The Effective Field Theory of nonsingular cosmology: II

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Abstract

Based on the Effective Field Theory (EFT) of cosmological perturbations, we explicitly clarify the pathology in nonsingular cubic Galileon models and show how to cure it in EFT with new insights into this issue. With the least set of EFT operators that are capable to avoid instabilities in nonsingular cosmologies, we construct a nonsingular model dubbed the Genesis-inflation model, in which a slowly expanding phase (namely, Genesis) with increasing energy density is followed by slow-roll inflation. The spectrum of the primordial perturbation may be simulated numerically, which shows itself a large-scale cutoff, as the large-scale anomalies in CMB might be a hint for.

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I. INTRODUCTION

Inflation is still being eulogized for its simplicity and also criticized for its past-incompleteness \cite{1\textsuperscript{[2]}}. A complete description of the early universe requires physics other than only implementing inflation.

To the best of current knowledge, the inflation scenario will be past-complete, only if it happens after a nonsingular bounce which is preceded by a contraction \cite{3\textsuperscript{[4]}\textsuperscript{[5]}}, or a slow expansion phase (namely, Genesis phase) with increasing energy density \cite{6\textsuperscript{[7]}\textsuperscript{[8]}}. These two possibilities will be called bounce-inflation and Genesis-inflation, respectively. Besides being past-complete, a bounce-inflation or Genesis-inflation scenario may explain the probable large-scale anomalies in cosmological microwave background (CMB) \cite{4\textsuperscript{[9]}}. The nonsingular Quintom bounce \cite{10\textsuperscript{[11]}\textsuperscript{[12]}}, the ekpyrotic universe \cite{14\textsuperscript{[15]}}\textsuperscript{[15]}, the Genesis scenario \cite{16\textsuperscript{[17]}\textsuperscript{[18]}\textsuperscript{[19]}}, and the slow expansion scenario \cite{20\textsuperscript{[21]}\textsuperscript{[22]}\textsuperscript{[23]}} have acquired intensive attention. In classical nonsingular (past-complete) cosmologies, the Null Energy Condition (NEC) must be violated for a period.

The ghost-free bounce models \cite{24\textsuperscript{[25]}\textsuperscript{[26]}\textsuperscript{[27]}\textsuperscript{[28]}} have been obtained in cubic Galileon \cite{29} and full Horndeski theory \cite{30\textsuperscript{[31]}\textsuperscript{[32]}}. However, recently, it has been proved in Ref. \cite{33} that there exists a “no-go” theorem for cubic Galileon, i.e., the gradient instability ($c_s^2 < 0$ with $c_s$ being sound speed of the scalar perturbations) is inevitable in the corresponding models. See also \cite{34} for the extension to the full Horndeski theory, and \cite{35} for the attempts to avoid “no-go” in Horndeski theory. Relevant studies can also be found in \cite{36\textsuperscript{[37]}\textsuperscript{[38]}}.

Recently, in Ref. \cite{39} (see also \cite{40}), we dealt with this issue in the framework of the Effective Field Theory (EFT) \cite{41\textsuperscript{[42]}\textsuperscript{[43]}\textsuperscript{[44]}}, which has proved to be a powerful tool. In EFT, the quadratic action of scalar perturbation could always be written in the form (see \cite{39} for detailed derivations)

\[
S^{(2)}_\zeta = \int d^4 x a^3 c_1 \left[ \dot{\zeta}^2 - c_s^2 \left( \frac{\partial \zeta}{a^2} \right)^2 \right],
\]

where we have neglected higher-order spatial derivatives of the scalar perturbation $\zeta$, the sound speed squared of scalar perturbation

\[
c_s^2 = \left( \frac{\dot{c}_3}{a} - c_2 \right) / c_1,
\]

with the coefficients $c_1$, $c_2$ and $c_3$ being time dependent parameters in general, and $c_1 > 0$ is needed to avoid the ghost instability. The condition for avoiding gradient instability is
\( c_3^2 \sim \dot{c}_3/a - c_2 > 0 \), which is usually integrated as
\[
\left. c_3 \right|_{t_f} - \left. c_3 \right|_{t_i} > \int_{t_i}^{t_f} ac_2 dt. \tag{3}
\]
The condition of satisfying the inequality is to have \( c_3 \) cross 0, which is hardly possible in models based on the cubic Galileon \cite{33,34}. However, we found that it can easily be satisfied by applying the EFT operator \( R^{(3)} \delta g^{00} \) (with \( R^{(3)} \) being the 3-dimensional Ricci scalar on spacelike hypersurface and \( \delta g^{00} = g^{00} + 1 \)), so that the gradient instability can be cured.

Though the integral approach (3) is simple and efficient, some details of curing the pathology might actually be missed. In this paper, based on the EFT, using a “non-integral approach”, we revisit the nonsingular cosmologies. We begin straightly with (2), and clarify the origin of pathology and show how to cure it in EFT with new insights into what is happening (Sec. II). To have practice in this clarification, we build a stable model of the Genesis-inflation scenario by using the \( R^{(3)} \delta g^{00} \) operator (Sec. III). As a supplementary remark, we discuss a dilemma in the Genesis scenario (Sec. IV).

II. RE-PROOF OF THE “NO-GO” AND ITS AVOIDANCE IN EFT

The EFT is briefly introduced in Appendix A. In the unitary gauge, the quadratic action of tensor perturbation \( \gamma_{ij} \) is (see \cite{39} for the derivation of Eqs. (4) to (8))
\[
S^{(2)}_{\gamma} = \frac{M_p^2}{8} \int d^4x a^3 Q_T \left[ \dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial_k \gamma_{ij})^2}{a^2} \right], \tag{4}
\]
where \( Q_T = f + \frac{2m_i^2}{M_p^2} > 0, c_T^2 = f/Q_T > 0, f \) and \( m_i^2 \) are coefficients defined in the EFT action (A4).

