Symplectic dynamics of the nuclear mean-field\(^1\)

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Collective and microscopic pictures of nuclear dynamics are related in the framework of time-dependent variational principle on symplectic trial manifolds. For symmetry breaking systems such manifolds are constructed by cranking, and applied to study the nuclear isovector collective excitations.

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\(^1\)in Proc. Int. School *Collective Motion and Nuclear Dynamics*, Predeal, Romania, 1995, Edited by A. A. Raduta, D. S. Delion, I. I. Ursu, World Scientific, Singapore, 1996, p. 251-259.
1 Introduction

The interplay between quantum and classical aspects in the nuclear collective dynamics is a long standing puzzle, similar to the one represented by the quantum behaviour of a macroscopic variable [1], and is not yet completely understood.

The phenomenological collective models are based essentially on the quantization of some simple classical systems (rigid body, liquid drop), with parameters obtained by fitting data [2]. Therefore, in this approach the nucleus is supposed to be almost frozen, because the model Hamiltonian depends on a small number of variables. Also, the Hilbert space constructed by quantization is rather artificial, because it does not account for all the observables of the many-nucleon system.

In the microscopical approach, the nuclear dynamics is represented in the full Hilbert space by the time-dependent Hartree-Fock (HF), or Hartree-Fock-Bogolyubov (HFB) equations [3]. These equations have static solutions corresponding to the ground state, while the excited states are obtained by the random phase approximation (RPA), or more generally, using requantization methods [4, 5]. A clear sign of collective behaviour, in the sense of the phenomenological models, appears when the ground state is non-invariant to a continuous symmetry group of the Hamiltonian (spontaneous symmetry breaking). This situation may appear for geometrical symmetries, like rotation, or dynamical, like particle-number, and the corresponding nuclei are known as deformed, respectively superfluid. For these nuclei, the inertial parameters of the collective motion may be calculated microscopically using the average energy of the ground state, shifted to a ”moving frame” by the symmetry generators (the ”cranking” method). However, the cranking is not related strictly to the symmetry breaking and is applied, for instance, to obtain the effective mass at fission [6].

The choice of the cranking operators is not arbitrary, because they should be in one-to-one correspondence with the canonical collective variables, up to a unitary change in the Hilbert space representation. This problem of correspondence is not simple, but a natural solution can be given if the cranking is applied at a more fundamental level, to generate both the collective phase space and the phenomenological Hamiltonian. The key object necessary for this purpose is represented by the shifted ground states, which play only a secondary role in the standard cranking calculation. These states are pa-
rameterized by the "shift" variables, and may be joined in trial manifolds (S), endowed with a classical phase space structure (symplectic form) induced from the many-body Hilbert space [7]. By construction, S should be considered both as collective phase space and as trial manifold for a time-dependent variational treatment of the microscopic Hamiltonian. Moreover, the Hamilton equations of motion for the collective variables are related kinematically to the evolution of the trial functions produced by the variational calculation. Therefore, the "artificial" quantization of the collective motion becomes useless, and can be replaced by requantization, to obtain directly the microscopic states which correspond to a particular collective motion.

It is interesting to remark that the standard RPA is included in this formalism as a limit case, reached when S is constructed without any correspondence to an intuitive model, but taking as generators all possible particle-hole (ph) or particle-particle (pp) operators in some finite basis. This nice example will be discussed in the next section. Then, Section 3 presents the symplectic trial manifolds for deformed and for superfluid systems, and two applications to the treatment of the isovector "scissors-like" collective excitations.

2 Symplectic dynamics on trial manifolds and the RPA

2.1 Symplectic dynamics and requantization

Let us assume that H is the many-body Hilbert space, H is the microscopic Hamiltonian, S = {⟨ψ⟩(X)} is a 2N-dimensional trial manifold of normalized functions, parameterized by the variables X = {x^i}, i = 1, 2N, and that the matrix ω^S = [ω^S_{ij}(ψ)],

$$\omega^S_{ij}(ψ) = 2\hbar \text{Im} \langle \partial_i ψ | \partial_j ψ \rangle,$$

is non-singular. Thus, ω^S defines a symplectic form on S, and the functional

$$\mathcal{J}[X] = \int_a^b \langle ψ | i\hbar \partial_t - H | ψ \rangle dt$$

is stationary for the solution X_t of the Hamilton equations

$$\sum_{j=1}^{2N} \dot{x}^j \omega^S_{jk}(ψ) = \frac{∂ \langle ψ | H | ψ \rangle}{∂ x^k}.$$  \hspace{1cm} (1)
Therefore, the solution of a time-dependent variational calculation within $S$ has the form $|\psi\rangle(X_t)$, where $X_t$ is a trajectory given by Eq. (1).

