Node Placement Optimization in Wireless Powered Communication Networks

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Abstract

The applications of wireless power transfer technology to wireless communications can help build a wireless powered communication network (WPCN) with larger throughput, higher robustness, and increased flexibility compared to the conventional battery-powered network. However, due to the fundamental differences in wireless information and power transmissions, many important aspects of conventional battery-powered wireless communication networks need to be redesigned for efficient operations of WPCNs. In this paper, we study the node placement optimization problem in WPCNs, where the wireless devices (WDs) harvest the radio frequency energy transferred by dedicated energy nodes (ENs) in the downlink, and use the harvested energy to transmit data to information access points (APs) in the uplink. In particular, we are interested in minimizing the network deployment cost with minimum number of ENs and APs by optimizing their locations, while satisfying the energy harvesting and communication performance requirements of the WDs. Specifically, we first study the minimum-cost placement problem when the ENs and APs are separately located, where an alternating optimization method is proposed to jointly optimize the locations of ENs and APs. Based on the obtained results, we further study the placement optimization when each pair of EN and AP are co-located and integrated as a hybrid access point, and propose an efficient algorithm to solve this problem. Simulation results show that the proposed methods can effectively reduce the network deployment cost and yet guarantee the given performance requirements, which is a key consideration in the future applications of WPCNs.

Index Terms

Wireless power transfer, wireless powered communication networks, energy harvesting, network planning, node placement optimization.

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Modern wireless communication systems, e.g., cellular networks and wireless sensor networks (WSNs), are featured by larger bandwidth, higher data rate and lower communication delays. The improvement on communication quality and the increased data processing complexity have imposed higher requirement on the quality of power supply to wireless devices (WDs). Conventionally, WDs are powered by batteries, which have to be replaced/recharged manually once the energy is depleted. Alternatively, the recent advance of radio frequency (RF) enabled wireless power transfer (WPT) provides an attractive solution to power WDs over the air [1]. By leveraging the far-field radiative properties of microwave, WDs can harvest energy remotely from the RF signals radiated by the dedicated energy nodes (ENs) [2]. Compared to the conventional battery-powered methods, WPT can save the cost due to manual battery replacement/recharging in many applications, and also improve the network performance by reducing energy outages of WDs. Currently, RF power in milliwatt (mW) level can be effectively transferred to a distance of more than 10 meters [3]. The energy is sufficient to power the activities of many low-power communication devices, such as sensors and RF identification (RFID) tags [3]. In the future, we expect more practical applications of RF-enabled WPT to wireless communications thanks to the rapid developments of many performance enhancing technologies, such as energy beamforming with multiple antennas [4] and more efficient energy harvesting circuit designs [5].

In a wireless powered communication network (WPCN), the operations of WDs, including data transmissions, are fully/partially powered by means of RF-enabled WPT [6]–[13]. A TDMA (time division multiple access) based protocol for WPCN is first proposed in [6], where the WDs harvest RF energy broadcasted from a hybrid access point (HAP) in the first time slot, and then use the harvested energy to transmit data back to the HAP in the second time slot. Later, [7] extends the single-antenna HAP in [6] to a multi-antenna HAP that enables more efficient energy transmission via energy beamforming as well as more spectrally efficient SDMA (space division multiple access) based information transmission as compared to TDMA. To further improve the spectral efficiency, [8] considers using full-duplex HAP in WPCNs, where a HAP can transmit energy and receive user data simultaneously via advanced self-interference cancelation techniques. Intuitively, using a HAP (or co-located EN and information AP), instead of two separated EN and information access point (AP), to provide information and energy access is
an economic way to save deployment cost, and the energy and information transmissions in the network can also be more efficiently coordinated by the HAP. However, using HAP has an inherent drawback that it may lead to a severe “doubly-near-far” problem due to distance-dependent power loss [6]. That is, the far-away users quickly deplete their batteries because they harvest less energy in the downlink (DL) but consume more energy in the uplink (UL) for information transmission. To tackle this problem, separately located ENs and APs are considered to more flexibly balance the energy and information transmissions in WPCNs [9]–[11]. In this paper, we consider the method using either co-located or separate EN and information AP to build a WPCN.

Most of the existing studies on WPCNs focus on optimizing real-time resource allocation, e.g., transmit signal power, waveforms and time slot lengths, based on instantaneous channel state information (CSI, e.g., [6]–[8]). In this paper, we are interested in the long-term network performance optimization based mainly on the average channel gains. It is worth mentioning that network optimizations in the two different time scales are complementary to each other in practice. That is, we use long-term performance optimization methods for the initial stage of network planning and deployment, while using short-term optimization methods for real-time network operations after the deployment. Many current works on WPCNs use stochastic models to study the long-term performance because of the analytical tractability, especially when the WDs are mobile in location. For instance, [9] applies a stochastic geometry model in a cellular network to derive the expression of transmission outage probability of WDs as a function of the densities of ENs and information APs. Similar stochastic geometry technique is also applied to WPT-enabled cognitive radio network in [10] to optimize the transmit power and node density for maximum primary/secondary network throughput. However, in many application scenarios, the locations of the WDs are fixed, e.g., a sensor network with sensor (WD) locations predetermined by the sensed objects, or an IoT (internet-of-things) network with static WDs. In this case, a practical problem that directly relates to the long-term performance of WPCNs, e.g., sensor’s operating lifetime, is to determine the optimal locations of the ENs and APs. Nonetheless, this important node placement problem in WPCNs is still lacking of concrete studies.

In conventional battery-powered wireless communication networks, node placement problem concerns the optimal locations of information APs only, which has been well investigated especially for wireless sensor networks using various geometric, graphical and optimization
algorithms (see e.g., [14]–[17]). However, there exist major differences between the node placement problems in battery-powered and WPT-enabled wireless communication networks. On one hand, a common design objective in battery-powered wireless networks is to minimize the highest transmit power consumption among the WDs to satisfy their individual transmission requirements. However, such energy-conservation oriented design is not necessarily optimal for WPCNs, because high power consumption of any WD can now be replenished by means of WPT via deploying an EN close to the WD. On the other hand, unlike information transmission, WPT will not induce harmful co-channel interference to unintended receivers, but instead can boost their energy harvesting performance [18]. These evident differences indicate that the node placement problem in battery-powered wireless communication networks should be revisited for WPCNs, to fully capture the advantages of WPT.

In this paper, we study the node placement optimization problem in WPCNs, which minimizes the deployment cost on ENs and APs given that the energy harvesting and communication performances of all the WDs are satisfied. Our contributions are detailed below.

1) We formulate the optimal node placement problem in WPCNs using either separated or co-located EN and AP. To simplify the analysis, we then transform the minimum-cost deployment problem into its equivalent form that optimizes the locations of fixed number of ENs and APs;

2) For the node placement optimization using separated EN and AP, we first exploit the problem structure by studying the method to optimize the locations of ENs given fixed AP locations. In particular, we propose an efficient greedy algorithm that places the ENs optimally one-by-one to tackle the non-convexity of the optimization problem. Based on the obtained results, we further propose an effective alternating optimization method that jointly optimizes the EN and AP placements;

3) For the node placement optimization using co-located EN and AP (or HAP), we extend the greedy EN placement method under fixed APs to solving the HAP placement optimization, which is achieved by incorporating additional considerations of dynamic WD-to-HAP associations during HAP placement. Specifically, a trial-and-error method is proposed to solve the WD-to-HAP association problem, which eventually leads to an efficient greedy HAP placement algorithm.
Due to the non-convexity of the node placement problems in WPCNs, all the proposed algorithms are driven by the consideration of their applicabilities to large-size WPCNs, e.g., consisting of hundreds of WDs and EN/AP nodes. Specifically, we show that the proposed algorithms for either separated or co-located EN and AP placement are convergent and of low computational complexity. Besides, simulations validate our analysis and show that the proposed methods can effectively reduce the network deployment cost to guarantee the given performance requirements. The proposed algorithms may find their wide application in the future deployment of WPCNs, such as wireless sensor networks and IoT networks.

