Quasiparticle current in ballistic $NcS'S$ junctions

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Abstract

Nonstationary properties of ballistic constrictions $NcS'S$ with disordered $S'S$ electrodes are analyzed theoretically. Amplitudes of Andreev and normal reflections at the constriction are related to the solutions of a stationary Green function problem in an inhomogeneous $S'S$ electrode in a dirty limit. This provides a generalization of the model of Blonder, Tinkham and Klapwijk for a spatially inhomogeneous case. The relation between quasiparticle current in $NcS'S$ junctions and energy spectrum of a $S'S$ proximity sandwich is found for arbitrary parameters of $S'$ and $S$ materials and of $S'S$ interface.

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I. INTRODUCTION

Tunnel junctions with high critical current density are presently a subject of extensive experimental investigation (see [1] and references therein). For such junctions the transparency of a tunnel barrier is not small, and therefore they do not fulfill the conditions of a standard tunnel theory. As was shown in [1], barriers in $\text{Nb} - \text{AlO}_x - \text{Nb}$ junctions with high $J_c$ are likely to be a series of constrictions, each having rather large transparency. Therefore, a small ballistic $\text{ScS}$ constriction is a suitable starting point to discuss more complicated models for high $J_c$ junctions.

The properties of $NcS$ constrictions are well understood in the framework of the model of Blonder, Tinkham and Klapwijk (BTK) [2]. In their approach, current through a constriction is fully determined by the amplitudes of normal and Andreev reflections at the $NS$ interface. The model assumes that both $N$ and $S$ metals are in thermal equilibrium and that both are spatially homogeneous. Later, the model was further developed by Klapwijk et al, and Octavio et al [3,4] (KBT and OTBK models) to treat $\text{ScS}$ constriction, in which the subharmonic structure on $I - V$ curves was explained to be due to multiple Andreev reflections at the constriction. Whereas the condition of thermal equilibrium is generally fulfilled for the constriction geometry, the other condition of spatial homogeneity of the superconducting electrodes is less general. An important case of an inhomogeneous system is the $SS'cS'S$ junction, where $S'$ is a superconductor with $T'_{c} < T$, ($T'_c = 0$ corresponds to the particular case of a normal metal).

Previous work on the generalization of the BTK approach to account for spatial inhomogeneity was started by Van Son et al [5]. Andreev reflection in the $NcN'S$ system (where $N'$ is a normal metal) was considered for a gradual variation of a pair potential near the normal metal - superconductor interface. To model Andreev reflection from the $N'$ region with proximity induced superconductivity, the existence of a spatially dependent pair potential $\Delta_{n'}(x)$ was assumed in $N'$. However, whereas the Cooper pair density in a normal region is indeed nonzero due to the proximity effect [6], the pair potential $\Delta_{n'}(x) = 0$, if $T'_c = 0$. 
Therefore a more consistent approach is needed to modify the Andreev amplitudes in the $NcN'S$ sandwich in comparison to the BTK case of $NcS$. Such a theory is necessary, in particular, to interpret experiments on point contact spectroscopy of proximity systems, like the one performed recently on a bilayer consisting of doped $Si$ backed with superconducting $Nb$.

It will be shown in this paper that the existence of a pair potential $\Delta_{n'}(x)$ in $N'$ is not a necessary condition for Andreev reflection at the $NcN'$ boundary. As is known, the spectrum in a normal region is modified due to the proximity effect, namely, bound states exist in $N'$ layer at energies below the energy gap of a superconductor, $\Delta_s$ [8,9]. The energy of the lowest bound state corresponds to an energy gap $\Delta_{gn'}$ induced in $N'$, which for a thin enough layer is close to $\Delta_s$. Therefore it is clear qualitatively that in $NcN'S$ systems Andreev reflection processes take place not only at the $N'S$ boundary, but also at the $NcN'$ boundary, because at $E < \Delta_{gn'}$ a quasiparticle can not penetrate into $N'$. This result however is not directly evident from Bogolubov de Gennes (BdG) equations, where the pair potential $\Delta_{n'}(x)$ plays the role of an effective scattering potential for nondiagonal scattering. To find such a potential for $N'S$ sandwiches, one should either solve a complete three-layer problem in the BdG equations, taking into account both $NN'$ and $N'S$ boundaries simultaneously, or to use the microscopic Green functions approach. For a disordered $S'S$ system the first approach would require knowledge of the full scattering matrix, while the second one is much more straightforward, and we shall use it in this paper to find the nondiagonal potential for tunneling into the $S'S$ bilayer and to calculate the quasiparticle current for the $NcS'S$ contact.