The quadratic action of the scalar perturbation \( \zeta \) is given by Eq. (1) with
\[
c_1 = \frac{Q_T}{4\gamma^2 M_p^2} \left[ 2M_p^4 Q_T \dot{f} \dot{H} - 2M_p^2 Q_T \left( 2f M_p^2 \dot{H} + \dot{f} M_p^2 - 4M_p^4 \right) \right. \left. -6\dot{f} M_p^2 m_3^3 + 3\dot{f}^2 M_p^2 + 3m_3^6 \right], \tag{5}
\]
\[
c_2 = f M_p^2, \tag{6}
\]
\[
c_3 = \frac{aM_p^2}{\gamma} Q_T Q_{\tilde{m}_4}, \tag{7}
\]
\[
\gamma = H Q_T - \frac{m_3^3}{2M_p^2} + \frac{1}{2} \dot{f}, \quad Q_{\tilde{m}_4} = f + \frac{2\tilde{m}_4}{M_p^2}, \tag{8}
\]
where $M_2^4$, $m_3^2$ and $\tilde{m}_4^2$ are coefficients defined in the EFT action (A4) and they could be time dependent in general.

Only if $c_1 > 0$ and $c_s^2 > 0$, the model is free from ghost and gradient instabilities, respectively. In nonsingular cosmological models based on the cubic Galileon [26][27][28], $c_1 > 0$ is not hard to obtain, as can be seen from Eq. (5), since the cubic Galileon contributes the $\frac{m_3^2(t)}{2}\delta K\delta g^{00}$ operator in EFT. However, since $c_3$ is also affected by $\frac{m_3^2(t)}{2}\delta K\delta g^{00}$ through $\gamma$, $c_s^2 < 0$ is actually inevitable, as will be demonstrated in the following.

Since $c_1 > 0$, the requirement of $c_s^2 > 0$ equals

$$
\left( H\gamma + \frac{Q_{\tilde{m}_4}}{Q_T} \gamma + \frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} \gamma - c_s^2 \frac{\gamma^2}{Q_{\tilde{m}_4}} - \dot{\gamma} \right) \frac{Q_T Q_{\tilde{m}_4}}{\gamma^2} > 0.
$$

(9)

Here, $Q_T \neq Q_{\tilde{m}_4}$ is required, which cannot be embodied by the Horndeski theory [30][31][32]. Thus whether $c_s^2 > 0$ or not is controlled by the parameter set $(H, \gamma, Q_T, c_s^2, Q_{\tilde{m}_4})$.

In the following, with condition (9), we will re-prove the \textquotedblleft no-go\textquotedblright\ theorem for the cubic Galileon, and clarify how to cure it in EFT. Different from the proof in [33][39][40], the re-proof is directly based on the derivative inequality instead of integrating it, which we called \textquotedblleft non-integral approach\textquotedblright. We assume that after the beginning of the hot \textquotedblleft big bang\textquotedblright or inflation, $\gamma = H > 0$, $\dot{\gamma} < 0$ and $Q_{\tilde{m}_4} = 1$.

A. Case I: initially $\gamma < 0$

Since initially $\gamma < 0$, $\gamma$ has to cross 0 from $\gamma < 0$ to $\gamma > 0$ at $t_\gamma$. The analysis below is also applicable for all cases with $\gamma$ crossing 0 from $\gamma < 0$ to $\gamma > 0$.

In the ekpyrotic and bounce models, initially $\gamma = H < 0$. In the Genesis model [16] and slow expansion model [21], $H > 0$ during the Genesis, but actually $\gamma = H - \frac{m_3^2}{2M_p^2} < 0$, as discussed in Sec. IV. Both belong to the Case I.

\textit{In the cubic Galileon case}, $f = Q_T = Q_{\tilde{m}_4} = 1$. Around $t_\gamma$, condition (9) is

$$
-\dot{\gamma} > 0.
$$

(10)

We see that $c_s^2 < 0$ is inevitable around $t_\gamma$, since $\dot{\gamma} > 0$. Thus the nonsingular models based on cubic Galileon is pathological, as first proved by LMR in [33].

\textit{In the EFT case}, around $t_\gamma$, condition (9) requires

$$
\left( \frac{\dot{Q}_T}{Q_T} \gamma + \frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} \gamma - c_s^2 \frac{\gamma^2}{Q_{\tilde{m}_4}} - \dot{\gamma} \right) Q_{\tilde{m}_4} > 0.
$$

(11)
We might have \( c^2_s > 0 \), only if (considering only the case where only one of \( Q_T \) and \( Q_{\bar{m}_4} \) is modified while the unmodified one is unity) around \( \gamma = 0 \)

\[
\frac{\dot{Q}_T}{Q_T} \gamma > \dot{\gamma},
\]

or \( Q_{\bar{m}_4} < 0 \), or \( \frac{\dot{Q}_{\bar{m}_4}}{Q_{\bar{m}_4}} \gamma > \dot{\gamma} + \frac{c^2_s \gamma^2}{Q_{\bar{m}_4}} \) (for \( Q_{\bar{m}_4} \geq 0 \)).

In solution (12), at \( t_\gamma, \gamma = 0 \) suggests \( Q_T = 0 \). Here, since \( \gamma = 0 \) at \( t_\gamma, c_1 \sim Q_T/\gamma^2 \) diverges.

One possibility of removing this divergence is that \( \gamma \sim (t - t_\gamma)^p \) and \( Q_T \sim (t - t_\gamma)^n \) around \( t_\gamma \), with \( n \geq 2p \) and \( p, n \) being constants. In Ijjas and Steinhardt’s model \([35]\), \( \gamma \sim t - t_\gamma \) while \( Q_T \sim (t - t_\gamma)^2 \), which belongs to this case.

