We consider hard exclusive production of exotic hadrons to study their internal structure. Revisiting the constituent-counting rule for the large-angle exclusive scattering, we discuss general features expected for the production cross section of exotic hadrons whose leading Fock states are given by multi-quark states other than the ordinary baryon ($qqq$) or meson ($q\bar{q}$) states. We take the production of $\Lambda(1405)$ as an example and propose to study its partonic configuration from the asymptotic scaling of the cross section, which is measurable at J-PARC. We also discuss the production of a pair of the light-hadrons such as $f_0(980)$ and $a_0(980)$ in $\gamma^*\gamma$ collisions in the framework of QCD factorization, in which the cross section is expressed as a convolution of the perturbative coefficients and the generalized distribution amplitudes (GDAs). We demonstrate how the internal structure of $f_0(980)$ or $a_0(980)$ can be explored by measuring the GDAs at $e^+e^-$ experiments such as the B-factories.

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1. Introduction

Despite its non-relativistic nature, the conventional quark model has been remarkably successful in classifying the hundreds of the observed hadrons into baryons and mesons. On the other hand, the underlying theory of the strong interaction, Quantum Chromodynamics or QCD, does not prohibit hadrons with other quark-gluon configurations such as tetraquarks, pentaquarks, glueballs, etc, to which we here refer as exotic hadrons. Actually, the discovery of “XYZ” states at the B-factories and other facilities in the last decade [1] has demonstrated the existence of exotic hadrons in the heavy-quark sector and has motivated extensive theoretical studies to understand their properties [2]. In the light-quark sector, there have been long-standing candidates for exotic hadrons. For example, \( \Lambda(1405) \) is hard to understand as an excited state of the ordinary \( uds \) combination since it is much lighter than the corresponding excited nucleon \( N(1535) \) and the possibility of being a \( \bar{K}N \) bound state has been discussed [3, 4, 5]. Likewise, the light scalar mesons obey an anomalous mass relation \( M(f_0) \approx M(a_0) > M(\kappa) \) instead of \( M(f_0) > M(\kappa) > M(a_0) \) which is expected from the conventional quark model, and could be understood as tetraquarks [6].

So far, the structure of exotic hadrons have been studied mainly in terms of hadronic observables such as mass, spin and decay width. Here, we consider the possibility of studying the internal structure by hard processes, in which the quarks and gluons are the relevant degrees of freedom. Since the exotic hadron candidates are unstable particles, we cannot use them as a target and need to find out what can imply the "exoticness" in the production processes. The inclusive production of exotic hadrons was studied in [7], in which it was demonstrated that the signature of a multi-quark configuration of \( f_0(980) \) can appear as the difference between the "favored" and "disfavored" fragmentation function. For the exclusive hard processes, some studies have already been performed for electro- and hadroproduction of \( \Theta^+ \) pentaquark [8], and the production of a hybrid meson in \( \gamma\gamma \) collisions [9]. In this work, we explore other possibilities for using exclusive processes as a means of studying the partonic structure of exotic hadrons. Firstly, we discuss the production of \( \Lambda(1405) \) in the large-angle exclusive scattering, \( \pi + p \rightarrow K + \Lambda(1405) \), in the light of the constituent-counting rule [10, 11]. Secondly, we discuss the exclusive pair production, \( \gamma^* + \gamma \rightarrow h + \bar{h} (h = f_0(980) \text{ or } a_0(980)) \), in the kinematical region where the process is described in terms of the GDAs [12, 13]. The details are found in [14, 15].

2. Exclusive production of \( \Lambda(1405) \)

In the framework of QCD factorization [16], the scattering amplitude of the \( 2 \rightarrow 2 \) hadronic process with a large momentum transfer is expressed schematically as

\[
M(a + b \rightarrow c + d)|_{s, t, u \gg \Lambda_{QCD}^2} \approx \phi_a \times \phi_b \times H(\alpha_s) \times \phi_c \times \phi_d,
\]

where \( s, t, u \) are the Mandelstam variables, \( \phi_i \) is a light-cone distribution amplitudes (LCDA) of a hadron \( i \) and \( H(\alpha_s) \) is the hard part of the scattering amplitude. Although the above expression holds as a general formula, it is difficult to predict the absolute value of the cross section in most cases because, (i) the LCDA's are not known except for the pion; (ii) a large number of Feynman diagrams contribute to the hard part. Nevertheless, the asymptotic scaling of the cross section can
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Figure 1: Cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ (left) and its scaling at high energies (right).

be predicted by the constituent-counting rule [10, 11]:

$$\left. \frac{d\sigma_{ab\to cd}}{dt} \right|_{s,|t|,|u| \gg \Lambda_{QCD}^2} = \frac{1}{s^{n-2}} f_{ab\to cd}(t/s),$$

where $f(t/s)$ is a dimensionless function of the scattering angle, and the number $n$ is defined by $n = n_a + n_b + n_c + n_d$ with the $n_i$ being the number of constituents in the leading Fock state of the hadron $i$. The constituent-counting rule can be understood from the dimensional counting of the hard part $H(\alpha_s)$ in eq.(2.1) [14] and has been confirmed in the experiments at BNL and JLab [17]. In addition, we found that the counting-rule holds for the production of the ground-state Lambda in $\pi^- + p \to K^0 + \Lambda$ with the scaling factor $n = 10.1 \pm 0.6$ [14]. In all of the above cases, the transition from the resonance region to the scaling region occur at $\sqrt{s} = 2 - 3\text{GeV}$.

