Astrophysical motivation for directed searches for a stochastic gravitational wave background

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The nearby universe is expected to create an anisotropic stochastic gravitational wave background (SGWB). Different algorithms have been developed and implemented to search for isotropic and anisotropic SGWB. The aim of this paper is to quantify the advantage of an optimal anisotropic search, specifically comparing a point source with an isotropic background. Clusters of galaxies appear as point sources to a network of ground based laser interferometric detectors. The optimal search strategy for these sources is a “directed radiometer search”. We show that the flux of SGWB created by the millisecond pulsars in the Virgo cluster produces a significantly stronger signal than the nearly isotropic background of unresolved sources of the same kind. We compute their strain power spectra for different cosmologies and distribution of population over redshifts. We conclude that a localised source, like the Virgo cluster, can be resolved from the isotropic background with very high significance using the directed search algorithm. For backgrounds dominated by nearby sources, up to redshift of about 3, we show that the directed search for a localised source can have signal to noise ratio more than that for the all sky integrated isotropic search.

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I. INTRODUCTION

Einstein’s theory of general relativity predicts gravitational waves (GWs) [1, 2]. The existence of GW was first confirmed from the observation that the period of the Hulse-Taylor binary pulsar system (PSR B1913+16) is reducing [3, 4]. While a “direct” detection of GW has not been possible yet, the first generation ground based Laser Interferometric detectors, LIGO [5, 6], Virgo [7, 8], GEO600 [9, 10] and TAMA300 [11, 12], have demonstrated that measurement of strain of the order $10^{-21}$ is now possible. However, in order to increase the probability of detection of GW sources to a realistic level, second generation detectors are being installed [13–15], which will have about ten times more strain sensitivity and offer a broader frequency range. Hence, they will be able to observe at least three orders of magnitude more volume, thereby enhancing the chances of detection in the next few years from few percent level to close to unity.

Sources of GWs can be broadly classified in three categories based on their duration and phase coherence:

1. Burst sources: Short duration sources with modelled (e.g., compact binary coalescence) or unmodelled waveforms (e.g., supernovae).
2. Continuous sources: long duration sources with phase coherence (e.g., spinning neutron stars).
3. Stochastic Background: long duration sources without phase coherence (e.g., a set of coalescing binaries or millisecond pulsars in a galaxy cluster).

Coalescing compact binary stars are the most promising sources of GWs, as the waveform from these sources can be modelled to a very high degree of accuracy, which make it possible to apply the techniques of matched filtering for digging out signal from noisy data from the detectors. However, GW astronomy promises a much broader spectrum of sources and, many, if not most, of these waveforms will not be known a priori. The stochastic gravitational wave background (SGWB), by definition, is one such type of sources.

SGWB is generated by an incoherent superposition of short and long term sources. These sources can be stochastic either in the time domain or in the frequency domain. In the time domain it is a pop-corn noise, a large set of events non-uniformly distributed over a certain interval of time, e.g., a population of supernovae or coalescing binaries in a galaxy cluster [16, 17]. In the frequency domain the background can be created by a “forest of emission lines”, narrow or broad, whose exact frequencies are not known, but the distribution can be modelled. For instance, a population of millisecond pulsars in a cluster of galaxies can create such a background [18].

Detection of these astrophysical SGWB can provide collective information about the corresponding sources which are not accessible by electro-magnetic (EM) observations. In particular, average physical properties of such sources, such as, mass asymmetry of neutron stars, equation of state, population distribution etc. can be probed via SGWB observations.

The best strategy to search for stochastic signals is by cross-correlating signals from different detectors. Over the last three decades algorithms have been developed and implemented to search for isotropic and different kinds of anisotropic SGWBs [19–36]. The signal-to-noise ratios (SNRs) expected from the optimised searches for...
these anisotropic SGWBs were also computed. However, the relative strengths of the expected isotropic and anisotropic backgrounds, based on the current knowledge of astronomy, and the corresponding relative SNRs, have not been studied in literature. This would, in turn, provide a firm justification for performing (or not performing) dedicated searches for different isotropic and anisotropic backgrounds.

The picture of the universe that we get from EM astronomy shows that the nearby universe is highly anisotropic, while at large scales it is fairly isotropic. Hence the true background will contain an isotropic as well as an anisotropic component. Our first task will be to compare to their relative strengths.

In the analysis of data from GW detectors, the searches for isotropic SGWBs are motivated from the fact that the primordial SGWB is statically isotropic (though different models for power spectral densities are still needed to search for different astrophysical sources which create the background). An anisotropic SGWB search will be justified in a situation provided either of the following criteria is satisfied:

1. the anisotropic search can confidently probe the anisotropy; that is, the search should be able to clearly distinguish the anisotropic component from the isotropic part;

2. the (optimal) anisotropic search has significantly large SNR as compared to the (suboptimal) isotropic search.

Because the nearby universe dominates the background, one would naively anticipate that the above criteria are always satisfied. However, as we show in this paper, this is not necessarily the case. Nearby sources may appear strong because of their proximity, but the distant sources are large in number. In general, the comparison depends on the population distribution of the sources at different frequencies and redshifts and the expansion history of the universe. Here we quantify the relative strengths of these backgrounds and discuss in which astrophysical situations the differences will be significant.

The specific case we consider in this paper is simple. We compare the relative strengths and detectability of a localised point source with those for an isotropic background created by similar sources with correct (optimal) and interchanged (suboptimal) filters. In our example the anisotropic part is created by a large number of Millisecond Pulsars (MSPs) in the Virgo cluster [18]. The Virgo cluster [37] is a localised source, which can be assumed to be a point source of SGWB. This is because the angular width of the Virgo cluster (few degrees) is comparable to the angular resolution of the network of present ground based Laser Interferometric GW detectors [29]. The isotropic part is created by all such MSPs in the rest of the universe, whose distribution is nearly isotropic. We first show that the total GW flux received from the Virgo cluster exceeds the flux from the same solid angle as that of the Virgo cluster from the integrated distant isotropic universe. This is similar to solving the “Olbers’ paradox” [38] problem in cosmology. However, this calculation would provide the comparison between the SNRs observed in different searches, if the searches had uniform frequency response and infinite bandwidth, which is of course impossible. To perform a realistic comparison, we evaluate the redshift integrated spectra of the background for different cosmological evolution and (simple) population distribution models. Combining these spectra with the frequency response of the search one can obtain the final observed SNRs. We compare the SNRs that one would observe for directed and isotropic searches for both the localised and isotropic components of the background. The numbers can then be used to reach the final conclusion on the effectiveness of different searches in different cosmological conditions.

