On the resolution limits of tunnel magnetoresistance sensors for particle detection

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Abstract. Elliptical magnetoresistance sensors are analyzed with respect to their capability of high spatial resolution of particle detection. The sensor response with respect to the particle position is investigated by solving the equation of static micromagnetism. We demonstrate how the shape of the sensor can be used to obtain information about the particle position. It is verified that the external fields applied to bring particles into saturation can be utilized to tune the measurement: increasing the external fields will increase the spatial resolution of the sensor but will also reduce the visibility field in which a particle can still be detected.

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1. Introduction

Magnetic particles have been thoroughly studied over recent decades because of their many promising applications. Due to their magnetic moment they can be manipulated by inhomogeneous external magnetic fields [1]–[3] enabling the possibility to guide them in e.g. microfluidic systems by electric [4, 5] or magnetic components [6]. Further, their magnetic stray field influences nearby magnetic material, giving rise to a way to detect them. Magnetoresistance sensors have been employed for the detection of magnetic particles in various applications in recent years [7]–[9]. The detection of moving particles in continuous flow devices was recently demonstrated by Loureiro et al [10] using spin valve sensors. Wang and Li [11] predicted a linear dependence of the obtained sensor signal with respect to the bead coverage, integrating the magnetic particle stray field components along the sensor area. However, in these situations it needs to be ensured that the dimension of the sensor is far larger than the size scale of the particles to be detected [12, 13]. Therefore, this configuration cannot be used in high spatial resolution detection devices to give additional information on the position of the particles. In previous work [14], elliptically shaped sensor elements with axis lengths of 400 and 100 nm were investigated. These samples consist of two ferromagnetic CoFeB-layers separated by an insulating MgO-barrier. The lower CoFeB-electrode is pinned via an artificial antiferromagnet along the short semiaxis. Due to high shape anisotropy the equilibrium state of the free upper layer is perpendicular to the bottom electrode. Such a configuration shows a linear relation between the tunnel magnetoresistance (TMR) ratio and an applied field along the short ellipse axis over a wide field range \( H \sim \pm 40 \text{ kA m}^{-1} \). This allows application of magnetic fields to bring particles close to saturation. In this work, we analyze the TMR-response of such a system with respect to the position of a magnetic particle. The main question is the spatial resolution limit, i.e. how exactly is a sensor capable of predicting the position of the particle? Further, we will discuss, how different measurements can be combined to retrieve more information dependent on the signal.

2. Theoretical model

The modeling of this system is schematically shown in figure 1: two ferromagnetic layers of thickness 4 nm are separated from each other by a tunneling barrier of thickness 2 nm. Since the magnetization distribution in the lower electrode is fixed, it can be written as \( \hat{m}_2 = M_S \hat{y} \) with \( M_S = 1194 \text{ kA m}^{-1} \) [15], the saturation magnetization of CoFeB. The configuration within the upper electrode can be calculated from the equation of static micromagnetism [16]

\[
\hat{m}_1 \times H_{\text{eff}} = 0
\]

with

\[
H_{\text{eff}} = \frac{2A}{\mu_0 M_S} (\nabla \hat{m}_1)^2 - \frac{\delta f_{\text{ani}}(\hat{m}_1)}{\delta \hat{m}_1} + H_{\text{demag}} + H_{\text{ext}}.
\]

The first summand in (2) corresponds to exchange energy within the magnetic material. The exchange constant \( A \) gives a measure for its strength; in the case of CoFeB a value of \( A = 2.86 \times 10^{-11} \text{ J m}^{-1} \) was reported [15]. High exchange contribution leads to a parallel alignment of the magnetization distribution or, more exactly, a small curvature of the components of the unit vector \( \hat{m}_1 \). Typical size scales on which changes of components of \( \hat{m}_1 \) can be observed...
Figure 1. (a) Schematic representation of the layer system with magnetic particles (dimensions shown do not match the actual sizes; the particles discussed have a diameter of 1 \( \mu \)m and are therefore larger than the sensor itself) and (b) equilibrium magnetic configuration without a particle stray field but with a homogenous field parallel to \( y \)-axis of 16 kA m\(^{-1}\). Due to layer coupling the magnetization direction in the free layer is not perfectly aligned with the geometrical easy axis. (c) Magnetization curve of the magnetic multicore particles MyOne\textsuperscript{TM} of a saturation magnetization of \( M_S = 120 \) kA m\(^{-1}\) and radius \( R = 0.5 \). (d) and (e) show the resulting in-plane magnetic fields of a particle at a distance of 0.562 \( \mu \)m with a magnetization perpendicular to the sensor plane and parallel to the \( x \)-axis, respectively.

