Complexity based Sensing Strategy for Spectrum Sensing in Cognitive Radio Networks

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Abstract

Spectrum sensing has attracted much attention due to its significant contribution to idle spectrum detection in Cognitive Radio Networks. However, specialized discussion is on complexity-based sensing strategy for spectrum sensing seldom considered. Motivated by this, this paper is devoted to complexity-based sensing strategy for spectrum sensing. Firstly, three efficiency functions are defined to estimate sensing efficiency of a spectrum scheme. Then a novel sensing strategy is proposed given sensing performance and computational complexity. After that, the proposed sensing strategy is extended to energy detector, Cyclostationary feature detector, covariance matrix detector and cooperative spectrum detector. The proposed sensing strategy provides a novel insight into sensing performance estimation for its consideration of both sensing capacity and sensing complexity. Simulations analyze three efficiency functions and optimal sensing strategy of energy detector, Cyclostationary feature detector and covariance matrix detector.

Keywords: Cognitive Radio Networks, spectrum sensing, sensing strategy, computational complexity

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1. Introduction

With the rapid development of wireless communication, artificial intelligence and Internet of Things, etc., wireless business is growing explosively [1]. As a result, the traffic of radio spectrum appears to be more and more crowded [2]. However, the utilization of registered spectrum seems abnormally low according to the investigation of Federal Communications Commission (FCC) [3]. Cognitive Radio (CR) [4-5] is presented to improve spectrum utilization by reusing the idle radio spectrum of registered band when it is unoccupied. The registered users are called primary users (PU) and the unlicensed users are named secondary users (SU) in a CR [6]. The essence of a CR is to communicate with its users for SU by means of the spectrum band of PU when there is no communication for PU at a certain time and in a certain space. Consequently, how to obtain the band state of PU works as a significant factor for a CR. This key technology behind CR is defined as spectrum sensing (SS) [7-9].

The main contribution of SS consists in the spectrum state judgment of observed band. Factually, SS is a kind of signal detection based on binary hypothesis [10]. In one case, both signal and noise exist. In another case, there is only noise at the receiver of a CR. Generally, three aspects are considered to estimate a SS scheme, detection probability, false alarm probability and available throughput [11]. A miss in detection will cause the interference with PU and a false alarm will reduce the spectrum efficiency [12]. The available throughput exhibits the communication capability of secondary network, which is inversely proportional to the false alarm probability under equivalent conditions [13].

Classical SS schemes include energy detector (ED) [14], Cyclostationary feature detector (CFD) [15], covariance matrix detector (CMD) [16] and cooperative spectrum sensing (CSS) [17]. ED is a widely accepted method for SS due to its low complexity and superior performance. However, it is sensitive to variational environment noise, especially in low SNR regimes. The main advantage of CFD lies in its higher sensing performance in strong noise conditions while it is at the expense of computational complexity. CMD is a blind detection scheme with no need for the prior knowledge of signal, channel and noise. Similarly, the complexity of CMD is extremely considerable compared with ED. CSS obtains the spectrum condition based on the joint judgement of multiple CR equipment placed in distinct positions. This is super helpful to solve hidden terminal problem and improves sensing performance as well.

The sensing strategy of spectrum sensing considers the primary factors that influence sensing performance and it determines the majorization direction and upper limit of sensing performance. Most alternative sensing strategy of spectrum sensing only contain part main elements of influencing sensing performance. This lowers the adaptability and robustness of cognitive system, which goes against the application and popularization of spectrum sensing in complex network environment. As a result, research on global sensing strategy that considers the whole primary factor influencing sensing performance deserves to be conducted.

Computational complexity of a scheme denotes the algorithm efficiency and implementation cost, which is a crucial factor that influences sensing performance of a CR. In consequence, sensing strategy with computational complexity is conducive to performance majorization of SS. To the best knowledge of the author, there is no related work about the discussion of complexity-based sensing strategy for SS. Motivated by this, this paper mainly discusses sensing strategy-based SS given both sensing performance and computational complexity. Firstly, three efficiency functions are defined to estimate sensing efficiency of a
spectrum scheme from different point of view. Then a novel sensing strategy is proposed
given sensing performance and computational complexity. After that the proposed sensing
strategy is extended to ED, CFD, CMD and CSS. Simulations validate the reasonability and
effectiveness of the proposed scheme.

The rest of this paper is organized as follows. Section 2 reviews the basic work of ED, CFD,
CMD and CSS. The main contribution of this paper is shown in Section 3, which contains
three efficiency functions, complexity-based sensing strategy and some application extensions.
Simulation experiments and result analysis are accomplished in Section 4. Finally, Section 5
concludes this paper.

