Acoustic wave front reversal in a three-phase media

N.I. Pushkina

Abstract

Acoustic wave front conjugation is studied in a sandy marine sediment that contains air bubbles in its fluid fraction. The considered phase conjugation is a four-wave nonlinear parametric sound interaction process caused by nonlinear bubble oscillations which are known to be dominant in acoustic nonlinear interactions in three-phase marine sediments. Two various mechanisms of phase conjugation are studied. One of them is based on the stimulated Raman-type sound scattering on resonance bubble oscillations. The second one is associated with sound interactions with bubble oscillations which frequencies are far from resonance bubble frequencies. Nonlinear equations to solve the wave-front conjugation problem are derived, expressions for acoustic wave amplitudes with a reversed wave front are obtained and compared for various frequencies of the excited bubble oscillations.

*Electronic address: N.Pushkina@mererand.com
I. INTRODUCTION

In the present paper two various phase conjugation processes are considered and compared: A. the process based on the stimulated Raman-type acoustic scattering by natural bubbles oscillations; B. the process which involves bubble oscillations (or better say bubble gating) either with zero frequency or bubble oscillations with double signal and pump waves frequency which does not coincide with a resonance bubble frequency. In all these cases two opposing pump waves with equal frequencies and a signal wave at an angle to them propagate in a sediment. If the difference frequency of the pump and signal waves is just equal to the natural frequency of bubble oscillations, there can take place nonlinear Raman-type scattering of the pump wave into the signal wave by these natural bubble oscillations. The same bubble oscillations can induce scattering of the second pump wave into the reversed signal wave only in case of much lower resonance bubble frequency than the frequencies of the pump waves in order the energy and momentum conservation laws be fulfilled for the interacting waves. The case B. involves four sound waves with the same frequency and bubble "oscillations" either with zero frequency or bubble oscillations with double sound waves frequency. This case for media containing bubbles with high resonance frequencies can be more advantageous for the wave front conjugation problem since such media are known [1, 2] to manifest significantly stronger acoustic nonlinear interactions than those with lower bubble resonance frequencies. Below these different cases are considered separately in detail.

II. THEORY

We consider first the acoustic phase conjugation associated with stimulated Raman-type scattering of sound by nonlinear bubble oscillations. This process is analogous to phase conjugation in nonlinear optics that involves nonlinear light scattering by hypersonic waves, that is stimulated Mandelstam-Brillouin scattering (see Ref. [3]). Let the signal and conjugate waves propagate along the x-axis at an angle $\theta$ to the propagation direction of the pump waves. Raman-type stimulated scattering of sound by bubble oscillations in three-phase marine sediments was studied in Ref. [4]. Dynamic equations obtained in this paper
are of the form

\[ \frac{\partial^2 p}{\partial t^2} - \frac{m}{\rho_0 G} \frac{\partial^2 p}{\partial x^2} - \nu \frac{\partial^2 \rho_s}{\partial t^2} = \frac{mn}{G} \frac{\partial^2 V}{\partial t^2} \]

\[ \frac{\partial^2 \rho_s}{\partial t^2}(1 - m) - \frac{k + (4/3)\mu}{\rho_{os}} \frac{\partial^2 \rho_s}{\partial x^2} - \nu \frac{\partial^2 p}{\partial x^2} = 0, \]  

(1)

where \( p \) is the pressure in the water, \( \rho_s \) and \( \rho_f \) are the solid and liquid phase densities, (the subscript "0" refers to the equilibrium values), \( m \) is the porosity of a sediment, \( n \) is the bubble concentration and \( V \) is the bubble volume, \( \nu = 1 - m - k/k_s \),

\[ G = \frac{1 - m}{k_s} + \frac{m}{k_f} - \frac{k}{k_s^2}, \]

where \( k_f, k_s \) and \( k \) are the bulk moduli of the fluid, mineral grains constituting the frame, and of the frame itself; \( \mu \) is the shear modulus of the frame.

