A kinetic theory of the pulsational mode of gravitational collapse in star-forming molecular clouds

Dawroichuh Challam
Department of Physics, Tezpur University, Tezpur, India

Pralay Kumar Karmakar (:pk@tezu.ernet.in)
Tezpur University  https://orcid.org/0000-0002-3078-9247

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Dawroichuh Challam, Pralay Kumar Karmakar

Department of Physics, Tezpur University, Napaam-784028, Tezpur, Assam, India

Abstract

A kinetic theory formulation of the pulsational mode of gravitational collapse (PMGC) in a complex unmagnetized self-gravitating partially ionized dense molecular cloud (DMC) is proposed. Applying a linear normal mode analysis, a quintic linear dispersion relation with a unique set of multi-parametric plasma-dependent coefficients is obtained. The reliability of the calculation scheme is validated in light of the various predictions available in the literature. It is then numerically analyzed in the parametric windows of judicious realistic input values. Our results indicate that the dust mass, equilibrium electron density, and equilibrium ion density act as destabilizing agencies to the PMGC evolution. In contrast, the dust charge number, equilibrium dust density, and dust temperature act as stabilizing agencies. The oscillatory and propagatory features of the PMGC are illustratively explained and comparatively validated in accordance with the observed astrophysical scenarios. This paper ends up with a brief highlight of the non-trivial implications and applications of the results actualizable in the self-gravitational collapse mechanism leading to varied structure formation processes in the mysterious astrocosmic universe.

1 Introduction

Dense molecular clouds (DMCs) composing the interstellar medium (ISM), which are self-gravitating partially ionized dusty plasmas, are well-known to be the ideal sites for active formation of stars, planets, and other bounded structures in galaxies [1-3]. These DMCs generally get fragmented into clumps and sub-clumps of size $\sim$10-10$^6$ pc, where 1pc $\approx$3×10$^{18}$ cm, and of mass $\sim$10-10$^4 M_{\odot}$, where $M_{\odot}$=10$^{33}$ g is the solar mass. Such local clump-like structures out of the global DMCs undergo canonical gravitational collapse, leading thereby to the structure formation dynamics [2-4]. Dust grains are understood to play many critical roles in the galactic evolution processes and are central to the chemistry of interstellar gas. The photoelectrons liberated from dust grains can dominate the heating of gas in the regions where ultraviolet starlight is present, and in the denser regions, the infrared emission from dust can be an important cooling mechanism. Dust grains also play an important role in interstellar gas dynamics, transporting radiation pressure from starlight to the gas, and coupling the magnetic field to the gas in the regions of low fractional ionization [1]. The constituent interstellar dust grains, comprise about 1% of the ISM mass, are composed mainly of graphite, silicate, and metallic derivative compounds. The normal mass ($m_d$) of a dust grain may vary typically from 10$^{-5}$ g in interplanetary space to 10$^{-12}$-10$^{-14}$ g in the
interstellar cloud [2]. Besides, there exist both heavier dust grains \((m_d \sim 10^{-2} \text{ g})\) and lighter \((m_d \sim 10^{-20} \text{ g})\) ones, as inferred from various astronomical observations reported in the literature [1-3].

Interstellar DMCs sometimes contains newborn stars, associated emission nebula, or even other ionizing sources. As a result, some parts of the clouds are always ionized. Due to a lower degree of the DMC ionization \((\sim 10^{-7})\), the dust grains are generally embedded in a weakly ionized plasma background [5-7]. When a significant fraction of radiation from the ionizing sources escapes without being absorbed by the in-falling matter from the accretion disk, the dust grains may pick up between \(10-10^3\) (or more) electronic charges from the ionized cloud [1, 2]. The wide range in the dust grain mass, size and charge, suggests that the dynamics of a dusty plasma can be studied in any of the following regimes: (a) Electromagnetic (EM) force \(\gg\) Gravitational Force (GF), (b) EM force \(\sim\) GF, and (c) EM force \(\ll\) GF. Plasma processes like radiation, heating, etc., are attributed to the first case, the second case has been shown to correspond to spoke formation in Saturn’s rings, and the last case corresponds to the formations of large-scale structures, such as stars and galaxies [4, 8]. The dust grains for which electromagnetic and gravitational forces compete, must be quite massive \((\sim 10^{-5} \text{ g})\) and large in size. In a collapsing molecular cloud, dust grains with such mass and size are expected to exist in the inner region of the protostellar disk, as the dust grains size always increases towards the central condensation [5].

