Electroreflectance investigations of quantum confined Stark effect in GaN quantum wells

A Drabinska, K Pakula, J M Baranowski and A Wysmolek

Institute of Experimental Physics, University of Warsaw, Hoza 69, Warsaw, Poland
e-mail: Aneta.Drabinska@fuw.edu.pl

Abstract. In this paper we present room temperature electroreflectance studies of GaN quantum wells (QWs) with different well width. The electroreflectance measurements were performed with external voltage applied to the structure therefore it was possible to tune the electric field inside QW up to its completely screening and furthermore even reversing it. The analysis of QW spectral lines showed the Stark shift dependence on applied voltage and well width reaching about 35 meV for highest voltage and widest well width. It was possible to obtain the condition of zero electric field in QW. Both broadening and amplitude of QW lines are minimal for zero electric field and increases for increasing electric field in QW. The energy transition is maximum for zero electric field and for increasing electric field it decreases due to Stark effect. Neither amplitude and broadening parameter nor energy transition does not depend on the direction of electric field. Only parameter that depends on the direction of electric field in QW is phase of the signal. The analysis of Franz-Keldysh oscillations (FKOs) from AlGaN barriers allowed to calculate the real electric field dependence on applied voltage and therefore to obtain the Stark shift dependence on electric field. The Stark shift reached from -12 meV to -35 meV for 450 kV/cm depending on the well width. This conditions were established for highest forward voltages therefore this is the value of electric field and Stark shift caused only by the intrinsic polarization of nitrides.

1. Introduction

Due to potential application for many optoelectronic systems [1-4], nitrides for several years are under advanced studies focused on the development and improvement of their optical properties. It has been predicted by theory and confirmed by experiment that nitrides in their wurtzite crystal structure are pyroelectric materials [5-8]. Large values of spontaneous polarization which are present without any external electric field (0.034 Cm⁻² for GaN and 0.09 Cm⁻² for AlN) are characteristic for these crystals. The orientation of this spontaneous polarization is always along the [000-1] direction and cannot be inverted by external electric fields. The orientation of the spontaneous polarization relative to the c-axis can only be controlled by adjusting the polarity of the material. The piezoelectric constants of group III nitrides are a factor of 5–20 larger than those of InAs, GaAs and AlAs. The macroscopic nonlinear pyroelectric polarization of wurtzite nitrides (both spontaneous and piezoelectric polarization) dramatically affects the optical and electrical properties of multilayered AlGaN/GaN hetero-, nanostructures and devices, due to the huge built-in electrostatic fields and bound interface charges caused by gradients in polarization at surfaces and heterointerfaces. The realization of undoped AlGaN/GaN heterostructures leads to the existence of a gradient in internal polarization and therefore it is relatively easy to obtain two-dimensional electron gases (2DEGs) in GaN/AlGaN structures [9]. Observations of Franz–Keldysh oscillations and Stark effects in InGaN/GaN and
AlGaN/GaN QW structures provide direct evidence for the presence of strong electric fields caused by gradients in polarization [10-13]. Controlling the electric field in QWs by external bias is crucial for the investigation of optical properties of quantum wells. Therefore the principle of these properties is the change of the absorption coefficient as a function of an applied voltage [12, 13]. In the case of quasi-two-dimensional semiconductor structures, i.e., quantum wells, the corresponding phenomenon is the quantum-confined Stark effect (QCSE) [15]. Although both effects have been extensively studied in GaAs-based structures [16], much less work has been done using GaN-based alloys.

2. Samples
The investigated structures were grown on the sapphire substrate in a horizontal, low pressure Metal-Organic Vapor Phase Epitaxy (MOVPE) reactor. On the 20 nm nucleation layer there were grown sequentially: Al$_{0.09}$Ga$_{0.91}$N layer and highly silicon doped Al$_{0.09}$Ga$_{0.91}$N layer, both 600 nm thickness. Next on the n-type layer there were grown: 10 nm back Al$_{0.09}$Ga$_{0.91}$N barrier, GaN QW and 20 nm Al$_{0.09}$Ga$_{0.91}$N top barrier. The structures were covered with very thin cap layer of low-temperature GaN. We present measurements of three structures with different QW thickness: 3 nm, 6 nm and 9 nm, marked in this paper as sample A, B and C respectively. The nominal concentration of dopants was $2 \times 10^{18}$ cm$^{-3}$ for narrower QWs and $2 \times 10^{19}$ cm$^{-3}$ for widest QW.

