Exponentially Large Extra Dimensions

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We show how the presence of a very light scalar with a cubic self-interaction in six dimensions can stabilize the extra dimensions at radii which are naturally exponentially large, \( r \sim \ell \exp[(4\pi^3/g^2)] \), where \( \ell \) is a microscopic physics scale and \( g \) is the (dimensionless) cubic coupling constant. The resulting radion mode of the metric becomes a very light degree of freedom whose mass, \( m \sim 1/(M_p r^2) \), is stable under radiative corrections. For \( 1/r \sim 10^{-3} \) eV the radion is extremely light, \( m \sim 10^{-33} \) eV. Its couplings cause important deviations from General Relativity in the very early universe, but naturally evolve to phenomenologically acceptable values at present.

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I. INTRODUCTION

Recent developments in both particle physics and cosmology appear to indicate the existence of important fundamental physics associated with the scale \( 10^{-3} \) eV.

- On the cosmological side, physics at this scale appears to be indicated by the recent observational evidence [1] for the existence of a ‘dark energy’ component to the universe (possibly a cosmological constant), having a negative pressure of order \( p \sim (10^{-3} \text{eV})^4 \).

- On the particle physics side several developments have made plausible the existence of interesting \( 10^{-3} \) eV physics. On the one hand, it has long been recognized that breaking supersymmetry at the TeV scale would imply masses of order \( m \sim (1\text{TeV})^2 / M_p \sim 10^{-3} \) eV for the gravitino, and other gravitationally-coupled particles. Here \( M_p \sim 10^{18} \) GeV is the (rationalized) Planck mass.

- More recently has come the recognition that extra dimensions can be much larger (and have much richer dynamics) than had hitherto been appreciated, and the realization that such large extra dimensions could help solve some long-standing problems like the hierarchy problem [2,3]. In particular, these dimensions could have radii as large as \( r \sim (10^{-3} \text{eV})^{-1} \).

Interest in all of these issues has been sharpened by the realization that the existence of new particles at this scale may have other observational consequences. The resulting modifications to Newton’s Law of Gravity may fall within reach of current and upcoming experiments. The dynamics of red giants and supernovae can be modified, and the resulting bounds can be important [4,5], but need not be fatal [6,7].

A crucial part of any large-extra-dimension scenario is a natural mechanism for generating a radius which is large compared to other microphysical scales. This is a particularly pointed requirement if the large ratio, \( (10^{-3} \text{eV})/(1\text{TeV}) \), is to be used to explain other hierarchies, like \( M_W / M_p \).

It is our purpose in this paper to propose a mechanism for generating such large dimensions. Although our proposal is not restricted to radii as large as \( r \sim (10^{-3} \text{eV})^{-1} \), it can satisfy the very restrictive phenomenological constraints which apply in this case. Furthermore, we argue that the energetics which chooses the value for the extra-dimensional radius in our mechanism can have attractive cosmological consequences when the extra dimensions are large.

Our proposal is based on the observation that large radii are naturally obtainable if the potential which governs the radion is logarithmic:

\[
\ell^4 V(r) = \left( \frac{\ell}{r} \right)^p [a_0 + a_1 \log \left( \frac{r}{\ell} \right) + \cdots] + \mathcal{O} \left( \left( \frac{\ell}{r} \right)^q \right)
\]

(1)

(where \( q > p \) and \( \ell \) is a microscopic length scale). Besides the usual runaway solution, \( r \to \infty \), the stationary condition \( dV/dr = 0 \) also admits the solutions:

\[
\frac{r}{\ell} \approx \exp \left( \frac{1}{p - a_0/a_1} \right),
\]

(2)

where the approximation becomes exact in the absence of higher powers of \( \log(r/\ell) \). This basic mechanism was proposed at a phenomenological level in the “Planck scale
quintessence” models [3]. In that context it was shown that the time for quantum tunneling out of the local min-
imum was exponentially larger than the age of the Universe [4].

Eq. (4) predicts $r$ is naturally exponentially large compared to $\ell$, if two conditions are satisfied:

1. $a_0$ and $a_1$ must have opposite signs; and

2. there is a modest hierarchy in the coefficients, $a_k$. For instance if $a_1 = -\hat{a}_1 \epsilon$, with $\epsilon < 1$ and $a_0 \neq a_1$ both positive and $\mathcal{O}(1)$, then $r/\ell = \mathcal{O}(\exp(a_0/(\hat{a}_1 \epsilon) + \mathcal{O}(1)) \gg 1$. Numerically, if $1/\ell \sim 1$ TeV and $a_0/(\hat{a}_1 \epsilon) \sim 35$ then $1/r \sim 10^{-3}$ eV falls into the interesting range.

We here propose a scenario which generates logarithmic radion potentials which very generically satisfy both of these two conditions. The scenario has the radion potential generated as the universe passes through a stage during which it is effectively six-dimensional, provided that there is at least one six-dimensional scalar field whose mass is of order $1/r$ and which has reasonably large, nonderivative, cubic self-interactions.

In six dimensions a cubic scalar self-interaction,

$$U_{\text{ren}}(\phi) = \frac{g}{3!} \phi^3,$$

is the only local interaction which has a dimensionless coupling constant. Because of this, in perturbation theory its renormalization gives it a logarithmic, rather than a power, dependence on $r$. In renormalization-group terms it is the only six-dimensional interaction which is not irrelevant in the infrared.

The logarithmic dependence of the low-energy coupling, $g(r)$, provides a natural way of obtaining logarithmic potentials for $r$. In the absence of light-scalar loops, for large $r$ the radion potential has the generic form

$$V(r) = \sum_{k=k_0}^{\infty} \frac{c_k}{r^k}.$$  \hspace{1cm} (4)

This can arise in particular examples in many ways. For instance, it arises when evaluating the classical action as a function of radius if six-dimensional gravity or supergravity is compactified on a sphere. Alternatively, it could dominantly arise as a quantum Casimir energy, such as in a toroidal compactification [11].

Scalar radiative corrections (in six dimensions) to this potential correct the constants $c_k$:

$$c_k = \sum_{l=k_0}^{\infty} c_k^{(l)} \left[ \frac{g^2(r)}{4\pi^3} \right]^l,$$

where $g(r) = g_0 + b g_0^3 \log(r/\ell) + \mathcal{O}(g_0^5)$.

Using eq. (6) in eq. (4) produces a logarithmic potential of the desired type, with several remarkable features:

- Eq. (6) automatically introduces the desired hierarchy (and so satisfies condition 2 above) by systematically suppressing higher powers of $\log r$ with the suppression factor $\epsilon \sim \alpha = g^2/(4\pi)^3$.

- The relative sign of the coefficients, $a_0$ and $a_1$, of the first two terms in eq. (6) depends crucially on the relative sign of the first few loop corrections $-c_{k_0}^{(l)}$ and $c_{k_0}^{(l+1)}$ and on the sign of $b$, the one-loop renormalization-group coefficient for $g$. Furthermore, given that the first term in $V$ is positive ($c_{k_0}^{(l)} > 0$, the signs of $a_0$ and $a_1$ are opposite, as required to generate a hierarchy (condition 1 above).

- Because the dependence of $V(r)$ on $r$ arises implicitly through the dependence of $V$ on $\alpha(r)$, the extremal value for $r = r_*$ corresponds to the coupling $\alpha_s = \alpha(r_s)$ which extremizes $V(r)$. Generically, if all constants $c_{k_0}^{(l)}$ are $\mathcal{O}(1)$, the minimum occurs when $\alpha(r_s) = \mathcal{O}(1)$, and so the precise value for $r = r_s$ at the minimum cannot be computed perturbatively in $\alpha(r_s)$. The hierarchy $r/\ell$ is nonetheless reliably predicted to be exponentially large, so long as $\alpha(r_s)$ differs significantly from $\alpha(\ell)$ (and either $\alpha(\ell)$ or $\alpha(r_s)$ lies within the perturbative regime) given the logarithmic running of $\alpha$.

Perturbative calculation of $r_*$ itself is also possible if the lowest-order coefficient, $c_{k_0}^{(l)}$, happens to be even modestly smaller than the others (as occurs in some examples below).

We present our argument in more detail in the subsequent sections. First, the next section gives more explicit expressions for the radion potential in a model consisting only of the metric and a very light scalar field. Since our mechanism expresses the explanation for large dimensions in terms of a light scalar field, we then follow with a discussion of how natural it is to find scalars with the required properties. We conclude with a brief discussion of the cosmological and phenomenological implications of the logarithmic potential, and the bounds which these may impose on model building.

\section{II. Radion Potentials and Light Scalars in 6 Dimensions}

In this section we compute explicitly the radion potential produced by a light scalar field for a simple compactification. To this end consider a model consisting of scalar fields, $\varphi_i$, and the six-dimensional metric, $G_{MN}$. We imagine this to be an effective six-dimensional theory obtained after integrating out all more massive degrees of freedom at scale $\ell$. The leading terms in the derivative expansion for this lagrangian have the form:
\[
\mathcal{L} = -\frac{1}{2f^4} \mathcal{R} - \frac{1}{2} G^{MN} \partial_M \varphi_i \partial_N \varphi_i - U(\varphi),
\]  
where \(\mathcal{R}\) denotes the scalar curvature built from the six-dimensional metric.

