Double transition in kinetic exchange opinion models with activation dynamics

Marcelo A. Pires and Nuno Crokidakis

1Departamento de Física, Universidade Federal do Ceará, Fortaleza, Ceará, Brazil
2Instituto de Física, Universidade Federal Fluminense, Niterói, Rio de Janeiro, Brazil

In this work, we study a model of opinion dynamics considering activation/deactivation of agents. In other words, individuals are not static and can become inactive and drop out from the discussion. A probability \( w \) governs the deactivation dynamics, whereas social interactions are ruled by kinetic exchanges, considering competitive positive/negative interactions. Inactive agents can become active due to interactions with active agents. Our analytical and numerical results show the existence of two distinct non-equilibrium phase transitions, with the occurrence of three phases, namely ordered (ferromagnetic-like), disordered (paramagnetic-like) and absorbing phases. The absorbing phase represents a collective state where all agents are inactive, i.e. they do not participate in the dynamics, inducing a frozen state. We determine the critical value \( w_c \) above which the system is in the absorbing phase independently of the other parameters. We also verify a distinct critical behaviour for the transitions among different phases.

This article is part of the theme issue ‘Kinetic exchange models of societies and economies’.

1. Introduction

Opinion dynamics is one of the hottest topics of research in statistical physics of complex systems [1–6]. One of the reasons for this interest is that even simple models can exhibit a complex collective behaviour that emerges from the interactions among individuals. Usually, those models exhibit phase transitions and rich
critical phenomena, which justifies the theoretical interest of physicists in the study of opinion dynamics.

In most opinion dynamics models, all agents are permanently active and have chances to interact with other agents [7–10]. However, many social environments especially online communities are not static in a way that agents can become inactive and withdraw from the discussion. In other words, social users do not concentrate on the discussion all the time, and they may lose interest and drop out of it [11,12]. However, these dormant agents can become active again following peers [13]. Some works have verified that the final average opinion depends significantly on the external influence and internal actions. Small external activation drives the initially inactive system to total consensus quickly, but large external deactivation is required to freeze the active dynamics [14]. This kind of activation dynamics leads to interesting results. The authors in [11] verified that if dissipation stays below a threshold value, the system evolves to a balance (paramagnetic) state where the average concentration of one opinion is equal to that of the other. On the other hand, it was verified that, in activation-like opinion dynamics that evolves on the top of complex networks, the topology of the network also evolves in time [13]. The authors verified that the model has power-law degree distribution, and clustering coefficients stay higher than results in Barabási–Albert networks. Another study revealed that under the impact of external circumstances, the population can evolve to distinct stationary states. On one hand, one opinion can finally be made dominant when the internal motivation is sufficiently large. On the other hand, without external activation, consensus is hardly reached in the system with interest decay [13].

On the other hand, we have the kinetic exchange opinion models (KEOM), that have been subject of study since the work of Lallouache–Chakrabarti–Chakraborti–Chakrabarti (LCCC) [15]. The LCCC model introduced a dynamic rule for opinion dynamics based on models of wealth exchange. The model considered continuous opinions in the range [-1.0,1.0], but discrete opinions were also considered in [16]. Several extensions for discrete and continuous opinions were also studied, considering, for example, the impact of agents' convictions [17], social temperature [18], inflexibility [19], non-conformist behaviours [20], dynamic individual influence [21], the presence of contrarian individuals [22], influencing ability of individuals [23,24], the relationship between coarsening and consensus [25], competition between noise and disorder [26], the analysis of noise-induced absorbing phase transitions [27] and the presence of distinct interaction rules [28]. The model was also considered in infinite dimensional lattices [29] (including applications to the 2016 presidential election in USA [30]), in triangular, honeycomb, and Kagome lattices [31], in quasi-periodic lattices [32], in modular networks [33] and in other complex networks [34].

Taking into account those two subjects, namely activation dynamics in opinion formation and kinetic exchange opinion models, we propose the inclusion of activation dynamics considering the KEOM social interaction rules. Co-evolution spreading was considered in other works [35–44]. Here we consider that the activation dynamics follows a contact-like process [45], and the social interactions are ruled by the KEOM discussed in [24].

This work is organized as follows. In §2, we discuss our co-evolution dynamics, and present the microscopic rules that govern our model. The analytical and numerical results are discussed in §3. Finally, the conclusion and final remarks are presented in §4.

