Singlet S-wave superfluidity of proton in neutron star matter

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The possible $^1S_0$ protonic superfluidity is investigated in neutron star matter, and the corresponding energy gap as a function of baryonic density is calculated on the basis of BCS gap equation. We have discussed particularly the influence of hyperon degrees of freedom on $^1S_0$ protonic superfluidity. It is found that the appearance of hyperons leads to a slight decrease of $^1S_0$ protonic pairing energy gap in most density range of existing $^1S_0$ protonic superfluidity. However, when the baryonic density $\rho_B > 0.377$ (or 0.409) fm$^{-3}$ for TM1 (or TMA) parameter set, $^1S_0$ protonic pairing energy gap is significantly larger than the corresponding values without hyperons. And the baryonic density range of existing $^1S_0$ protonic superfluidity is widen due to the appearance of hyperons. In our results, the hyperons not only change the EOS and bulk properties but also change the size and baryon density range of $^1S_0$ protonic superfluidity in neutron star matter.

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I. INTRODUCTION

The dense neutron star (NS) matter shows us an interesting subject to study the properties of nucleon matter with the density higher than nuclear saturation density $\rho_0$. There, it has pointed that the various new degrees of freedom, such as hyperons, quarks and their mixed phases, realize according to the density $[1,3]$. It is well known that NS matter has the properties of the strong degeneracy and exists the attractive interaction between two baryons, which are the conditions for the occurrence of superfluid states in Fermi systems. Thus, NS is already considered as the key laboratories of various superfluidity in nuclear matter $[4-13]$. In recent years, many studies have been focussing on $^1S_0$ protonic superfluidity. Since the protonic superfluid states as well as their pairing strength can greatly suppress the neutrino processes involving nucleons about $10^5$ – $10^6$ years of NS cooling phases, affect the properties of rotating dynamics, post-glitch timing observations and possible vertex pinning of NSs $[14-22]$.

The possibility of protonic superfluid states in NS matter is first suggested by Migdal in 1960 $[23]$. The interaction between two protons is the combination of strong repulsive short-range interaction and weaker attractive long-range interaction. In proton matter, when the interparticle distance is much larger than the range of the repulsive interaction, protons will condense into superfluid states due to the attractive interaction. For the size of $^1S_0$ protonic pairing energy gap, the calculations based on the microscopic theory have already carried out a great amount of work. All the numerical calculations yield qualitatively similar ranges for the appearance of $^1S_0$ proton pairing. However, obtaining the exact numerical results for $^1S_0$ protonic pairing energy gap and evaluating its quantitative influence on NS have been proved to be a hard problem, which is due to that there are many uncertainties about the proton-proton interaction (pp) interactions in NS matter, methods of approximation, paucity of experimental data in extreme conditions and so on. During this period, lots of researches of NS have been performed adopting various frameworks. Currently, many relativistic models call attention in researches on NS since they are well suited to describe NS in accord with the special relativity. The most commonly used among them is the relativistic mean field (RMF) model, extremely successful in nuclear matter studies $[24-26]$.

This article mainly does the following work. We study NS matter for two cases: (i)NS is made up of neutron, proton, electron and muon only (npe$\mu$), (ii)NS is composed of nucleons, hyperons (Λ and Ξ), electron and muon (npHe$\mu$). Our model excludes Σ hyperons on account of the remaining uncertainty of the form of Σ potential in nuclear matter at the saturation density $[27, 28]$. We use the RMF theory to describe the properties of NS. The $^1S_0$ protonic pairing energy gap is calculated by the Reid soft core (RSC) potential $[29, 30]$. We mainly focus on the influence of hyperon degrees of freedom on $^1S_0$ protonic pairing energy gap in NS matter.

II. THE MODELS

In the RMF theory, baryonic interactions are described by the exchanged mesons including isoscalar scalar and vector mesons $\sigma$ and $\omega$, an isovector vector meson $\rho$, two additional strange mesons $\sigma^*$ and $\phi$. The total lagrangian density of NS matter is $[24,26]$,

$$
L = \sum_B \bar{\psi}_B [i\gamma_\mu \partial^\mu - (m_B - g_{\sigma B} \sigma - g_{\sigma B} \sigma^* - g_{\rho B} \rho \cdot \tau \cdot \rho)^\mu] - g_{\omega B} \gamma_{\mu} \omega^\mu - g_{\phi B} \gamma_{\mu} \phi^\mu] \psi_B + \frac{1}{2} (\partial_{\mu} \sigma \partial^{\mu} \sigma - m_{\sigma}^2 \sigma^2) - \frac{1}{3} a \sigma^3 - \frac{1}{4} b \sigma^4 + \frac{1}{4} m_{\omega}^2 \omega_{\mu} \omega^\mu + \frac{1}{4} c_3 (\omega_{\mu} \omega^\mu)^2 + \frac{1}{2} m_{\rho}^2 \rho_{\mu} \rho^\mu - \frac{1}{4} F_{\mu\nu} F^\mu\nu - \frac{1}{4} G_{\mu\nu} G^\mu\nu + \frac{1}{2} (\partial_{\mu} \sigma^* \partial^{\mu} \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) - \frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} m_{\phi}^2 \phi_{\mu} \phi^{\mu} + \sum_l \bar{\psi}_l [i\gamma_\mu \partial^\mu - m_l] \psi_l. \tag{1}
$$

Here the field tensors of the vector mesons, $\sigma$ and $\omega$, are denoted by $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, and $G_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$.

