmion

We briefly review the 2D meron-pair (bimeron) in a two-component BEC [31–33]. For atoms with two hyperfine states such as $^{87}\text{Rb}$, the two condensates can be coherently coupled through a Rabi field. The 2D bimeron, which differs from the 2D skyrmion, is stabilized by Rabi coupling [32]. The cores of the two vortices are connected by a domain wall of the relative phase to form a vortex-molecule. Essentially, the coherent Rabi coupling plays the role of a transverse magnetic field that aligns the spin along the $x$-axis, leading to $\xi \to (1, 1)$ at a distance far from the core region. For the spin-1 polar BEC, two types of meron-pairs exist. One is called a double-pair skyrmion with a unit topological charge whereas the other is a meron-pair but with a null topological charge.

The OP for the 2D double-pair skyrmion can be expressed as

$$\left( \begin{array}{c} \psi_1(r) \\ \psi_0(r) \\ \psi_{-1}(r) \end{array} \right) = \sqrt{n(r)} \hat{U}^\dagger(r) \left( \begin{array}{c} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{array} \right),$$

where $\hat{U}(r) = \exp[-i\Lambda(r) (f_x \cdot n_x + f_y \cdot n_y)]$. It is illustrative to study the topological structure with the radial symmetric ansatz by assuming $\Lambda(0) = \pi$ and $\Lambda(\infty) = 0$ with $n = (n_x, n_y, 0) = (\cos \theta, \sin \theta, 0)$, which defines a map from the physical region $R^2$ to the OP space. The conventional 2D skyrmion OP in the spin-1 polar BEC is described by $\hat{U}(x, y, z) = e^{-i\phi} \psi_1(\mathbf{r}) = e^{-i\phi} \psi_{-1}(\mathbf{r})$ acting on the representative spinor $\zeta = (0, 1, 0)^T$. Therefore we can obtain the relation between $\psi_0$ of the double-pair skyrmion with the $\psi_1$ and $\psi_{-1}$ of the 2D skyrmion, $\psi_0^{\text{double-pair}} = -\frac{1}{\sqrt{2}} (\psi_1^{\text{skyrmion}} - \psi_{-1}^{\text{skyrmion}})$.
Figure 1 shows the density distributions of the ansatz wave function (3) with a uniform density profile. In contrast to the conventional 2D skyrmion, $|\psi_1|^2$ and $|\psi_-|^2$ is not concentrated in a toroidal region. We note that $|\psi_1|^2$ and $|\psi_-|^2$ vanish at an off-centered location, at which the $|\psi_0|^2$ reaches its maximum. The two points of the vanishing density are located at symmetric positions $x = \pm m$. The OP for the 2D skyrmion texture in a polar BEC is described by the spin rotation acting on the representative normalized spinor $z = (0, 1, 0)^T$. Therefore $\psi_0$ is in the central and the outermost region to fulfill the OP manifold of $S^2$. The 2D double-pair skyrmion forms another situation for the central and the outermost region filled by the $\psi_1$ and $\psi_-$. In other words, the representative spinor for this structure is $z = (-1/\sqrt{2}, 0, -1/\sqrt{2})^T$. This indicates that the different boundary condition is important for the topological structure.

In addition, the $\psi_1$ phase exhibits two clear $2\pi$ jumps along the two segments $[-\infty, -m]$ and $[m, \infty]$ on the $x$-axis. The phase is undefined at these two points and they correspond to a vortex–antivortex pair. In $\psi_-$, the phase jump occurs when one crosses the line connecting the vortex centers. It corresponds to a vortex–antivortex pair while connected by a domain wall $[-m, m]$.

The d texture is shown in figure 2(a). In the plane, all arrows point to $(-1,0,0)$ at a larger radius, whereas there is a pair of vortex cores in the plane which is highlighted by $d_z = 1$ (red) and $d_z = -1$ (blue). In comparison to the 2D skyrmion, $d_z = -1$ at the boundary instead of $d_z = -1$.

