Near- and Far-Field Communications with Large Intelligent Surfaces

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Abstract—This paper studies the uplink spectral efficiency (SE) achieved by two single-antenna user equipments (UEs) communicating with a Large Intelligent Surface (LIS), defined as a planar array consisting of $N$ antennas that each has area $A$. The analysis is carried out with a deterministic line-of-sight propagation channel model that captures key fundamental aspects of the so-called geometric near-field of the array. Maximum ratio (MR) and minimum mean squared error (MMSE) combining schemes are considered. With both schemes, the signal and interference terms are numerically analyzed as a function of the position of the transmitting devices when the width/height $L = \sqrt{N}A$ of the square-shaped array grows large. The results show that an exact near-field channel model is needed to evaluate the SE whenever the distance of transmitting UEs is comparable with the LIS’ dimensions. It is shown that, if $L$ grows, the UEs are eventually in the geometric near-field and the interference does not vanish. MMSE outperforms MR for an LIS of practically large size.

Index Terms—Intelligent reflecting surface, reconfigurable intelligent surface, metasurface, Massive MIMO, MIMO relays, power scaling law, near-field.

I. INTRODUCTION

Large Intelligent Surface (LIS) refers to arrays with a massive number of antennas in a compact space [1]. In its asymptotic form, it can be thought of as a spatially-continuous electromagnetic aperture that actively generates beamformed radio signals or receives them accordingly. Research on this topic is performed under many different names [2], among them: holographic MIMO [3]; reconfigurable intelligent surface [4]; and software-defined surface (SDS) [5]. A common practice in multiple antenna communications is to approximate the received electromagnetic wave with a plane wave. This approximation is valid when the terminal distance is much larger than the dimensions of the array and brings to the well-known geometric far-field approximation. This paper investigates the potential deficiencies of this approximation when an LIS of size comparable to (or larger than) the distance from the transmitting devices is considered. For brevity, we consider the deterministic line-of-sight propagation channel from [6], [7], which allows to study the so-called geometric near-field of the array by taking into account three fundamental aspects: 1) the varying distance to the antennas in the LIS; 2) the varying effective antenna areas; 3) the varying loss from polarization mismatch. Unlike [6], [7], this channel model is used to evaluate the uplink spectral efficiency (SE) achieved by two single-antenna user equipments (UEs) when communicating with an LIS of square geometry, using either maximum ratio (MR) or minimum mean squared error (MMSE) combining. Comparisons with the far-field approximation will show that the exact near-field model is unarguably needed when the distance of transmitting UEs is comparable with the LIS’ near-field channel model is needed to evaluate the SE whenever the LIS has comparable size to UE distances. We will also show that, as the LIS size grows, the UEs will eventually be in the geometric near-field and the interference will not vanish, which is different from what is conventionally considered in the Massive MIMO literature. Moreover, it will be shown that MMSE provides performance that is superior to MR for an LIS of practically large size. This makes it the preferred combining scheme.

II. SYSTEM MODEL

We consider the LIS shown in Fig. 1 consisting of $N$ antennas that each has area $A$. The antennas have size $\sqrt{A} \times \sqrt{A}$ and are equally spaced on a $\sqrt{N} \times \sqrt{N}$ grid. The antennas are deployed edge-to-edge, thus the total area of the LIS is $NA$. The LIS is centered around the origin in the $XY$-plane. If we number the antennas from left to right, row by row, according to Fig. 1, the $n$th antenna is located at $r_n = [x_n, y_n, 0]^T$ where

$$x_n = -\frac{(\sqrt{N} - 1)\sqrt{A}}{2} + \sqrt{A} \mod(n - 1, \sqrt{N}) \quad (1)$$

$$y_n = \frac{(\sqrt{N} - 1)\sqrt{A}}{2} - \sqrt{A} \left\lfloor \frac{n - 1}{\sqrt{N}} \right\rfloor \quad (2)$$

A. Channel model

The following lemma comes from [6] and extends prior work in [7] to provide a general way of computing channel gains to each of the $N$ antenna elements of the LIS.

Lemma 1. Consider a lossless isotropic antenna located at $s = [x_s, y_s, z_s]^T$ that transmits a signal that has polarization in the Y direction when traveling in the Z direction. The free-space channel gain $\chi_{s,n}$ at the $n$th receive antenna, located at $r_n = [x_n, y_n, 0]^T$, is given by (3) (at top of next page) where

$$\chi_{s,n} = \left\{ \sqrt{A}/2 + x_n - x_s, \sqrt{A}/2 - x_n + x_s \right\} \quad (4)$$
\[ \zeta_{n,r_n} = \frac{1}{4\pi} \sum_{x \in A_n} \sum_{y \in Y_{x,n}} \left( \frac{\frac{xy}{x^2 + 1}}{\sqrt{\frac{x^2}{x^2 + 1} + \frac{y^2}{y^2 + 1}}} + \frac{2}{3} \tan^{-1} \left( \frac{\frac{xy}{x^2 + 1}}{\sqrt{\frac{x^2}{x^2 + 1} + \frac{y^2}{y^2 + 1}}} \right) \right) \]  

