Mass Varying Neutrinos in Supernovae

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Abstract

We study limits for the mass varying neutrino model, using constraints from supernova neutrinos placed by the r-process condition, $Y_e < 0.5$. Also, we use this model in a supernova environment to study the regions of survival probability in the oscillation space parameter ($\tan^2 \theta$ and $\Delta m^2_{0}$), considering the channel $\nu_e \rightarrow \nu_s$ and $\bar{\nu}_e \rightarrow \bar{\nu}_s$. We observe that r-process nucleosynthesis does not put severe constraints in our model, although new regions of isocurves of probability appear when our parameterization includes an inversion in the hierarchy between the neutrino eigenvalues $m_1$ and $m_4$.

1 Introduction

Evidences from experimental data of type Ia supernovae (SNIa) [1, 2], cosmic microwave background (CMB) radiation [3, 4] and large scale structure (LSS) [5] point out that our universe is in accelerated expansion. One possible explanation for this dark energy issue (70% of the energy content of our universe) would be a cosmological constant ($\Lambda$), or simply, the non-zero vacuum energy, which is quite close to the critical cosmological energy density, $\rho_c \approx \rho_{\Lambda} \approx 4 \times 10^{-47}\text{GeV}^4$. However one critical point of $\Lambda$ is the necessity of a fine tuning, once that according to theoretical expectations, $\rho_{vac}$ is $10^{50} - 10^{120}$ larger than the magnitude allowed in cosmology. In reference [6], Dolgov emphasizes:

1. Why vacuum energy, which must stay constant in the course of cosmological evolution, or dark energy, which should evolve with time quite differently from the normal matter, have similar magnitude just today, all being close to the value of the critical energy density?

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If universe acceleration is induced by something which is different from vacuum energy, then what kind of field or object creates the observed cosmological behavior? Or could it be a modification of gravitational interactions at cosmologically large distances?

From the questions above, other candidates for dark energy have been proposed, such as: quintessence, K-essence, tachyon field, phantom field, dilatonic dark energy, Chaplygin gas (for excellent reviews see [7, 8]).

Neutrinos are powerful tools for astronomy investigation [8] and they play a crucial role in the mechanism of supernova explosion, according to the standard scenario [9]. Also, due to their peculiar features, supernova environments are potentially interesting to obtain information about possible new physics.

Based on the fact that the scale of the neutrino mass squared difference \((0.01 \text{ eV})^2\) is similar to the dark energy scale, the possibility of neutrinos coupling with a scalar field naturally arises. In this picture, the total energy of the fluid can vary slowly as the neutrino density decreases [10]. Consequently, the coupling with a scalar field causes a variation on the neutrino mass (dynamical field) with the neutrino density and perhaps with the baryonic matter [11]. This picture has been called Mass Varying Neutrino models, MaVaN. Bjalde et al. [12] showed that the proposed MaVaN models, in the non-relativistic regime, can be threatened by the strong growth of hydrodynamic perturbations associated with a negative adiabatic sound speed squared, many articles have treated neutrino oscillation via the mechanism of MaVaN [13, 14, 15, 16, 17, 18, 19, 20, 21]. We observe, nevertheless, that these articles focus on solar and atmospheric neutrinos, as well as observations from accelerators and reactors. However, the assumption of MaVaN mechanism in the neutrino evolution in supernovae environments has not yet been fully explored. In fact, the only article that connects MaVaN and supernovas is Li et al. [22] which uses data from a type Ia supernova to limit the interactions between neutrinos and scalar dark energy.

In this paper, therefore, we investigate the consequences of MaVaN mechanisms for the neutrino propagation in a supernova environment. We take into consideration that recent results from WMAP-7 [23] indicate that the number of species of relativistic neutrinos is equal to 4.34±0.87 which, in contrast with the standard model with 3 active neutrinos, suggests that there may be another species of neutrinos, for example, a sterile one \(\nu_s\), possibly associated with SU(2) singlets. These sterile neutrinos can present masses ranging from a few eV to values larger than the electroweak scale [24]. Therefore, a natural and potentially interesting investigation can arise from the analyzes of neutrino oscillations \(\nu_e \rightarrow \nu_s\) and \(\bar{\nu}_e \rightarrow \bar{\nu}_s\) in supernovae under the assumption that MaVaN models are implemented. We evaluate the consequences of such neutrino oscillations on the signal of \(\bar{\nu}_e\) and \(\nu_e\) in terrestrial detectors, as well as on supernova heavy-element nucleosynthesis.

