Multi-Wavelength Constraints on the Outflow Properties of the Extremely Bright Millisecond Radio Bursts from the Galactic Magnetar SGR 1935+2154

Shotaro Yamasaki*, 1, Kazumi Kashiyama2,3, and Kohta Murase4,5,6,7

1 Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel
2 Department of Physics, School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
3 Research Center for the Early Universe, School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
4 Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
5 Department of Astronomy & Astrophysics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
6 Center for Multimessenger Astrophysics, Institute for Gravitation and the Cosmos, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
7 Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto, Kyoto 606-8502 Japan

Accepted XXX. Received YYY; in original form ZZZ.

ABSTRACT

Extremely bright coherent radio bursts with millisecond duration, reminiscent of cosmological fast radio bursts (FRBs), were co-detected with anomalously-hard X-ray bursts from a Galactic magnetar SGR 1935+2154. We investigate the possibility that the event was triggered by the magnetic energy injection inside the magnetosphere, thereby producing magnetically-trapped fireball (FB) and relativistic outflows simultaneously. The thermal component of the X-ray burst is consistent with a trapped FB with an average temperature of ~ 200–300 keV and size of ~ 10^8 cm. Meanwhile, the non-thermal component of the X-ray burst and the coherent radio burst may arise from relativistic outflows. We calculate the dynamical evolution of the outflow, launched with an energy budget of 10^{39}–10^{40} erg comparable to that for the trapped FB, for different initial baryon load \( \eta \) and magnetization \( \sigma_0 \). If hard X-ray and radio bursts are both produced by the energy dissipation of the outflow, the outflow properties are constrained by combining the conditions for photon escape and the intrinsic timing offset \( \Delta t \) among radio and X-ray burst spikes. We show that the hard X-ray burst must be generated at \( r_X \geq 10^5 \) cm from the magnetar, irrespective of the emission mechanism. Moreover, we find that the outflow quickly accelerates up to a Lorentz factor of \( 10^5 \leq \Gamma \leq 10^3 \) by the time it reaches the edge of the magnetosphere and the dissipation occurs at \( 10^{13} \) cm. Our results imply either extremely-clean (\( \eta \geq 10^5 \)) or highly-magnetized (\( \sigma_0 \geq 10^9 \)) outflows, which might be consistent with the rarity of the phenomenon.

Key words: radio continuum: transients, stars: magnetars, stars: neutron

1 INTRODUCTION

Recently, one of the most prolific transient magnetars, SGR J1935+2154 (Israel et al. 2016) went into an intense bursting episode on April 27 2020, and hundreds of X-ray bursts were recorded in a few hours (Borghese et al. 2020; Younes et al. 2021). During this active phase, an extremely intense radio burst with millisecond duration, reminiscent of cosmological fast radio bursts (FRBs), was detected by radio telescopes CHIME/FRB (CHIME/FRB Collaboration et al. 2020) and STARE2 (Bochenek et al. 2020) on April 28 2020, strengthening the connection between FRBs and magnetars. Importantly, Insight/HXMT (Li et al. 2021a), Konus-Wind (Ridnaia et al. 2021), INTEGRAL/IBIS (Mereghetti et al. 2020) and AGILE (Tavani et al. 2021) independently detected an X-ray burst associated with the FRB-like radio burst (Mereghetti et al. 2020; Li et al. 2021a; Ridnaia et al. 2021; Tavani et al. 2021). The timing of the emissions is the same within the observational uncertainties and the radio burst detected by CHIME have a similar temporal structure to the X-ray burst. The X-ray burst is peculiar in that the spectrum is much harder than a typical SGR burst with comparable (or even higher) fluences and FRBs were not detected with many other X-ray bursts from the same source (Lin et al. 2020).

Theoretical interpretations of the April 28 event are broadly classified into two categories: “close-in” and “far-away” scenarios, depending on how close the radio emission is generated from the central engine (i.e., the magnetar). The former includes the curvature radiation in the open magnetic fields (e.g., Lu et al. 2020; Katz 2020; Yang & Zhang 2021), the plasma instability triggered by magnetic...
reconnection (Lyutikov & Popov 2020; Lyutikov 2020) and the low-altitude magnetospheric emission (Wadiasingh & Timokhin 2019; Wadiasingh & Chirenti 2020), whereas the latter invokes a maser-type instability at the shock between magnetar flare wind and the pre-existing material (e.g., Margalit et al. 2020; Yuan et al. 2020; Yu et al. 2021). The possibilities of generating double/multiple-peaked radio pulses by the quasi-periodic oscillation of magnetars (Wang 2020) or the scintillation effect (Simard & Ravi 2020) are also discussed. Whether the April 28 event as well as cosmological FRBs are generated by close-in or far-away models is subject to intense debate in the community, both from observational and theoretical aspects. Regarding cosmological FRBs, recent FAST observations on varying polarization angles in some repeaters (Luo et al. 2020) and discovery of recurrent bursts from FRB 121102 with too short separations down to milliseconds and too large energy budget (Li et al. 2021b) to accommodate the far-away (maser-type) models may prefer close-in (curvature-type) models. Meanwhile, it has also been pointed out that close-in models could have some theoretical flaws because realistic plasma effects are often neglected (e.g., Lyubarsky 2021).

In either the close-in and far-away models, the event is triggered by a deposition of magnetic energy in the magnetosphere, which may results in the formation of an electron/positron ($e^\pm$) plasma bubble confined to the stellar surface by the strong magnetic pressure (Thompson & Duncan 1995, 1996), so-called trapped fireball (FB), and also launching an outflow of relativistic plasma (or an expanding FB). In this paper, we aim to put general constraints on such FBs (i.e., the properties of the outflow responsible for the X-ray and/or radio bursts) with modest assumptions on the radiation mechanism, based on the multi-wavelength observations of the April 28 event. While the thermal component of the X-ray burst is consistent with a trapped FB, the non-thermal component of the X-ray burst and the coherent radio burst may arise from relativistic outflows. Regarding the origins of hard X-ray burst, we examine the two possibilities that (1) it is produced in the vicinity of the NS or the trapped FB (§3.1) and that (2) it arises from the relativistic outflow (§4.2). The second possibility is investigated by considering the evolution of outflows with different properties, assuming that the hard X-ray and coherent radio bursts have been produced due to some sort of energy dissipation inside the outflows, which broadly includes far-away models.

This paper is organized as follows. In §2, we summarize the key observational properties of the April 28 event. We constrain the total energy budget of the event in §3.1 by assuming that the thermal component of the X-ray spectrum is due to the trapped FB. In §3.2, we calculate the dynamical evolution of the outflow, which is likely responsible for the FRB-like burst and the non-thermal part of the hard X-ray burst spectrum. Constraints on the outflow properties are set from the general conditions required to generate the emission in §4 and our findings are summarized and implications are discussed in §5. Hereafter, we use $Q_s \equiv Q/10^5$ in cgs units.

## 2 KEY OBSERVED PROPERTIES OF APRIL 28 EVENTS FROM SGR 1935+2154

Here we review the key observed properties of the radio and X-ray bursts from SGR 1935+2154 on May 28 2020 (see also Table 1).

**SGR 1935+2154.** SGR 1935+2154 is one of the most prolific transient magnetars; the spin period and the spindown rate are measured to be $P_{\text{spin}} = 3.24$ s and $P' = 1.43 \times 10^{-11}$ s$^{-1}$, respectively (Israel et al. 2016). Accordingly, the surface dipole magnetic field strength is estimated as $B_p = 2.2 \times 10^{14}$ G. This magnetar has been recently in an active phase since April 27 2020 (Younes et al. 2021). The distance estimate is somewhat uncertain. SGR 1935+2154 is spatially associated with the supernova remnant (SNR) G57.2+0.8. Throughout this work, we adopt a source distance of 10 kpc, which is consistent with the different distance estimates between 6.7 kpc (Zhou et al. 2020) and 12.5 kpc (Kothes et al. 2018) in the literature.

