Training teachers: explorative activities measuring Planck's constant

R Capone¹, R De Luca¹, A De Santis², O Faella¹, O Fiore³ and A Saggese¹
¹ Dipartimento di Fisica “E. R. Caianiello”, Università di Salerno, via Giovanni Paolo II, I-84084 Fisciano (SA), Italy
² Istituto d’Istruzione Superiore “Tito Lucrezio Caro”, via Roma 28, I-84087, Sarno (SA), Italy
³ Istituto d’Istruzione Superiore “P. E. Imbriani”, Via S. Pescatori 155, Avellino I-83100, Italy
robertocapone69@gmail.com

Abstract. In recent years, Modern Physics has been included in the curricula of Italian secondary schools, so that many teachers are asking for specific didactical tools to allow their students to acquire the right skills to deal with final examinations. The Physics Department at University of Salerno (Italy) is, therefore, planning PLS (Piano Lauree Scientifiche / Scientific Degree Project) training courses to give some valid educational paths through new didactical strategies for Physics learning. Introducing quantum mechanics by experimental set-ups with commonly used materials is a rather interesting and challenging task. In fact, this practice allows teacher attending the course to reproduce the same experiments in their own class. Moreover, by using readily available material for the experimental set-up, a soft approach to Modern Physics can be achieved. In the present work, we propose the analysis of an experiment: the measurement of Planck’s constant, one of the most important fundamental quantities in Modern Physics, by means of Light Emitting Diodes (LEDs). The properties of LEDs can be explained by means of elementary quantum mechanics concepts. As a consequence, when describing the quantum behaviour of these commonly used physical systems, Planck’s constant needs to be considered. Therefore, to illustrate one possible way of measuring Planck’s constant to high-school students, we can make use of the current-voltage (I-V) characteristics of a LED. In fact, by opportuntely interpreting the measured I-V curves of these systems, by means of a linear fitting the value of Planck’s constant can be obtained.

1. Introduction

In recent years, Modern Physics has been included in the curricula of Italian High-School, so that many teachers are asking for specific didactical tools to allow their students to acquire the right skills to deal with final examinations. The Physics Department at University of Salerno (Italy) is organizing PLS (Piano Lauree Scientifiche / Scientific Degree Project) training courses in order to give some valid educational paths through new didactical strategies for Physics learning.

Introducing quantum mechanics by using experimental set-ups with commonly used materials is a rather interesting and challenging task. This training activities allow teachers who attend the course to reproduce the same experiments in their own class. Moreover, by using readily available material for the experimental set-up, a plain approach to Modern Physics can be achieved.

In the present work, we propose one of these experimental topics: the measurement of Planck’s constant ($\hbar$), one of the most important fundamental quantities in Modern Physics, by means of basic electric circuit with Light Emitting Diodes (LEDs). Photons emission mechanism can be explained by means of elementary quantum mechanics concepts. As a consequence, when the quantum behaviour of
these commonly used physical systems is described, Planck’s constant has to be considered. Moreover, a possible way of measuring Planck’s constant in an experiment suitable for high-school students is to use the current-voltage (I-V) characteristics of a LED. Therefore, by opportunely analysing the measured I-V curves of these systems, Planck’s constant can be obtained.

In this paper we discuss the results of the training activity in the case of the preparation of this didactical experiment through the Learning Action Research. Firstly, in the next section, the results regarding an interview on the educational needs of teachers about the Modern Physics topics is reported.

In the section 3, we firstly analyse the connection between the I-V characteristic of LED and the Energy Band Gap, (which is not straightforward). Nevertheless, a linear relation is finally found that can be used for the estimation of $h$. Moreover, the experimental set-up suitable for High-School laboratory is described. The estimation of Planck’s constant is made for LEDs emitting light of different wavelength. Finally, the issues found on this experience are discussed and a scheme of the experiment for students is outlined.

2. Motivations

Physics teaching is undergoing substantial changes in Italy. The required changes are not always followed by external efforts in supporting physics teachers in implementing new and effective didactical strategies. From a statistical analysis carried out in the region Campania (Italy) it can be argued that there exists a rather pronounced need, explicitly expressed by physics teachers, for developing new physics teaching approaches in High-School [1]. A statistical survey was conducted on a sample of 111 in service teachers from the Campania region in Southern Italy. As shown in Figure 1, the statistical sample is characterized by teachers with predominant female component with a long experience in teaching.
Most teachers believe that continuous training is very useful. The interview shows that most of the teachers want to train on the use of new teaching methods and on the use of multimedia tools. Furthermore, many believe that it is useful to keep up to date with the disciplinary contents (See the results of this interview shown in Figures 2-3).

