The $b \to s\gamma$ Decay in Supergravity with Radiatively Electroweak Breaking

MARCO AURELIO DÍAZ

Department of Physics and Astronomy
Vanderbilt University, Nashville, TN 37235

Abstract

It is analyzed the branching ratio $B(b \to s\gamma)$ in the context of minimal $N = 1$ supergravity with radiatively broken electroweak symmetry group. There is a strong dependence on supersymmetric parameters, but constraints on the charged Higgs mass in non-supersymmetric models are relaxed, due to large contribution from the chargino/up-type squark sector that interacts destructively with the Standard Model and the charged Higgs contributions. Large suppressions/enhancements of the branching fraction are found for large values of $\tan \beta$. 
The decay $b \to s\gamma$ is forbidden at tree level but induced in the Standard Model (SM) at one loop by $W$ and Goldstone bosons together with up-type quarks in the internal lines of the loop\cite{1}. The SM value of the branching ratio of this decay is $B(b \to s\gamma) \approx 4 \times 10^{-4}$ for $m_t = 140$ GeV and increases with $m_t$. In two-Higgs-doublets models, loops involving charged Higgs bosons and up-type quarks have to be added\cite{2}. The contribution from the charged Higgs boson in type II models (where one Higgs doublet couples to the up-type quarks and the other Higgs doublet couples to the down-type quarks) has the same sign as the SM contribution. In type I models (where only one Higgs doublet couples to the fermions) the charged Higgs boson contribution does not have a definite sign and decreases with $\tan \beta$. This fact is responsible for the strong constraints on type II models in comparison with type I models.

The first observation of the exclusive decays $B^0 \to K^*(892)^0\gamma$ and $B^- \to K^*(892)^-\gamma$ by the CLEO Collaboration\cite{3} is a strong evidence for the penguin diagrams at the quark level process $b \to s\gamma$. The ratio between the exclusive and the inclusive decays $\Gamma(B \to K^*\gamma)/\Gamma(b \to s\gamma)$ has been calculated\cite{4}, but with a high uncertainty: the predicted values range from 4% and 40%. This impose a lower bound on the inclusive branching ratio whose conservative estimation\cite{5} is $B(b \to s\gamma) > 0.65 \times 10^{-4}$. The latest experimental upper bound on the branching fraction for the inclusive decay mode $b \to s\gamma$, given by $B(b \to s\gamma) < 5.4 \times 10^{-4}$ at 90% c.l.\cite{6}, sets powerful constraints on the charged Higgs boson mass in two Higgs doublets models of type II\cite{7}. Other corrections have been calculated recently: next-to-leading logarithmic QCD-corrections\cite{8}, and electroweak corrections in the context of supersymmetry\cite{9} to the charged Higgs mass\cite{10} and to the charged Higgs-fermion-fermion vertex\cite{11}.

In supersymmetry, the contributions from charginos together with up-type squarks and from neutralinos and gluinos together with down-type squarks, have to be included\cite{12,13}. It was stressed that in this case, it is important the effect of loops involving supersymmetric particles\cite{14,15}. Here we study this effect in the context of the radiatively broken Minimal Supersymmetric Model\cite{16}, following ref.
Minimal $N = 1$ supergravity is characterized by the superpotential
\begin{equation}
W = h_U^{ij} Q_i U_j^c H_2 + h_D^{ij} Q_i D_j^c H_1 + h_E^{ij} L_i E_j^c H_1 + \mu \varepsilon_{ab} H_1^a H_2^b
\end{equation}
where $i, j = 1, 2, 3$ are indices in generation space, $\varepsilon_{ab}$ with $a, b = 1, 2$ is the antisymmetric tensor in two dimensions, and $\mu$ is the Higgs mass parameter. The $3 \times 3$ matrices $h_U, h_D$ and $h_E$ are the Yukawa couplings. The soft supersymmetry breaking terms are
\begin{equation}
\mathcal{L}_s = A_U^{ij} h_U^{ij} \tilde{Q}_i \tilde{U}_j^c H_2 + A_D^{ij} h_D^{ij} \tilde{Q}_i \tilde{D}_j^c H_1 + A_E^{ij} h_E^{ij} \tilde{L}_i \tilde{E}_j^c H_1 + B \mu \varepsilon_{ab} H_1^a H_2^b + \text{h.c.}
\end{equation}
plus a set of scalar and gaugino mass terms, which at the unification scale are
\begin{equation}
\mathcal{L}_m = m_0^2 \sum_i |S_i|^2 + \left[ \frac{1}{2} M_{1/2}(\lambda_1 \lambda_1 + \lambda_2 \lambda_2 + \lambda_3 \lambda_3) + \text{h.c.} \right]
\end{equation}
where $S_i$ are all the scalars of the theory and $\lambda_i, i = 1, 2, 3$ are the gauginos corresponding to the groups $U(1), SU(2)$ and $SU(3)$ respectively. In eq. (2) all the fields are scalar components of the respective superfields. The mass parameters $A$ and $B$ are of $O(m_0)$ and in some supergravity models they satisfy the following relation at the unification scale $M_X$: $A = B + m_0$ where $A$ is the common value for the $A_a^{ij}$ ($a = U, D, E$) parameters at $M_X$: $A_U^{ij} = A_D^{ij} = A_E^{ij} = A\delta^{ij}$.

