Mass Terms in Two-Higgs Doublet Models

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Abstract

We take a closer look at the mass terms of all renormalizable and CP conserving two-Higgs doublet models (THDM). We show how some of the dimension two parameters in the potential can be set equal to zero leading to relations among the tree-level parameters of the potential. The different versions of the THDM obtained give rise to different amplitudes for physical processes. We will illustrate this with two examples. The first one is the one-loop weak correction to the top decay width, $t \to bW$. The second one is the decay $h \to \gamma\gamma$ in the fermiophobic limit.

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1 Introduction

The generation of particle masses in the standard model (SM) is accomplished through the well known Higgs mechanism. In its minimal version, i.e., with one scalar doublet, there is only one free parameter in the scalar sector, the Higgs mass, $M_H$. After spontaneous symmetry breaking the two parameters in the scalar potential are replaced by the vacuum expectation value $v = 247 \text{ GeV}$ and by $M_H$, which remains unconstrained. Introducing another scalar doublet of complex fields makes the number of free parameters in the potential grow from two to fourteen. The number of scalar particles grows from one to four. If explicit CP violation is not allowed, the number of free parameters is reduced to ten. This number can be further reduced to seven by imposing either a global $U(1)$ symmetry or a $Z_2$ symmetry. We call the resulting potentials, $V_A$ and $V_B$ respectively. Some of the vertices in the scalar sectors of $V_A$ and $V_B$ are different and this gives rise to differences in the amplitudes for physical processes. This fact has often been overlooked in the literature.

Adding or subtracting mass terms to either of these potentials does not spoil the renormalizability of the model. In the first case it is well known that a softly broken theory remains renormalizable (see for instance [1]). In the second case the renormalization will introduce a counterterm with the same structure of the suppressed term. Again these are mass terms which are simply absorbed in the mass renormalization of the four scalar bosons. Hence, we have analyzed all possible CP-conserving and renormalizable models obtained by addition or subtraction of all available mass terms to $V_A$ and $V_B$. In some cases the resulting potentials have only six or even five free parameters. In paragraph II we discuss the potential. Then, to illustrate that they lead to different physical results we calculate in paragraph III the one-loop correction to the top quark decay $t \to bW$ and in paragraph IV the decay $h \to \gamma \gamma$ in the fermiophobic limit.

2 The potentials

To define our notation, we start with a brief review of the two-Higgs doublet models (THDM). Let $\Phi_i$ with $i = 1,2$ denote two complex scalar doublets with hyper-charge 1. Under $C$ the fields transform as $\Phi_i \to e^{i\alpha_i} \Phi_i^*$ where the parameters $\alpha_i$ are arbitrary. Defining the complete set of invariants $x_1 = \phi_1^\dagger \phi_1$, $x_2 = \phi_2^\dagger \phi_2$, $x_3 = \Re\{\phi_1^\dagger \phi_2\}$ and $x_4 = \Im\{\phi_1^\dagger \phi_2\}$ and choosing $\alpha_1 = \alpha_2 = 0$ we can write the most general $SU(2) \otimes U(1)$ invariant, $C$ invariant and renormalizable potential in the form

$$V = -\mu_1^2 x_1 - \mu_2^2 x_2 - \mu_3^2 x_3 + \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 + \lambda_4 x_4^2 + \lambda_5 x_1 x_2 + \lambda_6 x_1 x_3 + \lambda_7 x_2 x_3 . \quad (1)$$

In a previous paper [2] we have studied the different types of extrema for potential $V$. We have shown in [2] that there are two natural ways of imposing that a minimum with $CP$ violation never occurs, i.e., that the minimum of the potential is of the form

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} , \quad (2)$$

with $v_i$ real. The first one, denoted $V_A$, is obtained by setting $\mu_2^2 = \lambda_6 = \lambda_7 = 0$ in equation (1). This is the potential in Ref. [3]. It is invariant under the $Z_2$ transformation $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to -\Phi_2$. The second 7-parameter potential, denoted by $V_B$, is the potential obtained in the Minimal Supersymmetric Model (MSSM) and it corresponds to the conditions $\lambda_6 = \lambda_7 = 0$ and $\lambda_3 = \lambda_4$. Potential $V_B$ is invariant under a global $U(1)$ symmetry $\Phi_2 \to e^{i\alpha} \Phi_2$ except for the soft breaking term proportional to $\mu_2^2$.

