The counting approach to multidimensional poverty

The case of four African countries

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**Abstract:** This paper investigates the levels and evolution of poverty in Malawi, Mozambique, Tanzania, and Zimbabwe using the decomposability properties of poverty measures based on a counting approach. We compare poverty measures such as the Alkire and Foster index with alternative poverty indices that are sensitive to inequality.

Poverty is estimated using Demographic and Health Surveys for different years for Malawi (2004, 2010, and 2015), for Mozambique (2003 and 2011), Tanzania (2005, 2010, and 2015) and for Zimbabwe (2005, 2010, and 2015). Our findings show that one obtains insightful information when looking at the breadth and inequality components of multidimensional poverty.

**Keywords:** counting approach, multidimensional poverty measurement, Malawi, Mozambique, Tanzania, Zimbabwe

**JEL classification:** I32, D63

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1 Introduction

In September 2015, the adoption by the international community of Strategic Development Goal (SDG) 1.2 of ending poverty in all its forms everywhere incorporates an explicit multidimensional focus. Simultaneously, the SDGs encompass implicit or explicit goals addressing inequality. The recent adoption by the United Nations Development Programme (UNDP) of the so-called Multidimensional Poverty Index (MPI) illustrates the importance of taking multiple dimensions of poverty into account and addressing them. The MPI draws on the counting approach developed by Alkire and Foster (2011) and assesses poverty along the same dimensions as the Human Development Index (HDI). It not only counts the share of the population deprived in at least 30 per cent of the dimensions, but it also provides a snapshot of the breadth of poverty. This new measure, computed for over 100 countries using global standards such as the international monetary poverty line, is somehow similar to the calculation of monetary poverty.

However, recent literature stresses some conceptual and empirical weaknesses of the MPI. Among its conceptual shortcomings is its lack of sensitivity to inequality in the distribution of deprivations. The MPI does not provide any information on whether a decrease in poverty affects the poorest of the poor. Before Alkire and Foster (2011) suggested their measures, independent studies proposed several poverty measures using a counting approach. These papers suggested alternative ways of defining multidimensional poverty indices that comply with the basic axiomatic properties of multidimensional poverty indices based on continuous variables (Bossert et al. 2013).

Recently, Silber and Yalonetzky (2013) proposed a general framework that integrates ordinal variables into the measurement of multidimensional poverty. Drawing on this framework, Bérenger (2017) put special emphasis on the decomposability properties of the three ‘I’s of poverty of the main inequality sensitive poverty measures based on a counting approach. She showed the complementarities between several measures when analysing trends in poverty levels in Egypt and Jordan and in three South East Asian countries (Bérenger 2016).

Drawing on Bérenger (2017), the main goal of this paper is to make use of the decomposability properties of four main counting-based poverty measures found in the literature. We show how to use them in a way that complements the information provided by the MPI to assess the levels and trends in multidimensional poverty in Mozambique, Malawi, Tanzania, and Zimbabwe in the 2000s. We compare the results obtained when using poverty measures based on Alkire and Foster (2011), such as the UNDP’s MPI, and counting-based poverty measures sensitive to inequality. These include measures proposed by Chakravarty and D’Ambrosio (2006), by Rippin (2010), and the one suggested by Silber and Yalonetzky (2013). As these measures cover all deprived individuals, using a union approach, it is possible to manipulate them to adopt a more flexible approach that fits the identification approach of the MPI.

Section 2 presents four main multidimensional poverty indices based on the counting approach. Section 3 shows the results obtained when applying these multidimensional poverty indices to data from the Demographic and Health Surveys (DHS) of Malawi, Mozambique, Tanzania, and Zimbabwe. Section 4 gives concluding comments.
Multidimensional poverty using a counting-based approach with ordinal variables

In the counting approach, the way to summarize information amounts to determining a poverty line in each dimension and aggregating the dimensions for each individual, and then across individuals, in order to derive a summary measure of deprivations in multiple attributes. This order of aggregation preserves the essence of the multiple facets of poverty as it embeds the association between simultaneous deprivations experienced by the individual. Moreover, the central features of the counting approach are the use of binary variables and the comparison of individual achievements in a given dimension with a dimension-specific poverty line. Using solely binary indicators of well-being, the MPI summarizes the information by counting the number of dimensions in which an individual suffers from deprivations. The main innovative feature of Alkire and Foster (2011) is that they provided normative foundations for characterizing some properties of the index. Moreover, based on the extension of the Foster–Greer–Thorbecke (1984) indices, the MPI is able to deliver useful information to policy makers on the incidence and intensity of multidimensional poverty among those classified as poor people. However, the MPI does not provide any information on inequality among the poor.

Alkire and Seth (2014) addressed this issue recently. They proposed combining the MPI with a decomposable measure of inequality. Their suggestion would allow the analysis of inequality in deprivation counts among poor individuals. In addition, it would also be possible to look at disparity in poverty across population sub-groups. This separate measure echoes in some way the recent adoption by the World Bank of the mean income of the bottom 40 per cent of the population to account for inequality and promote the idea of shared prosperity. More recently, Alkire and Foster (2016) proposed an approach that incorporates sensitivity to inequality among the poor into the framework of their multidimensional poverty based on ordinal variables. They extend the MPI to a parametric class they called the M-gamma ($M_\gamma$) class, which is analogous to FGT alpha measures. However, other contributions have recommended the use of alternative methods when defining poverty indices based on a counting approach.

2.1 Some key features of the framework of the counting-based approach to poverty measurement

The designing of counting-based poverty measures involves two steps: the identification and aggregation steps.

Suppose that the relevant population consists of $n$ individuals. Let $X = (x_{i1}, \ldots, x_{im})$ be the vector of achievements (ordinal indicators). Let $X$ be the $n \times m$ matrix of these achievements so that $x_{ij}$ denotes the level of the $j^{th}$ attribute for individual $i$. Then, let $z = (z_1, \ldots, z_m)$ be the $m$-vector of specific deprivation cut-offs.

The identification step involves identifying first deprivation on each dimension and for each individual. The dimensionally poor are identified using a dichotomic function $\xi(x_i, z_j)$. It is equal to 1 if the value $x_{ij}$ of the attribute $j$ falls below the cut-off value $z_j$; otherwise it is equal to 0. This transformation generates a matrix of deprivations of $n \times m$ dimensions.

Since some deprivations are supposed to contribute more or less than others to the well-being of the individual, let $w=(w_1, \ldots, w_j, \ldots, w_m)$ be a vector of weights where $w_j$ denotes the weight of indicator $j$ such that $w_j \geq 0$ and $\sum_{j=1}^{m} w_j = 1$. 


Summing up the deprivations for each individual gives then a counting function defined as follows:

\[ c_i(x_j, z, w) = \sum_{j=1}^{m} \xi(x_{ij}, z_j) w_j . \]  

(1)

It expresses the individual deprivation score as a weighted sum of the dichotomic functions. This counting function enables identification of the multidimensionally poor individuals. Alkire and Foster (2011) introduced an identification approach called ‘intermediate’. It consists of using a cross-dimensional cut-off that defines a minimum number of deprivations (\( k \)) that an individual needs to be facing to be counted as multidimensionally poor. The identification function \( \psi^m \) is equal to 1 when an individual is deemed poor relative to the set of poverty lines \( z \) and the threshold \( k \). It means that the simultaneity of deprivations is required for someone to be included among the poor. Note that the cases where \( k=1 \) and \( k = \min(w_1, w_2, ..., w_m) \) correspond respectively to the intersection and union approaches.

With such an identification function, we know who is poor and how many poor there are. However, it might be interesting to find out how poor each poor individual is. Although with dichotomous functions there is no dimension-specific poverty line, we can at least find out in how many dimensions an individual is deprived. We can then define the individual poverty function as being equal to the product of the identification function and the breadth of poverty, which implies that it is a function of the poverty score \( c_i \).

Finally, we aggregate these individual poverty functions to define the overall poverty index as the average of these individual poverty functions. However, note that Aaberge and Peluso (2012) suggested expressing society’s poverty index as a function of the distribution of deprivations of those who are poor.

The poverty function is supposed to satisfy a set of desirable properties (see Silber and Yalonetzky (2013) and Bérenger (2017)). More specifically, the main issue in the literature related to the counting approach concerns the extension of the transfer principle so that it becomes possible to include the extent of inequality among poor individuals in the assessment of poverty. Since the counting approach neglects by construction the levels of achievement in the original variables, the only way to address inequality is to consider the distribution of deprivation scores among poor individuals.

