On a possible effective four-boson interaction and its implications at the upgraded LHC

Boris A. Arbuzov*† and Ivan V. Zaitsev†

M. V. Lomonosov Moscow State University, 1(2) Leninskie gory, Moscow 119991, Russian Federation
*E-mail: arbuzov@theory.sinp.msu.ru

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We consider the possibility of a spontaneous generation of four-fold effective interactions of electroweak gauge bosons $W^+$ and $B$. The conditions for the spontaneous generation are shown to lead to a set of compensation equations for parameters of the interaction. In the case of a realization of a non-trivial solution of the set, the important electroweak parameter $\sin^2 \theta_W$ is defined. The existence of two non-trivial solutions is demonstrated, which provide a satisfactory value for the electromagnetic fine structure constant $\alpha$ at scale $M_Z$: $\alpha(M_Z) = 0.007756$. There is a solution with the high effective cut-off being close to the Planck mass by order of magnitude. The most interesting solution corresponds to an effective cut-off of $\approx 10^2$ TeV. This solution gives a quite definite prediction for non-perturbative effects in the processes $p + p \rightarrow \bar{t}t(W^\pm, Z)$, which could be observed at the upgraded LHC.

1. Introduction

The standard model (SM) of particle interactions is considered quite a successful theory. It explains the phenomena in high energy physics experiments, and gives us an interpretation of their overall structure. However, we cannot currently tell whether SM is an all-sufficient theory. First of all, it cannot explain the lowest-energy gravitational interaction in coupling with other ones and in the same way as others. This causes theorists to attempt to build different SM extensions.

However, secondly, even if we leave such problems as scale hierarchy aside, we immediately face another, so to say, more prosaic problem. We cannot admit the standard model as a complete theory simply because there are too many external parameters we have to bring into it to provide its expository power. The number of these parameters (such as coupling constants and matter field mass ratios) has reached 29. Of course, we can hope that determination of their value will be supplied with the previously mentioned would-be extensions of the SM. But we see a striking contrast between the existing theory’s insularity and its wide experimental validation just in this insularity, on the one hand, and, on the other, the absence of any data confirmation of the necessary extensions at the present time and indeterminate perspectives in the foreseeable future.

Provided that the would-be extended theory will really be able to reduce the totality of all data to one general principle, we have to make considerable progress in experimental technology to validate this theory as well as the SM is validated now. This problem prompts us to make efforts in other directions. In particular, we can attempt to work out the necessary evaluations just within the existing

†These authors contributed equally to this work.
theory structure. And those minimal extensions, which we’ll have to build anyway, must rather be non-structural and deal not with new fields and particles, but with new types of effective interactions between known ones. The search region for such interactions may be, of course, indicated by the fact that the SM and general quantum field theory are less successful in describing lower-energy processes. We can ask ourselves whether there is some deep correlation between the failure of perturbation theory in describing such phenomena and the presence of special low-energy quantum effects, which are to be taken into account in some way. This situation patently corresponds with such effects as superconductivity and superfluidity, where classical local theory was powerless and analysis in terms of fundamental quantum theory equations was inaccessible also, but where the solution was found in the framework of non-perturbative contributions. We have no perturbative source of a “force” that binds electrons into Cooper pairs, but we can describe their behavior in this pairing.

The method of effective non-local interaction building, which we shall try to apply to the above-mentioned problem in this work, was developed from N. N. Bogoliubov’s compensation conception [1,2] developed for and successfully applied to superconductivity theory. Although in field theory it acquires some new specialities, the above-stated analogy seems to be quite encouraging for us. And on the other hand, we have also quite successfully applied this approach to a range of low-energy particle physics processes. The compensation approach was applied [3,4] to the problem of the spontaneous generation of effective interactions in quantum field theories. The most impressive effectiveness of the method was demonstrated in light meson physics, where spontaneously generated Nambu–Jona-Lasinio interaction [5,6] building allowed us [7] to predict the main particle properties with good precision using only fundamental QCD parameters, without bringing in external parameters. Also, applications to the composite Higgs particle problem [8] and to the spontaneous generation of the would-be anomalous three-boson interaction [9,10], to be discussed below, can be mentioned. Our aim in this work is to demonstrate the possibility of finding a solution for the fundamental SM parameter problem in terms of effective interactions. Correspondingly, we build a simple model, being guided by our previous experience in similar, but more advanced, models. Success in this attempt would be a really important step, hopefully opening the way to more sophisticated theories on this important subject that are closer to reality. But, at the same time, just with this simplified approach we shall present some predictions suitable for experimental study at the upgraded LHC.

