Updating spin dependent Regge intercepts

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We use new high statistics data from CLAS and COMPASS on the nucleon’s spin structure function at low Bjorken x and low virtuality, $Q^2 < 0.5$ GeV$^2$, together with earlier measurements from the SLAC E-143, HERMES and GDH experiments to estimate the effective intercept(s) for spin dependent Regge theory. We find $\alpha_{a_1} = 0.31 \pm 0.04$ for the intercept describing the high-energy behaviour of spin dependent photoabsorption together with a new estimate for the high-energy part of the Gerasimov-Drell-Hearn sum-rule, $-15 \pm 2\mu b$ from photon-proton centre-of-mass energy greater than 2.5 GeV. Our value of $\alpha_{a_1}$ suggests QCD physics beyond a simple straight-line $a_1$ trajectory.

I. INTRODUCTION

The high-energy behaviour of the spin dependent part of the photon-proton total cross section is important for determining the Gerasimov-Drell-Hearn sum-rule for polarised photon-proton collisions at low photon virtuality $Q^2 < 0.5$ GeV$^2$ and centre-of-mass energy $\sqrt{s} \geq 2.5$ GeV, together with earlier measurements from the E-143 experiment at SLAC [6], HERMES at DESY [7] and the GDH Collaboration in Bonn [8].

The large $s$ dependence of hadronic total cross-sections is usually described in terms of Regge exchanges [9, 10], e.g. summing the exchanges of hadrons with given quantum numbers that occur along Regge trajectories with slope (often taken as a straight line) related to the confinement potential. Regge phenomenology has considerable success in describing unpolarised high-energy scattering processes [11].

II. SPIN DEPENDENT REGGE THEORY

Let $\sigma_A$ and $\sigma_P$ denote the two cross-sections for the absorption of a transversely polarised photon with spin antiparallel $\sigma_A$ or parallel $\sigma_P$ to the spin of the target nucleon. The Regge prediction for the isovector and isoscalar parts of $(\sigma_A - \sigma_P)$ for a real photon, $Q^2 = 0$, with $s \to \infty$ is [12][13]:

$$\left(\sigma_A - \sigma_P\right)^{(p-n)} \sim \sum_i N_i^{(3)} s^{\alpha_{a_1} - 1}$$

$$\left(\sigma_A - \sigma_P\right)^{(p+n)} \sim \sum_i N_i^{(0)} s^{\alpha_{f_1} - 1} + N_g \ln \frac{s}{\mu^2} \frac{\ln s}{s} \quad \text{(1)}$$

Here, the $\alpha_i$ denote the Regge intercepts for isovector $a_1(1260)$ Regge exchange and the $a_1$-pomeron cuts [12]. The $\alpha_{f_1}$ denote the intercepts for the isoscalar $f_1(1285)$ and $f_1(1420)$ Regge trajectories and their $f_1$-pomeron cuts. The logarithm $\ln s/s$ term comes from two non-perturbative gluon exchange in the $t$-channel [13] with a vector short-range exchange-potential [14] and the mass parameter $\mu$ is taken as a typical hadronic scale. The coefficients $N_i^{(3)}, N_i^{(0)}$ and $N_g$ are to be determined from experiment.

If one makes the usual assumption that the $a_1$ Regge trajectories are straight lines parallel to the $\rho, \omega$ trajectories then one finds $\alpha_{a_1} \simeq -0.4$ for the leading trajectory, within the range of possible $\alpha_{a_1}$ values between -0.5 and zero discussed in Ref. [15]. Fitting straight line trajectories through the $a_1(1260)$ and $a_3(2030)$ states, the $a_1(1640)$ and $a_3(2310)$ states, and the $f_1(1285)$ and $f_2(2050)$ states yields near parallel trajectories with slopes 0.79 GeV$^{-2}$, 0.76 GeV$^{-2}$ and 0.78 GeV$^{-2}$ respectively. The two leading trajectories then have slightly lower intercepts, $\alpha_{a_1} = -0.25$ and $\alpha_{f_1} = -0.29$. With this value of $\alpha_{a_1}$ the effective intercepts corresponding to the $a_1$ soft-pomeron cut and the $a_1$ hard-pomeron cut are $-0.17$ and $+0.15$ respectively if one takes the soft pomeron with intercept $1.0808$ and hard pomeron proposed in Ref. [16] with intercept 1.4 as two distinct exchanges. Values of $\alpha_{a_1}$ close to zero could be achieved with curved Regge trajectories; $\alpha_{a_1} = -0.03 \pm 0.07$ is found in the model of Ref. [17]. For this value the intercepts of the $a_1$ soft-pomeron cut and the $a_1$ hard-pomeron cut are $\sim +0.05$ and $\sim +0.37$.