The Jeans instability of a dusty plasma has been extensively studied in the past by several authors in the framework of fluid theory [9-17] as well as kinetic theory [4, 18, 19]. Since the observed degree of ionization \((\sim 10^{-7})\) in interstellar DMCs is rather low, it is realistic to assume the presence of charged and neutral dust grains along with plasma particles in a molecular cloud. The Jeans collapse of a molecular cloud, which consists of electrically charged and neutral dust grains occurs essentially in the presence of an equilibrium electrostatic field. As a result, the resultant electrostatic repulsion among the grains will counteract the gravitational attraction, slowing or stopping the collapse altogether. The frictional coupling of the gravitational attraction (inwards) and electrostatic repulsion (outwards) of the massive charged grains constituting the DMCs gives rise to a new mode of the Jeans condensation, termed as the pulsational mode of gravitational collapse (PMGC), which is a purely hybrid mode composed of the gravito-electrostatic degrees of freedom [5, 6]. The PMGC dynamics has been extensively studied in the framework of multicomponent fluid model formalisms only in different astronomical circumstances [5, 6, 20-25]. In the recent past, the hybrid instability dynamics excitable in a strongly coupled magnetized self-gravitating conducting dusty plasma system has been investigated [15]. It has been mainly reported that the complex form of this hybrid instability criterion is not affected by the magnetic field unlike the radiative viscoelastic effects, radiative losses, and so forth.

In the present work, we report the model development of a theoretical formulation of the same PMGC in a self-gravitating unmagnetized DMC with the help of a self-consistent kinetic theory for the first time to the best of our knowledge. The formulation of the presented kinetic description is driven mainly by the fact that the phase speed of the cloud fluctuations in some interstellar situations become comparable to the thermal speeds of the constituent thermal species (electrons and ions). In other words, the fluid description is valid only in those circumstances where the fluctuations phase speed is much smaller than both the electronic and ionic thermal speeds; else, a kinetic theory is logically invoked. It hereby parallelly implicates that the hydrodynamic approximation is extensively useful for low-frequency fluctuation analysis; while the kinetic approximation is widely applicable for high-frequency phenomena [26, 27]. A linear normal mode analysis yields a quintic linear dispersion relation with a unique set of multi-
parametric coefficients. It is numerically analyzed in the parametric framework of judicious realistic input values. Our results indicate that the dust mass, equilibrium electron density, and equilibrium ion density act as destabilizing agencies to the PMGC evolutionary dynamics. The dust charge number and dust temperature, on the other hand, act as stabilizing agencies, and so forth. A brief highlight about the non-trivial implications and applications of our results actualizable in the self-gravitational collapse mechanism leading to the formation dynamics of varied astrostructures is finally outlined.

2 Physical model and mathematical formalism

In the present kinetic description, we consider a four-component dusty plasma model consisting of lighter electrons, ions; and heavier neutral dust and charged dust. All the constituent species are presumed to constitute a homogeneous distributed gaseous phase initially at least in the quasi-neutral hydrostatic equilibrium. The constituent electrons and ions are roughly approximated to behave as inertialess collisionless fluids on the bulk macroscopic domains. In partially ionized DMCs, as that under consideration (degree of ionization \( \sim 10^{-7} \)), the constitutive dust grains (identical solid micro-spheres each of radius \( a \)) accumulate charges from the background plasma environment. The contact electrification of the dust grains with the joint action of randomized electron-ion thermal currents imparts the grains negatively charged in the background plasma medium. This contact charging process is a dynamic one dominated mainly by the inertialess electronic flow currents against the ionic contributions. It is pertinent to understand that the grains can become positively charged with the help of photo-ionization processes in optically thin cloud media via the incident intense UV radiations emitted from nearby young hot stars acting as the ionizing sources [2, 28]. However, the considered DMC is optically thick and hence, only the negatively charged grains are accounted for as the constitutive element in our model setup.

