Adiabatic Pumping in Interacting Systems

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A dc current can be pumped through an interacting system by periodically varying two independent parameters such as magnetic field and a gate potential. We present a formula for the adiabatic pumping current in general interacting systems, in terms of instantaneous properties of the system, and find the limits for its applicability. This formula generalizes the scattering approach for noninteracting pumps. We study the pumped spin in a system that exhibits the two-channel Kondo effect as an application of the adiabatic pumping formula. We find that a quantized spin of $\hbar$ is transferred between the two channels as the temperature approaches zero, and discuss the non-Fermi liquid features of this system at finite temperatures.

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Introduction and Conclusions.—The scattering approach of Brouwer 1 for pumping through a finite, possibly disordered region of noninteracting electrons, which followed a work by Büttiker, Prêtre and Thomas 2 (BPT), tremendously enhanced the understanding of time dependent transport in mesoscopic systems. The derivation of the Brouwer formula 1 and its applications are well established 3,4, and recent studies considered special cases of interactions in quantum dots 5,6 and in Luttinger liquids 7,8. However, an apt formulation of the pumped current in the general case when interactions between the electrons are involved (beyond the Hartree level) has been lacking.

In this Letter we develop a generic formula [Eq. 2] which expresses the pumped current through a region of interacting electrons in the adiabatic limit. The pumped current is expressed in terms of an instantaneous linear response function, calculated at every step along the pumping trajectory. When the motion along the trajectory is sufficiently slow, the current can be found by integrating the contributions of each step [see Eq. 3]. We also rewrite Eq. 2 for the case of a quantum dot connected to noninteracting leads in terms of dot properties [Eq. 4], from which the noninteracting S-matrix formula 2 follows as a special case.

Finally, we consider as an example spin pumping in the vicinity of the two-channel Kondo (2CK) fixed point which is perturbed by two pumping parameters, magnetic field and channel anisotropy. Due to the non Fermi liquid (NFL) nature of the 2CK fixed point, the standard scattering approach cannot be applied straightforwardly. Using Eq. 2 we calculate the spin pumped from one channel into the other, as function of the parametric trajectory and temperature $T$. We find that as $T \to 0$ the pumped spin is quantized in units of $\hbar$ per period for trajectories which surround the NFL fixed point [paragraph following Eq. 4], while at finite temperatures this quantization is not accurate [Eq. 5]. At finite temperature, when the trajectory is sufficiently close to the fixed point, the temperature dependence of the pumped spin reflects the NFL physics of the 2CK [Eq. 10].

Generic adiabatic pumping formula.—We analyze a mesoscopic conductor which is described by an Hamiltonian $H_{X_1, X_2, \ldots}$ that depends at least on two external parameters $X_1(t)$ and $X_2(t)$ which are varied periodically and slowly in time. These parameters can be for example two metallic gates and/or an external magnetic field. By assumption the conductor may be divided into left $(L)$ and right $(R)$ contacts whose Hamiltonian does not depend on the parameters, and a central region whose Hamiltonian does depend on the parameters. Interactions may take place everywhere in the conductor.

In a pumping cycle the set of the parameters $X(t) = (X_1(t), X_2(t), \ldots)$ is varied periodically in time and defines a closed trajectory, $L$, in the parameter space. The change in the parameters may produce a current $J_j$ in contact-$j$ $(j = L, R)$ 2. To calculate $J_j$ at a point $X_0$ along the trajectory due to a small and slow change, $\delta X$, of one of the parameters, we assume that $\delta X = \delta X_0 e^{i\alpha_\tau}$. The current $J_j(\Omega)$ is then found by a linear response perturbation theory with respect to the infinitesimal perturbation $H' = \frac{\partial H}{\partial X_j} \delta X_0 e^{i\alpha_\tau}$. Including the possibility that the current operator, $\hat{J}_j$, may depend explicitly on $X$, we obtain the total response to linear order in $\delta X_0$:

$$J_j(\Omega) = \left\langle \left( \frac{\partial \hat{J}_j}{\partial X_j} \right) \cdot \frac{i}{\hbar} \int_{-\infty}^{0} d\tau e^{i\alpha_\tau} \left\langle \left[ \hat{J}_j(0), \frac{\partial H}{\partial X_j}(\tau) \right] \right\rangle \right|_{\Omega=0} \ .$$

with $\Omega^- = \Omega - i0^+$. Since Eq. 1 for the current is a first order expansion in the harmonic perturbation, the time evolution should be understood as $\hat{O}(\tau) = e^{iH_0 \tau} \hat{O} e^{-iH_0 \tau}$ and the quantum averages are performed with $(\Omega$ independent) instantaneous eigenstates of $H_{X_0}$ 10.

