Resilient Design of Robust Multi-Objectives PID Controllers for Automatic Voltage Regulators: D-Decomposition Approach

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ABSTRACT Resilient design of robust multi-objectives PID controllers via the D-decomposition method is presented in this paper for automatic voltage regulators (AVRs). The stabilizing interval of derivative gain ($k_d$) is analytically calculated by the Routh-Hurwitz criterion. The $k_p - k_i$ domain, for a fixed value of $k_d$, is decomposed in root invariant regions by mapping the stability boundary from the complex plane. Two regions, described by fixed damping isoclines, are assigned for pole-clustering in the open-left half plane (LHP). Other than regional pole clustering, gain and phase margins, as frequency domain specifications, are considered. Both robust stability and robust performance are considered by stabilizing a set of principle segment plants simultaneously. Optimal pole-placer PID controllers are computed analytically. If a robust control basin does exist for a specific compromise of control objective, the criterion of the maximum inscribed circle is considered to compute the maximum radius of controller resiliency. The merit of the proposed design is the simultaneous consideration of three control concerns, namely performance optimality, stability robustness and controller resiliency. Computation, validation, and simulation results are presented to show the simplicity and efficacy of the suggested method in tracing control basins (CBs) of all admissible PID controllers.

INDEX TERMS PID control, D-decomposition, optimality, robustness, regional pole-placement, gain and phase margins, control basin.

NOMENCLATURE

- $A$ System gain
- $t_s, E_\text{ss}$ Settling time and steady state error
- $\alpha, \xi$ Damping factor, $s^{-1}$ and damping coefficient
- $\alpha_{\text{max}}$ Maximum damping factor
- $\delta_p$ Uncertainty radius
- $\omega_b$ Upper value of frequency grid
- $\omega_g$ Frequency grid to be scanned
- $\Phi$ Empty set
- $\rho_c$ Controller resiliency radius
- $\theta_m$ Pre-specified phase margin
- $\vartheta$ System phase
- $A_m$ Pre-specified gain margin
- $C, s$ Complex plane and complex operator
- $D_{\alpha}(n)$ Basin $\alpha$—Hurwitz stabilizing PIDs
- $D_{\xi}(n)$ Basin $\xi$—Hurwitz stabilizing PIDs
- $D_{\omega}(n)$ Basin of all Hurwitz stabilizing PIDs
- $K_A$ Gain of the amplifier
- $k_d$ Derivative gain
- $k_i$ Integral gain
- $k_p$ Proportional gain
- $K_E$ Gain of the exciter
- $K_G$ Gain of the generator
- $K_S$ Gain of the sensor
- $M_p$ Maximum overshoot

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One of the principal control loops in electric power systems is the voltage control loop [1], [2]. Significant voltage deviations have serious effects on the performance of different equipment attached to the power network. Severe voltage dips/swells can cause electrical equipment to suffer complete damage. The reactive power generation is affected by the variation of voltage level and consequently, the variation in the shared reactive power influences the parallel operation of units. To deal with such problems, automatic voltage regulators (AVRs) are used to accommodate the synchronous generator voltage within the allowable limits. Parametric uncertainties of a power system model and yet the perturbations of the gains of controllers represent a challenge to guarantee stability robustness and optimal performance. The voltage control is crucial in the power system operation.

A. LITERATURE OVERVIEW

Different control techniques, such as robust, optimal, adaptive, sliding mode, and fuzzy logic control, are exploited for voltage regulation. In [3], sliding mode control is employed for voltage regulation in a micro-grid power system, where in [4], the sliding mode control is combined with an $H_\infty$ controller for voltage control in a two-level grid. However, the control signal frequently suffers from chattering when using sliding mode controllers. Furthermore, $H_\infty$ control leads to higher order controllers which may result in system improperness. Fuzzy logic control (FLC) is used for voltage regulation of a power distribution system as presented in [5]. In [6], FLC is used for Automatic voltage control of power transformers, where in [7], FLC and gain scheduling are employed for distributed voltage control in micro-grids. Nevertheless, the design of FLC systems implies a proper rule base for decision-making, fitting of scaling factors and adjustment of membership functions. In [8], an adaptive neuro-fuzzy inference system (ANFIS) is introduced to manage voltage in the distribution system. Various machine learning and sensitivity based approaches are proposed in [9]–[12] for the control of photovoltaic and wind power generation systems. An artificial neural network (ANN) controller for the regulation of voltage in distribution systems is presented in [13]. In [14], an ANN-based predictive controller to regulate the voltage of a buck converter is introduced. However, proper data training is crucial to acquire good performance of ANN-based controllers.

Model predictive control (MPC) is suggested for the regulation of the voltage of photovoltaic cell connected to a power distribution system in [15] and for robotic manipulator control in [16], while distributed MPC is utilized in multi-area power systems to maintain voltages within acceptable limits [17], [18]. A voltage optimization based on MPC is presented in [19], for distribution circuits incorporating distributed generators. An MPC-based centralized controller is presented in [20] to regulate voltage in a distribution system. The implementation of the MPC has challenges of high computational burden and fine-tuning with respect to the control horizon, prediction horizon and the weight factors of the cost function to provide good performance.

Adaptive control methods are widely used for voltage regulation of synchronous generators [21]–[24]. In [21], a fractional order adaptive controller is applied to an AVR system. In [22], an adaptive optimal control technique based on the policy iteration technique is presented for an AVR system. In [23], optimal excitation control is improved using a new FLC strategy with a coordinated design of a power system stabilizer and AVR system. In [24], a binary input-output fuzzy associative memories are used to design and implement an AVR control system. More adaptive control methods can be found in [25], [26] and the references therein.

