Determining the CP parity of Higgs bosons
at the LHC
in the $\tau$ to 1-prong decay channels

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Abstract

We propose a method for determining the CP nature of a neutral Higgs boson or spin-zero resonance $\phi$ at the CERN Large Hadron Collider (LHC) in its $\phi \rightarrow \tau^- \tau^+$ decay channel. The method can be applied to any 1-prong $\tau$-decay mode, which comprise the majority of the $\tau$-lepton decays. The proposed observables allow to discriminate between pure scalar and pseudoscalar Higgs-boson states and/or between a CP-conserving and CP-violating Higgs sector. We show for the decays $\tau \rightarrow \pi \nu_\tau$ that the method maintains its discriminating power when measurement uncertainties are taken into account. The method will be applicable also at a future linear $e^+e^-$ collider.

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I. INTRODUCTION

The major physics goal at the CERN Large Hadron Collider (LHC) is the search for Higgs bosons or other (spin-zero) resonances that pin down the mechanism of electroweak gauge symmetry breaking. (For reviews, see, e.g., [1, 2, 3, 4].) If such particles are found, then the next task would be the exploration of their properties. For electrically neutral spin-zero states this includes the determination of the parity ($P$) and charge conjugation times parity ($CP$) quantum numbers, respectively, which provide important information about the dynamics of these particles. There is an extensive literature on proposals of how to measure the $CP$ properties of Higgs bosons in their production and decay processes at hadron colliders or at a future linear $e^+e^-$ collider, including [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. (For a recent compilation and overview, see [23].)

A promising reaction in this respect is Higgs-boson decay into $\tau$ lepton pairs, where $\tau$-spin correlations discriminate between $CP$-even and -odd Higgs-boson states, and between a $CP$-conserving and $CP$-violating Higgs sector. For subsequent $\tau$-decays into three charged prongs it was shown in [17] that experimentally robust discriminating observables exist also for the LHC. In order to substantially increase the data sample in future experiments, one would like to employ for the measurement of the $CP$ properties of a Higgs boson also $\tau$ decays into one charged prong. However, for these modes the method proposed in [17] is not applicable at the LHC, because it requires the reconstruction of the $\tau^+$ rest frames. In this letter we construct observables that can also be applied to 1-prong $\tau$ decays. We demonstrate by simulations taking expected measurement uncertainties into account that, at the LHC, the $CP$ nature of a neutral Higgs boson – or any neutral spin-zero boson which decays into $\tau^-\tau^+$ pairs – can be determined with these observables.

II. OBSERVABLES

The analysis below applies to any neutral spin-zero resonance $h_j$, in particular to any neutral Higgs boson, with flavor-diagonal couplings to quarks and leptons $f$ (with mass $m_f$)

$$L_Y = -\sqrt{2}G_F^{1/2} \sum_{j,f} m_f (a_{jff} \bar{f}f + b_{jff} \bar{f}i\gamma_5 f) h_j,$$

where $G_F$ is the Fermi constant and $a_{jff}$ and $b_{jff}$ are the model-dependent reduced scalar and pseudoscalar Yukawa couplings. In the SM, $a_f = 1$ and $b_f = 0$. SM extensions where the couplings (1) appear include models with two Higgs doublets, such as the non-supersymmetric type II models and the minimal supersymmetric SM extension (MSSM) (see, e.g., [1, 2, 3, 23]). These models contain three physical neutral Higgs fields $h_j$ in the mass basis. If Higgs sector $CP$ violation (CPV) is negligibly small, then the fields $h_j$ describe two scalar states $h, H$ ($b_{jff} = 0$) and a pseudoscalar $A$ ($a_{jff} = 0$). In the case of Higgs sector CPV, the $h_j$ are $CP$ mixtures, that is, they have non-zero couplings $a_{jff}$ and $b_{jff}$ to quarks and
leptons (see, e.g., [24]) which lead to CP-violating effects in $h_j \rightarrow f\bar{f}$ already at the Born level.