It is interesting to note that if the couple $\mathcal{H}, H$ corresponds to the quantum harmonic oscillator, and $S$ is the manifold of the Glauber coherent states, then (1) represents the Hamilton equations for the classical oscillator.

The procedure of extracting information about the spectrum of $H$ from the orbits $|\psi\rangle(X_t)$ is called "requantization", and if the system of Eq. (1) is integrable, then the method GIPQ [4] (gauge-invariant periodic quantization) can be applied. According to this method, the periodic orbits $\gamma = \{X_t^\gamma\}$, 

$$X_t^\gamma = X_{t+T}\gamma,$$

should be quantized by a Bohr-Wilson-Sommerfeld (BWS) condition

$$\int_0^{T_\gamma} dt \langle \psi | i \partial_t | \psi \rangle = 2\pi n, \quad n = 0, 1, 2, \ldots.$$

This gives the energy spectrum, while the eigenstates corresponding to a quantized orbit $\gamma^n$ should be approximated by the time-average [5]

$$|\Omega_{\gamma^n}\rangle = \frac{1}{T_{\gamma^n}} \int_0^{T_{\gamma^n}} d\tau e^{i\Theta_t} |\psi\rangle(X_t^\gamma^n)$$

with $\Theta_t = \int_0^t dt' \langle \psi | i \partial_{t'} | \psi \rangle$ (the "Berry’s phase").

An extended version of GIPQ includes also the quantization of the invariant tori [8]. As it will be shown further, the physical applications strongly support the requantization by Eq. (3), though its geometrical meaning is not yet completely clear. The study of such integral representations ("quantization by membranes") represents an active field of research in mathematical physics [9].

### 2.2 The random phase approximation

We suppose now that the output of a static mean-field calculation (HF or HFB) provides the ground state $|g\rangle$ and a set of $2N$ operators $E_{\pm \alpha}$, $\alpha = 1, N$, $E_{-\alpha} = E_{\alpha}^\dagger$, so that $E_{-\alpha}|g\rangle = 0$. If the single-particle basis contains $n$ states, then in the HF case $|g\rangle \equiv c_1^\dagger c_2^\dagger c_3^\dagger \ldots c_A^\dagger |0\rangle$ is constructed by acting with $A$ fermion creation operators $c_h^\dagger$, $h = 1, A$, on the particle vacuum $|0\rangle$, and a possible choice is $E_{\alpha} = c_p^\dagger c_h$, with $1 \leq h \leq A$, $A < p \leq n$, and $N = A(n-A)$. 


If there are no "hole" states, and $|g\rangle \equiv |0\rangle$ is the particle (or quasiparticle in HFB) vacuum, then we may have $E_\alpha = c_{p_i}^\dagger c_{p_j}$, with $p_i < p_j = 1, n$, and $N = n(n - 1)/2$.

The operators $E_{\pm \alpha}$ can be used to generate a trial manifold $S^{RPA}$ represented by the set of functions

$$|\psi\rangle(Z) \equiv U(Z)|g\rangle,$$  \hspace{1cm} (4)

where

$$U(Z) = e^{\sum_{\alpha}(z_\alpha E_\alpha - z_\alpha^* E_{-\alpha})}$$  \hspace{1cm} (5)

is an unitary operator and $z_\alpha$ are $N$ complex variables. Therefore $S^{RPA}$ is parameterized by $2N$ real variables, denoted $x^i$, $i = 1, 2N$, so that for $i \leq N$, $x^i$ are $Re(z_\alpha)$, and for $N < i \leq 2N$, $x^i$ are $Im(z_\alpha)$.

The condition as $|g\rangle$, (or $x^i = 0$), to be a fixed point for Eq. (1) gives the static "mean-field" equations

$$\langle g|\left[H, E_{-\alpha}\right]|g\rangle = 0,$$  \hspace{1cm} (7)

automatically fulfilled in the HF or HFB case.