Amongst these control techniques, the proportional-integral-derivative (PID) controller is the most accepted controller in the industry thanks to its simple structure, low cost and uncomplicated implementation [27]–[30]. One of the main challenges of the application of PID controllers is the setting of controller gains to acquire satisfactory performance. Some conventional methods of tuning the gains of a PID controller, such as Ziegler Nichols (ZN) method, are commonly used [31]. Unfortunately, a good performance cannot be proved because the ZN technique relies mainly on constant basics applied to any system [32]. Routh-Hurwitz Criterion (RHC) is presented in [33], [34] for a systematic parameterization of robust stabilizing three-parameters power system stabilizers, where Kharitonov’s Theorem (KT) was used for guaranteeing Robust stability. In [35], a robust non-fragile PID based AVR control system is designed, while RHC and KT were dedicated for guaranteeing robust stability with perturbation in controller gains. Nevertheless, in [33]–[35], a robust performance was discarded. In [36], [37], the D-decomposition method is used for computing control basins (CBs) having all robust pole-placement power system stabilizers. Artificial intelligence (AI)
techniques are widely used to compute optimized controller gains that provide fine performance in different engineering applications. Among these AI-based methods, PID gains are adjusted using the anarotic society optimization (ASO) method [38]. An intelligent genetic algorithm (GA) is presented in [39] to optimize the PID gains of a blade-pitch controller. In [40], the PID gains are set using an FLC system tuned by GA. In [41] optimizing robust PI controllers are suggested for load frequency control (LFC) in an interconnected system by constrained population extremal optimization on a set of linear matrix inequalities (LMIs) expressing $H_{\infty}$ constraints. To design fractional order PID (FOPID) of an AVR system, a multi objectives extremal optimization (MOEO) is presented in [42], where three objective functions accounting for integral of absolute error (IAE), absolute steady-state error, and settling time are considered. An adaptive population extremal optimization-based PID neural network is presented in [43] for multivariable nonlinear control systems. Nonlinear optimal control has intensively been applied in diverse engineering applications [44]–[47]. In [48], particle swarm optimization (PSO) is used to optimize the gains of FOPID of an AVR system, while in [49], a combined PSO and gravitational search method is used to optimize these gains. A hybrid PSO and differential evolution algorithm is employed to compute optimal PID gains [50]. An improved PSO based PID is used to compute the position controller of a hydraulic system [51]. In [52], [53], a multi-objective non-dominated sorting GA II (NSGA II) optimization is used to tune the FOPID and PID controllers’ gains. Other than the single-objective optimization problems, multi-objective optimization has no universally adopted definition of “optimum”, which is considered as the main challenge of its applications [54]. In [55], the design of multi-objective PID controllers for AVR is achieved by a generalized Hurwitz approach which has limitations on the order of the system. Other optimization techniques are presented in [56], [57] to optimize controller gains for the AVR system.

Design of PID controllers for an AVR system by AI techniques often results in a unique set of optimistic gains. The simulation tests confirming the optimality of the designed controller are commonly performed for the model of the nominal plant. Furthermore, the tests demonstrating the stability robustness of these gains are often performed “adhoc” where no mathematical assessment is provided in the design phase. In practice, these gains may be imprecisely implemented and consequently, the performance of the controller is declined. The deviations between implemented and computed control gains may lead to the fragility of the controller.

B. MAIN CONTRIBUTIONS AND COMPARISONS WITH RELATED WORKS

Control basins (CBs) surrounding all possible PID gains are assigned in the space of such gains using a pair of frequency dependent polynomials, namely $k_p$ and $k_i$ subjected to an arbitrary value of $k_d$, where the coefficients of these polynomials are explicit functions of parameters of the model.

To avoid scanning unnecessary frequencies, swept-over frequency band $(0, \omega_h)$ is computed analytically. In this study, different basins are always labeled as $CB(x \geq x_o)$ in which the argument $(x \geq x_o)$ refers to the minimal value of the design objective, e.g. $CB(\alpha \geq \alpha_o)$ and $CB(\xi \geq \xi_o)$ encloses all PID gains that ensure root-clustering to the left of the line $s = -\alpha_o$ and a damping ratio of at least $\xi_o$, respectively. Similarly, in the frequency domain, the control basins $CB(g \geq g_o)$ and $CB(\theta \geq \theta_o)$ enclose all PID gains that guarantee the minimum phase margin and gain margin values of the closed loop system at $g_o$ and $\theta_o$, respectively. Shorty, any $CB(x \geq x_o)$ is written as $CB(x_o)$. The plot of any $CB(x_o)$ in either $k_p - k_i$ or $k_i - k_d$ plane can often be represented by a convex shape. So, the CB that guarantees dissimilar simultaneous control objectives is determined graphically by the intersection of the individual CBs as $CB(x_1, x_2, \cdots, x_n) = \bigcap_{i=1}^{m} CB(x_i)$. The enhancement of any control objective diminishes its accompanying CB, i.e., $CB(\xi) \subseteq CB(\xi_1), \forall \xi_1 \leq \xi$. As a CB diminishes into its lowest shape in space of PID gains, the upper limit of any control objective can be computed graphically. In brief, this procedure offers a preparatory step in respect of graphical calculating of optimal PID gains. Similarly, the effect of model parametric uncertainties is handled. On condition that these uncertainties are fully addressed by a set of $m$ plants then a robust control basin (RCB) is computed by $RCB(x) = \bigcap_{i=1}^{m} CB_i(x)$ for any control objective $(x)$. As the uncertainty radius increases, the RCB diminishes, where maximum uncertainty radius corresponds to the smallest value that satisfies $RCB(x) = \Phi$.

In this work, a resilient design of robust multi-objectives PID controller for an AVR is presented using the D-decomposition approach. In the proposed design, three control concerns (performance optimality, stability robustness, and controller resiliency) are accounted in a simultaneous manner rather than the contributions of [36], [37]. Note that the proposed method is considered universal even when applying to higher-order control systems. The main contributions are presented as follows.

- The proposed design accounts simultaneously for three control concerns namely performance optimality, stability robustness, and controller resiliency.
- Different design objectives, other than Hurwitz stability, are considered while their corresponding CBs are computed graphically.
- For performance optimality concerns, the considered performance indices are damping factor, damping coefficient, gain and phase margins.
- For robustness concern, model parametric uncertainties are accounted, based on Polyak’s corollary, where sixteen principle vertex plants are derived. The intersection of the CBs corresponding to these vertex plants represents the admissible set of robust controllers.
- For the controller resiliency, the maximum resilient robust PID controller is determined using the principle of the maximum-area circle that can be inscribed in the robust CB.
To ensure the efficacy of the proposed D-decomposition based design, it is compared with AI-based PID designs. This paper is organized as follows. Section II describes a dynamic model of an AVR system while Section III presents the D-decomposition and pole clustering. Section IV elaborates the nominal plant design of multi-objective PID controllers using the D-decomposition method. In Section V, the proposed robust design is presented. The computation, validation as well as simulation results are presented in Section VI, while Section VII provides conclusions.

II. DYNAMIC MODEL OF THE AVR SYSTEM

The essential role of the AVR system is to keep the voltage of the synchronous generator terminal in the allowed boundaries by adjusting the excitation voltage of the field circuit [1], [2]. In addition, supplementary signals can be used with the AVR system to improve the dynamic rotor angle stability [29]. These components are represented with their linear models where supplementary signals can be used with the AVR system. These regions are named as D-regions and labeled with their fixed number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, the CPP is subdivided into D-regions, which have a different number of LHP poles.