The paper is organised as follows. In section II we provide a brief review of the Olbers’ paradox and its more general form in terms of frequency spectrum. We compare the fluxes and the frequency spectra for a localised source and the isotropic background in section III. In section IV we compute the numerical results for the observed SNR for different combinations of sources and models. We discuss the results and future directions in section V.

II. OLBER’S PARADOX

A common observation is that the night sky is mostly dark. The stars and galaxies stand out in the night sky as the background is almost negligible. One might think that this is obvious, the observed sky should be dominated by the nearby universe, but it is not that straightforward. Naively the number of sources in the universe per unit solid angle increases exactly in the same way as their flux at the earth reduces, thereby compensating for the effect of the distance. To pose the problem mathematically, let us consider a solid angle $\Delta \Omega$. If the universe is homogeneous, the number of sources between distance $r$ and $r + dr$ that are contributing to the flux in this solid angle is given by $n r^2 \Delta \Omega dr$, where $n$ is the average number density of sources in the universe. However, the flux from these sources reduces as $1/r^2$. Hence the “effective” number of contributing sources placed at unit distance $dN_{eff}$, which would produce the same amount flux at the point of observation, is proportional to $n \Delta \Omega dr$, does not depend on $r$. That is, every distant part of the universe would contribute equally to this solid angle $\Delta \Omega$, which would make the night sky almost uniform bright in an isotropic universe and, infinitely bright in an infinite universe. The seeming incongruity of this prediction with our observations of the night sky is Olbers’ paradox. In terms of gravitational waves, Olbers’ paradox predicts that the anisotropic SGWB would be insignificantly small as compared to the isotropic part, as the latter is created by the distant universe which is much deeper than the local universe.
The solution to Olbers’ paradox becomes obvious when one includes the expansion of the universe, which introduces the additional decrease in flux due to the cosmological redshift $z$. Expansion of the universe not only reduces the energy of photons received by a factor of $1+z$, but also reduces the rate at which they arrive by the same factor. So overall the flux reduces by a factor of $(1+z)^2$ [39]. Thus, in the above calculation, the effective number of sources now becomes

$$dN_{\text{eff}} = \frac{n \Delta \Omega dr}{(1+z)^2}, \quad (2.1)$$

where $n$ is the comoving source number density$^1$ at redshift $z$. Since the redshift $z$ is a monotonically increasing function of coordinate distance $r$, the distant universe appears more and more dim. In practice, the total number of galaxies in the universe also reduces with redshift, which plays a role in reducing the above number even further. However, this has a weaker effect, because at high redshifts ($z \gtrsim 2$) the effective number density gets a very low weightage of $1/(1+z)^2$.

The solution to Olbers’ paradox for EM astronomy already implies that the same will hold for SGWB, as there is no difference between these two in this context. GW flux reduces in the same way as for EM waves [2, 40]. Hence, the extragalactic astrophysical GW background is created mostly by low-to-intermediate redshift sources.

The above resolution for total flux will, however, suffice if the detection scheme had flat frequency response over the whole infinite frequency range. This is, of course, never true in practice. So what is more relevant for studying the detectability of different backgrounds is a more general quantity than the flux, namely, the observed frequency spectra $S(f)$. The total flux is the integrated value of $S(f)$ over the entire frequency range. If a source with luminosity $L$ at a redshift $z$ had an intensity distribution of $J(f)$, whose integral over frequency range is unity, then one can show that the observed spectrum is,

$$S(f) = \frac{L J(f(1+z))}{4\pi r^2 a^2(t_0)(1+z)}, \quad (2.2)$$

where $a(t_0)$ is the cosmological scale factor [40] at the current epoch $t_0$. The above formula will be the starting point for the main strain power spectrum density (PSD) $H(f)$ calculation done in the next section.

### III. LOCALISED VS. THE ISOTROPIC BACKGROUND

In this section we compare the GW flux and power spectra generated by the Virgo cluster with those from the rest of the universe for the same kind of source. While the analytical framework we have developed here is valid for any kind of source, for the numerical evaluation we have used the milli-second pulsars (MSPs). This is because the strain power spectral density (PSD) of the SGWB created by MSPs in the Virgo cluster is available as a ready result [18]. Also note that the overall amplitude of the strain due to optimistic or pessimistic assumptions (e.g., about the mass-asymmetry of the neutron stars) do not matter in this work, as we are only interested in the relative strengths of the localised and isotropic background with the same sources.

A typical Milky Way like galaxy is expected to have at least 40,000 MSP [41]. Each of the MSPs are expected to emit narrow band GW signal [42], which would appear as a forest of emission lines on the frequency axis. In a galaxy cluster like the Virgo with $\sim 10^8$ MSPs, the forest would be so dense that it will essentially become a continuum. Using a population distribution model for the MSPs, the SGWB from the Virgo cluster was computed in Dhurandhar et al. [18]. Here we essentially integrate their results over different redshifts to get the PSDs, which in turn, gives the SNR for different searches.

It is important to emphasise that the GW background considered here is created by spinning neutron stars with mass distribution not symmetric about the spin axis and with period of few milliseconds, whether or not they emit “pulses”. The population models based on EM observations provide an estimate for MSPs, though there can be many more spinning neutron stars without any EM emission. In this paper, since we are interested in relative fluxes and SNRs, the numerical results only depend on the distribution of the sources, not on their total number. We assume the estimated distribution of MSPs is applicable to the whole set of spinning neutron stars. Had the total number of such sources been different, all the power spectra and the SNRs would have a different, but common, scaling factor, which does not affect the conclusion drawn from the numerical results.

