THE RELATIVISTIC IMPULSE APPROXIMATION FOR THE EXCLUSIVE ELECTRODISINTEGRATION OF THE DEUTERON

S. G. Bondarenko, V. V. Burov, E. P. Rogochaya,1 and A. A. Goy2

1Joint Institute for Nuclear Research, 141980, Dubna, Moscow region, Russia
2Far Eastern National University, 690950, Vladivostok, Russia

The electrodisintegration of the deuteron in the frame of the Bethe-Salpeter approach with a separable kernel of the nucleon-nucleon interaction is considered. This conception keeps the covariance of a description of the process. A comparison of relativistic and nonrelativistic calculations is presented. The factorization of the cross section of the reaction in the impulse approximation is obtained by analytical calculations. It is shown that the photon-neutron interaction plays an important role.

PACS numbers: 25.30.Dh

I. INTRODUCTION

Study of static and dynamic electromagnetic properties of light nuclei enables more deeply to understand a nature of strong interactions and, in particular, nucleon-nucleon (NN) interactions. Deuteron as a two nucleon system is a simplest object for the NN interaction investigation.

First experimental data got in 1960-th [1] were satisfactorily described in the framework of the nonrelativistic formalism. But when the precision of experiments and energies of particles taking part in the reaction increased nonrelativistic models failed. So it became clear that relativistic effects (which a priori are very important at large transfer momenta) are needed to include in the consideration.

Besides the extraction from experiments with light nuclei of the information about a structure of bounded nucleons requires to take into account relativistic kinematics of the reaction and dynamics of interaction. So the construction of a covariant approach and detailed analysis of relativistic effects in electromagnetic reactions with light nuclei are very
important. It can not be performed in the nonrelativistic picture. This understanding is an additional reason for construction of the relativistic approach.

The electrodisintegration of the deuteron at the threshold has been of interest of an investigation for a long time [2]-[7]. The reason is that the electrodisintegration is an essential instrument for study a structure of a two-nucleon system. First of all it is an electromagnetic structure. The deuteron has been used as a neutron target to get the information about neutron electromagnetic form factors. During last 20 years it has also been used to receive constraints on available realistic NN potentials. Analyzing of the electrodisintegration process we can clarify the role of non-nucleonic degrees of freedom. The deuteron is one of convenient candidates because complete calculations can in principle be performed.

The experimental results on the differential cross section derived from \((ed \rightarrow enp)\) reaction are available up to a momentum transfer of about 1 GeV [8]-[13]. This situation is very good for investigation of the deuteron structure at short distances with the allowance for some exotic effects which have not been earlier important. First of all these are the quark degrees of freedom (see [14], [15], for instance) but formerly it is necessary to take into account relativistic effects.

There are several approaches for the theoretical description of the deuteron and, in particular, the deuteron break-up reaction. One group uses a numerical solving of a relativistic wave equation for NN system based on the relativistic one-boson-exchange (OBE) model (see, for instance, [4]-[5]). Other one used a simple phenomenological approach by adding lowest-order relativistic corrections to the non-relativistic one-body current and including the kinematic wave function boost [2] or the covariant models based on the direct evaluation of those Feynmann diagrams which give the dominant contributions in the quasi-free region [16]. These is also the approach with the using the Paris potential [6].

The Bethe-Salpeter (BS) approach [17] can give a possibility to consider relativistic effects by consistent way [18]. In the paper the deuteron electrodisintegration within the covariant BS approach with the separable Graz II interaction kernel is presented. The exclusive differential cross section is calculated in the relativistic impulse approximation (RIA) with plane waves in final \(np\)-state.

The paper is organized as follows. In section II the relativistic kinematics of the reaction is considered, formula for the cross section is presented. Sections III and IV are devoted to the BS amplitude of the deuteron and \(np\)-pair. The relativistic consideration of the
hadron current in the BS formalism is defined in section V. In section VI the nonrelativistic approach is taken up. Factorization of the cross section is discussed in section VII. Then the results of our numerical calculations are presented in section VIII. Finally we review the results and outline further plans.

II. CROSS SECTION AND KINEMATICS

Let us consider the relativistic kinematics of the exclusive electrodisintegration of the deuteron. The initial electron \( l = (E, \mathbf{l}) \) collides with the deuteron in rest frame \( K = (M_d, \mathbf{0}) \) \((M_d \) is a mass of the deuteron). There are three particles in the final state, i.e. electron \( l' = (E', \mathbf{l}') \) and pair of proton and neutron. Neglecting an electron mass in one photon approximation we can express a squared momentum of the virtual photon \( q = (\omega, \mathbf{q}) \) via electron scattering angle \( \theta \)

\[
q^2 = -Q^2 = (l - l')^2 = \omega^2 - q^2 = -4|l||l'| \sin^2 \frac{\theta}{2}.
\]

(1)

np-pair is described by the invariant mass \( s = P^2 = (p_p + p_n)^2 \) which can be written through components of photon four-impulse:

\[
s = M_d^2 + 2M_d \omega + q^2.
\]

(2)

