ABSTRACT

Although some algebraic theories such as partial order set, lattice and their properties on fuzzy set had been studied, these algebraic theories on fuzzy automata (FA) have not been discussed at present. For saving the energy of the battery so as to make it play the best performance, this paper proposes first an algebraic system on a finite-state deterministic fuzzy automaton (FDFA), including the partial order set, lattice and its properties. Second, this paper constructs a detection model system to monitor the consumption of battery power, and the detection model consists of four parts, as well as including supervision function network and decision-making process. Third, this paper gives the whole dynamic process of consumption and consumption rate of battery power, as well as discusses the comparison of the detection accuracy of between the detection model and other detection methods in the battery power consumption by simulation. Through the homomorphism and the isomorphic mapping of the lattice, the partial state of the FDFA is studied to realize the function of the whole state. These research results will provide a theoretical reference, system modeling and application background for many departments such as control, detection, tracking, identification and so on.

INDEX TERMS

FDFA partial order set, FDFA lattice, detection model, homomorphism, isomorphism of lattice.

I. INTRODUCTION

Until now, we had studied the logicality, computational power, recognition language, state transition, robustness and composition of deterministic or fuzzy automata [1], [19], [21]. However, the algebraic systems of fuzzy automata, such as lattice sets and their properties, have not been discussed all along. The algebraic system of fuzzy automata has obvious advantages. Some parameters in the algebraic system have obvious physical meaning, which can reflect the real world and the objective things more realistically.

The literature [2] studied the fuzzy automata of lattice-valued uncertainties by comparing the behavior of lattice-valued fuzzy automata with the relationship of two equivalent languages, and discussed the robustness and composition of the set of complete lattice-valued fuzzy automata. In the literature [3], the invariant nature of the operation on the decomposition theorem, fuzzy subsystem, fuzzy root operator and sub-operator of the lattice-order fuzzy finite state automata (L-FFA) were discussed. In the literature [4], the residuated lattice-valued logic finite state automata (L-VFAs) and the lattice-valued non-deterministic Turing machine had been studied. The literature had given that the multi-band non-deterministic Turing machine and the single-band non-deterministic Turing machine was not equivalent on language recognition.
The literature [5] had studied the numerical simulation of the non-negative real number in the numerical application of the transformation system analysis with mark, simultaneously studied the semantic simulation of the transformation system with mark on the lattice-valued logic set, discussed the quantization transformation system with mark of the lattice-valued equivalence relation, gave the lattice-valued similarity relation among the states of the quantization transformation system, and mainly discussed the similarity between the fixed point and the logical feature. In the literature [6], [27], [29], the double labeling transformation system, the complete lattice, fuzzy transformation system and the multi-valued structure in the lattice-valued framework had been studied, and the lattice-valued similarity of the lattice-valued double labeling transformation systems to measure the compactness of the two systems had been discussed, at the same time, the robustness and the composition of lattice-valued similarity had been also discussed.

For monitoring the consumption process of battery power is from one energy state to another energy state at all times and saving the energy of the battery so as to make it exert the best performance, it needs to construct an appropriate monitor model, which should have the function of monitoring and saving the energy. Based on this problem’s requirement, this paper will propose an algebraic system on finite-state deterministic fuzzy automaton (FDFA), i.e., a lattice of FDFA which consists of a series of sub-model. Thus, the consumption and supplement of battery power can be monitored and controlled by the lattice of FDFA.

The literature [18] compared the lattice-valued (referred to as L-valued) regular tree grammars and L-valued top-down tree automata. They introduced two different ways, i.e., one was the alternating regular tree grammar, and the second way was L-valued alternating stack tree automaton (CASA). The literature [19] studied fuzzy finite automata in which all fuzzy sets were defined by membership functions whose codomain forms a lattice-ordered monoid L. They mainly introduced the notions of L-fuzzy regular expressions and give the Kleene theorem for NL-FFAs, and described the DL-FFAs by L-fuzzy regular expressions. The literature [20] study the concepts of homomorphism, fuzzy multiset transformation semigroup and coverings of fuzzy multiset finite automata and its theory. The literature [21] introduced the notion of factorizations of fuzzy sets in complete residuated lattices. Moreover, studied fuzzy partially orders, the upper and lower approximation operators and Alexander fuzzy topologies on factorizations of fuzzy sets. The literature [22] discussed the subautomaton, retrievability and connectivity of a fuzzy automaton, upper semilattices with fuzzy automata, a decomposition of a fuzzy automaton, a construction of a fuzzy automaton corresponding to a given finite partially ordered set (poset), and established an isomorphism between the poset of class of subautomata of a fuzzy automaton and an upper semilattice. The literature [3] presented and investigated fuzzy finite automata (L-FFAs for short) based on a more generalized structure L, discussed fuzzy successor and source operators which are shown to be closure operators on certain conditions are introduced and discussed, studied the decomposition theorem of a fuzzy finite automaton based on a lattice-ordered monoid.

In addition, the previous researches [11] had also studied the semigroup and its properties on the FDFA. However, this paper proposes an algebraic system on the FDFA, studies the partial order set, lattice and its properties such as homomorphism and isomorphism on the FDFA.

For studying the lattice of fuzzy automata, it is first to study the partial order set, lattice and its properties of FDFA in this paper, which is a base for presenting a detection model to be applied to test the battery power consumption. The lattice of FDFA can be flexible, fast and accurate to make the test to the consumption of battery power, which can provide a theoretical guidance and experience for reference for the latter part of the work. The use of the algebraic system of FDFA to accurately detect the battery power can be a good control to the equipment by using the battery. Thus, the efficiency to work and the reliability of data application can be improved. The blindness can be reduced. The running time of battery can be extended. The labor intensity of staff for operation of battery can be reduced. At the same time, a lot of resources will be saved.

II. LATTICE OF FUZZY AUTOMATA

A. PARTIAL ORDER PAIR OF FUZZY AUTOMATA

In here, an algebraic system of fuzzy automata (FA) is discussed, i.e., FA lattice. The FA lattice is not only used in the algebra itself, but also in the switch theory of FA, computational theory, FA automatic control and logic design.

From the definition of fuzzy cut set [7], [8], Q was a universal set, F was a fuzzy membership set, and the cut set of A set was \(A_\lambda = \{x \in Q | A(x) \geq \lambda, \lambda \in F\}\).

A partial order pair set is defined as follows:

Definition 1: If a pair set \((Q, F)\) and its pairwise binary relation are recorded as \((\subseteq, \subseteq)\), where \(\lambda \in F,\) and \(F\) is a fuzzy membership set. The following conditions are satisfied:

(i) For \(\forall a \in F,\) there is \(a \leq a \Rightarrow A_a \subseteq A_a,\) where \(A_a \subseteq Q\) (Reflexivity);

(ii) For \(\forall a, b \in F,\) \(a \leq b \Rightarrow A_b \subseteq A_a\) and \(b \leq a \Rightarrow A_a \subseteq A_b,\) then there is \(a = b \Rightarrow A_a = A_b,\) where \(A_a, A_b \subseteq Q\) (Anti - symmetry);

(iii) For \(\forall a, b, c \in F,\) \(a \leq b \Rightarrow A_b \subseteq A_a\) and \(b \leq c \Rightarrow A_c \subseteq A_b,\) then there is \(a \leq c \Rightarrow A_c \subseteq A_a,\) where \(A_a, A_b, A_c \subseteq Q\) (Transitivity).

According to [9], then \((\subseteq, \subseteq)\) is called a pair of partial order relation of \((Q, F),\) the pair set \((Q, F)\) with a partial order relation \((\subseteq, \subseteq)\) is called the partial order pair set, labeled as \((Q, F), (\subseteq, \subseteq)\).

