The modern industry is dominated by electric drives of the system frequency converter-induction motor, which employ integer-order controllers. Allowing the implementation and adjustment of fractional order controllers in the converter itself greatly expand their capabilities and, therefore, is relevant. This paper reports a procedure for the parametric synthesis of PFD*-controllers of fractional order, providing for the use of the desired forms of fractional order, as well as their practical implementation in the system frequency converter-induction motor. In this case, the control object is described by a transfer function of the fractional or integer order, derived on the basis of experimental results. The study results demonstrate the possibility of constructing new, as well as modernizing existing, electromechanical systems involving the fractional-order PFD*-controllers with an expanded range of dynamic properties that correspond to the desired forms of fractional order. The procedure for the parametric synthesis of a PFD*-controller has been theoretically substantiated, which was confirmed by applying the simulation and during field experiments concerning the system frequency converter-induction motor. The reported procedure is universal because it makes it possible to synthesize the PFD*-controller for standard forms of both the integer and fractional orders. It is clear that the range of the desired standard forms in the synthesis process can include all possible known forms, including those of fractional order. The result of this study allows us to argue that it is possible to apply the developed algorithm of actions for those engineering tasks that aim to build such systems for various industrial mechanisms. At the same time, no restrictions are imposed on the transfer function of the control object.

Keywords: frequency converter, induction motor, fractional order controller, system, parametric synthesis

1. Introduction

The procedure to synthesize electromechanical systems (EMS) involves a two-aspect issue. The first concerns the synthesis of controllers under a condition that the dynamic processes in the object of control are described by a known transfer function of the fractional or integer order. The second relates to the synthesis under a condition that the mathematical model of the dynamic processes of the object of control is not known. For an EMS of the integer-order, the first issue is almost resolved; it is based on the procedure of the structurally parametric synthesis of modal or subordinate regulation systems. The second issue for such systems can be resolved by intelligent methods, in particular, by the method of genetic algorithm (GA) provided the structure of the integer controller is specified. It is obvious that the parametric synthesis of such a controller is implied here. The task of EMS synthesis is considered to be completed once the dynamic processes in it correspond to those specified in advance. For an integer order EMS, the formalized representation of the desired dynamic processes corresponds to the standard forms of distribution of the poles of the transfer function on a complex plane. It should be noted that for an EMS of the fractional order, those can include the same standard forms of the integer order or the two forms of the fractional order proposed in works [1, 2]. Expressions for their transfer functions (TF) are as follows:

\[ W_{st}(s) = \frac{\omega_{o1}/K_1}{s^q + \omega_{o1}} \]  
\[ W_{s2}(s) = \frac{\omega_{o2}/K_2}{(s + \omega_{o2})^q} \]

where \( q \) is the fractional order of the characteristic polynomial, \( \omega_{o1}, \omega_{o2} \) is the mean-geometric root of the corresponding desired form, which determines the performance of the system, \( K_1 \) is the coefficient of feedback amplification based on the regulation coordinate.

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Regardless of the desired form of a fractional-order EMS, the controllers of such systems will almost always be of the fractional order. For the case when an object is described by
the integer-order TF, the structurally parametric synthesis of fractional order controllers for the desired forms of the integer and fractional order was reported in work [3]. It is obvious that the result of the structurally parametric synthesis of fractional order controllers necessitates their implementation in actual electric drives.

It is well known that the industry is dominated by the electric drives “Frequency Converter - Induction Motor” (FC-IM). They implement, specifically FC, the integer order controllers. Allowing the implementation of fractional order controllers in the converter itself greatly expands their capabilities and, therefore, is relevant. It is clear that this should be accompanied by an engineering procedure for setting their parameters. By analogy with an integer-order EMS with a PID controller, the most common case of a fractional-order controller may involve a fractional order $\text{PI}^\lambda$-$\text{D}^\mu$-controller with a TF

$$W_\lambda(s) = k_p + k_i s^{-\lambda} + k_d s^\mu. \quad (3)$$

On the other hand, if the fractional controller is built into a frequency converter, it can be considered together with the converter, in certain cases, as part of the control object. Then there is the issue related to the parametric synthesis of such a controller.

According to the recent research, the use of fractional order $\text{PI}^\lambda$-$\text{D}^\mu$-controllers in the EMS of FC-IM makes it possible to improve the quality of transition processes and increase the margin of stability, compared to similar systems, which employ classical (integer order) PID-controllers. The further application of $\text{PI}^\lambda$-$\text{D}^\mu$-controllers to manage various industrial processes instead of the integer-order PID-controllers can expand only if the set of scientific and applied tasks is resolved. The above set of scientific and applied issues is related to the synthesis and implementation of such systems, as well as methods of their configuration. However, at present, the task of the parametric synthesis of the fractional-order $\text{PI}^\lambda$-$\text{D}^\mu$-controllers, as well as the research into their capabilities for various automated EMS, require further development.

