A Nonlinear High-Resolution Beamforming for Plane Wave Ultrasound Flow Imaging

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Abstract—Inspired from the delay multiply and sum beamforming in B-mode imaging, this article presents a novel nonlinear beamforming technique to improve the sensitivity and resolution in high frame rate flow imaging. The proposed nonlinear high-resolution (NLHR) beamforming approach ensures proper beam width throughout the region of interest and synthesizes multiple high resolution sub-aperture beams at receive by employing a multiply and sum based beamforming technique. The proposed beamforming technique is investigated for a directional cross-correlation based velocity estimator as well as an autocorrelation based velocity estimator with typical parabolic flow profiles of different velocities and flow direction. The improvement in sensitivity and Doppler resolution is evaluated with complex pulsatile flows in simulations with different peak velocities and durations. Further the technique is validated using pulsatile flow phantom experiments. The sensitivity of NLHR beamforming is demonstrated by evaluating a sudden change in flow direction imposed during the phantom experiment. The spatial resolving capability is evaluated with an air bubble tracking experiment using flow phantom. Preliminary studies on the in-vivo carotid data further validates the enhanced sensitivity and resolution achieved by the proposed technique. Detailed results for each test case in simulation, phantom and in-vivo studies are attached as movies in the supplementary materials and are available online [Online Link].

Index Terms—High frame rate, high resolution, non-linear beamforming, plane wave, sensitivity, ultrasound, vector flow imaging

I. INTRODUCTION

ULTRASOUND Doppler technique is the most widely used non-invasive technique for flow imaging and has a great diagnostic value in investigating the vascular hemodynamics [1]. It uses high frequency non-ionizing sound waves to create multiple insonification of blood vessels which makes it the safest technique to quantize the complex flow dynamics. Traditional Doppler ultrasound-based flow imaging techniques are highly dependent on the beam-to-flow angle and least sensitive to the transverse flow component [1]-[3]. This makes it difficult to quantitate the complex flows associated with the superficial vasculature as they are immediately beneath the skin surface and the flow is purely transverse in most of the cases.

To address these limitations, a multi-dimensional estimation for velocity vectors, commonly known as vector flow imaging (VFI) has been proposed [4]. It provides an angle independent, absolute quantification of the flow rate and enables the dynamic multi-dimensional visualization of complex blood flow. In this regard, various methods like transverse oscillation [4], crossbeam Doppler [5], speckle tracking [6], vector triangulation [7], directional beamforming [8],[9] and its variants coupled with autocorrelation and cross-correlation based velocity estimation techniques were exploited [10]-[18]. Most of the above strategies make use of unfocused transmit schemes like diverging waves [10],[11], multi-angle steered plane waves [12]-[15] or single non-steered plane waves [16]-[18] to insonify the region of interest (ROI) at high frame rates. The directional beamforming technique in [8] and [9] focuses the received signals along the flow direction and employs cross-correlation to estimate the velocity magnitude. But this approach requires the knowledge of the flow angle with respect to the emitted beam for beamforming. In this regard, correlation and numerical triangulation-based techniques have been developed to estimate the flow angle [19], [20]. A real-time vector velocity estimator is proposed in [13] in which steered plane waves are used for insonification and directional beamforming-based approach is used to estimate the velocity profile. However, synthesizing the directional signals by rotating the grid would be difficult in the case of complex flows in which the flow is not in a well-defined direction. To address these limitations, vector triangulation-based flow imaging approaches are proposed in the literature [14]-[18]. However, the contrast, resolution and accuracy of the velocity estimation is mainly determined by the beamforming algorithm employed in the system.

Delay and sum (DAS) beamforming algorithm is the most commonly used technique in medical ultrasound imaging due to its inherent low complexity [21], [22]. Conventional DAS beamformer in ultrasound systems involves delaying the received signals and then summing it up to obtain the dynamically focused receive beams. Usually, the beamformer uses an apodization window to weight the delay compensated signal before summing. The apodization enhance the energy of the signal components from the desired grid point while suppressing the signal components from the undesired locations [23]. This effectively improves the signal-to-noise ratio (SNR) and provides a better beam at the output of the beamformer. But the conventional DAS technique offers limited imaging resolution, and low contrast at the output.

Delay multiply and sum, commonly known as DMAS algorithm is a nonlinear beamforming technique in B-mode...
ultrasound imaging which has a higher spatial and contrast resolution with a better clutter and noise rejection when compared to DAS [23, 24]. The DMAS is originally devised for confocal microwave imaging for the early detection of breast cancer [23]. In one of the recent studies reported in [24] the DMAS algorithm is modified to a filtered delay multiply and sum (F-DMAS) by employing an additional bandpass filter for B-mode imaging. The simulation and the in-vivo studies with F-DMAS algorithm has demonstrated a better-quality image with higher dynamic range and improved contrast resolution at the output [25]. This nonlinear technique is further investigated for plane wave and synthetic aperture high frame rate ultrasound imaging [25], [26] as well as for photoacoustic imaging [27], [28]. Similar to the formulation of DMAS, coherent flow imaging techniques based on short-lag spatial coherence beamforming for power doppler imaging is proposed in [29], [30]. Yet, the effort towards the application of DMAS technique for flow imaging is very limited in the literature.

With respect to the quality of the DMAS beamformed signal and a better point spread function compared to DAS as reported in [25], the DMAS beamforming could be leveraged for flow imaging for better resolution and sensitivity in VFI techniques. Improving the spatial resolving capability of VFI could help in understanding more localized flow patterns and velocity distribution in various pathological conditions where VFI is used for diagnosis and monitoring. Enhanced sensitivity in VFI towards highly dynamic and impulse components in velocity could probably help in identifying new biomarkers or pathological features in medical diagnostics.

