Discussion on constitutive relation of concrete creep and shrinkage by numerical calculation

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Abstract. In order to establish accurate and effective mechanical models of concrete creep and shrinkage by numerical calculation, the problems of parametric finite element analysis by APDL with ANSYS software and constitutive relations of creep and shrinkage by the creep criterion in the software are discussed. The accuracy of calculating creep coefficient and shrinkage strain by theoretical calculation and numerical simulation is analysed with an example. The calculation results show that the results from the two kinds of methods are in good agreement. It can get very good results to simulate the constitutive relationship of concrete creep and shrinkage by the software, achieving the basic function of accurately describing the long-term deformation effects of concrete. This function is the first and most important step in accurately simulating the long-term effects of complex constructions.

1. Introduction

When creep and shrinkage effects are not taken into account, a large number of mature research results have been obtained by using finite element software to analyze concrete problems. It is generally assumed that the elastic modulus of concrete $E_c$ is a constant, that is, the stress-strain linear relationship under instantaneous loading is maintained\cite{1-3}. Considering the long-term deformation, however, $E_c$ is the initial elastic modulus of concrete. The deformations caused by creep increase with the loading time, while the elastic deformations gradually reduce due to the total deformations constant, that is, the elastic deformations are converted to creep deformations. According to the relationship $\varepsilon = \sigma / E_c$, the stress reduces with the corresponding strain, named the stress relaxation. The changes of stress and strain are related to creep coefficient $\phi$ and the creep constitutive equation of concrete not only determined by $E_c$ but also by the creep characteristics of concrete\cite{4-5}. Only by matching the constitutive relation in the theory with the model parameters in the software can the accurate solution be obtained. Therefore, the validity of the results from numerical simulation depends on whether it can accurately describe the creep and shrinkage constitutive relationships of concrete using the finite element software\cite{6-14}.

In this paper, APDL, a high-level programming language of ANSYS software, is adopted for process control and the creep criterion in the software is used to establish creep and shrinkage constitutive relations of concrete. This function does not provide a model specifically for concrete creep, but simulates metal creep. In order to simulate concrete creep, the theory of metal creep should be considered. From the microscopic level, although the mechanism of metal creep and concrete creep is
different, their stress-strain constitutive relationship is the same to some extent. It is reasonable to calculate concrete creep according to the metal creep calculation method. And it is able to calculate the composite material section. Only need to calculate a single material of concrete or steel separately, the deformation and internal force of each material can be calculated and compared with the analytical calculation results.

2. Establishment of constitutive equations of concrete creep and shrinkage

There are 13 kinds of common stress-strain equations listed in the creep model of ANSYS. After the "MP" command is input to define the elastic modulus of concrete, the corresponding equation needs to be selected through the "TB" command and the coefficients in the equation are defined in the "Tbdata". According to the types and adaptability listed in the equation, the equation no. 11 in the model is adopted as the constitutive equation of concrete creep and shrinkage, as shown in equation (1).

\[ \varepsilon_{ct} + \varepsilon_{sh} = C_1 \cdot \sigma + t^{C_5-1} \cdot e^{C_5/T} / (C_3 + 1) + C_5 \cdot \sigma^{C_3-1} \cdot t^{C_3/T} \]  

Where \( \varepsilon_{ct} \) is the concrete strain caused by creep at any time for \( t>0 \), \( \varepsilon_{sh} \) is the concrete strain caused by shrinkage at any time for \( t>0 \), \( \sigma \) is the concrete stress at any time for \( t>0 \), \( T \) is the environment temperature at any time for \( t>0 \), \( C_1, C_2, C_3, C_4, C_5, C_6, C_7 \) are the constant coefficients.

By simplifying, the constitutive equations of concrete creep and shrinkage as shown in equation (2) and equation (3).

\[
\begin{align*}
\dot{\varepsilon}_{ct} &= C_1 \cdot \sigma \\
C_{1i} &= \frac{\varphi_{i}}{1 - \varphi_{i} \cdot \frac{1}{E_c}}, \quad i = 1 \\
C_{ii} &= \frac{\varphi_{i} - \varphi_{i-1}}{t_{i} - t_{i-1}} \cdot \frac{1}{E_c}, \quad i \geq 2 \\
\varepsilon_{sh} &= \frac{\varepsilon_{sh0}}{\varphi_{sh}}, \quad i=1 \\
C_{5i} &= \frac{\varphi_{i}}{t_{i} - t_{i-1}} \cdot \frac{\varepsilon_{sh0}}{\varphi_{sh}}, \quad i \geq 2
\end{align*}
\]

In each time step, the creep strain rate is assumed to be constant, as shown in figure 1. The smaller the time node, the closer to the real value. When the creep strain rate is large, a small time step should be taken to reduce the calculation errors. A creep strain rate of less than 0.1 will be obtained fairly accurate results. If the step size is too large, the solutions will become unstable or even converge. The upper limit of stability is 0.25. Therefore, the creep strain rate should be controlled within 0.25 in the calculation process.

