Applied calculus of fuzzy predicates for the formalization of knowledge

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Abstract. The formalization of knowledge remains one of the main problems of informatization. Special, applied knowledge can be streamlined, brought to a logical level and used to solve specific problems of a qualitative nature. The problems of applied fuzzy knowledge formalization from the point of view of mathematical logic and the theory of fuzzy sets are considered. Use of the applied calculus of fuzzy predicates of the first order was developed and proposed for the representation of knowledge and problem solving. The developed methods were used in the expert system for scientific research in the field of molecular spectroscopy, as well as other subject areas.

1. Introduction

The knowledge we use on a daily basis is presented in a meaningful way, they are imperfect, inaccurate, contradictory, etc., they contain non-formalizable concepts, for example, such as common sense.

Knowledge can be structured from one point of view or another. First of all, you can highlight special knowledge, which is relatively ordered and brought to a certain logical system. Such knowledge is called micro-knowledge. On the other hand, there is macro-knowledge, which is a synthesis of empirical knowledge and micro-knowledge, they have a lot of contradictions and illogic.

Macro-knowledge is difficult to represent formally, describe, use. Nevertheless, a person is usually free to manage macro-knowledge. From the point of view of logic, people’s thoughts have a qualitative character, they are fuzzy. They can be called macro-thoughts. When a person tries to find a solution to a complex task, his macro-thought intuitively breaks into essential and insignificant ones, the latter are discarded, and only the former are considered. In addition, macro-thinking is needed when processing large amounts of micro-information. However, it is difficult to separate the essential and discard insignificant information, everything can change depending on the behaviour and purpose of the person, i.e. goals determine the image of macro-thoughts.

In practice, knowledge has the fundamental property of fuzziness, uncertainty due to the complexity of the objects of formalization. At the same time, logical thinking is necessary to solve qualitative problems. The proposed solution to the problem is the applied calculus of fuzzy predicates developed by the authors.

2. Main methods

Table 1 below describes the methodologies for handling real knowledge.
Table 1. Methodologies for handling uncertainty, ambiguity, vagueness and inaccuracy.

| Class of obscurity | Used technology | Scientific disciplines underlying the methodology |
|--------------------|-----------------|-----------------------------------------------|
| A. Uncertainty, chance: | The theory of stochastic processes and decision theory, a measure of entropy | Probability theory |
| a) events and / or conditions of the environment due to chance; | | |
| b) phenomena that are not amenable to analysis and measurement with arbitrarily high accuracy | Uncertainty principle | Quantum mechanics |
| B. Fuzziness: | | |
| a) fuzziness as a result of the subjectivity or individuality of a person | The theory of fuzzy sets, the theory of subjective probabilities | Infinitely-significant logic of Lukasevich |
| b) vagueness or ambiguity in the processes of thinking and reasoning: | | |
| 1) unclear or inaccurate conclusion; | Theory of fuzzy or approximate reasoning | Fuzzy logic (propositional calculus, predicate calculus) |
| 2) ambiguity due to the complexity and (or) diversity of conclusions | Appeal to the approach that simulates the process of knowledge | Methods of artificial intelligence, supported by the theory of knowledge |
| C. Vagueness or ambiguity associated with natural languages: | Theory of fuzzy sets, fuzzy logic, modal logic | |
| a) the vagueness of the description or presentation; | Theory of fuzzy sets, fuzzy logic, modal logic | |
| b) ambiguity associated with the complexity and / or diversity of the semantics and structures of natural languages | Information semantics | |
| D. Vagueness or confusion in the presentation of drawings, paintings or scenes | Image Interpretation Technique | |
| E. Uncertainty due to structural complexity and diversity of information | Structural modeling techniques | Fuzzy Structural Modeling Techniques |

There are various formal approaches to the representation of knowledge.
In computer, in particular, expert systems for formalization of knowledge network, frame and production models, structurally combined by various methods are often used, for example, in a hierarchical network. At the same time, local knowledge (micro-knowledge) is logical in the form IF IF TO, i.e. product systems. For their formalization, Boolean algebra is used, or the calculus of first-order predicates.

