Thermally activated in-plane magnetization rotation induced by spin torque

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We study the role of thermal fluctuations on the spin dynamics of a thin permalloy film with a focus on the behavior of spin torque and find that the thermally assisted spin torque results in new aspects of the magnetization dynamics. In particular, we uncover the formation of a finite, spin torque-induced, in-plane magnetization component. The orientation of the in-plane magnetization vector depends on the temperature and the spin-torque coupling. We investigate and illustrate that the variation of the temperature leads to a thermally-induced rotation of the in-plane magnetization.

I. INTRODUCTION

We are witnessing a growing body of research on various phenomena related to the transfer of angular momentum by means of an electric current\textsuperscript{1}. The fact that the electron current carries/transfers spin is well-known, the interest to this phenomenon was fueled however by the experimental demonstrations that the electric current can strongly affect the magnetization dynamics in nanostructures\textsuperscript{2} with important consequences for technological applications such as steering magnetic domain walls\textsuperscript{3,4} and vortices, conception of high-frequency electrical oscillators\textsuperscript{5,6}, and the magnetization reversal in magnetic layers via exerting spin torque\textsuperscript{4}. The latter is achieved by a spin-polarized charge current and has been demonstrated for various magnetic nanostructures\textsuperscript{7–25} and magnetic tunnel junctions\textsuperscript{26}. While several microscopic mechanisms relevant for various nanosystems have been discussed, on the macroscopic level the effect of the spin-polarized current can also be described by the well-established macroscopic Landau-Lifshitz-Gilbert (LLG) equation\textsuperscript{27} upon including the appropriate spin-torque terms.

Another important factor, which can influence substantially the magnetization switching in nanostructures, is the effect of thermal fluctuations\textsuperscript{18} (here we refer to the very comprehensive recent overview\textsuperscript{17} and references therein for the details of the well-studied finite-temperature spin dynamics). This effect can be captured by including fluctuating Langevin fields into the LLG equation. Following the standard protocol\textsuperscript{15}, the magnetization trajectory can be identified as the average over the ensemble of noninteracting nanoparticles and described by the Fokker-Planck (FP) equation.

In this paper we consider the combined influence of thermal fluctuations and the spin torque terms previously derived for the case of the current-induced motion of a magnetic domain wall in a quasi-one-dimensional ferromagnet with easy-axis and easy-plane anisotropies\textsuperscript{14}. We show that such a torque term leads to an interesting physical phenomenon of thermally activated in-plane magnetization rotation.

We will show that in case of spin torque exerted by spin polarized current, orientation of the in-plane magnetization can be easily switched and controlled by thermal heating or thermal cooling of the system. Discovered effect may have promising applications based on controlling magnetization dynamics in nanostructures. A key issue in our result is the ratio between the thermal activation energy and the Zeeman energy of the magnetization vector in the external driving magnetic field. This means from the experimental viewpoint that our theoretical proposal can be easily implemented by tuning the amplitude of the external driving magnetic field. Thus, the in-plane magnetization vector can be controlled and switched by out-of-plane external magnetic field.

To obtain analytical solutions of the FP equation we develop a perturbation approach which substantially differs from the previously discussed methods\textsuperscript{19,25}. The advantage of our approach is that it allows for obtaining some analytical solutions with high accuracy in arbitrary order of the perturbation theory.

II. MODEL

The finite temperature magnetization dynamics in a thin ferromagnetic layer in the presence of a spin torque and an external magnetic field can be described by the following stochastic LLG equation\textsuperscript{14,18,26}:

\[
\begin{align*}
\frac{dM}{dt} &= \gamma_e M \times (H_{eff} + h(t)) \\
&- \gamma_s \lambda M \times [M \times (H_{eff} + h(t))] \\
&+ b M \times s + a M \times (M \times s) \\
&\equiv \mathcal{F} M + \mathcal{W}.
\end{align*}
\]

(1)

