Fizeau drag in graphene plasmonics

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Dragging of light by moving media was predicted by Fresnel1 and verified by Fizeau’s celebrated experiments2 with flowing water. This momentous discovery is among the experimental cornerstones of Einstein’s special relativity theory and is well understood3,4 in the context of relativistic kinematics. By contrast, experiments on dragging photons by an electron flow in solids are riddled with inconsistencies and have so far eluded agreement with the theory5–7. Here we report on the electron flow dragging surface plasmon polaritons8,9 (SPPs); hybrid quasiparticles of infrared photons and electrons in graphene. The drag is visualized directly through infrared nano-imaging of propagating plasmonic waves in the presence of a high-density current. The polaritons in graphene shorten their wavelength when propagating against the drifting carriers. Unlike the Fizeau effect for light, the SPP drag by electrical currents defies explanation by simple kinematics and is linked to the nonlinear electrodynamics of Dirac electrons in graphene. The observed plasmonic Fizeau drag enables breaking of time-reversal symmetry and reciprocity10 at infrared frequencies without resorting to magnetic fields11,12 or chiral optical pumping13,14. The Fizeau drag also provides a tool with which to study interactions and nonequilibrium effects in electron liquids.

Graphene offers an ideal medium15–18 for observing the plasmonic Fizeau drag, as it supports the propagation of highly confined, long-lived and electrically tunable SPPs. Crucially, graphene also withstands ultrahigh current densities (of the order of mA μm−1)19 so that the carrier drift velocity u can be comparable to the SPP group velocity. In the absence of current, the SPP dispersion follows ω(−q) = ω(q) (see Table 1), where ω and q are the frequency and wavevector of the SPPs, respectively. Under applied direct current, the SPP dispersion is predicted to depend20–24 on the relative orientation of SPP propagation and carrier flow, generating the plasmonic Fizeau effect that breaks the reciprocity of the system: ω(−q) ≠ ω(q). Here we report on exploiting the unique attributes of graphene to demonstrate the physics of the plasmonic Fizeau effect where SPPs are dragged by drifting Dirac electrons. After completing this work, we became aware of similar results by W. Zhao et al.25.

To explore the plasmonic Fizeau drag, we fabricated multi-terminal graphene devices schematically shown in Fig. 1b. Monolayer graphene (MLG) encapsulated in hexagonal boron nitride (hBN) was integrated into back-gated structures assembled on a Si/SiO2 substrate (285 nm of oxide). Gold SPP launchers16,18 were deposited directly on the graphene among drain electrodes such that the current-gating effect26 is the elementary charge and n is the carrier density. The drifting currents were predicted20–24 to induce a plasmonic Fizeau effect, which leads to an increase in λe when SPPs co-propagate with the carriers (Fig. 1a, right branch) and a decrease in λe for the counter-propagation scenario (Fig. 1a, left branch). Our detailed theory (Supplementary Information) corroborates this intuition but reveals additional complications. Specifically, the drifting carriers modify the electromagnetic response of the system in a quasi-relativistic way25,26 with the Lorentz corrections of the measurements.

We now describe how the frequency–momentum dispersion ω(q) of SPPs is affected by the electric current in the graphene channel. The direct observable of our nano-imaging experiments is the wavelength λe of SPPs, which is related to the real part of the SPP wavevector, q = 2π/λe (q = q1 + iq2). In realistic graphene devices, the square-root law of SPP dispersion is modified by phonon resonances in the hBN substrate16,18 but the dispersion relation ω(q) remains q1−q2 symmetric. Under applied direct current (d.c.) with density Jdc, the Dirac electrons supporting SPPs in graphene acquire a drift velocity u = Jdc/en, where e is the elementary charge and n is the carrier density. The drifting carriers were predicted20–24 to induce a plasmonic Fizeau effect, which leads to an increase in λe when SPPs co-propagate with the carriers (Fig. 1a, right branch) and a decrease in λe for the counter-propagation scenario (Fig. 1a, left branch). Our detailed theory (Supplementary Information) corroborates this intuition but reveals additional complications. Specifically, the drifting carriers modify the electromagnetic response of the system in a quasi-relativistic way25,26 with the Lorentz corrections of the measurements.

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factor $y = (1 - u^2/\eta^2)^{-1/2}$, where $v_F$ is the graphene Fermi velocity. In addition, the corrections of Fizeau shift in the second order of the drift velocity are not negligible and prompt nonlinearities discussed below.

Next, we explore the experimental parameter space of the plasmonic Fizeau effect by modelling the wavelength shift $\Delta \lambda_p/\lambda_p$ for our device at different gate voltages $V_g$, laser frequencies $\omega$ and drift velocities $u$ (Figs. 1c, d). Assuming a drift velocity of $u = 0.1v_F$, a value readily achievable in our structures, the SPP wavelength is shortened compared to $u = 0$ for plasmons counter-propagating with carriers (solid and dashed lines in Fig. 1c). The magnitude of the Fizeau shift $\Delta \lambda_p/\lambda_p$ increases at higher laser frequency $\omega$ and lower gate voltages $V_g$ (Fig. 1d). Choosing the lowest gate voltage $V_g = 30$ V seems favourable for generating the largest Fizeau shift. However, both the amplitude and the propagation length of SPPs rapidly diminish at low $V_g$, compromising the fidelity of the experimental data. Higher laser frequency also results in similar adverse effects. We therefore chose to conduct the measurement at $\omega = 890$ cm$^{-1}$ and a moderate gate voltage $V_g = 47$ V, corresponding to a carrier density $n = 2.9 \times 10^{12}$ cm$^{-2}$.

