Optimal Transmit Beamforming for Secrecy Integrated Sensing and Communication

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Abstract—This paper studies a secrecy integrated sensing and communication (ISAC) system, in which a multi-antenna base station (BS) aims to send confidential messages to a single-antenna communication user (CU), and at the same time sense several targets that may be suspicious eavesdroppers. To ensure the sensing quality while preventing the eavesdropping, we consider that the BS sends dedicated sensing signals (in addition to confidential information signals) that play a dual role of artificial noise (AN) for confusing the eavesdropping targets. Under this setup, we jointly optimize the transmit information and sensing beamforming at the BS, to minimize the matching error between the transmit beampattern and a desired beampattern for sensing, subject to the minimum secrecy rate requirement at the CU and the transmit power constraint at the BS. Although the formulated problem is non-convex, we propose an algorithm to obtain the globally optimal solution by using the semidefinite relaxation (SDR) together with a one-dimensional (1D) search. Next, to avoid the high complexity induced by the 1D search, we also present two sub-optimal solutions based on zero-forcing and separate beamforming designs, respectively. Numerical results show that the proposed designs properly adjust the information and sensing beams to balance the tradeoffs among communicating with CU, sensing targets, and confusing eavesdroppers, thus achieving desirable transmit beampattern for sensing while ensuring the CU’s secrecy rate.

Index Terms—Integrated sensing and communication (ISAC), physical layer security, transmit beamforming, secrecy rate, optimization.

I. INTRODUCTION

Integrated sensing and communication (ISAC) has been recognized as one of the potential key technologies towards future 5G and 6G wireless networks [1–3], in which sensing is integrated as a new functionality to enable emerging environment-aware applications such as auto-driving, industrial automation, and unmanned aerial vehicles (UAVs) [4]. As compared to conventional wireless networks with communication only, ISAC enables the dual use of wireless infrastructures and scarce spectrum/power resources for both communication and sensing, thus leading to enhanced system performance at reduced cost.

The emergence of ISAC introduces new data security issue for wireless networks, especially when information-bearing signals are reused for the purpose of sensing [5–7]. For example, to ensure the performance of both sensing and communication, the ISAC transmitters (e.g., cellular base stations (BSs)) need to focus their transmission (e.g., via transmit beamforming optimization [5–7]) towards both communication users (CUs) and targets. This, however, results in a severe information leakage issue, as the sensing targets can be untrusted and may eavesdrop the broadcast information signals [8].

To deal with this issue, the physical layer security (see, e.g., [9, 10]) has emerged as a viable solution to achieve perfectly secure information transmission in ISAC, by exploiting the difference between the legitimate communication channel and wiretap channels of eavesdropping targets [10]. In physical layer security, the secrecy rate is normally adopted as the performance metric, which is defined as the maximum transmission rate at which eavesdroppers are unable to decode any information [9]. Notice that the physical layer security is much easier to be implemented in ISAC systems than that in conventional secrecy communication systems. This is due to the fact that the ISAC transmitter can exploit the integrated sensing functionality to obtain the location and channel state information (CSI) of eavesdropping targets, thus resolving the eavesdroppers’ CSI issue faced in conventional secrecy communication systems.

To enhance the secrecy rate in ISAC systems while ensuring the sensing requirements, artificial noise (AN) has been widely employed to not only confuse the untrusted targets’ eavesdropping, but also play a dual role of sensing these targets [8]. For instance, the authors in [11] studied the secrecy ISAC system with one CU and one eavesdropping target, in which the transmit covariance matrices of both information signals and AN were jointly optimized to maximize the CU’s secrecy rate while ensuring the received signal-to-interference-plus-noise ratio (SINR) for sensing. The authors in [12, 13] further considered the scenario with multiple CUs and one eavesdropping target, in which the signal-to-noise ratio (SNR) at the eavesdropping target was minimized while ensuring the CUs’ individual SINR constraints and certain sensing beampattern requirements. Due to the non-convexity of the formulated problems, only sub-optimal beamforming/precoding designs were obtained in these prior works [11–13] under their respective setups.

