Two-body charmed baryon decays involving vector meson with

$SU(3)$ flavor symmetry

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Abstract

We study the two-body anti-triplet charmed baryon decays of $B_c \to B_n V$, with $B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c)$ and $B_n(V)$ the baryon (vector meson) states. Based on the $SU(3)$ flavor symmetry, we predict that $B(\Lambda^+_c \to \Sigma^+ \rho^0, \Lambda^0 \rho^+) = (0.61 \pm 0.46, 0.74 \pm 0.34)\%$, in agreement with the experimental upper bounds of $(1.7, 6)\%$, respectively. We also find $B(\Lambda^+_c \to \Xi^0 K^{*+}, \Sigma^0 K^{*+}, \Lambda^0 K^{*+}) = (8.7 \pm 2.7, 1.2 \pm 0.3, 2.0 \pm 0.5) \times 10^{-3}$ to be compatible with the pseudoscalar counterparts. For the doubly Cabibbo-suppressed decay $\Xi^+_c \to p\phi$, measured for the first time, we predict its branching ratio to be $(1.5 \pm 0.7) \times 10^{-4}$, together with $B(\Xi^+_c \to pK^*0, \Sigma^+ \phi) = (7.8 \pm 2.2, 1.9 \pm 0.9) \times 10^{-3}$. The $B_c \to B_n V$ decays with $B \simeq O(10^{-4} - 10^{-3})$ are accessible to the BESIII, BELLEII and LHCb experiments.

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I. INTRODUCTION

The two-body $B_c \rightarrow B_n V$ decays have not been abundantly measured as the $B_c \rightarrow B_n M$ counterparts, where $B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c)$ are the anti-triplet charmed baryon states, together with $B_n$ and $V(M)$ the baryon and vector (pseudo-scalar) meson states, respectively. For example, all Cabibbo-favored (CF) $\Lambda^+_c \rightarrow B_n M$ decays have been measured [1], including the recent BESIII observation for $\Lambda^+_c \rightarrow \Sigma^+ \eta'$ [2], whereas for the CF vector modes only $\Lambda^+_c \rightarrow pK^*0, \Sigma^+ \omega, \Sigma^+ \phi$ have absolute branching fractions [1]. In addition, the first absolute branching ratio for the $\Xi^+_c$ decays is $\Xi^+_c \rightarrow \Xi^-\pi^+$ [3], instead of any $\Xi^+_c \rightarrow B_n V$ decays.

Nevertheless, the $B_c \rightarrow B_n V$ decays are not less important than the $B_c \rightarrow B_n M$ counterparts. First, the participations of BESIII, BELLEII and LHCb Collaborations are expected to make more accurate measurements for $B_c \rightarrow B_n V$, such as $\Lambda^+_c \rightarrow \Sigma^+ \rho^0, \Lambda^0 \rho^+$, presented as $\mathcal{B}(\Lambda^+_c \rightarrow \Sigma^+ \rho^0, \Lambda^0 \rho^+) < (1.7, 6\%)$ due to the previous measurements [4]. Second, in the three-body $B_c \rightarrow B_n MM'$ decays, the $MM'$ meson pair is assumed to be mainly in the S-wave state [4]. However, the resonant $B_c \rightarrow B_n V, V \rightarrow MM'$ decay causes $MM'$ to be in the P-wave state, of which the contribution to the total $\mathcal{B}(B_c \rightarrow B_n MM')$ needs clarification. Note that $(S,P)$ denote $L = (0, 1)$ as the quantum numbers for the orbital angular momentum between $M$ and $M'$. Third, the three-body $\Xi^+_c$ decays can be measured as the ratios of $\mathcal{B}(\Xi^+_c \rightarrow B_n V)/\mathcal{B}(\Xi^+_c \rightarrow B_n MM')$. Particularly, the doubly Cabibbo-suppressed $\Xi^+_c \rightarrow p\phi$ decay is observed for the first time, with $\mathcal{B}(\Xi^+_c \rightarrow p\phi)/\mathcal{B}(\Xi^+_c \rightarrow pK^-\pi^+) = (19.8 \pm 0.7 \pm 0.9 \pm 0.2) \times 10^{-3}$ [5]. The information of $\mathcal{B}(\Xi^+_c \rightarrow B_n V)$ is hence helpful to determine $\mathcal{B}(\Xi^+_c \rightarrow B_n MM')$.

Since the study of $B_c \rightarrow B_n V$ is necessary, it is important to provide a corresponding theoretical approach. The factorization approach for the heavy hadron decays [6–8] seems applicable to $B_c \rightarrow B_n V$. Nonetheless, it has been shown that, besides the factorizable effects, there exist significant non-factorizable contributions in $B_c \rightarrow B_n M$ [9], such that the factorization approach fails to explain the data. In contrast, with both factorizable and non-factorizable effects [10–18], the $SU(3)$ flavor symmetry ($SU(3)_f$) approach can accommodate the measurements for $B_c \rightarrow B_n M$ [19–25], such as the purely non-factorizable $\Lambda^+_c \rightarrow \Xi^0 K^+$ decay [31]. In addition, the predicted values of $\mathcal{B}(\Lambda^+_c \rightarrow \Sigma^+ \eta')$ and $\mathcal{B}(\Xi^+_c \rightarrow \Xi^-\pi^+)$ are in agreement with the recent observations [2, 3, 22, 23]. Therefore, we propose to extend the $SU(3)_f$ symmetry to $B_c \rightarrow B_n V$, while the existing observations have been sufficient for the
numerical analysis. In this report, we will extract the \( SU(3)_f \) amplitudes, and predict the to-be-measured \( B_c \rightarrow B_n V \) branching fractions.

