Abstract. We have studied inflationary paradigm through an inflationary equation of state. With a single parameter equation of state as a function of the scalar field responsible for accelerated expansion, we find an observationally viable model satisfying all the constraints as laid down by the recent observations. The resulting model can efficiently cover a wide range of tensor-to-scalar ratio ranging from $r \sim \mathcal{O}(10^{-1})$ to $\mathcal{O}(10^{-6})$, other inflationary observables being consistent with the latest data. Nowadays ultimate eliminator between inflationary models is the tensor-to-scalar ratio, the model presented here is capable of keeping up with the future probes of tensor-to-scalar ratio at the same time having good agreement with other inflationary observables.
1 Introduction

More than four decades has elapsed since inflationary paradigm \([1–11]\) started its journey to resolve the puzzles of standard Big-Bang cosmology. Though Big-Bang theory has been complemented successfully, inflation has stayed as a paradigm due the non-existence of a unique compelling model. Recent observations \([12–15]\) have ruled out many inflationary models but still allowing plenty. As a consequence inflationary model building is still alive with added charm coming from availability of late time finespun data. The present and futuristic Cosmic Microwave Background Radiation (CMB) missions in the likes of BICEP2/Keck \([16]\), CMB-S4 \([17]\), LiteBird \([18]\) are promising to detect tensor-to-scalar ratio \(\mathcal{O}(10^{-4})\) which will further reduce the number of viable inflationary models.

In general inflationary paradigm is investigated by specifying inflaton potential which may have field theoretic motivation or may be of purely phenomenological origin. Recently inflationary scenario is examined from hydro-dynamical point of view \([19]\) by suitably selecting the equation of state parameter (EoS from now on) during observable inflation as a function of e-foldings. This EoS formalism is capable of explaining the whole inflationary scenario independent of any model and the observable outcomes can be confronted with the recent data at ease.

In this article we have studied single field inflationary paradigm with a phenomenological EoS which is a function of scalar field responsible for accelerated expansion employing Hamilton-Jacobi formulation. The EoS we are interested herein is given by

\[
1 + \omega = \frac{2}{3} \operatorname{sech}^p \left( \phi \mathcal{M}_p^{-1} \right)
\]  

where \(p\) is any dimensionless positive number. The Hamilton-Jacobi framework provides a simple one-to-one correspondence between inflationary EoS and Hubble parameter and the associated potential. We shall see that for any positive value of \(p\), above EoS has inflationary solution consistent with recent observations. We shall also see
that \( p \) is inversely proportional to logarithm of tensor-to-scalar ratio, which helps to determine a lower bound of the model parameter depending on present upper bound \( r < 0.064 \) [12]. We have found that the particular solution for \( p = 2 \) mimics \( \alpha \)-attractor class of inflationary models [20]. We also find that a single parameter inflationary EoS is consistent of latest data coming from various probes.

This article is organized as follows, in Sections 2 and 3 we have briefly reviewed the Hamilton Jacobi formalism and Mukhanov Parametrization of single field inflation respectively. In Section 4 we have discussed outcomes of the model under consideration. In Section 5 we have shown that for \( p = 2 \) our model represents a particular case of \( \alpha \)-attractor inflationary models. Finally we conclude in Section 6.

2 Hamilton Jacobi Formalism

Within the framework of Hamilton-Jacobi formulation of inflation, Friedmann equations can be rewritten as [21–29]

\[
\begin{align*}
[H'(\phi)]^2 - \frac{3}{2M_P^2}H(\phi)^2 &= -\frac{1}{2M_P^4}V(\phi) \\
\dot{\phi} &= -2M_P^2H'(\phi)
\end{align*}
\]

(2.1)

(2.2)

where prime and dot denote derivatives with respect to the scalar field \( \phi \) and time respectively, and \( M_P \equiv \frac{1}{\sqrt{8\pi G}} \) is the reduced Planck mass. We also have

\[
\frac{\ddot{a}}{a} = H^2(\phi)\left[1 - \epsilon_H\right]
\]

(2.3)

where \( \epsilon_H \) has been defined as

\[
\epsilon_H = 2M_P^2 \left(\frac{H'(\phi)}{H(\phi)}\right)^2.
\]

(2.4)

So inflation occurs when \( \epsilon_H < 1 \) and ends exactly at \( \epsilon_H = 1 \). So requirement for the violation of strong energy condition is uniquely determined only by \( \epsilon_H < 1 \). In general these equations are very complicated to solve without invoking to the particular form of the Hubble parameter.

