Non Unitarity at DUNE and T2HK with Charged and Neutral Current Measurements

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Abstract

Neutral current (NC) measurements play an important role in exploring the new physics scenarios at long baseline neutrino oscillation experiments. We have found that combining the NC measurements at Deep Underground Neutrino Experiment (DUNE) with its charged current (CC) measurements enhances the bounds on some of the Non Unitarity parameters. Combining DUNE with T2HK experiment improves the bounds further. We have shown that even in the averaged out regime of light sterile neutrino, the NC events are different from the heavy sterile case in the leading order. It is observed that NC measurements at DUNE provide much better constraints on $\alpha_{33}$ parameter than the CC measurements.

Keywords: Non Unitarity, Neutral Current, DUNE, T2HK

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I. INTRODUCTION

The three flavor neutrino oscillation framework consists of three mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$, two mass-squared differences $\Delta m^2_{31}$ and $\Delta m^2_{21}$, and the Leptonic Dirac CP phase $\delta_{cp}$. Although, most of the oscillation parameters have been determined to varying degrees of precision through a combination of accelerator, solar, reactor and atmospheric neutrino experiments, yet the leptonic CP phase $\delta_{cp}$ is still one of the least known parameters. Two long baseline experiments NO$\nu$A [1, 2] and T2K [3] has been taking data both in $\nu$ and $\bar{\nu}$ modes. Both the experiments have measured nearly same $\Delta_{32}$, they have disagreements in their measurements of $\sin^2 \theta_{23}$, hierarchy as well as Leptonic $\delta_{cp}$. T2K prefers near maximal values of $\sin^2 \theta_{23}$ in the higher octant (HO) while NO$\nu$A measures significantly higher value of $\sin^2 \theta_{23}$ in the HO [3, 4]. Although both the experiments prefer normal hierarchy (NH) over inverted hierarchy (IH), yet NO$\nu$A allows IH at 1$\sigma$. The tension between the data of the two experiments needs to explained and the whole neutrino physics community is waiting eagerly to get some answer. So neutrino physics is running in the precision era. In these days, the capabilities of the neutrino experiments have changed and the detectors are capable of precision measurements. The capabilities of such detectors can be used to probe new physics searches as well as its identification and disentanglement from the standard three neutrino paradigm.

Neutrino oscillation is one of the strongest hint of physics beyond the standard model. The smallness of neutrino mass is still to understand and there are different models to explain it. Non-unitarity (NU) of the neutrino mixing matrix [5–26] is yet another interesting departure from the standard three-neutrino paradigm. It appears in the theory because of the type-I seesaw mechanism [27–30] which gives masses to the neutrinos via exchange of fermionic messengers. Non unitarity of the leptonic mixing matrix may arise due to the effect of new physics at the high energy scale or at the low energy scale. In the high energy scenario, the non unitarity of the three flavor mixing matrix, which is also called indirect non-unitary effect [8, 10, 16, 23], is because of the mixing of heavy right-handed neutrinos. Such neutral heavy leptons are much heavier than the standard neutrinos. The production mechanism of such heavy neutral leptons are not same as light neutrinos and any physical transition from the standard neutrinos to them are kinematically forbidden. On the other hand, direct non-unitary effect can be seen at lower energy scale i.e. an energy which is
much below the electroweak breaking scale. In this case, the light SM gauge group singlet lepton mix with the standard neutrinos. They also take part in neutrino oscillations and hence possible to measure their signature in the neutrino experiments. It was the LSND experiment[31] which for the first time claimed oscillations driven by $\Delta m^2 \sim 1\text{ eV}^2$. It found excess in the positrons which could be explained in terms of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations driven by $\Delta m^2 \sim 1\text{ eV}^2$. This claim was also tested by MiniBooNE [32] experiment which ran in both the neutrino as well as antineutrino mode. If nature has such heavy sterile states, it can leads to non unitarity of the $3 \times 3$ leptonic mixing matrix. Because, this non unitarity is the generic nature of theories with heavy neutrinos irrespective of the range of heaviness. This formalism introduce a new non-unitary phase which being degenerate with the standard CP phase hampers the measurements at the far detector(s) of the long baseline experiments [5, 6, 12, 19–22, 24].

Most of the studies on non unitarity have been done considering the charge current (CC) measurements at the far detector of the long baseline neutrino experiments which generally measures $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\mu$ oscillations both in neutrino and anti-neutrino mode [12, 19–22, 24]. Recently, neutral current measurements have been explored at DUNE [33] in the context of one light sterile neutrino [34, 35]. Constraints on one light sterile neutrino has already been derived at DUNE and T2HK [36] and can be found in [37–40]. But non unitarity framework is much more general than one extra light sterile neutrino.

Here in this work, we have incorporated the neutral current measurements with the CC measurements to derive the model independent constraints on non unitary parameters. There already exists tight constraints on the non-uniatrity parameters that comes from weak interaction universality and lepton flavour violating processes (LFV) [12, 15]. There are also model independent direct bounds on NU parameters coming from zero distance neutrino oscillations experiments such as NOMAD [41, 42] and neutrino oscillation experiments[9, 13].