II. FORMALISM

To obtain the amplitudes for the two-body \( B_c \rightarrow B_n V \) decays, where \( B_{c(n)} \) is the singly charmed (charmless) baryon state and \( V \) the vector meson, we present the relevant effective Hamiltonian (\( \mathcal{H}_{eff} \)) for the tree-level \( c \) quark decays, given by

\[
\mathcal{H}_{eff} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i (V_{cs} V_{ud} O_{i} + V_{cq} V_{uq} O'_{i} + V_{cd} V_{us} O'_{i}) ,
\]

with \( q = d \) or \( s \), where \( G_F \) is the Fermi constant, \( V_{ij} \) are the CKM matrix elements, and \( c_{\pm} \) the scale-dependent Wilson coefficients. In Eq. (1), \( O^{(q,\prime)}_{\pm} \) are the four-quark operators:

\[
O_{\pm} = \frac{1}{2} \left[ (\bar{u}d)(\bar{s}c) \pm (\bar{sd})(\bar{uc}) \right] ,
\]

\[
O'_{\pm} = \frac{1}{2} \left[ (\bar{u}q)(\bar{q}c) \pm (\bar{qq})(\bar{uc}) \right] ,
\]

\[
O'_{\pm} = \frac{1}{2} \left[ (\bar{us})(\bar{dc}) \pm (\bar{ds})(\bar{uc}) \right] ,
\]

with \( (\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \). By neglecting the Lorentz indices, the operator of \( (\bar{q}_1 q_2)(\bar{q}_3 c) \) transforms as \( (\bar{q}_1 q_2)(\bar{q}_3 c) \) under the \( SU(3)_f \) symmetry, where \( q_i = (u, d, s) \) represent the triplet of 3. The operator can be decomposed as irreducible forms, which is accordance with \( (3 \times 3 \times 3)c = (\bar{3} + \bar{3'} + 6 + 15)c \). One hence has

\[
O^{(q,\prime)}_{-(+)} \sim O^{(q,\prime)}_{6(15)} = \frac{1}{2} \left[ (\bar{ud})(\bar{sd}) \mp (\bar{sd})(\bar{uc}) \right] c ,
\]

\[
O^{q}_{-(+)} \sim O^{q}_{6(15)} = \frac{1}{2} \left[ (\bar{u}q)(\bar{q}c) \mp (\bar{qq})(\bar{uc}) \right] c ,
\]

\[
O'^{q}_{-(+)} \sim O'^{q}_{6(15)} = \frac{1}{2} \left[ (\bar{us})(\bar{dc}) \mp (\bar{ds})(\bar{uc}) \right] c ,
\]

with the subscripts \( (6, 15) \) denoting the two irreducible \( SU(3)_f \) operators. By substituting \( O^{(q,\prime)}_{6(15)} \) for \( O^{(q,\prime)}_{-(+)} \), the effective Hamiltonian in Eq. (1) becomes

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[ c_+ \epsilon^{ijkl} H(6)_{lk} + c_+ H(15)_{lj}^{ij} \right] ,
\]

where the tensor notations of \( 1/2 \epsilon_{ijkl} H(6)_{lk} \) and \( H(15)_{lj}^{ij} \) contain \( O^{(q,\prime)}_{6(15)} \) and \( O^{(q,\prime)}_{15} \), respectively. In terms of \( (V_{cs} V_{ud}, V_{cd} V_{ud}, V_{cs} V_{us}, V_{cd} V_{us}) = (1, -s_c, s_c, -s_c^2) \) with \( s_c \equiv \sin \theta_c \), where \( \theta_c \) represents the well-known Cabbibo angle, we have \( H_{22}(6) = 2, H_{23,32}(6) = -2s_c, \)
$H^3_{12,21}(\Omega) = -H^3_{13,31}(\Omega) = s_c$, $H^{33}(6) = 2s_c^2$, and $H^3_{12,21}(\Omega) = -s_c^2$ as the non-zero entries \[15\]. Note that $n = 0, 1$ and $2$ in $s_n^2$ correspond to the Cabibbo-flavored (CF), singly Cabibbo-suppressed (SCS), and doubly Cabibbo-suppressed (DCS) decays, respectively. We also need $B_c$ and $B_n$ ($V$) to be in the irreducible representation of the $SU(3)_f$ symmetry, given by

\[
(B_c)_i = (\Xi^0, -\Xi^+_c, \Lambda^+_c),
\]

\[
(B_n)_j = \begin{pmatrix}
\frac{1}{\sqrt{6}} \Lambda^0 + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\
\Sigma^- & \frac{1}{\sqrt{6}} \Lambda^0 - \frac{1}{\sqrt{2}} \Sigma^0 & n \\
\Xi^- & \Xi^0 & -\sqrt{2} \Lambda^0
\end{pmatrix},
\]

\[
(V)_j = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\rho^0 + \omega) & \rho^- & K^{*-} \\
\rho^+ & -\frac{1}{\sqrt{2}} (\rho^0 - \omega) & \bar{K}^{*0} \\
K^{*+} & K^{*0} & \phi
\end{pmatrix}.
\]  \tag{5}

