1. Planar central configurations

Consider the complex plane \( \mathbb{C} \) and \( n \) real positive numbers \( m_1, m_2, \ldots, m_n \), with \( n \geq 3 \). We denote by \( \mathcal{X} \) the hyperplane in \( \mathbb{C}^n \) defined by the equation \( \sum_i m_i z_i = 0 \), where \( z = (z_1, z_2, \ldots, z_n) \) denotes a point in \( \mathbb{C}^n \). Let \( \mathcal{X} \) be the open subset of \( \mathcal{X} \) defined by the condition \( i \neq j \implies z_i \neq z_j \). That is, if \( \Delta_{ij} \) denotes the subspace of equation \( z_i = z_j \) and \( \Delta = \cup_{i<j} \Delta_{ij} \) the collision set, it is \( \mathcal{X} = \mathcal{X} \setminus \Delta \). It is the configuration space of \( n \) point particles in \( \mathbb{C} \) and center of mass in \( 0 \). An element in \( \mathcal{X} \) is called a configuration. Consider some real coefficients \( m_{ij} \), for \( i \neq j, i, j \in \{1, \ldots, n\} \) and a regular function \( \phi : \mathbb{R}_+ \to \mathbb{R} \) defined on the positive semiline of \( \mathbb{R} \). Without loss of generality we can assume \( m_{ij} = m_{ji} \) for all \( j \neq i \). Let \( \alpha \) be a real number and let \( U : \mathcal{X} \to \mathbb{R} \) be a potential function be of type

\[ U = \sum_{i<j} m_{ij} \phi(z_i - z_j), \]

where \( \phi(z) = |z|^\alpha + 2 \) or \( \phi(z) = \log |z| \) (in case \( \alpha = -2 \)). For example if \( m_{ij} = m_i m_j \) and \( \alpha = -3 \) then this is the potential of the Newtonian \( n \)-body problem with \( \phi(z) = |z|^{-1} \) or the Thompson vortex problem if \( \alpha = -2 \) and then \( \phi(z) = \log |z| \). The charged \( n \)-body problem is obtained by setting \( \alpha = -3 \) and \( m_{ij} = m_i m_j - q_i q_j \) where \( q_i \) are the electrostatic charges of the masses.

The \( n \)-body problem concerns the motion of \( n \) particles of masses \( m_i \) and potential \( U \); the Newton equations are therefore \( m_i \ddot{z}_i = \frac{\partial U}{\partial z_i} \). A central configuration is a configuration \( z \in \mathcal{X} \) such that there exists a non-zero real scalar \( k \) such that for \( i = 1, \ldots n \)

\[ km_i z_i = \frac{\partial U}{\partial z_i}. \]

Central configurations can be seen as critical points of the restriction of \( U \) to the ellipsoid in \( \mathcal{X} \) of equations \( \sum_i m_i |z_i|^2 = 1 \). Furthermore, they play an important role in the theory of periodic orbits in \( n \)-body problems because they yield homographic solution and are topological bifurcations of the energy and angular momentum level sets in \( \mathcal{X} \). Details and further reading on the topic can be found for example in [8, 11, 7, 13].

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Let \( \hat{f} : \mathcal{X} \to \mathcal{X} \) be the map defined by \( \hat{f}(z) = w \in \mathcal{X} \), where
\[
w_i = \sum_{j \neq i} m_i^{-1} m_{ij} |z_i - z_j|^\alpha (z_i - z_j).
\]

It is not difficult to see \( \frac{\partial U}{\partial z_i} = (\alpha + 2) m_i w_i \), so that a configuration \( z \in \mathcal{X} \) is central if and only if there is a real number \( \lambda \neq 0 \) such that \( \hat{f}(z) = \lambda z \), where \( \lambda = (\alpha + 2)^{-1} k \). Let \( \text{CC} \) denote the space of all the central configurations in \( \mathcal{X} \) and \( \text{CC}_1 \) its intersection with the ellipsoid \( E \subset \mathcal{X} \) of equation \( \sum_i m_i |z_i|^2 = 1 \). By the \( O(2) \)-invariance of \( U \), \( \text{CC}_1 \) is \( O(2) \)-invariant in \( \mathcal{X} \), where \( O(2) \) is the orthogonal group of the plane \( \mathbb{C} \). We are interested in the quotient spaces \( \text{CC}_1 / O(2) \) and \( \overline{\text{CC}} = \text{CC}_1 / SO(2) \).

Consider the projection \( p : \mathbb{C}^n \setminus \{0\} \to \mathbb{C}P^{n-1} \) onto the complex projective space. A point of \( \mathbb{C}P^{n-1} \) of homogeneous coordinates \( z_i \) is denoted by \( [z] = [z_1 : z_2 : \cdots : z_n] \). Then \( p \) projects \( \mathcal{X} \setminus \{0\} \) onto the hyperplane in \( \mathbb{C}P^{n-1} \) of equation \( \sum_i m_i z_i \), which we denote simply by \( \mathbb{C}P^{n-2} \). Let \( X \) be the image of \( \mathcal{X} \) in \( \mathbb{C}P^{n-2} \). With an abuse of terminology we will consider maps defined on open dense subsets of their domains. Such a subset of points in which the map is properly defined will be clear from the context. The map \( \hat{f} \) induces a map \( f : X \to \mathbb{C}P^{n-2} \). It is clear that a central configuration projects to a fixed point of \( f \).

**Lemma 1.** The projection \( p \) induces a homeomorphism \( \overline{\text{CC}} = \text{CC}_1 / SO(2) \approx \text{Fix}(f) \).

**Proof.** The map \( p \) induces a continuous map \( \text{CC} \to \text{Fix}(f) \), and a injective continuous map \( \overline{p} : \text{CC}_1 / SO(2) \to \text{Fix}(f) \). To see that it is surjective, consider a configuration \( z \) such that \( [z] = f[z] \), i.e. a configuration \( z = [z_1 : z_2 : \cdots : z_n] \) such that there exists \( \lambda \in \mathbb{C}^* \) and for every \( i = 1 \ldots n \)
\[
\lambda z_i = \sum_{j \neq i} m_i^{-1} m_{ij} |z_i - z_j|^\alpha (z_i - z_j).
\]

This implies
\[
\lambda m_i |z_i|^2 = \sum_{j \neq i} m_{ij} |z_i - z_j|^\alpha (|z_i|^2 - z_j \bar{z}_i)
\]
\[
\implies \lambda \sum_i m_i |z_i|^2 = \sum_{j \neq i} m_{ij} |z_i - z_j|^{2+\alpha},
\]
and therefore \( \lambda \) is real, i.e. \( [z] \) is a projection of a central configuration. Moreover, \( \overline{p} \) is a closed map and hence a homeomorphism. \( \square \)

Let \( I = \sum_i m_i |z_i|^2 \) the inertia of \( z \). In case \( \alpha \neq -2 \) (not the logarithmic case), the proof of the previous lemma implies that
(1)
\[
\lambda = 2 \frac{U}{I}
\]
and hence, if \( m_{ij} > 0 \) for all \( i, j \) that \( \lambda > 0 \). If \( \alpha = -2 \) the same holds, since
(2)
\[
\lambda = \frac{\sum_{i \neq j} m_{ij}}{I}.
\]

**Lemma 2.** If for every \( i \neq j \) the coefficient \( m_{ij} \) is not zero, and \( \alpha < -1 \), then \( \text{CC}_1 \) is compact.

**Proof.** Because \( p : E \to \mathbb{C}P^{n-2} \) is a proper map, it suffices to show that \( \overline{\text{CC}} \) is compact, hence that \( \text{Fix}(f) \) is compact. First, let \( X_2 \subset \mathbb{C}P^{n-2} \) be the subset of \( \mathbb{C}P^{n-2} \) consisting of all the points such that there is at most one pair of particles colliding \( z_i = z_j \). The map \( f \) can be extended continuously to \( X_2 \): If \( z \) tends to a point \( z' \) in \( X_2 \setminus X \) with a collision in \( i \) and \( j \), then the image of \( z \) tends to the point \( [w] = [w_1 : \cdots : w_n] \) with \( w_i = m_j m_{ij} \) and \( w_j = -m_i m_{ij} \) and otherwise 0. Let \( W_{ij} \) denote such point. Provided that \( m_{ij} \neq 0 \) such a point \( W_{ij} \) is in \( \mathbb{C}P^{n-2} \).
and it is different from $z'$ (we assume that the masses $m_i$ are positive and hence it cannot be that $m_i = -m_j$):

$$W_{ij} = [0 : \ldots : 0 : m_j : 0 : \ldots : 0 : -m_i : 0 : \ldots : 0].$$

Now suppose that $[z]$ tends to a multiple collision $[c]$ in $\mathbb{CP}^{n-2} \setminus X_2$. Let $\Gamma$ be the set of indexes $(i, j)$ such that the $i$-th particle collides with the $j$-th particle in $[c]$. It is not difficult to show as above that $f(z)$ tends to a subset of the projective subspace $\hat{c} \subset \mathbb{CP}^{n-2}$ spanned by the points $W_{ij}$ with $(i, j) \in \Gamma$. But $\hat{c}$ is a closed subspace which does not contain $[c]$, hence by continuity there is a neighborhood of $[c]$ in $\mathbb{CP}^{n-2}$ without fixed points of $f$. Therefore the collision set is contained in a fixed point free neighborhood, and hence $\overline{\text{CC}}$ is closed in $\mathbb{CP}^{n-2}$. Being a closed in a compact, it is compact (compare with the proof of Shub, for the Newtonian case [12], and with the estimates of Buck [4]).}

**Lemma 3.** Let $u: X \to \mathbb{R}$ be the map defined by $u[z_1 : \ldots : z_n] = UI^{-1/2}$ if $\alpha \neq -2$, or $u[z_1 : \ldots : z_n] = e^{U}I^{-1/2}\sum_{i<j}m_{ij}$ if $\alpha = -2$. Then $\overline{\text{CC}} = \text{Crit}(u)$, where $\text{Crit}(u)$ denotes the set of critical points of $u$.

*Proof.* In both cases $u$ is well-defined on $X$. Furthermore, $\text{CC}_1$ is the set of critical points of $U$ restricted to $E$; hence it is the set of critical points of $u\pi$, where $\pi: E \to X$ is the projection, restricted to $E$. The projection $\pi$ is a submersion, hence $\overline{\text{CC}} = \text{Crit}(u)$.

Let $C$ be the group of order 2 acting on $\mathbb{CP}^{n-2}$ by conjugation on coordinates. The space fixed by $C$ is the space of collinear configurations, and it is homeomorphic to $\mathbb{RP}^{n-2}$. Let $X^C$ be its intersection with $X$. The map $f$ is equivariant with respect to the action of $C$, therefore it induces a map $f^C: X^C \to \mathbb{RP}^{n-2}$; its fixed points, by lemma [1] are the collinear central configurations.

We list some known results. Some of them were proved in the Newtonian case ($m_{ij} = m_im_j$ and $\alpha = -3$), but the techniques worked in the same way in the general case. We understand that central configurations are counted in $\text{CC}/O(2)$: For every $n \geq 3$ there are exactly $n!/2$ collinear configurations (Moulton; see Smale [14] for a proof using critical point theory). If $n = 3$ then there is just one non-collinear central configuration. If the masses are equal, then there are exactly 19 non-collinear central configurations for $n = 4$ (Albouy [1]). For every $n$ there are at least $n-2$ non-collinear central configurations (McCord [4]). If the potential $U$ is a Morse function, there are at least $n!h(n)/2$ central configurations, where $h(n) = \sum_{i=3}^{n}1/i$ (McCord [4]). The Euler characteristic $\chi(X)$ is $(-1)^n(n-2)!$, hence by Morse theory in this case the alternating sum $\sum_{k}(-1)^k\nu_k = (-1)^n(n-2)!$, where $\nu_k$ denotes the number of critical points of index $k$.

## 2. Collinear configurations

We have seen that the collinear central configurations are the fixed points of $f^C: X^C \to \mathbb{RP}^{n-2}$. We show a proof of Moulton theorem using fixed point theory, instead of critical point theory. In some sense it is closer to the original proof of Moulton.

**Proposition 4.** If $m_{ij} > 0$ for every $i, j$ and $\alpha < -1$ then every fixed point of $f$ is isolated and its fixed point index is 1.

*Proof.* Let $E^C \subset X^C$ be the ellipsoid of equation $\sum_i m_i z_i$. Because $E^C \to \mathbb{RP}^{n-2}$ is a covering map and $\lambda > 0$ (by equations [1] and [2]), the map $f': E^C \setminus \Delta \to E^C$ defined by $w \mapsto p$, with $p_i = \frac{w_i}{\sqrt{I}}$, where as above $I = \sum_j m_j w_j^2$ and $w_j$ is defined in equation [12] is a lifting of $f$. Hence the Jacobian of $f$ at a point $[c] \in X^C$ is the same as the jacobian of $f'$ at a pre-image of $[c]$ in $E^C$. We are going to show that if $x \in E^C$ is a central configuration (that is, a fixed point for $f'$), then for every vector $v$ of the tangent space of $E^C$ in $x$ (endowed with the kinetic scalar product: $v \cdot v' = \sum_i m_i v_i v'_i$) the inequality $v \cdot D(f')v < 0$ holds, where $D(f')$ denotes the
differential of $f'$ at $x$. From this the claim follows, since all the eigenvalues of $D(f')$ need be negative. The map $f'$ is the composition of $\hat{f}$ and the projection $p$, hence $D(f') = D(p)D(\hat{f})$. The derivatives of $w_i$ are

$$\frac{\partial w_i}{\partial z_k} = \begin{cases} (\alpha + 1) \sum_{j \neq i} m_i^{-1} m_{ij} |z_i - z_j|^\alpha & \text{if } i = k \\ - (\alpha + 1) m_i^{-1} m_{ik} |z_i - z_k|^\alpha & \text{if } i \neq k. \end{cases}$$

Hence

$$\sum_{k=1}^{n} \frac{\partial w_i}{\partial z_k} = (\alpha + 1) \sum_{j \neq i} m_i^{-1} m_{ij} |z_i - z_j|^\alpha (v_i - v_j).$$

The derivatives of $p$ are

$$\frac{\partial p_i}{\partial w_k} = \begin{cases} I^{-3/2} \sum_{j \neq i} m_j w_j^2 & \text{if } i = k \\ - I^{-3/2} m_k w_i w_k & \text{if } i \neq k. \end{cases}$$

Hence

$$\sum_{i=1}^{n} m_i v_i \frac{\partial p_i}{\partial w_k} = m_k v_k \frac{\partial p_k}{\partial w_k} + \sum_{i \neq k} m_i v_i \frac{\partial p_i}{\partial w_k}.$$ 

Now, $v$ belongs to the tangent space at $x$ and if $x$ is a central configuration then $\sum_i m_i w_i v_i = 0$, therefore

$$\sum_{j \neq k} m_i v_i w_i = -m_k v_k w_k,$$

hence

$$\sum_{i \neq k} m_i v_i \frac{\partial p_i}{\partial w_k} = - I^{-3/2} \sum_{i \neq k} m_i v_i w_i m_k w_k = - I^{-3/2} m_k w_k.$$

Therefore

$$\sum_{i=1}^{n} m_i v_i \frac{\partial w_i}{\partial z_k} v_k < 0$$

is true. But by equation 3 the latter is equal to

$$\sum_{j \neq i} (\alpha + 1) m_{ij} |z_i - z_j|^\alpha (v_i - v_j) v_i = (\alpha + 1) \sum_{i < j} m_{ij} |z_i - z_j|^\alpha (v_i - v_j)^2,$$

which is negative because by assumption $\alpha + 1 < 0$ and $m_{ij} > 0$. This finishes the proof. 

