Seeding primordial black holes in multi-field inflation

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The inflationary origin of primordial black holes (PBHs) relies on a large enhancement of the power spectrum $\Delta_\zeta$ of the curvature fluctuation $\zeta$ at wavelengths much shorter than those of the cosmic microwave background anisotropies. This is typically achieved in models where $\zeta$ evolves without interacting significantly with additional (isocurvature) scalar degrees of freedom. However, quantum gravity inspired models are characterized by moduli spaces with highly curved geometries and a large number of scalar fields that could vigorously interact with $\zeta$ (as in the cosmological collider picture). Here we show that isocurvature fluctuations can mix with $\zeta$ inducing large enhancements of its amplitude. This occurs whenever the inflationary trajectory experiences rapid turns in the field space of the model leading to amplifications that are exponentially sensitive to the total angle swept by the turn, which induce characteristic observable signatures on $\Delta_\zeta$. We derive accurate analytical predictions and show that the large enhancements required for PBHs demand non-canonical kinetic terms in the action of the multi-field system.

Introduction.— Unlike their astrophysical counterparts, primordial black holes (PBHs) \cite{1,2} might have stemmed from large statistical excursions of the primordial curvature fluctuation $\zeta$ generated during the pre-Big-Bang era known as cosmic inflation \cite{3–7}. Once inflation is over, these fluctuations can induce overdense regions with matter that, collapsing under the pull of gravity, give birth to PBHs. In the correct abundance, these provide an excellent dark matter candidate \cite{8–12}, braiding early and late Universe dynamics.

Cosmic microwave background (CMB) observations over the range of scales $10^{-3} \text{ Mpc}^{-1} \lesssim k \lesssim 10^{-1} \text{ Mpc}^{-1}$, tell us that, once inflation was over, $\zeta$ ended up distributed according to a Gaussian statistics determined by a nearly scale invariant power spectrum $\Delta_\zeta(k)$ \cite{13,14}. If these properties persisted all the way up to the smallest cosmological wavelengths, large statistical fluctuations of $\zeta$ would constitute extremely rare events, preventing the formation of PBHs. It is by now well understood that inflationary PBHs, with an abundance compatible with dark matter \cite{15}, would require a strong violation of scale invariance for $k \gtrsim 10^8 \text{ Mpc}^{-1}$, in the form of a large enhancement of $\Delta_\zeta(k)$, by a factor of at least $10^7$, with respect to its CMB value.

Such an amplification can be achieved in several ways. Examples include: single-field models in which the inflaton potential is finely adjusted to have a plateau \cite{16,17}; single-field models with resonant backgrounds \cite{18}; models with light spectator fields \cite{19,20}; models where the inflaton shares an axion-like coupling with gauge fields \cite{21,22}; and models with resonant instabilities during the pre-heating inflaton’s decay \cite{23}. In this work, we are particularly interested in re-examining the inflationary enhancement of $\Delta_\zeta(k)$ within the paradigm of multi-field inflation \cite{24–27}. Ultraviolet complete frameworks (such as supergravity and string theory) unavoidably lead to models with a variety of fields charting multi-dimensional target spaces with curved geometries. The effective field theory description of these models, valid during inflation, includes potentially sizable interactions between $\zeta$ and other (isocurvature) fluctuations \cite{28–30}, an idea that, recently, has drawn considerable attention through the cosmological collider program \cite{31,32}.

The purpose of this letter is to show that a purely multi-field mechanism can indeed lead to enhancements of $\Delta_\zeta(k)$ large enough to produce PBHs abundantly. In multi-field inflation, the leading interaction between $\zeta$ and isocurvature fluctuations is found to be proportional to the angular velocity $\Omega$ with which the inflationary trajectory experiences turns in the target space. The total angle swept by the turning trajectory is of order $\delta \theta \sim \Omega \delta t$, where $\delta t$ is the duration of turn. Remarkably, we will derive an accurate analytical prediction for $\Delta_\zeta(k)$ generated by sudden turns, valid in the rapid turn regime \cite{28,29,33–40}, where $\Omega$ is much greater than $H$, the Hubble expansion rate during inflation. This will allow us to show that under a turn of total angle $\delta \theta$, the power spectrum is enhanced exponentially as