Lorentz invariant matrix element of the reaction (see Fig. 1) can be written as a product of lepton and hadron currents

\[
M_{fi} = -ie^2(2\pi)^4 \delta^4(K - P + q) < l', s'_e|j^\mu|l, s_e > \frac{1}{q^2} < np : (P, S_{nS})|J_\mu|d : (K, M) >, \tag{3}
\]

where \(< l', s'_e|j^\mu|l, s_e > = \bar{u}(l', s'_e)\gamma^\mu u(l, s_e)\) is an electromagnetic current (EM). Dirac spinor \( u(l, s_e) \) \((\bar{u}(l', s'_e))\) describes the initial (final) electron with spin projection \( s_e \) \((s'_e)\). The hadron current \(< np : (P, S_{nS})|J_\mu|d : (K, M) >\) is a transition matrix element from the initial deuteron \(|d : (K, M) >\) with total momentum \( K \), projection \( M \) to the final \( np\)-pair \(|np : (P, S_{nS}) >\) with total momentum \( P \) and spin \( S \), projection \( m_S \). The unpolarized cross section of the electrodisintegration of the deuteron can be easily written as a production of electron \( l^{\mu\nu} \) and hadron \( W^{\mu\nu} \) parts:

\[
\frac{d^5 \sigma}{dE'd\Omega'd\Omega_p} = \frac{\alpha^2}{8M_d(2\pi)^3 |l|} \frac{\sqrt{s}}{q^4} R l^{\mu\nu} W^{\mu\nu} \tag{4}
\]
with some kinematical factor $R$ which connects the final proton angle in the center-of-mass system (C.M.S.) (where the $np$-pair is rest) with the same in the laboratory system (L.S.) $p$:

$$R = \frac{p^2}{\sqrt{1 + \eta|p| - e_p \sqrt{\eta \cos \theta_p}}, \quad (5)$$

$e_p = \sqrt{p^2 + m^2}$, $\theta_p$ is an angle between the final proton and $Z$-axis, $m$ is a nucleon mass, and $\eta = q^2/s$.

Tensor of unpolarized leptons in (4) is expressed in a standard form

$$l_{\mu\nu} = \frac{1}{2} \sum_{s_e, s'_e} <l', s'_e|j_\mu^\dagger|l, s_e> <l, s_e|j_\nu|l', s'_e> = 2(l'_\mu l_\nu + l'_\nu l_\mu) + g_{\mu\nu}q^2 \quad (6)$$

and hadron tensor can be written as a production of hadron currents with averaging-out by the deuteron angle momentum

$$W_{\mu\nu} = \frac{1}{3} \sum_{M \& S_M} <d : (K, M)|J_\mu^\dagger|np : (P, S_m)| > < np : (P, S_m)|J_\nu|d : (K, M) > . \quad (7)$$

In most cases in order to average on initial and sum on final states it is convenient to introduce a helicity tensor which can be directly connected with structure functions (see, for example [2],[5],[7]). These quantities allow to calculate polarization and asymmetry observables easily and will be necessary in future (we didn’t calculate them in this work). Keeping in mind the Hermitian properties of the lepton and the hadron tensors the cross section can be rewritten as

$$\frac{d^5\sigma}{dE'd\Omega'd\Omega_p} = \frac{\sigma_{Mott}}{8M_d(2\pi)^3} \sqrt{s}R \times \left[ l_{00}^0 W_{00} + l_{++}^0 (W_{++} + W_{--}) + l_{+-}^0 2ReW_{+-} - l_{0+}^0 2Re(W_{0+} - W_{0-}) \right], \quad (8)$$

where $\sigma_{Mott} = (\alpha \cos \frac{\theta}{2}/2E \sin^2 \frac{\theta}{2})^2$ is Mott cross section for point-like particles and

$$l_{00}^0 = \frac{Q^2}{q^2}, \quad l_{0+}^0 = \frac{Q}{|q|\sqrt{2}} \sqrt{\frac{Q^2}{q^2} + tg^2 \frac{\theta}{2}}, \quad l_{++}^0 = \frac{1}{2} tg^2 \frac{\theta}{2} + \frac{Q^2}{4q^2}, \quad l_{+-}^0 = -\frac{Q^2}{2q^2}. \quad (9)$$

So the calculation of the cross section (8) comes to the calculation of the hadron tensor $W_{\mu\nu}$ which describes the NN interaction in the deuteron and is a main subject of our investigation.
III. BETHE-SALPETER AMPLITUDE OF THE DEUTERON

In the Bethe-Salpeter approach the deuteron as a system of two bounded particles can be described by the amplitude (BSA) $\Phi_M(k; K)$ which satisfies the Bethe-Salpeter equation. For the details of using formalism we refer to [18]. Here we present the BS vertex function for the $^3S_{1}^{++} - ^3D_{1}^{++}$ waves used in the calculations (for a rest deuteron $K = (M_d, 0)$):

$$\Gamma_M(k_0, k) = \Gamma_{^3S_{1}^{++}}(k)g_{^3S_{1}^{++}}(k_0, |k|) + \Gamma_{^3D_{1}^{++}}(k)g_{^3D_{1}^{++}}(k_0, |k|),$$  

(10)

where the spin-angular parts can be written as

$$\Gamma_a(k) = \frac{1}{\sqrt{8\pi^2|k|(m + \gamma_1)}}(m + q_2\gamma)\frac{1 + \gamma_0}{2}G_{aM}(k)(m - q_1\gamma),$$  

(11)

with

$$G_{aM}(k) = \begin{cases} 
\xi_M\gamma, & a = ^3S_{1}^{++} \\
\xi_M\gamma + 3/(2\sqrt{2}k^2)(q_1\xi_M)(q_1\gamma - q_2\gamma), & a = ^3D_{1}^{++} 
\end{cases}$$

The on-mass-shell four-vectors $q_1, q_2$ are connected with the relative momentum $k = (k_0, k)$ as $q_1 = (e_k, k), q_2 = (e_k, -k)$. For radial parts of the BS vertex function $g_a(k_0, |k|)$ we use the covariant Graz II kernel of $NN$-interaction.

