Massless Infinite Spin (Super)particles and Fields

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Abstract—A new twistor field formulation of a model of a massless infinite spin particle is proposed. A twistor infinite spin field is found, and its helicity decomposition is obtained. Twistorial equations of motion for infinite spin fields are derived in the cases of integer and half-integer helicities. The infinite integer spin field and infinite half-integer spin field are shown to form an $\mathcal{N}=1$ infinite spin supermultiplet. The corresponding supersymmetry transformations are presented. It is proved that the supersymmetry algebra is closed on-shell.

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1. INTRODUCTION

The symmetry principles formulated in terms of group theory play an essential role in theoretical and mathematical physics. Suffice it to say that the Standard Model is constructed on the basis of the gauge principle, which imposes strong restrictions both on the classical Lagrangian and on the scattering amplitudes. A fundamental contribution to the implementation of the gauge principle in quantum field theory was made by A. A. Slavnov [35, 36]. The present paper is devoted to some aspects of symmetry related to the Poincaré group.

Relativistic symmetry associates elementary particles with irreducible representations of the Poincaré group $\text{ISO}^\uparrow(1,3)$ (or its covering $\text{ISL}(2,\mathbb{C})$). A classification of the $\text{ISO}^\uparrow(1,3)$ unitary irreducible representations was given in [3, 44, 45]. Those unitary irreducible representations of the Poincaré group that are usually interesting from the physical point of view act in the space of states with nonnegative squared mass $m^2 \geq 0$ and nonnegative energy $E = k_0 \geq 0$ (here $k_0$ is the zero component of the 4-momentum of a particle).

To characterize these irreducible representations, we need to consider the corresponding irreducible representations of the Lie algebra $\text{iso}(1,3)$ with generators $\hat{P}_n$ and $\hat{M}_{nmk}$ (components of the momentum and angular momentum) and defining relations

\[ [\hat{P}_n, \hat{P}_m] = 0, \quad [\hat{P}_n, \hat{M}_{mk}] = i(\eta_{kn}\hat{P}_m - \eta_{mn}\hat{P}_k), \]
\[ [\hat{M}_{nmk}, \hat{M}_{k\ell}] = i(\eta_{nk}\hat{M}_{m\ell} - \eta_{mk}\hat{M}_{n\ell} + \eta_{m\ell}\hat{M}_{nk} - \eta_{n\ell}\hat{M}_{mk}), \]

where the metric tensor is $\|\eta_{mk}\| = \text{diag}(+1, -1, -1, -1)$.

There are two classes of physically interesting unitary irreducible representations (irreps) of the Poincaré group: massive irreps and massless irreps.

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1. **Massive irreps.** The algebra iso(1,3) has two Casimir operators $\hat{P}^n \hat{P}_n$ and $\hat{W}^n \hat{W}_n$, where

$$\hat{W}_n = \frac{1}{2} \varepsilon_{mnkr} \hat{M}^{mk} \hat{P}^r$$

are the components of the Pauli–Lubański vector, which satisfy the relations

$$\hat{W}_n \hat{P}^n = 0, \quad [\hat{W}_k, \hat{P}_n] = 0, \quad [\hat{W}_m, \hat{W}_n] = i \varepsilon_{mnkr} \hat{W}_k \hat{P}^r.$$

On the state space of massive irreducible representations, the Casimir operators are proportional to the identity operator $I$:

$$\hat{P}^n \hat{P}_n = m^2 I \quad (m^2 > 0), \quad \hat{W}^n \hat{W}_n = -m^2 j(j+1) I,$$

where the real number $m > 0$ is called a mass and the real number $j \in \mathbb{Z}_{\geq 0}/2$ is called a spin.

2. **Massless irreps.** The Casimir operators of iso(1,3) are

$$\hat{P}^n \hat{P}_n = m^2 = 0, \quad \hat{W}^2 = \hat{W}^n \hat{W}_n = -\mu^2.$$

In this case we have two possibilities:

(A) $\mu^2 = 0$ and

(B) $\mu^2 \neq 0$.

In the massless case (A) we obtain the usual massless helicity representations with

$$\hat{W}^2 = 0, \quad \hat{P}^2 = 0, \quad \hat{P}_n \hat{W}^n = 0 \quad \text{for } \mathbb{R}^{1,3} \quad \hat{W}_n = \Lambda \hat{P}_n,$$

where the central element $\Lambda \in$ iso(1,3) is called a helicity operator and its eigenvalues are $\Lambda = 0, \pm 1/2, \pm 1, \pm 3/2, \ldots$.

In the massless case (B) we have

$$\hat{W}^2 = -\mu^2, \quad \hat{P}^2 = 0, \quad \hat{P}_n \hat{W}^n = 0,$$

which corresponds to a massless irreducible representation of infinite (continuous) spin. To describe this representation, it is convenient to introduce variables $x_k$ and $y_m$ “canonically conjugate” to $\hat{P}_k$ and $\hat{W}_n$:

$$x = (x_0, x_1, x_2, x_3) \in \mathbb{R}^{1,3}, \quad y = (y_0, y_1, y_2, y_3) \in \mathbb{R}^{1,3}.$$

