Adaptive Background Compensation of FI-DACs with Application to Coherent Optical Transceivers

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Abstract—This work proposes a novel adaptive background compensation scheme for frequency interleaved digital-to-analog converters (FI-DACs). The technique is applicable to high speed digital transceivers such as those used in coherent optical communications. Although compensation of FI-DACs has been discussed before in the technical literature, adaptive background techniques have not yet been reported. The importance of the latter lies in their capability to automatically compensate errors caused by process, voltage, and temperature variations in the technology (e.g., CMOS, SiGe, etc.) implementations of the data converters, and therefore ensure high manufacturing yield. The key ingredients of the proposed technique are a multiple-input multiple-output (MIMO) equalizer and the backpropagation algorithm used to adapt the coefficients of the aforementioned equalizer. Simulations show that the impairments of the analog signal path are accurately compensated and their effect is essentially eliminated, resulting in a high performance transmitter system.

Index Terms—Frequency interleaving DAC, high-speed optical transmitter, background calibration, error backpropagation.

I. INTRODUCTION

INTENSITY modulation and direct detection in long haul and metro optical fiber links have been displaced by coherent transmission techniques [1], [2]. Next generation coherent transceivers will operate at symbol rates of $f_B = 128-150$ Gbd and beyond [3]. The main challenge in the design of transceivers for high-speed optical communications is achieving the large bandwidth (BW) and sampling rate required by the analog-to-digital and the digital-to-analog converters (ADC and DAC). One of the solutions proposed in recent literature is the use of frequency interleaving (FI) techniques [4], [5].

This paper focuses on FI-DACs in the context of applications to transceivers for digital communications, particularly high baud rate coherent optical transceivers. Although laboratory experiments with high baud rate optical transmission based on FI-DAC have been described in the technical literature [6], significant obstacles remain before this technology can be applied in commercial products. One of the main challenges is how to automatically compensate the impairments of the analog signal path. As it shall be discussed later, because of process tolerances, layout limitations, etc., errors may exist which, if left uncompensated, would introduce large distortions and severely degrade the performance of the system. Several techniques have been presented to compensate analog errors in FI architectures. In previous work [7], the compensation requires startup calibration which is done using foreground techniques. In coherent optical systems, the latter would imply the interruption of the communication to compensate the imperfections, which is undesirable. The compensation needs to be accurately tailored to the impairments, which are process, voltage, and temperature dependent and (slowly) time variant. The only way to achieve this at low cost and in a way that lends itself to high volume manufacturing is to use adaptive background compensation techniques. However, no adaptive background compensation techniques for FI-DACs have been reported so far in the technical literature.

The FI-DAC architecture considered in this paper is shown in Fig. 1. It is discussed in the context of its application to a high baud rate transmitter for coherent optical communications. The scheme of Fig. 1 corresponds to one polarization in a dual-polarization (DP) coherent optical transceiver. The transmit path is partitioned into two or more bands by DSP...
techniques. Without loss of generality, in this paper we assume
that it is decomposed into two bands. As shown in Fig. 2
each band is demodulated to baseline with two exponentials
$e^{j2\Omega_0 T_s}$ where $\Omega_0 = \omega_0 T_s$ with $\omega_0 = (2\pi f_B)/4$ and $1/T_s$
being the DSP sampling rate. The demodulated baseline
signals are first processed by lowpass filters (LPF) with
frequency response $G(e^{j\Omega})$, then they are downsampled
by a factor $\alpha$, and synthesized by DAs of lower bandwidth
and sampling rate than required by the full signal. After
synthesis, analog double quadrature mixers reconstruct the full
signal, which, after amplification by a modulator driver, is
used to control the Mach-Zehnder Modulator (MZM). The
reference clocks of the mixers and the DAs are assumed to
be properly synchronized. To process even larger BW signals,
this technique can be extended to more subbands, which would
require more DAs with similar characteristics as those just
described.