The amount of inflation is represented by e-foldings which is defined as

\[
N \equiv \ln \left(\frac{a_{\text{end}}}{a}\right) = \frac{1}{M_P} \int_{\phi_{\text{end}}}^{\phi} \frac{1}{\sqrt{2\epsilon_H}} \, d\phi.
\]

(2.5)

The e-folding has been defined in such a way that at the end of inflation \( N = 0 \) and \( N \) increases as we go back in time. In order to solve the big-bang puzzles and to comply with recent observations we need 50–70 e-foldings before the end of inflation. Another conventional parameter is

\[
\eta_H = 2M_P^2 \frac{H''(\phi)}{H(\phi)}.
\]

(2.6)
It’s worthwhile to mention here that $\epsilon_H$ and $\eta_H$ are not the usual slow-roll parameters, $\epsilon_H$ measures the relative contribution of the inflaton’s kinetic energy to its total energy, whereas $\eta_H$ determines the ratio of field’s acceleration relative to the friction acting on it due to the expansion of the universe [25]. Slow-Roll approximation applies when $\{\epsilon_H, |\eta_H| \ll 1\}$. Inflation goes on as long as $\epsilon_H < 1$, even if slow-roll is broken. The breakdown of slow-roll approximation drag the inflaton towards its potential minima and end of inflation happens quickly.

3 Mukhanov Parametrization

It is quite remarkable that the whole inflationary evolution can be represented efficiently by suitably picking the EoS as described in Ref.[19]. In this parametrization single field inflation can be studied model independently. Within the framework of Hamilton-Jacobi formulation, inflationary EoS is exactly given by

$$1 + \omega = \frac{2}{3} \epsilon_H. \quad (3.1)$$

For accelerated expansion we need $\omega < -\frac{1}{3}$. A cosmological constant would give us $\omega = -1$, but then we will have eternal inflation which is observationally forbidden. So we need an equation of state which will provide sufficient amount of inflation along with graceful exit from inflation. Given a model i.e. potential involved or the Hubble parameter, inflationary dynamics can be solved but with a specific EoS underlying scenario may be investigated model independently. In Ref.[19] inflationary paradigm has been investigated with the following parametrization of EoS

$$1 + \omega = \frac{\beta}{(N + 1)^{\delta}} \quad (3.2)$$

where $\delta$ and $\beta$ are positive constants, which captures a wide range of inflationary models with various observational predictions [30].

Considering $N$ as new time variable it is possible to express all the inflationary observables in terms of the EoS [31, 32], up to the first order in slow-roll parameters,

$$n_s - 1 \simeq -3 (1 + \omega) + \frac{d}{dN} \ln (1 + \omega) \quad (3.3)$$

$$\alpha_s \simeq 3 \frac{d}{dN} (1 + \omega) - \frac{d^2}{dN^2} \ln (1 + \omega) \quad (3.4)$$

$$r \simeq 24 (1 + \omega) \quad (3.5)$$

$$n_T \simeq -3 (1 + \omega) \quad (3.6)$$

$$\alpha_T \simeq 3 \frac{d}{dN} (1 + \omega). \quad (3.7)$$

To confront with the recent observational data above quantities are evaluated at the time of horizon crossing i.e. when there are 50-70 e-foldings left before the end of inflation.
4 Equation of State

It’s evident that a slowly varying EoS will produce sufficient amount of inflation along with graceful exit. In Ref.[19] single field inflation scenario has been investigated with EoS as a direct function of e-folding, instead, in this work we have examined inflationary paradigm with EoS which is a function of the scalar field responsible for inflation having the following form

\[ 1 + \omega = \frac{2}{3} \text{sech}^p \left( \phi \frac{\lambda}{M_P} \right) \]

(4.1)

where \( p \) is a dimensionless positive number. We have imposed the positivity restriction on the parameter \( p \) in order to get inflationary solutions. As \( p < 0 \) would imply decelerated expansion which is clear from Eq.(2.3). Apart from this positivity constraint the model parameter \( p \) is otherwise free. From Eq.(4.1) we see that \( \omega < -\frac{1}{3} \) as long as \( |\phi| > 0 \). So for this class of inflationary models, accelerated expansion goes on as long as \( |\phi| > 0 \) and ends exactly at \( \phi = 0 \). In Ref.[33] a generalization of the \( \alpha \)-Attractor T models has been proposed. Where inflationary scenario has been investigated with the following potential

\[ V = V_0 \left( 1 - \text{sech}^p \left( \lambda \phi / M_P \right) \right). \]

(4.2)

In this article our approach is somewhat different and we shall see that only \( p = 2 \) closely resembles \( \alpha \)-Attractor model for \( \alpha = 2/3 \).

