Fast rotation of neutron stars and equation of state of dense matter

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Abstract

Fast rotation of compact stars (at submillisecond period) and, in particular, their stability, are sensitive to the equation of state (EOS) of dense matter. Recent observations of XTE J1739-285 suggest that it contains a neutron star rotating at 1122 Hz (Kaaret et al., 2007). At such rotational frequency the effects of rotation on star’s structure are significant. We study the interplay of fast rotation, EOS, and gravitational mass of a submillisecond pulsar. We discuss the EOS dependence of spin-up to a submillisecond period, via mass accretion from a disk in a low-mass X-ray binary.

Key words: Dense matter, Equation of state, stars: neutron, stars: rotation, Pulsars, Low-mass X-ray binaries
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1. Introduction

Neutron stars, and more generally, compact stars (neutron stars, hybrid hadron-quark stars, quark stars) are the densest stellar objects in the Universe. Due to their compactness and strong gravity, compact stars can be very fast rotators. In the present paper we limit ourselves to a rigid rotation. Theoretical calculations show that compact stars could rotate at sub-millisecond periods (i.e., at frequency $f > 1000$ Hz: Cook et al. 1994; Salgado et al. 1994).

The quest for fast rotating compact stars has an interesting history. The first millisecond pulsar B1937+21, rotating at $f = 641$ Hz (Backer et al., 1982), remained the most rapid one for 24 years. In January 2006, discovery of a faster pulsar J1748-2446ad rotating at $f = 716$ Hz was announced (Hessels et al., 2006). However, sub-kHz frequencies are still too low to significantly affect the structure of massive neutron stars with $M > 1M_\odot$ (Shapiro et al., 1983; Haensel et al., 2007). Actually, pulsars B1937+21 and J1748-2446ad still rotate in a slow rotation regime, because their $f$ is significantly smaller than the mass shedding (Keplerian) frequency $f_K$. In the slow rotation regime rotational effects in neutron star structure, e.g., polar flattening, are $\propto (f/f_K)^2 \ll 1$, and hence not large. Rapid rotation regime for $M > 1M_\odot$ requires sub-millisecond pulsars with supra-kHz frequencies ($f > 1000$ Hz).

Exciting news came in December 2006. Kaaret et al. (2007) reported a discovery of oscillation frequency $f = 1122$ Hz in an X-ray burst from the X-ray transient, XTE J1739-285. Kaaret et al. (2007) wrote cautiously “this oscillation frequency suggests that XTE J1739-285 contains the fastest rotating neutron star yet found”. If confirmed, this would be the first detection of a sub-millisecond pulsar (discovery of a 0.5 ms pulsar in SN1987A remnant, announced in January 1989, was withdrawn one year later).

Fast rotation of compact stars is sensitive to the stellar mass and to the equation of state (EOS). Hydrostatic, stationary configurations of neutron stars rotating at given rotation frequency $f$ form a one-parameter family, labeled by the central density. This family - a curve in the mass - equatorial radius plane - is limited by two instabilities. On the high
central density side, it is instability with respect to axi-symmetric perturbations, making the star collapse into a Kerr black hole. The low central density boundary results from the mass shedding from the equator. In the present paper we discuss the dependence of rotation at \( f > 1000 \) Hz on the poorly known EOS, and derive constraints on the EOS of neutron stars which could result from future observations of stably rotating sub-millisecond pulsars.

The plan of the paper is as follows. In Sect. 1 we briefly describe EOSs used in our calculations. Numerical methods of solving equations of hydrostatic equilibrium of rigidly rotating compact stars in General Relativity, and criteria for their stability, are presented in Sect. 2. In Sect. 3 we consider the EOS dependence case of rotation at \( f = 1122 \) Hz. A systematic study of the EOS-dependence of fast rotation at \( f = 1000 - 1600 \) Hz is presented in Sect. 4. Reaching fast rotation via disk accretion in LMXBs and the EOS-dependence of the spin-up track is reviewed in Sect. 5. Finally, Sect. 6 summarizes briefly main results reported in the paper.

