Mesoscopic Simulation for Magnetized Nanofluid Flow Within a Permeable 3D Tank

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ABSTRACT The current analysis is carried out for the Al2O3 nanoparticles transportation through a permeable cubic geometry under the influence of the magnetic force through a hot cubic object. The physical phenomenon described in the basic equations together with the Maxwell’s equations. Then D3Q19 model is sketched for the discretization of the velocity vectors for using the 3D Lattice Boltzmann Method (LBM). The impact of Brownian motion in the Al2O3-H2O nanofluids is considered by taking the Koo–Kleinstreuer model into consideration. The nanoparticles transportation under the impacts of the buoyancy and Lorentz forces, and permeability is studied with LBM. Numerical simulations are performed for different values of magnetic parameter, Darcy’s and Rayleigh numbers. The validation of the applied technique is presented in Fig. 3 in the form of isotherms by comparing the obtained results with Calcagni et al.. For the enhancement of the heat transfer analysis a quantitative comparison of the current study is presented in Fig. 4. All the results for various parameters are presented in the form of isotherms. The obtained results show the efficient conduction for higher values of $Ha$.

The larger values of $Da$ show reduction in the boundary layer thickness. The average Nusselt number $Nu_{ave}$ enhances with higher values of the permeability parameter. The efficiency of the implemented technique is demonstrated in Table 4, where the Nusselt number is tabulated for distinct numbers of $Gr$ and $Ha$, and are compared with the available literature.

INDEX TERMS MHD, free convection, Al2O3-nanoparticles, LBM, porous medium, transportation.

I. INTRODUCTION The breakthrough of nanoparticles in the field of engineering and technology is started from the heat transfer analysis. In the performance of electronic devices, like computer chips; CPU, and other main boards, the heat produced reduce the performance of these devices. On large and small level processes the heat produced affect the performance of the machines [1]–[6]. To overcome this situation, in recent years for the purpose of cooling, the concept of nanoparticles is introduced, that further play a pivotal role in carrying out the heat produced. Nanoparticles are made of metal oxides and other elements whose dimension vary $(1 − 100)nm$ in size [7]–[11]. The dimension and shape of these particles matter a lot in the heat transfer analysis, and its impact is proved both theoretically and experimentally. The use of nanoparticles has now–a-days expanded to the treatment of different diseases, like tumor cells etc. [12]–[16].

Various investigations of magnetohydrodynamics flow for numerical and experimental purposes can be found in physics, and engineering. Particularly, for enhancement of heat transfer, MHD flows are taken into consideration, i.e. in microelectronic devices, and growth of crystals in liquid. Only two type of forces i.e. Lorentz and buoyancy forces occur in an electrically conducting free convective MHD flow inside an enclosure [17]. These two forces in interaction affect the mass and heat transfer, which is one of the big reason behind the study of the transport phenomena.
Keeping in view the applications of nanoparticles, almost two and a half decades ago Choi and Eastman [18] laid the foundation stone of the nanoparticles investigation. This pioneering work was followed by different researchers. Among all, Kang et al. [19] claimed the nanofluid correlation. For the nanoparticles analysis and heat transfer enhancement a Buoyancy-driven flow was performed by Khanafer et al. [20]. The sweetening of gas inside in an isothermal tower under the impact of nanofluid is analyzed by Farahbod [21], both theoretically and experimentally. In this work the theoretical and experimental results obtained are compared. Farahbod and Sara [22] also reported the theoretical and experimental results of the ZnO in drilling of oil. They reported the settling time, PH stability, turbidity of mud and the impact of various coagulants such as: Fe2(SO4)3, FeCl3 and Al2(SO4)3. The results obtained agreed with the experimental findings. The variation of Prandtl number inside a cavity with convective transfer of heat is presented by Moallemi and Jang [23].

