Detection Prestress Loss in Prestressed Concrete Slab using Modal Analysis

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Abstract. The paper presents numerical results of prestress loss in the prestressed hollow core slabs by using few natural frequencies. Loss of prestress has been investigated by the 3D finite element method, by using ANSYS software. Loss of prestress is modelled by decreasing of prestressing forces in the 7 wire strand that simulated damage or severing of prestress strands. The prestressing force in the strands is applied as an initial stress. The effectiveness of using frequency percentage changes as an indicator of the loss of prestress and effectiveness of the proposed method are investigated using the planning of experiments and response surface technique. After selection of equation of regression, the parametric studies are carried out in order to determine the influence of the slab length, concrete strength and initial stress on the frequency percentage changes and study the dependence of the percentage changes on these parameters. The present study shows that natural frequencies percentage change is dependent on the span of slab and compressive strength of concrete.

1. Introduction
Currently, the prestressed concrete elements are widely used in many civil constructions for the commercial and industrial buildings. Prestressing force in tendons is used to reduce cross-section and deflections of prestressed concrete structures. However, the prestressing force tends to decrease in the course of time due to time dependent deformation of concrete, steel relaxation in strands, creep of concrete and other factors. It is also known that the loss of the prestress force occurs due to the damage or severing of prestress strands. In this case, the prestressed concrete structure is considered as irreparable as it is seriously damaged upon the condition of the prestressing strands [1].

These losses of prestressing force lead to safety problem of prestressed concrete structures. In this case, the prestress loss estimation is very important because the concrete structure must keep effective prestressing force during the service life.

In the recent years, the development of damage detection techniques based on modal parameters was extensively studied. It is possible to identify damage of structure by comparing typical dynamic properties of the damaged and undamaged structures. This method is being widely used because of its ease of use. Many algorithms for damage detection were developed by many researchers over the three decades in the field of vibration based on damage detection [2, 3].

One of the methods of detection of a possible loss of prestressing force in prestressed concrete structure is the implementation of dynamic monitoring. Loss of prestress can change the stiffness of the structure, which causes changes of dynamic parameters. It is possible to identify the loss of
prestress by comparing frequencies of reference structures and frequencies of the structure with the reduced prestressing forces.

However, there are many disagreements about how the prestressing force in tendon affects natural frequencies of prestressed concrete structure:

1. The natural frequency decreases with the increasing prestressing force [4, 5].
2. The natural frequency increases with the increasing prestressing force [6, 7].
3. The prestressing force is not affected by natural frequency [8, 9].

Law, S. S. and Lu [4] studied the time-domain response of a prestressed Euler-Bernoulli beam under external excitation. Numerical simulation in compare with theoretical findings concluded that frequency decreases when prestressing force increases. Miyamoto et al. [5] studied the influence of the prestressing force introduced to the external tendons on the dynamic behaviour of a prestressed girder. According to the experimental results of dynamic test the formula for calculation of natural frequency was derived. The authors are founded that the natural frequency decreases with increase of the prestressing force.

Jang et al [6] tested post-tensioned concrete beams with bonded tendons. The authors founded an increase of the first natural frequency with increasing of prestressing force. Lu and Law [7] tested prestressed beam with seven-wire straight strand located at the centroid of beam. Test was conducted with and without prestressing force. It was observed that the prestressing force increase the first three natural frequencies. The results were compared with numerical simulation.

Noble et al. [8] tested post-tensioned concrete beam with zero eccentricity. The authors concluded that for a straight profiled prestress strand no change in natural frequencies was observed. Hamed and Frostig [9] developed a nonlinear analytical model for the dynamic behaviour of prestressed beams with bonded and unbounded tendons. Based on the derived governing equations it was founded that the magnitude of prestressed force does not affect the natural frequencies of bonded or unbounded prestressed beams.

Later, Breccolotti [10] analysed different sources of literature related to the problem of prestressed concrete beam. From the preparatory work performed by Brecoloti it was concluded that the increase of the modal frequency and prestressing force true for low values of prestressing level (5–10% of the ultimate compression strength of the section). In the opposite case, the frequency decreases when the prestressing force has higher value.

There are many parameters that affect the natural frequency of the prestressed concrete structure such as geometry, boundary conditions, profile of tendon, prestressing force, location of tendon and etc. It is difficult to consider all these parameters in the full test experiment.