The meson fields are seen as classical fields and field operators are instead of their expectation values in the RMF
approximation. The field equations derived from the Lagrange function are

\[ \sum_B g_{\sigma B} \rho_{SB} = m_\sigma^2 \sigma + a \sigma^2 + b \sigma^3, \]  
\[ \sum_B g_{\omega B} \rho_B = m_\omega^2 \omega_0 + c_3 \omega_0^3, \]  
\[ \sum_B g_{\rho B} \rho_B I_{3B} = m_\rho^2 \rho_0, \]  
\[ \sum_B g_{\sigma B} \rho_{SB} = m_\sigma^2 \sigma^*, \]  
\[ \sum_B g_{\phi B} \rho_B = m_\phi^2 \phi_0. \]

Here \( I_{3B} \) denotes baryonic isospin projection. \( \rho_{SB} \) and \( \rho_B \) are baryonic scalar and vector densities, respectively. They have the following form,

\[ \rho_{SB} = \frac{2J_B + 1}{2\pi^2} \int_0^{k_{FB}} \frac{m_B^2 k^2 dk}{\sqrt{k^2 + m_B^2}}, \]
\[ \rho_B = \frac{k_{FB}^3}{3\pi^2}. \]

Here \( J_B \) is baryonic spin projection, \( k_{FB} \) is baryonic Fermi momentum, \( m_B = m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^* \) is baryonic effective mass.

For NS matter consisting of n, p, Λ, Ξ\(^0\), Ξ\(^-\), e, μ, the charge neutrality condition is given by

\[ \rho_p = \rho_{\Xi^-} + \rho_e + \rho_\mu. \] (8)

The \( \beta \) equilibrium conditions are expressed as

\[ \mu_p = \mu_n - \mu_e, \quad \mu_{\Xi^-} = \mu_n + \mu_e \]
\[ \mu_\Lambda = \mu_{\Xi^0} = \mu_n, \quad \mu_\mu = \mu_e. \] (9)

At zero temperature the chemical potential of baryon and lepton are written by

\[ \mu_B = \sqrt{k_{FB}^2 + m_B^2} + g_{\omega B} \omega_0 + g_{\rho B} \rho_0 I_{3B} + g_{\phi B} \phi_0, \]
\[ \mu_{\Lambda} = \sqrt{k_{FL}^2 + m_{\Lambda}^2}. \] (10)

The \(^1S_0\) protonic pairing energy gap \( \Delta_p \) can be obtained by solving the BCS gap equation,

\[ \Delta_p(k) = -\frac{1}{4\pi^2} \int k^2 dk' \frac{V(k, k') \Delta_p(k')}{\sqrt{\varepsilon^2(k') + \Delta_p^2(k')}}, \] (11)

where \( \varepsilon(k') = E_p(k') - E_p(k_{FP}), E_p(k') = \sqrt{k'^2 + m_p^2} + g_{\omega p} \omega_0 + g_{\rho p} \rho_0 I_{3p} + g_{\phi p} \phi_0 \) is protonic single particle energy.

For the pp interaction, we adopt the RSC potential which is well suited for applying in NS matter. \( V(k, k') \) is defined the matrix element of \(^1S_0\) component of RSC potential in momentum space,

\[ V(k, k') = \langle k | V(^1S_0) | k' \rangle = 4\pi \int r^2 dr j_0(kr)V_{pp}(r) j_0(k'r). \] (12)

Here \( V_{pp}(r) \) is \(^1S_0\) pp interaction potential in coordinate space. It is expressed in the five-range Gaussian and depends on \( \rho_B \), asymmetry parameter \( \alpha = (\rho_n - \rho_p)/\rho_N \) and two-nucleon state \( \beta \) and \( \gamma \) as discussed in Refs.\[24, 30\],

\[ V_{NN}(r) = \sum_{i=1}^{5} c_i(\rho_N, \alpha, \beta, \gamma) e^{-r^2}. \] (13)
TABLE I. The coupling constants of the meson-nucleon of TM1 and TMA sets, the masses are unit of MeV\(^2\).\(^{2,3}\)

| Set     | \(m_\sigma\) | \(g_\sigma N\) | \(g_\omega N\) | \(g_\rho N\) | \(a\)  | \(b\)  | \(c\)  |
|---------|--------------|----------------|----------------|---------------|-------|-------|-------|
| TM1     | 511.198      | 10.0289        | 12.6139        | 4.6322        | 7.2325 | 0.6183 | 71.3075 |
| TMA     | 519.151      | 10.055         | 12.842         | 3.8           | 0.328 | 38.862| 151.59 |