$$\Delta_\zeta \sim \frac{e^{2\delta \theta}}{4(1 + 4\delta \theta^2)} \times \Delta_{\zeta,\text{CMB}},$$

where $\Delta_{\zeta,\text{CMB}}$ is its amplitude at CMB scales. Thus, in order to have a large enhancement of around $10^7$, the total angle $\delta \theta$ swept by a turning trajectory must be about $4\pi$. Since in models with canonical kinetic terms this is bounded as $\delta \theta < \pi$, we conclude that a large enhancement can only be achieved in models with curved target spaces (with non-canonical kinetic terms) as encountered in ultraviolet complete theories. As a consequence, our proposal opens up a new window of opportunity to test the existence of additional degrees of freedom interacting with the primordial curvature perturbation during inflation.
Multi-field inflation.— To set the context, we begin by reviewing some general aspects of multi-field inflation in the particular case of two fields. We consider a general action of the form

\[ S = S_{\text{EH}} - \int d^4 x \sqrt{-g} \left[ \frac{1}{2} \gamma_{ab} \partial_a \phi^o \partial^b \phi^b + V(\phi) \right], \]  
(2)

where \( S_{\text{EH}} \) is the Einstein-Hilbert term constructed from a spacetime metric \( g_{\mu\nu} \) with determinant \( g \) and \( \gamma_{ab} \) is a \( \sigma \)-model metric characterizing the geometry of the target space spanned by the fields \( \phi^a = (\phi^1, \phi^2) \).

Spatially flat cosmological backgrounds can be described by a line element of the form \( ds^2 = -dt^2 + a^2 dx^2 \), where \( a = a(t) \) is the usual scale factor. In these spacetimes, the background scalar fields \( \phi_0^a(t) \) satisfy the following equations of motion:

\[ D_t \phi_0^a + 3H \phi_0^a + \gamma^{ab} V_b(\phi_0) = 0, \]  
(3)

where \( H = \dot{a}/a \) is the Hubble expansion rate, \( V_a = \partial V/\partial \phi^a \), and \( \gamma^{ab} \) is the inverse of \( \gamma_{ab} \). In addition, \( D_t \) is a covariant derivative whose action on a vector \( A^a \) is given by \( D_t A^a = \dot{A}^a + \Gamma^d_{bc} \dot{\phi}^b \phi^c A^d \), where \( \Gamma^d_{bc} \) are Christoffel symbols derived from \( \gamma_{ab} \). Equation (3) must be solved together with the Friedmann equation \( 3H^2 = \frac{1}{2} \dot{\psi}^2 + V(\phi_0) \), where \( \phi_0^a \equiv \gamma_{ab} \phi_0^b \) (we work in units where the reduced Planck mass is 1). With appropriate initial conditions, this system yields a path \( \phi_0^a(t) \) with tangent and normal unit vectors defined as \( T^a \equiv \dot{\phi}^a / \dot{\psi} \) and \( N^a \equiv -\frac{1}{H} D_t T^a \), where \( \Omega = -N_t D_t T^a \) is the angular velocity which with the path bends. Successful inflation requires that \( H \) stay nearly constant. This is achieved by demanding a small first slow-roll parameter \( \epsilon \equiv -H/H^2 \ll 1 \). For simplicity, in what follows we assume that \( \epsilon \) can be treated as a constant, however, we allow for an arbitrary time-dependent angular velocity \( \Omega \). This corresponds to situations where the inflationary path can experience turns in the target space without affecting the expansion rate significantly. Our main conclusions will not depend on these assumptions.

