Numerical Analysis on the Similarity between Steel Ladles and Hot-water Models Regarding Natural Convection Phenomena

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(Received on July 5, 2001; accepted in final form on September 17, 2001)

The similarity between steel ladles and hot-water models regarding natural convection phenomena has been systematically analysed through examination of the numerical solutions of turbulent Navier–Stokes partial differential equations governing the phenomena in question. The numerical solutions have been obtained by using a computational fluid dynamics (CFD) simulation method. Key similarity criteria for non-isothermal physical modelling of steel ladles with hot-water models have been derived as

\[ \text{Fr}_m = \text{Fr}_p \quad \text{and} \quad (\beta \Delta T)_m = (\beta \Delta T)_p \]

where the subscripts \( m \) and \( p \) stand for the water model and the prototype steel ladle, respectively. Accordingly, appropriate conditions fulfilling the above criteria, such as model size, water temperature, time scale factor and the scale factor of boundary heat loss fluxes, have been proposed and discussed. As a result, water models with geometry scales between 1/5 and 1/3 and using hot-water of temperature higher than 45°C are appropriate for simulating natural convection phenomena in steel ladles.

KEY WORDS: similarity; model; prototype; water model; steel ladle; natural convection; CFD; numerical simulation.

1. Introduction

Water-model simulation of steel ladles, tundishes and continuous casting moulds is a convenient way to investigate flow phenomena in such high-temperature metallurgical vessels. Therefore, a large number of water-model applications for simulating these types of metallurgical vessels, especially under isothermal conditions, have been reported in the literature. In recent decades, researchers have been paying more and more attention to the transient thermal effect that may explain the difference between the results in prototype high-temperature metallurgical vessels and those obtained by the isothermal water modelling work. Accordingly, an increasing number of non-isothermal water modelling applications has emerged.

Hlinka and Miller11 were probably the first to use a hot-water and acrylic plastic system to simulate liquid steel and refractory systems. Both fluid flow and heat transfer in such systems were studied and the results could be applied, by direct scale-up, to the ladle-tundish systems. Lowry and Sahai3 investigated the residence time distributions (RTDs) for an industrial tundish by using a hot-water model tundish. The water modelling results confirmed that the density difference between the liquid steel in the tundish and the liquid steel coming from a new ladle significantly alters the flow pattern in the tundish. Barreto et al.31 modelled, both physically (using a 1/6-scale water model tundish) and mathematically (using a numerical method), steel flow and heat transfer under non-isothermal conditions with and without considering boundary heat losses. Damle and Sahai32 made a dimensional analysis on the turbulent Navier–Stokes equations describing fluid flow and heat transfer in tundishes. They proposed a single dimensionless number, \( \text{Tu} = \text{Gr}/\text{Re}^2 \), called “Tundish Richardson number”, as a criterion characterising the similarity between the model tundish and the full-scale industrial tundish. Wang et al.33 reported a demonstration of non-isothermal effects in tundishes, by using a hot-water model tundish with multiple-points temperature measurements. Their results showed that even a small temperature difference (say, 1°C) between the model ladle and the model tundish imposes a noticeable difference in flow patterns in the tundish. Based on RTD analyses of a 0.6-scale water model of the US Steel Gary Works No. 2 tundish, Sinha and Vassilicos34 reported that, compared to the isothermal water model, a generally improved agreement between the RTDs measured from the non-isothermal water model and from the actual tundish in the steel plant can be obtained when the effect of temperature gradient on flow is taken into consideration. Sheng et al.35,36 made a more systematic investigation on fluid flow and heat transfer in a 1/2-scale water model tundish under the influence of different input water temperatures. Flow patterns were both visualised physically and simulated mathematically. Temperature distributions were obtained both by measurements at 10 positions in the liquid bath using thermocouples and by numerical simulations. The authors reported that there is a substantial change in the flow patterns when hotter or cooler water is charged into the model tundish and concluded that the fluid flow in the steel tundish is controlled by both natural convection and forced convection, the interaction between which is governed by the dimensionless number \( \text{Gr}/\text{Re}^2 \).

The above reviewed investigations proved that the temperature difference between the incoming steel stream from a ladle and the steel melt already in the tundish has a marked influence on the fluid flow in the tundish. Therefore, this kind of thermal effect on fluid flow in tundishes cannot be neglected. Nevertheless, there is another kind of thermal effect on fluid flow, typically the natural convection phenomenon, which exists in most of steel la-
dles and becomes more pronounced during the holding per-
period before casting. However, relatively few non-isothermal
physical modelling investigations on steel ladles have been
reported. In fact, the natural convection flow in steel ladles,
resulting in temperature stratification of the melt bath, leads
to temperature variation in teeming steel stream that can exert
additional influences on fluid flow and heat transfer in
tundishes. Therefore, in order to define the behaviour of
steel stream temperature during teeming and, hence, attain a
closer simulation on steel ladle and tundish systems, studies
on the natural convection phenomenon in steel ladles and
its influence on the teeming stream temperatures are of pri-
mary importance. For this reason, the authors attempted to
use a hot-water model to simulate natural convection phe-
nomena in steel ladles so as to (1) investigate the thermal
stratification phenomenon and its effect on the teeming
stream temperature during casting and (2) validate mathe-
matical models previously developed for the same purpose.

