The mass and the coupling of the Dark Particle

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We argue that Dark Matter can be described by an interacting field theory with a mass parameter of the order of the proton mass and an interaction coupling of the order of the QED coupling.

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I. INTRODUCTION

We know that Dark Matter (DM) clumps around galaxies, tracing the clumping of matter. However, it does not seem to take part in stars and might not show up in our vicinity, at least not extensively. Can we get constraints from these facts? Are they consistent with a non interacting Dark Matter (DM) and Dark Energy (DE) model? We argue that we need a field theory in interaction in order to be able to explain such a phenomenology. Moreover, we believe that the strength of the interaction as well as the mass of the Dark Particle are restricted from general arguments.

Non interacting DM can be hardly understood in terms of field theories. Moreover, matter clumping depends crucially on friction: a frictionless matter cannot clump, it would just scatter as a consequence of gravity, while matter with dissipation falls into energy traps leading to structures. We thus need an interacting field theory describing DM. The fundamental question is how to entail interaction from the knowledge of astrophysical phenomenology.

Some works are quite definite on the requirement of DE and DM interaction. Several papers dealt with the matter of interaction in general [1] as well as [2] within the holographic model [3]. Recently it has been argued [4] that interactions should be present to explain the Bullet observations [5]. Here we give further arguments to support the picture of interacting DM, possibly with DE.

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II. GALAXY FORMATION

Let us suppose that Dark Matter scatters through some mechanism described by a cross section \( \sigma_{DM} \). In the case there is a Quantum Field Theory behind the scene, the fine structure constant corresponding to the coupling of DM, given by \( \alpha_{DM} \), should be related to the coupling \( g_{DM} \) of DM to some mediator via \( \alpha_{DM} = g_{DM}^2 \), up to a numerical factor of order of unity. We also suppose that the DM field is characterized by two mass parameters \( m_1 \) and \( m_2 \ll m_1 \) describing fields in interaction. The motivation is to describe a dark sector as similar as possible to our bright (or baryonic plus leptonic) matter.

Similarly to Thompson scattering, we assume that there is a relation between the above quantities given by

\[
\sigma_{DM} \approx \frac{\alpha_{DM}^2}{m_2^2}.
\]

In such a case, supposing the cooling of DM in very large scales to be due to some kind of bremsstrahlung process, the cooling time is [6]

\[
t_{cool} \approx (n \alpha_{DM} \sigma_{DM})^{-1} \left( \frac{T}{m_2} \right)^{1/2} = \frac{m_2^2}{\alpha_{DM}^4 n} \left( \frac{T}{m_2} \right)^{1/2},
\]

where \( n \) is the DM number density. This should be compared to the time of gravitational collapse,

\[
t_{grav} = \left( \frac{R^3}{GM} \right)^{1/2},
\]

where now \( M \) is the total mass and \( R \) the corresponding size. The temperature is \( T = \frac{GMm}{R} \); equating \( t_{cool} \) and \( t_{grav} \) as a condition for triggering structure formation and using \( n = \frac{N}{4\pi R^3} \) and \( N = M/m_1 \), we have

\[
R_{dark\ gal} \approx \frac{1}{4} \frac{\alpha_{DM}^3}{G(m_1 m_2)^{3/2}}.
\]

We can compare with the similar result of baryonic galaxy formation [6] (actually just replacing \( m_{1,2} \) by \( m_{p,e} \) and \( \alpha_{DM} \) by the fine structure constant \( \alpha \)). For \( R \approx R_{gal} \) we have \( (m_e = 10^{-3} m_p) \)

\[
\alpha_{DM} \approx 30 \frac{\sqrt{m_1 m_2}}{m_p} \alpha. \tag{1}
\]

Let us now examine the galaxy mass. Using \( M_{dark\ gal} \approx N m_1 \) and the temperature as \( T \approx \alpha_{DM}^2 m \approx \frac{GMm}{R} \) we find, for \( M_{dark\ gal} \approx 10 M_{bright\ gal} \)

\[
\alpha_{DM} \approx 3 \alpha, \quad \text{consequently,} \quad \sqrt{m_1 m_2} \approx 0.1 m_p. \tag{2}
\]

Therefore the Dark Matter coupling should differ from the QED fine structure constant by a factor of 3, approximately.
III. STAR FORMATION

Supposing that the dark particles are confined within a small region \( d \), their kinetic energy is of the order \( \frac{p^2}{m_2} \approx \frac{1}{m_2 d^2} \). Their energy is \( U \approx \frac{GM^2}{R} \), their thermal energy \( K \approx NT \) thus \( T \approx \frac{GMm_1}{R} \). The typical distance is gotten from \( d \approx RN^{-1/3} \), thus \( T \approx N^{2/3}Gm^2/d \). Arguing that we have a balance between thermal and kinetic energy \([6]\), we find \( T \approx N^{2/3}Gm^2/d - 1/(m_2d^2) \), whose maximum is achieved when

\[
d = d_{\text{dark star}} = 2N^{-2/3} \frac{1}{Gm_1^2m_2}
\]