We assume the scalar potential to have the form

\[
U(\varphi) = \frac{\mu^2}{2} \varphi_i \varphi_j + \frac{g_{ijk}}{3!} \varphi_i \varphi_j \varphi_k + \frac{\kappa_{ijkl}\ell^2}{4!} \varphi_i \varphi_j \varphi_k \varphi_l + \cdots
\]  
with the microscopic scale \(\ell\) setting the dimensions of all but two of the couplings in the scalar potential. The two exceptions are: (i) we assume there is no cosmological constant term in \(U(\varphi)\); and (ii) we assume the scalar masses are small: \(\mu_{ij} \lesssim 1/r \ll 1/\ell\). In this section we simply fine-tune the lagrangian to ensure these conditions are satisfied, but since our generation of the logarithmic potential is based on these choices, in the next section we address how difficult they are to arrange within models. Although we shall argue that assumption (ii) is simple to arrange in supersymmetric models, the tricky part is to have \(\mu\) be as small as \(O(1/r)\) without also finding the cubic term similarly suppressed, \(g_{ijk} \sim \ell^2/r^2\).

We consider in detail the two simplest cases. The first is a single real scalar, \(\varphi\), with self-coupling \(U = \frac{1}{2} \mu^2 \varphi^2 + \frac{1}{3} g \varphi^3 + \frac{1}{3} \kappa \ell^2 \varphi^4 + \cdots\). The second is a single complex scalar, \(\phi = (\varphi_1 + i\varphi_2)/\sqrt{2}\), with \(Z_2\) symmetry \(\phi \to \omega \phi\), where \(\omega^3 = 1\). The self coupling in this case is \(U = \mu^2 |\phi|^2 + \left(\frac{1}{2} g |\phi|^3 + c.c.\right) + \frac{1}{3} \kappa \ell^2 |\phi|^4 + \cdots\).

### A. One Loop Casimir Energy

Before searching for logarithmic corrections due to the cubic scalar coupling, we must first compute the potential of the form eq. (8) which is to be corrected. This we compute in a semiclassical expansion about the local minimum at \(\phi = 0\), whose existence is assured by our choice \(\mu^2 > 0\). Although this minimum is ultimately destabilized by the assumed cubic term, we assume the potential to be bounded below by virtue of the other terms in \(U(\phi)\), involving higher powers of \(\phi\). The detailed form of these higher terms do not play a role in the discussion which follows.

Our calculation also assumes the ground state geometry is flat: \(M^6 = R^4 \times T^2\), where \(R^4\) denotes flat Minkowski space and \(T^2\) is a torus, both of whose radii we denote by \(r\). (Although the torus has other moduli besides its radius, here we focus only on \(r\).) For the complex field, \(\phi\), we allow the possibility that the scalars satisfy twisted boundary conditions – \(\phi \to \omega \phi\), with \(\omega^3 = 1\) – about the cycles of the torus. Indeed this is our primary reason for considering the complex scalar case.

Under these circumstances the radion potential first arises as a one-loop Casimir energy for the fields living in the bulk. (The Casimir energy for fields on the brane do not depend directly on \(r\) in the absence of a nontrivial ‘warp factor’, such as we take to be the case here.) The contribution to this energy from a complex scalar is computed as a function of its boundary conditions in Appendix A, and is given by:

\[
V_1(r) = -\frac{1}{r^4} \int_0^\infty dx \, e^{-\beta x} \left[ e^{-\pi x (a^2 + b^2)} \times \theta_3(i \pi ax, -e^{-\pi x}) \right],
\]

where \(\beta = \mu^2 r^2/(4\pi)\), and \(\theta_3(z, q)\) denotes the usual Jacobi theta-function [11]. The constants \(a\) and \(b\) take the values 0, 1/3 or 2/3 depending on the type of twisted boundary condition which the scalar satisfies about each of the torus’ two nontrivial cycles. The choice \(a, b = 0\) corresponds to periodic boundary conditions about these cycles, while \(a, b = 1/3, 2/3\) corresponds to twisting by \(\omega\) or \(\omega^2\).

For real scalars no twist consistent with a cubic self-interaction is possible, and so the result is one half the expression of eq. (8) with \(a = b = 0\).

As is easily verified (see Appendix A), eq. (8) converges in both the ultraviolet and infrared, even if \(\mu \to 0\). If \(\mu r \gg 1\) then \(V_1\) falls exponentially as \(\mu r \to \infty\). If \(\mu \to 0\), then the potential takes the form of a power of \(r\): \(V_1 = c_4^{(1)}/r^4\) where the coefficient \(c_4^{(1)}\) is given by the integral in eq. (8) with \(\beta = 0\). Numerical integration gives the results shown in Table (1) below.

| \((a, b)\)   | \(c_4^{(1)}\) | \(c_4^{(2)}\) |
|-------------|----------------|----------------|
| (0, 0)      | -0.299         | -0.064         |
| (0, 1/3)    | -0.048         | 0.178          |
| (0, 2/3)    | -0.048         | 0.327          |
| (1/3, 1/3)  | 0.122          | 0.069          |
| (1/3, 2/3)  | 0.122          | 0.142          |
| (2/3, 2/3)  | 0.122          | 0.169          |

Table (1): The one- and two-loop Casimir-energy coefficients, \(c_4^{(1)}\) and \(c_4^{(2)}\).

To this should be added the contribution due to the graviton Casimir energy, as well as the Casimir energy due to any other six-dimensional particles. Because of the rapid falloff in the result as \(\mu r \to \infty\) it is clear that only the contribution of those degrees of freedom for which \(\mu r \lesssim 1\) is important for large \(r\). We consider these contributions in more detail in following sections, where the structure of the entire model is considered in more detail.

### B. Radiative Corrections

We next turn to the corrections to \(V_1(r)\) which dominate for large \(r\).
The first observation to be made is that only the cubic interaction of $U(\varphi)$ can contribute to $V(r)$ unsuppressed by further powers of $1/r$. This is because all other coupling constants have dimension of a positive power of length, and so are perturbatively nonrenormalizable.

To see how this works consider the potential $U(\varphi)$ for a real scalar. Imagine now scaling out powers of $r$ to make all couplings dimensionless, so the potential $U(\varphi)$ is written

$$U(\varphi) = \frac{(\mu r)^2}{r^2} \varphi^2 + \frac{9}{3!} \varphi^3 + \left( \frac{\kappa \ell^2}{r^2} \right) r^2 \varphi^4 + \cdots$$  \hspace{1cm} (10)

Once written this way it is clear that on dimensional grounds the Casimir energy can be written:

$$V = \frac{1}{r^4} v(\mu r, g, \kappa \ell^2 / r^2, \ldots),$$  \hspace{1cm} (11)

where $v$ is a dimensionless function of dimensionless arguments. Clearly each factor of the coupling $\kappa$ in a series expansion of $v$ is accompanied by a power of $\ell^2 / r^2$, implying a contribution which is further suppressed by powers of $1/r$ compared to the uncorrected term $[12]$.

For corrections to $V(r)$ which are not suppressed by more powers of $1/r$ we must consider graphs which involve only the dimensionless cubic scalar self-coupling $g$. The emergence of the logarithms can then be most easily seen in the limit as $\mu \to 0$, in which case it is revealed by a simple renormalization-group argument. For $\mu \to 0$ the dominant radion corrections can be written, on dimensional grounds, as:

$$V(r) = \frac{1}{r^4} \left[ A_0 + A_1(r/r_0)\alpha(r_0) + A_2(r/r_0)\alpha^2(r_0) + \cdots \right],$$  \hspace{1cm} (12)

where $\alpha = g^2 / (4\pi)^3$ is the six-dimensional loop-counting parameter, renormalized at an arbitrary renormalization point, $r_0$. The coefficients $A_k(r/r_0)$ can be dimensionless functions of $r/r_0$, although explicit calculation has just shown $A_0 = c_4^{(1)}$ to be a constant.

Since the dependence of $V$ on $r$ is tied to its dependence on $r_0$, and since $V$ cannot depend on $r_0$ at all, the $r$ dependence of the $A_k$’s can be related to the running of $\alpha$. That is, if:

$$r_0 \frac{d\alpha}{dr_0} = B \alpha^2 + O(\alpha^3),$$  \hspace{1cm} (13)

then the Callan-Symanzik equation, $r_0 dV/dr_0 = 0$, implies $dA_1/dr = 0$ and $rdA_2/dr = B A_1$. For renormalization schemes for which $B$ is $r$-independent, this implies:

$$V(r) = \frac{1}{r^4} \left[ c_4^{(1)} + c_4^{(2)} \alpha(r_0) + \alpha^2(r_0) \left[ c_4^{(3)} + B c_4^{(2)} \log \left( \frac{r}{r_0} \right) \right] + \cdots \right]$$

$$= \frac{1}{r^4} \left[ c_4^{(1)} + c_4^{(2)} \alpha(r) + c_4^{(3)} \alpha^2(r) + \cdots \right],$$  \hspace{1cm} (14)

where $\alpha(r)$ is the solution to the one-loop renormalization flow $r d\alpha/dr = B \alpha^2$:

$$\alpha(r) = \frac{\alpha_0}{1 - B \alpha_0 \log (r/\ell)}.$$  \hspace{1cm} (15)

As usual, this expression is accurate to leading order in $\alpha_0$, but to all orders in $\alpha_0 \log (r/\ell)$. Standard calculations give $B = +3/2$ for real scalars, and $B = -1/2$ for complex scalars, in six dimensions.

