Central Charge for 2D Gravity on $AdS_2$ and $AdS_2/CFT_1$ Correspondence

Mohsen Alishahiha$^a$ and Farhad Ardalan$^{a,b}$

$^a$ School of physics, Institute for Research in Fundamental Sciences (IPM)  
P.O. Box 19395-5531, Tehran, Iran

$^b$ Department of Physics, Sharif University of Technology  
P.O. Box 11365-9161, Tehran, Iran

Abstract

We study 2D Maxwell-dilaton gravity on $AdS_2$. We distinguish two distinctive cases depending on whether the $AdS_2$ solution can be lifted to an $AdS_3$ geometry. In both cases, in order to get a consistent boundary condition we need to work with a twisted energy momentum tensor which has non-zero central charge. With this central charge and the explicit form of the twisted Virasoro generators we compute the entropy of the system using the Cardy formula. The entropy is found to be the same as that obtained from gravity calculations. The agreement is an indication of $AdS_2/CFT_1$ correspondence.
1 Introduction

In 3D gravity the group of diffeomorphisms which preserves the condition that the metric be asymptotically $AdS_3$ is two copies of the Virasoro algebra with the central charge \[c = \frac{3l_3}{2G_3},\] (1.1)
where $l_3$ is the $AdS_3$ radius and $G_3$ is the three dimensional Newton’s constant.

This fact has been used to compute the entropy of three dimensional black hole. It has been shown \[2\] that one may use the Cardy formula with the above central charge and count the boundary degrees of freedom which agrees with the entropy of the black hole in the bulk. Therefore the symmetry is enough to find the entropy without knowing about the details of the dynamics.

Of course we now understand the reason behind this precise agreement and it is due to the AdS/CFT correspondence \[3\]. In three dimensions the lesson we have learned from the AdS/CFT correspondence is that “any consistent quantum gravity on an asymptotically $AdS_3$ spacetime is a 2D CFT living on the boundary of the $AdS_3$”.

Although AdS$_{d+1}$/CFT$_d$ have been understood for $d \geq 2$ mainly due to explicit examples, little have been known for the case of $d = 1$ (see however \[4–8\]). The lack of our knowledge of the holographic dual of $AdS_2$ is due to the special features of $AdS_2$ spacetime. First of all it has two boundaries. Secondly, unlike the higher order AdS space, the two dimensional AdS spacetime carries entropy. So far we do not have a concrete example of an $AdS_2$/CFT$_1$ in the context of string theory where we could identify both sides of the duality.

On the other hand the quantum gravity on $AdS_2$ geometry is important on its own right. Indeed the $AdS_2$ geometry is the factor which appears in the near horizon geometry of the extremal black holes in any dimension. Therefore understanding the gravity on $AdS_2$ might ultimately help us understand the origin of the black hole entropy in other dimensions.

To explore the $AdS_2$/CFT$_1$ correspondence one may utilize the experience of the $AdS_3$ case; namely, one could try to understand the asymptotic symmetry of $AdS_2$. In fact this has been done in several papers including \[4–7\]. In particular it has been shown \[4\] that the asymptotic symmetry group of an asymptotically $AdS_2$ geometry is one copy of the Virasoro algebra. Recently it has also been shown \[9\] that exactly the same argument as that for $AdS_3$ \[1\] can be made for quantum gravity with a $U(1)$ gauge field on $AdS_2$ leading to the following central charge\[1\]

\[c = 3kGQ^2l^4,\] (1.2)
where $l$ is the radius of $AdS_2$, $Q$ is the electric charge, $k$ is the level of the current which generates the $U(1)$ and $G$ is the two dimensional Newton’s constant.

\[1\]In order to compare the result with that in \[9\] one needs to use a unit in which $G = \frac{1}{l^4}$.
The aim of this article is to further study 2D Maxwell-dilaton gravity on $AdS_2$ background. In particular we would like to study the entropy of the corresponding solution and to see to what extent the information of the theory is encoded in a CFT whose central charge is given by (1.2). Our strategy is the 2D analog of [2]. Namely we will reproduce the black hole entropy obtained from gravity by making use of the Cardy formula with the central charge given by the asymptotic conformal diffeomorphism of the 2D theory. As observed in [9] in order to have a consistent boundary condition for gravity on $AdS_2$ coupled to gauge field, the energy momentum tensor has to be twisted. Having a non-zero central charge plus the fact that we can reproduce the entropy from this central charge provides a favorite indication of the $AdS_2/CFT_1$ correspondence showing that the corresponding holographic dual would be a chiral half of a 2D CFT.