The predicate is a propositional function $P(x_1,x_2,...,x_n)$, defined on individual variables $x_1,x_2,...,x_n$, the range of values of which are statements true or false (1 or 0). Such formalism reflects only the logical side of the issue. The other side of knowledge, vagueness and uncertainty, can be considered using the theory of fuzzy sets. The theory of fuzzy sets was proposed by L. Zade in 1965 [1]. The fundamental concept in the theory of fuzzy sets is the concept of the membership function.

Let $M$ be a set, $x$ be an element of $M$, then a fuzzy subset $A$ of $M$ is defined as a set of ordered pairs $\{(x,\mu_A(x))\}$, $\forall x \in M, \; \exists \mu_A(x)$ which is a characteristic membership function that takes its values in a well-ordered set $E$, which indicates the degree or level of belonging of an element $x$ to a subset $A$. A set $E$ is called a membership set. If $E = \{0,1\}$, then a fuzzy subset $A$ is treated as a normal subset.

3. Results
For an adequate representation of knowledge, a formal logical system was developed - the applied calculus of fuzzy predicates [2-6], combining the descriptive capabilities of the theory of fuzzy sets and the deductive predicate calculi.

Definition 1. A variable $z$ will be called fuzzy if it is defined on the set $M$ and its range of values is the set of fuzzy subsets $\{A_z\}$. Variable in the usual sense is a special case of a fuzzy variable, when each fuzzy subset consists of one element.

Definition 2. A fuzzy predicate is a function $G(z_1,z_2,...,z_n)$, defined on fuzzy variables $z_1,z_2,...,z_n$, the range of values of which is an assertion whose truth is estimated by values from the interval (0,1).

Definition 3. The interpretation of the formula $G$ for the calculus of fuzzy predicates includes a non-empty set of the subject domain $M$ and an indication of the values of the constants, functional and predicate symbols found in $G$.

Each constant is assigned a fuzzy subset of $M$ with the characteristic membership function $\mu(x)$, with $0 \leq \mu(x) \leq 1$.

Each $n$-local functional symbol is associated with a map from $M^n$ to the set of subsets $\{M \subseteq M$ and the set of their characteristic membership functions $\{\mu_F\}$ with $0 \leq \mu_F \leq 1$.

Each $n$-local predicate symbol is assigned a map of $M^n$ to the set of values of the truth function $\{\mu_P\}$, $0 \leq \mu_P \leq 1$.

Formula $G$, the predicate symbol $P$ is associated with the set of external values of the truth function $\{\mu_P\}$, $0 \leq \mu_P \leq 1$.

Every fuzzy variable or constant is a term $t$. The expression $F(t_1,t_2,...,t_n)$ is also a term if $t_1,t_2,...,t_n$ are terms. The fuzzy predicate $P(t_1,t_2,...,t_n)$, where $t_1,t_2,...,t_n$, are terms, is an elementary formula.

Definition 4. The value of the truth function of an elementary formula is equal to the algebraic product of the values of the characteristic membership functions of terms and the external truth value of a fuzzy predicate.

Definition 5. For two formulas $G$ and $F$, we define fuzzy logic operations as follows:

\[
G \land F = \min (G,F) \\
G \lor F = \max (G,F) \\
\neg G = 1 - G \\
G \rightarrow F = \neg G \lor F \\
G \sim F = (\neg G \lor F) \land (G \land \neg F)
\]

Every variable or individual (constant) is a term $t$. The expression $F(t_1,t_2,...,t_n)$ is also a term, if $t_1,...,t_n$ are terms. Predicate $P(t_1,...,t_n)$, where $t_1,...,t_n$ are terms, is an elementary formula.
An expression is a well-constructed formula (WCF), if it is an elementary formula or a formula of the form \( P, P_i \rightarrow P_j, P \lor P_j, P \land P_j \rightarrow P_j \), where \( P, P_i, P_j \) are WCF. If \( t \) is a variable in \( P_i \) and \( P_j \), then \( \exists P_i \) and \( \forall P \) are WCP, and the variable is called connected quantifier, \( P_i \) and \( P_j \) are the domains of the quantifier.