We next analyse the experimental evidence for the plasmonic Fizeau effect (Fig. 2), acquired from a representative device shown in Fig. 2a. Near-field signals were acquired at $T = 170$ K and $V_g = 47$ V by repeatedly scanning along the same line perpendicular to the launching edge while varying the current density. Individual line scans were assembled into a two-dimensional false-colour plot with position on the horizontal axis and current density on the vertical axis (Fig. 2b). In this representation, SPPs were excited by the gold launcher on the left of the field of view and propagated to the right, manifesting themselves as periodic oscillations$^{16,19}$ of the scattering amplitude signal (Fig. 2b). Somewhat enhanced plasmonic loss can be observed at the largest applied current densities, which is caused by Joule heating. In Fig. 2c, we show line profiles averaged over a range of $\pm 25$ $\mu$A $\mu$m$^{-1}$ extracted from Fig. 2b at different current densities. Damped sinusoidal functions were used to fit (Methods section Fitting method') the experimental data and the fitting results are displayed along with the raw data points (Fig. 2c).

Experimental line profiles at different current densities show a smooth evolution of the SPP wavelength with current density. For positive current densities, electrons flow towards the launcher and SPPs propagate away from the launcher. The counter-propagation of electrons and plasmons results in a clear reduction of the SPP wavelength, as evident from the comparison of the fitting results in Fig. 2d. The observed wavelength downshift is consistent with the $q < 0$ dispersion branch in Fig. 1a where the SPP wavelength $\varphi$ is enlarged in the counter-propagation setting. Notably, the wavelength minimally increased upon current polarity reversal, owing to the second-order correction to the Fizeau effect described below. We acquired data from multiple devices at 170 K and 60 K (Extended Data Fig. 7) which all showed the same trends as in Fig. 2b–d. The fact that the SPP wavelength depends on current polarity suggests that the SPP Fizeau effect breaks the $q = -q$ reciprocity in our nano-plasmonic device.

Having established qualitative indicators of the plasmonic Fizeau drag, we now examine the findings quantitatively. We fit every line profile (Methods sections 'Fitting method' and 'Monte Carlo simulation of regression coefficients') in datasets such as Fig. 2b and obtained the Fizeau shift $\Delta \lambda_p/\lambda_p$ as a function of current density $J_{dc}$ and drift velocity $u$ (Fig. 3a, b). The results reveal that the Fizeau shift reaches $-2.5\%$ at $J_{dc} = 0.7$ mA $\mu$m$^{-1}$ for the counter-propagation setting (Fig. 3a). For the co-propagation setting, the Fizeau shift is very small and shows nonlinear dependence on drift velocity. When reversing the polarity of the gate voltage, which switches the carrier type to holes, the observed dependence of Fizeau shift on hole drift velocity was similar to that for electrons (Fig. 3b). These observations indicate an ambipolar character of the Fizeau drag, which depends on the carrier drift velocity but not on the carrier type.

**Discussion**

A salient feature of the experimental data is that the Fizeau shifts in Fig. 3a, b are neither symmetric with the current direction nor linearly varying with the carrier drift velocity. The observed experimental trends can be understood based on a model which accounts for both linear and quadratic terms of the Fizeau shift as a function of $u $ $v_F$:

$$\frac{\Delta \lambda_p}{\lambda_p} = \varphi - \left(\frac{1}{4} \varphi \left(\frac{u}{v_F}\right)^2 \right), \quad \varphi = \frac{2\eta \lambda_p}{2\pi},$$

where $\varphi$ is the drag coefficient and $\varphi$ is the plasmon phase velocity. Note that $\varphi = v_F$ is reduced to the plasmon group velocity only when the dielectric screening is nondispersive. In our device, the screening from hBN boosts the magnitude of the Fizeau shift by suppressing $\varphi$. The drag coefficient $\varphi$ assumes a value in the interval $1/4 \leq \varphi \leq 1/2$ depending on $u$.
along the same line while varying the current density between ±0.8 mA μm−1.

The black arrow represents positive current densities. Data displayed in Fig. 3a, b. Provided equation (1) for the kinetic regime is plotted by the black solid lines in the measurement frequency on the quasiparticle collision rate $\Gamma_{ee}$ is small compared to $\Gamma_T$ in plasma and in GaAs semiconductors. Such systems have parabolic quasiparticle dispersions, which are invariant under a Galilean transformation. However, experimental condition, we estimate that $\Gamma_{ee} \sim T^2/|\hbar\gamma\sqrt{\pi}| < 1$ THz, which strongly suggests that $\eta = \frac{1}{4}$ should be appropriate, in agreement with the experimental data. It is also instructive to compare these results with previous works (Supplementary Table) on the Fizeau shift (often referred to as the Doppler shift) in plasma and in GaAs semiconductor structures. Such systems have parabolic quasiparticle dispersions, which are invariant under a Galilean transformation. However, the dashed arrows represent positions where the averaged line profiles in c were taken. A one-dimensional Fourier filter was applied for b only, to reduce visual noise. The circles are raw data; the solid lines are fitting results; the dashed line is a guide to the eye. The line profiles are shifted vertically for clarity. d. Fitted SPP line profiles without d.c. current (black) and with $J_0 = 0.69$ mA μm−1 (blue), illustrating a reduction of the SPP wavelength. a.u., arbitrary units.