Numerical modelling and research is an effective tool to predict preliminary behaviour of prestressed concrete structure and finding influence of prestressing force on natural frequency. Application of response surface methodology (RSM) allows significantly reduce the number of experiments and improve research effectiveness. The response surface method (RSM) is a combination of the mathematical and statistical methods used for modelling and analysing problems in which the response function depends on some variables and is presented in the form of a regression equation [11].

The aim of this study is to investigate the percentage change of natural frequency in prestressed concrete hollow core slab under various values of prestressing forces that simulate damage level in strand. Loss of prestress is modelled by decreasing of initial stress in the prestressed strand that simulated damage or severing of prestress strands occurred during the process of stretching and anchoring the tendons or other factors. The prestressing force in the strand is applied as an initial stress. The natural frequencies are numerically calculated by the 3D finite element method (FEM), by using ANSYS software. The effectiveness of using percentage changes in natural frequencies as indicators of the loss of prestress has been studied. The solution of this problem is carried out by the method of surface response. Applicability and effectiveness of the proposed method have been investigated by using various lengths of slabs and strength of concrete.
2. Design and methods

This section presents the cross section of the prestressed hollow core slab and finite element modelling. Response surface technique for parametric study is described.

2.1. Design of the prestressed hollow core slab

A typically prestressed hollow core slab that is generally used for flooring was chosen for numerical research. The cross–section dimensions are shown in Figure 1. The slab is 1200 mm in width with standard thicknesses of 200 mm, and the span length is 11000 mm. The slab is prestressed by seven–wire stress-relieved strands of 15 mm diameter located at a distance of 35 mm from the soffit and built by extrusion. The properties of the prestressed slab are outlined in Table 1.

![Figure 1. Cross–section dimensions of reinforced prestressed precast hollow core slab (dimensions in mm)](image)

Table 1. Material properties of concrete and steel strand

| Symbol                  | Concrete | Steel strand | Unit   |
|-------------------------|----------|--------------|--------|
| Young's modulus, $E$    | 4730     | 195          | [GPa]  |
| Poisson's ratio, $\nu$  | 0.2      | 0.3          | –      |
| Density, $\rho$         | 2400     | 7850         | kg/m³  |

where $f_c$ is ultimate compressive strength of concrete [12].

2.2. Finite element model

A three-dimensional (3D) finite element model of the prestressed hollow core slab is simulated by using commercial finite element software ANSYS16.0. Solid65 and Link180 elements are selected to represent concrete and 3D truss element, respectively. Solid65 element has three-dimensional concrete elements with eight nodes and three translational degrees of freedom at each node. The most important aspect of this element is the treatment of nonlinear material properties. This element is capable of plastic deformation, cracking in three orthogonal directions, and crushing. Link180 is suitable for modelling of the prestressing strands in the slab. This 3D spar element is a uniaxial tension-compression element with three translational degrees of freedom at each node and translations in the nodal x, y, and z directions. In the finite element model, Link180 element for the steel reinforcing strand is connected between the nodes of each adjacent concrete solid element and has the same length as the concrete elements along the span length.

The prestressed concrete slab is supported at the two shortest opposite edges. In the first edge, the bottom nodes are restricted in direction along the span length and height of slab. In the second edge, the bottom nodes are restricted only in span length direction. Some model simplifications have been carried out, namely the sidewalls of slab have been aligned. This allows decreasing the dimension of the finite element model. Figure 2 shows the fragment of the finite element mesh.

The procedure for performing a modal analysis of prestressed concrete slab in finite element program ANSYS is concerned by the effect of initial stress on the dynamic response of concrete structure and includes structural and modal analysis. At the first stage, a nonlinear static solution with prestress effect has been performed. At the second stage, the resulting stress field is used in a modal analysis.
The stresses are applied by using the “initial state” (INISTATE) command. This command has the ability to apply the prestressing force to the strand as an initial stress. The tension in the strand is transferred as the compression to the concrete structure through the perfect bond between the concrete and embedded link elements. The stresses are applied to all selected elements of strands and the resulting stress and strain fields are determined. The self-weight of prestressed slab is included in the finite element model and taken into account by providing the value of acceleration due to gravity (9.81 m/s²).

2.3. Response surface technique and plan of experiment

The numerical experiments are designed according to Full Factorial design (FFD). FFD of experiment is the most popular designs owing to their simplicity and relatively low cost. It is very useful for preliminary studies or in initial optimization steps.