#### A. Comparison of total Flux

In the context of Olbers’ paradox, the first quantities we compare are the total flux received from the Virgo cluster with that received on the average from the same solid angle as the cluster in an otherwise statistically isotropic universe. Instead of quoting numbers for flux, we again quote the effective number of Milky Way Equivalent Galaxies (MWEG) $N_{\text{eff}}$ placed at a distance of $1$ Mpc. We find this quantity to be more intuitive than flux. $N_{\text{eff}}$ is essentially flux $(F)$ in a different unit, related by the formula

$$F = \frac{L_{\text{MW}}}{4\pi} N_{\text{eff}}/(\text{Mpc})^2, \quad (3.1)$$

where $L_{\text{MW}}$ is the blue luminosity of a MWEG. Thus the ratio of fluxes is the same as the ratio of $N_{\text{eff}}$, which is

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1 If the universe was expanding but the structures did not evolve, the comoving number density would be a constant.
our primary interest in this paper.

Since the Virgo cluster is \( r_V = 16.5 \) Mpc (Luminosity distance) away and it has about \( N_V = 1500 \) MWEG \([37]\), the \( N_{\text{eff}} \) for Virgo cluster is

\[
N_{\text{eff}}^V = N_V \left( \frac{r_V}{\text{Mpc}} \right)^2 \approx 5.5. \tag{3.2}
\]

We now compute the average \( N_{\text{eff}} \) in a statistically isotropic universe from the solid angle subtended by the Virgo cluster \( \Delta \Omega_V = 0.012 \) steradian (\( \sim 50 \) sq. degree, equivalent to a circular region of radius \( \sim 8 \) degree). For this we would need to integrate over the radial distance coordinate \( r \) and account for cosmological redshift of the emitted waves.

According to general relativity, GWs propagate in the same way as EM waves along null geodesics \([2]\). In an expanding universe described by the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) metric \([39]\), along the null geodesics leading to an observer at the origin of the coordinate system the following condition is satisfied

\[
-c^2 dt^2 - a^2(t) dr^2 = 0. \tag{3.3}
\]

where \( t \) is the time-like coordinate, \( r \) is the space-like radial coordinate and \( a(t) \) is the so-called scale factor. Thus, if two pulses are emitted by a source at radial coordinate \( r \) at time \( t_1 \) and \( t_1 + \delta t_1 \), which reach the observer at time \( t_0 \) and \( t_0 + \delta t_0 \) respectively, one can write

\[
r = c \int_{t_1}^{t_0} \frac{dt}{a(t)} = c \int_{t_1 + \delta t_1}^{t_0 + \delta t_0} \frac{dt}{a(t)}. \tag{3.4}
\]

The above relation yields \( \delta t_0 = a(t_0)/a(t_1) \delta t_1 \). This implies that the rate at which the pulses are observed and energy is received is lower by a factor of \( a(t_0)/a(t_1) = 1 + z \), where \( z \) is the usual cosmological redshift. Moreover, due to the expansion of the universe the wavelengths are stretched and hence energy of each pulse is again reduced by the same factor of \( a(t_0)/a(t_1) = 1 + z \). Combination of these two factors implies that the flux received from a source of luminosity \( L \) at a redshift \( z \) is \([39]\)

\[
F = \frac{L}{4\pi r^2 a^2(t_0)(1 + z)^2}. \tag{3.5}
\]

If the comoving number density of MWEG in the universe in the Virgo solid angle \( \Delta \Omega_V \) is a function of redshift, given as \( n(z) \), the total flux received by the observer will be

\[
F_{\text{iso}} = \frac{L_{\text{MW}}}{4\pi} \int_0^{\infty} dr \frac{\Delta \Omega_V r^2 n(z)}{r^2 a^2(t_0)(1 + z)^2}. \tag{3.6}
\]

After simplifying, this can be written in terms of \( N_{\text{eff}} \) as

\[
N_{\text{eff}}^{\text{iso}} = \Delta \Omega_V \int_0^{\infty} dr \frac{n(z)}{a^2(t_0)(1 + z)^2}. \tag{3.7}
\]

In order to compute the above integral analytically or numerically, we need to relate the coordinate distance \( r \) with redshift \( z \). We use the standard relation \([43]\),

\[
dr = \left( \frac{c}{H_0} \right) dz/E(z), \tag{3.8}
\]

where, \( H_0 \) is the Hubble constant and the “Hubble parameter” in a flat universe,

\[
E(z) := \sqrt{\Omega_L + \Omega_m(1 + z)^3}. \tag{3.9}
\]

In this paper, we assume that co-moving number density of MWEG is not evolving with time, \( n(z) = n_0 \), where \( n_0 \) is the current galaxy number density of the universe in the units of MWEG/volume, but we do use redshift cut-offs. The rationale behind imposing such an assumption is twofold: first, our primary interest is not to get the numbers for a particular type of source, we want to show that there can be anisotropic SGWB created by sources in the nearby universe, with certain density distribution over redshift, for which the directed search and isotropic search can provide non-overlapping information. Second, to our knowledge there is no accurate model available for \( n(z) \) for sources that create a significant SGWB. Moreover, the co-moving matter density in the low-redshift universe can at most evolve slowly in the region where \( 1/(1 + z)^2 \) is not too small. So a nearly constant profile with a redshift cut-off \( z = z_{\text{max}} \) does indeed provide a reasonable model. Nevertheless, in the rest of this paper we derive general formulae valid for any arbitrary \( n(z) \), only to get the final results we use

\[
n(z) = \begin{cases} n_0 & \text{if } z \leq z_{\text{max}} \\ 0 & \text{otherwise}. \end{cases} \tag{3.10}
\]

Introducing the redshift cutoff discussed above and setting the scale factor at the current epoch to unity, \( a(t_0) = 1 \), the form of \( N_{\text{eff}}^{\text{iso}} \) becomes,

\[
N_{\text{eff}}^{\text{iso}} = n_0 \Delta \Omega_V \frac{c}{H_0} \int_0^{z_{\text{max}}} \frac{dz}{(1 + z)^2 E(z)}. \tag{3.11}
\]

We now evaluate the above expression in the specific cases. Here we consider only flat universe (\( \Omega_L + \Omega_m = 1 \)) and compute the results for two different cosmologies:

- Case I: Universe without dark energy

A flat matter dominated universe without dark energy (\( \Omega_L = 0, \Omega_m = 1 \)). This case allows us to get the results analytically. These results may not be very realistic, but they provide useful insights through their analytical forms.