Different from prior works, this paper studies a new secrecy ISAC scenario with one multi-antenna BS, one single-antenna...
CU, and multiple sensing targets, in which a portion of targets are untrusted suspicious eavesdroppers. Under this setup, we adopt the matching error between the exactly achieved transmit beampattern and a desired beampattern (with energy towards sensing targets) as the performance metric for sensing, and directly use the secrecy rate as the performance metric for secrecy communication. Our objective is to minimize the beampattern matching error for sensing by jointly optimizing the transmit information and sensing beamforming, subject to the minimum secrecy rate requirement at the CU and the transmit power constraint at the BS. Although the formulated problem is non-convex in general, we obtain its globally optimal solution by using the technique of semi-definite relaxation (SDR) together with a one-dimensional (1D) search, in which the tightness of SDR is rigorously proved. Next, to avoid the high complexity induced by the 1D search, we further develop two sub-optimal solutions based on zero-forcing (ZF) and separate beamforming designs, respectively. Finally, numerical results show that the proposed designs properly adjust the information and sensing beams to balance various tradeoffs among communicating with CU, sensing targets, and confusing eavesdroppers, thus achieving desirable transmit beampattern for sensing while ensuring the CU’s secrecy rate.

Notations: Vectors and matrices are denoted by bold lower- and upper-case letters, respectively. \( I \) and \( 0 \) represent an identity matrix and an all-zero vector/matrix with appropriate dimensions, respectively. For a square matrix \( A \), \( \text{Tr}(A) \) denotes its trace, and \( A \succeq 0 \) means that \( A \) is positive semi-definite. For an arbitrary-size matrix \( B \), \( \text{rank}(B) \), \( B^T \), and \( B^H \) denote its rank, transpose, and conjugate transpose, respectively. \( \mathbb{E}(\cdot) \) denotes the stochastic expectation, and \( \| \cdot \| \) denotes the Euclidean norm of a vector. \( \mathcal{CN}(\mu, \Sigma) \) denotes a circularly symmetric complex Gaussian (CSCG) random vector with mean vector \( \mu \) and covariance matrix \( \Sigma \). \( (x)^{+} \triangleq \max(x, 0) \).

II. SYSTEM MODEL

We consider a secrecy ISAC system, which consists of a BS equipped with a uniform linear array (ULA) with \( N \geq 1 \) antenna elements, a CU with one single antenna, and \( K \) sensing targets. Let \( \mathcal{K} = \{1, \ldots, K\} \) denote the set of targets, among which the first \( K_E \) ones (with \( K_E \leq K \)) are assumed to be untrusted or suspicious eavesdroppers, denoted by set \( \mathcal{K}_E \). We consider that the BS uses the linear transmit beamforming to send the confidential message \( s_0 \) to the CU, where \( s_0 \) is a CSCG random vector with zero mean and unit variance, i.e., \( s_0 \sim \mathcal{CN}(0,1) \), and \( w_0 \in \mathbb{C}^{N \times 1} \) denotes the corresponding transmit information beamforming vector. Besides the information signal \( s_0 \), the BS also sends dedicated sensing signal or equivalently AN \( s_1 \in \mathbb{C}^{N \times 1} \) to facilitate target sensing and confuse eavesdropping targets at the same time. Here, \( s_1 \) is an independent CSCG random vector with zero mean and covariance matrix \( S = \mathbb{E}(s_1s_1^H) \succeq 0 \), i.e., \( s_1 \sim \mathcal{CN}(0, S) \). Note that \( S \) is assumed to be of general rank, i.e., \( 0 \leq m = \text{rank}(S) \leq N \). This corresponds to the case with \( m \) sensing beams, each of which can be obtained via the eigenvalue decomposition (EVD) of \( S \). As a result, the transmitted signal by the BS is expressed as

\[
x = w_0s_0 + s_1.
\]

Suppose that the BS is subject to a total transmit power budget \( Q \). We thus have

\[
\mathbb{E}((||x||^2) = \text{Tr}(S) + ||w_0||^2 = Q.
\]