Then, as shown in [3, 44, 45], the massless infinite spin irreducible representations of the Poincaré group are realized in the space of wave functions $\Phi(x, y)$ satisfying the conditions

$$\frac{\partial}{\partial x_m} \frac{\partial}{\partial x_m} \Phi = 0, \quad \frac{\partial}{\partial x_m} \frac{\partial}{\partial y_m} \Phi = 0,$$

$$\frac{\partial}{\partial y_m} \frac{\partial}{\partial y_m} \Phi = \mu^2 \Phi, \quad -iy_m \frac{\partial}{\partial x_m} \Phi = \Phi.$$

(1.2)

This paper is devoted to some aspects of the theory of massless infinite (or continuous) spin unitary irreducible representations of the group ISL(2, C). Various problems related to the quantum-mechanical and field descriptions of such states have been considered in a wide range of works devoted to particles and fields of infinite (continuous) spin. The study of particle and field models with infinite spins was initiated in [10, 18] and then continued in [6, 7, 9, 20, 21, 29, 31–34]. The recent developments are discussed in [1, 2, 16, 19, 22–24, 30, 46] (see also the review papers [4, 8]). The investigations of infinite spin representations are motivated by the identical spectrum of states in the infinite spin theory [18] and higher spin theory [37–39] (see also the reviews [5, 40, 41]) and
by their potential relation to string theory (see [25] as well as the recent paper [42] and references therein) as candidates for quantum gravity.

In our recent papers [11, 12] we constructed a new model of an infinite (continuous) spin particle, which is a generalization of the twistor formulation of the model of the standard (with fixed helicity) massless particle [26–28] to massless infinite spin representations. As a result of a quantization procedure, we obtained infinite spin fields that demonstrate a transparent decomposition of continuous spin irreducible representations into an infinite sum of states with all helicities. We stress that in the massless case (B) the irreducible representations are not characterized by definite helicities. In addition, using the field twistor transform, we can now get a space–time–spinor description of infinite spin fields with integer or half-integer helicities that form a supermultiplet of infinite spins [10, 46].

This paper is based on the results obtained in [11, 12].

2. WIGNER–BARGMANN SPACE–TIME FORMULATION

The Wigner–Bargmann space–time formulation [3, 44, 45] of the irreducible infinite spin massless representation can be obtained by quantizing the particle model with the following Lagrangian:

$$\mathcal{L}_{\text{space–time}} = p_m \dot{x}^m + q_m \dot{y}^m + e_p m_p^m + e_1 p_m q^m + e_2 (q_m q^m + \mu^2) + e_3 (p_m y^m - 1), \quad (2.1)$$

where \(\{p_n (\tau), q_n (\tau)\}\) are momenta canonically conjugate to the coordinates \(\{x_n (\tau), y_n (\tau)\}\), \(\tau\) is the evolution parameter, and \(\dot{x}_k (\tau) := \partial_{\tau} x_k (\tau)\). The Lagrangian (2.1) yields the canonical Poisson brackets

\[\{x^m, p_n\} = \delta^m_n, \quad \{y^m, q_n\} = \delta^m_n\]

and first-class constraints

\[
\begin{align*}
T &:= p_m p^m \approx 0, \\
T_1 &:= p_m q^m \approx 0, \\
T_2 &:= q_m q^m + \mu^2 \approx 0, \\
T_3 &:= p_m y^m - 1 \approx 0,
\end{align*}
\]

which correspond to the Wigner–Bargmann equations (1.2). The variables \(e (\tau), e_1 (\tau), e_2 (\tau), \) and \(e_3 (\tau)\) are Lagrange multipliers for the constraints (2.2). Nonvanishing Poisson brackets of the constraints (2.2) are

\[\{T_1, T_3\} = -T, \quad \{T_2, T_3\} = -2T_1.\]

The action \(S_{\text{space–time}} = \int d\tau \mathcal{L}_{\text{space–time}}\) is invariant under the transformations generated by the quantities

\[P_m = p_m, \quad M_{mn} = x_m p_n - x_n p_m + y_m q_n - y_n q_m.\]

These charges form a Poincaré algebra with respect to the Poisson brackets. We see that the additional coordinates \(y^m\) in the arguments of these fields play the role of spin variables.

Now, using the constraints \(T \approx 0, T_1 \approx 0, T_2 \approx 0,\) and \(T_3 \approx 0,\) we obtain the relations

\[P_m p^m \approx 0, \quad W_m W^m = \frac{1}{2} M_{nk} M^{nk} P_m P^m - M_{nk} M^{nl} P^k P_l \approx -\mu^2,\]

where \(W_m = (1/2) \varepsilon_{mnkl} P^m M^{kl}\) are the components of the Pauli–Lubański pseudovector. Therefore, the model with Lagrangian \(\mathcal{L}_{\text{space–time}}\) indeed describes the massless particle with continuous spin. We note that the vectors \(q_m\) and \(W_m = \varepsilon_{mnkl} p^n q^l q^k\) do not coincide with each other and the components \(W_m\) are not, strictly speaking, canonically conjugate to \(y_m\).

After the canonical quantization, the constraints (2.2) yield the Wigner–Bargmann equations (1.2) for the continuous spin fields \(\Phi (x, y)\).
3. TWISTOR FORMULATION OF THE MODEL OF CONTINUOUS SPIN PARTICLES

Our aim is to reformulate the Wigner–Bargmann model with Lagrangian (2.1) in terms of twistor variables. More precisely, we construct a twistor particle model which is classically equivalent to the Wigner–Bargmann model with Lagrangian $L_{\text{space-time}}$.