A finite-state deterministic fuzzy automaton (FDFA) is a six-tuple \(M = (Q, \Sigma, \delta, q_0, G, F)\) [10], [11]. Where \(Q\) is a finite set of state; \(\Sigma\) is a finite set of input character; \(q_0 \in Q\) is an initial state; \(G\) is a fuzzy finite set of the final state of \(Q;\) \(F\) is a fuzzy membership set of the transition function;
$\delta : Q \times \sum_{\mu} \rightarrow Q$ is a transition function, that is, $\delta(q_i, a, \mu) = \{q_j\}$, where $q_i, q_j \in Q$, $a \in \sum$, $\mu \in F$; especially, the extended transition function $\delta^*$ : $Q \times \sum_{\mu} \rightarrow Q$.

Define $\delta^*$ $(q, \epsilon, \mu) = q$ and

$$\delta^*(q, \omega a, \mu) = \delta(q, \omega, \mu') , a, \mu) ,$$

where $\epsilon \in \sum_{\mu} \ast \omega \in \sum_{\mu} \ast \mu \in F$.

The degree $(L(FDFA))$$(\sigma_1, \ldots, \sigma_n)$ to which a FDFA $M$ accepts a language $\sigma_1 \ldots \sigma_n \in \sum_{\mu}$ is $L(FDFA)$, i.e.,

$$L(FDFA) = G(\delta^*, \sigma_1, \ldots, \sigma_n, \mu) .$$

Let $F = [0, 1]$. Let $G$ be a fuzzy set in $Q$, and $G_\lambda$ be a cut set of the set $G$. $G_\lambda = \{x \in Q| G(x) \geq \lambda, \lambda \in F\}$.

Obviously, $G_1 \subseteq Q$.

Define $\lambda G = \{G(x)| G(x) \geq \lambda, \lambda \in F, x \in Q\}$.

Obviously, $\lambda G \subseteq F$.

The pair set $(G_0, F)$ in FDFA and its pairwise binary relation $(\subseteq, \subseteq)$ satisfies the definition of the partial order. Where $G_0 \subseteq Q$, and $F$ is the fuzzy membership set. Since it satisfies:

(i) Reflexivity. For $\forall a \in F$, there is $a \leq a \Rightarrow G_a \subseteq G_a$, and $G_0 \subseteq Q$;

(ii) Anti-symmetry. For $\forall a, b \in F$, $a \leq b \Rightarrow G_a \subseteq G_b$, $b \leq a \Rightarrow G_a \subseteq G_b$, then there is $a = b \Rightarrow G_a = G_b$;

(iii) Transitivity. For $\forall a, b, c \in F$, $a \leq b \Rightarrow G_a \subseteq G_b$, $b \leq c \Rightarrow G_a \subseteq G_b$, then there is $a \leq c \Rightarrow G_a \subseteq G_b$.

By definition 1, $(\subseteq, \subseteq)$ is a pair of partial order relation of $(G_0, F)$.

**Definition 2:** If the element $(G_0, F)$ in FDFA $M$ has a partial order relation $(\subseteq, \subseteq)$, then it is called partial order FDFA, labeled as $\langle (G_0, F), (\subseteq, \subseteq) \rangle$ or $(M, (\subseteq, \subseteq))$.

**Example 1:** Assume the element $(G_0, F)$ in FDFA $M$ has the partial order relation $\langle \subseteq, \subseteq \rangle$, the fuzzy set $F = \{0.01, 0.03, 0.04, 0.09, 0.12, 0.16, 0.36, 0.48\}$, and the one $\leq$ of the partial order relation is the divisible relation, then the partial order set of the fuzzy set $(F, \subseteq)$ is shown in Figure 1.

![FIGURE 1. Partial order set of fuzzy set $(F, \subseteq)$](image)

**Definition 3:** Assume $(M, (\subseteq, \subseteq))$ is a partial order FDFA. For the element $(G_m, m)$ of $(G_0, F)$ in $M$, $\forall G_y \subseteq G_0$ and $\forall y \in F$, if there are $G_y \subseteq G_m$ and $m \leq y$, then $(G_m, m)$ is called the minimum element of $(G_0, F)$ or the minimum element of final state of the FDFA; For the element $(G_n, n)$ of $(G_0, F)$, $\forall G_y \subseteq G_0$ and $\forall y \in F$, if there are $G_n \subseteq G_y$ and $n \geq y$, then $(G_n, n)$ is called the maximum element of $(G_0, F)$ or the maximum element of final state of the FDFA.

**Theorem 1:** Assume $(M, (\subseteq, \subseteq))$ is a partial order FDFA. If the minimum element or maximum element of final state of FDFA exists, then it is unique.

**Proof:** Assume $(G_{y_1}, y_1)$ and $(G_{y_2}, y_2)$ all are the minimum element of final state of FDFA. Since $(G_{y_1}, y_1)$ is the minimum element of final state of FDFA, there are $G_{y_2} \subseteq G_{y_1}$ and $y_1 \leq y_2$. On the other hand, $(G_{y_2}, y_2)$ is also the minimum element of final state of FDFA, then there are $G_{y_1} \subseteq G_{y_2}$ and $y_2 \leq y_1, so$ there are $G_{y_1} = G_{y_2}$ and $y_1 = y_2$. Similarly, the case of the maximum element can be proved.

**Definition 4:** Assume $(M, (\subseteq, \subseteq))$ is a partial order FDFA. If $(G_T, T)$ is a subset of element $(G_0, F)$ of FDFA, that is, it satisfies $T \subseteq F$ and $\forall a \in T$, there are $G_a \subseteq G_0$, which is denoted by $G_T \subseteq G_0$. For $\forall a \in T$, if there are $a \leq u$ and $G_u \subseteq G_a$, then the element $(G_u, u)$ of FDFA $M$ is called an upper bound of $(G_T, T)$. For $\forall a \in T$, if there are $a \leq v$ and $G_v \subseteq G_a$, then the element $(G_v, v)$ of FDFA $M$ is called a lower bound of $(G_T, T)$.

**Definition 5:** Assume $(M, (\subseteq, \subseteq))$ is a partial order FDFA. Let $(G_T, T)$ be a subset of element $(G_0, F)$ of FDFA. For any $a$ of upper bound of $T$, if there are $a \leq u$ and $G_u \subseteq G_a$, then an upper bound $(G_u, u)$ of $(G_T, T)$ is called the minimum upper bound, labeled as $\sup \{a \in F| G_a = u \}$. Similarly, $\inf \{a \in F| G_a = v \}$ is called an upper bound of $(G_T, T)$.

**Proof:** Assume $(G_{y_1}, y_1)$ and $(G_{y_2}, y_2)$ are the upper bound of $(G_T, T)$, and $y_1 \leq y_2$. If for any $a$ of lower bound of $T$, if there are $a \leq v$ and $G_v \subseteq G_a$, then a lower bound $(G_v, v)$ of $(G_T, T)$ is called the maximum lower bound, labeled as $\inf \{a \in F| G_a = v \}$. Similarly, $\inf \{a \in F| G_a = v \}$ is called a lower bound of $(G_T, T)$.

B. LATTICE OF FUZZY AUTOMATA

The subset $(G_T, T)$ of element $(G_0, F)$ of partial order FDFA does not necessarily has the minimum upper bound or the maximum lower bound. However, some partial pair sets
(\(G_0, F\)), the subsets that consist of any two elements among them all have the minimum upper bound and the maximum lower bound.

**Definition 6:** Assume \(\langle M, (\leq, \leq) \rangle\) is a partial order FDFA. If the subset that consists of any two elements in \((G_0, F)\) of \(M\) has the minimum upper bound and the maximum lower bound, then \(M\) or \((G_0, F)\) regarding the partial order relation \((\leq, \leq)\) is called a lattice, labeled as the lattice \(\langle M, (\leq, \leq) \rangle\) or \((\langle G_0, F \rangle, (\leq, \leq))\).

