Anisotropic Compact Objects in Modified $f(R, T)$ gravity

S Dey\textsuperscript{1}, A Chanda\textsuperscript{2}, B C Paul\textsuperscript{3}

Department of Physics, University of North Bengal, Siliguri, Dist. Darjeeling 734 013, West Bengal, India
IUCAA Centre for Astronomy Research and Development, North Bengal
E-mail : \textsuperscript{1}sagardey1992@gmail.com, \textsuperscript{2}anirbanchanda93@gmail.com, \textsuperscript{3}bcpaul@associates.iucaa.in

Abstract. We obtain a class of anisotropic spherically symmetric relativistic solutions of compact objects in hydrostatic equilibrium in the $f(R, T) = R + 2\chi T$ modified gravity, where $R$ is the Ricci scalar, $T$ is the trace of the energy momentum tensor and $\chi$ is a dimensionless coupling parameter. The matter Lagrangian is $L_m = -\frac{1}{3}(2p_t + p_r)$, where $p_r$ and $p_t$ represents the radial and tangential pressures. Compact objects with dense nuclear matter is expected to be anisotropic. Stellar models are constructed for anisotropic neutron stars working in the modified Finch-Skea (FS) ansatz without preassuming an equation of state. The stellar models are investigate plotting physical quantities like energy density, anisotropy parameter, radial and tangential pressures in all particular cases. The stability of stellar models are checked using the causality conditions and adiabatic index. Using the observed mass of a compact star we obtain stellar models that predicts the radius of the star and EoS for matter inside the compact objects with different values of gravitational coupling constant $\chi$. It is also found that a more massive star can be accommodated with $\chi < 0$. The stellar models obtained here obey the physical acceptability criteria which show consistency for a class of stable compact objects in modified $f(R, T)$ gravity.
1. Introduction

General theory of Relativity (GTR) is a geometric theory of gravitation formulated on the concept that gravity manifests itself as the curvature of space-time. Although GTR is a fairly successful theory at low energy, it is entangled with some serious issues at ultraviolet and infrared limits. Some of the astronomical observational evidences namely, Galactic, extra Galactic and cosmic dynamics are not understood in the framework of GTR. The needle of hope points to the concept of the existence of exotic matter that represents the dark energy [1, 2] which we need if matter sector of GTR is to be modified. On the other hand a modification of the gravitational sector to fit the missing matter-energy of the observed universe is also another important area of present research. In the literature [3, 4, 5, 6] a number of theories of gravity with modification of the gravitational sector came up to understand the evolution of the observed universe as well as to solve some of the issues of non- renormalizability [7, 8] in GTR. In 1970, Buchdahl [9] using a non-linear function of Ricci scalar namely, \( f(R) \) gravity theory first introduced a modification of theory of gravity to explain some of the drawbacks in Friedmann- Lemaître-Robertson-Walker cosmological models. A higher derivative term in the gravitational action in the form \( R^2 \)-term was considered by Starobinsky [10] and found the existence of inflationary solutions in cosmology. Recently, Harko and his collaborators [11] introduced a more generalized form of gravity, the \( f(R) \)-gravity which consists of a self-assertive expression of the Ricci scalar \( R \) and the trace of the energy-momentum tensor \( T \) together introducing \( f(R, T) \)-theory of gravity. The modified theory is interesting as it is effectively accommodate the late time acceleration of the universe. Consequently, there is a spurt in research activities in understanding astrophysical objects of interest in the modified theory of gravity. It is known that the presence of an extra force perpendicular to the four velocity in the \( f(R, T) \) gravity helps test particles to follow a non-geodesic trajectory. It is shown [12] that for a specific linear form of \( f(R, T) \) -theory, say \( f(R, T) = R + f(T) \), the trajectory of the particles become a geodesic path. It is known [13] that that the \( f(R, T) \) theory of gravity pass solar system test satisfactorily. A number of cosmological models [15, 16, 17] are constructed in the \( f(R, T) \) theory of gravity which accommodates the observed universe successfully. Consequently, Moraes et al. [18] studied the equilibrium configuration of quark stars with MIT bag mode. It is shown [19] that an analytical stellar model for compact star in \( f(R, T) \) gravity may be obtained considering a correct form of the Tolman-Oppenheimer-Volkoff (TOV) equation. Deb et al. [20] analyzed both isotropic and anisotropic spherically symmetric compact stars and presented the graphical analysis of LMC X-4 star model. The effect of higher curvature terms present in \( f(R, T) \) gravity is probed in compact objects [21] making use of EoS given by polytropic and MIT bag model. The physical properties of a star in the above case can be derived knowing EoS, i.e., \( p = p(\rho) \) which is not yet known for a compact object at extreme terrestrial condition. In the absence of a reliable information of the EoS at very high densities, assumption of the metric potentials, based on the geometry has
been found to be a reasonable approach to construct a stellar model [22, 23, 24, 25]. The compact objects are stable objects at extreme terrestrial conditions. Thus the compact object can be probed alternatively, where for a given geometry the EoS can be predicted. The motivation of the present paper is to obtain relativistic solution for anisotropic compact stars with its interior space-time described by Finch-Skea (FS) geometry in a linear modified \( f(R, T) \) gravity and construct stellar models. FS metric originated to correct the Dourah and Ray [26] metric which is not suitable for compact object, Finch and Skea [27] modified the metric to describe relativistic stellar models. Subsequently, FS metric with a modification in 4- dimensions [28, 29, 30] and in higher dimensions [31, 32, 33] are considered to explore astrophysical objects. In compact objects the interior pressure may not be same in all directions, thus the study of the behaviour of anisotropic pressure for a spherically symmetric stellar model is important to explore. Ruderman [35] shown that at high density (> 10^{15} g/cm^3) nuclear matter object may be treated relativistically which exhibits the property of anisotropy. The reason for incorporating anisotropy is due to the fact that in the high density regime of compact stars the radial pressure (\( p_r \)) and the transverse pressure (\( p_t \)) are not equal which was pointed out by Camuto [36]. There are other reasons to assume anisotropy in compact stars which might occur in astrophysical objects for various reasons namely, viscosity, phase transition, pion condensation, the presence of strong electromagnetic field, the existence of a solid core or type 3A super fluid, the slow rotation of fluids etc. In this paper we construct relativistic stellar models and predict EoS in the framework of a linear \( f(R, T) \) gravity with isotropic or anisotropic fluid distribution.

