On Power Allocation for Distributed Detection with Correlated Observations and Linear Fusion

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Abstract—We consider a binary hypothesis testing problem in an inhomogeneous wireless sensor network, where a fusion center (FC) makes a global decision on the underlying hypothesis. We assume sensors’ observations are correlated Gaussian and sensors are unaware of this correlation when making decisions. Sensors send their modulated decisions over fading channels, subject to individual and/or total transmit power constraints. For parallel-access channel (PAC) and multiple-access channel (MAC) models, we derive modified deflection coefficient (MDC) of the test statistic at the FC with coherent reception. We propose a transmit power allocation scheme, which maximizes MDC of the test statistic, under three different sets of transmit power constraints: total power constraint, individual and total power constraints, individual power constraints only. When analytical solutions to our constrained optimization problems are elusive, we discuss how these problems can be converted to convex ones. We study how correlation among sensors’ observations, reliability of local decisions, communication channel model and channel qualities and transmit power constraints affect the reliability of the global decision and power allocation of inhomogeneous sensors.

Index Terms—Distributed detection, coherent reception, modified deflection coefficient, power allocation, correlated observations, linear fusion, parallel-access channel, multiple-access channel.

I. INTRODUCTION

The classical problem of binary distributed detection in a network consisting of multiple distributed sensors and a fusion center (FC), has a long and rich history. Each sensor (local detector) processes its single observation locally and passes its binary decision to the FC, that is tasked with fusing the binary decisions received from the individual sensors and deciding which of the two underlying hypotheses is true [1]–[3]. Motivated by the potential application of wireless sensor networks (WSNs) for event monitoring, researchers have further studied this problem and extended its setup, taking into account that bandwidth-constrained communication channels between sensors and the FC are error-prone, due to limited transmit power to combat noise and fading (so-called channel aware binary distributed detection [4]–[6]). Given each sensor makes its binary decision based on one local observation, they have investigated how the reliability of the final decision at the FC is affected by performance indices of local detectors (sensors) as well as wireless channel properties. Following these works, we consider channel aware binary distributed detection in a WSN with coherent reception at the FC [7]–[9]. In this paper, our goal is to study transmit power allocation, when each sensor has an individual transmit power constraint and/or all sensors have a joint transmit power constraint, such that the reliability of the final decision at the FC is maximized.

Power allocation for channel aware binary distributed detection in WSNs has been studied in [10], [11]. More specifically, [10] studied the power allocation that maximizes the J-divergence between the distributions of the received signals at the FC under two different hypotheses, subject to individual and total transmit power constraints on the sensors, with parallel access channel (PAC) and coherent reception at the FC (i.e., channel phases are known and compensated at the sensors). Leveraging on [10], [11] studied detection outage and detection diversity, as the number of sensors goes to infinity, and sensors have identical performance indices. Note that [10], [11] assume the sensors have uncorrelated observations under each hypothesis.

Power allocation in WSNs has also been studied for distributed estimation [13]–[22], where some works minimized the mean square error (MSE) of an estimator subject to certain transmit power constraints [14]–[22], while others minimized total transmit power subject to a constraint on the MSE of an estimator [13], [22]. These works, except [13], [19], [20], mainly focus on PAC with coherent reception at the FC. In [13], [19], [20], sensors and the FC are connected differently via a multiple-access channel (MAC), where the individual sensors send their signals simultaneously, albeit after channel phases are compensated at the sensors, and the FC receives the coherent sum of these transmitted signals. Most of these works assume the sensors’ observations are uncorrelated, with the exception of [14], [16], [22]. In [19], [20] sensors collaborate with each other by linearly combining their independent observations before sending to the FC.

For binary distributed detection in WSNs, [12] compared the detection performance using both PAC and MAC, with linear fusion rule and noncoherent reception at the FC (i.e., no channel phase compensation at the sensors), albeit without imposing any transmit power constraint. Assuming the sensors’ observations are uncorrelated under each hypothesis and the FC utilizes a linear fusion rule when using PAC, [12] showed that coherent MAC outperforms coherent PAC, whereas noncoherent PAC (MAC) outperforms noncoherent MAC (PAC) when sensors’ decisions are (un)reliable. Distributed detection with correlated observations has been studied assuming error-free [3], [23], [24] and erroneous communication channels [25]. The focus of these works though is on how to design optimal local and global decisions rules to improve the detection reliability at the FC, assuming sensors know the correlation among their observations. Different from [3], [23]–[25], we focus on how to optimally transmit the sensors’ decisions to

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the FC within certain transmit power constraints, with a linear fusion rule at the FC and assuming sensors are unaware of the correlation among their observations.

Our Contributions: We consider a binary hypothesis testing problem using $M$ sensors and a FC, where under $H_0$, sensors’ observations are uncorrelated Gaussian with covariance matrix $\sigma_0^2 I$ and under $H_1$ they are correlated Gaussian with a non-diagonal covariance matrix $\Sigma$. We relax the assumption in [3], [23], [25] that sensors know the correlation among their observations and consider a more practical scenario, where the sensors are unaware of such correlation. Sensors send their modulated binary decisions over nonideal fading channels, subject to individual and/or total transmit power constraints.

We consider PAC and MAC with coherent reception at the FC, assuming that channel phases are compensated at the sensors, we have 

$$
\sigma_k = \begin{cases} 
\sigma_{0_k} & \text{if } k \text{ is in } H_0 \\
\sigma_{1_k} & \text{if } k \text{ is in } H_1 
\end{cases}
$$

Also, an MDC-based optimization problem can lead into near-optimal solutions for its corresponding detection. We propose a transmit power allocation scheme, which maximizes modified deflection coefficient (MDC) of $T$. We choose MDC as the performance metric, since unlike detection probability and J-divergence that require the probability distribution function of $T$, obtaining MDC only needs the first and second order statistics of $T$, and often renders a closed-form expression [26], [28]. Also, an MDC-based optimization problem can lead into near-optimal solutions for its corresponding detection probability-based optimization problem with much less computational complexity [7], [26], [29]. We obtain the MDC of $T$ for coherent PAC and MAC in closed-forms that depends on the correlation among sensors’ observations. Considering three different sets of transmit power constraints, we investigate transmit power allocation schemes that maximize the MDC. Under the conditions that analytical solutions to our constrained optimization problems are elusive, we discuss how these problems can be converted to convex ones and thus can be solved numerically.

Paper Organization: Section II details our system model and three different sets of transmit power constraints. Section III derives the MDC of $T$ for coherent PAC and MAC in closed-form expressions. Section IV formulates three different sets of transmit power constraints, we investigate linear fusion rules that lead into near-optimal solutions for its corresponding detection. We propose a transmit power allocation scheme, which maximizes modified deflection coefficient (MDC) of $T$. We choose MDC as the performance metric, since unlike detection probability and J-divergence that require the probability distribution function of $T$, obtaining MDC only needs the first and second order statistics of $T$, and often renders a closed-form expression [26], [28]. Also, an MDC-based optimization problem can lead into near-optimal solutions for its corresponding detection probability-based optimization problem with much less computational complexity [7], [26], [29]. We obtain the MDC of $T$ for coherent PAC and MAC in closed-forms that depends on the correlation among sensors’ observations. Considering three different sets of transmit power constraints, we investigate transmit power allocation schemes that maximize the MDC. Under the conditions that analytical solutions to our constrained optimization problems are elusive, we discuss how these problems can be converted to convex ones and thus can be solved numerically.

