Single Longitudinal-Spin Asymmetries in Lepton-Pair Production at RHIC and J-PARC

Hiroshi YOKOYA

Department of Physics, Niigata University, Niigata 950-2181, JAPAN

Abstract

We study the single longitudinal-spin asymmetries in lepton-pair production with large transverse-momentum at RHIC and J-PARC experiments. The asymmetries in the azimuthal angular distribution of a lepton can arise from an absorptive part of production amplitudes. We revisit the one-loop calculation for the absorptive part of production amplitudes in perturbative QCD, and show that the asymmetries can be sizable at RHIC and J-PARC. Measurement of the asymmetries would test the one-loop prediction for the scattering phase of this process, and provide support for a study of the single transverse-spin asymmetries in the same kinematical region.

*) E-mail address: yokoya@nt.sc.niigata-u.ac.jp
Single longitudinal-spin asymmetries (SLSAs) in the Drell-Yan process can arise in the angular distribution of a lepton, when the lepton pair has non-zero transverse momentum ($q_T$). The asymmetries are naïvely $T$-odd, and thus result from the complex phase of the production amplitude. In the QCD collinear formalism at leading-twist level, a leading-order contribution to the complex phase comes from one-loop amplitudes with on-shell intermediate state. The one-loop calculation of the imaginary part of the Drell-Yan amplitudes has been performed in Refs. 1) and 2), where possibly large asymmetries are predicted. Related calculations have been also done for naïve-$T$-odd asymmetries in $W$-jet$^3$ and $Z$-jet$^4$ events at hadron colliders. Because naïve-$T$-odd asymmetries in hard processes have not been tested yet experimentally, measurement of such SLSAs is of great interest.

On the other hand, single transverse-spin asymmetries (STSAs) have been part of recent primary progresses in hadron-spin physics, since the discovery of the large STSAs in several experiments. STSAs require chirality flip of the amplitude, in addition to the scattering phase. In the collinear formalism, the STSAs call for higher-twist distributions to achieve the chirality flip, and pole of propagators to the scattering phase.$^5$–$^7$

For the small-$q_T$ case, there exists another approach to describe the STSAs, based on the formalism with transverse-momentum-dependent parton distributions.$^6$,$^8$,$^9$ Although an overlap region to be described simultaneously by the two formalism is pointed out,$^6$ it is meaningful to know which approach is effective at a certain kinematical point. In most of the experiments, the SLSAs and the STSAs can be measured at same kinematical points in one experiment. Therefore, measurement of SLSAs which have less ambiguity theoretically may become a key reference to that of STSAs where the unknown non-perturbative functions are involved.

In this paper, we revisit the results of Refs. 1) and 2) and give phenomenological studies for the SLSAs in the Drell-Yan process for the RHIC and J-PARC experiments. We consider productions of a lepton pair with large transverse-momentum, in collisions of longitudinally-polarized proton and unpolarized proton;

$$\vec{p} + p \rightarrow \ell^- + \ell^+ + X.$$  \hspace{1cm} (1)

Defining the spin-dependent cross section as $\Delta \sigma = (\sigma^+ - \sigma^-)/2$, where $\sigma^+$ ($\sigma^-$) denotes the cross section for the collision of polarized proton with positive (negative) helicity, the SLSAs are expressed in the lepton angular distributions as$^{1), 2)}$

$$\left( \frac{d\sigma}{dQ^2 dq_T^2 dy} \right)^{-1} \frac{d\Delta \sigma}{dQ^2 dq_T^2 dy d\Omega} = \frac{3}{16\pi} \left[ A_{L1} \sin 2\theta \sin \phi + A_{L2} \sin^2 \theta \sin 2\phi \right].$$  \hspace{1cm} (2)
$Q^2$ is the invariant mass of the lepton pair, $q_T$ is transverse momentum of the lepton pair with respect to the collision axis, and $y$ is the rapidity of the lepton pair in the center of mass frame of the protons where the $z$-axis is along the three momenta of polarized proton. The spin-independent cross section is defined as $\sigma = \frac{1}{2} (\sigma^+ + \sigma^-)$, and $d\Omega = d\cos \theta d\phi$ where $\theta, \phi$ are polar, azimuthal angles of a lepton $\ell^-$, respectively, in a rest frame of the lepton pair. Coordinates of the rest frame are fixed to the Collins-Soper frame,\(^{10}\) so the $z$ axis is taken to bisect the opening angle between $\vec{p}_p$ and $-\vec{p}_p$, and the $y$ axis is along the direction of $\vec{p}_p \times (-\vec{p}_p)$. The azimuthal angle is measured from the $x$ axis which lies in the scattering plane.