There might be several kinetic collisional momentum exchange processes among the diversified plasma constitutive species. It includes the momentum transfer between the neutral and charged dust \( (v_{cn}) \) and vice versa \( (v_{nc}) \); electron-dust collisions \( (v_{ed}) \) and vice versa \( (v_{de}) \); ion-dust collisions \( (v_{id}) \) and vice versa \( (v_{di}) \); electron-ion collisions \( (v_{ei}) \) and vice versa \( (v_{ie}) \); and so forth [4-6]. However, not all collisional processes are of equal importance in real astronomic configurations. For the ISM cloud under study \( (q_d/e = Z_d \sim 10^2) \), where \( q_d = Z_d e \) is the dust charge and \( e \) the electronic charge unit), collisional processes, such as electron-ion, electron-electron, and ion-ion can be neglected (because of small collision cross-sections) when compared with the dust charging processes which are governed by the electron-dust and ion-dust collisional processes [4, 5]. For our model, we ignored the dynamical behavior of the dust charge fluctuations. This is justified by the fact that for a cloud with the dust charge-to-mass ratio \( q_d/m_d \sim \sqrt{G} \) (where \( G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \) is the Newton universal gravitational constant), the electrostatic force on such constituent dust grains is on the order of the gravitational force leading to the possibility of gravitational condensation, so that \( \omega_j^{-1}/\tau_c >> 1 \) (where \( \omega_j^{-1} \) is the gravitational response time scale and \( \tau_c \) is the dust charging time scale) [6, 29, 30]. Hence, electron-dust and ion-dust collisional processes can be also be justifiably ignored. We consider here the collisional momentum exchange between the charged and neutral dust grains justifiably because of their high
 inertia and size. The binary collisional rates of the linear momentum transfer between the charged and the neutral dust grains, and vice versa [5, 6], are respectively given as

\[ V_{cn} \approx \pi a^2 n_{d0} v_{Td}, \]

\[ V_{nc} \approx \pi a^2 n_{d0} v_{Td}, \]

where \( \pi a^2 \) is the geometrical cross-section of the identical spherical dust grains. \( n_{d0}, n_{c0} \) is the equilibrium number density of the neutral (charged) dust grains. \( v_{Td} \) is the thermal velocity of the dust grains. The effects of tidal force field of remote gravitating objects are ignored. As the interstellar magnetic field is very small (~micro-Gauss); therefore, no magnetic effects are considered herein via the Lorentz force action [5]. All the heterogeneous constituent species are assumed to build up a quasi-hydrostatic homogeneous field-free equilibrium configuration at least initially through the lowest-order gravity-induced polarization effects. For simplicity, we assume that the neutral gas particles form the background that is weakly coupled with the collapsing cloud.

The equations describing the dynamics of the individual constitutive species composing the entire self-gravitating dusty plasma system in the proposed kinetic picture with all the generic astronomical notations [4, 18, 19, 26-28] are respectively given as

\[
\frac{\partial}{\partial t} + (v \cdot \nabla_v) \left\{ \frac{e}{m_e} \{ \nabla \phi(r,t), \nabla_v \} \right\} f_e(r,v,t) = 0,
\]

\[
\frac{\partial}{\partial t} + (v \cdot \nabla_v) \left\{ \frac{e}{m_i} \{ \nabla \phi(r,t), \nabla_v \} \right\} f_i(r,v,t) = 0,
\]

\[
\frac{\partial}{\partial t} + (v \cdot \nabla_v) \left\{ \nabla \psi(r,t), \nabla_v \right\} f_{dn}(r,v,t) = -\nu_{nc}(f_{dn} - f_{dn0}),
\]

\[
\frac{\partial}{\partial t} + (v \cdot \nabla_v) \left\{ \frac{q_d}{m_d} \nabla \phi(r,t) - \nabla \psi(r,t) \right\}. \nabla_v \right\} f_{dc}(r,v,t) = -\nu_{cn}(f_{dc} - f_{dc0}).
\]

Clearly, \( \alpha = e, i, dn, dc \) signifies the electrons, ions, neutral dust, and charged dust grains, respectively. Here, \( f_{\alpha}(r,v,t) \) is the distribution function of the \( \alpha^{th} \) species in the six-dimensional phase space define by \( (r,v) \). \( f_{\alpha 0}(v) \) is the equilibrium Maxwell-Boltzmann distribution to which the \( \alpha^{th} \) species relaxes over the zeroth-order collisional time scale. \( m_\alpha, T_\alpha, v_{T\alpha}, \) and \( \omega_{p\alpha} \) are the mass, temperature, thermal velocity, plasma frequency of the \( \alpha^{th} \) species, respectively. It is to mention here that Eqs. (5)-(6) depict the frictional coupling between the neutral and charged dust species and vice versa, respectively.
The electro-gravitational potential distributions sourced in the charge and material density fields of the constituent particles are described by the corresponding Poisson equations [5, 6, 20-22] given in the customary notations respectively as

\[ \nabla^2 \phi(r, t) = 4\pi \left\{ e(n_e - n_i) + q_d n_{dc} \right\}, \]  
\[ \nabla^2 \psi(r, t) = 4\pi G m_d (n_{dc} + n_{dn} - n_{d0}); \]

where \( n_{d0} = n_{dc0} + n_{dn0} \) models the Jeans swindle of the equilibrium gravitational force field. It is assumed that \( m_{dc} \approx m_{dn} = m_d \) for our model analysis.