### 2.2 Decomposability properties of the four main counting-based measures

Based on this framework, we consider four classes of counting-based poverty measures found in the literature. The first one is the Alkire and Foster (2011) measure, which corresponds to the so-called MPI published by the UNDP. The three others include measures developed by Rippin (2010), a class of social exclusion measures introduced by Chakravarty and D’Ambrosio (2006), and the measure developed by Silber and Yalonetzky (2013). While the measures of Alkire and Foster (2011) stress more the identification of the poor, these alternative measures take an implicit union approach to the identification of the poor and put a greater emphasis on the intensity and inequality in deprivations in the population. They include a measure of inequality that can appear explicitly through their decomposition.

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1 See Bérenger (2017) for a summary of the axiomatic properties of these measures.
Alkire and Foster (2011) ‘dimension-adjusted’ poverty measures

When the attributes of poverty are dichotomized variables, the poverty function is:

\[ P_0^{AF} = \frac{1}{n} \sum_{i=1}^{n} y_i^{AF}(x_i; z; k)c_i \]  

(2)

where \( c_i \) is given by (1).

It satisfies an array of desirable axioms, including decomposability and dimensional monotonicity properties. This measure is the adjusted headcount ratio used for the MPI and designated as \( M_0 \) by Alkire and Foster (2011). It is possible to express \( M_0 \) as \( M_0 = P_0^{AF} = HA \), i.e. as the product of the percentage of multidimensionally poor individuals (H) times the average deprivation share across poor individuals (A). However, \( P_0^{AF} \) is insensitive to the distribution of deprivations across individuals.

The Rippin (2010) class of ordinal poverty measures

\[ P_\gamma^{RI} = \frac{1}{n} \sum_{i=1}^{n} c_i \gamma \sum_{j=1}^{m} w_j \xi(x_{ij}; z_j) \]  

Here \( \gamma \) is a parameter of aversion to interpersonal inequality that takes into account the association between attributes.

\( P_\gamma^{RI} \) is also expressed as:

\[ P_\gamma^{RI} = \frac{1}{n} \sum_{i=1}^{n} c_i^{\gamma+1} \]  

(4)

This class of poverty measures is sensitive to the concentration of deprivation for \( \gamma \geq 0 \). Furthermore, it satisfies not only sub-group decomposability, but also factor decomposability, as (3) may be expressed as:

\[ P_\gamma^{RI} = \frac{1}{n} \sum_{j=1}^{m} w_j \sum_{i=1}^{n} \xi(x_{ij}; z_j)c_i^{\gamma} \]  

(5)

Unlike the Alkire-Foster (2011) measure, since \( c_i^{\gamma} \) acts as a weight function, the contribution of a given dimension to overall poverty is more sensitive when individuals deprived in that dimension cumulate deprivations in other dimensions.

Using the multiplicative decomposition of the FGT index developed by Aristondo et al. (2010), (4) can be decomposed into the three ‘I’s of poverty (Jenkins and Lambert 1997):

\[ P_\gamma^{RI} = H A^{\gamma+1} \left\{ I + \left[ (\gamma+1)^2 - (\gamma+1) \right] G E_{\gamma+1}(c) \right\} \]  

(6)
with $H$ the multidimensional headcount ratio, $A$ the intensity of deprivation among the poor, and $GE(c)$ the generalized entropy inequality index among the poor. This decomposition highlights the specific contribution of a change in one of these components to the overall change in the poverty index. It shows whether the poverty decrease reaches the poorest of the poor.

The Chakravarty and D’Ambrosio (2006) class of poverty measures

$$P_a^{CD} = \frac{1}{n} \sum_{i=1}^{n} c_i^a$$

(7)

By taking an implicit union approach, this class of measures complies with an axiom similar to the Pigou-Dalton transfer if $\alpha > 1$ and even for more general identification approaches. For $\alpha = 2$, $P_a^{CD}$ can be rewritten as the sum of $\left(p_0^{AF}\right)^2$ and the variance of the society deprivation scores $\sigma^2$:

$$P_2^{CD} = \left(p_0^{AF}\right)^2 + \sigma^2$$

(8)

Unlike the Rippin class of measures, $P_a^{CD}$ does not allow for factor decomposability. However, since $P_a^{CD}$ can also be viewed as a formulation of the FGT measure, when $\alpha = 2$, it can be also expressed as

$$P_2^{CD} = HA^2 \left[1 + \left(\frac{\sigma_p^2}{A^2}\right)\right]$$

(9)

with $\sigma_p^2$ the variance of deprivation scores among the poor. We note that the ratio $\frac{\sigma_p^2}{A^2}$ is somehow analogous to the square of the coefficient of variation of weighted deprivations among the poor.

The Silber and Yalonetzky (2013) measure

Drawing on the framework of Aaberge and Peluso (2012), Silber and Yalonetzky (2013) developed a social poverty function that can take into account different methods of identifying poor individuals. They consider

$$S(h) = \Pr(c_i \geq h)$$

for a number of deprivations $h$ and suggest the following social poverty function:

$$p^{ST}(x,z) = \frac{1}{m-k+1} \sum_{h=k}^{m} \Gamma(S(h))$$

(10)

where $\Gamma$ is a non-decreasing function taking the values $\Gamma(0) = 0$ and $\Gamma(1) = 1$. The first and second derivatives are $\Gamma' > 0$ and $\Gamma'' \leq 0$.

This class of measures corresponds to a union approach to poverty whenever $k = 1$. However, the choice of $k$ makes it possible to produce measures that identify poor individuals using an intermediate approach. Following Bérenger (2017), it is possible to generalize this measure and

\footnotesize

2 The axiom proposed by Chakravarty and d’Ambrosio (2006) is known as ‘non-decreasingness of marginal exclusion’.

3 Although expressions (4) and (7) are equivalent, they are not founded on the same concepts of inequality. Rippin (2010) introduces considerations of distributive justice at the identification step.
introduce weights. In this case, several individuals can then achieve the same deprivation scores with different combinations of deprivations depending on the weight vector. Let \( m' \) be the maximum number of non-zero deprivation.\(^4\)

Rank now the deprivation scores by rising levels of deprivation \( c_h \) with \( c_h \in [0,1] \) and \( h \) varying from 1 to \( m' \). A deprivation score equal to 1 indicates that an individual is deprived in every dimension. A cut-off value \( k \) implies then that we consider as multidimensionally poor all those individuals who have a deprivation score that is at least equal to \( c_k \). Let \( c \) refer to the vector of these deprivation scores. When identifying and counting the poor we will take into account the \((m'-k+1)\) values \( c \) which are not equal to 0.

We can then write the index \( P^{SY} \) as

\[
P^{SY} = \frac{m}{m' - k + 1} \sum_{h=k}^{m'} \omega_h \Gamma(S(h))
\]

In (11) \( \omega_h \) is defined as \( \omega_h = c_h - c_{h-1} \) and it is expressed as a function of the proportion of individuals whose deprivation score equals at least \( c_h \).

Note that the class \( P^{SY} \), which we defined in (11), may be decomposed as:

\[
P^{SY}(x,z) = \frac{m'}{m' - k + 1} \sum_{h=k}^{m'} \omega_h \left[ \Gamma[S(h)] - S(h) \right] + \frac{m'}{m' - k + 1} H \left[ A - c_{k-1} \right]
\]

In (12) \( A \) refers to the intensity of poverty while \( c_{k-1} \) corresponds to the smallest value of the deprivation score for not being considered as multidimensionally poor. On the right-hand side of (12) the first term measures the dispersion of the individual deprivations while the second term in brackets refers to the deprivation gap of the poor.

At the difference of the decomposition proposed by Rippin (2010), the dispersion component in (12) measures the spread of deprivation scores across the population. Note that this class of measures is not sub-group decomposable and does not allow computation of the contribution of the different dimensions to the overall level of poverty.

3 An empirical illustration: Malawi, Mozambique, Tanzania, and Zimbabwe in the 2000s

We now apply the multidimensional poverty measures presented in section 2 to four African countries classified as low-income countries that are among the least developed according to the HDI. Despite significant progress between 1990 and 2015, Mozambique ranked 181 out of 188 countries in 2015, followed by Malawi (170), Zimbabwe (154), and Tanzania (151). However, in the 2000s (2000–15), Mozambique became one of the fastest-growing economies in Southern Africa, with an average annual rate of growth of more than 7 per cent, close to that achieved by Tanzania (6.7 per cent) and higher than that of Malawi (5.4 per cent). By contrast, the average

\(^4\) Suppose \( m \) dimensions whose weights are given by the vector \( w = (w_1, \ldots, w_m) \) with \( \sum_{j=1}^{m} w_j = 1 \). In this case, \( m' \) is higher than the given number of dimensions \( m \).
annual growth rate was negative over the same period in Zimbabwe (-1.3 per cent). Because in those countries economic growth was mainly driven by the agriculture sector, and is vulnerable to climatic and price fluctuations, it is instructive to investigate the evolution of poverty in different periods separated by a few years.