Several conclusions may be drawn from eq. (14). First, it shows that the use of the renormalization group to resum all orders in $\alpha_0 \log (r/\ell)$ permits the inference of the coefficient of every power of $\log (r/\ell)$ to leading order in $\alpha_0 = \alpha(\ell)$. Furthermore, eq. (14) establishes that this leading log($r$) dependence is purely determined by the known one-loop renormalization group coefficient, $B$, and the two-loop vacuum energy coefficient, $c_4^{(2)}$, whose evaluation is our next task.

Before turning to this task, there is another lesson to be drawn from eq. (14) concerning the domain of validity of our conclusions. This equation shows that $V(r)$ depends logarithmically on $r$ only implicitly, due to its dependence on $\alpha(r)$. This implies the stationary point, $r_s$, of $V(r)$ occurs for $r_s$ satisfying the condition

$$-4 c_4^{(1)} - 4 c_4^{(2)} \alpha(r_s) + \alpha^2(r_s) \left[ B c_4^{(2)} - 4 c_4^{(3)} \right] + \cdots = 0,$$

which is satisfied by $\alpha(r_s) = O(1)$ (unless $c_4^{(1)} \ll |c_4^{(2)}|$), in which case $\alpha(r_s) \approx -c_4^{(1)}/c_4^{(2)} \ll 1$.

From these observations we see that

$$\frac{r_s}{\ell} = \exp \left[ \frac{1}{B} \left( \frac{1}{\alpha(\ell)} - \frac{1}{\alpha(r_s)} \right) \right],$$  \hspace{1cm} (16)

and so $r_s \gg \ell$ follows from the logarithmic running of $\alpha$, given a reasonably modest difference between $\alpha(r_s)$ and $\alpha(\ell)$, provided this running occurs within the perturbative regime $\alpha \ll 1$. There are two cases to consider:

- $B > 0$ (Real Scalars): For this choice $\alpha$ is asymptotically free, and so $r_s > \ell$ requires $\alpha(r_s) > \alpha(\ell)$. In this case all of the $c_4^{(1)}$’s and so also $\alpha(r_s)$ – can be $O(1)$, and so a large hierarchy is ensured for modestly small $\alpha(\ell)$. Although in this case the precise value of $r_s$ cannot be computed in perturbation theory, its order of magnitude is known reliably to be of order $\exp[1/B(\alpha)]$ compared to $\ell$.

- $B < 0$ (Complex Scalars): In this case $\alpha(r)$ falls as $r$ increases, so $r_s > \ell$ implies $\alpha(r_s) < \alpha(\ell)$. The use of the perturbative running of $\alpha$ therefore requires $\alpha(r_s) \ll 1$, and so consistency requires $c_4^{(1)}/c_4^{(2)}$ to be small and negative. Remarkably, we find below that this condition is satisfied for some twistings of the scalar on a torus.
Calculating $c_4^{(2)}$: The two-loop correction to the vacuum energy arises from the two-loop vacuum graph of Fig. (1), where the arrows denote the direction of charge flow for complex scalars. As computed in detail in Appendix B (for complex scalars), after renormalization this gives the following finite contribution to the radion potential:

$$V_2(r) = \frac{\alpha}{r^4} f_{ab}(\mu r),$$

$$\rightarrow c_4^{(2)}(a, b) \frac{\alpha}{r^4} (\mu \to 0),$$

(17)

where the function $f_{ab}(\mu r)$, which is symmetric in the twist numbers $(a, b)$, is computed in Appendix B. Numerical integration of the obtained expressions gives for the constant $c_4^{(2)}(a, b) = f_{ab}(0)$ the values presented in Table (1). The result for real scalars is one half the $(0, 0)$ entry for complex scalars.

Recall for complex scalars (for which $B < 0$) use of perturbation theory for the running of $\alpha$ requires $-\alpha(r_s) = c_4^{(1)}/c_4^{(2)}$ to be negative and small. This condition is satisfied for the two cases where there is a twist around only one cycle of the torus, with $\alpha(r_s) = 0.27$ for $(a, b) = (0, 1/3), (1/3, 0)$ and $\alpha(r_s) = 0.15$ for $(a, b) = (0, 2/3), (2/3, 0)$. Using $\alpha(\ell) = \infty$ and $B = -1/2$ in eq. (13) then gives $r_s/\ell = 1.7 \times 10^3$ for $(a, b) = (0, 1/3), (1/3, 0)$ and $r_s/\ell = 8.3 \times 10^5$ for $(a, b) = (0, 2/3), (2/3, 0)$.

Although it is remarkable that some choices of complex-scalar boundary conditions satisfy the consistency conditions and generate large $r_s/\ell$, we emphasize that other choices for $(a, b)$ may also be consistent and larger values of $r_s/\ell$ may be obtained, depending on the particle content of the model. This is because the total value for $c_4^{(1)}$ is obtained by summing the contributions of all of the massless six-dimensional states of the theory. In particular, a hierarchy as large as $r/\ell \sim 10^{15}$ (as would be required if $1/r \sim 10^{-3}$ eV) would require only $\alpha(r_s) = 0.055$ if $\alpha(\ell) = 1$.

Notice, however, that the two-loop contribution comes only from the scalars, and is positive for any nonzero twist, so any model with twisted complex scalars must have its particle content arranged to ensure $c_4^{(1)}$ is small and negative. Positive $c_4^{(1)}$ requires the use of untwisted scalars (real or imaginary), for which $c_4^{(2)} < 0$.

To summarize, we see that a very light six-dimensional scalar field with a cubic self-coupling term can naturally generate logarithmic radion potentials. For instance, in the case of a complex massless scalar ($\mu = 0$) compactified on a torus, and satisfying twisted boundary conditions about one or more of the torus’ cycles, the leading large-$r$ potential which is generated is given by eqs. (14) and (15), with $B = -1/2$ and $c_4^{(1)}$ and $c_4^{(2)}$ are given by Table (1). Notice that the leading coefficients of all powers of $\log r$ are determined by the running of the coupling $\alpha$, and are resummed by standard renormalization group arguments. The result for a real scalar may be obtained by halving the result for an untwisted complex scalar.

Furthermore, this logarithmic potential naturally has the features identified in the introduction to generate an exponentially large hierarchy for $r/\ell$, since higher powers of logarithms have systematically smaller coefficients. This is all the more remarkable given that the only choices which can be made are the boundary conditions which are satisfied by the scalar, and the massless particle content which contributes to the one-loop Casimir energy. In particular, the relative signs required for the existence of exponentially large stationary points for $V(r)$ need not have worked out as well as they did.

To this point, however, we are trading the puzzle of why $r/\ell$ is large for the puzzle of why there should be a light six-dimensional scalar field with cubic self-couplings. We now turn to a search for microscopic models which might be expected to have such scalars.

III. TOWARDS MICROSCOPIC MODEL BUILDING

In this section we ask what is required of a more microscopic model in order to provide the desired light scalar, whose couplings generate the logarithmic potential. Our purpose is twofold. We first intend to describe general features any such model must have, and to identify the naturalness issues which any such model must address. Although we examine several six-dimensional supergravity models in detail, we do not succeed in obtaining a completely natural candidate.

A. Model-Building Issues

The framework for more microscopic model-building depends crucially on how large are the radii which we

Figure 1: The Feynman graph which gives the two-loop contributions to the Casimir energy. To this must be added the one-loop contribution into which the one-loop self-energy counterterm is inserted.
are willing to contemplate. The two main options divide over whether \( r \) is larger or smaller than (1 TeV)\(^{-1}\). If \( r \leq (1 \text{ TeV})^{-1} \), considerable latitude exists because there are fewer constraints on the model-building. The intermediate-scale string [14] provides an attractive version of this scenario with \( 1/\ell \sim 10^{10} \text{ GeV}, 1/r \sim 1 \text{ TeV} \) and \( 1/(M_p r^2) \sim 10^{-3} \text{ eV} \). Here we will focus on the more ambitious option, with \( r \gg (1 \text{ TeV})^{-1} \), with an eye to applications for which \( 1/r \sim 10^{-3} \text{ eV} \). For radii this large, some general observations are immediate.

For radii larger than the weak scale, model-building must take place within the braneworld scenario, in which all (or most) standard-model particles are confined to a four-dimensional surface within the larger six-dimensional space [15]. (There may also be other particles confined to other branes, with which ordinary particles only couple indirectly, the exchange of ‘bulk’ states, which are free to move throughout the six dimensions. There is some freedom to choose what kinds of particles live in the bulk, but the bulk sector must include the graviton. In what follows we assume this framework to be true, although for simplicity, and in keeping with our earlier calculations with flat space, we imagine no cosmological constant in the six dimensions (the ADD scenario [2]), with gravitons not localized around the brane.