At the weak scale, the tree level Higgs potential is given by
\begin{equation}
V = m_{1H}^2 |H_1|^2 + m_{2H}^2 |H_2|^2 - m_{12}^2 (H_1 H_2 + \text{h.c.}) + \frac{1}{8} (g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g^2 |H_1^* H_2|^2
\end{equation}
where $m_{1H}^2 = m_i^2 + |\mu|^2$ ($i = 1, 2$) and $m_{12}^2 = -B \mu$. The two Higgs doublets mass parameters $m_1$ and $m_2$ satisfy $m_1 = m_2 = m_0$ at the unification scale. The three mass parameters in eq. (4) can be replaced by the $Z$ boson mass $m_Z$, the CP-odd
Higgs mass $m_A$, and the ratio between the vacuum expectation values of the two Higgs doublets $\tan \beta \equiv v_2/v_1$, according to the formulas

\[
m^2_{1H} = -\frac{1}{2} m^2_Z c_{2\beta} + \frac{1}{2} m^2_A (1 - c_{2\beta})
\]
\[
m^2_{2H} = \frac{1}{2} m^2_Z c_{2\beta} + \frac{1}{2} m^2_A (1 + c_{2\beta})
\]
\[
m^2_{12} = \frac{1}{2} m^2_A s_{2\beta}
\]

where $s_{2\beta}$ and $c_{2\beta}$ are sine and cosine functions of the angle $2\beta$. The previous relations are valid at tree level. The effects of the one-loop corrected Higgs potential may be important in some cases\(^{[18]}\), especially near $\tan \beta = 1$ when $m_{1H} = m_{2H} = m_{12}$ and the lightest neutral Higgs mass comes only from radiative corrections\(^{[19]}\).

The electroweak symmetry group is broken radiatively when the different parameters are evolved from the grand unification scale to the weak scale\(^{[20]}\). In ref. [21] can be found a typical solution of the renormalization group equations (RGE) in the spirit of ref. [22], but including the trilinear $A$ parameters and other Higgs mass parameters as well. The effects of the supersymmetric threshold are neglected. The RGE used are given in ref. [12] with the exception of the $A$ parameters, whose equations are taken from ref. [23]. The set of independent parameters is chosen to be $m_t, m_A$ and $\tan \beta$ at the weak scale, $M_{1/2}$ at the unification scale, and the sign of $\mu$ as a discrete parameter.

