Cosmologically viable gauge mediation

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Abstract

Gauge mediation provides us with a complete picture of supersymmetry breaking and its mediation within the effective field theories, and thus allows us to discuss consistencies with low-energy particle physics as well as cosmological observations. We study in detail the cosmological evolution of the pseudo-modulus field in the supersymmetry breaking sector and also the production of the gravitinos in the early Universe in a simple (but a complete) model of gauge mediation. Under fairly reasonable assumptions, it is found that there exists a parameter region where dark matter of the Universe is explained by both thermally and non-thermally produced gravitinos, while the baryon asymmetry of the Universe is generated through the thermal leptogenesis.
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1 Introduction

If supersymmetry is realized in nature, there should be a fermionic superpartner of the graviton, the gravitino. Since supersymmetry (SUSY) must be spontaneously broken, the gravitino is massive. Nevertheless, it becomes massless when we send the Planck scale to infinity, and thus the gravitino is the lightest superparticle under an assumption that the gravity is the weakest force to talk to the SUSY breaking sector. This discussion already motivates us to consider the gravitino as dark matter of the Universe, regardless of the hierarchy problem. The assumption that the gravity as the weakest communication is always true in gauge mediation scenarios, where various explicit models are known. We take one of the simple models of gauge mediation as an example, and demonstrate how such a scenario is consistent with observations of the Universe.

In linearly realized models of SUSY breaking, i.e., in the O’Raifeartaigh models, there is a chiral multiplet $S$ whose $F$-component acquires a vacuum expectation value (VEV). The fermionic component of $S$ is the Nambu-Goldstone fermion (the goldstino), and it is eaten by the gravitino in supergravity theory. The $S$ multiplet also contains a complex (or two real) physical scalar field which radiatively obtains a mass. Since the scalar component has no potential at tree level or at the supersymmetric level, it is called the pseudo-modulus, that may have cosmological importance. We will consider the dynamics of this field later in detail.

In the early Universe, the gravitinos are produced in various ways. The scattering of particles with superparticles in the thermal plasma, for example, produce gravitinos [1, 2, 3, 4, 5, 6, 7, 8, 9]. The fraction of the gravitino energy density, $\Omega_{3/2}$, from such thermal productions is proportional to the reheating temperature of the Universe, $T_R$. Since $\Omega_{3/2}$ is bounded by the total dark matter density $\Omega_{DM} \sim 0.2$, $T_R$ needs to be low enough, and that possibly causes a tension with scenarios of baryogenesis. For example, the thermal leptogenesis [10] requires $T_R \gtrsim 10^9$ GeV [11] which is too high in typical gauge mediation models. Gravitinos can also be produced non-thermally such as by the decays of the string moduli [14, 15, 16, 17, 18], the inflaton [19, 20, 21, 22, 23] and/or the pseudo-modulus field [24, 25, 26, 27, 28, 29, 30].

Once we set up a whole picture of a gauge mediation model, the gravitino abundance through the thermal scattering processes and also through non-thermal productions via the pseudo-moduli decays are calculable. For example, in Ref. [28, 29], a simple model of gauge mediation [31] has been considered, and indeed it has been found that the non-thermally

* Throughout this paper, the reheating refers to the decay of the inflaton.
generated gravitino can explain the dark matter abundance in a theoretically motivated parameter region.

In fact, the dynamics of the pseudo-modulus is somewhat complicated. As the theorem of Ref. [32] states, the O'Raifeartaigh-type model generally has multiple potential minima, and the one which gives the gaugino mass correctly is never the lowest one. Although the theorem does not apply to the model of [31], the situation is similar due to the existence of a supersymmetric vacuum where the messenger fields condense. An important assumption made in the studies of Refs. [28, 29] is that the field value of the pseudo-modulus after inflation is located far away from its origin so that it never approaches to the true minimum. Under this assumption, it has been found that the pseudo-modulus successfully settles down at the meta-stable SUSY breaking vacuum.

However, it has been pointed out recently that the above assumption is not necessary when we take into account the thermal effects on the potential [33, 34]. Once we start from the origin of the pseudo-modulus, the messenger fields are massless and thus they are thermalized. The thermal effects of the messenger fields contribute to the potential of the pseudo-modulus. A parameter region where the meta-stable minimum is selected along the thermal history was identified.

It is then natural to ask whether the gravitino abundance is consistent with observations in such a scenario. Since the messenger fields are abundant in the thermal plasma, it looks dangerous for the production of the goldstino component of the gravitino which is directly coupled to the messenger fields. In order to see if this scenario is viable, one needs to follow the cosmological history and calculate the gravitino abundance both from the thermal and non-thermal processes.

In this paper, we explicitly calculate the gravitino abundance in a scenario where the initial position of the pseudo-modulus is close to the origin where the thermal potential is minimized. We find various non-trivial behavior of the pseudo-modulus depending on model parameters. By numerically following the motion of the pseudo-modulus in the field space, it is found that the coherent oscillation of the pseudo-modulus eventually dominates the energy density of the Universe in a wide range of the parameter space and there the non-thermal production of the gravitinos by its decay can explain the right amount of dark matter. Moreover, it is found that the reheating temperature after inflation can be much higher than the one required by the thermal leptogenesis scenario, without having a trouble with the overproduction of the gravitinos. By considering the dilution of the baryon asymmetry
by the entropy production from the decays of the pseudo-modulus, the required reheating temperature is higher or comparable to $10^{12}$ GeV, with which one can explain both dark matter and baryon asymmetry of the Universe.

