Algebraic visualization difficulties of students in junior high school

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Abstract. Algebra has been considered as a key aspect in forming the base to courses in advanced Mathematics and Science coursework in Secondary and Post-Secondary Education. Failure for students in algebra has become a big barrier for students to proceed with education. We argue that for students to be successful in secondary school algebraic topics such as equations and functions, and in post-secondary courses such as Calculus and Algebra, they need to overcome difficulties that they have in the basic aspects of algebraic operations. The study aimed to explore and describe difficulties that students have in algebraic operations at junior secondary school. This study used descriptive case study and sampling is purposeful. 30 students were given test on algebraic operations. Their work was analyzed by identifying the difficulties students have when they factorize and simplifying algebraic expressions. Results showed that students have difficulties in visualizing algebraic forms and applying the associativity and distributive properties of algebraic expressions when they factorize or simplify algebraic terms. We conclude that algebraic visualization is a key aspect for success in Mathematics. Teachers are encouraged to investigate on difficulties of students in algebraic operations and provide instructional material that help students overcome certain learning obstacles.

1. Introduction

Basic algebra forms the foundation of further abstract algebra and bases for almost all the courses in Secondary School Mathematics and beyond. Algebra has been considered as a “gate keeper” to students’ further education and career [1]. The “gate” needs to be kept open for the benefit of the students and this can be done by identifying the difficulties that students have in completing tasks in algebra. Students have various kinds of difficulties in evaluating quadratic expressions. Some students lack expert strategies in solving algebraic structure sense problems [2]. They lack insights into the problem in applying quadratic concepts to evaluate complex non rational roots of equations. Also when students are used to one form of algebraic structure, they may struggle to think out of the box and represent some form of images and diagrams in form of algebraic expression. Thus students have difficulties in understanding the problem when faced with the question in a different context [3].

Algebraic “mathematization” is an issue where students have difficulties in converting real life story problems into mathematical tasks [4]. Students are used to mechanical procedures in solving problems without making sense of it. This is caused by difficulties in understanding words, phrases or sentences and formulating equations, schemes or diagrams [3]. Visual representations such as diagrams and schemes facilitate understanding (of fractions) and reduce difficulties that students may face later when dealing with abstract concepts [5]. Students also lack relational thinking, that is failure to understand the
meaning of the equal sign and the meaning of variables in an algebraic structure [6]. Equations are taken as some expressions that are calculated to make the other side be true. Students sometimes confuse multiplication with addition of algebraic terms. A comparison strategy is suggested as a method to avoid difficulties in mistaking addition for multiplication in solving equations, and vice-versa [1].

So much literature provide information on students’ difficulties on algebraic aspects such as equations, sets, matrices and calculus, but this research focuses on the basic aspects of algebra that form the foundation of all other algebraic and non-algebraic topics in secondary school and beyond. By algebraic visualization, we mean that seeing mathematics in the algebraic form and use algebraic concepts in solving tasks easily and correctly helps students overcome difficulties in early algebra. In this sense, we asked ourselves; what are the kind of difficulties that students show when operating algebraic expressions, which “close the gate” to their future career? The purpose is to identify those obstacles and deal with them before it’s too late.

2. Materials and methods

Thirty grade 8 students (boys and girls) of an average age of 14 years participated in this study. These students were from one of the Junior High Schools in West Java, Indonesia. Participants were selected purposefully to identify the kind of difficulties that students of the same grade have in common when operating algebraic expressions. Each participant completed a test on factorizing and simplifying algebraic terms. The task comprised of five questions, four of which are in variable form and one in numerical form, (Table 1). The task was supposed to be completed in 45 minutes. For the item such as factorize: students were not allowed to use any electronic device to evaluate the answer. The test was administered by the class teacher and the questions were written in Indonesian language, hence questions were made as short as possible with the help of a co-researcher to avoid language barriers since language expression was not really part of the study.