The structure functions for the SLSAs, $A_{L1,L2}$, are calculated as,

$$A_{L1,L2}(Q^2, q_T^2, y) = \frac{\sum_{a,b} \int dY \Delta D_{a/p}(x_+, \mu^2)D_{b/p}(x_-, \mu^2)g_{12}^{ab}(z, \cos \hat{\theta})}{\sum_{a,b} \int dY D_{a/p}(x_+, \mu^2)D_{b/p}(x_-, \mu^2)f_{12}^{ab}(z, \cos \hat{\theta})}, \quad (3)$$

where $D_{a/p}$ and $\Delta D_{a/p}$ are the unpolarized and longitudinally-polarized parton distribution functions (PDFs) of proton, respectively, $x^\pm = \sqrt{s} e^{\pm y}$, $\hat{s} = Q^2 + 2q_T^2 \sin^2 \hat{\theta}$,$\quad (4)$

and $z = Q^2 / \hat{s}$. $A_{L1,L2}$ are expressed in terms of weighted integrals of spin-dependent cross section as,

$$A_{L1,L2}(Q^2, q_T^2, y) = \left( \int d\Omega \omega_{1,2}(\theta, \phi) \frac{d\Delta \sigma}{dQ^2dq_T^2dyd\Omega} \right) \left( \frac{d\sigma}{dQ^2dq_T^2dy} \right), \quad (5)$$

where the weight functions are $\omega_1 = 5 \sin 2\theta \sin \phi$ and $\omega_2 = 5 \sin^2 \theta \sin 2\phi$.

In the leading order (LO), the hard part functions $f_{ab}$ and $g_i^{ab}$ in Eq. (3) have contributions from the annihilation subprocess $q\bar{q} \rightarrow \ell^-\ell^+g$ and the Compton subprocess $qg \rightarrow \ell^-\ell^+q$ ($\bar{q}g \rightarrow \ell^-\ell^+\bar{q}$). $f_{ab}$ are calculated from tree-level diagrams, and $g_i^{ab}$ from the one-loop diagrams, in the LO.

Here, we reproduce the results of Refs. 1) and 2), following the notation of Refs. 3) and 4). For the spin-independent functions,

$$f_{q\bar{q}}(z, \cos \hat{\theta}) = e_q^2 \frac{C_F}{N} \frac{1}{1 + 1/z} f_{1q\bar{q}}, \quad f_{q\bar{q}}(z, \cos \hat{\theta}) = e_q^2 \frac{T_F}{N} \frac{1}{1 + 1/z} f_{1q\bar{q}}, \quad (6)$$

with

$$f_{1q\bar{q}} = \frac{a^2 + b^2}{2ab(1 - c)}, \quad f_{1qg} = \frac{b^2 + (a + b - 2ab)^2}{2(1 - a)bc}, \quad (7)$$

and
and \( f_1^{qg} = f_1^{qg}(a \leftrightarrow b), f_1^{qg} = f_1^{qg}(a \leftrightarrow b) \), where

\[
\begin{align*}
    a &= \frac{2z}{1 + z - (1 - z) \cos \hat{\theta}}, \\
    b &= \frac{2z}{1 + z + (1 - z) \cos \hat{\theta}},
\end{align*}
\]

and \( c = a + b - ab \). \( e_q \) is the electromagnetic charge of quarks, and color factors are \( N = 3, C_F = 4/3, T_F = 1/2 \) and \( C_1 = -1/6 \).

For the spin-dependent functions,

\[
\begin{align*}
    g_{1,2}^{qg}(z, \cos \hat{\theta}) &= -e_q^2 \alpha_s \frac{C_F}{N} \frac{1}{1 + 1/z} f_{8,9}^{qg}, \\
    g_{1,2}^{qg}(z, \cos \hat{\theta}) &= -e_q^2 \alpha_s \frac{T_F}{N} \frac{1}{1 + 1/z} f_{8,9}^{qg},
\end{align*}
\]

where

\[
\begin{align*}
    f_8^{qg} &= \frac{c}{2\sqrt{1-c}} \left[ -C_F \frac{a}{b} + C_1 \frac{1}{1-a} \ln \frac{a}{c} \right] - (a \leftrightarrow b), \\
    f_9^{qg} &= \frac{\sqrt{c}}{2} \left[ -C_F \frac{a}{2b} - C_1 \frac{1}{1-a} \left( 1 + \frac{c}{c-a} \ln \frac{a}{c} \right) \right] + (a \leftrightarrow b), \\
    f_8^{qg} &= \frac{c-b}{2\sqrt{1-c}} \left[ -C_F \left( \frac{a}{b} + \frac{1+a}{2} \right) \right. \\
    &\quad \left. + C_1 \left\{ b - 1 + \frac{a}{c} \left( b + \frac{c-b}{c} \ln \frac{1}{1-c} \right) \right\} \right], \\
    f_9^{qg} &= \frac{c-b}{2\sqrt{c}} \left[ -C_F \left( \frac{a}{2b} + \frac{1+a}{2} \right) \right. \\
    &\quad \left. + C_1 \left\{ b + \frac{a}{c} \ln \left( \frac{1}{1-c} - \frac{1}{1-a} \right) \right\} \right].
\end{align*}
\]

Similarly for other subprocesses, \( f_8^{qg} = -f_8^{qg}(a \leftrightarrow b), f_9^{qg} = f_9^{qg}(a \leftrightarrow b) \), and

\[
\begin{align*}
    f_8^{qg} &= f_8^{qg}(a \leftrightarrow b) - C_1 \frac{a(1-a)}{\sqrt{1-c}}, \\
    f_9^{qg} &= -f_9^{qg}(a \leftrightarrow b) - C_1 \frac{c-a}{\sqrt{c}} \frac{1}{1-b} \left( 1 + \frac{b}{c-b} \ln \frac{b}{c} \right).
\end{align*}
\]

The functions for anti-quark and gluon scattering are \( f_i^{qg} = f_i^{qg}, f_i^{gq} = f_i^{gq} \) for \( i = 1, 8, 9 \).

