Topological Centrality and Its Applications

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Abstract—Recent development of network structure analysis shows that it plays an important role in characterizing complex system of many branches of sciences. Different from previous network centrality measures, this paper proposes the notion of topological centrality (TC) reflecting the topological positions of nodes and edges in general networks, and proposes an approach to calculating the topological centrality. The proposed topological centrality is then used to discover communities and build the backbone network. Experiments and applications on research network show the significance of the proposed approach.

Index Terms—Network structure, Centrality, Community, e-Science

1 INTRODUCTION

The rich get richer phenomenon exists in many complex networks like the World Wide Web. It is known that there are two ways for a node to become richer: connecting to more nodes; and, connecting to more important nodes.

We observe that a node may earn more if it connects to an important node than connects to many but less important nodes, and that both nodes and edges play an important role in forming network centrality.

Existing centrality measures focus on nodes. They cannot explain the topological characteristic of centrality. This paper is to explore a new network centrality called topological centrality.

Various centrality measures are defined in a graph $G = (V, E)$, where $V$ is the vertex set, $E$ is the edge set, $|V| = n$, and $|E| = m$.

The authority and hub reflect in-degree and out-degree characteristics of a node in the Web respectively [1]. The idea of HITS is that a good hub links to many authorities, while a good authority is linked by many good hubs. Nodes with the highest authority or hub in the Web graph act as authority centers and hub centers. The authority and hub of a node are calculated by:

$$a(i) = \sum_{(j,i) \in E} h(j)$$

$$h(j) = \sum_{(i,j) \in E} a(i),$$

where $a(x)$ and $h(x)$ are the authority and hub of node $x \in \{i, j\}$ respectively.

Degree centrality describes the degree information of each node [2] [3]. It is based on the idea that more important nodes are more active, that is, they have more neighbors in the graph. Degree centrality can be used to find the core nodes of a community; however, it only considers the hub characteristic and ignores the authority characteristic. Degree Centrality $C_D(v)$ for a vertex $v$ is calculated as follows:

$$C_D(v) = \frac{\deg(v)}{n - 1}.$$

Calculating degree centrality for all nodes $V$ in a graph takes $O(n^2)$ in a dense adjacency matrix representation of the graph. While in a sparse graph with edges $E$, the time complexity is $O(m)$. Similar to the degree centrality, an approach was proposed to improve the efficiency of information propagation in P2P network based on the in- and out-degrees of nodes [4].

Betweenness centrality describes the frequencies of nodes in the shortest paths between two indirectly connected nodes [2] [5] [6]. It is based on the idea that if more nodes are connected via a node, then the node is more important. Betweenness centrality can be used to find the edges between two communities in a complex network. Betweenness Centrality $C_B(v)$ for vertex $v$ is:

$$C_B(v) = \sum_{\substack{s \neq v \neq t \in V \atop s \neq t}} \frac{\sigma_{st}(v)/\sigma_{st}}{(n - 1)(n - 2)},$$

where $\sigma_{st}$ is the number of shortest geodesic paths from $s$ to $t$, and $\sigma_{st}(v)$ is the number of shortest geodesic paths from $s$ to $t$ that pass through a vertex $v$. The shortest paths between each pair of nodes in a graph can be found by Floyd-Warshall algorithm with time complexity $O(n^3)$ [7], so the time complexity of betweenness centrality is also $O(n^3)$. Betweenness centrality has been used to study community structure of social and biological networks [8].

Closeness centrality describes the efficiency of the information propagation from one node to the other nodes [2] [9] [10]. It is based on the idea that if a node can quickly reach others, then the node is central. Closeness centrality can be regarded as a measure of how long it will take information to spread from a given vertex to other reachable vertices in the network. Closeness
Centrality is defined as the mean geodesic distance (i.e., the shortest path) between a vertex $v$ and all other vertices reachable from $v$:

$$C_e(v) = \frac{n - 1}{\sum_{t \in V \setminus v} d_G(v, t)},$$

where $n \geq 2$ is the size of the network's connected component reachable from $v$. Calculating the closeness centrality for each node in the graph has time complexity $O(n^3)$.

Eigenvector centrality describes the importance of nodes according to the adjacent matrix of a connected graph [11]. It assigns relative scores to all nodes in the network based on the principle that connections to high-scored nodes contribute more to the score of a node than connections to low-scored nodes. PageRank is a variant of the eigenvector centrality measure [12].

Information centrality describes nodes’ influence on the network efficiency of information propagation [13]. The network efficiency is defined by

$$E_G = \frac{\sum_{i \neq j \in G} \epsilon_{ij}}{n(n - 1))} = \frac{1}{n(n - 1))} \sum_{i \neq j \in G} \frac{1}{d_{ij}},$$

where the efficiency $\epsilon_{ij}$ in the communication between two points $i$ and $j$ is equal to the inverse of the shortest path length $d_{ij}$. The information centrality of a vertex $i$ is defined as the relative drop in the network efficiency caused by the removal from $G$ of the edges incident with $v$:

$$C_I(v) = \frac{\Delta E}{E} = \frac{E[G] - E[G'_{v}]}{E},$$

where $G'_{v}$ indicates a network by removing the edges incident with node $v$ from $G$. Information centrality has been used to study the structures of communities in complex networks [14].

## 2 Topological Centrality

### 2.1 Definition

In a dynamic network, the weights of nodes and the weights of edges will influence each other and keep changing. Each time of influence between each pair of nodes is called one time of iteration. If the order of nodes’ weights keeps unchanging after many times of iteration, the network reaches the steady state and the nodes with the highest weights are called topological centers. An undirected graph may have one or more topological centers. The number of topological centers is decided by the graph structure. An undirected network may have one of the following structures.

1. A network with circular structure has $n$ ($n \geq 3$) topological centers as shown in Fig. 1a.
2. A network with symmetric structure has two topological centers as shown in Fig. 1b.
3. Otherwise, the network has a unique topological center as shown in Fig.1c.

In an undirected graph, the length of the shortest path between two nodes in a graph is the geodesic distance between them. Especially, if two nodes are unreachable, then their geodesic distance is $+\infty$. Geodesic distance can be used to find the nearest topological center of a node.