In the course of studying 2D Maxwell-dilaton gravity on $AdS_2$ we will encounter two distinctive cases depending on whether the corresponding $AdS_2$ solution can be lifted up into $AdS_3$ solution. These two models are given by the actions

$$S_1 = \frac{1}{8G} \int d^2x \sqrt{-g} \left( e^\phi (R + \frac{8}{l^2} - \frac{l^2}{4} F^2) \right),$$

$$S_2 = \frac{1}{8G} \int d^2x \sqrt{-g} \ e^\phi \left( R + \frac{2}{l^2} - \frac{l^2}{4} e^{2\phi} F^2 \right).$$

Both actions admit an $AdS_2$ vacuum solution. We note, however, that although in the second case the $AdS_2$ solution can be lifted up to three dimensional $AdS_3$ solution, in the first one it cannot. Our main conclusion is that for both cases we can reproduce the black hole entropy using the central charge of the twisted energy momentum tensor. Moreover we can show that in the first case approaching near horizon a CFT emerges which has the same central charge as that obtained by using the asymptotic symmetry. In other words in this case there may be a correspondence between the two “CFT’s”; one at infinity and the other at the horizon.

The paper is organized as follows. In section two we will study 2D Maxwell-dilaton gravity on $AdS_2$ based on the first action in (1.3). We will show that the central charge of the twisted energy momentum can be used to compute the entropy using Cardy formula. The entropy is the same as the black hole entropy obtained from gravity side. We will also study the model in terms of the near horizon modes where we show that at near horizon we get a CFT whose central charge is the same as that obtained from asymptotic symmetry. In section three we will do similar computations for the model based on the second action in (1.3). We will see that using the asymptotic symmetry one can read off the central charge of the twisted energy momentum tensor which can then be used to reproduce the black hole entropy correctly. The last section is devoted to the conclusions and discussions.
2 Type I 2D gravity

2.1 Central charge and entropy

In this section we study 2D Maxwell-dilaton gravity based on the action

\[ S_1 = \frac{1}{8G} \int d^2x \sqrt{-g} \left( e^\phi (R + \frac{8}{l^2}) - \frac{l^2}{4} F^2 \right). \] (2.1)

This model has recently been considered in [9] where the central charge of the CFT associate with the asymptotic symmetry of its AdS$_2$ solution was calculated. The authors of [9] noticed that the potential of the $U(1)$ gauge field is singular at the boundary and therefore the boundary condition is not respected under a conformal diffeomorphism. The resolution was to accompany the conformal transformation with a special $U(1)$ gauge transformation, generated by a current $j_{\pm}$, leading to the twisted energy momentum tensor [9]

\[ \tilde{T}_{\pm\pm} = T_{\pm\pm} \pm \frac{G^{1/2} Q l^2}{2} \partial_\pm j_{\pm}, \] (2.2)

where $T$ is the energy momentum tensor corresponding to the original conformal transformation with $c = 0$. The commutator of this twisted energy momentum tensor gives the charge $c = 3kGQ^2 l^4$.

Since the AdS$_2$ solution carries an entropy, it is natural to pose the question of whether this entropy can be reproduced using Cardy formula with central charge given by (1.2). In other words we should be able to compute the number of states of the CFT by evaluating the eigenvalue of $\tilde{L}_0$ coming from energy momentum (2.2) and then use Cardy formula with the central charge (1.2) to obtain the entropy of the AdS$_2$ solution.

Therefore it is important to first compute the entropy of the model using the gravity solution which we do utilizing the entropy function formalism [10], which only needs the information of the near horizon geometry. Since the geometry is AdS$_2$, we start from an ansatz respecting the SO(2,1) isometry of the AdS$_2$ solution,

\[ ds^2 = v \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad e^\phi = \eta, \quad F_{rt} = \frac{e}{l^2}, \] (2.3)

where $v, \eta$ and $e$ are constant to be determined by the equations of motion.

