A Round-Robin Packet Scheduler for Hierarchical Max-Min Fairness

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Abstract

Hierarchical link sharing addresses the demand for fine-grain traffic control at multiple levels of aggregation. At present, packet schedulers that can support hierarchical link sharing are not suitable for an implementation at line rates, and deployed schedulers perform poorly when distributing excess capacity to classes that need additional bandwidth. We present HLS, a packet scheduler that ensures a hierarchical max-min fair allocation of the link bandwidth. HLS supports minimum rate guarantees and isolation between classes. Since it is realized as a non-hierarchical round-robin scheduler, it is suitable to operate at high rates. We implement HLS in the Linux kernel and evaluate it with respect to achieved rate allocations and overhead. We compare the results with those obtained for CBQ and HTB, the existing scheduling algorithms in Linux for hierarchical link sharing. We show that the overhead of HLS is comparable to that of other classful packet schedulers.

I. INTRODUCTION

Packet scheduling plays a crucial role in the management of traffic flows, for prioritizing traffic, for flexible service differentiation, and for achieving performance metrics, such as flow completion times, throughput, and the tail of the delay distribution. This paper is concerned with packet scheduling methods that support traffic control at multiple aggregation levels. The need for such scheduling methods is largely driven by content providers that manage traffic within and between servers, clusters, and data centers. Increasingly, data centers rely on fine-grain traffic control at multiple levels of aggregation. The Google B4 inter data center network reports no less than five levels of traffic aggregation [1], [2]. Traffic control in support of a hierarchical distribution of available bandwidth is referred to as hierarchical link sharing.

As an example of link sharing, consider the hierarchy shown in Fig. 1. The top of the hierarchy, labeled as root, is a link with a fixed rate of 1000 (units are in Mbps). This bandwidth is to be divided between three traffic classes A, B, and C that each receive a minimum rate guarantee, as indicated in the figure. Traffic class A is further divided into classes A1 and A2, with guarantees of 100 and 200, respectively. Class B splits the bandwidth between B1 and B2 in the same fashion. Arriving packets are classified and mapped to leaf classes, which are the classes at the bottom level of the hierarchy.

Clearly, if the aggregate traffic from all leaf classes does not exceed the link capacity, every leaf class can obtain a rate equal to its arrival rate. Likewise, if the arrival rate of every leaf class exceeds its guaranteed rate, then each leaf class is limited to its guaranteed rate. The bandwidth allocation becomes less trivial when the aggregate arrival rate from all classes is larger than the link capacity, and some classes exceed their guaranteed rates, while others stay well below their guarantees. In this case, excess capacity left unused by some classes must be distributed equitably to classes that desire additional bandwidth.

Several packet scheduling algorithms that support class hierarchies with rate guarantees as shown in Fig. 1 are available, however, deployed or deployable algorithms show significant shortfalls while algorithms without such shortfalls are too complex to be deployable. This paper addresses this dichotomy by presenting a packet scheduler with provable link sharing properties and low computational complexity.

For a non-hierarchical setting, a bit-by-bit round-robin algorithm provides link sharing that satisfies a weighted version of max-min fairness [3]. However, bit-by-bit round robin assumes fluid flow traffic and is not implementable as a packet scheduler. Weighted-Fair-Queueing (WFQ) [4] has shown to have a strictly bounded deviation from the ideal bit-by-bit round robin [5]. The drawback of WFQ, which extends to some

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of its approximations \cite{6}, \cite{7}, is that it requires to maintain a priority queue that transmits packets in the order of assigned timestamps. Deficit-Round-Robin (DRR) \cite{8} is a packet-level round-robin scheduler for variable-sized packets, whose link sharing ability is inferior to WFQ, but with a simpler implementation. Due to the low complexity, Linux \cite{9} and line-rate switches \cite{10} generally realize link sharing with a round-robin scheduler, such as DRR.

For class hierarchies as in Fig. 1, Hierarchical Packet Fair Queueing (HPFQ) \cite{11} achieves link sharing by employing a cascade of hierarchically organized WFQ schedulers. Packets at the head of the queue of backlogged leaf classes engage in a virtual tournament, with one round of the tournament for each level of the class hierarchy. The tournament starts at the bottom of the hierarchy. In each round, the packet with the smallest timestamp at one level proceeds to the next level. The winner of the tournament is selected for transmission. While HPFQ achieves almost ideal link sharing, it involves a considerable overhead and has not been considered for deployments.\footnote{The claim in \cite{12} of realizing HPFQ by a hierarchy of PIFO queues is incorrect, as counterexamples are easily constructed when packet sizes are variable.}

Attempts to extend DRR to a class hierarchy have so far not resulted in practical scheduling algorithms. In \cite{13}, the class hierarchy is mapped to a flat hierarchy by interleaving classes according to their weight guarantees. This results in good fairness properties, but rounds grow prohibitively large which may result in excessive delays between packet transmissions for some classes. Other efforts in this direction, e.g., \cite{14}, \cite{15} make scheduling decisions in multiple stages, one per level in the class hierarchy, and thus inherit the drawbacks of HPFQ.

Class-based queuing (CBQ) \cite{16} and Hierarchical Token Bucket (HTB) \cite{17} are two packet schedulers for link sharing in class hierarchies that are actually deployed, even if the deployment is limited to Linux systems.\footnote{In the appendix, we provide supplemental information on the operation of CBQ and HTB.}

CBQ provides minimum bandwidth guarantees to traffic classes and distributes excess capacity to backlogged classes. CBQ measures the transmission rate of each class to identify traffic classes that are allowed to transmit, which are then served by a variant of DRR. HTB tries to improve the efficiency of CBQ by metering the transmission rates of classes with token bucket filters. Classes that exceed their rate guarantee can ‘borrow’ bandwidth from classes further up in the class hierarchy. HTB schedules packets with a set of DRR schedulers, where only one DRR scheduler is active at a time. In addition to link sharing, HTB also enforces rate limits. HTB has become the primary tool for scheduling and shaping of hierarchically structured traffic flows in Linux servers \cite{18}–\cite{20}.

CBQ and HTB implement rules that dictate when a class with need for additional bandwidth can transmit, however, with the rules it is not possible to determine (a priori) the allocated rates for a given traffic load. In contrast, the outcomes of schedulers such as HPFQ and hierarchical extensions of DRR

![Fig. 1. Link sharing hierarchy.](image-url)
schedulers satisfy a hierarchical version of max-min fairness, which ensures class guarantees as well as isolation between classes in the hierarchy.

Realizing hierarchical link sharing with round-robin schedulers is attractive, since it does not involve packet timestamps and priority queues, but has shown to be challenging. Extensions of DRR to class hierarchies has so far not resulted in a practical scheduling algorithm. On the other hand, CBQ and HTB systematically fail to isolate rate guarantees between classes in different branches of the class hierarchy. In particular, they allow classes to manipulate the rate allocations by reassigning rate guarantees in a subtree of the class hierarchy (see Subsec. VIII-C). Until now, there does not exist a round-robin packet scheduler for class hierarchies that can satisfy rate guarantees while isolating the allocations in different parts of the class hierarchy.

In this paper, we present Hierarchical Link Sharing (HLS), the first round-robin scheduler for hierarchical link sharing that ensures rate guarantees and isolation between classes, and that can run at high line rates. The rate allocation achieved by HLS satisfies a hierarchical version of max-min fairness. This allocation is strategy-proof, as defined in [21], in the sense that classes cannot improve their allocation through wrongful representation of their demand or the demand of their sub-classes. HLS is a non-hierarchical variant of DRR with a time-variable quantum for each class.

We have implemented HLS as a Linux kernel module [22]. We present experiments showing that HLS ensures rate guarantees for and isolation between classes, with an overhead that is comparable to other classful scheduler in the Linux kernel.

II. CLASS HIERARCHY: TERMINOLOGY

We introduce terminology needed to describe the relationships between classes in a class hierarchy. Fig. 2 depicts a class hierarchy as a rooted tree, where each node represents a class. The class at the top of the hierarchy, referred to as root, represents a network interface where the scheduling algorithm is active. Leaf nodes in the rooted tree represent leaf classes, which are shown as gray circles. As stated earlier, all traffic arrivals are mapped to leaf classes. Nodes that are neither the root nor a leaf node represent internal classes. If \( \mathcal{N} \) is the set of all classes, we denote by \( \mathcal{L} \) and \( \mathcal{I} \), respectively, the leaf classes and internal classes, with \( \mathcal{N} = \mathcal{L} \cup \mathcal{I} \cup \{ \text{root} \} \).

For class \( i \) in the figure, the incoming edge connects to its parent class \( p(i) \), and the outgoing edges connect to its child classes \( \text{child}(i) \). Other needed terms, such as \( \text{anc}(i) \), \( \text{sib}(i) \), \( \text{desc}(i) \), and \( \text{ldesc}(i) \) are defined in Table I and indicated by dashed boxes in Fig. 2.

| Term         | Description                                      |
|--------------|--------------------------------------------------|
| \( p(i) \)   | parent Next class on the path from \( i \) to root, with \( p(\text{root}) = \emptyset \) |
| \( \text{anc}(i) \) | ancestors Set of classes on the path from \( i \) to root (incl. root) |
| \( \text{sib}(i) \) | siblings All classes with the same parent as class \( i \) |
| \( \text{child}(i) \) | child classes Set of classes with \( i \) as parent |
| \( \text{desc}(i) \) | descendants Set of classes with \( i \) as ancestor |
| \( \text{ldesc}(i) \) | leaf descendants Set of leaf classes with \( i \) as ancestor |

**TABLE I**

NOTATION FOR CLASS HIERARCHY.

Fig. 2 is representative of the configuration of classful schedulers in Linux. In Linux traffic control [9], a scheduling discipline is referred to as a qdisc. Class hierarchies are specified using configuration commands and are built starting from the top of the hierarchy, which is called root qdisc. Filter expressions are used to map packets to leaf classes.

In a class hierarchy, each class is associated with a weight or with a rate guarantee. In Fig. 1 classes are assigned rate guarantees. Denoting the rate guarantee of class \( i \in \mathcal{I} \cup \{ \text{root} \} \) by \( g_i \), the guarantee must satisfy the superadditive property

\[
g_i \geq \sum_{j \in \text{child}(i)} g_j .
\]
The guarantee of the root class is the link capacity $C$, that is, $g_{\text{root}} = C$. There is an alternative specification of link sharing that is based on weights, where $w_i > 0$ is used to denote the weight of class $i$. If three sibling classes, say classes 1, 2, 3 have weights $w_1, w_2, w_3$, and all siblings are backlogged, the weights indicate that they will split the capacity made available to them as a group in the ratio $w_1 : w_2 : w_3$. Viewing link sharing in terms of weights is often more convenient, since it allows to express link sharing as dividing available bandwidth locally between siblings. In contrast, guarantees appear as global quantities with constraints across all classes. We emphasize that the concepts are equivalent. Guarantees that satisfy the superadditive constraints above can be viewed as weights, that is $w_i = g_i$ for each class $i$. Likewise, given the link capacity $C$ and weights $w_i$ for each class $i$, an absolute bandwidth guarantee of class $i$, $g_i$, is computed as

$$g_i = \prod_{j \in \text{anc}(i) \cup \{i\}} \frac{w_j}{\sum_{k \in \text{sib}(j)} w_k} \cdot C.$$  

Fig. 3 depicts the relationship between weights and guaranteed rates. In the following we will work with weights $w_i$, but ensure that they satisfy $w_i \geq \sum_{j \in \text{child}(i)} w_j$ for each class $i$. Then, we can use the terms weight and class guarantee interchangeably.
III. QUANTIFYING LINK SHARING GOALS

In a non-hierarchical setting, link sharing between classes can be achieved by fair queueing algorithms that approximate a bit-by-bit round robin, resulting in a max-min fair rate allocation. We define hierarchical max-min fairness as the result of applying (weighted) bit-by-bit round robin between each group of siblings in a class hierarchy. In [11], such a scheduler is referred to as Hierarchical Generalized Processor Sharing (HPGS). The allocation of this scheduler is also the targeted allocation of HPFQ scheduling. Expressions that quantify the solution of this allocation exist for a non-hierarchical setting, but are not available for class hierarchies. In the following we quantify both the non-hierarchical and hierarchical notions of fairness.

