Mechanism of Torque Ripple Generation by Time and Space Harmonic Magnetic Fields in Permanent Magnet Synchronous Motors

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Abstract—In this article, we investigate mechanism of torque ripple generation by time and space harmonic magnetic fields in permanent magnet synchronous motors to obtain advanced motor designs. The general expression between the torque ripples and harmonic air-gap flux densities in the motor is derived by using Maxwell stress tensor. Both the numerical and experimental verifications of this expression are carried out. Then, the major harmonic magnetic fields that produce the torque ripples are specified and the differences between the surface and interior permanent magnet synchronous motors are investigated. According to these investigations, the shape of the rotor surface of an interior permanent magnet motor is optimized. It is clarified that specific harmonic components of the torque ripples in interior permanent magnet synchronous motors can be reduced to be nearly zero by optimizing the rotor surface shape.

Index Terms—Air gaps, optimization methods, permanent magnet motors, torque, finite element methods.

NOMENCLATURE

- $B_r, B_\theta$: Radial and circumferential components of air-gap flux density
- $B$: A component of air-gap flux density ($B_r$ or $B_\theta$)
- $\theta$: Electric angle (stator coordinate system)
- $t$: Time
- $\omega$: Angular frequency
- $k$: Order of space harmonics
- $n$: Order of time harmonics
- $B_{k,n}$: Amplitude of harmonic components included in $B$
- $\phi_{k,n}$: Phase angle of harmonic components included in $B$
- $\alpha_{\text{stator}}$: Modulation factor of air-gap flux density by stator harmonics
- $\alpha_j$: j-th harmonic component of $\alpha_{\text{stator}}$
- $R$: Radius of air gap surface between stator and rotor
- $L$: Core length
- $\mu_0$: Vacuum permeability
- $T$: Total torque
- $T_m$: $m$-th harmonic component included in waveform of $T$
- $W_i$: Iron loss
- $\sigma_{\text{max}}$: Maximum von Mises stress

I. INTRODUCTION

Magnetic field in permanent magnet synchronous motors (PMSMs) includes various time and space harmonics, which are generated by stator and rotor harmonic magnetomotive forces (MMFs). These harmonics produce harmonic losses and torque ripples. There are many articles that dealt with the reduction of these ill effects. The specific harmonic losses can be reduced by reducing the corresponding harmonic magnetic fields due to the requirement of each motor application [1], [2]. It can be stated that the mechanism of harmonic loss generation is straightforward and relatively simple. On the other hand, it is considered that the mechanism of torque ripple generation is more complex and still unclear.

Several articles reported the torque ripple reduction of PMSMs by control techniques based on circuit models with d-q axis decomposition [3], [4]. However, the cogging torque components, which are included in the torque ripples, cannot be represented by the circuit models because it generates without armature current. In reference [5], the magnetic co-energy term is added to the usual d-q axis torque expression to take the effect of cogging torque into account. However, this term is not considered in the torque ripple minimization.

Also, there are many articles that report the reduction of cogging torques of PMSMs by improving the motor designs. For example, cogging torques of surface-mounted permanent magnet synchronous motors (SPMSMs) are often reduced by optimizing the shapes of permanent magnets (PMs) [6], [7]. This approach is very effective for SPMSMs, in which the harmonic magnetic field is directly determined by the surface shapes of the PMs. On the other hand, it is considered that the generation mechanism of cogging torques in interior permanent magnet synchronous motors (IPMSMs), which are widely used for variable speed/load applications, is more complex because the PMs are buried in the rotor core and the harmonic magnetic
fields are considerably affected by the magnetic saturation. Furthermore, the saturated area in the rotor core considerably varies with the armature current. Therefore, the harmonic components of torque ripples under load conditions often become considerably different from those of cogging torques under no-load condition. Consequently, in many articles, the reduction of torque ripples of IPMSMs is often carried out by trial-and-error process of stator and/or rotor designs.

The total torque including the torque ripples of PMSMs can also be expressed by air-gap flux density due to Maxwell stress tensor [8]-[10]. However, there is no article that reports the general relationship between the torque ripples and the harmonic magnetic fields generated by the stator and the rotor. In references [11] and [12], the magnetic fields in motors are decomposed into time and space harmonics. However, the torque ripples have not been derived from these harmonic components. We consider that the generation mechanism of cogging torques and torque ripples in PMSMs can be clarified and the observations for advanced motor designs can be obtained, if they are directly expressed by the harmonic magnetic fields.