Let us consider now a small amplitude vibrational periodic orbit $\gamma$ around $|g\rangle$ with the period $T$, so that all $x^k$ perform harmonic oscillations. This means

$$z_\alpha = X_\alpha e^{-i\Omega t} + Y_\alpha e^{i\Omega t},$$

with $\Omega = 2\pi/T$, and $T$, $X_\alpha$, $Y_\alpha$ unknowns which should be determined from the equations of motion. For the orbit $\gamma$,

$$U(Z_t^\gamma) = \exp(e^{-i\Omega t} B^\dagger - e^{i\Omega t} B)$$  \hspace{1cm} (6)

with

$$B^\dagger = \sum_{\alpha}(X_\alpha E_\alpha - Y_\alpha^* E_{-\alpha})$$

and in the linear approximation Eq. (1) reduces to the RPA-type equation

$$\langle g|[\left[H, B^\dagger\right] - \hbar\Omega B^\dagger, E_{\pm \alpha}]|g\rangle = 0.$$  \hspace{1cm} (7)

The standard particle-hole or particle-particle RPA are recovered when $|g\rangle$ is the HF or HFB ground state and $E_\alpha = c_{p_i}^\dagger c_{p_i}$, respectively $E_\alpha = c_{p_i}^\dagger c_{p_j}$. From Eq. (7) one obtains the "normal mode frequency" $\Omega$ and a one-parameter
family of periodic orbits having this frequency. As a parameter we may consider for instance the energy

\[ \mathcal{E} = \langle g | U(Z)^{-1} H U(Z) | g \rangle . \]

However, if \( \mathcal{E} \) is too large, the amplitudes \( X, Y \) increase and Eq. (7) fails in approximating Eq. (1).

Until now the whole discussion was about the classical dynamics on \( S_{RPA} \).

To establish the connection with the quantum many-body system is necessary to requantize the RPA periodic orbits according to Eq. (2). To perform this integral we will assume that exists a Hermitian operator \( W \), so that \([W, B^\dagger] = B^\dagger \), although the explicit form of \( W \) is not necessary. If \( W \) exists, then

\[ U^{-1} i \partial_t U = \Omega (U^{-1} W U - W) = \Omega (e^{-i\Omega t} B^\dagger + e^{i\Omega t} B + [B, B^\dagger] + ... ) . \]

By integrating this sum over a period, the terms linear in \( B, B^\dagger \) vanish, and the first non-vanishing term is \( 2\pi [B, B^\dagger] \). Thus, the BWS quantization gives

\[ \langle g | [B, B^\dagger] | g \rangle = n . \]

For \( n = 1 \) (the first excited state), this coincides with the RPA "normalization" condition, an interesting result proved before using path integrals [10].

After quantization, the state associated by Eq. (3) to \( \gamma \) can be easily obtained expanding \( U \) in powers of \( B^\dagger, B \), and retaining only the linear terms. The result has the familiar form

\[ |\Omega\rangle = B^\dagger |g\rangle , \]

but by contrast to the RPA assumption, the excitation operator \( B^\dagger \) acts on the uncorrelated ground state. However, on particular examples it can be shown that if the whole expansion of \( U \) is considered, then \( |\Omega\rangle = B^\dagger P_{RPA} |g\rangle \), with \( P_{RPA} \) a Hermitian operator which gives an approximate projection of \( |g\rangle \) on the vacuum \( |RPA\rangle \) of \( B \), defined by \( B |RPA\rangle = 0 \). Moreover, if \( |g\rangle \) is symmetry breaking and \( \gamma \) is a related rotational orbit, then Eq. (3) gives \( P_s |g\rangle \), with \( P_s \) a symmetry restoring projection operator [5].

6
3 Isovector excitations in symmetry breaking nuclei

3.1 The two rotor model

The prediction of the isovector angular rotational oscillations [11] (scissors vibrations) in deformed nuclei has been particularly stimulating for the experimental research on the nuclear magnetism, leading to the discovery of low-lying M1 states. These states have been observed in high resolution $(e,e')$ scattering experiments on rare earths [12], $fp$-shell nuclei [13], and in actinides [14]. Their apparent weak excitation in intermediate energy proton scattering [15] has supported the orbital character predicted by the two rotor model (TRM), but the highly fragmented structure has generated a long standing debate about their real origin. On one side were the phenomenological models supporting the TRM picture, like CSM [16], or IBA-II [17], while on the other were the microscopic RPA or QRPA calculations, indicating that the observed M1 excitations are produced by only few quasi-particle pairs. Not less important for this debate was the difficulty to decide if the states obtained by microscopic calculations correspond or not to angular vibrations. Therefore, the problem of finding the appropriate microscopic correspondent for a specific collective motion appears to be important.

