We study the neutral Higgs boson production at the CERN Large Hadron Collider (LHC) and the future $e^- e^+$ linear collider (ILC) in the two Higgs doublet model with CP violation. The CP-even and CP-odd scalars are mixed in this model, which affects the production processes of neutral Higgs bosons. We examine the correlation of the Higgs production at LHC and ILC and provide a strategy to distinguish the model from the CP conserving model and to determine the parameters of the Higgs sector.

I. INTRODUCTION

The Higgs boson is the only unobserved ingredient of the standard model (SM), which is responsible for the electroweak symmetry breaking (EWSB) and the generation of fermion masses. It is the principal motivation of the proposed future collider experiments to search for the Higgs boson and examine the underlying mechanism of the EWSB. The SM Higgs boson is expected to be discovered and its mass and production cross section to be measured at the CERN Large Hadron Collider (LHC) \cite{1,2}, while the complementary study on the detailed structure of the Higgs sector will be performed at the future $e^- e^+$ linear collider (ILC) \cite{2}.

Conventional wisdom is that the standard model is not the final theory but just an effective theory of the fundamental structure. The Higgs sector is generically extended when we consider the new physics beyond the SM because the new physics usually has larger symmetry than the SM and then more symmetry breaking is required than the minimal EWSB. The two Higgs-doublet (2HD) model is one of the simplest extensions of the Higgs sector, which consists of two scalar SU(2) doublets. In the 2HD model, the physical states are three neutral scalars and a pair of charged scalars after the Goldstone modes are eaten up by the $W$ and $Z$ bosons. When both Higgs doublets couple to all fermions, their Yukawa coupling matrices cannot be simultaneously diagonalized with the mass matrices of fermions. Then the off-diagonal matrix elements exist in general, which gives rise to the flavor-changing neutral current (FCNC) mediated by a neutral Higgs boson at tree level. Thus one has to constrain the Yukawa couplings arbitrarily, or introduce another symmetry to suppress the FCNC, e.g. an approximate flavor symmetry \cite{3}. The phenomenological implications of the FCNC to the collider signal and flavor physics have been widely studied \cite{4,5}. A natural flavor conservation (NFC) has been suggested in order to avoid the dangerous FCNC by imposing a discrete symmetry on the Higgs and fermion fields \cite{6}. We choose the discrete symmetry such that one Higgs doublet couples to up-type quarks and the other doublet couples to down-type quarks to get rid of the tree level FCNC. This is also the Higgs structure of the minimal supersymmetric standard model (MSSM). Such a discrete symmetry on the Higgs doublet fields forbids the Higgs sector to contain the CP violating terms. Therefore, the CP is the manifest symmetry of the theory and the physical states of neutral scalars are two CP-even Higgs bosons $H$ and $h$ and one CP-odd Higgs boson $A$.

Assuming the breakdown of the discrete symmetry in the Higgs sector, complex Higgs self-couplings exist in general, and consequently the explicit and/or spontaneous CP violation is allowed in the Higgs sector. In such a case $H$, $h$ and $A$ are no more CP eigenstates and the CP-even and CP-odd states are mixed. The Higgs boson couplings to gauge bosons and fermions also depend upon the mixing angle between the CP-even and CP-odd Higgs bosons and so does the production of neutral Higgs bosons at the future colliders. Even it is possible that the production cross section of the Higgs boson is suppressed to be missed at colliders in some parameter space although the Higgs boson mass is low enough to be produced at such colliders. However, if one neutral Higgs boson production may be suppressed,
other Higgs boson production processes will be enhanced instead according to sum rules of Higgs couplings and we can find the evidence of Higgs boson in general. Analysis to search for the neutral Higgs boson at the $e^- e^+$ collider in the CP violating 2HD model has been performed in the Refs. [3, 4]. In this work, we consider the Higgs sector with soft violation of the discrete symmetry in the 2HD model and discuss its implication for the neutral Higgs boson production at LHC and ILC.

This paper is organized as follows: In section 2, we briefly review the general 2HD model with non-zero CP violation and define the physical neutral Higgs bosons. Possible scenarios are examined in section 3 and correlation of LHC and ILC is discussed in section 4. Finally we conclude in section 5.

