A note on dark energy induced by D-brane motion

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In this note we study the possibility of obtaining dark energy solution in a D-brane scenario in a warped background that includes brane-position dependent corrections for the non-perturbative superpotential. The volume modulus is stabilized at instantaneous minima of the potential. Though the model can account for the existence of dark energy within present observational bound – fine-tuning of the model parameters becomes unavoidable. Moreover, the model does not posses a tracker solution.

I. INTRODUCTION

Study of high red-shifted supernovae and other cosmological observations [1] clearly indicate that the expansion of the universe is accelerating rather than slowing down as an expected result of gravitational pull. Within the framework of the standard cosmological model, this implies that 70 percent of the universe is composed of a new, mysterious dark energy [2, 3, 4] which counters the attractive force of gravity unlike any known form of matter or energy.

Known to be very homogeneous, not very dense and non-interacting other than gravitationally, the exact nature of dark energy is yet a matter of speculation. Dark energy appears to be the dominant component of the physical Universe, but there is no persuasive theoretical explanation for its existence or magnitude as on date.

The simplest possible explanation for dark energy is that it’s the ‘cost of having space’. In other words, the Universe is permeated by an energy density, constant in time and uniform in space. General arguments from the scale of particle interactions, however, suggest that if \( \Lambda \) is not zero, it would be expected to be \( 10^{120} \) times larger than what is observed.

An important step towards a realistic cosmological model based on string theory came from the realization that background fluxes can stabilize most of the moduli of string theory. It was shown in [5] that fluxes in warped compactifications, using a Klebanov-Strassler (KS) throat [6] can stabilize the dilaton and complex structure moduli of type IIB string theory compactified on an orientifold of a Calabi-Yau threefold. Infact, it was shown in [7] that all the closed string moduli can be stabilized by a combination of fluxes and non-perturbative effects. The non-perturbative effects are mainly responsible for stabilizing the Kahler moduli and they arise either from an Euclidean D3-brane wrapping a four cycle or from gauge dynamics of a stack of \( n \) D7-branes wrapping supersymmetrically a four cycle in the warped throat.

The D-brane physics finds interesting applications to early universe and late time cosmology. The efforts of constructing inflationary models exploiting D-brane dynamics is still under active consideration. Nevertheless it is important to remember that a viable inflation apart from meeting observational constraints also should be followed by a successful reheating. This severely constrains building inflationary models. Based on this it is not easy to build such a model in the frame work of D-brane dynamics [10, 11, 12]. However, it is simpler to build a dark energy model which is free from such constraints.

In this letter, we address the possibility of having a viable dark energy model from the effective scalar field potential \( V(\phi) \) obtained in Ref. [10] after the volume modulus is fixed at the instantaneous minimum.

II. EFFECTIVE FIELD DESCRIPTION

The inflaton potential can be obtained by performing string theoretic computations involving the details of the compactification scheme. In this setup, the inflation is realized by the motion of a D3-brane towards a distant static anti-D3-brane, placed at the tip of the throat. The position of the moving brane in the compactification manifold is
can be reduced to an effective one field potential when the field \( \sigma \) the condition \( V^{\ast} \sigma \) of the modulus is much larger than Hubble rate, the field has been readdressed and it has been observed that delicate fine tunings of the parameters are necessary to chase the dual B-cycle at the tip of the throat. The issue of volume modulus stabilization, thus needed to be re-analysed and \( M \) constant and \( r \) coordinates and one radial coordinate, the D3-scale of the supersymmetry breaking is given by and the anti-D3 brane. Note that the presence of the anti-D3- brane breaks the supersymmetry of the system and the superpotential as mentioned above and the usual D-term potential coming from the interaction between the D3-brane factor. The total potential that the inflaton field experiences is the sum of the potential (F-term) coming from the D3-brane and thus the superpotential for the nonperturbative effect gets corrected by an overall position dependent distance into the warped throat, that the presence of a D3-brane gives rise to a perturbation to the warp factor as given in [9] and requiring at least one of the four-cycles carrying nonperturbative effects to descend down a finite volume of this radial coordinate. The stability analysis for the trajectories in the angular directions has to be performed

