Weak Decays of Doubly-Heavy Tetraquarks $b\bar c q\bar q$

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We study the weak decays of exotic tetraquark states $b\bar c q\bar q$ with two heavy quarks. Under the SU(3) symmetry for light quarks, these tetraquarks can be classified into an octet plus a singlet: $3 \otimes \bar 3 = 1 \oplus 8$. We will concentrate on the octet tetraquarks with $J^P = 0^+$, and study their weak decays, both semileptonic and nonleptonic. Hadron-level effective Hamiltonian is constructed according to the irreducible representations of the SU(3) group. Expanding the Hamiltonian, we obtain the decay amplitudes parameterized in terms of a few irreducible quantities. Based on these amplitudes, relations for decay widths are derived, which can be tested in future. We also give a list of golden channels that can be used to look for these states at various colliders.

I. INTRODUCTION

Since the first discovery of X(3872) by Belle in 2003 [1], a large number of charmonium-like and bottomonium-like hadrons have been discovered in the past decade [2]. Many of these discovered states defy a standard quarkonium interpretation and likely have a pair of hidden flavored quarks, with the quark content $Q\bar Q q\bar q'$ (for a recent review, see Refs. [3, 4]). Here $Q$ represents a heavy bottom/charm quark and $q(q')$ denotes a light $u, d, s$ quark. Extensive theoretical studies have been carried out to explore their structures, productions and decays [5–30]. In 2016, the D0 collaboration has reported an evidence for the open-bottom tetraquark X(5568) [31], though it has not been confirmed by the other experimental groups [32–35]. Therefore the existence of open-flavored tetraquarks is an interesting question in hadronic physics, in particular the hadron spectroscopy.

Four-quark states with two different heavy quarks and two light quarks are of great interest since they can provide a unique platform to study strong interactions under two color static sources. In the diquark-antidiquark model [36], the four-quarks system $[bq][\bar c\bar q]$ with orbital angular momentum $L=0$ can have $J^P = 0^+$ [37]. Since the $0^+$ tetraquarks are lowest lying, their weak decays can provide unique insights to unravel their internal structure. In this paper, we adopt the SU(3) flavor symmetry to handle these weak decays. The SU(3) approach has been successfully applied into the B meson and heavy baryon decays [38–54] and a global picture consistent data has been established. In the SU(3) symmetry, the tetraquarks with two light quarks can form an octet and a singlet. In this work, we will concentrate on the octet, abbreviated as $X_{b\bar c8}$.

In the following, we will first construct the hadron-level effective Hamiltonian according to the irreducible representations of the SU(3) group. Expanding these Hamiltonian, we obtain the decay amplitudes parameterized in terms of a few SU(3) irreducible quantities. Based on the expanded amplitudes, relations for decay widths are derived, which can be examined in future. We also give a list of golden channels that can be used to look for these states at various colliders.

The rest of this paper is organized as follows. In Sec. II, we give the multiplets expressions under the SU(3) flavor symmetry. From Sec. III to Sec. IV, we mainly study the semi-leptonic and non-leptonic weak decays of the $X_{b\bar c8}$ states. In Sec. V, we will give a collection of the golden channels that can be used to discovery the doubly heavy tetraquarks in future experiments. we summarize in the last section.

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FIG. 1: Feynman diagrams for semileptonic decays of tetraquark $X_{bc\bar{s}}$. Panel (a) corresponds to the $b$ quark decay and panel (b) denotes the $\bar{c}$ quark decay.

II. PARTICLE MULTIPLETS

Based on the light flavor SU(3) symmetry, open-flavor tetraquark with the quark constituents $b\bar{c}q\bar{q}$ can form an octet and a singlet, of which the octet can be represented as

$$T_{bc\bar{s}} = \begin{pmatrix}
    T^{Bc}_{\pi^-} + \frac{T^{Bc}}{\sqrt{6}} \\
    T^{Bc}_{\pi^0} \\
    -\frac{T^{Bc}_{\pi^-}}{\sqrt{2}} - \frac{T^{Bc}_{\eta}}{\sqrt{6}} \\
    \frac{T^{Bc}_{\pi^0}}{\sqrt{2}} \\
    T^{Bc}_{K^-} \\
    -\frac{T^{Bc}_{K^0}}{\sqrt{2}} - \frac{T^{Bc}_{\eta}}{\sqrt{6}} \\
    -\frac{2}{\sqrt{6}} T^{Bc}_\eta
  \end{pmatrix}. \quad (1)$$

For simplicity, we will not consider the flavor singlet in this work. The decomposition can be reached by $3 \otimes \bar{3} = 1 \oplus 8$.

In the meson sector, light pseudoscalar mesons or vector mesons can also form an octet plus a singlet, generally, the octet is written as

$$M_8 = \begin{pmatrix}
    \pi^0 \\
    \pi^- \\
    -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \\
    \eta \\
    \pi^+ \\
    -\frac{\eta}{\sqrt{6}} \\
    -2\frac{\eta}{\sqrt{6}}
  \end{pmatrix}, \quad (2)$$

as the same quark content, vector meson octet will give the similar structure. Besides, we need the representation of bottom mesons which form an SU(3) anti-triplet given as: $B_i = \left( B^- , \bar{B}^0 , \bar{B}^\circ \right)$, and the representation of anti-triplet charmed mesons given as: $D_i = \left( D^0 , D^+, D^+_s \right)$, $\bar{D} = \left( \bar{D}^0 , D^- , D^-_s \right)$.

