Emergent nonlinear phenomena in a driven dissipative photonic dimer

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Collective effects leading to spatial, temporal or spatiotemporal pattern formation in complex nonlinear systems driven out of equilibrium cannot be described at the single-particle level and are therefore often called emergent phenomena. They are characterized by length scales exceeding the characteristic interaction length and by spontaneous symmetry breaking. Recent advances in integrated photonics have indicated that the study of emergent phenomena is possible in complex coupled nonlinear optical systems. Here we demonstrate that the out-of-equilibrium driving of a strongly coupled pair of photonic integrated Kerr microresonators (‘dimer’)—which, at the ‘single particle’ (that is, individual resonator) level, generate well-understood dissipative Kerr solitons—exhibits emergent nonlinear phenomena. By exploring the dimer phase diagram, we find regimes of soliton hopping, spontaneous symmetry breaking and periodically emerging (in)commensurate dispersive waves. These phenomena are not included in the single-particle description and are related to the parametric frequency conversion between the hybridized supermodes. Moreover, by electrically controlling the supermode hybridization, we achieve wide tunability of spectral interference patterns between the dimer solitons and dispersive waves. Our findings represent a step towards the study of emergent nonlinear phenomena in soliton networks and multimodal lattices.

Increasing the number of components in a dynamical nonlinear system often leads to the appearance of so-called emergent phenomena that are accompanied by the violation of underlying symmetries and even microscopic laws. Emergent phenomena are omnipresent and most often observed as the spontaneous self-organization of spatiotemporal patterns. The formation of galaxies, complex neural interaction in the human brain or collective dynamics in Bose–Einstein condensates can be understood as arising from emergent phenomena. Recently, advances in the manipulation of driven dissipative quantum systems have allowed the study of non-equilibrium phases in strongly interacting quantum matter and led to the discovery of time crystals. Particularly important characteristic phenomena associated with emergent dynamics are phase transitions and symmetry breaking in complex systems. Their properties are actively studied in different branches of physics including photonics. Emergent phenomena also occur in driven dissipative nonlinear optical systems out of equilibrium. For example, the complex self-organization of light in the form of dissipative solitons in active nonlinear optical cavities related to the phenomenon of mode locking has paved the way for efficient ultrashort pulse generation, which is the foundation of modern frequency metrology. More recently, spatiotemporal mode locking, that is, mode locking in the domain of multiple transverse mode families in addition to the longitudinal one, has been reported, which is an important example of the presence of the self-organization of light in more complex nonlinear optical systems.

The discovery of coherent localized light states, called dissipative Kerr solitons (DKSs), in continuous-wave-driven passive nonlinear microresonators has heralded a new generation of optical frequency combs that can now be integrated on-chip, providing a universal platform for various applications. Driven Kerr cavities are particularly attractive in the study of the nonlinear phases of driven dissipative nonlinear systems because of their ability to experimentally map out the two-dimensional stability chart spanned by the laser pump power and laser detuning and therefore to experimentally explore the rich and complex phase diagram of the DKS. Indeed, DKS dynamics has been extensively studied over the past years, providing an accurate understanding of physical phenomena ranging from breathers to soliton crystals. Further, perturbations to the exact model, such as Raman scattering or the influence of avoided mode crossings (AMX), have been largely explored. However, all these novel and recently reported underlying dynamical mechanisms can be well explained within the one-dimensional ‘single-particle’ (that is, an individual-resonator) Lugiato–Lefever equation (LLE) and its modifications. Thus, the study of emergent phenomena in driven nonlinear optical systems out of equilibrium has been mostly confined to cases with comparatively few degrees of freedom.

Here we report the study of emergent nonlinear phenomena of the DKS dynamics beyond the Lugiato–Lefever model by adding another spatial dimension. Our study fundamentally differs from all prior results, which were obtained by including perturbations into the ‘single-particle’ LLE. Instead, here we consider the dynamics resulting from two perfect (namely, ideal and unperturbed) coupled systems. By investigating DKS formation in the photonic dimer—a fundamental element of soliton lattices, we show surprisingly rich nonlinear dynamics related to the efficient photon transfer between the dimer supermodes. We study the case when DKSs are generated in both resonators and due to the underlying field symmetry; we refer to these structures as gear solitons (GSs). We show that the dimer phase diagram features states that are fundamentally inaccessible in single resonators. We numerically demonstrate the emergence of commensurate dispersive waves (DWs) whose periodic enhancement leads to the discretization of the soliton exis-
tence range, soliton hopping and an effect of symmetry breaking related to the discreteness of the system. We experimentally observe GS formation in a strongly coupled photonic dimer and probe its non-trivial dynamics. Strikingly and counter-intuitively, we observe that imperfect mode hybridization (that is, finite inter-resonator detuning) is required to initiate coherent dissipative structures. Moreover, on a practical level, the combination of a dimer with microresonator tuning enables the electronic control of hybrid DWs. Our results highlight the richness and complexity to be explored in the emergent nonlinear dynamics of multimodal resonator lattices.

