The study of hybrid composite plates under transverse sinusoidal load using ANSYS

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Abstract. Composite materials have been widely used in many applications but understanding their failure behavior is always challenging. For years, numerical investigation has been the effective alternative, even though the underlying theories related to composite materials are complicated. This study aims to investigate the improved strength of hybridization by comparing the failure behavior of glass-epoxy composite laminate and hybrid glass-graphite-epoxy laminate under transverse sinusoidal load. The composite and hybrid composite laminates with various aspect ratios were modelled using ANSYS. The plates were simply supported and transverse sinusoidal load applied until failure occurs. Initially, numerical validation was performed and the simulated results are found to be close to the 3D elasticity analytical results. In general, most simulated parameters produced error less than 2% and thus proving the reliability of the finite element models and implementation. The first ply failure (FPF) and last ply failure (LPF) curves were plotted to compare the failure trend of glass-epoxy composite laminate and its hybrid laminate. The results showed no significant difference in the FPF trend but hybridization could withstand up to 80% more normalized load in terms of the LPF. In conclusion, the present study has shed new knowledge about the failure behavior of hybrid laminate and thus, this knowledge could be applied in designing hybrid composite laminates.

1. Introduction

Composite materials have been widely used in many applications due to their tailorable properties [1] but understanding their failure behavior is always challenging. In precision engineering such as in aerospace, racing cars, pressure vessels, maritime and defense industries, the failure of components made of composite materials could be catastrophic [2]. Therefore, understanding the failure behavior of composite materials and designing reliable components made of composite materials is crucial.

Recently, hybridization of composite materials are being explored to produce even more flexible superior materials. By good understanding of its behavior, the main aim of designing hybrid composites is to obtain a material with balanced strength and stiffness, balanced bending and membrane mechanical properties, balanced thermal distortion stability, reduced weight and cost, improved fracture toughness and crack arresting properties and also other desirable properties. In short, hybridization makes it possible to obtain a viable compromise between mechanical properties and cost to meet specified design requirements [3].

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The most basic and realistic method to study about composite materials is by conducting experiments [4, 5] but producing composite laminate samples are very costly [5]. Hybrid composite laminates are even more expensive to produce as the sample preparation is more complex than normal composite laminate [6, 7]. Due to this, numerical approach has been the alternative choice to perform failure analysis of composite laminates [8]. Nevertheless, the underlying theories related to the mechanics of composite laminates are complicated [9-12].

For this, the use of commercially finite element software to perform failure analysis of composite laminates has becoming popular as it could avoid lengthy mathematical derivation and finite element programming [3][8][13,14]. Moreover, the capability and usability of the built-in failure criteria function provided by ANSYS has been explored and proven [15].

The construction of mechanical, aerospace, marine and automotive structures, generally requires high reliability. Thus, laminated composites are desirable due to their stiffness, strength, weight, and fatigue life [16]. For reliability assurance, the predictions of the failure process of laminated composite structures and the maximum loads that the structures can withstand before failure occurs have thus become an important topic of research. This is particularly true for the ply-by-ply failure analysis of laminated composite plates and hybrid laminates subjected to transverse loads, where until now, the information about their failure behavior is still lacking. Most of previous works have been concentrating on first-ply failure analysis of thin laminates under uniaxial and transverse loads. Not much work on failure of thin, moderately thick and thick composite laminates under sinusoidal load has been reported in the literature.

To overcome this discrepancy, this study aims to investigate the improved strength of hybridization by comparing the failure behavior of glass-epoxy composite laminate and hybrid glass-graphite-epoxy laminate under transverse sinusoidal load.

In this paper, a finite element simulation is presented for studying the deformation, first-ply failure and last-ply failure of laminated composite plates and its hybrid laminate. The accuracy of the proposed finite element implementation and model is validated via the comparison of the present finite element results with those reported in the literature.