Below we will use the following two-spinor conventions about notation. The totally antisymmetric tensor $\epsilon^{mnkl}$ has the component $\epsilon^{0123} = 1$. We use the set of $\sigma$-matrices $\sigma^n = (\sigma^0 \equiv I_2, \sigma^1, \sigma^2, \sigma^3)$ and the set of dual $\sigma$-matrices $\tilde{\sigma}^n = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$, where $\sigma^i$ are the usual Pauli matrices. We also use the standard van der Waerden twistor notation with dotted and undotted spinor indices and raise and lower them by means of the metrics $\epsilon_{\alpha\beta}, \epsilon_{\dot{\alpha}\dot{\beta}}$ and their inverses $\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}$ with components $\epsilon_{12} = -\epsilon_{21} = 1$. In particular, $(\tilde{\sigma}_m)^{\dot{\alpha}\beta} = \epsilon^{\dot{\alpha}\beta}(\sigma_m)_{\alpha\beta}$. The relation between the Minkowski four-vectors and spinor quantities is given by

$$A_{\alpha\beta} = \frac{1}{\sqrt{2}} A_m(\sigma^m)_{\alpha\beta}, \quad A^{\dot{\alpha}\dot{\beta}} = \frac{1}{\sqrt{2}} A_m(\tilde{\sigma}^m)^{\dot{\alpha}\dot{\beta}}, \quad A_m = \frac{1}{\sqrt{2}} A_{\alpha\beta}(\tilde{\sigma}_m)^{\dot{\alpha}\beta},$$

so that $A^m B_m = A_{\alpha\beta} B^{\dot{\alpha}\dot{\beta}}$.

In [11, 12] we proposed a twistor formulation of the model of the infinite (continuous) spin particle, which is described by the bosonic Weyl spinors

$$\pi_{\alpha}, \quad \pi_{\dot{\alpha}} := (\pi_{\alpha})^*, \quad \rho_{\alpha}, \quad \rho_{\dot{\alpha}} := (\rho_{\alpha})^*$$

(3.1)

and their canonically conjugate spinors

$$\omega^\alpha, \quad \omega^{\dot{\alpha}} := (\omega^\alpha)^*, \quad \eta^\alpha, \quad \eta^{\dot{\alpha}} := (\eta^\alpha)^*.$$  

(3.2)

The nonzero Poisson brackets of these spinors are

$$\{\omega^\alpha, \pi_{\beta}\} = \{\eta^\alpha, \rho_{\beta}\} = \delta^\alpha_\beta, \quad \{\omega^{\dot{\alpha}}, \pi_{\beta}\} = \{\eta^{\dot{\alpha}}, \rho_{\beta}\} = \delta^{\dot{\alpha}}_\beta.$$

The twistorial Lagrangian of the infinite (continuous) spin particle is written in the form [11, 12]

$$L_{\text{twistor}} = \pi_{\alpha} \dot{\omega}^\alpha + \pi_{\dot{\alpha}} \dot{\omega}^{\dot{\alpha}} + \rho_{\alpha} \dot{\eta}^\alpha + \rho_{\dot{\alpha}} \dot{\eta}^{\dot{\alpha}} + l\mathcal{M} + k\mathcal{U} + \ell\mathcal{F} + \overline{\mathcal{F}},$$

(3.3)

where $l(\tau), k(\tau), \ell(\tau)$, and $\overline{\mathcal{F}}(\tau)$ are the Lagrange multipliers for the constraints

$$\mathcal{M} := \pi^\alpha \rho_{\alpha} \pi_{\dot{\alpha}} - \frac{\mu^2}{2} \approx 0, \quad \mathcal{F} := \eta^\alpha \pi_{\alpha} - 1 \approx 0, \quad \overline{\mathcal{F}} := \pi_{\dot{\alpha}} \eta^{\dot{\alpha}} - 1 \approx 0,$$

$$\mathcal{U} := i(\omega^\alpha \pi_{\alpha} - \overline{\omega}^{\dot{\alpha}} \pi_{\dot{\alpha}} + \eta^\alpha \rho_{\alpha} - \overline{\eta}^{\dot{\alpha}} \rho_{\dot{\alpha}}) \approx 0.$$  

(3.4)

One can check that the first-class constraints (3.4) generate an abelian Lie group which acts in the phase space of spinors (3.1), (3.2) as follows:

$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \to \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}, \quad \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} \to \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} (e^{i\beta}, e^{i\dot{\beta}}),$$

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \to \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \to \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} (e^{-i\beta} - i\alpha e^{-i\dot{\beta}}) + \frac{2}{\mu^2}(\pi_{\alpha} \pi_{\dot{\beta}})(\pi_1 \pi_2) \begin{pmatrix} \gamma \\ 0 \end{pmatrix},$$

where $\beta(\tau), \gamma(\tau) \in \mathbb{R}$ and $\alpha(\tau) \in \mathbb{C} \setminus \{0\}$ are the parameters of the gauge group generated by the constraints (3.4). The transformations (3.5) and (3.6) should be supplemented with the complex conjugate ones.

