Research on Machining Error of Globoidal Cam Profile Based on Adaptive Algorithm

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Abstract: In order to effectively control the machining error of globoidal cam profile, an adaptive tool position error compensation algorithm is proposed. According to the space meshing principle and the rotation transformation tensor method, the actual working profile equation of the non-equalization diameter machining globoidal cam is established. The objective is to minimize the maximum deviation of the actual tool axis, and search the optimal tool offset and direction during machining to achieve adaptive compensation of the tool position. When the tool radius compensation amount \( \Delta r = 3 \text{mm} \), the theoretical value of the maximum normal error of the cam profile is close to the simulation value. The result validates the effectiveness of the tool position control technique of the adaptive algorithm for machining error compensation, thereby the machining accuracy of the globoidal cam is improved greatly.

1. Introduction

As the key component of the mechanical mechanism, the globoidal cam is not only of high precision, but also small in volume, lightweight, high transmission efficiency and long life, and is especially suitable for where is high speed, high precision and high efficiency [1]. Due to its unique structure, favorable applicability and ability to achieve any complex form of motion, globoidal cams play an indispensable role in light industrial production, automotive manufacturing, aerospace and automation machinery and other fields.

Globoidal cam is a precision mechanical element, and cam production is a complicated task [2]. At present, the NC machining methods of globoidal cam are generally divided into two categories, i.e. equalization diameter machining and non-equalization diameter machining[3] (also called un-equivalent machining, i.e., when the width of cam trough is bigger than that of machining cutter, the trough-outline needs to be machined at single side with small edge cutter). Equivalent machining is carried out on NC machine tool by generating method in equal diameter machining [4]. In non-equalization diameter machining, the tool diameter is smaller than the diameter of the roller, and the machining methods include the resembling freeform surface method, the tool position compensation method and the double enveloping method, Compared with equalization diameter machining, non-equalization diameter machining can improve machining efficiency and reduce production cost.

The machining error cannot be ignored when machining globoidal cam. The main factors that affect the error of machining precision are processing error, programming error, installation error and...
tool wear error. From the above way of processing error, we can see that there is originally theoretical error in the resembling freeform surface method and tool compensation method. Kong Mabin [5] proposed an optimization algorithm for calculating the tool position data of the single-sided NC machining of the curved cam with the minimum deviation between the trajectory surface of tool axis and equidistant surface of globoidal cam. Fuming Yin [6] analyzed the theoretical errors in the single side machining, and proposed the tool position control principle for the single side machining of the globoidal cam. J N Lee [7] analyzed the arch height error and chord height error caused by globoidal cam during machining, and proposed a way to reduce the error. From the perspective of the error effectiveness control and the efficiency of the algorithm, the overall error control is relatively small, and there is no need to consider the existence of other local errors in the cam profile processing. Therefore, in this paper, a new tool position control technique with diameter of cutters smaller than the roller size based on adaptive algorithm is proposed to effectively control the machining error of the cam profile.

2. The mathematical model of globoidal cam

The simplified model of the globoidal cam mechanism is shown in Figure 1, \(o-xyz\) is a fixed coordinate system fixed with the frame, where \(z\) axis is the axis direction of the driven disc, the direction of the \(x\) axis is directed to the center of the cam from the center of the driven disc. \(o_{1}-x_{1}y_{1}z_{1}\) is a dynamic coordinate system which is connected to the driven disc and rotates with it. The \(x_{1}\) axis coincides with the axis of the roller and turns around the ‘point \(O\)’ together with the roller, and \(\alpha_{1}\) is the angular displacement of the driven disc. \(o_{2}-x_{2}y_{2}z_{2}\) is the dynamic coordinate system which is connected to the cam and rotates around the \(y_{2}\) axis with the cam. The \(\alpha_{2}\) is the angle between the \(x_{2}\) and the \(x\) axis, \(c\) is the center distance of the driven disc to the globoidal cam center, the \(\beta\) is the contact angle of the roller and the cam at the contact point \(D\), and the \(n_{1}\) is the unit vector of the common line at the point \(D\) between roller and the globoidal cam.

![Figure 1. Simplified model of globoidal cam mechanism](image)

The contact point \(D\) on the roller in the coordinate system \(o_{1}-x_{1}y_{1}z_{1}\) the position vector can be expressed as

\[
R_{c} = (l_{0} + l, R\cos\beta, R\sin\beta)^{T}
\]

The \(l_{0}\) is the distance from the center of the driven disc to the inner end of the roller, and the \(l\) is the meshing depth, i.e. the distance from the inside end of the roller to the contact point \(D\).

