The \(\kappa\)-Weyl group and its algebra

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Abstract

The \(\kappa\)-Poincare group and its algebra in an arbitrary basis are constructed. The \(\kappa\)-deformation of the Weyl group and its algebra in any dimensions and in the reference frame in which \(g_{00} = 0\) are discussed.

1 Introduction

It is our great pleasure to contribute to this volume.

In last years we had an opportunity to collaborate with Prof. Jerzy Lukierski. Our common topic has been the deformed symmetries of space-time, mainly the so-called \(\kappa\)-Poincare algebra invented by Lukierski, Nowicki and Ruegg\(^1\). Apart from investigating the formal properties of \(\kappa\)-Poincare algebra and looking for its possible physical applications one of the main ideas of Prof. Lukierski is to extend the notion of \(\kappa\)-deformation to larger groups of space-time symmetries. This idea resulted in series of papers\(^2\)–\(^6\) devoted to the \(\kappa\)-deformation of SUSY extensions of the Poincare symmetry. The next step to be done is to look for \(\kappa\)-deformed conformal group / algebra. This problem has not been fully solved yet but some preliminary steps were already undertaken\(^7,8\).

Inspired by these papers and numerous discussions with Prof. Lukierski we attempt here to make a small step toward the solution of this problem.

Classically, the conformal group in four dimensions is nothing but \(SO(4,2)\). However, the standard (matrix) parametrization of \(SO(4,2)\) is not used, when \(SO(4,2)\) is viewed as conformal group. On the contrary, the conformal group is obtained from the action of \(SO(4,2)\) on light cone in six-dimensional space-time. But the light-cone coordinates are related in rather complicated way to Minkowski coordinates in four dimensions. Consequently, the standard parametrization of \(SO(4,2)\) is related to the “conformal” one by a complicated (even somewhere singular) change of group parameters. This poses no problem on the “classical” level. However, if we are passing to the “quantum” (i.e. deformed) case we are faced with typical ordering problems of quantization procedure. This gives some flavor of difficulties one meets trying to deform the conformal group.

In the recent paper\(^8\), Lukierski, Minnaert and Mozrzymas considered a new class of classical \(r\)-matrices on conformal algebras in three and four dimensions, which obey the classical Yang-Baxter equation and depend on dimensionfull parameter. An important observation concerning the \(d = 4\) case was that the classical \(r\)-matrices obtained by them depend only on generators belonging to Poincare subalgebra of conformal algebra. Due to the fact that they obey the classical Yang-Baxter equation (and not modified one) they provide \(r\)-matrices for any algebra, containing

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*Supported by KBN grant 2P30221706p02
Poincare algebra as subalgebra. One of the $r-$matrices considered in\textsuperscript{8} leads to the so called null-plane deformation of Poincare algebra found, by different methods, in Ref.\textsuperscript{9}. This deformation is similar to the standard $\kappa-$deformation. The only difference is in the choice of undeformed subalgebra which is the stability algebra of light-like fourvector instead of time-like one. However, this difference is significant: in the standard case the Schouten bracket is ad-invariant but does not vanish. Therefore, the relevant $r-$matrix does not provide automatically the $r-$matrix for any extension of Poincare algebra. Actually, the invariance is broken already after adjoining the dilatation generator $D$. Our aim here is to put the results of Ref.\textsuperscript{8} in more general setting. In sec.\textsuperscript{2} we review the properties of Poincare group for arbitrary chosen metric and discuss the Poisson structure on it. In sec.\textsuperscript{3} the quantization of this classical structure is performed. The bicrossproduct form of resulting quantum group allows us to find, by duality, the relevant algebra. The Weyl group and its algebra are constructed in sec.\textsuperscript{4}. Finally, sec.\textsuperscript{5} is devoted to some conclusions.

2 Relativity theory in on arbitrary coordinate system.

Let us consider the $n-$dimensional linearal metric space $M$ with metric tensor $g_{\mu\nu}$ ($\mu,\nu=0,1,...,n-1$) given by an arbitrary nondegenerate symmetric $n \times n$ matrix (not necessary diagonal).

