THE LANDAUER FORMULA: A MAGIC MANTRA REVISITED

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1. INTRODUCTION

a. Prolog

In 1957, Rolf Landauer published a different and – to some of the leading transport gurus of the epoch – iconoclastic interpretation of metallic resistivity[1]. Landauer envisioned the current, rather than the electromotive voltage, as the stimulus by which resistance is manifested[2]. The measured voltage is simply the macroscopic effect of the carriers’ inevitable encounters with the localized scattering centers within a conductor. Around any such scatterer the carrier flux resembles a diffusive flow, set up by the density difference between the upstream and downstream electron populations. In this purely passive scenario, energy dissipation does not enter.

The Landauer theory describes electron transport in an environment of scatterers that are purely elastic. As such it is not able to address the dynamical mechanisms of energy dissipation that characterize transport. This is despite the fact that Landauer theory, like any other description of conductance, must satisfy the fluctuation-dissipation (Johnson-Nyquist) theorem at some level. The theorem implies that dissipation is always present whenever there is resistance. It is an inescapable element in every theory of conductance. A properly formulated theory will describe dissipation explicitly, in physical detail.

This paper reviews the conceptual structure of the Landauer model of electron transport, in the light of what has long been accepted as the canonical description of conductance and fluctuations: quantum kinetics. Through some straightforward instances, we discuss the lack of a clearly discernible correspondence between Landauer theory on the one hand, and standard microscopics on the other.

From our microscopic critique it follows that one must consider, longer and
harder than before, how Landauer’s bold insight can be said to express its microscopic integrity. There is no question that Landauer, with his foresight and determination, inspired and guided the growth of mesoscopic physics as very few other personalities could have done. However, establishing a substantive connection with canonical microscopics remains a serious task, whether or not that fact is widely acknowledged.

b. The Rise of Landauer Theory

Landauer has recounted[3] how his ideas languished until their resurrection, and bold reinterpretation, by a new generation of less hide-bound theorists. The main early objection seemed to be his emphasis on the localized action of the scattering impurities in opposing current flow, in an era when the theoretical dogma held that local effects could never be individually probed; all that one could (and should) do was to compute a configurational average over ensembles of samples, with a spread of scattering-center distributions. A clear account of the configurational viewpoint is in the text by Doniach and Sondheimer[4].

All this changed radically in the ’eighties, with the advent of truly mesoscopic sample fabrication. It was now possible to study not bulk, coarsely grained ensembles, but individual samples with individual spatial arrangements of scatterers. Moreover, the phase coherence of the carriers could now be preserved over the much shorter lengths of the samples.

Overnight, the Landauer model came into its own. A new dogma was promptly declared (though not primarily by Landauer himself): transmission is all. One result of this sea change has been that standard, thoroughly established and powerful microscopic methods, such as Boltzmann kinetics and the quantum mechanical Fermi-liquid and Kubo theories, now languish in relative disuse within mesoscopics. Those papers that apply such methods to the subject tend to be obscured in the noise (we have reviewed several such works elsewhere[5]).

For one thing, the older approaches are much too labor-intensive to suit the milieu of instant results. For a second, it is taken – on faith alone – that they can do little else than corroborate Landauer’s far more compact phenomenology. For yet a third, all mesoscopic transport was now to occur solely by coherent elastic transmission, to all intents dissipationless (even when the fluctuation-dissipation theorem clearly states otherwise).

It is fair to say that the eclipse of standard microscopic methods in mesoscopics has no particular basis in logic. It is fashion-driven. Fashion dogma being ultimately less hard-wearing than good physics, the vogue circa 1957 did not last. Will the current one endure?1

Meanwhile, in other disciplines where short-range coherent phenomena are equally pivotal (such as nuclear matter), microscopic analysis remains not only indispensable but goes steadily from strength to strength, both in its power and sophistication. For an overview of modern developments in many-body quantum kinetics, see the volume

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1 Landauer reprinted his 1957 paper in J. Math. Phys. 37, 5269 (1996), with a self-explanatory commentary: “...IBM Journal of Research and Development, is not all that easily located in 1996. As a result the frequent citations to it often assign content to that paper which does not agree with reality.”
edited by Bonitz[6]. We now engage our subject technically.

