Bayesian network-based modal frequency–multiple environmental factors pattern recognition for the Xinguang Bridge using long-term monitoring data

He-Qing Mu, Zhen-Jie Zheng, Xiao-Huan Wu and Cheng Su

Abstract
Modal frequency is an important indicator reflecting the health status of a structure. Numerous investigations have shown that its fluctuations are related to the changing environmental factors. Thus, modelling the modal frequency–multiple environmental factors relation is essential for making reliable inference in structural health monitoring. In this study, the Bayesian network (BN)-based algorithm is developed for recognizing the pattern between modal frequency and multiple environmental factors. Different candidates of network structure of the BN are proposed to describe the possible statistical relations of different variables. In the BN-based pattern recognition, the learning phase conducts uncertainty quantification in both parameter and model levels; and the prediction phase makes inference under complete and incomplete observed information. Based on the long-term monitoring data, the most plausible network structure is selected, and its associated parameters are identified. The developed algorithm is then utilized for analyzing the long-term monitoring data (modal frequencies, temperature, humidity, wind speed and traffic volume) of the Xinguang Bridge (a 782-m three-span half-through arch bridge). It turns out that the selected network structure properly captures the pattern of modal frequency–multiple environmental factors.

Keywords
Bayesian network, environmental factor, model class selection, structural health monitoring

Introduction
Structural health monitoring is to infer structural health status given measurements of structural responses and environmental conditions, which has been developed and utilized for various types of infrastructures such as bridge,2–8 building structure, etc.9–16 The modal frequency is widely adopted as a global health status indicator in SHM.17,18 As monitored infrastructures are exposed to environmental conditions, evidence can be observed that the fluctuations of modal frequencies are related to the changing environmental factors such as the temperature, relative humidity, wind speed and traffic loading. Direct ignorance of the environmental effects in SHM possibly leads to misleading inference. Therefore, considerable efforts have been devoted to depicting the pattern between modal parameters and environmental factors. The modal frequency–multiple environmental factors pattern recognition contains the training/learning phase and prediction phase. Based on training SHM data
(modal frequencies and environmental factors), the training/learning phase is to select the model class and identify the parameters of modal frequency–multiple environmental factors relation. Once the model is trained, it can be utilized to predict the output data (modal frequencies) based on the given input data (multiple environmental factors) in the prediction phase. In practical SHM applications, the key is how to make accurate and robust predictions when the training input data only comprise a tiny fraction of all possible input data, which is known as generalization in pattern recognition. Cornwell et al. developed a linear model for daily temperature and modal frequencies, and they discovered that the first frequency of the Alamosa Canyon Bridge varied by 4.7% over the 24-h period. Peeters and Roeck utilized the ARX model to capture the relationship between temperatures (air temperature and stuctural temperatures) and modal frequencies by one-year monitoring data of the Z24-Bridge. Xia et al. modelled environmental conditions-modal parameters with a linear regression model by nearly two-year monitoring of a reinforced concrete slab outside the laboratory. It was found that the modal frequencies and damping ratios have strong correlation with temperature and humidity. Hua et al. modelled temperature–frequency with combined principal analysis and support vector regression technique. Liu et al. used the linear regression models to evaluate how the temperature variations influence the modal parameters. Zhou et al., Ni and Li et al. used the neural network to determine the correlation between the environmental conditions and the modal parameters. Yuen and Kuok inspected a set of model class candidates based on Bayesian probabilistic approach. Mu et al. developed a pattern recognition algorithm to select the relevance features in environmental conditions (temperature and relative humidity) and modal frequencies. Moser and Moaveni monitored a steel pedestrian bridge located on Medford. A fourth-order model, out of six regression models, without cross terms was selected as the best representative model for the relationship between the modal frequencies and the temperature. Zhang et al. used the Gaussian process regression technique to model the dependency between the bridge modal frequencies and the environmental along with operational conditions.

