One-Loop Vilkovisky-DeWitt Counterterms for 2D Gravity plus Scalar Field Theory

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Abstract

The divergent part of the one-loop off-shell effective action is computed for a single scalar field coupled to the Ricci curvature of 2D gravity \( c\phi R \), and self interacting by an arbitrary potential term \( V(\phi) \). The Vilkovisky-DeWitt effective action is used to compute gauge-fixing independent results. In our background field/covariant gauge we find that the Liouville theory is finite on shell. Off-shell, we find a large class of renormalizable potentials which include the Liouville potential. We also find that for backgrounds satisfying \( R = 0 \), the Liouville theory is finite off shell, as well.

August 1992

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1 Introduction and Outline

The subject of 2D gravity has recently been revived, primarily by the use of the $\sigma$-model representation of string theory. The powerful technology of string theory and conformal field theory has led to several breakthroughs, ranging from offering a consistent theory of quantum gravity, to the derivation of exact non-perturbative results with important conceptual consequences [1]. Paralleling these advancements there has been significant progress on the perturbative aspects of the subject as well [2,3,4).

As a starting point, the classical action that is often taken is one that reproduces Einstein’s field equations $R = 0$ in two dimensions [5]. This is done by the introduction of an auxiliary scalar field as a Lagrange multiplier, which is then elevated to a fully propagating field, due to the appearance of a kinetic term at the 1-loop level. Having necessitated the coupling of gravity to a scalar field, one is then faced with a plethora of candidate classical actions with different couplings and potential terms. In [3,6] it was shown that, in fact, by field redefinition one can reduce the number of arbitrary variables in the action to only one - the potential for the scalar field. In [3], using the machinery of $\sigma$-models, it was also argued that the Liouville theory defined by an exponential potential is renormalizable, and even finite for a special choice of couplings; this result is well motivated by string theory.

The work of [3] was done in the conformal gauge. Other gauges have been chosen by various authors [2,4] to study some of the same issues. The conformal field theory treatments are indeed consistent [2,4,7], but the perturbative results are plagued with gauge-dependent results [2], precluding the comparison of the results. This state of affairs is not unusual, and methods have been devised for eliminating such ambiguities.

If the issues of finiteness or renormalizability are to be addressed properly, one must be assured of the existence of a unique, gauge-independent, result. A quantity which in principle carries all of the information in a quantum field theory is the effective action, and Vilkovisky and DeWitt [8] have provided us with an algorithm for obtaining a unique effective action (also called the VD effective action.) Central in this plan is the introduction of a
metric and a connection on the manifold of fields. The presence of a gauge group $G$ in a theory calls for a $G$ invariant field metric and the modding out of the $G$-equivalent orbits on this space. The connection in a non-gauge invariant theory is given by the Christoffel symbols of the field metric, but for a gauge theory an additional non-local ($T$) term must be appended. The VD effective action is gauge-independent (as well as reparametrization invariant); the only caveat is the non-uniqueness of the field metric for some field theories. Counter terms computed to 1-loop using the VD effective action amount to the addition of terms (commonly called VD corrections) to the counter terms computed using the naive effective action. As examples of VD calculations, see [9,10,11]. In this paper we apply the VD procedure to the theory of 2D-gravity coupled to a scalar field.

An outline of the remainder of this paper is as follows. In section 2, the classical action is expanded about arbitrary values of the background fields, and the second-order piece is then modified to incorporate the 2-dimensional geometric identity $R_{\mu\nu} - (1/2)g_{\mu\nu}R = 0$. In the process of gauge-fixing the action, in section 3, we motivate the choice of a metric on the field configuration space. The divergent part of the 1-loop correction to this gauge-fixed action is also calculated. The divergent contribution of the ghost fields, at the 1-loop level, is found in section 4. Sections 5 and 6 are devoted to the VD corrections - the local (Christoffel), and the nonlocal ($T$) contributions, respectively. Finally, in section 7, we discuss some implications of the combined counter terms.

2 Perturbations of the classical action

As we discussed, a variety of 2-dimensional gravity+matter theories can be parametrized by a single potential appearing in the action. For such theories we wish to calculate the divergent part of the VD effective action to one-loop by using background field methods [10] (where MTW conventions are used). To make the calculation as simple as we can we use a particular DeWitt gauge that not only makes the contribution from the non-local part of the VD connection tractable, but also makes the remainder of the one-loop
operator minimal. The form of the classical action we start with is:

\[ S = -\int d^2x \sqrt{g} \left[ \frac{1}{2} \nabla^{\alpha} \bar{\phi} \nabla_\alpha \bar{\phi} + c \bar{\phi} \bar{R} + V(\bar{\phi}) \right]. \]  

(1)

By expanding \( S \) about the background values, \( \Phi \) and \( g_{\mu\nu} \), \( (\bar{\phi} = \Phi + \eta \) and \( g_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} ) \) the equations of motion for the background fields are easily read from the terms linear in \( \eta \) and \( h_{\mu\nu} \),

\[ \frac{\delta S}{\delta \bar{\phi}(x)} \bigg|_\Phi = -\sqrt{g(x)} \left[ -\Box \Phi + cR + V'(\Phi) \right], \]

\[ \frac{\delta S}{\delta g_{\alpha\beta}(x)} \bigg|_g = -\sqrt{g(x)} \left[ -\frac{1}{2} (\nabla^\alpha \Phi \nabla^\beta \Phi - \frac{1}{2} g^{\alpha\beta} \nabla^\gamma \Phi \nabla_\gamma \Phi) + c(\nabla^\alpha \nabla^\beta - g^{\alpha\beta} \Box) \Phi + \frac{1}{2} g^{\alpha\beta} V(\Phi) \right]. \]

