Dephasing of Electrons by Two-Level Defects in Quantum Dots

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The electron dephasing time $\tau_\phi$ in a diffusive quantum dot is calculated by considering the interaction between the electron and dynamical defects, modelled as two-level system. Using the standard tunneling model of glasses, we obtain a linear temperature dependence of $1/\tau_\phi$, consistent with the experimental observation. However, we find that, in order to obtain dephasing times on the order of nanoseconds, the number of two-level defects needs to be substantially larger than the typical concentration in glasses. We also find a finite system-size dependence of $\tau_\phi$, which can be used to probe the effectiveness of surface-aggregated defects.

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I. INTRODUCTION

Interference of the electron’s paths in a mesoscopic conductor results in various quantum phenomena such as the universal conductance fluctuation, persistent current, and weak localization. In all these phenomena, the dephasing time $\tau_\phi$ appears as a typical time scale over which the electronic trajectories have interference; weak localization correction to conductivity—for example, is conventionally used for the experimental determination of the dephasing time. In the moderate temperature range, experimentally-determined values of $\tau_\phi$ in diffusive metals are found to be in excellent agreement with the theoretical predictions of $\tau_\phi$ due to electron-electron interaction. While it is theoretically expected that $\tau_\phi \rightarrow \infty$ as $T \rightarrow 0$ in the absence of other external sources of dephasing, $\tau_\phi$ is found to saturate at low temperatures almost across all experiments, including the recent carefully-performed experiments. This severe discrepancy between theory and experimental observation of low temperature saturation has fast become a topic of controversy surrounding the question whether the ideal and the theory of zero-point fluctuations of the electromagnetic field created by the electron-electron interaction as a source of dephasing are tenable on general grounds. This poses a serious problem as zero temperature dephasing of electrons has been argued to be relevant to the problems of persistent current in normal metals, the low temperature metal/insulator, quantum-Hall/insulator and superconductor/insulator transitions, and transport through various normal-metal/superconductor hybrid junctions, but the most unsettling consequence is the negation of the fundamental premise upon which the theories—and hence our understanding—of metals and insulators are based: the many-body Fermi liquid picture.

Among various sustained efforts to find a zero temperature dephasing mechanism other than electron-electron interaction, dephasing due to dynamical defects inside the conductor has been recently argued to be important to the saturation problem. Low-energy excitations of the dynamical defects are usually modelled by two-level systems (TLS). Invoked some three decades ago, first by Anderson, Halperin and Varma, and also by Phillips, the tunneling model of TLS has been quite successful in explaining various anomalies in the acoustical, dielectric and thermodynamic properties of structural glasses and other amorphous solids.

Imry, Fukuyama and Schwab have recently suggested that the saturation behavior may have the same origin as the $1/f$ conductance noise, arising from the two-level defects. Zawadowski, von Delft and Ralph have argued that the apparent saturation of $\tau_\phi$ may be caused by the two-channel Kondo effect due to electron-TLS scattering. However, it was pointed out that hysteresis or switching behavior, expected from the effects of TLS, was not observed in experiments. In addition, various concentration-dependent Kondo-like bulk trends anticipated in these theories were also not observed in the experiments.

In this paper, we investigate the role of two-level defects in the dephasing of electrons in quantum dots. In the recent experiments from the Marcus group, Huibers and coworkers have observed the saturation of dephasing time in open quantum dots below 0.1 K along with a strong temperature dependence above 0.1 K. In addition to this experiment, saturation of $\tau_\phi$ in quantum dots has also been reported in other experiments, although the physical meaning of $\tau_\phi$ in these set-ups are not the same. If two-level defects are responsible for the saturation of dephasing time in this experiment, it is natural to suppose that just above 0.1 K the linear temperature dependence should be explained by two-level defects as well. Our calculations indeed show that the dephasing rate due to the TLS does have a linear temperature dependence. However, we find that the magnitude of the dephasing by two-level defects is too small to explain the experimentally observed dephasing time of nanoseconds. This implies that either other mechanisms are more effec-
tive or surface-aggregated two-level defects play a dom-
ninant role; defects on a disordered surface are likely to
have unusual distributions in their splitting energies; we
suggest that the surface defects can be experimentally
probed by measuring the size dependence of dephasing
time.

![Diagram of electron interferometry involving quantum dot]

FIG. 1. Two types of electron interferometry involving quantum dots: (a) Both paths are inside the dot. (b) One of the interfering path is outside the dot.

Consideration of two-level defects in quantum dots for
dominant role in dephasing is motivated by the
experimentally observed tell-tale signs of TLS in quan-
tum dots: hysteresis and switching behaviors, which
have been in fact detected in various quantum-dot ex-
periments. Unlike in the experiments on higher di-

dimensional diffusive metals. In a quantum dot, usually
the Thouless energy $E_T$ is the largest energy scale un-
like the diffusive metallic case. Therefore, the results
obtained in the diffusive metallic case cannot be ap-
plied (even after the appropriate dimensional considera-
tions) to quantum dots. Thouless energy is defined by
$E_T \equiv \hbar D/L^2$, where $D$ is the diffusion constant and $L$ is
the typical system size.