Inspired from the DMAS beamforming in B-mode imaging, this article presents a novel nonlinear beamforming technique to improve the sensitivity and Doppler resolution for flow imaging applications. We investigate the performance of the proposed nonlinear beamformer with a cross-correlation based velocity estimator and an autocorrelation based velocity estimator which are the common velocity estimation techniques in the literature. In this regard, the proposed beamforming technique is demonstrated for directional cross-correlation [8] and a triangulation-based single transmit dual angle multi receive (STDMR) scheme (which is proposed in one of our recent works [18]). To the best of the authors’ knowledge, this is the first effort towards the application of non-linear beamforming in flow imaging.

The rest of the article is organized as follows. The following section (Section II) explains the proposed nonlinear high-resolution beamforming technique for flow imaging and describes how it can be used in conjunction with triangulation based auto-correlation estimator and directional cross correlation estimator. Section III discusses the implementation details along with the simulation and experimental setup. Section IV presents the results obtained from the simulations and experimental investigations in comparison with DAS based VFI methods. Section V concludes the article and discusses some of the possibilities for the future work.

II. PROPOSED APPROACH

The proposed high resolution beamformer is a step towards the application of non-linear beamforming approach in VFI. A schematic of the proposed non-linear high resolution (NLHR) beamformer for flow imaging is as shown in Fig.1(a). The proposed beamformer comprises of five steps as illustrated in Fig. 1(b). The detailed description of each step is provided in the subsequent sections.

A. Nonlinear High-Resolution Beamforming for Flow Imaging

The first part of the NLHR beamforming deals with dynamic delay compensation of the received signals to enable channel-wise spatial focusing. The second part consists of the design of appropriate apodization function to form the directional weighted focused beams at left and right receive sub-apertures.

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Further, these beams are multiplied pairwise and summed before it is filtered to obtain the higher harmonic beamformed signals in the final step. These high-resolution beams are used to estimate the velocity magnitude and direction. In this work, non-steered plane waves are used to insonify the ROI at very high frame rates to aid the improvement in sensitivity and Doppler resolution. However, the proposed approach is also applicable to multi-angle plane waves, synthetic aperture and diverging wave transmit schemes. A detailed explanation of the approach is provided in the following subsections.

1) Time to Space Mapping

The received radio frequency (RF) echo signals, \( e_i[i,t] \) (where \( i \) represents the \( i^{th} \) channel), are delay compensated and mapped to a two-dimensional grid to ensure channel-wise spatial focusing. This process of mapping the received RF signals to a predefined spatial grid in the ROI is referred to as time to space mapping (TSM) and the delay compensated signal \( e[i,(x,z)] \) is referred to as the TSM signal/matrix from hereon and is of size \( N_c \times N_p \times N_t \), where \( N_c \) is the number of channels, \( N_p \) is the number of pixels and \( N_t \) is the number of frames.

2) Channel Directive Beam Synthesis

The TSM matrix can be interpreted as the channel-wise spatial representation of the received signals and hence a column of the TSM matrix is an \( N_c \)-element vector, \( e(x,z) \), that corresponds to one pixel. The samples in such a vector represents a single spatial point when viewed from \( N \) different receive elements or equivalently \( N \) different angles or directions. These vectors are then multiplied with the weight matrix, \( W_d \) of size \( N_c \times N_c \), to obtain the channel directive beams, \( S \) as given by (1):

\[
S_{(x,z)} = W_d^T \cdot e(x,z)
\]

where, each column of \( W_d \) is a Gaussian weight vector centered at each receive element. This step is to ensure better directional sensitivity and hence \( W_d \) is called as directivity weight matrix. It essentially converts the delay compensated channel data to the spatially focused channel beams. The size of the matrix \( S \) would be \( N_c \times N_p \times N_t \) as shown in Fig. 1. Thus, unlike traditional apodization, the channel beam synthesis as given by (2) creates \( N_c \) beam observations, that are equivalent to the focused observations from \( N \) different angles for each pixel. In this regard, the width of the Gaussian function is dynamically sized according to the time-of-flight (i.e., \( \text{width} = \frac{\text{time} \ - \ of \ - \ flight}{F_n} \) where \( F_n \) is the F-number), instead of depth as in traditional apodization, to ensure uniform beamwidth throughout the ROI.

3) Sub-Aperture Formation

The third step in the proposed NLHR beamforming technique is to apply suitable weight on to the channel directive beams to obtain multiple sub apertures to the left and right of the selected transmit element. These are referred to L-R aperture beams corresponding to different transmit-receive (Tx-Rx) angle \( \alpha \) and are given by (2) and (3):

\[
A_L(\alpha) = W_L(\alpha)^T \cdot S
\]

\[
A_R(\alpha) = W_R(\alpha)^T \cdot S
\]

where, \( S \) is the channel directive beams, \( W_L(\alpha) \) and \( W_R(\alpha) \) are the L-R apodization weight matrix whose columns are \( N \)-element weight vectors. A Gaussian weight vector for each spatial point is defined such that the maximum weight is given to the receive element that corresponds to the transmit-receive (Tx-Rx) angle \( \alpha \) under consideration. This essentially provides maximum weightage to the beam received at an angle \( \alpha \) and thus ensures the directionality of the L-R aperture beams corresponding to each pixel in the ROI. Unlike in the traditional DAS beamformer, the weighted signals are not directly summed in this approach and hence the signals obtained with (2) and (3) are regarded as alpha weighted beam focused L-R aperture signals. It should be noted that these signals are weighted in the direction defined by the \( \alpha \) and not the flow direction. This sub-aperture formation at receive enables the beamforming to be employed for most of the split aperture vector flow imaging techniques. The channel directive beam synthesis and sub-aperture formation is motivated by our previous work on dual stage apodization for VFI in [31].

