One PI and Wilsonian Actions in SUSY theories

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Abstract

The soft breaking terms in supersymmetric theories are calculated at some high scale characterizing the hidden supersymmetry breaking sector, and then evolved down to the TeV scale. These parameters are usually presented as the ones that should be compared to experiment. The physical parameters however are those occurring in the quantum effective (1PI) action - in particular the physical mass is the location of the pole in the full quantum propagator. Here we discuss the relation between the two and the possible existence of additive contributions to the gaugino mass. We argue that infra red effects which violate non-renormalization theorems are absent (for the 1PI action) if the calculation is done at a generic point in field space so that an effective IR cutoff is present. It follows that if a gaugino mass term is absent in the Wilsonian action it is absent in the 1PI action.

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# Introduction

In supersymmetric theories the source of supersymmetry breaking is identified with some hidden sector which is (typically) gravitationally coupled to the visible sector (i.e. some supersymmetric extension of the standard model). In such models the scale of the hidden sector is often close to the Planck scale (gauge mediation being an exception). In models derived from string theory this scale is identified with the cutoff scale at which string (or higher dimensional) physics becomes relevant. The hidden sector theory then determines a set of initial values for the soft breaking terms of the visible sector.

However these values are clearly not the relevant ones for comparison with experiment if supersymmetry is responsible for stabilizing the hierarchy between the weak scale and the Planck (or string or Kaluza-Klein (KK)) scale. The values of the soft parameters (typically determined in terms of the gravitino mass $m_{3/2}$), are used as initial values to solve the supersymmetric RG equations. This gives the values of these parameters at the TeV scale. These are then to be compared with the expected experimental values. Indeed it is only after RG evolution that the Higgs potential develops an instability resulting in the Higgs mechanism, since the initial values of the Higgs mass squared parameters are positive.

Nevertheless one may ask how accurate is this identification of the physical soft masses with what are essentially the running masses evaluated at the RG scale $\mu \simeq 1\text{TeV}$. The physical mass of say the stop is the position of the pole in the full quantum propagator of the stop. This must take into account not only the effects of high mass scales but also the effect of functionally integrating over all low energy scales as well. Nevertheless if indeed there is a physical stop at the TeV scale, then the difference between this pole mass and the RG evolved running mass, is expected to be small.

In this paper we will review this in some detail, and then discuss whether these expectations on the relation between the physics of the 1PI action and that coming from the Wilsonian action may be violated. One issue is whether the non-renormalization theorem of SUSY is violated in the 1PI action when there are massless particles in the theory. The other is the question of Weyl anomalies and naturalness arguments for fermion mass renormalization, and whether there could possibly be an additive contribution to the physical gaugino mass that is not present in the Wilsonian effective action. We show that this is not so in either of these cases.

## 1.1 Soft mass calculations in SUSY/SUGRA/String theories

The SUSY breaking which occurs in the hidden sector of supersymmetric effective field theories is transmitted to the visible sector, usually taken to be the minimally supersymmetric standard model (MSSM), by the (gravitational) couplings of the moduli to it. The formulae for the effective supersymmetry breaking soft term corrections to the MSSM were worked out in papers going back to the early eighties culminating in the papers \[1, 2\].

The “classical” formulae given in these works for the soft terms are expected to be valid at some high scale just below where the effective locally supersymmetric field theory gets replaced by a UV complete theory such as string theory. The structure of the effective theory is completely determined once the Kähler potential ($K$ a real scalar superfield), the superpotential ($W$ a chiral scalar superfield), and the gauge coupling function ($f$ a chiral superfield), are given in terms of the
fundamental (super) fields of the theory. In principle of course if the exact expressions for these, incorporating the effect of integrating out the quantum fluctuations at scales above the TeV scale and up to the effective UV cutoff are known, we could use the formulae of [1, 2] to compute the soft masses.

In practice of course what we do know (in various string theory based models for instance), are classical expressions corrected usually by some stringy ($\alpha'$) effects and string loop effects in $K$, as well as non-perturbative effects in the superpotential. In addition there are Weyl anomaly (a one-loop effect) contributions to $f$. Getting to this structure involved integrating out both string states (with mass scale $M_s = \sqrt{1/2\pi\alpha'}$) and Kaluza-Klein states ($M_{KK} \sim M_s/V^{1/6} < M_s$ with $V$ being the volume of the compactified space in string units). Thus we expect these values for $K, W, f$ and hence the corresponding expressions for the soft SUSY breaking terms obtained from [1, 2] to be valid at some scale just below this KK scale.