We consider a quasi-static channel model, in which the wireless channels remain unchanged over the time block of our interest, but may change from one block to another. Let \( g \in \mathbb{C}^{N \times 1} \) denote the channel vector from the BS to the CU. Accordingly, the received signal at the CU is expressed as

\[
y = g^Hw_0s_0 + g^Hs_1 + z,
\]

where \( z \sim \mathcal{CN}(0, \sigma_0^2) \) denotes the additive white Gaussian noise (AWGN) at the CU receiver. The received SINR at the CU is

\[
\gamma_0(w_0, S) = \frac{|g^Hw_0|^2}{g^HSc_0 + \sigma_0^2}.
\]

We consider the line of sight (LoS) channel from the BS to both trusted and untrusted targets, similarly as in prior works [12, 13]. Let \( \theta_k \) denote the angle of departure (AoD) from the BS to target \( k \). Accordingly, the steering vector with angle \( \theta_k \) is given as

\[
a(\theta_k) = \left[1, e^{j2\pi d \sin(\theta_k)}, \ldots, e^{j2\pi (N-1) d \sin(\theta_k)}\right]^T,
\]

where \( \lambda \) denotes the wavelength and \( d \) denotes the spacing between two adjacent antennas. With the LoS consideration, the channel vector from the BS to target \( k \) is denoted by \( h_k = \alpha_k a(\theta_k) \). Here, \( \alpha_k \) denotes the channel amplitude that is given by \( \alpha_k = \sqrt{2P_k} \), where \( P_k \) denotes the reference path loss at a distance of 1 meter, and \( D_k \) denotes the distance between the BS and target \( k \) in meters. The received signal at untrusted target \( k \in \mathcal{K}_E \) is denoted as

\[
y_k = h_k^Hw_0s_0 + h_k^Hs_1 + z_k,
\]

where \( z_k \sim \mathcal{CN}(0, \sigma_k^2) \) denotes the AWGN at the receiver of untrusted target \( k \). Accordingly, the received SINR at untrusted target \( k \) is

\[
\gamma_k(w_0, S) = \frac{|h_k^Hw_0|^2}{h_k^HSc_k + \sigma_k^2}.
\]

It is assumed that the BS perfectly knows the CSI \( g \) of the CU via channel estimation and feedback, and knows the CSI \( h_k \)'s of untrusted targets via efficient sensing. In this case, the achievable secrecy rate at the CU (in bits-per-second-per-Hertz, bps/Hz) is given by [9]

\[
r(w_0, S) = \min_{k \in \mathcal{K}_E} \left( \log_2(1 + \gamma_k(w_0, S)) - \log_2(1 + \gamma_0(w_0, S)) \right)^+.
\]

Next, we consider the targets sensing, for which the transmit beampattern gain is used as the performance metric. For any sensing angle \( \theta \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \), the beampattern gain \( P(\theta) \) is defined as the transmit signal power distribution at \( \theta \), i.e.,

\[
P(\theta) = \mathbb{E} \left( |a^H(\theta)(s_1 + w_0s_0)|^2 \right) = a^H(\theta)(S + w_0w_0^H)a(\theta).
\]
In particular, with the roughly known targets’ locations \( \theta_k \)'s, we define the desired beampattern \( \hat{P}(\theta) \) as

\[
\hat{P}(\theta) = \begin{cases} 1 & \exists k \in K, |\theta - \theta_k| < \Delta \theta, \\ 0 & \text{otherwise}, \end{cases}
\]

where \( \Delta \theta \) denotes the width of beampattern angle. Accordingly, we use the beampattern matching error as the sensing performance metric, which measures the difference between the actual transmit beampattern in angular domain versus the desired beampattern, i.e.,

\[
B((\hat{\theta}_m)_{m=1}^M, w_0, S, \eta) = \sum_{m=1}^M |\eta \hat{P}(\hat{\theta}_m) - a^H(\hat{\theta}_m)(S + w_0w_0^H)a(\hat{\theta}_m)|^2,
\]

where \( \{\hat{\theta}_m\}_{m=1}^M \) denote the \( M \) sample angles over \([-\pi, \pi]\) and \( \eta \) is a scaling factor to be determined. Notice that a larger value of \( M \) may lead to more accurate beampattern matching, but at the cost of increased computation complexity.

