Cosmological String Solutions in 4 Dimensions from 5d Black Holes

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Abstract

We obtain cosmological four dimensional solutions of the low energy effective string theory by reducing a five dimensional black hole, and black hole–de Sitter solution of Einstein gravity down to four dimensions. The appearance of a cosmological constant in the five dimensional Einstein–Hilbert action produces a special dilaton potential in the four dimensional effective string action. Cosmological scenarios implemented by our solutions are discussed.

To describe strings propagating in nontrivial background there are two different approaches. First, one can start with a conformal field theory, e.g. (gauged) WZW, Feigin-Fuchs or Liouville theory, and rewrite this as a string $\sigma$ model. Although the gauged WZW model is an exact conformal field theory it is possible to obtain the $\sigma$ model background only perturbatively. The reason is that in general the 2d gauge field can be eliminated only in an $\alpha'$ expansion. One example for which one got an exact result is the $SL(2,R)/U(1)$ WZW model \[4\]. In the second approach one starts with the low energy effective action and tries to find solutions in an $\alpha'$ expansion. In terms of both approaches some cosmological solutions have been obtained so far. Exact solutions, e.g., are given by the gauged WZW model based on $SL(2,R)/SO(1,1) \times R^2$ \[5\] or the combination of the $SU(2)$ WZW model with a Feigin-Fuchs part \[6\]. In the second or “phenomenological” approach cosmological solutions have been discussed in \[7\] and \[8\]. In all these solutions the metric is either time independent like the $SU(2)$ WZW in the string frame\[9\] or spatially flat or the dilaton is constant. In the present paper we use the second approach and find time dependent solutions (metric and dilaton) which are spatially not flat. We use the following procedure. We neglect all terms $O(\alpha'^2)$ and consider only curvature and dilaton terms. Then we will rewrite the effective string action in four dimensions (4d) as

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*This corresponds to a linear expanding universe in the Einstein frame (see \[3\])
a 5d Einstein-Hilbert action and look for solutions of this 5d theory. After reducing back to 4d we obtain a solution of the conformal invariance conditions (\(\beta\) equations) of the corresponding \(\sigma\) model.

So, restricting in the following on curvature and dilaton terms the 4d effective action in the lowest order in \(\alpha'\) is given by

\[
S = \frac{1}{2} \int d^4x \sqrt{|G|} e^{-2\phi} \left( R + 4(\partial\phi)^2 \right) ,
\]

where we assume that the central charge term is zero, i.e. there is also a decoupled conformal field theory describing the compactified dimensions. The action (1) is a special case (\(\omega = -1\)) of the more general Jordan-Brans-Dicke (JBD) theory,

\[
S = \frac{1}{2} \int \sqrt{|G|} (\eta R - \omega \eta^{-1}(\partial\eta)^2)
= \frac{1}{2} \int \sqrt{|G|} e^{-2\phi} (R - 4\omega(\partial\phi)^2)
\]

for \(\eta = e^{-2\phi}\). In this theory there are some observational restrictions coming from radar time delay and nucleosynthesis [8, 9]: \(|\omega|\) should be larger than a few hundreds which is certainly not the case for models motivated by string \(\sigma\) models (\(\omega = -1\)). A solution of this shortcoming could be provided by the incorporation of non-perturbative contributions like nontrivial dilaton potentials. However, the analogy between the string effective action and JBD theory is valid only at this level. If we couple further matter fields or if we consider higher order terms there is a crucial difference. In string theory matter fields couple directly to the dilaton field (via \(e^{-2\phi}\)), whereas in the JBD theory further matter fields are decoupled from the scalar field \(\eta\).

Let us now transform this 4d JBD action into a 5d Einstein-Hilbert (EH) action. First we define a five dimensional metric (not depending on the fifth coordinate),

\[
G_{\mu\nu} \rightarrow \tilde{G}_{ab} = \begin{pmatrix} e^{(\alpha+\beta)\phi} & \text{ } \\
\text{ } & e^{3\phi} G_{\mu\nu} \end{pmatrix},
\]

with

\[
\alpha = 2 \left( 1 \pm 3 \sqrt{1 + 2/3\omega} \right), \quad \beta = -2 \left( 1 \pm \sqrt{1 + 2/3\omega} \right),
\]

and Latin (Greek) indices are running from one to five (four). Using the five dimensional metric (3) and adding one dummy integration we get for the JBD action (2) the 5d EH action

\[
S \rightarrow \tilde{S} = \int d^5x \sqrt{|\tilde{G}|} \tilde{R} .
\]

