Rising and Sinking in Resonance: Mass Distribution Critically Affects Buoyancy-Driven Spheres via Rotational Dynamics

Jelle B. Will and Dominik Krug

Physics of Fluids Group and Max Planck Center Twente, J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500 AE Enschede, Netherlands

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We present experimental results for spherical particles rising and settling in a still fluid. Imposing a well-controlled center of mass offset enables us to vary the rotational dynamics selectively by introducing an intrinsic rotational timescale to the problem. Results are highly sensitive even to small degrees of offset, rendering this a practically relevant parameter by itself. We further find that, for a certain ratio of the rotational to a vortex shedding timescale (capturing a Froude-type similarity), a resonance phenomenon sets in. Even though this is a rotational effect in origin, it also strongly affects translational oscillation frequency and amplitude, and most importantly, the drag coefficient. This observation equally applies to both heavy and light spheres, albeit with slightly different characteristics for which we offer an explanation. Our findings highlight the need to consider rotational parameters when trying to understand and classify path properties of rising and settling spheres.

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A single particle settling or rising in a still fluid is one of the most intuitive and conceptually simple problems in fluid mechanics. However, the complexity arising from coupling between the motion of the body and the surrounding flow is intricate, and the resulting complex trajectories of a universal description alludes to the possibility that additional—yet largely unexplored—parameters may play a role. In fact, recently, the importance of rotational dynamics for spheres and 2D cylinders has been highlighted [14,27,28], showing that the moment of inertia (MOI, governed by the internal mass distribution) can affect the vortex shedding mode, the frequency and amplitude of oscillation, and the vertical velocity. The key physical mechanism behind this rotational-translational coupling is the Magnus lift force, which, in a still fluid, is given by \( F_m \sim \omega \times u \) [29], with \( \omega \) and \( u \) denoting particle angular and linear velocity vectors, respectively. It has been suggested that the dependence on particle MOI can be one of the factors contributing to the spread in particle drag coefficient as well as causing differences in oscillation amplitude [14], but conclusive evidence, in particular for spheres, is missing.

In this Letter, we systematically explore the effect of rotational dynamics on rising and settling spheres. To this end, we modify the rotational properties of the spherical particles in a controlled manner by introducing a center of mass (c.m.) offset \( \gamma = 2l/D \), where \( l \) is the distance along the unit vector \( p \) pointing from the c.m. to the geometrical center [see Fig. 1(a)]. Clearly, such an offset can also be expected to occur in a host of practical applications, where particle properties are rarely ever uniform. This concerns, for example, the falling of dandelion seeds [30] and snowflakes [31–35], the sedimentation behavior of sand
microplastic in the oceans [39]. Moreover, the practical relevance of this parameter was first noted by Jenny et al. [39], and on \( \mathbf{a}_c \), the acceleration of the center of mass.

For spheres, the geometric center and the center of pressure coincide. Therefore, the forcing term \( \mathbf{T}_f \) in Eq. (2) is solely due to skin friction, which, for \( \text{Re} \gtrsim 275 \) [45], provides an approximately periodic driving associated with the vortex shedding in the wake of the body [46]. Neglecting the additional dependence on \( \mathbf{a}_c \), the offset term acts as a restoring torque. Thus, Eq. (2) is similar to a periodically forced pendulum with a natural frequency \( f_p = \sqrt{5\gamma/D^2/2\pi} \) and the corresponding timescale \( \tau_p = f_p^{-1} \). The driving, due to vortex shedding, is characterized by \( \tau_v \sim D/U_b \), and on this basis, we define the ratio

\[
T = \frac{\tau_v}{\tau_p} = \frac{1}{2\pi} \sqrt{\frac{5\gamma}{1 - |\Gamma|}}. \tag{3}
\]

Note that \( T \) is entirely determined by particle properties. In relating translational \( (U_b) \) and dissipative \( (D/\tau_p) \) velocities, \( T \) corresponds to the inverse of the Froude number defined in [47] for falling strips. However, the definition in Eq. (3) is preferred, here, as it avoids a singularity at \( \gamma = 0 \).

To test the effect of variations in \( T \), laboratory experiments were performed for rising (blue) and settling spheres (red symbols) in a still fluid with systematic variations in \( \text{Ga}, \gamma \), and \( \Gamma \). An overview over the explored parameter range is shown in Fig. 1(c). Particles, \( D = 12–25 \text{ mm} \), were released to settle or rise in a large vertical water tank. After an initial transient (> 20D), the position and orientation of the spheres were tracked over a distance of \( \approx 30–80D \) using optical methods [48,49]. Details of the setup and the postprocessing of the data are provided in the Supplemental Material [50] that also includes movies of rendered trajectories at \( \text{Ga} = 1800 \).