In the following, $\phi$ denotes any of the neutral Higgs bosons $h_j$ just discussed or, more generally, a neutral spin-zero resonance. The observables discussed below for determining the $CP$ quantum number of $\phi$ in its $\tau$-decay channel may be applied to any Higgs production process $i \rightarrow \phi + X \rightarrow \tau^+ + \tau^- + X$. At the LHC, this includes the gluon and gauge boson fusion processes $gg \rightarrow \phi$ and $q_i q_j \rightarrow \phi q_i q_j$, respectively, and the associated production $t\bar{t}f$ or $b\bar{b}f$ of a light Higgs boson $\phi$. The spin of the resonance can be determined in standard fashion from the polar angle distribution of the $\tau$ leptons. In order to determine with these reactions whether $\phi$ is a scalar, a pseudoscalar, or a $CP$ mixture, one can use $\tau^\mp$ spin correlations. They lead to specific angular correlations among the charged particles (charged prongs) from $\tau^-$ and $\tau^+$ decay. A suitable set of observables involves the opening angle distribution between the charged prongs, $CP$-odd triple correlations and asymmetries [8, 10]. In order to exploit the full discriminating power of these observables, one must be able to determine the $\tau^\mp$ rest frames, i.e., the energies and three-momenta of the $\tau$ leptons. At the LHC, the reconstruction of the $\tau^\mp$ rest frames is possible for $\tau$ decays into 3 charged prongs, $\tau^- \rightarrow 2\pi^- \pi^+ \nu_{\tau}$ and likewise for $\tau^+$. For these channels, it was shown in [17] that one can discriminate, at the LHC, with these observables i) between a scalar, a pseudoscalar $\phi$, and a $CP$ mixture and also ii) between (nearly) mass-degenerate scalar and pseudoscalar Higgs bosons with $CP$-invariant couplings and one or several $CP$ mixtures.

In order to increase the statistics, one would like to exploit also $\tau$ decays into one charged particle. We consider here the case where both $\tau^-$ and $\tau^+$ decay into one charged prong. Then the determination of the $\tau^\mp$ four-momenta is not possible without further assumptions. However, for our purpose, it is not necessary to reconstruct the $\tau$ rest frames. As we will show below, one can construct discriminating observables in the zero-momentum frame (ZMF) of the two charged prongs from the $\tau^\mp$ decays which involve only directly measurable quantities, namely the momenta of the charged prongs and the impact parameter vectors defined below. For definiteness, we consider in the following the case where both $\tau^-$ and $\tau^+$ decay into a charged pion and a neutrino,

$$pp \rightarrow \phi + X \rightarrow \tau^- \tau^+ + X \rightarrow \pi^- \pi^+ + X.$$  

In order to put our approach into perspective, we first briefly recapitulate another method for determining the $CP$ nature of $\phi$ in this $\tau$ decay channel. It was pointed out a long time ago [6] that the distribution of the angle between the normal vectors of the $\tau^-$ and $\tau^+$ decay planes discriminates between a $CP = +1$ and $CP = -1$ Higgs boson. The formula of [6] can be generalized to the case where $\phi$ has arbitrary scalar and pseudoscalar couplings to $\tau$ leptons. We consider, in the $\phi$ rest frame, the decay

$$\phi \rightarrow \tau^-(\mathbf{k}^0) + \tau^+(-\mathbf{k}^0) \rightarrow \pi^- (\mathbf{p}^-) + \pi^+ (\mathbf{p}^+) + \nu_{\tau} + \bar{\nu}_{\tau}.$$  

Here $\mathbf{k}^0$ is the 3-momentum of the $\tau^-$ in the rest frame of $\phi$, and $\mathbf{p}^- (\mathbf{p}^+)$ is the $\pi^-(\pi^+)$...
3-momentum in the $\tau^-(\tau^+)$ rest frame. We shall take the $\tau^-$ direction $\hat{k}$ as $z$ axis both in the $\tau^-$ and the $\tau^+$ rest frame. Denoting the azimuthal angle of the $\pi^-(\pi^+)$ in the $\tau^-(\tau^+)$ rest frame by $\varphi_-(\varphi_+)$, one notices that $\varphi = \varphi_+ - \varphi_-$ is the angle between the normal vectors of the $\tau^- \to \pi^-$ and $\tau^+ \to \pi^+$ decay planes spanned by the above momentum vectors. Using the $\phi \to \tau^-\tau^+$ spin-density matrix \[10\] and the SM density matrix of polarized $\tau \to \pi\nu$ decay, we obtain:

$$\Gamma^{-1} \frac{d\Gamma}{d\varphi} = \frac{1}{2\pi} \left[ 1 - \frac{\pi^2}{16} (c_1 \cos \varphi + c_2 \sin \varphi) \right],$$

where $0 \leq \varphi < 2\pi$ and

$$c_1 = \frac{a_2^2 b_2^2 - b_2^4}{a_2^2 b_2^2 + b_2^4}, \quad c_2 = -\frac{2a_2 b_2}{a_2^2 b_2^2 + b_2^4}.$$ (5)

