Fuzzy-SPA method based quantitative risk assessment for tailings pond

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Abstract: Regarding characteristics of tailings pond and the requirements of regulations in China, a quantitative evaluation framework of tailings pond indicators was founded. The index system for tailings pond was divided into four parts, which were management capability, engineering designing, monitoring indexes and environmental impact. Furthermore, a set-pair analysis (SPA) based on fuzzy assessment method (Fuzzy-SPA) was presented for the quantitative risk assessment in this paper. It overcame some defects in traditional risk assessment methods and expanded the scope of current methods. Both qualitative and quantitative indexes can be used in the method, then a comprehensive risk score can be achieved. The proposed model was applied to assess a certain tailings pond in Benxi. The result indicated that this method used was feasible and rational. The work presented in this paper can be as reference through adequate selection of model parameters to the safety assessment work for the tailings ponds.

1. Introduction
Tailings pond must be managed and supervised according to relevant regulations and technical standards to ensure its safety because of its major hazards in the process of mine production. Tailings pond and its risk including environment problems is becoming an important research filed these years. YU et al combined the tailings own attributes on the comprehensive analysis of the tailings and put forward the online automatic detecting and early warning information security management technology and methods for safety monitoring and safety management [1]. M.Rico et al used the regression equation to predict the tailings flow which affected by run-out distance and other factors [2]. Mei used monte-carlo model to calculate the possibility of dam failure to improve the safety assessment technology of tailings dam[3]. Wang et al used the numerical and limit equilibrium method to analyze the tailings leakage and static stability and safety monitoring related problems which should be considered in design phase[4]. Yin et al used the laboratory physical model test and case to analyze the stability of tailings and got the results that tailings dam height should be less than originally planned [5]. Yang et al calculated the process of recovery in the tailings dam using ANSYS software on tailings recovery research and tailing dam stability calculation [6]. According to the two types of hazard theory presented by Professor Chen, the reason of unsafe system is the existent hazard [7]. The safety assessment was playing a more and more important role in ensuring the smooth functioning of the project and facility. Zheng et al developed one certain risk assessment software for the tailings dam foundation failure probability analysis and dam failure consequences assessment [8].To solve the subjectivity problem in the risk assessment, Zhu et al presented an uncertain AHP method and applied...
it in one tailings pond [9]. Li used fuzzy comprehensive assessment method to evaluate the dam break of two tailing pond [10].

To assess the risk level of a complicated system, several methods can be used, such as event tree analysis (ETA), fault tree analysis (FTA), fuzzy methods, and so on. However, these methods have a common defect, which was the probability of basic events were very difficult to obtain. The quantitative result were difficult to achieved without the original quantitative data. In these years, some risk assessment software or methods appeared and developed quickly [11]. However, most of the software focused on inherent risk only and the impact of safety management measures on the risk were ignored. To evaluate the comprehensive risk of a system, the likelihood and the consequence of an accident should be considered together.

For the above reasons, the triangular fuzzy method was applied to risk assessment for tailings pond safety. The weight of index was given based on its possibility and consequence [12]. The subjectivity was minimized by the set-pair analysis (SPA) method, which determined the membership function. Combining fuzzy logic and SPA, a new quantitative risk assessment model was presented to overcome the defects in above methods.

2. Fuzzy-SPA assessment method

2.1. Theory of SPA

The set-pair analysis (SPA) considers both certainties and uncertainties as an integrated certain-uncertain system and depicts the certainty and uncertainty systematically from three aspects as identity, discrepancy and contrary [13]. The basic idea of SPA is to analyze the features of a couple of sets (set-pair) and set up a connection degree of the two sets including identity degree, discrepancy degree and contrary degree under certain circumstances. SPA based researches have been conducted in many fields [13]. The connection degree of the set-pair was defined as follows:

$$\mu = a + bi + cj$$

Where $a$ was called identity degree, $b$ was called discrepancy degree, and $c$ was called contrary degree of the two sets under certain circumstances respectively. The $j$ was the coefficient of the contrary degree, which is specified as -1. The $i$ was the coefficient of the discrepancy degree, which was an uncertain value between -1 and 1.

2.2. Fuzzy-SPA assessment method

Because the fuzzy method has the advantage of handling uncertainty information, the combination of fuzzy method and other assessment methods were widely used to solve the uncertainly problems over the years. However, the determination of membership function was subjective by traditional fuzzy method. To minimize the subjectivity, the SPA method was used to provide membership function. The combination of fuzzy logic and SPA was named Fuzzy-SPA method, which combined the advantages of both methods and can provide more credible results.

The set of standard values of assessment grades and the set of factors’ values were seen as a set-pair. The grades of the assessing values were confirmed by the connection degree between the two sets. If a value was within a grade interval, the value was regarded identical with this grade. And for the adjacent grade, the value was regarded discrepant with it. While for other grades, the value was regarded contrary to them.