In the bounce model based on the cubic Galileon, Eq. (8) gives \( \gamma = H - \frac{m^4_4}{2M_p^2} \neq \dot{H} \). Generally, the NEC is violated when \( \dot{H} > 0 \), while the period of \( c^2_s < 0 \) corresponds to the phase with \( \gamma \simeq 0 \) and \( \dot{\gamma} > 0 \), these two phases do not necessarily coincide, see Eq. (8). As pointed out by Ijjas and Steinhardt \([35]\), it is the sign’s change of \( \gamma \) that causes the pathology. Here, we reconfirmed this point.

In solution (13), if \( Q_{\bar{m}_4} > 0 \), at \( t_\gamma, \gamma = 0 \) suggests \( Q_{\bar{m}_4} = 0 \); while if \( Q_{\bar{m}_4} < 0 \), since \( Q_{\bar{m}_4} = 1 \) eventually, \( Q_{\bar{m}_4} \) must cross 0 at \( t_{\bar{m}_4} \) (generally \( t_{\bar{m}_4} \neq t_\gamma \)), at which \( \dot{Q}_{\bar{m}_4} \gamma > c^2_T \gamma^2 \) must be satisfied. In both cases, \( Q_{\bar{m}_4} = 0 \) is required, as proposed by Cai et.al \([39]\) and Creminelli et.al \([40]\).

We see again the details of \( Q_{\bar{m}_4} \) crossing 0. In both the Genesis model and the bounce model, initially \( Q_{\bar{m}_4} = 1 \), so if \( Q_{\bar{m}_4} < 0 \) around \( t_\gamma, Q_{\bar{m}_4} \) must cross 0 twice. Thus it seems that \( \dot{Q}_{\bar{m}_4} \gamma > c^2_T \gamma^2 \) is hard to implement. However, with (2) and (7), one always could solve \( Q_{\bar{m}_4} \) for any given \( c^2_s \),

\[
Q_{\bar{m}_4} = \frac{\gamma}{aM_p^2} \int a \left( c_1 c^2_s + c_2 \right) dt,
\]

where \( Q_T = 1 \).
B. Case II: $\gamma > 0$ throughout

Since $\gamma > 0$ throughout, we must have $\dot{\gamma} \geq 0$ during some period initially\(^1\), otherwise $\gamma$ will diverge in the infinite past.

*In the cubic Galileon case*, condition (9) is

$$H\gamma - \gamma^2 - \dot{\gamma} > 0.$$  \hfill (15)

In the bounce model, $H < 0$ in the contracting phase, and in the Genesis model, $H \sim 0$ in the Genesis phase, both suggest $H\gamma - \gamma^2 - \dot{\gamma} < 0$ \(^2\). Thus $c_s^2 < 0$ is inevitable in the corresponding phases, so the nonsingular models based on the cubic Galileon is pathological.

We see the Genesis model in the cubic Galileon version again in detail. During the slow expansion (Genesis phase), $H \sim 1/(-t)^n$ with the constant $n > 1$. Thus

$$\frac{\dot{\gamma}}{H\gamma} \approx \frac{\dot{H}}{H^2} \sim (-t)^{n-1} \gg 1,$$  \hfill (16)

which implies $H\gamma \ll \dot{\gamma}$. Thus with (15), we see that $c_s^2 < 0$ is inevitable in the slow expansion phase. It seems that if $n = 1$, $H\gamma \ll \dot{\gamma}$ might be avoided. However, when $n = 1$, we have $H = p/(-t)$ and $a \sim 1/(-t)^p$ with constant $p$, thus $a \to 0$ in the infinite past. From (16), we see that $c_s^2 > 0$ requires $p = H^2/\dot{H} > 1$. Therefore, the universe is singular, or from another point of view, it is geodesically incomplete since the affine parameter of the graviton geodesics $\int_{t_i}^{t_f} adt$ is finite for $p > 1$ when $t_i \to -\infty$.

*In the EFT case*, condition (9) requires

$$\left(\frac{\dot{Q}_T}{Q_T} \gamma + \frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} \gamma - \frac{c_s^2 \gamma^2}{Q_{\tilde{m}_4}} + H\gamma - \dot{\gamma}\right) Q_{\tilde{m}_4} > 0.$$  \hfill (17)

We might have $c_s^2 > 0$, only if (considering only the case where either $Q_T$ or $Q_{\tilde{m}_4}$ is modified)

$$\frac{\dot{Q}_T}{Q_T} > c_s^2 \gamma - H + \frac{\dot{\gamma}}{\gamma},$$  \hfill (18)

or

$$\frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} < \frac{c_s^2 \gamma}{Q_{\tilde{m}_4}} - H + \frac{\dot{\gamma}}{\gamma} \quad \text{(initially $Q_{\tilde{m}_4} < 0$).}$$  \hfill (19)

\(^1\) Of course, in Case II, we could also have $\dot{\gamma} < 0$ during some period, but what we focus on is the period (i.e., $\dot{\gamma} \geq 0$) where pathologies appear.

\(^2\) In the case where $\gamma$ grows from 0 initially, (15) is also obeyed no more.
Generally, $-H\gamma + \dot{\gamma} > 0$, as in Genesis model and bounce model. Thus the solution (18) suggests $\dot{Q}_T > 0$, so that we will have $Q_T = 0$ in infinite past. Thus based on (12) and (18), it seems that though the pathology can be cured by applying $Q_T$, $Q_T = 0$ is inevitable. A model with (18) has been proposed by Kobayashi [34] ($Q_T \sim \frac{1}{(-t)^p}$, $p > n > 1$). During the Genesis $\gamma \sim H \sim 1/(-t)^n$, $n > 1$, (17) is
\[
\left(\frac{\dot{Q}_T}{Q_T}\right)^{-1} \frac{\dot{\gamma}}{\gamma} = n/p < 1.
\] (20)
Initially, $Q_T \sim \frac{1}{(-t)^p} = 0$.

