1. Introduction

In this paper, we consider the following problem:

\[
\begin{align*}
|u_t|^\alpha u_{tt} - \Delta u + \int_0^t h_1(t-s)\Delta u(s)ds + \left(|u|^p + |v|^q\right)|u_t|^{p-1}u_t &= f_1(u,v), \quad (x,t) \in \Omega \times (0,T), \\
|v_t|^\beta v_{tt} - \Delta v + \int_0^t h_2(t-s)\Delta v(s)ds + \left(|v|^\theta + |u|^\iota\right)|v_t|^{\theta-1}v_t &= f_2(u,v), \quad (x,t) \in \Omega \times (0,T), \\
u(x,t) &= v(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \\
u(x,0) &= u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \\
v(x,0) &= v_0(x), \quad v_t(x,0) = v_1(x), \quad x \in \Omega,
\end{align*}
\]

where \(k, l, \theta, \iota \geq 0; \ j, s \geq 1 \) for \( N = 1, 2 \), and \( 0 \leq j, s \leq (N + 2)/N - 2 \) for \( N \geq 3 \); and \( \eta \geq 0 \) for \( N = 1, 2 \), and \( 0 < \eta \leq (2N - 2)/N \) for \( N \geq 3 \); and \( h_i(\cdot) : R^+ \rightarrow R^+ \) (\( i = 1, 2 \)) are positive relaxation functions which will be specified later. \((|u|^\alpha + |v|^\beta)(\cdot)|^{\alpha-1}(\cdot), |v|^\theta + |u|^\iota \) is the degenerate damping term, and

\[
\begin{align*}
&f_1(u,v) = a_1 |u + v|^2(\alpha+1) (u + v) + b_1 |u|^p u, |v|^\beta + 2, \\
&f_2(u,v) = a_1 |u + v|^2(\alpha+1)(u + v) + b_1 |v|^\theta v, |u|^\igma^2.
\end{align*}
\]

The motivation of our problem firstly is by the initial boundary value problem for the quasilinear equation of the form

\[
|u_t|^\alpha u_{tt} - \Delta u + \int_0^t h(t-s)\Delta u(s)ds + g(u,u_t) = f(u).
\]
This type of problem is frequently found in some mathematical models in applied sciences, especially in the theory of viscoelasticity. Problem (3) has been studied by various authors, and several results concerning asymptotic behavior and blow-up have been studied (case \( \eta \geq 0 \)). For example, in the case \( (g(u, ut) = 0) \), problem (3) has been investigated in [1] and the author proved the blow-up result. In the case \( (g(u, ut) = 0) \), boundary value problem and in the presence of the dispersion term \((\Delta u)\), Liu [2] studied a general decay and blow-up of solutions. And, in [3], the authors applied the potential well method to indicate the global existence and uniform decay of solutions \((g(u, ut)) = 0 \) instead of \( \Delta u \). Furthermore, the authors obtained a blow-up result. In the case \( (g(u, ut) = |u|^m u) \), in [4], Wu studied a general decay of solution. Later, the same author in [5] considered the same problem but \((g(u, ut) = u) \) and discussed the decay rate of solution. Recently, in [6], the authors proved the existence of global solution and a general stability result.

There are several works in case \((\eta = 0) \), where the authors have studied the blow-up of solutions of problem (3) (for example, see [3, 7–12]).

For a coupled system, He [13] considered the following problem:

\[
\begin{cases}
|u|^\eta u_t - \Delta u + \int_0^t h_1(t-s)\Delta u(s)ds - \Delta u_t + g_1(u, u_t) = f_1(u, v), \\
|v|^\eta v_t - \Delta v + \int_0^t h_2(t-s)\Delta v(s)ds - \Delta v_t + g_2(v, v_t) = f_2(u, v),
\end{cases}
\]

where \( \eta > 0; j, s \geq 2; \) and \( g_1(u, u_t) = |u|^{j-2} u_t \) and \( g_2(v, v_t) = |v|^{s-2} v_t \). The author proved general and optimal decay of solutions. Then, in [14], the author investigated the same problem without damping term and established a general decay of solutions. Furthermore, the author obtained a blow-up of solutions. In addition, in problem (1) with \( \eta = 0 \), in [15], Wu proved a general decay of solutions. Later, in [16], Piskin and Ekinci established a general decay and blow-up of solutions with nonpositive initial energy for problem (1) case (Kirchhoff type).