However, before setting up such a physical modelling
system, it is necessary to first investigate the similarity be-
tween natural convection phenomena in the water model
and in the prototype steel ladle. This is to be made through
examination of non-dimensional governing equations from
which key criteria controlling the similarity between the
model and the prototype can be derived. Furthermore, the
key similarity criteria can be evaluated using the solutions
of the governing equations obtained by means of a com-
putational fluid dynamics (CFD) numerical method.

In general, through the present analysis on the similarity
between natural convection phenomena both in water mod-
els and in steel ladles, it is expected that the appropriate
modelling parameters may be determined, such as model
size, water temperature, time scale factor, and the scale
factor of boundary heat loss flux, based on which a non-
isothermal physical modelling system is to be established.

2. Theoretical Analysis

2.1. Modelling Phenomena and Governing Equations

The present work focuses on natural convection phe-
nomena taking place both in steel ladles and in hot-water mod-
els. These phenomena can be described by the same set of
turbulent Navier–Stokes type partial differential equations,
in the Cartesian tensor notation take the following forms:

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu_{\text{eff}} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\partial p}{\partial x_j} - \rho \beta \Delta T \gamma_j \ldots(1)
\]

for fluid flow, where the Boussinesq approximation of the
buoyancy force is adopted, and

\[
\frac{\partial (\rho C_T)}{\partial t} + \frac{\partial (\rho u_i C_T)}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \lambda_{\text{eff}} \frac{\partial T}{\partial x_j} \right) \ldots(2)
\]

for heat transfer.

Further, \(\mu_{\text{eff}}\) is defined as

\[\mu_{\text{eff}} = \mu_l + \mu_t\]

where \(\mu_l\) and \(\mu_t\) are, respectively, laminar (molecular) and
turbulent viscosities. In the same way, \(\lambda_{\text{eff}}\) is also defined as
the sum of laminar and turbulent heat conductivities, i.e.,

\[\lambda_{\text{eff}} = \lambda_l + \lambda_t\]

In addition, \(\mu_t\) and \(\lambda_t\) can be determined by using turbu-
rence models.

2.2. Modelling Criteria

According to the similitude theory, for geometrically
similar systems (model and prototype) in which the flows
are governed by the same type of mathematical equations,
the two flow phenomena can be considered similar if cer-
tain criteria are satisfied. Such criteria are normally repre-
sented with dimensionless numbers that characterise the
phenomena and can be obtained directly from the dimen-
sionless forms of the governing equations. Following the
method of Damle and Sahai,\(^4\) the dimensionless forms of Eqs. (1) and (2) are derived as follows:

\[
\frac{\partial (\rho^* u_i^*)}{\partial t^*} + \frac{\partial (\rho^* u_i^* u_j^*)}{\partial x_j^*} = \frac{\partial}{\partial x_i^*} \left( \mu_{\text{eff}} \frac{\partial u_i^*}{\partial x_j^*} + \frac{\partial u_j^*}{\partial x_i^*} \right) - \frac{\partial p^*}{\partial x_j^*} - \frac{g L}{U^2} \beta \Delta T \ldots(3)
\]

for fluid flow, and

\[
\frac{\partial (\rho^* T^*)}{\partial t^*} + \frac{\partial (\rho^* u_i^* T^*)}{\partial x_j^*} = \frac{\partial}{\partial x_i^*} \left( \lambda_{\text{eff}} \frac{\partial T^*}{\partial x_i^*} \right) \ldots(4)
\]

for heat transfer, where the variables with a “star” super-
script are the dimensionless variables defined as

\[
\rho^* = \frac{\rho}{\rho_{\text{ref}}} , \quad x_i^* = \frac{x_i}{L} , \quad T^* = \frac{T}{T_{\text{ref}}} , \quad \beta = \frac{\beta}{\beta_{\text{ref}}} , \quad u_i^* = \frac{u_i}{U} ,
\]

\[
\lambda_{\text{eff}} = \frac{\mu_{\text{eff}}}{\rho_{\text{ref}} U L} \quad \text{and} \quad \rho_{\text{ref}} = \frac{\rho - \rho_{\text{ref}}}{\rho_{\text{ref}} U^2} \ldots(7)
\]

In addition, \(\rho_{\text{ref}}, L, U, T_{\text{ref}}\), and \(\rho_{\text{ref}}\) are the reference values of
density, length, velocity, temperature and pressure, respec-
tively.

Under certain dimensionless initial and boundary condi-
tions, Eqs. (3) and (4) should have the same solutions both
for the water ladle (model) and for the steel ladle (proto-
type) if the following equalities are maintained simultane-
ously:

\[
\begin{align*}
\frac{\mu_{\text{eff}}}{\rho_{\text{ref}} U L} & = \frac{\mu_{\text{eff}}}{\rho_{\text{ref}} U L} \ldots(5) \\
\frac{\lambda_{\text{eff}}}{\rho_{\text{ref}} U L} & = \frac{\lambda_{\text{eff}}}{\rho_{\text{ref}} U L} \ldots(6)
\end{align*}
\]

and

\[
\begin{align*}
\frac{g L}{U^2} \beta \Delta T & = \frac{g L}{U^2} \beta \Delta T \ldots(7)
\end{align*}
\]

where the subscripts \(m\) and \(p\) stand for the model and the
prototype, respectively.