Subsequently, $\mathcal{H}_{\text{eff}}$ in Eq. \[1\] is enabled to be connected to the initial and final states in Eq. \[5\], such that we derive the amplitudes of $B_c \rightarrow B_n V$ as

\[
\mathcal{A}(B_c \rightarrow B_n V) = \langle B_n V | \mathcal{H}_{\text{eff}} | B_c \rangle = \frac{G_F}{\sqrt{2}} T(B_c \rightarrow B_n V),
\]  \tag{6}

instead of introducing the details of the QCD calculations for the hadronization. Explicitly, the $T$ amplitudes ($T$-amps) are given by \[15, 16\]

\[
T(B_c \rightarrow B_n V) = T(\mathcal{O}_0) + T(\mathcal{O}_{\mathcal{T}5}),
\]

\[
T(\mathcal{O}_0) = \bar{a}_1 H^{ij}(6) T_{ik}(B_n)^k_i (V)^l_j + \bar{a}_2 H^{ij}(6) T_{jk}(V)^k_i (B_n)^j_l \\
+ \bar{a}_3 H^{ij}(6) (B_n)^k_i (V)^l_j T_{kl} + \bar{h} H^{ij}(6) T_{ik}(B_n)^k_j (V)^l_i,
\]

\[
T(\mathcal{O}_{\mathcal{T}5}) = \bar{a}_4 H^{ij}_{jk}(\mathcal{T}5)(V)^l_i (B_n)^k_j (B_c)^l_i + \bar{a}_5 H(\mathcal{T}5)_{jk}(B_n)^k_j (B_c)^l_i (V)^l_i \\
+ \bar{a}_6 H(\mathcal{T}5)^i_{jk}(B_n)^k_i (V)^l_j (B_c)^l_i + \bar{a}_7 H(\mathcal{T}5)^i_{jk}(B_c)^l_j (B_n)^k_i (V)^l_i \\
+ \bar{h}' H^{ij}_{jk}(\mathcal{T}5)(B_n)^k_i (V)^l_i (B_c)^l_j,
\]  \tag{7}

where $T_{ij} \equiv (B_c)^k_e^{i,jk}$, and $(c_-, c_+)$ have been absorbed into the $SU(3)$ parameters ($\bar{a}_i, \bar{h}^{(\theta)}$). With the full expansion of $T$-amps in Table I, the two-body $B_c \rightarrow B_n V$ decays are presented with the $SU(3)_f$ parameters. Since $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$ actually mix with $\omega_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, the $(\bar{h}, \bar{h}')$ terms that are related to $(V)_l^I = \sqrt{2} \omega + \phi = \sqrt{3} \omega_1$ can contribute to the decays with $(\omega, \phi)$ only. In terms of the
TABLE I. The $T$-amps for the $B_c \to B_n V$ decays, where CF denotes the Cabibbo-favored processes, while SCS (DCS) the singly (doubly) Cabibbo-suppressed ones.

| $\Xi^0$ | CF T-amp | $\Xi^0$ | SCS T-amp | $\Xi^0$ | DCS T-amp |
| --- | --- | --- | --- | --- | --- |
| $\Sigma^+ K^-\bar{\Lambda}$ | $2(a_2 + \frac{3a_3 + a_4}{2})$ | $\Sigma^+ \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ | $\Sigma^- K^+ | -2(a_1 + \frac{3a_3 + a_4}{2})s_c$ | $p \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Xi^0 K^0$ | $-\sqrt{2}(a_2 + a_3 - \frac{3a_5 - a_7}{2})$ | $\Sigma^+ \rho^- | -2(a_1 + \frac{3a_3 + a_4}{2})s_c$ | $\Sigma^- K^+ | -2(a_1 + \frac{3a_3 + a_4}{2})s_c$ | $p \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Xi^0 \rho^0 | -\sqrt{2}(a_1 - a_3 - \frac{3a_5 - a_7}{2})$ | $\Sigma^- \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ | $\Sigma^0 K^0 | \sqrt{2}(a_1 + \frac{3a_3 + a_4}{2})s_c$ | $p \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Xi^0 \omega | -\sqrt{2}(a_1 - a_3 + 2h + \frac{3a_5 - a_7}{2})$ | $\Sigma^0 \rho^- | -2(a_3 + a_4 - \frac{3a_5 + a_6 - a_7}{2})s_c$ | $n \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Xi^0 \phi | a_2 + h + \frac{3a_5 + a_7}{2}$ | $\Sigma^0 \rho^- | -2(a_3 + a_4 - \frac{3a_5 + a_6 - a_7}{2})s_c$ | $n \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Xi^- \rho^- | 2(a_1 + \frac{3a_3 + a_4}{2})$ | $\Xi^- \rho^- | 2(a_1 + \frac{3a_3 + a_4}{2})s_c$ | $n \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |
| $\Lambda^0 K^0 | -\sqrt{2}(2a_1 - a_2 - a_3)$ | $p \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ | $\Lambda^0 K^0 | -\sqrt{2}(a_1 - a_2 - a_3)$ | $n \rho^- | -2(a_2 + \frac{3a_3 + a_4}{2})s_c$ |

