Fractional Hartley transform applied to optical image encryption

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Abstract. A new method for image encryption is introduced on the basis of two-dimensional (2-D) generalization of 1-D fractional Hartley transform that has been redefined recently in search of its inverse transform. We encrypt the image by two fractional orders and random phase codes. It has an advantage over Hartley transform, for its fractional orders can also be used as additional keys, and that, of course, strengthens image security. Only when all of these keys are correct, can the image be well decrypted. Computer simulations are also performed to confirm the possibility of proposed method.

1. Introduction
In recent years, with growing interests around the world, optical image encryption techniques have shown great potential in the field of optical information processing, and image encryption techniques have been proposed for optical image encryption by virtue of the arbitrary parameter selection, high computation speed, and extensive application. Some are done by Fourier transform (FT), Fresnel transform (FRT), or fractional Fourier transform (FRFT), which is the generalization of the FT, and compared with it, the fractional orders are the additional key when the original image is encrypted. Thus, we can add them in encryption for increasing degrees of freedom to make the decryption more secure. Some are base on Wavelet transform (WT) or fractional Wavelet transform (FWT). The latter has virtues of both FRFT and WT, and would strengthen the security by the key combination of fractional orders of FRFT and scaling factors of WT. Since Refregier and Javidi [1] proposed the double random phase encryption method in 1995, the method has been effectively and extensively used. Nowadays, most of these above-mentioned encryption systems often encrypt information with double random phase in the way of changing the information into noise-like pictures, and only when phase and others system parameters are correctly used in decryption, the image will be recovered originally. For instance, Unnikrishnan et al [2]. Proposed double random phase encoding technique in the FRFT domain. Situ and Zhang presented a novel encryption technique based on double random phase encoding in Fresnel by removing the lenses that are used in Fourier domain. Considering the fact that FT and WT have fractional forms (FRFT and FWT) with fractional orders that can improve the security, we expect a fractional form for Hartley transform and have been searching for it. Few researchers have presented fractional Hartley transform (FRHT) by summing the real and imaginary parts of the kernel of FRFT. Although FRHT defined in that way supplies a real output for a real input, it cannot satisfy the additive property.
More over, it does not have an inverse transform, and accordingly, we cannot recover the primary function directly, which restricts its applications. In this paper, we suggest a 2-D generalization of the 1-D FRHT, which has been redefined recently in another form by Pei and Ding. Then we propose a new optical encryption method based on it. It is different from those ever-proposed encryption methods. When the image is encrypted by this means, it takes efforts for FRFT or other transforms that we have known to recover the encryption image but in vain. Like many well-known transforms, it also has a number of potential applications. Compared with the conventional HT, it is superior in optical information processing, for FRHT can strengthen the information security to some extent with its additional fractional orders \( p_1 \) and \( p_2 \). Besides, with the new definition, the 2-D FRHT is a reversible transform analogous to FRFT. Although FRHT is not a real transform itself, it can be treated into special real form named fractional Hartley transform (SFRHT) with the exponent factor in the integral form of FRHT substituted by cosine factor in the relation \( \cos \varphi = \frac{\exp(i\varphi) + \exp(-i\varphi)}{2} \). For a real input, the output after the new transform SFRHT also turns out to be real and then it can be directly recorded on the intensity-only. Accordingly, the optical setup based on it is much simpler. It is possible that FRHT will become a useful information-processing tool in the future.

2. Fractional Fourier transform

The fractional Fourier transform in two-dimensional (2-D) is defined as [3]:

\[
\mathcal{F}^{p_1,p_2}(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_{p_1,p_2}(x, y; u, v) f(x, y) \, dx \, dy, \tag{1}
\]

with fractional kernel:

\[
B_{p_1,p_2}(x, y; u, v) = B_{p_1}(x, u) B_{p_2}(y, v), \tag{2}
\]

where:

\[
B_{p_1}(x, u) = \sqrt{1 - i \cot \varphi_1 2\pi} \exp \left[ \frac{i\pi \cot \varphi_1}{\lambda f} (y^2 + v^2) - 2\pi \frac{ux \csc \varphi_1}{\lambda f} \right], \tag{3}
\]

\[
B_{p_2}(y, v) = \sqrt{1 - i \cot \varphi_2 2\pi} \exp \left[ \frac{i\pi \cot \varphi_2}{\lambda f} (y^2 + v^2) - 2\pi \frac{vy \csc \varphi_2}{\lambda f} \right], \tag{4}
\]

\[
\varphi_1 = \frac{\pi}{p_1}; \varphi_2 = \frac{\pi}{2p_2}, \tag{5}
\]

where

\[
|\varphi_1| < \pi, |\varphi_2| < \pi. \tag{6}
\]

3. Fractional Hartley transform

Two-dimensional (2-D) fractional Hartley transform (HT) of a real function \( f(x, y) \) is defined as:

\[
H(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \cos [2\pi (ux + vy)] \, dx \, dy = \frac{\exp \left( \frac{i\pi}{4} \right)}{\sqrt{2}} \times \\
\left[ F(u, v) + \exp \left( -\frac{i\pi}{2} \right) F(-u, -v) \right] \tag{7}
\]
where \( \text{cas} = \cos + \sin \). Similar to the 1-D (Fractional Hartley transform, (FRHT) [4], the 2-D FRHT can be expressed as:

\[
H_{p_1,p_2}(u,v) = \frac{\sqrt{(1 - i \cot \phi_1)(1 - i \cot \phi_2)}}{2\pi} \times \exp \left[ i\pi \left( u^2 \cot \phi_1 + v^2 \cot \phi_2 \right) \right] \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ i\pi x^2 \cot \phi_1 \right] \exp \left[ i\pi y^2 \cot \phi_2 \right] \times \\
1 - i \exp \left[ \frac{i(\phi_1 + \phi_2)}{2} \right] \text{cas} \left( ux \csc \phi_1 + vy \csc \phi_2 \right) + \\
1 + i \exp \left[ \frac{i(\phi_1 + \phi_2)}{2} \right] \times \\
\text{cas} \left( -ux \csc \phi_1 - vy \csc \phi_2 \right) f(x,y) \, dx \, dy
\]