Our objective is to minimize the beampattern matching error in (11) by jointly optimizing the sensing and information beamforming design \((w_0, S)\), subject to the minimum secrecy rate constraint of \( R_0 \) and the transmit power constraint in (2). The secrecy rate constrained sensing beampattern matching problem is thus formulated as

\[(P1): \min_{w_0, S, \eta} \sum_{m=1}^M |\eta \hat{P}(\hat{\theta}_m) - a^H(\hat{\theta}_m)(S + w_0w_0^H)a(\hat{\theta}_m)|^2 \]

s.t. \( r(w_0, S) \geq R_0 \), \( \text{Tr}(S) + w_0w_0^H = Q, S \succeq 0 \).

Notice that problem (P1) is non-convex due to the non-convex objective function and the non-concave secrecy rate in (12a). We will solve problem (P1) optimally in Section III.

Before proceeding, we check the feasibility of problem (P1), which is equivalent to solving the following secrecy rate maximization problem (P2).

\[(P2): \max_{w_0, S} r(w_0, S) \]

s.t. \( \text{Tr}(S) + w_0w_0^H = Q, S \succeq 0 \).

Notice that problem (P2) has been optionally solved in [15]. Let \( R^* \) denote the optimal objective value or the maximum secrecy rate achieved in (P2). If \( R^* \geq R_0 \), then problem (P1) is feasible; otherwise, it is infeasible. Therefore, the feasibility of (P1) has been checked. In the sequel, we focus on the case when problem (P1) is feasible.

### III. Optimal Joint Beamforming Solution to Problem (P1)

This section presents the globally optimal joint beamforming solution to problem (P1) by using the technique of SDR together with a 1D search. Towards this end, we introduce \( W = w_0w_0^H \), where \( W \succeq 0 \) and \( \text{rank}(W) \leq 1 \). Accordingly, problem (P1) is equivalently reformulated as

\[(P3): \min_{W, S, \eta} \sum_{m=1}^M |\eta \hat{P}(\hat{\theta}_m) - a^H(\hat{\theta}_m)(S + W)a(\hat{\theta}_m)|^2 \]

s.t. \( \log_2 \left( 1 + \frac{g^H W g}{g^2 S g + \sigma_0^2} \right) \geq R_0, \forall k \in K_E \),

\( \text{Tr}(S) + \text{Tr}(W) = Q, \)

\( \text{rank}(W) \leq 1, \)

\( S \succeq 0, W \succeq 0 . \)

Next, we further introduce an auxiliary optimization variable \( \gamma_E > 0 \), which denotes the maximum SINR at the \( K_E \) eavesdropping targets. As a result, problem (P3) is further reformulated as

\[(P4): \min_{W, S, \eta, \gamma_E} \sum_{m=1}^M |\eta \hat{P}(\hat{\theta}_m) - a^H(\hat{\theta}_m)(S + W)a(\hat{\theta}_m)|^2 \]

s.t. \( h_k^H W h_k \leq \gamma_E (h_k^H S h_k + \sigma_0^2), \forall k \in K_E \), \( \gamma_E > 0 \), \( \beta = 2R_0(1 + \gamma_E) - 1 \), \( \text{rank}(W) \leq 1, \)

\( S \succeq 0, W \succeq 0 . \)

Let \( f(\gamma_E) \) denote the optimal objective value achieved by problem (P4.1) with given \( \gamma_E \). Accordingly, we solve problem (P4) by first solving problem (P4.1) under any given \( \gamma_E > 0 \) and then search over \( \gamma_E \) via a 1D search in problem (P4.2).

\[(P4.2): \min_{\gamma_E > 0} f(\gamma_E) .\]

In the following, we only need to focus on solving problem (P4.1). By using the SDR technique, we relax the rank constraint in (13c) and obtain the SDR version of (P4.1) as

\[(SDR.4.1): \min_{W, S, \eta} \sum_{m=1}^M |\eta \hat{P}(\hat{\theta}_m) - a^H(\hat{\theta}_m)(S + W)a(\hat{\theta}_m)|^2 \]

s.t. \( (13b), (13d), (14a), (14b). \)

It is observed that problem (SDR.4.1) is a convex quadratic semidefinite programing (QSDP) problem that can be optimally solved by convex solvers, such as CVX [16]. Let \( \bar{W}^*, \bar{S}^* \), and \( \bar{\eta}^* \) denote the obtained optimal solution to problem (SDR.4.1), where \( \text{rank}(W^*) \leq 1 \) may not hold in general. In this case, we need additional steps to construct rank-one solutions to problem (P4.1). In the following proposition, we show that the SDR is tight and accordingly construct an optimal rank-one solution to problem (SDR.4.1) and thus (P4.1).