II. THE MODEL

The general Higgs potential of the 2HD model is given by

$$V = \frac{1}{2}\lambda_1 (\phi_1^1 \phi_1^1)^2 + \frac{1}{2}\lambda_2 (\phi_2^1 \phi_2^1)^2 + \lambda_3 (\phi_1^1 \phi_1^2)(\phi_2^1 \phi_2^2) + \lambda_4 (\phi_1^1 \phi_2^1)(\phi_2^2 \phi_1^1)$$

$$+ \frac{1}{2}[\lambda_5 (\phi_2^1 \phi_2^1)^2 + H.c.] + [\lambda_6 (\phi_1^1 \phi_1^2)(\phi_1^1 \phi_2^2) + \lambda_7 (\phi_1^2 \phi_2^1)(\phi_1^1 \phi_2^1) + H.c.]$$

$$- m_{12}^2 (\phi_1^1 \phi_1^1) - m_{12}^2 (\phi_2^1 \phi_2^1) - [m_{12}^2 (\phi_1^1 \phi_2^1) + H.c.], \quad (1)$$

where $\lambda_5, \lambda_6, \lambda_7$ and $m_{12}^2$ are complex parameters and others are real. The discrete symmetry $\phi_1 \rightarrow -\phi_1$ or $\phi_2 \rightarrow -\phi_2$ is imposed in order to avoid a dangerous FCNC, which leads to the absence of $m_{12}^2, \lambda_6$ and $\lambda_7$. We allow soft violation of the discrete symmetry by the dimension 2 terms $m_{12}^2 \neq 0$ in this work for the explicit CP violation.

The potential is minimized with the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \left( 0 \ v_1 \right), \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \left( 0 \ v_2 e^{i\xi} \right), \quad (2)$$

where $v_1^2 + v_2^2 = v^2 = 4m_t^2/g^2$ and the phase $\xi$ leads to the spontaneous CP violation. The minimization of the potential (1) at $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ yields the relation

$$\text{Im}(m_{12}^2 e^{i\xi}) = v_1 v_2 \text{Im}(\lambda_5 e^{2i\xi}). \quad (3)$$

The global transform $\phi_i \rightarrow \phi_i e^{i\nu_i}$ preserves the potential invariant with the rephasing; $\lambda_5 \rightarrow \lambda_5 e^{-2i(\phi_2 - \phi_1)}$, $m_{12}^2 \rightarrow m_{12}^2 e^{-i(\phi_2 - \phi_1)}$ and $\xi \rightarrow \xi + \phi_2 - \phi_1$ while $\lambda_i (i = 1, \cdots, 4)$ and $m_{11,22}^2$ are not changed. Thus we can choose $\xi = 0$ indicating no spontaneous CP violation but the wholly explicit CP violation. Then the parameter $\text{Im} m_{12}^2$ can be replaced by $\text{Im} \lambda_5$, which plays the role of the order parameter of the CP violation.

The charged states are the Goldstone mode $G^\pm = \phi_1^1 \cos \beta + \phi_2^1 \sin \beta$ and the charged Higgs mode $H^\pm = -\phi_1^1 \sin \beta + \phi_2^1 \cos \beta$ with the mass $m_{H_H^\pm}^2 = (\text{Re} m_{12}^2/v_1 v_2 - \lambda_1 - \text{Re} \lambda_5) v^2/2$ and $\tan \beta = v_2/v_1$. In this paper, we concentrate on the neutral Higgs boson production and do not pay attention to the charged states. In order to pick out the physically allowed set of the Higgs potential parameters, however, we have to assume the charged Higgs boson mass in our numerical analysis.

The neutral states are defined by

$$G^0 = \sqrt{2}(\text{Im} \phi_1^0 \cos \beta + \text{Im} \phi_2^0 \sin \beta),$$

$$A^0 = \sqrt{2}(-\text{Im} \phi_1^0 \sin \beta + \text{Im} \phi_2^0 \cos \beta),$$

$$\varphi_1 = \sqrt{2}\text{Re} \phi_1^0,$$

$$\varphi_2 = \sqrt{2}\text{Re} \phi_2^0. \quad (4)$$

The mass matrix of neutral Higgs bosons is constructed as the form

$$\mathcal{M}^2 = v^2 \begin{pmatrix} R \sin^2 \beta + \lambda_1 \cos^2 \beta & (\lambda_3 + \lambda_4 + \text{Re} \lambda_5 - R) \frac{v_{1/2}}{v_1} \\ \frac{1}{2} \text{Im} \lambda_5 v^2 \sin \beta & R \cos^2 \beta + \lambda_2 \sin^2 \beta \end{pmatrix} \begin{pmatrix} \lambda_3 + \lambda_4 + \text{Re} \lambda_5 - R & \frac{v_{1/2}}{v_1} \\ \frac{1}{2} \text{Im} \lambda_5 v^2 \cos \beta & R - \text{Re} \lambda_5 \end{pmatrix}, \quad (5)$$

