Abstract. Spherical collapse of the Bose-Einstein Condensate (BEC) dark matter model is studied. The evolution of perturbed quantities like the density of the collapsed region and its expansion rate are calculated for two scenarios. Firstly, we consider the case of a sharp phase transition (which happens when the critical temperature is reached) from the normal dark matter state to the condensate one. In the second case studied we consider a smooth first order phase transition where there is a continuous conversion of “normal” dark matter to the BEC phase. We calculate in detail the perturbative quantities at nonlinear level presenting numerical results for the physics of the collapse for a wide range of the model’s space parameter i.e., the mass of the scalar particle $m_\chi$ and the scattering length $l_s$. We explicitly show the dependence of the transition redshift on $m_\chi$ and $l_s$ and show that the BEC model is almost indistinguishable from the usual dark matter scenario concerning nonlinear perturbations.

Keywords: dark matter theory, cosmological parameters from LSS, Bose-Einstein Condensation.
1 Introduction

It is widely accepted that dark matter is one of the main components of the universe. Due to the strong observational evidence corroborating its existence many different areas of physics have incorporated dark matter related investigations in their agenda. According to the standard cosmological model, dark matter composes around $\frac{1}{4}$ of the universe’s energy budget and $\frac{5}{6}$ of the total matter. Baryons represent the remaining fraction of the latter.

The crucial aspects of these studies concern the particle nature and the astrophysical/cosmological behavior of such component. At the particle level, candidates belonging to the WIMP (weakly interacting massive particles) category produce a viable model. Also, for the homogeneous, isotropic and expanding background, the dark matter ensemble should present a vanishing pressure in order to enable structure formation at the galactic level.

Although the success of the Cold Dark Matter (CDM) scenario it is important to mention some of its drawbacks. The theoretical clustering patterns (calculated via numerical simulations) of CDM particles at galactic level correspond to the NFW profile \cite{1} which is cuspy at the centre of the particle distribution. This seems to be in clear contradiction to the observed velocities in the central region of galaxies which demand a cored distribution. At the same time, the simulated distribution of satellites around typical Milk Way like galaxies shows an one order of magnitude excess of sub-structures which are not observed. Respectively, these two issues are known as the cusp-core problem and the missing satellites problem. See \cite{2} for a review. Even if baryonic physics in such simulations could eventually alleviate these problems, it is not clear so far whether or not CDM is the correct model for the dark matter phenomena.

One can argue that dark matter is a pathological manifestation of choosing Einstein’s general relativity (GR) as the gravitational theory. This suspect is the pillar of a research line in which modified gravity theories are invoked like, for example, MOND, f(R) and others. However, reliable experiments at the solar system level confirm GR predictions with great accuracy. Therefore, this fact seems to be powerful enough to keep GR as our standard description for gravitational interaction.

Since there are no confirmed evidences to abandon GR, dark matter remains being essential and therefore one needs new alternatives within this context. In this case, the possibilities are also vast. The classical ones were hot dark matter (HDM) and warm dark
matter (WDM) [3]. While the former has been ruled out due to the positive observation of galaxies below the jeans mass scale of relativistic dark matter particles, the latter is one of the leading rivals of CDM. Indeed, particles with masses $m \sim \text{keV}$ fit the WDM spirit. They are not so light as HDM particles and therefore allowing the existence of structures and, at the same time, not so heavy as CDM, in such a way that there would exist some suppression mechanism able to alleviate the small scale problems of the CDM paradigm. Models with a similar clustering dynamics as WDM are, for instance, the self interacting dark matter [4] and the viscous dark matter [5].

In this work we study a dark matter model which has a different nature. Let us assume 0–spin DM particles having therefore a bosonic distribution. As predicted and already observed in laboratory bosonic particles are able to condensate, occupying the same energy state and forming the so called Bose-Einstein condensates when their temperature reaches the critical value $T_{\text{crit}}$. Of course, this phenomena occurs under very controlled experimental situations but one might wonder in principle what happens if the same would happens on astrophysical scales.

Although quite hypothetical this description could serve as an effective approach for understanding dark matter as a cosmological scalar field $\phi$ whose dynamics is driven by some repulsive potential $V(\phi)$. This gives rise to the Bose-Einstein Condensate (BEC) dark matter model which has been widely studied [6–9]. The main idea is that normal, i.e., non-condensate, dark matter undergoes a phase transition at some critical redshift $z_{\text{crit}}$ during the universe’s evolution. Then, independently on the details of the transition, all the dark matter converts into the condensate state forming a BEC “fluid”.