The number density of the different components in our system is defined [4, 18, 19] as

\[ n_\alpha(r) = \int f_\alpha(r, v, t) dv. \]

The closure of the model setup is obtained with the help of the electrostatic and the self-gravitational Poisson equations, as given respectively by Eqs. (7)-(8), in a coupled closed form.

3 Perturbation analysis

We now perturb the distribution functions, population densities, electrostatic potential, and gravitational potential linearly relative to the defined homogeneous equilibrium as follows

\[ f_\alpha(r, v, t) = f_{\alpha0}(v) + f_{\alpha 1}(r, v, t), \]
\[ n_\alpha(r, t) = n_{\alpha 0} + n_{\alpha 1}(r, t), \]
\[ \phi(r, t) = \phi_0 + \phi_1(r, t), \]
\[ \psi(r, t) = \psi_0 + \psi_1(r, t); \]

where, \( f_{\alpha 1} << f_{\alpha 0} \), \( n_{\alpha 1} << n_{\alpha 0} \), \( \phi_1 << \phi_0 \) and \( \psi_1 << \psi_0 \). It may be noted that the existence of the finite non-zero equilibrium gravito-electrostatic potential values here is attributable mainly to the gravity-induced electrostatic polarization effects sourced in the constituent heterogeneous species having different masses in our model plasma configuration, and so forth. The basic physical insight behind such polarization effects is the mass-dependent stratification of the bipolar charged particles relative to the common reference center of the self-gravitating cloud. As a consequence, massive dust grains (with negative charge) settle closer to the center than the lighter species (with both positive protonic and negative electronic charge). It results in a strong potential gradient of electrostatic origin developed originally by the mass-dependent gravitational settlement of the dissimilar constitutive species [31-34]. The growth of the spatiotemporal fluctuations in the quasi-classic plane-wave approximation \( (kx >> 1) \) is adopted in a standard
Fourier form as: \( \exp(-i\omega t + k \cdot x) \). Application of Eqs. (10)-(13) in Eqs. (3)-(9) makes the system to evolve in the Fourier space \((\omega, k)\) instead of the coordination space \((t, x)\). So, Eqs. (7)-(8) in the wave space can respectively be cast as

\[
-k^2 \phi_1 = 4\pi e n_{e1} - 4\pi e n_{i1} + 4\pi q_d n_{dc1},
\]

\[
-k^2 \psi_1 = 4\pi e m_d n_{d1} + 4\pi e m_d n_{dc1}.
\]

Again, the expanded perturbed form of Eq. (9) for the different components composing the whole dusty plasma model can respectively be written as

\[
n_{e1} = \frac{e}{m_e} \phi_1 \int k \cdot \frac{\partial f_{e0}}{\partial \nu} \frac{d\nu}{(\omega - k \cdot v)},
\]

\[
n_{i1} = -\frac{e}{m_i} \phi_1 \int k \cdot \frac{\partial f_{i0}}{\partial \nu} \frac{d\nu}{(\omega - k \cdot v)},
\]

\[
n_{d1} = -\psi_1 \int k \cdot \frac{\partial f_{d0}}{\partial \nu} \frac{d\nu}{(\omega + iv_{nc} - k \cdot v)},
\]

\[
n_{dc1} = \left( \frac{q_d}{m_d} \phi_1 - \psi_1 \right) \int k \cdot \frac{\partial f_{dc0}}{\partial \nu} \frac{d\nu}{(\omega + iv_{cn} - k \cdot v)}.
\]

Assuming the local thermodynamical equilibrium, the background distribution function of the constituent particles can be described using the Maxwell-Boltzmann distribution law [14, 15, 26-28] given as

\[
f_{\alpha 0}(v) = \frac{n_{\alpha 0}}{(2\pi)^{3/2} v_{T\alpha}^3} \exp\left(-v^2/2v_{T\alpha}^2\right);
\]

where, \( n_{\alpha 0} \) is the equilibrium number density of the \( \alpha \)th species. Using Eq. (20) in Eqs. (16)-(19), one finds the following equations respectively

\[
n_{e1} = \frac{e n_{e0}}{m_e v_{Te}^2} \phi_1 W(\zeta_e),
\]

\[
n_{i1} = -\frac{e n_{i0}}{m_i v_{Ti}^2} \phi_1 W(\zeta_i),
\]
where, \( \zeta_e = \omega/kv_{Te} \), \( \zeta_i = \omega/kv_{Ti} \), \( \zeta_{dn} = (\omega + iv_{ne})/kv_{Td} \), \( \zeta_{dc} = (\omega + iv_{nc})/kv_{Td} \), and we have taken the assumption that \( T_{dn} \approx T_{dc} = T_d \) and \( V_{Tdn} \approx V_{Tdc} = V_T \). The function \( W(\zeta_a) \) is explicitly defined [28] as

\[
W(\zeta_a) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x}{x-\zeta_a} \exp\left(-\frac{x^2}{2}\right) dx. \tag{25}
\]