Before presenting our new results, we first recall the challenge of understanding the proton’s internal spin structure in high $Q^2$ deep inelastic scattering and $Q^2$ dependence of the intercepts $\alpha_i$ describing the asymptotic high-energy behaviour.

In deep inelastic kinematics the nucleon’s $g_1$ spin structure function is related to $(\sigma_A - \sigma_P)$ by

$$\left(\sigma_A - \sigma_P\right) \simeq \frac{4\pi^2 \alpha_{Q\text{QED}}}{p.q} g_1 \quad \text{(2)}$$

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where \( p \) and \( q \) are the proton and photon four-momenta respectively and \( \alpha_{\text{QED}} \) is the electromagnetic coupling. The Regge prediction for the isovector \( g_1^{p-n} = g_1^n - g_1^p \) at small Bjorken \( x = (Q^2/2p.q) \) is

\[
g_1^{p-n} \sim \sum_i N_i^{(3)} \left( \frac{1}{x} \right)^{\alpha_i} \tag{3}
\]

with all data taken at the same \( Q^2 \). Eq. (3) follows from

\[
s = (p + q)^2 = Q^2(1 - x) + M^2 \]

where \( M \) is the proton mass and \( s \approx Q^2/x \) in the small \( x \) limit. There is possible \( Q^2 \) dependence in the \( \alpha_i \) and \( N_i^{(3)} \). The COMPASS experiment found

\[
g_1^{p-n} \sim x^{-0.22 \pm 0.07} \tag{4}
\]

corresponding to an effective intercept \( \alpha_{\text{fit}}(Q^2) = 0.22 \pm 0.07 \) at \( Q^2 = 3 \text{ GeV}^2 \), with small \( x \) data down to \( x_{\text{min}} \sim 0.004 \) [18].

The isoscalar spin structure function \( g_1^{p+n} \approx 0 \) for \( x < 0.03 \) at deep inelastic \( Q^2 [19] \), in sharp contrast to the unpolarised structure function \( F_2 \) where the isosinglet part dominates through gluonic exchanges. The proton spin puzzle, why the quark spin content of the proton is so small ~ 0.3, concerns the collapse of the isoscalar spin sum structure function to near zero at this small \( x \). The spin puzzle is now understood in terms of pion cloud effects with transfer of quark spin to orbital angular momentum in the pion cloud [20], a modest polarised gluon correction \(-3\pi^2\Delta g/2\) with \( \Delta g \) less than about 0.5 at the scale of the experiments [19], and a possible topological effect at \( x = 0 \) [21].

The observed rise in \( g_1^{p-n} \) at deep inelastic values of \( Q^2 \) is required to reproduce the area under the fundamental Bjorken sum rule,

\[
\int_0^1 dx g_1^{(p-n)}(x, Q^2) = \frac{g_A^{(3)}}{6} C_{\text{NS}}(Q^2) \tag{5}
\]

Here \( g_A^{(3)} = 1.270 \pm 0.003 \) is the isovector axial-charge measured in neutron beta-decays and \( C_{\text{NS}}(Q^2) \) is the perturbative QCD Wilson coefficient, \( \approx 0.85 \) with QCD coupling \( \alpha_s = 0.3 \) [19]. The Bjorken sum-rule is connected to pion physics and chiral symmetry through the Goldberger-Treiman relation \( 2Mg_A^{(3)} = f_\pi q_{\pi NN} \) where \( f_\pi \) is the pion decay constant and \( q_{\pi NN} \) is the pion-nucleon coupling constant. The sum-rule has been confirmed in polarised deep inelastic scattering experiments at the level of 5% [18]. About 50% of the sum-rule comes from \( x \) values less than about 0.15. The \( g_1^{p-n} \) data is consistent with quark model and perturbative QCD predictions in the valence region \( x > 0.2 \) [22]. The size of \( g_A^{(3)} \) forces us to accept a large contribution from small \( x \) and the observed rise in \( g_1^{p-n} \) is required to fulfill this non-perturbative constraint.