The paper is organized as follows. In section 2 we set up a gauge mediation model which we use for the study of cosmology. The dynamics of the pseudo-modulus is studied in section 3 and the results are used in section 4 for the calculation of the gravitino abundance generated from scattering processes in the thermal bath. The non-thermal component is calculated in section 5.

2 A model of gauge mediation

We study the low-energy effective theory of O’Raifeartaigh type SUSY breaking model coupled with the messenger fields. After integrating out the massive fields, the Kähler potential and the superpotential are written as

$$
K = f^\dagger f + \bar{f}^\dagger \bar{f} + S^\dagger S - \frac{(S^\dagger S)^2}{A^2} + \cdots ,
$$

$$
W = m^2 S - \lambda S f \bar{f} + c,
$$

where $S$ is a gauge singlet field and $f$ and $\bar{f}$ are the messenger fields which carry standard model quantum numbers. We take the messenger fields $f$ and $\bar{f}$ to transform as $5$ and $\bar{5}$ under SU(5), and the messenger number $N = 1$ for simplicity. $\Lambda$ is the cutoff scale of the effective theory, which is typically the mass scale of the massive fields of the O’Raifeartaigh models. The $U(1)_R$ charge assignment is $R(S) = 2$ and $R(f \bar{f}) = 0$, whereas the constant term $c$ represents the R-symmetry breaking supergravity effect and contributes to the cosmological constant. We take $m$, $\lambda$, $c$ and $\Lambda$ real and positive by appropriate redefinition of the fields and the $U(1)_R$ transformation. There is a quantum correction to the Kähler potential from the interaction term $\lambda S f \bar{f}$,

$$
K_{1\text{-loop}} = - \frac{5 \lambda^2}{(4\pi)^2} S^\dagger S \log \frac{S^\dagger S}{A^2} ,
$$

at the one-loop level. As we shall see, if this radiative correction is too large, the SUSY breaking vacuum is destabilized, and so, $\lambda$ is bounded above.

Once we take into account the supergravity effects, a SUSY breaking vacuum appears [31],

$$
\langle S \rangle = \frac{\sqrt{3}}{6} \frac{\Lambda^2}{M_{\text{pl}}} , \quad \langle f \rangle = \langle \bar{f} \rangle = 0 ,
$$

5
where we have neglected the radiative correction to the Kähler potential. Here $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. The $F$-term of $S$ is

$$F_S = m^2,$$  \hspace{1cm} (5)

in this vacuum, and therefore SUSY is indeed broken. The messenger fields acquire a SUSY mass $M_{\text{mess}} = \lambda \langle S \rangle$ through the interaction with $S$. Note that $S$ develops a VEV along the real component, while the imaginary component vanishes in this vacuum. The constant term $c$ is fixed so that the cosmological constant is cancelled at the SUSY breaking vacuum,

$$c \simeq m_{3/2} M_{\text{pl}}^2,$$  \hspace{1cm} (6)

where the gravitino mass is given by

$$m_{3/2} = \frac{m^2}{\sqrt{3} M_{\text{pl}}},$$  \hspace{1cm} (7)

So far we have neglected the radiative correction (3), which could destabilize the SUSY breaking vacuum. In order to see its effect, let us study the mass terms of the $S$ field and the messenger fields (we take $\bar{f} = f$), including the correction:

$$V_{\text{mass}} = (S^\dagger S) M^2_S \begin{pmatrix} S \\ S^\dagger \end{pmatrix} + (f^\dagger f) M^2_f \begin{pmatrix} f \\ f^\dagger \end{pmatrix},$$  \hspace{1cm} (8)

\begin{align*}
M^2_S &\simeq \frac{m^4}{\Lambda^2} \begin{pmatrix} 4 & -\frac{15\lambda^2 M_{\text{pl}}^2}{8\pi^2} \\ -\frac{15\lambda^2 M_{\text{pl}}^2}{8\pi^2} & 4 \end{pmatrix}, \\
M^2_f &\equiv \begin{pmatrix} \lambda^2 \langle S \rangle^2 & -\lambda m^2 \\ -\lambda m^2 & \lambda^2 \langle S \rangle^2 \end{pmatrix},
\end{align*}  \hspace{1cm} (9)

where we have fixed the SUSY breaking vacuum as (4) for simplicity, and we have dropped terms proportional to $\lambda^2$ in the diagonal components of $M^2_S$. In order for the SUSY breaking minimum to be (meta)stable, $\det m^2_S > 0$ and $\det m^2_f > 0$ must be met, namely,

$$\frac{12m^2 M_{\text{pl}}^2}{\Lambda^4} < \lambda < \frac{4\sqrt{2}\pi}{\sqrt{15} M_{\text{pl}}}. \hspace{1cm} (11)$$

Note that $\lambda$ is bounded both above and below.