The QCD uncorrected amplitude for the decays $b \to s\gamma$ and $b \to s g$ are

\[
A^{\gamma,g}(m_W) = A^{\gamma,g}_{SM} + (f^+ f^-) A^{1\gamma,g}_{H^\pm} + (f^-)^2 \cot^2 \beta A^{2\gamma,g}_{H^\pm} + A^{\gamma,g}_{\chi^\pm} + A^{\gamma,g}_{\tilde{g}}
\]

where the form factors $f^\pm$ come from the renormalization of the charged Higgs boson coupling to a pair of fermions\(^{[11]}\). The different amplitudes $A$ can be found in ref. [12]. If we now run the scale from $m_W$ to $m_b$ and introduce the QCD corrections\(^{[24]}\), we get

\[
A^{\gamma}(m_b) = \eta \frac{16}{23} \left[ A^{\gamma}(m_W) + \frac{8}{3} A^g(m_W) (\eta \frac{2}{23} - 1) \right] + CA^{0}_{\gamma}
\]
where \( \eta = \alpha_s(m_b)/\alpha_s(m_W) \approx 1.83 \) and \( A_0^\gamma \) is given by

\[
A_0^\gamma = \frac{\alpha_W \sqrt{\alpha} V_{ts}^* V_{tb}}{2\sqrt{\pi}} \frac{m_2^2}{m_W^2}
\]

with \( C = 0.177, \alpha_W = g^2/4\pi, \) and \( \alpha = e^2/4\pi \). This last term proportional to \( C \) comes from mixing of four quark operators\([14]\).

In fig. 1 it is plotted the branching ratio of the inclusive decay \( B(b \to s\gamma) \) as a function of \( \tan \beta \). In this model it is not possible to get the correct electroweak symmetry breaking if \( \tan \beta > \approx m_t/m_b \). If \( \tan \beta \) increases the scalar mass parameter \( m_0 \) must grow with \( \tan \beta \) in order to get the necessary splitting \( m_{2H}^2 - m_{1H}^2 \), producing heavy squarks and consequently a suppressed contribution to the branching ratio from the chargino/up-type squarks loops. This in turn produce a large value for \( B(b \to s\gamma) \). On the other side, \( \tan \beta \) is bounded from below by \( \tan \beta > \approx 1 \), since it is not possible neither to get the correct electroweak symmetry breaking otherwise.

In fig. 2 we see the dependence of \( B(b \to s\gamma) \) on the mass of the CP-odd Higgs mass \( m_A \). For large values of \( m_A \) the contribution of the charged Higgs mass decreases since its mass increases. But also the scalar mass parameter \( m_0 \) increases with \( m_A \), thus making heavy squarks while keeping almost constant chargino and neutralino masses. This makes the magnitude of the chargino contribution (it has a negative sign with respect to the \( W \) and \( H^\pm \) contributions) to \( B(b \to s\gamma) \) decrease also with \( m_A \) but faster than the charge Higgs contribution. The net effect is a growing \( B(b \to s\gamma) \) with \( m_A \), and approaching to the SM value.

The dependence on the top quark mass of the branching ratio \( B(b \to s\gamma) \) can be seen in fig. 3. In the SM this branching ratio grows with the top quark mass and remains below the CLEO bound in the hole range of \( m_t \). In the SUSY–GUT model, for the parameters considered here, the branching ratio decreases with the top quark mass except for very large values of \( m_t \). This effect is due to a faster growing (and with opposite sign) chargino contribution compared to the charged
Higgs contribution. This can be seen also in ref. [14] taking into account that in the SUSY–GUT model, a change in $m_t$ implies a change in $m_0$, in opposition to the treatment in ref. [14] where both parameters are independent. If the top quark mass increases, the splitting $m_1^2 - m_2^2$ at the weak scale increases also, and to keep it constant $m_0$ must decrease. This in turn will decrease the absolute values of $m_1^2$ and $m_2^2$, so $\mu$ will grow in order to keep $m_1^2_{H}$ and $m_2^2_{H}$ unchanged.

In fig. 4 it is plotted the branching ratio $B(b \to s\gamma)$ as a function of the gaugino mass parameter $M_{1/2}$ (at the unification scale $M_X$). For small values of $M_{1/2}$ the mass of the lightest chargino and neutralino become too small. The relatively weak dependence of $B(b \to s\gamma)$ on $M_{1/2}$ appears because the masses of the charginos and up-type squarks grows slowly with $M_{1/2}$ and at a comparable rate.