Notations: Scalars, vectors and matrices are denoted by non-boldface lower, boldface lower, and boldface upper case letters, respectively. A Gaussian random vector $x$ with mean vector $\mu$ and covariance matrix $\Sigma$ is shown as $x \sim N(\mu, \Sigma)$. Transpose and complex conjugate transpose (Hermitian) of vector $a$ are denoted as $a^T$ and $a^H$, respectively. $\text{diag}(a)$ represents a diagonal matrix whose diagonal elements are the components of column vector $a$. $A \succ 0$ ($A \succeq 0$) indicates that $A$ is a positive (semi-)definite matrix. $a \succeq b$ ($a \succeq b$) indicates that each entry of $a$ is greater than (or equal to) the corresponding entry of $b$. $\Re\{x\}$ is the real part of $x$.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Our system model consists of an FC and $M$ distributed sensors with observation vector $x = [x_1, x_2, ..., x_M]^T$. The FC is tasked with solving the binary hypothesis testing problem $H_0 : x \sim N(0, \sigma_0^2 I), H_1 : x \sim N(0, \Sigma)$, where $\sigma_0$ is the variance under $H_0$ and $\Sigma$ is a non-diagonal covariance matrix under $H_1$ with diagonal entries different from $\sigma_0$, i.e., under $H_1$ (H_0) sensors’ observations are correlated (uncorrelated) Gaussian variables with different energy levels. Suppose sensor $k$, only based on its own observation $x_k$, makes a binary decision [5], [10], [12]. Also, an MDC-based optimization problem can lead into near-optimal solutions for its corresponding detection. We propose a transmit power allocation scheme, which maximizes modified deflection coefficient (MDC) of $T$. We choose MDC as the performance metric, since unlike detection probability and J-divergence that require the probability distribution function of $T$, obtaining MDC only needs the first and second order statistics of $T$, and often renders a closed-form expression [26], [28]. Also, an MDC-based optimization problem can lead into near-optimal solutions for its corresponding detection probability-based optimization problem with much less computational complexity [7], [26], [29]. We obtain the MDC of $T$ for coherent PAC and MAC in closed-forms that depends on the correlation among sensors’ observations. Considering three different sets of transmit power constraints, we investigate transmit power allocation schemes that maximize the MDC. Under the conditions that analytical solutions to our constrained optimization problems are elusive, we discuss how these problems can be converted to convex ones and thus can be solved numerically.

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Paper Organization: Section II details our system model and three different sets of transmit power constraints. Section III derives the MDC of $T$ for coherent PAC and MAC in closed-form expressions. Section IV formulates three different sets of constrained optimization problems and describes our approach to solve these problems. Section V presents our numerical results for different correlation values, sensors’ observations and communication channel qualities. Section VI concludes the paper.

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We consider coherent PAC and MAC with channel phase compensation at the sensors \([5, 10]\). Our goal is to find the transmit powers at sensors such that the MDC of \(T\) is maximized, subject to different sets of power constraints. We refer to these as the \(MDC\)-based transmit power allocation. We consider three different sets of transmit power constraints: (A) there is a total power constraint (TPC) such that \(\sum_{k=1}^{M} P_{tk} \leq P_{tot}\), where \(P_{tot}\) is the total transmit power budget among sensors, we refer to this set as TPC; (B) there is an individual power constraint (IPC) for each sensor such that \(0 \leq P_{tk} \leq P_{0k}\) as well as a TPC \(\sum_{k=1}^{M} P_{tk} \leq P_{tot}\), where \(P_{tot} < \sum_{k=1}^{M} P_{0k}\), we refer to this set as TIPC; (C) there are only IPCs for sensors such that \(0 \leq P_{tk} \leq P_{0k}\), we refer to this set as IPC.

Section III drives the MDC of \(T\) for coherent PAC and MAC. The \(MDC\)-based transmit power allocations under these three different sets of power constraints are discussed in Section IV.

### III. Deriving Modified Deflection Coefficient

Before delving into the derivations, we introduce the following definitions and notations. Consider the signal model in (1) and (2). We let \(a_k = \sqrt{P_k}\), \(w_k = \text{Re}(n_k)\), \(w = \text{Re}(n)\). We define the column vectors \(h = [h_1, ..., h_M]^T\), \(u = [u_1, ..., u_M]^T\), \(y = [y_1, ..., y_M]^T\), \(a = [a_1, ..., a_M]^T\), \(w = [w_1, ..., w_M]^T\), \(\mathbf{n} = [n_1, ..., n_M]^T\), \(\mathbf{p}_d = [p_{d1}, ..., p_{dM}]^T\), \(\mathbf{p}_f = [p_{f1}, ..., p_{fM}]^T\), \(\mathbf{u} = [u_1, u_2, ..., u_M]^T\), \(\mathbf{P} = [\mathbf{P}_1, ..., \mathbf{P}_M]^T\), \(\psi = [\psi_1, ..., \psi_M]\), \(\phi = [\phi_1, ..., \phi_M]\), and the square matrix \(H = \text{DIAG}\{h_i\}\).

We define the MDC of \(T\) as 

\[
MDC = \frac{(E[T|H, h] - E[T|0, h])^2}{E[T|H, h]},
\]

where \(E\{\cdot\}\) and \(\text{Var}\{\cdot\}\) are performed with respect to the channel inputs \(u_k\')s\) and the channels. To calculate \(E[T|H, h]\) for \(i = 0, 1\) and \(\text{Var}[T|H, h]\) in (3), we use the Bayes rule and the fact that \(h_i \rightarrow u_k \rightarrow y_k(y) \rightarrow u_0\) in PAC(MAC) form Markov chains for \(i = 0, 1\). Hence

\[
E[T|H, h] = \sum_u E[T|u, h]P(u|H, i),
\]

\(i = 0, 1\) (4)

\[
\text{Var}[T|H, h] = \sum_u \Delta P(u|H, i),
\]

and the sums are taken over all values of vector \(u\). To simplify \(\Delta\) in (5), we add and subtract \(E[T|u, h]\) to the terms inside the parenthesis in (5) and expand the products. We have

\[
\Delta = \frac{\{T - E[T|H, h]\}^2|u, h\} + \{E[T|u, h] - E[T|H, h]\}^2}{\{T - E[T|u, h]\}^2|u, h\} + 2E[T - E[T|u, h]|E[T|u, h] - E[T|H, h]|u|}
\]

We observe that the last term in (6) is zero. Thus \(\Delta\) in (6) is

simplified to \(\Delta = \Delta + \Delta'\). Using (4), (6) and (6), we derive the MDC in the following.