In the lattice \(\langle M, (\leq, \leq) \rangle\), for any two elements \((G_a, a)\) and \((G_b, b)\), assume \(a \land b \mid G_a \cup G_b\) denotes the minimum upper bound of \((G_a, a)\) and \((G_b, b)\), \(a \land b \mid G_a \cap G_b\) denotes the maximum lower bound of \((G_a, a)\) and \((G_b, b)\),

i.e., \(a \land b \mid G_a \cup G_b = \sup \{a, b \mid G_a, G_b\}\),
\[a \land b \mid G_a \cap G_b = \inf \{a, b \mid G_a, G_b\}\).

By \(a, b \mid G_a, G_b\), \(a \lor b \in F \mid G_a \cup G_b \leq G_0\) and \(a \land b \in F \mid G_a \cap G_b \leq G_0\) can be uniquely determined. Thus, \(\lor \mid U\) and \(\land \mid \cap\) can be seen as two binary operations of the lattice FDFA \(M\) or \((G_0, F)\).

**Definition 7:** The final state fuzzy set \((G_0, F)\) of FDFA \(M\) has two binary operations \(\lor \mid \cup\) and \(\land \mid \cap\) which satisfy (L1) \sim (L4), then \((\langle G_0, F \rangle, \lor \mid \cup, \land \mid \cap\) or \(\langle M, \lor \mid \cup, \land \mid \cap\) is called a lattice. \(\lor \mid \cup\) and \(\land \mid \cap\) are called a sum (union) and a product (intersection), respectively.

**Theorem 2:** Assume \(\langle M, (\leq, \leq) \rangle\) is a lattice. For \(a, b, c \in F\), accordingly, there is \(G_{a duke}, G_{c duke} \leq G_0\). If \(c \leq a \Rightarrow G_a \subseteq G_c\), then \(c \lor (a \land b) \leq a \land (b \lor c) \Rightarrow G_a \cap G_{b\lor c} \subseteq G_c \cup G_{a\land b}\).

**Proof:** Since
\[c \leq a \Rightarrow G_a \subseteq G_c, \quad a \land b \leq a \Rightarrow G_a \subseteq G_{a\land b},\]

then \(c \lor (a \land b) \leq a \Rightarrow G_a \subseteq G_c \cup G_{a\land b}\).

On the other hand, there are \(c \leq c \lor b \Rightarrow G_{c\lor b} \subseteq G_c\), and \(a \lor b \leq b \lor c \Rightarrow G_{a\lor c} \subseteq G_b \subseteq G_{a\lor b}\), then \(c \lor (a \land b) \leq c \lor b \Rightarrow G_{c\lor b} \subseteq G_c \cup G_{a\land b}\). Therefore,
\[c \lor (a \land b) \leq a \land (b \lor c) \Rightarrow G_a \cap G_{b\lor c} \subseteq G_c \cup G_{a\land b}\).

Assume \(\langle M, (\leq, \leq) \rangle\) is a partial order FDFA. Its final state fuzzy set is \((G_0, F)\). \((G_0, F')\) is a dual proposition that is obtained by exchanging all \((\leq, \leq)\) and \((\geq, \geq)\) in \((G_0, F)\). It can be concluded that \((G_0, F')\) about \((\geq, \geq)\) is also a partial set.

### III. CONSTRUCTION OF ENERGY SAVING DETECTION MODEL BY FDFA LATTICE

According to above these discussions on lattice of FDFA and its homomorphic mapping between two lattices, for detecting better consumption process of battery power so as to save the energy, the energy saving detection model by FA is constructed in here. The model can be used to detect the consumption and to make a decision by means of the homomorphic mapping between the lattices of FDFA. In the reduction layer, there is the reduction operator to consumption data of battery power. A homomorphic mapping is given by the lattice of FDFA in the homomorphic mapping layer. At the same time, a fuzzy inference algorithm by FA is given in the fuzzy inference layer, as well as the inference operator is also novel, because it adopts the fuzzy operator method instead of the usual detection operator method [12]–[17]. Compare it with the old method in the simulation. Figure 2 is the detection model of FA, which consists of four parts: data reduction, homomorphic mapping, fuzzy inference and decision-making for SOC detection. The simulation results show that the correct detection rate is 92.79%. The study to this detection model not only will develop the theory of FA, but also strengthen its combination with energy consumption detection, at the same time, promote its wide applications.

![Figure 2. Energy saving detection model by FDFA lattice.](image-url)

**A. REDUCTION ALGORITHM**

Neurons in the data reduction layer of this model are the reduction operators, such as game operators and rough set operations. Use the reduction operators to filter the redundant data. The reduction algorithm is:

For the reduction of spatiotemporal multi-sequence consumption data of battery power, a game competition reduction algorithm is proposed. The dimensions of the output grid of reduction are fixed, and the weights between the input operators and output operators of reduction are randomly initialized. Let \(t\) be the number of iterations of the algorithm, and assume \(t = 0\). The spatiotemporal multi-sequence consumption data of battery power are input to the competitive neurons in this layer. For each input value \(x_j\), the winning output neuron in this layer is selected for \(i^*\), that is, the output value of the nodes in the competitive neurons is minimized to be \(S_i^*\). Let \(N (i^*)\) be the nearest neighbor of the winning output neuron \(i^*\), which it is specified by the distance between the output neurons in this reduction layer. For each output

\[Q. Wu et al.: Construction of an Energy Saving Model Based on FA and Its Applications\]

VOLUME 8, 2020

54525
neuron $i \in \{N (i^n), i^n\}$ in this reduction layer, the weight is adjusted and updated based on the formula $w_k(t + 1) = f (w_k(t), \eta(t))$. Where $\eta(t) = \eta$ is the learning rate and $f$ is the weight adjustment function. The weights are normalized after being updated so that they are consistent with the input measurement criteria. Repeat the above steps, the number of iterations is set to be $t = t + 1$ until the shutdown criteria are satisfied. The final output values in this layer are the reduced data of the spatiotemporal sequence.

**B. HOMOMORPHIC MAPPING**

In the homomorphic mapping layer, $M$ neurons are the homomorphic mapping operators. The homomorphic mapping operator is used to deal better with the reduced data in the above layer. The relation between two lattices of FDFA is discussed as follows:

**Definition 8:** Assume $\langle M, \lor | U, \land | \cap \rangle$ and $\langle S, | + | U, \circ | \cap \rangle$ are two lattices of FDFA. If there is a mapping $f$ from $M$ to $S$ so as to make it satisfy the following the formula, for $\forall a, b \in F$, $G_a, G_b \in G_0$,

$$f(a \lor b) = f(a) + f(b),$$

accordingly, $f(G_a \cup G_b) = f(G_a) \cup f(G_b)$,

$$f(a \land b) = f(a) \circ f(b),$$

accordingly, $f(G_a \cap G_b) = f(G_a) \cap f(G_b)$. Then the mapping $f$ is called the homomorphic mapping from $M$ to $S$.

When the homomorphic mapping $f$ from $M$ to $S$ is a single mapping, it is called a single homomorphism from $M$ to $S$; when the mapping $f$ is a full mapping, it is called a full homomorphism from $M$ to $S$; when the mapping $f$ is a both single and full mapping, it is called the isomorphic mapping from $M$ to $S$, simply called as isomorphism. If there is an isomorphic mapping between the lattices $M$ and $S$, then $M$ and $S$ are called the isomorphism, labeled as $M \cong S$. The isomorphic lattices have the same characteristics.

