Transverse electric conductivity and dielectric permeability in quantum degenerate collisional plasma with variable collision frequency in Mermin’s approach

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Abstract

Formulas for transverse conductance and dielectric permeability in quantum degenerate collisional plasma with arbitrary variable collision frequency in Mermin’s approach are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector. For this purpose the kinetic Shrödinger–Boltzmann equation with collision integral of relaxation type in momentum space is applied. The case of degenerate Fermi plasma is allocated and investigated. The special case of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.

Key words: Klimontovich, Silin, Lindhard, Mermin, quantum collisional plasma, conductance, rate equation, density matrix, commutator, degenerate plasma.

PACS numbers: 03.65.-w Quantum mechanics, 05.20.Dd Kinetic theory, 52.25.Dg Plasma kinetic equations.

1. Introduction

In Klimontovich and Silin’s work \cite{1} expression for longitudinal and transverse dielectric permeability of quantum collisionless plasmas has been received.

Then in Lindhard’s work \cite{2} expressions has been received also for the same characteristics of quantum collisionless plasma.

By Kliewer and Fuchs \cite{3} it has been shown, that direct generalisation of formulas of Lindhard on a case of collisionless plasmas, is incorrectly. This lack for the longitudinal dielectric permeability has been eliminated in work of Mermin \cite{4} for collisional plasmas. In this work of Mermin \cite{4} on
the basis of the analysis of a nonequilibrium matrix density in $\tau$-approach
expression for longitudinal dielectric permeability of quantum collisional plasmas in case of constant frequency of collisions of particles of plasma has been announced.

For collisional plasmas correct formulas longitudinal and transverse electric conductivity and dielectric permeability are received accordingly in works [5] and [6]. In these works kinetic Wigner–Vlasov–Boltzmann equation in relaxation approximation in coordinate space was used.

In work [7] the formula for the transverse electric conductivity of quantum collisional plasmas with use of the kinetic Shrödinger–Boltzmann equation in Mermin’s approach (in space of momentum) has been deduced.

In work [8] the formula for the longitudinal dielectric permeability of quantum collisional plasmas with use of the kinetic Shrödinger–Boltzmann equation in approach of Mermin (in space of momentum) with any variable frequency of collisions depending from wave vector has been deduced.

In our work [9] formulas for longitudinal and transverse electric conductivity in the classical collisional gaseous (maxwellian) plasma with frequency of collisions of plasma particles proportional to the module particles velocity have been deduced.

Research of skin-effect in classical collisional gas plasma with frequency of collisions proportional to the module particles velocity has been carried out in work [10].

In our works [11] and [12] dielectric permeability in quantum collisional plasma with frequency of collisions proportional to the module of a wave vector has been investigated. The case of degenerate plasmas was studied in work [11]. The case of non-degenerate and maxwellian plasmas has been investigated in work [12].

Let’s notice, that interest to research of the phenomena in quantum plasma grows in last years [13] – [26].

In the present work formulas for transverse conductivity and dielectric permeability in quantum degenerate collisional plasma with arbitrary variable collision frequency in Mermin’s approach are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector.

For this purpose the kinetic Shrödinger–Boltzmann equation with collision integral of relaxation type in momentum space is applied. The case
of degenerate Fermi plasma is allocated and investigated. The special case of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.

1. Kinetic Schrödinger—Boltzmann equation for density matrix

Let the vector potential of an electromagnetic field is harmonious, i.e. changes as $A = A(r) \exp(-i\omega t)$.

We consider transverse conductivity. Therefore the following relation is carried out $\text{div} A(r, t) = 0$.

Communication between vector potential and intensity of electric field is given by following expression

$$A(q) = -\frac{i\gamma}{\omega} E(q).$$

The equilibrium matrix of density has the following form

$$\tilde{\rho} = \frac{1}{1 + \exp \frac{H - \mu}{k_B T}}.$$

Here $T$ is the temperature, $k_B$ is the Boltzmann constant, $\mu$ is the chemical potential of plasma, $H$ is the Hamiltonian. Further we will be consider dimensionless chemical potential of plasma $\alpha = \mu/k_B T$.

In linear approach Hamiltonian has the following form

$$H = \frac{(p - (e/c)A)^2}{2m} = \frac{p^2}{2m} - \frac{e}{2mc}(pA + Ap).$$

Here $p$ is the momentum operator, $p = -i\hbar \nabla$, $e$ and $m$ are the charge and mass of electron, $c$ is the velocity of light.