The profound effect variations in \( \gamma \) have on particle kinematics is exemplified in Fig. 2(a), where horizontal projections (XY plane) of drift corrected trajectories for \( \text{Ga} \approx 1800 \) (rising) case are shown. From these plots, it is obvious that the oscillation amplitude varies significantly with \( \gamma \) and even vanishes for the most extreme offset. Simultaneously, the shape of the oscillations also transitions from mostly planar to circular and then back to a more planar motion with additional precession as \( \gamma \) is increased. A similar behavior is observed across all \( \text{Ga} \) and \( \Gamma \) for rising particles. For \( \Gamma > 1 \), we observed a similar increase in

![FIG. 1. (a) Schematic of a sphere with c.m. offset. (b) The particle Frenet-Serret (TNB) coordinate system, with unit vectors \( \mathbf{T} \) (parallel to \( \mathbf{u} \), \( N \) (pointing in the direction of curvature of the path), and \( B \) (defined such that \( \mathbf{N} = \mathbf{B} \times \mathbf{T} \)). The angles \( \phi \) (azimuth) and \( \theta \) (elevation) uniquely define a vector in this space. (c) Explored parameter space. Grey shading indicates the resonance regime and \( T \) isocontours correspond to \( \Gamma = 1 \).]
amplitude but not the associated helical and precessing trajectories. Unlike reported at lower Ga [13], we did not encounter significant horizontal drift here.

As a first quantitative measure, we extract the frequency $f$ of the horizontal path oscillations. Sample results for three cases in the inset of Fig. 2(b) reveal that $f$ varies significantly with $\gamma$ with a remarkable sensitivity even at small offsets. All cases display a similar pattern relative to their respective pendulum frequency $f_p(\gamma)$ (dashed lines): At small $\gamma$, $f$ exceeds $f_p$, but the two quickly converge as the offset is increased resulting in a resonance ($f \approx f_p$) between the path oscillations (and hence, the vortex shedding) and the rotational dynamics of the particle. For offsets greater than those at resonance, $f_p$ quickly outgrows the shedding frequency, and path oscillations damp out (resulting in large variations in $f$ in this regime). Resonance occurs at different values of $\gamma$ for different particles. However, all data collapse when plotting $f/f_p$ against $\gamma$, as is done in the main panel Fig. 2(b). This confirms that $T$ is, indeed, the relevant parameter governing the behavior of particles with c.m. offset, and we identify the resonance range as $0.08 \lesssim T \lesssim 0.14$, where $f/f_p \approx 1 \pm 0.2$ (marked by grey shading in all figures). A similar lock-in phenomenon of the wake to object oscillations was observed earlier for forced translational oscillations of beams in a cross flow [51,52]. A key difference and a remarkable feature of the present results is, however, that, here, vortex shedding dynamics are governed by a parameter that is intrinsically rotational.

The resonance behavior revealed for the frequencies also has a direct imprint on other parameters, such as the normalized oscillation amplitude $\hat{\alpha}/D$ shown in Fig. 3(a) for both heavy and light particles. At $T = 0$, scatter in $\hat{\alpha}/D$ is considerable owing to variation in Ga, $I^*$, and $\Gamma$. However, these differences vanish and the variation of $\hat{\alpha}/D$ as a function of $T$ becomes remarkably similar across all cases tested, rendering this the dominant parameter once a small but finite offset ($\gamma > 0$) is introduced. Amplitudes are largest in the resonance band with a peak of $\hat{\alpha}/D \approx 1$ located at $T \approx 0.09$ for both rising and settling particles. Consistent with the observation in Fig. 2(a), path oscillations vanish at large $T$ in all cases, and it appears that the decrease in $\hat{\alpha}/D$ beyond resonance is steeper for larger values of $\Gamma$. While the resonant behavior in terms of $f/f_p$ and $\hat{\alpha}/D$ is very similar for heavy and light particles, remarkably, the same is not true for the drag coefficient $C_d = 4D[1 - \Gamma(g/3\langle v_z \rangle)^2]$ shown in Fig. 3(b). For rising spheres, there is almost a factor of 2 increase in $C_d$ in the resonance regime as compared to the $T = 0$ case. In contrast, the $C_d$ results appear virtually insensitive to any changes in $T$ for settling spheres. A clue pointing to the cause of this surprising behavior is given by the results for the rotational amplitude $\hat{\theta}_z$ in Fig. 3(c). The resonance peak for $\hat{\theta}_z$ is prominent at low $\Gamma$ reaching values even beyond 90°, but remains weak for $\Gamma > 1$. In all cases, the rotational amplitude vanishes for higher $T$, for which $f < f_p$. Indeed, the scaling $\hat{\theta}_z \sim T^{-2}$, which follows from a quasistatic assumption using $T_f \sim \rho_f D^4 U^2$ [53,54], appears to capture the decay of $\hat{\theta}_z$ with increasing $T$ well in this regime. Such a simple argument fails, however, to reproduce the prefactor properly for which the suggested $(\Gamma T)^{-1}$ dependence is weaker than the actual variation in the data. Dynamically, the rotation rate is more relevant than $\hat{\theta}_z$, and it further provides a more robust measure, even at $\gamma = 0$. Therefore, we additionally consider the mean rotation rate $\langle \omega \rangle$ in Fig. 3(d) and observe a good agreement between the trend of this quantity and that of $C_d$ as a function of $T$. This indicates that, instead of the path oscillation amplitude (which features a resonance peak even for $\Gamma > 1$), the particle drag correlates better with the rotational energy of the spheres.
In evaluating the nature of the rotational-translational coupling, it is useful to consider the Lagrangian Frenet-Serret coordinate system $[\mathbf{T}, \mathbf{N}, \mathbf{B}]$, which is defined with respect to the path of the sphere $[14,29,55]$. In Fig. 2(c), we have included histograms of the orientation of $\omega$ in the TNB coordinate frame corresponding to the sample trajectories displayed in Fig. 2(a). Especially for the resonance cases ($T = 0.096$ and $T = 0.137$), $\omega$ is found to align strongly with $\mathbf{B}$. This implies that the normal acceleration (along $\mathbf{N}$) is consistent with the direction of the Magnus lift force in this state, since $F_m \sim \omega \times \mathbf{u}$. In addition to the fact that no significant path oscillations are observed in the absence of particle rotation at high $T$ (Fig. 3), this underlines the crucial role rotational dynamics play for the path oscillations. The alignment between $\omega$ and $\mathbf{B}$ in the resonance range is slightly less pronounced at $\Gamma > 1$ (see Supplemental Material [50]) but remains a robust feature for all cases considered here. While light particles at $T$ outside resonance display distinct alignments away from $\mathbf{B}$, this is not observed at $\Gamma > 1$, as rotational amplitude quickly vanishes in those cases.