We are pleased to present our paper on fast rotation of neutron stars in the proceedings of the conference in honor of J.-P. Lasota. In 1990s Jean-Pierre became intrigued by a puzzling precision of “empirical formula” for the absolute upper bound on rotation frequency of compact stars, proposed by two of us (PH and JLZ) on the aftermath of ill fated “discovery” of 0.5 ms pulsar in SN 1987A ([Haensel & Zdunik, 1989]). Two joint papers resulted from our collaboration ([Lasota et al., 1996; Haensel et al., 1999]), some progress has been made, but the basic puzzle still remains to be solved.

2. Equations of state

In view of a high degree of our ignorance of EOS of dense matter at supranuclear densities \( (\rho > 3 \times 10^{14} \text{ g cm}^{-3}) \) we considered a broad set of theoretical models. The set of ten equations of state (EOSs) considered in the paper is presented in Fig. 1. The EOSs are also listed in Table 1, where the basic informations (label of an EOS, theory of dense matter, reference to the original paper) are also collected.

Two EOSs were chosen to represent a soft (BPAL12) and stiff (GN3) extreme cases. These two extreme EOSs should not be considered as “realistic”, but they are used just to “bound” the models from the soft and the stiff sides.

Four EOSs are based on realistic models involving only nucleons (FPS, BBB2, DH, APR). The next four EOSs are softened at high density either by the appearance of hyperons (GNH3, BGN1H1), or a phase transition (GMGS-Km, GMGS-Kp). A softening in the latter case is clearly visible in Fig. 1 at pressure \( P \sim 10^{35} \text{ dyn cm}^{-2} \).

Two EOSs, GMGS-Kp and GMGS-Km, describe nucleon matter with a first order phase transition to a kaon condensed state. In both cases the hadronic Lagrangian is the same. However, to get GMGS-Kp we assumed that the phase transition takes place between two pure phases and is accompanied by a density jump, calculated using the Maxwell construction. The GMGS-Km EOS was obtained assuming that the transition occurs via a mixed state of two phases (Gibbs construction). A mixed state is energetically preferred when the surface tension between the two phases is below a certain critical value. As the value of the surface tension is very uncertain, we considered both cases.

3. Calculation of stationary rotating configurations and their stability

We computed stationary configurations of rigidly rotating neutron stars in the framework of General Relativity by solving the Einstein equations for stationary axi-symmetric spacetime ([Bonazzola et al., 1993; Gourgoulhon et al., 1999]). Numerical computations have been performed us-
Table 1 Equations of state. N - nucleons and leptons only. NH - nucleons, hyperons, and leptons. Labels of EOSs are composed from first letters of names of authors of EOSs of the core. In all cases but FPS, the EOS of the crust is the DH one. For FPS EOS its own crust model is used.

| EOS       | model                               | reference              |
|-----------|-------------------------------------|------------------------|
| BPAL12    | N energy density functional         | Bombaci (1995)         |
| FPS       | N energy density functional         | Pandharipande & Ravenhall (1989) |
| GN3       | N relativistic mean field           | Glendenning (1985)     |
| DH        | N energy density functional         | Douchin & Haensel (2001) |
| APR       | N variational theory                | Akmal et al. (1998)    |
| BGN1H1    | NH, energy density functional       | Balberg & Gal (1997)   |
| GNHQm2    | NH + mixed baryon-quark state       | Glendenning (2000)     |
| BBB2      | N Brueckner theory                  | Baldo et al. (1997)    |
| GNH3      | NH relativistic mean field          | Glendenning (1985)     |
| GMGS-Km   | N + mixed nucleon-kaon condensed b) | Pons et al. (2000)     |
| GMGS-Kp   | N + pure kaon condensed b)          | Pons et al. (2000)     |

a A18δ+U1X* model of Akmal et al. [1998].
b GM+GS model with $U_K = −130$ MeV.

We calculated one-parameter families of stationary 2-D configurations for ten EOSs, presented in Fig. 1 and Table 1. Apart from fulfilling the equations of hydrostatic equilibrium, stationary configurations were required to be stable. Two instabilities were considered.

Mass-shedding. Stability with respect to the mass shedding from the equator implies that at a given gravitational mass $M$ the circumferential equatorial radius $R_{eq}$ should be smaller than $R_{max}$ which corresponds to the mass shedding (Keplerian) limit. The value of $R_{max}$ results from the condition that the frequency of a test particle at circular equatorial orbit of radius $R_{max}$ just above the equator of the actual rotating star is equal to the rotational frequency of the star. This stability condition sets the bound on our rotating configurations from the right side on $M−R_{eq}$ plane. It fixes the largest radius and the smallest central density, allowed for stable stationary configurations.