The corresponding temperature turns out to be

\[
T_{\text{dark star}} \approx N^{4/3}G^2m_1^4m_2
\]

We wish to impose that the star ignites. This is actually a very delicate point. Does it ignite? It would emit which kind of energy? In case we have a DE and DM interaction of the kind suggested elsewhere \([1, 2]\) the decay of DM into DE in average is not preferred (the interaction seems to be preferable in the other direction for entropic reasons). In case we first suppose that the physical process is the same as in the baryonic case, we need an energy \( \epsilon \approx \alpha^2_{DM}m_1 \) to ignite the star, in which case the condition becomes, for the number of particles,

\[
N_{\text{dark star}} \approx \left( \frac{\alpha_{DM}}{G} \right)^{3/2} \frac{1}{m_1^{9/4}m_2^{3/4}}
\]

Comparing with a baryonic (bright) star, \( N_{\text{dark star}} \approx \left( \frac{m_p}{m_1} \right)^{3/2} N_{\text{bright star}} \) The corresponding mass is of the order

\[
M_{\text{dark star}} \approx \left( \frac{m_p}{m_1} \right)^{1/2} M_{\text{bright star}}
\]

We also compute the radius of such a star,

\[
R_{\text{dark star}} \approx N_{\text{dark star}}^{-1/3} \frac{1}{Gm_1^2m_2}
\]

For the range of values \( m_1 \approx 3m_p \) and \( m_2 \approx 3m_e \), compatible with the previous constraints, we have \( N_{\text{dark star}} \approx 5N_{\text{bright star}} \), \( M_{\text{dark star}} \approx 0.6M_{\text{bright star}} \) and \( R_{\text{dark star}} \approx 0.05R_{\text{bright star}} \). Due to the difficulty in getting a decay of DM into DE (see \([2]\)), it is quite possible that such an object be forbidden as a star and its fate is a Black Hole, especially in view of the lack of radiation pressure: in case the star radiates just Dark Energy the pressure might be too small to maintain the small star, determining its fate as a compact object. An object of roughly the same mass and radius 20
times smaller is $10^4$ times denser, a fact which might lead us to Black Hole formation. This picture of an interacting theory but with supressed DE production is a correct one to explain the observed structure.

IV. PLANETS

For small objects we should have a mass $M = N m_1$ and $R \approx N^{1/3} a_0$, where $a_0 = \frac{1}{\alpha m_2}$ is the smallest quantum size of a molecule of dark matter. The gravitational energy is $E_g = \frac{GM^2}{R} \approx N^2 \frac{Gm_1^2}{R}$, while the total energy in the form of molecule binding energy is $E_0 = N \alpha^2 m_2$. For a stable configuration the gravitational energy should not be enough to crush matter. Therefore

$$E_g = N^2 \frac{Gm_1^2}{R(\approx N^{1/3} a_0)} = N^{5/3} \frac{Gm_1^2}{a_0}$$

$$= E_0 \text{ in order not to crush the configuration, in the compact limit} \quad (5)$$

We thus find the maximum of particles, $N_{\text{max}}$,

$$N_{\text{dark max}} \alpha_{DM}^2 m_2 \approx N_{\text{dark max}}^{5/3} Gm_1^2 \alpha_{DM} m_2$$

thus

$$N_{\text{dark max}} \approx \left[ \frac{\alpha_{DM}}{Gm_1^2} \right]^{3/2} \approx 0.2 N_{\text{bright max}} \quad (6)$$

The radius and mass are given by

$$R_{\text{dark planet}} \approx N_{\text{dark max}}^{1/3} a_0 = \frac{N_{\text{dark max}}^{1/3}}{\alpha_{DM} m_2} \approx 0.1 R_{\text{bright planet}} \quad (7)$$

$$M \approx N_{\text{max}} m_1 \approx 0.6 M_{\text{bright max}} \quad (8)$$

A planet has a reasonable density as compared to a usual baryonic planet.

V. CONCLUSIONS

We conclude that Dark Matter forms galaxies with approximately the same size as usual galaxies and a mass 10 times larger in case the DM particles are roughly as heavy as protons and electrons (up to a factor of a few) and the coupling roughly a few times the QED coupling. This is in accordance with observations [5] and previous interacting models based on interacting fluids [2, 3].
In such a case the substructures which can be formed are either Black Holes or small planets. This is quite consistent with (the lack of) further observations.

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