In Refs. [4–11], the Bogoliubov compensation principle [1,2] was applied to studies of the spontaneous generation of effective non-local interactions in renormalizable gauge theories. The method and applications are also described in full in Ref. [12].

In particular, Refs. [9,10] deal with an application of the approach to the electroweak interaction and the possibility of spontaneous generation of effective anomalous three-boson interactions of the form

\[-\frac{G}{3!} F \epsilon_{abc} W^a_{\mu\nu} W^b_{\nu\rho} W^c_{\rho\mu} ,\]

where

\[W^a_{\mu\nu} = \partial_{\mu} W^a_{\nu} - \partial_{\nu} W^a_{\mu} + g \epsilon_{abc} W^b_{\mu\nu} W^c_{\mu} , \]

with the uniquely defined form-factor \(F(p_i)\), which guarantees that the effective interaction (1) acts in a limited region of the momentum space. This was done in the framework of an approximate scheme, whose accuracy was estimated to be \(\sim 10\%–15\%\) [3]. The possible existence of effective interaction (1) leads to important non-perturbative effects in the electroweak interaction. This is usually called the anomalous three-boson interaction, and it has been considered for a long time
on phenomenological grounds \[13,14\]. Our interaction constant $G$ is connected with conventional definitions in the following way:

$$G = -\frac{g\lambda}{M_W^2},$$  \hspace{1cm} (2)$$

where $g \simeq 0.65$ is the electroweak coupling. The best limitations for parameter $\lambda$ read \[15\]

$$\lambda_Y = -0.022 \pm 0.019, \quad \lambda_Z = -0.09 \pm 0.06,$$  \hspace{1cm} (3)$$

where the subscript denotes a neutral boson being involved in the experimental definition of $\lambda$.

For the electroweak interaction we have \[9,10\] as conditions for a spontaneous generation of interaction (1) the following relations:

$$g(z_0) = 0.60366, \quad z_0 = 9.6175, \quad G = 0.000352 \text{ TeV}^{-2}. \hspace{1cm} (4)$$

Here, $z_0$ is a dimensionless parameter connected with the value of a boundary momentum, which is defined with the effective cut-off $\Lambda_0$ according to the following definition \[9,10\]:

$$\frac{2 G^2 \Lambda_0^4}{1024 \pi^2} = \frac{g^2 \lambda^2 \Lambda_0^4}{512 \pi^2 M_W^4} = z_0.$$  \hspace{1cm} (5)

Note that the solution of the analogous compensation procedure in QCD corresponds to $g(z_0) = 3.817$ \[11\], which gives a satisfactory description of the low-momentum behavior of the running strong coupling.

It is instructive to present in Fig. 1 the behavior of the form-factor $F(p, -p, 0)$, dependent on the momentum $p$, where

$$z = \frac{G^2 p^4}{512 \pi^2}$$  \hspace{1cm} (6)$$

and $F(z) = 0$ for $z > z_0$. As a rule, the existence of a non-trivial solution of a compensation equation imposes essential restrictions on the parameters of a problem. One example of these restrictions

![Fig. 1. The behavior of the form-factor for the electroweak theory.](image-url)
is the definition of coupling constant $g(z_0)$ in (4). It is advisable to consider other possibilities for spontaneous generation of effective interactions, and to find out which restrictions on physical parameters may be imposed by the existence of non-trivial solutions. In the present work we consider the possible definition of links between important physical parameters, first of all in relation to the fine structure constant $\alpha$.

2. Weinberg mixing angle and the fine structure constant

Let us demonstrate a simple model that illustrates how the well-known Weinberg mixing angle could be defined. Let us consider the possibility of a spontaneous generation of the following effective interaction of electroweak gauge bosons,

$$L^{\text{Weinberg}}_{\text{eff}} = -\frac{G_2}{8} W^{a}_{\mu} W^{a}_{\rho} W_{\rho \sigma} W_{\mu \sigma} - \frac{G_3}{8} W^{a}_{\mu} W^{a}_{\rho} B_{\rho \sigma} B_{\mu \sigma} - \frac{G_4}{8} Z_{\mu} Z_{\mu} W^{b}_{\rho \sigma} W^{b}_{\rho \sigma} - \frac{G_5}{8} Z_{\mu} Z_{\mu} B_{\rho \sigma} B_{\rho \sigma},$$

