Parity Violation in Neutron-Nucleus Collisions at Very Low Energies

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Abstract

We investigate parity violation in neutron-nucleus collisions at very low energy region \( (E \leq 100\text{eV}) \), including resonance states. We use the coupled channel formalism for resonances with radiative emission. The parity mixing between resonances with opposite parities is taken into account. Finally, our theory is applied to \( n^{-139}\text{La} \) collision and relevant remarks are given.
§1 Introduction

Large parity violation effects were experimentally found\textsuperscript{1} at rather many resonance states in medium to heavy nuclei that were produced by capture of very low energy neutrons (neutron every $E_n \leq 100\text{eV}$). Theoretical frameworks to deal with such cases have been presented\textsuperscript{2} assuming that the parity mixing parameter $\epsilon$ is not strongly energy dependent at and near the resonance region. In view of the large parity violation effect in resonances of a number of different nuclei, it would be desirable to investigate theoretically cases, where the parity mixing between resonances of same spin with opposite parity is large, while the parity mixing in potential scattering remains quite normal (small) (of the order of $10^{-6} \sim 10^{-7}$ in amplitude), so that the latter can be ignored as compared to the former. The detail of such cases shall be described in §2. Application to the neutron-$^{139}$La case shall be briefly discussed in §3 together with relevant discussions.

§2 General theory of parity-mixed nuclear resonances

We shall formulate the resonance scattering by Ref.3) formalism. We discuss the neutron-nucleus collision at very low energy region, including resonance states in $s$- and $p$-waves. To this end, we shall begin with the Schrödinger equation for the system, where there are no couplings between “resonances” (\textit{i.e.,} bound states) and scattering states of neutron-nucleus
system:

\[
\begin{pmatrix}
H_{D_s} & 0 & 0 & 0 \\
0 & H_{P_s} & 0 & 0 \\
0 & 0 & H_{P_p} & 0 \\
0 & 0 & 0 & H_{D_p}
\end{pmatrix}
\begin{pmatrix}
\phi_s \Phi_g \\
\Phi_s \\
\phi_p \Phi_g \\
\Phi_p
\end{pmatrix}
= E
\begin{pmatrix}
\phi_s \Phi_g \\
\Phi_s \\
\phi_p \Phi_g \\
\Phi_p
\end{pmatrix},
\]

where we divide channels into \(P\)(arent)-channels and \(D\)(aughter)-channels. (These four wave functions share the same total angular momentum, so that they can couple each other after introducing “channel coupling”, as we shall see below.) The parent channel \(\Phi_s\) and \(\Phi_p\) describe the bound state (wave functions which are for the discrete \(E\)). We assume that \(D\)-channels consist of two channels that describe the potential scatterings of the neutron \(n\) by the target nucleus in \(s\)-wave and \(p\)-wave, respectively. \(\Phi_g\) is the wave function of the ground state of the target nucleus \(^A\!Z\), excluding the total angular momentum part. \(\phi_s\) (or \(\phi_p\)) is the wave function describing the relative motion of the \(n + ^A\!Z\) system. The \(\phi_s\) (or \(\phi_p\)) also includes spin wave functions of \(n\) and \(^A\!Z\).

The Hamiltonians of \(D\)-channels which consist of neutron \(n\) and target nucleus \(^A\!Z\) are:

\[
\begin{align*}
H_{D_s} &= -\frac{1}{2m} \frac{1}{r} \frac{d}{dr} (r \frac{d}{dr}) + V_s + H_D^0, \\
H_{D_p} &= -\frac{1}{2m} \frac{1}{r} \frac{d}{dr} (r \frac{d}{dr}) + \frac{2}{r^2} + V_p + H_D^0,
\end{align*}
\]

where \(V_s\) and \(V_p\) are the potentials acting in the \(s\)- and \(p\)-states of \(n+^A\!Z\). \(H_D^0\) is the Hamiltonian for the target nucleus. The radiative capture processes exist even for very low energy neutrons. Therefore the potentials, \(V_s\) and \(V_p\), are the optical potentials that include imaginary parts. When the relative distance \(r\) between \(n\) and \(^A\!Z\) is large, \(\phi_s\) is asymptotically
given by

$$\phi_s \to \frac{\sin kr}{kr} + f_{s}^{pot} e^{i k r} \frac{e^{i kr}}{r}, \quad r \to \infty.$$  

