X-ray microlensing in the quadruply lensed quasar Q2237+0305

F. Zimmer\(^1\)*, R. W. Schmidt\(^1\) and J. Wambsganss\(^1\)

\(^1\) Astronomisches Rechen-Institut, Zentrum für Astronomie der Universität Heidelberg, Mönchhofstrasse 12-14, 69120 Heidelberg

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ABSTRACT

We use archival data of NASA’s Chandra X-ray telescope to compile an X-ray light curve of all four images of the quadruply lensed quasar Q2237+0305 (\(z_Q=1.695\)) from January 2006 to January 2007. We fit simulated point spread functions to the four individual quasar images using Cash’s C-statistic to account for the Poisson nature of the X-ray signal. The quasar images display strong flux variations up to a factor of \(\sim 4\) within one month. We can disentangle the intrinsic quasar variability from flux variations due to gravitational microlensing by looking at the flux ratios of the individual quasar images. Doing this, we find evidence for microlensing in image A. In particular, the time-sequence of the flux ratio A/B in the X-ray regime correlates with the corresponding sequence in the optical monitoring by OGLE in the V-band. The amplitudes in the X-ray light curve are larger. For the most prominent peak, the increase of the X-ray ratio A/B is larger by a factor \(\sim 1.6\) compared to the signal in the optical. In agreement with theory and other observations of multiply imaged quasars, this suggests that the X-ray emission region of this quasar is significantly smaller than the optical emission region.

Key words: gravitational lensing; micro - X-rays; galaxies - galaxies: quasars: individual Q2237+0305 - cosmology: observation.

1 INTRODUCTION

The quasar Q2237+0305 was discovered in 1984 (Huchra et al. 1985) during a spectroscopic survey of nearby galaxies. The spectrum of the nucleus of the barred spiral galaxy 2237+0305 (\(z_G=0.0394\)) was found to be superimposed by a quasar component at a redshift of \(z_Q=1.695\). The first high resolution observations of the system showed three images of the quasar (Tyson & Gorenstein 1985). This number was soon corrected by Yee (1988) who first observed all four known point like quasar images around the core of the galaxy. The images are arranged in a nearly symmetric way, hence the name ‘The Einstein Cross’. Spectroscopy proved that they are images of a single quasar (Schneider et al. 1988; Adam et al. 1989). The images are separated by up to 1.8" from each other, and are labeled A through D. After the prediction by Chang & Refsdal (1975) and initial theoretical studies (e.g., Kayser et al. 1986; Schneider & Weiss 1986; Paczynski 1986), microlensing was first detected in Q2237+0305 (Irwin et al. 1988; Wambsganss et al. 1990). In fact, this detection of ‘quasar microlensing’ was the first evidence for microlensing generally, including stellar microlensing in the galaxy which was not discovered until 1993 (Aubourg et al. 1993; Alcock et al. 1993; Udalski et al. 1993). Today ‘The Einstein Cross’ is one of the best studied multiply lensed quasars and there have been a lot of monitoring programs in the optical (Corrigan et al. 1991; Pen & et al. 1993; Østensen et al. 1994; Woźniak et al. 2000; Schmidt et al. 2002; Alcalde et al. 2002; Udalski et al. 2006) where much microlensing activity has been observed. The quasar has also been detected in the UV (Blanton et al. 1998), the NIR (Agol et al. 2000), the MIR (Agol et al. 2000) and the radio regime (Falco et al. 1996). However, there is no published light curve of Q2237+0305 in the X-ray regime yet. The X-ray emission of Q2237+0305 was first detected with ROSAT/HRI observations in 1997 (Wambsganss et al. 1999). Since then there have been other X-ray observations of Q2237+0305 like a single spectroscopic observation with XMM-Newton from 2002 (data set ID: 0110960101; PI: Watson) (Fedorova et al. 2008) and two Chandra observations (Dai et al. 2003). In this paper we study ten archival Chandra observations ranging from January 2006 until January 2007 and compile the first X-ray light curve of Q2237+0305.