Previously, a microscopic approach based on Green functions formalism was used to study the properties of $NcS$ and $NcN'S$ microcontacts without impurity scattering (clean limit). For the $NcS$ case, Zaitsev [10] has derived boundary conditions for the quasiclassical Eilenberger equations at the contact interface and has calculated the current through the contact at arbitrary transparency of the interface, thus providing a microscopic derivation of BTK model. Independently, the same derivation was done by Arnold [11] with another
method of retarded Green functions. As a particular case, properties of clean $NIN'S$ contacts with arbitrary barrier transparency were considered in [11]. More recently, detailed calculations for the $NcN'S$ case were done in the framework of the Arnold model in [12] with the additional assumption of small transparency barrier between $N'$ and $S$ layers.

In a case of strong disorder (dirty limit) nonequilibrium aspects got a lot of attention recently. In [13–16] the conductivity of a dirty $NN'S$ contact was studied in the model, where $N$ and $S$ electrodes are reservuars with fixed electrical potentials whereas the disordered contact region $N'$ is a one-dimensional bridge in nonequilibrium. Such a situation can be realized in disordered $NS$ contacts when the $N'$ layer simulates the interface region of the order of the inelastic mean free path. An enhancement of the zero bias conductance $\sigma(0)$ was predicted in this system.

In the present paper we consider a physically different situation: a ballistic constriction $NcS'S$ of the size smaller than the mean free path having disordered electrodes. In this case the potential drop takes place at the constriction, and therefore the electrodes are in thermal equilibrium. The ballistic condition is important. It should be compared with the opposite limit of disordered constriction of a size larger than a mean free path. The latter case was studied theoretically by Artemenko, Volkov and Zaitsev [17] for $ScS$ constrictions and more recently by Volkov [18] for $NcN'S$ constrictions with large transparency. It was obtained, in particularly, that due to disorder no conductance doubling is present at zero bias $V = 0$. As is shown in this paper, the conductance of the ballistic $NcN'S$ constriction is different in many respects as zero bias conductance doubling is being present for large transparency. The influence of the finite transparency of a constriction is also studied. Physically, our approach is closely related to the BTK one. We will give expressions for coefficients of Andreev and ordinary electron reflections at the ballistic constriction and their relation to the energy spectrum of the disordered $SS'$ system that is investigated in terms of Green functions. As a result, the simplicity and direct physical meaning of the BTK solutions are combined with the approach based on the selfconsistent solution of a stationary dirty limit Green functions problem.

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II. THE MODEL

Let us consider the boundary between $N$ and $S'S$ as a small constriction of size $a \ll \min(l_n, l_{s'})$, where $l_n, l_{s'}$ are mean free paths of $N$ and $S'$. The constriction is characterized by a transmission coefficient

$$D = \frac{4v_1 x v_2 x (v_1 x + v_2 x)^2 + 4H^2}{(v_1 x + v_2 x)^2 + 4H^2}$$

where $H$ is strength of repulsive potential $H\delta(x)$ located at the $NS'$ interface and $v_1 x, v_2 x$ are the components of the Fermi velocities of $N$ and $S'$ normal to the interface, respectively. We assume that $S'$ and $S$ metals are in the dirty limit $l_{s',s} \ll \xi_{s',s}$, whereas in $N$ there is no limitation on the mean free path.