**Proposition 1.** Based on the obtained optimal solution \( \bar{W}^*, \bar{S}^* \), and \( \bar{\eta}^* \) to problem (SDR.4.1) with \( \text{rank}(W^*) > 1 \) in
general, we can always construct an equivalent solution of $W^*$, $S^*$, and $\eta^*$ in the following, which is optimal for problems (SDR4.1) and (P4.1) with $\text{rank}(W^*) = 1$.

$$w_0^* = (g^H W^*)^{-1} W^* g,$$  \hspace{1cm} \text{(18a)}

$$W^* = w_0^* w_0^{*H},$$  \hspace{1cm} \text{(18b)}

$$S^* = \bar{W}^* + \bar{S}^* - W^*,$$  \hspace{1cm} \text{(18c)}

$$\eta^* = \bar{\eta}^*.$$  \hspace{1cm} \text{(18d)}

**Proof.** See Appendix A. \quad \Box

Suppose that the optimal solution of $\gamma_E$ in problem (P4.2) is given by $\gamma_E^*$. In this case, by solving problem (P4.1)/(SDR4.1) with given $\gamma_E^*$, the correspondingly constructed optimal solution of $W^*$, $S^*$, and $\eta^*$ in Proposition 1 becomes the optimal solution to problems (P4) and (P3). Accordingly, the corresponding $w_0^*$ from (18a), $S^*$, and $\eta^*$ are the obtained optimal solution to problem (P1).

**IV. SUBOPTIMAL Beamforming SOLUTIONS TO PROBLEM (P1)**

The preceding section proposed the globally optimal solution to problem (P1), which, however, suffers from high computational complexity due to the 1D search. To overcome this issue, this section presents two low-complexity designs to obtain high-quality sub-optimal solutions based on ZF and separate beamforming, respectively.

**A. ZF-Based Beamforming Design**

First, we consider the ZF-based beamforming design, in which the information beamforming vector $w_0$ is enforced to lie in the null space of all eavesdroppers’ channel vectors, i.e., $h_k^H w_0 = 0, \forall k \in K_E$, such that the eavesdroppers are not able to receive any confidential information from the BS. Notice that the ZF-based beamforming design only works when the number of transmit antennas $N$ at the BS is greater than the number of eavesdroppers $K_E$, i.e., $N > K_E$.

Let $H = [h_1, h_2, \ldots, h_{K_E}]^H$ denote the channel matrix from the BS to all the eavesdroppers, of which the singular value decomposition (SVD) is

$$H = U \Sigma V^H = U \Lambda [V_1 V_2]^H,$$  \hspace{1cm} \text{(19)}

where $\Lambda \in \mathbb{C}^{K_E \times N}$ has non-zero diagonal elements that correspond the singular values of $H$, $U \in \mathbb{C}^{K_E \times K_E}$ and $V \in \mathbb{C}^{N \times N}$ are both unitary matrices, and $V_1 \in \mathbb{C}^{K_E \times K_E}$ and $V_2 \in \mathbb{C}^{(N-K_E) \times N}$ consist of the first $K_E$ and the last $N-K_E$ right singular vectors of $H$, respectively. In order to ensure $h_k^H w_0 = 0, \forall k \in K_E$, we set

$$w_0 = V_2 \tilde{w}_0$$  \hspace{1cm} \text{(20)}

without loss of generality, where $\tilde{w}_0 \in \mathbb{C}^{(N-K_E) \times 1}$ is to be optimized. In this case, the secrecy rate becomes

$$\hat{r}(w_0, S) = \log_2 \left(1 + \frac{g^H V_2 \tilde{w}_0 w_0^{*H} V_2^H g}{\sigma_0^2 + g^H S g}\right).$$  \hspace{1cm} \text{(21)}

