Particle properties in the early universe from the contraction of the SM gauge group

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Abstract

The properties of elementary particles and their interactions at different stages of the evolution of the Universe, starting with the Planck energy \(10^{19}\) GeV, are presented. We assume that the Standard Model gauge group becomes simpler as the temperature of the universe increases. The description is based on the hypothesis that the high-energy (high-temperature) limit of the Standard Model is generated by the contraction of the gauge group. An explicit form of the Lagrangian is obtained for each stage of the evolution of the universe and is the basis for describing the properties of elementary particles. These properties change drastically in the infinite temperature limit: all particles lose mass, only massless neutral \(Z\) bosons and \(u\) quarks, as well as neutrinos and photons, survive. Electroweak interactions become long range and are mediated by neutral currents. All quarks are monochromatic.

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1 Introduction

The Standard Model is the modern theory of elementary particles and their interactions. It includes the Electroweak Model, which combines electromagnetic and weak interactions, and Quantum Chromodynamics (QCD), which describes strong interactions. The Standard Model describes the available experimental data quite well, and its adequacy has been convincingly confirmed by the recent discovery of the scalar Higgs boson in the experiments at the Large Hadron Collider. If one wants to investigate the properties and
interactions of particles beyond experimentally achieved energies, a possible way is to use the high-energy (high-temperature) limit of the Standard Model.

We assume that the gauge group of the Standard Model becomes simpler with increasing energy. Indeed, the Standard Model is a gauge theory based on a gauge group $SU(3) \times SU(2) \times U(1)$, which is a direct product of simple groups. Strong interactions of quarks are described by quantum chromodynamics with the $SU(3)$ gauge group and the characteristic temperature of 0.2 GeV. The electroweak model is based on the $SU(2) \times U(1)$ gauge group, responsible for electroweak interactions with a characteristic temperature of 100 GeV, while the $U(1)$ group is associated with long-range electromagnetic interactions. Due to the zero mass of the photon its characteristic temperature extends to the "infinite" Planck energy $10^{19}$ GeV. It follows from this observation that the gauge group of the theory of elementary particles becomes simpler with increasing temperature of the Universe. We assume that with further increase in the temperature, simplification of the gauge group of the Standard Model is described by its contraction.

The operation of contraction (or limiting transition) of groups [1], which, in particular, transforms a simple group into a nonsemisimple one, is well known in physics. The notion of contraction was extended to algebraic structures, such as quantum groups and supergroups, and to fundamental representations of unitary groups [2]. For a symmetric physical system, contraction of its symmetry group means a transition to the limiting state of the system. In the case of a complex physical system, which is the Standard Model, the study of its limiting states at any given limiting values of physical parameters allows a better insight into the behavior of the system as a whole. We will discuss the modified Standard Model with the contracted gauge group at the level of classical gauge fields.

In the broad sense of the word, deformation is the reverse of contraction. Nontrivial deformation of an algebraic structure means, generally speaking, its nonobvious generalization. A prominent recent example is quantum groups [3], i.e., such generalizations of Hopf algebras that are simultaneously noncommutative and noncocommutative, while previously Hopf algebras with only one of these properties were known. However, if a mathematical structure is first contracted, the initial structure can be reconstructed using deformation in the narrow sense, performed in the direction opposite to that of contraction.

We use this technique to reconstruct the evolution of elementary particles.
in the early Universe relying on the currently achieved knowledge. To this end, we consider the behavior of the Standard Model in the limit of the "infinite" temperature, which, according to our hypothesis, is generated by contraction of the $SU(2)$ and $SU(3)$ gauge groups \[4\]. Similar "infinitely" high temperatures could exist in the early Universe in the first instants after the Big Bang \[5, 6\]. It turns out that the gauge group contraction results in the Standard Model Lagrangian breaking down into a few terms with different powers of the zero-tending contraction parameter $\epsilon \to 0$. Since the average energy (temperature) in the hot Universe is related to its age, then moving forward in time, i.e., in the direction opposite to the high-temperature contraction, we come to the conclusion that after the birth of the Universe the elementary particles and their interactions pass through a number of stages in their evolution from the limiting state with the "infinite" temperature to the state described by the Standard Model. These stages of quark-gluon plasma formation and reconstruction of electroweak and color symmetries differ by the powers of the contraction parameter and consequently by the time of their origin. Based on the Standard Model contraction, we can classify these stages according to the "earlier-later" principle but cannot find the time elapsed after the birth of the Universe. To establish the absolute time, we use additional assumptions. The paper is an expanded and supplemented version of the report at the conference dedicated to the 110th anniversary of the birth of N.N. Bogolyubov \[7\].