Perturbative QCD evolution acts to push the weight of the distribution to smaller Bjorken \( x \) with increasing \( Q^2 \) with perturbative calculations predicting rising \( g_1^{p-n} \) at small \( x \) and deep inelastic \( Q^2 [23, 24] \). Regge phenomenology should describe the high-energy part of \( g_1 \) close to photoproduction and provide the input for perturbative QCD evolution at deep inelastic values of \( Q^2 \). One then applies perturbative QCD, typically above \( Q^2 > 1 \text{ GeV}^2 \). These perturbative QCD calculations involve DGLAP evolution and double logarithm, \( \alpha_s \ln^2 \frac{1}{x} \), resummation at small \( x \) [25], in possible combination with vector meson dominance terms at low \( Q^2 [26] \). For \( g_1^{p-n} \) with DGLAP evolution this approach has the challenging feature that the input and output (at soft and hard scales) are governed by non-perturbative constraints with perturbative QCD evolution in the middle unless the \( \alpha_i \) Regge input has information about \( g_A^{(3)} \) and chiral symmetry built into it. One possibility is a separate hard-exchange contribution (perhaps an \( \alpha_1 \) hard-ponomer cut) in addition to the soft \( \alpha_1 \) term [27].

### III. FITTING THE HIGH ENERGY SPIN ASYMMETRY

We next estimate the spin dependent Regge intercepts. Good statistics measurements of the spin asymmetry for photon-proton collisions \( A_1^y = (\sigma_A - \sigma_p)/(\sigma_A + \sigma_p) \) at large \( \sqrt{s} \) and low \( Q^2 \) have recently become available from the CLAS and COMPASS experiments, complementing earlier measurements from SLAC, HERMES and the GDH Collaboration. We make a Regge motivated fit to this data on \( \Delta \sigma = \sigma_A - \sigma_p = A_1^y (\sigma_A + \sigma_p) \) with the constraints \( \sqrt{s} \geq 2.5 \text{ GeV} \) where Regge theory is expected to apply [11] and \( Q^2 < 0.5 \text{ GeV}^2 \). Keeping \( Q^2 < 0.5 \text{ GeV}^2 \) is a compromise between keeping \( Q^2 \) as low as possible and including the maximum amount of data. This input data involves 18 points from COMPASS with \( \sqrt{s} \) between 11 and 15 GeV [5], 2 data points from HERMES with \( \sqrt{s} \) at 6.6 and 6.8 GeV [7], 7 points from SLAC E-143 with \( \sqrt{s} \) between 2.5 and 3.1 GeV [6], and 102 points from CLAS between 2.5 and 2.9 GeV [8]. This data is consistent with \( A_1^y \) being \( Q^2 \) independent in each experiment within the chosen kinematics. We also consider the highest energy single data point from the GDH photoproduction experiment with \( \sqrt{s} = 2.5 \text{ GeV} \) and \( Q^2 = 0 \) [8]. Data at higher \( Q^2 \) values between 0.5 and 1 GeV\(^2 \) are in principle sensitive to the extra effects of turning on DGLAP evolution and decay of higher-twist terms with increasing \( Q^2 \).