For a assessment object which had $n$ grades $Grade_1, Grade_2, \ldots, Grade_n$, and $m$ assessment indexes $p_1, p_2, \ldots, p_m$, the values of the indexes are $v_1, v_2, \ldots, v_m$. The connection degree between the value $v_k$ of the index $p_k$ and the grades was given by Eq.(2).

$$\mu_k = a_{k1}i_1 + a_{k2}i_2 + \cdots + a_{km}i_m$$

Where $a_{kj}$ ($k = 1, \ldots, m, j = 1, \ldots, n$) was the membership between $v_k$ and the $n$ grades. If the value was within the interval of a grade, $a_{kj}=1$. And for the adjacent grade, $a_{kj}\in(-1, 1)$. While for other grades, $a_{kj}=-1$. $i_j$ was the coefficient of $Grade_j$, $u_k$ was the connection degree. When $a_{kj}$ was seen as
membership, $i_j$ was the sign of certain grade, which was not an actual value. For the values of $m$ indexes, the connection degrees can be expressed by Eq. (3).

$$U = \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{pmatrix}$$

(3)

Also, if value $v_k$ was in the grade $Grade_i$ of index $p_k$, $a_{ij}$ can be determined by Eq. (4).

$$a_{ij} = \begin{cases} \frac{1}{2} - \frac{v_k - X_{k(i-1)l} - X_{k(i)l}}{X_{k(i)l} - X_{k(i-1)l}} & (j = i - 1) \\ 1 & (j = i) \\ \frac{1}{2} - \frac{X_{k(i)l} - v_k}{X_{k(i)l} - X_{k(i-1)l}} & (j = i + 1) \\ -1 & (j \geq i + 2 \text{ or } j \leq i - 2, 1 \leq j \leq n) \end{cases}$$

(4)

In Eq. (4), $X_{k(i)l}$ was the upper limit of $Grade_i$ of index $p_k$.

Figure 1 showed the membership $a_{ij}$ of different grades.

![Figure 1. SPA based membership function.](image)

If the weights of the $m$ indexes were $w_1, w_2, \ldots, w_m$, which were denoted by a vector $W$, the total connection degree can be calculated as follows:

$$R = W \times U = (w_1 \ w_2 \ \cdots \ w_m) \times \begin{pmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \end{pmatrix} \times \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{pmatrix}$$

(5)

It can also be denoted as:

$$R = W \times U = (r_1 \ r_2 \ \cdots \ r_n) \times \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{pmatrix}$$

(6)

$$r_k = \sum_{j=1}^{m} w_j a_{ij}(k = 1, 2, \cdots, n)$$

where $r_k$ indicated the total membership degree relating to $Grade_k$. To confirm the final assessment grade, the common fuzzy set method can be used. For example, according to the maximum membership degree rule, if $\max(r_1, r_2, \cdots, r_n) = r_k, k = 1, 2, \cdots, n$, the final assessment grade was $Grade_k$. 

3. Application of the model

3.1. Foundation of the tailings pond risk assessment index system

Factors influencing risk included engineering designing of the tailings pond and its actual situation, population density, etc. As is known, the risk of harm events occurrence is the combination of the possibility and consequence. Based on current regulations and structured division principles, an index system including four subsystems was presented. The four subsystems were respectively management capability, engineering designing, monitoring indexes, environment impact. The index system of risk assessment was shown in table 1.

| Factor                        | Content                                           | Weight | Value of the case |
|-------------------------------|---------------------------------------------------|--------|-------------------|
| Management capability         | Risk management structure                         | 0.114  | 85.4              |
|                               | Risk management philosophy                        | 0.219  | 86.1              |
|                               | Emergency management including plans, facilities, drilling | 0.296  | 63.7              |
|                               | Implementation of safety inspection                | 0.119  | 77.6              |
|                               | Risk analysis and management of hidden danger      | 0.252  | 81.2              |
| Engineering designing         | Sliding ability of tailings particles              | 0.072  | 75.3              |
|                               | The scale of deposited beach(length and height)    | 0.131  | 86.8              |
|                               | The stability of both embankment and starter dam   | 0.131  | 82.4              |
|                               | The height and slope ratio of the tailing dam       | 0.172  | 86.6              |
|                               | Collapsing, crack, marsh and subsidence conditions | 0.271  | 73.2              |
|                               | Flood protection standards and its facilities       | 0.152  | 77.5              |
|                               | Seepage condition including the height of the phreatic line | 0.071  | 80.9              |
| Monitoring indexes            | Monitoring the deformation of slope and dam        | 0.370  | 73.5              |
|                               | Monitoring the seepage conditions                  | 0.243  | 69.2              |
|                               | Monitoring the dry beach                          | 0.156  | 83.2              |
|                               | Monitoring hydrology and meteorology               | 0.039  | 81.3              |
|                               | The on-line monitoring system                      | 0.192  | 40.3              |
| Environment impact            | Site design for the earthquake intensity           | 0.178  | 85.0              |
|                               | Massif condition of beach slope                    | 0.362  | 75.8              |
|                               | Engineering geological conditions                  | 0.102  | 81.3              |
|                               | The distance between residential area and population density | 0.358  | 86.3              |

3.2. The determination of weight

The factor weight can be got by triangular fuzzy theory [12]. For a kind of harm event, its risk was determined by the consequence and the possibility, as is shown by Eq.(7).

\[ RF = RI \times RP \]  

where, \( RF \) represents risk, \( RI \) and \( RP \) respectively represent the consequence and the severity of the harm event.