Eqs. (1) are to be supplemented with the nonlinear equation for an individual bubble motion \[ 2 \]

\[ \ddot{V} + \omega_0^2 V + f\dot{V} - \alpha V^2 - \beta(2\dot{V}V + \dot{\dot{V}}^2) + \mu V^3 + \nu(V^2\dot{V} + \dot{\dot{V}}V) = \epsilon p, \]

(2)

where \( \omega_0 \) is the resonance bubble frequency. The coefficients in Eq. (2) are expressed through the equilibrium bubble volume \( V_0 \), its radius \( R_0 \) and the adiabatic index \( \gamma \),

\[ \alpha = \omega_0^2(1 + \gamma)/2V_0, \quad \beta = 1/6V_0, \quad \mu = (\gamma + 1)(\gamma + 2)\omega_0^2/6V_0^2, \quad \nu = (2/9)V_0^2, \]

\[ \epsilon = 4\pi R_0/\rho_{0f}, \quad f = \delta \omega_0, \]

\( \delta \) is the dimensionless absorption coefficient of bubble oscillations. The resonance bubble frequency \( \omega_0 \) is related to the equilibrium pressure in the pore water and the bubble radius as

\[ \omega_0^2 = 3\gamma P_0/\rho_{0f}R_0^2. \]

(3)

In Eqs. (1) nonlinear hydrodynamic terms are omitted since nonlinear acoustic processes in porous media are known to be governed mainly by nonlinear oscillations of bubbles contained in the pore water \[ 5, 6 \]. The only nonlinear term in Eqs. (1) that ensures the nonlinear process is the last term of the first equation which in fact is the sum of the linear \( V^l \) and the nonlinear part \( V^n \). Let’s denote the amplitudes of the opposing pump waves as \( P_1 \) and \( P_2 \), the amplitude of the signal wave \( P_3 \) and the conjugate wave amplitude \( P_4 \). Since as it was noted in Introduction we suppose the resonance bubble frequency \( \omega_0 \) nonlinearly excited by
the pump wave $P_1$ and the signal wave $P_3$ to be much less than the pump wave frequency $\omega$ we can put the signal-wave frequency $\omega_s \approx \omega$. The scattering of the pump wave $P_2$ on the same bubble oscillations gives the conjugate acoustic wave $P_4$ with practically the same frequency as that of the signal wave. For such a situation using Eqs. (1), (2) we arrive at the equations for the signal and conjugate waves

$$\frac{dP_3}{dx} = -Ea_1(a_1|P_1|^2P_3 + a_2P_1P_2P_4^*)$$
$$\frac{dP_4}{dx} = -Ea_2(a_2|P_2|^2P_4 + a_1P_1P_2P_3^*)$$

with

$$E = \frac{\rho f n e^3}{L \cos \theta \omega (\omega^2 - \omega_0^2)^2 \delta \omega^2}, \quad a_1 = \frac{\alpha - \beta (\omega^2 + \omega_0^2 + \omega \omega_0)}{\omega + 2\omega_0}, \quad a_2 = \frac{\alpha - \beta (\omega^2 + \omega_0^2 - \omega \omega_0)}{\omega - 2\omega_0},$$

$$L = 1 + \frac{\nu^2 \rho_f}{m \rho_s} K^{-1} \left( 1 + \frac{k + (4/3)\mu}{\rho_s c^2} K^{-1} \right), \quad K = 1 - m - \frac{k + (4/3)\mu}{\rho_s c^2}.$$

Eqs. (4) are derived in the so called fixed-field approximation that permits to neglect the changes in the amplitudes of the pump waves $P_1$ and $P_2$ due to nonlinearity their changes being only because of linear bubble oscillations that cause dispersion in the medium which is not taken into account here.

The solution to Eqs. (4) for equal intensities of the pump waves is of the form

$$\frac{P_4(0)}{P_3(0)} = \frac{a_1 a_2 (1 - e^{-Al})}{(a_1^2 + a_2^2 e^{-Al})},$$

where $A = E(a_1^2 + a_2^2)|P_{1,2}|^2$, $l$ is the interaction length.

