Entanglement-based quantum private comparison
protocol with bit-flipping *

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Abstract

By introducing a semi-honest third party, we propose in this paper a novel QPC protocol using \((n + 1)\)-qubit \((n \geq 2)\) Greenberger-Horne-Zeilinger (GHZ) states as information carriers. The parameter \(n\) not only determines the number of qubits contained in a GHZ state, but also determines the probability that the third party can successfully steal the participants’ data and the qubit efficiency. Our protocol uses the keys generated by quantum key distribution and bit-flipping for privacy protection, making both outsider and insider attacks invalid. In addition, our protocol does not employ any other quantum technologies (e.g., entanglement swapping and unitary operation) except necessary technologies such as preparing quantum states and quantum measurement, which can reduce the need for quantum devices. Furthermore, the GHZ states are prepared by participants rather than by the third party, which can reduce potential security risks.

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1 Introduction

Comparing two or more data to determine whether they are the same has a wide range of applications in information science, such as malware detection and clustering, patch generation and analysis, and bug search \[1,2\]. A natural question is how to complete the comparison if all the data are confidential. This problem is called “Tiersé problem” or “socialist millionaires’ problem”, which is originated from the “millionaires’ problem” raised by Yao in 1982 \[3–6\]. The solutions to this problem can also solve many problems in real life, such as secret bidding and auctions, secret ballot elections, e-commerce, data mining, and authentication \[3–7\].

Quantum private comparison (QPC) is the generalization of the solutions to the “Tiersé problem” in quantum mechanics. The difference between QPC and the classical solutions is that its security is based on the principles of quantum mechanics rather than computational complexity \[3–10\]. QPC has attracted wide attention from academia in recent decade because it can provide unconditional security for real-life information transactions \[3\]. A QPC protocol needs to introduce a semi-honest third party (conventionally called TP), who faithfully executes protocol processes to assist participants in completing private comparison, and will not collude with any participant, but he can steal the participants’ data in all possible ways \[3\]. In addition, a QPC protocol should satisfy two conditions \[3\]: 1) fairness: all participants get the comparison result at the same time without a specific order; 2) security: each participant’s data is confidential, and the other participants, TP and the attackers outside the protocol cannot successfully steal the participant’s data; if all participants’ data are the same, the participants know each other’s secret data.

In this paper, we propose a novel QPC protocol. We use Greenberger-Horne-Zeilinger (GHZ) states as information carriers, and introduce a semi-honest TP who assists the participants in completing the protocol without colluding with them. We use quantum key distribution (QKD) \[11–15\] to generate keys and combine the keys with bit-flipping to protect data privacy. In addition, unlike most existing protocols, our protocol only uses single-particle measurement technology instead of entanglement measurement, entanglement swapping, unitary operation and other technologies, which naturally reduces the need for expensive quantum devices. What is more, the GHZ states are prepared by participants rather than by TP, which can to some extent improve the security and efficiency of the protocol.

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We arrange the rest of this paper as follows. In Sec. 2, we first introduce the GHZ states used in our protocol, and then describe protocol steps in detail. Sec. 3 is devoted to analyze the protocol security, including the outsider attack and insider attack. Sec. 4 gives some useful discussions. We make a summary in Sec. 5.

2 Proposed quantum private comparison protocol

Let us now introduce the GHZ states. The canonical orthonormal $m$-qubit ($m \in \mathbb{N}_+$ and $m \geq 3$) GHZ states, as information carriers in our protocol, can be expressed as

$$\ket{G^k_2} = \frac{1}{\sqrt{2}} \left( \ket{B(k)} \pm \ket{B(2^m - k - 1)} \right),$$

where $k = 0, 1, \ldots, 2^{m-1} - 1$, and $B(k) = 0b_2b_3 \cdots b_m$ is the binary representation of $k$ in an $m$-bit string, thus $k = 0 \cdot 2^{m-1} + b_2 \cdot 2^{m-2} + b_3 \cdot 2^{m-3} + \cdots + b_m \cdot 2^0$. Obviously, they are orthonormal and complete.

Eq. [1] can be written in a more concise form as follows:

$$\ket{G^k_2} = \frac{1}{\sqrt{2}} \left( |0b_2b_3 \cdots b_m\rangle \pm |1\bar{b}_2\bar{b}_3 \cdots \bar{b}_m\rangle \right),$$

where a bar over a bit value indicates its logical negation.