A distinguishing feature of the 2D double-pair skyrmion that differs from the 2D skyrmion is the existence of the domain wall connecting the vortex-pair. Each half plane forms a half-skyrmion and constitutes a meron-pair. It is also distinct from the case of the meron-pair in the two-component BEC [32], where each component has only one vortex and the pseudospin $S$ orientations depend on the relative phase, which can be represented by an azimuthal angle $\arctan(S_y/S_x) = \theta_2 - \theta_1$. Here $\theta_1$ is the phase of the wave function $\psi_0$. The domain wall is the relative phase with a $2\pi$ difference. In our case $F = 0$, the spin texture is the d texture instead of the $F$ texture.
The phase of $\psi_{-1}$ in the spin-1 polar BEC is given by $\arg(\psi_{-1}) = \arctan(d_y/d_x)$, which determines the phase rotation in the horizontal plane of the spin texture [34].

2.2. Meron-pair with 0 charge
We can get an alternative meron-pair structure through the map $\hat{U}(r) = \exp[-i\lambda(r)(f_x \cdot n_x + f_y \cdot n_y)]$. As shown in figure 3, the density profile is the same as the 2D double-pair skyrmion, which also hosts double-pair vortices. But the difference is reflected in the phase distribution. By comparing figure 3(d) with figure 1(d), we note that both have two vortices. But in figure 3(d) the total phase winding along a loop encircling the two vortices is $4\pi$, differing to figure 1(d) where the phase winding is zero. The d texture and the corresponding distribution of the topological charge density is shown in figure 4. The d texture is different from the d texture of the double-pair skyrmion where it forms a half-skyrmion and an anti-half-skyrmion. The boundary is not fixed and the distribution of the topological charge is odd with respect to the x coordinate, indicating the creation of a vortex–antivortex pair with a vanishing total topological charge $Q = \int dq(r) = 0$. For the double-pair skyrmion, it forms a $1/2$ and $1/2$ meron-pair, where $1/2$ is the topological number of each half plane and the total topological charge is 1.

3. Double-pair skyrmion with a non-Abelian gauge field
In the literature on skyrmions, the stability and existence of a solution are usually taken to mean the energetic stability [35]. The 2D skyrmion is usually energetically unstable to expand to infinite size or shrink to infinitesimal size, resulting in a uniform spin texture in the spin-1 polar BEC [20]. A similar situation occurs with the 2D double-pair skyrmion in the spin-1 polar BEC, namely, the 2D double-pair skyrmion will decay in the presence of weak dissipations. To obtain a stable double-pair skyrmion, we incorporate a synthetic non-Abelian gauge potential into the condensates. The model Hamiltonian confined in a harmonic trap with a non-Abelian gauge field is given by
\begin{equation}
\mathcal{H} = \int \! \! \mathrm{d} \mathbf{r} \left( \Psi^\dagger \left[ \frac{1}{2m} \left( \mathbf{k}^2 + 2\gamma (k_x f_x + k_y f_y) \right) + V \right] \Psi + \left( \frac{c_0}{2} n^2 + \frac{c_1}{2} |\mathbf{F}|^2 \right) \right),
\end{equation}

where $\gamma$ characterizes the strength of SOC and $\hat{\mathbf{F}}$ is the spin-1 matrix.

For the SOC BEC, the Hamiltonian $\mathcal{H}$ is invariant under the simultaneous SO(2) global spin and space (spin-space) rotations. The concept of the helical modulation of the OP [36] provides a good method for understanding the stable skyrmions under the non-Abelian gauge field. Next, we demonstrate that the helical modulation of the OP due to the non-Abelian gauge field makes the double-pair skyrmion stable. Based on the theory in [36], the OP under a non-Abelian gauge field can be written with the rotation matrix

\[ \hat{V} = \exp(-i\varphi \mathbf{n} \cdot \mathbf{F}) \]

