Comments on regularization ambiguities and local gauge symmetries

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Abstract

We study the regularization ambiguities in an exact renormalized (1+1)-dimensional field theory. We show a relation between the regularization ambiguities and the coupling parameters of the theory as well as their role in the implementation of a local gauge symmetry at quantum level.

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1 Introduction

The study of anomalies in local gauge theories has been very important for the definition of the modern Standard model of elementary particles. The search for local gauge theories without gauge anomalies established the existence of equal number of families of leptons and quarks [1]. But, in the Standard model assumes the existence of a fundamental spin 0 particle, the Higgs boson [2], it allows to generate, in a gauge invariant way, mass for the vector bosons of the Electroweak sector via spontaneous symmetry breaking [3]. This procedure does not spoil the perturbative renormalizability of the model [4]. Nevertheless, to date there is no experimental evidence about the existence of this particle.

On the other hand, there is an alternative mechanism to generate mass, the so called dynamical mass generation. As it was shown first by Schwinger [5] (and at the first time it was suggested by Yang and Mills in their seminal paper [6]), it can generate mass for the vector boson at quantum level in a gauge invariant way. But, the dynamical mass generation can also be carried in a non-gauge invariant way, just as it happens, for example, in the chiral Schwinger model (CSM) [7, 8] or in the anomalous Schwinger model (ASM) [9, 10].

In quantum field theory is natural the arising of divergences when computing, for example, the correlation functions of a specific model. The calculus involved, sometimes, carries on to the rising of ambiguities linked to the prescription used to regularize these infinities [7, 8, 11].

In this paper we mixed both the dynamical mass generation and the regularization ambiguities such that a non-gauge invariant model at classical level can gain a local gauge symmetry at quantum level. In order to show it, we analyze a simple (1+1) dimensional model: a massive vector (Proca) field coupled to massless Dirac’s fermions known as the Thirring-Wess model [12]. As it is well-known this model does not have a local gauge symmetry at classical level, then, we compute the fermionic determinant in an external field with a generalized prescription which does not preserve the fermionic current, i. e., \( \partial_\mu \langle 0 | \bar{\psi} \gamma^\mu \psi | 0 \rangle A_\mu \neq 0 \) [13]. The generalized prescription introduces an ambiguity regularization parameter such as it happens in the CSM or ASM models. The fermionic determinant contributes with an additional mass for the vector field which is ambiguity dependent. Thus, the effective action depending in the value assumed for the ambiguity can gain or not a local \( U(1) \) symmetry \( (A_\mu \rightarrow A_\mu + \partial_\mu \alpha) \). Therefore, when we impose the local gauge symmetry in the effective action level we get to define the ambiguity parameter as a full function of the coupling constants, and the model is a well-known finite field theory. On the other hand, if we do not impose such local symmetry the model get still to be a consistent and renormalizable field theory.

Consequently, we can assert that this non–gauge invariant classical model suggest the possibility of generating a local gauge invariance by quantum effects.

The paper is displayed in the following way. In section 2, we quantize and analyze the massive vector field coupled to massless Dirac’s fermions in both situations, and the important results are analyzed in section 3.
2 The model

The Thirring-Wess model \[12\] is defined by the following Lagrangian density\footnote{In this paper, we use the natural units $\hbar = c = 1$ and the following conventions, the metric tensor is $g_{00} = 1 = -g_{11}$, the Levi-Civita tensor is $\epsilon^{01} = 1 = -\epsilon_{01}$. The $\gamma^\mu$ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, the $\gamma_5$ matrix is defined as $\{\gamma^\mu, \gamma_5\} = 0$, $\gamma_5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3$, and the chiral projectors are $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$. Also define the contraction $V^\mu = \epsilon^{\mu\nu}V_\nu$.}

$$L[\psi, \overline{\psi}, A] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \overline{\psi}( i \partial^\mu + e A^\mu ) \psi, \quad (1)$$

as it is clear, the mass term of vector field precludes the existence of local $U(1)$ gauge symmetry at classical level.