The application of magnetic field to the electrically conducting fluid is termed as MHD. This concept for the first time introduced by H. Alfven [24] in Magnetofluids, consist of a solution of salt-H2O and plasma. The potential difference under the earth’s magnetic field over Waterloo bridge in London for salty water is analyzed by M. Faraday in 1832 [25]. The current produced was too small in amount at that time to measure, but the analysis was termed as the Magneto-electric conduction. Analysis are carried out for the better performance of objects on the basis of shape factor and other physical properties. The MHD flow passing through a rectangular obstacle was numerically studied by Rudraiah et al. [26]. In their results they reported an inverse relation between the magnetic number and Nu. Ellahi [27] investigated the MHD non-Newtonian nanomaterial flow inside a tube, and found that the magnetic effect declines the velocity of the fluid. Furthermore, the thermal profile was found smaller compared to the velocity profile for non-constant viscosities. Sher Akbar [28] analyzed the generation of entropy inside a tube for the peristaltic flow and reporting that the rate of generation of entropy near the tube wall remains higher than that of its center. Abbas et al. [29] studied a free convective flow under the impact of thermal dispersion highlighting the impacts of saturated porous medium. The thermal stability of polyvinyl chloride under the influence of ZnO nanoparticles was analyzed by Farahbod et al. [30]. The kinematic performance of ZnO nanofluids inside a hot pipe was deliberated by Farahbod and Farahmand [31] suggesting enhanced heat transfer in an efficient way.

Benzi et al. [32] investigated the LBM with its applications, and particularly in the field of fluid mechanics describing different type of flows. Higuera et al. [33] studied the lattice gas dynamics with collision enhancement. They provided the linear stability of their numerical procedure and implemented the technique to different types of flow problems. All numerical methods in complex fluid flow phenomena are not always implementable. For such type of flows Lattice Boltzmann Method (LBM) work efficiently [34]. LBM is different from other numerical methods on its kinetic base [35]. One important property of this method is the linearity of the convection operator in the velocity filed. This important property makes it dominant on some CFD techniques, like, NS-equation solvers. Besides, the direct computation from the equation of state gives the pressure in LBM. Furthermore, the macroscopic and microscopic distribution related by a transformation in the phase space are greatly simplified, due to the minimal set of velocities in LBM [36]. Kefayati [37] studied the nanofluid flow inside a tank with a variable temperature in the presence of the magnetic effect. He observed a decline in the transfer of heat with the larger values of the Hartmann parameter. Mahmoudi et al. [38] deliberated the square cavity filled with Al2O3-water nanofluid under magnetic effects, and reported that a magnetic parameter is responsible to control the direction of the nanoparticles. A buoyancy induced MHD transport and heat enhancement inside a hot square object was also analyzed by Ozt top et al. [39] by using a numerical technique. They concluded that the higher values of the Hartmann number and the sinusoidal function amplitude reduce the heat transfer.

The MHD flow within a tank with the analysis of the heat migration was analyzed by Kahveci and Oztuna [40]. They proved that Nusselt number is showing an increasing behavior with variations in the inclination angle and reaches its peak value at high Rayleigh number. Hartmann parameter and volume fraction decrease the temperature profile for distinct numbers of Rayleigh parameter. Methri et al. [41] deliberated the laminar convection through a square cavity with entropy generation. They reported that smaller is the generation of entropy larger is the transfer of heat.

The three dimensional MHD nanofluid flow was analyzed by Sheikholeslami [42] using the LBM, and reported the optimal values of different parameters at fixed Hartmann number. He also provided that the generation of entropy declines with greater values of concentration of the particles. In the last few decades, heat transfer phenomena and its effectiveness analyzed by many researchers. One of the great factor behind this investigation is its use in the microelectronics, chemical production, and power station. Recently, for the heat transfer enhancement and its effectiveness a new subclass of fluid is introduced, termed as “nanofluid”. Ho et al. [43] experimentally investigated the water-Al2O3 nanofluid forced convective flow for cooling purposes in a microchannel. Mustafa et al. [44] scrutinized the nanofluid stagnation point flow towards a flexible plate. They concluded that the larger values of the stretching ratio decline the Nusselt number. The lattice Boltzmann equations (LBE) were studied by Succi et al. [45] for variable numerical techniques considering complex geometries in three dimensions. They reported their numerical results at low Reynolds number and proved their estimation by recovering the Darcy’s law. The LB technique for the non-equilibrium complex flows was also implemented by Montessori et al. [46]. They used this method to the three dimensional porous media and parallel plates.
The Al2O3 (alumina) nanofluid filled porous cubic cavity of length $L$ is considered as shown in Fig. 1. The bottom wall is hot located at $Z = \frac{L}{2} = 0$, while the top is cold, and is located at $Z = \frac{L}{2} = 1$. Other walls are assumed adiabatic. The magnetic field is used in such a way that it makes an angle $\theta_x = 90^\circ$ with the $x$–axis and $\theta_z = 90^\circ$ with the $y$–axis as exhibited in Fig. 1. Electromagnetic force $\vec{F}$ and current $\vec{J}$ are described by $\vec{F} = \sigma (\vec{V} \times \vec{B}) \times \vec{B}$ and $\vec{J} = \sigma (\vec{V} \times \vec{B})$ [42], [52].