Hence, we can present this Hamiltonian in the form of the sum two operators $H = H_0 + H_1$, where

$$H_0 = \frac{p^2}{2m}, \quad H_1 = -\frac{e}{2mc}(pA + Ap).$$

We take the kinetic equation for the density matrix in $\tau$– approximation

$$i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] + \frac{i\hbar}{\tau}(\tilde{\rho} - \rho). \quad (1.1)$$
Here $\nu = 1/\tau$ is the effective collisional frequency of plasma particles, $\tau$ is the characteristic time between two consecutive collisions, $\hbar$ is the Planck's constant, $[H, \rho] = H\rho - \rho H$ is the commutator, $\tilde{\rho}$ is the equilibrium matrix density.

Generally frequency of collisions $\nu$ should depend from electron momentum $\mathbf{p}$ (or a wave vector $\mathbf{k}$): $\nu = \nu(\mathbf{k})$.

Considering the requirement Hermitian character the equation (1.1) on the density matrix it is necessary to rewrite in the form

$$i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] + i\hbar \frac{\nu(\mathbf{k})}{2} (\tilde{\rho} - \rho) + (\tilde{\rho} - \rho)i\hbar \frac{\nu(\mathbf{k})}{2}.$$  

(1.2)

In linear approach on an external field we search for a density matrix in the form

$$\rho = \tilde{\rho}_0 + \rho_1.$$  

(1.3)

Here $\rho_1$ is the correction (perturbation) to the equilibrium density matrix, caused by presence of an electromagnetic field, $\tilde{\rho}_0$ is the equilibrium matrix of density, corresponds to the "equilibrium" Hamilton operator $H_0$.

We present the equilibrium matrix density $\tilde{\rho}$ in the following form

$$\tilde{\rho} = \tilde{\rho}_0 + \tilde{\rho}_1.$$  

(1.4)

We consider the commutator $[H, \tilde{\rho}]$. In linear approximation this commutator is equal

$$[H, \tilde{\rho}] = [H_0, \tilde{\rho}_1] + [H_1, \tilde{\rho}_0]$$  

(1.5)

and

$$[H, \tilde{\rho}] = 0.$$  

(1.6)

For commutators from right side of equality (1.5) we obtain

$$\langle \mathbf{k}_1|[H_0, \tilde{\rho}_1]|\mathbf{k}_2 \rangle = (\mathcal{E}_{\mathbf{k}_1} - \mathcal{E}_{\mathbf{k}_2}) \langle \mathbf{k}_1|\tilde{\rho}_1|\mathbf{k}_2 \rangle = (\mathcal{E}_{\mathbf{k}_1} - \mathcal{E}_{\mathbf{k}_2})\tilde{\rho}_1(\mathbf{k}_1 - \mathbf{k}_2),$$  

(1.7)

and

$$\langle \mathbf{k}_1|[H_1, \tilde{\rho}_0]|\mathbf{k}_2 \rangle = \langle \mathbf{k}_1|H_1\tilde{\rho}_0|\mathbf{k}_2 \rangle - \langle \mathbf{k}_1|\tilde{\rho}_0H_1|\mathbf{k}_2 \rangle =$$

$$= \frac{e}{2mc}(f_{\mathbf{k}_1} - f_{\mathbf{k}_2})(\mathbf{k}_1 + \mathbf{k}_2)\langle \mathbf{k}_1|A|\mathbf{k}_2 \rangle =$$

$$= \frac{e}{2mc}(f_{\mathbf{k}_1} - f_{\mathbf{k}_2})(\mathbf{k}_1 + \mathbf{k}_2)A(\mathbf{k}_1 - \mathbf{k}_2),$$  

(1.8)
where

\[ f_k = \frac{1}{1 + \exp(\beta E_k - \alpha)}, \quad \mathcal{E}_k = \frac{\hbar^2 k^2}{2m}, \quad p = \hbar k. \]

From relations (1.4)–(1.8) follows that

\[ \tilde{\rho}_1(k_1 - k_2) = -\frac{e\hbar}{2mc} \frac{f_{k_1} - f_{k_2}}{E_{k_1} - E_{k_2}} (k_1 + k_2) A(k_1 - k_2). \quad (1.9) \]

By means of equalities (1.3)–(1.5) we linearize the kinetic equation (1.2). We receive the following equation

\[ i\hbar \frac{\partial \rho_1}{\partial t} = [H_0, \rho_1] + 
\]

\[ + [H_1, \rho_0] + i\hbar \frac{\nu(k_1)}{2}(\tilde{\rho}_1 - \rho_1) + (\tilde{\rho}_1 - \rho_1)i\hbar \frac{\nu(k_2)}{2}. \quad (1.10) \]

