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CFBP Network-A Technique for Crack Detection

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Abstract

This paper describes the Cascade Forward Back Propagation (CFBP) network for crack detection in Euler Bernoulli beam like structure through the knowledge of changes in the natural frequencies and their measurements. A comparative study has been made between theoretical, experimental as well as the CFBP network analysis. In the theoretical analysis Finite element method has been used for the calculation of natural frequencies with the help of various shape functions as well as the stiffness matrix. The results of natural frequencies, using theoretical analysis, that has been trained to the CFBP network for further analysis. In this network there are three inputs and two outputs. The first three relative natural frequencies are the inputs and the relative crack depth and relative crack location are the outputs. After obtaining the results from the network analysis, it has been validated through experimental analysis. It is found that the results are in close agreement with each other.

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1. Introduction

Damage to structures may occur as a result of normal operations, accidents, deterioration or severe natural events such as earthquakes and storms. It is an important aspect to determine whether structural damage has occurred and, if so, to determine the location and extent of the damage. The information produced by a damage assessment process can play a vital role in the development of economical repair and retrofit program. The most common method of damage assessment is by visual inspection. But, this method when used for large and complex structures poses problems by means of accessibility and being imprecise. During the past few decades, a significant amount of research has been done in the area of damage assessment through the change in the dynamic response of a structure. Lee (2000) has developed a method to find the lowest four natural frequencies of the cracked structure by FEM. and the approximate crack location is obtained by using Armon's Rank-ordering method that uses the above four natural frequencies. The experimental investigation of the effects of cracks and damages on the integrity of structures is reported by Owolabi (2003). An analytical, as well as experimental approach has been developed by Nahvi (2005) to detect the crack in cantilever beams by vibration analysis. A new concept of nonlinear output frequency response functions (NOFRFs) has been introduced to detect cracks in beams using frequency domain information by Peng (2007). Saridakis (2008) has been introduced a model for the coupling effect of bending vibrations on the cracked shaft and then used to identify the rotational angle of the crack. A computational method on damage detection problems in structures was developed using neural networks by Haryanto and Agus (2009). Suresh et al. (2004) have presented a method by considering the flexural vibration in a cantilever beam with a transverse crack. The researchers are computed modal frequency parameter

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analytically for various crack locations and depths. After obtaining these values the parameters so obtained were used to train the neural network to identify the damage location and size. A linearized version of the model is considered by Little and Shaw (1978) which shows the capacity of the memory is related the number of synapses and not the number of neurons. A method was proposed by Loutridis et al. (2005) in which Forced vibration behavior and crack detection of cracked beams using instantaneous frequency is used to detect the crack in beam depending on instantaneous frequency (IF). The study monitored the dynamic behavior for the beam with breathing crack. They carried out the research under harmonic excitation with experimental and theoretical results. The observed data of the simulation an experimental test were analyzed using MATLAB. A relation between the depth of crack and the main difference of instantaneous frequency was established. The instantaneous frequency was found to be a good indicator for the size of crack. Various methodologies have been reviewed, which are used for crack detection and fault diagnosis by Thatoi et al. (2012) and Doebling et al. (1996). The analysis included changes in modal frequency, changes in mode shapes, and changes in flexibility coefficients.

2. Finite Element Vibration Analysis

Finite Element Analysis (FEA) was first developed in 1943 by R. Courant, who utilized the method of numerical analysis and minimization of vibrational calculus to obtain approximate solutions to vibration systems. FEA uses a complex system of points called nodes which make a grid called mesh. This mesh is programmed to contain the material and structural properties which define how the structure will react to certain loading conditions. Nodes are assigned at a certain density throughout the material depending on the anticipated stress levels of a particular area.

![](https://example.com/beam.png)

**Fig. 1.** Geometry of the cracked cantilever beam.

2.1. Finite Element Formulation Theory

The beam with a transverse edge crack is clamped at left end, free at right end and has uniform structure with a constant rectangular cross-section of 800 X 50 X 6 mm as shown in Fig.1. The Euler-Bernoulli beam model is assumed for the finite element formulation. The crack in this Finite particular case is assumed to be an open crack and the damping is not being considered in this theory. A cantilever beam with a transverse surface crack of depth \( a_1 \), location of crack at a distance \( L_c \) from the fixed end, beam of width \( B \), length \( L \) and height \( W \) is considered for the current research. The beam is subjected to axial force \( P_1 \) and bending moment \( P_2 \) as shown in Fig.1 which gives coupling with the longitudinal and transverse motion.

