The Littlest Higgs Model and One-Loop Electroweak Precision Constraints

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ABSTRACT

We present in this talk the one-loop electroweak precision constraints in the Littlest Higgs model, including the logarithmically enhanced contributions from both fermion and scalar loops. We find the one-loop contributions are comparable to the tree level corrections in some regions of parameter space. A low cutoff scale is allowed for a non-zero triplet VEV. Constraints on various other parameters in the model are also discussed. The role of triplet scalars in constructing a consistent renormalization scheme is emphasized.

The Standard Model requires a Higgs boson to explain the generation of fermion and gauge boson masses. Precision electroweak measurements suggest that the Higgs boson must be relatively light, \( m_H < 219 \text{ GeV} \). Currently, experimental data overwhelmingly support the SM with a light Higgs boson. The simplest version of the Standard Model with a single Higgs boson, however, has the theoretical problem that the Higgs boson mass is quadratically sensitive to any new physics which may arise at high energy scales. Little Higgs models are a new approach to understanding the hierarchy between the \( TeV \) scale of possible new physics and the electroweak scale. These models have an expanded gauge structure at the \( TeV \) scale which contains the Standard Model \( SU(2) \times U(1) \) electroweak gauge groups. The LH models are constructed such that an approximate global symmetry prohibits the Higgs boson from obtaining a quadratically divergent mass until at least two loop order. The Higgs boson is a pseudo-Goldstone boson resulting from the spontaneous breaking of the approximate global symmetry and so is naturally light. We present in this talk, which is based on the work done in Ref. [1], the one-loop electroweak precision constraints in the Littlest Higgs model (LLH) [2], which contains a gauged \([SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2\) symmetry as its subgroup. We include the logarithmically enhanced contributions from both fermion and scalar loops, and emphasize the role of triplet scalars in constructing a consistent renormalization scheme.

Precision electroweak measurements give stringent bounds on the scale of little Higgs type models. One of the strongest bounds comes from fits to the \( \rho \) parameter, since in the LLH model the relation \( \rho = 1 \) is modified at the tree level. While the Standard Model requires three input parameters in the weak sector, a model with \( \rho \neq 1 \) at tree level, such as the LLH model, requires an additional input parameter in the gauge-fermion sector, which can be taken to be the VEV of the Higgs triplet, \( v' \). Many of the familiar predictions of the Standard Model are drastically changed by the need for an extra input parameter [3,4]. We choose as our input parameters the muon decay constant \( G_\mu \), the physical \( Z \)-boson mass \( M_Z^2 \), the effective lepton mixing angle \( s_\theta^2 \) and the fine-structure constant \( \alpha(M_Z^2) \) as the four independent input parameters in the renormalization procedure. The \( \rho \) parameter, defined as, \( \rho \equiv M_W^2/(M_Z^2 s_\theta^2) \), where \( s_\theta^2 \) is the effective leptonic mixing angle at the \( Z \)-resonance, and the \( W \)-boson mass, which is defined through muon decay, are then
derived quantities. Since the loop factor occurring in radiative corrections, $1/16\pi^2$, is similar in magnitude to the expansion parameter, $v^2/f^2$, of chiral perturbation theory, the one-loop radiative corrections can be comparable in size to the next-to-leading order contributions at tree level. We compute the loop corrections to the $\rho$ parameter which are enhanced by large logarithms; we focus on terms of $O\left(\frac{1}{16\pi^2}\ln\left(\frac{M_Z^2}{Q^2}\right)\right)$, where $Q \sim M_Z$ and $M \sim f \sim O(TeV)$. At the one-loop level, we have to take into account the radiative correction to the muon decay constant $G_\mu$, the counterterm for the electric charge $e$, the mass counterterm of the Z-boson, and the counterterm for the leptonic mixing angle $s_\theta^2$. These corrections are collected in the quantity $\Delta r_Z$, 

$$s_\theta^2 c_\theta^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2 \rho} \left[1 - \frac{\Delta s_\theta^2}{s_\theta^2} + \frac{c^2}{\sqrt{2}G_\mu f^2} + \Delta r_Z\right],$$  

where

$$\Delta r_Z = -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta\alpha}{\alpha} - \left(\frac{c^2}{s^2} - s_\theta^2\right) \frac{\delta s_\theta^2}{s_\theta^2},$$  

We note that $\Delta r_Z$ defined in Eq. (2) differs from the usual $\Delta^r Z$ defined in the SM by an extra contribution due to the renormalization of $s_\theta^2$. The counter terms are given by

$$\frac{\delta G_\mu}{G_\mu} = -\frac{\Pi^{WW}(0)}{M_W^2} + \delta_{V-B}$$  

(3)

$$\delta M_Z^2 = \text{Re}\left(\Pi^{ZZ}(M_Z^2)\right)$$  

(4)

$$\frac{\delta s_\theta^2}{s_\theta^2} = \text{Re}\left[\left(\frac{c_\theta}{s_\theta}\right)\left[\frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} + \frac{v_e^2 - a_e^2}{a_e^2} S_A(m_e^2)\right.$$

$$\left. - \frac{v_e}{\Lambda Z^2} \left(\frac{\Lambda Z^2}{a_e^2} - \frac{\Lambda A^2}{a_e^2}\right)\right]\right]$$  

(5)

$$\frac{\delta\alpha}{\alpha} = \Pi^{\gamma\gamma'}(0) + 2\left(\frac{g^2_V - g^2_A}{Q_e}\right)\frac{\Pi^{WW}(0)}{M_Z^2}.$$  

(6)