### A. PAC

Considering the signal model in (1) and (2), we have \(\text{Re}(y_k) = a_k|h_k|u_k + w_k\) where \(w_k \sim N(0, \frac{Q}{2})\). We write \(T = a^T H(u + 1^T u)\). Therefore \(E[T|u, h] = a^T H|u\). Substituting \(E[T|u, h]\) into (4) and using the facts \(P_d = E[u|H_i] = \sum_u u P(u|H_i)\) and \(p_f = E[u|H_0] = \sum_u u P(u|H_0)\) we find

\[
E[T|H, h] = a^T H|p_d, \quad \text{and} \quad E[T|H, h] = a^T H|p_f.
\]

Next, we derive \(\Delta, \Delta'\) for \(\text{Var}[T|H, h]\). Since \(T - E[T|u, h] = 1^T u\), we find \(\Delta = E[1^T uu^T 1] = M \sigma^2\). Also, because \(E[T|u, h] - E[T|H, h] = a^T H|u - p_d\), we have \(\Delta' = a^T H|u - p_d| H|a\). Substituting \(\Delta = \Delta + \Delta'\) into (6) and using the facts \(\sum_u u P(u|H_i) = 1\)

\[
\sum_u (u - p_d)(u - p_d)^T P(u|H_i) = E[uu^T|H_i] - p_d p_d^T
\]

we reach to

\[
\text{Var}[T|H, h] = M \frac{\sigma^2}{2} + a^T H|\{P_d - p_d p_d^T\}|H|a.
\]

Substituting (7), (8) into (3) we have

\[
\text{MDC}(a) = a^T bb^T a \frac{a}{a^T K a + c}
\]

B. MAC

Considering the signal model in (1) and (2), we have \(\text{Re}(y) = \sum_{k=1}^{M} a_k|h_k|u_k + w\) where \(w \sim N(0, \frac{Q}{2})\). We write \(T = a^T H|u + w\). Therefore \(E[T|u, h] = a^T H|u\). Substituting \(E[T|u, h]\) into (4) and applying similar facts as stated above, we find

\[
E[T|H, h] = a^T H|p_d, \quad \text{and} \quad E[T|H, h] = a^T H|p_f.
\]

Next, we find \(\Delta, \Delta'\). Since \(T - E[T|u, h] = w\), we find \(\Delta = E[w^2] = \frac{\sigma^2}{2}\). Also, since \(E[T|u, h] - E[T|H, h] = a^T H|u - p_d\), we have \(\Delta' = a^T H|u - p_d| H|a\). Substituting \(\Delta = \Delta + \Delta'\) into (6) and using similar facts as stated above we reach

\[
\text{Var}[T|H, h] = \frac{\sigma^2}{2} + a^T H|\{P_d - p_d p_d^T\}|H|a.
\]

Substituting (11), (12) into (3) we have

\[
\text{MDC}(a) = a^T bb^T a \frac{a}{a^T K a + c}
\]
where

\[
b = |H|(p_d - p_f), \quad c = \frac{a_n^2}{2}, \quad K = |H|(p_d - p_a p_d^T)|H|
\]

Regarding the results in [10] and [13], a remark follows.

**Remark:** For both PAC and MAC, the MDC takes the following form

\[
\text{MDC}(a) = \frac{a^T b b^T a}{a^T K a + c}.
\]  

(14)

Vector \( b \) and matrix \( K \) are identical for PAC and MAC, whereas scalar \( c \), which captures the effect of the channel noises, is \( M \) times larger in PAC. Note that \( b \) and \( K \) depend on the channel amplitudes \( |H| \) and the local performance indices. Furthermore, \( K \) depends on the spatial correlation among sensors’ observations.

**IV. MDC-BASED TRANSMIT POWER ALLOCATION**

Recall \( P_k = P_t \theta_k \) where \( P_t \) is transmit power of sensor \( k \) and \( \theta_k \) captures the pathloss effect. Since \( \theta_k = \sqrt{P_t} \), we define \( a_k = \sqrt{P_t} \frac{a_k}{\sqrt{a_k^T a_k}} \). Let \( a_t = [a_1, ..., a_M]^T \), \( P_t = [P_{t1}, ..., P_{tM}]^T \), and \( \sqrt{\Theta} \) be the component-wise square root of \( \Theta = \text{DIAG}([\theta_1, ..., \theta_M]^T) \). We can rewrite (14) explicitly in terms of vector \( a_t \) as

\[
\text{MDC}(a_t) = \frac{a_t^T b b^T a_t}{a_t^T K a_t + c},
\]  

(15)

where \( b_t = \sqrt{\Theta} b \) and \( K_t = \sqrt{\Theta} K \sqrt{\Theta} \). In this section, we maximize the MDC in (15), with respect to \( a_t \), subject to different sets of power constraints specified in Section [II] (A) TPC, where \( a_t^T a_t \leq P_{t\text{tot}} \); (B) TIPC, where \( a_t^T a_t \leq P_{t\text{tot}} \) and \( 0 \leq a_t \leq \sqrt{P_0} \). We define vector \( P_0 = [P_{01}, ..., P_{0M}]^T \) and \( \sqrt{P_0} \) is the component-wise square root of \( P_0 \); (C) IPC, where \( 0 \leq a_t \leq \sqrt{P_0} \). Sections [IV-A] [IV-B] [IV-C] discuss the analytical solutions for MDC-based power allocations under these different sets of power constraints.

**A. Maximizing MDC in (15) under TPC**

The MDC-based transmit power allocation under TPC is the solution to the following problem

\[
\begin{align*}
\max_{a_t} & \quad \frac{a_t^T b b^T a_t}{a_t^T K a_t + c} \quad (O_1) \\
s.t. & \quad a_t^T a_t \leq P_{t\text{tot}} \\
& \quad a_t \succeq 0
\end{align*}
\]

We start with Lemma 1, which states that the solution to (O_1) satisfies TPC at equality.

**Lemma 1.** The maximum values of MDC in (15) are achieved when the inequality constraint \( a_t^T a_t \leq P_{t\text{tot}} \) turns into equality constraint.

**Proof:** Suppose \( a_{t1} \) maximizes MDC and \( a_{t2}^T a_t \leq P_{t\text{tot}} \). Define \( a_{t3} = \frac{a_{t2}}{\sqrt{a_{t2}^T a_{t2}}} \), which satisfies \( a_{t3}^T a_{t3} = \sqrt{P_{t\text{tot}}} \). We have

\[
\text{MDC}(a_{t2}) = \frac{a_{t2}^T b b^T a_{t2}}{a_{t2}^T K a_{t2} + c} \geq \frac{a_{t3}^T b b^T a_{t3}}{a_{t3}^T K a_{t3} + c} = \text{MDC}(a_{t3}),
\]

which contradicts the optimality assumption of \( a_{t1} \). i.e., the \( a_t \) that maximizes MDC must satisfy \( a_t^T a_t = P_{t\text{tot}} \).

When the inequality constraint in TPC is turned into equality constraint, we can rewrite MDC in (15) as

\[
\text{MDC}(a_t) = \frac{a_t^T b b^T a_t}{a_t^T Q_a a_t}, \quad \text{where } Q_a = K_t + \frac{c}{P_{t\text{tot}}},
\]  

(16)

Hence, \((O_1)\) reduces to

\[
\begin{align*}
\max_{a_t} & \quad \frac{a_t^T b b^T a_t}{a_t^T Q_a a_t} \quad (O_1') \\
s.t. & \quad a_t^T a_t = P_{t\text{tot}} \\
& \quad a_t \succeq 0
\end{align*}
\]

To analytically solve \((O_1')\), we use the result of Lemma 2 given below.

