THE EFFECT OF SYSTEMATICS ON POLARIZED SPECTRAL INDICES

I. K. WEHUSS1, U. FUSKELAND2, AND H. K. ERIKSEN2,3

1 Oxford Astrophysics, University of Oxford, DWB, Keble Road, Oxford OX1 3RH, UK; i.k.wehus@fys.uio.no
2 Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029, Blindern, N-0315 Oslo, Norway
3 Centre of Mathematics for Applications, University of Oslo, P.O. Box 1053, Blindern, N-0316 Oslo, Norway

Received 2012 August 22; accepted 2012 December 5; published 2013 January 17

ABSTRACT

We study four particularly bright polarized compact objects (Tau A, Vir A, 3C 273, and For A) in the 7 year Wilkinson Microwave Anisotropy Probe (WMAP) sky maps, with the goal of understanding potential systematics involved in the estimation of foreground spectral indices. First, we estimate the spectral index, the polarization angle, the polarization fraction, and the apparent size and shape of these objects when smoothed to a nominal resolution of 1° FWHM. Second, we compute the spectral index as a function of polarization orientation, α. Because these objects are approximately point sources with constant polarization angle, this function should be constant in the absence of systematics. However, for the K and Ka band WMAP data we find strong index variations for all four sources. For Tau A, we find a spectral index of \( \beta = -2.59 \pm 0.03 \) for \( \alpha = 30° \), and \( \beta = -2.03 \pm 0.01 \) for \( \alpha = 50° \). On the other hand, the spectral index between the Ka and Q bands is found to be stable. A simple elliptical Gaussian toy model with parameters matching those observed in Tau A reproduces the observed signal, and shows that the spectral index is particularly sensitive to the detector polarization angle. Based on these findings, we first conclude that estimation of spectral indices with the WMAP K band polarization data at 1° scales is not robust. Second, we note that these issues may be of concern for ground-based and sub-orbital experiments that use the WMAP polarization measurements of Tau A for calibration of gain and polarization angles.

Key words: cosmic background radiation – cosmology: observations – methods: statistical

Online-only material: color figures

1. INTRODUCTION

One of the central goals in contemporary observational cosmology is to detect the postulated background of primordial gravity waves predicted by inflation. The most direct observational signature of these gravity waves is a particular pattern in the polarization of the cosmic microwave background (CMB) known as B-modes. The amplitude of these gravity waves is typically parameterized in terms of the tensor-to-scalar ratio, \( r \) (see, e.g., Liddle & Lyth 2000 and references therein for a thorough review on inflation). During the last few years, many experiments have been planned, built, and fielded to measure \( r \), and the first relevant B-mode constraints have been already published by BICEP (\( r < 0.7 \); Chiang et al. 2010) and QUIET (\( r < 2.1 \); QUIET 2011, 2012). Other ground-based and sub-orbital experiments are expected to vastly improve on these limits in the very near future.

In order to make an actual detection of the inflationary gravity waves, it is widely believed that a sensitivity of \( r \lesssim 0.01 \) will be required. In terms of map-domain sensitivity, this corresponds to a signal with an rms of a few tens of nK. Thus, not only will exquisitely sensitive detectors be needed, but also detectors with extremely low systematics.

However, the single most problematic systematic for future B-mode experiments is likely not to come from the instrument itself, but rather from the sky: non-cosmological Galactic and extragalactic foregrounds, for instance synchrotron and thermal dust, radiate with a temperature of several microkelvin on large angular scales in the frequencies relevant for CMB measurements (e.g., Gold et al. 2011 and references therein). Therefore, in order not to be foreground-dominated, these foregrounds must very likely be suppressed by perhaps an order of magnitude or more. The only way to achieve this is by making multifrequency observations of the same fields of the sky, and exploit the different frequency dependency of the various components to separate out the cosmological CMB signal from the non-cosmological foregrounds.

As of today, a very large fraction of the information we have about polarized foregrounds on large angular scales comes from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment, and in particular the lowest frequency channel at 23 GHz (K band). This map is routinely used both for studies of foregrounds themselves and as ancillary data for other experiments. It is therefore critical to understand the systematic limitations inherent in these data. In this paper we measure the spectral indices of four particularly bright compact objects (Tau A, Vir A, 3C 273, and For A), with the goal of understanding some of the issues involved in spectral index estimation for CMB data in general: by considering high signal-to-noise objects with known properties, we have a clear a priori prediction, and deviations from these expectations would indicate either model problems or systematic errors.

