Abstract

A detailed analysis on the rare $\tau$ decay via $\tau \rightarrow \mu e\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\mu}$ in the string models with $E_6$ symmetry is reported. It is found that $\Gamma(\tau \rightarrow \mu e\bar{e}) \sim (6-7)\Gamma(\tau \rightarrow \mu\mu\bar{\mu})$ and these rates are in general about 1000 times less than that of $\Gamma(\mu \rightarrow e\gamma)$. It is also found that the out-going muon in $\tau \rightarrow \mu e\bar{e}$ is almost 100% right-handed polarized and the out-going electrons would be predominately parallel to each other. These decay processes may be accessible at the SSC.
1. Introduction  The existence of a huge number of the degenerate vacua makes it difficult to investigate the experimental consequences of the superstring theory. Nevertheless, progress has already been made in extracting the phenomenological implications of the heterotic string models [1,2,3], despite the lack of a theoretical framework to determine the true vacuum of the theory. Here, one takes a less fundamental attitude by imposing the phenomenological requirements that the theory must reduce to the standard model at low energies [4].

In this paper, we focus on a class of three-generation heterotic string models which have a symmetry structure
\[ E_6 \times (N = 1 \text{ Supergravity}) \times (\text{Hidden Sector}) \]  \tag{1.1}

at the compactification scale \( M_C \) close to the Planck scale \( M_{PL} = 2.4 \times 10^{18} \text{ GeV} \). This class of models arises naturally from the compactifications of the ten dimensional heterotic string [5] on the Calabi-Yau manifolds [6], and from the N=2 superconformal constructions [7], which allow for the breaking of the \( E_6 \) group to \([SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R\) by Wilson loops at \( M_C \). The massless multiplets are then in the \( 27, \overline{27} \) and singlet representations of \( E_6 \). In terms of the \([SU(3)]^3 \) quantum numbers, these states are:

\[
\begin{align*}
27_i &= L_i(1,3,3) \oplus Q_i(3,3,1) \oplus \overline{Q}_i^c(3,1,3); \\
\overline{27}_j &= 
\bar{L}_j(1,\overline{3},\overline{3}) \oplus \bar{Q}_j(\overline{3},3,1) \oplus \bar{Q}_j^c(3,1,\overline{3}),
\end{align*}
\]  \tag{1.2}

where \( i, j \) are generation indices, i.e., \( i = 1, 2, \cdots, n+3 \), \( j = 1, 2, \cdots, n \) for a three generation model. The notations for these massless multiplets, in terms of the Standard Model particle notation, are \( L = [l = (\nu, e); \ e^c; \ H; \ H'; \nu^c, \ N] \), \( Q = [q^a = (u^a, d^a); \ H_3^a \equiv D^a] \) and \( Q^c = [q^c_a = (u^c_a, d^c_a); \ H_3^{c_a} \equiv D_c^a] \), where \( l, H, H' \) and \( q^a \) are SU(2)\(_L\) doublets, \( e^c, u^c \) and \( d^c \) are the conjugate singlets, \( D^a \) and \( D_c^a \) are the color Higgs triplets of the SU(5) 5 and \( \overline{5} \) representations (\( a = 1, 2, 3 \) is the SU(3)\(_C\) index), \( \nu^c \) is a SU(5) singlet and \( N \) an O(10) singlet.

The breaking of the \([SU(3)]^3 \) into the standard model gauge group can be achieved by the VEV growth of the standard model singlet fields \( \nu^c \) and \( N \) at a lower scale \( M_1 \gtrsim O(10^{16}) \) GeV. In order to prevent too rapid decay of the proton, the structure of the couplings must
preserve matter parity [8] at $M_{\text{PL}}$. Then, it can be proved [9] that the spontaneous breaking of $[SU(3)]^3$ symmetry to the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$, triggered by the nonrenormalizable interactions and the supersymmetry breaking mass of $O(1 \text{ TeV})$, occurs at an intermediate scale $M_I > O(10^{16})$ GeV in a way such that the lowest lying extremum of the effective potential is the one that preserves simultaneously both $SU(2) \times U(1)$ and matter parity. This breaking is accomplished in one step in the sense that $N_i$ and $\nu^c_i$ must grow VEV’s at the same time and with the same order of magnitude $O(M_I)$.