In this case eqn. (3.11) becomes

\[
N_{\text{eff}}^{\text{iso}} = \Delta \Omega_V \frac{c}{H_0} \int_0^{z_{\text{max}}} \frac{dz}{(1 + z)^{7/2}} \tag{3.12}
\]

\[
= n_0 \Delta \Omega_V \frac{2}{5} \frac{c}{H_0} \left[ 1 - \frac{1}{(1+z_{\text{max}})^{5/2}} \right]. \tag{3.13}
\]
near the earth is given by, har et al. [18].

Thus, in some sense, the analytical treatment developed this depending on the spectrum of the source and frequency response of the detector, which will be the main consideration in the rest of the paper.

Note that these results and the above discussion do not involve anything special about GW, they are equally valid for EM observations. The only difference arises in the definition of the response function. In EM astronomy the detectors generally count photons, while the GW detectors measure strain. However, the strain response function of a GW detector can be converted to flux response function, which will have different frequency response, but the forms of the expressions remain the same. Thus, in some sense, the analytical treatment developed below for GW, can be extended in a straightforward manner to include EM detectors.

B. Comparison of Power Spectral Densities (PSDs)

Redshift not only reduces intensity, but it shifts power lower to frequencies, which essentially means that the further the sources are the lower the frequency where their power is concentrated. In this subsection we explicitly derive an expression for the strain PSD of the integrated SGWB generated at different cosmological distances and compare it with the Virgo-only PSD derived in Dhurandhar et al. [18].

Let $J(f)$ be the intensity distribution of the SGWB created by the sources (e.g., MSPs) in a MWEB. It can be shown that [39], if the source is at redshift $z$, the GW flux density near the earth is given by,

$$S(f) = \frac{L_{MW} J(f(1+z))}{4\pi^2 a^2(t_0)(1+z)}.$$  \hspace{1cm} (3.14)

SGWB strain signal $h_{ij}$ at a given point in space $x$ and time $t$ can be expanded in Fourier modes as

$$h_{ij}(t,x) = \sum_{A=+,x} \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega} \tilde{h}_A(f, \hat{\Omega}) e^{i f(t - \hat{\Omega} \cdot x/c)} ,$$ \hspace{1cm} (3.15)

where $\hat{\Omega}$ is the propagation direction of the wave, $f$ is the frequency and $e_{ij}^A(\hat{\Omega})$ is the symmetric-trace-free (STF) basis tensors. The Fourier components, $\tilde{h}_{+,x}(f, \hat{\Omega})$, have no correlations, for an SGWB us generated by a set of incoherent events, like the ones considered in this paper.

This can be expressed as

$$\langle \tilde{h}_A(f, \hat{\Omega}) \tilde{h}_A'(f', \hat{\Omega}') \rangle = \delta_{AA'} \delta(f-f') \delta^2(\hat{\Omega} - \hat{\Omega}') H(f) P(\hat{\Omega}),$$ \hspace{1cm} (3.16)

where $H(f)$ represents the shape of the frequency power spectrum of the background and $P(\hat{\Omega})$ is the direction dependent amplitude of the power spectrum. Note that, in general the $f$ and the $\hat{\Omega}$ components need not be separable. That is, the shape of the spectrum can be different in different directions. So, in general one should use a function of the form $P(\hat{\Omega}, f)$ instead of $H(f) P(\hat{\Omega})$. However, this is not a relevant issue in this paper, $P(\hat{\Omega})$ and $H(f)$ are indeed separable for the case we consider here. This is because we are considering either the Virgo cluster, localised in the direction $\hat{\Omega}_V$, for which the spectra of the other directions are zero, so without any loss of generality one can write $P(\hat{\Omega}, f) = \delta^2(\hat{\Omega} - \hat{\Omega}_V) H(f)$. For an isotropic background, by definition, the spectra is the same in every direction, $P(\hat{\Omega}, f) = H_{iso}(f)$. Thus, in both the cases $P(\hat{\Omega}, f)$ is separable by construction.

Also note that, $H(f)$ and $P(\hat{\Omega})$ can be normalised keeping the product fixed. So, without any loss of generality one can impose one extra condition that,

$$\int d\hat{\Omega} P(\hat{\Omega}) = 1,$$ \hspace{1cm} (3.17)

which means that $P(\hat{\Omega}) = \delta^2(\hat{\Omega} - \hat{\Omega}_V)$ for the Virgo cluster and $P(\hat{\Omega}) = 1/4\pi$ for an isotropic background.

The total energy density of GW is given by [2]

$$\rho_{GW} = \frac{c^2}{32\pi G} \langle \hat{h}(x,t) \hat{h}(x,t) \rangle ,$$ \hspace{1cm} (3.18)

where an overdot represents derivative with respect to time in the observer’s frame. We are interested in finding the GW flux density from a localised source, specifically the Virgo cluster. In the above equation, substituting $P(\hat{\Omega}) = \delta(\hat{\Omega} - \hat{\Omega}_V)$, eqn. (3.15) and eqn. (3.16), and comparing with the relation

$$\rho_{GW} = \frac{1}{c} \int df S(f) ,$$ \hspace{1cm} (3.19)

one can arrive at the formula,

$$S(f) = \frac{c^3}{32\pi G} f^2 H(f) .$$ \hspace{1cm} (3.20)
Finally, combining this with eqn. (3.14) one can finally write the expression for $H(f)$ for a $N_{\text{gal}}$ localised MWEG in a given direction as
\[
H(f) = N_{\text{gal}} \frac{8G L_{\text{MW}} J(f(1+z))}{c^3 f^2 r^2 a^2(t_0)(1+z)}.
\] (3.21)

For the Virgo cluster, $N_{\text{gal}} := N_V \approx 1500$ and $z \approx 0$. Hence,
\[
H_V(f) = N_V \frac{8G L_{\text{MW}} J(f)}{c^3 f^2 r^2}. \tag{3.22}
\]

However, the expression for expected $H_V(f)$ was already calculated in Dhurandhar et al. [18] for a given distribution of the pulsars $N(f)$ in a MWEG as,
\[
H_V(f) = \tilde{h}_0 f^4 N(f). \tag{3.23}
\]

The frequency independent constant $\tilde{h}_0$ is the typical strength of GW emitted by a $1.4M_\odot$ neutron star:
\[
\tilde{h}_0 \approx 7(\alpha^2) \times 10^{-34} \left(\frac{\epsilon}{10^{-8}}\right) \left(\frac{I}{1.1 \times 10^{45} \text{g cm}^2}\right).
\] (3.24)

where $\epsilon$ is the ellipticity of the neutron star, $I$ is the moment of inertia, $\alpha \leq 1$ is the orientation factor and $(\alpha^2)$ represents the average with respect to the inclination angle and the polarisation angle. For uniformly distributed source over the angles $(\alpha^2) = 2/5$. We use this relation to alleviate the complications in fixing the proportionality constants as discussed below.