Therefore, Eqs. (5) to (7) are the criteria controlling the
dynamic and thermal similarity between natural convection
phenomena in steel ladles and water models. Co-existence
of these equations together with consistent initial and
boundary conditions should guarantee a complete similarity
between the model and the prototype regarding the flow
phenomena of interest. However, it is hard to be sure that
these criteria could be satisfied simultaneously, because it
can be seen clearly that, for water and liquid steel, there is a
conflict between Eqs. (5) and (6), at least for laminar flows.
Nonetheless, according to Damle and Sahai,\(^4\) co-exis-
ence
of these two equations could be possible for fully turbulent flows. Even so, since Eqs. (5) and (6) contain such parameters as \( \mu_{\text{eff}} \) and \( \lambda_{\text{eff}} \) involving turbulence effects that are difficult to assess and control experimentally, these two criteria have to be sacrificed. Thus, Eq. (7) seems to be the only criterion that can possibly be satisfied under real conditions.

It should be mentioned here that Eq. (7) has already been applied as a key similarity criterion concerning thermal effect on fluid flow in tundishes and it has served very well.\(^2\)\(^-\)\(^7\) The dimensionless number involved in this equation has been derived for tundishes as

\[
Tu = \frac{q_L}{U^2} \beta \Delta T = \frac{Gr}{Re^2} \quad \text{(8)}
\]

where \( Tu \) is called the tundish Richardson number; \( "Gr" \) is the Grashof number; and \( "Re" \) is the Reynolds number.

Equation (7) arises from the buoyancy term of the Navier–Stokes equations that is the driving force on fluid flow by thermal effect. Therefore, this criterion should be effective not only for tundishes but also for steel ladles as well as any other liquid flow systems influenced by heat transfer, typically the natural convection phenomena. In the present modelling work, the natural convection phenomena in hot-water and in liquid steel are the major concern. Accordingly, in this work Eq. (7) is also taken as the key similarity criterion, but the relevant dimensionless number would be interpreted as the following:

\[
Fr = \frac{U^2}{q_L} \beta \Delta T \quad \text{(9)}
\]

where \( Fr \) is the “reference” Froude number defined as

\[
Fr = \frac{U^2}{q_L} \quad \text{(10)}
\]

Here we call Fr the “reference” Froude number because it is based on the reference velocity, \( U \), instead of real fluid velocity. Throughout this article, the same concept also applies to all the other dimensionless numbers involving velocity, such as “reference” Reynolds and Peclet numbers, as will be met later. Since the present article only deals with this type of dimensionless numbers, for simplicity of description, the term “reference” shall be omitted from now on.

According to Eq. (9), Eq. (7) can be rewritten as

\[
\left( \frac{\beta \Delta T}{Fr} \right)_m = \left( \frac{\beta \Delta T}{Fr} \right)_p \quad \text{(11)}
\]

Here it is already clear that one possible way to establish Eq. (11) is to keep Froude similarity together with thermal buoyancy similarity, i.e.,

\[
Fr_m = Fr_p \quad \text{(12)}
\]

and

\[
(\beta \Delta T)_m = (\beta \Delta T)_p \quad \text{(13)}
\]

Equations (12) and (13) are the alternative key similarity criteria based on which major modelling conditions, such as time scale factor and the scale factor of boundary heat loss flux, are obtained for the present study.

2.3. Modelling Conditions

2.3.1. Scale Factor of Time

Based on Froude similarity or Eq. (12) and considering \( (U/L)_m = (U/L)_p \), the time scale factor, \( f_t \), can be determined as

\[
f_t = \frac{t_m}{t_p} = \sqrt{f_o} \quad \text{(14)}
\]

where \( f_o = L_m/L_p \) is the scale factor of geometry.

2.3.2. Scale Factor of Boundary Heat Loss Flux

The boundary heat loss fluxes are the most important parameter to be considered in this physical modelling work. It is this parameter that causes natural convection phenomena. The scale factor of boundary heat loss flux, \( f_q \), can be estimated from the average bulk-cooling rate as

\[
f_q = \frac{q_m}{q_p} = \frac{\rho_m C_m V_m A_m \Delta T_m}{\rho_p C_p V_p A_p \Delta T_p} = \frac{\rho_m C_m V_m A_m}{\rho_p C_p V_p A_p} \frac{\Delta T_m}{\Delta T_p} \frac{f_o}{f_{qo}} \quad \text{(15)}
\]

Considering \( V_m V_p = f_q \), \( A_m A_p = f_{qo} \), and \( \Delta T_m / \Delta T_p = f_{qo} \) and introducing Eqs. (13) and (14), the above equation becomes

\[
f_q = \frac{q_m}{q_p} = \frac{\rho_m C_m \beta_m}{\rho_p C_p \beta_p} \sqrt{f_o} \quad \text{(16)}
\]

Equation (16) can be used to determine the model boundary heat loss flux.

2.3.3. Scale Factor of Geometry

If Eq. (7) is treated as the only similarity criterion for the physical modelling work, i.e., as long as Eqs. (14) and (16) are obeyed, there will be no further restriction on the geometry scale factor. This implies that there is some freedom in choosing the model size. However, if Eqs. (5) and (6) are disregarded, Eq. (7) itself does not guarantee a complete similarity between the model and the prototype. Therefore, the freedom in determining geometry scale factor may be limited and there may be an inappropriate geometry scale factor, probably relevant to Eqs. (5) and (6), with which a best possible similarity between the model and the prototype could be reached.