In solution (19), $\dot{Q}_{\tilde{m}_4}$ must cross 0 at $t_{\tilde{m}_4}$ to $Q_{\tilde{m}_4} > 0$, as pointed out by Cai et.al [39] and Creminelli et.al [40]. Around $t_{\tilde{m}_4}$, $\dot{Q}_{\tilde{m}_4} > c_T^2 \gamma$ must be satisfied.

In (17), if $Q_{\tilde{m}_4} > 0$ throughout,
\[
\frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} > \frac{c_T^2 \gamma}{Q_{\tilde{m}_4}} - H + \frac{\dot{\gamma}}{\gamma}
\] (21)
is obtained. Thus, similar to (18), we have $Q_{\tilde{m}_4} = 0$ (which definitely requires $\gamma = 0$) in the infinite past. In the Genesis model, $Q_{\tilde{m}_4} \sim 1/(-t)^p$ and $\gamma \sim 1/(-t)^n$ with $p > n$, since $\dot{Q}_{\tilde{m}_4}/Q_{\tilde{m}_4} > \dot{\gamma}/\gamma$. However, $p > n$ indicates $\dot{Q}_{\tilde{m}_4} < \gamma$ in the infinite past ($Q_{\tilde{m}_4} = 0$), which violates the inequality (21). Thus $Q_{\tilde{m}_4} > 0$ throughout seems unworkable.

| Nonsingular cubic Galileon models | Initially $\gamma < 0$ | $\gamma > 0$ throughout |
|-----------------------------------|------------------------|------------------------|
| Crossing 0 for $\gamma$?         | $\sqrt{\ }$           | $\times$               |
| $c_s^2 < 0$ is inevitable (“no-go”)? | $\sqrt{\ }$           | $\sqrt{\ }$           |
| Phase with $c_s^2 < 0$ (Pathological phase) | $\dot{\gamma} > 0$ around $\gamma \simeq 0$ | $H\gamma - \dot{\gamma} < \gamma^2$ |
| Curing pathology in EFT          |                        |                        |
| Conditions of $c_s^2 > 0$         | (11)                   | (17)                   |
| Applying $Q_T$                    | (12)                   | (18)                   |
| Applying $Q_{\tilde{m}_4}$       | (13)                   | (19)                   |

TABLE I: Pathology in nonsingular cubic Galileon cosmological models and its cure in EFT by either $Q_T$ or $Q_{\tilde{m}_4}$.
III. APPLICATION TO GENESIS-INFLATION

In this section, we will build a nonsingular model with the solution (19), in which the slow-roll inflation is preceded by a Genesis phase. A Genesis phase is a slowly expanding phase originating from the Minkowski vacuum with a drastic violation of NEC, i.e., $\epsilon \ll -1$, thus the energy density is increasing with the expansion of the universe and hence is free from the initial singularity [16][17] (see also [20]). As will be shown below, our model cannot only get rid of the pathology of instability, but also give rise to a flat spectrum with interesting features at large scales.

A. The setup of the model

The action of the model is
\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + M_p^2 g_1(\phi)X + g_2(\phi)X \Box \phi + g_3(\phi)X^2 - M_p^4 V(\phi) + \frac{\tilde{m}_4(t)^2}{2} R^{(3)} \delta g^{00} \right],
\]
where $X = -\nabla_\mu \phi \nabla^\mu \phi/2$, $\Box = \nabla_\mu \nabla^\mu$, and $\phi$ is a dimensionless scalar field, so dimensionless are $g_1(\phi)$, $g_2(\phi)$, $g_3(\phi)$ and $V(\phi)$.

Mapped into the EFT action (A4), (22) corresponds to
\[
f = 1, \quad (23)
\]
\[
\Lambda(t) = M_p^4 V - \frac{1}{2} g_2 \dot{\phi}^2 \left( 3H \dot{\phi} + \ddot{\phi} \right) + \frac{1}{4} g_3 \dot{\phi}^4, \quad (24)
\]
\[
c(t) = \frac{M_p^2}{2} g_1 \dot{\phi}^2 - \frac{1}{2} g_2 \dot{\phi}^2 \left( 3H \dot{\phi} - \ddot{\phi} \right) + \frac{1}{2} g_2 \dot{\phi}^4 + \frac{1}{2} g_3 \dot{\phi}^4, \quad (25)
\]
\[
M_2^2(t) = -\frac{1}{4} g_2 \dot{\phi}^2 \left( 3H \dot{\phi} + \ddot{\phi} \right) + \frac{1}{4} g_2 \dot{\phi}^4 + \frac{1}{2} g_3 \dot{\phi}^4, \quad (26)
\]
\[
m_3^3(t) = -g_2 \dot{\phi}^3, \quad (27)
\]
\[
m_4^2 = 0, \quad (28)
\]
\[
\tilde{m}_2^2 \neq 0. \quad (29)
\]
We can get the background equations

\begin{equation}
3H^2M_p^2 = \frac{M_p^2}{2}g_1\dot{\phi}^2 - 3g_2H\dot{\phi}^3 + \frac{1}{2}g_{2,\phi}\dot{\phi}^4 + \frac{3}{4}g_3\dot{\phi}^4 + M_p^4V, \tag{30}
\end{equation}

\begin{equation}
\dot{H}M_p^2 = -\frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{2}g_2H\dot{\phi}^3 - \frac{1}{2}g_{2,\phi}\dot{\phi}^2\dot{\phi} - \frac{1}{2}g_{2,\phi}\dot{\phi}^4 - \frac{1}{2}g_3\dot{\phi}^4, \tag{31}
\end{equation}