2. Model

We consider a fully connected population with $N$ agents. The agents can be classified as follows:

(I) Opinion: $o_i = +1$ if an agent $i$ supports opinion $+1$, $o_i = -1$ if $i$ supports opinion $-1$, $o_i = 0$ if $i$ is undecided/neutral.

(II) Activation status: $s_i = 1$ if an agent $i$ is active or $s_i = 0$ if $i$ is inactive.

Considering the social interactions, as explained below, we will follow the Biswas–Chatterjee–Sen (BCS) model [24]. In the BCS model, two agents $i$ and $j$ are randomly chosen, and they interact...
through the following rule:

$$o_i(t + 1) = o_i(t) + \mu_{ij} o_j(t).$$  

(2.1)

This expression shows how the opinion of a given agent $i$ in a given time step $t + 1$ is updated due to an interaction with another agent $j$. The first term in the right side of the equation indicates the tendency of agent $i$ to keep his/her current opinion in the time step $t$, but the opinion can be influenced by another agent $j$. The coupling $\mu_{ij}$ represents the strength of the pairwise interaction. Pairwise interaction strengths are annealed random variables distributed according to the binary probability density function (PDF)

$$F(\mu_{ij}) = p \delta(\mu_{ij} + 1) + (1 - p) \delta(\mu_{ij} - 1).$$  

(2.2)

In other words, the agents can exchange opinions with positive (+1) or negative (−1) influences, and $p$ quantifies the mean fraction of negative ones. Notice that, in equation (2.1), if the value of the opinion exceeds (falls below) the value 1 (−1), then it adopts the extreme value 1 (−1) [15]. The model defined by equations (2.1) and (2.2) presents only one parameter, $p$, and it undergoes an order–disorder phase transition at a critical value $p_c = 1/4$, i.e. the competitive positive/negative interactions are responsible for such ferromagnetic–paramagnetic transition [24].

In this work, we propose the inclusion of the above-mentioned activation state of agents (see above rule II) in the BCS model. In such a case, at each time step the following rules govern the dynamics of our model:

--- Select two agents $i$ and $j$;
--- If $i$ is active:
  
  • with probability $w$ apply the deactivation process: $s_i = 1 \rightarrow s_i = 0$.  
  (Rule 1).
  • with probability $1 - w$, the agents $i$ and $j$ follow the BCS model: $o_i(t + 1) = o_i(t) + \mu_{ij} o_j(t)$, where $\mu_{ij}$ follows the PDF of equation (2.2)  
  (Rule 2).
--- If $i$ is inactive:

  if $i$ and $j$ have the same opinion ($o_i = o_j$) and $j$ is active: apply the activation process $s_i = 0 \rightarrow s_i = 1$ with probability $(1 - p)g$.  
  (Rule 3).

As discussed in the Introduction, during a public debate there are people that are not active/open for discussion. But they can become active due to peers’ influence. As discussed in [11,14], active agents may gradually lose their interests in the discussion and drop out of it, which is related to our above Rule 1. In addition, our above Rule 2 is based on the fact that only active agents participate in the opinion dynamics. Finally, as also discussed in [11,14], active agents can motivate their inactive like-minded peers. If an inert agent holds the same opinion of an active neighbour and they have a positive interaction (with takes place with probability $1 - p$), then the inert agent is driven to be active with probability $(1 - p)g$. In other words, in order to have an activation with probability $g$, we considered as a necessary condition that the randomly chosen agents $i$ and $j$ have to interact positively, which occurs with probability $1 - p$, leading to the final probability $(1 - p)g$. These last sentences justify our above Rule 3.

3. Results

In a given time step $t$, let $x_{+1}(t)$, $x_0(t)$ and $x_{-1}(t)$ be the proportion of inactive agents with opinions +1, 0 and −1, respectively. Also, let $f_{+1}(t)$, $f_0(t)$ and $f_{-1}(t)$ be the proportion of active agents with opinions +1, 0 and −1, respectively. Following the analytical procedure of refs. [16, 17, 24] and the rules defined in the previous section (Rules 1, 2 and 3), we can write the master equations for the
evolution of the fractions of inactive agents as follows:

\[
\begin{align*}
\text{(Inactive with opinion +1)} & \quad \frac{dx_+}{dt} = wf_+ - (1-p)gx_+f_+ , \\
\text{(Inactive with opinion 0)} & \quad \frac{dx_0}{dt} = wf_0 - (1-p)gx_0f_0 \\
\text{(Inactive with opinion -1)} & \quad \frac{dx_-}{dt} = wf_- - (1-p)gx_-f_- .
\end{align*}
\]