Dephasing generally describes the loss of coherence or
suppression of interference. Accordingly, it is important
to know which kind of paths are considered before defin-
ing the typical time scale of the loss of interference along
these paths. In Fig. 1, we show two kinds of electron in-
terferometers involving quantum dots. In this paper, we
are concerned with pairs of time-reversed paths which
return to the origin in a diffusive system (Fig. 1. (a)).
These time-reversed paths enclose magnetic flux; their
interference manifests in the weak localization correction
to conductivity. These paths are chosen, for the
problem at hand, because their contribution to conduc-
tivity does not vanish even after disorder averaging. In

the interferometry discussed in this paper, change of the
mean conductance at a finite field from its zero field
value, $\delta g = \langle g \rangle_{B \neq 0} - \langle g \rangle_{B=0}$, can be used to ex-
tract the dephasing time from experimental data, where
$\langle \cdot \rangle$ means disorder averaging. Using the phenomelog-
ical random matrix theory (RMT) in Refs. [28,29], $\tau_\phi$ can
be defined, for instance, by the formula

$$\delta g \approx \frac{e^2}{\hbar} \left( \frac{N}{2N + \frac{2\pi}{\delta g}} \right),$$

where $N$ is the number of channels connected to a quan-
tum dot. Although, the formulae for the conductance
change are model dependent [30,31], the difference in the
equations in these models are not significant to the inter-
pretation of $\tau_\phi$ measured in experiments. In this work,
we will refer to $\tau_\phi$ which were obtained through $\delta g$ as
in Ref. [30] without discussing how $\delta g$ is related to $\tau_\phi$ any
further.

Our calculation of dephasing time is similar to the
calculation of dephasing time by Stern, Aharonov and
Imry [32]. Based on the interference of two time-revers-

trajectories, we calculate a typical time scale over which
the environmental state remains in the initial state. De-
phasing rate $1/\tau_\phi$ due to electron-electron interactions in
diffusive quantum dots has been calculated by Sivan and
coworkers [33].

$$\frac{1}{\tau_\phi} \approx \frac{\pi}{2} \left( \frac{\epsilon}{E_T} \right)^2, \quad \epsilon \gg k_B T;$$

where $\epsilon$ is the excitation energy of the particle and $\delta_1$ is
the mean level spacing. However, it should be noted that
the direct application of Eq. (2) to experimental data
is difficult, because it is not meaningful to estimate the
temperature dependence of $\tau_\phi$ by merely replacing $\epsilon$ with
$\sim k_B T$ in Eq. (2).

While the interference paths considered here are
inside the dot, as depicted in Fig.1.(a), there are experi-
ments [34,35] associated with other types of interfer-
ence involving one path through the dot and another out-
side the dot (Fig. 1. (b)). Interference appears as a peri-
odic Aharonov-Bohm oscillation rather than the fluctua-
tions in the electric current as in the case of Fig.1. (a).
In experiments, dephasing manifests in the non-ideality
of the visibility of the interference pattern. Theo-
retically, dephasing rate involved in the geometry shown
in Fig. 1. (b) is calculated as the level broadening [36,37].
In these considerations, level broadening is due to the
gauge fluctuations of a 0D-quantum state where the
electron dynamics inside the dot is neglected. There are
other ways to define dephasing time. In the work of Sivan
and coworkers [38], dephasing time is considered to be the
quasiparticle life time, measured in the tunneling experi-
ments. In the work of Bird and coworkers [39], conductance
fluctuation was used to measure $\tau_\phi$ at very strong mag-
netic fields. Nevertheless, we will refrain from discussing
these different views of dephasing time in this paper.
The organization of the paper is as follows. In section II, we describe how dephasing time is calculated in a general framework. In section III, interaction between the two-level defects and electrons is discussed. In section IV, the return probability is calculated in order to obtain \( \tau_0 \). In section V, we discuss dephasing by two-level defects with widely distributed energies. In section VI, dephasing by two-level defects with narrowly distributed energies are discussed.

II. INTERFERENCE AND DEPHASING OF PARTICLE’S TRAJECTORY

Let us consider the event that the electron is at the position \( r \) at an initial time \( t = 0 \), and it arrives at the position \( r' \) by diffusive motion after a time \( \tau_0 \). The environmental state changes from \( \eta \) to \( \eta' \) in this process; the corresponding probability amplitude of the event is \( \rho(r', r, \eta', \eta; \tau_0) \).

The description of the suppression of interference in electron’s paths by the electron-TLS interaction can be considered in two different approaches:

(i) The electron in the two different paths produces two different time-dependent electric fields on TLS, thereby TLS go to different states, which suppresses interference.

(ii) The fluctuating dipole moment of TLS produces the time-dependent electric field, thereby the electron in the two different paths gains random phase, which also suppresses interference.

In general, these two approaches are not equivalent, because the presence of the electron induces a back reaction from the TLS environment. However, in the presence of weak interaction between the particle and the environment, it is known that either the two description are equivalent, or at least they give the same dephasing rate up to the second order in the interaction. In this work, we use approach (i). Following the scheme of Chakravarty and Schmid, we use semiclassical approximation on particle’s trajectory and we consider quantum mechanical evolution of the TLS (environment) states. We further assume that the TLS environment does not influence the classical paths of the electron, therefore the diffusive electron motion comes from only static disorder. Under certain conditions, the two-level defects might be able to effectively change the semiclassical paths of the electrons. In that case, one may estimate \( \tau_0 \) by calculating electron-TLS inelastic scattering time. However, \( \tau_0 \) begins to lose its meaning as a dephasing time, since we lose the semiclassical picture of the electron’s path.

