Global stability of a stochastic Beddington-DeAngelis type predator-prey model with time delay and stage structure for prey incorporating refuge

Yanfang Fu¹ and Kai Zhou², ³
¹School of Natural Resources and Environment, Chizhou University, Chizhou, Anhui 247000, P. R. China
²School of Mathematics and Computer, Chizhou University, Chizhou, Anhui 247000, P. R. China
³Email: zk1984@163.com

Abstract. In this paper, we first propose a stochastic Beddington-DeAngelis type predator-prey model with time delay and stage structure for prey incorporating refuge. Then, we obtain the existence and uniqueness of the global positive solution for this model. By constructing proper Lyapunov function, we establish the sufficient criteria for the global stability of the positive equilibrium point. At the last, we performed some numerical simulations to verify the main results.

1. Introduction

There has been an increasing interest in population ecological models since the 1970s. Among these models, predator-prey systems are frequently proposed in the literature and have been studied widely by both applied mathematicians and ecologists. During these different models, the relationship between prey and predator is important. Various kinds of predator-prey models with different relationships, such as Holling type, ratio-dependent type and Leslie-Gower type etc., have been investigated extensively by the scholars [1-3]. Especially, Cantrell and Cosner [4] considered the following Beddington-DeAngelis type predator-prey system,

\begin{align}
    x'(t) &= x \left( b_1 - a_{11} x - \frac{a_{12} y}{1 + \beta x + \gamma y} \right), \\
    y'(t) &= y \left( \frac{a_{22} x}{1 + \beta x + \gamma y} - b_2 \right),
\end{align}

where \( x(t) \) and \( y(t) \) denote the prey’s and the predator’s total populations at time \( t \), respectively; and the coefficients are positive, which have the meanings below: \( b_1 \) denote the birth/death rate, \( a_{11} \) is the overcrowding rate, \( a_{12}, a_{22} \) are positive constants that describe the capture rate from each other population, \( \beta \) denotes the handling time on the feeding rate, \( \gamma \) is a constant describing the magnitude of interference among predators.

In the natural world, the growth process of many species is divided into two stages: one is the immature stage and another one is mature stage of their life, and each stage has different kinds of growth characters. Especially for some mammal preys, this division is reasonable since that the immature prey...
is not at risk of being attacked by the predator [5]. On the other hand, time delay seems unavoidable in actual models. For instance, Li and Li [6] considered the gestation of the predator and studied the predator-prey system with stage-structure for prey and time delay

\[
\begin{align*}
\dot{x}_1(t) &= ax_1(t) - r_1 x_1(t) - bx_1(t), \\
\dot{x}_2(t) &= bx_1(t) - r_2 x_2(t) - b_2 x_2(t) - \frac{a x_1^2(t) y(t)}{1 + mx_2^2(t)}, \\
y'(t) &= \frac{a x_1^2(t - \tau) y(t - \tau)}{1 + mx_2^2(t - \tau)} - ry(t).
\end{align*}
\]

Recently, many scholars have considered the species incorporating refuge in the dynamics of prey-predator model. Many researchers [7, 8] have paid attention to the prey-predator model with prey incorporating refuge, and mainly studied the impact of prey refuge on the dynamical properties of predator and prey species. Furthermore, the nature world is full of environmental noise and time delays, which are important components in an ecosystem, and many authors have studied delayed stochastic models in the predator-prey system, such as [9-11].