The outline of the paper is as follows: in section 2 we present the basic mathematical formulation of \( f(R, T) \) theory and the field equations. In section 3, a class of relativistic solutions are obtained for different parameters of the theory. In section 4 the constraints to obtain stellar models are presented. In section 5, general properties of compact stars, the stability of stellar models, energy conditions, mass to radius etc. are discussed. The EoS of matter inside the star is also predicted. Finally, we discuss the results in section 6.

### 2. The Gravitational action and the field equations in \( f(R, T) \) gravity

The gravitational action for modified theory of gravity is given by

\[
S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} \, d^4x + \int L_m \sqrt{-g} \, d^4x, \tag{1}
\]

where \( f(R, T) \) is an arbitrary function of the Ricci scalar \( (R) \) and \( (T) \) is the trace of the energy-momentum tensor \( T_{\mu\nu} \). The determinant of the metric tensor \( g_{\mu\nu} \) is given by \( g \) and \( L_m \) is the Lagrangian density of the matter part. We consider gravitation unit \( c = G = 1 \). The field equations for the modified gravity theory can be obtained by varying the action \( S \) with respect to the metric tensor \( g_{\mu\nu} \) which is given by,

\[
(R_{\mu\nu} - \nabla_\mu \nabla_\nu) f_R(R, T) + g_{\mu\nu} \square f_R(R, T)
\]
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$$-\frac{1}{2}g_{\mu\nu} f(R, T) = 8\pi T_{\mu\nu} - f_T(R, T) (T_{\mu\nu} + \Theta_{\mu\nu}),$$

(2)

where $f_R(R, T)$ denotes the partial derivative of $f(R, T)$ with respect to $R$, and $f_T(R, T)$ denotes the partial derivative of $f(R, T)$ with respect to $T$. $R_{\mu\nu}$ is the Ricci tensor, $\Box \equiv \sqrt{-g} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$ is the D’Alembert operator and $\nabla_\mu$ represents the co-variant derivative, which is associated with the Levi-Civita connection of the metric tensor $g_{\mu\nu}$. The energy momentum tensor $T_{\mu\nu}$ for perfect fluid changes the role in the $f(R, T)$-modified gravity because of the presence of $\nabla_\mu \nabla_\nu R$ and $(\nabla_\mu R)(\nabla_\nu R)$ and terms which originate from trace of the energy momentum tensor $T$ in the field equation. In the paper, we consider compact objects with anisotropic matter distribution in the modified gravity. The stress-energy tensors $T_{\mu\nu}$ and $\Theta_{\mu\nu}$ are defined as,

$$T_{\mu\nu} = g_{\mu\nu} L_m - 2 \frac{\partial L_m}{\partial g^{\mu\nu}},$$

(3)

$$\Theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$  

(4)

Using eq. (2) the covariant divergence of the stress-energy tensor can be written as

$$\nabla_\mu T_{\mu\nu} = \frac{f_T}{8\pi - f_T} \left[ (T_{\mu\nu} + \Theta_{\mu\nu}) \nabla^\mu ln f_T + \nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla^\mu T \right].$$

(5)

It may be mentioned here that the covariant derivative of the stress-energy tensor in $f(R, T)$ theory does not vanishes, which is different from the $f(R)$-theory. Consequently we describe an effective energy density and pressure which however leads $T_{\mu\nu}^{\text{eff}}; \mu = 0$.

In the modified gravity $f(R, T) = R + 2\chi T$, where $\chi$ is a coupling constant, the field eq. (2) can be represented as

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{\text{eff}}$$

(6)

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}^{\text{eff}}$ is the effective energy-momentum tensor. The energy-momentum tensor for anisotropic matter distribution is given by

$$T_{\mu\nu} = (\rho + p_t) u_\mu u_\nu - p_t g_{\mu\nu} + (p_r - p_t) v_\mu v_\nu,$$

(7)

where $v_\mu$ is the radial four-vector, while $u_\nu$ is four velocity vector, $\rho$, $p_r$ and $p_t$ are the energy density, the radial and tangential pressures respectively. Here, we consider the matter Lagrangian $L_m = -P$, where $P = \frac{1}{3}(2p_t + p_r)$. For anisotropic fluid $\Theta_{\mu\nu} = -2T_{\mu\nu} - g_{\mu\nu} P$, the effective energy-momentum tensor becomes

$$T_{\mu\nu}^{\text{eff}} = T_{\mu\nu} \left(1 + \frac{\chi}{4\pi}\right) + g_{\mu\nu} \frac{\chi}{8\pi} (T + 2P).$$

(8)

The above expression contains the original matter stress-energy tensor $T_{\mu\nu}$ and the curvature terms [11]. We consider $f(R, T) = R + 2\chi T$ and the eq.(5) becomes

$$\nabla_\mu T_{\mu\nu} = - \frac{\chi}{2(4\pi + \chi)} (g_{\mu\nu} \nabla^\mu T + 2\nabla^\mu (g_{\mu\nu} P)).$$

(9)

Now the effective conservation of energy equation is given by

$$\nabla_\mu T_{\mu\nu}^{\text{eff}} = 0.$$  

(10)
Thus the modified gravity allows a non-linear regime in addition to linear regime effectively. The motivation of the paper is to study the characteristics of gravitational dynamics in the compact objects having density greater than the nuclear density in the \( f(R,T) \)- theory of gravity which is an extension of both GTR and \( f(R) \)-gravity.