Let us follow the BTK notations. In the BdG equation formalism, the excitations are represented by a vector $\psi = \begin{pmatrix} f(x) \\ g(x) \end{pmatrix}$, where $f(x)$ describes electron-like excitations in a superconductor, and $g(x)$ describes hole-like (time-reversed) excitations. The BdG equations have the form:

$$i\hbar \frac{\partial f}{\partial t} = \left( -\frac{\hbar^2 \nabla^2}{2m} - \mu(x) + V(x) \right) f(x) + \Delta(x) g(x, t)$$
$$i\hbar \frac{\partial g}{\partial t} = -\left( -\frac{\hbar^2 \nabla^2}{2m} - \mu(x) + V(x) \right) g(x) + \Delta(x) f(x, t)$$

where $\mu(x), \Delta(x)$ and $V(x)$ are the electrochemical potential, the pair potential, and the ordinary potential, respectively.

In the absence of impurity scattering, the transmission and reflection coefficients for a quasiparticle incident from clean $N$ to clean $S$, are found in the BTK model by considering incoming, reflected and transmitted waves near the $NS$ boundary:

$$\psi_{inc} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq_1^+ x}, \quad hq_{1,2}^\pm = \sqrt{2m_{1,2}(\mu \pm \epsilon)}$$
\[ \psi_{\text{refl}} = a \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\tilde{q} x} + b \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\tilde{q} x} \]

\[ \psi_{\text{trans}} = c \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{i\tilde{q}_2 x} + d \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} e^{-i\tilde{q}_2 x} \] (4)

\[ 1 - v_0^2 = u_0^2 = \frac{1}{2} \left( 1 + \sqrt{\epsilon^2 - \Delta^2 / \epsilon} \right) \] (5)

where \( \Delta \) is a the bulk energy gap of \( S \), \( m_{1,2} \) are effective masses of the contacting metals and \( \epsilon \) is the quasiparticle energy.

Matching these solutions at the NS boundary, one can find the energy-dependent Andreev reflection coefficient \( A(\epsilon) \) and normal reflection coefficient \( B(\epsilon) \):

\[ A(\epsilon) = a a^* = \frac{|\eta|^2}{(1 + Z^2(1 - |\eta|^2))^2}, \]

\[ B(\epsilon) = b b^* = \frac{Z^2(1 + Z^2)(1 - |\eta|^2)^2}{(1 + |\eta|^2 Z^2)^2} \] (6)

\[ \eta(\epsilon) = v_0(\epsilon) \]

\[ \frac{u_0(\epsilon)}{u_0(\epsilon)} = \frac{\Delta / \sqrt{\epsilon^2 - \Delta^2}}{1 + E / \sqrt{\epsilon^2 - \Delta^2}} \] (7)

where \( Z \) is related to normal transmission coefficient \( D \) by: \( 1 + Z^2 = D^{-1} \). It is important to note that both \( A(\epsilon) \) and \( B(\epsilon) \) are controlled by one energy-dependent parameter \( \eta(\epsilon) \), \( |\eta|^2 \) being simply the ratio of probabilities for the excitation to be in hole-like, \( |v_0|^2 \), or in electron-like, \( |u_0|^2 \), states.

The solutions (5) for \( u_0, v_0 \) are applicable for a clean homogeneous BCS superconductor, and they are not valid in spatially inhomogeneous dirty \( S'S' \) sandwiches. To find the coefficients \( A(\epsilon) \) and \( B(\epsilon) \) in the latter case one needs to calculate \( \eta(\epsilon) \) as a solution of Gor’kov equations (GE) in the \( SS' \) system. For the ballistically clean constriction we can take the advantage that at a length scale smaller than the electron mean free path the solution of the GE can be written in the form of a combination of plane waves.

The GE in the region near the constriction, \( S' \), have the form [19]:

\[ \begin{cases} 2m_2(\epsilon'(x) + \mu) + \frac{\partial^2}{\partial x^2} \right) G_e(x, x') + 2m_2 \Delta'(x) F_e^*(x, x') = \delta(x - x') \\
2m_2(\epsilon'(x) - \mu) - \frac{\partial^2}{\partial x^2} \right) F_e(x, x') + 2m_2 \Delta'(x) G_e(x, x') = 0 \end{cases} \] (8)
Here \( E' \) and \( \Delta'(x) \) are the energy and pair potential renormalized by impurity scattering according to

\[
\epsilon'(x) = \epsilon + i \langle G_{\epsilon}(x, x) \rangle / 2\tau, \quad \Delta'(x) = \Delta(x) + \langle F_{\epsilon}(x, x) \rangle / 2\tau,
\]

\( \tau = (2\pi cV^2N(0))^{-1} \) is scattering time, \( c \) and \( V \) are impurity concentration and scattering potential, respectively. The brackets \( \langle ... \rangle \) denote angle averaging, and \( G_{\epsilon}(x, x') \) and \( F_{\epsilon}(x, x') \) are the normal and anomalous Green functions in energy representation.

