Large Tau and Tau Neutrino Electric Dipole Moments in Models with Vector Like Multiplets

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Abstract

It is shown that the electric dipole moment of the tau lepton several orders of magnitude larger than predicted by the standard model can be generated from mixings in models with vector like multiplets. The EDM of the tau lepton arises from loops involving the exchange of the W, the charginos, the neutralinos, the sleptons, the mirror leptons, and the mirror sleptons. The EDM of the Dirac tau neutrino is also computed from loops involving the exchange of the W, the charginos, the mirror leptons and mirror sleptons. A numerical analysis is presented and it is shown that the EDMs of the tau lepton and of the tau neutrino which lie just a couple of orders of magnitude below the sensitivity of the current experiment can be achieved. Thus the predictions of the model are testable in improved experiment on the EDM of the tau and of the tau neutrino.
1. INTRODUCTION

In the standard model the edm of the tau arises at the multiloop level and is extremely small\cite{1}, i.e., \( d_\tau < 10^{-34} \text{ecm} \). On the other hand the current experimental limit on the edm of the tau lepton\cite{2} is\footnote{For related papers where the upper limit on the edm of the tau is given to lie in the range \( 10^{-16} - 10^{-17} \) ecm see\cite{3,4}.}

\[
d_\tau < 1.1 \times 10^{-17} \text{ecm}. \tag{1}
\]

Thus an experimental test of the standard model prediction is beyond the realm of observability in any near future experiment since the theoretical values lies several orders of magnitude below the current experimental limits. A similar situation also holds for the edm of the tau neutrino where the current experimental limit on the edm of the tau neutrino is\cite{2} \( d_{\tau\nu} < 5.2 \times 10^{-17} \text{ecm} \),

\[
d_{\tau\nu} < 5.2 \times 10^{-17} \text{ecm}, \tag{2}
\]

while in the standard model extended by a singlet the edm again arises only at the multiloop level and is many orders of magnitude below the experimental limit. In this paper we investigate the possibility that the EDM of the tau and of the tau neutrino may be much larger by several orders of magnitude in models where there is a small mixing of the third generation leptons with a mirror in a vector like generation. Such a mixing may put the tau lepton EDM and the tau neutrino EDM with in the realm of observation with improved experiment. Thus vector like combinations are predicted in many unified models of particle interactions \cite{8,9} and their implications have been explored in many recent works \cite{10,16}. Such vector like combinations could lie in the TeV region and would be consistent with the current precision electroweak data.

In this work we allow for the possibility that there could be a tiny mixing of these vector like combinations with the sequential generations and these mixing affect very significantly the \( \tau \) lepton moments and also the tau neutrino moments. The implications of such mixings on the magnetic moment of the tau neutrino and on the anomalous magnetic moment of the tau were discussed in \cite{17}. Here we include the effects of CP phases (for a recent review of CP violation see \cite{18}) and discuss the enhancement of the leptonic EDMs due to the mixings with mirrors in the vector like generations. For the analysis here we will focus on the leptonic vector like multiplets. To simplify the analysis, we will assume that the mixings of the sequential generations with the ordinary heavy leptons in the vector like combinations
are small and thus will ignore it. The inclusion of such mixings will affect our overall results only by a small factor $\sim 2$. However, we are after much bigger effects, i.e., effects which are larger than the SM results by as much as a factor of $10^{15}$. Thus in the following analysis we will focus on the mixings of the sequential generations with the mirrors in the vector like combinations and show that they have huge effects. The mixing of the mirrors with the sequential leptons will introduce $V + A$ interactions for the ordinary leptons. Now there are very stringent constraints on such interactions for the first two generations and thus these mixings are effectively negligible and we suppress them in our analysis. On the other hand for the third generation leptons, a small mixing is possible and consistent with the current experimental constraints\cite{19}. We note in passing that a similar situation holds for the case of the third generation quarks \cite{20}.

The masses of the vector multiplets could lie in a large mass range, i.e., from the current lower limits given by the LEP experiment for color singlet states to the region in the several TeV mass range. If the mirror leptons are discovered at the LHC, the analysis here would be very relevant for planning of experiments for the discovery of the edms of the tau and of the tau neutrino. However, it is possible that the vector like multiplets have masses large enough that they might escape detection even at the LHC. This is specifically true for the leptonic vector multiplets since the discovery reach for them is typically much smaller at hadronic machines than for the color particles. However, even for this case the contribution of the mirrors to the edms can be huge as shown at the end of Sec.(4). Specifically it shown there that with the mirror masses in the TeV range, the edms of the tau neutrino and of the tau lepton can be $O(10^{14})$ larger than the Standard Model value and only a factor of $10^3$ smaller than the current sensitivity. An improvement in sensitivity of this magnitude is not necessarily outside the realm of future experiment. Further it is possible that a large edm for the tau neutrino could have astrophysical implications.

The outline of the rest of the paper is as follows: In Sec.(2) we give an analysis of the EDM of the tau lepton allowing for mixing between the vector like combination and the third generation leptons. Here the contribution to the edm of the tau arises from the exchanges of mirror neutrino, sneutrino-mirror sneutrino, from the third generation leptons and slepton-mirror sleptons along with W boson, chargino and neutralino exchanges. In Sec.(3) a similar analysis is given for the EDM of the tau neutrino with inclusion of the contributions arise from exchanges of the leptons from the third generation and from the mirrors, and also from the exchanges of the W bosons, charginos, sleptons and mirror sleptons.

A numerical analysis of sizes of the EDM of the tau lepton and of the tau neutrino are given in Sec.(4). In this section we also give a display of the EMDs on the phases and mixings. Conclusions are given in Sec.(5). Deductions of the mass matrices used in Sec.(2) and Sec.(3) are given in the Appendix.
FIG. 1: The loop contributions to the electric dipole moment of the tau via exchange of the $W$ boson and of tau neutrino and mirror neutrino denoted by $\nu_j$, via neutralino ($\tilde{\chi}^0_i$) and sleptons ($\tilde{\tau}_k$) exchange and via the exchange of charginos ($\tilde{\chi}^-_j$), sneutrinos and mirror sneutrinos denoted by ($\tilde{\nu}_k$).