To compute the SGWB from the isotropic part, one has to integrate over all the sources in the same solid angle as what the Virgo cluster subtends at the earth. Since the number of MWEG $dN_{\text{gal}}$ in solid angle $\Delta\Omega_V$ in radial coordinate range $r$ to $r+dr$ is $n(z)\Delta\Omega_V r^2 dr$, the net result can be obtained by integrating eqn. (3.21) as
\[
H_{\text{iso}}(f) = \Delta\Omega_V \int_0^\infty dr n(z) \frac{8G L_{\text{MW}} J(f(1+z))}{c^3 f^2 a^2(t_0)(1+z)}.
\] (3.25)

Then invoking the relation between $dr$ and $dz$, eqn. (3.8), one can arrive at the relation
\[
H_{\text{iso}}(f) = \frac{\Delta\Omega_V c}{H_0} \int_0^\infty dz \frac{8G L_{\text{MW}} J(f(1+z)) n(z)}{c^3 f^2 a^2(t_0)(1+z) E(z)}. \tag{3.26}
\]

Since we are primarily interested in comparing the relative strengths of the Virgo cluster and the isotropic background and we already have a formula for the Virgo cluster, we can hide the growing number of constants in the above equation, by dividing the above equation by eqn. (3.22). We can write
\[
\frac{H_{\text{iso}}(f)}{H_V(f)} = \frac{\Delta\Omega_V r_V^2 c}{N_V H_0} \int_0^\infty dz \frac{J(f(1+z)) n(z)}{a^2(t_0) E(z) E(z)}. \tag{3.27}
\]

This further reduces complications involving the proportionality constants when we expand the intensity distribution $J(f)$ in terms of the source number distribution $N(f)$, in fact, a considerable amount of details were presented in Dhurandhar et al. [18] in fixing those constants.

We know that GW intensity is proportional to the square of triple derivative of quadrupole moment. If there are $N(f)df$ sources in the GW frequency interval $f$ to $f+df$ in a Milky Way like galaxy, one should have $J(f) \propto N(f) f^6$, where the proportionality constant would be independent of frequency, and hence, redshift. Substituting this in the above equation, one can write
\[
H_{\text{iso}}(f) = H_V(f) \left(\frac{\Delta\Omega_V r_V^2 c}{N_V H_0}\right) \times \int_0^\infty dz (1+z)^5 \frac{n(z) N(f(1+z))}{E(z)} N(f).
\] (3.28)

where $H_V(f)$ is given by eqn. (3.23).

To proceed further we would need to introduce the number distribution of the sources explicitly. Qualitatively this happens because the frequency distribution of sources at different redshifts is dependent on the actual distribution and the cosmology.

As mentioned earlier, in this paper we are explicitly evaluating quantities for MSPs in galaxy clusters and the rest of the universe. From the radio survey of our Galactic disk, the number of pulsars is estimated to be at least 40,000. The population of these pulsars follow two different distributions, each distribution is Gaussian and has mean and standard deviation different from the others and they can be divided two regions separated by 50Hz. The distributions in each region may be approximated as log-normal distributions as stated below [18]. The probability that the log of pulsar rotation frequency is in the range $\log f_r$ and $\log f_r + df_r$ is given by

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\hline
z_{\text{max}} & \text{No dark energy} & \text{ACDM cosmology} \\
\hline
N_{\text{eff}}^{\text{iso}} & 0.0677 & 0.0770 & 0.0797 & 0.0821 & 0.0823 & 0.0846 & 0.1001 & 0.1049 & 0.1092 & 0.1096 \\
N_{\text{eff}}/N_{\text{eff}}^{\text{iso}} & 81.2 & 71.4 & 69.0 & 67.0 & 66.8 & 65.0 & 54.9 & 52.4 & 50.4 & 50.2 \\
\hline
\end{array}
\]

TABLE I: The first row of this table lists the effective number of MWEG sources $N_{\text{eff}}$ placed 1Mpc away from the detector, such that the flux is equivalent to all the sources present in the same solid angle as the Virgo cluster. The second row shows the ratio of the effective number of sources $N_{\text{eff}}$ in the Virgo cluster to $N_{\text{eff}}^{\text{iso}}$, which essentially quantifies how “bright” Virgo cluster would appear in the isotropic background, when observed with an instrument with a flat frequency response and infinite bandwidth. Note that $N_{\text{eff}}^{\text{iso}} = 5.5$. 

\[ P_1(\log f_r) \, d\log f_r = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(\log f_r - \log \mu_1)^2}{2\sigma_1^2}} \, d\log f_r, \]

(3.29)

\[ P_2(\log f_r) \, d\log f_r = \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(\log f_r - \log \mu_2)^2}{2\sigma_2^2}} \, d\log f_r, \]

(3.30)

where \( \mu_1 = 219\text{Hz}, \sigma_1 = 0.238, \mu_2 = 1.71\text{Hz}, \sigma_2 = 0.420 \) and \( f_r = \text{Pulsar spin frequency} = \text{half of gravitational wave frequency}. \) Following Dhurandhar et al. [18], here we consider a similar distribution of pulsars in the Virgo cluster and the rest of the universe. There are approximately 1500 galaxies in the Virgo cluster. The total number of pulsars in our galaxy can be taken as 10^8 for \( f_r < 50 \) and 40,000 for \( f_r > 50 \). So the distribution of pulsars including MSPs in Virgo cluster becomes

\[ N(f) df = (N_{\text{high}} P_1(\log f_r) + N_{\text{low}} P_2(\log f_r)) \frac{df_r}{f_r \ln 10}, \]

(3.31)

with \( N_{\text{high}} \approx 4 \times 10^7 \) for \( f_r > 50 \) and \( N_{\text{low}} \approx 10^{11} \) for \( f_r < 50 \). Note that the GW emission frequency \( f \) from the MSPs is twice the rotation frequency \( f_r \). For a bandwidth of \( \sim 10^3 \text{Hz} \), the number of pulsars in each mHz frequency bin is \( \sim 10 \) for \( f > 100 \text{Hz} \). In the low frequency regime this number is much larger. Thus, it is not possible to resolve the signal from each pulsar, the signals from the pulsars create a continuum.