We quantize the model by introducing the generating functional

$$Z[\eta, \overline{\eta}, J^\mu] = N \int dA_\mu d\psi d\overline{\psi} \exp \left( i \int dx \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu + \overline{\psi}( i \partial^\mu + e A^\mu ) \psi + + J_\mu A^\mu + \overline{\eta} \psi + \overline{\psi} \eta \right] \right) \quad (2)$$

where $\eta, \overline{\eta}$ are the external sources for the fermionic fields and $J^\mu$ is the correspondent source for the vector field. We consider as the first step, in the quantization procedure, the computing of the fermionic integration which gives

$$\text{det} \left( i \partial^\mu + e A^\mu \right) \exp \left( -i \int dx dy \, \overline{\eta}(x,y) G(x,y;A) \eta(y) \right). \quad (3)$$

The computation of the fermionic determinant will be carry out with a generalized prescription \[8, 13\] which does not necessarily preserve the fermionic current, and it gives the following contribution to the effective action for the massive vector field,

$$\text{det} \left( i \partial^\mu + e A^\mu \right) = \int dx \frac{1}{2} A_\mu(x) \left( \frac{e^2}{2\pi} (a + 1) g^{\mu\nu} - \frac{e^2}{2\pi} \square \right) A_\nu(x), \quad (4)$$

where we can see that the vector field get an additional mass, which is dynamically generated. The $a$ parameter characterizes the regularization ambiguities of the UV divergences which appear when compute the fermionic determinant.

Thus, after integration on the fermion fields in (2), it reads as

$$Z[\eta, \overline{\eta}, J^\mu] = \int dA_\mu \exp \left( i \int dx \frac{1}{2} A_\mu \left[ g^{\mu\nu} \left( \square + m^2 \right) - \left( \square + \frac{e^2}{2\pi} \right) \frac{\partial^\mu \partial^\nu}{\square} \right] A_\nu + J \cdot A \right) \times \exp \left( -i \int dx dy \, \overline{\eta}(x,y) G(x,y;A) \eta(y) \right) \quad (5)$$

where $m^2$ is the total mass of the vector field,

$$m^2 = m^2_p + \frac{e^2}{2\pi} (a + 1) = \frac{e^2}{2\pi} (b + 1) \quad (6)$$

and the $b$ parameter is defines as

$$b = a + 2\pi \frac{m^2_p}{e^2}. \quad (7)$$

$G(x,y;A)$ is the Green’s function of the Dirac’s equation,

$$(i \partial^\mu + e A^\mu) G(x,y;A) = \delta(x-y), \quad (8)$$

it can exactly be computed,

$$G(x,y;A) = \exp \left( -i e \int dz \, A_\mu(z) j_\mu^\nu(z,x,y) \right) P_+ G_F(x-y) + \exp \left( -i e \int dz \, A_\mu(z) j_\mu^\nu(z,x,y) \right) P_- G_F(x-y) \quad (9)$$
with $G_F(x-y)$ being the Green’s function of the free Dirac’s equation: $i\slashed{D}G_F(x-y) = \delta(x-y)$ and the contact current $j^\mu_F(z, x, y)$ is

$$j^\mu_F(z, x, y) = (\partial_\mu \mp \tilde{\partial}_\mu)[D_F(z-x) - D_F(z-y)],$$

where $D_F(x)$ is the Green’s function of the massless Klein–Gordon equation: $\Box D_F(x-y) = \delta(x-y)$.

At quantum level the Ward identity for the vectorial 1PI two-point function is given by

$$k_\mu \Gamma^{\mu\nu}(k) = \frac{e^2}{2\pi} (b-1) k^\nu,$$

then, we will have a local $U(1)$ gauge symmetry if and only if $b = 1$.

Therefore, we have to analyze the remaining vector quantization in the generating functional by distinguishing the case $b = 1$ (which we named as the quantum local gauge invariant model) from $b \neq 1$ case (named as the quantum non–gauge invariant model).