2. EDM OF THE TAU LEPTON

Fig.(1a) produces edm of the tau ($d_\tau$) through the interaction of the $W$ bosons with the tau and with the neutrino and its mirror, and we give here the relevant part of the Lagrangian which is

$$
\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}} W^-_{\mu} \sum_{j,k=1,2} \bar{\tau}_k \gamma^\mu [D^\nu_{L1j} D^{\tau*}_{L1k} P_L + D^\nu_{R2j} D^{\tau*}_{R2k} P_R] \nu_j + H.c. \quad (3)
$$

where $D^\nu_{L,R}$ are the diagonalizing matrices defined in the Appendix. These matrices contain phases and these phases generate the edm of the tau for the case of Fig.(1a).

Fig. (1b) produces a contribution to $d_\tau$ through neutralino exchange and the relevant interactions in this case are

$$
-\mathcal{L}_{\tau-\tilde{\tau}-\tilde{\chi}^0} = \sum_{j=1-4} \sum_{k=1-4} \bar{\tau}_1 [C_{jk} P_L + F_{jk} P_R] \tilde{\chi}^0_j \bar{\tilde{\tau}}_k + H.c., \quad (4)
$$

where

$$
C_{jk} = \sqrt{2} [\alpha_{\tau_j} D^\tau_{R11} \tilde{D}^\tau_{4k} - \gamma_{\tau_j} D^\tau_{R11} \tilde{D}^\tau_{3k} + \beta_{E\tau_j} D^\tau_{R12} \tilde{D}^\tau_{4k} - \delta_{E\tau_j} D^\tau_{R12} \tilde{D}^\tau_{2k}],
$$

$$
F_{jk} = \sqrt{2} [\beta_{\tau_j} D^\tau_{L11} \tilde{D}^\tau_{4k} - \delta_{\tau_j} D^\tau_{L11} \tilde{D}^\tau_{3k} + \alpha_{E\tau_j} D^\tau_{L12} \tilde{D}^\tau_{4k} - \gamma_{E\tau_j} D^\tau_{L12} \tilde{D}^\tau_{2k}] \quad (5)
$$

Fig.(1c) produces a contribution to $d_\tau$ through chargino exchange and the relevant interactions in this case are

$$
-\mathcal{L}_{\tau-\bar{\nu}-\tilde{\chi}^-} = \sum_{j=1-2} \sum_{k=1-4} \bar{\tau}_1 [K_{jk} P_L + L_{jk} P_R] \tilde{\chi}^-_j \bar{\tilde{\nu}}_k + H.c., \quad (6)
$$
where

\[ K_{jk} = -g_2 [D^\tau_{R11} \kappa_r U^*_j 1_k + D^\tau_{R12} \kappa_r U^*_j 1_k - D^\tau_{R11} \kappa_N U^*_j 2_k + D^\tau_{R12} \kappa_N U^*_j 2_k], \]

\[ L_{jk} = -g_2 [D^\tau_{L11} \kappa_r V^*_j 2_k + D^\tau_{L12} \kappa_N V^*_j 2_k - D^\tau_{L11} \kappa_N V^*_j 2_k + D^\tau_{L12} \kappa_N V^*_j 2_k]. \] (7)

Here \( U \) and \( V \) are the matrices that diagonalize the chargino mass matrix \( M_C \) so that

\[ U^* M_C V^{-1} = \text{diag}(m_{\chi_1}^+, m_{\chi_2}^+), \] (8)

and \( \kappa_N, \kappa_r \) etc that enter Eq.(7) are defined by

\[ (\kappa_N, \kappa_r) = \frac{(m_N, m_r)}{\sqrt{2M_W \cos \beta}}, \quad (\kappa_E, \kappa_\nu) = \frac{(m_E, m_\nu)}{\sqrt{2M_W \sin \beta}}. \] (9)

Using these interactions we have

\[ d_W^x = \frac{g_2^2}{32\pi^2 M_W^2} \sum_{j=1,2} m_\nu_j \text{Im}(D^\nu_{L11} D^\nu_{R12} D^\nu_{R21}) I_1 \left( \frac{m_\nu_j^2}{M_W^2} \right), \]

\[ d^x_\tau = -\frac{g_2^2}{16\pi^2} \sum_{j=1}^2 \sum_{k=1}^4 \frac{m_{\chi_j}^+}{m_\nu_k} \text{Im}(\eta_{jk} A(m_{\chi_j}^+/m_\nu_k)), \quad d^0_\tau = \frac{1}{2 \pi^2} \sum_{j=1}^2 \sum_{k=1}^4 \frac{m_{\chi_j}^0}{m_\tau_k} \text{Im}(\zeta_{jk} B(m_{\chi_j}^0/m_\tau_k)). \] (10)

where \( I_1(r), A(r) \) and \( B(r) \) are defined as follows

\[ I_1(r) = \frac{2}{(1-r)^2} [1 + \frac{1}{4} r + \frac{1}{4} r^2 + \frac{3}{2} \ln r], \quad A(r) = \frac{1}{2(1-r)^2} [3 - r + \frac{2}{(1-r)^2}], \quad B(r) = \frac{1}{2(1-r)^2} [1 + r + \frac{2}{(1-r)^2}]. \] (11)

and where

\[ \eta_{jk} = \left[ -D^\tau_{R11} \kappa_r U^*_j 1_k + D^\tau_{R12} \kappa_r U^*_j 1_k - D^\tau_{R11} \kappa_N U^*_j 2_k + D^\tau_{R12} \kappa_N U^*_j 2_k \right], \]

\[ \zeta_{jk} = 2 [\alpha_{\tau j} D^\tau_{R11} \tilde{D}_{1k}^\tau - \gamma_{\tau j} D^\tau_{R11} \tilde{D}_{3k}^\tau + \beta_{E r j} D^\tau_{R12} \tilde{D}_{4k}^\tau - \delta_{E r j} D^\tau_{R12} \tilde{D}_{2k}^\tau], \]

\[ \alpha_{E r j} = \beta_{E r j} = \gamma_{E r j} = \delta_{E r j} = \frac{g_2 m_{E} X_{1j}'}{2 m_{\nu} \sin \beta}. \] (12)