IV. BETHE-SALPETER AMPLITUDE OF THE np-PAIR

The BSA of the np-pair $\chi_{Sm_s}(p; \hat{p}, P)$ satisfies the inhomogeneous equation

$$\chi_{Sm_s}(p; \hat{p}, P) = \chi_{Sm_s}^{(0)}(p; \hat{p}, P) + \frac{i}{4\pi^3}S_2(p; P) \int d^4k V(p, k; P)\chi_{Sm_s}(k; p, P),$$  

(12)

with $\hat{p} \cdot P = 0$ and $\hat{p}^2 = -s/4 + m^2$ putting the outgoing particles onto the mass shell and $S_2(p; P) = S^{(1)}(P/2 + p)S^{(2)}(P/2 - p)$. The first term $\chi_{Sm_s}^{(0)}(p; \hat{p}, P)$ in Eq. (12) is an amplitude which describes the free motion of two nucleons:

$$\chi_{Sm_s}^{(0)}(p; \hat{p}, P) = (2\pi)^4\delta^{(4)}(p - \hat{p})\chi_{Sm_s}^{(0)}(\hat{p}, P),$$  

(13)

where

$$\chi_{Sm_s}^{(0)}(\hat{p}, P) = \sum_{m_1m_2} C_{Sm_s}^{m_1m_2} u_{m_1}(P_1)u_{m_2}(P_2),$$  

(14)
here four-momenta $p_1, p_2$ are in on-mass-shell form $p_1 = (e_p, \mathbf{p}_p), p_2 = (e_n, \mathbf{p}_n)$.

Neglecting second part in Eq. (12) we introduce the plane wave approximation (PWA). Thus the final nucleons are described by plane waves Eq. (14) and we can write in the matrix representation for the conjugated function

$$\chi^{(0)}_{Sm_s}(\hat{p}, P) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2e_n(m + e_n)}} \frac{1}{\sqrt{2e_p(m + e_p)}} (m - p_2\gamma) \left[ -\gamma_5 \right] \frac{1 + \gamma_0}{2} (m + p_1\gamma). \quad (15)$$

First (second) line in the brackets stands for $S = 0$ ($S = 1$) case and four-vector $\xi_{m_s}$ describes the polarization of the $np$-pair.

V. HADRÓN ELECTROMAGNETIC CURRENT

If the deuteron wave function is known we are able to write the matrix element of the hadron electromagnetic current with the BS amplitude using Mandelstam technique [19]

$$<np : (P, Sm_S)|J_\mu|d : (K, M) > = i \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \chi_{Sm_s}(p; \hat{p}, P) \Lambda_\mu(p, k; P, K) \Phi_M(k; K). \quad (16)$$

We consider the process of the electrodisintegration of the deuteron in RIA (see Fig. 2). In our further calculation only one-body currents are taken into account

$$\Lambda_\mu^{[1]}(p, k; P, K) = i(2\pi)^4 \left\{ \delta^{(4)}(p - k - \frac{q}{2}) \Gamma^{(1)}_\mu \left( \frac{P}{2} + p, \frac{K}{2} + k \right) S^{(2)} \left( \frac{P}{2} - p \right)^{-1} + \delta^{(4)}(p - k + \frac{q}{2}) \Gamma^{(2)}_\mu \left( \frac{P}{2} - p, \frac{K}{2} - k \right) S^{(1)} \left( \frac{P}{2} + p \right)^{-1} \right\} \quad (17)$$

(here the total and relative momenta are introduced: $P = p_1 + p_2, K = k_1 + k_2, k = \frac{1}{2}(k_1 - k_2), p = \frac{1}{2}(p_1 - p_2)$).

In this case the matrix element of the hadron current has the following form

$$<np : (P, Sm_S)|J_\mu|d : (K, M) > = i \sum_{\ell = 1, 2} \int \frac{d^4p}{(2\pi)^4} \chi_{Sm_s}(p; P) \Gamma_\mu^{(\ell)}(q) S^{(\ell)} \left( \frac{P}{2} - (-1)^\ell p - q \right) \Gamma_M \left( p + (-1)^\ell \frac{q}{2}; K \right). \quad (18)$$

Note that $\gamma_{NN}$- vertex was taken on-mass-shell

$$\Gamma_\mu^{(\ell)}(p', p) \longrightarrow \Gamma_\mu^{(\ell)}(q) = \gamma_\mu F_1^{(\ell)}(q^2) - \frac{1}{4m} (\gamma_\mu q_\gamma - q_\gamma \gamma_\mu) F_2^{(\ell)}(q^2). \quad (19)$$
Here $F_1^{(\ell)}$ ($F_2^{(\ell)}$) - Dirac (Pauli) form factor of the nucleon which obeys the next normalization conditions

\[
F_1^{(1)}(0) = 1, \quad F_2^{(1)}(0) = \kappa_p, \\
F_1^{(2)}(0) = 0, \quad F_2^{(2)}(0) = \kappa_n,
\]

(20)

$\kappa_p$ ($\kappa_n$) is the anomalous proton (neutron) magnetic moment.

Using (13) and integrating (18) over $p$ we obtain our basic PWA RIA (see Fig. 3) expression for the hadron current

\[
< np : (P, S_m) | J_\mu | d : (K, M) >= \\
i \sum_{\ell=1,2} \chi_{S_m}^{(0)}(\hat{P}, P) \Gamma_\mu^{(\ell)}(q) S^{(\ell)} \left( \frac{K}{2} - \hat{P} - (-1)^{\ell} \frac{q}{2} \right) \Gamma_M \left( \hat{P} + (-1)^{\ell} \frac{q}{2}; K \right).
\]

(21)

It has very simple form. And to get it we should just perform the analytical calculation of the trace. For this purpose we use the REDUCE system.