The dynamics of BEC systems is studied via the Gross-Pitaevskii equation which is a nonlinear Schrodinger equation [10]. From this starting point, the Madelung decomposition is used to transform the BEC dynamics into a set of fluid equations resulting in an effective positive pressure. With such fluid picture one is able to investigate astrophysical/cosmological problems. This procedure will be shown in more details in the next section.

The general aspects of this model concerning the background evolution and the linear perturbations are already very well understood [11–15]. But, in order to fully understand the final clustering patterns of the BEC dark matter model high resolution hydrodynamical/N-body simulations are still needed. However, a previous step is the study of the nonlinear gravitational collapse. Concerning the BEC dark matter model, recently Ref. [16] addressed the collapse of “already formed BEC condensates”, i.e., only the post-transition stage. Nevertheless, the situation can be more complicated.

The transition can also take place during the evolution of the collapsed region. Also, specially for galaxy cluster scales, the evolution of the background cosmological dynamics should be taken into account.

We will perform in this work a natural extension of Ref. [16] which has analysed the “free-fall” collapsed of a BEC dark matter sphere. However, we assume a more realistic cosmological scenario where dark matter coexist with baryons and a cosmological constant. Then, we address the correct case where the transition occurs during the nonlinear clustering process.

Fundamental quantities here are the condensate parameters, namely, the mass of the particle $m_\chi$ and the scattering length $l_s$. They determine the moment at which the phase transition takes place $z_{\text{crit}}$ and the speed of sound in the condensate fluid, for example. After the critical redshift $z_{\text{crit}}$ one can admit two different dynamics. The simplest case is to assume an abrupt transition, i.e., for $z < z_{\text{crit}}$ all dark matter obeys the Bose-Einstein dynamics. This
seems to be a reasonable approximation to the problem. This situation will be studied in section 3.

One can also assume the case in which the full conversion of all dark matter occurs in a finite time and it finishes at a redshift $z_{BEC} < z_{\text{crit}}$. Therefore, the phase transition lasts a finite time in which a mixture of “normal” and condensate dark matter make up the total matter component. We study in section 4 this case.

We present our results covering many order of magnitude in the model parameter space $10^{-6} \text{meV} < m_\chi < 10^4 \text{meV}; 10^{-12} \text{fm} < l_s < 10^{12} \text{fm}$. Interesting quantities to be found here are the final (at $z = 0$) value of the density contrast and the expansion rate and the redshift of the turnaround $z_a$, i.e., the moment at which the collapsed region detaches from the background.

In summary, this paper has the following structure. In the next section we develop the background dynamics of the BEC dark matter. We present in section 3 general equations for the spherical top-hat collapse formalism. This equations will be studied in more detail in sections 4 and 5 where, respectively, we address the case of abrupt transition and the usual phase transition. We conclude in the final section.

2 The background dynamics of the Bose-Einstein Condensate dark matter

In this work we always have a flat background dynamics composed by baryons, dark matter and a cosmological constant. This expansion rate reads

$$H^2 = \frac{8\pi G}{3} (\rho_b + \rho_{dm} + \rho_\Lambda).$$

The baryonic component is assumed to be pressureless $P_b = 0$ and therefore $\rho_b = \rho_{b0}(1 + z)^3$ where $\rho_{b0}$ is its density today at $z = 0$. Its value is such that $\rho_{b0} = \Omega_{b0} \rho_c$ where $\rho_c = 3H_0^2/8\pi G$ and we can safely adopt $\Omega_{b0} = 0.05$ according to nucleosynthesis constraints. The Hubble constant assumed here is $H_0 = 70 \text{Km/s/Mpc}$. We will also fix $\Omega_{dm0} = 0.25$ or equivalently $\Omega_\Lambda = 0.75$.

