Experimental - theoretical study of the stress-strain state of cable-stayed-rod systems of the dome type

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Abstract. The problem of determining forces and displacements in a spatial hinge-rod system with missing links under static loading is considered. Equilibrium equations and geometric relationships are written in matrix form. The calculated equations are not linear. To solve the equations, a technique is proposed that is not associated with assumptions regarding the structure of the system or the nature of the load, based on the fact that the displacements obtained by solving the linear part of the equations can be significantly improved by interpreting the nonlinear terms as a load, under which the system can be considered as linear. To test the theoretical assumptions and preconditions, an experimental study was carried out on a model made at a scale of 1:30 of a natural design. The analysis of the results obtained is presented.

1. Introduction
The cable-stayed-rod system of the dome type [1] is a spatial structure consisting of radially located, bending, rigid ribs and a pre-stressed cable-stayed network, limited by a cable-selection (flexible contour). The design scheme of the cable-stayed-rod system is a hinge-rod model of a variable, in particular instant-rigid type [4].

2. Formulation of the problem
The problem of determining the forces and displacements arising in a spatial hinge-rod system with missing (up to geometrically unchangeable) connections when the acting load changes is considered. The displacements are counted from a certain stable equilibrium state, called the initial one, which is considered given if the load and the corresponding system configuration and the forces in its elements (rods) are known. Any equilibrium state can be taken as the initial one: the state of prestrressing, or loading with a constant load, or one of the limiting states. It is assumed that the relative deformations of the rods are small compared to unity, and for the material of the rods a linear relationship between stresses and deformations is valid. These provisions with a sufficient degree of accuracy reflect the nature of the work of the materials used in cable-stayed structures.

All further reasoning is reduced to a prestressed spatial hinge-rod system, which is understood as a system of material points (hinge nodes), interconnected by linear elastic ties (rods). In order to define a hinge-rod system, it is necessary, first of all, to provide information about the incidence of its nodes and rods. The specified information about the nature of the links reflects the topological structure of the system. A graph is a visual way to describe the structure of a system. The incidence matrix of this graph is called a rectangular table A, which uniquely defines a graph with numbered vertices and edges, and, consequently, the structure of the hinge-rod system [3].
3. General solution construction

The equilibrium of a hinge-rod system of instant rigid type under the action of a load applied to it is described by the system of equations

\[ A \cdot X \cdot A^T \cdot Z = F \]  

(1)

where \( A \) - incidence matrix \([3]\); \( X, \Delta X \) - respectively, the diagonal matrix of linear forces and their increments in the rods of the system; \( Z, \Delta Z, \Delta F \) - three-column matrices, respectively, of the coordinates of the nodes, displacements of the nodes and the disturbing load along the coordinate axes; \( C \) - equilibrium matrix; \( B \) - quasi-diagonal matrix formed by three blocks of the form \( AXA^T \) (the dash denotes the transportation operation, the dash denotes the vector corresponding to the replacement of three-column matrices with a triple-length vector or a diagonal matrix by a vector consisting of elements of the main diagonal).

Moving on to considering the deformed state of the system, we transform equation (1), which describes the initial state, taking into account the change in the configuration and the efforts in the system caused by the increment in the load. Denoting by the symbol \( \Delta \) the increment of the corresponding quantities, we write the equilibrium equation

\[ A(X + \Delta X)A^T(Z + \Delta Z) = F + \Delta F \]  

(2)

Taking into account equation (1), we have

\[ A\Delta XAZ + AXA\Delta Z + A\Delta XA\Delta Z = \Delta F \]  

(3)

Equation (3) is nonlinear, the solution of which requires individual analysis to develop a rational solution method. In this regard, let us consider the question of linearization of equation (3). Since for a given cable-stayed-rod system of the dome type the possible displacements caused by the load are quite small, we can neglect the values of the nonlinear terms in (3), then we obtain a linear equation of the form:

\[ A\Delta XAZ + AXA\Delta Z = \Delta F \]  

(4)

or in a more compact form

\[ C\Delta X + B\Delta Z = \Delta F \]  

(5)

In equations (1), (2), the values of the corresponding quantities \( (X, Z) \) in the initial state of equilibrium under load \( F \). Calculation of the system according to the linear theory in accordance with the above-mentioned assumptions is possible only in the case when the rank of the matrix \( (C, B) \) is equal to the dimension of the load space. The linearization of equation (3) is based on the predominant use of the assumption that the initial form remains unchanged. Accordingly, after multiplying equations (5) by the matrix \( (C_1, D)^T \) (\( D \) - matrix orthogonal to \( C_1 \)) equilibrium conditions are reduced to the equations:

\[ C_1^TCA\Delta X = C_1^T\Delta F \]  

