Modeling and simulation of precursor diamagnetism in high critical temperature cuprates

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Abstract. In this work, it is developed a model to study the precursor diamagnetic response of a superconductor that is submitted to temperatures well above its critical temperature $T_c$. This approach is based on the Critical State Model that takes into account the effect of an applied magnetic field and a dependence of the average critical current density on the temperature of the sample. The results on the anomalous diamagnetic response are in good quantitative agreement with the precursor diamagnetism measurements. Modeling such phenomenon may be important to the understanding of the pseudogap region for High-Tc cuprates as well as to provide computational tools to any application using the magnetization near the critical temperature.

1. Introduction
While Low-Tc superconductors have a very well characterized metallic normal phase, the normal phase of High-Tc cuprates displays many non-conventional properties that remain to receive a complete explanation. Among the anomalous properties, that in many cases depends on the doping level, it is well known the measured gap that persists up to temperatures much higher than $T_c$ and it is known as the pseudogap [1,2,3]. The nature of this pseudogap is a matter of strong debate, and there is so far no consensus whether it is independent of superconductivity, or even if there are more than one pseudogap type [3,4].

As part of this issue, there is an anomalous high magnetic field dependence of the magnetization above the superconducting transition, which has been observed in several High-Tc cuprates by different groups. Although it does not remain at very large temperatures above $T_c$ compared with other properties, like the Nernst effect to mention just one [5], it is detected at relative temperatures on La$_{1.85}$Sr$_{0.15}$CuO$_2$ several orders of magnitude larger than similar results for Low-Tc superconductors[6]. Similarly, measurements on oriented powders of Y$_{1.3}$Ca$_{0.7}$Ba$_2$Cu$_3$O$_y$ have detected many different anomalous features [7], with a large magnetization enhancement, which depends on the doping level. Right after the first observations [8] it was clear that such large and non trivial magnetic response
could not be explained by classical scenarios like, for instance, the ones described by the Gaussian Ginzburg-Landau (GGL) theories [9] that predict nearly zero diamagnetic response above $T_c$.

On the other hand, several different experiments gave strong indications that these materials are highly inhomogeneous [10,11] with a strong intrinsic disorder in the doping level like, for instance, the stripe phase [12]. The use of the Critical State Model on a disordered superconductor with droplets of superconductivity below the pseudogap temperature $T^*$ was successful in reproducing qualitatively some of the experimental data on the magnetization above $T_c$ [13].

In this line of discussion, here it is implemented the Critical State Model of Kim [2] to simulate the anomalous precursor diamagnetism above $T_c$ on underdoped High-Tc systems, such as La$_{2-x}$Sr$_x$CuO$_2$. In addition to the inversely linear dependence on the applied magnetic field, it is inserted into the model a dependence of the average critical current density of the superconductor on the temperature, as measured in High-Tc superconductors [14]. The precursor magnetic response calculations on this system yielded excellent quantitative agreement with the experiments [6,15].

2. Modeling

The physical situation that was modeled and implemented in the simulations is represented in cylindrical coordinates as shown in figure 1. Figure 2 shows the grid that was used to map half of the superconductor’s section, to implement the model numerically.

As the system presents azimuthal symmetry and there is only the component $z$ of the homogeneous magnetic field, the vector potential associated with the applied field can be derived from its definition to yield:

$$\vec{A}_z(\rho,t) = \frac{P}{2} \vec{B}_z(t) \hat{\phi}$$

(1)

Considering the homogeneity of the applied field over the space, it can be assumed that the response of the superconductor may be symmetric in relation to the $x$-$y$ plane. Therefore, the potential vector due to the superconducting current density that flows as a reaction to the applied field, may be calculated using the Biot-Savart’s Law to result in the following expression:

$$\vec{A}_j(\vec{r},t) = \mu_0 \int_0^{a} d\rho \int_0^{b} dz' \Omega_{cil}(\vec{r}, \vec{r}') J(\vec{r}',t) \hat{\phi}$$

(2)

where $\vec{r} = (\rho, z)$, $\vec{r}' = (\rho', z')$ and the integral kernel is expressed by:
The total vector potential of the system is obtained by adding equation (1) with equation (2), which results:

$$A(\vec{r},t) = \mu_0 \int_{\vec{r}_0}^{\vec{r}} d\rho' \int_0^\rho d\rho'' Q_{cil}(\vec{r},\vec{r}'') J(\vec{r}'',t) + \frac{\rho'}{2} B_u(t)$$

The time derivative of (5) and substitution of Faraday’s Induction Law in its local formulation ($\partial A/\partial t = -E$), results in the equation below:

$$\frac{\partial J(\vec{r},t)}{\partial t} = -\mu_0^{-1} \int_{\vec{r}_0}^{\vec{r}} d\rho' \int_0^\rho d\rho'' Q_{cil}^{-1}(\vec{r},\vec{r}'') \left[ E(\vec{r}'',t) + \frac{\rho'}{2} \frac{dB_u(t)}{dt} \right]$$

At this point, the equations obtained are essentially classical in their treatment. The phenomenology of superconductivity may be introduced in this model by using an experimental exponential law between the local electrical field and the current density of the superconductor [16]:

$$\bar{E} = E_c \left( \frac{J}{J_{cm}(B,T)} \right)^n$$

where $E_c$ is the critical electrical field of the superconductor and $n=U_c/(k_B T)$, with $U_c$ corresponding to the critical activation energy, is the parameter that establishes the type of the superconductor. For $n=1$, the material behaves as an ohmic conductor.