Apart from the publications of the UNDP, there are only a few studies in these countries taking a multidimensional approach to poverty. Most of them rely on the Alkire and Foster (2011) approach. In a recent paper, Alkire and Housseini (2017) computed global MPIs for 46 countries in Africa and examined how multidimensional poverty changed over time in 19 sub-Saharan African countries, including Mozambique (2003–11), Malawi (2004–10), and Tanzania (2008–10). There are also country case studies, except for Malawi. Indeed, multidimensional poverty, using the Alkire and Foster (2011) approach, was analysed in the last National Poverty Assessment Report in Mozambique (2016) and by Cardoso et al. (2016), using census data (1997–2007) for Mozambique, and by Stoeffler et al. (2016) for Zimbabwe. We can also mention some studies using a first order stochastic dominance approach when analysing multidimensional poverty, such as that of Arndt et al. (2016) for Mozambique and Arndt et al. (2014) for Tanzania.

Since poverty measures based on the Alkire and Foster (2011) approach use a cross-dimensional cut-off, we carry out comparisons by modifying the identification function of the class of measures originally adopting a union identification approach.

### 3.1. Data description

UNDP (2010) used the Demographic and Health Surveys (DHS) to derive the MPI in several countries. They contain significant information on the living conditions for three different years in Malawi (2004, 2010, 2015) in Tanzania (2004, 2010, 2015) and Zimbabwe (2005, 2010, 2015) and for two years in Mozambique (2003, 2011). Table 1 presents the list of the indicators with the same dimensions as the HDI, namely education, health, and standard of living.
Table 1: List of dimensions and variables used to compute poverty measures

| Dimension      | Indicators                                                                 | Cut-off                                                                 | Relative weight |
|----------------|-----------------------------------------------------------------------------|--------------------------------------------------------------------------|-----------------|
| Education      | Child enrolment: Any school-aged child (6–15) not attending school          | 1/6                                                                      |                 |
|                | Years of schooling: No household member aged 10 years or older has completed 5 years of schooling | 1/6                                                                      |                 |
| Health         | Nutrition: One or more adults are underweight (in terms of BMI) or a child is undernourished (in terms of height for age) | 1/6                                                                      |                 |
|                | Mortality: Any child from a household who has died                           | 1/6                                                                      |                 |
| Standard of living | Water: No access to safe drinking water source within 30 minutes one-way distance from the residence | 1/18                                                                     |                 |
|                | Electricity: Household has no electricity                                     | 1/18                                                                     |                 |
|                | Sanitation: Household sanitation facility is not improved or shared          | 1/18                                                                     |                 |
|                | Floor: Household has rudimentary floor                                       | 1/18                                                                     |                 |
|                | Cooking fuel: Household cooks with dung, wood, charcoal, and other solid fuels | 1/18                                                                     |                 |
|                | Assets: Household does not own more than one radio, TV, telephone, bicycle, motorcycle, or refrigerator and does not own a car | 1/18                                                                     |                 |

Source: Author’s illustration based on DHS data.

In conformity with the methodology of the UNDP (2010), we adopted a nested weight structure, where each dimension has the same weight and each indicator for a given dimension also has the same weight.

3.2 Empirical results from the decomposition of poverty measures

Let us start with the approach of Alkire and Foster (2011) using various values of the cut-off $k$ which was defined previously. Of particular interest are the union and intermediate approaches with a threshold value of $k = 33\%$ (selected by UNDP 2010). Tables 2 and 3 present poverty estimates based on the Alkire and Foster (2011) measure ($P_{df}$) for Malawi and Mozambique and for Tanzania and Zimbabwe respectively.

Comparisons across countries shows that Mozambique, with 73.1 per cent of people being multidimensionally poor in at least 33 per cent of dimensions in 2011, registers the highest level of incidence of poverty, followed by Tanzania (63 per cent in 2015), Malawi (49.8 per cent in 2015), and Zimbabwe (36.4 per cent in 2015). The ranking remains almost the same over time, whatever the value of $k$ (see Tables 2 and 3).

As shown in Figure A1 in the Appendix, the incidence of poverty ($H$) reaches values close to 0 with $k = 99\%$ for Mozambique in 2011; $k = 84\%$ for Tanzania in 2015; $k = 77\%$ for Malawi in 2015, and $k = 67\%$ for Zimbabwe in 2015. This suggests that the degree of association between

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5 Information on adults’ nutrition refers to women of reproductive age. The nutritional indicator used is the Body Mass Index (BMI). An adult is considered to be undernourished if he or she has a BMI lower than 18.5 kg/m². Following the revised MPI introduced in the UNDP’s (2014) Human Development Report, the nutritional indicator used for children (0–59 months) is stunting (height for age) rather than underweight. As argued by Dotter and Klasen (2014), stunting is a better indicator than underweight as it is less sensitive to influences from the nutrition transition and is more responsive to the quantity as well to the quality of nutrition.
the dimensions of poverty is higher in Mozambique and Tanzania than in Malawi and Zimbabwe. As is evident from Figure A1, multidimensional poverty decreases over time, at the national level, in Malawi, Mozambique, and Tanzania. This is true whichever approach to multidimensional poverty we select. Assuming a cut-off $k$ equals 33%, we then observe that the incidence of poverty, which was equal to 76.9 per cent in 2004 decreased to 49.8 per cent in 2015 for Malawi. There was a similar reduction in poverty from 83.1 per cent in 2003 to 73.1 per cent in Mozambique, and from 76.7 per cent in 2004 to 63 per cent in 2015 for Tanzania (see Tables 2 and 3). By contrast, the evolution is blurred for Zimbabwe as revealed by the incidence curves (Figure 1d) and the contradictory conclusions drawn from using different values of $k$. In particular, poverty increases when adopting an extensive approach to poverty, while it decreases when focusing on people that cumulate simultaneous deprivation (for $k = 33\%$).
Table 2: Poverty measures of Alkire and Foster (2011)

|                | H               | RC(%)          | AARC(%)         | M₀          | RC(%)          | AARC(%)         | A          |
|----------------|-----------------|----------------|-----------------|-------------|----------------|-----------------|------------|
|                | 2004 2010 2015  | 2004–10 2010–15 2004–15 2004 2010 2015 2004–10 2010–15 2004–15 2004 2010 2015 | 2004–10 2010–15 2004–15 2004 2010 2015 | 2004–10 2010–15 2004–15 2004 2010 2015 | 2004–10 2010–15 2004–15 2004 2010 2015 |
| Malawi         |                 |                |                 |             |                |                 |            |
| National       | 0.990 0.991 0.986 | 0.10 -0.53 -0.04 | 0.476 0.403 0.332 | -15.31 -17.70 -3.23 | 48.111 40.702 33.676 |
| Urban          | 0.945 0.950 0.914 | 0.53 -3.76 -0.30 | 0.302 0.278 0.198 | -7.98 -28.71 -3.76 | 31.929 29.226 21.647 |
| Rural          | 0.998 0.999 0.998 | 0.05 -0.08 -0.003 | 0.508 0.427 0.355 | -15.98 -16.92 -3.22 | 50.916 42.756 35.549 |
| k=union        |                 |                |                 |             |                |                 |            |
| National       | 0.769 0.658 0.498 | -14.387 -24.292 -3.865 | 0.428 0.330 0.230 | -22.94 -30.13 -5.47 | 55.634 50.076 46.213 |
| Urban          | 0.419 0.361 0.177 | -13.748 -51.108 -7.549 | 0.202 0.168 0.074 | -16.75 -56.01 -8.73 | 48.177 46.503 41.841 |
| Rural          | 0.833 0.714 0.553 | -14.264 -22.555 -3.654 | 0.469 0.360 0.257 | -23.25 -28.65 -5.33 | 56.321 50.416 46.451 |
| Mozambique     |                 |                |                 |             |                |                 |            |
| National       | 0.988 0.981 0.988 | -0.68 -0.09 0.550 0.471 | -14.45 -1.93 | 55.705 47.782 |
| Urban          | 0.966 0.945 0.965 | -2.13 -0.27 0.380 0.301 | -20.79 -2.87 | 39.354 31.852 |
| Rural          | 0.999 0.998 0.998 | -0.13 -0.02 0.634 0.548 | -13.54 -1.80 | 63.460 54.940 |
| k=33%          |                 |                |                 |             |                |                 |            |
| National       | 0.831 0.731 0.731 | -12.10 -1.60 0.521 0.425 | -18.49 -2.52 | 62.750 51.186 |
| Urban          | 0.578 0.423 0.423 | -26.91 -3.84 0.312 0.217 | -30.61 -4.46 | 53.996 51.261 |
| Rural          | 0.955 0.871 0.871 | -8.83 -1.15 0.624 0.520 | -16.69 -2.26 | 63.383 59.719 |

Note: RC is the relative change in % and AARC is the average annualized relative change in %.

