Dynamics of a deformable body in a fast flowing soap film

Sungwhan Jung1, Kathleen Mareck1, Michael Shelley1, and Jun Zhang2,1

1Applied Mathematics Laboratory, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012, USA
2Department of Physics, New York University, 4 Washington Place, New York, New York 10003, USA

(Dated: October 7, 2018)

We study the behavior of an elastic loop embedded in a flowing soap film. This deformable loop is wetted into the film and is held fixed at a single point against the oncoming flow. We interpret this system as a two-dimensional flexible body interacting in a two-dimensional flow. This coupled fluid-structure system shows bistability, with both stationary and oscillatory states. In its stationary state, the loop remains essentially motionless and its wake is a von Kármán vortex street. In its oscillatory state, the loop sheds two vortex dipoles, or more complicated vortical structures, within each oscillation period. We find that the oscillation frequency of the loop is linearly proportional to the flow velocity, and that the measured Strouhal numbers can be separated based on wake structure.

PACS numbers: 47.32.ck, 47.54.De, 47.20.-k

The wake flow behind a rigid obstacle is a central object of study in fluid mechanics. When the oncoming flow velocity exceeds a threshold, vortices are shed behind the obstacle [1]. A typical wake is composed of successive eddies of alternating sign — the “von Kármán vortex street” — and is observed over a wide range of flow velocities and body shapes [2, 3]. The frequency of vortex shedding (f) is determined by the flow velocity (V) and the object size (d), whose relation is captured by the near constancy of the Strouhal number, \( St = d f / V \).

The dynamics of a rigid object which moves freely in the direction perpendicular to the flow is of interest in many industrial and biological applications [4, 5, 6]. Lateral motion of an object can be induced by interaction with the flow and is often called the vortex-induced vibration (VIV). At low flow velocities, the body starts to oscillate sideways with small amplitude (less than 0.4 times body diameter). Its associated wake structure is again a von Kármán vortex street. However, further increase of flow velocity causes the obstacle to oscillate in phase with the vortex shedding, and as a result, a series of dipoles are shed instead [7, 8].

Settling bodies or rising bubbles, where the balance of gravitational and drag forces set the velocity, also exhibit transitions as they interact with their wakes. For example, a slowly settling sedimenting sphere falls straight downwards but above a certain sedimentation velocity, the sphere’s motion becomes periodic and its trajectory a spiral or zigzag [9]. A deformable object, such as a droplet or bubble, can behave similarly even as its shape now changes [10, 11].

Finally, studies have shown the instability (and bistability) of slender deformable bodies to lateral oscillations in quasi-2D soap-film flows [12], and of heavy deformable sheets to lateral oscillations in fast 3D flows [13, 14, 15]. In these cases, the system corresponds to the flapping of a flag in a stiff breeze.

Flowing soap film provides a practical template upon which to study the dynamics of a nearly 2D flow [15, 16]. The experimental setup has been introduced earlier [13, 15, 16, 20]. In this work, we introduce a deformable closed body into a fast flowing soap-film. Two thin nylon wires (0.3 mm in diameter) separate at a nozzle (0.5 mm inner diameter) attached to the bottom of a reservoir. The reservoir contains soapy water maintained at a fixed pressure head, thus fixing the flux. A stopcock regulates the flow rate through the nozzle. The nylon wires extend downwards to a collection box 2.4 m below. Driven by gravity, the soap film flows downwards. Owing to air drag, a terminal velocity is reached approximately 60 cm below the nozzle with a velocity profile near the center close to being uniform (velocity differences are within 20% of the mean, over 60% of the span about the midline). From optical interference patterns, the film thickness is found to vary smoothly across the film by about 15% of its average thickness.

We use a thin rubber loop (0.2 mm thick) as the deformable structure. The loop wets into the soap-film and is supported from its inner side against the flow. The loop is much thicker when compared to the film thickness (0.003 mm) that the fluid presumably does not penetrate over the loop. Six loops of different circumferences (5–7.5 cm) are used. We find that for regimes studied here, the loop appears to undergo only bending deformations, and not stretching or compression, as its length shows no measurable increase or decrease. Currently, we do not understand what balance of effects sets the enclosed area of the loop, which is an important constraint on the possible dynamics. However, we do find that, once experimental conditions are fixed, and the loop is in a fixed state of dynamics, the enclosed area changes very little in time (e.g. \( \sim 5\% \) for a 5 cm loop). However, between different states or conditions, the enclosed area can vary by factors of two or three.

