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Remarks on the behaviour of financial market efficiency during the COVID-19 pandemic. The case of VIX

Bogdan Dima, Ştefana Maria Dima *, Roxana Ioan

East European Center for Research in Economics and Business (ECREB), West University of Timișoara, 16 J.H. Pestalozzi Street, 300115, Timișoara, Romania

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ABSTRACT

This paper investigates the Chicago Board Option Exchange Volatility Index’s (‘VIX’) response to the COVID-19 pandemic crisis, in terms of information efficiency. First, we estimate an Efficiency Index over rolling windows, based on closing levels, for a period between 1995-01-03 and 2020-12-30. Second, we check for the presence of deterministic chaos in efficiency series, by using the largest Lyapunov exponent and sample, as well as permutation entropy. However, we do not find that these estimators provide a clear evidence of a substantial change in VIX’s efficiency during 2020, in terms of deterministic chaos and irregular dynamics.

1. Introduction

The worldwide spread of COVID-19 has generated an unprecedented crisis, associated with more global human and social costs and a wider disruption of the economic activity than many of us may remember in modern times. As this shock is fully exogenous to the financial markets, this paper aims to investigate how these are reacting to the prolonged uncertainty surrounding the economic perspectives.

Consequently, we study the case of The Chicago Board Option Exchange Volatility Index (‘VIX’) market, for which we consider an information efficiency estimator. This index is based on the S&P 500 index options, and it is used to derive expected volatility by averaging the weighted portfolio of out-of-the-money puts and calls. The index represents a proxy for the ‘risk-neutral’ volatility of U.S. stock market expected over the coming 30 days. Usually called the ‘fear gauge’, it is widely employed by traders, analysts, and risk managers. VIX displays a significant predictive power for stock market returns, economic activity and financial instability and it reflects both stock market uncertainty and a variance risk premium (Bekaert and Hoerova, 2014). For that reason, it can be expected that, during the COVID-19 pandemic crisis, VIX will reflect increased uncertainty surrounding the future evolutions of financial markets and economy as a whole. And the already existing evidences show that “no previous infectious disease episode led to daily stock market swings that even remotely resemble the response in 2020 to COVID-19 developments” (Baker et al., 2020:3).

Thus, our study proposes a two-fold contribution to the market efficiency literature. First, it considers the dynamics of information

* Corresponding author.
E-mail address: stefana.dima@e-uvt.ro (Ş.M. DIMA).

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efficiency across several periods, revealing different market behaviors, in terms of stability and predictability. For a long period of
time, the ‘efficient market hypothesis’ (EMH) was widely accepted, as the ‘correct’ explanation for how markets work; when informa-
tion arises, it is quickly incorporated into the prices of securities without any significant delay. Hence, neither technical analysis,
nor the fundamental analysis, provides a support for portfolio management decisions. The reasoning behind this view is that, when
EMH applies, the future changes in prices reflect only future flows of information and are independent of the current changes in prices.
Since the nature and timing of future news is, by definition, unpredictable, then the subsequent price changes are also random. By the
beginning of the XXI century, EMH came under scrutiny. A more nuanced understanding of the markets’ mechanisms views stock
prices as, at least partially, predictable and past prices’ patterns and ‘fundamental’ variables as a prediction base for their future
evolutions (for some discussions and empirical evidences, see (Brown, 2011, Caporale et al., 2018, Gippel, 2012, Holtfort, 2019,
Malkiel, 2003, Shostak, 1997)). Although, at the moment, there is no single unified alternative to EMH, ‘behavioral finance’
(Hirshleifer, 2002, Kapoor and Prosad, 2017, Ritter, 2003), the ‘adaptive markets hypothesis’ (AMH) (Lo, 2004, 2005) or ‘fractal
market hypothesis’ (FMH) (Anderson and Noss, 2013, Peters, 1994) provide insights about the mechanisms that might push the
markets in (relative) inefficiency areas. By accounting for this stream of literature, the core argument of this paper is that a high prices’
predictability may be associated with lower levels of market information efficiency.