**VI. NONRELATIVISTIC FORMALISM**

The results obtained within the relativistic frame are compared with nonrelativistic calculations. In this case the nonrelativistic Graz II interaction kernel [20] for the deuteron consideration is used. The matrix element for the hadron current

\[
< S_m | j_\mu(q) | 1M >= \int dx \ e^{-iqx} < S_m | j_\mu(x) | 1M >
\]

is constructed according to a standard nonrelativistic description with using the translational invariance property

\[
j_\mu(x) = e^{iPx} j_\mu(0) e^{-iPx}.
\]

(23)

Here $P$ is a total deuteron impulse. The four-dimensional current is defined by the expression

\[
\hat{J}_\mu(0, r_1, r_2) = \left( \hat{\rho}(0, r_1, r_2), \hat{J}(0, r_1, r_2) \right)
\]

(24)

where

\[
\hat{\rho}(x, r_1, r_2) = \sum_{i=1,2} e_i \delta(x - r_i),
\]

\[
\hat{J}(x, r_1, r_2) = \sum_{i=1,2} \left\{ \frac{e_i}{2m_i} \left[ \psi_\beta^* (\nabla_i \psi_\alpha) - (\nabla_i \psi_\beta^*) \psi_\alpha \right] + \frac{e_i}{2m_i} \kappa_i \nabla_i \times [ \psi_\beta^* \sigma_i \psi_\alpha ] \right\} \delta(x - r_i).
\]
Here summation over nucleons $i$ in the deuteron is performed, $\kappa_i$ is the anomalous magnetic moment of the corresponding nucleon. The first term in $\hat{J}$ corresponds to an electric transition, the second one - to a magnetic transition. For the deuteron we use the following wave function

$$\psi_{1M}(\mathbf{p}) = \frac{1}{\sqrt{4\pi}} \chi_{1M} u(p) + \sum_{m\mu} C_{2m1\mu}^{1M} \mathcal{Y}_{2m}(\hat{\mathbf{p}}) \chi_{1\mu} w(p).$$

(25)

After some transformations in PWA (13) we get the hadron matrix elements:

for the charge operator:

$$<S_M|\hat{\rho}(0)|1M> = \frac{1}{m} \sum_{i=1,2} F^{(i)}_1(q^2) \left[ \frac{1}{\sqrt{4\pi}} \delta_{M,M_i} u(|\mathbf{p}_i|) + \sum_m C_{2m1m_i}^{1M} \mathcal{Y}_{2m}(\hat{\mathbf{p}}_i) w(|\mathbf{p}_i|) \right],$$

for the electric component of current operator:

$$<S_M|\hat{J}_{el}(0)|1M> = \frac{1}{m} \sum_{i=1,2} G^{(i)}_1(q^2) \left[ \frac{1}{\sqrt{4\pi}} \delta_{M,M_i} u(|\mathbf{p}_i|) + \sum_m C_{2m1m_i}^{1M} \mathcal{Y}_{2m}(\hat{\mathbf{p}}_i) w(|\mathbf{p}_i|) \right].$$

The most cumbersome component is the magnetic component of current operator. Therefore we separate cases for different values of final pair spin moment $S$:

$$<00|\hat{J}_{mag,\lambda}(0)|1M> = \frac{-q_z}{m\sqrt{2}} (-1)^{\lambda+1} C_{1\lambda 10}^{1\lambda} \sum_{i=1,2} G^{(i)}_M(q^2) \times$$

$$\frac{\delta_{M,-\lambda}}{\sqrt{4\pi}} u(|\mathbf{p}_i|) + \sum_{m'} C_{2m1-\lambda}^{1M} \mathcal{Y}_{2m'}(\hat{\mathbf{p}}_i) w(|\mathbf{p}_i|),$$

$$<1m_s|\hat{J}_{mag,\lambda}(0)|1M> = \frac{q_z}{m} C_{1\lambda 10}^{1\lambda} \sum_{i=1,2} G^{(i)}_M(q^2) \times$$

$$\left[ \frac{1}{\sqrt{4\pi}} C_{1M1\lambda}^{1m_s} u(|\mathbf{p}_i|) + \sum_{m'} C_{1\mu 1\lambda}^{1m_s} C_{2m'1\mu}^{1M} \mathcal{Y}_{2m'}(\hat{\mathbf{p}}_i) w(|\mathbf{p}_i|) \right],$$

where $\lambda$ is a cyclic component of the vector $\hat{J}$ and $\mathbf{p}_1 = \mathbf{p}_n$, $\mathbf{p}_2 = \mathbf{p}_p$. Using the presented expressions we construct the hadron tensor and calculate the cross section in the nonrelativistic case.

VII. FACTORIZATION OF THE CROSS SECTION

Let us consider the electrodisintegration of the deuteron supposing that an initial lepton collides only with the proton in the deuteron and the neutron is a spectator. In this case
the cross section is factorized on two parts, one is connected with the contribution of the neutron as a spectator and another with a proton contribution, the latter does not have interference terms between the $S$ and $D$ deuteron states.

A. Nonrelativistic case

The amplitude of the process can formally be presented as a production

$$\mathcal{M} = \chi_{m_{1}}^{+} \chi_{m_{2}}^{+} \hat{O} \Psi_{M}, \quad (26)$$

where spinors $\chi_{m_{1}}^{+}$, $\chi_{m_{2}}^{+}$ describe the outgoing $np$-pair, $\hat{O}$ corresponds to the interaction vertex, $\Psi_{M}$ is a wave function of the deuteron. Let us note that the vertex $\hat{O}$ stands in general for any one-particle interaction but in this paper describes $\gamma N N$-vertex.