In this case \( 1/r \sim 10^{-3} \text{ eV} \) is as large a radius as can be contemplated, due partly to the many bounds on modifications to Newton’s Law on scales larger than a millimetre [14], and partly to limits on particle emission in astrophysical environments like supernovae [4,5]. We believe models predicting radii close to \( 10^{-3} \text{ eV} \) are not yet ruled out in principle by these bounds, although they must be rechecked once specific models are proposed.

In any braneworld scenario with \( 1/r \sim 10^{-3} \text{ eV} \), there can be at most six dimensions, and the scale of physics on the branes themselves must be \( M_b \sim 1 \text{ TeV} \). This is because the four-dimensional Planck’s constant in these models is of order \( M_p^2 \sim M_b^2 + n r^n \), where \( n \) is the number of extra dimensions. For \( n = 2 \) (six dimensions) this states that \( M_b \sim (M_p/r)^{\frac{1}{2}} \sim 1 \text{ TeV} \), as claimed. For \( n \geq 3 \), \( M_b \) is unacceptably low: e.g. when \( n = 3 \), \( M_b \sim (M_p/r^3)^{\frac{1}{3}} \sim 1 \text{ GeV} \). (Of course it was this argument run backwards which led early workers to contemplate radii as large as we are considering.)

We seek models having very light, six-dimensional scalars whose cubic self-couplings are not systematically suppressed by powers of \( 1/r \). The first of these conditions is actually very easy to achieve, since the existence of very light six-dimensional scalars arises very naturally within the braneworld framework. To be six-dimensional the scalars must live in the bulk, and not be localized on the brane (as are ordinary particles, like photons and electrons). Such scalars would be sufficiently light if they were tied by a symmetry to another massless bulk particle, such as the graviton.

For instance, in any supersymmetric variant of the braneworld scenario (with millimetre-scale extra dimensions) supersymmetry must be directly broken at the brane scale, \( M_b \sim 1 \text{ TeV} \), and so cannot be hidden too much from ordinary matter in order to split supermultiplets by TeV scales. To the extent that bulk states only couple to the branes with gravitational strength – certainly true for the graviton supermultiplet – it follows that the mass splittings within the supermultiplets in the bulk are very small. They are small because they are suppressed by their weak gravitational couplings to the supersymmetry-breaking sector, and so are of order \( \Delta m \sim M_b^2/M_p \). Indeed, if \( M_b \sim 1 \text{ TeV} \) this mass splitting is precisely the size of interest to us: \( \Delta m \sim 10^{-3} \text{ eV} \).

In this way we see that bulk scalars in general, and in particular scalars that are tied to the graviton by supersymmetry, can be naturally expected to have masses which are sufficiently small. They are sufficiently light because the connection between \( M_p \) and \( r \) in six-dimensional models ensures the coincidence of the scales \( 1/r \) and \( M_b/M_p^2 \). Furthermore, the gravitational strength of their couplings would have hidden them from discovery before now.

We are led to ask: do scalars arise in plausible representations of six-dimensional supergravity? The answer is ‘yes’, including (in a particular sense, explained next) the graviton multiplet itself. Although strictly speaking the basic six-dimensional graviton supermultiplet consists of the sechsbein, gravitino and a Kalb-Ramond field [15,16]:

\[
\{ e_A, \psi_M, B_{MN}^+ \}, \tag{18}
\]

this multiplet does not admit a Lorentz-invariant action because the Kalb-Ramond field strength is self-dual. Consequently, six-dimensional supergravity lagrangians are written with the supergravity multiplet coupled to a Kalb-Ramond matter multiplet,

\[
\{ B_{MN}^-, \chi, \sigma \}, \tag{19}
\]

which contains an anti-selfdual Kalb-Ramond field plus a fermion and scalar. Thus the nonchiral gravity multiplet naturally contains a scalar, \( \sigma \), which plays the role of the dilaton in low-energy string compactifications.

Other spinless particles, which are tied by supersymmetry to other massless states in the bulk – like gauge bosons or chiral fermions – are also candidates for our light scalar, although they introduce somewhat more model-dependence than does the dilaton.

So far, so good: we have a light, six-dimensional scalar. The thorny issue for making models is obtaining a sizeable cubic self-interaction for these scalars. On one hand, the explicit calculation of the light-scalar couplings is difficult to perform, since both the light scalar and the
radion often have no potential energy at all unless supersymmetry is spontaneously broken. Since our understanding of the nature of supersymmetry breaking in brane models is poor, it is difficult to definitively decide how big the cubic (and other) couplings might be in the low energy theory, once the brane physics is integrated out.

On the other hand, it is very natural to assume that any cubic couplings obtained after supersymmetry breaking should be of the same order of magnitude as are the scalar masses themselves. For instance, to the extent that the scalar potential for a light scalar, \( \varphi \), is a supersymmetry-breaking effect, one might expect the entire potential to have the schematic form:

\[
U(\varphi) = \left( \frac{M_b}{M_p} \right)^2 \left[ \frac{k_1 M_b^2}{2} \varphi^2 + \frac{k_2}{3!} \varphi^3 + \frac{k_4}{4! M_b^2} \varphi^4 + \cdots \right],
\]

which expresses the suppression of all supersymmetry-breaking effects by \( 1/M_b^2 \). (Here the quantities \( k_i \) denote dimensionless \( O(1) \) numbers.) Although such a potential naturally gives a scalar mass of order \( \mu \sim M_b^2/M_p \sim 1/r \), it also predicts a cubic coupling of order \( g \sim (M_b/M_p)^2 \sim 1/(M_b r)^2 \). Unfortunately, such a coupling cannot give a purely logarithmic potential, since it only contributes to higher order in \( 1/r \). This is what happens in the simplest models of supersymmetry breaking, as we now see.

**B. Supergravity Models**

Consider a model consisting of the nonchiral supergravity multiplet, as above, together with an additional matter multiplet consisting of a gauge potential and its superpartner: \( \{A_M, \lambda \} \). With this model we take the dilaton, \( \sigma \), to be the six-dimensional light scalar whose cubic couplings are of interest.

Rather than constructing its coupling to a brane, for calculational simplicity we instead break supersymmetry by artfully choosing the manifold on which we compactify the theory. In what follows we consider two such compactifications. We first compactify on a sphere, and generate in this way a cubic scalar coupling. Although this coupling is too small to generate a logarithmic potential, for reasons which are much as indicated above, it is nonetheless instructive. Our second compactification is on a torus, where we break supersymmetry completely using a Scherk-Schwarz mechanism \[18\]. (That is, we break supersymmetry by assigning different boundary conditions to different members of a supermultiplet.)

With an eye to obtaining a positive radion potential, we require the dilaton to be antiperiodic about one of the cycles of the torus, and keep all of the rest of the fields periodic about both cycles. This compactification has the virtue of allowing many of the results of the previous section to be carried over in whole cloth.

The bosonic part of the supergravity action, coupled to these matter multiplets, is \[15,16\]

\[
\mathcal{L} = - \frac{1}{2 \kappa^2} R - \frac{1}{2} \partial_M \sigma \partial^M \sigma - \frac{1}{12} e^{2 \kappa \sigma} G_{MNP} G^{MNP} - \frac{1}{4} e^{\kappa \sigma} F_{MN} F^{MN} - \frac{2 q^2}{\kappa^4} e^{-\kappa \sigma},
\]

where \( G_{MNP} = 3 \partial_M B_{NP} + 3 \kappa F_{MNP} A_P \) is the Kalb-Ramond field strength, \( F_{MN} \) is the usual abelian field strength for \( A_M \), and all spinors carry a common charge, \( q \), under the gauge group.\[4\] Recall that in six dimensions the couplings have dimension \( \kappa \propto \ell^2 \) and \( q \propto \ell \).

**Compactification on a Sphere:** The equations of motion obtained from this action admit a solution consisting of constant (and arbitrary) \( \sigma \) (\( \partial_M \sigma = 0 \)), a gauge potential which is of the magnetic monopole form \( - \) with monopole number \( \pm 1 \) in two dimensions, and a metric which is the product between flat space and a two-sphere having radius \( r \)\[17\]:

\[
\hat{\mathcal{G}}_{MN} \equiv \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & \frac{1}{r^2} g_{mn} \end{pmatrix}.
\]

\( \hat{g}_{mn} \) is the metric on the (unit) two-sphere, and all other fields vanish: \( \hat{G}_{MNP} = \psi_M = \chi = \lambda = 0 \). This compactification is known as the Salam-Sezgin model \[17\]. The flatness of the four-dimensional metric is only possible when the monopole number is \( n = \pm 1 \), and this may be understood from the fact that the solution leaves one four-dimensional supersymmetry unbroken only with this choice for the monopole number.