In recent years, some other authors investigate the hyperbolic type system with degenerate damping terms (see [17–20]). Very recently, in the presence of the dispersion term \((\Delta u_t)\), our problem (1) has been studied in [21]. Under some restrictions on the initial datum and standard conditions on relaxation functions, the authors have established the global existence and proved the general decay of solutions.

Based on all of the abovementioned discussion, we believe that the combination of these terms of damping (memory term, degenerate damping, and source terms) constitutes a new problem worthy of study and research, different from the above that we will try to shed light on, especially the blow-up of solutions.

Our paper is divided into several sections: In Section 2, we lay down the hypotheses, concepts, and lemmas we need. In Section 3, we prove our main result. Finally, we give some concluding remarks in the last section.

2. Preliminaries

We prove the blow-up result under the following suitable assumptions:

(A1) \( h_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) are differentiable and decreasing functions such that

\[
h_i(t) \geq 0, 1 - \int_0^t h_i(s)ds = l_i > 0, \quad i = 1, 2.
\]

(A2) There exist a constants \( \xi_1, \xi_2 > 0 \) such that

\[
h'_i(t) \leq - \xi_i h_i(t), \quad t \geq 0, \quad i = 1, 2.
\]

Lemma 1. There exists a function \( F(u, v) \) such that

\[
F(u, v) = \frac{1}{2(p + 2)} |u f_1(u, v) + v f_2(u, v)|^2 + \frac{1}{2(p + 2)} [a_1 |u + v|^{2(p + 2)} + 2b_1 |uv|^{2(p + 2)}] \geq 0,
\]

where

\[
\frac{\partial F}{\partial u} = f_1(u, v),
\]

\[
\frac{\partial F}{\partial v} = f_2(u, v).
\]

We take \( a_1 = b_1 = 1 \) for convenience.

Lemma 2 (see [18]). There exist two positive constants \( c_0 \) and \( c_1 \) such that

\[
\frac{c_0}{2(p + 2)} |u|^{2(p + 2)} + |v|^{2(p + 2)} \leq F(u, v) \leq \frac{c_1}{2(p + 2)} |u|^{2(p + 2)} + |v|^{2(p + 2)}.
\]

Now, we state the local existence theorem that can be established by combining arguments of [13, 16].

Theorem 1. Assume (5) and (6) hold. Let

\[
\begin{cases}
-1 < p < \frac{4 - n}{n - 2}, \quad n \geq 3, \\
p \geq -1, \quad n = 1, 2.
\end{cases}
\]

Then, for any initial datum,

\[
(u_0, u_1, v_0, v_1) \in \mathcal{H}.
\]

Problem (1) has a unique solution, for some \( T > 0 \):
\[ u, v \in C\left([0, T]; H^2(\Omega) \cap H^1_0(\Omega)\right), \]
\[ u_t \in C\left([0, T]; H^1_0(\Omega) \cap L^{j+1}(\Omega)\right), \]
\[ v_t \in C\left([0, T]; H^1_0(\Omega) \cap L^{r+1}(\Omega)\right), \]

where
\[ \mathcal{H} = H^1_0(\Omega) \times L^2(\Omega) \times H^1_0(\Omega) \times L^2(\Omega). \]

Lemma 3. Assume (5), (6), and (10) hold; let \((u, v)\) be a solution of (1); then, \(E(t)\) is nonincreasing, that is,
\[ E(t) = \frac{1}{\eta + 2} \left[ \|u\|_{\eta_2}^2 + \|v\|_{\eta_2}^2 \right] \]
\[ + \frac{1}{2} \left( \left( 1 - \int_0^t h_1(s)ds \right) \|u_t\|_2^2 \right) \]
\[ + \left( 1 - \int_0^t h_2(s)ds \right) \|v_t\|_2^2, \]
\[ + \frac{1}{2} \left[ (h_1 o \nabla u)(t) + (h_2 o \nabla v)(t) \right] - \int_\Omega F(u, v)dx, \]

which satisfies
\[ E'(t) \leq \frac{1}{2} \left[ (h_1 o \nabla u)(t) + (h_2 o \nabla v)(t) \right] \]
\[ - \frac{1}{2} \left( h_1(t) \|u_t\|_2^2 + h_2(t) \|v_t\|_2^2 \right), \]
\[ - \int_\Omega (|u|^k + |v|^l)|u_t|^{j+1}dx - \int_\Omega (|v|^q + |u|^p)|v_t|^{r+1}dx, \]
\[ \leq 0. \]