| $\Xi^+ | CF T-amp | $\Xi^+ | SCS T-amp | $\Xi^+ | DCS T-amp |
| --- | --- | --- | --- | --- | --- |
| $\Sigma^+ K^0$ | $-2(a_3 - \frac{3a_4 + a_5}{2})$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | 2(a_1 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ K^0 | \sqrt{2}(a_1 - a_2)$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Xi^0 \rho^+ | 2(a_3 + \frac{3a_4 + a_5}{2})$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ K^0 | 2(a_1 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
| $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- K^+ | \sqrt{2}(a_1 - a_2)$ | $\Sigma^- \rho^- | \sqrt{2}(a_2 + a_3 - \frac{3a_4 + a_5}{2})s_c$ | $\Sigma^+ \rho^+ | \sqrt{2}(a_1 - a_2)$ |
measured by BELLE \cite{3}. In addition, the ratio of \( B \) measured to be relative to input for the CKM matrix elements. By using the equation of \cite{9} independent parameters to be extracted, whereas there exist 10 data points for the numerical analysis. To have a practical fit, we follow Refs. \cite{19, 22, 23, 25} to reduce the parameters. In \( \mathcal{H}_{\text{eff}} \propto c_-H(6) + c_+H(\bar{15}) \), since the QCD calculation at the scale \( \mu = 1 \) GeV leads to \((c_+, c_-) = (0.76, 1.78)\) in the naive dimensional regularization (NDR) scheme \cite{27, 28}, the ratio of \((c_-/c_+)^2 \simeq 0.17\) indicates the suppression of \( H(\bar{15}) \). We hence ignore \((\bar{a}_{4, 5, \ldots, \tau, h'})\).

On the other hand, \((\bar{a}_{1, 2, 3, h})\) from \( H(6) \) are kept for the fit, represented as
\[
\bar{a}_1, \bar{a}_2 e^{i\delta_2}, \bar{a}_3 e^{i\delta_3}, \bar{h} e^{i\delta_h},
\]
with the phases \(\delta_{a_{2, 3, h}}, \) and \(\bar{a}_1\) set to be relatively real.

### III. NUMERICAL ANALYSIS

For the numerical analysis, we collect (the ratios of) the branching fractions for the observed \( B_c \to BV \) decays in Table II where \( B(\Xi_c^+ \to pK^{*0}, \Sigma^0, \Sigma^+ K^{*0}) \) are in fact measured to be relative to \( B(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) \) \cite{1, 29}, recombined as \( R_{1,2}(\Xi_c^+) \). We obtain \( B(\Xi_c^0 \to \Lambda^0) \) from \( B(\Xi_c^0 \to \Lambda^0 \phi)/B(\Xi_c^0 \to \Xi^- \pi^+) \) \cite{1}, with the input of \( B(\Xi_c^0 \to \Xi^- \pi^+) \) measured by BELLE \cite{3}. In addition, the ratio of \( R(\Lambda_c^+) = (\Lambda_c^+ \to \Sigma^+ \rho^0)/B(\Lambda_c^+ \to \Sigma^+ \omega) \) comes from the data events in Ref. \cite{32}. Besides, \( s_c = 0.22453 \pm 0.00044 \) \cite{1} is the theoretical input for the CKM matrix elements. By using the equation of \( \bar{a}_1, \bar{a}_2 e^{i\delta_2}, \bar{a}_3 e^{i\delta_3}, \bar{h} e^{i\delta_h} \),

\[
\chi^2 = \sum_i \left( \frac{B_{\text{th}}^i - B_{\text{ex}}^i}{\sigma_{\text{ex}}^i} \right)^2 + \sum_j \left( \frac{R_{\text{th}}^j - R_{\text{ex}}^j}{\sigma_{\text{ex}}^j} \right)^2,
\]
we are able to obtain the minimum \( \chi^2 \) value, such that the \( SU(3)_f \) parameters can be extracted with the best fit. Note that \( B'(R^j) \) represents (the ratio of) the branching fraction, with the subscript \( \text{th} (\text{ex}) \) denoting the theoretical (experimental) input, while \( \sigma_{\text{th}}^{(j)}(\sigma_{\text{ex}}^{(j)}) \) stands for the experimental error. As the inputs in Eq. (10), \( B(R)_{\text{th}} \) come from the \( T \)-amps in Table II while \( B(R)_{\text{ex}} \) and \( \sigma_{\text{ex}} \) the data points in Table II. Subsequently, the global fit gives
TABLE II. The (ratios of) branching fractions of the $B_c \to B_n V$ decays. In column 2, the numbers are calculated with the extracted parameters, in comparison with the initial experimental inputs in column 3.