The rest of the paper is organized as follows. In Section II, we first introduce the system models of WPCN where the ENs and APs are either separated or co-located. Then, we formulate the optimal node placement problems for the above two cases in Section III. With either separated or co-located EN and AP, we propose efficient algorithms to the node placement optimization problems in Sections IV and V, respectively. In Section VI, simulations are performed to evaluate the performance of the proposed node placement methods. Finally, we conclude the paper in Section VII.

II. System Model

In this section, we first describe the system models of a WPCN where the ENs and APs are either separated or co-located in Sections II.A and II.B, respectively. Then, we introduce in Section II.C the key performance metric used in this paper.

A. Separated ENs and APs

For the case of separated ENs and APs, we consider in Fig. 1 a WPCN in $\mathbb{R}^2$ consisting of $M$ ENs, $N$ APs and $K$ WDs, whose locations are denoted by $2 \times 1$ coordinate vectors $\{u_i| i = 1, \cdots, M\}$, $\{v_j| j = 1, \cdots, N\}$, and $\{w_k| k = 1, \cdots, K\}$, respectively. We assume that the energy and information transmissions are performed on orthogonal frequency bands without interfering with each other. Specifically, the ENs are connected to stable power source and broadcast RF energy in the DL for the WDs to harvest the energy and store in their rechargeable batteries. At the same time, the WDs use the battery power to transmit information to the APs in the UL. The circuit structure of a WD to perform the above operations is also shown in Fig. 1.
In a transmission time slot of length $T$, the $M$ ENs transmit simultaneously on the same bandwidth in the DL, where each EN $i$ transmits

$$x_i(t) = \sqrt{P_0} s_i(t), \ t \in [0, T], \ i = 1, \cdots, M.$$  \hspace{1cm} (1)

Here, $P_0$ denotes the transmit power, $s_i(t)$ denotes the pseudo-random energy signal used by the $i$-th EN, which is assumed to be of unit power ($\mathbb{E}_t [|s_i(t)|^2] = 1$) and independent among the ENs ($\mathbb{E}_t [s_i(t)s_j(t)] = 0$ if $i \neq j$). The reason to use random signal instead of a single sinusoid tone is to avoid peak in transmit power spectrum density, for satisfying the equivalent isotropically radiated power (EIRP) requirement enforced by spectrum regulating authorities [1]. Notice that the energy beamforming technique proposed in [4] is not used in our setup, as it requires accurate CSI and DL symbol-level synchronization, which may be costly to implement in a highly distributed WPCN network considered in this work.

Accordingly, the received energy signal by the $k$-th WD is

$$y_k(t) = \sqrt{P_0} \sum_{i=1}^{M} \alpha_{i,k} s_i(t), \ k = 1, \cdots, K,$$  \hspace{1cm} (2)

where $\alpha_{i,k}$ denotes the equivalent baseband channel coefficient from the $i$-th EN to the $k$-th WD, which is assumed to be constant within a transmission block but may vary independently across different blocks. Let $h_{i,k} \triangleq |\alpha_{i,k}|^2$ denote the channel power gain, which follows a general
distribution with its mean determined by the distance between the EN and WD, i.e.,
\[
\mathbb{E}[h_{i,k}] = \beta |u_i - w_k|^{-d_D}, \quad i = 1, \cdots, M, \quad k = 1, \cdots, K,
\]
where \(d_D \geq 2\) denotes the path loss exponent in DL, \(\| \cdot \|\) denotes the \(l_2\)-norm operator, and \(\beta\) denotes a positive parameter related to the signal carrier frequency \([19]\). Then, each WD \(k\) can harvest an average amount of energy from the energy transmission within each block given by \([2]\)
\[
Q_k = \eta T \mathbb{E}[[y_k(t)]^2] = \eta \beta T P_0 \left( \sum_{i=1}^{M} h_{i,k} \right), \quad k = 1, \cdots, K,
\]
where \(\eta \in (0, 1]\) denotes the energy harvesting circuit efficiency. Let \(\lambda_k \triangleq \mathbb{E}[Q_k]/T\) denote the average energy harvesting rate over all the transmission blocks. We thus have
\[
\lambda_k = \eta \beta P_0 \cdot \sum_{i=1}^{M} |u_i - w_k|^{-d_D}, \quad k = 1, \cdots, K.
\]

In the UL information transmissions, we assume that each WD transmits data to only one of the APs. To make the placement problem tractable, the WD-to-AP associations are assumed to be fixed, where each WD \(k\) transmits to its nearest AP \(j_k\) regardless of the instantaneous CSI, i.e.,
\[
j_k = \arg \min_{j=1,\cdots,N} ||v_j - w_k||, \quad k = 1 \cdots, K.
\]
The data from different WDs are assumed to be sent over orthogonal channels, so that the received user signals are free from co-channel interference. Besides, for the simplicity of analysis, we assume no limit on the maximum number of WDs that an AP could receive data from. Then, the average power consumption rate for WD \(k\) is modeled as
\[
\mu_k = a_{1,k} + E_k \triangleq a_{1,k} + a_{2,k} |v_{j_k} - w_k|^{d_U}, \quad k = 1, \cdots, K,
\]
where \(a_{1,k}\) denotes the constant circuit power of WD \(k\), \(E_k\) denotes the average transmit power as a function of the distance between WD \(k\) and its associated AP \(j_k\) \([20], [21]\). Besides, \(a_{2,k} > 0\) is a parameter related to the signal carrier frequency and the encoding/decoding scheme used in the UL communication, while \(d_U \geq 2\) denotes the UL channel path loss exponent. Note that higher path loss \(d_U\) and larger WD-AP distance separation correspond to more average transmit power consumption needed in UL communication.
B. Co-located ENs and APs

A special case of the WPCN that we consider in Fig. 1 is when the ENs and APs are grouped into pairs and each pair of EN and AP are co-located and integrated as a hybrid access point (HAP), which corresponds to setting $M = N$ and $u_i = v_i$ for $i = 1, \cdots, M$. With the network model and HAP’s circuit structure shown in Fig. 2, a HAP transfers RF power in the DL and receives information in the UL simultaneously on different frequency bands. Although the use of HAPs is less flexible in placing the ENs and APs than with separated ENs and APs, the overall deployment cost is reduced, because the production and operation cost of a HAP is in general less than the sum-cost of two separate EN and AP.

For brevity, we reuse the notation $u_i, i = 1, \cdots, M$, to denote the location coordinates of the $M$ HAPs. Given other parameters unchanged, the expression of the average energy harvesting rate $\lambda_k$ of the $k$-th WD is the same as that in (5). Meanwhile, the average power consumption rate $\mu_k$ can be obtained from (7) by replacing $v_j$ with $u_i$ as follows.

$$\mu_k = a_{1,k} + a_{2,k} \| u_{ik} - w_k \| d_{uv}, \quad k = 1, \cdots, K,$$  

(8)
where $i_k$ is the index of the HAP that WD $k$ associates with, i.e.,

$$i_k = \arg \min_{i=1,\ldots,M} ||u_i - w_k||, \ k = 1, \cdots, K. \tag{9}$$

C. Performance Metric

With the above definitions, the net energy harvesting rates of the WDs in both cases of separate and co-located EN and AP are given by

$$\omega_k = \lambda_k - \mu_k, \ k = 1, \cdots, K. \tag{10}$$

In practice, the net energy harvesting rate can directly translate to the performance of device operating lifetime (see e.g., [22]). Specifically, given an initial battery level $C$, the average time before the $k$-th WD’s battery depletes is $-C/\omega_k$ when $\omega_k < 0$, and $+\infty$ when $\omega_k \geq 0$. In any case, a larger $\omega_k$ will lead to longer, if not infinite, average device operating lifetime, which is thus the key performance metric considered in this paper.

