Probing the Impact of WIMP Dark Matter on Universal Relations, GW170817 Posterior and Radial Oscillations

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ABSTRACT

In this study, we investigate the impact of Weakly Interacting Massive Particles (WIMPs) dark matter (DM) on $C$ – $\Lambda$ universal relations, GW170817 posterior and radial oscillations of neutron stars (NSs) by considering the interactions of uniformly trapped neutralinos as a DM candidate with the hadronic matter through the exchange of the Higgs boson within the framework of the Next-to-Minimal Supersymmetric Standard Model (NMSSM). The hadronic equation of state (EOS) is modeled using the relativistic mean-field (RMF) formalism with IOPB-I, G3, and QMC-RMF series parameter sets. Presence of DM softens the EOS at both the background and the perturbation levels that implies a small shift to the left in the posterior accompanied by a much larger jump in the left of the mass-radius curves with increasing DM mass. It is observed that EOSs with DM also satisfy the $C$ – $\Lambda$ universality relations among themselves but get slightly shifted to the right in comparison to that without considering DM. Additionally, we find that the inclusion of DM allows the mass-radius ($M$ – $R$) curves to remain consistent with observational constraints for HESS J1731-347, indicating the possibility of classifying it as a dark matter-admixed neutron star (DMANS). Moreover, we explore the impact of DM on the radial oscillations of pulsating stars and investigate the stability of NSs. The results demonstrate a positive correlation between the mass of DM and the frequencies of radial oscillation modes.

Key words: stars: neutron, (cosmology:) dark matter, (transients:) neutron star mergers, gravitational waves

1 INTRODUCTION

GW170817, the first discovery of gravitational waves (GWs) from the merger of binary-neutron-stars (BNS) et al. (2017a), has provided a new methodology to constrain an equation-of-state – a definitive relation between pressure and energy density – of NS Annala et al. (2018). A second signal GW190425, likely emitted during an NS binary coalescence, is originated from a much greater distance Abbott et al. (2020) and thus, much weaker than GW170817. However, the presence of neutron stars in the binary cannot be established beyond doubt. Nonetheless, possible upcoming detections of such a post-merger signal Torres-Rivas (2019) will put forward yet another macroscopic probe of NS physics, e.g. Baia et al. (2017); Bauswein (2015). The mass constraints put by the GW170817 data ($2.01 \pm 0.04 \leq M(M_\odot) \leq 2.16 \pm 0.03$) Rezzolla et al. (2018) and the experimental measured mass of PSR J0348+0432 (with mass $2.01 \pm 0.04(M_\odot)$ Antoniadis et al. (2013) propound that NSs should have the masses in the limit $\sim 2.0 M_\odot$. The Neutron Star Interior Composition Explorer (NICER) observation constrains the radius of an NS to be in the range (11.96 – 14.26 km) Miller et al. (2019). Recently, astronomers made an interesting discovery within the remains of a supernova explosion: a central compact object (CCO) has been seen and named HESS J1731-347 Doroshenko et al. (2022). This mysterious object shows a mass of about $0.7720 M_\odot$ and a radius estimated to be approximately $10.4^{+0.86}_{-0.78}$ km. The unexpected features of HESS J1731-347 have sparked a lively discussion among scientists, focusing on its true nature. The newly found object’s characteristics have raised an intriguing question: is it an NS with an extremely low mass, or could it be a strange star, displaying a more exotic EOS Horvath et al. (2023); Clemente et al. (2023); Das & Lopes (2023); Rather et al. (2023); Kubis et al. (2023); Huang et al. (2023); Sagun et al. (2023)? The nature of this compact object is still concealed in mystery, and further observations and thorough studies are needed to uncover its true identity.

The marvelous combination of low temperatures and supranuclear densities, unattended anywhere else in the Universe, makes NSs coveted and peerless laboratories for testing such extreme matter. With the establishment of GW astronomy on a firm observational footing, it is now plausible to comprehend and, better still, probe primal queries of nature. For example, recently in Ref. Chakravarti et al. (2020), the effect of introducing extra dimensions on NS properties has been studied, and the brane tension in the Brane-world model is constrained using the GW170817 posterior. Potential forthcoming BNS in-spirals, merger, and post-merger data can be used to test alterations in general relativity with finite-range scalar forces (in particular, $f(r)$ gravity) Sagunski (2018). The observation of GWs

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from the BNS merger can also shed light on a plausible model of dark matter (DM) that could yield some extra peaks in the post-merger frequency spectrum. These peaks can be detected in the foreseeable time GWs signal from NS mergers Ellis et al. (2018). Hence, NSs are a sensitive probe to detect and define the nature of DM particles.