(For similar considerations, see \[18, 19\].) Here $a_\tau$, $b_\tau$ are the couplings defined in \[1\]. In (5) the velocity $\beta_\tau$ may be put equal to 1. Eq. (4) includes the special cases of a pure scalar ($c_1 = 1$, $c_2 = 0$) and of a pure pseudoscalar ($c_1 = -1$, $c_2 = 0$), where the distribution is proportional to $1 \mp (\pi^2/16) \cos \varphi$. In the case of an ideal $CP$ mixture, $a_\tau = \pm b_\tau$, the distribution takes the form $(2\pi)^{-1}(1 \pm (\pi^2/16) \sin \varphi)$. While for a pure scalar or pseudoscalar $\phi$ the complete information on (4) is contained already in the range $0 \leq \varphi < \pi$, one must determine (4) in the complete interval $0 \leq \varphi < 2\pi$ in order to check for $CP$ violation \[18\]. For this aim, one must define/measure signed normal decay-plane vectors. If one cannot distinguish $\varphi$ from $2\pi - \varphi$, the resulting distribution of the angle $\varphi$ between the unsigned normal vectors is

$$\Gamma^{-1} \frac{d\Gamma}{d\varphi} = \frac{1}{\pi} \left( 1 - \frac{\pi^2}{16} c_1 \cos \varphi \right),$$

where here $0 \leq \varphi < \pi$. (Eq. (6) is obtained by adding (4) evaluated at $\varphi$ and at $2\pi - \varphi$.) In (6) the parity-odd term has averaged out.

At the LHC, it is extremely difficult - if not impossible - to measure the distributions (4), (6), because the determination of the $\phi$ and $\tau^+$ rest frames requires the reconstruction of the $\tau$ energies and momenta in the laboratory frame. Even for the simplest $\tau^-\tau^+$ decay channel \[2\] the energies of the tau leptons need still to be fixed, using the missing momentum $p_\text{miss}$ in the plane transverse to the proton beam. This leads to large uncertainties.

In order to proceed, we notice that the distributions (4) and (6) remain invariant – in the absence of detector cuts – when we switch from the $\tau^-\tau^+$ ZMF to another inertial frame, the $\pi^-\pi^+$ ZMF. Of course, the determination of the (un)signed decay-plane correlation in this frame requires knowledge of the $\tau^+$ momenta, too. As this is not feasible in general, we propose to use instead two observables in this frame, which can be unambiguously determined from quantities measured in the laboratory frame. The joint use of these observables avoids also the determination of a signed correlation. We construct these observables in the following way:

1) Consider the $\tau^+$ decays in the laboratory frame. For $\tau^- \to \pi^-$ the decay plane in this frame is shown in Fig. \[1\]. It is determined by the measured $\pi^+$ direction of flight and by the $\tau^-\tau^+$
production vertex $PV$, which is practically equal to the Higgs boson production vertex. This vertex is obtained from the visible tracks of the charged particles/jets produced in association with the Higgs boson $\phi$ [25]. The $\tau^+ \to \pi^+$ decay plane in the laboratory frame is obtained in analogous fashion. One can now determine the impact parameter vectors $n_\mp$ in the laboratory frame by projecting perpendicularly onto the $\pi^\mp$ directions from $PV$ – see Fig. 1. The pion momenta and the impact parameter vectors fix the normal vectors of these two decay planes. The distribution of the angle between these normal vectors or, alternatively, the distribution of the angle $\phi_{lab}$ between the vectors $n_-$ and $n_+$ shows already some sensitivity for discriminating between a scalar and a pseudoscalar boson – see the next section. 2) A much higher sensitivity can be achieved by determining the analogous correlations in the $\pi^- \pi^+$ ZMF. One can reconstruct this frame by a Lorentz boost from the laboratory frame with the measured pion 4-momenta $p^\mu_\mp = (E_\mp, p_\mp)$. The resulting $\pi^\mp$ energies and momenta are $E^*_\mp, p^*_\mp$ with $p^*_+ = -p^*_-$ (All quantities in this frame will be denoted by an asterisk.) However, the true decay planes in this frame can not be reconstructed, because the true impact parameter vectors in this frame can not be obtained from the measured laboratory-frame 3-vectors $n_\mp$. Instead we proceed as follows. Denoting the normalized impact parameter vectors in the laboratory frame by $\hat{n}_\mp$, we define the two space-like laboratory-frame 4-vectors $n^\mu_\mp = (0, \hat{n}_\mp)$. These vectors are boosted to the $\pi^- \pi^+$ ZMF, and we obtain $n^\mu_\mp = (n^\mu_0, n^\mu_\mp)$. Next we decompose the spatial parts $n^\mu_\mp$ into components parallel and perpendicular to the respective pion momentum $p_\mp^*$:

$$n^\mp_* = r_\perp \hat{n}^*_\perp + r_\parallel \hat{n}^*_\parallel,$$  \hfill (7)

where $r_\perp, r_\parallel$ are constants. In this way we obtain the unit vectors $\hat{n}^*_\perp, \hat{n}^*_\parallel$, which are orthogonal to $p^*_\mp$, respectively, for each event in a unique fashion. The angle, which takes the role of the