exceed the thickness of the magnetic films by several magnitudes. If we choose Cartesian coordinates as shown in figure 1 (\( z \)-axis perpendicular to the plane of the magnetic film), we can therefore simplify the spatial dependency of \( \hat{m}_1 \) to \( \hat{m}_1(x, y, z) = \hat{m}_1(x, y) \). The second summand in (2) refers to magneto-crystalline anisotropy; the energy functional \( f_{an} \) varies depending on the type of anisotropy discussed. \( H_{demag} \) and \( H_{ext} \) denote the demagnetization field of the upper layer itself and the external field, respectively. The latter decomposes into the particle stray field \( H_{part} \) (figure 1(d) and (e)), the stray field of the lower electrode and a homogeneous contribution which is applied to bring the magnetic particles into saturation (compare figure 1(c)). Additional to the stray field coupling, the two magnetic layers interact by
Néel-coupling \[17\]. This effect is due to a correlated surface roughness. Assuming a sinusoidal surface structure of period length \( \lambda \) and amplitude \( h \) and denoting by \( \langle \cdot, \cdot \rangle \) the Euclidean inner product, this is given by

\[
J_{\text{Néel}} = \frac{\mu_0 \pi^2 h^2}{\sqrt{2} \lambda} M_S^2 \exp \left( -\frac{2\pi \sqrt{d}}{\lambda} \right) \cdot \langle \hat{m}_1, \hat{m}_2 \rangle.
\] (3)

The system parameters \( \lambda \) and \( h \) can be determined from atomic force microscopy measurements. Typical values are \( \lambda = 30 \text{ nm} \) and \( h = 3 \text{ Å} \) which will also be chosen for the simulations \[18\]. To integrate equation (3) into (1) and (2), the variational derivative \( \delta J_{\text{Néel}} / \delta \hat{m}_1 \) has to be calculated.

Magnetic particles close to the sensor create a stray field \( H_{\text{part}} \) (figures 1(d) and (e)) influencing the magnetization distribution in the soft layer. Considering a homogenously magnetized spherical particle of magnetic moment \( m_{\text{part}} \) and center position \( r_{\text{part}} \), its stray field at \( r \) is given by the dipolar expression \[19\]

\[
H_{\text{part}}(r) = \frac{1}{4\pi} \cdot \frac{3 \langle m_{\text{part}}, \Delta r \rangle \cdot \Delta r - m_{\text{part}}}{|\Delta r|^5},
\] (4)

with \( \Delta r = r - r_{\text{part}} \). The magnetic moments of the particles depend on the applied magnetic field. In the calculations below magnetic MyOne\textsuperscript{TM} [20] are considered. These are magnetic multi-core particles of radius \( R = 0.5 \text{ \mu m} \) and a saturation magnetization \( M_S = 120 \text{ kA m}^{-1} \). The degree of saturation in respect to the applied field is shown in figure 1(c) and has been obtained by alternating gradient magnetometer measurements.

Equations (1)–(3) with a single particle at different positions \( r_{\text{part}} \) are solved by the finite-element method. As an initial configuration for \( \hat{m}_1 \), we choose the equilibrium state obtained as a solution of (1)–(3) with only the homogenous field applied. An example is shown in figure 1(b); it should be pointed that the magnetization is not perfectly aligned with the easy geometrical axis which is due to the coupling between the two magnetic layers. The TMR response is calculated along a grid with equidistant grid coordinates at the positions

\[
x_{\text{part}} = -1.5 \text{ \mu m} + (i - 1)0.2 \text{ \mu m}, \quad i = 1, \ldots, 16,
\]

\[
y_{\text{part}} = -1.5 \text{ \mu m} + (j - 1)0.1 \text{ \mu m}, \quad j = 1, \ldots, 31,
\]

\[
z_{\text{part}} = 0.562 \text{ \mu m}.
\] (5)

