Inverter harmonic perturbations rejection in renewable energy conversion systems applying a super-twisting algorithm

Daniel Memije\(^1\) \quad \text{Oscar Carranza}^2 \quad \text{Jaime J. Rodríguez}^1 \quad \text{Rubén Ortega}^2 \quad \text{Edgar Peralta}^3

\(^1\) Escuela Superior de Ingeniería Mecánica y Eléctrica, Instituto Politécnico Nacional, Ciudad de México, México
\(^2\) Escuela Superior de Computo, Instituto Politécnico Nacional, Ciudad de México, México
\(^3\) Engineering Department, Benemérita Universidad Autónoma de Puebla UPAEP, Puebla, México

Abstract

Grid background and dead-time harmonic perturbations pose challenges in maintaining power quality and stability in the grid integration of renewable energy conversion systems. Therefore, a super-twisting algorithm with the ability to reject harmonic perturbations on three-phase grid-tied inverters is proposed. The algorithm design is related to dead-time and grid background harmonics so that robust performance is achieved under these perturbations. To highlight the advantages of the proposed algorithm, it is compared against proportional-integral (PI) control in several dead-time and grid background harmonic sweeps. The maximum inverter current distortion obtained with PI control in simulation is 15.53%. However, by using the proposed algorithm instead, the maximum current distortion is reduced to 1.8%. Dead-time is experimentally swept by using a permanent magnet motor as a harmonic-free grid. Experimental results of grid background harmonics are obtained by injecting reactive energy into a grid with 2.19% voltage distortion. When PI control is used in the experimental setup, the maximum inverter current distortion obtained is 11.4%. However, with the proposed algorithm, the maximum current distortion is reduced to 1.5%, which complies with the standard IEEE 1547-2018.

1 | INTRODUCTION

Generally, any renewable energy conversion system (RECS) is tied to the grid by using a three-phase inverter as shown in Figure 1. However, grid-tied inverters are exposed to external perturbation (e.g. background harmonics \([1–9]\), grid impedance variations \([2, 10, 11]\), and internal perturbation (e.g. dead-time \([10–14]\) and saturation of reactors \([15]\)). Such perturbations pose challenges in maintaining power quality and system stability in the grid integration of RECS \([6]\).

Grid background harmonics are present in all electric power systems due to the non-linear loads demanding power from the grid \([4–7]\). The standard IEEE 519 \([16]\) stipulates limits for grid background harmonics at the point of common coupling. High levels of these harmonics have severe consequences on RECS, such as distortion of the inverter currents, equipment overheating, and failed operations of electronic equipment \([1–9]\). In addition, dead time, which prevents shoot-through in RECS inverters, also distorts the inverter currents \([10–13]\). Even with the availability of high-speed switching devices, dead-time effects cannot be ignored \([10]\). Consequently, dead-time and grid background harmonics can cause serious power quality problems in RECS \([1, 8, 11, 12]\). For these reasons, harmonic perturbations affecting the inverter in RECSs need careful consideration.

The total rated-current distortion (TRD), defined by Equation (2) in the standard IEEE 1547 \([17]\), permits to measure power quality in RECSs. In this standard, the maximum permissible TRD is 5%. Thus, RECSs should deliver high-quality power despite harmonic perturbations affecting the inverter.

Decreasing TRD is achieved by increasing the harmonic rejection properties of the power filter \([5, 18]\) or by increasing the bandwidth of the control system \([2, 3, 5]\). By using more efficient transistors, it is possible to increase the inverter switching frequency \([19–21]\). The switching frequency increase allows the use of power filters with smaller impedance and size, which is a trend at present with the introduction of high power density inverters \([22]\). However, the reduction of the filter impedance...
makes the inverter more susceptible to harmonic perturbations (this statement is explained in Section 2). Due to its potential to increase energy quality, the search for control algorithms that can reject harmonic perturbations is a topic of interest [2–8, 10–12].

Conventionally, proportional-integral (PI) regulators are used to control the RECS internal states. However, PI control has low harmonic perturbations rejection capacity. Consequently, PI control cannot reject completely the effects that dead-time and grid background harmonics produced on the inverter currents [4, 8, 9]. For instance, the study of a commercial solar inverter performed in [1] shows how a 3% total harmonic distortion (THD) of grid background voltage can induce up to 20% TRD in the inverter output currents.

There are several harmonic rejection algorithms for grid-tied inverters, which fall into two main categories: Selective and non-selective algorithms [9]. PI control with resonant action (PI+R) is the most popular selective algorithm [7, 8]. However, PI+R increases the computational burden because one resonant action is needed to reject each one of the unwanted harmonic components. PI control with repetitive action (PI+RC) is a non-selective algorithm that can reject all harmonic perturbation components with one controller [4, 23]. Nevertheless, PI+RC performance is degraded with grid frequency variations. Thus, PI+RC with frequency adapting capabilities is preferred, even though such enhancements increase the computational burden too [24–26].

Due to its fast dynamics and its robustness, an alternative to reject harmonic perturbations in three-phase grid-tied inverters is the sliding mode control (SMC). In [27], a grid-tied inverter working at unity power factor using SMC is presented. An adaptive integral SMC for a three-phase grid-tied inverter is proposed in [28, 29]. An adaptive fuzzy SMC for a three-phase active power filter is presented in [30], which permits a fast response and chattering reduction. A neutral-point-clamped inverter using SMC is presented in [31], where the power quality improvement due to SMC is reported. Space vector modulation based on SMC for a grid-tied inverter is designed in [32]. Although effective, all the aforementioned applications of SMC are based on first-order sliding mode theory, in which chattering is a severe complication. Commonly, chattering is attenuated by approximating the sign function at the expense of control robustness decrease [33].

Second-order SMC algorithms reduce the undesirable chattering effects, and they preserve control robustness [33]. The super-twisting SMC (ST-SMC) is one of the most popular second-order algorithms, as it does not rely on any time derivative. An ST-SMC is designed in [34, 35], where it is used in a single-phase inverter to deliver high-quality power into the grid. An application of an ST-SMC algorithm for a three-phase inverter is designed in [36], where the controller gains are adaptively computed in real time.

The main contribution in this study is the proposition of an ST-SMC as a non-selective harmonic perturbation rejection algorithm in three-phase grid-tied inverters. The ST-SMC is robust against grid frequency variations (an existing problem in repetitive control). Moreover, in contrast to resonant control, all harmonic perturbations are rejected by one ST-SMC. The ST-SMC is compared against PI control, in all experimental and simulated situation proposed. In simulation, the dead time is swept from 0 to 2 µs, and the grid background harmonic component is swept from the second to the 25th. In the experimental setup, the dead time is swept from 0.4 to 2 µs by using a permanent magnet synchronous motor (PMSM) as the experimental harmonic-free grid. Finally, a grid polluted with 2.19% THD is used experimentally. The results show how by using PI control under harmonic perturbations, the inverter TRD increases up to 15.58%, whereas the ST-SMC keeps the inverter TRD below 1.8% so that RECSs meet the standard IEEE 1547.