There is also a SUSY preserving vacuum at

$$\langle S \rangle_{\text{SUSY}} = 0, \quad \langle f \rangle_{\text{SUSY}} = \langle \bar{f} \rangle_{\text{SUSY}} = \sqrt{\frac{m^2}{\lambda}},$$  \hspace{1cm} (12)

and the messenger directions are tachyonic for

$$|S| < S_{\text{ct}} \equiv \sqrt{\frac{m^2}{\lambda}}.$$  \hspace{1cm} (13)
In the absence of the thermal corrections, therefore, $S$ must be outside this region throughout the evolution of the Universe since otherwise it would end up with the SUSY minimum.

Gaugino masses are calculated to be

$$m_\lambda = \frac{g^2}{(4\pi)^2} \frac{F_S}{\langle S \rangle} = \frac{g^2}{(4\pi)^2} \frac{2\sqrt{3}m^2 M_{pl}}{\Lambda^2}. \quad (14)$$

This simple model has three parameters $m$, $\Lambda$ and $\lambda$. In following analysis we fix the ratio $m/\Lambda$ so that the gaugino mass should be $O(100)\text{GeV}$. In particular, we fix the Bino mass $m_{\tilde{B}} \simeq 300\text{GeV}$. Once the gaugino mass is fixed, we can relate the gravitino mass to $\Lambda$ as

$$m_{3/2} \simeq 0.6\text{GeV} \left(\frac{m_{\tilde{B}}}{300\text{GeV}}\right) \left(\frac{\Lambda}{10^{16}\text{GeV}}\right)^2. \quad (15)$$

3 Pseudo-modulus dynamics

3.1 Vacuum selection

As the model breaks SUSY at a meta-stable state, we have to check whether the SUSY breaking minimum is actually selected in the cosmological evolution. The vacuum selection in meta-stable SUSY breaking models has been discussed in several literature for the ISS-type models [39, 40, 41, 42, 43, 44, 45] and the generalized O’Raifeartaigh models [46, 47, 48, 33, 34]. In particular, it was shown in Ref. [34] that the pseudo-modulus successfully moves from near the origin to the SUSY breaking minimum in the model of Eqs. (1) and (2), by virtue of the finite temperature potentials. Here we follow their discussion and clarify the parameter region where $S$ successfully reaches SUSY breaking vacuum.

We assume that the minimal supersymmetric standard model (MSSM) superfields and the messenger superfields are in thermal equilibrium in the early Universe. Although $S$ superfield is not in thermal plasma, its scalar potential receives thermal corrections due to interactions with the messenger fields. The minimum of the thermal potential is located at the origin, $\langle S \rangle = \langle f \rangle = \langle \bar{f} \rangle = 0$, rather than the SUSY breaking vacuum in Eq. (4). Thus, for a sufficiently high temperature, the potential minimum is close to the origin, and its real component will gradually approaches the SUSY breaking one as the temperature decreases, whereas the imaginary component remains stabilized at the origin.

The SM-like Higgs boson with mass $\sim 125\text{GeV}$ was recently found by the ATLAS [35] and CMS [36] experiments. For the Bino mass adopted in the text, the light Higgs boson mass may not be explained, unless the Higgs sector is extended or extra vector-like matter [37, 38] is introduced. Such extensions will not alter the following analysis significantly as we mainly focus on the SUSY breaking and mediation sectors.
During inflation, on the other hand, there is no thermal plasma. Since the inflaton potential largely breaks SUSY, the scalar potential of $S$ is modified through Planck-suppressed couplings. In particular, the $S$ field generically acquires a so-called Hubble-induced mass, and as long as the $U(1)_R$ remains a good symmetry during inflation, the origin of $S$ is close to the extremum of the potential. If the Hubble-induced mass is positive, therefore, $S$ is stabilized near the origin during inflation, and it likely remains there even after the inflation, which realizes the initial condition mentioned above.

Note that this initial condition is different from that of Ref. [29], where the $S$ field is assumed to be away from the origin in order not to fall into the SUSY preserving vacuum. In particular, it was pointed out that the imaginary part of $S$ needs to have a large initial value to avoid falling into the wrong vacuum, and the oscillation of the imaginary part is eventually the most important because of its relatively long life time. In the present case, since the initial position of $S$ is close to the origin after inflation, the imaginary component of $S$ stays at the origin throughout, which allows us to concentrate only on the real component of $S$.

The pseudo-modulus $S$ acquires a finite temperature potential from interactions with the messenger fields. The relevant terms along $f = \bar{f} = 0$ direction are calculated up to $O(S^3)$:

$$V_S = -\frac{2}{\sqrt{3}} \frac{m^4}{M_{pl}} (S + S^\dagger) + 4 \frac{m^4}{\Lambda^2} |S|^2 + \frac{5}{4} \lambda^2 T^2 |S|^2 - \frac{5}{3\pi} \lambda^3 T |S|^3.$$  \hfill (16) 

See Appendix A for the derivation. The potential minimum at temperature $T$ is therefore given by

$$S_{\text{min}}(T) \simeq \sqrt{3} \frac{\Lambda^2}{6} \frac{m_{S}^2}{M_{pl}} \frac{m_S^2}{m_S^2 + \frac{5}{4} \lambda^2 T^2},$$  \hfill (17) 

where

$$m_S = \frac{2m^2}{\Lambda}$$  \hfill (18) 

is the tree-level mass of $S$ at zero-temperature. We can see from this formula that the potential minimum is near the origin for a high temperature and moves toward the SUSY breaking one as the temperature decreases.

