**Abstract.** Starting from Heavy Ion Collisions with exotic nuclear beams at low energy, the reaction path is investigated in a transport theory based on a stochastic mean field approach. We focus on the interplay between reaction mechanisms, fusion vs. break-up in HIC with exotic nuclear beams at low energies. Fusion probabilities for reactions induced by 132Sn on 64,58Ni targets at 10AMeV are evaluated looking at the evolution of the phase-space quadrupole collective modes. The break-up events appear sensitive to the stiffness of the symmetry energy. 197Au + 197Au collisions at 15 and 23 AMeV are simulated to investigate the main modes of re-separation, ternary-quaternary partition, of a heavy nuclear system.

1. Introduction

We focus the attention on the fusion vs. deep-inelastic mechanism interplay for dissipative Heavy Ion Collisions (HIC) with exotic nuclear beams at low energies, just above the Coulomb Barrier (between 5 and 20 AMeV). The exotic nuclei production, in laboratory conditions, can open the exploration of new nuclear structure and dynamics aspects: unstable ion beams with large asymmetry, i.e. up to extreme ratios of neutrons (N) to protons numbers (Z), and with good intensities in this energy range will be soon available in Radioactive Ion Beam facilities as SPES [1] or SPIRAL2 [2]. The goal of this analysis is to pin down specific observables which are sensitive to the symmetry energy to learn about its poorly known density behaviour.

We show that the reaction dynamics with HIC undergoes a fusion/break-up path bifurcation at early times. This bifurcation due to Coulomb and angular momentum effects, is essentially ruled by dynamical fluctuations and it could be sensitive to differences in the repulsive symmetry term, because the neutron-rich neck connecting the two colliding partners can survive a long time producing in the case of break-up very deformed final fragments, eventually leading to ternary/quaternary fragmentation events.

Usually the observed non fusion processes were found to be binary deep-inelastic reactions at energies below 20MeV/nucleon and at peripheral impact parameters. Investigating heavy ions collisions and symmetric systems, it is not possible to describe fusion exit channel, even in central collisions, due to strong Coulomb repulsion: a ternary-quaternary partitioning of the system can be observed. Starting from LNS data [3] we can evaluate the fast ternary and quaternary breakup mechanism of a very heavy collision, \(^{197}\text{Au} + \^{197}\text{Au}\), at 15MeV/nucleon; the colliding system will break into three or four fragments of comparable mass in inelastic collisions, which are nearly aligned along a common reseparation axis.

The observed ternary and quaternary breakup reactions should not be mixed up with the very asymmetric projectile partitioning observed at the Fermi-energy domain, interpreted as neck fragmentation processes [4] and described by the stochastic Boltzmann-Nordheim-Vlasov (BNV) [5] and the Constrained Molecular Dynamics (CoMD-II) [6] models: the observed fragments seem to originate from the nearly symmetric non equilibrium breakup of the primary products of deep-inelastic reactions.
2. Reaction dynamics

The reaction path followed by HIC with neutron-rich (or exotic) nuclear beams at low energies is investigated within a transport theory based on a Stochastic Mean Field (SMF) approach, extension of the microscopic Boltzmann-Nordheim-Vlasov transport equation [7, 8].

\[
\frac{\partial f}{\partial t} + \frac{p}{m} \frac{\partial f}{\partial r} + \frac{\partial U}{\partial r} \frac{\partial f}{\partial p} = I_{\text{coll}}[f] + \delta I[f]
\]

(1)

The stochastic fluctuating term \(\delta I[f]\) is implemented by fluctuations of the spatial density [9] in the SMF model; it is essential to allow the growth of dynamical instabilities. The mean field is built from Skyrme forces:

\[
U_{n,p} = A \frac{P}{\rho_0} + B \left( \frac{P}{\rho_0} \right)^{\alpha+1} + C(p) \frac{P_n - P_p}{\rho_0} r_q + \\
\frac{1}{2} \frac{\partial C}{\partial \rho} \left( \rho_n - \rho_p \right)^2
\]