In terms of the Planck scale $M_P$, the KK scale is given by $M_{KK} \sim g_s^{1/2} M_P/V^{2/3}$, and depends on the string coupling $g_s$ as well as the volume of compact space, both of which are fixed in terms of internal fluxes, and choice of the gauge group(s) giving rise to non-perturbative effects. It is natural however to take this scale to be around the GUT scale (though it is extremely hard to actually produce an explicit working GUT model!). In any case even outside of string theory, in SUGRA based theories of SUSY breaking, the “classical” values of the soft masses coming from the original data of the effective SUGRA, are taken to be valid at some high scale close to the GUT scale. The effective Wilsonian action at the TeV scale is then obtained by using the “classical” data as initial values for solving the RG equations which take care of the quantum effects in running down to the latter scale. These are the soft masses that are compared to experiment.

However this procedure does not quite give the physical mass of a particle. This is defined as the position of the pole in the exact quantum propagator for the corresponding particle/field ($\phi$ say with running mass $M(\mu)$). i.e. it is the solution $p^2 = -M_{\text{phys}}^2$ of

$$
\Delta^{-1}(p) = p^2 + M^2(\mu) + M^2(\mu)\tilde{\Sigma}(\mu, \frac{M_i^2(\mu)}{\mu^2}) = 0.
$$

(1)

where $\Sigma \equiv M^2\tilde{\Sigma}$ is the sum of 1PI diagrams with two $\phi$ external lines and $g_i = g_i(\mu)$, $M_i = M_i(\mu)$ stand for the set of running couplings (dimensionless) and masses evaluated at the scale $\mu$. In writing this we have taken the input action to be the Wilsonian action $S_W(\phi_i)$ and the 1PI action is computed starting from this Wilsonian action.

In standard renormalization theory, the value of $M_{\text{phys}}$ is an experimental input, however here we are hoping to make predictions for a set of yet unobserved particles. What the Wilsonian procedure discussed earlier gives are values for the running masses and couplings evaluated at the TeV scale, which by assumption are supposed to be close to the values of the corresponding physical masses. If that is indeed the case, the former should be a good approximation to the latter and this is the implicit assumption of typical soft mass calculations in broken supersymmetry.

So in contrast to the procedure in standard renormalization theory, what is done is to compute the physical masses starting from the input masses and couplings given in $S_W$ valid at the scale

\footnote{Note that $V \gg 1$ for the effective 4D field theory derived from string theory via the low energy 10D SUSGRA to be valid.}

\footnote{See for example [3].}
\(\mu\). Also the parameters of the Wilsonian action which determine \(M^2_{\text{phy}}\) are fixed in terms of the initial values \(g_i^{(0)}, M_i^{(0)}\) which in turn are determined in terms of the gravitino mass and the data of the compactification, \(\{\sigma_r\}\) (Hodge numbers, flux integers) see equation (1). Hence the physical mass is given by

\[M_{\text{phy}} = m_{3/2}(\sigma_r)h(\sigma_r)\]

(with \(h\) a model dependent function of the string/SUGRA data) and is of course an RG invariant. In practice however what is usually calculated is \(M(\mu)\). The difference may be read off from (1);

\[\Delta M^2 \equiv M^2_{\text{phy}} - M^2(\mu) = M^2(\mu)\tilde{\Sigma}(\frac{M^2_{\text{phy}}}{\mu^2}, g_i(\mu), \frac{M^2_i(\mu)}{\mu^2}),\]

and is finite and is at least of one-loop \((g^2/4\pi)\) order. If indeed there are physical superpartner particles at the TeV scale then choosing the RG scale \(\mu\) at that scale will minimize the logarithms that enter into the calculation.

Now the RHS of (2) does not necessarily imply multiplicative renormalization - in fact a term of the form \(\tilde{\Sigma} \sim \frac{g^2}{16\pi^2}m^2_{3/2}\) would give an additive contribution to a scalar mass. In the case of interest namely when SUSY is softly broken we have an additive contribution which up to \(O(1)\) factors is (assuming there is no D-term SUSY breaking for simplicity),

\[\Delta M^2 \sim \frac{g^2}{16\pi^2}\text{Tr}M^2 = \frac{g^2}{16\pi^2}[2(n-1)m^2_{3/2} - 2F^i\bar{F}^jR_{i\bar{j}}].\]

Here \(R_{i\bar{j}} = K^{IJ}R_{i\bar{j}I\bar{J}}\) where the second factor is the Riemann tensor of the manifold of chiral scalars with \(i\bar{j}\) being tangent to the hidden sector directions (i.e. the SUSY breaking sector) and \(I\bar{J} = 1,\ldots,n\) being the observable sector directions. The value of the RHS of this equation and hence the accuracy of the Wilsonian mass is then a model dependent question.

1. In generic SUGRA hidden sector theories such as mSUGRA we have \(\Delta M^2 \sim \frac{g^2}{16\pi^2}m^2_{3/2}\). Since the classical soft mass at the UV scale is \(O(m_{3/2})\) this means that the correction in going from the Wilsonian to the physical mass is formally of the same order as the RG evolution in the Wilsonian mass.