**Lemma 2.** For \( Q > 0 \) the function \( f(x) = \frac{x^T b b^T x}{x^T Q x} \) is maximized at \( x^* = Q^{-1} b \) and its non-zero scales.

**Proof:** See Appendix A

To be able to use Lemma 2 to solve \((O_1')\), we need to examine whether symmetric matrix \( Q_a \) is positive definite. Note \( P_d - p_d a_p^T \) is positive definite since it is a covariance matrix. Thus \( K_t, K_i > 0 \). Also \( \frac{c}{P_{t\text{tot}}} = I > 0 \). Therefore \( Q_a > 0 \). To solve \((O_1')\), we find \( q = Q_a^{-1} b_t \). If \( q \geq 0 \), we let \( a_t^* = q \sqrt{P_{t\text{tot}}} \). But if all the entries of \( q \) do not have the same sign, we resort to numerical solutions. In particular, we turn the problem \((O_1')\) into a convex problem and solve it numerically. We discuss these numerical solutions in Section [IV-D].

- **Analytical Solution to \((O_1')\) with Independent Observations:** \( Q_a \) is given in [9] and \( K \) simplifies to \( K = |H| \text{DIAG}(p_d) - I - \text{DIAG}(p_d^2) |H| \). Let \( g_k = \sqrt{\theta_k} [h_k] \). It is easy to verify \( Q_a \) is a diagonal matrix with diagonal entries \( Q_{a_{kk}} = p_d (1 - p_d g_k^T g_k) + \frac{c}{P_{t\text{tot}}}, \quad k = 1, ..., M \), which is positive for \( p_d > p_f \). We observe \( q_k \approx (p_d - p_f) g_k^T g_k \) for large \( \frac{P_{t\text{tot}}}{c} \), whereas \( q_k \approx \frac{P_{t\text{tot}}}{c} (p_d - p_f) g_k \) for small \( \frac{P_{t\text{tot}}}{c} \). For homogeneous sensors where \( p_f = p_f \) and \( p_d = p_d \), we find the MDC-based power allocation strategy as \( q_k \approx \frac{1}{g_k} \) for large \( \frac{P_{t\text{tot}}}{c} \) (inverse water filling) and \( q_k \propto g_k \) for small \( \frac{P_{t\text{tot}}}{c} \) (water filling).

**B. Maximizing MDC in (15) under TIPC**

The MDC-based transmit power allocation is the solution to the following problem

\[
\begin{align*}
\max_{a_t} & \quad \frac{a_t^T b b^T a_t}{a_t^T K a_t + c} \quad (O_2) \\
s.t. & \quad a_t^T a_t \leq P_{t\text{tot}} \\
& \quad 0 \leq a_t \leq \sqrt{P_0}
\end{align*}
\]

While analytical solution to \((O_2)\) remains elusive, we find sub-optimal power allocation via solving the following optimization problem

\[
\begin{align*}
\max_{a_t} & \quad \frac{a_t^T b b^T a_t}{a_t^T Q_a a_t} \quad (O_2') \\
s.t. & \quad a_t^T a_t = P_{t\text{tot}} \\
& \quad 0 \leq a_t \leq \sqrt{P_0}
\end{align*}
\]

where \( Q_a \) is given in (16). Note that \((O_2')\) is identical to \((O_2)\), except that the inequality in TPC is turned into equality, i.e.,
the feasible set of \((O'_2)\) is a subset of the feasible set of \((O_2)\) and the objective function of \((O_2)\) is rewritten accordingly. Indeed, this sub-optimal solution is an accurate solution when \(\kappa = \frac{P_\text{to} - P}{P_\text{to} + P} \ll 1\) for \((O_2)\), as we show in the following. Examining \(K\) and \(K_t\) when \(\kappa \ll 1\), we can establish the following inequalities
\[
\begin{align*}
\min \quad & a_t^T K_a a_t \quad \text{(c)} \\
\text{s.t.} \quad & a_t^T a_t \leq P_\text{to} \\
& 0 \leq a_t \leq \sqrt{P_0} 
\end{align*}
\]
where \((c)\) is obtained noting that all entries of \(P_d - p_d p_d^T\) are less than 1, \((b)\) is found using Cauchy-Schwarz inequality, \((c)\) comes from the inequality constraint in \((O_2)\), and \((d)\) is due to \(\kappa \ll 1\). This implies that when \(\kappa \ll 1\), \((O_2)\) can be approximated as \((O'_2)\) in [17].

In Appendix B we show that the solution to \((O'_2)\) satisfies the equality \(a_t^* a_t = P_\text{to}\). This confirms that the solution to \((O'_2)\) (sub-optimal solution) is an accurate substitute for the solution to \((O_2)\) under the condition \(\kappa \ll 1\). To solve \((O'_2)\), we first ignore the box constraints of IPC and consider only TPC at equality. The problem solving strategy is similar to Section IV-B, we show below that, when \(\xi = \frac{1}{\sqrt{P_0} g^T g} \ll 1\), \((O'_3)\) can be approximated with \((O'_3)\) in (18).

\[
\begin{align*}
\min \quad & a_t^T K_a a_t + c \\
\text{s.t.} \quad & 0 \leq a_t \leq \sqrt{P_0} 
\end{align*}
\]

We show that \((O'_3)\) is not convex. In Appendix C we show that, despite this fact, the solution to Karush-Kuhn-Tucker (KKT) conditions for \((O'_2)\) is unique.

C. Maximizing MDC in \((O'_2)\) under IPC

The MDC-based transmit power allocation is the solution to the following optimization problem
\[
\begin{align*}
\max \quad & \frac{a_t^T b_j^T b_j a_t}{a_t^T K_a a_t + c} \\
\text{s.t.} \quad & 0 \leq a_t \leq \sqrt{P_0}
\end{align*}
\]

Examining \(K\) and \(K_t\) when \(\xi \ll 1\), we can establish the following inequalities
\[
\begin{align*}
& a_t^T K_a a_t \leq a_t^T \sqrt{\Theta} H \{H^T H \sqrt{\Theta} a_t = a_t^T g g^T a_t \leq \Theta a_t \} < a_t^T a_t \Theta (\xi) \\
& (a_t^T a_t) (g^T g) \leq 1^T P \Theta g^T g < c,
\end{align*}
\]

where \((a)\) is because all entries of \(P_d - p_d p_d^T\) are less than 1, \((b)\) is found using Cauchy-Schwarz inequality, \((c)\) comes from the inequality in IPC, and \((d)\) is due to \(\xi \ll 1\). This implies that when \(\xi \ll 1\), \((O_3)\) can be approximated with \((O'_3)\) in (18).
formulated in Section IV-B remains elusive. Hence, we have provided a sub-optimal solution, via solving \( (O_2) \) that is accurate solution when \( \gamma < 1 \). Similarly, we have derived a sub-optimal solution to \( (O_3) \), formulated in Section IV-C that is accurate solution when \( \xi < 1 \). In this section, we turn \( (O_1),(O_2),(O_3) \) into convex optimization problems, in order to solve them numerically using CVX program.