### Radio Observations

The radio burst from SGR 1935+2154 was detected independently by CHIME at 400–800 MHz and FAST at 1.4 GHz (CHIME/FRB Collaboration et al. 2020; Bochenek et al. 2020). The CHIME burst consists of two sub-bursts with widths of $\sim 5$ ms separated by $\sim 30$ ms, whereas the FAST2 burst has a single narrow spike with a width of 0.61 ms. According to the total fluence reported by STARE2, the radiated energy (isotropically equivalent) is estimated to be $E_{\text{radio}}^{\text{iso}} = (0.3-2.4) \times 10^{35}$ (d/10 kpc)$^2$ erg. The observed dispersion measures (DM) in both radio observations are consistent with a single value, $DM \sim 332.7$ pc cm$^{-3}$ (CHIME/FRB Collaboration et al. 2020; Bochenek et al. 2020), which is in agreement with sources in the Galactic plane. Except for the detection of other low-luminosity radio events, the FAST set stringent upper limits on the radio flux associated with many other X-ray bursts (Lin et al. 2020).

### X-ray Observations

There are four co-detections of the hard X-ray burst associated with the FRB-like radio burst (Li et al. 2021a; Ridnaia et al. 2021; Mereghetti et al. 2020; Tavani et al. 2021). The total duration of the burst is roughly 0.3–0.5 s. The X-ray light curves consist of a few narrow peaks with each sub-burst width $\lesssim 10$ ms (Li et al. 2021a; Ridnaia et al. 2021; Mereghetti et al. 2020), which is coincident with the radio-burst arrival times (see below). The X-ray spectrum extends up to 250 keV (Ridnaia et al. 2021; Li et al. 2021a) and is fitted by an exponentially-cutoff power law function with a typical peak energy $\epsilon_p \sim 50–100$ keV. This is unusually hard compared to other X-ray bursts with comparable (or even higher) fluence detected in the same (Younes et al. 2021) and past (Ridnaia et al. 2021; Li et al. 2021a; Mereghetti et al. 2020) bursting episodes. There is evidence for a temporal spectral hardening associated with two peaks of the burst (Mereghetti et al. 2020; Li et al. 2021a). The isotropic energy in the X-ray bands is $E_X^{\text{iso}} = (0.5-1.2) \times 10^{40}$ (d/10 kpc)$^2$ erg, which is $\sim 10^5$ times larger than the radio bands.

### Burst Arrival Time

The arrival time delay of a pulse with an observed frequency of $\nu$ with respect to a reference frequency $\nu_{\text{ref}}$ is

$$t_{\text{DM}}(\nu, \nu_{\text{ref}}) = k_{\text{DM}} \left( \frac{1}{\nu^2} - \frac{1}{\nu_{\text{ref}}^2} \right) \text{DM}, \quad (1)$$

where $k_{\text{DM}} \equiv e^2/(2\pi m_e c) \sim 4.15$ ms pc$^{-1}$ cm$^3$ GHz$^{-2}$ is the dispersion constant. The dispersion delay between CHIME and STARE2 is $t_{\text{DM}}(600$ MHz, 1.53 GHz) $\sim 3.25$ s, which is consistent with the observed time delay between the second CHIME sub-burst and the STARE2 burst (see Figure 1). In fact, the spectrum of the second CHIME sub-burst extends up to higher frequency (300 MHz), whereas the first CHIME sub-burst has an apparent spectral cutoff at $\lesssim 600$ MHz (CHIME/FRB Collaboration et al. 2020). Furthermore, the spiky temporal structure of the second CHIME
sub-burst resembles that of STARE2 burst. These all implies that the STARE2 burst may be of the same origin as the second CHIME sub-burst.

On the other hand, the dispersion delay between CHIME and the X-ray satellites is typical (600 MHz, c) ~ 3.84 s. Given this, the arrival times of first/second CHIME sub-bursts and the first/second peaks in the X-ray light curves are consistent within error of \( \Delta t_{\text{CHIME},X} = t_{X} - t_{\text{CHIME}} \leq 5 \) ms. Even if we additionally take into account the finite time-resolution of X-ray detectors (\( \leq 2 \) ms around burst peaks) and pulse width of CHIME sub-bursts (\( \sim 5 \) ms), most conservatively we get \( \Delta t_{\text{CHIME},X} = t_{X} - t_{\text{CHIME}} \leq 10 \) ms. Similarly, we obtain \( \Delta t_{\text{STARE2},X} = t_{X} - t_{\text{STARE2}} \leq 10 \) ms for the STARE2 burst and the X-ray second peak. In summary, the intrinsic time separation between X-ray and radio emission peaks is estimated to be no longer than \( |\Delta t_{X,\text{radio}}| \sim 10 \) ms.

3 THEORETICAL FRAMEWORK

Whatever the emission mechanism of the the radio burst is, (i) the event is likely to be triggered by an injection of energy into the magnetosphere, and (ii) the radio emission may arise from a relativistic outflow at a sufficiently large distance from the NS surface in order to avoid a significant scattering and absorption. The launch of a relativistic outflow might be accompanied by the formation of a trapped FB (Thompson & Duncan 1995). We first constrain the energy and size of the trapped FB from the X-ray data, and then calculate the expansion of the outflow, assuming that the energy and size of the outflow at launch are comparable to those of the trapped FB.

3.1 Trapped Fireball

Despite the peculiar light curve and unusually hard spectra of the X-ray burst (§2), it is possible that there might be an underlying trapped FB, partially contributing to the thermal part of the entire X-ray burst spectrum. Figure 2 compares the best-fit model of observed hard X-ray spectrum (Ridnaia et al. 2021) with the predicted spectra from the trapped FB emission (Lyubarsky 2002; Yamasaki et al. 2020) with an effective temperature of \( T_{\text{obs}} \sim 10 \) keV, which is consistent with the black body (BB) plus power-law spectral fitting result with a temperature of \( \sim 11 \) keV (Li et al. 2021a). One can clearly see that the thermal component of the observed spectra could be roughly described by these models (the excess in the non-thermal component will be discussed later in this section). Therefore, we assume that the thermal component of the X-ray burst spectrum might be interpreted as radiation from a trapped FB with a peak photon energy of \( E_{\text{obs}} \sim 3T_{\text{obs}} \sim 30 \) keV, where a factor of 3 reflects the Wien’s displacement law.

In the presence of a very strong magnetic field exceeding the critical quantum value \( B_{\text{crit}} = m_{e}c^{2}/\hbar(\nu_{\text{ref}}) \approx 4.4 \times 10^{13} \) G, the magnetic equilibrium pair number density is expressed as (Canuto & Ventura 1977)

\[
\frac{n_{e,\text{eq}}}{m_{e}}(T) = \frac{1}{\sqrt{2\pi}} \left( \frac{1}{T} \right)^{3/2} \frac{B}{B_{\text{crit}}} \left( \frac{T}{m_{e}c^{2}} \right)^{1/2} e^{-m_{e}c^{2}/T},
\]

where \( AC = h/(m_{e}c) \) is the electron Compton wavelength and the numerical factor \( (2\pi)^{1/2}/c^{3} \) \( \approx 8.1 \times 10^{29} \) cm\(^{-3}\). The energy transfer of the trapped FB under the strong magnetic field is governed by extraordinary-mode (X-mode) photons (Thompson & Duncan 1995; Lyubarsky 2002) with an effective Compton scattering cross section (Meszaros 1992)

\[
\sigma_{\text{eff}}(\nu) = \sigma_{T} \left( \frac{\nu}{m_{e}c^{2}} \right)^{2} \left( \frac{B}{B_{\text{crit}}} \right)^{-2},
\]

where \( \nu \) is the X-mode photon energy. The emergent spectrum is determined by the radiation spectrum at the depth corresponding to the mean free path of an X-mode photon

\[
l(T, \nu) \sim \frac{1}{n_{e,\text{eq}}(T) \sigma_{\text{eff}}(\nu)}.
\]

By solving the energy transfer equations across the trapped FB, Lyubarsky (2002) found that the emergent spectrum is well approximated by a modified BB with an effective temperature \( T_{\text{obs}} \).