III. Problem Formulation

In this paper, we assume that the locations of the WDs are known and study the optimal placement of ENs and information APs, which are either separated or co-located in their locations. This may correspond to a sensor network with sensor (WD) locations predetermined by the sensed objects, or an IoT network with static WDs. In particular, we are interested in minimizing the deployment cost given that the net energy harvesting rates of all the WDs are larger than a common prescribed value $\gamma$, i.e., $\omega_k \geq \gamma, \ k = 1 \cdots, K$, where $\gamma$ is set according to (10).

A. Separated ENs and APs

When the ENs and APs are separated, the total deployment cost is $c_1 M + c_2 N$ if $M$ ENs and $N$ APs are used, where $c_1$ and $c_2$ are the monetary costs of deploying an EN and an AP, respectively. To solve the minimum-cost deployment problem, let us first consider the following
feasibility problem:

\[
\begin{align*}
\text{Find} & \quad \mathbf{U}^M = [\mathbf{u}_1, \ldots, \mathbf{u}_M], \quad \mathbf{V}^N = [\mathbf{v}_1, \ldots, \mathbf{v}_N] \\
\text{subject to} & \quad \lambda_k \left( \mathbf{U}^M \right) - \mu_k \left( \mathbf{V}^N \right) \geq \gamma, \quad k = 1, \ldots, K, \\
& \quad b^l \leq \mathbf{u}_i \leq b^h, \quad i = 1, \ldots, M, \\
& \quad b^l \leq \mathbf{v}_j \leq b^h, \quad j = 1, \ldots, N,
\end{align*}
\]

(11)

where \( \{b^l, b^h\} \) specifies a feasible deployment area in \( \mathbb{R}^2 \). \( \lambda_k \) and \( \mu_k \) are functions of \( \mathbf{U}^M \) and \( \mathbf{V}^N \) given in (5) and (7), respectively. Evidently, if (11) can be efficiently solved for any \( M \) and \( N \), then the optimal node placement solution to the considered minimum-cost deployment problem can be easily obtained through a simple two-dimension search over the values of \( M \) and \( N \), i.e., finding a pair of feasible \( (M, N) \) that produces the lowest deployment cost \( c_1 M + c_2 N \).

Equivalently, (11) is feasible if and only if the optimal objective \( t^* \) of the following problem satisfies \( t^* \geq \gamma \) for any fixed \( M > 0 \) and \( N > 0 \),

\[
\begin{align*}
\text{maximize} & \quad t \\
\text{subject to} & \quad \lambda_k \left( \mathbf{U}^M \right) - \mu_k \left( \mathbf{V}^N \right) \geq t, \quad k = 1, \ldots, K, \\
& \quad b^l \leq \mathbf{u}_i \leq b^h, \quad i = 1, \ldots, M, \\
& \quad b^l \leq \mathbf{v}_j \leq b^h, \quad j = 1, \ldots, N.
\end{align*}
\]

(12)

Then, the key difficulty of solving the optimal deployment problem is to find efficient solution for problem (12).

B. Co-located ENs and APs

When the ENs and APs are integrated as HAPs, the total deployment cost is \( c_3 M \) if \( M \) HAPs are used. Here, \( c_3 \) denotes the cost of deploying a HAP, where in general \( c_3 < c_1 + c_2 \). Similar to the case of separated ENs and APs, the minimum-cost placement problem can be equivalently formulated as the following feasibility problem for any fixed number of \( M > 0 \) HAPs,

\[
\begin{align*}
\text{Find} & \quad \mathbf{U}^M = [\mathbf{u}_1, \ldots, \mathbf{u}_M] \\
\text{subject to} & \quad \lambda_k \left( \mathbf{U}^M \right) - \mu_k \left( \mathbf{U}^M \right) \geq \gamma, \quad k = 1, \ldots, K, \\
& \quad b^l \leq \mathbf{u}_i \leq b^h, \quad i = 1, \ldots, M.
\end{align*}
\]

(13)
where $\lambda_k (U^M)$ and $\mu_k (U^M)$ are given in (5) and (8), respectively. Equivalently, the feasibility of (13) can be determined by solving the following optimization problem

$$\begin{align*}
\text{maximize} & \quad t \\
\text{subject to} & \quad \lambda_k (U^M) - \mu_k (U^M) \geq t, \quad k = 1, \ldots, K, \\
& \quad b_l \leq u_i \leq b_h, \quad i = 1, \ldots, M,
\end{align*}$$

and then comparing the optimal objective $t^*$ with $\gamma$, to see whether $t^* \geq \gamma$ holds. In the following, we propose efficient algorithms to solve problems (12) and (14) in Sections IV and V, respectively, and evaluate their performance by simulations in Section VI.

IV. PLACEMENT OPTIMIZATION OF SEPARATED ENs AND APs

In this section, we study the node placement optimization for separately located EN and AP in problem (12). To understand the problem structure, we first study in Section IV.A the method to optimize EN placement assuming that the locations of APs are fixed in a WPCN, where an efficient greedy algorithm is proposed. Based on the proposed method, we then use an alternating optimization method to jointly optimize the locations of APs with those of ENs in Section IV.B. In addition, some alternative methods are considered in Section IV.C for performance comparison.

A. Optimal EN Placement with Fixed AP Location

We first consider the optimal EN placement problem when the locations of the APs are fixed, i.e., $v_j$’s are known. In this case, the WD-AP association $j_k$ is known for each WD $k$ from (6), and $\mu_k$’s can be calculated accordingly from (7). It is worth mentioning that the proposed algorithms under the fixed AP setup are also applicable to a wireless powered network of general power usage besides communication purpose, as long as the energy consumption rates $\mu_k$’s are known parameters. With $v_j$’s and $\mu_k$’s being fixed, we can rewrite (12) as

$$\begin{align*}
\text{maximize} & \quad t \\
\text{subject to} & \quad \varphi \cdot \sum_{i=1}^{M} ||u_i - w_k||^{-d_D} - \mu_k \geq t, \quad k = 1, \ldots, K, \\
& \quad b_l \leq u_i \leq b_h, \quad i = 1, \ldots, M,
\end{align*}$$

(15)
where $\varphi \triangleq \eta \beta P_0$. We can see that (15) is a non-convex optimization problem, because $||u_i - w_k||^{-d_D}$ is neither a convex nor concave function in $u_i$. As it currently lacks of effective method to convert (15) into a convex optimization problem, the optimal solution is in general hard to obtain. However, for a special case with $M = 1$, i.e., placing only one EN, the optimal solution is obtained in the following. By setting $M = 1$, (15) can be rewritten as

$$
\begin{align*}
\text{maximize} & \quad t \\
\text{subject to} & \quad ||u_1 - w_k||^{d_D} \leq \frac{\varphi}{t + \mu_k}, \quad k = 1, \ldots, K, \\
& \quad b^l \leq u_1 \leq b^h.
\end{align*}
$$

Although (16) is still a non-convex optimization problem (as $\varphi/(t + \mu_k)$ is not a concave function in $t$), it is indeed a convex feasibility problem over $u_1$ when $t$ is fixed. Therefore, the optimal solution of (16) can be obtained using a bi-section search method over $t$, whose pseudo-code is given in Algorithm 1.