4) Multiply and Sum (MAS) Operation

Once the alpha weighted beam focused L-R aperture signals are formed, each channel beam is multiplied with every other channel beam and then summed up to obtain the equivalent DMAS beamformed signal in VFI as given by (4) and (5). This constitutes the fourth step in the proposed NLHR beamformer.

\[
\hat{y}_L[x,z,\alpha] = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} A_L(\alpha)[i,(x,z)] \cdot A_L(\alpha)[j,(x,z)]
\]

\[
\hat{y}_R[x,z,\alpha] = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} A_R(\alpha)[i,(x,z)] \cdot A_R(\alpha)[j,(x,z)]
\]

It can be observed that the MAS operation is mathematically a cross correlation function between all the receive channels in which the self-product terms \( i = j \) are excluded. Also, it is to be noted that either \([A_L(\alpha)(i,x,z) \times A_L(\alpha)(j,x,z)]\) or \([A_R(\alpha)(i,x,z) \times A_R(\alpha)(j,x,z)]\) is considered in the summation of MAS operation as both the coefficients are equal in magnitude and hence the information is redundant. This will halve the corresponding correlation coefficient and reduces the number of multiplications required by a factor of 2 [24, 32]. The total number of multiplications required for an \( N \) channel receive aperture is given by (6).

\[
M_N = \frac{N^2 \ - \ N}{2}
\]

5) Band Pass Filtering

The frequency spectrum of the delay compensated directional focused beams of each channel will be a band centered around the center frequency of the insonified plane wave, \( f_0 \). But the frequency spectrum of MAS beamformed signals \( \hat{y}_L[x,z,\alpha] \) and \( \hat{y}_R[x,z,\alpha] \) will have two peaks, one centered at \( f_0 + f_0 \approx 0 \) and the other at \( f_0 + f_0 \approx 2f_0 \) because of the fact that (4) and (5) essentially multiplies two signals having almost similar frequency [24]. Now, a band pass filter (BPF) with a pass band centered around \( 2f_0 \) is used in the final step to remove the DC component. Thus, the frequency spectrum of filtered L-R
beamformed signals, \( \hat{y}_L[x,z,\alpha] \) and \( \hat{y}_R[x,z,\alpha] \), will be centered at \( 2f_0 \). In any Doppler technique, higher the transmit frequency, higher the Doppler shift and better the velocity estimate. In this work, since the harmonics induced by MAS is employed, it is expected to give a higher Doppler resolution in the velocity estimates.

B. Velocity Estimation

A truncated singular value decomposition filter as in the MUST toolbox [33] is employed for clutter suppression in the non-linear beamformed L-R signals. As shown in Fig. 1(a), the flow velocity vectors can be derived either by employing an autocorrelation-based estimator or a cross-correlation based estimator. Both the methods are described briefly in the following subsections.

1) Triangulation based Autocorrelation (TAC)

The mean slow time frequency of the clutter suppressed L-R signals, \( y_L[x,z,\alpha] \) and \( y_R[x,z,\alpha] \), is then determined using the classical lag-one autocorrelation approach [34]. Once the slow time frequency, \( f_s \) and \( f_R \), are estimated, the orthogonal components of the velocity can be estimated using the following equations (7) and (8) as in conventional triangulation [7] and that used in [17]:

\[
v \cos \theta = \frac{f_L(x,z,\alpha) + f_R(x,z,\alpha)}{(1 + \cos \alpha)} \frac{c}{2f'} \quad (7)
\]

\[
v \sin \theta = \frac{f_L(x,z,\alpha) - f_R(x,z,\alpha)}{(1 + \cos \alpha)} \frac{c}{2f'} \quad (8)
\]

where, \( f' = f_0 \) in case of DAS based beamforming and \( f' = 2f_0 \) in case of DMAS based beamforming because the center frequency of the DMAS based beamformed signal is around \( 2f_0 \). Unlike in [17] where only one pair of observation is considered for triangulation, the multiple sub-aperture formation in step 3 of the proposed approach provides multiple observations enabled by multiple \( \alpha \) at receive. The velocity magnitude and flow direction are calculated as the mean of all such observations.

2) Directional Cross-Correlation (DCC)

Directional cross-correlation based velocity estimation as in [8] is performed for the observations enabled by multiple \( \alpha \) at receive and the mean estimates among them is computed. However, to perform DCC, the beamforming must be done along the flow direction. This is done by rotating the flow grid and the time to space mapping of the received RF signals is done for the rotated spatial grid. Apart from this, no changes are required in the beamformer architecture to perform DCC based velocity estimation.

III. IMPLEMENTATION AND VALIDATION

In this section, the details on implementation of the proposed approach in MATLAB® as well as the simulation and experimental setup used for the evaluation and validation of the proposed approach are discussed.

A. Considerations to Avoid Aliasing

With the MAS operation discussed in Section II.A.4, the frequency spectrum of the beamformed signal would get shifted to \( 2f_0 \) and the Doppler frequency would also get doubled. This makes the DMAS based beamforming more prone to Doppler aliasing. However, in the proposed non-linear beamforming technique, this is addressed by resampling the received data along the axial direction as well in the temporal direction immediately after the acquisition to satisfy the Shannon-Nyquist sampling theorem. The size \( N_x \times N_y \times N_t \) in Fig. 1 represents the size of the received RF signals after resampling.