The crucial point of our analysis is the fact that MaVaN model can directly affect the relevant parameter for neutrino oscillations, namely, the squared mass difference \(\Delta m^2\). We propose a phenomenological MaVaN model that is more convenient to test using neutrinos from supernova. Such parameterization allows to fit the amplitude of the \(\Delta m^2\) variation, the position where the variation begins and also if the masses will increase or decrease along the

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1There are alternative scenarios to explain the acceleration, such as modified gravity, where occurs an introduction of quantum effects - higher curvature corrections - to the Einstein-Hilbert action.
neutrino trajectory. We use this parameterization to evaluate the constraints on the oscillation space parameter $\sin^2 2\theta \times \Delta m^2_0$ (where $\theta$ and $\Delta m^2_0$ are the mixing angle and the squared mass difference in vacuum, respectively) coming from the r-process nucleosynthesis condition, $Y_e < 0.5$, and by the limit for the average survival probability coming from SN1987A data, $\langle P_{ee} \rangle < 0.5$. Problems and criticisms to these MaVaN models were pointed by Peccei in [25].

We conclude that, with our MaVaN parameterization, significant modifications happen in the probability space parameter for anti-neutrinos when the mass modification is slow and for an inversion of hierarchy during the propagation. Also, for neutrinos, new regions of exclusion are also found and the r-process nucleosynthesis is not affected by the MaVaN assumption.

This article is organized as follows: section 2 presents the neutrino oscillation mechanism and the assumptions and approximations used in our work. Section 3 presents our proposition of a new parameterization of MaVaN model. Section 4 contains our results and related discussion. Finally section 5 summarizes our conclusions.

## 2 Neutrino Oscillation

Some comments about the assumptions for the neutrino oscillations we are interested in this article are in order. From the neutrino phenomenology we know the existence of two small scales, $\Delta m^2_{\text{atm,sun}}$ and two non-small mixing angles, $\theta_{21}$, $\theta_{23}$ and one small angle $\theta_{13}$. We are interested in oscillations between active electronic into sterile neutrinos as well as oscillations between active electronic into sterile anti-neutrinos in a supernova environment. For this we will include a the new scale, $\Delta m^2_0$ for neutrinos oscillations with sterile neutrinos. These new scale need to much bigger then the small scales for active neutrinos, $\Delta m^2_0 = 1 - 10^4$ eV$^2 > \Delta m^2_{\text{atm,sun}}$ to be compatible with constrains for sterile neutrinos. For an analysis of sterile neutrino oscillations in cosmological, astrophysical and terrestrial media with various mixtures and a wider range of $\Delta m^2$ see [26].

Such oscillations must be treated considering that both neutrino and anti-neutrino conversion can be resonantly enhanced in matter, the so-called Mikheyev-Smirnov-Wolfenstein (MSW) phenomena [27]. Nevertheless, for a convenient choice of the squared mass difference in vacuum, which we will denote $\Delta m^2_0$, we are allowed to neglect the conversion to sterile neutrinos in the inner core, for all values of the mixing angles we considered [28]. Furthermore, this same choice of the $\Delta m^2_0$ parameters allow neglecting $\nu - \nu$ forward-scattering contribution to the weak potentials in first approximation after neutrinos have escaped from the inner core of the star\(^2\).

