Study of Superoutbursts and Superhumps in SU UMa Stars by the Kepler Light Curves of V344 Lyrae and V1504 Cygni

Yoji Osaki
Department of Astronomy, School of Science, University of Tokyo, Hongo, Tokyo 113-0033
osaki@ruby.ocn.ne.jp

and

Taichi Kato
Department of Astronomy, Kyoto University, Sakyo-ku, Kyoto 606-8502
tkato@kusastro.kyoto-u.ac.jp

Abstract

We study the short cadence Kepler public light curves of SU UMa star s, V344 Lyr and V1504 Cyg extending for a period of well over two years by using power spectral analysis. We have determined the orbital period of V344 Lyr to be $P_{\text{orb}} = 0.087903(1)\text{ d}$. We have reanalyzed the frequency variation of the negative superhump in a complete supercycle of V1504 Cyg with additional data of the $O-C$ diagram, confirming its characteristic variation in accordance with the thermal tidal instability model. We present a new two-dimensional period analysis based on a new method least absolute shrinkage and selection operator (Lasso). The new method gives very sharp peaks in the power spectra and it is very useful for the study of frequency variation in cataclysmic variable stars. We have analyzed simultaneous frequency variation of the positive and negative superhumps. If appropriately converted, it is found that they vary in unison, indicating that they represent the disk radius variation. We have studied the frequency (or period) variations of the positive superhumps during superoutbursts. These variations are understood in a qualitative way by a combination of the disk radius variation and the variation of the pressure effects during superoutburst. A sudden excitation of oscillation with a frequency range near the negative superhump (which we call “impulsive negative superhump”) was observed in the descending branch of several outbursts of V344 Lyr. These events seem to occur just prior to the next superoutburst and to act as a “lead” of the impending superoutburst.

Key words: accretion, accretion disks — stars: dwarf novae — stars: individual (V344 Lyrae, V1504 Cygni) — stars: novae, cataclysmic variables

1. Introduction

Dwarf novae are eruptive variable stars, showing quasi-periodic outbursts with a typical amplitude of 2–5 mag and a typical recurrence time of weeks to months. They belong to a more general class of variable stars called the cataclysmic variable stars which are semi-detached close binary systems consisting of a white dwarf primary star and a red dwarf secondary star. The mass transferred from the Roche-lobe filling secondary star is not accreted directly to the white dwarf but it first forms an accretion disk around the primary white dwarf. The outburst of dwarf novae is now thought to be caused by a sudden brightening of the accretion disk. The outburst of ordinary dwarf novae is attributed to the thermal instability in accretion disks (e.g., see Cannizzo 1993, Lasota 2001 for review).

The SU UMa stars are one of subclasses of dwarf novae with a short orbital period, which exhibit a long outburst called "superoutburst" having a brighter maximum by about 1 mag with a typical duration of about 2 weeks beside a short normal outburst with a duration of a few days of ordinary dwarf novae. The most enigmatic feature of SU UMa stars is an appearance of so-called superhump during the superoutburst, a periodic photometric hump with a period longer than the orbital period by a few percent. The superhump mentioned above is also called the positive superhump because some of SU UMa stars and nova-like variable stars exhibit another periodic photometric hump with a period shorter than the orbital period by a few percent and the latter is called the negative superhump. General reviews of dwarf novae and SU UMa stars in particular are found in monographs dealing with the cataclysmic variable stars by Warner (1995), Hellier (2001).

The superoutbursts and superhumps in SU UMa stars are one of most intriguing phenomena in variable stars. The positive superhump is now thought to be produced by the eccentric precessing disk in which an accretion disk is deformed into eccentric form and its apsidal line precesses prograde and the positive superhump periodicity is produced by the synodic period between the progradely precessing disk and the orbiting secondary star. The (positive) superhump phenomenon is now well understood by the tidal instability in accretion disks; superhumps are produced by the periodic tidal stressing of the eccentric
precessing accretion disk, which is in turn produced by the tidal 3:1 resonance instability between accretion disk flow and the orbiting secondary stars (Whitehurst 1988, Hirose, Osaki 1990, Lubow 1991). On the other hand, the negative superhump is thought to be produced by the tilted disk in which an accretion disk is tilted from the binary orbital plane and its nodal line precesses retrograde and the negative superhump periodicity is produced by the synodic period between the retrograde precessing tilted disk and the orbiting secondary star.

As for the superoutburst phenomenon, Osaki (1989) has proposed so-called thermal-tidal instability model (abbreviated as TTI model) in which the ordinary thermal instability is coupled with the tidal instability [see, Osaki (1996) for a review of the TTI model]. Although the TTI model is fairly well accepted as a correct explanation of the superoutburst and supercycle phenomenon, objections to this model have been raised from time to time, and two alternative models have been proposed; an enhanced mass-transfer model (abbreviated EMT model) by Smak (1991) and a pure thermal instability model by Cannizzo (see, Cannizzo et al. 2010). Since we have already discussed about theoretical models for superoutbursts and superhumps in section 2 of Osaki, Kato (2013) (Paper I), we do not repeat them here.

The NASA Kepler mission (Borucki et al. 2010; Koch et al. 2010) to search for the earth-like planets around stars is a great success and besides its main objective it provides to the variable star community an unprecedented opportunity to study variable stars in great details and cataclysmic variable stars are no exception. Two SU UMa stars in the Kepler field of view are observed with the short cadence mode and they are V344 Lyr and V1504 Cyg. In paper I, we have studied the Kepler public light curve of one of SU UMa stars, V1504 Cyg, and demonstrated that the Kepler light curve has revealed clear evidence that the superoutburst in this star is caused by the tidal instability and the superoutburst and supercycle phenomenon in SU UMa stars are basically explained by the thermal-tidal instability model. In this paper we extend our previous study to another Kepler SU UMa star, V344 Lyr, together with some additional analysis of V1504 Cyg for data recently available to public.

V344 Lyr is a dwarf nova in the Kepler field and its Kepler light curve was first examined by Still et al. (2010) who showed that the superhumps found during the superoutburst continue to be detected during the following quiescent state and the next normal outburst. By using the power spectral analysis, Wood et al. (2011) have studied the positive and negative superhumps of this star by Kepler light curve of three quarters (June 2009 to March 2010). They have demonstrated that the ordinary superhump in the early phase of superoutburst is generated by viscous dissipation within the periodically flexing (eccentric) disk while in the later phase of the superoutburst the superhump signal (the so-called late superhump) is generated as the gas stream hot spot sweeps around the rim of the non-axisymmetric (eccentric) disk.

2. The Kepler Light Curves of V344 Lyr and V1504 Cyg

In this paper we examine the short cadence Kepler public data of V344 Lyr and also V1504 Cyg, extending for two years and a quarter from June 2009 to September 2011 for about 830 d from Barycentric Julian Date (BJD) 2455002 to 2455833. We use data of the Simple Aperture Photometry (SAP) whose count rates are converted to a relative magnitude, by mag = 13 − 2.5 log e where e is the Kepler electron count rate (electrons s$^{-1}$) and a constant of 13 is arbitrarily chosen just for convenience. The Kepler light curves of these two stars for about a period of 736 d have already been studied by Cannizzo et al. (2012) and we also note that our data for V344 Lyr in the early part used here overlap with those of Wood et al. (2011).

2.1. Global light curves and the power spectra of V344 Lyr and V1504 Cyg

We have first made two-dimensional power spectral analysis of the Kepler light curves of V344 Lyr and also of V1504 Cyg. In calculating the power spectra of V344 Lyr and V1504 Cyg, we have used the same method as that used in Paper I and its details should be consulted in Paper I. We show the overall power spectra together with its light curve for V344 Lyr in figure 1 and for V1504 Cyg in figure 2, respectively.

The Kepler data of V344 Lyr we have used here contain 7 superoutbursts and 6 supercycles and we summarize their main characteristics in table 1 as we did before for V1504 Cyg in Paper I. We also show them in table 2 for V1504 Cyg because of additional data for two quarters. The first column of tables 1 and 2 is the ordinal number while the next 3 columns are BJD date of the start of a supercycle (2), the start of the superoutburst (3), and its end (4), respectively, counted from BJD 2455002. The following 3 columns are lengths in days of a supercycle excluding superoutburst (5), of superoutburst duration (6), and of full supercycle (7). The next column (8) gives the number of normal outbursts within a supercycle while the last two columns (9) and (10) are comments on an appearance of negative superhumps and orbital humps in the power spectra.