When a network is in the steady state, the topological centrality (TC) of a node is the ratio of its weight to the largest weight of nodes. The topological centers have the largest weight of node 1. The topological centrality of an edge is the ratio of its weight to the largest weight of node.

The TC of a node reflects the geodesic distance from a node to its nearest topological center. The TC of an edge reflects the geodesic distance from the edge to its nearest topological center. The higher is the TC of a node/edge, the closer it is to the nearest topological center.

### 2.2 Calculating Topological Centrality

**Hypothesis 1.** The topological centrality of a node is positively influenced by the topological centrality degrees of its neighbor nodes.

Hypothesis 1 leads to the following characteristics:

1. a node connecting to nodes with higher TC degrees gets higher TC degree; and,
2. a node connecting to more nodes gets higher TC degree.

**Hypothesis 2.** If two nodes of an edge have higher TC degrees, then the edge has higher TC; and, if an edge has higher TC, then its two nodes also have higher TC degrees.

Hypothesis 2 leads to the following characteristics:

1. nodes closer to the topological center have higher TC degrees; and,
2. edges closer to the topological center have higher TC degrees. These characteristics reflect that nodes with higher TC degrees are incident with edges having higher TC degrees.

The two hypotheses can be represented by:

$$\begin{cases} \omega(n) \uparrow = \omega(n) + \sum g(\omega(\text{link}(n, n_i)) \uparrow, \omega(n_i) \uparrow) \\
\omega(l) \uparrow = f(\omega(l_s) \uparrow, \omega(l_t) \uparrow) \end{cases} \tag{1}$$

![Fig. 1. Three types of topological structures. The darker is the node, the higher the topological centrality is. The black nodes are the topological centers. Networks of circular structure have $n$ ($n \geq 3$) topological centers; network of symmetric structure has 2 topological centers; other networks have 1 topological center.](image)
where \( n \) is a node, \( n_i \) are neighbors of \( n \), \( \omega \) \((\text{link}(n, n_i)) \) is the weight of link between \( n \) and \( n_i \); \( l \) is a link, \( l_s \) and \( l_t \) are the source and target nodes of \( l \) respectively; \( f \) and \( g \) are two functions, and \( \triangledown \) means the positive correlative relations.

During the calculation process of TC degree, the weights of nodes and edges will increase after each time of iteration, but the descending order of weights of nodes will converge to the steady state. The weights of nodes can be normalized by dividing the largest weight of nodes. If the normalized weights of nodes converge, the descending order of nodes' weights will keep unchanging, and the edges' weights will also converge. The converged nodes' weights and edges' weights are the TC degrees of nodes and links respectively.

Normalization characteristics:

1. If the normalized weights of nodes converge, then the order of nodes by descending the weights of nodes will also converge. The normalization process does not change the order of weights of nodes. The difference is that the weights of nodes are mapped onto the interval \( (0, 1] \).
2. If the normalized weights of nodes converge, the weights of edges also converge. According to the definition of TC of an edge, the weights of edges are the sum of the weights of its two end nodes. Since the normalized weights of nodes converge, the weights of incident edges will also converge.
3. If the normalized weights of nodes converge, then the TC degrees of edges converge. It is also obvious, because the normalization of weights of edges is just to map the weights of edges onto the interval \( (0, 1] \), and keeps the order of weights of edges.

We propose the following approach to calculating the TC in a connected network. Suppose a connected graph \( G = (V, E) \) with \( n \) (\( n > 1 \)) nodes and \( m \) (\( m \geq n - 1 \)) edges, \( V = v_1, v_2, \ldots, v_n, E = e_1, e_2, \ldots, e_m \) and the corresponding adjacency matrix is \( A \). The element of \( A \) is \( a_{ij} \), and,

\[
a_{ij} = \begin{cases} 1 & \{i, j\} \in E \\ 0 & \{i, j\} \notin E \end{cases}
\]

The following formula implements the iterative calculation of topological centrality of nodes and edges, where \( \text{temp}_{\omega_i} \) and \( \omega_i \) are the weights of \( v_i \) before and after normalization, and \( \text{temp}_{\omega_{e(i,j)}} \) and \( \omega_{e(i,j)} \) are the weights of edge \( e(i, j) \) before and after normalization, and \( t \geq 0 \) is the iteration time.

\[
\begin{align*}
\text{temp}_{\omega_i}^{(t+1)} &= \omega_i^{(t)} + \sum_{j=1}^{n} a_{ij} \omega_{e(i,j)}^{(t)} \\
\text{temp}_{\omega_{e(i,j)}}^{(t+1)} &= \text{temp}_{\omega_i}^{(t+1)} + \text{temp}_{\omega_{e(i,j)}}^{(t+1)}
\end{align*}
\]  

(2)

The following formulas normalize the TC degrees of nodes and links.

\[
\begin{align*}
\omega_i^{(t+1)} &= \frac{\text{temp}_{\omega_i}^{(t+1)}}{\text{Max}_{j=1}^{n}\text{temp}_{\omega_i}^{(t+1)}} \\
\omega_{e(i,j)}^{(t+1)} &= \frac{\text{temp}_{\omega_{e(i,j)}}^{(t+1)}}{\text{Max}_{j=1}^{n}\text{temp}_{\omega_{e(i,j)}}^{(t+1)}}
\end{align*}
\]  

(3)

The iterative calculation terminates, if the following conditions are satisfied:

\[
\begin{align*}
\sum_{i=1}^{n}(\omega_i^{(t+1)} - \omega_i^{(t)})^2 < \epsilon_N \\
\sum_{j=1}^{m}(\omega_{e(j)}^{(t+1)} - \omega_{e(j)}^{(t)})^2 < \epsilon_M
\end{align*}
\]  

(4)

Algorithm 1 calculates the weights of nodes and links iteratively, where \( MAX, \epsilon_N \) and \( \epsilon_M \) control the times of iterative calculation.