TABLE II. The scalar coupling constants for hyperons in two sets, the masses are unit of MeV\(^2\).\(^{2,3}\)

| Set     | \(m_\omega\) | \(m_\rho\) | \(g_\sigma \Lambda\) | \(g_\sigma \Xi\) | \(g_\rho \Lambda\) | \(g_\rho \Xi\) |
|---------|--------------|------------|-----------------------|-----------------|------------------|---------------|
| TM1     | 783.0        | 770.0      | 6.2380                | 3.1992          | 3.7257           | 11.5092       |
| TMA     | 781.95       | 768.1      | 6.2421                | 3.2075          | 4.3276           | 11.7314       |

The critical temperature \(T_{cp}\) of \(^1S_0\) protonic superfluidity is given by its energy gap \(\Delta_p\) at zero temperature approximation\(^{[31]}\),

\[
T_{cp} \approx 0.66\Delta_p.
\]  

(14)

Combining Eqs.(1)-(6) with the charge neutrality and \(\beta\) equilibrium conditions, Eqs.(8) and (9), we can solve the system with a fixed \(\rho_B\). Thus, \(^1S_0\) protonic pairing energy gap \(\Delta_p\) and critical temperature \(T_{cp}\) can be obtained from Eqs.(11)-(14).

III. DISCUSSION

As mentioned above, the onset of \(^1S_0\) protonic superfluidity is determined by the energy gap function \(\Delta_p\). Theoretical calculation of \(\Delta_p\) is sensitively dependent on the model of pp interaction and many-body theory adopted. In this paper, we use the RSC potential as an example to analyze \(^1S_0\) protonic superfluidity in npe\(\mu\) and npHe\(\mu\) matter, respectively. We mainly focus on the influence of hyperon degrees of freedom on \(^1S_0\) protonic superfluidity. In order to make the results more clearly, we employ two successful parameter sets of TM1 and TMA to calculate separately \(^1S_0\) protonic pairing energy gap in NS matter. The parameter sets are listed in Table I and II. For the vector couplings of hyperons, we take the relations derived from SU(6) quark model(see Ref\([2,3,32]\) for details),

\[
\frac{2}{3}g_\omega N = g_\omega \Lambda = \frac{2}{3}g_\omega \Xi, \quad g_\rho N = g_\rho \Xi;
\]

\[
2g_\phi \Lambda = g_\phi \Xi = -\frac{2\sqrt{2}}{3}g_\omega N.
\]

FIG. 1. The EOS in npe\(\mu\) and npHe\(\mu\) matter for TM1 and TMA parameter sets.
FIG. 2. Baryonic fraction $Y_i = \rho_i/\rho_B$ as a function of baryonic density $\rho_B$ in np$\mu$ and npHe$\mu$ matter for TM1(top panel) and TMA (bottom panel) parameter sets.

FIG. 3. Protonic single-particle energy $E_p(k)$ vs baryonic density $\rho_B$ in np$\mu$ and npHe$\mu$ matter for TM1(top panel) and TMA (bottom panel) parameter sets.

FIG. 4. $^1S_0$ protonic pairing energy gap $\Delta_p(k)$ at the Fermi surface as a function of baryonic density $\rho_B$ in np$\mu$ and npHe$\mu$ matter for TM1(top panel) and TMA (bottom panel) parameter sets.
TABLE III. The peak values of $^1$S$_0$ protonic pairing energy gap $\Delta^p_{\text{max}}$ and the corresponding critical temperature $T_{cp}^{\text{max}}$, baryonic density $\rho_B$ in npe$\mu$ and npHe$\mu$ matter, for TM1 and TMA parameter sets.

| Set       | $\Delta^p_{\text{max}}$ | $T_{cp}^{\text{max}}$ | $\rho_B$  |
|-----------|--------------------------|------------------------|-----------|
| TM1 npe$\mu$ | 1.560                    | $1.060 \times 10^{10}$ | 0.174     |
| TM1 npHe$\mu$ | 1.528                    | $1.008 \times 10^{10}$ | 0.175     |
| TMA npe$\mu$  | 1.068                    | $7.049 \times 10^{9}$  | 0.169     |
| TMA npHe$\mu$ | 1.052                    | $6.943 \times 10^{9}$  | 0.170     |