Equations (5) and (6) resemble, respectively, equalities in “total” Reynolds and Peclet numbers between the model and the prototype, in which the following definitions are made:

\[
Re = \frac{\rho_{\text{eff}} U L}{\mu_{\text{eff}}} \quad \text{(17)}
\]

and

\[
Pe = \frac{\rho_{\text{eff}} C U L}{\lambda_{\text{eff}}} \quad \text{(18)}
\]

Here we call Re and Pe “total” Reynolds and Peclet numbers because they include both molecular and turbulent effects, according to the definitions of \( \mu_{\text{eff}} \) and \( \lambda_{\text{eff}} \). Of course, Re and Pe are also the “reference” dimensionless numbers because, like Froude number, they are also based on the reference velocity, \( U \). In addition, for a specific model or prototype, unlike Fr which is a constant, cf., Eq. (10), Re and Pe will vary with \( \mu_{\text{eff}} \) and \( \lambda_{\text{eff}} \) (actually \( \mu \) and \( \lambda \)) in the flow domain. As will be discussed later, it is this nature of Re and Pe that makes it possible to realise a nearly complete similarity between the model and the prototype.

According to Eqs. (17) and (18), Eqs. (5) and (6) become:

\[
\frac{1}{Re}_m = \frac{1}{Re}_p \quad \text{or} \quad Re_m = Re_p \quad \text{(19)}
\]

and
\[ \left( \frac{1}{Pe} \right)_m = \left( \frac{1}{Pe} \right)_p \quad \text{or} \quad Pe_m = Pe_p \quad \text{......(20)} \]

Further, the Peclet number is a product of Reynolds number and Prandtl number, i.e.,

\[ Pe = Re \cdot Pr \quad \text{.................(21)} \]

where Pr is the “total” Prandtl number defined as

\[ Pr = \frac{\mu t C_\text{eff}}{\lambda t} = \frac{\mu t + \mu_l}{\lambda t + \lambda_l} \quad \text{.................(22)} \]

Comparison between Eqs. (19) and (20) with consideration of Eq. (21) shows that the major difference between the two equations falls in Prandtl numbers. It is this difference that potentially causes problems in the non-isothermal physical modelling of steel ladles by using water models.

Using the thermal-physical properties of liquid steel and water given in Table 1, for laminar flows, i.e., \( \mu = 0 \) and \( \lambda = 0 \), calculations with Eq. (22) show that the Prandtl number of water is about 35 times as that of liquid steel, so that the conflict between Eqs. (19) and (20), namely Eqs. (5) and (6), is obvious. Introducing the same data from Table 1 into Eqs. (12), (19) and (20), it can be derived that the combined Froude–Reynolds similarity (i.e., Eqs. (12) and (19)) requires an almost full-scale \((f_\text{G}=1)\) water model; while the combined Froude–Peclet similarity (i.e., Eqs. (12) and (20)) needs to use a much smaller water model \((f_\text{G}=0.1)\).

On the other hand, for fully turbulent flows, in which the turbulence effect dominates over the laminar effect (i.e., \( \mu \gg \mu_l \) and \( \lambda \gg \lambda_l \)), \( Pr \) becomes virtually the turbulent Prandtl number. Many studies have shown that in a wide range of flow conditions the turbulent Prandtl number is almost constant at the level around 0.9.\(^{19}\) Therefore, it may be possible that Eqs. (12), (19) and (20), i.e., combined Froude–Reynolds–Peclet similarity, could exist altogether.

This situation would result in

\[ f_\text{G} = \left( \frac{\mu t_m}{\mu t_p} \right)^{\frac{2}{3}} \quad \text{.................(23)} \]

and

\[ f_\lambda = \left( \frac{\lambda t_m}{\lambda t_p} \right) \left( \frac{\rho_p \cdot C_p}{\rho_m \cdot C_m} \right)^{\frac{2}{3}} \quad \text{.................(24)} \]

Equations (23) and (24) reflect that, for fully turbulent flows, the appropriate geometry scale factor would be dependent on the turbulence intensities in the model and the prototype. It can be deduced from these equations that a geometry scale factor smaller than unity could be selected if \( \mu_l \) and \( \lambda_l \) of the water model were properly controlled and kept lower enough than those of the prototype steel ladle, e.g.,

\[ \frac{\lambda t_m}{\lambda t_p} = \frac{\mu t_m \cdot C_m}{\mu t_p \cdot C_p} \ll \frac{\rho_m \cdot C_m}{\rho_p \cdot C_p} \quad \text{.................(25)} \]

In this way, it is possible to use an appropriately down-scaled water model to simulate the natural convection phenomena in steel ladles. Unfortunately, as mentioned before, it is hard to foresee the turbulence parameters such as \( \mu_l \) and \( \lambda_l \) before making any experiments or mathematical simulations. Therefore, only when \( \mu_l \) and \( \lambda_l \) are known, it is possible to find out the lowest and best possible geometry scale factor. This needs to be explored in the present study by means of CFD mathematical modelling.