The \( z \)-coordinate \( z_{\text{part}} \) is chosen according to previous work \[13\] and decomposes into the particle radius \( R \) and an additional height value due to the topology of the sensor surface. The TMR ratio can be calculated from the solution for \( \hat{m}_1 \) by the following formula \[21\] evaluating the average angle between the magnetization vectors \( \hat{m}_1 \) and \( \hat{m}_2 \) in the top and bottom electrode, respectively

\[
\text{TMR} = p_m \frac{1 - \langle \delta m \rangle}{1 + p_m \langle \delta m \rangle}, \quad \text{with} \quad p_m = \frac{\max(\text{TMR})}{2 + \max(\text{TMR})}
\]

and

\[
\langle \delta m \rangle = \frac{1}{A_{\text{layer}}} \int_{A_{\text{layer}}} \langle \hat{m}_1, \hat{m}_2 \rangle \, dr.
\] (6)

\( \langle \delta m \rangle \) gives the average angle between the magnetization in the sensing free top electrode and the pinned bottom layer. Since the magnetization is constant in \( z \)-direction, the integration

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**Figure 2.** TMR-map for a homogeneous magnetic field of $16 \text{ kA m}^{-1}$ along the $y$-axis. Crossings of black lines correspond to grid nodes; the origin of the coordinate system coincides with the centre of the sensor (white ellipse). The pattern shows symmetries, though the symmetry axes do not coincide with the semiaxes of the ellipse but are turned by an angle $\alpha$ due to the coupling (stray field /Néel) of the electrodes.

can be restricted to any area cross-section parallel to the $x$–$y$-plane; $A_{\text{layer}}$ denotes the area of this cross-section. The parameter \(\max(\text{TMR})\) denotes the maximum TMR-value which is obtained for antiparallel magnetization alignment. For the sake of simplicity, it shall be set to \(\max(\text{TMR}) = 1\) in the following discussions. An example of a TMR-map for an external field $H_y = 16 \text{ kA m}^{-1}$ is presented in figure 2; the discrete data points have been extended to a surface by linear interpolation between the grid nodes. The response map shows a maximum value $\text{TMR}_{\max}$ close to the sensor position and two local minima $\text{TMR}_{\min}$ some distance from it. The response is symmetric to two axes, which are obtained by a rotation of the ellipse axes of an angle $\alpha$. This rotation originates from the coupling of the two magnetic layers. The sign and size of $\alpha$ depend on the interplay of the two coupling effects mentioned above: while Néel-coupling favors parallel alignment, the stray field interaction between the two layers leads to an antiparallel configuration. For sensors of area smaller than 1 $\mu\text{m}^2$, the stray field interaction dominates [22]. However, simulations show that $\alpha$ also depends on the external field applied (cf figure 4). This rotation of the symmetry axes is important to take into account if a high spatial resolution needs to be achieved.

**3. Simulation results**

As already mentioned in the introduction, the investigated system shows a linear relation between measured TMR ratio and applied field along the short ($y$-)axis of the ellipse [14]; we will therefore focus on external fields parallel to this axis for the moment. Similar calculations as presented in figure 2 are carried out for field values of $H_y = 8 \text{ kA m}^{-1}$, $24 \text{ kA m}^{-1}$, $40 \text{ kA m}^{-1}$ and $56 \text{ kA m}^{-1}$. The particle magnetization is chosen according to figure 1(c) and given by 0.31,
Figure 3. \( \Delta \text{TMR} \)-maps at the \( x \)- and \( y \)-positions given by (5): (a) at height \( z_{\text{part}} = 0.562 \, \mu m \) and \( H \) of 8, 24, 40 and 56 kA m\(^{-1} \) parallel to the \( y \)-axis and (b) at heights \( z_{\text{part}} = 0.562, 0.75, 1, 1.25 \) and 2 \( \mu m \) for \( H \) of 16 kA m\(^{-1} \) parallel to the \( y \)-axis. The two-dimensional plots present cross-sections of the surfaces along \( x = 0 \).