B. MAS Architecture and Computational Complexity

The MAS operation involves multiplication of every channel signal with every other channel signal and hence it is computationally costlier than the DAS technique. As discussed in Section II, redundant multiplications are not considered and thereby the number of multiplications is reduced considerably. However, the implementation of (4) and (5) can be simplified further by expanding the summation and factorizing it into \((N - 1)\) factors as shown in (9) [26], [35].

\[
P = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} X_i X_j \approx \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} X_i X_{N-1} + \left[X_i X_{N-1} + X_{N-1} X_{N-2}ight] X_{N-2} + \left[X_i X_{N-1} + X_{N-2} X_{N-3}ight] X_{N-3} + \cdots + X_1 X_2  
\]

where, \( X_i \) represents the directionally weighted focused beam corresponding to the channel \( i \) and \( P \) represents the MAS beamformed signal. With this simplification, the number of multiplications could be reduced from \( \frac{N^2-N}{2} \) to \((N - 1)\) for \( N \) channel receive aperture. A similar simplification is also reported in [32] and [35] where the authors show that the complexity of FDMAS beamforming is \( O(N) \). Since cross-channel multiplication in MAS results in power of the signal rather than amplitude, square root of the signal is taken prior to multiplication in a conventional DMAS beamformer [23], [32]. However, the signed square root operation is omitted for flow imaging as we are interested in local phase and frequency rather than the image contrast in B-mode imaging. This is because, the signed square root operation could alter the phase of a signal. Hence, the computational complexity of the conventional DMAS architecture is further reduced with the proposed MAS based non-linear beamforming for VFI applications as shown in Table I. The maximum number of additions and multiplications involved in proposed non-linear beamformer with an \( N \) channel receive aperture is obtained from (9). The factor of 4 that appears in the addition and multiplications for proposed MAS architecture accounts for the resampling of the

| TABLE I | THEORETICAL COMPARISON OF THE NUMBER OF MATHEMATICAL OPERATIONS REQUIRED FOR BEAMFORMING WITH DAS, CONVENTIONAL DMAS [24], SIMPLIFIED DMAS [32], [35] AND PROPOSED MAS ARCHITECTURE FOR VFI FOR N CHANNEL RECEIVE APERTURE. |
|----------------|------------------|------------------|------------------|
| Operations       | DAS              | DMAS I [24]      | DMAS II [32], [35] | Proposed MAS Architecture |
|------------------|------------------|------------------|------------------|------------------|
| Additions        | \( N - 1 \)      | \( \frac{N(N-1)}{2} - 1 \) | \( N - 2 \) | \( 4(N - 2) \) |
| Multiplications  | 0                | \( \frac{N(N-1)}{2} \) | \( N - 1 \) | \( 4(N - 1) \) |
| Square root Modulus | 0                | \( N \) | \( N \) | 0 |

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received data along the axial direction as well in the temporal direction to avoid Doppler aliasing.

C. Simulation Setup

The performance of the proposed approach is initially evaluated and validated with extensive simulations using Field II [36], [37] and MATLAB R2019a (The MathWorks Inc., Natick, MA, USA). Flow phantoms are generated using the Field II program (version 3.24) with the parameters as shown in Table II. A standard linear array probe with 128 active elements is modelled in Field II simulator and is used in the simulations without any transmit apodization. A single cycle sinusoid at 8 MHz is used for transducer excitation and Hanning weighted single cycle sinusoid are used as the transducer impulse response at the transmit and receive. Non-steered plane wave transmit is used for all the dataset in this paper. The simulation parameters closely follow the parameters chosen in [8].

The performance of the NLHR beamforming with TAC and DCC based velocity estimators is compared to the true/theoretical profiles and the corresponding velocity estimators with DAS beamforming. The DAS beamforming for TAC velocity estimator adopts the STDMR scheme in [18]. The DAS beamforming for DCC velocity estimator adopts the directional beamforming scheme reported in [8]. The correlation interval and sampling interval for DCC is chosen as \(-10\lambda:10\lambda\) and 0.1\(\lambda\), respectively, as in [8]. To have a fair comparison, the transmit-receive angles, correlation interval and other parameters are kept the same for DAS and NLHR beamforming. 16 frames of RF data are used for velocity estimation. It should be noted that the comparison of NLHR and DAS beamforming for DCC is only done for magnitude estimates as DCC estimates only provides the velocity magnitude.

Five different test cases were investigated with simulations for the performance evaluation.

Test case 1: To verify and validate the accuracy in velocity estimation, typical parabolic flow profiles, as given by (10) [8], having the peak velocities, \(V_p = 5, 10, 25\) and 50 cm/s is simulated for a transverse vessel of radius, \(R = 5\) mm with its axis at 25 mm depth from the probe surface. \[
V_p(r) = V_p \left[1 - \frac{r^2}{R^2}\right]
\]
(10)

where, \(r\) is the radial position in the vessel. The velocity magnitude and angles are estimated with TAC and DCC for NLHR beamforming and is compared with the corresponding estimates with DAS beamforming and the true profiles.

Test case 2: In the second test, dataset is simulated for a 5 mm radius vessel whose axis is inclined at \(-20^\circ, -10^\circ, 0^\circ, 10^\circ\) and \(20^\circ\) with the horizontal and having a peak velocity, \(V_p = 50\) cm/s. The performance comparison is done as in test case 1.

Test case 3: The third test aims to study the influence of correlation intervals in TAC and DCC estimators with DAS and NLHR beamforming. The correlation interval for DCC for comparison is chosen as \(-10\lambda:10\lambda\) and \(-5\lambda:5\lambda\), denoted by \(L=20\) and 10, respectively whereas the averaging interval for is chosen as \(k=16\) and 32. The correlation interval for TAC for comparison is chosen as \(k=16\) and 32 whereas the averaging interval is chosen as \(L=20\) and 10. The results for the 4 combinations of the \(k\) and \(L\) values are compared for DAS and NLHR beamforming with TAC and DCC estimators.

Test case 4: A multi-vessel phantom having two vessels with a diameter of 10 mm each is simulated. The vessels are oriented at 0\(^\circ\) and \(-10^\circ\) with the horizontal and are centered at 25 mm and 55 mm, respectively. The choice of vessel orientation for this study is driven by the fact that most of the peripheral vessels in the human vasculature are parallel to the skin surface (0\(^\circ\)) as we go deeper, the vessels would be inclined, and evaluated with the proposed approach to verify the spatial resolution improvement. The velocity profile for both the vessels are defined to be a complex Gaussian-Gradient profile as given by (11) with \(V_p=20\) cm/s.

\[
v_{grad}(r) = V_p \hat{g}(r)
\]
(11)

where, \(V_p\) is the peak velocity, \(r\) is the radial position in the vessel, \(R\) is the radius of the vessel, and \(\hat{g}\) represents the normalized gradient of a standard gaussian function. A theoretical plot of a gaussian-gradient flow is shown in Fig. 3(a). A gradient flow that resembles two closely placed narrow vessels having blood flow in opposite directions is chosen for this study as it would give better insights on the spatial resolving capability of the beamforming approach at two different depths. 16 frames of data at a pulse repetition frequency (PRF) of 6.2 kHz are used for velocity estimation. The results with NLHR based TAC estimator is compared to the theoretical profile and DAS based TAC estimator.