We may now perturb the system as \( \phi^a(x, t) = \phi_0^a(t) + T^a(t) \phi(x, t) + N^a(t) \psi(x, t) \), where \( \psi \) is the isocurvature fluctuation [11, 42]. In co-moving gauge \( (\psi = 0) \) we define the primordial curvature fluctuation \( \zeta \) by perturbing the metric as \( ds^2 = -N^2 dt^2 + a^2 \delta(\mathbf{x}) + \mathbf{N} dt^2 \), where \( \mathbf{N} \) and \( \mathcal{N} \) are the lapse function and the shift vector. Inserting these definitions back into (2) leads to an action for \( \zeta, \psi, \mathcal{N} \) and \( \mathbf{N} \). The lapse and shift functions respect constraint equations that can be solved in terms of \( \zeta \) and \( \psi \) order by order, finally leading to the Lagrangian \( \mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{iso}} \), with

\[ \mathcal{L}_{\text{kin}} = \frac{a^3}{2} \left[ (D_t \zeta)^2 - \frac{1}{a^2} (\nabla \zeta)^2 + \psi^2 - \frac{1}{a^2} (\nabla \psi)^2 \right], \]  
(4)

and \( \mathcal{L}_{\text{iso}} = -a^3 U(\psi) \), where \( U \) is a potential for the isocurvature field. In the previous expression, \( \zeta \equiv \sqrt{2} \zeta \) is the normalized version of \( \zeta \), and we have introduced a covariant derivative \( D_t \) defined as

\[ D_t \zeta \equiv \dot{\zeta} - \lambda H \psi, \quad \lambda \equiv 2 \Omega/H, \]  
(5)

reminiscent of the covariant derivative in (3). The background quantity \( \lambda(t) \), which is non-vanishing whenever the trajectory experiences turns in multi-field space, plays a prominent role in multi-field inflation: it couples \( \zeta \) and \( \psi \) at quadratic order in a way that cannot be trivially field-redefined away [43]. As we will show, it can also lead to a large enhancement of the amplitude of \( \zeta \).

Note that in (4) we have disregarded terms that are gravitationally suppressed. For instance, we are omitting terms multiplied by \( N - 1 = \zeta_c/\sqrt{2} \lambda H \), which is much smaller than 1 in the weak gravity regime. In what follows, we shall also disregard the self-interactions of \( \psi \) by setting \( U = 0 \). We will comment on the important role of \( U \) toward the end of this work.

Mild mixing between \( \zeta \) and \( \psi \).— In the particular limit \( \lambda \ll 1 \) the interaction between \( \zeta \) and \( \psi \) can be fully understood analytically [44]. Here, (4) can be split as

\[ \mathcal{L}_{\text{kin}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{mix}}, \]  

where \( \mathcal{L}_{\text{free}} \) is the free part of the theory, given by

\[ \frac{1}{a^2} \mathcal{L}_{\text{free}} = \frac{1}{2} \zeta^2 - \frac{1}{2 a^2} (\nabla \zeta)^2 + \frac{1}{2} \psi^2 - \frac{1}{2 a^2} (\nabla \psi)^2, \]  
(6)

and \( \mathcal{L}_{\text{mix}} \) is the interacting part, given by

\[ \mathcal{L}_{\text{mix}} \equiv -a^3 \lambda H \zeta_c \psi. \]  
(7)

In this limit, \( \zeta_c \) and \( \psi \) are massless scalar fields, interacting through the mixing term proportional to \( \zeta_c \psi \) coming from \( D_t \zeta_c \) in (4). In Fourier space, the solutions of the linear equations derived from the free theory (6) are

\[ \zeta_c^* \psi(k, t) = u(k, t) a_c(k) + u^*(k, t) a_c^*(\pi - k), \]  
(8)

\[ \psi^* \zeta_c(k, t) = u(k, t) a_\psi(k) + u^*(k, t) a_\psi^*(\pi - k), \]  
(9)

where \( a_{\zeta_c}(k) \) and \( a_{\psi}(k) \) are creation and annihilation operators (ensuring that \( \zeta_c \) and \( \psi \) are quantum fields) satisfying \( \{a_{\zeta_c}(k), a_{\psi}(q)\} = (2\pi)^3 \delta_{ab} \delta^{(3)}(k - q) \), with every other commutator vanishing. In addition, \( u(k, t) \) is the standard de Sitter mode function respecting Bunch-Davies initial conditions:

\[ u(t, k) = \frac{i H}{\sqrt{2k}} e^{ik\tau(t)} e^{-ik\tau(t)}, \]  
(10)