The time complexity of Algorithm 1 is \( O(MAX(n + m)) \). At the initializing stage, all the weights of nodes are assigned to 1. If the weights of edges are not given, then all the weights of edges are assigned 1. After the first iteration, the weight of a node in next iteration is the sum of weights of its neighbor nodes and its own weight; then the weights of edges are the sum of two end nodes. The values of weights of nodes become larger comparing to the initial values. The weights of nodes and edges are normalized by dividing the maximum weight of nodes and edges during each time of iteration.

Algorithm 1 has two termination conditions: one is the maximum iteration times \( MAX \); the other is the square deviation threshold of weight difference of nodes \( \epsilon_N \) and the square deviation threshold of weight difference of edges \( \epsilon_M \). After Algorithm 1 stops, the nodes with weights 1 are the topological centers. The weight of a node is topology centrality, and the larger is the weight of node, the closer the node is to the nearest topological center.

Table 1 makes a comparison between the topological centrality and other centrality measures.

| Centrality Measure                  | Time Complexity | About Node or Edge |
|-------------------------------------|-----------------|--------------------|
| degree centrality                   | \( O(n^2) \)    | node               |
| betweenness centrality              | \( O(n^2) \)    | node or edge       |
| closeness centrality                | \( O(n^2) \)    | node               |
| eigenvector centrality              | -                | node               |
| information centrality              | \( O(n^2) \)    | node               |
| topological centrality              | \( O(k(n + m)) \)| node and edge      |

2.3 Experiments

2.3.1 Convergence Experiment

We carry out experiments on several types of network to verify the convergence of the algorithm. Fig. 2 shows the experiment results of iterative TC calculation for nodes and links in different structured networks with different scales: (a) Watts-Strogatz small-world network with \( n = 1000 \) and \( m = 5000 \); (b) ring network with \( n = 1000 \) and \( m = 1000 \); (c) lattice network with \( n = 100 \) and \( m = 180 \); (d) full network with \( n = 30 \) and \( m = 435 \);
Algorithm 1 Calculating topological centrality degrees of nodes and edges

Require: node number \( n \), edge number \( m \), edges like \((\text{linknum}, \text{starNode}, \text{endNode}, \text{weight})\), limited iteration time \( M \times X \), deviation square limit of weight difference of nodes \( \epsilon_N \), deviation square limit of weight difference of links \( \epsilon_M \).

1. \( \text{nodeWeight}[1..n] \leftarrow 1, \text{count} \leftarrow 0, \text{nodeSum} \leftarrow n, \text{edgeSum} \leftarrow m \)
2. \( \text{while} \ (\text{count} < M \times X) \text{ and } ((\text{nodeSum} > \epsilon_N) \text{ or } (\text{edgeSum} > \epsilon_M)) \text{ do} \)
3. \( \text{oldNodeWeight}[1..n] \leftarrow \text{nodeWeight}[1..n] \)
4. \( \text{oldEdgeWeight}[1..m] \leftarrow \text{edgeWeight}[1..m] \)
5. \( \text{nodeWeight}[1..n] \leftarrow \frac{\text{nodeWeight}[1..n] + \sum_{\text{incident edge}} \text{edgeWeight} \times \text{nodeWeight}}{\max(\text{nodeWeight})} \)
6. \( \text{edgeWeight} \leftarrow \frac{\sum_{\text{node edge}} \text{nodeWeight}}{\max(\text{edgeWeight})} \)
7. \( \text{nodeSum} \leftarrow \sum_{i=1}^{n}(\text{nodeWeight}[i] - \text{oldNodeWeight}[i])^2 \)
8. \( \text{edgeSum} \leftarrow \sum_{i=1}^{m}(\text{edgeWeight}[i] - \text{oldEdgeWeight}[i])^2 \)
9. \( \text{count} \leftarrow \text{count} + 1 \)
10. \( \text{end while} \)
11. \( \text{return} \ \text{nodeWeight}[1..n] \text{ and } \text{edgeWeight}[1..m] \)

(d) Erdős-Rényi random graph with \( n = 1000 \), \( p = 0.02 \), and \( m = 10045 \). Experiment results show that the TC degrees of node and links can converge after many times of iteration, which is related to \( n \), \( m \), \( \epsilon_N \) and \( \epsilon_M \).

2.3.2 Comparison of Centrality Measures

Different centrality measures such as degree centrality, betweenness centrality, closeness centrality and information centrality are compared in [15]. Here we add two extra centrality measures: one is the PageRank of node as an instance of eigenvector centrality, the other is the topological centrality we proposed. The comparison is based on Fig. 3 which is a tree with 16 vertices. Table 2 shows different centrality degrees of vertices in Fig. 3. The experiment results show the following characteristics:

1. Degree centrality is a local centrality, and it only records the degrees of nodes without any global information. Nodes 1, 2, and 3 have degree 5, nodes 7 and 12 have the degree 2, and the other nodes have degree 1. Degree centrality is normalized by the number of edges 15.

2. Closeness centrality has similar result as information centrality. The difference is that the orders of nodes \{1, 3\} and \{7, 12\} are different. Information centrality degrees of vertex 1 and 3 are larger than 7 and 12. Because information centrality concentrates on the network efficiency. The influence on network efficiency by removing 1 and 3 is larger than that by removing 7 and 12.

3. PageRank result is far from other measures. Nodes 1 and 3 are two centers in PageRank, and node 2 have lower PageRank than nodes 1 and 3, because the authority of nodes 7 and 12 are divided into two parts, while nodes 1 and 3 have four neighbors which contributes all of their authority values to nodes 1 and 3 respectively. Nodes 7 and 12 have higher rank values than nodes 9, 10 and 11, because they have more neighbors.

4. Betweenness centrality reflects the frequencies of nodes occurring in the shortest paths between indirectly connected node pairs. However, betweenness centrality has the worst resolution of nodes. Node 2 has the highest betweenness centrality, nodes 1, 3, 7, and 12 have higher betweenness centrality, and the others have the same betweenness centrality 0.