In this work, we derive the model independent complementary bounds on NU parameters specially focusing the diagonal elements in the light of DUNE [33, 43] and T2HK [36] experiments. The non-diagonal NU parameters are already very tightly constrained [6, 15, 16]. We also discuss the role of neutral current measurements to constrain these NU parameters at DUNE. We have found that NC measurements at DUNE help us to constrain $\alpha_{33}$ parameter much better than the charged current process and hence it is possible to achieve better constraint on $\alpha_{33}$ over the existing direct bounds from the neutrino oscillation experiments.
We have then combined both DUNE and T2HK experiments to measure the bounds on the NU parameters and have found that the combining the two experiments can improve the results further.

This paper is organised as follow: In section II, we have reviewed the non-unitarity framework considered in this work. In that section, we have elaborately shown the effect of CC and NC measurements in neutrino oscillation probabilities. In section III, we have specified the details of the experiments considered in this work. In section IV, we have presented our results. The conclusions are drawn in section V.

II. NON UNITARITY FRAMEWORK

In presence of non unitarity due to heavy sterile neutrino, the mass basis ($|\nu_i\rangle$) remains orthogonal to each other, while the low energy effective flavor basis $^1(|\nu_\alpha\rangle)$, which is not orthogonal, can be represented as $^2$

$$|\nu_\alpha\rangle = N^\nu_\alpha |\nu_i\rangle,$$

where $N$ is a $3 \times 3$ general matrix and can be represented as $^2$

$$N = N^{NU} U = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix},$$

$U$ is the standard unitary PMNS mixing matrix and $N^{NU}$ contains the non unitarity part. Under the condition that all the diagonal element of $N^{NU}$ are unity and all the off-diagonal element vanishes, then $N$ becomes the standard PMNS mixing matrix. The diagonal elements ($\alpha_{11}, \alpha_{22}$ and $\alpha_{33}$) of $N^{NU}$ are real and the off-diagonal elements ($\alpha_{21}, \alpha_{31}$ and $\alpha_{32}$) are complex in general and can be expressed as $\alpha_{ij} = |\alpha_{ij}| e^{\phi_{ij}}$ for $i \neq j$. There are three new phases $\phi_{21}, \phi_{31}$ and $\phi_{32}$ that arises in the mixing matrix $N$ in presence of non unitarity. The new phases, specially $\phi_{21}$ can play an important role in the long baseline experiments such as DUNE and T2HK. It affects the standard $\delta_{cp}$ sensitivity of these experiments significantly $^{19, 21}$. Here in this section, we analysis the effect of non unitarity on

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1 For light sterile neutrino the flavor basis remains orthogonal i.e. $<\nu_\alpha |\nu_\beta> = \delta_{\alpha\beta}$.

2 Here we have not considered the normalization factor ($\frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}}}$) since it will cancel in the events calculations $^{44}$. 

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the neutrino oscillation probability.

In presence of non unitarity, the time evolution of the mass eigenstate in vacuum is:

\[
\frac{d}{dt} |\nu_i\rangle = H |\nu_i\rangle,
\]

where \(H\) is the free Hamiltonian in the mass basis and can be expressed as

\[
H = \begin{pmatrix}
0 & 0 & 0 \\
0 & \Delta m_{21}^2/2E & 0 \\
0 & 0 & \Delta m_{31}^2/2E
\end{pmatrix}
\]

where \(E\) is the energy of the neutrinos and \(\Delta m_{21}^2\) and \(\Delta m_{31}^2\) are the solar and atmospheric mass squared differences respectively. After time \(t(\equiv L)\), the flavor state can be written as

\[
|\nu_\alpha(t)\rangle = N^*_{\alpha i} |\nu_i(t)\rangle = N^*_{\alpha i} (e^{-iHt})_{ij} |\nu_j(t = 0)\rangle.
\]

Hence the transition probability from one flavor to another in presence of non unitarity can be written as:

\[
P(\nu_\alpha \to \nu_\beta) = |<\nu_\beta|\nu_\alpha(t)\rangle|^2 = |N^*_{\alpha i} diag(e^{-i\Delta m_{21}^2t/2E})_{ij} N_{\beta j}|^2
\]

Now, the transition probability for \(P_{\mu e}\) with non unitarity becomes [12]

\[
P_{\mu e} = (\alpha_{11}\alpha_{22})^2 P^{3\times3}_{\mu e} + \alpha_{11}^2|\alpha_{21}|P^I_{\mu e} + \alpha_{11}|\alpha_{21}|^2,
\]

where \(P^{3\times3}_{\mu e}\) is the standard oscillation probability and \(P^I_{\mu e}\) is the oscillation probability containing the new extra phase due to the non unitarity in the leptonic mixing matrix.