2 PROBLEMS STATEMENT

Figure 2(a) shows the vector control diagram of a grid-tied inverter with an L-filter, where PI regulators are used for the synchronous current control. Figure 2(b) shows the d-axis PI control system, where the grid impedance is neglected, \( e_d \) stands for harmonics perturbations, and \( i_d \) is the d-axis inverter current. Figure 2(b) is rearranged into Figure 2(c), where the
reference is set to zero (i.e. \( i_d^* = 0 \)) to illustrate the effect that \( e_d \) has on \( i_d \). Bode plots of the transfer function are shown in Figure 3(c), using the parameters listed in Appendix A, and the L-filter admittance for different inductance values are shown in Figure 3. Observe in Figure 3 how the perturbations can be attenuated by increasing any of the following: The L-filter inductance or the control system bandwidth. Although increasing the inductance will increase the inverter cost and weight [5, 18], the easiest solution is to increase the controller bandwidth, where increasing the bandwidth of a PI controller will decrease the system stability [8, 37]. This bandwidth limitation of PI controllers makes evident the need for control alternatives, other than just PI control, which is able to reject harmonic perturbations in grid-tied inverters [8].

3 | GRID-TIED INVERTER PERTURBATIONS AND UNCERTAINTIES

3.1 | Three-phase inverter general mathematical model

The inverter general mathematical model in the synchronous reference frame (dq frame) is given as follows:

\[
\frac{di}{dt} = \dot{A}i + Bv + e \tag{1}
\]

where \( \dot{A} \in \mathbb{R}^{2n \times 2n} \) is the state matrix, \( B \in \mathbb{R}^{2n \times 2} \) is the input matrix, \( i \in \mathbb{R}^{2n} \) is the state vector, \( v \in \mathbb{R}^{2} \) is the inverter output voltage vector, \( e \in \mathbb{R}^{2n} \) represents external perturbations, and \( n \) stands for the inverter filter order (e.g. for an L-filter: \( n = 1 \), for an LCL-filter: \( n = 3 \)). Uncertainty is included by assuming that \( \dot{A} \), and \( B \) are comprised by

\[
\dot{A} = \dot{\dot{A}} + \Delta A; \quad B = \dot{B} + \Delta B \tag{2}
\]

Also, vector uncertainty of \( i \), \( v \), and \( e \) is accounted for in the following expressions:

\[
i = \dot{i} + \Delta i; \quad v = \dot{v} + \Delta v; \quad \text{and} \quad e = \dot{e} + \Delta e \tag{3}
\]

where \( \dot{\dot{A}}, \dot{B}, \dot{i}, \dot{v} \) and \( \dot{e} \) are ideal matrices and vectors, which are all known by the control system. \( \Delta A, \Delta B, \Delta i, \Delta v \) and \( \Delta e \) represent the uncertainty and perturbations affecting the inverter, which are unknown by the control system. The description of Equations (2) and (3) is summarised in Table 1.

By substituting Equations (2) and (3) in (1), the inverter model is transformed as follows:

\[
\frac{di}{dt} = \dot{A}i + Bv + \varphi \tag{4a}
\]

\[
\varphi = e + \DeltaAi + \DeltaBi + \Delta Bv + \ddot{B}v + \Delta e - \frac{d\Delta i}{dt} \tag{4b}
\]

where \( \varphi \in \mathbb{R}^{2n} \) includes all the uncertainty and perturbations affecting the inverter. A common practice in control system design is to assume that \( \varphi \) is equal to zero. However, this assumption can be dangerous, as \( \varphi \) is never equal to zero. Vector \( \varphi \) is decomposed into the next two vectors:

\[
\zeta = \text{proj}_B \varphi; \quad \delta = \varphi - \zeta \tag{5}
\]

where \( \zeta \in \mathbb{R}^{2n} \) is the projection of all inverter perturbations and uncertainties onto the input matrix \( B \), and \( \delta \in \mathbb{R}^{2n} \) is orthogonal to \( B \). Vector \( \zeta \) includes all perturbations that can be rejected by SMC; hence, it is called: Matched perturbation vector. On the contrary, \( \delta \) represent all perturbations that cannot be rejected completely by SMC. For this reason, \( \delta \) is called an unmatched perturbation vector. SMC algorithms are robust only if the perturbations are in the form of \( \zeta \) (see [33]).

3.1.1 | Three-phase inverter with L-filter

The most interesting case for Equation (1) is when \( n = 1 \). This case describes an inverter with L-filter (see Figure 2). The matrices are (see [38]):

\[
\dot{A} = \begin{bmatrix} -R/L & \omega \\ -\omega & -R/L \end{bmatrix}; \quad \text{and} \quad B = \begin{bmatrix} 1/L & 0 \\ 0 & 1/L \end{bmatrix} \tag{6}
\]

where \( \omega \) is the grid rated frequency, \( R \) and \( L \) are the rated resistance and inductance of the L-filter. This inverter is interesting, as the matrix \( B \) is full rank. Thus, all perturbations affecting this inverter can be rejected, that is,

\[
\zeta = \text{proj}_B \varphi; \quad \text{and} \quad \delta = 0 \tag{7}
\]

By taking advantage of this property, the present study proposes an SMC algorithm for the inverter with an L-filter that presents robustness to all perturbation, such as parameter variation, background harmonics, dead time, and so forth.

3.1.2 | Three-phase inverter with LCL-filter

When \( n = 3 \), Equation (1) describes the inverter with LCL-filter as shown in Figure 4. The equations describing this inverter are presented in [38]. Due to its higher dynamics, the input matrix
### TABLE 1  System uncertainties, and perturbations

| Parameter | Description                                                                 |
|-----------|-----------------------------------------------------------------------------|
| \( \bar{A}, \bar{B} \) | Rated matrices                                                              |
| \( \bar{i} \) | States measurement                                                          |
| \( \bar{r} \) | External perturbation measurement                                            |
| \( \bar{e} \) | Ideal inverter output voltage                                               |
| \( \Delta \bar{A}, \Delta \bar{B} \) | Parameter uncertainty                                                       |
| \( \Delta \bar{i}, \Delta \bar{e} \) | Measurement errors                                                          |
| \( \Delta r \) | Inverter output voltage distortion                                           |

**Description**
- Rated system parameters
- Digital representation of the inverter states
- Digital representation of the inverter external perturbations
- Ideal inverter output voltage
- Difference between the rated parameters and real values
- Aliasing, sensor errors, noise
- Dead time, transistor voltage drop, inverter non-linearities

---

**FIGURE 4** Grid-tied inverter with LCL filter

### 4  SUPER TWISTING VECTOR CONTROL

#### 4.1  Control system definition

##### 4.1.1  Vector sliding surface

Vector sliding surface is defined in vector form as

\[
\mathbf{x} = \mathbf{i}^* - \mathbf{\bar{i}} = [\mathbf{x}^T \ \mathbf{y}^T]^T \tag{9}
\]

where \( \mathbf{\bar{i}} \) is a current measurement, \( \mathbf{i}^* \) is the control reference.