2. Methodology
This study adopts and adapts finite element modelling and simulation in investigating the deformation, first-ply failure and last-ply failure of laminated composite plates and its hybrid laminate using commercially available finite element software, ANSYS (ANSYS v18.1, 2018 SAS IP, Inc.). For better organization and clarity, the methodology adapted in this study is explained in two stages:

Stage I : Numerical Validation
Stage II : Failure Analysis

2.1. Stage I: Numerical Validation
In any research involving numerical approach, particularly in finite element modeling and simulation, the accuracy of the results obtained virtually using computers is very crucial. Therefore, numerical validation is essential to prove the accuracy of the computations before proceeding to further analyses. In this study, the accuracy of the present finite element implementation and model is validated via comparing the present simulated results with those established results reported in the literature.

For numerical validation, this study replicates the work conducted by Tasneem et al. [11] where they developed a finite element program to compute the displacements and stresses of composite laminates and compared to analytical results based on 3D elasticity theory. In this study, commercially finite element software, ANSYS has been used to model and simulate the same problem. In general, a square plate with 1m length subjected to transverse sinusoidal load was modelled using ANSYS. The plate was made of 4-layer composite laminate with the lay up of [0/90/90/0] and simply supported [11]. Using 8-noded shell elements (Shell 281), the plate was meshed into 16 (4 x 4) elements, representing only a quarter plate as the problem is axissymmetric. Table 1 shows the material properties according to the
work of Tasneem et al [11], where $E$ is the elastic modulus of the material, $v$ is the Poisson ratio and $G$ is the shear modulus.

**Table 1.** Material properties of the laminas for numerical validation [11].

| Properties | Values   |
|------------|----------|
| $E_1$      | 175 GPa  |
| $E_2 = E_2$ | 7 GPa    |
| $\nu_{12} = \nu_{12} = \nu_{12}$ | 0.25     |
| $G_{12} = G_{13}$ | 3.5 GPa  |
| $G_{23}$   | 1.4 GPa  |

The transverse sinusoidal load applied was $P(x,y) = P_0 (\sin \frac{\pi x}{a})(\sin \frac{\pi y}{a})$, where, $P_0 = 1000$ N. Once the model complete, the problem was solved using standard ANSYS solver. The displacements and stresses at aspect (length-to-thickness, $a/h$) ratio of $S=10$ and $S=100$ were determined and recorded. Obtaining results from the finite element procedure, the values of the maximum displacements and stresses were then normalized using equations following the work of Tasneem et al [11]. Finally, the present results are compared to established results reported in the literature [11,12]. This is presented in the results and discussion section. The percentage error was calculated using equation 1.

$$\%_{\text{error}} = \frac{3D \text{ Elasticity value} - \text{present value}}{3D \text{ Elasticity value}} \times 100\%$$

### 2.2. Stage II: Failure Analysis

For the present study, the failure analysis was performed on a composite laminate and its hybrid laminate. In general, a square composite plate with length, $a = 0.04$ m and various thickness, $h$, subjected to transverse sinusoidal load was modelled using ANSYS. Using 8-noded shell elements (Shell 281), the plate was meshed into 16 (4 x 4) elements, representing only the quarter plate as the problem is axissymmetric. The plate was made of 4-layer composite laminate with the lay up of [0/90/90/0]. Various aspect (length-to-thickness, $a/h$) ratio, $S$, were studied, ranging from 5 to 100 to incorporate the effect of thin, moderately thin and thick plate deformation characteristic. For the purpose of investigating the failure behavior of glass-epoxy composite laminate and hybrid glass-graphite-epoxy laminate, two lamination schemes were modelled. The composite laminate was represented by all the four layers made of glass-epoxy layers [GE/GE/GE/GE], while its hybrid composite laminate was obtained by adding graphite-epoxy layers, denoted as [GE/GrE/GrE/GrE]. The material properties of the glass-epoxy layer and graphite-epoxy layers are presented in table 2 and table 3 respectively.