In this work we show that all analog impairments in a
two-band FI-DAC can be modeled as a $4 \times 4$ multiple-input
multiple-output (MIMO) real filter defined by a $4 \times 4$ transfer
matrix $F(j\omega)$, followed by ideal quadrature modulators (see
Section II). Fig. 3 depicts a block diagram of the proposed
compensation architecture and an equivalent discrete time
model of the two-band FI-DAC based coherent optical transmis-
sion. To compensate the effects of the impairments $F(e^{j\Omega})$,
we introduce a MIMO adaptive equalizer called hereafter
Impairment Equalizer (IE) and defined by the transfer matrix
$H(e^{j\Omega})$, which includes the LPF responses $G(e^{j\Omega})$. Let
$e[n] = \hat{x}[n] - x[n]$ be the error between the reconstructed
($\hat{x}[n]$) and the original wideband ($x[n]$) full signal. This error
is measured at the input of the MZM (note that $\hat{x}[n]$ includes
all the impairments of the analog path up to the input of the
MZM). Then, the IE is adapted to minimize the mean squared
error (i.e., $E\{ |e[n]|^2 \}$) by using the least mean square (LMS)
algorithm. Towards this end, the digital backpropagation algo-
rithm [8, 9] is proposed to perform background compensation
of the channel impairments. This algorithm, in combination
with the estimated channel response $\hat{F}(e^{j\Omega})$, provides the
error samples required by the LMS algorithm to adapt the
coefficients of the IE. Computer simulations demonstrate that
the proposed IE architecture is able to compensate not only
DAs and mixer impairments, but also the amplitude and
phase distortions of the electrical paths (e.g., it acts as pre-
emphasis and/or compensator of time skew between in-phase
and quadrature (I&Q) components).

The rest of this paper is organized as follows. Section II
introduces a model of the channel impairments in FI-DAs for
a coherent optical transceiver. Section III describes the
proposed adaptive background compensation technique. Section IV
presents simulations and finally conclusions are drawn
in Section V.

II. MODEL OF ANALOG IMPAIRMENTS IN FI-DACS

Analog impairments drastically affect the performance of
any FI-DAC architecture, including the one presented here.
The most important ones for a two-band FI-DAC based
optical coherent transmitter are shown in Fig. 3 and include:
(i) distortion, bandwidth limitation, and (in-band) time
skew ($\tau_B$) caused by mismatches amongst electrical path
responses between DACs and the quadrature mixers (denoted
as $B_a^{(1/Q)}(j\omega)$ with $a \in \{1, 2\}$); (ii) gain and phase errors
(denoted, respectively, as $g_a^{(1/Q)}(j\omega)$ and $\delta_a^{(1/Q)}(j\omega)$ with $a, b \in \{1, 2\}$)
of quadrature mixers employed for the reconstruction of the
full analog signal; (iii) distortion, bandwidth limitations, and
time skew ($\tau$) caused by mismatches amongst the electrical path
responses going from the quadrature mixers to the optical
modulator (denoted as $G_a^{(1/Q)}(j\omega)$ with $a \in \{1, 2\}$ which
include the modulator drivers and any other components in
the signal path). The effect of the phase and gain errors in
the quadrature modulators is to create spurious terms that
cause interference in the adjacent band. As we shall show
later, this interference as well as the other impairments can
be compensated by the technique proposed in this paper.

Based on simple trigonometric identities and signal proces-
sing techniques, we show in the Appendix that the analog channel
model with impairments for one polarization described in
Fig. 4 can be reformulated as a $4 \times 4$ MIMO real channel
defined by a $4 \times 4$ transfer matrix $F(j\omega)$ with elements
$F_{u,v}(j\omega)$, $u, v \in \{1, \ldots, 4\}$, followed by ideal quadrature
modulators (see Fig. 5). Based on this result, we can derive
a simple discrete time model of the FI-DAC architecture for
application in an optical coherent transmitter as depicted in
Fig. 3. This formulation is important since it shows that
all the impairments in FI-DAs can be digitally compen-
sated by a MIMO compensator equalizer with transfer matrix
$H(e^{j\Omega})$. For example, for an ideal compensation, we get that
$H(e^{j\Omega})F(e^{j\Omega}) = G(e^{j\Omega})I_4$ where $I_4$ is the $4 \times 4$ identity
matrix.