Let us now examine the light curve and power spectra of V344 Lyr in figure 1 and table 1. Before we go into some details of the light curve, we first examine the orbital period of V344 Lyr. The orbital period of V344 Lyr was determined by Wood et al. (2011) to be $P_{\text{orb}} = 0.087904$ d (2.11 hr or 11.38 c/d in frequency) from the Q4 data (our SC No. 3 with 200–275 d). Since our data contain the signal of the orbital humps more on the other occasions with SC No. 4 (320–370 d), SC No. 5 (460–520 d), and

---

5 The origin of superhumps following the termination of the superoutburst is still in dispute (cf. Kato et al. 2009; also sub-

---

section 4.2 in Wood et al. 2011). We refer to “traditional late superhumps” here, i.e. superhumps whose phase is ~0.5 different from the superhumps during superoutbursts (see Kato et al. 2012 for the nomenclature.)
Fig. 1. Two-dimensional power spectrum of the Kepler light curve of V344 Lyr for the all data except Q14. (upper:) light curve; the Kepler data were binned to 0.02 d; (lower:) power spectrum. The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 2. Two-dimensional power spectrum of the Kepler light curve of V1504 Cyg for the all data except Q14. (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) power spectrum. The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
SC No. 6 (570–630 d), we re-determined the orbital period by the PDM method (Stellingwerf 1978) resulting in 0.0879049(11) d for SC No. 3, 0.0879024(15) d for SC No. 4, 0.0879014(36) d for SC No. 5, and 0.0878937(15) d for SC No. 6. Since the last value significantly differs from the first three, we determined from the first three that \( P_{\text{orb}} = 0.087903(1) \) d, which is consistent with that of Wood et al. (2011). More recently, Howell et al. (2013) reported a short (3.4 hr) radial-velocity study whose result is consistent with the photometric orbital period, although the inferred inclination (5–10°) appears to be too low to produce photometric orbital humps.

We note that the Kepler light curve of V344 Lyr and its power spectra show both similarity and difference with those of V1504 Cyg discussed in Paper I. The average supercycle length is about 114 d which is very close to that of V1504 Cyg (112 d). The strongest signal in the power spectra is that of positive superhumps with frequency around 10.8 c/d whenever a superoutburst occurs. Although the negative superhumps appear in the power spectra of V344 Lyr, these are more patchy (i.e., they come and go) than those of V1504 Cyg. We find them in the middle of SC No.1, SC No.2, and SC No.7.

One of the most important findings about V1504 Cyg in Paper I was that a strong correlation between an appearance of negative superhumps and the length of quiescent interval between two consecutive normal outbursts in V1504 Cyg, that is, an appearance of well visible negative superhump tended to reduce the frequency of occurrence of normal outbursts. This leads us to classify supercycles of V1504 Cyg into two types of Type L and Type S with or without negative superhumps. The same tendency is also seen in V344 Lyr for the SC No.1 and SC No.2 in figure 1 when the negative superhump signal appears. In particular a very strong signal of negative superhumps appears in the first 40 days and it apparently suppresses occurrence of normal outburst (as already noted by Cannizzo et al. 2012).

However the quiescence interval in SC No.3 is relatively long even though there is no visible signal of negative superhump. The frequency of normal outbursts in SC No.7 is not low even though the signal of negative superhumps is quite visible. Nevertheless the number of normal outbursts in SC No.7 is 8 and it is relatively small because it is 10 for SC No.4, 13 for SC No.5, and 10 for SC No.6, where no signal of the negative superhump is visible in the power

---

**Table 1.** Superoutbursts and supercycles of V344 Lyr. *

| (1) SC number | (2) start of SC† | (3) start of SO† | (4) end of SO† | (5) SC length excluding SO† | (6) SO duration‡ | (7) SC length‡ | (8) number of NO | (9) negative SH | (10) orbital hump |
|---------------|------------------|-----------------|----------------|-----------------------------|-----------------|----------------|----------------|----------------|------------------|
| 1             | –                | 55.5            | 73             | –                          | 17.5            | >70            | >4             | well visible    | no                |
| 2             | 73               | 161             | 178            | 88                         | 17              | 105            | 7              | well visible    | no                |
| 3             | 178              | 277             | 294            | 99                         | 17              | 116            | 7              | no              | visible           |
| 4             | 294              | 397             | 415            | 103                        | 18              | 121            | 10             | no              | visible           |
| 5             | 415              | 527             | 544.5          | 112                        | 17.5            | 129.5          | 13             | no              | visible           |
| 6             | 544.5            | 641.5           | 659.5          | 97                         | 18              | 115            | 10             | no              | well visible      |
| 7             | 659.5            | 743             | 760            | 83.5                       | 17              | 100.5          | 8              | well visible    | no                |
| 8             | 760              | –               | –              | –                          | –               | –              | >9             | partly visible   | partly visible    |

*Abbreviations in this table: supercycle (SC), superoutburst (SO), normal outburst (NO), superhump (SH).
†BJD–2455000.
‡Unit: d.

---

**Table 2.** Superoutbursts and supercycles of V1504 Cyg. *

| (1) SC number | (2) start of SC† | (3) start of SO† | (4) end of SO† | (5) SC length excluding SO† | (6) SO duration‡ | (7) SC length‡ | (8) number of NO | (9) negative SH | (10) orbital hump |
|---------------|------------------|-----------------|----------------|-----------------------------|-----------------|----------------|----------------|----------------|------------------|
| 1             | –                | 74.5            | 88.5           | 14                          | >88             | >8             | no             | no             | no                |
| 2             | 88.5             | 201             | 215            | 112.5                       | 14              | 126.5          | 10             | no             | no                |
| 3             | 215              | 312             | 325            | 97                          | 13              | 110            | 10             | later half     | partly visible    |
| 4             | 325              | 406.5           | 419            | 81.5                        | 12.5            | 94             | 6              | full           | no                |
| 5             | 419              | 516             | 530            | 97                          | 14              | 111            | 5              | full           | no                |
| 6             | 530              | 639             | 650            | 109                         | 11              | 120            | 10             | early part     | later part        |
| 7             | 650              | 750             | 763.5          | 100                         | 13.5            | 113.5          | 11             | no             | yes               |
| 8             | 763.5            | –               | –              | –                           | –               | –              | >7             | no             | yes               |

*Abbreviations in this table: supercycle (SC), superoutburst (SO), normal outburst (NO), superhump (SH).
†BJD–2455000.
‡Unit: d.
spectrum. The correlation between appearance of negative superhump and quiescence interval is rather weak in the case of V344 Lyr. Since this correlation is fairly well established in V1504 Cyg as well as in ER UMa (Oshrima et al. 2012, Zemko et al. 2013) and V503 Cyg (Kato et al. 2002, Kato et al. 2013), it is not clear whether V344 Lyr is an exceptional case or not.

Let us now examine the light curve and power spectra of V1504 Cyg for additional data. The newly added two quarters of the Kepler data basically confirm the finding of Paper I as seen in figure 2 and table 2, that is, the supercycle No. 7, which shows no signal of negative superhumps but instead a strong signal of the orbital hump, has the biggest number of normal outbursts of 11 in a supercycle, fitting with the classification of the Type S supercycle. Thus there is no need to revise our conclusion about V1504 Cyg with new data.

2.2. Frequency Variation of the Negative Superhumps during Supercycles

One of the most important findings in Paper I is that the frequency of the negative superhump (nSH) varies systematically during a supercycle in V1504 Cyg. As discussed in Paper I, if we adopt a tilted disk as the origin of the negative superhump, the frequency of negative superhump is given by a synodic frequency between a retrograde-precessing tilted disk and the orbiting secondary star, and it is written as

$$\nu_{\text{nSH}} = \frac{\nu_{\text{orb}}}{1 + \frac{3}{2} \frac{q}{\sqrt{1 + \frac{q}{A}}} \left( \frac{R_d}{A} \right)^{3/2} \cos \theta},$$  \hspace{1cm} (1)

where \(\nu_{\text{nSH}}\) and \(\nu_{\text{orb}}\) are frequencies of negative superhumps, and of the binary orbit, respectively, \(\theta\) is a tilt angle of the disk, \(q = M_2/M_1\) is the binary mass ratio, and \(A\) and \(R_d\) are the binary separation and the disk radius, respectively. Here we assume \(\cos \theta \simeq 1\) for a slightly tilted disk, as we did in Paper I. Observed frequency variation shown in figure 5 of Paper I, if interpreted as an indicator of the disk radius, agreed very well with the variation in disk radius predicted by the thermal-tidal instability model (Osaki 1989, Osaki 2005).

Recently Smak (2013) criticized our results on frequency variations of negative superhumps during normal outburst cycle. He stated that in our figure 5 the minima of \(\nu_{\text{nSH}}\) occurred \(\sim 3\) d \(\text{before}\) the initial rise to outburst maximum and the following increase of \(\nu_{\text{nSH}}\) till its maximum lasted for \(\sim 3\) d while the model calculations for \(R_d = 1 + 3d\) \text{before} and the model calculations for \(R_d = 1 + 3d\) \text{after} are the binary separation and the disk radius, respectively. Here we assume \(\cos \theta \simeq 1\) for a slightly tilted disk, as we did in Paper I. Observed frequency variation shown in figure 5 of Paper I, if interpreted as an indicator of the disk radius, agreed very well with the variation in disk radius predicted by the thermal-tidal instability model (Osaki 1989, Osaki 2005).