The equation which governs the flavor neutrino eigenstates evolution along their trajectory inside the supernova can be written as:

$$i \frac{\partial}{\partial r} \begin{bmatrix} \Psi_e(r) \\ \Psi_s(r) \end{bmatrix} = \begin{bmatrix} \phi_e(r) \\ \sqrt{\sigma} \end{bmatrix} \begin{bmatrix} \Psi_e(r) \\ \Psi_s(r) \end{bmatrix},$$  \hspace{1cm} (1)

where

$$\phi_e(r) = \frac{1}{4E}(\pm V(r)E - \Delta m^2_0 \cos 2\theta),$$  \hspace{1cm} (2)

\(^2\)Recently issues of self neutrino interactions, in the time we are considering (for the cooling stage), were discussed by A. Friedland in [29].
and
\[ \sqrt{\sigma} = \frac{\Delta m^2_0}{4E} \sin 2\theta. \] (3)

Under the assumptions described above, the matter potential can be written as follows
\[ V(r) = 2\sqrt{2}G_F \left[ N_{e^-}(r) - N_{e^+}(r) - \frac{N_n(r)}{2} \right]. \] (4)

In Eq. (2), the signal + is for neutrino, whereas the − signal is for anti-neutrinos. \( \Delta m^2_0 \) is the squared mass difference between two neutrinos mass eigenstates in vacuum; \( G_F \) is the Fermi coupling constant; \( \theta \) is the vacuum mixing angle and \( N_{e^-}(r) \), \( N_{e^+}(r) \) and \( N_n(r) \) are, respectively, the number densities of electrons, positrons and neutrons. Note that neutrinos (anti-neutrinos) undergo in a resonance when
\[ V(r) = \pm \frac{\Delta m^2_0}{2E} \cos 2\theta. \] (5)

The matter potential can be rewritten as
\[ V(r) = \frac{3G_F \rho(r)}{2\sqrt{2}m_N} \left( Y_e - \frac{1}{3} \right). \] (6)

where \( Y_e \) is the electronic fraction
\[ Y_e(r) = \frac{N_{e^-}(r) - N_{e^+}(r)}{N_p(r) + N_n(r)}, \] (7)

The spectrum of neutrino emission and it can be written as [30]:
\[ \frac{dN}{dE} = \frac{L}{F(\eta)T^4} e^{E/T(1 - \eta - \frac{1}{3})}. \] (8)

In the expression above, Eq. (5), \( \eta \) is the pinching factor and \( F(\eta) = \int_0^\infty dx x^3/(e^x - \eta + 1) \). For this spectrum \( \langle E \rangle/T \approx 3.1514 + 0.1250\eta + 0.0429\eta^2 + O(\eta^3) \). Typical values of \( \eta \) are \( \eta_{\nu_e} \sim 2, \eta_{\bar{\nu}_e} \sim 3 \) and \( \eta_x \sim 1 \) [31]. The average energy of neutrino species is \( \langle E_{\nu_e} \rangle \approx 11 \text{ MeV}, \langle E_{\bar{\nu}_e} \rangle \approx 16 \text{ MeV}, \langle E_{\nu_x} \rangle \approx 25 \text{ MeV} \) \( (x = \mu, \tau) \). To calculate the probability in regions that are not adiabatic, we numerically solve Eq. (1). And for regions that are adiabatic we use the analytical expression for the survival probability \( 0.5 + 0.5 \cos 2\theta \cos 2\tilde{\theta}_0 \) (\( \tilde{\theta}_0 \) is the matter mixing angle in the point of neutrino production). The calculation of average probability is done with a average over the spectrum of neutrino emission described in Eq. (8).

We discuss our MaVaN phenomenological model in next session.

3 Mass Varying Neutrino model

We will adopt a phenomenological approach in modeling the MaVaN mechanism [4]. We parameterize the effects of new physics directly in \( \Delta m^2 \), since we are interested in neutrinos oscillation and their implications on the oscillation probability and the r-process in the
supernova. We present a model in which $\Delta m_{\text{MaVaN}}^2 \equiv \Delta \tilde{m}^2$ can increase or decrease as the neutrino propagates in the supernova medium, with the following parameterization:

$$\Delta \tilde{m}^2 = \Delta m_0^2 - \frac{\delta}{1 + (n_\nu/n_\nu^0)^{-\eta}}, \quad (9)$$

where $n_\nu$ is the neutrino density in the supernova environment, and the parameters $n_\nu^0$ and $\eta$ control the profile of neutrino mass dependence.

The mass variation occurs around $n_\nu^0$, and for $n_\nu << n_\nu^0$ ($n_\nu >> n_\nu^0$) the value of $\Delta \tilde{m}^2$ tends to the asymptotic value of $\Delta m_0^2$ ($\Delta m_0^2 - \delta$). The parameter $\eta$ controls how fast the asymptotic values are achieved.