**Theorem 3:** Assume $f$ is the homomorphic mapping from the lattice $\langle M, \lor | U, \land | \cap \rangle$ to $\langle S, | + | U, \circ | \cap \rangle$, then $f$ is a keeping order mapping from $M$ to $S$. That is, for $\forall a, b \in F$, $G_a, G_b \in G_0$, when $a \leq b$, $G_b \subseteq G_a$, there are $f(a) \leq f(b)$ and $f(G_b) \subseteq f(G_a)$ in the final fuzzy set $(G_{S_0}, F_S)$ of $S$, where $\subseteq$ and $\subseteq'$ are a partial order relation of $M$ and $S$, respectively.

**Proof:** When $a \leq b$, $G_b \subseteq G_a$, there are $a \land b = a$ and $G_a \cap G_b = G_b$. So, there are $f(a \land b) = f(a)$ and $f(G_a \cap G_b) = f(G_b)$. Since $f$ is the homomorphic mapping from $M$ to $S$, then there are

$$f(a \land b) = f(a) \circ f(b),$$

$$f(G_a \cap G_b) = f(G_a) \cap f(G_b).$$

So $f(a) \circ f(b) = f(a)$,

$$f(G_a) \cap f(G_b) = f(G_b).$$

On the other hand, there are $a \lor b = b$ and $G_a \cup G_b = G_a$. So, there are $f(a \lor b) = f(b)$ and $f(G_a \cup G_b) = f(G_a)$. Since $f$ is the homomorphic mapping from $M$ to $S$, then there are

$$f(a \lor b) = f(a) + f(b),$$

$$f(G_a \cup G_b) = f(G_a) \cup f(G_b).$$

So $f(a) + f(b) = f(b)$,

$$f(G_a) \cup f(G_b) = f(G_a).$$

However, $f(a) \circ f(b) \leq f(a) + f(b)$ and $f(G_a \cap f(G_b) \subseteq f(G_a) \cup f(G_b)$ hold. Then, there are $f(a) \leq f(b)$, $f(G_b) \subseteq f(G_a)$. Then the mapping $f$ is both single and full mapping, there are $a \land b = a$ and $G_a \cap G_b = G_b$, i.e., $a \leq b$, $G_b \subseteq G_a$.

Assume $a \leq b$ and $G_b \subseteq G_a$ if and only if $f(a) \leq f(b)$ and $f(G_b) \subseteq f(G_a)$. Assume $a \land b = a$ and $G_a \cap G_b = G_b$. Then, there are $c \leq a$, $a \leq b$, $G_a \subseteq G_c$, $G_b \subseteq G_c$. Therefore,

$$f(a \land b) = f(c),$$

$$f(G_a \cap G_b) \subseteq f(G_a \cap G_b) = f(G_c),$$

$$f(c) \leq f(a), \quad f(c) \leq f(b), \quad f(G_a) \subseteq f(G_c),$$

$$f(G_b) \subseteq f(G_c),$$

then $f(c) \leq f(a) \circ f(b)$ and $f(G_b) \subseteq f(G_a \cap G_b)$ can be obtained. Assume $f(a) \circ f(b) = f(d)$. Then, there are $f(c) \leq f(d)$ and $f(G_a) \subseteq f(G_c)$. However, $f(d) \leq f(a)$, $f(d) \leq f(b)$, $f(G_a) \subseteq f(G_d)$, $f(G_b) \subseteq f(G_d)$, thus, there are $d \leq a$, $d \leq b$, $G_d \subseteq G_a$, $G_d \subseteq G_b$, $G_d \subseteq G_c$. Therefore, $d \leq a \land b = c$, $G_a \cap G_b \subseteq G_d$, then $f(d) \leq f(c)$ and $f(G_d) \subseteq f(G_a \cap G_b)$ can be obtained. So there are $d = f(c), f(G_d) = f(G_c)$. Therefore, $f(a \land b) = f(a) \circ f(b)$,

$$f(G_a \cap G_b) = f(G_a) \cap f(G_b).$$

Similarly, $f(a \lor b) = f(a) + f(b)$ and $f(G_a \cup G_b) = f(G_a) \cup f(G_b)$ can be proved.

The reduced battery power data in the previous layer are classified, that is, if they satisfy the homomorphism mapping between the lattices of FDFA, then they are input to the next layer for inference decision; if they don’t meet the homomorphism mapping, they are temporarily stored in this layer and are not input to the next layer.
C. SUPERVISION FUNCTION NETWORK DETECTION FOR SOC

In this paper, the current, voltage and state of charge (SOC) are discussed the relevant parameters of battery. In this paper, the SOC represents the remaining power capacity of the battery. Its value is defined as follows:

\[ V_{soc} = \frac{n}{N} \]  

where \( N \) denotes the total power capacity of the battery at the initial state; \( n \) denotes the remaining power capacity of the battery after a period of operation; \( V_{soc} \) denotes the value of SOC, which is the percentage of remaining power capacity of the battery after a period of operation.

In the homomorphic mapping layer shown in Figure 2, there is a supervision function network detection algorithm as follows:

In the supervision function network detection, the initial stage is FDFA homomorphic mapping detection, and then transition to the supervision function network detection. When a large error occurs during the execution of the supervision function, FDFA homomorphic mapping detection plays a leading role, and the supervision function network plays a regulating role. The detection principle is shown in Figure 3. The specific algorithm is as follows:

The detection vector function in the hidden layer is \( H = [H_1, H_2, \ldots, H_J]^T \). After a lot of experiments, \( H_m \) is defined as the following function, i.e.,

\[ H_m = \exp \left( -\frac{\|X(k) - c_m\|^2}{2b_m^2} \right) \]  

where \( m = 1, 2, \ldots, J \); \( X(k) \) is the input of this network, its center vector is \( c_m = [c_{1m}, c_{2m}, \ldots, c_{Jm}]^T \); the standard deviation that is the width vector is \( b_m = [b_{1m}, b_{2m}, \ldots, b_{Jm}]^T \). The number and size of vector elements \( c_m - b_m \) are the key to design network. This determination process is currently designed first and then checked. This paper debugs the network based on the state of the system under FDFA homomorphic mapping detection.

Let the weight vector be:

\[ W = [W_1, W_2, \ldots, W_J]^T \]  

The output of the supervision function network is:

\[ Y(k) = \sum_{m=1}^{J} (H_m \times W_m) \]  

In the current detection subnetwork of homomorphic mapping layer, there is a supervision function feedback network. The input \( X(k) \) is \( i_s(k) \), which is given by the operation of voltage outer loop. The output of the battery to be detected that is a feedback is the current flowing through the inductor, i.e., \( i_A(k) \), and the output \( Y(k) \) of the supervision function network is \( u(k) \). The battery at the feedback end of this supervision network is connected to the fuzzy inference layer of Figure 2, as shown in Figure 3.

The total output of supervision function detection network is

\[ u(k) = u_n(k) + u_h(k) \]  

The error index is:

\[ e(k) = \frac{[u_n(k) - u(k)]^2}{2} \]  

The weight adjustment algorithm of network is used as the gradient descent method, i.e.,

\[ \Delta W_m(k) = -\eta \frac{\partial E(k)}{\partial W_m(k)} = \partial [u_n(k) - u(k)] H_m(k) \]  

The weights are updated to:

\[ W(k) = W(k-1) + \Delta W(k) + \alpha [W(k-1) - W(k-2)] \]  

where \( \eta \in (0, 1) \) is the learning rate, which is used to reduce the oscillating trend in the learning process; \( \alpha \in (0, 1) \) is the inertia coefficient, which is used to accelerate the convergence speed of the weight update.

The supervision function network detector acts as a feedforward detector, and its role is to fit the inverse model of the detected object to achieve the effect of canceling its nonlinearity. The converter system has achieved good stability under...
the FDFA homomorphic mapping detection, and the supervision function network improves the tracking performance of the system and reduces the steady-state error of the system by compensating for the nonlinear detected object.