The characteristic polynomial of the studied PID-controlled AVR model is given by:

\[ f(s) = sD(s) + (k_d s^2 + k_p s + k_i)N(s) \]  

\[ f(s) = s^4 + 3s^3 + 2s^2 + (d_1 + n_0 k_d) s^2 + (d_0 + n_0 k_p)s + n_0 k_i \]

III. D-DECOMPOSITION AND POLE PLACEMENT

A. D-DECOMPOSITION: REVISITED

Consider an LTI system having the characteristic polynomial \( f(s, k_i) \) and depending on controller parameters \( k_i \), the stability boundary in the space of the controller parameters is given by:

\[ f(j\omega, k_i) = 0, \omega \in (0, \infty) \]

that is the instability boundary in the root plane, i.e., the imaginary axis is mapped into the space of the controller parameter. If \( k_i \in \mathbb{R}^2 \) (or \( k_i \in \mathbb{C} \)), then two equations are obtained, i.e., the real and imaginary parts of (5), in two variables and can define the parametric curve \( k_i(\omega) \), \( 0 < \omega < \infty \), which define the stability domain boundary. Furthermore, the \( k_i(\omega) \) curve subdivides the controller parameter plane (CPP) into root invariant regions. These regions have a fixed number of stable and unstable roots of the characteristic polynomial. These regions are named as D-regions and labeled with their inclusion of the poles in the left half plane.

A simple example is presented hereafter to demonstrate the application of the D-decomposition method. For a simple plant with a transfer function of \( G(s) = (s - 2)(s - 3)/(s + 1)(s^2 + 2s + 2) \), suppose that a PI controller, \( C(s) = k_p + k_i/s \), is used to stabilize this plant. The characteristic polynomial is expressed as:

\[ f(s, k_p, k_i) = s(s + 1)(s^2 + 2s + 2) + (k_p s + k_i)(s - 2)(s - 3) \]

The D-decomposition of the CPP \((k_p - k_i)\) is realized by the parametric curve, derived by substituting \( s = j\omega \), decomposing \( f(j\omega, k_p, k_i) \) into its real and imaginary components and equating them to zeros:

\[ k_p(\omega) = -\text{Re}(G^{-1}(j\omega)) = \frac{-8\omega^4 + 40\omega^2 - 12}{\omega^4 + 13\omega^2 + 36}, \]
\[ k_i(\omega) = -\text{Im}(G^{-1}(j\omega)) = \frac{8\omega^6 - 25\omega^4 + 34\omega^2}{\omega^4 + 13\omega^2 + 36} \]

The domain is also decomposed by the straight line \( k_i = 0 \), which is equivalent to \( \omega = 0 \) and this line represents the real root boundary (RRB) as shown in Figure 2a. It is noticed that the CPP is subdivided into D-regions, which have a different number of LHP poles. Clearly speaking, D(4), D(3), D(2), D(1) and D(0) refer that any controller gain pair within these regions results in four, three, two, one and zero stable roots of the characteristic polynomial, respectively. So, region D(4) represents the control basin of all stabilizing PI controllers. Generally, for a closed-loop characteristic polynomial of order \( n \) the stabilizing control basin is the region in the CPP that is labeled as \( D_0(n) \).
mial. This mapping is achieved by the parametric functions.

each to ensure a fast response. This is accomplished by replacing

k and polynomial and obtaining the two parametric functions

\(k\) and \(s\) verify that \(CB\) is not empty, i.e., \(D_{2}(n) \subset D_{a}(n)\) and \(D_{2}(n) \subset D_{a}(n)\) for \(0 < \xi_{1} < \xi_{2}\).

Simultaneous pole-placement in Region 2 and Region 3 necessitates an intersection of the two CBs to guarantee both damping factor and damping coefficient simultaneously, i.e., \(D_{\alpha-\xi}(n) = D_{d}(n) \cap D_{e}(n) \neq \emptyset\) (Not empty). Furthermore, it can be proved that \(D_{\alpha_{2}-\xi_{2}}(n) \subset D_{\alpha_{1}-\xi_{1}}(n)\), \(\forall \alpha_{2} > \alpha_{1}, \xi_{2} > \xi_{1}\). It is noticed that increasing the damping indices has the effect of decreasing the control basin \(D_{\alpha-\xi}(n)\) significantly. As the control basin reduces to a single point, optimality can be tackled, where optimal control pair \((k_{\alpha_{1}}^{*}, k_{\alpha_{2}}^{*})\) is obtained.

C. GAIN-PHASE MARGIN BASED DESIGN

In the case that the pre-specified \(A_{m}\) and \(\theta_{m}\) are considered, respectively, as minimum gain and phase margins, the CBs of PID gains (if any exists) which ensure that \(GM \geq A_{m}\) as well as \(PM \geq \theta_{m}\) need to be determined. The closed loop stability should be maintained subject to rising the gain of the loop by a factor of \(A_{m}\). Furthermore, an extra phase lag up to \(\theta_{m}\) can be added to the system prior to it turns to be unstable.

Subsequently, the admissible gains of PID must fulfill the next conditions:

1) \(f_{A}(s) = sD(s)+A(k_{d}s^{2}+k_{p}s+k_{i})N(s)\) is Hurwitz-stable for all \(A \in (1, A_{m})\)

2) \(f_{\theta}(s) = sD(s)+e^{-j\phi}(k_{d}s^{2}+k_{p}s+k_{i})N(s)\) is Hurwitz-stable for all \(\phi \in (0, \theta_{m})\)

As explained in subsection IV-A, the stability of \(f_{A}(s)\) and \(f_{\theta}(s)\) are studied in the same way as that of absolute stability, where for gain margin, \(N(s)\) is replaced by \(N(s) = AN(s)\), and in case of phase margin condition, a certain complex polynomial is obtained.