The charge $\delta Q(j, \Omega) = J_j(\Omega)/(i\Omega)$ entering the central region through contact $j$ is $\delta Q(j, \Omega) = e \frac{dn(j)}{dX} \delta X_0$, where $\frac{dn(j)}{dX}$, the emissivity into contact $j$, is given by:

$$\frac{dn(j)}{dX} = \lim_{\Omega \to 0} \frac{J_j(\Omega)}{i\epsilon \delta X_0} = \frac{1}{i\epsilon \delta X_0} \frac{d}{d\Omega} J_j(\Omega)_{\Omega=0} \ .$$

The last equality follows since in the static limit, $\Omega = 0$, no current flows through the contacts. This yields 10.
\[
\frac{dn(j)}{dX} = \lim_{\hbar \to 0} \frac{d}{dX} \int_{-\infty}^{0} d\tau e^{i\Omega \tau} \left\langle \frac{\partial H_{\delta X}^i(\tau)}{\partial X} \right\rangle. \tag{2}
\]

Notice that while here we take the limit \( \Omega \to 0 \), in practice the emissivity does not depend on \( \Omega \) for \( \Omega < 1/\tau_r \), where \( \tau_r \) is a characteristic relaxation time. The dependence of the emissivity on time is only through the location of \( \tilde{X}_0 \) on the trajectory \( L \).

Using Eq. (2) for the emissivity related to each parameter, the charge pumped per period, corresponding to a trajectory \( L \) in the parameter space, is given by

\[
Q(j) = \int_{L} d\tilde{X} \cdot \tilde{A}(j) = \int_{S} dX_1 dX_2 B(j), \tag{3}
\]

where \( \mathcal{S} \) is the area bounded by \( L \) and the effective “vector potential” and “magnetic field” are \( \tilde{A}(j) = e\frac{2dn(j)}{dX} \) and \( B(j) = \left( \vec{\nabla} \times \tilde{A}(j) \right)_3 = \frac{\partial}{\partial X^2} \frac{dn(j)}{dX} - \frac{\partial}{\partial X} \frac{dn(j)}{dX} \), respectively.

To determine the validity regime of Eqs. (2) and (3) we divide the trajectory \( L \) into elements of length \( \delta X \). The length, \( \delta X \), should be smaller than both the radius of curvature along the trajectory, \( r_c = \left| \frac{\dot{\vec{X}}}{|\dot{\vec{X}}|} \right| \), and \( \delta t \) must be longer than \( \tau_r \), so that after each step the system relaxes to a new equilibrium position determined by the Hamiltonian with the new parameters \( \tilde{X}_0 + d\tilde{X}/d\tau \delta t \). Combining both requirements we find that for every \( \tilde{X} \in L \), the length \( \delta X \) has to satisfy:

\[
\frac{d\tilde{X}}{d\tau} \tau_r \ll \frac{d\tilde{X}}{d\tau} \delta t = \delta X \ll \min \{r_2, r_c\}. \tag{4}
\]

Thus, formulas (2) and (3) are valid for

\[
\forall \tilde{X} \in L \quad \left| \frac{d\tilde{X}}{d\tau} \right| \leq \frac{1}{\tau_r} \min \{r_2, r_c\}. \tag{5}
\]

For a circular trajectory: \( \tilde{X}(t) = r_0 (\cos \Omega t, \sin \Omega t) \), one easily finds that \( r_c = r_0, \left| \frac{d\tilde{X}}{d\tau} \right| = \Omega r_0 \) and the condition becomes: \( \Omega_0 \ll \frac{1}{\tau_r} \min \{r_2, r_0, 1\} \).

