ON THE FREQUENCY EVOLUTION OF X-RAY BRIGHTNESS OSCILLATIONS DURING THERMONUCLEAR X-RAY BURSTS: EVIDENCE OF COHERENT OSCILLATIONS

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ABSTRACT

We investigate the time dependence of the frequency of X-ray brightness oscillations during thermonuclear X-ray bursts from several neutron star low-mass X-ray binaries. We find that the oscillation frequency in the cooling tails of X-ray bursts from 4U 1702–429 and 4U 1728–34 is well described by an exponential “chirp” model. With this model, we demonstrate that the pulse trains in the cooling tails of many bursts are highly phase coherent. We measure oscillation quality factors for the bursts from 4U 1728–34 and 4U 1702–429 as high as $Q = v_0/\Delta v_{FWM} \sim 4000$. We use this model of the frequency evolution to search sensitively for significant power at the harmonics and first subharmonic of the 330 and 363 Hz signal in the bursts from 4U 1702–429 and 4U 1728–23, respectively, but we find no strong evidence of significant power at any harmonic or the subharmonic. We argue that the high coherence of the oscillations favors stellar rotation as the source of the oscillations. The lack of a subharmonic in the bursts from both 4U 1728–34 and 4U 1702–429 suggests that in these sources, the burst oscillation frequency is indeed the stellar spin frequency. We briefly discuss the frequency evolution in terms of the rotational motion of an angular momentum–conserving thermonuclear shell. We discuss how the limits on harmonic content can be used to infer properties of the neutron star.

Subject headings: stars: individual (4U 1702–429, 4U 1728–34)—stars: neutron—stars: rotation—X-rays: bursts

1. INTRODUCTION

Millisecond oscillations in the X-ray brightness during thermonuclear bursts (“burst oscillations”) have been observed from six low-mass X-ray binaries (LMXBs) with the Rossi X-ray Timing Explorer (RXTE) (see Strohmayer, Swank, & Zhang 1998a for a review). The presence of large amplitudes near burst onset, combined with spectral evidence of localized thermonuclear burning, suggests that these oscillations are caused by rotational modulation of thermonuclear inhomogeneities (see Strohmayer, Zhang, & Swank 1997b). The asymptotic pulsation frequency in the cooling tails of bursts from 4U 1728–34 is stable over timescales of years, thus supporting a coherent mechanism such as the rotational modulation (Strohmayer et al. 1998b).

An intriguing aspect of these oscillations is the frequency evolution that is evident during many bursts. The frequency is observed to increase in the cooling tail, reaching a plateau or asymptotic limit (see Strohmayer et al. 1998b). However, Strohmayer (1999) has recently discovered an episode of spin-down in the cooling tail of a burst from 4U 1636–53. Evidence of frequency change has been seen in five of the six burst oscillation sources and appears to be commonly associated with the physical process responsible for the pulsations. Strohmayer et al. (1997a) have argued that this evolution results from angular momentum conservation of the thermonuclear shell. The thermonuclear flash expands the shell, increasing its rotational moment of inertia and slowing its spin rate. Near the burst onset, the shell is thickest, and thus the observed frequency is lowest. The shell then spins back up as it recouples to the bulk of the neutron star as it cools. This scenario is viable as long as the shell decouples from the bulk of the neutron star during the thermonuclear flash and then comes back into corotation with it over the $\approx 10$ s of the burst falloff. Calculations indicate that

the $\sim 10$ m thick preburst shell expands to $\sim 30$ m during the flash (see Joss 1978 and Bildsten 1995), which gives a frequency shift due to angular momentum conservation of $\approx 2v_{spin}(20 \, m/R)$, where $v_{spin}$ and $R$ are the stellar spin frequency and radius, respectively. For the several hundred hertz spin frequencies inferred from burst oscillations, this gives a shift of $\sim 2$ Hz, which is similar to that observed.