**Proposition 1.** The Wigner–Bargmann space–time (2.1) and twistor (3.3) models of the infinite (continuous) spin particle are equivalent at the classical level. The equivalence is provided by
the generalized Cartan–Penrose relations [26–28]
\[ p_{\alpha\dot{\beta}} = \pi_{\alpha} \pi_{\dot{\beta}}, \quad q_{\alpha\dot{\beta}} = \alpha_{\alpha} \pi_{\dot{\beta}} + \rho_{\alpha} \pi_{\dot{\beta}} \]
and the following generalized incidence relations [26–28]:
\[ \omega^\alpha = \pi_{\dot{\alpha}} x^{\dot{\alpha}} + \pi_{\dot{\alpha}} y^{\dot{\alpha}}, \quad \omega_{\dot{\alpha}} = x^{\dot{\alpha}} \pi_{\dot{\alpha}} + y^{\dot{\alpha}} \rho_{\dot{\alpha}}, \quad \eta^\alpha = \pi_{\dot{\alpha}} y^{\dot{\alpha}}, \quad \eta_{\dot{\alpha}} = y^{\dot{\alpha}} \pi_{\dot{\alpha}}. \]

The proof of this proposition is straightforward and is given in [11, 12].

4. QUANTIZATION OF THE TWISTOR MODEL AND TWISTOR FIELD
OF THE INFINITE SPIN PARTICLE

The quantization of the model is drastically simplified if we introduce new spinor variables by means of the Bogoliubov canonical transformations (cf. the gauge transformations (3.5) and (3.6)):
\[
\begin{pmatrix}
\pi_1 & \rho_1 \\
\pi_2 & \rho_2
\end{pmatrix} = \sqrt{M} \begin{pmatrix}
p_{1\alpha}^{(s)} & 0 \\
p_{2\alpha}^{(s)} & p_{1\alpha}^{(t)}
\end{pmatrix} \begin{pmatrix}
1 & p_{1\alpha}^{(t)} \\
0 & 1
\end{pmatrix},
\]
\[
\begin{pmatrix}
\eta_1 & \omega_1 \\
\eta_2 & \omega_2
\end{pmatrix} = \begin{pmatrix}
0 & z_1/\sqrt{M} \\
-t/\pi_1 & z_2/\sqrt{M}
\end{pmatrix} \begin{pmatrix}
1 & -p_{1\alpha}^{(t)} \\
0 & 1
\end{pmatrix} + \frac{s}{M} \begin{pmatrix}
\pi_1 & \rho_1 \\
\pi_2 & \rho_2
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix},
\]
where \( M = \mu/\sqrt{2} \) and new variables are defined by the expressions
\[
p_{\alpha}^{(s)} = \frac{\pi_{\alpha}}{\sqrt{M}}, \quad p_{\alpha}^{(s)} = \frac{\pi_{\alpha} \rho_{\alpha}}{M}, \quad p_{\alpha}^{(t)} = \frac{\rho_{\alpha}}{\pi_1},
\]
\[
\omega^\alpha = \frac{1}{\sqrt{M}} z^\alpha - \frac{1}{M} s p^\alpha - \frac{\delta^{\alpha 1}}{\pi_1} t p^{(t)}, \quad \eta^\alpha = \frac{1}{M} s p^\alpha + \frac{\delta^{\alpha 1}}{\pi_1} t.
\]

By means of complex conjugation we obtain the conjugate coordinates \( \pi_{\alpha}, \pi, \tilde{\pi} \) and their momenta \( p_{\alpha}^{(s)}, \pi_{\alpha}^{(s)}, \pi_{\alpha}^{(t)} \). The nonzero canonical Poisson brackets of the new variables are
\[
\{ z^\alpha, p_{\beta}^{(s)} \} = \delta_{\beta}^\alpha, \quad \{ \pi_{\alpha}, \pi_{\beta}^{(s)} \} = \delta_{\beta}^\alpha, \quad \{ s, p^{(s)} \} = \{ \pi, \pi^{(s)} \} = 1, \quad \{ t, p^{(t)} \} = \{ \tilde{\pi}, \pi^{(t)} \} = 1.
\]
In terms of the new variables (4.1), the constraints (3.4) of the spinor model (3.3) take the very simple form
\[
M': = p^{(s)} \pi^{(s)} - 1 \approx 0, \quad F': = t - 1 \approx 0, \quad \bar{F}': = \bar{t} - 1 \approx 0,
\]
\[
U': = \frac{i}{2} (z^\alpha p_{\alpha}^{(s)} - \pi_{\alpha}^{(s)} \pi_{\alpha}^{(s)} + i (s p^{(s)} - \pi \pi^{(s)})) \approx 0.
\]
After the canonical quantization \([\cdot, \cdot] = i \{ \cdot, \cdot \} \), these constraints turn into the equations of motion for physical states described by a wave function \( \Psi^{(c)} \):
\[
(p^{(s)} \pi^{(s)} - 1) \Psi^{(c)} = 0, \quad \frac{\partial}{\partial p^{(t)}} \Psi^{(c)} = \frac{\partial}{\partial \pi^{(t)}} \Psi^{(c)} = -i \Psi^{(c)}, \quad (4.3)
\]
\[
\left[ \frac{1}{2} \left( p_{\alpha}^{(s)} \frac{\partial}{\partial p_{\alpha}^{(s)}} - \pi_{\alpha}^{(s)} \frac{\partial}{\partial \pi_{\alpha}^{(s)}} \right) + p^{(s)} \frac{\partial}{\partial p^{(s)}} - \pi^{(s)} \frac{\partial}{\partial \pi^{(s)}} \right] \Psi^{(c)} = c \Psi^{(c)}, \quad (4.4)
\]
where the differential operators on their left-hand sides are quantum counterparts of the constraints (4.2). In equations (4.3) and (4.4), the wave function (or spinor field)
\[
\Psi^{(c)} (p_{\alpha}^{(s)}, \pi_{\alpha}^{(s)}, p^{(s)}, \pi^{(s)}, p^{(t)}, \pi^{(t)})
\]
is taken in the “momentum representation” and describes physical states, which form the space of the irreducible representation of the Poincaré group with continuous spin. The constant \( c \) is related
to the ambiguity of the operator ordering in equation (4.4). In other words, the constant \( c \) is an analog of the vacuum energy in the quantum oscillator model.