Test case 5: Two unique pulsatile flows were simulated with a PRF of 15.6 kHz to test the sensitivity of the proposed approach towards a quick change or the impulsive components in the flow. 1) A transverse vessel of radius 4 mm with a pulsatile flow in which five distinct impulses of different durations are present. 2) A transverse vessel of radius 4 mm with a pulsatile flow in which five distinct impulses having a peak velocity of 15, 20, 50, 80 and 100 cm/s. The results with NLHR based TAC estimator is compared to the theoretical profile and DAS based TAC estimator. The number of frames...
used for autocorrelation are kept same for both approaches.

D. Experimental Setup

The performance of the proposed approach is further evaluated with a detailed experimental investigation. The Verasonics Vantage 128 Research Ultrasound System (Verasonics Inc., Kirkland, WA, USA) employing 128 element L11-5v linear probe with a center frequency of 7.6 MHz is used for data acquisition. A 5-cycle sinusoid is used for transducer excitation for in-vitro and in-vivo experiments. The in-vitro experiments are performed using a commercial Doppler flow simulator from CIRS (CIRS Inc., Norfolk, VA, USA) that includes a peristaltic flow Pump (Model 769), Doppler fluid (Model 769DF) and a tissue mimicking Doppler ultrasound flow phantom (Model 069A) which has a peripheral vessel with a controlled flow velocity. The parameters used for experimental study is shown in Table II. A truncated singular value decomposition (SVD) filter available in the MUST toolbox [33] is used as the clutter filter.

The proposed technique is investigated with three test cases with pulsatile flow profiles in the flow phantom. For the first experiment, a typical pulsatile flow with a peak velocity of 50 cm/s is configured and the data is acquired using the Verasonics system at a frame rate of 8000 frames per second. In the second test case, we configured a similar pulsatile flow with a sudden reversal of direction imposed during the data acquisition. This aims to test the sensitivity of the proposed approach towards the impulsive and highly dynamic nature of flows. In the third test case, the Doppler fluid is seeded with some air bubbles by gently shaking the fluid and the same pulsatile flow is configured using the flow pump. The sensitivity and spatial localization capability are evaluated by tracking the air bubble over time with the proposed NLHR approach. The results are compared to that of DAS based method.

Further, the proposed beamforming technique is tested on an in-vivo carotid dataset captured from a healthy volunteer by following the principles outlined in the Helsinki Declaration of 1975, as revised in 2000. The instantaneous quiver plots and the velocity profiles along with the velocity estimates over time is compared to that of DAS based methods. For all the experimental studies, TAC estimator is used with NLHR beamforming and DAS beamforming.

E. Performance Metrics

The bias and standard deviation in velocity magnitude \((V_{\text{Bias}})\) and \((V_{\text{SD}})\) for the simulation and experimental studies, if not reported differently, are determined as:

\[
V_{\text{Bias}} = \frac{V_{M} - V_{T}}{V_{P}} \cdot 100 \quad V_{\text{SD}} = \frac{\text{Std}(V_{M})}{V_{P}} \cdot 100
\]

where \(V_{M}, V_{T}\) and \(V_{P}\) are the measured, the true (or theoretical), and the peak velocity values, respectively; \(\text{Std}(\cdot)\) denotes the standard deviation function.

The accuracy of angle estimation is determined using the bias \((A_{\text{Bias}})\) and standard deviation \((A_{\text{SD}})\) measures as defined in (13):

\[
A_{\text{Bias}} = A_{M} - A_{T} \quad A_{\text{SD}} = \text{Std}(A_{M})
\]

where \(A_{M}\) and \(A_{T}\) are the measured, the true (or theoretical), and the peak velocity values, respectively.

IV. RESULTS AND DISCUSSION

This section discusses the results obtained from the detailed simulation study and the experimental investigations. Detailed video results are attached as supplementary files and are available online [Online Link].

Fig. 2. Mean estimated velocity profiles with DAS and NLHR beamforming for DCC estimator (a), TAC estimator (b, c). Quantitative comparison of Bias and Standard Deviation (Std. Dev.) in velocity magnitude and flow angle for test case 1 with DCC (d), and TAC (e, f) estimators. Comparison plots for test case 2 with DCC (g), and TAC (h, i) estimators. Comparison plots for test case 3 with DCC (j), and TAC (k, l) estimators. The error bar is centered at median bias and the length of the error bar corresponds to one standard deviation.


A. Simulation Results

The estimated velocity for a transverse parabolic profile of peak velocity 50 cm/s with the proposed NLHR beamforming for DCC estimator compared with that of DAS beamforming shown in Fig. 2(a) and the comparison plots for velocity magnitude and angle with TAC estimator are shown in Fig. 2(b) and (c) respectively. The quantitative performance comparison of NLHR beamforming based velocity estimations with that of DAS based methods in terms of bias and standard deviation in the magnitude and angle estimates for test cases 1, 2 and 3 are shown in the plots in Fig. 2 column 2, 3, and 4 respectively. A slight underestimation in velocity magnitude when compared with DAS estimates are present for NLHR based estimates in most of the cases whereas the angle estimates are better with NLHR beamforming. However, in most of the cases, the standard deviation is similar, if not better. This shows the improvement in preciseness of the velocity estimates. The quantitative performance comparison for test case 3 with different correlation window sizes are shown in Fig. 2(j)-(l). It is observed that the NLHR based estimates are consistently better than that of DAS based estimates for all the correlation and averaging window sizes.