In what follows, we shall try to use the field \( \phi \) with the notations \( \phi = 0 \) which determines \( F(\phi) \) evolves along the instantaneous minima determined by the condition \( V, \phi = 0 \) which determines \( \sigma \) (equal to \( \sigma_\ast \)) in terms of the field \( \phi \). The two field potential \( V(\phi, \sigma) \), thus can be reduced to an effective one field potential when the field \( \sigma \) continues to remain in its instantaneous minimum \( \sigma_\ast(\phi) \) which evolves slowly. The effective single field potential then acquires the following form,

\[
V(\phi) = \frac{a}{3} \exp\left(-2w_0\sigma_\ast(\phi)\right) \frac{g(\phi)^{2/n}}{U(\phi, \sigma_\ast(\phi))^2} \left[ 2w_0\sigma_\ast(\phi) + 6 - 6\exp(w_0\sigma_\ast(\phi)) \right] |W_0| \frac{1}{g(\phi)^{1/n}} + \frac{3c}{n} \frac{\phi}{\phi_\mu} \left( \frac{\phi}{\phi_\mu} \right)^{3/2} \frac{1}{g(\phi)^2} - \frac{3}{n} \left( \frac{\phi}{\phi_\mu} \right)^3 \frac{1}{g(\phi)^2} + \frac{D(\phi)}{U(\phi, \sigma_\ast(\phi))^2}
\]

where the functions \( \sigma_\ast(\phi), g(\phi), U(\phi) \) and \( D(\phi) \) are given by,

\[
\sigma_\ast(\phi) \approx \left[ 1 + \frac{1}{n} \frac{1}{w_F} \left( 1 - \frac{1}{2w_F} \right) \left( \frac{\phi}{\phi_\mu} \right)^{3/2} \right],
\]

\[
g(\phi) = 1 + (\phi/\phi_\mu)^{3/2},
\]

\[
U(\phi) = \frac{2\sigma_\ast w_0}{a} - \frac{w_0}{3a} (\phi/\phi_\mu)^2 \phi_\mu^2,
\]

\[
D(\phi) = D_0 \left( 1 - \frac{27D_0}{64\pi^2(\phi/\phi_\mu)^4\phi_\mu^4} \right)
\]

with the notations

\[
c = \frac{9}{4nw_0\phi_\mu^2}, \quad w_F \equiv a\sigma_F, \quad w_0 \equiv a\sigma_0, \quad a = \frac{2\pi}{n}
\]

Here \( n \) designates the number of \( DT7 \) branes. The constants \( W_0, w_F \) and \( w_0 \) are constrained by the following relations

\[
3 \frac{|W_0|}{|A_0|} e^{\sigma_F} = 2a\sigma_F + 3,
\]

\[
3 \frac{|W_0|}{|A_0|} e^{w_0} = 2w_0 + 3 + s, \quad 1 < s < 3
\]

In what follows, we shall try to use the field \( \phi \) with effective potential [10] as a quintessence field.
III. DARK ENERGY

Before we get to numerics, we would like to emphasize the interesting features of the potential (Fig:1). As pointed out in Ref.[11], one can choose the parameters in the potential namely $\sigma_F$, $n$, $c$ and $D_0$ such that the potential looks flat for $\phi$ smaller than the point of inflection as $\phi$ moves toward zero ($A_0$ gives the over all scale in the potential and can be chosen as equal one). For $\phi$ approaching zero, the $D(\phi)$ term in the potential plays the deciding role as $V(\phi) \sim D(\phi) \sim -\left(\phi/\phi_\mu\right)^{-4}$ making $V(\phi)$ steep and large negative near the origin. $V(\phi)$ is positive and steep for $\phi \rightarrow \phi_\mu$. It further important to note that $D_0$ controls the height of the flat part of the potential which should mimic the cosmological constant like behavior; tuning $D_0$ can lead to the present day scale of dark energy. These are essential features that a model should exhibit in order to account for the late time acceleration.