In addition, another modern trend in the design and development of new EMS is the use of not only integrated but other optimization criteria as well. This requires that researchers aiming to solve the tasks of EMS analysis and synthesis should turn to new methods and tools, including such an intelligent method as a genetic algorithm (GA). The genetic algorithm is applied to find an extremum of complex functions. Given that the search space in such optimization problems is almost not limited, the resulting solution would suffice in practice, which sufficiently satisfies the meaning of a given problem. Together with the Fuzzy Logic and Neural Network packages implemented in the MATLAB software, GA is an effective tool for solving many problems.

The use of the fractional-order $\text{PI}^\lambda$-$\text{D}^\mu$-controllers in EMS makes it possible to improve the quality of dynamic processes and, in addition, to increase the margin of stability compared to similar EMS that employ the integer order controllers. However, to date, the task of implementing the fractional-order $\text{PI}^\lambda$-$\text{D}^\mu$-controllers, as well as studying their capabilities in automated EMS, continues to be relevant.

2. Literature review and problem statement

Work [4] considers an approach to the synthesis of fractional order $\text{PI}^\lambda$-$\text{D}^\mu$-controllers for the circuits in automated control systems based on a particle swarm method. However, the work did not address the task of identifying a real control object, in particular the FC-IM system, using a TF of the integer or fractional order, on the basis of experimental research. Based on practical experience, it was found that in practical tasks of synthesizing the fractional order controllers, the genetic algorithm operates slightly faster than the particle swarm method.

The authors of [5, 6] show a wide range of GA method applications, including the possibility to solve various optimization problems, in particular for the synthesis of controllers. However, they did not consider the task of identifying an actual control object by using a fractional order TF. The use of the GA method for the synthesis of fractional order controllers was not considered.

It should be noted that the use of fractional order controllers in the AC electric drives was considered in works [7, 8], which show the advantages of applying the fractional-order $\text{PI}^\lambda$-controllers in the system that controls a synchronous motor with permanent magnets. Owing to this, the authors derived the desired dynamic characteristics of the system while ensuring its robustness. However, the works did not address the issue of identifying an actual control object by using a TF of the integer or fractional order based on experimental research. In addition, the authors did not consider, in the synthesis of the relevant controllers, the desired forms of the fractional order in order to ensure the desired dynamic characteristics of electric drives.

Studies [9–11] show that in closed control systems of the speed and position of DC engines (DCE) using the fractional-order $\text{PI}^\lambda$-controllers in single-circuit systems is more effective compared to classical PI-controllers. However, the cited studies apply only to systems with DC engines. In addition, for such a class of EMS, the authors of [12, 13] compared different methods of evolutionary optimization to configure the controllers of both fractional and integer order, in the system that controls speed of the DCE. Three main procedures to synthesize the controllers were considered: a genetic algorithm, the particle swarm optimization, and differential evolution. The studies performed a comparison based on the quality indicators and stability of the developed system. However, the cited studies considered control over the speed of DCE only, which limits the application scope of their results.

The authors of work [14], when designing a new control system for an electric vehicle’s IM involving a $\text{PI}^\lambda$-controller, concluded that the properly synthesized and implemented fractional order controller ensures better characteristics than the classic PID-controller. In this work, however, the issue of identifying an actual control object by using a compact TF of the fractional order based on experimental research was not investigated. In the synthesis of fractional order controllers, the desired forms of fractional order to ensure the desired dynamic characteristics of electric drives were not considered.

Studies [15, 16] examined the approximation (replacement) of the high-order integer transfer function using simpler transfer functions of different types with three or five variable parameters. To this end, the particle swarm optimization method was used. The accuracy of such substitution was assessed by comparing transitional and frequency characteristics. It was proven that the approximation of a high-order TF, obtained experimentally, in the excitation current channel in an asynchronous machine, which op-
erates in the mode of a generator with self-excitation, is possible with the help of simpler models of fractional order. The studies addressed only the task of identifying an actual control object using a TF of the fractional order based on the results of experimental research. However, no issues related to the synthesis of controllers were considered.

There are cases of GA application as a universal optimization method, for the correction of a PI\( ^{D\mu} \)-controller’s parameters. It is shown in [17] that GA is effective at optimizing the parameters of integer order controllers. In the cited paper, the task to identify an actual control object by using a fractional order TF was not investigated. In addition, the authors neglected the issue of using a GA method for the synthesis of fractional order controllers for EMS. It should be noted that papers [18, 19] considered various options for the practical application of GA, including to solve optimization problems. However, the papers did not address the issue of identifying an actual control object using a TF of the fractional order and the synthesis of the relevant controllers.