Recently Smak (2013) criticized our results on frequency variations of negative superhumps during normal outburst cycle. He stated that in our figure 5 the minima of \(\nu_{\text{nSH}}\) occurred \(\sim 3\) d \(\text{before}\) the initial rise to outburst maximum and the following increase of \(\nu_{\text{nSH}}\) till its maximum lasted for \(\sim 3\) d while the model calculations for \(R_d = 1 + 3d\) \text{after} are the binary separation and the disk radius, respectively. Here we assume \(\cos \theta \simeq 1\) for a slightly tilted disk, as we did in Paper I. Observed frequency variation shown in figure 5 of Paper I, if interpreted as an indicator of the disk radius, agreed very well with the variation in disk radius predicted by the thermal-tidal instability model (Osaki 1989, Osaki 2005).

To confirm this further, we calculated the \(O-C\) diagram for the negative superhump of V1504 Cyg for a period BJD 2455450–2455490 including two normal outbursts, and figure 4 exhibits its light curve, the \(O-C\) diagram, and the amplitude of negative superhump from top to bottom, respectively. Following is the ephemeris of negative superhump maxima, from which \(O-C\) is counted,

$$\text{BJD(max)} = 2455449.9748 + 0.06806256E$$ \hspace{1cm} (2)

As seen from figure 4, the \(O-C\) diagram shows a cusp-like structure whenever an outburst occurs, indicating a jump in frequency of nSH. The peaks of cusp in \(O-C\) diagram (and therefore jump in frequency) occurred at around BJD 2455457 and BJD 2455480, corresponding to the rising branch of each outburst and a jump in frequency occurs with a short time scale less than a day and most likely less than 0.5 d. This is completely consistent with the prediction of the disk instability model.

The fact that a frequency jump occurs during a rising branch of an outburst strongly suggests that the outburst
is of “outside-in”, an unfavorable situation for Cannizzo’s pure thermal instability model for the superoutburst of SU UMa stars (Cannizzo et al. 2010) because an “inside-out” outburst is required to explain a short outburst in their model.

To study further the $O - C$ diagram for a complete supercycle, we have calculated it together with amplitude variation for V1504 Cyg from BJD 2455373 to 2455594, including a supercycle No. 5 which we studied in Paper I. Figure 5 illustrates the $O - C$ diagram for negative superhump with the same ephemeris of equation (2) and its amplitude variation together with the light curve and its frequency variation. The $O - C$ diagram for a complete supercycle in figure 5 shows a characteristic variation. If we make a smooth curve (a spline curve) passing at each cusp point corresponding to each normal outburst, the resultant curve exhibits a convex form, implying $P_{nSH} < 0$ in a long time scale during a supercycle, superimposed on it a short time-scale variation with a concave form between two consecutive normal outbursts, implying $P_{nSH} > 0$; a phenomenon already discussed as a variation in frequency of nSH and seen in the second panel of figure 5. This indicates a secular trend of expansion in the disk radius in a supercycle, superimposed on it a decrease in the disk radius between two consecutive normal outbursts in a shorter time-scale; a phenomenon exactly predicted by the thermal-tidal instability model.

2.3. Amplitude Variation of the Negative Superhump

Let us now turn our attention to amplitude variation. As seen in figure 5, the amplitude of negative superhump exhibits a characteristic variation during supercycle No. 5. The amplitude variation of nSH is, however, very complex because quite different phenomena affect on amplitudes of nSH. In the following discussions we adopt the standard interpretation of the negative superhump, that is, the negative superhump is produced by a tilted disk which precesses retrograde by the secondary tidal torque.

The light source of the negative superhump in quiescence is thought to be a release of kinetic energy of gas stream by hitting different radius of the disk as gas stream sweeps around a tilted disk (Wood et al. 2011). First of all, we take note that we still do not understand the basic mechanism that causes and maintains disk tilt although several different mechanisms have been suggested about the origin of disk tilt, such as the 3:1 resonance (Lubow 1992b), magnetic coupling between the disk and either the secondary star or the primary white dwarf (Murray et al. 2002), stream-disk interaction with variable vertical component of stream due to asymmetric irradiation.
of the secondary star (Smak 2009), and dynamical lift by the gas stream (Montgomery, Martin 2010). As seen in global power spectra in figure 1 for V344 Lyr and in figure 2 for V1504 Cyg, the negative superhumps come and go in a long time-scale (in the case of V1504 Cyg a time scale more than 300 d, much longer than a supercycle). This long time-scale variation in amplitude of nSH is mostly likely produced by variation in tilt angle $\theta$ (Montgomery 2009a), in other words, variation in amplitude of a tilt mode of the disk.

Let us now examine amplitude variation of nSH during a supercycle No. 5 of V1504 Cyg in figure 5. For a moment we assume that the tilt angle will stay more or less constant during this supercycle, although we must keep always in mind that the tilt angle can vary within a supercycle. From figure 5 we find that its amplitude increases whenever an outburst occurs. In particular, we
note that profile of amplitude variation during a normal outburst mimics outburst profile of the light curve in a smaller scale. Although it is generally thought that the light source of negative superhump is due to a release of kinetic energy of the gas stream, these observations suggest that the disk component may contribute to the light variation of negative superhump during an outburst besides the gas stream component. A possible origin of the disk component to the negative superhump light source during the superoutburst has already been discussed in the Appendix of Paper I. We also recall that Wood et al. (2009) noted that in their hydro-simulation of a tilted disk a negative superhump light signal was seen even when the gas stream has been completely shut off. It is well known that the main light source of the ordinary (positive) superhumps is due to the viscous dissipation of tidally stressed eccentric disk (or the periodically flexing disk in the words of Wood et al. 2011) in the main part of the superoutburst while it is due to the gas stream in the late stage of the superoutburst and in quiescence as it is called the late superhump.

We thus suggest that the amplitude increase of nSH during outbursts shown in figure 5 may be most likely produced by an addition of the disk component besides that of gas stream. To confirm this, we compare phase averaged light curves of nSH in quiescence, at superoutburst No. 4, and at a normal outburst which occurred around 389 in figure 6. As discussed already in paper I and shown in figure 7 and figure 8 of Paper I, the light curve of nSH in quiescence takes a saw-tooth like form with a rise time roughly twice the fall time in quiescence while it is more or less of sinusoidal form during a superoutburst. As seen in figure 6, the light curve in a normal outburst takes more or less a form just between these two. If we accept our assumption about an addition of disk component to nSH light source besides the gas stream component in outbursts, amplitude variation of nSH shown in figure 5 is most naturally explained as it faithfully follows variation of light curve.

In his recent paper, Smak (2013) has argued that our results on the disk radius variation were inconsistent with observed amplitudes of negative superhumps in superoutburst and in quiescence. He assumed that light source of nSH was solely due to gas stream both in a superoutburst and in quiescence. We think his criticism is irrelevant because light source of nSH is most likely different between these two phases.

As seen in figure 5, the amplitude of nSH is suppressed when a strong positive superhump signal appears at the start of superoutburst. An excitation of the positive superhump (pSH) seems somehow to suppress temporarily the amplitude of nSH, although the exact cause of suppression of negative superhumps is not known. We do not know yet whether it is due to some physical mechanism or it is simply due to an artifact of data analysis. Here we leave it as a problem to be solved in future.

2.4. Two-dimensional Period Analysis using a New Period Analysis Called “Lasso”

In paper I and figures 1 and 2 above, we have used two-dimensional discrete Fourier transform (2D DFT) method in order to make power spectra. Here we introduce another method of two-dimensional spectral analysis by using a new method of period analysis called least absolute shrinkage and selection operator (“lasso”, Tibshirani 1996) introduced to analysis of astronomical time-series data (Kato, Uemura 2012) which is very suitable to find peaks in power spectra (see also, Kato, Maehara 2013). The advantage of the lasso analysis is that peaks in power spectra are very sharp and thus it is very powerful in analyzing rapidly changing periods as in outbursting dwarf novae. There is, however, a set-back in this method in that the resultant powers are not linear in power amplitude. Thus this method and the Fourier transform method are complementary with each other.

In figure 7 we show the two-dimensional power spectra of individual supercycles for V1504 Cyg by using this method and in figure 8 we do the same ones for V344 Lyr. Since the results for the first six supercycles of V1504 Cyg shown in figure 7 overlap with those obtained by the discrete Fourier transform method given in figure 3 of Paper
I, we can compare the results of these two methods. Since much is common between figures 7 and 8, we only discuss figure 7 here. In the Lasso spectrum, nSH and orbital modulation are always seen as clear separate signals when nSH is present (e.g., BJD 2455310–2455420), while the Fourier spectrum can barely separate them. The Lasso spectrum clearly shows that the frequency of the nSH rises quickly when normal outbursts start, and then decreases gradually, and it gradually increases as the progress of the supercycle phase. This result is in very good agreement with the PDM analysis (figure 3). The same trend is only barely visible in the Fourier analysis. The strength of the orbital modulation also varies, and can be seen in the Lasso spectrum as a signal of variable strength, while the same trend is only barely seen in Fourier spectrum. Although PDM has an ability to measure the frequency of a single signal with a high precision, this method is not designed to deal with a combination of multiple signals. Although Fourier analysis, on the other hand, can handle a combination of multiple signals, it has a lower frequency resolution than in PDM, and its ability to detect the rapid variation of the frequency is limited. The Lasso analysis meets the both requirement (frequency precision and multiple signals), and our results clearly demonstrate that the Lasso period analysis is suitable for study of frequency variations when multiple signals are present, such as in positive and negative superhumps in SU UMa stars. We also added results for data of Kepler 14th quarter (Q14) recently released for public in the last panels of figure 7 and figure 8.