### 3. Mathematical Modelling

The major aim of the present work is to use numerical solutions of Eqs. (1) and (2) to study the similarity between steel ladles and hot-water models regarding natural convection phenomena. For this purpose, a numerical model for solving Eqs. (1) and (2) has been developed by using a commercial CFD simulation package, PHOENICS. With the developed numerical model, a series of computations were performed both for liquid steel and for hot-water, under presumed initial and boundary conditions. In the computations, the standard \( k–\epsilon \) two-equation turbulence model together with wall-functions was employed.\(^{16}\) The obtained numerical solutions were then used to examine the validity of the key similarity criteria, i.e., Eqs. (12) and (13), and to find out appropriate physical modelling conditions such as model size and water temperature.

In the present mathematical modelling work, the following general assumptions were made:

1. the steel ladle and the model ladle are simplified as cylinders with constant radii;
2. the flow and heat-transfer phenomena inside the ladles are axis-symmetrical, so that a two-dimensional modelling can be applied;
3. initially the liquid bath is quiescent and homogeneous in temperature \((T^* = 1)\);
4. non-slip condition is applied to all boundary walls;
5. zero-flux condition is considered along the centre axis;
6. shear-free condition is assumed at the top free surface;
7. heat is lost uniformly through ladle wall and bottom; however,
8. the top free surface is assumed to be covered by a slag layer that is thick enough to thermally isolate the liquid so that no heat loss is considered at this boundary.

A two-dimensional computation domain is defined as a slice of fluid volume bounded by ladle wall, bottom, centre axis, and top free surface; and a cylindrical-polar grid of 20 (radial)\(\times\)25 (axial) cells was applied to this domain. Table 1 gives the thermal-physical properties of liquid steel and water,\(^{11,12}\) used in the CFD simulations, and Table 2 lists the CFD simulation conditions. Mid-aged 107-tonne steel ladles of SSAB Luleå Steelworks (SSAB Tunnplåt AB in Luleå), in Sweden, are used as the prototype. The boundary heat loss flux of the prototype ladle is calculated based on an average bulk-cooling rate of 0.5°C/min during the holding period before casting. Accordingly, the cooling times and heat loss fluxes for model ladles of different scales are calculated based on Eqs. (14) and (16), respectively, as given in Table 2.

### 4. Experimental

In order to validate the CFD numerical model developed for this work, a 1/4-scale water model of SSAB 107-tonne steel ladles was established in the laboratory. Figure 1 schematically illustrates this physical model set-up. The water model is comprised of a cylindrical cooling chamber.
and a flat cooling chamber for simulating the ladle wall and bottom, respectively. The cooling chambers are made of 2-mm thick stainless steel sheet.

Table 3 gives the thermal-physical properties of the model shell material (stainless steel type: ASTM 304).13) Hot-water was used as the liquid bath simulating liquid steel bath in ladles, while cold water with controllable temperatures is introduced into the cooling chambers in directions shown in Fig. 1. T-type (copper-constantan) thermocouples (TCs) were employed to obtain the temperature information from the water model. 21 TCs were used for measuring the temperature profile in the water bath on a vertical plane bounded by side wall and centre axis. 7 TCs were used for measuring the temperature distribution in cold water along the height of the side cooling chamber. 4 TCs were used for measuring temperatures of water inflows and outflows of the cooling chambers. All the temperature signals were recorded into an HP34970A data logger for post processing. To avoid heat loss from the top free surface of the water bath, the free surface was covered with a light porous plastic plate that could float on the surface. In order to homogenise the hot-water bath, if needed, gas (pressurised air) could be blown into the water bath via the tuyere located at the centre of the bottom cooling chamber.

5. Results and Discussions

5.1. Comparison between Calculations and Measurements

As a means of verification, the CFD numerical model developed for the present study was first applied to the 1/4-scale water model established in the laboratory. The numerical simulations were made both on the hot-water bath and on the cold water flowing in cooling chambers (including heat conduction inside inner shells of the chambers in contacting with hot water). Figure 2 shows the comparisons between calculated and measured water temperatures at certain positions in the water model (cf., Fig. 1) for a simulation case. In this simulation case the hot-water bath had initially been stirred by blowing gas from the bottom. After stop of gas bubbling, the hot-water bath had a nearly homogenous temperature of 44.8°C. It was then left to cool for 6 min. Meanwhile, cold water with a constant temperature of 10°C was introduced into side and bottom cooling chambers at flowrates of 9 and 10 litres per minute, respectively. It can be seen from Fig. 2 that the agreement between calculated temperatures and measured ones is generally satisfactory. The CFD model gives quite good predictions on the temperatures in major portions of both the hot-water bath and the side cooling water, showing that the numerical model developed for the present study is generally feasible for use. It should be noted that the calculated temperatures at the positions close to the top free surface decrease relatively faster than the measured ones at the corresponding positions. The reason for this discrepancy will be explored in the future.

5.2. Examinations on the Similarity between Water Model and Steel Ladle

By means of CFD simulations, it is possible to use the numerical solutions to examine the similarity between the natural convection phenomena in hot-water models and in prototype steel ladles. The examinations are focused on the relative intensity of the differences between local parameters in the prototype steel ladle and those at the corresponding locations in the water model, which is defined as

\[
\text{Relative intensity} = \frac{1}{\bar{X}_p} \sqrt{\frac{1}{N} \sum_{n=1}^{N} (X_n)_m - (X_n)_p}^2
\]  

where, \(X\) is the parameter (representative dimensionless number) to be examined; \(\bar{X}_p\) is the average value of \(X\) in the flow field of the prototype steel ladle; \(n\) is the index number of a computational cell; and \(N\) is the total number of cells in a computation domain. Such a relative intensity represents the extent to which the two sets of parameters approach each other both in magnitudes and in distributions. The smaller the relative intensity is; the closer the two parameter sets are. As references, some typical values of the concerning parameters, \(\bar{X}_p\), are provided in Table 4.