(2)

where \(q = n, p\) and \(r_n = 1, r_p = -1\). The \(A, B\) and \(\alpha\) coefficients, characterizing the isoscalar part of the mean-field, are fixed to reproduce the saturation properties of symmetric nuclear matter \((\rho_0 = 0.145\,\text{fm}^{-3}, E/A = -16\,\text{MeV})\), with a compressibility modulus around 200 MeV. The time evolution of the semi-classical one-body distribution function \(f(r,p,t)\) is following a Boltzmann-Langevin evolution dynamics where two parameterizations are implemented for the density dependence of symmetry energy (Asy-stiff EOS, \(C(\rho)/\rho_0 = \left(\frac{32}{\rho_0}\right)\left(2\rho/\rho + \rho_0\right)\)), and Asysoft choice, \(C(\rho)/\rho_0 = 482 - 1638\rho\), where \(C(\rho)\) function give the potential part of the symmetry energy) [7].

3. Fusion dynamics for \(^{132}\text{Sn}\) induced reactions

A systematic and detailed fusion dynamics study, focusing on the symmetry energy dependence, will be possible with the availability of intense low energy radioactive beams with a different N/Z asymmetry. We perform the \(^{132}\text{Sn} + \ ^{64,58}\text{Ni}\) reactions at 10 AMeV for semi-peripheral impact parameters (from \(b = 4.5\) fm to \(b = 8.0\) fm, with \(\Delta b = 0.5\) fm) to study isospin and symmetry energy effects on the competition between fusion and break-up (deep-inelastic), exploiting the SMF approach where an in-medium dependent nucleon-nucleon cross section is employed in the collision integral, \(I_{\text{coll}}\), via the local density [10]. The cross section is set equal to zero for nucleon-nucleon collisions below 50 MeV of relative energy. In this way we avoid an increasing numerical noise due to spurious effects that may dominate in this energy range when the calculation time becomes too large. We used 100 test particles per nucleon for a better description of the phase space, and Gaussian phase space wave packets of test particles are considered to map the particle occupation at each time step.

We show that the reaction dynamics undergoes a fusion/break-up path bifurcation at very early times due to Coulomb and angular momentum effects. In such critical transition stage the final outcome is essentially ruled by dynamical fluctuations and it will be rather sensitive to differences in the neutron-repulsive symmetry term.

Fusion probabilities for reactions induced by \(^{132}\text{Sn}\) beam at 10AMeV are evaluated by the evolution of the phase-space quadrupole collective modes, to extract the fusion cross section at relatively early times: \(Q(t) = \langle 2z^2(t) - x^2(t) - y^2(t) \rangle\) the quadrupole moment in coordinate space averaged over the space distribution in the composite system and \(Q_K(t) = \langle 2p_z^2(t) - p_x^2(t) - p_y^2(t) \rangle\), the quadrupole moment in momentum space evaluated in a spatial region around the center of mass (z-axis coincides with the symmetry axis, the x-axis is on the reaction plane). The information on the final reaction path is deduced investigating the quadrupole fluctuations of the system at early times (200-300 fm/c), when the formation of composite elongated configurations is observed [11]. The method described can indicate when the reaction mechanism is changing into the SMF approach.

The \(Q(t)\) behaviour is different for more peripheral impact parameters (increasing trend, i.e. break-up) and for \(b = 5-6\) fm, where we have still a little oscillation in the time interval between 100 and 300
fm/c, indication of a fusion contribution. These observations can be interpreted assuming that starting from about $b = 5$ fm, we have a transition from fusion to a break-up mechanism, like deep-inelastic. The $Q(t)$-slope values should be associated with a quadrupole deformation velocity of the dinuclear system that is going to a break-up ($Q(t)$-slope > 0) or to a fusion ($Q(t)$-slope < 0) exit channel. When increasing impact parameter the quadrupole deformation in momentum space $Q_K(t)$, in a sphere of radius 3 fm around the center of mass, becomes more negative in the time interval between 100 and 300 fm/c. Since the angular momentum is large, the components perpendicular to the symmetry axis, which is rotating in reaction plane, are increasing leading to a separation of the deformed di-nuclear system. The break-up probability increases when the quadrupole moment in p-space is more negative [11].