where $R = \text{Re} m_{12}^2/v_1 v_2$. The non-zero off-diagonal elements $\mathcal{M}^2_{13}$ and $\mathcal{M}^2_{23}$ indicate the CP violation, since three neutral scalars are mixed and no more CP eigenstates. The parameter $\text{Im} \lambda_5$ is the only CP violating parameter in this model.
We diagonalize the mass matrix by the orthogonal transformation

$$M_d^2 = R M^2 R^\dagger,$$

(6)

where the orthogonal matrix $R$ can be parameterized by 3 Euler angles $\theta_a$, $\theta_b$, $\theta_c$

$$R = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_b & s_b \\
0 & -s_b & c_b
\end{pmatrix} \begin{pmatrix}
c_b & 0 & s_b \\
0 & 1 & 0 \\
-s_b & 0 & c_b
\end{pmatrix} \begin{pmatrix}
-s_a & c_a & 0 \\
c_a & s_a & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

$$= \begin{pmatrix}
-c_b s_a & c_a c_b & s_b \\
c_a c_c + s_a s_b s_c & s_a c_c - c_a s_b c_c & c_b s_c \\
-c_a s_c + s_a s_b c_c & -s_a s_c - c_a s_b c_c & c_b c_c
\end{pmatrix},$$

(7)

where $s_{a,b,c} = \sin \theta_{a,b,c}$ and $c_{a,b,c} = \cos \theta_{a,b,c}$. The choice of the angle $\theta_a$ is different from other angles to follow the convention of the mixing angle between CP-even Higgs bosons. Hereafter we set $\alpha \equiv \theta_a$ by convention. Then the physical states for neutral Higgs bosons $h_1, h_2, h_3$ are defined by

$$\begin{pmatrix}
h_1 \\
h_2 \\
h_3
\end{pmatrix} = R \begin{pmatrix}
\phi_1 \\
\phi_2 \\
A
\end{pmatrix}.$$

(8)

The CP-odd state $A$ is mixed with CP-even states $\phi_1, \phi_2$ and it indicates a manifest CP violation in the neutral Higgs sector.

The Yukawa couplings are given by with the discrete symmetry

$$\mathcal{L}_Y = -g^d_3 \tilde{Q}_L^i \phi_1 d_R^i - g^d_3 \tilde{Q}_L^i \phi_1 t_R^i - g^u_3 \tilde{Q}_L^i \tilde{\phi}_2 u_R^i + \text{H.c.},$$

(9)

where $\tilde{\phi}_2 = i \tau_2 \phi_2$. The relevant terms for the dominant production process $pp \rightarrow gg \rightarrow h$ at LHC is the top Yukawa coupling given by

$$\mathcal{L}_{Y_t} = -\frac{g}{2} \frac{m_t}{m_W} \sum_{i=1}^3 \left( \frac{(R^{-1})_{2i}}{\sin \beta} + i \frac{(R^{-1})_{3i} \cos \beta}{\sin \beta} \right) \bar{t}_L t_R h_i + \text{H.c.}$$

(10)

III. SCENARIOS FOR NEUTRAL HIGGS BOSON PRODUCTION

If $\text{Im} \lambda_5 = 0$, we can see that the mass matrix of Eq. (5) is reduced to that of the CP conserving case, where the CP-even and CP-odd Higgs bosons are separated. This is corresponding to the case of $\theta_b = \theta_c = 0$ in the matrix $R$. A few interesting scenarios for nonzero but limiting values of $\theta_b$ and $\theta_c$ are considered at each experiment in this section.