**Proposition 5.** If $m_{ij} > 0$ for every $i < j$ and if $\alpha < -1$ then the fixed point index of $f^C$ in each of the $n!/2$ components of $X^C$ is 1.

**Proof.** The space $X^C$ has $n!/2$ connected components (see for example Smale [14]). Consider, for every $t \in I$, the map defined by $f_t(z) = w_i$, with

$$w_i = \sum_{j \neq i} m_i^{-1} (tm_{ij} + (1 - t)m_i m_j) |z_i - z_j|^\alpha - 2 + 2t$$. 

It is the self-map corresponding to the collinear $n$-body problem with parameters $\alpha' = t \alpha - 2 + 2t$ and $m_{ij'} = tm_{ij} + (1 - t)m_i m_j$ and masses $m_i$. By lemma 2, Fix($f_i$) is compact for every $t \in I$. Therefore $f_i$ yields a compactly fixed homotopy from $f_1 = f$ to a map
$f_0 : X^C \to \mathbb{R}P^{n-2}$. The map $f_0$ is the self-map arising from the logarithmic potential collinear problem with masses $m_i$. Now we are going to define a compactly fixed homotopy $g_t$ from $f_0 = g_0$ to a map $g_1$ corresponding to the self-map of the problem with all the masses equal to 1 and the last mass equal to $m$. Let $m'_i = (1-t)m_i + t$ and $m''_i = (1-t)m_i + tm$; and let the hyperplane $\mathbb{R}P^{n-2}_t$ be defined by the equation $\sum_i m'_iz_i = 0$ in $\mathbb{R}P^{n-1}$ with homogeneous real coordinates $z_i$. By definition $X \subset \mathbb{R}P^{n-2} = \mathbb{R}P^{n-2}_0 = \mathbb{R}P^{n-2}_1$. Then there is a family of homeomorphisms $\varphi_t : \mathbb{R}P^{n-2}_0 \to \mathbb{R}P^{n-2}_1$. Let $G_t : \mathbb{R}P^{n-2}_t \setminus \Delta \to \mathbb{R}P^{n-2}_t$ be the self-map yielded by the $n$-body problem with masses $m'_i$, $m_{ij} = m_im_j$ and $\alpha = -2$. The composition $g_t = \varphi_t G_t \varphi_t^{-1}$ is a compactly fixed homotopy from $g_0 = f_0$ to a self-map conjugated to the self-map $h_m$ of the problem with logarithmic potential and masses $1, 1, \ldots, m$. Now we prove, somehow as in the proof of Moulton [10], that the fixed point index of $h_m$ in each component of $X^C$ is 1 for every $m$. By induction on the number of bodies $n$. If $n = 3$, then this is true. Otherwise, consider the map $h_m$. It is possible to let $m$ be equal to zero (the case of the infinitesimal mass), and everything that we have done in the previous section can be carried out literally. By looking at the proof of lemma [4], it is easy to see that if $m = 0$ then $h_0$ is compactly fixed, like in the case $m > 0$. Therefore $h_m$ is compactly fixed homotopic to $h_0$, and the fixed point indexes of $h_0$ coincide with the fixed point indexes of $h_m$. So consider the projection $\pi : \mathbb{R}P^{n-2} \to \mathbb{R}P^{n-3}$ which sends $[z_1 : \ldots : z_n]$ to $[z_1 : \ldots : z_{n-1}]$. It is well-defined, because if $m_n = 0$ then $\sum_i m_iz_i = 0 \implies m_1 + \ldots + m_{n-1} = 0$. The map $h_0$ induces a map $h'$ arising from the $(n-1)$-body problem with masses $1, \ldots, 1$ like in the following diagram.

$$
\begin{array}{ccc}
X & \xrightarrow{h_0} & \mathbb{R}P^{n-2} \\
\pi & \downarrow & \\
X' & \xrightarrow{h'} & \mathbb{R}P^{n-3}
\end{array}
$$

By the induction hypothesis the fixed point indexes of $h'$ in the $(n-1)!/2$ components of $X'$ are 1. There are a finite number of fixed point, being isolated and contained in a compact. Moreover $\pi : X \to X'$ is a fiber bundle with fibers equal to $\mathbb{R}$ minus $n-1$ points (the collisions). Consider a fixed point $[z_1 : \ldots, z_{n-1}]$ of $h'$. Let $F \approx \mathbb{R}$ be its pre-image under $\pi$. By equation [4] we have $w_i = \lambda z_i$, with $\lambda = n(n-1)/I$. Hence the induced map $h_0|F$ is

$$
(13) \quad x \in \mathbb{R} \mapsto \frac{I}{n-1} \sum_{j=1}^{n-1} (x - z_j)^{-1}.
$$

It is easy to see that $h_0|F$ has just one fixed point of index 1 in each of the $n$ components of $F$. By the product formula for the fixed point index [3], this means that the fixed point index of $h_0$ in each of the $n(n-1)!/2 = n!/2$ components of $X$ is 1. This completes the proof.

**Theorem 6.** If $\alpha < -1$ and the coefficients $m_{ij}$ are non-zero and have the same sign for every $i, j$, then there are exactly $n!/2$ collinear central configurations.

**Proof.** Assume $m_{ij} > 0$ for every $i, j$. Proposition [3] implies that the fixed point index of $f$ in every component $V$ of $X$ is 1. By proposition [4] and [5], in each component $V$ there are a finite number of fixed points of index 1. By the additivity property of the fixed point index, this implies that there is exactly one fixed point in each component, henceforth that there are exactly $n!/2$ collinear central configurations. If $m_{ij} < 0$ for every $i, j$, then apply the same argument to the map $-f$ obtained by taking $-U$ instead of $U$. Since $-f = f$, $\text{Fix}(f) = \text{Fix}(-f)$ hence the claim. \hfill \Box

3. **Three bodies**

The central configurations in the Newtonian 3-body problem are the three Euler collinear configurations and the equilateral Lagrange configuration. More generally, the case of three charged bodies has been done in [11] in case $\alpha = -3$. Here we give some bounds on the
number of solutions, assuming as above that \( \alpha < -1 \) (that is, the collisions are singularities for the field). The space \( X \) is homeomorphic to \( \mathbb{CP}^1 = S^2 \) minus three points (the three double collisions). The map \( f \) can be extended to \( S^2 \), since there are no triple collisions (apply the same argument in the proof of lemma \( \square \)). The action of the conjugation group \( C \) yields the reflection along the equator of \( S^2 \). A configuration is non-collinear if and only if it is not fixed by \( C \).

3.1. Collinear solutions. By the generalization of Moulton theorem \( \square \) if the coefficients \( m_{ij} \) have the same sign for every \( i, j \) then there are exactly 3 collinear central configurations. Otherwise, assume that some \( m_{ij} \) are positive and some negative and none is zero. Up to rearranging indexes and changing sign, we can suppose \( m_{12} > 0 \), \( m_{13} > 0 \) and \( m_{23} < 0 \), thus that \( f^C \) is compactly fixed homotopic to the map \( \varphi \) corresponding to the problem with \( m_1 = m_2 = m_3 = 1 \), \( m_{12} = 1 \), \( m_{13} = 1 \) and \( m_{23} = -1 \) and \( \alpha = -2 \). In real projective coordinates, we have

\[
\varphi[z_1 : z_2 : z_3] = [3z_1(z_2 - z_3) : -(z_1 - z_3)^2 : (z_1 - z_2)^2];
\]

in the affine chart \([t : 1 : -1 - t]\) the map \( \varphi \) can be written as a map \( \mathbb{R} \to \mathbb{R} \)

\[
t \mapsto -3\frac{t(2 + t)}{2t + 1}^2,
\]

which has degree 0. Thus the degree of \( f^C \) is equal to 0. This means that there is always at least one collinear central configuration. In this case, opposite to the case \( m_{ij} > 0 \), the number of fixed points might be greater than 1 (the fixed point index of \( f^C \)). According to \( \square \), for \( \alpha = -3 \) the fixed points can be any number from 1 to 5, depending on the coefficients.

Now consider the case in which some \( m_{ij} \) vanish. If all the \( m_{ij} \) vanish, then the map \( f \) is even not defined and the problem is totally degenerate. If two of the \( m_{ij} \)'s vanish, then the map \( f \) is equal to a constant, so that there is exactly one fixed point. In this case the three body problem reduces to a Kepler problem with a third mass which does not interact with the first two and there is not much physical relevance. So assume that only one of the \( m_{ij} \) vanishes: without loss of generality it is \( m_{23} \). The homotopies yielded by the variation of the coefficients now need not to be compactly fixed away from the collisions, but are defined also on the collision set. There are two cases: either \( m_{12} \) and \( m_{13} \) have the same sign, or not. If they have the same sign, then the map \( f^C \) up to sign is homotopic to the map \( f_3^C \) obtained by setting \( m_i = 1 \) and \( m_{12} = m_{13} = 1 \) and \( \alpha = -2 \). In coordinates, it is

\[
f_3 = [3z_1 : z_3 - z_1 : z_2 - z_1],
\]

which has affine form

\[
t \mapsto -3\frac{t}{2t + 1}.
\]

This map has degree \(-1\), therefore there are at least 2 fixed points. The only collision fixed by \( f_3 \) is the collision of the particle 2 with the particle 3, therefore there is always 1 fixed point of \( f^C \) corresponding to a central configuration. In case \( m_{12} \) and \( m_{13} \) have different signs, then \( f^C \) is homotopic to the map \( f_4^C \) obtained by setting \( m_i = 1 \) and \( m_{12} = -m_{13} = 1 \) and \( \alpha = -2 \). In coordinates, it is

\[
f_4 = [z_2 - z_3 : z_3 - z_1 : z_1 - z_2].
\]

This map has degree 1, and does not fix any collision; therefore the fixed point index is 0. Actually, what happens is that for some values of the coefficients there are no fixed points, for some other values there are.
3.2. Non-collinear solution. Let \( x_1 = |z_2 - z_3|, x_2 = |z_1 - z_3| \) and \( x_3 = |z_1 - z_2| \). We have
\[
U = m_{12}\phi(|z_1 - z_2|) + m_{13}\phi(|z_1 - z_3|) + m_{23}\phi(|z_2 - z_3|),
\]
that is
\[
U = m_{12}\phi(x_3) + m_{13}\phi(x_2) + m_{23}\phi(x_1),
\]
while
\[
I = m_1|z_1|^2 + m_2|z_2|^2 + m_3|z_3|^2 = (m_1 + m_2 + m_3)^{-1}(m_1m_2^2 + m_1m_3x_2^2 + m_2m_3x_1^2).
\]
The function \( u : X \rightarrow \mathbb{R} \) therefore induces function \( \bar{u} : X/C \rightarrow S^1 \) such that \( \bar{u}\pi = u \), where \( \pi : X \rightarrow \mathbb{RP}^2 \) denotes the map which sends \([z_1 : z_2 : z_3] \in X\) to \([x_1 : x_2 : x_3] \in \mathbb{RP}^2\). Let \( X_1 \) denote the set of non-collinear configurations in \( X \). The image \( \pi(X_1) \subset \mathbb{RP}^2 \) is homeomorphic to the quotient \( X_1/C \). Now, we can apply the same argument as in lemma 1 and show that the critical points of \( \bar{u} \) correspond to fixed points of the self-map \( X_1/C \rightarrow \mathbb{RP}^2 \) given by
\[
\bar{f}[x_1 : x_2 : x_3] = \left[ \frac{m_{23}}{m_2m_3}x_1^{\alpha+1} : \frac{m_{13}}{m_1m_3}x_2^{\alpha+1} : \frac{m_{12}}{m_1m_2}x_3^{\alpha+1} \right].
\]
Clearly, if any one of the \( m_{ij} \) vanishes then there are no such fixed points in \( X_1 \) (a configuration of type \([z : -z : 0]\) is always collinear). So in the affine chart the fixed points are the compatible solutions of the system of equations
\[
x_1^{\alpha} = \frac{m_{23}m_1}{m_1m_3}, \quad y_1^{\alpha} = \frac{m_{13}m_2}{m_1m_3}.
\]
If the \( m_{ij} \) do not have the same sign, then there are no solutions. In case of the Newtonian case we have \( m_{ij} = m_im_j \), and hence the solution is given as expected by the equilateral triangle (\( x_1 = x_2 = x_3 = 1 \)). Otherwise the solutions are either one (if the compatibility conditions on the sides of a triangle are fulfilled: \( x_i + x_j > x_k \) for each permutation \((i,j,k)\)), or none.

We can summarize the results in the following proposition.

**Proposition 7.** Given the coefficients \( m_{12}, m_{13} \) and \( m_{23} \), masses \( m_i > 0, i = 1,2,3 \) and \( \alpha < -1 \), if we define \( \bar{m}_1 = m_1m_{23}, \bar{m}_2 = m_2m_{13} \) and \( \bar{m}_3 = m_3m_{12} \), the number of planar non-collinear central configurations \( \#CC_1/O(2) \) is:
1. 1, if \( \bar{m}_i \) do not vanish, have the same sign, and for every permutation \((i,j,k)\)
   \[
   \left( \bar{m}_i \right)^{\frac{1}{\alpha}} + \left( \bar{m}_j \right)^{\frac{1}{\alpha}} > \left( \bar{m}_k \right)^{\frac{1}{\alpha}}
   \]
2. 0, otherwise.