The following theorem regarding the low energy spectrum below the intermediate scale was established in Ref. [2]:

**Theorem** For any string model having the symmetry structure of Eq. (1.1) and a matter parity invariance at the Plank scale with

(i) Breaking of $E_6 \to [SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R$ at a scale $M_C \sim M_{\text{PL}},$

(ii) Intermediate scale breaking of $[SU(3)]^3 \to SU(3)_C \times SU(2)_L \times U(1)_Y$ at a scale $M_I,$

where $M_{\text{PL}} > M_I > 10^{16}$ GeV, triggered by a mass $m < O(1 \text{ TeV}),$ and

(iii) VEVs of $N_i, \nu^c_i$ obeying $\sum \langle N_i \rangle \langle \nu^c_i \rangle = 0,$

then there exist, in addition to the three generations of light states of the Standard Model, always at least two new non-$E_6$ singlet light chiral multiplets given by

$$n_1 = (N_1 + \bar{N}_1)/\sqrt{2}, \quad \nu^c_2 = (\nu^c_2 + \bar{\nu}^c_2)/\sqrt{2}, \quad (1.3)$$

and in some cases four new non-$E_6$ singlet light chiral multiplets, with the additional possible light state given by

$$n_2 = \cos \theta N_2 + \sin \theta \bar{\nu}^c_1, \quad \bar{n}_2 = \cos \theta \bar{N}_2 + \sin \theta \nu^c_1. \quad (1.4)$$

Here $\tan \theta = \langle \nu^c_2 \rangle / \langle N_1 \rangle,$ and a basis has been chosen in generation space such that $\langle N_i \rangle = \langle N_1 \rangle \delta_{i1}$ and $\langle \nu^c_i \rangle = \langle \nu^c_2 \rangle \delta_{i2}.$ In addition there may also be a number of light $E_6$ singlet multiplets, $\phi_a.$

We will investigate the phenomenological consequences of this theorem in this paper. In Sec. 2., we discuss the interactions of these new particles with the standard model particles [10,2] and their implications on rare $\tau$ and $\mu$ decays. Sec. 3 briefly summarizes the results of Ref. [3] on the fermion family violating processes $\tau \to \mu \gamma$ and $\mu \to e\gamma.$ Sec. 4.
represents a detailed analysis on the decays of τ particle via $\tau \rightarrow \mu\mu\bar{\mu}$ and $\tau \rightarrow \mu e\bar{e}$ where the conclusion is reached that these decay processes may be accessible at the SSC. Sec. 5. is devoted to conclusions.

2. Low-Energy Effective Interactions and $\tau$ and $\mu$ Decays

The appearance of the new light particles can be regarded as the low energy remnants of the Planck scale physics dictated by superstring theory. Thus their detection, either direct or indirect, could be regarded as the confirmation of the dynamics induced by superstring theory. Since these new particles are Standard Model singlets and presumably have a large mass $\sim O(1 \text{ TeV})$ compared to the Standard Model particles, they are most likely to be detected indirectly by the new phenomena not predicted by the Standard Model, but which involve the Standard Model particles.