Fig. 3 | Quantitative analysis of the plasmonic Fizeau drag. a. The Fizeau shift, $\Delta \nu/F_p$, (circles), extracted from fitted line profiles as a function of current density and carrier drift velocity at $V_g = 47$ V and $T = 170$ K. Error bars represent ±1 standard deviation of the fitted Fizeau shift. Lines represent theoretical predictions of Fizeau shift for the kinetic regime (black solid line), the hydrodynamic regime (blue dashed line), and a 2D electron gas with parabolic dispersion (purple dashed line), respectively. b. As in a, at $V_g = 47$ V and $T = 170$ K. c. SPP dispersion at carrier density $n = 2.9\times 10^{12} \text{ cm}^{-2}$ calculated for the kinetic regime (Supplementary Information). The gap in the dispersion stems from phonon resonances in the hBN. Insets show enlarged views of the regions marked by the light blue boxes.
graphene quasiparticles are massless Dirac fermions characterized by a quasi-Lorentz invariance, with the speed of light replaced by \( v_d \). As indicated by the purple dashed lines in Fig. 3a, b, the Fizeau shift of plasmons in a Galilean-invariant system depends linearly on the drift velocity, with unit drag coefficient \( \eta = 1 \). The obvious discrepancy of these predictions with our experimental data vividly demonstrates that the Galilean invariance is broken in graphene.

Additional insights into the Fizeau shift can be obtained by placing our results in the context of the nonlinear electrodynamics of graphene. The current response at the plasmonic frequency \( \omega \), which can be expanded as \( j(\omega) = (\sigma + i \omega \varepsilon_0 \varepsilon_\text{r}) \varepsilon_\text{r} \varepsilon_{\text{ac}}(\omega) \equiv \sigma_{\text{ac}}(\omega) + \omega \varepsilon_\text{r} \varepsilon_{\text{ac}}(\omega) \equiv \sigma_{\text{ac}}(\omega) + \omega \varepsilon_{\text{ac}}(\omega) \), where \( \sigma_{\text{ac}}(\omega) \) is the n-th order nonlinear optical conductivity component, \( \varepsilon_{\text{ac}}(\omega) \) is the d.c. electric field that drives the static current \( \text{flow} \ u_{\text{dc}} = \varepsilon_\text{r} \varepsilon_{\text{ac}}(\omega) \), and \( \varepsilon_{\text{ac}}(\omega) \) is the inhomogeneous a.c. field from the launcher. All orders of \( \varepsilon_{\text{ac}} \) contribute to the effective a.c. conductivity \( \sigma_{\text{ac}}(\omega) \), which determines the Fizeau shift. Moreover, the Fizeau shift induced by \( \varepsilon_{\text{ac}} \) is a signature of \( (k + 1)\text{th}-order nonlinearity represented by the component \( \sigma_{\text{ac}} \).

Our data in Fig. 3 show that as the d.c. drive increases, the linear Fizeau shift acquires quadratic corrections: a manifestation of the third-order nonlinearity in \( \sigma_{\text{ac}} \).

### Outlook

The current-induced Fizeau drag of plasmons reveals novel aspects of interactions between infrared photons and Dirac electrons in graphene. Commonly, drag effects are understood as friction-like momentum transfer between two coupled sub-systems. Examples include Coulomb drag between spatially separated conductors, and drag effects between electrons and phonons in a crystal. Our data show that the notion of drag could be extended to the two constituents of a polaronic quasiparticle: a superposition of infrared photons and Dirac electrons. By ramping up the current in our platform, we solely perturbed the electronic constituent of the quasiparticles. The photonic component reacts by abiding to the rules of quasi-relativistic theory (Supplementary Information). However, the observed effect is not a mere consequence of relativistic kinematics. The plasmonic Fizeau drag in graphene is inherently a non-equilibrium and nonlinear phenomenon the magnitude of which depends on the dynamics of electron–electron, electron–phonon and electron–photon interactions of the Dirac electrons. A task for future experiments is to map the Fizeau drag for the entire SPP dispersion in order to optimize the infrared nonreciprocity for on-chip applications. In principle, plasmonic Fizeau drag offers means to probe the unique motional Fermi liquid effects and nonlocal effects. Fizeau drag experiments can also be extended to double-layer graphene and twisted bilayer graphene, potentially offering intriguing opportunities to probe the physics of Fermi liquid renormalization and strong correlations in electronic systems. Finally, by further enhancing the carrier drift velocity towards the plasmon velocity, Fizeau drag can be boosted, paving the way for plasmonic emission via plasmon instability and amplification.

### Online content

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41586-021-03640-x.
Methods

Device fabrication
Our devices involve monolayer graphene\textsuperscript{17} encapsulated in monoisotropic hexagonal boron nitride crystals\textsuperscript{48} (h\textsuperscript{BN}) with a global silicon gate, fabricated using a dry transfer technique\textsuperscript{49–51}. We first mechanically exfoliated graphene and hBN on an oxidized Si wafer, and searched for sizable, uniform flakes using optical contrast. Using a homemade transfer system, we picked up a thin (<10 nm) crystal of hBN as well as a thicker (>20 nm) neighbouring crystal with a polycarbonate (PC) membrane suspended over a thick (4 mm) polydimethylsiloxane (PDMS) polymer block mounted on a micromanipulator and heated to 100 °C. The thicker neighbouring flake was used as a marker to determine the position of the thin hBN, which, once on the PC, has extremely low optical contrast. The thin hBN was then used to pick up a monolayer graphene flake at room temperature. Finally, the two-layer stack was transferred onto the thick (>50 nm) hBN flake at elevated temperatures (160–170 °C)—a technique that we found resulted in clean, hydrocarbon-free van der Waals interfaces\textsuperscript{50}. We inspected the final heterostructure using atomic force microscopy (AFM) and selected flat, wrinkle-free areas for our device.