2.1 Prerequisites

Next, let us introduce three prerequisites for the proposed protocol.

1. Suppose that two participants, Alice and Bob, have secret data $X$ and $Y$ respectively; the binary representations of $X$ are $(x_1, x_2, \ldots, x_N)$, and $Y$ $(y_1, y_2, \ldots, y_N)$, where $x_j, y_j \in \{0, 1\} \ \forall j = 1, 2, \ldots, N, X = \sum_{j=1}^{N} x_j 2^{j-1}$, $Y = \sum_{j=1}^{N} y_j 2^{j-1}$ ($N \in \mathbb{N}_+$, and $N$ is usually a large number, just like the value of the millionaire’s wealth mentioned in the Introduction). With the help of the semi-honest third party (TP) who may behave badly but will not collude with either participant, Alice and Bob want to judge whether $X = Y$.

2. Alice(Bob) divides the binary representation of $X(Y)$ into $[N/n]$ groups,

$$G_a^1, G_a^2, \ldots, G_a^{[N/n]} (G_b^1, G_b^2, \ldots, G_b^{[N/n]}),$$

where $n \in \mathbb{N}_+$ and $2 \leq n \leq N$ throughout this protocol, and each group $G_i^j (G_i^j)$ includes $n$ bits ($i = 1, 2, \ldots, [N/n]$ throughout this protocol). If $N \mod n = l$, Alice(Bob) adds $l$ into the last group $G_a^{[N/n]} (G_b^{[N/n]})$.

3. Alice, Bob and TP agree on the following coding rules: $|0\rangle \leftrightarrow 0$ and $|1\rangle \leftrightarrow 1$; the coding rules are public.

2.2 Protocol steps

Let us now describe in detail the steps of the protocol: (the flow chart of the protocol is shown in fig. 1):

1. Step 1: key generation

Alice and Bob use QKD to generate the shared secret key sequence $\{K_{AB}^1, K_{AB}^2, \ldots, K_{AB}^{[N/n]}\}$ (indeed, if Alice and Bob are in the same place, they can directly generate the shared key sequence without using QKD). Similarly, Alice and TP generate the shared key sequence $\{K_{AC}^1, K_{AC}^2, \ldots, K_{AC}^{[N/n]}\}$. Bob and TP generate the shared key sequence $\{K_{BC}^1, K_{BC}^2, \ldots, K_{BC}^{[N/n]}\}$. Here, $K_{AB}^i, K_{AC}^i, K_{BC}^i \in \{0, 1\}$, and note that the keys generated by QKD are confidential and are always assumed to be secure in QPC. Otherwise, the security of the protocol can not be guaranteed and the design of the protocol can not be completed.

2. Step 2: encryption

Alice(Bob) computes $K_{AB}^i \oplus K_{AC}^i (\oplus K_{BC}^i)$ and denotes the computing results as $R_{AB}^i (R_{BC}^i)$ (i.e. $R_{AB}^i = K_{AB}^i \oplus K_{BC}^i, R_{BC}^i = K_{AB}^i \oplus K_{BC}^i$), where the symbol $\oplus$ denotes the module 2 operation (i.e. XOR operator) throughout this paper. Then, Alice(Bob) encrypts her(his) data $G_a^i (G_b^i)$ according to the value of $R_{AB}^i (R_{BC}^i)$.
Concretely, if $R_A^c = 1(R_B^c = 1)$, she (he) flips each bit in $G_a'(G_b')$ (e.g., 0101 → 1010), otherwise keeps $G_a'(G_b')$ unchanged. Finally, Alice (Bob) denotes her (his) encrypted data as $G_a'(G_b')$, that is,

$$G_a'(G_b') = \begin{cases} G_a'(G_b'), & \text{if } R_A^c = 0(R_B^c = 0); \\ -G_a'(G_b'), & \text{if } R_A^c = 1(R_B^c = 1), \end{cases}$$

(5)

where the symbol $\sim$ is a bitwise inverse operator. For example, if $G_a' = 0110$, then $-G_a' = 10010$.

