Application of Markov chains in managing human potentials

Nikolina Žajdela Hrustek1,∗, Damira Keček2 and Ines Polgar1

1 Faculty of Organization and Informatics
University of Zagreb, 42000 Varaždin, Croatia
E-mail: {nikolina.zajdela@foi.unizg.hr}, {ines.polgar@gmail.com}

2 University Center Varaždin, University North
42000 Varaždin, Croatia
E-mail: {dkecek@unin.hr}

Abstract. Human potentials make a unique foundation to every organization. Due to individuals’ differences which enable a business surroundings and create competitive advantage, it is necessary to coordinate them to the mission, vision and goals set by the organization in order to satisfy the needs for specific knowledge and skills, and effectively realize the defined business goals. Application of Markov chains enables prediction of random variables’ movements. This study shows, via a practical example, predicting the necessity for human potentials in an ICT company throughout a period of three years.

Keywords: human potentials, Markov chains, planning human potentials, predicting human potentials

Received: April 15, 2020; accepted: June 21, 2020; available online: July 07, 2020
DOI: 10.17535/crorr.2020.0012

1. Introduction

Key goal of every organization, whether profit or non–profit one, is to synchronize all resources with set goals, vision and mission, current economic and market situation, different technological, demographic and legislative changes, business strategy and workers’ knowledge and skills. By taking action on internal factors and quick adaptation to the external ones, the company is able to make right and cost-approved decisions. Organization’s leadership’s care on applying and keeping efficient employees is essential because they represent the critical point of company’s long-term success in surroundings of quick changes in technology and business trends. Keeping talented individuals depends on good stimulation and compensation, better motivation and earned promotion [15].

Dessler [10] quotes that strategic management of human potentials represents creating and executing policies and practices of people management that stimulate development of their competences and behavior, and serve to realize company’s strategic goals. Human capital today becomes more important to all employers, physical workers get less work, and the market focus is precisely on intellectual labor. Human resources represent potential for realizing competitors’ advantage if managed efficiently, since this is the resource that cannot be imitated by the competitors. One can say that companies are as effective and efficient as the people working in them [5].

Managing human potentials comprises two different terms which relate to human resources and human potentials. Term human resources considers physical and intellectual force, employees’ competences and skills used by the company to reach its goals, while human potentials

∗Corresponding author.

http://www.hdoi.hr/crorr-journal ©2020 Croatian Operational Research Society
mark employers’ potential which needs to be recognized and developed by the company in order to increase organization’s competitiveness [18]. According to [2], managing human potentials implicitly includes a cluster of managers’ activities and exercises linked with development and maintaining human potentials in terms which help organization’s success by using people’s talent efficiently and effectively in order to accomplish organization’s goals as successfully as possible. The way one manages human potentials in an organization effects employees’ everyday lives, which can be explained via employees’ life cycle, i.e. phases every employee goes through within an individual organization [10].

Connected to employees’ life cycle, the role of managing human potentials is reflected in the processes of employment, selection and familiarizing employees with demands of their work place, relevant people are found and chosen for unoccupied places and are given according support. Furthermore, in the processes connected to education and development, managing human resources implicates employees’ adequate training and further development and perfecting of employees’ skills necessary for carrying out work assignments. Likewise, implementation of discipline and control of employees’ behavior is integrated by human potential management. As mentioned before, it supports planning employees’ careers, changes in their status and carries out the termination of employment. Regarding development and implementation of work efficiency and rewarding, human potential management has the central role here. Human potential management is significant to employees as well as to the organization itself; with its help educated employees are created and implementation of set goals is ensured, while the same employees are enabled with support throughout their entire working active life within organization and said organization and its employees are connected by ensuring healthy mutual communication [18].

Regarding the fast development of technology, organizations will be expected to enable their employees work from home, i.e. their “virtual offices” and those in charge of managing human potentials will have to take into account that employees not physically present will have to be motivated to feel part of said organization [16, 24]. Human relationships and interactions still remain most important in terms of work relationship, and the team work efficiency greatly depends on the team’s members’ mutual confidence in each other. Successful teams need to pay great attention to building foundation of team work, it is necessary to organize chores for all members to know their duties in any given moment. Following up on newer technological trends is one of necessary assumptions in order to attract new talents who will contribute to innovativeness in said organization. Contemporary information and communication technologies (ICT), including first and foremost online portals and social networks, become new sources of employees to leaders of human potentials [20]. Concerning demographic structure of population, researches show that all types of future organizations will have to face the lack of man power. Employees’ education will be one of basic challenges in managing human potentials. Managers will face numerous and new challenges comprising areas of rewarding, compensation, motivation and employees’ disciplining. The last challenge concerns external factors which imply changes in surroundings related to globalization, organization’s flexibility, market insecurity and others [11, 14, 22].