The pair potential in (8) and (9) is given by the selfconsistency equation:

\[
\Delta(x) = gT \sum_{w_n} F_{\epsilon=-iw_n}(x, x)
\]

where \( g \) is the coupling constant and \( \omega_n = \pi T(2n + 1) \) is the Matsubara frequency. Note that in the particular case of \( T'_c = 0 \) the pair potential in \( S' \) is zero, \( \Delta(x) = 0 \), whereas \( F_{\epsilon}(x, x) \) is finite.

We note that \( \langle G_{\epsilon}(x, x) \rangle \) and \( \langle F_{\epsilon}(x, x) \rangle \) are quasiclassical Green functions of a dirty superconductor. They obey diffusion-like equations \([20][21]\) with the boundary conditions derived in \([22]\).

Let us consider the solutions of the GE in the \( S' \) region at distances less than \( \xi_{s'} \) from the constriction. In this region one can neglect variations of \( \Delta'(x) \) and \( \epsilon'(x) \) and to write the solutions of the linearized equations as a combination of plane waves:

\[
\begin{pmatrix}
G_{\epsilon}(x, x') \\
F_{\epsilon}(x, x')
\end{pmatrix} = C(x') \begin{pmatrix}
g(x) \\
f(x)
\end{pmatrix} e^{iq_2^+ x} + D(x') \begin{pmatrix}
f(x) \\
g(x)
\end{pmatrix} e^{-iq_2^- x},
\]

where the slowly varying functions \( g(x), f(x) \) determine amplitudes of electron like and hole like excitations. Substitution of the solution eq.(11) to the GE leads to the following linear system of equations for \( g(x) \) and \( f(x) \):

\[
L \begin{pmatrix}
g(x) \\
f(x)
\end{pmatrix} \equiv \begin{pmatrix}
2m_2 [\epsilon + i \langle G_{\epsilon}(x) \rangle / 2\tau + \mu] - q_2^2 & 2m_2 [\Delta + \langle F_{\epsilon}(x) \rangle / 2\tau] \\
2m_2 [\Delta + \langle F_{\epsilon}(x) \rangle / 2\tau] & 2m_2 [\epsilon + i \langle G_{\epsilon}(x) \rangle / 2\tau - \mu] + q_2^2
\end{pmatrix} \begin{pmatrix}
g(x) \\
f(x)
\end{pmatrix} = 0.
\]

\( L \)
Here $q_2^\pm$ is determined by the dispersion relation $DetL = 0$, which in the dirty limit $\Delta \tau \ll 1$ leads to the following result

$$q_2^\pm = \sqrt{2m_2 \left[ \mu \pm i \sqrt{\langle G_\epsilon(x) \rangle^2 + \langle F_\epsilon(x) \rangle^2} \right]}.$$ \hspace{1cm} (13)

Eq.(11) describes transmitted electron and transmitted hole waves in the $S'$ region near the constriction. Note that, in general, reflected electron and reflected hole waves are also present in $S'$ due to impurity scattering. However, the condition of a ballistic constriction, $a \ll l_{s'}$, made it possible to neglect these waves in eq.(11), because under this condition waves in $S'$ are scattered diffusively away from the contact, and there is a small probability for a wave, scattered at the distance $l_{s'}$ from the constriction, to reach it again.