3. Step 3: state preparation

According to the value of $G_a'(G_b')$, Alice (Bob) prepares the $(n+1)$-qubit GHZ state

$$\left| G(a_0^1, a_1^1, \ldots, a_n^1) \right> = \frac{1}{\sqrt{2}} \left( |0a_1a_2 \cdots a_n \rangle + |1a_1a_2 \cdots a_n \rangle \right)$$

$$\left( |G(b_0^1, b_1^1, \ldots, b_n^1) \right> = \frac{1}{\sqrt{2}} \left( |0b_1b_2 \cdots b_n \rangle + |1b_1b_2 \cdots b_n \rangle \right),$$

(6)

where $a_1a_2 \cdots a_n(b_1b_2 \cdots b_n)$ is the binary representation of $G_a'(G_b')$, hence

$$G_a' = a_1 \cdot 2^{n-1} + a_2 \cdot 2^{n-2} + \cdots + a_n \cdot 2^0$$

$$G_b' = b_1 \cdot 2^{n-1} + b_2 \cdot 2^{n-2} + \cdots + b_n \cdot 2^0.$$  

(7)

Subsequently, Alice (Bob) takes all the particles out from $\left| G(a_0^i, a_1^i, \ldots, a_n^i) \right>\left( \left| G(b_0^i, b_1^i, \ldots, b_n^i) \right> \right)$ to construct the sequence

$$a_0^0, a_1^1, a_2^2, a_3^3, \ldots, a_n^n, a_0^1, a_1^2, a_2^3, \ldots, a_n^n, a_0^2, a_1^3, a_2^4, \ldots, a_n^n, a_0^3, a_1^4, a_2^5, \ldots, a_n^n, \ldots, a_0^n, a_1^n, a_2^n, \ldots, a_n^n,$$

$$b_0^0, b_1^1, b_2^2, b_3^3, \ldots, b_n^n, b_0^1, b_1^2, b_2^3, \ldots, b_n^n, b_0^2, b_1^3, b_2^4, \ldots, b_n^n, b_0^3, b_1^4, b_2^5, \ldots, b_n^n, \ldots, b_0^n, b_1^n, b_2^n, \ldots, b_n^n,$$

(8)

and denotes it as $S_a(S_b)$.

4. Step 4: transmission

Alice (Bob) prepares a set of decoy photons, where each decoy photon is randomly chosen from the four states $|0\rangle, |1\rangle, |+\rangle, |−\rangle \ (|±\rangle = 1/√2 \ (|0\rangle \pm |1\rangle))$. Subsequently, Alice (Bob) inserts each decoy photon into $S_a(S_b)$ at a random position; the new generated sequence is denoted as $S_a'(S_b')$. Finally, Alice (Bob) sends $S_a'(S_b')$ to TP.

5. Step 5: eavesdropping checking

After confirming TP’s receipt of $S_a'(S_b')$, Alice (Bob) tells TP the positions and bases of the decoy photons. TP then measures the decoy photons with the bases announced, and tells Alice (Bob) the measurement outcomes. Based on the comparison between the initial states and the measurement outcomes of the decoy photons, they can judge whether there is eavesdropping in the quantum channels. The error rate exceeding the predetermined threshold will lead to the termination and restart of the protocol, otherwise the protocol will proceed to the next step.

6. Step 6: measurement and entanglement generation

TP performs single-particle measurements on each particle in $S_a$ and $S_b$ with $Z$ basis ($|0\rangle, |1\rangle$). That is, TP measures each particle marked by $a_0^i, a_1^1, \ldots, a_n^n(b_0^i, b_1^1, \ldots, b_n^n)$ in $S_a'(S_b')$. Then, according to the coding rules (see the third prerequisite of our protocol), TP denotes the binary numbers corresponding to the measurement result of the first particle marked by $a_0^i(b_0^i)$ as $M^i_a(M^i_b)$, and denotes the binary numbers corresponding to the measurement result of the remaining particles marked by $a_1^1, a_2^2, \ldots, a_n^n(b_1^1, b_2^2, \ldots, b_n^n)$ as $M^i_a(M^i_b)$.