In bursts where frequency drift is evident, the drift broadens the peak in the power spectrum and produces quality values of $Q = v_0/\Delta v_{FWM} \approx 300$. In some bursts, a relatively short train of pulses is observed during which there is no strong evidence of a varying frequency. A burst such as this from KS 1743–26, with 524 Hz oscillations, yielded the highest coherence of $Q \approx 900$ yet reported in a burst oscillation (see Smith, Morgan, & Bradt 1997).

In this Letter, we investigate the time dependence of the frequency observed in bursts from 4U 1728–34 and 4U 1702–429. We show that in the cooling tails of bursts, the pulse trains are effectively coherent. We show that with accurate modeling of the drift, quality factors as high as $Q \sim 4000$ are achieved in some bursts. We investigate the functional form of the frequency drift and show that a simple exponential “chirp” model works remarkably well. We use this model to search for significant power at the harmonics and first subharmonic of the strongest oscillation frequency in each source. Such searches are important in establishing whether the strongest oscillation frequency is the stellar spin frequency or its first harmonic, as appears now to be the case for 4U 1636–53 (see Miller 1999). The detection of harmonic signals, or limits on them, is also important in obtaining constraints on the stellar compactness (see Miller & Lamb 1998 and Strohmayer et al. 1998c). We note that Zhang et al. (1998) have previously reported on a model for the frequency evolution during a burst from Aql X-1.

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2. MODELING THE FREQUENCY DRIFT

To investigate the frequency evolution in the burst data, we use the $Z^2_+\$ statistic (see Buccheri et al. 1983)

$$Z^2_+ = 2/N \sum_{i=1}^{N} \left[ \sum_{j=1}^{N} \cos (k \phi_j) \right]^2 + \left[ \sum_{j=1}^{N} \sin (k \phi_j) \right]^2.$$  \hspace{1cm} (1)

Here $N$ is the total number of photons in the time series, $\phi_j$ are the phases of each photon derived from a frequency model, $\nu(t)$, viz., $\phi_j = 2 \pi \int_0^t \nu(t') dt'$, and $n$ is the total number of harmonics added together. For the burst oscillations, which are highly sinusoidal, we will hereafter restrict ourselves to $n = 1$. This statistic is particularly suited to event mode data, since no binning is introduced. $Z^2_+$ has the same statistical properties as the well-known Leahy normalized power spectrum, which, for a Poisson process, is distributed as $\chi^2$ with 2 degrees of freedom. All of the bursts discussed here were observed with the Proportional Counter Array (PCA) on board RXTE and sampled with 125 $\mu$s ($1/8192$ s) resolution.

For $\nu(t)$, we have investigated a number of functional forms, including $\nu(t) = \nu_0$ (a constant frequency), $\nu(t) = \nu_0 (1 + d_n t)$ (a linearly increasing frequency), and $\nu(t) = \nu_0 [1 - \delta_n \exp (-t/\tau)]$ (an exponential chirp). For a given data set and frequency model, we vary the parameters so as to maximize the $Z^2_+$ statistic. We then compare the maximum values from different models in order to judge which is superior in a statistical sense. Our aim is both to constrain the functional form of the frequency evolution and to determine whether the pulse train during all or a portion of a burst is coherent or not. We judge the coherence of a given model by computing the quality factor $Q = \nu_0/\Delta \nu_{\text{FWHM}}$ from the width of the peak in a plot of $Z^2_+$ versus the frequency parameter $\nu_0$. We also compare the peak width with that expected for a coherent pulsation in data of the same length. A pulsation in a time series of finite extent produces a broadened peak in a power spectrum. The well-known window function, $W(\nu) = |\sin(\pi \nu T)/\pi \nu|^2$, gives a width of $\Delta \sim 1/T$, where $T$ is the length of the data. We also confirm that for a successful frequency model, the integrated power under the $Z^2_+$ peak has to be consistent with that calculated by assuming no frequency evolution.