The unpolarised total cross-section, \( \sigma_{\text{tot}} = \sigma_A + \sigma_p \), measurements from HERA were found to be well described by a combined Regge and Generalized Vector Meson Dominance (GVMD) motivated fit in the kinematics \( Q^2 < 0.65 \text{ GeV}^2 \) and \( s \geq 3 \text{GeV}^2 [29, 30] \). The ZEUS Collaboration used the 4 parameter fit [28]

\[
\sigma_{\text{tot}}(s, Q^2) = \left( \frac{M_0^2}{M_A^2 + Q^2} \right) \left( A_R s^{\alpha_R - 1} + A_p s^{\alpha_p - 1} \right) \tag{6}
\]

to describe the low \( Q^2 \) region, also including fixed target data from the E665 Collaboration [31], with \( A_R = \)

\[
\sigma_{\text{tot}}^{(s,Q^2)}(s,Q^2) \approx \frac{4s^2\alpha_{\text{QED}}}{Q^2} F_2(x,Q^2)
\] (7)

where \( s \approx Q^2/x \). For \( Q^2 \) larger than 1 GeV\(^2\) the HERA data on \( F_2 \) seems to be well described by DGLAP evolution. Parametrising \( F_2 \sim A x^{-\lambda} \) at small \( x \) the effective intercept \( \lambda \) is observed to grow from 0.11 \pm 0.02 at \( Q^2 = 0.3 \) GeV\(^2\) to 0.18 \pm 0.03 at \( Q^2 = 3.5 \) GeV\(^2\), 0.31 \pm 0.02 at 35 GeV\(^2\) and increases with increasing \( Q^2 \) \cite{32,33}. The value 0.4 was found at the highest \( Q^2 \) motivating suggestions of a new hard pomeron \cite{35,36}.

Here, we first assume \( A_1^p \) to be \( Q^2 \) independent in the chosen kinematics with \( Q^2 < 0.5 \) GeV\(^2\). That is, we conjecture

\[
(\sigma_A - \sigma_P)^{\gamma^*p}(s,Q^2) = \left( \frac{M_0^4}{M_0^4 + Q^4} \right) (\sigma_A - \sigma_P)^{\gamma p}(s,0)
\] (8)

at large \( s \) and small \( Q^2 \) with the same value of \( M_0^4 \) in both Eqs.\( (6) \) and \( (8) \) and \( Q^2 \) independent values of the spin Regge intercepts \( \alpha_i \) at this low \( Q^2 \).

Second, we assume that the isoscalar deuteron asymmetry \( A_1^d \) can be taken as zero in first approximation. The deuteron data on \( A_1^d \) are consistent with zero in each experiment in our chosen kinematics \cite{147,148,149,150} (as well as in \( g_1^d \) measurements at deep inelastic \( Q^2 \) and low \( x < 0.03 \)) \cite{151}. This means that we set the normalisation factors \( N_0 = N_y = 0 \) in Eq.\( (1) \).

Third, we take \( \sigma_{\text{tot}} \) from a fit to unpolarised data. We assume that the errors on \( \sigma_{\text{tot}} \) can be neglected compared to the errors on \( A_1^p \). For the total photoproduction cross-section we take

\[
(\sigma_A + \sigma_P) = 67.7 \times s^{0.0808} + 129 \times s^{-0.4455}
\] (9)

in units of \( \mu \)b, which provides a good Regge fit for \( \sqrt{s} \) between 2.5 GeV and 250 GeV \cite{152}. The \( s^{0.0808} \) contribution is associated with gluonic pomeron exchange and the \( s^{-0.4455} \) contribution is associated with the isoscalar \( \omega \) and isovector \( \rho \) trajectories.

Our best fit of form \( (\sigma_A - \sigma_P) \propto N s^\alpha \) including all data is

\[
(\sigma_A - \sigma_P) = (35.3 \pm 3.6) \times s^{-0.693 \pm 0.04} \mu \text{b}
\] (10)

for \( \sqrt{s} \geq 2.5 \) GeV corresponding to an effective Regge intercept

\[
\alpha_{a_1} = +0.31 \pm 0.04
\] (11)

see Fig. 1. The \( \chi^2/\text{ndf} \) for the fit is 0.98. Statistical and systematic errors for each data point have been added in quadrature.