5. Topological centrality combines the degree information and neighbor weights information. It has the characteristics of degree centrality and PageRank. Node 2 is the topological center of the graph. Nodes 7 and 12 have higher TC degrees than nodes 9, 10 and 11 because they have extra neighbors. Nodes 1 and 3 follow nodes 9, 10 and 11, and then the left vertices. The order of node TC degrees confirms the geodesic distance between nodes and the topological centers correctly.

Fig. 3. A simple case (a tree with 16 nodes) for the comparison of centrality measures.

2.3.3 Topological Centrality Distributions on Research Network

Here DBLP dataset is used to study the structure and discover communities in heterogeneous networks. It contains part of metadata of papers provided by DBLP in XML formats. The number of papers is 664, 188, and the number of citation relations is 79, 128. The heterogeneous research network is based on the DBLP data set. The resource types are papers, researchers and conferences. The semantic links are authorOf between researcher and
Fig. 2. Topological centrality convergence experiments \( MAX = 100, \epsilon_N = \epsilon_M = 0.001 \): the left column lists networks of several structures; the middle column lists the node convergence records (x-axis is iteration times, and y-axis is normalized weights of nodes); and, the right column lists the link convergence records (x-axis is iteration times, and y-axis is normalized weights of links). (a) Watts-Strogatz small-world network with \( n = 1000 \) and \( m = 5000 \), and iteration time is 14; (b) ring network with \( n = 1000 \) and \( m = 1000 \), and iteration time is 2; (c) lattice network with \( n = 100 \) and \( m = 180 \), and iteration time is 17; (d) full network with \( n = 30 \) and \( m = 435 \), and iteration time is 2; (e) Edörs-Rényi random graph with \( n = 1000, p = 0.02, m = 10045 \), and iteration time is 17.
paper, coauthor between researchers, publishedIn between paper and conference/journal, and cite between papers.

The research network contains 1,084,198 semantic nodes and 2,153,385 semantic links. The iteration time limits are $MAX = 40$ and $\epsilon_M = \epsilon_N = 200$. The distribution of TC degrees of nodes is shown in Fig. 4. It shows that nodes with lower TC degree contain more resources than those with higher TC degree.

![Fig. 4. Topological centrality distributions.](image)

### 3 APPLICATION: DISCOVERING RESEARCH COMMUNITIES

#### 3.1 Research Community

Research communities are formed by relations among researchers, papers, projects, and research activities. Differences between research communities and graph-based communities are as follows.

1. Research communities are dynamically formed by research activities such as applying (e.g., funding and position), cooperating, publishing, and citing. Communities in general complex networks are viewed from connections (nodes within a community are linked more densely than nodes cross communities).

2. Research communities contain multiple types of nodes (researchers and papers can play different roles in research activities as discussed in [15]) and relations (e.g., coauthor relation and citation relation). There are no differences of nodes and edges in graph-based communities.

Among existing centrality measures, only the PageRank considers the influences between neighbor nodes, and the authority of a node is divided by its neighbors. However, PageRank does not reflect different influences of edges, that is, all the weights of edges are 1. In research network, collaborations between authority researchers are more important, and citations between authority papers are more important.

Topological centrality can well distinguish roles of different nodes in research network. (1) Nodes in a network elect the core nodes by a voting-like mechanism: a node connecting to more nodes is more probable to be the local core nodes. After a certain times of iterations, the local core nodes and the global topological centers are elected. The topological centers are the nodes connecting to the most core nodes with higher TC degrees. (2) Edges may play different roles on the mutual influence between the TC degrees of nodes. This confirms the phenomena of research communities: a researcher cooperating with authority researchers will be closer to the centers of a research community; a paper citing (citing may not be true) or is cited by authority papers will be more possible to be closer to the core papers on a research topic.

#### 3.2 Roles of Nodes

Nodes can play different roles according to topological positions in communities: core node, margin node, bridge node and mediated node.

1. Core nodes are usually hub or authority in the community;
2. Margin nodes belong to one community, and they have few connections to other nodes in the community;
3. Bridge nodes connect to two or more communities, and they usually have equal number of connections to two or more communities; and,
4. Other nodes except the core nodes, margin nodes and bridge nodes are mediated nodes.

The proposed topological centrality can be used to distinguish roles of nodes. For example, Fig. 5 contains three communities: $C_1 = \{1, 4, 5, 6, 7, 8\}$, $C_2 = \{2, 7, 9, 11, 12\}$ and $C_3 = \{3, 12, 13, 14, 15, 16\}$. Node 1, 2 and 3 are the core nodes of $C_1$, $C_2$ and $C_3$ respectively; Nodes 7 and 12 are bridge nodes; nodes 4, 5, 6 and 8 are margin nodes of $C_1$; nodes 9, 10 and 11 are margin nodes of $C_2$; and, nodes 13, 14, 15 and 16 are margin nodes of $C_3$.

Nodes can be classified by TC degrees.
Fig. 5. Distinguishing roles of nodes with topological centrality degrees.

1. If the TC degree of a node is larger than that of most of its neighbors, then the node is a core node;
2. If the TC degree of a node is no larger than the TC degrees of all of its neighbors, then the node is a margin node;
3. If the number of neighbors with lower TC degrees equals to the number of neighbors with higher TC degrees, then the node is a bridge node;
4. Otherwise, the node is a mediated node.

Let $\alpha = \#L(n)/\#N(n)$ and $\beta = \#H(n)/\#N(n)$, where $n$ is a node, $\#L(n)$ is the number of neighbors of $n$ with TC degrees lower than $n$, $\#H(n)$ is the number of neighbors of $n$ with TC degrees higher than $n$, and $\#N(n)$ is the neighbors of $n$, then role of $n$ is distinguished by

$$
\text{role}(n) = \begin{cases} 
\text{core node} & \alpha > \text{threshold(core)} \\
\text{margin node} & \alpha = 0 \\
\text{bridge node} & \alpha = \beta \\
\text{mediated node} & \text{otherwise}
\end{cases}
$$

Where $\text{threshold(core)} \in (0.5, 1]$ controls the number of core nodes.

