Further dual purpose evolutionary optimization of small wind turbine blades

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Abstract. Much work has been done to maximise the power extraction of wind turbine blades. However, small wind turbines are also required to be self starting and whilst blades designed for maximum power extraction can be optimised analytically, these blades often have poor starting performance. The numeric method of Differential Evolution is used here to maximise for both power and starting performance. Standard blade element theory is used to calculate the power coefficient, and a modified blade element method for starting time. The chord and twist of each blade element make up the genes for evolution. Starting times can be improved by a factor of 20 with only a small reduction in power coefficient. With the introduction of the tip speed ratio as an additional gene, up to 10% improvement in power coefficient was achieved. A second study was done in another case where analytical optimisation is not possible; the inclusion of tip losses. The inclusion resulted in only a small increase in the optimum chord in the tip region which becomes less noticeable at lower tip speed ratios.

1. Introduction
It is rare that optimization leads to analytical equations for the optimized quantity. For wind turbines, one of these rare examples is the equations of Burton et al. [1] for the optimal chord, $c$, and twist, $\phi$, of a wind turbine of $N$ blades operating at the Lanchester-Betz limit:

$$cC_l = \frac{16\pi}{9N\lambda\left[4/9 + (X + 2/(9X))^3\right]^2} \quad (1)$$

and

$$\tan\phi = \frac{2}{3X + 2/X} \quad (2)$$

where $C_l$ is the lift coefficient at the maximum lift:drag angle for the particular aerofoil section, $\lambda$ is the tip speed ratio and $X$ is the local speed ratio at radius $r$: $X = r\lambda/R$, where $R$ is the tip radius. These equations, which were derived by ignoring the drag, are used routinely for the design of maximally efficient blades that extract the most energy from the wind.

However, experimental and computational studies of the starting process [2], have shown a major problem with the use of (1) & (2) for small wind turbines: the optimized blades are the slowest to start.
Furthermore, for the reasons given by Wood [3], good starting performance is generally more critical for small rather than large turbines. It is, therefore, necessary to develop methods to allow a dual optimization of small blade design by trading some power-producing capability against improvements in starting performance. This should be achievable because starting torque is generated mainly near the hub [2], whereas it is well known that most power-producing torque comes from the tip region.

Hampsey & Wood [4] and Hampsey [5] reviewed various evolutionary strategies for wind turbine blade design. Hampsey used a differential evolution (DE) method of optimizing for both starting time and power production of a nominally 5 kW blade using a boundary integral computation method. This method used B-splines to represent the blade surface, and does not divide the blades into elements. The population’s ‘genes’ were the control points and knots. The main disadvantage was that the viscous drag had to be approximated. Wood [3] used a DE strategy, from Price et al. [6], with a blade element formulation where the ‘genes’ were the chord and twist of each element. This paper uses a similar method to Wood, with the introduction of an extra ‘gene’, the power-producing tip speed ratio \( \lambda_p \). For small wind turbines this is the tip speed ratio the rotor operates at during power production, with frequency varying with wind speed.

The main assumption of the analysis of starting is the experimental finding that a long “idling time” dominates [2]. Hence starting time in this work refers to this idling time. During this time initially high angles of attack on the blades reduce slowly as the blade angular velocity increases slowly from rest, justifying a quasi-steady analysis. It is further assumed that the wind does not slow down as it passes through the rotor and that all the aerodynamic torque, minus the resistive torque of the drive-train and generator, acts to accelerate the blade. Thus the basic blade element formulation results in an ordinary differential equation for the blade angular velocity which can be solved by standard methods: the standard fourth-order Runge-Kutta method is used here. The long idling time also means that the rotational kinetic energy of blade at the end of the starting process, which is usually and arbitrarily taken as the angular velocity where the blade angle of attack has fallen below about 30°, is small compared to the kinetic energy of the wind that has passed through the rotor during that time. This provides some justification for ignoring any energy extracted during starting. Full details on the calculation of starting time can be found in Wood [3].

Starting involves aspect of blade design, principally the rotational inertia, and the system dynamics, principally the resistive torque, that are ignored in the standard power-producing optimization. This has important consequence for a dual optimization: simply increasing the blade chord will increase the blade element torque but will also increase its contribution to the inertia.