We notice that the perturbation \( \rho_1 \sim \exp(-i\omega t) \), then from equation (1.10) we receive

\[ \hbar \omega \langle k_1|\rho_1|k_2 \rangle = \langle k_1|[H_0, \rho_1]|k_2 \rangle + \langle k_1|[H_1, \rho_0]|k_2 \rangle + 
\]

\[ + i\hbar \frac{\nu(k_1)}{2} \langle k_1|\tilde{\rho}_1 - \rho_1|k_2 \rangle + \langle k_1|\tilde{\rho}_1 - \rho_1|k_2 \rangle i\hbar \frac{\nu(k_2)}{2}. \]

We will enter the designation

\[ \tilde{\nu}(k_1, k_2) = \frac{\nu(k_1) + \nu(k_2)}{2} \]

and rewrite the previous equation in the form

\[ \hbar[\omega + i\hbar \tilde{\nu}(k_1, k_2)] \langle k_1|\rho_1|k_2 \rangle = \langle k_1|[H_0, \rho_1]|k_2 \rangle + \langle k_1|[H_1, \rho_0]|k_2 \rangle + 
\]

\[ + i\hbar \tilde{\nu}(k_1, k_2) \langle k_1|\tilde{\rho}_1|k_2 \rangle. \]

Using equalities (1.7) – (1.9), we will transform this equation to the following form

\[ \{ \mathcal{E}_{k_1} - \mathcal{E}_{k_2} - \hbar[\omega + i\hbar \tilde{\nu}(k_1, k_2)] \} \langle k_1|\rho_1|k_2 \rangle = 
\]

\[ = -\frac{e\hbar}{2mc} (f_{k_1} - f_{k_2}) \frac{\mathcal{E}_{k_1} - \mathcal{E}_{k_2} - i\hbar \tilde{\nu}(k_1, k_2)}{\mathcal{E}_{k_1} - \mathcal{E}_{k_2}} (k_1 + k_2) \langle k_1|A|k_2 \rangle. \]
Now from the previous equation we find

$$\langle k_1 | \rho_1 | k_2 \rangle = -\frac{e\hbar}{2mc} \Xi(k_1, k_2) (f_{k_1} - f_{k_2})(k_1 + k_2) \langle k_1 | A | k_2 \rangle. \quad (1.11)$$

Here

$$\Xi(k_1, k_2) = \frac{\mathcal{E}_{k_1} - \mathcal{E}_{k_2} - \hbar \bar{\nu}(k_1, k_2)}{(\mathcal{E}_{k_1} - \mathcal{E}_{k_2}) \{ \mathcal{E}_{k_1} - \mathcal{E}_{k_2} - \hbar \omega + \hbar \bar{\nu}(k_1, k_2) \}}.$$

In equation (1.11) we will put $k_1 = k$, $k_2 = k - q$. Then

$$\langle k_1 | \rho_1 | k_2 \rangle = \langle k | \rho_1 | k - q \rangle = \rho_1(q) =$$

$$= -\frac{e\hbar}{mc} \Xi(k, k - q) (f_k - f_{k-q}) kA(q). \quad (1.12)$$

Here

$$\Xi(k, k - q) = \frac{\mathcal{E}_k - \mathcal{E}_{k-q} - \hbar \bar{\nu}(k, k - q)}{(\mathcal{E}_k - \mathcal{E}_{k-q}) \{ \mathcal{E}_k - \mathcal{E}_{k-q} - \hbar \omega + \hbar \bar{\nu}(k, k - q) \}}.$$

2. Current density

The current density $j(q)$ is defined as

$$j(q, \omega) = e \int \frac{dk}{8\pi^3m} \left\langle k + \frac{q}{2} \left| \left( p - \frac{e}{c}A \right) \rho + \rho \left( p - \frac{e}{c}A \right) \right| k - \frac{q}{2} \right\rangle. \quad (2.1)$$

After substitution (1.3) in integral from (2.1), we have

$$\left\langle k + \frac{q}{2} \left| \left( p - \frac{e}{c}A \right) \rho + \rho \left( p - \frac{e}{c}A \right) \right| k - \frac{q}{2} \right\rangle =$$

$$= \left\langle k + \frac{q}{2} \left| p \rho_1 + \rho_1 p - \frac{e}{c}(A \tilde{\rho}_0 + \tilde{\rho}_0 A) \right| k - \frac{q}{2} \right\rangle.$$

It is easy to show, that

$$\left\langle k + \frac{q}{2} \left| p \rho_1 + \rho_1 p \right| k - \frac{q}{2} \right\rangle = 2\hbar k \tilde{\rho}_0(q),$$
\[ \langle \mathbf{k} + \frac{\mathbf{q}}{2} | \mathbf{A} \tilde{\rho}_0 + \tilde{\rho}_0 \mathbf{A} | \mathbf{k} - \frac{\mathbf{q}}{2} \rangle = \mathbf{A}(\mathbf{q}) \left[ \tilde{\rho}_0 \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) + \tilde{\rho}_0 \left( \mathbf{k} - \frac{\mathbf{q}}{2} \right) \right]. \]