A node is a core node because it connects to more nodes or more important nodes. A node is core node or not is decided by whether it has larger TC degrees than its neighbors. However, the topological centers of a connected network may be exceptions. In Fig. 5, node 2 is both the topological center and a core node, but the ellipse node in Fig. 6 is the topological center, and it is not a core node but a bridge node, although it has higher TC degree than all of its neighbors. So it is significant to distinguish the roles of topological centers. If the neighbors of a topological center are all core nodes, then, the topological center is a bridge node; else the topological center is a core node.

Researchers and papers may play such roles as source, authority, bee, hub and novice [15]. The source, authority, and hub may be core nodes; bee nodes are often bridge nodes; and the novice may be the margin nodes or bridge nodes.

In research network, a research group’s leader usually has more publications and cooperators. Correspondingly, they have more coauthor relations connecting to other researchers in the coauthor network. If each research group is regarded as a community, the research group’s leaders are the core nodes. The fresh students have few publications and cooperators, so they are the margin nodes in coauthor network. Visiting researchers and newly employed researchers are bridge nodes, because they have cooperators in different research communities. After the core nodes, the margin nodes and bridge nodes are distinguished, the left nodes are mediated nodes. Usually, mediated nodes only belong to one community.

In citation network, core nodes are the authority or hub papers having more citations than others; the margin nodes are the novice papers or newly published papers; and the bridge nodes connect two or more paper clusters. Each paper cluster may belong to a specific research topic or discipline.

Funding decision-making and research promotion need to evaluate researchers and their papers. Topological centrality can help distinguish the roles of researchers and papers, and the roles can be used to evaluate researchers and papers. TC degrees in the coauthor network help evaluate researchers, while TC degrees in citation network help evaluate papers.

In research network, roles of nodes will change year by year. In the coauthor network, a novice researcher may become an authority, a hub or even a bridge. With more papers published, the TC degree of a node in a coauthor network will become higher than its neighbors, and then the researcher become an authority or hub. Cooperating with researchers in different research groups or even different communities, a researcher becomes a bridge.

3.3 Discovering Communities by Roles

Tree in Fig. 3 can be a coauthor network or a citation network with directions of edges ignored. General community discovery algorithms like GN algorithm cannot discover its communities, because the betweenness of each edge is the same, and there is no way to choose the proper edge for deletion. However, nodes in the coauthor networks and citation networks play different roles, and communities can be discovered according to the roles of nodes.

The roles of nodes can be used to discover communities. One way is to find the core nodes, and then assign non-core nodes to the proper core nodes to form communities. Algorithm 2 discovers communities by finding core nodes for each non-core node.
The time complexity of algorithm 2 is $O(n(n+m))$. The number of core nodes can be controlled by setting the threshold of $|L(n)|/|A(n)|$. If there are more than one candidate core nodes, then the node should be classified into different communities, and the bridge nodes are often classified into several communities at the same time.

This way can discover communities globally in a network. If the number of communities is too many, the closely connected communities can be merged into larger communities. Closely connected communities may share many nodes and links, or there are many external connections between them. Suppose the number of communities is $k$, Algorithm 3 merges communities.

Another way is to find the core nodes first, and then expand from a node to form local communities. According to role of nodes, the community expansion needs to consider the following cases.

1. Forming local community according to core node. Algorithm 4 is for discovering local communities from a core node. A community may have more than one core node. If two communities share many common nodes and links, then the two communities can be merged into a larger community. This way can find the research groups in a coauthor network, and can find the specific topic related paper clusters in the citation network.

2. Form local community according to non-core node. To find local communities from a non-core node, it is necessary to find the core nodes connected to the node. Before finding communities of a non-core node, all the core nodes in the network should be found first. Then expand the local communities from the nearest core nodes connected to the non-core node respectively.

```latex
\begin{algorithm}
\caption{Merging communities}
\begin{algorithmic}
\Require the number of communities $k$;
\State Step 1. If the number of communities is less than $k$, then goto Step 4.
\State Step 2. Calculate the Jaccard similarity of node sets of each community pair. Suppose $A$ and $B$ are two communities, Jaccard similarity of $A$ and $B$ is calculated by
\[
\text{Jaccard}(A, B) = \frac{|A \cap B|}{|A \cup B|}.
\]
If all the Jaccard similarities of community pairs equal to 0, then goto Step 3; else, find the community pairs have the largest Jaccard similarity, and merge them into a larger community respectively. Goto Step 1.
\State Step 3. Count the external links between community pairs. An external link has two end nodes in two different communities respectively. If all the numbers of external link set equal to 0, then goto Step 4; else, find the community pairs have the maximum external links, and merge them into a larger community respectively. Goto Step 1.
\State Step 4. Stop merging communities.
\end{algorithmic}
\end{algorithm}
```

3. Finding local community of a set of nodes. Given a set of nodes, the local community can be found as follows.
   a) For each node, find the core nodes connected to it until the topological center is found; all the core nodes are added to $coreSet$.
   b) Building the subgraph containing these nodes and nodes in $coreSet$; and,
   c) Expanding the local community from the nodes in $coreSet$.

```latex
\begin{algorithm}
\caption{Expanding community from a core node}
\begin{algorithmic}
\Require A core node $c$ and a connected network $G$;
\State $\text{nodeQueue} \leftarrow \{c\}$, $\text{nodeSet} \leftarrow \{c\}$, $\text{linkSet} \leftarrow \{}$;
\While {$\text{nodeQueue} \neq \{\}$}
\State Fetch a node $x$ from $\text{nodeQueue}$;
\For {$y$ is the neighbor node of $x$}
\State Distinguish the role of $y$;
\If {$y \notin \text{nodeSet}$ and ($y$ is not a core node) and ($\text{nodeWeight}(y) < \text{nodeWeight}(x)$))}
\State $\text{nodeQueue} \leftarrow \text{nodeQueue} \cup y$;
\State $\text{nodeSet} \leftarrow \text{nodeSet} \cup y$;
\State $\text{linkSet} \leftarrow \text{linkSet} \cup \text{link}(x,y)$;
\EndIf
\EndFor
\EndWhile
\State \text{return } \text{linkSet}.
\end{algorithmic}
\end{algorithm}
```

Fig. 7 shows a segment of network with TC degrees of nodes. We can find a local community from a core node, a non-core node, and a set of nodes as follows.
1. Finding local community of core node $B$. The process is shown as Table 3.