(6)

\[ D^TBA\Delta Z = D^T\Delta F \]  

(7)

where \( C = C_1^T, C_1, T \) - matrices whose rank is equal to the rank of matrix \( C \),

\[ \Delta X_1 = T\Delta X \]  

(8)

\[ D^TC_1 = 0 \]  

(9)

From (6) the solution for \( \Delta X_1 \). In this case, the equilibrium part of the load

\[ \Delta F_1 = C_1(C_1^T\Delta F) \]  

(10)
As a solution to (7), the normal solution of the corresponding underdetermined system of equations

\[ \bar{Z}_2 = D(DBD)^{-1}D^T \Delta \bar{F} \]  

(11)

In this case \( \bar{Z}_2 \) - displacements generated by the nonequilibrium part of the load. Displacements caused by the action of the equilibrium part of the load

\[ \Delta \bar{X} = GC^i \bar{Z}_1 \]  

(12)

where \( G \) - diagonal stiffness matrix of bars.

According to (5) and (8)

\[ \bar{Z}_1 = C_1 (C_1^T C_1)^{-1} (TGT)^T \Delta \bar{X} \]  

(13)

where in

\[ \Delta \bar{X} = GT (TGT)^{-1} \Delta \bar{X}_1 \]  

(14)

The results are proved to be independent of the specific choice of matrices \( C_1, D \). Work of the balanced part of the load \( \bar{F}_1 \) on non-equilibrium movements \( \bar{Z}_2 \) and work of the non-equilibrium part of the load \( (\Delta \bar{F} - \bar{F}_1) \) on equilibrium displacements \( \bar{Z}_1 \) is equal to zero (Betty's theorems). Non-equilibrium movements do not cause changes in linear forces \( \Delta \bar{X} \). A partial refinement of the described linear approach is obtained by taking into account in the matrix B instead \( X, X + \Delta X \), where \( \Delta X \) obtained by (14), which corresponds to the quasilinear approach to solving the problem [2], [5], [7].

Figure 1. General view of the model.

The study of the developed algorithm and program showed the acceptability of the described linear approach for calculating a cable-stayed-rod system of a dome type in a fairly wide range of initial states and disturbing loads. Due to the physical content, the proposed method of bringing the load to equilibrium (MPNR) can be effectively used when choosing the main parameters of cable-stayed rod systems [6], [8]. An experimental study of the cable-stayed rod system was carried out to test the
theoretical assumptions and premises of the MPNR. The overall dimensions of the model, made on a scale of 1:30 natural construction, in the plan were $D = 2.36$ m (figure 1).

The span was modeled together with rigid and support contours. The section of the model was selected in accordance with the similarity theorem, taking into account the physical and mechanical properties, the materials used and the acting loads. The model, representing 8 cyclically symmetric sectors, consisted of a rigid contour, a cable-stayed network limited by a pick-up cable and a supporting contour in the form of tables. The rigid contour was a spatial rod system of meridianally located stiffeners made of channels № 5 connected to the central ring. Between the meridian stiffeners, a cable-stayed network was suspended, consisting of circular and radial cables made of rope wire ($d = 0.8$ mm). The selection rope is made of class A-1 reinforcing steel ($d = 4$ mm, SP 63.13330: M, 2012). The cable-stayed network, consisting of 35 rods, formed 12 internal and 16 contour nodes in each sector (figure 2).

![Figure 2. Plan of the sector of the model.
1 - stiffeners; 2 - rope selection; 3 - ring cables; 4 - radial cables.](image-url)

According to the calculation, a preliminary voltage was created in the network of each sector of the model. After prestressing, the shape of the cable-stayed network surface became close to a hyperbolic paraboloid. The study of the model was carried out on the static effect of the load, the model was loaded according to four schemes of temporary and constant loads. The design of the model allowed only angular load transfer. In this regard, the loading was carried out with concentrated loads suspended from the internal nodes of the cable-stayed set with the help of straps. For all loading schemes, floor loading was carried out up to a load of 24.5 N (five stages of 4.9 N each) in each node, as well as stage-by-stage unloading. To determine the deformations in the rope-selection, resistance strain gauges and an automatic strain gauge AID-1M were used. Displacements of five stiffeners were measured by deflection meters PAO - 6, displacements of nodes with a leveling compensator "KONI - 007", efforts in the cables by a deformer [9, 10]. Tables 1, 2, 3 show the experimental data, as well as the results of
theoretical studies obtained using the developed method of bringing the load to equilibrium (MPNR) and the finite element method (MKE). The load is applied in all sectors.