| (Ratio of) Branching fraction | This work  | Data     |
|-------------------------------|-----------|----------|
| $10^2 B(\Lambda_c^+ \to pK^{*0})$ | 1.9 ± 0.3 | 1.94 ± 0.27 [1] |
| $10^2 B(\Lambda_c^+ \to \Sigma^+ \omega)$ | 1.6 ± 0.7 | 1.69 ± 0.21 [1] |
| $10^3 B(\Lambda_c^+ \to \Sigma^+ \phi)$ | 3.9 ± 0.6 | 3.8 ± 0.6 [1] |
| $R(\Lambda_c^+)$ | 0.4 ± 0.3 | 0.3 ± 0.2 [32] |
| $10^3 B(\Lambda_c^+ \to \Sigma^+ K^{*0})$ | 2.3 ± 0.6 | 3.4 ± 1.0 [1] |
| $10^4 B(\Lambda_c^+ \to p\omega)$ | 11.4 ± 5.4 | 9.4 ± 3.9 [33] |
| $10^4 B(\Lambda_c^+ \to p\phi)$ | 10.4 ± 2.1 | 10.6 ± 1.4 [1] |
| $10^4 B(\Xi_c^0 \to \Lambda^0 \phi)$ | 8.4 ± 3.9 | 6.1 ± 2.2 [1, 3] |
| $R_1(\Xi_c^0)$ | (1.6 ± 0.2)s^2_c | (2.8 ± 1.0)s^2_c [1] |
| $R_2(\Xi_c^0)$ | (0.4 ± 0.1)s^2_c | (1.7 ± 1.2)s^2_c [1, 29] |

$(\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{h}) = (0.22 \pm 0.02, 0.23 \pm 0.04, 0.39 \pm 0.05, 0.16 \pm 0.01) \text{ GeV}^3$,

$(\delta a_2, \delta a_3, \delta \bar{h}) = (-85.5 \pm 13.0, 78.4 \pm 8.8, 99.3 \pm 7.7) \degree$,

$\chi^2/n.d.f = 6.3/3 = 2.1$,

where n.d.f represents the number of the degree of freedom. With the fit results in Eq. (11), we calculate the branching ratios, $R(\Lambda_c^+)$ and $R_{1,2}(\Xi_c^0)$ to be compared to their data inputs in Table III. Moreover, we predict the branching fractions for the $B_c \to B_n V$ decays, given in Table III.

IV. DISCUSSIONS AND CONCLUSIONS

With $\chi^2/n.d.f \simeq 2$ to present a reasonable fit, the approach based the $SU(3)_f$ symmetry is demonstrated to be reliable for $B_c \to B_n V$. Besides, our prediction

$$B(\Lambda_c^+ \to \Sigma^+ \rho^0, \Lambda^0 \rho^+) = (0.61 \pm 0.46, 0.74 \pm 0.34)\%,$$

agrees with the experimental upper bounds of $1.7\%$, respectively [1]. We also find

$$B(\Lambda_c^+ \to \Xi^0 K^{*+}, \Sigma^0 K^{*+}, \Lambda^0 K^{*+}) = (8.7 \pm 2.7, 1.2 \pm 0.3, 2.0 \pm 0.5) \times 10^{-3},$$

(13)
TABLE III. The numerical results of the $B_c \to B_n V$ decays, with $B_{B_n V} \equiv B(B_c \to B_n V)$.

| $\Xi_c^0$ | $\Xi_c^0$ | $\Xi_c^0$ | $\Xi_c^0$ |
|----------|----------|----------|----------|
| $10^3 B_{\Sigma^+ K^0}^+$ | 9.3 ± 2.9 | $10^4 B_{\Sigma^+ \rho^{-}}^+$ | 5.6 ± 1.8 | $10^5 B_{p\rho^{-}}^+$ | 3.6 ± 1.1 |
| $10^3 B_{\Sigma^0 K^+}^+$ | 2.7 ± 2.2 | $10^4 B_{\Sigma^- \rho^{+}}^+$ | 5.3 ± 0.7 | $10^5 B_{\Sigma^- K^{++}}^+$ | 2.5 ± 0.3 |
| $10^3 B_{\Xi^0 \rho}^+$ | 1.4 ± 0.4 | $10^5 B_{\Sigma^0 \rho}^+$ | 8.2 ± 6.7 | $10^5 B_{\Sigma^0 K^+}^+$ | 1.3 ± 0.2 |
| $10^3 B_{\Xi^0 \phi}^+$ | 1.0 ± 8.6 | $10^4 B_{\Sigma^0 \omega}^+$ | 1.0 ± 0.8 | $10^5 B_{n\rho}^+$ | 1.8 ± 0.6 |
| $10^4 B_{\Xi^0 \phi}^+$ | 1.5 ± 7.1 | $10^4 B_{\Sigma^0 \phi}^+$ | 2.4 ± 1.1 | $10^5 B_{n\omega}^+$ | 9.9 ± 1.6 |
| $10^3 B_{\Xi^- \rho}^+$ | 8.6 ± 1.2 | $10^4 B_{\Xi^- K^{++}}^+$ | 3.9 ± 0.5 | $10^5 B_{n\phi}^+$ | 3.7 ± 1.8 |
| $10^3 B_{\Lambda^0 K^+}^+$ | 4.6 ± 2.1 | $10^4 B_{\Xi^0 K^{+0}}^+$ | 6.3 ± 2.0 | $10^4 B_{\Lambda^0 K^{0+}}^+$ | 8.1 ± 7.2 |
| $10^4 B_{pK^-}^+$ | 3.0 ± 2.2 | $10^4 B_{nK^+}^+$ | 4.5 ± 3.4 | $10^4 B_{n\rho}^+$ | 9.2 ± 2.2 |
| $10^4 B_{\Lambda^0 \rho}^+$ | 0.1 ± 0.5 | $10^4 B_{\Lambda^0 \omega}^+$ | 0.1 ± 0.1 |

