Phase transition and thermodynamics of a hot and dense system in a scaled NJL model

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Abstract

The chiral phase transition of a hot and dense system of quarks is studied within a modified SU(3) NJL lagrangian that implements the QCD scale anomaly. The $u$- and $s$-quark condensates can feel or not the same chiral restoration depending on the considered region of the 3-dimension space ($T_c$, $\mu_{uc}$, $\mu_{sc}$). The temperature behaviour of the pressure and of the energy and entropy densities of the $u$- and $s$-quark system is investigated. At high temperature, the non-vanishing bare $s$-quark mass only modifies slightly the usual behaviour associated with an ideal quark gas.

Key words: NJL, SU(3) Thermodynamics, Hot and Strange Matter

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1 INTRODUCTION

It is generally accepted that QCD generates two phase transitions, a deconfinement one, corresponding to a transition of a hadron gas to a quark-gluon plasma and a chiral one, corresponding to the passage from a chiral symmetry breaking phase to a phase where chiral symmetry is restored. Lattice QCD calculations, based on the estimate of the Polyakov loops, show that the two transitions coincide in the pure gauge case. Lattice calculations including massive quarks [1] (albeit not yet in a perfect shape) seem to indicate that this situation persists [2–4] after the introduction of quarks. However the order of the transition is changing, from first order, for the pure gauge case, to second order with reasonable values of the quark masses. The case of the usually accepted values for the bare $u$, $d$ and $s$ masses seems to be close to the boundary separating first and second order transitions at zero quark chemical potential. Nothing is really known for non-zero chemical potential.

Whereas the deconfinement cannot be studied in existing satisfying models, the features of spontaneous chiral symmetry breaking and of its restoration can be studied in the $\sigma$-model or in the Nambu Jona-Lasinio (NJL) model [5]. The latter seems to be very successful in the description of many features of hadronic physics in the non-perturbative domain of QCD, despite of the absence of confinement and of renormalizability. It seems that the chiral aspects are much more important than the confinement features, for static properties at least. This model is thus well suited to study the chiral transition of an infinite uniform quark system, and the thermodynamics of the phases. The pure NJL model with SU(2) flavour has been used for this purpose by many authors [6–11]. Depending upon the unavoidable choice for free parameters left in this kind of model, the transition can be of first or second order [12], with a critical temperature of the order of 150-200 MeV at zero chemical potential. Most of these approaches are restricted to the classical (mean field) approximation. Let us nonetheless notice the works of Zhuang et al. [11] and of Blascke et al. [13], who calculated the thermodynamic potential to the order $\frac{1}{N_c}$, where $N_c$ is the number of colors, which amounts roughly to include the meson degrees of freedom. The latter seem to play an important role at low temperature only and does not affect significantly the position and the properties (order, discontinuities,...) of the transition [11]. The chiral transition has been studied for the SU(3) flavour case at zero chemical potential [14–17] and at non-zero light quark ($u,d$) chemical potentials [18].

Here, we want to make a systematic study of the chiral phase transition features for the SU(3) flavour case for any value of the light quark $\mu_u$ (taken equal to $\mu_d$) and of the strange quark $\mu_s$ chemical potentials. For this, we use an extension of the NJL model, which implements the trace anomaly of QCD through the introduction of a scalar dilaton field, and the axial anomaly.
The former allows the description of the gluon condensate and the latter is dictated by the low energy hadron phenomenology. The model as well as its predictions for the temperature and density dependences of the meson masses are described in ref. [17]. We will evaluate the thermodynamic potential in the mean field approximation within this model and make a systematic survey of the transition in the $T$, $\mu_u$ and $\mu_s$ spaces. The main thermodynamic quantities will be derived for each phase. We use the assumption of a vanishing current mass for the light quarks. The non-zero current $s$-quark mass constitutes the only flavour SU(3) symmetry breaking.

The paper is organized as follows. The NJL model used here is schematically described in section 2, as well as the associated action. Section 3 is devoted to the localisation of the chiral phase transition in the $T$, $\mu_u$ and $\mu_s$ space for two choices of the free parameters of the model. For the first one, the transition is only of first order, whereas the second one allows the possibility of a second order transition. Section 4 deals with the pressure, energy and entropy densities in the different phases. The limiting values in the chiral phase are particularly examined. Section 5 contains our conclusion.

2 THE A-SCALING NJL MODEL

The model used in this paper is extensively described in Ref. [17] under the name “A-scaling model”. We just recall some of the tools useful to the understanding of the present paper.

We start from the SU(3) effective Euclidean action:

$$I_{\text{eff}}(\varphi, \chi, \mu, \beta) = -Tr \chi \ln (-i\partial - W + m + \Gamma_a \varphi_a)$$

$$+ \int d^4x \frac{a^2 \chi^2}{2} \varphi_a \varphi_a + \int d^4x L_\chi + \int d^4x L_{UA(1)}$$

(1)

where the quark fields have been integrated out. The meson fields write:

$$\varphi_a = (\sigma_a, \pi_a), \quad \Gamma_a = (\lambda_a, i\gamma_5 \lambda_a) \quad a = 0, \ldots, 8$$

(2)

where $\lambda_a$ are the usual Gell-Mann matrices ($\lambda_0 = \sqrt{2} \mathbb{I}$) and the gluon condensate is represented by the scalar dilaton field $\chi \propto \langle G_{\mu\nu}^2 \rangle^{1/2}$. The lagrangian $L_\chi$ given by

$$L_\chi = \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{16} b^2 \left( \chi^4 \ln \frac{\chi^4}{\chi_G^4} - (\chi^4 - \chi_G^4) \right)$$

(3)
takes account of the QCD trace anomaly via the term $\chi^4 \ln \chi^4/\chi_0^2$ [22,23]. The $U_A(1)$ axial anomaly is introduced via the mass term:

$$\mathcal{L}_{U_A(1)} = \frac{1}{2} a^2 \xi \chi^2 \pi_0^2.$$  \hspace{1cm} (4)

In Eq. (1), the trace has been regularized by an ultra-violet cut-off which is $\chi$-dependent in order to preserve the scale invariance of the quark loop term. The chemical potentials are introduced via the vector field $W_\lambda = (-i\mu, \vec{0})$, where $\mu$ is a shorthand notation for $\mu = \text{diag}(\mu_u, \mu_d, \mu_s)$, if one neglects the isospin breaking ($\mu_u = \mu_d$). In the same way the quantity $m$ represents the diagonal matrix $m = \text{diag}(m_u, m_u, m_s)$ in flavor space.