5.2.1. Validity of Key Similarity Criteria

Firstly, it is essential to evaluate the validity of the key
similarity criteria, i.e., Eqs. (12) and (13), derived in this work. Since all the cooling times used in CFD simulations as listed in Table 2 are calculated based on Eq. (14), the first criterion, i.e., Froude similarity, is already satisfied. Therefore, it is only necessary to verify if Eq. (13) also holds true for both the models and the prototype, using the calculated temperatures.

**Table 4.** Average values of typical dimensionless numbers concerning natural convection in the prototype steel ladle ($X_p$).

| Cooling time (t) | 5 min | 10 min | 20 min | 30 min |
|-----------------|-------|--------|--------|--------|
| $Pr$            | 8.2714 x 10^8 | 8.2714 x 10^8 | 8.2714 x 10^8 | 8.2714 x 10^8 |
| $\beta \Delta T$ | 4.5087 x 10^2 | 6.6031 x 10^2 | 1.7515 x 10^3 | 2.5556 x 10^3 |
| $\alpha^*$      | 4.0835 | 2.5276 | 1.8603 | 1.5791 |
| $T^*$           | 1.0 | 1.0 | 1.0 | 1.0 |
| $Re$            | 604.66 | 150.43 | 191.09 | 220.99 |
| $Pe$            | 179.45 | 95.90 | 111.83 | 111.89 |

**Figure 3** shows the relative intensity of $\left(\frac{(\beta \Delta T)m}{(\beta \Delta T)p}\right)^2$, calculated using Eq. (26), in relation with model scale, initial water temperature and dimensionless cooling time. The smaller this relative intensity, the more valid is Eq. (13) and, hence, the better similarity between the model and the prototype. It can be seen from Fig. 3 that this criterion is mainly affected by model scale and cooling time, while initial water temperature has an insignificant effect on it. In the early period of cooling, big models are favourable for approaching Eq. (13), cf., Figs. 3(a) and 3(b); in the late period of cooling, however, smaller water models ($f_G<1/3$) would give better simulations to the steel ladles than the big ones, cf., Figs. 3(c) and 3(d). Considering the whole period of cooling, the appropriate model scale factor seems to be a value between 1/5 and 1/3. Too big models ($f_G \approx 1/2$) for longer cooling times ($t^*<0.667$) and too small models ($f_G<1/7$) for shorter cooling times ($t^*<0.333$) would not give better similarity to the prototype. Furthermore, for economic reasons, it is generally advisable to establish models that are as small as possible without significant loss in similarity to the prototype. Thus, Fig. 3 suggests that a model of scale as small as 1/5 may be applied without appreciable deviation from Eq. (13).

### 5.2.2. Closeness between Dimensionless Solutions

Secondly, it is also possible to convert the numerical solutions, i.e., velocity and temperature, into dimensionless forms by dividing them by their reference values. Then, the equality between the dimensionless solutions for the model and those for the prototype would signify a more general similarity between the two systems. In this work the reference values of velocity and temperature are defined as

$$U = \frac{H}{t_c} \quad \text{and} \quad T_{ref} = \bar{T}$$

It follows that the dimensionless solutions of velocity and temperature for the models and the prototype can be calculated, using numerical solutions, as

$$u^* = \frac{u^*}{H} \quad \text{and} \quad T^* = \frac{T}{\bar{T}}$$

where $u$ is the total velocity calculated from its local radial and axial components (actual numerical solutions) and $T$ is the local temperature. Then, if dynamic and thermal similarities exist between the model and the prototype there should be:

$$u^*_m = u^*_p \quad \text{and} \quad T^*_m = T^*_p$$

**Figure 4** illustrates the relative intensity of $(u^*_m-u^*_p)$, calculated using Eq. (26), in relation with model scale, initial water temperature and dimensionless cooling time. This figure shows that, for major time of cooling ($t^*<0.667$), bigger water models have dimensionless solutions that more closely resemble those of steel ladles. After that, however, too big a model (e.g., $f_G=1//2$) would not give better similarity than some smaller ones. Therefore, Fig. 4 also
suggests that models of medium sizes (1/5 ≤ f_G ≤ 1/3) would give a satisfactory dynamic similarity to the prototype. Again, it can be seen from this figure that initial water temperature has a negligible influence on the similarity in flow fields between the models and the prototype.

Figure 5 gives the relative intensity of \((\frac{T_m^* - T_p^*}{T_m^*})\), calculated using Eq. (26), in relation with model scale, initial water temperature and dimensionless cooling time. It can be seen from this figure that initial water temperature has a more significant influence on the thermal similarity between the model and the prototype than the model size does. Higher water temperature is very favourable for obtaining better thermal similarity. Nevertheless, it can also be seen from Fig. 5 that the magnitudes of the relative intensity of \((\frac{T_m^* - T_p^*}{T_m^*})\) are generally small, in the order of \(2^2\) to \(2^3\). This implies that hot water with a properly lower temperature (e.g., 45°C) could still be used in the models with a satisfactory thermal similarity to the prototype. Of course, as can be deduced from Fig. 5, water models with too low initial temperatures will not meet the required thermal similarity to the prototype steel ladle regarding natural convection phenomenon.

