Towards $p$-Adic Matter in the Universe

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Abstract
Starting from $p$-adic string theory with tachyons, we introduce a new kind of non-tachyonic matter which may play an important role in evolution of the Universe. This matter retains nonlocal and nonlinear $p$-adic string dynamics, but does not suffer of negative square mass. In space-time dimensions $D = 2 + 4k$, what includes $D = 6, 10, ..., 26$, the kinetic energy term also maintains correct sign. In these spaces this $p$-adic matter provides negative cosmological constant and time-dependent scalar field solution with negative potential. Their possible cosmological role is discussed. We have also connected non-locality with string world-sheet in effective Lagrangian for $p$-adic string.

1 Introduction
Observational modern cosmology has achieved significant progress by high precision experiments in the last decades. One of the greatest cosmological achievements was discovery of accelerated expansion of the Universe in 1998. If General Relativity is theory of gravity for the Universe as a whole then about 96% of its energy content is of unknown nature. This dark side of the Universe consists of about 23% of dark matter and 73% of dark energy. Dark matter is supposed to be responsible for anomaly large rotational velocities in the spiral galaxies. Dark energy has negative pressure and should govern the Universe accelerated expansion (as a recent review see [1]). Thus, according to this point of view, there is now only about 4% of visible matter which is described by the Standard Model of particle physics. Although dark matter and dark energy are well adopted among majority of scientists, they are not directly verified in the laboratory and still remain hypothetical forms of...
matter. Also General Relativity has not been so far confirmed at the cosmic scale. For these reasons, there is not yet commonly accepted theoretical explanation of the Universe acceleration. This situation has influenced also alternative approaches, mostly related to modification of General Relativity (a recent review in [2]).

While observational cosmology is in an arising state, theoretical cosmology is facing a big challenge. An exotic matter and modification of gravity are two alternative theoretical approaches. Since string theory is the best candidate for unification of matter elementary constituents and fundamental interactions, some theoretical ideas come from string theory and we consider the following one. According to the adelic product formula for scalar string amplitudes it follows that \( p \)-adic strings are at equal footing with ordinary strings (reviews on \( p \)-adic strings and adelic product formula can be found in [3,4]). Hence, if visible matter is composed of ordinary strings then there should be some matter made of \( p \)-adic strings. It is natural to assume that dark side of the Universe contains some kinds of \( p \)-adic matter. In \( p \)-adic string theory world-sheet has non-Archimedean (ultrametric) geometry and it should also somehow modify gravity. It is feasible that future theoretical cosmology will be a result of both modification of General Relativity and inclusion of new kinds of matter.

Inspired by \( p \)-adic string theory it has been already investigated some nonlocal modifications of General Relativity (see, e.g. [5] and references therein) and cosmology, see, e.g. [6,7] and references therein. In this article we consider some modification of open scalar \( p \)-adic strings as candidates for a new kind of matter in the Universe. In Section 2 we present various aspects of \( p \)-adic string theory necessary for comprehensive exposition. It also contains introduction of non-tachyonic \( p \)-adic matter. Section 3 is related to some adelic approaches to cosmology.

2 \( p \)-Adic Strings

\( p \)-Adic strings are introduced in 1987 by Volovich in his paper [8]. \( p \)-Adic string theory is mainly related to strings which have only world-sheet \( p \)-adic and all other their properties are the same with theory of ordinary strings [9]. Having exact Lagrangian at the tree level, \( p \)-adic scalar strings have attracted significant attention in string theory and nonlocal cosmology. To be more comprehensive and self consistent we shall first give a brief review of \( p \)-adic numbers and their applicability in modern mathematical physics.