## 2 Electroweak Model

The elementary particles of the Standard Model are as follows: scalar gauge bosons (photon $\gamma$, charged $W^{\pm}$ and neutral $Z^0$ weak bosons, gluons $A^k$, $k = 1, \ldots, 8$), a special particle $\chi$ (scalar Higgs boson), as well as vector particles, namely three generations of leptons \(\left(\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array}\right)\), \(\left(\begin{array}{c} \mu \\ \tau \end{array}\right)\), and three generations of quarks \(\left(\begin{array}{c} u \\ d \\ c \\ s \\ t \\ b \end{array}\right)\).

We will briefly describe the main features of the Electroweak Model according to \[8\]. The Lagrangian of the model is given by the sum of the boson, lepton, and quark Lagrangians $L = L_B + L_L + L_Q$. It is taken to be invariant under the action of the gauge group $SU(2) \times U(1)$ in the space $\mathbb{C}_2$:

\[ SU(2) : \mathbb{C}^2 = \mathcal{G}\mathbb{C}, \]
\[
\left( \begin{array}{c} z'_1 \\ z'_2 \end{array} \right) = \left( \begin{array}{cc} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{array} \right) \left( \begin{array}{c} z_1 \\ z_2 \end{array} \right), \quad |\alpha|^2 + |\beta|^2 = 1,
\]

\[U(1) : \bar{z}' = e^{i\omega/2}z, \quad \omega \in \mathbb{R}. \quad (1)\]

The \(U(1)\) group generator \(Y\) is proportional to unit matrix \(Y = \frac{1}{2}1\).

Generators of \(SU(2)\) group

\[T_1 = \frac{1}{2} \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) = \frac{1}{2}\tau_1, \quad T_2 = \frac{1}{2} \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right) = \frac{1}{2}\tau_2, \quad T_3 = \frac{1}{2} \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) = \frac{1}{2}\tau_3, \quad (2)\]

where \(\tau_k, \ k = 1, 2, 3\) are Pauli matrices are subject to commutation relations

\[ [T_1, T_2] = iT_3, \quad [T_3, T_1] = iT_2, \quad [T_2, T_3] = iT_1 \quad (3) \]

and form Lie algebra \(su(2)\).

The boson sector \(L_B = L_A + L_\phi\) includes the Lagrangian of the gauge fields

\[L_A = -\frac{1}{4}((F^1_{\mu\nu})^2 + (F^2_{\mu\nu})^2 + (F^3_{\mu\nu})^2) - \frac{1}{4}(B_{\mu\nu})^2, \]

\[F^1_{\mu\nu} = \partial_\mu A^1_\nu - \partial_\nu A^1_\mu + gA^2_\mu A^3_\nu, \quad F^2_{\mu\nu} = \partial_\mu A^2_\nu - \partial_\nu A^2_\mu + gA^3_\mu A^1_\nu, \]

\[F^3_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu + gA^1_\mu A^2_\nu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (4)\]

and the Lagrangian of the matter fields

\[L_\phi = \frac{1}{2}(D_\mu \phi)^\dagger D_\mu \phi - \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2, \quad \phi = \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right) \in \mathbb{C}_2. \quad (5)\]

