Supertubes connecting D4 branes

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We find and explore a class of dyonic instanton solutions which can be identified as the supertubes connecting two D4 branes. They correspond to a single monopole string and a pair of monopole-antimonopole strings from the worldvolume view point of D4 branes.

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1 Introduction

Recently there has been a considerable study of the so called supertube solutions. Originally Mateos and Townsend constructed supersymmetric configurations of D2-D0-F1 branes where D2 brane forms a single cylinder with D-particles and F-strings melted in [1]. The corresponding configurations in the matrix model have been found also [2, 3]. As a limit of the elliptic deformation of circular tubes, flat supersymmetric brane antibrane configurations are realized [4] and the wouldbe tachyons present in the ordinary brane-antibrane system become massless exhibiting the stability of the system [5]. In fact the cross sectional shape of the tubes is shown to be arbitrary in the transverse eight dimensions [6]. More recently, the DBI action has been explored to find the supersymmetric configuration for D2 supertubes ending on a D4 brane [7]. The higher dimensional analogue has also been studied [7, 8, 9] and there appeared other related works [10, 11, 12, 13].

In this note we are interested in tubular D2 connecting two D4 branes separated, say, in $x^9$ direction. As argued in Ref. [7], all the constituents of supertubes, i.e. D0, F1 and D2, may end on D4 branes and, hence, such configuration should be consistent with supersymmetries. Viewed from the worldvolume of D4, D0 appears as an instanton generating selfdual field strength and F-string extended along $x^9$ works as a localized electric source. One direction of D2 is extended in $x^9$ and the other spatial direction will in general form a curve in D4 worldvolume. Since this curve is magnetically charged, one may call it as a magnetic string from the view point of the D4 worldvolume. The instantons preserve only half of sixteen supersymmetries and F1/D2 breaks further half, so only 1/8 of 32 supersymmetries remain in general.

In this U(2) context of two D4 branes, one is ultimately interested in magnetic strings where the shape of curve on D4 is arbitrary, which correspond to tubes of arbitrary cross sections. However such general configurations seem involved for reasons described below. Therefore we shall focus on the monopole and antimonopole strings corresponding to supersymmetric flat D2-D2 connecting two D4 branes. This supersymmetric configuration is composed of a monopole string and an antimonopole string.

Simpler situation arises if one considers just one flat D2 (or a $\overline{D2}$) suspended between D4 branes, in which D0 and fundamental strings are melted in. As demonstrated below, such single D2 preserves actually 8 real supersymmetries becoming 1/4 BPS. When one brings such charged monopole strings and an antimonopole string together, one might naively expect that they cannot be BPS or static due to attraction between opposite charges. However this is not the case as in the supersymmetric brane antibrane systems [4, 5]. In spite of the presence of D4, still this monopole antimonopole
strings can be static preserving four real supersymmetries\cite{7,8}. Below we shall construct the super D2-anti D2 configuration in the context of D4 whose worldvolume fluctuations are described by super Yang-Mills theories.

For later comparison, we first briefly review here the key characteristics of supersymmetric tubular D2 branes using the Born-Infeld theory. One could consider the case of an arbitrary cross section with most general B-field. But such generality won’t be necessary because we are mainly concerned the flat D2 or anti-D2. Furthermore for the fully local characteristics of tubular branes, considering flat one suffices. The tubular configuration has a translational symmetry in one direction which we shall take $x^0 = z$ and the other spatial direction as $x^4 = \sigma$ where $\sigma$ and $z$ will be used for the worldvolume coordinates. Turning on $E = F_0z$ and $B = F_{\sigma z}$, the Born-Infeld action becomes

$$\mathcal{L} = -\sqrt{-\det(g + F)} = -\sqrt{1 - E^2 + B^2}$$  \hspace{1cm} (1)

The displacement $\Pi$ conjugated to $E$ becomes

$$\Pi = \frac{\partial \mathcal{L}}{\partial E} = \frac{E}{\sqrt{1 - E^2 + B^2}}.$$  \hspace{1cm} (2)