2.1. Linear and Exponential Frequency Drift

To begin, we demonstrate how a linear increase in frequency yields a significant improvement in the $Z^2_+$ statistic compared with a constant frequency model. We use the burst from 4U 1702-429 that was observed in 1997 July 26 at 14:04:19 UT, which we refer to as burst A (see Fig. 4 in Markwardt, Strohmayer, & Swank 1999). We used a 5.25 s interval during this burst to investigate the frequency evolution. In the top panel of Figure 1, we show results from our calculations of $Z^2_+$ for the constant frequency model (top panel) and the model with a linearly increasing frequency (bottom panel). In both cases, the ordinate corresponds to the frequency parameter $\nu_0$ defined in the models. For the linear frequency model, we found that $Z^2_+$ was maximized with $d_n = 1.264 \times 10^{-3}$ s$^{-1}$. By including the linear drift, we increased $Z^2_+$ from 88.48 to 271.4, a dramatic improvement of $\sim 183$ that was obtained with only 1 additional degree of freedom. The resulting $Z^2_+$ peak is also substantially narrower (see the top panel of Fig. 1), leaving no doubt that the pulsation frequency is increasing during this time interval.

The frequency evolution during the bursts can also be explored with dynamic power spectra. Several such spectra have been presented elsewhere (see Strohmayer et al. 1998a, 1998b). One striking behavior is that the pulsation frequency reaches an asymptotic limit in many bursts. Motivated by this behavior, we investigated a simple exponential chirp model with a limiting frequency, $\nu(t) = \nu_0 [1 - \delta_n \exp (-t/\tau)]$. This model has three parameters, the limiting frequency $\nu_\nu$, the fractional frequency change (or “bite”) $\delta_n$, and the relaxation timescale $\tau$. We fitted this model to burst A and found a maximum $Z^2_+$ of 342.9, which is an increase of 71.5 in $Z^2_+$ over the linear frequency model. This is also a dramatically significant improvement in $Z^2_+$. We fitted the peak in $Z^2_+$ versus $\nu_0$ obtained with the chirp model to a Gaussian in order to determine its width. The bottom panel of Figure 1 shows the resulting fit. The peak is well described by a Gaussian with a width of $\Delta \nu_{\text{FWHM}} = 0.201$ Hz, which gives $Q = \nu_0/\Delta \nu_{\text{FWHM}} = 1641$. We can compare the}
pare this with the width caused by windowing, which, for a 5.25 s interval, gives a width (FWHM) of \( \approx 0.17 \) Hz.

We used the chirp model to investigate a sample of bursts from 4U 1702−429 and 4U 1728−34. We do not present here a systematic description of all observed bursts, but rather we demonstrate the main results with several illustrative examples. A burst from 4U 1702−429 observed on 1997 July 30 at 12:11:58 UT (burst B) revealed an \( \approx 12 \) s interval during which oscillations were detected. Our results using the chirp model for this burst are summarized in Figure 2. The top panel shows a contour plot of the time evolution of the \( Z_i^2 \) statistic through the burst. It was computed by calculating \( Z_i^2 \) on a grid of constant frequency values using 2 s intervals, with a new interval starting every 0.25 s, i.e., assuming no frequency evolution. The burst count rate profile (solid histogram) and best-fitting exponential chirp models (thick solid line) are overlaid. The extent of the model curve defines the time interval used to fit the chirp model. The best model tracks the dynamic \( Z_i^2 \) contours remarkably well. The bottom panel of Figure 2 compares \( Z_i^2 \) with \( v_0 \), for the constant frequency (dashed curve) and chirp models (solid curve). We again fitted a Gaussian to the peak calculated with the chirp model, and we found a width of \( \Delta \nu_\text{FWHM} = 0.086 \) Hz, which yields \( Q = v_0/\Delta \nu_\text{FWHM} = 3848 \) for this burst. This compares with a width of \( \approx 0.071 \) for a windowed pulsation of 12.5 s duration.

We carried out similar analyses in order to investigate the frequency evolution in the bursts from 4U 1728−34. We again found that the chirp model provides a remarkably useful description of the frequency drift. Table 1 summarizes our results using the chirp model for several bursts from both 4U 1702−429 and 4U 1728−34.