Hence, expression for current density has the following form

\[ \mathbf{j}(\mathbf{q}, \omega, \bar{\nu}) = -\frac{e^2}{mc} \mathbf{A}(\mathbf{q}) \int \frac{d\mathbf{k}}{8\pi^3} \tilde{\rho}_0 \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) - \frac{e^2}{mc} \mathbf{A}(\mathbf{q}) \int \frac{d\mathbf{k}}{8\pi^3} \tilde{\rho}_0 \left( \mathbf{k} - \frac{\mathbf{q}}{2} \right) + \]

\[ + e\hbar \int \frac{d\mathbf{k}}{4\pi^3 m} \langle \mathbf{k} + \frac{\mathbf{q}}{2} | \rho_1 | \mathbf{k} - \frac{\mathbf{q}}{2} \rangle \].

First two members in this expression are equal each other

\[ \int \frac{d\mathbf{k}}{8\pi^3} \tilde{\rho}_0 \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) = \int \frac{d\mathbf{k}}{8\pi^3} \tilde{\rho}_0 \left( \mathbf{k} - \frac{\mathbf{q}}{2} \right) = \frac{N}{2}, \]

where \( N \) is the number density (concentration) of plasma.

Hence, the current density is equal

\[ \mathbf{j}(\mathbf{q}, \omega, \bar{\nu}) = -\frac{e^2 N}{mc} \mathbf{A}(\mathbf{q}) + e\hbar \int \frac{d\mathbf{k}}{4\pi^3 m} \mathbf{k} \langle \mathbf{k} + \frac{\mathbf{q}}{2} | \rho_1 | \mathbf{k} - \frac{\mathbf{q}}{2} \rangle. \quad (2.2) \]

The first composed in (2.2) is not that other, as calibration current density.

By means of obvious replacement of variables in integral from (2.2) expression (2.2) we can transform to the form

\[ \mathbf{j}(\mathbf{q}, \omega, \bar{\nu}) = -\frac{e^2 N}{mc} \mathbf{A}(\mathbf{q}) + e\hbar \int \frac{d\mathbf{k}}{4\pi^3 m} \mathbf{k} \langle \mathbf{k} | \rho_1 | \mathbf{k} - \mathbf{q} \rangle. \quad (2.3) \]

In the relation (2.3) subintegral expression is given by equality (1.12). Substituting (1.12) in (2.3), we receive the following expression for current density

\[ \mathbf{j}(\mathbf{q}, \omega, \bar{\nu}) = -\frac{e^2 N}{mc} \mathbf{A}(\mathbf{q}) - \]

\[ - \frac{e^2 \hbar^2}{4\pi^3 m^2 c} \int \mathbf{k} [\mathbf{k} \mathbf{A}(\mathbf{q})] \Xi(\mathbf{k}, \mathbf{k} - \mathbf{q}) (f_k - f_{k-q}) d\mathbf{k}. \quad (2.4) \]
Let’s direct an axis $x$ along a vector $\mathbf{q}$, and an axis $y$ we direct lengthways vector $\mathbf{A}$. Then the previous vector expression (2.4) can be rewrite in the form of three scalar

$$j_y(q, \omega, \bar{\nu}) = -\frac{e^2 N}{mc} A(q) - \frac{e^2\hbar^2 A(q)}{4\pi^3 m^2 c} \int k_y^2 \Xi(k, k - q)(f_k - f_{k-q})dk$$

and

$$j_x(q, \omega, \bar{\nu}) = j_z(q, \omega, \bar{\nu}) = 0.$$

Obviously, that

$$\int k_y^2 \Xi(k, k - q)(f_k - f_{k-q})dk = \int k_z^2 \Xi(k, k - q)(f_k - f_{k-q})dk.$$ 

Hence

$$\int k_y^2 \Xi(k, k - q)(f_k - f_{k-q})dk = \frac{1}{2} \int (k_y^2 + k_z^2) \Xi(k, k - q)(f_k - f_{k-q})dk = \frac{1}{2} \int (k^2 - k_x^2) \Xi(k, k - q)(f_k - f_{k-q})dk.$$ 

From here we conclude, that expression for current density we can present in the following invariant form

$$j(q, \omega, \bar{\nu}) = -\frac{Ne^2}{mc} A(q) -$$

$$-\frac{e^2\hbar^2}{8\pi^3 m^2 c} A(q) \int \Xi(k, k - q)(f_k - f_{k-q})k_\perp^2 dk,$$ 

