Three-dimension reconstruction based on spatial light modulator

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Abstract. Three-dimension reconstruction, known as an important research direction of computer graphics, is widely used in the related field such as industrial design and manufacture, construction, aerospace, biology and so on. Via such technology we can obtain three-dimension digital point cloud from a two-dimension image, and then simulate the three-dimensional structure of the physical object for further study.

At present, the obtaining of three-dimension digital point cloud data is mainly based on the adaptive optics system with Shack-Hartmann sensor and phase-shifting digital holography. Referring to surface fitting, there are also many available methods such as iterated discrete fourier transform, convolution and image interpolation, linear phase retrieval.

The main problems we came across in three-dimension reconstruction are the extraction of feature points and arithmetic of curve fitting. To solve such problems, we can, first of all, calculate the relevant surface normal vector information of each pixel in the light source coordinate system, then these vectors are to be converted to the coordinates of image through the coordinate conversion, so the expectant 3D point cloud get arise. Secondly, after the following procedures of de-noising, repairing, the feature points can later be selected and fitted to get the fitting function of the surface topography by means of Zernike polynomial, so as to reconstruct the determinand’s three-dimensional topography.

In this paper, a new kind of three-dimension reconstruction algorithm is proposed, with the assistance of which, the topography can be estimated from its grayscale at different sample points. Moreover, the previous stimulation and the experimental results prove that the new algorithm has a strong capability to fit, especially for large-scale objects.
1. Introduction
With the development of industry, the needs for improving quality and precision become increasingly urgent. Therefore, people work out some related technologies to solve this problem. Three-dimensional reconstruction, well known as an effectively solution and based on the measurement of analytes, can be divided into Contact and Non-contact. The contact measurement, which requires point-by-point scanning, has been gradually phased out considering its low efficiency. While, compared with the former, the non-contact measurement is now a hot topic in research field since it is simple and fast processing. Its theory can also simply described as following: via introducing various detectors, the system can obtain the information such as analytes’s images so as to obtain digital point cloud in further processing. By now, there are mainly three non-contact measurement methods: one is based on the spatial light modulator, one wavefront instrument and the last polarization imaging technology. Among them, the system based on spatial light modulator is the most common way to obtain point cloud, so this method is chosen for further study in reconstruction algorithm.

2. Principle of reconstruction
Three-dimension reconstruction of objects is composed of two parts. One of them is how to obtain the point cloud in the measurement system while the other is surface fitting. Surface fitting, always with the disorderly scattered points, has four normal fitting ways to establish model: the least-squares fitting method, the model fitting method, the backbone method and the separate algorithm. And the point cloud which is used to reconstruct surface feature need to be the height or the surface vectors of object. In this case, we choose phase which is linear with height as the reconstruction data. Then, with the data, an arithmetic is presented and proved to be feasible with experiment demonstration.

2.1. The acquisition of point cloud
Digital image reflecting the light intensity is described in the form of gray levels, so 8-bit digital figure correspond to the 256 grayscale pixel matrix. To make the gray levels change according to the cosine rule, the value in i row j column is expressed as follows:

\[ G(i, j) = 255 \times \left[ 1 + \cos\left(\frac{2\pi i}{T} + \phi_0\right) \right] / 2 \]  

where \( T \) is the change circle of pixel, and \( \phi_0 \) is the initial phase.

When applying cosine modulation virtual grating in phase-shifting phase measurement, the intensity distribution of deformable virtual raster image recorded in the CCD camera is as the formula:

\[ I(x, y) = A(x, y) + B(x, y) \cos\left[ \phi(x, y) + \theta(n) \right] \]  

where \( A(x, y) \) represents background intensity, \( B(x, y) \) on behalf of the cosine field stripes contrast and \( \phi(x, y) \) the phase function. Here \( (x, y) \) reflects the deformation of the stripes and the measured object height distribution. \( \theta(n) \) is a controlled amount of phase modulation with the parameter \( n \) the number of changes in the amount of phase modulation, in other words, number of phase shift. When \( n \) changes, the intensity distribution of the deformable virtual raster images changes while \( (x, y) \) is equal to the initial phase of the cosine function. By means of changing \( n \) to obtain pieces of the deformable virtual raster images, next get its initial phase according to the characteristics of the cosine trigonometric function, this is quite the essence of shift phase demodulation.

Expand the intensity distribution showed above:

\[ \phi(x, y) = -\arctan[t(x, y)/s(x, y)] = -\arctan \frac{\sum_{i=1}^{N_q} I_i(x, y) \sin \theta(i)}{\sum_{i=1}^{N_q} I_i(x, y) \cos \theta(i)} \]  

Through solving this equation, the phase map is gotten.