Source: Author’s calculation based on DHS data.
Table 3: Poverty measures of Alkire and Foster (2011)

|               | H            | RC(%)        | AARC(%)      | M0  | RC(%)        | AARC(%)      | A   |
|---------------|--------------|--------------|--------------|-----|--------------|--------------|-----|
| **Tanzania**  |              | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015|
| k=union       |              | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015|
| National      | 0.999        | 0.993 | 0.990 | -0.57 | -0.30  | -0.08 | 0.482 | 0.438 | 0.400 | -9.11 | -8.77  | -1.69 | 48.244 | 44.102 | 40.358 |
| Urban         | 0.996        | 0.973 | 0.971 | -2.33 | -0.18  | -0.21 | 0.319 | 0.286 | 0.255 | -10.30 | -10.70 | -1.99 | 31.990 | 29.380 | 26.285 |
| Rural         | 1.000        | 1.000 | 0.998 | -0.05 | -0.12  | -0.01 | 0.533 | 0.484 | 0.459 | -9.16 | -5.11  | -1.34 | 53.289 | 48.428 | 46.009 |
| k=33%         |              | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015|
| National      | 0.767        | 0.711 | 0.630 | -7.28 | -11.43 | -1.77 | 0.434 | 0.381 | 0.331 | -12.28 | -13.02 | -2.43 | 56.581 | 53.532 | 52.574 |
| Urban         | 0.459        | 0.401 | 0.327 | -12.70 | -18.24 | -2.77 | 0.223 | 0.188 | 0.152 | -15.43 | -19.09 | -3.39 | 48.501 | 46.983 | 46.493 |
| Rural         | 0.863        | 0.805 | 0.755 | -6.73 | -6.24  | -1.11 | 0.500 | 0.439 | 0.405 | -12.21 | -7.71  | -1.89 | 57.919 | 54.516 | 53.663 |
| **Zimbabwe**  |              | 2004 | 2010 | 2015 | 2005-10 | 2010-15 | 2005-15 | 2005 | 2010 | 2015 | 2005-10 | 2010-15 | 2005-15 | 2005 | 2010 | 2015|
| k=union       |              | 2004 | 2010 | 2015 | 2005-10 | 2010-15 | 2005-15 | 2004 | 2010 | 2015 | 2005-10 | 2010-15 | 2005-15 | 2004 | 2010 | 2015|
| National      | 0.880        | 0.917 | 0.911 | 4.20  | -0.65  | 0.35 | 0.285 | 0.270 | 0.259 | -5.30 | -4.10  | -0.96 | 32.386 | 29.436 | 28.415 |
| Urban         | 0.627        | 0.755 | 0.733 | 20.43 | -2.88  | 1.58 | 0.103 | 0.131 | 0.117 | 27.03 | -10.79 | 1.26 | 16.473 | 17.376 | 15.959 |
| Rural         | 0.990        | 0.985 | 0.986 | -0.51 | -0.06  | -0.04 | 0.364 | 0.328 | 0.319 | -9.82 | -2.98  | -1.33 | 36.780 | 33.339 | 32.925 |
| k=33%         |              | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015 | 2004-10 | 2010-15 | 2004-15 | 2004 | 2010 | 2015|
| National      | 0.433        | 0.389 | 0.364 | -10.05 | -6.41  | -1.71 | 0.201 | 0.173 | 0.161 | -13.98 | -6.78  | -2.19 | 46.490 | 44.459 | 44.280 |
| Urban         | 0.060        | 0.106 | 0.083 | 76.59 | -21.37 | 3.34 | 0.023 | 0.043 | 0.034 | 89.03 | -20.87 | 4.11 | 38.396 | 41.099 | 41.362 |
| Rural         | 0.596        | 0.509 | 0.483 | -14.51 | -5.11  | -2.07 | 0.279 | 0.228 | 0.215 | -18.33 | -5.67  | -2.57 | 46.846 | 44.753 | 44.492 |

Note: RC is the relative change in % and AARC is the average annualized relative change in %.

Source: Author’s calculation based on DHS data.
Let us now take a look at the different components of the Alkire and Foster (2011) approach and find out whether the decrease in the percentage $H$ of poor people was accompanied by a reduction in the intensity $A$ of poverty $M$.

More generally, adopting the union approach to the identification of the multidimensional poor contributes to inflating the level of poverty (Tables 2 and 3). This approach is difficult to justify when one wants to focus on deprivations that reflect poverty, and to distinguish and target the most extensively deprived individuals.

The analysis of the evolution of the components of poverty estimates based on the Alkire and Foster (2011) approach provides instructive information on the trends of poverty over time for each country and region of residence.

In Malawi, poverty (in relative terms) decreased more between 2010 and 2015 than between 2004 and 2010. This reduction in poverty covered both urban and rural areas, this being true for the whole period, but there were important differences between the two sub-periods (see Table 2). In rural areas, which includes the majority of the population, poverty is higher, whether we measure it via $H$, $A$ or $M$. As far as the changes are concerned, we observe that while the decrease in poverty was higher in rural areas than in urban areas during the first sub-period 2004–05, this was no longer the case between 2010 and 2015, irrespective of the identification approach adopted. In fact, urban areas registered an acceleration of the poverty decline between 2010 and 2015, as $M$ decreased by almost 56 per cent from its value of 2010, in comparison to the decline by 16.75 per cent during the period 2004–10. The results show that the high performance in urban areas was due to the joint impact of higher rates of decrease in $H$ and $A$ than the ones that occurred in rural areas. While a catch-up process was in place in rural areas during the first sub-period, this was no longer the case during the second sub-period, 2010–15, as the high performance recorded in urban areas contributes to widening the gap with rural areas, whatever the values of $k$.

In Mozambique, poverty declined at the national level between 2003 and 2011. However, the pace of poverty reduction was lower in comparison with Malawi in spite of a high level of multidimensional poverty (Table 2). Poverty estimates reveal striking disparities between urban and rural areas. The percentage of rural multidimensionally poor people is roughly three times the level registered in urban areas using $k=33\%$ (87.1 per cent vs. 42.3 per cent) This is due to the faster rate of decrease in poverty in urban than in rural areas according to $H$ and $M$. However, the poor in rural areas experienced a higher reduction in the number of their deprived dimensions ($A$) than in urban areas, as the contribution of the intensity effect represents more than 50 per cent (only roughly 12 per cent in urban areas) of the variation of $M$, using $k=33\%$.

In Tanzania, all poverty measures ($H$ and $M$) also declined at the national level but at different paces over the two sub-periods (Table 3) depending on the value of $k$. Hence, poverty falls more slowly between 2010 and 2015 than between 2004 and 2010 when using the union approach, while an opposite conclusion holds for $k=33\%$. However, the decomposition of poverty measures by area of residence gives us a clearer idea of the trends during these two sub-periods, particularly for rural areas. As is evident from Table 3, in rural areas poverty decreases at a slower rate than in urban areas during the two sub-periods, despite the fact that in rural areas there was a higher reduction in $A$ than in urban areas, whatever value of $k$ we select. In addition, between 2010 and 2015, the decline in rural poverty slows down, deepening the gap with urban areas.

Turning now to the case of Zimbabwe, we note that poverty levels are significantly lower than in the three other countries. There was a decline in poverty at the national level over the two sub-periods, but this trend is more ambiguous when adopting an extensive view of poverty (see Table 3). Indeed, the percentage increase in the multidimensionally poor between 2005 and 2010 (4.19
per cent) is compensated for by a higher decline in the share of deprivations among the poor, implying a relative decline in the adjusted head count ratio ($M_0$) by 5.3 per cent. In addition, the second sub-period witnesses a slowdown in poverty reduction, whatever the value of $k$.