It is seen from (5) that the conjugate wave amplitude can approach the same order of magnitude as that of the signal wave if $e^{-Al}$ is not close to unity. For rather typical sediment parameters listed for instance in Refs. [7–9] the value of the coefficient $A$ can become large enough only for very high bubble concentrations $nV \approx 10^{-1}$ and rather high pump-wave intensities, the frequencies being $\omega = 2\pi \times 10^4 s^{-1}$, $\omega_0 = 2\pi \times 10^3 s^{-1}$. With this we can conclude that wave-phase conjugation based on such Raman-type acoustic scattering is not favorable unlike a similar case of phase conjugation in nonlinear optics associated with Mandelstam-Brillouin scattering. This result is closely connected with the fact noted in Introduction that nonlinear acoustic interactions are usually more pronounced for high-resonance bubble frequencies (while in this case $\omega_0 \ll \omega$). This is reflected in Eq. (2) where
the nonlinear coefficients are inversely proportional to the equilibrium bubble volume, which can be expected since for the same oscillation amplitudes relative volume perturbations are higher for smaller equilibrium volume values. Since the resonance bubble frequency is directly connected with a bubble radius through Eq. (3), this frequency decisively influences the nonlinear sound interaction. In addition one can say that at a fixed quantity \( nV_0 \) a smaller equilibrium bubble volume \( V_0 \) means a higher bubble concentration \( n \) that definitely affects the nonlinear process.

From this point of view it would be preferable to use phase conjugation based on acoustic wave interactions not with natural bubble oscillations, but with induced bubble oscillations of appropriate frequencies (see also [2]) to fulfill energy-momentum conservation laws, at the same time resonance bubble frequencies being rather high. We shall study this type of phase conjugation in more detail. It splits into the following processes. As to the second order nonlinearity in Eq. (2) two nonlinear interactions can take place in this case. The first one: the two pump waves of frequency \( \omega \) generate bubble oscillations of frequency equal to \( 2\omega \) with zero wave vector, and this oscillation scatters on the signal wave of frequency \( \omega \) into the conjugate wave. The second process is somewhat different: the pump wave \( P_1 \) scatters on the signal wave into a bubble "oscillation" with zero frequency or better say into a bubble grating; and this bubble grating scatters the other pump way \( P_2 \) into the conjugate wave. As to the third order nonlinearity in Eq. (2) this nonlinearity mixes directly all the four waves with the same frequency \( \omega \) generating the conjugate wave. Taking into account these three physical processes and using Eqs. (1), (2) we arrive at the equations for the amplitudes of the signal and conjugate waves that are supposed to change slowly at a wavelength scale,

\[
\frac{dP_3}{dx} = -iF(a|P_1|^2P_3 + bP_1P_2P_4^*)
\]

\[
\frac{dP_4}{dx} = iF(a|P_2|^2P_4 + bP_1P_2P_3^*).
\]

These equations are similar to Eqs. (4), but with different coefficients,

\[
F = \frac{\rho_fnc\omega^3}{L\cos \theta \omega_0^3(\omega^2 - \omega_0^2)^4(\omega_0^2 - 4\omega^2)^2},
\]

\[
a = (\alpha - \beta \omega^2)^2(\omega_0^2 - 4\omega^2), \quad b = (\alpha - \beta \omega_0^2)^2(\omega_0^2 - 4\omega^2) + (\alpha - 3\beta \omega^2)^2\omega_0^2 + [(3/2)\mu - \nu \omega^2](\omega_0^2 - 4\omega^2)\omega_0^2.
\]