TP computes $M^i_a \otimes K^{i_A} \otimes K^{i_B}$, and denotes the computing results as $C_a(C_b)$. Then, according to the value of $C_a(C_b)$, TP performs the following operations on $M^i_a(M^i_b)$: If $C_a = 1(C_b = 1)$, TP flips each bit in $M^i_a(M^i_b)$, otherwise keeps $M^i_a(M^i_b)$ unchanged. Denotes the value of $M^i_a(M^i_b)$ after the operations as $M^i_a(M^i_b)$, then

$$M^i_a(M^i_b) = \begin{cases} M^i_a(M^i_b), & \text{if } C_a = 0(C_b = 0); \\ -M^i_a(M^i_b), & \text{if } C_a = 1(C_b = 1). \end{cases}$$

(9)

TP computes $M^i_a(M^i_b)$ and denote the computing results as $R^i_c$. If $R^i_c = 00 \cdots 0$ (i.e., the results are all 0), TP can conclude that $X = Y$, otherwise $X \neq Y$. Finally, TP publicly announces the comparison result to Alice and Bob.
Let us show that the output of our protocol is correct. That is, we will show that the value of $M^2_A \oplus M^2_B$ equals $G^2_A \oplus G^2_B$. For clarity, we list all possible intermediate computational results in Table 1. As can be seen from the table, $M^2_A \oplus M^2_B$ is always equal to $G^2_A \oplus G^2_B$ or $\neg G^2_A \oplus \neg G^2_B$. Next let us prove that the equation $G^2_A \oplus G^2_B = \neg G^2_A \oplus \neg G^2_B$ always holds. Indeed, we only need to prove that two bits satisfies the equation because XOR is a bitwise operator. Suppose there are two bits $p$ and $q$, where $p, q \in \{0, 1\}$. Due to $p \oplus q = (\neg p \land q) \lor (p \land \neg q)$, $\neg p \oplus \neg q = (\neg (\neg p) \land \neg q) \lor (\neg p \land \neg (\neg q)) = (p \land \neg q) \lor (\neg p \land \neg q) = p \oplus q$. Therefore, $G^2_A \oplus G^2_B = \neg G^2_A \oplus \neg G^2_B$.

3 Security analysis

In this section, we will analyze the security of the proposed protocol. First, we show that the external attacks (from the eavesdroppers outside the protocol) are invalid. Then, we show that the internal attacks, including those by participants and TP, are also invalid.

3.1 Outsider attack

Generally, the outsider attack refers to the attempt by someone outside the protocol to steal the participants’ secret data from a quantum channel. The famous attacks include the intercept-resend attack and measurement-resend attack. In our protocol, we use decoy photons to check the security of quantum channels (see Step 5), the idea of which is derived from QKD, and which has been proved unconditionally safe [12]. Indeed, it seems unimpressive to prove the security based on decoy photons in QPC. Nevertheless, the form of security proof for an attack varies with the quantum states and algorithms used in different protocols. In this case, it is necessary to give one or two security proofs. In what follows let us take the well-known entanglement-measurement attack as example to demonstrate that outsider attacks will not succeed. Concretely, an eavesdropper (conventionally called Eve) intercepts part or all of the particles transmitted between Alice(Bob) and TP in Step 4, and entangles them with the ancillary particles that she prepares beforehand, and then resends them to TP. Finally, by performing measurements on the ancillary particles, Eve can extract information carried by them.

Let us now analyze the case that Eve intercepts the particles sent from Alice to TP (as for the interception of the particles sent from Bob to TP, analysis can be carried out in the same way). Let us denote Eve’s unitary operator as $U$, without loss of generality, Eve’s entanglement action can be expressed as

$$U |0\rangle |e\rangle = \lambda_{00} |0\rangle |e_{00}\rangle + \lambda_{01} |1\rangle |e_{01}\rangle, \quad U |1\rangle |e\rangle = \lambda_{10} |0\rangle |e_{10}\rangle + \lambda_{11} |1\rangle |e_{11}\rangle,$$

(10)
meet the following conditions:

\[ U_{01} = U_{10} = 0, \lambda_{00} |e_{00}\rangle = \lambda_{11} |e_{11}\rangle. \]  

where \(|e_{00}\rangle, |e_{01}\rangle, |e_{10}\rangle, |e_{11}\rangle\) are the pure states determined only by \(U\), \(|e\rangle\) is an ancillary particle, and \(||\lambda_{00}\rangle^2 + ||\lambda_{10}\rangle^2 + ||\lambda_{11}\rangle^2 = 1\).