The tunneling motion in the TLS environment is to be described in a fully quantum mechanical way. To this end, we consider the time-dependent potential \( V(r(t)) \) exerted by the moving electron of the path \( r(t) \) on a two-level defect. The probability amplitude is given by

\[
\rho(r', r, \eta, \eta'; \tau_0) = \sum_j A_j(r', r; \tau_0) e^{i S_j} \langle \eta' | U_j(\tau_0) | \eta \rangle, \tag{3}
\]

where \( A_j \) and \( S_j \) are the corresponding amplitude and action of a classical electron’s trajectory labeled by \( j \). \( U_j(\tau_0) \) is a time-evolution operator (in the interaction picture) of the environmental state associated with the electron trajectory \( r_j(t) \)

\[
U_j(\tau_0) = \hat{T} \exp \left[ \frac{i}{\hbar} \int_0^{\tau_0} V_l(r_j(t), t) dt \right], \tag{4}
\]

where \( \hat{T} \) is the time-ordering operator and

\[
\hat{V}_l(r_j(t), t) = e^{\pi H_{\text{env}} t} \hat{V}(r_j(t)) e^{-\pi H_{\text{env}} t}. \tag{5}
\]

The probability \( P(r', r, \eta; \tau_0) \) of finding the particle at \( r' \) after time \( \tau_0 \), initially at \( r \) with the environment in the initial state \( |\eta\rangle \), is given by the sum of the absolute square of the probability amplitudes over the final states of the environment;

\[
P_{r,\eta}(r', r, \tau_0) = \int d\eta' |\rho(r', r, \eta', \eta; \tau_0)|^2 \tag{6}
\]

\[
= \sum_j |A_j(r', r; \tau_0)|^2 + \sum_{j \neq k} A_j A^*_k e^{i(S_j - S_k)} \langle \eta | U_j(\tau_0) U_j(\tau_0) | \eta \rangle, \tag{7}
\]

using the completeness relation for the environmental states.

The return probability \( P_{r,\eta}(\tau_0) \) of an electron is defined by the probability of finding the electron at position \( r \) after time \( \tau_0 \) which was initially at the same position with the environmental state \( |\eta\rangle \);

\[
P_{r,\eta}(\tau_0) = P_{\eta}(r', r, \tau_0)|_{r' \rightarrow r} \tag{8}
\]

\[
= \sum_j |A_j(r, r; \tau_0)|^2 \tag{9}
\]

\[
+ \sum_j |A_j|^2 |\eta\rangle \left[ U_j^\dagger U_j + U_j^\dagger U_j^T \right] |\eta\rangle + \cdots,
\]

where \( j^T \) denotes the time-reversed path of \( j \). The first term in Eq.(9) is termed as the classical return probability \( P^\text{class}_{r,\eta}(\tau_0) \). The second term comes from the interference of the pair of time-reversed paths. The remaining terms that do not appear in the above equation vanish upon ensemble averaging over disorder due to the random differences in its classical action \( S_j - S_k \) for \( k \neq j, j^T \). The coherent part \( P^\text{coh}_{r,\eta}(\tau_0) \) of the return probability is the second term in Eq.(8):

\[
P^\text{coh}_{r,\eta}(\tau_0) = \sum_j |A_j(r, r; \tau_0)|^2 \Re \langle \eta | U_j^\dagger(\tau_0) U_j(\tau_0) | \eta \rangle. \tag{10}
\]

Now, the dephasing time can be defined as a time scale for \( P^\text{coh}_{r,\eta}(\tau_0) \) to vanish with a decreasing function such as \( \exp(-\tau_0/\tau_0) \). But for the present purpose, the particular exponential form of time dependence is not needed.
III. ELECTRON-TLS INTERACTIONS

We consider two-level tunneling systems (TLS) as the environment for an electron in the quantum dot. Let us first consider TLSs which have asymmetry energy $\Delta$ and the tunnel splitting energy $\Delta_0$. The TLSs are assumed to be randomly distributed over the dot with their electric dipole moments randomly oriented. We will assume the dipole moment is not too strong so that we do not have to consider interaction among the TLS. The density of the TLS will be assumed to be not too high so that multiple scattering events between the electron and the dipoles can be neglected. Within this approximation, we calculate the return probability of an electron in the presence of a single TLS, thereby we extend the results to the case of many randomly distributed TLSs. The Hamiltonian of the TLS can be written in terms of the localized wave functions of the double well potential and also in terms of the eigenenergy basis

$$H_{TLS} = \frac{1}{2} \begin{pmatrix} \Delta & \Delta_0 \\ \Delta_0 & -\Delta \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix}$$

(11)

where $E = \sqrt{\Delta^2 + \Delta_0^2}$ and the transformation denoted by the arrow means localized wavefunction representation $\rightarrow$ eigen wavefunction representation. The dipole strength operator $\hat{p}$ is defined in the eigen wavefunction representation:

$$\hat{p} = p_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow p_0 \begin{pmatrix} \Delta/E & \Delta_0/E \\ \Delta_0/E & -\Delta/E \end{pmatrix}$$

(12)

where $p_0$ is the dipole moment when the particle is located in one of the wells of the defect potential. In the following sections, we will use the eigen wavefunction representation in which $H_{TLS}$ is diagonal. The TLS Hamiltonian will be used for the environment Hamiltonian $H_{env} = H_{TLS}$.