The interactions of these new particles can be determined by examining the mass matrix below $M_I$ and the Yukawa and gauge couplings. One finds then [10,2] from the $(27) \times (27)$ pieces in the superpotential,

$$W_{eff} = (\lambda^{(l)}_{pp} H' e_p^c l_p' + \lambda^{(u)}_{pp} H q_p^c u_p' + \lambda^{(d)}_{pp} H' q_p^c d_p')$$

$$+ \{[\lambda_p H l_p n_2 + \tilde{\lambda}_p H l_p \tilde{n}_2 + (\lambda_1 n_1 + \tilde{\lambda}_2 \tilde{\nu}_2^c) H H']$$

$$+ (\lambda^{(0)}_{ap} \phi_a^{(0)} H l_p + \lambda^{(e)}_{ap} \phi_a^{(e)} H' H)$$

$$+ (m_{1n_2} \tilde{n}_2 + m_{2n_2} n_2 + m_{3n_2} \tilde{n}_2)$$

$$+ (m_{4n_2} \nu_2^c + m_{5n_1} n_1 + m_{6\tilde{\nu}_2^c} \tilde{\nu}_2^c) + m_{ab} \phi_a \phi_b \} + W_{seesaw}, \quad (2.1)$$

and from the gaugino $(\lambda_L^{(-)})$ interaction

$$\mathcal{L}_{\text{gaugino}} = g_L U_{p\lambda}^\dagger l_p \gamma^0 (\lambda^{(g)}_{pp} e_p^c H^\dagger + \frac{1}{\sqrt{2}} (s\lambda_1 - c\tilde{\nu}_2^c) \tilde{l}_{p=1}^\dagger$$

$$+ (\lambda' n_2 + \tilde{\lambda}' \tilde{n}_2) H'^\dagger + H (\lambda n_2^\dagger + \tilde{\lambda} \tilde{n}_2^\dagger)] + h.c., \quad (2.2)$$

where $s = \sin \theta$ and $c = \cos \theta$. In eqs. (2.1) and (2.2), the $\lambda$’s are various effective coupling constants which are related to the elementary coupling constants by various unitary transformations onto the light fields. $\phi_a = (\phi_a^{(0)}, \phi_a^{(e)})$ are the C-odd, C-even E$^6$-singlet fields that remain light. The $H^\dagger$, $H'^\dagger$, $\tilde{l}$, etc., are scalar fields, e.g., $\tilde{l}_{p=1}$ is the slepton partner of $p = 1$ lepton, where $p = 1, 2, 3$ are the three light generations of the Standard Model.
The terms in the parentheses in Eq. (2.1) are just the low-energy Standard Model superpotential. Eq. (2.2) and the terms in the curly brackets in Eq. (2.1) are the interactions involving the new low-energy particles. These represent the new physics predicted by the heterotic string models that obey the Standard Model at low energies. There are also seesaw masses coming from $\nu_p$-Higgs boson-superheavy interactions, obtained by integrating out the heavy fields. The consequences of these terms for neutrino masses and neutrino oscillations were discussed in Ref. [1] where it was found that the experimental smallness of neutrino masses, $m_\nu < O(10 \text{ eV})$, leads to restrictions on Yukawa couplings that naturally explains the smallness of $m_e/m_\tau = O(10^{-3})$, and the $\nu_e,\mu \leftrightarrow \nu_\tau$ are the dominant oscillations. Here we are interested in the implications of these new particles and new couplings on the lepton family number violating processes such as $\tau$ and $\mu$ lepton decays channels.

The interactions of the new light particles discussed above can trigger lepton family number violating processes such as $\tau \to \mu \gamma$, $\mu \to e \gamma$, $\tau \to \mu \mu \bar{\mu}$, and $\tau \to \mu e \bar{e}$, not predicted by the Standard Model. Figure 1 demonstrates how $\tau \to \mu \gamma$ arises from the $n_1$ and $\hat{\nu}_2^c$ interactions. There are quite a few sources contributing to these processes: (i) $\nu_2^c$ and $n_1$ through the gaugino interactions Eq. (2.2):

$$L_g = -\frac{\alpha g_L}{\sqrt{2}} U_{p(-)}^\dagger l_p \gamma^0 \hat{\nu}_2^c \tilde{l}_p + s \frac{g_L}{\sqrt{2}} U_{p(-)}^\dagger l_p \gamma^0 n_1 \tilde{l}_p = 1,$$