Let us examine how the theory behaves by adding or subtracting all possible mass terms to the potential. We start by writing the potential in the form

$$V_{AB} = -\mu_1^2 x_1 - \mu_2^2 x_2 - \mu_3^2 x_3 - \mu_4^2 x_4 + \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 + \lambda_4 x_4^2 + \lambda_5 x_1 x_2 , \quad (3)$$
which allow us to study both potentials $V_A$ and $V_B$ at the same time. The minimum conditions are

$$
T_1 = v_1 \left[ -\mu_1^2 + \lambda_1 v_1^2 + \lambda_+ v_2^2 \right] - \frac{1}{2} \mu_3^2 v_2 = 0 \quad (4a)
$$
$$
T_2 = v_2 \left[ -\mu_2^2 + \lambda_2 v_2^2 + \lambda_+ v_1^2 \right] - \frac{1}{2} \mu_3^2 v_1 = 0 \quad (4b)
$$
$$
T_3 = \mu_1^2 v_2 = 0 \quad (4c)
$$
$$
T_4 = -\mu_2^2 v_1 = 0 \quad (4d)
$$

with $\lambda_+ = (\lambda_2 + \lambda_3)/2$. Obviously, conditions (4c,d) force $\mu_i^2 = 0$ because the vacuum configuration chosen in Eq. (2) implies a stable, CP-conserving minima. Since the $\mu_i^2$ term allows mixing between all neutral fields, its existence would only be possible in a theory with spontaneous CP violation. Each complex doublet $\phi_i$ can be written as

$$
\phi_i = \left[ \begin{array}{c} a_i^+ \\ (v_i + b_i + ic_i)/\sqrt{2} \end{array} \right] \quad (5)
$$

where $a_i^+$ are complex fields, and $b_i$ and $c_i$ are real fields. This, in turn, enables us to write the mass terms of potential $V_{AB}$ as:

$$
V_{AB}^{\text{mass}} = \left[ \begin{array}{c} a_1^+ \ a_2^+ \end{array} \right] M_a \left[ \begin{array}{c} a_1^- \\ a_2^- \end{array} \right] + \frac{1}{2} \left[ \begin{array}{cc} c_1 & c_2 \end{array} \right] M_c \left[ \begin{array}{c} c_1 \\ c_2 \end{array} \right] 
$$

$$
+ \frac{1}{2} \left[ \begin{array}{cc} b_1 & b_2 \end{array} \right] M_b \left[ \begin{array}{c} b_1 \\ b_2 \end{array} \right],
$$

with the matrices $M_a$, $M_b$ and $M_c$ defined as

$$
M_a = \frac{1}{2} \left[ \begin{array}{cc} -\mu_1^2 - v_1^2 \lambda_3 & -\mu_3^2 + v_1 v_2 \lambda_3 \\ -\mu_3^2 + v_1 v_2 \lambda_3 & -\mu_3^2 \end{array} \right] \quad (6a)
$$

$$
M_b = \left[ \begin{array}{cc} 2v_2^2 \lambda_4 + \frac{v_1^2}{2v_2} \mu_3^2 & v_1 v_2 (\lambda_4 + \lambda_5) - \frac{v_1^2}{2} \\ v_1 v_2 (\lambda_4 + \lambda_5) - \frac{v_1^2}{2} & 2v_2^2 \lambda_5 + \frac{v_1^2}{2} \mu_3^2 \end{array} \right] \quad (6b)
$$

$$
M_c = \frac{1}{2} \left[ \begin{array}{cc} v_2^2 (\lambda_4 - \lambda_3) + \frac{v_1^2}{v_2} \mu_3^2 & -v_1 v_2 (\lambda_4 - \lambda_3) - \mu_3^2 \\ -v_1 v_2 (\lambda_4 - \lambda_3) - \mu_3^2 & v_2^2 (\lambda_4 - \lambda_3) + \frac{v_1^2}{v_2} \mu_3^2 \end{array} \right]. \quad (6c)
$$