To proceed we need to evaluate the entropy function,

\[ E = 2\pi (Q e - f(e, v, \eta)), \] (2.4)

where

\[ f = \frac{v}{8G} \left[ \eta \left( -\frac{2}{v} + \frac{8}{l^2} \right) + \frac{e^2}{2v^2 l^2} \right] \] (2.5)
is the Lagrangian density evaluated for the ansatz. Extremizing the entropy function with respect to parameters $v, \eta$ and $e$ we get

$$ds^2 = \frac{4}{l^2} \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad e^\phi = GQ^2 l^4, \quad F_{rt} = 2GQ l^2.$$ (2.6)

The entropy is also given by

$$S_{BH} = 2\pi GQ^2 l^4$$ (2.7)

which can be recast in the suggestive form

$$S_{BH} = 2\pi \sqrt{\frac{1}{6} (3GQ^2 l^4)(2GQ^2 l^4)},$$ (2.8)

reminiscent of the Cardy formula

$$S = 2\pi \sqrt{\frac{c}{6} - 4\Delta_0(L_0 - \frac{c}{24})},$$ (2.9)

where $\Delta_0$ is the lowest eigenvalue of $L_0$. With this observation, the task is to see whether the CFT defined by the twisted energy momentum tensor (2.2) and central charge (1.2) can in fact reproduce the above entropy.

To proceed we note that the energy momentum tensor (2.2) satisfies the following commutator relation [9]

$$[\tilde{T}_-(t^-), \tilde{T}_-(s^-)] = -4\pi \partial_- \delta(t^- - s^-)T_- (s^-) + 2\pi \delta(t^- - s^-)\partial_- T_- (s^-) + \frac{\pi kGQ^2 l^4}{2} \partial^2 \delta(t^- - s^-).$$ (2.10)

This expression together with the definition of the twisted energy momentum tensor shows that upon mode expanding the energy momentum tensor, the Virasoro generators become

$$\tilde{L}_n = L_n + \frac{kGQ^2 l^4}{4} \delta_{n,0},$$ (2.11)

which satisfy a Virasoro algebra as follows

$$[\tilde{L}_n, \tilde{L}_m] = (n - m)\tilde{L}_{n+m} + \frac{1}{12}(3kGQ^2 l^4)(n^3 - n)\delta_{n+m,0}.$$ (2.12)

With the above expressions one can read the values of $\tilde{L}_0$ and $c$ and then plugging them into the Cardy formula (2.9) written in terms of the tilde quantities. One gets $c = 3kGQ^2 l^4$, $\tilde{L}_0 = \frac{kGQ^2 l^4}{4}$, $\tilde{\Delta}_0 = 0$, and

$$S = 2\pi GQ^2 l^4 \left( \frac{k}{4} \right).$$ (2.13)

Comparing the Cardy entropy with the black hole entropy (2.7) one has to set $k = 4$.

Thus to get the correct black hole entropy from the CFT the level of $U(1)$ is not a
free parameter. Of course we are familiar with such a phenomena; namely requiring to get correct black hole entropy may put a constraint on the level of affine algebra in the dual CFT (for example see [11–16]). So the central charge and $\tilde{L}_0$ of the CFT read

$$c = 12GQ^2 l^4, \quad L_0 = GQ^2 l^4.$$  

Therefore we would like to identify this central charge as the central charge of a CFT whose global $SL(2, R)$ symmetry is the isometry of the $AdS_2$ geometry. As a result with this particular number $k$ we see an exact agreement between gravity and CFT descriptions. This is in fact an strong indication in favor of the $AdS_2/CFT_1$ correspondence.

### 2.2 Near horizon modes

There is an alternative CFT living in the near horizon region of the black hole describing its entropy. In this approach one may identify the entropy as number of states at the horizon. To do this we follow [17] and make a change of variables in the action $S_1$,

$$2e^{2\phi} = \Phi^2 = q\Phi_0^2, \quad g_{\mu\nu} \to e^{\frac{2}{\Phi_0^2}} g_{\mu\nu},$$  

where $\Phi_0$ is the value of $\Phi$ at horizon, $\Phi_0^2 = 2GQ^2 l^4$, and $q$ is a free parameter and get

$$S_1 = \int d^2 x \sqrt{g} \left( \frac{1}{4} q\Phi_0^2 R + \frac{1}{2} (\nabla \phi)^2 + \frac{4q\Phi_0}{l^2} \phi e^{\frac{2}{\Phi_0^2}} - \frac{l^2}{4} e^{-\frac{2}{\Phi_0^2}} F^2 \right).$$  

Then integrating out the gauge field, using the Maxwell equations, leads to an effective potential for the scalar field $\phi$

$$S_1 = \int d^2 x \sqrt{g} \left( \frac{1}{4} q\Phi_0^2 R + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right).$$  

This is the action considered in [17] where it was shown that in the near horizon limit ($r \to 0$) the energy momentum tensor of the theory becomes traceless leading to a CFT with a specific central charge.