Eliminating \( P_3 \) from Eqs. (6) one gets the equation for \( P_4 \) as follows,

\[
\frac{dP_4^2}{dx^2} - 2iFa|P_2|^2\frac{dP_4}{dx} - F^2(a^2 - b^2)|P_1|^2|P_2|^2P_4 = 0.
\]
Eq. (7) can be simplified since in the approximation of a slowly varying amplitude the second derivative \(d^2 P_4/dx^2\) can be neglected, and the solution to Eq. (7) is of the form

\[
P_4(0) = P_3(0) - \frac{2P_1 P_2 a b (1 - e^{iB l})}{|P_2|^2 (a^2 + b^2 - 2a^2 e^{iB l})},
\]

with \(l\) being the interaction length and

\[
B = \frac{1}{2F} (a^2 - b^2)|P_1|^2.
\]

Let us compare the effect of wave phase conjugation in two cases described by the relations (5) and (8). Examine them from the point of view of frequency dependence. In both cases \(\omega\) is the frequency of the pump, signal and conjugate waves and \(\omega_0\) is the resonance bubble frequency. In case (5) \(\omega_0 \ll \omega\), while in case (8) the situation is just opposite, \(\omega_0 \gg \omega\). The quantity \(A\) determining essentially the phase conjugation in case (5) is proportional to \(\omega(\omega_0/\omega)^4\) while in case (8) the corresponding quantity \(B\) depends on frequency as \(\sim \omega\). This means that the effect of phase conjugation in Raman-scattering process is considerably less pronounced than in case of the induced bubble oscillations with frequencies equal either to zero or double frequency of the acoustic waves provided the resonance bubble frequency is much higher than the frequency of the sound waves. To perform a numerical estimate of the conjugate sound amplitude we shall use the following experimental data and the values of sediment parameters [7–9],

\[
\begin{align*}
\omega &= 2\pi \times 10^4 \text{s}^{-1}, & \omega_0 &= 2\pi \times 10^5 \text{s}^{-1}, & \rho_f &= 1 \text{g/cm}^3, & \rho_s &= 2.65 \text{g/cm}^3, & m &= 0.4, \\
c &\approx 1.7 \times 10^5 \text{cm/s}, & k &\approx 10^9 \text{dyn/cm}^2, & \mu &\approx 5 \times 10^8 \text{dyn/cm}^2, & k_s &\approx 3.6 \times 10^{11} \text{dyn/cm}^2, \\
\gamma &= 1.4, & nV_0 &= 10^{-5}, & P_1 &= P_2 \approx 10^5 \text{dyn/cm}^2, & P_0 &\approx 10^6 \text{dyn/cm}^2.
\end{align*}
\]

For the acoustic-wave frequency \(\omega \approx 2\pi \times 10^4 \text{s}^{-1}\) the amplitude damping coefficient \(\alpha \sim 4.0 \times 10^{-3} \text{cm}^{-1}\) [8] which means that the effective interaction length is of the order of \((200 - 300)\text{cm}\). The numerical estimates show that for the parameters listed above the conjugate wave amplitude \(P_4\) can approach the signal wave amplitude \(P_3\) by the order of magnitude at a distance within the attenuation length of the interacting acoustic waves.

III. CONCLUSION

The obtained results can be summarized as follows. Acoustic-wave phase conjugation based on nonlinear bubble oscillations contained in the pore water of marine sediments
is investigated. Two possible mechanisms of wave-front reversal are considered in detail. The first one is associated with Raman-type stimulated acoustic scattering on resonance-frequency bubble oscillations. The second one is based on the sound scattering by induced bubble oscillations which frequencies do not coincide with a resonance bubble frequency of the sediment. In the case of Raman-type sound scattering the natural frequency of bubble oscillations is to be much less than the acoustic-wave frequency in order energy-momentum conservation laws be fulfilled. While in the second case the resonance bubble frequency not involved directly in the scattering process can be much higher than the acoustic wave frequency. At the same time bubble oscillations with just high resonance frequencies are known to influence significantly nonlinear acoustic interactions. The performed numerical estimates confirm this fact. In the second case the amplitude of the reversed radiation can reach a measurable value for reasonable parameters of a sediment. While in case of the Raman-type scattering the reversed radiation amplitude is several orders of magnitude less for the same sound frequencies and intensities.
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