III. SEMI-LEPTONIC $T_{bc\bar{s}}$ DECAYS

In this section, we will discuss the possible semi-leptonic weak decay modes of the octet tetraquark $T_{bc\bar{s}}$. Considering the decay modes at quark level, $T_{bc\bar{s}}$ will hold both $b$-quark and $\bar{c}$-quark decays. For the $b$-quark, semi-leptonic weak decays are governed by

$$b \to c/\bar{u}\ell^- \bar{\nu}_\ell. \quad (3)$$

For the $\bar{c}$-quark, semi-leptonic decays are induced by

$$\bar{c} \to \bar{d}/\bar{s}\ell^- \bar{\nu}_\ell. \quad (4)$$

In the following, we will study the decays above in order.
TABLE I: Amplitudes for tetraquark $X_{bc8}$ decays into anti-charmed meson or a light meson.

| channel amplitude | channel amplitude |
|-------------------|-------------------|
| $X_{\pi}^{Bc} \rightarrow D^- l^- \bar{\nu}$ | $a_1 V_{ub}$ |
| $X_{Bc}^{K} \rightarrow D_s^- l^- \bar{\nu}$ | $a_1 V_{ub}$ |
| $X_{\pi}^{Bc} \rightarrow D^0 l^- \bar{\nu}$ | $a_1 V_{ub}$ |
| $X_{K}^{Bc} \rightarrow D^+ l^- \bar{\nu}$ | $a_1 V_{ub}$ |

| channel amplitude | channel amplitude |
|-------------------|-------------------|
| $X_{\pi}^{Bc} \rightarrow \pi^+ l^- \bar{\nu}$ | $a_2$ |
| $X_{K}^{Bc} \rightarrow \pi^0 l^- \bar{\nu}$ | $a_2$ |
| $X_{\pi}^{Bc} \rightarrow K^- l^- \bar{\nu}$ | $a_2$ |
| $X_{K}^{Bc} \rightarrow K^0 l^- \bar{\nu}$ | $a_2$ |
| $X_{\eta}^{Bc} \rightarrow \eta l^- \bar{\nu}$ | $a_2$ |

1. $b \rightarrow c/ul\,\nu \overline{\nu}$: decays into a meson and $\ell^-\bar{\nu}$

In $b$ quark decay, the general electro-weak Hamiltonian of the $b \rightarrow c/ul\,\nu \overline{\nu}$ transition can be expressed as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{q'b} q' \gamma^\mu (1 - \gamma_5) b \ell\gamma_\mu (1 - \gamma_5) \nu \ell \right] + \text{h.c.},$$

(5)

with $q' = u, c$, in which the electro-weak vertex is suggested to be a $V - A$ structure. As a contrast, the vertex forms a triplet representation $H_3'$ within SU(3) flavor symmetry, specifically $(H_3')^1 = 1$ and $(H_3')^{2,3} = 0$. At the hadron level, the transition can be included into the process that $X_{bc8}$ decays to a charmed meson and $\ell^-\bar{\nu}$. Following the SU(3) analysis, the Hamiltonian of hadronic level is constructed as

$$\mathcal{H}_{\text{eff}} = a_1 (T_{bc8})^j_3 (H_3')^j_3 D_1 \ell\nu,$$

(6)

here, the coefficient $a_1$ represents the non-perturbative parameter. For completeness, we give the corresponding Feynman diagram at quark level shown in Fig. 1(a). It is convenient to achieve the decay amplitudes given in Tab. I by expanding the Hamiltonian constructed above, in which all amplitudes are represented as $a_1$. Therefor, we can directly obtain the relations between different decay channels given as follows.

$$\Gamma(X_{\pi}^{Bc} \rightarrow D^- l^- \bar{\nu}) = 2\Gamma(X_{\pi}^{Bc} \rightarrow D^0 l^- \bar{\nu}) = \Gamma(X_{Bc}^{K} \rightarrow D_s^- l^- \bar{\nu}) = 6\Gamma(X_{\eta}^{Bc} \rightarrow D^0 l^- \bar{\nu}).$$

(8)

It is should be note that the phase space difference will provide corrections to these relations.

For the SU(3) singlet $b \rightarrow c$ transition, the process at the hadron level is that $X_{bc8}$ decays into a light meson octet and $\ell^-\bar{\nu}$. Consequently, the Hamiltonian at the hadron level is constructed as

$$\mathcal{H}_{\text{eff}} = a_2 (T_{bc8})^j_3 M_1^j \ell\nu.$$

(7)

One then obtain the amplitudes of different decay channels listed in Tab. I from which we derive that all the channels in the transition give the equal decay widths.

2. $\bar{c} \rightarrow \bar{d}/\bar{s}\ell^-\bar{\nu}$ decay into $B$ meson and $\ell^-\bar{\nu}$

In $\bar{c}$ quark decay, the electro-weak effective Hamiltonian are given as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cq} \bar{c} \gamma^\mu (1 - \gamma_5) q \ell\gamma_\mu (1 - \gamma_5) \nu \ell \right] + \text{h.c.},$$

(8)

where $q = d, s$, $V_{cd}$ and $V_{cs}$ are CKM matrix elements. Under the SU(3) symmetry, the $\bar{c} \rightarrow \bar{q}\ell^-\bar{\nu}$ transition can form an SU(3) triplet vertex, denoted as $H_3$ with $(H_3)^1 = 0$, $(H_3)^2 = V_{cd}$, $(H_3)^3 = V_{cs}$. Consistently, We construct the Hamiltonian at the hadron level as follows.

$$\mathcal{H}_{\text{eff}} = c_1 (T_{bc8})^j_3 (H_3)^j_3 B^\ell\ell\nu.$$

(9)
In B quark non-leptonic decays, the transitions can be classified into four different kinds in the light of CKM matrix:

\[ b \rightarrow c\bar{c}d/s, \quad b \rightarrow c\bar{d}u/s, \quad b \rightarrow u\bar{c}d/s, \quad b \rightarrow q_1\bar{q_2}q_3, \quad (10) \]

Here \( q_{1,2,3} \) represent the light quark.