In the absence of the thermal potential, $S$ would fall into the SUSY preserving vacuum if $|S| < S_{\text{cr}}$, since the messenger fields become tachyonic. For a sufficiently high temperature, however, $S$ does not fall into the SUSY preserving vacuum because the thermal effects lift the messenger direction. The messenger fields get thermal potentials through interactions
with the standard model gauge bosons,

\[ V_\ell = -\lambda m^2 (\ell \ell + \text{h.c.}) + \lambda^2 |\ell|^2 |\tilde{\ell}|^2 + \lambda^2 |S|^2 (|\ell|^2 + |\tilde{\ell}|^2) + \frac{T^2}{16} (3g^2 + g'^2) (|\ell|^2 + |\tilde{\ell}|^2), \quad (19) \]

\[ V_q = -\lambda m^2 (q \bar{q} + \text{h.c.}) + \lambda^2 |q|^2 |\bar{q}|^2 + \lambda^2 |S|^2 (|q|^2 + |\bar{q}|^2) + \frac{T^2}{16} (8g^2 + g'^2) (|q|^2 + |\bar{q}|^2), \quad (20) \]

where \( \ell \) and \( q \) denote the scalar components of the messenger field \( f \). One can see from the potential that the messenger direction becomes unstable at \( S \approx 0 \), when the temperature becomes lower than \( T_{cr} \):

\[ T_{cr} = 4m \sqrt{\frac{\lambda}{3g^2 + g'^2}}. \quad (21) \]

As \( S \) develops a VEV, the critical temperature goes down.

We also define a temperature \( T_S \) at which \( S \) field exits the region where the messenger direction is unstable at zero temperature, i.e.,

\[ S_{\text{min}}(T_S) = S_{cr}, \quad (22) \]

which gives

\[ T_S^2 \gtrsim \frac{8 \sqrt{3}}{15} \frac{m^3}{\lambda^{3/2} M_{pl}}. \quad (23) \]

In order for the \( S \) field to reach the SUSY breaking vacuum without falling into the SUSY preserving one, the condition

\[ T_S > T_{cr} \quad (24) \]

has to be satisfied\(^\dagger\), i.e., the \( S \) field should leave the dangerous region (the vicinity of \( S = 0 \)) before the messenger direction becomes tachyonic. This condition is converted to a constraint on the model parameters,

\[ \lambda < \left[ \frac{8 \sqrt{3}}{15} \frac{3g^2 + g'^2}{16} \frac{m}{M_{pl}} \right]^{2/5}. \quad (25) \]

We show the allowed region in Fig. \[\dagger\] From now on, we focus the discussion on the blue region where the pseudo-modulus successfully reaches the SUSY breaking vacuum.

\(^\dagger\)Precisely speaking, this is a sufficient condition, but we have confirmed that this is consistent with the numerical results.
Figure 1: SUSY breaking vacuum \( \langle S \rangle \) appears in the white and blue regions. Starting from the origin at high temperature, \( S \) field successfully reaches the SUSY breaking vacuum in the blue region. The model parameter \( m \) is taken to be \( m = 1.7 \times 10^{-7} \Lambda \) so that the Bino mass is fixed to be 300GeV.

3.2 Coherent oscillations

Let us examine the evolution of \( S \) direction more closely. We will find that there are various qualitatively different possibilities for the \( S \) motion after the inflation.

During inflation, we assume that \( S \) was stabilized near the origin by the positive Hubble-induced mass term. After inflation, the \( S \) follows the time-dependent minimum \( (17) \). When the thermal mass becomes comparable to the tree-level mass, the minimum \( S_{\text{min}}(T) \) quickly moves to the SUSY breaking minimum. This transition takes place at \( T \approx T_0 \), given by

\[
T_0 \equiv \frac{4}{\sqrt{5}} \frac{m^2}{\Lambda \lambda}.
\]

Whether or not the \( S \) field catches up the motion of the minimum depends on the competition between the effective mass of \( S \) and the friction caused by the expansion of the Universe. Here it is assumed that the messenger fields remain thermalized at \( T = T_0 \). Later we study the case where the messenger fields decouple from the thermal plasma before the temperature of the Universe goes down to \( T_0 \).
The dynamics of $S$ field is governed by the equation of motion
\[ \ddot{S} + 3H \dot{S} + \frac{\partial}{\partial S} V = 0. \]  
(27)
The effective mass of $S$ field is approximately given by the sum of the tree-level mass and the thermal mass,
\[ m_S^2(T) \equiv m_S^2 + \frac{5}{4} \lambda^2 T^2, \]  
(28)
where we have neglected the contribution from the radiative correction to the Kähler potential \[ [3], \] since it does not affect results. In the numerical calculations, the effect of the radiative correction is properly taken into account.

First let us consider the case that the Hubble parameter is larger than the effective mass at $T = T_0$, i.e., $H(T_0) > m_S(T_0)$. In this case, even if the potential minimum moves to the zero temperature value at $T = T_0$, $S$ is still trapped near the origin because of the large friction. At a later time when the Hubble parameter becomes comparable to the effective mass, $H \sim m_S$, $S$ leaves the vicinity of the origin and starts oscillations about the minimum. We define the temperature $T_{osc}$ as
\[ H(T_{osc}) = m_S(T_{osc}), \]  
(29)
where the temperature dependence of the Hubble parameter is given by $H(T) \sim T^2 M_{\text{pl}}$ and $H(T) \sim T^4 M_{\text{pl}}$ in the radiation and inflaton-matter dominated eras, respectively. The condition $H(T_0) > m_S(T_0)$ is equivalent to $T_0 > T_{osc}$.