Here \(H_{eff}\) is the effective magnetic field and \(h(t)\) is the random Langevin magnetic field related to the thermal fluctuations, \(a, b\) are the Slonczewski spin torque constants, \(\gamma_e\) is the gyromagnetic ratio for electrons, and \(\lambda\) is a phenomenological (Gilbert) damping constant. For convenience we introduce dimensionless quantities. Thus we deal with a normalized magnetization vector \(|M| = 1\), a dimensionless (rescaled) damping \(\lambda \rightarrow \lambda/|M|\), the torque constants \(a/\omega_0 \rightarrow \varepsilon a\), and \(b/\omega_0 \rightarrow \varepsilon b\). The dimensionless time \(t \rightarrow \omega_0 t\) is defined through the frequency of the Larmor precession in the...
effective field $\omega_0 = \gamma |\mathbf{H}_{eff}|$. The anisotropy field for the ferromagnetic system can be evaluated for thin film alloys of the permalloy class (Fe-Ni, Fe-Co-Ni) by using the formula\textsuperscript{27,28} $\beta_A = 2K_1/M_s$, where $K_1$ is the anisotropy coefficient and $M_s$ is the saturation magnetization. In particular, for a thin film\textsuperscript{26,28,29} Fe$_{50}$Co$_{25}$Ni$_{25}$ the saturation magnetization of the film is of the order of $M_s \approx 1025$ G, the anisotropy constant $K_1 \approx 4 \times 10^7$ erg/cm$^3$, the anisotropy field $\beta_A \approx 7.8$ Oe, and the anisotropy field in units of the frequency is $\omega_0 = \gamma \beta_A \approx 0.138 \times 10^9$ Hz ($\gamma = 1.755 \times 10^7$ Oe$^{-1}$sec$^{-1}$), while the Zeeman frequency in the reasonable strong external magnetic field is $\omega_0 = \gamma \mathbf{H}_0 \approx 17 \times (10^5 \div 10^6)$ MHz ($|\mathbf{H}_0| \approx 10^2 \div 10^3$ Oe). Since $\omega_0 > \omega_p$ we conclude that for the Fe-Co-Ni alloy, the dominating factor is the external magnetic field $\mathbf{H}_{eff} = (0, 0, H_0)$.

The components of the Langevin field $h_\alpha$, $\alpha = x, y, z$ obey the following correlation relations:

$$\langle h(t) \rangle = 0,$$  \hspace{1cm} $\langle h_\alpha(t)h_\beta(t') \rangle = 2\lambda T \delta_{\alpha\beta} \delta(t-t'), \tag{2}$$

where the averaging $(\langle \cdots \rangle)$ is performed over all possible realizations of the random field $\mathbf{h}(t)$. For the derivation of the stochastic Fokker-Planck equation we follow Ref.\textsuperscript{30} and use the functional integration method in order to average the dynamics over all possible realizations of random noise field. As shown in Ref.\textsuperscript{30} this method is quite general and straightforward, and for the case of a small coupling between the system and the bath it preserves previously obtained results\textsuperscript{28}.

We define the distribution function in the following form

$$f(\mathbf{h}, t) = \langle \pi(t, [\mathbf{h}]) \rangle_n, \quad \pi(t, [\mathbf{h}]) = \delta(\mathbf{N} - \mathbf{M}(t)). \tag{3}$$

Here $\mathbf{N}$ is the unit vector on the sphere, and we assume that the random field $\mathbf{h}(t)$ stands for a Gaussian noise with the associated functional:

$$F[\mathbf{h}(t)] = \frac{1}{Z_h} \exp \left[ -\frac{1}{2g} \int_{-\infty}^{+\infty} d\tau [\mathbf{h}^2(\tau)] \right]. \tag{4}$$

Here $Z_h = \int D\mathbf{h} F$ is the normalization factor and $D\mathbf{h}$ denotes the functional integration over all possible realizations of the random field $\mathbf{h}(t)$, and $g = 2\lambda T$. Note that for convenience we measure the temperature in units of the Larmor frequency $\omega_0 = \gamma |\mathbf{H}_{eff}|$. Therefore, the dimensionless temperature is defined via the expression $T \to k_B T / \omega_0 \hbar$.

Taking into account the relations\textsuperscript{30}.

$$\frac{\delta h_\alpha(\tau)}{\delta h_\beta(t)} = \delta_{\alpha\beta} \delta(t - \tau),$$

$$\int D\mathbf{h} \frac{\delta^n F[\mathbf{h}]}{\delta h_\alpha(t_1) \delta h_\beta(t_2) \cdots \delta h_\gamma(t_n)} = 0, \tag{5}$$

and following the standard procedure\textsuperscript{30}, we deduce from Eq.\textsuperscript{10} the following FP equation:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \mathbf{N}} \left\{ -\mathbf{N} \times \mathbf{H}_{eff} + \lambda \mathbf{N} \times \mathbf{N} \times \mathbf{H}_{eff} + \varepsilon_b \mathbf{N} \times \mathbf{s} + \varepsilon_a \mathbf{N} \times \mathbf{s} - \lambda T \mathbf{N} \times \mathbf{N} \times \frac{\partial}{\partial \mathbf{N}} \right\} f. \tag{6}$$

Solving for such a time-dependent FP equation is a difficult problem even in the absence of spin torque terms\textsuperscript{27,28}.