The heterostructures were covered by a polymethyl-methacrylate (PMMA) resist, and electron-beam lithography was used to define a protective mask in the shape of the device. We selectively etched away the exposed parts of the heterostructure with reactive ion etching (RIE) using plasma generated by Ar, CHF\textsubscript{3} and O\textsubscript{2} gases. We then performed a second round of electron-beam lithography and etching to define contact areas. These areas were first processed by a mild O\textsubscript{2} plasma to a second round of electron-beam lithography and etching to define using plasma generated by Ar, CHF\textsubscript{3} and O\textsubscript{2} gases. We then performed the exposed parts of the heterostructure with reactive ion etching (RIE) flat, wrinkle-free areas for our device.

Cryogenic scanning near-field imaging
Cryogenic near-field imaging was performed using a home-built scattering-type scanning near-field optical microscope operating at low temperature and ultrahigh vacuum based on Attocube scanner. We used a tapping-mode AFM operating at a frequency of ~275 kHz. A mid-infrared continuous-wave CO\textsubscript{2} laser (Access Laser) was focused on the sample, exciting the gold bar to launch propagating SPPs. The scattered light was collected by an off-axis parabolic mirror and recorded with a HgCdTe (MCT) detector. Using the pseudo-heterodyne interferometric method\textsuperscript{52}, we extracted the near-field signal at the third harmonic of the tip tapping frequency to suppress the background contribution. AFM topography is registered simultaneously with the near-field data. Data are collected at multiple lines in the middle of device where polaritons propagate with minimal extrinsic distortions.

Current and voltage appliance
We use one source meter (Keithley 2450) to source d.c. current through the whole device while maintaining the gate voltage using another source meter (Keithley 2450). The d.c. transport of the device is measured by a lock-in amplifier (SR 830) and the current-induced potential change at the gold launcher is monitored by a multimeter (Keithley DMM 6500).

Gate-dependent transport
Low contact resistance enables the application of high electric current through a graphene device and promotes an appreciable plasmonic Fizeau effect. We measured the contact resistance of a typical device using the two-terminal resistance method (Extended Data Fig. 1). As schematically shown in the inset of Extended Data Fig. 1, we sourced 100 nA of current through the graphene channel and measured the voltage drop across the entire device. The gate voltage dependence of the two-terminal resistance \( R_{\text{on}} \) shows a typical bell-shaped curve with a charge neutrality point at \( V_g = \pm 7.5 \) V. At high gate voltage \( |V_g| > 40 \) V where we acquired our Fizeau drag data, \( R_{\text{on}} \) is only 200–300 Ω. Considering that the sheet resistivity \( \rho \) of high-quality graphene devices at high gate voltage\textsuperscript{52} is of the order of 50 Ω, the total sheet resistance of our entire device is around 150 Ω. This suggests that the typical graphene–metal interface in our device has a resistance of less than 100 Ω. The ultra-low contact resistance enables us to drive a large current through the graphene channel using a small bias voltage, thereby mitigating the current-gating effects described in Methods section ‘Current-gating effect’.

Current-gating effect
The current-gating effect manifests itself as changes in the local carrier density caused by the spatial variation of the electrostatic potential in the graphene, owing to the biasing current. An appreciable current-gating effect is expected when high source/drain voltage is applied across the graphene. Since the SPP wavelength is sensitive to the local carrier density, it is imperative to evaluate the role of the current-gating effect in our Fizeau drag experiment.

To examine the magnitude of the current-gating effect, we measured the electrostatic potential of the launcher while sourcing large current (Extended Data Figs. 2, 3). Because the electrostatic potential close to the drain electrode changes minimally as a function of current, the current-gating effect is minimized close to the drain where we performed all of our plasmonic imaging experiments (Extended Data Fig. 2). Voltages on both the source electrode and the SPP launcher are recorded at the same time (Extended Data Fig. 3). The voltage on the SPP launcher depends linearly on the applied current with slope 150 mV/(mAμm), whereas the total two-terminal voltage reveals nonlinear behaviour. Standard graphene SPP dispersion implies that \( \lambda_p = \sqrt{V_g} \). We estimate the plasmon wavelength shift caused by the current-gating effect to be

\[
\frac{\Delta \lambda_p}{\lambda_p} = \frac{2}{V_g} \frac{\Delta V_g}{2} = \frac{1}{2} \frac{0.15 \text{V/(mAμm\textsuperscript{-1})}}{50 \text{V}} = 0.15\% / (\text{mAμm\textsuperscript{-1}}).
\]

Thus, the change in the SPP wavelength induced by the current-gating effect is much smaller than the 2% Fizeau effect observed in our measurements.

Large-area near-field images
In a typical near-field experiment, AFM topography and near-field images are registered simultaneously. Extended Data Fig. 4 shows representative near-field and AFM images without current at \( T = 170 \) K and \( V_g = 50 \) V. Near the graphene edge, we observed \( \Delta \lambda_p/2 \)-periodic fringes (top and bottom areas of the field of view in Extended Data Fig. 4c as well as Extended Data Fig. 4e, f). These characteristic \( \Delta \lambda_p/2 \)-periodic fringes are plasmonic waves excited by the near-field probe and reflected by the graphene edge. The \( \Delta \lambda_p/2 \) plasmons complete a ‘round trip’ and are subsequently out-coupled by the near-field probe. In contrast, SPPs close to the gold (left region of the field of view in Extended Data
Fig. 4a) are excited by the gold launcher and propagate to the right. The periodicity of Au-launched plasmon fringes equals the SPP wavelength \( \lambda_p \). In our Fizeau drag experiment, we focused on the \( \lambda_p \) fringe excited by the gold launcher.