Since both structural dynamical responses and environmental factors exhibit significant level of uncertainty, uncertainty quantification (UQ) is essential in modal frequency–multiple environmental factors pattern recognition. Bayesian probabilistic framework-based approach has attracted special attention as it provides a rigorous solution to UQ in both parameter level and model level. As the complexity of the pattern between modal frequency–multiple environmental factors grows with the number of the environmental factors in the inference process, a sophisticated graphical model-based tool, the Bayesian network (BN), is explored and developed for pattern recognition purpose. There are three advantages of the BN. First, its inference is based on Bayesian framework, so it possesses the capacity of UQ. Second, it utilizes a graphical interpretation to depict the casual or statistical dependent relationships between different variables and it is capable of directly identifying the conditional and joint probability distributions of the variables. Third, it can make inference under incomplete observed information (missing data), which is common in SHM. Due to these advantages, it has attracted attention in geospatial and structural engineering, such as ground-motion prediction, reliability analysis, and risk assessment.

In this study, the BN-based pattern recognition is performed based on one-year modal frequency and multiple environmental (temperature, relative humidity, wind speed, and traffic) monitoring dataset of a 782-m three-span half-through arch bridge over the Pearl River of Guangzhou City of China. The remaining parts of the paper are organized as follows. Dataset and BN structure candidates are firstly presented. Then, the BN-based pattern recognition is explored and developed. Finally, pattern recognition results of modal frequency–multiple environmental factors of the Xinguang bridge are presented and the prediction capability of the BN-based model is validated.

### Dataset and BN structure candidates

The monitored structure is the Xinguang Bridge (shown in Figure 1), which is a three-span half-through arch bridge with the mid span of 428 m, two side spans of 177 m each, and width of 37.62 m, over the Pearl River of Guangzhou City of China. It is the first bridge with a combination of the steel truss arch and the concrete triangular frame in China.

The monitoring period is from 1 January to 31 December of year 2014. As the operating time of sensors was set to be uniformly distributed from 00:00 to 23:59, more than one set of data can be achieved within one day. As the monitoring system requires regular maintenance, no record is measured for those maintenance days. Totally four types of data were collected for the environmental dataset: the temperature, the relative humidity, the wind velocity, and the traffic volume. Two weather sensors, located in the mid-span deck and the side-arch crown, measured the temperature and the relative humidity in the sampling time of 1 s. The average temperature $T$ (unit: $^\circ$C) and the average relative humidity $H$ (unit: %) were achieved by averaging the corresponding measured values
from the two weather sensors. Three wind sensors, located in the mid-span deck the main-arch crown, and the side-arch crown, measured the wind velocity in the sampling time of 1 s. The average wind velocity $W$ (unit: m/s) was achieved by averaging these three measured values. Two traffic volume cameras, located in the upstream and downstream traffic lanes, measured the number of vehicles passing through the bridge within 5 min. The traffic volume $Tr$ (unit: no. of vehicles) was achieved by summing up the cumulative number of vehicles of the three upstream traffic lanes and the three downstream traffic lanes. Figure 2 shows these four environmental factors of the Xinguang Bridge of year 2014. Forty accelerometers were installed and the corresponding locations of one side
of the bridge are shown in Figure 1. Note that ‘V’ and ‘H’ represent the vertical and horizontal directions of the sensors, respectively. The measured sampling frequency is 76.8 Hz, while the adjustable measured time spans are $120 \times n$ s with $n$ ranging from 1 to 10 for different measured times and days. The data-driven random subspace method is used to identify the modal frequency of the structure. In this study, the first and second modal frequencies ($f_1$ and $f_2$, unit: Hz) of the vertical mode of the Xinguang Bridge are achieved. Figure 3 shows the fluctuations of these two frequencies.