We work in the classical approximation so that the partition function writes:

$$Z \equiv Z_c = \exp \left[-I_{\text{eff}}^c (M_u, M_s, \chi_s)\right], \hspace{1cm} (5)$$

where $M_u (M_s)$ and $\chi_s$ denote respectively the constituent $u$-quark ($s$-quark) mass and the gluon condensate in the hot and dense system and are thus $\mu$- and $\beta$-dependent ($\beta = 1/T$). In the vacuum, the corresponding quantities are noted $M_0^u$, $M_0^s$ and $\chi_0$. These three quantities (in the vacuum or in the system) minimize the thermodynamical potential or equivalently the action (1), yielding the value $I_{\text{eff}}^c$ entering eq. (5). The relationship between $M_u$, $M_s$ and the stationary fields $\sigma_0^c$, $\sigma_0^8$ is given in [17]. We write:

$$I_{\text{eff}}^c (M_u, M_s, \chi_s) = I_{(\mu, \beta)}^c (M_u, M_s, \chi_s) + I_{(0, \infty)}^c (M_u, M_s, \chi_s) \hspace{1cm} (6)$$

where

$$I_{(\mu, \beta)}^c (M_u, M_s, \chi_s) = - \text{Tr} \ln (-i\partial - W + m + \Gamma a \varphi_a)^c$$

$$+ \text{Tr} \ln (-i\partial + m + \Gamma a \varphi_a)^c, \hspace{1cm} (7)$$

where the index $c$ indicates that the traces have to be evaluated with the stationary values of the fields. The contribution (7) to the action is finite and writes:

$$I_{(\mu, \beta)}^c (M_u, M_s, \chi_s) = - \beta \Omega \frac{2N_c}{2\pi^2 \beta} \sum_{i=u,s} a_i \int_0^\infty k^2 dk$$

$$\times \left\{ \ln (1 + \exp [-\beta (E_i + \mu_i)]) + \ln (1 + \exp [-\beta (E_i - \mu_i)]) \right\} \hspace{1cm} (8)$$

with $E_i = \sqrt{k^2 + M_i^2}$, $a_u = 2$, $a_s = 1$ and $\Omega$ the volume of the system. The second term of the r.h.s. of (6) corresponds to $\mu_u = \mu_s = 0$, $\beta = \frac{1}{T} = \infty$. It writes:
\[ I_{(0,\infty)}^c (M_u, M_s, \chi_s) = -Tr_{\Lambda \chi_s} \ln (-i \partial + m + \Gamma a \varphi_a)^c \]
\[ + \int d^4x \left\{ \frac{a^2 \chi_s^2}{2} \left[ (\sigma_0^s)^2 + (\sigma_8^s)^2 \right] + \frac{1}{16} b^2 \left[ \chi_s^4 \ln \left( \frac{\chi_s}{\chi_G} \right)^4 - (\chi_s^4 - \chi_G^4) \right] \right\} \]

which yields with a \( d^4k \) cut-off:

\[ I_{(0,\infty)}^c (M_u, M_s, \chi_s) = -\beta \Omega \frac{2N_c}{32\pi^2} \sum_{i=u,s} a_i \]
\[ \times \left\{ \left[ (\Lambda \chi_s)^4 - M_i^4 \right] \ln \left[ (\Lambda \chi_s)^2 + M_i^2 \right] - \frac{1}{2} (\Lambda \chi_s)^2 - M_i^2 \right\} \]
\[ + \beta \Omega \left\{ \frac{a^2 \chi_s^2}{4} \sum_{i=u,s} a_i M_i^2 \left( 1 - \frac{m_i}{M_i} \right)^2 + \frac{b^2}{16} \chi_s^4 \ln \left( \frac{\chi_s}{\chi_G} \right)^4 - (\chi_s^4 - \chi_G^4) \right\} . \]

(10)

Our model contains seven parameters: the bare quark masses \((m_u, m_s)\), the strengths \((a^2, b^2)\), the vacuum gluon condensate \(\chi_0\), the cut-off \(\Lambda\) and the parameter \(\xi\) of the axial anomaly. Five of these seven parameters are fixed to reproduce the vacuum masses of the pion \(m_\pi = 139\) MeV, of the kaon \(m_K = 496\) MeV, of the \(\eta'\) \(m_{\eta'} = 958\) MeV and of the glueball \(m_{GL} = 1.3\) GeV, together with the pion decay constant \(f_\pi = 93\) MeV. The two other parameters remain free and they have been chosen to be the vacuum constituent \(u\)-quark mass \(M_0^u\) (related to \(a^2\)) and the vacuum gluon condensate \(\chi_0\). We give results for two sets of parameters:

\[
M_0^u = 600\; \text{MeV} \quad , \quad \chi_0 = 125\; \text{MeV} \quad (11)
\]
\[
M_0^u = 300\; \text{MeV} \quad , \quad \chi_0 = 80\; \text{MeV} . \quad (12)
\]

This choice is driven by the fact that at \(\mu_u = \mu_s = 0\), the phase transition is of first order \((T_c = 168)\) MeV with the first set and of second order \((T_c = 150\) MeV) with the second set. Note that in the SU(2) case \([12]\), \(T_c\) was found to be 140 MeV with the second set, in agreement with the lattice data \([19]\). We then investigate how finite density can affect the order of the transition. Since we are interested in the determination of the parameters \((T_c, \mu_{uc}, \mu_{sc})\) at the phase transition, we choose to work in the limit \(m_u = 0\).
3 CHIRAL PHASE TRANSITIONS

The quark condensates are related to the constituent masses in the following way:

\[
\langle \bar{u}u \rangle \equiv \frac{1}{\beta \Omega} \frac{\partial I^c_{\text{eff}}}{\partial m_u} (M_u, M_s, \chi_s) = -\frac{1}{2} a^2 \chi^2_s M_u \quad (13)
\]

\[
\langle \bar{s}s \rangle \equiv \frac{1}{\beta \Omega} \frac{\partial I^c_{\text{eff}}}{\partial m_s} (M_u, M_s, \chi_s) = -\frac{1}{2} a^2 \chi^2_s (M_s - m_s) . \quad (14)
\]

The coupling between these two condensates comes through their coupling to the gluon condensate \( \chi_s \). The value of \( \chi_s \) is indeed coupled to those of \( M_u \) and \( M_s \) by the search of the minimum of the action (6) (see Eq. (1)). This coupling can be better visualized by looking at the “three coupled gap equations” (see [17]) that amount to minimize the action.

The chiral phase transitions happen when the quark condensates go to zero. There is a priori no reason why the two condensates should vanish at the same time. This is illustrated in Fig. 1 where the open circles represent the phase transition \( \mu_{\text{sc}} \) versus \( \mu_{\text{uc}} \), corresponding to a vanishing \( u \)-quark condensate at \( T_c = 0 \) (a), \( T_c = 100 \) MeV (b), \( T_c = 161 \) MeV (c), for the set of parameters (11). The transition for the \( s \)-quark is given by the crosses. At sufficiently small temperatures, one observes that the chiral transition of the \( u \)- and \( s \)-condensates are independent of one another (as in a pure NJL model without \( 't \) Hooft determinant), except in a small region of the space \( (\mu_{\text{sc}}, \mu_{\text{uc}}) \) where the coupling via the gluon condensate is sufficiently strong to force the two quark condensates to go to zero at the same time. When the temperature is high enough, only this region of strong coupling is present. Whatever the critical point of the three-dimension space \( (T_c, \mu_{\text{uc}}, \mu_{\text{sc}}) \), the transition is of first order. This is illustrated in Fig. 2 in the case of strong coupling (between the condensates) (a) \( T_c \approx 120 \) MeV, \( \mu_{\text{uc}} = \mu_{\text{sc}} = 250 \) MeV and of weak coupling (b) \( T_{\text{uc}} = 140 \) MeV, \( T_{\text{sc}} = 160 \) MeV, \( \mu_{\text{uc}} = 200 \) MeV, \( \mu_{\text{sc}} = 0 \) MeV). Within the present model, a discontinuity (transition) of a quark condensate is always accompanied by a discontinuity of the gluon condensate, in contradistinction with the suggestion of Ref. [20]. Note that \( \chi \neq 0 \) above the transition. Fig. 3 summarizes the previous results in the form of an artistic view. The space is divided into four regions: the inner one corresponds to quarks in their condense phase; the outer one can be seen as the quark-gluon plasma, the two remaining regions (extending along the axes to infinity) correspond to mixed phases with free \( u \) (\( s \))-quarks together with \( s \) (\( u \))-quarks in the hadronic phase if one assumes that the chiral transition corresponds to deconfinement. A remarkable prediction of the present model has to be singled out: if non-strange matter at sufficiently large up chemical potential is heated, the matter will transform...
first to a mixture of free light quarks surrounding strange hadrons (with total vanishing strangeness) before being transformed into a quark gas.

