S-Z power spectrum produced by primordial magnetic fields

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ABSTRACT

Primordial magnetic fields generated in the very early universe are one of the candidates for the origin of magnetic fields observed in galaxy clusters. After recombination, the Lorentz force acts on the residual ions and electrons to generate density fluctuations of baryons. Accordingly these fluctuations induce the early formation of dark halos which cause the Sunyaev-Zel’dovich (S-Z) effect in cosmic microwave background radiation. This additional S-Z effect due to primordial magnetic fields amplifies the angular power spectrum of cosmic microwave temperature anisotropies on small scales. This amplification depends on the comoving amplitude and the power law index of the primordial magnetic fields spectrum. Comparing with the small scale CMB observations, we obtained the constraints on the primordial magnetic fields, i.e., $B \lesssim 2.0 \mu$Gauss for $n = -2.9$ or $B \lesssim 1.0 \mu$Gauss for $n = -2.6$, where $B$ is the comoving amplitude of magnetic fields at $h^{-1}$ Mpc and $n$ is the power law index. Future S-Z measurements have the potential to give constraints tighter than those from temperature anisotropies and polarization of cosmic microwave background induced by the magnetic fields at the recombination epoch.

Key words: cosmology: theory – cosmic microwave background – large-scale structure of universe

1 INTRODUCTION

Many observations indicate the existence of large-scale magnetic fields associated with galaxies and galaxy clusters. These magnetic fields typically have strengths of a few $\mu$Gauss and large coherence lengths, i.e., a few kpc for galaxies and a few tens of kpc for galaxy clusters (Kronberg 1994). However, the origin of such magnetic fields is not understood clearly, while many generation processes have been proposed.

The generally accepted idea is an astrophysical dynamo scenario. Very tiny seed magnetic fields are generated in stars and supernova explosions by astrophysical processes such as Biermann battery, and the produced seed magnetic fields are amplified by the dynamo process in astrophysical objects. Finally these magnetic fields are spread into the inter-galactic medium by supernova winds or active galactic nuclei jets (Widrow 2002; Brandenburg & Subramanian 2005). However, there are two major problems remaining in this scenario. The first problem is the efficiency of the dynamo process in the expanding universe. Recent observations suggest the existence of $\mu$Gauss magnetic fields in high redshift galaxies (Kronberg et al. 1992). These galaxies may be dynamically too young to explain the existence of such magnetic fields by the dynamo process. The second problem concerns large coherence lengths. It is particularly difficult to explain observed magnetic fields with very large coherent scales in galaxy clusters (Kim et al. 1990, 1991).

Aside from this astrophysical scenario, there are alternative scenarios in which magnetic fields are generated in the early universe, e.g., inflation epoch or cosmological phase transitions such as QCD or electroweak. In these scenarios, there is the potential to obtain nano Gauss primordial magnetic fields. Such strength is sufficient to explain $\mu$Gauss magnetic fields observed at present without the dynamo process because the adiabatic compression due to the structure formation can easily
amplify primordial magnetic fields by a factor of \( \sim 10^3 \). However, if the seed magnetic fields generated in the early universe are too weak, the dynamo process is required even in these scenarios while the coherence length could be very large, unlike the astrophysical processes. For a detailed review, see Giovannini (2004).

If primordial magnetic fields existed in the early universe, these fields left traces of their existences in various cosmological phenomena, e.g., big bang nucleosynthesis (BBN), temperature anisotropies and polarization of cosmic microwave background (CMB), or large scale structure formations. From these traces, we can set observational constraints on primordial magnetic fields. These constraints give us clues to the origin of large scale magnetic fields, as well as when and how primordial magnetic fields were generated, because the strength and the coherence length of primordial magnetic fields depend on the generation process.

Let us first summarize BBN constraint. Since the primordial magnetic fields enhanced the cosmological expansion rate through the contribution of the energy density of primordial magnetic fields to the total energy density of the universe, the existence of primordial magnetic fields with sufficient strength may modify the abundance of light elements. The constraint on the magnetic field strength from BBN is \( B_0 \lesssim 7 \times 10^{-5}\text{ Gauss} \) where \( B_0 \) is the total comoving magnetic field strength (Cheng et al. 1996; Kerman et al. 1996).