and acts on an arbitrary $\psi(\mathbf{r}_0)$ around $\mathbf{n}$ by the angle $\varphi$. The non-Abelian gauge field favors the situation where the rotation axis $\mathbf{n}$ corresponds to the modulation vector $\mathbf{h}$. For the non-Abelian gauge field in our case, the rotation matrix $\hat{V} = \exp(-i\varphi [f_x \cdot n_x + f_y \cdot n_y])$. From the analysis of the single particle spectrum $E_0$ of ideal Bose gases in the thermodynamic limit, $E_0$ has a minimum line along $\sqrt{k_x^2 + k_y^2}$ as presented in [36]. Therefore, it turns out that the possible stable texture is the helical spin modulation with the vector $h|k_x, k_y, 0\rangle$ in the $x$-$y$ plane. This can result in the double-pair skyrmion texture with the appropriate $\varphi \sim k \cdot \mathbf{r}$ to fulfill the OP manifold according to the wave function (3) of the 2D double-pair skyrmion.

Our numerical simulations confirm that a 2D double-pair skyrmion is indeed created. Figure 5 displays the resultant density profile and phase distribution. It evidently reveals the topological structure of the double-pair vortices, just as in figure 1. This is done by propagating the Gross–Pitaevskii equations derived from equation (4) in imaginary time $[33, 38, 39]$ and taking a vortex-free Gaussian wave packet with a uniform spinor $\zeta = (1/\sqrt{2}, 0, -1/\sqrt{2})^T$ as the initial state. We have chosen the parameters $c_0 = 103$, $c_1 = 103/32.1$, which correspond to the parameters of $^{23}$Na and $\gamma = 1$.

### 4. Double-pair vortices with Rashba coupling

In the previous section, we took the non-Abelian gauge field $A$ as $A_x = f_x$ and $A_y = f_y$. In this section, we adopt the Rashba-type SOC, $A_x = f_x$ and $A_y = f_y$. The Hamiltonian is given by

\begin{equation}
\mathcal{H} = \int \! \! \mathrm{d} \mathbf{r} \left( \Psi^\dagger \left[ \frac{1}{2m} \left( \mathbf{k}^2 + 2\gamma (k_x f_x + k_y f_y) \right) + V \right] \Psi + \left( \frac{c_0}{2} n^2 + \frac{c_1}{2} |\mathbf{F}|^2 \right) \right).
\end{equation}

By propagating the Gross–Pitaevskii equations derived from equation (5) in imaginary time, we obtain a similar structure to the 2D double-pair skyrmion shown in figure 6, where the initial state is a vortex-free Gaussian wave packet with a uniform spinor $\zeta = (1/\sqrt{2}, 0, -1/\sqrt{2})^T$. We have also chosen the parameters $c_0 = 103$, $c_1 = 103/32.1$, which correspond to the parameters of $^{23}$Na and $\gamma = 1$.

The nature of the double-pair vortices of the meron-pair results in a total topological charge 0, introduced in section 2 and shown in figure 3. Like the 2D double-pair skyrmion, this type of meron-pair is also energetically unstable but can be created and stabilized in the spin-1 polar BEC with the Rashba-type SOC. As per the mechanism we adopted for the 2D double-pair skyrmion, the mechanism of the stable meron-pair can also be understood by the concept of helical modulation.
5. Summary

We have studied and proposed methods for creating two types of double-pair vortices in the antiferromagnetic spin-1 BEC. One has a total topological charge of 1, named a 2D double-pair skyrmion, which can be created by incorporating a non-Abelian gauge potential into spin-1 BEC. The other corresponds to a 1/2 and –1/2 meron-pair with a zero topological charge, which can be created by incorporating a Rashba-type SOC into spin-1 BEC. We believe that in the near future, experiments will also be able to produce these novel complicated structures. Finally, we note that similar configurations were investigated in other systems [40, 41].

Acknowledgments

This work is supported by the NSF of China under grant No.11604193, 11374036, and the Natural Science Foundation for Young Scientists of Shanxi Province, China (grant No.201601D202012).

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