\begin{equation}
0 = g_1\ddot{\phi} + 3g_1H\dot{\phi} + \frac{1}{2}g_{1,\phi}\dot{\phi}^2
- \frac{9g_2H^2\dot{\phi}^2}{M_p^2} - \frac{3g_2H\dot{\phi}^2}{M_p^2} - \frac{6g_2H\dot{\phi}\ddot{\phi}}{M_p^2} + \frac{2g_{2,\phi}\dot{\phi}\ddot{\phi}}{M_p^2} + \frac{g_{2,\phi,\phi}\dot{\phi}^4}{2M_p^2}
+ \frac{3g_3H\dot{\phi}^3}{M_p^2} + \frac{3g_3\dot{\phi}^2\dot{\phi}}{M_p^2} + \frac{3g_{3,\phi}\dot{\phi}^4}{4M_p^2}
+ M_p^2V_\phi, \tag{32}
\end{equation}

where “\(\phi = d/d\phi\)” and “\(\phi_\phi = d^2/d\phi^2\)”.

Initially, the universe is slowly expanding (in the Genesis phase), \(H \simeq 0\). We set \(V = 0\), \(g_1 = -f_1e^{2\phi}\), \(g_2 = f_2\) and \(g_3 = f_3\), see e.g. Ref. [7], with \(f_{1,2,3}\) being dimensionless constants. Thus with Eq. (30), we have \(\frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{4}g_3\dot{\phi}^4 = 0\), which suggests

\begin{equation}
e^{2\phi} = \frac{3f_3}{2M_p^2f_1}\dot{\phi}^2. \tag{33}\end{equation}

The solution is

\begin{equation}\dot{\phi} = \frac{1}{(-t)}, \quad t < 0. \tag{34}\end{equation}

Eq. (31) reads \(\dot{H} = \frac{f_3 - 2f_2}{4M_p^2}\dot{\phi}^4\). Thus we get

\begin{equation}H = \frac{f_3 - 2f_2}{12M_p^2} \frac{1}{(-t)^3} \tag{35}\end{equation}

after the integration. In principle, there could be a constant, i.e., \(H = \frac{f_3 - 2f_2}{12M_p^2} \frac{1}{(-t)^3} + \text{const}\), however, in that case we will have \(H \approx \text{const}\) initially, which is geodesically incomplete, see also [45].

Additionally, from Eq. (35), we have

\begin{equation}a(t) = e^{\int \dot{H}dt} = \exp \left( \frac{f_3 - 2f_2}{24M_p^2t^2} \right) \approx 1 + \left( \frac{f_3 - 2f_2}{24M_p^2t^2} \right), \tag{36}\end{equation}

while we set \(a(-\infty) = 1\).

During inflation, we set \(g_1 = 1\) and \(g_2 = g_3 = 0\), since we require that the inflationary phase is controlled by a simple slow-roll field\(^3\).

\(^3\) The behaviors of these \(g_i\)’s in the two phases can easily be matched together by making use of some shape functions [5][46].
B. The primordial perturbation and its spectrum

In the unitary gauge, the quadratic action of the scalar perturbation is presented in the form of Eq. (1). The coefficients $c_i$ are (substituting Eqs. (23) to (29) into (5)(6)(7))

$$c_1 = \frac{\dot{\phi}^2}{4M_p^2\gamma^2} \left[ 2\phi^2 M_p^2 \left( g_{2,\phi} + 2g_3 \right) - 2g_2 M_p^2 \left( 3H\dot{\phi} + \ddot{\phi} \right) + 3g_2^2 \dot{\phi}^4 \right] - \frac{\dot{H} M_p^2}{\gamma^2}, \quad (37)$$

$$c_2 = M_p^2, \quad (38)$$

$$c_3 = aM_p^2 Q \tilde{m}_4, \quad (39)$$

where

$$\gamma = H + \frac{g_2}{2M_p^2} \dot{\phi}^3, \quad Q \tilde{m}_4 = 1 + \frac{2\tilde{m}_4^2}{M_p^2}. \quad (40)$$

The sound speed squared $c_s^2$ of scalar perturbation is defined in Eq. (2). Here, when $\tilde{m}_4^2 \equiv 0$ or $Q \tilde{m}_4 = 1$, the sound speed squared of the scalar perturbation is reduced to

$$c_{s0}^2 = 1 + \frac{4\dot{\phi}^2 \left[ g_2 M_p^2 \left( \ddot{\phi} - H\dot{\phi} \right) + g_2^2 \dot{\phi}^4 + \ddot{\phi}^2 M_p^2 \left( g_{2,\phi} + g_3 \right) \right]}{4\dot{H} M_p^4 + \dot{\phi}^2 \left[ 2g_2 M_p^2 \left( 3H\dot{\phi} + \ddot{\phi} \right) - 3g_2^2 \dot{\phi}^4 - 2\ddot{\phi}^2 M_p^2 \left( g_{2,\phi} + 2g_3 \right) \right]}, \quad (41)$$

It is easy to see that $c_{s0}^2 = 1$ for inflation, since $g_2 = g_3 = 0$, but not for Genesis. However, using the operator $\frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00}$, we could always set $c_s^2 = 1$ in the Genesis phase, which requires $\tilde{m}_4^2 = -\frac{2M_p^2 (f_{2,\phi} + f_3)}{4f_{2,\phi} + f_3}$. This suggests $Q \tilde{m}_4 = -\frac{3f_3}{4f_{2,\phi} + f_3}$ is a constant at $| - t | \gg 1$, which is consistent with the solution (19).