The Fig. 6 of ref. [11] shows the $Q$ slope-$Q_K$ correlation plots for the systems $^{132}$Sn + $^{64}$Ni and the two asy-EOS. We can evaluate the normal curves of the displayed quantities and the relative areas for each impact parameter in order to select the events: break-up events will be located in the regions with both positive slope of $Q$ and negative $Q_K$. In this way, we can evaluate the fusion events for each impact parameter by the difference between the total number of events and the number of break-up cases [11]. In Figure 1 we present the fusion distribution plots. For the $^{132}$Sn + $^{64}$Ni system (left panel), we note a difference between the $\sigma$-fusion corresponding to the two different asy-EOS in the centrality transition region, with larger values for Asysoft, a 4-5% effect in the neutron rich system: 1128 mb (Asysoft) vs. 1078 mb (Asystiff), and even smaller, 1020 mb vs. 1009 mb, for the $^{58}$Ni target (Figure 1, right panel). However, through a selection in angular momentum, $130 \leq l \leq 180\ h\bar{h}$, we find that the Asysoft cross section curve is significantly above the Asystiff one, 10-15% in the case of the more neutron-rich system. We like to note that for the neutron-rich case, $^{132}$Sn+$^{64}$Ni, our absolute value of the total fusion cross section presents a good agreement with recent data, at lower energy (around 5 AMeV), taken at the ORNL [12].

The larger fusion probability obtained with the Asysoft choice seems to indicate that the reaction mechanism is regulated by the symmetry term at suprasaturation density, where the Asysoft choice is less repulsive for the neutrons [7, 15].

**Figure 1.** Angular momentum distributions of the fusion cross sections (mb) for the two reactions and the two choices of the symmetry term. For the $^{132}$Sn + $^{64}$Ni system (left panel), the results of PACE4 [13, 14] calculations are also reported, for different $l$-diffuseness.
A first transport approach analysis of symmetry energy effects on the structure of break-up events in peripheral collisions of $^{132}$Sn + $^{64}$Ni at 10 AMeV has been reported in ref. [16]. The neck dynamics on the way to separation is found influenced by the symmetry energy below saturation. This can be observed in the deformation pattern of the Projectile-Like (PLF) and Target-Like Fragments (TLF) [16]: larger deformations are seen in the Asystiff case for peripheral impact parameters, corresponding to a smaller symmetry repulsion at the low densities probed in the separation region.

4. Analysis of ternary events on $^{197}$Au + $^{197}$Au reaction

A study of the $^{197}$Au + $^{197}$Au reaction at 15AMeV and 23AMeV energy was carried out [3, 17, 19] by the CHIMERA Collaboration at Laboratori Nazionali del Sud (LNS). For this system at an energy of 15AMeV and at relatively small impact parameters, corresponding to strongly damped collisions [18], the data contain predominantly binary events but a fast break-up into three or four massive fragments of comparable mass has been experimentally revealed.

The ternary events are dominated by processes in which the heaviest fragment, close to the $^{197}$Au mass, is the remnant of the projectile fragment (PLF) or the target fragment (TLF), while the two fragments, F1 and F2, are induced from the complementary primary fragment breakup of the TLF or PLF: $^{197}$Au + $^{197}$Au → TLF + PLF → TLF + F1 + F2 or F1 + F2 + PLF. The mass spectra of F1 and F2 fragments differ significantly, that excludes the only pure statistical fission [3].

In the ternary partitioning case it is observed an alignment of the fragments close to the axis of the system re-separation into PLF and TLF: the PLF (or PLF) break-up is collinear with PLF-TLF separation axis. The smallest mass fragment, F1, has an energy greater than F2, as shown by the in-plane velocity distribution of fragments in ref. [3] (this excludes a purely statistical fission).

The partitioning results of a study of the same system at an energy of 23AMeV identified a variety of light, medium and heavy fragments.

An Intermediate-Mass Fragment (IMF) is observed with A less than 20 in addition to two main fragments, a TLF and a PLF. Emission of IMFs is a neck fragmentation process during the reseparation of the colliding system, that was observed in other experiments, but at higher energies. Ternary events are also observed going to a heavy fragment, TLF (or PLF) and two other fragments of comparable mass, originating from the reaction partner break-up or fission. The fig. 4 of ref. [19] of the two-dimensional transversal vs. longitudinal velocity distribution of three fragments, projected on the reaction plane, shows also the fragments F1 and F2 concentrated around a ‘Coulomb ring’ that indicated a sequential binary split mechanism of the primary PLF (i.e. after the TLF+PLF separation) in addition to a fragment alignment along the system re-separation axis. The experimental velocity distribution of three fragments could be decomposed in neck fragmentation, IMF, induced symmetric fission and F1-F2 breaking, like at 15AMeV.