This problem can be solved by an RPA calculation based on special trial manifolds $S^{rot}$, generated by cranking, instead of $S^{RPA}$ defined in Section 2.2. Let us denote by $G_x$ the group of rotations around the X axis, $L_x$ the orbital angular momentum operator, and by $|g⟩$ the axially-deformed ground state of the microscopic Hamiltonian $H$. Then, the intrinsic ground state of a system rotating around the X-axis with angular momentum $\mathcal{L}$ is given by the solution $|Z_\omega⟩$ of the variational equation

$$\delta⟨Z|H - \omega L_x|Z⟩ = 0 \ .$$

The set of functions $|Z_\omega⟩$ represents a curve in $\mathcal{H}$ containing $|g⟩$ and parameterized by the Lagrange multiplier $\omega$, or, implicitly, by the angular momentum

$$\mathcal{L} = ⟨Z_\omega|L_x|Z_\omega⟩ \ .$$
The action of $G_x$ moves this curve over a surface in $H$ which contains the states
\[ |\psi\rangle(\phi, \omega) = e^{-i \phi L_z / \hbar} |Z_\omega\rangle \]
and defines the trial manifold $S^{rot}$.

In arbitrary variables $\{q, p\}$, the symplectic structure of $S^{rot}$ is given by the 2-form $\omega^{rot}$,
\[ \omega^{rot}_{qp} = 2\hbar \text{Im} \langle \partial_q \psi | \partial_p \psi \rangle = \partial_\omega \langle Z_\omega | L_x | Z_\omega \rangle = 1 \]
proving that $\phi$ and $L$ are canonical, and $S^{rot}$ is the phase space of the classical (plane) rotor.

In the case of a deformed nucleus, $S^{rot}$ can be constructed separately for protons and neutrons, and the trial wave function corresponding to the total phase space $S^{rot}_{pn} = S^{rot}_p \times S^{rot}_n$ is
\[ |\psi\rangle(\phi_p, \phi_n, \omega_p, \omega_n) = e^{-i(\phi_p L^p_x + \phi_n L^n_x) / \hbar} |Z^p_{\omega_p} | Z^n_{\omega_n} \rangle . \]

Let us consider now a schematic nuclear Hamiltonian
\[ H = \sum_{\mu, \nu} (h_0)_{\mu \nu} c^\dagger_{\mu} c_{\nu} - \frac{\chi_0}{2} (Q_{is} Q^\dagger_{is} + b Q_{iv} Q^\dagger_{iv}) \]
consisting of a spherical oscillator term ($h_0$ is the one-body spherical oscillator Hamiltonian with frequency $\omega_0 = 41 A^{-1/3} \text{MeV}/\hbar$), and the quadrupole-quadrupole (QQ) interaction, with both isoscalar and isovector components ($b \approx -0.6$). Then, for $S^{rot}_{pn}$ and $H$, Eq. (1) takes the form of the Hamilton system of equations for two rotors [18], having the cranking moments of inertia $I_p, I_n$, and interacting by a restoring elastic potential $C_\chi(\phi_p - \phi_n)^2/2$.

Worth noting,
\[ C_\chi = 3\chi_0 (1 - b) \langle Q^p_0 | Q^n_0 \rangle \approx 9\delta^2 A \text{ MeV} \]
appears related to the microscopic QQ interaction ($Q^p_0 = \sqrt{5/16\pi} \sum_{p,n} (2z^2 - x^2 - y^2)_{p,n}$), by contrast to the TRM estimate
\[ C_{TRM} \approx 6\delta^2 A^{4/3} \text{ MeV} \]
\[ ^2 \text{F. Palumbo, “The scissors mode”, in Proc. Int. School Symmetries and semiclassical features of nuclear dynamics, Poiana Brasov, Romania, 1986, Springer (1987), p. 230.} \]
related to the symmetry energy ($\delta$ is the deformation parameter)\(^3\). Moreover, Eq. (2) is identical with the BWS condition for the two-rotor system, giving (for $n = 1$) the quantized angular oscillation amplitudes $a_\tau$ [18],

$$a_\tau = \frac{1}{I_\tau} \sqrt{\frac{2\hbar I_\tau}{\Omega_\chi}}, \quad \tau = p, n, \quad I_\tau = \frac{I_p I_n}{I_p + I_n},$$

while the excitation energy is $E_x \equiv \hbar \Omega_{X} = \hbar \sqrt{|C_X|/I_\tau}$.