When \(U\) is performed on the decoy states \(|+\rangle\) and \(|-\rangle\), one can get

\[
U |+\rangle |e\rangle = \frac{1}{\sqrt{2}} (A_{00} |0\rangle |e_{00}\rangle + A_{01} |1\rangle |e_{01}\rangle + A_{10} |0\rangle |e_{10}\rangle + A_{11} |1\rangle |e_{11}\rangle)
\]

\[
= \frac{1}{2} |+\rangle (\lambda_{00} |e_{00}\rangle + \lambda_{01} |e_{01}\rangle + \lambda_{10} |e_{10}\rangle + \lambda_{11} |e_{11}\rangle) + \frac{1}{2} |-\rangle (\lambda_{00} |e_{00}\rangle - \lambda_{01} |e_{01}\rangle + \lambda_{10} |e_{10}\rangle - \lambda_{11} |e_{11}\rangle),
\]  

(11)

and

\[
U |-\rangle |e\rangle = \frac{1}{\sqrt{2}} (A_{00} |0\rangle |e_{00}\rangle + A_{01} |1\rangle |e_{01}\rangle - A_{10} |0\rangle |e_{10}\rangle - A_{11} |1\rangle |e_{11}\rangle)
\]

\[
= \frac{1}{2} |+\rangle (\lambda_{00} |e_{00}\rangle + \lambda_{01} |e_{01}\rangle - \lambda_{10} |e_{10}\rangle - \lambda_{11} |e_{11}\rangle) + \frac{1}{2} |-\rangle (\lambda_{00} |e_{00}\rangle - \lambda_{01} |e_{01}\rangle - \lambda_{10} |e_{10}\rangle + \lambda_{11} |e_{11}\rangle).
\]  

(12)

From Eqs. 10, 11 and 12 if Eve wants to avoid introducing errors in the eavesdropping checking step, \(U\) must meet the following conditions:

\[
\lambda_{01} = \lambda_{10} = 0, \lambda_{00} |e_{00}\rangle = \lambda_{11} |e_{11}\rangle.
\]  

(13)
Next, let us demonstrate that entanglement-measurement attack is invalid to our protocol. Concretely, Eve entangles the \((n + 1)\)-qubit GHZ state \(G(a_{0}^{i}, a_{1}^{i}, \ldots, a_{n}^{i})\) prepared by Alice with ancillary particles, and then measures the ancillary particles to extract information. This is how Eve attempts to steal Alice’s data, which will be proved invalid below.

From Eqs. [10] [11] [12] and [13] if \(U\) acts on the single particle state \(|a\rangle\) where \(a \in \{0, 1\}\), then
\[
U |a\rangle |e\rangle = A_{a} |0\rangle |e_{a0}\rangle + A_{a1} |1\rangle |e_{a1}\rangle
\]
\[
= \begin{cases} 
A_{00} |0\rangle |e_{00}\rangle, & \text{if } a = 0; \\
A_{11} |1\rangle |e_{11}\rangle, & \text{if } a = 1,
\end{cases}
\]
hence we have
\[
U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} |0a_{1}a_{2} \cdots a_{n}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}
= U_{0} |0\rangle |e_{0}\rangle \otimes U_{1} |a_{1}\rangle |e_{a1}\rangle \otimes \cdots \otimes U_{n} |a_{n}\rangle |e_{an}\rangle
= A_{00} |0\rangle A_{a1} \cdots A_{a1} |a_{1}\rangle |e_{a1}\rangle \otimes \cdots \otimes A_{a1} \cdots A_{a1} |a_{n}\rangle |e_{an}\rangle
\]

(14)

where \(U_{i}(i \in \{0, 1, \ldots, n\})\) denotes the unitary operator acting on the particle \(|a_{i}\rangle\), and \(|e\rangle_{i}\) denotes the ancillary particle entangled on \(|a_{i}\rangle\). Similarly, we have
\[
U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} |1\lambda a_{2} \cdots a_{n}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}
= U_{0} |1\rangle |e_{0}\rangle \otimes U_{1} |\bar{a}_{1}\rangle |e_{\bar{a}1}\rangle \otimes \cdots \otimes U_{n} |\bar{a}_{n}\rangle |e_{\bar{a}n}\rangle
= A_{00} |1\rangle A_{a1} \cdots A_{a1} |\bar{a}_{1}\rangle |e_{\bar{a}1}\rangle \otimes \cdots \otimes A_{a1} \cdots A_{a1} |\bar{a}_{n}\rangle |e_{\bar{a}n}\rangle
\]