(3)

\[ Y_{x,n} = \left\{ \sqrt{A/2 + y_n - y_s}, \sqrt{A/2 - y_n + y_s} \right\}. \]  

(5)

Lemma 1 is important when quantifying the channel gain in the so-called geometric near-field of the array [4], [8] because it takes into account the three fundamental properties that make it different from the far-field: 1) the distance to the elements varies over the array; 2) the effective antenna areas vary since the element are seen from different angles; 3) the loss from polarization mismatch varies since the signals are received from different angles. We will use Lemma 1 in the remainder.

B. Signal model

We consider two single-antenna UEs that communicate with the LIS in Fig. 1 under the following assumption, shown in Fig. 2. This setup is sufficient to demonstrate a few key results.

Assumption 1. UE k for k = 1, 2 is located in the XZ-plane at distance \( d_k \) from the center of the array with angle \( \theta_k \in [-\pi/2, \pi/2] \). Both UEs send a signal that has polarization in the Y direction when traveling in the Z direction.

We denote by \( h_k = [h_{k1}, \ldots, h_{kN}]^T \in \mathbb{C}^N \) for \( k = 1, 2 \) the channel between UE k and the LIS. Particularly, \( h_{kn} = |h_{kn}|e^{-j\phi_{kn}} \) is the channel from the source to the nth receive antenna with \( |h_{kn}|^2 \in [0, 1] \) being the channel gain and \( \phi_{kn} \in [0, 2\pi) \) the phase shift. Following the geometry stated in Assumption 1 the two UEs are located at (see Fig. 2)

\[ s_1 = [x_{s1}, y_{s1}, z_{s1}]^T = [d_1 \sin(\theta_1), 0, d_1 \cos(\theta_1)]^T \]  

(6)

\[ s_2 = [x_{s2}, y_{s2}, z_{s2}]^T = [d_2 \sin(\theta_2), 0, d_2 \cos(\theta_2)]^T. \]  

(7)

From Lemma 1 the following corollaries are found.

**Corollary 1 (Exact model).** Under Assumption 1 the channel \( h_k = |h_{kn}|e^{-j\phi_{kn}} \) to the nth receive antenna is obtained as

\[ |h_{kn}|^2 = \zeta_{\text{nk}, r_n}, \quad \phi_{kn} = 2\pi \mod \left( \frac{|s_k - r_n|}{\lambda}, 1 \right). \]  

(8)

**Corollary 2 (Far-field approximation).** Under Assumption 1 if UE k is in the geometric far-field of the array, in the sense that \( d_k \cos(\theta_k) \gg \sqrt{N A} \), then \( h_k = |h_{kn}|e^{-j\phi_{kn}} \) is obtained as

\[ |h_{kn}|^2 = A \cos(\theta_k) \]  

(9)

\[ \phi_{kn} = 2\pi \mod \left( \frac{d_k - x_n \sin(\theta_k)}{\lambda}, 1 \right). \]

The received signal \( y \in \mathbb{C}^{M \times 1} \) at the LIS is \( y = s_1 h_1 + s_2 h_2 + n \) where \( s_i \sim N_\mathbb{C}(0, p_i) \) is the data signal from UE i

1Note that we assume throughout this paper that \( |s - r_n| \gg \lambda \), so the system does not operate in the reactive near-field of the transmit antenna (even if it is in the geometric near-field of the array). In fact, this assumption must be made to derive the expression in Lemma 1; see [7] for details.

**Fig. 2:** The two UEs are located in the XZ-plane at distances \( d_k \) for \( k = 1, 2 \) and have angles \( \theta_k \) for \( k = 1, 2 \).

and \( n \in \mathbb{C}^{M \times N} \) is thermal noise with i.i.d. elements distributed as \( N_\mathbb{C}(0, \sigma^2) \). We define the average received signal-to-noise ratio (SNR) of UE i as \( SNR_i = p_i / \sigma^2 \). The channels are deterministic and thus can be estimated arbitrarily well from pilot signals. Hence, perfect channel state information is assumed. The impact of imperfect knowledge of the interfering UE channel will be investigated in Section IV.D.