**Table 1.** Efforts in the middle ring cables (N).

| Load (N) | Node numbers |
|----------|--------------|
|          | 1      | 2      | 3      | 4      |
| Exp     | MPNR  | FEM   | Exp   | MPNR  | FEM   | Exp   | MPNR  | FEM   | Exp   | MPNR  | FEM   |
| 4.9     | 143.4 | 156.4 | 178.8 | 124.5 | 135.9 | 164.2 | 143.3 | 153.0 | 178.7 | 145.4 | 158.1 | 182.7 |
| 9.8     | 188.3 | 205.3 | 239.4 | 134.4 | 146.5 | 167.5 | 149.6 | 160.1 | 193.7 | 190.3 | 209.1 | 247.1 |
| 14.7    | 234.2 | 255.3 | 304.2 | 154.8 | 168.7 | 198.6 | 172.8 | 187.5 | 233.5 | 236.2 | 247.8 | 318.1 |
| 19.6    | 284.8 | 310.4 | 367.9 | 183.3 | 199.8 | 239.7 | 196.0 | 215.6 | 246.9 | 294.8 | 324.2 | 388.0 |
| 24.5    | 302.6 | 329.8 | 395.6 | 198.7 | 216.6 | 267.3 | 215.1 | 236.5 | 279.5 | 303.6 | 330.9 | 372.7 |

**Table 2.** Efforts in the middle radial cables (N).

| Load (N) | Node numbers |
|----------|--------------|
|          | 1      | 2      | 3      | 4      |
| Exp     | MPNR  | FEM   | Exp   | MPNR  | FEM   | Exp   | MPNR  | FEM   | Exp   | MPNR  | FEM   |
| 4.9     | 94.1   | 102.4 | 121.8 | 90.2   | 98.9  | 117.2 | 91.1   | 100.0 | 117.7 | 93.1   | 102.1 | 122.1 |
| 9.8     | 97.2   | 105.7 | 130.1 | 98.0   | 106.5 | 127.4 | 98.0   | 106.8 | 129.8 | 97.6   | 105.1 | 122.9 |
| 1.7     | 100.2  | 109.3 | 132.2 | 100.5  | 108.7 | 130.6 | 102.5  | 115.5 | 133.5 | 107.1  | 117.8 | 139.1 |
| 19.6    | 103.5  | 113.4 | 133.9 | 105.3  | 115.8 | 131.2 | 108.2  | 117.7 | 146.9 | 110.2  | 119.2 | 143.0 |
| 24.5    | 128.8  | 140.8 | 165.6 | 114.7  | 124.2 | 148.1 | 116.7  | 127.5 | 150.8 | 124.4  | 136.9 | 161.2 |

**Table 3.** Moving middle nodes in a circular direction (mm).

| Load (N) | Node numbers |
|----------|--------------|
|          | 1      | 2      | 3      |
| Exp     | MPNR  | FEM   | Exp   | MPNR  | FEM   | Exp   | MPNR  | FEM   |
| 4.9     | 0.151  | 0.164 | 0.196 | 0.181  | 0.199 | 0.238 | 0.161  | 0.177 | 0.209 |
| 9.8     | 0.233  | 0.254 | 0.303 | 0.262  | 0.288 | 0.346 | 0.243  | 0.267 | 0.316 |
| 14.7    | 0.412  | 0.453 | 0.536 | 0.483  | 0.532 | 0.638 | 0.422  | 0.464 | 0.548 |
| 19.6    | 0.643  | 0.702 | 0.836 | 0.653  | 0.718 | 0.862 | 0.664  | 0.731 | 0.863 |
| 24.5    | 0.987  | 1.105 | 1.283 | 1.402  | 1.146 | 1.375 | 0.993  | 1.093 | 1.291 |

Under the action of the temporary load of the 1st and 2nd stages, the movements of the nodes in the unloaded sectors are directed in the opposite direction of the line of action of the load. The forces in the ring cables exceed the forces in the radial cables by almost 2.5 times in the loaded sectors by several times.

**4. Conclusions**

The proposed method (iterative process) converges in a wide range of system parameters, as evidenced by the performed calculations. The convergence has a quadratic character, as a result of which, with the required degree of accuracy, two or three iterations are sufficient for practical calculations. The application of a temporary load to any sector causes a change in forces and displacements in all sectors, which confirms the spatial work of the structure. Under the action of temporary loads in the dome system, the forces are redistributed due to changes in geometry in individual sectors. The efforts in the cable-stayed ones vary according to the parabolic law.

Analysis of the results shows the discrepancy between the experiment and the developed method (MPNR) within 9-12%, comparison of the experiment with the MKE gives a discrepancy of 28-32%.
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