The s-wave potential scattering amplitude is given by

$$f_{s}^{pot} = e^{i \delta_s} \sin \delta_s / k.$$  \hspace{1cm} (2.3)

For very low energy neutrons, it is sufficient to retain only the first term in the effective range expansion;

$$k \cot \delta_s = -\frac{1}{a - i b} + \ldots .$$

The p-wave potential scattering amplitude $f_{p}^{pot}$ can be taken to be:

$$f_{p}^{pot} = e^{i \delta_p} \sin \delta_p / k \div 0$$

for very low energy neutrons.

Next we introduce (parity conserving) couplings, $V_{W_s}$ and $V_{W_p}$, between the bound states in $P$-channels and the continuous states of $D$-channels:

\begin{equation}
\begin{pmatrix}
H_{Ds} & V_{Ws} & 0 & 0 \\
V_{Ws} & H_{Ps} & 0 & 0 \\
0 & 0 & H_{Pp} & V_{WP} \\
0 & 0 & V_{WP} & H_{DP}
\end{pmatrix}
\begin{pmatrix}
\psi_s \Phi_g \\
\psi_s \\
\Psi_p \\
\psi_p \Phi_g
\end{pmatrix}
= E
\begin{pmatrix}
\psi_s \Phi_g \\
\psi_s \\
\Psi_p \\
\psi_p \Phi_g
\end{pmatrix}.
\end{equation}  \hspace{1cm} (2.4)

Now the $j$-wave scattering amplitudes $f_j$ include not only the potential scattering $f_{j}^{pot}$ but also the resonance contribution $f_{j}^{res}$. We can describe $f_j$ as follows:

\begin{equation}
\begin{align*}
f_j &= f_{j}^{pot} + f_{j}^{res}, \\
k f_{j}^{pot} &= e^{i \delta_j} \sin \delta_j, \\
k f_{j}^{res} &= -\frac{\Gamma_{0j}}{2} \frac{e^{2i \delta_j}}{E - E_j + \frac{1}{2} \Gamma_{0j}}.
\end{align*}
\end{equation}  \hspace{1cm} (2.5)

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where the total width $\Gamma_{0j}$ is the sum of the neutron width $\Gamma^n_{0j}$ and the radiation width $\Gamma^\gamma_{0j}$:

$$\Gamma_{0j} = \Gamma^\gamma_{0j} + \Gamma^n_{0j}$$

The neutron width $\Gamma^n_{0j}$ of $j$-wave is described in good approximation by:

$$\Gamma^n_{0j} = 2\pi \int d^3k \ |\langle \phi_j \Phi_g | V_{\mathcal{W} j} | \Phi_j \rangle|^2 \delta(E_k - E_j) \quad (j = s \text{ or } p). \quad (2.6)$$

The suffix 0 is attached to the width $\Gamma^x_{0j}$ to indicate that we are dealing with parity conserving cases.

Finally we introduce the parity violating nuclear potential (PVNP) $V_X$ which contains $W$ or $Z$ boson exchange contribution. Up to the first order in $V_X$, we find the following situation: the parity mixing between $s$-wave and $p$-wave in $D$-channels is not important (of the order of $10^{-6} \sim 10^{-7}$ in amplitudes). On the other hand, if the $s$-bound state $\Phi_s$ lies near the $p$-bound state $\Phi_p$ of the same spin (but opposite parity), the parity mixing between these two bound states $\Phi_s$ and $\Phi_p$ can be much enhanced. Therefore we may take the following Shrödinger equation:

$$\begin{pmatrix}
H_{Ds} & V_{Ws} & 0 & 0 \\
V_{Ws} & H_{Ps} & V_X & 0 \\
0 & V_X & H_{Pp} & V_{Wp} \\
0 & 0 & V_{Wp} & H_{Dp}
\end{pmatrix}
\begin{pmatrix}
\psi_s \Phi_g \\
\Psi_s \\
\psi_p \Phi_g \\
\Psi_p
\end{pmatrix} =
E
\begin{pmatrix}
\psi_s \Phi_g \\
\Psi_s \\
\psi_p \Phi_g \\
\Psi_p
\end{pmatrix}. \quad (2.7)$$