The equations of motion (4.3) can be solved explicitly in the form

\[
\Psi^{(c)} = \delta(p^{(s)}p^{(s)} - 1) e^{-i(p^{(s)} + p^{(s)})} \sum_{k=-\infty}^{\infty} e^{-ik\varphi} \tilde{\psi}^{(c+k)}(p^{(z)}, \bar{p}^{(z)}),
\]

where \( e^{i\varphi} := (p^{(s)}/\bar{p}^{(s)})^{1/2} \). Due to the constraint (4.4) the coefficient functions \( \tilde{\psi}^{(c+k)}(p^{(z)}, \bar{p}^{(z)}) \) satisfy the equations

\[
\frac{1}{2} \left( \frac{\partial}{\partial \rho^{(z)}} - \frac{\partial}{\partial \bar{\rho}^{(z)}} \right) \tilde{\psi}^{(c+k)} = (c+k) \tilde{\psi}^{(c+k)}.
\]

Now we can restore the dependence of the wave function (4.5) on the twistor variables. As a result we obtain the following statement.

**Proposition 2.** The twistor wave function which is a general solution of the equations of motion (4.3), (4.4) is represented in the form

\[
\Psi^{(c)}(\pi, \bar{\pi}, \rho, \bar{\rho}) = \delta((\pi\rho)(\bar{\pi}\bar{\rho}) - M^2) e^{-i(\rho_1/\bar{\rho}_1 + \bar{\rho}_1/\rho_1)} \bar{\Psi}^{(c)}(\pi, \bar{\pi}; \rho, \bar{\rho}),
\]

where we use the shorthand notation \( (\pi\rho) := \pi^\beta \rho_\beta, (\bar{\pi}\bar{\rho}) := \bar{\rho}_\beta \bar{\rho}^\beta, \) and

\[
\bar{\Psi}^{(c)}(\pi, \bar{\pi}; \rho, \bar{\rho}) = \psi^{(c)}(\pi, \bar{\pi}) + \sum_{k=1}^{\infty} (\pi\rho)^k \psi^{(c+k)} + \sum_{k=1}^{\infty} (\pi\rho)^k \bar{\psi}^{(c-k)}.
\]

The coefficient functions \( \psi^{(c\pm k)}(\pi, \bar{\pi}) \) obey the condition

\[
\Lambda \cdot \psi^{(c\pm k)}(\pi, \bar{\pi}) = -(c \pm k) \psi^{(c\pm k)}(\pi, \bar{\pi}),
\]

where

\[
\Lambda = -\frac{1}{2} \left( \frac{\partial}{\partial \alpha} - \frac{\partial}{\partial \bar{\alpha}} \right)
\]

is the helicity operator.

In view of condition (4.9), to describe the bosonic infinite spin representation related to all integer helicities, we put

\[
c = 0
\]

and therefore consider the twistor field \( \Psi^{(0)}(\pi, \bar{\pi}; \rho, \bar{\rho}) \). Note that the complex conjugate field \( \bar{\Psi}^{(0)} \) also has zero charge \( c = 0 \). Similarly, to describe the infinite spin representation related to half-integer helicities, we take for \( c \) the value

\[
c = -\frac{1}{2}.
\]

In view of condition (4.9), the corresponding wave function \( \Psi^{(-1/2)}(\pi, \bar{\pi}; \rho, \bar{\rho}) \) contains in its expansion only half-integer helicities. The complex conjugate field \( \bar{\Psi}^{(1/2)}(\pi, \bar{\pi}; \rho, \bar{\rho}) \) possesses the charge \( c = +1/2 \).

**Proposition 3.** The twistor wave function \( \Psi^{(c)}(\pi, \bar{\pi}; \rho, \bar{\rho}) \) defined in Proposition 2 describes the massless particles of infinite (continuous) spin:

\[
W^{\alpha\bar{\gamma}} W_{\alpha\bar{\gamma}} \cdot \psi^{(c)} = -\mu^2 \psi^{(c)},
\]

where \( W_{\alpha\bar{\gamma}} = (\sqrt{2})^{-1} W_{m}(\sigma^m)_{\alpha\bar{\gamma}} \) is the Pauli–Lubanski operator

\[
W_{\alpha\bar{\gamma}} = \pi_\alpha \bar{\pi}_\bar{\gamma} \Lambda + \frac{1}{2} \left[ \pi_\alpha \bar{\rho}_\bar{\gamma} \left( \bar{\pi}_\beta \frac{\partial}{\partial \rho_\beta} \right) - \rho_\alpha \bar{\pi}_\bar{\gamma} \left( \pi_\bar{\beta} \frac{\partial}{\partial \bar{\rho}_\bar{\beta}} \right) \right] + \frac{1}{2} \left[ (\bar{\rho}\bar{\pi})_{\alpha\beta} \frac{\partial}{\partial \rho_\beta} - (\pi\rho)_{\bar{\alpha}\bar{\beta}} \frac{\partial}{\partial \bar{\rho}_{\bar{\beta}}} \right].
\]
The expansion of the complex conjugate wave function where the component fields in the four-dimensional Minkowski space–time. Moreover, the quantity in the form \[ \Psi(c) \] in its expansion are also complex. Therefore, together with the field \( \Psi(c) \), we should consider its complex conjugate field \( (\Psi(c))^* := \overline{\Psi}(-c) \), which has the opposite charge \( c \rightarrow -c \).