Astrophysical constraints on the masses and radii of NSs have long been used to comprehend the microscopic properties of the matter that forms them. Since the first evidence of the "missing mass" problem Zwicky (1933, 2009), a lot of theoretical, experimental, and observational efforts have been made to solve the mystery of dark matter. The nature and origin of dark matter particles are still unknown, nevertheless, many dark matter candidates (for a review on dark matter candidates, see e.g. Taoso et al. (2008)) have been proposed and examined to constrain their properties. An enthralling solution to the dark matter case is given by weakly interacting massive particles (WIMPs) MUNOZ (2004) (and references therein), which satisfy the relic density of dark matter in the Universe (calculated by the WMAP Collaboration: 0.094 < relic-density < 0.129 Bennett et al. (2003); Spergel et al. (2003), and the PLANCK Collaboration: 0.120 ± 0.001 Planck Collaboration et al. (2020)). However, WIMPs are very arduous to detect, yet their interactions with nuclei through elastic MUNOZ (2004) or inelastic scattering Ellis et al. (1988); Ejiri et al. (1993); Starkman (1995) are being studied in several laboratories. Experiments for the direct detection of dark matter, such as LUX, XENON, and DAMA/LIBRA, impose stringent constraints on the parameter space of dark matter particles. The stiffest bound on the WIMP-nucleon spin-independent cross-section achieved by the LUX da Silva (2017), XENON et al. (2017b), and DAMA/LIBRA Belli (2011); Bernabei et al. (2019) collaborations are, respectively, $1.1 \times 10^{-46} \text{cm}^2$ for a $50 \text{GeV}/c^2$ WIMPs, $7.7 \times 10^{-47} \text{cm}^2$ for $35 \text{GeV}/c^2$ WIMPs, and $10^{47} \text{cm}^2$ for $100 \text{GeV}/c^2$ WIMPs at 90% C.L. The invisible Higgs decay width, provided by the ATLAS and CMS collaborations at the LHC, strongly constrains the dark matter below $m_H/2$ ($m_H$: Higgs mass), hence, kinematically favorable for Higgs boson to decay into pair of DM particle with mass $M_{DM} < 62.5 \text{GeV}$ Lopes (2022). Indirect searches for DM also constrain its mass. The mass of WIMP is constrained in the range of few $eV < M_{DM} < 1 \text{MeV}$ with the help of the data on the age, density, and observed structure of the present universe Steigman & Turner (1985). In Ref. Bringmann (2014), the thermal annihilation rate is constrained with the value $<v_{rel}> \lesssim 3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}$ for DM masses $M_{DM} \lesssim 100 \text{GeV}$ ($M_{DM} \lesssim 200 \text{GeV}$) assuming cardinal annihilation to $\mu^+\mu^-$ ($e^+e^-$) final states and for $M_{DM} \lesssim 55 \text{GeV}$ ($M_{DM} \lesssim 55 \text{GeV}$) considering chief annihilation into light quarks ($b\bar{b}$).