The creep coefficient can be determined by formula (2), but it cannot automatically calculate in the software. The call subroutine should be prepared according to the formula, or loop statement can be used in the command input formula calculation. By calling the programmed subroutines related to \( C_1 \), time-varying parameters about creep coefficient and \( C_1 \) can be obtained in a generated list. The final value of shrinkage strain and creep coefficient can be calculated according to reference[15]. The step size of time is the same as the creep coefficient equation, and the time-varying parameters about shrinkage and \( C_3 \) displayed in the list can be obtained according to the program compiled by ANSYS.
3. Analyses of an example
A concrete column is as shown in figure 2. A surface load of 1000N/mm² is applied to the top of the column. The concrete intensive grade is C25, with the elastic modulus of concrete $E_c = 3.5 \times 10^4$ N/mm², Poisson's ratio $\mu = 2.0$, annual average humidity RH=75%, load age $t_0=7$ day. Considering the effects of creep and shrinkage, please calculate the strain on the upper surface of the column by analytic and numerical methods, the period for 600 days.

The analytical calculation is programmed with Matlab, and the model established in ANSYS is shown in figure 3. Since it is adopted the same calculation formula through analytical calculation and ANSYS calculation, the results of creep coefficient and shrinkage strain are exactly the same, as shown in figure 4 and figure 5. Then the upper surface strain of the specimen is calculated. When creep and shrinkage are not considered, the instantaneous elastic strain is $\varepsilon = \sigma / E_c = 285.714 \times 10^{-4}$. The strain at other time is calculated according to different creep coefficient and shrinkage strain and the results are shown in the column ⑤ in table 1. The time-history diagram of the strain is obtain as shown in figure 6. The results of ANSYS are extracted from the program, where $\phi$, $\varepsilon_\phi$, C1 and C5 are obtained from the files generated by the subroutine. The total strain are listed in column ⑩ of table 1. The time history diagram of strain was obtained through the plotting function, as shown in figure 7. The load step is 10, and the data is...
extracted every 6 days. The total number is 600 days, starting from $t_0=7$, and the first time node is 13. By comparing the two calculation results in figure 6 and figure 7, the coincidence is very good.

![Image of creep coefficient comparison](image1)

![Image of shrinkage strain comparison](image2)

**Figure 4. Comparison of creep coefficient**  
**Figure 5. Comparison of shrinkage strain**

## Table 1. Comparison on strain values on the top surface of the specimen

| time /day | $\phi_1$ | $E_{eh}$ /$\times 10^4$ | elastic strain + $Crop$ /$\times 10^4$ | The total strain /$\times 10^4$ | $\phi_2$ | $E_{eh}$ /$\times 10^4$ | $C_t$ | $C_e$ | The total strain /$\times 10^4$ |
|----------|----------|-----------------|---------------------------------|-------------------------------|----------|-----------------|------|------|-------------------------------|
| 0        | 0.00     | 285.7140        | 285.7140                        | 0.00                          | 0.00     | 0.0000          | 0.0000       | 0.0000       | 285.7140                        |
| 13       | 0.94     | 554.8569        | 556.0269                        | 0.94                          | 1.17     | 0.0207          | 0.0822       | 554.7510       |                               |
| 19       | 1.15     | 615.1426        | 616.6426                        | 1.15                          | 1.50     | 0.0101          | 0.0449       | 615.2150       |                               |
| 25       | 1.30     | 655.9997        | 657.3997                        | 1.30                          | 1.74     | 0.0668          | 0.0302       | 655.9330       |                               |
| 31       | 1.41     | 687.1426        | 689.0726                        | 1.41                          | 1.93     | 0.0552          | 0.0233       | 687.2750       |                               |
| 37       | 1.50     | 712.8569        | 714.9469                        | 1.50                          | 2.09     | 0.0443          | 0.0191       | 712.9550       |                               |
| 43       | 1.57     | 734.8569        | 737.0769                        | 1.57                          | 2.22     | 0.0363          | 0.0162       | 734.7800       |                               |
| 49       | 1.64     | 753.7140        | 756.0440                        | 1.64                          | 2.33     | 0.0332          | 0.0141       | 753.7840       |                               |
| 55       | 1.70     | 770.5711        | 773.0011                        | 1.70                          | 2.43     | 0.0282          | 0.0125       | 770.6200       |                               |
| 61       | 1.75     | 785.7140        | 788.2340                        | 1.75                          | 2.52     | 0.0252          | 0.0112       | 785.7310       |                               |
| 67       | 1.80     | 799.4283        | 802.0283                        | 1.80                          | 2.60     | 0.0223          | 0.0102       | 799.4310       |                               |
| 73       | 1.84     | 811.9997        | 814.6697                        | 1.84                          | 2.67     | 0.0211          | 0.0093       | 811.9540       |                               |
| 79       | 1.88     | 823.4283        | 826.1583                        | 1.88                          | 2.73     | 0.0191          | 0.0086       | 823.4780       |                               |
| 85       | 1.92     | 833.9997        | 836.7897                        | 1.92                          | 2.79     | 0.0181          | 0.0079       | 833.7010       |                               |
| 91       | 1.95     | 843.9997        | 846.8397                        | 1.95                          | 2.84     | 0.0171          | 0.0074       | 843.6410       |                               |
| 97       | 1.99     | 853.4283        | 856.3183                        | 1.99                          | 2.89     | 0.0155          | 0.0069       | 852.9340       |                               |
| 103      | 2.02     | 861.9997        | 864.9297                        | 2.02                          | 2.93     | 0.0105          | 0.0064       | 861.9390       |                               |
| 151      | 2.21     | 916.2854        | 919.4754                        | 2.21                          | 3.19     | 0.0099          | 0.0042       | 916.0680       |                               |
| 205      | 2.35     | 957.1426        | 961.0740                        | 2.35                          | 3.36     | 0.0070          | 0.0029       | 957.5050       |                               |
| 253      | 2.45     | 984.5711        | 988.0411                        | 2.45                          | 3.47     | 0.0056          | 0.0022       | 984.4660       |                               |
| 301      | 2.52     | 1005.7140       | 1009.2540                       | 2.52                          | 3.54     | 0.0044          | 0.0018       | 1005.5500      |                               |
| 403      | 2.63     | 1038.2854       | 1041.9354                       | 2.63                          | 3.65     | 0.0031          | 0.0012       | 1038.2100      |                               |
| 505      | 2.71     | 1060.8569       | 1064.5769                       | 2.71                          | 3.72     | 0.0023          | 0.0008       | 1060.8500      |                               |
| 600      | 2.77     | 1077.7140       | 1081.4840                       | 2.77                          | 3.77     | 0.0010          | 0.0006       | 1077.5400      |                               |
4. Conclusions