On the other hand, if the Hubble parameter is already smaller than the effective mass at $T = T_0$, $H(T_0) < m_S(T_0)$, or equivalently $T_0 < T_{osc}$, the friction from the expansion of the Universe is small. This is the case if $\lambda$ is larger than the previous case. Then the $S$ field follows the time-dependent potential minimum and gradually reaches the SUSY breaking vacuum. The amplitude of oscillations is highly suppressed in this case. The suppression was first found in Ref. [49], in which the oscillation amplitude was shown to be exponentially suppressed in a limiting case\[ in the context of the cosmological moduli problem. This adiabatic suppression mechanism was recently examined more carefully in Ref. [50]. We show in Fig. 2 the typical evolution of $S$ in the above two cases.

There is yet another possibility. As the temperature of the Universe decreases and the value of the pseudo-modulus becomes sizable, the high temperature approximation in Eq. (16)\[ 
\[ \text{The initial condition adopted in Ref. [19] was given at an infinitely large Hubble parameter. There are various additional contributions in general, which are only power-suppressed [50]. We have numerically confirmed that the pseudo-modulus abundance is power-suppressed in our scenario.} \]
Figure 2: If $m_S(T_0) < H(T_0)$, $S$ starts coherent oscillations around $\langle S \rangle$ when $H \sim m_S$ (case A). The thermal mass increases as $\lambda$, and the adiabatic suppression takes place (case B). The bottom panel show the case of $m_S(T_0) > H(T_0)$, where one can see that $S$ follows the minimum $S_{\text{min}}(T)$ with suppressed oscillations. We set $\Lambda = 10^{16.5} \text{GeV}$.

breaks down at a certain point. The messenger fields become non-relativistic when the temperature of the Universe becomes comparable to the messenger mass. Then the finite temperature potential generated by messenger interactions gets suppressed by the Boltzmann factor $\sim e^{-\lambda S/T}$. We define the decoupling temperature $T_{\text{dec}}$ as

$$T_{\text{dec}} \equiv \lambda S(T = T_{\text{dec}}).$$

(30)

If the messenger fields decouple after $S$ field reaches the SUSY breaking vacuum, $T_{\text{dec}}$ is simply the messenger mass scale,

$$T_{\text{dec}} \simeq \lambda \langle S \rangle$$

(31)

for $T_{\text{dec}} < T_0$. If the decoupling occurs when $S$ is still on the way to the SUSY breaking
Figure 3: If the messenger fields become non-relativistic before $S$ reaches the zero temperature SUSY breaking vacuum $\langle S \rangle$, the thermal mass term quickly disappears and $S$ field begin to oscillate about $\langle S \rangle$. We set $\Lambda = 10^{16.5}$ GeV.

$vacuum$, $T_{\text{dec}}$ is calculated to be

$$T_{\text{dec}} \simeq \left[ \frac{8\sqrt{3}}{15} \frac{m^4}{\lambda M_{\text{pl}}} \right]^{1/3}$$

for $T_{\text{dec}} > T_0$. In this case, at $T = T_{\text{dec}}$, the position of the potential minimum instantly moves from $S_{\text{min}}(T)$ to the SUSY breaking vacuum, which triggers coherent oscillations about the minimum (see Fig. 3).

In summary, we have defined three temperatures: $T_0$, $T_{\text{osc}}$ and $T_{\text{dec}}$.

• The potential minimum quickly moves from the origin to the SUSY breaking vacuum at $T = T_0$.

• The Hubble parameter $H$ becomes comparable to the pseudo-modulus mass $m_S(T)$ at $T = T_{\text{osc}}$.

• The messenger fields become non-relativistic and disappear from the thermal plasma at $T = T_{\text{dec}}$.

The evolution of $S$, and therefore, its abundance, sensitively depends on the relations among these temperatures. The following three cases are shown in Fig. 4 for several reheating temperatures. Note that the abundance of $S$ is suppressed in the second case (white region), while it is significant in the other cases.

• $T_0 > T_{\text{osc}}$ (Green region) : $S$ starts coherent oscillation about $\langle S \rangle$ at $T = T_{\text{osc}}$.  

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Figure 4: The evolution of $S$ exhibits distinctive behavior depending on the model parameters. In the green region ($T_0 > T_{\text{osc}}$) $S$ starts coherent oscillations when $H \approx m_S(T)$. The $S$ abundance is suppressed by the adiabatic suppression mechanism in the white region ($T_{\text{osc}} > T_0 > T_{\text{dec}}$). In the blue region ($T_{\text{osc}} > T_{\text{dec}} > T_0$) the coherent oscillations are triggered when messenger fields decouple from thermal plasma and disappear. While $T_0$ and $T_{\text{dec}}$ are uniquely determined once we fix the model parameters, the value of $T_{\text{osc}}$ depends on $T_R$.