5. TWISTOR TRANSFORM FOR INFINITE SPIN FIELDS

In this section, we establish a correspondence between twistor fields and fields defined in the four-dimensional Minkowski space–time.

For further convenience we introduce the dimensionless spinor

\[ \xi_\alpha := M^{-1/2} \rho_\alpha, \quad \overline{\xi}_\alpha := M^{-1/2} \overline{\rho}_\alpha. \]

Then, the twistor wave function \( \Psi(c) \) of infinite integer-spin particle (4.7) for \( c = 0 \) can be represented in the form [11]

\[
\Psi^{(0)}(\pi, \overline{\pi}; \xi, \overline{\xi}) = \delta((\pi, \xi) - \overline{M}) e^{-ip_0/p_0} \widehat{\Psi}^{(0)}(\pi, \overline{\pi}; \xi, \overline{\xi}),
\]

\[
\widehat{\Psi}^{(0)} = \psi^{(0)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\xi, \pi)^k \psi^{(k)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\pi, \xi)^k \psi^{(-k)}(\pi, \overline{\pi}).
\]

In the expansion of \( \widehat{\Psi}^{(0)} \), all components \( \psi^{(k)}(\pi, \overline{\pi}) (k \in \mathbb{Z}) \) are in general complex functions (fields). Moreover, the quantity \( p_0/g_0 \) is expressed by means of the generalized Cartan–Penrose representations (3.7) in the spinorial form as

\[
\frac{g_0}{p_0} = \frac{\sqrt{M} \sum_{\alpha=\beta} (\pi, \xi)^\alpha \overline{\xi} + (\xi, \pi)^\alpha \overline{\pi}}{\sum_{\beta=\gamma} \overline{\pi} \overline{\pi}}.
\]

In the case \( c = -1/2 \), the wave function of the infinite half-integer spin particle is

\[
\Psi^{(-1/2)}(\pi, \overline{\pi}; \xi, \overline{\xi}) = \delta((\pi, \xi) - \overline{M}) e^{-ip_0/p_0} \widehat{\Psi}^{(-1/2)}(\pi, \overline{\pi}; \xi, \overline{\xi}),
\]

\[
\widehat{\Psi}^{(-1/2)} = \psi^{(-1/2)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\xi, \pi)^k \psi^{(-1/2+k)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\pi, \xi)^k \psi^{(-1/2-k)}(\pi, \overline{\pi}).
\]

The expansion of the complex conjugate wave function \( \overline{\Psi}^{(+1/2)} \) has the form

\[
\overline{\Psi}^{(+1/2)}(\pi, \overline{\pi}; \xi, \overline{\xi}) = \delta((\pi, \xi) - M) e^{ip_0/p_0} \widehat{\overline{\Psi}}^{(+1/2)}(\pi, \overline{\pi}; \xi, \overline{\xi}),
\]

\[
\widehat{\overline{\Psi}}^{(+1/2)} = \overline{\psi}^{(+1/2)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\xi, \pi)^k \overline{\psi}^{(+1/2+k)}(\pi, \overline{\pi}) + \sum_{k=1}^{\infty} (\pi, \xi)^k \overline{\psi}^{(+1/2-k)}(\pi, \overline{\pi}),
\]

where the component fields \( \overline{\psi}^{(r)}(\pi, \overline{\pi}) \) are the complex conjugates of the component fields \( \psi^{(-r)}(\pi, \overline{\pi}) \):

\[
(\psi^{(-1/2+k)})^* = \overline{\psi}^{(+1/2-k)}, \quad k \in \mathbb{Z}.
\]
5.1. The case of integer spins. In this case the U(1)-charge is zero, \( c = 0 \), and the space–time wave function is determined by means of the integral Fourier transform of the twistor field \( \Psi^{(0)}(\pi, \bar{\pi}; \xi, \bar{\xi}) \):

\[
\Phi(x; \xi, \bar{\xi}) = \int d^4 \pi \, e^{i p_{\alpha a} x^{\alpha a}} \Psi^{(0)}(\pi, \bar{\pi}; \xi, \bar{\xi}) = \int d^4 \pi \, e^{i \pi_{\alpha \bar{\alpha}} x^{\alpha a}} \Psi^{(0)}(\pi, \bar{\pi}; \xi, \bar{\xi}),
\]

where we have used the representation \( p_{a\bar{a}} = \pi_{\alpha \bar{\alpha}} \) and the integration is performed with respect to the measure

\[
d^4 \pi := \frac{1}{2} d\pi_1 \wedge d\pi_2 \wedge d\bar{\pi}_1 \wedge d\bar{\pi}_2 = d\phi \, d^4 p \, \delta(p^2)
\]

(here \( \phi \) is the common phase in \( \pi_{\alpha} \) which is not present in \( p_{a\bar{a}} = \pi_{\alpha \bar{\alpha}} \)).