Because of the photon energy dependence of the X-mode cross-section (Eq. [3]), only the Wein part of the BB spectrum with temperature \( T_{0} \) is effectively observable as an outgoing radiation with temperature \( T_{\text{obs}} \), and therefore in general \( T_{0} > T_{\text{obs}} \). For simplicity, let us consider an effective X-mode photosphere with a radius \( R_{0} \) and a uniform background FB temperature \( T_{0} \), emitting a radiation with a photon energy of \( E_{\text{obs}} \sim 30 \) keV. Assuming a uniform magnetic field of \( B = B_{p} \), the mean free path for X-mode photons is \( l(T_{0}, \nu_{\text{obs}}) \sim O(1) \) cm for \( T_{0}/m_{e}c^{2} \gtrsim 0.1 \), which is vanishingly small compared to the expected FB size \( R_{0} \).

In the above simple picture (Thompson & Duncan 1995, 1996; Lyubarsky 2002), the total energy of the trapped FB is dominated by the hot plasma component with radius \( R_{0} \) and temperature \( T_{0} \) as

\[
E_{X,\text{obs}} = \frac{4}{3} \pi R_{0}^{3} a T_{0}^{4},
\]

where \( a \) is the radiation constant. Meanwhile, the photon diffusion occurs only at the outermost surface with the observed luminosity

\[
L_{X} = 4\pi c R_{0}^{2} a T_{\text{obs}}^{4}.
\]

Combining Eqs. (5) and (6), the observed duration of the X-ray emission from the trapped FB is estimated as

\[
t_{X,\text{obs}} \sim \frac{E_{X,\text{obs}}}{L_{X}}.
\]
Table 1. Properties of the radio and hard X-ray burst associated with Galactic magnetar SGR 1935+2154 on 2020 April 28.

| Band   | Telescope   | Frequency           | Arrive Time UT (ν_{ref}) \(\nu^{a}\) | Total Duration | Total Fluence | Ref. | Energy \(b\) |
|--------|-------------|---------------------|--------------------------------------|----------------|--------------|------|------------|
| Radio  | CHIME       | 0.4–0.8 GHz         | 14:34:28.264 (0.6 GHz) 14:34:24.428 (o) 14:34:25.046 (1.53 GHz) 14:34:45.055 (o) | 40 ms \(d\) | 700 \(\gamma_{0}\) kJy ms | [1]  | \(3 \times 10^{34}\) erg |
|        | STARE2      | 1.28–1.53 GHz       | 14:34:28.264 (o) 14:34:24.428 (o) 14:34:25.046 (o) 14:34:45.055 (o) | 0.61 ms \(d\) | 1.5 \(\gamma_{0}\) M Jy ms | [2]  | \(2.4 \times 10^{35}\) erg |
| X/γ   | Insight-HXMT | 1–250 keV          | 14:34:24.29(2) (σ) \(f\) 14:34:24.28(1) (σ) \(f\) 14:34:24.43 (o) \(f\) | – 0.5 s | 7.1 \(\gamma_{0}\) \(10^{-7}\) erg cm\(^{-2}\) | [3]  | \(6 \times 10^{39}\) erg |
|        | Konus-Wind  | 20–500 keV         | 14:34:24.29(2) (σ) \(f\) 14:34:24.28(1) (σ) \(f\) 14:34:24.43 (o) \(f\) | – 0.3 s | 9.7 \(\gamma_{0}\) \(10^{-7}\) erg cm\(^{-2}\) | [4]  | \(1.2 \times 10^{39}\) erg |
|        | INTEGRAL    | 20–200 keV         | 14:34:24.29(2) (σ) \(f\) 14:34:24.28(1) (σ) \(f\) 14:34:24.43 (o) \(f\) | – 0.3 s | 5.2 \(\gamma_{0}\) \(10^{-7}\) erg cm\(^{-2}\) | [5]  | \(5 \times 10^{39}\) erg |
|        | AGILE       | 18–60 keV          | 14:34:24.44 (o) \(f\) 14:34:24.44 (o) \(f\) | ≤ 0.5 s | 5 \(\gamma_{0}\) \(10^{-7}\) erg cm\(^{-2}\) | [6]  | \(5.6 \times 10^{39}\) erg |

\(a\) Geocentric arrival time of the first peak at reference frequency \(ν = ν_{ref}\) with DM = 332.7 pc cm\(^{-3}\); \(b\) CHIME/FRB Collaboration et al. (2020) [2] Bochenek et al. (2020) [3] Li et al. (2021a) [4] Ridnaia et al. (2021) [5] Mereghetti et al. (2020) [6] Tavani et al. (2021); \(d\) Assuming a distance of 10 kpc; \(f\) The event consists of two sub-bursts with widths of ~ 5 ms separated by ~ 30 ms; \(g\) A single spiky burst.

Figure 3 shows the constrains on the trapped FB parameters. If we conservatively take \(E_{X,obs} = 10^{39}–10^{40}\) erg and \(ν_{obs} = 0.1–1\) s, the allowed parameter space for the FB radius and temperature are \(R_{0} \sim 10^{5}\) cm and \(T_{0} \sim 200–300\) keV, respectively. With a BB temperature of 200–300 keV, a good fraction (70%–87%) of the total energy is carried by photons with \(ε > m_{e}c^{2}\), and hence it is sufficient to keep the interior of trapped FB (except for the thin outer layer) optically-thick to pair production. Given the short duration of the emission compared to the spin period \(ν_{obs}/P_{spin} \leq 0.3\) as well as the relatively small FB size with respect to the NS, the FB may evaporate before being occulted due to the NS rotation.

As mentioned in §2, the spectrum of April 28 event is much harder than that of typical bursts from SGR 1935+2154 with comparable duration and total energy. The resonant cyclotron scattering may be responsible for the spectral hardening inside flaring magnetosphere. The magnetar magnetosphere is filled with \(e^{\pm}\) plasma both during flares and in the persistent state (Thompson et al. 2002; Beloborodov & Thompson 2007; Beloborodov 2013); one can easily see that the resonant cyclotron optical depth is unavoidably large. Therefore any outgoing radiation is reprocessed in the cyclotron resonance layer. In this case, a Doppler shift due to scattering on the bulk motions of the magnetospheric plasma could lead to formation of hard tails in thermal spectra. During the flare, a tremendous resonance radiation force keeps the plasma motion mildly relativistic (Yamasaki et al. 2020). As a result, under typical conditions for flaring magnetosphere, the degree of spectral hardening by a single scattering is at most twice in terms of observed photon energy and the single scattering model can successfully fit the observed intermediate flare (with \(L_{X} \sim 10^{40}–10^{41}\) erg s\(^{-1}\)) spectra from SGR 1900+14 (see, e.g., Figure 5 of Yamasaki et al. 2020). Figure 2 clearly indicates that the predicted spectrum from the trapped FB emission reprocessed by a single resonant cyclotron scattering cannot explain the hard spectral index of April 28 event. Hence, while most spectra of ordinary bursts from SGR 1935+2154 might be explained by this model, the formation of the extremely hard spectra of April 28 event by the same picture seems challenging unless one invokes an extremely dense magnetosphere that could lead to multiple resonant scatterings (see also Ioka 2020; Yang & Zhang 2021). Since a further exploration of such a possibility is outside the scope of this work, we just note that magnetospheric reprocession of the trapped FB emission or some alternative mechanisms may give rise to the observed hard X-ray spikes in §4.1.

3.2 Relativistic Outflow

Next we consider the relativistic outflows, which might be launched at the onset of the trapped FB formation and produce the radio burst and hard X-ray spikes. The intrinsic energy budget for launching relativistic outflows should be limited by the isotropic equivalent energy emitted by the trapped FB \(E_{X}^{iso} = 10^{40}\) erg. Given the small variability timescale for radio and X-ray bursts (∼ 10 ms), the maximum injected energy available for the outflow would be smaller than \(E_{X}^{iso}\). Thus, we conservatively set the initial outflow
energy to $E_{\text{flare}} \approx 10^{39}$ erg. In addition to the energy and the initial size, the dynamical evolution of the outflow depends on both the composition of the FB and the energy source for acceleration, which are highly uncertain. Thus, we consider a broad class of the outflow models in §3.2.1 and discuss its relevance to generation of coherent radio emission in §3.2.2.