- $T_{\text{osc}} > T_0 > T_{\text{dec}}$ (White region) : $S$ follows $S_{\text{min}}(T)$ and gradually reaches $\langle S \rangle$ without sizable oscillations.

- $T_{\text{osc}} > T_{\text{dec}} > T_0$ (Blue region) : The messenger fields decouple from thermal plasma when $S$ is on the way to $\langle S \rangle$. Coherent oscillations are triggered at $T = T_{\text{dec}}$. 


4 Thermal production of the gravitinos

In the framework of gauge mediation the dominant component of thermally produced gravitino is that of the longitudinal mode, the goldstino. The relic abundance is estimated to be \[ \Omega_{3/2}^{\text{th}} \approx 0.3 \left( \frac{\text{GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{\text{TeV}} \right)^2 \left( \frac{T_R}{10^5 \text{GeV}} \right). \] (33)

where \( T_R \) represents the reheating temperature after inflation, and it is assumed that there is no entropy production after the reheating. Note however that it is assumed implicitly in deriving this formula that the messenger mass scale is higher than \( T_R \). For \( T_R \) higher than the messenger scale, one needs to consider diagrams in which the messenger fields are in the external lines. As shown in Ref. [12], if \( T_R \) is higher than the messenger mass scale, or equivalently if the messenger fields once gets thermalized, the goldstino relic abundance is determined by the messenger mass scale \( M_{\text{mess}} = \lambda \langle S \rangle \) rather than the reheating temperature,

\[ \Omega_{3/2}^{\text{th}} \approx 0.8 \left( \frac{\text{GeV}}{m_{3/2}} \right) \left( \frac{m_{\tilde{g}}}{\text{TeV}} \right)^2 \left( \frac{M_{\text{mess}}}{10^5 \text{GeV}} \right) \quad (T_R > M_{\text{mess}}). \] (34)

The replacement of \( T_R \) by \( M_{\text{mess}} \) can be understood by looking at the temperature dependence of the goldstino reaction rate \( \Gamma(T) \):

\[ \Gamma(T) \sim \alpha_3 \lambda^2 T, \quad \text{for} \quad T \gtrsim M_{\text{mess}} \] (35)
\[ \Gamma(T) \sim \alpha_3 \frac{m_{\tilde{g}}^2}{m_{3/2}^2 M_{\text{pl}}^2} T^3 \quad \text{for} \quad T \lesssim M_{\text{mess}}. \] (36)

As \( \Gamma(T) \) has lower power dependence on \( T \) than the Hubble parameter \( (H \propto T^2) \) for \( T \gtrsim M_{\text{mess}} \), contributions to the goldstino production from MSSM fields decouple for \( T \gtrsim M_{\text{mess}} \). We should estimate the thermally produced gravitino abundance by Eq.(34) since the pseudo-modulus was at the origin and the messenger fields were inevitably thermalized in our scenario.

\footnote{It has been argued in Ref. [13] that there is a component of the interaction rate which still grows as \( T^3 \) at high temperatures, and thus the estimation of the gravitino abundance is sensitive to \( T_R \) in contrast to the conclusion of Ref. [12]. This is based on the observation that there are contact interactions between the gravitino and visible sector fields in the supergravity Lagrangian in the unitary gauge. However, we anticipate a cancellation of such contributions with loop diagrams at a high temperature since there is no such contribution in the goldstino picture.}
5 Pseudo-modulus decay and the gravitino abundance

In this section we examine the decays of the pseudo-modulus. So far we have assumed that there is no additional entropy production after reheating. If the oscillation energy of the pseudo-modulus dominates the Universe, however, a large amount of entropy is produced by the decay, and the pre-existing gravitino is diluted. Also the gravitinos are produced by the decay of the pseudo-modulus.

5.1 $S$-domination

The energy density of $S$ decreases more slowly than radiation, and so, if its lifetime is sufficiently long or the reheating temperature is high, $S$ may come to dominate the Universe and produce a large amount of entropy by the decay. Let us define the entropy dilution factor $\Delta$ as

$$\frac{1}{\Delta} \equiv \frac{s_{\text{inf}}}{s_S + s_{\text{inf}}} \simeq \text{Min} \left[ 1, \frac{s_{\text{inf}}}{s_S} \right],$$

where $s_{\text{inf}}$ and $s_S$ represent the entropy density produced from the inflaton and the pseudo-modulus, respectively. If $\Delta > 1$, the pre-existing gravitino abundance is diluted by a factor of $1/\Delta$. In this case, we can express the dilution factor in terms of the decay temperature of $S$, $T_d$, and its abundance as

$$\Delta \simeq \frac{s_S}{s_{\text{inf}}} = \frac{4}{3T_d} \cdot \left. \frac{\rho_S}{s_{\text{inf}}} \right|_{S \text{ decay}}.$$  

(38)

The pseudo-modulus abundance, $(\rho_S/s_{\text{inf}})$, can be estimated by following its evolution as discussed in Sec. 3. The decay temperature $T_d$ is defined as

$$T_d \equiv \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_{\text{pl}} \Gamma_S},$$

(39)

where $\Gamma_S$ is the decay width of $S$.