The number of collinear central configurations is:
1. If two of the \( \bar{m}_i \) vanish: 1.
2. If one of the \( \bar{m}_i \) vanishes and the sign of the left \( \bar{m}_j \) and \( \bar{m}_k \) is the same: at least 1.
3. If \( \bar{m}_i \neq 0 \) for \( i = 1,2,3 \) and the \( \bar{m}_i \) have the same sign: 3. Otherwise, if the sign of the \( \bar{m}_i \) changes: at least 1.

4. Computing central configurations

As a consequence of lemma 1, the central configurations are the solutions of the system of \( 2(n-2) \) equations in \( 2(n-2) \) unknown variables. Let \( A \) be the affine chart in \( \mathbb{CP}^{n-2} \) of
Let \( x = (x_1, \ldots, x_{n-2}) \) be the affine coordinates. In the chart \( A \) the map \( f \) is of the form \( x \mapsto w_i/w_{n-1} \), where \( w_i \) is the \( i \)-th component of \( f \) and it is evaluated on the point 
\[
[x_1 : x_2 : \cdots : x_{n-2} : 1 : -1 - \sum_{i=1}^{n-2} x_i].
\]
Thus the \( n-2 \) equations are 
\[
w_{n-1} x_i = w_i, \quad (i = 1, \ldots, n-2).
\]

I have computed the approximate solutions of such system for \( n = 3, \ldots, 10 \). Unfortunately, only in the case of \( n = 4 \) Albouy \([1, 2]\) proved that these are indeed solutions and the only solutions (actually, Morse equality \((14)\) below gives an alternate proof of the fact that a non-collinear solution with symmetry different from 3 and 4 exists). For \( n \geq 5 \) the fact that the system is solved up to the machine precision (in this case \( 10^{-15} \)) does not imply that the solutions actually exists. A computer-assisted proof was shown by Kotsireas \([3]\) for \( n = 4, 5 \).

A few words on the implementation: the program which computed the configurations in the appendix is written in FORTRAN 95, using the SLATEC F77 library, and partially in the language of MAPLE for post-processing. The algorithm is simple: a central configuration of equal masses can be ordered in a way that \(|z_1| \leq |z_2| \leq \cdots \leq |z_n| \). Therefore without loss of generality we can compute the configurations in the affine chart above. The root-finding subroutine starts by a random point until reaches a solution or a failure (there are different reasons for the failure, like approaching a collision or too many steps without progress). Since the norms of \( z_i \) are ordered, we can assume that \(|x_i| \leq 1\) for \( i = 1, \ldots, n-2 \). Therefore the random starting point can be chosen in the cube \([-1,1]^{2(n-2)}\). After a solution is found, the program stores it in a list, and computes the monic polynomial
\[
p(z) = \prod_{i=1}^{n} (z - z_i) = z^n + a_{n-2}z^{n-2} + \cdots + a_1 z + a_0.
\]

Given two solutions, they coincide up to permuting the particles, scaling, rotation or conjugation if and only if the coefficients \( a_i \) and \( a_i' \) of the corresponding polynomials are related by the equations
\[
b^i a_i = a_i'
\]
or
\[
b^i a_i = \bar{a}_i',
\]
where \( b \) is a suitable complex number. Then, if the configuration is not in the list of known configurations, it is added to it. After the program cannot find any new configuration for a long enough time (depending on the number of masses), the list of configurations found is piped to a MAPLE filter that cross-checks the solutions, computes the reduced potential \( U\sqrt{T} \), the critical point index, the fixed point index and the order of the isotropy group of the configuration (the symmetric group \( \Sigma_n \) acts on \( X \), therefore each solution has a isotropy group with respect to this action: in the tables we count this isotropy, in general; for a few configurations which are not symmetric with respect to a reflection, we denote the isotropy \( 1/2 \)).

The time spent by the program in computing the solutions is a tiny part, whenever compared to the time spent in searching for solutions once the list is complete. Unfortunately, not only these numerical experiments do not imply that the approximate solutions are solutions, but there is no proof of the fact that there are no extra solutions. Some hope that the list, at least for small \( n \), is complete, comes from the Morse equality
\[
(14) \sum_k (-1)^k \nu_k = \chi(X) = (-1)^n(n-2)!.
\]
Let $i(z)$ denote the size of the isotropy group of the central configuration $z \in CC_1/\text{SO}(2)$ with respect to the action of $\Sigma_n$, and $h(z)$ its Morse index. A consequence of equation 14 is the following equation.

\[
\sum_{z \in CC_1/\text{SO}(2)} \frac{(-1)^{h(z)}}{i(z)} = \frac{(-1)^n}{n(n-1)}.
\]

(15)

Therefore we can test equation 15 against the collected data, keeping in mind that in the tables the configurations are listed in $CC_1/O(2)$, hence if there is no axis of symmetry the configuration contributes to the sum twice. For example, for $n = 4$ we have

\[
\frac{1}{4} - 1 + \frac{1}{3} + \frac{1}{2} = \frac{1}{12};
\]

for $n = 5$,

\[
\frac{1}{5} + \frac{1}{4} - 1 + 1 - \frac{1}{2} = -\frac{1}{20};
\]

for $n = 6$,

\[
\frac{1}{5} + \frac{1}{3} - \frac{1}{6} - 1 + \frac{1}{2} + 1 - \frac{1}{3} - 1 + \frac{1}{2} = \frac{1}{30};
\]

for $n = 7$ analogously we obtain $-\frac{1}{42}$; for $n = 8$ we have the first examples of configurations without an axis of symmetry: they contribute with $\frac{1}{1/2} = 2$ instead of 1, and the sum gives $\frac{1}{56}$.

For $n = 9$, it gives the expected $-\frac{1}{72}$. Unfortunately, for $n = 10$ the sum is $-\frac{269}{90}$, thus there is a missing term +3. It is likely that the computation of the isotropy was broken somewhere or simply that there are some configurations missing.

5. Remarks

Remark 1. In case of the logarithmic potential, the collinear central configurations are the zeroes of the Hermite polynomials ([15], Theorem 6.7.3 pag. 141). This result can be in some sense extended to the planar central configurations as follows: the planar central configurations of $n$ bodies are the zeroes of degree $n$ polynomials $p(x)$ with complex coefficients such that $p(x) = 0 \implies p''(x) + \lambda \pi p'(x) = 0$, with $\lambda$ real non-zero parameter. That is, $p(x)$ is a polynomial of degree $n$ satisfying the differential equation $p'' + \lambda \pi p' + cp = 0$, where $c$ is a suitable function of $x$.

Remark 2. It is apparent from the data shown below that the fixed point index of a central configuration is likely to be equal to minus one to the power the critical point index. This is certainly true for the gradient map, but $f$ is not equal (nor homotopic) to the gradient map. This relation is true for collinear configurations (embedded in the plane), and it is likely that it is true for all the central configurations. Unfortunately I do not know any proof of it.

Remark 3. We see from the examples in the tables that if a central configuration has a rotational symmetry, then it has a symmetry axis. Thus the symmetry group of the configuration is either trivial or a dihedral group. In the tables there are no central configurations with a pure rotation symmetry. Is it true in general?

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Appendix

Tables of central configurations

| Central configurations for 3 bodies |
|-----------------------------------|
| $U\sqrt{7} = 3.00000000$          |
| Crit ind = 0                      |
| FP ind = 1                        |
| Isotropy = 3                      |
| $z_1 = (-0.50000000, 0.86602540)$ |
| $z_2 = (1.00000000, 0.00000000)$  |
| $z_3 = (-0.50000000, -0.86602540)$|

| Central configurations for 4 bodies |
|-----------------------------------|
| $U\sqrt{7} = 3.53553391$          |
| Crit ind = 1                      |
| FP ind = -1                       |
| Isotropy = 2                      |
| $z_1 = (0.00000000, 0.00000000)$  |
| $z_2 = (1.00000000, 0.00000000)$  |
| $z_3 = (-1.00000000, 0.00000000)$|

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Some central configurations for 5 bodies
Some central configurations for 6 bodies

$U\sqrt{T} = 15.65685425$
Crit ind = 0
FP ind = 1
Isotropy = 4
$z_1 = (.00000000, -.00000000)$
$z_2 = (-1.00000000, .00000000)$
$z_3 = (-.00000000, -1.00000000)$
$z_4 = (1.00000000, 0.00000000)$
$z_5 = (.00000000, 1.00000000)$

$U\sqrt{T} = 15.68397123$
Crit ind = 1
FP ind = 1
Isotropy = 1
$z_1 = (.12867234, .18881180)$
$z_2 = (.09578559, -.88501108)$
$z_3 = (-.85873077, -.23452283)$
$z_4 = (1.00000000, 0.00000000)$
$z_5 = (-.36572717, .93072211)$

$U\sqrt{T} = 17.12399663$
Crit ind = 2
FP ind = 1
Isotropy = 1
$z_1 = (.34137918, .02633026)$
$z_2 = (-.27690769, .20138312)$
$z_3 = (-.21269415, -.75194237)$
$z_4 = (1.00000000, 0.00000000)$
$z_5 = (-.85157734, .52422899)$
$z_6 = (1.00000000, 0.00000000)$

$U\sqrt{T} = 20.24094955$
Crit ind = 3
FP ind = -1
Isotropy = 2
$z_1 = (.00000000, -.00000000)$
$z_2 = (.47168469, -.00000000)$
$z_3 = (-.47168469, -.00000000)$
$z_4 = (1.00000000, 0.00000000)$
$z_5 = (-1.00000000, 0.00000000)$

$U\sqrt{T} = 26.50875757$
Crit ind = 0
FP ind = 1
Isotropy = 5
$z_1 = (-.00000000, 0.00000000)$
$z_2 = (.30901699, -.95105652)$
$z_3 = (-.80901699, .58778525)$
$z_4 = (-.80901699, -.58778525)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (.30901699, .95105652)$
$U \sqrt{I} = 26.85636352$

Crit ind = 0
FP ind = 1
Isotropy = 3

$z_1 = (.46417852, .80398078)$
$z_2 = (-.92835704, -.00000000)$
$z_3 = (.46417852, -.80398078)$
$z_4 = (-.50000000, -.86602540)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.50000000, .86602540)$

$U \sqrt{I} = 26.85645445$

Crit ind = 1
FP ind = -1
Isotropy = 6

$z_1 = (.50000000, -.86602540)$
$z_2 = (-1.00000000, -.00000000)$
$z_3 = (.50000000, .86602540)$
$z_4 = (-.50000000, .86602540)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.50000000, -.86602540)$

$U \sqrt{I} = 26.88681901$

Crit ind = 1
FP ind = -1
Isotropy = 1

$z_1 = (.30421317, .45080951)$
$z_2 = (-.82401445, .09016574)$
$z_3 = (.39197350, -.73040157)$
$z_4 = (-.49795663, -.73791542)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.37421558, .92734174)$

$U \sqrt{I} = 27.79833059$

Crit ind = 2
FP ind = 1
Isotropy = 2

$z_1 = (.33894038, -.00000000)$
$z_2 = (-.33894038, .00000000)$
$z_3 = (-.00000000, -.84172034)$
$z_4 = (.00000000, .84172034)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-1.00000000, 0.00000000)$

$U \sqrt{I} = 28.1161508$

Crit ind = 2
FP ind = 1
Isotropy = 1

$z_1 = (.36819982, .66656948)$
$z_2 = (-.18216036, .32683361)$
$z_3 = (-.68397183, -.39013941)$
$z_4 = (.13241025, -.77620471)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.63447788, .77294102)$
$U^\sqrt{T} = 29.59724379$
Crit ind = 3
FP ind = -1
Isotropy = 3
$z_1 = (-.19887188, -.34445619)$
$z_2 = (.39774375, .00000000)$
$z_3 = (-.19887188, .34445619)$
$z_4 = (-.50000000, -.86602540)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.50000000, .86602540)$

$U^\sqrt{T} = 30.99783846$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (.01360506, .06307395)$
$z_2 = (-.43717775, .21223104)$
$z_3 = (.48579169, .01314644)$
$z_4 = (-.5113510, -.70067240)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-.91108390, .41222097)$

$U^\sqrt{T} = 36.25298863$
Crit ind = 4
FP ind = 1
Isotropy = 2
$z_1 = (-.18383207, .00000000)$
$z_2 = (.18383207, .00000000)$
$z_3 = (.56459754, .00000000)$
$z_4 = (-.56459754, .00000000)$
$z_5 = (1.00000000, 0.00000000)$
$z_6 = (-1.00000000, -.00000000)$

$z_7 = (1.00000000, 0.00000000)$

Some central configurations for 7 bodies

$U^\sqrt{T} = 41.55339290$
Crit ind = 0
FP ind = 1
Isotropy = 6
$z_1 = (-.00000000, -.00000000)$
$z_2 = (-.50000000, -.86602540)$
$z_3 = (.50000000, -.86602540)$
$z_4 = (.50000000, .86602540)$
$z_5 = (-1.00000000, 0.00000000)$
$z_6 = (.50000000, .86602540)$

$U^\sqrt{T} = 42.32859664$
Crit ind = 1
FP ind = -1
Isotropy = 1
$z_1 = (.31194658, -.28006242)$
$z_2 = (-.39849482, .13018261)$
$z_3 = (.42399335, .73425006)$
$z_4 = (-.85245018, -.47481689)$
$z_5 = (.01487433, -.97565421)$
$z_6 = (1.00000000, 0.00000000)$
$z_7 = (-.49860928, .86610808)$
$U \sqrt{T} = 42.45998988$