2. DATA AND MODEL

Sky maps and processing. In this paper we consider the 7 year WMAP sky maps (Jarosik et al. 2011), coadded over years and pixelized at a HEALPix\(^4\) resolution of \( N_{\text{side}} = 512 \), corresponding to 7’ pixels. These data are available from LAMBDA,\(^5\) including all necessary ancillary data, such as beam profiles and noise model. Most of our analysis is performed with the K and Ka band data, although in one particular case we also consider the Q band data. All analyses are carried out in antenna

\(^4\) http://healpix.jpl.nasa.gov
\(^5\) http://lambda.gsfc.nasa.gov
temperature units, and given that we will consider objects with steep synchrotron-like spectra we adopt effective frequencies of 22.45 GHz ($K$ band), 32.64 GHz ($Ka$ band), and 40.50 GHz ($Q$ band), respectively (Page et al. 2003).

Before one can estimate spectral indices across frequencies, it is necessary to bring all maps to a common angular resolution. We therefore smooth all maps to an effective resolution of 1° FWHM by first deconvolving the instrument beam and then convolving with a Gaussian beam of the desired size. Note that the smoothing scale of 1° is a particularly common value adopted in the literature, and the results presented here are therefore of wide interest.

The estimation of uncertainties for all scalar quantities is done by forward Monte Carlo simulations. That is, we add smooth noise realizations to the actual WMAP data based on the provided noise model, evaluate each statistic for each simulation, and then compute the resulting standard deviation over the ensemble. Although there already is a noise component present in the WMAP data, this is identical for all simulations, and therefore does not contribute to the variance. We emphasize, though, that uncertainties estimated in this manner are only statistical in nature and do not account for systematic errors.

**Data selection.** In this paper we consider the four particularly bright point sources listed in Table 1. These were selected by thresholding the $K$ band polarization map, $P = \sqrt{Q^2 + U^2}$, at 100 $\mu$K, and discarding all regions that either show obviously extended features or have a strong background. This left us with Tau A as the only near-Galactic source, and three high-latitude sources (Vir A, 3C 273, and For A). For further details on the polarization properties of these objects, see, e.g., Aumont et al. (2010), Weiland et al. (2011), Fomalont et al. (1989), Ekers et al. (1983), and Rottmann et al. (1996).

Only pixels in a 1° radius disk around each source were kept for analysis, although we also tried 2° disks, obtaining consistent, but slightly more noisy, results. Note that a Gaussian beam of 1° FWHM ($\sim 2.35 \sigma$) has dropped off to 6% of its peak value at a distance of 1°, and most of the volume is therefore contained within this radius.

**Data model.** The low-frequency WMAP polarization observations are strongly dominated by synchrotron emission which has a sharply falling spectrum. We therefore approximate the total sky signal by a single power law, resulting in the following data model:

$$d_p = B_a \left( \frac{\nu}{\nu_0} \right)^\beta + n_p. \quad (1)$$

Here $d_p$ is an $N_{\text{pix}} \times 3$ matrix listing the temperature, $T$, and the Stokes $Q$ and $U$ parameters for all relevant pixels at frequency $\nu$ column wise, $B_a$ is a $3N_{\text{pix}} \times 3N_{\text{pix}}$ matrix denoting convolution with the common instrumental beam, $a$ denotes the true sky signal amplitude as measured at a reference frequency $\nu_0$, and $n_p$ is (smoothed) instrumental noise. All values are defined in antenna temperature units. Coordinates are defined according to the HEALPix convention (Górski et al. 2005).

### Table 1

| Object  | Longitude (deg) | Latitude (deg) | Size (arcmin) | FWHM (deg) | Ellipticity | Orientation (deg) |
|---------|-----------------|----------------|---------------|------------|-------------|-------------------|
| Tau A   | 184.56          | -5.78          | 7 $\times$ 5  | 0.985 $\pm$ 0.001 | 0.992 $\pm$ 0.001 | 0.142 $\pm$ 0.001 | 0.079 $\pm$ 0.001 | 54.0 $\pm$ 0.3 | 57.8 $\pm$ 1.5 |
| Vir A   | 283.78          | 74.49          | 8 $\times$ 6  | 0.91 $\pm$ 0.02 | 0.89 $\pm$ 0.05 | 0.17 $\pm$ 0.04 | 0.13 $\pm$ 0.06 | 76 $\pm$ 9 | 56 $\pm$ 33 |
| 3C 273  | 289.95          | 64.36          | <1            | 1.05 $\pm$ 0.02 | 0.88 $\pm$ 0.05 | 0.14 $\pm$ 0.03 | 0.22 $\pm$ 0.07 | 117 $\pm$ 7 | 97 $\pm$ 15 |
| For A   | 240.16          | -56.69         | 12 $\times$ 9 | 1.09 $\pm$ 0.02 | 1.09 $\pm$ 0.04 | 0.24 $\pm$ 0.02 | 0.24 $\pm$ 0.07 | 9 $\pm$ 3 | 5 $\pm$ 60 |