(7)

where we maintain the residual gauge invariance for the electromagnetic field. Here, indices $a, d$ correspond to charged $W$s, that is, they take values 1, 2, while index $b$ corresponds to three components of $W$ defined by the initial formulation of the electroweak interaction. Definition (7) corresponds to a convenient rule for Feynman rules for corresponding vertices, e.g. for the first term in, (7)

$$\Gamma G^{2}_{\mu \nu} (g_{\rho \sigma} (p q) - p_{\sigma} q_{\rho}),$$

(8)

where components of $W^{a}$ have indices $\mu, \nu$ and incoming momenta and indices $(p, \rho)$ and $(q, \sigma)$ refer to fields $W^{b}$. Let us remember the relation connecting fields $W^{0}, B$ with physical fields of the $Z$ boson and of the photon:

$$W^{0}_{\mu} = \cos \theta_{W} Z_{\mu} + \sin \theta_{W} A_{\mu},$$

$$B_{\mu} = - \sin \theta_{W} Z_{\mu} + \cos \theta_{W} A_{\mu}.$$

(9)

Thus, in terms of the physical states ($W^{+}, W^{-}, Z, A$), the would-be effective interaction (7) has the following form:

$$L^{\text{Weinberg}}_{\text{eff}} = -\frac{G_2}{2} W^{+}_{\mu} W^{-}_{\mu} W^{+}_{\rho \sigma} W^{-}_{\rho \sigma} - \frac{G_2}{4} W^{+}_{\mu} W^{-}_{\mu} \left(\cos^{2} \theta_{W} Z_{\rho \sigma} Z_{\rho \sigma}ight) + 2 \cos \theta_{W} \sin \theta_{W} Z_{\rho \sigma} A_{\rho \sigma} + \sin^{2} \theta_{W} A_{\rho \sigma} A_{\rho \sigma}\right) - \frac{G_4}{4} Z_{\mu} Z_{\mu} W^{+}_{\rho \sigma} W^{-}_{\rho \sigma} - \frac{G_4}{8} Z_{\mu} Z_{\mu} \left(\cos^{2} \theta_{W} Z_{\rho \sigma} Z_{\rho \sigma} + \sin^{2} \theta_{W} A_{\rho \sigma} A_{\rho \sigma} + 2 \cos \theta_{W} \sin \theta_{W} Z_{\rho \sigma} A_{\rho \sigma}\right) - \frac{G_5}{4} W^{+}_{\mu} W^{-}_{\mu} \left(\sin^{2} \theta_{W} Z_{\rho \sigma} Z_{\rho \sigma} + \cos^{2} \theta_{W} A_{\rho \sigma} A_{\rho \sigma} - 2 \cos \theta_{W} \sin \theta_{W} Z_{\rho \sigma} A_{\rho \sigma}\right) - \frac{G_5}{8} Z_{\mu} Z_{\mu} \left(\sin^{2} \theta_{W} Z_{\rho \sigma} Z_{\rho \sigma} + \cos^{2} \theta_{W} A_{\rho \sigma} A_{\rho \sigma} - 2 \cos \theta_{W} \sin \theta_{W} Z_{\rho \sigma} A_{\rho \sigma}\right)\right),$$

(10)

Interactions of type (10) were earlier introduced on phenomenological grounds in Refs. [16,17], and are subjects for experimental studies. Let us introduce an effective cut-off $\Lambda$ and consider the possibility of a spontaneous generation of interaction (7). In doing this, we proceed with the
add–subtract procedure, which was used throughout Refs. [3–10]. We start with usual form of the Lagrangian, which describes electroweak gauge fields \( W^a \) and \( B \):

\[
L = L_0 + L_{\text{int}},
\]

\[
L_0 = -\frac{1}{4} (W_{\mu\nu}^a W_0^{a\mu\nu}) - \frac{1}{4} (B_{\mu\nu} B^{\mu\nu}),
\]  
\[
(11)
\]

\[
L_{\text{int}} = -\frac{1}{4} (W_{\mu\nu}^a W_{\mu\nu}^a - W_0^{a\mu\nu} W_0^{a\mu\nu}),
\]  
\[
(12)
\]

\[
W_0^{a\mu\nu} = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,
\]

and \( W_{\mu\nu}^a \) is the well-known non-linear Yang–Mills field of \( W \) bosons. Then we perform the add–subtract procedure of expression (7):

\[
L = L'_0 + L'_{\text{int}},
\]

\[
L'_0 = L_0 - L_{\text{eff}},
\]  
\[
(13)
\]

\[
L'_{\text{int}} = L_{\text{int}} + L_{\text{eff}}.
\]  
\[
(14)
\]