**TABLE 3**

| Step | Node | nodeQueue | nodeSet | Expanded |
|------|------|-----------|---------|----------|
| 0    | B    | B         | B       | C, D, E  |
| 1    | C    | D, E      | B, C    |          |
| 2    | D    | E         | B, C, D | F, G, H  |
| 3    | E    | F, G, H   | B, C, D | E        |
| 4    | F    | G, H, I   | B, C, D | E, F     |
| 5    | G    | H, I      | B, C, D | F        |
| 6    | H    | I         | B, C, D | F, G, H  |
| 7    | I    | J         | B, C, D | E, F     |
| 8    | J    | B, C, D   | E, F, G | H, I     |

2. Finding local community of non-core node $F$ is to find the nearest core node $D$, then find the local community from $D$. The expansion process is shown in Table 4.

**TABLE 4**

| Step | Node | nodeQueue | nodeSet | Expanded |
|------|------|-----------|---------|----------|
| 0    | D    | D         | D, F    | G, H     |
| 1    | G    | H         | D, F    | G        |
| 2    | H    |           | D, F    | G        |

3. Finding local community of a node set $\{D, I, J\}$. $D$ is a core node, while $I$ and $J$ are two non-core nodes. If $D$ is the core node of the community containing $I$ and $J$, then $\{D, I, J\}$ forms the local community. However, $D$ is not the core node of the community containing $I$ and $J$. The possible core nodes of the community containing $D$ are $\{D, B, A\}$; the possible core nodes of the community containing $I$ and $J$ are the same, that is, $\{E, B, A\}$. Then, we can construct the subgraph containing node $D$, $I$ and $J$ and their possible core nodes $D, E, B$ and $A$ as shown in Fig. 8.

From the subgraph, we know that $B$ the nearest core node of the community containing $D, I$ and $J$. Then, we can expand from $B$ to find the local community containing node $D, I$ and $J$ as mentioned in case (1).

In research network, this way can find research team members of a researcher in a coauthor network and find topic-related papers of a paper in a citation network.

Given a set of papers, the coauthor relations form the coauthor network, and the citation relations form the citation network. After the TC degrees are calculated, the research groups can be discovered, and the papers can be clustered by citation relations. Researchers in the same communities may share the similar research interests, while papers in the same clusters are topic related. Topic-related papers can be recommended to researchers having similar research interests. Global communities show research groups and research topics in the paper set, while the local community expansion way help recommend papers in a large paper set to appropriate readers.

When making a funding decision, it is necessary to evaluate the status of a research group, cooperators, and publications. The discovered communities in coauthor network show the research groups of a research area, while the discovered communities in citation network show paper clusters in the research area. And, the roles of the researcher and his/her publications can be distinguished by TC degrees.

4 APPLICATION: DISCOVERING BACKBONE IN RESEARCH NETWORK

Given a set of research papers, research networks such as coauthor networks and citation networks can be constructed. Metadata of papers in computer science are often stored in Bibtext or XML files provided by online digital libraries such as Google Scholar, ACM Portal (http://portal.acm.org), IEEE digital library (http://ieeexplore.ieee.org), DBLP (http://www.informatik.uni-trier.de/~ley/db/) and Citeseer (http://citeseer.ist.psu.edu) etc.

4.1 Structures of Research Network

Researchers and the coauthor relation form the coauthor network. Coauthors of a paper formulate the motif [16]
of research network. A coauthor relation from A to B means that A and B are coauthors of the same paper, and A is before B in the author list.

Fig. 9 shows the structure of the coauthor network. With the directions of coauthor relations ignored, each motif describes the cooperation between authors of a paper: a loop for the sole author, an edge between two authors, a triangle for three authors, and a complete graph for \( n \) \((n > 3)\) authors. Coauthor network has three layers from local view to the global view: motif layer, module layer and global layer. Nodes’ degrees in coauthor network reflect the active degrees of researchers. The in-links reflect the hub characteristic, while out-links reflect authority.

![Global View](image1)

**Fig. 9.** Structure of coauthor network from local view to the global view: (1) the bottom layer contains the motifs; (2) the middle layer contains the modules combing one or more motifs; (3) the top layer contains the networks of modules.

Our first dataset collects papers of the International Semantic Web Conference (ISWC) from 2002 to 2007. The number of researchers and papers are 935 and 401 respectively. The number of coauthor relations is 2286. The number of citation relationship is 236, and citation relations are considered between the paper pairs both in ISWC. The number of authorOf relations is 1362. Fig. 12 shows the node TC degrees of the largest module of coauthor networks with a circular layout. The central nodes have higher TC degrees, and the topological nodes have the highest centrality. From a topological center to the margins, the TC degrees reduce to 0 step by step. If the number of nodes are very huge, the TC degrees are very small, and function \( \log() \) maps the TC from interval \((0, 1]\) to \((-21, 0]\), and the order of node TC keeps unchanging.

![Module layer](image2)

**Fig. 10.** Coauthor networks of ISWC data set: 147 modules, 935 researchers and 2286 coauthor relations.

Fig. 11 shows the modules in coauthor network of ISWC dataset. It contains 147 modules, 935 researchers and 2286 coauthor relations. Fig. 11 shows the largest module of Fig. 10. It contains 370 researchers and 1227 coauthor relations.

The number of coauthor relations between two researchers reflects the frequency of their cooperation. Node degrees in coauthor network reflect the active degrees of researchers. The in-links reflect the hub characteristic, while out-links reflect authority.

The density of a module is reflected by the frequency of cooperation between researchers. The average cooperation active degree between each pair of researchers, called cooperation density, can be used to assess the active degree of a research community. Cooperation density is the number of coauthor relations dividing the number of researchers.

**Theorem 1.** A module \( M \) of coauthor network has \( n \) researchers, the lower bound and upper bound of module density are within the range \([\frac{(n - 1)}{n}, n - 1]\).