The matrix elements \( \tilde{D}^\nu_{\tau \gamma} \) are the diagonalizing matrices of the sneutrino and slepton \( 4 \times 4 \) mass matrices (see the Appendix). The couplings that enter \( \zeta_{jk} \) in Eq.(12) are given by

\[ \alpha_{E r j} = \frac{g_2 m_{E} X_{1j}'}{2 m_{\nu} \sin \beta}, \quad \beta_{E r j} = e X_{1j}' + \frac{g_2}{\cos \theta_W} X_{2j}' \left( \frac{1}{2} - \sin^2 \theta_W \right), \]

\[ \gamma_{E r j} = e X_{1j}' - \frac{g_3}{\cos \theta_W} X_{2j}', \quad \delta_{E r j} = -\frac{g_2 m_{E} X_{4j}}{2 m_{\nu} \sin \beta}. \] (13)
and
\[ \alpha_{\tau j} = \frac{g_2 m_\tau X_{3j}}{2 m_W \cos \beta}, \quad \beta_{\tau j} = -e X'_{1j} + \frac{g_2}{\cos \theta_W} X'_{2j}(\frac{1}{2} + \sin^2 \theta_W), \]
\[ \gamma_{\tau j} = -e X'_{1j} + \frac{g_2 \sin^2 \theta_W}{\cos \theta_W} X'_{2j}, \quad \delta_{\tau j} = -\frac{g_2 m_\tau X_{3j}}{2 m_W \cos \beta}, \] (14)

where
\[ X'_{1j} = (X_{1j} \cos \theta_W + X_{2j} \sin \theta_W), \quad X'_{2j} = (-X_{1j} \sin \theta_W + X_{2j} \cos \theta_W), \] (15)

and where the matrix \( X \) diagonalizes the neutralino mass matrix so that
\[ X^T M_{\tilde{\chi}} X = \text{diag}(m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}, m_{\tilde{\chi}_3}, m_{\tilde{\chi}_4}). \] (16)

3. EDM OF THE TAU NEUTRINO

The edm of the tau neutrino receives contributions from the diagrams of Fig.(2). Using the interactions of Eq.(3) the contributions from the loop diagrams of Fig.(2a) and Fig.(2b) are as follows:
\[ d^{2(a)}_\nu = -\frac{g_2}{32 \pi^2 M_W^2} \sum_{j=1,2} m_{\tau j} I_m(D_{L11}^\nu D_{L1j}^\nu D_{R21}^\nu D_{R2j}^\nu) I_1\left(\frac{m_{\tau j}^2}{M_W^2}\right), \]
\[ d^{2(b)}_\nu = -\frac{g_2}{32 \pi^2 M_W^2} \sum_{j=1,2} m_{\tau j} I_m(D_{L11}^\nu D_{L1j}^\nu D_{R21}^\nu D_{R2j}^\nu) I_2\left(\frac{m_{\tau j}^2}{M_W^2}\right) \] (17)

where \( I_1(r) \) is given by Eq.(11) and \( I_2(r) \) is given by
\[ I_2(r) = \frac{2}{(1-r)^2} \left[ 1 - \frac{11}{4} r + \frac{1}{4} r^2 - \frac{3r^2 \ln r}{2(1-r)} \right] \] (18)

Similarly, the loop contributions of Figs (2c) and (2d) to \( d_\nu \) are given by
\[ d^{2(c)}_\nu = -\frac{1}{16 \pi^2} \sum_{j=1}^2 \sum_{k=1}^4 \frac{m_{\chi_j}^+}{m_{\tilde{\tau}_k}^2} I_m(S_{jk} T_{jk}^*) B\left(\frac{m_{\chi_j}^+}{m_{\tilde{\tau}_k}^2}\right), \]
\[ d^{2(d)}_\nu = \frac{1}{16 \pi^2} \sum_{j=1}^2 \sum_{k=1}^4 \frac{m_{\chi_j}^+}{m_{\tilde{\tau}_k}^2} I_m(S_{jk} T_{jk}^*) A\left(\frac{m_{\chi_j}^+}{m_{\tilde{\tau}_k}^2}\right), \] (19)

where \( S_{jk} \) and \( T_{jk} \) are given by
\[ S_{jk} = -g_2 [D^\nu_{R11} \kappa_{\nu} V_{j2} \tilde{D}^\tau_{1k} - D^\nu_{R12} V_{j1} \tilde{D}^\tau_{2k} + D^\nu_{R12} \kappa_{\nu} V_{j2} \tilde{D}^\tau_{2k}], \]
\[ T_{jk} = -g_2 [D^\nu_{L11} \kappa_{\nu} U_{j2} \tilde{D}^\tau_{3k} - D^\nu_{L12} U_{j1} \tilde{D}^\tau_{1k} + D^\nu_{L12} \kappa_{\nu} U_{j2} \tilde{D}^\tau_{2k}]. \] (20)

and \( A(r) \) and \( B(r) \) are as defined in Eq.(11).
FIG. 2: The loop contribution to the electric dipole moment of the tau neutrino ($\nu_\tau$) via exchange of the $W$ boson and of tau and mirror lepton denoted by $\tau_j$, and via exchange of the charginos ($\tilde{\chi}_j$), sleptons and of mirror sleptons denoted by ($\tilde{\tau}_k$).