The Figure 4 species the areas in which teachers believe it is useful to be trained. The need for innovating ways of approaching physics teaching is not only linked to the introduction of new concepts, but also to mastering new methodologies. However, when physics teachers are inquired on what contents they would like to be assisted, the almost unanimous answer was: “Modern Physics”. A specific need for laboratory-oriented activities emerged from the statistical analysis. This requirement is due to the lack of experimental activities based on available equipment in school laboratory in the scope of Modern Physics in Italian High-School. The Physics Department at University of Salerno (Italy) has planned...
and realized a PLS training course to provide valid educational paths in Physics learning, through improved didactical strategies.

3. Project

The project was conceived to propose some concepts in Modern Physics, relying on laboratory activities which could be reproduced in High-School labs. The methodology of Action-Research was used during this project. Teachers answer at the same questions they have asked themselves. In this way, a Learning-Action-Research context was set up.

One of the topics which triggered a great deal of attention from physics teachers attending the course was the activity related to the measurement of Planck's constant by the experimental determination of the I-V characteristics of light-emitting diodes. This activity is a crucial step in understanding the particle nature of light, moreover it proves that important information can be obtained without using expensive experimental equipment.

This topic is well known in the literature, [2-7] but the questions arising from the educated audience were very deep, especially after having referred the material reported on the subject in literature [8, 9]. Some of the participants suggested new ways of looking at the problem. In this way, all were engaged in a rather detailed analysis of the experimental activity. The outcome of this Learning-Action-Research process is illustrated below.

The frequency (ν) of the emitted photons is related to the energy gap (Eg) between the conduction and valence bands of the semiconductor used to make the LED junction. The Planck’s constant is given by

\[ h = \frac{E_g}{\nu}. \]

The aim is to obtain an equation where the \( E_g \) appears and thus it can be estimated by means of I-V measurements. Moreover, the resulting expression has to be simple enough by High-School students. The experimental set-up for performing the I-V measurements has to be easily assembled with the instruments supplied in Physics laboratories of Italian High-School and the experimental procedures has not to be difficult.

The I-V characteristics of LED (as well as a diode), is given by [10]:

\[ I = I_S \left[ e^{\left(\frac{V}{\eta V_T}\right)} - 1 \right] \]  

(1)

where \( I_S \) is the inversion saturation current, \( \eta \) is a parameter whose value is larger than 1 and which considers the recombination of electrons and holes in the junction area. Finally, \( V_T = \frac{k_B T}{e} \), where \( k_B \) is the Boltzmann constant, \( T \) is the absolute temperature and \( e \) is the electron charge.

\[ V [V] \]

\[ I [mA] \]  

\[ I = I_S \left( e^{\left(\frac{V}{\eta V_T}\right)} - 1 \right) \]

Figure 5. I-V characteristic of LED (continuous line) and its linear approximation (dotted red line).
In Figure 5, the I-V curve of LED is shown together with a linear approximation of the its branch at higher voltage (dotted red line). In the same figure, $V_E$ is defined as the voltage value extrapolated by the linear approximation of the high-current branch of LED I-V curve when $I = 0$, (Eq. (1) and see Figure 5).

The following remarks can be written down:

1. For determining the I-V function of a LED, $\eta$ and $I_g$ have to be obtained experimentally by means of Eq. (1). As explained afterward, $\eta$ only has to be experimentally estimated in order to obtain the Planck’s constant.

2. The I-V behaviour is nonlinear, however for $V > V_E$, I-V can be approximated by a linear function.

3. For a $V > V_E$, LEDs emit photons and the related branch in I-V is expected to depend on the energy gap $E_g$.

In other works, on this subject [2] $V_E$ is claimed to be the voltage such that $E_g = eV_E$. Planck’s constant was experimentally estimated by this method with an error larger than 10%.

By considering LED only in radiating regime, we may model the I-V relation as follows:

$$I = I_g e^{(V - V_g)/\eta V_T}$$ (2)

where $I_g$ is the current at the gap voltage $V_g = E_g/e$ and the Eq. (2) is valid for voltage drop $V > V_g$. In this way, LED is modelled as a device which emits photons and drives current only when its voltage drop is higher than $V_g$.

Near $V_g$, for $V \gtrsim V_g$, we may express the exponential function in terms of a first-order expansion as follows: $e^x \approx 1 + x$. In this way: $I \approx I_g \left[1 + (V - V_g)/\eta V_T \right]$ is now linearized. The above expression can be inverted, for $V > V_g$, by writing:

$$V = (V_g - \eta V_T) + \frac{\eta V_T}{I_g} I$$ (3)

which can be rewritten as:

$$V = V_E + R_e I$$ (4)

with $R_e = \eta V_T/I_g$.

For $I = 0$:

$$V_E = V_g - \eta V_T$$ (5)

Therefore, $V_E$ depends on $\eta$ and it cannot be considered equal to the $E_g/e$.