A. Max-min fair allocation

We formulate rate allocation for traffic classes with fixed-rate traffic at a link with fixed capacity \( C \). We define

\[
\begin{align*}
    r_i & \quad \text{Rate request of class } i, \\
    a_i & \quad \text{Rate allocation to class } i \ (a_i \leq r_i), \\
    w_i & \quad \text{Weight associated with class } i.
\end{align*}
\]

Rate requests are not made explicitly, but are determined by traffic arrivals from a class at the link and the resulting backlog. In a max-min fair allocation without weights, if a class is allocated less than it requests, it receives at least as much as any other class. As a consequence, two classes that do not satisfy their demand have the same allocation. Also, if the total demand exceeds the capacity then the entire link capacity is allocated. When specifying weights \( w_i \) for each class \( i \), the weighted max-min fair allocation is defined by the following rules:

(R1) If \( a_i < r_i \), then \( \frac{a_i}{w_i} \geq \frac{a_j}{w_j} \) for each class \( j \in \mathcal{N} \).

(R2) \( \sum_{j \in \mathcal{N}} a_j = \min\left(\sum_{j \in \mathcal{N}} r_j, C\right) \).

Rule (R1) states that, if a class is not allocated its entire request, then its allocation in proportion to its weight is as least as large as the (also proportional) allocation of any other class. The second rule simply ensures that either all requests are satisfied or all resources are allocated.

A weighted max-min fair allocation creates a set \( \mathcal{S} \) of satisfied classes, which receive their entire request \( (a_i = r_i) \), and a set \( \mathcal{N} \setminus \mathcal{S} \) of unsatisfied classes with \( a_i < r_i \). Rule (R1) implies that \( \frac{a_i}{w_i} = \frac{a_j}{w_j} \) for any two unsatisfied classes. The allocation is strategy-proof, since an unsatisfied class cannot increase its allocation by increasing or misrepresenting its request.

If there is at least one unsatisfied class \( i \), we define the fair share \( f \) as

\[ f = \frac{a_i}{w_i}, \]

which results in the allocation \( a_j = \min\{r_j, w_j f\} \).

Supposing that there exist unsatisfied classes, rule (R2) yields

\[ C = \sum_{j \in \mathcal{S}} r_j + \sum_{j \notin \mathcal{S}} w_j f. \]
Algorithm 1: Computing the fair share $f$.

**Input:** Link capacity $C$ and a set $N$ of classes with $r_i, w_i \geq 0$ for each class $i \in N$.

**Output:** Fair share $f$ from (1).

**Function** MaxMinFair $(N, \{r_i\}_{i \in N}, \{w_i\}_{i \in N}, C)$:

```plaintext
if $\sum_{i \in N} r_i \leq C$ then
    return $f \leftarrow \infty$
else
    $f_0 \leftarrow 0$
    $n \leftarrow 0$
    do
        $S^n \leftarrow \{i \mid r_i \leq w_i f_n\}$
        $f_{n+1} \leftarrow \frac{C - \sum_{i \in S^n} r_i}{\sum_{j \notin S^n} w_i}$
        $n \leftarrow n + 1$
    while $f_n \neq f_{n-1}$
    return $f \leftarrow f_n$
```

Solving for $f$ gives an expression for the fair share as

$$f = \frac{C - \sum_{j \in S} r_j}{\sum_{j \notin S} w_j}.$$  (1)

As long as there is at least one unsatisfied class, the fair share is uniquely defined. Even though the expression for $f$ is implicit, that is, $f$ is defined in terms of $S$, and $S$ is defined in terms of $f$, the fair share $f$ can be computed, e.g., by a water filling algorithm as given in Algorithm 1. The algorithm uses the fact that, with a fair share $f_n$, each class $i$ with $r_i \leq w_i f_n$ is satisfied. In the algorithm, the fair share is set to infinity when the total demand does not exceed the link capacity. The algorithm computes the fair share iteratively by initially assuming that no class with traffic is satisfied, and then labels classes as satisfied until the true fair share is obtained.

B. Hierarchical max-min fair allocation

Next consider a class hierarchy as given in Fig. 2. The requests and allocations of internal classes and the root consist of the total requests and allocations, respectively, of their child classes. That is, for each $i \in \mathcal{L} \cup \text{root}$,

$$r_i = \sum_{j \in \text{child}(i)} r_j, \quad a_i = \sum_{j \in \text{child}(i)} a_j.$$  (2)

With this notation, we can specify a max-min fair allocation for class hierarchies.

A hierarchical weighted max-min fair (HMM fair) allocation is defined by these two rules that hold for each $i \in \mathcal{L} \cup \mathcal{I}$.

(R1) If $a_i < r_i$, then $\frac{a_i}{w_i} \geq \frac{a_j}{w_j}$ for all $j \in \text{sib}(i)$.

(R2) $\sum_{j \in \mathcal{L}} a_j = \min(\sum_{j \in \mathcal{L}} r_j, C)$.

The rules are analogous to those for max-min fairness without a hierarchy. In essence, each parent allocates the capacity available to it to its child classes using the max-min fairness principle. According to (R1), if a class cannot satisfy its request, then its allocation relative to its weight is at least as large as the allocation of any of its siblings relative to the weight of that sibling. The second rule makes sure that all
available capacity is utilized. The allocation is strategy-proof for each group of siblings, since it satisfies max-min fairness from Sec. III-A and, therefore, is strategy-proof for the entire hierarchy. No class can obtain a larger allocation by increasing or misrepresenting its request.

Using the aggregation in (2) and rule (R2), we can make two observations for a class $i$:

(O1) If $a_i < r_i$ then $a_j < r_j$ for all $j \in \text{anc}(i)$.  
(O2) If $a_i = r_i$ then $a_j = r_j$ for all $j \in \text{desc}(i)$.

To explain (O1), if a class receives a smaller rate than it requests, it will try to get more capacity from its parent, which, in turn, will seek to acquire capacity from its own parent, and so forth. Hence, if the request of a class is not satisfied, the resources of all its ancestors will be exhausted, leaving them unsatisfied as well. Observation (O2) follows since requests and allocations of an internal class consist of the aggregated requests and allocations of their child classes.

If the rules for hierarchical max-min fairness are straightforward, the computation of the allocations to classes is much less so. The reason is that the capacity available at an internal class depends on the capacity available at its child class. We proceed by performing an induction over the levels of the hierarchy, starting at the top. The results of an HMM allocation are specified in the following theorem.

**Theorem 1.** Given a link with capacity $C$, and a class hierarchy where each class $i$ has a request rate $r_i$ and a weight $w_i > 0$. Define $a_{\text{root}} = \min\{r_{\text{root}}, C\}$. Then, the HMM fair allocation for each class $i \in \mathcal{I} \cup \mathcal{L}$ is

$$a_i = \begin{cases} \min (r_i, w_i f_{p(i)}) & \text{if } r_{p(i)} > a_{p(i)} \\ r_i & \text{otherwise} \end{cases},$$

where the fair share $f_k$ for each $k \in \mathcal{I} \cup \{\text{root}\}$ is

$$f_k = \frac{a_k - \sum_{j \in \mathcal{S}_k} r_j}{\sum_{j \notin \mathcal{S}_k} w_j},$$

and where $\mathcal{S}_k$ is defined as

$$\mathcal{S}_k = \{ j \in \text{child}(k) \mid r_j < w_j f_k \}.$$

**Proof.** We proceed by performing an induction over the levels of the hierarchy, starting at the top. The proof refers to rules (R1) and (R2) from Sec. III-B.

Consider the root class, where we have $a_{\text{root}} = \min\{r_{\text{root}}, C\}$. If $r_{\text{root}} = a_{\text{root}}$, then $a_n = r_n$ for every child $n \in \text{child}(\text{root})$. Moreover, with observation (O2), we have $a_j = r_j$ for all $j \in \text{desc}(\text{root}) = \mathcal{I} \cup \mathcal{L}$.

If $r_{\text{root}} > a_{\text{root}}$, there exists an $n \in \text{child}(\text{root})$ with $a_n < r_n$. Define $f_{\text{root}} = \frac{a_n}{w_n}$. By (R1), for every class $m \in \text{child}(\text{root})$ with $a_m < r_m$ we have $f_m = \frac{a_m}{w_m}$. We therefore have for each child class $n$ of the root that

$$a_n = \min\{w_n f_{\text{root}}, r_n\},$$

as well as $\mathcal{S}_{\text{root}} = \{ i \in \text{child}(\text{root}) \mid r_i < w_i f_{\text{root}} \}$. By rule (R2) we get

$$\sum_{n \notin \mathcal{S}_{\text{root}}} w_n f_{\text{root}} + \sum_{n \in \mathcal{S}_{\text{root}}} r_n = a_{\text{root}},$$

which gives (3) for $i = \text{root}$.

For the induction step, we consider a class $k$ and assume that the allocation has been computed for all ancestors $l \in \text{anc}(k)$. If the allocation of its parent $p(k)$ was such that $a_{p(k)} = r_{p(k)}$, then due to observation (O2), we get $a_k = r_k$. Now consider $a_{p(k)} < r_{p(k)}$. Then, the parent has computed a fair share $f_{p(k)}$ and the allocation to its child class $k$ was $a_k = \min\{w_k f_{p(k)}, r_k\}$. If $k$ is a leaf class ($k \in \mathcal{L}$), we are done. If $k$ is an internal class, there are two cases. If $r_k \leq w_k f_{p(k)}$, then $a_k = r_k$, and, by (O2), $a_l = r_l$ for each $l \in \text{desc}(k)$. If $r_k > w_k f_{p(k)}$, then there exists a class $l \in \text{child}(k)$ with $a_l < r_l$. Defining $f_k = \frac{a_l}{w_l}$, by (R1),
Algorithm 2: Computing fair shares for HMM fairness.

Input: Link capacity $C$, a set of $\mathcal{N}$ of classes with $r_i, w_i \geq 0$ for each class $i \in \mathcal{N}$, and a class hierarchy $M$ supporting the operations in Fig. 2.

Output: Fair shares $\{f_i\}_{i \in \mathcal{I} \cup \text{root}}$ from (3).

Function $\text{HierarchicalMaxMinTopDown}(\mathcal{N}, M, \{r_i\}_{i \in \mathcal{N}}, \{w_i\}_{i \in \mathcal{N}}, C)$:

\[
\begin{align*}
\text{foreach } i \in \mathcal{I} \cup \{\text{root}\} & \text{ do } \\
& r_i \leftarrow \sum_{j \in \text{ldesc}(i)} r_j \\
& L \leftarrow \{\text{root}\} \\
& a_{\text{root}} \leftarrow \min(r_{\text{root}}, C) \\
& \text{do } \\
& i \leftarrow \text{Select an element from set } L \\
& L \leftarrow L \setminus \{i\} \\
& \text{if } i \notin L \text{ then } \\
& \quad f_i \leftarrow \text{MaxMinFair}(\text{child}(i), \{r_j\}_{j \in \text{child}(i)}, \{w_j\}_{j \in \text{child}(i)}, a_i) \\
& \quad \text{foreach } j \in \text{child}(i) \text{ do } \\
& \quad \quad a_j \leftarrow \min(r_j, w_j f_i) \\
& \quad \quad L \leftarrow L \cup \{j\} \\
\end{align*}
\]

while $L \neq \emptyset$

return $\{f_i\}_{i \in \mathcal{I} \cup \{\text{root}\}}$

we have $a_j = w_j f_k$ for each $j \in \text{child}(k)$ with $a_j < r_j$. We also define $\mathcal{S}_k = \{i \in \text{child}(k) \mid r_i \leq w_i f_k\}$. With rule (R2) we obtain

\[
\sum_{n \notin \mathcal{S}_k} w_n f_k + \sum_{n \in \mathcal{S}_k} r_n = a_k ,
\]

where $a_k = w_k f_{p(k)}$. Solving the equation for $f_k$ gives (3) for $i = k$.