where the unitary matrices $U_{p(-)}^\dagger$ are defined from the projection of the SU(3)$_L$ gauginos, $\lambda^{(-)}_L$, onto the light lepton doublets, $\lambda^{(-)}_L = l_p U_{p(-)}^\dagger + \text{heavy fields}$. This interaction is always present regardless what the E$_6$ singlet couplings are. (ii) $n_2$ and $\bar{n}_2$ from $W_{\text{eff}}$

$$W_{n_2 \bar{n}_2} = \lambda_p H l_p n_2 + \bar{\lambda}_p H \bar{l}_p \bar{n}_2,$$

which is present if $n_2$ and $\bar{n}_2$ also remain light. (iii) E$_6$ singlets $\phi_a^{(0)}$ give

$$W_\phi = \lambda_{a(0)}^{(-)} \phi_a^{(0)} l_p H.$$

These are present provided $\phi_a^{(0)}$ remain light.

An estimate on the relative sizes of these sources is given in Ref. [3]. Thus for the entire reasonable range of $\varepsilon \equiv \tan \theta$, i.e., $10^{-3} < \varepsilon < 1$, the dominant effects come from $\hat{\nu}_2^c$.
and $n_1$ which are always present independent of $E_6$ singlet couplings. Since the $\hat{\nu}_2^c$ and $n_1$ couplings hold for all models satisfying the conditions of the Theorem stated above, these decay processes are essentially a universal prediction for all phenomenologically acceptable models of this type. We shall hereafter consider only contributions from $\hat{\nu}_2^c$ and $n_1$ and for simplicity we shall set their masses to a common value $\hat{m}$.

3. $\mu \to e\gamma$ and $\tau \to \mu\gamma$ Decays These processes were studied in Ref. [3], we briefly summarize the results here. We may parametrize the $\mu \to e\gamma$ decay by an effective Lagrangian

$$L = \frac{e}{4\hat{m}_\mu} F^{\alpha\beta} \bar{\mu} \sigma_{\alpha\beta} (a_R^{(\mu)} P_R + a_L^{(\mu)} P_L) e + h.c. \quad (3.1)$$

where $\mu(x)$ and $e(x)$ are the lepton fields. The coefficients $a_L$ and $a_R$ were determined to be

$$a_R^{(\mu)} \simeq \frac{\alpha}{8\pi \sin^2 \theta_W} \left[ \frac{m_\mu}{\hat{m}} \right]^2 L(x) \left[ \frac{m_e}{m_\tau} (1 + r^2)^{1/2} \right] \varepsilon^4$$

$$a_L^{(\mu)} = \left[ \frac{m_e}{m_\mu} \right] a_R^{(\mu)} \ll a_R^{(\mu)} \quad (3.2)$$

where $x = m_\varepsilon/\hat{m}^2$ and $L(x)$ is the loop integral

$$L(x) = \frac{1}{(1-x)^4} \left[ \frac{1}{3} + \frac{1}{2} x - x^2 + \frac{1}{6} x^3 + x \ln x \right] \quad (3.3)$$

and $r \sim 3.5$. The total decay rate is proportional to $a_{(\mu)}^2 \equiv 1/2(a_L^2 + a_R^2)$. A similar analysis for the $\tau \to \mu\gamma$ decay yields $a_{(\tau)} = (m_\tau^2/m_\mu^2)(\delta^2/\varepsilon)^{-1} a_{(\mu)}$, where $\delta^2$ is yet another parameter of the $E_6$ model and maybe determined from the electron and tau lepton mass ratio, $m_e/m_\tau = (\delta^2/\varepsilon)(1 + r^2)^{1/2}$ [1]. The following relation is then obtained