To apply Eq. (3), $|Z\rangle_\omega$ was approximated near $|g\rangle$ using a first order perturbative treatment of the cranking term. The ”scissors-like” state (not normalized) obtained from Eq. (3) has the form

$$|\Omega_\chi\rangle = \frac{1}{2\hbar} \frac{\Omega_\chi}{D} + 1 |a_p L_x^p - a_n L_x^n \rangle |g\rangle$$

(11)

with $D = |\delta|\omega_0$. This state gives the $B(M1)$ strength [18]

$$B(M1) = \frac{3}{4\pi\hbar} (g_p - g_n)^2 I_\tau D \mu_N^2.$$  

(12)

The comparison with the experiment is complicated by the ambiguities of separating the orbital and the spin strengths, and defining a reasonable sum over fragments. Assuming the dominance of the orbital strength at low energy, under 6 MeV in rare earths, and 4 MeV in actinides, the data are well reproduced by Eq. (12) with the irrotational moments of inertia. The range of the energy $E_x$ calculated by taking $I_\tau$ at the rigid or irrotational limits is not very large (below 2 MeV) and includes the data throughout all the mass regions investigated.

After normalization, the state $|\Omega_\chi\rangle$ becomes the same as the term independent of spin and pairing from the state $|ROT\rangle$, constructed before [19] to represent microscopically the scissors modes. However, this is not similar to an RPA state, because the ”excitation operator” $a_p L_x^p - a_n L_x^n$ is Hermitian. A quasiboson operator may be obtained if the cranking wave function $|Z_\omega\rangle$ is related to $|g\rangle$ by a unitary transformation. This problem is not easy, but it was solved recently [7] in terms of an unitary operator $U_\omega$ which relates the

\[^3\delta = \beta \sqrt{45/16\pi}. \] If $\langle Q_0^p \rangle_g = \langle Q_0^n \rangle_g$ then $\delta = 3\chi_0 c_0 (\langle Q_0^p \rangle_g / \hbar \omega_0) (c_0 = \sqrt{5/4\pi \hbar / m\omega_0})$. In $C_{\chi}$ [18], $\chi_0 c_0^2 = 96.3 A^{-5/3}$ MeV, which yields $\langle Q_{1s,0}^p \rangle_g \equiv \langle Q_{1s,0}^p \rangle_g + \langle Q_{1s,0}^n \rangle_g = 0.18 \delta A^{5/3} \text{ fm}^2$, half the liquid drop estimate $A R_0^2 \delta / \sqrt{5\pi} = 0.36 \delta A^{5/3} \text{ fm}^2$, with $R_0 = 1.2 A^{1/3}$ fm.
cranked anisotropic oscillator eigenstates to the eigenstates of a spherical harmonic oscillator. Thus, if $\omega$ is not larger than $\omega_s\sqrt{3}/2$, \(\omega_s = \sqrt{(\omega^2_y + \omega^2_z)/2}\), then we have

$$h_0 - \frac{\delta}{3}m\omega_0^2(2z^2 - x^2 - y^2) - \omega l_x = U_\omega h_s U_\omega^{-1}$$

with

$$U_\omega = e^{-i\lambda c_x} \exp(-i \sum_{k=x,y,z} \theta_k s_{2,k}) .$$

In the left-hand side of Eq. (13) the first two terms correspond to an anisotropic harmonic oscillator with the frequencies $\omega_x = \omega_y = \omega_0\sqrt{1 + 2\delta/3}$, $\omega_z = \omega_0\sqrt{1 - 4\delta/3}$, while

$$h_0 = \sum_{k=x,y,z} \hbar \omega_0(b_k^\dagger b_k + 1/2), \quad b_k^\dagger = \sqrt{m\omega_0/2\hbar}(x_k - ip_k/m\omega_0)$$