Data in Extended Data Fig. 4 attest that our device is free from cracks, wrinkles, bubbles and folds, as can be seen from our near-field images (Extended Data Fig. 4a). To minimize the role of these extrinsic factors in Fizeau drag experiment, we scanned along the exact same line in real space for each set of data. These extrinsic factors are not affected by current biasing and thus cannot contribute to the changes in SPP wavelength observed in the Fizeau drag experiment.

Besides the Fizeau effect of gold-launched plasmons studied in the current work, edge-reflected \( \lambda_p/2 \) plasmons (Extended Data Fig. 4) could reveal higher order Fizeau effects. Furthermore, current flow near the edges of graphene may be considerably different than its counterpart in the interior of graphene. These intriguing phenomena are beyond the scope of the current study and could motivate future explorations of the intricate interplay between the current flow and plasmon standing waves near graphene edges.

**Possible extrinsic factor: Joule heating**

The d.c. current could heat up the electronic system to \( T' = T(1 + C(\mu/\varepsilon)) \), where \( T \) is the lattice temperature and \( C \) is some constant. This effect primarily decreases the plasmon lifetime, but can also affect the plasmon wavelength. The Drude weight of graphene is known to be slightly temperature-dependent\(^ {28} \), \( D(n, T) = D(n, 0)[1 - (\pi^2/6)(T/\mu_0)^2] \) at \( T < \mu_0 \), where \( \mu_0 \) is the Fermi energy. Hence, the Joule heating leads to a reduction of plasmon wavelength by the relative amount

\[
\frac{\Delta \lambda_p}{\lambda_p} = \frac{D(n, T') - 1}{D(n, T)} - 1 = -\frac{\pi^2}{6} \frac{C_\mu^2 + C_\varepsilon^2}{C_\varepsilon^2 + C_\mu^2}
\]

For a rough estimate, we take \( T = 170 \) K and \( T' = 200 \) K at the experimentally accessible drift velocity \( u = 0.17\mu_0 \), which implies \( C = 6 \). The corresponding plasmon wavelength shift is \( \Delta \lambda_p/\lambda_p = -0.35 \). Such a heating effect may explain the small discrepancy between the measurements and the theoretical predictions of the Fizeau shift at the largest current in Fig. 3.

**Absence of mechanical distortion**

As described in the main text, we carried out near-field scans along the same line in the middle of the graphene channel (dark red region in Fig. 2a) while continuously changing the current. At different current densities, the topographic line profiles repeat themselves (Extended Data Fig. 5b), whereas the near-field line profiles clearly show a Fizeau shift (Extended Data Fig. 5d). These findings unequivocally support that our measurements are free from mechanical distortions and that the extracted Fizeau shift is reliable.

**Fitting method**

We used a damped sinusoidal function with far-field background

\[
y = A \sin(qx - B) \exp(-qz) + Cx + D,
\]

to model the line profiles of the near-field signals of propagating SPPs, where \( \mathbf{B} = \{A, B, C, D, q_x, q_z\} \) is the set of fitting parameters and the SPP wavelength is \( \lambda_p = 2\pi/q_z \). To estimate the parameters \( \mathbf{B} \) of the nonlinear model, we minimized the chi-squared value:

\[
\chi^2 = \sum_i \frac{y_{\text{exp},i} - y_i(\mathbf{B})^2}{\sigma_y^2},
\]

where \( y_{\text{exp}}, y_i(\mathbf{B}) \) are experimental and model-predicted signals, respectively, and \( \sigma_y^2 \) is the signal variance. We minimized \( \chi^2 \) using the Levenberg–Marquardt algorithm (LMA) and set the convergence criteria of LMA as \( \Delta^2 \chi^2 < 10^{-4} \). The reduced chi-squared value used to quantify the goodness of fit equals \( \chi^2/60 = 1 \), indicating a good fit. The denominator in the definition of the reduced chi-squared value is the number of pixels in a line profile minus the number of fitting parameters.

If the \( \mathbf{B} \) are Gaussian random variables, the covariance matrix is inversely proportional to the Hessian matrix \( V^\dagger \chi^2(\mathbf{B}) \), which is approximated to first order by the matrix product of Jacobians evaluated at the best-fit values of the parameters:

\[
\Sigma = \sigma_y^2 V(J^\dagger J)^{-1} + \sigma_y^2 \frac{\partial y}{\partial \mathbf{B}}.
\]

The standard deviations of the fitted wavelengths \( \sigma_i \) are the square roots of the corresponding diagonal terms in the covariance matrices of the various line profiles. We typically extract \( \sigma_i < 1 \) nm for our experiments, which corresponds to \( \approx 0.08 \) relative error for the fitted SPP wavelength. The error bars presented in Fig. 3 are 68% confidence intervals computed from \( \sigma_i \). Alternative estimates of confidence intervals using likelihood-based methods give similar results. The likelihood-based estimates include all wavelength estimates with \( \chi^2 \) value smaller than \( \chi^2_{\text{best fit}} + \chi^2_{(0.68, df)} \), where \( \chi^2_{(0.68, df)} \) is the 68% quantile of a chi-squared distribution with df degrees of freedom\(^ {25} \).