Crit ind = 1

FP ind = -1

Isotropy = 1

$z_1 = (-.46413573, .28708218)$

$z_2 = (.37471387, .39677156)$

$z_3 = (.47873707, -.67327333)$

$z_4 = (-.28952417, -.77373246)$

$z_5 = (-.96637766, -.25712684)$

$z_6 = (1.00000000, 0.00000000)$

$z_7 = (-.13341338, 1.02027888)$


$U \sqrt{T} = 42.49618473$

Crit ind = 2

FP ind = 1

Isotropy = 1

$z_1 = (.38450652, -.46240728)$

$z_2 = (-.54943511, -.29984561)$

$z_3 = (.39504259, .48551796)$

$z_4 = (-.2036196, .89597229)$

$z_5 = (-.84373160, .36398326)$

$z_6 = (1.00000000, 0.00000000)$

$z_7 = (-.18242044, -.98322062)$


$U \sqrt{T} = 42.60195114$

Crit ind = 2

FP ind = 1

Isotropy = 1

$z_1 = (-.30830344, -.17397349)$

$z_2 = (.34487831, -.97492791)$

$z_3 = (.62348980, .78183148)$

$z_4 = (-.90096887, .43388374)$

$z_5 = (.62348980, -.78183148)$

$z_6 = (1.00000000, 0.00000000)$

$z_7 = (-.95931901, -.28232434)$


$U \sqrt{T} = 42.68484275$

Crit ind = 2

FP ind = 1

Isotropy = 7

$z_1 = (-.22252093, .97492791)$

$z_2 = (-.22252093, -.97492791)$

$z_3 = (.62348980, .78183148)$

$z_4 = (-.90096887, -.43388374)$

$z_5 = (.62348980, -.78183148)$

$z_6 = (1.00000000, 0.00000000)$

$z_7 = (-.90096887, .43388374)$


$U \sqrt{T} = 43.29826801$

Crit ind = 2

FP ind = 1

Isotropy = 1

$z_1 = (.40301374, .14262935)$

$z_2 = (-.08849347, .41824868)$

$z_3 = (.37471387, -.69887152)$

$z_4 = (-.79143908, -.04524148)$

$z_5 = (.37471387, -.69887152)$

$z_6 = (1.00000000, 0.00000000)$

$z_7 = (-.52154178, .85322575)$
\[ U \sqrt{T} = 44.26667553 \]
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = ( -0.36116383, 0.16545203 ) \]
\[ z_2 = ( 0.39262204, -0.00511300 ) \]
\[ z_3 = ( -0.13252243, -0.44207852 ) \]
\[ z_4 = ( 0.23659975, 0.78926767 ) \]
\[ z_5 = ( -0.83509338, 0.55010821 ) \]
\[ z_6 = ( 1.00000000, 0.00000000 ) \]
\[ z_7 = ( -0.30044215, -1.0023809 ) \]

\[ U \sqrt{T} = 45.30890467 \]
Crit ind = 3
FP ind = -1
Isotropy = 2
\[ z_1 = ( -0.00000000, 0.00000000 ) \]
\[ z_2 = ( -0.48839825, 0.00000000 ) \]
\[ z_3 = ( 0.48839825, 0.00000000 ) \]
\[ z_4 = ( 0.00000000, -0.76203471 ) \]
\[ z_5 = ( 0.76203471, 0.00000000 ) \]
\[ z_6 = ( 1.00000000, 0.00000000 ) \]
\[ z_7 = ( -1.00000000, -0.00000000 ) \]

\[ U \sqrt{T} = 45.89112647 \]
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = ( -0.5012818, 0.13936080 ) \]
\[ z_2 = ( 0.50089913, -0.03169978 ) \]
\[ z_3 = ( -0.36593875, 0.34350202 ) \]
\[ z_4 = ( -0.56667787, -0.44643585 ) \]
\[ z_5 = ( 0.15246518, 0.70511215 ) \]
\[ z_6 = ( 1.00000000, 0.00000000 ) \]
\[ z_7 = ( -0.77087587, 0.63698540 ) \]

\[ U \sqrt{T} = 47.89025883 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = ( -0.02010407, 0.03184202 ) \]
\[ z_2 = ( 0.48991124, -0.00365264 ) \]
\[ z_3 = ( -0.21394680, 0.44074157 ) \]
\[ z_4 = ( -0.27911603, 0.44643585 ) \]
\[ z_5 = ( 1.5246518, -0.70511215 ) \]
\[ z_6 = ( 1.00000000, 0.00000000 ) \]
\[ z_7 = ( -0.77087587, 0.63698540 ) \]

\[ U \sqrt{T} = 47.89177651 \]
Crit ind = 5
FP ind = -1
Isotropy = 3
\[ z_1 = ( 0.00000000, 0.00000000 ) \]
\[ z_2 = ( -0.24860773, 0.43060122 ) \]
\[ z_3 = ( -0.49721546, 0.00000000 ) \]
\[ z_4 = ( -0.24860773, -0.43060122 ) \]
\[ z_5 = ( -0.50000000, 0.86602540 ) \]
\[ z_6 = ( 1.00000000, 0.00000000 ) \]
\[ z_7 = ( -0.50000000, -0.86602540 ) \]
Some central configurations for 8 bodies

1. \[ U \sqrt{T} = 50.74714105 \]
   - Crit ind = 4
   - FP ind = 1
   - Isotropy = 1
   - \[ z_1 = (-.17399740, .09130693) \]
   - \[ z_2 = (.19446170, .02822588) \]
   - \[ z_3 = (-.53865669, .19808028) \]
   - \[ z_4 = (.57387182, .00761282) \]
   - \[ z_5 = (-.11263064, -.65788396) \]
   - \[ z_6 = (1.00000000, 0.00000000) \]
   - \[ z_7 = (-.94304879, .33265446) \]

2. \[ U \sqrt{T} = 58.69057642 \]
   - Crit ind = 5
   - FP ind = -1
   - Isotropy = 2
   - \[ z_1 = (.00000000, .00000000) \]
   - \[ z_2 = (-.30469288, .00000000) \]
   - \[ z_3 = (.30469288, .00000000) \]
   - \[ z_4 = (-.62697699, .00000000) \]
   - \[ z_5 = (.62697699, .00000000) \]
   - \[ z_6 = (1.00000000, 0.00000000) \]
   - \[ z_7 = (-1.00000000, -.00000000) \]

3. \[ U \sqrt{T} = 61.20510193 \]
   - Crit ind = 0
   - FP ind = 1
   - Isotropy = 7
   - \[ z_1 = (.00000000, .00000000) \]
   - \[ z_2 = (-.62348980, .78183148) \]
   - \[ z_3 = (-.90096887, .4338374) \]
   - \[ z_4 = (-.22252093, -.97492791) \]
   - \[ z_5 = (-.90096887, -.4338374) \]
   - \[ z_6 = (.62348980, -.78183148) \]
   - \[ z_7 = (1.00000000, 0.00000000) \]
   - \[ z_8 = (-.22252093, .97492791) \]

4. \[ U \sqrt{T} = 61.66790403 \]
   - Crit ind = 1
   - FP ind = -1
   - Isotropy = 2
   - \[ z_1 = (-.28827131, -.19240111) \]
   - \[ z_2 = (.28827131, .19240111) \]
   - \[ z_3 = (-.49497029, -.74160556) \]
   - \[ z_4 = (.49497029, -.74160556) \]
   - \[ z_5 = (-.38363904, -.92348313) \]
   - \[ z_6 = (1.00000000, 0.00000000) \]
   - \[ z_7 = (1.00000000, 0.00000000) \]
   - \[ z_8 = (.38363904, .92348313) \]

5. \[ U \sqrt{T} = 61.72305434 \]
   - Crit ind = 1
   - FP ind = -1
   - Isotropy = 2
   - \[ z_1 = (-.32240266, .00000000) \]
   - \[ z_2 = (.32240266, .00000000) \]
   - \[ z_3 = (-.42002865, -.79225350) \]
   - \[ z_4 = (.42002865, .79225350) \]
   - \[ z_5 = (.42002865, -.79225350) \]
   - \[ z_6 = (-.42002865, .79225350) \]
   - \[ z_7 = (1.00000000, 0.00000000) \]
   - \[ z_8 = (-1.00000000, -.00000000) \]
\[ U \sqrt{I} = 62.41079049 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[ z_1 = (−29565305, −34710822) \]
\[ z_2 = (38769846, −23996822) \]
\[ z_3 = (−0.8289075, 52817029) \]
\[ z_4 = (−7.6455067, 56604705) \]
\[ z_5 = (5.5462624, 77287561) \]
\[ z_6 = (15.270456, −97396762) \]
\[ z_7 = (1.0000000, 0.0000000) \]
\[ z_8 = (−9.5201580, −30604889) \]

\[ U \sqrt{I} = 62.44858092 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[ z_1 = (38265292, 35399956) \]
\[ z_2 = (−38529668, 42622616) \]
\[ z_3 = (39501013, −41724006) \]
\[ z_4 = (−49373567, −45676435) \]
\[ z_5 = (0.0777213, −98766650) \]
\[ z_6 = (−98407872, 0.8446653) \]
\[ z_7 = (1.0000000, 0.0000000) \]
\[ z_8 = (0.0776789, 99607866) \]

\[ U \sqrt{I} = 62.45193681 \]
Crit ind = 3
FP ind = 1
Isotropy = 4
\[ z_1 = (41447962, 41447962) \]
\[ z_2 = (−41447962, 41447962) \]
\[ z_3 = (41447962, −41447962) \]
\[ z_4 = (−41447962, −41447962) \]
\[ z_5 = (0.0000000, −1.0000000) \]
\[ z_6 = (−1.0000000, 0.0000000) \]
\[ z_7 = (1.0000000, 0.0000000) \]
\[ z_8 = (0.0000000, 1.0000000) \]

\[ U \sqrt{I} = 63.18080222 \]
Crit ind = 3
FP ind = 1
Isotropy = 1
\[ z_1 = (−24626571, −21206535) \]
\[ z_2 = (43408722, 0.9069241) \]
\[ z_3 = (−0.2035026, 4507896) \]
\[ z_4 = (38075518, −72799675) \]
\[ z_5 = (−84097595, 19321853) \]
\[ z_6 = (−50518648, −78350193) \]
\[ z_7 = (1.0000000, 0.0000000) \]
\[ z_8 = (−2.0183401, 98857413) \]

\[ U \sqrt{I} = 63.18156622 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[ z_1 = (−19713350, 23819334) \]
\[ z_2 = (43883639, −0.7363464) \]
\[ z_3 = (−0.0971572, −44486519) \]
\[ z_4 = (35855637, 75648249) \]
\[ z_5 = (−81018292, −21078915) \]
\[ z_6 = (−59338484, 71697767) \]
\[ z_7 = (1.0000000, 0.0000000) \]
\[ z_8 = (−1.8697579, −98236452) \]
\( U \sqrt{T} = 63.40153688 \)
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = (0.00149563, -0.50606975) \]
\[ z_2 = (0.45035952, -0.23803573) \]
\[ z_3 = (-0.70461968, -0.17706579) \]
\[ z_4 = (0.47756151, 0.54769726) \]
\[ z_5 = (-0.10719337, 0.79944496) \]
\[ z_6 = (-0.66389803, 0.45808521) \]
\[ z_7 = (1.00000000, 0.00000000) \]
\[ z_8 = (-0.45350022, -0.89125616) \]

\( U \sqrt{T} = 63.46686913 \)
Crit ind = 3
FP ind = -1
Isotropy = 8
\[ z_1 = (0.00000000, -1.00000000) \]
\[ z_2 = (-0.00000000, 1.00000000) \]
\[ z_3 = (-0.70710678, 0.70710678) \]
\[ z_4 = (0.70710678, 0.70710678) \]
\[ z_5 = (0.70710678, -0.70710678) \]
\[ z_6 = (-0.70710678, -0.70710678) \]
\[ z_7 = (1.00000000, 0.00000000) \]
\[ z_8 = (-1.00000000, -0.00000000) \]

\( U \sqrt{T} = 63.78451455 \)
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = (-0.45615373, 0.03187986) \]
\[ z_2 = (0.42432105, 0.17042357) \]
\[ z_3 = (-0.0506108, 0.47702915) \]
\[ z_4 = (-0.2383072, -0.76155208) \]
\[ z_5 = (0.46068568, -0.65156106) \]
\[ z_6 = (-0.95167767, -0.30709870) \]
\[ z_7 = (1.00000000, 0.00000000) \]
\[ z_8 = (-1.06378353, 1.04087925) \]

\( U \sqrt{T} = 64.32981488 \)
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = (0.01339731, 0.10773738) \]
\[ z_2 = (-0.47070516, 1.46112207) \]
\[ z_3 = (0.49215468, 0.02637900) \]
\[ z_4 = (-0.44456437, -0.6275398) \]
\[ z_5 = (0.27899242, -0.71072936) \]
\[ z_6 = (0.09026948, 0.80633909) \]
\[ z_7 = (1.00000000, 0.00000000) \]
\[ z_8 = (-0.96954435, 0.24491580) \]

\( U \sqrt{T} = 65.05085183 \)
Crit ind = 4
FP ind = 1
Isotropy = 4
\[ z_1 = (-0.44458736, 0.00000000) \]
\[ z_2 = (0.44458736, -0.00000000) \]
\[ z_3 = (-0.00000000, -0.44458736) \]
\[ z_4 = (0.00000000, 0.44458736) \]
\[ z_5 = (1.00000000, 0.00000000) \]
\[ z_6 = (-0.00000000, -1.00000000) \]
\[ z_7 = (1.00000000, 0.00000000) \]
\[ z_8 = (0.00000000, 1.00000000) \]
$U \sqrt{T} = 65.55546559$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (.09786504, .21504289)$
$z_2 = (.51431659, .06073804)$
$z_3 = (-.29202953, .42770248)$
$z_4 = (-.29227507, -.64222807)$
$z_5 = (-.72158633, -.16367582)$
$z_6 = (.35055657, -.65160317)$
$z_7 = (1.0000000, 0.0000000)$
$z_8 = (-.65684727, .75402365)$

$U \sqrt{T} = 66.67340501$
Crit ind = 4
FP ind = 1
Isotropy = 1/2
$z_1 = (.05121110, -.13415293)$
$z_2 = (-.01715151, .42141469)$
$z_3 = (.51957506, -.01476313)$
$z_4 = (-.23702168, -.48253092)$
$z_5 = (-.73438594, .16843303)$
$z_6 = (.09355458, .92924616)$
$z_7 = (1.0000000, 0.0000000)$
$z_8 = (-.48867245, -.88764690)$

$U \sqrt{T} = 69.09626272$
Crit ind = 4
FP ind = 1
Isotropy = 2
$z_1 = (-.19242138, .00000000)$
$z_2 = (.19242138, .00000000)$
$z_3 = (.57720636, .00000000)$
$z_4 = (-.57720636, .00000000)$
$z_5 = (.00000000, .70497161)$
$z_6 = (.00000000, -.70497161)$
$z_7 = (1.0000000, 0.00000000)$
$z_8 = (-1.0000000, 0.00000000)$

$U \sqrt{T} = 69.98439855$
Crit ind = 4
FP ind = 1
Isotropy = 1
$z_1 = (.21086098, -.06693301)$
$z_2 = (-.14081889, -.17062363)$
$z_3 = (-.48046244, -.33127118)$
$z_4 = (.58333039, -.01761850)$
$z_5 = (-.48942863, .46925267)$
$z_6 = (.15655868, .65971800)$
$z_7 = (1.0000000, 0.00000000)$
$z_8 = (-.84004008, -.54252434)$