The power coefficient was calculated using standard blade element theory (BET), [1] with tip losses ignored initially as was done by Wood [3]. In the final section tip losses are included using a new correction factor introduced by Shen [7]:

\[
F_1 = \frac{2}{\pi} \cos^{-1} \left[ \exp \left( -g \frac{N(R-r)}{2rsin\phi} \right) \right]
\]

where \( g \) was given as:

\[
g = \exp[-0.125(N\lambda - 21)] + 0.1
\]

The following section describes the differential evolutionary strategy employed in the present study. The subsequent section details some computational results which are assessed partly by comparison to equations (1) & (2) above. The conclusions are given in the final section.

2. Differential Evolution for Optimisation

A range of evolutionary strategies have been applied to various aspects of blade optimization problems, differing primarily in their method of breeding a new population from the existing population. Usually the initial population is randomly generated and allowed to evolve over some
large number of generations. Through evaluating a measure of each blade’s fitness and keeping the best blades, the aim is achieving convergence.

In its normal form, [6], Differential Evolution (DE) generates a potential new member, or comparison vector, \( c_i \), for each member of the existing population, \( x_i \), by randomly swapping genes from \( x_i \) with genes from a trial vector, \( t_i \). This trial vector is constructed by the addition of a basis vector, \( b_i \), and a weighted difference vector as follows:

\[
t_i = b_i + w(u_i - l_i)
\]

where, \( w \) is the weighting factor and \( b_i, u_i \) and \( l_i \) are randomly chosen members of the current population. To decide whether to swap the gene or not, a random number, \( \text{rand} \), is compared to a crossover factor (\( CR \)) as follows:

\[
\text{FOR } j = 1, \text{number of genes} \quad \text{! for every gene}
\]

\[
\text{IF } (\text{rand} < \text{CR}) \quad \text{! swap gene with trial vector}
\]

\[
c_i(j) = t_i(j)
\]

\[
\text{ELSE} \quad \text{! swap gene with basis vector}
\]

\[
c_i(j) = b_i(j)
\]

Thus if the crossover condition is met for any element, the gene for that element is taken from the trial vector; otherwise the gene is taken from the basis vector. This method is slightly different to that set out by Price [6]; where if crossover was not met, the original gene was used for this element. However, this method did not lead to convergence to an optimal fitness front. Following Price, \( w \) was fixed at 0.8 and from the ranges of \( CR \) recommended by Price, \( CR = 0.1 \) was found to give the best results.

The fitness of \( c_i \) is determined by a single fitness score:

\[
\text{fitness}(c_i) = a \frac{C_p(c_i)}{\max(C_p)} + (1-a) \frac{\min(T_r)}{T_r(c_i)} \tag{5}
\]

where the factor, \( a \), is input by the user within the range \( 0 \leq a \leq 1 \). Equation (5) combines the two objective functions: the first is the standard power coefficient, \( C_p \) determined at a user-input wind speed, \( U_{\text{power}} \). The second is the inverse of the starting time, \( T_s \), required to reach a “final” tip speed ratio, \( \lambda_f \), at a “starting wind speed, \( U_{\text{starting}} \), both input by the user. The maximum \( C_p \) and the minimum \( T_s \) are determined each generation over the current population. Similarly the fitness of \( x_i \) is determined, and the vector with the highest fitness score survives to the next generation. The exception is when the vector reaches a maximum age, which was set to 20 generations. In this case \( c_i \) is always chosen.

Best results were obtained when the initial population was generated by assigning a constant random chord and a constant random pitch to each blade in the initial population.

3. Results
The parameters shown in table 1 were used for the initial blade optimization. These parameters are similar to [3] except that here the turbine had \( N = 3 \).

The results shown in this paper were obtained using 15 blade elements, each of which comprises a gene with two properties: chord and twist. Initial results are shown with \( \lambda_f = 10 \), and the expanded results are optimized with \( \lambda_f \) as the 16th gene in the range of 6-12. The population size was 2000, with a maximum blade age of 20 and the program was set to evolve over 200 generations. Typical run
times were just over 3 minutes on an AMD Athlon 3800+ running Matlab 7 and using the Microsoft C compiler for the mex executables.