The values of the fair shares in a class hierarchy can be computed with Algorithm 2. The algorithm starts at the top of the hierarchy and computes the fair share of the root. Then it proceeds to compute the fair share of children of the root and continues to traverse the class hierarchy in a top-down fashion (in no particular order) until a leaf class is reached. The algorithm uses the fair share computation from Algorithm 1.

The allocations we discussed are simplified in that arriving traffic and service are expressed in terms of rates. Without a hierarchy, expressions for max-min fair allocations for intermittent bursty traffic exist, but they require that traffic be shaped, e.g., by token buckets [5] or more general concave bounding functions [23].

To measure how well a scheduling algorithm realizes an HMM fair allocation for time-variable traffic, we will instead use a fairness metric that measures the deviation from the allocation of an ideal hierarchical bit-by-bit round-robin scheduler.

IV. THE HLS SCHEDULER

We next present the Hierarchical Link Sharing (HLS) scheduler which allocates rates according to the principle of HMM fairness. We have implemented HLS as a Qdisc in the Linux kernel [22].

The design of HLS departs from that of HTB and CBQ, which both track the transmission rates of classes using moving averages in CBQ and token buckets in HTB. If a class requires additional bandwidth, both HTB and CBQ allow the class to borrow bandwidth from other classes in a greedy fashion. (HTB and CBQ descriptions prefer the term ‘borrow,’ but the bandwidth so acquired is never returned.) Different from CBQ and HTB, HLS does not measure the transmission rates of classes. Instead, it gives transmission
quotas to classes such that HMM fairness is satisfied. Minimum rate guarantees follow as a consequence of achieving HMM fairness.

In HLS, each class \( i \) is associated with an integer weight \( w_i > 0 \), which can be set to the rate guarantee of the class (see Sec. II).

At its core, HLS is a non-hierarchical DRR scheduler with a time-variable quantum for each class, which we refer to as quotas. Each round of the round robin visits each class that is designated as active, one by one, in an arbitrary order. A leaf class is active if it is backlogged at the start of a round. An internal class is active if at least one of its child classes is active. We distinguish two kinds of rounds, main rounds and surplus rounds, where each main round may be followed up by one or more surplus rounds. The quota of a class is recomputed and assigned during a visit in a main round. If at the end of a main round some classes have unused quota, a surplus round is started, where the unused quotas are distributed to classes with a backlog. An additional surplus round is started if after the completion of a surplus round there is still unused quota left.

Each active leaf class is visited once per round (main or surplus). The determination of the set of active classes is done at the start of a round. If a class becomes idle during a round, it remains idle until the end of that round, even if there is an arrival to that class in the middle of the round. We use \( L_{\text{ac}} \) and \( I_{\text{ac}} \), respectively, to denote the set of active leaf and internal classes in a main or surplus round. We also use \( A = L_{\text{ac}} \cup I_{\text{ac}} \) to denote the set of all non-root active classes.

Each class \( i \) maintains a balance, denoted by \( B_i \), which maintains the number of bytes that the class is allowed to transmit (if \( i \in L \)) or that its leaf descendants are allowed to transmit (if \( i \in I \)) in the current round. The initial setting is

\[
B_i = \begin{cases} 
Q^* , & \text{if } i = \text{root} \\
0 , & \text{otherwise}
\end{cases},
\]

where \( Q^* > 0 \) denotes the total number of bytes from all classes that can be transmitted in a main or surplus round. In Section V-B, we address how to select \( Q^* \). In a main round, the root distributes its balance across its child classes, who, in turn, distribute their balance to their own child classes, and so forth. The root and active internal classes also maintain a residual, denoted by \( R_i \) (\( i \in I_{\text{ac}} \cup \{\text{root}\} \)), which contains permits for the transmission of bytes that were collected from descendants in the previous round, with initial setting \( R_i = 0 \).

In each round, all active internal classes recompute the number of bytes that a child class with weight set to one can transmit in the round, which is referred to as the fair quota and denoted by \( F_i \) for class \( i \). For a class \( i \in I_{\text{ac}} \cup \{\text{root}\} \), the fair quota is defined as

\[
F_i = \left\lfloor \frac{B_i}{w_i^{\text{ac}}} \right\rfloor, \tag{4}
\]

where

\[
w_i^{\text{ac}} = \sum_{k \in \text{child}(i) \setminus (L_{\text{ac}} \cup I_{\text{ac}})} w_k \tag{5}
\]

denotes the sum of the weights of the active child classes of class \( i \). The rounding by the floor function rounds the quantity to an integer multiple of unit byte to avoid floating point operations in the Linux kernel. The root class recomputes \( F_{\text{root}} \) only in a main round and sets \( F_{\text{root}} = 0 \) in all surplus rounds.

Before computing the fair quota, each class \( i \in I_{\text{ac}} \cup \{\text{root}\} \) updates its balance and residual. For the root class the update is

\[
B_{\text{root}} = B_{\text{root}} + R_{\text{root}} , \quad R_{\text{root}} = 0 \, , \tag{6}
\]

that is, the residual is added to the balance and then reset. For an active internal class, the update is

\[
B_i = B_i + R_i + w_i F_{p(i)} , \quad B_{p(i)} = B_{p(i)} - w_i F_{p(i)} , \quad R_i = 0 \, . \tag{7}
\]
Here, the balance of class $i$ is increased by $w_i F_p(i)$, and the balance of the parent $p(i)$ is reduced by the same amount. We refer to $w_i F_p(i)$ as the quota of class $i$. Also, the residual $R_i$ is added to the balance and then reset. Since the quota of an internal class depends on the fair quota of the parent class, the update of balances and computations of the quota is performed in a top-down fashion. Without the rounding in (4), every internal class would have a zero balance after the update. With rounding, the remaining balance of a class $i \in L_{ac}$ after the update of all its active child classes is bounded by $B_i < w_{ac}^i$. Note that the unit of $w_{ac}^i$ is in bytes, since it is the remainder of the integer division in (4).

Before an active leaf class $i \in L_{ac}$ transmits in a round, it performs the update

$$B_i = B_i + w_i F_p(i) , \quad B_p(i) = B_p(i) - w_i F_p(i) .$$

In the HLS Qdisc implementation, the update of balances in (6)–(8) and the computation of fair quotas in (4) is initiated by the leaf classes, which is illustrated in Fig. 4. In the figure, node $i$ represents an active leaf class. When this class is visited in the current round it requests the fair quota $F_p(i)$ from its parent. If the parent has not previously computed its quota in the current round, it sends a request for the fair quota $F_{gp(i)}$ to its own parent $gp(i)$ (we use $gp(i)$ to denote the grandparent of class $i$), and so forth. If the root is reached and the scheduler is in a main round, the balance $B_{root}$ and the fair quota $F_{root}$ is computed, and then $F_{root}$ is passed to $gp(i)$. In a surplus round the root returns $F_{root} = 0$. Next, the internal classes $gp(i)$ and $p(i)$ use the fair quotas from their respective parent to update their balances and compute their own fair quotas. In the last step, leaf class $i$ updates its balance. When the requests (steps 1–3 in Fig. 4) reach a class that already has computed its quota in the current round, no further upstream requests are made. In this fashion, each class $i \in L \cup \{root\}$ updates its quota only once and updates its balance at most $1 + |child(i)|$ times per round.

When an active leaf class $i$ is visited by the round robin, it updates its balance $B_i$ according to (8) before transmitting packets. If the packet at the head of the queue has length $L$ and $B_i \geq L$, the packet is transmitted, followed by the update

$$B_i = B_i - L , \quad B_{root} = B_{root} + L .$$

By increasing $B_{root}$ for each transmitted packet, the root class accrues a balance that will be distributed in the next main round. Class $i$ can continue transmitting packets as long as it has a sufficient balance. If the packet at the head of the queue has size $L$ and $B_i < L$, the scheduler turns to the next class in the round robin. If a leaf class $i \in L$ is served and has no more packets to transmit, it becomes *idle* and returns its balance $B_i$ to its parent with the update

$$R_p(i) = R_p(i) + B_i , \quad B_i = 0 .$$
An internal class becomes idle if all its children are idle. An idle internal class $i \in \mathcal{I}$ returns its balance $B_i$ and residual $R_i$ to its parent by computing
\[ R_{p(i)} = R_{p(i)} + B_i + R_i, \quad B_i = 0, \quad R_i = 0. \] (11)

Now we see the role of the residual. The residual $R_i$ of an internal class $i$ or the root collects the returned balances from child classes that became idle in the current round. The rationale for not adding the returned balance of an idle child class immediately to the balance of the parent is to prevent the returned balance from being used in the current round. Doing so would favor leaf classes that are visited later in the round robin. By adding the residual to the balance only at the start of a new round, we ensure that all descendants can obtain a portion of the unused balance.

HLS starts a new main round only if the sum of all quotas that has been distributed to classes has been used for transmissions. Note that a class does not use up its full quota only if it became idle in the current round. This results in the unused balance (‘surplus’) being added to the residual of the parent class. If this happens, the residuals accrued in a round will be distributed to descendant classes in subsequent surplus rounds. The condition to start a surplus round is that at least one internal class $i$ satisfies $B_i + R_i \geq w_{ac}^i$ where the unit of $w_{ac}^i$ in this case is in bytes instead of unitless, meaning that the class computes a nonzero quota in (4) using its balance and residual. A surplus round operates just like a main round. First, all backlogged classes are marked as active followed by a complete round robin of active classes with the updates from (6)–(11). The only difference to a main round is that $F_{\text{root}}$ is set to zero, meaning that no new quota is distributed from the root. If, at the end of a surplus round, there still exists an internal class $j$ with $B_j + R_j \geq w_{ac}^j$ another surplus round is started. This continues, until no internal class satisfies the condition, in which case a new main round is started.

With the updates of the balance counters in (6)–(11), the sum of balances and residuals of all classes satisfies the invariance
\[ \sum_{i \in \mathcal{N}} B_i + \sum_{i \in \mathcal{I} \cup \{\text{root}\}} R_i \equiv Q^*. \] (12)

Since balances are permits for transmission and the residuals are unused permits for transmission, maintaining the invariance ensures that the maximum amount of traffic transmitted in a round does not drift.

V. DIMENSIONING OF THE ROUND SIZE

The HLS scheduler begins a new main round whenever $Q^*$ bytes have been transmitted. There are two considerations for the selection of $Q^*$. On the one hand, $Q^*$ should not be selected too large, otherwise, the scheduler reacts too slowly to changes of the set of active leaf classes. On the other hand, if $Q^*$ is selected too small, the quotas that are passed down to leaf classes may not allow the transmission of any packet. In this section we derive a sufficient condition for a lower bound on $Q^*$. Our implementation of HLS uses this lower bound to adjust $Q^*$ dynamically at the start of a main or surplus rounds. We will simplify the derivation of the bound by rearranging the order in which HLS updates the balance and transmit packets from classes. Specifically, we consider that at the begin of each main or surplus round, HLS updates the quotas and balances for all classes, before transmitting any packets. As we will show in the next subsection, this modification does not alter the transmission schedule of HLS.

A. Replenishment Phase and Transmission Phase

Consider the class hierarchy from Fig. 1. Assume that only leaf classes $B_2$, $B_1$, and $A_1$ are backlogged and visited in this order. The sequence of quota and balance updates and transmissions of these classes in HLS is illustrated in Fig. 5(a). In the figure, quota and balance updates of a class are shown as gray boxes, and packet transmissions are shown as white boxes. The order follows from the description of HLS in Sec. IV. Following the recursive process illustrated in Fig. 4, a visit of class $B_2$ results in updates first
at the root class, then at B, and finally at B2. When B1 is visited next, only the quota and balance of B1 are updated, as the quotas of its ancestors (B and root) have already been computed. When class A1 is visited, the quotas of A and A1 are updated. As shown in Fig. 4, in this fashion HLS alternates between quota updates and packet transmissions as it visits active leaf classes in a round of the round robin.

