An algebraic metric for parametric stability analysis of power systems

Lewis Roberts, Student Member, IEEE, Alan Champneys, Keith Bell, Member, IEEE, Mario di Bernardo, Fellow, IEEE.

Abstract—An algebraic expression for the critical clearing time (CCT) metric is derived from direct methods for power system stability. Under the typical assumptions of direct methods, our algebraic expression is a strict lower bound to the true CCT. The formula has been designed to incorporate as many features of fault stability analysis as possible such as different fault locations and different post-fault network states. We demonstrate the performance of this metric on the so-called two machine infinite bus network by varying parameters using numerical continuation in the reduced bus matrix and the full bus matrix (before Kron reduction). Our metric is compared to two other expressions of the CCT which incorporate additional non-linearities in the model.

Index Terms—power system stability, stability metrics, swing equation, bifurcation theory, critical clearing time

I. INTRODUCTION

The complex dynamics of electric power systems have long been the subject of intense research particularly in the area of stability. Effective stability metrics provide control inputs and assist the system operator to ensure that a power system maintains synchrony after the network suffers a fault. A traditional transient stability metric for short circuit faults on a power network is the so-called critical clearing time (CCT) [1], [2]. The CCT provides an upper bound on the duration of a short circuit on a power network before it is removed - 'cleared' - by the action of protection mechanisms to isolate the faulted circuit such that the system will regain synchronisation once the fault is cleared. It provides a useful stability metric for power system design, however it is generally computed using numerical integration methods which have often been regarded as too slow for the metric to be useful in real time system analysis.

Currently, there are practical developments in power systems that promise to radically change power system dynamic behaviour. For example, the gradual substitution of power generated from large, synchronous machines by asynchronous machines or power fed via power electronic interfaces (e.g. wind farms, solar PV and HVDC interconnections to other systems), in addition to the changing nature of electrical loads [3]. As a consequence, there is value in articulating metrics that exploit theoretical, if simplified, descriptions of the system and can provide a deep understanding of the impact of a wide range of features of the network from parametric investigations. This can inform efforts to design strategies to mitigate possible instabilities in the system.

In the recent literature, alternative methodologies have been used to study stability when modelling a power system using the so-called swing equations [1], [4]. These include synchronisation [5], non-linear dynamics [6], bifurcation theory [7], passivity-based methods [8] and the computation of basins of attraction [9]. Direct methods [10] use swing equations in an energetic framework to provide a critical energy boundary for the whole system during a fault. Despite direct methods giving a conservative stability metric, their advantages include relatively quick computation, an analytical stability boundary, no need for further simplifications of a power system beyond the swing equation model and they can be applied to any system that can be parametrised. The system operator can use this metric for initial safety checks and to assess the stability margins of the system once a fault has been cleared.

One of the drawbacks of the direct methods is that it is difficult to predict when the system energy will cross the critical energy boundary because of the highly non-linear nature of the system dynamics. So-called fault trajectory sensitivity techniques [11]–[13] have been proposed to consider the effect of parameters on stability by linearising about the trajectory of a fault in state space with respect to a given parameter. Furthermore, a method for computing a so-called “direct CCT” has been proposed [14] which linearises about a specific fault trajectory with respect to the system energy itself. An estimate of the CCT under the perturbed system energy is found by extrapolation. However, to our knowledge, an algebraic CCT metric is not available. Our algebraic expression is derived by recasting the energetic metric used in the direct methods in terms of a metric in time by simplifying the energy functions and the fault dynamics. For systems that can be modelled using direct methods, our metric serves as a strict lower bound to the true CCT and can be applied to systems suffering a large fault at any location on a network of arbitrary size.

In general, a power network’s topology changes from its original structure when a fault is cleared. This is generally due to some switching action that isolates the region of the network that suffers the fault. Choosing the best strategy to quickly identify a need for and carry out this action is a crucial step in maintaining the stability and synchronisation of a power system. There is some uncertainty regarding the success and speed of protection actions, and, as a consequence, power flows may need to be restricted and more expensive, or higher carbon, power sources utilised. We argue that choices...
both in operation and the design of the system and its controls can be facilitated by parameter investigations of power system stability models such as the swing equation and applying quick but effective stability metrics to illustrate the effect of a given parameter value change. A rigorous study of the strategies available to the system operator could be provided in-part by the continuous variation of model parameters, which could possibly uncover optimal parameter values to maximise stability at the design stage or on-line.

The rest of this paper is organised as follows: In Section II we formulate the estimated CCT metric using the direct stability method and introduce the example two-machine infinite bus (TMIB) power network. By considering a lower bound on the estimated CCT we derive our analytical lower bound stability metric for an arbitrarily large network in Section III. The methodology for parameter investigations on the swing equations is presented in Section IV by considering a simplified TMIB network and the effect of varying maximum power flow capacity between generators. The results from a more tangible parameter study on a more realistic network is presented in Section V and finally, Section VI draws conclusions and suggests further work.

II. FAULT ANALYSIS USING ENERGY FUNCTIONS

A. Model description

We consider the classic swing equation model [1], [2] to describe the stability effects of transient faults on a power system with synchronous generation. The synchronous generators are modelled as transient reactances behind constant voltage sources and the loads are of constant impedance. This model can be written as a set of coupled one-dimensional ordinary differential equations (ODE), which describe the dynamics of the rotor angle for each synchronous generator

\[ \dot{\delta}_i = \omega_i \]

\[ \dot{\omega}_i = \frac{1}{M_i} (P_{mi} - P_{ei}(\delta) - P_{di}) \],

where \( \delta_i \) is the rotor angle of generator \( i \), \( \delta = [\delta_1, \ldots, \delta_n] \) is a vector of all rotor angles, \( M_i = \frac{2H_i}{\omega_0} \) is a lumped parameter, \( \omega_0 \) is the deviation of rotor angular speed relative to the system angular frequency \( \omega_0 = 2\pi f \) (where \( f \) is the grid frequency: 50 Hz in Europe), \( P_{mi} \) is the mechanical power input, \( P_{ei}(\delta) \) is the electrical power output, \( P_{di} \) is the power lost due to damping and \( H_i \) is the energy due to inertia.