* E-mail: fzimmer@ari.uni-heidelberg.de
2 CHANDRA X-RAY DATA

The data we use was taken by the Chandra X-ray Observatory [Weisskopf et al. 2000] in the period between January 9, 2006 (Julian date 2453745.0) and January 14, 2007 (Julian date 2454114.9)(PI: Kochanek). The exposure time was \(\sim 8\) ks per observation. In Table I we list the ten observation IDs, the exposure times and observation dates. The observation IDs are listed in order to reference the individual images. We obtained the data from the Chandra Data Archive. Figure II shows all ten observations. The brightness variabilities of the four images are already visible. The four quasar images are labeled in the common way from A to D.

2.1 Chandra configuration

All ten images were obtained with the ACIS-S detector in VFAINT mode. ACIS-S is one of the two detectors in the ACIS (Advanced CCD Imaging Spectrometer) [Garmire et al. 2003] instrument. In our data set Q2237+0305 was placed on the back-illuminated chip S3 (1024 \(\times\) 1024 pixel).

The VFAINT mode provides the observer with the event position in detector coordinates, the event amplitude, the arrival time, and the pixel values in a 5\(\times\)5 island (compared to the 3\(\times\)3 island in FAINT mode). In our case a subarray of only half of the ACIS-S array was used for the benefit of reducing the read-out time to 1.7 seconds avoiding pile-up effects.

2.2 Data processing

To process the data we use the CIAO [Fruscione et al. 2006] software package (version 4.2). All ten images are calibrated according to the latest Calibration Database (CALDB, version 4.2.0) offered by the Chandra X-ray Center. We reprocess the images using the standard level 1 event lists produced by the Chandra pipeline processing, including the latest gain maps and calibration products to create new level 2 event files. However, we do not apply the VFAINT background correction but only the FAINT correction as the photons marked as background events in the VFAINT correction clearly clump around the source location, indicating that in the 5\(\times\)5 island, real events of the closely separated four quasar PSFs have erroneously been flagged as background. In any case, this affected only about ten photons. Simulations (PIMMS v3.9) show that for the brightest image (image A) in the brightest exposure (6839) (see Figure III) pile-up was only 4\% using the spectrum by [Dai et al. 2003]. We also check for contaminating flares during the exposure of the single images and find that all ten images show flat light curves. We check the light curves for both, source photons (photons in the energy range between 0.5 keV and 8 keV) as well as background events (photon energies higher than 9.5 keV). Finally, we consider an energy range from 0.5 keV to 8 keV in this paper. The event files are converted into images with a pixel size of 0.492\ ''.

3 PHOTOMETRY

3.1 The Chandra PSF

For the purpose of doing photometry on each of the four images we created a suitable point spread function (PSF) in order to later use it for PSF-fitting. To account for Chandra’s special Wolter optics [Wolter 1952] that is comprised in the High Resolution Mirror Assembly (HRMA) [Weisskopf et al. 2000; Schwartz et al. 2000; Gaetz et al. 2000], and its special imaging characteristics as well as for the blended nature of the four quasar images (maximum separation of 1.8\ '') that requires a very precise spatial resolution, we create a special PSF. We use the two tools especially made for this scope: ChaRT (Chandra Ray Tracer) [Carter et al. 2003], a web interface to a raytrace code developed by the Chandra X-ray Center and MARX (Model of AXAF Response to X-rays, AXAF was Chandra’s name before launch) [Wise et al. 1997]. ChaRT allows to simulate HRMA PSFs at any off-axis angle and for any energy or spectrum.

3.1.1 Creating the PSF

We use ChaRT to simulate three monochromatic PSFs for the off-axis position of Q2237+0305 each with the maximum ray density available in order to get a good signal-to-noise ratio. We do not simulate pile-up effects. The photon energies of the PSFs are 0.5 keV, 2 keV and 8 keV. We choose the energies with respect to the spectral range and their relative contribution to the spectrum of Q2237+0305, which will be explained more detailed below. The output of ChaRT is processed with MARX which projects the rays onto the detector taking into account any detector effects. In addition to simulating the detector response, MARX uses the ray weights to account for the mirror effects, i.e., different efficiencies of different shells at different angles and energies.

3.1.2 Combining the PSFs

For the PSF fitting we calculate the sum of the three PSFs weighting each energy according to its fraction in the spectrum of Q2237+0305 as it is seen by Chandra. The weight of the single PSFs is estimated using XSPEC [Arnaud 1996] for a photon index of \(\Gamma = 1.90^{+0.05}_{-0.05}\) [Dai et al. 2003] and a galactic absorption of \(N_H = 5.5 \times 10^{20}\) cm\(^{-2}\) [Dickey & Lockman 1990]. The resulting relative weights of the PSFs are 1.000 for the 0.5keV PSF, 0.103 for the 2keV PSF and 0.008 for the 8keV PSF, respectively.