Now consider a modification to HLS, where each main and surplus round starts with a replenishment phase, in which the quotas and balances of all classes are updated according to (4)–(8). The replenishment phase is then followed by a transmission phase, which consists of a round robin that visits each active class and transmits from each active class, while updating the balance and residual according to (9)–(11). We will show that this modification does not alter the order of packet transmissions compared to the HLS scheduler described in Sec. IV. Fig. 5(b) illustrates the updates and transmissions for the example. Here, HLS replenishes every active class in the replenishment phase and then visits each active leaf class for the packet transmission in the transmission phase. We next show that rearranging the quota replenishments and packet transmissions in this manner does not change the behavior of HLS.

**Lemma 1.** The modified operation of HLS with a replenishment phase and transmission phase as described above does not change the order of packet transmissions in HLS.

**Proof.** Consider the unmodified HLS scheduler, where the packet transmission of some class i is immediately followed by the quota replenishment of another class j. During the packet transmission of class i, only the balances of class i and its ancestors are updated via (8), (10), and (11). Suppose we swap the packet transmission of class i and the quota replenishment of class j. In that case, the only differences observed by the quota replenishment of class j are the balances of class i and its ancestors. Upon computing the fair quota of an ancestor k of j, if class k is also an ancestor of class i, class k is already replenished, and HLS reuses $F_k$, which is unchanged by the swapping. If class k is not an ancestor of class i, then its balance is not altered and the computed $F_k$ remains the same. So, the quota replenishment of class j is not affected by the swapping.

Furthermore, as HLS replenishes class i before its packet transmission and the quota replenishment of class j, the balance $B_i$ of class i remains untouched by the swapping. Therefore, we can swap the quota

---

**Fig. 5.** Sequence of quota replenishments (gray) and transmissions (white) of classes.
replenishment of class \( j \) and the packet transmission of class \( i \) without changing the resulting transmission order of packets. By repeatedly swapping quota replenishments and packet transmissions of classes, we arrive at the modified HLS scheduler as described above.

For the remainder of Sec. V-B as well as Sec. VI and Sec. VII, we assume that HLS rearranges the quota replenishments and the packet transmissions into the replenishment phase and the transmission phase. Also, we use \( L_{i}^{\text{max}} \) to denote the maximum packet size of a leaf class \( i \in L \).

B. Selection of \( Q^{*} \)

We next present a lower bound on \( Q^{*} \), the number of bytes that are transmitted in a main round and subsequent surplus rounds. The lower bound ensures that there is at least one packet that can be transmitted in a main round, thus ensuring that the HLS scheduler is work-conserving. As a remark, a surplus round without a transmission is not an issue, since it will be followed by a main round.

**Theorem 2.** Setting

\[
Q^{*} = \sum_{i \in \mathcal{A}} w_{i} + \sum_{i \in \mathcal{L}_{\text{ac}}} L_{i}^{\text{max}},
\]

ensures that at least one packet can be transmitted in each main round.

Note that both terms of the summation depend on the set of active leaf classes.

**Proof.** We can ensure that, in any main round, at least one packet can be transmitted by satisfying the condition

\[
\sum_{i \in \mathcal{L}_{\text{ac}}} B_{i} \geq \sum_{i \in \mathcal{L}_{\text{ac}}} L_{i}^{\text{max}},
\]

at the end of the replenishment phase of the round. Then, by the pigeon hole principle, there is at least one active leaf class \( i \) with \( B_{i} \geq L_{i}^{\text{max}} \).

Consider the time at the end of the replenishment phase, before any packet transmission takes place. By Lemma 1, we can perform the updates of (6)–(8) for all classes at once. Since the residuals of internal classes are set to zero during the updates, we obtain from (12) that \( \sum_{i \in \mathcal{L}_{\text{ac}}} B_{i} = Q^{*} \). Since inactive classes have a balance of zero, positive balances appear only in active classes and the root. With this, we can write (13) as

\[
Q^{*} \geq \sum_{i \in \mathcal{L}_{\text{ac}} \cup \{\text{root}\}} B_{i} + \sum_{i \in \mathcal{L}_{\text{ac}}} L_{i}^{\text{max}}.
\]

Recall that, after updating the balances of all classes, the remaining balance of an internal class or the root satisfies \( B_{i} < w_{i}^{\text{ac}} \). Summing up we obtain

\[
\sum_{i \in \mathcal{L}_{\text{ac}} \cup \{\text{root}\}} B_{i} \leq \sum_{i \in \mathcal{L}_{\text{ac}} \cup \{\text{root}\}} w_{i}^{\text{ac}} = \sum_{i \in \mathcal{A}} w_{i}.
\]

Hence, by setting \( Q^{*} \) as given in the theorem, we ensure that (13) is always satisfied, meaning that there is at least one packet transmission in each main round.

We take advantage of the theorem in the Linux Qdisc implementation, where we adjust \( Q^{*} \) dynamically to the set of active leaf and internal classes at the start of each main or surplus round, using the residual \( R_{\text{root}} \). When a leaf class \( i \) becomes active we set \( R_{\text{root}} = R_{\text{root}} + L_{i}^{\text{max}} + w_{i} \), where \( L_{i}^{\text{max}} \) is set to the MTU of 1500 bytes. When a leaf class \( i \) becomes idle, we set \( R_{\text{root}} = R_{\text{root}} - L_{i}^{\text{max}} - w_{i} \). Similarly, when an internal class becomes active, we set \( R_{\text{root}} = R_{\text{root}} + w_{i} \), and set \( R_{\text{root}} = R_{\text{root}} - w_{i} \) when class \( i \) becomes idle. These updates to the root class may result in \( R_{\text{root}} < 0 \) and, after the update of (6), in \( B_{\text{root}} < 0 \). In this case, we set the fair quota of the root to \( F_{\text{root}} = 0 \) for the next round.
VI. Fairness Analysis

To evaluate how well HLS realizes an HMM fair allocation for time-variable traffic, we use a fairness metric that measures the deviation from the allocation of an ideal hierarchical bit-by-bit round-robin scheduler. The fairness metric is defined as follows.

**Definition 1.** A scheduling algorithm for a class hierarchy is HMM(\( \alpha \)) fair if in an arbitrary time interval \([t_1, t_2]\) where any two sibling classes \(i\) and \(j\) are backlogged,

\[
\left| \frac{D_i(t_1, t_2)}{w_i} - \frac{D_j(t_1, t_2)}{w_j} \right| \leq \alpha,
\]

where \(D_i(t_1, t_2)\) is the amount of traffic that class \(i\) or its leaf descendants transmit in the time interval \([t_1, t_2]\).

The left-hand side of the equation is the weighted difference between the number of bytes that two classes \(i\) and \(j\) transmit. Since the difference is zero in an ideal hierarchical bit-by-bit round-robin scheduler, the bound \(\alpha\) expresses how far a particular scheduling algorithm deviates from an ideal link sharing scheduler.

For our analysis of the fairness metric of HLS, we find it useful to use the concept of subtrees. The subtree of class \(i\), denoted by \(\text{stree}(i)\), consists of class \(i\) and its descendants, that is, \(\text{stree}(i) = \text{desc}(i) \cup \{i\}\). We also define the aggregate balance of subtree \(\text{stree}(i)\), denoted by \(A_i\), as the sum of the balances and residuals of all classes in \(\text{stree}(i)\), that is,

\[
A_i = \sum_{j \in \text{stree}(i)} B_j + R_j, \quad (14)
\]

For a leaf class \(i\), we obviously get \(\text{stree}(i) = i\) and \(A_i = B_i\).

In this section, we assume that \(Q^*\) is set to the lower bound given in Theorem 2.

A. Bound on Fairness of HLS

We now analyze the fairness metric of HLS. Throughout the analysis we assume that HLS operates as described in Sec. V-A, that is, with alternating replenishment and transmission phases. With Lemma 1, such a scheduler generates the same transmission schedule as the original HLS.

The following lemma expresses the number of bytes that a backlogged class \(i\) transmits in a given time interval in terms of the aggregate balance \(A_i\).

**Lemma 2.** If a non-root class \(i \in \mathcal{I} \cup \mathcal{L}\) is backlogged throughout a time interval \((t_1, t_2)\), then

\[
D_i(t_1, t_2) = A_i(t_1) - A_i(t_2) + w_i \sum_{r \in R(t_1, t_2)} F_i^{(r)} p(i), \quad (15)
\]

where \(A_i(t)\) is the value of \(A_i\) at time \(t\), \(R(t_1, t_2)\) is the set of main and surplus rounds that are started in the time interval \((t_1, t_2)\), and \(F_i^{(r)}\) is the fair quota of class \(i\) at round \(r\).

**Proof:** Consider a class \(i \in \mathcal{I} \cup \mathcal{L}\) that is continuously backlogged in time interval \((t_1, t_2)\). The values of \(B_j\) and \(R_j\) for class \(j \in \text{stree}(i)\) only change in one of the following scenarios:

1) Class \(j \in \mathcal{L}\) transmits a packet of size \(L\):
   The update for the transmission of a packet follows (9), which decreases the value of \(B_i\) by \(L\), which, in turn, decreases \(A_i\) by the same amount. So the value of \(A_i\) decreases from this scenario by the total number of bytes transmitted by any class \(j \in \text{stree}(i)\) during the interval \((t_1, t_2)\), that is, by \(D_i(t_1, t_2)\).
2) Quota replenishment of class \( i \) at the start of a main or surplus round:

For class \( i \), the quota replenishment of a round \( r \in R(t_1, t_2) \) increases \( A_i \) by \( w_i F_p^{(r)} \) according to (7) or (8). So, the total increase of \( A_i \) in \((t_1, t_2)\) from this scenario is given by \( \sum_{r \in R(t_1, t_2)} w_i F_p^{(r)} \).

3) Quota replenishment of class \( j \neq i \) at the start of a main or surplus round:

The quota replenishment of class \( j \) uses either (7) or (8). In this scenario, the sum of \( B_j + R_j + B_{p(j)} \) remains unchanged, and since \( p(j) \in stree(i) \), \( A_i \) also does not change its value.

4) Class \( j \) becomes inactive:

Since we assume that class \( i \) is active throughout the interval, \( j \neq i \). Following (10) and (11), \( B_j \) is set to zero and the balance of \( p(j) \in stree(i) \) is increased by the same amount. This update does not change \( A_i \).

Considering the changes from every scenario, we have

\[
A_i(t_2) = A_i(t_1) - D_i(t_1, t_2) + \sum_{r \in R(t_1, t_2)} w_i F_p^{(r)}.
\]

We then rearrange the terms to get (15).

Next, we present a set of lemmas that seek to bound the aggregate balance \( A_i \).

**Lemma 3.** For any time \( t \), let \( t' \) be the time at the end of the replenishment phase of the most recent main round prior to time \( t \). Then, for any child class \( i \) of the root class,

\[
A_i(t') \geq A_i(t),
\]

where \( A_i(t') \) and \( A_i(t) \), respectively, are the aggregate balance at times \( t' \) and \( t \).

**Proof.** Without loss of generality we only consider active classes. In the time interval \((t', t)\), the value of \( B_j \) or \( R_j \) for \( j \in stree(i) \) changes only in one the following situations:

1) Quota replenishment of a child of class \( j \) at the start of a surplus round:

As \( j \neq root \), HLS performs a quota replenishment on class \( k \in child(j) \), updating \( B_k, R_k \), and \( B_j \) using either (7) or (8). Since the sum \( B_k + R_k + B_j \) does not change by the updates, neither does \( A_i \).

2) Quota replenishment of class \( j \) at the start of a surplus round:

The quota replenishment of class \( j \) uses either (7) or (8). Since \( F_{root} = 0 \) in surplus rounds, the child classes of the root perform a quota replenishment only at the beginning of a main round, a time which is not in the interval \((t', t)\). Therefore, we have \( j \neq i \). For all other classes \( j \in stree(i) \), the sum of \( B_j + R_j + B_{p(j)} \) remains unchanged, and so does \( A_i \).