Apart from DM direct/indirect detections Gascon (2015); Kahlhoefer (2017); Gaskins (2016), constraints on DM parameters can also be seen by observations of NS properties Goldman (1989); Bertone (2008); Kouvaris (2008); de Lavalazz (2010); Kouvaris (2010, 2011); Leung (2011); McDermott (2012); Bramante (2013, 2014); Güver et al. (2014); Baryaktar (2017); Raj (2018); Bramante (2022); Ellis et al. (2018); Ellis (2018); Ivanovskiy (2020); Karkevandi (2022); Baym (2018); McKeen (2018); Gould et al. (1990); Hook & Huang (2018); Huang (2019). These studies are done by considering WIMPs trapped inside NS Goldman (1989), studying effects of self-interacting/annihilating DM on NS Kouvaris (2008, 2010); de Lavalazz (2010); Bramante (2013, 2014); Ellis (2018); Karkevandi (2022), the impact of charged massive DM particle on NSs Gould et al. (1990), probing axions with NS mergers Hook & Huang (2018); Huang (2019), and from the collapse of an NS due to accretion of non annihilating DM Leung (2011); Kouvaris (2010); McDermott (2012); Güver et al. (2014) etc. In Nelson et al. (2019); Karkevandi (2022), the DM parameters are constrained by considering DM halo around NS. The effects of DM (WIMPs trapped inside an NS core) on NS properties have been studied Panotopoulos (2017); Das (2019); Quddus et al. (2020) within the relativistic mean-field (RMF) formalism. Considering the mechanical model of the dark matter core inside an NS, it is shown in Ref. Ellis et al. (2018) that supplementary peaks may be observed in the power spectral density (PSD) of the GW emission following an NS-NS merger. In Ref. Das et al. (2020a), the unknown parameters of the DM EOS are fixed by using Bayesian parameter optimization. Recently, the mass of a Fermionic DM particle ($M_{DM} \sim 60 \text{GeV}$ for a $2M_\odot$ NS) with its fraction inside an NS is constrained by using data on the spatial distribution of DM in the Milky Way and following the two-fluid approach (DM and NS matter interact gravitationally) Ivanovskiy (2020). Moreover, in Ref. Karkevandi (2022), it is found that considering Bosonic DM inside the NS core and imposing constraints from NS observable mass and tidal deformability data by the LIGO/Virgo Collaboration, sub-GeV DM particles are favored with a low fraction below ~ 5%. In addition, another crucial microscopic property known as the oscillation of an NS plays a significant role in investigating the NS internal composition and serves as a valuable constraint for understanding the EOS. These oscillations can occur in two ways: radial and non-radial oscillations. Over the years, numerous studies have been proposed to elucidate the phenomenon of NS oscillation (Chandrasekhar 1964; Chandnugam 1977; Vaeth & Chandnugam 1992; Gondev et al. 1997; Gondev & Zdunik 1999; Kokkotas & Ruoff 2001; Kunjinurayil et al. 2022; Sen et al. 2023), each assuming different compositions of the NS. Therefore, in our DMANS model, the impact of dark matter on the oscillations is studied.

In this study, we have considered the presence of WIMPs trapped inside the NS core. The detection of theoretically predicted peaks in the post-merger frequency spectrum Ellis et al. (2018) may provide valuable insights into the nature of DM. However, at the present time, the LIGO-VIRGO-KAGRA detectors are still far from achieving the required sensitivity to fully solve the DM puzzle. As discussed, various theoretical studies Ellis et al. (2018); Ellis (2018); Ivanovskiy (2020); Karkevandi (2022); Kouvaris (2011); Panotopolous (2017); Das (2019); Quddus et al. (2020); Das et al. (2020a,b); Routaray et al. (2023a,a,b) lend support to the idea of a DM core existing inside an NS, but the DM parameters exhibit a significant degree of degeneracy. In this work, we present a posterior GW mass-radius contour, developed using data from BNS mergers, to assess which M-R plots (obtained at different WIMP masses) are consistent with the observational data. Additionally, we utilize the mass-radius constraint of the CCO HESS J1731-347 to determine its true nature within the DMANS model. The $C - \Lambda$ universality relations (where $C$ is the compactness of a star and $\Lambda$ is its dimensionless tidal deformability) have been discussed for the EOSs corresponding to these different WIMP masses to investigate whether this universality breaks down with DM presence (and on increasing its mass) or not.

2 EQUATION OF STATE OF DARK MATTER ADMIXED NEUTRON STAR

The presence of a DM core inside an NS with a mass fraction of approximately ~ 5% has significant effects on NS data Ellis (2018). The amount of DM present inside an NS is not universal; instead, it depends on the environment in which the NS was formed and its evolution history (age, initial temperature) Ellis (2018). For the usual evolution of an NS with a lifetime of approximately ~ 10 Gyr, the
The NMSSM introduces two additional Higgs bosons. It is noteworthy that in NMSSM, the spin-independent cross-section of DM-nucleon scattering can be as high as the one implied by the DAMA results. This is in contrast to the MSSM, where such high cross-sections are not achievable due to the absence of light Higgs bosons exchange. However, it should be mentioned that the DAMA results have faced significant criticism, and other similar experiments have not confirmed any signal yet.

In this study, we have considered the interaction of neutralino ($\chi$) - the lightest mass eigenstate of WIMP within the NMSSM - as one of the Fermionic DM candidates. We assume that the neutralino interacts with the hadronic part of the NS through the exchange of the Higgs field $h$. It is important to note that the obtained results and analyses remain valid for any Fermionic WIMP. The reason for considering the WIMP as a DM candidate is that a one-to-one relation is observed between the spin-independent direct detection rate and the DM relic density if the elastic scattering on nuclei occurs dominantly through Higgs exchange.