TLS dipole at the position $\mathbf{R}$ feels the electric field $\mathbf{E}(\mathbf{R})$ produced by the moving electron. The resulting interaction energy can be expressed by the operator $V(\mathbf{r}_j(t))$:

$$V(\mathbf{r}_j(t)) = -\hat{p} \mathbf{n} \cdot \mathbf{E}(\mathbf{R}) = -\hat{p} \mathbf{n} \cdot \nabla \mathbf{R} V_c(|\mathbf{R} - \mathbf{r}_j(t)|),$$

(13)

where $\hat{p}$ is the dipole moment operator for the TLS, which is along the direction of unit vector $\mathbf{n}$. $V_c$ is Coulomb interaction potential:

$$V_c(\mathbf{R} - \mathbf{r}_j(t)) = \frac{e}{\epsilon^* |\mathbf{R} - \mathbf{r}_j(t)|} \approx \frac{1}{L^d} \sum_{\mathbf{q}} v_{\mathbf{q}} e^{i \mathbf{q} \cdot (\mathbf{R} - \mathbf{r}_j(t))},$$

(14)

where $e$ is the electric charge, $\epsilon^*$ is the dielectric constant of the dot material, $L$ is the linear system size, and $d$ is the spatial dimensionality of the dot ($d=2$, 3). Here, $v_{\mathbf{q}}$ is the finite Fourier transform of Coulomb potential:

$$v_{\mathbf{q}} = \int_{\text{dot}} d\mathbf{r} e^{-i \mathbf{q} \cdot \mathbf{r}} \frac{e}{\epsilon^* |\mathbf{r}|}$$

$$= \frac{2\pi e}{c^* q^2} (2D, \ q \neq 0)$$

(16)

$$= \frac{4\pi e}{c^* q^2} (3D, \ q \neq 0)$$

(17)

We use the discrete values of $\mathbf{q} = \frac{2\pi}{L}(m,n,k)$ for the three-dimensional case, and $\mathbf{q} = \frac{2\pi}{L}(m,n)$ for the two-dimensional case where $l,m$, and $n$ are integers. By inserting Eq.(14) into Eq.(13), we get

$$\hat{V}(\mathbf{r}_j(t)) = \frac{1}{L^d} \rho \sum_{\mathbf{q}} (\mathbf{i} \mathbf{n} \cdot \mathbf{q} v_{\mathbf{q}} e^{-i \mathbf{q} \cdot \mathbf{R}} e^{i \mathbf{q} \cdot \mathbf{r}_j(t)}.$$

(18)

Derivation of the Eq.(18) is based on the semiclassical approximation of the electron’s motion and the unscreened dipole moment of TLS. From a pure quantum-mechanical point of view, one can consider the TLS-electron interaction similar to its treatment in metallic glasses. When the TLS has two positions $\mathbf{R} = \mathbf{R} \pm d/2$, the purely quantum mechanical TLS-electron interactions $\hat{V}_{QM}$ are written as

$$\hat{V}_{QM} = \frac{1}{L^d} \rho \sum_{\mathbf{q}} \frac{2i\mu_q e^{-i \mathbf{q} \cdot \mathbf{R}} \sin(\mathbf{q} \cdot \mathbf{d}/2) \sum_k c_k^* c_{k+\mathbf{q}}}{\epsilon^* q^2},$$

(19)

where $c_k^*$ ($c_k$) is the electron creation (annihilation) operator with momentum $k$, and $\mu_q$ is the Fourier transform of the ionic potential. However, the pure quantum-mechanical approach is not reliable, because the concept of dephasing becomes ambiguous as we leave the semiclassical approximation, which has been pointed out in Ref. Here, we merely get some useful informations by comparing the quantum mechanical Hamiltonian in Eq.(19) to our semiclassical potential in Eq.(18). Since $\mathbf{q} \cdot \mathbf{d} \ll 1$ in quantum dots (this is true—generally speaking, when the Fermi wavelength is much larger than $d$), $\sin(\mathbf{q} \cdot \mathbf{d}/2) \approx \mathbf{q} \cdot \mathbf{d}/2 = \mathbf{n} \cdot \mathbf{q} \frac{\mathbf{d}}{2}$. Eq.(19) can be understood as the interaction between the TLS-dipole coupled to an effective electric field produced by the electron.

One of the two differences between Eq.(18) and Eq.(19) is that the electron interacts with the ion through a screened interaction $\mu_q$ in Eq.(18) rather than the direct Coulomb interaction $v_{\mathbf{q}}$ as in Eq.(19). The specific form of $\mu_q$ is not known though it is expected to be less than $v_{\mathbf{q}}$. If a screened interaction $\mu_q$ is used, then the calculated dephasing rate $1/\tau_\phi$ would be smaller than that with the unscreened interaction $v_{\mathbf{q}}$. In this work, we will use $v_{\mathbf{q}}$ instead of $\mu_q$. The second difference, which is rather important, is the quantum mechanical nature of the electron motion in Eq.(19). $V(\mathbf{r}_j(t))$ in Eq.(19) is understood as $\langle \psi_j(\mathbf{r}, t) | V_{QM} | \psi_j(\mathbf{r}, t) \rangle$, where $\psi_j(\mathbf{r}, t)$ is the time-dependent wavefunction which describes a wavepacket corresponding to the trajectory $j$;
\[ e^{iq \cdot r_j(t)} \approx \langle \psi_j(r, t) | \sum_k c_k^{+} c_{k+q} | \psi_j(r, t) \rangle. \]  

(20)

At a finite temperature \( T \), the wave-packet state \( |\psi_j(r, t)\rangle \) will consist of mostly the eigenenergy states with energies limited within \( e_F \pm k_B T \). Therefore, the time dependence of \( e^{iq \cdot r_j(t)} \) will be limited by the frequency windows \( |\omega| < k_B T \). Though we use the semiclassical approach in Eq. (18), the frequency cut-off will be performed in our semiclassical calculation later.