Work [20] proposes a new approach to the design of the PI\( ^{D\mu} \)-controller in an asynchronous electric drive. In the work, the motor’s drive was modeled by using auto-regression with an input model whose parameters were experimentally identified using actual dynamic characteristics. Then, with the help of GA, the values of the parameters for such a controller were found. The experimental results show that the proposed controller significantly improves the dynamic characteristics of an asynchronous electric drive compared to the conventional PID-controller. However, in the work, the IM was modeled by an integer transfer function of the high order. Additionally, only the inverter was considered rather than the entire FC as a control object.

Study [21] proved that a fractional controller can more accurately operate a complex system than a traditional PID controller. A neural network of backpropagation was selected to optimize the controller’s parameters. The synthesis of an optimal PI\( ^{D\mu} \)-controller for the IM rotation speed control system by the evolutionary method of the cuckoo search was carried out in work [22]. The comparative analysis performed in these works concerning EMS with two types of controllers showed that the optimized fractional PI\( ^{D\mu} \)-controller ensures better indicators of damping the fluctuations in the coordinates of the system control. In addition, its application makes it possible to obtain better dynamic characteristics than the traditional PID-controller. The cited works do not show how to introduce fractional controllers into the FC–IM system.

Work [23] reports the results of developing a PI\( ^{D\mu} \)-controller of the fractional speed order for vector induction motors, where only possible changes in parameters and perturbation of load are considered. Based on modeling and experimental results, it was found that compared to the usual PID-controller, the proposed fractional-order PI\( ^{D\mu} \)-controller can ensure better robustness to the change in parameters and load perturbations. However, the cited work did not address the task of ensuring the specified dynamic characteristics of the system.

Paper [24] proposed using the transformation of Oustaloup of the first and second order to implement a fractional order controller for EMS. The possibility of such controllers functioning in actual time was proven. Thus, the authors created the preconditions for the synthesis and construction of fractional controllers in the FC–IM system. All this allows us to argue that the development of a procedure for the parametric synthesis of fractional order controller and their implementation in the FC–IM system is a relevant task.

Given a large body of research, a series of issues have remained unaddressed regarding the tasks to analyze, synthesize, and implement EMS, in particular FC–IM systems with the fractional order PI\( ^{D\mu} \)-controllers. Therefore, it is reasonable to conduct a study that could resolve these issues.

### 3. The aim and objectives of the study

The aim of this study is to develop a procedure for the parametric synthesis of the fractional-order PI\( ^{D\mu} \)-controllers and their implementation in the FC–IM system. This would make it possible to build new and modernize existing EMS with the fractional order PI\( ^{D\mu} \)-controllers with an expanded range of the dynamic properties that correspond to the desired forms of the fractional order.

To accomplish the aim, the following tasks have been set:
- to modernize a method of the parametric synthesis of PI\( ^{D\mu} \)-controllers in EMS by using a GA method to find the controllers’ parameters and identify the object of control;
- to devise a procedure for the parametric synthesis of PI\( ^{D\mu} \)-controllers in EMS based on the developed modernized method;
- to synthesize and implement the PI\( ^{D\mu} \)-controller for FC–IM as part of a typical FC, followed by an experimental study of the dynamic characteristics of the obtained EMS.

### 4. Materials and research methods

#### 4.1. The parametric synthesis of fractional-order PI\( ^{D\mu} \)-controllers using a genetic algorithm method

The structural and parametric synthesis of fractional controllers can produce [3] different TFs of the fractional order. Typically, they can be interpreted as a partial case of one or more, appropriately connected, PI\( ^{D\mu} \)-controllers. Therefore, one can immediately, by setting the structure of the universal PI\( ^{D\mu} \)-controller, proceed to the parametric synthesis of such controllers. Thus, it is possible to unify the type of controller to be introduced as a component in the object of control, in particular in FC. Should the result of the structural and parametric synthesis reveal that the TF differs from the PI\( ^{D\mu} \)-controller, it would be possible to implement it by adjusting the values (including zero) of parameters for the universal PI\( ^{D\mu} \)-controller.

For the parametric synthesis of EMS, an adaptive-based approach is suggested. It employs the transitional functions of the reference model and the actual system. Re-adjustment of the closed regulation system is carried out in accordance with the established parameters for PI\( ^{D\mu} \)-controllers based on the comparative analysis of these transitional functions. It is proposed to use the transfer functions (1), or (2) of the desired fractional form, as a reference model. In turn, the parametric synthesis of PI\( ^{D\mu} \)-controllers is carried out by the evolutionary GA method. The general functional scheme of such a procedure to synthesize a PI\( ^{D\mu} \)-controller is shown in Fig. 1.