We find that the peaks obtained with the chirp model are only modestly broader than those expected for a coherent pulsation of the same length. Some of this additional width is likely due to the fact that pulsations are not present during the entirety of each interval examined. It is also likely that the chirp model is not the exact functional form of the frequency evolution; this is suggested by the broader wings of the \( Z_i^2 \) peaks computed for several bursts; however, the success of such a simple model argues strongly that the pulsations during the cooling tails of these bursts are phase coherent.

3. HARMONICS AND SUBHARMONICS

Pulsations from a rotating hot spot can be used to place constraints on neutron star compactnesses (see Strohmayr et al. 1998c, Miller & Lamb 1998, and Miller 1999). The pulsation amplitude is constrained by the strength of gravitational light deflection. An observed amplitude places an upper limit on the compactness, \( GM/c^2R \), because stars that are too compact cannot achieve the observed modulation amplitude. Furthermore, an upper limit on the harmonic content places a lower limit on the compactness, since less compact stars produce more harmonic content, and at some limit, the harmonics should become detectable.

In some models for the kilohertz quasi-periodic oscillations (kHz QPOs) observed in the accretion-driven X-ray flux from neutron star LMXBs, the QPO frequency separation is closely related to the stellar spin frequency inferred from burst oscillations (see Miller, Lamb, & Psaltis 1998 and Strohmayr et al. 1996). In two sources, burst oscillations are seen with frequencies close to twice the kHz QPO frequency separation (Wijnands et al. 1997; Wijnands & van der Klis 1997; Mendez, van der Klis, & van Paradijs 1998). Miller (1999) has reported evidence of a significant 290 Hz subharmonic of the strong 580 Hz pulsation seen in 4U 1636−539 (Zhang et al. 1996), suggesting that the strongest signal observed during the bursts may actually be the first harmonic of the spin frequency, not the spin frequency itself. Based on these new results and the evidence of a beat-frequency interpretation, it is important to search for the subharmonic of the strongest signal detected during bursts.

We have shown that the frequency drift during the bursts can greatly smears out the signal power. We have also shown that simple models can recover a coherent peak. By modeling the drift, we can make a much more sensitive search for the harmonics. Moreover, we can coherently add signals from different bursts by first modeling their frequency evolution and then computing a total \( Z_i^2 \) by phase aligning each burst. We note that this procedure will also coherently add together power at any of the higher harmonics of the known signal. However, there will be a \( \pi \)-phase ambiguity of any signal at the first subharmonic (see Miller 1999).

We have coherently added the 330 Hz signals in all five bursts from 4U 1702−429 that were seen during our 1997 July observations (see Markwardt et al. 1999). We fitted the chirp model to the oscillations in each burst and then computed a total \( \Delta \nu_\text{FWHM} \) by phase aligning them. Figure 3 shows the results of this analysis. The top panel shows the total \( Z_i^2 \) power at 330 Hz obtained by adding the bursts coherently. The peak value is \( \approx 1400 \) and demonstrates that we have successfully added the bursts coherently. The highest power for any burst individually was \( \approx 487 \). The two lower panels show the power at the first and second harmonics of the 330 Hz signal. We find no evidence of a significant signal at these or higher harmonics. To search for a signal at the 165 Hz subharmonic, we computed a total \( Z_i^2 \) for each of the 16 different combinations of the phases from each of the five bursts. Since there is a twofold ambiguity when coherently adding a subharmonic signal from two bursts, with a total of five, we have \( 2^4 = 16 \) possible combinations. We found no significant power at the subhar-