4. NUMERICAL ANALYSIS

The mixing matrices between leptons and mirrors are diagonalized using bi-unitary matrices (see the Appendix). So we parametrize the mixing between $\tau$ and $E_\tau$ by the angles $\theta_L, \theta_R, \chi_L$ and $\chi_R$, and the mixing between $\nu$ and $N$ by the angle $\phi_L, \phi_R, \xi_L$ and $\xi_R$ where

$$D^\tau_L = \begin{pmatrix} \cos \theta_L & -\sin \theta_L e^{-i\chi_L} \\ \sin \theta_L e^{i\chi_L} & \cos \theta_L \end{pmatrix}, \quad D^\nu_L = \begin{pmatrix} \cos \phi_L & -\sin \phi_L e^{-i\xi_L} \\ \sin \phi_L e^{i\xi_L} & \cos \phi_L \end{pmatrix}, \quad (21)$$

and $D^\tau_R$ and $D^\nu_R$ can be gotten from $D^\tau_L$ and $D^\nu_L$ by the following substitution: $D^\tau_L \rightarrow D^\tau_R, \theta_L \rightarrow \theta_R, \chi_L \rightarrow \chi_R$, and $D^\nu_L \rightarrow D^\nu_R, \phi_L \rightarrow \phi_R, \xi_L \rightarrow \xi_R$. We note that the phases $\chi_{L,R}$ arise from the couplings $f_3$ and $f_4$ while the phases $\xi_{L,R}$ arise from the couplings $f_3$ and $f_5$ through the relations

$$\chi_R = \text{arg}(m_\tau f_3^* + m_E f_4^*), \quad \chi_L = \text{arg}(m_\tau f_4^* + m_E f_3),$$

$$\xi_R = \text{arg}(-m_\nu f_3^* + m_N f_5^*), \quad \xi_L = \text{arg}(m_\nu f_5^* - m_N f_3). \quad (22)$$

However, these four parameters are not independent since the input of three phases of $f_3$, $f_4$ and $f_5$ would produce these four parameters. For the case of lepton and neutrino masses arising from hermitian matrices, i.e., when $f_4 = f_3^*$ and $f_5 = -f_3^*$ we have $\theta_L = \theta_R, \phi_L = \phi_R, \chi_L = \chi_R = \chi$ and $\xi_L = \xi_R = \xi$. Further, here we have the relation $\xi = \chi + \pi$ and thus the $W$-exchange terms of the edms for tau neutrinos and tau leptons vanish. However, more generally the lepton and the neutrino mass matrices are not hermitian and they generate non-vanishing contributions to the EDMs. Thus the input
parameters for this sector of the parameter space are $m_\tau, m_E, m_\nu, m_N, f_5$ with $f_3, f_4$ and $f_5$ being complex masses with CP violating phases $\chi_3, \chi_4$ and $\chi_5$ respectively. For the slepton mass matrices we need the extra input parameters of the susy breaking sector, $\tilde{M}_L, \tilde{M}_E, \tilde{M}_\tau, \tilde{M}_\chi, \tilde{M}_N, A_r, A_E, A_\nu, A_N, \mu, \tan \beta$. The chargino and neutralino sectors need the extra two parameters $\tilde{m}_1, \tilde{m}_2$. We will assume that the only parameters that have phases in the above set are $A_E, A_N, A_r$ and $A_\nu$. These phases are $\alpha_E, \alpha_N, \alpha_\tau$ and $\alpha_\nu$ respectively. To simplify the analysis we set the phases $\alpha_\nu = \alpha_\tau = 0$. With this in mind the only contributions to the edm of the tau lepton and tau neutrino arise from mixing terms between the scalar matter - scalar mirrors, fermion matter - fermion mirror and finally between mirrors among themselves in the scalar sector. Thus in the absence of mirror part of the lagrangian, the edms of taus and neutrinos vanish. We can thus isolate the role of the CP violating phases in this sector and see the size of its contribution. The $4 \times 4$ mass matrices of sleptons and sneutrinos are diagonlized numerically. Thus the CP violating phases that would play a role in this analysis are

$$\chi_3, \chi_4, \chi_5, \alpha_E, \alpha_N.$$  

To reduce the number of input parameters we assume $\tilde{M}_a = m_0, a = L, E, \tau, \chi, \nu, N$ and $|A_i| = |A_0|, i = E, N, \tau, \nu$.

In Fig.(3), we give a numerical analysis of the edm of the tau lepton and discuss its variation with the parameter $\chi_3$ (left), with $|f_3|$ (middle) and with $\alpha_N$ (right). Regarding $\chi_3$ it enters $D^\nu, D^\tau, \tilde{D}^\nu$ and $\tilde{D}^\tau$ and as a consequences all diagrams in Fig.(1) that contribute to the edm of the tau are affected. The phase $\alpha_N$, however, enters only in the chargino exchange contribution since it enters $\tilde{D}^\nu$ and thus only the part of the tau edm arising from the chargino exchanged is affected by variations of $\alpha_N$. We note that the various diagrams Figs.(1a)- Fig.(1c) that contribute to the tau edm can add constructively or destructively in the latter case generating large cancellations reminiscent of the cancellation mechanism for the edm of the electron and for the neutron. Of course the desirable larger contributions for the tau edm occur away from the cancellation regions. The analysis of Fig.(3) shows that a tau edm as large $10^{-18} - 10^{-19} \text{ecm}$ can be gotten which is only about 2 orders of magnitude below the current experimental limits of Eq.(1). A similar analysis for the tau neutrino edm is given in Fig.(4). Here again one finds that the tau neutrino edm as large as $10^{-18} - 10^{-19} \text{ecm}$ can be gotten and again it lies only a couple of orders of magnitude below the current experimental limit of Eq.(2).

As discussed in Sec.(1) the scale of the vector like multiplets is unknown. They could lie in the sub TeV region but on the other hand they could also be several TeV size and escape direct detection even at the LHC. This is especially true for leptonic vector like multiplets since the discovery reach for color singlet leptonic states at hadronic machines is
typically much smaller than for the color particles. In this context it is then interesting to investigate the contributions to the tau lepton edm and to the tau neutrino edm from vector like leptonic multiplets in the TeV range. A comparison of these edms when the leptonic vector like multiplet lies in the sub TeV region vs in the TeV region is given in Table 1 below. It turns out that the dependence of the edms on the masses of the mirror leptons is a rather complicated one. Thus there are supersymmetric and non-supersymmetric contributions which have different dependence on the mirrors and mirror slepton masses. Thus in certain regions of the parameter space as the mirror leptons masses grow, the chargino and neutralino contributions are suppressed much faster than the W-exchange terms. In the susy contributions, both the couplings and form factors that contain the mirror lepton masses explicitly decrease as the mirror spectrum increases. In the W-exchange term, there is a competition between the couplings and form factors. The first term decreases while the latter increases and, in indeed a suppression of the edms occurs but here it is much slower than in the case of susy case. These phenomena are illustrated in the analysis of Table 1 which shows that the suppression of the W exchange terms in both tau and neutrino edms is much slower rate than the other components in this specific part of the parameter space.