2. In extended no-scale models (such as LVS models [4], [5]), there is a cancellation between the two terms on the RHS of (3) so that \(\Delta M^2 \sim \frac{g^2}{16\pi^2}m^2_{3/2}\). Nevertheless the point is that this suppression comes from the suppression of soft masses in these models so that \(M^2_{\text{cl}} \sim m^2_{3/2}/\mathcal{V}\), so the contribution due to running (i.e. \(M^2(\mu) - M^2_{\text{cl}}\)) is also proportional to \(\frac{g^2}{16\pi^2}m^2_{3/2}/\mathcal{V}\).

3. The third case is the sequestered one in which the gauginos get a mass in the UV because of the Weyl anomaly and is also therefore a one-loop effect. But the largest contribution to the scalar masses (as well as the scalar coupling \(A\) term) come from RG running. In this case \(M^2(\mu) \sim O\left(\frac{g^2}{16\pi^2}m^2_{3/2}\right), \Delta M^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2m^2_{3/2}\).

Actually though \(M^2(\mu) - M^2_{\text{cl}}\) is formally of the same order as \(\Delta M^2\) in so far as the leading terms are both one-loop effects, the former is the result of RG running over many decades (i.e.
integrating over the leading log contributions), and hence the numerical coefficients (what replaces the large logs in a naive perturbative contribution) dominate over a one-loop contribution with a small factor coming from the low energy loop integral in \( \Sigma \) i.e. \( \sim \ln(M(\mu)/\mu) \). Thus in all three cases the effect of the RG evolution will be larger than the loop corrections that go into \( \Delta M^2 \) in (2).

For fermionic masses of course there are naturalness arguments which imply that \( M_{\text{phys}} \) will be proportional to \( M \). These would apply if there are no terms (apart from the mass term) that break chirality. However there are several effects which might affect these arguments in SUSY theories with massless states. One is the issue of the breakdown of non-renormalization theories in SUSY theories with massless particles. The other is that of Weyl anomalies. We discuss these in the next two sections.

\section{2 IR effects and Chiral loops}

It has been claimed that the quantum effective action of global supersymmetric theories with massless particles violates the non-renormalization theorem for the superpotential \[6, 7\]. Let us revisit this issue since it is of relevance for the question under discussion.

The SUSY action for the Wess-Zumino (WZ) model is,

\[ S = \int d^8 z K(\Phi, \bar{\Phi}, V) + \left( \int d^6 z W(\Phi) + h.c. \right). \]

The WZ propagators \[8\] for chiral scalars are:

\[ G_{--}(z, z') = \langle \bar{\Phi}(z), \bar{\Phi}(z') \rangle = \frac{1}{16} D^2 D'^2 G_V(z, z'), \quad (4) \]
\[ G_{+-}(z, z') = \langle \Phi(z), \bar{\Phi}(z') \rangle = \frac{1}{16} \bar{D}^2 D'^2 G_V(z, z'), \quad (5) \]
\[ G_{-+}(z, z') = \langle \bar{\Phi}(z), \Phi(z') \rangle = \frac{1}{16} D^2 \bar{D}'^2 G_V(z, z'), \quad (6) \]
\[ G_{++}(z, z') = \langle \Phi(z), \Phi(z') \rangle = \frac{1}{16} \bar{D}^2 \bar{D}'^2 G_V(z, z'). \quad (7) \]

\( G_V \) is the solution (with Feynman boundary conditions) of

\[ (\Box - \frac{1}{4} \Psi(z) \bar{D}^2 - \frac{1}{4} \bar{\Psi}(z) D^2) G_V(z, z') = -\delta^8(z - z'), \quad (8) \]

where \( \Psi \equiv W_{\Phi \Phi} \) and \( D^2 \) is the square of the fermionic covariant derivative. Defining

\[ \Delta = (\frac{1}{4} \Psi(z) \bar{D}^2 + \frac{1}{4} \bar{\Psi}(z) D^2) \Box^{-1} \]

we observe that \( \Delta^2 = \mathcal{M}^2 \Box^{-1} \) with

\[ \mathcal{M}^2 \equiv \frac{\Psi}{4} \bar{D}^2 \Box^{-1} D^2 \bar{\Psi} - \frac{\bar{\Psi}}{4} + h.c.. \quad (9) \]
Thus we may write

$$G_V = \frac{1}{\Box - M^2} (1 + \Delta) \delta^8(z - z')$$

(10)

Note that we can write

$$M^2 = \Psi \bar{\Psi} \mathcal{P}_+ + \bar{\Psi} \Psi \mathcal{P}_- + O(D\Psi, D\bar{\Psi})$$