Theorem 1. For values of truth \( \mu_n \) WCF of applied logical calculus it is right that \( 0 \leq \mu_n \leq 1 \).

Definition 6. Two formulas \( G \) and \( F \) are equivalent (written as \( G = F \)) if and only if the truth values of \( G \) and \( F \) are the same for any interpretation. Based on definitions and properties, the following properties of fuzzy predicate calculus formulas can be proved: commutativity, associativity, idempotency, distributivity, de Morgan theorems.

Definition 7. A formula in applied logical calculus is in a predetermined normal form (PNF) if and only if it has the form \((\chi_1x_1)\ldots(\chi_nx_n)(F)\), where \( \chi=\{\forall,\exists\} \), and \( F \) is a formula that does not contain quantifiers, \((\chi_1x_1)\ldots(\chi_nx_n)\) is called the prefix, the \( F \)-matrix of the formula.

Due to the fact that formulas in applied logical calculus retain the basic properties of the classical predicate calculus, the well-known schemes of their transformations are preserved. To convert a formula to PNF, you must perform the following sequence of operations:
- Exclude bundles of ~ and →, Insert the negation sign inside the formula, rename the bound variables if necessary. Put the quantifiers to the beginning of the formula.
- It is known that the matrix of the formula \( F \) can be transformed into disjunctive (DNF) and conjunctive (CNF) normal forms using equivalent formulas. In practical applications, the quantifier-free form for representing formulas is used, as a rule.
- In practical applications, the quantifier-free form for representing formulas is used, as a rule.
- Definition 8. The formula \( G \) of applied calculus is satisfiable (non-contradictory) if and only if there is no interpretation \( I \) in which the truth value \( G \) is non-zero.
- Definition 9. An applied calculus formula is impracticable (contradictory) if and only if there is no interpretation \( I \) in which the truth value of \( G \) is not zero.
- Definition 10. The formula \( G \) of applied calculus is generally valid if and only if the truth value is not zero for all possible interpretations.
- Definition 11. A formula \( G \) is a logical consequence of the formulas \( F_1, F_2, \ldots, F_n \) if and only if for each interpretation \( I \), if \( F_1 \land F_2 \ldots \land F_n \) has a nonzero truth value, then \( G \) also has a truth value, non-zero in \( I \).

The task of the proof of the theorems is to clarify the question of the logical following of a formula \( G \) from a given set of formulas \( \{F_1, F_2, \ldots, F_n\} \), that is, clarifying the validity of the formula \((F_1 \land F_2 \ldots \land F_n) \rightarrow G\).

In practice it is more convenient to determine the impracticability, rather than universality, therefore we will consider the formula \( \neg((F_1 \land F_2 \ldots \land F_n) \rightarrow G) \), which is equivalent to the formula \( F_1 \land F_2 \ldots \land F_n \land \neg G \) and its impracticability. To establish the impracticability, it is necessary to prove that there is no such interpretation, at which simultaneously \( F_1, F_2, \ldots, F_n, \neg G \) are true. The procedure will lead to success if \( G \) follows from \( F_1, F_2, \ldots, F_n \), otherwise it may continue indefinitely.

In applied theory, \( G \) is a qualitative problem, and \((F_1, F_2, \ldots, F_n)\) is a system of logical expressions describing declarative knowledge, as well as conditions for solving a problem. To establish the impracticability of the formula \((F_1 \land F_2 \ldots \land F_n) \land \neg G \) in the applied calculus of fuzzy predicates, it is necessary to prove that there is no such interpretation that formulas \( F_1, F_2, \ldots, F_n, \neg G \) simultaneously have truth values different from zero.