A. the CERN Large Hadron Collider (LHC)

We expect that the SM Higgs boson will be discovered at LHC and its mass and the cross section will be measured. The dominant production channel of the neutral Higgs boson at LHC is the gluon fusion process, $pp \rightarrow gg \rightarrow h$. Since the cross section for the neutral Higgs boson production crucially depends upon Higgs couplings, the large deviation of the measured cross section from the SM prediction indicates the evidence of the new structure of the Higgs sector. Even it is possible that we cannot discover the lightest Higgs boson due to the suppressed coupling with some combination of $\alpha, \theta_b, \theta_c$ and $\tan \beta$ although the Higgs mass is small enough to be observed at LHC. Here, we examine a scenario where this dominant channel ($pp \rightarrow gg \rightarrow h$) is blind at LHC for three neutral Higgs bosons. The cross section for the lightest neutral Higgs boson is given by

$$\sigma(gg \rightarrow h_1) = \left( \frac{\cos^2 \alpha \cos^2 \theta_b}{\sin^2 \beta} + \frac{\cos^2 \beta \sin^2 \theta_c}{\sin^2 \beta} \right) \cdot \sigma_{SM}(m_{h_1}).$$

(11)

The orthogonality of the matrix $R$ gives rise to a sum rule for ratios of cross sections as

$$\sum_{i=1}^3 \frac{\sigma(gg \rightarrow h_i)}{\sigma_{SM}(m_{h_i})} = 1 + \frac{\cos^2 \beta}{\sin^2 \beta} > 1,$$

(12)
where $\sigma_{SM}(m)$ is the SM cross section with the Higgs mass $m$. The cross section $\sigma(gg \rightarrow h_1)$ vanishes in the limit of $\cos^2 \alpha \rightarrow 0$, $\sin^2 \theta_b \rightarrow 0$. In this limit, the cross section for $h_2$ is given by

$$\sigma(gg \rightarrow h_2) = \left(\frac{\cos^2 \theta_c}{\sin^2 \beta} + \frac{\cos^2 \beta \sin^2 \theta_c}{\sin^2 \beta}\right) \cdot \sigma_{SM}(m_{h_2}).$$

(13)

Thus if $\cos^2 \theta_c \rightarrow 0$ and $\tan \beta$ is sufficiently large, $\sigma(gg \rightarrow h_2)$ is also suppressed and we cannot find $h_2$ through $pp \rightarrow gg \rightarrow h$ process at LHC. Consequently neutral Higgs bosons are blind through this channel at LHC in such a scenario.

Let us test the above ‘blind’ scenario in more detail. Actually $m_3$ is not a free parameter in this model when other Higgs boson masses and mixing angles are fixed. In the Fig. 1, we show the value of $m_3$ with respect to $\tan \beta$ in the limit considered here. We find that $m_3$ is bounded above, and such a complete blind scenario is not plausible. In practice, we need not take a strict limit of $\cos \theta_2/\sin \theta_2 \rightarrow 0$, but it is enough if the signal is too small to be extracted from the large background of the hadron collider. We set the condition, $\cos^2 \alpha$, $\sin^2 \theta_c$, $\cos^2 \theta_c < 0.01$, in this analysis. Higgs boson masses $m_1$ and $m_2$ are varied up to 1 TeV which is the experimental bound for the SM Higgs boson to be found at LHC. For the numerical analysis, we demand the following constraints on the model parameters; (1) the perturbativity on the quartic couplings, $\lambda_i/4\pi < 1$, (2) the ordering of Higgs masses, $m_1 < m_2 < m_3$. If $\tan \beta$ is large, the $b$ quark Yukawa coupling is also large so that the contribution of $b$ quark loop becomes important. Including the contribution of $b$ quark loop contaminates the relations in our discussion. However, as we can see in Fig. 1, the favored value of $\tan \beta$ is less than 10, and we can safely assume that the top quark loop dominates in $pp \rightarrow gg \rightarrow h$ process.

B. the future $e^-e^+$ linear collider (ILC)

The phenomenology of the Higgs sector at the $e^-e^+$ linear colliders is governed by couplings of the Higgs bosons to gauge bosons. We write generalized $h_iZZ$ couplings here:

$$h_1ZZ \propto \sin(\beta - \alpha) \cos \theta_b,$$

$$h_2ZZ \propto \cos(\beta - \alpha) \cos \theta_c - \sin(\beta - \alpha) \sin \theta_b \sin \theta_c,$$

$$h_3ZZ \propto -\cos(\beta - \alpha) \sin \theta_c - \sin(\beta - \alpha) \sin \theta_b \cos \theta_c,$$

(14)

which are normalized by the SM coupling $g_mZ/\cos \theta_W$. Due to the orthogonality of $R$, these couplings satisfy a few sum rules, which can be found in the literatures [3, 4, 10].