### Table 1. Input parameters for initial blade optimisation

| Parameter                  | Value               |
|----------------------------|---------------------|
| $N$                        | $3$                 |
| $U_{\text{starting}}$      | $5 \text{ ms}^{-1}$ |
| $\lambda_f$                | $1.5$               |
| $R$                        | $1.5 \text{ m}$     |
| Maximum $c$                | $0.3$               |
| Minimum $c$                | $0.02$              |
| Moment of inertia of generator | $0 \text{kgm}^2$  |
| $\alpha$ in equation (5)  | $0.7 - 0.95$        |
| $\rho_b$                   | $550 \text{ kgm}^{-1}$ |
| $U_{\text{power}}$        | $10 \text{ ms}^{-1}$ |
| $\lambda_p$               | $10$                |
| $r_h$                      | $0.15$              |
| $\theta_p$                | $25^\circ$          |
| $\theta_p$                | $-5^\circ$          |
| $Q_r$                      | $0 \text{ Nm}$      |

The parameters that have not yet been explained are:

- The blade density, $\rho_b = 550 \text{ kgm}^{-3}$, comparable to the density of two species of Australian timber, Hoop pine and radiator pine, [8].
- The ‘final’ starting tip speed ration, $\lambda_f = 1.5$ set to ensure that the generic equations remained valid during starting by keeping $\alpha < \pi/6$. If this condition was not met, the blade was penalised by making $d\lambda/dt$ large and negative.
- The constraint on the chord was set based on the availability of good quality pine up to the maximum thickness.
- The maximum twist was set to avoid large changes in curvature in the transition section between the aerodynamic portion of the blade and the hub attachment.
- Minimum chord and twist were set based on machinability limits.

The three bladed rotor described in Table 1 has a 3m diameter and corresponds to a rated capacity of about 2kW at 10m/s. Initially the resistive torque of the generator, $Q_r$, was set to zero. The chosen airfoil section, the SG6043, was specially designed for small wind turbines, [9]. It has a large lift:drag ratio at a $Re = 2.2 \times 10^5$, typical of power producing blades. At this $Re$, the lift coefficient at maximum lift:drag, $C_l = 1.22$ at $\alpha = 5^\circ$. Results are displayed as optimal fitness fronts, defined as the subset of blades that have at least one objective function larger than that of every other blade. This typically comprises 1-3% of blades in the final population. In some sense, the front defines the “optimal” blades for a two-dimensional optimization. The front is obtained through varying the parameter $a$ in equation (5) such that a higher $a$ should result in better power producing blades and lower $a$ should result in faster starting blades. By varying $a$, the whole front can be mapped out. Figure 1 shows two optimal fitness fronts mapped out by varying $a$ between 0.7 and 0.95; the lower front is for $\lambda_p = 10$ and the higher front when $\lambda_p$ is set as an extra gene. The latter case will be discussed next paragraph. As expected, a higher values of $a$ gives a greater $C_p$, but a longer $T_s$. A more complete optimal fitness front was obtained using lower values of $a$, but these are not of interest for a practical blade design. About 20% non-dominated population is shown, which form a tight cluster for each value of $a$.

Wood [3] introduced the concept of the ‘superblade’, which takes advantage of the BET assumption that blade elements are independent of each other, by constructing a blade from the maximum torque producing elements of the entire final population. In this current work, the superblade is only shown for fixed $\lambda_p$ as the superblade concept only works for a constant $\lambda$ along the span of the blade. The superblades are shown on the optimal fitness front as the left most point of each run, i.e. the slowest starting blade. In figure 1, the three superblades coincide almost exactly at a $C_p$ of 0.492. This is within 0.5% of the $C_p$ of 0.490 calculated from equations (1) & (2). Whilst there is some trade off between power production potential and quick starting, the initial results for
Figures 2 and 3: Chord and twist distributions. At $\lambda_p = 10$; ○ superblade: $C_p = 0.492$ & $1/T_s = 0.266$, □ $a = 0.7 \lambda_p = 10$, ▽ $a = 0.85 \lambda_p = 10$; ◇ $a = 0.95 \lambda_p = 10$. ○ $a = 0.7 \lambda_p = 9.75$, □ $a = 0.85 \lambda_p = 6.00$, ▽ $a = 0.95 \lambda_p = 6.15.