To detect \( s_1 \) from y, the LIS uses the combining vector \( v_1 = h_1 / ||h_1|| \), and similarly for \( v_2 = h_2 / ||h_2|| \).

\[ \gamma_i = \frac{SNR_i |v_i^T h_1|^2}{SNR_2 |v_i^T h_2|^2 + ||v_i||^2} \]  

(10)

is the signal-to-interference-and-noise ratio (SINR). We begin by considering MR combining, defined as \( v_1 = h_1 / ||h_1|| \), leading to

\[ \gamma_i^{\text{MR}} = \frac{SNR_i |h_1|^2}{SNR_i |h_1|^2 + 1} = SNR_i ||h_1||^2 (1 - \alpha^{\text{MR}}) \]  

(11)

with \( \alpha^{\text{MR}} = \frac{SNR_2 |h_2|^2}{1 + SNR_2 |h_2|^2} \). The term

\[ SNR_2 \frac{|h_2|^2}{||h_1||^2} = SNR_2 \sum_{n=1}^N |h_{1n}|^2 \frac{e^{j(\phi_{1n} - \phi_{2n})}}{||h_1||^2} \]  

(12)

accounts for the interference generated by UE 2 whereas \( SNR_i ||h_i||^2 \) in (11) represents the received SNR in the absence of interference. Since MR does not do anything against the interference, the term \( \alpha^{\text{MR}} \) in (11) must be interpreted as the performance loss due to the presence of UE 2.

Instead of using the suboptimal MR combining, we note that \( \gamma_i \) in (10) is a generalized Rayleigh quotient with respect to \( v_1 \) and thus is maximized by MMSE combining:

\[ v_1 = \left( \sum_{i=1}^2 SNR_i h_i h_i^H + I_M \right)^{-1} h_1 \]  

(13)

leading to

\[ \gamma_i^{\text{MMSE}} = SNR_i ||h_1||^2 (1 - \alpha^{\text{MMSE}}) \]  

(14)
In the far-field case, it becomes

\[
\|h_k\|^2 = \xi_{d_k, \theta_k, N} = \sum_{i=1}^{2} \frac{B_k + (-1)^i \sqrt{B_k} \tan(\theta_k)}{6\pi (B_k + 1) \sqrt{2B_k + \tan^2(\theta_k) + 1 + 2(-1)^i \sqrt{B_k} \tan(\theta_k)}}
+ \frac{1}{3\pi} \tan^{-1} \left( \frac{B_k + (-1)^i \sqrt{B_k} \tan(\theta_k)}{2\sqrt{2B_k + \tan^2(\theta_k) + 1 + 2(-1)^i \sqrt{B_k} \tan(\theta_k)}} \right)
\]  

(15)

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+ \frac{1}{3\pi} \tan^{-1} \left( \frac{B_k + (-1)^i \sqrt{B_k} \tan(\theta_k)}{2\sqrt{2B_k + \tan^2(\theta_k) + 1 + 2(-1)^i \sqrt{B_k} \tan(\theta_k)}} \right)
\]  

(15)

Fig. 3: Behavior of the channel gain $\|h_1\|^2$ using either the exact model or the far-field approximation. The desired UE 1 has $\theta_1 = 2^\circ$ and is at a distance $d_1 = 25\lambda = 2.5$ m.

Fig. 4: Behavior of the interference gain $\|h_1 h_2\|^2$ using either the exact model or the far-field approximation. The desired UE 1 is at $\theta_1 = 2^\circ$ while the interfering UE 2 is at $\theta_2 = -2^\circ$. Both UEs are at a distance $d_1 = d_2 = 2.5$ m. Note that the valleys not always reaching their correct value, $-\infty$ dB, due to the limited numerical precision.

LIS larger than $L = 1$ m is already enough to notice the approximation gap, whereas $L \geq 10$ m is needed to approach the upper limit of 1/3 (this limit comes from considering the polarization mismatch loss [6] and differs from the 1/2 limit in [1]). This shows the importance of properly modeling the near-field.

A. Interference gain

We now analyze the normalized interference gain $\|h_1 h_2\|^2$.

We notice that the computation of a closed-form expression with the exact channel model is challenging while it takes the simple form with the far-field approximation [9, Eq. (12)]

\[\|h_1 h_2\|^2 = \frac{A \cos(\theta_2) \sin(\pi L \Omega / \lambda)}{4\pi d_2^2 \sin(\pi \sqrt{A \Omega / \lambda})^2}
\]  

(18)

with $L = \sqrt{NA}$ and $\Omega = \sin(\theta_2) - \sin(\theta_1)$. Fig. 4 plots the the normalized interference gain $\|h_1 h_2\|^2$ as a function of $L$, using either the exact model and the far-field approximation from Corollaries 1 and 2 respectively. We consider a setup with $\lambda = 0.1$ m in which the two UEs have different angles $\theta_1 = 2^\circ$ and $\theta_2 = -2^\circ$, but have the same distance from the LIS, given by $d_1 = d_2 = 2.5$ m. In line with the results of Fig. 3 the gap between the exact model and the far-field approximation is noticeable when $L \geq 1$ m. We notice that $L > 100$ m is needed with the exact model to approach its upper limit. This is more than 3 dB higher than the upper limit with the far-field approximation.
Fig. 5: SE behavior with MR and MMSE. The SE in the ideal interference-free case is reported as reference.