$$h_s = \sum_{k=x,y,z} \hbar \Omega_k(b_k^\dagger b_k + 1/2), \quad b_k^\dagger = \sqrt{m\omega_s/2\hbar}(x_k - ip_k/m\omega_s)$$

$$l_x = i\hbar(b_y b_z^\dagger - b_z b_y^\dagger) \quad c_x = b_y^\dagger b_z + b_z^\dagger b_y, \quad s_{2,k} = i((b_k^\dagger)^2 - (b_k^\dagger)^2)/4$$

and the parameters $\lambda$, $\theta_k$ of $U_\omega$ are given by

$$\tan 2\lambda = 2\omega/\omega_s \eta \quad \sinh \theta_k = \omega_s(1 - \omega^2_k/\omega^2_s)/2\Omega_k \quad \eta = (\omega^2_y - \omega^2_z)/2\omega^2_s$$

$$\Omega_x = \omega_x \quad \Omega^2_{y,z} = (\omega_s + \epsilon_y, \epsilon_z)^2 - (\omega_s \eta/2)^2$$

with $\epsilon_y = -\epsilon_z = \omega_s \eta/2 \cos 2\lambda$.

The operators $s_{2,k}$, $k=x,y,z$, are the "squeezing" generators, and they produce the transition from a spherical to a deformed basis. This transition corresponds to the "cranking" of $h_0$ by the term $\delta m\omega_0^2(2z^2 - x^2 - y^2)/3$, proportional to $Q_0$. Therefore, the unitary operator $U_0$ with $\theta_k$ as variables may be used to generate trial manifolds for the treatment of the quadrupole vibrations$^4$ ($E_x \sim 2\hbar \omega_0$). The commutation relations between $s_{2,k}$ and $b_k^\dagger b_k$ get closed to the su(1,1) ($\approx$sp(1,R)) algebra [20].

\footnote{details can be found in Section III of the article M. Grigorescu, N. Cărjan, Dissipative shape dynamics in the sd shell, Phys. Rev. C 54 706 (1996).}

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The operator $c_x$ generates the shift from a static frame to a frame rotating around the X-axis with the angular velocity $\omega$, and it appears as an "angle" operator conjugate to $l_x$. Analog operators, $c_y$, $c_z$ are associated to $l_y$ and $l_z$, and by commutation $c_x, c_y, c_z$ and $l_x, l_y, l_z$ generate an $su(3)$ algebra\(^5\). Similarly, $s_{2,x}, s_{2,y}, s_{2,z}$ and $l_x, l_y, l_z$ generate by commutation a $gl(3,R)$ algebra. This rather complicated set of algebras is included in the symplectic Lie algebra $sp(3,R)$ [21].

In the many-fermion case the one-body operators $h, h_0, h_s, l_k, c_k, s_k$ become particle-hole operators, denoted $H, H_0, H_s, L_k, C_k, S_k$, and may be used to write $|Z\rangle_\omega$ as

$$|Z\rangle_\omega = e^{-i\lambda C_x} \exp(-i \sum_{k=x,y,z} \theta_k S_{2,k}) |g\rangle_s.$$  

This form is especially suited to study large amplitude vibrations, but corrections to Eq. (11) appear already in the linear approximation [7]. If the dependence of $\theta_k$ on $\omega$ is neglected, then during the scissors vibration each $|Z\rangle_\omega$ in Eq. (9) changes in time only due to the factor of $C_x$, and Eq. (3) gives $|\Omega_\chi\rangle = B^\dagger_\chi |g\rangle$, with [7]

$$B^\dagger_\chi = \frac{1}{2\hbar}[a^p L^p_x - a^n L^n_x - \frac{i\hbar}{\omega_y - \omega_z}(a^p C^p_x - a^n C^n_x)].$$  

(15)

Unexpectedly, though $a^{p,n}$ are the same angular amplitudes as in Eq. (11), $B^\dagger_\chi$ is normalized in the RPA sense, $\langle g|[B_\chi, B^\dagger_\chi]|g\rangle = 1$. In fact, if the particle-hole excitations between different oscillator shells are neglected, then $\text{sgn}(\eta)i\hbar C_x |g\rangle \approx -L_x |g\rangle$, and $B^\dagger_\chi |g\rangle$ reduces to Eq. (11).

