Observational Constraints on Agegraphic Dark Energy

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In this paper, we use the Type Ia supernova data as well as the CMB and LSS data to constrain the agegraphic dark energy model recently proposed by Cai. Due to its peculiar nature, the parameter $n$ of this model cannot be well constrained by the SNIa data, while the other parameter $\Omega_{m0}$ can be constrained to be $0.34 \pm 0.04$. When combined with CMB and LSS data, the range of $1\sigma$ confidence level for $n$ is greatly narrowed, albeit still very large. The best fit result is $\Omega_{m0} = 0.28 \pm 0.02$, which is consistent with most observations like WMAP and SDSS, and $n = 3.4$, of which a meaningful range of confidence level can not be obtained due to the fact that the contours are not closed. Despite of this result, we conclude that for $n > 1$ this model is consistent with SNIa, CMB and LSS observations. Furthermore, the fitting results indicate a generalized definition for the agegraphic dark energy.

I. INTRODUCTION

The dark energy problem has become undoubtedly one of the most challenging problems in modern physics ever since the discovery of the accelerating expansion of the universe\cite{1}. Various theoretical models to explain the origin of the cosmic acceleration have been proposed (for a recent review, see \cite{2} and the references therein). Although the cosmological constant is the simplest one among other models, it suffers from the so-called coincidence problem and the parameter in this model has to be fine-tuned in order to be consistent with observations\cite{3}. One of the reasons for this awkwardness is that it identifies the cosmological constant with the vacuum energy of the quantum field theory in Minkowski spacetime. At the cosmological scales, however, the effect of gravity is significant and the above result may break down. A complete solution to the cosmological problem is expected to be given by a full theory of quantum gravity, which is unknown as yet. Luckily, the so-called Holographic Principle \cite{4} has shown some important features of quantum gravity. Based on this principle, Cohen et. al.\cite{5} suggested a relation between the IR cut-off and the UV cut-off in quantum field theory. That is, in a box of size $L$, the quantum zero-point energy, related to the UV cut-off, should not exceed the mass of a black hole of the same size. This leads to the holographic vacuum energy given by

$$\rho_{\Lambda} = \frac{3c^2M_p^2}{L^2},$$

where $M_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass and $3c^2$ is by convention a numerical factor. Although $L$ chosen as the Hubble scale $H_0^{-1}$ or the particle horizon can lead to an energy density consistent with current observation, the equation of state for the vacuum energy in these two cases is always larger than $-1/3$\cite{6,7}, therefore they cannot play the role of dark energy. Li\cite{7} proposed to choose $L$ as the future event horizon leading to the model of holographic dark energy which gives a correct equation of state. This model has been tested by observational data\cite{8-12} and is consistent with them. However, it is plagued on the fundamental level due to its assumption that the current properties of the dark energy is determined by the future evolution of the universe, which seems to violate causality. Moreover, it has been argued that this model can be inconsistent with the age of the universe\cite{13}.

Recently, Cai\cite{14} proposed a model, called 'agegraphic dark energy', based on the Károlyházy uncertainty relation\cite{15}

$$\delta t = \beta t_p^{2/3}t^{1/3},$$

where $\beta$ is a numerical factor of order one and $t_p$ is the Planck time. This relation arises from quantum mechanics combined with general relativity, and it imposes an upper limit of accuracy in any measurement.

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of distance \(t\) (with the speed of light \(c = 1\)) in Minkowski spacetime. Combined with the time-energy uncertainty relation in quantum mechanics, the energy density of the metric fluctuations in Minkowski spacetime is \[16, 17\]

\[
\rho_q \sim \frac{E_{\delta t}}{\delta t^3} \sim \frac{1}{t^2 \delta t^2},
\]

where \(E_{\delta t}\) is the energy fluctuation within a cell of size \(\delta t\). Based on this relation, Cai proposed the agegraphic dark energy as \[14\]

\[
\rho_q = \frac{3n^2 M_p^2}{T^2},
\]