The total electric power leaving generator \( i \) on the network is given by

\[ P_{ei}(\delta) = P_i(\delta) + \sum_{k \neq i} \bar{P}_{ik} \sin(\delta_i - \delta_k) \]

where \( \bar{P}_{ik} = |E_i||E_k|B_{ik} \) is the maximum active power flow between the generators \( i \) and \( k \), \( E_i = |E_i|e^{j\delta_i} \) is the internal voltage of generator \( i \) and \( B_{ik} \) is the susceptance of the network connection between node \( i \) and node \( k \). In order to apply Lyapunov direct methods to multi-machine systems the power dissipated at each generator \( i \)

\[ P_i(\delta) = E_i^2 G_{ii} + \sum_{k \neq i} |E_i||E_k|G_{ik} \cos(\delta_i - \delta_k) \]

is assumed to be a constant \( P_{ai} = P_i(\delta^*) \) (see Ref. [2]) where the superscript ’s’ denotes a stable equilibrium point; \( G_{ik} \) is the conductance between generators \( i \) and \( k \) and \( G_{ii} \) is the shunt conductance at bus \( i \). For a balanced network without an infinite bus [4] we have \( \sum_{i=1}^n P_{mi} - P_{ai} = 0 \).

The admittances \( Y_{ik} = G_{ik} + jB_{ik} \) are the elements of the symmetric reduced bus admittance matrix \( Y_{\text{red}} \in \mathbb{C}^{n \times n} \) where \( j^2 = -1 \). Kron reduction [15] is used to construct \( Y_{\text{red}} \) from a larger bus admittance matrix \( Y_{\text{BUS}} \in \mathbb{C}^{N \times N} \) (where \( N > 2n \)) which contains the full topology and load distribution (including the synchronous reactance) of a power network with \( n \) synchronous generators.

The power loss due to damping is modelled as in [1] by

\[ P_{di} = D_i \omega_i \]

where \( D_i \) is a constant of proportionality.

B. Fault analysis

1) Basic Definitions: We define a 2n-dimensional state space \( x = [\delta_1, \ldots, \delta_n, \omega_1, \ldots, \omega_n]^T = [\delta; \omega]^T \), where \( \delta = [\delta_1, \ldots, \delta_n] \), \( \omega = [\omega_1, \ldots, \omega_n] \) and \( n \) is the number of generators. The set of 2n one-dimensional ODEs for a power system, consisting of \( n \)-coupled equations of the form (1) for each generator is recast as

\[ \dot{x} = F(x, \lambda) \]

with solution

\[ x(t) = \Phi(t, \lambda; x(0)) \]

where

\[ \lambda = [P_{m1}, E_1, Y_{11}, \ldots, Y_{1,n-1}, D_1, H_1; \ldots; P_{mk}, E_k, Y_{kk}, \ldots, Y_{k,n-1}, D_k, H_k; \ldots; P_{mn}, E_n, Y_{nn}, H_n, f]^T \]

is a vector of all the system parameters where it is assumed \( Y_{ik} = Y_{ki} \). A stationary solution to (4) is denoted \( x^* \) and is such that \( F(x^*, \lambda) = 0 \).

2) Stability analysis of transient faults: The objective of fault analysis is to ensure that once a fault is cleared, the system will remain asymptotically stable and ideally will not require any further action from the system operator. To study this, we split the dynamics of the power system into three regimes: A swing equation of the form (1) models the dynamics before the fault occurs (pre-fault), during the fault (fault-on) and after the fault is cleared (post-fault). We will denote by \( \lambda_{\text{pre}} \), \( \lambda_{\text{on}} \) and \( \lambda_{\text{post}} \) the parameter values before, during and after the fault respectively (generically, we assume \( \lambda_{\text{pre}} \neq \lambda_{\text{on}} \neq \lambda_{\text{post}} \)).

In the context of dynamical systems, the chronology of fault dynamics is as follows: Pre-fault, the system is assumed to be balanced (i.e. power consumed = power supplied) and \( x^* \) is located at a stable equilibrium point \( x_{\text{pre}}^* = [\delta = \delta_{\text{pre}}^*; \omega = 0] \) where \( \delta_{\text{pre}}^* \in (-\pi/2, \pi/2)^n \) is a function of the parameter values \( \lambda_{\text{pre}} \). Say that at time \( t = 0 \), the power system suffers
a fault and the system evolves in state-space according to the 

\[ \dot{x}_{\text{on}}(t) = \Phi(t, \lambda_{\text{on}}; x_{\text{pre}}), \quad 0 < t \leq t_{\text{cl}}, \] 

where fault is cleared at \( t = t_{\text{cl}} \) and the post-fault system is engaged. The subsequent dynamics are 

\[ x_{\text{post}}(t) = \Phi(t, \lambda_{\text{post}}; x_{\text{on}}(t_{\text{cl}})), \quad t > t_{\text{cl}}, \]

where \( x_{\text{on}}(t_{\text{cl}}) = [\delta(t_{\text{cl}}); \omega(t_{\text{cl}})]^T \).

The primary aim of fault stability analysis is to ensure that the system returns to an asymptotically stable equilibrium point after the fault is cleared, i.e., for \( t > t_{\text{cl}} \). Given the parameter sets \( \lambda_{\text{pre}}, \lambda_{\text{fault}} \) and \( \lambda_{\text{post}} \), the time \( t_{\text{CCT}} \) is defined as the maximum time that the fault can be on-line before it is cleared at \( t_{\text{cl}} = t_{\text{CCT}} \) such that the system remains asymptotically stable in the post-fault regime. Formally, it is defined as the point at which the fault trajectory \( \{\delta(t); \omega(t)\} \) traverses the boundary of the region of attraction for the SEP of the system. This boundary is formed by the union of the stable manifolds of all of the UEPS in the system \( [10] \) and this definition of \( t_{\text{CCT}} \) is illustrated in Fig. 3. A critical clearing time metric is used to quantify both the absolute stability of a power system and to explore optimum values for the post-fault system parameters \( \lambda_{\text{post}} \) in order to maximise system stability given \( \lambda_{\text{pre}} \) and \( \lambda_{\text{fault}} \). It can be assumed that, in practice, only a few of the values in \( \lambda_{\text{post}} \) will differ from those in \( \lambda_{\text{pre}} \).

3) An energetic approach: If the dissipated power in a system can be assumed constant, the dynamics of a power network with zero damping at each generator \( (D_i = 0) \) can be exactly described using a Hamiltonian system with the function 

\[ H(x) = E_{\text{kin}}(\omega) + E_{\text{pot}}(\delta). \]

This Hamiltonian function quantifies the system energy and is the sum of the kinetic energy \( E_{\text{kin}}(\omega) \) and the potential energy \( E_{\text{pot}}(\delta) \) for a power system with \( n \) generators where 

\[ E_{\text{kin}}(\omega) = \sum_{i=1}^{n} \frac{1}{2} M_i \omega_i^2, \]

and

\[ E_{\text{pot}}(\delta) = -\sum_{i=1}^{n} (P_{mi} - P_{ai}) \delta_i - \sum_{i=1}^{n} \sum_{k>i}^{n} \bar{P}_{ik} \cos(\delta_i - \delta_k). \]

It can be used to construct an expression for the change in the system energy during fault conditions. Once a fault has been cleared, the total change to the system energy is 

\[ \Delta H_{\text{fault}}(t_{\text{cl}}) = H(x_{\text{on}}(t_{\text{cl}})) - H(x_{\text{pre}}), \]

where \( H \) is the Hamiltonian for the post-fault system (i.e. \( \lambda = \lambda_{\text{post}} \)).