2.2. Governing Equation of Free Vibration

The free bending vibration of an Euler-Bernoulli beam of a constant rectangular cross section is given by the following differential equation as given in:

\[
EI \frac{d^4 y}{dx^4} - m \omega_i^2 y = 0
\]

Where ‘\( m \)’ is the mass of the beam per unit length (kg/m), ‘\( \omega_i \)’ is the natural frequency of the \( i \)th mode (rad/sec), \( E \) is the modulus of elasticity (N/m²) and \( I \) is the moment of inertia (m⁴). By defining \( \lambda^4 = \frac{m \omega_i^2}{EI} \) equation is rearranged as a fourth-order differential equation as follows:
\[
\frac{d^4 y}{dx^4} - \lambda^4 y = 0
\]

The general solution to equation is
\[
y = A \cos \lambda_1 x + B \sin \lambda_1 x + C \cosh \lambda_1 x + D \sinh \lambda_1 x
\]

(3)

Where A, B, C, D are constants and 'λ i' is a frequency parameter. Adopting Hermitian shape functions, the stiffness matrix of the two-noded beam element without a crack is obtained using the standard integration based on the variation in flexural rigidity.

The element stiffness matrix of the un-cracked beam is given as:
\[
[K^e] = \int [B(x)]^T E I [B(x)] dx
\]

(4)

\[
[B(x)] = [H_1(x), H_2(x), H_3(x), H_4(x)]
\]

(5)

Where [H_1(x), H_2(x), H_3(x), H_4(x)] is the Hermitian shape functions defined as,

\[
H_1(x) = 1 - \frac{3x^2}{1^2} + \frac{2x^3}{1^3}
\]

(6a)

\[
H_2(x) = x - \frac{2x^2}{1} + \frac{x^3}{1^2}
\]

(6b)

\[
H_3(x) = \frac{3x^2}{1^2} - \frac{2x^3}{1^3}
\]

(6c)

\[
H_4(x) = -\frac{x^2}{1} + \frac{x^3}{1^2}
\]

(6d)

Assuming the beam rigidity EI is constant and is given by EI_0 within the element, and then the element stiffness is (6)

\[
[K^e] = \begin{bmatrix}
12 & 6I_l & -12 & 6I_l \\
6I_l & 4I_l^2 & -6I_l & 2I_l^2 \\
-12 & -6I_l & 12 & -6I_l \\
6I_l & 2I_l^2 & -6I_l & 4I_l^2
\end{bmatrix}
\]

(7)

\[
[K^e_c] = [K^e] - [K_c]
\]

(8)

Here,

\[K^e\] = Stiffness matrix of the cracked element

\[K^e\] = Element stiffness matrix

\[K_c\] = Reduction in stiffness matrix due to the crack

The matrix [K_c] is

\[
[K_c] = \begin{bmatrix}
K_{11} & K_{12} & -K_{11} & K_{14} \\
K_{12} & K_{22} & -K_{12} & K_{24} \\
-K_{11} & -K_{12} & K_{11} & -K_{14} \\
K_{14} & K_{24} & -K_{14} & K_{44}
\end{bmatrix}
\]

(9)

Where,

\[
K_{11} = \frac{12E(I_0 - I_c)}{L^4} \left[ \frac{2L^3}{L^2} + 3I_c \left( \frac{2L}{L^2} - 1 \right)^2 \right]
\]

(10a)
Here, 
\[ I_c = 1.5W \]

\[ I_0 = \frac{BW^3}{12} = \text{Moment of inertia of the beam cross section, } I_c = \frac{B(W-a)^3}{12} = \text{Moment of inertia of the beam with crack.} \]

It is supposed that the crack does not affect the mass distribution of the beam. Therefore, the consistent mass matrix of the beam element can be formulated directly as

\[
 \begin{bmatrix}
 M^c \\
 \end{bmatrix} = \frac{1}{L} \int_0^L \rho A [H(x)]^T [H(x)] \, dx
\]

\[
 \begin{bmatrix}
 M^c \\
 \end{bmatrix} = \frac{\rho A}{420} \begin{bmatrix}
 156 & 221 & 54 & -131 \\
 221 & 41^2 & 131 & -31^2 \\
 54 & 131 & 156 & -221 \\
 -131 & -31^2 & -221 & 41^2 \\
 \end{bmatrix}
\]