(11)

where

$$\mathcal{P}_+ = \frac{D^2 D^2}{16 \Box}, \quad \mathcal{P}_- = \frac{D^2 D^2}{16 \Box},$$

are respectively the chiral and anti-chiral projection operators. Let us now evaluate the (anti)
chiral propagators we have

$$G_{--}(z, z') = D^2 D^2 \frac{1}{\Box - M^2} (1 + \Delta) \delta^8(z - z')$$

$$= D^2 D^2 \frac{1}{\Box - \Psi \bar{\Psi}} \left( \Psi \frac{D^2}{4} \Box^{-1} + O(\bar{\Psi}, D\Psi, \bar{D}\bar{\Psi}) \right) \delta^8(z - z')$$

(12)

and

$$G_{++}(z, z') = \bar{D}^2 \bar{D}^2 \frac{1}{\Box - \Psi \bar{\Psi}} \left( \Psi \frac{D^2}{4} \Box^{-1} + O(\Psi, D\Psi, \bar{D}\bar{\Psi}) \right) \delta^8(z - z')$$

(13)

Let us note that in the WZ model

$$\Psi = m + \lambda \Phi,$$

(14)

and that to leading order in the derivative expansion (in terms of $D$) the effective mass term $\Psi \bar{\Psi}$
in the denominator of the propagators can be treated as a constant.

In the massless WZ theory it has been claimed that there is a violation of the non-renormalization theorem stemming from a two (chiral) loop contribution of the form

$$\int d^8 z \frac{D^2}{\Box} G(\Phi) = -\frac{1}{4} \int d^6 z \frac{D^2 D^2}{\Box} G(\Phi) = -4 \int d^6 z G(\Phi).$$

Such a contribution is supposed to come from the diagram in Figure 1.

In the calculation of this diagram in the literature (see [9] and [8] section 4.9.5) the propagator
(for the $m = 0$ theory) is taken to be $[12]$ but with $\Psi \bar{\Psi} \to m^2 = 0$ in the denominator. However
in actual fact even in the massless theory, in the calculation of the effective potential, there is
an effective infra-red regulator since $\Psi \bar{\Psi} = |\lambda \Phi|^2 \neq 0$ at a generic point in field space. What is
implicitly done in the literature is to expand this denominator in powers of $\lambda$ and keep just the
coupling constant independent term. Thus the propagator that is used is

$$G_{--}(z, z') = D^2 D^2 \frac{1}{\Box} \left( \Psi \frac{D^2}{4} \Box^{-1} + O(\bar{\Psi}, D\Psi, \bar{D}\bar{\Psi}) \right) \delta^8(z - z').$$
However given the fact that the effect in question is an infra red one arising in the constant field limit $(p = 0$ limit in momentum space), the natural IR cutoff provided by $\Psi \bar{\Psi}$ should not be ignored as being $O(\lambda)$. It should instead be treated as an IR cutoff. In other words in this infra-red situation (leading effectively to a result which is of the form $0/0$ for the loop diagram in the absence of a cutoff) the correct procedure should be to keep this natural IR cutoff. The above propagator should therefore be replaced by,

$$G_{--}(z, z') = D^2D'^2\frac{1}{\Box - |\lambda \Phi|^2} (\Psi \bar{\Psi}) \delta^8(z - z').$$

Given the potential for an infra-red ambiguity at $\Phi = 0$ it is crucial to keep the effective mass term in the denominator without expanding in powers of $\lambda$, and to the extent that we are ignoring derivative terms it is permissible to treat it as a constant effective IR regulator $|\lambda \Phi|^2 \equiv M^2$. With this propagator the above diagram gives rise to the following contribution to the 1PI action:\footnote{Except for the insertion of the effective IR cutoff the calculation is the same as that in [6, 7, 9].}

$$\Gamma^{(2)} = -\frac{\lambda^5}{12} \int d^4x \int d^4\theta \int \frac{d^4p_1 d^4p_2}{(2\pi)^8} \times \int d^4y_1d^4y_2 e^{ip_1(x-y_1)} e^{ip_2(x-y_2)} \Phi(x, \theta)^3 J(p_1, p_2) + \ldots \quad (15)$$

Note that the integration over $y_1, y_2$ gives $\delta^4(p_1)\delta^4(p_2)$. Here the ellipses represent derivative terms as well as non-chiral terms, and

$$J(p_1, p_2) = \int \frac{d^4k_1d^4k_2}{(2\pi)^8} \frac{k_1^2 p_1^2 + k_2^2 p_1^2 - 2(k_1.k_2)(p_1.p_2)}{\Omega(k, p)},$$

with

$$\Omega = k_1^2k_2^2((k_1 + k_2 - p_1 - p_2)^2 + M^2)[(k_1 - p_1)^2 + M^2][(k_2 - p_2)^2 + M^2].$$
Then we have for the integral $J \sim p^2/M^2$ so that the contribution to the superpotential in $\Gamma^{(2)}$ vanishes (since $p_1, p_2 = 0$ there) and thus there is no renormalization of the superpotential. This is just the non-renormalization theorem in action in the absence of infra-red issues. If on the other hand we had treated $M^2 = \lambda |\Phi|^2$ as a perturbation and expanded in $\lambda$ then we would have got the behavior $J \sim p^2/p^2$ resulting in a non-zero contribution to the superpotential at the two loop level.