However, these trends conceal a non-monotonic evolution of poverty according to the area of residence. As is evident from Table 3, poverty increases in urban areas between 2005 and 2010 and then declines during the second sub-period 2010–15, irrespective of the identification approach adopted. However, the decline in poverty during the second sub-period was not sufficient to recover at least the initial levels of 2005. The incidence of poverty ($H$) in 2015 is higher than in 2005, and the poor were poorer because they suffered from a higher number of deprivations, implying an increase of $M_0$ over the whole period. In contrast, there are clear continued reductions in multidimensional poverty rates in rural areas, where most of the population lives in 2015. However, the estimates tend to show a substantial slowdown in poverty reduction between 2010 and 2015, due to the lower variations in the incidence ($H$) as well as in the intensity of poverty ($A$). The variations in $H$ and $A$ were significantly lower than in urban areas during the same period, whatever the identification approach adopted. Therefore, as shown in Table 3, the magnitude of the gap between rural and urban poverty rates is illustrative of these contrasting developments. Although there was a clear decline in the disparities between rural and urban areas between 2005 and 2010, largely because of the increase in urban poverty, variations in poverty levels during the second sub-period show a rising trend in the rural and urban gap, even though the gap in 2015 is significantly lower than in 2005.

Let us now see what happens when we analyse the changes in poverty via indices of multidimensional poverty that take into account the impact of the spread of the deprivations across individuals.

The results of such an analysis appear in Tables 4 and 5. There, for each of the four countries, we give the values of the poverty measures belonging to the class of the Chakravarty and D’Ambrosio (2006) ($P^{CD}$) family of social exclusion indices ($\alpha=2$). In Tables 6 to 9 we present results derived from Rippin’s (2010) family of indices ($P^{RI}$), assuming that the parameter $\gamma$ is equal to 1.5. We also give the results obtained when adopting the approach of Silber and Yalonetzky (2013) ($P^{SY}$).

We cannot really compare these results with the values obtained for $M_0$ because these alternative approaches measure deprivation in the whole population while the Alkire and Foster (2011) index estimates it among the poor. Even when adopting the union approach, we cannot really make such a comparison because these measures depend on an inequality aversion parameter. If we select higher values of the parameter, we give a higher weight to greater levels of deprivation. Tables 4 and 5 for the Chakravarty and D’Ambrosio (2006) measures, and Tables A1 and A2 in the Appendix (Tables 4 and 5) for the Rippin (2010) measures, show that, for the union approach, poverty estimates decrease as $\alpha$ ($\gamma$) increases. We observe also that the rural/urban gap ratio rises when higher values of $\alpha$ or $\gamma$ are selected, this being true for each of the four countries. It thus appears that when we give a higher weight to the populations that are the most deprived in rural areas, we are probably focusing on individuals deprived in several dimensions, while this is apparently not the case in urban areas. This implies that there is a higher correlation between the deprivations in rural than in urban areas, at least among those who are the most deprived.

Straightforward conclusions cannot really be drawn when comparing these different measures. However, it is possible to take advantage of the information conveyed by the decomposition of these measures using the same identification as for the Alkire and Foster (2011) measure, which corresponds in our case to a value of $k = 33\%$.  

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Given that the Chakravarty and D’Ambrosio (2006) measure ($P^{CD}$) for $\alpha = 2$ can be expressed as the sum of the square of the average deprivation scores and of their variance, we observe that as the poverty measure decreases (from $k = \text{union}$ to $k = 33\%$), the contribution of its inequality component increases when emphasis is put on the multidimensionally poor $k = 33\%$ (see Tables 4 and 5).
Table 4: Poverty measures of Chakravarty and D’Ambrosio (2006) (P^{CD} with $\alpha=2$)

|          | $P^{CD}$ | RC(%) | AARC(%) | $\sigma^2$ | RC(%) |
|----------|----------|-------|---------|------------|-------|
| **Malawi** |          |       |         |            |       |
| National | 0.269    | 0.194 | 0.136   | -27.77     | -29.94|
|          | 0.042    | 0.032 | 0.026   | -3.92      | -2.92 |
| Rural    | 0.295    | 0.211 | 0.150   | -28.56     | -29.09|
|          | 0.037    | 0.028 | 0.024   | -2.81      | -2.29 |
| **k=union** |          |       |         |            |       |
| National | 0.26     | 0.18  | 0.11    | -31.41     | -36.15|
|          | 0.075    | 0.068 | 0.060   | -2.56      | -4.68 |
| Rural    | 0.28     | 0.19  | 0.13    | -31.94     | -34.91|
|          | 0.065    | 0.065 | 0.060   | -0.26      | -2.16 |
| **Mozambique** | 0.359    | 0.278 |         | -22.69     | -3.17 |
| National | 0.359    | 0.278 |         | -22.69     | -3.17 |
|          | 0.056    | 0.056 |         | -0.08      |       |
| Urban    | 0.199    | 0.138 |         | -31.06     | -4.54 |
|          | 0.055    | 0.047 |         | -4.09      |       |
| Rural    | 0.438    | 0.342 |         | -21.96     | -3.05 |
|          | 0.036    | 0.041 |         | 1.22       |       |
| **k=33%** |          |       |         |            |       |
| National | 0.353    | 0.268 |         | -24.18     | -3.40 |
|          | 0.081    | 0.087 |         | 1.66       |       |
| Urban    | 0.185    | 0.120 |         | -35.14     | -5.27 |
|          | 0.088    | 0.073 |         | -7.86      |       |
| Rural    | 0.436    | 0.335 |         | -23.09     | -3.23 |
|          | 0.046    | 0.065 |         | 4.28       |       |

Note: RC is the relative change in % and AARC is the average annualized relative change in %. Columns RC(%) display changes in the components expressed as a percentage of the value of poverty for the initial year of the period.

Source: Author’s calculation based on DHS data.
Table 5: Poverty measures of Chakravarty and D’Ambrosio (2006) (PCD with $\alpha=2$)

|                | 2004 | 2010 | 2015 | 2004–10 | 2010–15 | 2004–15 | 2004 | 2010 | 2015 | 2004–10 | 2010–15 | RC(%) |
|----------------|------|------|------|---------|---------|---------|------|------|------|---------|---------|-------|
| **Tanzania**   |      |      |      |         |         |         |      |      |      |         |         |       |
| National       | 0.278| 0.234| 0.204| -15.85  | -12.85  | -2.78   | 0.045| 0.042| 0.044| -1.30   | 0.93    |       |
| Urban          | 0.137| 0.115| 0.096| -15.73  | -16.41  | -3.13   | 0.035| 0.034| 0.031| -1.22   | -2.05   |       |
| Rural          | 0.321| 0.269| 0.248| -16.22  | -7.97   | -2.34   | 0.038| 0.035| 0.037| -0.77   | 0.70    |       |
| **Zimbabwe**   |      |      |      |         |         |         |      |      |      |         |         |       |
| National       | 0.266| 0.220| 0.188| -17.30  | -14.56  | -3.11   | 0.078| 0.075| 0.079| -1.007  | 1.45    |       |
| Urban          | 0.117| 0.095| 0.076| -18.41  | -20.26  | -3.83   | 0.067| 0.060| 0.053| -6.317  | -7.40   |       |
| Rural          | 0.313| 0.258| 0.235| -17.56  | -9.10   | -2.59   | 0.063| 0.066| 0.071| 0.747   | 1.96    |       |

| **k=33%**      |      |      |      |         |         |         |      |      |      |         |         |       |
| National       | 0.118| 0.103| 0.096| -13.08  | -6.16   | -2.02   | 0.037| 0.030| 0.029| -5.98   | -0.46   |       |
| Urban          | 0.023| 0.034| 0.029| 47.85   | -16.12  | 2.18    | 0.013| 0.017| 0.015| 19.71   | 5.89    |       |
| Rural          | 0.159| 0.131| 0.125| -17.55  | -5.07   | -2.42   | 0.027| 0.023| 0.023| -2.00   | -0.25   |       |

| **Zimbabwe**   |      |      |      |         |         |         |      |      |      |         |         |       |
| National       | 0.100| 0.082| 0.076| -18.23  | -7.41   | -2.74   | 0.059| 0.052| 0.050| -7.68   | -2.60   |       |
| Urban          | 0.009| 0.019| 0.015| 108.89  | -21.03  | 5.13    | 0.009| 0.017| 0.014| 93.81   | -17.28  |       |
| Rural          | 0.140| 0.108| 0.101| -22.44  | -6.43   | -3.15   | 0.062| 0.056| 0.055| -3.86   | -1.14   |       |

Note: RC is the relative change in % and AARC is the average annualized relative change in %. Columns RC(%) display changes in the components expressed as a percentage of the value of poverty for the initial year of the period.

