Supersymmetric Unification with Radiative Breaking of R-parity

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(September 23, 2018)

We show how R-parity can break spontaneously as a result of radiative corrections in unified N=1 supergravity models. We illustrate this with a concrete rank-four unified model, where the spontaneous breaking of R-parity is accompanied by the existence of a physical majoron. We determine the resulting supersymmetric particle mass spectrum and show that R-parity-breaking signals may be detectable at LEP200.

The possible role of supersymmetry in relation to the hierarchy problem and to the possible unification of fundamental interactions has attracted a lot of attention. Most phenomenological discussions have so far been made in the framework of the Minimal Supersymmetric Standard Model (MSSM). Such model assumes a discrete symmetry called R-parity, related to the spin (S), lepton number (L), and baryon number (B) accord-

able symmetry called R-parity, related to the spin (S), lepton number (L), and baryon number (B) according to \( R_p = (-1)^{3B+L+2S} \). Under this symmetry all standard model particles are even while their partners are odd. Conservation of B and L leads to R-parity con-

ervation and implies that SUSY particles must always be pair-produced, the lightest of them being absolutely stable.

Whether or not supersymmetry is realized with a con-

served R-parity is an open dynamical question, sensitive to physics at a more fundamental scale. On the other hand the phenomenological effects associated to R-parity violation may well be accessible to experimental verification. It is therefore of great interest to investigate alternative scenarios where the effective low energy theory does not exhibit a conserved R-parity.

As other fundamental symmetries, it could well be that R-parity is a symmetry at the Lagrangean level but is broken by the ground state. Such scenarios provide a very systematic way to include R parity violating effects, automatically consistent with low energy baryon number conservation and cosmological baryogenesis. They may provide an explanation of the of the deficit of solar neutrinos and the cosmological dark matter.

In this letter we show how R-parity can spontaneously break in N=1 supergravity unified models by virtue of radiative corrections, very much the same way as the electroweak symmetry. We first illustrate how this can happen in the case of rank-four unification, such as SU(5), where lepton number is an ungauged symmetry. In this case there is a physical Goldstone boson, the Majoron, associated to the spontaneous breaking of R-parity. Consistency with LEP measurements of the invisible Z width require that R-parity breaking be driven by \( SU(2) \otimes U(1) \) singlet vacuum expectation values (VEVS). In this case the Majoron is mostly singlet and does not couple to the Z. Here we perform a thorough study of the minimization of the scalar boson potential and present, as an example, the parameters of one of the R-parity-breaking minima we obtain. For this minimum we determine the resulting supersymmetric particle mass spectrum and show that R-parity-breaking signals may be accessible at LEP200.

Starting from some underlying N=1 unified supergravity model we consider the low energy theory characterized by the following \( SU(2) \otimes U(1) \) invariant superpotential:

\[
W = h_u u^c Q h_u + h_d d^c Q h_d + h_e e^c \ell h_d + h_0 h_u h_d \Phi + h_\nu \nu^c \ell h_u + h \Phi \nu^c S + \lambda \Phi^3
\] (1)

The first three terms are the usual ones that will be responsible for the masses of charged fermions and the fourth will give rise to the mixing of the Higgsinos. The last two terms involve gauge singlet superfields \((\nu^c, S)\) carrying lepton numbers -1 and 1, respectively. These singlets may arise in several extensions of the standard model and may lead to interesting phenomenological signatures of their own. Their existence ensures that the majoron will be essentially decoupled from the Z. The \( h_\nu \) term plays a crucial role in the phenomenology, as it will determine the strength of the R-parity violating interactions.

All terms in the superpotential in eq. (1) are cubic and conserve total lepton number as well as R-parity. The superfield \( \Phi \) has no lepton number. All couplings \( h_u, h_d, h_e, h_\nu, h \) are described by arbitrary matrices in generation space but for our present purposes it will be enough to assume that they are nonzero only for the third generation. We also assume all parameters to be real.