Second, our study aims to discover if efficiency’s trajectory becomes more chaotic during periods of major uncertainty, such as the
pandemic crisis.

In order to reach these aims, we consider a two-steps approach. In the first step, we assess the VIX market efficiency for a time span
covering both ‘business as usual’ and, respectively, significant financial and real turmoil periods. In a second step, we involve different
techniques to capture the potential presence of deterministic chaos in efficiency series itself. Since our analysis period includes a rare
and substantial exogenous-to-market shock (the pandemic crisis), we check if this type of shock impacts efficiency series’ predictability
differently than other crises generated by the market’s inner mechanisms (such as the ‘dot-com bubble’ or the 2007-2010 crisis).

We find that efficiency is, indeed, generated by a time-varying process. Also, we notice that the 2020 period does not reveal some
radical changes in efficiency series’ roughness.

2. Methodology

2.1. Measuring market efficiency

A first step entails the estimation of VIX’s market efficiency. With this aim, we employ the framework proposed by Krístoufek and
Vosvrda (Krístoufek and Vosvrda, 2013, Krístoufek and Vosvrda, 2014a, Krístoufek and Vosvrda, 2014b). According to this, an ‘Eff-
ciency Index’ (EI) at a moment 𝑡 can be defined as:

\[
EI_t = \sqrt{\sum_{i=1}^{n} \frac{\bar{M}_t^i - M^*_t}{R_i}} \tag{1}
\]

Here \(\bar{M}_t\) stands for an estimate of the 𝑖-th measure of efficiency, \(M^*_t\) is an expected value of the 𝑖-th measure for the efficient market
and \(R_i\) is a range of the 𝑖-th measure.

Relation (1) implies that this efficiency measure can be viewed as an estimator for the distance of the analyzed market from the
efficient one, based on various measures of market efficiency. So we consider two such measures: the Hurst exponent \(H\) - with an
expected value of 0.5 for the case of efficient market \((M^*_1=0.5)\) - and, respectively, the fractal dimension \(D\) - with an expected value of
1.5 \((M^*_D=1.5)\). Furthermore, \(R_{H}=R_{D}=1\). Hence, the maximum deviation from the efficient market is the same for both measures.

Moreover, two ways to quantify the efficiency are used here: one for detecting long-run memory in VIX’s returns’ series (the ‘Hurst
exponent’) and, respectively, one designed to capture the short-range memory (the ‘fractal dimension’). Since we are interested in the
inferred dynamics of market efficiency, we estimate EI not for the whole series, but at each moment 𝑡 from the analysis time span. So,
each measure is estimated over a specific ‘long’ (‘short’)-run rolling (‘overlapping’) window (a pre-set number of consecutive
observations).

2.2. Long-range memory: the Hurst exponent

The long-range dependency in a time series might be characterized as follows: a) in time domain, by power-law decay of auto-
correlation function; and b) in frequency domain, by a power-law divergence of spectrum close to the origin. The specific parameter is
the Hurst exponent, \(H\). For a stationary process, this parameter ranges between 0 and 1. At a reference level of 0.5, the autocorrelations
decay rapidly and there is no long-memory. Meanwhile, if \(H>0.5\) then the process is ‘persistent’ with strong positive correlations,
while at \(H<0.5\) the process is ‘anti-persistent’. However, empirical time series are not necessarily stationary. Hence, even though there
are available several estimators of \(H\) in both frequency and time domains, we employ the Multi-fractal De-trended Fluctuation Analysis
(MF-DFA) proposed by Kantelhardt et al. (Kantelhardt et al., 2002), due to its ability to detect, in a robust fashion, long-term cor-
rrelations in possible non-stationary time series. The key role in MF DFA is exercised by the \(q\)-th moment of fluctuation function for a
series \(x_t (j=1,2,...,N_i):\)
3.2. Detecting chaos in efficiency series: largest Lyapunov exponent and sample and permutation entropy

After the estimation of market efficiency, a next step is related to the analysis of its potential changes alongside different sub-periods. With this purpose, we investigate the possible presence of deterministic chaos in efficiency series as an underlying generative mechanism. In order to achieve this, we employ two main approaches: the largest Lyapunov exponent and two measures of efficiency entropy.