The densities $\rho_u$ and $\rho_s$ are related to the chemical potentials and temperature in the following way

$$\rho_i = -\frac{1}{\beta\Omega} \left( \frac{\partial I_{\text{eff}}^c}{\partial \mu_i} \right) = -\frac{N_c a_i}{\pi^2} \int_0^\infty k^2 (n_{+i} - n_{-i}) \, dk \quad i = u, s$$

(15)

where

$$n_{\pm i} = \frac{1}{1 + \exp \left\{ \frac{1}{\beta} \left( \sqrt{k^2 + M_i^2} \pm \mu_i \right) \right\}} \quad i = u, s.$$  

(16)

In Eq. (15), $\rho_u$ takes into account the $d$-quark contribution via $a_u = 2$. When the transition is of first order, there is a jump in the density at the transition. Just before the transition, the $\rho_i$'s are given by (15) and (16), with $M_i^2 \neq m_i^2 (\rho_i_{\text{bef}})$, while just after the transition, the densities ($\rho_i_{\text{aft}}$) increase up to values corresponding to $M_i = m_i$. A drawback of the present model is that at $T_c = 0$, $\rho_i_{\text{bef}} = 0$, so that the system can not exist in the hadronic phase. As $T_c$ departs from zero, $\rho_i_{\text{bef}}$ remains small. As an example, for $\mu_{uc} = \mu_{sc} = 250$ MeV, one has $\rho_{u_{\text{bef}}} = 0.362$, $\rho_{s_{\text{bef}}} = 0.050$ and $\rho_{u_{\text{aft}}} = 2.646$, $\rho_{s_{\text{aft}}} = 0.978$ where $\rho_0 = 0.51 \, \text{fm}^{-3}$. Within the present model, it is practically impossible to eliminate this drawback, as the only way would correspond to having small constituent masses in the hadronic phase while keeping large gluon condensate. Such a choice of parameters is not suitable [12] since it would yield too high $T_c$. A hope lies however in the inclusion of vector mesons which are expected to shift the critical densities up to acceptable values without modifying the qualitative features of the phase transition [25], as the latter are basically determined by the chiral mesons and their coupling to the condensates.

The previous results are not qualitatively changed drastically when shifting to the other choice of the parameters (Eq. (12)), except for the order of the transition which can become of second order at sufficiently low chemical potentials. This is illustrated in Fig. 4a. At $\mu_u = \mu_s = 0$, the $u$-quark condensate vanishes at $T_{uc} \approx 150$ MeV (140 MeV in SU(2)) while chiral transition happens at $T_{sc} \approx 200$ MeV for the $s$-quark. At higher chemical potentials, the transition becomes of first order but the chiral transitions for the $u$- and $s$-quarks remain decoupled (see Fig. 4b). Note, however, that a change of the gluon condensate is always associated with the transition of any quark condensate. Figs. 5 and 6 are the analogs of Figs. 1 and 3, respectively for the set (12). Figs. 1 and 5 are qualitatively similar except at high temperature where the two quark condensates are coupled whatever the couple $(\mu_{uc}, \mu_{sc})$
for the set (11) while the $u$-quark can only exist in the “deconfined” phase with a chiral transition at $\mu_{sc} \approx 220$ MeV whatever $\mu_{uc}$ for the strange quark, in the case of set (12). The results presented in Fig. 5 can be visualized in the 3-dimension space by use of the artistic view of Fig. 6.

4 THERMODYNAMICS

We now turn to the study of the thermodynamics of a hot and/or dense system. We will mainly focus on the behaviour with respect to the temperature of the pressure, the energy density and the entropy density. These three quantities are respectively given in the classical approximation by:

\[
P = \frac{1}{\beta} \ln \left( \frac{Z_c}{Z_c^0} \right) = -\frac{1}{\beta} \left[ \frac{\partial I^c_{\text{eff}}(M_u, M_s, \chi_s)}{\partial \Omega} - \frac{\partial I^c_{(0,\infty)}(M_u^0, M_s^0, \chi_0)}{\partial \Omega} \right]
\]

\[
\epsilon \equiv \frac{E}{\Omega} = -\frac{1}{\Omega} \left( \frac{\partial}{\partial \beta} \frac{1}{\beta} \sum_{i=u,s} \mu_i \frac{\partial}{\partial \mu_i} \right) \ln \left( \frac{Z_c}{Z_c^0} \right)
\]

\[
\epsilon = \frac{1}{\Omega} \left( \frac{\partial}{\partial \beta} - \frac{1}{\beta} \sum_{i=u,s} \mu_i \frac{\partial}{\partial \mu_i} \right) \left[ I^c_{\text{eff}}(M_u, M_s, \chi_s) - I^c_{(0,\infty)}(M_u^0, M_s^0, \chi_0) \right]
\]

\[
\epsilon = \frac{1}{\Omega} \left( \frac{\partial}{\partial \beta} - \frac{1}{\beta} \sum_{i=u,s} \mu_i \frac{\partial}{\partial \mu_i} \right) I^c_{(\mu,\beta)}(M_u, M_s, \chi_s)
\]

\[
\epsilon + 1 \frac{\beta}{\Omega} \left[ I^c_{(0,\infty)}(M_u, M_s, \chi_s) - I^c_{(0,\infty)}(M_u^0, M_s^0, \chi_0) \right]
\]

\[
s \equiv \frac{S}{\Omega} = \beta \left( P + \frac{E}{\Omega} - \sum_{i=u,s} \mu_i \rho_i \right)
\]

\[
s = -\frac{1}{\Omega} \left( 1 - \beta \frac{\partial}{\partial \beta} \right) I^c_{(\mu,\beta)}(M_u, M_s, \chi_s),
\]

where we have substracted the vacuum contribution $Z_c^0$ from the partition function in order to ensure vanishing pressure, energy and entropy in the vacuum.
Following the same prescription as in [21], we can write