where \( \tau(t) = -1/Ha(t) \), with \( a(t) \propto e^{Ht} \). If \( \lambda = 0 \) the dynamics of \( \zeta \) is of the single-field type, with a solution given by (8), leading to a power spectrum of the form

\[ \Delta_{\zeta, 0} = \frac{H^2}{8\pi^2 \epsilon}, \]  
(11)

with small slow-roll corrections breaking its scale invariance —recall that the dimensionless power spectrum
\( \Delta \zeta(k) \) is defined in terms of the 2-point correlation function of \( \zeta \) as \( \langle \zeta(k) \zeta(q) \rangle = (2\pi)^3 \delta^3(k-q) 2\pi^2 \Delta \zeta(k) \). On the other hand, if \( \lambda \) is small but non-vanishing, \( \psi \) sources \( \zeta \) via the mixing in (7), and one finds \( \zeta_c^2 = \dot{\zeta}_0^2 + 2\Delta \theta \dot{\psi}_0(k), \) \( \bar{\psi} = \dot{\psi}_0(k), \) \( \Delta \theta(k) = \frac{1}{2} \int_{t_0}^{t} \lambda H dt \) is the total angle swept by the turning trajectory felt by the mode from horizon crossing time \( t_{hc} = H^{-1} \ln(k/H) \) until the end of inflation. It immediately follows that the power spectrum of \( \zeta \) is given by \[ \Delta \zeta = \left[ 1 + 4\Delta \theta^2(k) \right] \times \Delta \zeta_{c,0}, \] which is larger than (11) by a factor \( 1 + 4\Delta \theta^2(k) \). Equation (12) shows that for small \( \lambda \), all superhorizon modes are equally enhanced while the turn is taking place, leading to the power spectrum (14) that has more amplitude on long wavelengths, independently of the form of \( \lambda(t) \). This is incompatible with a large enhancement of \( \Delta \zeta \) on a range of wavelengths shorter than those of the CMB, forcing us to consider the regime \( \lambda \gg 1 \), which is the subject of the rest of this letter.

**Strong mixing between \( \zeta \) and \( \psi \).** As we now show, we can study the system analytically in the case of brief turns, even when \( \lambda \) acquires values of order 1 or larger. Does this jeopardize the perturbativity of the system? \( \lambda \) comes exclusively from the kinetic term in (2) which, at quadratic order, gives rise to the covariant derivative \( D_t \zeta_c \). This implies that at higher orders \( \lambda \) will appear in the Lagrangian through operators of order \( (D_t \zeta_c)^2 \) gravitationally coupled to \( \zeta \). The condition granting that the splitting remains weakly coupled is \( \xi^{(2)}/\eta^{(2)} \sim \zeta_c/\sqrt{2\pi} H \ll 1 \), evaluated during horizon crossing. Given that at horizon crossing \( \zeta_c/\sqrt{2\pi} H \ll 1 \), the previous requirement is equivalent to the condition \( \Delta \zeta_c \ll 1 \), which is, by default, satisfied.

Hence, instead of splitting the theory as in (6) and (7), we may keep the mixing term into the full kinetic term of (4). Then, the equations of motion respected by the fluctuations are

\[
\frac{d}{dt} D_t \tilde{\zeta} + 3D_t \tilde{\zeta} + \frac{k^2 \lambda}{a^2} \tilde{\zeta} = 0, \\
\bar{\psi} + 3H \bar{\psi} + \frac{k^2 \lambda}{a^2} \bar{\psi} + \lambda H D_t \tilde{\zeta} = 0.
\]

(15)\( \quad \)

(16)

To proceed, let us consider the case in which \( \lambda \) consists in a top-hat function of the form

\[ \lambda(t) = \frac{\lambda_0}{2} \theta(t-t_1) - \theta(t-t_2), \]