In c̄ quark non-leptonic decays, the pronounced classifications are given as:

\[ \bar{c} \rightarrow \bar{s}d\bar{u}, \quad \bar{c} \rightarrow \bar{u}d\bar{d}/s\bar{s}, \quad \bar{c} \rightarrow \bar{d}s\bar{u}, \quad (11) \]

which are Cabibbo allowed, singly Cabibbo suppressed, and doubly Cabibbo suppressed respectively. In the following, we will study the \( X_{bcs} \) non-leptonic decays in these orders.
FIG. 3: Feynman diagrams for the $\bar{c}$-quark non-leptonic decays of tetraquark $X_{bc8}$.

The two-body processes are given in panels(a-h). The $\bar{c}$-quark decays have the similar structures with the $b$-quark decays. The panels(a-h) contribute to the process of $X_{bc8}$ decays into $B$ plus a light meson.

(I). \textit{b} $\rightarrow$ $c\bar{c}d/s$ transition: two-body decays into mesons

The operator of the $b \rightarrow c\bar{c}d/s$ transition can form an triplet under the SU(3) light quark symmetry, according to that we can write down the effective Hamiltonian of $X_{bc8}$ producing two mesons as follows.

$$H_{\text{eff}} = a_1(T_{bc8})^i_j (H_3)^j_l D_i J/\psi,$$

$$H_{\text{eff}} = a_2(T_{bc8})^i_j (H_3)^j_l D_k M^j_k + a_3(T_{bc8})^j_k (H_3)^k_l D_i M^i_l + a_4(T_{bc8})^j_k (H_3)^k_l D_i M^i_l,$$

with $(H_3)^2 = V_{cd}$ and $(H_3)^3 = V_{cs}$. Consistently, the corresponding Feynman diagrams are given in Fig. 2(a-d). In particular, the diagrams in Fig. 2(a,b) represent $X_{bc8}$ decays into $D$ and $J/\psi$ mesons, and the diagrams in Fig. 2(c,d) denote processes with $D$ and light mesons final states. Expanding the two Hamiltonian above, one obtains the decay amplitudes which are listed in Tab. [III]. In addition, the relations between the different decay widths are given as follows.

$$\Gamma(X_{\pi^-}^{B_c} \rightarrow K^- D^0) = \frac{1}{2} \Gamma(X_{\pi^-}^{B_c} \rightarrow K^- D^-) \equiv \Gamma(X_{\pi^-}^{B_c} \rightarrow K^0 D^-) = \frac{1}{2} \Gamma(X_{\pi^-}^{B_c} \rightarrow K^0 D^0),$$

$$\Gamma(X_{\pi^+}^{B_c} \rightarrow D^- D^0) = 2 \Gamma(X_{\pi^+}^{B_c} \rightarrow D^- D^-) = 2 \Gamma(X_{\pi^+}^{B_c} \rightarrow D^- D^0) = 2 \Gamma(X_{\pi^+}^{B_c} \rightarrow D^- D^0),$$

$$\Gamma(X_{K^0}^{B_c} \rightarrow \pi^- D^0) = 2 \Gamma(X_{K^0}^{B_c} \rightarrow \pi^- D^-) = 2 \Gamma(X_{K^0}^{B_c} \rightarrow \pi^- D^0) = 2 \Gamma(X_{K^0}^{B_c} \rightarrow \pi^- D^0),$$

$$\Gamma(X_{K^+}^{B_c} \rightarrow K^- D^0) = 2 \Gamma(X_{K^+}^{B_c} \rightarrow K^- D^-) = 2 \Gamma(X_{K^+}^{B_c} \rightarrow K^- D^0) = 2 \Gamma(X_{K^+}^{B_c} \rightarrow K^- D^0),$$

$$\Gamma(X_{K^0}^{B_c} \rightarrow \pi^0 D^-) = \Gamma(X_{K^0}^{B_c} \rightarrow \pi^0 D_s^-) = \Gamma(X_{K^0}^{B_c} \rightarrow \pi^0 D_s^-) = \Gamma(X_{K^0}^{B_c} \rightarrow \pi^0 D_s^-),$$

$$\Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^-) = \Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^-) = \Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^-) = \Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^-),$$

$$\Gamma(X_{K^0}^{B_c} \rightarrow \eta D^0) = \Gamma(X_{K^0}^{B_c} \rightarrow \eta D^-), \Gamma(X_{K^0}^{B_c} \rightarrow K^- D^0) = \Gamma(X_{K^0}^{B_c} \rightarrow K^- D^-),$$

$$\Gamma(X_{K^+}^{B_c} \rightarrow \pi^- D^-) = \Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^0), \Gamma(X_{K^+}^{B_c} \rightarrow \pi^+ D^-) = \Gamma(X_{K^+}^{B_c} \rightarrow K^0 D^-),$$