B. Spectral efficiency analysis

Fig. 5a plots the SE achieved by UE 1 with MR and MMSE as a function of $L$, using either the exact model or the far-field approximation from Corollaries 1 and 2. We consider the same setup of Fig. 3 that is, $\lambda = 0.1 \text{ m}$, $\theta_1 = 2^\circ$, $\theta_2 = -2^\circ$ and $d_1 = d_2 = 2.5 \text{ m}$. We set $p_1 = p_2 = 30 \text{ dBm}$ and $\sigma^2 = 0 \text{ dBm}$. The SE $\log_2(1 + \text{SNR}_1||h_1||^2)$ computed with the exact model in the ideal interference-free case is also reported as a reference. Fig. 5a shows that the SE saturates with both schemes as $L$ increases. However, while MMSE quickly converges to the interference-free case, the performance gap with MR is substantial with the exact model, while it asymptotically vanishes with the far-field model as in [9, Sec. III-D].

Fig. 5b plots the SE achieved by UE 1 with MR and MMSE as a function of $z = z_{s1} = z_{s2}$, using either the exact model or the far-field approximation. We consider an LIS of size $L = 6 \text{ m}$, and set $p_1 = p_2 = 30 \text{ dBm}$ and $\sigma^2 = 0 \text{ dBm}$. MMSE provides the same SE as in the interference-free case for $z \leq 10 \text{ m}$, whereas it is lower for larger values. The SE gap between MMSE and MR is relatively large for the values of $z$ of practical interest, i.e., smaller than tens of meters. MMSE converges to MR only when $z \geq 40 \text{ m}$.

Fig. 6: SE in bit/s/Hz achieved by UE 1 when the interfering UE 2 is transmitting at different locations over the $XZ$-plane. Both MMSE and MR are considered.

To gain further insights into the large performance gap between MMSE and MR in the near-field, Fig. 6 shows the SE in bit/s/Hz when the desired UE 1 is fixed at $\theta_1 = 2^\circ$ and $d_1 = 2.5 \text{ m}$ and the interfering UE 2 is transmitting from different locations over the $XZ$-plane, whose distance from the point-of-interest is measured in wavelengths. We assume $\lambda = 0.1 \text{ m}$ and use an LIS of size $L = 6 \text{ m}$. Fig. 6a reveals that the SE with MMSE is low only in an elliptic region around the point-of-interest, whose semi-major axis (along the $z$ direction) is roughly half-a-wavelength. This means that MMSE can efficiently reject any interfering signal that comes from a location that is at least half-a-wavelength away. On the contrary, Fig. 6b shows that a low SE is achieved with MR for most of the locations the interfering UE is transmitting from. This is because MR does not do anything against the interference. The region where the SE achieves its minimum value is still an ellipse but with a larger area. We see that the SE with both MMSE and MR ranges from $\approx 7$ to $1 \text{ bit/s/Hz}$ (a reduction of $86\%$), but while in the MMSE case the highest values are located in most of the observed area, that is not the case when using MR.
achieve with state-of-the-art solutions in wireless applications.

\[ r \]

distributed in a circle of radius \( r \) centered at the true position.

\[ \text{position differs from the true one by an error that is uniformly imperfectly known. Particularly, we assume that the estimated accuracy (having an estimation error of half-a-wavelength is enough for a} \]

\[ \text{reduction. This calls for estimation schemes with centimeter accuracy (having } \lambda = 10 \text{ cm}) \text{ which is far beyond what we can achieve with state-of-the-art solutions in wireless applications.} \]

\[ \text{Fig. 8 shows the average SE as a function of } \lambda \text{, expressed in wavelengths. We assume } \lambda = 0.1, L = 6 \text{ m and an inter-UE distance is } 17.5 \text{ cm} = 1.75 \lambda. \]

\[ \text{IV. SUMMARY} \]

This paper showed that a realistic assessment of the uplink SE achievable by two single-antenna UEs communicating with an LIS requires the use of an exact near-field channel model, whenever the distance of transmitting UEs is comparable with the LIS size, \( L \). It also showed that increasing \( L \) unboundedly does not guarantee the suppression of interference with MR combining, especially when the UEs are closely located in space. MMSE combining is still needed to efficiently suppress the interference. Particularly, the SE with MMSE quickly converges to the interference-free case when the UEs are transmitting from a distance of few meters and values of \( L \) of practical interest are considered, i.e., in the range \( 1 \leq L \leq 10 \) meters. However, the SE with MMSE deteriorates fast in the presence of channel estimation errors. Lastly, we showed that the polarization mismatch should not be ignored since it has a non-negligible impact on SE, especially when \( L \) grows.

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