In the presence of spin torque, the analytical consideration of the non-stationary FP equation becomes intractable. To proceed further we consider a particular configuration of the spin torque $s = (s, 0, 0)$\textsuperscript{14,15} and the driving external field $\mathbf{H}_{eff} = (0, 0, H_0)$ terms. Here for convenience, the amplitude of the renormalized magnetic field is set to one. We will look for the perturbation solution of Eq.\textsuperscript{6} and consider the case $\epsilon = 1/\omega_0 \ll 1$ ($\omega_0$ is the Larmor precession frequency in the external constant magnetic field) as a small parameter of the theory and look for a stationary solution of Eq.\textsuperscript{6} in form

$$f = C \exp \left[ \frac{1}{T} (\mathbf{N} \cdot \mathbf{H}_{eff}) + \varepsilon \psi(\mathbf{N}) \cdots \right]. \tag{7}$$

Here $\psi(\mathbf{N})$ is a function of the vector $\mathbf{N}$. A zeroth-order solution $f_0 = C \exp \left[ \frac{1}{T} (\mathbf{N} \cdot \mathbf{H}_{eff}) \right]$ corresponds to the solution in the absence of the spin torque. In the stationary case, inserting Eq.\textsuperscript{7} in Eq.\textsuperscript{6} and after straightforward calculations we obtain

$$f = C \exp \left[ \frac{1}{T} (\mathbf{N} \cdot \mathbf{H}_{eff}) + \frac{\varepsilon b}{2\lambda T^2} \mathbf{N} \cdot (\mathbf{s} \times \mathbf{H}_{eff}) + \cdots \right] + \cdots. \tag{8}$$

Here in Eq.\textsuperscript{8} we assume a high temperature limit, $\beta = 1/T = k_B/\gamma \mathbf{H}_0 \ll 1$, and therefore we can neglect higher order terms in the inverse temperature. For convenience, in the intermediate equations in what follows we set $\omega_0 = \gamma H_0 = 1$, $k_B = 1$, $\hbar = 1$. As we show below, the values of the temperature defines the limits of the application of the perturbation theory. We also neglect the higher order terms that are proportional to the small parameter $\varepsilon$.

### III. OBSERVABLE QUANTITIES

Using the distribution function\textsuperscript{8} we can evaluate the mean values of the components of the magnetization vector using the following parametrization: $M_x = \sin \theta \cos \varphi$, $M_y = \sin \theta \sin \varphi$, $M_z = \cos \theta$, $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$. In the absence of the spin torque, the distribution function takes on the following form

$$dw(\theta, \varphi) = f(\theta) d\Omega = Z^{-1}(\beta H_0) \exp(\beta H_0 \cos \varphi) d\Omega. \tag{9}$$

Here

$$Z(\beta H_0) = \int \exp(\beta H_0 \cos \varphi) d\Omega = \frac{4\pi}{\beta H_0} \sinh(\beta H_0), \tag{10}$$

is the partition function and $dw(\theta, \varphi)$ defines the probability that the magnetization vector $\mathbf{M}$ is oriented within a solid angle of the width $d\Omega = \sin \theta d\theta d\varphi$. Taking into account Eq.\textsuperscript{9}, we find $M_x = M_y = 0$ and

$$M_z = L(H_0/T), \tag{11}$$
where \( L(x) = \coth(x) - \frac{1}{x} \) is the Langevin function.