We explored the difficulties that students encounter in simplifying algebraic expressions especially when the form of the expressions changes from that that the students are familiar with. For example, students were asked to evaluate. We expected students to notice the relationship between the numbers; that they are two terms that can be added and then be factorized algebraically by applying the distributive property. We expected students to notice the quadratic form within the numerical expression and apply the distributive property in evaluating the answer. We also wanted to see how students use the distributive, commutative and associative properties in solving algebraic tasks. Students’ difficulties in solving tasks in algebraic operations were analyzed and coded according to the algebraic properties that were supposed to be applied (Table 1). We also recorded the number of students who obtained correct answers and those who did not. We took note of the areas of difficulties that students showed during operating the algebraic expressions and analyzed how they came up with certain solutions.

Table 1. Algebraic properties.

| Question | Algebraic Property                                      | Task                                      |
|----------|--------------------------------------------------------|-------------------------------------------|
| 1a)      | Carrying out order of operation in algebra             | Simplify: $2x + 12x ÷ 6 - 2x$             |
| 1b)      | Commuting algebraic terms/group like terms             | Simplify: $3xy - yz + 5yz - 6xy$         |
| 2        | Identifying common fraction in algebraic terms         | Simplify: $rac{x^2 + 3x + 2}{x+1}$     |
| 3        | Identifying an algebraic structure within numerical task | Evaluate: $145^2 - 147 	imes 143$       |
| 4        | Identifying a three term expression with rational factor | Factorize: $a^2 - 2b^2 - 2ab + ab$       |

3. Results and discussion

Table 2 below presents the general results of our study, showing number of students who obtained correct and incorrect answers for each item. Results have shown that (20/30) students have difficulties in carrying out the proper order of operation in algebra. Also some students (26/30) have showed
difficulties in identifying algebraic structures embedded in a numerical form. All 30 students failed to identify a three term quadratic expression in which the expression had rational factors. The most difficulty question for students was question 3 and 4. While question 2 was the easiest, shown by 29/30 students who got the answer correct. The results also show that most students have no difficulties in factorizing quadratic expressions which are in the form \( ax^2 + bx + c \), which is familiar structure to most of the students.

Table 2. Results of test.

| Question | Correct Answer | Incorrect Answer |
|----------|----------------|------------------|
| 1a)      | 10             | 20               |
| 1b)      | 26             | 4                |
| 2        | 29             | 1                |
| 3        | 4              | 26               |
| 4        | 0              | 30               |

Some students have difficulties in commuting algebraic terms following the proper order of operation. For a task such as Simplify: \( 2x + 12x \div 6 - 2x \), some students fail to recognize the existence of certain terms which need to be evaluated first before the others. Instead of first dealing with division, students tent to see thing in a superficial way. Most students are more comfortable with addition, of which they are supposed to take note of the proper order of operation. As of the case above, they were expected to operate division of \( 12x \div 6 \), followed by addition and lastly subtraction. Students lack adequate knowledge about the commutative property in algebra [1].

In some circumstances, students have difficulties in understanding the meanings and forms of algebraic fractions. Some learners tend to carryout operations that are incorrect. This is because students are familiar with cancellation of common factors without proper understanding of the circumstances that come with cancellation of common factors in the denominators; they end up just cancelling same variables in an improper situation, (Figure 1).

![Figure 1](image1.png)

**Figure 1.** Student’s work showing difficulty in identifying an algebraic expression in unfamiliar form. Variable x has been incorrectly cancelled on the bottom with one of the variable in the power of x.

In addition, learners lack algebraic visualization. In this sense, we mean that some learners fail to identify an algebraic structure to represent some numerical forms. For the item: Evaluate: \( 145^2 - 147 \times 143 \), students failed to identify that the value 145 can be represented by any variable \( a \) for example. In this case the numerical form changes to an algebraic form like \( a^2 - (a + 2)(a - 2) \). When simplified, the answer simply becomes 4. Such visualization is needed for students to in carrying out algebraic operations easily instead of spending time and energy doing long multiplication, (Figure 2a). In doing long multiplication, students may end up doing lots of
Students show some kind of mechanical thinking where they memorize procedures in obtaining answers without attaching meaning to the problem at hand. They lack expert strategies in solving certain quadratic tasks presented in several forms [7]. Teachers are encouraged to provide such materials that encourage students to develop mathematical critical and reasoning ability to make their learning easy. Figure 2 below is showing students’ difficulty in identifying an algebraic structure within a numerical task. Students resorted to long method multiplication probably learnt at primary school.