(2.5)

where

$$k_\perp^2 = k^2 - \left(\frac{kq}{q}\right)^2.$$ 

Considering kernel decomposition $\Xi(k, k - q)$ on fraction

$$\Xi(k, k - q) = \frac{1}{E_k - E_{k-q}} + \frac{\hbar \omega}{E_k - E_{k-q} - \hbar[\omega + i\bar{\nu}(k_1, k_2)]},$$

we present equality (2.5) in the following form

$$j(q, \omega, \bar{\nu}) = -\frac{Ne^2}{mc} A(q) - \frac{e^2\hbar^2}{8\pi^3 m^2 c} A(q) \int \frac{f_k - f_{k-q}}{E_k - E_{k-q}} k_\perp^2 dk -$$

$$-\frac{e^2\hbar^3 \omega}{8\pi^3 m^2 c} A(q) \int (f_k - f_{k-q})k_\perp^2 dk$$

$$\frac{1}{(E_k - E_{k-q})[E_k - E_{k-q} - \hbar[\omega + i\bar{\nu}(k_1, k_2)])}. (2.6)$$
First two members in the previous equality (2.6) do not depend on frequency $\omega$ also are defined by the dissipativity properties of a material defined by frequency of collisions $\nu(k)$. These members are universal parameters, defining Landau diamagnetism.

3. Transversal electric conductivity and dielectric permeability

Considering communication of vector potential with intensity of an electromagnetic field, and also communication of density of a current with electric field, on the basis of the previous equality (2.5) we receive the following expression of an invariant form for the transversal electric conductivity

$$\sigma_{tr}(q, \omega, \nu) = \frac{ie^2 N}{m\omega} \left[ 1 + \frac{\hbar^2}{8\pi^3 mN} \int \Xi(k, k - q)(f_k - f_{k-q})k_\perp^2 dk \right]. \quad (3.1)$$

Let’s take advantage of definition of transversal dielectric permeability

$$\varepsilon_{tr}(q, \omega, \nu) = 1 + \frac{4\pi i}{\omega} \sigma_{tr}(q, \omega, \nu). \quad (3.2)$$

Taking into account (3.1) and equality (3.2) we will write expression for the transversal dielectric permeability

$$\varepsilon_{tr}(q, \omega, \nu) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\hbar^2}{8\pi^3 mN} \int \Xi(k, q)(f_k - f_{k-q})k_\perp^2 dk \right]. \quad (3.3)$$

Here $\omega_p$ is the plasma (Langmuir) frequency, $\omega_p^2 = 4\pi e^2 N/m$.

From equality (3.3) it is visible, that one of equalities named a rule of sums is carried out (see, for example, [22], [27] and [28]) for the transversal dielectric permeability. This rule is expressed by the formula (4.200) from the monography [27]

$$\int_{-\infty}^{\infty} \varepsilon_{tr}(q, \omega, \nu)\omega d\omega = \pi \omega_p^2.$$

Let’s notice, that the kernel from subintegral expression from (3.1) can be we can present in the form of decomposition on partial fractions

$$\frac{\mathcal{E}_k - \mathcal{E}_{k-q} - i\hbar \nu(k, k - q)}{(\mathcal{E}_k - \mathcal{E}_{k-q})\{\mathcal{E}_k - \mathcal{E}_{k-q} - \hbar[\omega + i\nu(k, k - q)]\}} =$$
\[ \frac{i\tilde{v}(k, k - q)}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{1}{\varepsilon_k - \varepsilon_{k-q} + \hbar[\omega + i\tilde{v}(k, k - q)]} \]

Hence, for transversal electric conductivity and dielectric permeability we have explicit representations

\[ \sigma_{tr}(q, \omega, \tilde{v}) = \frac{ie^2 N}{m\omega} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\tilde{v}(k, k - q)}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{f_k - f_{k-q}}{\varepsilon_k - \varepsilon_{k-q}} k_\perp^2 dk \right. \]

\[ + \left. \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{(f_k - f_{k-q}) k_\perp^2 dk}{\varepsilon_k - \varepsilon_{k-q} - \hbar[\omega + i\tilde{v}(k, k - q)]} \right], \quad (3.2) \]

and

\[ \varepsilon_{tr}(q, \omega, \tilde{v}) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\tilde{v}(k, k - q)}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{f_k - f_{k-q}}{\varepsilon_k - \varepsilon_{k-q}} k_\perp^2 dk \right. \]

\[ + \left. \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{(f_k - f_{k-q}) k_\perp^2 dk}{\varepsilon_k - \varepsilon_{k-q} - \hbar[\omega + i\tilde{v}(k, k - q)]} \right]. \quad (3.3) \]