Inserting into this expression a complete set of pair states we can get the matrix element

$$\mathcal{M}_{\mu} = \sum_{m_{1}'} (\chi_{m_{1}}^{+} \hat{O}_{\mu} \chi_{m_{1}'}^{+}) \chi_{m_{1}'}^{+} \chi_{m_{2}}^{+} \Psi_{M} \quad (27)$$

which can be used to derive the hadron tensor. After evident transformations it can be written as

$$W_{\mu\nu} = \frac{1}{3} \sum_{m_{1}m_{2}M} \mathcal{M}_{\mu} \mathcal{M}_{\nu} = \frac{1}{3} \sum_{m_{1}m_{2}M} \left| \sum_{m_{1}'} \left[ \chi_{m_{1}}^{+} \hat{O} \chi_{m_{1}'} \right] \left[ \chi_{m_{1}'}^{+} \chi_{m_{2}}^{+} \Psi_{M} \right] \right|^{2}$$

$$= \frac{1}{3} \sum_{m_{1}m_{2}M} \sum_{m_{1}'} \left[ \chi_{m_{1}}^{+} \hat{O} \chi_{m_{1}'} \right]^{2} \left[ \chi_{m_{1}'}^{+} \chi_{m_{2}}^{+} \Psi_{M} \right] \left[ \Psi_{M}^{+} \chi_{m_{2}}^{+} \chi_{m_{1}'} \right].$$

Introducing the partial-wave decomposition for the deuteron:

$$\Psi_{M} = \sum_{lms_{1}s_{2}} C_{lms_{1}s}^{LM} C_{l1s}^{1s} \frac{1}{2} Y_{lm} \chi_{s_{1}} \chi_{s_{2}} u_{l}$$

and transforming the second term with the help of orthogonalization properties of the spinor $\chi$ and some relations for Clebsh-Gordan coefficients we can finally obtain the factorized expression

$$W_{\mu\nu} = \frac{1}{3} \sum_{l} |A_{\mu} A_{\nu}^{*}| \sum_{l} |u_{l}|^{2}, \quad (28)$$

where $A_{\mu} = \chi_{m_{1}}^{+} \hat{O}_{\mu} \chi_{m_{1}'}$ is a one-body interaction part and $l$ counts partial states of the deuteron.
Thus the double factorization is seen. The cross section is proportional to the sum of squared partial radial parts of the deuteron wave function multiplied by factorized interacting proton part.

### B. Relativistic case

In the relativistic case the matrix element of the deuteron electrodisintegration can be written schematically in the following form

\[ \mathcal{M} = \Psi_{\text{pair}} \otimes \hat{O} \otimes S \otimes \Gamma_M, \]  

where \( \Psi_{\text{pair}} \) is the wave function of the \( np \)-pair, \( \hat{O} \) is a vertex of interaction, \( S \) is a propagator of the nucleon, \( \Gamma_M \) is the vertex function of the deuteron.

Introducing the partial-wave decomposition of the deuteron vertex function in the L.S., considering the only proton interacting with a virtual photon, and supposing the PWA for the final \( np \)-pair we can present the comprehensive expression in the following form

\[ \mathcal{M}_\mu = \sum_{s_1s_2} C^{S_{ms_1}}_{\frac{1}{2}s_1\frac{1}{2}s_2} \bar{u}^{(1)}_{s_1}(p_1)\bar{u}^{(2)}_{s_2}(p_2)\Gamma^{(1)}_{\mu}(q)S_{1+}(k_1)\sum_{m_1'} u^{(1)}_{m_1'}(k_1)\bar{u}^{(1)}_{m_1}(k_1) \]

\[ \times \sum_{m_1m_2lmsm_s} C^{1M}_{lm_{ms_s}} C^{s_{ms_s}}_{\frac{1}{2}m_1\frac{1}{2}m_2} u^{(1)}_{m_1}(-k_1)u^{(2)}_{m_2}(-k_2)Y_{lm}(\hat{k})g_l(k_0,|k|), \]

where \( S_{1+}(k_1) = 1/(k_{10}-e_{k_1}) \) and \( \Gamma^{(1)}_{\mu}(q) \) vertex is described by Eq. (19). As it was assumed above only \( ^3S^+_1, \quad ^3D^+_1 \)-states are taken into account. Using orthogonalization properties of the bispinors and some relations for Clebsh-Gordan coefficients we can write

\[ \mathcal{M}_\mu = \sum_{s_1s_2m_1lmsm_s} C^{S_{ms_s}}_{\frac{1}{2}s_1\frac{1}{2}s_2} C^{1M}_{lm_{ms_s}} C^{s_{ms_s}}_{\frac{1}{2}m_1\frac{1}{2}m_2} Y_{lm}(\hat{k})g_l(k_0,|k|)S_{1+}(k_1)A^{(1)}_{\mu}(s_1, p_1; m, k_1), \]

with

\[ A^{(1)}_{\mu}(s_1, p_1; m, k_1) = \bar{u}^{(1)}_{s_1}(p_1)\Gamma^{(1)}_{\mu}(q)u^{(1)}_{m_1}(k_1) \]  

is a one-body photon-proton interaction part. Now we can derive the hadron tensor

\[ W_{\mu\nu} = \frac{1}{3} \sum_{MSms} \mathcal{M}_\mu \mathcal{M}_\nu. \]

Using once more properties of the Clebsh-Gordan coefficients and Dirac spinors we obtain the expression

\[ W_{\mu\nu} = C_d S p \{ (p_1\gamma + m)\Gamma^{(1)}_{\mu}(q)(k_1\gamma + m)\bar{\Gamma}^{(1)}_{\mu}(q) \} \]  

(31)
which involves the simply calculated trace and a function

$$C_d = \frac{1}{8\pi} \frac{1}{4e k_1 e p_\perp} S^2_{1+}(k) \sum_{l=0,2} |g_l(k, |k|)|^2$$

containing the structure of the deuteron. Performing the trace calculation we finally obtain the expression for the hadron tensor

$$W_{\mu\nu} = C_d \left( W^a_{\mu\nu} F_1^2(q^2) + W^b_{\mu\nu} F_1(q^2) F_2(q^2) + W^c_{\mu\nu} F_2^2(q^2) \right)$$

(32)

with

$$W^a_{\mu\nu} = 4 \left[ p_1 \mu k_1 \nu + k_1 \mu p_1 \nu + (m^2 - (p_1 \cdot k_1)) g_{\mu\nu} \right]$$