A setting influence is sent to the input of the PI\( ^{D\mu} \)-controller. At the output of the control object \( W_c(\lambda) \), a transitional function \( Y(t) \) is obtained, which is compared to the transitional function of the reference model \( Y^*(t) \), that is, to the desired dynamic characteristic of the system. The signal
from the output of the error assessment unit triggers the algorithm of the genetic algorithm, according to which the parameters of the $\text{PI}^\lambda \text{D}^\mu$-controller are derived to minimize the error, that is, to ensure the desired form of the original transition process.

2) the reference model is entered into the computer memory, the dynamic characteristics of which correspond to the desired fractional form;

3) the initial parameters of the $\text{PI}^\lambda \text{D}^\mu$-controller are set;

4) we start the process of searching for parameters of the fractional controller of the system using the GA method, which could ensure the desired transition process;

5) the iterative process of the GA method is terminated when the value of the selected functionality of quality $J$ becomes less than the specified value, or can be set manually;

6) after determining the parameters of the controller, it is necessary to configure the appropriate setting of the $\text{PI}^\lambda \text{D}^\mu$-controller.

This synthesis procedure was implemented in relation to the FC–IM electric drive. Here, the object of control is the FC–IM itself. Then there are two possible cases of EMS construction using FC–IM. The first case involves the possibility of implementing the functional scheme shown in Fig. 1, in actual time. In this case, there is no need to determine $W_d(s)$ because whatever the form of the transitional function of the actual object of control is, the GA method will ensure that the appropriate parameters of the $\text{PI}^\lambda \text{D}^\mu$-controller are defined, owing to which the desired dynamic characteristics are obtained. In the second case, the situation is considered when there is no possibility to implement the synthesis and configuration of the system in real time. Then the parameters of the controller are first synthesized on some computing device, and then the appropriate settings of the $\text{PI}^\lambda \text{D}^\mu$-controller are implemented. In this case, there must be a mathematical model of the control object.

Consider the second case in detail because in actual engineering tasks, due to various reasons, this situation is most commonly found. Fig. 3 shows the functional circuit of EMS in FC–IM, which implies feedback from the angular velocity of the motor ($\omega_m$).

The parameters of the controller are entered using the keyboard of the default FC, which traditionally includes the integer order controller.

For the case concerning the synthesis of the $\text{PI}^\lambda \text{D}^\mu$-controller, provided that the mathematical model of the dynamic processes of the object of control is unknown, it is necessary to first identify this object. Identification precedes the procedure of parametric synthesis.

For most objects of control, their mathematical model can be represented by the transfer functions of different order. Works [15, 16] consider the possibility of replacing (approximation) of the integer transfer function of a high order by means of the transfer functions of fractional order of different types with three, or with five variable parameters, namely:

- $\text{PI}^\lambda \text{D}^\mu$-controller
- $\text{PD}^\mu$-controller
- $\text{PI}^\lambda \text{D}^\mu$-controller
- $\text{PD}^\mu \text{D}^\lambda$-controller

For the case of the parametric synthesis of an integer PID-controller, or partial cases of the fractional PID-controller, for example, $\text{PI}^\lambda$, $\text{PD}^\mu$, a chromosome will have, respectively, only three genes.

4.2 A procedure of the parametric synthesis of $\text{PI}^\lambda \text{D}^\mu$-controllers

The process of the parametric synthesis of an integer PID-controller, or partial cases of the fractional PID-controller, has been proposed for the parametric synthesis of a $\text{PI}^\lambda \text{D}^\mu$-controller with its subsequent configuration. The sequence of activities in accordance with this procedure is as follows:

1) the parameters of the $\text{PI}^\lambda \text{D}^\mu$-controller (phenotype) are encoded using the chromosome so that the length of the chromosome is proportional to the number of parameters. The GA method functions faster at a small length of the chromosomes, so it is desirable, if possible, to minimize the number of configuration parameters and the range of their changes;
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\[ W(s) = \frac{k}{a_1s^2 + 1}. \]  \hspace{1cm} (4)

\[ W(s) = h_1s^3 + 1. \]  \hspace{1cm} (5)

Finding TF parameters (4) or (5) can be done by one of the intelligent methods. To preserve a single methodology for the parametric synthesis and identification of the object, the GA method was used. The features of this method are discussed above, and, in relation to the procedure of identification of the control object in Fig. 2, one must enter parameters corresponding to TF (4) or (5). It is obvious that for TF (4), the number of genes is three, and for TF (5) – five.