3. Modified Field Equations in \( f(R,T) \) gravity

We consider a spherically symmetric metric for the interior spacetime of a static stellar configuration given by

\[
 ds^2 = e^{2\nu(r)} dt^2 - e^{2\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]  

(11)

where \( \nu \) and \( \lambda \) are the metric potentials which are functions of radial coordinate \( r \) only. The non-zero components of the energy momentum tensors are given by

\[
 T^{0}_{0} = \rho(r),
\]

(12)

\[
 T^{1}_{1} = -p_r(r),
\]

(13)

\[
 T^{2}_{2} = T^{3}_{3} = -p_t(r),
\]

(14)

where \( p_r \) and \( p_t \) are radial and tangential pressures respectively. Using eqs. (6) - (8), the field equations can be rewritten as

\[
 e^{-2\lambda} \left( \frac{2\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi p_r^{eff},
\]

(15)

\[
 e^{-2\lambda} \left( \nu'' + \nu'^2 + \frac{\nu' - \lambda'}{r} - \nu' \lambda' \right) = 8\pi p_t^{eff},
\]

(16)

\[
 e^{-2\lambda} \left( \frac{2\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi \rho^{eff},
\]

(17)

where the prime (') is differentiation w.r.t. radial coordinate, \( \rho^{eff}, p_r^{eff} \) and \( p_t^{eff} \) are the effective density, radial pressure and tangential pressure. We get

\[
 \rho^{eff} = \rho + \frac{\chi}{24\pi} (9\rho - p_r - 2p_t) \]

(18)

\[
 p_r^{eff} = p_r - \frac{\chi}{24\pi} (3\rho - 7p_r - 2p_t) \]

(19)

\[
 p_t^{eff} = p_t - \frac{\chi}{24\pi} (3\rho - p_r - 8p_t). \]

(20)

To study the matter content inside the compact objects, the field eqs.(15)-(17) are used to determine the components of \( T_{\mu\nu} \) i.e. \( \rho, p_r \) and \( p_t \). Using eqs.(15) and (16) we get a second order differential equation which is

\[
 \nu'' + \nu'^2 - \nu' \lambda' - \frac{\lambda'}{r} - \frac{1}{r^2} + \frac{e^{2\lambda}}{r^2} = 2(4\pi + \chi) \Delta \ e^{2\lambda} \]

(21)

where \( \Delta = p_t - p_r \), which represents the measure of anisotropy in pressure. In terms of effective pressures we get

\[
 p_r^{eff} - p_t^{eff} = (1 + \frac{\chi}{4\pi}) \ \Delta
\]
which is related to the anisotropy measure. It is evident that for isotropic pressure \( i.e. \, p_r = p_t \) one finds isotropy in the effective pressure. It is also noted that the effective pressure difference vanishes even if \( p_r \neq p_t \) when \( \chi = -4\pi \).

In this section we adopt following transformations first proposed by Durgapal and Bannerji \[37\] on the matric potentials to obtain relativistic solutions

\[
A^2 y^2(x) = e^{2\nu(x)}, \quad Z(x) = e^{2\lambda(x)}, \quad x = Cr^2.
\]

where \( A \) and \( C \) are arbitrary constants. The above transformation reduces eq. (21) to a second order differential equation which is given by

\[
4x^2 \ddot{y} + 2x^2 \dot{z} \dot{y} + y \left[ x \dot{z} - z + 1 - \frac{2(4\pi + \chi)}{C} x \Delta \right] = 0 \tag{22}
\]

where the overdot denotes differentiation \( w.r.t. \) the variable \( x \).

### 3.1 Exact Relativistic Solutions

The eq. (22) is further simplified introducing \( Z(x) \) \[38\] as

\[
Z = \frac{1}{1 + x}. \tag{23}
\]

Note that the choice of \( Z \) is a sufficient condition for a static perfect fluid sphere which is regular at the center \[39\]. Eq.(22) can be expressed as

\[
4(1 + x) \ddot{y} - 2 \dot{y} + (1 - \alpha) y = 0 \tag{24}
\]

where \( \alpha = \frac{2\Delta(x+1)^2(4\pi+\chi)}{Cx} \). The measure of anisotropy is given by

\[
\Delta = \frac{\alpha x C}{2(4\pi + \chi)(x+1)^2} \tag{25}
\]

for \( \chi \neq -4\pi \) and \( C \neq 0 \). For \( \alpha = 0 \), one recovers Finch-Skea model with an isotropic pressure distribution. For anisotropic star, \( \Delta \) vanishes at the center (\( i.e. p_r = p_t \)), but away from the centre it is a regular solution which grows showing different patterns of evolution for both the pressures. For \( -1 < \alpha < 1 \), we substitute the following: \( X = 1 + x \) and \( y(x) = Z \) for simplicity in eq.(24) which yields

\[
4X \frac{d^2Z}{dX^2} - 2 \frac{dZ}{dX} + (1 - \alpha) Z = 0. \tag{26}
\]

Once again we introduce the following transformations: \( Z = w(X)X^n \) and \( u = X^\gamma \), where \( \gamma \) and \( n \) are real numbers. The above differential equation can be reduced to a standard Bessel equation. For \( \gamma = \frac{1}{2} \) and \( n = \frac{3}{4} \), the eq. (26) reduces to

\[
u^2 \frac{d^2w}{du^2} + u \frac{dw}{du} + \left[ (1 - \alpha) u^2 - \frac{3}{2} \right] w = 0. \tag{27}
\]

Now we consider further transformation from \( u \) to \( v \) variable as \( (1 - \alpha)^2 u = v \) in the eq.(27) which leads to a second order differential equation as follows

\[
v^2 \frac{d^2w}{dv^2} + v \frac{dw}{dv} + \left[ v^2 - \left( \frac{3}{2} \right)^2 \right] w = 0. \tag{28}
\]
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which is the Bessel equation of the order $\frac{3}{2}$. The general solution is given by

$$w = c_1 J_{\frac{3}{2}}(v) + c_2 J_{-\frac{3}{2}}(v)$$

where $c_1$ and $c_2$ are integration constants, $J_{\frac{3}{2}}(v)$ and $J_{-\frac{3}{2}}(v)$ are the Bessel functions, which can be written in terms of trigonometric functions. The general solution of the eq.(24) for modified FS-metric in four dimension [39] is given by

$$y(x) = (1 - \alpha)^{\frac{3}{4}} [(b - a \sqrt{(1 + C r^2)(1 - \alpha))}$$

$$\cos \sqrt{(1 + C r^2)(1 - \alpha)} + (a + b \sqrt{(1 + C r^2)(1 - \alpha))}$$

$$\sin \sqrt{(1 + C r^2)(1 - \alpha)}]$$

where, $a = c_1 \sqrt{\frac{2}{\pi}}$ and $b = - c_2 \sqrt{\frac{2}{\pi}}$ are arbitrary constants of the metric. We consider the metric potential of the modified 4-dimensional FS-metric as