\[ P = P_{\text{ideal gas}} - B(\beta, \mu_u, \mu_s) \]  
\[ \epsilon = \epsilon_{\text{ideal gas}} + B(\beta, \mu_u, \mu_s) \]  
\[ T_s = P_{\text{ideal gas}} + \epsilon_{\text{ideal gas}} - \sum_{i=u,s} \mu_i \rho_i \]

with

\[ P_{\text{ideal gas}} = -\frac{I_c^{(\mu, \beta)}(M_u, M_s, \chi_s)}{\beta \Omega} = \frac{N_c}{3\pi^2} \sum_{i=u,s} a_i \int_0^\infty \frac{k^4}{E_i} (n_{+i} + n_{-i}) \, dk \]

\[ \epsilon_{\text{ideal gas}} = \frac{1}{\Omega} \left( \frac{\partial}{\partial \beta} - \frac{1}{\beta} \sum_{i=u,s} \mu_i \frac{\partial}{\partial \mu_i} \right) I_c^{(\mu, \beta)}(M_u, M_s, \chi_s) \]
\[ = \frac{N_c}{\pi^2} \sum_{i=u,s} a_i \int_0^\infty k^2 E_i (n_{+i} + n_{-i}) \, dk \]

\[ B(\beta, \mu_u, \mu_s) = \frac{1}{\beta \Omega} \left\{ I_c^{(0, \infty)}(M_u, M_s, \chi_s) - I_c^{(0, \infty)}(M_u^0, M_s^0, \chi_0) \right\} . \]

The definition (25) differs from the one used in [17,22,23] where the bag pressure is defined as:

\[ B = \frac{1}{\beta \Omega} \left\{ I_c^{(0, \infty)}(0, 0, 0) - I_c^{(0, \infty)}(M_u^0, M_s^0, \chi_0) \right\} \]
\[ = \frac{1}{16} m_{GL}^2 \chi_0^2. \]

The temperature behaviour of \( P \) is exhibited in Fig. 7a, for the two sets of parameters (11) and (12) in the case \( \mu_u = \mu_s = 0 \). Due to the non-vanishing value of the strange quark bare mass \( m_s \), the temperature behaviour above \( T_c \) is not exactly the usual law in \( T^4 \). One can write (see Appendix A)

\[ P_{\text{ideal gas}} \approx \frac{7}{60} N_c \pi^2 T^4 - \frac{N_c}{12} m_s^2 T^2 - \frac{N_c}{8\pi^2} m_s^4 \ln \frac{m_s}{\pi T} \]
\[ + \frac{N_c}{16\pi^2} \left( \frac{3}{2} - 2\gamma \right) m_s^4 + ... \]

(27)
where $\gamma$ is the Euler constant. At the same time, $B$ becomes $T$-independent

$$B = \frac{1}{\beta \Omega} \left\{ I_c(0,\infty) (0,m_s,\chi_s) - I_c(0,\infty) \left( M_u^0, M_s^0, \chi_0 \right) \right\}, \quad (28)$$

where $\chi_s$ is the stationary value for $T > T_c$. Fig. 7a shows an excellent $T^4$ linear fit of $P$ above chiral restoration:

$$P \approx 3.40 \ T^4 - 2.097 \times 10^{-3}, \quad (29)$$

for the set of parameters (11) and

$$P \approx 3.40 \ T^4 - 1.195 \times 10^{-3}, \quad (30)$$

for the set (12). The coefficient of $T^4$ is not exactly equal to the coefficient of the same power in (27), which is equal to 3.454. In fact, due to the limited range of $T^4$ investigated in Fig. 7, the remaining terms in Eq. (27) change the apparent slope and the constant term of the curve. When a fit with the explicitly given terms of Eq. (27) plus an adjusted constant $C$ is performed, the latter turns out to be $C^{1/4} \approx 207$ MeV for set (11) and $C^{1/4} \approx 179$ MeV for set (12). These values are close to $B^{1/4} \approx 209$ MeV (set (11)) and $B^{1/4} \approx 183$ MeV (set (12)) obtained from a direct calculation using Eq. (28).

This underlines the consistency and the precision of our numerical results as well as the rapid convergence of expansion (27), although the expansion parameter $m_s/T$ is not very small. Moreover, note that the numerical values of $B^{1/4}$ given by (28) or extracted from the fits are not so far from the values of $B^{1/4}$ given by (26) ($B^{1/4} = 201$ MeV (set 11), 161 MeV (set 12)).

Below the phase transition, the temperature behaviour depends on the order of the transition. With the set (11) (first order), $\beta M_i \ (i = u, s)$ remain large, so that the $T$ behaviour is governed by the law

$$P_{\text{ideal gas}} \approx \frac{4N_c \beta^{-5/2}}{(2\pi)^{3/2}} \sum_{i=u,s} a_i M_i^{3/2} e^{-\beta M_i}, \quad (31)$$

It should be noted that the masses $M_i$ are decreasing with temperature, which makes the variation of the pressure more rapid than the $T^{5/2}$ factor in Eq. (31). The masses $M_i$ being larger with set (11) than with set (12), the pressure increases more slowly in the first case and the change of slope at the transition is more pronounced.

The $T^4$ law for $P_{\text{ideal gas}}$ does not contain the explicit gluon degrees of freedom
which would yield:

\[ P_{\text{ideal gas}} \approx \frac{\pi^2}{90} \left\{ 2N_g + \frac{7}{8}N_cN_f4 \right\} T^4 \approx 5.2 \ T^4. \]  

(32)

A first step in the inclusion of the gluon thermodynamics consists in going beyond the classical approximation by adding one loop corrections. One then has to add

\[ P_{\text{one loop}} = -\sum_j \int \frac{d^3k}{(2\pi)^3} \ln \left[ 1 - \exp \left( -\beta \sqrt{k^2 + m_j^2} \right) \right] \]

\[ = -\sum_j \frac{1}{3} \int \frac{dk}{2\pi^2} \frac{k^4}{\sqrt{k^2 + m_j^2}} \frac{1}{1 - \exp \beta \sqrt{k^2 + m_j^2}} \]  

(33)

where the sum runs over the 19 mesons: (9+1) scalar and 9 pseudo-scalar mesons. When chiral symmetry is restored, all the meson masses \( m_j \) (except for the dilaton) increase to infinity for increasing temperature so that the product \( \beta m_j \) remains large, and:

\[ P_{\text{one loop}}(\text{meson}) \approx \sum_j \frac{1}{3} \int \frac{dk}{2\pi^2} \frac{k^4}{\sqrt{k^2 + m_j^2}} e^{-\beta \sqrt{k^2 + m_j^2}} \]

\[ \approx \frac{\beta^{-5/2}}{(2\pi)^{3/2}} \sum_j m_j^{3/2} e^{-\beta m_j} \]  

(34)

in the non-relativistic limit. The exponential factor makes the one loop correction terms due to the 18 mesons vanishingly small compared to the \( T^4 \) term. The mass \( m_D \) of the 19th meson, the dilaton, remains finite above the phase transition so that \( \beta m_D \to 0 \), and

\[ P_{\text{one loop}}(\text{dilaton}) \approx \frac{\pi^2}{90} T^4 + \ldots \]  

(35)

where only one degree of freedom for the gluon is included.