(2)

Quadratic terms of the form \( \nabla^\mu \eta \nabla^\nu \hat{h}_{\mu\nu} \) and \( \Phi \nabla_\alpha \hat{h}^{\alpha\beta} \nabla^\mu \hat{h}_{\mu\beta} \) where \( \hat{h}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} h \) is the traceless part of the quantum field \( h_{\alpha\beta} \), appear in the expansion. Such expressions contribute non-minimal terms to the second variation of the action, i.e. to the non-gauge fixed one-loop operator. Because such non-minimal terms are difficult to deal with we choose to remove them by e.g., an appropriate choice of gauge. However, in 2D there is another way of removing some of these terms as well, and that is by the use of the 2-dimensional identity \( \bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} g_{\mu\nu} = 0 \). Expansion of this identity about background yields,

\[ (\Box - R) \hat{h}_{\alpha\beta} = 2 P_{\alpha\beta\gamma\delta} \nabla^\gamma \nabla^\sigma \hat{h}_\sigma, \]

where the trace-free projection tensor \( P_{\alpha\beta\gamma\delta} \equiv \frac{1}{2} (g_{\alpha\gamma} g_{\beta\delta} + g_{\alpha\delta} g_{\beta\gamma} - g_{\alpha\beta} g_{\gamma\delta}) \).

Upon multiplying by \( \Phi \) and \( \hat{h}^{\alpha\beta} \), partial integration results in the following identity that \( \Phi \) and \( \hat{h}^{\alpha\beta} \) must satisfy:

\[ \int \sqrt{g} \frac{1}{2} \Phi \hat{h}^{\alpha\beta}(\Box - R) \hat{h}_{\alpha\beta} + \Phi (\nabla^\alpha \hat{h}_{\alpha\gamma})(\nabla_\beta \hat{h}^{\beta\gamma}) + (\nabla^\alpha \Phi) \hat{h}_{\alpha\gamma} \nabla_\beta \hat{h}^{\beta\gamma} d^2x = 0. \]

(3)

Given the two distinct ways of removing the non-minimal terms - via gauge fixing or via the use of the above identity - we, for the sake of democracy, do not commit ourselves to either of these ways. Instead, we simply add the
identity (3), weighted by an arbitrary parameter $\xi$, from the second variation of the action. The result is

$$S^{(2)} \equiv \frac{1}{2} \phi^i \frac{\delta^2 S}{\delta \phi^i \delta \phi^j} \phi^j = - \int d^2 x \sqrt{g} \left\{ \frac{1}{2} g^{\mu \nu} \nabla_\mu \eta \nabla_\nu \eta - \nabla_\mu \Phi \dot{h}^\mu \nabla_\nu \eta \right\}$$

$$+ \frac{1}{2} \nabla_\mu \Phi \nabla_\nu \Phi (\dot{h}^\mu \dot{h}^{\alpha \nu} + \frac{1}{2} \dot{h}^{\mu \nu} - \frac{1}{4} g^{\mu \nu} \dot{h}_{\alpha \beta} \dot{h}^{\alpha \beta})$$

$$+ c \left[ - \nabla^\mu \eta \nabla^\nu \dot{h}_{\mu \nu} - \frac{1}{2} \eta \Box h + \frac{1}{2} \nabla^\mu \Phi \dot{h}^\alpha \nabla_\alpha \dot{h}_{\mu \nu} \right]$$

$$+ \frac{1}{4} (1 - \xi) \Phi \dot{h}_{\mu \nu} (\Box - R) \dot{h}^{\mu \nu} + \frac{1}{2} (1 - \xi) \Phi \nabla_\alpha \dot{h}^{\alpha \beta} \nabla^\mu \dot{h}_{\beta \mu}$$

$$+ \frac{1}{2} (2 - \xi) \nabla^\mu \Phi \dot{h}_{\mu \nu} \nabla_\alpha \dot{h}^{\alpha \nu}$$

$$- \frac{1}{4} \dot{h}^\nu \dot{h}_{\sigma \beta} \nabla^\sigma \nabla^\nu \Phi + \frac{1}{8} (3 \dot{h}_{\alpha \beta} \dot{h}^{\alpha \beta} + h^2) \Box \Phi$$

$$- \frac{1}{4} \dot{h}_{\alpha \beta} \dot{h}^{\alpha \beta} V(\Phi) + \frac{1}{2} V'(\Phi) \eta h + \frac{1}{2} V''(\Phi) \eta^2 \right\}, \quad (4)$$

where $\phi^i$ represents the set of quantum fields $\eta, h_{\alpha \beta}$. The addition of zero, i.e. equation (3), should not affect the final result, and as we shall see later, it does not.

3 Gauge fixing

A relatively simple choice of quantum gauge is implemented by the addition of the following to $S^{(2)}$:

$$S_{gf} = - \int d^2 x \frac{1}{2} \chi^\mu c_{\mu \nu} \chi^\nu, \quad (5)$$

with

$$c_{\mu \nu} = - \sqrt{g} c \Phi g_{\mu \nu} \delta (x - y),$$

$$\chi^\nu = \nabla_\mu \dot{h}^{\mu \nu} - \frac{1}{\Phi} \nabla^\nu \eta. \quad (6)$$

This choice of gauge would render $S^{(2)}$ minimal, i.e. all non-minimal terms would cancel, were it not for the reintroduction of the non-minimal terms.
through the identity (3). Below we will find another gauge which renders $S^{(2)} + S_{gf}$ minimal, where $S^{(2)}$ contains the identity (3) as given in (4). But first we use gauge (6) to motivate an essential ingredient in the VD effective action, a choice of metric on the field configuration space.