Poincare group $P$ is the group of inhomogenous transformations of the space $M$:

$$x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$$

where the matrices $\Lambda^\mu_\nu$ (Lorentz group) satisfy the condition:

$$g_{\mu\nu} = \Lambda^\alpha_\mu \Lambda^\beta_\nu g_{\alpha\beta}.$$ 

It is easy to see that the Poincare algebra $\tilde{P}$ reads:

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda} P_\mu - g_{\mu\lambda} P_\nu)$$

$$[M_{\mu\nu}, M_{\lambda\sigma}] = i(g_{\mu\sigma} M_{\nu\lambda} - g_{\nu\sigma} M_{\mu\lambda} + g_{\nu\lambda} M_{\mu\sigma} - g_{\mu\lambda} M_{\nu\sigma})$$

where

$$(M_{\alpha\beta})^\mu_\nu = i(\delta^\mu_\alpha g_{\nu\beta} - \delta^\mu_\beta g_{\nu\alpha}).$$

Now consider $r \in \wedge^2 \tilde{P}$ given as follows,\textsuperscript{10}:

$$r = \frac{i}{\kappa} M_{0\nu} \wedge P^\nu = r^{\mu\nu,\alpha} M_{\mu\nu} \wedge P_\alpha$$

(2.1)

where

$$r^{\mu\nu,\alpha} = \frac{i}{2\kappa} (\delta^\mu_\alpha g^{\nu\beta} - \delta^\nu_\beta g^{\mu\alpha})$$

and $\kappa$ is a real deformation parameter.

A calculation of Schouten bracket of $r$ with itself yields

$$[r, r] = \frac{i g_{00}}{\kappa^2} M_{\alpha\beta} \wedge P^\alpha \wedge P^\beta.$$ 

(2.2)

It is not difficult to see that $[r, r]$ is invariant, hence $r$ is defines a structure of a Poisson Lie group on $\mathcal{P}$, by the formula:

$$\{f, g\} = 2r^{\alpha\beta} (X^R_\alpha f X^R_\beta g - X^L_\alpha f X^L_\beta g)$$

(2.3)

where $X^R_\alpha, X^L_\beta$ are the right- and left-invariant vector fields.
It is easy to find the following expressions for the invariant vector fields:

\[
X^\alpha_L = \Lambda^\mu_\alpha \frac{\partial}{\partial \Lambda^\mu_\beta} - \Lambda^\mu_\beta \frac{\partial}{\partial \Lambda^\mu_\alpha} \\
X^\alpha_R = \Lambda^\mu_\alpha \frac{\partial}{\partial a^\mu} \\
X^{\alpha\beta}_R = \Lambda^\beta_\nu \frac{\partial}{\partial \Lambda^\alpha_\nu} - \Lambda^\alpha_\nu \frac{\partial}{\partial \Lambda^\beta_\nu} + a^\beta \frac{\partial}{\partial a_\alpha} - a^\alpha \frac{\partial}{\partial a_\beta} \frac{\partial}{\partial a_\alpha}.
\]

In the Lie algebra basis corresponding to the above vector fields we have the following relation between the generators of the Poincare algebra and the invariant vector fields:

\[
M^{\alpha\beta} = \hat{i}X^{\alpha\beta} \\
P^\alpha = X^\alpha.
\]

This enables us to calculate the Poisson brackets of the coordinate functions \(P\):

\[
\{\Lambda^\alpha_\beta, a^\sigma\} = \frac{-1}{\kappa}(\Lambda^\alpha_0 - \delta^\alpha_0)\Lambda^\sigma_\beta + (\Lambda_0^\beta - g_0^\beta)g^{\alpha\sigma} \\
\{a^\alpha, a^\sigma\} = \frac{1}{\kappa}(\delta_0^\alpha a^\sigma - \delta_0^\sigma a^\alpha) \\
\{\Lambda^\alpha_\beta, \Lambda^\mu_\nu\} = 0.
\]

(2.4)

3 The \(\kappa\)-Poincare group and \(\kappa\)-Poincare algebra in an arbitrary basis.

If we perform the standard quantizations of the Poisson brackets of the coordinate functions on \(P\) by replacing \(\{,\} \to \frac{1}{\hbar}[,,]\) one obtains the following set of commutation relations:

\[
[\Lambda^\alpha_\beta, a^\sigma] = \frac{-i}{\kappa}(\Lambda^\alpha_0 - \delta^\alpha_0)\Lambda^\sigma_\beta + (\Lambda_0^\beta - g_0^\beta)g^{\alpha\sigma} \\
[a^\alpha, a^\sigma] = \frac{i}{\kappa}(\delta_0^\alpha a^\sigma - \delta_0^\sigma a^\alpha) \\
[\Lambda^\alpha_\beta, \Lambda^\mu_\nu] = 0.
\]

(3.1)

This standard quantization procedure is unambiguous: there is no ordering ambiguity when quantizing the right-hand side of Eq.(2.4) due to the commutativity of \(\Lambda\)'s.