A second step in the description of the gluon thermodynamics would consist in including a temperature-dependent gluon potential \( V_\chi(T) \) as stipulated in [22,24], which could yield the right asymptotic behaviour (32).

Fig. 7b is the equivalent of Fig. 7a for \( \mu_u = \mu_s = 250 \) MeV. These values of the chemical potentials correspond to a strong coupling between \( \langle \bar{u}u \rangle \) and \( \langle \bar{s}s \rangle \) with the set (11) while they correspond to a weak coupling with the set (12). Even at high temperature, it is no more possible to perform a \( T^4 \) linear fit of
the results. Instead, the chemical potentials introduce a $T^2$ dependence in the pressure. In the chiral limit ($m_u = m_s = 0$), one would have

$$P = \frac{7}{60} N_c \pi^2 T^4 + \frac{N_c}{2} \mu^2 T^2 + \frac{N_c \mu^4}{4\pi^2} - B$$  \hspace{1cm} (36)$$

with $\mu \equiv \mu_u = \mu_s$. Due to $m_s \neq 0$, Eq. (36) has to be modified. However, the prescription used in the case $\mu = 0$ (see Appendix A) does not work here if $\mu_i > m_i$. From our results, one can see that the $T^4$ term is not modified by $m_s$, as expected, and the $T^2$ term is slightly decreased.

The temperature behaviour of the energy density (21), (24) is exhibited in Fig. 8a, for $\mu_u = \mu_s = 0$. At high $T$, one can write

$$\varepsilon_{\text{ideal gas}} \approx \frac{7}{20} N_c \pi^2 T^4 - \frac{N_c}{12} m_s^2 T^2 + \frac{N_c}{8\pi^2} m_s^4 \ln \frac{m_s}{\pi T} + \frac{N_c}{16\pi^2} \left(2\gamma + \frac{1}{2}\right) m_s^4 \ldots$$ \hspace{1cm} (37)$$

while the obtained results seem to show a $T^4$ linear behaviour :

$$\varepsilon \approx 10.269 \ T^4 + 1.620 \ 10^{-3}$$ \hspace{1cm} (38)$$

$$\varepsilon \approx 10.271 \ T^4 + 0.877 \ 10^{-3}$$ \hspace{1cm} (39)$$

for sets (11) and (12), respectively. When a fit is performed with the terms written in Eq. (37) plus an adjustable constant $C$, one obtains $C^{1/4} \approx 208$ MeV for set (11) and $C^{1/4} \approx 182$ MeV for set (12), in agreement with the results obtained from the $T$ behaviour of the pressure.

The first order phase transition and the strong coupling between $\langle \pi u \rangle$ and $\langle \overline{s}s \rangle$ (in the case of the set (11)) is well visualized by the sudden change of the energy value at $T_c$. For set (12), the weak coupling between the quark condensates and the presence of a second order phase transition for the $u$-quark (the $s$-quark still feeling a first order transition) is reflected by a slope breaking in $\varepsilon(T)$ at $T_{uc}$ and a small jump at $T_{sc}$.

Fig. 8b is equivalent of Fig. 8a in the case of a dense strange matter $\mu_u = \mu_s = 250$ MeV. Here again, the chemical potentials introduce a $T^2$ dependence which writes with $\mu \equiv \mu_u = \mu_u :

$$\varepsilon = \frac{7}{20} N_c \pi^2 T^4 + \frac{3}{2} N_c \mu^2 T^2 + \frac{3}{4} N_c \frac{\mu^4}{\pi^2} + B$$ \hspace{1cm} (40)$$
in the chiral limit. From our results, one sees that \( m_s \neq 0 \) only slightly affects the second term of (40). It also introduces an additional constant term.

For a pure ideal gas, the entropy density behaves as \( T^3 \) at \( \mu = 0 \). When the non-vanishing \( s \)-quark mass is taken into account, one can use Eqs. (22), (27) and (37) to get the three first terms of the expansion of \( T s \) in power of \( T^2 \). One then has for \( s \):

\[
s \approx \frac{7}{15} N_c \pi^2 T^3 - \frac{N_c}{6} m_s^2 T + \frac{N_c}{8\pi^2} m_s^4 T^{-1} + ..., \tag{41}
\]

where the dots represent more negative powers of \( T \). However, the results of Fig. 9a show that one can approximate (41) by a \( T^3 \) linear expression:

\[
s \approx 13.868 \ T^3 - 0.136 \ 10^{-1}, \tag{42}
\]

\[
s \approx 13.839 \ T^3 - 0.912 \ 10^{-2}, \tag{43}
\]

for sets (11) and (12) respectively. The coefficients of \( T^3 \) are not exactly equal to what would be obtained from combining Eqs. (29), (30), (38) and (39), although the numerical results are perfectly verifying Eq. (19). Similarly, the constants in Eqs. (42) and (43) are departing numerically from the second and third terms of Eq. (41). This reflects the limited meaning of simple fits like Eqs. (29), (38) and (42) in a restricted range of \( T^4 \). This remark would apply to any calculation. It would then be dangerous to try to extract a bag constant from for instance \( P \) versus \( T^4 \) curve by a simple fit.

Fig. 9b shows the temperature behaviour of \( s \) in the case of non-vanishing chemical potentials. Using (22), (36) and (40) together with the density after chiral restoration

\[
\rho \mu = \frac{N_c \mu^4}{\pi^2} + N_c \mu^2 T^2 \tag{44}
\]

one gets for an ideal gas:

\[
T s = \frac{7}{15} N_c \pi^2 T^4 + N_c \mu^2 T^2 \tag{45}
\]

expression which should be modified in order to take \( m_s \neq 0 \) into account. We observed that this non-vanishing \( m_s \) only slightly affects the value of the factor multiplying \( T^2 \). It should be noticed that above the transition, the entropy shows that all the available degrees of freedom are excited. However, as noted in Refs. [11,26], the massless Stefan-Boltzmann limit of QCD is not reached, because not all the gluon degrees of freedom are manifest in this model, and because the mass of the strange quark is not yet negligible.
5 CONCLUSION

We have investigated the properties of the chiral transition in an infinite system of quarks with the help of a SU(3) flavour NJL model, built to account for the axial and scale anomalies. There is some freedom in choosing the parameters of the model. We illustrated our calculations for two choices, corresponding to large and moderate constituent quark masses. With the first choice, the transition is always of first order. With the second choice, the transition can be of second order for small $u$ and $s$ chemical potentials. This is to be contrasted with many other NJL models (see e.g. Refs. [16,18]), where the precise form of the Lagrangian, the way of regularization and the choice of the parameters hardly allow for a first order transition. Let us mention that the chiral transition for the flavour SU(3) case is also studied in ref. [28] in a $\sigma$-model and in ref. [29] in a NJL model. These works however concentrate on the zero chemical potential case and on the modification of the meson masses. They include $\frac{1}{N_f}$ [28] and $\frac{1}{N_c}$ [29] terms.

An important feature of our results lies in the possibility of having coupled, i.e. simultaneous, chiral transitions for light and strange quarks. These transitions tend to become uncoupled if either the chemical potential $\mu_u$ or the chemical potential $\mu_s$ is large enough. We remind that, in our model, the $u$ and $s$ condensates are only coupled indirectly through their coupling to the gluon condensate. As a consequence the latter is always modified in a quark ($u$ or $s$) chiral transition.