Furthermore, the language variables for corresponding grades of \( RP \) and \( RI \) and their triangular fuzzy intervals are shown in table 2 and table 3[12]. Some factor only need consider \( RP \) or \( RI \). Some must consider them together.

| Description of the accident occurs | RP language     | Triangular fuzzy interval |
|------------------------------------|-----------------|---------------------------|
| frequently                         | high            | (0.7,0.9,1)               |
| sometimes                          | a little high   | (0.4,0.6,0.8)             |
| occasionally                       | a little low    | (0.2,0.4,0.6)             |
| hardly ever                        | low             | (0,0.1,0.3)               |
Table 3. The triangular fuzzy interval and its description of risk impact

| Description of the consequence | RI language  | Triangular fuzzy interval |
|-------------------------------|--------------|--------------------------|
| serious                       | catastrophic | (0.8,0.9,1)              |
| relatively serious            | serious      | (0.6,0.75,0.9)           |
| moderately                    | medium       | (0.3,0.5,0.7)            |
| little effect                 | little       | (0.1,0.25,0.4)           |
| almost no effect              | negligible   | (0,0.1,0.2)              |

The weight of the assessment factors can be got by Eq.(8) and Eq.(9).

\[
(RF_i) = \frac{\int_0^x xRF_i(x) dx}{\int_0^x RF_i(x) dx}
\]

\[
w_i = \frac{(RF_i)}{\sum_{i=1}^n (RF_i)}
\]

3.3. The determination of the value or interval for each index

In reference of Pan et al [12], four judgment grades were used, which were negligible, low, medium, high. The grade intervals for each index are listed in Table 4. The grades of low and medium can be defined as risk acceptable.

| Classification criteria       | I (95,100) | II (80,95) | III (60,80) | IV (0,60) |
|-------------------------------|------------|------------|-------------|-----------|
| The aggregate score t         |            |            |             |           |

The score interval and its risk classification of each index can be confirmed according to the relevant safety management regulations and technical requirements for tailing pond. For example, the criterion for sliding capability of tailings particles was shown in table 5.

Table 5. The criterion for sliding capability of tailings particles

| Average particle diameter / mm | Risk level |
|--------------------------------|------------|
| >0.50                          | negligible |
| 0.20~0.50                      | acceptable |
| <0.20                          | unacceptable |

The attribute value of each index was given by a expert team. Then the fuzzy judging attribute values of each index given by each expert were combined into one fuzzy judging attribute value \(x_i\). As is shown in the Eq.(10).

\[
x_i = \sum_{k=1}^i (\lambda_k x_i^k) = \lambda_1 x_1^1 \oplus \lambda_2 x_2^1 \oplus \cdots \oplus \lambda_k x_k^1 \oplus \cdots \oplus \lambda_i x_i^1
\]

where \(x_i^k\) means the triangular fuzzy judging attribute value of the \(i\)th index that given by the \(k\)th expert.

Finally, the risk grade of the tailings pond was determined by Fuzzy–SPA assessment method.

4. A case study

To verify the model above, a tailings pond in Benxi was chosen as an example. The physical model can be seen in figure 2.
Figure 2. physical model of the case

Using the model presented in the paper, a safety assessment team scored the tailings pond. The values given by different experts were listed in Table 1. Based on the model, the final assessment grade was “normal pond”.

The results by Fuzzy-SPA assessment method introduced were listed in Table 6.

| Assessment factor of the tailing pond | w          | membership degree               | Risk grade by Fuzzy-SPA | Risk grade by triangular fuzzy |
|-------------------------------------|------------|---------------------------------|-------------------------|-------------------------------|
| Management capability               | 0.245      | (-0.670, 0.308, 0.670, -0.489)  | III                     | III                           |
| Engineering designing               | 0.359      | (-0.509, 0.610, 0.679, -0.744)  | III                     | II                            |
| Monitoring indexes                  | 0.255      | (-0.923, 0.113, 0.669, 0.113)   | III                     | II                            |
| Environment impact                  | 0.141      | (-0.564, 0.848, 0.564, -0.848)  | II                      | II                            |

And the risk grade of the tailing pond can be got by Eq. (8).

\[ R = -0.662i_1 + 0.443i_2 + 0.658i_3 - 0.478i_4 \]

The result was grade III, “Medium, risk acceptable, but need improved” which was same to the result got by the triangular fuzzy method.

5. Conclusions
Based on Fuzzy-SPA theories, a quantitative risk assessment model for tailing pond was founded. The quantitative analysis and comparison of potential risk including risk probability and risk impact was realized. According to system hierarchy analysis, the performance of safety management including risk and emergency capability was depicted by an index system. The Fuzzy-SPA theories have obvious advantages to assess such a complicated system. The original information from different resources were analyzed in assessment phase and the comprehensive risk level of the tailing pond were obtained. The method presented in the paper can change qualitative linguistic variables to quantitative assessment results. Sorting the quantitative results, the defects requiring rectification in the system emerged.

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