In this work, we are mainly concerned about pulsars which emit in the frequency band of the ground based laser interferometers, which correspond to a time period of milliseconds. Hence we can only consider the distribution of MSPs, which is essentially dominated by \( P_1(\log f_r) \) of the bimodal pulsar distribution. Hence, from this point, we only consider the high frequency peak of the pulsar distribution and ignore the low frequency peak altogether.

Considering a log-normal distribution of sources presented in Eq. (19) of Dhurandhar et al. [18] (and ignoring the low frequency sources), one can then write

\[ \frac{N(f(1 + z))}{N(f)} = \frac{1}{1 + z} \frac{P(\log ((1 + z)f/2))}{P(\log (f/2))} \]

(3.32)

\[ = \frac{1}{1 + z} \exp \left[ -\log \left( \frac{(1 + z)f^2}{4\mu^2} \frac{\log(1 + z)}{2\sigma^2} \right) \right] \]

\[ = \frac{1}{1 + z} \left[ \frac{4\mu^2}{(1 + z)f^2} \right]^{\log(1 + z)/[2 \sigma^2 \ln 10]}. \]

Then the final expression, eqn. (3.28), becomes

\[ H_{\text{iso}}(f) = H_V(f) \left( \frac{\Delta \Omega_V r_V^2}{n_V} \right) \frac{c}{H_0} \times \]

(3.33)

\[ \int_{0}^{\infty} dz \, n(z) \left( 1 + z \right)^4 \frac{E(z)}{\Omega(z)} e^{-\log \left( \frac{(1 + z)f^2}{4\mu^2} \frac{\log(1 + z)}{2\sigma^2} \right)}. \]

Now, again, to proceed further we will have to introduce cosmology. As before, we use \( n(z) \) given by eqn. (3.10) for two different cosmological models.

- **Case I: Universe without dark energy**

As was in the case of total flux comparison, the justification for including this case is to have analytical handle without deviating too much from the realistic regime. Substituting \( E(z) = (1 + z)^{3/2} \) in eqn. (3.33) one gets

\[ H_{\text{iso}}(f) = H_V(f) \left( \frac{n_0 \Delta \Omega_V r_V^2}{n_V} \right) \frac{c}{H_0} \times \]

\[ \int_{0}^{\infty} dz \, (1 + z)^4 e^{-\log \left( \frac{(1 + z)f^2}{4\mu^2} \frac{\log(1 + z)}{2\sigma^2} \right)}, \]

(3.34)

where,

\[ y := \frac{7}{2} \sigma \ln 10 - \frac{1}{\sigma} \ln \left( \frac{f}{2\mu} \right), \]

(3.35)

\[ y_m := \frac{1}{\sigma} \ln(1 + z_{\text{max}}). \]

(3.36)

Here \( \mu \) and \( \sigma \) are the mean and standard deviation of the source distribution, and we use the standard definition of the error function

\[ \text{erf}(x) := \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt, \]

(3.37)

which happens to be an odd function:

\[ \text{erf}(-x) := \frac{2}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-t^2} dt = -\text{erf}(x). \]

(3.38)

In the limit \( z_{\text{max}} \rightarrow \infty \) (that is, \( y_m \rightarrow \infty \)), for finite values of \( f, \text{erf}((y_m - y)/\sqrt{2}) \rightarrow 1 \). Hence, in that case, one can write

\[ H_{\text{iso}}(f) = H_V(f) \left( \frac{n_0 \Delta \Omega_V r_V^2}{n_V} \right) \frac{c}{H_0} \times \]

(3.39)

\[ \left[ \sqrt{\pi/2} (\ln 10) \sigma e^{y_m^2/2} (1 + \text{erf}(y_m/\sqrt{2})) \right]. \]

Note that in this special case, \( H_{\text{iso}}(f)/H_V(f) \) is a monotonically decreasing function of frequency.

- **Case II: \( \Lambda \)CDM cosmology**

In this case we substitute eqn. (3.9) in eqn. (3.33) and then numerically integrate over redshift \( z \) for each frequency bin \( f \):

\[ H_{\text{iso}}(f) = H_V(f) \left( \frac{n_0 \Delta \Omega_V r_V^2}{n_V} \right) \frac{c}{H_0} \times \]

(3.40)

\[ \int_{0}^{\infty} dz \, (1 + z)^4 e^{-\log \left( \frac{(1 + z)f^2}{4\mu^2} \frac{\log(1 + z)}{2\sigma^2} \right)}. \]
In this paper, we use only one set of $H_V(f)$ taken from Dhurandhar et al. [18] which corresponds to the eccentricity $\epsilon = 10^{-5}$. Since $H_V(f) \sim \epsilon^2$, the PSDs and SNRs for other values of $\epsilon$ could be readily derived, though they would be redundant for this work, as here we are interested in the relative SNRs. Figure 1 shows the comparison between $H_V(f)$ and $H_{iso}(f)$ for different redshift cut-offs for two cosmological models: no dark energy (above) and $\Lambda$CDM (below). Qualitatively also the results are not surprising. The high redshifts sources move to lower frequencies. Hence, higher the redshift cut-off stronger the PSD at lower frequencies. Although only small changes can be noticed in the top and bottom panels of Figure 1, the differences in the plots are of the order of factor of few, which makes a significant difference in the overall observed SNR.

### IV. OBSERVED SIGNAL-TO-NOISE RATIO

Now we have all the details to answer the central questions—which component of the SGWB is more important in searches for a SGWB, does an optimal search make a significant difference?