To be precise the author of [17] starts from a $d$-dimensional spherically black hole and decompose the metric into two parts

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + h_{ij}(r)dx^i dx^j.$$  

Dimensionally reducting along the $x^i$’s he gets, after some manipulations, the above two dimensional action.

Although the considerations of [17] are for a non-extremal black hole where the leading behavior of $g$ in the near horizon limit is $g(r) \sim (r - r_h)$ with $r_h$ being the radius of the horizon, the procedure works for the extremal case as well where
\( g(r) = vr^2 \). The only difference is that in the non-extremal case the trace of energy momentum tensor vanishes exponentially when the horizon is approached while in our case the approach is power law.

Using the change of variable \( z = \frac{1}{r} \) (note that in this case horizon is at \( z \to \infty \)), the components of the energy momentum tensor of the action (2.17) read

\[
\tilde{T}_{00} = \frac{1}{4}((\partial_t \varphi)^2 + (\partial_z \varphi)^2) - \frac{q \Phi_0}{4}(\partial_z^2 \varphi - \frac{1}{2z} \partial_z \phi) + \frac{1}{z^2} V(\varphi),
\]
\[
\tilde{T}_{zz} = \frac{1}{4}((\partial_t \varphi)^2 + (\partial_z \varphi)^2) + \frac{q \Phi_0}{4}(-\partial_t^2 \varphi + \frac{1}{2z} \partial_z \phi) - \frac{1}{z^2} V(\varphi),
\]
\[
\tilde{T}_{0z} = \frac{1}{2}(\partial_t \partial_z \phi - \frac{q \Phi_0}{4} (\partial_z \partial_t \varphi - \frac{1}{2z} \partial_t \varphi)). \tag{2.19}
\]

Using the light-cone coordinates \( z_\pm = t \pm z \) the non-zero components of the energy momentum tensor are given by

\[
\tilde{T}_{\pm\pm} = (\partial_{\pm \varphi})^2 \pm \frac{1}{2} q \Phi_0 \partial_{\pm \varphi}, \tag{2.20}
\]

which has the same structure as (2.22) if we define \( T_{\pm\pm} = (\partial_{\pm \varphi})^2 \) and \( j_{\pm} = -q \sqrt{2} \partial_{\pm \varphi} \). It is also easy to find the Virasoro generators coming from this energy momentum tensor which are [18]

\[
\tilde{L}_n = L_n + \frac{c}{12} \delta_{n,0}, \tag{2.21}
\]

where \( c = 3\pi q^2 \Phi_0^2 \). Using the definition of \( \Phi_0 \) we get

\[
c = 6\pi q^2 G Q^2 l^4, \quad \tilde{L}_0 - \frac{c}{24} = \frac{G Q^2 l^4}{\pi q^2}. \tag{2.22}
\]

Note that for \( q = \frac{2}{\pi} \) these are exactly the same results we obtained using the asymptotic symmetry of the model.

We note also that the effect of the \( U(1) \) gauge transformation at asymptotic boundary condition reflects, upon integrating out the gauge field, a non-zero background charge in the near horizon limit description of the theory. Therefore in this case twisting at infinity is equivalent to a non-zero background charge at the horizon. The extra term in the energy momentum tensor in both cases appear in such a way as to give exactly the same \( c \) and \( L_0 \) (entropy). We conclude that there may be a one to one correspondence between these two “CFT’s” one at infinity and the other at the horizon (for discussions on these two CFT’s see for example [19]).

### 3 Type II 2D gravity

In this section we consider 2D Maxwell-dilaton gravity based on the following action

\[
S_2 = \frac{1}{8G} \int d^2x \sqrt{-g} \, e^\phi \left( R + \frac{2}{l^2} - \frac{l^2}{4} e^{2\phi} F^2 \right), \tag{3.1}
\]
which can actually be obtained from 3D pure gravity with cosmological constant by reducing to two dimensions along an $S^1$. This action has been used to study entropy of extremal black hole in the presence of higher order corrections (see for example [20]).