**Algorithm 1: Bi-section search for single EN placement.**

*input*: WD locations $w_k$'s, power consumption rates $\mu_k$'s

*output*: the optimal location of the EN $u_1^*$

1. Initialize: $LB \leftarrow 0$, $UB \leftarrow$ sufficiently large value;

2. repeat

3. $t \leftarrow (UB + LB)/2$;

4. if Problem (16) is feasible given $t$ then

5. $LB \leftarrow t$;

6. $\hat{u}_1 \leftarrow$ a feasible solution of (16) given $t$;

else

7. $UB \leftarrow t$;

end

8. until $|UB - LB|$ is sufficiently small;

9. Return $u^* \leftarrow$ the last feasible solution $\hat{u}_1$;

Since placing one EN optimally is solved, we have the potential to decouple the difficult EN placement problem (15) into $M$ relatively easy problems with $M > 1$. This motivates a greedy algorithm, which places the ENs iteratively one-by-one into the network. In each iteration, the newly placed EN optimizes the net energy harvesting rates of an expanding subset of WDs, until all the WDs are included. Specifically, we first separate the $K$ WDs into $M$ non-overlapping clusters (assuming $K \geq M$), denoted by $\{W_i|i = 1, \ldots, M\}$. This can be efficiently achieved with the well-known $K$-means clustering algorithm [23]. Then, in the $i$-th iteration ($i \geq 1$),
we obtain the optimal location of the \(i\)-th EN, denoted by \(u_i^*\), by maximizing the net energy harvesting rates of the WDs in the first \(i\) clusters as follows.

\[
\begin{align}
\text{maximize} & \quad t_i \\
\text{subject to} & \quad (t_i + \mu_k - \lambda_{i-1,k}) \cdot ||u_i - w_k||_D \leq \varphi, \ k \in \{W_1 \cup \cdots \cup W_i\}, \\
& \quad b^L \leq u_i \leq b^H,
\end{align}
\]

where \(\lambda_{i-1,k}\) denotes the accumulative RF power harvested at the \(k\)-th WD due to the \((i-1)\) previously deployed ENs, given by

\[
\lambda_{i-1,k} = \begin{cases} 
0, & i = 1, \\
\varphi \cdot \sum_{j=1}^{i-1} ||u_j^* - w_k||_D, & i > 1.
\end{cases}
\]

In each iteration, \(u_i^*\) can be efficiently obtained using the similar bi-section search method over \(t_i\) in solving (16). The only difference is that \(t_i + \mu_k - \lambda_{i-1,k} < 0\) may happen during the search over \(t_i\) for some \(k\), such that (17b) becomes a non-convex constraint. In this case, however, the
corresponding inequality in (17b) holds automatically, thus can be removed without affecting the result of feasibility test in consideration given the value of $t_i$.

The pseudo-code of the greedy algorithm is given in Algorithm 2 and illustrated in Fig. 3. In this example, we first divide the WDs into $M = 3$ clusters in Fig. 3(a), then place the 3 ENs one-by-one in Fig. 3(b)-(d). When placing the 1st EN or EN$_1$, the algorithm only considers the received energy of the WDs in the 1st cluster (shaded WDs in Fig. 3(b)) from the EN to be placed; for the 2nd EN or EN$_2$, it considers the received energy of the WDs in the 1st and 2nd clusters from the first 2 ENs; for the last EN or EN$_3$, it considers the received energy of all the WDs from the 3 ENs. The total number of feasibility tests performed is $M \log (2\delta/\sigma)$, while the time complexity of solving each convex feasibility test is in polynomial order of the number of WDs. Therefore, the overall time complexity is moderate even for a large-size network consisting of, e.g., hundreds of WDs.

**Algorithm 2:** Greedy algorithm for $M$ EN placement under fixed APs.

```plaintext
input : WD locations $w_k$'s, power consumption rates $\mu_k$'s;  
output: the optimal locations of $M$ ENs $\{u_1^*, \ldots, u_M^*\}$;  
1 Clustering the WDs into $\{W_i, i = 1, \ldots, M\}$;  
2 for $i = 1$ to $M$ do  
3 $\quad LB \leftarrow -\delta$, $UB \leftarrow \delta$, $\delta$ is sufficiently large;  
4 $\quad$ Update $\{\lambda_{i-1,k}, k = 1 \cdots, K\}$ using (18);  
5 $\quad$ repeat  
6 $\quad$ $t_i \leftarrow (UB + LB)/2$;  
7 $\quad$ Ignore the constraints in (17b) with $t_i + \mu_k - \lambda_{i-1,k} < 0, k = 1, \ldots, K$;  
8 $\quad$ if Problem (17) is feasible given $t_i$ then  
9 $\quad$$\quad$ $LB \leftarrow t_i$;  
10 $\quad$$\quad$ $u_i^* \leftarrow$ a feasible solution of (17);  
11 $\quad$ else  
12 $\quad$$\quad$$\quad$$UB \leftarrow t_i$;  
13 $\quad$ end  
14 $\quad$ until $|UB - LB| < \sigma$;  
15 end  
16 Return $\{u_1^*, \ldots, u_M^*\}$.  
```

B. Joint EN and AP Placement Optimization

Based on the proposed greedy algorithm for EN placement optimization under fixed APs, we further study in this subsection the joint optimization of the locations of APs with those of ENs.
With $v_j$'s being variables, $\mu_k$ is expressed as

$$\mu_k = a_{1,k} + a_{2,k} \min_{j \in \{1, \ldots, N\}} ||v_j - w_k||^{d_U}, \ k = 1, \ldots, K. \quad (19)$$

After substituting (5) and (19) into (12), we can formulate the joint EN-AP placement problem as follows.

$$\begin{align*}
\text{maximize} & \quad t, U^M, V^N \\
\text{subject to} & \quad \varphi \cdot \sum_{i=1}^{M} ||u_i - w_k||^{-d_D} - a_{2,k} ||v_{j_k} - w_k||^{d_U} \geq t + a_{1,k}, \ k = 1, \ldots, K, \\
& \quad b^l \leq u_i \leq b^h, \ b^l \leq v_j \leq b^h, \ i = 1, \ldots, M, \ j = 1, \ldots, N.
\end{align*} \tag{20}$$

where $j_k$ is the index of AP that WD $k$ associates with given in (6). Evidently, the optimization problem is highly non-convex because of the non-convex nature of function $||u_i - w_k||^{-d_D}$, and the minimum operator over convex functions in (6).

To solve the joint EN-AP placement problem, we propose Algorithm 3 as an alternating optimization method to iteratively update the locations of the ENs and APs. Specifically, we start with a random feasible AP placement $v_j^{(0)}$'s, from which $\mu_k$'s can be calculated using (19).

With $\mu_k$'s being fixed, we first optimize the locations of the $M$ ENs using the greedy algorithm proposed in Section IV.A. Then, we update the AP locations with the obtained EN placements (and thus $\lambda_k$’s calculated from (5)) by solving the following problem:

$$\begin{align*}
\text{maximize} & \quad t, U^M, V^N \\
\text{subject to} & \quad \lambda_k - a_{2,k} ||v_{j_k} - w_k||^{d_U} \geq t + a_{1,k}, \ k = 1, \ldots, K, \\
& \quad b^l \leq v_j \leq b^h, \ j = 1, \ldots, N.
\end{align*} \tag{21}$$

Notice that if $j_k$’s are known, (21) is a convex problem that is easily solvable. In practice, however, $j_k$’s are revealed only after (21) is solved and the optimal placement of APs is obtained. To resolve this conflict, a trial-and-error method is proposed (see lines 6-15 in Algorithm 3) to find feasible $j_k$’s. Conceptually, we first convert (21) into a convex problem by assuming a set of WD-AP associations, denoted by $j_k^{(b)}$, $k = 1, \ldots, K$, and then solve (21) for the optimal AP placement based on the assumed $j_k^{(b)}$’s. Next, we compare $j_k^{(b)}$’s with the actual WD-AP associations after the optimal AP placement is obtained using (6), denoted by $j_k^{(a)}$, $k = 1, \ldots, K$.