This tweaking of the blade is demonstrated in Figure 2 for the chord and Figure 3 for the pitch distributions. Shown are equations (1) & (2), the superblade for $a = 0.7$ and the ‘average’ optimized blade from the $\lambda_p = 10$, $a = 0.85$ run (the second set of results on these figures, with the optimized $\lambda_p$ will be discussed in the next paragraph). Each element of the average optimized blade is constructed by averaging each chord or pitch element for that radius from all blades on the optimal fitness front.
The average is taken to smooth out any scatter in the chord and twist distributions. The superblade has a chord and pitch distribution which closely follows that of equations (1) & (2). Along with the closeness in $C_p$, this can be seen as a ‘check’ on the DE optimization method. The optimized blade closely follows equations (1) & (2) near the tip, where the majority of power is produced, but the blade’s improved starting performance comes from the increase in chord and twist near the hub. As can be seen in the middle of the optimized blade, there is some scatter in the results. This is generally not a problem, as smoothing the distribution, by eye or with a polynomial fit produces similar starting and power performance to the scattered results.

In the second set of results in figures 1-3, $\lambda_p$ was optimized as a gene in the range of 6 to 12. When $a = 0.7$ there is no noticeable difference as the optimized $\lambda_p$ of 9.74 was close to the original $\lambda_p$ of 10. For $a = 0.85$, and 0.95 we have an improvement in power performance of almost 8%. This is at the expense of starting performance however, with starting performance comparable to equations (1) & (2) calculated at $\lambda_p = 10$. For comparison, equations (1) & (2) were recalculated using the optimized $\lambda_p = 6.00$ from the $a = 0.85$ run. This gave a comparable increase in $C_p$ demonstrating that the majority of the gains came from optimizing $\lambda_c$ alone. The optimal fitness front still gave an improvement over equations (1) & (2) in starting performance. The optimized $\lambda_p$ was reduced to 6.0 and 6.15 for $a = 0.85$ and 0.95 respectively and, as can seen in figures 2 and 3, this resulted in an increase in both chord and twist over the initial optimized distributions. The increase in chord has an advantage over the fixed $\lambda_p$ case, as a smooth optimum could only be obtained by relaxing minimum chord constraints. This results in a chord length at the tip of less than 0.03. As described in the last section, such a small chord could be difficult to machine accurately.

3.1. Effect of resistive torque.
A turbine as described operating at the conditions in table 1 would generate 2.08kW using the superblade, corresponding to power-producing torque of about 31Nm. A resistive torque of 0.5 Nm is negligible in comparison, and thus is ignored for power calculations. This figure is not negligible for starting however: compare to the resistive torque of 0.24Nm in the permanent magnet generator of the 500W experimental turbine used by Wright and Wood [2]. When this small resistive torque is introduced there is a large shift in the optimal fitness front. Note the change in the limits of the horizontal axis between figure 1 and figure 4. While the optimal fitness front has shifted, equations (1) & (2) have faired worse in terms of starting performance. A twenty-fold improvement over equations (1) & (2) is now possible without losing significant power performance. The BET optimized blade for the optimized $\lambda_p$ has a comparable improvement in starting time and almost 10% increase in power coefficient over $\lambda_p = 10$. Nevertheless, the DE indicates that a faster starting and more powerful blade is available.

3.2. Effect of the inclusion of Tip Losses.
Figure 7 shows an expected shift in the optimal fitness fronts when tip losses are included, with lower $C_p$, for comparable starting performance.

The respective chord and twist distributions are shown figures 8 and 9. If we compare these to the equivalent cases neglecting tip losses, i.e. figures 5 and 6 there is no significant difference in the distributions. The significant difference is in the reduction of $C_p$, which is more significant for the $\lambda_p$ optimized blades, due to the greater chord at the tip. Ignoring scatter due to Reynolds number effects, there is a slight increase in chord in the tip region. This is more apparent in the fixed $\lambda_p$ case due to the higher $\lambda_p$ which is to be expected from the dependence of equations (4) & (5) on $\lambda$ and can be seen as a check on the method. If the chord and twist distributions from figures 5 and 6 are recalculated with the inclusion of tip losses, the power coefficients for the $\lambda_p = 10$ case and the optimized $\lambda_p = 6.29$ case are reduced to 0.440 and 0.488 respectively. The difference in the lower $\lambda_p$ cases is negligible compared to a $C_p = 0.494$ for the optimized distributions in figures 8 and 9, so tip losses can be safely
Figure 4: Optimal fitness front with $Q_r = 0.5$ Nm. + Equations (1) & (2) at $\lambda_p = 10$, * Equations (1) & (2) at $\lambda_p = 6.00$, ○ $a = 0.7 \lambda_p = 10$, □ $a = 0.85 \lambda_p = 10$, ▽ $a = 0.95 \lambda_p = 10$, ● $a = 0.7 \lambda_p = 7.01$, ■ $a = 0.85 \lambda_p = 6.29$, ▼ $a = 0.95 \lambda_p = 6.00$.