Given the EoS it is now very simple to get the corresponding Hubble parameter which, within the Hamilton-Jacobi formulation, has the following exact form

\[ H = H_0 e^{\frac{1}{\sqrt{2}} \sinh \left( \phi \frac{\lambda}{M_P} \right)} \text{Hypergeometric} \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\sinh^2 \left( \phi \frac{\lambda}{M_P} \right) \right), \]

(4.3)

where \( \text{Hypergeometric} \) is Hypergeometric function. In Fig.1 we have shown the variation of inflationary Hubble parameter with the scalar field for six different values of the parameter \( p \). From the figure it is clear that apart from \( p = 0 \), the Hubble parameter varies slowly which essentially renders sufficient period of accelerated expansion. We have included \( p = 0 \) case in our plots for illustration purpose only.

Corresponding potential can be determined using Eq.(2.1) which is given by

\[ V = H_0^2 M_p^2 \left( 3 - \text{sech}^p \left( \phi \frac{\lambda}{M_P} \right) \right) e^{\frac{\sqrt{2}}{2} \sinh \left( \phi \frac{\lambda}{M_P} \right)} \text{Hypergeometric} \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\sinh^2 \left( \phi \frac{\lambda}{M_P} \right) \right). \]

(4.4)

Here again in Fig.2 we see that apart from \( p = 0 \), the potential is very flat during inflation and drop sharply near the end of inflation at \( \phi = 0 \).

The amount of inflation is represented by the number of e-foldings, \( N \), which can be exactly determined from Eq.(4.1) and we find that

\[ N = \frac{1}{\sqrt{2}} \sinh \left( \phi \frac{\lambda}{M_P} \right) \text{Hypergeometric} \left( \frac{1}{2}, \frac{1}{2}, -\frac{3}{4}, -\sinh^2 \left( \phi \frac{\lambda}{M_P} \right) \right). \]

(4.5)

In order to have satisfactory answer to the Big-Bang puzzles generally 50-70 e-foldings are required. In Fig.3 we have shown variation of \( N \) with the scalar field for different
Figure 1. The variation of Hubble parameter (in the units of $H_0$) with the scalar field for six different values of the model parameter, $p \in [0, 10]$.

Figure 2. Variation of potential involved in the units of $H_0^2M_P^2$ with the scalar field for six different values of the parameter $p$. 
Figure 3. Variation of e-foldings with the scalar field for six different values of the model parameter $p$.

values of the model parameter. From the figure it is obvious that sufficient amount of inflation is achievable for any positive value of the parameter.

Literally inflationary models are categorized into two broad classes depending on the excursion of inflaton, $\Delta \phi$, during observable inflation. One of them is small field inflation where $\Delta \phi < m_P$ and the large field modes where $\Delta \phi \geq m_P$. The energy scale of inflation is directly related to excursion of the inflaton. Accordingly, tensor-to-scalar ratio determines variation of the scalar field during observable inflation, as shown in Ref.[34], in this present context we have found

$$\Delta \phi = \frac{m_P}{8\sqrt{\pi}} (\phi_{CMB} - \phi_{end})$$

where $m_P = 2\sqrt{2\pi}M_P$ is the actual Planck mass, $\phi_{CMB}$ is the value of inflaton when the CMB scale left the horizon and $\phi_{end}$ is the value of inflaton at the end of inflation which is exactly zero for the model under consideration. For the large field models higher energy scale is required to match the current observations and as a result we get higher tensor-to-scalar ratio. In Fig.4 we have shown variation of inflaton excursion in the unit of Planck mass $m_P$ with the model parameter. From the figure we see that there is a small window where $\Delta \phi \geq m_P$ and for the rest of values of $p$, $\Delta \phi < m_P$. This allows us to address both the large and small scalar field models of within a single framework.

Now in order to confront this model with recent observations we shall adopt slow-
Excursion of the inflaton during observable inflation taking $N = 55, 60, 65$ has been plotted with the model parameter $p$. The horizontal line corresponds to $\Delta \phi = 1.0 \, \text{mp}$.

roll approximation which demands knowledge of another parameter given by

$$
\eta_H = \frac{1}{2} \text{sech}^p (\phi \text{M}_P^{-1}) \left(2 - \sqrt{2} p \, \tanh(\phi \text{M}_P^{-1}) \cosh^{\frac{p}{2}} (\phi \text{M}_P^{-1}) \right)
$$

$$
= \epsilon_H \left(1 - \frac{1}{\sqrt{2} p} \epsilon_H^{\frac{1}{2}} \left(1 - \epsilon_H^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)
$$

(4.7)

In Fig.5 and Fig.6 we have shown the variation of slow-roll parameters. From the figures it is obvious that the model has slow-roll region almost for every value of the model parameter, except for $p = 0$.