The objective of the present study is to establish the functional relationships between research parameters and frequency percentage change. The 4k factorial design is used to supply a relationship for frequency percentage change as mathematical function of research parameters, where \( k \) is the number of parameters and the base 4 represents the level of treatment for each considered parameter \([11]\). The plan of experiments is formulated for 3 parameters, concrete strength (\(X_1\)), initial stress (\(X_2\)), slab length (\(X_3\)), and 64 experiment points. 3D view of the plan of experiments is plotted in figure 3. The levels are reported in Table 1.

![Figure 3. Plan of experiment: 3D–view](image)
Table 2. Research parameters and levels

| Parameters          | Levels |
|---------------------|--------|
| Concrete strength [MPa] | 35     |
|                     | 40     |
|                     | 45     |
|                     | 50     |
| Initial stress [MPa]    | 0      |
|                     | 430    |
|                     | 860    |
|                     | 1290   |
| Slab length [m]        | 7      |
|                     | 8      |
|                     | 9      |
|                     | 10     |

3. Results and discussion

In this section analysis of numerical data by the response surfaces is carried out to investigate an influence of parameters on the natural frequency percentage change.

3.1. Natural frequency percentage change detection

Before numerical study and research of prestressed concrete slab with different values of initial stress and material properties, the finite element analysis is carried out to check validity of loss of prestress detection using natural frequencies.

At the first stage, a numerical model of slab has been calculated to compare the natural bending frequencies and to study the effectiveness of the proposed method for the calculation of the modal frequency percentage change, and to investigate the impact of prestressing on natural frequencies. Loss of prestress is modelled by decreasing the initial stress in the prestressed strand that is located in the centre of slabs. The ultimate tensile strength of steel strands $f_s$ is to 1850 MPa. The strand jacking stress is 70% of the ultimate tensile strength and equal 1290 MPa. The values of initial stress in strand are varying from 1290 MPa to 0 MPa which corresponds to the beginning of damage and loss of prestress, respectively. The strand is prestressed with 4 discrete levels to investigate the variation of natural frequencies. The results of these investigations are listed in Table 2 where the natural frequencies of the 1st and 2nd vertical bending modes are depicted, respectively.

The natural frequency percentage change has been calculated by using the ratio of corresponding natural frequencies for the prestressed concrete slab with the reference initial stress 1200 MPa and with reduced initial stress.

$$\Delta f = \left| \frac{f_i - f_{1290}}{f_i} \right| \times 100$$  (1)

where $f_{1290}$ – the natural frequency of prestressed concrete slab with initial stress 1290 MPa, Hz; $f_i$ – the natural frequency of prestressed concrete slab with reduced initial stress, Hz.

Table 3. Calculated frequencies and frequency percentage change

| Initial stress [MPa] | 1st Mode frequencies [Hz] | Frequency percentage change [%] | 2nd Mode frequencies [Hz] | Frequency percentage change [%] |
|----------------------|---------------------------|---------------------------------|---------------------------|---------------------------------|
| 1290                 | 2.82                      | 0                               | 14.47                     | 0                               |
| 860                  | 2.88                      | 2.1                             | 14.52                     | 0.3                             |
| 430                  | 2.94                      | 4.1                             | 14.57                     | 0.7                             |
| 0                    | 3.00                      | 6.2                             | 14.61                     | 0.1                             |

From the results given in Table 3 it is seen that natural frequencies are increasing with the initial stress in strand. The frequency percentage changes differ significantly depending on the mode shapes. Percentage change for second bending mode does not exceed 0.1%, while the percentage change obtained in the first bending mode gives comparably larger value – 6.2%. Further, the values of the first bending mode shape are used for calculation of frequency percentage change.

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3.2. Parametric study by Response surface method  

The numerical data obtained by the finite element calculations in the points of plan of experiments has been used to build the approximating functions by using the program EduOpt [13]. A third-order polynomial equation is given as follow:

\[
F(x) = 14.185 - 0.66 \cdot X_1 - 2.017 \cdot 10^{-3} \cdot X_2 - 1.469 \cdot 10^{-5} \cdot X_1 \cdot X_1 + 0.013 \cdot X_1 \cdot X_1 + 0.028 \cdot X_1 \cdot X_1 + 3.819 \cdot 10^{-7} \cdot X_2 \cdot X_2 + 8.456 \cdot 10^{-4} \cdot X_2 \cdot X_2 + 0.107 \cdot X_2 \cdot X_2 - 1.083 \cdot 10^{-4} \cdot X_1 \cdot X_1 \cdot X_1 + 1.453 \cdot 10^{-6} \cdot X_1 \cdot X_1 \cdot X_1 + 3.75 \cdot 10^{-4} \cdot X_1 \cdot X_1 \cdot X_1 + 6.550 \cdot 10^{-11} \cdot X_1 \cdot X_1 \cdot X_1 + 6.084 \cdot 10^{-9} \cdot X_1 \cdot X_1 \cdot X_1 + 2.46 \cdot 10^{-5} \cdot X_1 \cdot X_1 \cdot X_1 + 0.005 \cdot X_1 \cdot X_1 \cdot X_1
\]