A BN is a directed acyclic graphical (DAG) model, which is a graphical representation of the statistical relation between a set of random variables $X = (X_1, \ldots, X_{n_X})$. The structure of the BN is constructed by two types of components: a node and a directed arc. A node represents a random variable $X_i$. A directed arc pointing from node $X_j$ to node $X_i$ (like $X_j \rightarrow X_i$) represents that $X_i$ statistically depends on $X_j$. In this case, node $X_j$ is called a parent of $X_i$ and the collection of all the parent nodes of $X_i$ is denoted as $\pi(X_i)$. A node without any parent node is called a root node.

In modal frequency–multiple environmental factors pattern recognition, six nodes are required for six measured variables: $T, H, W, Tr, f_1, \text{and } f_2$. For inference purpose, discretization of data of each node is introduced. Over-finer discretization leads to the fluctuation of data probability, while over-coarser discretization leads to the masking of data probability. In this study, considering that there are 7811 points, Table 1 shows discretization of data and Figure 4 shows data histograms of different nodes.

Figure 5 shows the proposed candidates of network structure for modal frequency–multiple environmental factors pattern recognition. The candidates are proposed as follows: (1) Candidates $M_1$ and $M_3$ treat the four environmental factors ($T, H, W, Tr$) as the root nodes (i.e. the node without any parent node). As the four environmental factors are the parent nodes of the two modal frequencies ($f_1, f_2$), the directed arcs are added from the environmental factors to the modal frequencies. (2) Candidates $M_2$ and $M_4$ consider the correlation effect between the temperature $T$ and the relative humidity $H$, so they add the arc from $T$ to $H$. Note that adding the arc from $H$ to $T$ is a Markov equivalence to adding the arc from $T$ to $H$, so only one case of the arc linking between $T$ and $H$ needs to be considered in candidate construction. (3) Candidates $M_3$ and $M_4$ consider the correlation effect between $f_1$ and $f_2$, so they add the arc from $f_1$ to $f_2$. Finally, four candidates of network structure, $M_o, o = 1, \ldots, 4$, of the BN are proposed for modal frequency–multiple environmental factors pattern recognition.

**BN-based pattern recognition**

**Learning phase**

Let $\theta_{ik} = P(X_i = k | \pi(X_i) = j)$ denote the probability of node $X_i$, $i = 1, \ldots, n_X$, being in its $k$th state, $k = 1, \ldots, r_i$, given that its parent node collection $\pi(X_i)$ is in its $j$th state, $j = 1, \ldots, q_i$. It is worth to note that all the inferences...
Table 1. Discretization of data.

| Variable             | Notation | Unit | Number of bins | Bin boundaries                                                                 |
|----------------------|----------|------|----------------|-------------------------------------------------------------------------------|
| 1st frequency        | $f_1$    | Hz   | 14             | $0, \mu_{01} - 3\sigma_{01}, \mu_{01} - 2.5\sigma_{01}, \ldots, $           |
|                      |          |      |                | $\mu_{01} + 2.5\sigma_{01}, \mu_{01} + 3\sigma_{01}, +\infty$               |
| 2nd frequency        | $f_2$    | Hz   | 14             | $0, \mu_{02} - 3\sigma_{02}, \mu_{02} - 2.5\sigma_{02}, \ldots, $           |
|                      |          |      |                | $\mu_{02} + 2.5\sigma_{02}, \mu_{02} + 3\sigma_{02}, +\infty$               |
| Temperature          | $T$      | °C   | 10             | $-\infty, 0, 5, \ldots, 35, 40, +\infty$                                   |
| Relative humidity    | $H$      | %    | 10             | $0, 10, \ldots, 90, 100$                                                   |
| Wind speed           | $W$      | m/s  | 11             | $0, 1, \ldots, 9, 10, +\infty$                                             |
| Traffic loading      | $Tr$     | No. of vehicles | 19 | $0, 50, \ldots, 850, 900, +\infty$                                         |

Figure 4. Data histograms of different nodes.