Primordial magnetic fields produce CMB temperature anisotropies. Particularly, before recombination, primordial magnetic fields induce the vorticity of a baryon fluid by the Lorentz force. The induced vorticity generates CMB temperature anisotropies through the Doppler effect (Subramanian & Barrow 1998a). From the Wilkinson Microwave Anisotropy Probe (WMAP) data, the constraint on the primordial magnetic fields with 1Mpc–100Mpc is \( B_0 \lesssim 10^{-8}\text{ Gauss} \) (Mack et al. 2002; Lewis 2004; Tashiro et al. 2006; Yamazaki et al. 2006). Moreover, this vorticity generates CMB B-mode (parity odd) polarization as well as E-mode (parity even) polarization (Subramanian et al. 2003; Tashiro et al. 2006). In particular, B-modes are less contaminated by other sources than E-modes so that we expect to obtain stringent limits on the primordial magnetic fields by future observations of CMB B-modes.

After recombination, there are two main effects of primordial magnetic fields on the universe. One is the modification of the thermal evolution of baryons (Sethi & Subramanian 2003). Through the dissipation of primordial magnetic fields, primordial magnetic fields increase the baryon temperature after thermal decoupling of baryons from CMB. This dissipation is caused by the ambipolar diffusion and the direct cascade decay of small scale magnetic fields. The other effect is the generation of density fluctuations (Wasserman 1978; Kim et al. 1996; Gopal & Sethi 2003). The motion of ionized baryons induced by magnetic fields produces additional density fluctuations. These fluctuations induce density fluctuations of neutral baryons and dark matter through the gravitational force. The magnetic tension and pressure are more effective on small scales where the entanglements of magnetic fields are larger. Therefore, if primordial magnetic fields existed, it is expected that there is additional power in the density power spectrum, on small scales, which induces the early structure formation. These effects, modification of baryon thermal history and generation of additional density fluctuations, impact the reionization process. Therefore, it is possible to set constraints on primordial magnetic fields from the measurement of the optical depth (Sethi & Subramanian 2003; Tashiro & Sugiyama 2006a) and the observation of 21 cm lines (Tashiro & Sugiyama 2006b).

In this paper, we investigate the effect of primordial magnetic fields on the Sunyaev-Zel’dovich (S-Z) angular power spectrum. The S-Z effect occurs when CMB photons passing galaxy clusters are scattered by hot electron gas in galaxy clusters (Sunyaev & Zeldovich 1972). Due to the scattering, the CMB spectrum suffers distortion from the blackbody shape. The amount of distortion depends on the temperature and the number density of hot electron gas. In the low frequency limit, i.e., the Rayleigh-Jeans part, this distortion causes temperature decelment, which is observed as the temperature anisotropies in the CMB sky. Since the distribution of hot electron gas follows that of dark matter halos, the S-Z angular power spectrum traces the dark matter halo distribution which could be enhanced by primordial magnetic fields. Moreover, it is known that the S-Z effect is an ideal probe for the high redshift clusters/dark halos because the strength of the S-Z signal does not depend on redshift of the object, which is not the case for X-ray brightness temperature or the gravitational lensing effect. Since the primordial magnetic fields induce structure formation in the early epoch, we can conclude that the S-Z power spectrum can be used as a unique probe for the primordial magnetic fields.

This paper is organized as follows. In Sec. II, we discuss the density fluctuations due to primordial magnetic fields. In Sec. III, we summarize the calculation of the angular power spectrum of the S-Z effect. In Sec. IV, we show our results and discuss the constraint on primordial magnetic fields from the S-Z power spectrum. In Sec. V, we give the conclusion of this paper. Throughout the paper, we take 3-yr WMAP results for the cosmological parameters, i.e., \( h = 0.70 \) (\( H_0 = h \times 100 \text{ Km/s/Mpc} \)), \( T_0 = 2.725 \text{ K} \), \( \Omega_b = 0.044 \), \( \Omega_m = 0.26 \) (Spergel et al. 2007) and we assume \( \sigma_8 = 0.8 \). We normalize the value of the velocity of light to 1.