$$B(\tau \to \mu\gamma) = \left( \frac{m_\tau}{m_\mu} \right)^5 \left[ \left( \frac{m_\tau}{m_\mu} \right)^2 (1 + r^2)^{-1} \right] \left( \frac{\Gamma_\mu}{\Gamma_\tau} \right) B(\mu \to e\gamma), \quad (3.4)$$

which implies $B(\tau \to \mu\gamma) \simeq 2 \times 10^5 B(\mu \to e\gamma)$. Thus the theory predicts a definite relation between the two lepton number violating decays. As a consequence of the $\hat{\nu}_2^c$ and $n_1$ couplings (which arise from the gaugino couplings), the out-going leptons would be almost 100% right-handed polarized. This leads to a characteristic angular distribution of the out-going leptons relative to the spin of the initial lepton even if the spin of the out-going lepton is not measured.
4. $\tau \to \mu \mu \bar{\mu}$ and $\tau \to \mu e \bar{e}$ Decays

These processes arise when the out-going photon in Figure 1 converts into an electron-positron pair or a muon-anti-muon pair. These processes cannot be described by an effective Lagrangian as simple as Eq. (3.1) which was obtained utilizing the fact that all the particles involved, in particular the outgoing photon, are on shell. Here, the intermediate photon is not on-shell, but we may still use the fact that the in-coming tau particle and the out-going muon arising from the gaugino vertex are on shell. Then the $\tau - \mu - \gamma$ vertex can be written as:

$$f^\alpha(p, q) = -i \frac{e g_2 U^\dagger_{2(\pm)} U^\dagger_{3(\pm)}}{64 \pi^2 \hat{m}^2} \left[ P_L(\gamma^\alpha F_L + f^\alpha_L) + P_R(\gamma^\alpha F_R + f^\alpha_R) \right], \quad (4.1)$$

where $p$ and $q$ are four-momenta for the in-coming $\tau$ and the out-going $\mu$ respectively, $P_{L,R} = 1/2(1 \mp \gamma_5)$, and

$$F_L = 2m_\mu m_\tau (L_2(x) - L_3(x))$$

$$f^\alpha_L = -m_\mu \left[ (L_2(x) - \frac{2}{3} L_3(x))p^\alpha + (L_2(x) - \frac{4}{3} L_3(x))q^\alpha \right] \quad (4.2a)$$

$$F_R = (L_2(x) - \frac{4}{3} L_3(x))(m_\tau^2 + m_\mu^2) + \frac{2}{3} L_3 p \cdot q$$

$$f^\alpha_R = -m_\tau \left[ (L_2(x) - \frac{4}{3} L_3(x))p^\alpha + (L_2(x) - \frac{2}{3} L_3(x))q^\alpha \right] \quad (4.2b)$$

and $L_2$ and $L_3$ are loop integrals defined by

$$L_n(x) = \int_0^1 \frac{dy}{1 + (x-1)y}, \quad n = 2, 3, \quad x = \frac{m_\tau^2}{\hat{m}^2}. \quad (4.3)$$

Notice that $F_R$ and $f^\alpha_R$ are much larger than $F_L$ and $f^\alpha_L$.

The differential decay rate for the process $\tau \to \mu \bar{l}l$, where $l$ denotes either electron or muon, is given by

$$d\Gamma(\tau \to \mu \bar{l}l) = \frac{1}{2m_\tau} |\mathcal{M}|^2 \frac{d^3 \hat{q_1}}{2E(2\pi)^3} \frac{d^3 \hat{q_2}}{2E_1(2\pi)^3} \frac{d^3 \hat{q}}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p - q - q_1 - q_2), \quad (4.4)$$

where $\mathcal{M}$ is the amplitude. The total rate can be calculated from integrating over $q_1$ and $q_2$. In the rest frame of the in-coming $\tau$ lepton, the total decay rate is reduced into the following form:

$$d\Gamma(\tau \to \mu \bar{l}l) = \frac{m_\tau^5}{(4\pi)^3 \hat{m}^4} \left[ \frac{e^2 g_2^2 U^\dagger_{2(\pm)} U^\dagger_{3(\pm)}}{4(4\pi)^2} \right]^2 |\mathcal{M}'|^2 dE_1 dE_2 \delta(\cos \beta - f(E_1, E_2)) \theta(1 - E_1 - E_2 - m_\mu), \quad (4.5)$$
where all the energy and masses are scaled by \( m_\tau \), \( \beta \) is the angle between two out-going \( l \) particles, and \( f(E_1, E_2) = \frac{(1+2m_\tau^2-m_\mu^2-2(E_1+E_2)+2E_1E_2)/(2\sqrt{(E_1^2-m_\tau^2)(E_2^2-m_\tau^2)})}{(2\sqrt{(E_1^2-\mu)^2)-m_\tau^2)}} \). The prefactor in Eq. (4.5) can be written as

\[
A = \frac{m_\tau^5}{(4\pi)^3 m^4} \left[ \frac{e^2 g_2^2 U_{2(-)} U_{3(-)}^\dagger}{4(4\pi)^2} \right]^2 = \frac{1}{16(4\pi)^3} \frac{m_\tau^5}{m^4} \alpha^2 \alpha^2 (U_{2(-)} U_{3(-)}^\dagger)^2, \tag{4.6}
\]

where \( \alpha = g_2^2/4\pi \). The values of quantities in the above expression are \( U_{2(-)} U_{3(-)}^\dagger \sim 1 \), \( \alpha = 1/137 \) and \( \alpha^2 = 0.03322 \). We have

\[
d\Gamma(\tau \to \mu l\bar{l}) = \frac{3.4318 \times 10^{-11}}{m^4} |\mathcal{M}'|^2 dE_1 dE_2, \tag{4.7}
\]

where the three body phase space (the range of \( E_1 \) and \( E_2 \)) is determined by the \( \delta \)- and \( \theta \)-functions in Eq. (4.5) to be:

\[
(1 + 2m_\tau^2 - m_\mu^2 - 2(E_1 + E_2) + 2E_1E_2)^2 \leq 4(E_1^2 - m_\tau^2)(E_2^2 - m_\tau^2), \tag{4.8a}
\]

\[
E_1 + E_2 + m_\mu \leq 1. \tag{4.8b}
\]

For \( \tau \to \mu e \bar{e} \) decay, we have

\[
|\mathcal{M}'|^2 = \frac{1}{4(p - q)^4} (\text{tr} \gamma_1 \gamma_2 \gamma - m_e^2 \text{tr} \gamma_\alpha \gamma_\beta) [\text{tr}(p + 1) R^\alpha \gamma L^\alpha + \text{tr}(p + 1) (L^\beta \gamma L^\alpha + R^\beta \gamma R^\alpha)] \tag{4.9}
\]

where

\[
L^\alpha = \gamma^\alpha F_L + f^\alpha_L, \quad R^\alpha = \gamma^\alpha F_R + f^\alpha_R. \tag{4.10}
\]

After integrating over the phase space, we have

\[
\Gamma(\tau \to \mu e \bar{e}) = \frac{1.5501 \times 10^{-10}}{m^4} [(L_3 - 0.987L_2)^2 + 1.8237 \times 10^{-3}L_2L_3] \tag{4.11}
\]