The difference here from the standard ΛCDM model will be the dark matter dynamics. Before the transition takes place, at temperatures $T > T_{\text{crt}}$; or redshifts $z > z_{\text{crt}}$, DM behaves as an isotropic gas in thermal equilibrium. From kinetic theory the pressure of a non-relativistic gas in this regime is given by

$$p_{dm} = \frac{g}{3h^3} \int \frac{q^2 c^2}{E} f(q) dq \approx 4\pi \frac{g}{3h^3} \int \frac{q^4}{m} \sigma^2 \rho_{dm} \quad \text{with} \quad \sigma^2 = \frac{\langle v^2 \rangle}{3c^2},$$

where $g$ is the number of spin degrees of freedom, $h$ the Planck constant, $q$ the momentum of a particle with energy $E = \sqrt{q^2 c^2 + m^2 c^4}$ and distribution function $f$. A typical value for the velocity dispersion is $\sigma = 3 \times 10^{-6}$. In practice, since this quantity can be seen as the dark matter equation of state parameter $w_{dm} = p_{dm}/\rho_{dm}$ this value is consistent with the assumption of pressureless fluid usually adopted for CDM.

After dark matter’s conversion it obeys the condensate dynamics which is governed by the Gross-Pitaevskii equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_\chi} \nabla^2 \Psi + V(r, t) \Psi + g(|\Psi|)\Psi,$$
where \( m_\chi \) is the mass of the particle and \( V(r, t) \) is the trapping potential. The non-linearity term with only two-body interparticle interaction (quadratic) reads

\[
g(|\Psi|) = U_0 |\Psi|^2, \tag{2.4}
\]

where \( U_0 = 4\pi\hbar^2 l_s/m_\chi \). This definition has the fundamental parameters of the model, namely the scattering length \( l_s \) and the particle mass \( m_\chi \). Note that there appears some degeneracy for the \( U_0 \) parameter, i.e., there are infinity combinations of \( l_s \) and \( m \) capable to produce the same \( U_0 \) value. We discuss this degeneracy and the admissible numerical values of these parameters in the next sections.

In order to apply the Gross-Pitaevskii equation to astrophysical problems one proceeds with the so called Madelung decomposition. In this procedure, the wave function is replaced by

\[
\Psi = \sqrt{\rho(r, t)} e^{i\hbar S(r,t)} \tag{2.5}
\]

where \( \rho = |\Psi|^2 \) is the number density of the system and \( S \) is the velocity potential. The mass/energy density can be written in terms of the mass of each individual particle as \( \rho_\chi = m_\chi \rho \). Therefore, the BEC system can be described in terms of a hydrodynamical set of equations. This specific BEC-inspired fluid possesses an effective pressure

\[
p_\chi = \frac{2\pi\hbar^2 l_s}{m_\chi^3} \rho_\chi^2. \tag{2.6}
\]

A cosmological dark matter fluid with the above pressure leads to the background expansion

\[
H^2 = \frac{8\pi G}{3} (\rho_b + \rho_\chi + \rho_\Lambda). \tag{2.7}
\]

where \( \rho_\chi \) is the BEC dark matter density which is determined by the pressure (2.6) via the continuity equation.

More details will be discussed in sections 4 and 5. In fact, we will follow in this work the background expansion determined in ref. [7].

### 3 The nonlinear top-hat collapse

Here we present the basic equations that describe the evolution of a spherical collapsing matter region in an expanding background. This is the ideal technique for studying the clustering patterns of dark matter halos.

We will follow standard calculations presented in Refs. [17–20]. For general fluids, we define quantities such as

\[
\vec{v} = \vec{v}^0 + \vec{v}^p, \tag{3.1}
\]

\[
\rho^c = \rho (1 + \delta), \tag{3.2}
\]

\[
p^c = p + \delta p. \tag{3.3}
\]

They are respectively the velocity, density and pressure of the collapsed region. The background velocity expansion is given by \( \vec{v}^0 \) and is associated to the Hubble’s law. Peculiar motions are denoted by \( \vec{v}^p \). The total density within this spherical region under collapse \( \rho^c \) is written as the sum of the background density and the overdensity fraction \( \delta \rho \). The same happens to the pressure definition.
The rate at which the overdense region expands reads

\[ h = H + \frac{\theta}{3}(1 + z), \]  

(3.4)

where \( \theta = \nabla \cdot \vec{v} \).