Table 1:

| $m_{E}(TeV)$ | $m_{N}(TeV)$ | $d_{\tau}^{W e.cm}$ | $d_{\tau}^{+}e.cm$ | $d_{\tau}^{X0}e.cm$ | $d_{\nu}^{W e.cm}$ | $d_{\nu}^{X+}e.cm$ |
|--------------|--------------|----------------------|-------------------|-------------------|-------------------|-------------------|
| 0.1          | 0.2          | $6.5 \times 10^{-18}$ | $-3.4 \times 10^{-18}$ | $5.0 \times 10^{-19}$ | $3.7 \times 10^{-18}$ | $-2.4 \times 10^{-18}$ |
| 2.0          | 1.0          | $4.0 \times 10^{-20}$ | $-7.2 \times 10^{-22}$ | $3.0 \times 10^{-23}$ | $5.1 \times 10^{-20}$ | $-7.1 \times 10^{-22}$ |

Table caption: A sample illustration of the contributions to the electric dipole moments of $\nu_\tau$ and of $\tau$. The in puts are: $\tan \beta =5$, $|f_3|=90$, $|f_4|=120$, $|f_5|=75$, $m_0=150$, $|A_0|=100$, $\tilde{m}_1=75$, $\tilde{m}_2=150$, $\mu=130$, $\chi_3=-1.0$, $\chi_4=0.6$, $\chi_5=-0.8$, $\alpha_E=0.3$ and $\alpha_N=0.6$. All masses are in units of GeV and all angles are in radian.

The analysis given above shows that even if the vector like particles lie in the TeV range they could contribute a significant amount, i.e., $O(10^{-20}) ecm$ which is $O(10^{14})$ larger than what the Standard Model predicts and only three order of magnitude smaller than the current limits. The above results do not appear outside the realm of detection in future experiment with improved sensitivity. Further, the results above could have possible astrophysical implications.

5. CONCLUSION

In this paper we have considered extensions of the MSSM with vector like multiplets. We have specifically focused on the leptonic sector and considered mixings between the
sequential generation leptons and the mirrors in the vector like multiplets. For the first two
generations of leptons the $V - A$ structure of the weak interactions are very well established.
However, this is not the case for the third generation leptons. Thus for the third generation
leptons we consider small mixings of the tau lepton and of the tau neutrino with the mirrors
in the vector like generation. An analysis of the electric dipole moment of the tau lepton is
carried out in this framework. Further, we also compute the EDM of the tau neutrino. It
is found that the predictions of the EDMs in the model can be as large as just a couple of
orders of magnitude below the current experimental predictions. Thus an improvement in
experiment by this order of magnitude will begin to test the predictions of the model. These
results are very encouraging for the possible observation of the EDM of the tau lepton and
of the EDM of the tau neutrino in improved experiment.

Acknowledgments: This research is supported in part by NSF grant PHY-0757959 and
by PHY-0704067.

APPENDIX: MASS MATRICES OF LEPTONS AND SLEPTONS AND THEIR
MIRRORS

In this appendix we write down the mass matrices for the leptons, neutrinos and sleptons
and their mirrors that enter in the computations of the edms of the tau neutrino and
tau lepton discussed in the text of the paper. In deducing these matrices we need the
transformation properties of the leptons and their mirrors. Thus under $SU(3)_C \times SU(2)_L \times
U(1)_Y$ the leptons transform as follows

$$
\psi_L \equiv \left( \begin{array}{c} \nu_L \\ \tau_L \end{array} \right) \sim (1, 2, -1/2), \tau^c_L \sim (1, 1, 1), \nu^c_L \sim (1, 1, 0),
$$

(24)

where the last entry on the right hand side of each $\sim$ is the value of the hypercharge $Y$
deﬁned so that $Q = T_3 + Y$ and we have included in our analysis the singlet ﬁeld $\nu^c$. These
leptons have $V - A$ interactions. Let us now consider mirror leptons in the vector like
multiplets which have $V + A$ interactions (For previous works on mirrors see [21]). Their
quantum numbers are as follows

$$
\chi^c \equiv \left( \begin{array}{c} E^c_{\tau L} \\ N^c_L \end{array} \right) \sim (1, 2, 1/2), E_{\tau L} \sim (1, 1, -1), N_L \sim (1, 1, 0).
$$

(25)

We assume that the mirrors of the vector like generation escape acquiring mass at the GUT
scale and remain light down to the electroweak scale where the superpotential of the model
for the lepton part may be written in the form

\[
W = \epsilon_{ij}[f_1 \hat{H}_1 \hat{\psi}_L^c \hat{\tau}_R^c + f'_1 \hat{H}_2 \hat{\psi}_L^c \hat{\tau}_R^c + f_2 \hat{H}_1 \hat{\chi} \hat{N}_L + f'_2 \hat{H}_2 \hat{\chi} \hat{E}_R] \\
+ f_3 \epsilon_{ij} \hat{\chi} \hat{\psi}_L^c + f_4 \hat{\psi}_L^c \hat{E}_R + f_5 \hat{\psi}_L^c \hat{N}_L.
\]  \tag{26}

Mixings of the above type can arise via non-renormalizable interactions\[^9\]. Consider, for example, a term such as \(1/M_P \tilde{\nu}_L N_L \Phi_1 \Phi_2\). If \(\Phi_1\) and \(\Phi_2\) develop VEVs of size \(10^9-10^10\), a mixing term of the right size can be generated.