From these viewpoints, we have derived the general expression between the harmonic air-gap flux densities and torque ripples in [13] by using Maxwell stress tensor, to specify the harmonic fields that produce the torque ripples in IPMSMs, and to obtain advanced motor designs. In this article, this expression is applied to both the SPMSM and the IPMSM to clarify the difference in the generation mechanism of torque ripples. In addition, both the numerical and experimental verifications of the proposed expression are carried out. According to these investigations, the rotor surface shape of an IPMSM is optimized to reduce the specific components of torque ripples to be zero.

II. HARMONIC FIELDS AND TORQUE RIPPLE EXPRESSION

A. Harmonic Magnetic Fields in PMSMs

The air-gap flux density \( B \) in rotating machines can be decomposed into time and space harmonic components by using Fourier transformation, as follows [12]:

\[
B = \sum_{i=1}^{\infty} \sum_{n} B_{i,n} \cos(k \theta - n \omega t + \phi_{k,n})
\]  

where \( k \) and \( n \) are the space and time harmonic orders, respectively; \( B_{i,n} \) and \( \phi_{k,n} \) are the amplitude and phase angles of the harmonic component, respectively; \( \theta \) is the electric angle fixed to stator coordinates system, and \( t \) is the time. Forward harmonic rotational fields are expressed by positive \( n \), whereas the backward fields are expressed by negative \( n \). Both the radial and circumferential components can be expressed by (1) with different amplitudes and phase angles.

Then, let us discuss the major harmonic magnetic fields existing in PMSMs. Fig. 1 shows a standing wave produced by the stator and a traveling wave synchronized with the rotor. The traveling wave consists of fundamental rotational field and harmonic components produced by harmonic rotor MMFs. The sum of these components \( B_{\text{synchronized}} \) can be expressed, as follows:

\[
B_{\text{synchronized}} = \sum_{j=1}^{\infty} B_{j,\alpha} \cos(j \theta + \phi_{j,\alpha})
\]

The space harmonic orders \( k \) of these components is equal to the time harmonic orders \( n \) because they are synchronized. Therefore, both \( k \) and \( n \) are expressed by \( i (=1, 3, 5…) \) in (2).

The harmonic rotational fields caused by magnetic saturation are also included in (2).

On the other hand, the standing wave consists of phase-band harmonics and slot harmonics. It can be considered that the effects of these harmonics on the total air-gap flux density \( B \) do not vary with \( t \), whereas they vary with \( \theta \) due to the space harmonic order. Therefore, we assume following factor, which expresses the total effect of the stator harmonics on \( B \):

\[
\alpha_{\text{stator}} = \sum_{j=1}^{\infty} \alpha_j \cos(j \theta + \phi_{j,0})
\]

where \( j \) is the space harmonic order, which is determined by the numbers of phases and stator slots. For example, \( j=6, 12, 18… \) when the number of stator slots per pole pair is multiple of 6 and the number of phases is 3. \( \alpha_j \) is the amplitude of \( j \)-th harmonic components included in \( \alpha_{\text{stator}} \). Note that \( \alpha_{\text{stator}} \) includes both the effects of slot permeance distribution and winding configuration.

Fig. 2 shows an example of air-gap flux density waveform in PMSMs. This waveform can be expressed by \( B_{\text{synchronized}} \) modulated by \( \alpha_{\text{stator}} \). Therefore,

\[
B = \sum_{i} B_{ij} \left[ 1 + \sum_{j=1}^{\infty} \alpha_j \cos(j \theta + \phi_{j,0}) \right] \cos(i \theta - i \omega t + \phi_{i,\alpha})
\]

This expression can be expanded by mathematical formula of trigonometric functions, as follows:

\[
B = \sum_{i} B_{ij} \cos(i \theta - i \omega t + \phi_{i,\alpha}) + \sum_{j} \sum_{i} \frac{1}{2} B_{ij} \alpha_j \cos((j-i) \theta + i \omega t + \phi_{j,\alpha} - \phi_{i,\alpha}) + \sum_{j} \sum_{i} \frac{1}{2} B_{ij} \alpha_j \cos((j+i) \theta - i \omega t + \phi_{j,\alpha} + \phi_{i,\alpha})
\]
The first term is equal to (2). By comparing (1) and (5), it can be considered that the air-gap flux density of PMSMs includes following harmonic components:

a) Fundamental and harmonic rotational fields, which are synchronized with the rotor. They are expressed by the first term in (5) \((k=\eta=n=i)\)

b) The components generated by the modulation of above rotational fields by the stator standing wave. They are expressed by the second and third terms in (5).