The model described above is a very simple variant of the one proposed in ref. The matrices \( h_u \) and \( h_d \) in eq. (1) would be related if we take the unification group as SU(5) with minimum Higgs sector. This relation is not necessary in our analysis and our results apply also
to $SU(3) \otimes SU(2) \otimes U(1)$ string models where the gauge couplings unify by virtue of gravitational interactions. In this case there are no relations between the Yukawa parameters. For simplicity we neglect the $h_{\nu}$ coupling in the RGE. We will neglect, moreover, the bottom-quark Yukawa coupling, which is well justified provided $\tan \beta$ is not too large. First we write the RGE for the Yukawa couplings

$$16\pi^2 \frac{dh_u}{dt} = h_u \left( 6h_u^2 + h_0^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{9} g_1^2 \right)$$

(2)

$$16\pi^2 \frac{dh}{dt} = h \left( 3h^2 + 2h_0^2 + 18\lambda^2 \right)$$

(3)

$$16\pi^2 \frac{dh_0}{dt} = h_0 \left( h^2 + 4h_0^2 + 18\lambda^2 + 3h_0^2 - 3g_2^2 - g_1^2 \right)$$

(4)

$$16\pi^2 \frac{d\lambda}{dt} = \lambda \left( 3h^2 + 6h_0^2 + 54\lambda^2 \right)$$

(5)

where $t = \ln Q/M_U$ where $Q$ is the arbitrary RGE scale and $M_U$ is the unification scale. There are similar equations for the evolution of the corresponding cubic soft supersymmetry breaking parameters.

The soft-breaking mass parameters evolve according to:

$$8\pi^2 \frac{dM_{H_u}^2}{dt} = 3h_u^2(M_{H_u}^2 + M_Q^2 + M_{\nu}^2 + A_u^2) + h_0^2(M_{H_u}^2 + M_H^2 + M_\Phi^2 + A_0^2) - 3g_2^2M_S^2 - g_1^2M_1^2$$

(6)

$$8\pi^2 \frac{dM_{H_d}^2}{dt} = h_0^2(M_{H_d}^2 + M_H^2 + M_\Phi^2 + A_0^2) - 3g_2^2M_S^2 - g_1^2M_1^2$$

(7)

$$8\pi^2 \frac{dM_{\nu}^2}{dt} = 8\pi^2 \frac{dM_S^2}{dt} = h^2(M_{\nu}^2 + M_S^2 + M_\Phi^2 + A^2)$$

(8)

$$8\pi^2 \frac{dM_\nu^2}{dt} = 2h_0^2(M_{H_u}^2 + M_H^2 + M_\Phi^2 + A_0^2) + h^2(M_{\nu}^2 + M_S^2 + M_\Phi^2 + A^2) + 18\lambda^2(3M_\Phi^2 + A_0^2)$$

(9)

$$8\pi^2 \frac{dM_\nu^2}{dt} = -3g_2^2M_S^2 - g_1^2M_1^2$$

(10)

The $g_i$ are the $SU(3) \otimes SU(2) \otimes U(1)$ gauge couplings and the $M_i$ are the corresponding soft breaking gaugino masses. Similarly one can write the RGE corresponding to the evolution of the soft squark mass terms.

Note that RGE describing the evolution of the $\nu^c$ and $S$ soft supersymmetry breaking masses, given in eq. (8), are the same in the limit of negligible $h_\nu$. Moreover, the evolution of the stop supersymmetry breaking masses are the same as in the MSSM.

We now discuss the corresponding boundary conditions at unification. We assume that at the unification scale the model is characterized by one universal soft supersymmetry-breaking mass $m_0$ for all the scalars (the gravitino mass), except for the $SU(3) \otimes SU(2) \otimes U(1)$ singlets, and a universal gaugino mass $M_1/2$. Moreover we assume that there is a single trilinear soft breaking scalar mass parameter $A$. In other words we assume that

$$A_u = A = A_0 = A_{\nu} = A_{\lambda},$$

(11)

$$M_{H_u}^2 = M_{H_d}^2 = M_H^2 = M_\nu^2 = M_\Phi^2 = m_0^2,$$

(12)

$$M_{\nu}^2 = C_{\nu}m_0^2; M_S^2 = C_Sm_0^2; M_\Phi^2 = C_{\Phi}m_0^2,$$

(13)

$$M_3 = M_2 = M_1 = M_1/2$$

(14)

at $Q = M_U$. At energies below $M_U$ these conditions do not hold, due to the renormalization group evolution from the unification scale down to the relevant scale.