Notes. These beam parameters are derived in the coordinate system defined by the polarization angle of the respective source. Only statistical errors are included in the uncertainties, not systematic errors.

### 3. METHODS

Given the data and model described in Section 2, we estimate the polarization angle and fraction, the spectral index, and the apparent shape of each source. First, we note that each of the four objects considered here is well known in the literature, and has known polarization properties. Further, they are all known to be much smaller than 1° in angular dimensions (see Table 1 for precise details), and their polarization angles are known to be quite stable as a function of frequency (e.g., Weiland et al. 2011; Fomalont et al. 1989; Ekers et al. 1983; Rottmann et al. 1996 and references therein). For example, the polarization angle of Tau A is known to vary by only a few degrees over more than 10 decades in frequency (e.g., Aumont et al. 2010). We therefore assume that there is no real substructure within each source on the scales we consider.

**Polarization angle and fraction.** Since our objects effectively are point sources with constant polarization angle, there should (ideally) be a single well-defined coordinate system in which all signal is aligned with the Stokes $Q$ parameter. We search for this direction, $\alpha$, by minimizing the signal in the corresponding $U$ parameter,

$$\chi^2(\alpha) = \sum_p \frac{(-Q_p \sin 2\alpha + U_p \cos 2\alpha)^2}{\sigma} \quad (2)$$

where $Q_p$ and $U_p$ are the Stokes parameters in Galactic coordinates, and $\sigma$ is the noise level. Having rotated the data into the intrinsic polarization direction of the source, the polarization fraction is found simply by $\Pi = (Q(\alpha)/T)$.

**Observed ellipticity and FWHM.** Although we smooth the data to a common angular resolution, and therefore should expect the observations to have the desired FWHM, this is not true in practice due to beam asymmetries (Mitra et al. 2011; Wehus et al. 2009). To study the effective beam as a function of Stokes parameters, we rotate the original map by a rotation angle $\alpha$ into a new coordinate system $Q(\alpha) = Q \cos(2\alpha) + U \sin(2\alpha)$, and consider all angles between 0° and 90° in steps of 5°. Then, in this new coordinate system we fit an elliptical Gaussian, $g(Q_0, \text{FWHM}, \epsilon, \psi)$, to the $Q$ signal by minimizing

$$\chi^2 = \sum_p \left( \frac{Q_p(\alpha) - g(Q_0, \text{FWHM}, \epsilon, \psi)}{\sigma} \right)^2 \quad (3)$$

where $Q_0$ is the source amplitude, $\epsilon$ is the ellipticity, and $\psi$ is the direction of the semimajor axis. (Note that it is sufficient...
is given as insensitivity to absolute offsets in the data. The spectral index $\alpha$ all angles $Q$ to consider only the $K_a$ component, because we rotate through all angles $\alpha$. Thus, $\alpha = 45^\circ$ corresponds to $U$ in the original system.

Spectral indices. Finally, we estimate spectral indices for both $Q(\alpha)$ and $U(\alpha)$ using a standard temperature–temperature (TT) scatter plot approach. For a single pixel with noiseless data, this approach is simply defined by

$$
\frac{d_{Q_i}(p)}{d_{U_i}(p)} = \left( \frac{v_1}{v_2} \right)^{\beta} \Rightarrow \beta = \frac{\log[\frac{d_{Q_i}(p)}{d_{U_i}(p)}]}{\log[\frac{v_1}{v_2}]}.
$$

However, for multiple noisy observations more robust results are obtained by fitting a straight line, $y = ax + b$, to $d_{Q_i}$ as a function of $d_{U_i}$. An additional advantage of this method is its insensitivity to absolute offsets in the data. The spectral index is given as $\beta = \log a / \log(v_1/v_2)$. Since both $d_{Q_i}$ and $d_{U_i}$ have measured uncertainties, we adopt a variation on the method of least-squares (Petrolini 2011) that takes uncertainties in each variable into account when making the linear fit. As in the case of the beam parameters, we also compute the spectral index as a function of polarization angle.