Figures 5 & 6: Chord and twist distributions with $Q_r = 0.5$ Nm. At $\lambda_p = 10$; ○ superblade: $C_p = 0.491$ & $1/T_s = 0.005$, - Equations (1) & (2): $C_p = 0.490$ & $1/T_s = 0.005$, □ $a = 0.85$: $C_p = 0.471$ & $1/T_s = 0.259$. At $\lambda_p = 6.00$; - - Equations (1) & (2): $C_p = 0.519$ & $1/T_s = 0.130$, ■ $a = 0.85$: $C_p = 0.521$ & $1/T_s = 0.226$.

ignored in this case. In the higher $\lambda_p$ case with $C_p = 0.467$ the difference is small and on the edge of what could be confirmed experimentally.
Figure 7: Optimal fitness front with $Q_r = 0.5 \text{ Nm}$ and tip losses. + Equations (1) & (2) at $\lambda_p = 10$, * Equations (1) & (2) at $\lambda_p = 6.00$, ○ $a = 0.7$ $\lambda_p = 10$, □ $a = 0.85$ $\lambda_p = 10$, ▽ $a = 0.95$ $\lambda_p = 10$, ● $a = 0.7$ $\lambda_p = 8.04$, ■ $a = 0.85$ $\lambda_p = 6.99$, ▼ $a = 0.95$ $\lambda_p = 6.49$.

Figures 8 & 9: Chord and twist distributions with $Q_r = 0.5 \text{ Nm}$ and tip losses. At $\lambda_p = 10$; ○ superblade: $C_p = 0.485$ & $1/T_s = 0.005$, - Equations (1) & (2): $C_p = 0.456$ & $1/T_s = 0.005$, □ $a = 0.85$: $C_p = 0.467$ & $1/T_s = 0.236$. At $\lambda_p = 6.00$; - - Equations (1) & (2): $C_p = 0.486$ & $1/T_s = 0.103$, ■ $a = 0.85$: $C_p = 0.494$ & $1/T_s = 0.224$.

The tip regions of the distributions, particularly the twist, are not as smooth in figures 8 and 9 as the distribution in figures 2 and 3. The tip $Re$ typically is in the range of 150,000 - 300,000 and due to the nature of BET, there is nothing to say an individual blade element could operate at a significantly different $Re$ value to its neighbor. A different $Re$ in this range can result in a different optimum chord.
and pitch for negligible difference in power production. It was found that by ignoring Reynolds number effects in the power calculations, much of this ‘scatter’ disappears.

4. Conclusions
Differential evolution has been used to numerically optimise the chord and twist distributions of small wind turbine blades with the dual purpose of maximising power coefficient and minimising starting time. Further gains were achieved through optimising the turbines power producing tip speed. Through optimizing the chord and twist alone, an improvement of a factor of 20 in starting time was possible without significantly trading off any power production potential. When optimized in combination with the tip speed ratio, power coefficient was improved by 10% with similar gains in starting time to the previous case. The resulting chord and twist distributions were significantly different to analytical expressions obtained from blade element theory for a blade operating at the Lanchester-Betz limit, however the maximum power producing blade had chord and twist distributions closely match those of the analytical expressions.

A new tip loss correction technique was used to test if the subsequent numerical optimization yielded any significant differences in the shape of the chord and twist distributions. Whilst power coefficients were reduced as expected, there was only a slight increase in chord near the tip in the optimized blades, which was less noticeable at low tip speed ratios. The difference in power production from these differences would be negligible at low tip speed ratios, and at high tip speed ratios, small enough to be difficult to measure in practice.

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