![Figure 1: Definition of the impact parameter vector $n_-$ in the plane of the decay $\phi \to \tau^+ \to \pi^-$ in the laboratory frame. Here, $p_-$ is the measured $\pi^-$ momentum, $PV$ is the $\tau^-$ production vertex, and $k_-$ is the 3-momentum of the $\tau^-$.](image-url)
true angle between the unsigned normal vectors of the decay planes, Eq. (6), is defined by

$$\varphi^* = \arccos(\hat{n}_+ \cdot \hat{n}_-^*),$$

where $0 \leq \varphi^* < \pi$. In addition, the $CP$-odd and $T$-odd triple correlation $O^*_CP = \hat{p}_- \cdot (\hat{n}_+^* \times \hat{n}_-^*)$ turns out to be an appropriate tool for distinguishing between $CP$ invariance and $CP$ violation in Higgs-boson decay. Here $\hat{p}_-$ denotes the normalized $\pi^-$ momentum. As $-1 \leq O^*_CP \leq 1$, it is convenient to consider, alternatively, the distribution of the angle

$$\psi_{CP} = \arccos(\hat{p}_- \cdot (\hat{n}_+^* \times \hat{n}_-^*)).$$

We shall show in the next section that (8) and (9) are sensitive and robust observables for determining the $CP$ nature of a neutral Higgs boson.

III. RESULTS

As already emphasized above, the observables (8) and (9) can be applied to the $\tau$-decay channel of any Higgs-boson production process. The reason is that the normalized distributions of these variables do not depend on the Higgs-boson momentum if no detector cuts are applied. Furthermore we shall show for $\phi \to \tau^- \tau^+ \to \pi^- \pi^+$ that detector cuts have only a small effect on these distributions for Higgs masses larger than 200 GeV. Thus, our results will not change significantly if one considers a different Higgs production mode or if initial-state higher-order QCD corrections are taken into account. Therefore, we have computed in this analysis all distributions for the LHC reaction (2) with a Higgs boson production process at leading order. Specifically we have used $gg \to \phi$ and $b\bar{b} \to \phi$. For non-standard Higgs bosons $\phi$ and large $\tan \beta$, the latter production mode, respectively $gg \to b\bar{b}\phi$, is considered to be the most promising one in the search for the $\phi \to \tau\tau$ decay channel at the LHC [26, 27].

Fig. 2 (a) shows the distribution of the angle (8) for a scalar ($\phi = H$) and a pseudoscalar ($\phi = A$) Higgs boson, which is determined according to the procedure described above, in the absence of detector cuts. We checked for Higgs-boson masses $120 \text{ GeV} \leq m_\phi \leq 500 \text{ GeV}$ that this distribution is practically independent of $m_\phi$. Moreover, this distribution is practically identical to the distribution of the true decay-plane angle $\sigma^{-1}d\sigma/d\varphi^*_{true} = (\pi)^{-1}(1 + (\pi^2/16)\cos\varphi^*_{true})$ in the $\pi\pi$ ZMF (see (5)), which could be determined if the $\tau^+$ four-momenta in the laboratory frame were known.

Next we apply cuts on the $\pi^\pm$ pseudo-rapidities, $|\eta| \leq 2.5$, and on their transverse momenta, $p_T = \sqrt{p_x^2 + p_y^2} \geq 20 \text{ GeV}$, and recompute this distribution for various Higgs-boson masses. Fig. 2 (b) shows that it depends only very weakly on $m_\phi$, both for $\phi = H$ and $\phi = A$.