**Table 2.** Material properties for glass/epoxy [17].

| Elastic parameter | Strength data |
|-------------------|---------------|
| $E_1$             | $X_T$         |
| $E_2 = E_3$       | $X_C$         |
| $\nu_{12} = \nu_{12} = \nu_{12}$ | $Y_T$         |
| $G_{12} = G_{23} =$ | $Y_C$        |
| $G_{13}$         | $S$           |
| 19.94 GPa        | 700.11 MPa    |
| 5.83 GPa         | 570.37 MPa    |
| 0.29             | 69.67 MPa     |
| 2.11 GPa         | 122.12 MPa    |
| 68.89 MPa        |               |
Table 3. Material properties for graphite/epoxy [17].

| Elastic parameter | Strength data |
|-------------------|---------------|
| $E_1$             | $X_T$         |
| $E_2 = E_3$       | $X_C$         |
| $v_{12} = v_{13} = v_{12}$ | $Y_T$         |
| $G_{12} = G_{23} = G_{13}$ | $Y_C$         |
|                   | $S$           |
| 138 GPa           | 1450 MPa      |
| 10.6 GPa          | 1450 MPa      |
| 0.3               | 51 MPa        |
| 6.46 GPa          | 250 MPa       |
|                   | 93 MPa        |

The boundary condition was simply supported as shown in figure 1. The loading condition was transverse sinusoidal load applied, $P(x,y) = P_0(sin(\frac{\pi x}{a})) (sin(\frac{\pi y}{a}))$. To determine the failure load, $P_0$ was increased until the plate failed. Failure was predicted using the built-in failure criteria function in ANSYS; and Maximum Stress Failure Criteria was employed. First ply failure (FPF) load was recorded when at least one of the layer failed and last ply failure was recorded when all the layers failed at the minimum $P_0$. From the results obtained based on the failure analysis procedure, the FPF load and LPF load were then normalized using the equations 2a and 2b.

\[
\text{Normalized First Ply Failure (FPF load)} = \text{FPF load} \times \frac{S^2}{10^6} \quad (2a)
\]

\[
\text{Normalized Last Ply Failure (LPF load)} = \text{LPF load} \times \frac{S^2}{10^6} \quad (2b)
\]

3. Results and Discussion

3.1. Results from numerical validation

The results obtained from the finite element procedure are tabulated in table 4 for aspect ratio, $S = 10$ and table 5 for aspect ratio, $S = 100$; representing thick and thin laminates respectively. The values of the normalized maximum displacements and stresses for the present results are compared to established results reported in the literature [11]. For thin laminates, it could be seen that except for the stress in $yz$ plane, $\sigma_{yz}$, the error was found less than 2% for other stresses. This proves that the FE models and implementation are well validated. However, for thick laminates, it could be seen that except for stresses in $xx$, $xy$, $yz$ planes, ($\sigma_{zx}$, $\sigma_{zy}$, $\sigma_{zz}$), the error was found less than 2% for other stresses. This trend has been discussed well by Tasneem et al [11], where the present laminate model neglects the transverse shear generated by thick laminates. Observing table 4 and table 5, it could be concluded that the results obtained from the FE models and implementation could be accepted.
Table 4. Normalised Maximum Displacements, $w$ and stresses for Aspect ratio, $S = 10$.

| Maximum deflection and stresses | $w$ | $\sigma_{xx}$ | $\sigma_{yy}$ | $\sigma_{xy}$ | $\sigma_{yz}$ | $\sigma_{xz}$ |
|---------------------------------|-----|---------------|---------------|---------------|---------------|---------------|
| 3D Elasticity value [11,12]     | 0.743 | 0.559          | 0.401          | 0.0275        | 0.196         | 0.301         |
| Present Unnormalised value      | 1.0571E-06 | -48271        | -39876        | 2528.5        | 2156.7        | 3058.6        |
| Present Normalised values       | 0.7399 | 0.4826         | 0.3987        | 0.0252        | 0.2155        | 0.3058        |
| Error (%)                       | 0.417 | 13.667         | 0.573         | 8.363         | 9.949         | 1.595         |

Table 5. Normalised Maximum Displacement, $w$ and stresses for Aspect ratio, $S = 100$.

| Maximum deflection and stresses | $w$ | $\sigma_{xx}$ | $\sigma_{yy}$ | $\sigma_{xy}$ | $\sigma_{yz}$ | $\sigma_{xz}$ |
|---------------------------------|-----|---------------|---------------|---------------|---------------|---------------|
| 3D Elasticity value [11,12]     | 0.438 | 0.539          | 0.276          | 0.0216        | 0.141         | 0.337         |
| Present Unnormalised value      | 0.61690E-3 | -5379200      | -2709300      | 213280        | 15474         | 33598         |
| Present Normalised values       | 0.4318 | 0.5379         | 0.2710        | 0.0213        | 0.1545        | 0.3360        |
| Error (%)                       | 1.408 | 0.204          | 1.848         | 1.389         | 9.574         | 0.297         |