where \(T\) is the age of the universe defined as

\[
T = \int_0^t dt' = \int_0^a \frac{da}{H a} = \int_z^\infty \frac{dz}{(1 + z)H},
\]

and \(n\) is a parameter of order one representing some uncertainties, such as the species of quantum fields in the universe or the effect of curved spacetime (since equation (3) is derived for Minkowski spacetime). The form of this model mimics a model of holographic dark energy with \(L\) chosen as the age of the universe. It is not surprising since, as is mentioned in \[14\], the Károlyházy uncertainty relation is also a reflection of the entanglement between UV and IR scale in effective quantum field theory. Moreover, due to using the age of the universe instead of the future event horizon, the new model is free of the causality problem which undermines the holographic dark energy model. The effect of the interaction between the agegraphic dark energy and dark matter on the cosmological evolution has been investigated in \[18, 19\].

This paper is organized as follows. In Sec. II we introduce the agegraphic dark energy model and compare its features with those of the holographic dark energy model. In Sec. III we use data from Type Ia supernova as well as CMB and LSS observations to constrain the parameters of the model. Sec. IV is devoted to conclusions.

II. THE DARK ENERGY MODEL

In this section we review the agegraphic dark energy model proposed in \[14\]. Then we confront this model with the Type Ia supernova observation as well as a joint analysis together with CMB and large scale structure(LSS) data in the next section. Now we assume a spatially flat FRW universe with matter component \(\rho_m\) and the agegraphic dark energy \(\rho_q\). The Friedmann equation is

\[
H^2 = \frac{1}{3M_p^2}(\rho_q + \rho_m),
\]

where \(\rho_m = \rho_{m0}(1 + z)^3\). We define \(\Omega_q = \rho_q/3M_p^2H^2\) and \(\Omega_m = \rho_m/3M_p^2H_0^2 = \Omega_{m0}(1 + z)^3\) and recast equation (6) into

\[
E = \frac{H}{H_0} = \sqrt{\frac{\Omega_m}{1 - \Omega_q}}.
\]

Once the evolution of \(\Omega_q\) is known, \(E\) is determined. To find out how \(\Omega_q\) evolves with time(redshift), we combine equation (4) and (5) to get

\[
\int_z^\infty \frac{dz}{(1 + z)H} = \frac{n}{H \sqrt{\Omega_q}}.
\]

Taking derivative with respect to \(z\) in both sides of equation (8) leads to

\[
\Omega_q' = \frac{-1}{1 + z} \left(3 - \frac{2}{n} \sqrt{\Omega_q} \Omega_q(1 - \Omega_q)\right),
\]
where the prime denotes \( d/dz \). With the initial condition implied by setting \( z = 0 \) in equation (10)

\[
\Omega_{q0} + \Omega_{m0} = 1 ,
\]

we can solve this differential equation to obtain the evolution of \( \Omega_q \), and finally determine \( E(z) \). Here one point is worth particular mentioning. As mentioned in [14], adding a \( z \)-independent term \( \delta_q \) to the LHS of equation (8) leads to the same differential equation (9), and moreover, imposing the initial condition (10) cannot guarantee a vanishing \( \delta_q \) [27]. As a result, the solution to equation (9) leads to a dark energy density which should be written in a more general form as

\[
\rho_q = \frac{3n^2M_p^2}{(\delta_q + T)^2} ,
\]

where \( \delta_q \) may generally be a function of \( n \) as well as \( \Omega_{m0} \) and \( H_0 \), which can be calculated by

\[
\delta_q = \frac{n}{H(z)\sqrt{\Omega_q(z)}} - \int_z^{\infty} \frac{dz'}{(1+z')H(z')} .
\]

Note that although \( z \) explicitly emerges in the RHS of the above expression, \( \delta_q \) is independent of \( z \) due to the subtraction of the two terms.