There is a free parameter $\lambda$ in giving an $\ell_1$ term (cf. Kato, Uemura 2012). In producing figures 7, 8, we selected $\lambda$ which gives the best contrast of the signals against the background, i.e. most physically meaningful parameter. This $\lambda$ is close to the most regularized model with a cross-validation error within one or two standard deviations of the minimum (cf. R Sct for Kato, Uemura 2012). We also applied smearing of the signals between $\pm 3$ bins shifted by 0.5 d considering the width of the window.

2.5. Positive and negative superhumps and their (simultaneous) frequency variation

The positive and negative superhumps of V344 Lyr in Kepler data have already been discussed by Still et al. (2010) and in particular by Wood et al. (2011) in great details.

Here we study them from a different standpoint, in particular in relation to the supercycle. We note here that in this section and in the next subsection we deal with the positive superhump and the negative superhump as linear eigenmodes of an accretion disk (i.e., the eccentric mode and the tilt mode, respectively), and we discuss their eigen-frequency variations. We recognize very well that a linear approach has some limitation because observed superhumps are quite non-linear. In this approach the unperturbed disk, in which these two eigenmodes occur, is assumed to be a circular, coplanar Keplerian disk and all complications produced by non-linear effects, such as density waves, warps, are not taken into account in the unperturbed disk. Our approach is to adopt the simplest assumption of linear mode analysis and we then examine what this approach tells us.

As already pointed out by Still et al. (2010), the positive superhumps which first appeared with the start of a superoutburst continued to be seen during the following quiescence and the next normal outburst in V344 Lyr. Wood et al. (2011) demonstrated that the positive superhump signal was generated by viscous dissipation within the periodically flexing disk (i.e., eccentric precessing disk) in the early and main phase of superoutburst but in the later phase of superoutburst it was generated by impact of gas stream with non-axisymmetric disk rim (i.e., that corresponds to so-called late superhump).

As seen in figure 8 (for superoutbursts No. 1 and No. 6 of V344 Lyr, see, also figure 9), the frequency of the positive superhumps exhibits a characteristic variation with progress of a superoutburst and the following quiescence and the next normal outburst. When the superoutburst is just over and the cooling transition occurs at the outer disk-edge and the cooling wave starts to propagate inward, the nature of the superhump changes from that of the disk origin to that of gas stream as discussed by Wood et al. (2011). That is, the late superhump appears and a remnant eccentricity of the disk remains for a little while. In the case of V344 Lyr the remnant eccentricity remains during the following quiescence and the next one or two normal outbursts.

The positive superhumps are produced by the tidal instability at a strong resonance at 3:1 radius when a superoutburst starts (or rather the tidal instability and the superhumps initiate a superoutburst) and the large amplitude oscillations continue to be seen during the superoutburst. In this subsection we concentrate our discussions on the late superhump as we discuss the frequency variation of the positive superhumps during the superoutburst in the next subsection.

It is usual to define the fractional superhump period excess over the orbital period by $\epsilon$, which is written as

$$\epsilon = \left( P_{\text{SH}} - P_{\text{orb}} \right) / P_{\text{orb}}. \tag{3}$$

The quantity $\epsilon$ is positive for the positive superhump (i.e., $\epsilon_+ > 0$) and it is negative for the negative superhump (i.e., $\epsilon_- < 0$) by definition. Since the positive and negative superhump frequency deficiency or excess is related to the apsidal precession rate of the eccentric disk and the nodal precession rate of the tilted disk, respectively, it is convenient to introduce the superhump frequency deficiency or excess rate by

$$\epsilon^* = \frac{\nu_{\text{PR}}}{\nu_{\text{orb}}} = \frac{\nu_{\text{orb}} - \nu_{\text{SH}}}{\nu_{\text{orb}}}, \tag{4}$$

where $\nu_{\text{PR}}$ is a precession rate of disk, and the sign of $\epsilon^*$ is chosen in such a way that $\epsilon^* > 0$ if the precession is prograde and $\epsilon^* < 0$ if it is retrograde. These two $\epsilon$’s are related with each other by

$$\epsilon^* = \frac{P_{\text{orb}}}{P_{\text{SH}}} \epsilon = \frac{\epsilon}{1 + \epsilon}. \tag{5}$$
Fig. 7. Two-dimensional lasso power spectrum of the Kepler light curve of V1504 Cyg. (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum (log $\lambda = -3.9$). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 7. Two-dimensional lasso power spectrum of the Kepler light curve of V1504 Cyg (continued). (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum (log$\lambda = -3.9$). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 7. Two-dimensional lasso power spectrum of the Kepler light curve of V1504 Cyg (continued). (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum ($\log \lambda = -3.9$). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 8. Two-dimensional lasso power spectrum of the Kepler light curve of V344 Lyr. (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum ($\log \lambda = -4.8$). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 8. Two-dimensional lasso power spectrum of the Kepler light curve of V344 Lyr (continued). (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum \(\log \lambda = -4.8\). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 8. Two-dimensional lasso power spectrum of the Kepler light curve of V344 Lyr (continued). (upper:) light curve; the Kepler data were binned to 0.02 d, (lower:) lasso power spectrum ($\log \lambda = -4.8$). The width of the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 9. Variation in precession rates of positive and negative superhumps given by two $\epsilon^*$’s for the supercycle No. 7 of V344 Lyr. (Upper:) Light curve of V344 Lyr for a period of BJD 2455640–2455740. The Kepler data were averaged to 0.07 d bins. (Lower:) Absolute values of fractional superhump excesses (positive and negative) in frequency scale. The window width is indicated by a horizontal bar at the upper left corner and the error bars represent 1-σ errors in the periods.

Fig. 10. Variation in precession rates of positive and negative superhumps given by two $\epsilon^*$’s for a supercycle No. 2 of V344 Lyr (Upper:) Light curve of V344 Lyr for a period of BJD 2455555–2455660. The Kepler data were averaged to 0.07 d bins. (Lower:) Absolute values of fractional superhump excesses (positive and negative) in frequency scale. The window width is indicated by a horizontal bar at the upper left corner and the error bars represent 1-σ errors in the periods.

Fig. 11. Variation in precession rates of positive and negative superhumps given by two $\epsilon^*$’s for a supercycle No. 6 of V1504 Cyg. (Upper:) Light curve of V1504 Cyg for a period of BJD 2455515–2455620. The Kepler data were averaged to 0.07 d bins. (Lower:) Absolute values of fractional superhump excesses (positive and negative) in frequency scale. The window width is indicated by a horizontal bar at the lower left corner and the error bars represent 1-σ errors in the periods.
Fig. 12. Variation in precession rates of positive and negative superhumps given by two $\epsilon^*$’s for complete supercycles No. 4 and 5 of V1504 Cyg. (Upper:) Light curve of V1504 Cyg for a period of BJD 2455310–2455520. The Kepler data were averaged to 0.07 d bins. (Lower:) Absolute values of fractional superhump excesses (positive and negative) in frequency scale. The broken lines represent the trend of the value of $|\epsilon^*| \times 7/4$ at the peak of every normal outburst and its extrapolation to the time of the precursor peak of the next superoutburst. The window width is indicated by a horizontal bar at the lower left corner and the error bars represent 1-σ errors in the periods.

The precession rate of the eccentric disk, $\nu_{\text{PR}}$ in the late superhump is basically determined by the dynamical precession of the eccentric disk by gravitational field of the secondary star and it is given in the lowest order of expansion in the disk radius, $R_d$ (see, Osaki 1985) by

$$\epsilon^*_+ = \frac{\nu_{\text{PR}}}{\nu_{\text{orb}}} = 1 - \frac{\nu_{\text{SH}}}{\nu_{\text{orb}}} = \frac{3}{4} \frac{q}{\sqrt{1 + q}} \left( \frac{R_d}{A} \right)^{3/2}$$

where the plus sign in the subscript of $\epsilon^*_+$ signifies the positive superhump and $q = M_2/M_1$ is the mass ratio of the binary, $A$ is the binary separation as usual. Here $\nu_{\text{PR}}$ is the apsidal precession rate of an eccentric disk. Here we note that equation (7) is valid for the cold disk (i.e., in quiescence) because the pressure effects are unimportant in the cold disk and because the eigenfunction of the eccentric mode is confined to the outer part of the disk in the case of the cold disk, as will be discussed in the next subsection.