Axi-symmetric oscillations. Instability with respect to these oscillations determines the bound for most compact stars, with the smallest radius and the highest central density (i.e., from the left side on the $M−R_{eq}$ plane). This bound is determined by the condition:

$$\frac{\partial M}{\partial \rho_c} = 0.$$  (1)

For stable configurations:

$$\frac{\partial M}{\partial \rho_c} > 0.$$  (2)

In the opposite case, $(\partial M/\partial \rho_c) < 0$, a compact star star is doomed to collapse into a Kerr black hole.

4. An example: compact stars at 1122 Hz

In this section we present the parameters of the stellar configurations rotating at frequency 1122 Hz, a suggested rotation frequency of XTE J1739-285. For details and discussion see Bejger et al. [2007].

In Fig. 2 we show the $M(R_{eq})$ plots of stable stationary configurations rotating at $f = 1122$ Hz. We considered EOSs from Table 1. The relation between the calculated values of $M$ and $R_{eq}$ at the “mass shedding point” is extremely well approximated by the formula for the orbital frequency for a test particle orbiting at $r = R_{eq}$ in the Schwarzschild space-time of a spherical mass $M$ (which can be replaced by a point mass $M$ at $r = 0$). Let us denote the orbital frequency of such a test particle by $f_{Schw}(M, R_{eq})$. The locus of points satisfying $f_{Schw}(M, R_{eq}) = 1122$ Hz is represented by a dash line in Fig. 2. The points on the dash line satisfy the relation

$$f_{Schw}(M, R_{eq}) = 1122 \text{ Hz}.$$
Fig. 2. Gravitational mass, $M$, vs. circumferential equatorial radius, $R_{eq}$, for neutron stars stably rotating at $f = 1122$ Hz, for ten EOSs (Fig. 1). Small-radius termination by filled circle: setting-in of instability with respect to the axi-symmetric perturbations. Dotted segments to the left of the filled circles: configurations unstable with respect to those perturbations. Large-radius termination by an open circle: the mass-shedding instability. The mass-shedding points are very well fitted by the dashed curve $R_{\text{min}} = 15.52 \left(\frac{M}{1.4M_\odot}\right)^{1/3}$ km. For further explanation see the text.

This formula, obtained for the Schwarzschild metric, coincides with that for Newtonian gravity for a point mass $M$. As one can see, it passes through (or extremely close to) the open circles denoting the (numerically calculated) actual mass shedding (Keplerian) configurations. This is a quite remarkable property, in view of rapid rotation and strong flattening of neutron star at the mass-shedding point, visualized in Fig. 3.

Equation (3) implies a useful constraint for compact stars rotating at 1122 Hz:

$$\frac{1}{2\pi} \left(\frac{GM}{R_{eq}}\right)^{1/2} = 1122 \text{ Hz}.$$ (3)

5. Submillisecond pulsars

In this section we present results for compact stars rotating at sub-millisecond periods (supra-kHz frequencies) for a broad range of frequencies, 1000 - 1600 Hz. As in Sect. 4 calculations are performed for ten EOSs from Table 1. Mass vs equatorial radius relations for very fast rotating compact stars are presented in Fig. 5. The shape of the $M(R_{eq})$ curves is more and more flat as rotational frequency increases. For $f_{\text{rot}} = 1600$ Hz the curves $M(R_{eq})$ are almost horizontal. At that frequency the mass for each EOS is quite well defined. Moreover, curves for different EOSs are usually well separated. Both features are of a great practical importance. In principle, they are very useful for selecting ”true EOS”, provided one detects a very fast pulsar and is simultaneously able to measure its mass.

A dotted curve in every panel of Fig. 5 corre-
Fig. 5. $M$ vs $R_{\text{eq}}$ for stably rotating sub-millisecond compact stars. Notations as in Fig. 2. To indentify a curve corresponding to a specific EOS from Table 1, one has to use the sequence of open circles (ordered in the same way as in this figure) at the mass-shedding limit in Fig. 2.

$M = \frac{4\pi^2 f^2}{G} R_{\text{eq}}^3$, \hspace{1cm} (5)

used for a panel frequency (1000 Hz, 1400 Hz, 1600 Hz). Notice that Eq. (3) is a special case of Eq. (5). Equation (5) works very well in very broad range of rotational frequencies and EOSs (recently this formula has been tested by Krastev et al. (2007) for the frequency 716 Hz of PSR J1748-2446ad.)