Since the composition law is compatible with Poisson brackets, the above commutation rules are compatible with the following coproduct:

\[
\Delta \Lambda^\mu_\nu = \Lambda^\mu_\nu \otimes \Lambda^\nu_\nu \\
\Delta (a^\mu) = \Lambda^\mu_\nu \otimes a^\nu + a^\mu \otimes I.
\]

(3.2)

The antipode and the counit are given by:

\[
S(\Lambda^\mu_\nu) = \Lambda^\nu_\mu \\
S(a^\mu) = -\Lambda^\mu_\nu a^\nu \\
\varepsilon(\Lambda^\mu_\nu) = \delta^\mu_\nu \\
\varepsilon(a^\mu) = 0.
\]

(3.3)

If we define the \(*\)-operation in such away that \(\Lambda^\mu_\nu\) and \(a^\mu\) are selfadjoint elements, we conclude that the relations Eq.(3.1), (3.2), (3.3) define a Hopf \(*\)-algebra — the \(\kappa\)-Poincare group \(P_\kappa\). It
follows from the Eq.(3.1), (3.2), (3.3) that the form of the $\kappa$–Poincare group does not depend on the choice of the metric tensor $g_{\mu\nu}$. The differences between the various $\kappa$–Poincare group are related to the fact that $\Lambda_{0\beta}$ appearing in the first commutation relation of Eq.(3.1) are not the indipendent variables but are the linear combinations of the independent ones: $\Lambda_{0\beta} = g_{0\nu}\Lambda^\mu_{\beta}$.

It should be stressed, that the $\kappa$–Poincare group can be defined as a right-left bicrossproduct\textsuperscript{11,12}:

$$\mathcal{P}_\kappa = T^* \triangleleft C(L).$$

To see this it is sufficient to define the structure maps:

$$\beta(a^\mu) = \Lambda^\mu_\nu \otimes x^\nu$$

$$\Lambda^\mu_\nu \otimes x^e = -\frac{i}{\kappa}((\Lambda^\mu_0 - \delta^\mu_0)\Lambda^\nu_0 + (\Lambda_{0\nu} - g_{0\nu})g^{\mu\nu}).$$

Moreover, while $C(L)$ is the standard algebra of function defined over Lorentz group, $T^*$ is defined by the following relations:

$$[a^\mu, a^\nu] = i\frac{g^{\mu\nu} - \delta^{\mu\nu}a^0}{\kappa}$$

$$\Delta(a^\mu) = a^\mu \otimes I + I \otimes a^\mu$$

$$S(a^\mu) = -a^\mu$$

$$\varepsilon(a^\mu) = 0.$$

The bicrossproduct structure of the $\kappa$–Poincare group allows us to define the dual object, the $\kappa$–Poincare algebra $\tilde{\mathcal{P}}_\kappa$ as a left-right bicrossproduct:

$$\tilde{\mathcal{P}}_\kappa = T \triangleright U(\tilde{L})$$

where $T$ is dual to $T^*$ and $U(\tilde{L})$ is the universal enveloping algebra of the Lorentz algebra. The duality $T^* \iff T$ is defined by:

$$<a^\mu, P_\nu> = i\delta^\mu_\nu.$$.

The duality between the Lorentz group and algebra is defined in the standard way:

$$<\Lambda^\mu_\nu, M^{\alpha\beta}> = i(g^{\alpha\mu}\delta^\beta_\nu - g^{\beta\mu}\delta^\alpha_\nu).$$

The structure maps are defined by the following duality relations:

$$<t, M^{\alpha\beta} \triangleright P_\gamma> = <\beta(t), M^{\alpha\beta} \otimes P_\gamma>$$

$$<\Lambda \triangleleft t, M^{\alpha\beta}> = <\Lambda \otimes t, \delta(M^{\alpha\beta})>$$

here $t$ is arbitrary product of $a$’s while $\Lambda$ is an arbitrary product of $\Lambda$’s.