A similar procedure for constructing charged BH solutions is described in [10]. Before we use this procedure for the construction of new solutions we want to discuss some general
features of cosmological solutions. In cosmology one starts with the spatially isotropic
and homogeneous ansatz (Robertson-Walker),

\[ ds^2 = -d\tau^2 + K^2(\tau) \left[ dr^2 + f^2(r) \left( \sin^2 \theta d\phi^2 + d\theta^2 \right) \right] = -d\tau^2 + K^2(\tau)d\Omega^2_{3,\epsilon}, \]

\[ f(r) = \begin{cases} 
\sin r & \text{for } \epsilon = +1 \quad \text{(elliptic)} \\
r & \text{for } \epsilon = 0 \quad \text{(flat)} \\
\sinh r & \text{for } \epsilon = -1 \quad \text{(hyperbolic)}
\end{cases} \tag{6} \]

The spatial part has a constant curvature and its geometry is determined by \( \epsilon \): flat (\( \epsilon = 0 \)),
elliptic (\( \epsilon = +1 \)) or hyperbolic (\( \epsilon = -1 \)). The flat and hyperbolic cases correspond to open
universes whereas the elliptic case corresponds to a closed universe. The whole dynamics
of this metric is contained in the world radius \( K(\tau) \) which has to be determined by the
field equations.

First we demonstrate how to get a cosmological solution for \( \epsilon = 1 \). In this case the
spatial part of the space time is a \( S^3 \) manifold with \( K(\tau) \) as the time dependent radius.
The most general 5d metric respecting the \( S^3 \) symmetry is given by a Schwarzschild metric
which can be written as

\[ ds^2 = e^{\nu(t)} dx^2 - e^{\lambda(t)} dt^2 + t^2 d\Omega^2_{3,\epsilon=1} \tag{7} \]

where \( x \) is our fifth coordinate which the theory should not depend on and \( t \) corresponds
to the time in the 4d theory. In comparison to the usual Schwarzschild metric our time
corresponds to the radius and \( x \) to the time, however, with opposite signs in front of
\( dx^2 \) and \( dt^2 \). Because we have no matter in the 5d theory a nontrivial vacuum solution
satisfying the desired \( S^3 \) symmetry is given by a 5d Black Hole

\[ e^{-\lambda} = e^{\nu} = -1 + \frac{2m}{t^2}, \tag{8} \]

where \( m \) is an integration constant. The properties of this metric are well known. There
is a singularity at \( t = 0 \) and for \( m > 0 \) we have a horizon at \( t^2 = 2m \).

The generalization of this solution for arbitrary \( \epsilon \) is given by \( ^b \)

\[ e^{-\lambda} = C e^\nu = -\epsilon + \frac{2m}{t^2} \tag{9} \]

and a horizon appears therefore for \( \frac{2m}{\epsilon} > 0 \ (\epsilon \neq 0) \). Let us now perform the reduction
to the 4d theory. In terms of (3) it is easy to obtain the dilaton field and the 4d metric

\[ \nu = (\alpha + \beta)\phi \]

\[ ds^2 = e^{-\beta\phi} \left( -e^{\lambda} dt^2 + t^2 d\Omega^2_3 \right) \]

\[ = -\left( e^{\lambda} \right)^{\frac{\alpha + 2\beta}{\alpha + \beta}} dt^2 + \left( e^{\lambda} \right)^{\frac{\beta}{\alpha + \beta}} t^2 d\Omega^2_{3,\epsilon} \tag{10} \]

\(^b\)In this case we have to replace \( \sin r \) in \( d\Omega^2_{3,1} \) by \( \frac{\sin(\sqrt{\epsilon}r)}{\sqrt{\epsilon}} \) in \( d\Omega^2_{3,\epsilon} \).
Because the exponents of $e^\lambda$ in (10) are in general not integers (cf. (4)) it is impossible to perform analytically the integration $e^{\frac{a+2\beta}{2}} dt^2 = dt^2$. At the end we will discuss some special cases and present some numerical results. Furthermore, in order to have a real metric in (10) we have the restriction that (4) has to remain positive, i.e. $\frac{2m}{\Lambda} > \epsilon$.

