Accuracy based on simply* alpha open set in rough set and topological space

M. A. El Safty1 · M. El Sayed2 · S. A. Alblowi3

Accepted: 2 June 2021 / Published online: 12 July 2021
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2021

Abstract
Real-world applications now deal with enormous amounts of data, especially in the area of high dimensional features. In the present study, we provide an approach for the rough set which is simply* alpha open set (briefly, $M^*_a$ open set). This approach was used to introduce a new concept of separation axioms, from which fundamental properties and theorems of preservation were studied. The relationship between basic properties and preservation theorems was also discussed. New near continuous function definitions have also been developed and their characteristics have been discussed. Through an application presented in the work, the relationship between the simply* alpha open set and near continuous function was justified. The simply* alpha open set was studied by the rough set and accordingly new accuracies were obtained. Moreover, an accurate proposal was examined, which competes with that of the methods of Yao and Pawlak. To obtain the results, MATLAB software has been used.

Keywords Topology concepts · Rough set · Accuracy · Similarity · Simply* alpha open set

1 Introduction

Information about the world around is inaccurate and incomplete or uncertain. Granulation of information is very necessary to solve human problems, and thus have a very significant impact on the design and implementation of intelligent systems (Abualigaha et al. 2021; Abualigaha and Diabatb 2021). Decision making plays an important role in our daily life, there are many applications of decision making, such as (Alblowi et al. 2021; Alharthi and El Safty 2015). Topology is an important branch of mathematics which contains several subfields such as soft topology, algebraic topology, and differential topology (El Sayed 2017; Navalagi 2000). These subfields increase the limit of the topological applications (Alharthi and Safty 2015; Creco et al. 2006). The field of topology has many results and concepts, which can help us to discover the hidden information of data, composed of real-life applications (El Safty and Alkhathami 2020; Kozae et al. 2012). The topological methods are thus compatible with the processing static methods and geometric representation (Akiyama and Thuswaldner 2004). There exist many applications of topology such as information systems as well as other fields of topology applications; it is better to name these fields (Stone 1937).

The topological concepts include continuity, irresoluteness, compactness, connectedness, and continuous. The topological structure $\tau$ on a set $X$ is a general tool for constructing the basic concepts of topology (Abd El-Monsef 1980,2016; El Sayed 2006). Mashhour and Has-sanien (1983) further presented the concept of $\alpha$-continuous functions. In addition, Navalagi (Navalagi 2000) introduced the concept of $\alpha$-open sets. As topological spaces, $X$
and $Y$ are often used in the present analysis. Let $A$ be the $X$ subset. $\text{Int}(A)$ and $\text{cl}(A)$ are denoted by the interior and the closure of a set, respectively; the alpha interior and alpha closure of the set are denoted by $\alpha \text{int}(A)$ and $\alpha \text{cl}(A)$.

Pawlak’s (Pawlak 1991) foundation of the rough set theory was based on the forest chaos originated from insufficient and an incomplete information system. Rough set attribute reduction provides a filter-based technique by which knowledge can be compactly extracted from a domain, preserving the quality of the information while reducing the amount of knowledge involved. We assume only the category of attribute reduced is determined by the criterion of preserving the sure region as is provided by the entire set of attributes. It is necessary to note that the same statement can be used to research other reduced categories (Greco et al. 2010; Kozae et al. 2012; Lashin et al. 2005; Yao and Chen 2005).

In this paper, the authors defined the simply* alpha open set, simply* alpha normal and alpha simply alpha normal, $M^\alpha S - \alpha$ open set, $S - \alpha$ normal, $\alpha M^\alpha S - \alpha$ normal. The complement of all ($\alpha$-open “$\alpha o(X)$”, open “$o(X)$””, semi-open “$So(X)$”, and pre-open “$Po(X)$”) sets are ($\alpha$ – closed “$\alpha-\text{C}(X)$”, closed “$\text{C}(X)$”, semi-closed “$S\text{C}(X)$”, and pre-closed “$P\text{C}(X)$”) sets. The families of open sets of $X$’s (resp. simply alpha open and simply open set) are denoted by $(S \alpha o(X)$ and $S o(X))$. These topological concepts with their fundamental properties were studied in the current investigation in order to obtain a proposed accuracy.