Finally using the method described in\textsuperscript{19} we arrive at the following explicite form of the $\kappa$–Poincare algebra:

a) the commutation rules:

$$[M^{ij}, P_0] = 0$$

$$[M^{ij}, P_k] = i\kappa(\delta^{ij}_k g^{00} - \delta^{ij}_k g^{0j})(1 - e^{-\frac{P_0}{P_k}}) + i(\delta^{ij}_k g^{is} - \delta^{ij}_k g^{js})P_s$$

$$[M^{00}, P_0] = i\kappa g^{00}(1 - e^{-\frac{P_0}{P_k}}) + ig^{ik}P_k$$

$$[M^{00}, P_k] = -i\frac{\kappa}{2}g^{00}\delta^i_k(1 - e^{-\frac{P_0}{P_k}}) - i\delta^{ij}_k g^{0s}P_s e^{-\frac{P_0}{P_k}} +$$

$$+ ig^{0s}P_k (e^{-\frac{P_0}{P_k}} - 1) + i\frac{\kappa}{2}\delta^i_k g^{is}P_s P_k - i\frac{\kappa}{g^{is}P_s P_k} - i\frac{\kappa}{g^{is}P_s P_k}$$

$$[P_\mu, P_\nu] = 0$$

$$[M^{\mu\nu}, M^{\lambda\sigma}] = i(g^{\mu\sigma}M^{\nu\lambda} - g^{\nu\sigma}M^{\mu\lambda} + g^{\nu\lambda}M^{\mu\sigma} - g^{\mu\lambda}M^{\nu\sigma})$$
b) the coproducts:

\[ \Delta P_0 = I \otimes P_0 + P_0 \otimes I \]
\[ \Delta P_k = P_k \otimes e^{-\frac{P_0}{\kappa}} + I \otimes P_k \]
\[ \Delta M^{ij} = M^{ij} \otimes I + I \otimes M^{ij} \]
\[ \Delta M^{i0} = I \otimes M^{i0} + M^{i0} \otimes e^{-\frac{P_0}{\kappa}} - \frac{1}{\kappa} M^{ij} \otimes P_j \]

where \( i, j, k = 1, 2, 3, \ldots, n - 1. \)

Let us note that the \( \kappa \)-Poincare algebra and group in any dimensions and for the diagonal metric tensor were obtained in 14 and 15. However the duality between the \( \kappa \)-Poincare algebra and group was not discussed in these papers.

In the end of this section let us remark that the classical \( r \)-matrix Eq.(2.1), \( r = M_{0i} \wedge P^i \), does not modify the coproducts for the generators \( M^{ij} \), forming undeformed Lie subalgebra as well as the component \( P_0 \) of the fourmomentum. The algebra with the generators \( M^{ij}, P_0 \) describes the classical subalgebra of our \( \kappa \)-Poincare algebra.

4 The \( \kappa \)-Weyl group and algebra

The classical Weyl group \( W \) consists of the triples \((a, \Lambda, e^b)\), where \( a \) is a \( n \)-vector, \( \Lambda \) is the matrix of the Lorentz group in \( n \)-dimensions and \( b \in \mathbb{R} \), with the composition law:

\[ (a^\mu, \Lambda^\mu_\nu, e^b) \ast (a'^\nu, \Lambda'^\nu_\alpha, e^{b'}) = (\Lambda^\mu_\nu e^b a'^\nu + a^\mu, \Lambda^\mu_\nu \Lambda'^\nu_\alpha, e^b e^{b'}) \]

Its Lie algebra, the Weyl algebra \( \tilde{W} \) reads:

\[ [P_\mu, P_\nu] = 0 \]
\[ [M_\mu\nu, P_\lambda] = i(g_{\mu\lambda} P_\nu - g_{\mu\nu} P_\lambda) \]
\[ [M_\mu\nu, M_\lambda\sigma] = i(g_{\mu\sigma} M_\nu\lambda - g_{\nu\sigma} M_\mu\lambda + g_{\nu\lambda} M_\mu\sigma - g_{\mu\lambda} M_\nu\sigma) \]
\[ [M_\mu\nu, D] = 0 \]
\[ [P_\mu, D] = -iP_\mu \]

here \( M_\mu\nu, P_\mu \) are the generators of the Poincare algebra and \( D \) is the dilatation generator.