Specifically, we check if $j_k^{(a)} = j_k^{(b)}$, $\forall k$. If yes, we have obtained the optimal AP placement;
otherwise, we update $j_k^{(b)} = j_k^{(a)}$, $k = 1, \cdots, K$ and repeat the above process until $j_k^{(a)} = j_k^{(b)}$, $\forall k$. The convergence of the trial-and-error method is proved in Appendix. Intuitively, the trial-and-error method is convergent because the optimal value of (21) is bounded, while by updating $j_k^{(b)} = j_k^{(a)}$, we can always improve the optimal objective value of (21) in the next round of solving it. In addition, the overall alternating optimization method in Algorithm 3 converges because the objective value is non-decreasing as the iterations proceed.

**Algorithm 3: Alternating AP-EN placement optimization.**

```plaintext
input : $K$ WD locations $w_k$’s, $N$ initial AP locations $v_j^{(0)}$’s;
output: the optimal locations of $M$ ENs $\{u_1^*, \cdots, u_M^*\}$
the optimal locations of $N$ APs $\{v_1^*, \cdots, v_N^*\}$;

1. Initialize: $z_{-1} \leftarrow -\infty$, $z_0 \leftarrow -\infty$;
2. repeat
   3. Update previous optimal objective value $z_{-1} \leftarrow z_0$;
   4. Given $v_j^{(0)}$’s, calculate $j_k^{(b)}$’s and $\mu_k$’s using (6) and (19);
   5. Given $\mu_k$’s, optimize EN placement $u_i^*$’s using Algorithm 2 then update $\lambda_k$’s using (5);
   6. flag $\leftarrow 1$;
   7. while flag $= 1$ do
      8. Given $j_k^{(b)}$’s, solve (21) for optimal AP placement $v_j^*$’s;
      9. Given $v_j^*$’s, calculate $j_k^{(a)}$’s using (6);
      10. if $j_k^{(a)} \neq j_k^{(b)}$ for some $k$ then
           11. Update $j_k^{(b)} = j_k^{(a)}$, $k = 1, \cdots, K$;
        else
           12. Optimal AP placement is found, flag $\leftarrow 0$;
      end
   13. Update initial AP locations $v_j^{(0)} \leftarrow v_j^*$, $j = 1, \cdots, N$;
   14. $z_0 \leftarrow$ the current optimal objective value of (21);
   15. until $|z_0 - z_{-1}| < \sigma$;
16. Return $\{u_1^*, \cdots, u_M^*\}$, $\{v_1^*, \cdots, v_N^*\}$;
```

C. Alternative Methods

Besides the proposed alternating method for solving (20), we also consider in this subsection two alternative methods used as benchmark algorithms for performance comparison.

1) Local Searching: The local searching algorithm starts with a random deployment of the $M$ ENs and $N$ APs, i.e., $u_i$’s and $v_i$’s, and checks if the minimum net energy harvesting rate among the WDs, i.e.,

$$P_r \triangleq \min_{k=1,\cdots, K} \left( \varphi \cdot \sum_{i=1}^{M} ||u_i - w_k||^{-d_U} - a_{1,k} - a_{2,k} \min_{j=1,\cdots, N} ||v_j - w_k||^{-d_U} \right)$$

(22)
can be increased by moving any of the ENs and APs to a random neighboring location. If yes, it makes the move and repeats the random movement process. Otherwise, if $P_r$ cannot be increased, the algorithm has reached a local maximum and returns the current placement solution. Several off-the-shelf local searching algorithms are available, where simulated annealing is used in this paper. In particular, simulated annealing can improve the searching result by allowing the nodes to be moved to locations with decreased value of $P_r$ to reduce the chance of being trapped at local maximums. Besides, we can improve the quality of deployment solution using different initial node placements, which are obtained either randomly or empirically, and select the resulted solution with the best performance.

2) Sequential convex optimization (SCO): SCO is a combination of local searching and convex optimization methods. Similar to the above local searching algorithm, it iteratively updates the locations of the ENs and APs until a local maximum is reached. To apply the SCO method, we first equivalently express the joint EN-AP placement problem in (20) as follows.

$$\begin{align*}
\text{maximize} & \quad t \\
\text{subject to} & \quad \varphi \cdot \sum_{i=1}^{M} ||u_i - w_k||^{-d_D} - \mu_k \geq t, \quad k = 1, \ldots, K, \\
& \quad a_{1,k} + a_{2,k} ||v_j - w_k||^{d_U} \geq \mu_k, \quad k = 1, \ldots, K, \quad j = 1, \ldots, N, \\
& \quad b^l \leq u_i \leq b^h, \quad b^l \leq v_j \leq b^h, \quad i = 1, \ldots, M, \quad j = 1, \ldots, N,
\end{align*}$$  

(23)

where $\mu = [\mu_1, \ldots, \mu_K]$. Starting from the initial locations $U^{(0)} = [u_1^{(0)}, \ldots, u_M^{(0)}]$ and $V^{(0)} = [v_1^{(0)}, \ldots, v_N^{(0)}]$, SCO solves in the $l$-th iteration ($l \geq 1$) the linearized version of (23) by approximating the function $||u_i - w_k||^{-d_D}$ and $||v_j - w_k||^{d_U}$ by their first-order approximations within a box trust region around $U^{(l-1)}$ and $V^{(l-1)}$, i.e.,

$$\begin{align*}
\left[u_i^{(l-1)} - \sigma \cdot 1\right] & \leq u_i \leq \left[u_i^{(l-1)} + \sigma \cdot 1\right], \quad i = 1, \ldots, M, \\
\left[v_j^{(l-1)} - \sigma \cdot 1\right] & \leq v_j \leq \left[v_j^{(l-1)} + \sigma \cdot 1\right], \quad j = 1, \ldots, N,
\end{align*}$$  

(24)

where $\sigma$ is a small positive parameter and $1$ is a $2 \times 1$ all-one vector. Because the linearized (23) is a linear programming (LP) problem, the optimal solution $u_i^*$’s and $v_j^*$’s in each iteration can be easily obtained. The iteration repeats until a local maximum has reached. Similarly, we can improve the solution by selecting the best-performing solution using different initial points.
V. PLACEMENT OPTIMIZATION OF CO-LOCATED ENs AND APs

In this section, we proceed to study the node placement optimization problem (14) for the case of co-located ENs and APs. The problem is still non-convex due to which the optimal solution is hard to be obtained. Inspired by both Algorithms 2 and 3, we propose in this section an efficient greedy algorithm for HAP placement optimization.

A. Greedy Algorithm Design

The node placement optimization problem (14) is highly non-convex, because the expression of problem (14) involves non-convex function $||u_i - w_k||^{-d_D}$ in $\lambda_k (U^M)$ and minimum operator over convex functions in $\mu_k (U^M)$. Since its optimal solution is hard to obtain, a promising alternative is the greedy algorithm, which iteratively places a single HAP to the network at one time, similar to Algorithm 2 for solving (15) which optimizes the EN locations given fixed APs. However, by comparing problems (14) and (15), we can see that the algorithm design for solving (14) is more complicated, because each $\mu_k$ is now a function of $u_i$'s, instead of constant parameter in (15).