Diagonalizing the quadratic terms of $V_{AB}$ one obtains the mass eigenstates: 2 neutral CP-even scalar particles, $H$ and $h$, a neutral CP-odd scalar particle, $A$, the would-be Goldstone boson partner of the $Z$, $G_0$, a charged Higgs field, $H^+$ and the Goldstone associated with the $W$ boson, $G^+$. The relations between the mass eigenstates and the SU(2)$\otimes$U(1) eigenstates are:

$$
\left[ \begin{array}{c} H \\ h \end{array} \right] = R_\alpha \left[ \begin{array}{c} b_1 \\ b_2 \end{array} \right] \quad \left[ \begin{array}{c} H^+ \\ G^+ \end{array} \right] = R_\beta \left[ \begin{array}{c} a_1^+ \\ a_2^+ \end{array} \right] \quad \left[ \begin{array}{c} A \\ G_0 \end{array} \right] = R_\beta \left[ \begin{array}{c} c_1 \\ c_2 \end{array} \right] \quad (7a)
$$

with

$$
R_\alpha = \left[ \begin{array}{cc} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{array} \right] \quad R_\beta = \left[ \begin{array}{cc} -\sin \beta & \cos \beta \\ \cos \beta & \sin \beta \end{array} \right], \quad (7b)
$$

and $0 < \beta < \frac{\pi}{2}$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$. The particle masses and the angles are given by

$$
M_{H^+}^2 = \frac{1}{2} v^2 \left[ \lambda_3 - \frac{\mu_3^2}{v_1 v_2} \right] \quad (8a)
$$

$$
M_A^2 = \frac{1}{2} \mu_3^2 \frac{v_1^2}{v_1 v_2} + \frac{1}{2} (\lambda_4 - \lambda_3) v^2 \quad (8b)
$$
We will now consider the different possibilities regarding the mass terms of the potential. Ob-

The particular case was studied by Coleman and Weinberg [4] for the SM and by Inoue

we end up with a renormalizable model where all scalars have a definite mass which is related to

THDM [5]. In this model the scalars acquire a mass as a result of radiative corrections. Hence,

are given in terms of the physical masses and the angles

extra parameter can change the tree-level unitarity bounds obtained for

to this parameter. In fact, now, the potential has eight free parameters. The inclusion of this

show that there are additional contributions to the cubic and quartic scalar vertices proportional

with two zero mass terms have the same number of particles, the same number of rotation angles

When just one mass term is set equal to zero, we obtain the potential

A = \pm \sqrt{\left(\lambda_1 v_1^2 - \lambda_2 v_2^2 + \frac{1}{4} \mu_3^2 \left(\frac{v_2}{v_1} - \frac{v_1}{v_2}\right)\right)^2 + \left(v_1 v_2 (\lambda_3 + \lambda_5) - \frac{1}{2} \mu_3^2\right)^2} \hspace{1cm} (8d)

\tan 2\alpha = \frac{2 v_1 v_2 \lambda_+ - \frac{1}{4} \mu_3^2}{\lambda_1 v_1^2 - \lambda_2 v_2^2 + \frac{1}{4} \mu_3^2 \left(\frac{v_2}{v_1} - \frac{v_1}{v_2}\right)} \hspace{1cm} (8e)

\tan \beta = \frac{v_2}{v_1} \hspace{1cm} (8f)