Source: Author's calculation based on DHS data.
Similarly, Rippin’s (2010) measure allows a multiplicative decomposition into the incidence ($H$), the intensity ($A$), and the inequality of deprivations among the poor. Tables 6 and 7 present results of the decomposition of poverty measures based on Rippin’s (2010) approach for a value of the parameter of the degree of poverty severity $\gamma$ equal to 1.5. Rippin’s (2010) measures ($P^R$) decrease when the value of $k$ increases. This is also true for the inequality in deprivations among the poor, as shown by the values of $GE^*_2$ (Tables A1 and A2). As the value of $k$ increases, the contribution of the inequality component among the poor decreases, while the contribution of $H$ to the poverty measure increases (Tables 6 and 7).
Table 6: Variations of Rippin’s (2010) measure and their components for $\gamma = 1.5$

|        | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ |
|--------|-----------------|------------|-------------------|-------------------|-----------------|------------|-------------------|-------------------|-----------------|------------|-------------------|-------------------|
| Malawi | 2004–2010       | 2010–2015  |                   |                   | Malawi         | 2003–2011  |                   |                   | Mozambique     |            |                   |                   |
| Union  |                 |            |                   |                   |                |            |                   |                   |                |            |                   |                   |
| National | -33,03         | 0,10       | -34,17            | 1,62              | -34,80         | -0,53      | -37,73            | 5,26              | National       | -25,79     | -0,68             | -31,14            | 8,52            |
| Urban   | -19,75          | 0,53       | -19,84            | -0,42             | -52,31         | -3,76      | -52,78            | 4,93              | Urban          | -34,97     | -2,13             | -41,07            | 12,75           |
| Rural   | -33,80          | 0,05       | -35,38            | 2,39              | -33,96         | -0,08      | -36,96            | 4,84              | Rural          | -25,09     | -0,13             | -30,26            | 7,56            |
| $k=33\%$ |                 |            |                   |                   |                |            |                   |                   |                |            |                   |                   |
| National | -35,46          | -14,39     | -23,14            | -1,92             | -39,10         | -24,29     | -18,18            | -1,69             | National       | -26,68     | -12,10            | -17,20            | 0,73            |
| Urban   | -23,63          | -13,75     | -8,46             | -3,27             | -63,12         | -51,11     | -23,21            | -1,78             | Urban          | -37,58     | -26,91            | -12,19            | -2,75           |
| Rural   | -36,04          | -14,26     | -24,19            | -1,60             | -37,97         | -22,56     | -18,52            | -1,70             | Rural          | -25,76     | -8,83             | -20,18            | 2,02            |

Note: The sum of the variations of the components is not exactly equal to the variation of the measure since we have not used the logarithmic transformation of the measure. Rates of variations provide values that can be compared with values in Table 2.

Source: Author’s calculation based on DHS data.
Table 7: Variations of Rippin’s (2010) measure and their components for $\gamma = 1.5$

|        | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ |
|--------|----------------|-----------|------------------|-----------------|----------------|-----------|-----------------|-----------------|
| **Tanzania** |                |           |                  |                 |                |           |                  |                 |
| Union  |                |           |                  |                 |                |           |                  |                 |
| National | -18.76 | -0.57   | -20.10           | 2.256            | -14.24         | -0.30   | -19.90          | 7.39            |
| Urban   | -17.96 | -2.33   | -19.17           | 3.916            | -18.43         | -0.18   | -24.293         | 7.93            |
| Rural   | -19.23 | -0.05   | -21.27           | 2.631            | -9.04          | -0.12   | -12.027         | 3.52            |
| **Zimbabwe** |                |           |                  |                 |                |           |                  |                 |
| Union  |                |           |                  |                 |                |           |                  |                 |
| National | -19.72 | -7.28   | -12.93           | -0.56           | -15.33         | -11.43  | -4.415          | 0.01            |
| Urban   | -19.86 | -12.70  | -7.65            | -0.61           | -21.05         | -18.24  | -2.583          | -0.88           |
| Rural   | -20.10 | -6.73   | -14.05           | -0.33           | -9.78          | -6.24   | -3.866          | 0.09            |

$k=33\%$

|        | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ | $\Delta P_{RI}$ | $\Delta H$ | $\Delta A^{2.5}$ | $\Delta GE^{2.5}$ |
|--------|----------------|-----------|------------------|-----------------|----------------|-----------|-----------------|-----------------|
| **Tanzania** |                |           |                  |                 |                |           |                  |                 |
| National | -16.55 | 4.20     | -21.25           | 1.69            | -6.98          | -0.65   | -8.44           | 2.27            |
| Urban   | 63.92  | 20.43    | 14.27            | 19.12           | -18.09         | -2.88   | -19.15          | 4.31            |
| Rural   | -20.940 | -0.51   | -21.77           | 1.58            | -5.94          | 0.06    | -7.43           | 1.54            |
| **Zimbabwe** |                |           |                  |                 |                |           |                  |                 |
| National | -20.41 | -10.05  | -10.57           | -1.06           | -7.85          | -6.41   | -0.99           | -0.55           |
| Urban   | 122.97 | 76.59   | 18.54            | 6.51            | -21.56         | -21.37  | 1.61            | -1.82           |
| Rural   | -24.55 | -14.51  | -10.79           | -1.06           | -6.91          | -5.11   | -1.45           | -0.44           |

Note: The sum of the variations of the components is not exactly equal to the variation of the measure since we do not have used the logarithmic transformation of the measure. Rates of variations provide values that can be compared with values in Table 3.

Source: Author’s calculation based on DHS data.
Tables 8 and 9 show that the $P^{SY}$ may be decomposed into the sum of a function of the mean and of a measure of dispersion. We may also observe that the mean corresponds to the $M_0$ index in the case of the union approach, while it becomes a measure of the deprivations gap ratio in the population when selecting any other intermediate identification approach. In such a case, the measure corresponds to the smallest percentage (or amount) of deprivations that needs to be reduced in order to make sure the multidimensionally poor individuals move to a situation where they are just below the 1 cut-off value $k$. 


|                | p²sv   | RC(%) | Mean or gap | RC(%) | Gini MD | RC(%) |
|----------------|--------|-------|-------------|-------|---------|-------|
|                | 2004   | 2010  | 2015        | 2004  | 2010   | 2015  |
| **Malawi**     |        |       |             |       |        |       |
| Union          |        |       |             |       |        |       |
| National       | 0.592  | 0.503 | 0.422       | -15.06| -16.20 | 0.476 |
| Urban          | 0.406  | 0.373 | 0.273       | -8.08 | -26.94 | 0.302 |
| Rural          | 0.617  | 0.522 | 0.440       | -15.40| -15.60 | 0.508 |
|                |        |       |             |       |        |       |
|                | 2004–10| 2010–15| 2004–10| 2010–15| 2004–10| 2010–15|
| **Mozambique** |        |       |             |       |        |       |
| Union          |        |       |             |       |        |       |
| National       | 0.443  | 0.313 | 0.204       | -29.273| -34.75 | 0.296 |
| Urban          | 0.193  | 0.159 | 0.059       | -17.421| -62.60 | 0.114 |
| Rural          | 0.483  | 0.337 | 0.225       | -30.233| -33.22 | 0.334 |
|                |        |       |             |       |        |       |
| **Note:**      |        |       |             |       |        |       |
|                |        |       |             |       |        |       |
| **Source:**    | Author's calculation based on DHS data. | | | | | |
Table 9: Measure of Silber and Yalonetzky (2013) \( P^{SY} \) with \( \Gamma(S) = 2S - S^2 \)