**Proposition 4.** The field \( \Phi(x; \xi, \bar{\xi}) \) defined by the integral transformation (5.4) in the coordinate representation satisfies the four equations

\[
\partial^{\alpha \bar{\alpha}} \partial_{a\bar{a}} \Phi(x; \xi, \bar{\xi}) = 0, \quad \left( i \frac{\partial}{\partial \xi_{\alpha}} \frac{\partial}{\partial \xi_{\bar{\alpha}}} - M \right) \Phi(x; \xi, \bar{\xi}) = 0,
\]

\[
(i \xi_{\alpha} \partial_{a \alpha} \bar{\xi}_{\bar{\alpha}} + M) \Phi(x; \xi, \bar{\xi}) = 0, \quad \left( \xi_{\alpha} \frac{\partial}{\partial \xi_{\alpha}} - \bar{\xi}_{\bar{\alpha}} \frac{\partial}{\partial \bar{\xi}_{\bar{\alpha}}} \right) \Phi(x; \xi, \bar{\xi}) = 0.
\]

**Proof.** Use the integral transformation (5.4) and the equations of motion (4.12) and (4.13) for \( c = 0 \). \( \square \)

5.2. The case of half-integer spins. In this case the U(1)-charge equals \( c = -1/2 \). Then we use the standard prescription of the twistorial definition of space–time fields with nonvanishing helicities. Namely, we have to insert the twistor spinor \( \pi_{\alpha} \) in the integrand in the Fourier transform,

\[
\Phi_{\alpha}(x; \xi, \bar{\xi}) = \int d^4 \pi \, e^{i p_{\beta \bar{\beta}} x^{\beta \bar{\beta}}} \pi_{\alpha} \Psi^{(-1/2)}(\pi, \bar{\pi}; \xi, \bar{\xi}) = \int d^4 \pi \, e^{i \pi_{\alpha \bar{\alpha}} x^{\alpha a}} \pi_{\alpha} \Psi^{(-1/2)}(\pi, \bar{\pi}; \xi, \bar{\xi}),
\]

and obtain the external spinor index \( \alpha \). Then the complex conjugate twistor field with charge \( c = +1/2 \) is defined analogously:

\[
\Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) = \int d^4 \pi \, e^{-i \pi_{\beta \bar{\beta}} x^{\beta \bar{\beta}}} \bar{\pi}_{\bar{\alpha}} \bar{\Psi}^{(1/2)}(\pi, \bar{\pi}; \xi, \bar{\xi}).
\]

**Proposition 5.** The space–time fields \( \Phi_{\alpha}(x; \xi, \bar{\xi}) \) and \( \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) \) corresponding to the states with half-integer helicities satisfy the massless Dirac–Weyl equations

\[
\partial^{\alpha \bar{\alpha}} \Phi_{\alpha}(x; \xi, \bar{\xi}) = 0, \quad \partial^{\alpha \bar{\alpha}} \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) = 0
\]

and the integer spin equations

\[
(i \xi_{\beta} \partial_{\beta \alpha} \bar{\xi}_{\bar{\beta}} + M) \Phi_{\alpha}(x; \xi, \bar{\xi}) = 0, \quad (i \xi_{\beta} \partial_{\beta \alpha} \bar{\xi}_{\bar{\beta}} - M) \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) = 0,
\]

\[
(i \frac{\partial}{\partial \xi_{\beta}} \frac{\partial}{\partial \xi_{\alpha}} - M) \Phi_{\alpha}(x; \xi, \bar{\xi}) = 0, \quad \left( i \frac{\partial}{\partial \xi_{\beta}} \frac{\partial}{\partial \bar{\xi}_{\alpha}} + M \right) \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) = 0,
\]

\[
\left( \xi_{\beta} \frac{\partial}{\partial \xi_{\beta}} - \bar{\xi}_{\bar{\beta}} \frac{\partial}{\partial \bar{\xi}_{\bar{\beta}}} \right) \Phi_{\alpha}(x; \xi, \bar{\xi}) = 0, \quad \left( \xi_{\beta} \frac{\partial}{\partial \xi_{\beta}} - \bar{\xi}_{\bar{\beta}} \frac{\partial}{\partial \bar{\xi}_{\bar{\beta}}} \right) \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) = 0.
\]

**Proof.** Use the integral transformations (5.6), (5.7) and the equations of motion (4.12), (4.13) for \( c = \pm 1/2 \). \( \square \)

We stress that although the twistor fields \( \Psi^{(-1/2)}(\pi, \bar{\pi}; \xi, \bar{\xi}) \) and \( \bar{\Psi}^{(+1/2)}(\pi, \bar{\pi}; \xi, \bar{\xi}) \) have nonvanishing charges \( c = \mp 1/2 \), their integral transforms \( \Phi_{\alpha}(x; \xi, \bar{\xi}) \) and \( \Phi_{\bar{\alpha}}(x; \xi, \bar{\xi}) \) have zero U(1)-charge. This fact is crucial for forming infinite spin supermultiplets, as we will see below.
6. INFINITE SPIN SUPERMULTIPLICL

We unify the fields $\Phi(x; \xi, \bar{\xi})$ and $\Phi_\alpha(x; \xi, \bar{\xi})$ with integer and half-integer helicities into one supermultiplet. The fields $\Phi(x; \xi, \bar{\xi})$ and $\Phi_\alpha(x; \xi, \bar{\xi})$ contain the bosonic $\psi^{(k)}(\pi, \bar{\pi})$ and fermionic $\psi^{(k-1/2)}(\pi, \bar{\pi})$ component fields ($k \in \mathbb{Z}$) with all integer and half-integer spins, respectively.