$$e^{2\lambda(r)} = 1 + C r^2,$$

$$e^{2\nu(r)} = (1 - \alpha)^\frac{3}{4} A^2 [(b - a \sqrt{(1 + C r^2)(1 - \alpha))}$$

$$\cos \sqrt{(1 + C r^2)(1 - \alpha)} + (a + b \sqrt{(1 + C r^2)(1 - \alpha))}$$

$$\sin \sqrt{(1 + C r^2)(1 - \alpha)}]^2$$

where $C$, $a$, $b$, $A$ and $\alpha$ are the five unknowns. For $\alpha = 0$, the Finch-Skea solution obtained in GR for 4-dimensions with isotropic fluid is recovered [27]. The relativistic solution for $-1 < \alpha < 1$ obtained here is regular in the interior of the star which can be matched smoothly with the Schwarzschild exterior solution at the boundary. It can be used to construct stellar models determining the metric parameters $a$, $b$ and $C$ for given values of $\alpha$ and $\chi$. It may be mentioned here that for $\alpha \geq 1$, the stellar models are not stable. Consequently, we consider $-1 < \alpha < 1$ in the $f(R, T)$-modified gravity to construct stellar models for compact objects.

4. Analysis for Stellar Models

The following conditions [33] are imposed on the relativistic solutions for a physically realistic stellar configurations for compact objects in the modified gravity:

- At the boundary of a static star (i.e. at $r = b$), the interior space-time is matched with the exterior Schwarzschild solution. For the continuity of the metric functions at the surface, one consider

$$e^{2\nu(r)}|_{r=b} = \left(1 - \frac{2M}{b}\right)$$

$$e^{2\lambda(r)}|_{r=b} = \left(1 - \frac{2M}{b}\right)^{-1}$$

- The radial pressure ($p_r$) drops from its maximum value (at the center) to vanishing value at the boundary, i.e., at $r = b$, $p_{(r=b)} = 0$, the radius of the star $b$ can be
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- The causality condition is satisfied when the speed of sound $v^2 = \frac{dp}{d\rho} \leq 1$ which is also a condition for stable stellar configuration [34].
- The gradient of the pressure and energy-density should be negative inside the stellar configuration, i.e., $\frac{dp}{d\rho} < 0$ and $\frac{d\rho}{d\rho} < 0$.
- At the center of the star, $\Delta(0) = 0$ which implies zero radial and tangential pressure, $p_r(0) = p_t(0)$.
- The anisotropic fluid sphere must satisfy the following three energy conditions, viz., (a) null energy condition (NEC), (b) weak energy condition (WEC) and (c) strong energy condition (SEC) if it is made up of normal fluid.
- The adiabatic index: $\Gamma = \frac{\rho + p}{p} \frac{dp}{d\rho} > \frac{4}{3}$ required for ensuring stability of the stellar configuration [40].

There are three field equations and five unknowns, to solve the equations two adhoc assumptions are necessary for obtaining exact solutions. Thus to construct stellar models, the unknown metric parameters $a$, $b$, $C$ for a given mass $(M)$ and radius $(r = b)$ of a star are to be determined from the boundary conditions making use of permissible values of $\alpha$ and $\chi$ for a realistic stellar model. Alternatively, for a given mass we can predict the radius of the compact objects for values of the other parameters.

5. Physical Properties of compact stars for $-1 < \alpha < 1$

The physical features of anisotropic compact objects are studied for $-1 < \alpha < 1$. As the relativistic solutions are highly complex we analyze numerically the variations of the energy density, radial pressure, transverse pressures, energy conditions, anisotropy of pressure and stability for a given value of the model parameters. The graphical plots are important for predicting the EoS of the observed compact objects. We consider uncharged anisotropic stellar objects.

5.1. Density and Pressure of a compact objects in $f(R, T)$ gravity

In the $f(R, T)$ - modified gravity we determine physical parameters, namely, energy density ($\rho$), radial pressure ($p_r$) and tangential pressure ($p_t$). The metric potentials $e^{2\lambda(r)}$ and $e^{2\nu(r)}$ given by eqs. (30) and (31) are employed in eqs. (15) - (17) to determine the energy density ($\rho$), radial pressure ($p_r$) and tangential pressure ($p_t$) which are given by

$$\rho = \frac{C\left(\sin(\sqrt{\mu_1})(\mu_2 + h_3 \sqrt{\mu_1}) + \cos(\sqrt{\mu_1})(h_5 - a h_6 \sqrt{\mu_1})\right)}{g(r,a,b,C,\chi)},$$

$$p_r = \frac{C\left(h_3 \cos(\sqrt{\mu_1}) - h_4 \sin(\sqrt{\mu_1})\right)}{g(r,a,b,C,\chi)},$$

$$p_t = \frac{C\left(h_5 \sin(\sqrt{\mu_1}) + h_6 \cos(\sqrt{\mu_1})\right)}{g(r,a,b,C,\chi)},$$

where the denominator is denoted as $g(r,a,b,C,\chi) = 12 (\chi^2 + 6\pi \chi + 8\pi^2) (Cr^2 + 1)^2 (\sin(\sqrt{\mu_1})(a + b \sqrt{\mu_1}) + \cos(\sqrt{\mu_1})(b - a \sqrt{\mu_1})).$
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Figure 1. Radial variation of energy-density (\( \rho \)) in PSR J0348+0432 for \( \chi = 1 \) (Red), \( \chi = 3 \) (Blue), \( \chi = 5 \) (Green), \( \chi = 7 \) (Purple) and \( \chi = 10 \) (Black) (considering \( \alpha = 0.5 \)).