A way to measure the complexity in chaotic motion can be attained by highlighting the sensitivity of the dynamical behavior to the initial conditions given by two infinitely close initial states. The largest Lyapunov exponent (LLE) determines the predictability of the dynamical systems, by describing the divergence (or, opposite, the convergence) of two trajectories along certain directions in state space with similar initial conditions. The presence of at least one positive Lyapunov exponent can be viewed as an important evidence for chaos in the analyzed process.

In order to estimate the Lyapunov exponent of a time series, Eckmann and Ruelle (Eckmann and Ruelle, 1985) and Eckmann et al. (Eckmann et al., 1986) proposed a method based on nonparametric regression (the Jacobian approach). Even if, in principle, any nonparametric regression estimator can be employed for this method, in practice, one of the most used approaches is the Lyapunov exponent estimator based on neural networks proposed by Nychka et al. (Nychka et al., 1992) (see also (Ellner et al., 1991, McCaffrey et al., 1992, Shintani and Linton, 2004)). The estimation method is implemented by Sandebute and Escot (Sandebute and Escot, 2020).

Meanwhile, entropy can be viewed as one of the most important metrics for the assessment of a process complexity. Several entropy approaches are proposed in the literature, such as permutation entropy (PE), approximate entropy, sample entropy or fuzzy entropy. Sample entropy was proposed by Richman and Moorman (Richman and Moorman, 2000), as a modified version of permutation entropy (Pincus, 1991), aiming to correct for the self-match bias. Permutation entropy was first introduced by Bandt and Pompe (Bandt and Pompe, 2002). Each approach displays its distinctive advantages and limitations. For instance, permutation entropy has numerous benefits over other complexity measures, like its simplicity, non-parametric and free of restrictive parametric model assumptions. On the other hand, it is a single-scale-based approach and it does not account for the amplitudes when the process is mapped to permutation patterns. Consequently, we estimate over rolling windows both sample and permutation measures, in order to provide a comparison basis for the changes in entropy in each moment of the considered time horizon. The sample entropy method is implemented by Tomcala (Tomcala, 2018), whereas the permutation entropy estimation follows Sippel et al. (Sippel et al., 2019).
3. Data

Daily closing levels of VIX index are collected, from Yahoo Finance (https://finance.yahoo.com/) for a period between 1995-01-03 and 2020-12-30, by using R package ‘BatchGetSymbols’ (Perlin, 2020). We remove the non-available data ending with 6545 observations, on which we compute the daily returns of the index. Furthermore, in order to ride-off possible scale effects and to smooth out the outliers, the return values are transformed to their Z-scores (‘standard scores’). As a result, the data will show how many standard deviations below or above the population mean the raw score for each day is.

4. Results

Figure 1 displays the evolution of the EI for VIX market during the considered time span, while Table 1 reports the basic statistics for the series.

As the values of distribution parameters and the Jarque-Bera statistic for the full sample show, the empirical distribution of EI is asymmetrical with right fat-tails effects and platykurtic. Zivot and Andrews (Zivot and Andrews, 1992) test rejects the unit roots with drift null in favor of a stationary process with one-time break in both trend and intercept. In addition, there are no significant changes in these statistics for the 2020 sub-sample, when compared to the full sample, and the maximum deviation from efficiency is below that of the entire sample. Still, it is interesting to note that when Zivot and Andrews test is applied to this sub-sample, it shows that October 2020 includes a breakpoint in the efficiency series. It is worth mentioning that during the respective month, VIX had reacted to the upcoming results of the US presidential election, as well as to the second wave of Covid-19 infections, which has forced several countries to re-instate partial lockdowns. This was a period of substantial shocks due to the impossibility of most S&P 500 companies to remain fully operational during lockdown. Meanwhile, Zivot and Andrews test reflects shifts in VIX during ‘dot-com bubble’ or 2007-2010 financial and real turmoil.