IV. MULTI OBJECTIVES PID CONTROLLER (NOMINAL CASE DESIGN)

Consider the characteristic polynomial given in (4), where the sufficient and necessary conditions to ensure its Hurwitz stability require that all its coefficients \(c_{i} \in R^{+}\) and the inequalities resulting from Routh-Hurwitz tabulation are satisfied. Noticeably, the limits of the derivative gain \((k_{d})\) are independent of the pair \((k_{p}, k_{i})\) as imposed by \(d_{1} + n_{0}k_{d} > 0\) and \(d_{2}d_{3} - d_{4}(d_{1} + n_{0}k_{d}) > 0\). Hence, the stabilizing interval of \(k_{d}\) is computed explicitly as follows:

\[
k_{d} = (-d_{1}/n_{0}, -d_{1}/n_{0} + d_{2}d_{3}/n_{0}d_{4})
\]

(8)

A. STABILIZING PID CONTROLLERS SUBJECT TO FIXED VALUE OF \(k_{d}\)

If a fixed value of \(k_{d}\) is selected within (8), the design problem is reduced to compute the admissible set of \(k_{p} - k_{i}\) gains.
For the characteristic polynomial given by:
\[
f(s) = d_5s^5 + d_4s^4 + d_3s^3 + d_2s^2 + d_0s + n_0(k_ds^2 + k_ps + k_i) \tag{9}
\]
Substituting \( s = j\omega \) into (9) and decompose it into its real and imaginary parts, results in
\[
k_p(\omega) = -(1/n_0\omega)\mathrm{Im}\{B(j\omega)\},
k_i(\omega) = k_d\omega^2 - (1/n_0)\mathrm{Re}\{B(j\omega)\} \tag{10}
\]
where the real root boundary (RRB(\( \omega = 0 \)) is given by \( k_i = 0 \) and the complex root boundary (CRB) is given parametrically as follows:
\[
k_i(\omega) = \omega^2(-d_3\omega^2 + d_1 + n_0k_d)/n_0, \quad 0 < \omega < \infty
\]
\[
k_p(\omega) = (-d_4\omega^4 + d_2\omega^2 - d_0)/n_0, \quad 0 < \omega < \infty \tag{11}
\]

The RRB (\( k_i = 0 \)) intersect the CRB at nonzero frequency that is given by \( \omega_0 = \sqrt{d_1/d_3 + n_0k_d/d_3} \). Consequently, the frequency interval needed to fully trace the control basin (CB) is given by:
\[
\omega_r = (0, \sqrt{d_1/d_3 + n_0k_d/d_3}) \tag{12}
\]
If \( k_d \) is set to its limiting values, the CB is reduced to a point given by \( k_i = 0, \ k_p = -d_0/n_0 \) at \( k_d = -d_1/n_0 \) and it is diminished into a line segment given by \( k_d = d_0d_3/n_0d_4 \) and \( n_0k_d/d_3 \). The RBB and CRB intersect at a nonzero frequency given by \( \omega_0 = \sqrt{(c_2 + n_0k_2)/c_4} \), hence the swept-over frequency is given by:
\[
\omega_r = (0, \sqrt{(c_2 + n_0k_2)/c_4}) \tag{17}
\]

**Remark 1:** The width of the derivative gain interval (14) is reduced significantly by increasing the damping factor where at optimal damping factor the interval (14) is reduced to a single point given by \( k_d = -c_2/n_0 \). This occurs if and only if \( c_3 = 0 \) which in turn results in computing the maximum damping factor as given by
\[
\alpha_{\text{max}} = d_3/5d_4 - \sqrt{(d_3/5d_4)^2 - (d_2/10d_4)} \tag{18}
\]

**Remark 2:** The PID gains at the \( \alpha \)-optimal are computed analytically as follows [55]:
\[
\begin{bmatrix}
k_i^* \\
k_p^* \\
k_d^*
\end{bmatrix} = -
\begin{bmatrix}
0 \\
d_0/n_0 \\
d_1/n_0
\end{bmatrix}
\begin{bmatrix}
6\alpha_{\text{max}}^2 \\
15\alpha_{\text{max}}^4 \\
10\alpha_{\text{max}}^6
\end{bmatrix}
\begin{bmatrix}
0 \\
d_4/d_3 \\
d_2/d_2
\end{bmatrix}
\tag{19}
\]

**C. DESIGN OF \( \xi \)-POLE PLACER PID CONTROLLER**
If Hurwitz stability of the polynomial \( f(s - \alpha), \forall \alpha > 0 \) is maintained, then the roots of polynomial (9) lie to the left of the certain line \( s = -\alpha \) in the full complex s-plane. This shifted polynomial is formulated as:
\[
f(s - \alpha) = c_5s^5 + c_4s^4 + \cdots + c_0 + n_0(k_2s^2 + k_1s + k_0) \tag{13}
\]
where,
\[
c_5 = d_4, \ c_4 = -5ad_4 + d_3, \\
c_3 = 10a_d\alpha^2 - 4ad_3 + d_2 \\
c_2 = -10a_d\alpha^3 + 6a_\alpha^2d_3 - 3ad_2 + d_1, \\
c_1 = 5d_4\alpha^4 - 4d_3\alpha^3 + 3d_2\alpha^2 - 2d_1\alpha, \\
c_0 = -d_4\alpha^5 + d_3\alpha^4 - d_2\alpha^3 - d_1\alpha^2 - d_0\alpha, \\
k_2 = k_d, \ k_1 = k_p - 2a_k, \\
k_0 = \alpha^2k_\xi - \alpha k_p + k_i
\]
Similarly, stabilizing interval of \( k_2 \) is independent of the pair \( (k_1, \ k_0) \) and it explicitly given by:
\[
k_2 = (-c_2/n_0, \ -c_2/n_0 + c_3c_4/c_5) \tag{14}
\]
Substituting \( s = j\omega \) into (13) and decompose it into its real and imaginary parts, results in
\[
\omega^2(c_5\omega^3 - c_3\omega^2 + c_1 + n_0k_1) = 0 \tag{15}
\]
The RBB (\( \omega = 0 \)) is given by \( c_0 + n_0k_0 = 0 \). In terms of \( k_i \), RBB is rewritten as \( k_i = -c_0/n_0 - \alpha^2k_\xi + \alpha k_p \). While the CRB (\( 0 < \omega < \infty \)) is described by the following parametric curve:
\[
k_p(\omega) = \omega^2(-c_5\omega^2 + c_3)/n_0 - c_1/n_0 + 2\alpha k_d, \\
k_i(\omega) = \alpha^2\left(-c_4\omega^2 + (c_2 + n_0k_2)\right)/n_0 - c_0/n_0 - \alpha^2k_d + \alpha k_p \tag{16}
\]
Replacing each \( s \) by \( s_j = \phi \) into (9) and decompose it into its real and imaginary polynomials, results in
\[
f(s_j) = \frac{d_4e^{j\phi} s_j^5 + d_3e^{j\phi} s_j^4 + d_2e^{j\phi} s_j^3}{u_j + j\phi} + \frac{(d_1 + n_0k_d)e^{j\phi} s_j^2 + (d_0 + n_0k_p)e^{j\phi} s_j + n_0k_0}{u_2 + j\phi} u_0 \\
\]
\[
f(s_j) = \frac{u_5s_j^5 + u_4s_j^4 + \cdots + u_0}{f_0(s)} \tag{20}
\]
Substituting each $s$ by $s = j\omega$ and decompose into real and imaginaryparts, it gives

$$f_u(j\omega) = (u_4\omega^4 - u_2\omega^2 + u_0) + j\omega(u_5\omega^4 - u_3\omega^2 + u_1),$$

$$f_f(j\omega) = (-u_5\omega^5 + u_3\omega^3 - v_1\omega) + j\omega^2(v_4\omega^2 - v_2)$$

$$f(\omega e^{j(\pi/2+\phi)}) = \left( -u_5\omega^5 + u_4\omega^4 + v_3\omega^3 - u_2\omega^2 - v_1\omega + u_0 \right)$$

$$\frac{p(\omega, \xi)}{q(\omega, \xi)}$$

$$+ j\omega \left( u_5\omega^4 + v_4\omega^3 - u_3\omega^2 - v_2\omega + u_1 \right)$$