(17)

with a small width \( \delta t \equiv t_2 - t_1 \ll H^{-1} \). This profile describes a trajectory that experiences a brief turn of angular velocity \( \Omega = H\lambda_0/2 \) between the times \( t_1 \) and \( t_2 \). Now, if \( \delta t \ll H^{-1} \), during this brief period of time one can ignore the Hubble friction terms \( 3D_t \tilde{\zeta} \) and \( 3H \bar{\psi} \) in (15) and (16), and fully solve the effects due to \( \lambda \) as if the fluctuations were evolving in a Minkowski spacetime [45]. To be precise, before \( t_1 \), the solutions are exactly those dictated by (8) and (9). Between \( t_1 \) and \( t_2 \) the solutions, which we denote \( \Phi^a = (\zeta_\psi) \), are found to be of the form

\[
\tilde{\Phi}^a(k,t) = \left( A^a_\pm e^{i\omega_{\pm}t} + B^a_\pm e^{-i\omega_{\pm}t} \right) a_\zeta(k) + \left( C^a_\pm e^{i\omega_{\pm}t} + D^a_\pm e^{-i\omega_{\pm}t} \right) a_\psi(k) + h.c.(-k),
\]

(18)

where \( A^a_\pm, B^a_\pm, C^a_\pm \) and \( D^a_\pm \) are k-dependent amplitudes satisfying \( kA^c_\pm = \mp i\omega_\pm A^a_\pm \), \( kB^c_\pm = \pm i\omega_\pm B^a_\pm \), \( kC^c_\pm = \mp i\omega_\pm C^a_\pm \) and \( kD^c_\pm = \pm i\omega_\pm D^a_\pm \), and \( \omega_\pm \) are the respective frequencies, given by

\[ \omega_\pm = \sqrt{\frac{k^2 \pm \omega_0^2}{k_0^2}}, \]

(19)

where \( k_0 \equiv H e^{(t_1+t_2)/2} \) is the wave number of the modes that crossed the horizon when the turn took place. Finally, the solutions after \( t_2 \) are of the form

\[
\tilde{\Phi}^a(k,t) = \left[ E^a u(k,t) + F^a u^*(k,t) \right] a_\zeta(k) + \left[ G^a u(k,t) + H^a u^*(k,t) \right] a_\psi(k) + h.c.(-k),
\]

(20)

where again \( E^a, F^a, G^a \) and \( H^a \) are k-dependent amplitudes, while \( u \) is given in (10). The amplitudes shown in Eqs. (18) and (20) can be determined straightforwardly after imposing continuity of \( \zeta_c(k,t), D_t \tilde{\zeta}(k,t), \psi(k,t) \) and \( \bar{\psi}(k,t) \) at both \( t_1 \) and \( t_2 \).

Equation (18) shows that the coupling \( \lambda_0 \) induces a mixing of fields and modes. As soon as the turn starts, the fluctuation \( \tilde{\zeta}(k,t) \) consists of a linear combination of quanta created and destroyed by the ladder operators \( a_\zeta \) and \( a_\psi \). As a result, at the end of inflation, \( \zeta_c(k) \) contains two contributions, modulated by low and high frequencies:

\[
\tilde{\zeta}_c(k) = \frac{iH}{\sqrt{2k^3}} e^{2i\frac{k_0}{k^3} \sinh[2\Delta]} \sum_{\pm} \left\{ \frac{1}{2} \left[ \cos \left( \frac{\omega_{\pm}\delta N}{k_0} \right) - i\frac{k^2_0}{2k_0} + i\omega_{\pm} \frac{\omega_0}{k_0} \sin \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right] a_\zeta(k) \right. \\
- \left. i e^{2i\frac{k_0}{k} \sinh[2\Delta]} \left[ (k_0 + ik)^2 - \omega^2_0 \right] \frac{\omega_{\pm} \delta N}{k_0} \sin \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right] a_\zeta(k) \left[ \left( k_0 + ik \right)^2 \cos \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right. \\
- \left. (k_0 + ik)^2 \sin \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right] a_\zeta(k) \left[ \left( k_0 - ik \right)^2 \cos \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right. \\
- \left. (k_0 - ik)^2 \sin \left( \frac{\omega_{\pm}\delta N}{k_0} \right) \right] a_\zeta(k) \left\} + h.c.(-k),
\]

(21)
where \( \delta N \equiv H \delta t \) is the duration of the turn in e-folds. A similar solution can be obtained for \( \psi(k) \).