The final PSF is dominated by the 0.5 keV PSF, however, it has slightly broader wings. It is subsampled by a factor of two (corresponding to a binning of \(\sim 0.25\)\ '' per pixel) and trimmed to a size of \(\sim 25\ '' \times 25\ ''\) (102 px \times 102 px) to facilitate the shifting of the PSF. We use this PSF to construct the quadruply PSF configuration of our images.

3.2 PSF fitting

As the four images of Q2237+0305 are very close to each other (within a circle of 1.8\ ''diameter), their PSFs are blended and we need a sophisticated program to disentangle and isolate the individual fluxes. Thus, we choose GALFIT.
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Figure 1. All ten Chandra images of Q2237+0305 after recalibration and in the energy range between 0.5 keV and 8 keV as used here. Each image has its observation ID attached in the upper left. The brightest pixel of image A in January 2006 has 33 counts while the brightest pixel in image C has 6 counts. The greyscale is identical for each observation. Brightness variations in image A are clearly visible.

Table 1. The exposure times and starting dates of the Chandra observations of Q2237+0305 used here. The observation IDs are listed for future reference.

| Observation ID | Exposure Time/s | Start Date       |
|---------------|-----------------|------------------|
| 6831          | 7263.84         | 2006-01-09 23:39:56 |
| 6832          | 7936.11         | 2006-05-01 00:46:10 |
| 6833          | 7952.71         | 2006-05-27 09:45:50 |
| 6834          | 7937.77         | 2006-06-25 17:24:13 |
| 6835          | 7871.38         | 2006-07-21 12:03:49 |
| 6836          | 7927.81         | 2006-08-17 03:59:06 |
| 6837          | 7944.41         | 2006-09-16 04:50:17 |
| 6838          | 7984.25         | 2006-10-09 03:41:03 |
| 6839          | 7873.04         | 2006-11-29 20:08:05 |
| 6840          | 7975.95         | 2007-01-14 22:30:10 |

2.0.3 (Peng et al. 2002). GALFIT is a two-dimensional fitting program originally designed to extract structural components from galaxy images. Nevertheless, as GALFIT is able to fit several components simultaneously, i.e., quasar images, it is an appropriate choice in particular because it is able to fit user-provided PSFs. Compared to counting photons in circular apertures we can measure the flux of the whole PSF and obtain very precise limits of the order of 0.05″ on the position of the PSF on the quasar images. In our case, we fit four user-provided PSFs simultaneously in order to get the fluxes of each individual image A, B, C and D. The four PSFs have fixed relative positions while the absolute position of the template of the four PSFs is fitted. Here, the absolute position is the origin of the relative PSF positions listed in Table 2, i.e. the position of quasar image A. We use the relative positions obtained by Blanton et al. (1998) from UV data because these are minimally affected by galaxy light. Blanton et al. (1998) determined the astrometric properties of the lens system with the HST WFPC2 camera in the UV (F336W and F300W filters). We also adapt GALFIT for the X-ray regime to fit fluxes instead of magnitudes.

Table 2. Astrometry of the quadruply imaged quasar Q2237+0305: The table lists the relative positions of the four quasar images as presented by Blanton et al. (1998)

| Component | Right Ascension/″ | Declination/″ |
|-----------|-------------------|---------------|
| A         | 0.000 ± 0.0015    | 0.000 ± 0.0015 |
| B         | −0.671 ± 0.0015   | 1.697 ± 0.0015 |
| C         | 0.634 ± 0.0015    | 1.210 ± 0.0015 |
| D         | −0.866 ± 0.0015   | 0.528 ± 0.0015 |

3.2.1 The fitting algorithm

The fitting algorithm used in GALFIT is based on the Levenberg-Marquardt algorithm (Press et al. 1992; Bevington & Robinson 2003) that provides a numerical solution to the problem of minimising a function over a set
of nonlinear parameters of the function. By default, GALFIT and its implemented Levenberg-Marquardt algorithm is using a least-squares minimization that is based on the assumption of normally distributed photon counts. However, to cope with the purely Poissonian nature of the X-ray signal we fit the data by using a statistic that is based on a Poisson distribution. For this we adapt GALFIT to minimize Cash’s C-statistic (Cash 1979) (see Appendix) instead of the original χ²-statistic. Indeed, Humphrey et al. (2009) demonstrate that χ²-statistics can give rise to intrinsically biased parameter estimates and that the use of Cash’s C-statistic gives comparatively unbiased parameter estimates.