The operator $B^\dagger_\chi$ was obtained recently also by the canonical quantization of the TRM in relative coordinates [22], and it was proved to support the interpretation of all low-lying orbital $1^+$ excitations as a scissors mode.

It is interesting to note that if the dependence of $\theta_k$ on $\omega$ is taken into account\(^6\), then $B^\dagger_\chi$ will contain operators from $gl(3,R)$. Such terms are related to the excitation operator proposed long time ago by Hilton [23]. Instead of constructing $B^\dagger_\chi$ within the "angular momentum & shift" algebra, $su(3)$, he proposed a combination within the "angular momentum & squeezing" algebra, $gl(3,R)$, between $l_y$ and the "shear" generator $zp_x + xp_z \sim i(b^\dagger_z b^\dagger_x - b_z b_x)$.

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\(^5\)presented e.g. in math-ph/0007033.

\(^6\)M. Grigorescu, D. Rompf, W. Scheid, Dynamical effects of deformation in the coupled two-rotor system, Phys. Rev. C 57 1218 (1998).
3.2 The isovector Josephson oscillations

The combined effect of the proton-neutron interaction and breaking of the translational or rotational symmetries is related to the giant dipole resonance [24] or to the ”scissors modes”, respectively. However, there is one more important symmetry breaking in nuclei for which such type of isovector collective motion was not yet observed, and this appears when the nuclei are ”superfluid”.

The ground state of a superfluid system accounts for the $pp$ correlations produced by the pairing interaction, and is well approximated by a BCS function. For a single $j$-shell, the pairing Hamiltonian and the BCS function are

$$H_0 = \epsilon N - \frac{G}{4} P^\dagger P , \quad |BCS\rangle (\varphi, \rho) = e^{(zP^\dagger - z^* P)} |0\rangle ,$$

where $z = \rho e^{-i\varphi}$, $\varphi$ is the BCS ”gauge” angle, $N = \sum_{m=-j}^j c_m^\dagger c_m$ is the particle-number operator, and

$$P^\dagger = \sum_{m=-j}^j (-1)^{j-m} c_m^\dagger c_m^\dagger$$

is the pair creation operator.

The angle $\varphi$ is not a constant, and a superfluid system in the ground state performs a free gauge rotation with the angular velocity $\dot{\varphi} = \frac{2\epsilon_F}{\hbar}$, twice the Fermi frequency. In a nucleus, the proton and neutron systems are not isolated, but change particles until the Fermi energies $\epsilon_F^p, \epsilon_F^n$ become equal. Thus, we may see this as an indication about the existence of a phenomenological ”gauge restoring interaction”, which tends to fix the relative gauge angle $\varphi_p - \varphi_n$ at a constant value. If this is true, and there is an interaction between pairs of protons and neutrons, then oscillations of the protons against neutrons in the BCS gauge space should appear.

A Josephson-like proton-neutron interaction\(^7\)

$$H_{pn} = -\frac{\sigma}{4} (P_p^\dagger P_n + P_n^\dagger P_p)$$

may be related to the isospin symmetry breaking mean-field for a four particle interaction [25]. The problem of the mean-field created by the four particle interaction was considered also in M. Gerçeklioğlu, A model for the doublets of the $K^*=0^+$ states in deformed nuclei, Acta Phys. Slovaca 52 161 (2002).
interaction is not new, but previously [26] the main interest was for terms
\( \sim P_p^\dagger P_n^\dagger \), assumed to represent \( \alpha \) clusters, while terms like \( P_p^\dagger P_n \) were
neglected. Because \( H_{pn} \) does not commute with the isospin \( T_0 = (N_n - N_p)/2 \), it produces an “isorotational” term \( k_\sigma (N - Z)^2 \) in the total energy. This
means a term in the symmetry energy \( k_\sigma (N - Z)^2 \), \( (k_W = 28/A \text{ MeV}) \), from
the Weizsäcker mass formula with a ”dynamical” origin, beside the ”kinematic” one determined by the Pauli principle [27]. Thus, \( \sigma \) can be obtained
by a fit of the symmetry energy produced by the Hamiltonian
\[
H = H_p^0 + H_n^0 + H_{pn}
\] (17)
in a single j-shell. Considering the case of 1d\(_{3/2}\) nuclei, \( \sigma \) was estimated to be \( \approx 2.7/A \text{ MeV} \) [25]. However, an approximation of j-shells with high
degeneracies suggests a value about ten times larger [28].