The decay width of $S$ can be explicitly calculated. The $S$ field couples to the MSSM particles through loop diagrams of the messenger fields. The partial decay width of the two gravitino mode is suppressed compared to the MSSM particles. The interaction Lagrangians between $S$ and MSSM fields and calculations of decay widths can be found in Refs. [28, 29]. The main decay mode is $S \to b\bar{b}$ and $S \to hh + WW + ZZ$ for $2m_b < m_S < 2m_W$ and $2m_W < m_S \lesssim 1 \text{TeV}$ cases, respectively. The decay temperature is calculated to be $\mathcal{O}(0.1 - 10) \text{GeV}$ in the parameter region of our interest. (See Refs. [28, 29] for formulae and parameter dependences.)
Figure 5: The contour plot of the dilution factor $\Delta$, for $T_R = 10^5$, $10^7$, and $10^{12}$ GeV. The colored regions correspond to the regions where the pseudo-modulus dominates the Universe before the decay. In the case of $T_R = 10^{12}$ GeV, the oscillations of $S$ commence after the reheating, and the $S$ tends to dominate the Universe, producing larger entropy compared to the other two cases with lower $T_R$.

We evaluate the dilution factor numerically. We solve the equation of motion of the pseudo-modulus which provides the pseudo-modulus abundance, $\rho_S/s_{\text{inf}}$. For the potential at finite temperatures, we use the one in Eq. (44). The decay temperature is calculated by using the formulae in Refs. [28, 29]. The results are shown in Fig. 5. The colored regions ($\Delta > 1$) correspond to the regions where the oscillation energy dominates the energy density.
of the Universe. One can see that the $S$ abundance is indeed suppressed in the middle of each panel, because of the adiabatic suppression mechanism. The similarity with Fig. 4 is clear especially in the case of $T_R = 10^{12}$ GeV.

5.2 Non-thermally produced gravitino

Gravitinos are produced non-thermally by the rare decay $S \rightarrow \psi_{3/2}\psi_{3/2}$. The branching ratio of the gravitino mode, $B_{3/2}$, is calculated in Refs. 28, 29, and it is typically of $O(10^{-10} - 10^{-6})$ in the parameter regions of our interest.

If $S$ dominates the energy density of the Universe, non-thermal gravitino abundance is calculated as

$$\Omega_{3/2}^{NT} = \frac{3}{4} m_{3/2} \frac{T_d}{m_S} B_{3/2} \times \frac{2}{(\rho_c/s)_0},$$

where $(\rho_c/s)_0 \simeq 3.6 \times 10^{-9} h^{-2}$GeV. We will see later that there are parameter regions where the DM relic density is explained by the non-thermally produced gravitinos.

5.3 Total gravitino abundance

The relic density of the gravitinos is given by the sum of the thermally and non-thermally produced gravitinos. If the $S$ dominates the Universe before the decay, we need to take account of the entropy factor, which dilutes the pre-existing gravitinos produced at the reheating;

$$\Omega_{3/2}^{tot} = \frac{1}{\Delta} \Omega_{3/2}^{th} + \Omega_{3/2}^{NT}.$$  

Here the dilution factors are evaluated numerically as in Fig. 5. Note that $T_d$ is so low that the thermal production of gravitinos is negligible at the pseudo-modulus decay.

We show in Fig. 6 the contours of the total gravitino abundance for $T_R = 10^5, 10^7$ and $10^{12}$ GeV. Considering $O(1)$ uncertainties in the calculations of dilution factor and thermal gravitino productions, we show the highlighted region (in red) where the gravitino abundance $0.1 \leq \Omega_{3/2} \leq 1$ is obtained. We expect that somewhere in this region will provide the observationally consistent DM abundance, $\Omega_{DM} = 0.2$. For $T_R = 10^5$ GeV, the relic gravitino is mostly thermally produced one. On the other hand, for $T_R = 10^{12}$ GeV, there is an allowed region where the dilution factor $\Delta$ is about $10^3 - 10^6$. In this region, the observed DM abundance is a mixture of the thermally and non-thermally produced gravitinos. It
is crucial to take account of the fact that the gravitino abundance is independent of the reheating temperature as in Eq. (34).

We emphasize here that the contours of the gravitino abundance remain almost intact for \( T_R \gtrsim 10^9 \text{GeV} \). This is because the gravitino production rate is modified at a temperature above the messenger scale. This has a crucial impact on the leptogenesis as we shall see below.

The gravitino mass in the shaded region ranges from 10 MeV to a few GeV (See Eq. (15)). The thermally produced gravitinos therefore behave as cold dark matter. On the other hand, the non-thermally produced gravitinos have a relatively large velocity, which results in a free streaming length of \( \mathcal{O}(10 - 100) \text{ kpc} \) depending on the parameters [28, 29]. In our scenario, the DM consists of a mixture of thermally and non-thermally produced gravitinos for \( T_R \gtrsim 10^7 \text{GeV} \), and such partial suppression of the density perturbation at small scales may have an interesting impact on the large scale structure of the Universe.

### 5.4 Leptogenesis

Interestingly, in this model, it is possible to create the right amount of the baryon asymmetry through the thermal leptogenesis [10] while satisfying \( \Omega_{3/2} / 2 \lesssim 0.2 \).