The maximum correlation between the wavelength and any other fitting parameter \( \{A, B, C, D, q_z\} \) is 3.5%. This means that the additional fitting parameters have little effect on the optimal value and the variance of the extracted wavelength.

**Monte Carlo simulation of regression coefficients**

Here we justify the above-mentioned empirical estimates of \( \sigma_i \) with Monte Carlo simulations. Supposing there are experimental imperfections from signal noise, pixel size, and limited spatial resolution in a near-field measurement, then the empirical standard deviation of the wavelength \( \sigma_i = 1 \) nm may seem unrealistic. However, SPP line profiles observed in the experiments have up to six periods. The ‘redundant’ information in a periodic function enhances the precision of parameter estimates beyond the limited pixel size and spatial resolution of the near-field technique to support a statistically significant Fizeau shift. We demonstrate this by computing the variance \( \sigma_i \) and bias \( \Delta(\lambda - \lambda_p) \) of the least-squares wavelength estimate numerically for simulated line profiles that take into account all experimental imperfections, such as additive signal noise, positioner noise, limited spatial resolution and pixel size.

Consider equation (4) in the presence of noise and limited spatial resolution:

\[
\hat{y} = G(x, \text{SR})(A \sin(qx - B) \exp(-qz) + Cx + D) + \epsilon,
\]

where the limited positioning precision adds uncertainty to the spatial position \( \sigma_x \) such that the total noise is heteroscedastic and satisfies a normal distribution with an effective variance\(^ {26} \)

\[
\sigma_y^2 = \sigma_x^2 + \left( \frac{\partial y}{\partial x} \right)^2,
\]

\[\epsilon \sim N(0, \sigma_y^2 + \left( \frac{\partial y}{\partial x} \right)^2),\]

where \( \hat{y} \) is the empirical estimate of \( y \) with variance \( \sigma_y^2 \). We model the limited spatial resolution (SR) in a near-field experiment by a Gaussian filter, that is, convolution with a Gaussian function \( G(x) \) with a full-width at half-maximum given by SR. This has the effect of blurring sharp edges and peaks in the near-field signal. The spatial resolution of a near-field measurement is limited by the spatial extent of evanescent fields under the tip apex. The evanescent field averages the sample response weighted by distance from the tip apex in a qualitatively Gaussian fashion\(^ {27} \).

In Extended Data Fig. 6a, we show the distribution of wavelengths estimated by solving equation (5) with LMA in a Monte Carlo simulation.
of \( N = 10,000 \) random replications of equation (7) with \( SR = 20 \) nm, \( \alpha_t/\lambda = 10\% \), and \( \sigma_y = 1 \) nm (determined from scanner specifications). The true parameters in the simulations were \( \lambda_p = 2\pi/q = 118 \) nm, \( 1/q = 2.75\lambda_p \), \( A = 1 \), and \( B = C = 0 \). A few representative line profiles are shown in Extended Data Fig. 6b. These simulation parameters are extracted from typical experimental conditions. The distribution of the fitted wavelength is Gaussian (\( p \) value from Jarque–Bera test, <0.01) with a standard deviation \( \sigma_y = 0.71 \) nm \( \pm 1 \) nm, consistent with the empirical estimates from the covariance matrices.

We carefully studied the relationship between the error of the SPP wavelength estimate and various experimental conditions (Extended Data Fig. 6c, d). We can decompose the mean-squared error of the SPP wavelength estimate into bias (blue coloured lines in Extended Data Fig. 6c, d) and variance (red coloured lines in Extended Data Fig. 6c, d). We notice that the variance of the wavelength estimate is minimally affected by spatial resolution (Extended Data Fig. 6c.I) and pixel size (Extended Data Fig. 6c.II) under realistic experimental conditions. Limited spatial resolution will bias the wavelength estimate and the bias grows with the degradation of spatial resolution (blue line in Extended Data Fig. 6c.I). However, as long as SR < 30 nm, the bias and variance will be smaller than 1 nm. Since the spatial resolution is deterministic in this model, it alone cannot degrade the precision of the wavelength estimate. However, it does couple to random noise to increase the variance of the wavelength estimate (the Gaussian filter reduces \( f/f \)), as shown by the dark red line in Extended Data Fig. 6c.I. The pixel size of the line profiles has a vanishingly small effect on the wavelength estimate as long as the sampling is done well above the Nyquist rate set by the wavelength (Extended Data Fig. 6c.II).

We noticed that the variance of the wavelength estimate is strongly affected by the experimental signal noise (Extended Data Fig. 6d.I) and SPP propagation length (Extended Data Fig. 6d.II). The green curve in Extended Data Fig. 6d.I shows that with unlimited spatial resolution (\( SR = 0 \)) and \( \sigma_y = 0 \), the dependence of \( \sigma_x \) on \( \sigma_y \) has the linear form that is given by equation (6) for signal noise levels below 25%. At or above 25% noise, the LMA is unreliable, converging to many outliers such that the wavelength distribution is non-normal. As long as \( \sigma_y/\lambda < 10\% \) with \( 1/q \) = 2.75\( \lambda_p \), the estimated \( \sigma_y \) < 1 nm. In Extended Data Fig. 6d.II, both the variance and the bias of the wavelength estimate decrease with the increase of \( 1/q \). If we only observed a few SPP fringes with short propagation length, the error of our wavelength estimates would be much higher, approaching the limit set by the spatial resolution of the near-field technique.