(22)

$$p(\omega, \xi) = -u_5\omega^5 + u_4\omega^4 + v_3\omega^3 - u_2\omega^2 - v_1\omega + u_0$$

$$q(\omega, \xi) = u_5\omega^5 + u_4\omega^4 - u_3\omega^3 - v_2\omega^2 + u_1\omega$$

(23)

Solving $p(\omega, \xi) = 0$, and $q(\omega, \xi) = 0$ simultaneously to get explicit expressions of $k_p$ and $k_i$ as follows:

$$k_p(\omega, \xi) = (l_4\omega^4 + l_3\omega^3 + l_2\omega^2 + l_1\omega + l_0)/n_0,$$

$$k_i(\omega, \xi) = \omega^2(m_3\omega^3 + m_2\omega^2 + m_1\omega + m_0)/n_0$$

(24)

where:

$$l_4 = -(16\xi^4 - 12\xi^2 + 1)d_4,$$

$$l_3 = -4\xi(-2\xi^2 + 1)d_3,$$

$$l_2 = (-4\xi^2 + 1)d_2,$$

$$l_1 = 2\xi(d_1 + n_0d_k),$$

$$l_0 = -d_0,$$

$$m_3 = 4\xi(1 - 2\xi^2)d_4,$$

$$m_2 = -(1 - 4\xi^2)d_3,$$

$$m_1 = -2\xi d_2,$$

$$m_0 = d_1 + m_0d_k$$

The RRB ($\omega = 0$) is given by $k_i = 0$ while the CRB is described by the parametric curve given by (24). Herein, intersection of RRB and CRB depends mainly on the values of $k_d$ and $\xi$ considered in the design procedure. Intersection points are determined by the nonzero positive roots of the polynomial

$$m_3\omega^3 + m_2\omega^2 + m_1\omega + m_0 = 0$$

(25)

**Remark 3:** Depending on the value of $k_d$ and the damping coefficient, $\xi$ –parametric curve may have a self-intersection point (SIP). This SIP is computed numerically for fixed values of $k_d$ and $\xi$ by solving the following equations simultaneously:

$$l_3(\omega_4^4 - \omega_4^1) + l_2(\omega_3^3 - \omega_3^1) + l_1(\omega_2^2 - \omega_2^1)$$

$$+ l_0(\omega - \omega_1) = 0,$$

$$m_3(\omega_5^2 - \omega_5^1) + m_2(\omega_4^3 - \omega_4^1) + m_1(\omega_3^2 - \omega_3^1)$$

$$+ m_0(\omega_2^2 - \omega_2^1) = 0$$

(26)

**D. DESIGN OF PID CONTROLLER WITH GUARANTEED GAIN/PHASE MARGINS**

If the polynomial $f(s) = sD(s)+e^{-j\theta}(k_ds^2+k_ps+k_3N(s)$ is Hurwitz-stable $\forall \theta \in (0, \theta_m)$, then their roots lie in the complex s-plane in such a way that grantees $PM \geq \theta_m$.

Expanding this equation and decomposing it into its real and imaginary polynomials, result in:

$$f(s) = (d_4s^5 + d_3s^4 + d_2s^3 + d_1s^2 + d_0s)$$

$$+ e^{-j\theta}n_0(k_ds^2 + k_ps + k_i)$$

(27)

Explicitly;

$$k_p(\omega, \theta) = (z_4\omega^4 + z_3\omega^3 + z_2\omega^2 + z_1\omega + z_0)/n_0$$

$$k_i(\omega, \theta) = \omega(w_4\omega^4 + w_3\omega^3 + w_2\omega^2 + w_1\omega + w_0)/n_0$$

(28)

where;

$$z_4 = -d_4 \cos \theta,$$

$$z_3 = -d_3 \sin \theta,$$

$$z_2 = d_2 \cos \theta,$$

$$z_1 = d_1 \sin \theta,$$

$$z_0 = -d_0 \cos \theta,$$

$$w_4 = d_4 \sin \theta,$$

$$w_3 = -d_3 \cos \theta,$$

$$w_2 = -d_2 \sin \theta,$$

$$w_1 = d_1 \cos \theta + n_0d_k,$$

$$w_0 = d_0 \sin \theta$$

The RRB is given by $k_i = 0$, while CRB is determined by the parametric curve given by (28). The CRB intersect the RRB at certain frequency computed as the nonzero positive root of the polynomial

$$w_4\omega^4 + w_3\omega^3 + w_2\omega^2 + w_1\omega + w_0 = 0$$

(29)

**V. MULTI OBJECTIVES PID CONTROLLER (ROBUST DESIGN)**

The model parametric uncertainties are considered in this section as an extension of the preceding analyses, where the design of robust PID must cope with these uncertainties.

Since the characteristic equation coefficients depend simultaneously on time constants and the coefficients are dependent, the Kharitonov theorem could not be directly applied. Herein, the results of Polyak [58] is exploited to get a finite number of principle vertex polynomials, which describe parametric uncertainties in time constants sufficiently.

The parameters of the model, provided in Figure 1, are considered to be uncertain with a uncertainty radius $\delta_\rho$, where $0 < \delta_\rho < 1$. Every uncertain parameter ($\hat{\rho}$) with a nominal value of $\rho^0$ is bounded as $\rho^- \leq \hat{\rho} \leq \rho^+$, in which $\rho^- = (1 - \delta_\rho)\rho^0$ and $\rho^+ = (1 + \delta_\rho)\rho^0$. The system presented in Figure 1 contains cascade blocks of first order
FIGURE 3. D-decomposition and control basins subject to different values of $k_d$: (a) Root-invariant regions at $k_d = 0.5$; (b) Control basins subject to the range of $k_d = -0.151$ to 0.6148.

with uncertain time constants and gains. For the uncertain closed loop, the characteristic polynomial is given by:

$$
\hat{f}(s) = sD(s, \hat{T}_A, \hat{T}_E, \hat{T}_G) + (k_ds^2 + k_ps + k_i)N(s, \hat{K}_A, \hat{K}_E, \hat{K}_G, \hat{K}_S)
$$

(30)

where $\hat{T}_A \in [T_A^-, T_A^+]$, $\hat{T}_E \in [T_E^-, T_E^+]$, $\hat{T}_G \in [T_G^-, T_G^+]$, $\hat{T}_S \in [T_S^-, T_S^+]$, $\hat{K}_A \in [\hat{K}_A^-, \hat{K}_A^+]$, $\hat{K}_E \in [\hat{K}_E^-, \hat{K}_E^+]$, $\hat{K}_G \in [\hat{K}_G^-, \hat{K}_G^+]$, and $\hat{K}_S \in [\hat{K}_S^-, \hat{K}_S^+]$.