6. Spin up by accretion

It is commonly believed that fast (millisecond) pulsars are recycled old neutron stars, spun up to kHz frequencies via a long-time disk accretion in LMXBs. Such neutron stars have a weak magnetic field ($B < 10^{10}$ G), which does not affect the accretion flow. Therefore, the accretion disk extends down to the innermost stable circular orbit (ISCO).

In the present section we study some aspects of spin-up by accretion from the ISCO. We use the prescription given by Zdunik et al. (2002, 2005). Specific angular momentum per unit baryon mass of a matter element orbiting the neutron star at the ISCO, $l_{\text{isco}}$, is calculated by solving exact equations of the orbital motion of a particle in the space-time produced by a rotating neutron star (Appendix A of Zdunik et al. 2002).

Consider accretion of an infinitesimal amount of baryon mass $dM_B$ onto a rotating neutron star. As the star is assumed to spin up in a quasi-stationary manner, accretion of $dM_B$ leads to a new rigidly rotating configuration of mass $M_B + dM_B$ and angular momentum $J + dJ$, with

$dJ = x_l l_{\text{isco}} dM_B$. \hspace{1cm} (6)

Here, $x_l$ denotes the fraction of the angular momentum of the matter element, transferred to the star. The remaining fraction, $1 - x_l$, is assumed to be lost via radiation or other dissipative processes. Let us consider two values of $x_l$: $x_l = 1$ and $x_l = 0.5$. In Fig. 6 we plot the curves $M(R_{\text{eq}})$, corresponding to the spin-up from $f = 0$ to (final) $f_{\text{rot}} = 1400$ Hz. Calculations are performed for the DH EOS. Point F on this curve corresponds to the onset of instability with respect to axi-symmetric oscillations (con-
which can be spun-up to a given frequency are bounds on the initial mass of a non-rotating star. Let us consider several spin-up tracks, all terminating on the 1400 Hz mass-radius line. The curves starting at points A, B, C, and D are the tracks of accreting neutron stars defined by the Eq. (6), for cases C, D, and A, B, respectively. In order to reach the frequency 1400 Hz, one has to start with a non-rotating neutron star located in the segment CD (if the frequency 1400 Hz, one has to start with a non-rotating star. Left "triangle" was obtained for \( x_1 = 0.5 \), while the central shaded "triangle" (with points C and D) - for \( x_1 = 1 \).

The limits on the actual mass of rotating NS which was formed by disk accretion onto an initial static star are given by the three curves on the right in Fig. 7. These curves pass by three points E, F, and G in Fig. 6. The curve passing by the point E (green) is the bound resulting from the Keplerian limit of a rotating star. The (magenta) line passing by the point F is the boundary resulting from the instability with respect to axi-symmetric perturbations. Finally, curve passing through the point G is a locus of maxima on the mass-radius curve at a fixed final rotation frequency: point G is obtained for 1400 Hz. As the frequency of the final configuration increases, the mass at the Keplerian limit increases more rapidly, than that defined by the onset of the axi-symmetric instability at the maximum mass (at 1400 Hz: points F and G, respectively). For very high frequencies the maximum mass of the stars rotating at a fixed frequency is given by the value for Keplerian limit. In the considered case of the DH EOS, the point G disappears at frequency \( \gtrsim 1500 \) Hz (point \( G_{\text{max}} \)). For faster rotation the curve \( M(R_{\text{eq}}) \) is monotonous (no maximum between the both ends). However, the range of masses of stars rotating at so high frequency is very narrow. For \( f_{\text{rot}} > 1400 \) Hz it is smaller than 0.1\( M_\odot \) (see also discussion of Fig. 6 in Sect. 5).

7. Discussion and conclusions

Let us summarize main results reported in the present paper, and obtained using a large set of theoretical EOSs. The \( M(R_{\text{eq}}) \) curve for \( f \gtrsim 1400 \) Hz (rotation period \( \lesssim 0.7 \text{ ms} \)) is flat. Therefore, at such frequencies, for any given EOS the mass of a stably rotating compact star is quite well defined. Conversely, a measured mass of a compact star rotating at \( f \gtrsim 1400 \) Hz will allow us to unveil the actual EOS of dense matter. The "Newtonian" formula for the Keplerian frequency reproduces surprisingly well precise 2-D simulations and sets a firm upper limit on \( R_{\text{eq}} \) for a given \( f \). Finally, observation of \( f \gtrsim 1200 \) Hz sets stringent limits on the initial mass of a non-rotating star which was spun up to this frequency via accretion from a disk.