With this compactification a potential is generated for \( r \) and \( \sigma \) at tree level, due to the nonzero background values which are taken by the two-dimensional curvature scalar and electromagnetic field strength. The potential may be written as follows \[19\]:

\[
V(r, \sigma) = \frac{2 q^2 e^{-\kappa \sigma}}{\kappa^3 r^2} \left[ 1 - \frac{\kappa^2 e^{-\kappa \sigma}}{4 q^2 r^2} \right]^2.
\]

We see that the particular combination \( X = e^{\kappa \sigma}/r^2 \) appearing within the brackets has developed a potential, along whose minimum the potential vanishes. The combination \( Y = e^{-\kappa \sigma}/r^2 \) appearing as a prefactor in eq. \[23\] is then seen to be a modulus of the compactification, parameterizing a flat direction along the bottom of this potential.

\[1\] Since \( \psi_M \) and \( \lambda \) share the same chirality, while \( \chi \) has opposite chirality, this theory as it stands has anomalies. We imagine these to be cancelled either by the addition of more matter fields or through a Green-Schwarz shift of \( B_{MN} \), without affecting the rest of our analysis.
In principle, this has the form we seek. The field \( X \) is a six-dimensional scalar whose mass is naturally of order \( 1/r \). It also has a cubic self coupling, as measured by the third derivative of the potential in the \( X \) direction, evaluated at the minimum. Once the remaining supersymmetry is broken, lifting the potential’s degeneracy along the \( Y \) direction, one might hope to generate logarithmic terms in \( r \) along the lines described in the previous sections.

Unfortunately, the fly in the ointment is the size of the cubic coupling, which we see is of order \( 1/r^2 \). Although this model nicely illustrates the existence of light scalars, and the generation of a potential for them, it does not furnish an example of a loop-generated logarithmic potential.

**Toroidal Compactification:** An alternative is to compactify on a torus (or an orbifold if we should like to assign twisted boundary conditions to the scalar fields), which allows us to use the results of the previous section’s calculations for the radion potential. Toroidal compactifications are possible for the model in the case \( q = 0 \), in which case a classical solution is given by arbitrary, constant \( \sigma \) and a flat six-dimensional metric.

From the four-dimensional perspective, this vacuum solution preserves \( N = 2 \) supersymmetry and so no potential is generated for \( \sigma \) or \( r \) to any order in perturbation theory. In the absence of supersymmetry breaking, this can be seen explicitly at one loop as being due to the cancellation of the contributions of the various particles:

\[
V^\text{sup}(r) = \sum_n V_n^u(r) = 0,
\]

where \( V_n^u(r) \) denotes the contribution due to the \( n \)’th field, and the superscript ‘\( u \)’ is a reminder that the calculation is performed supersymmetrically, such as with all fields satisfying untwisted boundary conditions.

Suppose we now break the supersymmetry, by assigning twisted boundary conditions only to some members of a supermultiplet, labelled by \( \tilde{n} \). Then the above cancellation no longer obtains, leaving the one-loop result:

\[
V_1(r) = \sum_{\tilde{n}} V_{\tilde{n}}(r) + \sum_{n \neq \tilde{n}} V_n^u(r) = \sum_{\tilde{n}} \left[V_{\tilde{n}}^t(r) - V_{\tilde{n}}^u(r)\right],
\]

where the superscript ‘\( t \)’ denotes the result computed using twisted boundary conditions. In some circumstances the light scalar field itself can be among the twisted fields which break supersymmetry, such as for compactifications on orbifolds having a \( Z_3 \) symmetry.

In this model of supersymmetry breaking, the radiative-corrections computed in the previous section would directly apply, if there were only a nonzero cubic \( \sigma \) coupling. Although there is no potential for \( \sigma \) in the model as it stands, one can be generated by loop effects. Since our chosen method for supersymmetry breaking implies such a potential must vanish as \( r \to \infty \), this should lead once more to cubic couplings which are proportional to powers of \( 1/r \).

Although we do not have a model which circumvents this difficulty, we do see reasons to be hopeful one could be constructed. The basic problem is to construct a model for which the scalar mass is suppressed by a symmetry, but where its cubic self-interactions are not similarly suppressed. One line of model building which this suggests is to tie the scalar to massless spin-one particles by having it lie in a gauge multiplet, since gauge boson masses can be forbidden by unbroken gauge symmetries without also precluding their having cubic self-interactions. (Scalar moduli in toroidal compactifications indeed typically do fall into four-dimensional gauge multiplets of \( N = 2 \) supersymmetry [21].) In this case one might plausibly hope that supersymmetry protects the scalar mass more strongly than it does the scalar’s cubic couplings.

## IV. PHENOMENOLOGICAL CONSEQUENCES AND CONSTRAINTS

What is generic about these models is their prediction of extremely light fundamental scalars which are gravitationally coupled. The two fields which are generic to the models we consider are the radion, \( r \), and the light six-dimensional scalar (such as the dilaton, \( \sigma \), in the supersymmetric examples just considered). We now turn to a discussion of the phenomenologically relevant properties of these scalars, and of the physical signatures which follow from these.

### A. Masses and Couplings

The first question is the size of the scalar mass. We now compute this for the radion field, \( r \), although similar considerations may also apply for the six-dimensional scalar, depending on the model. Although we have discussed the radion potential in some detail in previous sections, to determine its mass from this we must also compute the radion kinetic terms. Since \( r \) begins its life as part of the six-dimensional metric (c.f. eq. (22)) this kinetic energy may be read off (at tree level) from the six-dimensional Einstein-Hilbert action.

A straightforward dimensional reduction of this action using the metric

\[
\mathcal{G}_{MN} = \begin{pmatrix}
\hat{g}_{\mu\nu}(x) & 0 \\
0 & \rho^2(x) h_{mn}(y)
\end{pmatrix},
\]

with \( \rho = r/\ell \), gives the result:
\[
\mathcal{L}_{\text{kin}} = -\frac{1}{2\ell^4} \int d^2 y \sqrt{g} \mathcal{R} \\
= -\frac{r^2}{2\ell^4} \sqrt{g} \left[ R(\hat{g}) - 2 \left( \frac{\partial r}{r} \right)^2 + \frac{\ell^2 R(h)}{r^2} \right],
\]

where we adopt the conventional normalization \( \int d^2 y \sqrt{h} = \ell^2 \).

The Einstein-Hilbert term may be canonically normalized by rescaling \( \hat{g}_{\mu\nu} = \rho^{-2} g_{\mu\nu} \), giving:

\[
\mathcal{L}_{\text{kin}} = -\frac{1}{2\ell^2} \sqrt{g} \left[ R(g) + 4 \left( \frac{\partial r}{r} \right)^2 + \ell^4 R(h) \right].
\]

From eq. (29) it is clear that the redefinition \( r = \ell e^{\xi/2} \) puts the kinetic term for \( \xi \) into canonical form. Adding this to an assumed logarithmic form for the potential, eq. (30), we have the four-dimensional radion-graviton dynamics relevant to cosmology given by:

\[
\frac{\mathcal{L}}{\sqrt{g}} = -\frac{1}{2\ell^2} R(g) - \frac{1}{2} (\partial \xi)^2 - V(\xi),
\]

\[
V(\xi) = e^{-\Lambda \xi} [a_0 + 2a_1 \xi/2 + \cdots + \mathcal{O} e^{q_0 \xi}],
\]

with \( \lambda = p/2 + 2 \) (so \( \lambda = 4 \) if \( V(r) \propto 1/r^4 \)).

We note here in passing that although an exponential potential, \( V(\xi) = \mathcal{V}_0 e^{-\Lambda \xi} \), follows generically (at tree level) from the assumption of a power-law potential, \( V(r) \sim 1/r^p \), the prediction \( \lambda = (p+4)/2 \) found above is not as robust. For instance, if the scalar potential were to mix \( \xi \) with another field – such as happened when \( r \) mixed with \( \sigma \) in the supergravity potential, eq. (23), of the previous section – then \( \lambda = (p+4)/\sqrt{2} \).

To see how this comes about, consider an extreme example, where the potential at very low energies (\( \sim 10^{-3} \) eV) is a function only of one combination of \( \xi \) and \( N \) other canonically-normalized fields, \( \varphi_i, i = 1, \ldots, N \):

\[
V \propto \exp[-\lambda_0 (\xi + \varphi_1 + \varphi_2 + \cdots + \varphi_N)].
\]

In this case the canonically-normalized field which appears in the potential is \( \xi = (\xi + \varphi_1 + \cdots + \varphi_N)/\sqrt{N+1} \) and so in terms of this variable the exponential is:

\[
-\lambda_0 \sqrt{N+1} \xi, \text{ leading to the prediction } \lambda = \lambda_0 \sqrt{N+1}.
\]

The supergravity case has precisely this form with \( \lambda_0 = \frac{1}{2}(p+4) \) and \( N = 1 \).

Regardless of the value found for \( \lambda \), the mass which results from these manipulations is extremely small, being of order

\[
m \sim \frac{1}{M_p r^2} \sim 10^{-33} \text{ eV}
\]

if \( 1/r \sim 10^{-3} \) eV, with the decisive suppression by \( M_p \) arising because of the radion’s kinetic term sharing a common origin with the four-dimensional Einstein-Hilbert action.

Before turning to the strong experimental constraints which any such scalar must satisfy, we make a brief aside to check whether such a small scalar mass is technically natural. That is, we ask if the small mass we have found is an artifact of the tree approximation, or if it is stable under quantum corrections and renormalization.