To derive the cubic and quartic vertices stemming from the potential it is convenient to rewrite

the \lambda_i in terms of the masses and angles. The results are:

\begin{align}
\lambda_1 &= \frac{1}{2 v^2 \cos^2 \beta} \left[ M_H^2 \cos^2 \alpha + M_h^2 \sin^2 \alpha - \frac{\mu_3^2}{2} \tan \beta \right] \hspace{1cm} (9a) \\
\lambda_2 &= \frac{1}{2 v^2 \sin^2 \beta} \left[ M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha - \frac{\mu_3^2}{2} \tan^{-1} \beta \right] \hspace{1cm} (9b) \\
\lambda_3 &= -\frac{2}{v^2} \left[ M_A^2 + \frac{\mu_3^2}{\sin(2\beta)} \right] \hspace{1cm} (9c) \\
\lambda_4 &= \frac{2}{v^2} \left[ M_A^2 - M_H^2 \right] \hspace{1cm} (9d) \\
\lambda_5 &= \frac{\sin(2\alpha)}{v^2 \sin(2\beta)} \left[ M_H^2 - M_h^2 \right] + \frac{2}{v^2} M_H^2 - \frac{\mu_3^2}{v^2 \sin(2\beta)} \hspace{1cm} (9e)
\end{align}

2.1 Potential \( V_A \)

Setting \( \mu_3^2 = 0 \) we obtain potential \( V_A \) with the extra soft breaking term \(-\mu_3^2 x_3\), i.e.,

\begin{align}
V_A &= -\mu_1^2 x_1 - \mu_2^2 x_2 - \mu_3^2 x_3 + \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 + \lambda_4 x_4^2 + \lambda_5 x_1 x_2 \hspace{1cm} (10)
\end{align}

We will now consider the different possibilities regarding the mass terms of the potential. Obviously, the conditions \( \mu_1^2 = \mu_2^2 = \mu_3^2 = 0 \) would lead to a potential with no mass scale. This particular case was studied by Coleman and Weinberg [4] for the SM and by Inoue et al for the THDM [5]. In this model the scalars acquire a mass as a result of radiative corrections. Hence, we end up with a renormalizable model where all scalars have a definite mass which is related to the gauge bosons masses. If \( \mu_3^2 = 0 \) one obtains a seven parameter potential where all vertices are given in terms of the physical masses and the angles \( \alpha \) and \( \beta \). However, if \( \mu_3^2 \neq 0 \), Eqs. (9) show that there are additional contributions to the cubic and quartic scalar vertices proportional to this parameter. In fact, now, the potential has eight free parameters. The inclusion of this extra parameter can change the tree-level unitarity bounds obtained for \( V_A \) [6].

When just one mass term is set equal to zero, we obtain the potential \( V_A \) described in [7]. Hence, the most interesting cases are the ones where just one mass terms is different from zero. Potentials with two zero mass terms have the same number of particles, the same number of rotation angles and the same number of vertices than the original potential. However, at tree-level, the seven parameters in the potential are no longer free. In fact, the number of free parameters is reduced to six by one of the following relations:

i) \( \mu_1^2 = \mu_2^2 = 0 \)

\begin{align}
\tan \beta &= -2 \frac{M_H^2 \cos^2 \alpha + M_h^2 \sin^2 \alpha}{\sin(2\alpha)(M_H^2 - M_h^2)} \hspace{1cm} (11a)
\end{align}
ii) $\mu_2^2 = \mu_3^2 = 0$

$$\tan \beta = -\frac{1}{2} \sin(2\alpha) \frac{M_H^2 - M_h^2}{M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha} , \quad (11b)$$

iii) $\mu_1^2 = \mu_2^2 = 0$

$$\tan^2 \beta = \frac{M_H^2 \cos^2 \alpha + M_h^2 \sin^2 \alpha}{M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha} . \quad (11c)$$

As we have already pointed out, these relations are not valid beyond tree-level. In fact, the renormalization scheme forces us to treat the angles and the masses as independent parameters.