Adiabatic pumping formula for quantum dots.—Next we consider a quantum dot coupled to noninteracting contacts, described by a parametric dependent version of the Hamiltonian discussed in Ref. (12),

\[
\delta \mathcal{H}_R = \sum_{k,\alpha \in L,R} \epsilon \kappa_\alpha c^\dagger_\kappa_\alpha c_\kappa_\alpha + H^{\text{int}}(d_n^a, d_n^b) + \sum_{k,\alpha \in L,R} V_{\alpha,n} (\tilde{X}) c^\dagger_\kappa_\alpha d_n^a + h.c. \tag{6}
\]

Here \( c^\dagger_\kappa_\alpha \) creates an electron with momentum \( k \) in channel \( \alpha \) belonging to contact \( j \), and \( \{d_n^a\} \) form a complete, orthonormal set of single-electron creation operators in the dot. In this case the current operator is \( \tilde{J}_j = \frac{e}{\hbar} \sum_{k,\alpha \in L,R} V_{\alpha,n}(\tilde{X}) c^\dagger_\kappa_\alpha d_n^a + h.c. \). A straightforward calculation shows that using Eq. (6) the emissivity can be written as \( (h = 1) \):

\[
\frac{dn(j)}{dX} = \frac{d}{d\Omega} \sum_{k,\alpha \in j} \int_{-\infty}^{\infty} d\omega V_{\alpha,n}(\tilde{X}) G_{\alpha,n,k}^\cdagger(\omega, \Omega) + c.c. \tag{7}
\]

where \( u = t_1 - t_1, \quad T = \frac{t_1 + t_1}{2} \),

\[
\delta \mathcal{X} G_{\alpha,n,k}^\cdagger(\omega, \Omega) = -e^{-i\Omega t} \int d\omega \int \delta \mathcal{X} G_{\alpha,n,k}^\cdagger(t_1, t_2, \Omega), \tag{8}
\]

and

\[
\delta \mathcal{X} G_{\alpha,n,k}^\cdagger(t_1, t_1, \Omega) = -i \int T \left\{ -i \int d\tau H(\tau) a(t_1) b(t_1) \right\}. \tag{9}
\]

Here \( G_{\alpha,n,k}^\cdagger(t_1, t_1, \Omega) \) is the Keldysh Green function related to the operators \( a, b = d_n^a, c_\kappa \). We note that the dependence of the Green function on \( \delta \mathcal{X} \Omega \) is through \( H' \) defined above Eq. (11).

Since the contacts are noninteracting, the summation over \( k \) in Eq. (8) can be carried out (12), and the emissivity can be written in terms of \( \delta \mathcal{X} G_{\alpha,n,k}^\cdagger \) only. Defining a vertex function

\[
\delta \mathcal{X} \tilde{G}(\epsilon, \Omega) = \tilde{G}(\epsilon - \frac{\Omega}{2}) \tilde{A}_X(\epsilon, \Omega) \tilde{G}(\epsilon + \frac{\Omega}{2}) \tag{10}
\]

where we used matrix notation both for the dot indices (bold letters) and for the Keldysh indices (hat, '), with

\[
\tilde{G}(G^r, G^\ell, G^0) \text{, we find } \tilde{G}(G^r, G^\ell, G^0) \tag{11}
\]

The arguments of \( f(\epsilon), G(\epsilon), \Gamma(\epsilon) \), and \( A_X(\epsilon, \Omega) \) were suppressed. \( \eta_j = \sum_{n=2}^{j} \int d\omega f(\omega) \left\{ -\frac{i}{\hbar} \text{Im} G_{\alpha,n,k}(\omega) \right\} \) is the equilibrium occupancy of contact \( j \) calculated in the presence of the dot,

\[
\Gamma_{\alpha,n}^0(\epsilon) = 2\pi \sum_{n=2}^{j} V_{\alpha,n}(\epsilon) \rho_{\alpha}(\epsilon) V_{\alpha,n}(\epsilon), \quad \rho_{\alpha}(\epsilon) \quad \text{is the bare density of states in channel} \ \alpha \ \text{and} \ V_{\alpha,n}(\epsilon) = V_{\alpha,n} \quad \text{for} \ \epsilon = \epsilon_{ka}. \quad \text{The matrix} \ \delta \mathcal{X} \Gamma_{\alpha,n}^0 \equiv \frac{\partial^2}{\partial \Gamma_{\alpha,n}^0} \frac{\partial}{\partial \Omega} \text{is antiderminant. In the noninteracting case one can show that expression} \ \frac{\partial n}{\partial \Omega} \ \text{without the first term,} \ -\frac{\partial n}{\partial \Omega}, \ \text{reduces to BPTs result. The BPTs emissivity contains the explicit derivations.} \tag{12}
\]
tive contribution $+\frac{\partial n}{\partial X}$, as they calculate the current deep inside the reservoir, while in this Letter the current defined above Eq. (4) is calculated at the entrance to the dot. An explicit derivative does not influence the pumping charge per period, and formally corresponds to a gauge transformation in the vector potential defined after Eq. (4). Notice that for an infinite flat band and energy independent tunneling couplings $\frac{\partial n}{\partial X} = 0$ even when interactions in the dot are included [17].