Here, \(P^{3\times3}_{\mu e}\) is given by:

\[
P^{3\times3}_{\mu e} = 4[\cos^2 \theta_{12} \cos^2 \theta_{23} \sin^2 \theta_{12} \sin^2 (\frac{\Delta m_{21}^2 L}{4E_{\nu}}) + \cos^2 \theta_{13} \sin^2 \theta_{13} \sin^2 \theta_{23} \sin^2 (\frac{\Delta m_{31}^2 L}{4E_{\nu}})]
\]

\[+ \sin(2\theta_{12}) \sin \theta_{13} \sin(2\theta_{23}) \sin(\frac{\Delta m_{21}^2 L}{2E_{\nu}}) \sin(\frac{\Delta m_{31}^2 L}{4E_{\nu}}) \cos(\frac{\Delta m_{31}^2 L}{4E_{\nu}} + \delta_{cp})\]

And

\[
P^I_{\mu e} = -2[\sin(2\theta_{13}) \sin \theta_{23} \sin(\frac{\Delta m_{31}^2 L}{4E_{\nu}}) \sin(\frac{\Delta m_{31}^2 L}{4E_{\nu}} + \phi_{21} + \delta_{cp})]
\]

\[+ \cos \theta_{13} \cos \theta_{23} \sin(2\theta_{12}) \sin(\frac{\Delta m_{21}^2 L}{2E_{\nu}}) \sin(\phi_{21})\]

Now from Eq. 5, we can draw the following points:
• At short distance \( \frac{\Delta m^2 L}{E} << 1 \), we will get non-zero transition \( \nu_\mu \rightarrow \nu_e \) in presence of NU if \( \alpha_{21} \) is not zero. Since at the short distance, the appearance probability does not depend on energy or length, the excess of \( \nu_e \) events exactly follow the \( \nu_\mu \) flux pattern in case of heavy sterile neutrino. But that is not the case for light sterile neutrino in general.

• If \( \alpha_{21} \sim 0 \), we will not get any excess of \( \nu_e \) events at short baseline experiments. But even if \( \alpha_{21} \) is very small, then at far detector, the appearance probability will be affected purely by \( \alpha_{11} \) or \( \alpha_{22} \). Therefore, the far detector at the long baseline will give us the unique capability to probe NU parameters than the short baseline.

In presence of matter the flavor eigenstates interact with the matter coherently and the free Hamiltonian gets modified. In presence of non unitarity, the interaction Lagrangian becomes:

\[
L_{\text{int}} = -\frac{g}{2\sqrt{2}}(W_\mu \bar{\nu}_i \gamma^\mu(1 - \gamma_5)N_{\alpha i} \nu_i) - \frac{g}{2\cos(\theta_W)}(Z_\mu \bar{\nu}_i \gamma^\mu(1 - \gamma_5)(N^\dagger N)_{ij} \nu_j) + \text{h.c.} \tag{8}
\]

Therefore in the mass basis, the total Hamiltonian \( (H_{\text{mat}}) \) \[44\] of the propagating neutrino is given by

\[
H_{\text{mat}} = \begin{bmatrix}
0 & 0 & 0 \\
0 & \Delta m^2_{21} / 2E & 0 \\
0 & 0 & \Delta m^2_{31} / 2E
\end{bmatrix} + N^T \begin{bmatrix}
V_{CC} + V_{NC} & 0 & 0 \\
0 & V_{NC} & 0 \\
0 & 0 & V_{NC}
\end{bmatrix} N^*, \tag{9}
\]

where \( V_{CC} = \sqrt{2}G_F n_e \) and \( V_{NC} = -\frac{1}{\sqrt{2}}G_F n_n \) are the charged current and neutral current matter potential respectively. Here \( n_e \) and \( n_n \) are the electron and neutron densities respectively \(^3\). The Hamiltonian \( H_{\text{mat}} \) is hermitian and we can diagonalize it by a unitary matrix \( (U_m) \) as:

\[
H_{\text{mat}} = U_m \begin{bmatrix}
a_1 & 0 & 0 \\
0 & a_2 & 0 \\
0 & 0 & a_3
\end{bmatrix} U^\dagger_m, \tag{10}
\]

\(^3\) We consider that the electron (\( n_e \)) and neutron densities (\( n_n \)) are same for DUNE and T2HK and also for simplicity we consider constant matter density \( (\rho = 2.95 \text{ gm/cc}) \) for our simulated results.
where \(a_1, a_2\) and \(a_3\) are the eigenvalues of \(H_{\text{mat}}\). Therefore, the transition probability \((\nu_\alpha \rightarrow \nu_\beta)\) becomes:

\[
P(\nu_\alpha \rightarrow \nu_\beta) = |<\nu_\beta|\nu_\alpha(t)>|^2 = |N_{\alpha i}^*(U_m diag(e^{-i a_1 t}, e^{-i a_2 t}, e^{-i a_3 t}) U_m^{\dagger})_{ij} N_{\beta j}|^2.
\]  

(11)