**D. FUZZY INFERENCE FOR DECISION-MAKING**

Neurons in fuzzy inference layer of FA detection model are some inference rules and membership functions operators. Use these operators to implement the detection for SOC (state of charge). Here, the mean and variance of the consumption of SOC are used as a reference index.

In this fuzzy inference layer, some features, such as the consumption, its mean and the variance of SOC, etc., will be extracted and inferred for decision-making of energy saving.

In this detection model system, the third layer is the inference rules of the FDFA lattice, which performs fuzzy reasoning operations. The rules are given as follows:

\[ R_1: \text{if } X \text{ is } A_1, \text{ then } Y = (B_1, f_1, \gamma_1); \]
\[ R_2: \text{if } X \text{ is } A_2, \text{ then } Y = (B_2, f_2, \gamma_2); \]
\[ \cdots \]
\[ R_n: \text{if } X \text{ is } A_n, \text{ then } Y = (B_n, f_n, \gamma_n); \]

**Fact:** if \( X \) is \( A^* \)

**Conclusion:** then \( Y \) is \( B^* \)

where \( A_j \in G_1, B_j \in G_2, G_1 \) and \( G_2 \) are the lattices of FDFA,, respectively. \( f_j \) denotes the degree of membership of \( j \)-th rule, \( \gamma_j \) is the threshold assigned to the \( j \)-th rule, \( j = 1, 2, \ldots, n \).

The fuzzy reasoning based on the FDFA lattice follows the following steps:

**Step 1:** A set of interval-valued fuzzy production rules and matching facts are given by FDFA lattice theory, expert experience of domain, functions and heuristic knowledge, etc.

**Step 2:** If \( A_j \in G_1, B_j \in G_2 \), when \( X = \{x_1, x_2, \ldots, x_n\} \) is a finite set, \( \lambda_j, \mu_i \in [0, 1] \) and \( \lambda_j + \mu_i = 1 \), \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) is a weight vector related to the discussed domain \( X \), where, \( \omega_i \in [0, 1], \sum \omega_i = 1, i = 1, 2, \ldots, n \), define

\[
f(A, B) = \sum_{i=1}^{n} \omega_i \left\{ 1 - \left( \lambda_i \left[ A^-_i(x_i) - B^-_i(x_i) \right]^2 + \mu_i \left[ A^+_i(x_i) - B^+_i(x_i) \right]^2 \right)^{1/2} \right\} \quad (9)
\]

Then \( f(A, B) \) is the degree of membership of interval-valued fuzzy sets \( A \) and \( B \). Where \( A^-_i(x_i) \) and \( A^+_i(x_i) \) represents the lower and upper approximate degree that \( x_i \) belongs to the \( A_i \), respectively, as well as \( A^-_i(x_i) \leq A^+_i(x_i) \). \( A = \{A_1, A_2, \ldots, A_n\}, B = \{B_1, B_2, \ldots, B_n\} \). According to the definition 4, the degree of membership vector \( \alpha_j = (\alpha_{j1}, \alpha_{j2}, \ldots, \alpha_{jn}) \) is calculated between the antecedent \( A_j \) on the \( j \)-th rule and the matching facts \( A^* \), where

\[
\alpha_{ji} = 1 - \left( \frac{\lambda_i \left[ A^-_i(x_i) - A^+_i(x_i) \right]^2 + \mu_i \left[ A^+_i(x_i) - A^+_i(x_i) \right]^2}{\lambda_i \left[ A^-_i(x_i) - A^+_i(x_i) \right]^2 + \mu_i \left[ A^+_i(x_i) - A^+_i(x_i) \right]^2} \right)^{1/2}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n.
\]

**Step 3:** According to the definition 5, in addition, according to the defined objective function and expert experience, the two is combined to obtain the degree of membership, then the comprehensive weight vector \( \omega_{ji} = (\omega_{j1}, \omega_{j2}, \ldots, \omega_{jn}) \) of \( A_j \) on the \( j \)-th rule is calculated.

In here, the objective function is defined as follows:

**Definition 9:** The ordered weighted vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) can be determined by the following formula:

\[
\omega_j = \frac{C_{j-1}^{n-1}}{\sum_{k=0}^{n-1} C_{n-1}^{k}}, \quad j = 1, 2, \ldots, n. \quad (10)
\]

Obviously, \( \sum_{j=1}^{n} \omega_j = 1 \). Due to \( \sum_{k=0}^{n-1} C_{n-1}^{k} = 2^{n-1} \), then \( \omega_j = \frac{C_{j-1}^{n-1}}{2^{n-1}}, j = 1, 2, \ldots, n \).

Based on the above discussion, the degree of membership formula (9) of interval-valued fuzzy set and the weighting operator method are given. According to the above definitions, the comprehensive degree of membership can be calculated between the two matched interval value sets according to the following step 4.

**Step 4:** According to the definition 6, the comprehensive degree of membership on \( j \)-th rule is calculated to be

\[
S_j (A^*, A_j) = F (\omega_j) = \sum_{i=1}^{n} \omega_{ji} \cdot \beta_{ji}, \quad \beta_{ji} \text{ is the i-th large data of } \omega_j.
\]

Then according to degree of membership of each rule given by the experts, we calculate the amendatory comprehensive degree of membership \( S'_j (A^*, A_j) = S_j (A^*, A_j) \cdot f_j \) which is correlated with a certain degree.

**Step 5:** If \( S'_j (A^*, A_j) \geq \lambda_j \), then the rule is aroused. Calculate the interval value degree of membership of inference result of the consequent by the formula

\[
D_j = \min \left\{ \left[ 1, 1 \right], \frac{D_j}{S'_j (A^*, A_j)} \right\}.
\]

Where, \( D_j \) denotes the interval value which the degree of membership belongs to. If more than one rule is aroused, the final inference result is calculated by the formula

\[
D' = \bigcup_{j=1}^{n} D_j. \quad \text{If } S'_j (A^*, A_j) < \lambda_j, \text{ then the rule is not aroused.}
\]

**Example 2:** Assume an interval-valued fuzzy inference of a FDFA lattice contains the following fuzzy production rules:

\[ R_1: \text{if } X \text{ is } A_1, \text{ then } Y = (B_1, f_1 = 0.94, \gamma_1 = 0.56); \]
\[ R_2: \text{if } X \text{ is } A_2, \text{ then } Y = (B_2, f_2 = 0.86, \gamma_2 = 0.55); \]
\[ R_3: \text{if } X \text{ is } A_3, \text{ then } Y = (B_3, f_3 = 0.96, \gamma_3 = 0.72); \]
\[ R_4: \text{if } X \text{ is } A_4, \text{ then } Y = (B_4, f_4 = 0.95, \gamma_4 = 0.62); \]

Where \( A_j \in G_1, B_j \in G_2, G_1 \) and \( G_2 \) are the lattices of FDFA,, respectively. \( f_j \in [0, 1] \) denotes the degree of membership of the \( j \)-th rule, \( \gamma_j \in [0, 1] \) is the threshold.
assigned to the j-th rule, \( j = 1, 2, \ldots, n \). Assume that the interval-valued degree of membership of each rule is:

\[
A_1 = \{(u_1, [0.45, 0.63]), (u_2, [0.68, 0.89]), (u_3, [0.25, 0.48]), (u_4, [0.35, 0.50])\}
\]

\[
A_2 = \{(u_1, [0.48, 0.67]), (u_2, [0.69, 0.82]), (u_3, [0.36, 0.58]), (u_4, [0.30, 0.48])\}
\]

\[
A_3 = \{(u_1, [0.76, 0.84]), (u_2, [0.52, 0.69]), (u_3, [0.61, 0.75]), (u_4, [0.66, 0.78])\}
\]

\[
A_4 = \{(u_1, [0.42, 0.70]), (u_2, [0.72, 0.96]), (u_3, [0.22, 0.48]), (u_4, [0.35, 0.54])\}
\]

\[
B_1 = \{(v_1, [0.32, 0.56]), (v_2, [0.15, 0.27]), (v_3, [0.24, 0.36])\}
\]

\[
B_2 = \{(v_1, [0.44, 0.67]), (v_2, [0.75, 0.86]), (v_3, [0.36, 0.49])\}
\]

\[
B_3 = \{(v_1, [0.63, 0.85]), (v_2, [0.33, 0.56]), (v_3, [0.42, 0.61])\}
\]

\[
B_4 = \{(v_1, [0.31, 0.53]), (v_2, [0.42, 0.61]), (v_3, [0.26, 0.38])\}
\]

where \( U = \{u_1, u_2, u_3, u_4\} \) and \( V = \{v_1, v_2, v_3\} \) are the different states of lattices \( G_1 \) and \( G_2 \), respectively.