2. ISSUES IN MESOSCOPIC TRANSPORT

a. Importance of the Landauer Formula in the Mesoscopic Literature

Landauer’s suggestion that conduction is at heart transmissive dominates the realm of mesoscopic physics[1]. His formula, derived for noninteracting electrons in a one-dimensional (1D) uniform conductor, has wide application to a variety of physical systems: quantum wires, quantum-Hall edge states, quantum point contacts, and carbon nanotubes. There is a current consensus that the physics and technology of transport in nanodevices are fully understood by the seminal work of Landauer and its extensions, and that his model supersedes all others.

We will clarify a number of important points, which have taken on a mythical significance in the electron-transport theory of mesoscopic conductors. Some of the arguments that we present may be at odds with the consensual Landauer formalism in terms of fundamental physics. In our microscopic formulation, the Landauer formula occurs as a highly idealized case. There is an important departure here: as for any standard microscopic treatment, our description of the physics of dissipation is explicit. So it must be. In Landauer-like descriptions, it remains a congenitally murky issue.

Let us begin with a simple derivation of the Landauer conductance, after Imry and Landauer[7]. A long, uniform 1D wire is connected to two large leads held at different chemical potentials $\mu_1$ and $\mu_2$, where all energies are measured from a global zero level. Electrons flow from the higher potential $\mu_1$ to the lower one $\mu_2$ (that is, the density $n(\mu)$ is assumed to increase monotonically).

A disturbance $\delta n$ of the equilibrium electron density across the ends of the wire defines the current-density response as

$$J = (-e)v \frac{\delta n}{2} \equiv (-e)\frac{v}{2}\frac{\partial n}{\partial E}\delta E = (-e)\frac{v}{2}\left(\frac{2}{\pi}D(E)\right)(\mu_2 - \mu_1),$$

where $v, E$ are the Fermi velocity and energy respectively; $D(E)$ is the 1D density of states. Note two points:

(i) the change in the physical carrier density $\delta n$ must be reduced by an ad hoc factor of two. That is because only “left-to-right” moving equilibrium carriers are understood to convey current from source to drain (an equal number of equilibrium electrons at the source flows the “wrong” way). Conversely, only “right-to-left” moving ones can carry the counter-current. Their net sum is $J$.

(ii) To expand perturbatively about the Fermi level, we must be in the extreme degenerate limit where all relevant energies are much less than $E$. The argument will not hold for classical ballistic carriers.

Next, by identifying $\mu_1 - \mu_2$ with the drop in electron potential $eV$ across the asymptotic source and drain leads, and by replacing $D(E)$ with its expression in terms of $v$, we get
\[ J = (-e)v \frac{D(E)}{\pi}(-eV) = e^2v \left( \frac{1}{\pi \hbar v} \right)V = \left( \frac{2e^2}{\hbar} \right)V. \]  

The Landauer conductance in 1D follows:

\[ G = \lim_{V \to 0} \left\{ \frac{J}{V} \right\} = \frac{2e^2}{\hbar} = 7.75 \times 10^{-5} \text{ Siemens.} \]

The important quantum ingredient is the one-dimensional density of states \( D(E) \). Its proportionality to \( 1/v \) cancels \( v \) in the numerator and reproduces the “universal” conductance quantum \( 2e^2/h \) irrespective of the length or material of the wire.

Landauer gave a wider interpretation of conductance quantization in terms of independent eigenchannels, through the transmission matrix. However, the real question is: What causes dissipation in a ballistic (collision-free) wire? That question remains without a definitive answer, though imaginative explanations have not been wanting[8-11].

The next serious point concerns the controversial use of several different chemical potentials within a driven system which is, ipso facto, (a) not in thermodynamic equilibrium[3], and (b) part of a topologically closed electrical circuit. The common claim that such a mismatch is absolutely required for any description of mesoscopic transport, is more than misleading. It is frankly incorrect, as Kamenev and Kohn have demonstrated through their completely orthodox quantum-kinetic derivation of Eq. (3)[12]. The thermodynamically erroneous idea of transport-induced mismatch in chemical potentials continues to propagate in the literature. It is widely employed in molecular-transistor studies; see for example Ref. [13].

b. Contact Effects in Quantized Conductance

Imry[8] has given an intuitive understanding of how the finite resistance of a collision-free ballistic wire arises; it is due to the bottleneck between the rich densities of states in leads and interfaces, and the sparse one for the few open channels that are available in the wire itself. He emphasizes the central role of the reservoirs in the process of dissipation. Since the power dissipated in conduction is \( P = IV = GV^2 \), it is certainly accessible in a sample whose resistance \( G^{-1} \) is of order 13 kΩ and no-one doubts its reality.