**Proof.** Suppose \( M \) is a connected digraph with \( n \) nodes. The lower bound of density: the number of edges is \( n - 1 \) at least, otherwise there will be some isolated researchers. So the lower bound density of \( M \) is \( \frac{(n-1)}{n} \). The upper bound of density: if there are at most one directed edge between two nodes, then the number of
edges in $M$ is $n(n-1)$ at most. So the upper bound density of $M$ is $n-1$. Therefore, the lower bound and upper bound of module density of module $M$ with $n$ nodes are within the range $[(n-1)/n, n-1]$. □

Citation network is a directed acyclic graph (DAG). Each paper has the fixed publishing time, and papers can only cite the papers already published, so there are no cycles in the citation network. Citation is direction sensitive, and it implies the time sequential relationship between two papers. Fig. 13a shows a module of citation network. Papers in the same module are topic related. Citation relations show the relevance between research papers, and paper communities can be discovered by citation relations. Citations in the community show the relevance between papers, while citations between paper communities show the relevance of research topics.

Fig. 13b shows the modules in citation network of ISWC dataset. It contains 36 modules, and the largest module contains 142 papers and 165 citation relations. All the citation relations are between papers published in ISWC. The connectivity density is less than the connectivity density of coauthor network. The citation density of a module reflects the relevance between the papers. The citation density is the number of citations dividing the number of papers.

4.2 Topological Centrality based Backbone Network

In a network, after roles of nodes are distinguished by the node TC degrees, core nodes and edges among them form a subgraph, called backbone network. The end nodes of edges in the backbone network are both core nodes. The backbone network consists of core nodes. It is useful for visualization and browsing, and can play the following roles in scientific research:

1. It helps display the research network of different levels. Each community can be represented by the core nodes in the backbone network. When a core node is focused, the detailed information of its local community can be browsed.

2. It shows the important researchers in a coauthor network. When a research community or research group is mentioned, the leaders of the community or the head of the research group are well known. Fig. 14 shows the backbone network of the largest module of the coauthor networks of ISWC dataset. The threshold of the core nodes is 0.5, and the threshold of the margin nodes is 0. It contains all of the core nodes and the coauthor relations among them. Most of the core nodes are connected, and this verifies the “rich club” phenomenon [17]: richer nodes are more possibly connected with other richer nodes. Some core nodes formulate the connected components alone, because the bridge nodes between them are non-core nodes.

3. Backbone network of coauthor network can be used to propagate information. Coauthor network
is a kind of social network. Core nodes are important during the information propagation because they have more impact in their communities. Suppose an invitation of PC members needs to be sent, the researchers in the backbone network should take the priority.

4. Papers formulate communities via the citation relations, and papers in a community share the same or relevant research topics. Core nodes are often important papers citing or are cited by more important papers. The backbone network of citation network helps find the development and history of a research area or a research topic. Core nodes and its neighbors reflect the main achievements at different research stages.

5. Paper publication venue network contains conferences and journals. Other research resources such as researchers, papers and publishers connect conferences and journals into a connected network. To find the citations among conferences and journals, the sub-network containing conferences, journals and papers can be built. If a super node represents the conference or journal containing papers, then citation relations in the super nodes and between different super nodes can be counted. The number of external citations reflects the relevance of conferences and journals.

![Fig. 14. Backbone network of the largest module of coauthor network of ISWC Dataset from 2002 to 2007.](image)

Similarly, the relevance of publishers’ businesses, projects, and institutions can be analyzed. The relevance of publishers is reflected by the relevance between books and papers published by them. The relevance among projects is reflected by the cooperation between researchers taking part in the projects and citations between papers supported by the projects. The relevance between institutions can also be reflected by the relevance between researchers and papers.

4.3 Evolution of Backbone Networks

Backbone networks can be used to study the development of scientific research. Backbone networks sorted by years reflect the evolvement of research networks. Similarly, the evolvement of backbone networks in citation network, paper venue network, and institution networks etc can be studied.

Fig. 15 shows the evolution of coauthor network of ISWC from 2002 to 2008. The coauthor networks are accumulated year by year, that is, the coauthor network of year \( n \) (2002 ≤ \( n \) ≤ 2008) contains the coauthor relations from year 2002 to year \( n \).

The evolvement of coauthor network reflects the history of ISWC. More and more researchers have taken part in the conference, while the nodes and links in backbone networks are also changing. The following characteristics in the evolvement of coauthor networks can be discovered:

1. New researchers in the coauthor network often cooperate with the researchers that have published papers in ISWC conference, because the scales of modules in coauthor networks become larger year by year.

2. Scientific researchers are tending to cooperate with others. The evolution graph shows that the isolated nodes enter the connected components step by step.

3. Core researchers are tending to cooperate with each other. The number of researchers in the largest modules of backbone networks becoming larger and larger. This reflects the “rich club” phenomenon [17] in scientific research.

4. Core researchers are active locally, and they have more cooperators than their neighbors. The roles of researchers in coauthor network are also changing: new researchers may become core researchers, while core researchers may become middle nodes or margin nodes.

5. The topological centers of the largest module are changing. The topological centers emerge through a voting-like mechanism. Table 5 shows the topological centers.

### TABLE 5

| Year | #Researcher | #Cooperation | Topological Center |
|------|-------------|--------------|--------------------|
| 2002 | 99          | 174          | Katia P. Sycara    |
| 2003 | 372         | 510          | Katia P. Sycara    |
| 2004 | 393         | 877          | Steffen Staab      |
| 2005 | 570         | 1310         | Steffen Staab      |
| 2006 | 733         | 1872         | Guus Schreiber     |
| 2007 | 897         | 2290         | Guus Schreiber     |
| 2008 | 1024        | 2647         | Guus Schreiber     |

5 Discussions

5.1 About the Topological Centrality

The TC degree of a node reflects the geodesic distance to the nearest topological center in the network. The value of TC degree has no definite explanation, but
Fig. 15. Evolvement of coauthor network of ISWC from 2002 to 2008: each row shows the coauthor network and its backbone network; the left column shows the coauthor network, while the right column shows the backbone network.