The covariant derivatives are

\[D_\mu \phi = \partial_\mu \phi - ig \left( \sum_{k=1}^{3} T_k A^k_\mu \right) \phi - ig' Y B_\mu \phi, \quad (6)\]

where \(T_k = \frac{1}{2}\tau_k, \ k = 1, 2, 3\) are the \(SU(2)\) generators, \(Y = \frac{1}{2}1\) is the \(U(1)\) generator, \(\tau_k\) are the Pauli matrices, and \(g\) and \(g'\) are the charges. The gauge fields

\[A_\mu(x) = -i \sum_{k=1}^{3} T_k A^k_\mu(x), \quad B_\mu(x) = -i B_\mu(x) \quad (7)\]
take their values in Lie algebras \( su(2), u(1) \) respectively, and their field strength tensors defined as follows

\[
F_{\mu\nu}(x) = F_{\mu\nu}^k(x) = \partial_{\mu} A_{\nu}^k(x) - \partial_{\nu} A_{\mu}^k(x),
\]

\[
B_{\mu\nu}(x) = \partial_{\mu} B_{\nu}(x) - \partial_{\nu} B_{\mu}(x).
\]

The new gauge fields

\[
Z_{\mu}(x) = \frac{1}{\sqrt{g^2 + g'^2}} (g A_{\mu}^3(x) - g' B_{\mu}(x)), \quad A_{\mu}(x) = \frac{1}{\sqrt{g^2 + g'^2}} \left( g' A_{\mu}^3(x) + g B_{\mu}(x) \right),
\]

\[
W_{\mu}^\pm(x) = \frac{1}{\sqrt{2}} \left( A_{\mu}^1(x) \mp i A_{\mu}^2(x) \right)
\]

are introduced instead of (7).

To generate mass for the vector bosons the special mechanism of spontaneous symmetry breaking is used. One of the \( L_B \) ground states

\[
\phi^{\text{vac}} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad A_{\mu}^k = B_{\mu} = 0
\]

is taken as a vacuum state of the model, and small field excitations \( v + \chi(x) \) with respect to this vacuum are regarded.

After spontaneous symmetry breaking the boson Lagrangian (4), (5) can be represented in the form

\[
L_B = L_B^{(2)} + L_B^{\text{int}} = \\
= \frac{1}{2} (\partial_{\mu} \chi)^2 - \frac{1}{2} m_{\chi}^2 \chi^2 + \frac{1}{2} m_Z^2 Z_{\mu} Z_\mu - \frac{1}{4} Z_{\mu\nu} Z_{\mu\nu} - \\
- \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + m_W^2 W_+^\mu W_-^\mu - \frac{1}{2} W_{\mu\nu}^+ W_{\mu\nu}^+ + L_B^{\text{int}},
\]

where \( F_{\mu\nu}(x) = \partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x) \), \( Z_{\mu\nu}(x) = \partial_{\mu} Z_{\nu}(x) - \partial_{\nu} Z_{\mu}(x) \), \( W_{\mu\nu}^\pm(x) = \partial_{\mu} W_{\nu}^\pm(x) - \partial_{\nu} W_{\mu}^\pm(x) \). As usual the second order terms describe the boson particles content of the model and higher order terms \( L_B^{\text{int}} \) are regarded as their interactions. So Lagrangian (11) includes charged \( W \) bosons with identical mass \( m_W = \frac{1}{2} g v \), massless photon \( A_{\mu} \), neutral \( Z \) boson with the mass \( m_Z = \frac{v}{2} \sqrt{g^2 + g'^2} \) and scalar Higgs boson \( \chi \), \( m_{\chi} = \sqrt{2\lambda v} \). All these particles are experimentally detected and have the masses: \( m_W = 80 \text{ GeV} \), \( m_Z = 91 \text{ GeV} \), \( m_{\chi} = 125 \text{ GeV} \).
The interaction Lagrangian $L_B^{int}$ looks as follows