If one tries to follow logically the physics of the classic derivation of \( G \) reproduced in Eq. (3), it is far from clear how dissipation in the reservoirs mediates the finite resistance for the ballistic channel, unless there is some manifest role for the reservoirs within the proof of the formula. There is none, of course; it is a case of “acting at a distance”, with the actors not even allowed within sight of the stage.

It is surprising, to say the least, that the result for \( G \) does not depend in any way on material parameters or the structure of the wire-lead contacts. After all, the Imry picture still asserts that the role of remote dissipation in the leads (via inelastic collisions) is central to ballistic transport. The fundamental incompleteness of this approach becomes evident.

There is universal agreement that dissipative effects are essential to stabilize transport, ballistic or not. In subsequent accounts of Landauer theory, they are in-
voked as an essential function of the sample’s contacts with the macroscopic leads. Yet the same theory makes no room for dissipative processes within the core mathematical derivation of conductance. In our view this is an unsatisfactory state of affairs for a serious mesoscopic transport model.

c. Experiments

Recently, in groundbreaking work, de Picciotto et al.\[14\] demonstrated electronic transport in a nearly ideal, ballistic quantum wire. The wire’s remarkably fine construction rendered electron flow intrinsically resistanceless. In this device any resistance appearing in two-terminal measurements\(^2\) is ascribed to the current contacts, or interfaces, of the wire with its reservoir leads. Irrespective of the perfectly resistanceless quantum wire connected to the ideal leads, the contact resistance per open channel is very near the rather substantial value of \((h/2e^2) \approx 12.9\ \text{kΩ}\).

The de Picciotto et al. study\[14\] is a tour de force. It goes to unprecedented lengths to fabricate perfectly ballistic 1D structures, minimally influenced by extraneous effects. Even so, the surprise is that their raw results for conductance quantization clearly show systematic deviations from the perfect Landauer values. This cannot be so, according to theory, unless the transmission in the sample is less than perfectly ballistic and/or dissipationless.

The above experiment, unquestionably the cleanest yet done, can be directly compared with the much earlier data from a pioneering study of 1D ballistic conductance: van Wees et al.\[15\]. This was the first to report perfect quantized conductance in integer units \(n(2e^2/h)\). A careful reader of Ref. \[15\] will note that, from the measured resistance, a lead resistance of 400 Ω was subtracted (an error of 3% in the value of \(h/2e^2\)). Once plotted, the thus-corrected conductance exhibited ideal quantized steps, right up to \(n = 12\).

Our central question is: What causes dissipation in the ballistic channel of van Wees et al.? If, as argued by Imry\[8\], the perfectly quantized resistance truly represents the whole of the ballistic wire’s contact “bottleneck” to its entire electrical environment, what is the physical significance of subtracting 400 Ω? If it were somehow part of the contact resistance to the sample, its removal would negate a contribution that should be integral to the perfectly quantized value. If not, then it must be an experimentally distinguishable effect lying beyond the all-inclusive Landauer-Imry explanation of the contact resistance. No circuit-specific origin for the 400 Ω was identified by van Wees et al.\[15\].

The point is that the contact resistance is not an additive lumped-circuit parameter (albeit it dissipates electrical power just as well). It cannot be analyzed as such. A mesoscopic device does not possess the well-defined boundaries to permit the additivity that we know from the world of discrete macroscopic components.

\(^2\) Two-terminal measurements probe both voltage and current across the macroscopic leads that feed the sample. Four-terminal measurements are those in which the voltage is measured at distinct probes on, or immediately adjacent to, the wire itself. See below.
3. FURTHER ANALYSIS

a. Two-Terminal versus Four-Terminal Measurements

After several confusing debates in the literature, see for example [8], the current consensus is that the meaning of conductance depends on the type of measurement that is done. Points a-c of the previous Section relate to two-terminal measurements, where the current leads are also the voltage leads. The measured resistance of a two-terminal device cannot reveal the intrinsic resistance of the wire (that is, its “true” value considered as a discrete component, presumably in series with the contact resistance).