\((k=j+i \text{ and } n=i \text{ for the second term})\)

\((k=j+i \text{ and } n=i \text{ for the third term})\)

Fig. 3 shows an example of predicted time and space harmonic orders included in the air-gap flux density of PMSMs, whose number of stator slots per pole pair is multiple of 6 and number of phases is 3. As the space harmonic order \(k\) is always defined to be positive, the signs of both \(k\) and \(n\) are reversed when \(j\) is negative. This figure will be confirmed in section III.

Note that additional harmonic fields will be superimposed on the stator flux density of PMSMs when \(n\) is not equal to \(k\). Therefore, the expression (8) can be written, as follows:

\[ T = \frac{R^2 L}{\mu_0} \sum_{k=0}^{\infty} \sum_{n_k=0}^{\infty} B_{r,k,n_k} B_{\theta,0,n_k} \left[ \cos(k \theta - n_k \omega t + \varphi_{r,k,n_k}) - \cos(n_k \omega t + \varphi_{\theta,0,n_k}) \right] d\theta \]

This is the general expression between torque and harmonic air-gap flux densities. For example, it implies that the \(m\)-th harmonic component included in the torque waveform \((m\)-th torque ripple) is generated by the product of the harmonic components of \(B_r\) and \(B_{\theta}\), whose space harmonic orders are identical to each other and the difference in the time harmonic orders \((n_k-n_n)\) is equal to \(\pm m\).

III. APPLICATION OF THEORY TO PMSMS

A. Numerical and Experimental Verifications

First, numerical and experimental verifications of the torque expression described in section II are carried out by applying the theory to a SPMSM, in which the mechanism of torque ripple generation is considered to be relatively simple. 2D nonlinear time-stepping finite element analysis (FEA) is applied to the calculation of harmonic fields and torques.

Table I lists the motor specification. Fig. 4 shows the cross section. The motor has one sintered Sm-Co magnet per pole. The numbers of phases and stator slots per pole pair are 3 and 6, respectively. This motor was manufactured by IEE-Japan rotating machine committee for the purpose of confirming the accuracy of measured and calculated cogging torques.

Fig. 5 shows the experimental and calculated cogging torque waveform (waveform of \(T\) at no load without armature current). The figure indicates not only the calculated result by
the Maxwell tensor method by (7), but also that by the nodal force method, which is often used in commercial software. In the nodal force method, the torque is calculated by the summation of the force acted on the nodes of the finite elements in the rotor region [14]. These calculated results are nearly identical to each other. The figure also indicates the experimental results by two different laboratories. It is confirmed that the calculated results are found to be in good agreement with those by (7) and the nodal force method.

Fig. 6 shows the amplitudes of the cogging torque \( T_6, T_{12}, \) and \( T_{24} \), which are the 6, 12, and 24\( \text{th} \) components extracted from the torque waveforms in Fig. 5. The results by (10) are also shown. In this expression, the torque ripples are obtained by the sum of the harmonic products whose \( (n-\nu r) \) is 6, 12, and 24, respectively. The results by (10) are also found to be in good agreement with those by (7) and the nodal force method.

Fig. 7 shows the time and space harmonic components included in \( B_r \) and \( B_\theta \) of the air-gap flux density in this case. These components can be calculated by using the method described in [12]. As it was predicted in Fig. 3, the space harmonic orders of the major harmonics are expresses as \( k=i \) \((i=1, 3, 5…, \text{synchronized harmonic rotational magnetic fields})\) and \( k=j \pm i \) \((j=6, 12, 18…, \text{the components generated by the modulation of } \nu \text{-th harmonic rotational field by } j \text{-th stator harmonic fields})\). Due to (10), the \( m \)-th cogging torque is generated by the sum of all the combination of harmonic \( B_r \) and \( B_\theta \), whose space harmonic orders are identical to each other and the difference in the time harmonic orders \( (n-\nu r) \) is equal to \( \pm m \). From these results, the theory described in section II is verified.

