Computer based original method employed to assess the force and the torque on the rudder's shaft

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Abstract. Appropriate dimensioning of the electric drives belonging to the steering gear of a ship is based on the maximum values of the torque applied on the rudder’s shaft. Evaluation of the moment may be done in several ways. The paper presents a method based on dimensionless coefficients given as diagrams. To express analytically the diagrams resulted from experimental studies, we use a digitization stage followed by an interpolation process based on spline functions. The output data of this stage consist of the spline functions coefficients stored in text files using the CSV format. An original application was developed, program which considers a general balanced rudder with a parameterised balance ratio and two directions of motion, ahead and astern. The output data of the application is the moment applied on the rudder’s shaft with respect to the angle of attack for the both directions of motion. We plan to continue the study by considering a general shape of the rudder, in this case being required a new algorithm. The results of the new approach may be tested using the output data of this study.

1. Introduction
The rudder is the main element of the manoeuvring equipment of a ship. It is useful to steer the ship, i.e. to maintain the direction of the ship and to turn it, if necessary. The ship manoeuvring system is very important in ports, straits, channels and in heavy traffic areas. The steering system failure may have terrible consequences; therefore, it is considered paramount for the ships’ safety. The rudder is remotely controlled from the wheelhouse aka the ship’s bridge, using an electrical drive system, [1] [2] [3]. A rudder consists of a vertical plate and the rudder’s vertical shaft located in the longitudinal plane of symmetry of the ship hull. The rudder may be powered hydraulically or electrically. By rotating the rudder, the pressure of the water stream produced by the ship’s motion is applied on the rudder’s plate. The rudder is connected to the ship hull through the shank, therefore the pressure applied on the rudder is producing the ship’s turn. To rotate the ship rudder with a given angle, a certain torque may be applied on the vertical shaft. The according force is usually produced by an electrical motor that is connected to the rudder by the use of a transmission mechanism, which may be either mechanical, or hydraulic. The rudder’s angle has a given range for both ship’s motions, i.e. ahead or astern. The dimensioning of the electrical motor which powers the steering system as well as its parameters in running conditions are depending on the force and the torque applied on the rudder’s shaft. It results that the calculus of the force and torque is important, along time being developed several methods to evaluate these values, with respect to the number of rudders, the shape of the...
rudder, the position of the propeller and other criteria. In this way, this topic was approached in books, [4] [5] and a series of scientific papers, [6] [7] [8].

2. Actual solutions
Let us denote by \( \alpha \) the angle between the rudder’s vertical plane and the ships’ motion direction. The water pressure force on the rudder’s plate, \( R \), is projected with respect to two system of axes, as it is presented in the following figure.

![Figure 1. Forces applied in the centre of pressure, A, on a rudder during the ship manoeuvring.](image)

The first system of axes is positioned with respect with the ship’s motion direction and by projecting the pressure force along these directions, it results the \( R_x \) drag force and the \( R_y \) lift force along the portside or starboard transversal directions, where:

\[
R_y = \sqrt{R_x^2 + R_y^2}.
\]  

(1)

The second system of axes is positioned with respect to the rudder’s direction and its perpendicular direction. By projecting the pressure force along this system, it results the \( R \) force which is useful to deduce the torque applied on the vertical shaft and the tangential force, \( T \), for which we have the relation:

\[
R_y = \sqrt{R_x^2 + T^2}.
\]

(2)

From the previous figure there may be deduced the following relations:

\[
R = R_x \cdot \sin(\alpha) + R_y \cdot \cos(\alpha),
\]

(3)

\[
T = R_x \cdot \cos(\alpha) - R_y \cdot \sin(\alpha).
\]

(4)

According to the Wing Theory, the normal force is:

\[
R = \left[ C_y \cdot \cos(\alpha) + C_x \cdot \sin(\alpha) \right] \cdot \frac{A \cdot V^2}{2},
\]