As discussed in Section II the analog path compensated by the
scheme proposed here includes the impairments of the modulator driver and the
interconnections among the FI-DAC, the driver, and the MZM. Said analog path may encompass components in different packages and printed circuit
board (PCB) interconnects.

Alternatively, the forward propagation algorithm [10] could be used. This
option will be described in detail in a future paper.
matrix and $G(e^{j\Omega})$ is the Fourier transform (FT) of the lowpass filter impulse response depicted in Fig. 1.

### Fig. 4. Analog impairments in a two-band FI-DAC based DP coherent optical transceiver (only one polarization is depicted).

### Fig. 5. Equivalent channel model of analog impairments in a two-band FI-DAC for one polarization in a DP coherent optical transmitter.

### Fig. 6. Low complexity architecture for estimating the equivalent channel model of analog impairments in a two-band FI-DAC for one polarization in a DP coherent optical transmitter.

### A. Channel Estimator (CE) Block

As we shall discuss in the next section, the proposed compensation scheme is based on the evaluation of the error $e[n] = \hat{x}[n] - x[n]$ and the background estimation of the analog path response with impairments, $F(e^{j\Omega})$. These operations are based on the samples of the output signal $\hat{x}(t)$ as depicted in Fig. 6. The feedback path includes buffers and track and holds (T&H) to support the bandwidth of the reconstructed full signal $\hat{x}(t)$. However, since channel impairments change slowly over time, the estimation algorithm does not need to operate at full rate. Therefore, low power, low speed (i.e., $1/(MT_c)$ with $M \gg 1$), medium resolution ADCs with adjustable sampling phase can be used. The estimation of the analog channel response $F(e^{j\Omega})$ can be achieved by using the LMS algorithm (LMS_CE) and the error between the DAC inputs (e.g., $s_1[n]$ and $s_2[n]$) and the samples of the reconstructed full signal as depicted in Fig. 6. Notice that CE block is also able to provide samples of the reconstructed signal at full rate for computation of the error $e[n]$. The response of the feedback path (i.e., T&H, ADC) could be initially estimated and removed from $F(e^{j\Omega})$ (the details are omitted due to space constraints).

### III. ADAPTAION OF THE IMPAIRMENT EQUALIZER (IE): ERROR BACKPROPAGATION ALGORITHM

The samples of the reconstructed full signal for one polarization can be expressed as (see Figs. 3 and 5):

\[ \hat{x}[n] = y_1[n]e^{j\Omega \alpha n} + y_2[n]e^{-j\Omega \alpha n}, \tag{1} \]

where $y_1[n] = y_1^{(I)}[n] + jy_1^{(Q)}[n]$ and $y_2[n] = y_2^{(I)}[n] + jy_2^{(Q)}[n]$ with components given by

\[ y[n] = F^{-1}\{F(e^{j\Omega})F\{s[n]\}\}, \tag{2} \]

where $F\{\cdot\}$ ($F^{-1}\{\cdot\}$) denotes the FT (inverse FT) operator, $y[n]$ is the $4 \times 1$ real vector defined by $y[n] = [y_1^{(I)}[n] \ y_1^{(Q)}[n] \ y_2^{(I)}[n] \ y_2^{(Q)}[n]]^T$ while $s[n] = [s_1^{(I)}[n] \ s_1^{(Q)}[n] \ s_2^{(I)}[n] \ s_2^{(Q)}[n]]^T$ is the $4 \times 1$ real vector with the digital samples at the DAC inputs.
Let $H_{u,v}(e^{j\Omega})$ with $u,v \in \{1,2,3,4\}$ be the $(u,v)$ transfer function of the $4 \times 4$ transfer matrix of the IE, $H(e^{j\Omega})$. We also define the real impulse responses $h_{u,v}[n] = F^{-1}\{H_{u,v}(e^{j\Omega})\}$ with $u,v \in \{1,2,3,4\}$. In this work we adopt the LMS algorithm to iteratively adapt the real coefficients of the set $h_{u,v}[n]$ in order to minimize the mean squared error (MSE) between the input and the reconstructed full samples (LMS$(t)$)