Then, we find the $s$-wave and $p$-wave scattering amplitudes, $f_s$ and $f_p$ in $D$-channels, are given by

$$\begin{pmatrix}
k f_s \\
k f_p
\end{pmatrix} =
\begin{pmatrix}
e^{i\delta_s} \sin \delta_s & 0 \\
0 & e^{i\delta_p} \sin \delta_p
\end{pmatrix}
+ \frac{-\frac{1}{2} \Gamma}{E - E_0 + \frac{1}{4} \Gamma}
\times
\begin{pmatrix}
\frac{\Gamma^s e^{2i\delta_s^0}}{\eta \sqrt{\Gamma^s \Gamma^\gamma_p e^{i(\delta_s^0 + \delta_p^0)}}} \\
\frac{\Gamma^\gamma_p e^{2i\delta_p^0}}{\Gamma^s \Gamma^\gamma_p e^{i(\delta_s^0 + \delta_p^0)}}
\end{pmatrix}. \quad (2.8)$$
at near the resonance energy $E_0$. Where $\Gamma = \Gamma_p^n + \Gamma_s^n + \Gamma_{\gamma}^n$, $\Gamma_p^n$ is $p$-wave neutron width, $\Gamma_s^n$ is $s$-wave neutron width and $\Gamma_{\gamma}^n$ is radiation width. Also notice that $\delta_j$ is the potential scattering phase shift for $j$-wave ($j = s$ or $p$), while $\delta_j^0$ is given by

$$e^{2i\delta_j^0} = \frac{(k \cot \delta_j - ik)^*}{k \cot \delta_j - ik}. \quad (2.9)$$

This result has never been stated clearly up to now. One finds

$$\delta_j^0 = \delta_j$$

for real $\delta_j$. However the existence of capture processes in $j$-waves leads complex $\delta_j$ and hence

$$\delta_j^0 \neq \delta_j$$

in general. When the capture is due only to radiative transitions, imaginary parts of $\delta_s$ and $\delta_p$ are proportional to $\alpha = \frac{e^2}{4\pi} \frac{1}{137} (\hbar = c = 1)$. So, one can use the approximation:

$$\delta_j^0 \approx \delta_j. \quad (2.10)$$

The phase factor $\eta$ ($|\eta| = 1$) should be $\pm 1$ if TRI holds, while $\eta^2 \neq 1$ if TRI does not hold. We notice that Eq.(2.8) is correct for not only the PVNP between $P$- channels in Eq.(2.7) but also in general for (short range) PVNP.

Though the effects of $V_X$ extend to all bound states and all continuum states in $P$-channels, the most important parity mixing effect may come from that between two closely lying $s$- and $p$- bound states with the same
spin as stated before. Therefore we may take into account only such a
parity mixing correction to the $p$-bound state $\Phi_p$:

$$\Phi_p \rightarrow \Phi_p + \frac{\langle \Phi_s | V_X | \Phi_p \rangle}{E_p - E_s} \Phi_s.$$  \hfill (2.11)

This second term is responsible to the $s$-wave neutron emission, whose
width $\Gamma^n_s$ is given by

$$\Gamma^n_s = 2\pi \int \left| \langle \phi_s | V_W | \Phi_s \rangle \right|^2 \frac{\langle \Phi_s | V_X | \Phi_p \rangle^2}{E_p - E_s} d^3k \delta(E_k - E_p)$$

$$= \Gamma^n_{0s} P_s,$$  \hfill (2.12)

where $\Gamma^n_{0s}$ is given by Eq.(2.6) ($j = s$) with the modification, $\delta(E_k - E_s)$ replaced by $\delta(E_k - E_p)$. The quantity $P_s$:

$$P_s = \left| \frac{\langle \Phi_s | V_X | \Phi_p \rangle}{E_p - E_s} \right|^2$$  \hfill (2.13)

represents the fraction of the $s$-wave resonance mixed in the $p$-wave (dominating) resonance.