To address these questions we review key results relevant for an anisotropic SGWB. A substantial amount of literature has been written in developing anisotropic searches [19–24, 28–31]. The key results from these papers can be summarised as follows: In order to search for an anisotropic background one generally divides the data into many segments with time intervals much greater than the light travel time delay between the detector sites (few tens of milliseconds), but small enough so that the earth can be considered stationary over that period. Generally the period is taken as few tens of seconds [35, 36]. The data from pairs of detectors are then correlated in the frequency domain with suitable filters to search for different signals. It can be shown that, in order to search for an unpolarised SGWB with angular power distribution $P(\hat{\Omega})$ and spectrum $H(f)$, the optimal statistic is [44]

$$S = \sum_{i=1}^{n} \int_{-\infty}^{\infty} df \frac{H(f) \gamma_P(t_i; f)}{P_1(t_i; |f|) P_2(t_i; |f|)} \tilde{S}_1(t_i; f) \tilde{S}_2(t_i; f)$$

$$= \delta T \sum_{i=1}^{n} \int_{-\infty}^{\infty} df \frac{H^2(f) \gamma_P(t_i; f)^2}{P_1(t_i; |f|) P_2(t_i; |f|)}$$

where $t_i$ is the segment time, $\tilde{S}_1(t_i; f)$ and $P_1(t_i; |f|)$ are respectively the short-term Fourier transform and noise PSD of the data from detector 1 and,

$$\gamma_P(t, f) := \sum_{A=+,-} \int d\Omega P(\hat{\Omega}) \times$$

$$F^A_1(\hat{\Omega}, t) F^A_2(\hat{\Omega}, t) e^{2\pi i f \hat{\Omega} \Delta x(t)/c}$$

is the general overlap reduction function expressed in terms of the separation vector between the detectors $\Delta x(t)$ and the antenna pattern functions $F^A_1(\hat{\Omega}, t)$. The bandwidth of the overlap reduction function for the isotropic search, $P(\hat{\Omega}) = 1/4\pi$, is very low, 0 to $\sim 60$ Hz, for the LIGO Hanford-Livingston (HL) baseline, as shown in figure 2, which excludes the most sensitive band of the ground based laser interferometric detectors. This unfortunate fact significantly limits the effectiveness in finding astrophysical SGWB generated in the local universe. It may be able to detect high frequency sources if they are appropriately redshifted. The directed search overlap reduction function, on the other hand, has an infinite bandwidth, which means that the search is only limited by the detector bandwidth, and hence can capture a wide range of high frequency sources.

Had the search been optimal, that is, if the model sky $P(\hat{\Omega})$ and PSD $H(f)$ matched the true counterparts $P_{true}(\hat{\Omega})$ and $H_{true}(f)$ respectively, the expectation value...
of the observed signal-to-noise ratio would be \[ \text{SNR}_{\text{true}} = 2\sqrt{\frac{\delta T}{\sum_{i=1}^{n} \int_{-\infty}^{\infty} df H^2(f) |\gamma_{\text{true}}(t_i, f)|^2}} \].

For a given background, this is the maximum observable average SNR. If, however, the model does not match the true sky, (expectation of) the observed SNR becomes

$$\text{SNR}_{\text{obs}} = 2\sqrt{\delta T} \times \frac{\sum_{i=1}^{n} \int_{-\infty}^{\infty} df \frac{H(f) \gamma_{\text{true}}(t_i, f)}{P_1(t_i; |f|) P_2(t_i; |f|)} H_{\text{true}}(f) \gamma_{\text{true}}(t_i, f) \sum_{i=1}^{n} \int_{-\infty}^{\infty} df \frac{H^2(f) |\gamma_{\text{true}}(t_i, f)|^2}{P_1(t_i; |f|) P_2(t_i; |f|)}}$$.

Using the above formulae we can now perform numerical evaluation for the following quantities to answer the questions we have raised:

1. **SNR$_V$**: SNR in a directed search for the Virgo cluster. We put $P_{\text{true}}(\Omega) = P(\Omega) = \delta(\hat{\Omega} - \hat{\Omega}_V)$, $H_{\text{true}}(f) = H(f) = H_V(f)$ (assuming that the Virgo cluster is nearly a point source for the resolution of the radiometer formed by the LIGO HL baseline).

2. **SNR$_{iso}$**: SNR obtained by the isotropic search for the isotropic background. We get this by setting $P_{\text{true}}(\Omega) = P(\Omega) = 1/4\pi$, $H_{\text{true}}(f) = H(f) = H_{iso}(f)$.

3. **SNR$_V$**: SNR obtained by doing a directed search when the actual background is isotropic. We use $P_{\text{true}}(\Omega) = 1/4\pi$, $P(\Omega) = \delta(\hat{\Omega} - \hat{\Omega}_V)$, $H_{\text{true}}(f) = H_{iso}(f)$, $H(f) = H_V(f)$. This number when compared to SNR$_V$, will quantify how much the directed search will be able to differentiate between the isotropic and anisotropic search and hence justify if the search is useful for probing the SGWB anisotropy in presence of an isotropic SGWB.

4. **SNR$_{iso}$**: SNR contributed by the Virgo cluster to the isotropic search. We put $P_{\text{true}}(\Omega) = \delta(\hat{\Omega} - \hat{\Omega}_V)$, $P(\Omega) = 1/4\pi$, $H_{\text{true}}(f) = H_V(f)$, $H(f) = H_{iso}(f)$. This number shows if the isotropic search will be able to detect the presence of a localised source. By comparing with SNR$_V$ one would be able to conclude if a dedicated directed search produces significantly better results.

Since we are using only one spectrum for the Virgo cluster for this relative study and the spectrum hardly depends on cosmology, we get only one value of SNR$_V$ in this paper, which happens to be SNR$_V = 0.6067$ for the LIGO HL baseline with an observation time of one

sidereal day, advanced LIGO design sensitivity “of zero-detuning of the signal recycling mirror, with high laser power” [45] and frequency range $10 - 2000$ Hz. The values of the other three SNRs, SNR$_{iso}$, SNR$_V$ and SNR$_{iso}^V$ are listed in Table II for different redshift cut-offs and cosmologies for the same detector pair and observation time. The table shows that when the universe has MSPs at high redshifts (younger universe) and it is statistically isotropic at large scales, the isotropic search performs well. However, if the background is dominated by nearby sources (older universe) a localised search can out perform an all sky isotropic search. Moreover, in all the cases the directed search for the isotropic background is much lower, by more than two orders of magnitude than the directed search for a localised source, signifying that the directed search is highly sensitive to the anisotropy of a background and would be able make a sky map, provided the background is stronger than the detector noise level (integrated over the observation time). Finally, the isotropic search for a localised background is even more suboptimal due to the mismatch in signal and model, the SNR is orders of magnitude below the optimal SNR, hence an isotropic search would not be able to detect the presence of a localised source.