3) Class \( j \in \mathcal{L} \) transmits a packet:

The update for the transmission of a packet follows (9), which decreases the value \( B_i \), which, in turn, decreases \( A_i \).

4) Class \( j \) becomes inactive:

If \( j \neq i \), following (10) and (11), \( B_j \) is set to zero and the balance of \( p(j) \) is increased by the same amount. This update does not change \( A_i \). If \( j = i \), then \( A_i \) becomes zero. This follows from (10) and (11), and the fact that \( p(i) \notin stree(i) \).

The claim follows since in all cases, the value of \( A_i \) either remains the same or decreases.

Lemma 3 implies that the maximum value of \( A_i \) occurs at the end of the replenishment phase of the main round.

**Lemma 4.** At the end of a main or surplus round, each leaf class \( i \in \mathcal{L} \) satisfies

\[
0 \leq B_i < L_i^{\text{max}}.
\]

**Proof.** We only need to consider classes that are active at the start of the main or surplus round. When HLS visits class \( i \) in the transmission phase, HLS transmits the packet at the head of the buffer from
class \( i \) as long as \( L_i \leq B_i \) where \( L_i \) is the size of the packet at the head of the buffer. So, the visit of class \( i \) ends when \( B_i < L_i \) or if class \( i \) becomes idle. In the first case, the claim follows from \( L_i \leq L_i^{\max} \). In the second case, we have \( B_i = 0 \) due to (10) and (11).

Lemma 5. At the start of a main round, before the replenishment phase, each leaf class \( i \in \mathcal{L} \) satisfies
\[
0 \leq A_i(t) < L_i^{\max},
\]
and each internal class \( i \in \mathcal{I} \) satisfies
\[
0 \leq A_i(t) \leq \sum_{j \in \text{desc}(i)} w_j + \sum_{j \in \text{ldesc}(i)} L_j^{\max},
\]

Note that the unit of \( w_j \) in the equation above is in bytes.

Proof. Consider the time at the start of a main round, which immediately follows the end of the previous main or surplus round. For \( i \in \mathcal{L} \), the lemma follows from Lemma 4. For a class \( i \in \mathcal{I} \), at the start of the round, it must hold that
\[
B_i + R_i < w_i^{ac},
\]
since otherwise next round will be a surplus round (instead of a main round). Recall that \( B_i + R_i \geq w_i^{ac} \) at the end of a main or surplus round is the condition to start a new surplus round. It then follows from (5) that
\[
B_i + R_i \leq \sum_{k \in \text{child}(i)} w_k.
\]

Then we derive
\[
A_i = \sum_{j \in \text{stree}(i)} (B_j + R_j) \\
= \sum_{j \in \text{stree}(i) \cap \mathcal{I}} (B_j + R_j) + \sum_{j \in \text{stree}(i) \cap \mathcal{L}} B_j \\
< \sum_{j \in \text{stree}(i) \cap \mathcal{I}} \sum_{k \in \text{child}(j)} w_k + \sum_{j \in \text{stree}(i) \cap \mathcal{L}} L_j^{\max} \\
\leq \sum_{j \in \text{desc}(i)} w_j + \sum_{j \in \text{ldesc}(i)} L_j^{\max}.
\]

The first line uses the definition in (14). We then split the sum over the flows in \( \text{stree}(i) \) into internal and leaf classes in the second line, where we use that leaf classes have no residual. The first term in the third line follows from (17), and the second term follows by applying Lemma 4. We arrive at the last line by rearranging the sums.

We now define \( \tilde{A}_i \) as an upper bound on \( A_i \) at the beginning of a main round. With Lemma 5, we have for every non-root class \( i \),
\[
\tilde{A}_i = \begin{cases} 
L_i^{\max} - 1, & i \in \mathcal{L}, \\
\sum_{j \in \text{desc}(i)} w_j + \sum_{j \in \text{ldesc}(i)} L_j^{\max}, & i \in \mathcal{I}.
\end{cases}
\]

We can use this bound to obtain an upper bound on \( A_i \) for a child class \( i \) of the root class, which holds at all times.

Lemma 6. Every class \( i \in \text{child}(\text{root}) \) satisfies at all times the bound
\[
A_i \leq \tilde{A}_i + w_i \max_{j \in \text{child}(\text{root})} \left\{ \frac{w_j + \tilde{A}_j}{w_j} \right\}.
\]

(18)
Proof. Consider a child class \( i \) of the root class. Let \( t' \) be the time right before the replenishment phase of a main round and \( t'' \) be the time when the quota replenishment is completed. We use \( A_i(t') \) and \( A_i(t'') \) to denote the aggregate balance of class \( i \) at these times. We derive

\[
A_i(t'') = A_i(t') + w_i F_{\text{root}} \leq \tilde{A}_i + w_i \frac{B_{\text{root}}(t')}{w^{ac}_{\text{root}}} \leq \tilde{A}_i + w_i \frac{A_{\text{root}}(t')}{w^{ac}_{\text{root}}}.
\]

The first line indicates added quota during the replenishment phase. The second line uses Lemma 5 and the definition of \( F_{\text{root}} \). We then relax the floor function. In the last line, we use \( B_{\text{root}}(t') \leq A_{\text{root}}(t') \) and drop the floor function.

Next we obtain a bound on \( A_{\text{root}}(t') \).

\[
A_{\text{root}}(t') = \sum_{j \in A} w_j + \sum_{j \in \mathcal{L}_{ac}} L^\text{max}_j = \sum_{c \in A} \sum_{j \in \text{stree}(c)} w_j + \sum_{c \in \text{child}(\text{root})} \sum_{j \in \mathcal{L}_{ac}} L^\text{max}_j 
\]

\[
\leq \sum_{c \in A} \left( \sum_{j \in A} w_j + \sum_{j \in \mathcal{L}_{ac}} L^\text{max}_j \right) 
\]

\[
= \sum_{c \in A} \left( \sum_{j \in \text{stree}(c)} w_j + \sum_{j \in \text{ldesc}(c)} L^\text{max}_j \right) 
\]

\[
\leq \sum_{c \in A} w_c + \tilde{A}_c.
\]

The first line follows because the total quota in the hierarchy is given by \( Q^* \), which we assumed is set to the lower bound given in Theorem 2. We split the summation in the second line. For the first term of the second line, note that for each \( j \in \mathcal{I}_{ac} \cup \mathcal{L}_{ac} \), there is exactly one (active) class \( c \in \text{child}(\text{root}) \cap A \) that is an ancestor of \( j \) (\( c \in \text{anc}(j) \)) or is itself class \( j \) (\( c = j \)). Conversely, every active class in \( \text{stree}(c) \) is also in \( \mathcal{I}_{ac} \cup \mathcal{L}_{ac} \). Therefore, the first terms in the first and the second lines are equivalent. The second term in the second line follows from similar considerations. In the third line, we relax the sum and combine similar terms, and then and rearrange the sums in the fourth line. In the last line, we apply the definition of \( \tilde{A}_c \).

With the bound on \( A_{\text{root}}(t') \) we continue to derive

\[
\frac{A_{\text{root}}(t')}{w^{ac}_{\text{root}}} \leq \sum_{j \in A \cap \text{child}(\text{root})} \frac{w_j + \tilde{A}_j}{w_j} \leq \max_{j \in \text{child}(\text{root}) \cap A} \frac{w_j + \tilde{A}_j}{w_j}.
\]
Using this inequality, we obtain for (19) that
\[ A_i(t'') \leq \tilde{A}_i + w_i \max_{j \in \text{child}(\text{root})} \frac{w_j + \tilde{A}_j}{w_j}. \tag{22} \]

Then for any time \( t \), let time \( t' \) be the time at the end of the replenishment phase of the most recent main round prior to time \( t \). From Lemma 3 and (22), it follows that
\[ A_i(t) \leq A_i(t') \leq \tilde{A}_i + w_i \max_{j \in \text{child}(\text{root})} \frac{w_j + \tilde{A}_j}{w_j}, \]
satisfying (18).

We continue with computing an upper bound of \( A_i \) for classes other than the children of the root class. We first define
\[ \text{rc}(i) = \begin{cases} i, & i \in \text{child}(\text{root}), \\ \text{rc}(p(i)), & \text{otherwise}, \end{cases} \]
which is the ancestor of class \( i \) that is a child of the root class (if \( i \notin \text{child}(\text{root}) \)), or class \( i \) itself (if \( i \in \text{child}(\text{root}) \)). With this definition we can provide a bound on the aggregate balance of classes that holds at all times.

**Lemma 7.** For any non-root class \( i \in I \cup L \), the bound
\[ A_i \leq \tilde{A}_{\text{rc}(i)} + w_{\text{rc}(i)} \max_{j \in \text{child}(\text{root})} \left\{ \frac{w_j + \tilde{A}_j}{w_j} \right\} \tag{23} \]
holds at all times.

As indicated in the lemma, we will denote the bound on the right hand side by \( \bar{A}_i \).

**Proof.** Due to (6)–(11), we have \( B_i \geq 0 \) and \( R_i \geq 0 \) for each non-root class \( i \in I \cup L \). Then, we can conclude from (14) that \( A_i \leq A_{\text{rc}(i)} \). Since \( \text{rc}(i) \in \text{child}(\text{root}) \), the claim follows by applying Lemma 6 to \( A_{\text{rc}(i)} \).

We now express an upper bound on the fairness metric for HLS.

**Theorem 3.** HLS is \( \text{HMM}(\alpha^{(\text{HLS})}) \)-fair with \( \alpha^{(\text{HLS})} = \max_{i \in N, j \in \text{sib}(i)} \alpha_{ij}^{(\text{HLS})} \), where
\[ \alpha_{ij}^{(\text{HLS})} = \frac{\bar{A}_i}{w_i} + \frac{\bar{A}_j}{w_j}. \]

**Proof.** Starting with Definition 1, we derive for two sibling classes \( i \) and \( j \) as follows:
\[ \left| \frac{D_i(t_1, t_2)}{w_i} - \frac{D_j(t_1, t_2)}{w_j} \right| = \left| \frac{A_i(t_1) - A_i(t_2)}{w_i} - \frac{A_j(t_1) - A_j(t_2)}{w_i} \right| \leq \left| \frac{A_i(t_1) - A_i(t_2)}{w_i} \right| + \left| \frac{A_j(t_1) - A_j(t_2)}{w_i} \right| \leq \frac{\bar{A}_i}{w_i} + \frac{\bar{A}_j}{w_j}. \]

In the first term, by applying Lemma 2 to \( D_i(t_1, t_2) \) and \( D_j(t_1, t_2) \), the rightmost term in (15) cancels out since \( i \) and \( j \) have the same parent node. The second line arrives from the property of the absolute function that \( |x - y| \leq |x| + |y| \). We arrive at the last line by applying Lemma 7 and the fact that \( A_i \geq 0 \).
Corollary 1. For a flat hierarchy, i.e., a hierarchy with no internal class, the value of $\alpha_{ij}^{(HLS)}$ becomes

$$\alpha_{ij}^{(HLS)} = \frac{L_{i}^{\max} - 1}{w_i} + \frac{L_{j}^{\max} - 1}{w_j} + 2 + 2 \max_{k \in \mathcal{L}} \frac{L_{k}^{\max} - 1}{w_k}.$$ 

This bound is identical to the bound for the non-hierarchical DRR scheduler with quantum $2 + 2 \max_{k \in \mathcal{L}} \frac{L_{k}^{\max} - 1}{w_k} \text{bytes}$ [8], which is no greater than $2L_{\max}^{\text{max}}$ where $L_{\max}^{\text{max}} = \max_{i \in \mathcal{L}} L_{i}^{\max}$ is the maximum packet size for the entire scheduler. For comparison, consider the fairness metric of the HDRR scheduler, which is derived in [13]. This is the only available bound available for hierarchical round-robin schedulers.

Theorem 4. HDRR is $\text{HMM}(\alpha^{(HDRR)})$-fair with $\alpha^{(HDRR)} = \max_{i \in \mathcal{N}, j \in \text{sib}(i)} \alpha_{ij}^{(HDRR)},$ where

$$\alpha_{ij}^{(HDRR)} = \frac{L_{i}^{\max}}{w_i} + \frac{L_{j}^{\max}}{w_j} + Q$$

where $Q$ is the quantum of the HDRR.