To find transmission and reflection probabilities for a given problem, one should compare the solutions (4) and (11) at distances smaller than mean free path $l_{s'}$ from the constriction. In this region one can neglect the small energy terms in (3) and (13) in comparison with chemical potential $\mu$, i.e. set $q_2^+ = q_2^- = \sqrt{2m_2 \mu}$ in the phases of all transmitted waves. Then, to find the probabilities $A(\epsilon)$ and $B(\epsilon)$ one should find the ratio $f(x)/g(x)$. For the dirty limit we obtain from eq.(12):

$$\frac{f(x)}{g(x)} = \frac{i \langle F_\epsilon(x) \rangle}{1 + \langle G_\epsilon(x) \rangle}.$$ \hspace{1cm} (14)

Finally, substituting $\eta(\epsilon)$ in eqs.(6),(7) by $f(x)/g(x)$ from eq.(14) and using the normalization condition for the Green functions $\langle G_\epsilon^2 \rangle + \langle F_\epsilon^2 \rangle = 1$, we find that the Andreev and normal reflection coefficients are directly related to the local energy spectrum of $S'$ near the constriction:

$$A(\epsilon) = \frac{|\langle F_\epsilon(0+) \rangle|^2}{|1 + 2Z^2 + \langle G_\epsilon(0+) \rangle|^2},$$ \hspace{1cm} (15)

$$B(\epsilon) = \frac{4Z^2(1 + Z^2)}{|1 + 2Z^2 + \langle G_\epsilon(0+) \rangle|^2},$$ \hspace{1cm} (16)

where $\langle G_\epsilon(0+) \rangle$ and $\langle F_\epsilon(0+) \rangle$ are quasiclassical Green functions in the vicinity of the contact in $S'$. The expressions (15) and (16) are the central result of the paper. They generalize the
corresponding BTK relations \[2\] for a spatially inhomogeneous case. For the latter case the Green functions are given by 
\[
\langle G_\epsilon(0+) \rangle = -i\epsilon/\sqrt{\Delta_0^2 - \epsilon^2}, \quad \langle F_\epsilon(0+) \rangle = \Delta_0/\sqrt{\Delta_0^2 - \epsilon^2},
\]
and it is easy to check, that the BTK relations follow from eqs.(15), (16).

The coefficients \(A(\epsilon)\) and \(B(\epsilon)\) given by eqs.(15,16) fully determine the quasiparticle current through \(NcS'S\) contact. Namely under the condition of thermal equilibrium in both electrodes the current through the constriction is expressed via \(A(\epsilon)\) and \(B(\epsilon)\) in complete analogy with the BTK result for the \(NcS\) constriction \[2\] :

\[
I(V) = \frac{R_0^{-1}}{(1 + Z^2)} \int_{-\infty}^{+\infty} \left[ f_0(\epsilon + eV) - f_0(\epsilon) \right] [1 + A(\epsilon) - B(\epsilon)] d\epsilon \tag{17}
\]

where \(R_0 = [2N_1(0)Se^2v_{F1}]^{-1}\) is the Sharvin resistance, \(S\) is contact area, \(N_1(0)\) and \(v_{F1}\) are the density of states per spin and Fermi velocity of \(N\) electrode, respectively (we have taken \(v_{F1} < v_{F2}\)). \(f_0\) is the Fermi distribution function and the brackets \(\langle \ldots \rangle\) denote angular averaging. The relations (15) and (16) show, that in the dirty limit local information is given by point contact measurements. In particular, the local density of states near the constriction is given in the usual way as \(N(\epsilon) = Re \{\langle G_\epsilon(0+) \rangle\}\). It is seen however from eqs.(15)-(17), that generally (for arbitrary \(Z\)) the current is not determined solely by \(N(\epsilon)\) like in a tunnel theory, but rather a crossover to the limit of a tunnel \(NIS'S\) junction takes place for small transparencies, like in \(NcS\) contacts \[2\]. We note, that the case of \(SS'S'cS'S\) ballistic constrictions can be considered in the same way, by extension of the OTBK model \[3\] with the help of eqs.(15), (16) for \(A(\epsilon)\) and \(B(\epsilon)\) \[23\].

Therefore, the problem is reduced to calculation of the functions \(\langle G_\epsilon(0+) \rangle\) and \(\langle F_\epsilon(0+) \rangle\) for the dirty \(SS'\) sandwich.

