Calculation of the $^{12}$C + $^{12}$C sub-barrier fusion cross section in an imaginary-time-dependent mean field theory

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INTRODUCTION&MOTIVATION

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An increase in the $^{12}$C + $^{12}$C fusion rate from resonances at astrophysical energies

A. Tumino1,2, C. Spitaleri2,3, M. La Cognata2, S. Cherubini2,3, G. L. Guardo2,4, M. Gulino1,2, S. Hayakawa2,5, J. Indelicato2, L. Lamia2,3, H. Petracca4, G. Pizzone3, S. M. r. Puglia2, G. G. rapisarda2, S. romano2,3, M. L. Sergi2, r. Spartá3, and L. Trache4

Status on $^{12}$C + $^{12}$C fusion at deep subbarrier energies: impact of resonances on astrophysical $S^*$ factors

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Eur. Phys. J. A (2020) 56:87

$$S^*(E_{\text{c.m.}}) = E_{\text{c.m.}} \sigma(E_{\text{c.m.}}) \exp(87.12 E_{\text{c.m.}}^{-1/2} + 0.46 E_{\text{c.m.}})$$

$$= S(E_{\text{c.m.}}) \exp(0.46 E_{\text{c.m.}})$$  \hspace{1cm} (1)
Feynman path integration in phase space

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Solve the Vlasov equation in imaginary time. Define collective variables R&P

\[
\begin{align*}
\{ R_A \} &= \int dr dp \{ r_p \} f(r, p; t) \\
\{ P_A \} &= \int dr dp \{ p_r \} f(r, p; t)
\end{align*}
\]

\[
\begin{align*}
\frac{dR_A}{dt} &= \frac{P_A}{m} \\
\frac{dP_A}{dt} &= F_A
\end{align*}
\]

in imaginary time \( t \to it \)

\[
\begin{align*}
\frac{dR^i_A}{dt} &= \frac{P^i_A}{m} \\
\frac{dP^i_A}{dt} &= -F_A
\end{align*}
\]

\( E_{\text{c.m.}} = 3.5 \text{ MeV} \)

NEWTONIAN DYNAMICS OF TIME-DEPENDENT MEAN FIELD THEORY

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Phys.Lett.B141(1984)9; 168B(1986)35.

Nuclear Physics A439 (1985) 353–370
The probability of fusion for the $l$ th-partial wave is given by $T_l = 1/(1 + \exp\{2A\})$, $A = \int_1^2 P \, dR$.

To take into account resonances modify the Bass potential as:

$$V_B \rightarrow V_B [1 + g(x, \gamma, \sigma)],$$

Analytical formula

$$S_0 = S_G e^{4\sqrt{2\mu Z_1 Z_2 e^2 R_N}/\hbar}. $$

$$S_G = \pi \hbar^2/(2\mu)$$

S. Kimura and A. Bonasera, Phys. Rev. C 76, 031602(R) (2007).
Last but not least, $S$ and $S^*$—what if we use the action $A$ instead?

\[ A = \frac{1}{2} \ln \left[ \frac{\pi \hbar^2}{2E_{cm} \sigma(E_{cm})} - 1 \right] \mid_{l=0} \]

Gamow limit

\[ A_G = e^2 \pi Z_T Z_P \sqrt{\frac{\mu}{2E_{CM}}} \].

S. Kimura and A. Bonasera, Phys. Rev. C 76, 031602(R) (2007).
Conclusions

The Neck model and the Vlasov approach in imaginary time give $S^* > 16 \text{MeVb}$ for $E_{cm} > 0.5 \text{MeV}$ (agrees with analytical formula as well)

Adding resonances is in some agreement with the THM

$l=0$ channel is dominant up to $E_{cm} = 3 \text{MeV}$

if the properties of the resonances (spin, width etc..) are confirmed then:

THANKS