Thus the evaluation of this diagram with this natural IR regulator gives zero for this purely chiral loop as would be expected for any chiral loop in the massive WZ theory. A similar statement applies to potential violations of the non-renormalization theorem for the gauge coupling function (which is expected to have quantum corrections only at one loop). These arguments will become relevant for the question addressed in the next section on whether there is an additive contribution to the gaugino mass in the 1PI action compared to the mass coming from the Wilsonian action.

### 3 Gaugino masses

Let us first ignore the Weyl anomaly contribution (KL) \[1,10\] coming from transforming to the Einstein frame from the Jordan frame in which off-shell supergravity is defined. To simplify the argument let us consider a SUGRA theory with no charged scalars, no FI terms and a single gauge group with a gauge coupling function which is just a constant.

The full $\mathcal{N} = 1$ supergravity with chiral scalar and gauge field couplings was first written down by Cremmer et al [11]. In Weyl spinor formalism it is given in appendix G equation (G.2) of Wess and Bagger (WB) [12] which we will use below. Since there are many terms in this action we will not write it down here but will refer to the relevant terms as given in this reference.

We first observe that the action has a chiral symmetry under the ("$\gamma_5$") transformation $\lambda \to i\lambda$ of the gaugino field, except for the breaking terms in lines 5 through 8, line 10 and lines 15,16, and lines 19,20. There is also the explicit (classical) gaugino mass term in line 24. But the latter as well as lines 6,7 and 15,16, 19 and 20, are zero for constant gauge coupling functions. The contribution of line 5 and 8 are zero under our assumption that there are no charged chiral scalar superfields. Thus under the stated conditions the only term which breaks the chiral symmetry of the gauginos is the dimension 5 term in line 10 i.e.

$$\frac{i}{4}\sqrt{g} [\psi_m \sigma^{ab} \sigma^m \bar{\lambda} + \bar{\psi}_m \sigma^{ab} \sigma^m \lambda] (F_{ab} + \hat{F}_{ab}),$$

with $F_{ab}, \hat{F}_{ab}$ being the gauge field strength and its dual defined in eqn (25.17) of WB. But this term (in the absence of non-zero background $F$) can only generate four fermi terms (and higher powers) and cannot give mass to the gauginos. Thus in this case a gaugino mass in the 1PI action can only arise as a consequence of the (super) Weyl anomaly - which contains the ordinary chiral anomaly that violates the above chiral symmetry, as well as terms related to it by SUSY.

Let us now relax the assumption that $f$ is a constant. In this case there will be additional terms violating the chiral symmetry $\lambda \to i\lambda$. But any such term (in addition to the classical mass term proportional to $F^i \partial_i f$ which vanishes when SUSY is unbroken) will only generate terms in the 1PI effective action that are proportional to $< \partial_i f >$ or higher point functions of this field. In particular such terms will not give rise to gaugino mass terms proportional to $m_{3/2}$ as in AMSB.
3.1 Anomaly Effects

The argument in the last subsection holds even in the presence of supersymmetry breaking and has nothing to do with tuning the cosmological constant to get flat space. Consider again the class of models where the classical gauge coupling is constant. The gaugino sector is independent of chiral scalars which are responsible for SUSY breaking and the fine-tuning of the cosmological constant (by suitably adjusting a constant term in the superpotential). In particular one can have broken SUSY in flat space with zero gaugino mass provided there is no anomaly in the chiral symmetry. Such an anomaly is the only way in which, in a situation where the classical gaugino mass is zero, a quantum one-loop mass is still generated.

The chiral symmetry (which is related to the super Weyl symmetry) is indeed anomalous. Under a Weyl transformation characterized by a chiral superfield transformation parameter \( \tau(x, \theta) \) (which for instance transforms \( \lambda \rightarrow e^{-3\tau}|0\lambda) \). The effect of this anomaly on the gauge coupling function super field is \[ f(\Phi) \rightarrow f(\Phi) + \frac{3c}{4\pi^2}\tau, \tag{16} \]