The most fascinating aspect of cosmological inflation is its ability to produce quantum mechanical seeds for cosmological perturbations observed in the large scale structure of the universe. Spectral tilt which measures the deviation from scale independence of the scalar curvature perturbation has been measured with great accuracy. Within the context of slow-roll inflation the first order expression for the spectral index is given by

$$
n_s \approx 1 - 4 \epsilon_H + 2 \eta_H
$$

$$
= 1 - 2 \epsilon_H - \sqrt{2} p \, \epsilon_H^{\frac{1}{2}} \left(1 - \epsilon_H^{\frac{1}{2}}\right)^{\frac{1}{2}}.
$$

(4.8)

In Fig.7 we have plotted the variation of spectral tilt, $\Delta n_s \equiv 1 - n_s$, with the model
Figure 5. Variation of first slow-roll parameter with the scalar field for six different values of the model parameter, $p$.

Figure 6. Variation of second slow-roll parameter with the scalar field for six different values of the model parameter.
Figure 7. The variation of spectral tilt with the model parameter \( p \) for three different values of \( N_{\text{CMB}} \). The shaded region corresponds to current 1-\( \sigma \) bound from Planck 2018.

parameter along with the current 1-\( \sigma \) bound on the spectral tilt from Planck 2018 [12]. We see that apart from very small values of \( p \), the model is consistent with current data. This also allows us to set a lower bound on the model parameter \( p \geq 0.535, 0.358, 0.288 \) for \( N_{\text{CMB}} = 55, 60, 65 \) respectively.

Another exciting feature of cosmological inflation is the production of primordial gravity waves through tensor perturbation, the tensor-to-scalar ratio up to first order in slow-roll parameters turns out to be

\[
 r \approx 16\epsilon_n. \tag{4.9}
\]

From Fig. 8 we recognize that as \( p \) increases tensor-to-scalar ratio decreases. The resent analysis from Planck suggests that \( r < 0.064 \) [12], which further tightens the lower limit on the model parameter depending on \( N_{\text{CMB}} \) and we find that \( p \geq 0.385, 0.352, 0.325 \) for \( N_{\text{CMB}} = 55, 60, 65 \) respectively. The upper bound on tensor-to-scalar ratio allows us to set the maximum value of the inflationary energy when the pivot scale exit the Hubble radius, \( V_0^{1/4} < 1.7 \times 10^{16} \) GeV. In Fig. 9 we have shown the variation of associated energy scale with the model parameter \( p \) and the energy scale decreases as \( p \) increases. In Fig. 10 we have plotted the logarithmic variation of tensor-to-scalar ratio with the spectral index.

The scale dependence of the spectral index is measured by the scalar running. The Planck 2018 analysis has set \( dn_s/d \ln k = -0.005 \pm 0.013 \) when the running of
Figure 8. The variation of tensor-to-scalar ratio with the model parameter $p$ for three different values of $N_{\text{CMB}}$.

Figure 9. The variation of inflationary energy scale (in GeV) with the model parameter for three different values of $N_{\text{CMB}}$. 

The logarithmic variation of tensor-to-scalar ratio with the scalar spectral index has been plotted for three different values of $N_{CMB}$. The running is ignored. So scalar spectral index is almost scale invariant and we have found a very small negative running $O(10^{-4})$ for most of the model parameter range as shown in Fig.11.

5 A Special Case: $p = 2$

In this section we shall present results for a particular value of the model parameter $p = 2$. Before producing expressions for the observable quantities, we first rewrite the number of e-foldings in terms of inflaton,

$$N = \frac{1}{\sqrt{2}} \sinh \left( \phi M_{P}^{-1} \right).$$

As a result the EoS can be rewritten as

$$1 + \omega = \frac{2}{3} \frac{1}{1 + 2N^2}.$$
Figure 11. Variation of the scalar running with the model parameter for three different values of $N_{\text{CMB}}$.

