1. Introduction

Madhu Sudan’s work spans many areas of computer science theory including computational complexity theory, the design of efficient algorithms, algorithmic coding theory, and the theory of program checking and correcting.

Two results of Sudan stand out in the impact they have had on the mathematics of computation. The first work shows a probabilistic characterization of the class NP – those sets for which short and easily checkable proofs of membership exist, and demonstrates consequences of this characterization to classifying the complexity of approximation problems. The second work shows a polynomial time algorithm for list decoding the Reed Solomon error correcting codes.

This short note will be devoted to describing Sudan’s work on probabilistically checkable proofs – the so called PCP theorem and its implications. We refer the reader to [29, 30] for excellent expositions on Sudan’s breakthrough work on list decoding, and its impact on the study of computational aspects of coding theory as well as the use of coding theory within complexity theory.

Complexity theory is concerned with how many resources such as time and space are required to perform various computational tasks. Computational tasks arise in classical mathematics as well as in the world of computer science and engineering. Examples of what we may call a computational task include finding a proof for a mathematical theorem, automatic verification of the correctness of a given mathematical proof, and designing algorithms for transmitting information reliably through a noisy channel of communication. Defining what is a ‘success’ when solving some of these computational tasks is still a lively and important part of research in this stage of development of complexity theory.

A large body of Sudan’s work, started while he was working on his PhD thesis, addresses the automatic verification of the correctness of mathematical proofs. Many issues come up: how should we encode a mathematical proof so that a computer can verify it, which mathematical statements have proofs which can be quickly verified, and what is the relation between the size of the description of the theorem

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and the size of its shortest proof which can be quickly verified. The work of Sudan sheds light on all of these questions.

2. Efficient proof checking

Let us start with the classic notion of efficiently checkable proofs, which goes back to the early days of computer science in the early seventies when the NP class was defined.

**Definition 1** The class $NP$ consists of those sets $L \subseteq \{0, 1\}^*$ for which there exists polynomial time verification algorithm $V_L$ and polynomial $p$ such that $x \in L$ if and only if there exists a $y_x \in \{0, 1\}^{p(|x|)}$ which makes $V_L(x, y_x) = TRUE$. We call $V_L$ the NP-verifier for the language $L \in NP$, and $y_x$ the NP-witness for $x$ in $L$.

One example of $L \in NP$ is the set of pairs $(G, k)$ where $k \in \mathbb{Z}$ and $G$ is a graph which contain a complete subgraph on $k$ vertices – the so called CLIQUE problem. The NP-witness for pair $(G, k) \in CLIQUE$ is the complete subgraph in $G$ of size $k$. Another example is the set of all logical formulas for which a truth assignment to its Boolean variables exists which makes it true – the SATISFIABILITY problem. The NP-witness for a logical formula $\phi$ is a particular setting of its variables which make the formula satisfiable. Graphs, logical formulas, and truth assignments can all be encoded as binary strings.

3. Probabilistic checking of proofs

In the eighties, extensions of the notion of an efficiently verifiable proof were proposed to address issues arising in disciplines involving interactive computation such as cryptography. The extensions incorporate the idea of using randomness in the verification process and allow a negligible probability of error to be present in the verification process. Variants of probabilistic proof systems include interactive proofs, public-coin interactive proofs, computational arguments, CS-proofs, Holographic proofs, multi-prover interactive proofs, memoryless oracles, and probabilistically checkable proofs. The latter three definitions are equivalent to each other although each was introduced under a different name.

By the early nineties probabilistically checkable proofs were generally accepted as the right extension for complexity theoretic investigations. The class PCP of sets for which membership can be checked by ”probabilistically checkable proofs” is defined as follows.

**Definition 2** Let $L \subseteq \{0, 1\}^*$. For $L$ in PCP, there exists a probabilistic polynomial time verification algorithm $V_L$

- if $x \in L$, then there exists a $O_x \in \{0, 1\}^*$ such that $Prob[V_L^{O_x}(x) = TRUE] > \frac{1}{2}$
- if $x \notin L$, then for all $O_x \in \{0, 1\}^*$, $Prob[V_L^{O_x}(x) = TRUE] < \frac{1}{2}$. 
The probabilities above are taken over the probabilistic choices of the verification algorithm $V_L$. The notation $V_L^{O_x}$ means that $V_L$ does not receive $O_x$ as an input but rather can read individual bits in $O_x$ by specifying their locations explicitly. We call $V_L$ the PCP-verifier for $L \in \text{PCP}$, and $O_x$ the PCP-witness for $x$ in $L$.