Equation (21) represents our main result. It shows the effect of the \( \zeta/\psi \) mixing resulting from a brief turn, valid for arbitrary values of \( \lambda \). An outstanding feature of (21) is that \( \omega_0 \) becomes imaginary for \( k < \lambda_0 k_0 \), implying an exponential enhancement of \( \zeta_c(k) \). Disregarding oscillatory phases, on scales \( k < \lambda_0 k_0 \) the fluctuation \( \zeta_c(k) \) displays an amplitude \( \propto e^{\sqrt{\lambda_0 k_0 k^2 \delta N / k_0}} \) which attains a maximum value at \( k_{\text{max}} = \lambda_0 k_0 / 2 \). This signals an instability during the turn affecting long wavelength modes (just as in the case with \( \lambda \ll 1 \)) but with its largest effect on scales around \( k_{\text{max}} \). As long as \( 2 \delta \theta \equiv \lambda_0 \delta N \gg 1 \) the instability generates a large enhancement of the power spectrum of \( \zeta \) at the end of inflation.

**PBHs from strong multi-field mixing.**— Let us now use the analytical insight offered by (21) to study the origin of PBHs from inflation due to multi-field effects.

Direct inspection of Eq. (21) tells us that on scales \( k \gg k_0 \lambda / 2, \zeta \) becomes

\[
\zeta(k) = i k^{-3/2} \Delta_{0}^{1/2} a_{\psi}(k) + \text{h.c.}(-k),
\]

from where it follows a conventional single-field scale invariant power spectrum as given by (11). This reveals that modes deep inside the horizon are not affected by the turn, as expected. On the other hand, on scales \( k \ll k_0 \lambda / 2 \) one has

\[
\zeta(k) = i k^{-3/2} \Delta_{0}^{1/2} \left[ a_{\psi}(k) + 2 \delta \theta a_{\psi}(k) \right] + \text{h.c.}(-k),
\]

implying that the power spectrum is given by \([1 + 4 \delta \theta^2] \times \Delta_{0}^{1/2} \), confirming that on long wavelengths all scales receive the same enhancement already shown in (14). Finally, around \( k \sim k_0 \lambda / 2 \) the curvature fluctuation is dominated by (disregarding the oscillatory parts)

\[
\frac{\zeta(k)}{\Delta_{0}^{1/2}} \sim i k^{-3/2} e^{\sqrt{\lambda_0 k_0 k^2 \delta N / k_0}} \left[ (1-i) a_{\psi}(k) + (1+i) a_{\psi}(k) \right] + \text{h.c.}(-k),
\]

implying that the power spectrum has a bump centered at \( k = k_0 \lambda / 2 \) of amplitude \( \Delta_{0}^{1/2} e^{2 \delta \theta^2} / 4 \), and width \( \Delta N_k \simeq \ln[4(\delta \theta^2 / \ln^2(16(\delta \theta^2))] \), where \( N_k = \ln(k/H) \) is the wavenumber in e-fold units. In summary we have:

\[
\frac{\Delta_{\zeta}}{\Delta_{0}^{1/2}} \sim \begin{cases} 
1 + 4 \delta \theta^2 & \text{if } k \ll k_0 \lambda / 2 \\
\frac{1}{4} e^{2 \delta \theta^2} & \text{if } k \sim k_0 \lambda / 2 \\
1 & \text{if } k \gg k_0 \lambda / 2
\end{cases}
\]

(25)

This behavior is quite distinctive and in principle it can be tested: for large \( \delta \theta \), the ratio between the long- and short-wavelength power spectrum determines the height of the bump. In particular, the enhancement of \( \Delta_{\zeta} \) with respect to its long-wavelength value (constrained by CMB observations) is predicted to be \( \sim e^{2 \delta \theta^2} / (4 \delta \theta^2) \). This indicates that to have an enhancement as large as \( 10^7 \), it is enough to have \( \delta \theta \sim 4 \pi \).