In general, an underestimation is observed with the velocity estimation with both beamforming and estimators. Hence for a fair comparison, a 10% bias compensation (i.e., 1.1*\(V_M\)) is empirically applied to all the estimates with both the estimators for DAS and NLHR beamforming. For angle estimation, the region where the velocity is less than 10% of \(V_p\) is excluded.

Fig. 3(a) shows the theoretical velocity profile for a simulated Gaussian-Gradient flow and Fig. 3(b), (c) shows a qualitative comparison of the vector flow images obtained with DAS and the proposed NLHR beamforming. An improvement in the velocity magnitude and angle estimation at the velocity transition region at the center of both the vessels is observed with the NLHR beamforming approach. The \(V_{Bias}\) for DAS beamforming and NLHR beamforming is observed to be -8.4% and -10.8% with a corresponding \(V_{SD}\) of 2.2% and 1.6% respectively for the top vessel. The \(V_{Bias}\) for the bottom vessel is observed as -10.6% and -10.6% with a \(V_{SD}\) of 4.4% and 4.2% for DAS and NLHR beamforming respectively. It is worth noting that \(V_{Bias}\) for NLHR has remained unaffected with the change in depth while \(V_{Bias}\) with DAS beamforming has increased by 2.2%. The \(A_{Bias}\) and \(A_{SD}\) for the top vessel were very close to zero for both DAS and NLHR based methods, but for the bottom vessel, the median value of \(A_{Bias}\) and \(A_{SD}\) along the central axial line is observed as 5° and 0.02° respectively for DAS and 0.02° and 0.02° for NLHR based method.

Fig. 4(a) shows a qualitative comparison of the vector flow images obtained with the proposed NLHR beamforming and DAS beamforming. Fig. 4(c) and (d) shows velocity variation over time for dataset with impulse of different duration and impulse of different peak velocities respectively. The proposed NLHR beamforming approach has shown better sensitivity to the subtle changes in the velocity and is evident from the initial two impulses having a peak velocity of 15 and 20 cm/s. Similarly, the sensitivity with respect to the duration of impulse is evident from the short duration impulses in Fig. 4(c).

B. Phantom Results

A correlation interval of 32 ms (corresponds to 256 frames of acquisition) is used for velocity estimation with DAS and NLHR based methods. Neither any bias compensation nor any spatial or temporal averaging is applied on the velocity estimates. Fig. 5 shows a comparison between DAS
beamforming and NLHR beamforming for a typical pulsatile flow dataset. A better smoothness and continuity in the velocity vectors are observed for NLHR beamforming and is evident from Fig. 5(a) and (b). The velocity magnitude and angle profile over time for NLHR beamforming doesn’t look smoother as compared to DAS owing to higher sensitivity and Doppler resolution associated with the proposed approach.

Fig. 6(a), (b) summarizes the result for the testcase in which a sudden reversal of the flow direction is imposed. The improvement in time and Doppler resolution could be observed from the enlarged view of the time of occurrence of direction reversal. The slope of the red curve as in Fig. 6(b) is observed to be double as that of the blue curve indicating the improvement in sensitivity by a factor of 2. The frames captured at three instances during the passage of an air bubble is presented in (i)-(iii) of Fig.6(c) and (d) for DAS and NLHR beamforming respectively. A spatially resolved tracking of the bubble is observed with the nonlinear beamforming which the DAS beamforming has failed to do.

**C. Preliminary in-vivo results**

The results from the preliminary in-vivo studies on the carotid dataset of a healthy individual is shown in Fig. 7. To demonstrate the advantage of the proposed approach, three different correlation window size, 16, 24 and 32 ms corresponding to 128, 192 and 256 frames of acquisition, respectively, are used for velocity estimation and comparison. DAS (1) and NLHR (1) corresponds to 16 ms window, DAS (2) and NLHR (2) corresponds to 24 ms window DAS (3) and NLHR (3) corresponds to 32 ms window. The transverse (Vx) and axial (Vz) components of velocity over time corresponding to each correlation window is shown in Fig. 7(a)-(c) for DAS and Fig.7(d)-(f) for proposed NLHR technique. A significant improvement in the velocity estimates is observed for NLHR (1), particularly during the atrial diastole when comparing with that of DAS (1). This is due to the doubling of Doppler frequency achieved with NLHR beamforming. In this regard, the signal corresponding to lower order Doppler shifts due to the low velocity movements in a correlation window of 16 ms, appears with a greater number of cycles (ideally, 2 times of that of DAS) that reduces the error with the autocorrelation estimator. The above reasoning is adequately supported with the stable and noise free estimates during the atrial diastole with DAS and NLHR corresponding to 24 ms and 32 ms correlation window. For a better comprehension, the quiver plot, axial and lateral velocity components along with the velocity-time trace obtained with DAS based TAC for a window size 32 ms and NLHR based TAC for a window size 24 ms are shown in Fig.7(g)-(l).

This validates the two major theoretical implications: 1) The improvement in the Doppler resolution due to the doubling of Doppler frequency with the MAS operation 2) a correlation window of small size would be sufficient for velocity estimation with NLHR beamforming that increase the sensitivity towards the impulse components in the velocity that are otherwise not captured with DAS based methods. The impulsive nature of the pulsatile flow is captured better by NLHR estimates as characterized by the quick rise of the
velocity towards the peak systole as in Fig. 7(d)-(f) when compared to Fig. 7(a)-(c). However, a slight underestimation is observed with NLHR estimates when compared to that of DAS estimates for all the correlation window size. Neither any bias compensation nor any spatial or temporal averaging is applied on the velocity estimates. The video result with more details of the in-vivo study is available as a downloadable movie.