§3 Parity mixing in the $p$-wave resonance

We shall apply the results of previous section for $p$-wave resonance.
In the case of parity mixing in an $s$-wave resonance (emitting $p$-wave
neutrons), the parity violation effects are damped down, because the po-
tential barrier suppresses $p$-wave neutron emission at low energies. On the
other hand, in the case of parity mixing in a $p$-wave resonance (emitting
$s$-wave neutron), the $s$-wave neutron emission is enhanced with respect
to the $p$-wave emission.
So far we have retained our discussion within the simple configuration: (single particle + core nucleus). We notice, again, that the results expressed in Eqs. (2.8) ~ (2.10) hold in general cases.

We introduce the reduced width \((\gamma_j)^2\), using the allowed maximum neutron width \(\text{i.e., Wigner’s width } \Gamma_j^{\text{Wigner}}\):

\[
\Gamma_j^n = \Gamma_j^{\text{Wigner}}|\gamma_j|^2. \tag{3.1}
\]

where \(\gamma_j\) is proportional to the probability amplitude to find the configuration, \(\phi_j \Phi_g\) [(single particle state) \(\times\) (the ground state target nucleus)], in the wave function of the resonance. [Notice the spatial wave functions of the resonances are described by bound state wave functions \(\Phi_s\) to a very good approximation (see Ref.3).] Therefore the bound state wave functions representing of \(s\)- and \(p\)-wave resonances for parity conserving case are written as,

\[
\begin{align*}
\Phi_p & = \gamma_p \psi_p \Phi_g + \cdots, \\
\Phi_s & = \gamma_s \psi_s \Phi_g + \cdots.
\end{align*} \tag{3.2}
\]

where \(\cdots\) describe the more complex configurations of bound states.

Next we switched on PVNP derived from the standard model. By using Eq.(2.11) matrix elements of \(V_X\) between various configurations may normally be small and have random signs. Especially the matrix elements of \(W^\pm\) boson exchange potential are unlikely to add up coherently. The only case of adding up the matrix element coherently is the one where \(Z^0\) boson exchange potential acts between \(\phi_s \Phi_g\) and \(\phi_p \Phi_g\) (see Appendices A and B). Let us assume such a matrix element dominates and see its consequence (its reasoning shall be presented in Appendix C).
Let us discuss $Z^0$ boson exchange contribution to the $p$-wave bound state. In view of Eqs. (2.11) and (3.2) the configuration $\phi_s\Phi_g$ in this resonance shall be

$$\gamma_p \frac{<\phi_s\Phi_g|V_X|\phi_p\Phi_g>}{E_p - E_s} \gamma_s.$$  

The parity mixed $p$-wave resonance is given by

$$\Phi_p \doteq \gamma_p [\phi_p + \gamma_s \frac{<\phi_s\Phi_g|V_X|\phi_p\Phi_g>}{E_p - E_s} \phi_s] \Phi_g + \cdots. \quad (3.3)$$

Then $p$- and $s$-wave neutron widths are given by

$$\Gamma^n_p = \Gamma^{Wigner}_p (\gamma_p)^2,$$

$$\Gamma^n_s = \Gamma^n_p \times \left[ \frac{<\phi_s\Phi_g|V_X|\phi_p\Phi_g>}{E_p - E_s} \right]^2 \gamma_s^2. \quad (3.4)$$

Let us apply the results obtained in the preceding discussion to the case of $n + ^{139}$La, where the large parity violation has been found at the $p$-wave (dominant) resonance, $^{140}$La$^*(0.734$eV). We have no information about the spins of this $p$-wave resonance as well as the $s$-wave resonance in $^{140}$La which lies on 49eV below the $p$-wave resonance. So we replace this energy difference, by the average level spacing of resonance with a definite spin-parity, $D \sim 100$eV as typical value of $(E_p - E_s)$. And the matrix element of $Z^0$ exchange $<\phi_s\Phi_g|V_X|\phi_p\Phi_g>$ is given by Appendices A and C.

$$|\frac{<\phi_s\Phi_g|V_X|\phi_p\Phi_g>}{D}|^2 \sim 10^{-3}. \quad (3.5)$$

If $\gamma_s^2$ is the same order of $\gamma_p^2 (\sim 10^{-5})$ for the $^{140}$La$^*$,

$$\frac{\Gamma^n_s}{\Gamma^n_p} \sim 10^{6-3-5} \sim 10^{-2}. \quad (3.6)$$
Then it is no wonder that $L-R$ asymmetry $A_L$ is order of $10^{-1}$ in helicity cross sections.