|               | 2004   | 2010   | 2015   | 2004–10 | 2010–15 | Mean or gap | 2004   | 2010   | 2015   | 2004–10 | 2010–15 | Gini MD | 2004–10 | 2010–15 |
|---------------|--------|--------|--------|---------|---------|-------------|--------|--------|--------|---------|---------|---------|---------|---------|
| **Tanzania**  |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| Union         |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| National      | 0.603  | 0.554  | 0.519  | -8.16   | -6.34   | 0.482       | 0.438  | 0.400  | -7.28  | -6.94   | 0.121   | 0.116   | 0.119   | -0.88   |
| Urban         | 0.423  | 0.387  | 0.352  | -8.41   | -9.03   | 0.319       | 0.286  | 0.255  | -7.76  | -7.89   | 0.104   | 0.102   | 0.097   | -0.65   |
| Rural         | 0.643  | 0.590  | 0.568  | -8.22   | -3.67   | 0.533       | 0.484  | 0.459  | -7.60  | -4.20   | 0.110   | 0.106   | 0.109   | -0.62   |
| **k=33%**     |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| National      | 0.459  | 0.388  | 0.342  | -15.47  | -11.85  | 0.306       | 0.251  | 0.213  | -11.94 | -9.74   | 0.153   | 0.137   | 0.128   | -3.525  |
| Urban         | 0.220  | 0.180  | 0.146  | -18.25  | -18.59  | 0.127       | 0.102  | 0.081  | -11.52 | -11.71  | 0.092   | 0.077   | 0.065   | -6.733  |
| Rural         | 0.515  | 0.437  | 0.406  | -15.15  | -7.10   | 0.361       | 0.296  | 0.268  | -12.71 | -6.44   | 0.153   | 0.141   | 0.138   | -2.439  |
| **Zimbabwe**  |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| Universe      |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| National      | 0.394  | 0.367  | 0.355  | -7.04   | -3.19   | 0.283       | 0.270  | 0.259  | -3.392 | -3.014  | 0.109   | 0.097   | 0.096   | -3.053  |
| Urban         | 0.163  | 0.200  | 0.181  | 23.06   | -9.73   | 0.103       | 0.131  | 0.117  | 17.145 | -7.068  | 0.060   | 0.069   | 0.064   | 5.919   |
| Rural         | 0.455  | 0.420  | 0.403  | -7.69   | -4.04   | 0.364       | 0.328  | 0.319  | -7.851 | -2.329  | 0.091   | 0.088   | 0.085   | -0.654  |
| **k=33%**     |        |        |        |         |         |             |        |        |        |         |         |         |         |         |
| National      | 0.188  | 0.149  | 0.139  | -20.62  | -6.84   | 0.106       | 0.084  | 0.078  | -11.240| -4.279  | 0.080   | 0.085   | 0.061   | -8.06   |
| Urban         | 0.015  | 0.041  | 0.028  | 175.98  | -32.25  | 0.008       | 0.021  | 0.014  | 91.898 | -17.050 | 0.007   | 0.020   | 0.013   | 84.06   |
| Rural         | 0.246  | 0.191  | 0.179  | -22.49  | -5.96   | 0.083       | 0.150  | 0.105  | 20.460 | -4.012  | 0.096   | 0.078   | 0.074   | -7.14   |

Note: Columns (RC %) display changes in the components expressed as a percentage of the value of poverty for the initial year of the period. Gini MD is Gini mean difference. Source: Author’s calculation based on DHS data.
While all these measures provide trends in poverty levels that are consistent with those highlighted using the Alkire and Foster (2011) approach, an analysis of the trends in the inequality component may differ according to the measure and to the identification approach used. Considering results obtained at the national level and by areas of residence, salient differences between $P^{CD}$, $P^{RI}$, and $P^{SY}$ are captured by the sign of the variation of the inequality components. In most cases, they occur when using the union approach and $P^{RI}$. This is due to the fact, that the Rippin (2010) measure uses a relative measure of inequality, which belongs to the class of generalized entropy indices. In addition, inequality is derived from the deprivation scores of individuals identified as poor and the average intensity of poverty ($A$). By contrast, the inequality components correspond to the Gini mean difference in the $P^{SY}$ measure and the variance in $P^{CD}$ for $\alpha = 2$ and provide a measure of inequality across all individuals and not only among the poor. Moreover, as mentioned in section 2, there is an equivalence between $P^{CD}$ and $P^{RI}$, when $\alpha = \gamma + 1$. With a value of the parameter $\alpha$ equal to 2, it is also possible to provide an equivalent additive decomposition of $P^{CD}$ as a function of the square of the coefficient of variation of weighted deprivations among the poor, making it possible to recover consistent conclusions regarding the trends in inequality derived from $P^{RI}$.

The results show that the decline in poverty in Malawi was accompanied by a decrease in the concentration of deprivations at the national level and both in rural and urban areas for $P^{CD}$ and $P^{SY}$ measures, whatever the value of $k$ (Tables 4 and 8) and for the $P^{RI}$ measure when using $k = 33\%$ (Table 6). For the overall poverty measure, the acceleration in the reduction in poverty during the second sub-period was reinforced by a larger percentage decline in inequality according to $P^{CD}$ and $P^{SY}$.

For Mozambique, the decomposition makes it clear that the lower performance recorded in rural areas than in urban areas was due to the counteracting effect of the increase in the inequality in deprivations in rural areas while inequality decreased in urban areas using the $P^{CD}$ and $P^{SY}$ measures, whatever the value of $k$ and $P^{RI}$ for $k = 33\%$.

As for the case of Tanzania, while poverty trends derived from $P^{CD}$, $P^{RI}$, and $P^{SY}$ are consistent with those shown using the Alkire and Foster (2011) approach, the evolution in the inequality component depends on the value of $k$ chosen and the poverty measure used (see Tables 5, 7, and 9). Inequality decreases among the urban poor during the second sub-period, whatever the poverty measure used, whereas a diverging evolution is observed during the two sub-periods in particular in rural areas, when comparing $P^{CD}$, $P^{RI}$, and $P^{SY}$.

Turning now to the case of Zimbabwe, as is evident from Tables 5, 7, and 9, the decrease in poverty in rural areas seems to benefit the poorest of the poor, as inequality decreased, whatever the identification approach selected, except for $P^{RI}$ which shows conflicting results using the union approach. In addition, the non-monotonic evolution of poverty during the two sub-periods in urban areas was associated with an increase in inequality using $k = 33\%$ and for the three poverty measures. It is worth mentioning that the deprivation gap ratio for the rural population amounts to 7.33 per cent, compared to a deprivation gap ratio of more than 25 per cent in 2011 for the rural population in Mozambique.

Finally, as mentioned previously, one of the advantages of the Alkire and Foster (2011) approach is that, after identifying those who are multidimensionally poor, it allows one to break down the aggregate poverty measure into the sum of the contributions of the different dimensions. This obviously is a very useful property for policy makers aiming at targeting the poor. Such a property does not hold for multidimensional poverty measures that are sensitive to the breadth of the distribution, with the exception of the Rippin (2010) family of measures. Let us therefore compare
the results obtained with the breakdown proposed by Alkire and Foster (2011) and that derived from the Rippin (2010) measures.

In Figure 1 we give the relative contributions of the ten indicators to the overall poverty index introduced by Alkire and Foster (2011), assuming the cut-off $k$ is equal to 33%, as well as the corresponding contributions based on the Rippin (2010) measure, assuming that $\gamma = 1.5$ and $k = 33\%$. It should be stressed that with the Alkire and Foster (2011) approach we can interpret the censored headcount ratios with the share of the poor in the whole population. In Figure 2 we show the relative change in the intensity of poverty in each dimension, in both urban and rural areas, using the Alkire and Foster (2011) index as well as the Rippin (2010) measure with $\gamma = 1.5$ and $k = 33\%$.

When selecting the Alkire and Foster (2011) measure, we observe that, in urban areas, the most important contribution to overall poverty is related to deprivations in the health dimension, this being true for each of the four countries. Hence, nutrition has the highest contribution in urban areas in Mozambique, Tanzania, and Zimbabwe, and child mortality in Malawi. In rural areas, education (years of schooling) has the highest contribution in Malawi and Mozambique while deprivation in nutrition contributes the most to overall poverty in Tanzania and Zimbabwe. Note that the contribution of health is particularly disproportionally high in comparison with the contributions of other dimensions in urban areas. The analysis of trends in the contributions of the different indicators makes it possible to identify indicators for which the decline in deprivation has been the highest. We find that the most important contribution to the decrease in poverty came from nutrition in Malawi, mortality in Mozambique, and education in Tanzania and Zimbabwe, in both rural and urban areas. However, the contribution of child mortality increased most in urban areas in Malawi and in rural areas in Zimbabwe. The lowest progress is observed for sanitation for Mozambique, both in urban and rural areas, and in nutrition for Tanzania in urban areas. As for the case of Zimbabwe, which registered an increase in poverty between 2010 and 2015, in urban areas there was an increase in the contributions of indicators related to access to basic services such as water, floor, and possession of assets.
Figure 1: Decomposition of the M<sup>0</sup> index and the Rippin (2010) measure by dimensions and by areas of residence using κ=33%

Figure 1a: Urban Malawi

Figure 1b: Rural Malawi

Figure 1c: Urban and rural Mozambique

Figure 1d: Urban Tanzania

Figure 1e: Rural Tanzania

Figure 1f: Urban Zimbabwe

Figure 1g: Rural Zimbabwe

Source: Author’s calculation based on DHS data.
Turning to the Rippin (2010) decomposition that takes into account the dispersion in the distribution of deprivation counts, the results do not differ significantly. The contributions of the indicators related to health and education remain the most important. However, the Rippin (2010) measure provides higher values for these relative contributions than the Alkire and Foster (2011) decomposition. Figure 2 presents, by area of residence and for each country, the variation in deprivation among the poor for each indicator and using the two decompositions previously mentioned.