In conclusion, our rigorous Monte Carlo simulation is consistent with the empirical analysis for the standard deviation of the wavelength estimate \( \sigma_y \). Even though we are limited to ~20 nm spatial resolution and -10 nm pixel size, as long as the signal noise \( \sigma_y \) is small and the SPP propagation length \( 1/q \) is large, we can extract very accurate \( \sigma_y < 1 \) nm wavelength estimates by fitting line profiles consisting of multiple SPP fringes.

**Hypothesis testing on the existence of Fizeau shift**

In Extended Data Fig. 6e, we set up a statistical hypothesis test based on the data in Fig. 3 to determine if we can reject a trivial model where there is no Fizeau shift. Consider a linear regression model for the Fizeau shift in a form reminiscent of equation (1):

\[
y = X\beta + \epsilon,
\]

\[
y = (\Delta\lambda_p)_i = \beta_0 + \beta_1 u_{x,k} + \beta_2 u_{x,k}^2,
\]

where the measurement error \( \epsilon = N(0, \sigma_y^2) \). Fitting to the data in Fig. 3a, b, we can obtain a least-square estimate of the coefficients \( \hat{\beta} = [-0.42, 12.31, -72.16] \). A linear hypothesis can be formulated by a set of \( N \) restrictions on the linear regression model. The restrictions are generally

\[ R\beta = q, \]

and the null \( (H_0) \) and alternative \( (H_1) \) hypotheses implied by the restrictions are as follows:

\[ H_0: \quad R\beta - q = 0, \]

\[ H_1: \quad R\beta - q \neq 0. \]

Given the least-squares estimate of the coefficients \( \hat{\beta} \), we can estimate the residual vector \( R\beta - q \). The question of statistical significance boils down to whether the deviation of \( R\beta - q \) from 0 can be explained by the sampling variability or whether it is significant. We can base a test of \( H_0 \) on the Wald statistic, which is chi-squared distributed with \( N \) degrees of freedom:

\[
W = (R\beta - q)^T [\Sigma (R\beta - q)]^{-1} (R\beta - q).
\]

\[
= \frac{1}{N\sigma_y^2} (R\beta - q)^T [R(X^T X)^{-1} R]^{-1} (R\beta - q).
\]

It is usually preferable to use the sample variance \( \sigma_y^2 \), an estimate of \( \sigma_y^2 \) with \( n \) observations and \( p \) parameters. In this case, the \( F \) statistic is used instead of the Wald statistic:

\[
F = \frac{W}{NS^2_{\Delta\lambda}} = \frac{1}{N\sigma_y^2} (R\beta - q)^T [R(X^T X)^{-1} R]^{-1} (R\beta - q).
\]

\[
= \frac{\chi^2[n-p]/(n-p)}{\chi^2[n-p]} = F(N, n-p).
\]

For example, in the case of a no-Fizeau-shift model, we would have the following \( N = 3 \) restrictions:

\[
\beta_0 = \alpha, \quad \beta_1 = 0, \quad \beta_2 = 0,
\]

which can be expressed as

\[
R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}.
\]

Pr(\( F[3, 217] \leq 2.65 \) = 0.95, so we can reject the null hypothesis that this trivial model is supported by the data with 95% confidence if \( F > 2.65 \). If the trivial model is rejected, then the Fizeau shift is statistically significant since we cannot attribute the current dependence to random noise.

We plot \( F \) as a function of \( s \) in Extended Data Fig. 6e. The standard deviation of the Fizeau shift estimated empirically from the data in Fig. 3 gives \( s = \frac{1}{n-p} \sum (y - X\hat{\beta})^2 = 0.74 \) nm which is consistent with the standard deviation from the covariance matrix of experimental line profiles in Methods section ‘Fitting method’ and with the Monte Carlo simulation in Methods section ‘Monte Carlo simulation of regression coefficients’. With this sample standard deviation and \( \alpha = 0 \), we compute an \( F \) statistic for the trivial model of 260.3 > 2.65 (purple solid line in Extended Data Fig. 6e). We can similarly entertain a model with no current dependence but a constant shift such that \( \alpha = \pm \Delta\lambda_p \). This test yields \( F = 159.8 > 2.65 \) (red solid line in Extended Data Fig. 6e). Statistical hypothesis testing thus suggests that the data are inconsistent to a high degree of certainty with a model that assumes that the data are randomly organized about a constant value (no dependence of the plasmon wavelength shift on the drift velocity). Thus, our experimental data support the existence of a significant Fizeau shift, under the estimated uncertainty.
Data availability
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Code availability
The code used to analyse data are available from the corresponding author upon reasonable request.

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D.A.B. and D.N.B. conceived and supervised the project. Y.D., L.X., S.Z., D.A.B. and D.N.B. designed the experiments. I.Y.P. and D.A.B. fabricated the devices. S.L. and J.H.E. provided the isotopic hBN crystals. Y.D., L.X., A.S.M. and R.P. performed the experimental measurements. Y.D., L.X., R.J., F.L.R., D.A.B. and D.N.B. analysed the experimental data. Z.S., M.M.F., A.J.M. and L.S.L. developed the theoretical analysis of the experimental data with input from P.J.-H., H.G. and Z.D. J.H.E. provided the isotopic hBN crystals. Y.D., L.X., A.S.M. and R.P. performed the experimental measurements. Y.D., L.X., R.J., F.L.R., D.A.B. and D.N.B. analysed the experimental data. Z.S., M.M.F., A.J.M. and L.S.L. developed the theoretical analysis of the experimental data with input from P.J.-H., H.G. and Z.D. Y.D., L.X., Z.S., M.M.F., D.A.B. and D.N.B. co-wrote the manuscript with input from all co-authors.