We would like to obtain the \( \kappa \)-deformation of the Weyl group and its Lie algebra. It is clear that the classical \( r \)-matrix for the Poincare algebra satisfying the classical Yang-Baxter equation (CYBE) is also a classical \( r \)-matrix for the Weyl algebra. From the Eq.(2.2) follows that our classical \( r \)-matrix Eq.(2.1) satisfies the CYBE iff \( g_{00} = 0 \). This means that our \( r \)-matrix defines a structure of a Poisson Lie group on the Weyl group only in the basis in which the metric tensor takes such a form that \( g_{00} = 0 \). We shall consider only these types of metrics.

In order to obtain the \( \kappa \)-deformation of the Weyl group we firstly find the invariant fields:

\[ X_{L}^{\alpha \beta} = \Lambda^\mu_\beta \frac{\partial}{\partial \Lambda^\alpha_\mu} - \Lambda^\mu_\alpha \frac{\partial}{\partial \Lambda^\beta_\mu} \]
\[ X_{L}^{\alpha} = e^b \Lambda^\mu_\alpha \frac{\partial}{\partial a^\mu} \]
\[ X_{L} = e^b \frac{\partial}{\partial e^b} \]
\[ X_{R}^{\alpha \beta} = \Lambda^\beta_\gamma \frac{\partial}{\partial \Lambda^\alpha_\gamma} - \Lambda^\alpha_\beta \frac{\partial}{\partial \Lambda^\gamma_\beta} + a^\beta \frac{\partial}{\partial a^\alpha} - a^\alpha \frac{\partial}{\partial a^\beta} \]
\[ X_{R}^{\alpha} = \frac{\partial}{\partial a^\alpha} \]
\[ X_{R} = a^\mu \frac{\partial}{\partial a^\mu} + e^b \frac{\partial}{\partial e^b}. \]
Then, using the Eq.(2.3) we calculate the Poisson brackets of the coordinate functions on \( W \) and perform the standard quantizations of the Poisson brackets, by replacing \( \{ , \} \rightarrow \frac{i}{\hbar} [ , ] \).

Finally we obtain the following set of commutation relations:

\[
\begin{align*}
[\Lambda^\alpha_\beta, a^\sigma] &= -\frac{i}{\kappa} ((e^b \Lambda^\alpha_0 - \delta^\alpha_0) \Lambda^\sigma_\beta + (\Lambda_0^\beta - e^b g_0^\beta) g^{\alpha \sigma}) \\
[a^\rho, a^\sigma] &= \frac{i}{\kappa} (\delta^\rho_0 a^\sigma - \delta^\sigma_0 a^\rho) \\
[\Lambda^\alpha_\beta, \Lambda^\mu_\nu] &= 0 \\
[\Lambda^\alpha_\nu, b] &= 0 \\
[a^\mu, b] &= 0 .
\end{align*}
\]

This standard quantization procedure is unambiguous: there is no ordering ambiguity. Since the composition law is compatible with the Poisson brackets, the above commutation rules are compatible with the following coproduct:

\[
\begin{align*}
\Delta \Lambda^\mu_\nu &= \Lambda^\mu_\alpha \otimes \Lambda^\alpha_\nu \\
\Delta a^\mu &= e^b \Lambda^\mu_\nu \otimes a^\nu + a^\mu \otimes I \\
\Delta b &= b \otimes I + I \otimes b. 
\end{align*}
\]

(4.1)

The antipode and the counit are given by:

\[
\begin{align*}
S(\Lambda^\alpha_\nu) &= \Lambda^\nu_\nu \\
S(a^\mu) &= -e^b \Lambda^\mu_\nu a^\nu \\
S(b) &= -b \\
\varepsilon(\Lambda^\alpha_\nu) &= \delta^\alpha_\nu \\
\varepsilon(a^\mu) &= 0 \\
\varepsilon(b) &= 0 .
\end{align*}
\]

(4.2)

We conclude that the Eq.(4.1), (4.2) define the Hopf algebra — \( \kappa \)-Weyl group \( \mathcal{W}_\kappa \).