$$\Gamma(X_{\pi^0}^{B_c} \rightarrow \pi^0 D^-) = \Gamma(X_{\pi^0}^{B_c} \rightarrow \pi^0 D^-), \Gamma(X_{\pi^0}^{B_c} \rightarrow \pi^0 D^-) = \Gamma(X_{\pi^0}^{B_c} \rightarrow \pi^0 D^-),$$
TABLE III: Tetraquark $X_{bc}$ decays into anti-charmed meson plus light meson or anti-charmed meson plus $J/\psi$.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $X_{bc}^{0} \rightarrow \pi^{0} D^{−}$ | $(a_{3} + a_{4}) V_{cd}^{*}$ | $X_{bc}^{0} \rightarrow \pi^{−} D_{s}^{−}$ | $a_{5} V_{cd}^{*}$ |
| $X_{bc}^{+} \rightarrow K^{0} D^{−}$ | $a_{4} V_{cd}^{*}$ | $X_{bc}^{0} \rightarrow \pi^{0} D^{−}$ | $\frac{1}{2}(a_{2} + 2a_{3} + a_{4}) V_{cd}^{*}$ |
| $X_{bc}^{+} \rightarrow \pi^{0} D_s^{−}$ | $a_{3} V_{cd}^{*}$ | $X_{bc}^{0} \rightarrow \pi^{0} D^{−}$ | $(a_{4} - a_{2}) V_{cd}^{*}$ |
| $X_{bc}^{0} \rightarrow K^{0} D^{−}$ | $-\frac{a_{2} V_{cd}^{*}}{\sqrt{3}}$ | $X_{bc}^{0} \rightarrow K^{−} D^{−}$ | $-\frac{a_{4} V_{cd}^{*}}{\sqrt{3}}$ |
| $X_{bc}^{+} \rightarrow K^{0} \bar{D}^{*}$ | $\frac{a_{2} V_{cd}^{*}}{\sqrt{3}}$ | $X_{bc}^{+} \rightarrow K^{+} \bar{D}^{−}$ | $a_{3} V_{cd}^{*}$ |
| $X_{bc}^{+} \rightarrow \pi^{+} D^{−}$ | $\frac{(a_{2} - a_{3}) V_{cd}^{*}}{\sqrt{3}}$ | $X_{bc}^{+} \rightarrow K^{+} D_{s}^{−}$ | $a_{2} V_{cd}^{*}$ |
| $X_{bc}^{+} \rightarrow K^{0} \bar{D}^{*}$ | $a_{4} V_{cd}^{*}$ | $X_{bc}^{+} \rightarrow \eta D^{−}$ | $(a_{2} + a_{3}) V_{cd}^{*}$ |
| $X_{bc}^{0} \rightarrow \pi^{+} \bar{D}^{*}$ | $-\frac{a_{2} V_{cd}^{*}}{\sqrt{3}}$ | $X_{bc}^{0} \rightarrow \eta D^{−}$ | $a_{2} V_{cd}^{*}$ |
| $X_{bc}^{0} \rightarrow K^{0} \bar{D}^{*}$ | $\frac{(a_{2} - a_{3}) V_{cd}^{*}}{\sqrt{3}}$ | $X_{bc}^{0} \rightarrow \pi^{−} D_{s}^{−}$ | $a_{2} V_{cd}^{*}$ |
| $X_{bc}^{+} \rightarrow \eta D^{−}$ | $(a_{2} + a_{3}) V_{cd}^{*}$ | $X_{bc}^{+} \rightarrow \pi^{+} D^{−}$ | $a_{3} V_{cd}^{*}$ |

\[ \Gamma(X_{bc}^{0} \rightarrow K^{0} D^{−}) = \Gamma(X_{bc}^{0} \rightarrow K^{−} D^{−}), \Gamma(X_{bc}^{0} \rightarrow K^{+} D_{s}^{−}) = \Gamma(X_{bc}^{0} \rightarrow K^{0} D_{s}^{−}), \]
\[ 2\Gamma(X_{bc}^{0} \rightarrow D^{−} J/\psi) = \Gamma(X_{bc}^{0} \rightarrow \bar{D}^{0} J/\psi) = \Gamma(X_{bc}^{0} \rightarrow D_{s}^{−} J/\psi) = 6\Gamma(X_{bc}^{0} \rightarrow D^{−} J/\psi), \]
\[ \Gamma(X_{bc}^{0} \rightarrow D^{−} J/\psi) = \Gamma(X_{bc}^{0} \rightarrow \bar{D}^{0} J/\psi) = \frac{3}{2} \Gamma(X_{bc}^{0} \rightarrow D_{s}^{−} J/\psi). \]

(II). $b \rightarrow c\bar{u}d/s$ transition: two body decays into mesons

The operator of the $b \rightarrow c\bar{u}d/s$ transition can form an octet according to the SU(3) symmetry, of which nonzero entry $(H_{8})_{1}^{2} = V_{ud}^{*}$ for the $b \rightarrow c\bar{u}$ transition, and $(H_{8})_{3}^{3} = V_{us}^{*}$ for the $b \rightarrow c\bar{u}s$ transition. As usual, the hadron-level effective Hamiltonian can be constructed as

\[ \mathcal{H}_{eff} = a_{6}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} J/\psi + a_{7}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} J/\psi, \]
\[ \mathcal{H}_{eff} = a_{8}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} J/\psi + a_{9}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} J/\psi + a_{10}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} J/\psi, \]
\[ \mathcal{H}_{eff} = a_{11}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} M_{l}^{k} + a_{12}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} M_{l}^{k} + a_{13}(T_{bc8})^{1}_{1}(H_{8})_{1}^{k} M_{k}^{l} M_{l}^{k}. \]
The relations for producing two light mesons become

\[ \Gamma(X_{\pi^+}^{Bc} \rightarrow \pi^0 J/\psi) = \Gamma(X_{\pi^0}^{Bc} \rightarrow \pi^- J/\psi) = 2 \Gamma(X_{\pi^0}^{Bc} \rightarrow K^- J/\psi), \]

\[ \Gamma(X_{\pi^+}^{Bc} \rightarrow \bar{K}^0 J/\psi) = 2 \Gamma(X_{\pi^0}^{Bc} \rightarrow \bar{K}^- J/\psi), \]

\[ \Gamma(X_{K^+}^{Bc} \rightarrow \pi^0 J/\psi) = \frac{1}{2} \Gamma(X_{K^0}^{Bc} \rightarrow \pi^- J/\psi). \]