Proof. By multiplying the first and second equations in (1) by \(u, v_t\) and integrating over \(\Omega\), we get
\[ \frac{d}{dt} \left\{ \frac{1}{\eta + 2} \left[ \|u\|_{\eta_2}^2 + \|v\|_{\eta_2}^2 \right] + \frac{1}{2} \left( \left( 1 - \int_0^t h_1(s)ds \right) \|u_t\|_2^2 \right) \right\} \]
\[ + \frac{1}{2} \left( h_2 o \nabla u)(t) - \int_\Omega F(u, v)dx \right\}, \]
\[ = - \int_\Omega (|u|^k + |v|^l)|u_t|^{j+1}dx - \int_\Omega (|v|^q + |u|^p)|v_t|^{r+1}dx + \frac{1}{2} \left( h_1 o \nabla u\right) - \frac{1}{2} h_1(t) \|u_t\|_2^2 + \frac{1}{2} \left( h_2 o \nabla u\right) - \frac{1}{2} h_2(t) \|v_t\|_2^2. \]

We obtain (14) and (15).

3. Blow-Up

In this section, we prove the blow-up result of solution of problem (1).

\[ \mathcal{H}(t) = -E(t) = -\frac{1}{\eta + 2} \left[ \|u\|_{\eta_2}^2 + \|v\|_{\eta_2}^2 \right] - \frac{1}{2} \left( \left( 1 - \int_0^t h_1(s)ds \right) \|u_t\|_2^2 + \frac{1}{2} \left( 1 - \int_0^t h_2(s)ds \right) \|v_t\|_2^2 \right) \]
\[ - \frac{1}{2} \left[ (h_1 o \nabla u)(t) + (h_2 o \nabla v)(t) \right] \]
\[ + \frac{1}{2(p + 2)} \left[ \|u + v\|_{2(p + 2)}^{2(p + 2)} + \|u\|_{2(p + 2)}^{2(p + 2)} \right]. \]
Theorem 4. Assume that (5), (6), and (10) hold, and suppose that \( E(0) < 0 \) and
\[
2(p + 2) > \max\{k + j + 1; l + j + 1; \theta + s + 1; q + s + 1\}.
\]
(18)

Then, the solution of problem (1) blows up in finite time.

Proof. From (14), we have
\[
E(t) \leq E(0) \leq 0.
\]
(19)

Therefore,
\[
\mathcal{H}'(t) = -E'(t) \geq \int_\Omega (|u|^k + |v|^l)|u_t|^\alpha + \int_\Omega (|v|^\theta + |u|^\varphi)|v_t|^\beta dx.
\]
(20)

Hence,
\[
0 < \alpha < \min\left\{ \frac{1}{2} - \frac{1}{\eta + 2} \right\} \left( \frac{2p + 3 - (k + j)}{2j(p + 2)} \right) \left( \frac{2p + 3 - (l + j)}{2j(p + 2)} \right) \left( \frac{2p + 3 - (\theta + s)}{2s(p + 2)} \right) \left( \frac{2p + 3 - (q + s)}{2s(p + 2)} \right) < 1.
\]
(24)

By multiplying the first and second equations in (1) by \( u, v \) and with a derivative of (23), we get
\[
\mathcal{H}'(t) = (1 - \alpha)\mathcal{H}^{-\alpha}\mathcal{H}'(t) + \frac{\varepsilon}{\eta + 1} \left( \|u\|^{\eta + \alpha} + \|v\|^{\eta + \beta} \right) + \varepsilon \int_\Omega \nabla u \int_0^t g(t-s)|\nabla u(s)|ds dx + \varepsilon \int_\Omega \nabla v \int_0^t h(t-s)|\nabla u(s)|ds dx
\]
\[
- \varepsilon \int_\Omega (|u|^k + |v|^l)|u_t|^\alpha u dx + \varepsilon \int_\Omega (|v|^\theta + |u|^\varphi)|v_t|^\beta v dx - \varepsilon (\|\nabla u\|_2^2 + \|\nabla v\|_2^2) + \varepsilon \int_\Omega \frac{\nabla u + \nabla v}{2} dx + \varepsilon \int_\Omega \frac{\nabla u + \nabla v}{2} dx
\]
(25)