A laser Doppler velocimeter (LDV; Model LDP-100, TSI Inc.) is used to record the upstream velocity V.
FIG. 1: Flow structures behind a 5 cm loop at 2.2 m/s flow velocity. The coupled fluid-structure system shows bistability: (a) the stationary state; the loop remains essentially motionless and its wake is a von Kármán vortex street. The loop is deformed by the flow into a teardrop shape. (b) the oscillatory state; the loop sheds two vortex dipoles within each oscillation period.

Micron-sized particles (TiO$_2$) are seeded into the flow for LDV measurements. Flow structures are visualized using interference patterns from monochromatic illumination (low-pressure sodium lamps operating at wavelength 585 nm). The movies of the wake flow together with the loop are recorded using a high speed camera at 1000 frames per second.

The interaction between the loop and the flow is quite complicated. In our experiments, we observe bistable states, one stationary and another oscillatory (see Fig. 1a and b), that co-exist over a range of flow velocities. At least in the conditions considered here, we do not observe spontaneous transitions between these two states. However, a transition from the stationary to the oscillatory state can be induced by externally perturbing the loop, or by abruptly changing the flow velocity.

In the stationary state, the loop behaves as a rigid hoop and has a teardrop shape with higher curvature on the top than on the sides (Fig. 1a). A characteristic length scale ($D$) of the loop, its width in the film, is 1 cm. The flow velocity ($V$) varies from 1.5 to 2.5 m/s and the kinematic viscosity ($\nu$) of soap film is approximately 0.04 cm$^2$/s. The frequency of vortex shedding ($f_s$) varies from 20 to 50 Hz. Based on these characteristic numbers, we estimate the Reynolds number ($Re$) and Strouhal number ($St_s$) for the system to be

$$ Re = \frac{V D}{\nu} \sim 5,000, \quad St_s = \frac{D f_s}{V} \sim 0.2, $$

where the subscript $s$ stands for vortex shedding, since the Strouhal number is calculated based on the vortical structure of the wake. Figure 1b shows the deforming body and its vortical wake using a 5 cm loop and flow velocity of 2.2 m/s. As can be seen, vortices of alternating sign are successively produced.

In the co-existing oscillatory state, shown in Fig. 1b at the same parameters as above, the loop now oscillates periodically in the horizontal direction, and the vortical wake behind it is quite different. For low flow velocity, two dipole pairs are shed during each oscillation period. Such a wake structure is also observed behind oscillating cylinders and is referred to as the 2P mode.

Figure 2 shows both the position of the loop at several time-points during one period of oscillation, and the path taken by its centroid. During the oscillation, the loop continuously changes its shape, and its centroid moves along a figure-eight trajectory (Fig. 2a). This figure-eight shape is due to the fact that the frequency of oscillation in the stream-wise direction is twice that in the transverse direction. This has been observed in the motions of a
and the closed ones are in the stationary state. The frequency relation between the oscillation frequency of the loop turning (or counter-clockwise) vortex is shed.

When the loop is at far right (or left), a clockwise oscillates in phase with that vortex shedding (Fig. 2b and c). When the loop is at far right (or left), a clockwise oscillates in phase with that vortex shedding (Fig. 2b and c). When the loop is at far right (or left), a clockwise oscillates in phase with that vortex shedding (Fig. 2b and c). When the loop is at far right (or left), a clockwise oscillates in phase with that vortex shedding (Fig. 2b and c).

By using loops of several different lengths, we find a linear relation between the oscillation frequency of the loop ($f_{loop}$) and a rescaled velocity $V \sqrt{\frac{\delta}{L}}$ (see Fig. 3), where $\delta$ is the film thickness, $a$ the thickness of the loop, and $L$ the loop length. Our results from loops of different lengths and differing flow velocities all collapse onto a single line with slope of about 0.27. This offset of this affine relation suggests a bifurcation to oscillation at a critical rescaled velocity of about 20, which is unfortunately beyond the reach of this experiment.