Now most of the upcoming super beam neutrino experiments will measure their flux through near detector measurements. Therefore, in presence of non unitarity the expected events at the near detector differ from the actual events by a factor of \(P(\nu_\alpha \rightarrow \nu_\alpha) = ((NN^{\dagger})_{\alpha\alpha})^2\). For DUNE and T2HK, the main source of neutrino is the muon neutrino and hence the above factor, i.e. the normalization factor, becomes \(((NN^{\dagger})_{\mu\mu})^2 = ((\alpha_{22})^2 + |\alpha_{21}|^2)^2\). Depending on the values of \(\alpha_{22}\) and \(|\alpha_{21}|\), we will get different muon events compared to the simulated events without NU at the near detector. If \(\alpha_{22} \sim 0.95\) and \(\alpha_{21} \sim 0\), then there will be a mismatch of around 20% events between the simulated and the actual events. Therefore, the near detector measurements can in principle put tight constraint on the non unitarity parameter \(\alpha_{22}\). Now, if we consider the near detector measurements, we can represent the transition probability [16], \(P_{\mu\alpha}\) as:

\[
P_{\mu\alpha} = \frac{R_\alpha}{R_\mu}
\]  

(12)

where \(R_\alpha\) and \(R_\mu\) are the events at the far and the near detector respectively. We can represent the transition probability as

\[
P_{\mu\alpha} = \frac{P(\nu_{\mu} \rightarrow \nu_\alpha)_{\text{far}}}{P(\nu_{\mu} \rightarrow \nu_\mu)_{\text{near}}} = \frac{|<\nu_\alpha|\nu_{\mu}(t)>|^2}{((NN^{\dagger})_{\mu\mu})^2}.
\]  

(13)

Therefore, if we consider the near detector information, then we have to use the normalization factor \(((NN^{\dagger})_{\mu\mu})^2\) in the probability expression. But that is not the case for the simulated flux at the source. In this analysis, we consider both the cases i.e. the simulated flux at the source and the near detector measurements.

In presence of non unitarity, both the charged current and the neutral current events get modified. The neutral current events for \(\nu_{\mu}\) beam in vacuum is proportional to

\[
N^{NC}_{\text{events}} \propto \sum_{j=1}^{3} \sum_{i=1}^{3} A(W \rightarrow \mu^+ \nu_i) exp(-i\Delta m^2_{1j} L/2E) A(\nu_i Z \rightarrow \nu_j) |^2
\]

\[
= \sum_{j=1}^{3} \sum_{i=1}^{3} N_{\mu i}^* exp(-i\Delta m^2_{1j} L/2E)(N^{\dagger} N)_{ji} |^2.
\]

(14)
In presence of matter the NC events will be proportional to

\[ N_{events}^{NC} \propto \sum_{k=1}^{3} \sum_{i,j=1}^{3} N_{\mu}^*(U_m \text{diag}(e^{-ia_1 t}, e^{-ia_2 t}, e^{-ia_3 t})U_m^\dagger)_{ij}(N^\dagger N)_{kj} \right| \right|^2, \]  

where \( U_m \) and \( a_i \)’s are defined in Eq. 10.

**Light Sterile case:**

The \( 3 \times 3 \) PMNS mixing matrix will also become non unitarity in presence of light sterile neutrino. As shown in [16], in presence of light sterile neutrino, the leading charged current transition probability among the active flavors will remain same as the non unitarity (due to the heavy sterile) if the effect of light sterile is averaged out in the detector. The active flavor states in presence of light sterile neutrino can be represented as

\[ |\nu_\alpha> = u_{al}^*|\nu_l> = \sum_{i=1}^{3} N_{\alpha i}^*|\nu_i> + \sum_{J=4}^{n} \Theta_{\alpha J}^*|\nu_J> \]  

where \( u \) is a unitary mixing matrix and its dimension (\( n \)) depends on the number of sterile neutrinos. \( N \) represents \( 3 \times 3 \) active-light sub-block of \( u \) and \( \Theta \) represents \( 3 \times n \) sub-block of \( u \) that mixes active and heavy states. The vacuum transition probability \( \nu_\alpha \rightarrow \nu_\beta \) in presence of light sterile neutrino is given by [16]

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |<\nu_\beta|\nu_\alpha(t)>|^2 = |u_{al}^* \text{diag}(e^{-i\Delta m^2_{l1} t/2E})_{ij} u_{\beta j}|^2 \]
\[ = \sum_{i,j=1}^{3} N_{\alpha i}^* \text{diag}(e^{-i\Delta m^2_{l1} t/2E})_{ij} N_{\beta j} + \sum_{J,K=4}^{n} \Theta_{\alpha J}^* \text{diag}(e^{-i\Delta m^2_{J1} t/2E})_{JK} \Theta_{\beta K}|^2 \]
\[ = \sum_{i,j=1}^{3} N_{\alpha i}^* \text{diag}(e^{-i\Delta m^2_{l1} t/2E})_{ij} N_{\beta j}|^2 + \mathcal{O}(\Theta^4) \]  

The cross terms will vanish in the limit \( \Delta m^2_{J1} L/2E >> 1 \) (where \( L \equiv t \)) since due to the finite energy resolution of the detector, the average over \( L/E \) will make \(< \text{sin}(\Delta m^2_{J1} L/2E) > = < \text{cos}(\Delta m^2_{J1} L/2E) > = 0 \). Therefore, if we neglect the correction corresponding to order \( (\Theta^4) \), then the leading order transition probability \( \nu_\alpha \rightarrow \nu_\beta \) will be same as Eq. 4.