We have studied the prediction for the branching ratio $B(b \to s\gamma)$ in the context of minimal \(N=1\) supergravity with a radiatively broken electroweak symmetry group. We found convenient to parametrize the model with the following independent parameters: $m_t$, $m_A$ and $\tan \beta$ at the weak scale, $M_{1/2}$ at the unification scale, and the sign of $\mu$ as a discrete parameter. It is clear that the experimental upper bound of the branching ratio of the decay $b \to s\gamma$ does not strongly constrain the charged Higgs mass, as it does to non-supersymmetric two-Higgs-doublet models of type II. The branching ratio $B(b \to s\gamma)$ lies below the CLEO bound and even below the SM value in some regions of the parameter space explored here, although the opposite also occurs. There is an important dependence on $\tan \beta$, and large suppressions/enhancements of $B(b \to s\gamma)$ with respect to the Standard Model are possible for large values of this parameter. We have include in the calculation QCD corrections and some electroweak radiative corrections as well. The later are more important at large values of $\tan \beta$.

Given the strong dependence of $B(b \to s\gamma)$ on the different supersymmetric parameters it is possible to rule out some regions of the parameter space and, because of this reason, we emphasize that an experimental measurement of this decay, or an improvement on the upper bound, will be an important way to test
supersymmetry.

**Note added:** When this work was completed, we received two preprints\(^{25}\) that calculate the branching ratio \(B(b \to s\gamma)\) in the MSSM with a radiatively induced breaking of the gauge group \(SU(2) \times U(1)\). In the DESY preprint, a strong dependence on \(\tan \beta\) is also observed when this parameter is large. In the CERN preprint, although the models considered have a more restricted parameter space than ours, the dependence of the branching ratio on \(\tan \beta\) is similar.

**ACKNOWLEDGMENTS**

Discussions with Howard Baer, Joseph Lykken, Xerxes Tata, and Thomas Weiler are gratefully acknowledged. I am thankful to the Particle Theory Group at Fermilab, where part of this work was completed. This work was supported by the U.S. Department of Energy, grant No. DE-FG05-8SER40226.

**REFERENCES**

1. T. Inami and C.S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981).

2. T.G. Rizzo, *Phys. Rev. D* **38**, 820 (1988); B. Grinstein and M.B. Wise, *Phys. Lett. B* **201**, 274 (1988); W.-S. Hou and R.S. Willey, *Phys. Lett. B* **202**, 591 (1988); T.D. Nguyen and G.C. Joshi, *Phys. Rev. D* **37**, 3220 (1988); C.Q. Geng and J.N. Ng, *Phys. Rev. D* **38**, 2857 (1988); D. Ciuchini, *Mod. Phys. Lett. A* **4**, 1945 (1989); B. Grinstein, R. Springer and M. Wise, *Nucl. Phys. B* **339**, 269 (1990); V. Barger, J.L. Hewett and R.J.N. Phillips, *Phys. Rev. D* **41**, 3421 (1990), and Erratum.

3. CLEO Collaboration, R. Ammar et al., *Phys. Rev. Lett.* **71**, 674 (1993).

4. T. Altomari, *Phys. Rev. D* **37**, 677 (1988); C.A. Dominguez, N. Paver and Riazuddin, *Phys. Lett. B* **214**, 459 (1988); N.G. Deshpande, P. Lo and
J. Trampetic, *Z. Phys. C* 40, 369 (1988); T.M. Aliev, A.A. Ovchinnikov and V.A. Slobodenyuk, *Phys. Lett. B* 237, 569 (1990); P.J. O'Donnell and H.K.K. Tung, *Phys. Rev. D* 44, 741 (1991); A. Ali and C. Greub, *Phys. Lett. B* 259, 182 (1991); A. Ali, T. Ohl and T. Mannel, *Phys. Lett. B* 298, 195 (1993).

5. J.L. Hewett and T.G. Rizzo, report No. ANL-HEP-PR-93-37 (June 1993), unpublished.

6. E. Thorndike, CLEO Collaboration, talk given at the 1993 Meeting of the American Physical Society, Washington D.C., April 1993.

7. J.L. Hewett, *Phys. Rev. Lett.* 70, 1045 (1993); V. Barger, M.S. Berger and R.J.N. Phillips, *Phys. Rev. Lett.* 70, 1368 (1993).