To get the mass matrices of the leptons and of the mirror leptons we replace the superfields in the superpotential by their component scalar fields. The relevant parts in the superpotential that produce the lepton and mirror lepton mass matrices are

\[
W = f_1 \hat{H}_1 \hat{\tau}_R^c + f'_1 \hat{H}_2 \hat{\nu}_L \hat{\nu}_R^c + f_2 \hat{H}_1 \hat{N}_R^c \hat{N}_L^c + f'_2 \hat{H}_2 \hat{E}_R \hat{E}_R \\
+ f_3 \hat{E}_R \hat{\tau}_L^c - f_3 \hat{N}_R^c \hat{\nu}_L^c + f_4 \hat{E}_R \hat{\nu}_R^c + f_5 \hat{E}_R \hat{N}_L^c.
\]  \tag{27}

The mass terms for the lepton and their mirrors arise from the part of the lagrangian

\[
\mathcal{L} = -\frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j + H.c.
\]  \tag{28}

where \(\psi\) and \(A\) stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, \((< H_1^1 >= v_1/\sqrt{2}\) and \(< H_2^2 >= v_2/\sqrt{2}\), we have the following set of mass terms written in 4-spinors for the fermionic sector

\[
-\mathcal{L}_m = \left( \hat{\tau}_R \hat{E}_R \right) \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 \\ f_3 & f'_2 v_2/\sqrt{2} \end{pmatrix} \begin{pmatrix} \tau_L \\ E_{\tau L} \end{pmatrix} + \left( \hat{\nu}_R \hat{\bar{N}}_R \right) \begin{pmatrix} f'_1 v_2/\sqrt{2} & f_5 \\ -f_3 & f_2 v_1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} + H.c.
\]

Here the mass matrices are not Hermitian and one needs to use bi-unitary transformations to diagonalize them. Thus we write the linear transformations

\[
\begin{pmatrix} \tau_R \\ E_{\tau R} \end{pmatrix} = D_R^\tau \begin{pmatrix} \tau_{1 R} \\ E_{\tau_{2 R}} \end{pmatrix}, \quad \begin{pmatrix} \tau_L \\ E_{\tau L} \end{pmatrix} = D_L^\tau \begin{pmatrix} \tau_{1 L} \\ E_{\tau_{2 L}} \end{pmatrix},
\]  \tag{29}

such that

\[
D_R^\dagger \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 \\ f_3 & f'_2 v_2/\sqrt{2} \end{pmatrix} D_L^\tau = \text{diag}(m_{\tau_1}, m_{\tau_2}),
\]  \tag{30}

and the same holds for the neutrino mass matrix so that

\[
D_R^{\nu L} \begin{pmatrix} f'_1 v_2/\sqrt{2} & f_5 \\ -f_3 & f_2 v_1/\sqrt{2} \end{pmatrix} D_L^{\nu} = \text{diag}(m_{\nu_1}, m_{\nu_2}).
\]  \tag{31}
Here $\tau_1, \tau_2$ are the mass eigenstates and we identify the tau lepton with the eigenstate 1, i.e., $\tau = \tau_1$, and identify $\tau_2$ with a heavy mirror eigenstate with a mass in the hundreds of GeV. Similarly $\nu_1, \nu_2$ are the mass eigenstates for the neutrinos, where we identify $\nu_1$ with the light neutrino state and $\nu_2$ with the heavier mass eigenstate. By multiplying Eq.(30) by $D^r_L$ from the right and by $D^r_R$ from the left and by multiplying Eq.(31) by $D^r_L$ from the right and by $D^r_R$ from the left, one can equate the values of the parameter $f_3$ in both equations and we can get the following relation between the diagonalizing matrices $D^r$ and $D^\nu$

$$m_{\tau r}D^r_{LR1}D^\nu_{LR1} + m_{\nu r}D^\nu_{LR2}D^r_{LR2} = -[m_{\nu 1}D^\nu_{LR1}D^r_{LR1} + m_{\nu 2}D^\nu_{LR2}D^r_{LR2}].$$

(32)

Next we consider the mixings of the charged sleptons and the charged mirror sleptons. We write the superpotential in terms of the scalar fields of interest as follows

$$W = -\mu \epsilon_{ij} \tilde{H}^i \tilde{H}^j + \epsilon_{ij}[f_1 H^i \tilde{\psi}^j_L \tilde{\tau}^*_L + f_1 H^i \tilde{\psi}^j_L \tilde{\nu}^*_L + f_2 H^i \tilde{\chi}^{ij} \tilde{N}_L + f_2 H^i \tilde{\chi}^{ij} \tilde{E}_L]$$

$$+ f_3 \epsilon_{ij} \tilde{\chi}^{ij} \tilde{\psi}^i_L + f_4 \tilde{\tau}^*_L \tilde{E}_L + f_5 \tilde{\nu}^*_L \tilde{N}_L.$$ 

(33)

The mass matrix of the slepton - mirror slepton comes from three sources, the F term, the D term of the potential and the soft susy breaking terms. Using the above superpotential and after the breaking of the electroweak symmetry we get for the mass part of the lagrangian $\mathcal{L}_F$ and $\mathcal{L}_D$ the following set of terms

$$- \mathcal{L}_F = (m^2_E + |f_3|^2) \tilde{E}_R \tilde{E}^*_R + (m^2_N + |f_3|^2) \tilde{N}_R \tilde{N}^*_R + (m^2_\nu + |f_4|^2) \tilde{\nu}_L \tilde{\nu}^*_L$$

$$+ (m^2_\nu + |f_5|^2) \tilde{\nu}_L \tilde{\nu}^*_L + (m^2_\tau + |f_2|^2) \tilde{\tau}_R \tilde{\tau}^*_R + (m^2_\tau + |f_3|^2) \tilde{\tau}_R \tilde{\tau}^*_L$$

$$+ (m^2_\nu + |f_3|^2) \tilde{\nu}_L \tilde{\nu}^*_L + \{-m_\nu \mu \tan \beta \tilde{\tau}_R \tilde{\tau}^*_L - m_N \mu \tan \beta \tilde{\tau}_L \tilde{\tau}^*_R - m_\nu \mu \cot \beta \tilde{\nu}_L \tilde{\nu}^*_R$$

$$- m_\nu f_5 (m_\nu f_3 - m_N f_3) \tilde{N}_L \tilde{\nu}^*_L + (m_N f_5 - m_\nu f_3) \tilde{N}_R \tilde{\nu}^*_R + h.c.\},$$