Equation (30) defines a family of characteristic polynomials, where it is noticed that there is no overlap between the intervals of time constants for any uncertainty radius. Furthermore, these intervals are set in an ascending order as $T_S^- < \hat{T}_A < \hat{T}_E < T_G^-$, i.e., $T_S^- < T_A^-, T_A^+ < T_E^-$, and $T_E^- < T_G^-$. Given the condition that no overlapping occurs, Polyak derived a set of vertex polynomials, which are indispensable to evaluate the stability of (30) [55], [58]. As dependent on the time constants, the set of principle vertices is defined as follows:

$$
\nu^1 = (T_S^-, T_A^-, T_E^-, T_G^-),
$$

(31)
While, as dependent on the block gains, this set is formulated as:

\[ \kappa_1 = (K_S^-, K_A^+, K_E^-, K_G^-), \]
\[ \kappa_2 = (K_S^+, K_A^+, K_E^+, K_G^+) \quad (32) \]

The corresponding principle vertex polynomials are presented as follows:

\[ f_{ij}(s, \upsilon^i, \kappa^j) = sD_p(s, \upsilon^j) + (k_d s^2 + k_p s + k_i)N_p(s, \kappa^j), \]
\[ i = 1, \ldots, 8, \quad j = 1, 2 \quad (33) \]

where; \( N_p(s, \kappa_1) = K_A K_E K_G s, N_p(s, \kappa_2) = K_A K_E K_G s \),
\[ D_p(s, \upsilon^j) = (1 + T_S s)(1 + T_A s)(1 + T_E s)(1 + T_G s), \]
\[ D_p(s, \upsilon^j) = (1 + T_S s)(1 + T_A s)(1 + T_E s)(1 + T_G s), \ldots, D_p(s, \upsilon^8) = (1 + T_S s)(1 + T_A s)(1 + T_E s)(1 + T_G s). \]

Consequently, the stability of this family of plants can be guaranteed only if the attained stability of these sixteen principle vertex polynomials is guaranteed in a simultaneous manner.

VI. COMPUTATION, VALIDATION AND SIMULATION RESULTS

Using the typical parameters the AVR system considered in [2], the transfer function \( E(s)/U(s) \) is given by:

\[ E(s)/U(s) = 10/(0.0004s^4 + 0.0454s^3 + 0.555s^2 + 1.51s + 1) \]

The resulting interval of the derivative gain is computed by (8) as \( k_d = (-0.151, 6.1483) \), where the swept-over frequency intervals are given by \( \omega_q = (0.5671\sqrt{1 + 6.6224k_d}) \) for an arbitrary value of \( k_d \). D-decomposition of \( k_p-k_i \) plane, for Reg. #1, is shown in Figure 3a subject to \( k_d = 0.5 \), and the corresponding CBs for different values of \( k_d \) is depicted in Figure 3b. Remarkably, CB is described by a unique point at \( k_d^{\min} = -0.151 \) and described by a line segment at \( k_d^{\max} = 6.1483 \).

For pole-clustering in Reg. #2 that is defined by \( s = -\alpha \), the resulting ranges of \( k_d \) at different values of damping factors are computed from (14) and are shown in Figure 4a. Samples of CB(\( \alpha \geq \alpha_o \)) that achieve different damping factors are computed subject to \( k_d = 0.2 \), and depicted in Figure 4b. On the other hand, if the damping factor is pre-specified \( (\alpha \geq 2.5s^{-1}) \), the corresponding control polyhedron is shown.
FIGURE 7. Multi-objective Control basins subject to $k_d = 0.25$: (a) CB($\alpha \geq 1.5, \xi \geq 0.5$) $\neq \Phi$; (b) CB($\alpha \geq 2, \xi \geq 0.62$) $= \Phi$; (c) CB($\alpha \geq 0.75, \xi \geq 0.3, \vartheta \geq 30^\circ$, GM $\geq 20$ dB); (d) CB($\alpha \geq 1.25, \xi \geq 0.6, \vartheta \geq 60^\circ$, GM $\geq 26$ dB).

in Figure 4c. Noticeably, for fixed $k_d$, the CB achieving maximum damping is reduced to a line segment. Also, for pre-specified damping, the CB obtained at a maximum value of $k_d$ is described by a line segment as well.

Similarly, in Reg. #3, $\xi$–Hurwitz CBs are computed for $k_d = 0.225$ and are depicted in Figure 5a, where it is noted that enhancing the damping ratio has the effect of diminishing the resulting CB accordingly. The controller polyhedron is depicted in Figure 5b for a fixed damping ratio ($\xi = 0.5$).

Remark: The guaranteed $\xi$ – based CBs have four phases that depend basically on the intersection between RRB and CRB. In some phases, the CRB parametric has a self intersection point (SIP).

The CBs guaranteeing different phase margins are depicted in Figure 6a subject to a fixed value of $k_d$ ($k_d = 0.2$), while the controller polyhedron that achieves phase margins greater than $60^\circ$ ($\vartheta \geq 60^\circ$) is shown in Figure 6b.

Enforcing two or more control objectives simultaneously requires that all the corresponding CBs do intersect, i.e., such CB that ensures dissimilar control objectives simultaneously is computed graphically using CB($\varphi_1 \geq \varphi_1^{o}$, $\varphi_2 \geq \varphi_2^{o}$, $\ldots$, $\varphi_n \geq \varphi_n^{o}$) = \bigcap_{i=1}^{n} CB($\varphi_i \geq \varphi_i^{o}$) if any exist.