(3.1) \hspace{1cm} (3.2) \hspace{1cm} (3.3)

In addition, we have the following master equations for the evolution of the fractions of active agents with opinions \(o = +1\) and \(o = -1\):

\[
\frac{df_+}{dt} = -wf_+ + (1-p)gx_+f_+ + (1-w)\left[-pf_+^2 - (1-p)f_+f_- + (1-p)f_+f_0 + pf_-f_0\right],
\]

and

\[
\frac{df_-}{dt} = wf_- - (1-p)gx_-f_- + (1-w)\left[-pf_-^2 - (1-p)f_-f_+ + (1-p)f_-f_0 + pf_+f_0\right].
\]

(3.4) \hspace{1cm} (3.5)

Additionally, we have the normalization condition:

\[
x_{+1} + x_{-1} + f_{+1} + f_{0} + f_{-1} = 1.
\]

(3.6)

In the steady state, we have for equations (3.1)–(3.3):

\[
0 = \frac{dx_+}{dt} = wf_+ - (1-p)gx_+f_+ \Rightarrow f_+ = 0 \text{ or } x_+ = \frac{w}{(1-p)g},
\]

\[
0 = \frac{dx_0}{dt} = wf_0 - (1-p)gx_0f_0 \Rightarrow f_0 = 0 \text{ or } x_0 = \frac{w}{(1-p)g}.
\]

(3.7) \hspace{1cm} (3.8)

and

\[
0 = \frac{dx_-}{dt} = wf_- - (1-p)gx_-f_- \Rightarrow f_- = 0 \text{ or } x_- = \frac{w}{(1-p)g}.
\]

(3.9)

Thus, the fraction of active agents, \(\rho = f_{+1} + f_0 + f_{-1}\), in the steady state is obtained from the normalization condition equation (3.6) written in the form \(f_{+1} + f_0 + f_{-1} = 1 - (x_{+1} + x_{0} + x_{-1})\). Considering the results (3.7), (3.8) and (3.9) for \(x_{+1}, x_{0}\) and \(x_{-1}\), respectively, we have

\[
\rho = 1 - \frac{3w}{(1-p)g}.
\]

(3.10)

The importance of such fraction \(\rho\) will be discussed in the following.

We are interested in the critical behaviour of the model. Thus, let us discuss the order parameter, \(m\). It is sensitive to the imbalance between extreme opinions \(+1\) and \(-1\). Notice that \(m\) plays the role of the ‘magnetization per spin’ in magnetic systems [17]. Since the order parameter can be defined as \(m = |f_{+1} - f_{-1}|\), we have

\[
0 = \frac{dm}{dt} \Rightarrow \frac{df_+}{dt} = \frac{df_-}{dt} \Rightarrow (f_+ - f_-)|p(f_{+1} + f_{-1}) + (1-2p)f_0| = 0.
\]

(3.11)

Then

\[
f_+ = f_- \Rightarrow m = 0,
\]

(3.12)

which represents a disordered state solution, or

\[
f_+ + f_- = \frac{1}{p} - 2p f_0.
\]

(3.13)

Using that \(f_+ + f_- = \rho - f_0\) we get

\[
f_0 = \frac{p}{1-p} \rho.
\]

(3.14)
Inserting equation (3.14) into equation (3.4) and using equation (3.7) we obtain
\[ f_{+1}^2 - \rho \frac{1 - 2p}{1 - p} f_{+1} + \rho^2 \frac{p^2}{(1 - p)^2} = 0. \] (3.15)

Thus,
\[ f_{+1} = \frac{\rho}{2(1 - p)} \left( 1 - 2p \pm \sqrt{1 - 4p} \right). \] (3.16)

Using that \( f_{-1} = \rho - f_0 - f_{+1} \) and equations (3.10) and (3.14) we arrive at
\[ m = |f_{+1} - f_{-1}| = \left( 1 - \frac{3w}{(1-p)^g} \right) \sqrt{1 - 4p}. \] (3.17)