**Pumping in the two channel Kondo effect.**—To demonstrate the new features of Eq. (2) which includes interactions, we study a specific example of pumping in a 2CK system at the exactly solvable Emery-Kivelson (EK) line [10]. The peculiarity of a symmetric 2CK problem is in its NFL behavior at low temperatures [17]. In the presence of external magnetic field, $B$, the Hamiltonian of the 2CK model is

$$H^{2\text{CK}} = \sum_{k\sigma j} \epsilon_k c_k^{\dagger} c_{k\sigma j} + \sum_{j,\lambda} J_{j\lambda} S^\lambda \cdot s_j^{\lambda} + g \mu_B H S_z.$$  

The index $j = 1, 2$ represents two channels, $\vec{S}$ is the impurity spin-1/2-operator, $s^\lambda_j$ is the spin density in channel $j$ in direction $\lambda = x, y, z$ near the localized spin.

In the present context we consider spin pumping from channel 1 into channel 2 by calculating the spin-flavor (sf) emissivity $\frac{ds_{1\rightarrow2}}{dx} = \frac{\hbar}{2} (\frac{ds_{1}(1)}{dx} - \frac{ds_{2}(2)}{dx})$, using Eq. (2) with $\tilde{J}_j$ replaced by $\tilde{J}_sf = -e \frac{\partial n_{sf}}{\partial t} = \frac{ie}{\hbar} [n_{sf}, H^{2\text{CK}}]$, where $n_{sf} = s_z(1) - s_z(2)$ is the spin difference between the channels. At the EK line $\tilde{J}_{1z} = \tilde{J}_{2z} = 2\pi \hbar e F$ the spin charge and flavor sectors commute with the spin-flavor (sf) sector that determines the evolution of $\tilde{J}_{sf}$.

In the presence of channel anisotropy in the spin flip processes, $\tilde{J}_{1 \perp} \neq \tilde{J}_{2 \perp}$, the Hamiltonian of the SF sector in a Nambu notation: $\Psi_i^\dagger = (\psi_i^\dagger, \psi_i)$, $i = d, sf$ takes the form of a Majorana resonance level (MRL) model

$$H_{h,\Delta}^{\text{MRL}} = \frac{1}{2} (iv_F \int_{-\infty}^{\infty} dx \Psi_i^{\dagger}(x) \tilde{\Psi}_d(x) \frac{\partial \Psi_d(x)}{\partial x}) + \frac{\Gamma}{2} [\hbar \Psi_d^{\dagger} \tilde{\Psi}_d + \Psi_d^{\dagger} \tilde{\Psi}_d + \Psi_d^{\dagger} \tilde{\Psi}_d (0) \tilde{\Psi}_d (0)],$$

where $\tilde{\Psi} = \sqrt{\Gamma/(2\pi \rho)} (\cos(\theta/2) \tau_z + i \sin(\theta/2) \tau_y)$, $\Gamma \equiv \Gamma_1 + \Gamma_2$, $\Gamma_{1(2)} = \rho \tilde{J}_{1(2)}^{\dagger}/4a$, $a$ can be considered as the lattice spacing, $\hbar = 2g \mu_B H \Gamma$, $\rho = (2\pi \hbar e F)^{-1}$ is the density of states of the chiral fermion field $\Psi_d$, and $\tilde{\tau}$ are the pauli matrices. The operator that describes transition of spin between the channels is $n_{sf} = \int_{-\infty}^{\infty} dx \psi_{sf}^{\dagger}(x) \psi_{sf}(x)$ [18] and $\psi_d$ is a local fermion operator.

The unique properties of the 2CK system can be seen by taking $\theta = \pi/2$ and $h = 0$ in the MRL model. In order to describe pumping in the vicinity of this point we choose as pumping parameters $X_1 = h$ and $X_2 = \Delta = (\Gamma_1 - \Gamma_2)/\Gamma = \cos\theta$.