The quantity $S^{(2)} + S_{gf}$, with $\xi = 0$, can be written in the form

$$-\frac{1}{2} \phi^i \left[ -K_{ij} \Box + L_{ij}^\mu \nabla_\mu + M_{ij} \right] \phi^j,$$

which is exactly what is meant by a minimal second order operator (referring to the quantity in the brackets). For this gauge choice $K_{ij}$ reads

$$K_{ij} = \sqrt{g(x)} \left( 1 + \frac{c^2}{2} \Phi^{1/2} \frac{1}{2}cg^{\gamma\delta} \gamma \Phi \delta^\alpha \Phi \right) \delta(x - y),$$

where $\gamma \equiv \frac{1}{2}c(\xi - 1)$. Equation (8) can be used as a metric on the space of fields defined by $\phi^i$; however, there is an ambiguity in defining this metric, for example it could be defined by $S^{(2)}$ alone; in which case the $\eta\eta$-component would be 1. Motivated by this ambiguity, we will take the configuration space metric to be

$$G_{ij} = \sqrt{g(x)} \left( \frac{\Theta(\Phi)}{2}cg^{\alpha\beta} \frac{1}{2}cg^{\gamma\delta} \right) \delta(x - y),$$

where $\Theta(\Phi)$ is an arbitrary function of $\Phi$, and $\xi \neq 0$. The most general local Killing metric on the field space (the gauge-symmetry group being given by the diffeomorphisms, see Eqn. 20) is similar to this but with $c$ replaced by an arbitrary function of $\Phi$, and the $\eta\eta$-component $[a_1(\Phi)P^{\alpha\beta\mu\nu}(a_2)]$ containing two more arbitrary functions of $\Phi$, where $P_{\alpha\beta\gamma\delta}(a_2) \equiv \frac{1}{2}(g_{\alpha\gamma}g_{\beta\delta} + g_{\alpha\delta}g_{\beta\gamma} - a_2(\Phi)g_{\alpha\beta}g_{\gamma\delta})$. Using this general field metric in the VD theory introduces non-minimal terms that cannot be removed by a choice of gauge; this problem may be overcome in a separate article. Here we concentrate on field metrics of the form (9).

For reasons that will become clear (see section 6) we choose to work in the DeWitt gauge, defined by

$$\frac{\delta \chi^\mu_{DW}}{\delta \phi^i} = -(c^{-1})^{\mu\nu} \nabla_\nu G_{ji},$$

5
where $\nabla^j$ are the gauge group generators [see Eqn. (20)]. Clearly the DeWitt gauge (assumed linear and homogeneous in the quantum fields) is determined uniquely by the choice of the field metric $G_{ij}$ and $c_{\mu\nu}$. For the metric in (3) and with
\begin{equation}
 c_{\mu\nu} \equiv 2\sqrt{g}\gamma\Phi g_{\mu\nu}\delta(x - y),
\end{equation}
the solution is
\begin{equation}
 \chi^{\nu}_{DW} = \nabla^\mu \hat{h}^{\mu\nu} + \frac{\nabla^\nu \Phi}{\Phi} \hat{h}^{\mu\nu} + \frac{c}{2\gamma\Phi} \nabla^\nu \eta - \frac{\Theta}{2\gamma\Phi} (\nabla^\nu \Phi) \eta - \frac{c}{4\gamma\Phi} (\nabla^\nu \Phi) h.
\end{equation}
Adopting this gauge results in a minimal gauge-fixed one-loop operator. In this minimal gauge, $S^{(2)} + S_{gf}$ can again be written in the form (7), with $K$ given by (8), and $L$ and $M$ given as follows:

\begin{align}
 L^\mu_{\Phi(x)\Phi(y)} &= 0, \\
 L^\mu_{\Phi(x)g_{\alpha\beta}(y)} &= -L^\mu_{g_{\alpha\beta}(x)\Phi(y)} \\
 &= \sqrt{g} \left\{ (1 - \frac{c}{\Phi} - \Theta) P^{\alpha\beta\mu\nu} \nabla^\nu \Phi + \frac{c^2}{4\gamma\Phi} g^{\alpha\beta} \nabla^\mu \Phi \right\} \delta(x - y), \\
 L^\mu_{g_{\alpha\beta}(x)g_{\gamma\delta}(y)} &= \sqrt{g} \left\{ \frac{c}{2} + \gamma \right\} \left\{ P^{\alpha\beta\gamma\delta} - P^{\gamma\delta\alpha\beta} \right\} \nabla^\nu \Phi \delta(x - y), \\
 M_{\Phi(x)\Phi(y)} &= \sqrt{g} \left\{ V''(\Phi) + \frac{c}{4\gamma} (\nabla^\mu \Phi)^2 \right\} \delta(x - y), \\
 M_{\Phi(x)g_{\alpha\beta}(y)} &= M_{g_{\alpha\beta}(x)\Phi(y)} \\
 &= \sqrt{g} \left\{ \frac{c}{2\Phi} \nabla^\gamma \Phi \nabla^\delta \Phi \right\} \left\{ \Theta + \frac{c}{\Phi} \right\} \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ g^{\alpha\beta} \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
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 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
 &+ \left\{ \Theta - \frac{c}{2\Phi} \right\} \nabla^\gamma \Phi \nabla^\delta \Phi \\
\[ + P^{\gamma \rho \sigma} \left[ \left( \frac{1}{4} - \frac{c}{2\Phi} \right) \nabla_{\rho} \Phi \nabla_{\sigma} \Phi \right] g^{\alpha \beta} + P^{\alpha \beta \gamma \delta} \left[ \frac{3}{4} c \Box \Phi - \frac{1}{4} (\nabla \Phi)^2 - \frac{V}{2} + \gamma \Phi R \right] + g^{\alpha \beta} g^{\gamma \delta} \left[ \frac{c}{4} \Box \Phi + \frac{c^2}{8\gamma \Phi^2} \right] \delta(x - y). \]