Below we shall consider $E > 0$ case without loss of generality. The Hamiltonian may be arranged as

$$\mathcal{H} = \sqrt{(1 + B^2)(1 + \Pi^2)} = \sqrt{(1 - |B|\Pi)^2 + (|B| + \Pi)^2} \geq |B| + \Pi.$$  \hspace{1cm} (3)

The BPS equation

$$|B|\Pi = 1$$  \hspace{1cm} (4)

leads to the condition $E = 1$. The Gauss law $\partial_z \Pi = 0$ then implies that $B$ should be independent of $z$ but an arbitrary function of $\sigma$, i.e. $B = B(\sigma)$. The $B$ field describes the spatial density of D0 branes melted into D2 because $\frac{1}{2\pi} \int d\sigma B$ counts the total number of D0 branes while $\Pi$ is the linear density of F-string extended in $z$ direction as the electric flux $\int d\sigma \Pi$ counts the total number of string charges. Further there is nonvanishing momentum density produced by the field. By evaluating Poynting vector, one finds that the field momentum density is nonvanishing with $P_\sigma = 1$. The key local characteristics of supertubes are summarized as follows. 1) The F-string density $\Pi$ is related to D0 density by $\Pi = |B|^{-1}$; as a result, the field momentum density satisfies $P_\sigma = 1$. 2) $E = 1$. 3) $\mathcal{H}_{D2} = |B| + |\Pi|$.

In the discussion of the monopole strings, we shall see that the above key structures are precisely reproduced. Since the Born-Infeld analysis involves highly nonlinear terms, why the super Yang-Mills theories that are at most quadratic in their field strengths, have a capacity to do so is not clear. But note that all the above four characteristics are faithfully reproduced in the matrix model description\cite{2,3,4,8}. The matrix model is closely related to the (noncommutative) Yang-Mills theories and this could partially explain the reasons.
2 Field theory set-up

We are interested in the five dimensional super Yang-Mills theory with a single adjoint scalar field \( \phi = X^9 \) turned on. The relevant bosonic part of Lagrangian is

\[
\mathcal{L} = -\frac{1}{4g^2} \text{Tr} \left( F_{MN} F^{MN} + 2D_M \phi D^\mu \phi \right)
\]  

(5)

where \( F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N] \). We use the notation \( A_M = A^a_M T^a \) where \( \text{Tr} T^a T^b = \delta^{ab}/2 \) and \( T^a = \sigma^a/2 \) for the SU(2) case. The energy functional can be bounded\[\]

\[
\mathcal{E} = \frac{1}{2g^2} \int d^4x \text{Tr} \left\{ (F_{0\mu} \mp D_\mu \phi)^2 + (B_i - F_{i4})^2 + (D_0 \phi)^2 \right\} + \frac{4\pi^2}{g^2} \nu \pm Q^E
\]  

(6)

where \( B_i = \epsilon_{ijk} F_{jk}/2 \).

\[
\nu = \frac{1}{4\pi^2} \int d^4x \text{Tr} B_i F_{i4}
\]  

(7)

\[
Q^E = \frac{1}{g^2} \int d^4x \partial_\mu \text{Tr} (F_{0\mu} \phi).
\]  

(8)

The selfdual equation for the dyonic instanton is

\[
D_0 \phi = 0
\]  

(9)

\[
B_i = F_{i4}
\]  

(10)

\[
F_{\mu 0} \mp D_\mu \phi = 0
\]  

(11)

The above equations and the Gauss law \( D_\mu F_{\mu 0} + i[\phi, D_0 \phi] = 0 \) imply

\[
D_\mu^2 \phi = 0
\]  

(12)

The selfdual equation for the instanton alone preserves 8 real supersymmetries. The presence of the electric and the scalar components breaks further half. Thus configurations satisfying the above BPS equations in general preserve 4 real supersymmetries.

While we can obtain a field configuration with \( D_0 \phi \neq 0 \) by the Lorentz boost of the above configurations, we will restrict our discussion to the configurations such that \( D_0 \phi = 0 \).