2.1 \( p \)-Adic Numbers and Their Applicability

\( p \)-Adic numbers are discovered by Kurt Hansel in 1897 as a new tool in number theory. In modern approach to introduce \( p \)-adic numbers one usually starts with the
field $\mathbb{Q}$ of rational numbers. Recall that according to the Ostrowski theorem any non-trivial norm on $\mathbb{Q}$ is equivalent either to the usual absolute value $| \cdot |_\infty$ or to a $p$-adic norm ($p$-adic absolute value) $| \cdot |_p$. A rational number $x = p^\nu \frac{a}{b}$, where integers $a$ and $b \neq 0$ are not divisible by prime number $p$, by definition has $p$-adic norm $|x|_p = p^{-\nu}$ and $|0|_p = 0$. Since $|x + y|_p \leq \max\{|x|_p, |y|_p\}$, $p$-adic norm is a non-Archimedean (ultrametric) one. As completion of $\mathbb{Q}$ with respect to the absolute value $| \cdot |_\infty$ gives the field $\mathbb{Q}_\infty \equiv \mathbb{R}$ of real numbers, by the same procedure using $p$-adic norm $| \cdot |_p$ one gets the field $\mathbb{Q}_p$ of $p$-adic numbers (for any prime number $p = 2, 3, 5 \cdots$). Any number $0 \neq x \in \mathbb{Q}_p$ has its unique canonical representation

$$x = p^\nu \sum_{n=0}^{+\infty} x_n p^n, \quad \nu \in \mathbb{Z}, \quad x_n \in \{0, 1, \cdots, p-1\}, \quad x_0 \neq 0. \quad (1)$$

$\mathbb{Q}_p$ is locally compact, complete and totally disconnected topological space. There is a rich structure of algebraic extensions of $\mathbb{Q}_p$.

There are many possibilities for mappings between $\mathbb{Q}_p$. The most elaborated is analysis related to mappings $\mathbb{Q}_p \to \mathbb{Q}_p$ and $\mathbb{Q}_p \to \mathbb{C}$. Usual complex valued functions of $p$-adic argument are additive $\chi_p(x) = e^{2\pi i (x)_p}$ and multiplicative $|x|^s$ characters, where $(x)_p$ is fractional part of $x$ and $s \in \mathbb{C}$ (for many aspects of $p$-adic numbers and their analysis, we refer to [3, 4, 10, 11]).

The field $\mathbb{Q}$ of rational numbers, which is dense in $\mathbb{Q}_p$ and $\mathbb{R}$, is also important in physics. All values of measurements are rational numbers. Any measurement is comparison of two quantities of the same kind and it is in close connection with the Archimedean axiom. Set of rational numbers obtained in the process of repetition of measurement of the same quantity is naturally provided by usual absolute value. Hence, results of measurements are not rational numbers with $p$-adic norm but with real norm. It means that measurements give us real and not $p$-adic rational numbers. Then the following question arises: Being not results of measurements, what role $p$-adic numbers can play in description of something related to physical reality? Recall that we already have similar situation with complex numbers, which are not result of direct measurements but they are very useful. For example, in quantum mechanics wave function is basic theoretical tool which contains all information about quantum system but cannot be directly measured in experiments. $p$-Adic numbers should play unavoidable role where application of real numbers is inadequate. In physical systems such situation is at the Planck scale, because it is not possible to measure distances smaller than the Planck length. It should be also the case with very complex phenomena of living and cognitive systems. Thus, we expect inevitability of $p$-adic numbers at some deeper level in understanding of the content, structure and evolution of the Universe in its parts as well as a whole. The first steps towards probe of $p$-adic levels of knowledge is invention of relevant mathematical methods and construction of the corresponding physical models. A brief overview of $p$-adic mathematical
physics is presented in [12]. It includes both $p$-adic valued and real (complex) valued functions of $p$-adic argument. In the sequel we shall consider $p$-adic strings, non-tachyonic $p$-adic matter and its some possible role in modern cosmology.

2.2 Scattering Amplitudes and Lagrangian for Open Scalar $p$-Adic Strings

Like ordinary strings, $p$-adic strings are introduced by construction of their scattering amplitudes. The simplest amplitude is for scattering of two open scalar strings. Recall the crossing symmetric Veneziano amplitude for ordinary strings

$$A_\infty(a,b) = g_\infty^2 \int_{\mathbb{R}} |x|^{a-1} |1-x|^{b-1} d\omega x = g_\infty^2 \frac{\zeta(1-a)}{\zeta(a)} \frac{\zeta(1-b)}{\zeta(b)} \frac{\zeta(1-c)}{\zeta(c)}, \quad (2)$$

where $a + b + c = 1$. The crossing symmetric Veneziano amplitude for scattering of two open scalar $p$-adic strings is direct analog of (2) [9], i.e.