| $\Xi_c^+$ | $\Xi_c^+$ | $\Xi_c^+$ |
|----------|----------|----------|
| $10^2 B_{\Sigma^+ K^0}^+$ | 10.1 ± 2.9 | $10^3 B_{\Sigma^0 \rho}^+$ | 1.9 ± 0.6 | $10^5 B_{\Sigma^0 K^{++}}^+$ | 5.0 ± 0.7 |
| $10^2 B_{\Xi^0 \rho}^+$ | 9.9 ± 2.9 | $10^3 B_{\Sigma^+ \rho}^+$ | 1.9 ± 0.6 | $10^5 B_{\Sigma^+ K^{+0}}^+$ | 9.9 ± 1.3 |
| $10^4 B_{\Sigma^+ \omega}^+$ | 8.2 ± 5.9 | $10^5 B_{p\rho}^+$ | 7.1 ± 2.2 |
| $10^3 B_{\Sigma^+ \phi}^+$ | 1.9 ± 0.9 | $10^4 B_{p\omega}^+$ | 3.9 ± 0.6 |
| $10^4 B_{\Xi^0 K^{++}}^+$ | 9.6 ± 7.9 | $10^4 B_{p\phi}^+$ | 1.5 ± 0.7 |
| $10^3 B_{pK^+}^+$ | 7.8 ± 2.2 | $10^4 B_{n\rho}^+$ | 1.4 ± 0.4 |
| $10^3 B_{\Lambda^0 \rho}^+$ | 7.1 ± 1.7 | $10^5 B_{\Lambda^0 K^{++}}^+$ | 3.2 ± 2.9 |

| $\Lambda_c^+$ | $\Lambda_c^+$ | $\Lambda_c^+$ |
|----------|----------|----------|
| $10^2 B_{\Sigma^0 \rho}^+$ | 6.1 ± 4.6 | $10^4 B_{p\rho}^+$ | 3.5 ± 2.9 | $10^4 B_{pK^+}^+$ | 1.6 ± 0.5 |
| $10^2 B_{\Sigma^+ \rho}^+$ | 6.1 ± 4.6 | $10^4 B_{n\rho}^+$ | 7.0 ± 5.8 | $10^4 B_{nK^+}^+$ | 1.6 ± 0.5 |
| $10^2 B_{\Xi^0 K^{++}}^+$ | 8.7 ± 2.7 | $10^3 B_{\Sigma^0 K^{++}}^+$ | 1.2 ± 0.3 |
| $10^2 B_{\Lambda^0 \rho}^+$ | 7.4 ± 3.4 | $10^3 B_{\Lambda^0 K^{++}}^+$ | 2.0 ± 0.5 |

To be compatible with the pseudo-scalar counterparts. According to Table II, we obtain

\[
T(\Lambda_c^+ \to \Sigma^0 \rho^+) + T(\Lambda_c^+ \to \Sigma^+ \rho^0) = 0,
\]

\[
T(\Lambda_c^+ \to \Sigma^+ K^{*0}) - \sqrt{2}T(\Lambda_c^+ \to \Sigma^0 K^{*+}) = -2a_5 s_c,
\]

8
\[ T(\Lambda_c^+ \rightarrow n\rho^+) - \sqrt{2}T(\Lambda_c^+ \rightarrow p\rho^0) = -(\bar{a}_4 + \bar{a}_6)s_c, \]
\[ T(\Lambda_c^+ \rightarrow nK^{*+}) + T(\Lambda_c^+ \rightarrow pK^{*0}) = -2(\bar{a}_4 + \bar{a}_5)s_c^2. \quad (14) \]