In the SU(2) super Yang-Mills theories of two D4 branes, solutions of the selfdual equation as well as \( E_\mu = D_\mu \phi \) have already appeared in the literature[15, 16, 17]. (See Ref. [18] for a more recent discussion on dyonic calorons.) They involve instantons (D0’s) as well as electric charges of fundamental strings stretched to the \( x^9 \) direction. The separation \( h \) of the two D4 branes is

\[\text{.}^1 i, j, k = 1, 2, 3 \ \text{and} \ \mu, \nu = 1, 2, 3, 4.\]
described by the vacuum expectation value of scalar field \( \langle \phi \rangle = \frac{\sigma}{2} h \). These dyonic instantons solve precisely the same BPS equations for the supertubes connecting separated D4 branes. The \( N \) dyonic instanton solution in the ’t Hooft ansatz reads

\[
A_\mu = \frac{\sigma_a}{2} \tilde{\eta}^{a\mu\nu} \partial_\nu G,
\]

\[
\phi = X^9 = \frac{\sigma_3}{2} \frac{h}{G}
\]

(13)

where \( \tilde{\eta}^{a\mu\nu} \) is the antiselfdual ’t Hooft tensor. The function \( G \) is given by

\[
G = 1 + \sum_{n=1}^{N} \frac{\rho_n^2}{|x - x_n|^2}.
\]

(14)

where \( \rho_n \) is for the size and \( x^n_\mu \) represents the position of \( n \)-th instanton. This SU(2) configuration certainly carries electric charges and the number of D0 is \( N \). However the configurations do not carry magnetic string charges. Since \( F_{\mu\nu} \sim 1/r^4 \), the system carries at most dipole moment of monopole charges. Thus, for example, one monopole string discussed below does not follow straightforwardly by arranging them over a line uniformly. Moreover considering \( N = 1 \) dyonic instanton, the throat connecting two D2 branes has a topology \( R \times S^3 \), which is not the geometry of tubular D2 branes.

There is well known way to get ordinary monopole solution independent of \( x^4 \) from above instanton solution\[19\]. First arrange all the instanton over \( x^4 \) axis with equal spacing and make all the size equal i.e. \( \rho_n = \rho \). In the limit where the size parameter goes to infinity, the above solution \( A_\mu \) becomes, up to a gauge transformation, one BPS monopole solution independent of \( x^4 \), where \( A_4 \) plays the role of scalar\[19\]. In this limit, \( \phi \) \( (= X^9) \) vanishes and the limiting solution does not correspond to a D2 (between D4 branes) presented below. As we shall see below the limit where \( \rho \) goes to infinity corresponds to sending massless antimonopole to infinity. In doing so, the magnetic charge as well as the electric charge become different, so there is a change in their main physical content.

In short, what we like to argue here is that the above collection of the dyonic instanton does not carry D2 charges. Further they do not satisfy the key characteristics of tubular D2 branes. For example if one arranges them on a straight line on D4 uniformly with equal spacing \( \Delta \), then the instanton number density is proportional \( 1/\Delta \). The total electric charge is again proportional to \( 1/\Delta \). This kind of configuration then does not satisfy the relation, say, \( \Pi B = 1 \) in general. The momentum density \( \mathcal{P} = 1 \) is not produced either. Instead they carry only nonvanishing angular momentum.

The solutions given by the ’t Hooft ansatz could be interpreted as the collapsed supertubes.
The configurations beyond the ’t Hooft ansatz may have a tube structure but it remains to be seen whether this is true. For this reason, we shall construct the flat charged D2 or super D2 connecting D4 branes, which are simpler examples of supertubes.

3 Single monopole string

Let us start with the magnetic monopole-like string which has uniform instanton density along the \( x^4 \) direction. The configuration is given by \( A^a_i = V^a_i \) and \( A^a_4 = V^a_4 \) where

\[
V_i(r, u) \equiv \epsilon^{aij} \frac{a^j}{2} \left( 1 - \frac{u r}{\sinh u r} \right)
\]

\[
V_4(r, u) \equiv -\frac{a^a}{2} \left( u \coth u r - \frac{1}{r} \right)
\]

It is selfdual as \( B_i = F_{i4} \). The asymptotic value of \( V_4 \) can be gauged away and so there is no gauge symmetry breaking. The instanton number density per unit length along the \( x^4 \) direction

\[
\mathcal{I} = \frac{1}{4\pi^2} \int d^3 x \text{Tr} B_i F_{i4}
\]

for the above configuration is \( \mathcal{I} = \frac{\pi}{2\pi} \). This is a purely instanton string which has a long range tail.