8. M. Misiak, *Nucl. Phys.* B393, 23 (1993).

9. M.A. Díaz, *Phys. Lett. B* 304, 278 (1993).

10. J.F. Gunion and A. Turski, *Phys. Rev. D* 39, 2701 (1989); 40, 2333 (1989); A. Brignole, J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett. B* 271, 123 (1991); M. Drees and M.M. Nojiri, *Phys. Rev. D* 45, 2482 (1992); A. Brignole, *Phys. Lett. B* 277, 313 (1992); P.H. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett. B* 274, 191 (1992); M.A. Díaz and H.E. Haber, *Phys. Rev. D* 45, 4246 (1992).

11. M.A. Díaz, *Phys. Rev. D* 48, 2152 (1993); M.A. Díaz, *The Fermilab Meeting DPF’92*, ed. by C. Albright et al., World Scientific, page 1194.

12. S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* B353, 591 (1991).

13. S. Bertolini, F. Borzumati and A. Masiero, *Nucl. Phys.* B294, 321 (1987); S. Bertolini, F.M. Borzumati, and A. Masiero, *Phys. Lett. B* 192, 437 (1987); T.M. Aliev and M.I. Dobroliubov, *Phys. Lett. B* 237, 573 (1990).

14. R. Barbieri and G.F. Giudice, *Phys. Lett. B* 309, 86 (1993).
15. N. Oshimo, *Nucl. Phys.* B404, 20 (1993); J.L. Lopez, D.V. Nanopoulos, and G.T. Park, *Phys. Rev. D* 48, 974 (1993); Y. Okada, *Phys. Lett. B* 315, 119 (1993); R. Garisto and J.N. Ng, *Phys. Lett. B* 315, 372 (1993).

16. J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs Hunter’s Guide* (Addison-Wesley, Reading MA, 1990).

17. H.P. Nilles, *Phys. Rep* 110, 1 (1984); H.E. Haber and G.L. Kane, *Phys. Rep.* 117, 75 (1985).

18. S. Kelley, J. Lopez, D. Nanopoulos, H. Pois and K. Yuan, *Nucl. Phys.* B398, 3 (1993).

19. M.A. Díaz and H.E. Haber, *Phys. Rev. D* 46, 3086 (1992).

20. M. Drees and M.M. Nojiri, *Nucl. Phys.* B369, 54 (1992).

21. M.A. Díaz, report No. VAND-TH-93-11, presented at the Workshop on Physics at Current Accelerators and the Supercollider, Argonne National Laboratory, Argonne Il, June 2-5 1993.

22. G.G. Ross and R.G. Roberts, *Nucl. Phys.* B377, 571 (1992).

23. N.K. Falck, *Z. Phys. C* 30, 247 (1986).

24. S. Bertolini, F. Borzumati and A. Masiero, *Phys. Rev. Lett.* 59, 180 (1987); N.G. Deshpande, P. Lo, J. Trampetic, G. Eilam and P. Singer, *Phys. Rev. Lett.* 59, 183 (1987); B. Grinstein, R. Springer and M.B. Wise, *Nucl. Phys.* B339, 269 (1990); P. Cho and B. Grinstein, *Nucl. Phys.* B365, 279 (1991); M. Misiak, *Phys. Lett. B* 269, 161 (1991); A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* B370, 69 (1992); M. Misiak, *Nucl. Phys.* B393, 23 (1993); K. Adel and Y.-P. Yao, report No. UM-TH-93-20 (August 1993), unpublished.

25. J.L. Lopez, D.V. Nanopoulos, G.T. Park and A. Zichichi, report No. CERN-TH.6979/93, July 1993, unpublished; F.M. Borzumati, report No. DESY 93-90, August 1993, unpublished.
FIGURE CAPTIONS

1) Branching ratio of the inclusive decay $B(b \rightarrow s\gamma)$ as a function of $\tan \beta$ for the SM and for the SUSY-GUT model. It is also displayed the CLEO upper bound.

2) Same than fig. 1 but as a function of $m_A$.

3) Same than fig. 1 but as a function of $m_t$.

4) Same than fig. 1 but as a function of $M_{1/2}$.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9311228v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9311228v1