The equation of motion of $\zeta$ is

$$u'' + \left( c_s^2 k^2 - \frac{z''}{z} \right) u = 0, \quad (42)$$

with $u = z\zeta$, $z = \sqrt{2}a^2 c_1$, the prime denotes the derivative with respect to the conformal time $\tau = \int dt/a$. The initial state is the Minkowski vacuum, thus $u = \frac{1}{\sqrt{2c_s k}} e^{-ic_s k \tau}$ for $\zeta$ modes deep inside the horizon. The power spectrum of $\zeta$ is

$$P_R = \frac{k^3}{2\pi^2} \left| \frac{u}{z} \right|^2. \quad (43)$$

In the following, we will analytically estimate the spectrum of the scalar perturbation. We set $c_s^2 = 1$ throughout for simplicity, which could be implemented by using $Q \tilde{m}_4(t)$, as will be illustrated by the numerical simulation.
In the Genesis phase, substituting Eqs. (34), (35) into (5), we have

\[ c_1 = \frac{108f_3M_p^4}{(4f_2 + f_3)^2} (-t)^2. \] (44)

Thus

\[ z = \frac{6\sqrt{6f_3M_p^2}}{4f_2 + f_3} (-t) \cdot \exp \left( \frac{f_3 - 2f_2}{24M_p^2 t^2} \right). \] (45)

Then it is straightforward to obtain \( z''/z \approx \frac{(f_3 - 2f_2)^2}{\tau^2 M_p^2} \approx \frac{0}{\tau}, \) where \( \tau = \int \frac{1}{a} dt \approx t. \) Thus the solution of Eq. (42) is

\[ u_1 = \frac{\sqrt{-\pi \tau}}{2} \left[ C_{11} \cdot H_{1/2}^{(1)}(-k\tau) + C_{12} \cdot H_{1/2}^{(2)}(-k\tau) \right], \] (46)

where \( C_{11} \) and \( C_{12} \) are functions of \( k, H_{\nu}^{(1)} \) and \( H_{\nu}^{(2)} \) are the first and the second kind Hankel functions of \( \nu \)-th order, respectively. The initial condition \( u = \frac{1}{\sqrt{2k}} e^{-ik\tau} \) indicates

\[ C_{11} = i, \quad C_{12} = 0. \] (47)

In the inflation phase, \( c_1 = \epsilon M_p^2, \) thus \( z = \sqrt{2\epsilon a^2 M_p^2}. \) We set \( \epsilon \ll 1 \) as a constant during inflation. Then \( z''/z \approx (2 + 3\epsilon) / \tau^2. \) The solution of Eq. (42) is

\[ u_2 = \frac{\sqrt{-\pi \tau}}{2} \left[ C_{21} \cdot H_{\nu_2}^{(1)}(-k\tau) + C_{22} \cdot H_{\nu_2}^{(2)}(-k\tau) \right] \] (48)

with \( \nu_2 \approx 3/2 + \epsilon. \)

We require that \( u_1(\tau_m) = u_2(\tau_m) \) and \( u'_1(\tau_m) = u'_2(\tau_m), \) with \( \tau_m \) approximately corresponding to the beginning time of inflation phase, and we obtain

\[ C_{21} = -i \frac{e^{-ik\tau_m}}{4} \frac{\sqrt{-\pi}}{2k\tau_m} \left[ 2k\tau_m H_{\nu_2-1}^{(2)}(-k\tau_m) + (2\nu_2 - 1 - 2ik\tau_m) H_{\nu_2}^{(2)}(-k\tau_m) \right], \] (49)

\[ C_{22} = i \frac{e^{-ik\tau_m}}{4} \frac{\sqrt{-\pi}}{2k\tau_m} \left[ 2k\tau_m H_{\nu_2-1}^{(1)}(-k\tau_m) + (2\nu_2 - 1 - 2ik\tau_m) H_{\nu_2}^{(1)}(-k\tau_m) \right]. \] (50)

The power spectrum of \( \zeta \) is given by

\[ P_{\zeta} = P_{\zeta}^{inf} \cdot |C_{21} - C_{22}|^2, \quad k \ll aH, \] (51)

where \( P_{\zeta}^{inf} = \frac{H_2^{2inf}}{8\pi^2 M_p^6} \cdot \left( \frac{k}{aH} \right)^{3-2\nu_2} \) is the power spectrum of scalar perturbation modes that exit horizon during inflation. We see that for the perturbation modes exiting horizon in the Genesis phase, \( -k\tau_m \ll 1, |C_{21} - C_{22}|^2 \approx (-k\tau_m)^2, \) thus \( P_{\zeta} \sim k^2 \) is strong blue-tilted, while for the perturbation modes exiting horizon in the inflation phase, \( -k\tau_m \gg 1, |C_{21} - C_{22}|^2 \approx 1, \) thus \( P_{\zeta} \sim k^{3-2\nu_2} = k^{-2\epsilon} \) is flat with a slightly red tilt.
Tensor perturbation is unaffected by the $R^{(3)}\delta g^{00}$ operator. Its quadratic action is given in Eq. (4) with $Q_T = 1$ and $c_T^2 = 1$. The spectrum of primordial GWs can be calculated similarly, see also Ref. [6]. Since $z''_{T}/z_T = a''/a$, we have

$$P_T = P_{T}^{inf} \cdot |C_{21} - C_{22}|^2, \quad k \ll aH,$$

where $P_{T}^{inf} = \frac{2H_T^2}{\pi^2 M_p^2} \cdot \left(\frac{k}{aH}\right)^{3-2
u_2}$ is the power spectrum of tensor perturbation modes that exit horizon during inflation. Thus the spectrum of primordial GWs has a shape similar to that of the scalar perturbation.