$U \sqrt{T} = 72.08640113$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (-.15015379, .15813350)$
$z_2 = (.21675677, .02385085)$
$z_3 = (-.14511054, -.39649642)$
$z_4 = (.58937904, .00592407)$
$z_5 = (-.48942863, .46925267)$
$z_6 = (-.30079550, -.82188612)$
$z_7 = (1.0000000, 0.00000000)$
$z_8 = (-.76375739, .64550340)$
Some central configurations for 9 bodies

- $U\sqrt{T} = 77.32149683$
- Crit ind = 5
- FP ind = -1
- Isotropy = 1
  - $z_1 = (0.06621569, -0.04371522)$
  - $z_2 = (-0.29642382, -0.10215784)$
  - $z_3 = (0.31315096, -0.1548482)$
  - $z_4 = (-0.08902634, 0.62612575)$
  - $z_5 = (0.63322067, -0.00486084)$
  - $z_6 = (-0.60676966, -0.18117015)$
  - $z_7 = (1.00000000, 0.00000000)$
  - $z_8 = (-0.96036751, -0.27873689)$

- $U\sqrt{T} = 88.48350923$
- Crit ind = 6
- FP ind = 1
- Isotropy = 2
  - $z_1 = (-0.12845093, -0.00000000)$
  - $z_2 = (-0.12845093, -0.00000000)$
  - $z_3 = (0.39058071, 0.00000000)$
  - $z_4 = (-0.39058071, -0.00000000)$
  - $z_5 = (0.67131756, -0.00000000)$
  - $z_6 = (-0.67131756, -0.00000000)$
  - $z_7 = (1.00000000, 0.00000000)$
  - $z_8 = (-1.00000000, 0.00000000)$

- $U\sqrt{T} = 85.90069864$
- Crit ind = 0
- FP ind = 1
- Isotropy = 1
  - $z_1 = (-0.27202755, 0.14390992)$
  - $z_2 = (0.28973718, -0.15327879)$
  - $z_3 = (-0.24618043, 0.86277978)$
  - $z_4 = (0.57473480, 0.68896565)$
  - $z_5 = (-0.15320370, 0.93279998)$
  - $z_6 = (-0.85733689, -0.39819672)$
  - $z_7 = (0.56265924, -0.8268893)$
  - $z_8 = (1.00000000, 0.00000000)$
  - $z_9 = (-0.89838264, 0.47520867)$

- $U\sqrt{T} = 85.90099418$
- Crit ind = 1
- FP ind = -1
- Isotropy = 1
  - $z_1 = (-0.30525333, 0.07170100)$
  - $z_2 = (0.30225296, -0.08344927)$
  - $z_3 = (-0.21785509, -0.85303320)$
  - $z_4 = (0.56690970, 0.70454393)$
  - $z_5 = (-0.15965544, 0.89010050)$
  - $z_6 = (-0.86406779, -0.42329589)$
  - $z_7 = (0.55809940, -0.7806979)$
  - $z_8 = (1.00000000, 0.00000000)$
  - $z_9 = (-0.87754040, 0.47950271)$
\[ U \sqrt{I} = 86.09428612 \]
Crit ind = 0
FP ind = 1
Isotropy = 8
\[
\begin{align*}
z_1 &= (.0000000, -.0000000) \\
z_2 &= (.7071067, .7071067) \\
z_3 &= (.7071067, -.7071067) \\
z_4 &= (-.7071067, .7071067) \\
z_5 &= (-.7071067, -.7071067) \\
z_6 &= (1.0000000, 0.0000000) \\
z_7 &= (-.0000000, -1.0000000) \\
z_8 &= (.0000000, 1.0000000) \\
z_9 &= (-.0000000, 1.0000000) \\
\end{align*}
\]

\[ U \sqrt{I} = 86.12713056 \]
Crit ind = 1
FP ind = 1
Isotropy = 1
\[
\begin{align*}
z_1 &= (.0618299, -.0546070) \\
z_2 &= (.5003768, .4419234) \\
z_3 &= (.5944342, -.6021997) \\
z_4 &= (-.5241168, .8108313) \\
z_5 &= (-.6791932, -.5998507) \\
z_6 &= (-.9232059, .0846931) \\
z_7 &= (.3000551, .9265952) \\
z_8 &= (1.0000000, 0.0000000) \\
z_9 &= (.1235899, .9923334) \\
\end{align*}
\]

\[ U \sqrt{I} = 86.47395899 \]
Crit ind = 1
FP ind = 1
Isotropy = 1
\[
\begin{align*}
z_1 &= (.3135669, .1533035) \\
z_2 &= (.0412038, -.4646992) \\
z_3 &= (.3918458, .2527421) \\
z_4 &= (-.9736403, -.0691367) \\
z_5 &= (-.5434218, .810831) \\
z_6 &= (-.5374598, -.824864) \\
z_7 &= (.3208668, .9307567) \\
z_8 &= (1.0000000, 0.0000000) \\
z_9 &= (.6141721, -.7891720) \\
\end{align*}
\]

\[ U \sqrt{I} = 86.47616352 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[
\begin{align*}
z_1 &= (-.2576878, .1202717) \\
z_2 &= (.4236326, .2949241) \\
z_3 &= (.0459899, -.5415304) \\
z_4 &= (-.5786407, .7648254) \\
z_5 &= (-.9578435, -.0477126) \\
z_6 &= (-.2511371, .9526063) \\
z_7 &= (-.5688748, -.8043073) \\
z_8 &= (1.0000000, 0.0000000) \\
z_9 &= (.6422505, -.7664975) \\
\end{align*}
\]
$U\sqrt{I} = 86.47673336$
Crit ind = 2
FP ind = 1
Isotropy = 3

$z_1 = (-.38278598, .19117010)$
$z_2 = (.02583483, -.42708743)$
$z_3 = (.35695115, .23591733)$
$z_4 = (-.99270843, -.12054037)$
$z_5 = (-.50000000, -.86602540)$
$z_6 = (.39196319, .91998090)$
$z_7 = (-.50000000, .86602540)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (.60074524, -.79944053)$

$U\sqrt{I} = 86.65509034$
Crit ind = 2
FP ind = 1
Isotropy = 1

$z_1 = (-.19882309, -.33371308)$
$z_2 = (.38752437, .02683099)$
$z_3 = (-.20927537, .34034142)$
$z_4 = (.46829354, -.76157883)$
$z_5 = (-.90674832, -.3175861)$
$z_6 = (.43753246, .79483739)$
$z_7 = (.45127355, -.89238567)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (.52730304, .85742638)$

$U\sqrt{I} = 86.65524794$
Crit ind = 3
FP ind = -1
Isotropy = 3

$z_1 = (-.19536055, -.33374430)$
$z_2 = (.39072110, .00000000)$
$z_3 = (-.19536055, .33374430)$
$z_4 = (.45028680, -.77991962)$
$z_5 = (-.90057361, -.00000000)$
$z_6 = (.45028680, .77991962)$
$z_7 = (-.50000000, -.86602540)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.50000000, .86602540)$

$U\sqrt{I} = 86.75347800$
Crit ind = 2
FP ind = 1
Isotropy = 1

$z_1 = (-.21338975, 2.4088355)$
$z_2 = (.43000417, .2442878)$
$z_3 = (-.29454041, -.39738652)$
$z_4 = (.52773439, -.59572933)$
$z_5 = (.25032729, .91345257)$
$z_6 = (-.93697539, -.13833458)$
$z_7 = (-.64259266, .72538630)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.12060343, -.99270077)$
$U\sqrt{T} = 87.31115245$
Crit ind = 2
FP ind = 1
Isotropy = 1
$z_1 = (-.15563009, -.17669118)$
$z_2 = (.46867893, .12656096)$
$z_3 = (.06638029, .48090672)$
$z_4 = (.4830739, -.65204361)$
$z_5 = (-.70809404, .39910769)$
$z_6 = (-.21765902, -.84699397)$
$z_7 = (-.81273909, -.32284579)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.12624437, .99199917)$

$U\sqrt{T} = 87.38701579$
Crit ind = 2
FP ind = 1
Isotropy = 1
$z_1 = (.13410007, .34882068)$
$z_2 = (.45967777, .26278972)$
$z_3 = (.51733513, -.11281157)$
$z_4 = (-.03233323, -.78562587)$
$z_5 = (.55027709, -.56164808)$
$z_6 = (.19757768, .94733171)$
$z_7 = (-.78128946, .57101759)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.74247464, -.66987418)$

$U\sqrt{T} = 87.43994782$
Crit ind = 3
FP ind = 1
Isotropy = 1
$z_1 = (.40108031, -.24383634)$
$z_2 = (-.19384974, .42748550)$
$z_3 = (.43082152, .42694065)$
$z_4 = (-.59651209, .10981267)$
$z_5 = (-.59126367, .71137549)$
$z_6 = (.08740410, .92087368)$
$z_7 = (.28832043, -.93401181)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.82600087, -.56366885)$

$U\sqrt{T} = 87.95358020$
Crit ind = 3
FP ind = 1
Isotropy = 2
$z_1 = (-.00000000, .00000000)$
$z_2 = (-.48850023, -.00000000)$
$z_3 = (.48850023, .00000000)$
$z_4 = (.35781420, -.72203871)$
$z_5 = (-.35781420, -.72203871)$
$z_6 = (.35781420, .72203871)$
$z_7 = (-.35781420, .72203871)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-1.00000000, -.00000000)$
$U\sqrt{T} = 88.11971644$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (-.46565385, .07468209)$
$z_2 = (.03524249, .47026597)$
$z_3 = (.47088716, -.17335105)$
$z_4 = (.05952884, -.49823852)$
$z_5 = (-.85103731, -.38396410)$
$z_6 = (.57070119, .73891346)$
$z_7 = (-.58796793, .7445931)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.23170058, -.97278715)$

$U\sqrt{T} = 88.49177887$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (-.01729065, .42892275)$
$z_2 = (.44906469, -.12341822)$
$z_3 = (-.43767328, -.15916422)$
$z_4 = (.02024314, -.50216426)$
$z_5 = (.5157036, .70256114)$
$z_6 = (-.57542762, .71857500)$
$z_7 = (-.99675519, -.08049279)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (.04211855, -1.04418139)$

$U\sqrt{T} = 88.69024481$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (.01237317, -.44743339)$
$z_2 = (-.45315425, -.20571993)$
$z_3 = (.46383082, -.18036194)$
$z_4 = (.47985453, .52084357)$
$z_5 = (-.50790561, -.49352840)$
$z_6 = (.02435102, .88057156)$
$z_7 = (-.99847172, -.05526503)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (.02782408, -1.00616324)$

$U\sqrt{T} = 88.72533804$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (.02121607, .36421618)$
$z_2 = (.48567000, .17025371)$
$z_3 = (-.46261725, .22549269)$
$z_4 = (.03997521, -.68625425)$
$z_5 = (.54885312, -.60616203)$
$z_6 = (-.61552162, -.53833567)$
$z_7 = (.05561141, .95468072)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-.99323652, .11610865)$
$U \sqrt{T} = 88.73030585$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (0.03900758, -0.27443147)$
$z_2 = (-0.44102761, -0.24185949)$
$z_3 = (0.4905379, -0.10938804)$
$z_4 = (-0.10389374, -0.73092750)$
$z_5 = (0.49201444, 0.61207277)$
$z_6 = (-0.64308099, 0.45073078)$
$z_7 = (1.2641932, -0.88940261)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-0.96039288, -0.27864944)$

$U \sqrt{T} = 89.01859330$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (0.07226767, 0.65345149)$
$z_2 = (-0.60203943, 0.26414011)$
$z_3 = (0.5297176, 0.38931139)$
$z_4 = (-0.60203943, -0.26414011)$
$z_5 = (0.7226767, -0.65345149)$
$z_6 = (0.5297176, -0.38931139)$
$z_7 = (-0.50000000, -0.86602540)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-0.50000000, 0.86602540)$

$U \sqrt{T} = 89.03021429$
Crit ind = 4
FP ind = 1
Isotropy = 1
$z_1 = (0.50207893, -0.30690695)$
$z_2 = (0.08193762, -0.58271893)$
$z_3 = (0.5812149, -0.31057303)$
$z_4 = (0.51764241, 0.4136316)$
$z_5 = (0.6731257, 0.69881593)$
$z_6 = (-0.66795302, 0.21613295)$
$z_7 = (-0.51823249, 0.78941776)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-0.50000000, 0.86602540)$

$U \sqrt{T} = 89.08114579$
Crit ind = 3
FP ind = -1
Isotropy = 1
$z_1 = (-0.50318413, 0.00830395)$
$z_2 = (-0.05752836, -0.49995371)$
$z_3 = (0.47660923, -0.20492269)$
$z_4 = (-0.25054987, 0.45373999)$
$z_5 = (0.50631842, 0.55857312)$
$z_6 = (-0.91732003, -0.36252906)$
$z_7 = (-0.27097695, -0.94840735)$
$z_8 = (1.00000000, 0.00000000)$
$z_9 = (-0.99790513, 0.99519575)$
\[ U\sqrt{T} = 89.08122317 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (0.06171642, -0.51195322) \]
\[ z_2 = (-0.51503818, 0.02531188) \]
\[ z_3 = (0.47760377, -0.21629087) \]
\[ z_4 = (-0.24956547, 0.46109021) \]
\[ z_5 = (0.50633856, 0.54355493) \]
\[ z_6 = (-0.91308565, -0.36272577) \]
\[ z_7 = (-0.29716291, -0.93647722) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.07080655, 0.99749007) \]

\[ U\sqrt{T} = 89.09024068 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (0.45109676, 0.07097413) \]
\[ z_2 = (0.11615896, -0.44776520) \]
\[ z_3 = (0.11699761, 0.47529901) \]
\[ z_4 = (-0.38990407, -0.29592635) \]
\[ z_5 = (-0.65053231, -0.42757401) \]
\[ z_6 = (-0.10247011, 0.98408560) \]
\[ z_7 = (-0.94400673, -0.29626996) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (0.39664697, -0.91797123) \]

\[ U\sqrt{T} = 89.77034759 \]
Crit ind = 4
FP ind = 1
Isotropy = 9
\[ z_1 = (0.17364818, -0.98480775) \]
\[ z_2 = (-0.50000000, 0.86025400) \]
\[ z_3 = (0.17364818, 0.98480775) \]
\[ z_4 = (0.76044444, -0.64278761) \]
\[ z_5 = (-0.50000000, -0.86025400) \]
\[ z_6 = (-0.93969262, 0.34202014) \]
\[ z_7 = (0.76044444, 0.64278761) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.93969262, -0.34202014) \]