The sample outputs from the finite element simulation are shown in figure 2. Figure 2 displays the typical contour for the displacement in z-direction, $w$. Since only the quarter of the plate was modelled due to axissymmetry of the problem, the maximum deflection of the plate is actually occurred at the middle of the plate. Similar finding has been reported in established results reported in the literature [11][12]. Nevertheless, unlike using finite element programming [11][12], the present study has the advantage to view and display the deformation contour directly, as these outputs are provided by the commercial available finite element software, such as ANSYS. In finite element analysis, the visualization of deformation and stress distribution is very useful when designing and modifying physical components made of composite laminates before finalizing the component design. In this study, such information could enhance understanding about the deformation behavior and stress distribution of the glass-epoxy composite laminate and its hybrid laminate under transverse sinusoidal load.
Figure 2. Sample outputs from finite element simulation displaying the typical displacement contour, $w$, (in z-direction) of the quarter plate from (a) top view, (b) rotated isometri view and (c) selected angle view to highlight the maximum deflection of the composite laminate.

3.2. Failure load and failure curves

The results obtained from the failure analysis using finite element procedure are tabulated in table 6.1, table 6.2, table 7.1 and table 7.2 for various aspect ratio, $S = 10$ to $S = 100$. Table 6.1 tabulates the FPF and LPF loads for glass-epoxy composite laminates. Figure 3 highlights the trend of the normalized FPF and LPF curves under transverse sinusoidal load for various aspect ratio, representing from thin to thick laminates. The trend of the FPF and LPF curves are typical for a composite plate as shown earlier by Tolson and Zabaras [18].

**Table 6.1.** FPF load for Glass-epoxy [GE/GE/GE/GE] composite laminate.

| Ply thickness, $t_i$ (mm) | Load Applied (MPa) | Normalized Load, FPF* (MPa) |
|----------------------|-------------------|-----------------------------|
| 2                    | 21,608,160        | 540.204                     |
| 1                    | 5,610,990         | 561.099                     |
| 0.5                  | 1,418,645         | 567.458                     |
| 0.2                  | 227,750           | 569.375                     |
| 0.1                  | 56,982            | 569.82                      |

**Table 6.2.** LPF load for Glass-epoxy [GE/GE/GE/GE] composite laminate.

| Ply thickness, $t_i$ (mm) | Load Applied (MPa) | Normalized Load, LPF* (MPa) |
|----------------------|-------------------|-----------------------------|
| 2                    | 37,876,270        | 946.90                      |
| 1                    | 9,833,345         | 983.3345                    |
| 0.5                  | 2,486,655         | 994.662                     |
| 0.2                  | 399,209           | 998.0225                    |
| 0.1                  | 99,851            | 998.51                      |
Figure 3. FPF (a) and LPF (b) curves for Glass-epoxy [GE/GE/GE/GE] composite laminate.

More interestingly for the present study when considering hybrid composite laminates, the FPF and LPF loads for glass/graphite-epoxy [GE/GrE/GrE/GrE] composite laminates are presented in table 7.1 and table 7.2. Figure 4 highlights the trend of the hybrid composite laminates normalized FPF and LPF curves under transverse sinusoidal load for various aspect ratio, which again representing from thin to thick laminates. In general, the trend of the FPF and LPF curves are similar to typical composite laminate as shown in figure 3. Nevertheless, what interesting to find is that the values increased for the LPF load.