**C. Numerical simulation**

In the numerical calculation, we set

$$g_1(\phi) = \frac{f_1 e^{2\phi}}{1 + f_1 e^{2\phi}} \tanh \left[q_1(\phi - \phi_0)\right],$$

$$g_{2,3}(\phi) = f_{2,3} \left(1 - \tanh \left[q_{2,3}(\phi - \phi_0)\right]\right)/2,$$

$$V(\phi) = \frac{\lambda}{2} (\phi - \phi_1)^2 \left(1 + \tanh \left[q_4(\phi - \phi_2)\right]\right)/2,$$

with $f_{1,2,3,4}$, $q_{0,1,2}$ and $\lambda$ being dimensionless constants. When $\phi \ll \phi_0$, we have $g_1 = -f_1 e^{2\phi}$, $g_2 = f_2$ and $g_3 = f_3$, which brings a Genesis phase (36), while $\phi \gg \phi_0$, we have $g_1 = 1$ and $g_2 = g_3 = 0$, the slow-roll inflation will occur with $V(\phi) \sim \phi^2$. When $\phi \ll \phi_2$, $V(\phi) \approx 0$, while $\phi \gg \phi_2$, $V(\phi) \approx \frac{\lambda}{2} (\phi - \phi_1)^2$. We do not require $\phi_0 = \phi_2$ but $\phi_0 > \phi_2$.

We start the simulation at $t_i \ll -1$, and we set

$$\dot{\phi}(t_i) = \frac{1}{(-t_i)}, \quad \phi(t_i) = \frac{1}{2} \ln \left[\frac{3f_3}{2f_1 M_p^2 (-t_i)^2}\right],$$

and

$$a(t_i) = 1, \quad H(t_i) = \frac{f_3 - 2f_2}{12M_p^2} \frac{1}{(-t_i)^3}.$$

We show the evolution of $\phi$ and $\dot{\phi}$ in Fig. 1, and the evolution of $a$, $H$ and $\epsilon$ in Fig. 2. In Fig. 3(a), $c_1$ is plotted, and $c_1 > 0$ is satisfied. In Fig. 3(b), we see that $\gamma$ does not cross 0, which implies that, in the Genesis phase, $c_{s0}^2 < 0$ (see Fig. 4(a)), as proved in Sec. IV. By including the operator $R^{(3)}\delta g^{00}$, we could have $c_s^2 > 0$, and so cure the gradient
instability. The spectrum of the scalar perturbation can be simulated numerically, which is plotted in Fig. 5. The spectrum obtained has a cutoff at large scale $k < k^*$ and is nearly scale-invariant for $k > k^*$, as displayed in Eq. (51).

FIG. 1: The evolution of $\phi$ and $\dot{\phi}$, while we set $f_1 = 5, f_2 = -0.23, f_3 = -13f_2, q_1 = 1, q_2 = 0.2, q_3 = 0.2, q_4 = 2, \lambda = 4 \times 10^{-4}, \phi_0 = 7, \phi_1 = 22.7 \text{ and } \phi_2 = 5.2$.

![Graph showing the evolution of a, H, and epsilon](image)

(a) $a$ and $H$  
(b) $\epsilon$

FIG. 2: The evolution of $a$, $H$ and $\epsilon$, while we set $f_1 = 5, f_2 = -0.23, f_3 = -13f_2, q_1 = 1, q_2 = 0.2, q_3 = 0.2, q_4 = 2, \lambda = 4 \times 10^{-4}, \phi_0 = 7, \phi_1 = 22.7 \text{ and } \phi_2 = 5.2$.

IV. THE DILEMMA OF $\gamma$ IN THE GENESIS SCENARIO

In the Genesis scenario based on the cubic Galileon [16], (also [21]), we have

$$\gamma = H + \frac{f_2}{2M_p^2} \phi^3 = \frac{f_3 + 4f_2}{12M_p^2} \phi^3$$

(58)

during the Genesis, where $f_2 < 0$. 

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FIG. 3: The evolution of $c_1$, $\gamma/H$ and $\epsilon$, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$.

FIG. 4: The sound speed of the scalar perturbation with and without $\tilde{m}_4^2$, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$. 
FIG. 5: The spectrum $P_R$ of the scalar perturbation, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$, $\phi_2 = 5.2$ and $k_*$ corresponds to the comoving wave number of the perturbation mode which exits horizon around the beginning of inflation.

In Ref. \cite{16}, $f_3 = -f_2$, which suggests $\gamma = \frac{f_2}{4M_p^2}\dot{\phi}^3 < 0$. Thus if a hot “big bang” or inflation ($\gamma = H > 0$) starts after the Genesis phase, $\gamma$ must cross 0 at $t_\gamma$ ($c_{s0}^2 < 0$ around $t_\gamma$, which may be cured by applying $Q_{\tilde{m}_a}$). It is obvious that when $\gamma = 0$, $c_1$ in (1) will be divergent. Though this divergence might not be a problem, it will affect the numerical simulation for perturbations \cite{47}\cite{48}, unless $Q_T/\gamma^2$ is finite at $t_\gamma$, as in Ijjas and Steinhardt’s model \cite{35}.

In the model of \cite{7}, the Genesis is followed by Galileon inflation \cite{49}. Though $f_3 = -f_2$ and $\gamma = \frac{f_2}{4M_p^2}\dot{\phi}^3 < 0$ in the Genesis phase, one might also have $\gamma < 0$ for Galileon inflation, since $g_2 \neq 0$ in (22) during inflation. Thus it seems that $\gamma$ might not necessarily cross 0. However, after inflation, $\gamma$ crossing 0 is still inevitable.

In our model, the Genesis is followed by the slow-roll inflation, $\gamma = H > 0$ for inflation. To not cross 0, initially $\gamma$ must satisfy $\gamma > 0$. In the Genesis phase, this suggests $f_3 > -4f_2$. Thus we will have $\gamma > 0$ throughout. However, for the cubic Galileon, the expense is

$$c_{s0}^2 = 1 - \frac{4f_2 + 4f_3}{3f_3} < 0$$

during the Genesis. Here, this pathology is cured in EFT by applying (19).
V. CONCLUSION

Based on the EFT of cosmological perturbations, we revisit the nonsingular cosmologies, using the “non-integral approach”. By doing this, we could have a clearer understanding of the pathology in nonsingular Galileon models and its cure in EFT.