To maximize the secrecy rate, the beamformer $w_0$ can be set as $\tilde{w}_0 = \sqrt{Q_0 \frac{V_2^H g}{\|V_2^H g\|}}$, and accordingly we have

$$w_0 = \sqrt{Q_0} V_2 \frac{V_2^H g}{\|V_2^H g\|},$$  \hspace{1cm} \text{(22)}

where $Q_0 \geq 0$ is the optimal transmit power for information signals. Hence, by substituting the ZF beamforming design of $w_0$ in (22) in problem (P1), the secrecy rate constrained sensing beampattern matching problem is recast into

\begin{align*}
\text{P5:} & \quad \min_{q_0, S, \eta} \sum_{m=1}^{M} |\tilde{H}(\bar{\eta}_m) - a^H(\bar{\eta}_m) (S + Q_0 w_0 w_0^H) a(\bar{\eta}_m)|^2 \\
\text{s.t.} & \quad Q_0 \|V_2^H g\|^2 \geq (\sigma_0^2 + g^H S g) (2 R_0 - 1), \\
& \quad \text{Tr}(S) + Q_0 = Q, S \succeq 0, \\
& \quad w_0 = V_2 \frac{V_2^H g}{\|V_2^H g\|}. 
\end{align*}

Problem (P5) is also a convex QSDP problem that can be optimally solved by CVX. Let $\tilde{Q}_0$, $\tilde{S}$, and $\tilde{\eta}$ denote the obtained optimal solution to problem (P5). Accordingly, based on (22) the ZF-based beamforming solution is obtained as

$$w_0 = \sqrt{\tilde{Q}_0} V_2 \frac{V_2^H g}{\|V_2^H g\|}, S = \tilde{S}, \eta = \tilde{\eta}.$$  \hspace{1cm} \text{(24)}

**B. Separate Beamforming Design**

In this subsection, we propose another sub-optimal beamforming design, in which the information beamforming vector $w_0$ and the sensing covariance matrix $S$ are designed separately. In this approach, we first design the information beamforming vector $w_0$ to achieve the secrecy rate with the minimum required power, and then use the remaining power for sensing signals $S$.

First, we design the information beamforming $w_0$. Without sensing signals or AN, the secrecy rate is given as

$$\hat{r}(w_0) = \min_{k \in K_E} \left(\log_2 (1 + \gamma_0(w_0)) - \log_2 (1 + \gamma_k(w_0))\right),$$  \hspace{1cm} \text{(25)}

where $\gamma_0(w_0) = \frac{|g^H w_0|^2}{\sigma_0^2}$ denotes the SNR at the CU and $\gamma_k(w_0) = \frac{|h_k^H w_0|^2}{\sigma_k^2}$ denotes the SNR at eavesdropping target $k \in K_E$. In this case, the information beamforming vector $w_0$ is designed by minimizing the transmit power $\|w_0\|^2$ while ensuring the minimum secrecy rate of $R_0$, i.e.,

\begin{align*}
\text{P6:} & \quad \min_{w_0} \|w_0\|^2 \\
\text{s.t.} & \quad \hat{r}(w_0) \geq R_0, \\
& \quad \|w_0\|^2 \leq Q. 
\end{align*}

It is observed that problem (P6) is a typical multiple-input single-output (MISO) secrecy communication problem and has been optimally solved in [15]. Let $w_0^*$ denote the obtained optimal solution beamforming to problem (P6). Next, we design the sensing covariance matrix $S$ without interfering with the confidential information signals reception at the CU, i.e., $g^H S g = 0$. Without loss of generality, we define $Q_2 = I - g g^H/\|g\|^2$, and accordingly set

$$S = Q_2 S Q_2^H,$$  \hspace{1cm} \text{(27)}
where $\bar{S} \in \mathbb{C}^{N \times N}$ and $\bar{S} \succeq 0$. By substituting $w_0^*$ and $S$ in (27) to problem (P1), we have the following beampattern matching error minimization problem.

\begin{equation}
\begin{aligned}
&\min_{S, \eta} \sum_{m=1}^{M} \eta P(\bar{\theta}_m) - a^H(\bar{\theta}_m)(Q_2 \bar{S} Q_2^H + w_0^* w_0^{H*}) a(\bar{\theta}_m) \\
&\text{s.t.} \quad \bar{S} \succeq 0, \\
&\quad \text{Tr}(Q_2 \bar{S} Q_2^H) + \|w_0^*\|^2 = Q.
\end{aligned}
\end{equation}

(28)

Problem (P7) is also a QSDP problem that can be optimally solved by CVX. Let $\hat{S}^*$ denote the optimal solution to problem (P7). Then $S^* = Q_2 \hat{S}^* Q_2^H$ and $w_0^*$ become the obtained separate beamforming designs.