In addition, Fig. 5 also illustrates that, in the early period of cooling (e.g., \(t^* \approx 0.333\)), big water models give better simulations of the prototype than the small ones; however, in the late period of cooling (e.g., \(t^* \approx 0.667\)), some smaller models (1/7 ≤ f_G ≤ 1/3) have better thermal similarity than too big (f_G = 1/2) or too small (f_G = 1/10) ones. Therefore, considering the whole cooling period, the optimum geometry scale factor still seems to fall in the range between 1/5 and 1/3.

5.2.3. Reynolds and Peclet Similarities

According to the above results and discussions, there is an optimum geometry scale factor in the range between 1/5 and 1/3 for which a best possible similarity between the
water model and the steel ladle could be reached. As mentioned earlier, this optimum geometry scale factor may be relevant to two other similarity criteria, i.e., Eqs. (5) and (6), which have been ignored in the present numerical simulations. If this judgement holds true, the relationship between model size and similarity, as revealed by Figs. 3 to 5, may well be explained. Fortunately, with the help of CFD numerical modelling, knowledge about local values of turbulence parameters such as $\mu_t$ and $\lambda_t$ (hence, $\mu_{eff}$ and $\lambda_{eff}$) makes it possible to examine the extent to which Eqs. (5) and (6), i.e., Reynolds and Peclet similarities, are approached under the condition that merely Eq. (7) is controlled to be satisfied.

As analysed before, fully turbulent flows are the presupposition for establishing a combined Reynolds and Peclet similarity that necessitates identical Prandtl numbers for both the model and the prototype. Therefore, first of all, the state of turbulence should be evaluated for the model and the prototype. This can be made through examination of the equality of total Prandtl numbers, defined by Eq. (22), between the two flow systems.

Figure 6 shows the relative intensity of $(Pr_m - Pr_p)$, calculated using Eq. (26), in relation with model scale and dimensionless cooling time.
ty for laminar flows, showing that in the early stage ($t^* \leq 0.167$) laminar flow is still dominating the natural convection. This judgement is also supported by Fig. 6 which shows appreciable differences in Prandtl numbers between the prototype and the models when $t^* \leq 0.167$. This result is not surprising because in this work all CFD simulations are started with a presumed initially quiescent liquid bath, and time is needed for the development of turbulence. After this initial period, optimum model size giving the best Peclet similarity is found in the range between 1/5 and 1/3, as depicted by Fig. 8 when $t^* = 0.333$. This could explain the existence of an upper limit in choosing the appropriate model size.

5.2.4. Comparison between Flow Patterns in the Models and the Prototype

Figure 9 illustrates a comparison between natural convection flow patterns in the prototype steel ladle and in the water models of different geometry scales. (All the models use the hot water with the same initial temperature of 45°C.) This comparison can provide a qualitative examination on the similarity in flow fields between the models and the prototype. It can be seen from Fig. 9 that the flow patterns in the models enveloped by the dashed lines look more similar to the flow pattern in the prototype, at the corresponding cooling times, than the rest flow patterns outside the envelop. The enveloped flow patterns depict that bigger models ($f_G \geq 1/3$) only give a better similarity for shorter cooling time ($t^* \leq 0.333$) corresponding to less than 10 min holding time for the prototype steel ladle; smaller models ($f_G \leq 1/7$) only give a better similarity for longer cooling time ($t^* \geq 0.667$) corresponding to more than 20 min holding time for the prototype steel ladle. If considering the whole cooling period, i.e., a 30 min holding time for the prototype steel ladle, medium sized models (e.g., $f_G = 1/4$ and $f_G = 1/5$) will have flow patterns similar to that in the prototype in a wider range of holding time. This result nearly supports the conclusion for determining the appropriate model size discussed in the previous sections.

5.3. Prediction of Steel Temperature by Use of Water Models

Finally, by means of CFD numerical modelling, one can directly assess the individual temperatures at certain positions in the prototype and at the corresponding positions in the models. By scaling-up the water temperatures, via Eq. (13), predictions of steel temperatures using the water mod-

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Fig. 9. Comparison of flow patterns in prototype steel ladle with those in water models of different sizes with an initial temperature of 45°C.

Fig. 10. Comparison between calculated temperatures along the centre axis of prototype steel ladle and those predicted from water models via Eq. (13).
els can be realised. Thus, a verification of the accuracy of such predictions will provide a further means of evaluating the thermal similarity between the models and the prototype.

Figure 10 shows the temperature distributions along the centre axis of the liquid bath calculated for prototype steel ladle and predicted by using water models with an initial water temperature of 45°C. As seen, for models of all sizes studied in this work, a quite satisfactory approximation of the temperature distribution to the prototype can be achieved with most of discrepancies falling within ±2°C. This further proves that the key similarity criteria, Eqs. (12) and (13), are generally effective for predicting the steel temperature in industrial prototype ladles with use of hot-water models and hot water with an initial temperature of 45°C can be utilised for this purpose. Furthermore, it can also be seen carefully from this figure that models that are too small (e.g., \( f_0 \leq 1/7 \)) slightly overestimate the temperatures in the upper part of the bath, while models that are too big (e.g., \( f_0 = 1/2 \)) give some appreciable over estimations in the lower part of the bath. Therefore, an appropriate model scale would be in the range of 1/5 to 1/3, so that an even better simulation accuracy can be achieved.