Physical model of coupled Kerr resonators

Nonlinear dynamics in the photonic dimer can be described using a Hamiltonian formalism. Here we consider a tight binding model with Kerr nonlinearity such that the corresponding Hamiltonian can be written as

$$\hat{H}_d = -\hbar J \sum_{\mu} \hat{a}_{1,\mu}^\dagger \hat{a}_{2,\mu} + \hat{a}_{2,\mu}^\dagger \hat{a}_{1,\mu} + \hat{H}_1 + \hat{H}_2,$$

where $J$ denotes the evanescent coupling between the resonators. We note that in contrast to the Bose–Hubbard dimer case, each resonator considered here has an infinite set of optical bosonic modes (for example, $\hat{a}_{1,\mu}$), designated by the mode index $\mu$ and distributed according to the microresonator dispersion relation $D_{\omega}(\mu) = \omega_s - (\omega_s + \mu D)$ (Fig. 1d,e), where $D$ is the mode spacing, $\omega_s$ and $\omega_p$ stand for the frequency of the $p$th and $0$th modes, respectively. As the resonators have almost identical free-spectral ranges (FSRs), hybridization occurs among all the considered modes of both resonators. In the DKS state, cavity dispersion and nonlinearity are exactly balanced for each mode $\mu$ such that the parabolic dispersion profile collapses into a straight line (Fig. 1d,e). Further, $\hat{H}_{1,2}$ represent the conventional single-resonator Hamiltonian that can be expressed as

$$\hat{H}_i = \hbar \sum_{\mu} \omega_{\mu} \hat{a}_{\mu}^\dagger \hat{a}_{\mu} - \frac{\hbar g_K}{2} \left( \sum_{\mu} (\hat{a}_{\mu} + \hat{a}_{\mu}^\dagger) \right)^4,$$

where the single-photon Kerr shift is given by $g_K = \omega_0/c_n V_0 \sqrt{\alpha}$, where $c$ denotes the speed of light in a vacuum, $V_0$ is the effective mode volume and $\alpha$ is Planck's constant; further, $n_0$ and $n_1$ are the linear and nonlinear refractive indexes, respectively. Dissipation is introduced using the standard approach for open quantum systems, where we assume the regime that $J \gg \kappa$, where $\kappa$ denotes the cavity decay rate of an individual mode designated by its creation operators $\hat{a}_{\mu}^\dagger$ and $\hat{a}_{\mu}$, respectively. An alternative classical description is provided in Methods.

We next introduce the notion of orthogonal cavity supermodes (Fig. 1b). We refer to the supermodes with a higher frequency as asymmetric (AS) and with a lower frequency as symmetric (S). The hybridized supermodes can be expressed as

$$\hat{a}_{\mu} = \alpha \hat{a}_{1,\mu} + \beta \hat{a}_{2,\mu},$$

$$\hat{a}_{\mu} = \beta \hat{a}_{1,\mu} - \alpha \hat{a}_{2,\mu},$$

with coefficients $\alpha, \beta = \sqrt{1 \pm d/\sqrt{2}}$ and the normalized inter-resonator detuning $d = \delta/\Delta \omega$ (which accounts for detuning