(33)

$$W^b_{\mu\nu} = 2 \left[ k_1 \mu q_\nu - q_\mu k_1 \nu - p_1 \mu q_\nu + q_\mu p_1 \nu \right]$$

$$W^c_{\mu\nu} = \left[ \left( -q^2 m^2 - q^2 (p_1 \cdot k_1) + 2(p_1 \cdot q)(k_1 \cdot q) \right) g_{\mu\nu} + (m^2 + (p_1 \cdot k_1)) q_\mu q_\nu 
- \left( (k_1 \cdot q)(p_1 \mu q_\nu + q_\mu p_1 \nu) + (p_1 \cdot q)(k_1 \mu q_\nu + q_\mu k_1 \nu) - q^2 (p_1 \mu k_1 \nu + k_1 \mu p_1 \nu) \right) \right] / m^2.$$ 

Let us note here in the expressions Eqs. (31,33) the four-vector $k_1$ has the on-mass-shell form $k_1 = (e k_1, k_1)$ in differ with $k_1 = (k_{10}, k_1)$ in the Fig. 2.

Thus we see that the factorization of the electrodisintegration cross section exists both in nonrelativistic and relativistic cases. The necessary conditions for this are the plane-wave approximation for the final np-pair, the neutron in the deuteron is supposed to be a spectator (the one-body type of the interaction in the vertex $\hat{O}$) and only positive-energy states for the deuteron are taking into account. As for the second condition the type of one-body interaction does not play any role but only spin-one-half particle is scattered. The third condition means the $P$ waves in the deuteron (namely $^3P_l^{+\pm}$ and $^1P_l^{+\pm}$) destroy the factorization.

**VIII. RESULTS AND DISCUSSION**

We present here the results of the calculation of the deuteron electrodisintegration cross section in the relativistic plane wave impulse approximation with the separable Graz II rank III kernel of interaction. In our calculations we follow the conditions of real experiments and we distinguish eight sets of experimental data. Let us mark these sets as SaclayI, SaclayII (see [8], Table 3); SaclayIII ([9], Table 1); BonnI, BonnII (see [10], Table 3); BonnIII,
Bonn$_{IV}$, Bonn$_{V}$ (see [13], Tables 5,3,4, respectively). The kinematical conditions for all sets of the experiment are shown in the Table I.

First of all we illustrated the influence of the spectator neutron on the cross section (see Figs. 4, 5, 6). It is seen that it increases with the increasing of the neutron momentum and reaches 50% in the Saclay$_{III}$ kinematic range. One can see that the cross section of the deuteron electrodisintegration versus $\sqrt{s}$ changes not so strong, nevertheless the contribution of the spectator neutron is not negligible. Let us note that this contribution changes sign in the Saclay$_{III}$ kinematic region (see Fig. 4). In order to understand the origin of this behavior we present on the Fig. 7 partial contributions of the S- and D-states for this kinematical region versus neutron momenta. We found that the D-state plays an important role and then it is naturally to ask what happens if we change the magnitude of the D-state. On the Figs. 8, 9, 10 we can see that for different magnitudes of the D-states the cross section changes distinctly (especially for the kinematic region [9]) but the ratio $\delta = (\sigma_{p+n} - \sigma_p)/\sigma_{p+n}$ is not changed at all (see Fig. 11). It means that the difference is mainly connected with the spectator neutron contribution. Thus we can make a conclusion that the experimental data within the kinematics from Saclay$_{III}$ [9] can supply the good test for various models of NN interactions in the deuteron.

To check the influence of the relativistic effects we present the results of the relativistic and nonrelativistic calculations for various experimental conditions, see figures 12-14. It was shown that relativistic effects play very important role even for small transfer momenta $Q^2$ (see Table I). We can stress that at the Fig. 14 the difference mounts to the order of value. The next step is to take into account final state interactions, $P$-waves and to study the influence of nucleon (on-shell and off-shell) form factors on the deuteron disintegration.

**IX. SUMMARY**

In the presented paper we have considered the electrodisintegration of the deuteron in the Bethe-Salpeter approach. It is realized for a two-nucleon system by using the multipole expansion with the spinor structure of two nucleons. The separable ansatz for the interaction kernel has provided a manageable system of linear homogeneous equations for deriving the BS amplitude.

Then we have switched to the using of the covariant revision of the Graz II separable
potential with the summation of several separable functions.

The reaction of the deuteron electrodisintegration served as a testing ground for the method under investigation and helped to outline both strong and weak points of the approach. The analysis has proved the technique to be very promising, even if we find an evident discrepancies with experimental data at this stage of development. Several items can be suggested for the program of further theoretical study. First of all it is necessary to take into account the final state interaction for the $np$-pair. Then we need to consider the negative-energy states for the BS amplitude and calculate the contribution of $P$ waves to the electrodisintegration (see, [18] and [21]). After that we will be able to calculate different asymmetries of the $(ed \rightarrow enp)$ process which can give new qualitative information about the structure of the deuteron.

\textbf{X. ACKNOWLEDGMENTS}

We wish to thank our collaborators K. Yu. Kazakov, A. V. Shebeko, S. Eh. Shirmovsky, D. V. Shulga for their contribution to the presented paper. We would like to thank Professor H. Toki and Professor D. Blaschke for their interest to this work and fruitful discussions.