2. Related Work

The basic work on ED, CFD, CMD and CSS is reviewed in this section.

Assume the signal \( r(t) \) at the receiver of CR is an additive combination of clean
signal \( x(t) \) and noise \( w(t) \), then
\[
\begin{align*}
    r(t) &= x(t) + w(t), \\
    x(t) &= \sum_{i=1}^{M} \phi_i h_i(t) x_i(t),
\end{align*}
\]
where \( x(t) \) and \( w(t) \) are independent and identically distributed stationary random process with
mean zero and variance \( \sigma_x^2, \sigma_w^2 \), respectively. \( M \geq 1 \) denotes the number of primary users
and \( \phi \in \{0, 1\} \) signifies the absence or presence of the \( i \)th PU. Additionally, \( h_i(t) \) indicates the
channel gain of the \( i \)th PU with mean zero and \( |h_i(t)| \) follows Nakagami distribution.

After sampling at the rate of \( f_s \), the received signal based on binary hypotheses \( H_0 \) can be
written as
\[
\begin{align*}
H_0 : r(n) &= w(n) \\
H_1 : r(n) &= \sum_{i=1}^{M} \phi_i h_i(n) x_i(n) + w(n),
\end{align*}
\]
where \( H_0 \) denotes the absence of PU; \( H_1 \) means the presence of PU.

2.1 ED

Assume sensing duration is \( \tau \) at a sensing event, then the corresponding number of samples
reaches \( N = \tau f_s \). As a result, the average energy at the receiver of CR [18] could be described as
\[
E(r) = \frac{1}{N} \sum_{n=1}^{N} |r(n)|^2.
\]
The false alarm probability and detection probability [19] are respectively formulated as
\[
\begin{align*}
    P_f &= Q((\frac{\gamma - 1}{\sigma_x^2} - 1) \sqrt{\tau f_s}), \\
    P_d &= Q((\frac{\gamma - 1}{\sigma_x^2} - \frac{\tau f_s}{2\gamma + 1})), \\
    Q(x) &= \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt,
\end{align*}
\]
where $\varepsilon$ is the presupposed threshold; $\gamma = \frac{\sigma_s^2}{\sigma_n^2}$ represents received signal-to-noise ratio (SNR) of a CR; $Q(x) = 1 - \Phi(x)$ denotes complementary distribution function of standard Gaussian $\Phi(x)$.

The throughput of SU is defined as $C_0$ and $C_1$ in the absence and presence of PU, respectively. Since a length of $\tau$ period out of the total frame time $T$ is used for sensing [20], the achievable throughputs of the SUs under these scenarios are, respectively, given as

$$R_0(\tau, \varepsilon) = C_0 P(H_0)(1 - \frac{\tau}{T})(1 - P_f(\tau, \varepsilon)),$$

$$R_1(\tau, \varepsilon) = C_1 P(H_1)(1 - \frac{\tau}{T})(1 - P_d(\tau, \varepsilon)),$$

where $P(H_0)$ and $P(H_1)$ respectively denote the absence and presence probability of PU in the observed bands. We assume SU could immediately stop communicating with its users ($C_i \approx 0$) as soon as PU resumes the observed band in this paper. As a result, the average achievable throughput of SU [20] could be reformulated as

$$R(\tau, \varepsilon) = C_0 P(H_0)(1 - \frac{\tau}{T})(1 - P_f(\tau, \varepsilon)).$$

### 2.2 CFD

A continuous time random process $r(t)$ is said to be second-order Cyclostationary if its mean and autocorrelation show periodicity in time [21]. Mathematically this means that for a cyclic period $T$, $r(t)$ [21] can be represented as

$$E[r(t)] = E[r(t + T)],$$

$$R(r, t) = E[r(t)r(t + \tau)] = E[r(t + T)r(t + \tau + T)],$$

for all $t$ and $\tau$.

The Fourier representation of (13) [22] could described as

$$R(r, \tau) = \sum \alpha R^\alpha_r(\tau)e^{j2\pi\alpha \tau}.$$  

The Fourier coefficients of (14) [23] can be expressed as

$$R^\alpha_r(\tau) = \lim_{N \to \infty} \sum_{t=0}^{N-1} R(r, \tau)e^{-j2\pi\tau},$$

where $\alpha$ is named as the cyclic frequency of $R(r, \tau)$ and $R^\alpha_r(\tau)$ is called the cyclic autocorrelation function (CAF) [24].