If to enter designations

\[ J_{\tilde{v}} = \frac{\hbar^2}{8\pi^3 m N} \int \frac{i\tilde{v}(k, k - q)}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{f_k - f_{k-q}}{\varepsilon_k - \varepsilon_{k-q}} k_\perp^2 dk \]

and

\[ J_{\omega} = \frac{\hbar^2 \omega}{8\pi^3 m N} \int \frac{1}{\omega + i\tilde{v}(k, k - q)} \cdot \frac{(f_k - f_{k-q}) k_\perp^2 dk}{\varepsilon_k - \varepsilon_{k-q} - \hbar[\omega + i\tilde{v}(k, k - q)]}, \]

then expression (3.2) for electric conductivity and (3.3) for dielectric permeability will be transformed to the following form

\[ \sigma_{tr}(q, \omega, \tilde{v}) = \frac{ie^2 N}{m\omega} (1 + J_{\tilde{v}} + J_{\omega}) \quad (3.4) \]

and

\[ \varepsilon_{tr}(q, \omega, \tilde{v}) = 1 - \frac{\omega_p^2}{\omega^2} (1 + J_{\tilde{v}} + J_{\omega}). \quad (3.5) \]
Let’s notice, that in case of constant frequency of collisions we have
\( \bar{\nu}(k, k - q) = \nu \) and formulas (3.4) and (3.5) will be transformed in

\[
\frac{\sigma_{tr}(q, \omega, \nu)}{\sigma_0} = \frac{i \nu}{\omega} \left( 1 + \frac{\omega I_\omega + i \nu I_\nu}{\omega + i \nu} \right)
\]

and

\[
\varepsilon_{tr}(q, \omega, \nu) = 1 - \frac{\omega^2 \nu}{\omega^2} \left( 1 + \frac{\omega I_\omega + i \nu I_\nu}{\omega + i \nu} \right).
\]

Here

\[
I_\nu = \frac{\hbar^2}{8 \pi^3 m N} \int \frac{f_k - f_{k-q}k^2}{\mathcal{E}_k - \mathcal{E}_{k-q}} dk
\]

and

\[
I_\omega = \frac{\hbar^2}{8 \pi^3 m N} \int \frac{(f_k - f_{k-q})k^2}{\mathcal{E}_k - \mathcal{E}_{k-q} - \hbar(\omega + i \nu)} dk.
\]

Let now \( \nu(k) \equiv 0 \), i.e. \( \bar{\nu}(k, k - q) = \nu(k + q, k) \equiv 0 \). In this case
dielectric permeability equals

\[
\varepsilon_{tr}(q, \omega) = 1 - \frac{\omega^2}{\omega^2} \left[ 1 + \frac{\hbar^2}{8 \pi^3 m N} \int \left( \frac{1}{\mathcal{E}_k - \mathcal{E}_{k-q} - \hbar \omega} - \frac{1}{\mathcal{E}_{k+q} - \mathcal{E}_k - \hbar \omega} \right) f_k k^2 d k \right].
\]

Let’s return to the case of variable frequency of collisions. Integrals \( J_\nu \) and \( J_\omega \) we can transform to the following form

\[
J_\nu = \frac{i \hbar^2}{8 \pi^3 m N} \int \left[ \frac{\bar{\nu}(k, k - q)}{[\omega + i \bar{\nu}(k, k - q)]\{\mathcal{E}_k - \mathcal{E}_{k-q}\}} - \frac{\bar{\nu}(k + q, k)}{[\omega + i \bar{\nu}(k + q, k)]\{\mathcal{E}_{k+q} - \mathcal{E}_k\}} \right] f_k k^2 d k \quad (3.8)
\]

and

\[
J_\omega = \frac{\omega \hbar^2}{8 \pi^3 m N} \int \left[ \frac{1}{[\omega + i \bar{\nu}(k, k - q)]\{\mathcal{E}_k - \mathcal{E}_{k-q} - \hbar(\omega + i \nu(k, k - q))\}} - \frac{1}{[\omega + i \bar{\nu}(k + q, k)]\{\mathcal{E}_{k+q} - \mathcal{E}_k - \hbar(\omega + i \nu(k + q, k))\}} \right] f_k k^2 d k \quad (3.9)
\]
4. Degenerate plasma

Instead of the vector $k$ we will enter the dimensionless vector $K$ by following equality $K = \frac{k}{k_F}$, $k_F = \frac{p_F}{\hbar}$, where $k_F$ is the Fermi wave number, $p_F = mv_F$ is the electron momentum on the Fermi surface, $v_F$ is the electron velocity on the Fermi surface.