The example of this La case can be extended to other medium heavy nuclei bombarded by very low energy neutrons (see Table I and II). Marked enhancement in parity violation effects, e.g., $A_L \sim (10^{-2} \sim 10^{-1})$ found for many nuclei now seems to be reasonable at least as far as their magnitude is concerned. The enhancement is due to the ratio $(kR)^2$ of the penetration factors and the coherent nature of natural $Z^0$ exchange force.

The signs of $A_L$ are (+) (see Table I) for so many other nuclear cases. It is difficult to explain this fact from our theory since $A_L$ is proportional to

$$<\phi_s \Phi_g | V_X | \phi_p \Phi_g > \gamma_s, \quad (3.7)$$

which does not seem to be always the same sign. We expect that the example of $A_L < 0$ shall be found more.

We use the known $s$-wave resonance energies listed in the Table I for calculating $E_p - E_s$ in Eq. (3.7), which lie nearest to each $p$-wave resonance. But strictly speaking, we must emphasize the fact that these $p$- and $s$-wave resonances sharing the same spin are not yet shown experimentally.
Appendix A: Parity violating by $Z^0$ exchange

In this appendix A, we evaluate the matrix element of parity violating effect by neutral weak boson $Z^0$ exchange in the standard model on the simplest (one particle) shell model nuclear structure theory. The neutral current Hamiltonian density is

$$H = \frac{g^2}{2M_{Z^0}^2}(J_\mu^0 J^{\mu 0} + h.c.),$$  \hspace{1cm} (A.8)

where $J^0$ is neutral current of $Z^0$. We extract parity violating part, i.e. (vector $\times$ axial vector part) from Eq.(A.1). We assume that the core nucleus has no spin. (See Fig.1.)

The explicit form of the parity violating Hamiltonian density is

$$H_{PV} = -\frac{G_A}{G_V} \frac{g^2}{16 M_{Z^0}^2} [\phi^*_\text{core} 2T \phi^*_{\text{core}} \phi^*_{\text{val}} \frac{2T_3 + 1}{2} \frac{\sigma \cdot (P - P')}{2m} \frac{t_3}{2} \phi_{\text{val}}] - 4 \sin^2 \theta_W \phi^*_\text{core} 2T \phi^*_{\text{core}} \phi^*_{\text{val}} \frac{\sigma \cdot (P - P')}{2m} \frac{t_3}{2} \phi_{\text{val}},$$  \hspace{1cm} (A.9)

where $G_A$ (or $G_V$) is axtial vector (or vector) coupling constant, $T$ and $t$ are isobaric spin of core nucleus and valence nucleon respectively, and $T_3$ and $t_3$ are the third component of $T$ and $t$ respectively. $\theta_W$ is Weinberg angle. $\phi_{\text{core}}$ (or $\phi_{\text{val}}$) is the wave function describing core nucleus of $^{139}\text{La}$ (or valence nucleon).

We obtain the matrix element of $s_{1/2}$-$p_{1/2}$ mixing.

$$< s_{1/2} | H_{PV} | p_{1/2} > \simeq -1.7 \text{eV},$$  \hspace{1cm} (A.10)
and when $E_s = -48.6$eV ($^{139}$La case)

$$P_s \equiv \left| \frac{< s_{1/2} | H_{PV} | p_{1/2} >}{E_p - E_s} \right|^2 \simeq 4.2 \times 10^{-3}. \quad (A.11)$$
Appendix B: The asymmetry parameter from $s - p$ mixing

In low energy neutron - nucleus reaction, the sign of $L - R$ asymmetry parameter is given in Table I.

Now we attempt to explain large asymmetries. In this low energy ($E_n \leq 10\text{eV}$) region, it is sufficient to consider $s_{1/2}$, $p_{1/2}$ - and $p_{3/2}$-waves.

The effective range expansion at low energies can be used for non-resonant regions.

\[ k \cot \delta^\ell_J = -\hat{a}^\ell_J + \cdots, \quad (B.12) \]

where $\hat{a}^\ell_J \equiv a^\ell_J(1 - ika^\ell_J)$, $J$ is the total angular momentum, $\ell$ is the orbital quantum number ($\ell = 0, 1, 2, \cdots$) and $a$ is the scattering amplitude. Notice that $a^\ell_J$ is the complex amplitude.