Figure 5. Proposed candidates of network structure for modal frequency–multiple environmental factors pattern recognition.
and variables are conditional on the proposed network structure \( \mathcal{M}_o \), \( o = 1, \ldots, 4 \), but it is not reflected in some notation due to symbolic simplicity. For an independent and identically distributed complete dataset \( D = \{ D_1, \ldots, D_N \} \), the likelihood function conditional on \( \theta = \{ \theta_{ijk}, i = 1, \ldots, n_i, j = 1, \ldots, q_i, k = 1, \ldots, r_i \} \) of a candidate of network structure \( \mathcal{M}_o, \ o = 1, \ldots, 4 \) can be written as

\[
P(\mathcal{D} | \theta, \mathcal{M}_o) = \prod_{i=1}^{n_i} \prod_{j=1}^{q_i} \prod_{k=1}^{r_i} \theta_{ijk}^{N_{ijk}}
\]  

(1)

where \( N_{ijk} \) is the cumulative number of data points belonging to the category of \( X_i = k \) and \( \pi(X_i) = j \), and it can be obtained by

\[
N_{ijk} = \sum_{l=1}^{N} \chi(i,j,k; D_l, \mathcal{M}_o)
\]

(2)

with the characteristic function \( \chi(i,j,k; D_l, \mathcal{M}_o) \)

\[
\chi(i,j,k; D_l, \mathcal{M}_o) = \begin{cases} 
1 & \text{if } X_i = k \text{ and } \pi(X_i) = j \text{ in } D_l \\
0 & \text{otherwise}
\end{cases}
\]

(3)

The product Dirichletian distribution, the conjugate prior with global and local independence,\(^{53}\) is introduced

\[
p(\theta | \mathcal{M}_o) = \prod_{i=1}^{n_i} \prod_{j=1}^{q_i} \prod_{k=1}^{r_i} \Gamma(\chi_{ijk}) \theta_{ijk}^{\chi_{ijk}-1}
\]

(4)

where \( \Gamma(\cdot) \) is the Gamma function; and \( \chi_{ijk} = \sum_{k=1}^{r_i} \chi_{ijk} \) with \( \chi_{ijk} \) being the hyperparameters, representing the number of virtual samples belonging to the category of \( X_i = k \) and \( \pi(X_i) = j \). A popular choice for the hyperparameter is \( \chi_{ijk} = 1 \), that is, a uniform prior over the parameters of \( X_i \) for each value combination of \( \pi(X_i) \).\(^{54}\)

Finally, the posterior distribution can be obtained by Bayes’ theorem

\[
p(\theta | \mathcal{D}, \mathcal{M}_o) \propto p(\theta | \mathcal{M}_o) P(\mathcal{D} | \theta, \mathcal{M}_o) = \prod_{i=1}^{n_i} \prod_{j=1}^{q_i} \prod_{k=1}^{r_i} \frac{\Gamma(N_{ij} + \chi_{ijk})} {\Gamma(N_{ijk} + \chi_{ijk})} \theta_{ijk}^{N_{ijk} + \chi_{ijk}-1}
\]

(5)

where \( N_{ij} = \sum_{k=1}^{r_i} N_{ijk} \). It can be observed that the posterior distribution \( P(\theta | \mathcal{D}, \mathcal{M}_o) \) is again a product Dirichletian distribution. The posterior expectation \( \tilde{\theta}_{ijk} \), mode \( \hat{\theta}_{ijk} \) and variance \( \text{Var}(\theta_{ijk}) \) can be derived as

\[
\tilde{\theta}_{ijk} = \frac{N_{ijk} + \chi_{ijk}} {\sigma_j}
\]

(6)

\[
\hat{\theta}_{ijk} = \frac{N_{ijk} + \chi_{ijk} - 1} {\sigma_j - r_i}
\]

(7)

\[
\text{Var}(\theta_{ijk}) = \frac{(N_{ijk} + \chi_{ijk})(\sigma_j - (N_{ijk} + \chi_{ijk}))} {\sigma_j(\sigma_j + 1)}
\]