$$A_p(a,b) = g_p^2 \int_{\mathbb{Q}_p} |x|^{a-1}_p |1-x|^{b-1}_p d\omega x = g_p^2 \frac{1-p^{a-1}}{1-p^{-a}} \frac{1-p^{b-1}}{1-p^{-b}} \frac{1-p^{c-1}}{1-p^{-c}}. \quad (3)$$

Integral expressions in (2) and (3) are the Gel’fand-Graev-Tate beta functions on $\mathbb{R}$ and $\mathbb{Q}_p$, respectively [10]. Note that here, by definition, ordinary and $p$-adic strings differ only in description of their world-sheets – world-sheet of $p$-adic strings is presented by $p$-adic numbers. Kinematical variables contained in $a, b, c$ are the same real numbers in both cases. It is worth noting that the final form of Veneziano amplitude for $p$-adic strings is rather simple and presented by an elementary function.

It is remarkable that there is an effective field description of the above open $p$-adic strings. The corresponding Lagrangian is very simple and at the tree level describes not only four-point scattering amplitude but also all higher (Koba-Nielsen) ones. The exact form of this Lagrangian for effective scalar field $\varphi$, which describes open $p$-adic string tachyon, is

$$\mathcal{L}_p = \frac{n^D}{g^2} \frac{p^2}{p-1} \left[ -\frac{1}{2} \varphi p^{-\frac{\square}{2m^2}} \varphi + \frac{1}{p+1} \varphi^{p+1} \right], \quad (4)$$

where $p$ is any prime number, $D$ – spacetime dimensionality, $\square = -\partial^2 + \nabla^2$ is the $D$-dimensional d’Alembertian and metric has signature $(- + ... +)$ [13][14]. This is nonlocal and nonlinear Lagrangian. Nonlocality is in the form of infinite number of spacetime derivatives

$$p^{-\frac{\square}{2m^2}} = \exp \left( -\frac{\ln p}{2m^2} \square \right) = \sum_{k \geq 0} \left( -\frac{\ln p}{2m^2} \right)^k \frac{1}{k!} \square^k \quad (5)$$
Figure 1: The 2-adic string potential $V_2(\phi)$ (on the left) and 3-adic potential $V_3(\phi)$ (on the right) of standard Lagrangian (4), where potential is presented by expression (6) with $\frac{m_D}{g^2} = 1$.

and it is a consequence of the fact that strings are extended objects.

The corresponding potential $V(\phi)$ for Lagrangian (4) is $V_p(\phi) = -\mathcal{L}_p(\Box = 0)$, which the explicit form is

$$V_p(\phi) = \frac{m_D}{g^2} \left[ \frac{1}{2} \frac{p^2}{p-1} \phi^2 - \frac{p^2}{p^2-1} \phi^{p+1} \right]. \quad (6)$$

It has local minimum $V_p(0) = 0$. If $p \neq 2$ there are two local maxima at $\phi = \pm 1$ and there is one local maximum $\phi = +1$ when $p = 2$.

The equation of motion for the field $\phi$ is

$$p^{-\frac{1}{2p-1}} \phi = \phi^p \quad (7)$$

and it has trivial solutions $\phi = 0$ and $\phi = 1$, and another $\phi = -1$ for $p \neq 2$. There are also inhomogeneous solutions in any direction $x^i$ resembling solitons

$$\phi(x^i) = p^{\frac{1}{2p-1}} \exp \left( -\frac{p-1}{2p \ln p} m^2 (x^i)^2 \right). \quad (8)$$

There is also homogeneous and isotropic time-dependent solution

$$\phi(t) = p^{\frac{1}{2p-1}} \exp \left( \frac{p-1}{2p \ln p} m^2 t^2 \right). \quad (9)$$

Solution (9) (and analogously (8)) can be obtained using identity

$$e^A e^{B t^2} = \frac{1}{\sqrt{1-4AB}} e^{\frac{B t^2}{1-4AB}}, \quad 1-4AB > 0. \quad (10)$$
Note that the sign of the above field solutions \( \varphi(x^i) \) and \( \varphi(t) \) can be also minus \((-\)) when \( p \neq 2 \). Various aspects of \( p \)-adic string theory with the above effective field have been pushed forward by papers \([15, 16]\).