## 2 DENSITY FLUCTUATIONS DUE TO PRIMORDIAL MAGNETIC FIELDS

In this section, we calculate the density fluctuations produced by primordial magnetic fields. Let us make some assumptions about primordial magnetic fields at first. Since the length scales which we are interested in are large, the back-reaction of the
fluid velocity is small. Therefore, it is an assumption in this paper that primordial magnetic fields are frozen in cosmic baryon fluids,

\[ B(t, x) = \frac{B_0(x)}{a(t)^2}, \]

where \( B_0(x) \) is the comoving strength of primordial magnetic fields and \( a(t) \) is the scale factor which is normalized as \( a(t_0) = 1 \) at the present time, \( t_0 \). For simplicity, we assume that primordial magnetic fields are statistically homogeneous and isotropic and have the power law spectrum with the power law index \( n \),

\[ \langle B_0(k_1) B_0^*(k_2) \rangle = \delta(k_1 - k_2) \left( \frac{k_1 k_2}{k_i^2} \right) B_n^2 \left( \frac{k}{k_n} \right)^n, \]

where \( \langle \cdot \rangle \) denotes the ensemble average, \( B_0(k) \) are Fourier components of \( B_0(x) \), \( k_n \) is the wave number of an arbitrary normalized scale and \( B_n \) is the magnetic field strength at \( k_n \).

Our interest is to constrain the magnetic field strength on a certain scale in the real space. Therefore, we have to convolve the power spectrum with a Gaussian filter transformation of a comoving radius \( \lambda \), in order to get the magnetic field strength in the real space,

\[ B_\lambda^2 \equiv \langle B_0(x) B_0(x) \rangle_{\lambda} = \frac{2}{(2\pi)^3} \int d^3k \langle |B_0(k)|^2 \rangle \exp\left(-\frac{\lambda^2 k^2}{2}\right)^2. \]

Substituting Eq. (2) to Eq. (3), we can associate \( B_\lambda \) with \( B_n \),

\[ B_\lambda^2 = \frac{B_n^2}{2\pi^2 \lambda^3} (k_n \lambda)^{-n} \Gamma((n+3)/2). \]

We take \( h^{-1} \) Mpc as \( \lambda \) throughout our paper.

Primordial magnetic fields produce vorticity in a cosmic fluid. This vorticity is damped by the interaction between electrons and photons around the recombination epoch. This damping causes the dissipation of primordial magnetic fields and causes a sharp cutoff on the power spectrum of primordial magnetic fields. The cutoff scale \( 1/k_c \) after the recombination epoch is decided by (Jedamzik et al. 1998; Subramanian & Barrow 1998a),

\[ k_c^{-2} = V_A^2 \int_{t_r}^{t_0} \frac{L_e}{a^2(t)} dt, \]

where \( t_r \) is the recombination time and \( L_e \) is the mean free path of photons, which is described with the electron number density \( n_e \) and the Thomson cross section \( \sigma_T \) as \( L_e = 1/n_e \sigma_T \). In Eq. (5), \( V_A \) is the effective Alfvén velocity at the cutoff scale, \( V_A = B_c / \sqrt{4\pi \rho_i} \), where \( \rho_i \) is the radiation energy density and \( B_c \) is the effective magnetic fields at the cutoff scale, which is obtained by smoothing primordial magnetic fields. In the case of the power-law spectrum of primordial magnetic fields, the \( B_c \) is given by (Mack et al. 2002)

\[ B_c = B_\lambda \left( \frac{k_n}{k_\lambda} \right)^{(n+3)/2}. \]

Assuming the matter dominated epoch, we can obtain the relation between \( k_c \) and \( B_\lambda \) as

\[ k_c = \left[ 143 \left( \frac{B_c}{1 \text{ nG}} \right)^{-1} \left( \frac{h}{0.7} \right)^{1/2} \left( \frac{h^2 \Omega_b}{0.021} \right)^{1/2} \right]^{2/n+5} \text{ Mpc}^{-1}. \]