Though we have gone through a labyrinthine path, these equations represent a simple result. The expression in (7), with \( K, L \) and \( M \) given by (8), (13) and (14) simply represents \( S^{(2)} + S_{gf} \) in a very specific gauge, i.e. the DeWitt gauge associated with the field metric (9). The one-loop correction in the naive effective action theory is given by

\[
W^{(1)}_{S+gf} = \frac{i}{2} \text{Tr} \ln \left( G^{ki} \left[ - \frac{1}{2} \{ K_{ij}, \Box \} + \frac{1}{2} \{ L^\mu_{ij}, \nabla_\mu \} + M_{ij} \} \right) \right.
\]

\[
= \frac{i}{2} \text{Tr} \ln \left\{ -G^{km} K_{mi} \left[ \delta_j^i \Box + L^\mu_{ij} \nabla_\mu + M^k_j \right] \right\}
\]

\[
= \frac{i}{2} \text{Tr} \ln \left\{ -G^{-1} K \left[ \Box + \hat{L}^\mu \nabla_\mu + \hat{M}' \right] \right\},
\]

(15)

plus a ghost term. In the above \( G^{-1} = G^{km} \) is the inverse of (9), and \( \hat{L}^\mu \equiv - K^{-1} L^\mu + K^{-1} \nabla^\mu K \) and \( \hat{M}' \equiv - K^{-1} M - (1/2) K^{-1} (\nabla_\mu L^\mu) + (1/2) K^{-1} (\Box K) \).

The divergent part of (15) is:

\[
W^{\text{div}}_{S+gf} = \frac{i}{2} \text{Tr} \ln \left[ \hat{L}^\mu \nabla_\mu + \hat{M}' \right] \bigg|_{\text{div}}
\]

\[
= - \frac{1}{4\pi \epsilon} \int d^2x \sqrt{g} \text{ tr} \left[ - \frac{R}{6} + \frac{1}{4} \hat{L}^\mu \hat{L}^\nu - \hat{M}' \right],
\]

(16)

where dimensional regularization \( (\epsilon \equiv 2 - n) \) has been used to express the irreducible divergence as a simple pole. From (8), (13), and (14) we find

\[
\text{tr} \hat{L}^\mu \hat{L}^\nu = - \frac{2c(\nabla \Phi)^2}{\gamma \Phi^2}.
\]

1Due to an oversight on our part, only the first terms in these equations were included in an original rough draft of this work. We would like to express our gratitude to F.D. Mazzitelli who found this important error. Without his contribution our results would have been in error.
\[ \text{tr } \hat{M}' = (\nabla \Phi)^2 \left[ \frac{c^2}{4 \gamma^2 \Phi^2} + \frac{c}{2 \gamma \Phi^2} - \frac{2}{\Phi^2} - \frac{\Theta}{\gamma \Phi} \right] + (\Box \Phi) \left[ \frac{2}{\Phi} + \frac{1}{c} \right] - \left[ \frac{2V'}{c} - \frac{V}{\gamma \Phi} + 2R \right]. \] (17)

Substitution of these into (16) gives
\[ W_{S+gf}^{\text{div}} = -\frac{1}{4\pi\epsilon} \int d^2x \sqrt{g} \left\{ \frac{4}{3} R + \frac{2V'}{c} - \frac{V}{\gamma \Phi} \right. \]
\[ + \left. (\nabla \Phi)^2 \left[ -\frac{c^2}{4 \gamma^2 \Phi^2} - \frac{c}{\gamma \Phi^2} + \frac{2}{\Phi^2} + \frac{\Theta}{\gamma \Phi} \right] - (\Box \Phi) \left[ \frac{2}{\Phi} + \frac{1}{c} \right] \right\}. \] (18)

This is to be combined with a ghost contribution and a VD correction. As it stands now the divergent part is clearly gauge and field metric dependent (the \( \gamma \) and \( \Theta \) terms).

4 The ghost contribution

In this section we shall compute the divergent contribution of the ghost fields to the effective action. The ghost operator \( \hat{Q}^{-1} \) is defined by
\[ (Q^{-1})_{\mu}^{\nu} = \nabla_{\nu} \frac{\delta \chi_{\mu}}{\delta \phi^{i}}, \]
which in the DeWitt gauge is simply related to the vertical part of the field metric \( N_{\mu\nu} \),
\[ N_{\mu\nu} \equiv \nabla_{i} G_{ij} \nabla_{\mu}^{i}, \]
\[ (Q^{-1})_{\mu}^{\nu} = -(c^{-1})^{\mu\nu} N_{\sigma\nu}. \] (19)