Next we discuss about $n + ^{139}\text{La}$ reaction case. The resonance of $^{140}\text{La}$ due to the $p$-wave absorption at the energy (0.734 eV) exhibits large parity mixing. It is natural to consider that the opposite parity mixing occurs between $p_{1/2}$ and $s_{1/2}$. The parity violating $p_{1/2}$ part is described as

\[ |"p_{1/2}" > = |p_{1/2} > + \epsilon |s_{1/2} >, \quad (B.13) \]

where $\epsilon$ is mixing parameter.

The Breit-Wigner resonance amplitudes for partial waves are ;

\[ A^{p_{3/2}}_{\text{res}} = \frac{1}{k(E - E_0 + \frac{i}{2}\Gamma)}, \quad (B.14) \]
\[
A_{\text{res}}^{p_{1/2}} = \frac{1}{k} \frac{-\frac{1}{2} \Gamma_n^{p_{1/2}}}{E - E_0 + \frac{i}{2} \Gamma},
\]
\[
A_{\text{res}}^{s_{1/2}} = \frac{1}{k} \frac{-\frac{1}{2} \Gamma_n^{s_{1/2}}}{E - E_0 + \frac{i}{2} \Gamma},
\]
and
\[
A_{\text{res}}^{\text{mix}} = \frac{1}{k} \frac{-\frac{1}{2} \sqrt{\frac{\Gamma_n^{s_{1/2}} \Gamma_n^{p_{1/2}}}}}{E - E_0 + \frac{i}{2} \Gamma},
\]
respectively where \(\Gamma_n^{s_{1/2}}\) (or \(\Gamma_n^{p_{1/2}}\)) is \(s\)- (or \(p\))-wave neutron width, \(E_0\) is resonance energy and \(\Gamma\) is total width \((\Gamma = \Gamma_n^{p_{1/2}} + \Gamma_n^{s_{1/2}} + \Gamma_\gamma)\).

Then the relevant \(S\) matrix can be expressed as
\[
\left( S - \frac{1}{2ik} \right) \begin{pmatrix}
|p_{3/2}J; M> \\
|p_{1/2}J; M> \\
|s_{1/2}J; M>
\end{pmatrix} = \begin{pmatrix}
A_{\text{res}}^{p_{3/2}} & 0 & 0 \\
0 & A_{\text{res}}^{p_{1/2}} & \eta A_{\text{res}}^{\text{mix}} \\
0 & \eta^* A_{\text{res}}^{\text{mix}} & (-\frac{2J+1}{16} \hat{a}_J) + A_{\text{res}}^{s_{1/2}}
\end{pmatrix} \begin{pmatrix}
|p_{3/2}J; M> \\
|p_{1/2}J; M> \\
|s_{1/2}J; M>
\end{pmatrix},
\]
where \(J\) is the spin of the resonance, \(M\) is the third component of \(J\) and \(\hat{a}_J\) is \(s\)-wave scattering amplitude with total angular momentum \(J\) (\(\hat{a}_J\) does not exist for \(J = 2\) or 5). As is stated in the text, the \(p\)-wave potential scattering is negligible. \(\eta\) is the phase factor (see \(\S 2\)).

We derive the asymmetry parameter \(A_L\) for (total) cross sections, \(\sigma_R\) and \(\sigma_L\),
\[
A_L = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L},
\]
assuming the spin of resonance \(J\) to be \(2, \cdots, 5\).

By using the optical theorem, the \(L - R\) asymmetry parameter \(A_{L(J)}\) are easily found. The case of \(J = 5\) or 2 gives no asymmetry, \(i.e. A_{L(J=2or5)} =\)
0. We find

\[ A_{L(J=3)} = \frac{-Im[\eta 7A_{mix}^{res}]}{Im[-(7\hat{a}_3 + 9\hat{a}_4) + 7A_{res}^{s_{1/2}} + 7A_{res}^{p_{1/2}} + 7A_{res}^{p_{3/2}}]} \text{ for } J = 3 \] (B.20)

