Neutral Scalar Higgs Masses and Production Cross Sections in an Extended Supersymmetric Standard Model

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ABSTRACT

Upper bounds on the three neutral scalar Higgs masses are considered in the supersymmetric standard model with a gauge singlet Higgs field. When the lightest Higgs is singlet-dominated the second lightest Higgs is shown to lie near or below the theoretical upper bound on the lightest Higgs mass. We also consider detectability of these Higgs bosons at a future $e^+e^-$ linear collider with $\sqrt{s} \sim 300$ GeV and show that at least one of the neutral scalar Higgs has a production cross section larger than 0.04 pb.
In the standard model of the elementary particle physics one of the most important subjects both experimentally and theoretically is to explore the Higgs sector. In addition to establishing the symmetry breaking mechanism, an investigation of the Higgs sector gives us a clue to physics beyond the standard model. This might be the case for the supersymmetric (SUSY) standard model whose Higgs sector has distinct features. In particular, a relatively light neutral Higgs is one of the general consequences of the SUSY models. Since no definite upper bound exists for the masses of the superpartners, the Higgs sector could be the first check point of the SUSY models.

In the minimal supersymmetric standard model (MSSM) it is well-known that we can calculate the upper bound on the lightest neutral Higgs mass as a function of the top and the stop masses. The upper bound is about 120 GeV for a top mass of 150 GeV and a stop mass of 1 TeV, and therefore exceeds the search limit of the LEP II experiment [1].

If we consider extended versions of the SUSY standard model, the situation is a little different. The simplest and interesting extension is to include a gauge singlet Higgs field. In this case we can introduce a new coupling among two Higgs doublets and a singlet in the superpotential which induces a new quartic term in the tree level Higgs potential. Therefore, the tree level mass relations themselves are modified and there is no upper bound on the lightest neutral Higgs mass without specifying the strength of the new coupling constant. However, if we require that none of dimensionless coupling constants blow up below the grand unification scale, then we can determine an upper bound on the lightest neutral Higgs mass [2, 3]. Taking account of the top and stop loop effects the upper bound is obtained as $130 \sim 150$ GeV for a reasonable range of the top mass and the 1 TeV stop mass [4, 5].

Although we can set a fairly strong mass bound for the lightest neutral Higgs, it may not be produced since it can be singlet-dominated in some region of parameter space and has a reduced coupling to the $Z^0$ boson [5]. In such a case the bounds for the heavier Higgs masses become important.

In this letter we derive upper bounds on the heavier neutral Higgs masses and show that the second lightest one obeys a similar but somewhat weaker mass bound if the lightest Higgs is dominated by a singlet component. We also discuss the detectability of the neutral Higgses in a future $e^+e^-$ linear collider like JLC-I [6] and show that at least one of the Higgses has a production cross section for $e^+e^- \rightarrow Z^0h$ mode larger than 0.04 pb at $\sqrt{s} = 300$ GeV.

We consider here a SUSY standard model with a gauge singlet Higgs field. In order to obtain a general consequence of this model we introduce all possible terms allowed by the gauge symmetry and renormalizability in the superpotential and the SUSY soft breaking
terms. The superpotential is given by
\[ W = \lambda_1 H_1 H_2 N + \frac{1}{6} \lambda_2 N^3 + \mu_1 H_1 H_2 + \frac{1}{2} \mu_2 N^2, \] (1)
then the tree level scalar potential is given by
\[ V = m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_N^2 |n|^2 \\
+ [\lambda_1 A_1 n h_1 h_2 + \frac{1}{6} \lambda_2 A_1 n^3 + B_1 \mu_1 h_1 h_2 + \frac{1}{2} B_2 \mu_2 n^2 + h.c.] \\
+ (|h_1|^2 + |h_2|^2) |\lambda_1 n + \mu_1|^2 + \lambda_1 h_1 h_2 + \frac{1}{2} \lambda_2 n^2 + \mu_2 n|^2 \\
+ \frac{g_2^2}{8} (h_1^\dagger \tau^a h_1 + h_2^\dagger \tau^a h_2)^2 + \frac{g'^2}{8} (|h_1|^2 - |h_2|^2)^2, \] (2)
where, \( H_1, H_2 \) are doublet superfields and \( N \) is a gauge singlet one, and \( h_1, h_2 \) and \( n \) are used for the scalar component. We have to include the top and stop loop effect to the above potential. For the degenerate stop case the one loop potential is given by
\[ V^{(1 \text{ loop})} = \frac{3}{16 \pi^2} \left[ (m^2 + y_t^2 |h_2|^2)^2 \left( \ln \frac{m^2 + y_t^2 |h_2|^2}{\mu^2} - \frac{1}{2} \right) \\
- y_t^4 |h_2|^4 \left( \ln \frac{y_t^2 |h_2|^2}{\mu^2} - \frac{1}{2} \right) \right], \] (3)
where \( m^2 \) is a SUSY breaking mass for the squark, \( \mu \) is a renormalization scale and \( y_t \) is the top Yukawa coupling constant. In the case with the large left-right stop mixing the above formula should be extended appropriately.