If we forget for a moment about our general theory, we can check by explicit calculation that our structure is self consistent (Jacobi identities, the relations \( [\Delta a, \Delta b] = \Delta [a, b] \)) iff \( g_{00} = 0 \) or \( b = 0 \).

For example:

\[
[\Lambda^\alpha_\beta, [a^\rho, a^\sigma]] + [a^\sigma, [\Lambda^\alpha_\beta, a^\rho]] + [a^\rho, [a^\sigma, \Lambda^\alpha_\beta]] = \frac{1}{\kappa^2} g_{00} (1 - e^{-2b}) (g^{\alpha \sigma} \Lambda^\rho_\beta - g^{\alpha \rho} \Lambda^\sigma_\beta) .
\]

It is easy to see that the \( \mathcal{W}_\kappa \) has a right-left bicrossproduct structure:

\[
\mathcal{W}_\kappa = T^* \bowtie C(A)
\]

where \( C(A) \) is the standard algebra of functions defined over group \( A \). The group \( A \) consists of the pairs \((\Lambda, e^b)\), where \( \Lambda \) is a matrix of the Lorentz group and \( b \in R \) and with the composition law:

\[
(\Lambda, e^b) \ast (\Lambda', e^{b'}) = (\Lambda \Lambda', e^{b + e^{b'}}).
\]

\( T^* \) is defined by the relations:

\[
\begin{align*}
[a^\rho, a^\sigma] &= \frac{i}{\kappa} (\delta^\rho_0 a^\sigma - \delta^\sigma_0 a^\rho) \\
\Delta a^\mu &= a^\mu \otimes I + I \otimes a^\mu \\
S(a^\mu) &= -a^\mu \\
\varepsilon(a^\mu) &= 0 .
\end{align*}
\]

(4.3)
To see this it is sufficient to define the structure maps as follows:

\[
\beta(\alpha^\mu) = e^b \Lambda^\mu_\alpha \otimes a^\alpha
\]

\[
\Lambda^\mu_\nu \lhd a^\epsilon = -\frac{i}{\kappa}((e^b \Lambda^\mu_\alpha - \delta^\mu_\alpha)\Lambda^\epsilon_\nu + (\Lambda_{\mu\nu} - e^b g_{\mu\nu})g^{\epsilon\rho})
\]

\[
e^b \rhd a^\epsilon = 0.
\]

This right-left bicrossproduct structure of the \( \kappa \)-Weyl group allows us to define the \( \kappa \)-Weyl algebra \( \hat{W}_\kappa \) as a left-right bicrossproduct structure:

\[
\hat{W}_\kappa = T \bowtie U(\hat{A})
\]

where \( T \) is dual to \( T^* \) and \( U(\hat{A}) \) is the universal enveloping algebra of the Lie algebra \( \hat{A} \) of the group \( A \).

The duality \( T^* \iff T \) is defined by:

\[
\langle a^\mu, P_\nu \rangle = i\delta^\mu_\nu.
\]

The duality between the group \( A \) and the algebra \( \hat{A} \) is defined in the standard way:

\[
\langle \Lambda^\mu_\nu, M^{\alpha\beta} \rangle = i(g^{\alpha\mu}\delta^\beta_\nu - g^{\beta\mu}\delta^\alpha_\nu)
\]

\[
\langle b, M^{\alpha\beta} \rangle = 0
\]

\[
\langle \Lambda^\mu_\nu, D \rangle = 0
\]

\[
\langle b^a, D \rangle = i\delta_{\nu,1}.
\]

The structure maps are defined by the following duality relations:

\[
\langle t, M^{\alpha\beta} \rhd P_\gamma \rangle = \langle \beta(t), M^{\alpha\beta} \otimes P_\gamma \rangle
\]

\[
\langle t, D \rhd P_\gamma \rangle = \langle \beta(t), D \otimes P_\gamma \rangle
\]

\[
\langle \Gamma \lhd t, M^{\alpha\beta} \rangle = \langle \Gamma \otimes t, \delta(M^{\alpha\beta}) \rangle
\]

\[
\langle \Gamma \lhd t, D \rangle = \langle \Gamma \otimes t, \delta(D) \rangle
\]

here \( t \) is arbitrary product of \( a \)'s while \( \Gamma \) is an arbitrary product of \( \Lambda \)'s.