The given facts are:

\[
A^* = \{(u_1, [0.46, 0.78]), (u_2, [0.55, 0.66]), (u_3, [0.86, 0.94]), (u_4, [0.60, 0.78])\}. 
\]

The reasoning will be performed according to the reasoning steps mentioned above, and the reasoning results of system will be obtained.

According to the formula (1), the degree of membership vector is calculated to be \( a_j = (a_{j1}, a_{j2}, a_{j3}, a_{j4}) \), \( 1 \leq j \leq 4 \) between the antecedent \( A_j \) on the j-th rule and the matching facts \( A^* \), where \( a_{ji} = F(A^*(u_i), A_j(u_i)) \). For simplicity, we take \( \lambda_j = \frac{1}{3}, \mu_i = \frac{1}{3}, 1 \leq i \leq 4 \), and get

\[
a_1 = (0.91, 0.71, 0.44, 0.74), \\
a_2 = (0.93, 0.85, 0.21, 0.70), \\
a_3 = (0.75, 0.97, 0.77, 0.95), \\
a_4 = (0.94, 0.78, 0.41, 0.75). 
\]

Then, based on the method of the combination of the defined functions and expert experience to obtain a new degree of membership, for simplicity, the experts’ attitudes to the elements of the domain are the same, then the comprehensive weight vector of degree of membership vector \( a_j \) on the j-th rule is calculated to be \( w_j = (0.125, 0.375, 0.375, 0.125) \), \( j = 1, 2, 3, 4 \). Then, the comprehensive degree of membership is calculated between the antecedent \( A_j \) on the j-th rule and the given facts \( A^* \) to be \( S_1 = 0.71, S_2 = 0.72, S_3 = 0.86, S_4 = 0.74. \) Then according to degree of membership of each rule given by the experts, the amendatory comprehensive degree of membership which is correlated with a certain degree is calculated to be \( S_1' = 0.67, S_2' = 0.62, S_3' = 0.83, S_4' = 0.61. \)

Because \( S_1' = 0.67 > \gamma_1 = 0.56 \), the rule \( R_1 \) is aroused, so the reasoning result of the rule \( R_1 \) is:

\[
D'_1 = \min \left\{ \left[ (1, 1), \frac{D_1}{0.67} \right] \right\} = \{(v_1, [0.48, 0.84]), (v_2, [0.22, 0.40]), (v_3, [0.52, 0.81])\}
\]

Because \( S_2' = 0.62 > \gamma_2 = 0.55 \), the rule \( R_2 \) is aroused, so the reasoning result of the rule \( R_2 \) is:

\[
D'_2 = \min \left\{ \left[ (1, 1), \frac{D_2}{0.62} \right] \right\} = \{(v_1, [0.71, 1]), (v_2, [1, 1]), (v_3, [0.58, 0.79])\}
\]

Because \( S_3' = 0.83 > \gamma_3 = 0.72 \), the rule \( R_3 \) is aroused, so the reasoning result of the rule \( R_3 \) is:

\[
D'_3 = \min \left\{ \left[ (1, 1), \frac{D_3}{0.83} \right] \right\} = \{(v_1, [0.76, 1]), (v_2, [0.40, 0.67]), (v_3, [0.51, 0.73])\}
\]

Because \( S_4' = 0.61 < \gamma_4 = 0.62 \), the rule \( R_4 \) is not aroused.

In summary, the final result of interval-valued fuzzy inference of the FDFA lattice is:

\[
D' = \bigcup_{j=1}^{3} D'_j = \{(v_1, [0.76, 1]), (v_2, [1, 1]), (v_3, [0.58, 0.81])\}.
\]

**E. DECISION-MAKING FOR SOC**

In this detection model system, the four layer is the decision-making unit for SOC, which performs a decision-making action or judgment result.

In order to verify the feasibility and effectiveness of the detection model system based on FDFA lattice shown in Figure 2 and Fuzzy 3 constructed above, its practical application in power detection of battery is given as follows:

Figure 3 is a model system of FDFA lattice detection. In Figure 3, the input is connected to the inside of the homomorphic mapping layer in Figure 2, and the output is the feedback terminal of the battery, which is directly connected to the fuzzy inference layer in Figure 2. The workflow of this part includes four steps of execution units. The first step is the acquisition unit based on FDFA lattice, which mainly collects information on power consumption through the homomorphic mapping. The second step is the calculation unit to calculate the error of voltage and value of SOC. The third step is an inference unit that verifies whether the battery is currently healthy by calculating the feedback current, which is connected to the fuzzy inference layer in Figure 2. The fourth step is a decision unit that uses the parameters-triggering feature of battery to make a decision on the state of charge, such as continuing work, working while starting to charge or add electrolyte to the battery, or stopping the operation of battery with an alarm, dialing the worker phone, etc.
The process diagram is shown in Figure 4. The Figure mainly expresses the flow of information in the system, wherein the direction of the arrow represents the direction of transmission of the information. The actual experimental results show that the FDFA lattice detection model detector has a small amount of computation and good real-time performance.

The decision-making is implemented by the energy saving detection model based on homomorphic mapping of FDFA lattice as follows: When \( e(k) > \delta \) in the formula (6), that is, when the SOC of the battery is an interval value, such as the interval value in Example 2 above, then the detection model starts to charge the battery or add electrolyte to the battery and keep the battery working, or stop the operation of battery and make an alarm reminder. Where, \( \delta \) is a given threshold; \( \gamma_1, \gamma_2 \) the lower limit and the upper limit of the fuzzy inference interval in Subsection D of Section III, respectively, i.e., the lower limit and the upper limit of the threshold interval.

The relationship between the SOC and the capacity degrade is given as follows: the less the value of SOC is, the more the capacity degrade is. At this time, it is necessary to detect whether the energy of the battery reaches the minimum lower limit according to the FDFA energy saving model shown in Figure 2. Further, when the SOC is less than a given lowest threshold, the operation of battery stops and FDFA detection model makes an alarm.

### IV. APPLICATIONS OF DETECTION MODEL BY FDFA

The consumption process of battery power is from one energy state to another energy state at all times, and the set that consists of any two energy states of the battery has the minimum upper bound and the maximum lower bound in any a time period. So the battery’s energy consumption is a lattice of FDFA. Thus, the consumption and supplement of battery power can be monitored and controlled by the above detection model based on lattice of FDFA.