To gain insight into the intrinsic resistance, one has to make a four-terminal measurement by putting two extra voltage leads (probes) to the wire located between the reservoir contacts. These voltage probes must be noninvasive; they cannot interfere with the passage of current in the wire. The voltage drop between these probes, divided by the current through the wire, should give the resistance of the wire.

In the Landauer-Büttiker formulation [8,9,10] this resistance is given by $G^{-1}$, where now

$$G = \frac{2e^2}{h} \frac{T}{1 - T}. \tag{4}$$

The transmission factor $T$ gives the transparency of the barrier represented by the wire. In this picture ballistic transmission, or ideal reflectionless elastic “scattering” ($T = 1$), produces ideal (infinite) conductance, as envisaged long ago by Bloch and others.

Realization of the foregoing measurement scenario – the attainment of $T = 1$ – was the express goal of de Picciotto et al.[14]. They reported the vanishing of the intrinsic four-probe resistance, over a wide range of gate voltage. As we have already noted in Sec. 2, the experiments of de Picciotto et al., show a two-terminal resistance that is nonuniversal; that is, $T < 1$, while the four-terminal resistance is ideal (zero), implying $T = 1$. A primitive understanding of Landauer theory might lead to the conclusion that the two-terminal and four-terminal measurements jointly harbor a contradiction.

b. Scattering Theory and Transmission

The core of the Landauer theory is the assertion that transmission is conductance. Transmission is to be computed from the quantum mechanics of elastic single-particle scattering\(^3\). A problem arises when the wire size is of the same order as the inelastic scattering length. In that case the transmission properties of the wire are strongly affected by the leads. The current-carrying state has a dissipative component, spoiling its coherence. Even when the wire is perfectly ballistic, its

\(^3\)At this simple level of description, any attempt to include inelasticity (dissipation) destroys the unitarity of the Schrödinger evolution. The wave function no longer conserves probability.
contacts with the leads will induce a certain amount of decoherence. Therefore the zero-resistance state of the four-probe results (see, for example, Fig. 2 of Ref.[14]) remains unexplained.

In principle, scattering theory is microscopic. In practice, its use in Landauer-based models is as a prop to phenomenology, rather than the reverse as in most other models. The elements of the scattering matrix are not extracted microscopically, but are left for some separate calculation to feed into the Landauer theory. In thus “outsourcing” its physics, such a framework does little to explain the key details of real devices.

An important point is that the scattering theory invoked by Landauer, Büttiker, and Imry is a one-particle theory which considers only elastic scattering and ignores altogether the inelastic processes vital to the stability of mesoscopic transport. We recall that dissipation is the unique outcome of inelastic scattering, and that dissipation through any resistance is mandatory (the Johnson-Nyquist theorem). On that score the occurrence of a finite resistance in a family of models that are purely elastic, and in which the action of dissipation in the leads (conceded by all to be essential) is so vestigial as to be totally invisible in the formalism, is an astonishing result.

4. MICROSCOPIC RESOLUTION

Dissipation is always a many-body problem. It is mediated by electron-phonon and electron-electron interactions, and many others besides. These processes unfold side by side with, say, impurity or barrier scattering which are well described as elastic one-body effects.

Whether classical or quantum, unless there is an external agent, a particle continues to travel unhindered. If there are correlations among the particles, their freedom and independence are at least partially lost. This dynamic is readily followed within Kubo’s formulation of linear-response theory: microscopic correlations in the current response will produce, directly and transparently, a finite conductivity[16].

a. Linear Response and the Many-Body Kubo Formula

The Kubo theory is a theory of the electrons’ full density matrix, not of the far more restrictive set of one-body wave functions. We briefly consider the core structure of the Kubo conductance formula. Complete details, including the explicit microscopic construction of the formula from both elastic and inelastic scattering processes, are in Ref.[16]. The electrical conductivity is given by

\[ \sigma(t) = \frac{n e^2}{m^*} \int_0^t C_{vv}(t) dt \]  \hspace{1cm} (5)