In presence of the light sterile neutrino, the neutral current events also change from the standard events. Only the active flavors participate in the neutral current events. Therefore
the neutral current events for $\nu_\mu$ beam in vacuum is proportional to

$$N_{\text{events}}^{NC} \propto \sum_{j=1}^{n} \left| \sum_{i=1}^{n} A(W \rightarrow \mu^+ \nu_i) \exp(-i\Delta m_{12}^2 L/2E) A(\nu_i Z \rightarrow \nu_j) \right|^2$$

$$= \sum_{j=1}^{n} \left| \sum_{i=1}^{n} u_{\mu i}^* \exp(-i\Delta m_{12}^2 L/2E)( \sum_{\rho=e,\mu,\tau} u_{ij\rho}^\dagger u_{i\rho}) \right|^2$$

$$= \sum_{\rho=e,\mu,\tau} P(\nu_\mu \rightarrow \nu_\rho). \quad (18)$$

Now from Eq. 18, we can write

$$N_{\text{events}}^{NC} \propto \sum_{j=1}^{n} \left| \sum_{i=1}^{n} u_{\mu i}^* \exp(-i\Delta m_{12}^2 L/2E)( \sum_{\rho=e,\mu,\tau} u_{ij\rho}^\dagger u_{i\rho}) \right|^2$$

$$= \sum_{j=1}^{n} \left| \sum_{i=1}^{n} N_{\mu i}^* \exp(-i\Delta m_{12}^2 L/2E)( \sum_{\rho=e,\mu,\tau} N_{ij\rho}^\dagger N_{i\rho}) \right|^2 + \sum_{j=1}^{n} \left| \sum_{i=1}^{n} \sum_{\rho=e,\mu,\tau} \Theta^\dagger_{j\rho} N_{i\rho} \right|^2 \quad (19)$$

Due to the presence of $\Theta^2$ term in Eq. 20, the neutral current events will not remain same as Eq. 14 in the leading order. Therefore, the NC analysis will be different for light and heavy sterile case in the leading order. Here, in the rest of the paper, we consider light sterile analysis for the NC and CC measurements.

### III. EXPERIMENTAL AND SIMULATION DETAILS

In this work we have presented our results considering DUNE and T2HK experiments. The specifications of these experiments are as follow:

#### A. DUNE

DUNE [33] is a proposed future super-beam experiment at Fermilab, U.S capable to establish the existence of CPV in the leptonic sector. The facility is also capable to resolve
the issues like mass hierarchy and the octant of $\theta_{23}$ in the neutrino sector. Here, the optimized beam of 1.07 MW - 80 GeV proton will deliver $1.47 \times 10^{21}$ protons-on-target (POT) per year. The far detector of the setup will be placed at the Homestake mine in South Dakota. It is a Liquid Argon (LAr) detector of mass 40 Kt and the baseline is of 1300 km. The experiment will run for 7 years divided equally between neutrinos and anti-neutrinos. It corresponds to a total exposure of $4.12 \times 10^{23}$ kt-POT-yr. All the details of the experiment like CC signal and background definitions, detector efficiencies are taken from [45]. The details of the NC events at DUNE are taken from [46]. The assumed detection efficiency related to the NC event is 90%. We have used the migration matrices to reproduce the NC event spectra correctly. In a NC event, since the outgoing (anti-)neutrino carries away a fraction of the incoming energy, hence due to this missing energy, the reconstructed visible energy is somehow less than the total incoming energy. Hence, using a gaussian energy resolution function can not give accurate result. So, we have used the migration matrices from [47]. For the NC analysis, we take 5% and 10% signal and background normalization errors respectively. All other details regarding the backgrounds and the NC measurements are taken from [35].

B. T2HK

The Hyper-Kamiokande (HK) [36, 48, 49] is the upgraded version of the Super-Kamiokande (SK) [50] program in Japan. In this project, the fiducial mass of the SK detector will be increased by about twenty times. HK will have two 187 kt third generation Water Cherenkov detector modules which will be placed near the current SK site. The detector will be placed at a baseline of 295 km from the J-PARC proton accelerator research complex in Tokai, Japan. T2HK has almost similar physics goals as DUNE such as measuring neutrino mass hierarchy, octant of $\theta_{23}$, measuring the leptonic CP phase etc.