(15)

note here that we use \(A_{a1} |\epsilon_{a1}\rangle = \lambda_{a1} |\epsilon_{a1}\rangle\) (see Eq. [13] in above equation. From Eqs. [15] and [16] we have
\[
U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} \left|G(a_{0}^{i}, a_{1}^{i}, \ldots, a_{n}^{i})\right| |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}
= U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} \left|0a_{1}a_{2} \cdots a_{n}\rangle + |1\lambda a_{2} \cdots a_{n}\rangle\right| |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}
= \frac{1}{\sqrt{2}} \left(U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} |0a_{1}a_{2} \cdots a_{n}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}ight)
+ \frac{1}{\sqrt{2}} \left(U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} |1\lambda a_{2} \cdots a_{n}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}\right)
= \frac{1}{\sqrt{2}} \left(A_{00} A_{a1} \cdots A_{a1} |0a_{1}a_{2} \cdots a_{n}\rangle |e_{a1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{an}\right)
+ \frac{1}{\sqrt{2}} \left(A_{00} A_{a1} \cdots A_{a1} |1\lambda a_{2} \cdots a_{n}\rangle |e_{a1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{an}\right)
= A_{00} A_{a1} \cdots A_{a1} \left|G(a_{0}^{i}, a_{1}^{i}, \ldots, a_{n}^{i})\right| |e_{a1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{an}\)
\]

(16)

From the equation, there is no error is introduced iff the ancillary particles and the intercepted particles are in product states. Therefore, the entanglement-measurement attack is invalid to our protocol.

The above conclusion can also be obtained by mathematical induction. From Eqs. [15] [16] and [17] it is only necessary to prove that Eq. [15] holds. The proof process is divided into three steps:

1. When the quantum state is the single particle state \(|a_{i}\rangle\), we have \(U |a_{i}\rangle |e\rangle = A_{a_{i}} |a_{i}\rangle |e_{a_{i}}\rangle\) (see Eq. [14]).

2. Suppose that when the GHZ state is \(|0a_{1}a_{2} \cdots a_{n-1}\rangle\), the following equation holds,
\[
U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n-1} |0a_{1}a_{2} \cdots a_{n-1}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n-1}
= A_{00} A_{a_1} \cdots A_{a_{n-1}} |0a_{1}a_{2} \cdots a_{n-1}\rangle |e_{a_1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{a_{n-1}}
\]

(18)

3. Then, when the state is \(|0a_{1}a_{2} \cdots a_{n}\rangle\), we have
\[
U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n} |0a_{1}a_{2} \cdots a_{n}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n}
= U_{0} \otimes U_{1} \otimes \cdots \otimes U_{n-1} |0a_{1}a_{2} \cdots a_{n-1}\rangle |e_{0}\rangle_{1} \otimes \cdots \otimes |e\rangle_{n-1} \otimes U_{n} |a_{n}\rangle |e_{a_{n}}\rangle
= A_{00} A_{a_1} \cdots A_{a_{n-1}} |0a_{1}a_{2} \cdots a_{n-1}\rangle |e_{a_1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{a_{n-1}} \otimes A_{a_{n}} |a_{n}\rangle |e_{a_{n}}\rangle
= A_{00} A_{a_1} \cdots A_{a_{n}} |0a_{1}a_{2} \cdots a_{n}\rangle |e_{a_1}\rangle_{1} \otimes \cdots \otimes |e\rangle_{a_{n}}
\]

(19)
We now would like to analyze briefly the security in the presence of quantum channel noise. Experiments in recent years show that the quantum bit error rate (QBER) caused by noise is between 1% and 10%, where the quantum channels used in the experiments include optical fibers and free-space [16,23]. However, the QBER caused by intercepting and measuring a decoy photon is 25%. That is, participants can detect the attacks of Eve with the probability of 25% using only one decoy photon. Further, if Eve intercepts and measures l decoy photons, the probability of detecting her attacks is \(1 - (3/4)^l\), which will get closer and closer to 1 with the increase in l. Obviously, the QBER caused by Eve’s interference to decoy photons is much higher than that caused by the noise. Therefore, Eve cannot eavesdrop successfully under the cover of channel noise as long as a threshold is set reasonably in advance according to the noise, which also shows the effectiveness of detecting eavesdropping with decoy photons.