The gauge generators \( \nabla_{\alpha}^{\Phi} \) and \( \nabla_{\alpha}^{g_{\mu\nu}} \) are the generators of infinitesimal coordinate changes of the scalar and the gravitational fields, respectively:
\[ \nabla_{\mu(x)}^{\Phi(y)} = (\nabla_{\mu} \Phi)_{y} \delta(x - y), \]
\[ \nabla_{\mu(x)}^{g_{\alpha\beta}(y)} = (g_{\mu\alpha} \nabla_{\beta} + g_{\mu\beta} \nabla_{\alpha})_{y} \delta(x - y). \] (20)
The ghost contribution to the one-loop effective action is

$$ W_Q^{(1)} = -i \text{Tr} \ln(\hat{Q}^{-1}) , \quad (21) $$

and from (9), (19), (20), and (11) we find

$$ (Q^{-1})^{\alpha(x)}_{\nu(y)} = -c^{\alpha\mu} N_{\mu
u} , $$

$$ = \left\{ \delta^\alpha_\nu \Box + \left[ \delta^\alpha_\nu \nabla^\sigma \Phi - \left( \frac{c}{\gamma} + 2 \right) \frac{1}{\Phi} g^{\alpha\mu} \nabla_{[\mu} \delta^\sigma_{\nu]} \right] \nabla_\sigma 
+ \frac{R}{2} \delta^\alpha_\nu - \frac{\Theta}{2\gamma \Phi} \nabla^\alpha \Phi \nabla_\nu \Phi + \frac{c}{2\gamma \Phi} \nabla^\alpha \nabla_\nu \Phi \right\} \delta(x - y) . \quad (22) $$

This is a minimal operator, the divergent part of whose Trace-log is found from (14) to be

$$ W_Q^{\text{div}} = \frac{1}{2\pi\epsilon} \int d^2 x \sqrt{g} \left\{ -\frac{4}{3} R 
+ \left[ -\frac{c}{2\gamma} \left( 1 + \frac{c}{4\gamma} \right) + \frac{\Theta \Phi}{2\gamma} \right] \frac{(\nabla \Phi)^2}{\Phi^2} - \frac{c}{2\gamma} \Box \Phi \right\} . \quad (23) $$

## 5 Christoffel symbols

As discussed in section (1), in order to obtain a unique effective action at one-loop Vilkovisky and DeWitt direct us to modify the gauge-fixed quantum action $S^{(2)} + S_{gf}$ by adding

$$ -\frac{1}{2} \Gamma^i_{jk} \phi^i \phi^k S_j , \quad (24) $$

where $\Gamma^i_{jk}$ is the connection on the configuration space constructed from the metric (4) and the gauge (12). Due to the presence of the equations of motion $S_j \equiv \frac{\delta S}{\delta \Phi_j}$ [see Eqn. (2)] , this correction vanishes on-shell, but is frequently non-trivial off-shell. For a gauge theory the connection is given in two parts:

$$ \Gamma^i_{jk} = \left\{ \begin{array}{c} i \\ j \\ k \end{array} \right\} + \tau^i_{jk} , \quad (25) $$
where $\phi^i = \eta(x)$, $h_{\mu\nu}(x)$ in condensed notation. The local contribution is given by the Christoffel symbols $\{\}$, and the non-local contribution given by $\mathcal{T}$ will be the subject of the next section. For the choice of our metric (4), the Christoffel symbols are given as follows:

\[
\begin{align*}
\begin{bmatrix}
\eta(x) \\
\eta(y) \\
\eta(z)
\end{bmatrix} & = \frac{\Theta}{2c} \delta(y - x)\delta(z - x), \\
\begin{bmatrix}
\eta(x) \\
\eta(y) \\
h_{\alpha\beta}(z)
\end{bmatrix} & = 0, \\
\begin{bmatrix}
\eta(x) \\
h_{\mu\nu}(y) \\
h_{\alpha\beta}(z)
\end{bmatrix} & = -\frac{\gamma\Phi}{2c} P_{\mu\nu\alpha\beta} \delta(y - x)\delta(z - x), \\
\begin{bmatrix}
h_{\mu\nu}(x) \\
\eta(y) \\
h_{\alpha\beta}(z)
\end{bmatrix} & = \frac{1}{2c} \frac{\Theta^2}{c} + \Theta' g_{\mu\nu} \delta(y - x)\delta(z - x), \\
\begin{bmatrix}
h_{\mu\nu}(x) \\
\eta(y) \\
h_{\alpha\beta}(z)
\end{bmatrix} & = \left[ \frac{1}{2\Phi} P_{\mu\nu}^{\alpha\beta} + \frac{\Theta}{4c} g_{\mu\nu} g^{\alpha\beta} \right] \delta(y - x)\delta(z - x), \\
\begin{bmatrix}
h_{\mu\nu}(x) \\
h_{\alpha\beta}(y) h_{\gamma\delta}(z)
\end{bmatrix} & = -\frac{1}{4} \left[ P_{\mu\nu}^{\gamma\delta} g^{\alpha\beta} + P_{\mu\nu}^{\alpha\beta} g^{\gamma\delta} \right] - \frac{1}{2} \left[ P_{\mu\nu}^{\alpha(\gamma \delta)\beta} + P_{\mu\nu}^{\gamma(\alpha \beta)\delta} \right] \\
& \quad - \frac{1}{2c} \left[ 1 + \frac{c}{\Theta} - \frac{\Theta\Phi}{c} \right] g_{\mu\nu} P_{\alpha\beta\gamma\delta} \delta(y - x)\delta(z - x). \quad (26)
\end{align*}
\]

Because the Christoffel connection symbols are local functions of the background, as are the equations of motion, they will contribute only to $M_{ij}$ and not to $K_{ij}$ or $L^i_{ij}$ in (13) and hence only through the trace $\text{tr}(M')$ to (14). The addition to (17) is

\[
\Delta\text{tr}(\hat{M}') = -(K^{-1})^{ij} \left\{ \begin{array}{c} k \\ j \\ i \end{array} \right\} \frac{\delta S}{\delta \Phi^k},
\]

\[
= \left[ -\frac{2\Theta}{c} + \frac{1}{c} + \frac{1}{\Phi} \left( \frac{c}{\gamma} + 1 \right) \right] \square \Phi \\
+ \left[ \frac{2\Theta}{c^2} - \frac{1}{c\Phi} \left( \frac{c}{\gamma} + 1 \right) \right] V - \frac{V'}{c} - R, \quad (27)
\]
which then adds a term

\[ W^{\text{div}}_\{i\} = \frac{1}{4\pi \epsilon} \int d^2x \sqrt{g} \Delta \text{tr}(\hat{M}'), \]  

(28)
to the effective action’s divergent part (18). The remaining non-local (T) contribution is somewhat less straightforward to evaluate.