We find

\[ A_{L(J=4)} = \frac{-Im[\eta 9A_{mix}^{res}]}{Im[-(7\hat{a}_3 + 9\hat{a}_4) + 9A_{res}^{s_{1/2}} + 9A_{res}^{p_{1/2}} + 9A_{res}^{p_{3/2}}]} \text{ for } J = 4 \] (B.21)

Of course, the above discussions can be extended for more general cases. For any target nucleus, the asymmetry parameter can be derived from a parity mixed resonant state.
Appendix C: Estimates of $\left< \Phi_p | V_X | \Phi_s \right>$

In general the wave functions $\Phi_s$ and $\Phi_p$ of two resonances of opposite parities may consist of $N$ configurations $\Phi_n^J$ (assumed to be normalized).

$$
\Phi_s = \sum_{n=1}^{N} C_n^s \Phi_n^s,
$$

$$
\Phi_p = \sum_{n=1}^{N} C_n^p \Phi_n^p,
$$

where

$$
< \Phi_n^J | \Phi_m^{J'} > = \delta_{J,J'} \delta_{nm}, \quad \sum_n |C_n^J|^2 = 1 \ (J = s \text{ or } p) 
$$

(C.22)

Let $D (\sim 100\text{eV})$ be the average level spacing between resonances which have the same spin parity. Then $N$ would be of the order of

$$
N \approx \frac{(\text{several MeV})}{D} \sim \text{several } 10^4. 
$$

(C.23)

Therefore on the average

$$
< |C_n^J|^2 >_{av} \sim \frac{1}{N} 
$$

(C.24)

is expected (of course, subject to rather large fluctuation).

Now let us discuss the matrix elements of $P$-violating forces between $\Phi_n^s$ and $\Phi_n^p$. For clarity, we use parity violating forces derived from the standard model.

(a) The configuration that the $Z^0$ exchange effects are maximal (appendix A)

This is where $\Phi_n^J$ is the product of single valence orbitals $\phi_J$ and the residual core $\Phi_c$

$$
< \phi_p \Phi_c | V_{Z^0\text{exchange}} | \phi_s \Phi_c >. 
$$

(C.25)
When the residual core $\Phi_c$ is the ground state, it was discussed in Appendix A. For $^{139}$La, the value of Eq. (C.4) is -1.7 eV.

(b) The configurations where the $W^\pm$ exchange contribution is large

Let the $\Phi^J_n$ be the product of single orbital $\phi_p(\phi_s)$ and residual core wave function $\Phi^J_{core}$, where $\phi_p$ and $\phi_s$ belong to an iso-multiplet;

$$\begin{align*}
\Phi^s_n &= \phi_s\Phi^s_{core}, \\
\Phi^p_n &= \phi_p\Phi^p_{core}.
\end{align*}$$

(C.26)

And $\phi_s$ is, say, an $s$-wave neutron and $\phi_p$ is a $p$-wave neutron, then we can estimate the above matrix element of $W^\pm$ exchange contribution to be:

$$<\Phi^p_n|V_{W^\pm exchange}|\Phi^s_n> \simeq 0.01 \text{eV}.$$  (C.27)

Notice that this value is much smaller as compared to the case (a).

(c) The other configurations

Other configurations of $\Phi^J_n$ than above two cases (a) and (b) have much smaller matrix elements of parity violating forces (hadron exchange potentials with weak ($Z$ or $W$) vertex corrections).

$$|<\Phi^p_n|V_X|\Phi^s_n>| \ll 0.01 \text{eV}.$$  (C.28)

Since amplitudes $C^{ss}_{n}C^{m}_{n}n^p$ would have random sign in general, each contribution $C^{ss}_{n}C^{m}_{n} < \Phi^p_m|V_X|\Phi^s_n >$ in $< \Phi^p_p|V_X|\Phi^s_s >$ shall cancel out substantially. Also note that

$$|C^J_n|^2 \sim <|C^J_n|^2>_{av} \sim \frac{1}{N}.$$
Then the contribution to matrix elements in the case of (b) and (c) (estimation from the random walk theory) shall be at most

\[ | ⟨ \Phi_s | V_X | \Phi_p ⟩ | \leq \sqrt{N} \sqrt{|C_{n}^{m}|^2 |C_{m}^{n}|^2} | ⟨ \Phi_p | V_{exchange} | \Phi_s ⟩ |_{av} \]

\[ \approx \frac{1}{\sqrt{N}} | ⟨ \Phi_p | V_{exchange} | \Phi_s ⟩ |_{av}. \]

(C.29)

This is evidently smaller than case (b), i.e., the value \( \sim 0.01\text{eV} \).