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Fig. 1 | DKS formation in single- and coupled-ring resonators. a, Single-resonator (particle) case. DKS generation with a monochromatic pump at frequency $\omega_p$ (green). Resonance in the linear transmission trace is represented by a single Lorentzian dip. The spectrum of a single DKS generated in the resonator is shown by discrete orange lines having a hyperbolic secant profile. b, Mode splitting on transition to the dimer case. Resonator modes initially separated by the inter-resonator detuning $\delta$ are hybridized in the dimer configuration and form a two-step ladder with the separation $\Delta \omega = \sqrt{4J^2 + \delta^2}$. c, Two strongly coupled resonators (dimer case). Simultaneous spatiotemporal self-organization in both cavities forms the GS. The hyperbolic secant spectrum is modified by two symmetrically spaced Fano shapes. The output spectrum of the pumped resonator is shown with blue lines, while the one of the auxiliary is indicated with red ones. Orientation of the Fano shapes depends on the sign of the inter-resonator detuning. d, Cascaded FWM in the particle case leading to the formation of a DKS. e, FWM pathways between the supermodes that take place in the photonic dimer. Odd interband (2, dashed line) and even interband (3, solid line) FWM pathway leading to the emergence of DWs.
between the resonator modes, as shown in Fig. 1b. The frequency splitting of the normalized modes is given as $\Delta \omega_0 = \sqrt{4\eta^2 + \delta^2}$.

Introducing a change of variables from equation (3) into equation (1), we find a set of nonlinear coupling terms representing the interaction between the supermodes. With the notation $i \bar{\gamma}_\sigma \bar{a}_\mu \bar{a}_{\mu'} \bar{a}_{\sigma'} \bar{a}_{\mu'\mu''}$, where $\sigma = \{a,s\}$ denotes the antisymmetric and symmetric modes, respectively, and the last index implies the conservation of momentum in the nonlinear interaction between the modes $\mu, \mu'$ and $\mu''$, the resulting Hamiltonian (see Supplementary Section 1 for details) can be expressed as follows:

$$H = \hbar \sum_\mu \left[ i \omega_\mu \left( \bar{a}_{\mu} \bar{a}_{\mu} + \bar{a}_{\mu} \bar{a}_{\mu} \right) + \frac{\hbar}{2} \left( \bar{a}_{\mu} \bar{a}_{\mu} - \bar{a}_{\mu} \bar{a}_{\mu} \right) \right]$$
$$- \hbar \sum_\mu \bar{a}_{\mu} \bar{a}_{\mu} \left[ \frac{1}{2} \left( 1 + d^2 \right) \left( k_{\mu \mu', \mu''}^{a, a, a} + k_{\mu \mu', \mu''}^{s, s, s} \right) \right]$$
$$- d \sqrt{1 - d^2} \left( k_{\mu \mu', \mu''}^{a, a, a} + k_{\mu \mu', \mu''}^{s, s, s} \right)$$
$$+ \frac{1}{2} \left( 1 - d^2 \right) \left( k_{\mu \mu', \mu''}^{a, a, a} + 4 k_{\mu \mu', \mu''}^{s, s, s} \right)$$

GSs can be generated in both supermodes; however, the modification of the spectral profile has been observed only in the AS case, that is, by tuning over the upper parabola on the dispersion relation. Indeed, the phase-matching conditions can be fulfilled when the solitonic line crosses the lower (S-supermode) parabola, which creates efficient four-wave mixing (FWM) pathways between the AS and S supermodes. The presence of such phase-matched interactions makes the nonlinear dynamics of the photonic dimer remarkably more rich and complex than in the single-resonator case. These nonlinear processes can be divided into two categories according to the number of photons from each of the supermodes involved: even and odd. Remarkably, the even and odd processes have different efficiencies depending on the normalized inter-resonator detuning, as per equation (4).

The GS is maintained in the AS supermodes by the cascaded FWM processes similar to the single-resonator case (Fig. 1d) represented by the term $k_{\mu \mu', \mu''}^{a, a, a}$ shown in equation (4). Since the photons stay within the same supermode family, we refer to this process as even and intraband. Interband FWM processes are shown in Fig. 1c. Process number 2 is odd since two photons annihilated in the AS supermode create two photons: one in the AS and one in the S supermode, while process number 3 is even. They are represented by $k_{\mu \mu', \mu''}^{a, a, a}$ and $k_{\mu \mu', \mu''}^{s, s, s}$ in equation (4), respectively. Interband FWM generates DW when the solitonic line is in the vicinity of an S supermode. DWS are represented by two symmetrically spaced Lorentzian profiles in the S-supermode spectrum. The maxima of the Lorentzians occur exactly at the mode where the crossing occurs. Interference between the hyperbolic secant (in AS supermodes) and Lorentzians (in S supermodes) results in a Fano-shaped spectrum, as shown in Fig. 1c. Remarkably, a similar spectral feature has been numerically discovered in a $PT$-symmetric dimer, where the appearance of the Fano structures was interpreted as the result of modulation instability.