When the scalar field takes nonzero expectation value asymptotically

\[
\langle \phi \rangle = \frac{\sigma^3}{2} h
\]

with positive \( h > 0 \), the gauge symmetry is spontaneously broken to \( U(1) \) subgroup. The nontrivial scalar and \( A_0 \) fields of the BPS equations (10) and (12) in this background are

\[
\phi = \alpha \frac{h}{u} V_4, \quad A_0 = \beta \frac{h}{u} V_4
\]

with \( \alpha, \beta = \pm 1 \) independently. Since the gauge invariant \( U(1) \) magnetic field becomes asymptotically \( \frac{2}{\hbar} \text{Tr} (B_i \phi) \approx \alpha i \hat{r}/r^2 \), the total magnetic flux on the transverse three dimensions is \( 4\pi \alpha \). The instanton string now appears as magnetic or antimonopole string depending on the sign \( \alpha \).

Also the gauge invariant electric field becomes asymptotically \( 2 \text{Tr} (F_{i0} \phi) = \alpha \beta \frac{h}{u} i \hat{r}/r^2 \) and so the electric charge density per unit length becomes \( 2\pi \alpha \beta h g^2 u \) and so its magnitude is proportional to \( h \) and its sign depends on the sign \( \alpha \beta \). The linear momentum density along the \( x^4 \) direction is

\[
\mathcal{P}_4 = -\frac{1}{g^2} \int d^3 x \text{Tr} F_{i4} F_{i0}
\]

which is \( \mathcal{P}_4 = -\frac{2\pi}{g^2} \beta h \). The energy density per unit length of this dyonic instanton string is

\[
\mathcal{H}_4 = \frac{2\pi}{g^2} \left( u + \frac{h^2}{u} \right).
\]
Thus our string configuration carries the instanton density, magnetic charge, electric charge density and linear momentum density along the string. The eigenvalues of the matrix scalar field \( \phi = X_9 \) describe the deformation of D4 branes along the transverse ninth direction. The zeros of the field \( \phi \) are the place where two D4 branes meet. For our string configuration the zeros lie along the line \( r = 0 \). Thus our solution can be interpreted as a flat charged D2 connecting two D4 branes. As discussed in the introductory part, this particular configuration preserves eight real supersymmetries instead of four. This enhancement of supersymmetries occurs due to the fact \( A_4 \) is proportional to \( \phi \), which may be directly checked using the gaugino variation.

The free parameters of our dyonic-instanton-monopole string are \( u \) and two signs \( \alpha, \beta \). Depending on \( \alpha \), the magnetic charge can be positive or negative. The sign of the electric charge depends on \( \alpha \beta \). The sign of the linear momentum depends on \( \beta \).

To compare the flat charged D2 appearing in the above with that of the Born-Infeld description, first we evaluate the D0 brane density from the viewpoint of the D2 worldvolume. Of course the D2 is extended in \( x^4 \) and \( x^9 \) directions. Since D2 has a length \( \left( 2\pi l_s^2 \right) \frac{h}{2} \) in \( x^9 \), the D0 brane density may be evaluated as

\[ n_{D0} = \frac{T}{h} = \frac{u}{2\pi h}. \]  

Hence from \( \frac{1}{2\pi} B = n_{D0} \), we get

\[ B = \frac{u}{h}. \]  

On the other hand, the F-string density is proportional to the linear electric charge density along \( x^4 \) direction and given by

\[ q_{F1} = \frac{1}{2\pi h} \int d^3 x \partial_i \text{Tr} E_i \phi = \Pi = \frac{h}{u}, \]  

reproducing \( B \Pi = 1 \).