From this potential we can calculate the Higgs mass matrix. In the present case physical states are three neutral scalars, two pseudoscalar and one charged Higgs pair. It is convenient to eliminate \( m_{11}^2, m_{22}^2, m_N^2 \) by the minimization conditions of the potential and use three vacuum expectation values, i.e. \( v_1 = \langle h_1^0 \rangle, v_2 = \langle h_2^0 \rangle, x = \langle n \rangle \) as independent parameters. Here \( h_1^0 \) and \( h_2^0 \) are the neutral components of \( h_1 \) and \( h_2 \) respectively, and for simplicity we assume that these v.e.v.'s are real.

The most useful upper bound is obtained by the (1,1) component of the neutral scalar matrix \( M^2 \) in the basis \( (\phi_1, \phi_2, \phi_3) \) where \( \phi_2 \) does not have any vacuum expectation value:
\[ \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ n \end{pmatrix}, \] (4)
\[ ^1 \text{In eq.(1) we have eliminated the linear term of } N \text{ by a field redefinition of } N. \text{ In the scalar potential we can in principle include a linear SUSY soft breaking term. But this does not make any difference in the following discussion.} \]
\[ ^2 \text{We can also add the radiative correction due to the } \lambda_1 \text{ and } \lambda_2 \text{ coupling constants but the effect is not so large.} \]

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with $\tan \beta = v_2/v_1$. The upper bound of the lightest neutral scalar $S_1$ is given by

$$m^2_{S_1} \leq M^2_{11} = m^2_2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \delta,$$

where $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV and $\delta$ represents the top and stop effects

$$\delta = \frac{3}{4\pi^2} \frac{m^4_t}{v^2} \ln \frac{m^2_{stop}}{m^2_t}.$$

The maximum value of $\lambda_1$ is determined by requiring that none of the dimensionless coupling constants blow up below the GUT scale ($10^{16}$ GeV). In figure 1 we present the upper bound of eq.(5) in the parameter space of the top mass and $\tan \beta$. The corresponding figure for the MSSM is also shown. We can see that the maximum value of the lightest Higgs is about 135 GeV which is realized in the parameter region of a large $\tan \beta$ and large $m_t$ or of a small $\tan \beta$ and small $m_t$. Note that the $\delta$ term in eq.(5) becomes large in the former case, on the other hand the $\lambda_1$ term in eq.(5) becomes more important in the latter case. This should be compared to the MSSM case where the maximal value is given only in the large top mass region.

As is explained before it is important to determine upper bounds on the heavier neutral Higgs masses because the lightest one may be dominated by the singlet. We can derive an upper bound on the second lightest neutral mass in the following way. Denoting the second lightest and the heaviest neutral scalar Higgs by $S_2$ and $S_3$ respectively, the orthogonal diagonalization matrix $V$ is given by

$$\begin{pmatrix} \phi_1 - v \\ \phi_2 \\ \phi_3 - x \end{pmatrix} = V \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}.$$

It is a matter of a simple algebra to show that the second lightest eigenvalue satisfies the following inequality:

$$m^2_{S_2} \leq \frac{M^2_{11} - V^2_{11} m^2_{S_1}}{1 - V^2_{11}},$$

where $V_{11}$ is the (1,1) component of the diagonalizing matrix. $V_{11}$ is directly related to the following quantity:

$$V^2_{11} = \frac{\sigma(e^+e^- \rightarrow Z^0 S_1)}{\sigma(e^+e^- \rightarrow Z^0 h_{SM})} \bigg|_{m_{S_1} = m_{h_{SM}}}.$$

where $\sigma(e^+e^- \rightarrow Z^0 h_{SM})$ is the production cross section for the standard model Higgs and $\sigma(e^+e^- \rightarrow Z^0 S_1)$ is the corresponding production cross section for the lightest neutral Higgs boson in the present model. In figure 2 we show a contour plot of the upper bound on the second lightest Higgs mass in the space of the lightest neutral Higgs mass and
In this figure we take the top mass to be 150 GeV and the stop mass to be 1 TeV and the resulting upper bound of the lightest Higgs mass is 132 GeV which is realized at \( \tan \beta = 1.9 \). We can see that the upper bound on the second lightest Higgs mass can be 10% larger than \( M_{11} \) in the region where \( V_{11}^2 \lesssim 0.2 \). This means that even when the lightest neutral Higgs is singlet-dominated, and \( V_{11}^2 \) is thereby reduced, we still can expect at least one doublet-like Higgs below, say, 150 GeV\(^3\). On the other hand, \( m_{S_2} \) is not constrained in the limit of \( V_{11} \rightarrow 1 \).