Evaluate: $145^2 - 147 \times 143$

Figure 2a. A representative example of students trying to obtain answer by long multiplication.  

Figure 2b. A representative example of students’ difficulties in understanding the distributive property.

Although some answers end up being correct, methods and procedures that students sometimes use are worthy to be taken note of. A close assessment of students’ work is essential rather than only looking at the final answer [8]. Some students have shown difficulties in understanding the meaning behind common factors. Once they see similar figures, they cancel without understanding reasons behind the cancellation. From the same task, in Figure 2b, the learner eliminated 1 and 4 by simply cancelling them and remains with the figures that gave him/her the correct answer. Some learners have a tendency of using incorrect procedures and incorrect arithmetic models to solve tasks. Teachers need to emphasize how students should use the distributive property to evaluate tasks that are in algebraic forms [9].

There is need for Mathematics teachers to identify the kind of scaffolding that will help students overcome such kind of difficulties. In some cases students show difficulties in identifying familiar concepts presented to them in a different form besides the structure that they are used to. Their thinking remains focused on the forms that they are familiar with [10]. Once the forms change, many students become confused and they leave some tasks uncompleted. Figure 3 is showing difficulty that students have in factorizing quadratic forms with both variables $a$ and $b$ raised to the power of 2. Some students failed to combine like terms and re-arrange expression in such a way that they become three term expressions of the form $ax^2 + bx + c$ which is familiar and can be factorized to get rational roots. Students lack resilience in trying different ways of completing task. They have difficulties in solving problems that have different context, [3].
Figure 3. Student’s work showing difficulty in identifying an algebraic expression in unfamiliar form.

In addition the knowledge of fractions and associative and distributive properties of algebraic expression are essential prerequisites for overcoming difficulties in algebraic operations [9]. There is need for teachers to provide learning materials that facilitate students’ ability to be resilient in completing mathematical tasks instead of quitting tasks before completing them. In this way, student’s understanding is facilitated as they can identify each other’s mistakes. Students also need to be creative in carrying out algebraic operations and in any other topic in mathematics [10]. To overcome problems that may be faced by students in secondary school, expects suggest that students should be introduced to algebra early at lower grades to reduce the gape in shifting from arithmetic to algebra [11]. Authors have several suggestions for teaching strategies that would improve students’ algebraic performances later at secondary school. These include strategies that emphasizes understanding of fractions when learners are still at lower grades [12]. We suggest suitable pedagogical strategies such as group work so that students discuss and share ideas. It is therefore important for schools to create learning environments and learning materials that help improve students’ algebraic competence, creativity critical thinking and all other mathematical abilities.

4. Conclusion

Based on the findings and discussion above, we conclude that, some junior high school students have difficulties in the use of distributive and associative properties of algebraic operations. It has also been observed that, often students lack algebraic visualization. This means to say, they face problems in operating algebraic expressions once the expression has been manipulated a bit such that the expression is presented in a different form. We also noted that some difficulties in operating algebraic expressions are a result of inadequate prior knowledge on concepts such as fractions, variables and constants in algebraic expressions. Indeed understanding and mastery of algebraic concepts is essential for success in Mathematics. If students were to succeed in school and beyond, in the 21st century then algebraic knowledge is critical for them. Indeed, the activities done here revolve around showing the several ways on how students fail mathematics, algebra being the “gate keeper” of the life of the students. Keeping students’ difficulties in algebraic operations unknown will surely have negative implications on their education achievement, which is like closing the “gate” of life for them. The teacher is a scientist in the classroom, who should consistently investigate the issues within the classroom. Such issues include finding the difficulties students have in learning and try to help them overcome these difficulties in some way. Teachers are also encouraged to emphasize and investigate on students’ prior knowledge which will be necessary in the learning of new concepts.

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