By fixing the constants and parameters as,

$$w_F = 9.9956, \quad w_0 = 10.1, \quad n = 8$$

$$a = \frac{2\pi}{n}, \quad \phi_\mu = 0.25, \quad W_0 = 3.496 \times 10^{-4}$$

the effective single field potential as a function of $\phi/\phi_\mu$ and $D_0$ can be expressed as a Taylor series expansion around any particular value of $\phi/\phi_\mu$ in the flat region of the potential. The potential is flat around $\phi/\phi_\mu = 0.4$. The flat part of potential can be expressed as,

$$V\left(\frac{\phi}{\phi_\mu}, D_0\right) \approx -1.6544 \times 10^{-11} + 0.0015 D_0 - 0.6444 D_0^2$$

(9)

Since the value of $D_0$ is expected to be of the order of $10^{-8}$ from string theory point of view, one can observe that by fine tuning of $D_0$, it is possible to realize a cosmological constant within present observational bound. It corresponds to the case when $D_0 \approx (1.6544 \times 10^{-11})/0.0015$. It is further desirable that the field energy density should mimic the background being subdominant soon after the commencement of the radiative regime and it should take over the background energy density at late times thereby alleviating the fine tuning and coincidence problems. In what follows we shall investigate the cosmological viability of D-brane scenario in context with dark energy.

The Friedmann equation and field equation

$$H^2 = \frac{1}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi) + \rho_m\right),$$

(10)

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

(11)
can be cast as first order equations

\[ \frac{dx}{dN} = \frac{y}{H(x)}, \]  

(12)

\[ \frac{dy}{dN} = -3y - \frac{U(x)}{H(x)} \]  

(13)

\[ H(x) = \phi_\mu \sqrt{\frac{1}{3} \left( \frac{y^2}{2} + U(x) + \phi_\mu^2 \rho_m \right)} \]  

(14)

where \( x = \phi/\phi_\mu, \ y = \dot{\phi}/\phi_\mu, \ U = V/\phi_\mu^2, \ N = \ln a \) and \( \rho_m = \rho_m^0 e^{-3N} \) is the matter energy density.

We numerically investigated the dynamics described by the single field potential (14); our results are depicted in Figures 2 and 3. We show the evolution of field energy and background (matter) energy densities versus \( N \) along with the dimensionless density parameter.

![Figure 2: Plot of field energy and matter densities versus N in case of overshoot(left) and undershoot(right).](image)

The right and left figures in the Fig. 2 display the cases of undershoot and overshoot respectively. In both the cases the field energy density continues scaling faster than the background energy density till the field rolls along the steep part of the potential, it then freezes mimicking the cosmological constant like behavior. When the matter density becomes comparable to field energy density, it begins evolving slowly and takes over the background to account for the dark energy. It is clear that the model under consideration does not possess tracker solution. For the tracker to exist, it is necessary that the field potential remains steep close to the exponential potential for most of the history of universe and becomes shallow only at late times. Unlike the tracker solution, the present scenario exhibits dependence on the initial conditions. Thus apart from the tuning of the model parameters, the adjustment of the initial conditions is necessary to obtain the observed accelerated expansion.

To summarize, we have examined the possibility of late acceleration using single field D-brane potential. This scenario was applied to inflation earlier; it is perfectly legitimate to apply the same to late time acceleration with little change of parameters. We have shown that de-Sitter is a late time attractor of the model. The present value of the cosmological constant can be obtained by fine tuning the value of the constant \( D_0 \). The absence of a tracker solution gives rise to additional dependency on the initial conditions of the field \( \phi \).

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FIG. 3: The evolution of dimensionless density parameter $\Omega$ with $N$.

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