Energy conservation is also required for the collapsing region. Therefore, each component \( i \) obeys a separate equation of the type

\[ \dot{\delta}_i = -3H(c_{eff,i}^2 - w_i)\delta_i - [1 + w_i + (1 + c_{eff,i}^2)\delta_i] \frac{\theta}{a} \]  

(3.5)

where the energy density contrast is defined as

\[ \delta_i = \left( \frac{\delta\rho}{\rho} \right)_i, \]

(3.6)

and the effective speed of sound is computed following \( c_{eff,i}^2 = (\delta p / \delta \rho)_i \). Note that overdot means derivative with respect to the cosmic time \( t \).

The dynamical evolution of the homogeneous spherical region will be governed by the Raychaudhuri equation

\[ \dot{\theta} + H\theta + \frac{\theta^2}{3a} = -4\pi Ga \sum_i (\delta\rho_i + 3\delta p_i). \]

(3.7)

For a cosmological model composed by \( N \) distinct fluids one has to solve \( N+1 \) equations. One of the type 3.5 for each fluid and, since we adopt the top-hat profile, one single equation for the velocity potential \( \theta \) which is sourced by the density fluctuations of the \( N \) fluids.

Since we will use the standard ΛCDM universe as our reference model here we show its equations for the spherical collapse. Both the baryonic and the dark matter component are assumed to be pressureless fluids. Therefore, we can write down

\[ \dot{\delta}_b = -(1 + \delta_b) \frac{\theta}{a}, \]

(3.8)

\[ \dot{\delta}_{dm} = -(1 + \delta_{dm}) (1 + \sigma^2) \frac{\theta}{a}, \]

(3.9)

\[ \dot{\theta} + H\theta + \frac{\theta^2}{3a} = -4\pi G \left[ \rho_b \delta_b + \rho_{dm} \delta_{dm}(1 + \sigma^2) \right]. \]

(3.10)

Note that there is no equation of clustering of the cosmological constant since it is treated as a background quantity. Therefore, it influences this set of equations only via the expansion rate \( H \equiv H(\rho_b, \rho_{dm}, \Lambda) \). In order to numerically solve (3.8-3.10) one usually specify the initial conditions for \( \delta_b, \delta_{dm} \) and \( \theta \) at the redshift of decoupling \( z_{dec} \sim 1000 \) from which one can treat baryons as an independent fluid.

### 4 Abrupt phase transition

The temperature \( T_{crit} \) sets the beginning of the BEC phase transition. This is in fact a process which takes some finite time \( \Delta t \) until all the normal dark matter has been converted into the BEC phase. As estimated in [7] \( \Delta t \) is of order of \( 10^6 \) years. Although the latter value is
parameter dependent, it is in general indeed an almost negligible fraction of the universe’s lifetime. Therefore, the assumption that at \( z_{crt} \) there is an instantaneous conversion to the BEC phase seems to be plausible and it will be considered in this section.

For \( z > z_{crt} \) the dark matter equation of state calculated in (2.2) reads

\[
p_{dm} = \sigma^2 \rho_{dm},
\]

where \( \sigma^2 \equiv \langle \mathbf{v}^2 \rangle / 3c^2 \). Applying this to the continuity equation one finds

\[
\rho_\chi = \rho_{crt} \left( \frac{1 + z_{crt}}{1 + z} \right)^{3(1 + \sigma^2)}, \quad z \geq z_{crt},
\]

where \( z_{crt} \) is the redshift at the transition point and \( \rho_{crt} \equiv \rho(z_{crt}) \).

For \( z < z_{crt} \) the effective equation of state of the BEC dark matter is

\[
p_\chi = u_0 \rho_\chi^2, \quad u_0 \equiv \frac{2\pi h^2 l_s}{m_\chi^3},
\]

and again, using the continuity equation we find

\[
\rho_\chi \left( \frac{1 + \omega_{crt}}{1 + z_{crt}} \right)^3 - \omega_{crt}, \quad z \leq z_{crt},
\]

where \( \rho_\chi \) is a continuous function at \( z_{crt} \) and \( \omega_{crt} \equiv \rho_{crt} / \rho_{crt} = \sigma^2 \). At this point the continuity of the pressure (see discussion in [7]) sets

\[
\sigma^2 \rho_{crt} = u_0 \rho_{crt}^2 \Rightarrow \rho_{crt} = \frac{\sigma^2}{u_0},
\]

which, of course, depends on the model parameters. From this definition,

\[
\rho_{\chi 0} = \frac{\rho_{crt}}{(1 + \omega_{crt})(1 + z_{crt})^3 - \omega_{crt}} \Rightarrow (1 + z_{crt})^3 = \frac{\Omega_{\chi 0} + \omega_{crt}}{1 + \omega_{crt}},
\]

where \( \Omega_{\chi 0} = 0.25 \) is the today’s fractionary dark matter energy density parameter.