4. RESULTS

In Table 1 we list the position and apparent (beam-convolved) size and shape of each of the four objects under consideration. The polarization fraction and angles, and spectral indices are tabulated in Table 2. Images of Tau A are shown in Figure 1, both for $K$ band (left column) and $K_a$ band (right column), and for Stokes $Q$ and $U$ parameters. In order to highlight the beam differences between these cases, we have first adopted a coordinate system which is offset by 22.5 from the intrinsic polarization direction of Tau A. This ensures a significant signal-to-noise in both $Q$ and $U$. Second, the color scale is tuned to highlight the tails of the instrumental beam, and scaled properly.

Table 2

| Object | Polarization Fraction | Polarization Angle (deg) | Spectral Index |
|--------|-----------------------|--------------------------|---------------|
|        | $K$ | $K_a$ | $K$ | $K_a$ | $\beta_T$ | $\beta_P$ |
| Tau A  | 6.17 ± 0.01 | 6.48 ± 0.04 | 88.43 ± 0.03 | 87.6 ± 0.1 | -2.280 ± 0.001 | -2.33 ± 0.01 |
| Vir A  | 3.4 ± 0.1 | 5.1 ± 0.8 | -27 ± 1 | -24 ± 4 | -2.62 ± 0.01 | -2.5 ± 0.3 |
| 3C 273 | 5.8 ± 0.2 | 4.4 ± 0.6 | 52.7 ± 0.6 | 44 ± 2 | -2.27 ± 0.01 | -2.8 ± 0.2 |
| For A  | 6.7 ± 0.2 | 6.7 ± 0.5 | -2.6 ± 0.7 | -5 ± 2 | -2.90 ± 0.02 | -2.6 ± 0.2 |

Notes. Uncertainties on polarization fraction and angles include only statistical errors; uncertainties on spectral indices additionally include an estimate of systematic errors.
bands; the red points (for Tau A only) are computed from the $Ka$ bands. The black points are computed from the $K$ bands; and the red points (for Tau A only) are computed from the $Ka$ and $Q$ bands. (A color version of this figure is available in the online journal.)

![Figure 4.](image_url) **Figure 4.** $K$ vs. $Ka$ band scatter plots for Tau A for three different polarization orientations. Note that the slopes are different for each direction, corresponding directly to different effective spectral indices. Further, there is no sign of either instrumental noise or background, demonstrating that the results are highly robust against such effects. (A color version of this figure is available in the online journal.)

between the two frequencies taking into account the spectral index of Tau A.

The main results of this paper are shown in Figures 2 and 3. The first figure shows the beam parameters for Tau A as a function of polarization orientation, and the second shows the spectral index as a function of polarization direction for all four sources. In the latter, the black points indicate the spectral index computed from $K$ and $Ka$ bands, and (for Tau A only) the red points show the spectral index between $Ka$ and $Q$ bands. Figure 4 shows a subset of the Tau A TT plots that are used for the $K - Ka$ calculations, corresponding to $\alpha = 30^\circ$, $50^\circ$, and $75^\circ$, respectively.

As seen from the results shown in Figure 3, the polarized spectral index as measured by WMAP between $K$ and $Ka$ bands at $1^\circ$ angular scale depends strongly on the coordinate system in which the index is computed. For Tau A, the derived index varies between, say, $\beta = -2.6$ for a rotation angle of $\alpha = 30^\circ$ and $\beta = -2.0$ for $\alpha = 50^\circ$. This effect is statistically highly significant, and it is robust with respect to instrumental noise and background levels (see Figure 4). It therefore indicates the presence of a real systematic effect not taken into account in the present analysis.

To understand these structures in greater detail, we construct an elliptical Gaussian model of Tau A based on the parameters listed in Tables 1 and 2 at $K$ and $Ka$ bands, and estimate the spectral index from the resulting noiseless model, as for the real data. The results are shown in Figure 5. Clearly, the model faithfully reproduces the observed structures. The only difference is a slight vertical offset, which is due to the fact that the measured Tau A parameters. For the black curve we have used the same beam and polarization angle parameters for $Ka$ as for $K$, corresponding to an ideal instrument. For the other curves we change one parameter at a time to the measured $Ka$ value. (A color version of this figure is available in the online journal.)

![Figure 5.](image_url) **Figure 5.** Comparison of the observed (points) and simulated (dashed line) spectral index for Tau A. The simulation is based on a noiseless point source observed with the same beam and polarization parameters as measured for Tau A. (See Tables 1 and 2 for full specification.)