Combining the factors mentioned above, we propose the following stochastic predator-prey model with time-delay and stage structure for prey incorporating refuge

\[
\begin{align*}
\dot{x}_1 &= \left( ax_2 - r_1 x_1 - a_1 x_1^2 - bx_1 \right) dt + \sigma_1 x_1(x_1 - x_1^*) dB_1(t), \\
\dot{x}_2 &= \left[ bx_1 - r_2 x_2 - a_2 x_2^2 - \frac{a_1(1-m)x_1 y}{1 + a_1(1-m)x_2 + a_2 y} \right] dt + \sigma_2 x_2(x_2 - x_2^*) dB_2(t), \\
\dot{y} &= \left[ -r_2 + \frac{a_2(1-m)x_2(t - \tau)}{1 + a_2(1-m)x_2(t - \tau) + a_2 y(t - \tau) - a_3 y} \right] dt + \sigma y(y - y^*) dB(t),
\end{align*}
\]

where \(x_1(t), x_2(t)\) and \(y(t)\) denote the densities of immature prey species, mature prey species and predator species at time \(t\) respectively, \((x_1^*, x_2^*, y^*)\) is the positive equilibrium state of the homologous deterministic system for (2). For biological meaningful, we assume that the coefficients are positive and mention the meanings of coefficients below: \(a\) is the birth rate of immature prey species proportioned to the existing mature prey species; \(b\) is the transformation rate from the immature prey to mature prey; \(r_1, r_2, r\) denote the death rates of immature prey, mature prey and predator species respectively; \(a_i, (i=1,2,3)\) denote the overcrowding rate of immature prey, mature prey and predator species; \(a_i(1-m)\) is the capturing rate of the predator, while \(\frac{a_2}{a_3}\) is the conversion rate of nutrients into the production of the predator; \(\tau\) is the time delay; \(m\in[0,1)\) is refuge rate to prey, and \((1-m)x_2\) leaves the prey species available to the predator species. In practice, we usually estimate the death rates \(r_1, r_2, r\) by an average value plus an error term, which obeys a normal distribution and is dependent on how much the current population sizes differ from the equilibrium state (see e.g. [12]). Therefore, we replace the rates \(r_1, r_2, r\) by an average value plus a random fluctuation term \(\sigma_i (x_i - x_i^*) B_i(t)\) , \(r + \sigma (y - y^*) B(t)\), where \(\sigma_i, \sigma\) represent the intensities of the noises, \(B_i(t), B(t)\) are independently standard Brownian motions, and obtain the model (2) above. We will give the global stability result for system (2) in the next section.
2. Main results

For the significance of biology and the verification of global stability, we should firstly give the existence and uniqueness of the global positive solution for system (2).

**Lemma 2.1.** For system (2), if \( \sigma_1, \sigma_2, \sigma > 0 \), then there is a unique positive local solution \( (x_1(t), x_2(t), y(t)) \) on \([0, \tau)\) almost surely (a.s.) for any initial value \( x_{10}, x_{20}, y_0 \in C([-\tau, 0], \mathbb{R}^+) \), where \( C([-\tau, 0], \mathbb{R}^+) = \{\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \phi_3) : \|\phi_3\| = \max_{\theta \in [-\tau, 0]} \|\phi(\theta)\| \} \) is a continuous function space.

The proof of this lemma is similar to that in Liu and Wang [13], so we omit it here.

**Lemma 2.2.** For any given initial value \( x_{10}(\theta) > 0, x_{20}(\theta) > 0, y_0(\theta) > 0, \theta \in [-\tau, 0] \), system (2) has an unique global solution \( (x_1(t), x_2(t), y(t)) \) and it will remain in \( \mathbb{R}^+ \) with probability one.

**Proof.** Choose a positive number \( k_0 \) large enough such that \( x_{10}, x_{20}, y_0 \) lying within the interval \( \left[ \frac{1}{k_0}, k_0 \right] \). For every integer \( k > k_0 \), define

\[
\tau_k = \inf \left\{ t \in [0, \tau) : x_1(t) \notin \left( \frac{1}{k}, k \right) \text{ or } x_2(t) \notin \left( \frac{1}{k}, k \right) \text{ or } y(t) \notin \left( \frac{1}{k}, k \right) \right\}.
\]