The relations for producing the charmed meson and anti-charmed meson become

\[ \Gamma(X_{\pi^+}^{Bc} \rightarrow D^0 D_s^-) = \frac{1}{2} \Gamma(X_{\pi^+}^{Bc} \rightarrow D^+ D_s^-). \]

The relations for producing two light mesons become

\[ \Gamma(X_{\pi^-}^{Bc} \rightarrow \pi^- \pi^-) = 2 \Gamma(X_{K^-}^{Bc} \rightarrow K^- \bar{K}^-) = 2 \Gamma(X_{K^0}^{Bc} \rightarrow K^0 \pi^-) = 2 \Gamma(X_{\pi^0}^{Bc} \rightarrow \pi^0 \pi^-), \]

\[ \Gamma(X_{\pi^+}^{Bc} \rightarrow \pi^- K^-) = \frac{1}{2} \Gamma(X_{K^-}^{Bc} \rightarrow K^- \bar{K}^-) = \Gamma(X_{K^0}^{Bc} \rightarrow \bar{K}^- K^-), \]

\[ \Gamma(X_{\pi^0}^{Bc} \rightarrow \pi^- \bar{K}^0) = 3 \Gamma(X_{\pi^+}^{Bc} \rightarrow \pi^0 \bar{K}^0). \]

The operator \((\bar{u}b)(\bar{q}c)\) can form an anti-symmetric 3 and a symmetric 6 representations. In the transition \(b \rightarrow uc\bar{s}\), the nonzero components of the anti-symmetric tensor \(H''_3\) and the symmetric tensor \(H_6\) are given respectively as

\[ (H''_3)^{13} = -(H''_3)^{31} = V_{cs}, \quad (H_6)^{13} = (H_6)^{31} = V_{cs}^* \]

In the transition \(b \rightarrow uc\bar{d}\), the nonzero components can be obtained by interchanging the subscripts 2 \(\leftrightarrow 3\), and replacing \(V_{cs}\) by \(V_{cd}\). Therefore, the effective Hamiltonian at the hadron level for \(X_{bcs}\) producing two mesons is constructed as

\[ \mathcal{H}_{eff} = a_{16}(T_{bcs})^i_j (H_3)^{ij} (D_i D_k) + a_{17}(T_{bcs})^i_j (H_6)^{ij} (D_i D_k). \]

Also the Feynman diagrams corresponding with the Hamiltonian above are given in Fig. 2(a-d). One then deduce the decay amplitudes for different channels shown in Tab. 1 which leads to the relations for decay widths as

\[ \Gamma(X_{\pi^+}^{Bc} \rightarrow D^- D_s^-) = \frac{1}{2} \Gamma(X_{K^-}^{Bc} \rightarrow D^- D_{s0}), \]

\[ \Gamma(X_{\pi^-}^{Bc} \rightarrow D^- D^-) = 2 \Gamma(X_{K^-}^{Bc} \rightarrow D^- D^-), \]

\[ \Gamma(X_{K^+}^{Bc} \rightarrow D^- D^0) = 2 \Gamma(X_{K^0}^{Bc} \rightarrow D^- D^0). \]
TABLE IV: Tetraquark $X_{bcar{a}}$ decays into $J/\psi$ plus light meson or charmed meson plus anti-charmed meson or two light mesons.

| Channel                              | Amplitude                                         | Channel                              | Amplitude                                         |
|--------------------------------------|--------------------------------------------------|--------------------------------------|--------------------------------------------------|
| $X_{bc0}^{J/\psi 0} \rightarrow \pi^+ J/\psi$ | $\frac{g_{bc0}^{J/\psi 0}}{\sqrt{2}}$           | $X_{bc0}^{K^- J/\psi}$              | $g_{bc0}^{K^- J/\psi}$                          |
| $X_{bc0}^{J/\psi 0} \rightarrow \pi^0 J/\psi$ | $\frac{g_{bc0}^{J/\psi 0}}{\sqrt{2}}$           | $X_{bc0}^{K^- J/\psi}$              | $g_{bc0}^{K^- J/\psi}$                          |
| $X_{bc0}^{J/\psi 0} \rightarrow \eta J/\psi$  | $\frac{g_{bc0}^{J/\psi 0}}{\sqrt{2}}$           | $X_{bc0}^{K^- J/\psi}$              | $g_{bc0}^{K^- J/\psi}$                          |
| $X_{bc0}^{J/\psi 0} \rightarrow \pi^- J/\psi$  | $\frac{g_{bc0}^{J/\psi 0}}{\sqrt{2}}$           | $X_{bc0}^{K^- J/\psi}$              | $g_{bc0}^{K^- J/\psi}$                          |
| $X_{bc0}^{J/\psi 0} \rightarrow K^0 J/\psi$   | $\frac{a_6 V_{us}^*}{\sqrt{2}}$                 | $X_{bc0}^{J/\psi 0} \rightarrow K^+ J/\psi$ | $g_{bc0}^{K^+ J/\psi}$                          |
| $X_{bc0}^{J/\psi 0} \rightarrow K^- J/\psi$   | $\frac{a_6 V_{us}^*}{\sqrt{2}}$                 | $X_{bc0}^{J/\psi 0} \rightarrow K^- J/\psi$ | $g_{bc0}^{K^- J/\psi}$                          |

