Charm Coalescence at relativistic energies

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The \( \frac{e}{\bar{e}} \) yield at midrapidity at the top RHIC (relativistic heavy ion collider) energy is calculated within the statistical coalescence model, which assumes charmonium formation at the late stage of the reaction from the charm quarks and antiquarks created earlier in hard parton collisions. The results are compared to the new PHENIX data and to predictions of the standard models, which assume formation of charmonium exclusively at the initial stage of the reaction and their subsequent suppression. Two versions of the suppression scenario are considered. One of them assumes gradual charmonium suppression by comovers, while the other one supposes that the suppression sets in abruptly due to quark-gluon plasma formation. Surprisingly, both versions give very similar results. In contrast, the statistical coalescence model predicts a few times larger \( \frac{e}{\bar{e}} \) yield in the most central collisions.

A study of open and hidden charm production in nucleus-nucleus (\( A + A \)) collisions at RHIC (relativistic heavy ion collider) BNL is expected to shed light upon an important physical question of the space-time history of the charmonium formation. The standard “suppression” approach is based on the idea of Matsui and Satz\textsuperscript{1}: charmonia are formed at the early stage of \( A + A \) reaction, the further evolution leads exclusively to their suppression due to interaction with initial nucleons from the colliding nuclei, secondary comoving hadrons, and/or deconfined medium.

The idea of thermal \( \frac{e}{\bar{e}} \) production\textsuperscript{2} triggered the development of an alternative charmonium formation scenario, the statistical coalescence model (SCM)\textsuperscript{3,4}. Hidden charm mesons are assumed to be created at hadronization near the point of chemical freeze-out due to coalescence of charm quarks \( c \) and antiquarks \( \bar{c} \) produced at the initial stage. The distribution of \( c \)'s and \( \bar{c} \)'s over different open and hidden charm species is given by the laws of equilibrium statistical mechanics. The SCM describes remarkably well the centrality dependence of the \( \frac{e}{\bar{e}} \) yield\textsuperscript{7} as well as the transverse spectra\textsuperscript{6} at SPS.

A combination of the standard and SCM approaches\textsuperscript{6} as well as a nonthermal \( c\bar{c} \) coalescence\textsuperscript{5} have been also considered.

The preliminary RHIC data\textsuperscript{5} on the \( \frac{e}{\bar{e}} \) rapidity density and its centrality dependence in Au+Au collisions at \( \sqrt{s} = 200 \) GeV from the PHENIX Collaboration have been already discussed in Refs.\textsuperscript{11,12}. The final data\textsuperscript{12}, which became available recently, differ essentially from the preliminary ones. Because of low statistics, the data have rather large errorbars. The most likely value of \( \frac{e}{\bar{e}} \) yield in the most central collisions is not reported. Instead, only 90\% confidence level upper limit is given. Still, the data appeared to be able to exclude the most extreme versions of the nonthermal coalescence scenarios\textsuperscript{5}.

The aim of the present paper is to check whether SCM is tolerated by the data and compare its predictions to the standard suppression models.

Let two nuclei \( A \) and \( B \) collide at impact parameter \( b \). The number of produced \( \frac{e}{\bar{e}} \) mesons is given within the standard scenario by\textsuperscript{14}

\[
\langle \frac{e}{\bar{e}} \rangle_{AB} = \sigma_{\frac{e}{\bar{e}}}^{NN} \cdot \int d^2s \cdot \frac{T_A(|\vec{s}|)}{T_B(|\vec{s} - \vec{b}|)} \cdot S(\vec{b}, \vec{s}),
\]

where \( \sigma_{\frac{e}{\bar{e}}}^{NN} \) is the cross section of \( \frac{e}{\bar{e}} \) production in nucleon-nucleon (\( N + N \)) collisions, \( T_A(B) \) is the nuclear thickness function related to the nucleon density in the nucleus, and \( S(\vec{b}, \vec{s}) < 1 \) is a factor responsible for the \( \frac{e}{\bar{e}} \) suppression.