For \( |\zeta_a| \ll 1 \), we have

\[
W(\zeta_a) = i \sqrt{\frac{\pi}{2}} \zeta_a \exp(-\zeta_a^2) + 1 - \zeta_a^2 + \frac{\zeta_a^4}{3}; \tag{26}
\]

whereas, in the opposite limit, namely \( |\zeta_a| \gg 1 \), \( W(\zeta_a) \) takes the form given as

\[
W(\zeta_a) = i \sqrt{\frac{\pi}{2}} \zeta_a \exp\left(-\frac{\zeta_a^2}{2}\right) - \frac{1}{\zeta_a} - \frac{3}{\zeta_a^2}. \tag{27}
\]

Assuming the condition \( v_{Td} \ll (\omega/k) \leq v_{Te}, v_{Ti} \), we find that

\[
|\zeta_e| \ll 1, \quad |\zeta_i| \ll 1, \quad |\zeta_{dn}| \gg 1, \quad |\zeta_{dc}| \gg 1. \tag{28}
\]

We now apply all the above conditions in Eqs. (21)-(24) along with the quasi-neutrality condition for the analysis of the proposed plasma fluctuations. A rigorous method of elimination and simplification results in a generalized linear quintic dispersion relation given as

\[
D(\Omega, K) \equiv \Omega^5 + A_4\Omega^4 + A_3\Omega^3 + A_2\Omega^2 + A_1\Omega + A_0 = 0; \tag{29}
\]

where, \( \Omega = \omega/\omega_J \) is the Jeans normalized angular frequency and \( K = k/k_J \) is the Jeans normalized angular wavenumber of the fluctuations. The different involved coefficients actively appearing in Eq. (29) are respectively given as

\[
A_4 = -iD_4/D_5, \quad A_3 = D_3/D_5, \quad A_2 = -iD_2/D_5, \quad A_1 = D_1/D_5, \quad A_0 = -iD_0/D_5;
\]
where,
\[
D_5 = \sqrt{\frac{\pi}{2} \Omega_{pd}^3 \left( \frac{1}{\lambda_e^3 \Omega_{pe}} + \frac{1}{\lambda_i^3 \Omega_{pi}} \right)} \frac{1}{K^3},
\]

\[
D_4 = A - 2D_5 \left\{ F_{nc} + F_{cn} \right\},
\]

\[
D_3 = 2A \left\{ F_{nc} + F_{cn} \right\} + D_5 \left\{ \Omega_{jn}^2 + \Omega_{jc}^2 - F_{nc}^2 - F_{cn}^2 - 4F_{nc} F_{cn} \right\},
\]

\[
D_2 = A \left\{ \Omega_{jn}^2 + \Omega_{jc}^2 - F_{nc}^2 - F_{cn}^2 - 4F_{nc} F_{cn} \right\}
- 2D_5 \left[ F_{cn} \left( \Omega_{jn}^2 - F_{nc}^2 \right) - F_{nc} \left( \Omega_{jc}^2 - F_{cn}^2 \right) \right] - \Omega_{pd}^2,
\]

\[
D_1 = 2A \left[ F_{cn} \left( \Omega_{jn}^2 - F_{nc}^2 \right) - F_{nc} \left( \Omega_{jc}^2 - F_{cn}^2 \right) \right]
+ D_5 \left[ F_{cn}^2 \left\{ \Omega_{jn}^2 - \Omega_{jn}^2 \right\} - F_{nc}^2 \Omega_{jc}^2 \right] - 2F_{nc} \Omega_{pd}^2,
\]

\[
D_0 = A \left[ F_{cn}^2 \left\{ \Omega_{jn}^2 - \Omega_{jn}^2 \right\} - F_{nc}^2 \Omega_{jc}^2 \right] + \left\{ F_{nc}^2 - \Omega_{jn}^2 \right\} \Omega_{pd}^2;
\]

\[
A = 1 + \Omega_{pd}^2 \left( \frac{1}{\lambda_e^2} + \frac{1}{\lambda_i^2} \right) \frac{1}{K^2},
\]

\[
F_{nc} = \frac{v_{nc}}{\omega_j}, \quad F_{cn} = \frac{v_{cn}}{\omega_j},
\]

\[
\Omega_{pe} = \frac{\omega_{pe}}{\omega_j}, \quad \Omega_{pi} = \frac{\omega_{pi}}{\omega_j}, \quad \Omega_{pd} = \frac{\omega_{pd}}{\omega_j}, \quad \Omega_{jn} = \frac{\omega_{jn}}{\omega_j}, \quad \Omega_{jc} = \frac{\omega_{jc}}{\omega_j},
\]

\[
\lambda_e = \frac{\lambda_{De}}{\lambda_{Dd}}, \quad \lambda_i = \frac{\lambda_{Di}}{\lambda_{Dd}}.
\]