#### 4.1.2  Vector sign function

Vector sign function is defined from Equation (9) as

\[
\text{sign}(\mathbf{x}) = \begin{cases} 
\frac{x}{|x|} & \text{if } x \neq 0 \\
0 & \text{otherwise}
\end{cases} \tag{10}
\]

#### 4.1.3  Absolute value square root

Absolute value square root is defined from Equation (9) as

\[
\sqrt{|\mathbf{x}|} = \begin{bmatrix} \sqrt{|x|} & 0 \\ 0 & \sqrt{|y|} \end{bmatrix} \tag{11}
\]

#### 4.1.4  ST-SMC algorithm

Based on Equations (1), (3), (9)–(11), a vector form of the ST-SMC is proposed as (see Figure 5)

\[
\frac{d}{dt} \begin{bmatrix} \mathbf{i} \\ \mathbf{u} \end{bmatrix} = \begin{cases} 
\mathbf{A}\mathbf{i} + \mathbf{B} (\bar{\mathbf{v}} + \Delta \mathbf{v} - \mathbf{e}) + \mathbf{k}\mathbf{P}\text{sign}(\mathbf{x}) \\
k_1 x + k_2 \text{sign}(\mathbf{x}) + \bar{\mathbf{e}} + \mathbf{u}
\end{cases} \tag{12a}
\]

where \( A, B \) are given by Equations (2) and (6), \( \mathbf{i} \) is the inverter current, \( e \) is the grid voltage, \( \bar{\mathbf{v}} \) is the ideal inverter voltage, \( \Delta \mathbf{v} \) is the dead-time distortion, \( k_p, k_i, k_1, k_2 \) are controller gains, \( u \) is the control system integral action, and \( \bar{e} \) is the measurement of grid voltage. Here, \( \bar{e} \) increases the ST-SMC performance.

A PI control with the feedforward of \( \bar{e} \) is a common practice [37], but this alternative is not enough to reject grid harmonics because it does not consider dead-time distortion and the analogue-to-digital conversion (ADC) process errors [40, 41].

Dc voltage and inverter currents are also processed by an ADC (see Figure 5). The phase lock loop shown in Figure 5 is the same as in [42]. A PI controller regulates the dc voltage (designed also in [42]), in which a low-pass filter (LPF) rejects the noise introduced by the ADC. For stability reason, LPFs are not used for the currents; instead, the sampling frequency is twice the switching frequency to avoid aliasing [41].
4.2 Controller stability demonstration

The stability demonstration of Equation (12) is carried out by defining the potential and kinetic energy of Equation (9). The objective of this demonstration is to determine the controller gains that guarantee the convergence to zero of Equation (9) in the presence of perturbations and uncertainties so that robust control is achieved. The definition of the potential energy will be obtained from the second derivative of Equation (9).

The first time derivative of Equation (9), substituting Equation (12(a)), is expressed as follows:

$$\frac{dx}{dt} = \frac{d\bar{i}}{dt} + \frac{d\Delta i}{dt} - A_1 - B_1(\bar{v} + \Delta v - e) \quad (13)$$

By substituting $\bar{v}$ in Equation (13) for Equation (12(b)), it gives as a result:

$$\frac{dx}{dt} = -B_1 \left[ k_1 \bar{F} + k_1 f(x) + u + u \right] \quad (14a)$$

$$\mathbf{i} = \Delta v + \Delta e + \mathbf{e} \quad (14b)$$

$$\mathbf{e} = -B_1^{-1} \left( \frac{d\bar{i}}{dt} + \frac{d\Delta i}{dt} - A_1 \right) \quad (14c)$$

where $\mathbf{i}$ is the matched perturbations vector, $\mathbf{e}$ stands for the unknown uncertainties, and $f(x) = \sqrt{|x|} \text{sign}(x)$. Observe how $\mathbf{i}$ contains all the perturbations that affect the inverter.

By taking the time derivative of Equation (14(a)), substituting Equation (12(a)), a second-order matrix differential equation is obtained:

$$\frac{d^2x}{dt^2} + B_1 \left[ k_1 \bar{F} + k_1 f(x) \right] \frac{dx}{dt} + \rho = 0 \quad (15a)$$

$$\rho = B_1 \left( k_1 \bar{F} + k_1 \text{sign}(x) + \frac{d\zeta}{dt} \right) \quad (15b)$$

where $f(x)$ is the Jacobian matrix of $f(x)$, given by the following positive definite matrix:

$$f(x) = \begin{bmatrix} \frac{1}{\sqrt{|x|}} & 0 \\ 0 & \frac{1}{\sqrt{|y|}} \end{bmatrix} \quad \forall \ x, y \neq 0 \quad (16)$$

4.2.1 Kinetic energy

Kinetic energy is defined as the following quadratic form:

$$K = \frac{1}{2} \frac{dx}{dt}^T \frac{dx}{dt} > 0 \quad (17)$$

4.2.2 Potential energy

Potential energy is defined as the line integral of Equation (15(b)), that is,

$$U = B_1 \int_{(0,0)}^\infty \left( k_1 l + k_2 \text{sign}(l) + \frac{d\zeta}{dt} \right) dl \quad (18)$$

where $l$ is a dummy variable used for integration purposes.

By assuming that $\zeta$ depend only on time, it can come out of the integral. Thus, the evaluation of Equation (18) yields:

$$U = B_1 \left( k_1 x^T + k_2 \text{sign}(x) + x^T \frac{d\zeta}{dt} \right) \quad (19)$$

where $\|x\|_1$ is the 1-norm [42].

4.2.3 Lyapunov function

The Lyapunov function is defined as the addition of Equations (17) and (19), which gives as a
result

$$E = \frac{1}{2} \frac{dx}{dt}^T \frac{dx}{dt} + \left( \frac{k_i}{2} x^T x + k_2 x_1 + \frac{d\xi}{dt}^T x \right) B^T$$  \hspace{1cm} (20)

To be a valid Lyapunov function, Equation (20) must fulfill two conditions: It must be positive definite and its time derivative

$$\frac{dE}{dt} = - \frac{dx}{dt}^T H [k_p + k_i ] \frac{dx}{dt}$$  \hspace{1cm} (21)

must be negative definite [33]. Expression (21) is obtained by replacing Equation (15) in the time derivate of Equation (20). Since $k_p, k_i, k_1, k_2$ multiply positive values, it is sufficient that $k_p, k_i, k_1, k_2 > 0$ so that the system is stable. But the ST-SMC must perform robustly in the presence of harmonic perturbations; thus, the gains $k_p, k_i, k_1, k_2$ will be designed carefully next.