To compare the bounds of HLS and HDRR, consider the hierarchy in Fig. 1 where we set the weight of a class to its rate guarantee. The maximum packet size of each leaf class $i$ is set to $L_{\max} = 1500$ bytes. For HDRR, we also set its quantum $Q$ to the maximum packet size of 1500 bytes. Computing the bounds we obtain

$$\alpha^{(HLS)} = 103.5 \text{ bytes},$$
$$\alpha^{(HDRR)} = 1522.5 \text{ bytes}.$$ 

Here, HLS clearly has a better fairness metric. In general, due to the very different operation of HLS and HDRR, it is not feasible to show that HLS always has a better fairness metric. In fact, for deep hierarchies, the fairness metric of HDRR can be better than that of HLS.

VII. TRANSMISSION GAP

For hierarchical round-robin scheduling algorithms, we can define a second performance metric which expresses the elapsed time between visits of a given class by the scheduler. The transmission gap of a round-robin scheduling algorithm expresses the delay incurred by a packet at the head of the transmission buffer at the end of a visit of its class. The transmission gap expresses how long this packet has to wait until the round-robin scheduler returns to its class. Note that transmission gap also provides a bound on the delay until a class that becomes backlogged is visited by the scheduler. As before, we assume that $Q^*$ is set to the lower bound given in Theorem 2.

Definition 2. A round-robin based scheduling algorithm has a transmission gap of $\gamma$ if after visiting an arbitrary (leaf) class $i$ at time $t$, which remains backlogged after the visit, the next visit to class $i$ is guaranteed to occur at or before time $t + \gamma$.

The transmission gap and packet delay are related in the sense that, if $\gamma$ is the transmission gap for a scheduling algorithm, and $d_{\max}$ is the maximum delay experienced by a packet, then $d_{\max} \geq \gamma$.

For HLS, the transmission gap is as given in the following theorem.

Theorem 5. An HLS scheduler with link capacity $C$ has a transmission gap of

$$\gamma^{(HLS)} = \frac{2}{C} \left( \sum_{i \in \mathcal{L} \cup \mathcal{L}} w_i + \sum_{i \in \mathcal{L}} L_{i}^{\max} \right).$$  

Proof. We assume that the computation time in the quota replenishment phase is negligible and that HLS transmits packets for the entire duration of a main or surplus round.
Consider a main or surplus round $r$. Let $t_{s}^{(r)}$ be the time at the start of round $r$ after its replenishment phase, and $t_{e}^{(r)}$ be the time at the end of round $r$. Let $L_{i}^{(r)}$ be the number of bytes that a leaf class $i$ transmits in round $r$, and let $L^{(r)}$ be the total number of bytes that HLS transmits in round $r$. That is,

$$L^{(r)} = \sum_{i \in L_{ac}^{(r)}} L_{i}^{(r)},$$

where $L_{ac}^{(r)}$ is the set of active leaf classes at the beginning of round $r$. Finally, let $Q^{*}(t)$ and $B_{i}(t)$ be the values of $Q^{*}$ and $B_{i}$ at time $t$, respectively.

In the interval $(t_{s}^{(r)}, t_{e}^{(r)})$, the balance $B_{i}$ for each active leaf class $i$ is only updated using (9) due to a packet transmission or using (10) when class $i$ becomes idle. If class $i$ remains active after the visit in round $r$, it follows that

$$L_{i}^{(r)} = B_{i}(t_{s}^{(r)}) - B_{i}(t_{e}^{(r)}).$$

Otherwise, class $i$ becomes idle at some time $t_{idle} \leq t_{e}^{(r)}$ when the visit ends, and

$$L_{i}^{(r)} = B_{i}(t_{s}^{(r)}) - B_{i}(t_{idle}).$$

In both cases, $L_{i}^{(r)} \leq B_{i}(t_{e}^{(r)})$. With $Q^{*}$ set to the lower bound from Theorem 2 and with $B_{i}(t) \geq 0$ and $R_{i}(t) \geq 0$ at all times $t$, we obtain

$$L^{(r)} = \sum_{i \in L_{ac}} L_{i}^{(r)}$$

$$\leq \sum_{i \in L_{ac}} B_{i}(t_{s}^{(r)})$$

$$\leq Q^{*}(t_{e}^{(r)})$$

$$\leq \sum_{i \in I \cup L} w_{i} + \sum_{i \in L} L_{i}^{\text{max}}.$$

With the link capacity $C$ and due to our assumption that HLS transmits packets for the entire duration of round $r$, we get

$$\frac{t_{e}^{(r)} - t_{s}^{(r)}}{C} = \frac{L^{(r)}}{C} \leq \frac{1}{C} \left( \sum_{i \in I \cup L} w_{i} + \sum_{i \in L} L_{i}^{\text{max}} \right). \quad (25)$$

Consider a class $i$ that HLS finishes visiting at time $t_{v}^{(r)}$ in some main or surplus round $r_{1}$ and remains backlogged after the visit. Let $r_{2}$ denote the (main or surplus) round that follows round $r_{1}$. Since class $i$ is backlogged after the visit in round $r_{1}$, HLS visits class $i$ again in round $r_{2}$. Let the time that HLS visits class $i$ in round $r_{2}$ be $t_{v}^{(r_{2})}$.

From the definition of $t_{s}^{(r)}$ and $t_{e}^{(r)}$, and from our assumption that the computation time of each replenishment phase is negligible, we obtain the relationship

$$t_{s}^{(r_{1})} \leq t_{e}^{(r_{1})} = t_{s}^{(r_{2})} \leq t_{e}^{(r_{2})}.$$

Furthermore, since the time $t_{e}^{(r_{1})}$ is a time within the transmission phase of round $r_{1}$, and $t_{v}^{(r_{2})}$ is a time within the transmission phase of round $r_{2}$, it follows that

$$t_{s}^{(r_{1})} \leq t_{v}^{(r_{1})} \leq t_{e}^{(r_{1})} \quad \text{and} \quad t_{s}^{(r_{2})} \leq t_{v}^{(r_{2})} \leq t_{e}^{(r_{2})}.$$

We then combine the inequalities together to obtain

$$t_{s}^{(r_{1})} \leq t_{v}^{(r_{1})} \leq t_{e}^{(r_{1})} = t_{s}^{(r_{2})} \leq t_{v}^{(r_{2})} \leq t_{e}^{(r_{2})}. \quad (26)$$
We now compute the upper bound for the duration between the visits to class $i$ in round $r_1$ and $r_2$. That is,

\[
\begin{align*}
\hat{t}_w^{(r_2)} - \hat{t}_w^{(r_1)} &\leq t_e^{(r_2)} - t_s^{(r_1)} \\
&\leq \left( t_e^{(r_2)} - t_s^{(r_2)} \right) - \left( t_e^{(r_1)} - t_s^{(r_1)} \right) \\
&\leq \frac{2}{C} \left( \sum_{i \in I \cup L} w_i + \sum_{i \in L} L_i^{\max} \right) \\
&= \gamma^{(\text{HLS})}.
\end{align*}
\]

We arrive at the first two lines by applying (26). The third and last lines follow from (25) and the definition of $\gamma^{(\text{HLS})}$, respectively. As such, after HLS finishes visiting class $i$ at time $t_v^{(r_1)}$, HLS guarantees to visit class $i$ again at time $t_v^{(r_2)} \leq t_v^{(r_1)} + \gamma^{(\text{HLS})}$.

For comparison, we also compute the transmission gap of the HDRR scheduler from [13]. Similar to DRR, HDRR keeps track of a deficit counter $d_i$ for each leaf class $i$. Initially, the value of $d_i$ for each leaf class $i$ is set to zero. When HDRR visits a leaf class $i$ for transmission, it increases $d_i$ by a fixed quantum $Q$ and transmits a packet at the head of the queue for that class. At each transmission for a packet of size $L$, HDRR subtracts $L$ from $d_i$. The visit to class $i$ stops when $d_i < L$ where $L$ is the size of the packet at the head of the queue. If class $i$ becomes idle, HDRR sets $d_i$ to zero. Note that these are the only scenarios where the value of $d_i$ changes. We now compute the number of bytes that HDRR may transmit for a class $i$ given the number of times that HDRR visits that class.

**Lemma 8.** Given an HDRR scheduler with quantum $Q$. If a leaf class $i$ is visited $k$ times in a time interval $(t_1, t_2)$ and HDRR does not visit class $i$ at time $t_1$ or $t_2$, then

\[
kQ - L_i^{\max} \leq D_i(t_1, t_2) \leq kQ + L_i^{\max},
\]

**Proof.** Let $d_i(t)$ be the value of $d_i$ at time $t$. In HDRR, the deficit counter $d_i$ for class $i$ only changes when:

1) HDRR visits class $i$:
   - Each visit increases $d_i$ by a fixed quantum $Q$. Since there are $k$ visits in the time interval $(t_1, t_2)$, the total increase is $kQ$.
2) Class $i$ transmits a packet:
   - When class $i$ transmits a packet of size $L$, it subtracts $L$ from $d_i$. In $(t_1, t_2)$, $d_i$ is therefore decreased by the number of bytes that class $i$ transmits in the interval, which is given by $D_i(t_1, t_2)$.

Combining the two cases, we obtain

\[
d_i(t_2) = d_i(t_1) + kQ - D_i(t_1, t_2).
\]

Since HDRR does not visit class $i$ at time $t_1$, there are two possible scenarios:

1) HDRR does not visit class $i$ at all prior to time $t_1$:
   - In this case, the value of $d_i(t_1)$ equals its initial value, which is zero.
2) HDRR visits class $i$ at least once before time $t_1$:
   - Let $t'$ be the time right after the most recent visit to class $i$ prior to time $t_1$, and so $d_i(t_1) = d_i(t')$ because there is no change to $d_i$ in the interval $(t', t_1)$. If class $i$ becomes idle after the visit ends at time $t'$, then $d_i(t') = 0$. Otherwise, class $i$ remains backlogged after time $t'$, and it holds that $0 \leq d_i(t') < L$ where $L$ is the size of the head of the queue at time $t'$.

In both cases, it follows that

\[
0 \leq d_i(t_1) \leq L_i^{\max}.
\]
By using the same consideration for $t_2$, we obtain
\[
0 \leq d_i(t_2) \leq L_i^{\text{max}}.
\] (30)
We then combine (28), (29), and (30) to arrive at (27).

In order to compute the transmission gap of HDRR, we also make use a result from [13].

**Lemma 9 ([13])**. For an HDRR scheduler, if class $i$ is backlogged, then class $i$ is visited at least once every $\beta_i$ visits where
\[
\beta_i = \prod_{j \in \text{anc}(i)} \sum_{k \in \text{child}(j)} w_k.
\]

**Theorem 6.** An HDRR with link capacity $C$ and quantum $Q$ has a transmission gap of
\[
\gamma^{(\text{HDRR})} = \frac{1}{C} \left( \max_{i \in \mathcal{L}} (\beta_i - 1)Q + \sum_{j \in \mathcal{L}} L_j^{\text{max}} \right),
\]
where $\beta_i$ is as given in Lemma 9.