If we assume that $\sin \theta_b \sim 0$ and $\cos \theta_c \sim 0$, $h_2$ is decoupled and identified with the CP-odd Higgs boson $A$. In the limit that $\cos \theta_b \sim \cos \theta_c \sim 0$, $h_1$ is decoupled to be $A$. In both cases, Higgs-gauge couplings $g_{h_iZZ}$ and $g_{h_ih_jZ}$ go close to those of the CP conserving case. Thus these limiting cases are similar to the CP conserving case except for the possibility that the lightest Higgs boson may be the CP-odd Higgs boson.

More interesting scenario is obtained by taking the limit $\sin \theta_c \rightarrow 0$. In this case, the off-diagonal elements of the mass matrix of neutral Higgs bosons become

$$M_{12}^2 = s_a c_b s_b (m_2^2 - m_3^2),$$

$$M_{23}^2 = -c_a c_b s_b (m_3^2 - m_2^2).$$

(15)

Comparing the ratio $M_{12}^2/M_{23}^2$ from Eq. (15) with the same ratio given in Eq. (5) leads to $\tan \beta = -\tan \alpha$ and thus $\beta = -\alpha$. The CP violating parameter $\text{Im} \lambda_5$ is directly related to $\theta_b$ and Higgs masses,

$$\text{Im} \lambda_5 = \sin 2\theta_b \frac{m_2^2 - m_3^2}{\eta^2}.$$  

(16)

If we additionally assume that $\sin \theta_b$ is close to 1, the lightest Higgs boson decouples to be the CP-odd Higgs and heavier Higgses $h_2$ and $h_3$ are mixed with each other. The $g_{h_iZZ}$ and $g_{h_ih_jZ}$ couplings are given by

$$h_1ZZ \propto \epsilon_c \sin(\beta - \alpha),$$

$$h_2ZZ \propto \cos(\beta - \alpha),$$

$$h_3ZZ \propto -\sin(\beta - \alpha),$$

(17)
where \( c_{\epsilon} = \cos \theta_{\epsilon} \). This limit may look like the CP conserving case since the CP-odd Higgs decouples. However, we see that the ratio \( |g_{zzxz}/g_{zzxx}| = 1/\tan(\beta - \alpha) \), while \( |g_{zzx}/g_{zzxx}| = \tan(\beta - \alpha) \) for the CP conserving case. It may be a clue to discriminate this scenario from the CP conserving model in the gauge-Higgs sector without the manifest observation of the CP asymmetry.[1]

IV. INTERPLAY BETWEEN LHC AND ILC

It is natural that the first evidence of the Higgs boson will be discovered at LHC before ILC. Thus we can assume that the lightest Higgs boson mass and corresponding cross section will be measured at LHC without loss of generality. We define \( \Delta \) to be the ratio of the cross section of \( pp \to gg \to h \) in our model to the SM cross section of corresponding \( m_h \),

\[
\Delta \equiv \frac{\sigma(pp \to gg \to h_1)}{\sigma_{SM}(m_{h_1})},
\]

where \( \sigma_{SM}(m_{h_1}) \) is the SM cross section of \( pp \to gg \to h \) channel with the Higgs boson mass \( m_{h_1} \). The mixing angle \( \theta_{h} \) is expressed in terms of \( \Delta \) and other angles such as

\[
\cos^2 \theta_{h} = \frac{\Delta \sin^2 \beta - \cos^2 \beta}{\cos^2 \alpha - \cos^2 \beta},
\]

when \( \cos^2 \alpha \neq \cos^2 \beta \). If the measured cross section deviates from the SM prediction, \( \Delta \neq 1 \), it directly indicates a new structure of the Higgs sector.

The most promising channel at ILC is the Higgsstrahlung process \( e^- e^+ \to Zh_1 \). In the CP conserving model, the CP-odd Higgs boson \( A \) does not couple to the gauge boson. So the observation of \( ZA \) production is a direct evidence of the CP violating model in principle. However, it is hard to tag \( A \) or \( h \) in the Higgsstrahlung process due to its spin 0 nature. Moreover in our case, the produced neutral Higgs boson is not a pure CP eigenstates but a mixed one. Thus it is impossible to find the \( ZA \) production in our model. Let us first assume that we have the minimal information of the Higgs sector from LHC at the moment when the \( e^- e^+ \) linear collider starts running: \( e.g. \) the lightest Higgs mass \( m_{h_1} \) and the ratio of the cross section \( \sigma(pp \to gg \to h_1)/\sigma_{SM} \) have been already determined. The cross section of \( e^- e^+ \to Zh_1 \) is