In our case we choose a slightly modified form as implemented in XSPEC [Arnaud 1996]¹:

\[
C = -2 \sum_{i=1}^{N} y_i \cdot \log(\tilde{y}_i(z, a_1, ..., a_M)) - \tilde{y}_i(z, a_1, ..., a_M) + y_i - y_i \cdot \log(y_i)
\]

Here \(y_i\) is the \(i\)-th datapoint that is fitted to the model \(\tilde{y}_i(z, a_1, ..., a_M)\) with \(M\) adjustable parameters \(a_j\). The C-value is an indicator for the goodness-of-fit and is smallest for the best-fitting model.

3.2.2 The fitting method

To fit the four PSFs and thereby estimate the fluxes of the four images, it is sufficient to choose relatively coarse estimates for the initial fluxes and the position. GALFIT then typically converges after a few iterations. It determines the PSF template position with an accuracy of \(\frac{\alpha''}{100}\) of a pixel (0.05′′). The convergence is robust against changes in the initial conditions. Nevertheless, we confirm the fitting results by manually stepping through the six-dimensional (four fluxes and two coordinates) parameter space.

Once the best fit is found, we use the fluxes provided by GALFIT. The method was carried out on all ten Chandra images. Figure 2 shows the raw Chandra image (ID 6838) and its best-fitting model. It is notable how well the Einstein Cross is reproduced and how good the model reflects the relative brightness of the four quasar images compared to the raw image.

To estimate the error on the fluxes we manually determine the 68% contours for each quasar image flux in each epoch using the \(\Delta C = 1\) contour with respect to the minimum C-value found for the best-fit.

3.3 Robustness of the fit

With the implementation of Cash’s C-statistic in GALFIT we found an ideal way of fitting the X-ray data. The nature of our signal is clearly Poissonian and this is reflected in Cash’s statistic. For example, the total number of counts for observation 6831 is less than 300 with 33 counts in the brightest pixel. One of the crucial facts is that the Poisson distribution is discrete since the counts in one pixel can only be a natural number. The continuous normal distribution does not have this limitation, furthermore the normal distribution even allows negative values, an aspect that clearly rules out χ²-based fitting of low number counts. The asymmetric Poisson distribution assures that the fitting takes better care of pixels with a low number of counts (< 3 counts) and therefore provides an accurate description of the pixels. This is very important because all of the quasar image PSFs have pixel counts in exactly that count regime.

4 RESULTS

The final fluxes for all ten observations and all four lensed quasar images in the 0.5 keV - 8 keV band are shown in Table 3. The visualization of the data is shown in Figure 3 where we present the X-ray light curve that shows the flux variations for the ten epochs spread over one year of quasar image A (circle), B (diamond), C (square) and D (cross).

Although the light curve is only sparsely sampled it shows flux variabilities in a wide range and for each of the four quasar images. Most prominent are the variations in image A and the general and very steep increase in the fluxes of all four images for the last two epochs. As we are looking at images originating from a single quasar, variations in all four images are due to processes in the quasar itself because the time-delay is expected to be of the order of a few hours only (see below). However, stars in the lensing galaxy may also induce brightness variations through microlensing which hence are different for each of the quasar images. In the following we will separate both types of variability.

4.1 Microlensing

Microlensing in the optical is a well known phenomenon in Q2237+0305 (e.g. recent studies including Kochanek (2004); Anguita et al. (2008); Eigenbrod et al. (2008b); Pointdexter & Kochanek (2010)). Thus we here analyse the X-ray light curve to find evidence for X-ray microlensing in ‘The Einstein Cross’.