In the language of critical phenomena [45], one can rewrite equation (3.17) as
\[ m \sim (p_{ca} - p)^{\beta_{cp}} (p_{ci} - p)^{\beta_{ising}}, \] (3.18)

where \( \beta_{cp} = 1 \) and \( \beta_{ising} = 1/2 \) and
\[ p_{ci} = \frac{1}{4}, \quad p_{ca} = 1 - \frac{3w}{g}. \] (3.19)

Equation (3.18) predicts the occurrence of two distinct non-equilibrium phase transitions. The critical point \( p_{ci} \) is the same for the BCS model, i.e. \( p_{ci} = 1/4 \). For this first critical point, the critical behaviour suggests an Ising-like exponent \( \beta = \beta_{ising} = 1/2 \), as in the BCS model [24]. On the other hand, the second critical point is related to the activation dynamics, and it depends on the parameters \( g \) and \( w \), \( p_{ca} = 1 - \frac{3w}{g} \). For this second transition, the critical behaviour suggests a contact process-like exponent \( \beta = \beta_{cp} = 1 \). Another form to see this second transition is also considering as another order parameter the fraction of active agents, equation (3.10), written in the form \( \rho \sim (p_{ca} - p)^{\beta_{cp}} \). We will discuss those points in more detail in the following.

Based on the rules defined in §2, we performed Monte Carlo simulations of the model, in order to confirm our analytical predictions. From the simulations, we can obtain \( m \) through the definition
\[ m = \left\langle \frac{1}{N} \sum_{i=1}^{N} a_i \right\rangle. \] (3.20)

where \( \langle \ldots \rangle \) denotes average over disorder or configurations, computed at the steady states. Usually in opinion dynamics models, \( m \) is called collective opinion. In figure 1, we exhibit numerical results for \( w = 0.0, g = 1.0 \) (a) and \( w = 0.1, g = 0.8 \) (b). The lines for the order parameter \( m \) were obtained from our analytical result, equation (3.17), whereas the symbols were obtained from the simulations. In addition to the order parameter \( m \), obtained numerically from equation (3.20) (blue circles in figure 1), we also measured in the simulations the fraction of active agents (red stars in figure 1). This last result is compared with the analytical result obtained from equation (3.10).

First of all, we exhibit in figure 1a the behaviour of \( m \) and \( \rho \) for the special case \( w = 0 \), for which we have no deactivation and thus we have to recover the results of [24]. In such a case, for the order parameter \( m \) we observed the non-equilibrium phase transition at \( p_c = 1/4 \) studied in [24], and the fraction of active individuals is \( \rho = 1 \) for all values of \( p \), as expected since we have no deactivation. On the other hand, in figure 1b we exhibit the case \( w = 0.1 \) and \( g = 0.8 \). In such a case, we observed the abovementioned two distinct transitions: one for the order parameter \( m \) and another for the fraction of active agents \( \rho \). The critical points are obtained from equation (3.19), i.e. the first critical point is not modified by the deactivation process and we have \( p_{ci} = 1/4 \). However, due to the dynamics of activation/deactivation, the maximum value of the order parameters is less than 1, as previously indicated by our analytical results, equation (3.17). This is due to the presence of deactivated agents, i.e. we have now \( x_{+1} > 0, x_{-1} > 0 \) and \( x_0 > 0 \), which leads to smaller values of the active fractions \( f_{+1} \) and \( f_{-1} \) and consequently we have lower values of the collective ordering measure \( m \), even for \( p = 0 \). On the other hand, the process of
deactivation induces a second critical point that depends on \( w \) and \( g \), given by \( p_{ca} = 1 - (3w/g) \). For the parameters we considered for figure 1, we have \( p_{ca} = 0.625 \). The two non-equilibrium critical points are indicated by vertical lines in figure 1.

Notice that the two transitions are of distinct nature. The analytical results of equations (3.10), (3.18) and (3.19) suggest two distinct critical exponents \( \beta \). One of them is related to the behaviour of the magnetization near the critical point \( p_{ci} = 1/4 \), i.e. we have \( \beta = \beta_{\text{ising}} = 1/2 \). This transition is a usual ferromagnetic-paramagnetic transition. Indeed, we obtained a paramagnetic (disordered state) solution, equation (3.12). In addition, we can observe in figure 1 the usual finite-size effects for numerical results regarding ferromagnetic–paramagnetic phase transitions for values of \( m \) near \( p = p_{ci} \). On the other hand, for the second-phase transition observed for \( \rho \) at \( p = p_{ca} = 1 - 3w/g \), the analytical results suggest an active-absorbing phase transition, since the critical exponent is \( \beta = \beta_{p} = 1 \) [45]. Indeed, we also obtained an analytical result \( f_{+1} = f_{0} = f_{-1} = 0 \), see equations (3.7), (3.8) and (3.9), i.e. it is the absorbing state where \( \rho = 0 \). In this case, the fraction of active agents needs to be zero even in the computer simulations, which in fact we observed (see the red stars in figure 1b).