(34)

and

$$- \mathcal{L}_D = \frac{1}{2} m^2_Z \cos^2 \theta_W \cos 2 \beta \{\tilde{\nu}_L \tilde{\nu}^*_L - \tilde{\tau}_L \tilde{\tau}^*_L + \tilde{E}_R \tilde{E}^*_R - \tilde{N}_R \tilde{N}^*_R\}$$

$$+ \frac{1}{2} m^2_Z \sin^2 \theta_W \cos 2 \beta \{\tilde{\nu}_L \tilde{\nu}^*_L + \tilde{\tau}_L \tilde{\tau}^*_L - \tilde{E}_R \tilde{E}^*_R - \tilde{N}_R \tilde{N}^*_R + 2 \tilde{E}_L \tilde{E}^*_L - 2 \tilde{\tau}_R \tilde{\tau}^*_R\}.$$  

(35)

Next we add the general set of soft supersymmetry breaking terms to the scalar potential so that

$$V_{\text{soft}} = \tilde{M}_E \tilde{\psi}^*_L \tilde{\psi}^*_L + \tilde{M}_N \tilde{\chi}^{ij} \tilde{\chi}^{ij} + \tilde{M}_\nu \tilde{\psi}^*_L \tilde{\psi}^*_L + \tilde{M}_\nu \tilde{\tau}^*_L \tilde{\tau}^*_L + \tilde{M}_E \tilde{\nu}^*_L \tilde{\nu}^*_L + \tilde{M}_N \tilde{\nu}^*_L \tilde{\nu}^*_L$$

$$+ \epsilon_{ij} \{f_1 A_\tau H^i \tilde{\psi}^j_L \tilde{\tau}^*_L - f_1 A_\nu H^i \tilde{\psi}^j_L \tilde{\nu}^*_L + f_2 A_N H^i \tilde{\chi}^{ij} \tilde{N}_L - f_2 A_E H^i \tilde{\chi}^{ij} \tilde{E}_L + h.c.\}.$$  

(36)

From $\mathcal{L}_{F,D}$ and by giving the neutral Higgs their vacuum expectation values in $V_{\text{soft}}$ we can produce the the mass matrix $M^2_t$ in the basis ($\tilde{\tau}_L, \tilde{E}_L, \tilde{\tau}_R, \tilde{E}_R$). We label the matrix elements...
of these as \((M^2_\nu)_{ij} = M^2_{ij}\) where

\[
\begin{align*}
M^2_{11} &= \bar{M}^2_E + m^2_\nu + |f_3|^2 - m^2_Z \cos 2\beta (\frac{1}{2} - \sin^2 \theta_W), \\
M^2_{22} &= \bar{M}^2_E + m^2_\nu + |f_4|^2 + m^2_Z \cos 2\beta \sin^2 \theta_W, \\
M^2_{33} &= \bar{M}^2_\tau + m^2_\tau + |f_4|^2 - m^2_Z \cos 2\beta \sin^2 \theta_W, \\
M^2_{44} &= \bar{M}^2_\tau + m^2_\tau + |f_3|^2 + m^2_Z \cos 2\beta (\frac{1}{2} - \sin^2 \theta_W), \\
M^2_{12} &= M^2_{21} = m_E f_3^* + m_\tau f_4, \\
M^2_{13} &= M^2_{31} = m_\tau (A^*_\tau - \mu \tan \beta), \\
M^2_{14} &= M^2_{41} = 0, \\
M^2_{24} &= M^2_{42} = m_E (A^*_E - \mu \cot \beta), \\
M^2_{34} &= M^2_{43} = m_E f_4 + m_\tau f_3^*. 
\end{align*}
\]

(37)

Here the terms \(M^2_{11}, M^2_{13}, M^2_{21}, M^2_{33}\) arise from soft breaking in the sector \(\tilde{\tau}_L, \tilde{\tau}_R\). Similarly the terms \(M^2_{22}, M^2_{24}, M^2_{32}, M^2_{44}\) arise from soft breaking in the sector \(\tilde{E}_L, \tilde{E}_R\). The terms \(M^2_{12}, M^2_{21}, M^2_{23}, M^2_{32}, M^2_{14}, M^2_{41}, M^2_{34}, M^2_{43}\), arise from mixing between the staus and the mirrors. We assume that all the masses are of the electroweak size so all the terms enter in the mass\(^2\) matrix. We diagonalize this hermitian mass\(^2\) matrix by the unitary transformation \(\bar{D}^\dagger M^2_\nu \bar{D} = \text{diag}(m^2_{\tilde{\tau}_1}, m^2_{\tilde{\tau}_2}, m^2_{\tilde{\nu}_3}, m^2_{\tilde{\nu}_4})\). There is a similar mass\(^2\) matrix in the sneutrino sector. In the basis \((\tilde{\nu}_L, \tilde{\nu}_R, \tilde{\nu}_R, \tilde{\nu}_R)\) we can write the sneutrino mass\(^2\) matrix in the form \((M^2_\nu)_{ij} = m^2_{ij}\) where

\[
\begin{align*}
M^2_{11} &= \bar{M}^2_E + m^2_\nu + |f_3|^2 + \frac{1}{2} m^2_Z \cos 2\beta, \\
M^2_{22} &= \bar{M}^2_\tau + m^2_\tau + |f_3|^2, \\
M^2_{33} &= \bar{M}^2_\nu + m^2_\nu + |f_5|^2, \\
M^2_{44} &= \bar{M}^2_\nu + m^2_\nu + |f_3|^2 - \frac{1}{2} m^2_Z \cos 2\beta, \\
m^2_{12} &= m^2_{21} = m_\nu f_5 - m_N f_3^*, \\
m^2_{13} &= m^2_{31} = m_\nu (A^*_\tau - \mu \cot \beta), \\
m^2_{14} &= m^2_{41} = 0, \\
m^2_{23} &= m^2_{32} = 0, \\
m^2_{24} &= m^2_{42} = m_\tau A^*_\tau - \mu \tan \beta, \\
m^2_{34} &= m^2_{43} = m_\nu f_4 - m_\nu f_3^*. 
\end{align*}
\]

