Fractal analysis of the multiple fracture patterns in the low-carbon steel

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Abstract. We studied the process of damages accumulation at tension of low carbon steel specimens with lateral notch. The fractal dimension of microcrack patterns in a plastic zone and angular parameter of microcracks distribution have been estimated. It is established that these parameters change in opposite phase. The analogy to the similar dependences estimated according to seismic activity is drawn.

1. Introduction

The authors of [1-3] were admittedly the first to establish self-similarity of multiple fracture patterns for a wide range of materials (metals, polymers, and rocks) under conditions of tension, creep, fatigue, and superplasticity. The self-similarity was confirmed when a unique curve for the distribution of defects was plotted based on the normalization, which corresponds to the relation

\[
\frac{N}{N_{\text{max}}} \sim \left( \frac{l}{l_{\text{max}}} \right)^a
\]  

(1)

The power law describing the distribution of faults in the Earth's crust found by many researchers (in particular, by the authors of [4]) confirms self-similarity of the microcrack pattern and Eq. (1) at the global level of development of the multiple fracture process. The tangent of the inclination angle of these cumulative distributions was accepted in [5] as the fractal dimension (D) of the microcrack patterns observed in the Earth's crust. It was assumed that fractal dimension can be estimated from relation

\[
D = \frac{3b}{c},
\]  

(2)

where \(b\) is the exponent in Guttenberg-Richter power equation \((N \sim E^b)\) for the relation between the number of seismic events \((N)\) and their energy \((E)\) (or magnitude \(M = \log E\)) and \(c\) is a constant equal to \(~3/2\). This means that the fractal dimension of the fracture pattern is proportional to the seismic parameter \(b\) (\(b\) value) estimated from the tangent of the inclination angle of the distribution of accumulated number of seismic events by magnitude.

The objective of this study included estimation of fractal dimension and parameter \(b\) of the real pattern of fractures of metal specimens and establishment of the correlation between them.
Let us introduce the following notations for easier description of the results: $b_c$, $b_{AE}$ and $b_S$ are exponents in the power laws describing cumulative distributions of microcracks (or faults), acoustic signals, and seismic events, respectively; $D_C$, $D_{AE}$, and $D_S$ are fractal dimensions estimated from the data of the analysis of real damage, acoustic, or seismic responses to this damage, respectively.

Relation (1) was confirmed in [7, 8], where acoustic parameter $b_{AE}$ was estimated on the rock specimens by the method of the acoustic emission. However, in the first case [7], fractal dimension was accepted as the angular coefficient of cumulative curves of microcracks distribution by length $b_C$; in the second case [8], as the fractal (correlation) dimension of the epicenters of acoustic events $D_{AE}$. In both cases, positive correlation of the estimated parameters was found (although the authors of [8] observed it only at the initial stage of the deformation of samples) corresponded in the accepted notations to relations:

$$b_c = 3 \frac{b_{AE}}{c} \quad \text{and} \quad D_{AE} = 3 \frac{b_{AE}}{c}.$$

### 2. Results and discussion

In this work, we estimated $b_C$, the parameter of real microcracks pattern based on the method of replicas obtained at the polished surface of samples at different stages of loading. We also estimated fractal dimension $D_C$ (corresponding to these patterns) measured by the least squares using relation [9]:

$$N = \lim_{R \to x} BR^{-D},$$

where $N$ is the number of squares with side $R$ containing black pixels (microcracks were colored black) and $B$ is a constant. Variation in the area of the covered square is no less than two orders of magnitude. The cumulative curve of the distribution of microcrack length ($L$) used for determining parameter $b_C$ corresponded to the power relation ($N = BL^{b_C}$) over the interval of variation in the microcracks length no less than by 1.5 orders of magnitude.

The investigations were carried out with flat notched specimens of model material (low-carbon steel) under conditions of static and cyclic loading. The thickness of the specimens ranged from 4 to 16 mm. The replicas allowed us to study the zone of the localization of fracture and the microcracks patterns in the zone at each stage of the process. After this, the replicas were analyzed using an optical microscope equipped with digital video camera, and the obtained patterns of multiple fractures were processed by the image analysis program. The maximal number (1500 - 10000) of microcracks measured on each sample depended on the degree of damage of the material at the each stage of loading. The results of measurements were used for plotting cumulative curves of the distribution of microcracks by length in the following coordinates: total number of microcracks ($N$) with the length greater and equal to the current length versus current length of microcracks ($L$).

Figure 1 shows a typical deformation curve of samples with highlighted points, corresponding to them zones of fracture localization, and patterns of microcracks in zones. The curves of variations in parameters $b_C$ and $D_C$ with deformation are also shown. It is seen that the zone of localization of fractures is already formed at the linear region of the deformation curve. At this initial stage of loading numerous slip bands that are developed both at the boundaries of the structural elements and within them were found. Microcracks are absent. Increase of strain leads to increase of the density of slip bands and appearance of defect of the next rank - pores formed in the slip bands or places of stress concentrators (scratches at the polished surface of the sample). When the yield stress is exceeded and the related size of the localization zone increases, the defect of the third rank appears as microcracks formed by the merging of pores along the slip bands.

Figure 1 also suggests a decrease in the $b_C / D_C$ ratio with the increase in strain accompanied by the variation in parameters $b_C$ and $D_C$ in the opposite phase, unlike in-phase correlation of these parameters corresponding to Eq. (2). Such variation in parameters $b_S$ and $D_S$ was observed in [11, 12]. In [8], such variation in $b_{AE}$ and $D_{AE}$ was observed at the final stage of the fracture of specimens.

In this case, decrease in parameter $b_C$ is caused by merging of microcracks, while increase in the fractal dimension $D_C$ is caused by increase in the total area of fractures related to increase in the length and opening of microcracks. Investigation of fractures of specimens with different thicknesses...
confirmed the noted variation of these parameters with deformation and demonstrated that the dependence of the fractal dimension on parameter $b_C$ corresponds to a linear dependence $D_C = 2.86 - 0.67b_C$ (correlation coefficient 0.95) for all tested samples. A similar linear dependence was also found in the analysis of interrelation between the correlation fractal dimension $D_S$ and parameter $b_S$ estimated from the data of seismic activity: $D_S = 2.3 - 0.73b_S$ (correlation coefficient 0.76) [11] and $D_S = 2.7 - 1.39b_S$ (correlation coefficient 0.85) [12].

Fig. 1. Deformation dependences of stress in net-section of samples with notch, parameters $b_C$ and $D_C$ and their ratio $\frac{b_C}{D_C}$, zones of localization of fractures in the notch tip, and patterns of microcracks in zones.

Thus, it was established that:

- The kinetics of multiple fractures observed in steel specimens is similar to one on a global level of fracture process development. It confirms the ideas of self-similarity of fracture process of solids.
- For the multiple fracture patterns the dependence of the fractal dimension on $b$ parameter corresponds to the reverse linear relation $D = 2.86 - 0.67b$.

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