Similarly, for \( \tau \to \mu \mu \bar{\mu} \), we have

\[
|\mathcal{M}'|^2 = \frac{1}{2} \left[ \frac{A(q_1, q_2)}{(p - q_2)^4} + \frac{A(q_2, q_1)}{(p - q_1)^4} - \frac{B(q_1, q_2) + B(q_2, q_1)}{(p - q_1)^2(p - q_2)^2} \right], \tag{4.12}
\]
where

\begin{align}
A(q_1, q_2) &= \frac{1}{2} \text{tr}(\slashed{q} - m_\mu)\gamma_\alpha(\slashed{q}_1 + m_\mu)\gamma_\beta \\
&\quad \times \text{tr}\left[(\slashed{p} + 1)(L_2^2 P_R + R_2^2 P_L)(\slashed{q}_2 + m_\mu)(P_L L_2^\beta + P_R R_2^\beta)\right] \\
B(q_1, q_2) &= \frac{1}{2} \text{tr}\left[(\slashed{p} + 1)(L_2^2 P_R + R_2^2 P_L)\right. \\
&\quad \times \left(\slashed{q}_2 + m_\mu)\gamma_\beta(\slashed{q}_1 - m_\mu)(\slashed{q}_1 + m_\mu)\right. \\
&\quad \times (P_L L_2^\beta + P_R R_2^\beta)\right] \\
\end{align}

(4.13a)

(4.13b)

where \(q_1\) and \(q_2\) are the momenta of the out-going muons and \(q\) is that of the out-going anti-muon, and \(L_i^\alpha\) and \(R_i^\alpha\) are given by Eq. (4.10) by replacing \(q\) by \(q_i\) for \(i = 1, 2\). After integrating over the phase space, we have

\[
\Gamma(\tau \to \mu\mu\bar{\mu}) = 3.8217 \times 10^{-11} \left[ (L_3 - 0.9235L_2)^2 + 5.8093 \times 10^{-3}L_2L_3 \right].
\]

(4.14)

The decay rates \(\Gamma(\tau \to \mu e\bar{e})\) and \(\Gamma(\tau \to \mu\mu\bar{\mu})\) as functions of \(x = m_e^2/\hat{m}^2\) are listed in Table 1. We see that for the physically interesting range of \(x\), i.e., \(0.01 < x < 100.0\), \(\Gamma(\tau \to \mu e\bar{e})\) is about \(6 \sim 7\) times \(\Gamma(\tau \to \mu\mu\bar{\mu})\). These decay rates are about 1000 times less than that of \(\Gamma(\mu \to e\gamma)\), and therefore have much less chance to be detected. One nice feature about \(\tau \to \mu e\bar{e}\) is that the out-going muon is almost 100\% right-handed polarized which may be used as a distinguishing feature for its detection. Figure 2 and Figure 3 are Dalitz plots for the processes \(\tau \to \mu e\bar{e}\) and \(\tau \to \mu\mu\bar{\mu}\) respectively. Figure 4 draws the differential event rate of \(\tau \to \mu e\bar{e}\) as the function of the angles between the out-going \(e\bar{e}\) pair (normalized to 1000 events). One finds that about 55\% of events have the \(e\bar{e}\) pair coming out with angles less than \(5^\circ\) between them, and about 97\% are forward with angles between the \(e\bar{e}\) pair being less than \(90^\circ\). One concludes than that the out-going \(e\bar{e}\) pair will be predominately parallel to each other. Thus, for \(\hat{m} = 0.5\) TeV and \(x = 0.04\) (corresponding to a selectron mass of 100 GeV which is in accord with the experimental lower bound of \(m_e \gtrsim 65\) GeV [11]), Eqs. (4.3), (4.11) and (4.14) imply that \(B(\tau \to \mu e\bar{e}) = 7.16 \times 10^{-11}\) and \(B(\tau \to \mu\mu\bar{\mu}) = 9.25 \times 10^{-12}\). The SSC with a luminosity of \(\mathcal{L} = 10^{33} - 10^{34}\) cm\(^{-2}\)s\(^{-1}\) and a B-production cross section of \(\sigma_B \sim 0.5\) mb will produce \(5 \times 10^{11}\) to \(5 \times 10^{12}\) \(\tau\) events per year. Therefore, one would expect to have about 30 to
300 events of $\tau \rightarrow \mu e\bar{e}$ and about 4 to 40 events of $\tau \rightarrow \mu\mu\bar{\mu}$ at the SSC per year. With an event acceptance of 10% and the cleanness of the three lepton events, we conclude that the rare $\tau$ decay modes, $\tau \rightarrow \mu e\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\mu}$, may be accessible at the SSC.