At the very initial stage, charmonia experience absorption, \( S = S_{\text{abs}} \), by sweeping nucleons of the colliding nuclei (see, for instance, Refs.\textsuperscript{11,12}). Bound \( c\bar{c} \) states are assumed to be absorbed in the so-called preresonance state, before the final hidden charm mesons are formed. The absorption cross section is therefore taken to be the same for all charmonia. The value \( \sigma_{\text{abs}} = 4.4 \) mb\textsuperscript{14} follows from the most recent SPS data analysis and is close to the theoretical prediction of Ref.\textsuperscript{17}. We assume that the same value of \( \sigma_{\text{abs}} \) prevails also at RHIC energies.

Those charmonia that survive normal nuclear suppression are subjected to the comover\textsuperscript{15,18} or quark-gluon plasma (QGP) suppression\textsuperscript{19}. Both suppression scenarios describe successfully the centrality dependence of the \( \frac{e}{\bar{e}} \) yield in Pb+Pb collisions at the SPS.

In the comover approach, an additional suppression factor appears: \( S = S_{\text{abs}} \cdot S_{\text{co}} \). The suppression factor \( S_{\text{co}} \) depends on the density of comovers and on an effective cross section \( \sigma_{\text{co}} \) of \( \frac{e}{\bar{e}} \) dissociation by comovers (averaged over all comover species and all charmonium states contributing to the \( \frac{e}{\bar{e}} \) yield through their decays and also over particle momenta in the medium). The value \( \sigma_{\text{co}} = 0.65 \) mb\textsuperscript{20} from fits of new SPS NA50 data\textsuperscript{21} will be used in our analysis. We assume that the value of \( \sigma_{\text{co}} \) remain the same also at RHIC. The charmo-
nium suppression at RHIC becomes, however, stronger, due to the higher comover density.

There are two reasons for increasing the comover density at RHIC relative to SPS. The multiplicity of produced secondary hadrons per unit rapidity interval at midrapidity increases by a factor of about 1.5 from $\sqrt{s} = 17$ GeV to $\sqrt{s} = 200$ GeV already in elementary nucleon-nucleon collisions. Additionally, the deviations from the wounded nucleon model becomes stronger at higher energies. This increases the comover density in central nucleus-nucleus collisions. The centrality dependence of the number of light-flavored hadrons per unit pseudorapidity interval in $\text{Au}+\text{Au}$ collisions at RHIC can be parametrized as \[ dN^{\text{AuAu}}_h/dy \] at $y = 0$ [22], $N_p(b)$ is the number of participants and $N_{\text{coll}}(b)$ is the number of collisions. Both are calculated in the Glauber approach.

Calculating the centrality dependence of the $J/\psi$ suppression, it is convenient to introduce an effective participant density in the plane transverse to the collision axis:

$$n_p^\ast(\vec{b}, \vec{s}) = [(1 - x)n_p(\vec{b}, \vec{s}) + 2x n_c(\vec{b}, \vec{s})]. \quad (3)$$

Here $n_p(\vec{b}, \vec{s})$ and $n_c(\vec{b}, \vec{s})$ are, respectively, the densities of nucleon participants and collisions in the transverse plane: $N_p(b) = \int d^2s \ n_p(\vec{b}, \vec{s})$ and $N_{\text{coll}}(b) = \int d^2s \ n_{\text{coll}}(\vec{b}, \vec{s})$. Note that the multiplicity of light-flavored hadrons [22] is proportional to $N_p^\ast(b) = \int d^2s \ n_p^\ast(\vec{b}, \vec{s})$. Motivated by this fact, we assume that the comover density in the transverse plane, which is needed to calculate $S^{\text{eff}}$, is proportional to $n_p^\ast$.

The cross section of $J/\psi$ production per unit rapidity interval at midrapidity $d\sigma_{J/\psi}^{NN}/dy \bigg|_{y=0}$ is the only free parameter of our fit. The PHENIX data on the $J/\psi$ multiplicity in $p+p$ and $\text{Au}+\text{Au}$ collisions [36] are fitted simultaneously. The best fit, $\chi^2/\text{ndf} = 2.0$ [37], is reached at $B_{e^+e^-}^{J/\psi} \cdot d\sigma_{J/\psi}^{NN}/dy \bigg|_{y=0} = 4.9 \times 10^{-2} \mu b$ ($B_{e^+e^-}^{J/\psi}$ is the branching ratio of $J/\psi$ decays into electron positron pair). The result is shown in Fig. 11.