(38)

As in the charged slepton sector here also the terms \(m^2_{11}, m^2_{13}, m^2_{31}, m^2_{33}\) arise from soft breaking in the sector \(\tilde{\nu}_L, \tilde{\nu}_R\). Similarly the terms \(m^2_{22}, m^2_{24}, m^2_{42}, m^2_{44}\) arise from soft breaking in the sector \(\tilde{\nu}_L, \tilde{\nu}_R\). The terms \(m^2_{12}, m^2_{21}, m^2_{23}, m^2_{32}, m^2_{14}, m^2_{41}, m^2_{34}, m^2_{43}\), arise from mixing between the physical sector and the mirror sector. Again as in the charged lepton sector we assume that all the masses are of the electroweak size so all the terms enter in the mass\(^2\) matrix. This mass\(^2\) matrix can be diagonalized by the unitary transformation \(\bar{D}^\dagger M^2_\nu \bar{D} = \text{diag}(m^2_{\tilde{\nu}_1}, m^2_{\tilde{\nu}_2}, m^2_{\tilde{\nu}_3}, m^2_{\tilde{\nu}_4})\). The physical tau and neutrino states are \(\tau \equiv \tau_1, \nu \equiv \nu_1\), and the states \(\tau_2, \nu_2\) are heavy states with mostly mirror particle content. The states \(\tilde{\tau}_i, \tilde{\nu}_i; i = 1 - 4\) are the slepton and sneutrino states. For the case of no mixing these limits are as follows: \(\tilde{\tau}_1 \rightarrow \tilde{\tau}_L, \tilde{\tau}_2 \rightarrow \tilde{E}_L, \tilde{\tau}_3 \rightarrow \tilde{\tau}_R, \tilde{\tau}_4 \rightarrow \tilde{E}_R, \tilde{\nu}_1 \rightarrow \tilde{\nu}_L, \tilde{\nu}_2 \rightarrow \tilde{\tilde{\nu}}_L, \tilde{\nu}_3 \rightarrow \nu_R, \tilde{\nu}_4 \rightarrow \tilde{\tilde{\nu}}_R\). The couplings \(f_3, f_4\) and \(f_5\) can be complex and thus the matrices \(D^r_{L,R}\) and \(D^\nu_{L,R}\) will have
complex elements that would produce electric dipole moments through their arguments discussed in the text of the paper. Also the trilinear couplings $A_{\nu,\tau,E,N}$ could be complex and produce electric dipole moment through the arguments of $\tilde{D}_\nu$ and $\tilde{D}_\tau$. We will assume for simplicity that this is the only part in the theory that has CP violating phases (For a recent review of CP violation see [18]). Thus the $\mu$ parameter is considered real along with the other trilinear couplings in the theory. In this way we can automatically satisfy the constraints on the edms of the electron, the neutron and of Hg and of Thallium.

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FIG. 3: Left: An exhibition of the dependence of $d_\nu$ on $\chi_3$ when $\tan \beta = 10$, $m_N = 100$, $|f_3| = 50$, $|f_4| = 70$, $|f_5| = 90$, $m_0 = 100$, $|A_0| = 150$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$, $\chi_4 = 0.4$, $\chi_5 = 0.6$, $\alpha_E = 0.5$, $\alpha_N = 0.8$, and $m_E = 300, 250, 200, 150, 100$ (in ascending order at $\chi_3 = 0$). Middle: An exhibition of the dependence of $d_\nu$ on $|f_5|$ when $\tan \beta = 10$, $m_N = 120$, $m_E = 100$, $|f_3| = 80$, $|f_5| = 60$, $m_0 = 150$, $|A_0| = 100$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$ GeV $\chi_4 = 0.3$, $\chi_5 = 0.7$, $\alpha_E = 0.4$, $\alpha_N = 1.0$, $\alpha_f = 0$, $\alpha_N = 0.4$, 0.8, 1.2 (in ascending order). Right: An exhibition of the dependence of $d_\nu$ on $\alpha_N$ when $\tan \beta = 20$ $m_N = 100$, $|f_3| = 70$, $|f_4| = 50$, $|f_5| = 80$, $m_0 = 120$, $|A_0| = 130$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$, $\chi_4 = 0.5$, $\chi_5 = 0.6$, $\chi_5 = 0.7$ and $\alpha_E = 0.6$, and $m_E = 180, 130, 80$ (in ascending order at $\alpha_N = 0$). Masses in GeV and angles in rad here and in figures below.

FIG. 4: Left: An exhibition of the dependence of $d_\nu$ on $\chi_3$ with the input $\tan \beta = 10$, $m_E = 120$, $|f_3| = 60$, $|f_4| = 80$, $|f_5| = 100$, $m_0 = 100$, $|A_0| = 170$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$, $\chi_4 = 0.2$, $\chi_5 = 0.7$, $\alpha_E = 0.6$ and $\alpha_N = 0.4$, and $m_N = 300, 250, 200, 150, 100$ (in ascending order at $\chi_3 = 0$). Middle: An exhibition of the dependence of $d_\nu$ on $|f_3|$ with the input $\tan \beta = 10$, $m_N = 120$, $m_E = 100$, $|f_4| = 80$, $|f_5| = 60$, $m_0 = 150$, $|A_0| = 100$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$ GeV and the phases $\chi_4 = 0.3$, $\chi_5 = 0.7$, $\alpha_E = 0.4$ and $\alpha_N = 1.0$, and $\chi_3 = 0.4$, 0.8, 1.2 (in ascending order). Right: An exhibition of the dependence of $d_\nu$ on $\alpha_E$ with the input $\tan \beta = 20$ $m_E = 120$, $|f_3| = 80$, $|f_4| = 60$, $|f_5| = 90$, $m_0 = 100$, $|A_0| = 120$, $m_1 = 50$, $m_2 = 100$, $\mu = 150$, $\chi_3 = 0.4$, $\chi_4 = 0.8$, $\chi_5 = 0.7$ and $\alpha_N = 0.5$, and $m_N = 190, 140, 90$ (in ascending order at $\alpha_E = 0$).