To convert the fit results in Eqs. \( (9-11) \) into a prediction for the asymmetry \( A_1^p \) as a function of \( x \), it is important to note that \( s \approx Q^2/x \) at large centre-of-mass energy and take into account that experimental measurements in different \( x \) bins are typically taken at different \( Q^2 \) values. For example, the COMPASS measurements using a 160 GeV muon beam at \( \langle x \rangle = 0.000052 \) were taken at \( Q^2 = 0.0062 \) GeV\(^2\) whereas their measurements at \( \langle x \rangle = 0.0020 \) were taken at \( Q^2 = 0.33 \) GeV\(^2\) \cite{37}, a factor of 53 greater in \( Q^2 \). Within each \( x \) bin taken separately, \( Q^2 \) was varied over a more limited factor of about 5 and the experimental uncertainties too large to make definite conclusions about possible \( Q^2 \) dependence within individual \( x \) bins. All of our COMPASS points with \( Q^2 < 0.5 \) GeV\(^2\) are in the range \( \sqrt{s} \) between 11 and 15 GeV. One expects \( A_1^p \) to vanish in the small \( x \) limit, which follows in this data when all points are shifted to the same \( Q^2 \) by dividing out the factor \( (Q^2)^{\alpha_{a_1} + 1.0808} \sim (Q^2)^{-0.77} \) from Eqs. \( (9-11) \).

IV. DISCUSSION

It is very interesting that the intercept in Eq.\( (11) \) is close to the value found in deep inelastic scattering, \( \alpha_{a_1}(Q^2) = 0.22 \pm 0.07 \) at \( Q^2 = 3 \) GeV\(^2\) in Eq.\( (4) \). Our new low \( Q^2 \) value signifies either the presence of a hard exchange, perhaps involving an \( a_1 \) hard pomeron cut, or a curved Regge trajectory instead of just a simple straight-line \( a_1 \) Regge trajectory.

More valuable experimental input could come from the proposed future electron-ion-collider which could extend the experimental data up to \( \sqrt{s} \) values between 40 GeV and 140 GeV \cite{38,39} – that is, up to an order of magnitude higher in \( \sqrt{s} \) than the present highest centre-of-mass energy COMPASS data. Estimates for the expected asymmetries are given in \cite{40}. The fit values in Eqs. \( (10,11) \) suggest low \( Q^2 \) asymmetries \( A_1^p = (1.7 \pm 0.5) \times 10^{-3} \) at \( \sqrt{s} = 40 \) GeV and \( A_1^p = (2.5 \pm 1.0) \times 10^{-4} \) at \( \sqrt{s} = 140 \) GeV.
Taking the fit values in Eq. (10), we estimate the high-energy contribution to the Gerasimov-Drell-Hearn sum-rule from $\sqrt{s} \geq 2.5$ GeV to be
\[ \int_{s_0}^{\infty} \frac{ds}{s - M^2} (\sigma_p - \sigma_A) = -15 \pm 2 \, \mu b. \tag{12} \]

This determination compares with previous estimates: $-15 \pm 10 \mu b$ for $\sqrt{s} \geq 2.5$ GeV based on an extrapolation of lower energy photoproduction data which also gave $\alpha_{s_0} = 0.42 \pm 0.23 \ [38]$, $-25 \pm 10 \mu b$ from an early estimate using lower statistics low $Q^2$ data (pre CLAS and COMPASS) for $\sqrt{s} \geq 2.5$GeV [39], and $-26 \pm 7 \mu b$ for $\sqrt{s} \geq 2$ GeV [40] from early Regge fits to low $Q^2$ data. The new result in Eq. (12) is a factor of 3.5 times more accurate than the previous most accurate determination. The corresponding integral from threshold up to $\sqrt{s} = 2.5$ GeV has been extracted from proton fixed-target experiments with photon energy up to 2.9 GeV. One finds $226 \pm 5 \mu b$ S [38]. Combining this number and the result in Eq. (12) gives
\[ \int_{M^2}^{\infty} \frac{ds}{s - M^2} (\sigma_p - \sigma_A) = 211 \pm 13 \, \mu b \tag{13} \]

for the Gerasimov-Drell-Hearn sum-rule. This value compares with the sum-rule prediction $2\pi^2 \alpha_Q \kappa^2 / M^2 = 205 \, \mu b$ with $\kappa = 1.79$ the proton’s anomalous magnetic moment [12].

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