The neutrino evolution with MAVAN is described by changing $\Delta m_0^2 \rightarrow \Delta \tilde{m}^2$ in Eqs.(1,2,3,4,5) for neutrinos and anti-neutrinos.

Our parameterization is very versatile in reproducing the behavior of any specific models which includes MaVaN. For instance, the model presented in [15] is very well reproduced by choosing $\delta = 10 \text{ eV}^2$, $\eta = 1$ and $n_\nu^0 = 34500 \text{ cm}^{-3}$.

Clearly, if $\delta = 0$ we have the “standard” scenario, in which neutrinos have a constant mass. According to the model, if $\delta > \Delta m_0^2$, then we can obtain $\Delta \tilde{m}^2 < 0$ somewhere inside the supernova. This tell us that the hierarchy is being inverted in some point of the propagation.

Figures [1] shows the evolution of $\Delta \tilde{m}^2$ with the supernova radius ($r$) by Eq. (9) for some parameters choices.

4 Results and Discussion

It is possible to constraint the oscillation parameters, such as the mixing angle and the mass-squared difference, from the SN1987A data [33, 34, 35]. One of them comes from the spectrum of the observed events, probably dominated by $\bar{\nu}_e$. From the observed energy spectrum of SN1987A, the analysis from Kamiokande indicate that the temperature of $\bar{\nu}_e$ is lower than the expectation [36]. The second constraint is based that the expanding envelope driven by thermal neutrino wind of the supernova is a possible site of heavy nuclei formation beyond iron (r-process nucleosynthesis). Since the $\nu_e$ and $\bar{\nu}_e$ conversion into a sterile state (or even into an active state) are different, the fraction of neutrons to protons, determined by $\bar{\nu}_e + p \rightarrow e^+ + n$ and $\nu_e + n \rightarrow e^- + p$, is modified by the oscillation mechanism in a radius of about a hundred kilometers. Related to this constraint, a very simple bound is $Y_e < 0.5$, which can be extracted from Eq. (17) simply considering a very neutron-rich environment [37, 38]. In [38], it is shown that $Y_e$ has a dependence on the neutrino oscillations probabilities by

$$Y_e \sim \frac{1}{1 + P_\nu(E_{\nu_e})/P_\bar{\nu}(E_{\bar{\nu}_e})}, \quad (10)$$

given in Eq. (17) of [38], where $P_\nu$ and $P_\bar{\nu}$ are, respectively, the survival probabilities of antineutrinos and neutrinos. From this expression, we clearly see that it is possible to constraint the oscillation parameters from $Y_e < 0.5$. Also in [36], it is also discussed another constraint based on the fact that the first or the second event of Kamiokande could be related with $\nu_e$. 

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We divide this section into two parts. The first one concerns the analysis of the oscillation of active electronic (anti)neutrinos to sterile (anti)neutrinos. In the second section we discuss the possible limits imposed by r-process nucleosynthesis in the oscillation space parameter. We emphasize that we are seeking changes caused by our MaVaN model, presented in section 3.

4.1 (Anti)neutrinos

Here we calculate the survival probability ($P_{ee}$) for the $\bar{\nu}_e \rightarrow \bar{\nu}_s$ channel of oscillation. For this purpose the calculation of this survival probability is done analytically for regions where the neutrino propagation is adiabatic. When there is a non-adiabatic region we calculate numerically using Eq. (1). We emphasize that the MaVaN model changes the regions where resonances occur, since there is a modification of $\Delta \tilde{m}^2$ along the neutrino path and these values enter in place of $\Delta m^2_0$ in Eq. (5). The electronic potential profile we have used is for a supernova 1 second after supernova bounce ($t_{pb} > 1s$), where the size of the neutrinosphere is approximately equal to the core size and the supernova is in the stage where heavy nuclei start to form. Based on the data from SN1987A and neutrino oscillation, several articles, such as [38], discuss that the survival probability cannot be less than 0.5. In this way it is possible to find regions of exclusion in the oscillation space parameters ($\Delta m^2_0$, $\tan^2 \theta$). We explore here $0 \leq \theta \leq \pi/4$, called light side, and $\pi/4 \leq \theta \leq \pi/2$, called the dark side, which
can be interpreted for neutrino oscillation with \( 0 \leq \theta \leq \pi/4 \). So our results can also be extended for the neutrino oscillation case, where dark side of anti-neutrinos will be light side for neutrinos and vice versa.