**Proof.** For any backlogged leaf class $i$, let $t_1$ be the time at the end of a visit to class $i$ where class $i$ remains backlogged after the visit, and let $t_2$ be the time of the subsequent visit to class $i$ after time $t_1$. Let the $V$ be the set of visits between $t_1$ and $t_2$, and $M$ be the set of classes that are visited in the time interval $(t_1, t_2)$. Furthermore, let $V_j \subset V$ be the set of visits to class $j \in M$ in the interval $(t_1, t_2)$. From the definition, $\{V_j\}_{j \in M}$ is pairwise disjoint, and
\[
V = \bigcup_{j \in M} V_j.
\]

Let $D(t_1, t_2)$ be the number of bytes that HDRR transmits in the interval $(t_1, t_2)$. Since HDRR has the link capacity of $C$ and it is guaranteed to be backlogged in the time interval $(t_1, t_2)$ due to class $i$ being backlogged in the interval, it follows that
\[
D(t_1, t_2) = C(t_2 - t_1).
\] (31)
Consider,
\[
D(t_1, t_2) = \sum_{j \in M} D_j(t_1, t_2)
\leq \sum_{j \in M} |V_j|Q + L_j^{\text{max}}
= |V|Q + \sum_{j \in M} L_j^{\text{max}}
\leq (\beta_i - 1)Q + \sum_{j \in \mathcal{L}} L_j^{\text{max}}.
\]
The first line comes from the definition of $D(t_1, t_2)$ and the fact that HDRR only visits classes in $M$ in the interval $(t_1, t_2)$. We then apply Lemma 8 in the second line and rearrange the summation in the third line. We then apply Lemma 9 and the fact that $M \subseteq \mathcal{L}$ in the last line. By applying (31) and the definition of $\gamma^{(\text{HDRR})}$, we obtain
\[
t_2 \leq t_1 + \frac{(\beta_i - 1)Q + \sum_{j \in \mathcal{L}} L_j^{\text{max}}}{C}
\leq t_1 + \gamma^{(\text{HDRR})}.
\]
Therefore, after class $i$ is visited at time $t_1$, it is guaranteed to be visited again at time $t_2 \leq t_1 + \gamma^{(\text{HDRR})}$.

\[\Box\]
For the class hierarchy in Fig. 1 with a maximum packet size \( L^{\text{max}} = 1500 \) bytes for all classes, and a link with rate \( C = 1 \) Gbps, we obtain

\[
\gamma^{(\text{HLS})} = 0.146 \text{ ms},
\]

\[
\gamma^{(\text{HDRR})} = 3600 \text{ ms}.
\]

We observe that HLS has a significantly smaller transmission gap compared to HDRR. The difference of the transmission gaps is exacerbated with a larger class hierarchy. Consider a class hierarchy consisting of a complete binary tree with \( l \) levels where each left child class has its weight set to 3 and the right child class has its weight set to 7. We again assume \( C = 1 \) Gbps and \( L^{\text{max}} = 1500 \) bytes. Table II shows the transmission gaps of HLS and HDRR for the range \( 4 \leq l \leq 11 \). It is apparent that the transmission gap of HDRR becomes unsustainable for large class hierarchies.

| \( l \) | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( |L| \) | 8   | 16  | 32  | 64  | 128 | 256 | 512 | 1024 |
| \( \gamma^{(\text{HLS})} \) (in ms) | 0.096 | 0.19 | 0.39 | 0.77 | 1.55 | 3.09 | 6.18 | 12.37 |
| \( \gamma^{(\text{HDRR})} \) (in ms) | 12   | 120  | 1200 | 12000 | 120002 | 1200003 | 12000006 | 120000012 |

**TABLE II**

Transmission gaps of HLS and HDRR for a binary tree hierarchy with \( l \) levels.

**VIII. Evaluation**

We have implemented HLS as a kernel module in Linux kernel 4.15.0-101-generic [22]. A description of the implementation is available in [24]. Here we present measurement experiments of the HLS Qdisc in Linux and compare them with measurements of the existing link sharing schedulers in Linux, CBQ and HTB. The experiments are conducted on Emulab [25], a network testbed for network experiments.

**A. Experimental Setup**

The topology of the experiments involves three Linux servers as shown in Fig. 6 designated as *traffic generator*, *scheduler*, and *traffic sink*. Each server is a Dell PowerEdge R430 with two 2.4 GHz 8-Core CPUs, 64 GB RAM, a dual-port/quad-port 1GbE PCI-Express NICs, and a dual-port/quad-port Intel X710 10GbE PCI-Express NICs. The servers run Ubuntu 18.04LTS.

Fig. 6. Network topology for experiments.

The traffic generator and the scheduler are connected by a 10 Gbps Ethernet link, and the scheduler and the traffic sink are connected by a 1 Gbps Ethernet link. Routing tables of all servers are set up statically so that all traffic is routed from the traffic generator to the traffic sink. The link sharing schedulers are configured at the egress of the 1 Gbps interface at the scheduler node. The traffic generator uses FIFO scheduling. With this setup we can saturate the outgoing link at the scheduler without overloading its CPUs.

The traffic generator sends UDP/IPv4 datagrams with a length of 1000 bytes, where destination port numbers are mapped to classes at the scheduler node. Our graphs plot the transmission rates of traffic classes using jumping windows with length 0.2 s. The rate at which the traffic generator sends packets is such that it ensures that each active leaf class is permanently backlogged at the egress of the scheduler node.
B. Experiment 1: Validation of HMM fairness

In this experiment, which is a scaled version of an experiment in [16], [26], we show that HLS quickly converges to an HMM fair allocation when the set of active flows changes. The class hierarchy of the experiment is as shown in Fig. 1 for a 1 Gbps link, but with the following rate guarantees:

| Class: A B A1 A2 B1 B2 |
|------------------------|
| Rate guarantee:        |
| 700 300 300 400 100 200 |

In the experiment, all leaf classes are initially active and transmit packets with a fixed packet size of 1000 B. At certain time intervals, one class becomes idle, in the following sequence:

| Interval (in seconds): | [4, 7] | [11, 14] | [19, 22] |
|------------------------|--------|----------|----------|
| Inactive class:        | B1     | A1       | A2       |

The throughput of the classes is shown in Fig. 7. The plot for each class is labeled. Dashed gray lines show the aggregate traffic of the internal classes A and B. The dashed line with label ‘Total’ indicates the aggregate traffic from all classes, which is at or close to the link capacity of 1 Gbps.

Fig. 7(a) shows the HMM fair allocations from Theorem 1. The measured rates for HLS in Fig. 7(b) show that HLS satisfies HMM fairness for all classes at all times. When all leaf classes are active, they each obtain their class guarantees. If one class drops out, the sibling class consumes the guarantee of its sibling.

Figs. 8(a)–8(c) show the results of this experiments for CBQ. Fig. 8(a) has the measurements of the CBQ Qdisc from Emulab. Figs. 8(b) and 8(c) show the results of ns2 simulations. using the variants formal link sharing and top-level. The data shows that each variant satisfies the guarantees of leaf classes at all times. The guarantees of the internal classes A and B are satisfied when all leaf classes are sending traffic. When this is not the case, the throughput of one of the internal classes may fall below its guarantee, while the other class grabs the remaining bandwidth. Interestingly, the Linux qdisc implementation shows smaller violations of rate guarantees than the ns2 simulations.

Lastly, we present measurements of the HTB Qdisc in Fig. 8(d). For this experiment, HTB satisfies the allocation of HMM fairness.

C. Experiment 2: Isolating class guarantees

This experiment illustrate the need for isolating class guarantees, and the inability of the existing link sharing schedulers CBQ and HTB to realize isolation between classes. The experiment uses the class hierarchy from Fig. 1 (in Sec. I). In addition to the guarantees shown in the figure, we vary the guarantees of classes A1, A2, B1, B2 to evaluate three scenarios, labeled as ‘L’, ‘M’, ‘H’, which stands for low, medium, and high differences between the guarantees. The guarantees in the scenarios (in Mbps) are as follows:

| Class:          | A1 | A2 | B1 | B2 | A  | B  | C  |
|----------------|----|----|----|----|----|----|----|
| Low ‘L’         | 140| 160| 140| 160| 300| 300| 400|
| Medium ‘M’      | 100| 200| 100| 200| 300| 300| 400|
| High ‘H’        | 60 | 240| 60 | 240| 300| 300| 400|

The ‘M’ scenario corresponds to the guarantees shown in Fig. 1. The guarantees of classes A, B, and C are the same in all three scenarios, and are as shown in Fig. 1.

In the experiment, three leaf classes (A1, B2, C) generate traffic. In the middle of the experiment, in the interval [10, 20] s, class C pauses transmissions. Since leaf classes A1 and B2 compete with each other at the level of their respective parent classes A and B, their allocation should be determined by the guarantees of the parents. If this is the case, A1 and B2 receive the same allocation in all three scenarios.

Fig. 9(a)–9(c) show the measured throughput of the HLS Qdisc. In all three scenarios, the throughput of active classes corresponds to the HMM fair allocation. When all three classes are active (in [0, 10] s
Fig. 7. Experiment 1: HMM fairness.

(a) HMM fair allocation.

(b) HLS Qdisc (Linux).

Fig. 8. Experiment 1: HMM fairness (CBQ and HTB).

(a) CBQ qdisc (Linux).

(b) CBQ ns2 simulation (formal link sharing).

(c) CBQ ns2 simulation (top-level).

(d) HTB qdisc (Linux).
and $[20, 25] \text{s}$), they split the allocation in the ratio $3 : 3 : 4$, according to the guarantees of classes $A, B, C$. When class $C$ drops out, $A1$ and $B2$ split the capacity evenly, since $A$ and $B$ have the same guarantee.

The second row of graphs in Fig. 9 presents measurements of the HTB Qdisc. First note that, for all scenarios the minimum rate guarantees of internal and leaf classes are maintained at all times. When all three classes are active, they have the same allocation as HLS. However, when class $C$ drops out, classes $A1$ and $B2$ do not split the freed up link capacity evenly. Instead, the throughput appears to depend on the guarantees of the active leaf classes $A1$ and $B2$. As seen in Figs. 9(e) and 9(f), by increasing the guarantee of class $B2$ and decreasing that of $A1$, the allocation becomes more lopsided.

The throughput in the scenarios under CBQ, depicted in the last row of graphs in Fig. 9, shows that CBQ allocates rates in a similar fashion as HTB. That is, sharing of bandwidth at the level of internal classes does not respect the guarantees of the internal classes. As with HTB, the allocation appears to be again determined by the guarantees of the leaf classes.

The observed link sharing of CBQ and HTB indicates a lack of isolation between classes in the hierarchy. Here, class $B$ can manipulate its allocation by bundling its traffic in a single descendant class, at the cost of class $A$. The HMM fair allocation of HLS does not allow this to happen.
Fig. 10. Class hierarchy in Experiment 3.

Fig. 11. Experiment 3. Non-determinism. With CBQ, stopping and resuming transmissions results in different rate allocations.

D. Experiment 3: Non-determinism

This experiment shows that, for a given input, the CBQ qdisc does not uniquely determine a rate allocation. We consider the class hierarchy in Fig. 10 where active classes \((A, B, C, D)\) are indicated by a shaded background. In the experiment, classes \(A, B_1, C_1\) repeatedly transmit for \(4\) s and then pause for \(0.5\) s. This creates an on-off pattern which is repeated several times. The transmission pauses are sufficiently long so that the backlog at the scheduler is fully cleared.

Fig. 11(a) presents the allocations of the CBQ qdisc. We note again that the minimum guarantees of leaf and internal classes are never violated. The main observation is that when classes resume transmissions after a pause, the link sharing settles to two different allocations. In both outcomes, class \(C_1\) is always kept at \(250\) Mbps, the minimum guarantee of its parent class. In one of the outcomes, the allocation of class \(A\) is close to its minimum guarantee, while class \(B_1\) consumes all of the remaining capacity. In the other outcome, \(B_1\) receives the minimum guarantee given to its parent, and \(A\) consumes all other bandwidth. No equitable sharing between classes occurs in either outcome.

We have observed non-deterministic rate allocations in other scenarios, including situations where rate allocation change spontaneously without pausing traffic sources (e.g., see Fig. 9(h) at \(\sim 13\) s). This non-determinism indicates that the allocation of the CBQ qdisc is under-determined, that is, multiple allocations satisfy its link sharing guidelines. We have not observed non-deterministic allocations of CBQ in ns2 simulations of this experiment.