We believe that the work done in this research is crucial as it presents a proposal that uses the attributes as compared with the prior works usage of objects and it also discusses properties of the simply* alpha open set as shown in Fig. 1. We also introduced a proposed accuracy that depends upon the simply* alpha open set. In conclusion, our survey outlines a new model that gets major accuracies, competing with Pawlak and Yao methods. The results were given using the MATLAB programme.

The paper is structured as follows: the basic concepts of the simply* alpha open set (briefly, $M^\alpha S - \alpha$ open set) were explored in section two and section three. In section four, some proposed concepts were simply* alpha normal. Also, we introduced a new concept to calculate the degree of accuracy, which has been applied in section five, and section six concludes and highlights future scope.

2 Preliminaries

The notes and definitions that were used in this study are provided below:

Definition 2.1 El Sayed (2006) If $f : (X, \tau) \rightarrow (Y, \sigma)$ is defined:

i. Alpha-irresolute (briefly, $\alpha$-irresolute), if $f^{-1}(G) \in \alpha o(Y)$ for each $G \in \alpha o(Y)$.

ii. Irresolute if $f^{-1}(G) \in So(X)$ for each $G \in So(Y)$.

iii. Continuous if $f^{-1}(G) \in \tau$ for each $G \in \sigma$.

iv. Simply continuous if $f^{-1}(G) \in S^M o(X)$ for each $G \in \sigma$.

Definition 2.2 Abd El-Monsef (1980) A subset $A$ of a topological space $(X, \tau)$ is referred to as:

i. Alpha open set ($\alpha$–open) if $A \subseteq \text{int}(cl(A))$.

ii. Pre-open is defined if $A \subseteq \text{int}(cl(A))$.

iii. Regular open if $A = \text{int}(cl(A))$.

iv. Simply open set if $A = G \cup N$ where $G$ is open and $N$ is nowhere dense.

v. Alpha interior ($\alpha$–int $(A)$) is the union of all alpha open set which is contained in $A$.

vi. Alpha closure ($\alpha$–cl(A)) is the intersection of all alpha closed set which contains a set $A$.

Definition 2.3 El Sayed (2006) A topological space $(X, \tau)$ is called:

i. Normal space if for every $F_1, F_2 \in \tau^C$, $F_1 \cap F_2 = \phi$, then there exist $U, V \in \tau, U \cap V = \phi$, such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

ii. Continuous if for every $F_1, F_2 \in \alpha\text{o}(X), F_1 \cap F_2 = \phi$, then there exist $U, V \in \alpha o(X), U \cap V = \phi$, such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Definition 2.4 Pawlak (1991) $(U, R)$ is an information system, $E \in R$, let $xR$ be an after set defined by $xR = \{ y \in U : xRy \}$, $E$ is dispensable in $R$ if IND $(R) = IND(R\{-E\})$, (Reduct), but IND $(R) \neq IND(R\{-E\})$, then $E$ is indispensable in $R$ (Core) and $R$-lower, $R$-upper approximations of $X$ as:

$$\text{R}(X) = \cup \{ Y \in U/R, Y \subseteq X \}, \text{R}(X) = \cup \{ Y \in U/R, Y \cap X \neq \phi \}.$$
3 simply* continuity of soft multifunction

In the following section, we introduce some types of simply x-open set, simply* x-open set, simply* x-closed and simply* alpha limit point.

**Definition 3.1** Let a topological space \((X, \tau)\), \(E \subseteq X\) is called simply x-open set if \(x\text{int}(x\text{cl}(E)) \subseteq x\text{cl}(x\text{int}(E))\).