As discussed in Paper I, the precession rate of a tilted disk over the orbital frequency is given (see, Larwood 1998) by

$$\epsilon^*_+ = \frac{\nu_{\text{PR}}}{\nu_{\text{orb}}} = 1 - \frac{\nu_{\text{SH}}}{\nu_{\text{orb}}} = \frac{3}{4} \frac{q}{\sqrt{1 + q}} \left( \frac{R_d}{A} \right)^{3/2} \cos \theta,$$

where $\nu_{\text{PR}}$ is the nodal precession rate of a tilted disk, and the negative sign signifies its retrograde nature. If we compare two expressions of $\epsilon^*$ for positive and negative superhumps, we find

$$\frac{\epsilon^*_+}{|\epsilon^*-|} \simeq \frac{7}{4}.$$  

Here we assumed $\cos \theta \simeq 1$ for a slightly tilted disk.

One of the most interesting aspects in the power spectrum of V344 Lyr is that the positive superhump signal (the late superhump) is seen together with the negative superhump signal during quiescence just after a superoutburst and the next two normal outbursts in supercycle No. 7, i.e., in a period between days BJD 2455670 to 2455680. This means that the disk was eccentric and tilted simultaneously. A question then naturally arises how these two different superhumps are generated simultaneously because both light variations of the late superhump and of the negative superhump in quiescence are thought to be produced by the impact of gas stream. Although the exact
mechanism is not known yet, it is most likely that a part of
gas stream left from the Lagrangian point collides with the
rim of an eccentric disk, producing the positive superhump
signal while other part of gas stream spills over the rim
and arrives in the inner part of a tilted disk, producing the
negative superhump signal. In fact, Montgomery (2012)
has found that the positive and negative superhumps ap-
ppear simultaneously in her numerical simulations.

In paper I we have demonstrated that the frequency
variation of the negative superhump is a good indicator of
the disk-radius variation and its variation thus obtained
fits very well with a prediction of the thermal-tidal in-
stability model. It will be very interesting if we analyze
frequency variations of the two different superhump sig-
als. We thus study the frequency variations of positive
superhumps and negative superhumps during supercycle
No. 7 in V344 Lyr for a period from day BJD 2455640 to
2455740.

To do so, we have used a sliding window with a width
of 2 d and time step of 0.5 d and we have calculated fre-
cquencies by using PDM in obtaining the periods. We illus-
trate the frequency deficiency (or excess) over the orbital
frequencies by using PDM in obtaining the periods. We illus-
rate the frequency deficiency (or excess) over the orbital
frequency for the negative and positive superhumps thus
obtained in figure 9 simultaneously where the results, \(e^*_+\),
for the positive superhumps are shown in filled squares
and those for the negative superhumps are shown in filled
circles by converting them to those of \((7/4) \times |e^*_+|\) in order
to show within one figure.

Let us first look at the variation in frequency excess of
the negative superhumps \(|e^*_+|\) in figure 9. The absolute
value of \(e^*_+\) increases whenever a normal outburst occurs
and it decreases during quiescence. Its mean level in one
normal outburst cycle increases with advance of supercy-
cle phase which is in a good agreement with that of V1504
Cyg. One exception is that occurring in the second nor-
mal outburst from the last. This is due to the impulsive
negative humps which will be discussed separately in sub-
section 2.7 below.

The variation in \(e^*_+\) for the late superhump in quies-
cence and the first and the second next normal outbursts
just after a superoutburst No. 6 exhibits a same pattern
with that of negative superhump, that is, an increase when
a normal outburst occurs and it decreases during quies-
cence. When these two oscillations overlap, two \(e^*\) values
agree fairly well with each other if converted by equation
(9) as seen in figure 9.

We think that an agreement of these two variations is
not just chance coincidence but it demonstrates that both
the frequency variations of the late superhump and of the
negative superhumps represent the disk-radius variation
and it strengthens our conclusion of Paper I, supporting
the thermal-tidal instability model.

Figure 10 illustrates the same type of figure but for the
supercycle No. 2 of V344 Lyr, basically confirming the
above mentioned results. Figure 11 is another example
for the supercycle No. 6 of V1504 Cyg, showing a similar
pattern. However, points based on the nSH are systematic-
ically higher than those based on the pSH in an interval
BJD 2455530–2455535 and this point will be discussed in
the Appendix.

So far we have used equation (8) for the nodal preces-
sion rate of a tilted accretion disk which was obtained
by Larwood (1998) and where a proportionality constant
appearing in equation (8) is 3/7. However different au-
thors derived slightly different expressions for this co-
efficient because of slightly different assumptions. For in-
stance, Montgomery (2009b) obtained 15/32 instead of
3/7 for this coefficient [see, equation (38) of Montgomery
(2009b)]. Montgomery’s expression is larger by a factor
1.09 than 3/7.

In order to take this effect into account, here we intro-
duce a correction factor \(\eta\) to allow for a different expres-
sion of the nodal precession rate in such a way that the
numerical coefficient in equation (8) is modified as

\[
\frac{v_{nPR}}{v_{orb}} = -\frac{3}{7} \sqrt{\frac{q}{1+q}} \frac{R_d}{A} \theta^{3/2} \eta \cos \theta, \tag{10}
\]

Then we have \(e^*_+ / |e^*_+| \approx \eta^{-1} 7/4\) for equation (9). In the
Appendix we will discuss a possible value for \(\eta\) by taking
into account for a different mass distribution in the disk.

As shown in the appendix, difference of the correction
factor from 1 is rather small, typically less than 10% but
it could be as large as 20% in some cases.

2.6. Frequency Variations of the Positive Superhumps
during Superoutbursts

The interpretation of the frequency (or period) varia-
tions of the positive superhumps during superoutbursts is
much difficult and so we discuss them separately from
those in quiescence and in normal outbursts. Extensive
survey of period variations of superhumps in SU UMa
stars in a form of the \(O-C\) diagram have been accumu-
lated in a series of papers (I–IV) by Kato et al. (2009),
Kato et al. (2010), Kato et al. (2012), Kato et al. (2013).
Data based on the Kepler light curves of V1504 Cyg and
V344 Lyr supplement these observations in particular for
SU UMa stars of longer orbital periods and of high mass
transfer systems.

2.6.1. Factors Contributing to the Frequency Variations
of the Positive Superhumps

The superhump periodicity (and hence frequency) of
the positive superhump is produced by a synodic pe-
riod between a precessing eccentric disk and the orbit-
ing secondary star, which is given by \(v_{pSH} = v_{orb} - v_{nPR}\).
Theoretical precession rate of an eccentric disk in relations
with superhumps of SU UMa stars has been discussed by
Osaki (1985), Hirose, Osaki (1990), Lubow (1991), Hirose,
Osaki (1993), and Goodchild, Ogilvie (2006) among oth-
ers. Lubow (1992a) showed that the precession rate of
eccentric disk consisted of three different terms, which is
written by

\[
v_{nPR} = v_{dyn} + v_{pressure} + v_{stress}, \tag{11}
\]

where the first term, \(v_{dyn}\), represents a contribution to
disk precession due to tidal perturbing force of the sec-
dary, giving rise to prograde precession, the second
term, \(v_{pressure}\), does that due to pressure effect (or an
effect of finite thickness of the disk), giving rise to retrograde precession, and the last term, $\nu_{\text{stress}}$, does that due to wave-wave interaction, which arises when the eccentric mode grows or decays in time, giving rise to either retrograde or prograde precession. The third term may become important when eccentricity (and thus superhump) grows rapidly in the initial stage of superoutburst or when it decays in the final stage of a superoutburst, but Lubow (1992) argued that it is much smaller than the first two terms. In what follows, we concentrate our discussion to the first two terms.

### 2.6.2. Eigenfrequency Expression of the Frequencies of the Positive Superhumps

The precession rate of eccentric disk was discussed in terms of an eigenfrequency of an eccentric mode in an accretion disk (the fundamental mode of $m = 1$ in a thin disk, which hereafter we call the superhump mode) by Hirose, Osaki (1993) and Goodchild, Ogilvie (2006). Hirose, Osaki (1993) treated this as a free disk mode while Goodchild, Ogilvie (2006) did it as a complex eigenmode so that growth and decay of the mode could be discussed together with frequency. Since we are here interested only in frequency of eccentric mode and since we do not go into the problem of its excitation, we adopt discussion by Hirose, Osaki (1993) in what follows.

As shown by equation (32) in Hirose, Osaki (1993), the eigenfrequency of $m = 1$ mode is written in a variational form as

\[
\nu_{\text{PR}} = \frac{\int_{r_i}^{R_d} \nu_{\text{pr}}(r)X(r)2\,dr - \int_{r_i}^{R_d} f(r)\,dX(r)/dr\,dr}{\int_{r_i}^{R_d} g(r)\,X(r)2\,dr}
\]

where $r$, $r_i$, and $R_d$ are radial distance from the central star in the disk, that of the inner edge, and of the outer edge of the disk, respectively, $X(r)$ is an eigenfunction of the superhump mode (see figure 13), and $g(r)$ is a weighting function which increases with radius $r$, while $\nu_{\text{pr}}(r)$ is the local precession rate of a ring with radius $r$ which is given by

\[
\nu_{\text{pr}}(r)/\nu_{\text{orb}} = \frac{3}{4} \frac{q}{\sqrt{1+q}} (r/A)^{3/2}
\]

and $f(r)$ is a function proportional to square of local sound speed, (i.e., local temperature).