![Figure 6.](image_url) **Figure 6.** Effect of various systematics on the measured spectral index of Tau A between $K$ and $Ka$ bands. The simulated $K$ band map is always based on the measured Tau A parameters. For the black curve we have used the same beam and polarization angle parameters for $Ka$ as for $K$, corresponding to an ideal instrument. For the other curves we change one parameter at a time to the measured $Ka$ value.

(A color version of this figure is available in the online journal.)

We can now use this model to understand the relative importance of the various systematic effects. To do so, we start out with an ideal model, adopting the observed $K$ band parameters also for $Ka$ band, and set the $Ka$ parameters one-by-one to their true values. The results from this exercise are shown in Figure 6. Here we see that the most important systematic by
far is the detector angle, and this effect alone reproduces the signal seen in Figure 5 very well. The second most important effect is the beam ellipticity, which is at least three to four times smaller than the detector angle effect over most of the well-sampled regions of the polarization orientation. Other effects are small compared to these two.

5. CONCLUSIONS

We have studied four particularly bright polarized point sources in the 7 year WMAP data, with the goal of understanding the effect of systematics on polarized spectral index estimation. This topic is important for at least two reasons. First, the WMAP polarization sky maps represent the best currently available full-sky measurements of the polarized foregrounds at CMB frequencies. As a result, they play a critical role in the analysis and optimization of existing and future B-mode experiments. Second, many ground-based and sub-orbital experiments use the WMAP polarization measurements of Tau A directly as a calibration source for both detector angles and absolute gain.

In this paper, we have found that the observed polarized spectral index of the relevant sources depends sensitively on the coordinate system in which the index is estimated. For example, the spectral index of Tau A is \( \beta = -2.59 \pm 0.03 \) for a coordinate system rotated by 30° relative to the intrinsic polarization direction of the source, while it is \( \beta = -2.03 \pm 0.01 \) in a coordinate system rotated by 50°. The most significant contributor to this effect is the slightly different polarization angles of the K and Ka band detectors, with some smaller contribution coming from beam asymmetries. Experiments that, directly or indirectly, use the K and Ka band measurements of Tau A as a calibrator source should take into account these systematic uncertainties when performing their analyses.

Finally, we note that the test described in this paper is very simple to implement, only takes a few CPU seconds to run, and has a very direct and intuitive interpretation. We therefore expect other experiments to find it useful as a test of their own systematics, in particular when applied to Tau A.

This project was supported by the ERC Starting Grant StG2010-257080 and a Leverhulme visiting professorship for H.K.E. I.K.W. acknowledges support from ERC grant 259505. Some of the results in this paper have been derived using the HEALPix (Górski et al. 2005) software and analysis package.

REFERENCES

Aumont, J., Conversi, L., Thum, C., et al. 2010, A&A, 514, A70
Chiang, H. C., Ade, P. A. R., Barkats, D., et al. 2010, ApJ, 711, 1123
Ekers, R. D., Goss, W. M., Wellington, K. J., et al. 1983, A&A, 127, 361
Fomalont, E. B., Ebner, K. A., van Breugel, W. J. M., & Ekers, R. D. 1989, ApJ, 346, 17
Gold, B., Odegard, N., Weiland, J. L., et al. 2011, ApJS, 192, 15
Górski, K. M., Hivon, E., Banday, A. J., et al. 2005, ApJ, 622, 759
Jarosik, N., Bennett, C. L., Dunkley, J., et al. 2011, ApJS, 192, 14
Liddle, A. R., & Lyth, D. H. 2000, in Cosmological Inflation and Large-Scale Structure, ed. A. R. Liddle & D. H. Lyth (Cambridge: Cambridge Univ. Press), 414
Mitra, S., Rocha, G., Górski, K. M., et al. 2011, ApJS, 193, 5
Page, L., Barnes, C., Hinshaw, G., et al. 2003, ApJS, 148, 39
Petrolini, A. 2011, arXiv:1104.3132
QUIET Collaboration 2011, ApJ, 744, 111
QUIET Collaboration 2012, ApJ, 760, 145
Rottmann, H., Mack, K.-H., Klein, U., & Wielebinski, R. 1996, A&A, 309, L19
Wehus, I. K., Ackerman, L., Eriksen, H. K., & Groeneboom, N. E. 2009, ApJ, 707, 343
Weiland, J. L., Odegard, N., Hill, R. S., et al. 2011, ApJS, 192, 19