Indeed, security against external attacks is rooted in the security of QKD, hence we would not like to give more discussion. Next, let us analyze insider attacks.

### 3.2 Insider attack

In 2007, Gao et al. [24] first pointed out that malicious participants’ attacks are usually more powerful and deserve more attention. In what follows we consider the insider attack from two aspects. One is that one malicious participant tries to steal the secret data from another. The other is that TP attempts to steal the secret data from one or both participants.

#### 3.2.1 Attacks from TP

Without loss of generality, we assume that TP manages to steal Alice’s secret data. Throughout our protocol, Alice’s secret data \(G_a^i\) is essentially encrypted by \(K_{AB}^i\) and \(K_{AC}^i\), in which the value of \(K_{AB}^i\) is unknown to TP, thus TP can guess \(G_a^i\) with the successful probability of 1/2. In this case, TP can guess Alice’s data \(X\) with the successful probability of \(1/2^{[N/n]}\) (see the second prerequisite of our protocol), where \(1/2^{[N/n]}\) decreases with the increase in \([N/n]\). Therefore, the probability that TP guesses Alice’s secret data can be made as small as \(1/2^{[N/2]}\) and as large as 1/2 by varying the value of \(n\). Note that \(N\) is usually a large number (see the first prerequisite of our protocol), and the larger the value of \(N\), the longer the bit length of its binary representation. Even if the value of \(N\) is small, Alice and Bob can increase it in many confidential ways. For example, they can agree in advance on a secret positive integer \(M\) (or generate it as an additional key in the first step of the protocol), whose value is confidential, and then they can calculate \(N \times M\), or \(N^M\). In this way, just make sure that \([N/n]\) is big enough so as to make \(1/2^{[N/n]}\) small enough. Of course, actual situations should be taken into account in the way to increase the value of \(N\). After all, the larger the value of \(N\), the more quantum resources the protocol consumes.

#### 3.2.2 Attacks from one dishonest participant

Let us assume that Alice is dishonest and tries to steal Bob’s data because of the same role that two participants play. Throughout our protocol, there are no qubits exchanged between Alice and Bob. Therefore, only by intercepting the particles that Bob sends to TP and measuring these particles after Bob tells TP the positions and bases of the decoy photons, Alice can have the chance to steal Bob’s data. In Step 2, Bob decides whether to flip the value of \(G_b^i\) according to the value of \(K_{AB}^i \oplus K_{BC}^i\); the value of \(K_{BC}^i\) is unknown to Alice. In this case, Alice can guess the value of \(G_b^i\) with the probability of 1/2. Similar to the analysis above, it is easy to see that Alice’s probability of guessing Bob’s data \(Y\) is also \(1/2^{[N/n]}\). Additionally, Alice’s attack will of course be detected as an external attacker during eavesdropping checking even if she intercepts Bob’s particles and replaces them with false particles, because at this time she has no idea of the positions and bases of the decoy photons. Therefore, Alice’s attacks will not succeed.

### 4 Discussion

In this section, we will first calculate qubit efficiency, and then make a comparison between our protocol and some other existing ones.

Qubit efficiency is defined as \(\eta = c/t\), where \(c\) denotes the number of compared classical bits, and \(t\) the number of consumed particles, excluding the decoy photons and those consumed in the process of the key generation using QKD [6]. In our protocol, two \((n+1)\)-qubit GHZ states \((n \in N_+\) and \(2 \leq n \leq N\) are used to achieve the comparison of \(n\) bits of classical information (i.e., the comparison of \(G_a^i\) and \(G_b^i\) requires two \((n+1)\)-qubit GHZ states). Therefore, the qubit efficiency of our protocol is \(\frac{n}{2n+2}\). Obviously, \(\frac{n}{2n+2}\) increases with the increase in \(n\), hence we have

\[
\frac{1}{3} \leq \frac{n}{2n+2} \leq \frac{N}{2N+2} < \frac{1}{2}\tag{20}
\]
and

\[
\lim_{N \to +\infty} \frac{N}{2N + 2} = \frac{1}{2},
\]

(21)

which means \(\frac{N}{2N + 2}\) will get closer and closer to 1/2 with the increase in \(N\).