In the Critical State Model proposed by Kim [2], the critical current density of the superconductor is considered inversely proportional to the local magnetic induction as presented in the equation:

$$J_{cm}(B,T) = \frac{J_{cm}(T)}{1 + \frac{B}{\alpha B^*}}$$

where $B^*$ is the critical magnetic induction which indicates that the superconducting sample is completely fulfilled with currents and $\alpha$ is a constitutive parameter of the superconductor associated with the pinning force that bends the magnetic flux vortices in the material. The influence of the temperature in the critical current is inserted in the model by using dependence with the exponent 3/2, which has been measured on YBCO thin films [14]:

$$J_{cm}(T) = J_{cm0} \left( 1 - \mu \frac{T}{T_c} \right)^{3/2}$$

where $J_{cm0}$ is the average current density for temperatures well below the critical temperature of the material and $\mu$ is a factor that must be adjusted from the experiments.

Taking equations (7) and (8) together with equation (6), will lead to the following equation for the time dependent behavior of the current density inside the superconductor:

$$J(\vec{r},t) = -\mu_0^{-1} \int_{\vec{r}_0}^{\vec{r}} d\rho' \int_0^\rho d\rho'' Q_{cil}^{-1}(\vec{r},\vec{r}'') \left[ E_c \left( \frac{J}{J_{cm}(T)} \right)^n \left( 1 + \frac{B}{\alpha B^*} \right)^n \right] \left[ \text{sign}(J) + \frac{\rho'}{2} \frac{dB_u(t)}{dt} \right] dt + J(\vec{r},t-dt)$$
Equation (10) can only be numerically evaluated. Here it was used the Method of Moments [17], with all the initial conditions zero. Further details of its numerical and computational implementation are presented in [18].

With all values of the current density for each point of the superconductor’s grid (Figure 2) calculated, the total magnetization of the sample is directly obtained by:

$$M(t) = \frac{2\pi \int_0^r d\rho \int_0^a dz J(r,z,t) \rho^2}{Vol}$$

(11)

where $Vol$ is the volume of the superconducting sample.

3. Results

Figure 3 shows some measured magnetization curves of a cylindrical La$_{2-x}$Sr$_x$CuO$_2$ underdoped sample ($x=1$), as reported in [6]. The radius of the sample is 3 mm and its height is 0.2 mm. The critical temperature of the superconductor is $T_c = 26.3$ K, and for temperatures above $T_c$, the sample presents an anomalous precursor diamagnetic response, particularly high for low applied magnetic fields. The magnetization calculated with the model presented in this work is seen in figure 4.

To obtain these results the equations presented in section 2 were first used to estimate the average current densities that the sample might have, as the temperature varies from $T = 27.3$ K to $T = 28$ K. For these simulations, it was considered a grid of 40 points to map the cylindrical section of the superconductor. The sample was modeled with 0.5 mm of height, instead of 0.2 mm, because the relation between the height and the radius of the superconductor ($2b/a$) would be too small and this compromise the numerical stability of the integral kernel (4). The parameter $n$ of the exponential characteristic equation (7), was chose to be 11, which is near to the values usually considered for High-Tc cuprates [19]. The critical magnetic induction $B^*$ was set equal to 0.03 T, which is close to the value taken by González et al [13].

In figure 5, it can be seen the four adjusted average current densities (indicated by the circles), which were obtained by comparison between the respective magnetization curves (calculated by the model) and the four experimental curves shown in figure 3. The model used in the simulations considered that $J_{cm}$ has a dependence only on the applied magnetic induction. Once these values of $J_{cm}$
were found, it was used equation (10) to estimate the parameters $J_{\text{cm0}}$ and $f_t$ that could best reproduce the dependence of the current density with the temperature.

![Figure 5](image.png)

Figure 5. Circles: adjusted values of the average current densities, obtained from comparison between the magnetization curves calculated by the model and the experimental curves. Lines: estimation of the dependence of the current density with the temperature. The best fit indicates that the maximum temperature occurs at $T_{\text{max}} = 28.6$ K.

The parameters of the curve that fits the four points in figure 5 are $J_{\text{cm0}} = 1.04 \times 10^7$ A/m$^2$ and $f_t = 0.9196$, which yielded a maximum temperature $T_{\text{max}} = 28.6$ K. This indicates that if the sample is submitted to this temperature, all the material is in the normal state and the diamagnetic response would completely cease. As shown in figure 3, this result seems to confirm the tendency of the experiments.

The value obtained for $T_{\text{max}}$ is much higher than the usual GL fluctuation effect, as shown by Cabo et al [6], but much lower than other properties, like the Nernst signal [5] that persists up to $T = 100$ K, for the underdoped ($x=0.1$) compound. One possible explanation is that, although the system needs a critical current around regions in order of few millimeters, as displayed in figure 1, the vortices develop themselves in nanoscale regions. Consequently, the charge inhomogeneities present in most underdoped samples generate small droplets of superconductivity [7,11,15] which can hold some vortices, say, near 100 K. As the temperature decreases, these droplets increase in size and others appears in such a way that the superconducting volume increase continuously. In this scenario, at $T = 28.6$ K, there are superconducting regions with the order of magnitude of the cylinder (figure 1) that exhibit a measurable magnetic response.

4. Conclusions

A Critical State Model, with a dependence on the magnetic induction and the temperature for the critical current density, and a methodology to simulate the precursor diamagnetism in underdoped High-Tc samples, were implemented. When compared with experiments, the calculations of the model showed great quantitative accordance. This model, together with the methodology developed, can be used to obtain the dynamical behavior of the current density inside High-Tc cuprates, for temperatures above $T_c$.

The predictions of the model could also be used to confirm a dependence of the critical current density on the temperature, with an exponent 3/2, even for temperatures much higher than the critical temperature (8.75 % above $T_c$). Although these results are in accordance with a system that considers a high inhomogeneous charge distribution for underdoped cuprates samples, further investigations are demanded to confirm this scenario.

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