Fig. 8 shows the distributions of \( B_r \) and \( B_\theta \) at \( t=0 \). The included space harmonics are also shown. It is observed that all the harmonic components of \( B_r \) are \( \pm \text{sine curves} \), whereas those of \( B_\theta \) are \( \pm \text{cosine at } t=0 \). These facts implies that the phase angle \( \phi_{r,0} \) in (8)-(10) under the no-load condition is 90 or 270 deg, whereas \( \phi_{0,0,0} \) is 0 or 180 deg, for any harmonic orders.

B. Components of Cogging Torque and Torque Ripples

Then, let us discuss the major harmonic fields that produce the cogging torques and torque ripples. The harmonic fields of an IPMSM are also investigated and the results are compared with those of the SPMSM described in section III-A.

Table II lists the specification of the IPMSM. Fig. 9 shows...
the cross section. The motor has 3 sintered Nd-Fe-B magnets per pole. The number of stator slots per pole pair is 12. This motor is designed for variable speed/load applications.

Fig. 10 shows the variation in torque waveforms of the SPMSM and the IPMSM with load. The armature current of SPMSM under the load condition is set to be 3 A, whereas that of the IPMSM is 90 A, which is one of the typical condition of this motor. It is observed that both the phase angle and the amplitude of the torque ripples under the load condition varies from those of the cogging torques under the no-load condition, particularly in the case of the IPMSM.

Fig. 11 shows the phasor diagrams of the components that produce $T_e$, $T_{12}$, and $T_{24}$ decomposed by the proposed expression (10). In this figure, the components in (10) are expressed by the blue circles, whose distance from the origin corresponds to the amplitude, whereas the angle corresponds to the phase angle of each component. The sum of these components is also shown by the red squares, whose distance from the origin is equal to the result of Fig. 6 in the case of the SPMSM under the no load condition. It is clarified that the cogging torques and torque ripples are generated by considerable number of the combinations of harmonic $B_r$ and $B_\theta$ in both the SPMSM and the IPMSM. There are 4 major components in each order $T_m$, as follows:

$$T_m \approx \frac{\pi^2 L}{\mu_0} B_{r\text{,antist},1} B_{\theta\text{antist},1} \cos(\phi_{r\text{,antist},1} + \phi_{\theta\text{antist},1} - \phi_{\text{antist},1})$$ (11)

As discussed in section II-A, $(B_{r\text{,antist},1}, B_{\theta\text{antist},1})$ is produced by rotor harmonics, which are determined by the rotor shape and permeability. On the other hand, $(B_{r\text{,antist},1,1,1}, B_{\theta\text{antist},1,1,1})$ is produced by the modulation of fundamental rotational field $B_{r,1}$ by the $m$-th stator standing wave, which is phase-band harmonics or slot harmonics. It is observed that $T_e$ of the IPMSM under the no-load condition is zero, whereas that under the load condition is not negligible. The reason is that all the components of $T_e$ in the IPMSM are generated only by the phase-band harmonics, which are produced by three phase currents, because the number of stator slots per pole pair is 12.

Fig. 11 also indicates that the phase angles of all the torque components under the no-load condition are 90 or 270 deg. The reason is that these phase angles are expressed in (10) as the difference between the phase angles of harmonic $B_r$ and $B_\theta$ $\phi_{r,kr,n} - \phi_{\theta,k,n,\theta}$. As shown in Fig. 8, $\phi_{r,kr,n}$ under the no-load condition is 90 or 270 deg, whereas $\phi_{\theta,k,n,\theta}$ is 0 or 180 deg. Therefore, $\phi_{r,kr,n} - \phi_{\theta,k,n,\theta}$ becomes 90 or 270 deg. On the other hand, these phase angles considerably vary with load. This variation is caused by the variation in $\phi_{r,kr,n}$ and/or $\phi_{\theta,k,n,\theta}$.