The maximum change in energy \( \Delta H_{\text{max}} = \Delta H_{\text{fault}}(t_{\text{cl}} = t_{\text{CCT}}) \) that a power system can withstand without losing stability is particular to the specific fault that occurs on the system. However, a conservative estimate for the maximum energy \( \Delta H_{\text{max}} \) that a fault can contribute to the post-fault system can be found algorithmically using direct methods \([10]\). In this paper we choose the closest unstable equilibrium point (UEP) method \([10]\) to provide this estimate because, while a more conservative measure, it provides an energy bound for a wider range of faults on the system. It is defined as

\[ \Delta H_{\text{max}} = E_c - H(x_{\text{pre}}), \]

where

\[ E_c = E_{\text{pot}}(\delta^u) = \min\{E_{\text{pot}}(\delta^u_1), \ldots, E_{\text{pot}}(\delta^u_m)\}. \]

The point \( \delta^u \) is the so-called closest UEP where

\[ S = \{\delta^u_1, \ldots, \delta^u_m\}, \quad \omega_i = 0 \text{ for all } i = 1, \ldots, m, \]

is the set of all unstable equilibria of \((4)\) where \( \lambda = \lambda_{\text{post}} \).

An estimate for the critical clearing time \( t_{\text{CCT}} \) can be found by equating \((10)\) and \((11)\) giving

\[ \dot{H}(x_{\text{on}}(t)) = E_c, \]

and then solving for \( t \). The solution, which we denote \( t_{\text{CCT}} \), is rarely analytic and can be computed using numerical integration methods if the fault trajectory \((6)\) is known. For networks where direct methods can be applied \( t_{\text{CCT}} \leq t_{\text{CCT}} \) where equality is only valid for lossless networks.

C. An example network

The so-called two machine infinite bus (TMIB) system is used for the parameter stability investigations in Section IV. The ODE in the form \((4)\) for this system is given by

\[ \begin{align*}
\dot{\delta}_1 &= \omega_1 \\
\dot{\delta}_2 &= \omega_2 \\
\dot{\omega}_1 &= \frac{1}{M_1} \left[ (P_{m1} - P_{a1}) - \bar{P}_{13} \sin(\delta_1) - \bar{P}_{12} \sin(\delta_1 - \delta_2) \right] \\
\dot{\omega}_2 &= \frac{1}{M_2} \left[ (P_{m2} - P_{a2}) - \bar{P}_{23} \sin(\delta_2) - \bar{P}_{12} \sin(\delta_2 - \delta_1) \right]
\end{align*} \]

where damping has been neglected. The power flow terms in the equation are illustrated in the schematic of the reduced network for a TMIB system in Fig. 1. After Kron reduction there are three interconnected nodes in the network: two nodes
connected to synchronous generators of the form (1) and an infinite bus at bus 3. The infinite bus models the rest of a larger network by setting its internal voltage $E_3$ and rotor angle $\delta_3$ to be constant in time. Without loss of generality we can set $\delta_3 = 0$ and use it as a reference point for the other two rotor angles.

The energetic approach to fault analysis is visualised in Fig. 2 for a lossless TMIB system. The expressions for kinetic and potential energy in the Hamiltonian function (7) for this system are

$$E_{\text{kin}}(\omega_1, \omega_2) = \frac{1}{2} M_1 \omega_1^2 + \frac{1}{2} M_2 \omega_2^2,$$

$$E_{\text{pot}}(\delta_1, \delta_2) = -(P_{m1} - P_{a1}) \delta_1 - (P_{m2} - P_{a2}) \delta_2 - P_{13} \cos(\delta_1) - P_{23} \cos(\delta_2) - P_{12} \cos(\delta_1 - \delta_2).$$

where (16) is plotted as a surface in 3-dimensions in Fig. 2. The critical energy change estimate $\Delta \tilde{H}_{\text{max}}$ is given exactly by the difference in energy between the level sets $E_{\text{pot}}(\delta_1^1, \delta_2^1) = E_{\text{pot}}(\delta_{\text{pre}})$ and $E_{\text{pot}}(\delta_1^2, \delta_2^2) = E_c = E_{\text{pot}}(\delta_1^1)$ indicated in the figure where $\delta_1^1$ is the closest UEP.

III. AN ALGEBRAIC STABILITY METRIC

We present and derive here an algebraic stability metric, denoted with a slight abuse of notation as $\tilde{t}_{\text{CCT}}$ which is purely a function of the network parameters $\lambda_{\text{pre}}, \lambda_{\text{post}}$ on an arbitrarily large system. This metric is formulated by considering (14) and replacing the energetic Hamiltonian in (7) with a new function which is strictly greater than the Hamiltonian. In addition, the dynamics of the rotor angles during the fault are approximated as constant but different accelerations. These steps were taken such that the algebraic metric satisfies the inequality

$$\tilde{t}_{\text{CCT}} < t_{\text{CCT}} \leq t_{\text{CCT}},$$

for systems that can be modelled using direct methods. The relationship between these three critical time metrics is illustrated in Fig. 3. Although these approximations may seem cumbersome, and will detract significantly from the true dynamics of the system, we remind the reader that this metric is designed to provide an instant illustration of the stability of a power system. In addition, in the proceeding two sections we demonstrate how an algebraic metric can be used to conduct quick parametric enquiries of the stability of power systems modelled using the multi-machine swing equations.

An analytical lower bound on $\tilde{t}_{\text{CCT}}$ which experiences a three-phase to ground fault at a given bus (on a balanced system such that it can be modelled by means of a single phase equivalent) can be found by altering the expression (14) such that we solve

$$h_{\text{alt}}(t) = E_c,$$

where $h(t) \equiv \mathcal{H}(x_{\text{on}}(t))$ and

$$h_{\text{alt}}(t) \geq h(t),$$

is a polynomial function where $h_{\text{alt}}(0) = h(0)$.