We clarify the application of the operator $\bar{m}_4^2 R^{(3)} \delta g^{00}/2$ in EFT, which is significant for curing the gradient instability. We show that if $Q_{\bar{m}_4} < 0$ around $\gamma = 0$ is adopted to cure the gradient instability, in solution (13) (with $\gamma < 0$ and $Q_{\bar{m}_4} = 1$ initially), $Q_{\bar{m}_4}$ must cross 0 twice; while in solution (19) (with $\gamma > 0$ throughout), initially $Q_{\bar{m}_4} < 0$ must be satisfied, $Q_{\bar{m}_4}$ will cross 0 to $Q_{\bar{m}_4} > 0$ at $t_{\bar{m}_4}$, and crosses 0 only once. Thus at a certain time, $Q_{\bar{m}_4}$ meeting 0 is required, as pointed out first by Cai et.al [39], and also by Creminelli et.al [40].

We also clarify that in the bounce model with $\gamma < 0$ initially, $c_s^2 < 0$ will occur in the phase with $\gamma \approx 0$ and $\dot{\gamma} > 0$, while the NEC is violated when $\dot{H} > 0$ (bounce phase), these two phases do not necessarily coincide. As pointed out by Ijjas and Steinhardt [35], it is the sign’s change of $\gamma$ that causes $c_s^2 < 0$. Here, we verify this point. In Genesis model [16][7], and also [21], the case is similar, as discussed in Sec. IV.

The nonsingular model with the solution (19) ($\gamma > 0$ throughout) has not been studied before. In Sec. III, we design such a model, in which a slow expansion phase (namely, the Genesis phase) is followed by slow-roll inflation. Under the unitary gauge, since $\dot{\gamma} > 0$ and $\gamma > 0$ (not crossing 0), the evolution of primordial perturbations can be simulated numerically. The simulation displays that the spectrum acquires a large-scale cutoff, as expected in Ref. [6].

We conclude that, based on EFT, not only a stable nonsingular cosmological scenario may be built without getting involved in unknown physics, but also the phenomenological possibilities of its implementation are far richer than expected (see also recent [50][51] for the higher spatial derivative operators).

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Appendix A: EFT of cosmological perturbations

With the ADM line element, we have

\[ g_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \vspace{10pt} \\
                        N_i & h_{ij} \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} -N^{-2} \frac{N^i}{N^2} & \frac{N^j}{N^2} \vspace{10pt} \\
                                      \frac{N^i}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}, \]  

(A1)

and \( \sqrt{-g} = N \sqrt{h} \), where \( N_i = h_{ij} N^j \). The unit one-form tangent vector is defined as \( n^\nu = n^0 (dt/dx^\mu) = (-N, 0, 0, 0) \) and \( n^\nu = g^{\mu\nu} n_\mu = (1/N, -N^i/N) \), which satisfies \( n_\mu n^\mu = -1 \).

On the hypersurface, the induced 3-dimensional metric is \( H_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \), thus

\[ H_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \vspace{10pt} \\
                        N_i & h_{ij} \end{pmatrix}, \quad H^{\mu\nu} = \begin{pmatrix} 0 & 0 \vspace{10pt} \\
                                      0 & h^{ij} \end{pmatrix}. \]  

(A2)

The extrinsic curvature is \( K_{\mu\nu} \equiv \frac{1}{2} \mathcal{L}_n H_{\mu\nu} \), where \( \mathcal{L}_n \) is the Lie derivative with respect to \( n^\mu \). The induced 3-dimensional Ricci scalar \( R^{(3)} \) associated with \( H_{\mu\nu} \) is

\[ R^{(3)} = R + K^2 - K_{\mu\nu} K^{\mu\nu} - 2\nabla_\mu (K n^{\nu} - n^{\nu} \nabla_\nu n^\mu). \]  

(A3)

Without higher-order spatial derivatives, the EFT reads [39]

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right. \right. \]
\[ + \frac{M_4^2(t)}{2} (\delta g^{00})^2 - \frac{m_3^2(t)}{2} \delta K \delta g^{00} - m_4^2(t) \left( \delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu} \right) \]
\[ + \left. \frac{\bar{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \right] + S_m[g_{\mu\nu}, \psi_m], \]  

(A4)

where \( \delta g^{00} = g^{00} + 1, \delta K_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu} H \) with \( H \) being the Hubble parameter. The coefficient set \((f, c, \Lambda, M_2, m_3, m_4, \bar{m}_4)\) specifies different theories and could be time-dependent in general 4. A particular subset \((m_4 = \bar{m}_4)\) of EFT (A4) is the Horndeski theory. \( S_m[g_{\mu\nu}, \psi_m] \) is the matter part, which is minimally coupled to the metric \( g_{\mu\nu} \).

To obtain the quadratic actions for scalar and tensor perturbations, we will work in the unitary gauge, thus we set

\[ h_{ij} = a^2 e^{2\xi} (e^\gamma)_{ij}, \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij}. \]  

(A5)

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4 Different conventions of the nomenclatures of these coefficients were adopted during the development of the EFT of cosmological perturbations (see e.g., [41][42][43][44]). Here, we follow the convention used in Refs. [43][44].
Then we follow the standard method first used by Maldacena [52], it is straightforward
(though tedious) to obtain the quadratic actions of scalar perturbation $\zeta$ and tensor pertur-
bation $\gamma_{ij}$, as exhibited in Eqs. (1) and (4), respectively (see [39] for detailed derivations).

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