Solving for $M_W^2$ and $\rho$ iteratively, we obtain a prediction for the physical W-boson mass

$$M_W^2 = \frac{1}{2} \left[a(1 + \Delta \hat{r}) + \sqrt{a^2(1 + \Delta \hat{r})^2 + 4a}\Pi^{WW}(0)\right]$$  

(7)

where $a \equiv \pi\alpha(M_Z^2)/\sqrt{2}G_\mu s_\theta^2$, and $\Delta \hat{r}$ is defined as

$$\Delta \hat{r} = \Delta r_Z - \frac{\Delta s_\theta^2}{s_\theta^2} + \frac{c^2}{\sqrt{2}G_\mu f^2} - \frac{\Pi^{WW}(0)}{M_W^2}.$$  

(8)

We find that the one-loop contribution to $\Delta r_Z$ due to the SU(2) triplet scalar field, $\Phi$, scales as $1/[16\pi^2(1/v^2)(v'/v)^2M_\Phi^2]$. In the limit $v' = 0$ while keeping $f$ fixed, which is equivalent to turning off the coupling $\lambda_{\Phi hh}$ in the Coleman-Weinberg potential, the one loop contribution due to the SU(2) triplet, $\Delta r_Z^s$, vanishes. The large $f$ limit of the scalar one-loop contribution, $\Delta r_Z^s$, vanishes depending upon how the limit $f \to \infty$ is
As $f$ approaches infinity, the parameter $\mu^2$ (thus $v^2$) can be kept to be of the weak scale by fine-tuning the unknown coefficient in the mass term $\mu^2$ in the Coleman-Weinberg potential while all dimensionless parameters remain of order one. The scalar one-loop contribution in this limit does not de-couple because $M^2_{\Phi}$ increases as $f^2$ which compensates the $1/f^2$ suppression from $v'^2/v^2$. In this case, the SM Higgs mass $m_H$ is of the weak scale $v$. On the other hand, without the fine-tuning mentioned above, $v$ can be held constant while varying $f$, if the quartic coupling $\lambda^4_h$ (thus $\lambda^2_{\Phi^2}$) approaches infinity as $f^2/v^2$. This can be done by taking $a \sim f^2/v^2$ while keeping $a'$ finite and $s$ and $s'$ having specific values. The scalar one-loop contribution then scales as

$$\Delta r^s_Z \sim \frac{1}{v^2} \left( \frac{v'}{v} \right)^2 M^2_{\Phi} \sim \left( \frac{1}{v^2} \right) \left( \frac{\lambda_{h^4}}{\lambda_{\Phi^2}} \right)^2 \frac{v^2}{f^2} \lambda_{\Phi^2} f^2 \rightarrow \frac{\lambda^2_{h^4}}{\lambda_{\Phi^2}}.$$ (9)

Since the coupling constant $\lambda_{\Phi^2}$ must approach infinity in order to keep $v$ constant as we argue above, the scalar one-loop contribution $\Delta r^s_Z$ thus vanishes in the limit $f \to \infty$ with $v$ held fixed and no fine tuning. In this case, $m_H \sim \mu$ scales with $f$.

We analyze the dependence of the W-boson mass, $M_{W_L}$, on the mixing between $SU(2)_1$ and $SU(2)_2$, described by $s'$, the mixing between $U(1)_1$ and $U(1)_2$, described by $s$, the mixing parameter in $t-T$ sector, $x_L$, and the VEV of the $SU(2)$, $v'$. The predictions for $M_{W_L}$ with and without the one-loop contributions for $f = 2$ TeV is given in Fig. 1 which demonstrates that a low value of $f$ ($f \sim 2$ TeV) is allowed by the experimental restrictions from the $W$ and $Z$ boson masses, provided the VEV of the $SU(2)$ triplet scalar field is non-zero. This shows the importance of the $SU(2)$ triplet in placing the electroweak precision constraints. In order to have experimentally acceptable gauge boson masses, however, the parameters of the model must be quite finely tuned, regardless of the value of the scale $f$. On the other hand, the prediction for $M_{W_L}$ is very sensitive to the values of $s'$ as well as $v'$. The non-decoupling of the $SU(2)$ triplet scalar field shown in Fig. 2 implies the importance of the inclusion of the scalar one-loop contributions in the analyses. In the region below $f = 4$ TeV, where the tree level corrections are large, the vector boson self-energy is about half of the size of the tree level contributions, but with an opposite sign. (Other one-loop contributions roughly cancel among themselves in this region). Due to this cancellation between the tree level correction and the one-loop correction, there is an allowed region of parameter space with low cutoff scale $f$. Fig. 2 also shows that the tree level contribution of the LH model get smaller as $f$ increases, as is expected. In order to be consistent with experimental data, the triplet VEV $v'$ must approach zero as $f$ goes to infinity.

Our results emphasize the need for a full one loop calculation.

1. References

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Figure 1: Prediction for $M_{W_L}$ as a function of the mixing angle $s'$ at the tree level and the one-loop level. Also plotted is the correlation between $M_Z$ and $s'$ for fixed $s$, $v'$ and $f$. The cutoff scale $f$ in this plot is 2 TeV, the $SU(2)$ triplet VEV $v' = 3.4$ GeV, the mixing angle $s = 0.22$, and $x_L = 0.4$.

Figure 2: The tree level correction, $\Delta_{\text{tree}}$, the fermionic and scalar contributions to the one loop correction, $\Delta r_Z^f$ and $\Delta r_Z^S$, the total one loop correction, $\Delta \hat{r} - \Delta_{\text{tree}}$, and $\Pi^{WW}(0)/M_Z^2$ as functions of the cutoff scale $f$ at fixed $s$, $s'$, $x_L$ and $v'$. 