Then

$$(k^2 - k_x^2) d^3k = k_F^5 (K^2 - K_x^2) d^3K = k_F^5 K_\perp^2 d^3K,$$

where

$$K_\perp^2 = K^2 - K_x^2 = K_y^2 + K_z^2.$$

Further we will consider the case of degenerate plasmas. Then we have

$$\left(\frac{mv_F}{\hbar}\right)^3 \equiv \left(\frac{p_F}{\hbar}\right)^3 \equiv k_F^3 = 3\pi^2 N.$$

Hence,

$$(k^2 - k_x^2) d^3k = 3\pi^2 N m^2 v_F^2 \frac{\hbar^2}{h^2} (K^2 - K_x^2) d^3K.$$

Absolute Fermi–Dirac’s distribution $f_k$ for degenerate plasma transforms into Fermi distribution $f_k = \Theta(k) \equiv \Theta(E_F - \mathcal{E}_k)$. Here $\Theta(x)$ is the Heaviside step function,

$$\Theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases}$$

Here $E_F = \frac{p_F^2}{2m}$ is the electron energy on Fermi surface.

Calculation of transversal electric conductivity can be spent on any of formulas (5.4) - (5.6). Let’s calculate the integrals entering into expression (5.6). The first integral is special case of the second. Therefore at first we will calculate the second integral. Energy $\mathcal{E}_k$ we will express through Fermi’s energy. We have

$$\mathcal{E}_k = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 k_F^2}{2m} K^2 = \frac{p_F^2}{2m} K^2 = E_F K^2 \equiv \mathcal{E}_K.$$

In the same way we receive

$$\mathcal{E}_{k-q} = \frac{\hbar^2 (k_F K - q)^2}{2m} = \frac{\hbar^2 k_F^2}{2m} \left(K - \frac{q}{k_F}\right)^2.$$
Further we introduce dimensionless wave vector $Q = \frac{q}{k_F}$. Then
\[ \mathcal{E}_{k-q} = \frac{\hbar^2 k_F^2}{2m} (K - Q)^2 = \mathcal{E}_F(K - Q)^2 = \mathcal{E}_{K-Q}. \]

We notice that
\[ \mathcal{E}_K - \mathcal{E}_{K-Q} = \mathcal{E}_F K^2 - \mathcal{E}_F(K - Q)^2 = \mathcal{E}_F[2K_xQ - Q^2] = 2Q \mathcal{E}_F(K_x - \frac{Q}{2}). \]

Besides
\[ \mathcal{E}_K - \mathcal{E}_{K-Q} - \hbar[\omega + i\tilde{\nu}(K, K - Q)] = 2\mathcal{E}_F Q (K_x - \frac{z^-}{Q} - \frac{Q}{2}), \]
\[ \mathcal{E}_{K+Q} - \mathcal{E}_K - \hbar[\omega + i\tilde{\nu}(K + Q, K)] = 2\mathcal{E}_F Q (K_x - \frac{z^+}{Q} + \frac{Q}{2}), \]
where
\[ z^\pm = x + iy^\pm, \quad x = \frac{\omega}{k_F v_F}, \]
\[ y^- = \frac{\tilde{\nu}(K, K - Q)}{k_F v_F}, \quad y^+ = \frac{\tilde{\nu}(K + Q, K)}{k_F v_F}. \]

Let’s consider the special case, when frequency of collisions is proportional to the module of a wave vector
\[ \nu(k) = \nu_0 |k|. \]

Then
\[ \tilde{\nu}(k, k - q) = \frac{\nu(k) + \nu(k - q)}{2} = \frac{\nu_0}{2} (|k| + |k - q|), \]
and
\[ \tilde{\nu}(k + q, k) = \frac{\nu(k + q) + \nu(k)}{2} = \frac{\nu_0}{2} (|k + q| + |k|). \]

The quantity $\nu_0$ we take in the form $\nu_0 = \frac{\nu}{k_F}$, where $k_F$ is the Fermi wave number, $k_F = \frac{mv_F}{\hbar}$, $\hbar$ is the Planck’s constant, $v_F$ is the Fermi electron velocity. Now we have
\[ \nu(k) = \frac{\nu}{k_F} |k|. \quad (4.1)\]
Let’s notice, that on Fermi’s surface, i.e. at \( k = k_F \): \( \nu(k_F) = \nu \). So, further in previous formulas frequency collisions according to (4.1) it is equal

\[
\bar{\nu}(k, k - q) = \frac{\nu}{2k_F}(|k| + |k - q|) = \frac{\nu}{2}(|K| + |K - Q|) = \bar{\nu}(K, K - Q),
\]

\[
\bar{\nu}(k + q, k) = \frac{\nu}{2k_F}(|k + q| + |k|) = \frac{\nu}{2}(|K + Q| + |K|) = \bar{\nu}(K + Q, K).
\]

