Longitudinal boost-invariance of charge balance function in hadron-hadron and nucleus-nucleus collisions

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Using Monte Carlo generators of the PYTHIA model for hadron-hadron collisions and a multi-phase transport (AMPT) model for nucleus-nucleus collisions, the longitudinal boost-invariance of charge balance function and its transverse momentum dependence are carefully studied. It shows that the charge balance function is boost-invariant in both \( p+p \) and \( \text{Au}+\text{Au} \) collisions in these two models, consistent with experimental data. The balance function properly scaled by the width of the pseudorapidity window window is independent of the position or the size of the window and is corresponding to the balance function of the whole pseudorapidity range. This longitudinal property of balance function also holds for particles in small transverse momentum ranges in the PYTHIA and the AMPT default models, but is violated in the AMPT with string melting. The physical origin of the results are discussed.

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I. INTRODUCTION

Charge balance function (BF) has been widely used as an effective exploring for the hadronization scheme in hadron-hadron collisions at the ISR energies \[1\] and \( e^+ + e^- \) annihilations at PETRA energies \[2\]. Recently, the charge BF gains special attentions in clocking hadronization at relativistic heavy-ion collisions. A narrowing of the BF is suggested as a signature for delayed hadronization \[3,4\].

The dependence of the BF on centrality and system size has been reported by several relativistic heavy-ion experiments \[3,6\]. However, most of the current heavy-ion experiments are limited by the pseudorapidity window \[3,5,7\]. The experiments indicate that charge balance of produced particles is invariant under longitudinal boost over the whole pseudorapidity range of produced particles, in spite of the non-boost-invariance of the single-particle density. Moreover, the BF of the whole pseudorapidity range can be deduced from the BF properly scaled by the width of rapidity windows \[3\].

The STAR experiment has full 4\(\pi\) acceptance and excellent momentum resolution \[3\]. It is found in the experiment that the BF in \( \pi^+p \) and \( \text{K}^+p \) Collisions at 22 GeV is invariant under longitudinal boost over the whole rapidity range of produced particles, in spite of the non-boost-invariance of the single-particle density. Moreover, the BF of the whole rapidity range can be deduced from the BF properly scaled by the width of rapidity windows \[3\].

The STAR experiment covers a finite but relative wide pseudorapidity range. The scaling property of the BF in Au+Au collisions at 200 GeV is further observed in the experiment \[10\]. This scaling property of the balance function is also found in different \( p_T \) ranges of final state particles.

These results from both hadron-hadron and nuclear collisions indicate that charge balance of produced particles in strong interactions is boost-invariance in longitudinal phase-space, in contrary with the single particle density.

Therefore, it is interesting to see if those properties are taken into account in the models which are successfully described hadron-hadron and nuclear collisions, and how they associate with the mechanisms of particle production in the models.

II. CHARGE BALANCE FUNCTION AND IMPLEMENT MODELS

Charge balance function measures how the conserved electric charge compensate in the phase space, i.e., how the surrounding net charge are rearranged if the charge of a selected point changes \[4\]. In high energy collisions, the production of charged particles are constrained by charge balance in the phase space. The BF therefore provides a direct access to collision dynamics.

The BF has been originally defined in terms of a combination of four kinds of charge-related conditional densities in pseudorapidity \[1\].

\[
B(\eta_1|\eta_2) = \frac{1}{2} \left[ \rho(+, \eta_1|+, \eta_2) - \rho(+, \eta_1|+, \eta_2) + \rho(-, \eta_1|+, \eta_2) - \rho(-, \eta_1|-, \eta_2) \right],
\]

where the notation \( \rho(a, \eta_1|b, \eta_2) \) represents the ratio \( \rho_{ab}(\eta_1, \eta_2)/\rho_0(\eta_2) \) with \( a, b \) standing for + or - charged particles. Projecting to pseudorapidity difference \( \delta \eta = \eta_1 - \eta_2 \) in an pseudorapidity window \( \eta_w \), it becomes \[3,5\].

\[
B(\delta \eta|\eta_w) = \frac{1}{2} \left[ \frac{(n_{++}(\delta \eta)) - (n_{++}(\delta \eta))}{\langle n_{++} \rangle} + \frac{(n_{--}(\delta \eta)) - (n_{--}(\delta \eta))}{\langle n_{--} \rangle} \right],
\]
where \( n_{ab}(\delta \eta) \) is the total number of pairs of opposite charged particles with pseudorapidity difference \( \delta \eta \) in the pseudorapidity window \( \eta_w \). \( n_+ \) and \( n_- \) are the number of positively and negatively charged particles in the window \( \eta_w \), respectively. \langle \cdots \rangle \) is the average over the whole event sample.