\[ U\sqrt{T} = 89.80038558 \]
Crit ind = 4
FP ind = 1
Isotropy = 1/2
\[ z_1 = (-0.42056000, -0.04089290) \]
\[ z_2 = (0.4348807, 0.49370389) \]
\[ z_3 = (0.49524522, 0.1915505) \]
\[ z_4 = (-0.38871013, -0.37004111) \]
\[ z_5 = (-0.7297367, 0.35474966) \]
\[ z_6 = (0.31844974, -0.74593769) \]
\[ z_7 = (0.04918008, 0.99878993) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.74532331, -0.70952683) \]
\[ U \sqrt{T} = 90.35161303 \]
Crit ind = 3
FP ind = -1
Isotropy = 1/2
\[ z_1 = (1.2337959, -0.20777092) \]
\[ z_2 = (-0.09340433, 0.37049034) \]
\[ z_3 = (0.53682685, -0.05190529) \]
\[ z_4 = (-0.19087397, -0.50766709) \]
\[ z_5 = (-0.74035031, -0.05807770) \]
\[ z_6 = (0.38995145, 0.72090338) \]
\[ z_7 = (-0.58223954, 0.63397411) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.4328975, -0.89996684) \]

\[ U \sqrt{T} = 90.39185803 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (-0.11834655, -0.17807590) \]
\[ z_2 = (-0.1936543, 0.29143823) \]
\[ z_3 = (0.53893775, -0.04320452) \]
\[ z_4 = (-0.1688527, -0.51361298) \]
\[ z_5 = (0.35474100, 0.68877230) \]
\[ z_6 = (-0.77240590, -0.06031246) \]
\[ z_7 = (0.48977159, 0.73695870) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.38727711, -0.92196336) \]

\[ U \sqrt{T} = 90.63286156 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (-0.14835923, -0.28890376) \]
\[ z_2 = (0.52892225, -0.10458042) \]
\[ z_3 = (-0.2317830, -0.49080267) \]
\[ z_4 = (-0.06157639, 0.72358322) \]
\[ z_5 = (-0.55219856, 0.47163659) \]
\[ z_6 = (-0.73641997, -0.06110931) \]
\[ z_7 = (0.47873624, 0.56290348) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.38264451, -0.81272713) \]

\[ U \sqrt{T} = 90.91991870 \]
Crit ind = 4
FP ind = 1
Isotropy = 1/2
\[ z_1 = (-0.13471742, -0.21684020) \]
\[ z_2 = (-0.42850131, 0.02859559) \]
\[ z_3 = (0.53524801, -0.06348966) \]
\[ z_4 = (-0.21466210, -0.51191955) \]
\[ z_5 = (-0.18922652, -0.69795918) \]
\[ z_6 = (0.41396844, 0.61878212) \]
\[ z_7 = (-0.82615592, 0.36845212) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.42538800, -0.92153961) \]
\[ U \sqrt{I} = 92.29991200 \]
Crit ind = 4
FP ind = 1
Isotropy = 4
\[
\begin{align*}
\mathbf{z}_1 &= (-0.00000000, 0.00000000) \\
\mathbf{z}_2 &= (-0.51728060, -0.00000000) \\
\mathbf{z}_3 &= (-0.00000000, 0.51728060) \\
\mathbf{z}_4 &= (0.00000000, -0.51728060) \\
\mathbf{z}_5 &= (0.51728060, -0.00000000) \\
\mathbf{z}_6 &= (-1.00000000, -0.00000000) \\
\mathbf{z}_7 &= (-0.00000000, 1.00000000) \\
\mathbf{z}_8 &= (1.00000000, 0.00000000) \\
\mathbf{z}_9 &= (-1.00000000, 0.00000000)
\end{align*}
\]

\[ U \sqrt{I} = 92.34608471 \]
Crit ind = 5
FP ind = -1
Isotropy = 1
\[
\begin{align*}
\mathbf{z}_1 &= (0.08578407, -0.10830058) \\
\mathbf{z}_2 &= (0.3121065, 0.45867627) \\
\mathbf{z}_3 &= (-0.45363895, 0.07463060) \\
\mathbf{z}_4 &= (-0.10990984, -0.52935404) \\
\mathbf{z}_5 &= (-0.54045734, -0.1420315) \\
\mathbf{z}_6 &= (-0.93402566, -0.14872325) \\
\mathbf{z}_7 &= (0.06906875, 0.94326669) \\
\mathbf{z}_8 &= (1.00000000, 0.00000000) \\
\mathbf{z}_9 &= (-0.22894636, -0.97343904)
\end{align*}
\]

\[ U \sqrt{I} = 92.72728347 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[
\begin{align*}
\mathbf{z}_1 &= (-0.18156806, 0.9150316) \\
\mathbf{z}_2 &= (0.19762125, 0.04780829) \\
\mathbf{z}_3 &= (-0.5632148, 0.14384477) \\
\mathbf{z}_4 &= (0.58127411, -0.01195036) \\
\mathbf{z}_5 &= (0.25024974, -0.66690904) \\
\mathbf{z}_6 &= (-0.39537577, -0.59251212) \\
\mathbf{z}_7 &= (0.08491125, 0.73687005) \\
\mathbf{z}_8 &= (1.00000000, 0.00000000) \\
\mathbf{z}_9 &= (-0.97379104, 0.22744453)
\end{align*}
\]

\[ U \sqrt{I} = 94.52190449 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[
\begin{align*}
\mathbf{z}_1 &= (-0.9781710, 2.4112496) \\
\mathbf{z}_2 &= (-0.23333894, 1.1516238) \\
\mathbf{z}_3 &= (-0.42101256, 0.4166920) \\
\mathbf{z}_4 &= (0.59160760, 0.01176230) \\
\mathbf{z}_5 &= (-0.23621007, -0.62099702) \\
\mathbf{z}_6 &= (0.3375374, -0.6126596) \\
\mathbf{z}_7 &= (-0.65554208, -0.23634559) \\
\mathbf{z}_8 &= (1.00000000, 0.00000000) \\
\mathbf{z}_9 &= (-0.74720848, 0.66458972)
\end{align*}
\]
\[ U \sqrt{T} = 95.02122139 \]
Crit ind = 5
FP ind = -1
Isotropy = 1
\[ z_1 = (0.20547598, -0.11904493) \]
\[ z_2 = (-0.19662305, 0.06262960) \]
\[ z_3 = (-0.09389802, -0.46272563) \]
\[ z_4 = (0.58770608, -0.00524878) \]
\[ z_5 = (-0.54318034, 0.22445734) \]
\[ z_6 = (0.14330677, 0.70552615) \]
\[ z_7 = (-0.18194269, -0.89573805) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.92075373, 0.39014430) \]
Some central configurations for 10 bodies

\[ U \sqrt{T} = 101.34912671 \]
Crit ind = 6
FP ind = 1
Isotropy = 3
\[ z_1 = (-1.12628169, -0.21872630) \]
\[ z_2 = (0.25256338, -0.00000000) \]
\[ z_3 = (-1.12628169, 0.21872630) \]
\[ z_4 = (-0.30464978, -0.52766891) \]
\[ z_5 = (-0.30464978, 0.52766891) \]
\[ z_6 = (0.60929957, -0.00000000) \]
\[ z_7 = (-0.50000000, -0.86602540) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.50000000, 0.86602540) \]

\[ U \sqrt{T} = 103.66479887 \]
Crit ind = 6
FP ind = 1
Isotropy = 1
\[ z_1 = (0.1768472, 0.05809232) \]
\[ z_2 = (0.32864409, 0.01567635) \]
\[ z_3 = (-0.26416172, -0.19614094) \]
\[ z_4 = (-0.12230136, -0.40174615) \]
\[ z_5 = (0.64312990, 0.00483452) \]
\[ z_6 = (-0.53134300, 0.36237281) \]
\[ z_7 = (-0.24128096, -0.79258071) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.83037166, 0.55720993) \]

\[ U \sqrt{T} = 111.62059518 \]
Crit ind = 6
FP ind = 1
Isotropy = 1
\[ z_1 = (-0.12450125, -0.05539299) \]
\[ z_2 = (0.22370088, 0.00000000) \]
\[ z_3 = (-0.22370088, -0.00000000) \]
\[ z_4 = (-0.45498388, -0.00000000) \]
\[ z_5 = (0.45498388, 0.00000000) \]
\[ z_6 = (-0.70503464, 0.00000000) \]
\[ z_7 = (0.70503464, 0.00000000) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-0.97099118, -0.23911529) \]

\[ U \sqrt{T} = 126.52336886 \]
Crit ind = 7
FP ind = -1
Isotropy = 2
\[ z_1 = (-0.00000000, 0.00000000) \]
\[ z_2 = (0.22370088, 0.00000000) \]
\[ z_3 = (-0.22370088, -0.00000000) \]
\[ z_4 = (-0.45498388, 0.00000000) \]
\[ z_5 = (0.45498388, 0.00000000) \]
\[ z_6 = (-0.70503464, 0.00000000) \]
\[ z_7 = (0.70503464, 0.00000000) \]
\[ z_8 = (1.00000000, 0.00000000) \]
\[ z_9 = (-1.00000000, -0.00000000) \]
\[ U \sqrt{T} = 115.77936446 \]
Crit ind = 1
FP ind = -1
Isotropy = 1
\[ z_1 = (-2.0460617, -19816824) \]
\[ z_2 = (0.1053035, .31289606) \]
\[ z_3 = (0.46355894, -0.35505428) \]
\[ z_4 = (-0.72131483, .52135683) \]
\[ z_5 = (-0.46621515, -0.84797073) \]
\[ z_6 = (0.65891218, .70870370) \]
\[ z_7 = (-0.13056375, .96719188) \]
\[ z_8 = (-.95932848, -.17943218) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (0.31369729, -94952304) \]

\[ U \sqrt{T} = 116.30138460 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[ z_1 = (-0.12793203, -0.10300742) \]
\[ z_2 = (0.48221243, -0.35140630) \]
\[ z_3 = (-0.24042499, 0.54608703) \]
\[ z_4 = (0.49798779, .049996634) \]
\[ z_5 = (-0.41473254, -0.76746237) \]
\[ z_6 = (-0.83828000, -0.2412954) \]
\[ z_7 = (0.26647350, -0.9163855) \]
\[ z_8 = (-0.83807287, .45591905) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (0.21336869, 97697175) \]

\[ U \sqrt{T} = 116.46027730 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[ z_1 = (0.00194923, 0.01279499) \]
\[ z_2 = (0.53319856, -0.41608320) \]
\[ z_3 = (-0.63291002, -0.23843413) \]
\[ z_4 = (-0.39578514, 0.63093402) \]
\[ z_5 = (-0.56570769, 0.48445683) \]
\[ z_6 = (-0.4757897, -0.81654750) \]
\[ z_7 = (0.20321930, -0.91873926) \]
\[ z_8 = (0.14683510, 0.96384218) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (0.21336869, 97697175) \]

\[ U \sqrt{T} = 116.65839606 \]
Crit ind = 2
FP ind = 1
Isotropy = 3
\[ z_1 = (0.00000000, 0.00000000) \]
\[ z_2 = (-0.66929145, 0.25741444) \]
\[ z_3 = (0.55757317, 0.45091618) \]
\[ z_4 = (0.11171828, -0.70833062) \]
\[ z_5 = (0.20917868, 0.97787744) \]
\[ z_6 = (0.74227736, -0.67009277) \]
\[ z_7 = (-0.95145604, -0.30778467) \]
\[ z_8 = (-0.50000000, -0.86602540) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.50000000, 0.86602540) \]
\[ U \sqrt{T} = 116.70250556 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[
z_1 = (-0.7677422, 16477168) 
\]
\[
z_2 = (0.4746930, 2300901) 
\]
\[
z_3 = (-0.48145070, -21549778) 
\]
\[
z_4 = (0.04363077, -79787594) 
\]
\[
z_5 = (0.58283574, -54663714) 
\]
\[
z_6 = (-3.7069069, 79557082) 
\]
\[
z_7 = (-0.89056585, 27594682) 
\]
\[
z_8 = (0.30157205, .85937176) 
\]
\[
z_9 = (1.0000000, 0.0000000) 
\]
\[
z_{10} = (-0.64324641, -76565923) 
\]

\[ U \sqrt{T} = 116.76781698 \]
Crit ind = 3
FP ind = 1
Isotropy = 3
\[
z_1 = (-0.0000000, 0.0000000) 
\]
\[
z_2 = (-0.86652788, -32027532) 
\]
\[
z_3 = (0.71063050, -59029749) 
\]
\[
z_4 = (0.15589737, .91057282) 
\]
\[
z_5 = (0.5589737, -91057282) 
\]
\[
z_6 = (0.71063050, 59029749) 
\]
\[
z_7 = (-0.86652788, 32027532) 
\]
\[
z_8 = (-0.5000000, -86602540) 
\]
\[
z_9 = (1.0000000, 0.0000000) 
\]
\[
z_{10} = (-0.5000000, -86602540) 
\]

\[ U \sqrt{T} = 116.77034759 \]
Crit ind = 4
FP ind = 1
Isotropy = 9
\[
z_1 = (0.0000000, -0.0000000) 
\]
\[
z_2 = (0.76604444, -64278761) 
\]
\[
z_3 = (-0.93969262, -34202014) 
\]
\[
z_4 = (1.7364818, 98480775) 
\]
\[
z_5 = (-0.5000000, 86602540) 
\]
\[
z_6 = (-0.5000000, 86602540) 
\]
\[
z_7 = (0.76604444, 64278761) 
\]
\[
z_8 = (-0.93969262, 34202014) 
\]
\[
z_9 = (1.0000000, 0.0000000) 
\]
\[
z_{10} = (1.7364818, -98480775) 
\]

\[ U \sqrt{T} = 116.77729075 \]
Crit ind = 2
FP ind = 1
Isotropy = 1
\[
z_1 = (0.13193015, -4.4834690) 
\]
\[
z_2 = (-0.42977943, -1.8360323) 
\]
\[
z_3 = (0.13193015, 4.4834690) 
\]
\[
z_4 = (0.42977943, 1.8360323) 
\]
\[
z_5 = (-0.41798441, -8.8684214) 
\]
\[
z_6 = (0.41798441, 8.8684214) 
\]
\[
z_7 = (-0.63047322, 7.7129880) 
\]
\[
z_8 = (0.63047322, -7.7129880) 
\]
\[
z_9 = (1.0000000, 0.0000000) 
\]
\[
z_{10} = (-1.0000000, 0.0000000) 
\]
| Case          | U√I         | Crit ind | FP ind | Isotropy | Z1     | Z2     | Z3     | Z4     | Z5     | Z6     | Z7     | Z8     | Z9     | Z10    |
|--------------|-------------|----------|--------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1            | 116.84701909 | 3        | -1     | 1        | (.33598940, 21881579) | (-.27474161, 292000050) | (.42753311, -.39692534) | (-.50922512, -.28467731) | (.52084881, .83162882) | (-.30962777, .93113942) | (-.48138401, -.86956189) | (.26231831, -.95867490) | (1.00000000, 0.00000000) | (-.97169111, .23625490) |
| 2            | 117.12687289 | 2        | 1      | 1        | (.10887508, -.12254814) | (.50719270, .19738442) | (.13628727, .52690685) | (-.59946262, .49166252) | (.55883908, -.53739687) | (.03992178, -.84921141) | (-.84765096, -.63333303) | (.56549020, -.63650716) | (1.00000000, 0.00000000) | (-.11775388, .99304281) |
| 3            | 117.36759860 | 2        | 1      | 1        | (.44209690, .32120220) | (-.16866599, .51971608) | (.54646182, -.00000000) | (.44209690, -.32120220) | (.16866599, -.51971608) | (.80901699, .58778525) | (.80901699, -.58778525) | (.30901699, .95105652) | (1.00000000, 0.00000000) | (.30901699, -.95105652) |
| 4            | 117.39457924 | 2        | 1      | 1        | (-.36600022, -.06290818) | (.05616945, .30709480) | (.12851332, -.48712966) | (.48940978, -.11953680) | (.31854535, .84647286) | (.85218393, .30284100) | (.70751585, -.56478154) | (.55167999, .71777855) | (1.00000000, 0.00000000) | (.01838280, -.99983102) |
$U \sqrt{T} = 117.45234024$