In Figs. 3 (a), (b) we have plotted, both for $\phi = H$ and $\phi = A$, the dependence of the $\varphi^*$-distribution on the cut on the $\pi^\pm$ transverse momenta and on $\eta$, respectively, for $m_\phi = 200 \text{ GeV}$. While there is a relatively weak dependence on $p_T^{min}$, the dependence on $\eta_{max}$ is negligibly small. We checked for $120 \text{ GeV} \leq m_\phi \leq 500 \text{ GeV}$ that this feature holds true also for other Higgs-boson masses.
Figure 2: (a) The distribution of $|\phi^*|$ in the $\pi\pi$ ZMF for a scalar and pseudoscalar Higgs boson without detector cuts. (b) Dependence of the $|\phi^*|$ distribution on the Higgs-boson mass if detector cuts are applied.

Figure 3: (a) Dependence of the $|\phi^*|$-distribution in the $\pi\pi$ ZMF on the required minimal transverse pion momentum (a), and on the maximal pseudo-rapidity (b). In the latter case, the curves lie on top of each other.

Performing studies for the distribution $\sigma^{-1}d\sigma/d\phi^*_{true}$ of the true decay-plane angle $\phi^*_{true}$ analogous to Fig. 2(b) and Figs. 3(a), (b) we find that it is practically identical to $\sigma^{-1}d\sigma/d\phi^*$ also in these cases. This demonstrates that the angle $\phi^*$ is a very efficient variable for discriminating between a $CP$-even and $CP$-odd Higgs-boson in a wide range of masses $m_\phi$.

One may wonder whether the angle $\phi_{lab} = \arccos(n_+ \cdot n_-)$ between the impact-parameter vectors $n_+$ and $n_-$ in the laboratory frame is already sensitive to the Higgs-boson parity. In Fig. 4 the distribution of this angle is plotted for a scalar and a pseudoscalar boson with mass $m_{H,A} = 120$ GeV and $500$ GeV, respectively. Fig. 4 shows that $\phi_{lab}$ has some sensitivity: in the case of $H$ decay the events peak around $\phi_{lab} = 120^\circ$, while for $A$ decay the maximum of the distribution is near $\phi_{lab} = 60^\circ$. It is gratifying that the dependence of the distributions
on the mass of the Higgs boson is not very strong. For light states the distance between the maxima of the $A$ and $H$ distributions is somewhat smaller than in the case of heavy states. This is due to the fact that lighter Higgs bosons have, for a specific production reaction, a larger average velocity in the laboratory frame than heavy states. The larger the speed of the Higgs boson, the more the discriminating power of the $\phi_{lab}$ distribution will be diminished.

A comparison of Fig. 4 with Figs. 2, 3 shows that the distribution of the angle $\phi^*$ in the $\pi\pi$

Figure 4: The distribution of the angle $\phi_{lab}$ in the laboratory frame for a scalar and a pseudoscalar Higgs boson with mass $m_{H,A} = 120$ GeV and $m_{H,A} = 500$ GeV.

ZMF is, nevertheless, more sensitive to the parity of a Higgs boson than the distribution of $\phi_{lab}$. Therefore we continue to analyze the former.

According to SM extensions, it is not unlikely that some of the Higgs-boson states are (nearly) mass-degenerate. These states cannot be resolved in the $\tau-$pair invariant mass spectrum. Suppose there is a scalar and a pseudoscalar Higgs boson $H$ and $A$, respectively, with nearly degenerate masses which both contribute to the reaction (2). The resulting distribution of the angle $\phi^*$ (or of the true decay-plane angle $\phi^*_{true}$) will have a shape somewhere between the scalar and pseudoscalar extremes shown in Figs. 2, 3 depending on the relative reaction rates. If such a distribution would be found in an experiment one could, however, not infer its origin. It could also be due to the production of one (or several) $CP$ mixture(s) $f$ with mass(es) $m_f \approx m_{H,A}$. This is shown in Fig. 5(a), where the $\phi^*$ distribution is plotted for two scenarios$^3$. Case (i): Production and decay of both a scalar and a pseudoscalar Higgs boson with couplings such that the respective reaction rates for (2) are equal, $\sigma_H = \sigma_A$. This leads to a flat $\phi^*$ distribution. (Recall that interferences of the $H$ and $A$ scattering amplitudes neither contribute to this distribution nor to $\sigma$.) A distribution with the same shape is, however, generated by a $CP$ mixture with scalar and pseudoscalar couplings to $\tau$ leptons of equal strength, $|a_\tau| = |b_\tau|$. This can be understood with the unsigned correlation (6). The angle

$^3$ For simplicity we consider in the following only one $CP$ mixture $\phi$. The case of several mass-degenerate Higgs boson states with $CP$-violating couplings does not change our conclusions.
$\phi^*_{true}$ is equally distributed for these couplings. Case (ii): Here the $\phi^*$ distribution is shown for $H$ and $A$ exchanges with couplings such that $a_H = 2a_A$. This scalar-like distribution has the same shape as the one that originates from the decay of a $CP$ mixture with couplings $|a_\tau| = \sqrt{2}|b_\tau|$. (Again, inserting these couplings into (6) results in a $\phi^*_{true}$ distribution which is practically identical to the one shown in Fig. 5(a).) For comparison we have also plotted in Fig. 5(a) again the distributions due to a pure scalar and a pure pseudoscalar boson.