It is obvious that \( \{\tau_k\} \) is an increasing sequence, and we let \( \tau_\infty = \lim_{k \to \infty} \tau_k \). Then \( \tau_\infty \leq \tau_\infty \) a.s. To show the existence of the global solution, it suffices to verify \( \tau_\infty = \infty \). If this statement is false, there are constants \( T > 0, \varepsilon \in (0, 1) \) such that \( P\{\tau_\infty \leq T\} > \varepsilon \). Thus, there exists an integer \( k_1 \geq k_0 \) such that \( P\{\tau_k \geq T\} \geq \varepsilon, k \geq k_1 \). Define

\[
V(x_1, x_2, y) = \left( \sqrt{x_1} - 1 - 0.5 \ln x_1 \right) + \left( \sqrt{x_2} - 1 - 0.5 \ln x_2 \right) + \left( \sqrt{y} - 1 - 0.5 \ln y \right).
\]

If \( (x_1(t), x_2(t), y(t)) \in \mathbb{R}^+ \), by Itô’s formula, we have

\[
dV = LV(x_1, x_2, y)dt + \frac{\sigma_1\left(\sqrt{x_1} - 1\right)(x_1 - x_1^2)}{2}dB_1(t) + \frac{\sigma_2\left(\sqrt{x_2} - 1\right)(x_2 - x_2^2)}{2}dB_2(t) + \frac{\sigma_3\left(\sqrt{y} - 1\right)(y - y^2)}{2}dB(t),
\]

where \( LV(x_1, x_2, y) \) is defined by

\[
LV = \frac{\sqrt{x_1} - 1}{2x_1} \left( ax_2 - r_1x_1 - a_1x_1^2 - bx_1 \right) + \frac{\sigma_1^2(2 - \sqrt{x_1})}{8}(x_1 - x_1^2)^2 + \frac{\sigma_2^2(2 - \sqrt{x_2})}{8}(x_2 - x_2^2)^2 + \frac{\sigma_3^2(2 - \sqrt{y})}{8}(y - y^2)^2
\]

\[
< 0.5 \left( \frac{a_2}{\sigma_2} + r_1 > b + a_1 x_1 \right) + 0.25 \sigma_1^2 \left( x_1^2 + x_1 x_1^3 + (x_1^2) \right) + 0.5 \left( \frac{b x_2}{\sqrt{x_2}} + r_2 + a_2 x_2 + \frac{a_2(1 - m)x_2}{\sigma_2} \right) + 0.25 \sigma_2^2 \left( x_2^2 + x_2 x_2^3 + (x_2^2) \right) + 0.5 \left( \frac{r + a_2}{\sigma_2} \sqrt{y} + a_3 y \right) + 0.25 \sigma_3^2 \left( y^2 + y y^2 + (y^2) \right)
\]

\[
= K,
\]

where \( K \) are positive constants due to the positivity of the coefficients of system (2). Therefore,

\[
dV \leq Kdt + \frac{\sigma_1\left(\sqrt{x_1} - 1\right)(x_1 - x_1^2)}{2}dB_1(t) + \frac{\sigma_2\left(\sqrt{x_2} - 1\right)(x_2 - x_2^2)}{2}dB_2(t) + \frac{\sigma_3\left(\sqrt{y} - 1\right)(y - y^2)}{2}dB(t).
\]

The following certification is similar to Liu and Wang [13], so we leave out the proof.

**Theorem 2.3.** Denote
\[ D_1 = -x_1 \left( a_1 - \frac{\sigma_1^2}{2} x_1 \right), \]
\[ D_2 = -x_2 \left( a_2 - \frac{\sigma_2^2}{2} x_2 - \frac{a_1 (1-m) [1+ \omega_1 (1-m) x_1^2 + 2 \omega_1 y]}{2[1+ \omega_1 (1-m) x_1^2 + \omega_1 y]} \right), \]
\[ D_3 = -a_3 + \frac{\sigma_3^2}{2} y^2 + \frac{a_3 (1-m) [1+ \omega_2 (1-m) x_2^2 + \omega_2 y]}{2[1+ \omega_1 (1-m) x_2^2 + \omega_2 y]}, \]
\[ D_4 = \frac{a_2 (1-m) [1+ \omega_2 y]}{2[1+ \omega_1 (1-m) x_2^2 + \omega_2 y]}, \]
\[ D_5 = \frac{a_2 (1-m) [1+ \omega_2 (1-m) x_2^2 + \omega_2 y]}{2[1+ \omega_1 (1-m) x_2^2 + \omega_2 y]} . \]