To understand the detailed mechanism of this variation, the variation in the harmonic magnetic fields with load is investigated. Fig. 12 shows the phasor diagram of harmonic fields that cause two major components of $T_{12} (B_{r,1,1,1}B_{\theta,1,1,1} + B_{r,1,1,1}B_{\theta,1,1,1})$. In the SPMSM, $B_{r,1,1,1}$ and $B_{r,1,1,1}$, which are produced by harmonic rotor MMF, are almost constant with load. Only the phase angles of $B_{r,1,1,1}$ and $B_{r,1,1,1}$, which are produced by the modulation of fundamental $B_{r,1}$ by the stator standing wave, are changed. This variation is mainly caused by the variation in $B_{r,1,1,1}$ by the armature reaction under the load condition. On the other hand, in the case of the IPMSM, not only $B_{r,1,1,1}$ and $B_{r,1,1,1}$, but also $B_{r,1,1,1}$ and $B_{r,1,1,1}$ vary with load. This variation must have been caused by the variation in the rotor permeability distribution with load. Fig. 13 shows the calculated relative permeability distribution of the IPMSM. It is observed that the distribution considerably varies with load, particularly at the rotor core surface. As a result, the rotor harmonics varied. On the other hand, in the SPMSM, it is considered that the rotor harmonics are not considerably changed with load because there is no core at the rotor surface. It can be stated that the mechanism of the torque ripple generation in IPMSMs is more complex than SPMSMs because of the variation in all the harmonic magnetic fields with load.

IV. TORQUE RIPPLE REDUCTION BY MOTOR OPTIMIZATION

A. Strategy of Torque Ripple Reduction

Finally, the reduction of the torque ripples in the IPMSM is investigated by optimizing the motor shape. From the observations obtained in section III, three strategies can be considered to reduce the $m$-th torque ripples of the IPMSM, as follows:
**Fig. 10.** Variation in torque waveforms with load.

**Table:**

| SPMSM | IPMSM |
|-------|-------|
| **No load (Cogging torque)** | **Loaded (Torque ripple)** |
| ![Graph](image1.png) | ![Graph](image2.png) |
| **No load (Cogging torque)** | **Loaded (Torque ripple)** |
| ![Graph](image3.png) | ![Graph](image4.png) |
| **Components** | **Total** |
| ![Graph](image5.png) | ![Graph](image6.png) |
| **Components** | **Total** |
| ![Graph](image7.png) | ![Graph](image8.png) |
| **Components** | **Total** |
| ![Graph](image9.png) | ![Graph](image10.png) |

**Fig. 11.** Components of cogging torques and torque ripples.

**Fig. 12.** Harmonic fields that cause major torque ripple components.
a) Decrease the rotor harmonics \((B_{p,m=1,n=1}, B_{p,m=1,n=1})\) by optimizing rotor shape.
b) Decrease \(m\)-th standing wave produced by the stator.
c) Decrease the sum of components in the \(m\)-th torque ripple in Fig. 11 by optimizing rotor shape and/or stator-slot shape.

In this article, the strategy (c) with the modification of rotor surface shape is adopted because the rotor harmonics in IPMSMs significantly vary with the air gap. It is expected that the torque ripple can be reduced by slight modification of the rotor surface shape without considerable deteriorations of the other important motor characteristics.

### B. Conditions of Automatic Optimization

Then, the automatic rotor surface optimizations are carried out to decrease the torque ripples under frequent driving conditions in the variable speed/load application.

Fig. 14 shows the rotor surface points, whose radial and circumferential coordinates are selected as the design variables. The circumferential coordinates of \(P_1\) and \(P_{11}\) are fixed to be 0 and 22.5 deg, respectively to keep the rotor shape symmetric.

Table III lists the estimated characteristics and driving conditions. The condition A is a typical low speed and low torque condition (the same condition in Figs. 10 to 13), whereas the condition B is the instantaneous maximum torque condition. The average torque \((T_0)\), and the torque ripples \((T_{12}, T_{24})\) are estimated under these conditions. The torque ripple reduction is particularly required under the condition A because the driving term of the analyzed motor around this condition is very long. On the other hand, the condition C is the maximum speed condition under flux weakening control with rated armature current. The iron loss \(W_i\) and the maximum von Mises stress \(\sigma_{\text{max}}\) by centrifugal force are estimated under this condition because they considerably increase with the rotational speed.

Fig. 15 shows the calculation system for the automatic shape optimization [15]. Both the 2D stress FEA (under the condition A) and the 2D electromagnetic field FEA (under the condition A, B, and C) are carried out due to the rotor shape determined by Rosenbrock’s method. Then, the torques, iron loss, and maximum von Mises stress are fed back to Rosenbrock’s method as the objective function or constraint conditions in order to determine the optimized shape.

### C. Results of Optimizations

First, the objective function is set to be the total torque ripple under condition A, which is defined by the difference between the maximum and minimum torques in the torque waveform. In this case, the constraint conditions are imposed, as follows:

a) \(T_0\) under both the conditions A and B should be larger than 97% of those of the initial design, respectively.
b) \(T_{12}, T_{24}\) under the condition B should be less than 130% of those of the initial design, respectively.
c) \(W_i\) under the condition C should be less than that of the initial design.
d) \(\sigma_{\text{max}}\) under the condition C should be less than 300 MPa.