In this way, from Eq. (5):

$$\frac{E_g}{e} = V_g = V_E + \eta V_T$$ (6)

and finally, Planck’s constant is given by:

$$h = \frac{E_g}{\nu} = \frac{e}{\nu} (V_E + \eta V_T)$$ (7)

or by using the wavelength $\lambda$:

$$h = \frac{\lambda e}{c} (V_E + \eta V_T)$$ (8)

For determining $h$, $V_T$ is evaluated at room temperature, whereas $\eta$ and $V_E$ have to be experimentally obtained.

In order to estimate, $\eta$ and $V_E$ the experimental setup shown in Figure 6 has been prepared. The LED was connected to a resistor $R$ and a DC power supply. A digital voltmeter and an ammeter were
connected as shown in Figure 6 for measuring respectively the voltage drop V across the LED and the current I in the circuit.

![Figure 6. Measuring apparatus to find the voltage vs. current characteristics of a LED. An electromotive force generator provides a voltage $V_0$ to the circuit composed of a LED and a resistor $R$ in series. The voltage is measured directly on the LED by means of a digital voltmeter $V$ whereas the ammeter measures the current $A$.](image)

Experimental data were first gathered by using a LED with wavelength $\lambda = 460$ nm. Successively, we considered LEDs of various wavelengths.

For a given LED, $N$ different experimental points by means of current and voltage measurements were taken: $(V_n, I_n)$, for $n = 1$ to $N$.

3.1 Experimental estimation of $\eta$ from I-V characteristic of LED

As the measured current in the circuit is in the order of micro Ampere, for the experimental points $(V_j, I_j)$, let’s impose $I_j = I_S e^{V_j/\eta V_T}$, a simplified functional dependence of Eq. (1), because the last term can be neglected ($I_S \approx 10^{-21}$ A).

By defining $x_j = \ln \left( \frac{I_j}{I_0} \right)$ and $y_j = \frac{V_j}{V_T}$ where $I_0 = 1.00 \mu A$ is a normalization constant, we have the following relation:

$$y_j = \eta x_j + b$$  \hspace{1cm} (9)

with $b = -\eta \ln \left( \frac{I_S}{I_0} \right)$

In this way,

$$I_S = I_0 e^{-b/\eta}$$

Therefore, a best fit analysis, based on the least square method, can be performed to determine $\eta$ and $I_S$. In particular, for the blue led used in our experiment, $\eta = 3.60 \pm 0.04$.

In Figure 7, the experimental curve is shown by blue dots symbols. Data were fitted by last square method and the resulting curve is shown as blue continuous line.

3.2 Experimental estimation of $V_E$ from high-current ($V_j, I_j$), points

The experimental data related to a current larger than 0.2 mA, was used to fit the regime where the LED is radiating. The experimental points were fitted by using the linear function given in Eq. (4). and
the values of $V_E$ can be in this way obtained. In Figure 7, the red continuous line represents the linearization of the I-V characteristic of LED at high current.

![Figure 7. Current (I) vs. Voltage (V) curve for a p-n diode with wavelength $\lambda=460$ nm. The blue dots represent the experimental points; the blue full-line curve is an exponential best fit of the same data; the red lines represent the linearized p-n junction model.](image)

The following values of the parameters are found in the case of the blue LED:

\[
V_E = (2.586 \pm 0.002)V;
\]
\[
R_e = (183 \pm 3)\Omega.
\]

3.3 Estimation of Planck’s constant

Obtained experimentally the values of $\eta$ and $V_E$. The Planck’s constant can be evaluated by using Eqs. (6) and (8).

In the case of the blue LED, $\lambda = 460$ nm and

\[
V_g = V_E + \eta V_T = 2.679V
\]

The Planck’s constant is experimentally estimated as:

\[
h = \frac{\lambda e V_g}{c} = 6.587 \times 10^{-34} \text{ J s}.
\]

From errors analysis, $\Delta h = 0.15 \times 10^{-34} \text{J s}$ results, that corresponds to a relative error of about 2.3%. Finally,

\[
h = (6.59 \pm 0.15) \times 10^{-34} \text{ J s}
\]

We have also considered LEDs emitting light at different wavelengths. The I-V characteristics of five additional LEDs were analysed: ultraviolet InGaN, $\lambda = 400$ nm, green $\lambda \approx 560$ nm; yellow $\lambda \approx 600$ nm; red $\lambda \approx 630$ nm; infrared GaAlAs/GaAs $\lambda = 400$ nm. For all these LEDs, Planck’s constant was estimated with the same procedure used for the blue LED. The results are summarized in Table 1.
During the course it was noted that the measured value of Planck’s constant in some case could not be considered acceptable.