Before discussing the solution (10) in detail we consider a possible generalization of the 5d theory. The simplest extension is given by adding a cosmological constant, i.e.

$$\tilde{S} \rightarrow \tilde{S} = \int d^5x \sqrt{|\tilde{G}|} \left( \tilde{R} - \Lambda \right).$$

(11)

Again we look for a “static” BH solution and find for arbitrary $\epsilon$

$$e^{-\lambda} = C e^{\nu} = -\epsilon + \frac{2m}{t^2} + \frac{\Lambda}{12} t^2.$$  

(12)

For $\epsilon = 1$ this solution corresponds to the known 5d Schwarzschild - de Sitter metric [12] (after interpreting $x$ as time and $t$ as radius). The constant $C$ can be eliminated by a constant rescaling of $x$ or equivalently by a constant shift in the dilaton (cf. (10)), i.e. the constant part of the dilaton ($\phi \sim \phi_0 + \phi(t)$) is fixed by the $x$ scale. Another useful parameterization is given by

$$e^{-\lambda} = \frac{\Lambda}{12} \frac{(t^2 - t_+)(t^2 - t_-)}{t^2} \quad \text{with} \quad t_{\pm} = \frac{6\epsilon}{\Lambda} \left( 1 \pm \sqrt{1 - \frac{2m\Lambda}{3\epsilon^2}} \right).$$

(13)

where $t_{\pm}$ are the two horizons (BH and de Sitter) of the Schwarzschild - de Sitter metric. Both horizons coincide at the critical limit $3\epsilon^2 = 2m\Lambda$. In order to get a real 4d metric we have here (as for $\Lambda = 0$) the restriction that $e^\lambda > 0$. If we reduce the 5d action (11) in terms of (3) to the 4d theory we observe that the cosmological constant produces a dilaton potential in four dimensions

$$\tilde{S} = \frac{1}{2} \int d^5x \sqrt{|\tilde{G}|} \left( \tilde{R} - \Lambda \right) \quad \rightarrow \quad S = \frac{1}{2} \int d^4x \sqrt{|G|} e^{-2\phi} \left( R - 4\omega(\partial\phi)^2 - \Lambda e^{\beta\phi} \right).$$

(14)

The corresponding 4d metric and dilaton are again given by (10). It is easy to check that these background fields fulfill the corresponding $\bar{\beta}$ equations ($\omega = -1$)

$$R_{\mu\nu} + 2D_{\mu} \partial_{\nu} \phi + \Lambda \frac{\beta}{4} e^{\beta\phi} G_{\mu\nu} = 0$$

$$-D^2 \phi + 2(\partial\phi)^2 - \Lambda \frac{\beta + 2}{4} e^{\beta\phi} = 0$$

(15)

with $\beta = -2(1 \pm \sqrt{\frac{1}{3}})$. The structure of the dilaton potential is similar to the higher genus contributions. The main difference, however, is that in our case the power of $e^\phi$ is real whereas in the higher genus contribution it is an integer [13].

To get contact with standard cosmology let us now consider the solution (12) in the Einstein frame which is defined by the Weyl transformation

$$G^{(E)}_{\mu\nu} = e^{-2\phi} G_{\mu\nu}$$

(16)

It is especially easy if one used suitable computer programs like MATHEMATICA/MATHTENSOR.
and for the effective action \((\ref{eq:14})\) we find

\[
S = \int d^4x \sqrt{|G^{(E)}|} \left[ R^{(E)} + 2\omega(\partial\phi)^2 - \Lambda e^{(\beta+2)\phi} \right].
\] \hfill (17)

In this parameterization we have the standard Einstein-Hilbert term as gravitational part and the matter part is given by the dilaton terms. The corresponding energy–momentum tensor is

\[
T^\text{matter}_{\mu\nu} = 2 \left( \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 G^{(E)}_{\mu\nu} \right) - \frac{1}{2} \Lambda e^{(\beta+2)\phi} G^{(E)}_{\mu\nu}.
\] \hfill (18)

In this frame the 4d metric is given by

\[
ds^2_E = -e^{\lambda/2} dt^2 + t^2 e^{-\lambda/2} d\Omega_3^2.
\] \hfill (19)

In contrast to the string frame (10) the metric does not contain the root expressions (4).