The comover model has been historically referred to as a “hadronic” model. One might doubt whether the extrapolation of this model to the RHIC energies is legal. Indeed, the estimated energy density is extremely high even at SPS, so that hadrons can hardly preserve their individuality. The authors of the comover approach do not insist, however, on its hadronic interpretation (see, for instance [22]). We do not therefore make any assumptions about the nature of the comoving medium. We merely consider the comover model as an extreme scenario, which assumes a gradual increase of the charmonium suppression with growing energy density, without any abrupt changes of the absorptive properties of the medium.

The QGP scenario of Ref. [19] represents the opposite extreme: the charmonium suppression sets in, as soon as the energy density exceeds some threshold value. The excited charmonia, which contribute about 40% to the total $J/\psi$ yield, are suppressed at lower energy densities than directly produced $J/\psi$’s. We have updated the fit [19] to the SPS data (new NA50 data [21] were added) using corrected values of the parameters, $\sigma_{abs} = 4.4 \pm 0.5 \mu b$ and $\sigma_{J/\psi}^{NN}/\sigma_{DY}^{NN} \approx 43.1$, reported recently [16]. Our results are $n_1 = 2.99 \text{ fm}^{-2}$ and $n_2 = 3.86 \text{ fm}^{-2}$. Here $n_1$ ($n_2$) is the participant density in the transverse plane, corresponding to the threshold energy density at which excited charmonia ($J/\psi$’s) are fully suppressed.

Extrapolating to RHIC energies, one again has to take into account that the number of produced hadrons per unit rapidity and, consequently, the energy density of the produced medium grows with the collision energy and centrality. Due to the deviation from the wounded nucleon model [22], the charmonium suppression sets in, when the effective participant density $n_p^\ast(\vec{b}, \vec{s})$ rather than the usual $n_p(\vec{b}, \vec{s})$ exceeds the threshold values. The number of secondary hadrons per effective participant pair at $\sqrt{s} = 200$ is higher than that at the SPS by a factor of about 1.5. The critical energy density at RHIC is reached, therefore, at lower effective participant density: $n_1^\ast = n_1/1.5 \approx 2.0 \text{ fm}^{-2}$ and $n_2^\ast = n_2/1.5 \approx 2.6 \text{ fm}^{-2}$.

Similarly as in the comover model, the $J/\psi$ production cross section is the only free parameter in the fit of the RHIC data. The minimum $\chi^2/\text{ndf} = 2.2$ is obtained at $B_{e^+e^-}^{J/\psi} \cdot d\sigma_{J/\psi}^{NN}/dy \bigg|_{y=0} = 5.0 \times 10^{-2} \mu b$. The best fit of the QGP suppression scenario is also shown in Fig. 1.

As was already noted above, the extrapolation of the standard suppression models from SPS to RHIC energies was based on the assumption that the value of the normal nuclear absorption cross section $\sigma_{abs}$ does not change essentially with the collision energy. Other viewpoints are also possible. The $J/\psi$ nuclear suppression mechanism at RHIC may be completely different from that at SPS [22]. This does not improve the agreement of the standard suppression models with the data, if the nuclear suppression becomes stronger at RHIC [26]. It is not excluded, however, that the nuclear suppression may be even weaker [22]. A $J/\psi$ measurements in $d + \text{Au}$ collisions would clarify this point.

Now we will check whether the statistical coalescence model [4] can be tolerated by the new data. In the SCM [3], the total charm content of the final hadron system equals the number of $c$ and $\bar{c}$ created at the initial stage of $A+A$ reaction. Statistical laws control only the distribution of $c$ and $\bar{c}$ among different hadron states in terms of the hadron gas (HG) model parameters: temperature
The centrality dependence of charmonium production at RHIC, the situation is different: the total \((4\pi)\) multiplicity of light hadrons are approximately proportional to the number of participants, while at midrapidity, it grows faster [see Eq. (2)]. The centrality dependence of charmonium production at different rapidities should, in this case, be also different. To compare the SCM prediction to the PHENIX data, which are related to the J/\(\psi\) yield at midrapidity \(dN_{J/\psi}/dy\), one has to derive a formula for the \(J/\psi\) yield in a finite rapidity interval.