The NMSSM is a simple extension of the MSSM with the addition of a singlet supermultiplet. It offers several advantages over the MSSM:

- It provides a solution to the hierarchy problem while preserving the favorable properties of the MSSM.
- It addresses the $\mu$ problem.
- Compared to the MSSM, NMSSM introduces two additional Higgs bosons.

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In this work, we propose to analyze the effect of this region of the parameter space on the properties of NSs. The interaction Lagrangian density for the neutralino interacting with the hadronic matter (NM) ($\phi$) through Higgs exchange is given as follows.

$$L = L_{\text{had.}} + \bar{\chi} \left[ i\gamma^\mu \partial_\mu - M_{DM} + y h \right] \chi + \frac{1}{2} \partial_{\mu} h \partial^\mu h - \frac{1}{2} M_h^2 h^2 + f \frac{M_h}{v} \phi \bar{\psi} \psi,$$

where $L_{\text{had.}}$ is the Lagrangian density for the hadronic matter (for details, see Ref. Quddus et al. [2020]). The factor $f$ represents the Higgs-nucleon coupling parameter (Cline [2013, 2015]), with a central value $f = 0.3$ considered based on lattice computations.

Figure 1. (color online) Universality relation for an NS corresponding to the chosen EOSs without DM is presented in the panel (a) of the figure, followed by the same in the presence of DM with $M_{DM} = 4$ GeV (b), $M_{DM} = 8$ GeV (c), and $M_{DM} = 24$ GeV (d), respectively. The black bold line is the fitted curve of $C - \Lambda$ relation when there is no DM presence. The sequence of $C - \Lambda$ points deviate more as more massive DM is added, but does not break the qualitative nature of the universal behavior. The fitted curves are represented by black dashed lines in Panels (b-d).
In this context, $M_N$ and $M_p$ denote the masses of the nucleon and Higgs boson, respectively, and $v = 246$ GeV corresponds to the Higgs vacuum expectation value. Within NMSSM, a light Higgs boson with mass $M_H = 40$ GeV is chosen to satisfy the condition that the Yukawa coupling (interaction strength of the Higgs field with the DM particles) remains in the perturbative regime ($y < 1$). On changing the DM particle mass, the coupling coefficient $y$ is calculated by following the Eq.

$$\sigma(\chi \phi \to \chi \phi) = \frac{\mu^2}{\pi v^2 m^2_{DM}} f^2 M^2_{\chi}$$

(2)

where $\mu$ is the reduced mass of a dark matter-nucleon system, and $\sigma$ is the spin-independent cross-section of dark matter interaction with nucleon, satisfying the DAMA results Andreas et al. (2008).

The EOS of NM can be derived within effective theories that incorporate appropriate degrees of freedom. Various approaches, such as non-relativistic Skyrme-type density functional theory Tondeur et al. (1984), perturbative QCD Kurkela (2010); Gorda (2016), and the relativistic mean-field formalism (RMF) Walecka (1974), are used to generate EOSs of nuclear matter. In this study, we have generated EOSs of pure hadronic matter using the RMF formalism. These EOSs correspond to the parameter sets IOBP-I Kumar (2018), G3 Kumar et al. (2017), QMC-RMF1, QMC-RMF2, QMC-RMF3, and QMC-RMF4 Alford et al. (2022). The QMC-RMF series of parameter sets are specifically developed to model nuclear matter across a wide range of temperatures and densities that are relevant for neutron stars and neutron star mergers. These interactions take into account the uncertainties provided by chiral effective field theory ($\chi$EFT) for neutron matter and are consistent with contemporary astrophysical constraints on the EOS Alford et al. (2022).

By considering the RMF EOS for the hadronic part of the NS and imposing charge neutrality and chemical equilibrium conditions, the EOS of a dark matter-admixed NS can be determined using mean-field approximations and the variational principle. The detailed formalism can be found in Ref. Qudus et al. (2020), but for completeness, we provide here the expressions for the energy density and pressure of the dark matter-admixed neutron star.

$$E_{NS} = E_{had.} + E_{DM} + E_l$$
$$P_{NS} = P_{had.} + P_{DM} + P_l$$

(3)

Where

$$E_l = \sum_{l=e, \mu} \frac{2}{(2\pi)^3} \int_0^{k_l} d^3k \sqrt{k^2 + m^2_l}$$

$$P_l = \sum_{l=e, \mu} \frac{2}{(2\pi)^3} \int_0^{k_l} d^3k \frac{k^2}{\sqrt{k^2 + m^2_l}}$$