Exploring the phase diagram of the photonic dimer

Even though weakly coupled and size-mismatched microring resonators have been recently studied in the context of DKS synchronization and enhancement of the nonlinear conversion efficiency, the deployment of large nonlinear photonic lattices requires an understanding of the dynamics of strongly coupled and uniform systems of nonlinear cavities.

To investigate the phase diagram of the strongly coupled nonlinear photonic dimer, we perform numerical simulations of the coupled-mode equations (see Methods as well as Extended Data Fig. 1 for details). Figure 2a shows a schematic phase diagram. The initial dynamics is found to be similar to the single-resonator case. We observe the formation of primary combs followed by coidal waves (Turing rolls) in both resonators (dark-blue area in Fig. 2a,b). Figure 2c,d shows the underlying evolution of the intracavity power (spatiotemporal diagram) and power spectral density (PSD) of the pumped resonator. The appearance of the spectral components enhanced due to the supermode interaction can be seen as converging distinct lines, as shown in Fig. 2d.

Substantial divergence from the single-resonator case is observed in the soliton existence range. Stable solitons are strongly perturbed by the periodic emergence of DWS resulting from the interband FWM interactions. The soliton existence range is represented by the green area shown in Fig. 2a. The DW maxima are shown by vertical magenta and orange lines. The interaction between supermodes leads to a periodic increase in the average intracavity power in both resonators (Fig. 2b). High-amplitude DWS perturb the GS state, which can lead to its decay or decrease in the number of solitons. The strength of this effect directly depends on the number of solitons, and therefore, the system is naturally forced towards the single GS state where the perturbation is the weakest. Since the values of the laser detuning at which the resonant enhancement of the DW occurs are related to the discretization of the lower parabola, the soliton steps are discretized, too, according to the position of the resonances. The phase diagram is averaged over ten realizations, and the values of the laser detuning at which the single soliton state decays most probably are plotted.

The difference between the two DW maxima (magenta and orange) is shown in Fig. 2c. The maxima depicted in magenta correspond to the cases when the solitonic line on the dispersion relation exactly crosses the points on the lower parabola (Fig. 2g, solid line). The resonant interaction becomes efficient, which leads to an increase in the average intracavity power. A further change in the pump laser detuning brings GS between two discrete points on the parabola of the S supermodes, where it demonstrates an unstable behaviour. This instability leads to symmetry breaking during which the GS acquires a group velocity. The sign of the group velocity varies arbitrarily from one realization to another. The additional group velocity corresponds to a tilt of the solitonic line on the nonlinear dispersion relation (Fig. 2g, green and grey dashed lines). We found that the group velocity self adjusts to a value needed for the dispersionless soliton line to reconnect with the discrete points on the S parabola. The fulfilment of the resonance condition, as previously, leads to an increase in the average intracavity power, as depicted by the orange lines in Fig. 2a,b. The optical spectrum and intracavity power profiles of the two different maxima of the DW amplitude are shown in Fig. 2e (I and II). The change in the group velocity of GS leads to asymmetry in the wings of the spectrum and a displacement of the Fano-shaped peaks by one mode number (dashed line in Fig. 2e, II). The decomposition of the cavity field into the supermodes, according to equation (3), demonstrates a clear separation of the GS and DWS belonging to different supermodes (Fig. 2e, I (bottom)).

After reaching a critical value of laser detuning, at higher values of pump power, the system exhibits periodic energy exchange between the coupled resonators while being in the GS state. This dynamic regime is referred to as soliton hopping and is indicated by the blue area shown in Fig. 2a,b. This process is similar to the so-called self-pulsation effect discovered in single-mode dimers. However, in the multimodal case, the presence of solitons allows for
the synchronous pulsation of a large number of longitudinal modes, which enables the hopping of the spatiotemporal structure as a whole, emphasizing the quasi-particle nature of the DKSs. The soliton hopping regime is studied by extracting the complex amplitude from both resonators at point III, as shown in Fig. 2c,d, and numerically propagating it with fixed laser detuning (Fig. 2f). The average power in the coupled resonators oscillates in the counter phase. In this regime, the GS is strongly projected onto the S supermode, which leads to periodic constructive and destructive interference. The soliton hopping frequency coincides well with the gap between the hybridized parabolas in the dispersion relation. PSDs and intracavity powers at the maximum power in the pumped resonator and auxiliary resonator are shown in Fig. 2e,f. The nonlinear dispersion relation is numerically reconstructed in Fig. 2g,h, showing the symmetry-breaking mechanism and its effect on the soliton hopping state.
the maximum power in the auxiliary resonator are shown in Fig. 2f (plots 1 and 2, respectively). Additional sidebands often associated with breathing are observed in the optical spectra. The distance between the sidebands decreases with an increase in the mode number. This is similar to the case of Kelly sidebands that appear during the periodic perturbation of a solitonic state. The origin of the sidebands can be easily understood by analysing the corresponding nonlinear dispersion relation (Fig. 2h). We numerically reconstruct the nonlinear dispersion relation by taking a double Fourier transform of the spatiotemporal diagram. Here the periodic soliton hopping is represented by a set of equally spaced lines (ladder) separated by \( \Delta \omega \), which corresponds to the hopping frequency. The sidebands exactly appear at the points where the ladder crosses the S and AS parabolas.