By ignoring the parameters in \( H(15) \), we obtain
\[ \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0\rho^+; \Sigma^+\rho^0) = (6.1 \pm 4.6) \times 10^{-3}, \]
\[ \mathcal{B}(\Lambda_c^+ \rightarrow p\rho^0) = \frac{1}{2}\mathcal{B}(\Lambda_c^+ \rightarrow n\rho^+) = (3.5 \pm 2.9) \times 10^{-4}, \]
\[ \mathcal{B}(\Lambda_c^+ \rightarrow nK^{*+}, pK^{*0}) = (1.6 \pm 0.5) \times 10^{-4}, \quad (15) \]

which respect the isospin symmetry. We also get
\[ \frac{1}{\sqrt{2}}T(\Lambda_c^+ \rightarrow pK^{*0}) - \frac{1}{s_c}T(\Lambda_c^+ \rightarrow p\rho^0) = T(\Lambda_c^+ \rightarrow \Sigma^0\rho^+), \]
\[ \frac{1}{\sqrt{2}}T(\Lambda_c^+ \rightarrow pK^{*0}) + \frac{1}{s_c}T(\Lambda_c^+ \rightarrow p\rho^0) = \sqrt{3}T(\Lambda_c^+ \rightarrow \Lambda^0\rho^+), \quad (16) \]

which lead to
\[ \mathcal{B}(\Lambda_c^+ \rightarrow p\rho^0) \simeq \frac{s_c^2}{2}[3.6\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0\rho^+) + 1.3\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0\rho^+) - 1.1\mathcal{B}(\Lambda_c^+ \rightarrow p\bar{K}^{*0})], \quad (17) \]

where the pre-factors (3.6,1.3,1.1) have taken into account the differences for \( |\bar{p}_{\text{cm}}| \) in Eq. (8).

It is interesting to note that the \( \Lambda_c^+ \rightarrow p\pi^0 \) decay has a similar relation to that in Eq. (17), where \( (\rho, \bar{K}^{*0}) \) are replaced by \( (\pi, \bar{K}^0) \). However, the relation for \( \Lambda_c^+ \rightarrow p\pi^0 \) causes \( \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^0) \simeq 5 \times 10^{-4} \), disapproved by the data [1]. This indicates that, even though the ignoring of \( H(15) \) is viable, the possible interferences between \( H(6) \) and \( H(15) \) might give sizeable contributions to some decay modes [25]. In this work, since the fit still accommodates the data, it is not clear which of the \( \Lambda_c^+ \rightarrow B_n V \) decays receives sizeable interferences between \( H(6) \) and \( H(15) \). Like the \( \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^0) \) case, the precise measurement of \( \mathcal{B}(\Lambda_c^+ \rightarrow p\rho^0) \) can test the ignoring of \( H(15) \). For the \( \Xi_c^+ \) decays, we obtain
\[ B(\Xi_c^+ \rightarrow \Sigma^+\bar{K}^{*0}, \Xi^0\rho^+) = (10.1 \pm 2.9, 9.9 \pm 2.9) \times 10^{-2}, \]
\[ B(\Xi_c^+ \rightarrow p\bar{K}^{*0}, \Sigma^+\phi) = (7.8 \pm 2.2, 1.9 \pm 0.9) \times 10^{-3}. \quad (18) \]

With \( f_{\tau_{B_c}} \equiv \tau_{\Xi_c^+}/\tau_{\Lambda_c^+} \simeq 2.2, B(\Xi_c^+ \rightarrow \Sigma^+\bar{K}^{*0}, \Xi^0\rho^+) \simeq (2 - 4)f_{\tau_{B_c}} \mathcal{B}(\Lambda_c^+ \rightarrow p\bar{K}^{*0}) \) is found to be in accordance with \( |\bar{a}_3|^2 \simeq (2 - 4)|\bar{a}_1|^2 \), which can be tested by more accurate measurements.

By means of \( \mathcal{B}(B_c \rightarrow B_n V, V \rightarrow MM') = \mathcal{B}(B_c \rightarrow B_n V)\mathcal{B}(V \rightarrow MM') \), the resonant contribution to the total \( \mathcal{B}(B_c \rightarrow B_n MM') \) can be investigated, where \( MM' \) from the
vector meson decay are in the P-wave state. On the other hand, the theoretical study of $B_c \to B_n M M'$ needs $M M'$ to be mainly in the S-wave state [4]. Using $\mathcal{B}(\rho^0(+) \to \pi^+\pi^-(0)) \simeq 100\%$ [1] and the predictions for $\mathcal{B}(\Lambda_c^+ \to \Sigma \rho, \Lambda^0 \rho^+)$, we obtain

$$\begin{align*}
\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \rho^0, \rho^0 \to \pi^+\pi^-) &= (6.1 \pm 4.6) \times 10^{-3}, \\
\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \rho^+, \rho^+ \to \pi^+\pi^0) &= (6.1 \pm 4.6) \times 10^{-3}, \\
\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \rho^+, \rho^+ \to \pi^+\pi^0) &= (7.4 \pm 3.4) \times 10^{-3},
\end{align*}$$

which are within the total branching ratios of $(4.42 \pm 0.28, 2.2 \pm 0.8, 7.0 \pm 0.4) \times 10^{-2}$ [1], respectively, showing that the P-wave contributions from $V \to M M'$ are indeed minor to these decays. By putting $\mathcal{B}(\Xi_c^+ \to p\phi) = (1.5 \pm 0.7) \times 10^{-4}$ into the measured ratio of $\mathcal{B}(\Xi_c^+ \to p\phi)/\mathcal{B}(\Xi_c^+ \to pK^-\pi^+) = (19.8 \pm 0.7 \pm 0.9 \pm 0.2) \times 10^{-3}$ [3], we obtain $\mathcal{B}(\Xi_c^+ \to pK^-\pi^+) = (0.8 \pm 0.4)\%$, which is a little smaller than the predicted value of $(1.7 \pm 0.5)\%$ [4].