If \( a_1 > \frac{\sigma_1^2}{2} x_1^* \), \( D_2 + D_4 < 0 \), \( D_3 + D_5 < 0 \),

then the equilibrium \((x_1^*, x_2^*, y^*)\) of (2) is almost surely globally asymptotic stable, i.e., for any given initial value \((x_{10}(\theta), x_{20}(\theta), y_{0}(\theta)) \in R^3, \theta \in [-\tau, 0]\), the system of equation (2) satisfies

\((x_{1}(t), x_{2}(t), y(t)) \to (x_1^*, x_2^*, y^*)\) a.s. as \( t \to \infty \).

**Proof.** Due to the stability theory of stochastic differential equation, we only need to find a Lyapunov function \( V(z) \) satisfying \( LV(z) \leq 0 \) and the identity holds if and only if \( z = z^* \) (see e.g., [14,15]), where \( z = z(t) \) is the solution of the \( n \)-dimensional stochastic differential equation

\[ dz(t) = f(t, z(t))dt + g(t, z(t))dB(t), \]

\( z^* \) is the equilibrium state of (4) with \( g(t, z(t)) \), and

\[ LV(z) = V_i(z)f(t, z) + 0.5 \text{trace}[g^T(t, z)V_{zz}(z)g(t, z)] . \]

Define

\[ V_1(x_1) = c_1 \left( x_1 - x_1^* - x_1^* \ln \frac{x_1}{x_1^*} \right), \quad V_2(x_2) = c_1 \left( x_2 - x_2^* - x_2^* \ln \frac{x_2}{x_2^*} \right) , \]

where \( c_1, c_2 \) are positive constants to be determined later. If \((x_1(t), x_2(t), y(t)) \in R^3\), making use of Itô’s formula yields

\[ LV_1(x_1) = c_1 \left( x_1 - x_1^* \right) \left( ax_2 - \eta_1 x_1 - a_1 x_1^2 - b \omega_1 x_1 \right) + \frac{c_1 \sigma_1^2}{2} \left( x_1 - x_1^* \right)^2 \]
\[ = c_1 \left( x_1 - x_1^* \right) \left( ax_2 - \eta_1 x_1 - a_1 x_1^2 + a_1 x_1 x_1^* \right) + \frac{c_1 \sigma_1^2}{2} \left( x_1 - x_1^* \right)^2 \]
\[ = c_1 \left( x_1 - x_1^* \right) \left( \frac{a}{x_1} \left( x_1 x_2 - x_1^* x_2 \right) - a_1 x_1 \left( x_1 - x_1^* \right) \right) + \frac{c_1 \sigma_1^2}{2} \left( x_1 - x_1^* \right)^2 \]
\[ = c_1 \left( x_1 - x_1^* \right) \left( x_2 - x_2^* \right) - \frac{x_2}{x_1} \left( x_1 - x_1^* \right)^2 \]
\[ - c_1 \left( a_1 - \frac{\sigma_1^2}{2} \right) \left( x_1 - x_1^* \right)^2 , \]