Note that before the phase transition we have \( c_s^2 = \sigma^2 = \omega_{crt} \). After this point the equation of state parameter and the adiabatic (\( c_s^2 = \partial p / \partial \rho \)) speed of sound associated to this fluid reads, respectively,

\[
\omega_\chi(z) = u_0 \rho_\chi(z), \quad c_{s,\chi}^2 = 2u_0 \rho_\chi(z) = 2\omega_\chi(z).
\]

Concerning the perturbed region the effective speed of sound is actually given by the expression

\[
c_{eff,\chi}^2 \equiv \frac{\delta p_\chi}{\delta \rho_\chi} = \frac{p_\chi' - p_\chi}{\rho_\chi' - \rho_\chi} = \frac{w_\chi (1 + \delta_\chi)^2 - 1}{\delta_\chi} = w_\chi (2 + \delta_\chi),
\]

from which one can expand for small values of \( \delta \) finding \( c_{eff}^2 \rightarrow c_s^2 \) as expected.

Since a crucial issue in this model is the determination of the moment at which the transition happens in Fig. 1 we show the dependence of \( z_{crt} \) (4.6) on the model parameters \( m_\chi \) and \( l_s \). The solid line sets the parameter values for which the transition happens today at \( z = 0 \). Therefore, only for the parameters values below the solid line the BEC dark
Figure 1. The redshift of the phase transition ($z_{\text{crit}}$) in the parameters plane $l_s \times m$. The solid line sets the parameters in which the transition happens today at $z = 0$. The long dashed line sets the parameters for which the transition takes place around the decoupling time $z_{\text{crit}} = 1000$.

matter model is able to leave some imprint on the observations. Note, for example, that the configuration $(m_{\chi}, l_s) = (10^{-4} \text{ meV}, 10^5 \text{ fm})$ is an acceptable one. However, in this case, it would be impossible to probe the bosonic nature of dark matter since the transition will happen in a far future. On the other hand, over the long-dashed line the transition happens at the time of photon-baryon decoupling. In principle, $z_{\text{crit}} < 1000$ is also allowed but this could cause some damage to the CMB anisotropies. Although this issue has not yet been investigated in detail we prefer to avoid such cases and therefore let us consider only $0 < z_{\text{crit}} < 1000$ which correspond to the gray region in this plot. The short-dashed line corresponds $z_{\text{crit}} = 10$ and it is shown to guide the reader on how $z_{\text{crit}}$ evolves in this plane.

The meaning of the mass of the dark matter particle is quite clear. But in the cosmological context what does mean the scattering length $l_s$?

Typical BEC experiments work with values in the range $10^6 \text{ fm} < l_s < 10^9 \text{ fm}$. For these values the condition $z_{\text{crit}} > 0$, i.e., assuring that the transition has already occurred, is satisfied for masses $m > 10 \text{ meV}$ and $100 \text{ meV}$, respectively. We also note that by extrapolating the lines to lighter particles, as for example ultra-light masses of order $m \sim 10^{-22} \text{ meV}$, as many works in the literature to indicate to be the correct one, would require almost negligible $l_s$ values which could be much smaller than the Planck length ($l_{\text{plk}} \sim 10^{-20} \text{ fm}$).

It is also worth noting that the space parameter indicated by the gray region is consistent with the stability of BEC dark matter halos as calculated in Ref. [21]. However, see also a related discussion on the non-stability of BEC halos in Ref. [22].

In order to solve for the evolution of the perturbed quantities during the collapse we adopt the following strategy. We solve numerically the $\Lambda$CDM equations taking initial conditions at a redshift $z_i = 1000$ and with the values $\delta_{dm}(z_i) = 3.5 \times 10^{-3}$, $\delta_b(z_i) = 10^{-5}$ and $\theta(z_i) = 0$ [19, 20]. This set of equations is evolved until the critical redshift $z_{\text{crit}}$. At this point, the quantities $\delta_{dm}(z_{\text{crit}})$, $\delta_b(z_{\text{crit}})$ and $\theta(z_{\text{crit}})$ are used as initial conditions for the BEC dark matter equations, which uses 4.8, from the critical redshift to $z = 0$.