### 4.2.4 Linear gains $k_p$ and $k_i$

The linear controller gives stability to the system when the error is big [36]. In order to design it, the next two expressions, taken from [42], are used:

$$k_i = \omega_c \sqrt{\frac{\omega_c^2 L^2 + R^2}{\tan^2 \left[ BM - \frac{\pi}{2} + \tan^{-1} \left( \frac{\omega_c L}{R} \right) \right] + 1}}$$ \hspace{1cm} (22)

$$k_p = \left( \frac{k_i}{\omega_i} \right) \tan \left[ MF - \frac{\pi}{2} + \tan^{-1} \left( \frac{\omega_i L}{R} \right) \right]$$ \hspace{1cm} (23)

where $\omega_c$ is the crossover frequency that sets the controller speed of response, given in radians per second, and $BM$ is the phase margin that sets the controller stability, given in radians. $k_p, k_i$ are designed by assuming that $\omega_c = 1000 \pi$ [rad/s], $BM = \pi/3$ [rad], and using the L-filter parameters listed in Appendix A.

### 4.2.5 Gain $k_1$

There is not yet a direct relationship between the gain $k_1$ and the inverter current distortion. However, it has been observed that increasing the value of $k_1$ increases the chattering frequency to higher values, and thus improving the algorithm performance. However, selecting a high value for $k_1$ will increase chattering amplitude, so a compromise between these two phenomena has to be done. The proposed gain $k_1$ that gives good results is listed in Appendix A.

### 4.2.6 Gain $k_2$

After $x$ reaches zero, it is necessary to maintain it at zero all the time so that robustness is assured. Since all values in Equation (21) are positive except the time derivative of $\xi$, the gain $k_2$ is designed to overcome this time derivative. This is achieved by taking the worst-case scenario, that is, when the time derivative of $\xi$ is at its maximum negative value, which is expressed mathematically by the following inequality:

$$k_2 \| x \|_1 + \frac{d\xi}{dt}^T x \geq \left[ k_2 - \omega \sum_{n=-\infty}^{\infty} n \| Z_n \|_2 \right] \| x \|_1 > 0$$ \hspace{1cm} (24)

where $Z_n$ is the $n$th harmonic component of $\xi$, $\omega$ is the grid angular frequency, and $\| \cdot \|_2$ is the 2-norm [43]. The vector $Z_n$ is comprised of dead-time harmonic components ($D_n$), the grid background harmonics components ($G_n$), and the unknown perturbation harmonic components ($K_n$). As a result, the gain $k_2$ is computed as

$$k_2 = \omega \sum_{n=-M}^{M} n \| D_n \|_2 + \| G_n \|_2 + \| K_n \|_2$$ \hspace{1cm} (25)

where vectors $D_n$, $G_n$ are defined in Appendix B, and the infinite sum has been approximated by a finite value $M$. The parameter $M$ is set as 25 in this study. The first $25^{th}$ dead-time harmonic upper bound is obtained using Equation (B15) in Appendix B with the parameters listed in Appendix A as

$$\omega \sum_{n=-25}^{25} n \| D_n \|_2 = 149649$$ \hspace{1cm} (26)

An upper bound for the grid background harmonics is obtained by using Equations (B16) and (B17) in Appendix B with the parameters given in Appendix A, which gives as result:

$$\omega \sum_{n=-25}^{25} n \| G_n \|_2 = 134353$$ \hspace{1cm} (27)

The term $\| K_n \|_2$ is the unknown perturbations harmonic components 2-norm, given by Equation (14(c)). Even though it is unknown, this perturbation is assumed upper bounded. Therefore, an upper bound value is proposed as follows:

$$\omega \sum_{n=-25}^{25} n \| K_n \|_2 = 40998$$ \hspace{1cm} (28)

Finally, by adding Equations (26) to (28), the gain $k_2$ is obtained. The value of $k_2$ is given in Appendix A.

### 5 Simulation Results

Simulations are carried out in MATLAB, where the simulated inverter is comprised of 6 Insulated Gate Bipolar Transistor (IGBT), and it is switched at 40 kHz using asymmetrical regular sample of Pulse Width Modulation (PWM) [44]. The control system is digitally implemented using C code, sampled at 80 kHz. In order
to observe the complete effect of grid background harmonics on the inverter currents, the feedforward correction term $\gamma$, see Figure 5, is assumed to be equal to zero for all the simulations. In all the following simulations the inverter injects 15 amperes of $q$-axis current (reactive power) into the grid because the $d$-axis current regulates the dc-link voltage (see Figure 5). The system parameters and control gains are listed in Appendix A.

### 5.1 Grid polluted with 5th harmonic component

#### 5.1.1 PI control results

PI control is simulated using the vector control diagram shown in Figure 2(a) and the grid voltages shown in Figure 6(a). Observe in Figure 6(a) that after the 0.7 s, the grid voltages are polluted with 5% THD of negative sequence 5th harmonic component. The inverter currents obtained are shown in Figure 6(b), which shows two cases: When energy is injected into a grid polluted with harmonics and when the grid voltages are harmonic free. Figure 6(c) shows the harmonic spectrum of the currents injected into a harmonic-free grid, in which the TRD is only 1.6%. Figure 6(d) shows the harmonic spectrum of the currents when the grid is polluted with 5% THD. Compare Figures 6(c) with (d) and observe how the TRD increases from 1.6% to 15.53% because of the grid background harmonics.

#### 5.1.2 ST-SMC algorithm results

The proposed ST-SMC algorithm is simulated using the vector control diagram shown in Figure 5. The grid voltages used are shown in Figure 7(a), which are polluted with 5% THD of negative sequence 5th harmonic component. The inverter currents obtained are shown in Figure 7(b), which shows two cases: When PI control is used and when the ST-SMC algorithm is used. Observe how, as opposed to PI control, the ST-SMC rejects completely the effects of grid background harmonics on the currents. Thus, the inverter TRD is reduced from 15.53% to 1.90%. Also, observe in Figure 7(b) how the inverter currents recover their sinusoidal shape. Now, compare Figures 7(c) with (d) and observe how the harmonics are completely rejected by the ST-SMC.