The Hamiltonian of the system during the fault is written as

$$\mathcal{H}_{\text{on}}(x_{\text{on}}(t)) = \mathcal{H}_{\text{on}}(x_{\text{pre}}),$$

where the subscript ‘on’ indicates that the parameters $\lambda_{\text{on}}$ have been used for both the Hamiltonian function and the fault dynamics. In accordance with conservative systems, this function is a constant in time and this property is used to recast the expression for $\mathcal{H}(x_{\text{on}}(t))$ by considering the trivial relation

$$\mathcal{H}(x_{\text{on}}(t)) = \mathcal{H}(x_{\text{on}}(t)) - \mathcal{H}_{\text{on}}(x_{\text{on}}(t)) + \mathcal{H}_{\text{on}}(x_{\text{pre}}),$$

Fig. 2. (colour online) An illustration of $\Delta \tilde{H}_{\text{max}}$ in rotor angle state space. Equation (16) is plotted as a surface in 3-dimensions and the estimate $\Delta \tilde{H}_{\text{max}}$ is given exactly by the difference in energy between the level sets $E_{\text{pot}}(\delta_1, \delta_2) = E_c = E_{\text{pot}}(\delta_1^1)$ and $E_{\text{pot}}(\delta_1, \delta_2) = E_{\text{pot}}(\delta_{\text{pre}})$ where $\delta_1^1$ is the closest UEP. $\delta_{\text{pre}}$ is the pre-fault stable equilibrium point and $\delta_{\text{post}}$ is the stable equilibrium point to (15) in this domain, plotted for completeness.

Fig. 3. (colour online) This figure illustrates the definitions of the three CCT metrics: (i) $t_{\text{CCT}}$ when the full fault trajectory (6) - dash-dot line intersects a stable manifold ($W_1$ or $W_2$) of an unstable equilibrium point (see 16) of the post-fault system in the form (4) where 3 is not constant. (ii) $\tilde{t}_{\text{CCT}}$ is defined using the direct methods by solving (14) and in the figure it is where the fault trajectory (dashed line) intersects the boundary of the energy function where $E$ is assumed constant. (iii) $t_{\text{CCT}}$ is our algebraic metric, the solution to (16) in $\delta$-space (18) is given by the ellipse for a TMIB system and the fault trajectory (dotted line) is approximated using a constant but unique acceleration for each generator.
resulting in the succinct expression
\[
\mathcal{H}(\mathbf{x}_{\text{on}}(t)) = \sum_{i=1}^{n} (P_{ai} - P_{ai}^{\text{on}}) \delta_{\text{on},i}(t)
- \sum_{i=1}^{n} (\bar{P}_{ik} - \bar{P}_{ik}^{\text{on}}) \cos(\delta_{\text{on},i}(t) - \delta_{\text{on},k}(t))
+ \mathcal{H}_{\text{on}}(\mathbf{x}_{\text{pre}}).
\]
\hspace{1cm} (22)

In (22), it is assumed that the governor control systems for the mechanical input power \( P_{mi} \) in each generator are not able to act quickly enough to change the parameter during (or immediately after) the fault; therefore \( P_{mi}^{\text{pre}} = P_{mi}^{\text{on}} = P_{mi} \) throughout this analysis. In order to find the values of the parameters \( \bar{P}_{ik}, P_{ai}, \bar{P}_{ik} \) and \( P_{ai} \) the fault analysis techniques in [17] and [2] are used where the internal generator voltage magnitudes \( |E_{j}| \) are assumed to be constant, i.e. the dynamics of an exciter are not considered.

The Hamiltonian (22) in this form can be subjected to some adaptations in order to construct an appropriate functional expression for \( h_{alt}(t) \). Given that the demand is modelled using constant impedance loads, the total electrical load of a network reduces during a fault because voltages can be assumed to fall across the network. Therefore, it is reasonable to assume that in general \( \bar{P}_{ik} - \bar{P}_{ik}^{\text{on}} \geq 0 \) and \( P_{ai} - P_{ai}^{\text{on}} \geq 0 \), however their moduli are taken to ensure strict positive quantities such that (19) is satisfied. Also, we can replace the cosine terms with the decreasing function
\[
1 - \frac{1}{2} \Delta \delta_{\text{on},ik}^{2}(t) \leq \cos(\Delta \delta_{\text{on},ik}(t)).
\]

This substitution, together with imposing the initial condition \( h_{alt}(0) = h(0) \) allows us to construct the candidate function
\[
h(t) \leq h_{alt}(t) = \sum_{i=1}^{n} |P_{ai} - P_{ai}^{\text{on}}| (\delta_{\text{on},i}(t) - \delta_{\text{pre},i}) + \sum_{i=1}^{n} \frac{1}{2} (\bar{P}_{ik} - \bar{P}_{ik}^{\text{on}})^2 (\Delta \delta_{\text{on},ik}^{2}(t) - \Delta \delta_{\text{pre},ik}^{2}) + \mathcal{H}(\mathbf{x}_{\text{pre}}),
\]
\hspace{1cm} (23)

where \( \Delta \delta_{\text{on},ik}(t) = \delta_{\text{on},i}(t) - \delta_{\text{on},k}(t) \) and \( \Delta \delta_{\text{pre},ik} = \Delta \delta_{\text{on},ik}(0) \).

In order to make (23) an explicit function of time, the evolution of the rotor angles must also be modelled as a function of time. However, the dynamics of the rotor angles during a fault are modelled using equations (4) which are trigonometrically non-linear. The relative acceleration between two rotor angles \( i \) and \( k \) is given by
\[
\frac{d^2}{dt^2} \Delta \delta_{\text{on},ik} = \ddot{\delta}_{\text{on},i} - \ddot{\delta}_{\text{on},k}
\]
\[
= \frac{1}{M_i} \left( P_{mi} - P_{ai} - \sum_{l=1}^{n} P_{il}^{\text{on}} \sin(\delta_{\text{on},i} - \delta_{\text{on},l}) \right)
- \frac{1}{M_k} \left( P_{mk} - P_{ak} - \sum_{m=1}^{n} \bar{P}_{km}^{\text{on}} \sin(\delta_{\text{on},k} - \delta_{\text{on},m}) \right),
\]
\hspace{1cm} (24)

and this cannot be integrated analytically so an approximation for the functions \( \Delta \delta_{\text{on},ik}(t) \) and \( \delta_{\text{on},i}(t) \) is developed. In addition, equation (23) is only valid if \( \Delta \delta_{\text{on},ik}(t) \geq \Delta \delta_{\text{pre},ik} \) and \( \delta_{\text{on},i}(t) \geq \delta_{\text{pre},i} \), so we model the dynamics of the rotor angles during a fault as a constant positive acceleration. Therefore, the rotor angles are given by the polynomial functions
\[
\frac{1}{2} u_i t^2 + \delta_{\text{pre},i} \geq \delta_{\text{on},i}(t)
\]
\hspace{1cm} (25)

and
\[
\frac{1}{2} u_i t^2 + \Delta \delta_{\text{pre},ik} \geq \Delta \delta_{\text{on},ik}(t)
\]
\hspace{1cm} (26)

where the acceleration terms \( u_i, u_k \geq 0 \) and the initial condition \( \delta_{\text{on},i}(0) = 0 \) holds for all \( i \). By taking a crude, physically inaccessible but, crucially, valid upper bound on (24), the acceleration values used in this paper are
\[
u_{ik} = \left| \frac{P_{mi} - P_{ai}}{M_i} - \frac{P_{mk} - P_{ak}}{M_k} \right| + \sum_{l=1}^{n} \bar{P}_{il}^{\text{on}} M_l + \sum_{m=1}^{n} \bar{P}_{km}^{\text{on}} M_m.
\]

and
\[
u_i = \frac{1}{M_i} \left( P_{mi} - P_{ai} + \sum_{l=1}^{n} \bar{P}_{il}^{\text{on}} \right).
\]