The work is supported in part by the Russian Foundation for Basic Research, grant No.05-02-17698a.
[1] M. Crossaux, Phys. Rev. 127, 613 (1962).
[2] T. Wilbois, G. Beck, H. Arenhovel, Few-Body Syst. 15, 39 (1993).
[3] G. Beck, T. Wilbois, H. Arenhovel, Few-Body Syst. 17, 91 (1994).
[4] W. W. Buck, F. Gross, Phys. Rev. D20, 2361 (1979).
[5] V. Dmitrasinovic, F. Gross, Phys. Rev. C40, 2479 (1989).
[6] V. V. Kotlyar, Yu. P. Melnik, A. V. Shebeko, Part. Nucl. 26, 192 (1995).
[7] G. I. Gakh, A. P. Rekalo, Egle Tomasi-Gustafsson, Ann. Phys. 319, 150 (2005).
[8] M. Bernheim et al., Nucl. Phys. A365, 349 (1981).
[9] S. Turck-Chieze et al., Phys. Lett. 142B, 145 (1984).
[10] H. Breuker et al., Nucl. Phys. A455, 641 (1986).
[11] M. Bernheim et al., Phys. Rev. Lett. 46, 402 (1981).
[12] S. Auffret et al., Phys. Rev. Lett. 55, 1362 (1985).
[13] B. Boden et al., Nucl. Phys. A549, 471 (1992).
[14] T.S. Cheng, L.S. Kisslinger, Nucl. Phys. A457, 602 (1986).
[15] V. V. Burov, S. M. Dorkin, V. N. Dostovalov, Z. Phys. A: Atoms and Nuclei 315, 205 (1984);
    V. V. Burov, V. K. Lukyanov, Nucl. Phys. A463, 263 (1987).
[16] Jr. J. Adam, E. Truhlik, D. Adamova, Nucl. Phys. A492, 556 (1989).
[17] E. E. Salpeter, H. A. Bethe, Phys. Rev. 84, 1232 (1951).
[18] S. G. Bondarenko et al., Prog. Part. Nucl. Phys. 48, 449 (2002).
[19] S. Mandelstam, Proc. Roy. Soc. Lond. A233, 248 (1955).
[20] L. Mathelitsch, W. Plessas, M. Schweiger, Phys. Rev. C26, 65-76 (1982).
[21] S. G. Bondarenko et al., Part. and Nucl., Lett., 2, 17 (2005).
Figure 1: One photon approximation.
Figure 2: Relativistic impulse approximation.
Figure 3: Plane wave approximation.
Figure 4: The electrodisintegration cross section versus the neutron momentum for three kinematics of the experiments at Saclay. Solid and dashed lines correspond to the calculations with and without neutron contribution (upper plot). Bottom plot shows the relative neutron contribution in the corresponding experimental regions. The experimental data regions were taken from [8](SaclayI,II) and [9](SaclayIII).
Figure 5: The same as in previous figure but versus pair invariant mass $\sqrt{s}$ for the kinematical conditions were taken from [10] (Bonn I, II).
Figure 6: The same as in previous figure. The kinematical conditions were taken from [13](BonnIII,IV,V).
Figure 7: The contributions of the spectator neutron versus the outgoing neutron momentum to the electrodisintegration cross section of the deuteron partial $S$-, $D$-states are shown for the experimental sets from [9](SaclayIII).
Figure 8: The contributions of the deuteron partial D-state to the electrodisintegration cross section versus neutron momenta are shown for three sets of the experiments [8](Saclay I, II), [9] (Saclay III).
Figure 9: The contributions of the deuteron partial D-state to the electrodisintegration cross section versus pair invariant mass $\sqrt{s}$ are shown for the conditions of the experiments [10] (Bonn I, II).
Figure 10: The same as in the previous figure but for [13] conditions (Bonn\textsubscript{III,V}). We omitted curves for the Bonn\textsubscript{IV} case because they are very close to Bonn\textsubscript{V}.
Figure 11: The contribution of the spectator neutron versus neutron momenta to the electrodisintegration cross section for different deuteron $D$-states for conditions of the experiment SaclayIII [9]. In the first picture solid (dashed) line stands for $p+n$- ($p$-) contribution with different $D$-states in the deuteron: $p_D = 4\%$ for lower line and $p_D = 6\%$ for upper line.
Figure 12: The relativistic and nonrelativistic calculations for conditions of the experiments SaclayI,II and SaclayIII. Experimental data are taken from [8] and [9].
Fig. 1: One photon approximation.

Fig. 2: Relativistic impulse approximation.

Fig. 3: Plane wave approximation.

Fig. 4: The electrodisintegration cross section versus the neutron momentum for three kinematics of the experiments at Saclay. Solid and dashed lines correspond to the calculations with and without neutron contribution (upper plot). Bottom plot shows the relative neutron contribution in the corresponding experimental regions. The experimental data regions were taken from [8](SaclayI,II) and [9](SaclayIII).

Fig. 5 The same as in previous figure but versus pair invariant mass $\sqrt{s}$ for the kinematical conditions were taken from [10] (BonnI,II).

Fig. 6: The same as in previous figure. The kinematical conditions were taken from [13](BonnIII,IV,V).

Fig. 7: The contributions of the spectator neutron versus the outgoing neutron momentum to the electrodisintegration cross section of the deuteron partial $S$, $D$-states are shown for the experimental sets from [9](SaclayIII).

Fig. 8: The contributions of the deuteron partial $D$-state to the electrodisintegration cross section versus neutron momenta are shown for three sets of the experiments [8](SaclayI,II), [9] (SaclayIII).

Fig. 9: The contributions of the deuteron partial $D$-state to the electrodisintegration cross section versus pair invariant mass $\sqrt{s}$ are shown for the conditions of the experiments [10] (BonnI,II).
Fig. 10: The same as in the previous figure but for [13] conditions (Bonn_{III,V}). We omitted curves for the Bonn_{IV} case because they are very close to Bonn_{V}.

Fig. 11: The contribution of the spectator neutron versus neutron momenta to the electrodisintegration cross section for different deuteron D-states for conditions of the experiment Saclay_{III} [9]. In the first picture solid (dashed) line stands for p + n- (p-) contribution with different D-states in the deuteron: p_{D} = 4\% for lower line and p_{D} = 6\% for upper line.