We have studied in great detail the parameter space $m_{\chi}$ and $l_s$ and although the BEC
dark matter model indeed yields to a distinct dynamics at nonlinear level, this difference is, in practice, almost negligible. We show in the left panel of Fig. 2 this feature where the expansion of the collapsed region is shown. The solid red line represents the standard cosmology while the dashed black line was calculated for a mass $m_\chi = 20$ meV and a scattering length $l_s = 10^6$ fm. With this choice the transition occurs at $z_{crt} = 3.19$ as seen in the vertical dashed line. Both curves are in practice indistinguishable. The effective speed of sound is plotted in the right panel of Fig. 2. This shows the reason there are no significant changes in the evolution. We remark again that this result is not due to the specific choice $m_\chi = 20$ meV and $l_s = 10^6$ fm. It is a general feature of the model.

5 Smooth phase transition

We deal now with the situation in which there is a gradual conversion of “normal” dark matter into the condensed phase which starts at a redshift $z_{crt}$ and is finished at a redshift $z_{BEC}$. This is indeed the more realistic case. The dynamics shown in this section was also developed for the first time in Ref. [7].

As mentioned in the last section the estimated duration $\Delta t = t(z_{BEC}) - t(z_{crt})$ of this transition is of order $\Delta t \sim 10^6$ years which means a small fraction of the universe’s lifetime $t_U \sim 10^{10}$ years [7]. However, $\Delta t$ depends on the model parameters $l_s$ and $m_\chi$. We calculate here again $\Delta t$ for some values $l_s$ and $m_\chi$ and plot it in Fig. 3. In the right panel of this figure, there is a maximum value $\Delta t_{\text{max}} = 3.4 \times 10^9$ years assuming, for instance, a mass $m_\chi = 1$ meV and $l_s \sim 3.1 \times 10^2$ fm. There are of course other combinations of $l_s$ and $m_\chi$ which produces similar $\Delta t$ values. The lower values for $\Delta t$ we have found are $\sim 10^6$ years. Therefore, this analysis shows that contrary to previous estimations, the phase transition can last a non-negligible fraction of the universe’s lifetime.

As we will see below, the background dynamics and the evolution of the perturbation for the smooth phase transition differs significantly from the abrupt case studied in the last
section. Then, one can expect that now we can observe some distinguishable feature of the BEC dark matter nonlinear collapse.

Let us now develop the dynamics during the smooth phase transition. Before the transition starts we have the same dynamics of a isotropic non-relativistic gas as described in the last section by equations (4.1) and (4.2).

During the phase transition we can define the fraction of converted dark matter as

\[ f(z) = \frac{\rho(z) - \rho_{crt}}{\rho_{BEC} - \rho_{crt}} \],

where \( \rho(z) \) is the dark matter density along the transition, \( \rho_{crt} \) is the dark matter density before the transition and \( \rho_{BEC} \) its value afterwards. The function \( f(z) \) is defined in such a way that at \( z_{crt} \) we have \( f(z_{crt}) = 0 \). When the dark matter has fully converted to the BEC phase \( f(z_{BEC}) = 1 \).

Using (5.1) into the continuity equation and integrating it from \( z_{crt} \) to \( z \geq z_{BEC} \) we find

\[ f(z) = \frac{1 + \omega_{crt}}{\Omega_{BEC} - 1} \left[ \left( \frac{1 + z}{1 + z_{crt}} \right)^3 - 1 \right]. \]

(5.2)

Then, the dark matter density evolution becomes now

\[ \rho_{\chi} = \rho_{crt} \left( \frac{1 + z}{1 + z_{crt}} \right)^3(1 + \sigma^2), \quad z \geq z_{crt}, \]

(5.3)

\[ \rho_{\chi} = \rho_{crt} \left\{ 1 + (1 + \omega_{crt}) \left[ \left( \frac{1 + z}{1 + z_{crt}} \right)^3 - 1 \right] \right\}, \quad z_{crt} \geq z \geq z_{BEC}, \]