(8)

The expression (8) can be written as:

\[
H_{p_1,p_2}(u,v) = \frac{1 - \exp \left[ \frac{i(\phi_1 + \phi_2)}{2} \right]}{2} \mathcal{F}_{p_1,p_2}(u,v) + \\
\frac{1 + \exp \left[ \frac{i(\phi_1 + \phi_2)}{2} \right]}{2} \mathcal{F}_{p_1,p_2}(-u,-v)
\]

(9)

because FRHT can be defined in terms of the fractional Fourier transform, one can say that the FRHT meets all properties of fractional Fourier transform. The FRHT has a periodicity of 2 with respect to fractional orders.

4. **Mathematical formulation of encryption and decryption**

For increasing security we implement two phase masks to the image real encryption in conjunction with the implementation of the FRHT. The coordinates of the input image and the transformed plane are \( (x,y) \) and \( (U,y) \), respectively, and the distributions of the random phase mask are \( P(x,y) \) and \( q(u,y) \), [5]. Under these conditions, the complex distribution obtained after the transformation can be written as:

\[
g(u,v) = H_{p_1,p_2}[f(x,y)p(x,y)]
\]

(10)

The result in terms of transformation is multiplied by the second phase mask, arranged as follows:

\[
E(u,v) = q(u,v)g(u,v)
\]

(11)

Since FRHT has the additive property, it is reversible, with orders \(-p_1\) and \(-p_2\) is the inverse transform of 2-D FRHT with orders \(p_1\) and \(p_2\). In this case, the original image recovered after these two transformations, namely:

\[
f(x,y) = \|H^{-p_1-p_2}[E(u,v)q^*(u,v)]\|
\]

(12)

Where \(*\) is the conjugate operator.
5. **Computer simulation of the encryption and decryption technique**

The digital simulation encryption and decryption was implemented on the platform of Matlab® v.7.7, due to their facilities and high performance in handling matrices (images). The encrypted images are 8 bits grayscale images. The test image is in the fig. 1(a). After digitally implementing equations (9), (10) and (11), [6 – 7], it was possible to encrypt the input image, obtaining an image with a completely random distribution, as shown in fig. 1(b) and 1(c), the distribution of encrypted image intensities (both its magnitude and phase) varies by changing the keys (the fractional orders FRHT and random phase masks).

When performing the decryption process with the correct keys, we were able to recover the original image without any loss as shown in fig. 1(d).

![Input image](image1a.png) ![Encrypted image (Magnitude)](image1b.png) ![Encrypted image (Phase)](image1c.png) ![Output image](image1d.png)

**Figure 1:** Encryption and Decryption Technique, figure (a, d) taken from Matlab file exchange [8].

If the keys used in the decryption process are equal to the keys used in the encryption process, the image be recovered, as shown in the fig. 1(d); the reversibility of numerical algorithm of transform is important to the field of optical information processing, in this test when the phase mask is correct and the fractional orders in the process of FRHT decryption are correct with respect to the values of the encryption process, found that the recovered image not differs of the image original.

And finally, if the fractional orders are not correct but the second random phase mask used in the decryption process is equal to that used in the encryption process, the image not recover, as shown in fig. 2(a) and if the fractional orders are correct but the second random phase mask used in the decryption process is not equal to that used in the encryption process, the image
not recover, as shown in fig. 2(b)

![Output image](image1)

![Output image](image2)

Figure 2: Decoding technique with mask values and incorrect orders.

Images missing squared error (MSE, Mean Square Error) and Signal to Noise Ratio (SNR, Signalto Noise Ratio) between the input image and our image are deciphered calculated to validate the reliability of digital simulation:

\[
MSE = \frac{1}{M \cdot N} \sum_{x=1}^{M} \sum_{y=1}^{N} [I(x, y) - I'(x, y)]^2
\]

(13)

\[
SNR = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} [I(x, y)]^2}{\sum_{x=1}^{M} \sum_{y=1}^{N} [I(x, y) - I'(x, y)]^2}
\]

(14)

Where \( I(x, y) \) and \( I'(x, y) \) is the input image and our image decoded in the pixel \((x, y)\), respectively, and \( M \cdot N \) is the image size. The MSE between the test image (Fig. No. 1 (a)) and the decoded image correctly shown in Fig. No. 1 (d) is 0, the corresponding MSE between Fig. No. 2 (a) and the test image (Figure No 1 (a)) is 4018.6 and the MSE between Fig. No. 2 (b) and the image of testing is 6704.7. The SNR between the test image and the image correctly decoded shown in Fig. No. 1 (d) is \( \infty \), the corresponding SNR between Fig. No. 2 (a) and test image SNR is between 5.0297 and Fig. No. 2 (b) and test image is 3.0146.

6. Conclusions

The encryption and decryption of images using the 2-D fractional Hartley transform has been presented in this research article. Addition to the encryption methods based on fractional Fourier transform is encontratra useful technique for image encryption based onthe FRHT, which adds random phase mask and the fractional orders as the key encryption scheme proposed. Some results of computer simulations were given to verify the validity of the proposed encryption method and were also evaluated the MSE and the SNR of the recovered image and test image.

References

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