In the case of the high temperature limit \( H_0/T < 1 \), that means for \( T > \gamma_c H_0 h/k_B \), we have \( M_z = M \approx \gamma_c H_0 h/3k_B T \). In the case of low temperatures, \( T < \gamma_c H_0 h/k_B \), we have \( M_z = M = 1 \). For the square components of the magnetization we have

\[
\overline{M_2} = M_y^2 = \frac{L(\beta H_0)}{\beta H_0}, \quad M_z^2 = 1 - 2L(\beta H_0)/\beta H_0. \tag{12}
\]

We see that Eq. (12) conserves the magnetization vector \( M_x^2 + M_y^2 + M_z^2 = 1 \). For the dispersion we have

\[
(\Delta M_i)^2 = (M_i - M_0)^2, \quad i = x, y, z,
\]

\[
(\Delta M_z)^2 = (\Delta M_y)^2 = \frac{2}{\beta H_0} L(H_0/T), \tag{13}
\]

\[
(\Delta M_x)^2 = 1 - \frac{2L(H_0)}{\beta H_0} L(H_0/T) - L^2(H_0/T).
\]

By using the explicit form of solutions \( \overline{U} \) and partition function \( \overline{V} \) we can evaluate the mean energy of the system:

\[
\overline{U} = -\frac{\partial}{\partial \beta} \ln Z(\beta H) = -\gamma_c H_0 h L \left( \frac{\gamma_c H_0 h}{k_B T} \right), \tag{14}
\]

and the heat capacity

\[
C_V = \left( \frac{\partial \overline{U}}{\partial T} \right)_V = k_B \left[ 1 - \frac{(4\pi)^2}{Z^2} \left( \frac{\gamma_c H_0 h}{k_B T} \right) \right]. \tag{15}
\]

If the spin distance terms are taken into account the results are changed. The distribution function takes the form

\[
f = C \exp \left[ \alpha \cos \theta + \varepsilon \delta \sin \theta \cos \phi - \varepsilon \eta \sin \theta \sin \phi \right],
\]

\[
\alpha = \beta H_0, \quad \delta = \frac{as}{\lambda T}, \quad \eta = -\frac{bs}{2\lambda T^2}. \tag{16}
\]

The expressions for magnetization components in this case are quite involved and are presented in the appendix. For the particular case of \( \varepsilon^2(\delta^2 + \eta^2) < 1 \), i.e. for

\[
\left( \frac{1}{T} \right)^2 < 2 \left( \frac{a}{b} \right)^4 + \frac{2a^2 \lambda^2}{b^2 s^2} - 2 \left( \frac{a}{b} \right)^2 \tag{17},
\]

the expressions for the mean components of the magnetization vector simplifies to

\[
\overline{M_x}(H_0, T) \approx -\frac{as}{\lambda H_0 \omega_0} L(H_0/T),
\]

\[
\overline{M_y}(H_0, T) \approx \frac{bs}{2\lambda H_0 \omega_0} L(H_0/T), \tag{18}
\]

\[
\overline{M_z}(H_0, T) \approx L(H_0/T) - \frac{1}{2\lambda H_0 \omega_0} \left( a^2 s^2 \frac{H_0^2}{\lambda^2 H_0^2} + b^2 s^2 \frac{1}{2H_0^2 T^2} \right)
\]

\[
\times \left( 3L(H_0/T) + \frac{H_0}{T} L^2(H_0/T) - \frac{H_0}{T} \right),
\]

and

\[
\overline{M_z}^2(\omega_0, H_0, T) \approx \frac{T}{H_0} L(H_0/T). \tag{19}
\]

From Eq. (19) it is easy to see that the normalization condition holds, \( M^2 = 1 \). Equation (17) defines the minimum values of the temperature, for which the solutions (18), (19) are still valid. In particular, taking into account that \( \omega_0/bs \gg 1 \), from (17) we obtain

\[
T > T_{cr}, \quad T_{cr} \approx \frac{h}{k_B} \sqrt{\frac{\omega_0}{bs \gamma_c H_0}}. \tag{20}
\]

Equation (20) shows that the temperature, above which our approach is valid, increases with the amplitudes of external field \( \omega_0 = \gamma_c H \) or of the torque \( bs \). The meaning of (18) is straightforward. The torque leads to a formation of transversal components \( M_{xp}(\omega_0, H_0, T) \) while the external field tries to align the magnetization along the \( z \) axis.

Taking typical values of the parameters for the thin film Fe_{25}-Co_{25}-Ni_{50} such as: \( \omega_0 = \gamma_c H_0 \approx 17 \times 10^4 \text{MHz} \), \( |H_0| = 10^4 \text{Oe} \), \( \lambda = 10^{-4} \) for the maximal value of the critical threshold temperature \( T_{cr} \) we have \( T_{cr} < 70K \).