A few comments are in order.

For each bit of $O_x$ read, we charge $V_L$ for the time it takes to write down the address of the bit to be read. The requirement that $V_L$ runs in polynomial time implies then that the length of the PCP-witness for $x$ is bounded by an exponential in $|x|$. A verifier may make an error and accept incorrectly, but the probability of this event can be made exponentially (in $|x|$) small by running a polynomial number of independent executions of $V_L$ and accepting only if all executions accept. In light of the above, we argue that probabilistically checkable proofs capture what we want from any efficiently checkable proof system: correct statements are always accepted, incorrect statements are (almost) never accepted, and the verification procedure terminates quickly.

Are probabilistically checkable proofs more powerful than the deterministic NP style proofs? Developments made in a sequence of beautiful papers [32, 24, 5], finally culminated in the result of Babai et al. [5] showing that indeed $\text{PCP} = \text{NEXP TIME}$.

By the separation of the non-deterministic time hierarchy, it is known that $\text{NP}$ is strictly contained in $\text{NEXP TIME}$. Thus indeed, the probabilistic checking of proofs is more powerful than the classical deterministic one (at least when the verifier is restricted to polynomial time).

Soon after the power of PCP verifiers was characterized, a finer look was taken at the resources PCP verifiers use. Two important resources in classifying the complexity of language $L$ were singled out [10]: the amount of randomness used by the PCP verifier and the number of bits it reads from the PCP-witness (the latter number is referred to as the query size of $V_L$).

**Definition 3** Let $\text{PCP}(r(n), q(n))$ denote class of sets $L \in \text{PCP}$ for which there exists a PCP verifier for $L$ which on input $x \in \{0, 1\}^n$ uses at most $O(r(n))$ random bits and reads at most $O(q(n))$ bits of the witness oracle $O_x$. \footnote{The class $\text{NEXP TIME}$ is defined exactly in the same manner as $\text{NP}$ except that the verifier $V_L$ has exponential time and the witness may be exponentially long.}

Obviously, $\text{NP} \subset \text{PCP}(0, \cup, n^\omega)$ as an NP verifier is simply a special case of the PCP verifier which does not use any randomness. Starting with scaling down the result of [5] it was shown (or at least implied) in a sequence of improvements [1, 11, 2] that $\text{NP} \subset \text{PCP}(\log n, \text{poly}(\log n))$. These results successively lowered the number of bits that the PCP-verifier needs to read from the PCP-witness, but it seemed essential for the correctness of the verification procedure that this number should be a function which grows with the size of the input.

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such as cryptography. The extensions incorporate the idea of using randomness in the verification process and allow a negligible probability of error to be present in the verification process. Variants of probabilistic proof systems include interactive proofs \cite{14}, public-coin interactive proofs \cite{3}, computational arguments\cite{4}, CS-proofs \cite{26}, Holographic proofs \cite{6}, multi-prover interactive proofs \cite{7}, memoryless oracles \cite{14}, and probabilistically checkable proofs \cite{10, 2}. The latter three definitions are equivalent to each other although each was introduced under a different name.

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4. The PCP theorem

In a breakthrough, which has since become known as the PCP theorem, Sudan and his co-authors characterized the class $NP$ exactly in terms of $PCP$. They showed that $NP$ contains exactly those languages in which a $PCP$-verifier can verify membership using only a constant query size and using logarithmic (in the instance size) number of coins. Moreover, there exists a polynomial time procedure to transform an $NP$-witness of $x$ in $L$ into a PCP-witness of $x$ in $L$.

**Theorem 6** \cite{16} $NP = PCP(\log n, 1)$

On an intuitive level, the PCP theorem says that there exist a probabilistic verifier for proofs of mathematical assertions which can look only at a constant number of bit positions at the proof and yet with some positive probability catch any mistake made in a fallacious argument.