Primordial magnetic fields affect motions of ionized baryons by the Lorentz force even after recombination (Wasserman 1978). Although the residual ionized baryon rate to total baryons is small after recombination, the interaction between ionized and neutral baryons is strong in those redshifts that we are interested in. Therefore, we can assume baryons as a MHD fluid. Using the MHD approximation, we can write the evolution equations of density fluctuations with primordial magnetic fields as,

\[ \frac{\partial^2 \delta_b}{\partial t^2} = -2 \frac{\partial}{\partial a} \frac{\partial \delta_b}{\partial t} + 4\pi G (\rho_b \delta_b + \rho_\delta \delta_\delta) + S(t, x), \]

\[ S(t, x) = \nabla \cdot \left( (\nabla \times B_\lambda(x)) \times B_\lambda(x) \right), \]

\[ \frac{\partial^2 \delta_\delta}{\partial t^2} = -2 \frac{\partial}{\partial a} \frac{\partial \delta_\delta}{\partial t} + 4\pi G (\rho_b \delta_b + \rho_\delta \delta_\delta), \]

where \( \rho_b \) and \( \rho_\delta \) are the baryon density and the dark matter density, and \( \delta_b \) and \( \delta_\delta \) are the density contrast of baryons and dark matter, respectively. The source term in Eq. (10) is only the gravitational potential like that in the standard cosmology case, without primordial magnetic fields, while other source term caused by magnetic fields is added in Eq. (8). The solutions of Eqs. (8) and (10) can be given by
\[ \delta_p = D_{Sp}(t_0) \delta_p(t_0) + D_{Mp}(t_0) t_0^2 S(t_0, x), \]

where \( p \) denotes \( b \) for baryons and \( d \) for dark matter. Here \( D_{Sp}(t) \) corresponds to the growth rate of each component in the case of the \( \Lambda \)CDM cosmology without primordial magnetic fields and involves both the growing and decaying modes of primordial fluctuations, which are proportional to \( t^{2/3} \) and \( t^{-1} \) in the matter dominated epoch, respectively. Meanwhile, \( D_{Mp}(t) \) describes the growth rate of density fluctuations produced by primordial magnetic fields. Assuming the matter dominated epoch, we can write \( D_{Mp} \) as

\[ D_{Mb}(t) = \left[ \frac{9}{10} \left( \frac{t}{t_0} \right)^{2/3} - \frac{9 \Omega_b}{\Omega_m} \left( \frac{t}{t_0} \right)^{-1/3} + \frac{3}{5} \left( \frac{t}{t_0} \right)^{-1} - \left( \frac{3}{2} - 9 \frac{\Omega_b}{\Omega_m} \right) - 3 \log \left( \frac{t}{t_0} \right) \right]. \]

\[ D_{Md}(t) = \left[ \frac{9}{10} \left( \frac{t}{t_0} \right)^{2/3} - 9 \left( \frac{t}{t_0} \right)^{-1/3} + \frac{3}{5} \left( \frac{t}{t_0} \right)^{-1} + \frac{15}{2} - 3 \log \left( \frac{t}{t_0} \right) \right]. \]

We plot the growth rates, \( D_{Mb} \) and \( D_{Md} \), during the matter dominated epoch in Fig. 1. We also show the growth rate of the density contrast for total matter, \( \delta_t = (\rho_b \delta_b + \rho_d \delta_d)/(\rho_b + \rho_d) \). In this figure, we normalize growth rates as \( D_{Mb} = 1 \) at \( z = 1 \). The figure shows that the density fluctuations of baryons are produced by the Lorentz force at first, while the density fluctuations of dark matter follow those of baryons gravitationally. The growth rates of both fluctuations are proportional to \( 1 + z \).

Next, we calculate the power spectrum of the density fluctuations. Taking the assumption that there is no correlation between primordial magnetic fields and primordial density fluctuations for the sake of simplicity, we can describe the power spectrum as

\[ P_b(k) = P_{Sp}(k) + P_{Mp}(k) \equiv \langle |\delta_{Sp}(k)|^2 \rangle + \langle |\delta_{Mp}(k)|^2 \rangle, \]

where \( \delta_{Sp}(k) \) and \( \delta_{Mp}(k) \) are Fourier components of each density contrast. The power spectrum \( P_{Mp}(k) \) is written as

\[ P_{Mp}(k) = \left( \frac{\Omega_b}{\Omega_m} \right)^2 \frac{t_0^2}{4\pi \rho_0 a^3(t_0)} D_{Mp}(t_0) t_0^2 I^2(k), \]

where

\[ I^2(k) \equiv \langle |\nabla \cdot (\nabla \times B_0(x)) \times B_0(x)|^2 \rangle. \]