2. Importance of planning human potential

Planning human potential enables for future businesses and their demands to link themselves with key employees, but also to prevent problems which could eventually endanger the organization’s competitive position. With planning human potential becoming increasingly important and present activity of each organization’s leadership during the process of strengthening organization’s competitive abilities, there are many reasons which provide importance to said activity. Among other things, Bahtijarević [1] emphasizes: inadequate supply on the market for work (man) power, human potentials becoming critical resource, time necessary for education
and development, employment related costs, development and maintenance, costs comprising ineffective use of human potentials, technological changes, demographic changes, emphasis on career management, emphasis on productivity, service quality, increased use of computers.

Companies that execute human potential planning are much more flexible and enterprising. Human potential planning leads to more efficient use, as well as to human potential development, which results in considerable influence on other programs and functions within the company’s organizational structure. Analyses of human potential activities show that their efficiency primarily depends on answers to the questions on the number of people, their profiles and types of skills that are considered necessary. Adequate planning of human potentials has been manifested as leading to greater pleasure and development of more successful relations among employees due to the fact that said employees can individually take part in planning their own career and development. By planning human potentials, one keeps the best workers in the company, maintains management’s continuity, provides information on needs and eventually avoids costs of losing employees [14].

Human potential function is responsible for employment and development of work force necessary in all organizational units. As such, it can affect successfullness of each individual organizational unit by having the correct employment strategy, choice and improvement of adequate employees for specific jobs in each individual organizational unit. Human potential department is responsible for health and safety of all employees in an organization. Besides being in contact with legislation such as employment and equality, leaders of human potential function should be acquainted with all changes regarding health and safety rules and regulations in order to avoid possible inconveniences which could be caused by employees or executives, and could eventually lead to disturbances and decrease in productivity. Organization’s activity performance can, among other things, be affected by laws, regulations and agreements of various organizations, such as unions; non-fulfilling their needs can in some cases lead to industrial disputes and strikes, which can in return affect employees’ productivity and organization’s profitability [23]. Also, associated with human potential planning, some research show that human potential planning must be closely linked to the business strategy in order to achieve positive results from the financial aspect [4, 8, 9, 17].

Regarding the impossibility of predicting future events to completely eliminate all risks within an organization and with purpose of reducing business failures to minimum, theoreticians have developed various mathematical and statistical modules with purpose of partial prediction of certain events and risks happening. Markov chains is one in a number of methods with wide specter of application which is used in predicting the movements and needs for human potential. Foundation of this method’s application is in analyzing current movements of random variables with goal of predicting its future movement.

3. Modelling management of human potential via Markov chains

Predictions via Markov chains are represented in different branches or areas, but this section will provide examples of their application in predicting human potential.

Modelling management of human potentials relates mostly to predicting future behavior of employees. Belhaj and Tkioiat [3] presented the module of predicting employees in hierarchical system of human resources dependent on time which divides employees in homogeneity subgroups, and elements in each subgroup belong to the same group of marks. Due to this hierarchical system, said module supposes that promotion is executed only for the next higher mark, which is the reason why the suggested matrix is of super diagonal crossing. The suggested module is inspired by real development of human potential, i.e. limitation of employees’ progress, hence the authors consider it appropriate that each individual is assigned to a subgroup where they will develop their abilities. Likewise, a numeric module is also presented which presents differences
between management’s expectations and results of predictions, which opens the possibility of managers reducing the gap between expectations and real-life results [3].