In the limit of a strong field and low temperatures, \( T > \gamma_c H_0 h/k_B \), we obtain \( M_z(\omega_0, T) \approx 1 \). From (18) we also see that

\[
\overline{M_z}(\omega_0, T) \approx \frac{2a}{b} k_B T \gamma_c H_0 h. \tag{21}
\]

The meaning of Eq. (21) is that we can rotate the magnetization’s transversal component in the plane via (cf. Fig.1).

Using (A1) we find

\[
\overline{U} = -\frac{\partial}{\partial \beta} \ln \left( \frac{\sin(\beta H_0)}{\beta H_0} \right) \tag{22}
\]

\[
-\frac{\partial}{\partial \beta} \ln \left( 1 + \frac{\varepsilon^2}{2} (\delta^2 + \eta^2) \frac{L(\beta H_0)}{\beta H_0} \right).
\]

and for the mean energy we obtain:

\[
\overline{U} = -\gamma_c H_0 h L \left( \frac{\gamma_c H_0 h}{k_B T} \right) - \overline{U}. \tag{23}
\]
We can evaluate now the heat capacity

$$C_V = \frac{\partial U}{\partial T} = k_B \left\{ \left( 1 - \frac{\left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2}{\sinh^2 \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)} \right) + \frac{3 b^2 s^2}{4 \lambda^2} \omega_0 \frac{\hbar}{k_B T} L \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right) - \frac{1}{2 \omega_0^2} \frac{b^2 s^2}{\lambda^2} \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2 \left( 1 - \frac{\left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2}{\sinh^2 \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)} \right) + \frac{1}{\omega_0^2} \left( \frac{a^2 s^2}{\lambda^2} + \frac{b^2 s^2}{\lambda^2} \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2 \right) \times \frac{\left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2}{\sinh^2 \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)} \right\}.$$  

and the change of the heat capacity due to the spin torque

$$\delta C_V = k_B \left\{ \frac{3 b^2 s^2}{4 \lambda^2} \omega_0 \frac{\hbar}{k_B T} L \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right) - \frac{1}{2 \omega_0^2} \frac{b^2 s^2}{\lambda^2} \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2 \left( 1 - \frac{\left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2}{\sinh^2 \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)} \right) + \frac{1}{\omega_0^2} \left( \frac{a^2 s^2}{\lambda^2} + \frac{b^2 s^2}{\lambda^2} \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2 \right) \times \frac{\left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)^2}{\sinh^2 \left( \frac{\gamma_c H_0 \hbar}{k_B T} \right)} \right\}.$$  

IV. NUMERICAL RESULTS

Let us inspect the temperature dependence of the mean values of the magnetization components using the analytical results derived in the previous section. We note again that the analytical solutions Eq. (18), Eq. (19) contain first and second order terms. First order terms correspond to the solution in the absence of spin torque and are valid for arbitrary values of temperature while second order terms are defined for temperatures $T > T_{cr}$ see Eq. (20). Since we measure temperature in units of $T_{cr}$, the solution obtained using perturbation theory is not well-defined in the vicinity of $T \approx 1$. Therefore, we expect to see a slight loss of smoothness of the magnetization curves in the vicinity around this area. However, our main finding of thermally activated in-plane magnetization rotation (see Eq. (20)) is well defined for arbitrary values of the temperature. Fig.1 shows the rotation of the in-plane component of the magnetization induced by the change of the temperature and is plotted using Eq.(18).

Note, that the expression for the $M_z(H_0, T)$ in Eq. (18) contains two terms. The first term recovers the result obtained without the spin torque Eq (11) and is defined for arbitrary temperatures. While the second term in Eq. (18) is the contribution of the perturbation theory and therefore according to Eq. (20) is defined only for temperatures above $T_{cr}$. We should take this into account when plotting $M_z(H_0, T)$ using.
FIG. 4. Same as Fig.3 for the magnetization component $M_y(T)$. Slight loss of the smoothness of the magnetization curve in the vicinity $T \sim 1$ is connected to the fact that perturbation solution is not well-defined in the area which is marked out by dashed lines.

FIG. 5. Dependence of the magnetization component $M_z^2(T)$ on the temperature for different values of the spin torque constant $a$. Area in which perturbation theory is not well defined is marked out by two dashed lines. Temperature unit is given by $\frac{\hbar}{2 \gamma H_0}$, with $\omega_0 = \gamma H_0$, $\omega_0 = 1$, and $\lambda = 1$.

FIG. 6. The same as Fig.5 but for $M_y^2(T)$. Area in which perturbation theory is not well defined is marked out by two dashed lines.