Crit ind = 3
FP ind = -1
Isotropy = 1/2

$z_1 = (-24212045, 0.02307857)$
$z_2 = (1.0188239, -5.1425662)$
$z_3 = (0.99284176, 5.3548682)$
$z_4 = (5.0838046, -20.331307)$
$z_5 = (-6.9968120, -5.0678906)$
$z_6 = (-8.7121475, 20.068530)$
$z_7 = (-4.4905568, 8.1013401)$
$z_8 = (6.4438355, -6.6937430)$
$z_9 = (1.0000000, 0.0000000)$
$z_{10} = (-0.8541610, -1.01440025)$

$U \sqrt{T} = 117.48830277$

Crit ind = 3
FP ind = -1
Isotropy = 1

$z_1 = (-0.00287904, -1.7687418)$
$z_2 = (0.48974852, -11.5037401)$
$z_3 = (-4.9323431, -0.9907457)$
$z_4 = (0.0103689, 6.2276017)$
$z_5 = (-5.6455547, 6.6939098)$
$z_6 = (5.8603978, 6.5058043)$
$z_7 = (3.6691956, -7.9826839)$
$z_8 = (-3.9270569, -7.8590370)$
$z_9 = (1.0000000, 0.0000000)$
$z_{10} = (-0.9947024, 0.3254609)$

$U \sqrt{T} = 117.54057407$

Crit ind = 3
FP ind = -1
Isotropy = 1

$z_1 = (-0.01731857, -2.6329857)$
$z_2 = (0.47647778, 1.4706592)$
$z_3 = (-4.9163598, 0.8339873)$
$z_4 = (5.32008059, -5.1766572)$
$z_5 = (-4.4779369, -5.8132905)$
$z_6 = (5.0753382, -8.7470102)$
$z_7 = (3.5682040, 8.3241908)$
$z_8 = (-4.6277990, 7.7856953)$
$z_9 = (1.0000000, 0.0000000)$
$z_{10} = (-0.99138446, -1.3098414)$

$U \sqrt{T} = 117.78576236$

Crit ind = 3
FP ind = -1
Isotropy = 1

$z_1 = (-0.04555508, -2.7408273)$
$z_2 = (-0.41215287, 2.82709582)$
$z_3 = (0.48147256, 1.3426703)$
$z_4 = (0.8319133, 5.0052175)$
$z_5 = (-7.9262802, -4.3637754)$
$z_6 = (5.4923873, -6.4893699)$
$z_7 = (-1.4915490, -0.89739245)$
$z_8 = (-9.4623429, -3.2348210)$
$z_9 = (1.0000000, 0.0000000)$
$z_{10} = (1.08822541, 1.01572302)$
U√I = 117.91094722
Crit ind = 3
FP ind = -1
Isotropy = 1/2
z₁ = (-.31468019, .07786138)
z₂ = (.08987796, .49353153)
z₃ = (-.21518188, -.47028249)
z₄ = (.49776277, .19279000)
z₅ = (.53013295, -.51960736)
z₆ = (-.74741094, .51743154)
z₇ = (-.86051796, -.32902493)
z₈ = (.03110998, -.97247709)
z₉ = (1.00000000, 0.00000000)
z₁₀ = (-.11092700, 1.00977742)

U√I = 118.35557043
Crit ind = 3
FP ind = -1
Isotropy = 1/2
z₁ = (.05738884, .16824714)
z₂ = (-.05055782, -.4104318)
z₃ = (.51812877, .04398856)
z₄ = (-.35140250, .38948954)
z₅ = (-.77069382, -.35557043)
z₆ = (.31294871, .78974150)
z₇ = (.47328430, -.7136364)
z₈ = (-.45037307, -.78754262)
z₉ = (1.00000000, 0.00000000)
z₁₀ = (-.73872342, .67999425)

U√I = 118.39761429
Crit ind = 3
FP ind = -1
Isotropy = 3
z₁ = (.00000000, -.00000000)
z₂ = (-.24963802, -.43238573)
z₃ = (-.24963802, .43238573)
z₄ = (.49927604, .00000000)
z₅ = (-.83387462, .00000000)
z₆ = (.41693731, -.72215661)
z₇ = (.41693731, .72215661)
z₈ = (-.50000000, -.86602540)
z₉ = (1.00000000, 0.00000000)
z₁₀ = (-.50000000, .86602540)

U√I = 118.41208145
Crit ind = 4
FP ind = 1
Isotropy = 1
z₁ = (.04129108, .08720932)
z₂ = (-.18495617, -.39063889)
z₃ = (.51544303, .02393743)
z₄ = (-.30814747, .41388362)
z₅ = (-.81278668, -.10578469)
z₆ = (.43327838, -.69576032)
z₇ = (.35619456, .75230497)
z₈ = (-.40656308, -.85868638)
z₉ = (1.00000000, 0.00000000)
z₁₀ = (-.83375366, .77353494)
\( U \sqrt{T} = 119.24821560 \)
Crit ind = 4
FP ind = 1
Isotropy = 1
\[
\begin{align*}
 z_1 &= (0.45004980, \ 0.00769943) \\
 z_2 &= (0.16313482, \ 0.41951297) \\
 z_3 &= (0.29275709, \ -0.47610918) \\
 z_4 &= (-0.34520518, \ 0.43956803) \\
 z_5 &= (-0.41912354, \ -0.63220202) \\
 z_6 &= (-0.73824928, \ -0.17415404) \\
 z_7 &= (-0.88718823, \ 0.4710169) \\
 z_8 &= (1.00000000, \ 0.00000000) \\
 z_9 &= (0.00000000, \ 0.00000000) \\
 z_{10} &= (0.34633318, \ 0.93807465) \\
\end{align*}
\]

\( U \sqrt{T} = 119.24821560 \)
Crit ind = 4
FP ind = 1
Isotropy = 1
\[
\begin{align*}
 z_1 &= (-0.36687191, \ 0.30119729) \\
 z_2 &= (0.45141137, \ 0.14677390) \\
 z_3 &= (-0.09770592, \ 0.51773973) \\
 z_4 &= (-0.55272003, \ -0.24172499) \\
 z_5 &= (0.4260753, \ -0.42654005) \\
 z_6 &= (0.16338208, \ -0.90576231) \\
 z_7 &= (-0.48225275, \ -0.78392050) \\
 z_8 &= (-0.93122178, \ 0.36445301) \\
 z_9 &= (1.00000000, \ 0.00000000) \\
 z_{10} &= (0.34493518, \ 1.02778392) \\
\end{align*}
\]

\( U \sqrt{T} = 119.28946483 \)
Crit ind = 5
FP ind = -1
Isotropy = 1
\[
\begin{align*}
 z_1 &= (0.08895979, \ -0.85999487) \\
 z_2 &= (0.48413176, \ 0.1471957) \\
 z_3 &= (-0.38219358, \ 0.32923559) \\
 z_4 &= (-0.60994724, \ -0.21356201) \\
 z_5 &= (0.46703891, \ -0.44667552) \\
 z_6 &= (0.11982494, \ 0.85499838) \\
 z_7 &= (-0.48225275, \ -0.78392050) \\
 z_8 &= (0.20543142, \ 0.94867885) \\
 z_9 &= (1.00000000, \ 0.00000000) \\
 z_{10} &= (-0.91049238, \ 0.41352584) \\
\end{align*}
\]

\( U \sqrt{T} = 119.49891253 \)
Crit ind = 4
FP ind = 1
Isotropy = 2
\[
\begin{align*}
 z_1 &= (-0.13740419, \ -0.53748254) \\
 z_2 &= (0.13740419, \ 0.53748254) \\
 z_3 &= (0.5026771, \ 0.23471804) \\
 z_4 &= (-0.5026771, \ -0.23471804) \\
 z_5 &= (-0.2457843, \ -0.41095489) \\
 z_6 &= (-0.4957843, \ -0.41095489) \\
 z_7 &= (0.18549139, \ 0.98264589) \\
 z_8 &= (-0.18549139, \ 0.98264589) \\
 z_9 &= (0.00000000, \ 0.00000000) \\
 z_{10} &= (-0.00000000, \ 0.00000000) \\
\end{align*}
\]
\[ U\sqrt{T} = 119.80126934 \]
Crit ind = 3
FP ind = -1
Isotropy = 1
\[ z_1 = (-.16304321, .22540977) \]
\[ z_2 = (.16795299, -.23219761) \]
\[ z_3 = (.55389940, -.06503618) \]
\[ z_4 = (-.11162132, -.54642011) \]
\[ z_5 = (-.71666075, -.24548118) \]
\[ z_6 = (.55748247, .60379916) \]
\[ z_7 = (-.72754948, .39502725) \]
\[ z_8 = (-.14742686, .81464108) \]
\[ z_9 = (1.0000000, 0.00000000) \]
\[ z_{10} = (-.31303325, -.94974217) \]

\[ U\sqrt{T} = 120.51872081 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (.15415837, .53540050) \]
\[ z_2 = (-.38685020, .40095563) \]
\[ z_3 = (.56763856, .31568703) \]
\[ z_4 = (-.64938150, .01324801) \]
\[ z_5 = (1.0000000, 0.00000000) \]
\[ z_{10} = (-.24338662, .97939215) \]

\[ U\sqrt{T} = 120.52398426 \]
Crit ind = 5
FP ind = -1
Isotropy = 1
\[ z_1 = (.51209715, -.26477259) \]
\[ z_2 = (.15592999, -.55506870) \]
\[ z_3 = (.51531976, .33299932) \]
\[ z_4 = (-.43018267, -.43747671) \]
\[ z_5 = (.61257768, -.44002571) \]
\[ z_6 = (-.33229727, -.68089231) \]
\[ z_7 = (-.88367170, -.46810718) \]
\[ z_{10} = (-.20189558, -.97940705) \]

\[ U\sqrt{T} = 120.52510744 \]
Crit ind = 4
FP ind = 1
Isotropy = 1
\[ z_1 = (.17169194, .25938933) \]
\[ z_2 = (-.43317640, -.13687846) \]
\[ z_3 = (.04324393, -.45225202) \]
\[ z_4 = (.55299992, .08101856) \]
\[ z_5 = (-.14120573, .54005569) \]
\[ z_6 = (.52147843, -.68826149) \]
\[ z_7 = (-.83729923, .21112469) \]
\[ z_8 = (-.48633082, -.73474050) \]
\[ z_9 = (1.0000000, 0.00000000) \]
\[ z_{10} = (-.39070205, .92051720) \]
$U\sqrt{T} = 120.85403438$
Crit ind = 4
FP ind = 1
Isotropy = 1/2
$z_1 = (.15979498, -.21579313)$
$z_2 = (-.0026936, .4144985)$
$z_3 = (-.43077239, -.07680436)$
$z_4 = (.55469278, -.05892093)$
$z_5 = (-.12275544, -.55821831)$
$z_6 = (.47011301, .69992355)$
$z_7 = (-.46702842, .70605364)$
$z_8 = (-.90667378, .07126380)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.25467138, -.98165411)$

$U\sqrt{T} = 121.20754739$
Crit ind = 4
FP ind = 1
Isotropy = 1/2
$z_1 = (.17917236, .27268883)$
$z_2 = (-.0026936, .27268883)$
$z_3 = (-.55469278, -.05892093)$
$z_4 = (.47011301, .69992355)$
$z_5 = (-.46702842, .70605364)$
$z_6 = (-.90667378, .07126380)$
$z_7 = (.55469278, -.05892093)$
$z_8 = (-.90667378, .07126380)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.25467138, -.98165411)$

$U\sqrt{T} = 121.24506424$
Crit ind = 4
FP ind = 1
Isotropy = 2
$z_1 = (.19050513, .00000000)$
$z_2 = (.19050513, .00000000)$
$z_3 = (-.58077135, -.00000000)$
$z_4 = (.58077135, -.00000000)$
$z_5 = (.31799622, .67513226)$
$z_6 = (.31799622, .67513226)$
$z_7 = (.31799622, .67513226)$
$z_8 = (.31799622, .67513226)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (1.00000000, -.00000000)$

$U\sqrt{T} = 121.39001611$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (.18544606, .23177205)$
$z_2 = (.24336532, -.40666237)$
$z_3 = (-.45036225, .14920342)$
$z_4 = (-.10144921, .55178760)$
$z_5 = (.56060201, .02206546)$
$z_6 = (.41948543, -.52427643)$
$z_7 = (-.88527978, -.11138215)$
$z_8 = (.08533141, -.88815013)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.21930633, .97564256)$
$U \sqrt{I} = 121.61633134$
Crit ind = 5
FP ind = -1
Isotropy = $1/2$
$z_1 = (1.8047524, -29758745)$
$z_2 = (1.2685212, -0.49655047)$
$z_3 = (-0.32532203, -0.42451113)$
$z_4 = (-0.15976940, 0.34115099)$
$z_5 = (0.5510746, 0.10929046)$
$z_6 = (-0.68193485, 0.19071147)$
$z_7 = (0.5896754, -0.69232761)$
$z_8 = (-0.83275486, -0.41987848)$
$z_9 = (1.0000000, 0.00000000)$
$z_{10} = (-0.44822120, 0.90152733)$