\[
\sigma(e^+ e^- \to h_1Z) = f_1^2 \sigma_{SM}(m_{h_1})
\]

where \( f_i \) are the normalized \( h_i ZZ \) couplings given in Eq. (14) and \( \sigma_{SM}(m_{h_i}) \) is the SM cross section for \( e^+ e^- \to h_iZ \) process with corresponding Higgs mass. For given \( m_{h_1} \) and \( \Delta \), we can rewrite

\[
f_1^2 = \sin^2(\beta - \alpha) \frac{\Delta \sin^2 \beta - \cos^2 \beta}{\cos^2 \alpha - \cos^2 \beta},
\]

assuming \( \cos^2 \alpha \neq \cos^2 \beta \). In Fig. 2, we plot the cross section \( \sigma(e^- e^+ \to Zh_1) \) with respect to the measured \( \Delta \). Values of \( \tan \beta \) is assumed to be fixed since \( \tan \beta \) can be measured in separate processes, \( e.g. \) charged Higgs sector, while \( \alpha \) is varied.

Next we assume that we have already measured cross sections for 2 channels at LHC, \( \sigma(gg \to h_1) \) and \( \sigma(gg \to h_2) \), and, however, measured only one cross section at ILC. This is a very likely assumption since the CM energy of ILC is much lower than LHC. In such a scenario, we can critically discriminate the CP violating model with CP conserving model. In the CP conserving model, \( \theta_b, \theta_c = 0 \), the cross sections at LHC are

\[
\Delta_{LHC1} \equiv \frac{\sigma(gg \to h_1)}{\sigma_{SM}(m_{h_1})} = \frac{\cos^2 \alpha}{\sin^2 \beta}, \quad \Delta_{LHC2} \equiv \frac{\sigma(gg \to h_2)}{\sigma_{SM}(m_{h_2})} = \frac{\sin^2 \alpha}{\sin^2 \beta},
\]

while the cross section at the ILC is given by

\[
\Delta_{ILC} \equiv \frac{\sigma(e^- e^+ \to h_1Z)}{\sigma_{SM}(m_{h_1})} = \sin^2(\beta - \alpha).
\]

Then these observables satisfy the relation

\[
\Delta_{LHC2}(1 - \Delta_{ILC}) = (\sqrt{\Delta_{LHC1}} \Delta_{ILC} - 1)^2,
\]
which is the surface on $(\Delta_{LHC1}, \Delta_{LHC2}, \Delta_{ILC})$ plane. Since this relation is derived in the CP conserving two Higgs doublet model, a deviation of any observable from this relation indicates the CP violating model and/or more complex Higgs structure. In Fig. 3, we show a contour plot on this relation. We assume that the measurement of $\Delta_{LHC1}$ agrees with the SM prediction. The contour is corresponding to $\theta_b = \theta_c = 0$ and we plot the departure with $\theta_b \neq 0$ and $\theta_c = 0$ case. The $\theta_c \neq 0$ and $\theta_b = 0$ provides a line perpendicular to this plane.

V. CONCLUDING REMARKS

The neutral Higgs boson production processes have been explored together at LHC and ILC in the context of the two Higgs-doublet model with non-zero CP violation. Because of the spin 0 nature, it is hard to get the direct CP violating signals from the neutral Higgs sector. Instead, by investigating the combined production rate at LHC and ILC, we can find out if the CP violating signal to indicate the scalar-pseudoscalar mixing. We suggest a few limiting cases which shows characteristic phenomenology in the neutral Higgs boson production and a strategy to determine the Higgs mixing angles at ILC with the data obtained from LHC.

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FIG. 1: In the blind scenario at LHC, the allowed values of $m_{h_3}$ with respect to $\tan \beta$. 

![Graph showing allowed values of $m_{h_3}$ with respect to $\tan \beta$.]
FIG. 2: Predicted cross section of $e^-e^+ \rightarrow Zh_1$ process with respect to $\Delta$ obtained from LHC. $\tan \beta$ and the mixing angle $\alpha$ is varied. The Higgs mass is assumed to be 130 GeV and the dotted horizontal line is corresponding cross section predicted by the SM.
FIG. 3: Deviation from the CP conserving limit in ($\Delta_{LHC2}, \Delta_{ILC}$) plane. $\Delta_{LHC1} = 1$ is assumed.