After Fourier transform for CAF, the cyclic spectral density (CSD) [25] could be obtained

$$S^\alpha_r(f) = \sum_{\tau=0}^{\infty} R^\alpha_r(\tau)e^{-j2\pi f \tau},$$

Assume that the CSD of clean signal and noise are respectively defined as $S^\alpha_s(f)$ and $S^\alpha_w(f)$ [26], then

$$S^\alpha_r(f) = S^\alpha_s(f) + S^\alpha_w(f).$$

On the basis mentioned above, the false alarm probability and detection probability of CFD [27] could be expressed as

$$P_f = P(S^\alpha_s(f) > 0 | H_0),$$

$$P_d = P(S^\alpha_s(f) > 0 | H_1).$$
2.3 CMD

$L$ consecutive samples are considered in CMD and we define the following vectors [28]:

\[
\begin{align*}
    r(n) &= [r(n) \ r(n-1) \ \ldots \ r(n-L+1)]^T, \\
    s_i(n) &= [s_i(n) \ s_i(n-1) \ \ldots \ s_i(n-L+1)]^T, \\
    n(n) &= [n(n) \ n(n-1) \ \ldots \ n(n-L+1)]^T.
\end{align*}
\]

The statistical covariance matrices of $r(n)$ could be formulated as

\[
R_n = E[r(n)r^T(n)],
\]

Let us define $\alpha_{mn}$ as the $m$th row and $n$th of matrix $R$, then

\[
T_1 = \frac{1}{L} \sum_{n=1}^{L} \sum_{i=1}^{L} |\alpha_{mn}|.
\]

At hypothesis $H_0$, $T_i = 0(m \neq n)$ since noise is an independent and identically distributed random process. That is, $T_i = T_2$ at hypothesis $H_0$; $T_i > T_2$ at hypothesis $H_1$. Therefore, the ratio of $T_i/T_2$ is regarded as one possible scheme to compare with a presupposed threshold $\lambda_i$ for the presence or absence of PU [28]. However, $T_i = 0(m \neq n)$ is always false in practice on the account of the limited samples.

Finally, the false alarm and detection probability [28] could be severally exhibited as

\[
P_{fa} = \Phi\left(\sqrt{\frac{N_s}{2}} \left[ \frac{1}{\lambda_i} (1 + (L-1)\sqrt{2/N_s}) - 1 \right]\right),
\]

\[
P_d = \Phi\left(\sqrt{\frac{N_s}{2}} \left[ \frac{1}{\lambda_i} + \frac{\lambda_i^2 \gamma}{\lambda_i (\gamma+1) - 1} \right]\right),
\]

\[
\lambda_i = \frac{2 \sum_{l=1}^{L} (L-l)|\alpha_l|}{1 + (L-1)\sqrt{2/N_s}},
\]

\[
\alpha_l = E[r(n)r(n-l)]/\sigma^2_l,
\]

where $N_s$ is the available sample number, $\Phi(x) = 1 - Q(x)$; $\lambda_i$ is the presupposed threshold for given false alarm probability; $\lambda_L$ denotes the overall correlation strength among the consecutive $L$ samples.

2.4 CSS

We assume that both detection probability and false alarm probability are identical for each node in CSS. In addition, the node number is considered as $K$. Then when AND fusion rule [29] is considered for CSS, the false alarm probability and detection probability are shown as

\[
P_{fa}(N,v) = (P_{fa})^K,
\]

\[
P_d(N,v) = (P_d)^K.
\]
If OR fusion rule [30] works in CSS, then the false alarm probability and detection probability can be given as

\[ P_f(N, \varepsilon) = 1 - (1 - P_f)^K. \] (33)

\[ P_d(N, \varepsilon) = 1 - (1 - P_d)^K. \] (34)

When N-out-of-K fusion rule [31] is applied to CSS, the overall detection probabilities and false alarm probabilities are respectively rewritten as

\[ P_f(N, \varepsilon) = \sum_{i=N}^{K} \binom{K}{i} P_f^i (1 - P_f)^{K-i}. \] (35)

\[ P_d(N, \varepsilon) = \sum_{i=N}^{K} \binom{K}{i} P_d^i (1 - P_d)^{K-i}. \] (36)

where \( \binom{K}{i} \) is the basic combination expression and \( \binom{K}{i} = C_k^i \).

3. Main Work and Contribution

Based on ED, CFD, CMD and CSS, a novel sensing strategy is proposed to estimate their performance given both sensing performance and computational complexity in this section. Then the sensing strategy is applied to ED, CFD, CMD and CSS for optimal performance estimation.

3.1 Definitions

Firstly, some indispensable definitions are provided for convenience to give the novel sensing strategy proposed in this paper.