Figure 2. Radial variation of radial pressure (\( p_r \)) in PSR J0348+0432 for \( \chi = 1 \) (Red), \( \chi = 3 \) (Blue), \( \chi = 5 \) (Green), \( \chi = 7 \) (Purple) and \( \chi = 10 \) (Black) for \( \alpha = 0.5 \).

\[
\begin{align*}
\mathbf{h}_0 &= \chi (Cr^2(\alpha + 3) + 12) + 12\pi (Cr^2 + 3), \\
\mathbf{h}_1 &= -(\alpha - 1)(Cr^2 + 1), \\
\mathbf{h}_2 &= \chi (-2Cr^2(\alpha - 3) - 3(\alpha - 5) + 12\pi (Cr^2 + 3), \\
\mathbf{h}_3 &= a\sqrt{\mathbf{h}_1} (Cr^2(\alpha + 3) + 12\pi (Cr^2 + 1)) + b \left( \chi (2Cr^2(3 - 5\alpha) - 9\alpha + 9) - 12\pi (2\alpha - 1) Cr^2 + 1) \right), \\
\mathbf{h}_4 &= a(\chi (2Cr^2(5\alpha - 3) + 9(\alpha - 1)) + 12\pi (2\alpha - 1)(Cr^2 + 1)) + b\sqrt{\mathbf{h}_1}(Cr^2(\alpha + 3) + 12\pi (Cr^2 + 1)), \\
\mathbf{h}_5 &= a(\chi (2Cr^2(3 - 2\alpha) - 9\alpha + 9) - 12\pi (Cr^2(\alpha - 1) + 2\alpha - 1)) + b\sqrt{\mathbf{h}_1}(Cr^2(5\alpha - 3) \chi + 12\pi (Cr^2(\alpha - 1) - 1)), \\
\mathbf{h}_6 &= a\sqrt{\mathbf{h}_1}(Cr^2(3 - 5\alpha) \chi - 12\pi (Cr^2(\alpha - 1) - 1)) + b(\chi (2Cr^2(3 - 2\alpha) - 9\alpha + 9) - 12\pi (Cr^2(\alpha - 1) + 2\alpha - 1)).
\end{align*}
\]

The radial variation of the energy density (\( \rho \)), radial pressure (\( p_r \)) and tangential pressure (\( p_t \)) are plotted for PSR J0348+0432 in Figs. (1), (2) and (4) respectively for \( \alpha = 0.5 \) with different \( \chi \). It is evident that the physical quantities are maximum at the origin which however, decrease monotonically away from the centre. As \( \chi \) is increased the values of the physical parameters decreases. Similarly, the radial variation of radial pressure (\( p_r \)) and tangential pressure (\( p_t \)) for different \( \alpha \) for \( \chi = 1 \) are plotted in Figs. (3) and (5) respectively. It is noted that as \( \alpha \) increases, the radial pressures and tangential pressure decreases which are positive and regular at the origin with maximum values. Thus the model is free from physical and mathematical singularities. It is also
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**Figure 3.** Radial variation of radial pressure ($p_r$) in PSR J0348+0432 for $\alpha = 0$ (Gray), $\alpha = 0.1$ (Yellow), $\alpha = 0.2$ (Pink), $\alpha = 0.3$ (Cyan), $\alpha = 0.4$ (Brown) and $\alpha = 0.5$ (Red) for $\chi = 1$

**Figure 4.** Radial variation of transverse pressure ($p_t$) in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$

evident that the radial variation of energy density gradient and radial pressure gradient for different values of $\chi$ are plotted in Figs. (6) and (7) respectively, which are found negative and it increases as $\chi$ is decreased for $\alpha = 0.5$.

### 5.2. Anisotropic Star

The anisotropy of a compact star which is determined by the difference of tangential and radial pressures is obtained from eqs. (35) and (36) as follows:

$$\Delta = p_t - p_r = \frac{C^2 r^2 \alpha}{2(\chi + 4\pi)(Cr^2 + 1)^2}.$$  \hspace{1cm} (37)

An isotropic stellar model can be obtained for $\alpha = 0$ in 4-dimensions, it is evident that in modified gravity it always permits anisotropic star unless $\chi = -4\pi$ which follows from eq. (37). In GTR, it is known that FS metric does not permit anisotropic compact star in a 4-dimensional geometry, but recently it is shown that a higher dimensional extension of the Finch-Skea geometry permits an anisotropic star \[32\]. As the structure of $f(R,T)$-gravity is interesting found that anisotropic star is always permitted in a
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4-dimensional FS metric. In Fig. (8), we plot the radial variation of $\Delta$ for different $\chi$ values with a given $\alpha$. It is found that for $\chi > 0$, $\Delta$ is positive $i.e.$, $p_t > p_r$ which in turn implies that the anisotropic stress is directed outwards, hence there exists a repulsive gravitational force that allows the formation of super massive stars.

The radial variation of $\Delta$ for different values of $\alpha$ in the range $(-1.0$ to $0.5)$ is drawn in Fig. (9) with $\chi = 1$. It is evident that when $\chi = 0$ it corresponds to isotropic star ($\alpha = 0$) in four dimensions but when $\alpha < 0$ one gets $\Delta < 0$ for positive values of $\chi$ with $|\chi| \neq 4\pi$. The anisotropy increases as $\alpha$ increases but it decreases if $\alpha$ is more negative. It is also noted that for negative values of $\alpha$ one gets $\Delta < 0$, where the radial pressure is greater than the tangential one, $i.e.$, $p_r > p_t$. We also note that when $\alpha = -0.5$, we get a situation where $\Delta < 0$ in the range $-1 > \chi > -3.5$ for PSR J0348+0432. Similarly, $\Delta < 0$ is recorded for $\alpha = -0.2$, in the range $-1 > \chi > -4.4$ and for $\alpha = -1.0$, in the range $-1 > \chi > -1.4$. Thus it is clear that this negative range of $\chi$ varies with negative $\alpha$ values which permits compact objects with $p_t > p_r$. However, our model is not allowed for negative $\chi$ values with positive $\alpha$. 

**Figure 5.** Radial variation of transverse pressure ($p_t$) in PSR J0348+0432 for $\alpha = 0$ (Gray), $\alpha = 0.1$ (Yellow), $\alpha = 0.2$ (Pink), $\alpha = 0.3$ (Cyan), $\alpha = 0.4$ (Brown) and $\alpha = 0.5$ (Red) for $\chi = 1$

**Figure 6.** Radial variation of energy-density gradient ($\frac{\rho}{\rho t}$) in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$
Figure 7. Radial variation of pressure gradient ($\frac{dp}{dr}$) in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$.