In order to decrease the number of screw dislocations, ensure high resistivity of undoped layers and ensure high quality and flatness of interfaces, two of the structures were grown on modified low-temperature AlN nucleation layer. It has been shown that the 3D or 2D growth mode of AlGaN layers depends predominantly on the growth conditions of the low temperature (LT) nucleation layer [17]. Commonly used 3D growth mode is achieved on LT GaN or AlN nucleation layer. Growth of a successive layer begins from separated sites, where individual 3D crystallites are formed. Threading dislocations present in crystallites bend on their facets, which reduces the quantity of dislocations. However, slight crystallographic misorientations between crystallites lead to the creation of new dislocations during coalescence of the crystallites. As a result, edge and mix dislocations appear at similar densities of about $10^9$ cm$^{-2}$. Modification of growth conditions of LT AlN nucleation layer, especially reduction of their growth rate, leads to drastic changes in properties of the layer. Growth of successive AlGaN layer at high temperature starts evenly on whole surface retaining atomic flatness. Thus growth at high temperature occurs only by 2D mode. Therefore, it is possible to grow a very thin AlGaN layers directly on top of LT nucleation layer. Such layers contain large number ($10^{10}$ cm$^{-2}$) of edge dislocations, and relatively small number (less than $10^8$ cm$^{-2}$) of mix dislocations. The widest QW was grown on the standard GaN buffer.

3. Experimental technique
Modulation spectroscopy is the branch of optical spectroscopy that deals with the measurement and interpretation of changes in optical spectra caused by modification of measurements condition. The general concept of modulation spectroscopy is that an optical spectrum that contains of many broad lines can be reduced to a series of derivative-like lines [18-21]. In this work we present only one particular modulation i.e. electric field modulation. In the case of electro-modulation reflectivity (EMR) the measured parameter is a periodic variation in reflectivity caused by small periodic changes of electric field in the sample. The benefits of this technique is that it gives very sharp, sensitive to build-in electric field, lines even at room temperature.

Electroreflectance (ER) is one of the method of implementing EMR. It requires at least two contacts on the structure and modulation voltage is applied between them so the internal electric field is modulated. In this paper we used Schottky barrier electroreflectance (SBER), in which one of the contacts is a semitransparent gold Schottky contact evaporated on the structure surface. Melting indium into a side of the sample made ohmic contacts to the n-doped Al$_{0.09}$Ga$_{0.91}$N. In this case the electric field parallel to the growth direction is modulated. The ER is measured in the depleted region which extends from the surface through the QW and barriers to the n-doped Al$_{0.09}$Ga$_{0.91}$N.
The reflectivity was measured with a monochromatic light obtained from a Xenon lamp. The light of intensity $I_0$ was focused on the semitransparent Schottky electrode and the reflected light was detected by silicon detector. The intensity signal consists of dc value $I_0 R(E)$ and ac value $I_0 \Delta R(E)$, measured by a voltmeter and lock-in amplifier synchronized with the modulation source, respectively. Dividing both signals eliminates dependence on the intensity of the light source, and yields to the quantity $\Delta R/R(E)$. Except the modulation voltage it is also possible to apply constant voltage in this method. This allows to change the electric field in depletion layer and furthermore to measure the dependence of optical properties of QW and barriers on electric field.

4. Results and discussion
The electroreflectance maps (figure 1) were measured at room temperature for energy range 3.3 – 3.8 eV for sample A and 3.3 – 4.0 eV for samples B and C and voltage range -1.0 – 0.7 V, -1.5 – 0.8 V and -1.5 – 1.0 V for samples A, B and C respectively. For energies higher than 3.6 eV for all samples characteristic vanishing oscillations are clearly observed. These are Franz-Keldysh oscillations from AlGaN barriers. For all three samples the period of this oscillations increases with increasing reverse bias. For energies lower than 3.6 eV transitions in QW are observed. The position of these lines changes with applied voltage. This lines vanishes from ER spectrum for voltage -1.1 and -0.7 V for sample B and C respectively.