Table 7.1. FPF load for Glass/Graphite-epoxy [GE/GrE/GrE/GrE] hybrid composite laminate.

| Aspect Ratio, S | 5    | 10   | 20   | 50   | 100  |
|----------------|------|------|------|------|------|
| Ply thickness, $t_i$ (mm) | 2    | 1    | 0.5  | 0.2  | 0.1  |
| Load Applied (MPa) | 22,608,490 | 5,666,640 | 1,417,780 | 226,897 | 56,726 |
| Normalized Load, FPF* (MPa) | 565.2123 | 566.665 | 567.112 | 567.243 | 567.26 |

Table 7.2. LPF load for Glass/Graphite-epoxy [GE/GrE/GrE/GrE] hybrid composite laminate.

| Aspect Ratio, S | 5    | 10   | 20   | 50   | 100  |
|----------------|------|------|------|------|------|
| Ply thickness, $t_i$ (mm) | 2    | 1    | 0.5  | 0.2  | 0.1  |
| Load Applied (MPa) | 73,630,740 | 18,408,126 | 4,602,077 | 736,340 | 184,084 |
| Normalized Load, LPF* (MPa) | 1840.768 | 1840.813 | 1840.831 | 1840.85 | 1840.84 |
Figure 4. FPF (a) and LPF (b) curves for Glass/Graphite-epoxy [GE/GrE/GrE/GrE] hybrid composite laminate.

Combining all the results, figure 5 highlights the difference of the failure behavior of composite laminate and its hybrid laminate. In general, the similar trend is observed for all curves but the LPF curves for the hybrid laminate [GE/GrE/GrE/GrE] has increased about twice the original glass-epoxy laminates [GE/GE/GE/GE].

From figure 5, it could be observed that as the curves reach to aspect ratio of $S = 20$ and ply-thickness of $t = 0.5$mm (thick plate), the curves begin to stable (converging to a value) and this trend maintains until it reaches the aspect ratio of $S = 100$ (thin plate). The reason for these changes from aspect ratio, 5 to 20, could possibly be due to their higher interaction ultimate strength tolerate more pressure [16].

Figure 5. The comparison of normalized FPF and LPF Loads between glass-epoxy [GE/GE/GE/GE] and hybrid glass/graphite-epoxy [GE/GrE/GrE/GrE] laminates.

Nevertheless, when considering the FPF curves, no significant difference could be observed. This is because the first ply failure still occur on the glass-epoxy layer, which means upon certain load, the glass-epoxy layer fails but the strength of the graphite layer has protected the whole laminate from failing...
earlier. This could signify that the adding of the graphite layer has increased the durability of the laminate from total failure, even though failure already occurred on the glass-epoxy layers.

The glass layers are observed lower in Young’s Modulus as shown presented in table 2. Therefore, it is observed in table 6.1 and table 6.2 that the maximum of FPF and LPF loads reached 998 MPa and 567 MPa based on Maximum Stress Failure Criterion.

As comparison to glass-epoxy laminated composite, the graphite-epoxy layers are observed to sustain higher failure loads due to its higher longitudinal and transverse Young’s modulus (table 3) compared to glass-epoxy layers. From table 7.1 and table 7.2, the last ply failure are observed to reach 1840 MPa based on the same Maximum Stress Failure Criterion. Compared to Graphite-epoxy, layer of glass-epoxy is going to fail first as they are weaker in terms of layer strength, when comparing their properties in table 2 and table 3. This can be concluded that, the failure of laminate is subjected to higher stresses due to higher longitudinal Young’s modulus, hence, making the failure occurred in high pressure.

Regardless the results obtained, the present study has demonstrated a useful technique using commercially available finite element software, ANSYS in investigating the failure behavior of composite laminates and its hybrid laminate. Moreover the capability to visualize the deformation behavior and stress distribution promptly as this is essential in design work. Therefore, this becomes a significant advantage of the present study compared to finite element programming [9][11][12][18]. More importantly, new knowledge about finite element simulation and its validity, as well as understanding the failure behavior of composite materials have been presented well in this study. The technique, results and case studies presented could be replicated and further explored to solve other kinds of hybrid composite materials under various loading conditions.

4. Conclusion
This paper presents the implementation of finite element analysis and simulation in investigating the failure behaviour of glass-epoxy [GE/GE/GE/GE] composite laminate and hybrid glass-graphite-epoxy [GE/GrE/GrE/GrE] laminate under transverse sinusoidal load. As the finite element models have been validated and showing most errors of less than 2%, this proves that the results from finite element simulation are reliable which makes it an effective alternative to conducting expensive and tedious experiments. In terms of failure trend of both glass composite and its hybrid laminate, the simulated results show that there is no significant difference in the first ply failure (FPF) trend but in terms of the last ply failure (LPF), the hybrid laminate could withstand up to 80% more normalized load. As a conclusion, it can be said that the present study has shed new knowledge about the failure behavior of hybrid laminates and thus, this knowledge could be applied in designing hybrid composite laminates.