### B. Naturalness

We now argue that masses as incredibly small as those of eq. (32) can be stable under radiative correction, in models such as were considered in the previous sections [24]. This remarkable stability may be seen most easily by integrating out physics at successive scales, and asking how large the contributions to the scalar mass must be as these scales are integrated out.

Consider first the contribution from energies much smaller than \( 1/r \). In this energy range the effective theory is four-dimensional, and so the standard analysis for a 4D scalar mass applies. Since the canonically normalized radion, \( \xi \), couples with strength \( 1/M_p \) to everything in the effective 4D theory, we expect loops to generically generate mass terms in the potential which are of order \( \delta V \lesssim \Lambda^4 (\xi/M_p)^2 \), with \( \Lambda \) representing the largest energy scale relevant to the effective theory. Since use of a 4D theory presupposes \( \Lambda \lesssim 1/r \), we expect loop effects from these low scales to alter the radion mass by an amount \( \delta m^2 \sim \Lambda^2/M_p \sim 1/(M_p r^2) \). Since these are of the same order as the mass itself, such corrections do not destabilize the radion mass.

Potentially more dangerous are quantum effects from scales in the range \( 1/r \lesssim \Lambda \lesssim M_b \). Naively one might imagine regarding the effective theory in this case as a complicated 4D theory, involving many Kaluza-Klein states, and so again apply the 4D analysis just described. Using \( \Lambda \sim M_b \sim \text{TeV} \) would then lead one to the usual expectation \( \delta m \sim M_b^2/M_p \sim 1/r \), and so that scalar masses as small as \( m \sim 1/(M_p r^2) \) must be fine-tuned.

The 4D argument is misleading, however, because it hides many symmetries which restrict the form which any UV-sensitive corrections must take. Specifically, the effective theory for scales \( \Lambda > 1/r \) is really six-dimensional, and the radion – being a component of the 6D metric – is subject to the strong constraints of 6D locality and general covariance. The implications of these conditions are hidden in a KK analysis because these conditions are difficult to enunciate in terms of the metric’s KK modes.

In the 6D theory there are two kinds of UV-sensitive terms involving the 6D metric, \( G_{MN} \), which can arise in the effective action. Those due to virtual bulk states may be written as local 6D effective interactions like

\[
S_{\text{bulk}} = \int d^6 x \sqrt{\hat{g}} \left[ A \mathcal{L}^6 + B \mathcal{L}^4 \mathcal{R} + C \mathcal{L}^2 \mathcal{R}^2 + \cdots \right],
\]

(33)
where the ellipses represent other curvature squared terms, plus terms involving higher derivatives. $A, B$ and $C$ are dimensionless constants. By contrast, those involving only virtual brane-bound states are localized at the position of the branes, and have the form

$$S_{\text{brane}} = \sum_b \int_{\Sigma_b} d^4 y \sqrt{\gamma} \left[ a_b \Lambda^4 + \cdots \right], \quad (34)$$

where $\gamma_{\mu\nu}$ is the induced metric on brane $\Sigma_b$ and the omitted terms can involve both extrinsic and intrinsic curvatures. $a$ is a dimensionless constant.

Suppose, first, that the extra-dimensional metric were roughly spherical, and so $R \sim 1/r^2$. In this case $S_{\text{bulk}}$ would imply a radion potential of the form

$$V(r) \sim A \Lambda^6 r^2 + B \Lambda^4 + C \Lambda^2 / r^2 + \cdots. \quad (35)$$

As stated earlier, like all other workers we put aside the cosmological constant problem, and so take the six-dimensional cosmological term to vanish: $A \sim 0$. The vanishing of the $B$ term requires only that the extra dimensions have the topology of a torus.

(In passing we remark that within a supersymmetric theory a small bulk cosmological constant might not be so hard to arrange. In the models of interest we have supersymmetry in the bulk space, with supersymmetry broken at scale $\ell$ on the branes on which we live. Since all bulk-brane couplings are gravitational in strength, we saw that supersymmetry-breaking interactions of the four-dimensional gravity multiplet are suppressed by powers of $1/M_p \propto 1/r$. On dimensional grounds, any supersymmetry-breaking mass splittings within the four-dimensional gravity multiplet are at most of order $M_p^2 / M_p \sim 1/r$, leading one to expect the vacuum energy to be $AA^6 \sim 1/r^6$.)

Once the cosmological constant is removed, the quadratically-divergent curvature-squared term is seen to give a dangerous contribution to the radion mass, since it predicts $\delta m \sim \Lambda / (M_p r) \gg m$. This contribution does not arise for the toroidal geometries we are using, however, because these are flat: $R = 0$. The naturalness question in this language then becomes the question whether quantum corrections can allow the extra-dimensional curvature to satisfy $R \ll 1/r^2$. We now argue that, for six-dimensional spaces, this can be so.

In the absence of a bulk cosmological constant, the most UV-sensitive quantum contributions are to the brane tensions. Although the cosmological constant fine-tuning requires $\sum_b a_b \Lambda^4 \sim 0$, it does not require the tension on each brane to separately vanish. Furthermore, our knowledge of the spectrum of observable particles on our brane suggests $a_0 = O(1)$ for our brane at least. One might worry that large tensions like these must generate large extra-dimensional curvatures once they are considered as sources for the gravitational field.

We now argue that this does not happen. The key point is that even a large $\Lambda^4$ type vacuum energy on the brane does not produce any curvature in the extra dimensions. This is because in 6D this is NOT a cosmological constant, but is instead a delta function, localized at the brane position. As a result, we must ask to what extent using this as a source forces us into having a curved metric in the two extra dimensions. It is a special property of 6D that point 3-brane sources do not induce curvature in the transverse dimensions. This is because the gravitational field of any straight system with co-dimension 2 (e.g., a cosmic string in 4D or a 3-brane in 6D) is strictly flat. The gravitational influence of such a source arises because of the conical defect which is induced at the position of the brane, rather than from local curvature.

We see that the radion mass is protected from receiving large corrections for two reasons. First, like all of the metric’s KK modes, it must remain massless if $1/r \to 0$ because in this limit it is tied to the massless 4D graviton by an unbroken symmetry: 6D Lorentz invariance. For flat compactifications this suppresses the contributions from loops in the effective 6D theory – i.e., for $\Lambda > 1/r$ to be of order $\delta V(r) \sim 1/r^4$. The radion then acquires a further mass suppression by $1/M_p$ because its kinetic term is enhanced by $M_p^2$ relative to the potential. (The same is not true for other KK modes because these acquire both their mass and their kinetic terms from the 6D Einstein action, leading to no relative enhancement of the kinetic term relative to the mass term.)

C. Kinetic Term Corrections

Although they do not change the order of magnitude of the mass inferred, radiative corrections to the kinetic terms do have an important impact on our later confrontation with experimental bounds, and so we pause to consider them briefly here.

We have seen that $\alpha(r)$ corrections can modify the potential at large $r$ in an important way, by adding logarithmic corrections in $r$. The same is true for the radion kinetic terms, which are also dominated for $r \gg \ell$ by logarithmic corrections arising due to powers of $\alpha(r)$. Including these corrections, the relevant kinetic terms in the effective theory at large scales have the form:

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2 r^2} \sqrt{g} A^2 [\alpha] \left[ R(g) + 4 B^2 [\alpha] \left( \frac{dr}{r} \right)^2 \right], \quad (36)$$

where both $A$ and $B$ have the series expansion $1 + O(\alpha)$.

In general, the function $B$ changes the field redefinition, $\xi(r)$, which is required to canonically normalize the radion kinetic terms. For general $A$ and $B$ we have

$$\xi = \frac{2}{\ell} \int_{\ell}^{r} \frac{F[\alpha(r')]}{r'} \, dr',$$  \quad (37)
where $F^2 = B^2 + \frac{3}{2} (A'/A)^2 \beta^2$, $A' = dA/\alpha$ and $\beta = r d\alpha/dr$.

$A$ and $B$ are independent of $r$ only to next-to-lowest order, where $A \approx 1 + a_0 r$ and $B \approx 1 + b_0 r$, and so to this order $\xi$ is still logarithmically related to $r$, but we have $\lambda \approx (\frac{2}{3} p + 2)[1 - b_0 r]$. Beyond this order $\xi$ need not be strictly logarithmic in $r$, and so the functional form of the potential $V(\xi)$ changes in a more complicated way. We shall see that this has important implications for the cosmology of the radion field.

D. Experimental Constraints

We see there are three important mass scales for the bulk sector of these models: the microscopic scale, $1/\ell$; the Kaluza-Klein mass scale, $1/r$; and the radion mass scale, $1/(M_p r^2)$. These imply potentially interesting modifications of gravity on a variety of scales. For instance, the intermediate-scale scenario $1/\ell \sim 10^{10}$ GeV and $1/r \sim 1$ TeV — puts $1/(M_p r^2) \sim 10^{-3}$ eV into the millimetre range. Alternatively, for large extra dimensions $1/r \sim 10^{-3}$ eV and $1/M_p r^2 \sim 10^{-33}$ eV.