The isotropic Gaussian static of primordial magnetic fields makes the nonlinear convolution Eq. 10 rewritten as

\[ I^2(k) = \int dk_1 \int d\mu B_0^2(k_1) \left( \frac{1}{k - k_1} \right) (2k^5 k_1^3 \mu + k^4 k_1^4 (1 - 5\mu^2) + 2k^3 k_1^5 \mu^3), \]

where \( \mu = k \cdot k_1/|k||k_1| \). Note that the range of integration of \( k_1 \) in Eq. 17 depends on \( k \) because we assume that the power spectrum has a sharp cutoff below \( 1/k_c \) so that \( k_1 < k_c \) and \( |k - k_1| < k_c \) must be satisfied.

We introduce an important scale for the evolution of density perturbations, i.e., magnetic Jeans length. Below this scale, the magnetic pressure gradients, which we do not take into account in Eq. 3, counteract the gravitational force and prevent further evolution of density fluctuations. The magnetic Jeans scale is evaluated as \( \text{Kim et al. 1996} \).
$S-Z$ power spectrum produced by primordial magnetic fields

Figure 2. Mass dispersion $\sigma$ for different primordial magnetic fields. The dotted, solid and dashed lines represent $\sigma$ for primordial magnetic fields with $B_\lambda = 3.0$ nGauss, $B_\lambda = 2.0$ nGauss, and $B_\lambda = 1.0$ nGauss, respectively. Their power law indices are $n = -2.9$. We also plot $\sigma$ for primordial magnetic fields with different power law indices; for $n = -2.6$ and $B_\lambda = 1.0$ nGauss as the dashed-dotted-dotted line and for $n = -2.3$ and $B_\lambda = 1.0$ nGauss as the dashed-dotted line. For a comparison, we give $\sigma$ in the case without primordial magnetic fields as the thin solid line.

$$k_{\text{ML}} = \left[ 13.8 \left( \frac{B_\lambda}{\text{nG}} \right)^{-1} \left( \frac{h^2 \Omega_m}{0.18} \right)^{1/2} \right]^{2/n+5} \text{Mpc}^{-1}. \quad (18)$$

For simplicity, we assume that the density fluctuations do not grow below the scale, although the density fluctuations below the scale are, in fact, oscillating like the baryon oscillation.

In Fig. 2 we show the mass dispersion $\sigma$, which is calculated from the power spectrum of dark matter by

$$\sigma^2(M) = \int dk k^2 P_d(k) W(kR), \quad (19)$$

where $R$ is the scale which corresponds to mass $M$ and $W(x)$ is the top hat window function. Here we normalized the primordial power spectrum at $\sigma_8 = 0.8$. The power law index of $\sigma$ does not depend on that of primordial magnetic fields. This independence is brought by the sharp cutoff of magnetic fields and the nonlinear term given by Eq. (17). We can analytically estimate Eq. (17) in the limit of $k/k_c \ll 1$ as $P^\prime(k) \sim \alpha B_\lambda^{2n+10} k^{2n+7} + \beta B_\lambda^2 k^4$ where $\alpha$ and $\beta$ are coefficients which depend on $n$ [Kim et al. 1996]. Here we employ the fact that the cutoff scale $k_c$ is proportional to $B_\lambda^{-1}$ as is shown in Eq. (7). The former term dominates if $n > -1.5$, while the latter dominates for $n < -1.5$. Accordingly, if magnetic fields have a power law index smaller than $-1.5$, the power law index of $\sigma$ does not depend on that of magnetic fields. This power law index of $\sigma$ for such magnetic fields is about 7 and the amplitude of $\sigma$ is decided by their cutoff scale. Therefore, we can find that the spectra of the primordial magnetic fields with $n = -2.9$ and $B_\lambda = 3.0$ and with $n = -2.3$ and $B_\lambda = 1.0$, or with $n = -2.9$ and $B_\lambda = 2.0$ and with $n = -2.6$ and $B_\lambda = 1.0$, are very similar in Fig. 2 because the magnetic fields of these pairs have almost the same cutoff scales as those in Eqs. (4) and (7). We also show $\sigma_8$ for different power law indices of primordial magnetic fields in Fig. 3. In this figure, we plot $\sigma_8$ as the functions of $B_\lambda$. The more blue spectrum primordial magnetic fields have, the more amplitude of $\sigma_8$ they produce, even if magnetic fields have the same strength at a given scale, for example, $h^{-1}$ Mpc in this paper.