The velocity and temperature distribution functions denoted respectively by $f$ and $g$ are the two key factors of the LB model. The domain of the fluid is uniformly discretized into Cartesian cells. The cells are chosen in such a way that the distribution functions are of fixed number in each cell. The distribution functions can be directly computed from the LB model, for which the streaming and collision is given as [20], [42]:

$$
\begin{align*}
-\Delta t \tau^{-1}_c \left[ f_i (x, t) - f_i^{eq} (x, t) \right] + f_i (x, t) & = 0 \\
+ \Delta t c_i F_k & = f_i (x + \Delta t c_i, t + \Delta t) \\
\Delta t \tau^{-1}_c & \left[ -g_i (x, t) + g_i^{eq} (x, t) \right] \\
& = -g_i (x, t) + g_i (x + \Delta t c_i, t + \Delta t).
\end{align*}
$$

Here $c_i$, $\tau_c$, $\Delta t$ demonstrate the lattice discrete velocity, time for velocity relaxation and the lattice time step. Thermal diffusivity and kinematic viscosity are defined by $\alpha = c_s^2 \left( \tau_c - \frac{1}{2} \right)$ and $\nu = c_s^2 \left( \tau_c - \frac{1}{2} \right)$ respectively, for $\tau > \frac{1}{2}$ in both the cases.

In the three dimensional flow, the particle is restricted to the 19 possible directions, where one dimension is kept at rest. The velocity components constituted in this way are referred to as microscopic velocities and are denoted by $c_i$, where $i = 0 \ldots 18$.

$$
c_i =
\begin{pmatrix}
0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & -1 & 1 & 1 \\
-1 & -1 & 1 & 0 & -1 & -1 & 1 & 1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\
-1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0
\end{pmatrix}
$$

The equilibrium distributions $g_i^{eq}$ and $f_i^{eq}$ demonstrate the velocity and temperature profiles respectively. The use of BGK model directly leads to the following equilibrium distributions [20]:

$$
\begin{align*}
\frac{f_i^{eq}}{w_i} & = \frac{w_i \rho}{2 c_s^2} \left[ \frac{1}{2} \left( c_i u^2 \right) + \frac{1}{2} \left( c_i u^2 \right) + 1 + \frac{c_i u^2}{c_s^2} \right], \\
\frac{f_i^{eq}}{w_i} & = \frac{w_i \rho}{2 c_s^2} \left[ \frac{1}{2} \left( c_i u^2 \right) + \frac{1}{2} \left( c_i u^2 \right) + 1 + \frac{c_i u^2}{c_s^2} \right], \\
\frac{g_i^{eq}}{w_i} & = \frac{w_i T}{c_s^2} \left[ 1 + \frac{c_i u^2}{c_s^2} \right],
\end{align*}
$$

They compared their results with the analytical and numerical techniques and found their results in good agreement for different regimes. The heat transfer analysis and magnetic field impacts of various nanofluids can be found in the references [47]–[50].

Inspired from above and keeping in view the effectiveness of LBM, here we will analyze and simulate the Al2O3 nanoparticle movement through a porous cavity subject to the magnetic effect. The basic equations coupled with the Maxwell’s equations with the Koo–Kleinstreuer model [51] will be taken into account for simulation purposes, together with the D3Q19 discretization of the velocity vectors.