Markov chain is also used in planning armed forces in the United States of America. Basic challenge in the military case is understanding what motivates soldiers to stay or leave the army. In conducted research, Markov chain of military personnel was constructed and appraised via three-year-long data of the US Ministry of Defense. The advantage of applying Markov chain in this case is enabling description of military personnel dynamics through time and answering business relevant questions. Also, Markov chain enables simple calculation of different statistics on individual or total level. On individual level, it can be used to describe probable progress made by military personnel in certain career phase, while on total level it can gain information on total progress rates and behavior of separation, which simultaneously presents key contribution in development of maintenance programme. Markov chain was used to build stochastic dynamic programming module of final level to research whether decision on staying or leaving depends on financial stipulation. By implementing Markov module, it is possible to choose one stipulation option in order to keep the soldiers. Since the Ministry of Defense’s budget is limited and divided on military personnel through payment grades dependent on soldiers’ skills, one possibility is to determine optimal maintaining policy directed on individuals with high skills marks motivated by high financial stipulations, and said financial stipulations would increase according to time in service. Contrarily, by removing budget limitations, one can determine minimal cost necessary to accomplish optimal maintaining goals [26].

Continuation of this study describes Markov chains methodology via practical example of an ICT company from Cakovec, Međimurje County (Croatia). Based on employees’ movement (income and outgo) within three main company’s departments in one year, along with application of described methodology, prediction on number of employees in individual departments for the period of next three years has been made. Markov chains methodology, which assumes that in the analyzed period there will be no changes, has been applied to the short-term, three-year prediction of the number of employees in each department given that human resources subject to change.

4. Markov chains methodology

This study monitors discrete-time stochastic processes with the Markovian property which says that behavior of processes in near future, with condition to present and past, is equal to behavior of the processes in near future, with condition to present only. Such processes are called Markov chains [6, 12, 25]. Continuation provides basic mathematical notations of Markov chains applied in this study for the prediction of human potentials of a service company.

A sequence of random variables \((X_t, t \in \mathbb{N}_0)\) is called Markov chain with a state space \(S = \{i, j, i_0, i_1, \ldots, i_{t-1}\}\) if it has Markovian property

\[
P(X_{t+1} = j | X_t = i, X_{t-1} = i_{t-1}, \ldots, X_0 = i_0) = P(X_{t+1} = j | X_t = i), \quad t \in \mathbb{N}_0, 1 \leq i, j \in S. \quad (1)
\]

The temporal moment \(t + 1\) here represents future and \(t\) current, while \(0, \ldots, t - 1\) represent past moments. Consequently, Markovian property says that future state depends only on current state, not on the way the process came to be to what it is currently. Markov chain is supposed to be time homogeneous, meaning the probability of Markov chain passing from state \(i\) to state \(j\) depends on states \(i\) and \(j\) alone, not on the moment of the passing, i.e.

\[
P(X_{t+1} = j | X_t = i) = P(X_1 = j | X_0 = i), \quad t \in \mathbb{N}_0, 1 \leq i, j \in S. \quad (2)
\]

Transition probabilities (2) are \(p_{ij}\) and usually presented in a transition probability matrix
for which holds:

(i) \( p_{ij} \geq 0 \quad \forall \ i, j \in S \), i.e. elements of transition probability matrix are non-negative and

(ii) \( \sum_{j \in S} p_{ij} = 1 \quad \forall \in S \), i.e. elements in each row must sum to 1.

Transition probability matrix \( P \) shows transition probabilities from one state to another in one step of Markov chain. Transition probabilities from state \( i \) to state \( j \) in \( n \) steps is noted as

\[
p_{ij}^{(n)} = P(X_{t+n} = j | X_t = i), \quad i, j \in S, \quad t, n \in N_0.
\] (4)

Transition probability matrix in \( n \) steps is \( P^{(n)} \).
In order for Markov chain to be described completely, besides knowing the transition probability matrix, one also needs to know the distribution of random variable \( X_0 \). Distribution of random variable \( X_0 \)

\[
p^{(0)} = (p_1^{(0)}, p_2^{(0)}, \ldots)
\] (5)

is also called the initial state vector, where

\[
p_i^{(0)} = P(X_0 = i), \quad i \in S
\] (6)

represents the probability that the process in its initial state is in state \( i \).

The state vector after \( n \) steps becomes vector

\[
p^{(n)} = (p_1^{(n)}, p_2^{(n)}, \ldots),
\] (7)

where

\[
p_i^{(n)} = P(X_n = i), i \in S
\] (8)

denotes probability that the process will achieve the state \( i \) in \( n \) steps.