### III. PROXIMITY EFFECT IN THE DIRTY \(SS'\) SANDWICH

The proximity effect in dirty \(S'S\) sandwiches was studied previously in \[22,24\] for the case of arbitrary transparency of the \(S'/S\) interface, where the case of a thin \(S'\) layer was considered. Now we first generalize the results of \[22,24\] to arbitrary \(S'\) layer thickness and then, using these solutions, calculate the quasiparticle current of a \(NcS'S\) contact.
The angle averaged quasiclassical Green functions in the dirty $S'S$ bilayer satisfy the equation:

$$i\varepsilon F_{s',s}(x) + \frac{D_{s',s}}{2} \left( G_{s',s}(x) \frac{\partial^2 F_{s',s}(x)}{\partial x^2} - F_{s',s}(x) \frac{\partial^2 G_{s',s}(x)}{\partial x^2} \right) - \Delta_{s',s}(x) G_{s',s}(x) = 0,$$

where we have omitted the angular brackets $\langle ... \rangle$ for the functions $G_{s',s}, F_{s',s}$ in $S'$ and $S$ regions respectively. Here $D_{s',s}$ is diffusion coefficient, $\Delta_{s',s}$ is the order parameter. To be more specific, we shall discuss below the particular case of $\Delta_{s'} = 0$, i.e. of $T_{c'} = 0$ ($NS$ sandwich). The generalization to the case of nonzero $T_{c'}$ is straightforward \cite{25} and does not change our results qualitatively. The latter case will be discussed separately elsewhere.

It is convenient to rewrite eq.(18) using the notations $G(\epsilon, x) = \cos \theta(\epsilon, x), F(\epsilon, x) = \sin \theta(\epsilon, x)$. Then eq.(18) takes the following form in $N$ and $S$ regions:

$$\xi_{N,S}^2 \theta''_{N,S}(x) + i\epsilon \sin \theta_{N,S}(x) + \Delta_{N,S}(x) \cos \theta_{N,S}(x) = 0,$$

with the boundary conditions at the $NS$ interface ($x = 0$) \cite{22}:

$$\gamma_B \xi_{N} \theta'_{N} = \sin(\theta_{S} - \theta_{N}),$$

$$\gamma \xi_{N} \theta'_{N} = \xi_{S} \theta'_{S},$$

in the bulk of the $S$—layer

$$\theta_{S}(\infty) = \arctan(i\Delta_0(T)/\epsilon),$$

as well as at the $N$ metal free surface ($x = -d_N$):

$$\theta'_{N}(-d_N) = 0.$$

The selfconsistency equation for the order parameter in $S$ region has the form:

$$\Delta_{s}(x) \ln T_{c} + 2 T_{c} \sum_{n} \left[ \frac{\Delta_{s}(x)}{\omega_{n}} - \sin \theta_{s}(x, \epsilon = i\omega_{n}) \right] = 0.$$

The parameters $\gamma_B$ and $\gamma$

$$\gamma_B = \frac{R_B}{\rho_N \xi_N}, \gamma = \frac{\rho_S \xi_S}{\rho_N \xi_N}$$

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have simple physical meanings: $\gamma$ is a measure of the strength of the proximity effect between the $S$ and $N$ metals, whereas $\gamma_B$ describes the effect of the boundary transparency between these layers. Here $\rho_{N,S}, \xi_{N,S} = \sqrt{D_{N,S}/2\pi T_c}$ and $D_{N,S}$ are normal state resistivities, coherence lengths and diffusion constants of $N$ and $S$ metals, respectively, while $R_B$ is the product of the resistance of the $NS$ boundary and its area. We have normalized $\epsilon$ and $\Delta(x)$ to $\pi T_c$, where $T_c$ is the critical temperature of the bulk $S$.