We start with \( (O_2) \), in which we wish to minimize
\[
\frac{1}{MDC(a_i)} = \frac{a_i^2}{(b_i a_i)^2 + c},
\]
under TIPC. Let \( x_a = \frac{a_i}{b_i a_i} \) and \( t_a = \frac{1}{b_i a_i} \). Therefore \( b_i^t a_i = 1 \) and \( a_i = \frac{t_a}{t_a} \). Employing these definitions, \( (O_2) \) can be rewritten in the following equivalent form
\[
\begin{align*}
\min_{\substack{z_a, t_a}} & \quad x_a^T K_a x_a + c t_a^2 \\
\text{s.t.} & \quad x_a^T x_a \leq \mathcal{P}_{tot} t_a^2 \\
& \quad x_a \leq t_a \sqrt{\mathcal{P}_0} \\
& \quad 0 \leq x_a \\
& \quad b_i^T a_i x_a = 1
\end{align*}
\] (21)

We can reformulate \( (O_2) \) as
\[
\begin{align*}
\min_{z_a} & \quad z_a^T D_a z_a \\
\text{s.t.} & \quad z_a^T \begin{bmatrix} I & 0 \\
0 & 0^T & -\mathcal{P}_{tot}
\end{bmatrix} z_a < 1 \\
& \quad \begin{bmatrix} I & 0 \end{bmatrix}^T z_a \geq 0 \\
& \quad [b_i, 1]^T z_a = 1, \quad \gamma \geq 0
\end{align*}
\]
where
\[
\begin{align*}
D_a = \begin{bmatrix} K_a & 0 \\
0 & c
\end{bmatrix}
\end{align*}
\]

Examining \( (O_2) \), we realize that it is a quadratic programming (QP) convex program since \( D_a \succeq 0 \) and hence it can be solved using CVX program. One can take similar steps to turn \( (O_1) \) and \( (O_3) \) into a problem whose optimal solution can be found using CVX program. In particular, we formulate \( (O_{1a}),(O_{1b}) \) via deleting the second inequality constraint corresponding to IPC and \( (O_{3a}),(O_{3b}) \) by removing the first inequality constraints corresponding to TPC from \( (O_2) \), \( (O_3) \), respectively. Since \( D_a \succeq 0 \), \( (O_{1a}) \) and \( (O_{3a}) \) are also QP convex problems.

V. Numerical Results

In this section, through simulations, we corroborate our analytical results. We study the effect of correlation between sensors’ observations on the MDC, the performance improvements achieved by the MDC-based transmit power allocations (we refer to as “DPA”), and the impact of different sensing and communication channels on DPA. For our simulations, we consider the signal model \( \mathcal{H}_k : z_k = s_k, \mathcal{H}_z : x_k = s_k + z_k \) for \( k = 1, ..., M \), where \( z_k \sim \mathcal{N}(0, \sigma_z^2) \) and \( s_k \sim \mathcal{N}(0, \sigma_s^2) \) is a sample of an external Gaussian signal source \( s \sim \mathcal{N}(0, \sigma_s^2) \). We assume \( \sigma_s^2 = \frac{\gamma^2}{d_{sk}^2} \) where \( d_{sk} \) is the distance between sensor \( k \) and \( s \) and \( \epsilon_k \) is the pathloss exponent. We assume \( z_k \) and \( s_k \) are mutually uncorrelated, however, \( s_k, s_k \) are correlated. Let \( s = [s_1, s_2, ..., s_M]^T \) have covariance matrix \( K_s = \mathbb{E}\{ss^T\} \). We assume \( K_{sij} = \rho_{ij} \sqrt{\sigma_s^2 a_{ij}^2} \) where \( \rho_{ij} = \rho_{dij}, 0 \leq \rho \leq 1 \) is the correlation at unit distance and depends on the environment and \( d_{ij} \) is the distance between sensors \( i \) and \( j \). Each sensor employs an energy detector that maximizes \( p_{d_k} \), under the constraint \( p_{f_k} < 0.1 \). Sensors are deployed at equal distances from each other, on the circumference of a circle with diameter 5m on the x-y plane, where the coordinate of its center is \((0, 0, 0)\). For sensing part, we assume \( M = 8, \epsilon_2 = 2, \sigma_s^2 = 5 \text{dBm}, \sigma_d^2 = -70 \text{dBm}, \) and for communication part we let \( \sigma_n^2 = -70 \text{dBm}, G = -55 \text{dB} \).}

Performance of DPA when \( p_{d_k} \)'s and pathloss are identical: Suppose the coordinates of signal source \( s \) and the FC, respectively, are \((0, 0.3m), (0, 0, -10m)\). With this configuration, \( p_{d_k} = 0.6615, \forall k \) and pathloss are identical. We assume \( h_k \sim \mathcal{N}(0, 1), \forall k \) and we average over 10,000 number of channel realizations to obtain the results. We explore the MDC enhancements achieved by DPA and compare the MDC values with those of obtained by uniform power allocation (we refer to as “UPA”), in which sensors transmit at equal powers.

Fig. 1(a) compares optimal power allocation (OPA), which finds the sensors’ powers that maximize \( P_D \) under the constraint \( P_F < \beta_F \), DPA and UPA, for linear fusion rule and the optimal LRT rule. To find OPA with both linear and the LRT rules and DPA with the LRT rule, we use brute force search to find the power values, and we simplify the network and only consider \( s_1 \) and \( s_5 \). We assume \( \rho = 0.1, \beta = 0.1 \) and plot \( P_D \) versus \( P_{tot} \) under TPC for PAC. Furthermore, we observe that at low \( P_{tot} \), they are close to each other but as \( P_{tot} \) increases, they diverge and DPA outperforms UPA but performs worse than OPA. Fig. 1(b) compares OPA, DPA and UPA, given linear fusion rule. We observe that at low \( P_{tot} \), they are close to each other but as \( P_{tot} \) increases, they diverge and DPA outperforms UPA but performs worse than OPA. Fig. 1(c) shows DPA, UPA and MAC, given linear fusion rule. We observe that at low \( P_{tot} \), MAC is close to DPA and UPA.

Figs. 2 and 3 show \( P_D \) and maximized MDC versus \( P_{tot} \), respectively, under TPC for PAC and MAC, \( \rho = 0.1, 0.9, \) and \( \beta_F = 0.05 \). Comparing Figs. 2 and 3, we observe that the MDC and \( P_D \) follow similar trends. Hence, to make our computations faster and less complex, in the rest of this section we only calculate the MDC. From Fig. 3, we note that the MDC increases by increasing \( P_{tot} \) or by decreasing \( \rho \). Comparing PAC and MAC, we note that MAC outperforms PAC at low \( P_{tot} \), whereas PAC converges to MAC at high \( P_{tot} \). These are due to the facts that, at low \( P_{tot} \), the effect of communication channel noise characterized by \( c \) in the MDC expression of PAC is \( M \) times larger than that of MAC (see equations [10] and [11]), and thus MAC outperforms PAC. However, at high \( P_{tot} \), this difference in \( c \) values is negligible and hence PAC converges to MAC. We also observe that, at low \( P_{tot} \), the performance gaps corresponding to DPA and UPA are negligible, despite the fact that sensors experience different communication channel fading. This is because at low \( P_{tot} \) the dominant effect of communication channel noise...
renders the decisions of sensors equally important to the FC, regardless of the channel realizations and the actual (different) decisions. On the other hand, at high $\mathcal{P}_{\text{tot}}$ the performance gaps corresponding to DPA and UPA are significant. Note that this performance gap in MAC is wider than that of PAC. This is expected, since the larger $c$ value in PAC undermines the differences between sensors and narrows the performance gap between DPA and UPA. As $\rho$ increases, the chances that sensors make similar decisions increase and therefore the performance gaps between DPA and UPA shrink.