II. DESCRIPTION OF THE GEOMETRY

The Al2O3 (alumina) nanofluid filled porous cubic cavity of length “$L$” is considered as shown in Fig. 1. The bottom wall is hot located at $Z = \frac{L}{2} = 0$, while the top is cold, and is located at $Z = \frac{L}{2} = 1$. Other walls are assumed adiabatic. The magnetic field is used in such a way that it makes an angle $\theta_x = 90^\circ$ with the $x$–axis and $\theta_z = 90^\circ$ with the $y$–axis as exhibited in Fig. 1. Electromagnetic force $\vec{F}$ and current $\vec{J}$ are described by $\vec{F} = \sigma (\vec{V} \times \vec{B}) \times \vec{B}$ and $\vec{J} = \sigma (\vec{V} \times \vec{B})$ [42], [52].

III. SOLUTION METHODOLOGY

A. MESOSCOPIC TECHNIQUE

In recent years, LBM is widely implemented for the simulation of the complex fluids. This method strongly depends on the kinetic equations, mesoscopic and microscopic models. The key idea behind the mechanism of the LBM is to treat the fluids as source of large number of particles colliding with each other. Hence, the basic equations can be constituted for the collision of this type of molecular interaction with the help of the Boltzmann transport equation.
\[ w_i = \begin{cases} 
\frac{1}{36} i = 7 : 18; \\
\frac{1}{18} i = 1 : 6; \\
\frac{1}{3} i = 0. 
\end{cases} \tag{7} \]

where, \( \rho \) is the density of the fluid lattice, \( w_i \) is the weighting function and \( c_S = \frac{\Delta v}{2} \) is the lattice speed.

Now the basic Navier-Stokes equations for the given LBM model can be derived using the Chapman-Enskog procedure. Interested readers are referred to the reference [53] for more detail.

The Forcing terms given in equation (1) are presented as follow [20], [42]:

\[ F = F_y + F_z + F_x, \tag{8} \]

where,

\[ F_x = 3w_i\rho \left[ -\sin(\theta_i) \left( -\sin^2(\theta_i)v\cos(\theta_i) + u\sin(\theta_i)\sin^2(\theta_i) \right) \\
- \left( \cos^2(\theta_i)u + \frac{\cos(\theta_i)}{2}\sin(2\theta_i)w \right) \right] - \rho w_i\mu BB (3), \]

\[ F_y = 3Aw_i\rho \left[ -0.5w\sin(\theta_i)\sin(\theta_i) + \cos^2(\theta_i) + v \sin^2(\theta_i) + 0.5u\sin^2(\theta_i)\sin(2\theta_i) \right] \\
- 3\rho v BB w_i, \tag{9} \]

and

\[ F_Z = 3\rho w_i \left[ g_i \beta (T - T_m) + A \cos(\theta_i) \left( -w\sin^2(\theta_i)\cos(\theta_i) + \frac{u}{2}\cos(2\theta_i) \right) \\
+ \left( \sin(2\theta_i) \right) \left( -w\sin(\theta_i)\sin^2(\theta_i) \right) + 3A \sin(\theta_i) \right] \]

\[ - 3w_i\rho BB. \]

Here, \( Ha = B_0 L (\mu/\alpha)^{-0.5} \) is the Hartmann parameter, \( Da = \frac{K}{\mu} \) is the Darcy’s parameter, and \( A = Ha^2 \mu L^{-2}, BB = \frac{\mu}{\rho} \) are other constants in terms of Darcy’s and Hartmann numbers. Also \( K \) is the medium permeability, \( B_0 \) is the magnetic field strength and \( \mu \) is the dynamic viscosity.

Here, the radiation factor is ignored due the Boussinesq approximation in the natural convection. Furthermore, the characteristic speed of the velocity is chosen such that it does not exceed the fluid speed of sound. To compute the macroscopic variables, we use the following relations for the momentum, temperature, and flow density [42]:

\[ \rho u = \sum_i c_i f_i, \tag{10} \]

\[ T = \sum_i g_i, \tag{11} \]

\[ \rho = \sum_i f_i. \tag{12} \]

**Table 1. Thermo-Physical properties used for water and nanoparticles [54].**

|       | \( \rho \) | \( C_p \) | \( k \) | \( d_p \) | \( \sigma \) |
|-------|------------|------------|--------|---------|---------|
| Pure water | 997.1      | 4179       | 0.613  | -       | 0.05    |
| \( Al_2O_3 \) | 3970       | 765        | 25     | 47      | 10-12   |