For a numerical estimate, we use the GRV98 (NLO \( \overline{\text{MS}} \) scheme) parameterization\(^{11}\) for the unpolarized PDFs, and the AAC03 parameterization\(^{12}\) for polarized PDFs. We set the scale of PDFs and the strong coupling constant \( \alpha_s \) to \( \mu = Q \). In Fig. II, we show the numerical estimates of these asymmetries for RHIC \( \vec{p}\vec{p} \) collisions at \( \sqrt{s} = 200 \text{ GeV} \) and \( Q = 5 \) GeV. We plot the asymmetries \( A_{L1} \) (left) and \( A_{L2} \) (right) for three different values of \( q_T \): 1 GeV (dot-dashed), 3 GeV (dashed) and 5 GeV (solid). The absolute magnitude of the asymmetries increases with \( q_T \), and becomes largest in large rapidity region, \( A_{L1} \sim 5\% \) and \( A_{L2} \sim 3.5\% \). For small \( q_T \), the asymmetries are predicted to be small over the whole \( y \) range.
In the forward region (positive $y$), the asymmetries mainly come from subprocesses with polarized quarks. On the other hand, in the backward region (negative $y$) the asymmetries receive dominant contributions from subprocesses with a polarized gluon.

In Fig. 2, we show the numerical estimates of these asymmetries for J-PARC experiment with polarized-proton at $\sqrt{s} = 10$ GeV and $Q = 2$ GeV. The asymmetries for $q_T = 1$ GeV (dot-dashed), 1.5 GeV (dashed) and 2 GeV (solid) are plotted. The asymmetries amount to $A_{L1} \sim 5.5\%$ and $A_{L2} \sim \pm 5\%$ in the large $|y|$ region for $q_T = 2$ GeV. For the J-PARC case, the absolute magnitude of the asymmetries in the backward region are almost the same as those in the forward region. This is because the asymmetries are proportional to $\Delta g/g$ in the backward region, and the mean values of $x_+$ are around $\langle x_+ \rangle_{y=-1} \sim 0.3-0.5$ at J-PARC, depending on $q_T$, while $\langle x_+ \rangle_{y=-3} \sim 0.01-0.03$ at RHIC. The asymmetries may be used to constrain the parameterizations of polarized PDFs. In the backward lepton-pair production, $\Delta g/g$ at around $\langle x \rangle \sim 0.01$ (0.3) is tested at RHIC (J-PARC).

Finally, we make some remarks about the estimated asymmetries. Since our predictions are based on the LO calculation, there exists a significant ambiguity in the choice of the scale of the PDFs and $\alpha_s$. The asymmetries are $\mathcal{O}(\alpha_s)$, and therefore increase with decreasing scale of $\alpha_s$. In case we take the scale as $\mu = q_T$, the asymmetries increase at most by 30%-50%, uniformly in $y$. QCD higher-order corrections have been known to change the $q_T$ distribution of production cross-sections, especially for small-$q_T$ region ($q_T \ll Q$) by the multiple soft-gluon emissions. However, even though we do not proceed to the small-$q_T$
region, it is expected that these effects largely cancel out in the lepton’s angular asymmetries, as far as the QCD collinear formalism is valid.\textsuperscript{14)}

For the production of lepton pairs with small $q_T$, the asymmetries are described by a formalism with transverse-momentum-dependent parton distributions.\textsuperscript{15)} In this formalism, only the asymmetry $A_{L2}$ is generated, while $A_{L1}$ remains zero\textsuperscript{*).

In conclusion, we have studied the single longitudinal-spin asymmetries in lepton-pair production at RHIC and J-PARC. The asymmetries in the azimuthal angular distribution of a lepton arise from the absorptive part of the production amplitude, when the lepton pair has non-zero transverse momentum. We re-analyzed the asymmetries in leading-order for the RHIC and J-PARC experiments, and showed that they can be sizable for large $q_T$ and at large forward or backward rapidity. This would be a good experimental test for the scattering phase of the production amplitude, and the comparison with the one-loop calculation in the collinear formalism may provide phenomenological supports for a study of the single transverse-spin asymmetries at the same kinematical region.

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\textsuperscript{*}) The asymmetry $A_{L2}$ is proportional to a product of chiral-odd distributions $h_1^1$ and $h_{1L}^1$, where the former is $T$-odd (Boer-Mulders function). See Ref. 15) for the definition of these functions.
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