### IV. The Return Probability

For the simplicity of calculation, we suppose that the TLS is initially in the state \( |-\rangle \), an eigenstate of the Hamiltonian in Eq. (11) with the eigenenergy \( E_- \) (the case with \( |+\rangle \) can be calculated in a similar way). The time evolution operator \( U_j(t) \) (\( U_j(t) (-\rangle > = c_+ \langle | \) + \( c_- \langle |> | \) - \( \langle | \) - \( \rangle > \)) of the TLS corresponding to the electron path \( j \) can be written as

\[ \begin{pmatrix} c_+(t) \\ c_-(t) \end{pmatrix} = \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_0^t dt' \mathbf{V}_1(r_j(t)) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \]  

(21)

where

\[ \mathbf{V}_1(r_j(t)) = \begin{pmatrix} V_j(r_j(t'), t') & \langle + | V_j(r_j(t'), t') \rangle \\ \langle - | V_j(r_j(t'), t') \rangle & \langle - | V_j(r_j(t'), t') \rangle - \langle + | V_j(r_j(t'), t') \rangle \end{pmatrix}. \]  

(22)

To find the corresponding time evolution of the TLS states for the time-reversed paths, \( (U_j^r(t)|-\rangle = d_+(t)|+\rangle + d_-(t)|-\rangle \) one obtains a similar form by using \( r_j(t) = r_j(\tau_0 - t) \). Expanding \( \langle - | U_j^r(\tau_0) U_j^r(\tau_0) | - \rangle = c_+^r(\tau_0) d_+(\tau_0) + c_-^r(\tau_0) d_-(\tau_0) \) up to the second order in interaction \( \mathcal{V} \), we get

\[ \text{Re} \langle - | U_j^1(\tau_0) U_j^r(\tau_0) | - \rangle = 1 + \frac{1}{\hbar^2} \int_0^{\tau_0} dt \int_0^{\tau_0} dt' \left[ \cos(\Omega(t + t' - \tau_0)) - \cos(\Omega(t - t')) \right] \times \langle + | \mathcal{V}(r_j(t)) | + \rangle + \langle - | \mathcal{V}(r_j(t)) | + \rangle \]  

(23)

where \( \hbar \Omega = E_+ - E_- = \sqrt{\Delta^2 + \Delta^2}, \) and we used the relation \( \mathcal{V}(r_j(t))^\dagger = \mathcal{V}(r_j(t)) \) in the last equality.

Now from Eq. (18), Eq. (23) and Eq. (10), one can get the coherent return probability in

\[ P_{r}^{coh}(\tau_0) = \sum_j |A_j(r, r; \tau_0)|^2 \times \frac{1}{3 \hbar^2 L^2 d} \int_0^{\tau_0} dt \int_0^{\tau_0} dt' \left[ \cos(\Omega(t + t' - \tau_0)) - \cos(\Omega(t - t')) \right] \times \sum_q q^2 |v^2_q| \sum_j |A_j(r, r; \tau_0)|^2 e^{iq \cdot (r_j(t') - r_j(t))}. \]  

(24)

Here, we omitted the subscript \( \eta \) in \( P_{r}^{coh} \) for the TLS state, because both of the initial states of the TLS \( |\eta\rangle = |\pm\rangle \) give rise to the same expression. \( P_{r}^{coh}(\tau_0) \) in Eq. (24) is the value averaged over the TLS position. Here, we used the disorder average over TLS \( \langle \exp[i\mathbf{R} \cdot (\mathbf{q} + \mathbf{q'})] \rangle = \delta_{\mathbf{q}, -\mathbf{q'}} \). The factor 3 in Eq. (24) comes from the average over the orientation of the TLS dipoles.

In the case of \( \sqrt{D} \tau_0 > l \) (\( l \) is the mean free path), the sum over the classical paths which appears in Eq. (24) can be written as a path integral using the Wiener measure \[\mathcal{D}[\mathbf{x}(\tau)]\]. The path integral can be calculated as

\[ \sum_j |A_j(r, r; \tau_0)|^2 e^{iq \cdot (r_j(t') - r_j(t))} \]

\[ = \int_{\mathcal{D}[\mathbf{x}(\tau)]} D[\mathbf{x}(\tau)] \]

\[ \exp\left( -\frac{1}{4D} \int_0^{\tau_0} dt \left| \dot{\mathbf{x}}(\tau) \right|^2 \right) \exp(iq \cdot (x(t) - x(t'))) \]

\[ = \frac{1}{V} \sum_{|q'| < \pi/l} \exp(-D|q'|^2 \tau_0) e^{-D(|q'|^2 - 2q' \cdot \mathbf{q})|t-t'|}, \]

where we used the boundary conditions of a quantum dot by a rectangular box of the volume \( V = L \times L \times (L \text{ or } a < L \text{ for 2D dots}) \). Several remarks are in order. Eqs. (23) are valid only in diffusive regimes. The summation over the discretized momentum variables \( \mathbf{q} \) and \( \mathbf{q}' \) is understood to be limited by \( \pi/l \). The contributions from ballistic regime, which are supposed to be small when \( \sqrt{D} \tau_0 > l \), are neglected in this work. The last equality in the Eqs. (23) is obtained for \( \mathbf{r} = 0 \). To simplify the calculation, from here on, let us consider the return probability of the particle at the origin \( \mathbf{r} = 0 \).