In Fig. 11(b) and 11(c) we show the outcomes for this experiment for HTB and HLS, respectively. Both create unique allocations. The results for HTB in Fig. 11(b) show link sharing between classes \(A\) and \(B_1\), while class \(C_1\), similar to CBQ, is kept at the guarantee of its parent class. The allocation of
HLS in Fig. 11(c) is HMM fair in each ‘on’ phase. Classes \(A, B1,\) and \(C1\) share the link capacity in the ratio 4:5:5, according to the ratio of the guarantees of classes \(A, B,\) and \(C.\)

E. Experiment 4: Overhead

We next measure the processing overhead of HLS and compare it to that of CBQ and HTB. Since all schedulers are implemented as Linux Qdiscs, they share the performance limitations of the Qdisc framework, in particular, the single Qdisc lock. In order to move the bottleneck of the experimental setup to per-packet processing, we replace the 1 Gbps link in Fig. 6 between the scheduler and the traffic sink by a 10 Gbps link, and we send small packets. We verified that the servers that run the traffic generator and traffic sink are not bottlenecks in the experiment.

The experiment uses the NetPerf TCP-RR [27], similar to an experiment in [19, Fig. 6]. TCP senders and receivers send 1-byte packets in each direction in a ping-pong fashion. One round of the ping-pong is called a transaction. The traffic from each TCP sender is mapped to a separate leaf class at the scheduler node (in Fig. 6). The performance metric is the total number of completed transactions per second.

We consider two class hierarchies: a perfect binary tree and a flat hierarchy. In a binary tree, with \(N\) leaf classes, the total number of classes, including the root, is \(2N - 1.\) In the flat hierarchy, the root class has \(N\) children which are all leaf classes. For HLS we set the weight of every class to one in both scenarios. For HTB and CBQ, the rate guarantees are divided evenly between the leaf classes. In addition to the hierarchical scheduling algorithms HLS, HTB, and CBQ, we also include measurements with FIFO scheduling. Since FIFO is a classless scheduler, outcomes are not sensitive to the class hierarchy.

Fig. 12(a) shows the transactions per second as a function of the number of leaf classes for the binary tree hierarchy. All schedulers show roughly the same performance. The number of transactions initially increases linearly with the leaf classes and saturates at around 300K transactions (100K = 10^5). Since HTB limits the number of levels in the class hierarchy, the binary tree hierarchy cannot be increased beyond 128 leaf classes. The fact that FIFO sometimes has worse results than the hierarchical schedulers indicates the degree of randomness in experiments that involve a large number of TCP flows.

Fig. 12(b) depicts the results for the flat hierarchy. Here, the number of transactions initially increases and plateau at around 300K transactions, similar to binary tree scenario. After around 256 leaf classes, however, the performance of the classful schedulers declines. A comparison with FIFO points to a performance bottleneck that arises during packet classification, which impacts all classful schedulers in the same way. The classification compares the packet destination port to the port associated to each leaf class, and the number of comparisons grows linearly with the number of leaf classes.
The experiment leads us to conclude that the HLS Qdisc does not incur a performance penalty, compared to HTB and CBQ. In fact, since the schedulers perform similarly to FIFO, none of the hierarchical schedulers presents a bottleneck.

We emphasize that the outcome of this experiment is sensitive to the configuration of filters that map packets to traffic classes. In Fig. 12, the mapping of packets to classes is done progressively. Each internal node in the hierarchy has (two) filter expressions for mapping traffic to its child classes. Alternatively, the mapping can be performed at the root Qdisc for all leaf classes. If this is done, the results show a precipitous drop of completed transactions when the number of leaf classes exceeds 100.

IX. RELATED WORK

There are several reasons for the recent surge of interest in shaping and scheduling algorithms. First, the increased flexibility of recent programmable packet switches has enabled customization of scheduling algorithms to application requirements [12], [28], [29]. Second, the Ultra-Reliable Low-Latency Communication service category in 5G networks, which guarantees latencies below 1 ms has led to standardization efforts by the IEEE (for Layer-2) and by the IETF (for Layer-3) for compatible protocol frameworks [30] and traffic control algorithms [31]. Third, an increased demand for fine-grain control of traffic in data centers has created a need for advanced packet scheduling methods at servers [1], [2].

These efforts benefit from an intense period of research in the 1990s that created many of the scheduling and shaping algorithms in use today [4], [8], [16], [32]. Recent research on packet scheduling has put emphasis on generality, e.g., PIFO [12], UPS [33], and efficient implementations, for example, Carousel [19], Eiffel [20], Loom [34], CQ [35].

Most relevant to our paper is the claim in [12] of realizing HPFQ by a hierarchy of PIFO queues. However, the claim holds only when packets have a fixed size. For variable-sized packets and classes with different weights the arrival of a packet may require changing the relative order of packets in the PIFO buffers. By design, PIFO does not support reshuffling buffered packets.

BwE [18] performs a centralized rate allocation for hierarchically organized inter-data center traffic, which computes end-to-end max-min fair rate allocations for a network setting, which are enforced by HTB ceiling rates. Interestingly, in [18, Sec.9] it is argued that fair queueing is not suitable since ‘weights are insufficient for delivering user guarantees.’ By showing the equivalence of rate guarantees and weights in Sec. XI, our paper shows that the above statement requires a correction.

We have not included HFSC [36], [37] in this paper, even though it is another hierarchical scheduler available in Linux. HFSC is hybrid scheduler that deterministically guarantees service curves and shares excess bandwidth with fairness objectives. As pointed out in [36] it is, in general, not possible to simultaneously guarantee the service curves of HFSC and its fairness criteria. Since HFSC resolves conflicting guarantees by giving priorities to service curves, the role of link sharing is limited. We also note that HFSC realizes so-called ‘lower service curves’ [38]. Rate guarantees of these service curves share a drawback with the VirtualClock scheduler [39], where a class that is served above its guaranteed rate for some time, may be later served at a rate below its guarantee. Differently, fair scheduling algorithms realize ‘strict service curves’ [38], which ensure rate guarantees for every time interval where a class is backlogged.

X. CONCLUSIONS

We presented a round-robin scheduler for hierarchical link sharing that ensures rate guarantees and isolation between classes, and which is suitable for supporting high line rates. The presented HLS scheduler resolves shortcomings of deployed hierarchical link sharing algorithms when distributing excess capacity to traffic classes. The link sharing in HLS is strategy-proof in that a class that needs more bandwidth cannot increase its allocation by increasing its transmissions or misrepresenting the class hierarchy of its descendants. We have shown that the implementation of HLS does not create a performance bottleneck. In future work, we will extend the Qdisc implementation of HLS to also enforce maximum (ceiling) rates.
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Since available documentation for Linux schedulers often focuses on configuration issues, we provide conceptual descriptions of HTB and CBQ. First, we briefly discuss two building blocks:

**DRR scheduler.** The scheduler maintains one FIFO queue for each class. Queues with a backlog are visited in a round-robin fashion. Each class has a counter for the number of bytes that the class can transmit. When a packet is transmitted, the counter is decremented by the packet size. In a visit by the round robin, a class can transmit as long as it has packets and the counter does not become negative. At the start of a new round of the round robin, the counters are incremented by the quantum which is set proportional to the class guarantee.

**Token Bucket.** A token bucket has two parameters, a bucket size $b$ and a rate $r$. The bucket content is a counter, which is initialized to $b$. When a packet of size $L$ is transmitted, $L$ tokens are removed from the bucket. If there are less than $L$ tokens in the bucket, the packet has to wait until $L$ tokens are available. Tokens are added at a constant rate of $r$, but the bucket content may not exceed $b$.

**CBQ**

The following description of CBQ is based on the Linux qdisc code [40] and a reference implementation of CBQ in the ns2 simulator [41]. The implementations sometimes deviate from the description in [16, Appendix A].

The main components of a CBQ scheduler, shown in Fig. 6 are (1) an estimator, (2) a regulator, and (3) a scheduler. The estimator computes the transmission rate of each class using an exponential moving average and determines if a class is serviced below or above its guaranteed rate. The class is underlimit in the former case and overlimit in the latter. The regulator uses the output of the estimator to tag classes as regulated or unregulated. Regulated classes are not considered for transmission. This is different from [16, Appendix A], which describes CBQ as having two schedulers, one for regulated and for unregulated classes. The scheduler runs Weighted Round Robin, which is essentially identical to DRR, to transmit packets from unregulated classes, where the quantum of each class is determined from the class guarantee. The regulator inspects the underlimit/overlimit state in the class hierarchy. A class is unregulated (1) if it is underlimit or (2) if it has an ancestor that is underlimit and that has no underlimit descendants with a backlog. The first rule ensures that a class receives its rate guarantee. The second rule determines when a class can borrow bandwidth in excess of its guarantee. In [16] these rules are referred to as formal link sharing guidelines.

To reduce the complexity involved in applying the formal link sharing guidelines, CBQ proposes two approximations, which are referred to as ancestor-only and top-level link sharing guideline. The Linux implementation of CBQ only supports top-level link sharing.
Since HTB supports both minimum class guarantees and upper bounds on the rate of a class, it is both a scheduler and a shaper. Possibly because of its name, discussions of HTB often focus on its shaping operation, and it is frequently characterized as a pure traffic shaper. The following description, which is based on the HTB qdisc code [42], is simplified in parts.

In HTB, each class has two token buckets, an assured bucket and a ceiling bucket. The assured and ceiling buckets of class $i$ are filled at rate $AR_i$ and $CR_i$, respectively, with $CR_i \geq AR_i$. For an internal class $i$, reasonable choices should satisfy

$$AR_i \geq \sum_{j \in \text{child}(i)} AR_j \quad \text{and} \quad CR_i = \max_{j \in \text{child}(i)} CR_j.$$

In the experiments we set $CR_i$ to the link capacity $C$ for all classes, which effectively disables traffic shaping. For transmission of a packet with size $L$ from a class, $L$ tokens are removed from both the assured and ceiling buckets, as well as from the buckets of the ancestors of the class.

The filling level of the token buckets determine the state of a class, where each state is associated with a color:

| State                                           | Color |
|-------------------------------------------------|-------|
| Assured bucket has tokens                       | Can send | green |
| Assured bucket is empty, but ceiling bucket has tokens | Can borrow | yellow |
| Ceiling bucket is empty                         | Can’t borrow | red |

A green class can always transmit and a red class cannot transmit. A yellow class can transmit by borrowing tokens if its parent is green. If the parent is also yellow, the class tries to borrow from the next ancestor. This continues until an ancestor is reached that is either green or red. If a red ancestor is reached, the class cannot transmit. If a green ancestor is reached, the class can transmit, and tokens are removed from the ceiling buckets of the class and its ancestors, as well as from the assured buckets of green ancestors.

Consider Fig. 14 as an example, which depicts a class hierarchy where the states of the classes are indicated by the shading of the circles. Among the leaf classes, classes 1 and 4 can transmit because they are green, and class 3 cannot because it is red. Class 2 is yellow, and goes out to borrow bandwidth. It cannot borrow from its parent or grandparent, because they are both yellow. However, it can borrow from the root, its great-grandparent, and is allowed to transmit. (In a workconserving scheduler, the root class is always green.) Class 5, which is also yellow, has a red parent and is therefore blocked from transmission.

Fig. 14. States of classes in HTB.
HTB organizes green classes into groups and prioritizes the groups. (HTB also permits the configuration of class priorities, which we do not discuss here.) The grouping of green classes is based on their position in the class hierarchy. A leaf class $i$ is assigned $\text{level}(i) = 0$, and an internal class $i$ is assigned
\[
\text{level}(i) = 1 + \max_{j \in \text{child}(i)} \text{level}(j).
\]
Classes with the same level are placed in the same group, and the group with the lowest level becomes the top group. HTB maintains one DRR scheduler for each group, where only the DRR scheduler of the top group is active. In any of the DRR schedulers, the quantum of a class is set proportional to its assured rate.

As long as there are green leaf classes, they are served in a DRR fashion. If there is no green leaf class, then the DRR scheduler for the green classes at level 1 becomes active. This scheduler performs a DRR over the yellow leaf classes with a green parent. If there is no green class at level 1, then the DRR scheduler for the green classes at level 2 is activated. This DRR scheduler serves yellow leaf classes with a yellow parent and a green grandparent. And so on. After each transmission, all classes are checked for a change of their color, which may also cause a change of the top group.