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Extended Data Fig. 1 | Gate-dependent transport of a typical device.
Two-terminal resistance $R_{2pt}$ of a typical device as a function of the back-gate voltage $V_g$ at $T = 170$ K. Inset shows the transport measurement configuration, where a source meter (Keithley 2450) was used to source gate voltage, and a lock-in amplifier (SR 830) was used to measure the resistance of the entire device.
Extended Data Fig. 2 | Experimental configuration for measuring current-gating effects. Voltage is applied across the source/drain electrode and SPP imaging is performed close to the drain. The black streamlines symbolize the d.c. current. The voltmeter measures the electrostatic potential of the SPP launcher as a function of the biasing current.
Extended Data Fig. 3 | Current–voltage characteristics of a typical device. The black symbols represent the source–drain voltage while sourcing current through the device. The blue symbols represent the simultaneously measured voltage on the SPP launcher, representing the current-gating effect induced by the biasing current. Inset shows a magnified view of the low-voltage region.
Extended Data Fig. 4 | Characteristic real-space nano-infrared and topography images. 

a, Near-field image taken in the vicinity of a gold launcher at \( T = 170 \text{ K} \) and \( V_g = 50 \text{ V} \). Gold-launched \( \lambda_p \) fringes and tip-launched \( \lambda_p/2 \) fringes near the graphene edge are clearly visible. Dashed rectangles mark the regions magnified in c and d.

b, AFM topography image taken simultaneously with the near-field image in a. The graphene region is uniform with minimal topographic variations.

c, d, Magnified near-field images near graphene edges, showing the tip-launched and edge-reflected SPP fringes with \( \lambda_p/2 \) periodicity.
Extended Data Fig. 5 | Real-space nano-infrared and topography line profiles under current. a–d, Representative results for simultaneously taken topography (a, b) and near-field (c, d) data as a function of current density at $T = 170\,\text{K}$ and $V_g = -50\,\text{V}$. a, AFM topography collected in the vicinity of a gold launcher on the left of the field of view. The 2D plot is assembled from AFM line profiles measured at different current densities while scanning along the same line in real space. Red and black arrows and dashed lines indicate positions where the averaged line profiles in b are acquired. b, Averaged line profiles of AFM topography for current densities of $\pm 0.75\,\text{mA/\mu m}^2$, whose topography signals are essentially the same. One of the line profiles is shifted vertically for clarity. c, Near-field data, taken simultaneously with a. A standard one-dimensional Fourier filter was applied here to reduce noise. d, Averaged line profiles of the near-field signal in c for the same current densities as the topography data in b. A Fizeau shift is clearly visible.
**Extended Data Fig. 6 | Uncertainty analysis for fitted SPP wavelength.**

**a.** Distribution of the least-squares estimate of the SPP wavelength in equation (7) generated by Monte Carlo simulation. **b.** Examples of typical simulated SPP line profiles used for analysis in **a.**

**c.** I: Dependence of variance (bright and dark red lines) and bias (blue line) of wavelength estimate on spatial resolution (SR). Bright red line corresponds to zero signal noise ($\sigma_y/A = 0$). Dark red and blue lines correspond to $\sigma_y/A = 10\%$. II: Dependence of bias and variance in wavelength estimate on pixel size in units of wavelength. The pixel size has minimal effect on $\sigma_y$ as long as one samples above the Nyquist rate, as indicated by the vertical green dashed line.

**d.** I: Strong dependence of error in wavelength estimate on signal noise $\sigma_y$. The variance of the wavelength estimate (bright red, dark red and green lines) will increase roughly linearly with $\sigma_y$ until about 25%, and the bias (bright and dark blue lines) is less than 1 nm for SR = 20 nm; II: Dependence of error in wavelength estimate on SPP propagation length 1/$q_z^2$. Both the variance (bright and dark red lines) and the bias (blue line) of the wavelength estimate improve with 1/$q_z^2$, even more so when there is positioning noise $\sigma_x$ (dark red line).

**e.** Assessing the statistical significance of the Fizeau shift using an $F$ test. Solid red and purple lines represent the dependence of $F$ statistics on the sample wavelength-shift standard deviation $s_{\Delta \lambda}$. Purple line assumes $\alpha = 0$ and red line assumes $\alpha$ is finite (see text). Cyan shaded region corresponds to $F$ statistics that reject the null hypothesis of no Fizeau shift ($F > F_{crit} = 2.65$). Vertical dashed line corresponds to wavelength-shift standard deviation $s_{\Delta \lambda}$ estimated from data in Fig. 3a, b.
Extended Data Fig. 7 | Additional datasets revealing plasmonic Fizeau drag in graphene. Other representative data when scanning along the same line at different d.c. currents (averaged ±25 μA μm⁻¹ for each profile) for different gate voltages, temperatures and devices. Within a set of polariton line profiles, the first polariton fringes are aligned to enable better visual inspection of Fizeau shifts. Line profiles are shifted vertically for clarity. Within each panel, the fitted line profiles of the smallest and largest current densities are shown in the lower panel for visual comparison. The images of the devices are near-field scattering amplitude measured at 170 K without gating. a, Device 1, T = 170 K, Vg = -47 V; b, Device 1, T = 170 K, Vg = +47 V; c, Device 1, T = 170 K, Vg = -60 V; d, Device 1, T = 60 K, Vg = +60 V; e, Device 2, T = 170 K, Vg = +50 V; f, Device 3, T = 60 K, Vg = +60 V.