Now let us formulate compensation equations for the would-be interaction (7). We are to require that, considering the theory with Lagrangian \( L'_0 \) (13), all contributions to four-boson connected vertices corresponding to interaction (7) are summed to zero. That is, the undesirable interaction part in the would-be free Lagrangian (13) is compensated. Then we are left with interaction (7) only in the proper place (14). We would formulate these compensation equations using experience acquired in the course of the application of the method to the Nambu–Jona-Lasinio interaction and the triple weak boson interaction (1). As is demonstrated in Sect. 3.3 of [12], the first approximation for the problem of spontaneous generation of the Nambu–Jona-Lasinio interaction takes the form-factor \( F(p) \) to be a step function \( \Theta(\Lambda^2 - p^2) \), and only horizontal diagrams of the type presented in Fig. 2 are taken into account. The next approximation, described in detail in [4] and in Chap. 5 of [12], also includes vertical diagrams, and form-factor \( F(p) \) is uniquely defined as a solution of the set of compensation conditions in terms of standard Meijer functions. We have demonstrated that the first approximation gives satisfactory results, and the next one serves for its specification. In the present work we just use the first approximation.

So, let us introduce the effective cut-off \( \Lambda \), which is a subject for definition by solution of the problem, and use just a step function \( \Theta(\Lambda^2 - p^2) \) for the effective form-factor.

In this way, we have the following set of compensation equations, which corresponds to the diagrams presented in Fig. 2:

\[
-x_2 - 2F_W x_2^2 - (1 - a^2) F_Z x_3 x_4 - a^2 F_Z x_2 x_4 = 0,
\]  
\[
(15)
\]

\[
-x_3 - 2F_W x_2 x_3 - a^2 F_Z x_2 x_5 - (1 - a^2) F_Z x_3 x_5 = 0,
\]

\[
-x_4 - 2F_W x_2 x_4 - a^2 F_Z x_4^2 - (1 - a^2) F_Z x_3 x_4 = 0,
\]

\[
-x_5 - 2F_W x_3 x_4 - a^2 F_Z x_4 x_5 - (1 - a^2) F_Z x_5^2 = 0,
\]

\[
F_W = 1 - \frac{2M_W^2}{\Lambda^2} \left( L_W - \frac{1}{2} \right),
\]
Fig. 2. Diagram representation of set (15). Simple lines represent $W$, dotted lines represent $B$, and lines consisting of black spots represent $Z$.

\[
F_Z = 1 - \frac{2M_Z^2}{\Lambda^2} \left( L_Z - \frac{1}{2} \right),
\]

\[
x_i = \frac{3 G_i \Lambda^2}{16 \pi^2}, \quad L_W = \ln \frac{\Lambda^2 + M_W^2}{M_W^2}, \quad L_Z = \ln \frac{\Lambda^2 + M_Z^2}{M_Z^2}, \quad a = \cos \theta_W.
\]

The factor 2 in these equations corresponds to the sum by weak isotopic index $\delta_a = 2$, $a = 1, 2$.

We have the following solutions of set (15) in addition to the evident trivial one, $x_2 = x_3 = x_4 = x_5 = 0$:

\[
x_3 = x_5 = 0, \quad x_2 = -\frac{1 + a^2 F_Z x_4}{2 F_W}; \quad (16)
\]

\[
x_3 = x_5 = 0, \quad x_2 = -\frac{1}{2 F_W}, \quad x_4 = 0; \quad (17)
\]

\[
x_2 = x_4 = 0, \quad x_3 = \frac{a^2}{2(1 - a^2) F_W}, \quad x_5 = -\frac{1}{(1 - a^2) F_Z}; \quad (18)
\]

\[
x_2 = x_4 = -\frac{1}{2 F_W}, \quad x_3 = \frac{a^2}{2(1 - a^2) F_W}, \quad x_5 = -\frac{1}{(1 - a^2) F_Z}; \quad (19)
\]

\[
x_2 = -\frac{1}{2 F_W}, \quad x_4 = x_3 = x_5 = 0; \quad (20)
\]

\[
x_2 = x_4 = 0, \quad x_5 = -\frac{1}{(1 - a^2) F_Z}; \quad (21)
\]

\[
x_2 = -\frac{1}{2 F_W}, \quad x_4 = 0, \quad x_3 = \frac{a^2}{2(1 - a^2) F_W}, \quad x_5 = -\frac{1}{(1 - a^2) F_Z}; \quad (22)
\]
Fig. 3. Diagram representation of set (25, 26). Simple lines represent $W$, dotted lines represent $B$, wavy lines represent a photon, and lines consisting of black spots represent $Z$. Thick lines represent the Higgs scalar.

\[
x_2 = -\frac{1}{2 F_W}, \quad x_4 = x_5 = 0; \quad (23)
\]
\[
x_2 = x_4 = -\frac{1 + (1 - a^2)F_Z x_5}{2 F_W + a^2 F_Z}, \quad x_3 = x_5. \quad (24)
\]

Note that the absence of some $x_i$ in a solution means that this $x_i$ is arbitrary.