| Burst | \( T_\text{burst} \) (UT) | \( \nu_b \) (Hz) | \( \delta_\nu \) (\( \times 10^{-4} \)) | \( \tau \) (s) | \( Q \) |
|-------|-----------------|-------------|-------------|--------|------|
| A     | 1997 Jul 26 (14:04:19) | 329.851 ± 0.1 | 7.7 ± 0.32  | 1.880 ± 0.25 | 1641 |
| B     | 1997 Jul 30 (12:11:58) | 330.546 ± 0.02 | 4.8 ± 0.31  | 4.016 ± 0.07 | 3848 |
| C     | 1996 Feb 16 (10:00:45) | 364.226 ± 0.05 | 6.6 ± 0.14  | 3.520 ± 0.28 | 4535 |
|       | 1997 Sep 9 (06:42:56) | 364.102 ± 0.05 | 5.9 ± 0.22  | 1.845 ± 0.15 | 2023 |
We performed a similar analysis using four bursts from 4U 1728–34 that showed strong oscillations in their cooling tails; again we found no significant harmonic or subharmonic signals. The 90% confidence upper limits on the signal power, $Z_2^2$, at the first harmonic in the bursts from 4U 1702–429 and 4U 1728–34 are 5.8 and 1.8, respectively. These correspond to lower limits on the ratio of power at the fundamental to power at the first harmonic, $h$, of 242 and 556, respectively.

4. DISCUSSION

In this work, we have concentrated on the pulsations in the cooling tails of bursts. Bursts also show pulsations during the rising phase (Strohmayer et al. 1997b). We have not yet been able to show that the pulsations that begin near the burst onset can be phase-connected to those in the cooling tail with a simple model. To fully address this interesting question, we will need more sophisticated modeling than we have employed here. We will address this question in future work.

With the chirp model, we find magnitudes of the frequency shift, $\nu_0 \delta n$, of $\sim$2–3 Hz. These values are consistent with simple estimates that are based on angular momentum conservation using theoretical values for the pre- and postburst thickness of bursting shells (Bildsten 1995). For the frequency relaxation timescale, $\tau$, we find a range of values from 1.7 to 4 s. Interestingly, different bursts from the same source can show markedly different decay timescales. For example, the two bursts from 4U 1728–34 summarized in Table 1 show similar values for $\nu_0$ and $\delta$, but have decay timescales $\tau$ that differ by almost a factor of 2. Of these two bursts, burst C had both a substantially greater peak flux and fluence. This seems consistent with the idea that the frequency increase is due to the hydrostatic settling of the shell as it radiates away its thermal energy; however, the study of more bursts is required to establish such a connection firmly.

If the angular momentum conservation argument is correct, it implies the existence of a shear layer in the neutron star atmosphere. In the chirp model, the total amount of phase shearing is simply $\phi_{\text{shear}} = \nu_0 \delta \tau (1 - e^{-T/T_0})$, where $T$ is the length of the data interval. For the bursts examined here, we find $\phi_{\text{shear}} \sim 4$–8, so that the shell “slips” this many revolutions over the underlying neutron star during the duration of the pulsations. The dynamics of this shear layer are no doubt com-
plex. Given the physical conditions in the shell (the shear flows are characterized by a large Reynolds number), it is likely that the dissipation of the shear velocity and the recoupling will be dominated by turbulent momentum transport. Magnetic fields may also play a role as well. Shear layers can be unstable to Kelvin-Helmholtz instability; however, Bildsten (1998) has suggested that the shear may be stabilized by either thermal buoyancy or the mean molecular weight contrast. We urge new theoretical investigations to explore the mechanisms of recoupling in order to determine if such a shear layer can survive long enough to explain the persistence of pulsations for ~10 s as well as the observed relaxation timescale.

For the rotational modulation of a hot spot, the ratio, $h$, of the signal power at the fundamental to that at the first harmonic is a function of the stellar compactness (see Miller & Lamb 1998), so that the measurement of $h$ can be used to constrain the compactness. More compact stars have less harmonic content in their pulses and therefore larger $h$. Since the pulsations in the cooling tails of the bursts are likely caused by a broad brightness anisotropy on the neutron star surface and not a point spot, it will require more realistic modeling of such an emission geometry in order to use the limits on harmonic content derived here to constrain the stellar compactness. We will perform such modeling in future work.

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