The scaled NJL model studied here allows the possibility of having first order transitions, with important jumps of the thermodynamical quantities. This is an interesting feature in view of the search of the quark-gluon plasma in heavy ion collisions. However, as we explained in section 3, the model should be made more realistic through, e.g. the introduction of vector mesons. It should be checked how the discontinuities will be modified by this introduction.

We also investigated the equation of state for the chiral phase. The latter is very close to the relativistic free gas one. Departures from the massless Stefan-Boltzmann limit comes from the non-vanishing mass of the $s$ quark in the chiral phase and from the “bag” constant, which can be identified as due to the chiral symmetry breaking of the vacuum (regardless of the non-zero current mass of the strange quark). Next steps in our investigations would involve the inclusion of the mesons degrees of freedom, which can be viewed as the quantum fluctuations of the quark fields [17], the study of their influence on the equation of state, especially in chiral asymmetric phase and the possibility of including all the gluon degrees of freedom.
Acknowledgements

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Appendix

A High temperature expansions for a hot ideal gas

We give explicitly the expansion of thermodynamical quantities at zero chemical potential and high temperature.

a) Pressure

At high $T$, the pressure of an ideal gas of massless $u$ quarks and massive $s$ quarks is given by

$$P_{\text{ideal gas}} = P_{\text{ideal gas}}(u) + P_{\text{ideal gas}}(s)$$  \hspace{0.5cm} \text{(A.1)}

or

$$P_{\text{ideal gas}} = 2 \frac{N_c}{3 \pi^2} \left\{ 2 \int_0^\infty \frac{k^3}{1 + e^{\beta k}} dk + \int_0^\infty \frac{k^4}{E_s} \frac{1}{1 + e^{\beta E_s}} dk \right\}$$  \hspace{0.5cm} \text{(A.2)}

with $E_s = \sqrt{k^2 + m_s^2}$, $m_s$ being the bare $s$-quark mass. The first term of the r.h.s. of (A.2) has the usual $T^4$ behaviour

$$P_{\text{ideal gas}}(u) = \frac{7}{90} N_c \pi^2 T^4.$$  \hspace{0.5cm} \text{(A.3)}

The second term can be estimated in the following way :

$$P_{\text{ideal gas}}(s) = \frac{2 N_c}{3 \pi^2} m_s^4 \lim_{\epsilon \to 0} \int_1^\infty \frac{(y^2 - 1)^{3/2}}{1 + e^\epsilon \epsilon m_s \beta y} dy$$  \hspace{0.5cm} \text{(A.4)}

where the infinitively small positive quantity $\epsilon$ allows to regularize the $T$ independent term of $P_{\text{ideal gas}}(s)$ (see below). One then has :
\[ P_{\text{ideal gas}}(s) = \frac{2N_c}{3\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\varepsilon} \int_1^\infty \left( y^2 - 1 \right)^{3/2} e^{-nm_s\beta y} dy \]

\[ = \frac{2N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\varepsilon} K_2(nm_s\beta) \left( \frac{n^2}{n^2m_s^2\beta^2} \right) \] (A.5)

Writing the modified Bessel function \( K_2(z) \) as an ascending series in power of \( z \) [27]

\[ K_2(z) = 2z^{-2} \left( 1 - \frac{z^2}{4} \right) \]

\[ + \frac{1}{8} z^2 \sum_{k=0}^{\infty} \frac{\psi(k+1) + \psi(k+3)}{k!(2+k)!} \left( \frac{z^2}{4} \right)^k - \ln \frac{z}{2} I_2(z), \] (A.6)

with \( \psi \) being the Digamma function and with the other modified Bessel function \( I_2(z) \)

\[ I_2(z) = \frac{z^2}{4} \sum_{k=0}^{\infty} \frac{1}{k!(k+2)!} \left( \frac{z^2}{4} \right)^k, \] (A.7)

one may write (A.5) as a double sum which is absolutely convergent, owing to the convergence factor \( e^{-n\varepsilon} \). One can easily control the first terms of the expansion

\[ P_{\text{ideal gas}}(s) = p_4 T^4 + p_2 T^2 + p_\ell \ln T \]

\[ + p_0 + p_\ell T^{-2} \ln T + p_{-2} T^{-2} + \ldots \] (A.8)

One has

\[ p_4 = \frac{4N_c}{\pi^2} \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} \frac{e^{-n\varepsilon}(-1)^{n+1}}{n^4} = \frac{4N_c}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7}{180} N_c \pi^2 \] (A.9)

\[ p_2 = \frac{N_c}{\pi^2} m_s^2 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} \frac{e^{-n\varepsilon}(-1)^n}{n^2} = -\frac{N_c}{\pi^2} m_s^2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = -\frac{N_c}{12} m_s^2 \] (A.10)

\[ p_\ell \ln T = \frac{N_c}{4\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-)^n e^{-n\varepsilon} \ln \frac{nm_s\beta}{2} \]

\[ = \frac{N_c}{4\pi^2} m_s^4 \left[ \ln \frac{m_s\beta}{2} \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-)^n e^{-n\varepsilon} + \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-)^n e^{-n\varepsilon} \ln n \right] \]

\[ = \frac{N_c}{4\pi^2} m_s^4 \left[ -\frac{1}{2} \ln \frac{m_s\beta}{2} + \lim_{\varepsilon \to 0} \frac{d}{dx} \sum_{n=1}^{\infty} (-)^n e^{-n\varepsilon} n^x \bigg|_{x=0} \right] \]
\[ p_0 = \frac{N_c}{8\pi^2} m_s^4 \left( -2\gamma + \frac{3}{2} \right) \lim_{\varepsilon \to 0} \frac{1}{1 + e^{-\varepsilon}} = \frac{N_c}{16\pi^2} m_s^4 \left( -2\gamma + \frac{3}{2} \right), \]  

(A.12)

where \( \gamma \) is the Euler constant,

\[ p_{-2} T^{-2} \ln T = \frac{N_c}{48\pi^2} m_s^6 T^{-2} \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^n \left( \psi (1) + \psi (3) \right) e^{-n\varepsilon} \ln \frac{nm_s^2}{2} \]

\[ = \frac{N_c}{48\pi^2} m_s^6 T^{-2} \left[ 7 \ln \frac{m_s^2}{2} \zeta (-2) - \frac{d}{dx} \left[ (1 - 2^{1+x}) \zeta (-2 - x) \right]_{x=0} \right] \]

\[ = -\frac{7N_c}{48\pi^2} m_s^6 T^{-2} \zeta' (-2), \]  

(A.13)

as the Riemann function \( \zeta (s) \) vanishes for \( s = -2 \). The \( p_{-2} \) term vanishes, since it involves the sum

\[ \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^n e^{-n\varepsilon} n^2 = \zeta (-2) = 0. \]  

(A.14)