6 The T contribution

To evaluate the non-local (T) part of the connection’s contribution to the effective action (divergent part) we simply make use of the technique outlined by Barvinsky-Vilkovisky. This technique relies on having used the DeWitt gauge, which we have. We do not present any of the derivation here but only apply it. In general (T)’s contribution to the effective action can be written as a sum of traces of products of two operators \( U_1 \) and \( U_2 \), (see Sec. 5.8 of [10])

\[ W^{(1)}_T = -\frac{i}{2} \left( \text{Tr} U_1 - \text{Tr} U_2 \right) - \frac{i}{4} \text{Tr}(U_1)^2 + \mathcal{O}[\langle S, i \rangle^3], \]  

(29)

where

\[ U^\mu_{\nu} \equiv N^{\mu\beta} \nabla^i (\mathcal{D}_i \nabla^k) S_{,k} N^{\sigma\gamma} \epsilon_{\gamma\nu}, \]  

(30)

\[ U^\mu_{2\nu} \equiv N^{\mu\beta} S_{,k} (\mathcal{D}_i \nabla^k) G^{ij} (\mathcal{D}_j \nabla^l) S_{,l} N^{\sigma\gamma} \epsilon_{\gamma\nu}, \]

and where \( G^{ij} \) is the propagator for the gauge fixed one-loop operator including the Christoffel term, i.e. \( S + S_{gf} \) plus (24) where only the Christoffel connection is used. The other terms have been introduced before except \( \mathcal{D}_i \) and it is the field covariant derivative operator with only the Christoffel symbol as its connection,

\[ \mathcal{D}_i \nabla^k_{\sigma(x)} = \frac{\delta \nabla^j_{\sigma(x)}}{\delta \Phi^i} + \nabla^j_{\sigma(x)} \left\{ \begin{array}{c} k \\ j \\ i \end{array} \right\}. \]  

(31)

The \( U_1 \) and \( U_2 \) operators act on the space of infinitesimal gauge transformations and not on the \( \phi^i \) field space. Their only contribution to the divergent part of (23) in 2D is simply

\[ W^{\text{div}}_T = -\frac{i}{2} \text{Tr} U_1^{\text{div}}. \]  

(32)
That this is the case is shown by a dimensional argument\textsuperscript{1}: Divergent contributions come from traces of operators of order $O[l^{-d}]$, where $d = 2$ here. Since $S_j$ is already of order $O[l^{-2}]$ and all other terms are at least of order $O[l^0]$, (32) is the only contribution. Because $U_1$ need only be evaluated to order $O[l^{-2}]$ we can write $[N^{\mu\nu} \equiv (N_{\mu\nu})^{-1}$, see Eqn.(22)],

$$N^{\mu(x)\nu(y)} = \frac{1}{(\sqrt{g} 2\gamma \Phi)_x g^{\mu\nu} 1 \not{\Box}_{xy}} + O[l^{-1}],$$

and hence

$$W_{\text{div}}^T = \frac{i}{2} \int dxdydz \frac{1}{\sqrt{g(z) 2\gamma \Phi(z)}} g^{\mu}(z) \left[ \nabla^i_{\mu(z)} (D_i \nabla^j_{\nu(z)}) S_{ik} \right] \frac{\delta^\nu_r \delta^\tau_s}{\not{\Box}_{xy} \not{\Box}_{yz}} [\text{div}].$$

Equation (33) can now be evaluated by use of the following coincidence limit, derivable from the general algorithms outlined in [10]:

$$\nabla^i_{\mu(z)} (D_i \nabla^j_{\nu(z)}) S_{ij} \equiv \left[ T_{\mu\alpha}^{\alpha\beta} \nabla_{\alpha} \nabla_{\beta} \right] z \delta(z - x) + O[l^{-3}],$$

thereby defining the tensor $T_{\mu\alpha}^{\alpha\beta}$. Equation (33) can now be evaluated by use of the following coincidence limit, derivable from the general algorithms outlined in [10]:

$$\nabla_{\alpha} \nabla_{\beta} \frac{\hat{1}}{\not{\Box}_2} \delta(z - w) \bigg|_{w \rightarrow z} = -\frac{i}{4\pi \epsilon} \sqrt{g(z) g_{\alpha\beta}(z)} \hat{1}.$$ 

Consequently,

$$W_{\text{div}}^T = \frac{1}{8\pi \epsilon} \int d^2z \sqrt{g} \frac{T_{\mu\alpha}^{\alpha\beta} z}{2\gamma \Phi(z)}.$$ 

Computation of the trace of $T_{\mu\alpha}^{\mu\alpha}$ is all that remains. We note that since $O[S_1] = O[l^{-2}]$ and $O[\nabla^\Phi_{\mu}] = O[l^{-1}]$ the terms $D_\Phi \nabla^\Phi_{\mu}$ and $D_\Phi \nabla_{\mu\nu}^{\mu\mu}$ do not contribute to $T$, and only some terms in $D_{g_{\mu\nu}} \nabla^\Phi_{\sigma}$ and $D_{g_{\mu\nu}} \nabla_{\sigma\lambda}^{\mu\nu}$ do. The former can be found to be

$$D_{g_{\mu\nu}(w)} \nabla^\Phi_{\sigma(x)} = -\delta(z - w) \left[ \frac{\gamma \Phi}{c} P^{\mu\nu,\sigma\tau} \nabla_{\tau} \right] w \delta(w - x),$$

\textsuperscript{1}In this notation, $O[\cdot]$ signifies the background field dimension, and it is $O[l^0]$ for all of $\gamma$, $\Phi$, $g_{\mu\nu}$, $c$, and $\Theta$. 