Using Eq. (2) one finds that the spin flavor emissivity is given by $\tilde{A} = \frac{\hbar}{2\pi} \frac{-(-\Delta, h)}{\tau^2 (\Delta, h) - 4\tilde{\epsilon}(\Delta, h) - 4\tilde{\epsilon}(h, \Delta)}$, where

$$\tilde{\epsilon} = \frac{h}{2\pi} \frac{r^2 (-\Delta, h) - 4\tilde{\epsilon}(\Delta, h) - 4\tilde{\epsilon}(h, \Delta)}{16\tilde{\epsilon}^2 + 8\tilde{\epsilon}^2 (2 - r^2) + r^4}.$$  

Here $\tilde{\epsilon} = \epsilon/\Gamma$, $r = \sqrt{\Delta^2 + \tau^2}$, and $\frac{d \tau}{dx} \equiv (\frac{\partial r}{\partial x}, \frac{\partial \tilde{\tau}}{\partial x}).$

**FIG. 1:** The effective magnetic field $B$ in the $(h, \Delta)$ parameter space. For $T \ll \Gamma$ the effective magnetic field has a peak of weight $h$ at $(h, \Delta) = (0, 0)$ and the shape of the peak is temperature independent if plotted against the scaled parameters $(h/\sqrt{8\pi T/\Gamma}, \Delta/\sqrt{8\pi T/\Gamma})$. Each of the pumping trajectories $\mathcal{L}_1 : (h/h_0)^2 + (\Delta/\Delta_0)^2 = 1$ (full line) and $\mathcal{L}_2 : (h + 3h_0/h_0)^2 + (\Delta/\Delta_0)^2 = 1$ (dashed line) is plotted for $T_1 \ll \Gamma \Delta_0^2$ and for $T_2 \gg \Gamma \Delta_0^2$ in the scaled parameter space, leading to four different curves $\mathcal{L}_i^j$, $i, j = 1, 2$ (all trajectories are anticlockwise). Denoting the area bounded by $\mathcal{L}_i^j$ as $\mathcal{S}_i^j$, we see that while $\mathcal{S}_1^1$ contains the entire area of the peak of $B$, corresponding to pumping a spin of exactly $h$ from channel 1 to 2, the intersection of the area of the peak with $\mathcal{S}_2^1$ is empty. On the other hand, the relative area of the peak contained in $\mathcal{S}_2^2$ is approximately $\frac{2\pi \hbar e F}{\tau^2}$, corresponding to the pumped spin $\sim h \Delta / \tau^2$. The anomalous power is a manifestation of the NFL behavior.

At $T = 0$ Eq. (4) gives $\tilde{A} = \frac{h}{2\pi} \frac{(\Delta, h)}{h^2 + \Delta^2}$, (which is $\frac{h}{2\pi} \frac{\tilde{A}}{\tau^2}$ in polar coordinates) – the vector potential corresponding to an effective magnetic field $B = h \delta^2 (h, \Delta)$ perpendicular to the plane of the trajectory. Thus, in a pumping period encircling the NFL point $(h, \Delta) = (0, 0)$ anticlockwise, a total spin of exactly $h$ is transferred from channel 1 to channel 2 [10]. The magnetic field $B$ can be obtained analytically for any finite temperature. We discuss below the different regions of $B$ as a function of $T$.

For $T/\Gamma \ll 1$, namely for temperatures smaller than the Kondo scale, the pumping vector potential can be approximated by $\tilde{A}(\epsilon) = \frac{h}{2\pi} \frac{\tilde{B}^2}{\epsilon^2}$, which after integration over $\epsilon$ gives $\tilde{A} = \frac{h}{2\pi} \frac{\tilde{B}}{\epsilon^2} \psi_1 (1 + \epsilon^2 / \Gamma / 2 \pi)$, where $\psi_1$ is the trigamma function. (We have neglected the third term in

\[\tilde{A} = \frac{h}{2\pi} \frac{\tilde{B}^2}{\epsilon^2} \psi_1 (1 + \epsilon^2 / \Gamma / 2 \pi)\]
The temperatures $T$ for very large temperatures requires a temperature dependence of the pumped spin, using the function $F_1(x) = \frac{1}{\pi} \frac{\partial}{\partial y} \langle \psi_1 (1/2 + y) \rangle_{|y=\pi^2}$. This means that the effective magnetic field of strength $\sim \hbar T/(16T)$ is concentrated in a circle of radius $\sim \sqrt{T}/ \Gamma$, and decays strongly as $h_{2} \times T(16T)^2$, as depicted in Fig. 1.