Then, the classical return probability \( P_{r}^{class}(\tau_0) \) for \( \mathbf{r} = 0 \) is given by

\[ P_{r}^{class}(\tau_0) = \sum_j |A_j(r, r; \tau_0)|^2 \]

\[ = \frac{1}{V} \sum_{|q'| < \pi/l} \exp(-D|q'|^2 \tau_0). \]  

(26)

Inserting Eq. (25) into Eq. (24), we get the coherent part of the return probability

\[ P_{r}^{coh}(\tau_0) = \frac{1}{V} \sum_{q'} \exp(-D|q'|^2 \tau_0) \times \left[ 1 + \frac{|q|}{3h^2 L^2 d} \int_0^{\tau_0} dt \int_0^{\tau_0} dt' \left[ \cos(\Omega(t + t' - \tau_0)) - \cos(\Omega(t - t')) \right] \times \sum_q q^2 |v^2_q| \sum_j |A_j(r, r; \tau_0)|^2 e^{iq \cdot (r_j(t') - r_j(t))}. \]  

(27)

where we used the change of variables of \( t^+ = t + t' \) and \( t^- = t - t' \). We restrict to the ergodic regime \( \tau_0 > \tau_D \), therefore significant contribution comes only from \( q' = 0 \). Then, we obtain
\[ P_{\text{coh}}(\tau_0) = \frac{1}{V} \left[ 1 + \sum_q \left( \frac{1}{\Omega^2 + (Dq^2)^2} \right) \right] \]\[ \left[ \cos \Omega (t^+ - \tau_0) - \cos \Omega t^- \right] \]
\[ \frac{1}{2\pi} \int d\omega \exp \left( i\omega (t^-) \right) \] (28)

The frequency of the time-dependent electric field produced by the electron is not infinitely large but has a cut-off. By assuming the electron to be in equilibrium with other electrons at temperature \( T \), the high frequency cut-off of \( \omega \) is given by \( k_B T \) \((\omega < k_B T)\); this is true at temperatures that are not too low. Note that because of the finite size of the system, \( g\nu_q = 0 \) for \( q = 0 \).

Therefore, there is no divergence at low frequencies and the low-frequency cut-off of \( \omega \) does not play an important role. By integrating Eq. (28) with the condition of \( |\omega| < k_B T \), we obtain the coherent part of the return probability \( P_{\text{coh}} \), which decays as \( \propto 1 - \tau_0/\tau_\phi + \cdots \). We have now defined \( \tau_\phi \) as the dephasing time. The dephasing rate \( 1/\tau_\phi \) from a randomly distributed TLS with an asymmetry energy \( \Delta \) and a tunnel splitting \( \Delta_0 \) is given by

\[
\frac{1}{\tau_\phi(\Delta, \Delta_0)} = \frac{2\langle |\hat{p}|^2 \rangle}{3\hbar^2 L^2} \sum_q q^2 \frac{\Omega^2}{\Omega^2 + (Dq^2)^2} = \frac{2\hbar^2 \Delta_0^2}{3\hbar^2 D(\Delta^2 + \Delta_0^2)} \sum_{0 < |q| < \pi/l} v_q^2 L^{-2d}. \] (30)

We used \( \Omega_T \ll 1 \) in the second equality in Eq. (30). \( 1/\tau_\phi \) obtained above is valid only when \( k_B T > \hbar \Omega \), while in the other case \( P_{\text{coh}}(\tau_0) \) is an oscillating function of \( \tau_0 \) with a small amplitude. Therefore \( 1/\tau_\phi \) \((k_B T < \hbar \Omega)\) is negligible.

By inserting Eq. (16) and Eq. (17) into Eq. (30) when \( k_B T > \sqrt{\Delta^2 + \Delta_0^2} \), we get

\[
\frac{1}{\tau_\phi(\Delta, \Delta_0)} = \frac{2\hbar^2 \Delta_0^2}{3\hbar^2 D(\Delta^2 + \Delta_0^2)} \] (31)

where \( d = 2, 3 \) is the spatial dimension of the quantum dot, and

\[
\Xi_2 = \sum_{0 < m^2 + n^2 < \langle L/l \rangle^2} \frac{1}{m^2 + n^2} \] (32)

\[
\Xi_3 = \frac{1}{\pi^2} \sum_{0 < m^2 + n^2 + k^2 < \langle L/l \rangle^2} \frac{1}{m^2 + n^2 + k^2}. \] (33)

V. DEPHASING BY TWO-LEVEL DEFECTS WITH WIDELY DISTRIBUTED ENERGIES

We can generalize \( 1/\tau_\phi \) to the case where the TLSs are distributed with a distribution function \( f(\Delta, \Delta_0) \),

\[
\frac{1}{\tau_\phi} = \frac{1}{\tau_\phi(\Delta, \Delta_0)} f(\Delta, \Delta_0). \] (34)