This leads us to the final expression of the (now non-Hamiltonian) function
\[
h_{alt}(t) = \mathcal{H}(\mathbf{x}_{\text{pre}}) + \sum_{i=1}^{n} |P_{ai} - P_{ai}^{\text{on}}| \frac{1}{2} u_i t^2 + \sum_{i=1}^{n} \frac{1}{2} \nu_{ik} t^2 \left( \frac{1}{8} u_i^2 t^4 + \frac{1}{2} u_i \Delta \delta_{\text{pre},ik} t^2 \right)
\]
\hspace{1cm} (27)

which is a quadratic in \( t^2 \). Now (18) can be written as
\[
\alpha t^4 + \beta t^2 - \gamma = 0,
\]
\hspace{1cm} (28)

with the coefficients
\[
\alpha = \sum_{i=1}^{n} \frac{1}{8} \nu_{ik} |u_i|^2 > 0,
\]
\[
\beta = \sum_{i=1}^{n} \frac{1}{2} \nu_{ik} |u_i| \Delta \delta_{\text{pre},ik} + \sum_{i=1}^{n} \frac{1}{2} |P_{ai} - P_{ai}^{\text{on}}| u_i,
\]
\[
\gamma = \mathcal{E}_c - \mathcal{H}(\mathbf{x}_{\text{pre}}) = \Delta \mathcal{H}_{\text{max}} > 0
\]

as functions of the power network parameters. The solution of (28) and thus the expression for our lower bound for the CCT is given by
\[
cct = \left( \frac{-\beta \pm \sqrt{\beta^2 + 4 \alpha \gamma}}{2 \alpha} \right)^{\frac{1}{2}}.
\]
\hspace{1cm} (29)

A purely imaginary value for \( cct \) is produced if a post-fault network with an inadequate energy boundary is selected such that \( \mathcal{E}_c < \Delta \mathcal{H}_{\text{max}} \). In order to ensure that this metric is always a lower bound, the smallest positive root in the numerator of (29) is taken; a positive root will always exist due to the coefficients \( \gamma \) and \( \alpha \) being strictly positive for an appropriate post-fault network.
IV. PARAMETRIC STABILITY ANALYSIS

A. Implementation details

The stability of a power network is not only dependent on the type or duration of a fault but also on the choice of system parameters. Optimal regions of parameter space that increase the stability of a power system can be identified by the continuous variation of system parameters. Here, we find optimum parameter regions by using a combination of bifurcation theory and the conservative techniques outlined in Section II-B. These methods are introduced by considering first a hypothetical lossless power network, a more realistic network will then be discussed in Section V. Our new analytical stability metric \( \hat{t}_{\text{CCT}} \) is compared to the CCT estimate \( t_{\text{CCT}} \) (for this system) and the energy margin metric \( \Delta H_{\text{max}} \).

The stationary points for a TMIB system modelled by the ODEs in (15) are located as the bifurcation parameter \( B_{12} \) is varied by using the continuation software AUTO\(^{2}\). We have not attempted to rigorously prove that the solution branches in Fig. 4 contain all the possible solutions which include the stable equilibrium point and its associated unstable equilibria within one period; this has been an unresolved topic of research as indicated in [19]. However, no other solutions were found numerically for this system when performing an exhaustive search over state space using the root finding algorithm `fsolve` from the Scipy\(^{3}\) library in the Python\(^{1}\) programming language. Therefore, without further analysis, we assume that only in a TMIB system can all the necessary stationary points be found using this continuation method.

B. An optimum critical clearing time

The effect of the maximum power flows on stability between nodes 1 and 2 in a simplified TMIB system is investigated. This is analogous to changing the coupling strengths in a complex network such as the Kuramoto model [19] which has been previously applied to power systems [5]. Specifically, a simplified TMIB system (see Fig. 1) with no shunt conductances and lossless lines such that \( P_{a1} = P_{a2} = 0 \) is considered, where the maximum power flow from either machine to the infinite bus is set to be identical (\( P_{13} = P_{23} \)). The susceptance \( B_{12} \) is chosen as the bifurcation parameter and varying it will change the stationary solutions and critical energy value \( E_c \). The associated element in the reduced admittance matrix is \( Y_{12} = jB_{12} \) and any variation of elements in the reduced matrix will result in new values for all elements in the full bus admittance matrix \( Y_{\text{BUS}} \) due to Kron reduction. In the interest of studying a system with some asymmetry, the mechanical input powers are different (\( P_{m1} \neq P_{m2} \)). In order to ensure that the network is always operating within its capacity, \( P_{m1} < P_{mk} \) for all \( (i, k) \) during the pre-fault and post-fault. In order to simplify the fault analysis, we assume that the pre-fault and post-fault systems are identical (i.e. \( \delta^\text{pre} = \delta^\text{post} \)) and that the generator voltages remain constant throughout. All these parameters are non-dimensionalised to make the analysis clearer in a similar way to [2]. For this system, the values of the parameter vectors in the form \( \lambda = [P_{m1}, P_{m2}, P_{12}, P_{13}, P_{23}, D_1, D_2, M_1, M_2]^T \) are

\[
\lambda_{\text{pre}} = \lambda_{\text{post}} = [0.5, 0.3, B_{12}, 1.0, 1.0, 0.0, 0.0, 1.0, 1.0]^T
\]

where \( |E_1| = |E_2| = |E_3| = 1.0 \) and the inertia parameters are set to unity to suppress the effect of inertia on the results. A solid three phase to ground fault at bus 1 is considered on the network such that the voltage magnitude \( |E_1| = 0.0 \). Therefore, the fault parameters \( \lambda_{\text{on}} \) are identical to those in \( \lambda_{\text{pre}} \) with the exception of \( P_{23}^{\text{on}} = 0 \) and we set \( P_{23}^\text{off} = 0.5 \). The maximum power flow \( P_{23}^\text{off} < P_{23} \) in order to model the smaller operational network during a fault conditions.