**B. WORKING FLUID**

For the simulation of the nanofluid flow LBM can be implemented after introducing the nanoparticles impact in the flow regime. The effective heat capacitance of the nanofluid \( (\rho C_p)_n \), density \( (\rho f)_n \), electrical conductivity \( (\sigma)_{nf} \), and thermal expansion \( (\rho\beta)_n \) are presented as under [38], [42], [52]:

\[ \frac{(\rho C_p)_n}{(\rho C_p)_f} = 1 + \phi \left[ \frac{(\rho C_p)_S}{(\rho C_p)_f} - 1 \right], \tag{13} \]

\[ \frac{(\rho f)_n}{(\rho f)_f} - 1 = \phi \left[ \frac{\rho f}{\rho f} - 1 \right], \tag{14} \]

\[ \frac{(\rho\beta)_n}{(\rho\beta)_f} = 1 + \phi \left[ \frac{(\rho\beta)_S}{(\rho\beta)_f} - 1 \right], \tag{15} \]

\[ \frac{1}{3} \left( \frac{\sigma_{nf}}{\sigma_f} - 1 \right) = (\Delta - 1) [(\Delta + 2) - \phi (\Delta - 1)]^{-1} \tag{16} \]

where, \( \Delta = \frac{\alpha_S}{\alpha_f} \), and \( \phi \) denotes the volume fraction. These properties are explained in Table 1 for both nanoparticles and water.

The Brownian motion impact is explained by Koo and Kleinstreuer [55], [56] as given below:

\[ k_{\text{effect}} = k_{\text{stat}} + k_{\text{Brown}}, \tag{17} \]

where

\[ \frac{1}{3} \left( \frac{k_{\text{stat}}}{k_f} - 1 \right) = (\phi) \left( \frac{k_p}{k_f} - 1 \right) \left[ \left( \frac{k_p}{k_f} + 2 \right) - \phi \left( \frac{k_p}{k_f} - 1 \right) \right]^{-1}, \tag{18} \]

and

\[ k_{\text{Brown}} = 5 \times 10^4 \left( \frac{\rho f c_p f_{\phi} (\phi, T) \phi}{d_p \rho f} \right). \tag{19} \]

The KK model [51], is extended by Li [54], by introducing a new empirical function \( g' \) composed of the nanomaterial volume fraction \( \phi \), temperature \( T \) and diameter \( d_p \) and is defined as:

\[ g' (\phi, d_p, T) = \text{Ln} (T) \left[ b_1 + b_3 \text{Ln} (\phi) + \text{Ln} (d_p)^2 b_5 + \text{Ln} (d_p) b_7 + b_6 + b_{10} \text{Ln} (d_p)^2 \right] \]

\[ + \left[ b_3 \text{Ln} (\phi) + \text{Ln} (d_p) b_7 + b_6 + b_{10} \text{Ln} (d_p)^2 \right]. \tag{20} \]
Here the coefficient $b_i$ for $i = 1 \rightarrow 10$, depend on the nanoparticles type, and are given in Table 2. The interfacial thermal resistance $R_f (R_f = 4 \times 10^{-8} km^2/W)$ is introduced, and a new term $k_{\text{effect.p}}$ is used in equation (14) instead of $k_p$, which is given by

$$k_{\text{effect.p}} = d_p \left( R_f + \frac{d_p}{k_p} \right)^{-1}. \hspace{1cm} (21)$$

Finally, the modified Koo–Kleinstreuer model known as KKL model [51], and the dynamic viscosity take the form:

$$k_{\text{Brown}} = \left( \rho_f c_p f \prime \left( \varphi, d_d, T \right) \sqrt{\frac{k_f T}{d_p}} \right) 5 \times 10^4 \hspace{1cm} (22)$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^2} + \frac{\mu_f}{\text{Pr} k_f k_{\text{Brown}}}. \hspace{1cm} (23)$$

For the comparison of the heat transfer enhancement, Nusselt numbers can be given as [52]:

$$Nu_{\text{loc}} = - \frac{k_{nf}}{k_f} \frac{\partial T}{\partial Z} \bigg|_{aZ=0} \hspace{1cm} Nu_{\text{ave}} = \int_0^1 \int_0^1 Nu \, dY \, dX \hspace{1cm} (24)$$