By inserting Eq. (33) into the above equation, we get

\[
\frac{1}{\tau_\phi} = \frac{L^d}{D} \frac{L^d}{3\hbar^2 e^{\frac{2d}{2}}} S(T), \] (35)

where

\[
S(T) = \int d\Delta \int d\Delta_0 \frac{\Delta_0^2}{\Delta^2 + \Delta_0^2} f(\Delta, \Delta_0) \theta(k_B T - \sqrt{\Delta^2 + \Delta_0^2}), \] (36)

where \( a \) is the thickness of the dot in case \( d = 2 \). It is interesting to note that at \( d = 2 \), the dephasing time does not depend on the dot area.

To calculate \( \tau_\phi \), we use the standard tunneling model for the two-level defects. The essential postulate in this theory is the uniform distribution of the tunneling parameter \( \lambda \) associated with the tunnel splitting \( \Delta_0 \propto e^{-\lambda} \). The energy distribution function \( f(\Delta, \Delta_0) \) in this case is written as

\[
f(\Delta, \Delta_0) = \frac{\bar{P}}{\Delta_0}. \] (37)

Furthermore, it is also assumed that \( \Delta_0 \) has a nonzero minimum value \( \Delta_{0,\text{min}} \). By applying this distribution, we find;

\[
S(T) = \bar{P} \int_{\Delta_{0,\text{min}}}^{k_B T} d\Delta_0 \int_{-\sqrt{(k_B T)^2 - \Delta_0^2}}^{\sqrt{(k_B T)^2 - \Delta_0^2}} d\Delta \frac{\Delta_0}{\Delta^2 + \Delta_0^2} \] (38)

where

\[
\mathcal{F}(z) = \int_1^z x^{-1} \sqrt{x^2 - z^2} dx \] (40)

Here, the above expression is valid for \( k_B T < \Delta_{0,\text{max}} \), which is an realistic and common assumption for the temperature below 1K. Note that \( \mathcal{F}(z) \sim z \ln z \) when \( z >> 1 \), therefore in the case of \( k_B T >> \Delta_{0,\text{min}} \), we expect the following temperature dependence

\[
\frac{1}{\tau_\phi} \propto T \ln(\frac{k_B T}{\Delta_{0,\text{min}}}), \] (41)

which is closer to \( T \) rather than \( T^2 \).

Now let us estimate \( \tau_\phi \) quantitatively. We consider the experiments by Huibers and coworkers\[2\] on two-dimensional ballistic semiconductor quantum dots. The quantum dots in the experiments are in the ballistic regime, while our \( \tau_\phi \) is for diffusive quantum dots. However, since the dephasing time is in the ergodic regime \((\tau_\phi > \tau_D)\), the results for diffusive dots should be applicable to the chaotic quantum dots in the ballistic
regime. The diffusion coefficient is obtained through the ergodic time scale and \( D \sim (E_{Th}/\delta_1)(\hbar/2m^*)^2 \), where \( m^* \) is the effective mass of the electron (for GaAs, \( m^* = 0.067m_e \)), and \( E_{Th}/\delta_1 \sim 30 \) for Ref. For ballistic dots, the Thouless energy is given by \( \hbar v_F/L \). For GaAs, \( \hbar^2 e^2/m^*e^2 \sim 10nm \). A reasonable size of the dipole moment is \( p_0 \sim e \times 10^{-10}m \). The thickness of the two-dimensional quantum dot is roughly \( a \sim 10nm \). By putting together \( \Xi_2 \) and \( \ln(k_BT/\Delta_{0,ln}) \), which are roughly \( 1-10 \), into Eq. (53), we find
\[
1/\tau_0 \sim (10^{-16} - 10^{-15})m^3s^{-1}P_0k_BT. \tag{42}
\]
In order to obtain \( \tau_0 \sim 1ns \) near \( T = 0.1K \), the average concentration should be \( P \sim (10^{48} - 10^{49})J^{-1}m^{-3} \). Although this number is not completely unreasonable, it is too large to be expected from well-textured semiconductors used in the experiments. For comparison, we note that glassy materials possess a typical TLS concentration of \( P \sim 10^{45} - 10^{46}J^{-1}m^{-3} \).

One may anticipate a different temperature dependence which might show the saturation of \( \tau_0 \) by considering the dissipative two-level system due to TLS-phonon interactions or incoherent two-level systems due to TLS-TLS interactions. However, it is very difficult to expect that the dephasing rate is enhanced by several orders of magnitude by such interactions.

The large magnitude of \( \bar{\Delta} \) may be possible if a large number of two-level defects aggregate on the surface of the quantum dots. This possibility can be experimentally checked by varying the system size and the dimensionality. Using our results,
\[
1/\tau_0 \propto L^{d-2}a^{3-d}. \tag{43}
\]
For example, for a 2D quantum dot, the dephasing rate \( 1/\tau_0 \) by “intrinsic” two-level defects will increase as the thickness \( a \) of the dot increases, whereas it will decrease with \( a \) for surface defects.