Using (19) and (20), the momentum and the energy density per unit area of D2 are evaluated as

\[ P_{D2} = \frac{P_4}{h} = \frac{2\pi}{g^2} = 1 \]

\[ H_{D2} = \frac{H_4}{h} = \frac{2\pi}{g^2} \left( \frac{u}{h} + \frac{h}{u} \right) = |B| + |\Pi|, \]  

where we use \( g^2 = 2\pi \) and choose \( \beta = -1 \). Furthermore, the worldvolume E-field is reproduced to be \( E_9 = 1 \) noting \( E_i \partial_i (-dr) = E_9 dX^9 \). Hence all the key characteristic properties of the charged D2 in the Born-Infeld descriptions are reproduced in the above Yang-Mills theory configuration.

\[ \text{For simplicity, we use the convention } 2\pi l_s^2 = 2\pi g_s l_s = 1 \text{ in the Born-Infeld description where } l_s \text{ and } g_s \text{ are respectively the string length scale and the string coupling. In this convention, the D2 tension } T_{D2} = g_s (2\pi)^2 l_s = 1 \text{ and the four dimensional Yang-Mills coupling } g^2 = g_s (2\pi)^2 l_s = 2\pi. \]
One can imagine various superpositions of our configurations with different parameters, keeping the selfduality of the four dimensional gauge field. Some of them would describe many parallel super charged D2 branes. Finding such configurations would be daunting in general. The simplest one would be having two parallel strings with identical parameters. This solution would be independent of $x^4$ direction and so can be obtained in principle from the BPS two monopole field configuration.

4 Supersymmetric monopole-antimonopole strings

To argue that certain limit of dyonic instanton configurations are supertubes connecting two D4 branes, we want to show that one can superpose two parallel instanton strings of opposite magnetic charge. This would correspond to the supersymmetric D2 anti-D2 branes connecting two D4 branes. Obviously such a configuration would depend on $x^4$ coordinate nontrivially. It seems very hard to construct such field configuration in general. Let us try to introduce monopole string of instanton density $u/2\pi$ of positive electric charge and antimonopole string of instanton density $v/2\pi$ and positive electric charge. Then their fourth linear momentum densities have opposite signs, and we choose the Lorentz frame along $x^4$ direction so that their sum vanishes. In large separation of two such strings, the $x^4$ dependence becomes weaker and one should be able to identify one as magnetic string and another as antimagnetic string. In short separation the $x^4$ dependence becomes prominent and it becomes hard to distinguish two strings.

To find such a configuration, we regard that the instanton density is the sum $(u + v)/2\pi$ at large separation. Thus the instanton charge per unit length is

$$\frac{1}{L} = \frac{u + v}{2\pi} \quad (25)$$

As the instanton number over the length $L$ is one and so we want to require that the field configuration is periodic under shift $x^4 \rightarrow x^4 + L$ up to a gauge.

Instanton configurations which are periodic in $x^4$ direction can be regarded as calorons, or periodic instantons. Such configurations have been found with trivial symmetry breaking [19] or with nontrivial Wilson loop symmetry breaking [20, 21, 22]. The 1/4 BPS configurations involving calorons have been also studied [18]. Here we start from what is known. The Wilson loop symmetry breaking can be recasted so that the gauge field is not periodic but $A_M(x^4 + L) = U A_M(x^4) U^\dagger$ where $U = e^{i2\pi u L}$

The method to construct the solution is the Nahm’s construction. The detail of the construction is given in Refs. [21, 22]. The key parameters are the separation $D$ between two strings. In the
transverse three dimensions, the position parameters of two strings are
\[ \mathbf{x}_1 = (0, 0, x_1), \quad \mathbf{x}_2 = (0, 0, x_2) \] (26)
with the separation parameter \( D = x_2 - x_1 \). The field configuration at the position \((\mathbf{x}, x^4)\) is expressed in terms of two coordinates
\[ \mathbf{y}_1 = (\mathbf{r} - \mathbf{x}_1), \quad \mathbf{y}_2 = (\mathbf{r} - \mathbf{x}_2), \] (27)
and dimensionless parameters
\[ s_1 = u \mathbf{y}_1, \quad s_2 = v \mathbf{y}_2. \] (28)