In order to determine the values of the Yukawa couplings and of the soft breaking scalar masses at low energies we first run the RGE from the unification scale $M_U \sim 10^{16}$ GeV down to the weak scale. In doing this we randomly give values at the unification scale for the parameters of the theory. The range of variation of these parameters at the unification scale is as follows

$$10^{-2} \leq h_\nu^2/4\pi \leq 1$$

$$10^{-3} \leq h^2/4\pi; h_0^2/4\pi; A^2/4\pi \leq 1$$

$$10^{-7} \leq h_\nu^2/4\pi \leq 1$$

$$-3 \leq A/m_0 \leq 3$$

$$0 \leq m_{1/2}/m_0 \leq 2$$

(15)

After running the RGE we have a complete set of parameters, Yukawa couplings and soft-breaking masses $m_i^2(RGE)$ to study the minimization.

The full scalar potential along neutral directions may be written at low energies as

$$V_{total} = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + V_D + V_{SB} + V_{RC}$$

(16)

where $z_i$ denotes any one of the neutral scalar fields in the theory, $V_D$ are the usual $D$-terms, $V_{SB}$ the SUSY soft breaking terms and $V_{RC}$ are the one-loop radiative corrections.

Because of the complexity of the problem we do not do it directly, solving the non-linear extremization equations for the VEVs. We use, instead, the procedure developed
in terms of solving the extremum equations for the soft mass-squared parameters in terms of the VEVS, which are linear. To do this we have to give values to the VEVS. We do this in the following way:

1. The value of $v_u$ is determined from $m_{top} = h t v_u$ for $m_{top} = 175 \pm 15$ GeV. If $v_u$ determined in this way is too high we go back to the RGE and choose another starting point.
2. $v_d$ and $\tan(\beta)$ are then determined by $m_W$.
3. $v_L$ is obtained by solving approximately the corresponding extremum equation.
4. We then vary randomly $m_0$, $v_R$, $v_S$, $v_\phi$ in the range $100 \text{GeV} \leq m_0 \leq 1000 \text{ GeV}$ and $10 \text{GeV} \leq v_R; v_S; v_\phi \leq 1000 \text{ GeV}$.

After doing this, for each point in parameter space, we solve the extremum equations for the soft breaking masses, which we now call $m_i^2$. Then we calculate numerically the eigenvalues for the real and imaginary part of the neutral scalar mass-squared matrix. If they are all positive, except for the Goldstone bosons, the point is a good one. If not, we go to the next random value. After doing this we end up with a set of points for which:

1. The Yukawa couplings and the gaugino mass terms are given by the RGE.
2. For a given set of $m_i^2$ each point is also a solution of the minimization of the potential that breaks R-Parity.
3. However, the $m_i^2$ obtained by the minimization of the potential differ from those obtained from the RGE $m_i^2(RGE)$.

Our next goal is to find which solutions for $m_i^2$ that minimize the effective low-energy potential have the property that they coincide with the $m_i^2(RGE)$ obtained, for a given unified theory, from the RGE, namely

$$m_i^2 = m_i^2(RGE) \ \forall i$$

(17)

To do that we define a function

$$\epsilon = \max \left( \frac{m_i^2}{m_i^2(RGE)}, \frac{m_i^2(RGE)}{m_i^2} \right) \ \forall i$$

(18)

Defined in this way it is easy to see that we have always $\epsilon \geq 1$ the equality being what we are looking for. We are then all set for a minimization procedure. We want, by varying the parameters, to get $\epsilon$ as close to 1 as possible. Before we move on we have to clarify what are our parameters in the minimization. At first we assumed universality and our $\epsilon$ was a function of $h_t^U, h_t^U, h_t^U, h_t^U, A_t, m_0, m_{1/2}, v_R, v_S, v_\phi$, and the allowed range for these parameters was as specified above.

With these conditions we used the MINUIT package to find the minimum of $\epsilon$. We should add that we have also enforced that we get a solution that it is both a solution of the minimization of the potential and lower than other trivial minima. After sampling a few million points we did not find any solution with $\epsilon < 1.1$. We then decided to relax the universality condition on the soft mass-squared parameters at the unification scale. Indeed, deviations from universality, are a generic feature of soft-breaking terms obtained from 4-dimensional string models. For definiteness, we adopted a very conservative and unnecessary restriction of keeping universality for the MSSM scalars but allowed the $SU(2) \otimes U(1)$ singlet masses to vary away from universality. To be more precise we defined

$$\eta_S = \frac{m_S^2}{m_0^2} ; \ \eta_{\psi} = \frac{m_{\psi}^2}{m_0^2} ; \ \eta_\phi = \frac{m_\phi^2}{m_0^2}$$

(19)

and allowed $\eta_S, \eta_{\psi}$ and $\eta_\phi$ to vary from $1/4$ to 10. Finally we also allowed a variation of the top quark mass within present experimental errors.