The emergence of sidebands in coupled nonlinear systems has been extensively investigated in the past, but the treatment of coupled microresonators operating in the solitonic regime has been lacking.

**Experimental results**

**Experimental observation of GSs on the Si\(_3\)N\(_4\) platform.** Experimental investigation of the phenomena described above requires microresonators with anomalous dispersion, ultralow loss and exceptional uniformity, which are fabricated with the photonic Damascene reflow process on Si\(_3\)N\(_4\) (ref. 42). The DKSs were generated in a pair of 181 GHz resonators (Fig. 3a) with a frequency-dependent evanescent coupling of \( J/2\pi = 3–6 \) GHz having intrinsic and external coupling loss rates of \( \kappa_0, \kappa_e \approx 2 \pi \times 25 \) MHz by laser tuning (Fig. 3b,c). Linear spectroscopy of Si\(_3\)N\(_4\) samples is shown in Extended Data Fig. 2. Further details of the experimental setup are provided in Methods.

Figure 3d shows the power of the light generated in the dimer as the pump laser frequency is reduced, crossing the AS resonance. Similar to the single-resonator case, a chaotic modulation instability state is observed, which collapses to a low-noise GS state as the laser crosses the resonance from blue to red detuning. At low pump power (200 mW in the waveguide), a manifold of multisoliton GS can be accessed (Fig. 3e,f, II). At high pump power (900 mW in the waveguide), the multisoliton states rapidly collapse to a single GS state due to their mutual disruption in the presence of strong resonant DW formation in the S supermode. We observe two types of dual DW in the hybridized supermodes: Fano shaped and Lorentzian shaped. Fano-shaped DWs are continuously observed during the tuning scan, while Lorentz-shaped DWs resonantly appear at fixed detuning (Fig. 3e, III). We investigate the coherence between the DW and soliton using absolute-frequency measurements of each frequency component of the spectrum.

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**Fig. 3 | Kerr comb reconstruction of the photonic dimer states.** a, Microscopy image of coupled Si\(_3\)N\(_4\) resonators. Arrows indicate the propagation direction of the soliton in the resonators. b, c, GS spectra recorded at the bus (b, blue) and drop (c, red) waveguides. d, Power of the light generated by the nonlinear interaction \( P_{\text{gen}} \) while the pump laser is scanned across the antisymmetric resonance. Colours correspond to those shown in Fig. 2a, e. Optical spectra of the generated light at different pump laser detuning values. I, modulation instability; II, three GS states; III, stable GS with offset DW; IV, stable GS with commensurate DW. The blue region indicates comb reconstruction. f, Kerr comb reconstruction of different states showing the frequency offsets of the comb lines versus their relative mode numbers (lighter colours correspond to higher comb line power). I, modulation instability follows the parabolic dispersion profile; II and III, resonant DW appears with the carrier-envelope offset from the main soliton comb; IV, GS line with a commensurate DW signature. More details of the comb reconstruction procedure can be found in Methods as well as Extended Data Fig. 3.
Fig. 4 | Electrical control of GSs and DW interference. a, Microscopy image of a chip containing two coupled Si$_3$N$_4$ resonators partially covered by gold microheaters. b, Optical spectra recorded at the drop waveguide at different inter-resonator detunings $\delta$ (colour code follows that in d). c, Linear measurements of the transmission. d, Position of the spectral maxima associated with the DWs ($f_{ew}^*$) relative to the frequency of the pump laser ($f_{pump}$) as a function of the inter-resonator detuning $\delta$. e, Zoomed-in image of the second spectrum from the top in b showing that the energy of the comb lines remains similar over a wide range of frequencies. Colours in all the plots are preserved.