$$h_{u,v}^{(k+1)} = h_{u,v}^{(k)} - \beta \nabla h_{u,v}[e[n]]^2, \quad u,v \in \{1,...,4\}, \quad (3)$$

where $k$ denotes the number of iteration, $h_{u,v} = [h_{u,v}[0], h_{u,v}[1], ..., h_{u,v}[L_h - 1]]^T$, $L_h$ is the number of coefficients of the filters, $\beta$ is the adaptation step, and $\nabla h_{u,v}[e[n]]^2$ is the gradient of the MSE with respect to the real vector $h_{u,v}$. The computation of the latter is not trivial since $e[n]$ is not the error at the output of the IE (see Fig. 3).

To get the proper error samples to adapt the coefficients of the filters as expressed in eq. (3), we propose to use the backpropagation algorithm widely used in machine learning [8, 9]. Towards this end, we first generate the demodulated band errors $e_1[n] = e[n]e^{j\Omega_{un}}$ and $e_2[n] = e[n]e^{-j\Omega_{on}}$. These errors, in combination with eq. (2) and the estimated channel response $\hat{F}(e^{j\Omega})$, are used to get the backpropagated errors $\hat{e}_1[n]$ and $\hat{e}_2[n]$ (see [8]–[10] for a detailed description of the backpropagation technique). Finally, based on these backpropagated errors we can estimate the gradient $\nabla h_{u,v}[e[n]]^2$ as usual in the classical LMS algorithm.

Since channel impairments change slowly over time, the coefficient adaptation does not need to operate at full rate, and subsampling can be applied. The latter allows implementation complexity to be significantly reduced. Additional complexity reduction is enabled by: 1) strobing the algorithms once they have converged, and/or 2) implementing them in firmware in an embedded processor, typically available in coherent optical transceivers.

IV. SIMULATIONS

We investigate the performance of the proposed background calibration technique in a two-band FI-DAC based DP coherent optical system by using computer simulations. We assume 16-QAM modulation with a symbol rate of $f_B = 1/T = 128$ Gbd in a back-to-back optical channel. The oversampling factor used in the DSP blocks is $T/T_s = 2$. We consider 8-bit resolution DACs with sampling rate of 128 GS/s (i.e., $\alpha = 2$ in Fig. 1) and nominal BW of $B_0 = 32$ GHz, which is half of what would be needed to process the input signal band in a non-interleaved architecture. The electrical analog path responses $B_{1/2}(j\omega)$ with $a \in \{1,2\}$ in Fig. 3 are simulated with third-order Butterworth lowpass filters with nominal BW $B_0$. Ideal feedback channel is assumed. The number of taps of the impairment equalizers $h_{u,v}$ is $L_h = 21$. The subsampling factor of the feedback ADC is $M = 128$. Other details of the DSP blocks are omitted due to space limitations (see 11 and references therein for details of typical DSP blocks). We focus on the optical signal-to-noise ratio (OSNR) penalty at a bit-error-rate (BER) of $10^{-3}$, which is computed using an ideal software coherent receiver.

3See 11 for a definition of OSNR penalty.

V. CONCLUSIONS

An FI-DAC architecture with adaptive background compensation of the analog signal path errors for coherent optical transceivers has been presented. Simulations show the effectiveness of the proposed technique, which results in the elimination of the penalty caused by the DAC frequency response and the gain and phase errors in the mixers, as well as other
impairments. Although the technique was presented in the context of a transmitter for coherent optical communications, it is more general and can be used in other applications.