From the findings of the BF at NA22 [9] and STAR experiments [10], the BF is boost-invariant in the whole rapidity range in hadron-hadron collisions and may be in nuclear collisions as well. In the case, the properly scaled BF is corresponding to the BF of the whole pseudorapidity range and is deduced by

\[
B_s(\delta \eta) = \frac{B(\delta \eta|\eta_w)}{1 - \frac{\delta \eta}{|\eta_w|}} \quad \text{(3)}
\]

where \( |\eta_w| \) is the width of pseudorapidity window.

The PYTHIA 5.720 [11] is well set up for \( p+p \) collisions. It is a standard Monte Carlo generator with string fragmentation as hadronization scheme. Two versions of a multi-phase transport (AMPT) model [12] are used to study \( Au+Au \) collisions. One is the AMPT default and the other one is the AMPT with string melting. In both versions, the initial conditions are obtained from the HIJING model, and then the scattering among partons are given by ZPC. In the AMPT default model, the partons recombined with their parent strings when they stop interacting, and the resulting strings are converted to hadrons using the Lund string fragmentation model, while in the AMPT model with string melting, quark coalescence is used in combining partons into hadrons. The dynamics of the hadronic matter is described by ART model.

It is commonly believed that in relativistic heavy ion collisions, the charge ordering during the string fragmentation in elementary collisions is no longer valid, and it should be replaced by the quark-coalescence mechanism in hadronization [13]. So it is interesting to see whether the boost-invariance of the BF is sensitive to the mechanisms of hadronization.

In this paper, we firstly study the boost-invariance of the BF for \( p+p \) collisions at \( \sqrt{s} = 22 \text{ GeV} \) and \( \sqrt{s} = 200 \text{ GeV} \) using the PYTHIA, and for \( Au+Au \) collisions at \( \sqrt{s} = 200 \text{ GeV} \) using two versions of the AMPT. The transverse momentum dependence of longitudinal scaling property of the BF is then examined in the models. The obtained results are compared with corresponding experimental data and discussed.

**III. BOOST-ININVARIANCE AND LONGITUDINAL SCALING OF THE BF**

In order to demonstrate directly whether the BF is invariant under a longitudinal Lorentz transformation over the whole rapidity in hadron-hadron collisions, we choose four equal size \(|\eta_w| = 3\) pseudorapidity windows locating at different positions \((-3, 0), (-2, 1), (-1, 2)\) and \((0, 3)\). The results for \( p+p \) collisions at \( \sqrt{s} = 22 \text{ GeV} \) and \( \sqrt{s} = 200 \text{ GeV} \) are shown in Fig. 1(a) and (b) respectively. The statistic errors are smaller than the markers. It is clear that the BF measured in four windows are approximately identical to each other at two incident energies. This indicates that the charge compensation is essentially the same in any longitudinally-Lorentz-transformed frame for \( p+p \) collisions in the PYTHIA model, consistent with the data from NA22 experiment. These results show that the string fragmentation mechanism implemented in PYTHIA well describes the production mechanisms of charged particles and their charge balance in longitudinal phase space.

Fig. 1(c) and (d) are the scaled balance function \( B_s(\delta \eta) \) at two incident energies. They are deduced from directly measured \( B(\delta \eta|\eta_w) \) at six different pseudorapidity windows, \((-0.8, 0.8)\) (open circles), \((1, 3)\) (open triangles), \((-3.1)\) (open squares), \((-2.4, 2.4)\) (open diamonds), \((0, 3)\) (open crosses), and \((-2, -1)\) (open stars). From the figures we can see that all the \( B_s(\delta \eta) \) deduced from different windows are coincide with each other within errors, as expected from boost-invariance of the BF [3]. The solid down triangles in the same figures are the BF of the whole pseudorapidity range, \( B(\delta \eta|\eta_w) \). It is close to the scaled balance function \( B_s(\delta \eta) \). These results indicate that the scaled BF is in fact corresponding to the BF of the whole pseudorapidity range \( B(\delta \eta|\infty) \) [3].

It is then interesting to see whether the boost-invariance of the BF is held in nucleus-nucleus collisions. STAR experiment only observe the boost-invariance of
The longitudinal property of boost invariance of BF holds for particles in different $p_T$ ranges. Fig. 3 shows the BF for $p+p$ collisions at $\sqrt{s} = 22$ GeV and $\sqrt{s} = 200$ GeV from PYTHIA in three transverse momentum bins $(0 < p_T < 0.2)$, $(0.2 < p_T < 0.4)$, and $(p_T > 0.2)$ GeV/c, respectively. These $p_T$ bins are selected to make the multiplicity in each bin comparable. The result shows that the points at a given $\delta \eta$ in a restricted $p_T$ interval are approximately coincide with each other, i.e., the boost-invariance of the BF hold in small $p_T$ ranges. It indicates that particles produced at different $p_T$ ranges are also boost-invariant for hadron-hadron collisions in the PYTHIA model.