We will find probabilities in the context of the MaVaN parameterization, given by Eq. (9). We consider four sets of \((\eta, n_\nu^0)\) values for each \(\delta\) (in eV\(^2\)) which will assume values of 2, -2, 20, -20. They are: \((0.5, 1 \times 10^{28} \text{ cm}^{-3})\), \((0.8, 1.5 \times 10^{30} \text{ cm}^{-3})\), \((3.0, 1 \times 10^{32} \text{ cm}^{-3})\) and \((10.5 \times 10^{32} \text{ cm}^{-3})\). All these sets ensure that \(\Delta \tilde{m}^2\) reaches the vacuum value inside the supernova. Also they are only illustrative, i.e., another values could be used for similar results.

We start our analysis with the curve evolution set \((\eta, n_\nu^0) = (0.5, 1 \times 10^{28} \text{ cm}^{-3})\) for \(\delta = 20\) eV\(^2\). In Fig. 2 we note the presence of a new area in the region of \(\tan^2 \theta \approx 10^{-6} - 10^6\) and around \(\Delta m_0^2 \approx 1 - 10\) eV\(^2\). Above this region the result is close to the situation without MaVaN (\(\delta = 0\)). In Figs. 2-5 the thicker curves represent the case without MaVaN. The presence of new regions of probability is explained by the fact that \(\Delta \tilde{m}^2\) reaches negative values, i.e., \(\tilde{m}_4 < \tilde{m}_1\), and this creates new regions of resonance, which can be seen in Fig. 3 where resonance points are represented by the crossings between neutrino potential curve and the resonances curves, which are calculated by Eq. (3). We emphasize that, until \(r \approx 13.3\) km (bottom part of Fig. 3), for the situation without MaVaN and normal hierarchy (\(\tilde{m}_4 > \tilde{m}_1\)), the first region of the potential is an important region for resonances of anti-neutrinos for \(0 \leq \theta \leq \pi/4\). The top part of Fig. 3 shows also the important region of resonances when \(\pi/4 \leq \theta \leq \pi/2\).

However, if \(\Delta \tilde{m}^2\) is negative in some region inside the supernova, up to three resonances may occur for \(\tilde{\nu}_e\). It is precisely the creation of new resonant points that will generate new regions in the parameter space for a given value of probability.

We also note in Fig. 2 that for very similar values of \(\Delta m_0^2\) and a single value of \(\tan^2 \theta\), there may be very different values of survival probability. Fig. 3 shows the electronic potential \(V_e\) (solid line) and the resonance curves for \(\Delta m_0^2 = 1\) eV\(^2\) (dashed line) and \(\Delta m_0^2 \approx 1.4\) eV\(^2\) (dotted line), for a value of \(\tan^2 \theta \approx 2.5 \times 10^{-5}\) and \(\delta = 2\) eV\(^2\). These values are merely illustrative and other values could have been chosen. For \(\Delta m_0^2 \approx 1.4\) eV\(^2\) we notice that there are two consecutive resonances, keeping the survival probability practically constant. For \(\Delta m_0^2 = 1\) eV\(^2\) the survival probability changes when the electronic anti-neutrino crosses the first resonance, as it is expected. After the electronic anti-neutrino propagates and crosses the second resonance, the probability fall is interrupted at the exact moment when the transition of hierarchy occurs from a negative value to the positive one, resulting in a different value of probability compared with the \(\Delta m_0^2 \approx 1.4\) eV\(^2\) case.

In Fig. 4, which represents the state of evolution \((0.8, 1.5 \times 10^{30} \text{ cm}^{-3})\) for \(\delta = 20\) eV\(^2\), we see the reduction of the effect, since the change in hierarchy is brought further back and some regions of new resonances are lost. For \(\delta = 2\) eV\(^2\), the effects are not so relevant, since the durability of the inversion of hierarchy is small.