Fig. 4 shows the MDC maximized under TIPC versus $\mathcal{P}_{\text{tot}}$ for PAC and MAC, $\bar{\mathcal{P}} = 30$ mW, and $\rho = 0.1, 0.9$. Similar to Fig. 3, the MDC increases by increasing $\mathcal{P}_{\text{tot}}$ or decreasing $\rho$ and MAC outperforms PAC. We also compare the MDC obtained from solving (O2) and (O3), in which we have the inequality constraint (I) $|\alpha_i| \leq \mathcal{P}_{\text{tot}}$ and the equality constraint (E) $|\alpha_i| = \mathcal{P}_{\text{tot}}$ respectively. We observe that at low $\mathcal{P}_{\text{tot}}$, there is no performance gap corresponding to “DPA with E” and “DPA with I”, whereas at high $\mathcal{P}_{\text{tot}}$, the performance of “DPA with E” degrades from that of “DPA with I”. This performance degradation in MAC is due to the increasing interference of sensors’ decisions at the FC when sensors are assigned higher transmit power. At very high $\mathcal{P}_{\text{tot}}$ the performance of “DPA with E” reduces to that of UPA. This is because the maximum value that $\mathcal{P}_{\text{tot}}$ can assume is $M\mathcal{P}$.

Hence, at very high $\mathcal{P}_{\text{tot}}$ we have $|\alpha_i|^2 = M\mathcal{P}$, implying that $a_{t_k, \forall k} = \sqrt{M\mathcal{P}}$, Fig. 5 shows the MDC maximized under IPC versus $\mathcal{P}$ for PAC and MAC and $\rho = 0.1, 0.9$. We note that at low $\mathcal{P}$, the performances of DPA and UPA are similar, since $a_{t_k, \forall k} = \sqrt{\mathcal{P}_{\text{tot}}}$ across sensors increase: for the case when the MDC is maximized under IPC, the resulting $\mathcal{P}_{\text{tot}}$ is right below sensor $S_1$. With this configuration, $\mathcal{P}_{\text{tot}} = 0.6615$, $\forall k$, whereas the pathloss are different (note that the pathloss corresponding to $S_1$, $S_2$ and $S_3$ are the three smallest). We observed that $\mathcal{P}_{\text{tot}}$’s for different $\rho$ values remain the same. Hence, in this part we focus on $\rho = 0.1$. DPA is shown in figures 8 and 9 for MAC and PAC, respectively. We observe that, for both PAC and MAC under TPC or TIPC, sensors with larger pathloss are assigned higher $\mathcal{P}_{t_k}$ (we refer to as inverse water filling). Examining the case when the MDC is maximized under IPC and $\bar{\mathcal{P}} = 4$ mW, we have $\mathcal{P}_{t_k} = \bar{\mathcal{P}}, \forall k$ (UPA) in PAC, whereas sensors with larger pathloss are assigned higher $\mathcal{P}_{t_k}$ (inverse water filling) in MAC. This is due to the fact that the $c$ value in MAC is smaller and therefore, the effective received signal-to-noise ratio in MAC is larger, leading to variations of $\mathcal{P}_{t_k}$’s across sensors. To investigate more the effect of different pathloss on DPA, we move the FC further from the sensors and change its coordinate to $(2.5m, 0, -10m)$, to effectively increase the pathloss between all the sensors and the FC (and decrease received power at the FC), while still $S_1$, $S_2$ and $S_3$ have the three smallest pathloss. We observe that in TPC and TIPC sensors with smaller pathloss are assigned higher $\mathcal{P}_{t_k}$ (we refer to as water filling), whereas in IPC, $\mathcal{P}_{t_k} = \bar{\mathcal{P}}, \forall k$ (we have UPA).

VI. CONCLUSION

We considered a channel aware binary distributed detection problem in a WSN with coherent reception and linear fusion rule at the FC, where observations are correlated Gaussian and sensors are unaware of such correlation when making decisions. Assuming that the sensors and the FC are connected via PAC or MAC, we studied power allocation schemes that maximize the MDC at the FC. Our numerical results suggest that when MDC-based power allocation and optimal transmit power allocation are employed at low $\mathcal{P}_{\text{tot}}$, the resulting $P_{D_0}$ is very close for both linear fusion rule and the LRT rule. For homogeneous sensors with identical pathloss, MAC outperforms PAC at low $\mathcal{P}_{\text{tot}}$ under TPC and TIPC (low $\bar{\mathcal{P}}$ under IPC), whereas PAC converges to MAC at high $\mathcal{P}_{\text{tot}}$. Compared with equal power allocation, performance enhancement offered by the MDC-based power allocation is more significant in MAC and this improvement reduces as correlation increases. For inhomogeneous sensors with identical pathloss, sensors with more reliable decisions are assigned higher powers. As
correlation increases, the variations of power across sensors increase: sensors with more (less) reliable decisions, are assigned higher (lower) powers. For homogeneous sensors with different pathloss, power allocations are invariant as correlation changes. At low (high) received power at the FC, sensors with smaller (larger) pathloss are assigned higher powers under TPC and TIPC.

APPENDIX

A. Proof of Lemma 2

Consider $Q = DAD^T$, where $A = \text{DIAG}([\lambda_1, ..., \lambda_M]^T)$ and $\lambda_k$’s are the positive eigenvalues of $Q$ and columns of $D$ are the eigenvectors of $Q$. We can rewrite $f(x)$ as $f(x) = \frac{|x^T(DAD^T)x|^2}{x^TD^TAD^T x} = \bar{a}x^Tb_0^T x$, where $\bar{x} = \sqrt{\Lambda}D^T x$ and $b_0 = (D\sqrt{\Lambda})^{-1}b_t$. Using the Rayleigh Ritz inequality [50], we find $f(x) \leq \lambda_{\text{max}}(b_0b_0^T)$ and the equality is achieved when $\bar{x}$ is the corresponding eigenvector of $\lambda_{\text{max}}(b_0b_0^T)$. Since $b_0b_0^T$ is rank-one with the eigenvalue $|b_0|^2$ and the eigenvector $b_0$, we have $f(x) \leq b_0^T b_0 = b_0^T Q b_0$ and the equality is achieved at $\bar{x} = b_0$ or $x = Q^{-1}b_0$ and its non-zero scales.