It is worth remarking that the two sub-optimal beamforming designs in Sections IV-A and IV-B do not require the computational-heavy 1D search in the optimal solution, thus significantly reducing the computational complexity.

V. NUMERICAL RESULTS

This section provides numerical results to validate the performance of our proposed joint information and sensing beamforming designs for secrecy ISAC. In the simulation, the BS is deployed with $N = 8$ antenna elements, and there are $K = 8$ targets located at angles $-10^\circ, 10^\circ, -30^\circ, 30^\circ, 80^\circ, -80^\circ, -50^\circ$, and $50^\circ$. The two targets located at $-30^\circ$ and $30^\circ$ are assumed to be untrusted eavesdroppers. We set the noise powers at the CU and all the eavesdroppers are identical to be $-60$dBm, i.e., $\sigma_0^2 = \sigma_k^2 = -60$dBm, $\forall k \in \mathcal{K}_E$, and the path loss from the BS to the CU and the eavesdroppers to be $-70$dB. Furthermore, the transmit power budget at the BS is set to be $Q = 20$dBm. We choose $M = 201$ angles for $\{\bar{\theta}_m\}$, which are uniformly sampled in $[-\pi/2, \pi/2]$. We also set the beampattern width as $\Delta \theta = 5^\circ$. The normalized spacing between two adjacent antennas is set as $d = 0.5$. The LoS model is considered for the channel from the BS to the CU.

Fig. 1 shows the normalized beampattern gains achieved by the proposed optimal beamforming design solution, where the CU is located at $\theta_0 = 0^\circ$ and the secrecy rate threshold is set as $R_0 = 3$bps/Hz. For illustration, we also show the beampattern gain solely achieved by information signals (i.e., $a^H(\theta) w_0^* w_0^{H*} a(\theta)$) and that solely achieved by sensing signals (i.e., $a^H(\theta) \hat{S}^* a(\theta)$). For comparison, we also consider the sensing only benchmark, which corresponds to the design in problem (P1) with $R_0 = 0$ and $w_0 = 0$. It is observed that the beampattern gain achieved by the information signals is around $30$dB lower at angles of $-30^\circ$ and $30^\circ$. This is for the purpose of preventing the information leakage as the eavesdropping targets are located at these two angles. It is also observed that the beampattern gain achieved by the sensing signals is around $70$dB lower at $0^\circ$. This is in order to not interfere with the CU at that angle. By combining them, the joint information and sensing signals are observed to achieve similar beampattern gains as those by the sensing only benchmark. This shows the effectiveness of the proposed optimal solution.

Fig. 2 shows the beampattern matching error versus the secrecy rate threshold $R_0$ under different designs, where the

\begin{figure}
\centering
\includegraphics[width=\columnwidth]{fig1}
\caption{The normalized beampattern gains achieved by the proposed optimal beamforming design solution, where the CU is located at $\theta_0 = 0^\circ$ and the secrecy rate threshold is $R_0 = 3$bps/Hz. Here, the beampattern gain by the information signals is defined as $a^H(\theta) w_0^* w_0^{H*} a(\theta)$, and that by the sensing signals is defined as $a^H(\theta) \hat{S}^* a(\theta)$.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\columnwidth]{fig2}
\caption{The beampattern matching error versus the secrecy rate threshold $R_0$ under different designs.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\columnwidth]{fig3}
\caption{The normalized beampattern gains under different designs, where the CU is located at $\theta_0 = 60^\circ$, and the secrecy rate threshold is $R_0 = 3$bps/Hz.}
\end{figure}
CU is located at $\theta_0 = 60^\circ$. It is observed that as $R_0$ becomes large, the achieved beampattern matching errors for the three proposed designs increases, and the performance gap between the optimal solution versus the sub-optimal ones becomes more significant. It is also observed that as $R_0$ becomes small, such performance gap becomes small, and all the three designs approach the beampattern gain error lower bound by the sensing only benchmark.