Figure 2 shows additional results for \( m \) and \( \rho \) for \( g = 1 \) and typical values of the deactivation probability \( w \). In figure 2a, we exhibit the curves for \( m \) and \( \rho \) for \( w = 0.2 \). As previously discussed, the maximum value of the magnetization decreases for increasing values of \( w \), as predicted by equation (3.17). In addition, the second critical point \( p_{ca} \) decreases its value and becomes near the first critical point \( p_{ci} \). One can also see from equation (3.19) that both critical points \( p_{ci} \) and \( p_{ca} \) coalesce for a given value of \( w \). Taking \( p_{ci} = p_{ca} \), we obtain that such a value of \( w \) is given by \( w = \frac{1}{4}g \). For the special case \( g = 1 \), this equation gives us \( w = 0.25 \). For this value we have \( p_{ci} = p_{ca} = 1/4 \), which is exhibited in figure 2b. The other result, \( w = 0.3 \) suggests that there is a critical value of the deactivation dynamics above which there is no phase transition, and the system is in an absorbing state for all values of \( p \). This critical value \( w_{c} \) can be obtained from equation (3.19), considering \( p_{ca}(w_{c}) = 0 \). In this case, we obtain

\[
w_{c} = \frac{1}{3}g.
\]  

(3.21)

For the special case \( g = 1 \), equation (3.21) gives us \( w \approx 0.33 \). In figure 2c, we exhibit results for a value near such critical value \( w_{c} \), namely \( w = 0.3 \). We yet observe a transition, but the values of the order parameters \( m \) and \( \rho \) are quite small, indicating the proximity of the limiting case of occurrence of phase transitions, in agreement with equation (3.21).
To summarize the results, we exhibit in figure 3 the phase diagram of the steady states of the model in the plane $p$ versus $w$, for $g = 1$. Notice the critical value $w_c \approx 0.33$, above which the system is in the absorbing state for all values of $p$. The Ordered (O) and Disordered (D) phases are also shown, and the transition between such phases is observed for the constant value $p_{ci} = 1/4$. The other boundary is obtained by the second critical point $p_{ca} = 1 - 3w/g$.

4. Final remarks

During a public debate, individuals may abandon the discussion while others may become interested in an ongoing discussion by the influence of peers. To model such a situation, we consider an activation dynamics coupled to opinion dynamics. The activation/deactivation dynamics is ruled by a contact-like process, whereas the social interactions are governed by kinetic exchanges. The results show that such coupled dynamics undergoes multiple transitions, namely: (a) from an ordered state to a disordered state; (b) from a disordered state to an absorbing state; (c) from an ordered state straight to an absorbing state. Our mean-field results suggest that the transition (a) takes place in the Ising universality class, whereas (b–c) occurs in the universality class of the contact process. The absorbing state means that all the individuals stop participating in the discussion, so the debate fades out. Our results show that even with a small interest decay rate $w$ the dynamics can be trapped in the absorbing state if the disagreement rate $p$ is too high. On the other hand, for sufficient high values of the deactivation rate $w$ the system is always in the
Figure 3. Phase diagram of the model in the plane \( p \times w \). Lines were obtained from equation (3.19) with the activation rate \( g = 1 \). Acronyms: O, Ordered phase; D, Disordered phase; A, Absorbing phase. We see the presence of multiple transitions: \( O \rightarrow D \rightarrow A \) as well as \( O \rightarrow A \). For \( w > w_c \approx 0.33 \) there is no transition, as discussed in the text. Even with a small interest decay rate \( w \) the dynamics can enter in the absorbing state if the disagreement rate \( p \) is high enough. (Online version in colour.)

absorbing phase, independently of the values of the other parameters. In future works, it would be interesting to consider a networked extension of the model studied here as well as additional social features such as plurality and polarization [10].

Data accessibility. This article has no additional data.

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