In order to obtain statements about the evolution of the (4d) universe we have to bring the solution into the form (6), where the world radius \(K(t(\tau))\) is given by

\[
K^2 = t^2 (e^\lambda)^{\frac{\beta}{\alpha+\beta}} \quad \text{(in the string frame),}
\]

\[
K^2_{(E)} = t^2 e^{-\lambda/2} \quad \text{(in the Einstein frame),}
\] \hfill (20)

\[
e^{-\lambda} = -\epsilon + \frac{2m}{t^2} + \frac{\Lambda}{12} t^2
\]

and the function \(t(\tau)\) is a solution of the differential equation

\[
t^2(\tau) = (e^\lambda)^{\frac{\alpha+2\beta}{\alpha+\beta}} \quad \text{(in the string frame),}
\]

\[
t^2(\tau) = e^{-\lambda/2} \quad \text{(in the Einstein frame).}
\] \hfill (21)

If there are no horizons (in the 5d theory) it is useful to fix the integration constant of (21) via \(t(0) = 0\), i.e. the singularity (big bang) appears at \(\tau = 0\). In this case \(t\) runs from zero to infinity. The cases where there are horizons (defined by \(e^{-\lambda} = 0\)) we have to restrict ourselves to regions with \(e^{-\lambda} > 0\) and the horizons correspond either to the beginning or to the end of the universe. Extremal values of \(K\) possibly occur at \(\frac{dK}{d\tau} = 0\). Provided that \(\dot{t} \neq 0\) this is equivalent to

\[
-\epsilon + (1 - 2\delta) \frac{2m}{t^2} + \frac{\Lambda}{12} (1 + 2\delta) t^2 = 0,
\] \hfill (22)

where \(\delta = -\frac{\beta}{2(\alpha+\beta)} = \frac{1}{4} (\sqrt{3} \pm 1)\) in the string frame, and \(\delta = \frac{1}{4}\) in the Einstein frame. Since we were not able to solve (21) in general we are going to consider now the asymptotic behavior of the world radius.

1. \(t^2 \ll \min(2m, \frac{1}{|\Lambda|})\), \(m > 0\): We emphasize that this limit makes sense only if \(\alpha' \ll t^2\) is still valid since we have neglected higher order corrections in \(\alpha'\). The zeroth order asymptotic behavior of \(K\) is given by

\[
K^2(\tau) \sim \tau^{\frac{3}{\alpha+\beta}}, \quad K^2_{(E)}(\tau) \sim \tau^{\frac{1}{2}}
\]

\[
e^{2\phi} \sim \tau^{-(1+\sqrt{3})}, \quad e^{2\phi_{(E)}} \sim \tau^{\frac{3}{\alpha+\beta}}.
\] \hfill (23)
Although the dilaton is not transformed if we go to the Einstein frame we have to perform different time redefinitions in both frames. For $\epsilon = \Lambda = 0$ (23) is an exact expression. This is just the solution which has already been given by Mueller in [7]. Another interesting asymptotic region is

2. $t \rightarrow \infty (\Lambda \geq 0)$: In this limit we find the following behavior

$$K^2(\tau) \sim \begin{cases} \tau^{\pm 2\sqrt{3}} & \Lambda \neq 0 \\ \tau^2 & \Lambda = 0 \end{cases}, \quad K^2_E(\tau) \sim \begin{cases} \tau^6 & \Lambda \neq 0 \\ \tau^2 & \Lambda = 0 \end{cases}$$

$$e^{2\phi} \sim \begin{cases} \tau^{3 \pm \sqrt{3}} & \Lambda \neq 0 \\ 1 & \Lambda = 0 \end{cases}, \quad e^{2\phi_E} \sim \begin{cases} \tau^{\pm 2\sqrt{3}} & \Lambda \neq 0 \\ 1 & \Lambda = 0 \end{cases} \tag{24}$$

For vanishing potential ($\Lambda = 0$) we have to restrict ourselves on $\epsilon = -1 (e^\lambda > 0)$. In this case we get the remarkable consequence that in the asymptotic limit our solution is in both frames a flat space time ($K = \tau$) with constant dilaton.

If there are horizons in the five dimensional theory we have to consider also the asymptotics near those horizons.