It is worth noting that Lagrangian (4) is written completely in terms of real numbers and there is no explicit dependence on the world sheet. However, (9) can be rewritten in the following form:

\[
\mathcal{L}_p = \frac{m^D}{g^2} \frac{p^2}{p - 1} \left[ \frac{1}{2} \varphi \left( \int_{\mathbb{R}} \left( \int_{Q_p \setminus \mathbb{Z}_p} \chi_p(u) |u|_{\mathbb{Z}_p}^{\frac{2}{p^2 - 1}} \, du \right) \tilde{\varphi}(k) \chi_\infty(kx) \, d^4k \right] + \frac{1}{p + 1} \varphi^{p+1} \right],
\]

(11)

where \( \chi_\infty(kx) = e^{-2\pi i kx} \) is the real additive character. Since \( \int_{Q_p \setminus \mathbb{Z}_p} \chi_p(u) |u|^{s-1} \, du = \frac{1 - p^{-s}}{1 - p^{-1}} = \Gamma_p(s) \) and it is present in the scattering amplitude (3), one can say that expression \( \int_{Q_p \setminus \mathbb{Z}_p} \chi_p(u) |u|_{\mathbb{Z}_p}^{2} \, du \) in (11) is related to the \( p \)-adic string world-sheet.

### 2.3 New Kind of Matter, Which Has Origin in \( p \)-Adic Strings

Since there are infinitely many primes \( p \), in principle it can be infinitely many kinds of \( p \)-adic strings. We suppose that for all but a finite set \( \mathcal{P} \) of primes \( p \) these \( p \)-adic strings are in their local potential minimum, i.e. \( \varphi_p = 0 \), and consequently \( \mathcal{L}_p = 0 \), for all \( p \notin \mathcal{P} \). This can be a result of tachyon condensation. Further we suppose that in the remaining finite set of strings there was a transition \( m^2 \rightarrow -m^2 \) (transition from tachyon to no-tachyon state), what could be a result of some quantum effect which was happened before process of tachyon condensation was finished. For simplicity, we shall assume two kinds of such strings and denote their set by \( \mathcal{P} = \{ q, \ell \} \). In the sequel for these strings in the above expressions (4) - (11) one has to replace \( m^2 \) by \(-m^2\) and \( m^D \rightarrow (-1)^{\frac{D}{2}} m^D \), where spacetime dimensionality \( D \) is even. Note that there is transition \( m^D \rightarrow -m^D \) for critical dimensions \( D = 26 \) and \( D = 10 \), but for \( D = 4 \) there is no change of sign. The corresponding potentials for \( p = 2 \) and \( p = 3 \), and \( D = 2 + 4k \), are presented at Figure 2. To make distinction with tachyons we denote these new scalar strings by \( \phi_p, p \in \mathcal{P} \).

The related new Lagrangian is

\[
L_p = (-1)^{\frac{D}{2}} \frac{m^D}{g^2} \frac{p^2}{p - 1} \left[ - \frac{1}{2} \varphi \frac{p_{2m^p}}{\mathbb{Z}^p} \varphi + \frac{1}{p + 1} \varphi^{p+1} \right]
\]

(12)

with the corresponding potential

\[
V_p(\phi) = (-1)^{\frac{D}{2}} \mathcal{V}_p(\phi) = (-1)^{\frac{D}{2}} \frac{m^D}{g^2} \left[ \frac{1}{2} \frac{p^2}{p - 1} \varphi^2 - \frac{p^2}{p^2 - 1} \varphi^{p+1} \right].
\]

(13)
The equation of motion for scalar strings $\phi_p$ is

$$p \frac{\Box}{m^2} \phi_p = \phi_p^p$$  \hspace{1cm} (14)

and it has trivial solutions $\phi_p = 0$ and $\phi_p = 1$, and another $\phi_p = -1$ for $p \neq 2$. What is local maximum and minimum depends on dimensionality $D$. When $D = 2 + 4k$, solution $\phi_p = 0$ is a local maximum, and $\phi_2 = +1$ and $\phi_p = \pm 1, p \neq 2$ are local minima. For any dimensionality, nontrivial solutions become now

$$\phi_p(x) = p \frac{1}{2p} \exp \left( -\frac{p - 1}{2p} m^2 (x^2)^2 \right),$$  \hspace{1cm} (15)

$$\phi_p(t) = p \frac{1}{2p} \exp \left( -\frac{p - 1}{2p} m^2 t^2 \right).$$  \hspace{1cm} (16)

