Forecasting methods in engineering

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Abstract. Forecasting in engineering is one of the most important topics when it comes to optimisation, which is related to energy savings, material savings, increasing efficiency, and appropriate and correct decisions at the level of a company, institution, city, or region. Moreover, forecasting is indirectly related to cost savings and the sustainable development of society and environment. In the energy industry (electricity, natural gas, heat load), there are requirements to balance the supply and demand. Markets are very dynamic, and for this reason forecasting is more challenging. Forecasting errors are usually penalized drastically. However, a well-developed forecasting approach represents a competitive advantage, and so a company may significantly reduce expenditure and increase profits. Many publications on forecasting have been published during recent years. Long-term forecasting methods offer many opportunities for strategic planning and optimal scheduling, whereas a short-term forecasting approach would help attain optimal daily operations and the maximum utilisation of the company’s resources. Although different forecasting techniques can be used, the major conclusions are that exponential smoothing methods are the simplest and the least expensive. Distinguished by their simplicity, their forecasts are comparable to forecasts of more complex statistical time series models. In this paper, the forecasting performance of Additive, Multiplicative, and Extended Holt-Winters methods were analysed. We also analyse whether the data format influences the choice of the forecasting method: is the most accurate method for monthly data also the best method for quarterly data?

1. Introduction

Forecasting in engineering is very important topic when it comes to efficient processes, cost reduction, efficient logistic, reduction of energy consumption, sustainable development [1] and key drivers of the transition toward a low carbon society [2]. A main purpose of forecasting is a balancing of the supply and demand. Forecasting is widely used in different branches in the energy industry, for example, there are different methods for prediction of natural gas consumption [3], electricity energy consumption [4,5,6], electric load [7], heat load [8,9,10], cooling [11] and building energy consumption [12,13].

Although there are various forecasting methods, we decided to analyse Additive, Multiplicative and Extended Holt-Winters approach [14,15,16] with and without additional optimisation of initial values. The aim of this article was the identification of the best forecasting methods for monthly and quarterly long- and short-term forecasting. In other words, we focused on the best forecasting methods for monthly forecasting and then explored if the same forecasting methods were also the best choice for quarterly data. A real seasonal time series from the M4-Competition were analysed. The forecasting accuracy was determined taking into account Theil’s U-statistics.
The paper is organised as follows. In Section 2 the forecasting methods are presented, and in Section 3 the data and methodology are described. The results are presented in Section 5 and research findings in Section 6.

2. Forecasting methods
The Holt-Winters method of exponential smoothing involves trend and seasonality and is based on three smoothing equations: an equation for level, for trend, and for seasonality. The decision as to which method to use depends on time series characteristics: the additive method is used when the seasonal component is constant, and the multiplicative method is used when the size of the seasonal component is proportional to the trend level [17].

2.1. Holt-winters’ additive procedure (AHW)
The basic equations for the AHW method are:

\[ L_t = \alpha (Y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + b_{t-1}) \]  
\[ b_t = \beta (L_t - L_{t-1}) + (1 - \beta)b_{t-1} \]  
\[ S_t = \gamma (Y_t - L_t) + (1 - \gamma)S_{t-s} \]  
\[ F_{t+m} = L_t + b_t m + S_{t-s+m} \]

where \( L_t \) – the estimation of variable in time \( t \), \( Y_t \) – the observed value, \( b_t \) – the trend estimation of time series in time \( t \), \( S_t \) – the estimation of seasonality in time \( t \), \( F_{t+m} \) – the forecast in time \( t \) for \( m \) period ahead, \( \alpha, \beta, \gamma \) – smoothing parameters in the interval \([0, 1]\), \( m \) – the number of forecasted periods, and \( s \) – the duration of seasonality (for example, number of months or quarters in a year).

For initialisation of the additive method initial values of variable \( L_1 \), trend estimation \( b_1 \) and seasonality estimation \( S_1 \) are needed. To determine initial estimates we need at least one whole data season (that is, \( s \) data). Initialisation of variable \( L_s \) is calculated with the formula:

\[ L_s = \frac{1}{s} (Y_1 + Y_2 + \cdots + Y_s) \]  

For trend initialisation, it is more suitable if we use two whole seasons (that is, \( 2s \) data):

\[ b_s = \frac{1}{s} \left( \frac{Y_{s+1} - Y_1}{s} + \frac{Y_{s+2} - Y_2}{s} + \cdots + \frac{Y_{2s} - Y_s}{s} \right) \]

Seasonal indices are calculated as differences between the observed value and variable estimation \( L_s \):

\[ S_1 = Y_1 - L_s, S_2 = Y_2 - L_s, \ldots, S_s = Y_s - L_s \]

The method is proved to be (regarding costs and calculation itself) comparable with more complex methods (for example Box-Jenkins); in some cases the results gained with the Holt-Winters were even better than more complex methods [17].