In this analysis we have consider a beam power of 1.3 MW and the $2.5^0$ off-axis flux for T2HK. The total fiducial mass considered is 374 kt which is due to two tank each of 187 kt. We have assumed a total run time of 10 years. Within these 10 years, neutrino will run for 2.5 years while anti-neutrino will run for 7.5 years. The assumed energy resolution is $15%/\sqrt{E}$. We have matched the number of events used in this work with the TABLE III and TABLE IV of ref. [48]. The signal normalization error in $\nu_\mu(\bar{\nu}_\mu)$ disappearance and
appearance channel are 3.9% (3.6%) and 3.2% (3.6%) respectively. The background and energy calibration errors assumed in this work are 10% and 5%, respectively for all channels.

Throughout the analysis, we have fixed the true values or the best fit values of the neutrino oscillation parameters as given in [51] unless stated. We fix the true values of the solar and the reactor mixing angles at $\theta_{12} = 33.82^\circ$ and $\theta_{13} = 8.61^\circ$ respectively. Assumed true value of the atmospheric mixing angle is $\theta_{23} = 49.7^\circ$. The true value of the leptonic CP phase is fixed at $\delta_{cp} = 217^0$. The mass square differences considered in this work are $\Delta m^2_{21} = 7.39 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{31} = 2.525 \times 10^{-3}$ eV$^2$ respectively. The 3$\sigma$ bounds on the NU parameters are taken from the neutrino experiments only and can be found at [21]. We have prepared a non unitarity code for this work which is consistent with MonteCUBES’s [52] non unitarity engine. The results presented in this work are generated by incorporating our non-unitarity code with GLoBES [53, 54].

IV. RESULTS

In this section, we present our results for DUNE and T2HK experiments. First, we discuss the effect of non unitarity on neutrino oscillation at the probability level and then at the $\chi^2$ level.

A. Probability Plots

In Fig.1 and Fig.2, we show the effect of NU parameters on both the appearance and the disappearance probabilities at DUNE in presence of matter effect. We consider one NU parameter at a time to disentangle the effect of a particular parameter from the rest but in the $\chi^2$ analysis we consider all the parameters. Again, to incorporate the near detector measurements, we have to consider the normalization factor $^4((NN^\dagger)_{\mu\mu})^2 = (\alpha_{22}^2 + |\alpha_{21}|^2)^2$, in the transition probability as in Eq.13. But that is not the case for the simulated flux. We have shown the probability plots both with and without the normalization factor. Wherever we use the normalization factor, we specify it in the plots.

$^4$ Only $\alpha_{22}$ and $\alpha_{21}$ will arise in the normalization factor for $\nu_{\mu}$ beam. Therefore, we consider the normalization factor for $\alpha_{22}$ and $\alpha_{21}$. We consider only one NU parameter at a time while generating the probability plots. Hence there is no difference between with and without normalization for other NU parameters.
FIG. 1: Effect of diagonal NU parameters on appearance and disappearance channels considering one parameter at a time. All the non diagonal parameters are kept at zero.

The plots are shown for $\alpha_{11} = \alpha_{22} = 0.95$ and $\alpha_{33} = 0.9$.

We have shown the effects of the diagonal NU parameters on appearance and disappearance channels in Fig. 1. The red line corresponds to the standard 3\nu oscillation probability. The purple line corresponds to the case with $\alpha_{11} = 0.95$. It is seen that $\alpha_{11}$ has significant effect on appearance channel. The cyan solid (dashed) line show the effect of $\alpha_{22}$ with (without) the normalization factor. In the appearance channel, $\alpha_{22}$ has significant effect irrespective of the normalization. On the other-hand, normalization abates the effect of $\alpha_{22}$ on the disappearance channel. But the effect of $\alpha_{22}$ without the normalization is significant in the disappearance channel. Therefore, if we consider the simulated flux, then both the appearance and disappearance channels will get affected by $\alpha_{22}$. The effect of $\alpha_{33}$ is very small on both the appearance and the disappearance channel and hence constraining it by these channels is not very fruitful.

In Fig. 2, we show the effect of non diagonal NU parameters on oscillation probability allocating zero value to the diagonal parameters. Since with each non-diagonal parameters, there is a phase associated, we have shown the probability plots for a fixed value of the phase $\phi_{ij}$. In the left of the top panel, we show the variation of $\alpha_{21}$ while in the right of the top panel, variation of $\alpha_{31}$ and $\alpha_{32}$ has been shown. In the lower panel, we have shown the effect of all the three non diagonal parameters on disappearance channel. Even a small value of
\( \alpha_{21} \) can change \( P(\nu_\mu \rightarrow \nu_e) \) oscillation probability significantly almost for all the values of energy. The effect of the normalization factor is negligible in this case. If the phase \( \phi_{21} \) is allowed to vary for a given value of \( \alpha_{21} \), the probability deviates from the standard 3\( \nu \) case specially around the oscillation maxima. The other two non-diagonal parameters \( \alpha_{31} \) and \( \alpha_{32} \) has negligible effect on the appearance channel. But for larger values of \( \alpha_{31} \) (say around 0.1), we can see significant deviation from the standard case and has large phase dependency. From the plots in the lower panel, it is observed that the non diagonal parameters do not affect the measurements of the disappearance channel significantly. From all these results, we can draw the following conclusions:

- Effects of \( \alpha_{11} \) on the appearance channels are large compared to the disappearance channel.
The effect of normalization is crucial for \( \alpha_{22} \) parameter. Depending on the normalization condition both the appearance and disappearance channel will contribute.