**Definition 1**: For a certain detection probability \( P_d \), if the number of the multiplication and addition operations needed is expressed as \( O(P_d) \), we define the efficiency function of a CR to protect PU as

\[ \eta_d = \frac{P_d}{O(P_d)}. \] (37)

**Definition 2**: If a false alarm occurs with probability \( P_f \) for a CR and its corresponding complexity is with \( O(P_f) \), then we define the efficiency function of a false alarm as

\[ \eta_f = \frac{P_f}{O(P_f)}. \] (38)

**Definition 3**: For a certain available throughput \( R(\tau, \varepsilon) \) of CR with sensing complexity \( O(R(\tau, \varepsilon)) \), the communication efficiency of secondary network is defined as

\[ \eta_R = C \frac{1-P_f}{O(R(\tau, \varepsilon))}. \] (39)

\[ C = C_o P(H_o)(1 - \frac{\tau}{T}). \] (40)
Definition 4: For given $P_d$, $P_f$ and $R(\tau, \varepsilon)$, the overall efficiency function for a CR is given as

$$\eta = \eta_d + \eta_R - \eta_f = \frac{P_d}{O(P_d)} + C \frac{1-P_f}{O(R(\tau, \varepsilon))} - \frac{P_f}{O(P_f)}.$$  \hfill (41)

Note that detection probability is beneficial to secure communication of PU and available throughput indicates communication capacity of SU while false alarm probability is to the disadvantage of a CR. Consequently, the operation is described as shown in (40). Additionally, a higher $\eta_d$ denotes a better protection efficiency to PU while $\eta_R$ indicates the communication efficiency in secondary network. For convenience, $C = 1$ in this paper. Consequently, (40) can be rewritten as

$$\eta = \frac{P_d}{O(P_d)} + \frac{1-P_f}{O(R(\tau, \varepsilon))} - \frac{P_f}{O(P_f)}.$$  \hfill (42)

3.2 Novel sensing strategy

In this section, given detection probability, false alarm probability, achievable throughput and computational complexity, a novel sensing strategy is proposed. It is defined as

$$F = \eta_{\text{max}}$$

$$= \left[\frac{P_d}{O(P_d)} + \frac{1-P_f}{O(R(\tau, \varepsilon))} - \frac{P_f}{O(P_f)}\right]_{\text{max}}.$$  \hfill (43)

Theorem 1: For ED, sensing strategy $F$ reaches maximum when

$$\varepsilon = \varepsilon_1 = \left[\frac{1}{2} + \frac{1}{4} + \frac{\gamma}{2} + \frac{2\gamma + 1}{\gamma\sqrt{N}} \ln 2\sqrt{2\gamma + 1}\right] \sigma_n^2,$$  \hfill (44)

and

$$F_{\text{max}} = \frac{P_d + 1 - 2P_f}{2N + 2} \mid \varepsilon = \varepsilon_1.$$  \hfill (45)

Proof: For ED,

$$O(P_d) = O(R(\tau, \varepsilon)) = O(P_f) \approx 2N + 2.$$  \hfill (46)

According to (6), (7) and (43),

$$\frac{\partial F}{\partial \varepsilon} = \frac{\partial P_d}{\partial \varepsilon} - 2 \frac{\partial P_f}{\partial \varepsilon}.$$  \hfill (47)

Then

$$\frac{\partial F}{\partial \varepsilon} \geq 0 \Leftrightarrow \frac{\partial P_d}{\partial \varepsilon} \geq 2 \frac{\partial P_f}{\partial \varepsilon} \Leftrightarrow \frac{\varepsilon}{\sigma_n^2} - \frac{\sigma_n^2}{\sigma_n^2} \geq \frac{1}{2} \left[2 + \frac{1}{2} + \frac{\gamma}{2} + \frac{2\gamma + 1}{\gamma\sqrt{N}} \ln 2\sqrt{2\gamma + 1}\right] \sigma_n^2.$$  \hfill (48)
For CFD, the proposed sensing strategy could be further described as

\[
F = \frac{P_d}{O(P_d)} + \frac{1 - P_f}{O(R(\tau, \varepsilon))} - \frac{P_f}{O(P_f)}_{\text{max}}
\]

\[
= \frac{1}{2N^2 - 1}(P_d + 1 - 2P_f)_{\text{max}}.
\]

(49)

**Theorem 2**: For CMD, the optimization \( \lambda_i \) that maximizes

\[
F = \frac{P_d}{O(P_d)} + \frac{1 - P_f}{O(R(\tau, \varepsilon))} - \frac{P_f}{O(P_f)} \text{ consists in}
\]

\[
\lambda_i = \frac{-b + \sqrt{4ac}}{2a},
\]