For sets of \((\eta, n_\nu^0)\) equal to \((3.0, 1 \times 10^{32} \text{ cm}^{-3})\) and \((10.5 \times 10^{32} \text{ cm}^{-3})\) there is a great coincidence of the survival probabilities with the situation without MaVaN. This happens because the evolution of \(\Delta \tilde{m}^2\) is so fast in both sets that it reaches rapidly \(\Delta m_0^2\) and kills the possibility of creation of new resonances. Since the curves of MaVaN and \(\delta = 0\) practically match, we do not show them here.

Now we present the results when \(\delta\) assumes a negative value. In Fig. 5 we show the
results for $\delta = -20\ \text{eV}^2$ and $(\eta, n_0^0) = (0.5, 1 \times 10^{28}\ \text{cm}^{-3})$, since for these values we get the most significant modification. We do not show the other situations, because there is a complete match among the curves of MaVaN and without MaVaN. For $\delta = -20\text{eV}^2$ the effect is more significant, especially for values of small $\Delta m_0^2$, because the effect of a higher $\delta$ is more important in these regions, since the resonance position can be modified or simply removed by MaVaN effects. Here the change of hierarchy does not happen, then we do not expect any new resonances and, therefore, very different and unusual regions defined by the probability curves will disappear, since the resonance regions of anti-neutrinos are still for $r < 13.3$ km (light side) and $r > 13.3$ km (dark side), when $\delta$ is negative. Again we see that as $\Delta m^2$ reaches faster the value of $\Delta m_0^2$, the probability curves will coincide for the MaVaN and the usual situation, $\delta = 0$. We do not show for $\delta = -2\ \text{eV}^2$ since the effect is very small and can be neglected.

### 4.2 r-process

The production of heavy nuclei in supernova is still an open problem. The region between the protoneutron star and the escaping shock wave a few seconds after the bounce may be a good site for this process with high entropy and an excess number of neutrons. From $\beta$ reactions, $\bar{\nu}_e p \rightarrow n e^+$ and $\nu_e n \rightarrow p e^-$, a modification in the $n/p$ fraction could happen if one considers neutrino oscillation \[37, 39\]. We will analyze what happens with this condition in
the context of our MaVaN parameterization and for $\bar{\nu}_e \rightarrow \bar{\nu}_s$ and $\nu_e \rightarrow \nu_s$. We limit our analysis of neutrino flavor conversion to about $r < 50$ km, i.e., a region where there is not any shock wave and has a higher "influence" on anti-neutrinos. This radius is approximately the region where the r-process nucleosynthesis happens - for a review in this subject see [40].

Since the analysis is limited to $r \approx 50$ km, when there is a slow evolution of $\Delta \tilde{m}^2$ to the vacuum value, i.e., for the sets $(0.5, 1 \times 10^{28}$ cm$^{-3}$) and $(0.8, 1.5 \times 10^{30}$ cm$^{-3}$), no resonances will happen for electronic anti-neutrinos since there is an inversion of hierarchy considering $\delta = 2, 20$ eV$^2$. However, for neutrinos, there are resonances, but all of them are adiabatic with different initial conditions, so, the r-process with the MAVAN mechanism always give $Y_e < 0.5$, independent of oscillation parameters. Even for $\delta = -2, -20$ eV$^2$ where the initial conditions are modified, the $Y_e < 0.5$ is also maintained.

For the sets, $(3.0, 1 \times 10^{32}$ cm$^{-3}$) and $(10.5 \times 10^{32}$ cm$^{-3}$), considering any $\delta = 2, 20, 2, -20$ eV$^2$, where $\Delta \tilde{m}^2$ reaches very fast $\Delta m^2$, we obtain approximately the same Fig. 7 of reference [38], since, as we commented before, these situations are very similar to the situation without MaVaN ($\delta = 0$).

The evolution of $\Delta \tilde{m}^2$ represented by the dotted line in Fig. 4 and its implication in the r-process nucleosynthesis and oscillation probabilities were discussed in [41].
Figure 4: Isocurves of survival probability ($P_{ee}$). The dashed represents $P_{ee} = 0.1$, the solid one $P_{ee} = 0.5$ and the dotted curve $P_{ee} = 0.7$. The curve evolution set $(\eta, n^0_\nu) = (0.8, 1.5 \times 10^{30} \text{ cm}^{-3})$ for $\delta = 20 \text{ eV}^2$. In this figure, the thicker curves represent the case without MaVaN ($\delta = 0$).