According to the reference [8] and the principle of conjugate surface meshing, the relative velocity between the two surfaces is orthogonal to each other at the common normal. Then the unit normal vector of the contact point \(D\) on the roller in the dynamic coordinate \(o_{1}-x_{1}y_{1}z_{1}\) is:

\[
\mathbf{n} = \frac{\partial R_{x}}{\partial \beta} \times \frac{\partial R_{y}}{\partial \beta} = \begin{pmatrix} 0 \\ \cos\beta \\ \sin\beta \end{pmatrix} = (0, \cos\beta, \sin\beta)^{T}
\]

Let \(F_{1}\) and \(F_{2}\) be the angular velocity vectors of the roller and the cam in the fixed coordinate system \(o-xyz\). Then, at the contact point \(D\), the relative speed of the meshing surface is:
\[ v = F_1 \times R_1 - F_2 \times (R_2 - c) \]  

(3)

Where \( c = (c, 0, 0)^T \).

The meshing equation of the conjugate surface between the work surface of the roller and the profile of the globoidal cam is as follows:

\[
\tan \beta = \pm \frac{(l + l_0)}{c - (l + l_0) \cos \omega_1} \left( \frac{\omega_1}{\omega_2} \right)
\]

(4)

In the formula, the positive sign is a left-handed cam, minus is a right-handed cam, and \( \omega_1 \) is a driven disc angular velocity, and \( \omega_2 \) is a globoidal cam angular velocity.

3. Determination of non-equalization Machining Tool Position of Globoidal Cam

The profile of globoidal cam at the division is an unfolding space curved surface. The tool can't process the curve with the same shape no matter how it is compensated. Then the profile will have errors. Thus it is not possible to process the globoidal cam groove by means of tool compensation when machining the plane cam. If the tool radius is smaller than the roller radius for machining, no matter how to control the tool position, there still be a programming error on the cam profile, but the machining error can be smaller by a certain tool position control method. Thus, the positional relationship between the theoretical contact line and the actual contact line determines the error size of the cam profile. The proper tool position control method is adopted so that the closer the two curves are, the better the degree of coincidence is, and the error of the convex profile surface is smaller.

The tool compensation method is the realization of the generating machining of cam profile through the cutter offset, the aim is to minimize the machining error. The machining principle of the tool position compensation method is shown in Fig. 2, where \( R \) is the radius of the roller, \( r \) is the tool radius, A and D are the contact points of the tool and the roller, the curve MN (i.e. the actual contact line) is the contact line between the tool and the cam profile, curve PQ (i.e. the theoretical contact line) is the contact line between the roller and the cam profile, B is the intersection of the theoretical contact line and the actual contact line. When the diameter of the tool is smaller than the diameter of the roller, it is offset by a compensation amount of \( \Delta = R - r \), and the roller and cutter are tangent at AC. The actual contact line in machining is PQ, then the tool can only reach to point B, while other points on the MN are in under-cut state and occurs under-cut error [9]. In this way, although the theoretical contact line and the actual contact line are on different cylinders, the two contact line is intersected at a point B at half the length of the roller contact line, and the normal error of the profile surface is 0, then the error of the two contact line is minimized.

![Figure 2. Tool compensation diagram](image)

4. Optimization algorithm of adaptive compensation tool position

For the determination of the position of the tool center after the tool position compensation, according to the rotation transformation tensor method and the principle of differential geometry, the coordinate
of the tool center in the cam coordinate system is obtained through the coordinate transformation of the calculated values in the roller coordinate system. As shown in Figure 3, \( o_1-x_1y_1z_1 \) is a dynamic coordinate system which is connected to the driven disk and rotates with it. The \( x_1 \) coincides with the axis of the roller and turns around the ‘o’ point together with the roller, and the \( \alpha_1 \) is the angular displacement of the driven disc. \( o_2-x_2y_2z_2 \) is a dynamic coordinate system that is fixed with the cam and rotates with the cam around the \( y_2 \), where \( \alpha_2 \) is the angle between \( x_2 \) and the \( x \). \( W' \) is a point on the axis of the roller, also known as the tool center position before compensation. Point \( W \) is the tool center position after compensation, and assume \( l \) is the vertical distance cross section of the tool center to the \( o_1 \) of the \( o_1-x_1y_1z_1 \) coordinate system. In each process, the amount of cutters is set to a fixed value, that is, \( l' \).

![Figure 3. Tool center coordinate for compensation processing](image)

In the roller coordinate system, the vector of the \( W' \) tool center position before compensation is \( t_1 \), and it is also a function of the feed parameter \( l' \) for each machining process. Then, the tool center point \( W \) for tool position compensation is

\[
t_2'(\alpha_2)_j = t_1'(l') + \Delta_r \cdot n(l, \alpha_2)
\]