where \( c = T(G) - \sum_r T(r) = T(G) \) since the second term, the sum over matter representations is absent if there is no charged matter as in our simplified case. Here \( T(G) \) is the trace of a squared generator in the adjoint representation of the gauge group. KL \[10\] fix the superfield \( \tau \) by demanding that the transformation takes one from the Jordan frame (which is the natural frame in which off shell supergravity is formulated), to the Einstein frame so that \( 2\tau + 2\bar{\tau} = -\frac{\hat{K}(\Phi, \bar{\Phi})}{3}|_H \). The instruction on the LHS here is to keep only the chiral and the anti-chiral terms in the component expansion of \( K \) and is in effect the analog of Wess-Zumino gauge fixing\[4\]. The effect of this anomaly then is to give additional terms to both the gauge coupling and and the gaugino mass\[5\],

\[
\frac{1}{g_{\text{phys}}^2} = \Re f - \frac{3T(G)}{16\pi^2}K|_0, \tag{17}
\]

\[
\frac{2M}{g_{\text{phys}}^2} = \frac{1}{2} F^i \partial_i f - \frac{3T(G)}{16\pi^2}F^i K_i|_0. \tag{18}
\]

In SQCD coupled to supergravity with neutral chiral scalars breaking SUSY and a classical gauge coupling constant which is field independent, the first term on the RHS of the gaugino mass equation \[18\] will be zero, but there will nevertheless be a mass term that is generated by the Kähler-Weyl anomaly. This term however also vanishes if supersymmetry is unbroken (i.e. \( F = 0 \)) as needs to be the case in AdS supersymmetry. It also satisfies the criterion that in the Wilsonian effective action UV effects should not break the structure of the supergravity action given in appendix G of \[12\]. In other words unless one is claiming that there is an anomaly in local supersymmetry the general structure of the local supergravity action should be preserved\[6\], with the

\[4\]Note that this is a superfield relation and in particular should be used to fix the F-term of \( \tau \) as well as scalar and fermionic components. This follows from the fact that the Jordan frame superconformal factor that needs to be removed is the superfield \( e^{K/3} \).

\[5\]We ignore an additional (NSVZ) anomaly term coming from redefining the gauge kinetic term to get canonical normalization for it.

\[6\]For a detailed discussion of these issues see \[13\].
appropriate perturbative corrections to the expressions for the Kähler potential, the superpotential and the gauge coupling function.

This is in contrast to the claims in the AMSB literature [14, 15], where an additional term 
\[ \frac{3T(G)}{16\pi^2} e^{K/2} W_0 |\lambda\lambda \right] \] 

is said to be needed on the RHS of (18). Note that this term is independent of the factor \( \partial_t f \) and therefore has nothing to do with possible corrections to gaugino mass in the 1PI action coming from non-mass terms in the Wilsonian action proportional to this factor.

There is also the possibility in principle that IR effects in the 1PI action can violate arguments based on the Wilsonian action as in the above discussion. However as we’ve argued in the previous section such violations are spurious. Thus our conclusion is that if the gaugino mass in the Wilsonian effective action (after including the Weyl anomaly contribution) is zero (as would be the case if SUSY is unbroken), then the physical gaugino will have zero mass.

3.2 Non-perturbative effects

In a recent paper [16] it has been argued that there is a non-perturbative effect coming from gaugino condensation which requires that above the scale of the condensing gauge group (say \( \Lambda_c \)) one should have added a “anomaly mediated” gaugino mass counter term in the effective theory. If true this claim would appear to violate the above argument. So let us examine it.

The argument proceeds from the observation that when there is a non-perturbatively generated superpotential \( W_{np} \), well below the scale \( \Lambda \), we should be able to identify the term

\[ -3W_{np}W_0^* + h.c. \]  

(19)

in the potential with \( W_0 \) being a constant in the superpotential. It is then claimed that the existence of this term requires adding the “anomaly-mediated” gaugino mass counter term to the high-energy theory,

\[ \mathcal{L}_{\lambda\lambda} = \frac{1}{2} \frac{3T(G)}{16\pi^2} e^{K/2} W_0 |\lambda\lambda \right] + h.c. \]  

Firstly, below the scale of the condensing gauge group (but above the SUSY breaking soft mass scale) obviously there are no gauginos anymore that pertain to the gauge group under consideration. Since the degrees of freedom of this gauge group have been integrated out, they are irrelevant to the low energy phenomenology. This is in contrast to the claim of AMSB which posits a contribution (proportional to the gravitino mass) to the mass of a gaugino which survives in the low energy theory.