#### 5.1.3 Grid harmonic sweep

A harmonic sweep is carried out by polluting the grid voltages with 5% THD of harmonics in the order between the 2nd and the 25th. Positive and negative sequence components are considered because they are present in the system in the case of the appearance of unbalanced harmonics. The results of the harmonic sweep with PI control are presented in Figure 8(a), where the inverter output current TRD is different for different grid harmonic components. A maximum 15.53% and a minimum 5% TRD are observed. Figure 8(b) shows the inverter output current TRD when ST-SMC is used. The proposed algorithm is...
able to suppress any grid harmonic component, and it keeps the TRD below 1.8%. However, the negative sequence 2nd harmonic component surprisingly produces the highest TRD. Therefore, the 2nd harmonic component may be more difficult to reject than any other harmonic components of the grid voltages.

5.2 Dead-time harmonic rejection

The PI controller is compared against the ST-SMC algorithm under 1 μs of dead time. In this simulation, the inverter is switched at 40 kHz, and it is tied to a harmonic-free grid with voltages as shown in Figure 9(a). The inverter output currents are shown in Figure 9(b) at the instant when the control structure is changed from PI control to the proposed ST-SMC. Observe in Figure 9(b) how the ST-SMC algorithm rejects completely the effects of dead-time harmonics on the inverter currents. Thus, the inverter TRD is reduced from 7.61% to 1.44%; also, compare Figures 9(c) with (d).

Figure 10 shows a dead-time sweep from 0 to 2 μs, where the ST-SMC algorithm is compared against PI control. Observe how in both situations the inverter TRD increases as the dead-time period increases. Note how with PI control, the inverter TRD increases to such a degree that 12.79% is reached when deadtime is equal to 2 μs. On the contrary, observe how the ST-SMC keeps the TRD lower than 1.56% for all dead-time periods.

6 EXPERIMENTAL RESULTS

6.1 Experimental setup

The laboratory equipment used to obtain experimental results is shown in Figure 11. A connection diagram is shown in Figure 12, where the power flow and the control signals are illustrated. This setup is comprised of two back-to-back converters (BTBC), one digital signal processor (DSP) model TMS320F28379D, and two PMSM. The BTBC 1 is comprised of two identical three-phase two-level POWEREX converters, model POW-R-PAK PP75T120. The BTBC 2 is comprised of two identical three-phase two-level Unidrive converters, model SP3201. The two converters in the BTBC 1 are programmed with the proposed ST-SMC, using grid voltage-oriented vector control and using asymmetrical regular sample PWM switching at 40 kHz. The flowchart of the system is shown in Figure 12.
Figure 13, where test 1 consist of a dead-time sweep, and test 2 is the reactive power injection into the grid. The control system is digitally implemented using C code sampled at 80 kHz. All the integrators and filters are digitally implemented using the Tustin approximation [41]. The laboratory grid voltage is used as a three-phase source. The PMSM 1 is used as a harmonic-free grid, and the PMSM 2 is controlled by the BTBC 2 to rotate at 900 rpm so as to obtain 60 Hz three-phase voltages at the PMSM 1 terminals.

6.2 Experimental rejection of dead-time harmonics

In this experiment, the PMSM 2 is set to rotate at 900 rpm so that a harmonic-free back-electromotive force (BEMF) is induced in PMSM 1 terminals (see Figure 14(a)). This BEMF is used as a harmonic-free three-phase grid. The line-to-line rms voltage of this BEMF is 77 V, and its THD is less than 1% (see Figure 14(b)).

The BEMF magnitude of PMSM 1 is different from the grid voltage used in simulations. However, according to Equation (B2) in Appendix B, dead-time distortion only depends on dc voltage, switching frequency and dead-time period. Therefore, the experiment and simulation results will be approximately the same due to dc voltage, switching frequency and dead-time period used to obtain experimental and the simulations results are the same.

Active power equal to 1.15 kW is injected into PMSM 1 so that the experimental currents are equal to the simulation currents. The dead-time period is equal to 1 μs. The system parameter used is listed in Appendix A. The currents resulting from this experiment are shown in Figure 15(a), which shows two cases: Active power injection with PI control, and active power injection with the proposed ST-SMC. The harmonic spectrum of the currents that result from the use of PI control is shown in Figure 15(b), where the TRD obtained is 6.73%. Observe in Figure 15(b) how there are strong 5th, 7th, 11th, and 13th harmonics components present, in addition to some 2nd, 4th, 6th, and 8th harmonic components. The appearance of even
harmonic components in the inverter currents is mainly caused by asymmetries in the system, such as impedance unbalance, voltage unbalance, and so forth [14]. Although similar, this result contradicts what simulations predicted because only odd harmonics appear in the simulation results shown in Figure 9(c). When the proposed ST-SMC algorithm is used instead of PI control, the inverter TRD is reduced from 6.73% to 1.04% (see Figure 15(c)). Observe also in Figure 15(c) how the ST-SMC algorithm rejects odd and even harmonic components, which is one benefit of the ST-SMC algorithm. Now, compare Figures 15(c) with 9(d) and observe the similarity between experimental and simulation results.

6.2 Experimental dead-time sweep

Figure 17 shows an experimental dead-time sweep from 0.4 to 2 µs, where active power equal to 1.15 kW is injected into PMSM 1. Observe how in both control algorithms, the TRD increases as dead-time increases. However, for PI control, the TRD increases to such a degree that 11.4% is reached when the dead-time period is equal to 2 µs. As opposed to PI control, ST-SMC keeps the TRD below 1.5% for all dead-time tested. This result shows the excellent performance of the proposed ST-SMC under dead-time harmonic distortion. Compare Figures 16 with 10 and observe the similarity. This result validates Expression (B2) in Appendix B.

6.3 Experimental step response

Figure 17 presents the inverter synchronous currents responding to a step control reference. The results obtained using the ST-SMC algorithm are shown in Figure 17(a), where the inverter currents respond in 1 ms. The results obtained using PI control are shown in Figure 17(b), where the inverter currents respond in 3 ms. Oscillations, as a consequence of using PI control under dead-time effects, are present in the inverter currents shown in Figure 17(b). The proposed ST-SMC algorithm is only three times faster than the PI controller. However, dead-time harmonics are rejected because of their robustness property. Therefore, in Figure 17(a), only dc current is obtained. The inverter currents in abc reference frame using the ST-SMC algorithm and PI control are shown in Figures 18(a) and (b), respectively. Note how the currents in Figure 18(b) are distorted, while the currents shown in Figure 18(a) are sinusoidal. Low current distortion is the main benefit of the ST-SMC algorithm, although some high-frequency chattering is present in the currents shown in Figures 17(a) and 18(a). 6.5 Experimental grid background voltage rejection

The laboratory grid voltage polluted with harmonics is used as a three-phase source to demonstrate the experimental
rejection of grid background harmonics. Figure 19(a) shows these grid voltages, where some distortion is visible. Figure 19(b) shows the harmonic spectrum of the grid voltages; note how the 3rd, 5th, and 7th unbalanced harmonic components are present. In this experiment, the dead time is reduced to 0.4 µs (minimum dead time safe for the inverter used), and the switching frequency is reduced to 20 kHz. This is done to reduce the dead time as much as possible so that dead-time effects are negligible in the inverter currents. Reactive power equal to 2 kVAR is injected into the grid.