We denote these models by $V_A^i$, $V_A^{ii}$ and $V_A^{iii}$. In $V_A^i$, $V_A^{ii}$ one can readily see that $\sin(2\alpha) < 0$, that is, the rotation angle $\alpha$ is now bounded to be in the interval $-\frac{\pi}{4} < \alpha < 0$. This, in turn, implies that all Yukawa couplings have a definite sign. In $V_A^{iii}$ $\tan \beta$ is in the range

$$\frac{M_h}{M_H} < \tan \beta < \frac{M_H}{M_h} , \quad (12)$$

which means that values of $\tan \beta$ close to zero or infinity are not allowed.

In the limit $M_H >> M_h$ all equations (11,a,b,c) are reduced to

$$\tan \beta \approx -\frac{1}{\tan \alpha} . \quad (13)$$

This condition implies that all couplings between the fermions and the lightest Higgs, $h$, are the SM ones. The remaining Yukawa couplings will be proportional to either $\tan \beta$ or $\cot \beta$. The couplings between the scalars and the gauge bosons are such that $\cos(\alpha - \beta) \approx 0$ and $\sin(\alpha - \beta) \approx -1$. Hence, in this limit the lightest Higgs, $h$, resembles very much the SM Higgs boson. It can only be distinguished from the SM Higgs through the couplings to the other scalars. On the other hand the heavier CP-even Higgs, $H$, couples very weakly to the gauge bosons- it is almost bosophobic.

In the limit where the CP-even Higgs are almost degenerated ($M_H \approx M_h$) the values of $\tan \beta$ can be quite different in the three models. In $V_A^i$ $\tan \beta$ tends to be very large while in $V_A^{ii}$ it is close to zero. Finally, in $V_A^{iii}$ $\tan \beta \approx 1$.

2.2 Potential $V_B$

Setting $\lambda_3 = \lambda_4$ and $\mu_3^2 \neq 0$ we obtain potential $V_B$, i.e.,

$$V_B = -\mu_1^2 x_1 - \mu_2^2 x_2 - \mu_3^2 x_3 + \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 [x_3^2 + x_4^2] + \lambda_5 x_1 x_2 , \quad (14)$$

Unlike $V_A$, when all mass terms are different from zero there is no extra parameter. Again, if all mass terms are zero, masses are generated through radiative corrections as in potential $A$. If $\mu_3^2 = 0$ and $\mu_1^2, \mu_2^2 \neq 0$ the CP-odd scalar becomes massless at tree-level.

Forcing $\mu_3^2 \neq 0$ we obtain the following relation:

i) $\mu_1^2 = 0$

$$M_A^2 = \frac{1}{2 \sin^2 \beta} \left[ M_H^2 \cos^2 \alpha + M_h^2 \sin^2 \alpha + \frac{1}{2} \sin(2\alpha) \tan \beta (M_H^2 - M_h^2) \right] , \quad (15a)$$

ii) $\mu_2^2 = 0$

$$M_A^2 = \frac{1}{2 \cos^2 \beta} \left[ M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha + \frac{\sin(2\alpha)}{2 \tan \beta} (M_H^2 - M_h^2) \right] , \quad (15b)$$
iii) $\mu_1^2 = \mu_2^2 = 0$

\[ \tan^2 \beta = \frac{M_H^2 \cos^2 \alpha + M_h^2 \sin^2 \alpha}{M_H^2 \sin^2 \alpha + M_h^2 \cos^2 \alpha}, \quad (15c) \]

and either Eq. (15a) or (15b). In this last case, at tree-level, the number of free parameters in the potential is reduced from seven to five.