In Fig. 4a, the modulus of the stationary solutions \( |x^\ast| \) to (15) is plotted against \( B_{12} \). In an undamped system, the “stable” stationary point is technically a centre node (see [20]) but we will continue to refer to it as the stable node for clarity. At \( B_{12} = 0 \) the generators are completely decoupled from each other and operate as two separate single machine infinite bus (SMIB) systems [1]. The stable solutions for each of these SMIB systems are \( \delta_1 = \arcsin(0.5) \) and \( \delta_2 = \arcsin(0.3) \) which gives \( |x^\ast| = 0.606 \) at \( B_{12} = 0.0 \) and this is found in the figure. In the limit \( B_{12} \to +\infty \) the system can be modelled as one combined SMIB with mechanical input power \( P_m = P_{m1} + P_{m2} \) and maximum electrical power output \( P = P_{13} + P_{23} \). Therefore, the two solutions remaining on the right-hand-side of Fig. 4a, are tending towards \( |x^\ast| = \sqrt{2} \arcsin(0.4) = 0.582 \) (blue line) and \( |x^\ast| = \sqrt{2} (\pi - \arcsin(0.4)) = 3.861 \) (thick black line). The colour of a solution branch indicates the local stability that is obtained by linearising system (15) and computing the eigenvalues of the Jacobian matrix at the stationary points. The specific eigenvalue ranges that characterise the stability of each branch colour are found in Table I. Note that these are the solutions for a system with no damping and purely imaginary eigenvalues can be found. The closest UEP, defined in (12) is indicated by a thicker branch line.

\[
\begin{array}{cccc}
\text{Branch Label} & \lambda_1 & \lambda_2 & \lambda_3 \\
A & ju_A & -ju_A & jv_A & -jv_A \\
B & u_B & -u_B & jv_B & -jv_B \\
C & ju_C & -ju_C & v_C & -v_C \\
D & u_D & -u_D & v_D & -v_D \\
\end{array}
\]

**TABLE I**

**EIGENVALUE STABILITY FOR THE SOLUTION BRANCHES IN THE BIFURCATION DIAGRAMS IN FIGS. 4 AND 8 AND THE STATIONARY POINTS IN FIG. 5** where \( u_A, v_A, u_B, v_B, u_C, v_C, u_D, v_D \in \mathbb{R} \setminus \{0\} \) and \( j^2 = -1 \).

At a critical value \( B_{\text{crit}} = 0.6712 \) (Fig. 4, vertical dashed line) there is a degeneracy where two distinct unstable points in the set \( S \) given by (13) satisfy the minimum critical energy in (12). This degeneracy is observed as a discontinuity at \( B_{\text{crit}} \).
in the closest UEP $[\delta_1^c]$ (thick line in Fig. 4) located between the fold points $L_2$ and $L_3$. Figure. 5 illustrates the change in the closest UEP as $B_{12}$ is varied over a small range from $B_{12} = B_{\text{crit}} - \epsilon$ to $B_{12} = B_{\text{crit}} + \epsilon$ where $\epsilon$ is small. In each sub-figure of Fig. 5 the stationary points of (15) are plotted using the same colour coding found in the legend of Fig. 4 and the level set $E_{\text{pot}}(\delta_1, \delta_2) = E_c$ (black line) is observed to intersect the closest UEP.

In Figure 4b we plot three further quantities as a function of $B_{12}$: the conservative energy metric $\Delta \mathcal{H}_{\text{max}}$, the critical clearing time $t_{\text{CCT}}$, and the new lower bound CCT metric $t_{\text{CCT}}$ proposed in Section IV. By inspection, all three functions are $C^0$ continuous in the given domain but are not differentiable at $B_{\text{crit}}$. There is a maximum point in both CCT metrics at $B_{\text{crit}}$ which, interestingly, is not evident in the energy metric. Instead the critical energy change $\Delta \mathcal{H}_{\text{max}}(B_{12})$ for the power system experiences a step change in the gradient at $B_{\text{crit}}$ but the gradient is never negative in this domain.

V. A 3-GENERATOR EXAMPLE

A. Implementation details

In order to provide a more practical view of the results in Section IV.B we perform a parameter study on a 9 bus, 3 generator power network found in [17]. A schematic of this network is provided in Fig. 6. All parameter values for this network are taken from [17] with the exception of the shunt loads whose values are given in the figure caption. The shunt load parameters are altered such that direct methods are able to be applied before any bifurcation analysis is carried out. The alteration involves reducing their conductance values. The bifurcation parameters we choose are the conductive and susceptive parts of shunt load $C$ ($Y_C = G_C + jB_C$) where one is held constant as the other is varied. This parameter was chosen to be able to draw comparisons with the results from the simplified network in Section IV where the power flow between the two generators is varied; as such the generator at bus 3 is modelled as an infinite bus.

The fault analysis method in [4], [17], recently summarised in [21] for power networks with constant impedance loads, is employed here. This technique computes a reduced admittance matrix for each of the pre-fault, fault-on and post-fault systems by performing Kron reduction on their corresponding bus admittance matrices. The mechanical input powers $P_{m1}$ and $P_{m2}$ for each generator are found by performing a pre-fault load-flow on the system using the pre-fault rotor angles in [17] and the full pre-fault bus admittance matrix. The specific fault we consider is a three-phase to ground fault close to bus 7 on the line connecting buses 5 and 7. The post fault network is identical to the pre-fault network except the line connecting buses 5 and 7 is switched out.

The stationary points for each value of the bifurcation parameters in Fig. 8a and Fig. 7b are obtained by the following method: A stable stationary point denoted by
The fault we consider occurs on the line 5-7 close to bus 7, and the post fault network has line 5-7 switched out.

\[
x^* = [\delta_1^*, \delta_2^*, \omega_1 = 0, \omega_2 = 0]
\]

for the set of ODE’s

\[
\begin{align*}
\dot{\delta}_1 &= \omega_1 \\
\dot{\delta}_2 &= \omega_2 \\
\dot{\omega}_1 &= \frac{1}{M_1} (P_{m1} - P_{e1}(\delta)) \\
\dot{\omega}_2 &= \frac{1}{M_2} (P_{m2} - P_{e2}(\delta))
\end{align*}
\]

is found using the root finding algorithm fsolve, where \(\delta_i^* \in [-\pi, \pi]\) for \(i = 1, 2\). This point belongs to the lower (blue) branch of the bifurcation diagrams in the lower panels of Figs. 7 and 8. By definition, this stationary point is also a stationary point for the system of ODEs in (15). The other unstable equilibria associated with this stable equilibrium \(x^*\) are found by treating \(B_{12}\) from the reduced admittance matrix \(Y_{\text{red}}\) as a bifurcation parameter. Once the bifurcation branches are found, we extract the stationary points at the value of \(B_{12}\) found in \(Y_{\text{red}}\). The local stability of the stationary points obtained are found by computing the eigenvalues of the Jacobian matrix for the system (15) and their ranges are found in Table 1. Notice that there are negative values for \(G_C\) in Fig 8 which is intended to model electrical power injection onto the grid at the distribution level.