It is obvious that the first and the second terms in the numerator of equation (12) represent the dynamical effect and the pressure effect in equation (11), respectively. As for the dynamical effect, if we neglect for a moment the dynamical effect and the pressure effect in equation (11), respectively.

As for the dynamical effect, if we neglect for a moment the dynamical effect and the pressure effect in equation (11), respectively. The dynamical effect is sensitive to the eigenfunction which is in turn very sensitive to the disk temperature. These two effects work for reduction of the precession rate in a sense that higher the disk temperature (figure 13 left), the lower the eigen-frequency (i.e., the precession rate). As a matter of fact, the second effect dominates over the first effect as demonstrated by Hirose, Osaki (1993).

If the disk is cold, the eigenfunction is well confined in the outermost region of the disk (figure 13 right) and thus the frequency is given by the local precession rate of the outermost part of the disk, which justifies equation (7) for pSH in quiescence. (Here we note once more that our treatment is of linear mode analysis in which the unperturbed disk is assumed to be a circular, coplanar, Keplerian disk and any complicated phenomena which may arise by non-linear effects such as spiral density waves are not taken into account.) In the previous subsection we have demonstrated that frequencies of the negative superhump and the positive superhump vary in unison in quiescence and in normal outbursts when they are represented in terms of $\epsilon^*$.

To summarize briefly, the frequency of positive superhumps is reduced when the disk is hot and the superhump oscillation spreads to a large range of the radius (large eigenfunction even in small disk radii). To the contrary, the frequency of positive superhumps can take a large value when the superhump oscillation is confined to the outermost region, either when the disk is cold or when the superhump oscillation has not enough time to spread appropriately after its excitation.

### 2.6.3. Frequency Variation of Positive Superhumps during Superoutburst: Observations

Let us now examine observed variations in frequency (or period) of pSH during the superoutburst. As seen in figure 9, the quantity $\epsilon^*$ representing the disk precession rate (or fractional excess of the positive superhump period) starts from its highest value at the start of the superoutburst but it decreases very rapidly in initial phase but later more slowly, approaching to a constant small value during the main part of the superoutburst. As the end of the superoutburst draws near, the nature of the superhump changes from the ordinary superhump to the late superhump, $\epsilon^*$ increases to a new local maximum. When the plateau stage of the superoutburst ends and the rapid decay of light curve starts, $\epsilon^*$ decreases continuously to quiescence. If the remnant eccentricity survives further, $\epsilon^*$ decreases in quiescence and it increases when the next outburst occurs. Our results basically confirm the period variation of the positive superhump found in V344 Lyr by Wood et al. (2011).

### 2.6.4. Frequency Variation of Positive Superhumps during Superoutburst: Interpretation

We now present our interpretation of observed variation in the precession rate, $\epsilon^*$. Our interpretation is still in a speculative state and more observations and theoretical investigation are needed to clarify the variation of superhump periodicity. Since our discussion is confined only to qualitative one, we discuss it sometime in terms of the positive superhump period and sometime in terms of the
Fig. 13. Schematic illustration of eigenfunctions of the superhump mode $m = 1$. Darker colors represent larger values in eigenfunction and larger eccentricities (displacement). The eccentricity shown here is important only in relative sense with different radii and its absolute value has no meaning because of linear mode analysis. The eigenfunctions of these figures are for the case of $q = 0.15$. (Left:) hot disk ($c_0 = 0.1$) where $c_0$ is the dimensionless sound velocity. The eigenfunction takes large values for a large range of the disk radius, and the eccentricity wave is present in a wider region of the disk. ($\omega = 0.0289$ where $\omega$ is the dimensionless eigenfrequency which is equivalent to $\epsilon^*_0$). (Right:) cold disk ($c_0 = 0.005$). The eigenfunction takes large values only near the outer edge of the disk, and the eccentricity wave is trapped in the outermost region of the disk. ($\omega = 0.0492$). In the left (hot) case, a larger portion of the disk becomes elliptic and the precession rate of the entire disk is smaller due to the contribution of slower local precession rates in the inner radii.

disk precession rate, $\epsilon^*$. We first note that the disk radius variation and the pressure effect discussed above are two major causes responsible for the period variation of the positive superhump. The main objective of this subsection is to clarify how these two effects make an interplay.

We interpret that the highest value of $\epsilon^*$ at the start of the superoutburst (i.e., the stage A of superhump in Kato et al.’s classification) is most likely given by that corresponding to the precession rate at the 3:1 resonance radius in the outer edge. Its following rapid decrease in the SH period is then understood as due to propagation of the eccentricity wave to the inner part of the disk by which a larger portion of the disk is involved to determine the overall precession rate of the disk. That is, the pressure effects have decreased the SH period appreciably. After the superoutburst reaches its maximum, the superhump period (and so the precession rate $\epsilon^*$) decreases with much slower pace toward the end of the superoutburst. This slow decrease in the pSH period is understood as due to a slow decrease in the disk radius as the mass and angular momentum of the disk are depleted during the plateau stage of the superoutburst by mass accretion and strong tidal removal of angular momentum of the disk as the thermal-tidal model indicates. As the superoutburst approaches to its end, the disk begins to cool and it brings about an ease of the pressure effects which makes the eigenfunction of the superhump mode to be more concentrated in the outer part. This in turn brings about a rapid increase in the superhump period when the late superhump starts. Once the cooling transition occurs at the outer edge of the disk, the cooling front propagates inward, extinguishing the superoutburst. The period variation now becomes simple, basically due to the disk radius variation in the cold state as discussed in the preceding subsection. The variation of pSH period during the superoutburst can be understood qualitatively in the framework of the thermal-tidal instability model.

2.6.5. Change of the Disk Radius during a Supercycle as Seen from the Variations of the Negative Superhump Periods

This general picture can be confirmed from figure 12 which exhibits variations in the two $\epsilon^*$ and which covers two supercycles No. 4 and 5 of V1504 Cyg. We first note that the variations in the nSH frequency in the supercycle No. 4 (from BJD 2455325 to BJD 2455406) represented by $|\epsilon^*_n| \times 7/4$ show a secular increase in disk radius with the
advance of supercycle phase upon which superimposed are a sudden increase in the disk radius when a normal outburst occurs and its gradual decrease during quiescence. If we connect the value of $|c^*|\times 7/4$ at the peak of every normal outburst by a spline curve and extrapolate it to the time of the precursor peak, we find this value is about 0.055 (broken lines in figure 12). If we take into account the correction factor $\eta \approx 0.94$ for the hot disk, this value becomes about 0.058. In either case, its estimated value agrees fairly well with the value of $c^*$ for the positive SH at the start of the superoutburst No. 4. We suggest that these two values represent two precession rates corresponding to the 3:1 resonance disk radius.

### 2.6.6. Separation of the Variable Radius and Pressure Effect

The frequency of the negative superhump decreases monotonically during the superoutburst No. 4 (BJD 2455406–2455419) as seen both in figure 3 and figure 12. This indicates a monotonic decrease in the disk radius because the nodal precession rate of a tilted disk depends only on the disk radius during the hot viscous plateau stage as the density distribution is self-similar as discussed in the Appendix and the correction factor discussed in equation (10) is $\eta \approx 0.94$

We now compare these two $c^*$ during the superoutburst No. 4 in figure 12. The values shown in red square corresponding to the pSH are systematically smaller than those shown in green filled-circle corresponding to nSH, roughly speaking about 30%. This difference is due to the pressure effects discussed above. By this way we can now separate the pressure effects on the positive superhump periodicity from that of the disk radius variation during a superoutburst if the positive SH and negative SH signals coexist. This basically confirms our interpretation presented above on the frequency variation of the positive superhump during a superoutburst.

### 2.7. Impulsive (failed) negative superhumps

One of the most important outstanding problems in V344 Lyr and V1504 Cyg is the problem of origin of the negative superhump, in other words, a problem of the origin of the disk tilt, how the disk tilt is produced and how it is maintained.

Observationally the negative superhump in V1504 Cyg first appeared around BJD 2455280 and its amplitude grew until the middle of SC No. 4 (around BJD 2455350) and then it kept its large amplitude over 200 d until the late stage of SC No. 6 and it finally disappeared around BJD 2455610 before the next superoutburst.

In the case of V344 Lyr, a strong signal of negative superhump was visible from the start of observations (at BJD 2455003) but it was weakened before the first superoutburst. However a weak signal continued to the next supercycle (SC No. 2) but it disappeared around BJD 2455140 before the next superoutburst. After a long interval of more than 500 d a new signal of the negative superhump again appeared in the early phase of SC No. 7 around BJD 2455670 and it stayed almost near the next superoutburst but it disappeared before it. However a weak signal of negative superhump was still visible in the early part of SC No. 8 but it finally disappeared in its later phase. From these observations it will be difficult to tell what initiates the disk tilt and what quenches it finally.