The proof of the PCP theorem is deep, beautiful, and quite complex. It brings together ideas from algebra, error correcting codes, probabilistic computation, and program testing.

Although the PCP theorem establishes a complexity result, its proof is algorithmic in nature, as it is a transformation of an NP-witness and a deterministic NP-verifier for $L \in NP$ into a PCP-witness and an PCP-verifier for $L$. As such it uses methods from the design of computer algorithms and the design of error correcting codes. Several excellent expositions of the proof appeared \cite{28}.

In a very strong sense, the act of transforming an NP witness into a PCP witness is similar to transforming a message into an error correcting code word. The analogy being that a code word is an encoding of a message which enables

\[ O(g(n)) = cf(n) \text{ s.t. there exists a constant } c \text{ such that } g(n) \leq cf(n) \text{ for all } n \text{ sufficiently large} \]
error detection in spite of noise injected by an adversary, and a PCP witness is an encoding of a proof which enables detection with high probability of an error in spite the best efforts to hide it made by a cheating pretend-to-be prover.

Yet, the act of classic decoding of a code word is very different than the act of checking the correctness of a PCP witness. Whereas in error correcting codes one attempts to recover the entire original message from the corrupted code word if too much noise has not occurred; here we only want to verify that the PCP-witness is a proper encoding of a valid NP-witness (of the same fact) which would have convinced an NP-verifier to accept. It suffices to read only a constant number of bit positions to achieve the latter task, whereas the decoding task depends on reading the entire code word.

One of the subsequent contributions of Sudan, involves constructing a new type of locally testable codes [11, 17]. Locally testable codes are error-correcting codes for which error detection can be made with probability proportional to the distance of the non-codeword from the code, based on reading only a constant number of (random) symbols from the received word. A related concept is that of locally decodable codes [23, 18] which are error correcting codes which may enable recovery of part of the message (rather than the entire message) by reading only part of the corrupted code word.

5. PCP and hardness of approximation

The intellectual appeal of the PCP theorem statement is obvious. What is much less obvious and what has been the main impact of the PCP theorem is its usefulness in proving NP hardness of many approximate versions of combinatorial optimization problems. A task which alluded the theoretical computer science community for over twenty years.

Shortly after the class $NP$ and the companion notion of an $NP$-complete and $NP$-hard problems were introduced, Karp illustrated its great relevance to combinatorial optimization problems in his 1974 paper [22]. He showed that a wide collection of optimization problems (including the minimum travelling salesman problem in a graph, integer programming, minimum graph coloring and maximum graph clique suitably reformulated as language membership problems) are $NP$-complete. Proving that a problem is $NP$-complete is generally taken to mean that they are intrinsically intractable as otherwise $NP = P$.

In practice this means there is no point in wasting time trying to devise efficient algorithms for NP-complete problems, as none exists (again if $NP \neq P$). Still these problems do come up in applications all the time, and need to be addressed. The question is, how? Several methods for dealing with NP-completeness arose in the last 20 years.

One technique is to devise algorithms which provably work efficiently for particular input distribution on the instances (“average” instances) of the NP-complete problems.

\footnote{A set is $NP$-hard if any efficient algorithm for it, can be used to efficiently decide every other set in $NP$. An NP set which is NP-hard is called NP-complete. By definition, every $NP$-complete language is as hard to compute as any other.}
problems. It is not clear however how to determine whether your application produces such input distribution.

Another direction has been to devise approximation algorithms. We say that an approximation algorithm $\alpha$-approximates a maximization problem if, for every instance, it provably guarantees a solution of value which is at least $\frac{1}{\alpha}$ of the value of an optimal solution; an approximation algorithm is said to $\alpha$-approximate a minimization problem if it guarantees a solution of value at most $\alpha$ of the value of an optimal solution.

Devising approximation algorithms has been an active research area for twenty years, still for many NP-hard problems success has been illusive whereas for others good approximation factors were achievable. There has been no theoretical explanation of this state of affairs. Attempting to prove that approximating the solution to NP-hard problems is in itself NP-hard were not successful.