We find that the Fano-shaped hybrid DW is bound to the soliton, that is, it has identical repetition rate $f_{rep}$ and carrier-envelope offset frequency $f_{o en}$ and it constitutes a commensurate continuation of the primary soliton frequency comb, facilitating their exploitation in metrology$^{44}$. In contrast to the single-particle LLE intuition, the Lorentz-shaped hybrid DW is actually generated at a different $f_{sw}$ (Fig. 3f, III) and hence constitutes an incommensurate secondary frequency comb. Numerical studies (Methods as well as Extended Data Fig. 4) reveal that the cause of the emergence of incommensurate DWs is the frequency dependence of the coupling rate $J$ inherent in the evanescent coupling scheme (Extended Data Fig. 5). Strikingly, the spectrum still reveals multisoliton interference patterns—an indication of a fully coherent comb in the single-particle LLE$^{25}$—which, however—in the dimer case—does not always lead to a fully coherent optical comb (Fig. 3c, II). It is noteworthy that the DWs continue the spectral line pattern of multisoliton and even soliton crystal states (Supplementary Fig. S2). The observation of commensurate and incommensurate DWs in the photonic dimer motivates a deeper analysis of equation (1). The prefactors of even and odd interband FWM transitions reveal that for vanishing normalized detuning $d=0$, the odd interband transition, which inherently preserves the frequency offset of the soliton in the DW, vanishes as well. Strikingly and counter-intuitively, our numerical studies show that a degree of asymmetry either in dissipation (that is, asymmetric external coupling $\kappa_+/(2\pi)$) or a finite inter-resonator detuning $\delta$ is required for the formation of fully coherent dissipative structures in the S modes.

Electrical control of hybrid DWs. Having studied emergent non-linear phenomena in the nonlinear dimer, we show some practical relevance of the additional control provided by this system. Since the Fano shape in the optical spectrum originates from the interference of the soliton in the AS-supermode family and DWs in the S-supermode family, its position is determined by the intersection of the solitonic line with the S parabola given by $\mu_{DW} = \sqrt{\delta^2 + \frac{\omega_s^2}{\Delta^2}}$, where $\delta = \omega_L - \omega_s$. Therefore, the position of the enhanced spectral lines can be readily tuned by thermally varying the inter-resonator detuning $\delta$ (ref. 46). We suppress the appearance of incommensurate DWs by operating the device with strongly asymmetric external coupling. Thermal tuning up to 19 GHz is achieved via integrated gold microheaters (Fig. 4a), which corresponds to the tuning of DW positions over a range of 10 THz. While Fano-shaped$^{26}$ and single-mode DWs$^{35}$ have been observed before, the photonic dimer case presents the opportunity to electrically control the Fano line-shape, that is, the spectral regions where constructive interference takes place. In particular, when the value of $\delta$ is negative, the Fano shapes are oriented such that the resulting solitonic spectra are flat over a broad spectral range. This unique feature, together with the demonstrated tunability, makes the soliton dimer particularly suitable for telecommunication-related applications$^{47}$.

Conclusion

In conclusion, by studying linearly coupled driven nonlinear micro-resonators, we have observed emergent nonlinear dynamics that is not contained in a single-particle LLE. The observations range from resonant Fano DWs and soliton hopping to the surprising relation between the phase coherence of dissipative structures and imperfect mode hybridization. These observations cannot be predicted nor anticipated from the single-particle LLE, and they indicate the rich nonlinear dynamics that can occur when increasing the
system size further to larger lattices. Therefore, the current work represents a step in the development of integrated soliton lattices. Experimental and theoretical approaches demonstrated here—including the representation of the hybridized dispersion relation, spectral analysis of the FWM pathways and thermal tuning of the Fano resonances—can serve as a foundation to further study large one- and higher-dimensional soliton lattices. Our study underscores Anderson’s statement that ‘more is different’ and extends it to driven dissipative nonlinear cavities, which—due to recent progress in photonic materials and fabrication—can now be arranged in extended lattices. The latter are of strong current interest in the frameworks of nonlinear all-optical computing and neural network implementations, topological photonics, digitally programmable chromatic dispersion systems and \(PT\)-symmetric systems. Finally, we anticipate the possibility of creating global Bloch solitons in a lattice of coupled resonators containing DKSs in each of them.