(8)

\[
\sigma_j = \sum_{k=1}^{r_i} (N_{ijk} + \chi_{ijk})
\]

(9)

In order to select the most plausible network structure, Bayesian probability is utilized as the relative plausibility measure of the aforementioned four candidates \( \mathcal{M}_o, \ o = 1, \ldots, 4 \). Given the dataset \( \mathcal{D} \), the model
probability \( P(M_o|D) \) is evaluated as

\[
P(M_o|D) = \frac{p(D|M_o)P(M_o)}{\sum_{o=1}^{4} p(D|M_o)P(M_o)}, \quad o = 1, \ldots, 4
\]  

(10)

where \( P(M_o) \) is the prior probability of \( M_o \) and the uniform prior is considered \( P(M_o) = 1/4 \). The evidence \( p(D|M_o) \) can be evaluated by the theorem of total probability

\[
p(D|M_o) = \int P(D|\theta, M_o)p(\theta|M_o)d\theta \approx \log p(D|\tilde{\theta}, M_o) - \frac{\Theta}{2}\log N, \quad o = 1, \ldots, 4
\]  

(11)

where \( \tilde{\theta} \) is the posterior expectation of the parameters in equation (6), and \( \Theta = \sum_{i=1}^{n_x} \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} 1 \) is the number of effective parameters of \( M_o \). Here, the Bayesian informative criterion (BIC) is introduced for the integral evaluation as it is accurate in the large samples case. Finally, the optimal network structure is selected as

\[
\hat{M} = \arg \max_{M_o} P(M_o|D), \quad o = 1, \ldots, 4
\]  

(12)

**Prediction phase**

In the case of complete observation information, given a full-observed sample \( D_{N+1} \in \mathbb{R}^{n_x} \), the posterior predictive probability \( P(D_{N+1}|D, M_o) \) can be obtained by considering the compound distribution of likelihood function \( P(D_{N+1}|\theta, M_o) \), with the uncertain parameters distributed according to posterior PDF \( p(\theta|D, M_o) \), as follows

\[
P(D_{N+1}|D, M_o) = \int P(D_{N+1}|\theta, D, M_o)p(\theta|D, M_o)d\theta
\]

\[
= \prod_{i=1}^{n_x} \int \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} \chi(i,j,k; D_{N+1}, M_o)\theta_{ijk}p(\theta|D, M_o)d\theta
\]

\[
= \prod_{i=1}^{n_x} \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} \chi(i,j,k; D_{N+1}, M_o)\tilde{\theta}_{ijk}
\]  

(13)

In contrast to the traditional inference requiring complete information of observation, the BN is capable to make prediction on the target node with incomplete observed information of other nodes. That is, without knowing the unobserved node \( x^{uo}(M_o) \in \mathbb{R}^{n_{uo}} \), based on the information of the observed node \( x^{o}(M_o) \in \mathbb{R}^{n_o} \), the predicted value of the target node \( x^p(M_o) \in \mathbb{R}^{n_p} \) can be calculated, where \( n_o + n_{uo} + n_p = n_x \). In this case, the BN makes prediction based on \( x^p(M_o) \) by maximizing the following conditional probability with respect to \( x^p(M_o) \)

\[
\hat{x}^p(M_o) = \arg \max_{x^p} P(X^p = x^p|X^o = x^o, M_o)
\]  

(14)

The above conditional probability can be factorized as

| Table 2. BIC and probability results of different proposed candidates of network structure. |
|----------------------------------|----------------|----------------|----------------|----------------|
| \( M_1 \) | \( M_2 \) | \( M_3 \) | \( M_4 \) |
| BIC | \(-3.4333E+06\) | \(-3.4323E+06\) | \(-2.5242E+07\) | \(-2.5241E+07\) |
| \( P(|M_o|D) \) | 0.00 | 1.00 | 0.00 | 0.00 |

BIC: Bayesian informative criterion.
Note: Bold values signifies the most plausible network structure.
Figure 6. Predicted histogram of $f_1$ with incomplete observed information of the most plausible model $M_3$. 