Then we consider the equation of state of the dark energy. From the conservation equation

\[
\dot{\rho}_q + 3H(1 + w_q)\rho_q = 0 ,
\]

we derive

\[
w_q = -1 - \frac{\dot{\rho}_q}{3H\rho_q} ,
\]

which leads to

\[
w_q = -1 + \frac{2}{3n} \sqrt{\Omega_q} .
\]

The quantum fluctuations can serve as dark energy for \( w_q < -1/3 \), that is, \( n > \sqrt{\Omega_q} \). For example, if we take \( \Omega_{q0} = 0.73 \), as indicated by WMAP [20] for LCDM, \( n_0 \) should be larger than 0.85 to behave as the dark energy. In fact, in order to drive the cosmic acceleration, \( n_0 \) should be even larger given the fact that currently the matter component is not negligible and the total effective equation of state should be taken into account. In addition, it is shown in [14] that \( \rho_q \) becomes dominant in the future, namely \( \Omega_q \rightarrow 1 \). Therefore, the universe will eternally accelerate if \( n > 1 \); for \( n < 1 \), current cosmic acceleration would be a transient process and the expansion would finally slow down in the future. We note that the same phenomenon may appear in the braneworld scenario as discussed in [21]. The evolution of \( w_q \) is shown in figure 2. It is interesting to compare this model with the holographic dark energy, for which we use a subscript ‘h’ to denote its corresponding quantities. For \( \Omega_h \), its evolution is determined by

\[
\Omega'_h = \frac{-1}{1+z} (1 + \frac{2}{c} \sqrt{\Omega_h})\Omega_h(1 - \Omega_h) ,
\]

with the same initial condition as (10). It is the underlined part that indicates the difference in the two models (if we identify \( n \) with \( c \)). For illustration, the evolution curves for both \( \Omega_q \) and \( \Omega_h \) with respect to \( z \) are plotted in figure 2. Note that equation (9) and (16) only depend on one parameter \( n \) and \( c \) respectively. The figure shows that for \( \Omega_q \), the curve moves downwards as \( n \) increases and asymptotically tends to a fixed position as a lower bound; for \( \Omega_h \), the curve moves upward as \( c \) increases and ends up asymptotically at a fixed position as an upper bound. The asymptotic behavior of the fractional energy density for large \( c \) or \( n \) in the two models can easily be seen from the corresponding differential equations:

\[
\Omega'_q = \frac{-1}{1+z} 3\Omega_q(1 - \Omega_q) ,
\]

\[
\Omega'_h = \frac{-1}{1+z} (1 + \frac{2}{c} \sqrt{\Omega_q})\Omega_q(1 - \Omega_q) ,
\]
FIG. 1: The evolution of $\Omega_q$ (left) and $\Omega_h$ (right) with respect to redshift $z$ with varying parameters $n$ and $c$. Here we assume $\Omega_{m0} = 0.27$. For $\Omega_q$ the traces asymptotically tend to a fixed curve (denoted by the red +’s) as a lower bound when $n$ increases; for $\Omega_h$ the traces asymptotically tend to a fixed curve (denoted by the red +’s) as an upper bound when $c$ increases. Of course the values of the parameter much larger than 1 is not physical, here we just use them as an illustration for the asymptotic behavior of the corresponding differential equations.

and

$$
\Omega_h' = -\frac{1}{1+z} \Omega_h (1 - \Omega_h). 
$$

(18)

As for the equation of state, for holographic dark energy

$$
w_h = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_h}. 
$$

(19)

Considering its future evolution, for $c \geq 1, w_h > -1$ forever, while for $c < 1, w_h$ may cross the phantom divide and end up with $w_h < -1$. This is a significant difference from the agegraphic dark energy, where $w_q$ can never be less than $-1$. In addition, as $z$ grows, $w_h \to -1/3$ whereas $w_q \to -1$ implying the agegraphic dark energy behaves like a cosmological constant at early time. This difference in high redshift region may exert different influence on the evolution of the early universe, such as the matter fluctuation and the formation of large scale structure. Further analysis based on cosmological perturbation theory together with the CMB observation may shed some light on distinguishing these two models. The evolution of the equation of state for both models is plotted in figure 2 for comparison.