Previously selfconsistent solutions of the boundary value problem (19)-(24) for the $NS$ sandwich at arbitrary values of $\gamma$ and $\gamma_B$ were studied only for a thin $N$ layer, $d_N \ll \xi_N$, in \cite{24-26}. In particular, the densities of states in $N$ and $S$ layers $N_{N,S}(\epsilon) = \text{Re} \left( \cos \theta_{N,S}(\epsilon) \right)$ were discussed. It was shown for this case that for any values of $\gamma$ and $\gamma_B$ a superconducting state is induced with finite energy gap $\Delta_{gN}$. When both layers are thin $d_{N,S} \ll \xi_{N,S}$, the results of the well known McMillan tunnel model \cite{27} of the proximity effect can be reproduced \cite{26}. Then, in the language of the McMillan model, the existence of a finite gap in the considered diffusive limit is due to finite inverse lifetime of quasiparticles in thin $N$ layer, $\Gamma_N \equiv \tau_N^{-1} \sim (\hbar v_{FN}/2d_N)D$, where $D$ is the angle averaged $NS$ boundary transparency. As a result, there is a finite minimal energy of quasibound states in $N$ and therefore, a nonzero energy gap is induced. For finite thickness $d_N$ the densities of states in a dirty $NS$ sandwich were discussed previously only in the framework of rigid boundary conditions, i.e. for $\gamma/\gamma_B \ll 1$, in \cite{13-15}.

In the general case of arbitrary $d_N, d_S, \gamma$ and $\gamma_B$ the solutions should be determined selfconsistently together with the spatial dependence of the order parameter from the selfconsistency equation. The results depend essentially on the parameters $\gamma, \gamma_B$, but from lifetime considerations it is clear qualitatively, that the above conclusion about a finite gap should hold also for any finite thickness of $N$. Below we will calculate the density of states in $N$ from the solution of eqs.(19)-(24) and will demonstrate the reduction of the gap in $N$ with increase of $d_N$.

Taking advantage of the condition $\Delta_N = 0$ one can integrate the eq.(19) in $N$ region and with the help of boundary conditions (20) obtain
\[ \cos \theta_N(0) - \cos \theta_N(-d_N) = \frac{\sin^2(\theta_S(0) - \theta_N(0))}{2i \epsilon \gamma_B}. \] (25)

The analytic solution of eq. (25) is simplified only in a limiting case of small \( \gamma/\gamma_B \) ratio, when one can take \( \theta_S(0) = \theta_S(\infty) \) in the first approximation. Then for sufficiently low energies \( \epsilon \ll 1 \), it follows from (25) that \( \theta_N(0) \simeq \theta_S(0) = \arctan(i \Delta_0/\epsilon) \), and as a result we obtain the BCS expression with a gap \( \Delta_0 \) for the density of states in this energy range:

\[ N_N(\epsilon,0) = \text{Re} \left( \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta_0^2}} \right). \]

For large \( N \) layer thickness, \( d_N \gg \xi_N \), one can substitute \( \cos \theta_N(d) = 1 \) in eq.(25) and find the asymptotic behavior of \( N_N(\epsilon) \) at the NS boundary:

\[ N_N(\epsilon) = \sqrt{\frac{\epsilon}{\pi T_c} \gamma_B}, \quad \frac{\epsilon}{\pi T_c} \gamma_B \ll 1. \] (26)

For the case of arbitrary \( N \) layer thickness and arbitrary values of the parameters \( \gamma, \gamma_B \) the boundary value problem (19)-(24) was solved numerically. The results of the calculations of the densities of states in \( N \) at \( x = 0 \) and \( x = -d_N \) at low temperatures \( T \ll T_c \) are presented in Figs.1 and 2 for a number of \( d_N/\xi_N \) ratios. As is seen from Fig.1 at \( x = -d_N \) (free surface of \( N \)) two peaks exist in \( N(\epsilon) \) for small \( \gamma \) values, provided that \( d_N/\xi_N \leq 1 \). The first peak is at \( \epsilon = \Delta_{gN} \) and the second one at \( \epsilon = \Delta_0 \). It is also seen from comparison of Figs.1 and 2, that the second peak at \( \epsilon = \Delta_0 \) is smeared out with the increase of \( d_N \) as well as with the increase of \( \gamma \). On the other hand, the peak at \( \epsilon = \Delta_{gN} \) becomes more pronounced at large \( d_N \). Dotted lines show the behavior of \( N(\epsilon) \) in \( N \) at the boundary with \( S \) \( (x = 0) \). The asymptotic behavior given by eq.(26) should take place at large \( d_N \). It is important to note, that the energy gap \( \Delta_{gN} \) is preserved for all \( d_N \), going to zero rather slowly. This is consistent with the qualitative picture of the gap being proportional to inverse lifetime in \( N \), which in the considered diffusive approximation is given by \( \tau_N^{-1} \sim (\hbar D_N/d_{N}^2)D \). The results of study of the dependence of \( \Delta_{gN} \) on the parameters of the \( NS \) sandwich will be presented in more detail elsewhere.