Consequently there is a possibility that the \( Z^0 \) exchange potential shall be dominating. If it is the case, \( ⟨ \phi_s | V_{exchange} | \phi_p ⟩ \) may be order of \( C_n^{m} C_p^{n} \times 1.7\text{eV} \), where we could equate \( |C_n^{m} C_p^{n}| \) with the geometric mean of reduced widths \( \sqrt{\gamma_s^2 \gamma_p^2} \).

The \( \rho^0 \) or \( \omega \) meson exchange potential (parity violating), in which the one vertex is due to strong interactions (and parity conserving) and the other vertex (containing parity violation) is weakly corrected, contributes also to the matrix elements \( ⟨ \Phi_p | V_X | \Phi_s ⟩ \), whose magnitude shall be a similar order of magnitude as that of the \( Z^0 \) exchange potential, Eq.(C.4). Therefore the matrix element of parity violating potentials could typically be represented by the \( Z^0 \) exchange potential.
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Table I

The experimental results\textsuperscript{4}) of $A_L$ in low energy neutron - nucleus collision processes are shown. Here $E_p$ is $p$-wave resonance energy and $E_s$ is the nearest $s$-wave resonance energy.

| target nucleus | $E_p$[eV]   | $E_s$[eV]   | $A_L$             |
|----------------|-------------|-------------|-------------------|
| $^{81}_{35}$Br($\frac{3}{2}$)$^-$ | 0.88 ± 0.01 | 101.10 ± 0.14 | + 0.021 ± 0.001   |
| $^{93}_{41}$Nb($\frac{9}{2}$)$^+$   | 35.9 ± 0.1  | 119.2 ± 0.2  | + 0.003 ± 0.005   |
| $^{108}_{46}$Pd(0)$^+$              | 2.69 ± 0.01 | 33.10 ± 0.17 | + 0.002 ± 0.002   |
| $^{124}_{50}$Sn(0)$^+$              | 62.0 ± 0.1  | −20          | + 0.002 ± 0.004   |
| $^{139}_{57}$La($\frac{9}{2}$)$^+$ | 0.734 ± 0.005 | −48.63 | + 0.098 ± 0.003 |
| $^{93}_{41}$Nb($\frac{9}{2}$)$^+$   | 42.3 ± 0.1  | −105.39      | − 0.000 ± 0.006   |
| $^{111}_{48}$Cd($\frac{1}{2}$)$^+$ | 4.53 ± 0.03 | −4           | − (0.013 + 0.007, −0.004) |
Table II

This Table II shows the $p$-wave reduced width $\gamma_p^2$ from experimental data and the matrix elements of Eq. (3.3) $<\phi_p|V_X|\Phi_g>$ for example of Table I.

| target nucleus | $\gamma_p^2$ | $<\phi_p|V_X|\Phi_g>$ [eV] |
|----------------|-------------|-----------------------------|
| $^{81}_{35}$Br($\frac{3}{2}^-$) | $5.8 \times 10^{-5}$ | $-0.1$ |
| $^{93}_{41}$Nb($\frac{5}{2}^+$) | $2.1 \times 10^{-4}$ | $-1.2$ |
| $^{108}_{46}$Pd(0)$^+$ | $8.8 \times 10^{-4}$ | $-1.4$ |
| $^{124}_{50}$Sn(0)$^+$ | $1.1 \times 10^{-2}$ | $-1.6$ |
| $^{139}_{57}$La($\frac{7}{2}^+$) | $4.1 \times 10^{-5}$ | $-1.7$ |
| $^{93}_{41}$Nb($\frac{7}{2}^+$) | $1.3 \times 10^{-4}$ | $-1.2$ |
| $^{111}_{48}$Cd($\frac{1}{2}^+$) | $1.3 \times 10^{-4}$ | $-1.4$ |
Fig. 1. $Z^0$ boson exchange interaction of nucleon-nucleus reaction [in the simplest (one particle) shell model].