Let \(\xi_{\Delta y} < 1\) is the probability that a \(c\) quark, produced in a nucleus-nucleus collision, has rapidity within the interval \(\Delta y\). The probability distribution of the number \(k_c\) of \(c\) quarks inside the interval \(\Delta y\) for events with fixed total \((4\pi)\) number of \(c\bar{c}\) pairs is given by the binomial law:

\[
f(k_c|N_{c\bar{c}}) = \frac{N_{c\bar{c}}!}{k_c! (N_{c\bar{c}} - k_c)!} \xi_{\Delta y}^k_c (1 - \xi_{\Delta y})^{N_{c\bar{c}} - k_c}.
\]

The probability distribution of the number \(k_c\) of \(c\bar{c}\) pairs inside the interval \(\Delta y\) is assumed to be independent of \(k_c\). It conforms to the same binomial law. Event-by-event fluctuations of the number of \(c\bar{c}\) pairs \(N_{c\bar{c}}\) created at the early stage of \(A + A\) reaction in independent nucleon-nucleon collisions, are Poisson distributed:

\[
P(N_{c\bar{c}}; \bar{N}_{c\bar{c}}) = \exp \left( -\bar{N}_{c\bar{c}} \right) \frac{\left( \bar{N}_{c\bar{c}} \right)^{N_{c\bar{c}}}}{N_{c\bar{c}}!}.
\]

The probability distribution of \(c\bar{c}\) coalescence is proportional to the product of their numbers and inversely proportional to the system volume. The proportionality coefficient depends on the thermal densities of the open and hidden charm, and is the same as in the case of the total charmonium yield [4].

The average \(J/\psi\) multiplicity at fixed values of \(k_c\) and \(k_{\bar{c}}\) is therefore given by the formula [28]

\[
\langle J/\psi \rangle_{k_c, k_{\bar{c}}} \approx k_c k_{\bar{c}} \frac{n_{total}^{J/\psi}}{(n_O/2)^2} \frac{1}{V_{\Delta y}}.
\]

[Deriving Eq. (6) we used the fact that the thermal number of hadrons with hidden charm is much smaller than that with open charm.] Folding Eq. (6) with the binomial and Poisson distributions one gets

\[
\langle J/\psi \rangle_{\Delta y} \approx \xi_{\Delta y}^2 \bar{N}_{c\bar{c}} \left( \bar{N}_{c\bar{c}} + 1 \right) \frac{n_{total}^{J/\psi}}{(n_O/2)^2} \frac{1}{V_{\Delta y}},
\]

where \(n_O\) is the thermal density of all open charm hadrons and \(n_{total}^{J/\psi}\) is the total thermal \(J/\psi\) density (with decay contributions from the excited charmonium states included). Both \(n_O\) and \(n_{total}^{J/\psi}\) are calculated in the grand canonical ensemble with the QGP hadronization parameters \(T, \mu_B, V_{\Delta y}\) found from fitting the data of light-flavored hadron yields in the rapidity interval \(\Delta y\).
The average number of $c\bar{c}$ pairs $\overline{N}_{c\bar{c}}$ is, however, related to their total $(4\pi)$ yield. In Au+Au collisions at $\sqrt{s} = 200$ GeV, the yield of light-flavored hadrons at midrapidity is fitted within the hadron gas model with $T = 177$ MeV and $\mu_B = 29$ MeV [30]. The centrality dependence of the volume is calculated from

$$V_{\Delta y=1} = \frac{1}{n_{ch}(T, \mu_B)} \approx \frac{1.2}{\Delta y} \frac{dN_{ch}^{AuAu}}{d\eta},$$

(8)

[the coefficient 1.2 is needed to recalculate the number of particles per unit pseudorapidity ($\eta$) interval to that per unit rapidity ($y$) interval [22]. Here $n_{ch}$ is the charged hadron density calculated in the HG model.

The average number of the initially produced $c\bar{c}$ pairs is proportional to the number of binary nucleon-nucleon collisions: $\overline{N}_{c\bar{c}} = N_{coll}(b) \sigma_{c\bar{c}}^{NN}/\sigma_{inel}^{NN}$. The charm production cross section, $\sigma_{c\bar{c}}^{NN}$, has been measured at RHIC by the PHENIX Collaboration [31]. The result is consistent with PYTHIA calculations: $\sigma_{c\bar{c}}^{NN} \approx 650$ $\mu$b.