There are a number of methods for proving theorems in first-order predicate calculus, the most famous of which are based on the principle of Robinson's resolutions. To apply the principle, logical expressions (formulas) are reduced to a pre-normal form (PNF), that is, they are represented as \((\chi_1x_1)\ldots(\chi_nx_n)F\), where \( \chi \in \{\forall,\exists\} \), and \( F \) is a formula in conjunctive normal form (CNF). For the transformation, known identities of predicate logic are used, then with skelletization, the existence quantifiers \( \exists \) are eliminated, the universal quantifiers \( \forall \) are omitted, the conjunction signs \( \land \) are replaced with commas, as a result we have many clauses \( S \).
Theorem 2. Let \( G \) be a formula of applied calculus in a preliminary form, that is, 
\[ G = \gamma(x_1, \ldots, x_n)[x_1, \ldots, x_n], \] 
and \( F[x_1, \ldots, x_n] \) is the matrix of the formula \( G \), containing the variables \( x_1, \ldots, x_n \) 
and represented in conjunctive normal form. We denote the set of disjuncts of the matrix \( F[x_1, \ldots, x_n] \) as \( S \). 
Formula \( G \) is contradictory if and only if \( S \) is contradictory.

In the first-order predicate calculus, a permutation is a finite set \( \gamma \) of the form 
\[ \gamma = \{(t_1/x_1, \ldots, t_n/x_n) \} \] 
where \( x_i \) is a variable, \( t_i \) is a term. We will denote the formula \( G \), for which all occurrences of \( x_i \) variables 
are replaced by \( t_i \) as \( G_t \). The set \( \{G_{t_i}\} \) is called unifiable if there exists a permutation called a unifier 
such that \( G_{t_1} = G_{t_2} = \ldots = G \). There is a unification algorithm that always finds the most general unifier in 
a finite number of steps.

In applied logical calculus, \( x_i \) and \( t_i \) are terms, and if \( x_i \) is a constant, then \( t_i \) is also a constant. In the 
general case, \( x_i \) and \( t_i \) are sets of odd disjuncts with membership functions \( \mu_{x_i}(t_i) \) \( \mu_{x_i}(x_i) \), then a 
substitution is a finite set of \( \alpha \) pairs of the form \( \alpha = \{(t_1/x_1, \{ \mu_{x_1}(1), \mu_{x_1}(1) \} \}), \ldots, \{t_n/x_n, \{ \mu_{x_n}(t_n), \mu_{x_n}(t_n) \} \} \} \)

A set \( \{G_{\alpha}\} \) is called unifiable if there exists a substitution \( \alpha \), for which the truth values of each 
formula from \( \{G_{\alpha}\} \) are nonzero. The consequence of this definition is that none of the products 
\( \mu_{x_1}(t_1) \mu_{x_1}(t_1) \) should be zero.

If at substitution of \( \alpha \) at least one product is zero, then we will define such a substitution \( \alpha_x \) as 
degenerate, while it is a set of subsets \( \alpha_x = \{(t_1/x_1, \{ \mu_{x_1}(1), \mu_{x_1}(1), \mu_{x_1}(1) \} \}), \ldots, \{t_n/x_n, \{ \mu_{x_n}(t_n), \mu_{x_n}(t_n), \mu_{x_n}(t_n) \} \} \} \).

The degenerate substitution describes replacing the constant \( c \) with the constant \( r \), the value of the 
truth function of the disjunction \( \mu_{x_i}(c) \) is replaced by \( \mu_{x_i}(r) \), and the membership function of fuzzy 
variables are replaced from \( x_i, \mu_{x_i}(c) \) to \( \mu_{x_i}(r) \).

It is clear that with a degenerate substitution, the uncertainty of knowledge increases. This can be 
expressed either in decreasing the truth value of the disjunction \( \mu_{x_i}(r) \) or in expanding the domains where 
\( 0 < \mu_{x_i}(r) < 1 \) for the characteristic membership functions of fuzzy variables. The nature of the changes 
depends on the nature of the tasks being solved. Most often, reducing the value of \( \mu_{x_i}(r) \) is impractical; 
it seems more reasonable to save the values of \( \mu_{x_i}(r) \) even due to the large expansion of the regions 
\( 0 < \mu_{x_i}(r) < 1 \). The source of information about the nature of changes is that part of knowledge, which is 
described as patterns.