where we have
\[
J_1 = \varepsilon \int_0^t h_1(t-s)ds \int_\Omega \nabla u \cdot (\nabla u(s) - \nabla u(t))dx ds
\]
\[
+ \varepsilon \int_0^t h_1(s)ds \|\nabla u\|_2^2,
\]
(26)

\[
J_2 = \varepsilon \int_0^t h_2(t-s)ds \int_\Omega \nabla u \cdot (\nabla u(s) - \nabla u(t))dx ds
\]
\[
+ \varepsilon \int_0^t h_2(s)ds \|\nabla v\|_2^2,
\]
(27)

From (25), we find that
\[
\mathcal{H}'(t) \geq \varepsilon \left( \frac{1}{2} - \frac{1}{\eta + 2} \right) \left( \frac{2p + 3 - (k + j)}{2j(p + 2)} \right) \left( \frac{2p + 3 - (l + j)}{2j(p + 2)} \right) \left( \frac{2p + 3 - (\theta + s)}{2s(p + 2)} \right) \left( \frac{2p + 3 - (q + s)}{2s(p + 2)} \right) \left( \frac{\varepsilon}{\eta + 1} \right) \left( \|u\|^{\eta + \alpha} + \|v\|^{\eta + \beta} \right)
\]
(28)

At this point, we use Young’s inequality; for \( \delta > 0 \),
\[
XY \leq \frac{\delta^a X^a}{\alpha} + \frac{\delta^{-\beta} X^\beta}{\beta}, \alpha, \beta > 0, \frac{1}{\alpha} + \frac{1}{\beta} = 1.
\]
(29)

We get that for \( \delta_1, \delta_2 > 0 \),
by Young's inequality, we find that for $\delta_3, \delta_4 > 0$,

\[
\int_{\Omega} |v^\theta| |v|^{s+1} dx \leq \frac{\delta_3}{l + j + 1} \|v^\theta\|_{l}^{s+1} + \frac{s + 1}{\delta_4} \|v^\theta\|_{q}^{s+1}.
\]

Hence,

\[
H^s(t) \int_{\Omega} (|u|^k + |v|^l) |u|^j dx \leq H^s(t) \|u\|_{k+j+1}^{s+1} + \frac{\delta_3}{l + j + 1} \|v^\theta\|_{l}^{s+1} + \frac{s + 1}{\delta_4} \|v^\theta\|_{q}^{s+1}.
\]

Since (10) holds, we obtain the following by using (22) and (24):

\[
\|u\|^{j+1} |u| \leq \frac{j^{j+1}}{j + 1} |u| |u|^{j+1},
\]

\[
|v|^{j+1} |v| \leq \frac{s^{j+1}}{s + 1} |v| |v|^{j+1}.
\]

Hence, we have

\[
J_3 \leq \frac{\epsilon}{j + 1} \int_{\Omega} (|u|^k + |v|^l) |u|^{j+1} dx
\]

\[
+ \epsilon \frac{j^{j+1}}{j + 1} \int_{\Omega} (|u|^k + |v|^l) |u|^{j+1} dx,
\]

\[
J_4 \leq \frac{\epsilon}{j + 1} \int_{\Omega} (|v|^\theta + |u|^\theta) |v|^{j+1} dx
\]

\[
+ \epsilon \frac{s^{j+1}}{s + 1} \int_{\Omega} (|v|^\theta + |u|^\theta) |v|^{j+1} dx.
\]

Therefore, using (21) and by setting $\delta_1, \delta_2$ so that

\[
\frac{j^{j+1}}{j + 1} = \frac{\delta_3}{2},
\]

\[
\frac{s^{j+1}}{s + 1} = \frac{\delta_4}{2},
\]

and substituting in (28), we get

\[
\mathcal{H}(t) \geq [(1 - \alpha) - \epsilon \kappa] \mathcal{H}_0^1 + \epsilon \left( \int_{0}^{t} \left( |u|^k + |v|^l \right) |u|^{j+1} dx \right) + \epsilon \left( \int_{0}^{t} \left( |v|^\theta + |u|^\theta \right) |v|^{j+1} dx \right) + J_5,
\]
for some positive constants \(c_i, i = 1, \ldots, 4\). By using (24) and the algebraic inequality

\[
B \leq (B + 1) (B + b), \quad \forall B > 0, 0 < \zeta < 1, b > 0,
\]

we have, \(\forall t > 0\),

\[
\|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} \leq \alpha + \|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} + \|H(t)\|.
\]