It is natural to expect that the individual components $\psi^{(k)}(\pi, \bar{\pi})$ and $\psi^{(k-1/2)}(\pi, \bar{\pi})$ of these fields should form an on-shell $\mathcal{N} = 1$ higher spin supermultiplet. Therefore, the bosonic (even) $\Phi(x; \xi, \bar{\xi})$ and fermionic (odd) $\Phi_\alpha(x; \xi, \bar{\xi})$ fields themselves should form an on-shell $\mathcal{N} = 1$ infinite spin supermultiplet containing an infinite number of conventional supermultiplets.

Similar to the Wess–Zumino supermultiplet (see, e.g., [17, 43]), we write the supersymmetry transformations of the fields $\Phi$ and $\Phi_\alpha$ in the form

$$\delta \Phi = \varepsilon^\alpha \Phi_\alpha, \quad \delta \Phi_\alpha = 2i\bar{\varepsilon}^\beta \partial_{\alpha\beta} \Phi,$$

(6.1)

where $\varepsilon_\alpha$ and $\bar{\varepsilon}_\dot{\alpha}$ are the constant odd Weyl spinors. The commutators of these transformations are

$$(\delta_1 \delta_2 - \delta_2 \delta_1) \Phi = -2ia^{\beta\dot{\beta}} \partial_{\beta\dot{\beta}} \Phi, \quad (\delta_1 \delta_2 - \delta_2 \delta_1) \Phi_\alpha = -2ia^{\beta\dot{\beta}} \partial_{\beta\dot{\beta}} \Phi_\alpha + 2ia^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \Phi_{\beta\dot{\beta}},$$

(6.2)

where $a_{\alpha\dot{\beta}} := \varepsilon_\alpha \bar{\varepsilon}_{2\dot{\beta}} - \varepsilon_{2\alpha} \bar{\varepsilon}_{\dot{\beta}}$. As we see, the superalgebra (6.2) is closed on-shell on the generator

$$P_{\beta\dot{\beta}} = -i\partial_{\beta\dot{\beta}}$$

due to the Dirac–Weyl equations of motion (5.8). Moreover, the whole system of equations of motion (5.5), (5.8), (5.9) is invariant with respect to the supersymmetry transformations (6.2).

Using the inverse integral Fourier transforms, we rewrite (6.2) as supersymmetry transformations for the twistor fields $\Psi(0)(\pi, \bar{\pi}; \xi, \bar{\xi})$ and $\Psi(-1/2)(\pi, \bar{\pi}; \xi, \bar{\xi})$ in the momentum representation:

$$\delta \Psi^{(0)} = \varepsilon^\alpha \pi_\alpha \Psi^{(-1/2)}, \quad \delta \Psi^{(-1/2)} = -2\bar{\varepsilon}^\dot{\alpha} \pi_{\dot{\alpha}} \Psi^{(0)}.$$

(6.3)

For the bosonic $\psi^{(k)}(\pi, \bar{\pi})$ and fermionic $\psi^{(-1/2+k)}(\pi, \bar{\pi})$ twistor components at all $k \in \mathbb{Z}$ we have

$$\delta \psi^{(k)} = \varepsilon^\alpha \pi_\alpha \psi^{(-1/2+k)}, \quad \delta \psi^{(-1/2+k)} = -2\bar{\varepsilon}^\dot{\alpha} \pi_{\dot{\alpha}} \psi^{(k)}.$$

(6.4)

The bosonic field $\psi^{(k)}$ and fermionic field $\psi^{(-1/2+k)}$ at fixed $k \in \mathbb{Z}$ describe massless states with helicities $(-k)$ and $(1/2 - k)$, respectively. Thus, the infinite-component supermultiplet of infinite spin stratifies into an infinite number of levels with pairs of the fields $\psi^{(k)}$, $\psi^{(-1/2+k)}$ at fixed $k \in \mathbb{Z}$. The supersymmetry transforms the bosonic and fermionic fields into each other inside a given level $k$. The boosts of the Poincaré group transform the levels with different $k$ and therefore mix the fields with different values of $k$.

Finally, we point out that a superfield description of an infinite spin supermultiplet was presented in the recent paper [13].

7. SUMMARY AND OUTLOOK

Let us summarize the obtained results.

1. We have presented a new twistor formulation of the model of massless infinite spin particles and fields.

2. We have derived the helicity decomposition of twistor infinite spin fields and constructed the field twistor transform to define the space–time infinite (continuous) spin fields $\Phi(x; \xi, \bar{\xi})$ and $\Phi_\alpha(x; \xi, \bar{\xi})$. 

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We have found the equations of motion for $\Phi(x; \xi, \bar{\xi})$ and $\Phi_\alpha(x; \xi, \bar{\xi})$ and showed that these fields form the $\mathcal{N} = 1$ infinite spin supermultiplet.

A natural question arises about the status of such fields in Lagrangian field theory as well as about the possibility to construct a self-consistent interaction of such fields. One of the commonly used methods for this purpose is the BRST approach, which was used in the case of continuous spin particles in [1, 2, 9, 16, 22, 23]. In the recent paper [16] the covariant Lagrangian formulation of the infinite integer-spin field was constructed by using the methods developed in [14, 15].

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