3 ANGULAR POWER SPECTRUM OF THE S-Z EFFECT

The angular power spectrum of the $S-Z$ effect is obtained through the halo formalism by many authors, e.g., Cole & Kaiser (1988); Makino & Suto (1993); Komatsu & Kitayama (1999); Komatsu & Seljak (2002). The angular power spectrum is given by

$$C_l = g_{\nu}^2 \int_0^{z_{\text{rec}}} \frac{dV}{dz} \int dM \frac{dn(M,z)}{dM} |y_l(M,z)|^2,$$  \hspace{1cm} (20)

where $g_{\nu}$ is the spectral function of the $S-Z$ effect which is $g_{\nu} = -2$ in the Rayleigh-Jeans limit, $V(z)$ is the comoving volume, $n(M,z)$ is the comoving number density of the dark matter halo with mass $M$ at redshift $z$, and $y_l(M,z)$ is the 2-D Fourier transform of the projected Compton $y$-parameter. Presently, we are interested in multipoles higher than $l = 300$, and neglect the halo-halo correlation term in Eq. (20).

For calculating $dn(M,z)/dM$ in Eq. (20), we adopt the Press-Schechter theory [Press & Schechter, 1974].
Following Komatsu & Seljak (2002), we set these assumptions, the radial profile depends on the dark matter profile, taking the three assumptions: the gas pressure and the dark matter potential reach the hydrostatic equilibrium; the gas density follows the dark matter density in the outer parts of dark halos; and the equation of state of gas is polytropic $P \propto \rho^{\gamma}$ for primordial magnetic fields with $\gamma = 1.137 + 8.94 \times 10^{-2} \ln(c/5) - 3.68 \times 10^{-3} (c - 5)$, where $c$ is a non-dimensional radius. The effect of primordial magnetic fields is taken into account through $\sigma(M, z)$ which is obtained from Eq. (19) in the former section.

The 2-D Fourier transform component $y_l$ is given in terms of the radial profile of the Compton $y$-parameter $y(x)$ through the Limber approximation,

$$y_l = \frac{4 \pi r_s}{l_s^2} \int_0^\infty dx x^2 y(x) \frac{\sin(lx/l_s)}{lx/l_s}.$$  \hspace{1cm} (22)

where $x$ is a non-dimensional radius $x \equiv r/r_s$ where $r_s$ is a scale radius which characterizes the radial profile, and $l_s$ is the multipole corresponding to $r_s$. The scale radius $r_s$ is associated to the virial radius with the concentration parameter $c$. Following Komatsu & Seljak (2002), we set

$$c \approx \frac{10}{1 + z} \left[ \frac{M}{M_{\odot}(0)} \right]^{-0.2},$$  \hspace{1cm} (23)

where $M_{\odot}(0)$ is a solution to $\sigma(M) = \delta_c$ at the redshift $z = 0$.

As the radial profile $y(x)$, we adopt the results of Komatsu & Seljak (2002). They obtained $y(x)$ based on the NFW dark matter profile, taking the three assumptions: the gas pressure and the dark matter potential reach the hydrostatic equilibrium; the gas density follows the dark matter density in the outer parts of dark halos; and the equation of state of gas is polytropic $P_{\text{gas}} \propto \rho_{\text{gas}}^{\gamma}$ where $P_{\text{gas}}$, $\rho_{\text{gas}}$ and $\gamma$ are the gas pressure, the gas density and the polytropic index. According to these assumptions, the radial profile $y(x)$ is written as

$$y(x) \equiv \frac{\sigma_T k_B}{m_e} n_e(x) T(x) = \frac{\sigma_T k_B}{m_e} n_e(0) T(0) y_{\text{gas}}(x),$$  \hspace{1cm} (24)

where the gas profile $y_{\text{gas}}(x)$, the central number density $n_e(0)$ and the central temperature $T(0)$ are represented as

$$y_{\text{gas}}(x) = \left\{ 1 - 3 \frac{x}{r_{\text{vir}}} \left[ \ln(1 + c) - \frac{1}{1 + c} \right] - \frac{1}{x} \right\}^{1/(\gamma - 1)},$$  \hspace{1cm} (25)