\[
L_B^{int} = \frac{g m_Z}{2 \cos \theta_W} (Z_\mu)^2 \chi - \lambda \nu \chi^2 + \frac{g^2}{8 \cos^2 \theta_W} (Z_\mu)^2 \chi^2 - \frac{\lambda}{4} \chi^4 -
- \frac{1}{2} W_\mu^+ W_\mu^- + m_W^2 W_\mu^+ W_\mu^- 
- 2i g (W^+ W_\nu^- - W^\mu_- W^-) (F_{\mu \nu} \sin \theta_W + Z_{\mu \nu} \cos \theta_W) - 
- \frac{i}{2} e [A_\mu (W^\mu_\mu W_\nu^- - W^\mu_- W^\nu_\mu) - A_\nu (W^\mu_\mu W_\mu^- - W^\mu_- W^\mu_\mu)] + 
+ g W^\mu_\mu W^- \mu \chi - \frac{i}{2} g \cos \theta_W [Z_\mu (W^\mu_\mu W_\mu^- - W^\mu_- W^\mu_\mu) - 
- Z_\nu (W^\mu_\mu W_\mu^- - W^\mu_- W^\mu_\mu)] + \frac{g^2}{4} (W^\mu_\mu W_\mu^- - W^\mu_- W^\mu_\mu)^2 + 
+ \frac{g^2}{4} W^\mu_\mu W^- \mu \chi^2 - \frac{e^2}{4} \left\{ \left[ (W^\mu_\mu)^2 + (W^-_\mu)^2 \right] (A_\mu)^2 - 
- 2 (W^\mu_\mu W_\mu^- + W^\mu_- W^-_\mu) A_\mu A_\nu + \left[ (W^\mu_\mu)^2 + (W^-_\mu)^2 \right] (A_\mu)^2 \right\} - 
- \frac{2 g^2}{4} \cos \theta_W \left\{ \left[ (W^\mu_\mu)^2 + (W^-_\mu)^2 \right] (Z_\mu)^2 - 
- 2 (W^\mu_\mu W_\mu^- + W^\mu_- W^-_\mu) Z_\mu Z_\nu + \left[ (W^\mu_\mu)^2 + (W^-_\mu)^2 \right] (Z_\mu)^2 \right\} - 
- \epsilon g \cos \theta_W \left\{ W^\mu_- W^-_\mu A_\nu Z_\nu + W^\mu_\mu W^-_\mu A_\mu Z_\mu - 
\frac{1}{2} (W^\mu_- W^-_\mu + W^\mu_\mu W^-_\mu) (A_\mu Z_\nu + A_\nu Z_\mu) \right\}. \tag{12}
\]

The fermion sector is represented by the lepton $L_L$ and quark $L_Q$ Lagrangians. For first-generation particles, the lepton Lagrangian is written in the form

\[
L_{L,e} = L_L^i \bar{i} \tau \mu L_i + e^+_\mu i \tau \mu D_\mu e_r - h_e [e^+_\mu (\phi^\dagger L_i) + (L_i^\dagger \phi) e_r], \tag{13}
\]
where \( L_l = \left( \begin{array}{c} \nu_l \\ e_l \end{array} \right) \in \mathbb{C}_2 \) is the \( SU(2) \) doublet, \( e_r \) is the \( SU(2) \) singlet, \( \tau_0 = \tilde{\tau}_0 = 1 \), \( \tilde{\tau}_k = -\tau_k \), \( h_e \) is the Yukawa coupling, \( e_r, e_l, \nu_l \) are the two-component Lorentz spinors. (We restrict ourselves to considering only particles. To take into account antiparticles, the fields must be four-component Dirac bispinors). Here \( D_{\mu} \) are the covariant derivatives of the lepton fields

\[
D_{\mu}L_l = \partial_{\mu}L_l - i \frac{g}{\sqrt{2}} \left( W_{\mu}^+ T_+ + W_{\mu}^- T_- \right) L_l - i \frac{g}{\cos \theta_w} Z_{\mu} \left( T_3 - Q \sin^2 \theta_w \right) L_l - i e A_{\mu}Q L_l,
\]

\[
D_{\mu}e_r = \partial_{\mu}e_r - ig' Q A_{\mu} e_r \cos \theta_w + ig' Q Z_{\mu} e_r \sin \theta_w,
\]

where \( T_{\pm} = T_1 \pm i T_2 \), \( Q = Y + T_3 \) is the generator of the \( U(1)_{em} \) electromagnetic subgroup, \( Y = \frac{1}{2} \) is the hypercharge, \( e = gg'(g^2 + g'^2)^{-1/2} \) is the electron charge, and \( \sin \theta_w = eg^{-1} \).