When the second lightest Higgs as well as the first one has a reduced coupling to the \( Z^0 \) boson neither of them can be produced\(^4\). Then we need an upper bound of the heaviest neutral Higgs boson mass. A simple generalization of eq.(8) gives us the following relation for the heaviest neutral Higgs mass:

\[
m_{S_3}^2 \leq \frac{M_{11}^2 - (V_{11}^2 + V_{12}^2)m_{S_1}^2}{1 - (V_{11}^2 + V_{12}^2)} .
\]

Then the contour plot for the upper bound of the heaviest neutral Higgs boson mass is the same as the previous one provided that the x-axis is replaced by \( V_{11}^2 + V_{12}^2 \).

In order to show usefulness of the above formulas we consider the Higgs detection in a future \( e^+ e^- \) linear collider with \( \sqrt{s} \sim 300 \) GeV. The neutral scalar Higgses can be produced by either \( e^+ e^- \rightarrow Z^0 S_i \) process or \( e^+ e^- \rightarrow A^0_i S_j \) process where \( A^0_i \) is a pseudoscalar Higgs. Since the availability of the latter process depends on the mass of the pseudoscalar boson we only consider here the former process as a production mechanism of the neutral Higgs. Furthermore the decay modes of the neutral scalar Higgs are quite parameter-dependent. Although the standard model Higgs of the intermediate mass decays predominantly into a \( b \bar{b} \) pair, it is known in the MSSM case that the main decay mode of the neutral Higgs may be an invisible neutralino pair depending on parameters\(^5\).\(^6\). A similar situation can be realized in the present model. Even in such a case the recoil mass distribution measurement in the process \( e^+ e^- \rightarrow Z^0 S_i \) is available to show an evidence for the neutral Higgs. Therefore, we assume that the neutral Higgs boson can be detected if the production cross section for \( e^+ e^- \rightarrow Z^0 S_i \) mode is large enough. The production cross section for \( S_i \) is obtained from that for the standard model Higgs of the same mass by multiplying the \( V_{1i}^2 \) factor. The standard model cross section is given by\(^7\)

\[
\sigma_{SM} = \frac{\pi a^2(1 + (1 - 4 \sin^2 \theta_W)^2)(\lambda + 12m_Z^2/s)\sqrt{\lambda}}{192s \sin^4 \theta_W \cos^4 \theta_W} \left(\frac{1}{1 - m_Z^2/s}\right)^2 ,
\]

\(^{3}\)It is pointed out in ref.\(^8\) that the mass of the second lightest Higgs satisfies the upper bound of the lightest Higgs mass in the limit that the lightest Higgs becomes the pure singlet. This is because the mass matrix for the neutral scalar Higgses becomes block diagonal.

\(^{4}\)In this case the scalar-pseudoscalar-\( Z^0 \) coupling may become significant, and the scalar-pseudoscalar pair production is possible if the pseudoscalar boson is light enough.
where \( \lambda \) is

\[
\lambda = \left[ 1 - \frac{(m_Z + m_h)^2}{s} \right] \left[ 1 - \frac{(m_Z - m_h)^2}{s} \right],
\]

and \( m_h \) is the Higgs mass.

Let us now consider the detectability of the three neutral scalar Higgses. We would like to know whether at least one of the three Higgses can be discovered at an \( e^+ e^- \) linear collider with a given integrated luminosity. For a given set of \( V_{11}^2, V_{12}^2 \) and \( m_{S_1} \), we can determine the upper bounds on \( m_{S_2} \) and \( m_{S_3} \) from eqs. (8) and (10), then we can calculate the lower bounds of the production cross section for the second and third Higgses as well as the production cross sections for the lightest one. By denoting the above three values of the cross sections as \( \sigma_{2\text{min}}, \sigma_{3\text{min}}, \sigma_1 \) we then define the following quantity:

\[
\sigma(V_{11}^2, V_{12}^2) \equiv \min_{m_{S_1}} \{ \max(\sigma_1, \sigma_{2\text{min}}, \sigma_{3\text{min}}) \}.
\]

(13)

The meaning of this value is that for a given point of the parameter space \( (V_{11}^2, V_{12}^2) \) at least one of the three Higgses has always a production cross section larger than \( \sigma(V_{11}^2, V_{12}^2) \).