APPENDIX

In this Appendix we derive the channel model of Fig. 5. With a proper design, it is possible to assume that the two cosine (and sine) signals used in each quadrature mixer have the same gain and phase [1], i.e., $A_{a,1} = A_{a,3} = 1 + \delta_a$, $A_{a,2} = A_{a,4} = 1 - \delta_a$, $\phi_{a,1} = \phi_{a,3} = \phi_a/2$, and $\phi_{a,2} = \phi_{a,4} = -\phi_a/2$ with $a \in \{1,2\}$. Note that $\delta_a$ and $\phi_a$ represent the gain and phase imbalance of the quadrature mixer for band $a$, respectively. Then, based on the complex model proposed in [12], the modulator output of band $a$ denoted as $z_a(t) = z_a^{(1)}(t) + jz_a^{(Q)}(t)$ (see Fig. 4), can be expressed as

$$z_a(t) = \hat{s}_a(t)p_a(t), \quad a \in \{1,2\},$$

where $\hat{s}_a(t) = \hat{s}_a^{(1)}(t) + j\hat{s}_a^{(Q)}(t)$ is the mixer input, and $p_a(t) = k_{a,1}e^{j\omega t} + k_{a,2}e^{-j\omega t}$

with complex constants given by

$$k_{1,1} = \frac{1 + \delta_1}{2}e^{j\phi_1/2} + \frac{1 - \delta_1}{2}e^{-j\phi_1/2},$$

$$k_{1,2} = \frac{1 + \delta_1}{2}e^{-j\phi_1/2} - \frac{1 - \delta_1}{2}e^{j\phi_1/2},$$

$$k_{2,1} = \frac{1 + \delta_2}{2}e^{j\phi_2/2} - \frac{1 - \delta_2}{2}e^{-j\phi_2/2},$$

$$k_{2,2} = \frac{1 + \delta_2}{2}e^{-j\phi_2/2} + \frac{1 - \delta_2}{2}e^{j\phi_2/2}.$$

In the absence of gain and phase errors, notice that $p_1(t) = e^{j\omega t}$ and $p_2(t) = e^{-j\omega t}$. Fig. 9 shows the equivalent complex-valued model of the quadrature mixer with impairments. The quadrature mixer outputs $z_a^{(1/Q)}(t)$ are transmitted up to the optical modulator through electrical paths (which include the modulator drivers) with mismatch responses $C_a^{(1/Q)}(j\omega)$ (see Fig. 3). From eq. (4) and Fig. 10 the complex signal at the MZM input can be expressed as

$$\hat{x}_a(t) = z_a^{(1)}(t) \otimes c_a^{(1)}(t) + j z_a^{(Q)}(t) \otimes c_a^{(Q)}(t),$$

where symbol $\otimes$ denotes the convolution operation and $c_a^{(1/Q)}(t) = F^{-1}\{C_a^{(1/Q)}(j\omega)\}$. Since $\hat{x}_a(t) = 0.5[z_a(t) + z_a^{*}(t)]$ and $j\hat{x}_a(t) = 0.5[z_a(t) - z_a^{*}(t)]$, we can get

$$\hat{x}_a(t) = z_a(t) \otimes u_a(t) + z_a^{*}(t) \otimes \overline{u}_a(t),$$

(11)

where $u_a(t) = 0.5[c_a^{(1)}(t) + c_a^{(Q)}(t)]$ and $\overline{u}_a(t) = 0.5[c_a^{(1)}(t) - c_a^{(Q)}(t)]$ (see Fig. 10).

Fig. 10. Complex-valued channel model of the mixer-MZM electrical paths as shown in the block diagram on the right side of Fig. 11. Moreover, taking into account that $\hat{x}_a(t)e^{j\omega t} \otimes u(t) = [\hat{x}_a(t) \otimes u(t)e^{j\omega t} + \hat{x}_a(t) \otimes u(t)e^{-j\omega t}]$, it is possible to exchange the order of the modulator and filter blocks $u_a(t)$ and $\overline{u}_a(t)$. Then, grouping the signals properly, the analog channel for the subband $a$ can be reduced to a $2 \times 4$ MIMO real channel followed by two ideal quadrature mixers as depicted on the right side of Fig. 11. Notice that the response of the DAC-mixer electrical paths (i.e., $B_a^{(1/Q)}(j\omega)$ in Fig. 3) can be easily included within this MIMO channel model. Finally, the model just described is applied to the two subbands which are combined resulting in the $4 \times 4$ MIMO real channel model of Fig. 5.

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