The velocity-velocity correlation function is

\[ C_{vv}(t) = \frac{\langle v(t)v(0) \rangle}{\langle v(0) \rangle^2} \sim \exp(-t/\tau_m) \]  \hspace{1cm} (6)

where the expectations are taken in the equilibrium state (this gives the leading linear term in the expansion of the nonequilibrium response), and \( \tau_m \) characterizes
the dominant decay of the correlations. This parameter includes, on an equal footing, the microscopic contribution from every physically relevant collision mechanism[16]. In the long time limit,

$$\sigma \to \frac{n e^2 \tau_m}{m^*}. \quad (7)$$

This is the celebrated Drude formula. In one dimension, the density in terms of the Fermi wavenumber is $n = 2k_F/\pi$. The conductance over a sample of length $L$ becomes

$$G \equiv \frac{\sigma}{L} = \frac{2k_F e^2}{\pi \hbar m^* \tau_m} = \frac{2e^2}{\hbar} \left( \frac{2\hbar k_F}{L m^* \tau_m} \right) \equiv \frac{2e^2}{\hbar} T_K, \quad (8)$$

in which the transmission coefficient $T_K$ (K for Kubo) is now proportional to the ratio of the effective scattering length $v_F \tau_m$ to length $L$.

All of the dissipative many-body effects have been embedded within $\tau_m$, as well as any elastic impurity scattering. The interface physics is incorporated into the microscopic Kubo conductance as fully and directly as the physics of the device itself.

There is nothing in Eq. (8) that precludes conformity with the Landauer formula, Eq. (3). Unfortunately, the reverse is not true. This is plain from Eqs. (1) and (2), its usual phenomenological derivation. Quite unlike the analytical structure of the Kubo formula, there is no provision for the many-body physics at the interfaces that explicitly generates the dissipation. Kubo, by contrast, includes all of the physical detail that this problem demands.

b. The Landauer Formula without Landauer’s Assumptions

The microscopically consistent Kubo derivation of Eq. (3) by Kamenev and Kohn[12] is a landmark. It lays to rest the myth that response theory can work only within the thermodynamically unfounded leaky-reservoir paradigm (where a mismatch of source and drain chemical potentials is fancied to accompany the current). However, their derivation was for a closed mesoscopic 1D loop not subject to the major dissipative effects that govern an open mesoscopic system. We now show that the Kubo formula also works just as well for open systems, and that it recovers the ideal Landauer-conductance expression as a (very) special case of Eq. (8) “in the open”.

Let us consider a simple model for the behavior of $T_K$. The wire is uniform; so are the driving field and carrier distribution within. At a distance $L$ apart lie the interfaces where the current is, in effect, injected and extracted. $L$ is not a geometrically defined quantity. Rather, it characterizes the dynamical processes for the open mesoscopic system (somewhat abstractly but quite specifically; see below). Note that it is the injection and extraction of the current that explicitly energizes the system. There is no appeal to chemical-potential differences in any way, shape, or form.

The interfaces are regions of strong elastic scattering with impurities in the leads (relaxation time is $\tau_{el}$) and strong dissipative interactions with the background modes.
excited by the influx and efflux of carriers from the current source (relaxation time is \(\tau_{in}\))^4. The scattering mechanisms are independent, so that the Matthiessen rule applies:

\[
\frac{1}{\tau_m} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{in}}.
\]  

(9)

The mean free path (MFP) associated with the elastic collisions is obviously \(L\), since by hypothesis that is the size of the impurity-free region. Therefore \(\tau_{el} = L/v_F\). By the same token the MFP for inelastic scattering cannot be greater than \(L\), though it may be less. Hence

\[
\tau_{in} \leq \tau_{el} = L/v_F.
\]  

(10)

We conclude that

\[
\mathcal{T}_K = \frac{2v_F}{L} \left( \frac{\tau_{el}\tau_{in}}{\tau_{el} + \tau_{in}} \right) = \frac{2\tau_{in}}{\tau_{in} + L/v_F} \leq 1.
\]  

(11)

It is the explicit competition between the elastic processes in the mesoscopic system (whose very fabrication guarantees that the characteristic length \(L\) will be equal to the elastic MFP), and the dissipative processes (hopefully restricted to the current injection/extraction areas bounding \(L\), but also liable to intrude into the interior) that determines the physical, and measurable, transmission through the sample. A full-scale Kubo analysis would clarify the physics in all its microscopic detail. Nevertheless, the essence of it is already in Eqs. (8)-(11).