| Channel                              | Amplitude                                         | Channel                              | Amplitude                                         |
|--------------------------------------|--------------------------------------------------|--------------------------------------|--------------------------------------------------|
| $X_{bc0}^{D^0 D^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         | $X_{bc0}^{D^0 D^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         |
| $X_{bc0}^{D^+ D^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         | $X_{bc0}^{D^+ D^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         |
| $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         | $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         |
| $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         | $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         |
| $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         | $X_{bc0}^{K^- K^-}$                  | $\frac{(a_8 + a_10) V_{ud}^*}{\sqrt{2}}$         |

(IV). Charless $b \rightarrow q_1 \bar{q}_2 q_3$ transition: two body decays into mesons

The charless tree level operator $(\bar{q}_1 b)(\bar{q}_2 q_3)$ ($q_i = d, s$) can be decomposed into a triple $H_3$, an antisymmetric sextet $H_6$ and a traceless symmetric $H_{15}$ in upper indices, while the charless penguin level operator behave as the triplet $H_3$. 
TABLE V: Tetraquark \( X_{bc8} \) decays into two anti-charmed mesons.

| Channel                      | Amplitude                        | Channel                      | Amplitude                        |
|------------------------------|----------------------------------|------------------------------|----------------------------------|
| \( X_{bc}^* \rightarrow D^- D^- \) | \( 2(a_{16} + a_{17}) V_{cd} \) | \( X_{bc}^* \rightarrow D^- D_s^- \) | \( (a_{16} + a_{17}) V_{cd} \) |
| \( X_{bc}^* \rightarrow D^- D_s^- \) | \( \sqrt{2}a_{16} V_{cd} \)     | \( X_{bc} \rightarrow D^- D_s^* \) | \( (a_{16} + a_{17}) V_{cd} \)   |
| \( X_{bc}^* \rightarrow D^- D^*_s \) | \( 2(a_{17} - a_{16}) V_{cd} \) | \( X_{bc}^* \rightarrow D^- D_s^- \) | \( (a_{16} + a_{17}) V_{cd} \) |
| \( X_{bc}^* \rightarrow D^- D_s^- \) | \( 2(a_{16} + a_{17}) V_{cd} \) | \( X_{bc}^* \rightarrow D^- D_s^* \) | \( (a_{16} - a_{17}) V_{cd} \) |

For the \( \Delta S = 0 \rightarrow d \) decays, the nonzero components of these irreducible tensors are given as

\[
(H_3)^2 = 1, \quad (H_{15})^2 = -(H_{15})^1 = \frac{1}{2}(H_{15})^2 = -(H_{15})^3 = 1,
\]
\[
2(H_{15})^1 = 2(H_{15})^2 = -3(H_{15})^1 = -6(H_{15})^2 = -6(H_{15})^3 = 6.
\] (15)

For the \( \Delta S = 1 \rightarrow s \) decays, the nonzero entries in the irreducible tensor \( H_3, H_{15}, H_{17} \) can be obtained from Eq. (15) with the exchange 2 \( \leftrightarrow 3 \). Accordingly, the hadron-level effective Hamiltonian for \( X_{bc8} \) decays into mesons is constructed as

\[
H_{eff} = b_1(T_{bc8})\mathcal{H}_1^k M_1^k + b_9(T_{bc8})\mathcal{H}_3 D_1 M_1^k + b_9(T_{bc8})\mathcal{H}_5 D_1 M_1^k + b_6(T_{bc8})\mathcal{H}_7 D_1 M_1^k
\]
\[
+ b_7(T_{bc8})\mathcal{H}_8 D_1 M_1^k + b_8(T_{bc8})\mathcal{H}_2 D_1 M_1^k + b_9(T_{bc8})\mathcal{H}_4 D_1 M_1^k
\]
\[
+ b_{10}(T_{bc8})\mathcal{H}_5 D_1 M_1^k + b_{11}(T_{bc8})\mathcal{H}_6 D_1 M_1^k + b_{12}(T_{bc8})\mathcal{H}_7 D_1 M_1^k
\]
\[
+ b_{13}(T_{bc8})\mathcal{H}_8 D_1 M_1^k.
\] (16)

In the two-body decays of the transition, the decay amplitudes are given in Tab. VII for the transition \( b \rightarrow d \) and Tab. VIII for the transition \( b \rightarrow s \). We obtain no direct relation of these decay widths.