The results of studying the synthesized EMS in FC–IM

The control object selected was a typical FC, the TWERD MFC 710 type, which drives an IM of the type 4AHB2P1000L. The FC parameters: \( P_l = 3.0 \text{ kW} \); encoder: Kubler 8.3620.546E.1024; DPS: P22, \( P_l = 1 \text{ kW} \), \( n_l = 1,500 \text{ rpm} \), \( U_l = 220 \text{ V} \), \( I_l = 5.9 \text{ A} \). The IM settings: \( P_l = 4.0 \text{ kW} \), \( n_l = 420 \text{ rpm} \), \( U_l = 380 \text{ V} \), \( I_l = 8.7 \text{ A} \), \( \cos \phi = 0.84 \), \( \eta = 84 \% \). The FC–IM was loaded using a DC machine in a generator mode. It should be noted that the FC control system of the type TWERD MFC 710 employs the MODBUS interface. Owing to it, it is possible to control the frequency converter in any automatic control system. If necessary, one can use a programmable logic controller (PLC), microcontroller, computer, or specialized microprocessor device to control such FC.

The designed test bench is shown in Fig. 4.

Similarly, experimental studies were conducted to launch the electric drive to other steady speed values, namely 750 rpm, 900 rpm, and 1,500 rpm.

The approximation of the obtained experimental transitional processes by a fractional TF was carried out using the GA particles.

It should be noted that the oscillogram of the transition process in the FC–IM system that was initially acquired, for example, for a speed of 1,200 rpm (Fig. 5), must be prepared for approximation. To this end, one needs to perform the following activities:

- match the beginning of the acquired transition process to the time equal to zero;
- to convert the scale of the transition process from the range of 0–5 V, at which the Arduino Mega 2560 (Italy) board operates, to the range of 0–10 V, at which the MFC1000/10 board (Poland) operates, which implements the fractional PI\(_{D\alpha}\)-controller and FC MFC 710, by multiplying each point of the resulting transition process by the appropriate scale factor 2.

The transitional process of IM speed prepared for approximation corresponds to curve 1 in Fig. 6.

The selected FC–IM system with such parameters is widely used in machine electric drives where the speed of working tools is most often used that corresponds to motor rotation frequencies from 750 rpm to 1,500 rpm. Therefore, it was in this range of changes in motor speed that our experimental studies were conducted.

For the case of an unknown mathematical model of the control object, in the first stage of synthesizing a fractional order controller, it is necessary to identify this object. Therefore, initially, the transitional process of launching the open FC–IM system was experimentally obtained. The oscillogram of the corresponding speed transition process is shown in Fig. 5. It reflects the process of launching the electric drive to a steady speed at 1,200 rpm. The information was acquired at the output of a speed sensor using the Board Arduino Mega 2560 (Italy) and was recorded in memory.

\[ W(s) = \frac{4.101}{0.961s^{0.303} + 1}. \]  \hspace{1cm} (6)

Fig. 6 (curve 2) shows the transitional start-up process of FC–IM obtained for the object of control approximated
by expression (6). The rms deviation ($\sigma_n$) of the resulting approximating transitional function from the experimental one is 0.132. This value can be slightly reduced if one uses TF (5).

It should be noted that such identification of the object of control can be used during the structural and parametric synthesis of fractional controllers when the TF of the object of control or its parts are not known.

Following the identification of the control object, it is necessary to proceed to the parametric synthesis of a PI$^\lambda$D$^\mu$-controller.

We have synthesized the PI$^\lambda$D$^\mu$-controller by a GA method for the above FC–IM. We shall impose a requirement that the dynamic process of starting the closed EMS FC–IM should be characterized by the following parameters: overshooting $\delta=0$ %, the time of reaching 0.95 of the steady speed $t_{0.95}=3.01$ s. According to [1], these parameters are ensured by the desired form (1) with the following parameters $q=1.0$ and $\omega_{oc}=1$ s$^{-1}$. It is obvious that this is an integer order TF. Thus, it is necessary to solve the problem using a fractional controller – to translate the fractional-order PT-IM into a closed integer-order EMS.

The MATLAB software environment, based on the application of the optimization package Optimization Tool, the corresponding GA program is implemented.

Fig. 7 shows the results of the iterative process, which occurs during the synthesis of the parameters of a PI$^\lambda$D$^\mu$-controller for EMS in accordance with the desired dynamic process. We compare the results of two transitional processes at each iteration using the $J$ quality functionality, where

$$J = \sum_{i=1}^{n} |y_i - Y_i|,$$

The assigned population size is 100.