The QMD model reproduces some evidences of the reaction; both TLF and the dynamical breakup of PLF into two fragments of comparable masses are correctly located in the velocity space, but it fails to reproduce the collinear emission experimentally observed [3].

In this work, we tried to reproduce the dynamical break up. The mechanism is investigated in a transport theory based on a SMF model, where the Asystiff parameterization for the density dependence of the symmetry energy is implemented. In order to map the particle occupation at each time step, triangular phase space wave packets of test particles are considered. The in-medium nucleon-nucleon cross section is fixed at 0.14fm$^{-3}$ density in the collision integral. We run 100 events for each set of macroscopic initial conditions and we obtain the 70% of ternary events over this ensemble.

In Figure 2 (left) we present a theoretical simulation of the $^{197}$Au + $^{197}$Au system at 15 AMeV for b=4fm, left panel, and b=6fm, right panel, impact parameters. The reaction simulation is very sensitive to the test particles number and to the surface term used in the code, especially because of strong Coulomb repulsion. 100 test particles per nucleon have been employed in simulations for an accurate description of the mean field dynamics. Through a qualitative study of the time evolution of the space density distributions projected on the reaction plane, the reaction mechanism seems to be heavily
dominated by the quadrupole and octupole modes. The neutron-rich neck connecting the two partners survives a rather long time, producing very deformed primary PLF/TLF especially for more central impact parameters, that lead to three or four massive fragments of comparable size.

**Figure 2.** Time evolution of the space density distributions for the $^{197}$Au + $^{197}$Au collisions at 15 AMeV beam energy, for $b=4$ and $b=6$fm impact parameter.

**Figure 3.** Like in Fig.2 but for $^{197}$Au + $^{197}$Au system at 23 AMeV beam energy, $b=5$ and $b=6$fm centrality.

Assuming that the ternary events have a larger probability than quaternary events, we try in a simple way to evaluate the breaking fragment masses from the PLF or TLF residues with large deformation (with large quadrupole and octupole component): first we evaluate the center of the larger agglomerate inside the deformed PLF or TLF, then we obtain the nucleon number $N_{2\text{part}}$ integrating from the left extreme to the center (with fragment orientation as shown in figure 4).

**Figure 4.** Scheme to evaluate the breaking fragment masses from the PLF or TLF (see text).

We can evaluate the $F_1$ and $F_2$ mass for each event by $F_2 = 2N_{2\text{part}}$ and $F_1 = N_{\text{tot}} - F_2$. Figure 5 shows the reconstructed mass spectrum of lightest, called $F_1$, and heaviest fragments, $F_2$, obtained by this method, for two relatively small impact parameters where we observe almost all break up events.
Figure 5. Reaction $^{197}$Au+ $^{197}$Au semi-peripheral at 15AMeV. The mass spectrum of mass numbers of fragments F1 and F2, for b=4fm and b=5fm impact parameters, reconstructed by the method describe above.

The mass distribution presents a good agreement with recent data at 15AMeV (shown in ref. [3], fig.4b), taken at LNS. In particular we can reproduce the distance between the F1 and F2 centroids and the ratio between the centroid and the variance of the distribution. This seems to be a nice evidence that the ternary partitioning in comparable masses comes from semi peripheral impact parameters. We underestimate the fragment mass, by about 20 units, with respect to the experimental data [3]. It is probably due to the overestimation of the evaporated spurious particles in the code because the calculation time for break up is very large at low energy especially for b=4fm and b=6fm. Indeed the evaluated time are 1250fm/c and 900fm/c, respectively. The final time choice is evaluated when the symmetry axis of the system is next to the deformed fragment axis, as shown by the experimental evidence, because the alignment is crucial to reproduce the velocities of the PLF and TLF fragments.