The two cases, $H + A$ exchange versus exchange of a $CP$ mixture, can be disentangled with the $CP$ angle $\psi^*_{CP}$. In Fig. 5(b) the distribution of this angle is shown for $CP$-invariant and $CP$-violating Higgs-boson couplings. If $CP$ is conserved, as it is the case for $H$, $A$, or $H + A$ exchange, the expectation value of the $CP$-odd triple correlation associated with $\psi^*_{CP}$ is zero. That is, the distribution of this observable and, likewise, that of $\psi^*_{CP}$ is symmetric, the latter one with respect to $\psi^*_{CP} = 90^\circ$. In fact, as Fig. 5(b) shows, $H$, $A$, or $H + A$ exchange leads to an essentially flat distribution. On the other hand, for a $CP$ mixture the distribution of the $CP$ angle is asymmetric with respect to $\psi^*_{CP} = 90^\circ$. Fig. 5(b) shows the case of an ideal $CP$ mixture with couplings $a_\tau = -b_\tau$ and the case of a $CP$ mixture where $b_\tau = -5a_\tau$. Notice that the case $a_\tau = -5b_\tau$ yields the same $\psi^*_{CP}$ distribution. This scalar-like Higgs boson can be distinguished from the pseudoscalar-like boson ($b_\tau = -5a_\tau$) by means of the $\phi^*$ distribution.

In addition to the $\psi^*_{CP}$ distribution, one may use the asymmetry

$$A_{CP} = \frac{N(\psi^*_{CP} > 90^\circ) - N(\psi^*_{CP} < 90^\circ)}{N_> + N_<}$$

(10)

in order to discriminate between $CP$-conserving and $CP$-violating Higgs-boson exchanges.

How robust are the distributions of $\phi^*$ and $\psi^*_{CP}$ with respect to measurement uncertainties expected at the LHC? In order to study this question with Monte Carlo methods, we have accounted for the expected measurement uncertainties by “smearing” the relevant quantities with a Gaussian according to $\exp\left(-\frac{1}{2}(X/\sigma)^2\right)$, where $X$ denotes the generated quantity (coordinate in position space, momentum component, energy) and $\sigma$ its expected standard
To obtain a rough idea about the length scales involved in the measurement one may assume for a moment that the Higgs boson is produced at rest in the laboratory frame. Then the energy of each τ lepton is \( m_\phi / 2 \). For a 2-body decay of a τ lepton into a π and a neutrino, the energy of the π in the τ-rest frame is \( E_\pi = m_\tau / 2 \). If one assumes that the pion is emitted transversely to the τ direction, then the angle between the τ and the π in the laboratory frame is \( \angle_{\text{lab}}(k, p) \approx 29 \text{ mrad}, 17 \text{ mrad}, \text{ and } 7 \text{ mrad} \) for \( m_\phi = 120 \text{ GeV}, 200 \text{ GeV}, \text{ and } 500 \text{ GeV} \), respectively. If one assumes that the decay length of a certain \( \tau \to \pi \nu \) event is given by the average τ decay length, \( c \tau_\tau = 87 \mu m \), the length of the impact-parameter vector \( n \) in the laboratory frame is \( |n| \approx 80 \mu m \) for the three Higgs-boson masses. In view of this fact and in view of the relatively large value of \( |n| \) our method works for a large range of Higgs masses. This rough estimate also indicates the resolution that must be achieved in an experiment for the primary vertex and the tracks of the pions.

The length of \( n \) depends on the decay length of the τ lepton. For decay lengths shorter than the one used above, \( |n| \) will be smaller and therefore smearing will affect the distributions \( \phi^* \) and \( \Psi^*_{CP} \) in a stronger fashion. Our proposed distributions lose their discriminating power for \( \tau \)-decay events with very short decay lengths. Using the exponential decay law of the τ leptons in their rest frame, we found in our numerical simulations that, for instance in the case of \( m_\phi = 200 \text{ GeV} \), a minimum decay length of \( l_\tau^\text{min} = 2 \text{ mm} \) is required for both τ leptons in order to obtain reasonable results. In an experiment such a cut could be realized by applying a minimum cut on \( |n| \). Due to this requirement the number of \( \tau^- \tau^+ \) events decreases approximately by a factor of 2. On the other hand, such a cut might be experimentally advantageous to separate the \( \tau \tau \) events from the background.