(V). \( \bar{c} \rightarrow \bar{q}1q2q3 \) transition: two body decays into mesons

Under the flavor SU(3) symmetry, the operator \( \bar{c}q_1q_2q_3 \) transforms as \( \bar{3} \otimes 3 \otimes \bar{3} = \bar{3} \otimes \bar{3} \otimes \bar{3} \otimes [3 \otimes \bar{3}] \). Following the classifications mentioned before, the Cabibbo allowed transition to be \( \bar{c} \rightarrow \bar{s}d\bar{u} \), and the nonzero tensor components are given as

\[
(H_6)^2_{31} = -(H_6)^3_{13} = 1, \quad (H_{15})^2_{31} = (H_{15})^3_{13} = 1.
\] (17)

In the singly Cabibbo suppressed transition \( \bar{c} \rightarrow \bar{u}d\bar{d} \) and \( \bar{c} \rightarrow \bar{u}s\bar{s} \), the combination of tensor components are given as

\[
(H_6)^2_{31} = -(H_6)^3_{13} = (H_6)^2_{12} = -(H_6)^2_{21} = \sin(\theta_C),
\]
\[
(H_{15})^3_{31} = (H_{15})^2_{12} = -(H_{15})^2_{21} = \sin(\theta_C).
\] (18)

while for the doubly Cabibbo suppressed transition \( \bar{c} \rightarrow \bar{d}s\bar{u} \), we have

\[
(H_6)^3_{21} = -(H_6)^1_{12} = \sin^2(\theta_C), \quad (H_{15})^3_{21} = (H_{15})^3_{12} = \sin^2(\theta_C).
\] (19)

Therefore, it is convenient to construct the hadron-level effective Hamiltonian for \( X_{bc8} \) decays into mesons as

\[
H_{eff} = f_3(T_{bc8})\mathcal{H}_1^k M_1^k + f_4(T_{bc8})\mathcal{H}_3 D_1 M_1^k + f_5(T_{bc8})\mathcal{H}_7 D_1 M_1^k
\]
TABLE VII: Tetraquark $X_{bcs}$ decays into anti-charmed and light mesons induced by the charmless $b \to d$ transition.

| channel | amplitude |
|---------|-----------|
| $X_{bcs}^{Bc} \to D^- \pi^-$ | $b_4 + b_6 + b_9 + b_{10} - 2b_{12} + 3b_{13} + 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^0 \pi^- \frac{1}{2} \sqrt{2} (b_4 + b_5 + 2b_6 - b_7 - b_8 + b_9 + b_{10} - 5b_{11} - b_{12} - 5b_{13} - 3b_{14})$ |
| $X_{bcs}^{Bc} \to D^- \eta \frac{1}{3} \sqrt{3} \left( b_4 + b_5 + b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14} \right)$ |
| $X_{bcs}^{Bc} \to D^+ K^0 \frac{1}{3} \sqrt{3} \left( b_4 + b_5 + b_6 + b_8 + b_{10} - b_{12} - 3b_{14} \right)$ |
| $X_{bcs}^{Bc} \to D^0 K^0 \frac{1}{3} \sqrt{3} \left( b_4 + b_5 + b_6 + b_8 + b_{10} - 3b_{12} + 3b_{13} + 3b_{14} \right)$ |
| $X_{bcs}^{Bc} \to D^- K^+ \frac{1}{3} \sqrt{3} \left( b_9 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14} \right)$ |
| $X_{bcs}^{Bc} \to D^- \pi^0 \frac{1}{3} \sqrt{3} \left( b_4 + b_5 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14} \right)$ |

TABLE VIII: Tetraquark $X_{bcs}$ decays into anti-charmed and light mesons induced by the charmless $b \to s$ transition.

| channel | amplitude |
|---------|-----------|
| $X_{bcs}^{Bc} \to D^- K^- \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 + b_9 + b_{10} + b_{11} + 3b_{12} + 3b_{13}$ |
| $X_{bcs}^{Bc} \to D^- K^0 \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 + b_9 + b_{10} + 3b_{12} + 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^0 K^0 \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 + b_9 + b_{10} - 3b_{12} + 3b_{13} + 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^- \pi^- \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^- \eta \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^- \pi^0 \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14}$ |
| $X_{bcs}^{Bc} \to D^- \pi^0 \frac{1}{4} \sqrt{2} b_4 + b_5 + b_6 - b_7 - b_8 + 3b_9 + 3b_{10} + 5b_{11} + 3b_{12} - 3b_{13} - 3b_{14}$ |

\[
\begin{align*}
\Gamma(X_{bcs}^{Bc} \to D^- \pi^-) & = \Gamma(X_{K^-}^{Bc} \to B^- K^-), \\
\Gamma(X_{bcs}^{Bc} \to B^- \pi^0) & = \Gamma(X_{\eta}^{Bc} \to B^- \eta), \\
\end{align*}
\]

As usual, the corresponding Feynman diagrams are given in Fig. 3. Expanding the Hamiltonian above to obtain decay amplitudes, and listed in Tab. VIII One deduces the relations between different decay widths given as

\[
\Gamma(X_{bcs}^{Bc} \to D^- \pi^-) = \Gamma(X_{K^-}^{Bc} \to B^- K^-), \Gamma(X_{bcs}^{Bc} \to B^- \pi^0) = \Gamma(X_{\eta}^{Bc} \to B^- \eta),
\]
TABLE VIII: Tetraquark $X_{bc8}$ decays into a B meson and light meson.