Fig. 3 shows the normalized beampattern gains under different designs, with $R_0 = 3.5$bps/Hz and $\theta_0 = 60^\circ$. It is observed that the beampattern gains by the two sub-optimal designs well match those by the optimal solution and sensing only benchmark in most sensing angles. This validates the effectiveness of the proposed designs.

VI. CONCLUSION

This paper studied the joint transmit information and sensing beamforming design for secrecy ISAC system with one CU and multiple untrusted and trusted targets. Our objective was to minimize the sensing beampattern matching error, while ensuring the secrecy rate requirement. We proposed the globally optimal solution to the highly non-convex problem, via the SDR and 1D search, in which the tightness of SDR is proved. We also proposed two sub-optimal solutions based on ZF and separate beamforming, respectively. Numerical results showed that at the proposed solutions, the information beams are designed towards the CU and the trusted targets, while the sensing beams are designed towards both untrusted and trusted targets, thus increasing the sensing performance while ensuring the CU’s secrecy rate. How to extend the proposed designs to setups with imperfect CSI, multiple CUs, and/or multiple BSs is interesting research directions worth future pursuing.

APPENDIX A

PROOF OF PROPOSITION 1

First, it is observed from (18c) that $S^* + W^* = W^* + \tilde{S}^*$. As a result, $W^*$, $S^*$, and $\eta^*$ achieve the same objective value for problem (SDR4.1) as that achieved by $\tilde{W}^*$, $\tilde{S}^*$, and $\tilde{\eta}^*$, and $W^*$, $S^*$, and $\eta^*$ satisfy the constraint in (13b).

Next, based on $\tilde{W}^* \succeq 0$, we have $\tilde{W}^* = W^* H$. Based on this, for any $v \in \mathbb{C}^{N \times 1}$, it follows that

$$
\begin{align*}
  v^H(\tilde{W}^* - W^*)v &= v^H \tilde{W}^*v - v^H W^* g^H g v \\
  &= \frac{1}{g^H g} (v^H W^* g v - v^H W^* g v) \\
  &= \frac{g^H g}{g^H g} (||a||^2 - ||b||^2 - ||a||^2 - ||b||^2) \geq 0, 
\end{align*}
$$

where $a = W^* v \in \mathbb{C}^{N \times 1}$, $b = \tilde{W}^* g \in \mathbb{C}^{N \times 1}$, and inequality (a) holds because of the Cauchy-Schwarz inequality. Accordingly, we have $\tilde{W}^* - W^* \succeq 0$. By using this together with (18c), we have $S^* \succeq \tilde{S}^* \succeq 0$. Therefore, constraint (13d) holds for $S^*$ and $W^*$.

Furthermore, we prove that $W^*$, $S^*$, and $\eta^*$ satisfy the constraints in (14a) and (14b). On one hand, it is clear from (18b) and (18c) that $g^H W^* g = g^H \tilde{W}^* g$ and $g^H \tilde{S}^* g = g^H S^* g$, and therefore, constraint (14b) holds. On the other hand, based on $W^* - W^* \succeq 0$ and $S^* \succeq \tilde{S}^* \succeq 0$, it follows that

$$
\begin{align*}
  h_k^H W^* h_k &\leq h_k^H \tilde{W}^* h_k \\
  &\leq \gamma_E (h_k^H S^* h_k + \sigma_k^2) \\
  &\leq \gamma_E (h_k^H \tilde{S}^* h_k + \sigma_k^2), \forall k \in K_E.
\end{align*}
$$

As a result, constraint (14a) is satisfied.

By combining the results above, it is proved that $W^*$, $S^*$, and $\eta^*$ are optimal for problem (SDR4.1). Notice that rank($W^*$) = 1 with $W^* = w^*_k w_k^H$. Therefore, $W^*$, $S^*$, and $\eta^*$ are also optimal for problem (P4.1). This thus completes the proof.

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