Then, following the reasoning of the approach, we assume that the Higgs scalar corresponds to a bound state consisting of a complete set of fundamental particles. Here we study the would-be effective interaction (7, 10) of the electroweak bosons, so we take into account just these bosons as constituents of the Higgs scalar. There are two Bethe–Salpeter equations for this bound state, because the constituents are either $W^a, W^a$ or $Z, Z$. These equations are presented in the two rows of Fig. 3. In the approximation of the very large cut-off $\Lambda$ these equations have the following form using the notation of (15):

\[
- 3 x_2 (2 F_W + a F_Z) - \frac{x_3 (1 - a^2)}{a} - \frac{3 \alpha_{ew}}{16 \pi} \left[ \frac{a^2 (a^6 - a^4 - 5 a^2 + 1)}{1 - a^2} L_W + \right.
\]
\[
\left. \frac{(1 + a^2)(1 - 3 a^2)}{a^2 (1 - a^2)} L_Z - \frac{(1 - a^2 - a^4)(1 - a^2)}{a^2} \right] + \quad (25)
\]
\[
- \frac{3 \alpha_{ew} M_W^2}{32 \pi} \left[ \frac{3 M_H^2}{(M_H^2 - M_W^2)^2} \ln \left( \frac{M_H^2}{M_W^2} \right) - \frac{3}{M_H^2 - M_W^2} - \frac{8}{M_W^2} \right] = \frac{1}{B_W},
\]
\[
- x_4 (2 F_W + a F_Z) - \frac{x_5 (1 - a^2)}{a} - \frac{\alpha_{ew} a^2}{4 \pi} +
\]
\[
\frac{3 \alpha_{ew} M_Z^2}{32 \pi a^4} \left[ \frac{3 M_H^2}{(M_H^2 - M_W^2)^2} \ln \left( \frac{M_H^2}{M_Z^2} \right) - \frac{3}{M_H^2 - M_Z^2} - \frac{8 M_W^2}{M_Z^2} \right] = \frac{1}{a^2 B_Z},
\]

\[
B_W = F_W + \frac{M_H^2}{2 \Lambda^2} L_W - \frac{13}{12}, \quad B_Z = F_Z + \frac{M_H^2}{2 \Lambda^2} L_Z - \frac{13}{12},
\]

\[
\alpha_{ew} = \frac{\alpha_0}{1 + \frac{5 \alpha_0}{6 \pi} \ln \frac{\Lambda^2}{M_Z^2}}, \quad \alpha_0 = 0.0337, \quad a = \cos \theta_W (\Lambda),
\]

\[
1 - a^2 = \frac{\alpha}{\alpha_0} \left( \frac{1 + \frac{5 \alpha_0}{6 \pi} \ln \frac{\Lambda^2}{M_Z^2}}{1 - \frac{5 \alpha}{6 \pi} \ln \frac{\Lambda^2}{M_Z^2}} \right), \quad \alpha = \frac{\alpha^2(M_Z)}{4 \pi} = \alpha(M_Z) = 0.007756.
\]

Now we look for solutions of set (15, 25, 26, 27) for variables \(x_2, x_3, x_4, x_5, \) and \(\Lambda\) that give an appropriate value for \(\alpha(M_Z) = 0.007756,\) according to relation (27). We use these values for the physical masses:

\[
M_W = 0.0804 \text{ TeV}, \quad M_Z = 0.0912 \text{ TeV}, \quad M_H = 0.1251 \text{ TeV}.
\]

We have studied the solutions of the set of equations and have come to the conclusion that only solutions (16), (21), and (24) of compensation set (15) give the necessary value \(\alpha(M_Z) = 0.007756.\) For the first option (16) there are two solutions that satisfy our conditions. Namely, the following ones, where \(a\) and \(x_4\) are just solutions of the set and \(x_2\) is defined by relation (16):

\[
\Lambda = 5.226 \times 10^2 \text{ TeV}, \quad x_2 = -0.3238, \quad x_4 = -0.4865, \quad a = 0.8511;
\]

\[
\Lambda = 8.687 \times 10^6 \text{ TeV}, \quad x_2 = -0.3160, \quad x_4 = -0.7113, \quad a = 0.7192.
\]

These solutions define coupling constants of effective interaction (7) again for the two solutions

\[
G_2 = -6.24 \times 10^{-5} \text{ TeV}^{-2}, \quad G_4 = -9.376 \times 10^{-5} \text{ TeV}^{-2};
\]

\[
G_2 = -2.2045 \times 10^{-33} \text{ TeV}^{-2}, \quad G_4 = -4.962 \times 10^{-33} \text{ TeV}^{-2}.
\]