The fluid kinetic energy ($E_c$) is given by:

$$E_c = \frac{1}{2} \left[ (u)^2 + (v)^2 + (w)^2 \right]. \hspace{1cm} (25)$$

IV. VALIDATION AND MESH INDEPENDENCY

The size of the grid is chosen by performing various meshes for test. The grid independence is observed at $(81 \times 81 \times 81)$ as shown in Table 3. At this point the change in $Nu_{\text{ave}}$ is observed insignificant at the bottom wall. The convergence of our approach/technique is achieved with the following
Influence of MnF on migration of nanomaterial when $Ra = 10^3$.

$$\max_{\text{grid}} |\gamma^{n+1} - \gamma| \leq 10^{-7}. \quad (26)$$

where $\gamma$ represents the variables $T$, $U$, $V$, and $W$.

It was observed here that the change in the size of the grid does not change the output result. The variation in grid was performed and the result is displayed in Table 3, for average Nusselt number. The implemented code accuracy and correctness are displayed in Figures 3-4, and in Table 4.

For validation, the current code is matched with the published work. The results of Khanafe et al. [20] are compared with the current results for distinct numbers of the radiation parameters, as presented in Fig. 3.
A quantitative comparison of this study is carried out in Fig. 2, where a comparison of the obtained results is made with those of the Khanafer et al. [20].

The impact of the magnetic field over the porous enclosure is presented in Table 4. A comparative study is also carried out with numerical analysis of the Rudraiah et al. [26]. The compassion is carried at different values $Gr$ and $Nu$.

V. RESULTS AND DISCUSSION
A convective MHD alumina flow with a cubic obstacle inside a permeable cubic cavity is analyzed. LBM is used for the numerical study of the basic equations. These computations are performed for different numbers of $(Ha = n(10);$ for $n = 0, 2, 4, 6$), Rayleigh number $(Ra = 10^n$ for $n = 3, 4, 5$), Darcy number $(Da = 0.001$ to 100) and the fraction
TABLE 2. Values of Coefficients of nanofluid defined in equations (20) [51].

| Coefficient values | $Al_{2}O_{3}$ – Water |
|--------------------|-----------------------|
| $a_{10}$           | -0.999063481          |
| $b_{2}$            | -0.2354329626         |
| $b_{3}$            | -3.9225289283         |
| $b_{4}$            | -34.532716906         |
| $b_{5}$            | -298.19819084         |
| $b_{6}$            | 0.176919300241        |
| $b_{7}$            | 4.17455552786E-02      |
| $b_{8}$            | 0.6955745084          |
| $b_{9}$            | 6.115637295           |
| $b_{10}$           | 52.813488759          |

TABLE 3. Various grid tests along the hot wall for $(Nu_{ave})$ at $Da = 100$, $Ra = 10^5$, $Ha = 60$ and.

| Size of the mesh | 51x51 | 61x61 | 71x71 | 81x81 | 91x91 |
|------------------|-------|-------|-------|-------|-------|
| $Nu_{ave}$       | 2.231171 | 2.244939 | 2.254852 | 2.256422 | 2.255132 |

TABLE 4. Variation in $(Nu_{ave})$ for $Pr = 0.733$ at distinct numbers $Ha$ and $Gr$.

| $Ha$ | Rudraiah et al. [26] | Present |
|------|----------------------|---------|
| 10   | 2.266726              | 2.261544 |
| 50   | 1.099454              | 1.083047 |

of nanofluid volume ($\phi = 0.0$ and $0.04$). The impact of Rayleigh and Hartmann parameters is shown by streamlines, isotherms and isokinetic energy in Figs. 3-6. Symmetry of the boundary conditions is reflected in all the contours. The only forces affecting the nanofluid in the enclosure are the Lorentz and buoyancy forces. It is with noticing that the buoyancy favors the natural convection here, whereas the Lorentz forces works against it. For $Ha^{2}/Ra \approx 1$ one of the two forces are significant. The effectiveness of the buoyancy forces and Lorentz forces are significant under the conditions $Ha^{2}/Ra \ll 1$ and $Ha^{2}/Ra \gg 1$, respectively.