5 Conclusions and Outlook

In this paper we proposed a new phenomenological parameterization for the variation of the relevant neutrino oscillation parameters generally present in MaVaN models. This parameterization has the advantage of being very versatile and easily adaptable to any theoretical and phenomenological approach derived from MaVan models. Considering the formulation of Eq. (9), we have analyzed the modification in the neutrino survival probability for the channels $\bar{\nu}_e \rightarrow \bar{\nu}_s$ and $\nu_e \rightarrow \nu_s$ compared to the case $\delta = 0$, which corresponds to the absence of MaVaN. In general, we observe that new regions in the neutrino oscillation parameter space appear when $\delta$ assumes positive values. In particular, we observe this behavior for $\delta = 20 \text{ eV}^2$, and $(\eta, n^0_\nu)$ equals to $(0.5, 1 \times 10^{28} \text{ cm}^{-3})$ and $(0.8, 1.5 \times 10^{30} \text{ cm}^{-3})$. When the amplitude gets smaller, then the effects also will be smaller. On the contrary, when $\delta$ assumes negative values, no significant modification, if we compare with the positive case, happens.

In both cases cited before, the r-process nucleosynthesis condition $Y_e < 0.5$ is always guaranteed and, consequently, no significant new constraints to the oscillation parameter space appear from the enforcement of this condition.

Furthermore, when the evolution is settled by $(3.0, 1 \times 10^{32} \text{ cm}^{-3})$ and $(10.5 \times 10^{32} \text{ cm}^{-3})$ no significant modifications were introduced with respect to standard, where no MaVaN is present, since the evolution of $\Delta \tilde{m}^2$ quickly reaches $\Delta m^2_0$, approximately mimicking
\[ \Delta m_0^2 (\text{eV}^2) \]
\[ \tan^2 \theta \]

Figure 5: Isocurves of survival probability \( P_{ee} \). The dashed represents \( P_{ee} = 0.1 \), the solid one \( P_{ee} = 0.5 \) and the dotted curve \( P_{ee} = 0.7 \). The curve evolution set \( (\eta, n_0^\nu) = (0.5, 1 \times 10^{28} \text{ cm}^{-3}) \) for \( \delta = -20 \text{ eV}^2 \). In this figure, the thicker curves represent the case without MaVaN (\( \delta = 0 \)).

\( \delta = 0 \) probabilities. In the last section, we pointed out that these sets will reproduce the Fig. 7 of \cite{38} for the r-process.

We point out that these kind of studies, without the MaVaN approach, were made in the past by \cite{37, 38}. The reaction \( \bar{\nu}_e p \) shows a cross section much larger than the reaction \( \nu_e e \), so the number of events expected in future detectors and detected in SN1987 from the inelastic scattering is bigger than the elastic scattering. The direction of motion of the charged lepton on the incident neutrino is well preserved in \( \nu e \) collisions. For reactions \( \bar{\nu}_e p \), the angular distribution is practically isotropic (there is a backward asymmetry). For elastic scattering \( \nu e \) it is expected that the distribution of the signal to be forward-peaked. Motivated by the anomaly of the directionality of detected events in Kamiokande-II and IMB for SN1987A, it was proposed that some of the events detected were due to electronic neutrinos. However, such events had an unexpectedly high energy, but this energy should be smaller since the neutrino would carry out a part of it after the elastic scattering. Moreover, this process is extremely forward-peaked, especially for neutrinos with high energy, and hence such event candidates do not fit well the data from SN1987A (especially for IMB) - for a recent analysis of events detected from SN1987A see \cite{42}.

For the next galactic supernova we expect a much larger amount of events coming from elastic scattering, then we consider that a good understanding of what happens to electronic neutrinos inside the supernova, in the context of neutrino oscillations, is crucial to understand the signal that will be detected. Needless to say that, for anti-neutrinos, the sample data...
will be larger and with higher statistics the probability to find new phenomena will increase.

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