Hence, quantities \( z^\pm \) are equal

\[
z^\pm = x + iy\rho^\pm, \quad y = \frac{\nu}{k_Fv_F},
\]

\[
\rho^\pm = \frac{1}{2}\left(\sqrt{K^2_x + K^2_y + K^2_z} + \sqrt{(K_x \pm Q)^2 + K^2_y + K^2_z}\right).
\]

Now integrals \( J_\nu \) and \( J_\omega \) are accordingly equal

\[
J_\nu = \frac{3iy}{8\pi Q} \int \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right) f_K K_x^2 d^3 K, \quad (4.2)
\]

and

\[
J_\omega = \frac{3x}{8\pi Q} \int \left( \frac{1}{(x + iy\rho^-)(K_x - z^-/Q - Q/2)} - \frac{1}{(x + iy\rho^+)(K_x - z^+/Q + Q/2)} \right) f_K K_x^2 d^3 K. \quad (4.3)
\]

Three-dimensional integrals (4.2) and (4.3) after passing to polar coordinates in a plane \((K_y, K_z)\) are easily reduced to the double

\[
J_\nu = \frac{3iy}{4Q} \int_{-1}^{1} dK_x \int_{0}^{\sqrt{1-K_x^2}} \left( \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right) r^3 dr, \quad (4.4)
\]
\[
J_\omega = \frac{3x}{4Q} \int_{-1}^{1} dK_x \int_{0}^{1} \frac{1}{(x + iy\rho^-)(K_x - z^-/Q - Q/2)} \left( -\frac{1}{(x + iy\rho^+)(K_x - z^+/Q + Q/2)} \right) r^3 dr.
\]

Let’s notice, that in case of constant frequency of collisions \( \rho^\pm = 1 \) and formulas (4.4) and (4.5) pass in the following

\[
J_\omega = \frac{3x}{16(x + iy)} T(Q, z), \quad T(Q, z) = \int_{-1}^{1} \frac{(1 - K_x^2)^2 dK_x}{(K_x - z/Q) - (Q/2)^2},
\]

\[
J_\nu = \frac{3iy}{16(x + iy)T(Q, 0)}, \quad z = x + iy.
\]

By means of these expressions we receive known formulas for electric conductivity and dielectric permeability of quantum collisional degenerate plasmas with constant frequency of collisions of particles [7]

\[
\sigma_{tr} = \sigma_0 \frac{iy}{x} \left[ 1 + \frac{3}{16} \frac{x T(Q, z) + iy T(Q, 0)}{x + iy} \right]
\]

and

\[
\varepsilon_{tr} = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{3}{16} \frac{x T(Q, z) + iy T(Q, 0)}{x + iy} \right].
\]

On Figs. 1 – 8 we will present comparison real and imaginary parts of dielectric function.

5. Conclusion

In the present work formulas for the transversal electric conductivity and dielectric permeability into quantum collisional plasma are deduced. Frequency of collisions of particles depends arbitrarily on a wave vector. For this purpose the kinetic equation with integral of collisions in form of relaxation model in momentum space is used. The case of degenerate Fermi plasma is allocated and investigated. The special case of frequency of collisions proportional to the module of a wave vector is considered. The graphic analysis of the real and imaginary parts of dielectric function is made.
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Fig. 1. Real part of dielectric function, $x_p = 1, Q = 1$. Curves 1,2,3 correspond to values $y = 0.5, 0.3, 0.1$.

Fig. 2. Imaginare part of dielectric function, $x_p = 1, Q = 1$. Curves 1,2,3 correspond to values $y = 0.5, 0.3, 0.1$. 
Fig. 3. Real part of dielectric function, $x_p = 1, y = 0.1$. Curves 1,2,3 correspond to values $x = 0.9, 1.0, 1.1$.

Fig. 4. Imaginary part of dielectric function, $x_p = 1, y = 0.01$. Curves 1,2,3 correspond to values $x = 0.9, 1.0, 1.1$. 
Fig. 5. Real part of dielectric function, $x_p = 1, y = 0.1$. Curves 1,2,3 correspond to values $x = 0.1, 0.2, 0.3$.

Fig. 6. Imaginare part of dielectric function, $x_p = 1, y = 0.1$. Curves 1,2,3 correspond to values $x = x = 0.1, 0.2, 0.3$. 
Fig. 7. Real part of dielectric function, $x_p = 1, y = 0.1$. Curves 1,2,3 correspond to values $x = 0.9, 1.0, 1.1$.

Fig. 8. Imaginary part of dielectric function, $x_p = 1, y = 0.1$. Curves 1,2,3 correspond to values $x = 0.9, 1.0, 1.1$. 