The reason why the isotropic background, which only dominates in a small range of low frequencies, can still compete with the Virgo cluster, is illustrated in figure 2 obtained for the $\Lambda$CDM cosmology with $z_{\text{max}} = 2$. The top left panel shows $H(f)$ for isotropic and Virgo backgrounds; the top right panel shows $H_{\text{true}} H_{\text{obs}}/(P_1(f) P_2(f))$; the bottom left panel shows the overlap reduction function for the two cases; the bottom right plot shows essentially the final integrand which provides the SNR. Even though $H_{iso}(f)$ seems to be dominating over $H_V(f)$ in this case, the large bandwidth of the directed search equalises the observed SNR in either cases when the model matches the true signal. As shown in Table II, SNR$_V \approx $ SNR$_{iso}^V$ for $z_{\text{max}} = 2$.

For the purpose of completeness, we also include numerical results for the Hanford-Virgo and Livingston-Virgo detector baselines. We do this for a few realistic values of $z_{\text{max}}$ for $\Lambda$CDM cosmology and Advanced Virgo design sensitivity [46]. The results are shown in Table III. Clearly, for the LIGO-Virgo baselines the results are similar to the LIGO HL baseline. However, isotropic search with the LIGO-Virgo baselines has better high frequency sensitivity [47] than the LIGO HL baseline. This provides comparatively higher SNR to the isotropic search even for low redshift cut-offs.

V. CONCLUSION

Over the past two decades the search algorithms for both isotropic and anisotropic stochastic gravitational wave background have been developed and applied to data from the ground based Laser Interferometric GW detectors. In this work we showed that both kinds of
Table II: The (expected) SNR for different combinations of source and model power spectra and sky model of the SGWB observed with LIGO Hanford-Livingston baseline. For comparison, directed search SNR for the same baseline for the Virgo cluster is $\text{SNR}_V = 0.6067$. Thus, in this case, the directed search for Virgo cluster exceeds the SNR of the all sky integrated search for an isotropic background if the background is dominated by sources below a redshift of $z_{\text{max}} \sim 3$. The total observation time is taken as one sidereal day, frequency range $10^{-2} - 2000$ Hz and Advanced LIGO design noise PSD of “zero-detuning of the signal recycling mirror, with high laser power” [45].

| $z_{\text{max}}$ | No dark energy | $\Lambda$CDM cosmology |
|------------------|----------------|------------------------|
|                  | 1              | 2                      | 3        | 10       | $\infty$ | 1         | 2         | 3        | 10       | $\infty$ |
| $\text{SNR}_{iso}^{\text{V}}$ | 0.1049 | 0.2678 | 0.4775 | 2.7007 | 9.2317 | 0.1514 | 0.4446 | 0.8431 | 5.2149 | 17.8521 |
| $\text{SNR}_{V}^{\text{V}}$    | 1.27e-4 | 1.84e-4 | 2.02e-4 | 1.97e-4 | 1.92e-4 | 1.75e-4 | 2.74e-4 | 3.08e-4 | 3.10e-4 | 3.00e-4 |
| $\text{SNR}_{iso}^{\text{V}}$ | 7.37e-4 | 4.17e-4 | 2.57e-4 | 4.42e-5 | 1.26e-5 | 7.01e-4 | 3.74e-4 | 2.22e-4 | 3.60e-4 | 1.02e-5 |

FIG. 2: This figure shows how different quantities combine to yield little more SNR for directed search Virgo cluster (dashed line) and isotropic search (solid line) with $z_{\text{max}} = 2$. Top left shows $H(f)$ for isotropic background and Virgo cluster; Top right the frequency spectrum part of the optimal search filter; Bottom left are the overlap reduction function. Bottom right is the final integrand in the expression for SNR.

searches are important for different backgrounds and one search cannot replace the other. However, for detection purposes, one can conclude that the isotropic search is more effective when the sources are distributed uniformly across the sky and up to a high redshift, while the directed search is more efficient when most of the sources are in the nearby universe and their distribution is anisotropic, with one (or few) localised sources.

We show that the directed search is highly sensitive to the anisotropy of a background which is useful for mak-
Moving one step further, since the above mentioned ratio also depends on the SGWB spectrum emanating from a M Weg, one can even aim to test the theoretical models for that spectrum, when the data quality becomes good and one has an independent handle on the redshift cut-off or \( n(z) \).

The analyses in this paper are restricted to either highly localised or isotropic backgrounds, which are, by their intrinsic nature, suboptimal. They are currently performed this way due to the lack of reliable models of the anisotropic sky. It may however be possible to gather information from electromagnetic astronomy and construct more optimal filters to perform search for anisotropic backgrounds. Here one may even consider using different spectra for different directions, as the composition of sources in different direction may be different. However, such an analysis does not exist in literature and, also, it may not be very straightforward to develop such an analysis in a computationally viable way. In such cases also, a comparative study similar to this paper needs to be performed, to decide if such an effort is worthwhile.

In this paper we computed most of the numerical results for the LIGO Hanford-Livingston (HL) baseline, which is sufficient to justify the importance of different searches. For academic interest, we also included results for longer baselines including the Virgo detector. However, for longer baselines the resolution of the radiometer is considerably high and the assumption that the Virgo cluster is a point source breaks down. So to get more rigorous SNR estimations, one should consider the finite size of the Virgo cluster and, perhaps, use a model for the mass distribution.

The goal of this paper is not to address the issue of template mismatch and suggest a solution. However, since some of the SNRs in the tables are negative, we must caution the readers that template mismatch can cause negative values of the statistic in specific frequency bands and eventually lead to wrong upper limits when integrated over all frequencies. The absolute value of the estimator may be one way to address the issue, though care must be taken to ensure that the statistic is indeed estimating the desired quantity. This issue can be ignored when one is trying to probe a background that is by far the strongest in the sky in the frequency band considered. But that may not be the case in general and a more rigorous strategy must be developed in order to alleviate this problem, perhaps by fitting all the SGWB components together.

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