III. CONSTRAINTS FROM TYPE IA SNE OBSERVATION

Now we perform the $\chi^2$ statistics to constrain the parameters $(n, \Omega_{m0})$ of the model in question with the goldset of 182 SNIa data compiled by Riess et.al.\[22\]. The observations of supernovae measure the apparent magnitude $m$, which is related to the luminous distance $d_L$ of an object (SN) at redshift $z$ as

$$
m(z) = M + 5 \log d_L(z) + 25, 
$$

(20)

where $M$ is the absolute magnitude, which can generally be considered to be the same for Type Ia supernovae as the standard candles. In a flat universe

$$
d_L = H_0^{-1}(1 + z) \int_0^z \frac{dz'}{E(z')}, 
$$

(21)
where the Hubble scale \( H_0^{-1} = 2997.9 h^{-1} \text{Mpc} \). In order to constrain the parameters, we compute the distance modulus \( \mu = m - M \), and minimize the quantity \( \chi^2 \) defined by

\[
\chi^2(n, \Omega_m) = \sum_{i} \frac{[\mu_{\text{obs}}(z_i) - \mu_h(z_i; n, \Omega_m)]^2}{\sigma_i^2},
\]

(22)

where \( \sigma_i \) is the observational uncertainty. Assuming that the errors are Gaussian, the likelihood is \( \mathcal{L} \propto e^{-\chi^2/2} \). In figure 3 we plot the contours of confidence level at 68.3\%, 95.4\% and 99\% in the \((n, \Omega_m)\) plane with \( h \) marginalized. The best fit values corresponding to the minimum of \( \chi^2 \) are \( n = 39 \) and \( \Omega_{m0} = 0.34 \), which are denoted by a red star on the plot. As we see, the parameter \( \Omega_{m0} \) is well constrained, and the range \( n < 1 \) is outside the 3\( \sigma \) confidence level. This indicates that an accelerated expansion in our model is compatible with current observations. However, the contours are not closed from above within the given range, and the point of the best fit value is on the border of the upper limit of \( 1 < n < 39 \). It seems that the range of \( n \) is not large enough to encompass the best fit value. We point out that it is the particularity of this model that makes the parameter \( n \) cannot be well constrained by SN data. As an illustration, we use equation (17) to calculate the \( \chi^2_{\text{min}} \) for \( n \to \infty \). The result is \( \chi^2_{\text{min}}|_{n=\infty} = 158.7422 \) with the best fit \( \Omega_{m0} = 0.34 \). Comparing this with \( \chi^2_{\text{min}} = 158.8526 \) within the range \( 1 < n < 39 \), the difference is negligible given the fact that \( n \) spans such a large range from 39 to \( \infty \). This implies that the parameter \( n \) within such a large range is always consistent with SN data. We further illustrate this in figure 4. As we see, in the low redshift region, the curves corresponding to different values of \( n \) are indistinguishable. For high redshift, however, we see that for \( n < 1 \) (e.g. \( n = 0.6 \) in the plot) the curve predicted by the model is remarkably not consistent with the data points, whereas for \( n > 1 \), all the curves, including the curve for \( n = \infty \)(the green line), are confined within a very narrow bunch which seems not significantly inconsistent with the data. Therefore, this model cannot be well constrained by using SN data alone.