VI. DEPHASING BY TWO-LEVEL DEFECTS WITH A NARROW ENERGY DISTRIBUTION

Low-energy excitations exist in semiconductor crystals due to the tunneling of impurity ions between equivalent interstitial lattice sites. Due to the crystal fields, definite positions are preferred and a wide distribution of excitation energies is not expected; in glasses, the wide distribution arises because of structural disorder. However, defects on the surface may result in a wider distribution of energies because of surface roughness. A single tunnel-splitting energy implies a narrow distribution of relaxation times such that the standard tunneling model, applicable to structural glasses, as discussed in the previous section, is not valid.

In this section, we consider a well defined tunnel-splitting energy \( \Delta_0 \) rather than a wide distribution. The asymmetry may be uniformly distributed with a gaussian width \( \Delta_1 \), usually determined from the experimental data. The distribution function is defined as
\[
f(\Delta) = n_{TLS} \frac{1}{\Delta_1 \sqrt{\pi}} e^{-\Delta^2/\Delta_1^2}. \tag{44}
\]
\( n_{TLS} \) is the TLS density.

The function \( S(T) \) defined in Eq. (24) in the expression for the dephasing rate \( 1/\tau_0 \) is simplified to
\[
S(T) = \int d\Delta \frac{\Delta_1^2}{\Delta^2 + \Delta_0^2} f(\Delta) \theta(k_BT - \sqrt{\Delta^2 + \Delta_0^2}). \tag{45}
\]
Note that the variable \( \Delta_0 \) is not integrated over, in contrast to the case for the standard tunneling model; and the final result depends on \( \Delta_0 \). Evaluation of the above integral yields
\[
S(T) \sim n_{TLS} (k_BT >> \Delta_0 >> \Delta_1) \sim \frac{\Delta_0}{\Delta_1} n_{TLS} \ (k_BT >> \Delta_1 >> \Delta_0). \tag{46}
\]
If temperature is larger than the energy scales of TLS, then it is possible to obtain saturation or temperature-independent dephasing rate \( 1/\tau_0 \). In realistic systems, \( \Delta_0 \) is usually a small fraction of \( \Delta_1 \); thus the experimentally relevant limit is the second case, \( \Delta_1 \gg \Delta_0 \), in expression (47). The dephasing rate can now be obtained:
\[
\frac{1}{\tau_0} \sim (10^{-16} - 10^{-15})n_{TLS} \frac{\Delta_0}{\Delta_1} m^3s^{-1}. \tag{48}
\]
For \( \tau_0 \) to be on the order of 1 ns, the two-level defect density should be
\[
n_{TLS} \sim \frac{\Delta_1}{\Delta_0} (10^{24} - 10^{25})m^{-3}. \tag{49}
\]
Now let us estimate \( n_{TLS} \) for a typical single-crystal system. Single crystal silicon structures have been studied in this context in the temperature range of interest, below 1 K down to 5 mK. Both acoustic dissipation and heat capacity measurements on silicon resonators by Kleiman, Agnolet and Bishop (see the corresponding estimates by Phillips and Keyes) find that the TLS density, \( n_{TLS} \sim 10^{23}m^{-3} \), with an estimated value of \( \Delta_1/\Delta_0 \sim 100 \). Now using the same value for \( \Delta_1/\Delta_0 \) in the expression (48), the order-of-magnitude estimate of TLS density is found to be \( n_{TLS} \sim 10^{26} - 10^{27}m^{-3} \). The required density needs to be at least three orders of magnitude higher than the typical concentration in the silicon structures to result in a TLS-induced dephasing time \( \tau_0 \sim 1 ns \). This is an unreasonably large number, even for the typical intentionally-doped semiconducting structures/heterostructures for the effects of two-level systems have not been done, in the temperature range of interest for dephasing. Recent studies on semi-insulating gallium arsenide resonators suggest that the typical TLS density is comparable to that in silicon.
VII. CONCLUSION

We have calculated the dephasing time by assuming the presence of two-level defects inside diffusive quantum dots. The temperature dependence of $1/\tau_\phi$ is found to be roughly $\sim T$ for widely distributed two-level defects in the standard tunneling model. We find that to explain the size of the experimentally-observed dephasing times, we need a large number of two-level defects. This number is substantially larger than that found in glassy materials (almost by three orders of magnitude). Therefore, it is hard to believe that the electron dephasing is dominated by the intrinsic two-level defects at low-temperatures. We have also calculated $\tau_\phi$ from a distribution of narrow energy two-level defects, and we find a regime of temperature independent $\tau_\phi$. However, the required number of two-level defects is too large as in the case of widely distributed TLS. The system size dependence obtained in our calculation can be used to check the possibility of surface defects which are probably effective. Because of the large surface-to-volume ratio in quantum dots, it may be reasonable to assume that most of the defects are surface-aggregated. It will be interesting to estimate $\bar{P}$ or $n_{TLS}$ required for the observed low-temperature charge noises of quantum dots and compare to the values from dephasing time. Unfortunately, we are not aware of any quantitative theory for the quantum-dot charge noise arising from the two-level defects.

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Note added.- In a recent paper, Aleiner, Altshuler, and Galperin have analyzed the relevance of TLS for electron dephasing. Although they use a different approach and evaluate $\tau_\phi$ for different systems (metals not quantum dots), their conclusions are similar to ours—that is, a substantially large concentration of TLS, $\bar{P}$, much larger than the typical values in metallic glasses is required for the quantitative explanation of the saturation observed in experiments on metallic wires by two-level systems.

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