To better understand the relation between oscillation frequency and flow velocity, we propose a simple model for the oscillations of an elongated loop with longitudinal length $L_t$ driven by a “lift force” in the direction perpendicular to the stream. The lift force ($F$) is taken as proportional to $\rho V^2 L_t \delta$ where $\rho$ is the density of fluid, $V$ the fluid velocity. Hence, $F = (1/2) C_L \rho V^2 L_t \delta$, where $C_L$ is a lift coefficient. Typically, $L_t$ is proportional to the loop circumference, $L$. At an angle $\theta$ inclined to the flow stream, $C_L$ is proportional to $\sin \theta$. For small $\theta$, $\sin \theta \approx x_{cm}/y_{cm}$ where $(x_{cm}, y_{cm})$ is the center of mass (centroid) location. Therefore, we approximate the lift force as $F = m \ddot{x}_{cm} = (1/2) \rho V^2 L_t \delta \ddot{x}_{cm}/y_{cm}$, where $\ddot{x}_{cm}$ is the acceleration in the transverse direction and $m$ is the total body mass. In this experiment, the mass of the (wetted) loop is much greater than that of the enclosed fluid. Hence, we assume that the total body mass ($m$) is proportional to $\rho_L L a^2$ where $\rho_L$ is the density of the loop. Also, the $y$-component of the centroid, $y_{cm}$, and the length $L_t$ are assumed to be proportional to the length of the loop if the body is elongated due to the flow. With the trivial solution for the $x$-component of centroid as $x_{cm} = C e^{\omega t}$, we obtain an expression for the oscillation frequency: $\omega = 2\pi f_{loop} = V \sqrt{\frac{\delta}{a L}}$. This is consistent with our observations and underlies our rescaling of the data in Fig. 3. Put differently, this is simply the oscillation frequency of a hanging pendulum where the gravitational force is replaced by a drag force.

As the flow velocity increases, a more complicated mode in the oscillatory state can be observed (left panel in Fig. 4). In this case, the loop sheds more than four vortices over a single period of oscillation. We refer to this wake structure as a flag-like mode. We use this terminology because the wake now resembles more that behind a flapping flag (see Fig. 1), and because the body itself looks elongated and “flag-like”. This is because the enclosed area is now smaller in relation to $L^2$ than for the body in the 2P mode. To characterize the loop oscillation and the wake structure, we again define a Strouhal number, now using the oscillation frequency of the loop itself, or $St_L = A f_{loop}/V$ where $A$ is the outer amplitude of oscil-
FIG. 5: Enclosed area (normalized by $L^2/4\pi$) and amplitude (normalized by $L$) of the oscillating loop for 6 cm loop. Area and amplitude abruptly change as the wake structure transits from 2P modes (open symbols) to flag-like modes (closed symbols). As the velocity increases further, the amplitude decreases due to the large drag force.

This work is supported by DOE Grant No. DE-FG02-88ER25053.

[1] D. J. Tritton, *Physical Fluid Dynamics* (Van Nostrand, 1977), 1st ed.
[2] M. V. Dyke, *An album of fluid motion* (Parabolic Press, Stanford, 1982), 1st ed.
[3] C. H. K. Williamson, Annu. Rev. Fluid Mech. 28, 477 (1996).
[4] C. H. K. Williamson and R. Govardhan, Annu. Rev. Fluid Mech. 36, 413 (2004).
[5] T. Sarpkaya, J. Fluids Struct. 19, 389 (2004).
[6] J. C. Liao, D. N. Beal, G. V. Lauder, and M. S. Triantafyllou, Science 302, 1566 (2003).
[7] R. Govardhan and C. H. K. Williamson, J. Fluid Mech. 420, 85 (2000).
[8] D. Brika, and A. Laneville, J. Fluid Mech. 250, 481 (1993).
[9] I. Nakamura, Phys. Fluids 19, 5 (1976).
[10] D. G. Karamanev and L. N. Nikolov, AIChE J. 38, 1843 (1992).
[11] J. Magnaudet and I. Eames, Annu. Rev. Fluid Mech. 32, 659 (2000).
[12] P. C. Duineveld, J. Fluid Mech. 292, 325 (1995).
[13] J. Zhang, S. Childress, A. Libchaber, and M. Shelley, Nature 408, 835 (2000).
[14] M. Shelley, N. Vandenberghe, and J. Zhang, Phys. Rev. Lett. 94, 094302 (2005).
[15] Y. Watanabe, S. Suzuki, M. Sugihara, and Y. Sueoka, J. Fluids Struct. 16, 529 (2002).
[16] L. Huang, J. Fluids Struct. 9, 127 (1995).
[17] S. Taneda, J. Phys. Soc. Jpn. 24, 392 (1968).
[18] Y. Couder, J. M. Chomaz, and M. Rabaud, Physica D 37, 384 (1989).
[19] M. A. Rutgers, X.-L. Wu, and W. B. Daniel, Rev. Sci. Instrum. 72, 3025 (2001).
[20] S. Alben, M. Shelley, and J. Zhang, Nature 420, 479 (2002).
[21] W.-J. Kim and N. C. Perkins, J. Fluids Struct. 16, 229 (2002).
[22] L. D. Landau and E. M. Lifschitz, *Fluid Mechanics, Course of Theoretical Physics Vol. 6* (Pergamon Press, Oxford, 1987), 2nd ed.