B. Proving that solution of $(O_2^t)$ satisfies TPC at the equality

Consider $(O_2^t)$ and let $\mu$ and $\psi$, be the Lagrange multipliers corresponding to $a_k^2 a_k \leq P_{tot}$, $a_k \geq \sqrt{P_0}$ and $a_k \geq 0$, respectively. The KKT conditions are

$$-2\mu b_0 b_0^T a_t = -2\mu a_t + \psi - \phi_k = 0, \quad k = 1, ..., M \quad (22)$$

$$\mu (a_k^2 a_k - P_{tot}) = 0, \quad \mu \geq 0, \quad \mu a_k^2 a_k \leq P_{tot} \quad (23)$$

$$\psi(k a_t - \sqrt{P_0}) = 0, \quad \psi \geq 0, \quad a_t \leq \sqrt{P_0} \quad , \quad \phi(k a_t = 0, \quad \phi \geq 0, \quad a_t \geq 0 \quad (24)$$

We show $\mu \neq 0$. Substituting $\mu = 0$ in (22), we have $-2\mu b_0 b_0^T a_t = \phi_k - \psi_k < 0$. Note that $\psi_k$ and $\phi_k$ cannot be both positive, since from (24) it is infeasible to have $a_t = \sqrt{P_0}$ and $a_t = 0$. Therefore $\psi_k$ or $\phi_k$ must be zero. Since $\phi_k - \psi_k < 0$, we conclude that $\phi_k = 0$ and $\psi_k > 0$. Now, from (24) we have $a_t = \sqrt{P_0}$, leading to $a_k^2 a_k = \sum_{k=1}^M P_0 \geq P_{tot}$, which contradicts (23). Therefore, $\mu \neq 0$ and we have $a_k^2 a_k = P_{tot}$.

C. Analytical Solution of $(O_2^t)$

Since $a_k^2 a_k = P_{tot}$, the objective function in $(O_2^t)$ reduces to $P_{tot} - a_k^2 a_k$. Let $\mu$ and $\psi$ and $\phi$ be the Lagrange multipliers corresponding to $a_k^2 a_k = P_{tot}$, $a_k \geq \sqrt{P_0}$ and $a_k \geq 0$. The KKT conditions are

$$2\mu a_t - a_{tk}^2 + \psi_k - \phi_k = 0, \quad k = 1, ..., M \quad (25)$$

$$\mu (a_k^2 a_k - P_{tot}) = 0, \quad \mu \geq 0, \quad \mu a_k^2 a_k \leq P_{tot} \quad (23)$$

$$\psi_k(a_k - \sqrt{P_0}) = 0, \quad \psi \geq 0, \quad a_k \leq \sqrt{P_0} \quad , \quad \phi(k a_t = 0, \quad \phi \geq 0, \quad a_t \geq 0 \quad (24)$$

Solving the KKT conditions for $a_{tk} > 0$ yields

$$a_{tk} = \begin{cases} \frac{a_{tk}^2}{2\mu}, & \text{for } \mu \geq \frac{a_{tk}^2}{2\sqrt{P_0}} \\ \sqrt{P_0}, & \text{for } \mu < \frac{a_{tk}^2}{2\sqrt{P_0}}. \end{cases} \quad (25)$$

To find positive $\mu$ and consequently $a_{tk}$, suppose sensors are sorted such that $a_{tk}^2 \geq a_{tk+1}^2$. Assume for $1 \leq m \leq M$ we have $a_{tk} = \sqrt{P_0}$ and $a_{tk+1} = \sqrt{P_0}$. Substituting (25) into $\sum_{j=1}^m a_{tk}^2 = P_{tot}$ and solving for $\mu$, we find $\mu = \frac{\sum_{j=m+1}^M a_{tk+1}^2}{P_{tot} - \sum_{j=1}^m P_0}$, $\mu = \frac{a_{tk+1}^2}{\sqrt{P_0}} \leq \mu \leq \frac{a_{tk}^2}{\sqrt{P_0}}$, the above assumption is valid, and we substitute $\mu$ in (25) to calculate $a_{tk+1}$. Otherwise, we increase $m$ by one and repeat the procedure, until we reach $\mu$ that lies within the proper interval. Although $(O_2^t)$ is not convex, we show below that the KKT solution in (25) is unique. Suppose the solution in (25) is not unique, i.e., there exist $1 \leq m, m' \leq M$, $m' \geq m + 1$ such that

$$a_{tk} = \begin{cases} \frac{a_{tk}^2}{2\mu}, & \text{for } 1 \leq j \leq m, \mu \geq \frac{a_{tk}^2}{2\sqrt{P_0}} \\ \sqrt{P_0}, & \text{for } 1 \leq j \leq m, \mu < \frac{a_{tk}^2}{2\sqrt{P_0}} \end{cases} \quad (26)$$

Also, $a_{tk}$ can be obtained from (26) by substituting $j, m, \mu$ with $j', m', \mu'$ respectively. Since $m' \geq m + 1$, from (26), we have $\mu' \geq \frac{a_{tk+1}^2}{\sqrt{P_0}}$. On the other hand, due to sensor ordering, we have $\mu' \leq \frac{a_{tk+1}^2}{\sqrt{P_0}}$. Applying the mediant inequality, we obtain $\sum_{j=m+1}^{m'} a_{tk}^2 \geq \sum_{j=m+1}^{m'} a_{tk+1}^2$. We observe that two different fractions are greater than or equal to the $a_{tk+1}^2 \geq \sqrt{P_0}$. Hence, using the mediant inequality and definition of $\mu'$, we find

$$\frac{\sum_{j=m+1}^{m'} a_{tk}^2 - \sum_{j=m+1}^{m'} a_{tk+1}^2}{4(P_{tot} - \sum_{j=1}^{m+1} P_0) - \sum_{j=m+1}^{m'} P_0) } = \mu' \geq \frac{a_{tk+1}^2}{4\sqrt{P_0}}$$

However, this inequality contradicts the one in (26) when $j, \mu$ are replaced with $j', \mu'$. Hence, our assumption regarding the existence of $m, m'$ is incorrect and the solution in (25) is unique.

D. Proving that solution of $(O_2^t)$ is equal to IPC upper limit

Consider $(O_2^t)$ and let $\psi$, $\phi$, be the Lagrange multipliers corresponding to $a_k \geq \sqrt{P_0}$ and $a_k \geq 0$, respectively. The KKT conditions are

$$-2\mu b_0 b_0^T a_t = -2\mu a_t + \psi - \phi_k = 0, \quad k = 1, ..., M \quad (22)$$

$$\mu (a_k^2 a_k - P_{tot}) = 0, \quad \mu \geq 0, \quad \mu a_k^2 a_k \leq P_{tot} \quad (23)$$

$$\psi(a_k - \sqrt{P_0}) = 0, \quad \psi \geq 0, \quad a_k \leq \sqrt{P_0} \quad , \quad \phi(k a_t = 0, \quad \phi \geq 0, \quad a_t \geq 0 \quad (24)$$

Solving the KKT conditions for $a_{tk} > 0$ yields

$$a_{tk} = \begin{cases} \frac{a_{tk}^2}{2\mu}, & \text{for } \mu \geq \frac{a_{tk}^2}{2\sqrt{P_0}} \\ \sqrt{P_0}, & \text{for } \mu < \frac{a_{tk}^2}{2\sqrt{P_0}} \end{cases} \quad (25)$$