$D$-dimensional solution of (14) is product of solutions (15) and (16) (see also [17]), i.e.

$$\phi_p(x) = p \frac{1}{2p} \exp \left( -\frac{p - 1}{2p} m^2 (x^2)^2 \right), \quad x^2 = -t^2 + \sum_{i=1}^{D/2} \frac{1}{p} x_i^2.$$  \hspace{1cm} (17)

Lagrangian for this collection of two distinct strings $\{\phi_q, \phi_\ell\}$ is of the form

$$L_{\phi} = \sum_{p \in \mathcal{P}} L_p = \sum_{p \in \mathcal{P}} (-1)^p \frac{m^D}{g_p} p^2 \left[ -\frac{1}{2} \phi_p \frac{\Box}{m^2} \phi_p + \frac{1}{p + 1} \phi_p^{p+1} \right].$$  \hspace{1cm} (18)

String field $\phi_\ell$ in (18) we shall consider in its vacuum state $\phi_\ell = +1$ or $-1$ with Lagrangian

$$L_\ell = -V_\ell(\pm 1) = (-1)^{\ell + 1} \frac{m^D}{g^2} \frac{\ell^2}{2(\ell + 1)} \sim -\Lambda$$  \hspace{1cm} (19)

related to the cosmological constant $\Lambda$ (prime index $\ell$ may remind $\Lambda$). Note that vacuum state $\phi_\ell = \pm 1$ is stable only in spaces with dimension $D = 2 + 4k$.

Field $\phi_q$ corresponds to time-dependent solution (16) in dimensions which satisfy $(-1)^q \frac{q}{2} = -1$, and the form of the corresponding potential is presented at Figure 2. As a simple example, one can take $D = 6$ as respective solution of condition $(-1)^q \frac{q}{2} = -1$. For the case $D = 6$, or any other $D = 2 + 4k$, we have ($q$ may remind quintessence)

$$\phi_q(t) = q \frac{1}{2q} \exp \left( -\frac{q - 1}{2q} m^2 t^2 \right)$$  \hspace{1cm} (20)
which corresponds to potential of the form at Figure 2. At the moment $t = 0$ (the big bang) the field $\phi_q$ has its maximum which is $\phi_q(0) = q^{2q-1}$ and it is a bit larger than 1. Then by increasing of time $\phi_q(t)$ is decreasing and $\phi_q(t) \to 0$ as $t \to +\infty$. The situation is symmetric with respect to transformation $t \to -t$.

If we consider $\phi_q(t)$ in spaces of dimension $D = 4k$, and in particular $D = 4$, then we face by two problems. First, the kinetic energy term has not correct sign. Second, the position of field at moment $t = 0$ is $\phi_q(0) = q^{2q-1} > 1$ and it should have rolling to $-\infty$, instead of to 0, what contradicts to the time dependence (20) of the field (see also Fig. 1).

3 Adelic Cosmological Modelling

In the preceding section we have seen that the fields of $p$-adic and real numbers can be obtained by completion of the field of rational numbers, and that $\mathbb{Q}$ is dense in $\mathbb{Q}_p$ as well as in $\mathbb{R}$. This gives rise to think that it should exist some way for unification of $p$-adic and real numbers. A unified and simultaneous treatment of $p$-adic and real numbers is through concept of adeles. Adelic formalism is a mathematical method how to connect $p$-adic with ordinary real models.

3.1 Adeles and Their Applicability

An adele $\alpha$ is an infinite sequence made od real and $p$-adic numbers in the form

$$\alpha = (\alpha_\infty, \alpha_2, \alpha_3, \ldots, \alpha_p, \ldots), \quad \alpha_\infty \in \mathbb{R}, \ \alpha_p \in \mathbb{Q}_p,$$

(21)

where for all but a finite set $\mathcal{P}$ of primes $p$ it has to be $\alpha_p \in \mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\}$. $\mathbb{Z}_p$ is ring of $p$-adic integers and they have $v \geq 0$ in (1). The set $\mathbb{A}_\mathbb{Q}$ of all completions
of \( \mathbb{Q} \) in the form of the above adeles can be presented as

\[
\mathbb{A}_\mathbb{Q} = \bigcup_{\mathcal{P}} A(\mathcal{P}), \quad A(\mathcal{P}) = \mathbb{R} \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p \times \prod_{p \notin \mathcal{P}} \mathbb{Z}_p.
\]  

Elements of \( \mathbb{A}_\mathbb{Q} \) satisfy componentwise addition and multiplication and form the adele ring.