2.2. The arithmetic of reconstruction
As to the essence of three-dimensional reconstruction, we mainly take grid-class method, parameter-class of surface reconstruction and implicit surface fitting method to get the data point cloud after the...
surface fitting. Generally the way of forming parameters of polynomial fitting on the surface is most adopted for basis-function’s simplicity. By changing the number of polynomial basis-function and the coefficient vector, rich and expressive shapes can be got. More than that, since basis-function is infinitely differentiable, the surface can be smooth sufficiently and it will be much easy to calculate the function value and the derivative values. For the most part, polynomial function are used as the basis-function, as well as polynomial’s base. In addition, calculating the coefficients of basis-functions is very easy when carrying out surface fitting.

2.2.1. Bicubic spline interpolation arithmetic

The spatial light modulator process images by the method of phase shift, obtaining the phase distribution of continuous coordinates via unwrapping envelope. Phase value at each point \((x, y)\) can be set as:

\[
p(x, y) = \sum_{i=0}^{n} C_i f_i(x, y)
\]  

Bicubic interpolation is an extension of cubic interpolation for interpolating data points on a two-dimensional regular grid. The interpolated surface is smoother than corresponding surfaces obtained by bilinear interpolation or nearest-neighbor interpolation. If typical value \(\{r_{ij}\}\) is given in the coordinate \(\{(x_i, y_j)\}\), \((i=0,1,2,…; n; j=1,2,…,m)\), the spline function \(s (x, y)\) can be written in the form of tensor produce:

\[
s(x, y) = \sum_{i-k}^{n-1} \sum_{j-l}^{m-1} c_{ij} N_{i,k}(x)N_{j,l}(y)
\]  

when both \(k\) and \(l\) equal to 3, the equation is called bicubic spline interpolation arithmetic.

The continuation of data in spline surface is often required to be given at first with the boundary constraint condition. Bicubic spline surface usually refers to the subdivision cubic polynomial which belongs to \(C^2(\Omega)\) in rectangular grid. In order to create a smooth surface, different from the curve fitting, bicubic interpolation requires not only the intensity value, but also its derivative in any direction along \(x, y,\) or \(xy\). So the interpolation condition is set as:

\[
\begin{align*}
s_i(x_j, y_j) &= p(x_j, y_j), 0 \leq i, j \leq n \\
\frac{\partial s_i}{\partial x}(x_j, y_j) &= \frac{\partial p}{\partial x}(x_j, y_j), 0 \leq j \leq n, i = 0, n \\
\frac{\partial s_i}{\partial y}(x_j, y_j) &= \frac{\partial p}{\partial y}(x_j, y_j), 0 \leq j \leq n, i = 0, n \\
\frac{\partial^2 s_i}{\partial x \partial y}(x_j, y_j) &= \frac{\partial^2 p}{\partial x \partial y}(x_j, y_j), i, j = 0, n
\end{align*}
\]  

In general, The solution of bicubic spline function which meet the above conditions is unique, but when it comes to the condition that the number of equations great than unknown quantity, the solution of over-determined equation does not exist. So we can only seek the solution through approaching these points that is the closest to solution in some sense. Here we adopt the method called minimum 2 multiplication. It requires the square sum of deviation between the real value and the measurements minimum, that is:

\[
E = \sum_{i=1}^{N} (p(u_i) - p_i)^2
\]  

Where \(u_i\) represents the surface coordinates \((x_i, y_i)\).

To obtain the minimum value \(E\), it requires:
\[
\frac{\partial E}{\partial c_{ij}} = 0
\]  

(8)

According to the equation above, the value of \( c_{ij} \) (\( i=1,2,\ldots,n \), \( j=1,2,\ldots,n \)) can be obtained from selected feature points, so is the fitting function \( p(u) \), with which we can move forward to the phase of all the coordinates. Furthermore, because of the linear relationship between phase and height, the three-dimensional shape of analytes can be restored so as to achieve three-dimensional reconstruction.

3. Results of experiment

The five object phase shifting images modulated by spatial light modulator, after comparing with the conference images, are then demodulated to the phase map (Fig.1). Since the phase is discontinuous at the period of cosine function, the phase map is unwrapped with the threshold equivalent to 3. After that, the unwrapped phase map (Fig.2) which reflects the phase as gray scale is obtained.

As the unwrapped phase map has 1310720 pixels, to improve the computing speed and the accuracy of surface fitting, the pixels outside the target are cut out. After that, the data is used to compute a bicubic spline approximation.

In Fig.3 and Fig.4, 267000 point data is used to fit, while 99 point data is chosen in Fig.5 and Fig.6.
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