Therefore, the estimation of VIX’s efficiency seams ‘locally’ sensitive to such shocks.

However, the shape of EI, as depicted in Figure 1, suggests that there might be several sub-periods with larger deviance from efficiency. A Bai and Perron test (Bai and Perron, 1998, Bai and Perron, 2003) as implemented by Zeileis et al. (Zeileis et al., 2002, Zeileis et al., 2003), supports this idea.

Based on BIC information criteria, this test detects for EI a three breaks model. A first identified break can be found in 2006, in the aftermath of the 2007-2010 financial and real crisis, while a second one occurs at its conclusion in 2010. A third break is located in 2014, after the market recovery period. With the notable exception of the first segment, the confidence intervals for these estimates are relatively narrow. Yet, in this framework, 2020 does not look like a period with major structural changes in VIX’s market efficiency.

In order to provide more insights, we then turn our attention towards the analysis of EI roughness. Figure 2 shows the evolution of the largest Lyapunov exponent and of the considered measures of efficiency entropy, while Table 3 and Table 4 report their main statistics.

Several evidences can be emphasized here. First, it appears that none of the LLE values is positive. Hence, LLE fails to provide evidence that the dynamics of EI is chaotic (of course, supposing that other usual conditions -e.g., phase space compactness- are fulfilled). In other words, there is no clear sensitive dependence on initial conditions for EI trajectory and local stability manifests itself. Nevertheless, the existence of negative values for LLE should be interpreted with caution since local exponents are not invariant under nonlinear changes. Furthermore, the estimation method does not necessarily detect high-dimensional and intermittent chaos.

Second, lowest values of LLE occur in 2014, while the highest ones build up during 2004-2005 pre-crisis periods and a break is
present in 2012. Meanwhile, during 2020, LLE values are somehow lower than in 2019, with no clear emerging trend for this period. However, the volatility of LLE estimates declines almost three times compared with the full sample. A significant decline in LLE volatility also occurs during 2007-2010 (and, to a lesser extent, during ‘dot-com bubble’).

Third, the values of permutation entropy remain close to 1, for the entire analysis period, suggesting that EI data can be driven by a non-Gaussian random process. Although sample entropy infers a significant irregularity in EI series, its values are placed in a relatively

Table 1
Basic statistics for EI

| Statistics                  | Full sample | 2000-2002 sub-sample | 2006-2008 sub-sample | 2009-2011 sub-sample | 2020 sub-sample |
|-----------------------------|-------------|-----------------------|----------------------|----------------------|-----------------|
| Maximum                     | 0.583       | 0.407                 | 0.521                | 0.402                | 0.393           |
| Minimum                     | 0.000       | 0.013                 | 0.008                | 0.000                | 0.004           |
| Mean                        | 0.148       | 0.145                 | 0.144                | 0.149                | 0.139           |
| Standard deviation          | 0.083       | 0.074                 | 0.083                | 0.074                | 0.080           |
| Skewness                    | 0.875       | 0.501                 | 0.852                | 0.523                | 0.766           |
| Kurtosis                    | 4.000       | 2.788                 | 3.513                | 2.937                | 3.054           |
| Jarque-Bera statistic       | 1098.30     | 32.861                | 99.785               | 34.617               | 24.692          |
| Zivot and Andrews root test | -37.578 (Critical) | -13.421             | -17.030              | -12.130              | -17.820         |
| Potential break point at:   | 2014-11-20  | 2002-08-22            | 2007-10-22           | 2010-06-10           | 2020-10-06      |

Notes: The Zivot and Andrews test is implemented in R package ‘urca’ (Pfaff, 2008).