Substituting (40) and (42) in (38), we get

\[
\|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} \leq c_0 \left( \|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} + \|H(t)\| \right),
\]

Hence, by fixing \(\delta_3, \delta_4 > 0\), we get

\[
\|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} \leq M_1 + M_2 \left( 1 + \frac{1 + \delta_3^{l+1/j+1}}{l + 1} + \frac{\delta_3^{l+1/j+1}}{l + 1} \right) \left( \|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} + \|H(t)\| \right).
\]

for some constants \(M_1, M_2 > 0\). Now, for \(0 < a < 1\), from (17),

\[
J_5 = \frac{\epsilon}{\|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)}} \left( \|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} + 2\|\nabla u\|_{L^2(\mathbb{R}^d)} \right)
\]

\[
+ \epsilon (p + 2) (1 - a) \left( 1 - \int_0^t g(s) ds \right) \|\nabla u\|_{L^2}^2 + \epsilon (p + 2) (1 - a) \left( 1 - \int_0^t h(s) ds \right) \|\nabla v\|_{L^2}^2
\]

Substituting in (33) and by using (9), we get

\[
\|u\|_{L^2(\mathbb{R}^+)\cap L^2(\mathbb{R}^d)} \leq C (X^\gamma + Y^\gamma), \quad X, Y > 0, \gamma > 0,
\]

where \(d = 1 + (1/\|\|)(0)\). Also, since

\[
(X + Y)^\gamma \leq C (X^\gamma + Y^\gamma), \quad X, Y > 0, \gamma > 0,
\]
\( \mathcal{K}'(t) \geq [(1-a) - \varepsilon \kappa] \| \mathcal{H}'(t) \| + \varepsilon \left\{ \frac{2 \varepsilon (p + 2) (1 - a)}{\eta + 2} + \frac{1}{\eta + 1} \right\} \left( \| u_t \|_{\eta+2}^{\eta+2} + \| v_t \|_{\eta+2}^{\eta+2} \right), \)

\[ + \varepsilon \left\{ (p + 2) (1 - a) \left( 1 - \int_0^t h_1(s)ds \right) - \left( \frac{1}{2} \int_0^t h_1(s)ds \right) \right\} \| \nabla u \|_2^2 \]

\[ + \varepsilon \left\{ (p + 2) (1 - a) \left( 1 - \int_0^t h_2(s)ds \right) - \left( \frac{1}{2} \int_0^t h_2(s)ds \right) \right\} \| \nabla v \|_2^2 \]

\[ + \varepsilon \left\{ (p + 2) (1 - a) - \frac{1}{2} \right\} \left( h_1 \nabla u_h + h_2 \nabla v \right) + \varepsilon c_0 a - (M_3 C_1(\kappa) + M_4 C_2(\kappa)) \left( \| u \|_{2(p+2)}^2 + \| v \|_{2(p+2)}^2 \right) \]

\[ + \varepsilon [2(p + 2)(1 - a) - (M_3 C_1(\kappa) + M_4 C_2(\kappa)) \| \mathcal{H}(t) \| , \]

where

\[ M_3 = M_1 \left( 1 + \frac{l \delta_3^{(l+j+1/\eta)}}{l + j + 1} + \frac{(j + 1) \delta_3^{(l+j+1/\eta)}}{l + j + 1} \right) > 0, \]

\[ M_4 = M_2 \left( 1 + \frac{u \delta_4^{(u+p+1/\eta)}}{u + s + 1} + \frac{(s + 1) \delta_4^{(u+p+1/\eta)}}{u + s + 1} \right) > 0. \]

