Search for pairing-vibration states of even Ca isotopes in $^{40}$Ca+$^{208}$Pb transfer reactions

S. Szilner, L. Corradi, B.R. Behera, E. Fioretto, A. Gadea, A. Latina, A.M. Stefanini
M.Trotta, A.M. Vinodkumar, Y.W. Wu

Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Legnaro, I-35020 Legnaro, Italy

S.Beghini, G.Montagnoli, F.Scarlassara
Dipartimento di Fisica, Università di Padova, and Istituto Nazionale di Fisica Nucleare, I-35131 Padova, Italy

G.Pollarolo
Dipartimento di Fisica Teorica, Università di Torino, and Istituto Nazionale di Fisica Nucleare, 10125 Torino, Italy

F. Haas, E. Caurier, F. Nowacki
Institut de Recherches Subatomiques, (IN2P3-CNRS-Université Louis Pasteur), F-67037 Strasbourg Cedex 2, France

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Multi-nucleon transfer reactions in $^{40}$Ca+$^{208}$Pb have been studied at several bombarding energies close to the Coulomb barrier. Light reaction products have been identified in mass and charge with a time-of-flight spectrometer. The energy spectra of the inclusive two neutron pick-up channel show a population in a narrow region of excitation energies which corresponds to the predicted energy of pairing vibration states in $^{42}$Ca.

I. INTRODUCTION

In the nuclear medium, the ability of nucleons to form $0^+$ pairs has been realized very early from the study of nuclear masses, but the recognition that these pairs of nucleons play an important role in the description of nuclear spectra, in particular in correlating spectra of neighbouring nuclei, is more recent [1–3]. The development of the pair model received considerable inputs from the extensive experimental work on transfer with light ions, in particular $(p,t)$ and $(t,p)$ reactions [4,5]. The use of heavy-ions was perceived [6] as a very promising tool for the study of pair rotation and/or pair vibration degrees of freedom since one could envisage the exchange of many nucleons among the reactants. Unfortunately, the lack of sufficient mass, charge and energy resolution postponed the experimental studies to recent times, when high resolution spectrometers became available and the clear identification of transfer products up to at least four neutron pick-up and four proton stripping has been achieved [7–10]. In these measurements, attempts to identify pair transfer modes were based on total or differential cross sections, but no unambiguous contribution of these degrees of freedom could be drawn.

Systematic studies of $(p,t)$ and $(t,p)$ reactions led to the identification of the calcium region as the only known region where the cross sections for the population of the excited $0^+$ states is larger than the ground state. Those states have been recognised as multi (addition and removal) pair-phonon states [4]. Nuclear structure and reaction dynamics studies attribute this behavior to the influence of the $p_{1/2}$ orbital that gives a much larger contribution to the two-nucleon transfer cross section than the $f_{7/2}$ orbital, which dominates the ground state wave function.

In the present work we report on a study of multi-nucleon transfers in $^{40}$Ca+$^{208}$Pb. Since both nuclei are recognised as pair vibrational, they provide an ideal tool to study multi pair-phonon excitations by looking at $Q$-value distributions for the different isotopes. Since this system is closed shell, both in proton and neutron, it provides also an excellent opportunity for a quantitative comparison with a theoretical model whose results depend strongly on the form factors defining the transfer process [11,12].

II. RESULTS AND DISCUSSION

The experiment was performed using the Tandem + ALPI accelerator of the Laboratori Nazionali di Legnaro. A $^{40}$Ca beam was accelerated onto a $^{208}$Pb target (200 μg/cm²) at several bombarding energies close to the Coulomb barrier: $E_{lab}$= 250, 236 and 225 MeV. Light reaction products were detected with the spectrometer PISOLO [10,13], that makes use of two micro-channel plate detectors for time-of-flight signals and a $\Delta E/E$ ionization chamber for nuclear charge and energy determination. The good mass and charge resolution of $\Delta A/A \simeq 1/120$ and $\Delta Z/Z \simeq 1/60$, respectively, allowed a clear separation between different channels. Taking into account the intrinsic energy resolution of the detector and the energy straggling in the target the obtained energy resolution was $\simeq 2$ MeV. With a total solid angle of $\simeq 3$ msr, transfer cross sections up to the pick-up of seven neutrons and the stripping of seven protons have been extracted. In the present paper we focus on pure neutron transfer channels.
The comparison of the data is done with the Complex-WKB model [12,14,15] that calculates the outcome of the reaction, namely total kinetic energy loss, differential and total cross sections, taking into account the transfer of single nucleons by using the well known one-particle transfer form factors. This model solves in an approximate way the system of coupled equations which determine the exchange of particles between the nuclei by treating the particle transfer degrees of freedom as independent and in the harmonic approximation.