$$n_e(0) = 3.01 \left( \frac{M}{10^{14} M_{\odot}} \right) \left( \frac{r_{\text{vir}}}{1 \text{ Mpc}} \right)^{-3} \left( \frac{\Omega_b}{\Omega_m} \right) \frac{c^2}{\gamma y_{\text{gas}}(c)(1 + c)} \left[ \ln(1 + c) - \frac{c}{1 + c} \right]^{-1} \text{cm}^{-3},$$  \hspace{1cm} (26)

$$T(0) = 0.88 T_{\odot} \left( \frac{M}{10^{14} M_{\odot}} \right) \left( \frac{r_{\text{vir}}}{1 \text{ Mpc}} \right)^{-1} \text{keV}.$$  \hspace{1cm} (27)

Here, the polytropic index $\gamma$ and the mass temperature normalization factor at the center $\eta_c$ are given by

$$\gamma = 1.137 + 8.94 \times 10^{-2} \ln(c/5) - 3.68 \times 10^{-3} (c - 5),$$  \hspace{1cm} (28)

$$\eta_c = 2.235 + 0.202 (c - 5) - 1.16 \times 10^{-3} (c - 5)^2.$$  \hspace{1cm} (29)
4 RESULTS AND DISCUSSION

First, we calculate S-Z power spectra for different magnetic field strength with $n = -2.9$. We plot the results on Fig. 4. For references, we give the S-Z power spectra for the case of $\sigma_8 = 0.8$ and $\sigma_8 = 0.9$ without primordial magnetic fields. We find the effect of primordial magnetic fields arises on small scales. Although primordial magnetic fields with $2.0 \mu G$ amplify $\sigma_8$ to 0.9 by the generation of additional density fluctuations (see Fig. 3), the S-Z power spectrum for $2.0 \mu G$ magnetic fields is much different from that in the case of $\sigma_8 = 0.9$ without magnetic fields on small scales. Therefore, the CMB observation on small scales has the potential to resolve the degeneracy of $\sigma_8$ between the primordial density fluctuation and the additional density fluctuation by primordial magnetic fields.

The amplification of the S-Z power spectrum on small scales is due to the early formation of dark halos which is induced by the additional blue spectrum of the density fluctuations by primordial magnetic fields. Since the electron density in Eq. (24) is more dense in the early universe than in the late universe because of the cosmological expansion, the S-Z power spectrum is more affected by high redshift structures than other observations of mass distributions, for example, gravitational lensing. Therefore, the early halo formation contributes to the amplification of the S-Z power spectrum on small scales. We can see this contribution in Fig. 5 where we show the redshift distribution of $C_l$ for given $l$ modes. In large $l$ modes, there are enhancements in the tail part on the side of high redshifts which come from the density fluctuations generated by primordial magnetic fields, although the peak position is not changed, compared to the redshift contributions in the case without primordial magnetic fields.

Fig. 6 shows the S-Z angular power spectra for different power law indices of primordial magnetic fields. We choose $B_\Lambda = 1.0$ nGauss for all plotting cases. Comparing to Fig. 4, we find that the spectrum of primordial magnetic fields for $n = -2.6$ and $B_\Lambda = 1.0$ nGauss is similar to that for $n = -2.9$ and $B_\Lambda = 2.0$ nGauss. This is because, in the case of $n < -1.5$, the spectrum of density perturbations caused by primordial magnetic fields depends more strongly on the cutoff scale of magnetic fields than on the power law index, as mentioned in Sec. 2. Magnetic fields with $n = -2.6$ and $B_\Lambda = 1.0$ nGauss and with $n = -2.9$ and $B_\Lambda = 2.0$ nGauss have almost the same cutoff scales so that they have similar S-Z angular power spectra, even though their power law indices are different.

Although we computed the power spectra in the case of $n < -1.5$, we will give some comment on the case of $n \geq -1.5$. In such a case, the power spectral index of the density fluctuations generated by primordial magnetic fields depends on $n$. Therefore, the obtained S-Z power spectra with different $n$ are different, even though the cutoff scales of primordial magnetic fields are the same. The S-Z spectrum becomes steep if $n$ increases.