According to modern knowledge, all known leptons and quarks form three generations. The next two lepton generations are introduced in a way similar to (13). They are left \( SU(2) \)-doublets

\[
\left( \begin{array}{c} \nu_\mu \\ \mu \end{array} \right)_l, \quad \left( \begin{array}{c} \nu_\tau \\ \tau \end{array} \right)_l, \quad Y = -\frac{1}{2}
\]

and right \( SU(2) \)-singlets: \( \mu_r, \tau_r, Y = -1 \). In addition to \( u \) and \( d \) quarks of the first generation there is \( (c, s) \) and \( (t, b) \) quarks of the next generations, whose left fields

\[
\left( \begin{array}{c} c_l \\ s_l \end{array} \right)_l, \quad \left( \begin{array}{c} t_l \\ b_l \end{array} \right)_l, \quad Y = \frac{1}{6}
\]

are described by the \( SU(2) \)-doublets and the right fields are \( SU(2) \)-singlets: \( c_r, t_r, Y = \frac{2}{3} \), \( s_r, b_r, Y = -\frac{1}{3} \). Their Lagrangians are introduced in a way similar to (13). Full lepton and quark Lagrangians are obtained by summing over all generations. In what follows we will discuss only the first generations of leptons and quarks.

In terms of electron and neutrino fields the lepton Lagrangian (13) is written in the form

\[
L_L = e_1^\dagger i \tilde{\tau}_\mu \partial_{\mu} e_1 + e_2^\dagger i \tilde{\tau}_\mu \partial_{\mu} e_2 - m_e(e_1^\dagger e_1 + e_2^\dagger e_2) +
\]

\[
+ \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_1^\dagger \tilde{\tau}_\mu Z_{\mu} e_1 - ee_1^\dagger \tilde{\tau}_\mu A_{\mu} e_2^\dagger g' \cos \theta_w e_1^\dagger \tilde{\tau}_\mu A_{\mu} e_2 +
\]

\[
+ g' \sin \theta_w e_1^\dagger \tilde{\tau}_\mu Z_{\mu} e_2 + \nu_1^\dagger i \tilde{\tau}_\mu \partial_{\mu} \nu_1 + \frac{g}{2 \cos \theta_w} \nu_1^\dagger \tilde{\tau}_\mu Z_{\mu} \nu_1 +
\]
where \( m \) make up a conjugate representation of the SU(2) and quark masses. For particles all fields are written as spinors. The covariant derivatives of the quark fields are:

\[
D_\mu Q_l = \left( \partial_\mu - ig \sum_{k=1}^{3} \frac{\tau_k}{2} A_\mu^k - ig' \frac{1}{6} B_\mu \right) Q_l,
\]

\[
D_\mu u_r = \left( \partial_\mu - ig \frac{2}{3} B_\mu \right) u_r, \quad D_\mu d_r = \left( \partial_\mu + ig \frac{1}{3} B_\mu \right) d_r.
\]