2.2. Extended holt-winters’ procedure (EHW)
The EHW method differs from AHW only in the equation for level (1); all other equations remain the same as with the AHW (2–7). The equation for level now contains an additional smoothing parameter \( \delta \):

\[ L_t = \alpha Y_t - \delta S_{t-s} + (1 - \alpha)(L_{t-1} + b_{t-1}) \]

This method allows us to smooth the seasonal factors more or less than the AHW method, depending on the value of the parameter \( \delta \) [17].
2.3. Holt-winters’ multiplicative procedure (MHW)

The basic equations for the MHW method are as follows:

\[
L_t = \alpha(Y_t/S_{t-s}) + (1 - \alpha)(L_{t-1} + b_{t-1})
\]

\[
b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}
\]

\[
S_t = \gamma(Y_t/L_t) + (1 - \gamma)S_{t-s}
\]

\[
F_{t+m} = (L_t + b_t) \cdot S_{t-s+m}
\]

The second of these equations (10) is identical to the second equation (2) of AHW. The only differences in the other equations are that the seasonal components are now in the form of products and ratios instead of being added and subtracted.

3. Data and methodology

We used a real seasonal time series from the M4-Competition. The M-Competitions are empirical studies that compare the performance of a large number of major time series methods using recognised experts who provide forecasts for their method of expertise [18].

In our study, we analysed five monthly series that refer to industry: M26726, M26727, M26730, M26734, and M26735. For simplicity, we used notation 26 m, 27 m, 30 m, 34 m, and 35 m, respectively (see figure 1). Whereas data for these time series are given for different time periods, we selected consecutive data for a period of 138 months (11 years and 6 months) for unification and easier mutual comparison. The obtained data of each time series were split into initialiseation (the first two years, from period 1 to 24), fitting (period from 25 to 120), and testing set (period from 121 to 138). A ratio between initialiseation, fitting, and testing set was chosen regarding previous papers [19,20] and experiences. Later on, accumulated quarterly data for each monthly series was prepared and split into initialiseation (from period 1 to 8), fitting (period from 9 to 40), and testing set (period from 41 to 46). We used the notation 26q, 27q, 30q, 34q, and 35q, respectively (figure 2). The fitting set was used for method learning. With the testing set we checked a time series learning ability.

![Figure 1. Plots of monthly time series.](image1)

![Figure 2. Plots of quarterly time series.](image2)

We calculated the forecasting values for the testing subset and then compare these values to independent-real data. In case of the short-term forecasting approach, we calculated the forecasting value \(F_{t+m}\) for one month \((m=1)\) ahead (monthly short-term forecasting) and one quarter \((m=1)\) ahead (quarterly short-term forecasting), where \(t\) represents an arbitrary time point from the testing set. For the long-term forecasting approach, we calculated the forecasting value \(F_{t+m}\) for 18 months \((m=18)\) ahead (monthly long-term forecasting) and six \((m=6)\) quarters ahead (quarterly long-term forecasting), where \(t\) represents last month/quarter in the fitting set.

For the evaluation of the forecasting methods we applied three forecasting accuracy measures, Mean
Squared Error (MSE), Mean Absolute Percentage Error (MAPE), and Theil’s U-statistics:

\[
MSE = \frac{1}{N} \sum_{t=1}^{N} (Y_t - F_t)^2, \quad t = 1, 2, 3, ... N
\]

\[
MAPE = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{Y_t - F_t}{Y_t} \right| \times 100 \quad \% , \quad t = 1, 2, 3, ... N
\]

\[
U = \left( \frac{\sum_{t=1}^{N-1} \left( \frac{F_{t+1} - Y_{t+1}}{Y_t} \right)^2}{\sum_{t=1}^{N} \left( \frac{Y_{t+1} - Y_t}{Y_t} \right)^2} \right)^{1/2}
\]

where \( Y_t \) represents the actual value, \( F_t \) the forecasted value, and \( N \) the number of samples. As MSE penalizes errors proportional to their squares, minimising MSE leads to smoothing parameters that produce fewer large errors at the expense of tolerating several small errors. Of course, the lower values of MSE, MAPE, and Theil’s U-statistics represent a better forecasting performance.