We denote these models by $V_{B}^{i}$, $V_{B}^{ii}$ and $V_{B}^{iii}$. Similarly to the previous case, Eqs. (15a) and (15b) could also imply a restriction on the allowed values of $\alpha$. In fact, since $M_A > 0$, depending on the values of $M_H$, $M_h$ and $\tan \beta$, some values of $\alpha$ can be excluded. For the sake of simplicity, we have chosen $M_A$ as the dependent variable in these two relations. If a process does not depend heavily on the value of $M_A$, it is likely that $V_{B}^{i}$ and $V_{B}^{ii}$ give the same result as the original $V_B$.

Let us now look at $V_{B}^{iii}$. Equation (12) obtained for $V_A$ still holds in this case. In the limit $M_H >> M_h$ we have

\[ \tan \beta \approx \frac{1}{\tan \alpha} ; \quad M_A \approx M_H \quad (16) \]

and $0 < \alpha < \pi/2$. All conclusions on the behaviour of the lightest Higgs boson for $V_A$ also apply in this case. On the other hand, if $M_H \approx M_h$, then

\[ \tan \beta \approx 1 ; \quad M_A \approx M_H \approx M_h \quad (17) \]

and $-\pi/2 < \alpha < \pi/2$.

### 3 The Decay $t \rightarrow b W^+$

The electroweak corrections to the decay $t \rightarrow b W^+$ of order $\alpha^2$ were evaluated in Refs. [8] and [9]. However, the renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix was done in such a way that the final result was gauge dependent [10]. Recently, using a renormalization prescription introduced before [11], we have evaluated this decay width [12] in the SM. A similar calculation in the THDM was done by Denner and Hoang [13], but again it suffers from the same gauge dependence anomaly. Despite the fact that the contributions of the $\delta V_{tb}$ counterterm is small we repeat here the calculation [14] using the renormalization prescription proposed in Refs. [7] and [11]. Skipping the details of the calculation that can be found in Ref. [12], we present the results in terms of the parameter

\[ \delta = 2 \frac{\Re[T_0 T_1^+]}{|T_0|^2}, \quad (18) \]

where $T_0$ and $T_1$ are the tree-level and one-loop amplitudes respectively. Hence, including one-loop corrections, the decay width is

\[ \Gamma_1 = \Gamma_0 [1 + \delta], \quad (19) \]

where $\Gamma_0$ is the tree-level decay width.

The parametric space of the THDM is very large. Hence, it is not possible to show the results for a systematic scan of this space. It is more interesting to present the qualitative trend and to illustrate the differences that occur for the different versions of the model. In general $\delta$ is larger than in the SM by a factor of order two or three. In Fig. 1 we show $\delta$ as a function of $\alpha$ for the models of type A, using $M_h = 100 \, GeV$ and $M_H = 600 \, GeV$. In this case, the three models $V_{A}^{i,ii,iii}$ give almost the same result. This is a simple consequence that they are not far from the limit given by Eq. (13). On the same figure we also plot the value of $\delta$ calculated with $V_A$ for two values of $\beta$. Now, it is possible to obtain results that are almost one order of magnitude larger than in the SM ($\delta_{SM} = 4.46\%$ [12]). Furthermore, this is possible for values of masses and angles that do not give a large contribution to $\rho = M_Z^2 / (M_Z^2 \cos^2 \theta_W)$ due to the Higgs bosons. We only consider regions of the parameter space which give $|\Delta \rho| < 2 \times 10^{-3}$.

In Fig. 2 we plot $\delta$ for type B models. In most cases values of $\delta$ of the order of 10% are obtained, but one does not need to go to an extreme corner of the parametric space to obtain even
larger values. The curves for models $V_A^{i,ii,iii}$ are not plotted in the entire range of $\alpha$. This is due to two reasons. Some values of $\alpha$ are excluded by Eqs. (15) while others are excluded by our $\Delta\rho$ criteria.

One should notice that for the top decay there are no contributions due to the cubic scalar vertices. This means that, for this process, $V_A$ and $V_B$ give exactly the same results. Hence, the differences illustrated in Figs. 1 and 2 are entirely due to the restrictions induced by Eqs. (11) and (15).