**Definition 3.2** A subset \(E\) of a topological space \((X, \tau)\) is called an simply* x-open (for short, \(M^\ast_x\) x-open) set if \(E \in \{X, \emptyset, G \cup N : G\) is a proper x-open set, and \(N\) is a nowhere dense set\}. The family of all simply* x-open set of set \(X\) is denoted by \(S\ x\text{o}(X)\). The complement of simply* x-open set is said to be simply* x-closed (for short, \(M^\ast_x\) x-closed) set.

The following Fig. 1 presents the relationships between simply* alpha open sets and some other types of open sets.

**Example 3.1** Let \(X = \{b, c, d, a\}\),
\[
\tau = \{X, \phi, \{a, c, d\}, \{c, d\}, \{a\}\}.
\]

Then, we have \(M^\ast_x\ x\text{o}(X) = \{X, \phi, \{a, c, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}\), \(M^\ast_x\ x\text{o}(X) = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}\). Then, we have the set \(\{b\} \in S\ x\text{o}(X)\) but \(\{b\} \notin M^\ast_x\ x\text{o}(X)\) and \(\{b, c, d\} \in M^\ast_x\ x\text{o}(X)\) but \(\{b, c, d\} \notin S\ x\text{o}(X)\).

**Example 3.2** Let \(X = \{a, b, c, d\}\),
\[
\tau = \{X, \phi, \{a, c, d\}, \{a, c, d\}\}.
\]

Then, we have \(M^\ast_x\ x\text{o}(X) = \{X, \phi, \{b, c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}\), \(S\ x\text{o}(X) = \{X, \phi, \{a\}, \{a, d\}, \{b, c, d\}, \{a, b, c\}\}\). Then, we have the set \(\{b, d\} \in M^\ast_x\ x\text{o}(X)\) but \(\{b, d\} \notin S\ x\text{o}(X)\).

**Remark 3.1** If \((X, \tau)\) is a topological space, the class of simply x-open sets “\(M^\ast_x\ x\text{o}(X)\)” of \(X\) and the class of simply closed sets of “\(M^\ast_x\ x\text{c}(X)\)”.

**Definition 3.3** For \(p \in X\) and \((X, \tau)\) is a topological space; \(p\) is called simply* alpha limit point (briefly \(M^\ast_x\) x-limit point of \(E\) if for every \(M^\ast_x\ x\)-open set \(G\) containing \(p\) contains a point of \(E\) other than \(p\). The set of all \(M^\ast_x\ x\)-limit points of \(E\) is called simply* alpha-derived set of \(E(M^\ast_x\ x - d(A))\).

4 simply* alpha normal

Here, we discussed some types of normality and regularity based on simply* open set and simply alpha open set.

**Definition 4.1** For a topological space \((X, \tau)\) is called:

- i. Alpha simply* alpha-normal space, \(\forall F_1, F_2 \in x\text{c}(X), F_1 \cap F_2 = \phi \rightarrow \exists U, V \in S^M_x\ x\text{o}(X)\) such that \(U \cap V = \phi\) and \(F_1 U \text{ and } F_2 V\).

- ii. simply* alpha-normal space, \(\forall F_1, F_2 \in S^M_x\ x\text{c}(X), F_1 \cap F_2 = \phi \rightarrow \exists U, V \in S^M_x\ x\text{o}(X)\) such that \(U \cap V = \phi\) and \(F_1 U \text{ and } F_2 V\).

**Remark 4.1** The next Fig. 2 shows the relationship between the new types of normality, which are mentioned in the previous definition, alpha simply alpha normal space properly contains alpha simply* alpha normal, alpha normal and simply* alpha normal.

The next example No. 4.1 illustrates that the effects of in Fig. 2 need not to be reversible.

**Example 4.1** Let a topological space \((X, \tau)\), \(X = \{b, c, a\}, \tau = \{X, \phi, \{b, c\}, \{c\}\}\), then \((X, \tau)\) is a simply* normal but not x-normal, since there exist \(\{a\}, \{b\} \in x\text{c}(X)\) but there is no disjoint x-open sets containing them.

**Proposition 4.1** For \((X, \tau)\) is a topological space then:

- i. Every alpha simply* alpha normal space is alpha simply alpha normal space.