Fig 16 shows the variation in \(T_0, T_{12}, T_{24}\), and \(T_0\) with optimization iterations. It is observed that all the torque ripples under the condition A are reduced by this optimization, whereas the other characteristics satisfy the constraint conditions. In this case, the total torque ripple is reduced to be nearly 1/3 of that of the initial design.

Next, three additional optimizations, whose objective functions are \(T_0, T_{12}\) and \(T_{24}\) under the condition A, are individually carried out. It is because the reduction of specific order torque ripple is often strongly desired due to the requirement of the mechanical system in each motor.
application. In this case, the final shape in the optimization of Fig. 16 is used as the initial design. The constraint conditions (a)-(d) are also imposed.

Fig. 17 shows the variation $T_6$, $T_{12}$, and $T_{24}$ with optimization iterations. Fig. 18 shows the torque waveforms of the initial and optimized motors. It is observed that each order torque ripple is reduced to be nearly zero when it is selected as the objective function. It implies that an arbitrary order torque ripple can be zeroed by optimizing the rotor surface shape under the specific driving condition due to the requirement of each application. On the other hand, Fig. 17 also indicates the tradeoff between the objective order torque ripple and the others. $T_{24}$ increases when $T_6$ or $T_{12}$ is selected as the objective function, whereas $T_{12}$ increases when $T_{24}$ is the objective function.

However, these torque ripples are still significantly smaller than those of the initial design.

Fig. 19 shows the von Mises stress distribution of the optimized rotor and the decomposed torque ripple components under the condition A. In all the optimized motors, two dents are generated around the edges of the outside PM under the constraint condition of $\sigma_{\text{max}}$. It is also confirmed that the phasors of $T_6$, $T_{12}$, and $T_{24}$ (red squares) are zeroed in each optimization from the initial value in Fig. 11 (right side, IPMSM, loaded), as follows:

a) $T_6$ is zeroed mainly by the variation in the phase angles of the components $B_{o,5,5}B_{o,5,5}$ and $B_{o,5,5}B_{o,5,1}$, which become nearly opposite to those of $B_{o,7,7}B_{o,7,7}$ and $B_{o,7,7}B_{o,7,1}$.

b) $T_{12}$ is zeroed mainly by the decrease in the amplitude of $B_{r,0,13}B_{o,13,13}$ and $B_{r,13,13}B_{o,13,1}$, while the increase in those of $B_{r,0,11}B_{o,11,1}$ and $B_{r,0,11}B_{o,11,1}$. The amplitudes of these components become nearly equal to each other, respectively.

c) $T_{24}$ is zeroed by both the variation in phase angles and amplitudes of the included components.

V. CONCLUSION

The mechanism of torque ripple generation by time and space harmonic magnetic field in PMSMs is investigated to obtain advanced motor designs. The harmonic fields that produce the torque ripples are specified by deriving the general expression between torque ripples and harmonic air-gap flux densities. The numerical and experimental verifications of the proposed torque ripple expression are carried out. Then, the difference of the torque ripple generation between SPMSMs and IPMSMs are discussed. From the observation by the proposed expression, the rotor surface optimization of an IPMSM is carried out to reduce the total and specific order torque ripples. As a result, following knowledges are obtained:

a) $n$-th order torque ripple is generated by the product of the harmonic components of $B_r$ and $B_{o}$ whose space harmonic orders are identical to each other and the difference in the time harmonic orders is equal to $\pm n$.

b) There are considerable number of harmonic $B_rB_{o}$ combinations that produce the torque ripple. The major components are the product of harmonic rotor MMFs and the harmonics generated by the modulation of fundamental field by stator standing waves.

c) The harmonic fields caused by rotor MMFs in SPMSMs are almost constant with load, whereas all the harmonic fields of IPMSMs vary with load because of the variation in rotor core saturation. As a result, the mechanism of the torque ripple generation in IPMSMs is more complex than SPMSMs.

d) An arbitrary order torque ripple of IPMSMs under specific driving condition can be zeroed by optimizing the rotor surface shape without considerable deteriorations of the other
important motor characteristics. Further work will be required to consider the fluctuation of the manufacturing, which affects the torque ripples.

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