Figure 8. Radial variation of anisotropy parameter ($\Delta$) in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$.

Figure 9. Radial variation of anisotropy parameter ($\Delta$) in PSR J0348+0432 for $\alpha = -1.0$ (Red, DotDashed), $\alpha = -0.7$ (Brown, DotDashed), $\alpha = -0.5$ (Cyan, DotDashed), $\alpha = -0.3$ (Pink, DotDashed), $\alpha = -0.1$ (Yellow, DotDashed), $\alpha = 0$ (Gray), $\alpha = 0.1$ (Yellow), $\alpha = 0.2$ (Pink), $\alpha = 0.3$ (Cyan), $\alpha = 0.4$ (Brown) and $\alpha = 0.5$ (Red) for $\chi = 1$. 
5.3. Stability of the Stellar Model

5.3.1. **Herrera cracking concept** The stability of a stellar model is studied numerically plotting the radial variation of the square of the radial speed of sound \( v_r^2 = \frac{dp}{d\rho} \) and square of the transverse speed of sound \( v_t^2 = \frac{dp}{d\rho} \) separately in Figs. (10) and (11) respectively. It is found that a stable configuration of anisotropic compact object can be accommodated. Herrera and Abreu [41] pointed out that for a physically stable stellar system made of anisotropic fluid distribution the difference of square of the sound speeds should maintain its sign inside the stellar system. Accordingly, in a potentially stable region, square of the radial sound speed should be greater than the square of the tangential sound speeds. Hence, according to Herreras cracking conjecture the required condition \( |v_r^2 - v_t^2| \leq 1 \) is found to satisfy. We plot variation of \( |v_r^2 - v_t^2| \) w.r.t. \( r \) in Fig. (12) and it is found that the condition is found to satisfy \( |v_r^2 - v_t^2| \leq 1 \) for different values of \( \chi \) with \( \alpha = 0.5 \).

5.3.2. **Adiabatic index** The stiffness of the EoS for given energy density is characterised by adiabatic index which has significant importance for understanding the
stability of relativistic as well as non-relativistic compact objects. Chandrasekhar began
the study of the dynamical stability against infinitesimal radial adiabatic perturbation
of the stellar system. It is estimated that the magnitude of the adiabatic index should
be greater than $\frac{4}{3}$ in the interior of a dynamically stable stellar object. For anisotropic
fluid distribution the adiabatic index is given by,

$$\Gamma = \frac{\rho + p_r \frac{dp_r}{d\rho}}{p_r}.$$  \hfill (38)

The radial variation of the adiabatic index is plotted in Fig. (13) for different values of
$\chi$. The stellar models obtained here are found to have dynamical stability as $\Gamma \geq \frac{4}{3}$.
The stellar models are stable against infinitesimal radial adiabatic perturbations. In
Fig. (14) we plot the radial variation of adiabatic index ($\Gamma$) for different values of the
parameter $0 < \alpha < 0.5$ with $\chi = 1$. We note acceptable range $0 < \alpha < 0.3$ for $\chi = 0.5$
and $0 < \alpha < 0.5$ for $\chi = 1, 1.5, 2$. Thus on increasing $\chi$ the acceptable range of $\alpha$
remains same for anisotropic star. Thus in $f(R,T)$ modified gravity we get an upper
bound on $\alpha$ for $\chi > 0$.

5.4. **Energy conditions of the stellar model in the $f(R,T)$ gravity**

The energy conditions play a crucial role in determining the observe normal or exotic
nature of matter inside the stellar model. The energy conditions are null (NEC),
dominant (DEC), strong (SEC) and weak energy conditions (WEC). in an anisotropic
fluid distribution are expressed as follows:

$$NEC: \rho \geq 0,$$
$$WEC1: \rho + p_r \geq 0, \quad WEC2: \rho + p_t \geq 0,$$
$$SEC: \rho + p_r + 2p_t \geq 0,$$
$$DEC1: \rho - p_r \geq 0, \quad DEC2: \rho - p_t \geq 0.$$  \hfill (42)
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Figure 13. Radial variation of $\Gamma$ in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) (considering $\alpha = 0.5$)

Figure 14. Radial variation of $\Gamma$ in PSR J0348+0432 for $\alpha = 0.1$ (Yellow), $\alpha = 0.2$ (Pink), $\alpha = 0.3$ (Cyan), $\alpha = 0.4$ (Brown) and $\alpha = 0.5$ (Red) (considering $\chi = 1$)

The evolution of all the energy conditions against the radial coordinate $r$ for the compact stellar structure is studied here for different $\chi$ with $\alpha = 0.1$ in $f(R,T)$-gravity. These are shown graphically in the Figs. (15)- (19).

5.5. Stellar Mass - Radius Relation

For a static spherically symmetric stellar models with anisotropic fluid Buchdahl found a limit on the mass to radius ratio, i.e. $\frac{2M}{R} < \frac{8}{9}$ [42]. In this section we analyze graphical behaviour of the mass- radius relation for different values of the parameters. The effective mass is given by

$$m(r) = \int_0^r 4\pi r'^2 \rho dr'.$$

We consider PSR J0348+0432 with observed mass equal to $M = 2.01 \pm 0.04 \, M_\odot$. Now plotting the observed mass in the mass-radius curve in Fig.(20), it is found that one can predict the variation of the size of a compact object for different $\chi$ values. In Fig.(20) it is shown that for a given mass of known object, the radius increases for the increasing values of $\chi$, thus the compactness factor of the star decreases. Thus, we can state that
for lower values of $\chi$ we can find more dense object comparatively. The mass function is regular at the center of the compact stellar structure. As it is not yet measured the radius of a star accurately, many aspects of a compact object may be understood once the mass and radius are determined accurately.