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and the latter can be cast in the form
\[
\mathcal{D}_{g_{\mu\nu}(w)} \nabla_{\sigma(x)} g_{\kappa\lambda}(z) \equiv \delta(z - w) \left[ t_{\kappa\lambda}^{\mu\nu}_{\sigma\tau} \nabla_{\tau(x)} \right] \delta(w - x) + \mathcal{O}[l^{-1}],
\]
with
\[
t_{\kappa\lambda}^{\mu\nu}_{\sigma\tau} = 2 \delta_{\sigma\tau}^{\mu\nu} \delta_{\kappa\lambda}^{\mu\nu} - \delta_{\kappa\lambda}^{\mu\nu} \delta_{\sigma\tau}^{\mu\nu} - [1 + \frac{\gamma}{c} - \frac{\Theta \gamma}{c^2} \Phi] g_{\kappa\lambda} P_{\sigma\tau}^{\mu\nu} + \frac{1}{2} (g_{\sigma\tau} P_{\kappa\lambda}^{\mu\nu} + g_{\kappa\lambda} P_{\sigma\tau}^{\mu\nu}) - (P_{\kappa\lambda}^{\mu\nu} \delta_{\tau}^{\nu} + P_{\kappa\lambda}^{\nu\tau} \delta_{\sigma}^{\nu}).
\]
The trace is then given by
\[
T_{\mu\nu}^\mu^\nu = 2 \left[ \frac{\gamma}{c} P_{\mu\nu}^{\mu\nu} \delta S_{\mu\nu} - t_{\kappa\lambda}^{\mu\nu}_{\sigma\tau} \delta g_{\kappa\lambda} \right],
\]
and by (34) leads to
\[
W_\text{div}^{\text{div}} = \frac{1}{8\pi\epsilon} \int d^2 x \sqrt{g^2} \left[ \Box \Phi - cR - V' \right.
+ \left. \left( \frac{1}{\Phi} - \frac{\Theta}{c} \right)(c\Box \Phi - V) \right].
\]
Note that this quantity vanishes on shell, as it should and that boundary terms have been kept.

7 Conclusions and discussion

At this point we combine (18) and (23) to get the counter term as computed using the naive effective action theory,
\[
W_\text{div}^{\text{div}} = W_\text{div}^{\text{div}}_{S + g_f} + W_\text{div}^{\text{div}}_Q
= -\frac{1}{2\pi\epsilon} \int d^2 x \sqrt{g} \left[ 2R + \frac{V'}{c} - \frac{1}{2\gamma \Phi} V \right.
+ \left. \left( \frac{\nabla \Phi}{\Phi^2} - \frac{1}{2} \left( \frac{2}{\Phi} - \frac{c}{\gamma \Phi} + \frac{1}{c} \right) \Box \Phi \right) \right],
\]

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where no divergent surface terms have been dropped. Note the absence of themetric term $\Theta$ and the presence of $\gamma$ in this equation. We suspect that the$\gamma$ appears much like gauge parameters appear in the off-shell naive effectiveaction through quantum gauge fixing. Even though it appears in the actionbefore gauge fixing through the 2D identity (3), it also enters in the gaugefixing term (12), and, overall, it disappears on shell. Use of a non-minimalgauge would be required before the source of the $\gamma$-dependence could be decided. The $\Theta(\Phi)$ term could conceivably appear because it was in thegauge fixing term, but it doesn’t.

The two VD correction terms (28) and (35) combine to give

$$\Delta W_{VD}^{\text{div}} = W_{T}^{\text{div}} + W_{T}^{\text{div}} = -\frac{1}{4\pi \epsilon} \int d^2 x \sqrt{g}(\Theta c - \frac{c}{\gamma \Phi})(\Box \Phi - V),$$

(37)

which contains both $\Theta$ and $\gamma$, and properly vanishes on shell. Combining(36) and (37) yields our final result for the divergent part of the “unique”1-loop effective action:

$$W_{VD}^{\text{div}} = -\frac{1}{2\pi \epsilon} \int d^2 x \sqrt{g} \left[ 2R + \frac{V'}{c} - \frac{\Theta V}{2c^2} + \frac{(\nabla \Phi)^2}{\Phi^2} - \frac{1}{2} \left( \frac{2}{\Phi} - \frac{\Theta - 1}{c} \right) \Box \Phi \right],$$

(38)

where, again, all divergent surface terms have been kept.