This article presents a discussion about establishing accurate and effective mechanical models of concrete creep and shrinkage by numerical calculation. The accuracy of calculating creep coefficient and shrinkage strain by theoretical calculation and numerical simulation is analysed with an example. The calculation results show that the results from the two kinds of methods are in good agreement. It can get very good results to simulate the constitutive relationship of concrete creep and shrinkage by the software, achieving the basic function of accurately describing the long-term deformation effects of concrete. This function is the first and most important step in accurately simulating the long-term effects of complex constructions.

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References

[1] Ban, H., Uy, B., Pathirana, S.W., et al. (2015) Time-dependent behaviour of composite beams with blind bolts under sustained loads. Journal of Constructional Steel Research, 112:196-207.
[2] Nguyen, Q.H., Hjiaj, M. (2016) Nonlinear Time-dependent behavior of composite steel-concrete beams. Journal of Structural Engineering, 142: 151-157.
[3] Gattesco, N., Macorini, L., Fragiacoimo, M. (2010) Moment redistribution in continuous steel-concrete composite beams with compact cross section. Journal of Structural Engineering, 136:193-202.
[4] Han, C.X., Zhou, D.H., Yang, Y.H., et al. (2016) Use of an algebraic constitutive equation to calculate the stress redistribution due to creep and shrinkage of composite beams. Journal of Harbin Engineering University, 37:1041-1049.
[5] Virtuoso, F., Vieira, R. (2004) Time dependent behaviour of continuous composite beams with flexible connection. Journal of Constructional Steel Research, 60:451-63.
[6] FIP-MC 2010 Fib model code for concrete structures 2010[S]. Switzerland:the Ernst & Sohn publishing house,2013.
[7] Francis, T.K. (2011) Accurate time-dependent analysis of concrete bridges considering concrete creep, concrete shrinkage and cable relaxation. Eng Struct 33(1):118–126.
[8] Altoubat, S.A., Lange, D.A. (2001) Creep, shrinkage, and cracking of restrained concrete at early age. ACI Mater J 98(4):323–331.
[9] Kim, Y., Trejo, D., Hueste, M., et al. (2011) Experimental study on creep and durability of high-early-strength self-consolidating concrete for precast elements. ACI Mater J 108(2):128–138.
[10] Pan, Z. F., Lü, Z.T. (2011) Experimental study on creep and shrinkage of high-strength plain concrete and reinforced concrete. Adv Struct Eng 14(2):235–248.
[11] Saliba, J., Loukili, A., Grondin, F., et al. (2014) Identification of damage mechanisms in concrete under high level creep by the acoustic emission technique. Mater Struct 47(6):1041–1053.

[12] Wang, J. (2011) Study on creep behavior of prestressed concrete beams with pretensioned bent-up tendons. J Zhengzhou Univ (Eng Sci) 36(2):25–29.

[13] Xue, W., Liu, T., Wang, W., et al. (2011) Creep behavior of high-speed railway prestressed concrete girders for 500 days. ACI Struct J 108(4):497–504.

[14] Huang, J. (2011) Behavior of an Integral Abutment Bridge in Minnesota, US. Struct Eng Int 21(3):320–331.

[15] Gara, F., Ranzi, G., Leoni, G. (2006) Time analysis of composite beams with partial interaction using available modelling techniques: a comparative study. Journal of Constructional Steel Research, 62:917-30.