### 3.2.1 Outflow Models

We consider three outflow models: (i) leptonic outflow composed of $e^\pm$ pairs and photons, (ii) baryonic outflow composed of $e^\pm$ pairs, baryons and photons and (iii) magneto-leptonic (or simply, magnetic) outflow composed of cold $e^\pm$ pairs loaded with large Poynting flux. We use the theory of an adiabatic FB (Paczynski 1986; Goodman 1986) to track the dynamical evolution of these outflows (see Appendix A). The evolution of leptonic outflow is uniquely determined for a given set of initial outflow parameters, such as size, temperature and bulk Lorentz factor (or energy), whereas the latter two outflows are characterized by additional model parameters.

The baryonic outflow is characterized by the baryon loading parameter $\eta$ defined as a ratio of radiation flux to matter energy flux. The magnetic outflow is described by means of its initial magnetization parameter $a_0$ defined as a ratio of Poynting flux to matter energy flux at the magnetosonic point where the outflow attains a velocity of $\Gamma_0 = \sqrt{\frac{a_0}{\eta}}$ and starts to evolve further (see Appendix for further details). In order to accelerate the magnetic outflow efficiently, a strong dissipation may be important. We adopt a classic model proposed by Drenkhahn 2002 in the context of gamma-ray bursts (GRBs), in which the toroidal magnetic field with alternating polarity (so-called striped wind model; Kennel & Coroniti 1984; Lyubarsky & Kirk 2001) decays into kinetic energy above the light cylinder [$r_c = cP_{\text{spin}}/(2\pi) \approx 10^{15}$ cm for SGR 1935+2154]. With the assumption that the outflow is highly dominated by magnetic energy and that the thermal energy is negligible we derive the dynamical evolution at $r > r_c$. We adopt a classic model proposed by Drenkhahn 2002 in the context of gamma-ray bursts (GRBs), in which the toroidal magnetic field with alternating polarity (so-called striped wind model; Kennel & Coroniti 1984; Lyubarsky & Kirk 2001) decays into kinetic energy above the light cylinder [$r_c = cP_{\text{spin}}/(2\pi) \approx 10^{15}$ cm for SGR 1935+2154]. With the assumption that the outflow is highly dominated by magnetic energy and that the thermal energy is negligible we derive the dynamical evolution at $r > r_c$.

Based on the trapped FB properties estimated in §3.1, we fix the initial non-magnetic outflow radius and temperature to be $r_0 = R_0 \approx 10^5$ cm and $T_0 \approx 200$ keV respectively, so that $E_{\text{flare}} = \frac{4}{3\pi}\frac{\sigma}{\Gamma_0^3}a_0^2T_0^5$. The initial density of non-magnetic outflow is set to the thermal equilibrium value, which only depends on $T_0$. Meanwhile, the evolution of the magnetic outflow is calculated from $r = r_c$. Since we use the cold approximation, the initial density is determined by equating the initial kinetic energy to $E_{\text{flare}}$. The black curves in Figures 4–6 show the dynamical evolution of each outflow. The evolution of leptonic outflow is uniquely determined, whereas for baryonic and magnetic outflows we show the evolution with characteristic values of $\eta$ and $a_0$. To summarize, the terminal bulk Lorentz factor that each outflow attains is

$$
\Gamma_\infty \approx \begin{cases} 
4.4 \times 10^2 \frac{r_0}{a_0} & \text{(Leptonic)} \\
\min(\eta, \eta_{\text{heavy}}) & \text{(Baryonic)} \\
\frac{\eta_{\text{mild}}}{\eta_{\text{heavy}}} \frac{T_0}{a_0} & \text{(Magnetic)}
\end{cases}
$$

where $\Theta_0 = T_0/m_e c^2$ is the dimensionless initial FB (outflow) temperature and hereafter we use a notation $\Theta_0 = \Theta_0/0.4$, corresponding to $T_0 = 200$ keV. We cover the baryonic outflows in heavy ($\eta < \eta_{\text{heavy}}$) and mild ($\eta_{\text{heavy}} < \eta < \eta_{\text{mild}}$) load regimes, where $\eta_{\text{heav}} \approx 60r_0, \Theta_0$ and $\eta_{\text{mild}} \approx 1.5 \times 10^5 \Theta_0 r_0$ are the critical values (see Appendix A).

One concern regarding the early evolution of the outflow is the possible disturbance by the large-scale magnetic field of the magnetar. Given a dipole magnetic field $B \propto r^{-3}$, the background magnetic pressure at an altitude $h$ above the NS surface is $P_B = B^2/(8\pi) \approx 4 \times 10^{12} \rho_B^2 \theta_h^2$ erg cm$^{-3}$, whereas the total pressure of the non-magnetic outflow with initial temperature $T_0$ is $P_{\text{nf}} = a_0^2 T_0^5 \approx 3 \times 10^{13} \Theta_0^2$ erg cm$^{-3}$. Namely, $P_B \approx P_{\text{nf}}$ at an altitude $h \leq h_c \approx 3 \times 10^6 \rho_{1.5}^{1/2} \theta_h^{1/2} \Theta_0$ cm. While a lepton outflow is barely affected by the background magnetic field because it continues to accelerate up to much larger distance $r_{\infty} = \Gamma_\infty r_0 \approx 4.4 \times 10^7 r_0^{5/4} \Theta_0$ cm compared to $h_c$, it may significantly modify the early evolution of baryonic outflows with low acceleration efficiency $r_{\infty} \approx \eta_{\text{mild}} r_0 \approx 6.0 \times 10^6 r_0^{5/4} \Theta_0$ cm, which is almost comparable to $h_c$. In this respect, our estimate on $\Gamma_\infty$ could be slightly overestimated. The situation might be more complicated for cold magneto-leptonic outflows due to the absence of the radiation pressure. Nevertheless, such uncertainties must be

---

3 Note that the definition of $r_0$ here is different from the conventional one that defines it when the flow is static ($\Gamma_0 = 1$).

4 One caveat of models with strong dissipation, however, may be that it is not clear whether a highly ordered magnetic field can be maintained at

---

The FRB generation site in order for the synchrotron maser mechanism to operate.
sub-dominant relative to the assumption that the flow starts to evolve at \( r = r_0 \) with significant acceleration \( \Gamma_0 = \sigma_0^{1/2} \). Therefore, we neglect the potential modification of inner outflow evolution by the background magnetic field hereafter.

### 3.2.2 Plasma cutoff frequency

It is often assumed that the GHz coherent emission is generated by coherent charge bunches through, e.g., curvature or synchrotron maser processes. In the case of curvature radiation, the emission is often thought to be triggered by magnetic reconnection in the vicinity of NS (e.g., Katz 2016; Kumar et al. 2017; Ghisellini & Locatelli 2018; Lu & Kumar 2018; Yang & Zhang 2018; Katz 2020; Lu et al. 2020). In the case of the synchrotron maser emission, the emission occurs at relativistic shocks propagating in the pre-existing media, such as nebula (Lyubarsky 2014; Murase et al. 2016; Waxman 2017), steady magnetar wind (Beloborodov 2017), or past flare-driven ejecta (Metzger et al. 2019; Beloborodov 2020; Margalit et al. 2020; Yuan et al. 2020; Yu et al. 2021).

In either case, one of the important constraints for localizing the radio emission region comes from the plasma cutoff effect. The waves have cutoff frequencies \( \omega_{\text{cutoff}} \) (measured in the plasma frame) below which they become evanescent. In general, the cutoff frequency is conveniently expressed in terms of the plasma frequency \( \omega_p \) defined in the rest frame as

\[
\omega_p \equiv \zeta \sqrt{\frac{4 \pi n'_e e^2}{m_e}}, \tag{9}
\]

where \( n'_e \) is the comoving number density of electrons in region which is responsible for the generation of waves. We include all the uncertainties associated radiation mechanisms and plasma conditions in the fudge factor \( \zeta \), representing both the possible relativistic effects and the specific treatment of the shock. Throughout this work, for simplicity, we set \( \zeta = 1 \) and leave the parameter dependence to keep generality.