The result of the iterative process is:

$$W_p(s) = 2.821 + 3.366s^{0.031} + 1.028s^{1.014}.$$  \hspace{1cm} \text{(7)}

Fig. 8 shows the simulated transitional functions of the desired form (curve 1), as well as the transitional functions of the closed EMS with the synthesized controller (curve 2). The control object is characterized by curve 3.

The practical implementation of such a controller using FC of the type MFC710 with a fractional PI$^\lambda$D$^\mu$-controller is not problematic.

Another variant of the parametric synthesis of a PI$^\lambda$D$^\mu$-controller with the following parameters of the desired dynamic process was also considered: $\delta=7.3$ % and $t_{0.95}=1.92$ s. This is provided by the desired form with the parameters $q=1.2$ and $\omega_{oc}=1$ s$^{-1}$ [1]. This variant of synthesis is very similar to the previous one, but here the desired form is described by a TF of the fractional order.

Accordingly, Fig. 11 shows the results of the iterative process, which occurs during the parametric synthesis of parameters for a PI$^\lambda$D$^\mu$-controller in a given fractional EMS.

When the synthesis process is over, the following transfer function of the PI$^\lambda$D$^\mu$-controller is obtained

$$W_p(s) = 3.349 + 3.763s^{-\lambda} + 0.619s.$$  \hspace{1cm} \text{(8)}

We have checked the performance of the synthesized FC–IM system in an extended range of speeds of 400–1500 rpm based on the results of appropriate experimental studies for the same parameter values of the desired dynamic processes. The results from physical experiments are shown in Fig. 10.

The results obtained have fully confirmed the solution to the set task.

**Fig. 9 (curve 1) shows the oscillogram of the transitional process of FC–IM speed, derived from a physical experiment, and curve 2 represents the desired form.**

**Fig. 10 (curve 1) shows the oscillogram of the transitional process of FC–IM speed, derived from a physical experiment, and curve 2 represents the desired form.**

**Fig. 11 (curve 1) shows the oscillogram of the transitional process of FC–IM speed, derived from a physical experiment, and curve 2 represents the desired form.**

**Fig. 12 (curve 1) shows the oscillogram of the transitional process of FC–IM speed acquired from a physical experiment, and curve 2 represents the desired form.**
from a physical experiment, and curve 2 represents the desired fractional form.

We have conducted similar studies of the closed EMS in FC–IM for different values of steady speed for the same parameter values of the desired dynamic processes of the fractional order. The results from physical experiments are shown in Fig. 13.

Modernizing a method of the parametric synthesis of PI\(^{\lambda D^\mu}\)-controllers implies the use of additional standard forms of the fractional order (1), (2). Provided that the \(q\) parameter is an integer, we come to the standard forms of the integer order (by Butterworth or a binomial form). Thus, it is evident that integer forms are a partial case of the fractional order forms. This expands the range of EMS dynamic characteristics. In the process of the parametric synthesis of a PI\(^{\lambda D^\mu}\)-controller, such forms as the reference model are laid down in the synthesis procedure, which is represented in Fig. 2.

On the other hand, when the mathematical model of the control object is unknown, it is identified according to data on the transition function. For the case of AC EMS, in particular FC–IM, the integer order TF \(W_0(s)\) is approximated by the fractional-order TF, for example (4) or (5). Even in the case of simple expression (4), where there are only three unknown parameters, the rms deviation of the resulting approximating transitional function from the experimental one is 0.132. The search for these three parameters involves the same method as is used during the synthesis of a PI\(^{\lambda D^\mu}\)-controller, namely: by a GA method. Thus, the parametric synthesis of the controller and the identification of the control object involve a single methodological basis.

Works [20–23] used a PI\(^{\lambda D^\mu}\)-controller in the FC–IM system, but, to identify the control object, the authors applied a high-order integer transfer function, which is much more complex than the proposed fractional order TF. Therefore, the parametric synthesis of the controllers themselves is greatly complicated. In addition, the dynamic characteristics of such systems do not correspond to the desired. They are obtained in accordance with the accepted parameters of fractional controllers, and not in accordance with the desired fractional or integer forms.

The procedure of the parametric synthesis of a PI\(^{\lambda D^\mu}\)-controller implies the execution of six steps. The main provisions of this procedure are substantiated in the previous chapters. They are formulated in such a way
that the proposed procedure for the synthesis of fractional controllers can be used to solve a wide range of engineering problems of this type. For example, it shows how this is done in relation to the FC–IM system, where the requirements for dynamic processes to be ensured are specified. If necessary, the proposed procedure can be supplemented with the identification of the control object. It should be noted that these six points relate specifically to the synthesis of the PI\(^\lambda\)D\(^\mu\)-controllers, considered in this work; therefore, such a procedure is not reported by other literary sources.