The SCM is applicable only to large systems: $N_p > 100$ in Pb+Pb at the SPS [2, 3]. Therefore, PHENIX's $p+p$ point and the leftmost Au+Au point, corresponding to $N_p \approx 30$, cannot be used in the SCM fit procedure. From this reason, we restrict ourselves only to a rough estimation of the SCM prediction for the $J/\psi$ yield at midrapidity at the top RHIC energy.

We fix the charm production cross section in nucleon-nucleon collisions at its PYTHIA value, $\sigma_{c\bar{c}}^{NN} = 650$ $\mu$b. There is no experimental data for the value of $\xi_{\Delta y=1}$, but one can roughly estimate it assuming approximately the same rapidity distribution of the open charm and $J/\psi$'s in $p+p$ collisions. This leads to $\xi_{\Delta y=1} \approx 0.3$. The charm rapidity distribution in Au+Au collisions can be broader than in $p+p$ due to rescattering of $c$ and $\bar{c}$ by sweeping nucleons. This will not change our result essentially, however. The estimation of the total charm production cross section is based on the single electron measurement at midrapidity. Extrapolation to the total phase space has been done assuming that the charm rapidity distribution does not change from $p+p$ to Au+Au. The charm production rate per binary collision at midrapidity was found to be independent of the centrality (at least within the present measurement accuracy). This means that the value of the total charm production cross section would grow with the centrality, if there were a broadening of the rapidity distribution. Both effects, the decreasing of $\xi_{\Delta y=1}$ and the increasing of $\sigma_{c\bar{c}}^{NN}$, nearly cancel each other in Eq. (7) and the prediction of SCM does not change significantly.

The result is shown in Fig. 1. The SCM dependence of the $J/\psi$ rapidity density per binary collision on the centrality is almost flat at $N_p \gtrsim 100$, in contrast to the total $J/\psi$ yield, where a $J/\psi$ enhancement is expected [32]. This difference appears because the hadronization volume at midrapidity, $V_{\Delta y=1}$, grows with the centrality faster than the total volume $V$.

In conclusion, we have compared predictions of three different models for $J/\psi$ production at the top RHIC energy $\sqrt{s} = 200$ GeV. None of the models are favored and none is excluded by the data. The statistical coalescence model predicts a few times larger $J/\psi$ production rate than the standard suppression models. Hopefully, measurements during the next Au+Au run will be able to clarify whether charmonia are formed only at the initial stage of the reaction (the standard suppression models) or production at the late stage via $c\bar{c}$ coalescence (SCM) is dominant. A crucial test for the SCM would be a measurement of the centrality dependence of $\psi'$ to $J/\psi$ ratio in Au+Au collisions. It should be constant (excluding the peripheral collision region) and equal to the value in equilibrium HG, if SCM is valid.

The two standard charmonium suppression models, the gradual suppression by comovers and the abrupt suppression by QGP, give quite similar predictions. High quality data with small errorbars are needed to clarify, which of the models describes adequately the charmonium suppression process, if contribution from $c\bar{c}$ coalescence is not significant.

What is the charm production mechanism at SPS energies? Here both the standard suppression models [1] and SCM [32], as well as their combination [32] are also able to reproduce the data. However, SCM requires an essential (by the factor of about 3.5) enhancement of the open charm in Pb+Pb collisions. There is an indirect experimental evidence for such an enhancement [32] and its possible mechanism has been considered [34]. Still, only a direct experimental verification [35] can give the final answer.

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[36] The $J/\psi$ multiplicity in $p + p$ is related to the cross section by the standard formula $N_{pp}^{J/\psi}/dy|_{y=0} = 1/\sigma_{pp}^{\text{inel}} d\sigma_{NN}/dy|_{y=0}$, where $\sigma_{pp}^{\text{inel}}$ is the total inelastic $p + p$ cross section.
[37] Only statistical errors are taken into account in the calculation of $\chi^2$. The most likely value of the $J/\psi$ yield in the most central collisions is not reported. Therefore, this point is not used in our fit procedure.
[38] This differs from Ref. [10], where exact equality, $k_c = k_{\bar{c}}$, within the chosen interval $\Delta y$ is assumed. In fact, net charm is exactly zero only in the total system. In any finite rapidity interval, event-by-event fluctuations with $k_c \neq k_{\bar{c}}$ are possible.
[39] At RHIC, the strangeness as well as all other conserved charges, excluding charm, can be safely considered in the grand canonical ensemble.