The PCP theorems of Sudan and others, starting with the work of Feige et. al. \cite{10}, has completely revolutionized this state of affairs. It is now possible using the PCP characterization of NP to prove that approximating many optimization problems each for different approximation factors is in itself NP-hard. The mysteries of why it is not only hard to solve optimization problems exactly but also approximately, and why different NP-hard problems behave differently with respect to approximation have been resolved.

The connection between bounding the randomness and query complexity of PCP-verifiers for NP languages and proving the NP hardness of approximation was established in \cite{10,1} for the Max-CLIQUE problem (defined below). It seemed at first like an isolated example. The great impact of Sudan et. al.’s \cite{1} theorem was in showing this was not the case. They showed that proving characterization of NP as \textit{PCP}(log$n$,1) implies the NP hardness of approximation for a collection of NP-complete problems including Max-3-SAT, Max-VERTEX COVER, and others (as well as improving the Max-CLIQUE hardness factor).

The basic idea is the following: A PCP type theorem provides a natural(?) optimization problem which cannot be efficiently approximated by any factor better than 2 as follows. Fix a PCP-verifier $V_L$ for an NP language $L$ and an $x$. Any candidate PCP-witness $O_x$ for $x$ defines an acceptance probability of $V_L^{O_x}(x)$. The gap of 1/2 in the maximum acceptance probability for $x \in L$ versus $x \notin L$ (which exists by the definition of PCP) implies that it is NP-hard to 2-approximate the maximum acceptance probability of $V_L$. In other words, the existence of a polynomial time algorithm to 2-approximate the acceptance probability of $x$ by $V_L$ would imply that $NP=P$.

For different optimization problems, showing hardness of approximation is done by demonstrating \textit{reductions} from variants of the above optimization problem. These reductions are far more complex than reductions showing NP-hardness for exact problems as one needs to address the difference in in-approximability factors of problems being reduced to each other.

Moreover, these new NP-hardness results have brought on a surge of new research in the algorithmic community as well. New approximation algorithms have been designed which at times have risen to the task of meeting from above the
approximation factors which were proved using PCP theorems to be best possible (unless \( NP = P \)) . This has brought on a meeting of two communities of researchers: the algorithm designers and complexity theorists. The former may take the failure of the latter to prove NP hardness of approximating a problem within a particular approximation factor as indication of what factor is feasible and vice versa.

This radical advance is best illustrated by way of a few examples. Finding the exact optimal solution to all of the following problems is \( \mathcal{NP} \)-complete. Naive approximation algorithms existed for a long time, which no one could improve. They yield completely different approximation factors. For some of these problems we now have essentially found optimal approximation problem. Any further advancement will imply that \( NP \) problems are efficiently solvable.

**Max-CLIQUE**: Given a finite graph on \( n \) vertices, find the size of the largest complete subgraph. A single vertex solution is within factor \( n^{0.999} \) of optimal. More elaborate algorithms give factor \( n^{-0.999} \). This problem was the first one to be proved hard to approximate using PCP type theorem [10]. It is now known that achieving a factor of \( n^{0.999} \) is \( \mathcal{NP} \)-hard for every \( \epsilon > 0 \) [19].

**Max-3-SAT**: Given a logical formula in conjunctive normal form with \( n \) variables where there is at most 3 literals per clause, determine the maximal number of clauses which can be satisfied simultaneously by a single truth assignment. A simple probabilistic algorithm satisfies \( \frac{7}{8} \) of the clauses. It is now known [20] that achieving a factor \( 7/8 - \epsilon \) for \( \epsilon > 0 \) approximation factor is \( \mathcal{NP} \)-hard even if the formula is satisfiable. At the same time [21] has shown an algorithm which matches the \( 7/8 \) approximation factor when the formula is satisfiable.

**Min-Set Cover**: Given a collection of subsets of a given finite universe of size \( n \), determine the size of the smallest subcollection that covers every element in the universe. A simply greedy algorithm, choosing the subsets which maximizes the coverage of as many yet uncovered elements as possible, yields a factor \( \ln n \) from optimal. It is now known that approximation by a factor of \( (1 - \epsilon) \ln n \) is \( \mathcal{NP} \)-hard for every \( \epsilon > 0 \) [9].

We point the reader to a collection of papers and expositions by Sudan himself [31] on these works as well as exciting further developments.

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