(5)

where \( C_y \) is a dimensionless coefficient of the \( R_y \) lift force, \( C_x \) is a dimensionless coefficient of the \( R_x \) drag force, \( V \) is the ship’s speed \( \left[ m/s \right] \), \( A \) is the rudder’s area \( \left[ m^2 \right] \), \( \rho \) is the sea water density, \( \rho = 1025 \text{ kg/m}^3 \).
The values of the $C_y$ and $C_z$ coefficients depend on the rudder’s ratio of width to length (aspect ratio), $\lambda$, the relative thickness, $t$, the angle of attack, $\alpha$, and others.

The results of the experimental research studies were used to present diagrams of the $C_x$ and $C_y$ coefficients for rudders of various aspect ratios and for a wide range of aerodynamic shapes. These coefficients are experimentally measured and they are expressed as diagrams, as it is presented in the following figure. They depend on the hydrodynamic characteristics of the rudder, the relative dimensions of the rudder, the rudder angle and others.

Figure 2. $C_x$, $C_y$ and $C_p$ coefficients for two case studies.

Because the experimental studies were run for the $\lambda_1$ aspect ratio, the previous coefficients must be adjusted for the real aspect ratio, $\lambda_2$, using the relations:

$$C_{x2} = C_{x1} + \frac{C_y^2}{\pi} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right),$$  \hspace{1cm} (6)

$$\alpha_2 = \alpha_1 + 57.3 \cdot \frac{C_y}{\pi} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right),$$  \hspace{1cm} (7)

while the $C_y$ and $C_m$ coefficients are not modified.
Usually, a NACA hydrodynamic profile is denoted NACA00XY, this means that the profile in question is a NACA profile, having \( t = 0.XY \).

The moment on the rudder shaft must be known in order to design the electrical component of the steering equipment. In this way, the calculus schemes in the following figure are used to deduce the moment arm of the \( R \) pressure force.

Figure 3. Calculus scheme used to deduce the lever arm of the \( R \) pressure force for a ship moving ahead and astern in two cases: ordinary rudder and balanced rudder.

For a rectangular shape ordinary rudder, the moment on the rudder’s shaft is:

\[
M_{shft} = R \cdot x,
\]

where \( x = X_p \) is the distance from the centre of pressure, A, to the rudder’s shaft, when the ship moves ahead.

The location of the centre of pressure is modifying with respect to \( \alpha \), the attack angle. Let us define a dimensionless coefficient, \( C_p \), related to the location of the centre of pressure:

\[
C_p = \frac{X_p}{b}\]

(9)

The values of the centre of pressure coefficient, \( C_p \), are experimentally measured for various aspect ratios, with respect to the attack angle, \( \alpha \). For the cases presented in figure 2, the variation of the \( C_p \) coefficient is expressed as a diagram. The distance from the leading edge of the rudder to the centre of pressure is \( X_p = C_p \cdot b \), with respect to the current direction of motion. By the use of the (8) and (9) relations it results the moment:

\[
M_{shft} = R \cdot C_p \cdot b
\]

applied on the ordinary rudders’ shaft.

For the balanced rudders, the shaft is located at distance \( b_1 \) with respect to the front edge of the rudder and the moment is computed using the lever arm \( x = X_p - b_1 \), i.e.

\[
M_{shft} = R \cdot (C_p \cdot b - b_1)
\]

(11)
By replacing (3) in (10), it results:

\[ M_{\text{shift}} = \left[C_y \cdot \cos(\alpha) + C_x \cdot \sin(\alpha) \right] \cdot \rho \cdot \frac{A \cdot V^2}{2} \cdot C_p \cdot b. \]  

(12)

Let us define the \( C_m \) dimensionless coefficient as:

\[ C_m = \left[C_y \cdot \cos(\alpha) + C_x \cdot \sin(\alpha) \right] \cdot \frac{M_{\text{shift}}}{\rho \cdot A \cdot V^2}, \]  

(13)

and the previous relation becomes:

\[ M_{\text{shift}} = C_m \cdot \rho \cdot b \cdot \frac{A \cdot V^2}{2}. \]  

(14)

\( C_m \) is designated the moment’s coefficient and it expresses the moment’s modification with respect to \( \alpha \), the attack angle. This coefficient is also experimentally measured and it is also given as diagrams. However, between the dimensionless coefficients there is the relation (13) which connects them.