With the relevance of the driving via the Magnus force established, it is then possible to analyze the phase relation between a forcing parameter and a system response. We do so by evaluating the phase angle $\Delta \Phi$ between the projections of the acceleration $\mathbf{a}$ and of the Magnus lift force $F_m$ along an arbitrary horizontal direction. By definition, particle acceleration lags behind Magnus lift forcing for $\Delta \Phi < 0$ and vice versa for $\Delta \Phi > 0$. The results for $\Delta \Phi$ in Fig. 3(e) display a collapse as a function of $T$ with a zero crossing (at $T \approx 0.12 \pm 0.01$) within the resonance band. The latter is in line with the findings in Fig. 2(c) and implies an enhancement of path oscillations through $F_m$. Therefore, a key feature of the alignment between $\omega$ and $\mathbf{B}$ in the resonance regime, $F_m$ strongly aligns with the direction of normal acceleration $\mathbf{N}$, such that translational accelerations due to other forces also amplify the effect in this case.

Finally, to put our results into perspective, we compare them to compiled literature data in terms of $C_d$ vs $\text{Re}$ in Fig. 4. The range of $C_d$ in the present measurements is seen to cover the full spread in the literature data with matching bounds, indicating that, at least at this level, the dynamics explored here are comparable to those encountered (nominally) without c.m. offset. The fact that, here, this variation arises from altering only the rotational dynamics is testament to the crucial importance of related parameters such as $I^*$ and $\gamma$. Therefore, incorporating these appears necessary for a complete description of the problem. Moreover, there is a longstanding notion [20], with mention already by Newton [8], that high levels of $C_d$ are associated with large path amplitudes $\dot{a}/D$. This is clearly at odds with our results at $\Gamma > 1$ (but, also, with findings by others...
[13,17,49,56]), where \( C_d \) remains low even though \( \dot{a}/D \) is significant. Our analysis suggests that \( C_d \) is, instead, more closely related to particle rotations.

In summary, we have provided strong evidence for how critically the overall behavior of free rising or sinking spheres in the vortex-shedding regime is related to their rotational dynamics. The revealed sensitivity to c.m. offsets as small as \( \gamma = 0.5\% \) is remarkable, and therefore, this parameter is likely to play a role in many practical cases. In particular, it might affect the behavior of spheroidal bubbles [64], which are known to display spiral or zigzag motion when rising in a contaminated fluid [65–67]. In that case, a c.m. offset might arise due to the fact that surfactants are swept to the back of the bubble by the flow and we estimate (assuming \( \Gamma \rightarrow 0 \) and \( \Gamma^* = 1 \)) that \( \gamma \approx 5\% \) would suffice to reach a \( \mathcal{T} \) value in the resonance regime. Clearly, the present findings are also useful to tailor particle behavior. In the future, it will be of particular interest to broaden the investigation to turbulent flow. Given how easily and effectively their resonance behavior can be tuned, c.m. spheres may be efficient means to “shape” turbulence by very selectively enhancing specific frequencies in the flow.

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j.b.will@utwente.nl
d.j.krug@utwente.nl

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