The \( \phi \to \tau^- \to \pi^- \) decay plane is illustrated in Fig. 1 and the \( \phi \to \tau^+ \to \pi^+ \) plane may be drawn analogously. In order to simulate the uncertainties in the experimental determination of the production/decay vertex \( PV \) and of the \( \pi^\pm \) tracks, we vary the position of \( PV \) along and transverse to the beam axis with \( \sigma_z^{PV} = 30 \mu m \) and \( \sigma_{tr}^{PV} = 10 \mu m \), respectively. The track of a charged pion is smeared at the intersection point of the impact-parameter vector \( n \) and the pion momentum \( p \). We vary this point within a circle of radius \( \sigma_{tr}^{PV} = 10 \mu m \) transverse to the π track. The angular resolution of the π track is smeared by \( \sigma_\pi = 1 \text{ mrad} \) around the track. Moreover, the energy of the π is varied by \( \Delta E_\pi / E_\pi = 5\% \). These values appear to be realistic for the LHC experiments [25, 28]. As mentioned before, we apply also a minimum cut on the τ decay length, which is \( l_\tau \geq 2 \text{ mm} \) for \( m_\phi = 200 \text{ GeV} \).

Taking this smearing into account, the distribution of the angle \( \phi^* \) is displayed in Fig. 6(a) for a scalar and a pseudoscalar Higgs boson with mass \( m_\phi = 200 \text{ GeV} \). For comparison, this figure contains also the unsmeared distributions already shown in Fig. 2. The solid lines show the distributions using the smearing parameters stated above. While the simulated uncertainties diminish the difference between the curves for \( A \) and \( H \) somewhat, they are still clearly separated. This holds true also for other Higgs-boson masses, as we have checked.
Figure 6: Distributions (a) for $\phi^*$ and (b) for $\psi_{CP}^*$, taking into account measurement uncertainties. In both figures the dashed lines show the distribution without smearing, while the solid lines include all smearing parameters as stated in the text.

for $120 \text{ GeV} \leq m_\phi \leq 500 \text{ GeV}$. Thus we conclude that $\phi^*$ is an appropriate observable to distinguish between a scalar and a pseudoscalar Higgs boson at the LHC.

Fig. 6(a) shows that smearing affects the $H$ distribution stronger than the distribution for $A$. If the smearing parameters are chosen to be very large, the distributions will peak at $\phi^* \rightarrow 0^\circ$ and will be depleted for large $\phi^*$; i. e., the distribution for $H$ will approach the one for $A$. This is because for large values of $|\mathbf{n}_\perp|$ and $|\mathbf{n}_\parallel|$ the minimum distance between the $\pi^-$ and the $\pi^+$ tracks becomes negligibly small compared to $|\mathbf{n}_\parallel|$ and therefore $\mathbf{n}_\perp$ and $\mathbf{n}_\parallel$ will be almost parallel in the $\pi\pi$ ZMF.

The distribution of the $CP$ angle $\psi_{CP}^*$ is displayed in Fig. 6(b). In the case of production and decay of one or several (mass-degenerate) $CP$ eigenstates ($H, A, A+H$, etc.), the distribution of $\psi_{CP}$, which is a horizontal line, is not affected by any smearing. The dotted line is the unsmeared distribution due to the decay of an ideal $CP$ mixture ($a_\tau = -b_\tau$), already displayed in Fig. 5(b). The solid line includes the uncertainties stated above. These uncertainties decrease the discriminating power of $\psi_{CP}$ somewhat, but this observable is clearly the appropriate tool for distinguishing between Higgs-boson states with $CP$-conserving and $CP$-violating couplings to $\tau$ leptons.