- The effect of \( \alpha_{33} \) is very small on both the appearance and disappearance channel.

- Out of the three non diagonal parameters, \( \alpha_{21} \) affects the appearance probability significantly. The effect enhances in presence of the phase. None of these parameters has any noticeable effect on disappearance probability.

In Fig. 3, we show the variation of \( P_{ee} \) as well as NC measurements with energy as a function of NU parameters. \( P_{ee} \) oscillation probability plays an important role in the background. With the normalization\(^5\), although the effect of \( \alpha_{11} \) on \( P_{ee} \) is negligible, yet the probability changes drastically for the same value of \( \alpha_{11} \) if the normalization is switched off. Effect of \( \alpha_{31} \) is small compared to \( \alpha_{11} \) but it shows CP dependency. From the right panel of Fig. 3, we observe that in presence of \( \alpha_{22} \) and \( \alpha_{33} \), \( P_{NC} \) oscillation probability decreases significantly from unity. But with normalization factor (as \( \alpha_{22}^4 \) is in the denominator) the NC probability becomes greater than unity. Therefore, when we consider both the parameters \( \alpha_{22} \) (with

\(^5\) For \( P_{ee} \) channel the normalization factor is \( \alpha_{11}^4 \).
norm) and \( \alpha_{33} \) simultaneously there is a cancellation between \( \alpha_{22} \) and \( \alpha_{33} \) as shown by the pink line in the right panel of Fig.3.

In the next subsection, we present our sensitivity plots to constraint the NU parameters.

**B. \( \chi^2 \) analysis**

To quantify the effect of non unitarity at DUNE and T2HK, we have performed the \( \Delta \chi^2 \) analysis. We define the \( \Delta \chi^2 \) as:

\[
\Delta \chi^2 \simeq \sum_i \sum_j \frac{[N_{ij}^{\text{true}}(\text{Standard}) - N_{ij}^{\text{fit}}(\text{NU})]^2}{N_{ij}^{\text{true}}(\text{Standard})},
\]

where, \( N_{ij}^{\text{true}}(\text{Standard}) \) stands for the true events corresponding to standard three neutrino oscillation paradigm and \( N_{ij}^{\text{fit}}(\text{NU}) \) represents the events corresponding to the new physics i.e. non unitarity. In the fit, we have marginalized over all the standard neutrino oscillation parameters in their 3\( \sigma \) allowed ranges. The standard CP phase (\( \delta_{\text{CP}} \)) is marginalized over the full range. In addition to that, we have also marginalized over all the NU parameters in the ranges : \( \alpha_{11} \in [1, 0.95], \alpha_{22} \in [1, 0.96], \alpha_{33} \in [1, 0.76], \alpha_{21} \in [0, 0.026], \alpha_{31} \in [0, 0.098] \) and \( \alpha_{32} \in [0, 0.017] \). The unknown CP phases \( \phi_{ij} \) are marginalized over the full range i.e. \( \phi_{ij} \in [0^\circ, 360^\circ] \). In this way, we choose the minimum \( \Delta \chi^2 \) for a selective NU parameter by marginalizing over all the standard as well as the remaining NU parameters.

In Fig.4, 5 and 6, we show the capability of DUNE, T2HK and the combination of them, to probe the diagonal and non-diagonal NU parameters. The results are shown for two specific cases: with normalization factor (w norm) and without normalization factor (w/o norm). The plots captioned as ‘w norm’ means that the norm factor is used for both background and signal. The term ‘w/o \( \nu_e \) BG norm’ stands for the case where the norm factor is not used for \( \nu_e \) (and \( \bar{\nu}_e \)) background, but used for all other backgrounds. The term ‘w/o norm’ stands for the cases where norm factor is not used for both background and signal. In the top panel of Fig.4 we have shown the sensitivity of \( \alpha_{11} \) (upper panel) and \( \alpha_{22} \) (lower panel) both for DUNE and T2HK. We have presented the results for CC measurements at T2HK and then for the combination of it with CC and NC measurements at DUNE, named as ‘COMB’. In Fig.5, we have shown the constraints for \( \alpha_{21} \) and in Fig.6, obtained constraints are shown for \( \alpha_{33} \) both at DUNE and T2HK. We draw the following
conclusions from this analysis:

FIG. 4: Constraints on $\alpha_{11}$ and $\alpha_{22}$ at DUNE and T2HK using CC measurements. We have also combined CC measurements at T2HK with the CC+NC measurements at DUNE. We call these combined results as ‘COMB’.