As follows, for every \( j \in S \) is

\[
p_j^{(n)} = \sum_{i \in S} P(X_n = j, X_0 = i) = \sum_{i \in S} P(X_n = j | X_0 = i) \cdot P(X_0 = i) = \sum_{i \in S} p_i^{(0)} p_{ij}^{(n)}
\] (9)

Equation system (9) in its matrix notation has the form

\[
p^{(n)} = p^{(0)} \cdot P^{(n)}
\] (10)

Therefore, along with familiar transition probability matrix and initial state vector, considering (10), the state vector can be determined at any step \( n \) [7, 21].

More valuable information is a long-run behavior of a Markov chain. Namely, a common question arising in Markov chain is what is the long-run probability that the process will be in each state after a large number of transitions, independent of the initial state. As it is not possible to predict long-run probabilities for all Markov chains, but for irreducible and ergodic Markov chain [13], in the continuation states of a Markov chain are classified. If state \( j \) is
accessible from state $i$ and state $i$ is accessible from state $j$, then states $i$ and $j$ are said to communicate. A Markov chain is said to be irreducible if all states communicate. A state is said to be a recurrent state if, upon entering this state, the process will return to this state again. If there are two consecutive numbers $s$ and $s + 1$ such that the process can be in state $i$ at times $s$ and $s + 1$, the state is said to have period 1 and is called an aperiodic state. If all states are recurrent and aperiodic, Markov chain is said to be ergodic.

To understand long-run behaviour of Markov chain, the following result is essential. For any irreducible and ergodic Markov chain

$$\lim_{n \to \infty} p_{ij}^{(n)} = \pi_j > 0,$$

where $\pi_j$ are called the steady–state probabilities of the Markov chain and for every $j \in S$ satisfy following equations

$$\pi_j = \sum_{i \in S} \pi_i p_{ij} \quad \text{and} \quad \sum_{j \in S} \pi_j = 1.$$  

Therefore, after a large number of transitions, the probability that process is in state $j$ tends to the value $\pi_j$, independent of the probability distribution of the initial state. [13, 19, 25]

5. Predicting human potentials in service activities

According to methodology described in previous section, the continuation provides examples of Markov chains application in predicting human potentials in an ICT service company from Medimurje County, Croatia. The said company has three departments: Programming, Executive Department and Manager and General Business. Programming hires employees in charge of programming and designers. Executive Department hires engineers and managers subordinate to executive director. Manager and General Business hires a manager, their assistant and consultant, several individual employees in certain functions, accountant, human potential manager, secretary and a cleaner.

Table 1 presents inflow and outgo of workers within the ICT company’s three departments throughout a year. On July 31, 2018 Programming had 25 employees, Executive Department 16 employees, and Manager and General Business had one manager and 20 employees. At the beginning of period surveyed, the company had 62 employees. Throughout the year surveyed, employees were restructured due to working on a new project, which resulted in 12 employees switching their place of work due to maximization of work effect. Furthermore, 50 employees remained in same positions. On July 31, 2019 Programming had 26 employees, Executive Department 18 employees, and Manager and General Business had one manager and 17 employees. At the end of period surveyed, the company had 62 employees.

| Movement of workers’ numbers | Programming | Executive | Manager and General Business | Total |
|------------------------------|-------------|-----------|------------------------------|-------|
| State (July 2018)            | 25          | 16        | 21                           | 62    |
| Outgo of workers             | 3           | 3         | 6                            | 12    |
| Remained in same positions   | 22          | 13        | 15                           | 50    |
| Inflow of new workers        | 4           | 5         | 3                            | 12    |
| State (July 2019)            | 26          | 18        | 18                           | 62    |

Table 1: Inflow and outgo of workers within the ICT company’s three departments
Table 2 presents the structure of workers’ inflow and outgo within the said company throughout period surveyed, starting July 31, 2018 to July 31, 2019. Columns show the inflow of workers due to transition to other departments, and rows present outgo of workers due to transition to other departments.