With these modifications our $\epsilon$ is now a function of $h_t^U, h_t^U, h_t^U, h_t^U, A_t, m_0, m_{1/2}, v_R, v_S, v_\phi, \eta_S, \eta_{\psi}, \eta_\phi$ and $m_{top}$, and MINUIT was able to find solutions with $\epsilon$ as close to 1 as we wanted.

Here we present for one specific case the values at the unification scale as well as the low energy values and the low energy spectrum. The starting values at the unification scale are the following:

$$A = 2.99,$$

$$m_0 = 143.6 \text{ GeV},$$

$$C_{\psi} = 0.869 ; \ C_S = 0.742 ; \ C_\phi = 1.204,$$

$$M_{1/2} = 0.907 \text{ m}_{0},$$

$$\frac{h_t^2}{A_t} = 0.03 ; \ \frac{h_t^2}{A_t} = 0.015 ; \ \frac{h_t^2}{A_t} = 1.2 \times 10^{-7},$$

$$\frac{h_t^2}{A_t} = 0.032 ; \ \frac{h_t^2}{A_t} = 0.0064$$

(22)

(23)

(24)

With these values we get the following particle mass spectrum at low scale

$$m_t = 174 \text{ GeV}; \ m_{t1} = 295 \text{ GeV}; \ m_{t2} = 435 \text{ GeV},$$

$$m_{\chi^0_1} = 78 \text{ GeV}; \ m_{\chi^0_2} = 250 \text{ GeV},$$

$$m_{\nu_e} = 65 \text{ KeV}; \ m_{\chi^0_1} = 43 \text{ GeV}; \ m_{\chi^0_2} = 83 \text{ GeV},$$

$$m_{\chi^0_2} = 221 \text{ GeV}; \ m_{\chi^0_1} = 251 \text{ GeV},$$

$$m_h = 69 \text{ GeV}; \ m_H = 161 \text{ GeV}; \ m_A = 198 \text{ GeV}$$

(21)

(22)

(23)

(24)

(25)

The shape of the scalar potential close to this minimum can be displayed as a function of the relevant VEVS, for example the R-parity violation VEVS $v_{\psi}$ and $v_\phi$ (fig. 1) or the electroweak breaking VEVS $v_u$ and $v_d$. We have also checked that the R-parity minimum is lower than trivial minima, for which electroweak and/or R-parity are
unbroken, and that at all scales the traditional bound for no colour breaking

\[ |A_u|^2 \leq 3 (m^2_{Q_u} + m^2_{u_d} + m^2_\nu) \]  

(26)
is satisfied.

We see that, in this example, the lightest CP-even Higgs boson, the lightest chargino and the lightest neutralino can all be produced at LEP200. Moreover, since R-parity is broken, the lightest neutralino decays. Moreover, typically this decay happens in the detector, as can be seen from fig 2.

In our model the value of \( m_\nu \) determines the rates for all R-parity-violating couplings. Since the value of \( m_\nu \) for this solution is relatively small (65 KeV), the most likely site for the violation of R-parity will be in the decay of the lightest neutralino which would arise as the final stage of the cascade decays of the other supersymmetric particles. Note that the above minimum is just one out of many. There are others with light SUSY spectra, for which \( m_\nu \) lies in the tens of MeV range. In the latter case R-parity violation would show up not only through the decay of the lightest neutralino, but might also be observable at LEP100, e.g. through the single production of charginos, as proposed earlier [8].

Before concluding we wish to comment on the issue of the universality of soft-breaking masses. The solutions with light supersymmetric mass spectrum that we have obtained have non-universal values at unification. We do not know if this is a necessary feature of the model. Were this to be confirmed by further studies, we would regard it as an interesting clue to relate R-parity-breaking with physics at the Planck scale in the string context. Indeed, deviations from universality are a generic feature of soft-breaking terms obtained from 4-dimensional strings [1].

This work was supported by DGICYT grants PB92-0084 and HP-53 and by a Valencia-Yerphi exchange (A.I.). We thank Carlos Muñoz for discussions.

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