### 2.1 The $b = 1$ case

In this way the possibility to get a quantum local gauge theory from the model turns into a reality. We first set $b = 1$ in the generating function, such procedure leads us up to an equation for the ambiguity parameter $a$ as a function of the coupling constants of the classical field theory,

$$a = 1 - 2\pi \frac{m^2_P}{e^2},$$

at this level we can say that the ambiguity parameter $a$ is related to the weak and strong coupling regime of the model, i.e., $m^2_P < e^2$ and $m^2_P > e^2$. And consequently, the vectorial mass, would be given for

$$m^2 = \frac{e^2}{\pi},$$

it is well-defined. And, we recover the well-known local gauge invariant Schwinger model.

This way, the requirement to get a local gauge symmetry at quantum level in the effective action for the vector field leads us to find a highly physical meaning for the ambiguity parameter. We find a way of turning the ambiguity parameter, which was known to be unrelated to the theory, an element of the full quantized theory. This open a possibility to redefine an anomalous gauge model or non–renormalizable model (in the usual sense) to becomes a well defined and renormalizable field theory and, therefore, to be capable to predict physical phenomena.

Now, it is worthwhile to clarify that when $b = 1$ the theory is a gauge theory, then it has to be quantized considering its local gauge symmetry. By example, we can follow the Faddeev-Popov technique for gauge theories, whose finality is to lead the functional integration well–defined.

### 2.2 The $b \neq 1$ case

Now, we analyze the quantum non–gauge invariant model or $b \neq 1$ case. In this situation, the Thirring–Wess model is exactly equal, in all aspects, at quantum level to the anomalous Schwinger model studied in references, but in this case the $a$ ambiguity parameter from the cited references is replaced by the $b$ parameter. In these references the authors made a detailed study of the ASM model, such as semi–perturbative renormalizability, determination of non–perturbative character of UV divergences, also its exact renormalization by summing the regularized semi–perturbative expansion and the consequences on the physical properties of the model.

Thus, from equation, we can compute all Green’s functions of the non-gauge invariant model. For example, the vectorial propagator is straightforwardly computed, it yields (in momentum space) (see for example)

$$i G_{\mu\nu}(k) = \frac{1}{k^2 - m^2} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - \frac{2\pi}{e^2(b-1)} \frac{k_\mu k_\nu}{k^2},$$

the transversal part of vector field has a modified mass which is

$$m^2 = m^2_P + \frac{e^2}{2\pi} (a + 1) = \frac{e^2}{2\pi} (b + 1).$$

with $b \geq -1$ to avoid tachyonic particles. As we can notice, the mass remains indefinite due the explicit dependence in the regularization ambiguity.
3 Discussion and conclusion

We have studied a simple $(1+1)$-dimensional field theory which does not have a local gauge symmetry at classical level i.e. the Thirring-Wess model. Because, it is not a local gauge model the fermionic determinant can be computing by using a generalized prescription to regularize the ultraviolet divergences. This generalized procedure leads to the appearing of regularization ambiguities during the quantization process (read as computation of the functional integration) characterized by the $a$ parameter. Thus, upon the fermionic integration, the effective action for the Proca field can would get a local gauge symmetry if we choose an special value for the ambiguity $a_{(12)}$. The resulting gauge theory is the well-known Schwinger model [5].

On the other hand, if we left the ambiguity parameter arbitrary the effective action is non local gauge invariant, and the resulting theory is exactly equivalent to the anomalous Schwinger model studied in [9, 10, 15, 16].

The possibility of exploring the ambiguity parameter and get a local symmetry at quantum level it was also observed in the massless Thirring model [17], where such requirement allows to found a new phase to the model. Thus, it leads us to fix the ambiguity parameter as a explicit function of the coupling constants of the theory.

The model studied has the possibility to be extended to higher dimensions but in this case the results will be only of perturbative character. Therefore, it can would be interesting to know, at least, the 1-loop properties of the $(1+2)$-D model.

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