Figure 2: Relative variation of deprivation by dimension among the poor
The results obtained make it possible to highlight some differences between the two decompositions. We observe, for instance, that while deprivation related to child mortality increased among the poor in urban areas in Malawi, according to Alkire and Foster (2011), the Rippin (2010) decomposition shows that deprivation decreased. This is probably due to the fact that the poor cumulated fewer deprivations in the various dimensions in 2015 than was the case in 2004. We also observe an increase in the deprivation related to sanitation in Mozambique, irrespective of the decomposition, and both in urban and rural areas. Similarly, the low performance in nutrition and schooling in urban areas in Tanzania leads to an increase in deprivation among the poor for these dimensions, this being true for both decompositions. Finally, we also observe that the increase in poverty in urban areas in Zimbabwe was characterized by a higher level among the poor of deprivations related to access to basic services. While poverty declined in rural areas in Zimbabwe between 2005 and 2015, the poor experienced a higher level of deprivations related to child mortality.

4 Conclusion

Drawing on Bérenger (2017), our main goal was to compare results obtained when using the poverty measures proposed by Alkire and Foster (2011), such as the UNDP’s MPI, and counting-based poverty measures, which are sensitive to inequality. These include measures proposed by Chakravarty and D’Ambrosio (2006), by Rippin (2010), and the one suggested by Silber and Yalonetzky (2013). As the latter cover all deprived individuals using a union approach, we adopted a more flexible approach that fits the identification approach of the MPI. In that way, we showed how they could complement the information provided by the MPI with regard to inequality.

Poverty was estimated using Demographic and Health Surveys for three different years for Malawi (2004, 2010, and 2015), Mozambique (2003 and 2011), Tanzania (2005, 2010, and 2015), and Zimbabwe (2005, 2010, and 2015), by considering deprivations in education, health, and standard of living.

Our findings indicate that Mozambique shows the highest level of poverty, followed by Tanzania and Malawi, while Zimbabwe is the least poor. At the national level, all countries experienced a reduction in their multidimensional poverty, regardless of the poverty measures used. There are striking differences in the pace of poverty reduction among the four countries. Malawi registered the highest performance followed by Mozambique, Tanzania, and Zimbabwe.

When examining the evolution of poverty over time for each country, the use of various poverty measures provided insightful information on the evolution of the breadth and inequality components of poverty. In Malawi, the reduction in poverty between 2004 and 2015 was
accompanied by a decrease in the concentration of deprivations at the national level and both in rural and urban areas, though progress has been more in favour of the urban poor. By contrast, in Mozambique the lower performance registered in rural areas was due to an increase in inequality among the poor during the period 2003–11. In the case of Tanzania, while the poverty decrease reached the poorest of the poor in urban areas, the results are less conclusive for the rural poor. Finally, in Zimbabwe, poverty declined in rural areas during the two sub-periods, while urban poverty increased between 2005 and 2010 but decreased between 2010 and 2015. The decrease in poverty in rural areas seems to have benefited the poorest of the poor, since inequality decreased, while it increased in urban areas. We also exploited the dimensional decomposability of the Alkire and Foster (2011) measure and that of the Rippin (2010) measure to provide a better understanding of the main drivers of poverty trends in each country. The results showed that the most important contribution to the decrease in poverty came from nutrition in Malawi, mortality in Mozambique, and education in Tanzania and Zimbabwe, in both rural and urban areas. The increase in poverty in urban areas in Zimbabwe was characterized by a higher level among the poor of deprivations associated with access to basic services.

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Appendix

Figure A1: Multidimensional poverty headcount ratios

**Figure A1a: Malawi**

- MW 2004
- MW 2010
- MW 2015

**Figure A1b: Mozambique by area of residence**

- MZ 2003 Rural
- MZ 2003 Urban
- MW 2003 Urban
- MW 2011 Urban

**Figure A1c: Tanzania**

- TZ 2004
- TZ 2010
- TZ 2015

**Figure A1d: Zimbabwe**

- ZW 2009
- ZW 2010
- ZW 2015

Source: Author’s calculation based on DHS data.
Table A1: Rippin’s (2010) measure and inequality component for $\gamma = 1.5$

|                     | $\text{PI} (\gamma = 1.5)$ | $\text{GE}_{2.5}$ |
|---------------------|-----------------------------|-------------------|
|                     | 2004 | 2010 | 2004–15 | 2004 | 2010 | 2015 |
| **k=union**         |      |      |         |      |      |      |
| National            | 0.211 | 0.142 | 0.092  | -7.26 | 1.330 | 1.351 | 1.422 |
| Urban               | 0.089 | 0.071 | 0.034  | -8.36 | 1.636 | 1.629 | 1.709 |
| Rural               | 0.234 | 0.155 | 0.102  | -7.25 | 1.266 | 1.296 | 1.359 |
| **k=33%**           |      |      |         |      |      |      |
| National            | 0.206 | 0.133 | 0.081  | -8.14 | 1.159 | 1.136 | 1.117 |
| Urban               | 0.079 | 0.060 | 0.022  | -10.88 | 1.167 | 1.129 | 1.109 |
| Rural               | 0.229 | 0.146 | 0.091  | -8.06 | 1.154 | 1.136 | 1.116 |
| **Mozambique**      | 2003 | 2011 | 2003–11 | 2003 | 2011 |
| Union               |      |      |         |      |      |
| National            | 0.301 | 0.223 |         | -3.66 | 1.315 | 1.427 |
| Urban               | 0.154 | 0.100 |         | -5.24 | 1.646 | 1.856 |
| Rural               | 0.373 | 0.279 |         | -3.55 | 1.163 | 1.251 |
| **k=33%**           |      |      |         |      |      |
| National            | 0.298 | 0.218 |         | -3.81 | 1.150 | 1.158 |
| Urban               | 0.148 | 0.092 |         | -5.72 | 1.192 | 1.160 |
| Rural               | 0.372 | 0.276 |         | -3.65 | 1.127 | 1.150 |

Source: Author’s calculation based on DHS data.

Table A2: Rippin’s (2010) measure and inequality component for $\gamma = 1.5$

|                     | $\text{PI} (\gamma = 1.5)$ | $\text{GE}_{2.5}$ |
|---------------------|-----------------------------|-------------------|
|                     | 2004 | 2010 | 2004–15 | 2004 | 2010 | 2015 |
| **Tanzania**        |      |      |         |      |      |      |
| Union               |      |      |         |      |      |      |
| National            | 0.220 | 0.179 | 0.153  | -3.23 | 1.364 | 1.395 | 1.498 |
| Urban               | 0.096 | 0.079 | 0.065  | -3.59 | 1.674 | 1.739 | 1.878 |
| Rural               | 0.259 | 0.209 | 0.190  | -2.76 | 1.249 | 1.282 | 1.327 |
| **k=33%**           |      |      |         |      |      |      |
| National            | 0.215 | 0.172 | 0.146  | -3.45 | 1.156 | 1.156 | 1.156 |
| Urban               | 0.087 | 0.070 | 0.055  | -4.08 | 1.157 | 1.150 | 1.140 |
| Rural               | 0.255 | 0.203 | 0.184  | -2.93 | 1.156 | 1.152 | 1.153 |
| **Zimbabwe**        | 2005 | 2010 | 2015 | 2005–15 | 2005 | 2010 | 2015 |
| Union               |      |      |         |      |      |      |
| National            | 0.081 | 0.067 | 0.063  | -2.50 | 1.540 | 1.567 | 1.602 |
| Urban               | 0.012 | 0.020 | 0.016  | 2.99  | 1.739 | 2.072 | 2.161 |
| Rural               | 0.111 | 0.088 | 0.082  | -2.92 | 1.366 | 1.387 | 1.409 |
| **k=33%**           |      |      |         |      |      |      |
| National            | 0.072 | 0.057 | 0.053  | -3.05 | 1.132 | 1.120 | 1.114 |
| Urban               | 0.006 | 0.013 | 0.010  | 5.75  | 1.043 | 1.111 | 1.091 |
| Rural               | 0.101 | 0.076 | 0.071  | -3.47 | 1.132 | 1.120 | 1.115 |

Source: Author’s calculation based on DHS data.