For a balanced rudder, the moment applied on the rudder’s shaft is computed using the relation

\[ M_{\text{shift}} = \rho \cdot b \cdot \frac{A \cdot V^2}{2} \cdot \left(C_m - f_{\alpha} \cdot k \right), \]  

(15)

where \( f_{\alpha} = C_y \cdot \cos(\alpha) + C_x \cdot \sin(\alpha) \) and \( k = \frac{A_1}{A} \) is the balance ratio, for which \( A_1 \) is the area of the balanced region, while \( A \) is the total area of the rudder.

If the ship moves astern, the centre of pressure has a different location, as it is presented in figure 3, and the moment applied on the rudder’s shaft is computed using one of the relations:

\( ^o \) for an ordinary rudder

\[ M_{\text{shift}} = R \cdot \left(b - X_p \right) = \rho \cdot b \cdot \frac{A \cdot V^2}{2} \cdot \left(f_{\alpha} - C_m \right) \]  

(16)

\( ^o \) for a balanced rudder

\[ M_{\text{shift}} = R \cdot \left(b - X_p - h_1 \right) = \rho \cdot b \cdot \frac{A \cdot V^2}{2} \cdot \left[f_{\alpha} \cdot (1 - k) - C_m \right]. \]  

(17)

The calculus relations of the moment applied on the rudder’s shaft were verified starting from the basic aspects of the theory. Once implemented, various tests were run, the results being also extensively verified.

3. Discussion

The previous calculus relations were used in order to create an algorithm which outputs the value of the moment applied on the rudder’s shaft.

The diagrams of the \( C_x, C_y \) and \( C_p \) dimensionless parameters were digitized, the result being presented in figure 2. The points along each diagram were used to approximate the curve by the use of the data processor presented in [9], based on spline functions.
Figure 4. Output of the data interpolation processor.

As it is presented in the previous figure the results of the spline interpolation processor consist of: an image of the interpolation solution, automatically generated code in C++, Java and GNU Octave and a text file which uses the CSV format to store the main information regarding the spline functions. In the next figure there may be noticed the following columns: index of the current interval, horizontal coordinate of the left side of the current interval, horizontal coordinate of the right side of the current interval, coefficients of the third degree polynomial function.

Figure 5. Interpolation data for the $C_x$ dimensionless parameter.
As mentioned in [9] if a curve is divided in ‘N’ intervals, its approximation with spline functions may be expressed as

\[ f(x) = \sum_{i=1}^{N} \left[ H(x-x_i) - H(x-x_{i+1}) \right] \left[ A_i \cdot (x-x_i)^3 + B_i \cdot (x-x_i)^2 + C_i \cdot (x-x_i) + D_i \right] \tag{18} \]

where \( H(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases} \).

The interpolation problem was solved using an original software based on GNU Octave. The previously mentioned algorithm was implemented and it resulted a C++ code.

The input information is structured as case studies, for each case study being considered the following sets of data: rudder’s geometry, range of the angles of attack, ship’s speed and the names of the files where are stored the data regarding the spline functions which approximate of the dimensionless coefficients.

The output consists of the values of the moment that loads the rudder shaft with respect to the attack angle. The output data are stored in a text file using the CSV format, in this way being very easy to create the diagrams and to do the interpretation of the output data by the use of a spreadsheet application that may be found in any operating system.