While predicting the number of workers in individual departments after the period of three years and based on data from Tables 1–2, Markov chain application begins with finding an initial state vector which elements are equal to the ratio of number of employees in every department surveyed at the end of period surveyed (July 31, 2019) and total number of employees in the company. Initial state vector for the company surveyed is:

\[ p^{(0)} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \end{bmatrix} \]

i.e. Programming employs 40 %, Executive Department 30 %, and Manager and General Business 30 % of ICT company’s workers.

| Departments                | Programming | Executive | General Business | Total |
|----------------------------|-------------|-----------|------------------|-------|
| Programming                | –           | 1         | 2                | 3     |
| Executive                  | 2           | –         | 1                | 3     |
| Manager and General Business| 2           | 4         | –                | 6     |
| Total                      | 4           | 5         | 3                | 12    |

Table 2: Employees’ inflow and outgo structure in ICT company

Transition probability matrix is formed in the next step. Elements of the main diagonal in transition probability matrix are obtained by dividing the number of employees remaining working the same job with the number of employees from the starting state, while the other elements of transition probability matrix are obtained via combination of data from Table 2 and the initial state. Transition probability matrix is

\[ P = \begin{bmatrix} 0.880 & 0.040 & 0.080 \\ 0.125 & 0.813 & 0.063 \\ 0.095 & 0.190 & 0.714 \end{bmatrix} \]

According to equation (10), the state vector is

\[ p^{(3)} = \begin{bmatrix} 0.44 & 0.33 & 0.23 \end{bmatrix}, \]

the elements of which indicate, on one hand, the increase in employees’ number in ICT company’s Programming Department from 40% to 44%, in Executive Department from 30% to 33% while, on the other hand, department Manager and General Business will decrease in number from 30% to 23%. Expressed in an absolute numbers, Programming will need 28 employees, Executive Department 20 of them, while Manager and General Business will hold 14 employees necessary. Results obtained are not surprising since the present is marked with fast development of technology which could make programmers considered as one of the most wanted professions in business surroundings.

According to calculated prediction of human potentials for the period of three–years, the ICT company’s management can plan employees’ payment costs in advance, which will be closely connected to employees’ profession. Furthermore, since 44% of employees are programmers, the said ICT company can get connected to trade schools and/or universities in order to contribute to the greater interest among the young for ICT professions, while by enabling scholarships, following up on the best students or providing them with work experience they can in advance provide human potentials of the highest quality. Likewise, ICT company can also plan its future
projects according to the number of specific professions necessary for starting a project team. As Markov chain in this case study is irreducible and ergodic, steady-state probabilities were calculated according to equations (12)–(13). Along with a non-changing number of employees in departments, Programming would have the highest number of employees, no less than 49%, while 31% would be in Executive Department and 20% in Manager and General Business.

6. Conclusion

Throughout their lifecycle, organizations are gradually directed to product quality and buyers, while human potentials obtain a role growing in power in acquiring a defined mission, vision and goals. The choice of employees, their education and motivation alone contribute to the increase in work efficiency. The present is subjected to fast changes, people are required to use their own intellectual potentials at their maximum and be responsible for their own work. This role is taken over by the human potential manager with their assignments and activities and contributes to development and maintaining human potentials with goal of using them efficiently and effectively in achieving organizational goals. In order to make adequate decisions linked to planning and managing human potentials, managers of departments ie. organizational units have numerous statistically-mathematical methods at their disposal. One of them is Markov chain which considers random variables and predicting future movement of variables based on data from previous periods. Markov chain is suitable to use in different disciplines, while in this study it has been applied in predicting the internal offer of human potentials.

Predicting human potential in a practical example was presented via Markov chain for an ICT company for the period of three years. Based on results acquired, one comes to conclusion about the existence of probability of the greatest need for human potentials arising in Programming with programmers as their major employees, following up by the Executive Department and the least need in Manager and General Business. Results acquired by prediction can be used in multiple ways. Some with the purpose of acquiring new employees by making decisions on establishing collaboration with trade schools and/or universities in order to provide potential employees. Furthermore, based on results, ICT company can plan the costs of future employees according to payment assigned to professions required. Likewise, it is possible to plan on working on new projects according to number of specific professions necessary for team formation. One can conclude that Markov chain application is possible in various business areas, depending on types of problems and requirement of not too high investments, while its application can contribute to a better planning and, finally, reaching better business decisions with long-term higher profit and company’s growth.