The multiplicative group of ideles \( \mathbb{A}_\times \mathbb{Q} \) is a subset of \( \mathbb{A}_\mathbb{Q} \) with elements \( \eta = (\eta_{\infty}, \eta_2, \eta_3, \cdots, \eta_p, \cdots) \) where \( \eta_{\infty} \in \mathbb{R}^\times = \mathbb{R} \setminus \{0\} \) and \( \eta_p \in \mathbb{Q}_p^\times = \mathbb{Q}_p \setminus \{0\} \) with the restriction that for all but a finite set \( \mathcal{P} \) one has that \( \eta_p \in \mathbb{U}_p = \{x \in \mathbb{Q}_p : |x|_p = 1\} \), i.e. \( \mathbb{U}_p \) is multiplicative group of \( p \)-adic units. The entire set of adeles, related to \( \mathbb{Q}^\times = \mathbb{Q} \setminus \{0\} \), is

\[
\mathbb{A}_\times \mathbb{Q} = \bigcup_{\mathcal{P}} A^\times(\mathcal{P}), \quad A^\times(\mathcal{P}) = \mathbb{R}^\times \times \prod_{p \in \mathcal{P}} \mathbb{Q}_p^\times \times \prod_{p \notin \mathcal{P}} \mathbb{U}_p.
\]  

A principal adele (idele) is a sequence \( (x, x, \cdots, x, \cdots) \in \mathbb{A}_\mathbb{Q} \), where \( x \in \mathbb{Q} \) (\( x \in \mathbb{Q}^\times \)). \( \mathbb{Q} \) and \( \mathbb{Q}^\times \) are naturally embedded in \( \mathbb{A}_\mathbb{Q} \) and \( \mathbb{A}_\times \mathbb{Q} \), respectively. This concept of principal adeles gives way to present rational numbers together with their nontrivial norms. Adeles are a generalization of principal adeles so that it takes into account all completions of \( \mathbb{Q} \) and has well-defined mathematical structure.

Space of adeles (ideles) has its adelic (idelic) topology. With respect to their topology \( \mathbb{A}_\mathbb{Q} \) and \( \mathbb{A}_\times \mathbb{Q} \) are locally compact topological spaces. There are adelic-valued and complex-valued functions of adelic arguments. For various mathematical aspects of adeles and their functions we refer to books \([10, 18]\) and for their applications in mathematical physics to \([3, 4, 19]\).

Ideles and adeles are introduced in the 1930s by Claude Chevalley and André Weil, respectively. \( p \)-Adic numbers and adeles have many applications in mathematics. Since 1987, they have employed in \( p \)-adic mathematical physics.

Adelic connection of \( p \)-adic and real properties of the same rational quantity can be well illustrated by the following two simple examples:

\[
|x|_\infty \times \prod_{p \in \mathbb{P}} |x|_p = 1, \quad \text{if} \ x \in \mathbb{Q}^\times, \tag{24}
\]

\[
\chi_\infty(x) \times \prod_{p \in \mathbb{P}} \chi_p(x) = 1, \quad \text{if} \ x \in \mathbb{Q}, \tag{25}
\]

where \( \mathbb{P} \) is set of all primes and

\[
\chi_\infty(x) = \exp(-2\pi ix), \quad \chi_p(x) = \exp(2\pi i \{x\}_p) \tag{26}
\]

with \( \{x\}_p \) as fractional part of \( x \) in expansion with respect to base \( p \).
More complex connection, but also very significant, is the Freund-Witten product formula for string amplitudes [20]:

\[ A(a, b) = A_\infty(a, b) \prod_p A_p(a, b) = g_\infty^2 \prod_p g_p^2 = \text{const}. \]  

(27)

which connects \( p \)-adic Veneziano amplitudes (3) with their real analog (2). Formula (27) follows as a consequence of the Euler product formula for the Riemann zeta function applied to \( p \)-adic string amplitudes (3). Main significance of (27) is in the fact that scattering amplitude for real string \( A_\infty(a, b) \), which is a special function, can be presented as product of inverse \( p \)-adic amplitudes, which are elementary functions. Also, this product formula treats \( p \)-adic and ordinary strings at the equal footing. It gives rise to suppose that if there exists an ordinary scalar string then it should exist also its \( p \)-adic analog. Moreover, \( p \)-adic strings seem to be simpler for theoretical investigation and useful for cosmological investigations.