The CB that guarantees pole-placement in a D-shape region that is bounded by $\alpha \geq 1.5s^{-1}$ and $\xi \geq 0.5$, is shown in Figure 7a, while the CB that guarantee pole-placement in a D-shape region bounded by $\alpha \geq 2s^{-1}$ and $\xi \geq 0.62$ does not exist as depicted in Figure 7b. Meeting pole-placement and frequency domain specification simultaneously is shown in Figure 7c where $\alpha \geq 0.75$, $\xi \geq 0.3$, $\vartheta \geq 30^\circ$, and GM $\geq 20$ dB. Increasing these specifications results in diminishing the CB that meets these requirements as shown in Figure 7d, where $\alpha \geq 1.3s^{-1}$, $\xi \geq 0.62$, PM $\geq 60^\circ$, and GM $\geq 26$ dB. Using D-decomposition, one can simply decide if a set of control objectives can be met simultaneously or not.

A. ROBUSTNESS ANALYSIS

The adopted nominal model parameters are proposed to be uncertain within $\delta_{\rho} = 10\%$ in this study. As presented in Section V, The set of principle vertex polynomials are computed as described below:

$$N_1(s) = 7.29, \quad N_2(s) = 13.31,$$

$$D_1(s) = 2.62 \times 10^{-4}s^4 + 0.03s^3 + 0.45s^2 + 1.359s + 1,$$
FIGURE 8. Robust control basins: (a) derivative gains as function of damping factors; (b) robust CB subject to $k_d = (0.15, 0.25)$ and $\alpha \geq 1s^{-1}$; (c) robust CBs subject to various $\alpha$ values; (d) robust CBs subject to various $\xi$ values.

FIGURE 9. Multi-objective robust control basins subject to $\delta_p = 10\%$ and $k_d = 0.1785$. (a) robust control basins for $\alpha \geq 1s^{-1}$, $\xi \geq 0.5$, $\vartheta \geq 56^\circ$ and $A \geq 20$ dB; (b) intersected robust control basin achieving all the designed objectives indicating the radius of controller resiliency.
that the increase of the accompanying CB is shown in Figure 8c. It is remarked active gain and RCB(α)
attainable attain the required ranges of α considerably. Further, the RCB is shrunk considerably by
upon the shrinking of the RCB to a single point, which
sixteen vertex principle plants are obtained.

For the α—robust control basins RCB(α ≥ α_o), the required ranges of α—stabilizing k_d to get the highest attainable α are given in Figure 8a. The existence of such RCBs requires the existence of robust α—stabilizing derivative gain and RCB(α) = \bigcap_{i=1}^{16} CB_i(α) ≠ Φ.

For example, the RCB(α ≥ 1) is shown in Figure 8b. The effect of increasing the radius of uncertainty (δ_p) on the accompanying CB is shown in Figure 8c. It is remarked that the increase of δ_p results in shrinking the resulting RCB considerably. Further, the RCB is shrunk considerably by rising the α and ξ as given in Figures 8c and 8d, respectively. So, optimal robust gains of PID controllers can be obtained upon the shrinking of the RCB to a single point, which can be tackled graphically. Similarly, for a fixed value of k_d subject to fixed ρ, robust control basins for gain margin and phase margin are determined graphically. The constraint of the existence of such RCB is given by RCB(φ_i ≥ φ_{i0}) = \bigcap_{i=1}^{16} CB_i(φ_i ≥ φ_{i0}) ≠ Φ. Furthermore, a compromise of damping factor and ratio, phase margin and gain margin is considered to obtain robust control basins which
guarantee robust pole placement subject to certain k_d and ρ, i.e., RCB(α ≥ α_o, ξ ≥ ξ_o, θ ≥ θ_m, A ≥ A_m) = RCB(α ≥ α_o) \bigcap RCB(ξ ≥ ξ_o) \bigcap RCB(θ ≥ θ_m) \bigcap RCB(A ≥ A_m) if any exists. A robust CB is shown in Figure 9, in which a set of control objectives are met in a simultaneous manner, where α ≥ 1 s^{-1}, ξ ≥ 0.5, θ ≥ 56º, A ≥ 20 dB, subject to δ_p = 10% and k_d = 0.1785. It is worth noted that the increase of either the uncertainty or the optimality decreases the resiliency of the obtained gain of the controller.

B. VALIDATION AND TRANSIENT RESPONSE
To manifest the effectiveness of the proposed robust PID design, the poles of the closed loop system, gain and phase margin values, and time domain response are compared with other PID designs in this section. Figure 10 shows the locations of the dominant closed loop poles of the introduced PID controller compared with these of ABC-based

\begin{align*}
D_2(s) &= 3.21 \times 10^{-4}s^4 + 0.03s^3 + 0.45s^2 + 1.361s + 1, \\
D_3(s) &= 3.92 \times 10^{-4}s^4 + 0.04s^3 + 0.48s^2 + 1.381s + 1, \\
D_4(s) &= 4.79 \times 10^{-4}s^4 + 0.05s^3 + 0.56s^2 + 1.461s + 1, \\
D_5(s) &= 5.86 \times 10^{-4}s^4 + 0.06s^3 + 0.68s^2 + 1.661s + 1, \\
D_6(s) &= 4.80 \times 10^{-4}s^4 + 0.06s^3 + 0.67s^2 + 1.659s + 1, \\
D_7(s) &= 3.92 \times 10^{-4}s^4 + 0.05s^3 + 0.64s^2 + 1.639s + 1, \\
D_8(s) &= 3.21 \times 10^{-4}s^4 + 0.04s^3 + 0.54s^2 + 1.559s + 1.
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{Bode plots of the sixteen vertex plants with the proposed PID controller.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure12.png}
\caption{Time response of terminal voltage when subjected to a unit step change of V_{ref} for the proposed design of PID, ABC-based PID [2], NSGA II-based PID [53], and MOEO-based PID controllers [42].}
\end{figure}
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FIGURE 13. Time responses of terminal voltage subjected to a unit step change of $V_{ref}$ for the proposed design of PID with: (a) uncertainty of the nominal model time constants with $\delta_\rho = 10\%$ (b) perturbation of the gains of the proposed PID controller with $\pm 8\%$ (c) simultaneous uncertainty of time constants with $\delta_\rho = 10\%$ and perturbation of the gains of the proposed PID controller with $\pm 8\%$.

FIGURE 14. Block diagram of a PID-controlled AVR system with a disturbance.

FIGURE 15. Time response of terminal voltage when subjected to a unit step change of $V_{ref}$ and an instantaneous unit step disturbance for the proposed design of PID, ABC-based PID [2], NSGA II-based PID [53], and MOEO-based PID controllers [42].

PID [2], NSGA II-based PID [53] and MOEO-based PID controllers [42] for uncertainty radius of $\delta_\rho = 10\%$ and perturbation PID controller gains by $\pm 8\%$, simultaneously. It is remarkable that the proposed PID controller guarantees regional pole-placement with a lowest $\alpha$ of $1 s^{-1}$ and $\xi$ of 0.5.