Similar to the greedy algorithm in Section IV.A, we first separate the $K$ WDs into $M$ non-overlapping clusters, denoted by $\{W_i|i = 1, \cdots, M\}$, and add to the network a HAP in each iteration. Specifically, in the $i$-th iteration, given that the previous $(i - 1)$ HAPs are fixed, we obtain the optimal location of the $i$-th HAP, denoted by $u_i^*$, by maximizing the net energy harvesting rates of the WDs in the first $i$ clusters. To simplify the notations, we also use $\lambda_{i-1,k}$ as in Section IV.A to denote the accumulative RF harvesting power of the WD $k$ from the previously placed $(i - 1)$ HAPs, which can be calculated using (18). Besides, let $\mu_{i-1,k}$ denote the energy consumption rate of the $k$-th WD after the first $(i - 1)$ HAPs have been placed, where

$$\mu_{i-1,k} = \begin{cases} +\infty, & i = 1, \\ a_{1,k} + a_{2,k}\min_{j=1,\cdots,i-1}||u_j^* - w_k||^{d_D}, & i > 1. \end{cases}$$

(25)

Notice that the only difference between placing the $i$-th HAP and the $i$-th EN in Section IV.A is that $\mu_k$ is now a function of $u_i$ instead of a given constant. By substituting (25) into (17), the
optimal location of the \( i \)-th HAP is obtained by solving the following problem

\[
\begin{align*}
\text{maximize} & \quad t_i \\
\text{subject to} & \quad (t_i + \mu_{i,k} - \lambda_{i-1,k}) \cdot \| u_i - w_k \|_{d_D} \leq \varphi, \quad k \in \{ \mathcal{W}_1 \cup \cdots \cup \mathcal{W}_i \}, \\
& \quad \mathbf{u}' \leq u_i \leq \mathbf{u}^h,
\end{align*}
\] (26a)

where

\[
\mu_{i,k} = \min (\mu_{i-1,k}, a_{1,k} + a_{2,k} \| u_i - w_k \|_{d_U}).
\] (27)

From (27), we can see that a WD may change its association to the \( i \)-th HAP, if the newly placed HAP is closer to the WD than all the other \((i-1)\) HAPs that have been previously deployed. Evidently, the minimum operator in (27) renders problem (26) non-convex even if \( t_i \) is fixed. In the following, we apply the similar trial-and-error technique as that in Section IV.B to solve problem (26) optimally.

### B. Optimal Solution to Problem (26)

The basic idea to obtain the optimal solution of (26) is to convert the problem into a convex problem given \( t_i \), and then use simple bi-section search over \( t_i \). The convexification of (26) is achieved by a trial-and-error method similar to that used for finding feasible WD-AP associations proposed in Algorithm 3. That is, we first make a set of assumptions on whether the WDs change their associations after the \( i \)-th HAP is added, i.e., assuming either \( \mu_{i-1,k} < a_{1,k} + a_{2,k} \| u_i^* - w_k \|_{d_U} \) or \( \mu_{i-1,k} \geq a_{1,k} + a_{2,k} \| u_i^* - w_k \|_{d_U} \) for each \( k \). Then, for each constraint on \( k \) in (26b), we separate our discussions in the following four cases given a fixed \( t_i \).

1) **Case 1**: If we assume that WD \( k \) does not change its WD-HAP association after the \( i \)-th HAP is placed into the WPCN, or equivalently \( \mu_{i-1,k} < a_{1,k} + a_{2,k} \| u_i^* - w_k \|_{d_U} \), we can replace the corresponding constraint in (26b) with

\[
(t_i + \mu_{i-1,k} - \lambda_{i-1,k}) \cdot \| u_i - w_k \|_{d_D} \leq \varphi.
\] (28)

With a fixed \( t_i \), (28) is a convex constraint if \( t_i + \mu_{i-1,k} - \lambda_{i-1,k} > 0 \).

2) **Case 2**: If we still assume \( \mu_{i-1,k} < a_{1,k} + a_{2,k} \| u_i^* - w_k \|_{d_U} \), while \( t_i + \mu_{i-1,k} - \lambda_{i-1,k} \leq 0 \) holds, we can safely drop the constraint in (26b) without changing the feasible region of \( u_i \).
3) Case 3: On the other occasion, if we assume that WD \( k \) changes its WD-HAP association, or \( \mu_{i-1,k} \geq a_{1,k} + a_{2,k} |u_i^* - w_k|^{d_U} \), the corresponding constraint in (26b) becomes
\[
(t_i + a_{1,k} + a_{2,k}) |u_i - w_k|^{d_U} - \lambda_{i-1,k}) \cdot |u_i - w_k|^{d_D} \leq \varphi,
\]
which can be further expressed as
\[
||u_i - w_k||^{d_U+d_D} + \frac{t_i + a_{1,k} - \lambda_{i-1,k}}{a_{2,k}} ||u_i - w_k||^{d_D} - \frac{\varphi}{a_{2,k}} \leq 0.
\]
Notice that, given a fixed \( t_i \), (30) is a convex constraint if \( t_i + a_{1,k} - \lambda_{i-1,k} \geq 0 \).

4) Case 4: Otherwise, if we assume \( \mu_{i-1,k} \geq a_{1,k} + a_{2,k} |u_i^* - w_k|^{d_U} \) and \( t_i + a_{1,k} - \lambda_{i-1,k} < 0 \) holds, (30) is a non-convex constraint, as the left-hand-side (LHS) of (30) is the difference of two convex functions. Nonetheless, we show that (30) can still be converted into a convex constraint in this case. Let us first consider a function
\[
z(x) = x^{d_U+d_D} + \frac{t_i + a_{1,k} - \lambda_{i-1,k}}{a_{2,k}} x^{d_D} - \frac{\varphi}{a_{2,k}},
\]
where \( x \geq 0 \) and \( t_i + a_{1,k} - \lambda_{i-1,k} < 0 \). We calculate the first order derivative of \( z(x) \) and find that \( z \) increases monotonically when
\[
x > \left[ -\frac{(t_i + a_{1,k} - \lambda_{i-1,k}) d_D}{a_{2,k} (d_U + d_D)} \right]^{1/d_U} \triangleq \tau_{i,k},
\]
and decreases monotonically if \( x \leq \tau_{i,k} \). Notice that \( \tau_{i,k} > 0 \) and \( z(0) = -\frac{\varphi}{a_{2,k}} < 0 \) always hold.

To better visualize the function value, we plot \( z(x) \) in Fig. 4, where we can see that although \( z(x) \) is not a convex function, \( z(x) < 0 \) can still be equivalently expressed as \( x < \theta_{i,k} \), with \( \theta_{i,k} \) being some positive number satisfying \( z(\theta_{i,k}) = 0 \). The value of \( \theta_{i,k} \) can be efficiently obtained using many off-the-shelf numerical methods, such as the classic Newton’s method or bi-section search method. A close comparison between the LHS of (30) and \( z(x) \) in (31) shows that, by letting \( x \triangleq ||u_i - w_k|| \), we can equivalently express (30) as a convex constraint
\[
||u_i - w_k|| \leq \theta_{i,k},
\]
when \( t_i + a_{1,k} - \lambda_{i-1,k} < 0 \) holds.

To sum up, given a fixed \( t_i \), we tackle the \( k \)-th constraint in (26b) using one of the following methods:

1) Replace by (28) if assuming \( \mu_{i-1,k} < a_{1,k} + a_{2,k} |u_i^* - w_k|^{d_U} \) and \( t_i + \mu_{i-1,k} - \lambda_{i-1,k} > 0 \);

2) Drop the constraint if assuming \( \mu_{i-1,k} < a_{1,k} + a_{2,k} |u_i^* - w_k|^{d_U} \) and \( t_i + \mu_{i-1,k} - \lambda_{i-1,k} \leq 0 \);
3) Replace by (30) if assuming $\mu_{i-1,k} \geq a_{1,k} + a_{2,k}||u_i^* - w_k||^{d_U}$ and $t_i + a_{1,k} - \lambda_{i-1,k} \geq 0$;
4) Replace by (33) if assuming $\mu_{i-1,k} \geq a_{1,k} + a_{2,k}||u_i^* - w_k||^{d_U}$ and $t_i + a_{1,k} - \lambda_{i-1,k} < 0$.