### 3.2 Some Adelic Cosmological Investigations

The first consideration of \( p \)-adic gravity and adelic quantum cosmology was in [21]. It was introduced an idea of the fluctuating number fields at the Planck scale giving rise to \( p \)-adic valued as well as real valued gravity. Using Hartle-Hawking approach, it was shown that the wave function for the de Sitter minisuperspace model can be presented as an infinite product of its \( p \)-adic counterparts.

Since adelic generalization of the Hartle-Hawking proposal was serious problems in minisuperspace models with matter, further developments of adelic quantum cosmology were done (see [22] and references therein) using formalism of adelic quantum mechanics [23]. It was shown that \( p \)-adic effects in adelic approach yield some discreteness of the minisuperspace and cosmological constant.

Possibility that the universe is composed of real and some \( p \)-adic worlds was considered in [24]. In the present paper we adopted approach that \( p \)-adic worlds are made of non-tachyonic \( p \)-adic matter.

Let us also mention research on \( p \)-adic inflation [25], and investigation of non-local cosmology with tachyon condensation by rolling tachyon from a false local vacuum to a stable one (see, e.g., [26, 27, 28, 29] and references therein).

### 4 Concluding Remarks

In the present article we have introduced a non-tachyonic \( p \)-adic matter which has origin in open scalar \( p \)-adic strings. Formally the corresponding Lagrangian was obtained replacing \( m^2 \) by \(-m^2\) in Lagrangian for \( p \)-adic string. In space-time dimensions \( D = 2 + 4k \) the kinetic energy term has correct sign and stable negative local
vacua. For this case there is decreasing time dependent field solution of the equation of motion and negative cosmological constant. This $p$-adic matter interacts with ordinary matter by gravity and should play some role in the dark side of the universe. In particular, the negative cosmological constant can change expansion to contraction and provide bouncing in cyclic universe evolution. These cosmological aspects are under consideration. If $p$-adic matter would be produced at the LHC experiment in CERN, then its first signature should be in the form of missing mass (energy) in the final state, because it interacts with ordinary matter only through gravitational interaction.

In the case of gravity with Friedmann-Lemaître-Robertson-Walker (FLRW) metric the d’Alembertian is $\Box = -\partial_t^2 - 3H\partial_t$, where $H$ is the Hubble parameter $H(t) = \frac{\dot{a}}{a}$. Then equation of motion contains this operator $\Box$ and time dependent solution (20) for a constant $H$ is

$$
\phi_q(t) = q^{\frac{1}{2(q-1)}} \exp \left(-\frac{3\ln q}{2} \frac{H}{m^2} \partial_t \right) \exp \left(-\frac{q-1}{2q\ln q} m^2 t^2 \right)
$$

$$
\exp \left(-\frac{q-1}{2q\ln q} m^2 \left(t - \frac{3\ln q}{2} H m^2 \partial_t \right) \right).
$$

(28)

Note that equation of motion (14) can be formally obtained from (7) by partial replacement $p \rightarrow \frac{1}{p}$ in the following two ways. $(i)$ In the LHS of (7) replace $p$ by $\frac{1}{p}$ and $\phi$ by $\phi$. $(ii)$ In the RHS of (7) replace $p$ by $\frac{1}{p}$ and $\phi$ by $\phi^p$. In [30], the equation

$$
e^{-\beta \Box} \Phi(x) = \sqrt{k} \Phi^k, \quad 0 < k < 1, \quad (\beta > 0)
$$

was considered and it corresponds to the case $(ii)$ when $k = \frac{1}{p}$.

We have also emphasized that results of measurements are rational numbers with norm in the form of the familiar absolute value, i.e. they are real rational numbers and not $p$-adic ones. In Lagrangian we have made some connection between nonlocality and $p$-adic valued world-sheet.

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