Where \( n(l, \alpha_2) \) is the direction of compensation, \( \Delta_r \) is the radius compensation. In the process of error control, the direction of the tool position compensation is generally the normal vector direction at any point on the theoretical contact line, and \( n(l, \alpha_2) \) is a function of the parameter \( l \) along the axis of the tool and the cam angle \( \alpha_2 \). According to the coordinate transformation matrix \( M_{i0} \) from the coordinate system \( o_1-x_1y_1z_1 \) to the \( o-xyz \) of the fixed coordinate system, then position of the cutter center in the cam processing is as follows:

\[
t_2'(\alpha_2)_j = M_{i0} \cdot t_1'(l')_j
\]

The key of the cutter position compensation in the non-equalization diameter machining of the globoidal cam is the selection of the direction of compensation. The key to the tool position compensation used in non-equal diameter machining of cambered camber surface is the selection of the tool position compensation direction, and the best compensation amount is the difference between the radius of the roller and the tool. In the machining process, if the compensation amount \( \Delta_r \) and the compensation direction \( n(l, \alpha_2) \) are properly restricted, the normal error \( \Delta_n \) is a binary function of the compensation amount \( \Delta_r \) and parameter \( l \) of the compensation direction for any cam angle \( \alpha_2 \), then the minimum value of the normal error is used as the objective function, and the adaptive tool position optimization compensation of the tool location is performed to obtain the optimal parameter value. The optimization algorithm can reasonably control the selection of the tool location and the compensation amount at a certain point on the contact line. In order to effectively shorten the optimization compensation time, for selection of compensation direction, a small section of the middle position of the radial effective length of the roller is considered as the domain of the compensation parameter, i.e. \( l \in (l_0/2, l_0) \). For compensation amount, a small section of difference \( \Delta_r \) between the
tool and roller radius can be selected as a domain, i.e. $\Delta \in C(\Delta_r, \sigma)$. Therefore, the cutter location compensation optimization algorithm is transformed into the optimization problem of two-dimension nonlinear constraint interval.

5. **Comparison between theoretical error and simulation**

According to normal error calculation of globoidal cam by chapter 3, now analyzes the calculation error of a cam profile. The parameters are as follow: $c=180\text{mm}$, $77\text{mm} \leq l \leq 97\text{mm}$, $R = 20\text{mm}$, roller width $20\text{mm}$, cam dwell angle $240^\circ$, number of disc rollers 8, the period law of disc selects the modified sine acceleration. Set the different radius compensation amount $\Delta_r$, then the precision value of the normal error of the cam profile caused by the tool error and the approximate value of the simulation experiment are shown in Table 1. It can be seen from table 1 that the profile error is proportional to $\Delta_r$.

| $\Delta_r$/mm | $\delta_{\text{max}}$/mm |
|---------------|--------------------------|
|               | precision | approximation |
| 1             | 0.00354   | 0.00359       |
| 2             | 0.00697   | 0.00687       |
| 3             | 0.01024   | 0.01024       |
| 6             | 0.02198   | 0.02142       |
| 9             | 0.03367   | 0.03254       |

According to the result of table 1 and adaptive optimization algorithm to determine the tool position display that the difference between the precision value of the maximum normal error and the approximation value is very small, and the optimal compensation occurs when $\Delta_r = 3\text{mm}$. The relationship between the optimal compensation position through adaptive optimization algorithm and the cam angle is shown in Fig. 4.

![Figure 4. Relationship between optimal compensation position and cam angle](image)

It can be seen from table 1 that the error of the profile error increases with the increase of tool radius compensation, and the maximum error is linear. In order to effectively reduce the normal error of the profile caused by the tool and maintain the accuracy, the tool radius compensation $\Delta_r = 3\text{mm}$ is selected, and the influence of the profile normal error by the tool is shown in fig.5, 6.
In Figure 5, when the radius compensation of tool radius \( \Delta_r = 3 \text{mm} \), the error of the roller axis section and edge of the cutter is larger, the intermediate error is 0, the trajectory surface of the roller axis is concave, and the modified drum roller can be used to carry out the meshing transmission. Meanwhile, in fig. 6, when the tool radius compensation \( \Delta_r = 3 \text{mm} \), the cam angle is 80°, then the maximum normal error of the cam is \( \Delta_n = 0.015154 \text{mm} \) and occurs only once.

In order to verify the algorithm and simulation results presented in this paper, the variable milling function in the CAM module of UG8.5 is used. The drive type is "surface drive", the drive surface generated a 5-axis CNC machining G code through the NURBS approximation surface and performed toolpath generation by experimental simulation, shown in figure 7, simulation results verified the feasibility of the algorithm.

**Figure 7** Adaptive optimization algorithm for tool rail generation

### 6. Conclusion

(1) The new tool position control method is adopted to compensate the profile machining error, which greatly reduces the machining error of the cam profile caused by the tool, then the machining precision of the cam profile and the processing technology are improved.
(2) By the method deduced of calculating the normal error of globoidal cam, the error analysis of the profile machining of the globoidal cam can be more effective in real time, and the error can be controlled within the permission range in time.

(3) This method can enlarge the use range and service life of tool with proper cost.

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