Here, \( \lambda_{Dd} = \left[ T_a / \left( 4\pi n_a e_a^2 \right) \right]^{1/2} \) is the plasma Debye length associated with the \( \alpha \)th species. \( \omega_{jn} = \left[ 4\pi G_m n_{d0} \right]^{1/2} \) and \( \omega_{jc} = \left[ 4\pi G_m n_{c0} \right]^{1/2} \) are the Jeans frequencies associated with the neutral and charged dust grains, respectively. The analytic construct of Eq. (29) seems to depict the possibility of excitation of five distinct modes. It usually includes the electron-plasma oscillations, ion-plasma oscillations, charged dust-plasma oscillations, Jeans mode, and the
pulsational hybrid mode resulting from the coupling of Jeans mode and the bulk acoustic mode. We are interested in a kinetic formalism of the considered plasma instability which makes all the conventional modes, as indicated above, to go Landau-damped except the coupled pulsational mode, which is a hybridized mode developed out of the bulk acoustic mode and the gravitational Jeans mode in the low-frequency regime. As a consequence, by neglecting the effects of the Landau damping caused by the non-linear wave-particle (and wave-wave) interaction processes (as given by Eq. (30)), the linear dispersion relation (Eq. (29)) reduces to a simplified form as

$$\left(\frac{\omega}{\omega_j}\right)^4 + 2\left(i\frac{\nu_{cn}}{\omega_j}\right)\left(\frac{\omega}{\omega_j}\right)^3 + \left\{1 - \frac{\nu_{cn}^2}{\omega_j^2} - \left(\frac{\omega_{pd}^2}{\omega_j^2}\right)\right\}\left(\frac{1}{1 + \eta}\right) k^2 = 0. \quad (40)$$

It is herein assumed that $\nu_{cn} >> \nu_{wc}$, $k\lambda_D << 1$, and $\eta = n_{d0}/n_{d0}$. The above simplified dispersion relation (Eq. (40)) derived in the kinetic fabric is identical to the one (Eq. (19)), as already derived in the fluid model framework as seen in the literature [6]. The matching of the two distinctly obtained dispersion relations put forward a reliability check up on our current calculation schemes.

In order for further analysis of the pulsational instability dynamics in the current kinetic model, Eq. (29) is expressed as

$$D(\Omega, K) \equiv D_r(\Omega, K) + iD_i(\Omega, K), \quad (41)$$

where,

$$D_r = D_4 \Omega^4 + D_2 \Omega^2 + D_0, \quad (42)$$

$$D_i = D_5 \Omega^5 + D_3 \Omega^3 + D_1 \Omega. \quad (43)$$

The Jeans-normalized real frequency ($\Omega_r$) and the Jeans-normalized imaginary frequency ($\Omega_i$) are calculated, respectively from $D_r(\Omega = \Omega_r, K) = 0$ and $\Omega_i = -D_i(\Omega_r, K)/\partial D_r/\partial \Omega_r$. Use of $D_r$ and $D_i$ from Eqs. (42)-(43), respectively, yields

$$\Omega_r = \left[\left(-X_1 \pm \sqrt{X_1^2 - X_2}\right)/2\right]^{1/2}, \quad (44)$$

$$\Omega_i = -\left[\left(D_5 \Omega_r^5 + D_3 \Omega_r^3 + D_1\right)/(4D_4 \Omega_r^4 + 2D_2)\right], \quad (45)$$
where,

\[
X_1 = \left( \Omega_{Jn}^2 + \Omega_{Jc}^2 - F_{nc}^2 - F_{cn}^2 - 4F_{nc}F_{cn} \right) + \left( \Omega_{pd}^2 / \Delta \right)
\]

(46)

\[
X_2 = 4 \left[ F_{cn}^2 \Omega_{Jn}^2 - F_{nc}^2 \Omega_{Jc}^2 - \left( F_{nc}^2 - \Omega_{Jn}^2 \right) \left( \Omega_{pd}^2 / \Delta \right) \right]
\]

(47)

We now construct a numerical illustrative platform to explore the physical insights of the PMGC in the considered DMC characterized by Eqs. (44)-(45).