Figure 20(a) shows the inverter currents in which two cases are shown: When PI control is used and when ST-SMC algorithm is used. When PI control is used, the inverter currents present an 8.28% TRD due to grid background harmonics. Observe how in contrast to PI control, the ST-SMC rejects the grid background harmonics efficiently, and thus the inverter TRD is reduced from 8.28% to 0.88%. The harmonic spectrum of the inverter currents, obtained using PI control, is shown in Figure 20(b). The harmonic spectrum of the inverter currents obtained using PI control is shown in Figure 20(c). Compare Figures 20(b) and (c) and see how odd and even harmonics are rejected. The even harmonics in the inverter currents are severally penalised by the standard IEEE 1547 [17]. Thus, the rejection of even harmonics is an important advantage of the proposed ST-SMC. This result shows how by using the proposed ST-SMC, it is possible to deliver high-quality energy into the grid.

7 CONCLUSION

The proposed ST-SMC presented robust performance under the appearance of background harmonics, grid frequency variations, dead-time harmonics, and system parameter variations. By using this algorithm as the inverter current control loop, in simulations and experiments, the power quality of a RECS was increased in comparison to PI control. With PI control, the inverter output current TRD increased up to 15.5%. In contrast, the proposed ST-SMC algorithm increased the energy quality by keeping the TRD of the inverter output current lower than 1.8%
in the presence of 5% THD of individual grid harmonics as dictated by the standard IEEE 519. Also, excellent experimental performance was achieved in the presence of inverter switching dead-time harmonics, meeting the standard IEEE 1547. Based on the experimental results obtained in this study, the energy delivered by RECs three-phase inverters, equipped with an L-filter and the proposed ST-SMC algorithm, can be increased despite the presence of dead time, grid background voltage, and unknown uncertainties. The results obtained were independent of the L-filter inductance used, so an L-filter with low inductance can be used in high power density inverters. Additionally, since the ST-SMC is independent of the system frequency, this algorithm can be used to control the torque and flux in PMSM drives, where the harmonic perturbations are of variable frequency. Finally, the study of harmonic perturbations rejection capability of the ST-SMC in LCL grid-tied inverter is left as a future work.

REFERENCES

1. Zong, X., Gray, P.A., Lehn, P.W.: New metric recommended for IEEE standard 1547 to limit harmonics injected into distorted grids. IEEE Trans. Power Delivery 31(3), 963–972 (2016), https://doi.org/10.1109/TPWDRD.2015.2403278.

2. Xu, J., et al.: Adaptive feedforward algorithm without grid impedance estimation for inverters to suppress grid current instabilities and harmonics due to grid impedance and grid voltage distortion. IEEE Trans. Ind. Electron. 64(9), 7574–7586 (2017), https://doi.org/10.1109/TIE.2017.2711523.

3. Jafarian, H., Kim, N., Parkhideh, B.: Decentralized control strategy for AC-stacked PV inverter architecture under grid background harmonics. IEEE J. Emerging Sel. Top. Power Electron. 6(1), 84–93 (2016), https://doi.org/10.1109/JESTPE.2015.2773079.

4. Geng, H., et al.: Fast repetitive control with harmonic correction loops for shunt active power filter applied in weak grid. IEEE Trans. Ind. Appl. 55(3), 3198–3206 (2019), https://doi.org/10.1109/TIA.2019.2895570.

5. Liu, Y., et al.: An efficient and robust hybrid damper for LLC- or LLCLS-based grid-tied inverter with strong grid-side harmonic voltage effect rejection. IEEE Trans. Ind. Electron. 63(2), 926–936 (2016), https://doi.org/10.1109/TIE.2015.2478738.

6. Sinvula, R., Abo-Al-Ez, K.M., Kahn, M.T.: Harmonic source detection methods: A systematic literature review. IEEE Access 7, 74283–74299 (2019), https://doi.org/10.1109/ACCESS.2019.2921149.

7. Hu, J., et al.: Dynamic modeling and improved control of DFIG under distorted grid voltage conditions. IEEE Trans. Energy Convers. 26(1), 163–175 (2011), https://doi.org/10.1109/TEC.2010.2071875.

8. Xu, H., Hu, J., He, Y.: Operation of wind-turbine-driven DFIG systems under distorted grid voltage conditions: Analysis and experimental validations. IEEE Trans. Power Electron. 27(5), 2354–2366 (2012), https://doi.org/10.1109/TPEL.2011.2174255.

9. Kang, S., Kim, K.: Sliding mode harmonic compensation strategy for power quality improvement of a grid-connected inverter under distorted grid condition. IET Power Electron. 8(8), 1461–1472 (2015), https://doi.org/10.1049/iet-pelc.2014.0833.

10. Khaligh, A., et al.: Dead-time distortion in generalized selective harmonic control. IEEE Trans. Power Electron. 23(3), 1511–1517 (2008), https://doi.org/10.1109/TPEL.2008.291162.

11. Shen, Z., Jiang, D.: Dead-time effect compensation method based on current ripple prediction for voltage-source inverters. IEEE Trans. Power Electron. 34(1), 971–983 (2019), https://doi.org/10.1109/TPEL.2018.2820727.

12. Chen, L., Peng, F.Z.: Dead-time elimination for voltage source inverters. IEEE Trans. Power Electron. 23(2), 574–580 (2008), https://doi.org/10.1109/TPEL.2007.915767.

13. Moore, D.C., Odavie, M., Cox, S.M.: Dead-time effects on the voltage spectrum of a PWM inverter. IMA J. Appl. Math. 79(6), 1061–1076 (2014), https://doi.org/10.1093/imanum/drt006.

14. Wu, C.M., Lau, W.H., Chung, H.S.H.: Analytical technique for calculating the output harmonics of an H-bridge inverter with dead time. IEEE Trans. Circuits Syst. I: Fundam. Theory Appl. 46(5), 617–627 (1999), https://doi.org/10.1109/81.762927.

15. Wu, T., et al.: Direct digital control of single-phase grid-connected converters with LCL filter based on inductance estimation model. IEEE Trans. Power Electron. 34(2), 1851–1862 (2019), https://doi.org/10.1109/TPEL.2018.2830821.