5.6. Class of Stellar Models with EoS

The different physical parameters $a$, $b$, $C$ of Finch-Skea metric given by eqs. (35) and (36) are determined using the boundary conditions, satisfying the criterion for a physically realistic stellar object. We tabulated different metric parameters in Tables-1 and 2 for PSR J0348 + 0432 which admits different class of stellar model in $f(R, T)$ gravity. The value of $C$ is calculated for a particular stellar object which is independent of $\chi$ and $\alpha$. Considering $C = 0.009664$, we determined the parameters for PSR J0348 + 0432 whose observed mass $M = 2.01 \pm 0.04 M_\odot$ and radius, $R = 11$ km. In Table-1, values of $a$ and $b$ for different $\chi$ at $\alpha = 0.5$ and in Table-2, the variations of $a$ and $b$ are displayed for different $\alpha$ taking $\chi = 1$. We also tabulated parameters for different known sources namely, Vela X-1, 4U 1820-30, Cen X-3, LMC X-4, SMC X-1 with their precise estimated mass. For $\chi = 1$ and $\alpha = 0.5$ we estimated the permissible radii. It may be pointed out here that if the values of the parameters $\chi$ and $\alpha$ are taken different then for a given mass one estimates the radii which is different from the estimated value in the Table-2. As the radius of a star can not be measured precisely, we can predict the radius in the models. The predicted radii in the modified gravity with FS-geometry permits very compact objects namely, neutron stars, strange stars.

5.7. Equation of State

The variation of the energy-density and radial pressure are plotted in Figs. (1) and (2) from which we determine functional form by best fitting the curve. Here we determine the best fit relation between $\rho$ and $p_r$, the expressions so obtained for different $\chi$ values have been listed in Table-1 for a given $\alpha$. In Fig.(21) we plot $p_r$ vrs. $\rho$ for PSR J0348+0432 with different $\chi$ for a given $\alpha$. It is found that EoS for PSR J0348+0432 is non-linear for $\chi = 1$ and $\chi = 3$, but linearity develops in Fig. (21) as the values of $\chi$ increases. It is shown that quadratic fitting of the EoS curve is better than a linear one for lower value of $\chi$. Thus the MIT Bag model representing the EoS in a compact star is not suitable in a compact object with FS geometry, it predicts a non-linear EOS.

6. Discussion

In the paper we obtain a class of relativistic solutions for compact objects in hydrostatic equilibrium in a modified gravity $f(R, T) = R + 2\chi T$. Since the field equations are highly complex we adopt a technique to project the field equation in a second order differential equation. The anisotropic stellar models are constructed here. We analyze
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Figure 15.

WEC 1

Figure 16.

WEC 2

Figure 17.

DEC 1

Figure 18.

DEC 2

Figure 19.

SEC

Figure 20. Mass - Radius relation in PSR J0348+0432 for $\chi = -3$ (Blue Dashed) and $\chi = -1$ (Red Dashed) taking $\alpha = -0.5$, $\chi = 0$ (Brown) $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$
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$$\chi = 1$$ $b = 0.21193$, $a = 0.31475$

$$p_r = -0.0000627 + 0.2174\rho - 38.72\rho^2$$

$$\chi = 3$$ $b = 0.22385$, $a = 0.26512$

$$p_r = -0.0000526 + 0.2139\rho - 24.96\rho^2$$

$$\chi = 5$$ $b = 0.23221$, $a = 0.23033$

$$p_r = -0.00004291 + 0.20162\rho$$

$$\chi = 7$$ $b = 0.23840$, $a = 0.28104$

$$p_r = -0.00004107 + 0.21631\rho$$

$$\chi = 10$$ $b = 0.24515$, $a = 0.17649$

$$p_r = -0.00003744 + 0.23221\rho$$

Table 1. Physical parameters for PSR J0348+0432 in $f(R, T)$ gravity for $\alpha = 0.5$.

| $\chi$ | $b$  | $a$  | EoS                    |
|--------|------|------|------------------------|
| 1      | 0.21193 | 0.31475 | $p_r = -0.0000627 + 0.2174\rho - 38.72\rho^2$ |
| 3      | 0.22385 | 0.26512 | $p_r = -0.0000526 + 0.2139\rho - 24.96\rho^2$ |
| 5      | 0.23221 | 0.23033 | $p_r = -0.00004291 + 0.20162\rho$ |
| 7      | 0.23840 | 0.28104 | $p_r = -0.00004107 + 0.21631\rho$ |
| 10     | 0.24515 | 0.17649 | $p_r = -0.00003744 + 0.23221\rho$ |

Table 2. Anisotropy and Metric parameters for PSR J0348+0432 in modified gravity for $\chi = 1$.

| $\alpha$ | $b$  | $a$  | $\alpha$ | $b$  | $a$  |
|----------|------|------|----------|------|------|
| 0.1      | 0.26914 | 0.28295 | -0.1     | 0.28035 | 0.30121 |
| 0.2      | 0.25967 | 0.27915 | -0.3     | 0.28273 | 0.32754 |
| 0.3      | 0.24723 | 0.28104 | -0.5     | 0.27767 | 0.35765 |
| 0.4      | 0.23148 | 0.29136 | -0.7     | 0.26633 | 0.38909 |
| 0.5      | 0.21193 | 0.31475 | -1.0     | 0.23950 | 0.43558 |

Table 3. Numerical values of physical parameters for different compact object for $\alpha = 0.5$ and $\chi = 1$ where $b$ represents the predicted radius of the pulsars.

| Stars    | Mass ($M_\odot$) | $b$  | $a$  | $C$    | $b$ (km.) |
|----------|------------------|------|------|--------|----------|
| Vela X-1 | $1.77 \pm 0.08$  | 0.24665 | 0.31616 | 0.00783 | 10.88    |
| 4U 1820-30 | $1.58 \pm 0.06$ | 0.26769 | 0.31601 | 0.00716 | 10.52    |
| Cen X-3  | $1.49 \pm 0.08$  | 0.27895 | 0.31616 | 0.00783 | 10.88    |
| LMC X-4  | $1.29 \pm 0.05$  | 0.29876 | 0.31439 | 0.00631 | 9.926    |
| SMC X-1  | $1.04 \pm 0.09$  | 0.3250  | 0.31172 | 0.00569 | 9.301    |