![Diagram](image1)

**FIG. 1.** In (a) the angle and Q-value integrated cross sections of the pure neutron pick-up channels for the $^{40}$Ca+$^{208}$Pb reaction at $E_{\text{lab}}$=236 MeV (dots) are shown as a function of the mass number together with the theoretical (CWKB) prediction (histogram), the curve corresponds to a Poisson distribution with an average number of transferred particles $\langle n \rangle = 1.1$. In the other frames the Q-value integrated angular distributions for the quasi-elastic (qe), one-neutron (+1n) and two-neutron (+2n) pick-up channels (dots) are shown together with the theoretical calculation (curves).

The experimental total cross sections and the angular distributions at $E_{\text{lab}} = 236$ MeV are shown in Fig. 1 together with the theoretical predictions. In order to cover the full range of transferred energies, the single-particle form factors governing the exchange of nucleons were calculated by including all single-particle levels above the Fermi surface and a full shell below. Those levels are shown in Fig. 2 for neutrons. For the real part of the optical potential we used the empirical potential of Ref. [16]. For the imaginary part the model uses only a volume term with a small diffusivity ($a \sim 0.4$ fm) since the depopulation of the entrance channel is taken explicitly into account by the coupling to the transfer channels. This potential gives an adequate estimation of the total reaction cross section as can be inferred from the good description of the quasi-elastic angular distribution.

The bell shaped distributions of the differential cross sections show the grazing character of the reaction. The integrated cross sections decrease smoothly as a function of the number of transferred neutrons, and the overall trend of the data is reproduced by calculations. We notice that the total cross section for the even calcium isotopes is in agreement with a Poisson distribution, with an average number of transferred neutrons $\langle n \rangle = 1.1$, that underlines the harmonic character of the degrees of freedom determining the evolution of the reaction. However from the observed behavior of the inclusive total cross sections alone, as was clear from previous experiments, it is difficult to draw unambiguous conclusions on the role played by a pair transfer mode. As we will discuss below, the excitation functions, corroborated from the knowledge of the excitation spectra and from the results of reaction and shell model calculations, provide valuable information in this respect.

![Diagram](image2)

**FIG. 2.** Neutron single-particle levels for projectile and target. The shaded areas indicate the occupied levels. The spectroscopic factors have been chosen to be unity for all the levels.

For the bombarding energy of 236 MeV we show in Fig. 3 the total kinetic energy loss (TKEL) distributions measured at the scattering angle of $\theta_{\text{c.m.}} = 95^\circ$ for channels up to $+4n$ in comparison with the theoretical prediction calculated for a partial wave close to the grazing one. In the figure we indicate with an arrow the ground to ground state Q-value. As can be appreciated, the multi-
nucleon transfer channels (we recall that the optimum $Q$-value for all neutron transfer channels is close to 0) display a well defined maximum that is shifted to high energy losses, leaving the ground states unpopulated. By looking at the final population of the single particle levels we can infer that the maxima for the $+2n$ and $+4n$ channels are essentially due to two and four neutrons in the $p_{3/2}$ orbital. This fact together with the known low energy spectra of $^{42,44}$Ca [18] suggest that these maxima correspond to the excited $0^+$ states that were identified with the pair mode [4,5]. Using $^{40}$Ca as the ground state of the pair vibrational model these states are interpreted as the mode $(4,1)$ in $^{42}$Ca and $(4,2)$ in $^{44}$Ca where the first number in parentheses refers to the pair-removal ($n_{s}$) and the second to the pair addition ($n_{a}$) mode. We recall that the pair vibrational model assigns the energies with the formula $E(n_{s},n_{a}) = h\omega_{p}(n_{s}+n_{a})$, $\omega_{p}$ being the pair frequency. In the case of calcium $h\omega_{p}$ is $\approx$ 3 MeV. To strengthen this assignment, for the $+2n$ channel we show in Fig. 4 the spectra taken at the three bombarding energies; here arrows label the position of the $0^+$ states.