and

\[ LV_2(x_2) = c_2 \left( x_2 - x_2^* \right) \left( b x_2 - r_2 x_2 - a_2 x_2^2 - \frac{a_1 (1-m) x_1 y}{1+ \omega_1 (1-m) x_1^2 + \omega_1 y} \right) + \frac{c_2 \sigma_2^2}{2} \left( x_2 - x_2^* \right)^2 \]
\[ = c_2 \left( x_2 - x_2^* \right) \left( \frac{b}{x_2} \left( x_2 - x_2^* \right) - x_2 \left( x_2 - x_2^* \right) - a_2 x_2 \left( x_2 - x_2^* \right) + a_1 (1-m) x_2 \right) \times \left[ \frac{a_1 (1-m) y}{1+ \omega_1 (1-m) x_1^2 + \omega_1 y} \left( y - y^* \right) \right] + \frac{c_2 \sigma_2^2}{2} \left( x_2 - x_2^* \right)^2 . \]
\[
\begin{align*}
&\leq c_2 \left( x_1 \right) + \left( x_2 \right) \leq D_1 \left( x_1 \right) + D_2 \left( x_2 \right) + \frac{a_1 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \left[ \left( x_1 \right) \left( x_2 \right) \right] \left( y \right)
\end{align*}
\]

where we have used the inequality: \( |p| |q| \leq \frac{p^2 + q^2}{2} \) for \( p, q \in R \) in the process of amplifying \( LV_2 \left( x_2 \right) \).

In order to eliminate the cross items in \( LV_1 \left( x_1 \right) \) and \( LV_2 \left( x_2 \right) \), we choose \( c_1 = \frac{x_1}{a}, c_2 = \frac{x_2}{b} \), then we have

\[
LV_1 \left( x_1 \right) + LV_2 \left( x_2 \right) \leq D_1 \left( x_1 \right) + D_2 \left( x_2 \right) + \frac{a_1 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \left[ \left( x_1 \right) \left( x_2 \right) \right] \left( y \right)
\]

Furthermore, we define

\[
V_{ij} \left( y \right) = y - y^* - y \ln \frac{y}{y^*}, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( x_{ij} \left( s \right) - x_{ij}^* \right)^2 ds, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( y \left( s \right) - y^* \right)^2 ds.
\]

Then we have

\[
LV_3 \left( y \right) = \left( y - y^* \right) \left( -a_{33} \left( y - y^* \right) + \frac{a_2 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \left( y \left( t \right) - y^* \right) \right)
\]

Then we have

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\]

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where we have used the inequality: \( |p| |q| \leq \frac{p^2 + q^2}{2} \) for \( p, q \in R \) in the process of amplifying \( LV_2 \left( x_2 \right) \).

In order to eliminate the cross items in \( LV_1 \left( x_1 \right) \) and \( LV_2 \left( x_2 \right) \), we choose \( c_1 = \frac{x_1}{a}, c_2 = \frac{x_2}{b} \), then we have

\[
LV_1 \left( x_1 \right) + LV_2 \left( x_2 \right) \leq D_1 \left( x_1 \right) + D_2 \left( x_2 \right) + \frac{a_1 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \left[ \left( x_1 \right) \left( x_2 \right) \right] \left( y \right)
\]

Furthermore, we define

\[
V_{ij} \left( y \right) = y - y^* - y \ln \frac{y}{y^*}, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( x_{ij} \left( s \right) - x_{ij}^* \right)^2 ds, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( y \left( s \right) - y^* \right)^2 ds.
\]

Then we have

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LV_3 \left( y \right) = \left( y - y^* \right) \left( -a_{33} \left( y - y^* \right) + \frac{a_2 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \right)
\]

so

\[
LV_3 \left( y \right) = \left( y - y^* \right) \left( -a_{33} \left( y - y^* \right) + \frac{a_2 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \right)
\]

Furthermore, we define

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V_{ij} \left( y \right) = y - y^* - y \ln \frac{y}{y^*}, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( x_{ij} \left( s \right) - x_{ij}^* \right)^2 ds, \quad V_{ij} \left( t \right) = \int_{t}^{\infty} \left( y \left( s \right) - y^* \right)^2 ds.
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\]

so

\[
LV_3 \left( y \right) = \left( y - y^* \right) \left( -a_{33} \left( y - y^* \right) + \frac{a_2 \left( 1 \right)}{2 \left( 1 + a_2 \right)} \right)
\]
\[
\begin{aligned}
&\frac{a_2(1-m)\omega_2 x_2^*}{2[1+\alpha_1(1-m)x_2^*+\omega_2 y_2^*]}[(y(t-\tau)-y^*)^2+(y-y^*)^2] \\
&= -a_3 + \frac{\sigma^2}{2} y_2^* + \frac{a_2(1-m)(1+\omega_2 x_2^*+\omega_2 y_2^*)}{2[1+\alpha_1(1-m)x_2^*+\omega_2 y_2^*]}(y-y^*)^2 \\
&+ D_4(x_2(t-\tau)-x_2^*)^2 + D_5(y(t-\tau)-y^*)^2.
\end{aligned}
\]