![Figure 1. Electroreflectance signal as a function of energy and applied voltage of sample A – (a), sample B – (b) and sample C – (c).](image)

4.1. Fitting procedure
The perturbation, due to the modulation of electric field, destroys the translational symmetry and hence the free charge carriers in the crystal can be accelerated. The lineshape of ER signal from layers can be expressed as a function of broadening parameter ($\Gamma$), transition energy ($E_g$), electric field ($F$) and effective mass in field direction ($\mu_||$) using following formula [22, 23]:

\[
\frac{\Delta R}{R} = \text{Re} \left[ A e^{i\phi} \frac{H(z)}{(E-i\Gamma)^2} \right],
\]  

(1)
where \( H(z) = 2\pi e^{i\pi/3} A_i(z) Ai'[z e^{-2i\pi/3}] + z e^{-2i\pi/3} Ai(z) Ai[z e^{-2i\pi/3}] + i\sqrt{z} \),
\[ z = \frac{E_e - E + i\Gamma}{\hbar \theta} \quad \text{and} \quad \left( \frac{\hbar \theta}{2\mu} \right)^2 = \frac{(eFh)^2}{2\mu} . \]

For carriers in bound states, like quantum wells, the effective mass along the field direction is infinitive and electro-optic energy \((\hbar \theta)\) vanishes. These types of particles are confined in space and the electric field is not able to accelerate them. In this case the effect of an electric field applied along direction of confinement is very different that it was described previously. For example, considering the square quantum well, the effect of the electric field along the confinement direction adds a linear potential, which tilts the quantum well and the barriers, thus changing the shape of the potential well. The wave functions are mixed in such a way, that electrons and holes are shifted in opposite directions, but they are still confined. In addition, their energy of the state is changed as well. In high electric field the quantum well may be perturbed in a way that tunneling may take place. In this case, the measured ER signal hardly depends on electric field and can be approximated by the formula:

\[ \frac{\Delta R}{R} = \text{Re} \left[ A e^{i\phi} \left( E - E_e + i\Gamma \right)^2 \right] \]

(2)

Figure 2. Fit of electroreflectance signal as a function of energy and applied voltage of sample A – (a), sample B – (b) and sample C – (c). QW lines were fitted using equation (2) and FKO using equation (1). Fitting procedure used one, two and three QW spectral lines for samples A, B and C respectively.

For all three samples for each voltage there was fitted theoretical lineshapes (from one to three QW lines for energy lower than 3.6 eV and FKOs for energies higher 3.6 eV). Since the lineshape of each transition is complicated, depends on several parameters (4 for QW transitions and 5 for transition in barriers) and individual lines overlap with each other, the fitting procedure is tricky. First, fitting was performed separately for FKO and QW. For samples B and C, were there was more than one QW lines, fit was performed step by step, adding individual QW lines starting from the lowest energies. Finally with well established initial parameters, there was performed the final fit of spectrum for each voltage. The results of the fitting are presented in figure 2.
4.2. QW lines

Due to the spontaneous and piezoelectric polarization positive and negative polarization charge is present on both AlGaN/GaN interfaces. Although the direction of the electric field in the outer barrier is established by the surfaces states the internal electric field in QW is opposite to the one in the inner barrier and to the electric field build by the applied bias. Although internal polarization can not be changed by applying external bias, it is possible to add external electric field that can compensate this internal electric field. Furthermore by applying high external bias it is possible to obtain the net electric field opposite to the polarization induced field.

For low reverse bias the electric field in QW is opposite to the one in barriers (figure 3a), therefore QW lines shift into blue with increasing reverse bias and reaches maximum for bias -1.0 V and -0.7 V for samples B and C respectively (figure 1b and c). For sample A it is not possible to establish voltage for maximum QW transition energy, since it was not reached in experiment (figure 1a), but it is greater than maximum reverse voltage (-1.0 V). This voltage is sufficient enough to completely compensate electric field in QW (figure 3b) The decrease of screening voltage with increasing well width is due to increasing depletion layer width and different voltage-electric field dependence for each sample. For zero electric field, for samples B and C the electro-reflectance signal from QWs vanishes. Higher reverse biases builds opposite electric field to the QWs so the lines appear again in spectrum but with opposite phase. Increasing reverse bias increases electric field in the QW therefore lines shift back into red (figure 1b and c). This time electric field is in the same direction than the one in barriers (figure 3c).

Figure 3. The schematic band structure for III-nitride QW (considering its pyroelectricity i.e. spontaneous and piezoelectric polarization) without external electric field (a) and in external electric field opposite to the pyroelectric field (b) and (c). (b) presents the case when external electric field has the same value that fields induced by pyroelectric polarization therefore the net electric field in QW is zero.