In figure 3 we show the value for \( \sigma(V_{11}^2, V_{12}^2) \) in the space of \( V_{11}^2 \) and \( V_{12}^2 \) for \( \sqrt{s} = 300 \) GeV, \( m_t = 150 \) GeV and \( m_{\text{stop}} = 1 \) TeV. The minimum of the cross section is 0.046 pb. This corresponds to about 30 events of \( e^+ e^- \to Z^0 S_i \) followed by \( Z^0 \to e^+ e^-, \mu^+ \mu^- \) in integrated luminosity of 10 fb\(^{-1} \) which will be attained at JLC-I after the operation of one third of a year\(^5 \). Since the production cross section of the standard model Higgs of the same mass as the lightest neutral Higgs’ upper bound (132 GeV) is 0.17 pb, the production cross section is reduced by a factor 4 at the worst case and therefore four times as large integrated luminosity is required to cover the whole parameter space. In the above analysis we made essential use of eqs. (8) and (10) where the upper bounds of the second and the third Higgs masses are given in term of \( V_{11}^2 \) and \( V_{11}^2 + V_{12}^2 \). Suppose that neutral Higgses are not found below the lightest upper bound for an enough integrated luminosity. This means that the lightest one has a reduced coupling to the \( Z^0 \) boson, therefore we get an upper bound on \( V_{11}^2 \). Then, from the figure 2, we can determine an upper bound on the second lightest Higgs mass. If the second Higgs is also not found up to its upper bound, we can use a similar argument and determine upper bounds on \( V_{12}^2 \) and the third Higgs mass. Since we now know the upper bound of the third Higgs mass and the lower bound of \( V_{13}^2 \) from the relation \( V_{13}^2 = 1 - V_{11}^2 - V_{12}^2 \), we can determine whether the third Higgs can be discovered for the given integrated luminosity.

In figure 4 we show the minimum value of \( \sigma(V_{11}^2, V_{12}^2) \) in the \( (V_{11}^2, V_{12}^2) \) space, denoted by \( \sigma_{\text{min}} \), as a function of \( \sqrt{s} \) for various top masses and a stop mass of 1 TeV. We can see

\(^5\) A more realistic estimation taking account of detection efficiency will increase the necessary integrated luminosity to about 30 fb\(^{-1} \) [6, 10].
that we always have a neutral scalar Higgs with a production cross section larger than 0.04 pb around $\sqrt{s} = 300$ GeV. This is encouraging for a future $e^+e^-$ linear collider like JLC-I since independently of parameters in the model at least one of the neutral scalar Higgses is detectable at the level of one year of operation and this can be a definitive test of this model.

Finally, we make comments on several possible extensions. The value of $M_{11}$ may be changed by the left-right stop mixing and the $\lambda_1, \lambda_2$ loop effects. However once the value of $M_{11}$ is specified the argument after eq.(7) does not change. Since these effects do not modify $M_{11}$ significantly, we expect a similar conclusion on the production cross section of the neutral scalar Higgses. We may also consider some other extensions of the Higgs sector. When the v.e.v’s become complex or more additional singlets are included, the dimension of the relevant mass matrix increases. A straightforward generalization of the above analysis is possible in such extensions.

The authors would like to thank K. Hikasa, T. Kawagoe, A. Miyamoto and T. Yanagida for useful discussions and comments. This work is supported in part by the Grant-in-aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.
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Figure Captions

Fig.1: (a) The contours of the lightest Higgs mass upper bound in the space of the top mass and tan \( \beta \) for the model with a gauge singlet Higgs. The blow up region is excluded since the top Yukawa coupling blows up below the GUT scale even if \( \lambda_1 = \lambda_2 = 0 \). (b) The corresponding contours for the MSSM. The blow up region is not shown in this figure.

Fig.2: The contours of the second light Higgs mass upper bound for \( m_t = 150 \) GeV and \( m_{\text{stop}} = 1 \) TeV.

Fig.3: The contours of \( \sigma(V_{11}^2, V_{12}^2) \) in the \( V_{11}^2-V_{12}^2 \) plane. We take \( \sqrt{s} = 300 \) GeV, \( m_t = 150 \) GeV and \( m_{\text{stop}} = 1 \) TeV.

Fig.4: The minimum cross section \( \sigma_{\text{min}} \) for the top masses, \( m_t = 120, 150 \) and 180 GeV, and \( m_{\text{stop}} = 1 \) TeV.
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