| channel                        | amplitude            | channel                        | amplitude            |
|--------------------------------|----------------------|--------------------------------|----------------------|
| $X_{c8}^{Bc} \rightarrow B^- K^0$ | $-3f_5 + f_6 - f_7 + f_10$ | $X_{c8}^{Bc} \rightarrow B^- K^0$ | $-3f_5 + f_6 - f_7 + f_10$ |
| $X_{c8}^{Bc} \rightarrow B^- K^+$ | $-3f_5 + f_6 + f_7 + f_10$ | $X_{c8}^{Bc} \rightarrow B^- K^0$ | $-3f_5 + f_6 + f_7 + f_10$ |
| $X_{c8}^{Bc} \rightarrow B^- K^0$ | $2f_5 + f_6 + f_7 + f_10$ | $X_{c8}^{Bc} \rightarrow B^- K^0$ | $2f_5 + f_6 + f_7 + f_10$ |
| $X_{c8}^{Bc} \rightarrow B^- \pi^-$ | $f_4 + f_5 + f_7 + f_9$ | $X_{c8}^{Bc} \rightarrow B^- \pi^0$ | $f_4 + f_5 + f_7 + f_9$ |
| $X_{c8}^{Bc} \rightarrow B^- \eta$ | $f_4 + f_5 + f_7 + f_9$ | $X_{c8}^{Bc} \rightarrow B^- \eta$ | $f_4 + f_5 + f_7 + f_9$ |
| $X_{c8}^{Bc} \rightarrow B^- K^+$ | $f_4 + f_5 + f_7 + f_9$ | $X_{c8}^{Bc} \rightarrow B^- K^0$ | $f_4 + f_5 + f_7 + f_9$ |
| $X_{c8}^{Bc} \rightarrow B^- K^0$ | $f_4 + f_5 + f_7 + f_9$ | $X_{c8}^{Bc} \rightarrow B^- K^0$ | $f_4 + f_5 + f_7 + f_9$ |

\[
\Gamma(X_{c8}^{Bc} \rightarrow B^- \pi^+) = \Gamma(X_{K^0}^{Bc} \rightarrow B^- K^+) = \Gamma(X_{c8}^{Bc} \rightarrow B^- K^0) = \Gamma(X_{K^0}^{Bc} \rightarrow B^- K^0),
\]
\[
\Gamma(X_{c8}^{Bc} \rightarrow B^- K^0) = \Gamma(X_{B^-}^{Bc} \rightarrow B^- K^0) = \Gamma(X_{K^0}^{Bc} \rightarrow B^- K^0) = \Gamma(X_{B^-}^{Bc} \rightarrow B^- K^0).
\]

V. GOLDEN DECAY CHANNELS

In this section, we will discuss the golden channels to reconstruct the $X_{bc8}$ and give an estimate of the decay branching fractions. In our analysis given in the previous sections, the final meson can be replaced by its corresponding counterpart with the same quark constituent but different quantum numbers. For instance, one may replace $\mathbf{B}^-$ by $\mathbf{K}^0$.

Golden decay channels must satisfy the following criteria.

- Branching fractions: For charm quark decays, one should use the Cabibbo allowed decay modes, while for bottom quark, the quark level transition $b \rightarrow c \bar{d}$ or $b \rightarrow c \bar{e}s$ gives the largest branching fractions.

- Detection efficiency: At hadron colliders like LHC, charged particles have better chances to be detected than...
The nonperturbative effects are parametrized into a few quantities (order $10^{3}$ $X_{\psi}$ $V$).

Some comments are appropriate to our SU(3) analysis. The typical branching fraction for charm quark decay in $X_{b\bar{s}\bar{s}}$ modes with $b$ or even smaller, due to the weak decay of bottom meson is needed. So the branching fraction for the decay $B_{c}^{+} \rightarrow K^{+} \pi^{-}$, $X_{\psi} \rightarrow \pi^{0}$, $\eta$, $\phi$, $\rho^{\pm}(\rightarrow \pi^{\pm}\pi^{0})$, $K^{*\pm}(\rightarrow K^{\pm}\pi^{0})$ and $\omega$, but keep the modes with $\pi^{\pm}, K^{0}(\rightarrow \pi^{+}\pi^{-}), \rho^{0}(\rightarrow \pi^{+}\pi^{-})$.

The two-body decay modes that can be used to reconstruct the $X_{b\bar{s}\bar{s}}$ are collected in Tab. IX and Tab. X.

Some comments are appropriate to our SU(3) analysis. The typical branching fraction for charm quark decay in Tab. IX is at a few percent level. On the experiment side, to construct the bottom meson, another factor, at the order $10^{-3}$ or even smaller, due to the weak decay of bottom meson is needed. So the branching fraction for the decay chains to reconstruct the $X_{b\bar{s}\bar{s}}$ might reach the order $10^{-5}$, or smaller.

If the $b$ quark decay first in $X_{b\bar{s}\bar{s}}$, the typical branching fraction is at the order $10^{-3}$. The final states which include the $J/\psi$ or $D$ meson such as $X_{bc}^{D} \rightarrow K^{-}D^{+}$ would introduce a factor $10^{-3}$ to reconstruct. So the branching fraction of these channels may also reach the order $10^{-5}$.

The channels with two light mesons such as $X_{bc}^{D} \rightarrow \pi^{-}\pi^{-}$ require the annihilation of the two heavy quarks. But since the CKM matrix element $V_{cb}$, these channels might have sizable decay branching fractions.

### VI. CONCLUSIONS

Tetraquarks with the quark content $[bq][\bar{c}\bar{q}]$ are of great research interest but have not been discovered yet. In this paper, we have systematically studied the weak decays of the doubly-heavy tetraquarks $X_{b\bar{s}\bar{s}}$ under SU(3) flavor symmetry, which include the semileptonic and nonleptonic $b$ and $\bar{c}$ quark decays. The $\bar{c}$ quark decays are dominant, of which the typical branching fraction is at a few percent level.

Using the building blocks in SU(3), we construct the effective Hamiltonian at the hadron level for their weak decays. The nonperturbative effects are parametrized into a few quantities $(a_{i}, b_{j}, ...)$. Therefore, one can easily derive the
decay amplitudes, based on which relations between different channels can be obtained. Finally, we give a list of the golden channels which is useful to search for the $X_{bcs}$ state tetraquarks in future experiments.

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