The literature [23] first discussed online internal resistance detection method. Secondly, established the relationship between the measured internal resistance and the LiBs capacity by linear fitting. Finally, applied the capacity through internal resistance conversion in SOC estimation. This method can effectively enhance the SOC estimation accuracy regardless of temperature change and battery degradation by the estimation. The literature [24] studied the commercial 16 Ah C/NMC Li-ion cells during cycling at 5 degrees C at a rate of 1C between 2.7 V and 4.2 V (namely between 0 and 100% of state of charge (SOC), with significant performance fading after 50 cycles only, performed it up to 4000 cycles at 45 degrees C with the same commercial cells, and performed the monitoring of the potential of each electrode during cycling through the successful introduction of lithium metal as reference electrode into the commercial cell. In the literature [25], to overcome an accurate and reliable detection for SOC (state-of-charge), the literature proposed a new on-line SOC detection method with a radial basis function neural network, used the physical values related to the degradation degree as input signal in the neural network, investigated the detection accuracies for different sized batteries and various degradation states. It is because the characteristics of the battery greatly change due to its degradation. Moreover, an automobile has many driving patterns, which are unknown beforehand. Thus it is not easy to detect the SOC analytically.

At present, there are some disadvantage in the existing literatures [12]–[17], [25]–[33] on the detection methods for battery power, because these methods only detected the state of charge, but couldn’t achieve automatic charging or add electrolyte to the battery automatically.

However, this paper can solve this problem by FDFA energy saving model proposed shown in Figure 2. The detection method proposed in this paper is based on the lattice and its properties such as homomorphism and isomorphism on the FDFA, which can perform the reasoning, calculation and decision-making to achieve energy saving by charging the battery or adding electrolyte to the battery automatically.

In the experiment, the detection process of the hardware connection that is corresponding to the above model proposed is shown in Figure 5 to Figure 8.

After the battery is used, the loss of battery is normal. There is a direct relationship between the loss amount of battery and the frequency of the use of the battery. The loss of battery refers to that its actual capacity becomes smaller after the battery is used in a long period, which is lower than the
nominal capacity. Since the lithium-ion battery overcharges or over discharges in the charge and discharge process, electrolyte decomposition, formation of SEI coating, the dissolution of active substances and other factors will lead to the loss of battery capacity. The consumption process of battery power can be detected by using the above detection model, as shown in Figure 6. In Figure 6, the red in tanks of SOC denotes the residual SOC (state of charge), and the white denotes the empty.

This paper gives the detection method for resources by using the above detection model based on lattice of FDFA, which is through the state transition process of the detection model simulator and its working principle, then the throughout process of electricity movement and consumption of battery power can be tracked, as shown in Figure 7.

By experiment, the results show that this detection model can support multi-cluster, achieve multi-battery types, carry out tracking under different operating environment and detect calculation. This detection model can make the working efficiency of FA and the reliability of FA system improve, and extend the cycle of a variety of application software that the middleware of FA supports. These features can not only delay the aging of the design of FA, but also control better the stable operating environment of FA system.

In here, a 6-channel detection model constructed is utilized to detect the quantity of electricity. Through the state transition process and the working principle of the 6-channel detection model, the consumption of battery power can be put on display, as shown in Figure 8.

In the use of the detection model given by this paper to detect the consumption of battery power, it is found that the consumption rate of battery power is 50% and 20% at temperatures 60°C and 30°C, respectively, when the running time of battery is about 1000 times, as shown in Figure 9. The utilization rate of consumption of battery power is more than 90% by detection. The effective detection rate of this detection method by detection model constructed is improved averagely to be about 9.45% than that of other detection methods for consumption of quantity of electricity.

In this paper, the detection method by the detection model constructed is applied in consumption tracking of battery
This paper carries out the experiments for comparison of several detection methods based on 120 times of consumption of battery power at 20 different temperatures. Each category of data includes a total of 6 times of consumption of battery power. The simulating detection time of each SOC is about 30s. 60 sets of SOC data were used for training in experiment, and the remaining 60 sets of SOC data are taken as the test samples. Divide consumption into the units in experiment. The simulating time of each unit is about 20ms. The correct detection rate can be obtained by letting the number of units of detecting correctly be divided by the total number of units, i.e.,

\[ R = \frac{l}{N} \]

where \( R \) indicates the correct detection rate; \( l \) denotes the number of SOC units by detecting correctly; \( N \) indicates the total number of SOC units.

The above experimental process was repeatedly carried out 30 times, and then their average is taken. Compare the accuracy of detection of the detection model constructed and existing detection algorithms \([12]–[17]\) in the detection of SOC. The experimental results show that the correct average detection rate of the detection model constructed is 92.79%, that of Everest method is 59.93%, that of Lu great master is 18.48%, and that of hardware detection is 5.69%, as shown in Figure 10. Moreover, the detection speed of the detection model proposed is 1.98s. However, that of Everest method is 4.76s, that of Lu great master is 3.29s, that of hardware detection is 5.14s.

The practical meaning for the experiment is discussed as follows:

Through the detection method proposed in this paper for detection of SOC, the more reasonable and effective consumption of battery power can quickly be detected based on the experimental results directly. The proposed detection method not only can quickly, accurately and effectively detect the consumption of battery power, but also has strong anti-interference ability. At the same time, each different stage on consumption can be distinguished, so that the useful environment for battery will be rationally, orderly, efficiently and step-by-step implemented on follow-up work, which it provides strong theoretical methods and experimental support for the battery protection and other decisions.

According to the above evaluation criterion, as can be seen from Figure 10, in the detection for consumption of battery power, the test results show that the detection model constructed by the lattice of FDFA for detecting consumption of battery power not only has a faster detection speed, lower occupancy rate of resources, but also a better display effect of power. Its accuracy of detection is the highest, which is higher than the corresponding value of other methods, and its running time is minimal. The standard deviation is less than the corresponding value of other methods. These test results indicate that the predicted effects are relatively stable. The comparison of the detection accuracy between the detection model constructed by the lattice of FDFA and other detection methods \([14]–[33]\) in the consumption tracking of battery power for resources is shown in Figure 11.
The established detection model reduces the processing time to detect consumption process of battery power, watch directly efficiency, and ensures the best use of valid battery power.

(2) The throughout process of electricity movement by the detection model is given, thus the detected valid battery power is more targeted, fast, accurate and effective.

(3) Define the lattice and its properties of FDFA, and give the detection method by lattice of FDFA for battery power. According to the detection method, it takes into account the characteristics of electricity movement, so it is not only suitable for the detection of storage cell, but also suitable for high voltage device. Therefore, this detection method is fast, accurate, strong in anti-interference ability, and more detailed in detection for storage cell. It has particularity and is also universal so that it can provide a strong guarantee to implement more orderly and efficient decision for the reasonable and effective use of battery.

However, the disadvantages of the proposed detection method are that a large number of sample experiments, debugging feature properties of lattice of FDFA and effective establishment of detection model are required.

V. CONCLUSION

In this paper, an algebraic system for the finite state deterministic fuzzy automata is proposed, that is, the definition of the partial order set, the lattice and its corresponding properties, some corresponding theorems and detection model constructed are also given. At the same time, the application of detection model in detection of consumption of battery power has been discussed. Through the homomorphic mapping and the isomorphic mapping of lattice of FDFA, the function that the overall state of FDFA can be achieved by its partial state transition has been studied. For the use of lattice of FDFA in detection of consumption of battery power, a detection model is built, which monitors the consumption of battery power. Simultaneously, the whole dynamic process, the quantity of consumption and the rate of consumption of battery power are given. The comparison by simulation between the accuracy of the detection method by detection model constructed and that of other detection methods for consumption of battery power is also discussed.

Now, there are some questions, that is, how to use a neural network for training lattice of fuzzy automata? How many orders network is necessary to derive lattice of fuzzy automata? How to build the model of neural network and design its algorithms and stability for simulating a variety of lattice systems of fuzzy automata? In order to extract a more general lattice of fuzzy automata, through the training of neural networks, whether is there a uniform standard or not? To solve these problems, many scholars will need to study together in the future work.