The distribution of the variables $\phi^*$ and of $\psi_{CP}^*$ can be determined in completely analogous fashion also for the other 1-prong $\tau$ decays, that is, $\phi \rightarrow \tau^- \tau^+ \rightarrow f_1^- f_2^+ + \text{neutrals}$, where $f_1^-, f_2^+$ denote either a charged lepton from $\tau^+ \rightarrow e^\mp \mu^\mp$, or a charged pion from $\tau^+ \rightarrow \rho^\mp$ and $\tau^+ \rightarrow \pi^\mp 2\pi^0$. For $\phi = H, A$ the distribution of the true decay-plane angle $\phi_{true}^*$ in the $f_1 f_2$ ZMF is given by $\sigma^{-1} d\sigma/d\phi_{true}^* = (\pi)^{-1} [1 + (\pi^2/16) \kappa_1 \kappa_2 \cos \phi_{true}^*]$, where the numbers $\kappa_1, \kappa_2$ signify the $\tau$-spin analyzer quality of the respective charged prong. We recall that for the decays of 100% polarized $\tau$ leptons we have the angular distribution $d\Gamma(\tau^+ \rightarrow f^+) \propto 1 \pm \kappa_f \cos \theta_f$, where $\theta_f$ is the angle between the $\tau$ spin vector and the direction of the charged prong in the $\tau$ rest frame. While the spin analyzer quality factor is maximal, $\kappa_\tau = 1$, for
\( \tau \rightarrow \pi \nu_\tau \), it is considerably smaller for the other 1-prong decays. For \( \tau \rightarrow l = e, \mu \) it is \( \kappa_l = -0.33 \), and for a charged pion from \( \tau \rightarrow \rho \) it is only \( \kappa \simeq -0.07 \). The spin analyzer quality of a charged lepton and of the charged pion from \( \rho, a_1 \) can, however, be significantly enhanced by appropriate energy cuts, i.e., by taking into account only charged prongs above some suitably chosen minimum energy in the \( f_1 f_2 \) ZMF. The gain in spin-analyzing power outmatches by far the loss in statistics. In this way, one can achieve an effective correlation coefficient \( \kappa_{eff} \simeq 0.8 \) while reducing the number of 1-prong \( \tau \) decays that can be used in the analysis from \( N_1 \) to \( N_{eff} \simeq 0.54 N_1 \). Likewise, the discriminating power of the \( CP \)-odd spin correlation underlying the distribution of \( \psi_{CP}^* \) is maintained by these cuts. A detailed account will be given elsewhere [29].

Finally, we make a crude estimate of how many events are needed in order to distinguish between i) a scalar and pseudoscalar Higgs boson and/or ii) between \( CP \)-conserving and \( CP \)-violating states, assuming \( m_\phi = 200 \) GeV. As to i), we define an asymmetry

\[
A_{\phi^*} = \frac{N(\phi^* > 90^\circ) - N(\phi^* < 90^\circ)}{N_\geq + N_\leq}.
\]  (11)

From Fig. 6 (a) we obtain from the smeared distributions \( A_{\phi^*}^H = 0.31 \) and \( A_{\phi^*}^A = -0.42 \). Taking into account an effective \( \tau \)-spin analyzing coefficient for 1-prong decays \( \kappa_{eff} = 0.8 \), these asymmetries are reduced by a factor of about 0.64. Thus, for distinguishing \( H \) from \( A \) with 5 s.d. significance requires about 120 1-prong events. Concerning ii), the result of Fig. 6 (b) implies that for an ideal \( CP \) mixture the \( CP \) asymmetry defined in Eq. (10) takes the value \( A_{CP} = -0.37 \) while it is zero for pure \( H, A \) and degenerate \( H \) and \( A \) intermediate states. Thus, about 400 1-prong events will be needed to establish this \( CP \)-violating effect at the 5 s.d. level. This should be feasible, depending on the masses and couplings of \( \phi \), after several years of high luminosity runs at the LHC.

**IV. CONCLUSIONS**

We have proposed a method for determining the \( CP \) nature of a neutral Higgs boson or spin-zero resonance \( \phi \) at the LHC in its \( \tau \) pair decay channel. The method can be applied to any 1-prong decay mode of the \( \tau \) lepton. It requires the measurement of the energy and momentum of the charged prong \( (\pi^\pm, e^\mp, \mu^\mp) \) from \( \tau^\pm \) decay and the determination of the \( \tau^- \tau^+ \) production vertex with some precision. The distributions of the angles \( \phi^* \) and \( \psi_{CP}^* \) allow to discriminate between pure scalar and pseudoscalar states and/or between a \( CP \)-conserving and \( CP \)-violating Higgs sector. For the decays \( \tau \rightarrow \pi \nu_\tau \) we have shown that the variables \( \phi^* \) and \( \psi_{CP}^* \) maintain their discriminating power when measurement uncertainties are taken into account. The smearing parameters that we used in our simulations indicate the precision which should eventually be achieved by the LHC experiments. Our method could, of course, be applied also at a future \( e^+e^- \) linear collider where Higgs-boson production and decay would take place in a much cleaner environment.
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