Table 2
Bai and Perron test: regression coefficients in a three-break model

|          | \( \hat{\delta}_1 \) | \( \hat{\delta}_2 \) | \( \hat{\delta}_3 \) | \( \hat{\delta}_4 \) |
|----------|-----------------------|-----------------------|-----------------------|-----------------------|
| \( \hat{\delta}_1 \) | 0.140                 | 0.157                 | 0.123                 | 0.171                 |
|          | (0.003)               | (0.005)               | (0.003)               | (0.005)               |
| Corresponding breakpoint estimates | \( \hat{T}_1 \) | \( \hat{T}_2 \) | \( \hat{T}_3 \) | \( \hat{T}_4 \) |
| \( \hat{T}_1 \) | 2006.10.18            | 2010.10.06            | 2014.11.05            | (2002.11.21-2009.09.11) |
| \( \hat{T}_2 \) | (2010.06.17-2011.09.15) | 2014.11.05            | (2013.12.24-2014.12.19) |

Notes: Standard errors are in () and 97.5% confidence intervals in {}. The standard errors are estimated utilizing a kernel HAC estimator with a quadratic spectral kernel, pre-whitening using a VAR(1) model and an AR(1) approximation for the automatic bandwidth selection. The confidence intervals are derived from the distribution of the \( \text{argmax} \) functional of a process composed of two independent Brownian motions with different linear drifts and scales (see (Zeileis and Kleiber, 2005)).

Figure 2. Check for chaos in EI: LLE, sample entropy and permutation entropy
Notes: All the estimations are performed on a rolling (overlapping) window with a length of 1000 consecutive observations. The LLE estimation follows the implementation from the R package ‘DChaos’ (Sandubete and Escot, 2020); the sample entropy method is implemented in R package ‘TSEntropies’ (Tomcala, 2018), while the permutation entropy is implemented in package ‘statcomp’ (Sippel et al., 2019).
Table 3
Basic statistics for ILE

| Statistics                           | Full sample | 2000-2002 sub-sample | 2006-2008 sub-sample | 2009-2011 sub-sample | 2020 sub-sample |
|--------------------------------------|-------------|-----------------------|----------------------|----------------------|-----------------|
| Maximum                              | -0.124      | -0.157                | -0.141               | -0.159               | -0.195          |
| Minimum                              | -0.486      | -0.331                | -0.278               | -0.284               | -0.248          |
| Mean                                 | -0.205      | -0.218                | -0.179               | -0.188               | -0.223          |
| Standard deviation                   | 0.055       | 0.044                 | 0.020                | 0.020                | 0.016           |
| Skewness                             | -1.556      | -0.662                | -1.137               | -2.094               | 0.313           |
| Kurtosis                             | 6.736       | 2.060                 | 6.762                | 9.060                | 1.610           |
| Jarque-Bera statistic (chi-square distributed with two degrees of freedom) | 5387.60     | 82.585                | 607.71               | 1790.30              | 24.388          |
| (Critical values: (p = 0.000)         | (p = 0.000) | (p = 0.000)           | (p = 0.000)          | (p = 0.000)          | (p = 0.000)     |
| Zivot and Andrews (Zivot and Andrews, 1992) unit root test with potential break occurred in both intercept and linear trend | -16.162     | -18.664               | -17.827              | -20.067              | -3.211 (Critical values: (p = 0.000)         |
| Potential break                      | point at: 2012.07.10 | point at: 2002.01.23 | point at: 2008.06.19 | point at: 2010.01.27 | point at: 2020.05.20 |