Online content
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Methods

Numerical reconstruction of the dimer phase space. The phase diagram is reconstructed by superimposing ten intracavity power traces and thereby identifying the range of parameters that corresponds to the diagram regions. The coupling coefficient $J$ is considered linear and, at first, frequency independent. The angle between the geometrical centre of the resonators and the bus waveguide is chosen to be $\pi$, which corresponds to a simple vertical arrangement. The inter-resonator detuning $\delta$ is included by adding a correction to the integrated dispersion. The noise is taken on the level of $10^9$ photons per mode with uniformly distributed random fluctuation. Other simulation parameters are taken from experimental measurements of real Si$_3$N$_4$ devices. Nonlinear propagation in the photonic dimer can be described by two coupled LLEs. Let $A(t,0)$ and $B(t,0)$ be slowly varying complex field envelopes (functions of time $t$ and a polar angle $\theta$). Equation (5) is not solved analytically. The linear part in the single-mode description can be solved and gives insights into the processes. It can be written as

$$\frac{\partial u}{\partial t} = Mu + s, \quad u = (A, B), \quad \theta = (\delta_0, \gamma_0, \theta_0^0)$$

where $\delta_0 = \delta_1 + \delta_2$ represent the total loss rate in the pumped and axial cavities, respectively, composed of internal losses represented by $\kappa$, and losses due to the coupling $\kappa_{1,2} = \delta_0 = \omega_1 - \omega_2$ is the pump laser detuning from the cavity resonance, $D$ describes the deviation of the resonant frequencies from the equidistant grid defined by the FWM, $s$ is the Kerr shift per photon and $s_0$ is the pumping rate. Further, $\theta = \theta_0$, where $\theta_0$ is the angle between the centres of the rings and bus waveguide.

Experimental setup. The experimental setup for the characterization of nonlinear frequency mixing and DKS generation in photonic dimers is shown in Extended Data Fig. 3a. It combines the experimental setup for Kerr comb reconstruction as well as dissipative soliton generation and phase modulation response measurements. A widely tunable external cavity diode laser (ECDL) is passed through a fibre-coupled phase modulator, amplifier in an erbium-doped fibre amplifier and coupled to a photonic chip. The laser is tuned into antisymmetric resonance via the piezo-tuning method. Light from the chip is either retrieved at the transmission or drop waveguide ports. The pump light is reflected by a tunable fibre Bragg grating, redirected by an optical circulator and impinged onto a fast photodiode. The relative detuning of the pump laser and Kerr-shifted resonance is determined by interrogating the phase modulation response $S_{\phi}$ of the photonic dimer using the phase modulator driven by a vector network analyzer and receiving the signal from the photodiode. The optical spectra generated by nonlinear interaction in the photonic dimer are analysed using an optical spectrum analyzer. The intensity fluctuations of the optical spectra can be used to measure the intensity fluctuations to determine the optical coherence function.

To directly measure the nonlinear dispersion relation of the soliton states and emergent nonlinear phenomena, we utilize the Kerr comb reconstruction method, that is, we reconfigure the frequency-club-assisted diode laser spectrograph as a heterodyne optical spectrum analyzer by superimposing the output of the photonic dimer with the scanning laser on a balanced photodetector. The spectral resolution is 4 MHz, determined by a 2 MHz low-pass filter. A high dynamic range and logarithmic response is achieved by inserting a multistage logarithmic amplifier (Analog Devices 8307) after the low-pass filter. Our data analysis and reconstruction of the Kerr comb is shown in Extended Data Fig. 3b–d. To extend the spectral range of Kerr comb reconstruction, we employ two widely tunable ECDL lasers (Santec TSL-550) with overlapping wavelength ranges (1,500–1,630 and 1,350–1,505 nm). Each laser scan takes about 20 s to complete and we continuously record the signals from the optical spectrum analyzer and vector network analyzer to ensure that the Kerr comb reconstruction is consistently performed. The passive stability of the pump laser (Topica CT151) and fibre coupler is sufficient to retain the state of the soliton during a scan. The DKS itself can be used to precisely select the consecutive laser scans, which avoids the necessity of a stable transfer laser. The calibrated grating spectrograph (Extended Data Fig. 3b) is used to calibrate both the global frequency offset and the amplitude of the optical spectrum obtained by Kerr comb reconstruction. We ensure that the comb reconstruction is consistently performed. The passive stability of the pump laser (Topica CT151) and fibre coupler is sufficient to retain the state of the soliton during a scan. The DKS itself can be used to precisely select the consecutive laser scans, which avoids the necessity of a stable transfer laser. The calibrated grating spectrograph (Extended Data Fig. 3b) is used to calibrate both the global frequency offset and the amplitude of the optical spectrum obtained by Kerr comb reconstruction. We ensure that the comb reconstruction is consistently performed. The passive stability of the pump laser (Topica CT151) and fibre coupler is sufficient to retain the state of the soliton during a scan. The DKS itself can be used to precisely select the consecutive laser scans, which avoids the necessity of a stable transfer laser.
ECDL laser pumping the photonic dimer. This limitation could be overcome by phase locking the pump laser to the commercial frequency comb (Menlo Systems FC1500), which is also used for frequency calibration.