0.57, 0.69 and 0.76 times the saturation value, respectively. To ease the comparison between different maps, we introduce a relative ratio \( \Delta \text{TMR} \) corresponding to the normalized relative resistance change. Denoting by \( \text{TMR}_{\text{part}} \) and \( \text{TMR}_{\text{stack}} \) the TMR-ratios with and without bead, respectively, this is given by [14]

\[
\Delta \text{TMR} = \frac{\text{TMR}_{\text{part}} - \text{TMR}_{\text{stack}}}{\text{TMR}_{\text{stack}} + 1}.
\] (7)

The results are shown in figure 3(a). An increasing external field value leads to an increasing change effect at the centre of the sensor. The measurable \( \text{TMR}_{\text{part}} \)-values, however, decrease as shown in figure 4. Therefore, high fields increase the resolution of the sensor, but decrease the area in which a particle can be detected. This is similar to the behavior of optical lenses.
Figure 4. Sensor characteristics for different external fields along the $y$-axis. For high fields the maximum and minimum measurable TMR$_{\text{part}}$-values decrease. The angle $\alpha$ increases and reaches zero close to $H_y = 56 \text{ kA m}^{-1}$.

If a critical field value is exceeded, the response of the sensor changes (figure 3(a), $H_y = 56 \text{ kA m}^{-1}$); the maximum $\Delta$TMR-value does not increase any further, but decreases together with TMR$_{\text{part}}$. An analysis of the angle values shows that this point coincides with $\alpha = 0$ (figure 4). This behavior is due to two reasons: while the external magnetic field increases linearly, the particle moment reaches saturation (figure 1(c)) and therefore, the external field will overcome the particle influence at a certain point. On the other hand, the sensing free top layer reaches saturation and becomes less sensitive. Thus, it should be pointed out that ideal detection conditions can be obtained by adjusting the external magnetic field to the particles to be detected and the sensor chosen for the detection.

Additionally, figure 3(b) shows TMR-maps for particles magnetized by an external field of $16 \text{ kA m}^{-1}$ parallel to the $y$-axis for different particle heights. The response of the sensor decreases rapidly with distance. To ensure detection a higher field value according to figure 3(a) might be essential for experimental measurements. In the following, we will always assume a particle very close to the sensor and set $z_{\text{part}} = 0.562 \mu\text{m}$ for all following discussions.

The obtained TMR-maps can be used to estimate the position of the magnetic particle. Here, we will focus on the distance $d$

$$d = \sqrt{x_{\text{part}}^2 + y_{\text{part}}^2}, \quad z_{\text{part}} = 0.562 \mu\text{m} \quad (8)$$

between the particle and the ellipse center of the top electrode. As can be seen in figure 2 a certain TMR-value corresponds to several particle positions and can therefore only give an upper and a lower bound for the distance. To analyze these bounds, we divide the interval $[\text{TMR}_{\text{min}}, \text{TMR}_{\text{max}}]$ into $N$ equally sized sub-intervals $\{\Delta M R_i\}_{i=1,...,N}$. For our analysis the size of $\Delta M R_i$ is basically arbitrary; it only needs to introduce a proper class division of the data points along the discrete grid nodes; for the data analysis we choose $N = 100$. In the experimental situation, however, the minimal size of $\Delta M R_i$ will correspond to the achievable exactness of the measurement. The map is divided according to the assignment of each map point to the
Figure 5. Plots for upper and lower $d$-bounds obtained from the TMR-maps for external magnetic fields of $16 \text{ kA m}^{-1}$ along the positive coordinate axes. For a measured $\Delta \text{TMR}$-value, the distance between particle and sensor centre can be estimated according to the given bounds. For the given intervals, the estimation gaps indicate the range of $\Delta \text{TMR}$ where there is no estimation possible because not all corresponding values can be found on the grid chosen. Jumps in the TMR-surface can be attributed to a finite mesh resolution.

corresponding interval $\Delta M R_i$. This defines a relation between $\Delta M R_i$ and the distance $d$ of the regarded map point to the center of the sensor. A $\Delta M R_i$-$d$-plot is presented in figure 5 (blue markers). Due to the construction each $\Delta M R_i$-interval corresponds to a set of $d$-values. A single measurement can therefore only give an upper and a lower bound for the distance between particle and sensor; the two lines of data points show these bounds. At a given $\Delta \text{TMR}$-value each distance value in between is possible. However, if additional measuring directions are taken into account, further information can be obtained.