We can check \( \Delta_{\zeta}(k) \) resulting from (21) against numerical solutions of (15) and (16). The results are shown in Fig. 1, where the numerical (solid) and analytical (dashed) solutions are plotted with respect to \( N(k) \equiv \ln(k/k_0) \), for the cases \( \delta N = 0.1 \) and \( \delta N = 1 \), with values of \( \lambda_0 \) chosen such that \( \Delta_{\zeta} \) receives an enhancement of order \( \sim 10^7 \). As expected, we find a good agreement for the case \( \delta N = 0.1 \). On the other hand, for \( \delta N = 1 \) the analytical result still constitutes a good prediction of \( \Delta_{\zeta}(k) \). In fact, together with a WKB approximation, the method leading to Eq. (25) should allow one to obtain solutions for more general functions \( \lambda(t) \) describing brief turns of duration \( \delta N \ll 1 \).

A salient feature of the enhancement shown in Fig. 1 is the rapid growth of the power spectrum over the limited range \( \Delta N_k \simeq \ln[4(\delta \theta^2 / \ln^2(16(\delta \theta^2))] \), evading known limitations in achieving steep enhancements in single-field models [46]. Together with (21), one can now compute the abundance of PBH as a function of their mass and the parameters \( \lambda_0 \) and \( \delta N \), along the lines of [47]. The band structure of \( \Delta_{\zeta}(k) \) resulting from the oscillatory phases in (21) should lead to characteristic signatures that could be tested with the help of constraints on PBHs: a polychromatic mass spectrum exhibiting periodic gaps.
An ultraviolet complete example.— We now offer a concrete action of the form (2), where the inflationary trajectory can experience sudden turns while \( \epsilon \) is kept small and constant, with \( U(\psi) = 0 \). This can be easily achieved within the context of holographic inflation [48–50], where the potential \( V \) in (2) is determined by a “fake” superpotential \( W(\phi) \) as \( V = 3W^2 - 2\gamma^a_{\phi b}W_aW_b \), with \( W_a = \partial W/\partial \phi^a \). Here, the background solutions respect Hamilton-Jacobi equations:

\[
\dot{\phi}^a = -2\gamma^a_{\phi b}W_b, \quad H = W. \tag{26}
\]

It turns out that \( U \) is related to \( W \) in the following way [51]:

\[
\partial^2_\phi U|_{\psi=0} = 6HW_{NN} - 4W^2_{,NN} + 2\dot{W}_{NN}, \tag{27}
\]

where \( W_{NN} \equiv N^a N^b \nabla_a W_b \). We can now define suitable expressions for \( \gamma_{ab} \) and \( W \) such that \( \epsilon \) and \( \lambda \) have any desired time dependence. For instance, let us take fields \( (\phi^1, \phi^2) = (\phi, \chi) \) and consider the following form for \( \gamma_{ab} \):

\[
\gamma_{ab} = \begin{pmatrix}
0 & \epsilon f(\phi, \chi) \\
0 & 1
\end{pmatrix}, \tag{28}
\]

with non-vanishing Christoffel symbols \( \Gamma^\chi_{\phi \phi} = f_\phi, \Gamma^\phi_{\phi \phi} = -e^{2f} f_\chi \) and \( \Gamma^\phi_{\chi \phi} = \Gamma^\chi_{\phi \phi} = f_\chi \). If we now take a superpotential \( W \) that only depends on \( \phi \), then (26) implies that \( \dot{\phi} = -2e^{-2f}W_\phi \), and \( \dot{\chi} = 0 \) regardless of the location in field space. Thus, assuming that \( \phi > 0 \), the tangent and normal vectors are given by \( T^a = (e^{-f}, 0) \) and \( N^a = (0, 1) \). An immediate consequence of this is that the turning rate is \( \Omega = -N_a D_t T^a = \dot{\phi} e^f f_\chi \), and therefore \( \lambda H = 2\dot{\phi} e^f f_\chi \). Also, it follows that \( W_{NN} = 0 \) and, thanks to (27), \( \partial^2_\phi U|_{\psi=0} = 0 \). Given that this result is independent of \( \chi \), and that \( \psi \) is a perturbation along the \( \chi \) direction, then \( U = 0 \) exactly. Next, notice that \( f \) can be expanded as

\[
f(\phi, \chi) = \sum_n \frac{1}{n!} (\chi - \chi_0)^n f_n(\phi). \tag{29}
\]