V. CONCLUSION AND FUTURE SCOPE

This article presents a novel nonlinear beamforming technique to improve the sensitivity and Doppler resolution for high frame rate flow imaging. The proposed beamformer is demonstrated DCC and TAC based velocity estimators. The accuracy of velocity magnitude and angle estimates are observed to be better with reduced size of correlation window when compared to that of DAS beamforming counterparts. However, a small underestimation is observed in the velocity magnitude estimates as compared to that of DAS beamforming when the correlation window is made longer. The pulsatile flow simulations with different peak velocities and durations demonstrate the enhanced sensitivity of the proposed approach towards the impulse components in the flow velocity. It is further validated with a pulsatile flow phantom experiment having a sudden reversal of flow direction. The air bubble tracking experiment shows that the proposed approach provides a significantly better spatial localization in the velocity vectors than that of DAS beamforming. Owing to the doubling of doppler shift because of MAS operation in the beamformer, a remarkable improvement in the low velocity estimates is observed for the preliminary in-vivo studies when compared to DAS estimates with a lower correlation window size. This implies that a smaller correlation window would be sufficient for velocity estimation with NLHR beamforming and hence increasing the sensitivity towards the impulse components in the velocity that are otherwise not captured with DAS based methods. VFI algorithms with enhanced sensitivity towards the highly dynamic and impulse velocity components could probably help in identifying new biomarkers or pathological features in medical diagnostics. In this regard, the future work would include the investigations of the proposed approach in various pathological scenarios.

It should be noted that the advantages of the proposed approach are at the cost of increased computational complexity when compared to DAS beamforming. As per the theoretical comparison of the required computations shown in Table I, the proposed approach requires $4N - 1$ multiplications and an additional $3(N - 2) - 1$ additions when compared to that of DAS beamforming. In view of this, a detailed hardware study to optimize the computational requirements imposed by the MAS operation is foreseen. Future developments of this work also include the analysis of this approach for power Doppler, synthetic aperture imaging and diverging wave vector flow imaging at high frame rates. As the importance of portable ultrasound systems are magnified in the emergency and ambulatory scenarios, our efforts would be to build upon the proposed technique to realize a portable VFI system.

SUPPLEMENTARY MATERIALS

The detailed video results associated with the various test cases are attached as supplementary files and are available online at shorturl.at/fzCF9.

Fig. 7. Results obtained with in-vivo carotid experiment. Velocity-time trace of $V_x$ and $V_z$ obtained with DAS (a)-(c) and NLHR beamforming (d)-(f) for different correlation window size. Quiver plot obtained with DAS beamforming (g) and proposed NLHR beamforming (h) overlaid on the DAS beamformed B-mode image. Mean estimated velocity profiles with DAS based TAC with a correlation window of $32\,ms$ (i) and NLHR based TAC with a correlation window of $24\,ms$ (j) during a peak systole for the axial line passing through the point shown in (g) and (h), respectively. Comparison of estimated velocity magnitude (k) and flow angle (l) over time obtained with DAS and NLHR beamforming. The plots in (a)-(f) and (k)-(l) are estimated for the corresponding point shown in (g) and (h). No bias compensation is applied on any of the plots.
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REFERENCES