What is the optimum outcome for Eq. (11), and what does it yield for the conductance? The maximum value of \(\mathcal{T}_K\) is unity, and it is attained precisely when

\[
\tau_{in} = \tau_{el} = L/v_F.
\]  

(12)

In other words, no inelastic events intrude into the core of the wire; they all occur at the interfaces. From Eq. (8) one easily discerns the corresponding value of \(G\) for this open, maximally ballistic 1D wire. It is the Landauer conductance \(2e^2/h\).

Which assumptions have been made in common with Landauer? There are two.

- That the channels available in 1D transport are discrete, and sufficiently apart in energy that each can be treated independently;  
- That the conductor is internally uniform over most of its operative length.

What have we NOT assumed on the way to Landauer’s rightfully celebrated result?

- That transport is exclusively due to an energy drop between carrier reservoirs at different (effective) densities;  
- That coherent elastic transmission is the sole mechanism that should be included in the formula;

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4 From the viewpoint of a carrier inside the 1D wire, its dynamical evolution in the presence of the open leads is Markovian, but over a length scale set by \(L\) if the wire is truly collisionless (ballistic).
• That dissipation in an open conductor, though acknowledged to be vital (simply to save the Johnson-Nyquist theorem), is a remote asymptotic effect in the reservoirs with no immediate role in conduction.

c. Quantum Kinetics and Quantized Conductance

Both the Kubo and Landauer formulae, though radically different in philosophy, presuppose an external driving force that is weak enough to let one linearize the nonequilibrium transport equation. In practical mesoscopic devices, there is no guarantee that typical internal fields are small enough to justify linear response. As a prosaic example, we cite the case of high-mobility transistors with a strongly quantum-confined heterojunction channel. At optimum operation in a microwave-amplifier circuit, such structures must reliably sustain electric fields of order 50 kV/cm over their active region. The upper bound for linear response is much lower, at best 3 kV/cm in GaAs.

There is an evident need to extend transport theory to the nonlinear regime, while retaining all the quantum effects (not least, many-body correlations) that impart unique properties to mesoscopic structures, making them desirable for novel applications. This is done most naturally within a quantum kinetic approach[17,18]. We briefly review our basic kinetic treatment of nonequilibrium transport which recovers all the results of the previous section at low fields, and also allows one to study the problem well away from linear response.

We examine the electron-transport equation in a 1D uniform wire. In steady state, with driving field \( E \), the electron distribution function \( f_k \) in wave-vector space \( \{k\} \) obeys

\[
\frac{eE}{\hbar} \frac{\partial f_k}{\partial k} = -\frac{1}{\tau_{\text{in}}(\epsilon_k)} \left( f_k - \frac{\langle \tau_{\text{in}}^{-1} f \rangle}{\langle \tau_{\text{in}}^{-1} f_{\text{eq}} \rangle} f_{\text{eq}} \right) - \frac{1}{\tau_{\text{el}}(\epsilon_k)} \frac{f_k - f_{-k}}{2}. \tag{12}
\]

The scattering times \( \tau_{\text{in}}(\epsilon_k) \) and \( \tau_{\text{el}}(\epsilon_k) \) are in general energy-dependent. Equation (12) has some essential properties:

1. **Thermodynamic consistency.** Only one chemical potential enters the problem, through the equilibrium Fermi distribution

\[
f_{k}^{\text{eq}} = 1/\{1 + \exp[(\epsilon_k + \epsilon_i - \mu)/k_B T]\}
\]

(where \( \epsilon_i \) is the threshold energy for the \( i \)th 1D subband and \( k_B T \) is the thermal energy). This is the only place at which dependence on the chemical potential \( \mu \) comes in.