16. 519–2014–IEEE Recommended Practice and Requirements for Harmonic Control in Electric Power Systems (Revision of IEEE Std 519–1992), pp. 1–29. IEEE, New York (2014)

17. 1547–2018–IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces (Revision of IEEE Std 1547–2003), pp.1–138. IEEE, New York (2018), https://doi.org/10.1109/IEEESTD.2018.8332112.

18. Ertasgin, G., et al.: Low-pass filter design of a current-source 1-ph grid-connected PV inverter. In: 2016 57th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON), Riga, pp. 1–6 (2016), https://doi.org/10.1109/RTUCON.2016.7763111.

19. Otsikawa, K., Isho, J., Ponniran, A.B.: Minimization of passive components in multi-level flying capacitor DC-DC converter. IEEE J. Ind. Appl. 5(1), 10–11 (2016)

20. Imai, K., et al.: Real-time estimation control of inductance parameters using dust core materials for PWM inverter. In: 2018 International Power Electronics Conference (IPEC-Nagoya 2018 – ECCE Asia), Nagoya, pp. 3363–3368 (2018) https://doi.org/10.23919/IPEC.2018.8507764.

21. Shi, K., et al.: Zero-voltage-switching SIG-MOSFET three-phase four-wire back-to-back converter. IEEE J. Emerging Sel. Top. Power Electron. 7(2), 722–735 (2019), https://doi.org/10.1109/JESTPE.2019.2900232.

22. Kim, K.-A., et al.: Opening the box: Survey of high power density inverter techniques from the little box challenge. CPSS Trans. Power Electron. Appl. 2(2), 131–139 (2017), https://doi.org/10.24295/CPSTPEA.2017.00013.

23. Xie, N., et al.: Novel hybrid control method for APF based on PI and FRC. J. Eng. 2019(16), 3002–3006 (2019), https://doi.org/10.1049/joe.2018.8399.

24. Chen, D., Zhang, J., Qian, Z.: An improved repetitive control scheme for grid-connected inverter with frequency-adaptive capability. IEEE Trans. Ind. Appl. 60(2), 814–823 (2013), https://doi.org/10.1109/TIA.2012.2205364.

25. Liu, Z., et al.: Virtual variable sampling discrete fourier transform based selective odd-order harmonic repetitive control of DC/AC converters. IEEE Trans. Power Electron. 33(7), 6444–6452 (2018), https://doi.org/10.1109/TPEL.2017.2764020.

26. Zhao, Q., et al.: A frequency adaptive PIMR-type repetitive control for a grid-tied inverter. IEEE Access 6, 65418–65428 (2018), https://doi.org/10.1109/ACCESS.2018.2878416.

27. Silva, J.E.: Sliding-mode control of boost-type unity-power-factor PWM rectifiers. IEEE Trans. Ind. Electron. 46(3), 594–603 (1999), https://doi.org/10.1109/41.767067.

28. Bag, A., Subudhi, B., Ray, P.K.: An adaptive sliding mode control scheme for grid integration of a PV system. CPSS Trans. Power Electron. Appl. 3(4), 362–371 (2018), doi: 10.2495/CPSTPEA.2018.00035.

29. Pahari, O.P., Subudhi, B.: Integral sliding mode improved adaptive MPPT control scheme for suppressing grid current harmonics for PV system. IET Renewable Power Gener. 12(16), 1904–1914 (2018), https://doi.org/10.1049/iet-rpg.2018.5215.

30. Cao, D., Fei, J.: Adaptive fractional fuzzy sliding mode control for three-phase active power filter. IEEE Access 4, 6645–6651 (2016), https://doi.org/10.1109/ACCESS.2016.2586958.

31. Sebaaly, F., et al.: Sliding mode fixed frequency current controller design for grid-connected NPC inverter. IEEE J. Emerging Sel. Top. Power Electron. 4(4), 1397–1405 (2016), https://doi.org/10.1109/JESTPE.2016.2586378.
APPENDIX A

1. System parameters

where the low pass filter used is

\[ G(s) = \frac{\omega_f^2}{s^2 + 2\omega s + \omega_f^2} \]  

\[ (A1) \]

APPENDIX B: Harmonic perturbation analysis

### TABLE A1 System parameters and controller gains

| Parameter       | Value | Units | Description                  |
|-----------------|-------|-------|------------------------------|
| Grid \( V_{LL} \) | 140   | [V]   | Grid line-to-line voltage    |
| Grid \( \omega \) | 377   | [rad/s] | Grid frequency               |
| Inverter \( V_d \) | 250   | [V]   | DC-link voltage              |
| \( C \)         | 6.6   | [mF]  | DC-link capacitance          |
| \( P_{rot} \)   | 2     | [W]   | Rated power                  |
| \( R \)         | 0.15  | [Ω]   | L-filter resistance          |
| \( L \)         | 1.2   | [mH]  | L-filter inductance          |
| \( T_s \)       | 25    | [s]   | Switching period             |
| \( T_d \)       | 1     | [s]   | Proposed dead time           |
| Permanent magnet synchronous motor | | | |
| \( R \)         | 0.15  | [Ω]   | Stator resistance            |
| \( L \)         | 2     | [mH]  | Stator inductance            |
| \( K_s \)       | 0.0855| [V/\( \bar{s} \)] | Voltage constant |
| Super-twisting (ST) sliding mode control (SMC) linear gains | | | |
| \( \alpha \)     | 500   | [Hz]  | Corner frequency             |
| PM              | 60    | [deg] | Phase margin                 |
| \( k_p \)       | 3.1898|       | Proportional gain            |
| \( k_i \)       | 6329.9|       | Integral gain                |
| ST-PMC non-linear gains | | | |
| \( k_1 \)       | 25    |       | Proportional gain            |
| \( k_2 \)       | 325000|       | Integral gain                |
| Dc link voltage proportional–integral (PI) control | | | |
| \( \alpha \)     | 30    | [Hz]  | Corner frequency             |
| PM              | 60    | [deg] | Phase margin                 |
| \( k_{pDC} \)   | 1.918 |       | Proportional gain            |
| \( k_{iDC} \)   | 206.23|       | Integral gain                |
| Second-order filter for dc voltage measurement | | | |
| \( \omega_f \)  | 250   | [Hz]  | Corner frequency             |
| \( c \)         | 1/\sqrt{2} |       | Damping coefficient          |

**How to cite this article:** Memije D, Carranza O, Rodríguez JJ, Ortega R, Peralta E. Inverter harmonic perturbations rejection in renewable energy conversion systems applying a super-twisting algorithm. *IET Renew Power Gen.* 2021;15:1483–1497. https://doi.org/10.1049/rpg2.12128