Legend

--- : True value

: Updated Probability
Figure 7. Predicted histogram of \( f_2 \) with incomplete observed information of the most plausible model \( \mathcal{M}_2 \).

\[
P(X^p = x^p | X^o = x^o, \mathcal{M}_o) = \frac{P(X^p = x^p, X^o = x^o, \mathcal{M}_o)}{P(X^o = x^o, \mathcal{M}_o)}
\]
\[
= \frac{\sum_{X^{o^c} = x^{o^c}} P(X = \mathbf{\theta}, \mathcal{M}_o)}{\sum_{X^o = x^o} \sum_{X^{o^c} = x^{o^c}} P(X = \mathbf{\theta}, \mathcal{M}_o)}
\]

with

\[
P(X = \mathbf{\theta}, \mathcal{M}_o) = \prod_{i=1}^{n} \prod_{j=1}^{d_i} \prod_{k=1}^{r_i} \overline{\theta}_{ijk}
\]

where \( \overline{\theta}_{ijk} \) is the posterior expectation of the parameters.
Table 3. Root-mean-square errors (RMSE) on predicted frequencies with different training and test data.

| Case                                                                 | RMSE          | Candidate |  \( M_1 \)   |  \( M_2 \)   |  \( M_3 \)   |  \( M_4 \)   |
|----------------------------------------------------------------------|---------------|-----------|---------------|---------------|---------------|---------------|
| I. Training data: Full; Test data: Full; Incomplete: No          | \( f_1 \)    | 3.3826E-03| 3.3826E-03    | 3.5040E-03    | 3.5109E-03    |
|                                                                | \( f_2 \)    | 3.3826E-03| 3.3826E-03    | 3.5040E-03    | 3.5109E-03    |
| II. Training data: Full; Test data: Full; Incomplete: Yes         | \( f_1 \)    | 3.4124E-03| 3.4114E-03    | 3.5159E-03    | 3.5134E-03    |
|                                                                | \( f_2 \)    | 6.5680E-03| 6.5674E-03    | 6.9220E-03    | 6.9239E-03    |
| III. Training & test data: Leave-one-out cross validation; Incomplete: Yes | \( f_1 \)    | 3.9722E-03| 3.9721E-03    | 4.0505E-03    | 4.0544E-03    |
|                                                                | \( f_2 \)    | 7.7573E-03| 7.7562E-03    | 7.9677E-03    | 7.9666E-03    |

*Incomplete: Each test point is randomly selected to be an incomplete point with some probability. An incomplete point means that each component of the four environmental factors is with some probability to be ‘missing’ in prediction.

*Note:* Bold values signifies the most plausible network structure.

Figure 8. Predicted to measured ratios and normalized residuals by \( M_2 \) of Case I of Table 3.

Pattern recognition results

Table 2 shows BIC and probability results of different proposed candidates of network structure. The probability of the most plausible network structure \( M_2 \) is almost equal to 1; and the probability values of \( M_1 \) and \( M_2 \) are higher than those of \( M_3 \) and \( M_4 \). Recall Figure 5 of different network structures, the probability results reflect that the predictions can be improved by considering the correlation between the temperature and relative humidity. On the other hand, the improvement of predictions is not significant by considering the dependency between \( f_1 \) and \( f_2 \). In this case, the BN prefers model \( M_2 \) as it accounts the dependency between \( T \) and \( H \) and avoid considering the unnecessary dependency between \( f_1 \) and \( f_2 \). Figure 6 shows predicted histogram of \( f_1 \) with incomplete observed information of the most plausible model \( M_2 \). Note that the vertical dash line is the true observed
value of $f_1$, while the grey histogram is the updated probability of different states of $f_1$ given the available observed information. For example, the middle right subplot “Observed: T, H, W” represents that the temperature, relative humidity, and wind speed are observed, while the traffic volume is not observed. With the increasing of the information gain from the environmental effects, the probabilities of different states of $f_1$ are being updated. It can be seen that the prediction of the BN can be improved with increasing observed information and the accurate prediction can be achieved even though the observed information is not complete (for “Observed: T, H, W,” the traffic volume is not observed). In the same fashion as Figure 6, Figure 7 shows predicted histogram of $f_2$ with incomplete observed information of the most plausible model $M_2$. Again, with increasing observed information, the probability of the optimal predicted state of $f_2$ is increasing and it approaches to 1 when all the four environmental factors are observed. These two figures have successfully demonstrated the appealing feature of the BN that it is capable to make prediction under incomplete observed information, and its prediction accuracy can be improved with increasing observed information.