Using the solutions \( \theta_N(\epsilon,-d_N) \), one can calculate the reflection coefficients \( A(\epsilon) \) and \( B(\epsilon) \) and then the quasiparticle current for the \( NcNS \) contact from eqs.(15)- (17). To be more specific, let us discuss here the case of a thin \( N \) layer, \( d_N/\xi_N \ll 1 \). Then the
number of parameters is reduced from $\gamma, \gamma_B$ and $d_N$ to the following set: $\gamma_m = \gamma d_N/\xi_N$ and $\gamma_{BN} = \gamma_B d_N/\xi_N$ [22].

Fig. 3 shows the results of the calculations of the reflection coefficients $A(\epsilon)$ and $B(\epsilon)$ for a number of $Z$ values. It is seen that for $Z \neq 0$ a characteristic two-peak structure exists for both $A(\epsilon)$ and $B(\epsilon)$, which is directly related to the two-peak structure of $N_N(\epsilon)$ discussed above, the first peak being at $\epsilon = \Delta g_N$ and the second one at $\epsilon = \Delta_0$.

The zero-temperature conductance of $NcNS$ junctions calculated according to eq.(17) is shown in Fig.4 for the same parameters. Again the two-peak structure is present in $dI(V)/dV$ at voltages $\Delta g_N$ and $\Delta_0$. In accordance with the arguments given above, the peak at $\Delta_0$ would be smeared out quite easily by any pair-breaking process, i.e. by large $\gamma_m$ values (spatial gradients in $S$), or large $d_N$. Then such a spectroscopy will show only a proximity induced energy gap in $N$ with almost no signatures of $\Delta_0$. The position of the first peak at $eV = \Delta g_N$ can be used to study properties of the $NS$ interface for any given material combination. The appearance of such a conductance peak at low bias for $Z \neq 0$ is a consequence of the given model, which follows directly from the structure of the densities of states in the $N$ region at $x = -d_N$, as is seen from Figs.1 and 2.

IV. CONCLUSIONS

In conclusion, the BTK model is generalized for a spatially inhomogeneous case of $NcS'S$ ballistic constrictions with disordered $S'S$ electrodes. The expressions for the amplitudes of Andreev and normal reflection are given, which allow to calculate a quasiparticle current for arbitrary parameters of $S'$ and $S$ materials and their interface, if the conditions of the dirty limit are fulfilled. An energy gap in $S'$ is always present, even for finite thickness of the $S'$ layer. The magnitude of this gap is studied as a function of the parameters of the $S$ and $S'$ materials, as well as of the transparency of the $SS'$ interface. It is shown, that the conductance of ballistic $NcS'S$ junctions reflects proximity induced energy gap in $S'$ and, under certain conditions, also the bulk gap of the superconductor $S$. 
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FIGURES

FIG. 1. The densities of states in the $N'$ layer of the $N'S$ sandwich, normalized to their normal-state values, at the free surface (solid lines) and at the $N'S$ boundary (dashed lines) for different thicknesses $d_N/\xi_N = 10$ (curve 1), 2 (2), 1 (3) and 0.5 (4).

FIG. 2. The densities of states in the $N'$ layer of the $N'S$ sandwich, normalized to their normal-state values, at the free surface (solid lines) and at the $N'S$ boundary (dashed lines). Thicknesses $d_N/\xi_N$ are the same as in Fig.1.

FIG. 3. Probabilities of Andreev reflection, $A(\epsilon)$ (solid lines), and of normal reflection, $B(\epsilon)$ (dashed lines), for the ballistic $NcN'S$ constriction with $\gamma_m = 0.1$ and $\gamma_{BN} = 1$.

FIG. 4. Zero-temperature conductance of the ballistic $NcN'S$ constriction with the same parameters as in Fig.3.