TC degrees have close relation with the authority of nodes. Authoritative nodes have higher TC degrees than its neighbors. The authority of a node reflects the importance of a node in information propagation. The TC degrees are explainable in communities. Core nodes have higher TC degrees than their neighbors. Isolated resources have less influence in the global society.

Backbone networks can help study relations between resources of different types. Backbone network of heterogeneous research networks connects important resources in a research topic and important resources may be researchers, papers, conferences, journals, institutions and publishers etc. This helps find and recommend information. Furthermore, related information can be displayed by an interactive visualization based browser.

In general complex networks, edges have no semantics. While in semantics-rich networks, edges have semantic relations. Weights of nodes are affected by their neighbors, and different relations have different effects. So it is necessary to consider the influences of relations on the topological centrality calculation. Relations can be assigned with different weights and participate the iterative calculation as shown in Eq. (5), where \( r \) is the relation of link \( e(i, j) \), \( \omega_r \) is the weight of \( r \) that affects the calculation of TC in each iteration:

\[
\begin{cases}
    \text{temp} \omega_i^{(t+1)} = \omega_i^{(t)} + \sum_{j=1}^{n} a_{ij} \omega_r \omega_{e(i,j)}^{(t)} \\
    \text{temp} \omega_{e(i,j)}^{(t+1)} = \text{temp} \omega_{i}^{(t+1)} + \text{temp} \omega_{j}^{(t+1)}
\end{cases}
\]

(5)

Where \( r \) is the relation of link \( e(i, j) \), \( \omega_r \) is the weight of \( r \), which affects the calculation of TC in each iteration.

An important characteristic is that the original topological centers may change when we merge two networks into one by certain links and recalculate the topological centers in the new network. For example, if we merge the coauthor network with the citation network by the \texttt{authorOf} semantic links, the topological centers of the new network may not be simply the sum of the topological centers in the coauthor network and those in the citation network. Recalculation of topological centers can synthesize more relations, so this can more accurately evaluate nodes. For example, authors can be evaluated by more factors (e.g., number of publications, number of co-authors, number of citations) in the new network than in the old networks. If applications require to keep the old topological centers in the new network and avoid recalculation, we can adopt the following strategy: find the relations (e.g., \texttt{authorOf}) between the old topological centers and then compose the corresponding old topological centers to form new topological centers. Such an integrated topological centers can provide semantic relevant information services (e.g., the authority author and his/her high impact papers can be obtained at the same time) for applications in large network.
5.2 Related Works

General community discovery approaches are based on the connections between vertices in a network. A fast community discovery algorithm in very large network was proposed with approximate linear time complexity \(O(n \log^2 n)\), where \(n\) is the number of nodes [18]. The general methods like GN algorithm can be used to discover communities in weighted networks by mapping them onto unweighted networks [19].

Research and learning resources form a network, and the connections are the relations among resources. Different from the communities in general complex networks, semantic communities in the relational network were discovered according to the roles of relations during reasoning on relations [20].

Many works are on the collaboration networks and citation networks of scientific research. Most of them focus on the characteristics of collaboration networks. For the structure of social science collaboration network, disciplinary cohesion from 1963 to 1999 was studied [21]. The structure of scientific collaboration networks including the shortest paths, weighted networks, and centrality was studied [22] [23] [24]. Coauthor relations were used to study the collaborations between researchers especially the mathematician, and the distribution of relations between papers of Mathematical Review against the number of authors was studied [25] [26]. Relations between researchers were analyzed in Erdős collaboration graph, and the shortest path lengths between researchers were studied [27].

Evolutions of the social networks of scientific collaborations in mathematics and neuro-science were studied [28]. The research result shows that the social network of collaboration network is scale-free; and, the node separation decreases with the increase of connections.

Social network in academic research can be extracted from the webpages and paper metadata provided by the online databases [29]; furthermore, relations among researchers are mined in academic social networks [30]. Social structure in scientific research was studied based on the citations [31].

Citation relations between scientific papers, and the citation distribution of papers was studied [32] [33] [34], and shows that some papers are not cited at all, most papers are cited once, while a little part of papers covers the references of most papers in a research area.

Resources in research networks are ranked in Object level. Research resources were ranked by popRank approach considering the mutual influences between relevant resources [35]. Object based ranking approach can help search and recommend different resources such as papers, conferences, journals and researchers etc.

Researchers and papers are often ranked in coauthor network and citation network respectively. A co-ranking framework of researchers and papers was proposed, in which researchers and papers were ranked in a heterogeneous network combining the coauthor network and citation network by coauthor relations [36].

Our approach is different from the existing approaches in the following aspects:

1. We distinguish the roles of nodes by topological centrality, and then discover the communities by roles of nodes. Global communities and local communities are discovered based on the roles of nodes. So our approach is based on role rather than only on the connections. Although the topological centrality degrees of nodes and edges are calculated considering connections between nodes, the topological centrality degrees of neighbor nodes have influences on each other at the same time. The role based community discovery approach is fit for the research networks, and can discover communities in tree-like networks that are hard to discover by general community discovery approaches such as GN algorithm.

2. We have built the backbone networks for coauthor networks and citation networks, and the evolution characteristics of backbone networks have been studied. PageRank algorithm can also find the local core nodes, but it has no way to connect most of the core nodes into a backbone network, because it is hard to choose the connecting nodes between the core nodes by the PageRank values. While topological centrality degrees of nodes can choose the core nodes and connect them into a connected backbone network as more as possible, because the core nodes include the community central nodes and important nodes connecting different communities. The backbone network construction approach is also based on the topological centrality. The approach can be applied not only in the research networks with single resource type but also those with multiple resource types.

6 Conclusion

This paper first proposes the notion of topological centrality and the calculation approach to reflect the topological positions of nodes and edges in a network, and then studies its applications in discovering communities and building the backbone network in scientific research networks. Research communities can be discovered according to the roles of nodes distinguished by topological centrality degrees. We also propose an approach to building the backbone network by using the topological centrality. Experiments on real research network and simulation networks show the feasibility and effective of our approaches.

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