3. $t \rightarrow \text{“horizon”} (\frac{2m}{\epsilon} > 0, \Lambda = 0)$: In the previous cases we admitted $t$ to run from zero to infinity. However, if there are horizons in the five dimensional theory we have to care that our solution is real ($e^\lambda > 0$). First we will consider the case $\epsilon = -1$. The metric is real if $t \in (\sqrt{\frac{2m}{\epsilon}}, \infty)$. We fix the integration constant of (21) via

$$t(0) = \sqrt{\frac{2m}{\epsilon}}, \tag{25}$$

which ensures that $\tau$ runs from zero to infinity. With (25) we obtain the behavior

$$K^2(\tau) \sim \tau^{\pm \frac{2}{\sqrt{3}}}, \quad K^2_E(\tau) \sim \tau^{\frac{2}{3}}$$

$$e^{2\phi} \sim \tau^{-(1 \pm \sqrt{3})}, \quad e^{2\phi_E} \sim \tau^{\pm \frac{2 \sqrt{3}}{3}}. \tag{26}$$

In the other case ($\epsilon = +1$) the metric is real if $t \in (0, \sqrt{\frac{2m}{\epsilon}})$, i.e. the lifetime of the universe is finite. In that case the behavior at zero is given by (23) and near the horizon by

$$K^2(\tau) \sim (\tau_0 - \tau)^{\pm \frac{2}{\sqrt{3}}}, \quad K^2_E(\tau) \sim (\tau_0 - \tau)^{\frac{2}{3}}$$

$$e^{2\phi} \sim (\tau_0 - \tau)^{-(1 \pm \sqrt{3})}, \quad e^{2\phi_E} \sim (\tau_0 - \tau)^{\pm \frac{2 \sqrt{3}}{3}}, \tag{27}$$

where $\tau \rightarrow \tau_0$ at the “horizon”.

We will not discuss every possible parameter constellation in detail but collect the solutions in table [ ] for the string frame. With the given data the same can be done for the Einstein frame very easily.

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*Here the restriction on $\Lambda = 0$ is done in order to simplify the discussion, a consideration of $\Lambda \neq 0$ is also possible (cf. table [ ]).*
| $\epsilon$ | $m$ | $\Lambda$ | “horizon” | extrema possible for | $t \to 0$ | $t \to \text{“horizon”}$ | $t \to \infty$ |
|---|---|---|---|---|---|---|---|
| 0 | $> 0$ | 0 | no | – | $\tau^+ \frac{\sqrt{3}}{2}$ | – | $\tau^+ \frac{\sqrt{3}}{2}$ |
| 0 | 0 | $> 0$ | no | – | $\tau^\pm \sqrt{3}$ | – | $\tau^\pm \sqrt{3}$ |
| 0 | $> 0$ | $> 0$ | no upper sign | $\tau^+ \frac{\sqrt{3}}{2}$ | – | $\tau^+ \frac{\sqrt{3}}{2}$ | $\tau^+ \sqrt{3}$ |
| 0 | $< 0$ | $> 0$ | $t = (-\frac{24m}{\Lambda})^\frac{1}{4}$ | – | – | $\tau^\pm \frac{\sqrt{3}}{2}$ | $\tau^+ \sqrt{3}$ |
| 0 | $> 0$ | $< 0$ | $t = (-\frac{24m}{\Lambda})^\frac{1}{4}$ lower sign | $\tau^+ \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | – |
| $-1$ | $> 0$ | 0 | no upper sign | $\tau^+ \frac{\sqrt{3}}{2}$ | – | $\tau$ | – |
| $-1$ | 0 | $> 0$ | no | – | $\tau$ | – | $\tau^{+ \sqrt{3}}$ |
| $-1$ | $> 0$ | $> 0$ | no upper sign | $\tau^+ \frac{\sqrt{3}}{2}$ | – | $\tau^+ \sqrt{3}$ | $\tau^+ \sqrt{3}$ |
| $-1$ | $> 0$ | $< 0$ | $t^2 = t_+$ both sign | $\tau^+ \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | – |
| $-1$ | $< 0$ | $> 0$ | $t^2 = t_-$ | – | – | $\tau^\pm \frac{\sqrt{3}}{2}$ | $\tau^\pm \sqrt{3}$ |
| $* - 1$ | $< 0$ | $< 0$ | $t^2 = t_-$ upper sign | – | $\tau^+ \frac{\sqrt{3}}{2}$ | – | – |
| $* - 1$ | $< 0$ | $< 0$ | $t^2 = t_+$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | $(\tau_0 - \tau)^{\pm \sqrt{3}}$ | – |
| $-1$ | 0 | $< 0$ | $t^2 = -\frac{12}{\Lambda}$ upper signs | $\tau$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | – |
| 1 | 0 | $> 0$ | $t^2 = 2m$ | – | $\tau^+ \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – |
| $* 1$ | $> 0$ | $> 0$ | $t^2 = t_+$ lower sign | $\tau^+ \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | – |
| $* 1$ | $> 0$ | $> 0$ | $t^2 = t_+$ upper sign | – | $\tau^\pm \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \sqrt{3}}$ | – |
| 1 | $> 0$ | $< 0$ | $t^2 = t_-$ lower sign | $\tau^+ \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \frac{\sqrt{3}}{2}}$ | – | – |
| 1 | $< 0$ | $> 0$ | $t^2 = t_+$ upper sign | – | $\tau^\pm \frac{\sqrt{3}}{2}$ | $(\tau_0 - \tau)^{\pm \sqrt{3}}$ | – |