Corresponding Hubble parameter now can be exactly determined using the EoS parametrization in the Hamilton-Jacobi formalism. In the present context we have found that

$$H = H_0 \exp \left[ \frac{1}{\sqrt{2}} \tan^{-1} \left( \sinh (\phi M_P^{-1}) \right) \right]$$

$$= H_0 \exp \left[ \frac{1}{\sqrt{2}} \tan^{-1} \left( \sqrt{2}N \right) \right]. \quad (5.3)$$

The associated inflaton potential

$$V = H_0^2 M_P^2 \left[ 3 - \text{sech}^2 (\phi M_P^{-1}) \right] e^{\sqrt{2} \tan^{-1} (\sinh (\phi M_P^{-1}))}$$

$$= H_0^2 M_P^2 \left[ 3 - \frac{1}{1 + 2N^2} \right] e^{\sqrt{2} \tan^{-1} (\sqrt{2}N)}.$$ \quad (5.4)

The slow-roll parameters in this special case turns out to be

$$\epsilon_H = \frac{1}{1 + 2N^2}, \quad \eta_H = \frac{1 - 2N}{1 + 2N^2}. \quad (5.5)$$

The excursion of inflaton during observable inflation turn out to be

$$\Delta \phi = \frac{m_P}{8\sqrt{\pi}} (\phi - \phi_{\text{end}})$$

$$= \frac{m_P}{8\sqrt{\pi}} \sinh^{-1} \left( \sqrt{2}N \right). \quad (5.6)$$
Figure 12. Variation of the inflaton potential in the units $H_0^2M_P^2$ with $N$.

From the above expression it is obvious that for $p = 2$ inflaton variation is sub-Plankian and it corresponds to small field inflation.

The amplitude of curvature perturbations within the slow-roll approximation is given by

$$P_R \approx \frac{1}{8\pi^2M_P^2} \frac{H^2}{\epsilon_H}$$

$$= \frac{H_0^2}{8\pi^2M_P^2} (1 + 2N^2) \exp \left[ \sqrt{2} \tan^{-1} \left( \sqrt{2}N \right) \right]$$  (5.7)

$$= \frac{V_0}{8\pi^2M_P^4} (1 + 2N^2) \exp \left[ \sqrt{2} \tan^{-1} \left( \sqrt{2}N \right) \right]$$  (5.8)

where in the last line we have defined the inflationary energy scale $V_0 \equiv H_0^2M_P^2$. The Eq.(5.7) we can be inverted to get the value of Hubble parameter during inflation

$$H_0 \sim 2\sqrt{2}\pi M_P \frac{P_R^{\frac{1}{2}}}{1 + 2N^2}^{-1/2} \exp \left[ -\frac{1}{\sqrt{2}} \tan^{-1} \left( \sqrt{2}N \right) \right] .$$  (5.9)

Current Planck 2018 upper bound on the Hubble parameter during inflation is $H_0 < 2.7 \times 10^{-5} \, M_P$. For the model under consideration we have found that $H_0 < 1.75613 \times 10^{-6} \, M_P$, a order lower than present bound.
The spectral index turns out to be

\[ n_s \approx 1 - \frac{2 + 4N}{1 + 2N^2} \]
\[ \approx 1 - \frac{2}{N} \]  
(5.10)

The tensor-to-scalar ratio is now

\[ r \approx \frac{16}{1 + 2N^2} \]
\[ \approx \frac{8}{N^2} \]  
(5.11)

From the above outcomes it is clear that the particular case for \( p = 2 \) very closely resembles \( \alpha \)-Attractor class of inflationary models with \( \alpha = \frac{2}{3} \) which is special as it corresponds to unit moduli space curvature.

6 Conclusion

Inflation remains the best tool to explain early universe scenario, in fact it is the only mechanism that is compatible with recent observations when combined with the Big-Bang. Precise data of late has eliminated many inflationary models but still allowing plenty of them. As a consequence more precision is required towards a specific compelling model of inflation.

In this present analysis we have found that a simple single parameter EoS has quite remarkable fit to the latest data. Not only that, the prediction from this particular inflationary EoS can efficiently explain wide range of tensor-to-scalar ratio which many existing inflationary models fails to address. The EoS for a specific value of the model parameter mimics the \( \alpha \)-Attractor class of inflationary models with \( \alpha = \frac{2}{3} \).

The present and future observational probes are targeting the primordial gravity waves, whose detection would certainly help to discriminate between inflationary models. The tensor-to-scalar ratio being the most powerful up-to-date eliminator and the model discussed here is capable of producing almost any value of \( r \), it might have the endurance and vision to emerge as the winner in long term.

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