If part of the flux variations are due to microlensing, the pairwise flux ratios of the quasar images should not be constant. We therefore have to consider that the flux of each image \(X\) at time \(t\) is a result of different factors.

\[
F^X(t) = \mu_{\text{macro}}^X \cdot \mu_{\text{micro}}^X(t) \cdot Q(t)
\]

Hence the brightness is affected by:

- \(Q(t)\): The time-dependent flux of the quasar as the source of the X-ray radiation. It is the same for each quasar image.
- \(\mu_{\text{macro}}^X\): The magnification factor that originates from the macro lens model and its local lensing parameters \(\kappa\) and \(\gamma\). It therefore depends on which quasar image we are looking at.
- \(\mu_{\text{micro}}^X(t)\): The variable magnification factor due to microlensing changes with time and is different for each quasar image.

We do not need to correct for time-delays between the images as they are negligible [Schneider et al.].

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¹ Please refer to the XSPEC manual for a more detailed derivation.
As we are interested in microlensing we form the pairwise magnification ratios, for each pair of quasar images. The arguments given above and the fact that the correlated variations especially in the ratio \( \frac{B}{C} \) do not support random fluctuations as the cause, leads to the conclusion that we have observed microlensing induced variabilities. However, by comparing the observed flux ratios with the macro magnification ratios, we find that all quasar images may be affected by microlensing because their ratios are not consistent with the MIR data \( \frac{Y}{X} \). The flux ratios can also be affected by substructures in the lensing halo, however, due to the higher mass scale the time-scale of the variations induced by substructures is much longer (due to the higher mass scale) than the time-scale considered in our analysis (Metcalf 2005; Xu et al. 2010).

Focussing on the ratio of image A to image B and comparing it to OGLE (Woźniak et al. 2000a; Udalski et al. 2004) observations (V-band, restframe UV) obtained at the same time, we find that the maximum as well as the subsequent flattening of the ratio in the X-ray regime coincides with the ratio \( \frac{A}{B} \). However, the evidence for microlensing is most prominent in image A and the ratio \( \frac{A}{B} \) which is why we only focus on this quasar image and image ratio, respectively.

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Figure 3. The final X-ray light curve of the four images of Q2237+0305 in the 0.5 keV - 8 keV band as obtained with the C-statistic based GALFIT. The figure shows the variations with time for all four lensed images. The datapoints and errorbars have been connected with shaded bands for each quasar image. A sudden increase of flux due to intrinsic variations in ‘the central engine’ of the quasar can be seen for the last two epochs. The corresponding data can be found in Table 3.

with the optical observations. This is shown in Figure 5 where the ratio in the X-rays is overplotted with the OGLE data. For the most prominent peak around Julian date 2453937.5, the increase since Julian date 2453856.0 of the X-ray ratio A/B is larger by a factor \( \sim 1.6 \) compared to the signal in the optical. Furthermore, also the A-B difference light curve in the UV (Eigenbrod et al. 2008b) for that period shows similar characteristics. However, the events are less distinct at UV and optical wavelengths which is exactly what we expect as the UV and the optical emitting regions are thought to be larger than the X-ray emitting region (Wambsganss & Paczynski 1991, Rauch & Blandford 1991, Dai et al. 2003, Pooley et al. 2006, Kochanek et al. 2007). The inset in Figure 5 shows theoretical light curves for two Gaussian sources crossing a straight caustic. The radii are different by a factor of 6.7.

4.2 Intrinsic variations

The fact that there is strong evidence for microlensing in image A and D does not explain the high flux rates for the last two epochs and the flux rate drop for the third observation as the microlensing in image A had its peak at JD 2453937.5 (July 2006). These fluctuations can only be explained by intrinsic variations of the quasar itself. X-ray variability on such short time scales of weeks indicates that the emission generation takes place in a relatively compact region and that the emission processes, that make quasars so luminous, are highly variable. Such strong variations are not seen in the optical OGLE light curve. Merely, the minimum at JD \( \sim 2454010 \) can be seen.

5 SUMMARY AND OUTLOOK

In this paper we have analysed archival Chandra data of the gravitationally lensed quasar Q2237+0305. We compiled an X-ray light curve for all four images. The data set comprises ten epochs ranging from January 2006 to January 2007. Because of the blended nature of the four images (see Figure 1) it was necessary to simultaneously fit appropriate PSFs to all four images in order to obtain proper photometry. For this, we used a simulated PSF which accounts for the optical properties of the Chandra observatory and the spectrum of the quasar X-ray emission. The fitting was accomplished by a two-dimensional fitting algorithm and a grid search by minimising Cash’s C-statistic. Finally we analysed the light curve and found evidence for microlensing...
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Figure 4. Pairwise X-ray count rate ratios of the four quasar images A through D of Q2237+0305 in order to identify microlensing. The ratios are calculated according to Equation 3. The horizontal lines mark the MIR flux ratios (Agol et al. 2000), i.e. the macro magnification ratios.