Table 1: Collection of the solutions in the string frame. ($*m\Lambda < \frac{3}{2}$)

Finally we give the result of numerical calculations done in some special cases. Figure 1 is an example for a closed universe ($\epsilon = 1$) with finite lifetime and vanishing dilaton potential. The expansion starts at $\tau = 0$, reaches the maximum at $t^2(\tau) = m$, (cf. (22)) and shrinks to zero size at the “horizon” $t^2(\tau) = 2m$. The corresponding “lifetime” of the universe is according to (24) given by

$$\tau = \int_0^{\sqrt{2m}} \left( \frac{t^2}{2m - t^2} \right)^\frac{1}{2} dt = \sqrt{\frac{2m}{\pi}} \Gamma\left(\frac{q + 1}{2}\right) \Gamma\left(\frac{1 - q}{2}\right)$$

(28)

where: $q = \frac{1}{2}(1 \mp \sqrt{3})$ in the string frame and $q = \frac{1}{2}$ in the Einstein frame. Figure 2 shows an open (hyperbolic) universe expanding forever with decreasing velocity. The asymptotic limit is a flat space time with a constant dilaton. In figure 3 the “big bang” appears as an implosion at $t = 0$, the universe shrinks until it reaches it’s minimum at $t^4(\tau) = \frac{24m\delta - 1}{\Lambda 2\delta + 1}$
Figure 1: $K(\tau)$ in the Einstein frame for $\epsilon = 1, m = 50, \Lambda = 0$

Figure 2: $K(\tau)$ in the string frame for $\epsilon = -1, m = 50, \Lambda = 0$

Figure 3: $K(\tau)$ in the string frame for $\epsilon = 0, m = 50, \Lambda = 1/m$

(cf. (22)) and then expands with increasing velocity. This is an example for an open universe which is spatially flat ($\epsilon = 0$). The asymptotic behavior is given by (24) and we see that the dilaton potential ($\Lambda \neq 0$) accelerates the expansion in both frames.

To summarize, in the present paper we have obtained various cosmological solutions of the low energy effective action of string theory. Although the method presented is very simple (reduction of a 5d Black Hole solution) our solution has as far as we know not been obtained before. The reason is that we did not use the standard parameterization of the Robertson-Walker metric (6) and it seems to be impossible to solve (21) analytically. However, in most cases it is possible to get some impression about the features of our solutions. We have investigated the asymptotic behavior of the world radius near zero, near the horizons of the corresponding 5d Black Hole, and in the infinite future. For vanishing dilaton potential and $\epsilon = -1$ we obtained a flat universe in the large time limit. In the case where we were able to get an analytic expression ($\Lambda = \epsilon = 0$) in the Robertson-Walker parameterization (8) our result coincides with Mueller’s solution [7]. Finally we should mention that the cosmological scenarios implemented by our solutions have not to be taken too seriously since one can not expect to get quantitative exact
results in such a simple model (pure dilaton, graviton system). Therefore it would be very useful to take into account additional matter. Unfortunately the incorporation of matter is not straightforward because this simple reduction procedure works only for the pure dilaton graviton system. On the other hand it should be possible by duality and/or O(d,d) transformations to construct new solutions in the 5d theory and perhaps it is possible to interpret the corresponding 4d fields in the context of string theory.

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