5. Conclusions

In conclusion, string models with $E_6$ symmetry are possible viable models from the low-energy phenomenological viewpoints. The rare $\tau$-decays via $\tau \rightarrow \mu e\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\mu}$, which may be accessible at the SSC, give yet one more channel to confront these models and hence superstring theory with experiments.

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Figure Caption

Figure 1. Decay $\tau \rightarrow \mu \gamma$ arising from intermediate $\hat{\nu}_c \bar{c}$ and $n_1$. This decay is also induced by $\phi_a$, $n_2$ and $\bar{n}_2$ fermions. Additional diagrams exist with the photon emitted by the initial and final fermions. The decays $\tau \rightarrow \mu e\bar{e}$ and $\tau \rightarrow \mu \mu \bar{\mu}$ arise when the out-going photon converts into $e-\bar{e}$ pair or $\mu-\bar{\mu}$ pair.

Figure 2. Dalitz plot of $\tau \rightarrow \mu e\bar{e}$ (in the rest frame of $\tau$). The variables along the two axises are $(p - q_1)^2$ and $(p - q_2)^2$ where $p$, $q_1$ and $q_2$ are the momenta of $\tau$ and the out-going $e$ and $\bar{e}$, respectively. $x = m^2_\bar{e}/\hat{m}^2$ is taken to be 1.

Figure 3. Dalitz plot of $\tau \rightarrow \mu \mu \bar{\mu}$ (in the rest frame of $\tau$). The variables along the two axises are $(p - q_1)^2$ and $(p - q_2)^2$ where $p$, $q_1$ and $q_2$ are the momenta of $\tau$ and the two out-going $\mu$,s, respectively. $x = m^2_\bar{e}/\hat{m}^2$ is taken to be 1.

Figure 4. The differential event rate of $\tau \rightarrow \mu e\bar{e}$ as the function of the angles between $e\bar{e}$ pair, normalized to 1000 events, for $x = m^2_\bar{e}/\hat{m}^2 = 1$. One sees that the $e\bar{e}$ pair would mostly come out parallel to each other, e.g., 55% of them will have an angle between $e\bar{e}$ pair less than $5^\circ$.

Table 1. Decay Rates as Functions of $x = m^2_\bar{e}/\hat{m}^2$, with $\hat{m}$ in GeV.

| $x = m^2_\bar{e}/\hat{m}^2$ | $\Gamma_{\tau \rightarrow \mu e\bar{e}} \times \hat{m}^4$ | $\Gamma_{\tau \rightarrow \mu \mu \bar{\mu}} \times \hat{m}^4$ | Ratio |
|----------------|-----------------|-----------------|-------|
| 0.0001        | $2.44 \times 10^{-11}$ | $1.51 \times 10^{-11}$ | 1.61  |
| 0.001         | $1.81 \times 10^{-11}$ | $6.43 \times 10^{-12}$ | 2.81  |
| 0.01          | $1.33 \times 10^{-11}$ | $2.21 \times 10^{-12}$ | 6.00  |
| 0.1           | $6.67 \times 10^{-12}$ | $8.41 \times 10^{-13}$ | 7.93  |
| 1.0           | $9.91 \times 10^{-13}$ | $1.46 \times 10^{-13}$ | 6.78  |
| 10.0          | $3.02 \times 10^{-14}$ | $4.90 \times 10^{-15}$ | 6.17  |
| 100.0         | $3.88 \times 10^{-16}$ | $6.40 \times 10^{-17}$ | 6.05  |
| 1000.0        | $4.00 \times 10^{-18}$ | $6.65 \times 10^{-19}$ | 6.03  |