From the definition of parameters in the experimental work [19],

\[
L_{\text{eff}} = -\frac{\alpha^2 a^W_0}{8 \Lambda^2} A_{\mu \nu} A_{\mu \nu} W^+ \rho^- W^+ \rho^- - \frac{\alpha^2 g^2 k^W_0}{\Lambda^2} A_{\mu \nu} Z_{\mu \nu} W^+ \rho^- W^+ \rho^-,
\]

and from (7) we have

\[
\frac{a^W_0}{\Lambda^2} = \frac{2 G_2}{g^2}, \quad \frac{k^W_0}{\Lambda^2} = \frac{G_2 \cos \theta_W}{2 g^4 \sin \theta_W}.
\]

Results (31, 32) lead to the following prediction for parameters \(a^W_0\) and \(k^W_0\) for the two solutions:

\[
\frac{a^W_0}{\Lambda^2} = -0.000147 \text{ TeV}^{-2}, \quad \frac{k^W_0}{\Lambda^2} = -0.000142 \text{ TeV}^{-2};
\]

\[
\frac{a^W_0}{\Lambda^2} = -1.044 \times 10^{-32} \text{ TeV}^{-2}, \quad \frac{k^W_0}{\Lambda^2} = -1.13 \times 10^{-32} \text{ TeV}^{-2}.
\]
Comparing the last two expressions and taking from the experimental work [19] the following limitations,

$$-21 \text{ TeV}^{-2} < \frac{a_W}{\Lambda^2} < 20 \text{ TeV}^{-2}, \quad -12 \text{ TeV}^{-2} < \frac{k_W}{\Lambda^2} < 10 \text{ TeV}^{-2}, \quad (37)$$

we see that the predictions (35, 36) are well inside the boundaries of limitations (37). The most recent limitations [20] at 7–8 TeV, which essentially improve results for $a_W$,

$$-1.1 \text{ TeV}^{-2} < \frac{a_W}{\Lambda^2} < 1.1 \text{ TeV}^{-2}, \quad (38)$$

also do not contradict estimates (35, 36). Of course, the second solution (36) gives a negligibly small value, whereas the first one (35) needs further improvement in its precision in order to be able to check it.

The second solution (21) of the set of compensation equations gives the following solution:

$$x_2 = x_4 = 0, \quad x_3 = -4.21777, \quad x_5 = -5.95333, \quad a = -0.87338, \quad \Lambda = 0.3646 \text{ TeV}, \quad G_2 = G_4 = 0, \quad G_3 = -1670 \text{ TeV}^{-2}, \quad G_5 = -2360 \text{ TeV}^{-2}. \quad (39)$$

The third solution (24) of the set of compensation equations gives two solutions with the same cut-off. We have the following sets of parameters:

$$x_2 = x_4 = -1.72596, \quad x_3 = x_5 = 3.9589, \quad a = -0.876955, \quad \Lambda = 0.1068 \text{ TeV}, \quad G_2 = G_4 = -7970 \text{ TeV}^{-2}, \quad G_3 = G_5 = 18270 \text{ TeV}^{-2}; \quad (40)$$

$$x_2 = x_4 = -0.864885, \quad x_3 = x_5 = -2.61273, \quad a = 0.876955, \quad \Lambda = 0.1068 \text{ TeV}, \quad G_2 = G_4 = -3992 \text{ TeV}^{-2}, \quad G_3 = G_5 = -12060 \text{ TeV}^{-2}; \quad (41)$$

Solutions (39, 40, 41) evidently contradict limitations (37, 38) due to very low values for the cut-off $\Lambda$.

There is also solution (30) with a very large cut-off $\Lambda$. It is remarkable that this solution corresponds to the cut-off being of the order of magnitude of the Planck mass $M_{Pl} = 1.22 \times 10^{19} \text{ TeV}$. Of course, the effective coupling constants $G_i$ in this case are extremely small. This possibility may serve as an explanation of the hierarchy problem [21]. Indeed, with this solution the actual values for the masses of $W$, $Z$, and $H$ and the value $\alpha(M_Z)$ may be reconciled with the effective cut-off being defined by the gravitational Planck mass. So the actual relation between the electroweak scale and the gravity scale may acquire at least a qualitative interpretation.