Finally using the method described in\(^{13}\) we arrive at the following explicite form of the \( \kappa \)-Weyl algebra:

a) the commutation rules:

\[
[M^{ij}, P_0] = 0
\]

\[
[M^{ij}, P_k] = i\kappa(\delta^i_kg^{0i} - \delta^i_kg^{0j})(1 - e^{-\frac{\rho_0}{2\kappa}}) + i(\delta^i_kg^{is} - \delta^i_kg^{sj})P_s
\]

\[
[M^{00}, P_0] = i\kappa g^{00}(1 - e^{-\frac{\rho_0}{2\kappa}}) + ig^{ij}P_k
\]

\[
[M^{00}, P_k] = -i\kappa g^{00}\delta^i_k(1 - e^{-\frac{\rho_0}{2\kappa}}) - i\delta^i_kg^{0s}P_se^{-\frac{\rho_0}{\kappa}} +
\]

\[
+i\delta^i_kg^{0s}P_sP_k - \frac{i}{2\kappa}\delta^i_kg^{rs}P_rP_s - \frac{i}{\kappa}g^{0s}P_sP_k
\]

\[
[D, P_0] = i\kappa(1 - e^{-\frac{\rho_0}{2\kappa}})
\]

\[
[D, P_i] = iP_ie^{-\frac{\rho_0}{2\kappa}} + \frac{i}{2}\kappa g^{00}g_{0i}(1 - e^{-\frac{\rho_0}{2\kappa}})^2 + ig_{00}g^{0s}P_s(1 - e^{-\frac{\rho_0}{2\kappa}}) + \frac{i}{2\kappa}g_{0s}g^{rs}P_rP_s
\]

b) the coproducts:

\[
\Delta D = D \otimes I + I \otimes D - g_{0i}M^{00} \otimes (1 - e^{-\frac{\rho_0}{2\kappa}}) - \frac{1}{\kappa}g_{0i}M^{ik} \otimes P_k
\]

\[
\Delta P_0 = I \otimes P_0 + P_0 \otimes I
\]

\[
\Delta P_k = P_k \otimes e^{-\frac{\rho_0}{2\kappa}} + I \otimes P_k
\]

\[
\Delta M^{ij} = M^{ij} \otimes I + I \otimes M^{ij}
\]

\[
\Delta M^{00} = I \otimes M^{00} + M^{00} \otimes e^{-\frac{\rho_0}{2\kappa}} - \frac{1}{\kappa}M^{ij} \otimes P_j.
\]
5 Conclusions

We have constructed the $\kappa$–Poincare group resulting from Poincare group formulated in an arbitrary basis. The quantization is unambiguous due to the absence of ordering problems. The resulting quantum group has a bicrossproduct structure. Using this and the methods developed in\textsuperscript{13} we were able to construct the relevant $\kappa$–Poincare algebra. The Schouten bracket of the classical $r$–matrix we have used appeared to be proportional to the $g_{00}$ component of the metric tensor. Therefore in the reference frame chosen in such a way that $g_{00} = 0$ the relevant Poisson structure can be extended to any group containing Poincare group as a subgroup. This was used to define the Poisson structure on the Weyl group, which allowed us to construct $\kappa$–deformation of this group. Again we obtained a bicrossproduct structure which allowed us to construct the relevant $\kappa$–Weyl algebra.

The above construction seems to us to be a proper introductory step toward the definition $\kappa$–deformed conformal group. One can attempt to quantize the Poisson structure on conformal group resulting from the same $r$–martix we used in the case of the Weyl group hoping that the ordering problems could be overcome in some way (as for example in the case of $\kappa$–Poincare supergroup\textsuperscript{2–6}). As a next step one try to construct the relevant algebra. This might be more difficult as the bicrossproduct structure is lacking in the case of conformal group.

An alternating way of attacking the problem would be to try to incorporate on the quantum level the property of conformal group that it can be obtained from Weyl group by adding (in a special way) the operation of inversion.

6 References

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