4 The fermiophobic limit

In the so-called fermiophobic limit [15, 16], the lightest Higgs decouples from the fermions at tree level. This is accomplished in the THDM by setting $\alpha = \pi/2$ in model I, as defined in Ref. [7]. In Ref. [17] we have studied all possible decays of a fermiophobic Higgs. The dominant decay, for the LEP energies is $h \rightarrow \gamma \gamma$ [18]. We have showed before [15, 16] that this decay width is different in models $A$ and $B$ due to a difference in the $hH^+H^-$ vertex. Similarly the same is true for some of the models derived from $V_A$ and $V_B$. Models $V_A^{i}$ and $V_A^{ii}$ do not allow a fermiophobic Higgs. In $V_A^{i}$ $\alpha = \pi/2$ leads to $M_h = 0$ and in $V_A^{ii}$ it forces $M_H = 0$. In $V_A^{iii}$ equation (11c) is reduced to

$$\tan \beta = \frac{M_h}{M_H}.$$ (20)

Then

$$\frac{1}{2} < \sin^2(\alpha - \beta) < 1$$ (21)

where the value $\sin^2(\alpha - \beta) \approx 1$ is obtained when $M_h << M_H$.

A detailed study of this process using either $V_A$ or $V_B$ was done before [16]. So there is no need to repeat it here. What remains to be done is to discuss the changes induced by the more
Figure 2: δ as a function of α for models $V_B$ and $V_B^{i,ii,iii}$. In model $V_B^{iii}$ β is given by Eq. (15c)

constrained potentials $V_A^{i,ii,iii}$ and $V_B^{i,ii,iii}$. To do this we denote by $R_A$ the ratio of the $h \to \gamma\gamma$ widths calculated in models $V_A^{iii}$ and in model $V_A$. For small values of β $R_A$ is of the order one. However, for larger β, β = π/3 for instance, $R_A \approx 4$. This could be relevant when analysing the experimental data to exclude some mass regions.

In potential B a fermiophobic Higgs is always allowed. In this limit Eqs. (15) can be written as

i) $V_B^i$

$$\tan^2 \beta = \frac{M_h^2}{2M_A^2 - M_h^2},$$  \hspace{1cm} (22a)

ii) $V_B^{ii}$

$$\tan^2 \beta = \frac{2M_A^2 - M_H^2}{M_H^2},$$ \hspace{1cm} (22b)

iii) $V_B^{iii}$

$$\tan \beta = \frac{M_h}{M_H}, \hspace{1cm} M_A^2 = \frac{M_H^2 + M_h^2}{2}$$ \hspace{1cm} (22c)

Again, defining similar ratios $R_B$ for potentials B, we show in Fig. 3 their variation with $M_h$, for β = π/3. Now the enhancement is even larger. On the contrary, for smaller values of β, β = π/12 for instance, $R_B$ varies between 0.7 e 1.2.
5 Conclusion

We have studied the different versions of a renormalizable and CP-conserving THDM. In general the different potentials of these models fall into two main categories. They are either invariant under $Z_2$ or under a global $U(1)$. In any of these cases the symmetry can be softly broken with the addition of mass terms. Alternatively if some of the mass terms are set to zero one obtains constraining relations among the parameters of the potential. Despite the fact that these models are very similar they can give substantial different predictions for the amplitude of physical processes. This point is illustrated with two examples. The first one is a calculation of the one-loop weak corrections to the top quark decay $t \rightarrow bW$. In general the THDM give loop contributions that are larger than in the SM by a factor of two or three. For some values of the parameters, it is even possible to obtain one order of magnitude enhancement of the SM result. The second example is the decay $h \rightarrow \gamma\gamma$ for a fermiophobic Higgs. Now the results are even more sensitive to the different potentials since the loop diagrams depend particularly on the values of $hH^+H^-$ vertex, which is different for $V_A$ and $V_B$. This study is complementary to the previous ones [16, 17].

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