This seems surprising since almost all models of dark energy can be constrained significantly by SN data. There would be no surprise, however, once we recall the asymptotic behavior of \( \Omega_q \) with increasing \( n \). In figure 4 it shows that the larger \( n \) is, the less difference it makes between two neighboring curves. Therefore, as \( n \) grows, changes in the quantity \( \chi^2 \) become less significant. In fact there is also a similar asymptotic behavior in the holographic dark energy, where the \( \Omega_h \) becomes insensitive to the change of \( n \) as \( n \) grows large. In that case, the best fit value \( c = 0.2 \approx 0.9 \) [9-12] lies in the range where \( \Omega_h \) is sensitive to varying \( n \). Thus the holographic model can be well constrained by SN data. Moreover, figure 4 also shows that the range where \( \Omega_h \) is sensitive to \( c \) is just that where \( \Omega_q \) is insensitive to \( n \), and vice versa. Thus, if we assume that the evolution of dark energy in both models should not be different significantly in the same range, we find that unfortunately the possible ‘best fit’ value for \( n \) falls in the range of insensitivity.
FIG. 3: 68.3%, 95.4% and 99% confidence contours for the parameters \((n, \Omega_m)\). It is obvious that the range for \(n < 1\) is outside the 3\(\sigma\) confidence contour.

FIG. 4: Illustration for the weakness of using SN data to constrain the agegraphic dark energy model. Here we assume \(\Omega_m = 0.28\) and \(h = 0.62\).

In order to improve the situation, we combine the constraints from CMB\(^{20}\) and LSS\(^{25}\) observations. The CMB shift parameter is the most model-independent parameter we can extract from the CMB data, which is given by\(^{23}\)

\[
R = \sqrt{\Omega_m} \int_0^{z_{rec}} \frac{dz}{E(z)},
\]

(23)
where \(z_{\text{rec}} = 1089\) represents the redshift of recombination. This parameter relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at \(z_{\text{rec}}\) and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations. Here we assume the value \(R = 1.70 \pm 0.03\) given by Wang and Mukherjee[24]. The \(\chi^2\) value is

\[
\chi^2_{\text{MB}} = \frac{(R - 1.70)^2}{0.03^2}.
\]

For LSS data, we use the measurement of the baryon acoustic peak (BAO) in the distribution of SDSS luminous red galaxies (LRG’s)[25]. This peak is the imprint left by the cosmic perturbation in early universe on the late-time non-relativistic matters, and it can provide an independent constraint on dark energy models. We use here the parameter \(A\) defined as

\[
A = \sqrt{\Omega_{m0} E(z_1)^{-1/3} \left( \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3}},
\]

where \(z_1 = 0.35\) is the typical LGR redshift. The observational value is given by Eisenstein et al[26] as \(A = 0.469 \pm 0.017\). And the \(\chi^2\) value is

\[
\chi^2_{\text{BAO}} = \frac{(A - 0.469)^2}{0.017^2}.
\]

Now we perform a joint analysis by minimizing the combined quantity

\[
\chi^2 = \chi^2_{\text{SN}} + \chi^2_{\text{MB}} + \chi^2_{\text{BAO}}.
\]

The result is plotted in figure 5 with the red dotted line for SN data, the blue dashed line for the shift parameter \(R\), the green dash dotted line for parameter \(A\) and the black solid line for the combined contours. We can see the contour of 1\(\sigma\) is closed, although the range is still large. The best fit values are \(\Omega_{m0} = 0.28\) and \(n = 3.4\). Accordingly, the current equation of state is \(w_{\phi0} = -0.83\), which is consistent with the WMAP observation[21].

The likelihood functions for \(\Omega_{\text{m0}}\) and \(n\) are shown in figure 6. As we can see, the \(\Omega_{\text{m0}}\) likelihood has a near-gaussian shape, therefore we can give the best fit at 1\(\sigma\) by integrating to 68.3% of the total area under the curve. The results are \(\Omega_{\text{m0}} = 0.33 \pm 0.04\) for using SN data alone and \(\Omega_{\text{m0}} = 0.28 \pm 0.02\) for the joint analysis. For \(n\), however, the likelihood curve is highly asymmetric and, what is more, the total area is divergent. Thus we cannot extract a range for the best fit \(n = 3.4\) with definite statistic meaning. As a consequence, although we use the terms such as 68.3%, 95.4% and 99% confidence level, they are not valid in a strict sense. Here we just use them by convention to denote the contours corresponding to \(\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}} = 2.30, 6.17\) and 9.21 respectively.