We have shown that by adjusting the value of \(n\), we can determine the probability that TP can guess the participants’ secret data \((1/2^{\lceil N/n \rceil})\), the probability that a malicious participant can guess another’s data, and determine the qubit efficiency \((n/(2n + 2))\). Of course, the value of \(n\) also determines how many qubits that a \((n + 1)\)-qubit GHZ state contains, thus determines the difficulty of preparing the state. Generally, the more qubits a quantum state contains, the more difficult it is to prepare and manipulate the quantum state, with many challenges still remain in the preparation and manipulation of multi-particle entanglement [25–28]. Fortunately, a series of significant progress has been made in the preparation and manipulation of multi-particle GHZ states in recent years. Recent research results show that 18-qubit entangled GHZ states have been successfully prepared experimentally [28].

In Table 2, we make a comparison between our protocol and the existing ones proposed in Refs. [29–41]. In our protocol, the GHZ states are prepared by participants rather than by TP, which to some extent, improves the security and efficiency of the protocol. Indeed, in order to achieve the purpose of private comparison, the participants will not prepare fake quantum states. Nevertheless, in most existing QPC protocols, the quantum states acting as information carriers are prepared by TP, in which case the protocol will face more security risks because the more work TP undertakes in the protocol, the more chance he has to counterfeit. A typical case is that he can prepare fake quantum states to steal the participants’ data, which has proved to be effective in attacking some protocols [42, 43]. In addition, it is known that preparing and measuring devices for quantum states are necessary in QPC, in which the prepared quantum states act as information carriers and measurements are usually used to extract information contained in the quantum states. In our protocol, except for the necessary devices for preparing quantum states, only single-particle measurement technology is used without any additional technology (e.g. entanglement measurements and unitary operations). Therefore, our protocol has advantages over many existing protocols from security and device consumption.

Table 2: Comparison between our protocol and some existing ones

| Reference | [29] | [30] | [31] | [32] | [33] | [34] | [35] | [36] | [37] | [38] | [39] | [40] | [41] | Our protocol |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------------|
| QKD       | ×    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓          |
| Unitary operation | ✓   | ×    | ✓    | ✓    | ×    | ×    | ×    | ×    | ✓    | ×    | ×    | ×    | ×    | ✓          |
| Entanglement swapping | ×   | ✓    | ×    | ×    | ✓    | ×    | ×    | ×    | ✓    | ×    | ×    | ×    | ×    | ×          |
| Entanglement measurement | ×   | ✓    | ×    | ×    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓    | ✓          |
| Qubit efficiency | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2  | 1/2 ≤ η < 1/2 |

5 Conclusion

We have proposed a novel QPC protocol, in which the entanglement correlation of the \((n + 1)\)-qubit GHZ state and bit-flipping play key roles in the comparison of participants’ secret data. We have shown that TP and Eve cannot steal any useful information about the participants’ data, and that Alice and Bob cannot successfully steal each other’s data. Our protocol uses single-particle measurement technology to extract information contained in quantum states, which is easier to implement than entanglement measurements under existing technical conditions. Moreover, the quantum states are prepared by participants rather than by TP, which makes it unnecessary for our protocol to verify the authenticity of the quantum states. What is more, participants can choose an appropriate value for \(n\) according to actual situations. That is, the participants can determine the value of \(n\) based on how many qubits of the GHZ state can be prepared by their own devices. Indeed, the larger the value of \(n\), the higher the qubit efficiency, but this also means the increase of the probability that TP can guess the participants’ data although the probability is very small. Regardless of these, our protocol has some flexibility because of the selectivity that the participants have in the preparation of the GHZ states.

Admittedly, the advantages of our protocol, both from the quantum states used and qubit efficiency, may not be impressive. Nevertheless, we propose a new encryption and decryption method for QPC, which we believe is enlightening for the design of QPC protocols. We also wish that the method can find applications in other branches of QSMC, especially GHZ-based protocols.
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