At $T \sim \Gamma$ the peak size approaches unity and ceases to be circularly symmetric due to the second term in Eq. (3).

For $T > \Gamma$ the effective magnetic field becomes practically independent of $-1 \leq \Delta \leq 1$, and given by

$$B(h, T) = \frac{\hbar \Gamma}{16T} F_2 \left( \frac{\hbar \Gamma}{T} \right),$$

(8)

with $F_2(x) = \frac{\pi}{\sqrt{2}} \text{Re} \left[ \psi_1 \left( \frac{1}{2} \pm \frac{x}{\pi \sqrt{2}} \right) \right]$. Since $\int_{-\infty}^{\infty} dx F_2(x) = 8$, the weight of the pumping peak is again $h$, however the peak is very wide $\sim T/\Gamma \gg 1$.

The total pumped spin is obtained by performing the integral $\int_{-\infty}^{\infty} d\Gamma B(r)$, where $S$ is the area contained in the trajectory $L$. We can easily estimate the temperature dependence of the pumped spin, using the structure of $B(r)$ described in Eqs. (7) and (8).

Consider for example, $L_1$, an elliptic pumping trajectory $(h/h_0)^2 + (\Delta/\Delta_0)^2 = 1$ where $\Delta_0 \ll 1 < h_0$ (see Fig. 1). At $T = 0$ it encircles the origin and therefore $s_{L_1}^{\perp}(T = 0) = h$. At low temperatures $\sqrt{T/\Gamma} \ll \Delta_0$, taking into account the tail of the peak of $B$, we obtain

$$s_{L_1}^{\perp}(T) = \hbar \left( 1 - \frac{T}{T_0} \right)^2, \quad T_0 = \frac{\Gamma}{4\pi} \sqrt{\frac{6h_0^2 \Delta_0^2}{\hbar^2 + \Delta_0^2}}.$$  

(9)

This is expected from the FL behavior along the trajectory (no anomalous exponents appear). At higher temperatures when $\Delta_0 \ll \sqrt{T/\Gamma} \ll 1$ we find

$$s_{L_1}^{\perp}(T) = c \hbar \sqrt{T/\Gamma} \Delta_0,$$  

(10)

with $c$ of order unity. Here the anomalous exponents ($\sim T^{-1/2}$) of the NFL point become apparent. In Fig. 1 the magnetic field is plotted in terms of scaled parameters $(h/\sqrt{8\pi T/\Gamma}, \Delta/\sqrt{8\pi T/\Gamma})$. Using this scaled parameters, as long as $T \ll \Gamma$, the shape of the peak is temperature independent, however each trajectory $L$ acquires a temperature dependence $L^T$. Let us consider the temperatures $T_1$ and $T_2$ satisfying $\sqrt{T_1/\Gamma} \ll \Delta_0$ and $\Delta_0 \ll \sqrt{T_2/\Gamma} \ll 1$ respectively. We see that the area bounded by $L_1^{T_1}$ contains the entire peak of $B$ while the area bounded by $L_1^{T_2}$ contains only a one dimensional cut of the peak, (whose radius scales as $\Gamma/T$ for the bare parameters) explaining the anomalous behavior for $T_2$.

For $1 \ll T/\Gamma < h_0$ we find $s_{L_1}^{\perp}(T) = h \Delta_0$. Finally for very large temperatures $1 \ll h_0 \ll T/\Gamma$ we have $s_{L_1}^{\perp}(T) = h \pi h_0 \Delta_0 \Gamma/(16T)$. Such area law [$O(\pi h_0 \Delta_0 \Gamma)$] is expected in this regime, since $B$ is practically constant for $h_0, \Delta_0 \ll T/\Gamma$.

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[9] Here $J_j \equiv -e \frac{\partial N_j}{\partial t}$, and $N_j$ is the number of electrons in contact $j$.
[10] In particular the first term in Eq. (11) is $\Omega$ independent, and vanishes after differentiation. Notice that when the term $(\gamma/\epsilon)$ does not vanish, a counter term from the commutator in Eq. (11) cancels it so that at $\Omega = 0$ the total current is zero.
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