After processing all the $K$ constraints in (26b), we can convert (26) into a convex feasibility problem given a set of WD-HAP association assumptions and a fixed $t_i$. Accordingly, the optimal solution of (26) under the assumptions, and thus the optimal placement of the $i$-th HAP ($u_i^*$), can be efficiently obtained using a bi-section search method over $t_i$. Similar to the trial-and-error technique used in Algorithm 3, we check if the obtained $u_i^*$ satisfies all the assumptions made. If yes, we have obtained the optimal solution of (26). Otherwise, we switch the violating assumptions, then follow the above constraint processing method to resolve (26) for a new $u_i^*$, and repeat the iterations until all the assumptions are satisfied. The above trial-and-error method converges. The proof follows the similar argument as given in the Appendix, which proves the convergence of the trial-and-error method used for solving problem (21). Thus, this proof is omitted here.

C. Overall Algorithm

Since the optimal placement of a single HAP can be obtained via solving (26), we can iteratively place the $M$ HAPs into the WPCN. The pseudo-code of the revised greedy algorithm
is presented in Algorithm 4. For example, Fig. 5 illustrates the detailed steps taken to place the 2nd HAP, or HAP₂ (total 3 HAPs while the first HAP, or HAP₁ is already placed). Specifically, we first assume in Fig. 5(a) that all the WDs in the 1st cluster associate with HAP₁, and the WDs in the 2nd cluster associate with HAP₂, after the new HAP is added into the network. Then, we obtain in Fig. 5(b) the optimal placement of the HAP₂ based on the association assumptions made. However, the obtained location of HAP₂ results in a contradiction with the association assumption made on WD₁ (assumed to be associated with HAP₁). Therefore, we change the association assumption of WD₁ to HAP₂, and recalculate the optimal placement solution for HAP₂ (Fig. 5(c)). In Fig. 5(d), the newly obtained location of HAP₂ satisfies all the association assumptions, thus the optimal placement of this HAP is achieved. Following the similar argument in the Appendix, the association assumption update procedure converges, because the optimal objective value of (26) is non-decreasing upon each association assumptions update (lines 10 – 27 of Algorithm 4). After obtaining the optimal location for the HAP₂, the optimal location of the 3rd HAP, HAP₃, can also be obtained using the similar procedures as above.
Algorithm 4: Greedy algorithm for HAP placement optimization.

input : $K$ WD locations $w_k$’s
output: the optimal locations of $M$ HAPs $\{u^*_1, \ldots, u^*_M\}$

1. Cluster the WDs into $\{W_i, i = 1, \ldots, M\}$;
2. for $i = 1$ to $M$ do
   3. for each WD $k$ do
      4. Update $\lambda_{i-1,k}$ and $\mu_{i-1,k}$ using (18) and (25);
      5. Assume WD $k$ satisfies condition (a) or (b):
         (a) $\mu_{i-1,k} < a_{1,k} + a_{2,k}||u^*_i - w_k||_{d_U};$
         (b) $\mu_{i-1,k} \geq a_{1,k} + a_{2,k}||u^*_i - w_k||_{d_U}.$
   6. end
   7. StopFlag ← 0;
   8. repeat
      9. $LB ← -\delta, \ UB ← \delta, \ \delta$ is sufficiently large;
      10. repeat
          11. $t_i ← (UB + LB)/2$;
          12. Given $t_i$, convert (26) into a convex problem using the procedures in Section V.B;
          13. if Problem (26) is feasible given $t_i$ then
              14. $LB ← t_i; \ u^*_i ←$ a feasible solution of (26);
          15. else
              16. $UB ← t_i$;
          17. end
      18. until $|UB - LB| < \sigma$;
      19. if all the $K$ assumptions are valid then
          20. StopFlag ← 1; the $i$-th HAP location $← u^*_i$;
      21. else
          22. StopFlag ← 0;
          23. For each WD violating the assumption, switch the assumption from (a) to (b), or (b) to (a);
      24. end
   25. until StopFlag = 1;
   26. end
27. Return the HAP locations $\{u^*_1, \ldots, u^*_M\}$.

VI. SIMULATION RESULTS

In this section, we use simulations to evaluate the performance of the proposed node placement methods. All the computations are executed by MATLAB on a computer with an Intel Core i5 2.90-GHz CPU and 4 GB of memory. In the DL energy transmission, we assume that $d_D = 2.2$, $\eta = 0.9$, and $\beta = 5.49 \times 10^{-4}$ (this corresponds to 915 MHz carrier frequency). The transmit power of each EN is $P_0 = 2W$ (Watt). In the UL information transmission, we assume that $d_U = 2$, the circuit power $a_{1,k} = 0.1mW$ and $a_{2,k} = 10^{-5}W/m^2$ (square meter) for $k = 1, \ldots, K$. Besides, all the WDs, ENs and APs are placed within a $24m \times 24m$ box region specified by $b^l = (0, 0)^T$ and $b^h = (24, 24)^T$. Unless otherwise stated, each point in the following figures is an average performance of 20 random WD placements, each with $K = 60$ WDs uniformly
placed within the box region.

A. Separated EN and AP Deployment

We first evaluate the performance of the proposed alternating optimization method (Algorithm 3) for placing separated ENs and APs. Without loss of generality, we consider $N = 8$ APs and show in Fig. 6 the minimum net energy harvesting rate $P_r$ in (14) achieved by Algorithm 3 when the locations of APs are jointly optimized with those of different number of ENs ($M$). Evidently, a larger $P_r$ indicates better system performance. Besides the alternating optimization algorithm (AltOpt), local searching algorithm (LS), and sequential convex optimization (SCO) introduced in Section IV, we consider two additional benchmark placement methods as follows.

- Cluster center method (CC): separate the WDs into $M$ clusters and place an EN at each of the cluster centers. Similarly, separate the WDs into $N$ clusters and place the $N$ APs at the cluster centers;
- Optimize only EN locations: the AP locations are set as the same initial AP locations $v_j^{(0)}$, used in Algorithm 3 for joint AP-EN placement, while the EN placement is optimized based
on the AP locations using Algorithm 2.

Evidently, we can see that the proposed alternating optimization has the best performance among the methods considered. The SCO method has the worst performance, even worse than the simple CC method, because its linear approximations are too loose in (23). We can also see that significant performance gain can be obtained through jointly optimizing the placements of ENs and APs compared to optimizing EN placement only. The local searching method has relatively good performance when $M$ is small, but the performance gap increases with $M$, due to the increasing probability of being trapped at local maximums with a larger $M$. In practice, Fig. 6 can be used to evaluate the deployment cost of each algorithm. For instance, when $P_r \geq 0$ is required, we see that the joint AP-EN placement optimization on average needs 8 ENs, the LS method needs 10 ENs, optimizing EN placement only requires 12 ENs, and the CC method needs 15 ENs, while the SCO needs 20 ENs, with the same number of information APs deployed (i.e., $N = 8$). The above results validate the effectiveness of the proposed joint AP-EN placement optimization method in Algorithm 3.
B. Co-located EN and AP Deployment

Next, we evaluate in Fig. 7 the performance of the proposed greedy node placement optimization for the case of co-located ENs and APs (Algorithm 4). The value of $P_r$ achieved by Algorithm 4 is plotted against different number of HAPs used ($M$). In particular, we compare its performance with that of local searching (LS) and the cluster center (CC) placement method, i.e., the HAPs are placed at the $M$ cluster centers. From Fig. 7, we can see that the proposed greedy algorithm in Algorithm 4 has the best performance among the methods considered. Interestingly, the simple CC method has relatively good performance compared to that of the greedy algorithm, which in general needs only $2 - 3$ more HAPs than that of the greedy method under any $P_r$ requirement. An intuitive explanation is that the doubly-near-far phenomenon for co-located EN and AP renders the HAPs to be optimally placed around each cluster center. The LS method improves upon the CC method by setting the cluster centers as the initial searching points. When the system requires $P_r \geq 0$, the greedy algorithm, LS method and CC method need 11, 12 and 13 HAPs, respectively.