The natural frequency then can be calculated from the relation.

\[
 [-\omega^2 [M] + [K] [q]] = 0
\]

Where,
\[ q = \text{displacement vector of the beam} \]

3. The Inverse Tool: Artificial Neural Network

In order to determine the crack parameters from the frequency data we take the help of artificial intelligence in the form of neural network. The structure of a neural net is very similar to the exact biological structure of a human brain cell. In order to be precise, neural network can be stated as a network model whose functionality is similar to that of the brain. In other words a neural network is at first trained to recognize a predefined pattern or an already known relationship from certain pre-found values. It works by taking certain number of inputs and computing the output after carefully adjusting the weights which are attached with the input values to differentiate these input values on the basis of importance and priority in processing. These weight values are utilized to obtain the final output. For example, if we have two inputs \( x_1 \) and \( x_2 \) then a simple neural net can be designed and the net input can be found out as, 
\[
 y_{in} = + x_1 w_1 + x_2 w_2,
\]
where, \( x_1 \) and \( x_2 \) are the activations of the input neurons \( X_1 \) and \( X_2 \) i.e., the output of the input signals. The output \( y \) of the output neuron \( Y \) can be obtained by applying activations over the net input, i.e., the function of the net input, \( y = f(y_{in}) \). Output = Function (net input calculated). The function to be applied over the net input is called an activation function.
A neural network is classified on the basis of the model’s synaptic interconnections, the learning rule adapted and the activation functions used in the neural net. Based on the synaptic interconnections we choose a multi-layer perceptron model for our research purpose. Now, depending on the process of learning a neural network is classified as Supervised Learning network, Unsupervised Learning network and Reinforced Learning network. Supervised learning process requires a set of already known values to train the network and hence find out the output. Since we have a set of values obtained after monitoring the vibrational characteristics of the cracked beam and subjecting it to finite element modeling we utilize the values to train our network. The activation function that we have chosen is the tansigmoid hyperbolic activation function. Finally, we arrived at the conclusion of using the Cascade Forward Back-Propagation Multilayer Perceptron model in our research purpose and analyze the results thus obtained.

3.1. The CFBP Network

As stated earlier we are using a CFBP network for our analysis. The structure of CFBP network is shown in Fig. 2. In this network the input values are calculated after every hidden layer is back-propagated to the input layer and the weights adjusted subsequently. The input values are directly connected to the final output and a comparison occurs between the values obtained from the hidden layers and the values obtained from the input layers.

Various authors have presented about the effectiveness of CFBP network [Sahoo et al. (2008), Gopikrishnan and Santhanam (2011), Badde et al. (2013)]

The Algorithm followed in the following paper is given as:
1. Initialize the predefined Input Matrix
2. Initialize the desired output or target matrix
3. Initialize the network by using the net = newcf (Input, Output, Hidden layers, Transfer Function, Training algorithm, Learning Function, Performance Function)
4. Define the various training parameters such as number of epochs, number of validation checks, maximum and minimum gradient, etc.
5. Test the new found weights and biases for accuracy.
6. Using the weights and biases determine the unknown results.

The initial weight and bias values are taken as 0.

A Cascade back propagation neural network controller has been developed for detection of the relative crack location and relative crack depth (Fig. 2). The neural network has got three input parameters and two output parameters.

The inputs to the neural network controller are as follows:
- fn1=Relative First Natural frequency, fn2=Relative Second Natural frequency, fn3=Relative Third Natural frequency

The outputs to the neural network controller are as follows:
- rcd= Relative Crack depth, rcl= Relative Crack Location

![image](image-url)  
Fig. 2. Structure of CFBP Network
4. Experimental Analysis

The experiment has been conducted in two ways. The pictorial view of experimental setup 1 and setup 2 are shown in the Fig. 3. In the first figure a cracked cantilever beam is rigidly clamped to the concrete foundation base. The free end of the Cantilever beam is excited with a vibration exciter. The vibration exciter is excited by the signal from the function generator. The signal is amplified by a power amplifier before being fed to the vibration exciter. The natural frequency is measured from the function generator at the point of resonance under the excitation. In the second figure the same cantilever beam is taken into consideration. The free end of the cantilever beam is excited freely with the help of thumb and allowed to vibrate freely. The amplitude of vibration of un cracked and cracked cantilever beam is taken by the vibration pick up and is fed to the digital storage oscilloscope. The vibration signatures are analyzed graphically in the oscilloscope and the natural frequency of the beam is calculated.