(50)

\[
a = [(L-1) \frac{2}{N, \pi} - \frac{\gamma}{\gamma + 1} \lambda_i, \{2 + (L-1) \frac{2}{N, \pi} + \frac{\gamma}{\gamma + 1} \lambda_i\}],
\]

(51)

\[
b = \frac{4}{N, \varepsilon} \ln \{\frac{2(\gamma + 1)[1 + (L-1) \frac{2}{N, \pi}] + \gamma + 1 \lambda_i\},
\]

(52)

\[
c = 2[(L-1) \frac{2}{N, \pi} + \frac{\gamma}{\gamma + 1} \lambda_i\}.
\]

(53)

The optimal sensing performance could be exhibited as

\[
F_{\text{max}} = \frac{1}{LN, \varepsilon} (P_d + 1 - 2P_f) | \varepsilon = \lambda_i.
\]

(54)

**Proof**: For CMD,

\[
O(P_d) = O(R(\tau, \varepsilon)) = O(P_f) \approx LN, \varepsilon.
\]

(55)

Based on (26)-(30) and (43), we can obtain

\[
\frac{\partial F}{\partial \lambda_i} = \frac{1}{LN, \varepsilon} (\frac{\partial P_d}{\partial \lambda_i} - 2 \frac{\partial P_f}{\partial \lambda_i}).
\]

(56)

Due to

\[
\frac{\partial P_d}{\partial \lambda_i} = \frac{N}{\sqrt{2\pi \lambda_i^2}} \exp\left(-\frac{t_1^2}{2}\right),
\]

(57)

\[
\frac{\partial P_f}{\partial \lambda_i} = \frac{\sqrt{\frac{N}{2\pi}}[1 + \frac{2(L-1)}{N, \pi}]}{\sqrt{2\pi \lambda_i^2}} \exp\left(-\frac{t_2^2}{2}\right),
\]

(58)

\[
t_1 = \sqrt{\frac{N}{2\pi}} \left[ \frac{1}{\lambda_i} (1 + (L-1) \frac{2}{N, \pi}) - 1\right],
\]

(59)

\[
t_2 = \sqrt{\frac{N}{2\pi}} \left[ \frac{1}{\lambda_i} + \frac{\lambda_i \gamma}{\lambda_i (\gamma + 1) - 1}\right],
\]

(60)

then
\[
\frac{\partial F}{\partial \lambda_1} \leq 0 \iff \frac{\exp(-t_1^2/2)}{\exp(-t_2^2/2)} \leq \frac{2[1+(L-1)\sqrt{\frac{2}{N,\pi}}]}{1+\frac{\gamma}{\gamma+1} \lambda_1}.
\] (61)

(59) could be further simplified as
\[
\frac{\partial F}{\partial \lambda_1} \leq 0 \iff \frac{t_1^2 - t_2^2}{2} \geq \frac{2[1+(L-1)\sqrt{\frac{2}{N,\pi}}]}{1+\frac{\gamma}{\gamma+1} \lambda_1}
\] (62)

\[
\frac{t_1^2 - t_2^2}{2} = \ln \frac{2[1+(L-1)\sqrt{\frac{2}{N,\pi}}]}{1+\frac{\gamma}{\gamma+1} \lambda_1}
\] is the function on \( \lambda_1 \). By solving this linear equation in two unknowns, the conclusion in Theorem 2 can be reached.

### 3.3 Extensions to CSS

**Theorem 3**: For AND fusion rule-based CSS, the sensing performance is optimal when
\[
\epsilon = \epsilon_2 = \left[ \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2}{N} + \frac{2\gamma + 1}{2N} \ln 2\sqrt{2\gamma + 1}} \right] \sigma_n^2,
\] (63)

and the corresponding performance could be formulated as
\[
F_{\text{max}} = \frac{(P_d)^k + 1 - 2(P_f)^k}{2KN + 2K} \bigg|_{\epsilon = \epsilon_2}.
\] (64)

**Proof**: The proof of this theorem can refer to the demonstration in Theorem 2.

Note that for AND fusion rule-based CSS,
\[
O(P_d(N,\epsilon)) = O(R(\tau,\epsilon)) = O(P_f(N,\epsilon)) = 2KN + 2K.
\] (65)