The same study for Au+Au 200 GeV collisions from the two versions of the AMPT are presented in the upper and lower panels of Fig. 4, respectively. Where four $p_T$ bins are, $(0.15, 0.4)$, $(0.4, 0.7)$, $(0.7, 1)$ and $(1, 2)$ GeV/c. From the upper panel of the figure, we can see that the BF of different pseudorapidity windows in each $p_T$ bin are close to each other, in consistent with the data from STAR experiment [10]. However, in the AMPT with string melting, as shown in the lower panel of the figure, where the BF of different pseudorapidity windows are not as close to each other as those in the upper panel.

This is because in the AMPT with string melting, each parton in the evolution of nuclear collision has its own freeze-out time, which last a very long period after the interaction of two nucleons [18]. The particles in the same transverse-momentum range are not freeze-out simultaneously with well balanced charge, and therefore the longitudinal boost-invariance of the BF in small $p_T$ ranges is violated. In the AMPT default, the partons recombined with their parent strings immediately after they stop interacting, and converted to hadrons. So the charge balance of the produced particles in the same $p_T$ ranges is preserved and boost-invariance of the BF keeps.

V. SUMMARY

In the paper, we systematically study the longitudinal boost-invariance of charge balance function and its $p_T$ dependence for $p+p$ and Au+Au collisions using PYTHIA the AMPT models. It shows that charge balance function is boost-invariance in both hadron-hadron and nuclear interactions, in contrary to the single particle density. As
expected, this boost-invariance of the BF make the BF properly scaled by window size is independent of window and corresponds to the BF of the whole (pseudo)rapidity range. Therefore, the BF is a good measure free from the restriction of finite longitudinal acceptance.

It is further show that the boost invariance of the BF in specified $p_T$ range is valid in PYTHIA for hadron-hadron collisions and the AMPT default for Au+Au collisions. While the AMPT with string melting fails to reproduce this property due to the different schemes at hadronization. So the $p_T$ dependence of the longitudinal property of the BF may be served as a sensitive probe for charge balance in hadronization mechanism.

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[1] D. Drijard et al., Nucl. Phys. B 155 269 (1979); B 166, 233 (1980); I. V. Ajinenko et al., ibid. C 43, 37 (1989).
[2] R. Brandelik et al., Phys. Lett. B 100, 357 (1981); M. Althoff et al., Z. Phys. C 17, 5 (1983); H. Aihara et al.,
Phys. Rev. Lett. 53, 2199 (1984); 57, 3140 (1986); P. D. Acton et al., Phys. Lett. B 305, 415 (1993).

[3] S. A. Bass, P. Danielewicz and S. Pratt, Phys. Rev. Lett. 85, 2689 (2000).

[4] S. Jeon and S. Pratt, Phys. Rev. C 65, 044902 (2002).

[5] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 90, 172301 (2003); Gary. D. Westfall (for STAR Collaboration), J. Phys. G. 30, S345 (2004).

[6] C. Alt et al. (NA49 Collaboration), Phys. Rev. C 71, 034903 (2005); C. Alt et al. (NA49 Collaboration), arXiv: 0705.1122.

[7] K. Adcox et al. (PHENIX Collaboration), Phys. Rev. Lett. 89, 082301 (2002).

[8] T. A. Trainor, hep-ph/0301122.

[9] M. R. Atayan et al. (NA22 Collaboration), Phys. Lett. B 637, 39 (2006).

[10] Li Zhiming, Li Na, Liu Lianshou and Wu Yuanfang, Int. J Mod. Phys. E 16, 3347 (2007).

[11] T. Sjöstrand, Comp. Phys. Commun. 82, 74 (1994).

[12] Z. W. Lin, C. M. Ko, B. A. Li, B. Zhang and S. Pal, Phys. Rev. C 72, 064901 (2005).

[13] A. Bialas, Phys. Lett. B 579, 31 (2004); R. C. Hwa and C. B. Yang, Phys. Rev. C 70, 024904 (2004).

[14] R. C. Hwa and Y. Wu, Phys. Rev. C 60, 054904 (1999).

[15] M. Asakawa, S. A. Bass, B. Müller and C. Nonaka, Phys. Rev. Lett. 101, 122302 (2008).

[16] F. Grassi, Y. Hama, and T. Kodama, Phys. Lett. B 355, 9 (1995); Y. M. Sinyukov, S. V. Akkelin, and Y. Hama, Phys. Rev. Lett. 89, 052301 (2002).

[17] M. Asakawa, S. A. Bass, B. Müller and C. Nonaka, Phys. Rev. Lett. 101, 122302 (2008).

[18] Yu Meiling, Du Jiaxin, and Liu Lianshou, Phys. Rev. C (Rapid communication) 74, 044906 (2006).