The analysed exponential smoothing methods deal with smoothing parameters that are determined according to the past data. The smoothing parameters were determined by minimising the MSE and MAPE. The starting values of the smoothing parameters were set to 0.5. The minimising problem was solved by using Solver (Microsoft Office Excel 2010). The additional optimisation (notation method-init) of the initial values (for level, trend, and seasonality) has been performed.

4. Results

4.1. MSE, MAPE, and Theil’s U-statistics

Table 1 shows MSE, MAPE, and Theil’s U-statistics results of short-term forecasting for fitting and testing sets for monthly and quarterly data of time series M26726 obtained with six different methods. As expected, MSE values for quarterly data are much higher than for monthly data (since monthly data are smaller than the corresponding accumulated quarterly data), but Theil’s U-statistics for quarterly data are smaller than for monthly data, which means that forecasts of quarterly data are more accurate. It is further noted that the size of the U-MSE and U-MAPEs, regardless of the fitting or test set, are about the same (see figures 3 and 4). We can see that on average U-MSE values are slightly lower than the U-MAPE value, especially for the test set, which means that the forecasts obtained with minimising MSE are slightly more accurate than those obtained by minimising MAPE. Because we came to similar conclusions in the long-term approach (all results can be obtained upon request), in the continuation of the article, we will present in detail only the results of U-statistics obtained with MSE minimisation.

Table 1. Results for short-term forecasting of time series M26726.

|       | MSE  | U-MSE | MSE  | U-MSE | MAPE | U-MAPE | MAPE  | U-MAPE |
|-------|------|-------|------|-------|------|--------|-------|--------|
|       | (fit)| (fit) | (test)| (fit) | (fit)| (test) | (fit) | (test) |
| S_26m-AHW | 24,292.67 | 0.7228 | 33,332.78 | 0.7055 | 1.22 | 0.7563 | 1.38 | 0.6643 |
| S_26m-AHW-init | 9,077.67 | 0.4429 | 17,062.15 | 0.5073 | 0.83 | 0.5069 | 1.12 | 0.5675 |
| S_26m-MHW | 25,808.78 | 0.7415 | 35,973.53 | 0.7329 | 1.22 | 0.7698 | 1.46 | 0.6893 |
| S_26m-MHW-init | 9,337.49 | 0.4515 | 15,873.97 | 0.4908 | 0.74 | 0.4614 | 0.97 | 0.5289 |
| S_26m-EHW | 18,459.70 | 0.6371 | 25,618.28 | 0.6191 | 1.01 | 0.6225 | 1.42 | 0.6545 |
| S_26m-EHW-init | 8,690.37 | 0.4329 | 15,984.94 | 0.4922 | 0.71 | 0.4507 | 0.92 | 0.5289 |
| S_26q-AHW | 207,531.70 | 0.3571 | 730,794.23 | 0.6584 | 0.99 | 0.3415 | 2.82 | 0.6995 |
| S_26q-AHW-init | 113,607.43 | 0.2773 | 288,901.03 | 0.4221 | 0.84 | 0.2687 | 2.13 | 0.5767 |
| S_26q-MHW | 221,263.96 | 0.3676 | 750,706.93 | 0.6676 | 1.08 | 0.3591 | 2.74 | 0.7115 |
| S_26q-MHW-init | 149,639.57 | 0.3201 | 308,717.39 | 0.4361 | 0.90 | 0.3156 | 2.31 | 0.6765 |
| S_26q-EHW | 186,568.20 | 0.3402 | 673,765.81 | 0.6287 | 0.90 | 0.3242 | 2.81 | 0.7062 |
| S_26q-EHW-init | 94,678.46 | 0.2467 | 237,144.36 | 0.3801 | 0.77 | 0.2543 | 1.11 | 0.3957 |
4.2. Forecasts for M26726

The best-obtained fitting result of the long- and short-term approaches was reached by using the EHW-init method (figure 5), minimum fitting Theil’s U-statistics value for long-term was 0.23, and for short-term approach was 0.247. If we only consider monthly data, the minimum Theil’s U-statistics was 0.421, obtained by the AHW-init method and long-term approach. For the testing set (figure 6), the minimum Theil’s U-statistics was 0.38, obtained by EHW-init method for quarterly data and the short-term approach. As the U-statistics for monthly and quarterly data, regardless long or short-term approach, were the smallest for the EHW-init method, this method was selected as the best method.