In this work we also report the partitioning study results of the same $^{197}$Au + $^{197}$Au system at the higher bombarding energy of 23AMeV. In addition to the partitions observed at low energies, an emission of intermediate mass fragments is also observed in the data at 23AMeV [19]. In Figure 3 (right) an increasing instability in the neck region is observed. The plot shows that a process of neck fragmentation during re-separation of the colliding system plays an important role, in which one of the two fragments is systematically enriched in mass at b=6fm. Increasing the beam energy and impact parameter, the statistical spontaneous fission is more favored while this effect could be not alive at 15AMeV energy because the system returns to a more spherical compound fragment. This can also be associated with enhanced quadrupole and reduced octupole modes that leads to a PLF (or TLF) statistical breaking. The observed strong instability could lead to lose the preferential collinear emission because we could have different fragment breaking times. This could also explain why the F1 and F2 fragment data are concentrated around a “Coulomb ring”.

The velocity distribution of the primary PLF/TLF is reconstructed in figure 6 at 23 AMeV, b=6fm centrality, where the evaluated time is 750 fm/c. The simulated primary fragments are localized in a region of velocity space like in the data, that proves the good selection of the macroscopic initial condition set, in particular that the fragments originate from deep inelastic collisions in a region of semi-peripheral impact parameters.
Figure 6. Velocity distribution of primary PLF and TLF fragments from partitioning of the $^{197}$Au$+^{197}$Au system at 23AMeV energy beam and for b = 6fm centrality (in the laboratory reference frame as a function of the longitudinal and transversal components).

The mass spectrum of fragments F1 and F2, for b=5fm impact parameter, reconstructed by the method and approximations describe above is reported in left panel of figure 7. We notice an analogous behavior as observed at 15AMeV, especially for the centroid position, and a statistical event component between the F1 and F2 centroids, caused by the enhanced number of events with only quadrupole deformation. In the right panel, we report the spectra analysis excluding the fragments with only quadrupole component (we do not consider the statistical fission).

Figure 7. Reaction $^{197}$Au$+^{197}$Au at 23AMeV for b=5fm impact parameter. The mass number spectrum of fragments F1 and F2 reconstructed by the method describe above. Left panel: events with quadrupole and quadrupole + octupole components. Right panel: quadrupole + octupole events (see text).

In addition, we repeated the analysis using the following approximations for b=5fm and b=6fm impact parameter: we identified the more energetic fragments with F1 and considered only the fragments with large quadrupole and octupole mode into the mass spectrum.
Figure 8. The weighted average mass number spectra of $^{197}$Au+$^{197}$Au collisions at 23AMeV energy beam between b=5 and b=6fm impact parameters.

The figure 8 shows the weighted average spectra between b=5 and b=6fm. We notice that, with a simple impact parameter and energy selection, the plot reproduces the smooth behavior of the fragment masses, with a preferential emission direction as seen in the experimental data, especially at 15AMeV.

Figure 9. N - Z plot of PLF/TLF primary fragments and reconstructed fragments, by the method described in the text, at b = 5fm and b=6fm.

Let us move to discuss the fragment isotopic properties. The N - Z plot, figure 9, shows a linear behavior, with the same slope, for primary (region with N ≥ 70 and Z ≥ 50) and reconstructed fragments at b = 5fm. The lightest fragment exhibits a slightly larger slope at b=6fm. It seems that in this case, owing to the strong Coulomb repulsion in the composite system, protons are transferred to the low-density neck region, from which the lightest fragment emerges. On the other hand, the N/Z ratio seems to be more equilibrated at b=5fm, where the latter effect is not observed.

Conclusions and perspectives
The reaction study by stochastic BNV approach of collisions with exotic systems at 10AMeV beam energy, from fusion to dissipative binary events, showed that the fusion vs. break-up probability is influenced by the neutron repulsion because the observed densities are above the normal value during
the approaching phase in the connecting region. Larger fusion cross sections are observed in asy-Soft case because we have a smaller symmetry energy value at supra-saturation densities. Our analysis is based on the fluctuations of quadrupole moments, in phase space, that develop during the di-nuclear phase and that essentially determine the final reaction channel. Isospin effects selecting the semi-peripheral impact parameters at low collision energy are revealed. The symmetry term study below and above saturation density can open new interesting perspectives for new experiments. We suggest the observable of the ternary/quaternary events probability too. We tested the ternary/quaternary reaction dynamics in the $^{197}$Au + $^{197}$Au collisions and the alignment along the axis of reseparation of PLF + TLF system in the first reaction phase. This study showed a larger ternary and quaternary partitioning probability at small impact parameters, where the characteristic collinear emission of the reseparation fragments is also observed, especially at 23AMeV, due to the increasing instability in the neck region.

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