**Theorem 4**: As for OR fusion rule-based CSS, the optimal sensing performance arrives as
\[
\epsilon = \left[ \frac{1}{2} + \sqrt{\frac{1}{2} + \frac{(2\gamma + 1)\ln 2}{N\gamma} + \frac{(2\gamma + 1)(K+1)\ln \sqrt{2\gamma + 1}}{2N\gamma}} \right] \sigma_n^2.
\] (66)

and its sensing performance could be written as
\[
F_{\text{max}} = \frac{2(1-P_f)^k - (1-P_d)^k}{2KN + 2K}.
\] (67)

**Proof**: For OR fusion rule-based CSS,
\[
O(P_d(N,\epsilon)) = O(R(\tau,\epsilon)) = O(P_f(N,\epsilon)) = 2KN + 2K.
\] (68)

On the basis of (6)-(8) and (33)-(34),
\[
\frac{\partial F}{\partial \epsilon} = K(1-P_d)^k \frac{\partial P_d}{\partial \epsilon} - 2K(1-P_f)^k \frac{\partial P_f}{\partial \epsilon},
\] (69)

then
\[
\frac{\partial F}{\partial \epsilon} \geq 0 \iff \left| \frac{\partial P_d}{\partial \epsilon} \right| \geq \left| \frac{1-P_f}{1-P_d} \right|^{k-1} \frac{\partial P_f}{\partial \epsilon}.
\] (70)
By reason of

\[ \frac{1-P_f}{1-P_d} \approx \sqrt{2\gamma+1}, \quad (71) \]

(69) could be rewritten as

\[ \frac{\partial F}{\partial \varepsilon} \geq 0 \\iff \exp(x_1 - x_2) \geq \frac{2\ln 2\sqrt{2\gamma+1} + (K-1)\ln\sqrt{2\gamma+1}}{4}, \quad (72) \]

\[ x_1 = \frac{1}{2}(\varepsilon - 1)\sqrt{\frac{f_i}{\gamma}}, \quad (73) \]

\[ x_2 = \frac{1}{2}(\varepsilon - 1)\frac{\sqrt{f_i}}{2\sqrt{\gamma+1}}. \quad (74) \]

(72) could be further simplified as

\[ \frac{2(\varepsilon - 1)^2}{4} - \frac{1}{2} \geq \frac{(2\gamma+1)\ln 2}{N\gamma} + \frac{(2\gamma+1)(K+1)\ln\sqrt{2\gamma+1}}{2N\gamma}. \quad (75) \]

That is

\[ \varepsilon \geq 1 + \sqrt{\frac{\gamma}{2} + \frac{1}{4} + \frac{(2\gamma+1)\ln 2}{N\gamma} + \frac{(2\gamma+1)(K+1)\ln\sqrt{2\gamma+1}}{2N\gamma} \sigma^2_n}. \quad (76) \]

**Theorem 5:** With regard to N-out-of-K fusion rule-based CSS, the optimal threshold \((\varepsilon = \varepsilon^*)\) exists that maximizes the sensing strategy in (41).

**Proof:** Based on (6)-(8) and (35)-(36), we obtain

\[ \frac{\partial P_f(N, \varepsilon)}{\partial \varepsilon} = \sum_{i=1}^{K} \left( \frac{K}{i} \right) \frac{P_d}{P_f} \left( 1 - \frac{P_d}{P_f} \right)^{i-1} \left( 1 - \frac{P_i}{P_f} \right)^{K-i}, \quad (77) \]

\[ \frac{\partial P_i(N, \varepsilon)}{\partial \varepsilon} = \sum_{i=1}^{K} \left( \frac{K}{i} \right) \frac{P_f}{P_d} \left( 1 - \frac{P_f}{P_d} \right)^{i-1} \left( 1 - \frac{P_i}{P_d} \right)^{K-i}. \quad (78) \]

From (41), we have

\[ \frac{\partial F}{\partial \varepsilon} \geq 0 \\iff \frac{\frac{\partial P_f(N, \varepsilon)}{\partial \varepsilon}}{\frac{\partial P_i(N, \varepsilon)}{\partial \varepsilon}} \geq \frac{2}{\varepsilon}. \quad (79) \]

According to (76)-(78), we can get

\[ \frac{\partial F}{\partial \varepsilon} \leq 0 \\iff \frac{\frac{\partial P_f}{\partial \varepsilon}}{\frac{\partial P_i}{\partial \varepsilon}} \geq \frac{2}{\varepsilon}. \quad (80) \]

The solution to (80) can be evaluated numerically (\(\varepsilon = \varepsilon^*\)). The solution is exactly the optimal detection threshold. Then the optimal sensing performance could be denoted as

\[ F_{\max} = \frac{P_f(N, \varepsilon) + 1 - 2P_f(N, \varepsilon)}{2KN + 2K} | \varepsilon = \varepsilon^*. \quad (81) \]