Fig. 12: The relativistic and nonrelativistic calculations for conditions of the experiments Saclay_{I,II} and Saclay_{III}. Experimental data are taken from [8] and [9].

Fig. 13: The same as in Fig. 12 for conditions of the experiments Bonn_{I,II}. Experimental data are taken from [10].

Fig. 14: The same as in Fig. 12 for conditions of the experiments Bonn_{III-V}. Experimental data are taken from [13].

Table I: Kinematical conditions of the experiments under consideration. All quantities are in laboratory system: \theta_{qe} is an angle between the beam and the virtual photon, p_{n} - neutron momentum, \theta_{n} - angle between neutron and virtual photon, p_{p} - proton momentum, \theta_{p} - angle between proton and virtual photon, \theta_{pe} - angle between beam and proton. \sqrt{s} - 2m - kinetic energy of the np-pair, \omega and q_{z} are the components of the virtual photon four-momentum q = (\omega, 0, 0, q_{z}). If value max is not stated it is equal to upper min.
Figure 13: The same as in Fig. 12 for conditions of the experiments \textit{Bonn I,II}. Experimental data are taken from \cite{10}.
Figure 14: The same as in Fig. 12 for conditions of the experiments Bonn$_{III−V}$. Experimental data are taken from [13].
| S_I | S_{II} | S_{III} | B_I | B_{II} | B_{III} | B_{IV} | B_V |
|-----|--------|---------|-----|-------|--------|-------|-----|
| $E, \text{GeV}$ | 0.500 | 0.500 | 0.560 | 1.464 | 1.569 | 1.2 | 1.2 | 1.2 |
| $E', \text{GeV}$ | min 0.395 | 0.352 | 0.360 | 1.175 | 1.118 | 0.895 | 0.895 | 0.895 |
| max 0.800 | 0.800 | 0.800 | 1.464 | 1.569 | 1.2 | 1.2 | 1.2 |
| $\theta, ^\circ$ | 59 | 44.4 | 25 | 21 | 21 | 20.15 | 20.15 | 20.15 |
| $p_n, \text{GeV}$ | min 0.005 | 0.165 | 0.294 | 0.314 | 0.500 | 0.126 | 0.197 | 0.197 |
| max 0.350 | 0.350 | 0.550 | 0.660 | 0.773 | 0.564 | 0.423 | 0.488 |
| $\theta_n, ^\circ$ | min 101.81 | 172.07 | 153.01 | 60.53 | 74.60 | 142.32 | 155.72 | 165.36 |
| max 37.78 | 70.23 | 20.81 | 62.49 | 63.52 | 93.96 | 136.09 | 112.86 |
| $\theta_{qe}, ^\circ$ | min 48.79 | 44.74 | 33.06 | 61.94 | 45.57 | 59.56 | 51.52 | 51.25 |
| max 37.39 | 29.49 | 25.57 | 25.57 | 25.57 | 25.57 | 25.57 | 25.57 |
| $p_p, \text{GeV}$ | min 0.451 | 0.514 | 0.557 | 0.466 | 0.681 | 0.525 | 0.620 | 0.622 |
| max 0.276 | 0.403 | 0.306 | 0.664 | 0.791 | 0.834 | 0.929 | 0.889 |
| $\theta_p, ^\circ$ | min 0.622 | 2.54 | 13.86 | 35.82 | 45.12 | 8.42 | 7.52 | 4.47 |
| max 51.03 | 54.90 | 140.28 | 61.68 | 60.90 | 42.40 | 18.41 | 30.40 |
| $\theta_{pe}, ^\circ$ | min 49.41 | 47.28 | 46.92 | 97.77 | 90.68 | 68.00 | 44.00 | 56.00 |
| max 99.81 | 99.64 | 173.35 | 99.08 | 90.39 | 90.39 | 90.39 | 90.39 |
| $\sqrt{s}, \text{GeV}$ | min 1.929 | 1.993 | 2.057 | 1.9675 | 2.1375 | 1.98 | 2.04 | 2.04 |
| max 2.2125 | 2.3325 | 2.28 | 2.28 | 2.28 | 2.28 | 2.28 | 2.28 |
| $\sqrt{s}-2m, \text{GeV}$ | min 0.051 | 0.115 | 0.176 | 0.090 | 0.260 | 0.101 | 0.161 | 0.161 |
| max 0.335 | 0.455 | 0.401 | 0.401 | 0.401 | 0.401 | 0.401 | 0.401 |
| $Q^2, \text{GeV}^2$ | min 0.192 | 0.101 | 0.038 | 0.257 | 0.255 | 0.154 | 0.145 | 0.145 |
| max 0.206 | 0.209 | 0.106 | 0.106 | 0.106 | 0.106 | 0.106 | 0.106 |
| $\omega, \text{GeV}$ | min 0.105 | 0.148 | 0.200 | 0.162 | 0.348 | 0.148 | 0.210 | 0.210 |
| max 0.422 | 0.568 | 0.476 | 0.476 | 0.476 | 0.476 | 0.476 | 0.476 |
| $q_z, \text{GeV}$ | min 0.450 | 0.350 | 0.279 | 0.532 | 0.613 | 0.420 | 0.435 | 0.435 |
| max 0.620 | 0.729 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 | 0.577 |

Table I: Kinematical conditions of the experiments under consideration. All quantities are in laboratory system: $\theta_{qe}$ is an angle between the beam and the virtual photon, $p_n$ - neutron momentum, $\theta_n$ - angle between neutron and virtual photon, $p_p$ - proton momentum, $\theta_p$ - angle between proton and virtual photon, $\theta_{pe}$ - angle between beam and proton. $\sqrt{s}-2m$ - kinetic energy of the $np$-pair, $\omega$ and $q_z$ are the components of the virtual photon four-momentum $q = (\omega, 0, 0, q_z)$. If value max is not stated it is equal to upper min.