a. **Bound on $\alpha_{11}$**: It is observed from the upper panel of Fig. 4 that both DUNE and T2HK give lose constraints when we use the norm factor in the measurements. But, if the norm factor is not used in the $\nu_e$ (and $\bar{\nu}_e$) background only, then we can see a significant enhancement in the sensitivity in both DUNE and T2HK. This enhancement is because of the decrease in $\nu_e$ (and $\bar{\nu}_e$) background due to the exclusion of the norm factor. It also can
be confirmed from the probability plots in Fig.3. So it is important to point that $\nu_e$ (and $\bar{\nu}_e$) background is the main channel to constraint $\alpha_{11}$ parameter. From the plots we observe that DUNE can exclude all values of $\alpha_{11} \leq 0.94$ while T2HK can exclude all $\alpha_{11} \leq 0.95$ at $3\sigma$ CL. So it is seen that T2HK can give slightly better constraints on $\alpha_{11}$ compared to DUNE. Combinations of the two experiments can improve the constraint further and at $3\sigma$ CL, it can exclude all $\alpha_{11} \leq 0.965$.

b. **Bound on $\alpha_{22}$**: We observe from the lower panel of Fig. 4 that the bound is very poor with norm for both DUNE and T2HK. Even their combinations with the norm is not improving the bounds. But without the norm, the bounds are improving significantly and adding NC with CC at DUNE is enhancing the bounds. It can rule out $\alpha_{22} \leq 0.978$ at $3\sigma$ CL. Finally, when we combine both the experiments we get better constraints which is $\alpha_{22} \leq 0.988$ at $3\sigma$ CL. It is observed from Fig.1 that disappearance probability for $\alpha_{22}$ decreases significantly if the norm factor is not applied. Therefore, the constraints on $\alpha_{22}$ is mainly coming from the disappearance channels.

![Constraints on $\alpha_{21}$ at DUNE and T2HK using the CC measurements. We also show the effects of combining CC and NC measurements at DUNE. Then we combine DUNE (CC+NC) with the CC measurements at T2HK.](image)

**FIG. 5:** Constraints on $\alpha_{21}$ at DUNE and T2HK using the CC measurements. We also show the effects of combining CC and NC measurements at DUNE. Then we combine DUNE (CC+NC) with the CC measurements at T2HK.

c. **Bound on $\alpha_{21}$**: From Fig.5, we observe that use of the normalization factor is not affecting the bounds on $\alpha_{21}$ like $\alpha_{11}$ and $\alpha_{22}$. But at DUNE, adding NC with CC, improves the constraints slightly. Combining T2HK with DUNE is improving the constraints further.
and at $3\sigma$, the combination can exclude all $\alpha_{21} \leq 0.04$ \footnote{On $\alpha_{21}$, tighter constraints can be achieved in short baseline experiments at Fermilab and related details can be found in \cite{55}.}.

![FIG. 6: Constraints on $\alpha_{33}$ at DUNE and T2HK using CC measurements. We have also combined CC measurements at T2HK with the CC+NC measurements at DUNE.](image)

**d. Bound on $\alpha_{33}$:** From Fig. 6, it is observed that NC measurements at DUNE is capable to improve the bounds on $\alpha_{33}$ compared to the CC measurements. Without the norm factor, combining NC with CC measurements at DUNE constraints $\alpha_{33}$ such that at $3\sigma$ CL all values of $\alpha_{33} \leq 0.92$ are excluded. Use of the norm factor alleviates the sensitivity as the marginalization over $\alpha_{22}$ cancels the effect of $\alpha_{33}$ as shown by the pink line in the right panel of Fig. 3. In case of T2HK, CC measurements do not improve the bounds on $\alpha_{33}$. DUNE CC measurements give better bounds on $\alpha_{33}$ than T2HK both with and without the norm factor due to the large matter effect. Combination of this with CC+NC measurements at DUNE slightly improves the bounds. The bound on $\alpha_{33}$ that comes from the combination is $\alpha_{33} \leq 0.925$ at $3\sigma$ CL.

**V. SUMMARY AND CONCLUDING REMARKS**

In this work, we attempt to derive constraints on non unitarity parameters at DUNE and T2HK specially focusing the NC measurements at DUNE. We have calculated the NC events in presence of both heavy and light sterile neutrino and have found that even in the averaged
out regime of light sterile neutrino, the NC events are different from heavy sterile case in the leading order. In this analysis, we have found that $\nu_e$ background is the most dominant component in the measurements of $\alpha_{11}$ and hence this parameter will be better bounded by this background than the signal. In case of $\alpha_{22}$, NC measurement helps in enhancing the bounds further. We have also found that combining both DUNE and T2HK can improve overall bounds on all the NU parameters. Finally, we have found that NC measurements at DUNE helps in deriving better bounds on $\alpha_{33}$ over the CC measurements.

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