With (A.3), (A.8)–(A.14), the high temperature behaviour of the ideal gas pressure writes

\[ P_{\text{ideal gas}} = \frac{7}{60} N_c \pi^2 T^4 - \frac{N_c}{12} m_s^2 T^2 - \frac{N_c}{8\pi^2} m_s^4 \ln \frac{m_s}{\pi T} + \frac{N_c}{16\pi^2} \left( \frac{3}{2} - 2\gamma \right) m_s^4 \]

\[ - \frac{7N_c}{48\pi^2} m_s^6 T^2 \zeta' (-2) + \ldots \]  

(A.15)

b) Energy density
Using the same technical procedure as before, one finds:

$$\varepsilon_{\text{ideal gas}} = \varepsilon_{\text{ideal gas}} (u) + \varepsilon_{\text{ideal gas}} (s)$$  \hspace{1cm} (A.16)

with

$$\varepsilon_{\text{ideal gas}} (u) = \frac{4N_c}{\pi^2} \int_0^\infty \frac{k^3}{1 + e^{\beta k}} \, dk = 3P_{\text{ideal gas}} (u) = \frac{7}{30} N_c \pi^2 T^4 \hspace{1cm} (A.17)$$

$$\varepsilon_{\text{ideal gas}} (s) = \frac{2N_c}{\pi^2} \int_0^\infty k^2 E_s \frac{1}{1 + e^{\beta E_s}} \, dk$$

$$= \frac{2N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \int_1^\infty \frac{[(y^2 - 1)^{3/2} + (y^2 - 1)^{1/2}]}{1 + e^{\varepsilon m_s \beta y}} \, dy$$

$$= \frac{2N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^\infty e^{-\varepsilon} (-1)^n + \frac{3K_2 (nm_s \beta)}{n^2 m_s^2 \beta^2} + \frac{K_1 (nm_s \beta)}{nm_s \beta}$$

$$= 3P_{\text{ideal gas}} + \frac{2N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^\infty e^{-\varepsilon} (-1)^n + \frac{3K_2 (nm_s \beta)}{n^2 m_s^2 \beta^2} + \frac{K_1 (nm_s \beta)}{nm_s \beta}. \hspace{1cm} (A.18)$$

With [27]

$$K_1 (z) = \frac{1}{z} - \frac{z}{4} \sum_{k=0}^\infty \frac{\psi (k + 1) + \psi (k + 2)}{k! (k + 1)!} \left( \frac{z^2}{4} \right)^k + \ln \frac{z}{2} I_1 (z) \hspace{1cm} (A.19)$$

and

$$I_1 (z) = \frac{z}{2} \sum_{k=0}^\infty \frac{1}{k! (k + 1)!} \left( \frac{z^2}{4} \right)^k \hspace{1cm} (A.20)$$

one finds:

$$\varepsilon_{\text{ideal gas}} (s) = \varepsilon_4 T^4 + \varepsilon_2 T^2 + \varepsilon_\ell \ln T + \varepsilon_0$$

$$+ \varepsilon_{\ell^{-2}} T^{-2} \ln T + \varepsilon_{-2} T^{-2} + ... \hspace{1cm} (A.21)$$

with

$$\varepsilon_4 = 3p_4 = \frac{7}{60} N_c \pi^2 \hspace{1cm} \text{(A.22)}$$

$$\varepsilon_2 = 3p_2 + \frac{N_c}{\pi^2} m_s^2 \lim_{\varepsilon \to 0} \sum_{n=1}^\infty \frac{e^{-\varepsilon} (-1)^n + 1}{n^2} = - \frac{N_c}{12} m_s^2 = p_2 \hspace{1cm} (A.23)$$
\[\varepsilon \ln T = 3p_\ell \ln T + \frac{N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} e^{-n\varepsilon} (-1)^{n+1} \ln \frac{nm_s\beta}{2}\]

\[= \frac{N_c}{8\pi^2} m_s^4 \ln \frac{m_s\beta}{\pi} = -p_\ell \ln T,\]  

(A.24)

where we have used (A.11),

\[\varepsilon_0 = 3p_0 - \frac{2N_c}{4\pi^2} m_s^4 (\psi (1) + \psi (2)) \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\varepsilon} \]

\[= \frac{N_c}{16\pi^2} m_s^4 \left( 2\gamma + \frac{1}{2} \right),\]  

(A.25)

\[\varepsilon_{T-2} T^{-2} \ln T = 3p_{T-2} T^{-2} \ln T + \frac{2N_c}{8\pi^2} m_s^6 T^{-2} \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\varepsilon} n^2 \ln \frac{nm_s\beta}{2}\]

\[= 3p_{T-2} T^{-2} \ln T + \frac{7N_c}{8\pi^2} m_s^6 T^{-2} \zeta' (-2)\]

\[= \frac{7N_c}{16\pi^2} m_s^6 T^{-2} \zeta' (-2) = -3p_{T-2} T^{-2} \ln T.\]  

(A.26)

The term \(\varepsilon_{T-2}\) vanishes because it is proportional to \(\zeta' (-2)\). Gathering the results, one obtains:

\[\varepsilon_{\text{ideal gas}} = \frac{7}{20} N_c \pi^2 T^4 - \frac{N_c}{12} m_s^2 T^2 + \frac{N_c}{8\pi^2} m_s^4 \ln \frac{m_s}{\pi T} + \frac{N_c}{8\pi^2} m_s^4 \left( \gamma + \frac{1}{4} \right)\]

\[+ \frac{7}{16} \frac{N_c}{\pi^2} m_s^6 \zeta' (-2) T^{-2} + ...\]  

(A.27)

c) Entropy density

Using (22), (A.15) and (A.27), one gets:

\[Ts = 2 \frac{N_c}{\pi^2} m_s^4 \lim_{\varepsilon \to 0} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\varepsilon} \frac{K_3 (nm_s\beta)}{nm_s\beta}\]

\[= \frac{7}{15} N_c \pi^2 T^3 - \frac{N_c}{6} m_s^2 T + \frac{N_c}{8\pi^2} m_s^4 T^{-1} + \frac{7}{24} \frac{N_c}{\pi^2} m_s^6 T^{-3} \zeta' (-2) + ...\]  

(A.28)

\[s = \frac{7}{15} N_c \pi^2 T^3 - \frac{N_c}{6} m_s^2 T + \frac{N_c}{8\pi^2} m_s^4 T^{-1} + \frac{7}{24} \frac{N_c}{\pi^2} m_s^6 T^{-3} \zeta' (-2) + ...\]  

(A.29)
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Figure captions

Figure 1.
Chiral phase transition $\mu_{sc}$ versus $\mu_{uc}$ for $T = 0$ MeV (a), $T = 100$ MeV (b), $T = 161$ MeV (c). The free parameters of the model are $M^0_u = 600$ MeV, $\chi_0 = 125$ MeV. The line with the crosses (open circles) corresponds to a vanishing $\bar{s}s$ ($\bar{u}u$) condensate.

Figure 2.
Temperature behaviour of the constituent $u$- and $s$-quark masses and of the gluon condensate for $\mu_u = \mu_s = 250$ MeV (a) and $\mu_u = 200$ MeV, $\mu_s = 0$ MeV (b) with $M^0_u = 600$ MeV, $\chi_0 = 125$ MeV.

Figure 3.
Chiral first order phase transition $(T_c, \mu_{uc}, \mu_{sc})$ : artistic view.