4.3. Forecasts for M26727

According to long and short-term approaches, the EHW-init method was the best fitting method for
monthly and quarterly data (see figure 7). The same conclusions could be made for the testing set (figure 8) if the long and short-term approaches are compared, but we noticed that the best method for monthly data is MHW-init.

4.4. Forecasts for M26730

From figure 9 we observed that, for all six methods, obtained values of Theil’s U-statistics were almost equal for the long and short-term approaches, so the forecasts on the fitting set are equally precise regardless of whether we use a long or short-term approach. In this case, the EHW-init was chosen as the best method. For the testing set (figure 10), the best method for the long-term approach was MHW, and for the short-term approach MHW-init was best.

4.5. Forecasts for M26734

Similar to the previous time series, the EHW-init method was the best method for the fitting set, regardless of the approach (figure 11). An interesting development for the testing set (figure 12) was that the init methods (with the exception of the EHW method) improved forecasts for the short-term approach, and worsened forecasts for the long-term approach. The best method on the test set for the long-term approach was AHW (Theil’s U-statistics was 0.43), and for the short-term approach EHW-init (Theil’s U-statistics was 0.123).

4.6. Forecasts for M26735

The best fitting result for the long- and short-term approaches was again attained by the EHW-init method (figure 13). If the results for the long- and short-term approaches for testing set (figure 14) were compared, the best method for the long-term was MHW, and for the short-term approach was EHW. For monthly data, the minimal Theil’s U-statistics were reached by AHW-init, and for quarterly data by the EHW method.
5. Research findings

In this section, the testing results of monthly and quarterly, long- and short-term forecasting, obtained by described forecasting methods, are presented.

The best forecasting methods of monthly and quarterly data, which were identified as the most appropriate methods for the industrial forecasting implementation, are collected in Table 2. Considering that Theil’s U-statistics was selected as the main performance measure, the AHW, MHW, MHW-init, and EHW-init method were recognised as the best method in general (all these methods appeared 4-times as the best), whereas the EHW method was not very applicable for our application. Furthermore, MHW was the best method for long-term, and MHW-init for short-term monthly and quarterly forecasting. AHW and MHW-init were very appropriate for monthly, but EHW-init was the most effective for quarterly long- and short-term forecasting. Moreover, when we focused on a very specific combination regarding the accumulated data (monthly, quarterly) and forecasting approach (long-, short-term), the best forecasting method for every combination was identified. AHW and MHW were very suitable for monthly-long-term forecasting, MHW-init for the monthly-short-term combination, MHW and EHW-init for quarterly-long-term forecasting, and EHW-init for quarterly-short-term forecasting. Theil’s U-statistics of quarterly data was very often lower than Theil’s U-statistics for a monthly series. A very similar finding may be observed for the short-term forecasting approach, which was much more accurate than a long-term approach.

|       | M26726 | M26727 | M26730 | M26734 | M26735 |
|-------|--------|--------|--------|--------|--------|
| L     | S      | L      | S      | L      | S      |
| M     | AHW    | MHW-init | MHW    | MHW-init | MHW    | AHW    | AHW-init | AHW-init | AHW-init |
| Q     | EHW-init | EHW-init | EHW-init | EHW-init | EHW-init | EHW-init | MHW      | EHW      |

6. Conclusions

This study deals with different forecasting methods, taking into account monthly and quarterly data as well as long- and short-term forecasting approaches. The key results can be summarised as follows:

- MHW was recognised as the best method for long-term forecasting,
- MHW-init was the best method for short-term forecasting,
- AHW and MHW-init were the best methods for monthly data,
- EHW-init was the most suitable forecasting method for quarterly data,
- Theil’s U-statistics of quarterly data was lower than Theil’s U-statistics of a monthly series,
- Short-term forecasting was much more accurate than long-term forecasting.

The result shows that the data format influences the choice of forecasting method. More specifically, if we want to predict quarterly data, we have to use other methods than in cases where we analyse...
monthly data. As was already emphasized, EHW-init was the most accurate forecasting method for quarterly data. Although the forecasting is much more accurate in the case of quarterly data in comparison to monthly data, different forecasting (daily, weekly, monthly, etc.) are very important for short- and long-term planning, which lead to more efficient operations and better business output.

However, long-term forecasting is very important for strategic decision, i.e. production, warehousing, transportation, and logistics in general. In contrast, short-term forecasting help us to manage rapid changes, for instance in consumption choice.

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