Secondly the argument is based on reasoning which is not valid for the off-shell formulation of supergravity (which is what one needs at the quantum level). In the off-shell formulation (i.e. before integrating out auxiliary fields), the theory is linear in the superpotential. Indeed as long as supersymmetry is not broken explicitly, the superpotential dependent part of the superspace action (with chiral superspace measure \( \mathcal{E} \)) must take the form\(^7\)

\[ \int d^6 z \mathcal{E} C^3 W(\Phi). \]  

\(^7\)It is convenient for the subsequent discussion to keep explicitly the chiral scalar compensator in the action as in [10].
The total superpotential is now a sum of the classical and non-perturbative terms,

\[ W = W_{cl}(\Phi) + W_{NP} = W_{cl}(\Phi) + Ae^{-\frac{3\pi^2}{8b_c}f_c(\Phi)}. \]  
\[ (21) \]

For simplicity we’ve assumed in the above that there are no matter fields charged under the condensing gauge group \( G_c \) (with gauge coupling function \( f_c \)). Its 1 loop beta function coefficient is \( b_c = 3T(G_c) \). The term \( (19) \) then arises in the usual fashion from the term \(-3|W|^2\) that comes in the potential once one eliminates the auxiliary fields to get the on-shell supersymmetric action.

The manner in which the second term of \( (21) \) arises from a condensing gauge group in the supergravity context was discussed first in [17] and elaborated on in [18]. Let us recapitulate the argument as presented in the latter reference. Above the confining scale \( \Lambda_c \) of \( G_c \) the action has an explicit superspace gauge field kinetic term

\[ \int d^6\theta \mathcal{E}_1 \frac{1}{2} f_c(\Phi) W_{cW_c} + h.c. \]

Well below the scale \( \Lambda_c \) the \( G_c \) degrees of freedom need to be integrated out. This gives an effective action \( \Gamma \) defined schematically by

\[ e^{-\Gamma(\Phi, \bar{\Phi}, C, \bar{C})} \approx \int d(\text{gauge}) \exp \left\{ -\frac{1}{4} \int f_c(\Phi) - \frac{b_c}{8\pi^2} \ln C |W|^2 W^c + h.c. \right\}, \]
\[ (22) \]

where we included the KL anomaly contribution\(^8\) corresponding to\(^8\) in the gauge coupling function. Since SUSY should not be broken by this procedure, we expect \( \Gamma \) to have the general form of a superspace action and in particular should develop a superpotential. Given the general argument that any superpotential should come with a factor \( C^3 \), we see that the corresponding term in \( \Gamma \) will be (a superspace integral of)

\[ C^3 W_{NP} = C^3 A_a \exp \left( -3\frac{8\pi^2}{b_a} f_a(\Phi) \right). \]
\[ (23) \]

The total superpotential below \( \Lambda_c \) is then given by \( (21) \). Thus there is absolutely no need to add any kind of counter term proportional to \( W_{cl} \) to the gaugino mass in the UV theory above the scale \( \Lambda_c \). Indeed the addition of such a term would be a violation of the principle that the Wilsonian action at the two derivative level must preserve the general structure of a SUGRA action given in appendix G of WB [12]. It should also be stressed that once the auxiliary fields are solved for in terms of the propagating fields, the Einstein-Kähler frame Lagrangian will be exactly as given in equation G2 of WB [12] with the classical superpotential being replaced everywhere by the sum of the classical and non-perturbative terms \( (21) \). In particular this would mean of course an additional contribution to the gravitino mass so that line 21 of equation G2 of WB will become

\[ -ee^{K/2} \{ (W_{cl} + W_{NP}) \bar{\psi}_a \sigma^{ab} \bar{\psi}_a + h.c. \}. \]
\[ (24) \]

### 3.3 Linearized SUGRA

In another recent paper [19] the authors have given what on the face of it appears to be yet another argument for an additive term in the expression for the gaugino mass. However this term appears

\(^8\)The crucial assumption here is the quasi-locality of this contribution to \( \Gamma \) which enables us to define its derivative expansion and then focus on its two derivative action which should be of the standard supergravity form.
only because the linearized formalism in which the authors work ignores a term coming from the chiral density $E$ (which in the authors’ formalism has been set to unity). In other words, the correct superspace term corresponding to their equations (28) and (30) should have the form, in the notation of WB [12] (for a constant gauge coupling function such as the one implicit in these two equations), $c \int d^2 \theta E \mathcal{W} W$. The authors only calculate the F-term of $W^2$ but ignore (since for them $E = 1$) the F-term of the chiral density which exactly cancels the term that they calculate. Indeed this is why in equation G2 of WB, when the gauge coupling function is a constant, there is no gaugino mass term. The penultimate line in this equation is zero, whether or not one is in flat space (with $E = 1$). The point is the action must first be calculated in a generally covariant and supersymmetric fashion, before one specializes to flat space thus breaking general covariance. The authors are therefore in manifest contradiction with the Cremmer et al Lagrangian [11] given in equation G2 of [12].