In this stage, we take \( a > 0 \) small enough so that

\[ \lambda_1 = (p + 2)(1 - a) - 1 > 0, \]

and we assume that

\[ \max \left\{ \int_0^t h_1(s)ds, \int_0^t h_2(s)ds \right\} < \frac{(p + 2)(1 - a) - 1}{((p + 2)(1 - a) - (1/2))} \]

\[ = \frac{2 \lambda_1}{2 \lambda_1 + 1} \]

\[ (49) \]

By (9), for some \( \beta_1 > 0 \), we obtain

\[ \mathcal{K}'(t) \geq \beta_1 \left\{ \mathcal{H}(t) + \| u_t \|_{\eta+2}^{\eta+2} + \| v_t \|_{\eta+2}^{\eta+2} + \| \nabla u \|_2^2 + \| \nabla v \|_2^2 + (h_1 \nabla u) + (h_2 \nabla v) + \| u \|_{2(p+2)}^2 + \| v \|_{2(p+2)}^2 \right\}. \]

\[ (53) \]

Next, using Hölder’s and Young’s inequalities, we have
where \((1/\mu) + (1/\theta) = 1\).
We take \(\mu = (\eta + 2)(1 - \alpha)\), to get
\[
\theta \frac{\eta + 2}{1 - \alpha} = (\eta + 2)(1 - \alpha)(\eta + 2) - 1 \leq 2(p + 2).
\]
(57)
Subsequently, by using (24), (22), and (39), we obtain
\[
\|u\|_{L_2^{((\eta/2)(1 - \alpha))(2p + 1)}} \leq d\left(\|u\|_{L_2^{((\eta/2)(p + 1))}} + \Omega(t)\right),
\]
\[
\|v\|_{L_2^{((\eta/2)(p + 1))}} \leq d\left(\|v\|_{L_2^{((\eta/2)(p + 1))}} + \Omega(t)\right), \forall t \geq 0.
\]
(58)

Hence, by substituting (59) into (23), we get
\[
\mathcal{K}^{(1/1 - \alpha)}(t) = \left(\|u\|^{\eta/2} + \|v\|^{\eta/2}\right)^{(1/1 - \alpha)},
\]
which we can use to study the emergence of these terms in the system.

Subsequently, by using (24), (22), and (39), we obtain
\[
\|u\|_{L_2^{((\eta/2)(1 - \alpha))(2p + 1)}} \leq d\left(\|u\|_{L_2^{((\eta/2)(p + 1))}} + \Omega(t)\right),
\]
\[
\|v\|_{L_2^{((\eta/2)(p + 1))}} \leq d\left(\|v\|_{L_2^{((\eta/2)(p + 1))}} + \Omega(t)\right), \forall t \geq 0.
\]
(58)

From (53) and (60), we get
\[
\mathcal{K}'(t) \geq \lambda \mathcal{K}^{(1/1 - \alpha)}(t),
\]
where \(\lambda > 0\), and this depends only on \(\beta\) and \(c\).

By integration of (61), we obtain
\[
\mathcal{K}'^{(a - 1/\alpha)}(t) \geq \mathcal{K}^{(-a/1 - \alpha)}(0) - \lambda (a/1 - \alpha)t
\]
(62)
Hence, \(\mathcal{K}(t)\) blows up in time
\[
T \leq T^* = \frac{1 - \alpha}{\lambda \alpha \mathcal{K}(a/1 - \alpha)}(0).
\]
(63)
Then, the proof is completed.

4. Conclusion

The objective of this work is the study of the blow-up of solutions for a quasilinear viscoelastic system with degenerate damping. This type of problem is frequently found in some mathematical models in applied sciences, especially in the theory of viscoelasticity. What interests us in this current work is the combination of these terms of damping (memory term, degenerate damping, and source terms), which dictates the emergence of these terms in the system.

In the next work, we will try using the same method with the same problem in addition to other damping terms (dispersion term, Balakrishnan–Taylor damping, and delay term).

Data Availability

No data were used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] H. Song, "Global nonexistence of positive initial energy solutions for a viscoelastic wave equation," Nonlinear Analysis, vol. 125, pp. 260–269, 2015.
[2] W. Liu, "General decay and blow-up of solution for a quasilinear viscoelastic problem with nonlinear source," Nonlinear Analysis: Theory, Methods & Applications, vol. 73, no. 6, pp. 1890–1904, 2010.
[3] G. Liang, Y. Zhaoqin, and L. Guonguang, "Blow up and global existence for a nonlinear viscoelastic wave equation with strong damping and nonlinear damping and source terms," Applied Mathematics, vol. 6, pp. 806–816, 2015.
[4] S. T. Wu, "General decay of solutions for a viscoelastic equation with nonlinear damping and source terms," Acta Mathematica Scientia, vol. 318, pp. 1436–1448, 2011.
S. T. Wu, “General decay of energy for a viscoelastic equation with damping and source terms,” *Taiwanese Journal of Mathematics*, vol. 16, no. 1, pp. 113–128, 2012.