We would draw attention to the low cut-off case also. The value of $\Lambda$ in (29) is close to the boundary value of the momentum in the problem of the spontaneous generation of the anomalous triple $W$ interaction (1). Indeed, the value of the electroweak gauge constant $g$ at this boundary (4) is

$$g(\Lambda) = 0.60366. \quad (42)$$

Then the following relation is to be fulfilled:

$$\frac{g(\Lambda)^2}{4\pi} = \alpha_{ew}, \quad (43)$$

where $\alpha_{ew}$ is defined in (27). Then this relation is an equation for parameter $\Lambda$. The solution of this equation gives

$$\Lambda = 7.91413 \times 10^2 \text{ TeV}. \quad (44)$$
We see that this value is of the same order of magnitude as the value $5.2262 \times 10^2$ TeV in solution (29).

Now we could formulate the results in a rather different manner. We have two interesting values for the possible cut-off $\Lambda$. The low value (44), which is compatible with previous results [9,10] by the order of magnitude, and the Planck mass. Let us consider the set of equations (16, 25, 26) for these values of the cut-off. Earlier, we fixed the actual value for the electromagnetic constant $\alpha(M_Z)$ and calculated values for the cut-off (29, 30). Now we fix $\Lambda$ and calculate $\alpha(M_Z)$. In this way, for the value in (44) and the Planck mass we obtain, respectively,

$$\alpha(M_Z)_{44} = 0.00792, \quad \alpha(M_Z)_{Pl} = 0.00790.$$  

Both values differ from the actual value $\alpha(M_Z) = 0.007756$ by 2%. Thus it might be possible to interpret the results in (45) as a calculation of the value of $\alpha$ with this precision. Just contributions of the order of magnitude of a few % are expected at the next approximation in the development in powers of $\alpha_{ew}$.

Of course, there is the trivial solution of set (15): all $x_i = 0$, which gives no additional information. However, we also have quite informative non-trivial solutions.

The problem of the choice of the genuine solution is of course essential. The answer is connected with the problem of the stability of the solutions. There are also possibilities of phase transitions between different solutions. These problems are very difficult and need extensive additional studies.

3. Experimental implications

The effective interaction (10) leads to effects in inclusive reactions:

$$p + p \rightarrow W^+ + W^- + W^\pm (Z, \gamma) + X.$$  

Unfortunately, with the values for the effective coupling constants $G_2$ and $G_4$ for the preferred solution (29, 31), one could hardly hope to achieve the necessary precision even at the upgraded LHC.

However, there is the possibility of an enhancement of the effect in processes involving $t$-quarks due to large $m_t$. Let us consider the would-be contribution of interaction (10, 29) to the vertex

$$\frac{G_W t}{2} W^b_{\mu \nu} W^b_{\mu \nu}.$$  

The effective coupling for this vertex is defined by the diagrams presented in Fig. 4. We use the diagram representation of a $\bar{t}tWW$ vertex. Dotted lines represent $Z$ bosons, simple lines represent $W^b$. The $t$-quarks are on the left.
same cut-off $\Lambda$ (29) in the calculation of this diagram, and we obtain

$$G_{Wt} = -\frac{g^2(\Lambda) M_t(\Lambda)}{24 M_W^4} \left(2 x_2 + a^2(\Lambda) x_4\right) = 4.25 \times 10^{-8} \text{ GeV}^{-3},$$

(48)

where we take the parameters (29, 42) and for $M_t(\Lambda)$ we use the standard evolution expression

$$M_t(\Lambda) = M_t \left(1 + \frac{7 a_s(M_t)}{4\pi} \ln \frac{\Lambda^2}{M_t^2}\right)^{\frac{2}{7}},$$

(49)

where $M_t = 173.2 \text{ GeV}$ is the table value for the $t$-quark mass [15]. Let us consider $p + p \rightarrow \bar{t} t W^\pm (Z)$ processes. With the value (48) we have an additional contribution of the new effective interaction (47) to the cross section $\sigma_{tW}$ of the process,

$$p + p \rightarrow \bar{t} t + (W^\pm, Z) + X;$$

(50)

for $\sqrt{s} = 8 \text{ TeV}$, we have the following estimate:

$$\Delta \sigma_{tW} \text{(8 TeV)} = 103.5 \text{ fb}.$$ 

(51)

For the same process with the negative $W$ we have

$$\Delta \sigma_{tW^-} \text{(8 TeV)} = 28.0 \text{ fb}.$$ 

(52)

For the process $p + p \rightarrow \bar{t} t + Z$ we have the following contribution:

$$\Delta \sigma_{tZ} \text{(8 TeV)} = 47.2 \text{ fb}.$$ 

(53)

These results, as well as the subsequent ones, were obtained using the CompHEP package [22]. The recent CMS result at $\sqrt{s} = 8 \text{ TeV}$ [23] for these processes are

$$\sigma_{tW} \text{(8 TeV)} = 170^{+110}_{-100} \text{ fb}, \quad \sigma_{tZ} \text{(8 TeV)} = 200 \pm 90 \text{ fb}.$$ 

(54)

The results in (54) are compatible with the would-be additional contributions (51, 53) as well as with the standard model. There is no data for process (52) in [23].