**B. Results**

In the upper panels of Figs. 7 and 8 the domains of the bifurcation parameters \(B_C\) and \(G_C\) respectively are constrained by two conditions: (i) the energy margin \(\Delta H_{\text{max}} \geq 0\) and (ii) that there exists a stable solution to (30). The valid domains for each bifurcation parameter are limited by constraint (i) for the lower bound (plots do not extend to this bound for clarity) and constraint (ii) for the upper bound.

In a similar fashion to the results for the illustrative example, we plot the critical energy change for the system \(\Delta H_{\text{max}}\) and the critical clearing time estimates \(t_{\text{CCT}}\) and \(t_{\text{CCT}}\) as functions of the bifurcation parameters in the lower panels of Figs. 7 and 8. In addition we plot the true \(t_{\text{CCT}}\) using a simple algorithm that uses a binary search to find the maximum duration which the fault can be left on-line such that the rotor angles do not diverge once the fault is cleared. There are two different scales to facilitate observing the functions in the lower panels of Figs. 7 and 8. The energy change \(\Delta H_{\text{max}}\) should be read using the right-hand y-axis labels and the energy margin metric \(\Delta H_{\text{max}}\) should be read using the right y-axis.

In the bifurcation diagram in Fig. 7 notice that there are two discontinuities in the closest UEP (thick line) at \(B_C = -3.65\) and 5.63 which are both located between two distinct pairs of fold points. In Fig. 7 there is a discontinuous change in the gradient of \(\Delta H_{\text{max}}\) at both discontinuities in the closest UEP. The maximum point of the algebraic \(t_{\text{CCT}}\) seems to coincide with the discontinuity in the closest UEP at \(B_C = -3.65\), however, unlike the results for the hypothetical system in Section IV the maximum point in \(t_{\text{CCT}}\) does not coincide with any point of significance in the bifurcation plot. The maximum of \(\Delta H_{\text{max}}\) does coincide with the discontinuity in the closest UEP at \(B_C = 5.63\) which further shows that the energy margin is not the best metric to use to quantify stability. Notice that \(t_{\text{CCT}}\) emulates the general profile of the function.
in the reduced network. If this term cannot be assumed to be small then the use of direct methods to measure stability becomes invalid. Traditionally, in order to validate using direct methods, the transfer conductances $G_{ij}$ in the reduced network matrix $Y_{red}$ are assumed to be small or zero \cite{10}. However, even for a network with lossless lines, Kron reduction invokes complications in which a large shunt load conductance in full bus matrix will increase the absolute values of $G_{ij}$ \cite{23}. This is why it is common for the ‘lossless network’ assumption to be employed such that all $G_{ij}$ in the reduced admittance matrix are set to zero.

### VI. Discussion

In this paper we have presented a new analytical stability metric $\hat{\iota}_{CCT}$ designed to be a concrete lower bound on the true CCT ($\iota_{CCT}$) of an arbitrarily large power system experiencing a three-phase to ground fault at any one of the network buses. In the hypothetical power system in Section IV we considered a lossless system (i.e. $P_i(\delta) = 0$ and $D_i = 0$ for all $i = 1, \ldots, n$) where the direct methods are fully valid. We showed that our metric successfully captures the key characteristics (e.g. matching the locations of maximum points) of the function for $\hat{\iota}_{CCT}$ as the effective susceptance $B_{12}$ between two generators is varied as well as demonstrating that the inequality (17) is satisfied for this lossless system. However, when considering lossy systems, our metric suffers from the same drawbacks that all direct methods have, which concern the path dependent term $P_i(\delta)$ modelling the dissipation of power in the shunt loads and transmission lines in the whole system. However, even when used on lossy systems, new stability metrics can still have significant value and our algebraic metric was designed with this in mind. The limitations of our new metric were investigated by studying a more realistic test system in Section V. Our metric is valid in principle for power systems with or without an infinite bus, however the numerical results in this paper are for networks with an infinite bus where the dynamics are well behaved due to an asymptotically stable region of attraction. In future work we shall extend our numerical results to larger systems without an infinite bus.

In section V, the stability of the more realistic test power system was analysed using our algebraic metric when the shunt load C was varied. The conductive and susceptive part of the shunt load were considered separately and the results showed that the performance of our algebraic metric $\hat{\iota}_{CCT}$ and the CCT derived from direct methods $\iota_{CCT}$ are dependent on the bifurcation parameter chosen. Given that the shunt loads are found in the full bus matrix, this is a more tangible parameter to vary but it was a more tedious exercise because Kron reduction of the network had to be performed every time this parameter was varied in order to accurately find the stationary points to (15). When $B_C$ is a bifurcation parameter, for suitably low conductances held constant, our metric was relatively successful as a lower bound on the CCT for a large part of the domain of $B_C$. Variation of $B_C$ can represent, for example, a network owner’s installation of reactive compensation, a measure that is known to contribute

---

*Fig. 8. (colour online) The bifurcation diagram in (a) plots the modulus of the stationary solutions $|x^*|$ to the ODE (15) as a function of the bifurcation parameter $G_C$. The stability of the solution branches are colour coded using the legend in Fig. 4 and the eigenvalue ranges for each colour are found in Table I. The thicker line indicates the closest UEP $|x^*_U|$ = $|\delta^*_U|$. In (b) the CCT metrics $\hat{\iota}_{CCT}$, $\iota_{CCT}$ and $\hat{\iota}_{CCT}$ should be read using the left y-axis and the energy margin metric $\Delta H_{max}$ should be read using the right y-axis.*
not only to voltage regulation but also transient stability [24]. However, when the conductive part of the load \( G_C \) was varied for a constant susceptance, our metric was only successful when \( |G_C| \) is small. This is unsurprising given that our metric is derived using the energy functions found in the direct methods and relies on the assumption that the conductances in the reduced admittance matrix are small. The Kron reduction process itself is a limiting factor for direct stability methods applied to lossy power networks [25] and it has been proved in [26] that a general analytical energy function for studying transient stability on lossy power systems is non-existent. A possible avenue of further research could be to construct an algebraic CCT using the structure preserving model [27] for power systems.

Another drawback is regarding the critical energy boundary \( \mathcal{E}_c \). In this paper we chose the closest UEP method to compute the energy boundary \( \mathcal{E}_c \) so that it is valid for any fault on a power system. For the TMIB network we have considered, we used numerical continuation of the parameter \( B_{12} \) in the reduced admittance matrix to search for the stationary points and subsequently define the closest UEP. An exhaustive algorithmic search for the stationary points was also conducted in order to confirm that all stationary points were found by numerical continuation alone, but this was not proved. For larger systems it is increasingly difficult to identify all bifurcation branches in state space and algorithmic validation of the closest UEP in large system is itself a significant area of research [25]. However, an advantage of our lower bound algebraic stability metric is that it is independent of the method chosen to calculate a critical energy boundary and is not restricted to the closest UEP method.