(4)

are the energy density and pressure for the leptonic part of an NS with lepton mass $m_l$ and fermion momentum $k_l$.\n
$$E_{DM} = \frac{2}{(2\pi)^3} \int_0^{k_{DM}} d^3k \sqrt{k^2 + (M^2_{DM})^2} + \frac{1}{2} M^2_{h0}$$

$$P_{DM} = \frac{2}{(2\pi)^3} \int_0^{k_{DM}} d^3k \frac{k^2}{\sqrt{k^2 + (M^2_{DM})^2}} - \frac{1}{2} M^2_{h0}$$

(5)

are the expressions for energy density and pressure for the DM interaction with the hadronic matter of an NS through the exchange of Higgs field with DM fermi momentum $k_{DM} = 0.06$ GeV Das (2019); Qudus et al. (2020) (assuming baryonic density of NS about $10^3$ times greater than the DM density inside NS). The energy density and pressure for the hadronic part of NS are given as the following:

$$E_{had.} = \frac{2}{(2\pi)^3} \int d^3k E^2_l(k) + \rho_b W + \frac{m^2_f \Phi^2}{g^2_\phi} \left( \frac{1}{2} + \frac{\kappa_3 \Phi}{3! M} \right)$$

$$+ \frac{\kappa_4 \Phi^2}{4! M^2} - \frac{1}{2} \eta_1 \Phi \frac{W^2}{g^2_\phi} \left( 1 + \eta_1 \Phi + \frac{\eta_2 \Phi^2}{2 M^2} \right)$$

$$+ \frac{1}{4! \frac{\nu^4 W^4}{g^4_\phi}} + \frac{1}{2} \eta_3 \Phi \frac{W^2}{g^2_\phi} \left( 1 + \eta_1 \Phi + \frac{\eta_2 \Phi^2}{2 M^2} \right)$$

$$- \Lambda_{\omega}(R^2 \times W^2) + \frac{m^2_\rho}{g^4_\rho} (D^2)$$

$$P_{had.} = \frac{2}{3(2\pi)^3} \int d^3k \frac{k^2}{E^2_l(k) + \rho_b W + \frac{m^2_f \Phi^2}{g^2_\phi} \left( \frac{1}{2} + \frac{\kappa_3 \Phi}{3! M} \right)$$

$$+ \frac{\kappa_4 \Phi^2}{4! M^2} - \frac{1}{2} \eta_1 \Phi \frac{W^2}{g^2_\phi} \left( 1 + \eta_1 \Phi + \frac{\eta_2 \Phi^2}{2 M^2} \right)$$

$$+ \frac{1}{4! \frac{\nu^4 W^4}{g^4_\phi}} + \frac{1}{2} \eta_3 \Phi \frac{W^2}{g^2_\phi} \left( 1 + \eta_1 \Phi + \frac{\eta_2 \Phi^2}{2 M^2} \right)$$

$$+ \Lambda_{\omega}(R^2 \times W^2) - \frac{1}{2} \frac{m^2_\rho}{g^4_\rho} (D^2)$$

(6)

Where $\Phi$, $D$, $W$, and $R$ are the redefined fields for $\sigma$, $\delta$, $\omega$, and $\rho$ mesons (see Qudus et al. (2020)), while $E^2_l(k) = \sqrt{k^2 + M^2_l}$ $(i = p, n)$ is the energy with effective mass $M^2_l = k^2 + M^2_f$, and $k$ is the momentum of the nucleon. The quantities $\rho_b$ and $\rho_\delta$ in the above equation have their usual meanings as the baryonic and iso-scalar densities, respectively. Due to the presence of Higgs field, the effective mass of nucleon get modified as $M^*_{ipt} = M_i + g_\Phi \sigma - g_\omega \delta - g_\rho \rho_\delta$ while the DM effective mass is $M^*_{DM} = M_{DM} - \gamma h_0$, where $h_0$ is the time component of Higgs field. Putting Eqn. 3 in the Tolman–Oppenheimer–Volkoff (TOV) equations Oppenheimer (1939); Tolman (1939) gives the mass-radius profile of NS.