In the case of maser emission at far zone, electromagnetic (EM) waves follow the well-known dispersion relation in the non-magnetized plasma with a cutoff at \( \omega_{\text{cutoff}} = \omega_p \), but the treatment of the shocked region becomes important for an appropriate estimate of plasma frequency. As seen in §3.2.1, we calculate the dynamical evolution of a single outflow (\( \Gamma \) and \( n'_e \)) without deceleration, which may differ from the exact quantitative dynamics of decelerating outflow shells that produce internal shocks. Nevertheless, we can reasonably assume that the most efficient internal shock with a large contrast between shell Lorentz factors and comparable densities is generated at each radius \( r \). We assume that the upstream (downstream) of the shock is cold (hot), and the maser emission is produced by the cold upstream plasma at the shock front. In this case, the apparent plasma frequency in the observer frame for maser-type scenarios is evaluated by (Plotnikov & Sironi 2019, see also Iwamoto et al. 2017, 2019)

\[
\omega_{p,\text{obs}} \equiv \Gamma \omega_p \max \{ 1, \sigma^{1/2} \}, \tag{10}
\]

where the coefficient of 3 appearing in the original formula is neglected for simplicity. Here, again, there is an uncertainty in the treatment of bulk Lorentz factor depending on the shock models. But, as this is small ~ 1, it can be absorbed by the fudge factor \( \zeta \) in Eq. (9).

### 4 CONSTRAINTS ON RELATIVISTIC OUTFLOW AND EMISSION REGION

Based on the outflow models outlined in §3.2, we aim to obtain general constraints on the properties of the outflow that is responsible for the generation of radio and hard X-ray bursts from SGR 1935+2154.

#### 4.1 Coherent Radio Burst

Radio emission suffers from various constraints when escaping from the system without significant attenuation, and there is a radio compactness problem when the radio emission originates from relativistic outflows. For example, Murase et al. (2017) investigated whether radio emission can coincide in region with X-ray and gamma-ray emission in light of FRB 131104 (DeLaunay et al. 2016). Radio waves can propagate only when their frequencies are higher than the plasma cutoff frequency and they also suffer from the induced Compton scattering within the outflows and ambient environments (e.g., Murase et al. 2016). Here we focus on the plasma cutoff condition for the radio wave propagation:

\[
\omega_{p,\text{obs}} \left( \frac{\nu_{\text{radio}}}{c} \right) \leq \omega_{\text{obs}}, \tag{11}
\]

where \( \omega_{p,\text{obs}} = \omega_p \) is the apparent plasma frequency in the observer frame. We set the observed radio frequency to \( \nu_{\text{obs}} = \nu_{\text{obs}}/(2\pi) \approx 1 \text{ GHz in mind of CHIME and STARE2} \). Depending on the radial evolution of the observed plasma frequency, the above condition sets a limit on the radio-emitting radius \( r_{\text{radio}} \).

Another constraint comes from the intrinsic timing of radio and X-ray bursts. When there is a bulk motion with a Lorentz factor of \( \Gamma \), the comoving size of the region responsible for the generation of emission can be larger by a factor of \( \Gamma^2 \). Given the intrinsic time delay \( \Delta t_{\text{X,radio}} \leq 10 \text{ ms (see §2)} \), the radio (or X-ray) photons should be emitted at

\[
r_{\text{radio}}(X) \leq \Gamma^2 c \Delta t_{\text{X,radio}} \tag{12}
\]

which gives an upper limit on the radio (or X-ray) emitting radius. Since the time delay between X-ray and radio emissions generally depends on the emission mechanisms and initial FB size, it could be much shorter. Also, when there is little or no time delay between X-ray and radio emission as predicted by some models (e.g., Metzger et al. 2019; Margalit et al. 2020; Yuan et al. 2020), the time delay argument (Eq. [12]) could be less constraining. In this sense, the above limit is most conservative.

Given relativistic outflow models (§3.2.1) and maser-type
emission (§3.2.2), the plasma frequency argument (Eq. [11]) sets the lower limit on the radio-emitting radius

\[ r_{\text{radio}} \gtrsim r_{\text{cutoff}} \]

\[ \begin{align*}
3.7 \times 10^{13} & \frac{5^{1/4}}{0.5} \Theta_0^2 \zeta^2 v_{\text{obs}}^{-1} \text{ cm}, \\
1.1 \times 10^{14} & \frac{v_{\text{radio}}^{-1}}{0.5} \text{ cm}, \\
2.9 \times 10^{14} & \frac{r_{\text{flare},39} v_{\text{obs}}^{-1} \text{ cm}}{0.5},
\end{align*} \]

where \( r_{\text{cutoff}} \) is the plasma cutoff radius defined by \( v_{\text{p,obs}}(r_{\text{cutoff}}) = v_{\text{obs}} \). Next, the time delay argument (Eq. [12]) suggests that the radio emission be emitted at

\[ r_{\text{radio}} \lesssim \Gamma_0^2 c \Delta \nu_{\text{radio}} \]

\[ \begin{align*}
\Delta \nu_{\text{radio}} & \approx \left( \frac{5.8 \times 10^{13}}{0.5} \frac{\Theta_0^2}{\zeta^2} \frac{v_{\text{radio}}}{10 \text{ ms}} \right), \\
3.0 \times 10^{14} & \left( \min \frac{\eta_{\text{heavy}}}{\eta_{\text{mild}}} \right), \\
3.0 \times 10^{14} & \frac{\sigma_0^2}{0.2} \text{ cm},
\end{align*} \]

which gives an upper limit on the radio-emitting radius. Here we set \( \Gamma = \Gamma_0 \) to make the radial constraints most conservative.

The allowed region for the radio emission, as well as dynamical evolution, are indicated by the vertical green shaded regions in Figures 4–6. One can see from Figure 4 that the allowed locale of radio emission from a leptoic outflow is constrained to within somewhat narrow regions at \( r_{\text{radio}} \sim 10^{12} \text{–} 10^{14} \text{ cm} \). On the other hand, the evolution of bulk Lorentz factor of baryonic and magnetic outflows strongly depends on the initial degree of baryon load (\( q \)) and magnetization (\( \sigma_0 \)). Figures 5 and 6 demonstrate how these parameters affect the conditions of Eq. (13) and Eq. (14). From the left panels of Figures 5 and 6, it is apparent that heavily baryon-loaded and weakly magnetised outflows are not compatible with observed time delay due to the modest acceleration. Meanwhile, although the maximum acceleration is also limited in the mild-load regime (\( \eta_{\text{heavy}} < \eta_{\text{mild}} \)), the left panel of Figure 5), there is an allowed range for radio emission site because of smaller plasma frequency. Similarly, the higher initial magnetization \( \sigma_0 \) results in the faster acceleration, which broadens the allowed range of emission region. As a consequence, the initial properties

\[ \begin{align*}
\eta & \geq 6.2 \times 10^{13} \frac{5^{1/4}}{0.5} \Theta_0 \zeta^2 v_{\text{obs}}^{-1} \left( \frac{\Delta \nu_{\text{radio}}}{10 \text{ ms}} \right)^{-2}, \\
\sigma_0 & \geq \left( 99 r_{0.5}^{-1/6} \frac{\Delta \nu_{\text{radio}}}{10 \text{ ms}} \right) \left( 1/3 \right)^{1/3} v_{\text{radio}}^{-1/3} \frac{\left( \Delta \nu_{\text{radio}} \right)}{10 \text{ ms}} \right)^{-1/3} \frac{1}{0.5},
\end{align*} \]

are required for each outflow to keep the consistency with arguments on the plasma cutoff frequency and the observed time delays between X-ray and radio emission, i.e., \( r_{\text{cutoff}} \leq \Gamma_0^2 c \Delta \nu_{\text{radio}} \). In Eq. (16), we use an approximation \( \Gamma_{\infty} = \sigma_0 \Delta \psi X + \sigma_0 \sim \sigma_0 \Delta \psi X \) for simplicity.