2. **Microscopic conservation.** The leading, inelastic collision term on the right of Eq. (12) has a restoring contribution proportional to the expectation

\[
\langle \tau_{\text{in}}^{-1} f \rangle = \int_{-\infty}^{\infty} \frac{2dk}{2\pi} \tau_{\text{in}}^{-1}(\epsilon_k) f_k.
\]

The inelastic collision term respects continuity and gauge invariance. Last, the second term on the right of Eq. (12) represents the elastic collisions, which work to restore symmetry to the nonequilibrium distribution \( f_k \).
Figure 1. Conductance of a one-dimensional ballistic wire, computed from Eq. (14) within the quantum kinetic model. We show $G$ as a function of chemical potential $\mu$. The conductance is in units of the Landauer quantum, and $\mu$ is in units of thermal energy. $G$ exhibits strong shoulders as $\mu$ crosses the two subband energy thresholds in succession. (These are set at $\varepsilon_1 = 5k_BT$ and $\varepsilon_2 = 17k_BT$ in this simulation.) Well above each threshold, the electrons in that subband are degenerate. The conductance tends to a well defined quantized plateau. Solid line: $G$ for an ideal ballistic channel: the Landauer limit. Dot-dashed line: the collision-time ratio $\tau_{\text{in}}/\tau_{\text{el}}$ of the upper subband is set to the non-ideal value of 0.8. Note how the increased inelastic scattering brings down the upper plateau. Dotted line: as above, with $\tau_{\text{in}}/\tau_{\text{el}} = 0.6$. The departure from ideality is more pronounced. Such effects cannot be predicted from the conventional phenomenology of the Landauer formula.

The equation can be solved systematically. For the special case that the collision times are independent of the electronic band energy $\varepsilon_k$, there is an exact solution (not only for $f_k$ but also for the much richer nonequilibrium current correlation, $C_{vv}(t)$). The response behavior parallels that for the Kubo analysis. Indeed, all of the results Eqs. (5)-(11) are recovered[19].

d. A Worked Example

We end with a presentation of the conductance obtained from our kinetic model. First, recall that the common derivation of the Landauer conductance assumes a highly degenerate electron band; that is, we are in the zero-temperature limit. If the band is even marginally full, then the full $G$ comes out. If the band is empty (the only other possibility at zero temperature), then there is no conduction and $G = 0$.

In a real experimental situation at finite temperature, such as in de Picciotto et al.[14] or van Wees et al.[15], the carrier density in a 1D subband is controlled via a gate above the wire. The electron population duly undergoes a continuous transition, from a low-density classical regime to a high-density Fermi-Dirac one. While there is no provision for this classical-to-quantum crossover in the standard treatments of the
Landauer conductance, it is no problem at all within the Kubo or kinetic treatment. It is sufficient to note that, classically, the elastic mean free path is a function, not of the Fermi velocity, but of the thermal velocity $v_{th} = \sqrt{2k_B T/m^*}$. In general, we must replace $\tau_{el}$ with the expression

$$\tau_{el}(n, T) = \frac{L}{\bar{v}} \equiv L\frac{n}{\langle v|f^{eq}\rangle}.$$  

(13)

In the classical limit, $\bar{v} = v_{th}$. In the degenerate limit, $\bar{v} = v_F$, as in Eq. (10). We can then extend Eq. (8) for $G$ to all accessible regimes of density at finite temperature:

$$G = G_0\left(\frac{hn}{2m^*\bar{v}}\right)\left(1 - \frac{1}{1 + \tau_{in}/\tau_{el}}\right),$$

(14)

where $G_0 = 2e^2/h$ is the Landauer conductance quantum, and where $v_F$ has been replaced with its equivalent expression in 1D: $v_F = \hbar k_F/m^* = hn/4m^*$.

When the system is at low density ($\mu - \varepsilon_i \ll k_B T$) the conductance vanishes with $n$. When the system is degenerate ($\mu - \varepsilon_i \gg k_B T$) the conductance reaches a plateau, which is ideally quantized when $\tau_{in} = \tau_{el}$. In between, it rises smoothly as the chemical potential (and density) is swept from much below the subband threshold $\varepsilon_i$ to much above it. The result is depicted in Figure 1.

In Fig. 1 we see the conductance of a 1D ballistic wire computed from Eq. (14), with full temperature dependence, as a function of chemical potential. The threshold steps at the two subband plateaux are clear. Also clear is the progressive loss of ideality as the inelasticity in the problem is increased. Nonetheless, the characteristic Landauer steps survive robustly, even when the height of the steps no longer corresponds to perfect ballistic transport inside the body of the wire.