### IV. CONCLUSION AND DISCUSSION

In this paper, we briefly reviewed the agegraphic dark energy model based on the Károlyházy uncertainty relation, and then compared it with the holographic dark energy, which is motivated by the holographic principle. In fact both models based on some principles that relate the IR and UV cut-offs in effective quantum field theory, therefore they have similar features. Apart from the essential distinction in physical motivations, the apparent difference lies in that for the agegraphic dark energy the IR cut-off is chosen as the age of the universe and the equation of state \(w_{\phi} > -1\) forever, whereas for the holographic dark energy the length measure is the future event horizon and \(w_{\phi}\) can cross \(-1\). When using the Type Ia supernova data to constrain the model, we find that although \(\Omega_{\text{m0}}\) can be well constrained, the parameter \(n\) is unbounded from above. After a joint analysis together with the CMB shift parameter \(R\) and the BAO \(A\) parameter, \(\Omega_{\text{m0}}\) is enhanced from \(\Omega_{\text{m0}} = 0.33 \pm 0.04\) to \(\Omega_{\text{m0}} = 0.28 \pm 0.02\). The situation for \(n\) is essentially unchanged. As a result, this model can be consistent with current SNIa data as well as the CMB and LSS data for \(n > 1\), which is just the requirement of this model for an accelerated expansion, as is mentioned in[14].
FIG. 5: Confidence contour plot of 68.3%, 95.4% and 99% with the red dotted line for SN data, the blue dashed line for CMB, the green dash dotted line for BAO, and the black line for the joint analysis. We marginalized the nuisance parameter $h$. The best fit values are $\Omega_{m0} = 0.28, n = 3.4$

As we mentioned before, the solution to equation (9) corresponds to the general definition for the agegraphic dark energy (11). With the best fits from the joined analysis, we calculate $\delta_q$ by equation (12). By definition, $\delta_q$ is independent of $z$, therefore choosing arbitrarily any $z$ will lead to the same result. Here we choose the value from a large range $0 \leq z \leq 1000$. From figure 7 we may safely draw the conclusion that $\delta_q \simeq 3.0$ (in unit of $H_0^{-1}$) and the variation of $\delta$ is due to the errors from numerical method. For comparison, we also calculate the corresponding quantity in the holographic dark energy model, where

$$
\delta_h = \frac{c}{H(z)\sqrt{\Omega_h(z)}} \frac{1}{(1 + z)} \int_{-1}^{z} \frac{dz'}{H(z')}.
$$

We assume the best fits $\Omega_{m0} = 0.29$ and $c = 0.91$ from [12]. Figure 8 shows $\delta_h \lesssim 0.0007$. Such a tiny quantity may well be neglected in practical consideration. But here, $\delta_q$ is of the same order as the current age of the universe ($\sim \frac{1}{H_0}$), and therefore has to be taken into account. Such a term can be understood as a result of generalizing the energy density of the quantum fluctuations in Minkowski spacetime to the cosmological scenario. More fundamental interpretation for $\delta_q$ and the general definition of the agegraphic dark energy needs further studies.

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FIG. 6: The likelihood for $\Omega_{m0}$ and $n$. The black line corresponds to the joint analysis, and the blue dotted line corresponds to using only SN data. Note that for the likelihood function for $n$, the plot asymptotically tends to a horizontal line as $n \rightarrow \infty$, which means the area under the plot is divergent. This actually implies that $n$ cannot be well constrained by these observations.

FIG. 7: For agegraphic dark energy, $\delta_q$ in unit of $H_0^{-1}$ calculated with $z$ from the range [0,1000]. $\Omega_{m0} = 0.28$, $n = 3.4$.

FIG. 8: For holographic dark energy, $\delta_h$ in unit of $H_0^{-1}$ calculated with $z$ from the range [0,1000]. $\Omega_{m0} = 0.29$, $c = 0.91$. 
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[27] It is easy to see that the same situation also exists in the holographic dark energy model.