- ii. Every simply* alpha normal space is alpha simply* alpha normal space

**Proofs**

- i. Let \(F_1, F_2 \in x\text{c}(X), F_1 \cap F_2 = \phi\), since \((X, \tau)\) alpha simply* alpha normal space, then there exists a disjoint simply* alpha open set \(U, V \in S^M_x\ x\text{o}(X)\) such that \(F_1 \subseteq U\) and \(F_2 \subseteq V\) and since every simply* alpha open set is simply alpha open set, i.e.

\[
S^M_x\ x\text{o}(X) \rightarrow S^M_x\ x\text{o}(X) \rightarrow \alpha S^M_x\ x\text{o}(X) \leftarrow \alpha S^M_x\ x\text{o}(X)
\]

\[
\downarrow
\]

\[
M^\ast_x\ x\text{o}(X) \rightarrow M^\ast_x\ x\text{o}(X) \rightarrow \alpha M^\ast_x\ x\text{o}(X) \leftarrow \alpha M^\ast_x\ x\text{o}(X)
\]

Fig. 2 Illustration of some relationships of topological concepts
Definition 5.1 For the information system $(X, R, S o(X))$ is a $S o(X)$-approximation space associated with relation $R$ over a set $X$ and $E \subseteq X$, simply* $\alpha$-lower and simply* $\alpha$-upper are defined: $B \subseteq E = \bigcup \left\{ G, G \in S o(X), G \subseteq E \right\}$, $B^{-} (E) = \bigcap \left\{ F, F \in S o(X), F \supseteq E \right\}$, respectively. The accuracy of the simply* $\alpha$-lower and simply* $\alpha$-upper approximations of $E$ in $(X, R, S o(X))$ is defined by $\mu(E) = \frac{\theta(B^{-})}{\theta(B)}$, where $|l|$ denotes the cardinality of the set.

Definition 5.2 $\forall B \subseteq E, R_B \subseteq U \times U$ is defined by $w_{RB}z = \frac{\sum_{i=1}^{N} (|w_i| - |z_i|)}{|B|} \leq \alpha$, where $|B|$ is the cardinality of $B$ and $\alpha$ represents any number and where $\alpha$ is calculated by an expert of the field, for example, let $B = \{ E_2 \}$, $|B| = 1$, $w_{R1z} = |(i(w) - i(z))/1 \leq \alpha$.

5.1 Data gathering

Using the data of the securities business of market (Al-blowi et al. 2021), the application of the method is conducted where $u = \{ y_1, y_2, ..., y_{10} \}$ denotes 10 listed companies, the attributes of companies.

$b = \{ b_1, b_2, ..., b_6 \}$ and $D = \{ d \} =$ (decision of investment).

The coding is shown by converting a value from 0 to 1 as: $S_{new} = (S_{old} - S_{min})/(S_{max} - S_{min})$, and dividing the interval $[0, 1]$ into 3 parts, the consequence of Table 1 discretion using the $E$-means clustering is the following Table 2.

The coding attributes obtained by the following Algorithm are:

```python
function [M] = coding(xapp, code);
    [nl,nc] = size(xapp);
    M = zeros(nl,nc);
    for i = 1:nc
        M(:,i) = (xapp(:,i) - min(xapp(:,i)))/max(xapp(:,i) - min(xapp(:,i)));
    end
    [I,J] = find(M >= ((code-1)/code) & M <= 1);
    for i = 1:length(I)
        M(I(i),J(i)) = code;
    end
    for i = 1:(code-1)
        [I,J] = find(M >= ((i-1)/code) & M < ((i)/code));
        for t = 1:length(I)
            M(I(t),J(t)) = i;
        end
    end
```

with relation $R$ over a set $X$ and $E \subseteq X$, simply* $\alpha$-lower and simply* $\alpha$-upper are defined: $B \subseteq E = \bigcup \left\{ G, G \in S o(X), G \subseteq E \right\}$, $B^{-} (E) = \bigcap \left\{ F, F \in S o(X), F \supseteq E \right\}$, respectively. The accuracy of the simply* $\alpha$-lower and simply* $\alpha$-upper approximations of $E$ in $(X, R, S o(X))$ is defined by $\mu(E) = \frac{\theta(B^{-})}{\theta(B)}$, where $|l|$ denotes the cardinality of the set.