Figure 1 shows the cross section of the exclusive $\Lambda(1405)$ production, $\pi^+ + p \to K^0 + \Lambda(1405)$. For this process, there is only one experimental data [18] which is shown in the left panel of Figure 1. The data is roughly consistent with the theoretical estimate based on the chiral unitary model [19]. The solid line in Figure 1 is obtained by extrapolating the experimental data by assuming the leading Fock component of $\Lambda(1405)$ to be a five-quark state, so that the total number of constituents is given by $n = 2 + 3 + 2 + 5 = 12$ and the scaling rule that $s^{10} d\sigma/dt = \text{constant}$. The comparison of the cross sections anticipated when the $\Lambda(1405)$ is a three-quark state and a five-quark state is shown in the right panel. The result indicates that those two cases can be clearly distinguished if we have enough data from experiments, and such experiments are possible using the high-momentum beam at J-PARC [20].

3. Pair production of $f_0(980)$ and $a_0(980)$ in the GDA kinematics

The cross section of the hadron pair production $\gamma^* + \gamma \to h + \bar{h}$ can be described as a production of a parton pair at the short distance followed by the non-perturbative transition of the parton pair into the hadron pair when the virtuality of the virtual photon $Q^2$ is large enough and the invariant mass of the hadron pair $W^2$ is not too large [12, 13]. The latter part can be parameterized in terms of the GDAs, which is defined in analogy of of the LCDAs for a single hadron production. We now
consider only at the leading order of $\alpha_s$, so that there appears only the quark GDAs:

$$
\Phi^h_q(z, \zeta, W^2) = \left. \frac{dx}{2\pi} e^{iP \cdot x} \langle h(p)\bar{h}(p')\bar{q}(x)q(0)\rangle \right|_{x^+ = \zeta_+ = 0},
$$

where $P = (p + p')/2$, $W^2 = (p + p')^2$ and $\zeta = P^+ / P^+$ is the momentum fraction of a hadron $h$ in the final $hh$ pair. Also, the variable $z$ has a meaning of a momentum fraction carried by a quark in the intermediate quark pair. Although this process is obtained by the $s$-$t$ crossing of the deeply virtual Compton scattering (DVCS), it is not straightforward to relate the GDAs with the corresponding non-perturbative functions, generalized parton distributions (GPDs), which appear in DVCS \[21\]. This is because the GDAs in the physical region corresponds to the GPD in the unphysical region \[15\]. In this work, we take a simple model for the GDAs (\(h = f_0(980)\) or \(a_0(980)\)) as

$$
\Phi^h_q(z, \zeta, W^2) = N_{h(q)} z^\alpha (1 - z)^\beta (2z - 1) \zeta (1 - \zeta) F_{h(q)}(W^2),
$$

which is consistent with the symmetry relations: $\Phi^h_q(1 - z, 1 - \zeta, W^2) = \Phi^h_q(1 - z, \zeta, W^2) = -\Phi^h_q(z, \zeta, W^2)$ for the charge-conjugation even part of the GDAs \[13\]. Here $F_{h(q)}(W^2)$ is the quark form factor of the energy-momentum tensor which is related to the GDAs by a momentum sum rule \[22\]. Now we fix the overall factor $N_{h(q)}$ such that the sum of the second moments in $z$ amounts to 0.5 as the usual nucleon PDFs and parameterize the form factor as \[23\]

$$
F_{h(q)}(W^2) = \frac{1}{\left[1 + (W^2 - 4m_h^2)/\Lambda^2\right]^{n-1}},
$$

where $\Lambda$ is the cutoff parameter and $n$ is the number of the constituents of the hadron $h$. Note that the complex phase from the final state interaction, etc., needs not be included because the bremsstrahlung process can be neglected in this case, and only a single amplitude contributes to the cross section \[3\].

The left panel of Figure 2 shows the differential cross sections, $d\sigma / dQ^2 dW^2$, for the process \(e + \gamma \rightarrow e + h + \bar{h}\) \[15\] as a function of $Q^2$ at a fixed value of $W = 2.0$ GeV. Each line is calculated...
using the GDAs of eq.\([\ref{eq:3.2}]\) with \((\alpha, \beta) = (1, 1), (2, 2), (3, 3)\) and \(\Lambda = 1.0\, \text{GeV}\) in eq.\([\ref{eq:3.3}]\). The result is quite sensitive to the small-\(z\) and large-\(z\) behavior of the GDAs. In the right panel of Figure \([\ref{fig:2}]\) we also show the same cross sections as a function of \(W^2\) at a fixed \(Q^2 = 10\, \text{GeV}^2\) with various values of the cutoff \(\Lambda\) and \(n\). Similarly to the results in the previous section, the cross sections assuming the ordinary \(q\bar{q}\) state and those with the tetra-quark \(qq\bar{q}\bar{q}\) state show quite a different behavior as a function of \(W^2\), so that one can say that these observables could be used to explore the internal structure of the scalar mesons.

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