The most recent data at $\sqrt{s} = 8 \text{ TeV}$ of both ATLAS [28] and CMS [29] collaborations are:

$$t \bar{t} W : \sigma = 369^{+100}_{-91} \text{ fb}, \quad t \bar{t} Z : \sigma = 176^{+58}_{-52} \text{ fb} \text{ (ATLAS)};$$

(55)

$$t \bar{t} W : \sigma = 382^{+117}_{-107} \text{ fb}, \quad t \bar{t} Z : \sigma = 242^{+65}_{-56} \text{ fb} \text{ (CMS)}.$$ 

(56)

For the process $pp \rightarrow t \bar{t} W$ with both charge signs we have in new data results that agree with both the SM value ($\sim 232 \text{ fb}$) and the predicted one ($\sim 363 \text{ fb}$). However, one sees that the data of both collaborations slightly favor the last predicted value.

1 We have been persuaded that the interference of contributions of effective interaction (47) with the SM terms is negligible.

2 For the results for $\sqrt{s} = 7 \text{ TeV}$, see [24].
Table 1. SM results for cross sections of processes $p + p \rightarrow \bar{t}tV$ at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV, and predictions for additional contributions due to effective interaction (47).

| Channel | $\sigma_{SM}$ fb, 8 TeV | $\Delta \sigma$ fb, 8 TeV | $\sigma_{SM}$ fb, 14 TeV | $\Delta \sigma$ fb, 14 TeV |
|---------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\bar{t}tW^+$ | $161^{+19}_{-12}$ | 103.5 | $507^{+147}_{-111}$ | 1257 |
| $\bar{t}tW^-$ | $71^{+11}_{-15}$ | 28.0 | $262^{+40}_{-60}$ | 355 |
| $\bar{t}tZ$ | $197^{+22}_{-33}$ | 47.2 | $760^{+74}_{-64}$ | 578 |

Let us note that the additional contributions $\Delta \sigma(\bar{t}tW, Z)$ increase with the energy, and for the updated energy of the LHC of $\sqrt{s} = 14$ TeV they become

$$\Delta \sigma_{\bar{t}tW^+}(14 \text{ TeV}) = 1257 \text{ fb},$$

$$\Delta \sigma_{\bar{t}tW^-}(14 \text{ TeV}) = 355 \text{ fb},$$

$$\Delta \sigma_{\bar{t}tZ}(14 \text{ TeV}) = 578 \text{ fb}.\quad (57)$$

Our predictions are compared with the SM calculations [25–27] in Table 1.\(^3\)

We have already noted that the results for $\sqrt{s} = 8$ TeV do not contradict the current data (54, 55, 56). As for $\sqrt{s} = 14$ TeV, we see from the table that the most promising process for testing the present results at the upgraded LHC is $p + p \rightarrow \bar{t}t W^\pm$. Indeed, the total additional contribution to the production of charged $W$ with a top pair is around 1.6 pb, which more than twice exceeds the corresponding total SM value. Note that we do not include in the table the process $p + p \rightarrow \bar{t}t \gamma$, because the effect here is significantly less pronounced. Namely, for $\sqrt{s} = 13$ TeV we have $\sigma_{SM} = 1.744 \pm 0.005$ pb [27], whereas the effect of interaction (47) is calculated to be $\Delta \sigma = 0.125$ pb. Let us also note that our estimations of the effect might have an accuracy of around 10% according to the experience of applications of the approach to several examples (see Ref. [12]).

Provided the prediction is confirmed, the first non-perturbative effect in the electroweak interaction would be established.

4. Conclusion

To conclude, let us draw attention to the the results in view of the compensation approach to the problem of the spontaneous generation of an effective interaction. First of all, the results are obtained exclusively due to application of this approach. We would emphasize that the existence of a non-trivial solution of the compensation conditions always imposes strong restrictions on the parameters of the problem. We see such restrictions in both the problems of the spontaneous generation of the Nambu–Jona-Lasinio interaction [7] and the triple anomalous weak boson interaction [9,10]. In the present work, such conditions for existence of the interaction (7) are shown to define the Weinberg mixing angle, which leads to results (45) for the electromagnetic coupling constant.

It is also worth mentioning that the would-be effective interaction under consideration leads to the significant experimental effects discussed in Sect. 3, which may be either proved or disproved in forthcoming studies at the LHC.

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\(^3\) The result for $\sigma_{SM}(\bar{t}tZ)$ in the fourth column corresponds to $\sqrt{s} = 13$ TeV.
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