**Fig. 1** exhibits the flow chart of the proposed sensing strategy. Firstly, four efficiency functions \((\eta_d, \eta_f, \eta_k, \eta_t)\) are defined to describe sensing efficiency of spectrum sensing schemes (ED, CFD, CMD and CSS), where \(\eta_d\) denotes efficiency function of a CR to protect...
PU, $\eta_f$ expresses efficiency function of a false alarm, $\eta_R$ shows communication efficiency of secondary network and $\eta$ remarks the overall efficiency function of a spectrum sensing scheme. Then given sensing capacity and sensing complexity, the optimal sensing strategy is obtained based on the maximization of the overall efficiency function $\eta$. Finally, $\eta$ is applied to classical spectrum sensing schemes and the corresponding optimal sensing performance is obtained and demonstrated therein.

**Fig. 1.** The flow chart of the proposed sensing strategy

### 4. Simulation and discussion

In this section, simulations based on MATLAB platform are made to examine the performance of the proposed efficiency functions in (37)-(39) and the novel sensing strategies in (43). Orthogonal Frequency Division Multiplexing (OFDM) signal [32] is chosen as the test signal (PU) for its generality and popularity in wireless communication networks, whose center frequency is denoted as $f_c = 1.0 \times 10^8$ hertz and sampling frequency is denoted as $f_s = 3.0 \times 10^8$ hertz. The sampling point at the receiving terminal of a CR is $N = f_c \tau = 1000$ without special instructions. The sensing performance is obtained under various signal-noise-ratio (SNR) in AWGN, Rayleigh and Rician channel, respectively. The Doppler shift in both Rayleigh and Rician channel is set as 250, the Rice factor in the Rician channel is fixed as $K = 3$. In addition, the simulation values are magnified 1000 times in amplitude for convenience.
1. Generate OFDM signal
\[
PSDU = \text{round}\left(\text{rand}\left(1, 8 \cdot TXVECTOR.LENGTH\right)\right);
\]
\[
sig = \text{transmitter}\left(PSDU, TXVECTOR\right);
\]
2. Carrier modulation
\[
sig1 = \text{real}\left(\text{sig} \cdot \exp\left(j \cdot 2 \cdot \pi \cdot fc / fs \cdot (0 : \text{length}\left(\text{sig}\right) - 1)\right)\right);
\]
3. for SNR
   \[\text{existence} = 0; \text{inexistence} = 0;\]
   for statistics number
5. Noise production: \[
\text{Noise}_\text{awgn} = \text{randn}\left(1, N\right)
\]
6. average power of noise: \[
\text{AvgPower}_\text{Noise}_\text{awgn} = \text{sum}(\text{abs}\left(\text{sig1}^2\right)) / N / \text{SNR};
\]
   \[
\text{std}_\text{Noise}_\text{awgn} = \sqrt{\text{AvgPower}_\text{Noise}_\text{awgn}};
\]
   \[
\text{Noise}_\text{awgn} = \text{std}_\text{Noise}_\text{awgn} \cdot \text{Noise}_\text{awgn};
\]
7. Normalized signal sample: \[
\text{norm}_\text{signal}_\text{awgn} = \text{sig1} / \text{std}_\text{Noise}_\text{awgn};
\]
8. Normalized Noise sample: \[
\text{norm}_\text{noise}_\text{awgn} = \text{Noise}_\text{awgn} / \text{std}_\text{Noise}_\text{awgn};
\]
9. Normalized received signal in \(H_1\) case:
10. \[
\text{Rev}_\text{sig}_\text{awgn} = \text{norm}_\text{signal}_\text{awgn} + \text{norm}_\text{noise}_\text{awgn};
\]
11. Compute energy value of Normalized received signal and Normalized Noise sample
   \[
   E_{E_1} = \text{dot}(\text{Rev}_\text{sig}_\text{awgn}, \text{Rev}_\text{sig}_\text{awgn}) / N;
   \]
   \[
   E_{E_0} = \text{dot}(\text{norm}_\text{noise}_\text{awgn}, \text{norm}_\text{noise}_\text{awgn}) / N;
   \]
12. Compare \(E_{E_1}\) and \(E_{E_0}\) with their corresponding optimal sensing threshold
   \[
   \Delta_{\text{optimal}};
   \]
13. If \(E_{E_1} > \Delta_{\text{optimal}}| H_1\)
   \[
   \text{existence} = \text{existence} + 1;
   \]
14. If \(E_{E_0} > \Delta_{\text{optimal}}| H_0\)
15. \[
   \text{inexistence} = \text{inexistence} + 1;
   \]
16. End if
17. End if