One principal conclusion stands out. In an open mesoscopic ballistic conductor, the close interplay of elastic and dissipative scattering dominates the form and behavior of the conductance. On its own, neither collision mode can encapsulate the relevant physics. They must be allowed to act in concert, as they do in nature.

5. EPILOG

In this paper we have shown that the Landauer quantized-conductance formula, foundational to so much of mesoscopic transport, possesses a microscopic validity and scope well beyond the popular rationale in which it has been clothed for so long. For an open mesoscopic system, there is no escaping the direct – indeed vital – interplay between elastic one-body scattering on the one hand, and inelastic many-body scattering on the other. Theories that favor the former at the expense of the latter, without adducing valid microscopic reasons for doing so, court a serious distortion of the physics.

Were it not for Landauer’s intuition and his formula, mesoscopics in the last two decades would have fared very differently. His legacy has truly been one of unprecedented progress. It would therefore be a cause for concern if, for the sake of fashion, inadequately reasoned casuistic phenomenologies were to win out in the
theoretical arena over the microscopically based analytic methods first put in place by Maxwell and Boltzmann, and culminating in the contributions of Born, Fermi, Landau, and others. Such a reversal of values does not serve progress in the long term.

One criterion matters for continued reliance on any mesoscopic formula. It is not whether the formula is fashionable, nor whether it is easily finessed for yet one more quick publication opportunity. The only question that counts is whether the model has a rational and clearly traceable origin within canonical microscopics, the bedrock of modern condensed-matter physics. If the answer is “yes”, that formula will be far more likely to keep on performing well.

REFERENCES

[1] R. Landauer, IBM J. Res. Dev. 1, 223 (1957); Phil. Mag. 21, 863 (1970).
[2] See e.g. A. D. Stone and A. Szafer, IBM J. Res. Dev. 32, 384 (1988).
[3] R. Landauer, in Coulomb and Interference Effects in Small Electronic Structures, ed. by D. C. Glattli, M. Sanquer and J. Tran Than Van (Editions Frontières, Gif-sur-Yvette, 1994) p. 1.
[4] S. Doniach and E. H. Sondheimer, Green Functions for Solid State Physicists (W. A. Benjamin, Reading, MA, 1974).
[5] F. Green and M. P. Das, in Condensed Matter Theories Vol. 17, ed. by M. P. Das and F. Green, (Nova Science Publ., New York, 2003).
[6] M. Bonitz (editor), Progress in Nonequilibrium Green’s Functions (World Scientific, Singapore, 2000).
[7] Y. Imry and R. Landauer, Rev. Mod. Phys. 71, S306 (1999).
[8] Y. Imry, Introduction to Mesoscopic Physics second edition (Oxford University Press, Oxford, 2002).
[9] D. K. Ferry and S. M. Goodnick, Transport in Nanostructures (Cambridge University Press, Cambridge, 1997).
[10] S. Datta, Electronic Transport in Mesoscopic Systems (Cambridge University Press, Cambridge, 1997).
[11] J. H. Davies, The Physics of Low Dimensional Semiconductors: an Introduction, (Cambridge University Press, Cambridge, 1998).
[12] A. Kamenev and W. Kohn, Phys. Rev. B 63, 155304 (2001).
[13] P. Dample, T. Rakshit, M. Paulsson, and S. Datta, Preprint cond-mat/0206328
[14] R. de Picciotto, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Nature 411, 51 (2001); see also A. Chang, Nature 411, 39 (2001).
[15] B. J. van Wees, J. van Houten, C. W. J. Beenaker, J. G. Williams, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. 60, 848 (1988).
[16] R. Kubo, M. Toda, and N. Hashitsume, Statistical Physics II: Nonequilibrium Statistical Mechanics, second edition (Springer, Berlin, 1991).
[17] F. Green and M. P. Das, J. Phys.: Condens. Matter 12, 5233 (2000).
[18] F. Green and M. P. Das, J. Phys.: Condens. Matter 12, 5251 (2000).
[19] F. Green and M. P. Das, Fluctuation and Noise Letters 1, C21 (2001)