The observed duration of the burst emission that each outflow predicts can be estimated by

\[ \Delta t \sim \frac{r}{c \Delta \psi X}. \]

Considering the observed duration of interest \( \Delta t = 1 \text{–} 100 \text{ ms} \), we show the allowed emission region in the lower panels of Figures 4–6. By examining whether it overlaps with the radio-emitting region, one finds that the leptoic outflow, mildly-loaded baryonic outflow are in principle compatible with radio observations. Meanwhile, heavily baryon-loaded and magnetized outflows cannot reproduce sufficiently short duration of radio bursts due to their weak or delayed acceleration.

\[ \text{Figure 4. Dynamical evolution of electron number density in the plasma rest frame (upper panel, right-hand-side axis), plasma frequency in the observer frame (upper panel, left-hand-side axis) and bulk Lorentz factor (lower panel) of the leptoic outflow with } E_{\text{burst}} = 10^{39} \text{ erg} (r_0 = 10^3 \text{ cm and } T_0 = 200 \text{ keV}). \text{ The allowed radii for X-ray and radio emission are indicated by shaded regions in the upper panel. The region corresponding to the observed burst duration of } 1 \text{–} 100 \text{ ms is indicated by vertical lines in the lower panel. For radio emission Eq. (13) and Eq. (14) are used while for X-ray emission Eq. (21) and Eq. (24) are used. We assume } \Delta \nu_{\text{radio}} = 10 \text{ ms when deriving the time delay constraints and the duration is evaluated by means of Eq. (17),} \]

\[ \text{4.2 Hard X-ray Bursts from Relativistic Outflow} \]

The existence of non-thermal component in the observed X-ray spectra implies that the source is optically thin to Thomson scattering on \( e^\pm \) pairs, which is often the case with the prompt emission of GRBs. An inevitable source for such pairs is the annihilation of photons with rest-frame energy above \( n_{\gamma} \gamma^2 \). The scattering optical depth for created pairs is expressed as \( \delta \gamma = \frac{\sigma T f_{\text{th}} N'}{\pi^2 \gamma X} \), where \( N' \) is the total number of emitted photons in the rest frame of the outflow and \( f_{\text{th}} \) the fraction of photons that create pairs. We approximate the energy-averaged cross section as \( \sigma_0 \) for simplicity. The observer-frame quantities, \( \theta_X \) and \( \nu_X \), are the geometric opening angle (relative to the outflow direction of motion) within which

\[ \text{6 Since the maximum observed photon energy } \sim 250 \text{ keV (Ridnaia et al. 2021) of the hard X-ray counterpart to the radio burst on April 28 is well below } n_{\gamma} \gamma^2, \text{ the opacity to } \gamma \gamma \text{ pair production provides less stringent constraints on the beaming of the outflow.} \]
most of the photons propagate and radial distance of X-ray emission region, respectively. The total number of photons is related to observed quantities by \( N' \approx \left( L_{\text{iso}}^{10} \delta t_X / \epsilon_p \right) / \delta t_\text{D} \left( \theta, \Gamma \right) \), where \( L_{\text{iso}}^{10} \), \( \delta t_X \), \( \epsilon_p \) are the isotropic equivalent X-ray luminosity, the variability timescale (corresponding to the observed peak width of X-ray burst spikes), and the peak energy of photons in observed \( vF_v \) spectra, respectively. Here \( \delta t_\text{D} \equiv 1 / \left[ \Gamma (1 - \beta \cos \theta) \right] \) denotes the Doppler factor corresponding to a Lorentz factor \( \Gamma \) (and velocity \( \beta \)) and observer viewing angle \( \theta \), which is measured from the center of the X-ray beam. The angular variation of the Doppler factor depends on the product \( \Gamma \theta \) and the size of X-ray emission region satisfies: \( \theta_X \sim \max \left( 1 / \Gamma, \theta \right) \). Generally, the radial distance of X-ray emission region \( r_X \) is limited by the variability timescale \( \delta t_X \). Here we conservatively assume

\[
 r_X \sim \Gamma \delta t_\text{D} \left( \theta, \Gamma \right) \delta t_X, \tag{19}
\]

which is true at least inside the beam with angle \( 1 / \Gamma \) regardless of specific dissipation mechanisms (Piran 1999) and indeed gives the loosest limit on the pair creation optical depth even outside the beam (Matsumoto et al. 2019). The relativistic beaming effect can also significantly change the pair-creation criteria and we define the energy threshold of photons which can self-annihilate as \( \epsilon_p \). Here we can directly determine the magnetic field behind the shock as \( B^2 \approx 4 \pi r_X \Gamma^2 U' \), where \( U' = m_e c^2 \) is the internal energy density of upstream material. For magnetic outflow, we can directly determine the magnetic field behind the shock as \( B^2 \approx 4 \pi r_X \Gamma^2 U' \). The synchrotron cooling Lorentz factor of outflow material is given by (Sari et al. 1998)

\[
 \gamma_c = \frac{6 \pi m_e c^3}{\sigma_T B^2 \Gamma t}, \tag{22}
\]

where \( t \sim r / (\Gamma^2 c) \) is the dynamical timescale of the flow in the

---

**Figure 5.** Same as Figure 4 but for baryonic outflows in the heavy-load \( \eta < \eta_{\text{heavy}} \) (left) and mild-load \( \eta_{\text{heavy}} < \eta < \eta_{\text{mild}} \) (right) regimes. In the limit of extremely weak baryon load \( \eta \rightarrow \infty \), the dynamical evolution of the outflow asymptotically approaches to that of a pure leptonic one shown in Figure 4. Note that the upper limit on the radii due to the fast cooling scales as \( B \) (Eq. (24)) and could be much smaller than shown here (we take an extreme limit \( \epsilon_B = 1 \)).
The Lorentz factor of electrons at the internal shock may be estimated by assuming that a fraction $\xi_e$ of the total internal energy goes into random motions of the electrons:

$$y_m \sim \xi_e \, \langle \varepsilon_e \rangle^{-1} \Gamma,$$

where $\langle \varepsilon_e \rangle = \xi_e \leq 1$ is the fraction of electrons that undergo acceleration (Eichler & Waxman 2005). Here we take $\xi_e = 1$, considering the maximum acceleration expected for an internal shock inside the (magneto-)leptonic outflow. Meanwhile, for baryonic acceleration (Eichler & Waxman 2005). Here we take $\xi_e = 1$, considering the maximum acceleration expected for an internal shock inside the (magneto-)leptonic outflow. Meanwhile, for baryonic acceleration

$$\text{ERO} \sim \xi_e \langle \varepsilon_e \rangle^{-1} \Gamma,$$

where we have used an analytic expression for the evolution of magnetic outflow (see Appendix A). Hence, this could be considered as an upper limit on the X-ray emission radius. Clearly, the leptonic outflow cannot keep a high radiation efficiency far outside the magnetosphere. In Eq. (24), the possible uncertainty stemming from the treatment of bulk Lorentz factor used in Eqs. (22) and (23), which depends on the detail of the shock model is neglected here.

By combining the available constraints on X-ray and radio emission with the duration constraint (Eq. [17]), one finds that the leptonic outflow is excluded since it is unable to explain the X-ray burst duration. Due to the same reason, mildly-loaded baryonic outflows are also excluded. In contrast to the non-magnetic cases, high-$\eta_0$ flows are marginally consistent with observations, albeit with somewhat long duration ($> 10$ ms) for radio emission.

5 SUMMARY AND DISCUSSION

In this work, we constrained the outflow properties associated with the unique April 28 event from SGR 1935+2154 consisting of radio and X-ray bursts. The event is likely to be triggered by sudden eruptions of magnetic energy of $\sim 10^{39}$–$10^{40}$ erg into the magnetosphere, which would generate FB plasmas. As a consequence, a relativistic outflow might be launched at the onset of the trapped FB formation. In this case, the hard X-ray burst can be explained as a mixture of thermal and non-thermal emission. We showed that the thermoelectromagnetic of X-ray burst spectrum is consistent with a trapped FB with temperature of a few hundred keV and size of $\sim 10^5$ cm.

On the other hand, non-thermal radiation, including the non-thermal component of X-ray burst and the coherent radio burst, may arise from the relativistic outflow at large distances from NS ($r_X \sim 10^8$–$10^{10}$ cm and $r_{\text{radio}} \gtrsim 10^{11}$–$10^{12}$ cm) to avoid absorption/scattering by the outflow itself. We calculated the dynamical evolution of the outflow so that its initial conditions are consistent with the inferred properties of the trapped FB. By assuming that these emissions are both produced by the energy dissipation at the internal shocks of the outflow, we show that any outflows should be accelerated up to bulk Lorentz factor of order $\sim 10^2$ at the outer edge of magnetosphere.