**Data availability**

Source data are provided with this paper. All data that support the plots within this paper and other findings of this study are available from the corresponding authors upon reasonable request as well as at https://doi.org/10.5281/zenodo.4291973.

**Code availability**

Numerical codes used in this study are available from the corresponding authors upon reasonable request.

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**Author contributions**

T.J.K. initiated the study. A.T. and K.K. developed the idea and performed theoretical and numerical analysis with the assistance of H.G. J.R., K.K. and M.C. performed the experiments and data analysis with the assistance of C.S. and A.T. Metal heaters were fabricated by S.H. R.N.W. and J.L. fabricated the Si₃N₄ samples. P.S. and T.J.K. supervised the project. A.T. and J.R. wrote the manuscript with contributions from all the authors.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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Extended Data Fig. 1 | Numerical reconstruction of the dimer phase space. (left) Superposition of interactivity power traces. For every value of the pump power there are 10 interactivity power traces superimposed. (right) Three examples for three different pump powers: 0.9, 1.2 and 1.5 W.
Extended Data Fig. 2 | Linear spectroscopy of the photonic dimers. (a,b) Normalized frequency dependent transmission of top and bottom waveguides of a photonic dimer. (c) Superimposed echellogram of photonic dimer normalized waveguide transmissions. Successive transmission lines are recessed by the cavity free spectral range of 181.8 GHz, starting at 186 THz in the bottom and ending at 203 THz in the top. Avoided mode crossings (AMX), that is scattering into transverse higher-order dimer modes, are much stronger on the symmetric dimer mode (S mode, left parabola) than on the anti-symmetric dimer modes (AS, right parabola). (d,e) Zoomed in transmission traces of top and bottom waveguide transmissions with fitted coupling, detuning and loss parameters.
Extended Data Fig. 3 | Experimental setup for Kerr combs reconstruction of the photonic dimer solitons. (a) External cavity diode laser (ECDL); phase modulator (PM); erbium doped fiber amplifier (EDFA); Fiber polarization controller (FPC); Optical circulator (CIRC); Vector network analyzer (VNA); Fiber Bragg grating (FBG); Optical spectrum analyzer (OSA); Electrical spectrum analyzer (ESA); low-pass filter (LP); Logarithmic amplifier (LA); Sampling oscilloscope (OSC). (b) Optical spectrum measured with the grating-based optical spectrum analyzer. (c) Calibrated Kerr comb reconstruction trace of long wavelength (red) and short wavelength (green) ECDL scans. (d) Zoom into the overlap region of the two laser scans. Grey area indicates spectral region highlighted in the panel below. (e) Kerr comb reconstruction spectrogram. The offset frequency marks the frequency difference of the incommensurable dispersive wave from the soliton frequency comb. (f) Zoom into offset regions around incommensurable dispersive waves (top and bottom) and soliton frequency comb. The spectral precision of Kerr comb reconstruction is limited by slow drifts of the pump laser and photonic dimer frequency.
Extended Data Fig. 4 | Numerical investigation of the influence of the inter-resonator coupling dependence on the mode number. Emergence of incommensurate dispersive waves is observed when the inter-resonator detuning is equal to zero which corresponds to the enhanced efficiency of the even inter-mode processes. (a) Power spectral density of the intraresonator field. (b) Supermode decomposition. (c) Reconstructed nonlinear dispersion relation.
Extended Data Fig. 5 | Frequency dependent cavity dissipation and coupling rates of the photonic dimers. Frequency-dependent results of photonic dimer fitting. (a,c) Loaded cavity loss rates $\kappa_{AS}/2\pi$ and external $\kappa_{S}/2\pi$ of symmetric and antisymmetric mode families for sample 1 and 2. (b,d) Linear coupling rate $J$ and relative detuning $\delta$ of fundamental resonator modes for samples 1 and 2.