For all measured samples for energies below 3.6 V, using equation (2), there were fitted QW lineshapes. Fitting one QW transition for sample A, two for sample B and three for sample C gave the best fitting results. The fitted energy positions, broadening parameters and amplitude of the sample B are presented in figure 4 a, b and c respectively. For screening voltage (zero electric field in QW), for both transitions, the amplitude and broadening parameter are minimal. Increasing electric field in barriers causes an increase of both: the amplitude and broadening parameter of fitted lines. During the change of direction of electric field in QW the modulation of voltage has not changed. This results in the inversion of phase of ER signal after the inversion of the electric field. For sample A and C the dependence is very similar. Square dependence for each QW transition was fitted to obtain the value of screening voltage ($U_0$) and the energy transition ($E_0$) for zero electric field for B and C samples. Fitted screening voltage are -1.0 V and -0.7 V for sample B and C respectively. Transition energies for sample B are 3.432 eV, 3.465 V and for sample C – 3.413 eV, 3.441 eV, 3.482 eV. For sample A in ER measurement the conditions of screening electric field wasn’t reached therefore we can only say that screening voltage is over -1 V of reverse voltage and transition energy is larger than 3.464 eV. However the energy dependence on bias suggests that these parameters are very close to the “zero electric field” case.
To obtain the Stark shift of each transition there was subtracted energy transition for “zero electric field” ($E_0$) from the fitted energy transition for each voltage. Since the value of Stark shift does not depend on the direction of electric field the Stark shift was presented (figure 5) as a function of absolute value of the difference between the actual voltage and screening voltage ($U_0$). The value of Stark shift depends on the well width and for 1.6 V reaches -14 meV, -20 meV and -35 meV for sample A, B and C respectively.

Figure 5. Stark shift dependence on voltage applied to QW for sample A (black squares), B (red circles) and C (blue triangles).

4.3. Franz-Keldysh oscillations from barriers
Although the exact expression describing Franz-Keldysh oscillations is complicated (equation (1)) it is relatively easy to evaluate the value of electric field without full analyze of the spectrum. Due to periodic dependence of ER signal as a function of $(E-E_g)^{3/2}$ it is possible to calculate the electric field ($F$) from the period ($T$) of FKOs using the following expression:
\[ T = \frac{3\pi eFh}{\sqrt{8\mu_0}} \]

Unfortunately for the higher reverse voltage amplitude of FKOs from barriers is very small and only two-three oscillations are observed, therefore obtained value of electric field is not accurate. However it is clearly visible for all samples that the electric field in barriers, the same as the period of oscillations, increases with increasing reverse voltage. Analysis of FKOs in all samples showed that electric field increases linearly with the applied voltage. Linear dependence of electric field on voltage can be understood assuming the constant width of depletion layer. The best fit to the formula:

\[ F = -\frac{U - U_B}{d}, \]

where \( U_B \) is Schottky barrier height and \( d \) is depletion layer width, gives the depletion layer width 33 nm, 36 nm and 39 nm for samples A, B and C respectively. This agrees with nominal widths of depletion layer for each sample. Obtained Schottky barrier height was equal 1.95 V.

The presence of intrinsic polarization (spontaneous and piezoelectric) results in electric field jump on each interface in heterostructures. On the first AlGaN/GaN interface, the electric field in QW is reduced comparing to the one in barrier. From analysis of QW lines it was possible to obtain value of voltage for which the electric field in QW is zero. Therefore, for that voltage, the value of electric field in barriers gives the difference of intrinsic polarization between AlGaN and GaN layers. Obtained the drop of polarization was 0.0059 C/m\(^2\) for sample B and 0.0052 C/m\(^2\) for sample C. The spontaneous polarization in GaN (0.034 C/m\(^2\)) and AlGaN (0.039 C/m\(^2\)) is the same for both samples, therefore the difference in these values (0.0007 C/m\(^2\)) is caused by the piezoelectric polarization. The difference in piezoelectric polarization between individual samples is most probably caused by the different growth mode (2D mode for sample B and 3D for sample C) which produced different stress distribution.

More accurate analyses of Franz-Keldysh oscillations are possible after fitting data with equation (1). First of all the fitting procedure showed that for forward and low reverse voltage the best fit gives fitting with two sets of FKOs with the same energy transition and different electric field (figure 6a), whereas for higher reverse voltage it was enough to fit only one set of FKO (figure 6b).

![Figure 6](image_url)

**Figure 6.** Fit of sample B FKO region using equation (1) for 0.8 V (a) and -0.2 V (b). Points represent experimental data, solid red line is the best fit (two set of FKOs for 0.8 V and one set of FKOs for -0.2 V). For 0.8 V with blue line is presented the best fit using only one set of FKOs.