As already noticed and discussed in details by Wood et al. (2011), a sudden excitation of oscillation with a frequency range of negative superhump occurred in the descending branch of several normal outbursts in V344 Lyr but the oscillation did not grow to a full negative superhump [see also figure 27 of Wood et al. (2011)].

Here we discuss this kind of events by calling it as an impulsive (failed) negative superhump to distinguish it from the “ordinary” negative superhump discussed in the main part of this paper. The ordinary negative superhump has been quite stable (sometimes continued over 300 d) while the impulsive negative superhump was short-lived (less than several days) and it never developed to a full-scale negative superhump (so we added an adjective “failed”). Wood et al. (2011) suggested from this type of events that the disk tilt could be generated by the impulsive coupling between an intensified disk magnetic field and the field of the primary or secondary star.

Here we suggest another possibility for the impulsive negative superhump. The impulsive negative superhump is seen in the power spectrum of V344 Lyr as a signal which appears near the frequency of negative superhump but it quickly moves to higher frequency and disappears. It occurs in the descending branch of a normal outburst [figure 14; figure 16 in Wood et al. (2011)]. This quick variation of the frequencies can be illustrated as follows: in the normal outbursts shown in figure 14, the negative superhumps appeared as signals with a longer period, and then switched to ones with a shorter period [0.0839(1) d] in outburst No. 2. [Here the outburst number is that used by Wood et al. (2011) and by Cannizzo et al. (2012) and we also adopt this number in this paper.] In outburst No. 19, the signal started with a period [0.0879(2) d] close to the orbital period, and then the period smoothly decreased to 0.0794(2) d. The latter period is astonishingly short, corresponding to $c^* = 11\%$. There is some indication that these impulsive negative superhump in outburst No. 2 also started with a period close the orbital period.

As already noted by Wood et al. (2011), this kind of event occurred during outbursts Nos. 2, 10, 17, and 19. From figure 8, we add to this list those outbursts of Nos. 28, 42, 54, and 62, and possibly Nos. 29, 30, 41, 43, 63. We also find the impulsive negative superhump in three normal outbursts just prior to a superoutburst which occurred at around BJD 2456193 in recently released Kepler data of V344 Lyr shown in the last panel of figure 8. In V1504 Cyg, we found the similar impulsive negative superhump in a “failed superoutburst”, which occurred two outbursts before a superoutburst (figure 77 in Kato et al. 2012). We could not find strong evidence for the impulsive negative superhump in other normal outbursts, but instead we find some sign of the failed positive superhump in normal outbursts just prior to a superoutburst as already discussed in Paper I.
It would be worth noting that negative superhumps were reported twice in normal outbursts of V1159 Ori (Patterson et al. 1995). These negative superhumps had period deficit (−6.0−7.9% in $\epsilon^*$) unusually large for this orbital period (0.0624 d). These values were so unusual (see table 2 of Montgomery 2009b or table 3 of Olech et al. 2009), not all the literature lists this detection as negative superhumps. This unusual period deficit may be understood if observed features were the impulsive negative superhump.

As seen in the global light curve of V344 Lyr (see figure 8), we find that all these normal outbursts accompanied with the impulsive negative superhump listed above occurred just prior to the next superoutburst and they look like a kind of lead for an impending superoutburst. There is no such event in the early phase of a supercycle. In fact, all seven superoutbursts observed in V344 Lyr we have studied were preceded by one, two or three normal outbursts accompanied with the impulsive negative superhump. The outbursts with possible impulsive negative superhump in V1159 Ori also occurred in the later stage of the supercycles, though not immediately before the next superoutbursts. Although it is still speculative, we suggest that the impulsive negative superhump observed in V344 Lyr may be excited by the 3:1 resonance for the tidal inclination (tilt) instability discussed by Lubow (1992b). As shown by him, the condition for the resonant excitation of the tidal inclination (tilt) instability at 3:1 resonance is very similar to that for the ordinary 3:1 resonance tidal eccentric instability but the tilt instability occurs in a slightly smaller disk radius than that of the ordinary eccentric instability, that means that the tidal tilt instability occurs slightly earlier than the tidal eccentric instability. This fits very well with the picture presented above.

However there remains a problem for this explanation, as the growth rate for the tilt instability is much lower than the tidal eccentric instability as pointed out by Lubow (1992b) and it is not at all clear if the tilt of disk could be produced within a time-scale of the outburst duration. Here we leave it as a future problem to be solved. As for a question whether or not the impulsive negative superhump could be a mechanism to produce the ordinary negative superhump if some other condition could be met, we do not know its answer. As a matter of fact, the impulsive negative superhump, that occurred during outbursts No. 2 and No. 62, rather worked for quenching the existing negative superhump.
3. Summary

We have analyzed the short-cadence Kepler light curves of two SU UMa stars, V344 Lyr and V1504 Cyg extending for an interval of well over two years (from BJD 2455003–2455832) to study their superoutbursts and superhumps. The followings are our major findings.

(1) We have made two-dimensional power spectral analysis for the Kepler light curves of V344 Lyr as well as V1504 Cyg. The light curve of V344 Lyr and its power spectra show both similarity and difference with those of V1504 Cyg studied in Paper I. The negative superhump signals seen well in V1504 Cyg were found in V 344 Lyr as well but they were more patchy (i.e., come and go) as compared with those of V1504 Cyg. The correlation between an appearance of the negative superhump and the reduction of outburst frequency, which is well established in the case of V1504 Cyg, exists in the case of V344 Lyr but it is much weaker. As for V1504 Cyg, additional Kepler data basically confirm the findings of Paper I.

(2) We have presented detailed analysis of frequency variation of the negative superhump for a complete supercycle No. 4 of V1504 Cyg because Smak (2013) criticized our results in Paper I. We have shown by using the $O-C$ diagram that a frequency jump of the negative superhump takes place during a rising branch of an outburst with a short time scale most likely less than 0.5 d exactly as predicted by the disk instability model. We also find that amplitudes of the negative superhump vary systematically during a supercycle in such a sense that its amplitude follows the same pattern with the light curve with a smaller scale. We suggest that these variations are caused by an addition of the disk component during outbursts to the superhump light source besides that of gas stream component as evidenced by the phase-average light profiles of the negative superhump during quiescence, superoutburst, and normal outburst.

(3) We have presented a new two-dimensional power spectral analysis based on a new method called “Lasso” for individual supercycles of V1504 Cyg and V344 Lyr. The new method gives very sharp peaks in the power spectra and thus it is very suitable for the study of frequency variation in cataclysmic variable stars.

(4) We have analysed frequency variations of the positive and negative superhumps, in particular simultaneous variations of these two superhumps. If they are appropriately converted, it is found that they vary in unison. This suggests strongly that these variations represent disk radius variation during the supercycle of SU UMa stars, supporting the thermal-tidal instability model for the superoutburst and supercycle of SU UMa stars.

(5) We have examined frequency (or period) variations of the positive superhumps during superoutbursts in V1504 Cyg and V344 Lyr. Two different factors contribute to the frequency variations of the positive superhumps: the disk radius variation and the pressure effects. As for the pressure effects, the higher the disk temperature the slower the precession rate, because the eccentricity wave propagates in a wider range of disk radii. Observations of the positive superhump during a superoutburst of V344 Lyr exhibit a characteristic variation in that its period (or its precession rate) decreases rapidly from its highest value in its start and then it decreases more slowly during the plateau stage of superoutburst, reaching a local minimum and it again increases to a local maximum near the end of the superoutburst when the nature of the superhump changes from the disk origin to the gas stream origin (i.e., the late superhump). By using the frequency variation of the negative superhump, we can separate the pressure effects on the frequency (or period) variation of the positive superhump during superoutbursts from the effects of disk radius variation.

(7) As already noted by Wood et al. (2011), a sudden excitation of oscillation with a frequency range near the negative superhump occurred in the descending branch of several outbursts in V344 Lyr but it damped within a few days. This phenomenon was observed exclusively in V344 Lyr and we called it as an impulsive (failed) negative superhump to distinguish from the ordinary negative superhump. These events seem to occur in outbursts just prior to the next superoutburst. We speculate that the impulsive negative superhump might be excited by the 3:1 resonance for the tidal inclination (tilt) instability first discussed by Lubow (1992b) and it seems to act as a kind of “lead” of the impending superoutburst.

We thank the Kepler Mission team and the data calibration engineers for making Kepler data available to the public. We are grateful for the referee, Dr. M. Montgomery, for her constructive comments to our paper.