[1] Nelson, T. R., and D. H. Pretorius. “The Doppler sign: where does it come from and what does it mean?” American Journal of Roentgenology vol. 151, no. 3, pp. 439-447, 1988.
[2] Tortoli, P., Gabriele Guidi, and P. Pignoli. “Transverse Doppler spectral analysis for a correct interpretation of flow sonograms.” Ultrasound in medicine & biology vol. 19, no. 2, pp. 115-121, 1993.
[3] Newhouse, V. L., et al. “Three-dimensional vector flow estimation using two transducers and spectral width.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 41, no.1, pp. 90-95, 1994.
[4] Jensen, Jørgen Arendt, and Peter Munk. “A new method for estimation of velocity vectors.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 45, no. 3, pp. 837-851, 1998.
[5] Fox, Martin D. “Multiple crossed-beam ultrasound Doppler velocimetry.” IEEE Transactions on Sonics and Ultrasonics vol. 25, no.5, pp. 281-288, 1978.
[6] Trahey, Gregg E., John W. Allison, and Olaf T. Von Ramm. “Angle independent ultrasonic detection of blood flow.” IEEE Transactions on Biomedical Engineering vol. 12 pp. 965-967, 1987.
[7] Overbeck, John R., Kirk W. Beach, and D. Eugene Strandness jr. Vector Doppler: Accurate measurement of blood velocity in two dimensions. Ultrasound in medicine & biology, vol. 18, no.1, pp. 19-31, Jan. 1992.
[8] Jensen, Jørgen Arendt. “Directional velocity estimation using focusing along the flow direction. I: Theory and simulation.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 50, no. 7, pp. 857-872, Jul. 2003.
[9] Jensen, Jørgen Arendt, and Rasmus Bjerringaard. “Directional velocity estimation using focusing along the flow direction. II: Experimental investigation.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 50, no. 7, pp. 873-880, Jul, 2003.
[10] Jensen, Jørgen Arendt. “Estimation of high velocities in synthetic aperture imaging: I. Theory.” IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, vol. 66, no. 6, pp. 1024-1031 Mar. 2019.
[11] Jensen, Jørgen Arendt. “Estimation of high velocities in synthetic aperture imaging: II: Experimental investigation.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 66, no. 6, pp. 1032-1038, Mar. 2019.
[12] Fadnes, Solveig, et al. “Robust angle-independent blood velocity estimation based on dual-angle plane wave imaging.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 62, no.10, pp. 1757-1767, 2015.
[13] Ricci, Stefano, Luca Bassi, and Piero Tortoli. “Real-time vector velocity assessment through multigate Doppler and plane waves.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 61, no. 2, pp. 314-324, Feb. 2014.
[14] Yiu, Billy YS, Simon SM Lai, and C. H. Alfred. “Vector projective imaging: Time-resolved dynamic visualization of complex flow patterns.” Ultrasound in medicine & biology, vol. 40, no. 9 pp. 2295-2309, Sep. 2014.
[15] Yiu, Billy YS, and C. H. Alfred. “Least-squares multi-angle Doppler estimators for plane-wave vector flow imaging.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 63, no. 11, pp. 1733-1744, 2016.
[16] Lenge, Matteo, et al. “Plane-wave transverse oscillation for high-frame-rate 2-D vector flow imaging.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 62, no.12 pp. 2126-2137, 2015.
[17] Ricci, Stefano, et al. “Real-time blood velocity vector measurement over a 2-D region.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 65, no.2, pp. 201-209, 2017.
[18] Madhavanunnii A. N. and Mahesh Raveendranathna Panicker “Triangulation based vector flow imaging with non-steered plane waves for transverse flows”, in Proc. SPIE Imaging: Ultrasonic Imaging and Tomography, vol. 11319, 2020, pp. 1131905.
[19] Korbek, Jacob, and Jørgen Arendt Jensen. “Estimation of velocity vector angles using the directional cross-correlation method.” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 53, no. 11, pp. 2036-2049, Dec. 2006. [Online]. Available: https://core.ac.uk/download/pdf/192980163.pdf
[20] Hoyos, Carlos Armando Villagómez, et al. “Accurate angle estimator for high-frame-rate 2-D vector flow imaging.” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 63, no. 6, pp. 842-853, Apr. 2016.
[21] Perrot, Vincent, et al. “So you think you can DAS? A viewpoint on delay-and-sum beamforming.” Ultrasonics vol. 111, pp. 106309, 2021.
[22] Parker, Kevin J. “Correspondence: apodization and windowing functions” IEEE transactions on ultrasonics, ferroelectrics, and frequency control vol. 60, no. 6, pp. 1263-1271, Jun. 2013. [Online]. Available: http://www.ece.rochester.edu/People/faculty/parker_kevin/assets/pdf/apodizationTUFFFF2013.pdf
[23] Lim, Hooi Been, et al. “Confocal microwave imaging for breast cancer detection: Delay-multiply-and-sum image reconstruction algorithm.” IEEE Transactions on Biomedical Engineering, vol. 55, no. 6, pp. 1697-1704, May 2008.
[24] Matrone, Giulia, et al. “The delay multiply and sum beamforming algorithm in ultrasound B-mode medical imaging.” IEEE transactions on medical imaging, vol. 34, no. 4, pp. 940-949, Nov. 2014.
[25] Matrone, Giulia, et al. “High frame-rate, high resolution ultrasound imaging with multi-line transmission and filtered-delay multiply and sum beamforming.” IEEE transactions on medical imaging, vol. 36, no. 2, pp. 478-486, Oct. 2016.
[26] Mozaffarzadeh, Moein, et al. “Double-stage delay multiply and sum beamforming algorithm applied to ultrasound medical imaging.” Ultrasound in medicine & biology, vol. 44, no. 3, pp. 677-686, 2018.
[27] Mozaffarzadeh, Moein, et al. “Double-stage delay multiply and sum beamforming algorithm: Application to linear-array photoacoustic imaging.” IEEE Transactions on Biomedical Engineering, vol. 65, no. 1, pp. 31-42, Apr. 2017.
[28] Park, Jongin, et al. “Delay-multiply-and-sum-based synthetic aperture focusing in photoacoustic microscopy.” Journal of biomedical optics, vol. 21, no. 3, pp. 036010, Mar. 2016. [Online]. Available: https://www.spiedigitallibrary.org/journalArticle/Download?fullDOI=1.01117/JBO.21.3.036010.
[29] Dahl, Jeremy J., et al. “Coherent flow imaging: A power Doppler imaging technique based on backscatter spatial coherence.” 2013 IEEE International Ultrasonics Symposium (IUS). IEEE, 2013.
[30] Li, You Leo, and Jeremy J. Dahl. “Coherent flow power Doppler (CFPD): Flow detection using spatial coherence beamforming.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 62, no.6, pp.1022-1035, 2015.
[31] Madhavanunnii A. N. and Mahesh Raveendranathna Panicker “Directional beam focusing based dual apodization approach for improved vector flow imaging”, in Proc. IEEE ISBI, Iowa city, 2020, pp. 300-303.
[32] Ramalli, Alessandro, et al. “High dynamic range ultrasound imaging with real-time filtered-delay multiply and sum beamforming.” in Proc. IEEE IUS, Sep. 2017, pp. 1-4.
[33] Posada D, Poréé J, Pellissier A, Chayer B, Tournoux F, Cloutier G, Garcia D. Staggered multiple-PRF ultrafast color Doppler. IEEE Trans Med Imaging, vol. 35, pp. 1510-1521, 2016.
[34] Kasai, Chihiro, et al. “Real-time two-dimensional blood flow imaging using an autocorrelation technique.” IEEE Transactions on sonics and ultrasonics, vol. 32, no.3, pp. 458-464, May 1985. [Online]. Available: http://bme.elektro.dtu.dk/31545/documents/kasai_et_al_1985.pdf
[35] G. Malamal and M. R. Panicker, “Towards A Pixel-Level Reconfigurable Digital Beamforming Core for Ultrasound Imaging,” in IEEE Transactions on Biomedical Circuits and Systems, vol. 14, no. 3, pp. 570-582, Jun. 2020.
[36] Jensen, Jørgen Arendt, and Niels Bruun Svendsen. “Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers.” IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 39, no. 2, pp. 262-267, Mar. 1992.
[37] Jensen, Jørgen Arendt. "Field: A program for simulating ultrasound tomography", in 10th Nordicbaltic Conference on Biomedical Imaging, vol. 4, Supplement 1, Part 1: pp. 351-353, 1996.