A field redefinition of \( \phi \) allows one to set \( f_0(\phi) = 0 \). Having done so, along the specific path \( \chi = \chi_0 \), one finds \( \dot{\phi} = -2W_\psi \) and \( \lambda = \sqrt{8\epsilon f_1(\phi(t))} \). As a consequence, \( \epsilon = 2W^2_\phi / W^2 \) and the expansion rate depends exclusively on \( W(\phi) \), remaining unaffected by the turning rate \( \lambda \). One can finally define \( W(\phi) \) and \( f_1(\phi) \) to obtain desired expressions for \( \epsilon \) and \( \lambda \).

Discussion and conclusions.— The coupling \( \lambda \) parametrizes how non-geodesic an inflationary trajectory is in field space (i.e. how fast it is turning). When \( \lambda = 0 \) the dynamics of \( \zeta \) is of the single-field type, meaning that its evolution is dictated only by the expansion rate \( H \) and slow-roll parameters. Genuine multi-field inflation happens as long as \( \lambda \neq 0 \), in which case \( \zeta \) mixes with the isocurvature fluctuation \( \psi \) and, if present, other scalar degrees of freedom.

It is already well-known that sudden turns of the inflationary trajectories produce features in the power spectrum [29]. Here we have focussed on the generation of large enhancements of the power spectrum of \( \zeta \), over a limited range of scales, compatible with PBHs constituting a sizable fraction of dark matter. Noteworthily, we obtained accurate analytical solutions for \( \zeta \) valid in the regime of large coupling \( \lambda \). As far as we know, this is the first example of a full analytical solution in the rapid turn regime with a time dependent coupling.

As we have already seen, large enhancements of the power spectrum (of order \( 10^7 \)) require rapid turns [recall our discussion after Eq. (14)]. However, there is a second requirement: a non-trivial field space geometry [as in (28)]. To appreciate this, consider for instance the task of enhancing the power spectrum in canonical multi-field inflation as a result of a turn taking place during a period of \( \delta N \) e-folds, at a constant rate \( \Omega \). This implies \( \lambda = 2\delta \theta/\delta N \). Nevertheless, because in canonical models \( \delta \theta < \pi \), one has a maximum size for \( \lambda \), given by \( \lambda_{\text{max}} = 2\pi/\delta N \). Thus, if the turn takes place over a long period \( \delta N \gg 1 \), then one necessarily falls within the slow turn regime that, as already stated, cannot produce large enhancements. On the other hand, if \( \delta N \) is of order 1 or smaller, one falls within the rapid turn regime, which is well described by our analytical estimates of (25). Nonetheless, as we have learned from (25), to achieve enhancements of order \( 10^7 \) one needs \( \delta \theta \sim 4\pi \), therefore excluding canonical models. Of course, a canonical multi-field model can still display large enhancements of \( \Delta \zeta \) if the expansion rate assists in amplifying \( \zeta \), as found in single-field models.

To keep the present discussion simple, we have ignored the potential \( U \). Turning it on has two important effects that do not alter our main findings but instead enrich the phenomenology associated to our mechanism. First, a non-vanishing \( U \) would introduce a mass for \( \psi \), the so called entropy mass \( \mu \equiv (\partial^2_\phi U|_{\psi=0})^{1/2} \). This modifies both the dispersion relations shown in (19) and the amplitudes in (18) leading to the solution (21), altering the details of our estimates in (25). Another effect is the generation of non-Gaussianity, along the lines of Refs. [52–54]. A large \( \lambda \) will necessarily induce non-Gaussian deformations on the statistics of \( \zeta \) that could change the details of PBH formation [55–61]. This direction is worth studying and is left for future work.

To summarize, multi-field inflation is able to enhance the primordial power spectrum with distinctive signatures. This in turn can seed primordial black holes sensitive to the details of the ultraviolet theory wherein inflation is realized, opening a new window into the physics of the early Universe at scales far smaller than the CMB.

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