Appendix 1. Nodal Precession Rate of a Tilted Disk for Different Mass Distribution in the Disk

It is known that the nodal precession rate of a tilted disk is determined by the mass distribution within the disk and its general expression is written as (see Larwood 1998)

$$\omega_{nPR} = 2\pi \nu_{nPR} = \frac{3}{4} \frac{GM_2}{A^3} \int \frac{\Sigma r^3 dr}{\int \Sigma \Omega r^3 dr} \cos \theta,$$

where $\omega_{nPR} = 2\pi \nu_{nPR}$ is angular frequency of nodal precession, $A$ is the binary separation, $\Omega = \sqrt{GM_1/r^3}$ is the Keplerian angular rotation rate of the disk, and integrations should be performed from the inner edge $r_1$ to the outer edge $R_d$ of the disk. If we adopt a simple power-law form with $\Sigma(r) = \Sigma_0(r/R_d)^n$ for the mass distribution within the disk where $n$ is a power law index, we can perform integrations in equation (A1), to obtain

$$\nu_{nPR} = \frac{3}{4} \frac{2.5 + n}{4 + n} \frac{q}{\sqrt{1 + q}} (R_d/A)^{3/2},$$

where we assumed $R_d > r_1$ and $n > -1.5$.

If we denote the coefficient appearing in equation (A2) by $C_1 = (2.5 + n)/(4 + n)$, we can now discuss it for several different mass distribution of the disk. For instance, if we assume a constant surface density $\Sigma = \text{const}$, we obtain $15/32$ for the coefficient of equation (A2) and we recover...
Montgomery’s expression. In the same way, if we assume $n = -1/2$, we recover the original expression of equation (8). As discussed by Osaki (1989), the surface density distribution of the disk in quiescence in its start (i.e., at the end of outbursts which applies for a normal outburst as well as for a superoutburst) is approximately given by $\Sigma \propto r$ and we then find $c_1 = 21/40$. In the same way for a hot quasi-steady disk corresponding to the plateau stage of a superoutburst of SU UMa stars, the surface density distribution is approximately given by $\Sigma \propto r^{-3/4}$ and we find $c_1 = 21/52$. We summarize these results for a correction factor $\eta$ in table 3. The correction factor $\eta$ for different mass distributions is rather small, typically less than 10%.

On the other hand, since the correction factor $\eta$ in the case of quiescence at its start amounts for about 20%, some more discussion is needed. Although we know fairly well mass distribution of the disk in quiescence at its start, we do not know how mass distribution changes within the disk during quiescence. First of all, we do not know what proportion spills over the rim and arrives at the inner part in a tilted disk. Furthermore we do not know how viscous diffusion changes the mass distribution within the disk during quiescence. Nevertheless, we have some restriction for the mass distribution within the disk during quiescence. It is well known in the disk instability model as represented by an S-shaped thermal equilibrium curve, there are two critical surface densities, $\Sigma_{\min}$ and $\Sigma_{\max}$, where $\Sigma_{\min}$ is that below which no cold state exists, and $\Sigma_{\max}$ is that above which no cold state exists. The radius dependence of these two critical ones is approximately linear, i.e., $\Sigma_{\min}(r), \Sigma_{\max}(r) \propto r$. As shown by Osaki (1989), the surface density distribution in quiescence at its start $\Sigma_{qstart}$ is approximately given by $\Sigma_{qstart} \simeq 2\Sigma_{\min}$. Then the density distribution begins to change by mass addition from the gas stream and by viscous diffusion. On the other hand the local surface density $\Sigma(r)$, which starts from $2\Sigma_{\min}$ must be limited by a condition $\Sigma(r) < \Sigma_{\max}(r)$ during quiescence. Thus we expect that surface density, $\Sigma(r)$ will most likely be limited in a range between $\Sigma_{\min}$ and $\Sigma_{\max}$. In such a case we do not expect any extremely different mass distribution during quiescence from that of its start. From these considerations we assume for the moment $\eta \sim 1.22$ during quiescence.

In paper I we converted the variation in the frequency of nSH for V1504 Cyg to variation in the disk radius by using equation (8) and by assuming a binary mass ratio $q = 0.2$. Since a proportional constant in the precession rate of a tilted disk appears in equation (10) by a combination with $\eta q/\sqrt{1+q}$, all discussion remains still valid even when a correction factor $\eta = 1.22$ is applied if we assume the binary mass ratio $q \simeq 0.16$ instead of $q = 0.2$. A new value for binary mass ratio $q \simeq 0.16$ is more appropriate for the observed orbital period of V1504 Cyg.

An introduction of correction factor $\eta = 1.22$ in comparison between the positive and negative SH frequencies in figure 9 makes an agreement poor by decreasing the data of negative SH by about 20%. On the other hand, if the same correction factor is applied to figure 11 for the SC No. 6 of V1504 Cyg, a better agreement is obtained between these two $\epsilon^*$ values. It is not clear at present moment how serious a disagreement in the case of the SC No. 7 of V344 Lyr is, and we leave it as a problem to be solved in future.

References

Borucki, W. J., et al. 2010, Science, 327, 977
Cannizzo, J. K. 1993, in Accretion Disks In Compact Stellar Systems, ed. C. Wheeler (Singapore: World Scientific), p. 6
Cannizzo, J. K., Smale, A. P., Wood, M. A., Still, M. D., & Howell, S. B. 2012, ApJ, 747, 117
Cannizzo, J. K., Still, M. D., Howell, S. B., Wood, M. A., & Smale, A. P. 2010, ApJ, 725, 1393
Goodchild, S., & Ogilvie, G. 2006, MNRAS, 368, 1123
Hellier, C. 2001, Cataclysmic Variable Stars: how and why they vary (Berlin: Springer-Verlag)
Hirose, M., & Osaki, Y. 1990, PASJ, 42, 135
Hirose, M., & Osaki, Y. 1993, PASJ, 45, 505
Howell, S. B., et al. 2013, AJ, 145, 109
Kato, T., et al. 2013, PASJ, 65, 23
Kato, T., et al. 2009, PASJ, 61, S395
Kato, T., Ishioka, R., & Uemura, M. 2002, PASJ, 54, 1029
Kato, T., & Maehara, H. 2013, PASJ, in press (arXiv astro-ph/1303.1237)
Kato, T., et al. 2012, PASJ, 64, 21
Kato, T., et al. 2010, PASJ, 62, 1525
Kato, T., & Uemura, M. 2012, PASJ, 64, 122
Koch, D. G., et al. 2010, ApJL, 713, L79
Larwood, J. 1998, MNRAS, 299, L32
Lasota, J.-P. 2001, New Astron. Rev., 45, 449
Lubow, S. H. 1991, ApJ, 381, 259
Lubow, S. H. 1992a, ApJ, 401, 317
Lubow, S. H. 1992b, ApJ, 398, 525
Montgomery, M. M. 2009a, MNRAS, 394, 1897
Montgomery, M. M. 2009b, ApJ, 705, 603
Montgomery, M. M. 2012, ApJL, 753, L27
Montgomery, M. M., & Martin, E. L. 2010, ApJ, 722, 989
Murray, J. R., Chakrabarty, D., Wynne, G. A., & Kramer, L. 2002, MNRAS, 335, 247
Ohshima, T., et al. 2012, PASJ, 64, L3
Olech, A., Rutkowski, A., & Schwarzenberg-Czerny, A. 2009, MNRAS, 399, 465
Osaiki, Y. 1985, A&A, 144, 369
Osaiki, Y. 1989, PASJ, 41, 1005
Osaiki, Y. 1996, PASP, 108, 39
Osaiki, Y. 2005, Proceeding of the Japan Academy, Series B, 81, 291
Osaiki, Y., & Kato, T. 2013, PASJ, in press (arXiv astro-ph/1212.1516)
Patterson, J., Jablonski, F., Koen, C., O'Donoghue, D., & Skillman, D. R. 1995, PASP, 107, 1183
Smak, J. 2009, Acta Astron., 59, 419

Table 3. A correction factor $\eta$ for different mass distribution in the disk.

| $n$  | $-3/4$ | $-1/2$ | 0   | 1   |
|------|--------|--------|-----|-----|
| $c_1$ | 21/52  | 3/7    | 15/32 | 21/40 |
| $\eta$ | 0.94  | 1      | 1.09 | 1.22 |
Smak, J. 2013, Acta Astron., 63, 109
Smak, J. I. 1991, Acta Astron., 41, 269
Stellingwerf, R. F. 1978, ApJ, 224, 953
Still, M., Howell, S. B., Wood, M. A., Cannizzo, J. K., & Smale, A. P. 2010, ApJL, 717, L113
Tibshirani, R. 1996, J. R. Statist. Soc. B, 58, 267
Warner, B. 1995, Cataclysmic Variable Stars (Cambridge: Cambridge University Press)
Whitehurst, R. 1988, MNRAS, 232, 35
Wood, M. A., Still, M. D., Howell, S. B., Cannizzo, J. K., & Smale, A. P. 2011, ApJ, 741, 105
Wood, M. A., Thomas, D. M., & Simpson, J. C. 2009, MNRAS, pp 2110–2121
Zemko, P., Kato, T., & Shugarov, S. 2013, PASJ, in press (arXiv astro-ph/1212.5940)