Bode plots of the sixteen principle vertex plants are presented to emphasize that the requirements of both gain and phase margins are simultaneously met. Figure 11 shows the Bode plots of the sixteen vertex plants with the introduced PID controller, where it is noticed that the proposed PID controller guarantees $GM \geq 20$ dB and $PM \geq 56^\circ$. A comparison of the attained minimum GM and PM of the sixteen vertex plants, for different PID controller designs such as the PID based on artificial bee colony (ABC) [2], teaching-learning based optimization (TLBO) [59], non-dominated sorting genetic algorithm (NSGA) II [53], future search algorithm (FSA) [35] and multi objectives extremal

| PID based type | Minimum GM | Minimum PM |
|----------------|------------|------------|
| ABC-PID        | 14.93 dB   | 30.4°      |
| TLBO-PID       | 22.73 dB   | 58.6°      |
| MOEO-PID       | 16.7 db    | 41.8°      |
| FSA-PID        | 20.9 dB    | 52.8°      |
| NSGA-II-PID    | 10.96 dB   | 19.95°     |
| Proposed PID   | 22.77 dB   | 58.3°      |
TABLE 2. Comparisons of the steady state and transient performance evaluation indices.

| PID based type | $k_p$  | $k_i$  | $k_d$ | $M_p$% | $t_s$ | $E_{ss}$ | FOD  | IAE | ITAE |
|----------------|--------|--------|-------|--------|-------|---------|------|-----|------|
| ABC-PID        | 1.652  | 0.408  | 0.365 | 26.956 | 1.576 | 0.01238 | 0.6993 | 0.2931 | 0.277 |
| TLBO-PID       | 0.5302 | 0.4001 | 0.1787| 0.5926 | 0.6206| -0.00045| 0.09461| 0.2634| 0.0776|
| MOEO-PID       | 0.8503 | 0.7473 | 0.3874| 6.7019 | 0.899 | 6.34E-05 | 0.30608 | 0.1888 | 0.0865|
| FSA-PID        | 0.645  | 0.473  | 0.225 | 1.0192 | 0.4397| -0.00037| 0.06119 | 0.2192 | 0.0563|
| NSGA II-PID    | 2.7666 | 0.4991 | 0.5008| 42.684 | 1.343 | 0.011372| 0.7282 | 0.2932 | 0.2466|
| Proposed PID   | 0.5364 | 0.42779| 0.1785| 1.0904 | 0.5817| -0.000343| 0.08594 | 0.2629 | 0.0861|

Optimization (MOEO) controllers [42], is presented in Table 1. It is noticed that the required objectives are achieved using the proposed PID controller, thanks to the adopted D-decomposition approach.

It is important to note that the most existing approaches tune the PID controller using different optimization methods. Unlike these optimization based methods, the proposed method presents an analytical, step by step procedure for obtaining the required control objective such as damping factor, damping ratio, gain margin and phase margin. Accordingly, all the robust PID gains are determined, within the robust control basin, to get the required objective rather than obtaining a unique set of PID controller gains using optimization methods.

Subject to a unit step change in $V_{ref}$, the comparison of the responses of the terminal voltage of the nominal system are presented in Figure 12. Comparisons of the transient and steady state performance indices are presented in Table 2. The time domain response are quantified by performance indices like IAE, ITAE and figure of demerit [60]. It can be remarked the privileged terminal voltage response when using the proposed PID controller. To prove robust and resilient performance of the proposed PID design, time responses with uncertain time constants of $\delta_T = 10\%$ are presented in Figure 13 and with perturbed gains of the proposed PID controller of $\pm 8\%$ are presented in Figure 13a, while Figure 13b presents the response for simultaneous uncertainty of time constants and perturbation of the gains of the proposed PID controller. In other words, resilient (non-fragile) controller refers that the controller gains can be uncertain with a certain percentage ($8\%$ in this work) without affecting the performance of the controller.

To emphasize the privilege of the proposed PID controller, different scenarios of disturbance are applied to the AVR model, as follows:

- An instantaneous unit step disturbance is applied to the AVR model, as shown in Figure 14, for a period of 0.1 s at the $t = 5$ s. The responses of the terminal voltage of the nominal system are presented in Figure 15.
- An impulsive disturbance is initiated by a change of 0.1 of the initial value of the fourth state, while keeping the other initial states at zero. The effect of such impulse disturbance on the terminal voltage of the nominal system is shown in Figure 16.
- The effect of applying an additive white noise to the measured signal is shown in Figure 17 in which the terminal voltage of the nominal system is presented.

FIGURE 17. Comparisons of the response of the terminal voltage when subjected an additive white noise of the measured signal of the AVR model.

- The effect of applying an input fluctuation to the AVR model is shown in Figure 18 in which the terminal voltage of the nominal system is shown.

As shown in the studied disturbance scenarios, the proposed PID design yields an enhanced performance for the AVR model compared to the other controllers.
VII. CONCLUSION
In this study, D-decomposition is considered to synthesize resilient robust multi-objective PID controllers for the AVR system. The main contributions, observations and recommendations are summarized. The CB of a specific control objective is systematically traced in the space of PID gains using a pair of frequency polynomials, namely \( k_p \) and \( k_i \) for an arbitrary value of \( k_d \). The coefficients of these polynomials are expressed explicitly as functions of model parameters and the considered performance index like damping factor, damping coefficient, GM and PM. Swept-over frequency band (0, \( \omega_B \)) is computed analytically to avoid scanning unnecessary frequencies. Different control objectives are simultaneously met if their accompanying CBs intersect in a common region. Also, it is observed that enhancing any performance index has the effect of shrinking the resulting CB accordingly. Optimal performance can be tackled graphically where the performance index reaches its peak when the accompanying CB diminishes into a point. Based on Polyak’s corollary, sixteen principle vertex plants are derived to account for all model parametric uncertainties. The admissible set of robust controllers is obtained if the CBs corresponding to these vertex plants do intersect. The maximum resilient robust PID controller is determined using the principle of the maximum-area circle that can be inscribed in the robust CB. The proposed design accounts simultaneously for three control concerns namely performance optimality, stability robustness and controller resiliency. Comparisons with AI-based PID designs are carried out to ensure the effectiveness of the proposed D-decomposition based design. Finally, it is strongly recommended to address further performance indices in the time domain like IAE, and ITAE using D-decomposition in order to control the time response precisely. It is worth noting that continuous-time PID controllers are simply applied as described in control literature. In turn, digital PID controllers are employed in the industry; therefore, it is recommended to apply the proposed methodology in the digital PID design. In the future work, the D-decomposition method will be extended to design a multi-objective PID controller for multiple AVR while considering renewable generation uncertainty.

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