Despite its drawbacks, our algebraic lower bound stability metric has the potential to inform optimal fault management strategies to increase system stability. Its key advantage is that under suitable parameter selection and variation, the metric can be computed instantly once all the system parameter values for pre-fault, fault-on and post-fault have been collected. This feature of stability metrics could be of use due to system dynamics becoming more unpredictable from the changing nature of loads [3] and generation [29–31] under the constraint of limited power flow through transmission lines. Furthermore, optimisation techniques could be applied to algebraic metrics to find optimal regions in parameter space that could translate into tangible changes to the power system for increasing stability.

REFERENCES

[1] P. S. Kundur, Power system stability and control. McGraw-Hill, 1994.
[2] J. Machowski, J. W. Bialek, and J. R. Bumby, Power system dynamics: stability and control. Wiley, 2008.
[3] K. Yamashita, S. Djokic, J. Matevosyan, F. Resende, L. Korunovic, Z. Dong, and J. Milanovic, “Modelling and aggregation of loads in flexible power networks and status of the work of cigré wg c4. 605,” in Power Plants and Power Systems Control, vol. 8, no. 1, 2012, pp. 405–410.
[4] M. A. Pai, Energy function analysis for power system stability. Kluwer, 1999.
[5] F. Dörfler and F. Bullo, “Synchronization and transient stability in power networks and non-uniform Kuramoto oscillators,” SIAM J. Control Optim., vol. 50, no. 3, pp. 1616–1642, 2012.
[6] Y. Susuki, I. Mezić, and T. Hikihara, “Coherent swing instability of power grids,” Journal of Nonlinear Science, vol. 21, no. 3, pp. 405–439, Feb. 2011.
[7] V. Agarapu and B. Lee, “Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system,” IEEE Transactions on Power Systems, vol. 7, no. 1, pp. 424–431, 1992.
[8] M. Galaz, R. Ortega, A. S. Bazanella, and A. M. Souza, “An energy-shaping approach to the design of excitation control of synchronous generators,” Automatica, vol. 39, pp. 111–119, 2003.
[9] Y. Hasegawa and Y. Ueda, “Global basin structure of attraction of two degrees of freedom swing equation system,” International Journal of Bifurcation and Chaos, vol. 9, no. 8, pp. 1549–1569, 1999.
[10] H.-D. Chiang, Direct methods for stability analysis of electric power systems: theoretical foundation, BCU methodologies, and applications. Wiley, 2011.
[11] M. J. Laufenberg and M. A. Pai, “A new approach to dynamic security assessment using trajectory sensitivities,” in Power Industry Computer Applications, 1997, 20th International Conference on, 1997, pp. 272–277.
[12] A. A. Fouad and S. E. Stanton, “Transient stability of a multi-machine power system Part I: Investigation of system trajectories,” IEEE Transactions on Power Apparatus and Systems, vol. 100, no. 7, pp. 3408–3416, 1981.
[13] I. A. Hiskens and M. A. Pai, “Trajectory sensitivity analysis of hybrid systems,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 47, no. 2, pp. 204–220, 2000.
[14] T. B. Nguyen and M. A. Pai, “Dynamic security-constrained rescheduling of power systems using trajectory sensitivities,” IEEE Transactions on Power Systems, vol. 18, no. 2, pp. 848–854, 2003.
[15] F. Dörfler and F. Bullo, “Kron reduction of graphs with applications to electrical networks,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 60, no. 1, pp. 150–163, Jan. 2013.
[16] H.-D. Chiang, C.-c. Chu, and G. Cauley, “Direct stability analysis of electric power systems using energy functions: Theory, applications, and perspective,” in Proceedings of the IEEE, vol. 83, no. 11, 1995, pp. 1497–1529.
[17] P. M. Anderson and A. A. Fouad, Power System Control and Stability, 2nd ed. IEEE Press, 2002.
[18] F. Dörfler and F. Bullo, “Synchronization in complex networks of phase oscillators: A survey,” Automatica, vol. 50, no. 6, pp. 1539–1564, 2014.
[19] S. Strogatz, “From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators,” Physica D: Nonlinear Phenomena, no. 1-4, pp. 1–20, Sep. 2000.
[20] ———, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering. Westview Press, 2001.
[21] A. Gajduk, M. Todorovski, and L. Kocarev, “Stability of power grids: An overview,” The European Physical Journal Special Topics, vol. 223, no. 12, pp. 2387–2409, Jun. 2014.
[22] J. Sloatweg and W. Kling, “Impacts of distributed generation on power system transient stability,” IEEE Power Engineering Society Summer Meeting, vol. 2, pp. 862–867, 2002.
[23] M. Ribbens-Pavella and F. Evans, “Direct methods for studying dynamics of large-scale electric power systems survey,” Automatica, vol. 21, no. 1, pp. 1–21, 1985.
[24] B. Delfino, G. Denegri, M. Invernizzi, and P. Pinetti, “Estimating first swing stability of synchronous machines as affected by saturation and controls,” Energy Conversion, IEEE Transactions on, vol. 3, no. 3, pp. 636–646, 1988.
[25] M. Ribbens-Pavella and F. Evans, “Direct methods for studying dynamics of large-scale electric power systemsA survey,” Automatica, vol. 21, no. 1, pp. 1–21, Jan. 1985.
[26] H.-D. Chiang, “Study of the existence of energy functions for power systems with losses,” IEEE transactions on circuits and systems, vol. 36, no. 11, pp. 1423–1429, 1989.
[27] A. R. Bergen and D. J. Hill, “A structure preserving model for power system stability analysis,” Power Apparatus and Systems, IEEE Transactions on, no. 1, pp. 25–35, 1981.
[28] C. W. Liu and J. S. Thorp, “A novel method to compute the closest unstable equilibrium point for transient stability region estimate in power systems,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 44, no. 7, pp. 630–635, 1997.
[29] H. Urdal, R. Ierna, J. Zhu, C. Ivanov, A. Dahresobh, and D. Rostom, “A fast and efficient method to find the closest UEP in large power systems,” in 20th International Conference on Electrical Power Quality and Utilisation (EPQU), 2014, pp. 1–6.
[30] T. B. Nguyen, M. A. Pai, and E. Muljadi, “Impact of Wind Power Plants on Voltage and Transient Stability of Power Systems,” in IEEE Energy2030, no. November, 2008.
[31] D. Gautam, V. Vittal, and T. Harbour, “Impact of Increased Penetration of DFIG-Based Wind Turbine Generators on Transient and Small Signal Stability of Power Systems,” *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1426–1434, 2009.