A third-order polynomial equation has been used, because second-order polynomial equation does not give satisfactory accuracy for research function. This response surface is verified by the finite element solutions in the points different from the points taken in the plan of experiments. Examples of finite element verification of the response surfaces of prestressed hollow core slab are presented in figure 4, where a very good correlation is observed for the approximations and finite element solutions.

![Figure 4](image)

**Figure 4.** Accordance between approximations function and control points

After selection of equation of regression, the parametric studies are carried out study the influence of parameters on frequency percentage change. The view of the response surfaces is shown in Figures 5-6. In these figures, one parameter kept fixed and the other two are varied between the maximum and minimum design bounds.

![Figure 5](image)

**Figure 5.** Dependency on the frequency change: a-strength is 35 MPa; b-strength is 50 MPa;

Figure 5 shows the frequency change dependency on the initial stress in strand and slab length. The data of Figure 5 demonstrate a frequency percentage changes when the strength of concrete is 35 and
50 MPa. It can be seen that the value of the frequency changes significantly linear increases with an increase of stress in strand. The frequency change presents parabolic behaviour when stress in strand is minimal. In this case the frequency change begins to grow with an increase of slab length. The maximum frequency change reaches 6.9% and 5.1% for concrete when strength is 35MPa and 50 MPa, respectively. The maximal difference between two strength of concrete is 1.8%.

The dependence of the concrete strength and initial stress in strand on the frequency change is illustrated in Figure 6. It can be observed that the smallest values of the percentage changes correspond to the shortest length of slab. The maximum frequency change reaches 6.9% when slab length is 10 m. It can be seen that the value of the maximum frequency significantly increases with an increase of the initial stress. In other cases, an increase of the strength of concrete reduces the value of the percentage changes. The difference in the percentage change between two lengths reaches almost 4.6%. It is seen that the modal frequency percentage change is significantly dependent on the length of slabs.

![Figure 6. Dependency on the frequency change: a-slab length is 7 m; b- slab length is 10 m.](image)

It is assumed that the frequency percentage change must exceed 5% in order to detect the loss of prestress. If the difference does not exceed 5%, the detection is impossible. This is related to the fact that the modal frequencies are low at each level of loading. Table 4 shows the initial stress in strand required for loss of prestress detection with frequency percentage change of 5%.

It is seen that the slab of 8, 9 and 10 m in length had the smallest frequency percentage change in comparison with slab of 10 m (less than 5%). The data of Table 4 demonstrate that the prestressed concrete slab with minimal strength of concrete has more capability to detect loss of prestress with a much lower reduction of initial stress in strand than the prestressed concrete slab with the greatest elastic modulus.

| Length of slab [m] | Strength of concrete [MPa] |
|--------------------|---------------------------|
|                    | 35 | 40 | 45 | 50 |
| 7                  | −  | −  | −  | −  |
| 8                  | −  | −  | −  | −  |
| 9                  | −  | −  | −  | −  |
| 10                 | 325| 230| 130| 50 |

4. Conclusion
The present investigation of the frequency percentage change in the prestressed concrete hollow core slab under various values of prestressing forces that simulate damage level in strand has been carried out. The natural frequencies have been numerically calculated by the 3D finite element method
(FEM), by using ANSYS software. The effectiveness of using percentage changes in modal analysis as indicators of the loss of prestress has been studied. Applicability and effectiveness of the proposed method have been investigated by using various span lengths of slabs and strength of concrete. Response surface method has been used for parametric study. Based on the discussion of the results obtained by numerical simulation, the following conclusions can be made:

- The results of the analyses have shown that the modal frequency percentage change is dependent on the span length of the prestressed concrete hollow core slab and strength of concrete.
- The maximal initial stress in strand for the required frequency percentage change with 5% has been determined. It is seen that prestressed hollow core slab with frequency percentage change of 5% is available for structure with minimal strength and maximal length.
- The numerical results of this investigation demonstrate the effectiveness to detect loss of prestressing force in prestressed concrete slabs by using modal frequency percentage change.

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