4 Results

This paper reveals the development of a kinetic model theory to understand the PMGC stability dynamics in the gravito-electrostatically bounded DMCs founded in the approximation of plane-parallel geometry. A quintic dispersion relation (Eq. (29)) is obtained with the help of the Fourier technique in order to see the PMGC dynamics in an interstellar cloud. An analytic inspection reveals that the propagatory features of the PMGC are controlled by a unique set of various plasma-dependent multi-parametric involved coefficients. It is then numerically analyzed to explore the mode features illustratively as presented in Figs. 1-7.

In Fig. 1, we display the profile patterns of the Jeans-normalized (a) real frequency part ($\Omega_r$), (b) its zoomed part ($\Omega_r$), and (c) imaginary frequency part ($\Omega_i$) of the PMGC with variation in the Jeans-normalized wave number ($K$) for different values of the dust grain mass ($m_d$). The various lines link to $m_d = 2 \times 10^{-8}$ g (red, solid line), $m_d = 3 \times 10^{-8}$ g (blue, dashed line), and $m_d = 4 \times 10^{-8}$ g (black, dotted line), respectively. The different parametric input values for the numerical analysis adopted from reliable astronomical literature [5, 6, 16-23] used here are $m_e = 9.1 \times 10^{-28}$ g, $m_i = 1.67 \times 10^{-24}$ g, $n_{e0} = 1 \times 10^{6}$ cm$^{-3}$, $n_{i0} = 5 \times 10^{6}$ cm$^{-3}$, $n_{d0} = 4$ cm$^{-3}$, $n_{dc0} = 2$ cm$^{-3}$, $T_e \approx T_i \approx 1$ eV, $T_d \approx 2 \times 10^{-2}$ eV, and $Z_d = 100$. It is interestingly seen that the real frequency part (Figs. 1a and b) decreases gradually with an increase in $m_d$, and vice-versa. It implicates that $m_d$ plays a decelerating role in the PMGC propagatory dynamics. In contrast, the growth rate increases with an increase in $m_d$, and vice-versa (Fig. 1c). It is attributable to the fact that, as $m_d$ increases, the cloud becomes heavier and the inward gravitational pull increases, and vice-versa. As a result, it can be inferred that $m_d$ acts as a destabilizing agency to the PMGC evolutionary dynamics. This is against the previous result reported for a viscoelastic cloud configuration [21], where $m_d$ acts as a stabilizing agency to the PMGC evolutionary dynamics.

In Figs. 2-3, we display the same as Fig. 1, but with fixed $m_d = 2 \times 10^{-8}$ g for different $Z_d$-values and $n_{d0}$-values, respectively. We see that $\Omega_r$ increases gradually with an increase in both $Z_d$ and $n_{d0}$, and vice-versa (Figs. 2a and 3a). This implicates that both $Z_d$ and $n_{d0}$ play accelerating roles to the propagatory PMGC dynamics. It is further seen that $\Omega_r$ decreases with enhancement in both $Z_d$ and $n_{d0}$, and vice-versa (Figs. 2b and 3b). Thus, we can infer that both $Z_d$ and $n_{d0}$ act as stabilizing agencies to the PMGC evolutionary dynamics. Physically this is due to the fact that the enhancement of $Z_d$ and $n_{d0}$ increases the strength of the electrostatic interaction, thereby, preventing the cloud to collapse against the inward gravitational pull. If this result is compared with findings in previous literature [22], it is found that for a viscoelastic cloud fluid, $Z_d$ and $n_{d0}$
act as destabilizer and stabilizer, respectively. Thus, we can conclude that $n_{e0}$ acting as a stabilizer of the PMGC is a common feature in both the kinetic and fluid frameworks.

In Figs. 4-5, we display the same as Fig. 1, but with fixed $m_d = 2 \times 10^{-8} \text{ g}$ for different $n_{e0}$-values and $n_{i0}$-values, respectively. It is seen that $\Omega_r$ decreases, with an increase in both $n_{e0}$ and $n_{i0}$, and vice-versa. It implicates that both $n_{e0}$ and $n_{i0}$ play decelerating roles to the propagatory PMGC dynamics (Figs. 4a and 5a). It is further seen that enhancement in both $n_{e0}$ and $n_{i0}$, results in the increase of $\Omega_r$, and vice-versa (Figs. 4b and 5b). The restoring electrostatic force (outward) fails to stabilize the collapsing cloud against the long-range gravitational force (inward). Thus, we can infer that both $n_{e0}$ and $n_{i0}$ act as destabilizing agencies to the pulsating cloud responsible for active star formation processes via the canonical gravitational collapse mechanism. If this result is compared with the closely correlated findings available in the literature [22], it is found that $n_{e0}$ and $n_{i0}$ conjugationally act as destabilizers and stabilizers for a viscoelastic cloud fluid. It enables us to conclude that $n_{e0}$ acting as a destabilizing agency to the self-gravitating DMC is a common feature realistically experience in both the kinetic and fluid frameworks.