(5.4)

\[ \rho_{\chi} = \rho_0 \frac{(1 + z)^3}{(1 + \omega_0) - \omega_0(1 + z)^3}, \quad z \leq z_{BEC}. \]

(5.5)
We still have to determine the redshift \( z_{\text{BEC}} \) when the phase transition is over. With the condition \( f(z_{\text{BEC}}) = 1 \) inserted in (5.2) we find

\[
\left[ \frac{\Omega_{\text{BEC}}}{\Omega_{\text{crit}}} - 1 \right] \left( \frac{1 + z_{\text{crit}}}{1 + z_{\text{BEC}}} \right)^3 = (1 + \omega_{\text{crit}}),
\]

and using \( z = z_{\text{BEC}} \) in the expression (5.5) for the condensated dark matter density we have

\[
\frac{\Omega_{\text{BEC}}}{\Omega_{\text{crit}}} = \frac{\Omega_{\text{crit}}(1 + z_{\text{BEC}})^3}{(1 + \omega_0) - \omega_0(1 + z_{\text{BEC}})^3}.
\]

Eqs. (5.6) and (5.7) can now be solved, leading to a solution for \( z_{\text{BEC}} \) and \( \Omega_{\text{BEC}} \).

As said before, during the phase transition both non-condensated and condensated dark matter coexist and the dark matter pressure is constant and has the same value for both components in the interval \( z_{\text{BEC}} \leq z \leq z_{\text{crit}} \), as given by Eq. (4.5). We will assume that the same happens for the collapsed pressure \( p^c \), which allow us to find the constraint

\[
1 + \delta_{\sigma}^c = (1 + \delta_{B}^c)^2,
\]

where we used the expression \( \rho_\chi(z) = \rho_\sigma(z) + \rho_B(z) \), which compared with Eq. (5.2) allows us to identify \( \rho_\sigma(z) = \rho_{\text{crit}}(1 - f(z)) \) as the non-condensated dark matter density and \( \rho_B(z) = \rho_{\text{BEC}} f(z) \) as density of the condensated state.

The continuity of dark matter fluid pressure enable us to treat both components as one single fluid also at perturbative level. In this case, the effective fluid sound velocity during the phase transition becomes

\[
c_{\text{eff}}^2 = \frac{p_{\text{crit}} - p_{\text{BEC}}}{\rho_\chi \delta_\chi} = \sigma^2 \rho_{\text{crit}} \delta_{\text{crit}} \frac{\delta_{\text{BEC}}}{\rho_\chi \delta_\chi} = \omega(z) \frac{\delta_{\text{BEC}}}{\delta_\chi},
\]

where \( \omega(z) = \omega_{\text{crit}} \rho_{\text{crit}} / \rho_\chi(z) \) is the equation of state parameter for the dark matter fluid during the phase transition. After the phase transition is completed, i.e., when \( z \leq z_{\text{BEC}} \), the effective fluid sound velocity of the BEC dark matter will be

\[
c_{\text{eff}}^2 = \frac{p_{\text{crit}} - p_{\text{BEC}}}{\rho_\chi \delta_\chi} = \sigma^2 \rho_{\text{crit}} \delta_{\text{crit}} \frac{\delta_{\text{BEC}}}{\rho_\chi \delta_\chi} = \omega(z)(2 + \delta_\chi),
\]

where \( \omega(z) = \omega_{\text{crit}} \rho_\chi(z) / \rho_{\text{crit}} \) is the equation of state parameter for the dark matter after the phase transition.

Since the velocity dispersion \( \sigma^2 \) for the dark matter particles before the BEC phase transition is small the same assumptions made on \( z_{\text{crit}} \) in the previous section is still valid here, and we will consider values for the model parameters \((m_\chi, t_b)\) such that \( 0 < z_{\text{crit}} < 1000 \). We will also consider only cases where \( z_{\text{BEC}} \geq 0 \).

This set of equations is evolved until the critical redshift \( z_{\text{crit}} \) assuming the same initial conditions at \( z_i \) as before. At this point, the quantities \( \delta_{\text{dm}}(z_{\text{crit}}), \delta_B(z_{\text{crit}}) \) and \( \theta(z_{\text{crit}}) \) are used as initial conditions for the phase transition perturbed Eqs. (3.5) and (3.7) with the suitable background parameters. This set of equations is again evolved until the \( z_{\text{BEC}} \), and the quantities \( \delta_{\text{dm}}(z_{\text{BEC}}), \delta_B(z_{\text{BEC}}) \) and \( \theta(z_{\text{BEC}}) \) are used as initial conditions for the BEC dark matter perturbed equations.