We will redo our previous section’s computations for the action (3.1). To start with note that this action also has an $AdS_2$ solution. Following our discussion in the previous section we start with the following ansatz

$$ds^2 = v \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad e^\phi = \eta, \quad F_{rt} = \frac{e}{l^2},$$

where $v, \eta$ and $e$ are constant to be determined by the equations of motion and use the entropy function formalism to find

$$ds^2 = \frac{l^2}{4} \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right), \quad e^\phi = \sqrt{4GQl^2}, \quad F_{tr} = \sqrt{\frac{1}{16GQl^2}}$$

with the entropy,

$$S_{BH} = 2\pi \sqrt{\frac{GQl^2}{4G}}.$$  

Following our discussions in the previous section the goal is to reproduce this entropy using the number of states of a CFT which can be defined by asymptotic symmetry of the $AdS_2$ solution.

To proceed we note that the above solution, unlike the solution in the previous section, has the underling symmetry of $AdS_3$. In other words one may lift the two dimensional solution to three dimensions as follows

$$ds^2_{(3)} = ds^2_{(2)} + l^2 e^{2\phi} (dz + A_\mu dx^\mu)^2,$$

which for our solution, defining $y = \sqrt{16GQl^2} z$, becomes

$$ds^2_{(3)} = \frac{l^2}{4} \left( dy^2 + 2r^2 dy dt + \frac{dr^2}{r^2} \right),$$

clearly the $AdS_3$ written in the $S^1$ fibered over $AdS_2$ coordinates. So we get the $AdS_3$ with an identification or the solutions is lifted up to the extremal BTZ black hole.

Taking into account the isometry of $AdS_3$ which is $SL(2,R)_R \times SL(2,R)_L$, our two dimensional solution can be thought of as an $SL(2,R)_L$ quotient of $AdS_3$ [21]. Therefore we are left with just the $SL(2,R)_R$ symmetry of $AdS_3$ which under the reduction reduces to the $SL(2,R)$ isometry of $AdS_2$ plus a gauge transformation [4]. Following the arguments of [9] one can see that the conformal diffeomorphism in two dimensions will not respect the boundary conditions of the gauge field, necessitating an extra $U(1)$ gauge transformation leading to a twisted energy momentum tensor [4]

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm \partial_{\pm} j_{\pm},$$

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where \( j_\pm \) is the appropriately normalized current associated to the Kaluza-Klein \( U(1) \) gauge symmetry, and \( T \) is the \( AdS_2 \) energy momentum tensor with zero central charge.

The central charge of the twisted quantum theory crucially depends on the correct normalization of the \( U(1) \) current. In our case there is a short cut in calculating the central charge. One may mode expand the twisted energy momentum to get the Virasoro generators \( \tilde{L}_n \). On the other hand the twisted energy momentum tensor is equal to right handed energy momentum tensor of \( SL(2,\mathbb{R})_R \) isometry of \( AdS_3 \) whose Virasoro generators are \( L^{(3)}_n \) which satisfy

\[
[L^{(3)}_n , L^{(3)}_m] = (n - m) L^{(3)}_{n+m} + \frac{c^{(3)}}{12} (n^3 - n) \delta_{n+m,0},
\]

with \( c^{(3)} = 3l/2G_3 \). Now taking into account \( L^{(3)}_n = \tilde{L}_n \), and the fact that \( G_3 = lG \) the algebra for the twisted Virasoro generators reads

\[
[\tilde{L}_n , \tilde{L}_m] = (n - m) \tilde{L}_{n+m} + \frac{1}{12} (\frac{3}{2G}) (n^3 - n) \delta_{n+m,0},
\]

giving the central charge of the twisted energy momentum tensor as

\[
c = \frac{3}{2G}.
\]

The entropy can now be computed from the Cardy formula and found to be exactly as that obtained from gravity computations. This again is a direct indication in favor of \( AdS_2/CFT_1 \) correspondence.

4 Conclusions and discussions

In this paper we have studied 2D Maxwell-dilaton gravity on \( AdS_2 \) geometry. We have seen that there are two distinctive models of 2D Maxwell-dilaton gravity defined by actions \( S_1 \) and \( S_2 \) in (1.3).