Notes: Same specification as for Table 1

Table 4
Basic statistics for sample entropy and permutation entropy

(a) Sample entropy

| Statistics                           | Full sample | 2000-2002 sub-sample | 2006-2008 sub-sample | 2009-2011 sub-sample | 2020 sub-sample |
|--------------------------------------|-------------|-----------------------|----------------------|----------------------|-----------------|
| Maximum                              | 2.011       | 1.859                 | 1.820                | 1.804                | 1.749           |
| Minimum                              | 1.533       | 1.742                 | 1.656                | 1.646                | 1.705           |
| Mean                                 | 1.759       | 1.813                 | 1.734                | 1.722                | 1.728           |
| Standard deviation                   | 0.091       | 0.030                 | 0.046                | 0.035                | 0.010           |
| Skewness                             | 0.088       | -0.527                | 0.275                | 0.178                | -0.176          |
| Kurtosis                             | 2.753       | 2.113                 | 1.633                | 2.517                | 2.224           |
| Jarque-Bera statistic (chi-square distributed with two degrees of freedom) | 20.945      | 59.420                | 68.344               | 20.07                | 7.625           |
| (p = 0.000)                          | (p = 0.000) | (p = 0.000)           | (p = 0.000)          | (p = 0.000)          | (p = 0.000)     |
| Zivot and Andrews (Zivot and Andrews, 1992) unit root test with potential break occurred in both intercept and linear trend | -3.389      | -6.043                | -4.187               | -3.027               | -4.495 (Critical values: (p = 0.000)         |
| Potential break                      | point at: 2014.12.31 | point at: 2002.01.14 | point at: 2007.01.24 | point at: 2010.07.02 | point at: 2020.05.27 |

(b) Permutation entropy

| Statistics                           | Full sample | 2000-2002 sub-sample | 2006-2008 sub-sample | 2009-2011 sub-sample | 2020 sub-sample |
|--------------------------------------|-------------|-----------------------|----------------------|----------------------|-----------------|
| Maximum                              | 0.999       | 0.996                 | 0.996                | 0.999                | 0.96            |
| Minimum                              | 0.988       | 0.988                 | 0.991                | 0.992                | 0.994           |
| Mean                                 | 0.995       | 0.991                 | 0.993                | 0.995                | 0.995           |
| Standard deviation                   | 0.002       | 0.002                 | 0.001                | 0.001                | 0.000           |
| Skewness                             | -0.702      | 1.031                 | 0.270                | -0.215               | -0.801          |
| Kurtosis                             | 2.788       | 2.538                 | 2.702                | 1.550                | 4.261           |
| Jarque-Bera statistic (chi-square distributed with two degrees of freedom) | 458.97      | 140.11                | 11.972               | 72.029               | 43.626          |
| (p = 0.000)                          | (p = 0.000) | (p = 0.003)           | (p = 0.000)          | (p = 0.000)          | (p = 0.000)     |
| Zivot and Andrews (Zivot and Andrews, 1992) unit root test with potential break occurred in both intercept and linear trend | -3.739      | -4.774                | -4.135               | -4.114               | -4.793 (Critical values: (p = 0.000)         |
| Potential break                      | point at: 2002.01.14 | point at: 2002.03.01 | point at: 2007.08.03 | point at: 2010.07.07 | point at: 2020.05.28 |

Notes: Same specification as for Table 1

wide range, between 1.53 and 2.01. In addition, the empirical distribution of the sample entropy deviates less from normal distribution for 2020 sub-sample. The levels of both estimators clearly increase in 2020 compared to 2019, but resemble those specific to other periods (like the 2007-2010 crisis).
Fourth, all three indicators show a local breakpoint in May 2020. 
Fifth, permutation entropy is the only indicator placing a global break in January 2002, while the other two locate such break during the post-2010 recovery period of 2012-2014. Meanwhile, there are some substantial differences between the behaviour of EI during the ‘dot-com’ and, respectively, the 2007-2010 crisis.

Furthermore, we consider an alternative approach to the assessment of EI changes in the 2020 predictability. Namely, we select 1999-2019 as a reference period and we estimate, for it, the global values of LLE, sample and permutation entropies. Then, we compute the relative deviations during each trading day of out-of-sample 2020 period such as:

$$\Delta X_{i/\text{ref}} = \frac{|X_i - X_{i/\text{ref}}|}{X_{i/\text{ref}}} \times 100$$

Here, $X_i$ (with $i=1,2,3$) is the corresponding value of the EI roughness measure “$i$” during current day “$t$”, while $X_{i/\text{ref}}$ is the global in-sample corresponding value.