As pointed out in the earlier works [13][21] by the author, the only way an additive gaugino mass contribution (i.e. one that does not vanish when the the F-term of chiral scalars is zero), can arise is if there is an anomaly in local supersymmetry. Indeed preserving general covariance as well as supersymmetry is crucial for getting correct results. If there is no anomaly in either general covariance or supersymmetry, then the Wilsonian effective action (including the effects of integrating out high scales) should respect the structure of the classical theory. This means that the action is still given in terms of a Kähler potential ($K$) a superpotential $W$ and a gauge coupling function $f$ with appropriate loop and non-perturbative corrections. What we have argued in this section is that the physical masses (position of poles in propagators) will be perturbatively related to those in the Wilsonian action. In particular there cannot be an additive contribution to the gaugino mass as compared to the corresponding mass term in the Wilsonian action.

### 4 Phenomenological issues and Conclusions

In phenomenology based on high scale supersymmetry breaking in the hidden sector of a SUGRA model, the (classical) masses and couplings calculated at that scale are used as initial conditions for RG evolution down to the TeV scale. In effect these running masses and couplings are computed as Wilsonian parameters - and are then compared to data at the TeV scale for scalar as well as gaugino masses. But these are obviously not the actual parameters in the 1PI action - in particular the physical mass is the position of the pole (or its real part for a unstable particle) in the two point function. However the difference between the Wilsonian mass and the 1PI mass is a higher order effect and is smaller than the effect of running. The point of running down to the TeV scale is of course the expectation that the physical (pole) mass is near that scale and that the (small) difference can be computed in perturbation theory.

In standard renormalization theory mass (and coupling) parameters are input data from experiment. For instance the physical electron mass (defined as the zero of the inverse propagator) is an experimental input which goes into the renormalized Lagrangian. Similarly the coupling is defined at some scale in terms of a cross section at that scale - in QED for example this would be fixed by

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9In the traditional argument for AMSB given in [14][15] for instance a similar problem arises because of the linearized formalism in which the authors work. For detailed discussion see [13]. For a discussion of other issues that may be missed in the linearized formalism as compared to the full non-linear supergravity see [20].
the Thompson cross section. To compute the cross section at some high scale, one would use the beta function equations to evolve the coupling and mass parameter to the appropriate scale using these physical inputs as initial conditions. These new parameters are then used in a perturbative calculation of a cross section at that high scale.

In making predictions for SUSY soft masses and couplings we must necessarily follow the inverse procedure. Firstly of course we need to assume that low energy (TeV scale) supersymmetry exists! This means that we expect experiments to yield partners to the standard model particles at the this scale. But experiments give us the value of physical i.e. pole masses for the lightest superparticle (LSP), the stop, the gluino, etc. The theory of supersymmetry breaking however has a natural scale which is typically the scale at which we can construct a SUGRA coming say from string theory. In most scenarios this is around the string/GUT scale. The theory then gives a value for the masses etc. which are then used as initial conditions for evolving down to the TeV scale. Assuming that SUSY is relevant for solving the hierarchy problem, one evolves the RG equations down to a scale of say a TeV. Thus one expects the (hopefully) measured masses - the pole masses in the S-matrix - to be given by the running masses at this scale, up to small perturbative corrections. To put it another way the Wilsonian masses should be the leading approximation to the actual physical masses in the quantum effective action, with any differences being perturbatively small.

We have addressed in this paper two issues that are relevant to this procedure. The first is that in computing the 1PI action one should work at a generic point in field space. The (non-zero) field then acts as an effective IR regulator so that terms which in the absence of a regulator violate the non-renormalization theorems, are in fact absent. Thus we expect all the constraints of supersymmetry that are present in the Wilsonian effective action to be present in the local terms (such as the mass terms) of the 1PI action. Secondly we’ve argued that the physical (pole) mass of fermions (in particular gauginos) cannot acquire an additive contribution as compared to the mass term in the Wilsonian effective action. Finally we addressed some claims in two recent papers which appear to get a contribution to the gaugino mass that does not fit the structure of the generally covariant and supersymmetric effective action.

Let us reemphasize the main message of this note. In deriving a quantum effective action for locally supersymmetric theories, one should keep natural infrared cutoffs and not break general covariance by working in some fixed metric background. Failure to do so can give rise to spurious effects, i.e. ones which are absent in the generally covariant and supersymmetric action such as the so-called AMSB term in the gaugino mass.\footnote{It is a curious fact that in string models with no-scale like structure for the Kaehler potential there is a cancellation between this AMSB term and the KL term - i.e. the second term in (18). Since practically all of string phenomenology is based on such Kaehler potentials, and given that these are the only UV completions that we know of for theories for the MSSM soft masses, this would mean that if one imposed the criterion that a sensible theory of SUSY breaking should have an UV completion, then as far as we know the adherents of the AMSB term must admit that the whole Weyl anomaly issue is spurious. On the other hand it has long been the contention of this author \cite{21,13} that since only the KL term is present so that there is no cancellation, the Weyl anomaly is indeed very relevant for a discussion of SUSY phenomenology in string theory.}
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