In sum, within the framework of the $SU(3)_f$ symmetry, we have studied the $B_c \to B_n V$ decays. We have predicted $\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \rho^0, \Lambda^0 \rho^+) = (0.61 \pm 0.46, 0.74 \pm 0.34)\%$, in agreement with the experimental upper bounds of $(1.7, 6)\%$, respectively. It has also been shown that $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^{*+}, \Sigma^0 K^{*+}, \Lambda^0 K^{*+}) = (8.7 \pm 2.7, 1.2 \pm 0.3, 2.0 \pm 0.5) \times 10^{-3}$. For the $\Xi_c^+$ decays, we have obtained $\mathcal{B}(\Xi_c^+ \to \Sigma^+ K^{*0}, \Xi^0 \rho^+) = (10.1 \pm 2.9, 9.9 \pm 2.9) \times 10^{-2}, \mathcal{B}(\Xi_c^+ \to pK^{*0}, \Sigma^+ \phi) = (7.8 \pm 2.2, 1.9 \pm 0.9) \times 10^{-3}$ and $\mathcal{B}(\Xi_c^+ \to p\phi) = (1.5 \pm 0.7) \times 10^{-4}$. The predicted $\mathcal{B}(B_c \to B_n V)$ can be compared to the future measurements by BESIII, BELLE II and LHCb.

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[1] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D **98**, 030001 (2018).
[2] M. Ablikim et al. [BESIII Collaboration], [arXiv:1811.08028] [hep-ex].
[3] Y.B. Li et al. [Belle Collaboration], [arXiv:1811.09738] [hep-ex].
[4] C.Q. Geng, Y.K. Hsiao, C.W. Liu and T.H. Tsai, [arXiv:1810.01079] [hep-ph].
[5] R. Aaij et al. [LHCb Collaboration], [arXiv:1901.06222] [hep-ex].
[6] A. Ali, G. Kramer and C.D. Lu, Phys. Rev. D **58**, 094009 (1998).
[7] C.Q. Geng, Y.K. Hsiao and J.N. Ng, Phys. Rev. Lett. 98, 011801 (2007).
[8] Y.K. Hsiao and C.Q. Geng, Phys. Rev. D 91, 116007 (2015).
[9] H.J. Zhao, Y.K. Hsiao and Y. Yao, arXiv:1811.07265 [hep-ph].
[10] X.G. He, Y.K. Hsiao, J.Q. Shi, Y.L. Wu and Y.F. Zhou, Phys. Rev. D 64, 034002 (2001).
[11] H.K. Fu, X.G. He and Y.K. Hsiao, Phys. Rev. D 69, 074002 (2004).
[12] Y.K. Hsiao, C.F. Chang and X.G. He, Phys. Rev. D 93, 114002 (2016).
[13] X.G. He and G.N. Li, Phys. Lett. B 750, 82 (2015).
[14] M. He, X.G. He and G.N. Li, Phys. Rev. D 92, 036010 (2015).
[15] M.J. Savage and R.P. Springer, Phys. Rev. D 42, 1527 (1990).
[16] M.J. Savage, Phys. Lett. B 257, 414 (1991).
[17] G. Altarelli, N. Cabibbo and L. Maiani, Phys. Lett. 57B, 277 (1975).
[18] X.G. He, Y.J. Shi and W. Wang, arXiv:1811.03480 [hep-ph].
[19] C.D. Lu, W. Wang and F.S. Yu, Phys. Rev. D 93, 056008 (2016).
[20] W. Wang, Z.P. Xing and J. Xu, Eur. Phys. J. C 77, 800 (2017).
[21] D. Wang, P.F. Guo, W.H. Long and F.S. Yu, JHEP 1803, 066 (2018).
[22] C.Q. Geng, Y.K. Hsiao, Y.H. Lin and L.L. Liu, Phys. Lett. B 776, 265 (2018).
[23] C.Q. Geng, Y.K. Hsiao, C.W. Liu and T.H. Tsai, Phys. Rev. D 97, 073006 (2018).
[24] C.Q. Geng, Y.K. Hsiao, C.W. Liu and T.H. Tsai, Eur. Phys. J. C 78, 593 (2018).
[25] C.Q. Geng, C.W. Liu and T.H. Tsai, Phys. Lett. B 790, 225 (2019).
[26] A.J. Buras, hep-ph/9806471.
[27] S. Fajfer, P. Singer and J. Zupan, Eur. Phys. J. C 27, 201 (2003).
[28] H.n. Li, C.D. Lu and F.S. Yu, Phys. Rev. D 86, 036012 (2012).
[29] J.M. Link et al. [FOCUS Collaboration], Phys. Lett. B 571, 139 (2003).
[30] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D 97, 091101 (2018).
[31] M. Ablikim et al. [BESIII Collaboration], Phys. Lett. B 783, 200 (2018).
[32] Y. Kubota et al. [CLEO Collaboration], Phys. Rev. Lett. 71, 3255 (1993).
[33] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D 97, 091101 (2018).
[34] D. Wang, arXiv:1901.01776 [hep-ph].