We see that the rotation amplitude depends on the ratio between the spin torque constants $a/b$ and for $a/b > 1$ has a maximum. The temperature dependence of the mean values of the in-plane magnetization components $M_x$, $M_y$ is shown in Fig.2, Fig.3, and Fig.4. We see that the maximal values of $M_z$ decreases with the increase of the spin torque component $a$ (see Fig.4).

Now we present square components of the magnetization plotted using Eq. (19). See Fig.5, Fig.6 and Fig.7.

The general conclusion is that the asymmetry between the spin torque coefficients $a$, $b$ has more important consequences for the thermal rotation of the magnetization in the $xy$ plane and the mean values of the magnetization components $M_{x,y,z}$ however, it is less evident for the mean values of the square of the components $M_{x,y,z}^2$. Finally, we show the temperature dependence of the dispersion for the $M_z$ component of the magnetization. We see that for different ratios between the spin torque constants $a/b$, the values of the dispersion are different. At higher temperatures all these different values merge together.

Additionally, we perform full numerical finite-temperature calculations based on the solution of the stochastic LLG equation by means of the Heun method which converges in quadratic mean to the solution interpreted in the sense of Stratonovich. Exact numerical solution of the stochastic LLG equation is important since analytical results are obtained in the framework of perturbation theory and therefore are valid for the temperatures above critical temperature $T > T_{cr}$ only. In order to observe dependence of the magnetization components on the temperature and spin torque constants, we numerically solve stochastic LLG equation Eq. (1) and generate random trajectories on the sufficiently large time interval until magnetization components after relaxation process reaches stationary regime. In the stationary regime, values of the magnetization components are time independent and depend on the temperature and spin torque parameters.

FIG. 7. The same as Fig.5 but for $M_z^2(T)$.
only. Therefore after averaging results over the ensemble of random trajectories for the magnetization components we obtain mean values which we can compare to the mean values of the magnetization components obtained via the solution of stationary Fokker-Plank equation Eq. (8). In Fig. 9 the rotation of the magnetization denoted by the angle $\theta$ reproduces the analytical results of Eq. (21). As we see, depending on the ratio between the spin torque constants $a/b$ maximal values of the observed rotational angle $(\Delta \theta)_{\text{max}} \approx \pi/2$ is in a good quantitative and qualitative agreement with the analytical results presented in the Fig. 1. Fig. 10 shows all three magnetization components for the chosen values of the spin current. These results are in full agreement with the analytical results depicted in Figs. 2-4, which predict a decay of the magnetization with increasing temperatures. We have deviation between analytical and numerical results only in the area below critical temperature $T < T_{cx} \approx 1$ where perturbation theory used in analytical calculations is not defined. Our full numerical results supplement for the low temperature case, i.e. for $T < 1$ in dimensionless units, or in real (non-scaled) units $T < 70$ [K]. The numerics can go beyond the range of validity of the perturbation theory. The numerically accurate results for the magnetization are smooth. Fig. 11 additionally presents the zero temperature equilibrium from which for certain value of the ratio $a/b$ the non-zero temperature calculations start. In particular Fig. 11 defines equilibrium ground state of the system for the zero temperature. This zero temperature ground state depends on the torque parameters. Finally, in Fig. 12 we show the effect of the magnetization rotation calculated for each time step for the averaged values of the squared projections of the magnetization, i.e. $\mathcal{M}_x^2$.

V. CONCLUSION

In this work we studied the thermally assisted spin-torque and its influence on the magnetization dynamics in a thin permalloy film and presented results for Fe$_{25}$Co$_{25}$Ni$_{50}$. We found that the spin torque term leads to nontrivial dynamical effects in the finite temperature magnetization dynamics. Assuming the spin torque terms to be small compared to the Larmor precessional term we developed a perturbational approach to the Fokker-Planck equation and obtained analytical expression for the distribution function including the spin torque terms. In particular, we proved that the spin torque term leads to the formation of a non-vanishing in-plane magnetization component. The ratio between the mean values of the components $M_x$ and $M_y$ defines the orientation of the in-plane magnetization vector $\hat{M}_x = \frac{M_x}{\sqrt{M_x^2 + M_y^2}}$. We find that the orientation of the in-plane magnetization depends on the ratio between the spin torque constants $a/b$ and between the temperature and the amplitude of the external magnetic field $T/H_0$. Therefore, changing the temperature leads to a thermally induced rotation of the in-plane magnetization vector. We name this as “thermally activated in-plane magnetization rotation”. We found that if from the two spin torque terms $\varepsilon_b[\hat{M}, \vec{s}^\perp]$, $\varepsilon_a[\hat{M}, \vec{s}^\parallel]$ the last term is the dominant one $a > b$ the effect of the thermally activated in-plane magnetization rotation is enhanced (cf. Fig.1).
Fit of $M_x$ using eq. (18)
Fit of $M_y$ using eq. (18)