In the thermal leptogenesis, there is an upper bound on the baryon asymmetry for a fixed reheating temperature [51, 52, 11]:

\[
\eta_B \lesssim 5 \times 10^{-11} \left( \frac{T_R}{10^9 \text{GeV}} \right). \tag{42}
\]

This often causes a tension with the gravitino overproduction. In the presence of the late-time entropy production, one can increase the reheating temperature because the thermally produced gravitinos are diluted. However, this does not solve the tension because both the baryon asymmetry and the conventional formula of the graviton abundance [33] are proportional to \( T_R \).

In our scenario, the reheating temperature can be much higher than \( 10^9 \text{ GeV} \) without having a problem of gravitino overproduction, because the gravitino abundance becomes independent of the reheating temperature for sufficiently high \( T_R \). Recall that the contours of the gravitino abundance in Fig. 6 are almost the same for \( T_R \gtrsim 10^9 \text{GeV} \).

By comparing Fig. 5 with Fig. 6, we see that the size of the dilution factor \( \Delta \) is about \( \sim 10^3 - 10^6 \) in the region where \( \Omega_{3/2} \simeq 0.1 - 1 \). Therefore, if the reheating temperature is higher than \( \sim 10^{12} \text{GeV} \), even though it is partially diluted, the observed amount of baryon
Figure 6: The contour plot of the total gravitino abundance $\Omega_{3/2}^{\text{tot}}$. Red regions correspond to DM consistent regions $0.1 \leq \Omega_{3/2} \leq 1$. In $T_R = 10^5 \text{GeV}$ case thermally produced gravitino comes dominantly from the MSSM sector. The contribution from the messenger sector becomes comparable to that of the MSSM sector in $T_R = 10^7 \text{GeV}$. In $T_R = 10^{12} \text{GeV}$ case the thermal component consists of the messenger sector contribution. Non-thermal components from the decay of the pseudo-modulus field are also included in the figures.

Asymmetry can be generated by thermal leptogenesis. This is one of the distinctive features of our scenario.
6 Summary

We have followed the cosmological evolutions of the SUSY breaking pseudo-modulus field and the messenger fields in a simple gauge mediation model which break SUSY at a metastable state. We adopt an initial condition such that the pseudo-modulus field is close to the origin, stabilized by thermal potential via interactions with the messenger fields in thermal plasma. Such an initial condition can be naturally realized if the $U(1)_R$ symmetry remains a good symmetry during inflation and $S$ is stabilized near the origin due to the positive Hubble-induced mass. We have found that this simple gauge mediation model with the initial condition is cosmologically viable in a sense that the pseudo-modulus can settle down at the correct SUSY breaking minimum and that the gravitino relic abundance can explain the DM of the Universe. Furthermore, thermal leptogenesis is possible without the gravitino overproduction.

We have numerically calculated the total gravitino relic abundance, both the thermally produced one taking account of the entropy dilution and the non-thermally produced one, and shown that the observed DM density can be explained for $m_{3/2} = \mathcal{O}(10)\text{MeV} - \mathcal{O}(1)\text{GeV}$.

The messenger fields play a crucial role in the scenario. The messengers acquire a thermal mass when the pseudo-modulus stays near the origin, which prevents the messengers to fall into the SUSY preserving minimum. At the reheating, the gravitino production rate is modified if the messenger fields are in the thermal plasma. This has a great impact on the gravitino abundance; it becomes independent of the reheating temperature of the Universe. It is this fact that enables thermal leptogenesis to create the right amount of baryon asymmetry without overproduction of gravitinos.

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A Pseudo-modulus potential at a finite temperature

The finite temperature effective potential up to one-loop is given by

\[ V = V_{\text{tree}} + V_1 + V_{\text{thermal}}, \]  

(43)

where \( V_{\text{tree}} \) is the classical potential calculated from Eq.(1) and (2) and \( V_1 \) is the zero-temperature one-loop potential calculated from Eq.(3). Finite temperature one-loop correction is

\[ V_{\text{thermal}} = \frac{T^4}{2\pi^2} \left[ \int_0^\infty dx x^2 \sum_i \log[1 - e^{-\sqrt{x^2 + (M_S^2)^i}/T}] \right. \]

\[ -2 \int_0^\infty dx x^2 \sum_r \log[1 + e^{-\sqrt{x^2 + (M_F^2)^r}/T}] + 3 \int_0^\infty dx x^2 \sum_a \log[1 - e^{-\sqrt{x^2 + (M_V^2)^a}/T}] \left. \right], \]

(44)

where the three terms represent the contributions from real scalar fields \( \phi_i \), Weyl fermions \( \psi_r \) and vector bosons \( A_\mu^a \) with the eigenvalues of the squared mass matrices \( (M_S^2)^i \), \( (M_F^2)^r \) and \( (M_V^2)^a \). For a high temperature limit of \( T \gg M_S, M_F, M_V \), the potential can be expanded as

\[ V_{\text{thermal}} = -\frac{\pi^2 T^4}{90} \left( 7 \frac{N_F}{8} N_B \right) + \frac{T^2}{24} \left[ \text{Tr}(M_S^2) + 3\text{Tr}(M_V^2) + \text{Tr}(M_F^2) \right] \]

\[ - \frac{T}{12\pi} \left[ \text{Tr}(M_S^3) + 3\text{Tr}(M_V^3) \right] + \cdots. \]

(45)

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