In Fig. 4 we show the expansion of the collapsed region for the smooth phase transition model, where the solid red curve represents the standard \( \Lambda \text{CDM} \) model and the black dashed
curve represents the BEC model, for $m_\chi = 20$ meV in the left panel, $m_\chi = 10$ meV in the right panel and $l_s = 10^6$ fm in both cases. The dashed vertical lines shows the initial and the end point of the transition phase. In the left panel, $z_{\text{cr}} = 3.19$ and $z_{\text{BEC}} = 1.43$ and $z_{\text{cr}} = 1.10$ and $z_{\text{BEC}} = 0.45$ in the right panel. These intervals correspond to $2.40 \times 10^9$ years and $3.38 \times 10^9$ years. As in the abrupt transition model there are no major difference between CDM and BEC dark matter.

The evolution of the non-linear density perturbations are shown in Fig. 5, where $\delta_{\text{dm}} \equiv \delta \rho_{\text{dm}} / \rho_{\text{dm}}$ is the dark matter density contrast. Again, the red curve represents the standard $\Lambda$CDM model, while the black dashed curve shows the behavior of the BEC model for $m_\chi = 20$ meV in the left panel, $m_\chi = 10$ meV in the right panel and $l_s = 10^6$ fm in both cases.
The curves are again indistinguishable.

The redshift of turnaround $z_{\text{ta}}$ is the one which marks the instant when perturbed region starts to decrease its physical radius. This happens when $h = 0$, i.e., $z_{\text{ta}} = z(h = 0)$. For $\Lambda$CDM model $z_{\text{ta}}^{\Lambda\text{CDM}} = 0.2113$ and for the cases seen in both panels of Fig. 4 we have $|z_{\text{ta}}^{\text{BEC}} - z_{\text{ta}}^{\Lambda\text{CDM}}| \approx 10^{-4}$.

6 Conclusions

We have studied the nonlinear clustering properties of the Bose-Einstein dark matter model. In this scenario, bosonic dark matter particles are able to undergo a phase transition as their temperature reaches the critical one $T_{\text{crit}}$ which corresponds to some critical redshift $z_{\text{crit}}$. The main questions here are: i) how does $z_{\text{crit}}$ depend on the fundamental model parameters $m_\chi$ (the particle mass) and $l_s$ (the scattering length)? and ii) what is the background and perturbative dynamics during the phase transition?

Fig. 1 shows in detail the expected degeneracy of $z_{\text{crit}}$ values in the $l_s \times m_\chi$ plane, i.e., for a given $z_{\text{crit}}$, there are infinity admissible parameter configurations. This result identifies the parameters values for which $z_{\text{crit}} > 0$ and therefore are able to leave imprints on large scale structure observations. At the same time, if the actual parameters values of the BEC model lie in the region $z_{\text{crit}} < 0$ then the bosonic nature of the dark matter particles can not be accessed via cosmological observables.

Our strategy was to identify specific signatures of the BEC dark matter nonlinear clustering. Since there is a positive pressure associated to the BEC dark fluid one can expect that the corresponding effective speed of sound will modifies somehow the agglomeration rate. We tried to understand this process via both the abrupt and the smooth phase transition approaches. In the former scenario the dark matter dynamics changes suddenly at $z_{\text{crit}}$. In the latter, there is a continuous conversion from the “normal” to the BEC phase. Although we showed that the smooth transition can indeed last a quite significant fraction of the universe lifetime. Then, it seems that this case could eventually lead to a remarkable dynamics. However, in both approaches of the phase transition we could not identify any relevant difference between the BEC model and the standard CDM model. This is mostly due to the fact that the model parameters leading to $z_{\text{crit}} < 0$ produce almost negligible $c_{\text{eff}}$ values. On one hand, this guarantees that the nonlinear clustering patterns of the BEC model are very similar to the CDM model. On the other hand, this eliminates the nonlinear perturbative study as a possible technique to probe the bosonic nature of dark matter particles.

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