Millimetre Scales

As the above examples show, very different choices for $\ell$ and $r$ imply modifications to gravity at scales of order $10^{-3}$ eV. The implications for experiments probing sub-millimetre range forces can differ dramatically depending on the nature of the microscopic physics. In the most extreme case $1/r \sim 10^{-3}$ eV and any observed modifications to Newton’s Law on these scales would signal the transition to six-dimensional gravitational physics. Searches for such deviations are now starting to probe forces within this interesting range.

The exact kinds of signals experiments searching for these deviations should expect to see depend on the precise nature of the couplings of the relevant states to ordinary matter. Unfortunately these are difficult to cleanly predict, since they depend on the scenario involved. Furthermore, if the modifications are due to the onset of 6-dimensional physics, then predictions are also hampered by the large size of $\alpha(r)$ whose growth at low energies underlies our mechanism for generating large radii. One generically expects deviations from the equivalence principle, and from the $1/r$ falloff of the gravitational potential, at scales smaller than a millimetre.

Very Long-Range Forces

Since the masses of some of the lightest states, like the radion, can be incredibly small, $\sim 10^{-33}$ eV, their couplings are very strongly constrained. Such small masses make the radion’s Compton wavelength, $1/m$, of order $10^{26}$ m, permitting them to mediate extremely long-ranged forces. We will argue here that the properties of the scalar predicted by our mechanism for radius stabilization can evade these bounds, and do so in an interesting way.

If the tree-level action of eqs. (27) and (29) were the whole story, our model would be ruled out. Since ordinary matter, trapped as it is on a four-dimensional brane within the six dimensions of the bulk, couples to the radion only indirectly through $g_{\mu\nu}$, these expressions, with the field redefinition $\Phi = r^2$ show that the radion behaves precisely as a Brans-Dicke scalar, with coupling parameter $\omega = -\frac{1}{3}$. Although Brans-Dicke scalars can be phenomenologically acceptable, even if they are massless, solar-system tests require their couplings must be strongly suppressed relative to gravity, with current constraints requiring $\omega \gtrsim 3,000$.

The story is much more interesting once the radiative corrections due to $\alpha(r)$ are included, since these imply $\omega$ becomes a function of $r$. For weak coupling, for instance, $\omega(\Phi) = -\frac{3}{2} + \omega_0 r_0 \log \Phi + \cdots$.

The recognition that $\omega$ is a function of $\Phi$ is crucial when comparing with experiments, since it implies in particular that $\omega$ is a function of time as the universe evolves cosmologically. Since the best limits only apply at the current epoch, the constraints are satisfied so long as $\omega(t)$ approaches sufficiently large values sufficiently quickly as the universe evolves towards the present. Better yet, the evolution of $\omega(\Phi)$ in general scalar-tensor theories has been found to generically be attracted towards large $\omega$ during cosmological evolution [26], indicating that current bounds can be generically satisfied without making unreasonable assumptions about the cosmological initial conditions.

As we report in more detail in a companion publication [27], we have applied the analysis of ref. [26] to realistic cosmological evolution, using the functions $\omega(r)$ and $V(r)$ which are plausibly obtained within our scenario. We find that current constraints on post-Newtonian gravity and constraints from cosmology can all be satisfied, and that it is also easy to arrange the energy density of the scalar field to be currently starting to dominate the energy density of the universe, making the radion a natural microscopic realization of the ‘Planck scale quintessence’ described in [8].

Accelerator Signals

In the event that $1/\ell \sim$ TeV and $1/r \sim 10^{-3}$ eV experimental signals can be expected at future collider experiments, due to reactions where bulk states are emitted and so appear as missing energy. These reaction rates are not $M_p$ suppressed due to the cumulative contribution of the numerous KK levels of the six-dimensional fields. Although it is beyond the scope of this paper to provide detailed calculations of the signals expected for the emission of the six-dimensional scalars we consider here, these
should add to and closely resemble the signals which have been computed for the emission of six-dimensional metric states \( \mathbb{R}^2 \mathbb{R}^2 \) and six-dimensional vector states \( \mathbb{R}^2 \).

V. SUMMARY

Our purpose has been to propose a mechanism for naturally generating exponentially large radii, within a plausible scenario of extra-dimensional physics. Besides making the usual assumption that there is no large microscopic cosmological constant, our mechanism requires the following three ingredients:

1. The universe must pass through a phase during which there are effectively six large dimensions;
2. A very light – mass \( m \lesssim \mathcal{O}(1/r) \) – spinless particle must be present within the six-dimensional effective theory;
3. The light scalar must have a cubic self interaction in the effective six-dimensional theory, whose coupling \( g \) is not itself already suppressed by powers of \( 1/r \).

Under these circumstances we have shown that the renormalization of the one marginal six-dimensional coupling, \( g \), generically generates a potential energy which depends logarithmically on the radius, \( r \), of the 2 extra dimensions.

Besides generically having a runaway minimum, with \( r \to \infty \), this potential also has a minimum with \( r/\ell \) exponentially large, where \( \ell \) is the length scale of the microscopic six-dimensional theory. The existence of this minimum for perturbatively small couplings requires specific relative signs between the contributions of successive loops in \( V(r) \). Within the context where the radion potential is generated as a Casimir energy (such as due to the light six-dimensional scalar) we have shown that the required signs can be obtained.

In order to determine how natural our above three requirements are, we explored several kinds of six-dimensional models explicitly. We have found that supersymmetry can naturally assure items 1 and 2 in the list above, but assuring item 3 is more difficult. In the models explored, supersymmetry both protects the scalar mass and its cubic coupling to be of order \( 1/r \), violating item 3. Unfortunately, it is difficult to exhaustively explore the potentials which can be generated for such scalars given the present poor understanding of how supersymmetry breaks.

The same features of the radion potential which allow it to have naturally large extrema, also allow the radion to evade all experimental constraints, despite the extremely small masses which are possible: \( m \sim 10^{-33} \text{ eV} \). The evasion of these bounds arises through the cosmological evolution of the radion’s effective couplings, and so rely crucially on the existence of the same radiative corrections which generate the exponentially large radii. The role played by these corrections is to convert the radion from an ordinary Brans-Dicke scalar to a more general scalar-tensor theory for which the couplings to matter evolve cosmologically to small values.

If the leading contribution to the radion energy is positive (as may be arranged by adjusting the model’s particle content), the cosmological evolution of this very light radion suggests its interpretation as the source of quintessence, accounting for the present-day cosmological constant. We have verified that it is possible to construct a working quintessence model based on this picture, \([27]\), and believe that a more systematic exploration of the cosmological implications of our radion-stabilization mechanism would be very fruitful.

VI. ACKNOWLEDGEMENTS

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VII. APPENDIX A: THE ONE LOOP CASIMIR ENERGY

In this appendix we provide expressions for the one-loop Casimir energy (as a function of radius) of a complex scalar field \( \phi \) with mass \( \mu \) on the six-dimensional product of four-dimensional Minkowski space with a torus. The interaction Lagrangian \( \mathcal{L}_{\text{int}} = (g/3!)(\phi^3 + c.c) \) is invariant under \( Z_3 \) rotations \( \phi \to w\phi \) where \( w \) is a cubic root of unity, i.e. \( w^3 = 1 \). We compute the Casimir energy for fields which satisfy the corresponding \( Z_3 \) invariant boundary conditions about the torus’ two nontrivial cycles. The momenta of the scalar in the toroidal directions are then \( (n + a, m + b) \) in units of \( 2\pi/r \), where the constants \( a \) and \( b \) can take the values 0 for strictly periodic boundary conditions and 1/3 or 2/3 in the twisted cases.

The one loop vacuum energy for such a complex scalar, is given (for Euclidean momenta) by

\[
\Lambda_1 = \frac{1}{r^2} \sum_{mn=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \log \left( k^2 + \mu_{mn}^2 \right),
\]

\[
= -\frac{1}{r^2} \int_0^{\infty} \frac{dy}{y} \int \frac{d^4k}{(2\pi)^4} \sum_{mn=-\infty}^{\infty} \exp \left[ -y(k^2 + \mu_{mn}^2) \right],
\]

(38)
where
\[ \mu_{mn}^2 = \mu^2 + \left( \frac{2\pi}{r} \right)^2 [(m + a)^2 + (n + b)^2]. \]  

(39)

In the second of eqs. (38) the integral over \( k \) is now gaussian while the sums can be performed explicitly in terms of Jacobi theta-functions. In particular we will make use of

\[ \theta_3(z, q) = \sum_{n=-\infty}^{\infty} q^{n^2} e^{2inz} \]  

(40)

where customary \( q = \exp(i\pi\tau) \). For the sums in eqs. (38) we then have

\[ \sum_{n=-\infty}^{\infty} e^{-\pi x(n+a)^2} = e^{-\pi x a^2} \theta_3(i\pi ax, e^{-\pi x}) \]  

(41)

with \( x = 4\pi y/r^2 \). Using these in eq. (38), and integrating the result over the torus to obtain a four-dimensional effective potential

\[ V(r) = \int d^2z \Lambda = 2r^2 \Lambda, \]  

(42)

leads to

\[ V_1(r) = -\frac{1}{r^4} \int_0^\infty \frac{dx}{x^3} \ e^{-\beta x} e^{-\pi x(a^2+b^2)} \]  

\[ \times \ \theta_3(i\pi ax, e^{-\pi x}) \theta_3(i\pi bx, e^{-\pi x}), \]  

(43)

where \( \beta = \mu^2 r^2/(4\pi) \) and \( a, b = 0, 1/3 \) or \( 2/3 \) according to the boundary conditions which are appropriate.