5. Results and Discussions

Early detection of damage in beam type structural elements is very essential to avoid a major failure or accident. For non-destructive testing of cracked cantilever beam, vibration based methods make a good approach. Vibration based methods use the fact that due to the presence of the crack, there is a change in the flexibility which affects the natural of the structural element. The natural frequencies of the cracked cantilever beam at different locations with different depths are derived using Finite element analysis. Then these frequencies are trained in the Cascade Forward Back Propagation Neural Network. For this a single hidden layer was used with 13 neurons. The TANSIG Hyperbolic tangent sigmoid transfer function or activation function was used to calculate the final output of the neural net. The TANSIG transfer function is given as:

\[
t\text{ansig (n)} = \frac{2}{1 + \exp(-2n)} - 1
\]

The Levenberg-Marquardt (trainlm) training process was followed to train the neural network. Training stops when any of these conditions occurs:
- The maximum number of EPOCHS (repetitions) is reached.
- The maximum amount of TIME has been exceeded.
- Performance has been minimized to the GOAL.

The Division of training data was done using the Random (Dividerand) method. The number of iterations provided was 200. The gradient was set at a maximum value of 1 and minimum value of 0. The performance or goal or maximum number of error checks was set to be 150. There was no constraint on the amount of time for which the training program ran. From the huge results we selected some of the values to validate the result.
Table 1. Input data and the Comparison of Results.

| First three relative natural frequencies | Theoretical | Experimental | CFBP  |
|----------------------------------------|-------------|--------------|------|
| fn1 | fn2 | fn3 | rcd | rcl | rcd | rcl | rcd | rcl |
| 0.8142 | 0.9537 | 0.9266 | 0.3167 | 0.125 | 0.315 | 0.124 | 0.316 | 0.125 |
| 0.8635 | 0.9737 | 0.9335 | 0.3 | 0.1875 | 0.299 | 0.185 | 0.298 | 0.190 |
| 0.9013 | 0.9813 | 0.9470 | 0.2834 | 0.25 | 0.282 | 0.260 | 0.281 | 0.245 |
| 0.9315 | 0.9867 | 0.9523 | 0.2667 | 0.3125 | 0.267 | 0.315 | 0.268 | 0.313 |
| 0.9544 | 0.9888 | 0.9664 | 0.25 | 0.375 | 0.248 | 0.377 | 0.245 | 0.372 |
| 0.9692 | 0.9905 | 0.9757 | 0.2334 | 0.4375 | 0.234 | 0.438 | 0.233 | 0.440 |
| 0.9839 | 0.9917 | 0.9845 | 0.2167 | 0.5 | 0.219 | 0.505 | 0.214 | 0.512 |
| 0.9908 | 0.9946 | 0.9855 | 0.2 | 0.5625 | 0.203 | 0.569 | 0.21 | 0.561 |
| 0.9964 | 0.9967 | 0.9993 | 0.1834 | 0.625 | 0.182 | 0.623 | 0.182 | 0.629 |
| 0.9986 | 0.9980 | 0.9994 | 0.1667 | 0.6875 | 0.165 | 0.679 | 0.165 | 0.687 |

The results of relative natural frequencies along with the various results obtained for relative crack depth and relative crack location for ten experiments have been mentioned in the Table 1. In this table also a comparison result among theoretical, experimental as well as for the proposed network has been stated. It has been found that, they are in good agreement with each other. Using the methodology crack location and crack depth can be found efficiently.

By taking the data obtained from various analysis, a comparison graph has been plotted, which is shown in Fig.4. In the first figure the comparison has been shown for the crack depth and in the second figure the comparison has been shown for the crack location.

Figure 5 describes about the performance and regression analysis of the current network (CFBP) which is used in our analysis. In the performance plot, it shows the relationship between the trained, tested and validated points against a threshold value and the regression plot shows the individual results.
6. Conclusion
Comprehensive investigations of the effects of cracks on the vibration signatures of a dynamically vibrating uniform cracked cantilever beam have been presented in this paper. The vibration analysis has been done using theoretical, experimental and CFBP analysis. The network controller developed using back propagation algorithm to predict the crack location and crack depth by taking relative deviation of first three natural frequencies as inputs. The neural network controller predicted results are reasonably acceptable and is in agreement with the experimental data as well as theoretical analysis.

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