![Figure 6. Screen shot of the original application.](image)

As it was presented in [10], a method to easily develop computer based instruments is to use the previously created libraries, in this case being created new C++ header files used to handle the spline functions data. In this way, the new application was smaller, i.e. easier to control, and the library of header files was extended. A screen shot of the application is presented in the previous figure. There were used the MinGW compiler and the Eclipse environment.

The original application was run for several values of the balance ratio. There was considered that the ship moves ahead with 20 knots and astern with 5 knots, the values of the speeds (in knots) being given as an input data.
The results are presented in figure 8. As it can be noticed in figure 2, \( C_p \) is not defined for \( \alpha \in (0^\circ \ldots 5^\circ) \), therefore we had to solve an extrapolation problem for this range of values. However, the maxim values of the moment which loads the rudder’s shaft are reached for significant larger values of the attack angle.

There should be also mentioned that the dimensionless coefficients have different values for the ahead, respectively astern direction of the rudder. However, the speed astern of the ship is very small in comparison with the speed ahead and it is rarely used, in most cases for manoeuvring purposes.

![Diagram showing moment applied on the rudder shaft for various balance ratios](image)

**Figure 7.** Diagram which presents the \( M_{shift} \) moment applied on the rudder’s shaft with respect to the \( \alpha \) angle of attack for various balance ratios.

4. Conclusions
The rudder is a significant component of the ship for navigation safety and transport efficiency, being an important research topic over the years, [11]. The paper presents an original method based on experimental data expressed in an analytical way, in order to compute the moment applied on the rudder’s shaft. The values of the moment are used for the design of the electrical drives of the ship steering gear.

At present the rectangular rudder case was solved, a future research direction being dedicated to the rudders having an arbitrary shape. In this way, there may be developed another component of the analytical general model of a ship.

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5. References
[1] Freidzon I-R 1979 Electric drives for naval mechanisms (Bucharest: Technics Publishing House)
[2] Dordescu M and Gheorghiu S 2015 Naval electric drive systems (Constanta: Nautica Publishing House)
[3] Dordescu M and Gheorghiu S 2015 Naval electric drives (Constanta: Nautica Publishing House)
[4] Sgrumala M and Bidoae I 1978 Design and construction of the small ships (Bucharesta: Technics Publishing House)
[5] Ion C. Ionita and Jimbu Apostolache 1986 On-board naval equipment, Construction and utilization (Bucharest: Technics Publishing House)
[6] Benedict K, Kirchhoff M, Gluch M, Fischer S, Schaub M, Baldauf M and Klaes S 2014 TRANSNAV the International Journal on Marine Navigation and Safety of Sea Transportation 8 1
[7] Zelazny K 2014 TRANSNAV the International Journal on Marine Navigation and Safety of Sea Transportation, 8 3
[8] Liu J, Hekkenberg R-G, Rotteveel E and Hopman H 2017 Hydrodynamic characteristics of multiple-rudder configurations, Ships and Offshore Structures, 12 6
[9] Oanta E, Panait C, Lazaroiu G and Dascalescu A-E 2014 Scientific.Net - Advanced Materials Research (ModTech2014) 1036 1017-1022
[10] Oanta E, Raicu A and Panait C 2017 IOP Conference Series: Materials Science and Engineering 227 012084
[11] Liu J and Hekkenberg R-G 2017 Ship and Offshore Structures, 12(4) 495-512
[12] Oanta E, Panait C, Lepadatu L, Tamas R, Batrinca G, Nistor C, Marina V, Iliadi G, Sontea V, Marina V and Balan V 2010 Mathematical models for inter-domain approaches with applications in engineering and economy, MIEC2010, ANCS Ro-Md scientific research project
[13] Oanta E, Panait C, Nicolescu B, Dinu S, Pescaru A, Nita A and Gavrila G 2007 Computed aided advanced studies in applied elasticity from an interdisciplinary perspective ID1223 CNCSIS Romania research project