Both barriers have the same Al content, therefore the spontaneous polarization in back and top barrier is the same. Assuming the same stress in both layers (QW is too thin to produce significant stress comparing to Al\(_2\)O\(_3\) substrate and 1200 nm of AlGaN layers), the value of electric field in both barriers should be the same. However if the quantum state is occupied and 2DEG is present in QW, it produces additional electric field opposite in top and back barrier. Therefore for forward and low
reverse voltage QW is occupied. This results in two values of electric field in barriers. For higher reverse voltage the electric field in back barriers is large enough to produce sharp triangular potential barrier (figure 3c) and allow the tunneling of carriers from the QW. For higher reverse voltage the tunneling is so effective that QW is no longer occupied. This is confirmed also in luminescence measurement, in which the intensity of the PL peak dramatically decreases when sufficient reverse voltage is applied.

4.4. Quantum Confined Stark effect

Analysis of Franz-Keldysh oscillations showed that electric field in barriers depends linearly on the voltage for all investigated samples. Since electric field in QW is shifted by intrinsic polarization, but the dependence on bias is the same, it was possible to translate the voltage dependence of the Stark shift in QWs (figure 5) to its electric field dependence (figure 7). Maximum electric field applied to the QW was about 450 kV/cm and for this electric field the Stark shift was -14 meV, -20 meV and -35 meV for sample A, B and C respectively.

![Figure 7. Stark shift dependence on electric field in QW for sample A (black squares), B (red circles) and C (blue triangles). Fitted with equation (3) parabola dependencies are shown as a black, red and blue lines for sample A, B and C respectively.](image)

For electron confined in a QW, drift in an electric field $F$ is impossible. Instead its wave function $\psi(z)$ is moved by the field to the wall of the QW and squeezed. The Schrödinger equation should be completed with a potential $V = -eFz$. In the case of a rectangular QW of width $L$, with infinite barriers, the solution is a sum of the Airy functions [24]. The resulting energy eigenvalue ($W_{\text{QW}}$) is lowered by an energy of the order of $eFL$ (about 0.1 eV). The calculated dependence was nonlinear. In the range $eFL<10^5W_{\text{QW}}$, it can be approximated with expression:

$$E - E_g - W_{\text{QW}} = -AF - BF^2,$$

(3)

where $A = \alpha \frac{eL}{W_{\text{QW}}}$, $B = \beta \left( \frac{eL}{W_{\text{QW}}} \right)^2$, $\alpha = 0.0090$ and $\beta = 0.0086$.

The above dependence was fitted to the Stark shift obtained from ER analysis (figure 7). The fitted coefficients are presented in Table 1.

|          | $A \times 10^{-29}$ [Cm] | $B \times 10^{-55}$ [C^2m^2] | $B/A^2$ |
|----------|--------------------------|-----------------------------|---------|
| SAMPLE A | 3.18±1.35                | 0.88±0.56                   | 87±66   |
| SAMPLE B | 3.28±0.94                | 2.68±0.36                   | 249±79  |
| SAMPLE C | 7.00±1.06                | 3.15±0.45                   | 64±13   |

According to equation (3) quantity $B/A^2$ should be constant for all samples and equal to 122. Experimentally obtained values for sample A and C are lower and for sample B is bigger than the
expectations. However both A and B samples agrees with this value. In measured QWs the highest electric fields are of the order of 450 kV/cm, for which for the widest quantum well $eF L \sim 10^4 W_{QW}$, therefore the limits of validity of equation (3) approximation is reached. This can explain the observed discrepancies.

5. Conclusions
We have presented results of the electroreflectance measurements on three Al$_{0.09}$Ga$_{0.91}$N/GaN quantum wells with different well width (3, 6 and 9 nm). The electroreflectance measurements were performed at room temperature with external voltage applied to the structure. Analysis of Franz-Keldysh oscillations from barriers showed that applied voltage built over 800 kV/cm of electric field for highest reverse voltage. The electric field have been built in all layers in depletion region, which extended over QW and both barriers. The presence of intrinsic polarization (spontaneous and piezoelectric) results in electric field jump on each interface in heterostructures. On the first AlGaN/GaN interface, the electric field in QW is reduced comparing to the one in barrier (for zero external electric field it is even inverted) and on the second GaN/AlGaN interface is again increased, so the electric field in both barriers is the same. Additional carriers in QW produces extra electric field which modifies the electric field in barriers (to one of the barriers its added to the other subtracted). This results in two set of Franz-Keldysh oscillations (of the same energy transition and different electric field) observed for forward and low reverse voltage. Screening voltage (voltage for which the electric field in QW is completely screened) in ER measurement was found to change from -0.7 V to over -1.0 V for different samples. The electric field value in barriers for screening voltage allowed to obtain the difference of total polarization between AlGaN and GaN layers equal 0.0059 C/m$^2$ and 0.0052 C/m$^2$ for samples B and C respectively. Screening voltage of QWs and electric field dependence on bias in barriers allowed to obtained the value of electric field in QWs for the whole voltage range.