3 RADIAL OSCILLATIONS

To analyze the equations governing the radial oscillations of NS, we introduce the variable $\delta r(r, t)$, representing the time-varying radial displacement of a fluid element.

$$\delta r(r, t) = X(r)e^{i\omega t}$$

(7)

The variable $X(r)$ represents the amplitude, while $\omega$ denotes the circular frequency of the standing wave solution. The linearized perturbation equation is mathematically represented as shown by Kokkotas & Ruoff (2001); Sen et al. (2023).

$$c^2 \frac{X''}{X} + \left( c^2 \right)' - Z \frac{4\pi G}{c^4} \gamma \rho e^{\psi} - v^2 \frac{X'}{X'}$$

$$+ \frac{\nu}{c^2} \frac{X'}{X'} = 0,$$

(8)

The variable $c^2$ represents the square of the speed of sound, while $\gamma$ denotes the adiabatic index. Here, $c^{2 lim}(R) = \left( 1 - \frac{2\Omega M}{c^2 R} \right)^{-1}$ is
Figure 2. (color online) $M - R$ curves of an NS corresponding to the chosen parameter sets in the absence of DM core (a) and in the presence of DM with $M_{DM} = 4$ GeV (b), $M_{DM} = 8$ GeV (c), and $M_{DM} = 24$ GeV (d). The 90% and 50% C.L. posteriors for the primary (blue) and secondary (green) components of GW170817 are shown bounded by solid lines and dashed lines, respectively, in each panel. The dashed rectangular box shows the observational bound for HESS J1731-347.

The boundary conditions for the equations governing oscillations must be formulated to ensure zero displacement at the center ($\delta r (r = 0) = 0$), while also guaranteeing that the Lagrangian perturbation of pressure becomes zero at the surface ($\Delta P (r = R) = 0$).

Considering the mentioned boundary conditions, the displacement function can be rewritten as follow,

$$\zeta = r^2 e^{-\nu} X.$$  \hspace{1cm} (9)

By introducing the above new variable, the equation (8) can be reformulated as a Sturm-Liouville differential equation that possesses a self-adjoint characteristic,

$$\frac{d}{dr} \left( H \frac{d\zeta}{dr} \right) + \left( \omega^2 W + Q \right) \zeta = 0,$$  \hspace{1cm} (10)

where

$$r^2 H = (P + E) e^{4+3\nu} c_s^2,$$

$$r^2 W = (P + E) e^{3+\nu},$$

$$r^2 Q = (P + E) e^{4+3\nu} \left( (\nu')^2 + \frac{4}{r} \right) + \frac{8\pi G}{c^4} e^{2\nu} P.$$  \hspace{1cm} (11)

The equation labeled as (10) represents the fundamental equation governing radial oscillations. In this equation, the quantity $\Delta P$ takes a simple expression given by $\Delta P = -r^{-2} e^{\nu} (P + E) e_s^2 \epsilon'.$

Furthermore, it is worth noting that Equation (10) can be expressed...
in the Sturm-Liouville form. In this representation, the function $\zeta_n$ exhibits $n$ nodes between the surface and the center and possesses discrete eigenvalues denoted as $\omega_n^2$. These eigenvalues follow a specific structure,

$$\omega_0^2 < \omega_1^2 < \ldots < \omega_n^2 < \ldots$$

The solution represented by Equation (7) indicates that oscillations exhibit harmonic behavior and remain stable when the frequency $\omega$ is real. However, the star will experience instability when the frequency associated with the node is imaginary.

4 RESULTS AND DISCUSSIONS

In the literature, it is well known that there exist EOS independent relations Yagi & Yunes (2013); Yagi (2013); Yagi & Yunes (2017) among NSs. These relations hold true under various circumstances, as long as we exclude cases where abrupt phase transitions occur within the star Yagi & Yunes (2017). Of particular interest to us are the $C - \Lambda$ universal relations, also referred to as Universal relations of the First kind. This relation can be expressed through the coefficients $c_n$ of the polynomial fit

$$C = \sum_{n=0}^{4} c_n (\log_{10} \Lambda)^n$$

(12)

between $C$ and $\log_{10} \Lambda$. In the presence of DM, both $C$ and $\Lambda$ undergo changes. The panels of Fig. 1 display the variation of $C - \Lambda$ across different stellar configurations for four distinct values of DM particle masses. Interestingly, we observe that the sequence of $C - \Lambda$ points deviates more as more massive DM is added, but does not break the qualitative nature of the universal behavior. This enables us to fit $C - \Lambda$ for each case of DM mass, and the corresponding fitting coefficients are presented in Table 1.