The resulting gauge field is nontrivial superposition of two monopole strings,
\[ A_\mu = -i \overline{\partial_\mu} N \frac{2}{2N^2} + i C_1 \overline{\partial_\mu} C_1 + i C_2 \overline{\partial_\mu} C_2 + C_1 \overline{V}_\mu(y_1, u) C_1 + C_2 \overline{V}_\mu(y_2, v) C_2 \] (29)
where the normalization factor
\[ N = 1 + \frac{2D}{\mathcal{M}} \left[ N(y_1, u) (\cosh s_2 - (\hat{y}_2)_3 \sinh s_2) + N(y_2, v) (\cosh s_1 + (\hat{y}_1)_3 \sinh s_1) \right] \] (30)
with \( N(r, u) = \sinh ur/r \) and \( \mathcal{M} = 2 \left( \cosh s_1 \cosh s_2 + \hat{y}_1 \cdot \hat{y}_2 \sinh s_1 \sinh s_2 - \cos \frac{2\pi x^4}{L} \right) \). The matrices are
\[ C_1 = i \sqrt{\frac{2DN(y_1, u)}{N}} \frac{B_1}{\mathcal{M}} \left( e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_2} Q_+ + e^{\frac{i}{2} \hat{x} \cdot \mathbf{s}_2} Q_- \right) e^{-\frac{i}{2} \mathbf{s}_3 x^4} \] (31)
\[ C_2 = i \sqrt{\frac{2DN(y_2, v)}{N}} \frac{B_2}{\mathcal{M}} \left( e^{\frac{i}{2} \hat{x} \cdot \mathbf{s}_1} Q_+ + e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_1} Q_- \right) e^{\frac{i}{2} \mathbf{s}_3 x^4} \] (32)
where
\[ B_1 = e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_4} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_3} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_2} - e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_4} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_3} e^{\frac{i}{2} \hat{x} \cdot \mathbf{s}_2} \] (33)
\[ B_2 = e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_4} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_3} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_1} - e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_4} e^{-\frac{i}{2} \hat{x} \cdot \mathbf{s}_3} e^{\frac{i}{2} \hat{x} \cdot \mathbf{s}_1} \] (34)
and \( Q_\pm = (1 \pm \sigma_3)/2 \).

The solution of the scalar field equation is
\[ \phi = \frac{h}{N} \sigma_3 + \frac{H}{u} C_1 V_4(y_1, u) C_1 - \frac{H}{v} C_2 V_4(y_2, v) C_2 \] (35)
where \( H = \frac{\mu Dh}{1 + \mu D} \) with \( \mu = uv/(u + v) \). Notice that this is a nonlinear superposition of the scalar fields for each magnetic monopoles.

Some properties of the instanton part are explored in Refs. [21, 18]. The above configuration has gauge singularities at \((\mathbf{x} = 0, x^4 = nL)\) with an integer \( n \). These gauge singularities can be removed to the spatial infinity. The topological charge can be found at the singularities.
Far from the monopole core region \((s_1, s_2 \gg 1)\) and \(r \gg D\), we can neglect exponentially small terms. In the asymptotic region, the gauge field approaches

\[
A_\mu \to \mathcal{O} \left( \frac{1}{r^2} \right),
\]

and the \(U(1)\) magnetic field \(2\pi B_i \hat{\phi}\) falls off like \(1/r^3\). This implies that the total net magnetic charge vanishes and the system is composed of a monopole and an antimonopole. For the electric charge densities, a straightforward evaluation leads to

\[
\phi \to \left( 1 + \frac{q_1 + q_2}{r} + \frac{\hat{r} \cdot (q_1 \mathbf{x}_1 + q_2 \mathbf{x}_2)}{r^2} \right) \frac{\sigma^3}{2} h + \mathcal{O} \left( \frac{1}{r^3} \right)
\]

where

\[
q_1 = \frac{\mu D}{1 + \mu D} \frac{h}{u} \quad q_2 = \frac{\mu D}{1 + \mu D} \frac{h}{v}.
\]

As for the BPS configuration \(A_0 = \phi\), the electric charge densities per unit length then become \(q_1\) and \(q_2\) for the monopole and the antimonopole strings respectively. These charges increase from zero to a finite value when \(\mu D\) increases from zero to infinity.

The expression in (37) shows that the charges \(q_1\) and \(q_2\) are located \(\mathbf{x}_1\) and \(\mathbf{x}_2\) respectively in the three space. Compared with the single string case, the zeros of the field \(\phi\) would not be exactly on the monopole string position. When the separation is very large, one can calculate the \(\phi\) field around each monopole string and see how its zero gets modified. One can show easily that the shift is at most of order \(1/D^2\) and so essentially the zeros of the \(\phi\) field lie along \(\mathbf{x}_1\) and \(\mathbf{x}_2\). This shows that \(D4\) branes are connected at the monopole and antimonopole strings as expected for the supertube connecting two \(D4\) branes.