The decreasing values of $Ra$ favors the heat transfer enhancement, as is clear from the shape of the isotherms. Stream function takes its maximum value at under the variations of the intensity of the cavity. The distortion of the isotherms is caused due to the stronger natural convection. It means that $Ha$ is the increasing function of the stream function; which implies the faster flow, as shown in the figures. The Higher values of the $Ra$ lead to increases in the convection rate of the current due to the dominant buoyancy forces over the viscous forces. The circulation is influenced pushing the fluid (cold) downwards and convection becomes dominant in the heat transfer mode. The mode of transfer is pure conductive, for all values of $Ra$ with the augmenting amounts of $Ha$; due to the buoyancy and Lorentz forces interaction that decreases the fluid flow, and become dominant over the convection rate. The convection augments due to the increasing values of the Rayleigh number; it means that a larger magnetic field is required to suppress the flow convection rate. But, higher strength of the magnetic field disturbs the layers thus reducing the convection rates in mechanism.
of the heat transfer. The Hartmann number $Ha$ decreases the maximum strength of the stream function. For smaller values of the Rayleigh number the kinetic energy representing a wall elongated in the $x$-direction, due to the strength of the $Ha$. When $Ra$ goes up to $10^5$, new smaller cells are produced and the original cell is elongated in the $y$-axis. Besides this, it can also be seen that the kinetic energy enhances with the larger values of $Ha$.

The impact of $Nu$, is graphed for different values of the $Ha$ and Darcy’s number. Concentric circles are obtained considering lower values of $Ha$ and higher values of $Da$.which become flattened on both surfaces as $Ha$ assumes greater values with $Da$ increasing up to 100. Similarly the velocity profiles both at the $x$-axis and $z$-axis are described. The $x$-axis velocity is almost similar at the extreme values of the Hartmann number, while the $z$-axis velocity profiles are elongated in the vertical axis with higher values of $Ha$. Furthermore, the thickness of the boundary layer reduces with the larger values of $Ha$. On the other hand larger values of $Ra$ enhance the Nusselt number. Thus there is a direct relation between Nusselt number and $Ra$. Impact of $φ$ on behavior of nanofluid flow at the range $(φ = 0.0 \text{ and } 0.04)$ are shown in Fig. 6. It is obvious that thermal conductivity always increase with the addition of Nano powder and base fluid. It is observed that the nanofluid flow behavior change with change in volume fraction.

Fig. 7 shows the impact of $Ra$ and $Da$ over the $Nu_{ave}$. Physically, at $z = 0$ the fluid flows in the $xy$-plane, which is assumed a hot surface. The direct relation between the fluid temperature and $Ra$ further points in the direction of enhanced Nusselt numbers $Nu_{ave}$. The parameter $Da$ however assumes higher values for lower values of $L$ and is climaxed at $z=0$. It is thus fairly concluded that the higher values of these numbers at the bottom surface due to the temperature further augments $Nu_{ave}$. When the magnetic field strength is increased, the isotherms declines that further declines the $Nu_{ave}$. Hence increasing $Ha$ declines that further the profile of $Nu_{ave}$. Hence increasing $Ha$ leads to a decline in the profile of Nusselt number, $Nu_{ave}$.

\[
Nu_{ave} = 2.17 + 0.43 \log (Ra) - 0.16 (Ha) (Da) -0.27Ha + 0.23Da \log (Ra) +0.37 \log (Ra))^2 + 0.18Da -0.38Ha \log (Ra)
\]  

(27)

VI. CONCLUSION

The Lorentz force impact over a porous cubic enclosure is analyzed with the help of LBM. In simulation the characteristics of the nanomaterial and the impact of the Brownian motion is considered. The properties of the nanofluids are simulated with the help of the KKL model. Impacts of $Ra$ nanoparticles; volume fraction, and $Ha$ on the hydrothermal profile are analyzed and displayed with graphs and tables. The importance of the magnetic field inside the cavity to control the convection is proved with the variations in the Nusselt number, Hartmann number and Rayleigh number. Results have shown that $Nu$ vary directly with the variation in $Ra$, and inversely related to the Hartmann number. And hence we obtained that the convection inside the square can be controlled by the MHD implemented. Furthermore, the heat transfer enhances due to the increasing values of the Hartmann number.
COMPEalin INTERESTS

The authors declare no competing interests.

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