6. Conclusions

Through a systematic study on the similarity between 107-tonne steel ladles and hot-water models with different scales and water temperatures, the following conclusions can be drawn:

1) it is convenient and economic to use CFD numerical methods with computers to simulate both the prototype steel ladles and the water models so as to find out appropriate modelling conditions such as model size and water temperature;

2) dimensional analysis on the governing equations suggests that the similarity between natural convection phenomena in the model and the prototype can be approached by simultaneously fulfilling the following key criteria:

\[
Fr_m = Fr_p \quad \text{and} \quad (\beta \Delta T)_m = (\beta \Delta T)_p;
\]

3) water models of scales between 1/5 and 1/3 are appropriate for use to simulate natural convection phenomena in prototype steel ladles, and these medium sized models can also provide a better Peclet similarity between the model and the prototype;

4) models with hotter water are favourable for achieving better thermal similarity to the steel ladles; however,

5) warm water with an initial temperature of 45°C can be used in non-isothermal physical modelling of steel ladles with a satisfactory accuracy.

Acknowledgements

The authors acknowledge the Computer Assisted Materials and Process Development Association (CAMPADA), Sweden, for financial support for this study. We also extend our thanks to Dr. Dongyuan Sheng of MEFOS, Sweden, for fruitful suggestions and discussions on this work and to Dr. Carl-Erik Grip of SSAB Tunnplåt AB in Luleå, Sweden, for providing plant data and valuable suggestions in establishing the water model.

Nomenclature

- \( A \): Heat transfer area (m²)
- \( C \): Thermal capacity (J/kg K)
- \( f_0 \): Geometry scale factor (–)
- \( f_t \): Scale factor of boundary heat flux (–)
- \( f_r \): Time scale factor (–)
- \( Fr \): Froude number (–)
- \( g \): Gravitational acceleration (m/s²)
- \( H \): Height of liquid bath (m)
- \( L \): Reference length (m)
- \( n \): Index number of a cell in a computation domain (–)
- \( N \): Total number of cells in a computation domain (–)
- \( p \): Pressure (Pa)
- \( Pe \): Peclet number (–)
- \( Pr \): Prandtl number (–)
- \( q \): Heat flux (W/m²)
- \( R \): Radius of liquid bath (m)
- \( Re \): Reynolds number (–)
- \( t \): Time (s)
- \( t_c \): Total cooling time (s)
- \( T \): Temperature (K)
- \( \bar{T} \): Average temperature of liquid bath (K)
- \( \Delta T \): Difference of temperature from the initial temperature (K)
- \( u \): Velocity (m/s)
- \( U \): Reference velocity (m/s)
- \( V \): Volume of liquid bath (m³)
- \( x \): Cartesian co-ordinate (m)
- \( X \): Representative dimensionless number (–)
- \( \bar{X} \): Average value of the representative dimensionless number (–)
- \( \beta \): Thermal expansion coefficient (1/K)
- \( \lambda \): Thermal conductivity (W/m K)
- \( \rho \): Density (kg/m³)
- \( \mu \): Viscosity (Pa s)

Subscripts

- eff: Effective quantity containing both laminar and turbulent effects
- \( i \) and \( j \): Cartesian indices
- \( l \): Laminar flow quantity
- \( m \): Model parameter
- \( p \): Prototype parameter
- ref: Reference quantity
- \( t \): Turbulent flow quantity

Superscript

- *: Dimensionless quantity

REFERENCES

1) J. W. Hlinka and T. W. Miller: Iron Steel Engr., 8 (1970), 123.
2) M. L. Lowry and Y. Sahai: ISS Transactions, 14 (1993), 17.
3) J. de J. Barreto S., M. A. Barron Meza and R. D. Morales: ISIJ Int., 36 (1996), 543.
4) C. Damle and Y. Sahai: ISIJ Int., 36 (1996), 681.
5) J. Wang, C. Dai, L. Zhou, Y. Zhang, Z. Li and Z. Xiao: Acta Metall. Mater. Trans. B, 33 (1997), 509.
6) A. K. Sinha and A. Vassilicos: Ironmaking Steelmaking, 25 (1998), 387.
7) D.-Y. Sheng, C.-S. Kim, J.-K. Yoon and T.-C. Hsiao: ISIJ Int., 38 (1998), 843.
8) D.-Y. Sheng and L. Jonsson: Metall. Mater. Trans. B, 30B (2000), 979.
9) Turbulence, Vol. 12, ed. by P. Bradshaw, Springer-Verlag, New York, (1976), 244.
10) B. E. Launder and D. B. Spalding: Comp. Meth. Appl. Mech. Eng., 3 (1974), 269.
11) J. Chen: Handbook of Diagrams and Data for Steelmaking, Publish House of Metallurgical Industry, Beijing China, (1984), 391.
12) R. C. Weast: Handbook of Chemistry and Physics, 57th ed., CRC Press, USA, (1977), F-5.
13) Technical data book of Avesta Sheffield Group, INF. 10100 GB, Avesta Sweden, (2000), 8.