REFERENCES

[1] Y. Li and Q. Wang, “The universal fuzzy automaton,” Fuzzy Sets Syst., vol. 249, pp. 27–48, Aug. 2014.
[2] H. Pan, Y. Li, Y. Cao, and P. Li, “Nondeterministic fuzzy automata with membership values in complete residuated lattices,” Int. J. Approx. Reasoning, vol. 82, pp. 22–38, Mar. 2017.
[3] J. Jin, Q. Li, and Y. Li, “Algebraic properties of L-fuzzy finite automata,” Inf. Sci., vol. 234, pp. 182–202, Jun. 2013.
[4] L. Wu, D. Qiu, and H. Xing, “Automata theory based on complete residuated lattice-valued logic: Turing machines,” Fuzzy Sets Syst., vol. 208, no. 5, pp. 43–66, Dec. 2012.
[5] H. Pan, Y. Li, and Y. Cao, “Lattice-valued simulations for quantitative transition systems,” Int. J. Approx. Reasoning, vol. 56, no. 4, pp. 28–42, Jan. 2015.
[6] H. Pan, Y. Cao, M. Zhang, and Y. Chen, “Simulation for lattice-valued doubly labeled transition systems,” Int. J. Approx. Reasoning, vol. 55, no. 3, pp. 797–811, Mar. 2014.
[7] E. T. Lee and L. A. Zadeh, “Note on fuzzy languages,” Inf. Sci., vol. 1, no. 4, pp. 421–434, Oct. 1969.
[8] L. Zadeh, “Fuzzy languages and their relation to human and machine intelligence,” Electron. Res. Lab., Univ. California, Berkeley, Berkeley, CA, USA, Tech. Rep. ERL-M, Nov. 1971, pp. 302–313, vol. 1, no. 1.
[9] P. Liu, L. Xiao, Y. Tang, Y. Chen, L. Ye, and Y. Zhu, “Study on the reduction roasting of spent LiNiCoMnO2 lithium-ion battery cathode materials,” J. Thermal Anal. Calorimetry, vol. 136, no. 3, pp. 1323–1332, May 2019.
[10] Q. E. Wu, T. Wang, X. M. Pang, Y. X. Huang, and J. S. Li, “Discussion of relation of fuzzy automata,” J. Inf. Decis. Sci., vol. 2, no. 4, pp. 349–357, Jul. 2007.
[11] Q. E. Wu, M. K. Guang, H. Chen, and L. J. Sun, “Semigroup of fuzzy automata and its application for fast accurate fault diagnosis on machine and anti-fatigue control,” Appl. Intell., vol. 50, no. 1, pp. 278–293, Jan. 2020, doi: 10.1007/s10489-019-01611-4.
[12] Q. Luo, X. Fang, Y. Sun, L. Liu, J. Ai, C. Yang, and O. Simpson, “Surface defect classification for hot-rolled steel strips by selectively dominant local binary patterns,” IEEE Access, vol. 7, pp. 23488–23499, Feb. 2019.
[13] D. He, K. Xu, and P. Zhou, “Defect detection of hot rolled steels with a new object detection framework called classification priority network,” Comput. Ind. Eng., vol. 128, pp. 290–297, Feb. 2019.
[14] P. Liu, L. Xiao, Y. Tang, Y. Chen, L. Ye, and Y. Zhu, “Study on the reduction roasting of spent LiNiCoMnO2 lithium-ion battery cathode materials,” J. Thermal Anal. Calorimetry, vol. 136, no. 3, pp. 1323–1332, May 2019.
[15] R. Xu and D. Wunsch, “Survey of clustering algorithms,” IEEE Trans. Neural Netw., vol. 16, no. 3, pp. 645–678, Sep. 2005.
[16] P. Aldag, “A simplified strategy for managing power and services for edge facilities,” Battery Power Digit. Mag., vol. 8, no. 1, pp. 7–40, Dec. 2017.
[17] S. Wu and L. Shu, “Maximum principle for partially-observed optimal control problems of stochastic delay systems,” J. Syst. Sci. Complex., vol. 30, no. 2, pp. 316–328, Apr. 2017.
[18] M. Ghorani and M. M. Zahedi, “Alternating regular tree grammars in the framework of lattice-valued logic,” Iranian J. Fuzzy Syst., vol. 13, no. 2, pp. 71–94, Dec. 2016.
[19] Y. Li and W. Pedrycz, “Fuzzy finite automata and fuzzy regular expressions with membership values in lattice-ordered monoids,” Fuzzy Sets Syst., vol. 156, no. 1, pp. 68–92, Nov. 2005.

[20] B. K. Sharma, S. P. Tiwari, and S. Sharan, “On algebraic study of fuzzy multisets finite automata,” Fuzzy Inf. Eng., vol. 8, no. 3, pp. 315–327, Sep. 2016.

[21] J. M. Ko and Y. C. Kim, “Algebraic and topological structures on factorizations of fuzzy sets,” J. Intell. Fuzzy Syst., vol. 30, no. 3, pp. 1709–1718, Jun. 2016.

[22] S. P. Tiwari, V. K. Yadav, and A. K. Singh, “On algebraic study of fuzzy automata,” Int. J. Mach. Learn. Cybern., vol. 6, no. 3, pp. 479–485, Jun. 2015.

[23] Y. Bao, W. Dong, and D. Wang, “Online internal resistance measurement application in lithium ion battery capacity and state of charge estimation,” Energies, vol. 11, no. 5, pp. 1–11, Dec. 2018.

[24] B. P. Matadi, S. Geniès, A. Delaille, C. Chabrol, E. de Vito, M. Bardet, J.-F. Martin, L. Daniel, and Y. Bultel, “Irreversible capacity loss of li-ion batteries cycled at low temperature due to an untypical layer hindering li diffusion into graphite electrode,” J. Electrochem. Soc., vol. 164, no. 12, pp. A2374–A2389, 2017.

[25] Y. Morita, S. Yamamoto, S. H. Lee, and N. Mizuno, “On-line detection of state-of-charge in lead acid battery using radial basis function neural network.” Asian J. Control, vol. 8, no. 3, pp. 268–273, Apr. 2006.

[26] Y. Li and Y. Zhang, “Robust infrared small target detection using local steering kernel reconstruction,” Pattern Recognit., vol. 77, pp. 113–125, May 2018.

[27] O. D. R. Filho and G. L. de Oliveira Serra, “Recursive fuzzy instrumental variable based evolving neuro-fuzzy identification for non-stationary dynamic system in a noisy environment,” Fuzzy Sets Syst., vol. 338, no. 1, pp. 50–89, May 2018.

[28] K. Ahmad and E. Salari, “Small dim object tracking using frequency and spatial domain information,” Pattern Recognit., vol. 58, no. 10, pp. 227–234, Oct. 2016.

[29] X. K. Huang, Q. G. Li, and Q. M. Xiao, “The L-ordered semigroups based on L-partial orders,” Fuzzy Sets Syst., vol. 339, no. 2, pp. 31–50, Nov. 2018.

[30] D. E. George and A. Unnikrishnan, “Tracking of manoeuvring targets using fuzzy information fusion filter,” Int. J. Image Data Fusion, vol. 9, no. 2, pp. 115–130, Apr. 2018.

[31] B. Mihailović, V. M. Jerković, and B. Malešević, “Solving fuzzy linear systems using a block representation of generalized inverses: The group inverse,” Fuzzy Sets Syst., vol. 353, pp. 66–85, Dec. 2018.

[32] Q. Huangpeng, H. Zhang, X. Zeng, and W. Huang, “Automatic visual defect detection using texture prior and low-rank representation,” IEEE Access, vol. 6, pp. 37965–37976, Aug. 2018.

[33] X. Zhang, M. Kano, M. Tani, J. Mori, J. Ise, and K. Harada, “Prediction and causal analysis of defects in steel products: Handling nonnegative and highly overdispersed count data,” Control Eng. Pract., vol. 95, Feb. 2020, Art. no. 104258.