Furthermore, by examining the intrinsic timing offset between radio and X-ray burst spikes with $\lesssim 10$ ms, we constrain the initial degree of baryon load and magnetization, showing that $\eta \gtrsim 6 \times 10^5$ and $\eta_0 \gtrsim 100$, respectively. The former constraint translates into an upper-limit on the total baryon mass of $mb \lesssim 1.8 \times 10^{14}$ g, which is many orders of magnitude smaller than that inferred from the afterglow observation of historical giant flare from SGR 1806–20: $mb \sim 10^{20}$–$10^{23}$ g (Nakar et al. 2005) or $mb \gtrsim 10^{24}$ g (Granot et al. 2006). A more precise time coincidence between the radio and the X-ray burst spikes (say $\Delta t_{\text{X,radio}} \lesssim 1$ ms), if confirmed by a joint radio-X-ray timing analysis, would place stringent constraints on the baryon load and initial magnetization of the outflow. We defer the investigations of more realistic but complicated outflow models that include both baryons and magnetization within the GRB context (e.g., Gao & Zhang 2015) for future works.

Our results may have important implications for why the hard X-ray burst and coherent radio burst seen in April 27 event is rarely observed. An interesting possibility is that the high radiation efficiency far outside the magnetosphere. In Eq. (24), the possible uncertainty stemming from the treatment of bulk Lorentz factor used in Eqs. (22) and (23), which depends on the detail of the shock model is neglected here.

By combining the available constraints on X-ray and radio emission with the duration constraint (Eq. [17]), one finds that the leptonic outflow is excluded since it is unable to explain the X-ray burst duration. Due to the same reason, mildly-loaded baryonic outflows are also excluded. In contrast to the non-magnetic cases, high-$\eta_0$ flows are marginally consistent with observations, albeit with somewhat long duration ($> 10$ ms) for radio emission.

ACKNOWLEDGEMENTS

We thank the anonymous referee for useful comments that improved the manuscript. We also thank Anna Ridnaia for sharing detailed information of the hard X-ray burst detected by Konus-Wind, Yuri Lyubarsky for a useful conversation on the evolution of magnetized outflow and Kunihito Ioka for useful comments on the effect of global magnetic field that can affect the early evolution of the outflow. The work of SY was supported by the Institute for Cosmic Ray Research and the advanced ERC grant TReX. The work of K.K. is supported by KAKENHI No. 20K04010. The work of K.M. is supported by the NSF Grant No. AST-1908689, No. AST-2108466 and No. AST-2108467, and KAKENHI No. 20H01901 and No. 20H05852.

DATA AVAILABILITY

This is a theoretical paper that does not involve any new data.
Figure 6. Same as Figure 4 and 5 but for magneto-leptonic outflows with initial degree of magnetizations $\sigma_0 = 10$ (left) and $\sigma_0 = 10^3$ (right). The outflow energy is set to $E_{\text{out}} = 10^{50}$ erg at $r = n_e$. The radial evolution of magnetization parameter $\sigma$ is also shown in the lower panels.

REFERENCES

Arons J., Barnard J. J., 1986, ApJ, 302, 120
Beloborodov A. M., 2013, ApJ, 777, 114
Beloborodov A. M., 2017, ApJ, 843, L26
Beloborodov A. M., 2020, ApJ, 896, 142
Beloborodov A. M., Thompson C., 2007, ApJ, 657, 967
Beskin V. S., Kuznetsova I. V., Rafikov R. R., 1998, MNRAS, 299, 341
Bochenek C. D., Ravi V., Belov K. V., Hallinan G., Kocz J., Kulkarni S. R., McKenna D. L., 2020, arXiv e-prints, p. arXiv:2005.10828
Borghese A., Coti Zelati F., Rea N., Esposito P., Israel G. L., Mereghetti S., Tiengo A., 2020, ApJ, 902, L2
CHIME/FRB Collaboration et al., 2020, Nature, 587, 54
Canuto V., Ventura J., 1977, Fundamentals Cosmic Phys., 2, 203
Chen F. F., 1984, Introduction to Plasma Physics and controlled fusion, 2nd edition (Plenum Press, New York)
Contopoulos I., Kazanas D., 2002, ApJ, 566, 336
DeLauyard J. J., et al., 2016, ApJ, 832, L1
Drenkhahn G., 2002, A&A, 387, 714
Drenkhahn G., Spruit H. C., 2002, A&A, 391, 1141
Eichler D., Waxman E., 2005, ApJ, 627, 861
Gao H., Zhang B., 2015, ApJ, 801, 103
Ghisellini G., Locatelli N., 2018, A&A, 613, A61
Goodman J., 1986, ApJ, 308, L47
Grant J., et al., 2006, ApJ, 638, 391
Grimsrud O. M., Wasserman I., 1998, MNRAS, 300, 1158
Ioka K., 2020, ApJ, 904, L15
Israel G. L., et al., 2016, MNRAS, 457, 3448
Iwamoto M., Amano T., Hoshino M., Matsumoto Y., 2017, ApJ, 840, 52
Iwamoto M., Amano T., Hoshino M., Matsumoto Y., Niemiec J., Ligorini A., Kohzari O., Pohl M., 2019, ApJ, 883, L35
Katz J. L., 2016, ApJ, 826, 226
Katz J. L., 2020, MNRAS, 499, 2319
Kennel C. F., Coroniti F. V., 1984, ApJ, 283, 710
Kirsten F., Snelders M. P., Jenkins M., Nimmo K., van den Eijnden J., Hessels J. W. T., Gawroński M. P., Yang J., 2021, Nature Astronomy, 5, 414
Komissarov S. S., Vlahakis N., Königl A., Barkov M. V., 2009, MNRAS, 394, 1182
Kothes R., Sun X., Gaensler B., Reich W., 2018, ApJ, 852, 54
Kumar P., Lu W., Bhattacharyya M., 2017, MNRAS, 468, 2726
Li C. K., et al., 2021a, Nature Astronomy, 5, 378
Li D., et al., 2021b, Nature, 598, 267
Lin L., et al., 2020, Nature, 587, 63
Lithwick Y., Sari R., 2001, ApJ, 555, 540
Lu W., Kumar P., 2018, MNRAS, 477, 2470
Lu W., Kumar P., Zhang B., 2020, MNRAS, 493, 1397
Luo R., et al., 2020, Nature, 586, 693
Lyubarsky Y. E., 2002, MNRAS, 332, 199
Lyubarsky Y., 2014, MNRAS, 442, L9
Lyubarsky Y., 2021, Universe, 7
Lyubarsky Y., Kirk J. G., 2001, ApJ, 547, 437
Lyutikov M., 2020, arXiv e-prints, p. arXiv:2006.16029
Lyutikov M., Popov S., 2020, arXiv e-prints, p. arXiv:2005.05093
Margalit B., Beniamini P., Sridhar N., Metzger B. D., 2020, ApJ, 899, L27
Matsumoto T., Nakar E., Piran T., 2019, MNRAS, 486, 1563
Mereghetti S., et al., 2020, ApJ, 898, L29
Meszaros P., 1992, High-energy radiation from magnetized neutron stars
Metzger B. D., Margalit B., Sironi L., 2017, MNRAS, 461, 1498
Murase K., Mészáros P., Fox D. B., 2017, ApJ, 836, L6
Nakar E., 2007, Phys. Rep., 442, 166
Nakar E., Piran T., Sari R., 2005, ApJ, 635, 516
Paczynski B., 1986, ApJ, 308, L43
Piran T., 1999, Phys. Rep., 314, 575
Plotnikov I., Sironi L., 2019, MNRAS, 485, 3816
Ridnaia A., et al., 2021, Nature Astronomy, 5, 372
Sari R., Piran T., Narayan R., 1998, ApJ, 497, L17
Shemi A., Piran T., 1990, ApJ, 365, L55
The initial FB is at rest in pair equilibrium due to its high temperature with its size $r = r_0$. It immediately expands and cools down to the electron rest mass energy, and then $n_e$ begins to deviate from $n_{e,eq}$. The pair annihilation dominates the pair process since the number of pair-creating high-energy photons decreases as the FB cools. Eventually, the FB reaches the photospheric radius $r_{ph} \sim 2.5 \times 10^6$ cm $r_{0.5} \Theta_0$ at which the optical depth to electron scattering becomes an order of unity. When the FB becomes optically thin, photons begin to leak freely out of the photosphere. However, they still continue to supply the radiation energy to pairs, which accelerates pairs up to the coasting radius $r_{co} \sim 1.1 \times 10^9$ cm $r_{0.5} \Theta_0$. The photons cease to inject the radiation energy to pairs, and the FB begins to freely coast at constant speed $c_{\gamma} = r_{co}/r_{1} \sim 1.1 \times 10^3 r_{0.5} \Theta_0$. At this stage, the pair annihilation no longer occurs due to the small number density. As a result, the total number of pairs conserves and the pair density evolves as $\propto r^{-2}$. The number density of the pair at the coasting phase has an analytical form (Yamasaki et al. 2019):

$$n'_e(r) = 5.5 \times 10^{30} \frac{Z^3}{A_0} \frac{\Theta_0^2}{\Phi_{eq}} r^{-2} \text{ cm}^{-3},$$