Since $b_t \geq 0$, $a_t \geq 0$, $a_t \neq 0$, we find $-2\mu b_0 b_0^T a_t = \phi_k - \psi_k < 0$. Note that $\psi_k$ and $\phi_k$ cannot be both positive, since from (27) it is infeasible to have $a_t = \sqrt{P_0}$ and $a_t = 0$. Therefore either $\psi_k$ or $\phi_k$ must be zero. Since $\phi_k - \psi_k < 0$, we conclude that $\phi_k = 0$ and $\psi_k > 0$. Now, from (27) we have $a_t = \sqrt{P_0}$, or equivalently, $a_1 = \sqrt{P_0}$. 
E. Regarding Analytical Solution to (O3) with Independent Observations

First, we show that at least one of \( a_{tk} \)'s in (20) is equal to \( \sqrt{P_0} \). Suppose \( a_{tk} < \sqrt{P_0}, \forall k \). From (20), we have \( a_{tk} = \frac{b_{ij}^t}{K_{ij}|k|} \eta \) or equivalently \( \eta = \frac{a_{ij}^t K_{ij} |k| a_{tk}}{\sum_{b_{ij}^t} b_{ij}^t a_{tk}} = a_{ij}^t K_{ij} a_{tk} + c \frac{b_{ij}^t a_{tk}}{b_{ij}^t a_{ij}^t} \), since \( c > 0 \). However, this violates the definition of \( \eta \) in (19) and contradicts our initial assumption. Hence, there should be at least one \( a_{tk} = \sqrt{P_0} \). Next, we show that, although (O3) is not convex, the KKT solution in (20) is unique. Suppose there exist \( b_{ij}^t \) such that \( K_{ij}|k| \geq \ldots \geq K_{ij}|1| \), i.e., \( a_{tk} \geq \ldots \geq a_{tk} \), where at least \( a_{tk} = \sqrt{P_0} \). Assume that the solution in (20) is not unique, i.e., there exist two indices \( m, m' \in \{1, \ldots, M\} \), \( m' \geq m + 1 \) such that

\[
 a_{tk} = \begin{cases} 
 \frac{b_{ij}^t}{K_{ij}|j|} \eta, & \text{for } m + 1 \leq j \leq M, \ \eta \leq \frac{K_{ij}|j|}{b_{ij}^t}, \sqrt{P_0} \\
 \frac{1}{\sqrt{P_0}}, & \text{for } 1 \leq j \leq m, \ \eta > \frac{K_{ij}|j|}{b_{ij}^t}, \sqrt{P_0}, 
\end{cases}
\]

where \( \eta = \frac{\sum_{m} K_{ij}|j| + c}{\sqrt{P_0} \sum_{m} b_{ij}^t} \). Also, \( a_{tk} \) can be obtained from above by substituting \( j, m, m' \) with \( j', m', \eta' \), respectively. Since \( m' \geq m + 1 \), (28), we find \( \eta \leq \frac{K_{ij}|m'|}{b_{ij}^t}, \sqrt{P_0} \). On the other hand, due to sensor ordering, we have \( b_{ij}^t \geq \frac{K_{ij}|m' + 1|}{b_{ij}^t} \). Applying the mediant inequality, we obtain

\[
 \frac{\sum_{m} K_{ij}|j| + c}{\sqrt{P_0} \sum_{m} b_{ij}^t} \leq \frac{K_{ij}|m'|}{b_{ij}^t}, \sqrt{P_0}. \]

We observe that two different fractions are less than or equal to \( \frac{K_{ij}|m'|}{b_{ij}^t}, \sqrt{P_0} \).

\[
 \frac{\sum_{m} K_{ij}|j| + c + \sum_{m + 1} K_{ij}|j|}{\sqrt{P_0} \sum_{m + 1} b_{ij}^t} = \eta' \leq \frac{K_{ij}|m'|}{b_{ij}^t}, \sqrt{P_0}. 
\]

However, this inequality contradicts the one in (28) when \( j, \eta \) are replaced with \( j', \eta' \). Hence, our assumption regarding the existence of two indices \( m, m' \) is incorrect and the solution in (20) is unique.

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Fig. 1. $P_{D_0}$ under TPC versus $P_{tot}$ for a 2-sensor PAC with identical $p_{d_k}$'s and pathloss and $\rho = 0.1$: (a) Linear fusion rule, (b) LRT fusion rule.
Fig. 2. $P_{D_0}$ under TPC versus $P_{tot}$.

Fig. 3. Maximized MDC under TPC versus $P_{tot}$. 
Fig. 4. Maximized MDC under TIPC versus $P_{\text{tot}}$ and $ar{P} = 30$ mW.

Fig. 5. Maximized MDC under IPC versus $ar{P}$. 

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Total transmit power (mW)
Fig. 6. DPA in MAC with different $p_{d_k}$’s and identical pathloss: (a) Maximized MDC under TPC, $\rho = 0.1$; (b) Maximized MDC under TPC, $\rho = 0.9$; (c) Maximized MDC under TIPC, $\rho = 0.1$, $\bar{P} = 30$ mW; (d) Maximized MDC under TIPC, $\rho = 0.9$, $\bar{P} = 30$ mW; (e) Maximized MDC under IPC, $\rho = 0.1$; (f) Maximized MDC under IPC, $\rho = 0.9$. 
Fig. 7. DPA in PAC with different $p_d$’s and identical pathloss: (a) Maximized MDC under TPC, $\rho = 0.1$; (b) Maximized MDC under TPC, $\rho = 0.9$; (c) Maximized MDC under TIPC, $\rho = 0.1$, $P = 30\text{mW}$; (d) Maximized MDC under TIPC, $\rho = 0.9$, $P = 30\text{mW}$; (e) Maximized MDC under IPC, $\rho = 0.1$, (f) Maximized MDC under IPC, $\rho = 0.9$. 

(a) Maximized MDC under TPC, $\rho = 0.1$; (b) Maximized MDC under TPC, $\rho = 0.9$; (c) Maximized MDC under TIPC, $\rho = 0.1$, $P = 30\text{mW}$; (d) Maximized MDC under TIPC, $\rho = 0.9$, $P = 30\text{mW}$; (e) Maximized MDC under IPC, $\rho = 0.1$, (f) Maximized MDC under IPC, $\rho = 0.9$. 

(a) Total power: 30mW
(b) Total power: 120mW
(c) Total power: 240mW
(d) Total power: 30mW
(e) Total power: 120mW
(f) Total power: 240mW
Fig. 8. DPA in MAC with identical $p_{d_k}$'s, different pathloss and $\rho = 0.1$: (a) Maximized MDC under TPC, (b) Maximized MDC under TIPC, $\bar{P} = 30\, \text{mW}$, (c) Maximized MDC under IPC.

Fig. 9. DPA in PAC with identical $p_{d_k}$'s, different pathloss and $\rho = 0.1$: (a) Maximized MDC under TPC, (b) Maximized MDC under TIPC, $\bar{P} = 30\, \text{mW}$, (c) Maximized MDC under IPC.