References

[1] Bahtijarević-Šiber, F. (1999). Management ljudskih potencijala. Zagreb: Golden marketing.
[2] Bahtijarević-Šiber, F. (2014). Strateški menadžment ljudskih potencijala. Zagreb: Školska knjiga.
[3] Belhaj, R. and Tkiouat, M. (2013). A Markov model for human resources supply forecast dividing the HR system into subgroups. Journal of Service Science and Management, 6(3), 211–217. doi: 0.4236/jssm.2013.63023
[4] Bird, A. and Beechler, S. (1995). Links between business strategy and human resource management strategy in US–based Japanese subsidiaries: An empirical investigation. Journal of International Business Studies, 26(1), 23–46. doi: 10.1057/palgrave.jibs.8490164
[5] Bloomberg D. J., LeMay S. and Hanna J. B. (2006). Logistika. Zagreb: Mate d.o.o.
[6] Brémaud, P. (2000). Markov chains: Gibbs fields, Monte Carlo simulation and queues. Texts in Applied Mathematics book series, vol. 11. New York: Springer. doi: /10.1007/978-1-4757-3124-8
[7] Brooks, S., Gelman, A., Jones, G. L. and Meng, X. L. (2011). Handbook of Markov Chain Monte Carlo. New York: Chapman and Hall/CRC. doi: 10.1201/b10905
[8] Chowhan, J. (2016). Unpacking the black box: Understanding the relationship between strategy, HRM practices, innovation and organizational performance. *Human Resource Management Journal*, 26(2), 112–133. doi: 10.1111/1748-8583.12097

[9] Cooke, F. L. (2001). Human resource strategy to improve organizational performance: A route for firms in Britain? *International Journal of Management Reviews*, 3(4), 321–339. doi: 10.1111/1468-2370.00071

[10] Dessler G. (2015). *Upravljanje ljudskim potencijalima* (12. izdanje). Zagreb: Mate d.o.o.

[11] Dilworth J. B. (2000). *Operations management: Providing value in goods and services*. Orlando: Harcourt, Inc.

[12] Grinstead, C. M. and Snell, J. L. (1997). *Introduction to probability*. American Mathematical Society.

[13] Hillier, F. S. and Lieberman, G. J. (2000). *Introduction to operations research*. New York: McGraw–Hill.

[14] Jambrek, I. and Penić, I. I. (2008). Upravljanje ljudskim potencijalima u poduzećima–ljudski faktor, motivacija zaposlenika kao najvažniji čimbenici uspješnosti poslovanja poduzeća. *Zbornik Pravnog fakulteta Sveučilišta u Rijeci*, 29(2), 1181–1206. https://hrcak.srce.hr/40704

[15] Kalakota R. and Robinson M. (2002). e–Poslovanje 2.0 – Vodić ka uspjehu. Zagreb: Mate d.o.o.

[16] Kolaković, M. (2005). Novi poslovni modeli u virtualnoj ekonomiji i njihov utjecaj na promjene u transportnoj logistički i upravljanju lancem opskrbe. *Zbornik Ekonomskog fakulteta u Zagrebu*, 3(1), 195–210. https://hrcak.srce.hr/26221

[17] Michie, J. and Sheehan, M. (2005). Business strategy, human resources, labour market flexibility and competitive advantage. *The International Journal of Human Resource Management*, 16(3), 445–464. doi: 10.1080/0958519042000339598

[18] Obradović, V., Samardžija, J. and Jandrić, J. (2015). *Menadžment ljudskih potencijala u poslovnim praksi*. Zagreb: Plejada.

[19] Privault, N. (2018). *Understanding Markov chains: Examples and applications*. Singapore: Springer. doi: 10.1007/978-981-13-0659-4

[20] Relja, R. and Šuljug, Z. (2010). Novi oblici rada u umreženom društvu. *Informatologia*, 43(2), 143–149. https://hrcak.srce.hr/55855

[21] Resnick, S. I. (1992). *Adventures in stochastic processes*. Boston: Birkhäuser. doi: 10.1007/978-1-4612-0387-2

[22] Robbins S. P. and Junge A. T. (2009). Organizacijsko ponašanje. Zagreb: Mate d.o.o.

[23] Rowbotham, F., Galloway L. and Masoud, A. (2012). *Operations management in context*. London: Routledge. doi: 10.4324/9780080409240

[24] Schniederjans M. J., Cao Q. and Triche J. H. (2014). *E-commerce operations management*. New Jersey: World scientific publishing.

[25] Winston, W. L. and Goldberg, J. B. (2004). *Operations research: Applications and algorithms* (4th edition). Belmont, CA: Thomson Brooks/Cole.

[26] Zais, M. and Zhang, D. (2016). A Markov chain model of military personnel dynamics. *International Journal of Production Research*, 54(6), 1863-1885. doi: 10.1080/00207543.2015.1108533