The indicated maxima of the spectra are compatible with the excitation of $0^+$ states in $^{42}$Ca at $\approx$ 6 MeV, leaving room for the mutual excitation of the yrast states in $^{206}$Pb [19]. We notice here that the spectra widen with the increase of the bombarding energy with a slight shift of the maxima reflecting the energy dependence of the optimum $Q$-value. Both features are well reproduced by the theoretical calculations indicating that the used single particle levels cover the full $Q$-value ranges spanned by the reaction.

Since the illustrated experimental results and reaction calculations point to a selective feeding of $0^+$ states dominated by a pair of neutrons in the $p_{3/2}$ orbital we felt that it would be interesting to calculate, for $^{42}$Ca, the strength distribution of these peculiar $0^+$ states over all others $0^+$ states. This has been performed in the framework of large scale shell model (SM) calculations by using the same model space and interaction as in a recent publication concerning various spectroscopic features of calcium isotopes [24]. To extract the channel strength function we used the Lanczos method by using as pivot state $\psi_1 = (a_{p_{3/2}}^+ a_{p_{3/2}}^+) |0^+\rangle$ that corresponds to the creation of two neutrons, coupled to 0, on the $|0^+\rangle$ ground state of $^{40}$Ca. The valence space, used in these SM calculations consists of a $^{28}$Si inert core and of the $2s\frac{1}{2}$, $1d\frac{3}{2}$, $1f\frac{7}{2}$ and $2p\frac{3}{2}$ subshells for both protons and neutrons. The ground state of $^{40}$Ca is thus described by a $(np-nh)$ configuration with $n$ up to 12. Through the Lanczos iterative procedure, and using the overlap relation between the obtained eigenstates of the SM Hamiltonian and the pivot state $\psi_1$, we can calculate [25,26] the strength function for these peculiar $0^+$ states. The strength distribution $S(E)$, shown in the lower panel of Fig. 4, displays, clearly, a strong concentration near $\approx$ 6 MeV of excitation energy, an energy very close to that of a configuration where a $p_{3/2}$ neutron pair is coupled to a closed shell of $^{40}$Ca ground state ($E \sim 5.9$ MeV). This calculation demonstrates the dominant contribution of the $p_{3/2}$ orbitals and the predicted very narrow energy distribution suggests its interpretation as a pair mode.

![FIG. 3. Total kinetic energy loss distribution of the quasi-elastic (qe) and neutron pick-up channels (histograms) together with the CWKB calculations (curves) measured at $E_{lab} = 236$ MeV. Arrows indicate the ground to ground state $Q$-value. The data and calculations have been normalised arbitrarily.](image-url)
FIG. 4. Experimental (histograms) and theoretical (curves) total kinetic energy loss distributions of the two neutron pick-up channels at the indicated energies. The arrows correspond to the energies of \(0^+\) states in \(^{42}\text{Ca}\) with an excitation energy lower than 7 MeV [18]. The group of states at 5.87 (collecting most of the strength), 6.02, 6.51 and 6.70 MeV were strongly populated in \((t,p)\) reactions [20-22]. The same excitation energy range was strongly populated in the \(^{40}\text{Ca}(^{14}\text{C},^{12}\text{C})^{42}\text{Ca}\) reaction [23]. Bottom panel shows the strength function \(S(E)\) from SM calculations (see text) after convoluting with Gaussian of two different widths: 300 keV and 1.5 MeV (close to the experimental energy resolution - curve). The represented strength function has been obtained after 200 Lanczos iterations to allow a correct convergence of all eigenstates.

III. SUMMARY

Multi-neutron transfer reactions, at several bombarding energies, have been studied in \(^{40}\text{Ca}+\text{^{208}Pb}\). Due to the peculiar properties of the \(f-p\) shell, this system allows us to interpret the total kinetic energy spectra as being dominated by the excitation of pair modes in even calcium isotopes. The population of these channels is in agreement with the harmonic properties of these modes. This experiment demonstrates the importance of pursuing these studies with heavy ions even if, within the present energy resolution, a definite assignment of these states is not possible. The large measured cross sections would allow the interesting study of the decaying pattern of these pair modes.

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