Also, we obtain Table 3 after the cancellation of symmetry; the objects are $U = \{ Y_1, Y_2, ..., Y_8 \}$ which denote 8 mentioned companies. The attributes are $B = \{ B_1, B_2, ..., B_6 \}$, as shown in Table 3.

When removing $B_6$, we had the objects $Y_4$ and $Y_3$ which are the same, and when $B_3$ was removed, we got $Y_5$ and $Y_8$ which are the same. Similarly, removing $B_4$, we obtained $Y_4$ and $Y_6$ which are the same. We noticed that $IND (B) \neq IND (B - \{ B_1 \})$, $IND (B) \neq IND (B - \{ B_2 \})$ and $IND (B) \neq IND (B - \{ B_4 \})$. Then $B_1, B_3$ and $B_4$ are indispensable. Also, removing $B_2, B_3$ and $B_6$, we had $IND (B) = IND (B - \{ B_2 \})$, $IND (B) = IND (B - \{ B_3 \})$, and $IND (B) = IND (B - \{ B_6 \})$. Then, $B_2, B_5$, and $B_6$ were called superfluous, as shown in Table 4.

Then, $\{ B_1, B_3, B_4 \}$ were called the core, and $B_2, B_5$, and $B_6$ were called the superfluous.
Now, we discuss the following data in Table 5. After classification of Table 5, we got the final Table 6.

(i) One attributes case: Let \( C = \{B_2\} \), \( |C| = 1 \), \( xR_1y \leftrightarrow |i(x) - i(y)|/1 \leq \alpha \), as follows in Table 7.

**Discussion 1** when \( \alpha \leq 0 \), we got the intelligence system that follows: \( xR_1y = \{(h_1, h_1), (h_1, h_3), (h_2, h_2), (h_3, h_1), (h_3, h_3), (h_4, h_4)\} \), then \( h_1R_1 = \{h_1, h_3\} \), \( h_2R_1 = \{h_2\} \).
We obtained all the accuracies as follows in Table 8 and Table 9, simply* alpha method and Yao, Pawlak’s methods and the proposed method are $\mu_{11} = \frac{\text{int}(H)}{\text{cl}(H)}$, $\mu_{12} = \frac{L(H)}{U(H)}$, and $\mu_{13} = \frac{B(H)}{2}$ as shown in Table 8 and Table 9, respectively.

Using the methods of Pawlak and Yao methods, we obtained the accuracies as shown in Table 8.

As given in Table 9, we had the accuracy model via a simply* alpha open set.
Yao and Pawlak accuracies are shown in Table 8, but the proposed accuracy was given in Table 9. Consequently, it is obvious that our proposed accuracy is better than Yao and Pawlak accuracies, and for one attribute, it was noted that the proposed accuracy was the best of the others.

6 Conclusions

The present research clearly indicated that the accuracies of the information system are the function of the best attributes of the life information. simply* alpha open set method shows the best accuracy. When the simply* alpha open set method was used to introduce application on the information system of the business securities of the marketing, we get the best proposed accuracy when comparing other objects with other attributes, which can also be used in other sciences. Moreover, these results prompt us to safely express these concepts to be applied to other different areas of advanced topology. The rough set techniques were presented in the form of classification or decision rules obtained from a set of the previous applications. Additionally, our approach provided a new insight into the problem of attribute reduction, and also suggested more semantic properties preserved by an attribute reduction. Consequently, our method provides more flexibility to the decision-maker to choose which is suitable for him. We also obtained a proposed accuracy that depends upon the simply* alpha open set, which was found to be better than that of Yao and Pawlak accuracies. In the future, based on some topological studies, we will further expand the research content of this paper. Also, our approach has been used to help discover the most important symptoms of Coronavirus (Covid-19).