The implicit idea behind relation (5) is that the mechanisms of market efficiency normally display a certain long-run stability. So, a ‘global’ reference level of the efficiency series can be identified, with ‘local’ specific values converging towards it. Meanwhile, substantial shocks (like the pandemic one) can temporarily disrupt these mechanisms and, consequently, push current predictability of the efficiency away from its ‘global’ level, as markets go through areas with lower stability. The results are reported in Figure 3. It appears that the largest deviations from the global characteristic values of 1999-2019 occur for LLE, while the lowest deviations are registered for permutation entropy. In June and July of 2020, the shifts in entropy measures reach minimal values; whereas the highest values are registered during March, November and December 2020.

5. Conclusions

In a nutshell, neither the LLE nor the entropy estimators provide compelling evidence for a substantial change in the Efficiency Index during 2020, in terms of deterministic chaos and irregular dynamics. Obviously, the 2020 sub-sample is too short as to derive an ultimate conclusion on the pandemic’s impact on financial markets. Nonetheless, the results suggest no fundamental change in market leading mechanisms or in investors’ decisions. A possible explanation of these results may be related to the fact that, during high-uncertainty periods, investors turn their attention towards volatility-trading financial assets (including those linked to VIX index) and, thus, the market dynamic becomes more demand-driven. This leads to corresponding adjustments in markets’ functional mechanisms and it influences the fundamental determinants of its trajectory.

Another possible explanation may be connected to investors’ asymmetric responses to public policy measures (with restrictions on commercial activity and social distancing). Whereas the decrease of US GDP at an annual rate reaches a record-breaking level of 32.9%, in the second quarter of 2020 (according to Bureau of Economic Analysis’ U.S. Economy at a Glance), investors and companies from different sectors of the economy respond in a non-uniform manner, by displaying heterogeneous beliefs and by adopting a wide range of corporate-response decisions.

Several practical implications of these findings can be emphasized here: i) if market information efficiency is a time-varying process (as it is the corresponding degree of efficiency predictability) then both fundamental and technical analysis trading strategies may work (with various probabilities of success from one period to another). Broadly, in terms of portfolio management, investors can consider a diversification approach based on the differences between markets’ current stance of efficiency; ii) the VIX index displays a significant predictive power for economic activity and financial markets’ evolutions. Thus, changes in this market efficiency may in fact reveal investors’ expectations about the future prospects of the economic environment; iii) it appears that different triggering determinants of financial instability periods impact market efficiency in a non-uniform manner. Hence, a simple extrapolation of past efficiency evolutions may not be enough to derive sound predictions for efficiency dynamics during current periods of turmoil; iv) the financial markets respond positively to 2020 fiscal stimulus packages, Fed’s quantitative easing measures and to other COVID-19 containment measures (lockdowns, travel bans). Such reaction is translated into a moderate adjustment in the efficiency’s level

![Figure 3. Relative changes in LLE, sample entropy and permutation entropy measures (2020 versus the reference period 1999.04.07-2019.12.31)](image-url)
and its degree of predictability. Nevertheless, one cannot exclude the presence of some permanent effects on financial markets and capital flows (including those manifested in the mechanisms through which the new information is incorporated into financial assets prices).

Nevertheless, further research is required for a better understanding of: a) how market efficiency’s roughness estimators adjust to large out-of-the-market shocks; and b) the consequences of such adjustments.

CRediT authorship contribution statement

Bogdan DIMA: Conceptualization, Methodology, Software, Formal analysis, Supervision. Ștefania Maria DIMA: Conceptualization, Investigation, Writing - original draft, Writing - review & editing. Roxana IOAN: Conceptualization, Data curation, Formal analysis.

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