**Fig. 2.** Coding flow of the proposed sensing strategy

**Fig. 2** shows coding flow of the proposed sensing strategy. It is noted that **Fig. 2** takes AWGN channel for example, where PSDU denotes the generated random number, \(TXVECTOR.LENGTH = 2000\) and \(TXVECTOR.DATARATE = 6\). In addition, \(f_c = 1.0 \times 10^6\) hz denotes the carrier frequency and \(f_s = 3.0 \times 10^6\) hz denotes the sampling frequency at the receiver of CR.

**Fig. 3** gives \(\eta_f\) comparisons of various schemes under different SNRs in various channels when \(\eta_{\text{max}}\) is maximum. The \(\eta_f\) of ED in AWGN, Rayleigh and Rician channel are all zero. Consequently, there is only one label in **Fig. 3**. So is **Fig. 4** or **Fig. 5**. From **Fig. 3**, ED has the lowest \(\eta_f\) score and CFD takes second place while CMD has the highest \(\eta_f\) score under equal conditions. This is due to that the energy value of received signal is more sensitive than its corresponding Cyclostationary feature and correlation for spectrum sensing given sensing
capacity and sensing complexity, which indicates that ED has the highest efficiency to
discover idle spectrum while this capacity of CMD is the worst.

Fig. 3. $\eta_c$ comparisons of various schemes under different scenarios

Fig. 4. $\eta_d$ comparisons of various schemes under different scenarios
Fig. 5. \( \eta_R \) comparisons of various schemes under different scenarios

\( \eta_d \) comparisons of various schemes are made under different scenarios in Fig. 4 when sensing performance is optimal according to (45), (49) and (54). In contrast to the case in Fig. 3, the \( \eta_d \) of ED is the worst while CMD has the highest \( \eta_d \) score. The reason for these is that ED suffers from the influence of noise uncertainty. It signifies that CMD has the highest efficiency to protect PU from possible interference of SU and this capacity of ED is inferior compared with the others.

Fig. 5 provides \( \eta_R \) comparisons for ED, CFD and CMD under various scenarios when sensing performance is optimal according to (45), (49) and (54). For convenience, \( C_0 P(H_o)(1 - \frac{T}{T'}) \) is regarded as 1 in the same condition for ED, CFD and CMD. Thus, Fig. 5 works as a supplement of Fig. 3. From Fig. 5, the \( \eta_R \) of ED is the highest, which manifests the superior communication efficiency of ED with its users. Compared with ED, the \( \eta_R \) of CMD and CFD declines sharply. It states clearly that the higher protection efficiency of CMD and CFD for PU is at the expense of their communication efficiency with corresponding users.

To sum up, the proposed efficiency functions in (37)-(39) offer deep insight into the sensing performance of different spectrum sensing scheme given false alarm probability, detection probability, available throughput and computational complexity. On this basis, some significant conclusions are drawn:

1) In the respect of communication efficiency in secondary networks, ED is the optimal selection.
2) CFD and CMM are adept at protecting PU from possible interference of SU.
3) Multistage detection given the combination of ED and CFD (or CMM) may consider both communication efficiency of secondary networks and PU protection.

![Graph showing comparisons of various schemes under different scenarios](image)

**Fig. 6.** $\eta_{\text{max}}$ comparisons of various schemes under different scenarios. From Fig. 6, ED is a SS scheme with the highest efficiency overall; CMD takes the second place while CFD has the lowest efficiency within the compared schemes. This indicates that ED is the optimal selection for SS in terms of both PU protection and communication capacity in secondary network. The computational complexity of CMD and CFD are relatively higher than ED, therefore their efficiency is lower than ED.

In summary, the sensing strategy in (43) provides a new perspective to estimate various schemes of SS, where both sensing performance and computational complexity is considered. As a result, this works as a global sensing strategy that considers the whole primary factors that influence sensing performance and gives a superior majorization selection for SS.

## 5. Conclusions

In this paper, three efficiency functions are provided to estimate the sensing efficiency of a spectrum sensing scheme given sensing performance and computational complexity. On this basis, a novel sensing strategy is proposed and demonstrated to detect idle spectrum with a higher efficiency. Furthermore, the proposed sensing strategy is applied to energy detector, Cyclostationary feature detector, covariance matrix detector and cooperative spectrum detector. Some significant conclusions are reached therein. Moreover, simulations also validate the effectiveness of the proposed scheme.
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