Author contributions This paper is written through the efforts of all contributions. The individual contributions and obligations of all authors can be summarized as follows: the idea of the whole article was put forward by El Sayed, and Alblowi, along with doing the paper preparations. El Safty analysed the existing work of simply* alpha open set, and revised the paper.

Funding We have no funding for this article.

Availability of data and materials Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Declarations

Conflict of interests The authors declare that they have no conflict of interests.

Human and animal rights This article does not contain any studies with human participants or animals performed by the author.

References

Abd El-Monsef ME (1980) Studied on Some pre topological concepts. Ph.D. Dissertation, University of Tanta
Abd El-Monsef ME, Kozae AM, El-Bably MK (2016) Generalized covering approximation space and near concepts with some applications. Appl Comput Informat 12(1):51–69. https://doi.org/10.1016/j.aci.2015.02.001
Abualigaha L, Diabat A (2021) A novel hybrid antlion optimization algorithm for multi-objective task scheduling problems in cloud computing environments. Clust Comput 24:205–223
Abualigaha L, Diabat A, Mirjalilid S, Abd EM, Gandomih AH (2021) The arithmetic optimization algorithm. Comput Methods Appl Mech Eng 376:1–38
Akiyama S, Thuswaldner JM (2004) A survey on topological properties of tiles related to number systems. Geom Dedic 109(1):89–105. https://doi.org/10.1007/s10711-004-1774-7
Alblowi SA, El Sayed M, El Safty MA (2021) Decision making based on fuzzy soft sets and its Application in COVID-19. Intelli Auto Soft Comput. https://doi.org/10.32604/iasc.2021.018242
Alharthi TN, El Safty MA (2015) Attribute topologies based similarity.Cogent Math J 3:1–11. https://doi.org/10.1080/23311835.2016.1242291
Creco S, Inuiuchi M, Slowinski R (2006) Fuzzy rough sets and multiple-premise gradual decision rules. Int J Approx Reason 41:179–211. https://doi.org/10.1016/j.ijar.2005.06.014
El Sayed M (2006) Relation between Some Types of Continuity in Topological space. M.Sc. Dissertation, Tanta University, Egypt
El Sayed M (2017) Soft simply open sets in soft topological space. J Comput Theor Nano Sci 14:4100–4103. https://doi.org/10.1166/jctn.2017.6792
El Safty MA, Alkhathami AM (2020) A topological method for reduction of digital information uncertainty. Soft Comput 24:385–396. https://doi.org/10.1007/s00500-019-03920-9
Greco S, Matarazzo B, Slowinski R (2010) On topological dominance-based rough set approach. Transactions on Rough Sets XII. Lecture Notes in Computer Science. 6190:21–45. Springer-Verlag Berlin Heidelberg 2010. https://doi.org/10.1007/978-3-642-14467-7_2
Kozae AM, Elsafy M, Swealam M (2012) Neighbourhood and reduction of knowledge. AISS 4:247–253. https://doi.org/10.4156/aiss
Lashin EF, Kozae AM, Khadra AA, Medhat T (2005) Rough set theory for topological spaces. Int J Approx Reason 40:35–43. https://doi.org/10.1016/j.ijar.2004.11.007
Maslouhi AS, Hassanein IA, ElDeen SN (1983) A note on α-continuous and open mappings. Acta Math Hungar 41:213–218. https://doi.org/10.1007/BF01961309
Navalagi GB (2000) Definition bank in general topology. Topology Atlas preprint, 449. URL http://at.yorku.ca/i/d/e/b/75
Pawlak Z (1991) Rough Sets: Theoretical aspects of reasoning about data. Rough Sets 9:1–237. https://doi.org/10.1007/978-94-011-3534-4_7
Stone MH (1937) Applications of the theory of Boolean rings to general topology. Trans Am Math Soc 41:375–381. https://doi.org/10.1090/S0002-9947-1937-1501905-7
Yao YY, Chen Y (2005) Subsystem based generalizations of rough sets approximations in LNCS. Found Intell Syst 3488:210–218. https://doi.org/10.1007/11425274_22

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.