These two monopole strings carry opposite linear momentum and so their sum vanishes. The monopole and the antimonopole strings carry respectively instanton number densities \(I_1 = \frac{u}{2\pi}\) and \(I_2 = \frac{v}{2\pi}\). As in the case of the single monopole string, the worldvolume B-fields of \(D2\) and \(\overline{D2}\) may be evaluated as \(B_1 = u/h\) and \(B_2 = -v/h\) where the extra (−) sign for \(B_2\) comes from the fact that we are considering \(\overline{D2}\) with \(D0\)’s melted in. From the expressions of the electric charges densities, the linear F-string densities are identified as \(\Pi_1 = q_1\) and \(\Pi_2 = q_2\). The total energy density from the viewpoint of \(D2-\overline{D2}\) worldvolume may be checked to be \(\mathcal{H}_2 = |B_1| + |\Pi_1| + |B_2| + |\Pi_2|\). Thus the key characteristic properties of the supersymmetric \(D2\) and \(\overline{D2}\) are reproduced except the relation \(\Pi_a = |B_a|^{-1} \frac{\mu D}{1 + \mu D}\) for \(a = 1, 2\). This additional factor \(\mu D/(1 + \mu D)\) in the F-string densities is resulted from the effect of the \(D4\) branes to the supersymmetric \(D2\) and \(\overline{D2}\) system. For the large separation, \(\mu D/(1 + \mu D)\) approaches one and the effect disappears as expected.

A few comments are in order. The gauge field and the scalar field is not invariant under \(x^4 \to \)
$x^4 + L$. It transforms as

$$(A_\mu, \phi)(x^4 + L) = e^{-i\sigma^3 u L}(A_\mu, \phi)(x^4)e^{i\sigma^3 u L}$$

We can make a nonsingle-valued gauge transformation $U = e^{-i\sigma^3 u x^4}$ to get periodic gauge and scalar fields, in which case the $A_4$ field would have nontrivial expectation value

$$\langle A_4 \rangle = \frac{\sigma^3}{2} u.$$  

In the compact $x^4$ case, this expectation value becomes the Wilson loop which cannot be gauged away [21, 22].

In the limit either $u$ or $v$ vanishes, the above solution becomes the conventional caloron solution found before [19]. A single caloron is made of a massive monopole and massless monopole when $D$ is large. The electric charge would increase linearly with the string separation $D$.

In short distance separation with $D \ll L$, the caloron solution becomes basically a periodic array of well separated instantons. The zeros of the $\phi$ field are expected to change from two separated line shape to a periodic array of isolated regions of zeros. When $D = 0$, the instantons get very small and so the positions of zeros would be a periodic array of the instanton positions. We do not know how this transition occurs exactly.

5 Conclusion

We have studied the various aspects of supertubes connecting D4 branes. Especially we found and explored the dyonic instanton configuration which corresponds to dyonic monopole and antimonopole strings. They are identified as the supersymmetric D2-D2 connecting two D4 branes. Compared to the case of supersymmetric D2-D2 without D4 branes, the effect of D4 branes appears in the F-string densities by the factor $\mu D/(1 + \mu D)$ depending on the separation $D$ of D2 and $\overline{D2}$.

The dyonic instantons found by the ’t Hooft ansatz have only isolated zeros and so could not be identified with supertubes connecting two D4 branes. For these solutions, the angular momentum of the selfdual dyonic instanton is antiselfdual. This is not true for our single monopole string, in which case the angular momentum with respect to any reference point has both selfdual and antiselfdual component. Clearly our solutions are outside the ’t Hooft ansatz family.

While our examples for supertubes have infinite instanton number, we wonder a possibility that supertubes may appear even with finite instanton number for generic dyonic configurations beyond the ’t Hooft ansatz.
Finally, the study of moduli dynamics of the charged monopole string would be quite interesting including the interactions between the charged monopole and antimonopole strings.

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