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References
[1] Zheng W Kassapoglou C and Zheng L 2019 Tailoring of AP-PLY composite laminates for improved performance in the presence of delaminations Compos. Struct. 211, p. 89-99.
[2] Mali M Samsudin A H Mahmud J Hussain A K and Alansary M D Failure Analysis of Composite Laminates Under Biaxial Tensile Load Due to Variations in Lamination Scheme 2017 J. Mech. Eng. 4, 5 p. 167-182.
[3] Zhang J 2012 Hybrid Composite Laminates Reinforced With Glass and Carbon Mater. Design. 30, p. 75-80.
[4] Mahmud J Hussain A K Rahimi N and Rahim M A 2013 Failure analysis of composite laminate based on experiment-simulation integration J. CREAM. 2, p. 7-22.
[5] Jeannin T Gabrion X Ramasso E and Placet A 2019 About the fatigue endurance of unidirectional flax-epoxy composite laminates, Compos. Part B: Eng. 165, p. 690-701.
[6] Belgacem L, Ounas D, Olay J A V and Amado A A 2018 Experimental investigation of notch effect and ply number on mechanical behavior of interply hybrid laminates (glass/carbon/epoxy) Compos. Part B: Eng. 145, p. 189-196.

[7] Muflikhun M A, Yokozeki T and Aoki T 2019 The strain performance of thin CFRP-SPCC hybrid laminates for automobile structures Compos. Struct. 220, p. 11-18.

[8] Rahimi N, Musa M, Hussain A K and Mahmud J 2012 Finite Element Implementations to Predict the Failure of Composite Laminates Under Uniaxial Tension Adv. Mater. Res. 499, p. 20-24.

[9] Liew K M, Pan Z Z and Zhang L W 2019 An overview of layerwise theories for composite laminates and structures: Development, numerical implementation and application Compos. Struct. 216, p. 240-259.

[10] Goswami Y 2016 Stress and failure analysis of inter-ply hybrid laminated composite using finite element method J. Eng. and Technol. 3, 7 p. 2391-2396.

[11] Tasneem P, Khalid A Z and Al-Jahwari F K S 2010 Effects of Boundary Conditions in Laminated Composite Plates Using Higher Order Shear Deformation Theory Appl. Compos. Mater. 17, p. 499–514.

[12] Lee J D 1982 Three Dimensional Finite Element Analysis of Damage Accumulation in Composite Laminate, Comput. and Struct. 15, 3 p. 335-350.

[13] Rahimi N, Rahim M A, Hussain A K and Mahmud J 2012 Evaluation of failure criteria for composite plates under tension Proc. of 2012 IEEE Symposium on Humanities, Sci. and Eng. Research (SHUSER 2012) p. 849-854.

[14] Banerjee S and Sankar B V 2014 Mechanical properties of hybrid composites using finite element method based micromechanics Compos. Part B: Eng. 58, p. 318-327.

[15] Rahimi N, Hussain A K, Meon M S and Mahmud J 2012 Capability Assessment of Finite Element Software in Predicting the Last Ply Failure of Composite Laminates Procedia Eng. 41, p. 1647-1653.

[16] Manders P W and Bader G M 1981 The strength of hybrid glass/carbon fibre composites J. Mater. Sci. 16, 8 p. 2233–2245.

[17] Noh N N, Samsudin A, Hand Mahmud J 2017 Failure Analysis of Glass/Epoxy and Graphite/Epoxy Laminates due to the Effect of Variation in Lamination Scheme and Angle of Fibre Orientation Mater.s Sci. Forum. 889, p. 36-44.

[18] Tolson S and Zabaras N 1991 Finite Element Analysis of Progressive Failure in Laminated Composite Plate Comput.s and Struct. 38, p. 361-372.