Analysis of QWs lines dependence showed that both broadening and amplitude of lines is minimal for screening voltage and increases for increasing electric field in QW. The energy transition is maximum for screening voltage and for increasing electric field it decreases due to Stark effect. Neither amplitude and broadening parameter nor energy transition does not depends on the direction of electric field. Only parameter that depends on the direction of electric field in QW is phase of the signal. The Stark shift depends on the well width and electric field and reached from -12 meV to -35 meV for 450 kV/cm depending on the sample. This conditions were established for highest forward voltages therefore this is the value of electric field and stark shift caused only by the intrinsic polarization.

References
[1] Razeghi M and Rogalski A 1996 J. Appl. Phys. 79 7433
[2] Pulfray D L, Kudek J J, Nener D B, Parish G, Mishra U K and Tarsa E J 1999 phys. stat. sol. (a) 176 169
[3] Yang W, Nohava T, Krishnankutty S, Torreano R, McPherson S and Marsh H 1998 Appl. Phys. Lett. 73 1086
[4] Teke A, Yun F, Dogan S, Reschchikov M A, Le H, Liu X Q, Morkoç H, Zhang S K, Wang W B and Alfano R R 2003 Solid State Electronics 47 1401
[5] Bernardini F, Fiorentini V and Vanderbilt D 1997 Phys. Rev. B 56 R10024
[6] Zoroddu A, Bernardini F, Ruggerone P and Fiorentini V 2001 Phys. Rev. B 64 45208
[7] Ambacher O 1998 J. Phys. D: Appl. Phys. 31 2653
[8] Ambacher O, Majewski J, Miskys C, Link A, Hermann M, Eickhoff M, Stutzmann M, Bernardini F, Fiorentini V, Tilak V, Schaff B and Eastman L F 2002 J. Phys.: Condens. Matter 14 3399
[9] Jena D, Heikman S, Speck J S, Gossard A, Mishra U K, Link A and Ambacher O 2003 Phys. Rev. B 67 153306
[10] Wetzel C, Takeuchi T, Yamaguchi S, Katoh H, Amano H and Akasaki I 1998 Appl. Phys. Lett. 73 1994
[11] Grandjean N, Massies J and Leroux M 199 Appl. Phys. Lett. 74 2361
[12] Drabinska A, Baranowski J M, Pakula K, Caban P and Strupinski W 2009 phys. stat. sol. (a) 206 816
[13] Drabinska A, Korona K P, Pakula K and Baranowski J M 2007 phys. stat. sol. (a) 204 459
[14] Hosea T J C 2006 phys. stat. sol. (b) 189 531
[15] Miller D A B, Chemla D S, Damen T C, Gossard A C, Wiegmann W, Wood T H and Burrus C A 1984 Phys. Rev. Lett. 53 2173
[16] Miller D A B, Chemla D S and Schmitt-Rink S 1986 Phys. Rev. B 33 6976
[17] Pakula K, Baranowski J M and Borysiuk J 2007 Crystal Research and Technology 42 1176
[18] Cardona M, Shaklee K L and Pollak F H 1967 Phys. Rev. 154 696
[19] Aspnes D E and Studna A A 1973 Phys. Rev. B 7 4605
[20] Gay J G and Klauder L T 1968 Phys. Rev. 172 811
[21] Yin X and Pollak F H 1991 Appl. Phys. Lett. 59 2305
[22] Aspnes D E 1967 Phys. Rev. 153 972
[23] Hall D J, Hosea T J C and Lancefield D 1997 J. Appl. Phys. 82 3092
[24] Miller D A B, Chemla D S, Damen T C, Gossard A C, Wiegmann W, Wood T H and Burrus C A 1985 Phys. Rev. B 32 1043