Figure 4.
Temperature behaviour of the constituent $u$- and $s$-quark masses and of the gluon condensate for $\mu_u = \mu_s = 0$ MeV (a) and $\mu_u = \mu_s = 250$ MeV (b) with $M^0_u = 300$ MeV, $\chi_0 = 80$ MeV.

Figure 5.
Chiral phase transition $\mu_{sc}$ versus $\mu_{uc}$ for $T = 0$ MeV (a), $T = 100$ MeV (b), $T = 130$ MeV (c) and $T = 165$ MeV (d), with $M^0_u = 300$ MeV, $\chi_0 = 80$ MeV. Same conventions as for Fig. 1.

Figure 6.
Chiral phase transition $(T_c, \mu_{uc}, \mu_{sc})$, summarizing the results presented in Fig. 5 : artistic view.

Figure 7.
Equation of state for $\mu_u = \mu_s = 0$ MeV (a) and $\mu_u = \mu_s = 250$ MeV (b). Approximated $T^4$ linear fits above $T_c$ are indicated in (a) while quadratic fits in $T^2$ are required for part (b). Results are shown for $M^0_u = 600$ MeV, $\chi_0 = 125$ MeV (crosses) and $M^0_u = 300$ MeV, $\chi_0 = 80$ MeV (full dots).

Figure 8.
Energy density versus $T^4$ for $\mu_u = \mu_s = 0$ MeV (a) and $\mu_u = \mu_s = 250$ MeV
Figure 9.

Entropy density versus $T^3$ for $\mu_u = \mu_s = 0$ MeV (a) and $\mu_u = \mu_s = 250$ MeV (b). Approximated $T^3$ linear fits, above $T_c$ are indicated in (a) while quadratic fits for $T_s$ are required for part (b). Same conventions as in Fig. 7.
$M_0^u = 600 \text{ MeV}, \chi_0 = 125 \text{ MeV}$ (Modèle A)

hadronic phase $u$

quarks $s$

quarks $u$ & $s$

hadronic phase $u$ & $s$

$\rho_u = \rho_s = 0$

$T = 0 \text{ MeV}$

$\mu_{uc}$ (MeV)

$\mu_{sc}$ (MeV)
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{\textbf{Graphical Representation} \(\langle \bar{u}u \rangle = \langle \bar{s}s \rangle\) (c)\textbf{.}
\(\mu_{se}\) (MeV) vs. \(\mu_{ue}\) (MeV) with quarks \(u \& s\) and hadronic phase \(u \& s\).
\(T = 161\textnormal{MeV}\)\textbf{.}}
\end{figure}
Modèle A

\[ M_u^0 = 600 \text{ MeV}, \quad \chi_0 = 125 \text{ MeV} \]

\[ \mu_u = \mu_s = 250 \text{ MeV} \quad (a) \]

\[ \langle \bar{u}u \rangle / \langle \bar{u}u \rangle_0 \]

\[ \langle \bar{s}s \rangle / \langle \bar{s}s \rangle_0 \]

\[ \frac{\chi_s}{\chi_0} \]

T (GeV)
Modèle A

\( \mu_u = 200 \text{ MeV}, \mu_s = 0 \text{ MeV} \)
$M_0^0 = 300\text{ MeV}, \chi_0 = 80\text{ MeV}$

Modèle A

$\mu_u = \mu_s = 0\text{ MeV}$ (a)

$\chi_s \chi_0$

$\langle \bar{u}u \rangle \langle \bar{u}_0u \rangle$

$\langle \bar{c}s \rangle \langle \bar{c}_0s \rangle$
Modèle A

$\mu_u = \mu_s = 250 \text{ MeV}$ (b)

$\chi_s/\chi_0$

$\langle \bar{u}u \rangle/\langle \bar{u}u \rangle_0$

$\langle \bar{s}s \rangle/\langle \bar{s}s \rangle_0$

$T$(GeV)
$M_u^0 = 300 \text{ MeV}, \chi_0 = 80 \text{ MeV} \ (\text{Modèle A})$

- Hadronic phase $u$
- Quarks $s$
- Quarks $u$ & $s$
- Hadronic phase $u$ & $s$
- $\rho_u = \rho_s = 0$
- $T = 0 \text{ MeV}$
\( T = 100 \text{ MeV} \)

- Hadronic phase: \( \mu_{sc} \) and \( \mu_{uc} \)
- Quarks: \( s \) and \( u \)
\[ T = 130 \text{ MeV} \]

- Quarks u & s
- Hadronic phase u & s
- Hadronic phase s
$T = 165$ MeV

- quarks $u$ & $s$
- quarks $u$
- hadronic phase $s$
(a) $\mu_u = \mu_s = 0$ MeV

$M_u^0 = 300$ MeV, $\chi_0 = 80$ MeV

$P \approx 3.40 T^4 - 1.195 \times 10^{-3}$

$M_u^0 = 600$ MeV, $\chi_0 = 125$ MeV

$P \approx 3.40 T^4 - 2.097 \times 10^{-3}$
(b) $\mu_u = \mu_s = 250\text{ MeV}$

$M_0^u = 300\text{ MeV}, \chi_0 = 80\text{ MeV}$

$P \approx 3.45 T^4 + 8.51 \times 10^2 T^2 - 9.201 \times 10^4$

$M_0^u = 600\text{ MeV}, \chi_0 = 125\text{ MeV}$

$P \approx 3.45 T^4 + 8.385 \times 10^2 T^2 - 1.734 \times 10^3$
(a) $\mu_u = \mu_s = 0$ MeV

$M^0_u = 600$ MeV, $\chi_0 = 125$ MeV

$\varepsilon \approx 0.162 \times 10^{-2} + 10.269 T^4$

$M^0_u = 300$ MeV, $\chi_0 = 80$ MeV

$\varepsilon \approx 0.8771 \times 10^{-3} + 10.271 T^4$
(b) $\mu_u = \mu_s = 250$ MeV

$\epsilon \approx 10.364 T^4 + 0.270 T^2 + 2.592 \times 10^{-3}$

$M_u^0 = 600$ MeV, $\chi_0 = 125$ MeV

$\epsilon \approx 10.360 T^4 + 0.272 T^2 + 1.806 \times 10^{-3}$

$M_u^0 = 300$ MeV, $\chi_0 = 80$ MeV
(Modèle A)

(a) $\mu_u = \mu_3 = 0$ MeV

$M_u^0 = 300$ MeV, $\chi_0 = 80$ MeV

$s \approx 13.839 T^3 - 0.912 \times 10^{-2}$

$M_u^0 = 600$ MeV, $\chi_0 = 125$ MeV

$s \approx 13.868 T^3 - 0.136 \times 10^{-1}$
(b) $\mu_u = \mu_s = 250$ MeV

$M_u^0 = 300$ MeV, $\chi_0 = 80$ MeV

$s \approx 13.814 T^3 + 0.1701 T - 1.110 \times 10^3 / T$

$M_u^0 = 600$ MeV, $\chi_0 = 125$ MeV

$s \approx 13.813 T^3 + 0.1676 T - 1.919 \times 10^3 / T$