Since the S-Z power spectrum has a strong dependence on the cutoff scale of primordial magnetic fields, we obtain the constraint on the cutoff scale by comparing with the observed CMB data on small scales. For example, using the ACBAR data at $l = 2500$ (Kuo et al. 2007), we obtain $k_c \lesssim 95$ kpc. This limit corresponds to $B_\Lambda \lesssim 2.0$ nGauss for $n = -2.9$ and $B_\Lambda \lesssim 1.0$ nGauss for $n = -2.6$. This result is comparable with other constraint given by other effects of primordial magnetic fields on CMB temperature and polarization anisotropies caused by primordial magnetic fields, e.g., (Yamazaki et al. 2008).

5 CONCLUSION

We investigated the effect of primordial magnetic fields on the S-Z power spectrum. Primordial magnetic fields generate additional density fluctuations after recombination, so as to induce the early dark halo formation. The generated dark halos
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Figure 5. Distribution of the redshift contribution of the S-Z angular power spectrum for given $l$ modes. Primordial magnetic fields have $n = -2.9$ and $B_\lambda = 1.0$. The solid, the dashed, the dotted, and the dashed-dotted lines represent the distributions for $l = 500$, $l = 1000$, $l = 5000$ and $l = 10000$, respectively. For a comparison, we plot the distributions for the case without primordial magnetic fields as thin lines.

Figure 6. S-Z angular power spectra for different power law indices. We choose $B_\lambda = 1.0$ in all plots. The solid, dotted, dashed-dotted, and dashed lines show the S-Z spectra for magnetic fields with $n = -2.9$, $n = -2.6$ and $n = -2.3$, respectively. The S-Z angular power spectrum without primordial magnetic fields for $\sigma = 0.8$ and $\sigma = 0.9$ are shown as the thin solid and thin dashed lines, respectively. For reference, we plot primordial CMB temperature angular power spectrum and ACBAR data.

in the early universe amplify the S-Z power spectrum on small scales. We found that the amplification depends on the cutoff scale of primordial magnetic fields. Therefore, comparing our calculated results with present CMB observational data on small scales, we obtain the constraint on the cutoff scale of primordial magnetic fields, $k_c \lesssim 95$ kpc. This constraint is equivalent to $B_\lambda \lesssim 2.0$ nGauss for $n = -2.9$ or $B_\lambda \lesssim 1.0$ nGauss for $n = -2.6$ at $h^{-1}$ Mpc. The smaller the interesting scale of the S-Z power spectrum goes to, the larger the enhancement by the primordial magnetic fields becomes. Therfore we can expect that the future S-Z measurements can give constraints tighter than those from CMB temperature anisotropies and polarization induced by the magnetic fields at the recombination epoch.

The small scale CMB observations, e.g., CBI (Mason et al. 2003), BIMA (Dawson et al. 2002) and ACBAR (Kuo et al. 2007) detected an excess of temperature anisotropies from the small scale temperature anisotropy than what was expected from the WMAP results. This excess corresponds to the S-Z effect with $\sigma_8 = 1.0$ (Bond et al. 2005). However, this high value conflicts with the WMAP result, $\sigma_8 = 0.8$, which is obtained from the large scale temperature anisotropies (Spergel et al. 2007). The existence of primordial magnetic fields may resolve this discrepancy, because the density fluctuations generated by primordial magnetic fields do not affect at large scales but add a blue spectrum on small scales.

The S-Z power spectrum depends on the electron density profile in dark halos. For obtaining a highly accurate constraint on primordial magnetic fields, we need a detailed study on the effect of primordial magnetic fields on the electron density profile. However, we ignored this effect in this paper. One possible effect of magnetic fields is brought by the pressure of magnetic fields. The magnetic field pressure prevents electron gas from falling into the gravitational potential well of dark matter. The modification of the electron density profile can be detected by the S-Z effect, if there are magnetic fields with several $\mu$Gauss in a halo (Zhang 2004), such magnetic field strength is easily obtained from primordial magnetic fields with order of nano Gauss by adiabatic contraction in the halo formation. We will study the effect on the S-Z effect due to primordial
magnetic fields, and the consistent constraint on those fields, considering effects other than the density fluctuations generation of primordial magnetic fields, in the future.

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