Another type of permutations needed to solve problems of an applied nature is called absorption 
substitution. Such substitutions are necessary when solving problems with arithmetic changes of 
quantitative quantities. We will define it as follows: \( \alpha = \{(t_1/x_1, \{ \mu_{x_1}(1), \mu_{x_1}(t_1), \mu_{x_1}(t_1) \} \}), \ldots, \{t_n/x_n, \{ \mu_{x_n}(1), \mu_{x_n}(t_n), \mu_{x_n}(t_n) \} \} \} \).

If the product is \( 0 < \mu_{x_i}(t_1) \mu_{x_i}(t_1) < 1 \) for all \( t_i \), we convert it to the form 
\( \max(\mu_{x_i}(t_1) \mu_{x_i}(t_1))/\mu_{x_i}(t_1) \mu_{x_i}(t_1) \) and the value \( \mu = \max(\mu_{x_i}(t_1) \mu_{x_i}(t_1)) \) will be called the 
external value of the membership function for the substitution \( \alpha \).

We will call a formula a letter or its negation, a disjoinst - a disjunction of a letter.

For first-order predicate logic, the concept of resolvent is introduced as follows.

Let \( C_1 \) and \( C_2 \) be two disjuncts that have no common variables, \( L_1 \) and \( L_2 \) be two letters in \( C_1 \) and \( C_2 \), 
respectively. If \( L_1 \) and \( L_2 \) have the most common unifier \( \gamma \), then the disjoinst \( C_0 = (C_{1\gamma} \lor L_{1\gamma}) \lor (C_{2\gamma} \lor L_{2\gamma}) \) is called the resolvent \( C_1 \) and \( C_2 \). The letters \( L_1 \) and \( L_2 \) are called cut letters.

We define the resolvent in the calculus of fuzzy predicates.

Let \( C_1 \) and \( C_2 \) be two clauses that have no common variables, \( L_1 \) and \( L_2 \) two letters with truth functions 
\( \mu_{t_1} \) and \( \mu_{t_2} \) in \( C_1 \) and \( C_2 \). If \( L_1 \) and \( L_2 \) have the most common unifier \( \alpha \), then the disjoinst 
\( C_{\alpha} = \{ \mu_{t_1}(1) \mu_{t_2}(1) \} \lor (C_{1\alpha} \lor L_{1\alpha}) \lor (C_{2\alpha} \lor L_{2\alpha}) \) will be called the resolvent \( C_1 \) and \( C_2 \) in terms of fuzzy predicates.

Theorem 3. Let two clauses \( C_1 \) and \( C_2 \), not having common variables, and their truth functions \( \mu_1 \) 
and \( \mu_2 \) be given. Then, for the truth function \( \mu_0 \) of the resolvent \( C_0 \) of disjuncts \( C_1 \) and \( C_2 \), obtained 
by the substitution \( \alpha \), the inequality \( \mu_0 \geq \mu_{t_1} \mu_{t_2} \) is valid.
Theorem 4. Let two clauses $C_1$ and $C_2$ given no common variables, their truth functions $\mu_1$ and $\mu_2$, and there exist a substitution $\alpha$ such that $C_{1\alpha} = L\lor C_{1,0}, C_{2\alpha} = \neg L\lor C_2$, resolution $C_0 = C_{1\alpha}\lor C_{2,0}$. If at least one of the truth functions is $\mu_1^{1,0}$ or $\mu_2^{1,0} \geq 0.5$, then for the truth function $\mu_0$ of the resolvent $C_0$, the inequality $\mu_0 \geq \min(\mu_{1\alpha}, \mu_{2\alpha})$ holds.

To describe the membership functions, two approaches are possible. The first is expert judgment. The specific type of membership functions for each subject area is determined on the basis of various additional assumptions about the properties of these functions (piecewise linear approximation, exponential, quadratic, characteristics such as symmetry, monotony, continuity of the first derivative, etc.) taking into account the specific uncertainty, real situation and the number of degrees of freedom in functional dependence.