(A3)

which is valid for $r > r_{co}$. Consequently, the plasma cutoff radius $r_{cutoff}$ at which $\gamma_{\text{pair}} = \phi_{\text{obs}}$ is

$$r_{cutoff} \sim 2.3 \times 10^{13} \frac{Z}{A_0} \frac{\Theta_0^2}{\phi_{\text{obs}}},$$

(A4)

Figure 4 shows the overall evolution of leptonic FB.

**A2 Baryonic Wind**

Provided that the FB outflow forms in the vicinity of the NS surface, it is expected that some amount of baryons might be contaminated, which was most likely the case for SGR 1806–20 giant flare in 2004 (Granot et al. 2006). This might affect the radial evolution of FB with respect to the pure-leptonic case (e.g., Grisrud & Wasserman 1998; Nakar et al. 2005). Conservation of baryon number and energy reads

$$\dot{M} = r^2 \rho \frac{\Gamma \beta c}{\gamma_{\text{pair}}} = \text{const},$$

(A5)

$$L = r^2 \frac{(U' + P') \Gamma^2 \beta c}{\gamma_{\text{pair}}} = \text{const},$$

(A6)

where $\rho'$, $U'$ and $P'$ are the rest mass density, the total energy density and the total pressure, respectively. In case of baryonic wind, $\rho' = A m_p n'$, where $n'$ is the comoving baryon number density with mass number $A$ (atomic number $Z$) and $m_p$ being the proton mass. The magnitude of bulk Lorentz factor is limited by the total entropy per baryon in the FB as

$$\eta \equiv \frac{L}{Mc^2} = \frac{(U' + P') \Gamma}{A m_p n'},$$

(A7)

We can see that the adiabatic evolution ($\Gamma \propto r$ and $T' \propto r^3$) breaks up when the kinetic energy begins to dominate the radiation energy. This transition takes place when $U' + P' \sim A m_p n' c^2$ with a corresponding radius $\Gamma M = \eta (r_0/\Gamma_0)$, above which the Lorentz factor stays constant ($\Gamma_{co} = \eta$). This critical value of $\eta$ is obtained as

$$\eta_{\text{heavy}} \sim 140 \left( \frac{Z}{A} \right)^{1/4} \left( \frac{r_{0.5} \Theta_0}{\text{cm}} \right)^{3/4} \Theta_0.$$  

(A8)

by simply setting $\Gamma M = r_{ph}$, where the Thomson optical depth is approximated as $\tau_T \approx Z n' \sigma_T / \Gamma$, taking into account baryon-associated electrons. An outflow with $\eta > \eta_{\text{heavy}}$ becomes optically thin before reaching coasting radius (i.e., $\Gamma M < r_{ph}$), the coasting...
Lorentz factor becomes \( \eta_r \) at \( r > r_M = \eta_r r_0 \). Therefore, the bulk Lorentz factor evolves as

\[
\Gamma(r) = \Gamma_0 \begin{cases} \left( r/r_0 \right)^{r/r_0} & \text{for } r < r_M, \\ \min[\eta_r, \eta_{\text{heavy}}]^{-1} & \text{for } r > r_M, \end{cases}
\]

where \( r_M = r_0 \min[\eta_r, \eta_{\text{heavy}}] \). We consider here the case of relatively high-load FB with \( \eta \lesssim 10^3 \), for which the number density of positrons becomes negligible compared to that of both electrons and baryons (i.e., \( n_e^+ \approx Z n_p^+ \) assuming the charge neutrality). In this case, pair annihilation does not occur anymore and the electron number density conserves:

\[
\partial_t \left( r^2 n_e^+ \Gamma \right) = 0,
\]

where LHS represents the net pair creation rate. Therefore, setting \( b_{\text{bar}} \) (i.e., \( n_e^+ \)) becomes negligible compared to both the electrons and \( \eta_{\text{heavy}} \).

The plasma cutoff radius \( r_{\text{cut}} \) at which
\[
\eta = \eta_{\text{mild}} = 3.8 \times 10^4 (Z/\Lambda) \theta, \quad \text{and } \eta_{\text{weak}} \approx \eta_{\text{mild}} \text{ at } r = r_{\text{ph}},
\]

where \( \Lambda = 1 \) and \( Z/\Lambda = 1 \).

Although not covered in this work, for completeness, we briefly describe the weak load case. The weakly-loaded baryon outflow evolution (\( \eta < \eta_{\text{mild}} \)) can be characterized by the additional critical value of \( \eta = \eta_{\text{weak}} \) when \( m_r n_e^+ - m_p n^+ \) at \( r = r_{\text{ph}} \) (hence \( \eta_{\text{weak}} \approx r_{\text{ph}}^{-2} m_r / m_p \)).

The plasma cutoff radius \( r_{\text{cut}} \) at which \( \eta = \eta_{\text{obs}} \) is

\[
r_{\text{cut}} = 7.1 \times 10^{-15} r_0 \eta_2 \theta^{-1} \left( \eta < \eta_{\text{heavy}} \right) \left( \eta_{\text{obs}} > \eta < \eta_{\text{mild}} \right).
\]

The above evolution is true up to the second critical point with \( \eta = \eta_{\text{mild}} \approx 3.8 \times 10^4 (Z/\Lambda) \theta, \) where \( n_e^+ \approx Z n_p^+ \) at \( r = r_{\text{ph}} \).

In the absence of dissipation, the bulk Lorentz factor of a magnetised outflow grows as \( \Gamma \approx 10 \) due to the balance between the electromagnetic and centrifugal forces up to the fast magnetosonic surface, beyond which there is little acceleration (Becker et al. 1998; Contopoulos & Kazanas 2002; Komissarov et al. 2009). We set the initial flow velocity to the Alfven four-velocity

\[
U_A = B^0 / (4 \pi r_0 \rho_0^2 c^2)^{1/2} = c_0^{1/2} (B_0^2 \rho_0^2 / 4 \pi r_0^2)^{1/2},
\]

where \( \rho_0 = 1/2 \) and \( B_0^2 / 4 \pi \) is the magnetic field at the initial radius \( r = r_0 \). Since the dissipation only sets in at \( r_{\text{diss}} \sim r_{\text{ph}} / \epsilon = 10 c_0^{-1} r_0 \) where \( c_0 \) is the rest mass density at the initial radius \( r = r_0 \). Therefore, we can safely neglect the dynamical evolution before passing the fast magnetosonic point (Drenkhahn 2002), unless an extremely high magnetization \( (\sigma_0 \gg 1000) \) is considered. The initial pair number density is determined at \( r = r_0 \) by the following condition:

\[
E_{\text{bare}} = (1 + \sigma_0) I_2^{3/2} 4 \pi \rho_0^2 c^2 r_0^2 r_{\text{cut}},
\]

where \( \Gamma_0 = \beta / 2 \) and \( \eta_{\text{mild}} \) is the saturation radius where the acceleration ends. We confirm that this gives a very good approximation of Lorentz factor during the acceleration phase for \( \sigma_0 \gg 1 \). Assuming that the flow is in the acceleration phase, we the cutoff radius for maser-type emission as

\[
r_{\text{cut}} \approx 2.9 \times 10^{14} r_0^{-1/12} c_0^{1/2} I_{14}^{1/12} 9^{-1/2} \eta_{\text{mild}} \eta_{\text{obs}}^{-1} \text{cm},
\]

which is remarkably independent of \( \sigma_0 \) and \( \epsilon \).