In the present paper we limited ourselves to hadronic stars, built exclusively or predominantly...
of hadrons. We did not discuss our results obtained for hypothetical compact stars built of a self-bound quark matter, called quark stars or strange stars. Some of results for quark stars with and without crust were briefly reported in Ref. [Bejger et al. (2007)]. Generally, most of general features and relations obtained for hadronic stars hold also for quark stars. However, in some cases one notices systematic difference between rapidly rotating hadronic and quark stars. Our results will be described in a forthcoming publication.

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References

Akmal, A., Pandharipande, V.R., Ravenhall, D.G., 1998, Phys.Rev. C, 58, 1804
Backer, D.C., Kulkarni, S.R., Heiles, C., et al., 1982, Nature, 300, 615
Balberg, S., Gal, A., 1997, Nucl. Phys. A., 625, 435
Baldo, M., Bombaci, I., Burgio G.F. 1997, A & A, 328, 274
Bejger, M., Haensel, P., Zdunik, J. L., 2007, A&A, 464, 49
Bombaci, I., 1995, in: Perspectives on Theoretical Nuclear Physics, ed. by I. Bombaci, A. Bonaccorso, A. Fabrocini, et al. (Pisa: Edizioni ETS), p. 223
Bonazzola, S., Gourgoulhon, E., Salgado, M., Marck J.-A., 1993, A & A, 278, 421
Cook, G. B., Shapiro, S.L., Teukolsky, S.A., 1994a ApJ, 424, 823
Douchin, F., Haensel, P., 2001, A & A, 380, 151
Glendenning, N. K. 1985, ApJ, 293, 470
Glendenning, N. K., 2000, Compact Stars: Nuclear Physics, Particle Physics, and General Relativity, Springer, New York
Glendenning, N. K., Moszkowski, S.A., 1991, Phys. Rev. Lett., 67, 2414
Glendenning, N.K., Schaffner-Bielich, J., 1999, Phys. Rev. C, 60, 025803
Haensel, P., Zdunik, J.L., 1989, Nature, 340, 617
Haensel, P., Lasota, J.-P., Zdunik, J.L., 1999, Astron. Astrophys., 344, 151
Haensel, P., Potekhin, A.Y., Yakovlev, D.G., 2007 Neutron Stars I. Equation of State and Structure, (Springer, New York)
Hessels, J.W.T., Ransom, S.M., Stairs, I.H., Freire, P.C.C., Kaspi, V.M., Camilo, F., 2006, Science, 311, 1901
Gourgoulhon, E., Haensel, P., Livine, R., Paluch, E., Bonazola, S., Marck, J.-A., 1999, A&A, 349, 851 Kaaret, P., Prieskorn, Z., in’t Zand, J.J.M., Brandt, S., Lund, N., Mereghetti, S., Goetz, D., Kuulkers, E., Tomsick, J.A., 2007, ApJ Letters, 657, 97
Kraastev P.G., Li B., Worley A., 2007, arXiv:0709.3621
Lasota, J.-P., Haensel, P., Abramowicz, M.A., 1996, ApJ, 456, 300
Pandharipande, V.R., Ravenhall, D.G., 1989, in: Proc. NATO Advanced Research Workshop on nuclear matter and heavy ion collisions, Les Houches, 1989, ed. M. Soyeur et al. (Plenum, New York, 1989), 103
Pons, J.A., Reddy, S., Ellis, P.J., Prakash, M., Lattimer, J.M., 2000, Phys. Rev. C, 62, 035803
Salgado, M., Bonazzola, S., Gourgoulhon, E., Haensel, P., 1994, A & A 108, 455
Shapiro, S.L., Teukolsky, S.A., Wasserman, I., 1983, ApJ, 272, 702
Zdunik, J. L., Haensel, P., Gourgoulhon, E., 2002, A&A, 381, 933
Zdunik, J. L., Haensel, P., Bejger, M., 2005, A&A, 441, 207