Both actions admit an \( AdS_2 \) vacuum solution. In both cases to maintain the consistent boundary conditions of the fields in the theory the conformal diffeomorphism in the boundary at infinity has to be accompanied by a special \( U(1) \) gauge transformation. In other words the theories are well-described by the twisted energy momentum tensor defined by

\[
\tilde{T}_{\pm\pm} = T_{\pm\pm} \pm A \partial_\pm j_\pm,
\]

where \( A \) is a constant depending on the normalization of the \( U(1) \) current \( j_\pm \). Although the Virasoro algebra of \( T \) has zero central charge, as expected from 2D quantum gravity, the twisted energy momentum tensor leads, after fixing the constant \( A \) correctly, to the non-zero central charge given by

\[
c = 12GQ^2l^4,
\]

\[
genesis = \frac{3}{2G}.
\]
for the models based on $S_1$ and $S_2$, respectively; which can be used in the Cardy formula to compute the number of states of the CFT. It was then observed that the resultant entropy is equal to the entropy of the $AdS_2$ solution obtained from gravity calculations. This precise agreement can be thought of as a strong indication of $AdS_2/CFT_1$ correspondence. In particular it might be a sign that the holographic dual of gravity on $AdS_2$ is a chiral half 2D CFT. In fact one may go further and claim that any consistent 2D quantum gravity on an $AdS_2$ is dual to a chiral half 2D CFT generalizing the situation of $AdS_3$.

Although these models show certain similarities, they have significant differences. For example the central charge of the first model depends on the detail of the solution considered, while in the second case it only depends on the two dimensional Newton’s constant. In the first model even though we have a gauge field, the solution can not be lifted to an $AdS_3$ geometry. In fact there are indications that the model may be obtained from a higher dimensional extremal black hole. Actually following [17] we started from a four dimensional extremal black hole with near horizon geometry $AdS_2 \times M_2$ and reducing along the $M_2$ manifold we found an effective Maxwell-dilaton gravity on $AdS_2$. In this approach the entropy is associated with the near horizon modes where a new CFT emerges. We have seen that the emergent CFT has the same central charge as that obtained using asymptotic symmetry. Moreover we have observed that the energy momentum tensor has the form of a twisted energy momentum tensor where the extra twist term can be interpreted as a non-zero background field (or Louville theory). Indeed as far as the energy momentum tensor, central charge and entropy are concerned there is a correspondence between the two CFT’s; one at infinity and the other at the horizon. This observation requires further study.

On the other had the $AdS_2$ solution of the second case was lifted to an $AdS_3$ solution, exhibiting a direct connection between the two and three dimensional theories. In particular we were able to relate the central charge of the twisted energy momentum tensor to the Brown and Henneaux-like central charge.

Taking into account this direct connection between the two and three dimensional theories, one may pose a question whether this relation helps us to understand some aspects of these theories using the other one. In particular it has recently been observed that upon adding the Chern-Simons terms to the three dimensional gravity we are led to a chiral gravity [22]. It is then interesting to study the resultant chiral gravity in the 2D theory. Actually a two dimensional Chern-Simons term can be added to our theory. In our notation the Chern-Simons action is given [23]

$$S_{cs} = \frac{1}{32G\mu} \int d^2x \ e^{2\phi} \left( lR e^{\mu\nu} F_{\mu\nu} + l^2 e^{2\phi} e^{\mu\nu} F_{\mu\rho} F^{\rho\delta} F_{\delta\nu} \right).$$

(4.3)

It is easy to show that a model based on $S_2 + S_{cs}$ still has an $AdS_2$ solution. The corresponding central charge can be computed and is seen to get corrected. The correction of the central charge depends on the sign of electric field $e$. Using entropy
function formalism, we get
\[
c = \frac{3}{2G}(1 - \frac{1}{l\mu}), \quad \text{for } e > 0 \\
c = \frac{3}{2G}(1 + \frac{1}{l\mu}), \quad \text{for } e < 0.
\]  
(4.4)

A chiral gravity in two dimensions, may be also defined the same as in three dimensions. We note, however, that a priori there is no reason from two dimensional point of view to force the relation $\mu l = 1$. Nevertheless since the theory is related to three dimensional theory, one would expect to find some tachyonic modes leading us to a particular value for $\mu$. Work on this question is in progress.

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