In order to compare the prediction capability of different network structure candidates, prediction capability tests are implemented. Table 3 shows root-mean-square errors (RMSEs) on predicted frequencies with different training and test data. Three cases with different training data, test data, and data incompleteness are considered. Case I: The training data and test data are both the full observation dataset (the original dataset with 7811 points). Case II: The training data and test data are both the full observation dataset, but each test point is randomly selected to be an incomplete point with some probability. An incomplete point means that each component of the four environmental factors is with some probability to be ‘missing’, so the ‘missing’ component will not be utilized in frequency prediction. Here, the probability of random incompleteness is 0.05, and for a selected incomplete point, each component of the four environmental factors ($T$, $H$, $W$, $Tr$) is with $1/3$ probability to be ‘missing’. Case III: The training data and test data are constructed using leave-one-out cross validation. That is, for each implementation, one point is selected as the test data, while the remaining points are the training data. Random
incompleteness is considered in this case as that in Case II. From Table 3, it is obvious that the optimal network structure $M_2$ possesses the smallest RMSE in predicing both $f_1$ and $f_2$. This reconfirms the probability results of the BN in Table 2. Figure 8 shows predicted to measured ratios and normalized residuals by $M_2$ of Case I of Table 3. Most of the predicted to measured ratios are within 0.99 to 1.01, indicating that the most plausible model $M_2$ gives very accurate predictions on two frequencies. The normalized residual is within 0.98 to 1.02, reconfirming the high prediction capability of $M_2$. Figures 9 and 10 show the predicted to measured ratios and normalized residuals by $M_2$ of Case II and III, respectively, of Table 3. Again, the accuracy results by $M_2$ for frequency prediction can be observed. Finally, it can be concluded that the optimal network structure $M_2$ is capable to precisely recognize the pattern of modal frequency–multiple environmental factors.

**Conclusion**

In this study, the BN-based algorithm is developed for recognizing the pattern between modal frequency–multiple environmental factors of the Xinguang Bridge based on long-term monitoring data (model frequencies, temperature, humidity, wind speed, and traffic volume). Taking the advantages of the BN approach, the uncertainty is quantified in both parameter and model levels in learning phase; and the inference is made under both complete and incomplete observed information in the prediction phase. Based on the monitoring data, the results of the most plausible network structure indicate that consideration of the correlation between the temperature and relative humidity can improve prediction, while consideration of the correlation between two frequencies cannot. The appealing feature of the BN for making prediction under incomplete observed information is demonstrated. The performances of different network structure are evaluated by the full training dataset along with full test dataset, the full training dataset along with full test dataset considering random incompleteness of test points, and the leave-on-out cross validation considering random incompleteness of test points. The positive evaluation results of the most plausible network structure confirm that the BN-based approach is capable to precisely recognize the pattern of modal frequency–multiple environmental factors. The proposed algorithm can

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**Figure 10.** Predicted to measured ratios and normalized residuals by $M_2$ of Case III of Table 3.
be utilized in structural condition assessment. The predicted residuals of modal frequencies can be calculated and compared with a prescribed threshold. The predicted residuals being larger than the threshold can be treated as an alert for structural health, and the corresponding actions (further data analysis, engineering judgement and/or safety inspection) need to be taken.

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