Figure 21. EoS in PSR J0348+0432 for $\chi = 1$ (Red), $\chi = 3$ (Blue), $\chi = 5$ (Green), $\chi = 7$ (Purple) and $\chi = 10$ (Black) for $\alpha = 0.5$.
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the stellar models numerically and predicted the EoS of matter inside the compact objects assuming a modified Finch-Skea metric. For $\chi = 0$ it corresponds to GR and it represents isotropic stellar configuration [27]. In the modified gravity it is found that stellar models represent anisotropic uncharged compact objects always unless $\chi = -4\pi$. It is also found that realistic stellar models are permitted for a given range of values of anisotropy $-1 < \alpha < +0.5$ which are stable. We study the physical features of the compact stars of known observed mass. As the EoS in a compact object at extreme condition of density is not yet known we adopted a different technique, considering a modified Finch-Skea ansatz in the framework of modified gravity the EoS is predicted with different values of the model parameters. For a known stars its mass is precisely known but not its radius, consequently the parameters of the model are determined analytically to find the probable radius and the EoS of the matter inside the compact objects. A precise measurement of radius will be helpful for

(i) The radial variation of energy density, radial pressure and tangential pressure plotted in Figs. (1) - (4) show that they are maximum at the origin which however decrease away from the centre. The radius of the star is determined from the condition that the radial pressure vanishes at the surface. The coupling parameter $\chi$ in the gravitational action is playing an important role to accommodate anisotropic compact objects. We note that both the central density and pressure decreases as $\chi$ is increased. We note that as $\chi$ is decreased it accommodates a more dense star.

(ii) There is no physical and mathematical singularities as the radial variation of the radial pressure ($p_r$) and tangential pressure ($p_t$) shown in Fig. (2) - (5) are positive and regular at the origin.

(iii) An isotropic stellar configuration is obtained for $\alpha = 0$ in eq. (31). For PSR J0348+0432, the radial variation of $\Delta$ for different $\chi$ in Fig.(8) shows that $\Delta > 0$ i.e. $p_t > p_r$ for $\chi > 0$ which implies that the anisotropic stress is directed outwards. There exists a repulsive gravitational force that allows the formation of super massive star in this case.

(iv) For $\chi = 1$ the plot of radial variation of $\Delta$ for different values of $\alpha$ in Fig. (9) shows that it admits stellar models with $p_r > p_t$ indicating the formation of ultra compact objects. It is also noted that the range $0.5 < \alpha < 1$ is not suitable as no stable configuration allowed. In $f(R, T)$ gravity, the anisotropy is small near the centre which however attains maximum value at the surface. For PSR J0348 + 0432 we determine the values of $\chi$ for which $\Delta < 0$. It is found that (i) $\alpha = -1.0$ in the range $-1.4 < \chi < -1$, (ii) $\alpha = -0.5$ in the range $-3.5 < \chi < -1$, (iii) $\alpha = -0.2$, in the range $-4.4 < \chi < -1$, thus as $\alpha$ is decreased the lower value of $\chi$ is increased.

(v) The radial variation of the adiabatic index $\Gamma$ plotted in Fig.(13) shows that the
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stellar models are stable as it satisfies the Buchdhal limit $\chi > \frac{4}{3}$. A class of relativistic solutions are obtained here for anisotropy lying in the range $0 < \alpha < 0.5$ which permits stable stellar models evident from the Fig.(14). The $f(R,T)$-gravity with modified FS-metric ansatz permits anisotropic star in four dimensions, it is different from that of GTR result where it accommodates stars with isotropic pressure. We obtain upper bounds on anisotropy $\alpha$ for different $\chi$ for anisotropic stars.

(vi) All the energy conditions, *viz.*, (a) Null energy condition (NEC), (b) Weak energy condition (WEC) and (c) Strong energy condition (SEC) drawn in Figs. (15) - (19) are satisfied. Thus no exotic matter required for building stellar models.

(vii) The mass-radius relation of PSR J0348+0432 plotted in Fig.(20) for different values of $\chi$ shows that for a given mass of the compact object, the radius increases for an increasing value of $\chi$. Thus for lower $\chi$, the models accommodates very compact object as the compactness factor $\left(\frac{M}{b}\right)$ increases (where $b$ is the radius of a star).

(viii) We constructed stellar models for PSR J0348 + 0432 without pre-assuming EoS. Instead we assume modified FS metric ansatz to determine EoS with the metric coefficients $a$, $b$ and gravitational coupling parameter $\chi$ for an anisotropic configuration with $\alpha = 0.5$. The probable EoS are tabulated in Table-1, it is evident that both linear and quadratic EoS are obtained. The numerical fitting of the pressure and density curves show that the goodness of fit for the quadratic fitting is better than that of the linear one for lower values of $\chi$. However, the linear EoS obtained here are different from that corresponds to MIT bag model [20, 21]. The EoS for matter interior to a compact star in modified gravity is predicted here which is non-linear for massive star. In Table-2, we displayed $a$ and $b$ for different anisotropy ($\alpha$) with $\chi = 1$ for a stable stellar configuration. The anisotropy lies in the range $-1.0 < \alpha < 0.5$ for $\chi = 1$ in a stable stellar model.

(ix) For observed masses of the pulsars namely, *Vela X-1, 4U 1820-30, Cen X-3, LMC X-4, SMC X-1*, we determine the values of $a$, $b$ $C$ with $\chi = 1$ stable anisotropic models are shown for anisotropy $\alpha = 0.5$. A class of relativistic solutions are obtained for different anisotropic pressure inside the star. The predicted radius of the above pulsars for the parameters are displayed in Table-3. However, varying the values of $a$, $b$ $C$, it is possible to obtain stable anisotropic stellar models with different coupling parameter and anisotropy. It is possible to estimate the corresponding radius which lies in the range $(10 \sim 14)$ km. for a stable neutron star.

Thus a class of new relativistic solutions are found in $f(R,T)$ gravity with FS ansatz which are useful for building stellar models. The precise measurement of radius of a neutron star in future will be useful to accept the modification incorporated in the gravitational action for building stellar models which can dig out information on the matter inside the star at extreme terrestrial condition. It is found that the EoS for a compact object with modified FS ansatz is non-linear.
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