Fig.1 shows the EOS, pressure $P$ versus energy density $\varepsilon$ in npe$\mu$ and npHe$\mu$ matter, for parameter sets TM1 and TMA. From Fig.1 one can see that at low densities the EOSs are unchanged in npe$\mu$ and npHe$\mu$ matter for two parameter sets. However, as baryonic density increases, the $\Lambda$, $\Xi$, EOS becomes softer in npe$\mu$ and npHe$\mu$ matter. From Fig.1 one can see that at low densities the EOSs are unchanged in npe$\mu$ and npHe$\mu$ matter for two parameter sets. This is because that the occurrence of hyperons suppresses protonic fraction due to the conditions of the charge neutrality and $\beta$ equilibrium (see Eqs.(8) and (9)) in NS matter. Then according to Eq.(7), when hyperons appear in NS, $k_{FP}$ becomes smaller than the corresponding values in npe$\mu$ matter. Fig.3 shows protonic single particle energy $E_p$ as a function of baryonic density $\rho_B$ in npe$\mu$ and npHe$\mu$ matter, for parameter sets TM1 and TMA. In Fig.3, one can see that $E_p$ in npHe$\mu$ matter is also less the corresponding values in npe$\mu$ matter for two parameter sets. This is due to the decrease of protonic Fermi momentum in npHe$\mu$ matter (see Fig.2).

Up to now, we do not known $^1$S$_0$ protonic pairing energy gap increase or decrease, if hyperons appear in NS matter. As the uncertainty of the pp interaction, we calculate $\Delta_p$ on the basis of the RSC potential. We concentrate on the influence of hyperon degrees of freedom on $^1$S$_0$ protonic pairing energy gap in NS matter. The peak values of $^1$S$_0$ protonic pairing energy gap $\Delta^p_{\text{max}}$ and the corresponding critical temperature $T_{cp}^{\text{max}}$, baryonic density $\rho_B$ in npe$\mu$ and npHe$\mu$ matter are listed in Table III, for the TM1 and TMA parameter sets. As shown in Table III, the appearance of hyperons makes $\Delta^p_{\text{max}}$ and $T_{cp}^{\text{max}}$ decrease which will inevitably cause the cooling rate of NS changing. In Fig.4, we show $^1$S$_0$ protonic pairing energy gap $\Delta_p$ as a function of baryonic density $\rho_B$ in npe$\mu$ and npHe$\mu$ matter, for parameter sets TM1 and TMA. As seen from Fig.4, the protonic pairing energy gap $\Delta_p$ as the function of $\rho_B$ is a typical bell-shaped curve from zero to a maximum value to zero again. The change of $\Delta_p$ in behavior is due to the reduction of mean interparticle distance. From Fig.4, we also can see that $^1$S$_0$ protonic superfluidity appears within the baryonic density range of $\rho_B = 0.0 - 0.649$ (0.685) fm$^{-3}$ and $\rho_B = 0.0 - 0.953$ (0.853) fm$^{-3}$ for parameter sets TM1 (or TMA) in npe$\mu$ and npHe$\mu$ matter, respectively. It is clear from these data that the appearance of hyperons makes baryonic density range of $^1$S$_0$ protonic superfluidity widen. Such baryonic density ranges can cover or partially cover the cores of NSs and are highly relevant to the direct Urca processes involving nucleons which play a leading role in NS cooling. In particular, from Fig.4, the appearance of hyperons also leads to $^1$S$_0$ protonic pairing energy gap in npHe$\mu$ matter obviously larger than the corresponding values in npe$\mu$ matter when baryonic density $\rho_B \geq 0.377$ (0.409) fm$^{-3}$ for TM1 (or TMA) parameter set (see the two dots shown in Fig.4). According to Eq.(14), the critical temperature of $^1$S$_0$ protonic superfluidity positively increases which could further suppress the cooling rate of NS.

IV. CONCLUSION

We study the influence of hyperon degrees of freedom on the size and baryonic density range of $^1$S$_0$ protonic pairing energy gap by adopting the RMF and BCS theories in NS matter. It is shown that the appearance of hyperons makes the peak values of $^1$S$_0$ protonic pairing energy gap and critical temperature decrease. And baryonic density range of existing $^1$S$_0$ protonic superfluidity is widen from $0.0 - 0.649$ (0.685) fm$^{-3}$ in npe$\mu$ matter to $0.0 - 0.953$ (0.853) fm$^{-3}$ for TM1 (or TMA) parameter sets in npHe$\mu$ matter. In addition, when the baryonic density $\rho_B \geq 0.377$ (0.409) fm$^{-3}$ for TM1 (or TMA) parameter set, the appearance of hyperons leads to $^1$S$_0$ protonic pairing energy gap obviously larger than the corresponding values in npe$\mu$ matter which could further suppress the cooling rate of NSs. In our results, the appearance of hyperons in NS matter not only changes the EOS and bulk properties but also changes the
properties of $^1S_0$ protonic superfluidity.

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