Note that the parameter $\gamma \equiv \frac{1}{2} c (\xi - 1)$ appearing in (36), which representsthe 2D identity discussed above, disappears from the VD-corrected result(38), as anticipated. The $\gamma$-dependence in (36) is in fact consistent withother results in the literature [2,4]. Upon setting $\gamma = -\frac{1}{2} c$, i.e. $\xi = 0$,which amounts to not including the identity at all, we find these same termsin [2] and [4]. However, the agreement is not complete; the $(\nabla \Phi)^2$ andthe $(\Box \Phi)$ terms do not agree with those of [2], though they do agree withthe revised version of [4]. Differences may be due to the use of differentquantum gauge conditions. In fact, there is very little overlap in the choiceof gauge conditions used in [2] and in this calculation, and as a result thereare expected discrepancies. Further, having said that the $\gamma$-dependence inthe $V/(2\gamma \Phi)$ term agrees with other independent calculations, the fact thaton-shell (i.e. $\Box \Phi = V/c$) the $\gamma$-dependence in (36) vanishes, lends furtthersupport to the correctness of our result.
The VD-corrected effective action (38) is constructed to circumvent the problems of gauge-dependent results alluded to above. Of course, as can be seen from (37), equations (36) and (38) coincide when on-shell, but for the off-shell case it is the latter which is designed to be gauge-independent and unique. “Uniqueness” refers to quantum gauge fixing independence, and it does not in any way rule out the possibility of having results that depend on the choice of the configuration-space metric. Given an action and a field metric, one can choose fields to put one or the other in a canonical form but ordinarily not both. To standardize both there must be some significant relation between these two structures. The Θ-dependence in (38) is a reflection of this relationship, or lack thereof. Congenially, though, the Θ-dependence in (38) does actually vanish on shell, as one expects.

There are several natural choices of Θ that one may consider. The classical action, when expanded about background values of the fields, contains a $\frac{1}{2} \eta \Box \eta$ term [see Eqn.(1)] which suggests $\Theta = 1$ as the $\eta \eta$-component of the field metric. A non-trivial choice $\Theta = 1 + c^2/(2\gamma \Phi)$, emerges from the gauge-fixed action [see (8)]. Note that this choice itself depends on the value of the parameter $\gamma$. Clearly $\Theta = 1$ is a preferred choice.

To find out what potentials give rise to on-shell finiteness, we first partially integrate (38), keeping the surface terms, to obtain

\[ W_{\text{div}}^{\text{VD}} = -\frac{1}{2\pi \epsilon} \int d^2 x \sqrt{g} \left[ 2R + \frac{V'}{c} - \frac{0 V}{2c^2} + \frac{\Theta - 1}{2c} \Box \Phi - \Box \log \Phi \right]. \]

Substituting the equations of motion $R = (V/c - V')/c$ and $\Box \Phi = V/c$, the Θ-dependence vanishes, leaving us with

\[ W_{\text{div}}^{\text{VD}} = -\frac{1}{2\pi \epsilon} \int d^2 x \sqrt{g} \frac{1}{c} \left[ \frac{3}{2} \Box \Phi - V' - 2c \Box \log \Phi \right], \]

where all quantities take their on-shell values. The last term, albeit a total derivative, makes it difficult to evaluate the integrand without knowledge of the explicit solutions. Discarding surface terms now allows us to find the potentials for which the theory is finite on-shell, i.e. for which (40) vanishes,
modulo surface terms. Since \( V \) itself is a total derivative on shell \( (c \Box \Phi) \), if it satisfies
\[
-V' = \frac{\alpha}{c} V ,
\]
whose solution is the form
\[
V = \mu \exp\left\{ -\frac{\alpha \Phi}{c} \right\} ,
\]
where \( \mu \) and \( \alpha \) are constants, then the theory is finite. We conclude that the Liouville theory is finite on shell.

Off-shell, our gauge-fixing independent VD effective action becomes significant. For example, we also find potentials for which (39) is zero off-shell (again modulo surface terms), but only for the specific case of a flat metric \( (R = 0) \). In this case, with the choice \( \Theta = \text{constant} \), we again find the potentials (41). As a special case, this makes the Liouville theory finite, off-shell as well. It is worth noting that this off-shell result follows from the VD effective action (39) and not from the naive one (36), since the latter contains the non-surface term \( \Box \Phi / \gamma \Phi \).

The renormalizability of these models depends on the choice of \( V(c_i, \Phi) \), as well, where \( c_i \) are coupling constants in \( V \). It is easy to see that the renormalizations
\[
\begin{align*}
    c & \rightarrow c , \\
    c_i & \rightarrow c_i + \Delta c_i \\
    \Phi & \rightarrow \Phi - \frac{1}{\pi \epsilon c} , \\
    g_{\mu \nu} & \rightarrow g_{\mu \nu} \left[ 1 + \Theta \right. \frac{1}{4 \pi \epsilon c^2} \left. \right] ,
\end{align*}
\]
absorb the infinities in (39) into the “wave functions” \( \Phi \) and \( g_{\mu \nu} \); note that under these redefinitions \( \Theta \) drops out. In this case we find
\[
V' = 2\pi \epsilon c \Delta c_i \frac{\partial V}{\partial c_i} \equiv -\frac{\partial V}{\partial b} ,
\]
whose solution is
\[
V = c^2 \mathcal{A}\left( \frac{\Phi - b}{c} \right) ,
\]
where $A$ is an arbitrary function of the specified argument, and $b$ is yet another parameter whose renormalization (accompanying 42) is

$$b \rightarrow b - \frac{1}{2\pi \epsilon c}.$$  

It is clear that the Liouville potential is also in the family of renormalizable potentials (with the $\mu$ in (41) being renormalized as $\mu \rightarrow \mu(1 - \alpha/2\pi \epsilon c^2)$). What has emerged that differs from the conformal gauge analysis is that a much wider family of potentials are renormalizable.

Acknowledgements

We have benefited from useful discussions with D. Birmingham, S. Odintsov, N. Mohammedi, and B. Whiting whom we gratefully acknowledge. F. Mazzitelli and A. Tseytlin are thanked for bringing to our attention crucial errors in a rough draft of this paper. Our gratitude is expressed to H.T. Cho for pointing out the relevance and suggesting the calculation of the Vilkovisky-DeWitt corrections. This work was supported by the Department of Energy, and the Southern Association for High Energy Physics (SAHEP) funded by the Texas National Research Laboratory Commission (TNRLC).

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