Table 1. Experimental estimation of Planck’s constant by using different LEDs.

| \( \lambda \) [nm] | \( V_g \) [Volt] | \( h \times 10^{-34} \) [J . s] |
|------------------|-----------------|------------------|
| 940              | 1.054           | 5.29 ± 0.15      |
| 628.1            | 1.668           | 5.71 ± 0.19      |
| 596.4            | 1.773           | 5.69 ± 0.19      |
| 558.1            | 1.857           | 5.66 ± 0.21      |
| 460              | 2.679           | 6.59 ± 0.15      |
| 400              | 2.911           | 6.22 ± 0.16      |

The material used for building LED can be a semiconductor with direct bandgap or indirect bandgap. The blue LED are built in InGaN which is a direct bandgap semiconductor. Conversely, the green yellow and red LEDS are realized with indirect bandgap semiconductors. We argue that if the LED is made with indirect band semiconductor the radiative process is more complicated and the I-V curve is not straightforward related to bandgap. For experiment performed by students, blue LED should be used in order to have a good estimation of Planck’s constant.

3.4 Proposed scheme for the experimental measurement of Planck’s constant for High School students

For the determination of the Planck’s constant can be used the experimental setup shown in Figure 6 by using blue LEDs only. Students can be introduced to a qualitative description of optical transition in semiconductors and in particular on the relation between energy band gap and frequency of emitted photons.

In order to simplify the experimental procedure, the value of \( \eta \) of LED can be previously measured by teacher or technical staff of labs. In this way, students know the \( \eta \) values of LED.

Students can be introduced to I-V characteristic of LED in the high current regime and how this I-V branch can be related to the photon emission process.

The experiment can be performed measuring several \((V_g, I)\) points at high current, (after the I-V knee) and the linear fit of the experimental data can be computed by using (Eq. (4)). In this way, \( h \) can be determined via Eq. (8).

Students can experiment which values of current are suitable for the best estimation of \( h \). Students can discuss and analyse the effect of LED warming and understand which experimental procedures can affect negatively the right estimation of \( h \).

Several topics in Modern Physics can be explored by students at basic level: Quantum Mechanics \((E = h\nu)\), Solid State Physics, (electronic bands, semiconductors, optical emission), Electronics, (nonlinear devices, p-n junction, nonlinear I-V characteristics). Moreover, by measuring Planck’s constant, students can verify as Quantum Mechanics is at the basis of macroscopic devices as LEDs and it can explain macroscopic properties of materials; students can realize it is not all confined to explain the microscopic world of atoms and elementary particles only.

4. Conclusions

The experimental measurement of Planck’s constant has been elaborated by analysing the I-V characteristics of LEDs. In particular, for voltages at which LEDs emits photons, the I-V curve has been linearized, including in the equation the energy bandgap of the semiconductor as \( eV_g \). An experimental procedure has been planned in order to be suitable for High School students. The experiments carried out with in service teachers attending the training course have led us to successfully estimated the
Planck’s constant in the case of blue LEDs. These kinds of LED are hence suggested for laboratory activities on Modern Physics.

The adopted training model is well suited to meet the needs to acquire a better approach to introduce quantum Physics in High-School. Efficiency of this approach is mainly due to experimental paths, giving the opportunity to experiment “hands on” quantum mechanics. This didactical activity could be a best practice for all High-School. Moreover, this training offers to everybody the opportunity to design a personalized project development, depending on specific classroom requirements.

5. References
[1] Capone R 2017 PhD thesis Università di Salerno.
[2] Zhou F and Cloninger T 2008 The Physics Teacher 46(7), 413-415, DOI: 10.1119/1.2981288
[3] Indelicato V, La Rocca P, Riggi F, Santagati G and Zappalà G 2013 European Journal of Physics 34(4), 819, DOI:10.1088/0143-0807/34/4/819
[4] Maria Rute de Amorim e Sá Ferreira André and Paulo Sérgio de Brito André 2014 Science in School 28, 28-33.
[5] Planinšič G and Etkina E 2014 The Physics Teacher 52(2), 94-99, DOI: 10.1119/1.4862113
[6] Etkina E and Planinšič G 2014 The Physics Teacher 52(4), 212-218, DOI: 10.1119/1.4868933
[7] Cecchetti A and Fantini A 2015 World Journal of Chemical Education 3, 87-92, DOI: 10.12691/wjce-3-4-2
[8] Hermann F and Schätzle D 1996 American Journal of Physics 64, 1448, DOI: 10.1119/1.18404
[9] Morehouse R 1998 American Journal of Physics 66, 12, DOI: 10.1119/1.19034
[10] Ashcroft N W and Mermin N D 1976 Solid State Physics Saunders College Philadelphia.