At last, we define

\[V(t) = V_1(x_1) + V_2(x_2) + V_3(y) + D_4 V_4(t) + D_5 V_5(t).
\]

From (5) and (6), we can easily obtain that

\[LV(t) = LV_1(x_1) + LV_2(x_2) + LV_3(y) + D_4(x_2-x_2^*)^2 + D_5(y-y^*)^2
\]

\[-D_4(x_2(t-\tau)-x_2^*)^2 - D_5(y(t-\tau)-y^*)^2
\]

\[\leq D_4(x_1-x_1^*)^2 + (D_2+D_4)(x_2-x_2^*)^2 + (D_3+D_5)(y-y^*)^2.
\]

Let \(|Z-Z^*| = ||x_1-x_1^*||,|x_2-x_2^*||,|y-y^*||^T\), then we have

\[LV(t) \leq |Z-Z^*|^T \begin{pmatrix} D_4 & 0 & 0 \\ 0 & D_2 + D_4 & 0 \\ 0 & 0 & D_3 + D_5 \end{pmatrix} |Z-Z^*|.
\]

Obviously, if (3) holds, we can conclude that \(LV(t)<0\) along all trajectories in \(R^3_+\) except \((x_1^*,x_2^*,y^*)\). Then the conclusion follows.

3. Numerical simulations

In this section, we will use the Milstein method mentioned in Higham [16] to verify the main results.

For system (2), we choose the coefficients as below:

\[a = 0.6, \eta_1 = 0.4, \eta_2 = 0.2, \tau = 0.1, b = 0.8, a_{31} = a_{32} = 0.5, a_{41} = a_{42} = 0.5, \omega_1 = \omega_2 = 0.5, \tau = 0.5.
\]

Firstly, we set \(m = \sigma_1 = \sigma_2 = \sigma = 0\), then we obtain Figure 1. In this case, we consider the determine-stic system of (2) without prey incorporating refuge. By calculation, we have \(x_1^* = 0.1179, x_2^* = 0.2474, y^* = 0.0184\). In Figure 1, we see that the positive equilibrium is globally asymptotically stable.

Consequently, we set \(m = 0.1, \sigma_1 = 0.3, \sigma_2 = \sigma = 0.5\), then we have Figure 2. From the numerical simulation, we notice that the density of prey species increases in the presence of prey refuge, and the density of predator species decreases. And the positive equilibrium \((x_1^*,x_2^*,y^*)\) is also globally asymptotically stable.

**Figure 1.** System (2) with \(m = \sigma_1 = \sigma_2 = \sigma = 0\). **Figure 2.** System (2) with \(m = 0.1, \sigma_1 = 0.3, \sigma_2 = \sigma = 0.5\).
4. Conclusions
In this paper, we consider the global asymptotic stability of a stochastic Beddington-DeAngelis type predator-prey system with time delay and stage structure for prey incorporating refuge. Sufficient conditions for the global asymptotic stability of the system are established. Theoretical results disclose that time delay has no effect for the global stability of equilibrium state in some cases, and the overcrowding coefficients \( \alpha_i (i=1,2,3) \) are crucial in the stability of the positive equilibrium state \( (x_1^*, x_2^*, y^*) \). The result is amusing and important because from the biological point of view, a globally stable positive equilibrium means that the community is stable in which all species could coexist.

It is also meaningful to consider the impact of jumps and impulsive perturbations on the stochastic models, which is worthwhile for us to study further, and we will proceed it in the future work.

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