**Dead-time harmonics**

During dead time, both transistor in one inverter leg are deactivated, so the inverter voltage is determined by the phase current sign, \( \delta_\alpha \), where \( \alpha = a, b, c \). A voltage loss occurs when \( \delta_\alpha > 0 \) and a gain of voltage when \( \delta_\alpha < 0 \). For simplicity, \( \delta_\alpha \) is assumed to change sign only twice in each period. Dead time is related to the inverter voltage by

\[ v_{abc} = \bar{v}_{abc} + \Delta v_{abc} \]  

\[ (B1) \]

where \( v_{abc} \) is the three-phase vector of the inverter voltages, which is measured from the phases to the half dc-bus voltage (phase voltage), \( \bar{v}_{abc} \) is the ideal PWM phase voltage, and \( \Delta v_{abc} \) is the dead-time phase voltage. Fourier analysis of Equation (B1) is complex [13]; thus, an approximate Fourier analysis, similar to...
where $T_d$ is the dead-time period, $T_s$ is the switching period, $v_{dc}$ is the dc-bus voltage, $\Delta \phi$ is equal to 1 if $\dot{i}_c > 0$ and $-1$ otherwise (where $\chi = a, b, c$). The complex Fourier series coefficients of phase $a$ in Equation (B2) are given by

$$C_a = \frac{T_d v_{dc}}{T_s \pi} \left[ \frac{\theta}{j} e^{-j \Delta \phi T_d} - \frac{\pi + j}{j} e^{-j \Delta \phi T_d} \right]$$

$$C_a = \frac{2 j e^{-j \Delta \phi T_d} T_s v_{dc}}{\pi T_i} \left[ 1 - (-1)^n \right]$$

where $\theta$ is the phase delay of the inverter currents. Fourier coefficients of phase $b$ and $c$ are given by Equation (B3) displaced by a factor $e^{\pm j 2\pi n/3}$. Based on this assumption and using Equation (B3), Expression (B2) has the following equivalent form:

$$\Delta v_{abc} = \sum_{n=-\infty}^{\infty} C_n e^{j \omega n t} \left[ 1, e^{-j \frac{2\pi n}{3}}, e^{j \frac{2\pi n}{3}} \right]^T$$

The Clark transform is used to give:

$$\Delta v_{\alpha\beta} = T_{\alpha\beta} \Delta V_{abc}; \quad T_{\alpha\beta} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 - 1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

where $\Delta v_{\alpha\beta}$ is the dead-time distortion in $\alpha\beta$ reference frame, which is given by the following expression:

$$\Delta v_{\alpha\beta} = \sum_{n=-\infty}^{\infty} C_n e^{j \omega n t} \left[ \sqrt{\frac{2}{3}} \left( 1 - \cos \left( \frac{2\pi n}{3} \right) \right), -\sqrt{2} j \sin \left( \frac{2\pi n}{3} \right) \right]$$

In order to take Equation (B6) into the $dq$ frame, the Park transform is given in the complex form as the following matrix:

$$T_{dq} = \frac{1}{2} \begin{bmatrix} e^{j \omega n t} & -j e^{-j \omega n t} \\ j (e^{j \omega n t} - e^{-j \omega n t}) & e^{j \omega n t} + e^{-j \omega n t} \end{bmatrix}$$

By multiplying Equations (B6) and (B7), then dead-time voltage in $dq$ frame is obtained as follows:

$$\Delta v = T_{dq} \Delta v_{\alpha\beta} = \sum_{n=-\infty}^{\infty} C_n \Delta_n e^{j \omega n t}$$

where $\Delta v$ is the $dq$ dead time, and $\Delta_n$ is the following vector:

$$\Delta_n = \frac{1}{2} \begin{bmatrix} a_n e^{j \omega n (n+1) t} + b_n e^{-j \omega n (n-1) t} \\ j a_n e^{j \omega n (n+1) t} - j b_n e^{-j \omega n (n-1) t} \end{bmatrix}$$

with the parameters $a_n$ and $b_n$ expressed as follows:

$$a_n = \frac{\sqrt{6}}{3} \left( 1 - \cos \left( \frac{2\pi n}{3} \right) \right) - \sqrt{2} \sin \left( \frac{2\pi n}{3} \right)$$

$$b_n = \frac{\sqrt{6}}{3} \left( 1 - \cos \left( \frac{2\pi n}{3} \right) \right) + \sqrt{2} \sin \left( \frac{2\pi n}{3} \right)$$

Expression (B11) is divided into positive ($\Delta v^+$) and negative ($\Delta v^-$) sequence dead-time voltages, that is:

$$\Delta v^+ = \frac{1}{2} \sum_{n=-\infty}^{\infty} C_n a_n e^{j \omega n t}$$

$$\Delta v^- = \frac{1}{2} \sum_{n=-\infty}^{\infty} C_n b_n e^{j \omega n t}$$

where the sub-indices in Equations (B12a) and (B12b) have been shifted by $-1$ and by $+1$, respectively, without affecting the final result. By adding Equation (B12a) and (B12b), the final expression for the dead-time harmonics in $dq$ frame is obtained as

$$\Delta v = \sum_{n=-\infty}^{\infty} D_n e^{j \omega n t}$$

where $D_n$ represent the $dq$ dead-time coefficients given by

$$D_n = \frac{1}{2} \left( C_{n+1} a_{n+1} + C_{n+1} b_{n+1} \right)$$

which has a 2-norm equal to

$$||D_n||_2 = \frac{1}{\sqrt{2}} \left( ||C_{n+1}|| a_{n+1} + ||C_{n+1}|| b_{n+1} \right)$$

**Grid background harmonics**

Limits for grid harmonic components are listed in the standard IEEE 519 [16]. This standard states that for systems operating below 1 kV, the maximum THD expected is 8% with a maximum 5% individual THD for each component. In this study, a 10% THD with 5% of the 5th, 7th, 11th and 13th harmonic components are considered. This is expressed as

$$G_n = \frac{V_{LL}}{\sqrt{6}} \begin{cases} 0.05 \delta_8 \text{ for } \omega n = 5, 7, 11, \text{ and } 13 \\
0 \text{ otherwise} \end{cases}$$

where $G_n$ represent the grid background harmonics components, $V_{LL}$ is the grid rated line-to-line rms voltage, and $\delta_8$ is the...
\( n^{th} \) harmonic phase angle. The \( dq \) grid voltage decomposition is obtained by substituting \( C_n \) in Equation (B4) for \( G_n \) in Equation (B16) and exactly following the same procedure described from Equations (B4) to (B15), which yields the 2-norm of the grid background harmonics, which is expressed by

\[
\|G_n\|_2 = \frac{1}{\sqrt{2}} \left( |G_{n-1}| |a_{n-1}| + G_{n+1} |b_{n+1}| \right) \quad (B17)
\]

where \( G_n \) is the vector that contains the grid background harmonics Fourier coefficients in \( dq \) reference frame.