Since the frequency of gravitational waves depends on the chirp mass and mass ratio, the mass $M$ of a neutron star can be inferred from the observed phases of GWs. Similarly, the tidal deformability $\Lambda$ of the neutron star can also be inferred from the GWs phase during the inspiraling process. Therefore, $(M, \Lambda)$ can be considered as primary GW observables. Subsequently, the $C - \Lambda$ universal relations are utilized to convert the raw $M - \Lambda$ posterior into $M - R$ posteriors et al. (2017a), which are extensively used in EOS inference. In the presence of dark matter, the modified nature of the $C - \Lambda$ function results in a shift in the posterior in the $M - R$ plane, making the EOSs softer, and the $M - R$ curves shift to the left (Fig. 2). This shift also affects the posteriors (including the 90% and the 50% confidence levels) as shown in Fig. 2. A somewhat similar effect was observed in Chakravarti et al. (2020), where the presence of extra dimensions resulted in a shift of the $M - R$ curves and posteriors to the right. The effects of changing EOS due to dark matter are not only limited to the background properties of neutron stars but also extend to the perturbations, leading to variations in $C$ for a given value of $\Lambda$ (Fig. 1). Consequently, the posteriors shift accordingly in the $M - R$ plane (Fig. 2). It is clearly evident that the shifting is more with higher DM mass. Another crucial observation from Fig. 2 is that the mass-radius profile satisfies the observational bound for HESS J1731-347 when dark matter is considered. Specifically, for $M_{DM} = 24$ GeV, all the $M - R$ curves satisfy this constraint. As a result, our study also refers the nature of HESS J1731-347 as a dark matter admixed neutron star Sagun et al. (2023).

In Fig. 3, we investigate the relationship between the eigenfrequencies and central $\rho_c$ for the $f$-mode of radial oscillation. To

| $M_{DM}$ (GeV) | $c_0$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\chi^2_{(1e-6)}$ |
|--------------|-------|-------|-------|-------|-------|-----------------|
| 0            | 0.34791 | -0.06568 | -0.00358 | 0.00136 | -6.90e-05 | 6.0516         |
| 4            | 0.35042 | -0.06785 | -0.00232 | 0.00114 | -5.82e-05 | 6.4963         |
| 8            | 0.35242 | -0.06950 | -0.00140 | 0.00140 | -5.07e-05 | 6.8681         |
| 24           | 0.35821 | -0.07416 | -0.00140 | 0.00136 | -3.31e-05 | 7.9838         |

Figure 3. For different DM mass, the radial $f$-mode frequency is shown for G3, IOPB, and QMC-RMF series model with varying the central energy density ($\rho_c$).

Figure 4. Behaviour of the $f$-mode frequency as a function of mass is plotted for different DM mass.
accomplish this, we vary the mass of the DM using different models. As depicted in the corresponding figure, the stability limit is attained when the density increases and reaches the critical density, regardless of the EOS. At this critical point, the star reaches its maximum mass and becomes unstable, as indicated by a zero eigenvalue for the f-mode (Kokkotas & Ruoff 2001). However, as the mass of DM increases, the critical density also arises due to the softening of the EoS, resulting in higher frequency oscillations. When the density of the star exceeds the critical density, its oscillations exhibit exponential growth. Consequently, the star loses its ability to return to its initial state, ultimately undergoing gravitational collapse and forming a black hole.

Figure 4 displays how the frequency of the f-mode changes with different NS masses, considering the presence and absence of DM, using various models. We observe that the behavior of the f-mode rapidly approaches zero precisely at the point where the NS reaches its maximum mass. This finding aligns with the stability criteria $\partial M/\partial \rho_c > 0$, proposed by Harrison et al. (1965); Shapiro & Teukolsky (1983), which indicate that the NS remains stable as its mass changes with respect to the $\rho_c$.

5 SUMMARY AND CONCLUSION

The primary objective of this work is to investigate the impact of WIMP dark matter on the $C - \Lambda$ universal relation, GW170817 posteriors, and radial f-mode oscillation of NSs. We consider interactions of uniformly trapped neutralinos (the lightest WIMP) as a dark matter candidate with the hadronic matter of NSs through the exchange of the Higgs boson, which is the lightest CP-even eigenstate of the NMSSM. The hadronic part of the EOS is modeled using RMF with IOPB-I, G3, and QMC-RMF series of parameter sets. The presence of dark matter inside the NS core results in reduced values of its properties, making the EOSs softer. The findings of this study can be summarized as follows:

- The EOSs with dark matter satisfy the universality relation of the first kind, the $C - \Lambda$ universal relations, but they slightly shift to the right with increasing dark matter mass compared to the $C - \Lambda$ universal relation when no dark matter is considered. However, the deviated curves satisfy universality relation among themselves.
- A change in the mass of dark matter results in a small shift to the left in the posterior, accompanied by a much larger jump to the left in the $M - R$ curves.
- The incorporation of dark matter into the analysis allows the M-R curve to remain consistent with the mass-radius constraints imposed by HESS J1731-347. Therefore, this study contributes to determining the characteristics of HESS J1731-347, suggesting the possibility that it may be classified as a dark matter-admixed neutron star.
- We further investigated the radial oscillations of pulsating stars by solving the Sturm-Liouville equations for the perturbations and imposing necessary boundary conditions. This allowed us to accurately determine the frequencies associated with the mode. The calculations for the fundamental f-mode were done with and without the presence of dark matter, while systematically varying the dark matter mass. The numerical findings indicate that the inclusion of dark matter within NSs leads to a reduction in the stiffness of the EOS, resulting in a decrease in the maximum mass of these stars. Additionally, the inclusion of dark matter leads to an increase in the frequencies of pulsating objects. In conclusion, there exists a positive correlation between the mass of the dark matter and the frequencies of the radial oscillation modes.

- Finally, we verified the stability criterion of NSs by studying the f-mode as a function of their mass.

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DATA AVAILABILITY

The EOSs used in this article can be inferred from their original articles cited above. Fitting coefficients for $C - \Lambda$ universality relations are given in the text.

REFERENCES

Abbott B. P., et al., 2020, The Astrophysical Journal Letters, 892, L3
Alarcón J., Martin Camalich J., Oller J., 2013, Annals of Physics, 346, 413
Alford M. G., Brodie L., Haber A., Tews I., 2022, Phys. Rev. C, 106, 055804
Alvarez-Ruso L., Ledwig T., Camalich J. M., Vicente-Vacas M. J., 2013,
Physical Review D, 88, 054507
Amare J., et al., 2020, Journal of Physics: Conference Series, 1468, 012014
Andreas S., Hambye T., Tytgat M. H. G., 2008, Journal of Cosmology and
Astroparticle Physics, 2008, 034
Annala E., Gorda T., Kurkela A., Vuorinen A., 2018, Physical Review Letters,
120, 172703
Antoniadis J., et al., 2013, Science, 340
Baiotti L., Rezzolla L., 2017, Reports on Progress in Physics, 80, 096901
Baryakhtar M., 2017, Physical Review Letters, 119
Bauswein A., 2015, Physical Review D, 91
Baym G., 2018, Physical Review Letters, 121
Belli P., 2011, Physical Review D, 84
Bennett C. L., et al., 2003, The Astrophysical Journal Supplement Series,
148, 1
Bernabei R., et al., 2019, Nuclear Physics and Atomic Energy, 20, 317
Bertone G., 2008, Physical Review D, 77
Bramante J., 2013, Physical Review D, 87
Bramante J., 2014, Physical Review D, 89
Bramante J., 2022, Physical Review Letters, 128
Brayeur L., 2012, Physical Review Letters, 109
Bringmann T., 2014, Physical Review D, 90
 Buttazzo D., Panch P., Rossi N., Strumia A., 2020, Journal of High Energy
Physics, 2020, 137
Chakravarti K., Chakraborty S., Phukon K. S., Bose S., SenGupta S., 2020,
Classical and Quantum Gravity, 37, 105004
Chandrasekhar S., 1964, apj, 140, 417
Channugum G., 1977, apj, 217, 799
Clemente F. D., Drago A., Pagliara G., 2023, Is the compact object associated
with HESS J1731-347 a strange quark star? (arXiv:2211.07485)
Cline J. M., 2013, Physical Review D, 88
Cline J. M., 2015, Physical Review D, 92
Das A., 2019, Physical Review D, 99
Das H. C., Lopes L. L., 2023, Anisotropic Strange Stars in the Spotlight: Unveiling
Constraints through Observational Data (arXiv:2306.00326)
Das A., Malik T., Nayak A. C., 2020a, arXiv:2011.01318
Das H. C., Kumar A., Kumar B., Biswal S. K., Nakatsukasa T., Li A., Patra S. K.,
2020b, Monthly Notices of the Royal Astronomical Society, 495, 4893
Doroshenko V., Suleimanov V., Pühlhofer G., Santangelo A., 2022, Nature
Astronomy, 6, 1444
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