In order to confirm the correctness of the decisions taken, we have synthesized and experimentally investigated PI\(^\lambda\)D\(^\mu\)-controllers for FC–IM. The implementation of such EMS required the identification of the control object. The approximation of TF by expression (6) allowed an adequate representation of FC–IM during the synthesis of a fractional speed controller in a closed EMS. If one needs a higher approximation accuracy, it is possible to use expression (5). For the case of approximating the control object using TF (6), the PI\(^\lambda\)D\(^\mu\)-speed controllers of the closed FC–IM system for two different desired forms were synthesized. For \(q=1\), TF (7) of the controller was obtained, and for \(q=1.2\), TF (8) of the controller was obtained. The simulation and experimental results are represented in Fig. 8, 10, and Fig. 12, 13, respectively: they confirm the correctness of the synthesis of such EMS and the possibility of implementing the synthesized PI\(^\lambda\)D\(^\mu\)-controllers using the MFC1000/10 board (Poland). It should be noted that not all typical FCs are equipped with such boards. Therefore, this should be considered when selecting a converter as part of an EMS of the fractional order. Such synthesized and implemented fractional EMS should be analyzed in terms of their treatment of perturbations. It is in this area that our further work is planned.

### 7. Conclusions

1. We have modernized a method for the parametric synthesis of fractional-order PI\(^\lambda\)D\(^\mu\)-controllers for EMS. The modernization implies that a GA method employed as a reference model the desired standard form of the fractional or integer order. Thus, the search for the parameter values for controllers is purposeful. Owing to a large number of standard forms, the implementation of a wide range of the dynamic characteristics of a fractional-order EMS is ensured. A given synthesis method implies the use of identification of the control object by applying a fractional TF. Therefore, the mathematical model of such an object is greatly simplified, which, together with its approximation by the GA method, helps reduce the time to achieve the result of a parametric synthesis.

2. The use of the modernized method for the parametric synthesis of fractional controllers to build the relevant EMS involves a certain algorithm of actions, which is represented by the devised procedure of six steps together with the identification of the control object. Thus, an engineering procedure has been developed to implement this particular method for the parametric synthesis of fractional-order PI\(^\lambda\)D\(^\mu\)-controllers for EMS.

3. For a typical FC–IM electric drive, a fractional speed controller was synthesized: the simulation and experimental studies of a given EMS were conducted. The results confirmed the applicability of the proposed modernization of the method of parametric synthesis. Ensuring any desired dynamic characteristic of the created system was carried out by changing the parameters of PI\(^\lambda\)D\(^\mu\)-controllers using a keyboard (Fig. 3) of the MFC1000/10 board (Poland). These parameters were found on the basis of the results from identifying the FC–IM as a control object, and the synthesis of the controller in accordance with the desired form of random order. A typical TWERD MFC 710 converter is equipped with such a board and this made it possible to use the synthesized fractional controllers in the actual FC–IM system.

### References

1. Marushchak, Y., Kopchak, B. (2015). Synthesis of Automatic Control Systems by Using Binomial and Butterworth Standard Fractional Order Forms. Computational problems of electrical engineering, 5 (2), 89–94.

2. Lozynskyy, O., Lozynskyy, A., Kopchak, B., Paranchuk, Y., Kalenyuk, P., Marushchak, Y. (2017). Synthesis and research of electromechanical systems described by fractional order transfer functions. 2017 International Conference on Modern Electrical and Energy Systems (MEES). doi: https://doi.org/10.1109/mees.2017.8248877

3. Kopchak, B., Marushchak, Y., Kushner, A. (2019). Devising a procedure for the synthesis of electromechanical systems with cascade-enabled fractional-order controllers and their study. Eastern-European Journal of Enterprise Technologies, 5 (2 (101)), 65–71. doi: https://doi.org/10.15587/1729-4061.2019.177320

4. Kopchak, B. (2015). Synthesis of automatic control systems by a particle swarm optimization method using butterworth fractional standard forms. 2015 16th International Conference on Computational Problems of Electrical Engineering (CPEE). doi: https://doi.org/10.1109/cpee.2015.7333342

5. Hall, M. (2012). A Cumulative Multi-Niching Genetic Algorithm for Multimodal Function Optimization. International Journal of Advanced Research in Artificial Intelligence, 1 (9), 6-13. doi: https://doi.org/10.14569/ijarai.2012.019002