FIG. 10. (Color online) Ensemble-averaged (over 100 realizations) magnetization components calculated for the parameters listed in the caption of Fig. 9. Good agreement between numerical and analytical results is evident. We have deviation between analytical and numerical results for the component $M_y$ only in the area below critical temperature $T < T_{cr} \approx 1$ where perturbation theory used in analytical calculations is not defined. Our full numerical results supplement for the low temperature case, i.e. for $T < 1$ in dimensionless units, or in real (non-scaled) units $T < 70$ [K].

FIG. 11. (Color online) Illustration of the relaxation of the magnetization from the initially chosen arbitrary states to the zero temperature equilibrium state. In case of zero temperature relaxation of the magnetization vector is connected to the phenomenological damping constant $\lambda$. As we see due to the spin torque terms transversal components of the magnetization vector are different from the zero in the equilibrium. Initial state is chosen as $M_z(t=0) = -1$. a) $a/b = 0.1$, b) $a/b = 0.2$, c) $a/b = 1$, d) $a/b = 10$; other parameters are as those listed in the caption of Fig. 9.

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Here we give some expressions for the components of the magnetization state is

\[ \text{rotation of temperature (} T \in [0 : 7]) \text{ calculated from trajectories averaged for each temperature at quasi-equilibrium after relaxation time } \tau \text{. Number of averaging is 100, } a/b = 1. \text{ Initial magnetization state is } \{ M_x, M_y, M_z \} (t = 0) = \{ 0, 1, 0 \}.

VI. APPENDIX

We use the partition function

\[ Z(\alpha) = 4\pi \frac{\sinh \alpha}{\alpha} \left( 1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\alpha) \right), \quad (A1) \]

Here we give some expressions for the components of the magnetization

\[
\begin{align*}
M_x &= \varepsilon_\delta \frac{L(\beta H_0)}{\beta H_0} \frac{1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)} \\
&\approx \varepsilon_\delta \frac{L(\beta H_0)}{\beta H_0} \left( 1 - \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} \frac{L(\beta H_0)}{\beta H_0} \right), \\
M_y &= -\varepsilon_\eta \frac{L(\beta H_0)}{\beta H_0} \frac{1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)} \\
&\approx -\varepsilon_\eta \frac{L(\beta H_0)}{\beta H_0} \left( 1 - \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} \frac{L(\beta H_0)}{\beta H_0} \right), \\
M_z &= \frac{L(\beta H_0)}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)}
\end{align*}
\]

\[
\begin{align*}
M^2_x &= \frac{L(\beta H_0)}{\beta H_0} \frac{1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)} \\
&\approx L(\beta H_0) \left( 1 - \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} \frac{L(\beta H_0)}{\beta H_0} \right) \\
&+ \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2 L(\beta H_0)^{-1}} \frac{L(\beta H_0) - 1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)}
\end{align*}
\]

For the modulus squares of the magnetization components we find

\[
\begin{align*}
M^2_y &= \frac{L(\beta H_0)}{\beta H_0} \frac{1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)} \\
&\approx \frac{L(\beta H_0)}{\beta H_0} \left( 1 - \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} \frac{L(\beta H_0)}{\beta H_0} \right) \\
&+ \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2 L(\beta H_0)^{-1}} \frac{L(\beta H_0) - 1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)}
\end{align*}
\]

\[
\begin{align*}
M^2_z &= \frac{L(\beta H_0)}{\beta H_0} \frac{1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)} \\
&\approx \frac{L(\beta H_0)}{\beta H_0} \left( 1 - \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} \frac{L(\beta H_0)}{\beta H_0} \right) \\
&+ \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2 L(\beta H_0)^{-1}} \frac{L(\beta H_0) - 1}{1 + \frac{\varepsilon^2 (\delta^2 + \eta^2)}{2} L(\beta H_0)}
\end{align*}
\]

where \( \alpha = \beta H_0 = \gamma_e H_0 \hbar / k_B T \).

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