Similarly, Fig. 6 displays the same as Fig. 2, but with fixed $m_d = 2 \times 10^{-8} \text{ g}$ for different $T_d$-values. It is seen that $T_d$-enhancement results in an increase of $\Omega_r$ in the domain $K<0.4$; whereas, in the domain spurned by $K>0.4$, the $T_d$-enhancement results in a decrease of $\Omega_r$, and vice-versa (Fig. 6a). Therefore, in the $K$-space defined by $K<0.4$ ($K>0.4$), the $T_d$-enhancement plays an accelerating (decelerating) role to the PMGC. From Fig. 6b, we see that the $T_d$-enhancement results in a decrease in $\Omega_r$ and vice-versa. Thus, we can infer that $T_d$ acts as a stabilizing agency to the pulsating cloud, thereby preventing the cloud from inward collapsing into local cloudlets. Physically, this happens due to the fact that $T_d$-enhancement exerts the thermal force (outward) in an anti-cloud-centric direction, thereby preventing it from collapsing against the gravity (inward).

Lastly, Fig. 7 shows the color spectral profile of the PMGC in a phase space defined by $\Omega_r$, $\Omega_i$, $K$. It spectrally demonstrates the instability evolution exactly characterized by the said color map. The various inputs used here are the same as Fig. 1, but with fixed $m_d = 2 \times 10^{-8} \text{ g}$. It clearly gives a composite picture of the PMGC evolution on a color mapping footing so as to illuminate the underlying physical insights.

5 Conclusions

A kinetic model formalism is proposed to study the PMGC dynamics in a complex unmagnetized self-gravitating partially ionized DMC. The DMC is initially assumed to be in a quasi-neutral hydrostatic homogeneous equilibrium configuration subject to no zeroth-order force field effects. The model is allowed to undergo a small-scale perturbation relative to the defined equilibrium. A systematic application of linear normal mode analysis transforms the slightly perturbed DMC into a generalized quintic dispersion relation involving a unique set of variable complex plasma multi-parametric coefficients. A constructive numerical analysis is systematically carried out to explore the microphysical insights of the PMGC fluctuation dynamics in the framework of realistic astrophysical parametric windows. This PMGC is indeed the most relevant normal mode developed due to the long-range gravito-electrostatic coupling mechanism. The stabilizing and destabilizing factors are illustratively discussed and compared with the literature for validation. The main conclusive remarks drawn from this kinetic investigation can be presented as follows:
1. This paper reports a kinetic model development to explore the PMGC stability dynamics for the first time in the fabric of linear normal mode analysis.
2. The reliability of our kinetic calculation scheme is ensured with the help of a matching fluid-acoustic dispersion relation comparison with the literature in the absence of any kind of wave-particle interactions in particular.
3. It is numerically and graphically seen that the dust mass, equilibrium electron, and ion densities play decelerating and destabilizing roles to the PMGC dynamics.
4. The dust-charge and the equilibrium dust density play accelerating and stabilizing roles to the evolution of the PMGC dynamics.
5. In the $K$-space defined by $K<0.4$ ($K>0.4$), the dust temperature plays an accelerating (decelerating) role to the PMGC; however, as a stabilizing agent for both the $K$-regimes.

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FIG. 1. Profile of the Jeans-normalized (a) real frequency part ($\Omega_r$), (b) its zoomed part ($\Omega_r$), and (c) imaginary frequency part ($\Omega_i$) in the space of the Jeans-normalized angular wavenumber ($K$) for different $m_d$ values as shown.
FIG. 2. Same as Fig. 1, but for different $Z_d$ values as shown.

FIG. 3. Same as Fig. 1, but for different $n_{d0}$ values as shown.
FIG. 4. Same as Fig. 1, but for different $n_{e0}$ values as shown.

FIG. 5. Same as Fig. 1, but for different $n_{i0}$ values as shown.
FIG. 6. Same as Fig. 1, but for different $T_d$ values as shown.

FIG. 7. Profile of the Jeans-normalized imaginary frequency part ($\Omega_i$) with the conjoint variation in the Jeans-normalized angular wavenumber ($K$) and Jeans-normalized real frequency part ($\Omega_r$).