6. Malhotra, R., Singh, N., Singh, Y. (2011). Genetic Algorithms: Concepts, Design for Optimization of Process Controllers. Computer and Information Science, 4 (2), 39–54. doi: https://doi.org/10.5539/cis.v4n2p39

7. Zheng, W., Wang, X., Pi, Y. (2015). Study of the fractional order proportional integral controller for PMSM based on differential evolution algorithm. 2015 IEEE Advanced Information Technology, Electronic and Automation Control Conference (IAEAC). doi: https://doi.org/10.1109/iaeac.2015.7428547

8. Lino, P., Maione, G., Salvatore, N., Stasi, S. (2016). Fractional-order PI control of PMSM drives in nested loops. Conference: ICFDA 2016 – IEEE International Conference on Fractional Differentiation and its Applications. Novi Sad, 333–342.

9. Ruszewski, A., Sobolewski, A. (2013). Position control of DC motor using fractional order controller. Archives of Electrical Engineering, 62 (3), 505–516. doi: https://doi.org/10.2478/aae-2013-0041
10. Leuzzi, R., Lino, P., Maione, G., Stasi, S., Padula, F., Visioli, A. (2014). Combined fractional feedback-feedforward controller design for electrical drives. ICFDA'14 International Conference on Fractional Differentiation and Its Applications 2014. doi: https://doi.org/10.1109/icfda.2014.6967380
11. Copot, C., Muresan, C., Keyser, R. (2013). Speed and position control of a dc motor using fractional order PI-PD control. In Proc. 3rd International Conference on Fractional Signals and Systems – FSS 2013. Ghent.
12. Ahuja, A., Aggarwal, S. (2014). Design of fractional order PID controller for DC motor using evolutionary optimization techniques. WSEAS Transactions on systems and control, 9, 171–182.
13. Ahuja, A., Tandon, B. (2014). Design of Fractional Order PID controller for dc motor using Genetic Algorithm. TELKOMNIKA Indonesian Journal of Electrical Engineering, 12 (12). doi: https://doi.org/10.11591/telkomnika.v12i12.6470
14. Bendjedia, M., Tehrani, K. A., Azzouz, Y. (2014). Design of RST and Fractional order PID controllers for an Induction motor drive for Electric Vehicle Application. 7th IET International Conference on Power Electronics, Machines and Drives (PEMD 2014). doi: https://doi.org/10.1049/cp.2014.0445
15. Kopchak, B. (2017). Approximation accuracy of electromechanical systems high order objects using different types of fractional order transfer functions. 2017 XIIIth International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH). doi: https://doi.org/10.1109/memstech.2017.7937544
16. Kopchak, B., Kopchak, M. (2018). Application of fractional order transfer function with zero and pole in approximation of electromechanical systems high order objects. 2018 XIV-Th International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH). doi: https://doi.org/10.1109/memstech.2018.8365694
17. Burakov, M. V. (2008). Geneticheskiy algoritm: teoriya i praktika. Sankt-Peterburg: GUAP, 164.
18. Haupt, R. L., Haupt, S. E. (2003). Practical genetic algorithms. John Wiley & Sons. doi: https://doi.org/10.1002/0471671746
19. Konak, A., Coit, D. W., Smith, A. E. (2006). Multi-objective optimization using genetic algorithms: A tutorial. Reliability Engineering & System Safety, 91 (9), 992–1007. doi: https://doi.org/10.1016/j.ress.2005.11.018
20. Saleem, A., Soliman, H., Al-Ratrout, S., Mesbah, M. (2018). Design of a fractional order PID controller with application to an induction motor drive. TURKISH JOURNAL OF ELECTRICAL ENGINEERING & COMPUTER SCIENCES, 26 (5), 2768–2778. doi: https://doi.org/10.3906/elk-1712-183
21. Deng, W., Zhao, H., Yang, X., Li, X., Dong, C. (2016). An optimized fractional order PID controller for suppressing vibration of AC motor. Journal of Vibroengineering, 18 (4), 2205–2220. doi: https://doi.org/10.21595/jve.2016.16652
22. Thammarat, C., Puangdownreong, D. (2019). Design of Fractional Order PID Controller for Induction Motor Speed Control System by Cuckoo Search. International journal of circuits, systems and signal processing, 13, 92–96.
23. Chang, Y., Wu, C., Lin, H., Hsu, C., Liao, G. (2009). Design of fractional-order PID controller for vector-controlled induction motors. In Proc. of the 9th WSEAS international conference on Robotics, control and manufacturing technology (ROCOM’09). Hangzhou, 142–147.
24. Kopchak, B. (2016). Development of fractional order differential-integral controller by using Oustaloup transformation. 2016 XII International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH). doi: https://doi.org/10.1109/memstech.2016.7507521