From our calculated data, we can estimate the degree of accuracy to which the distance $d$ between particle and sensor center can be determined by combining different measurement directions. As a simplification, we will assume that the particle is stationary during all measurements. As can be seen from figure 3(a), employing different field values along the $y$-axis will give rise to several difficulties: the $\Delta \text{TMR}$-response changes at every position in the same manner; additional information on the particle position can therefore only be caused by a varying angle $\alpha$. This effect, however, is hard to exploit since significant angle perturbations require fields lying in the range of particle and sensor saturation. This reduces the visibility field as already explained above by a strong amount leading to decreased spatial resolution.
Figure 6. Error estimates according to the simulation data of figures 3 and 5. (a) Upper and lower bound for each grid point; a particle at the given coordinate leads to a corresponding distance estimation. (b) The maximum error $\Delta d$ from the actual distance. The grey level coincides with the plane $\Delta d = 0.2 \mu m$. The subplot (c) shows a possible sensor array where the sensor positions are chosen according to (b) so that a detection with a precision of $\Delta d = 0.2 \mu m$ is achieved.

Instead, we combine bound estimations for applying external magnetic fields parallel to the $x$- and $z$-axes. Thus, we obtain upper and lower bounds $d_{up}^H$, $d_{down}^H$, $d_{up}^{Hz}$, $d_{down}^{Hz}$, respectively, according to figure 5. The best estimation for the distance $d$ is given by the interval

$$I = [d_{down}, d_{up}] = \left[ \max_{H_x, H_y, H_z} d_{down}^H, \min_{H_x, H_y, H_z} d_{up}^H \right],$$

(9)

where the minimum and the maximum need to be taken over all three measuring directions. For the evaluation only $\Delta TMR$-values are taken into account, which cannot be found along the boundary of the grid. Based on our calculation no estimation is possible for these; values are given in figure 5. The distance estimates for the combined detection, $d_{up}$ and $d_{down}$, are shown in figure 6(a). The upper and lower bounds almost coincide close to the sensor, while the interval size increases with increasing particle-sensor distance $d$. Certain features (two local maxima for the lower bound) can be found due to the combined measurement increasing the accuracy of the position detection. The maximum absolute error

$$\Delta d = \max_{\delta=\text{up, down}} |d - \delta|$$

(10)

is presented in figure 6(b). Along an area of a radius of about $0.6 \mu m$ around the sensor center, the device predicts a distance with an error less than $0.2 \mu m$.

In a similar manner, instead of the distance, detection methods to determine each single spatial coordinate can be performed. It should be pointed out that these calculations can be used
for the construction of sensor arrays with a predefined resolution. If e.g. a distance measurement below an error threshold of $\Delta d = 0.2 \, \mu m$ is to be realized on the base of the elliptical sensors discussed, the sensors should be arranged according to figure 6(c) in a hexagonal lattice structure with lattice parameters as shown.

4. Conclusion

We have investigated elliptic double-layer TMR-sensors with respect to their adaptability for high spatial resolution detection. Assuming a particle does not change its position during the detection process, we have shown that the distance along the sensor plane between the particle and the center of the sensor can be estimated up to within an error smaller than $0.2 \, \mu m$ in a region of diameter of $1.2 \, \mu m$ around the ellipse with semiaxes of 200 and 50 nm. The measuring principle can easily be extended to the determination of the spatial coordinates of the particle. It was also found that extensive knowledge of the system is necessary for high resolution. In detail, we found a rotation of the TMR-symmetry to the geometric symmetry due to layer coupling that cannot be omitted on the size scale of the sensors investigated here. It is further possible to adjust the measurement resolution by the strength of the applied external field. In general, high fields enable a high spatial resolution but only along a small visibility field. This observation can help to adjust the setup to different measuring tasks: if the question is to decide whether there is a particle at all somewhere in the range of the sensor, small fields should be applied. If additional information on its position is required, the measuring field needs to be increased. The limit for maximum applied fields is given by the limit of the linear response range.

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