The interaction \( H_{pn} \) contributes to the symmetry energy in all nuclei, but
in the superfluid nuclei it produces also a restoring potential \( C_\sigma (\varphi_p - \varphi_n)^2/2 \)
for the BCS angles. This potential can be related to \( H_{pn} \) by a treatment
similar to the one applied to the QQ interaction responsible for the orbital
scissors modes. Indeed, the BCS functions define symplectic manifolds \( S^{BCS} \)
which can be parameterized by \( \varphi \) and \( \rho \), or by the canonical variables \( \varphi \) and
\[
p = \langle BCS|N|BCS\rangle/2 = (j+1/2) \sin^2 2\rho
\].

In these variables
\[
\omega_{\varphi p}^{BCS} = 2h \text{Im} \langle \partial_{\varphi} BCS(\varphi, \rho)|\partial_p BCS(\varphi, \rho) \rangle = h \partial_p \langle (BCS|N/2|BCS) \rangle = h
\].

For a proton-neutron system the trial manifold will be represented by the
product \( S^{BCS}_p \times S^{BCS}_n \), and the collective motion determined for \( H \) by Eq.
(1) shows the occurrence of isovector ”gauge-angles” vibrations [25]. For a
half-filled shell, the fixed point in (1) corresponds to the ground state of \( H \),
\[
|g\rangle = e^{\frac{i}{2}(P^\dagger + N^\dagger - P - N)}|0\rangle
\] (18)
and the gap parameter \( \Delta = G\langle g|P^\dagger g\rangle/2 \) is the same both for protons and
neutrons. The restoring potential has the constant \( C_\sigma = 2\sigma (\Delta/G)^2 \), and the
excitation energy for the isovector oscillations is \( E_\sigma = h\Omega_\sigma = 4\sqrt{2k_W C_\sigma} \) [25].

The excitation operator defined by Eq. (3) has the form
\[
B^\dagger_\sigma = \frac{1}{2j+1} \sqrt{\frac{E_\sigma}{\sigma}} [T_0 - \frac{\sigma(2j+1)}{4E_\sigma}(P^\dagger - P - N^\dagger + N)]
\] (19)
and by accident, is the same as the one provided by the standard QRPA [29]. However, the RPA vacuum defined by $B_σ|RPA⟩ = 0$ exists only if $j + 1/2$ is an even integer [28].

The states generated by $B^\dagger$ are isovector monopoles, and correspond to Josephson oscillations between the proton and neutron superfluids\(^8\). Such oscillations might be excited by the Coulomb interaction in electron scattering [28], or by the current of pairs between the two superfluids produced in pion double charge-exchange (DCX) reactions. The importance of Josephson-type correlations in DCX reactions was proved first by shell-model calculations [30], suggesting that the "scissors modes" in gauge space discussed here are worth of experimental investigation.

4 Summary

In this lecture I presented a microscopical approach to the collective motion, based essentially on the time-dependent variational principle and GIPQ re-quantization (Section 2.1), but which is peculiar by the choice of the trial functions. The trial manifolds are supposed to have the phase-space (symplectic) structure of a specific collective model, and for the symmetry breaking nuclei are constructed using the cranking procedure [7]. The Hamilton equations of motion (Eq. (1)) appear by a constrained variational calculation in the Hilbert space, rather than by semiclassical approximations ($\hbar \to 0$). This formalism includes the standard RPA or QRPA (Section 2.2), and was applied with success to the treatment of the collective isovector excitations (Section 3). It solves the problems of the inertial parameters, restoring force constants, and of the microscopic analog for a particular collective motion (here the scissors vibrations). Moreover, isovector vibrations in the BCS gauge space of the superfluid nuclei were predicted. The variational formulation is appropriate to account for the coupling between a quantum system and a thermal environment [31]. Therefore, the study of giant resonances using a Langevin form of Eq. (1), with noise and memory-friction terms in the right-hand side, appears highly interesting.

\(^8\)At $σ \ll G$ an odd-even effect in the total number of pairs can appear (M. Grigorescu, Low-lying excitations in superconducting bilayer systems, cond-mat/9904242, or High-Tc Update 13 No. 10, May 15 (1999), or Can. J. Phys. 78 119 (2000)).
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