variations in quasar image A (see Figure 4). The X-ray microlensing signal in image A coincides with the signal in the optical OGLE light curve. Assuming that this parallel behaviour is caused by the same process, i.e. one source that is microlensed, the amplitude of the microlensing signal is a direct measure for the source size in the particular wavelength regime (Wambsganss & Paczynski 1991, Wambsganss 1998, Kochanek 2004, Anguita et al. 2008, Eigenbrod et al. 2008b). As the microlensing signal in the X-ray regime is much more prominent than in the optical, this suggests that the X-ray emission region is much smaller than the optical emission region of the quasar (Pooley et al. 2004, Dai et al. 2010). In a future paper (Zimmer et al. 2010, in prep.) we will make use of this effect to measure the size of the X-ray and the optical emission region in Q2237+0305. While the time-delays in Q2237+0305 are negligible the intrinsic variations have to be considered in the analysis. Figure 3 clearly shows how strong these variations are in the X-ray regime. We do not find such strong fluctuations in the optical. This indicates that the mechanism powering the quasar is variable and leads to brightness variations by a factor of $\sim 4$ on time-scales of less than 30 days.

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Figure 5. Flux ratios of images A and B of Q2237+0305 in the X-ray regime in the 0.5 keV - 8 keV band (green diamonds) and the optical V-band (red circles) from January 2006 (2453745.0) to January 2007 (2454114.9). We applied an additional systematic error of 0.03 mag to the OGLE data. The inset shows theoretical light curves for two Gaussian sources crossing a caustic, their radii differ by a factor of 6.7.
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\[ p(y_i) = \frac{\tilde{y}_i(z, a_1, ..., a_M)^{y_i} \cdot e^{-\tilde{y}_i(z, a_1, ..., a_M)}}{y_i!} \] (A1)

So the probability of the whole data set is the product over all datapoint probabilities \( p(y_i) \).

\[ P = \prod_{i=1}^{N} p_i(y_i) \] (A2)

To maximize the data set probability by estimating the model parameters \( a_1, ..., a_M \) one can also minimize \(-2 \cdot \log(P)\):

\[ -2 \cdot \log(P) = -2 \cdot \sum_{i=1}^{N} y_i \cdot \log(\tilde{y}_i(z, a_1, ..., a_M)) - \tilde{y}_i(z, a_1, ..., a_M) - \log(y_i!) \] (A3)

leading to the definition of \( C \):

\[ C = -2 \cdot \sum_{i=1}^{N} y_i \cdot \log(\tilde{y}_i(z, a_1, ..., a_M)) - \tilde{y}_i(z, a_1, ..., a_M) \] (A4)

The term \(- \log(y_i!\) can be left out as it does not depend on the model parameters and is therefore just a constant offset. For our implementation in GALFIT we chose a slightly modified form as implemented in Xspec: (Arnaud 1996):

\[ C = -2 \cdot \sum_{i=1}^{N} y_i \cdot \log(\tilde{y}_i(z, a_1, ..., a_M)) - \tilde{y}_i(z, a_1, ..., a_M) + y_i - \log(y_i) \] (A5)

This translates to the following function to be minimized by GALFIT:

\[ C = -2 \cdot \sum_{x=1}^{n_x} \sum_{y=1}^{n_y} \text{flux}_{x,y} \cdot \log(\text{model}_{x,y}) - \text{model}_{x,y} + \text{flux}_{x,y} - \text{flux}_{x,y} \cdot \log(\text{flux}_{x,y}) \] (A6)

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APPENDIX A: CASH’S C-STATISTIC

In comparison to \( \chi^2 \)-statistic Cash’s C-statistic (Cash 1979) is based on the assumption that each datapoint \( y_i \) is a random variable out of a Poissonian distribution with an expected value of \( \tilde{y}_i(z, a_1, ..., a_M) \). Therefore, it has the probability:

\[ p(y_i) = \frac{\tilde{y}_i(z, a_1, ..., a_M)^{y_i} \cdot e^{-\tilde{y}_i(z, a_1, ..., a_M)}}{y_i!} \] (A1)