From the asymptotic forms for the theta functions

\[ \theta_3(z, e^{-\pi x}) = 1 + \cos z e^{-\pi x} + \cdots \]  

(44)

as \( x \to \infty \) and

\[ \theta_3(z, e^{-\pi x}) = \frac{1}{\sqrt{x}} e^{-\sin^2 z/x} \left[ 1 + \mathcal{O}(e^{-\pi/x}) \right], \]  

(45)

as \( x \to 0 \), we see that the one-loop vacuum energy converges in the infrared \( (x \to \infty) \) even if \( \mu \) vanishes. For twisted boundary conditions this convergence is exponential, reflecting the absence of exactly massless four-dimensional modes in this case.

On the other hand, the vacuum energy diverges in the ultraviolet \( (x \to 0) \), but this divergence is independent of \( r \) and so may be absorbed into a renormalization of the six-dimensional cosmological constant. The finite, \( r \)-dependent result is obtained by subtracting the result for \( r \to \infty \), giving eq. [3].

VIII. APPENDIX B: THE TWO-LOOP CASIMIR ENERGY

In this appendix we compute the contribution to the Casimir energy coming from the two-loop graph of Fig. (1), plus the graph in which a wavefunction and mass renormalization counterterm is inserted into the one-loop Casimir energy. We show that the result converges, as expected on general grounds, in the infrared and ultraviolet, and evaluate the constant \( c_4^{(2)} \) which controls its overall size.

Our starting point is the contribution to the four-dimensional energy density coming from the evaluation of Fig. (1):

\[ V_2^{\text{Fg}^2}(r) = -\frac{g^2}{6r^2} \sum_{njkl} \int_0^\infty \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \]  

\[ \times \ \frac{1}{(p^2 + \mu_{nj}^2)(q^2 + \mu_{kl}^2)((p + q)^2 + \mu_{l+k+n}^2)} \]  

As before, \( \mu_{mn}^2 = \mu^2 + (2\pi/r^2)[(n + a)^2 + (l + b)^2] \), where \( a, b = 0, 1/3, 2/3 \) reflect the scalar boundary conditions and \( n, l \) are integers. Use of the identity \( 1/X = \int_0^\infty ds e^{-sx} \) for each propagator permits the performance of the gaussian integrals. Using the basic result eq. (41), we can then write the quadruple sum in eq. (46) as

\[ \sum_{njkl} e^{-\mu_{nj}^2 - \mu_{kl}^2 - \mu_{l+k+n}^2} = e^{-\beta(s+t+u)} S_a S_b, \]  

(47)

where \( \beta = \mu^2 r^2/(4\pi) \) and we define the function \( S_a \) by:

\[ S_a = e^{-\pi(u+0)a^2} \sum_{n=-\infty}^{\infty} e^{-\pi(n+a)^2} - 2\pi au(n+a) \]  

\[ \times \theta_3(i\pi(u+2au+at), e^{-\pi(u+t)}). \]  

(48)

The function \( S_b \) is defined correspondingly.

Combining everything gives the following contribution to \( V(r) \):

\[ V_2^{\text{Fg}^2} = -\frac{g^2}{6r^4(4\pi)^3} \int_0^\infty \frac{ds \ dt \ du}{(st + su + ud)^2} \]  

\[ \times e^{-\beta(s+t+u)} S_a S_b. \]  

(49)

Now the first question concerns divergences. The infrared limit corresponds to taking \( s, t, u \) large. For \( u + t \gg 1 \) we then obtain from the asymptotic form eq. (44) and the defining equation eq. (41)

\[ S_a = e^{-\pi(s+t+4u)a^2} \theta_3(i\pi a(s+2u), e^{-\pi(s+u)}). \]  

(50)

Since \( S_a \) thus goes at worst to unity in this limit, we see that the integral converges in the infrared, even if \( \mu \to 0 \).

It has several sources of divergence in the ultraviolet. The divergence when all three variables, \( s, t \) and \( u \), vanish is removed by subtracting the result for \( r \to \infty \),
implying that it is removed by renormalizing the six-dimensional cosmological constant. Using the asymptotic form eq. (53) for \( x \to 0 \), one finds for \( u + t \ll 1 \)

\[
S_a = \frac{1}{\sqrt{u + t}} \theta_3(i\pi aw, e^{-\pi w}) \left[ 1 + \mathcal{O}(e^{-\pi/(t+u)}) \right]. \tag{51}
\]

where \( w = (su + st + tu)/(u + t) \). The result, after subtracting the large-\( r \) limit, is therefore obtained from eq. (53) by the replacement

\[
S_a S_b \to \frac{1}{st + su + tu}. \tag{52}
\]

Although this subtraction renders finite the limit where \( s, t, u \) all vanish with fixed nonzero ratios, it does not cure the ultraviolet divergence when two of the variables \( s, t, u \) vanish with the third held fixed. This divergence cancels with the result obtained when the counterterms for wavefunction and mass renormalization are inserted into the one-loop Casimir energy.

To see how this works, notice that an evaluation of the one-loop scalar self-energy for the torus is

\[
\Pi(p^2) = \frac{g^2}{4\pi^2} \int_0^{\infty} dt \; dt \; du \; \sum_{i} \int \frac{d^4 k}{(2\pi)^4} e^{-t(k^2 + \mu^2) - u((k-p)^2 + \mu^2)}
\]

The integrals and sums here are of the same kind as before and we obtain

\[
\Pi(p^2) = \frac{g^2}{4\pi^2} \int_0^{\infty} dt \; dt \; du \; e^{-\beta(u+t)} e^{-\left(\frac{s}{u+t}\right)^2} \times e^{-\pi(s^2 + b^2)} \theta_3(ixa, e^{-x}) \theta_3(ixb, e^{-x}) \tag{54}
\]

where \( x = u + t \). For an on-shell renormalization scheme the mass and wavefunction counterterms are now found by expanding this in powers of \( p^2 \). The resulting integrals are dominated by the contributions from the region where \( u + t \ll 1 \) and gives:

\[
\delta Z = \frac{g^2}{(4\pi)^3} \int_0^{\infty} dt \; dt \; du \; \frac{ut}{(u+t)^2} e^{-\beta(u+t)} \tag{55}
\]

\[
\mu^2 \delta Z + \delta \mu^2 = -\frac{g^2}{(4\pi)^2} \int_0^{\infty} dt \; dt \; du \; e^{-\beta(u+t)} \tag{56}
\]

where we eventually will take the regulator \( \epsilon \to 0 \) in the ultraviolet limit. Denoting \( \xi(p^2) = \delta Z(p^2 + \mu^2) + \delta \mu^2 \), the insertion of these counterterms into the one-loop Casimir energy gives

\[
V_{2t} = -\frac{1}{2(4\pi)^2} \int_0^{\infty} ds \sum_{i} \int \frac{d^4 p}{(2\pi)^4} \xi(p^2) e^{-s(p^2 + \mu^2)}
\]

\[
= \frac{g^2}{6r^4(4\pi)^3} \int_0^{\infty} ds \; dt \; du \; e^{-\beta(s+t+u)} T_{ab}(s, t, u) \tag{57}
\]

where we have symmetrized the result with respect to permutations of \( s, t, u \) and the function \( T_{ab} \) is defined by

\[
T_{ab} := \left[ 1 - \frac{2ut}{s(u+t)} \right] \frac{1}{s^2(t+u)^3} \times \left[ e^{-\pi(s^2 + b^2)} \theta_3(ixa, e^{-\pi s}) \theta_3(ixb, e^{-\pi s}) - \frac{1}{s} \right] + \text{cyclic permutations of } s, t, u. \tag{58}
\]

The two-loop contribution to the radion potential is obtained by summing \( V_{2t}^2(r) \) with the result, eq. (53), from Fig. (1). It is both ultraviolet and infrared finite, and when evaluated at \( \mu = 0 \) gives expression (17) of the text.
introduce a dependence of $V(r)$ on nominally irrelevant interactions. Although generically power-law infrared divergences do not occur in six uncompactified dimensions, they can exist for compactifications where the scalar field has zero modes (i.e. states whose mass, $\mu$, is much smaller than $1/r$). The power-law infrared divergences as $\mu \to 0$ can then be understood within the effective 4d theory applicable to distances much larger than $r$, where they arise because the cubic self-interaction has positive mass dimension, $g_3 \sim g/r$, and so is super-renormalizable. Such an interaction can lead to contributions to $V(r)$ involving positive powers of $g_3/\mu$, which are singular like a power of $1/\mu$ as $\mu \to 0$. We believe they do not change our conclusion regarding the irrelevance of all other interactions at large $r$, however, since they do not introduce positive powers of $r$ into any results. Of course, these complications can be completely avoided for scalars satisfying twisted boundary conditions on the torus. (We thank S. Shenker for conversations on this point.)

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