$U \sqrt{I} = 121.73019826$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (0.20043614, -0.34694902)$
$z_2 = (0.54550336, -0.3961549)$
$z_3 = (-0.15154803, -0.52430950)$
$z_4 = (0.3029828, 0.45463979)$
$z_5 = (-0.65475704, -0.22535760)$
$z_6 = (0.10945340, 0.72331183)$
$z_7 = (-0.68127726, 0.6649831)$
$z_8 = (-0.39057700, 0.67607720)$
$z_9 = (1.0000000, 0.00000000)$
$z_{10} = (-0.49953186, 0.86629552)$

$U \sqrt{I} = 122.14138716$
Crit ind = 5
FP ind = -1
Isotropy = 10
$z_1 = (-0.29915039, -0.92485804)$
$z_2 = (0.29915039, 0.92485804)$
$z_3 = (-0.30704090, 0.93321303)$
$z_4 = (-0.30704090, -0.93321303)$
$z_5 = (0.79540494, 0.57662090)$
$z_6 = (-0.79540494, -0.57662090)$
$z_7 = (-0.81057176, 0.58563933)$
$z_8 = (0.81057176, -0.58563933)$
$z_9 = (1.0000000, 0.00000000)$
$z_{10} = (-1.0000000, 0.00000000)$

$U \sqrt{I} = 122.20145746$
Crit ind = 4
FP ind = 1
Isotropy = 1
$z_1 = (-0.15706989, -0.18525420)$
$z_2 = (0.21190421, -0.11868729)$
$z_3 = (0.58345777, -0.03393597)$
$z_4 = (-0.53481533, -0.23568450)$
$z_5 = (-0.12131727, 0.67245022)$
$z_6 = (-0.60484308, 0.41858759)$
$z_7 = (0.42043678, 0.60355914)$
$z_8 = (0.13920266, -0.7158726)$
$z_9 = (1.0000000, 0.00000000)$
$z_{10} = (-0.93095886, -0.34944773)$
$U\sqrt{T} = 123.4564660$
Crit ind = 5
FP ind = -1
Isotropy = 1/2
$z_1 = (0.20642092, -0.443143)$
$z_2 = (-0.18356418, -0.11677307)$
$z_3 = (-0.28870543, 0.36264396)$
$z_4 = (0.58834980, -0.01142357)$
$z_5 = (-0.50326076, -0.33157480)$
$z_6 = (0.31579124, 0.66764273)$
$z_7 = (0.20619397, -0.72477439)$
$z_8 = (-0.50152009, 0.75872783)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-0.83970547, -0.56003726)$

$U\sqrt{T} = 123.93098067$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (0.20254995, 0.10365132)$
$z_2 = (-0.22146311, 0.05219358)$
$z_3 = (-0.06436945, 0.53040583)$
$z_4 = (0.58330422, -0.02718410)$
$z_5 = (-0.57287412, -0.11312839)$
$z_6 = (0.37407282, -0.62239953)$
$z_7 = (-0.21433930, 0.09380856)$
$z_8 = (-0.11590943, 0.95509651)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-0.97097158, -0.23919487)$

$U\sqrt{T} = 124.62108602$
Crit ind = 4
FP ind = 1
Isotropy = 1/2
$z_1 = (-0.06358922, -0.26742154)$
$z_2 = (0.25701312, -0.11071653)$
$z_3 = (-0.24156129, 0.27769907)$
$z_4 = (0.60644564, 0.03007160)$
$z_5 = (-0.32697037, -0.51433731)$
$z_6 = (0.36067633, 0.60287574)$
$z_7 = (0.7530824, 0.08939479)$
$z_8 = (-0.26070769, 0.7263844)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-0.57489828, -0.81915755)$

$U\sqrt{T} = 124.63531469$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (-0.6196784, 0.26742154)$
$z_2 = (0.25672625, 0.09719014)$
$z_3 = (-0.16073578, -0.30091712)$
$z_4 = (-0.31611035, 0.52045729)$
$z_5 = (0.60835116, 0.02665341)$
$z_6 = (-0.34567613, -0.64242043)$
$z_7 = (-0.72616160, -0.06985111)$
$z_8 = (-0.38974452, -0.72964962)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-0.55603345, 0.38115991)$
\[ U \sqrt{T} = 124.98614209 \]
Crit ind = 5
FP ind = -1
Isotropy = 1
\[ z_1 = (0.26061298, 1.6873320) \]
\[ z_2 = (-0.5131068, -0.30619802) \]
\[ z_3 = (0.59848032, 0.5135838) \]
\[ z_4 = (-0.36591196, 0.47636627) \]
\[ z_5 = (-0.3334932, -0.67901381) \]
\[ z_6 = (-0.47864910, -0.48277010) \]
\[ z_7 = (-0.70607415, -0.3451651) \]
\[ z_8 = (0.45093986, 0.5441283) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.67473795, 0.73805738) \]

\[ U \sqrt{T} = 125.34057708 \]
Crit ind = 5
FP ind = -1
Isotropy = 1/2
\[ z_1 = (-0.06514342, 0.25416241) \]
\[ z_2 = (0.26052708, 0.11965160) \]
\[ z_3 = (-0.41378240, -0.10073840) \]
\[ z_4 = (0.60604612, 0.03331037) \]
\[ z_5 = (-0.32144990, 0.52045714) \]
\[ z_6 = (-0.15007423, -0.66974957) \]
\[ z_7 = (0.38720107, -0.58415416) \]
\[ z_8 = (-0.75492823, -0.41238754) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.54839610, 0.83944816) \]

\[ U \sqrt{T} = 126.00626395 \]
Crit ind = 6
FP ind = 1
Isotropy = 2
\[ z_1 = (-0.21585256, 0.00000000) \]
\[ z_2 = (-0.21585256, 0.00000000) \]
\[ z_3 = (0.00000000, -0.48587379) \]
\[ z_4 = (-0.00000000, 0.48587379) \]
\[ z_5 = (-0.59775051, -0.00000000) \]
\[ z_6 = (0.59775051, 0.00000000) \]
\[ z_7 = (0.00000000, -0.91876248) \]
\[ z_8 = (0.00000000, 0.91876248) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-1.00000000, -0.00000000) \]

\[ U \sqrt{T} = 126.75698146 \]
Crit ind = 6
FP ind = 1
Isotropy = 1
\[ z_1 = (-0.06572443, -0.25029013) \]
\[ z_2 = (0.25233307, -0.05738359) \]
\[ z_3 = (0.05475730, 0.43148845) \]
\[ z_4 = (-0.40795287, 0.15084793) \]
\[ z_5 = (0.61174538, -0.01324919) \]
\[ z_6 = (-0.27096121, -0.54862370) \]
\[ z_7 = (0.10917732, 0.86916405) \]
\[ z_8 = (-0.82123669, 0.30485428) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.46213786, -0.88680810) \]
$U\sqrt{T} = 128.70332084$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (.00573478, .05595694)$
$z_2 = (.31830305, .02400155)$
$z_3 = (-.30673645, .08845912)$
$z_4 = (.63984576, .00679357)$
$z_5 = (-.62516646, .13643916)$
$z_6 = (.23080065, -.63260089)$
$z_7 = (-.35437270, -.57288903)$
$z_8 = (.07080311, .69085878)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.97921173, .20284079)$

$U\sqrt{T} = 129.67705293$
Crit ind = 6
FP ind = 1
Isotropy = 1
$z_1 = (-.21549709, -.12820455)$
$z_2 = (.24334850, .06047239)$
$z_3 = (-.11348145, .27597682)$
$z_4 = (.60535941, .01565640)$
$z_5 = (-.44126666, -.4171541)$
$z_6 = (-.24742444, .60171428)$
$z_7 = (.27285022, -.66354752)$
$z_8 = (-.71073990, -.70345490)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.39314859, .95610249)$

$U\sqrt{T} = 131.0943219$
Crit ind = 5
FP ind = -1
Isotropy = 1
$z_1 = (.04720259, .14389133)$
$z_2 = (.3507521, .06483067)$
$z_3 = (-.23156332, .25071266)$
$z_4 = (-.19709874, -.60083140)$
$z_5 = (.64578098, .01900093)$
$z_6 = (-.5094147, .39783275)$
$z_7 = (-.60213010, -.28305726)$
$z_8 = (.31746061, -.58472287)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.80568576, .59234319)$

$U\sqrt{T} = 131.25534651$
Crit ind = 6
FP ind = 1
Isotropy = 1
$z_1 = (.00062249, -.00371874)$
$z_2 = (-.20796499, -.10085778)$
$z_3 = (-.23156332, .25071266)$
$z_4 = (-.07608838, .45454929)$
$z_5 = (.60853891, -.20811887)$
$z_6 = (.64314159, .00140373)$
$z_7 = (.11234929, -.67117063)$
$z_8 = (-.14205667, .84864143)$
$z_9 = (1.00000000, 0.00000000)$
$z_{10} = (-.94548663, -.32560991)$
\( U \sqrt{I} = 132.42878391 \)
Crit ind = 6
FP ind = 1
Isotropy = 1/2
\[ z_1 = (0.04302483, -10614423) \]
\[ z_2 = (0.34019163, -0.04124347) \]
\[ z_3 = (-0.23295418, -25160089) \]
\[ z_4 = (-0.28425666, 28542275) \]
\[ z_5 = (0.64962848, -0.01236400) \]
\[ z_6 = (-0.47546165, -0.44706045) \]
\[ z_7 = (0.22837805, 0.62778194) \]
\[ z_8 = (-0.53036331, 0.2232106) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.73818719, -0.67711271) \]

\( U \sqrt{I} = 139.07007346 \)
Crit ind = 7
FP ind = -1
Isotropy = 1
\[ z_1 = (-0.02121674, 0.03343510) \]
\[ z_2 = (-0.14245716, -0.28928072) \]
\[ z_3 = (0.32366529, -0.00511110) \]
\[ z_4 = (-0.16701630, -0.00000000) \]
\[ z_5 = (0.43258942, -0.50799405) \]
\[ z_6 = (1.00000000, 0.00000000) \]
\[ z_{10} = (0.54846504, 0.86431665) \]

\( U \sqrt{I} = 139.07675434 \)
Crit ind = 8
FP ind = 1
Isotropy = 3
\[ z_1 = (0.00000000, 0.00000000) \]
\[ z_2 = (0.16701630, 0.28928072) \]
\[ z_3 = (0.32388902, 0.56099224) \]
\[ z_4 = (-0.00000000, -0.00000000) \]
\[ z_7 = (-0.32388902, 0.56099224) \]
\[ z_8 = (-0.50000000, 0.86602540) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-0.50000000, -0.86602540) \]

\( U \sqrt{I} = 139.14558600 \)
Crit ind = 6
FP ind = 1
Isotropy = 2
\[ z_1 = (0.13291577, -0.00000000) \]
\[ z_2 = (-0.13291577, 0.00000000) \]
\[ z_3 = (-0.39994776, 0.00000000) \]
\[ z_4 = (0.39994776, 0.00000000) \]
\[ z_5 = (0.00000000, -0.63201170) \]
\[ z_6 = (0.00000000, 0.63201170) \]
\[ z_7 = (-0.67891260, 0.00000000) \]
\[ z_8 = (0.67891260, 0.00000000) \]
\[ z_9 = (1.00000000, 0.00000000) \]
\[ z_{10} = (-1.00000000, 0.00000000) \]
\( U \sqrt{T} = 140.69033840 \)
Crit ind = 6
FP ind = 1
Isotropy = 1
\( z_1 = (1.1446285, .05446140) \)
\( z_2 = (1.0878211, 1.09002569) \)
\( z_3 = (1.0473000, .02235877) \)
\( z_4 = (1.35498999, .18727850) \)
\( z_5 = (1.38762761, .48174623) \)
\( z_6 = (1.1545948, -.59872937) \)
\( z_7 = (1.68208315, .00736631) \)
\( z_8 = (1.61743168, .28758761) \)
\( z_9 = (1.0000000, 0.00000000) \)
\( z_{10} = (-.91100409, .41239733) \)

\( U \sqrt{T} = 143.29732130 \)
Crit ind = 7
FP ind = 1
Isotropy = 1
\( z_1 = (1.14593811, -.03644447) \)
\( z_2 = (1.11338186, -.09658580) \)
\( z_3 = (1.0920007, -.01107367) \)
\( z_4 = (1.35379968, -.20589586) \)
\( z_5 = (1.10140125, .39712686) \)
\( z_6 = (1.68340646, -.00383875) \)
\( z_7 = (1.61743168, .28758761) \)
\( z_8 = (1.0000000, 0.00000000) \)
\( z_{10} = (-.91100409, .41239733) \)

\( U \sqrt{T} = 154.51403602 \)
Crit ind = 7
FP ind = 1
Isotropy = 1
\( z_1 = (1.00339091, -.03644447) \)
\( z_2 = (1.22081422, -.06227227) \)
\( z_3 = (1.22895247, -.01474856) \)
\( z_4 = (1.44900569, -.10258980) \)
\( z_5 = (1.46053091, -.00000000) \)
\( z_6 = (1.0619153, .58006606) \)
\( z_7 = (1.69265415, -.15048416) \)
\( z_8 = (1.70880850, -.00240138) \)
\( z_9 = (1.0000000, 0.00000000) \)
\( z_{10} = (.77791720, -.20899271) \)

\( U \sqrt{T} = 173.66904306 \)
Crit ind = 8
FP ind = 1
Isotropy = 2
\( z_1 = (-.9826571, .00000000) \)
\( z_2 = (.9826571, .00000000) \)
\( z_3 = (-.29739841, .00000000) \)
\( z_4 = (.29739841, -.00000000) \)
\( z_5 = (.50521414, .00000000) \)
\( z_6 = (.50521414, -.00000000) \)
\( z_7 = (.73150788, -.00000000) \)
\( z_8 = (.73150788, .00000000) \)
\( z_9 = (1.0000000, 0.00000000) \)
\( z_{10} = (-1.0000000, -.00000000) \)

Dipartimento di Matematica del Politecnico di Milano, Piazza Leonardo da Vinci 32 – 20133 Milano (I), Current address: Max-Planck-Institut für Mathematik, Vivatsgasse, 7 - 53111 Bonn (DE)
E-mail address: ferrario@mate.polimi.it