The values of the parameters of functions are determined by experts on the basis of literary data, personal experience and other information. For example, for the recognition system of the structures of molecules for each possible fragment of the molecule, possible values of the intervals of characteristic frequencies of the IR spectra are set [4-6].

The second approach involves the use of training procedures. Both the experts themselves and specialized programs can be trained; the use of neural networks is promising for a number of applications [7-11].

Membership functions play a crucial role in the representation of knowledge, and in assessing the truth of solutions in fuzzy-logical systems.

The variety of applied tasks of a qualitative nature is very large. These include tasks of decision-making, assessment of objects and phenomena, systematization and classification, planning, management, explanation, training, recognition, determination of conformity, prediction, interpretation, control, identification, etc [12-16]. Nevertheless, subject to certain conditions and restrictions, one can consider the question of their systematization and classification [3].

We will confine ourselves to the consideration of applied knowledge, for which there exists a domain of premises (causes) and a domain of results (consequences). The links between the premises and the results can be logical and structural, concrete and indirect, explicit and hidden, objective and subjective, definite and fuzzy.

We represent the described model as follows: $P \Longrightarrow R$, where $P$ is the domain of premises, $R$ is the result area, $\Longrightarrow$ is a generalized causal operator. We assume that the problem includes the formulation of $F$ and the conditions of the solution of $C$. It is obvious that $P, R, F$ and $C$ can contain known and unknown values, conventionally we call them constants and variables. Within the framework of the proposed model, the following classification of applied tasks of a qualitative nature is possible:

1. The statement of the problem includes mainly variables and belongs to the results area, $F \in R$. The conditions of the problem include constants and relate to the domain of premises, $C \in P$. This task belongs to the class of forecasting tasks. As a result of solving the problem, the values of the variables contained in the formulation of Problem $F$ are set.

2. The formulation of the problem includes mainly constants and belongs to the results area, $F \in R$. The conditions of the problem include mainly constants and relate to the domain of premises, $C \in P$. We assign this task to the class of interpretation tasks. As a result of solving the problem, the values of the variables contained in the formulation of problem $F$ and, possibly, in conditions $C$ are established.

In the formalism of calculus of fuzzy predicates, the generalized causal operator is replaced by implication. In such a formulation, the prediction and interpretation tasks have a solution, since clauses from the pairs $\neg F$ and $R$, $P$ and $C$ have the same shape, but different signs, and when resolving, empty clauses will be formed.

The areas of the forecasting and interpretation tasks overlap and it is impossible to draw a clear boundary between them (today’s weather forecast today can be used to interpret the conditions for its compilation).

3. The formulation of the problem includes mainly variables and belongs to the field of premises, $F \in P$. The conditions of the problem include mainly constants and relate to the results area, $C \in R$. We
assign this task to the class of identification tasks. As a result of solving the problem, the values of the variables contained in the formulation of the problem \( F \) and, possibly, in the domain of premises \( P \).

In the formalism of fuzzy predicates calculus, the identification problem has no solution, since clauses of the pairs \( \neg F \) and \( \neg P \), \( R \), and \( C \) have the same shape and the same signs; when resolving, it is impossible to form an empty clause.

In science and technology, such tasks are often referred to as inverse. To solve them, it is necessary to transform the described form of knowledge representation into the inverse, that is, from the \( P \Rightarrow R \) form to the \( R \Rightarrow P \) form, intuition, and common sense. Such tasks are difficult to formally solve.

4. Conclusion
An applied calculus of fuzzy predicates has been developed for the representation of knowledge and the solution of qualitative problems by the method of proving theorems.

For the practical application of this theory and methods, the main issues are the development and structuring of a system of fuzzy predicates that adequately reflect the subject area, and a description of the membership functions of predicates, variables and constants. This requires the joint efforts of specialists in the subject area as well as in knowledge engineering [4-7,16].

The developed methods were tested in systematization and formalization of knowledge for the development of expert systems in the field of molecular spectroscopy [17], in the field of small business management and a number of other applications [2,4,6,19-21]. The theory, methods and practical implementation of fuzzy logic find wide practical application (see the list of references).

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