Although the greedy algorithm and the LS method perform closely, they differ significantly in the computational complexity. To see this, we plot in Fig. 4 the CPU time of the two methods as $M$ increases, where each point on the figure is normalized against the CPU time achieved by the respective method when $M = 4$. Clearly, we can see that the complexity of the greedy algorithm increases linearly with $M$, where the CPU time increases approximately 6 times when $M$ increases from 4 to 24. The LS method, however, has a much faster increase in complexity with $M$, where the CPU time increases by around 45 times when $M$ increases from 4 to 24. Therefore, even in a large-size WPCN with large $M$, the computation time of the proposed greedy algorithm is still moderate, while this may be extremely high for the LS method, e.g., couple of minutes versus several hours for $M = 50$. Furthermore, unlike the LS method that is sensitive to the selection of initial search points and the settings of searching parameters, the performance of the proposed greedy algorithm is robust in different network topologies. From the above discussions, we can conclude that the proposed greedy algorithm is the most cost-effective method for EN placement problem among the schemes that we considered.
C. Case Study: Separated Versus Co-located EN and AP

Finally, we present a case study to compare the cost of node placement achieved by using either separated or co-located EN and AP. In particular, we consider a WPCN in Fig. 9 where 60 WDs (dots in the figure) are uniformly placed within the $24m \times 24m$ box region. Here, we assume the deployment costs for using an EN, AP and HAP are $c_1 = 0.7$, $c_2 = 1$ and $c_3 = 1.4$, respectively. As an illustrating example, we consider a performance threshold $\gamma = 0$ and show in Fig. 9 the node locations solution produced by both the separated and co-located EN and AP placement optimization methods (Algorithms 3 and 4) that achieve the lowest deployment cost. For the separated EN and AP case, the solution is obtained by enumerating a set of feasible $(M, N)$ pairs and selecting the one with the minimum cost, while a bi-section search method over $M$ is used for the case of co-located EN and AP. In this particular setup, the separated EN and AP method deploys 12 ENs (triangles in Fig. 9) and 4 APs (circles), with the minimum cost equal to 12.4 units, while the co-located EN and AP method uses 11 HAPs (stars), with the minimum cost equal to 15.4 units.
Fig. 9. Schematic of the WD locations and the minimum-cost node deployment solutions using separated/colocated EN and AP ($\gamma = 0$).

Fig. 10. Comparison of the minimum deployment costs achieved by separated/colocated EN and AP placement optimization methods, where in (a) $c_3 = 1.4$ and in (b) $c_3 = 1.05$. 
In Fig. 10(a), we show the minimum deployment costs achieved by the two methods under different performance requirement $\gamma$. The number of nodes used by both methods are also marked in the figure. That is, $M$ ENs and $N$ APs for the separated node case and the number of HAPs for the co-located case. With $\{c_1, c_2, c_3\} = \{0.7, 1, 1.4\}$, we can see that using separated EN and AP is strictly better than co-located HAPs. However, when the deployment cost of a HAP is decreased from $c_3 = 1.4$ to 1.05, we can see in Fig. 10(b) that using co-located EN and AP for node deployment is better. Notice that we do not intend to claim using separated EN and AP is better than co-located case, or vice versa. Rather, we show that a minimum-cost deployment plan can be efficiently obtained using the proposed methods. In practice, the choice of using either separated or co-located EN and AP depends on a joint consideration of the node deployment costs, the WD locations and the performance requirement.

VII. CONCLUSIONS

In this paper, we have studied the node placement optimization problem in WPCN, which minimizes the node deployment cost while satisfying the energy harvesting and communication performances of wireless devices. Specifically, we first studied the optimal placement problem using separate EN and information AP, where an efficient alternating optimization method is proposed to jointly optimize the locations of ENs and information APs by tackling the non-convexity of the optimization problem. Based on the same algorithm approach, we further proposed a greedy algorithm to solve the node placement optimization problem when co-located EN and AP is used. Simulation results showed that the proposed algorithms can effectively reduce the network deployment cost, while requiring low computational complexity even for large-size WPCNs.

APPENDIX

PROOF OF CONVERGENCE OF THE TRIAL-AND-ERROR METHOD IN SOLVING (21)

Let $\{l^{(l)}, v_j^{(l)}, j = 1, \cdots, N\}$ denote the optimal solutions of (21) calculated from the $l$-th ($l \geq 1$) set of assumptions made on the WD-AP associations, denoted by $j_k^{(l)}$, $k = 1, \cdots, K$. Let $\mathcal{K}^{(l)}$ denote the set of WDs to which the optimal solution $v_j^{(l)}$’s contradict with the WD-AP assumptions (we consider only $\mathcal{K}^{(l)} \neq \emptyset$, because otherwise the algorithm has reached its
optimum), i.e.,
\[
\mathcal{K}^{(l)} : k = 1 \cdots, K, \quad ||v_{j_k^{(l)}} - w_k|| > \min_{j=1,\cdots,N} ||v_{j}^{(l)} - w_k||
\]
(34)

According to the proposed trial-and-error method, \(j_k^{(l+1)}\) is set as
\[
j_k^{(l+1)} = \begin{cases}
    j_k^{(l)}, & k \notin \mathcal{K}^{(l)}; \\
    \arg\min_{j=1,\cdots,N} ||v_{j}^{(l)} - w_k||, & k \in \mathcal{K}^{(l)},
\end{cases}
\]
(35)
for \(k = 1, \cdots, K\).

Let \(\hat{t}^{(l+1)}\) denote the minimum net energy harvesting rate among all the WDs given the updated WD-AP association \(j_k^{(l+1)}\)'s and the current AP locations \(v_{j}^{(l)}\)'s, i.e.,
\[
\hat{t}^{(l+1)} = \min_{k=1,\cdots,K} \left( \lambda_k - a_{1,k} - a_{2,k} ||v_{j_k^{(l+1)}} - w_k||^{d_U} \right).
\]
(36)

We can see that \(\hat{t}^{(l+1)} \geq t^{(l)}\) because the update of WD-AP associations in (35) does not increase the energy consumption rate of any WD achived by assuming \(j_k^{(l)}\), \(k = 1 \cdots, K\). Besides, \(\left\{\hat{t}^{(l+1)}, v_{j}^{(l)}, j = 1, \cdots, N\right\}\) is a feasible solution of (21) under the association assumption \(j_k^{(l+1)}\)'s. Therefore, the optimal solution \(\left\{t^{(l+1)}, v_{j}^{(l+1)}, j = 1, \cdots, N\right\}\) calculated from the association assumption \(j_k^{(l+1)}\)'s will lead to \(t^{(l+1)} \geq \hat{t}^{(l+1)} \geq t^{(l)}\). In other words, the optimal objective of (21) is non-decreasing in each trial-and-error update of WD-AP associations. This, together with the fact that the optimal value of (21) is bounded, leads to the conclusion that the proposed trial-and-error method is convergent.

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