H. Yang, S. Fang, F. Liang, and M. Li, “A general stability result for second order stochastic quasilinear evolution equations with memory,” *Boundary Value Problems*, vol. 62, pp. 1–16, 2020.

A. M. Alghamdi, S. Gala, M. A. Ragusa, and Z. Zhang, “A regularity criterion for the 3D density-dependent MHD equations,” *Bulletin of the Brazilian Mathematical Society, New Series*, vol. 52, no. 2, pp. 241–251, 2020.

A. Mohammad Alghamdi, S. Gala, S. Gala, C. Qian, and M. Alessandra Ragusa, “The anisotropic integrability logarithmic regularity criterion for the 3D MHD equations,” *Electronic research archive*, vol. 28, no. 1, pp. 183–193, 2020.

A. Choucha, D. Ouchenane, and S. Boulaaras, “Blow-up of a nonlinear viscoelastic wave equation with distributed delay combined with strong damping and source terms,” *Journal of Nonlinear Functional Analysis*, vol. 31, 2020.

H. Song and D. Xue, “Blow up in a nonlinear viscoelastic wave equation with strong damping,” *Nonlinear Analysis: Theory, Methods & Applications*, vol. 109, pp. 245–251, 2014.

H. Song and C. Zhong, “Blow-up of solutions of a nonlinear viscoelastic wave equation,” *Nonlinear Analysis: Real World Applications*, vol. 11, no. 5, pp. 3877–3883, 2010.

A. Zarai, A. Draifia, and S. Boulaaras, “Blow-up of solutions for a system of nonlocal singular viscoelastic equations,” *Applicable Analysis*, vol. 97, no. 13, pp. 2231–2245, 2018.

L. He, “On decay and blow-up of solutions for a system of equations,” *Applicable Analysis*, vol. 100, no. 11, pp. 2449–2477, 2019.

L. He, “On decay of solutions for a system of coupled viscoelastic equations,” *Acta Applicandae Mathematica*, vol. 167, no. 1, pp. 171–198, 2020.

S.-T. Wu, “General decay of solutions for a nonlinear system of viscoelastic wave equations with degenerate damping and source terms,” *Journal of Mathematical Analysis and Applications*, vol. 406, no. 1, pp. 34–48, 2013.

E. Piskin and F. Ekinci, “General decay and blow up of solutions for coupled viscoelastic equation of Kirchhoff type with degenerate damping terms,” *Mathematical Methods in the Applied Sciences*, vol. 42, no. 16, pp. 5468–5488, 2019.

M. M. Cavalcanti, V. N. Domingos Cavalcanti, and J. Ferreira, “Existence and uniform decay for a non-linear viscoelastic equation with strong damping,” *Mathematical Methods in the Applied Sciences*, vol. 24, no. 14, pp. 1043–1053, 2001.

D. Ouchenane, K. Zennir, and M. Bayoud, “Global nonexistence of solutions for a system of nonlinear viscoelastic wave equation with degenerate damping and source terms,” *Ukrainian Mathematical Journal*, vol. 65, no. 7, 2013.

E. Piskin and F. Ekinci, “Blow up of solutions for a coupled Kirchhoff-type equations with degenerate damping terms, applications and applied Mathematics,” *International Journal*, vol. 14, no. 2, pp. 942–956, 2019.

N. Mezouar and S. Boulaaras, “Global existence and decay of solutions of a singular nonlocal viscoelastic system with damping terms,” *Topological Methods in Nonlinear Analysis*, vol. 1, 2020.

F. Ekinci, E. Piskin, S. M. Boulaaras, and I. Mekawy, “Global existence and general decay of solutions for a quasilinear system with degenerate damping terms,” *Journal of function Spaces*, vol. 2021, Article ID 4316238, 2021.