The quark Lagrangian (18) in terms of \( u \) and \( d \) quarks fields can be written as:

\[
L_Q = d_l^\dagger i\bar{\tau}_\mu \partial_\mu d_l + d_r^\dagger i\bar{\tau}_\mu \partial_\mu d_r - m_d(d_d^\dagger d_l + d_l^\dagger d_r) - \frac{e}{3} d_l^\dagger \bar{\tau}_\mu A_\mu d_l -
\]

\[
-\frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d_l^\dagger \bar{\tau}_\mu Z_\mu d_l - \frac{1}{3} g' \cos \theta_w d_l^\dagger \tau_\mu A_\mu d_l +
\]

\[
+ \frac{1}{3} g' \sin \theta_w d_l^\dagger \tau_\mu Z_\mu d_r + u_l^\dagger i\bar{\tau}_\mu \partial_\mu u_l + u_r^\dagger i\bar{\tau}_\mu \partial_\mu u_r - m_u(u_u^\dagger u_l + u_l^\dagger u_r) +
\]

\[
+ \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \bar{\tau}_\mu Z_\mu u_l + \frac{2e}{3} u_l^\dagger \bar{\tau}_\mu A_\mu u_l +
\]

\[
+ \frac{g}{\sqrt{2}} \left[ u_l^\dagger \bar{\tau}_\mu W^+_\mu d_l + d_l^\dagger \bar{\tau}_\mu W^-_\mu u_l \right] + \frac{2}{3} g' \cos \theta_w u_l^\dagger \tau_\mu A_\mu u_r -
\]

\[
- \frac{2}{3} g' \sin \theta_w u_l^\dagger \bar{\tau}_\mu Z_\mu u_r,
\]

where \( m_e = h_e v / \sqrt{2} \) and \( m_u = h_u v / \sqrt{2} \), \( m_d = h_d v / \sqrt{2} \) represent electron and quark masses.
3 Electroweak Model at High Energies

There are two ways to describe the action of a contracted group in a space. The traditional way is to act by a matrix group with real or complex elements on vectors with similar components

\[
\begin{pmatrix}
z_1' \\
z_2'
\end{pmatrix} = \begin{pmatrix}
\alpha & \beta \\
-\epsilon^2 \bar{\beta} & \bar{\alpha}
\end{pmatrix}
\begin{pmatrix}
z_1 \\
z_2
\end{pmatrix},
\]

\[
\det u(\epsilon) = |\alpha|^2 + \epsilon^2 |\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1.
\] (21)

In the limit \(\epsilon \to 0\), this matrix has the form

\[
u(0) = \begin{pmatrix}
\alpha & \beta \\
0 & \bar{\alpha}
\end{pmatrix}, \quad \alpha = e^{i\gamma}, \quad \gamma \in R
\]

and obviously belongs to the group \(E(2)\).

We will introduce a contracted group \(SU(2; \epsilon)\) and corresponding space \(C_2(\epsilon)\) by the consistent rescaling of the group \(SU(2)\) and the space \(C_2\)

\[
\begin{pmatrix}
z_1' \\
\epsilon z_2'
\end{pmatrix} = \begin{pmatrix}
\alpha & \epsilon \beta \\
-\epsilon \bar{\beta} & \bar{\alpha}
\end{pmatrix}
\begin{pmatrix}
z_1 \\
\epsilon z_2
\end{pmatrix},
\]

\[
\det u(\epsilon) = |\alpha|^2 + \epsilon^2 |\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1.
\] (22)

Our approach is based on the action of matrices with elements depending on the contraction parameter \(\epsilon\) on vectors, the components of which also depend on this parameter. The contraction parameter tending to zero is convenient for physical applications, but mathematically it can be equal to the nilpotent unit \(\iota\), which has the property \(\iota \neq 0\), but \(\iota^2 = 0\). Then the contracted matrix

\[
u(\iota) = \begin{pmatrix}
\alpha & \iota \beta \\
-\iota \bar{\beta} & \bar{\alpha}
\end{pmatrix}, \quad \alpha = e^{i\gamma}, \quad \gamma \in R.
\]

will not be diagonal, but will have nilpotent elements. This approach is detailed in [5] (formulae (1.17)–(1.24) and section 1.3.2, which describes the \(SU(2; \epsilon)\)).

After contraction the group \(SU(2; \epsilon = 0)\) is isomorphic to Euclid group \(E(2)\). The contracted space \(C_2(\epsilon = 0)\) is split on the base spanned by the \(\{z_1\}\) coordinate and the fiber spanned by the \(\{z_2\}\) coordinate. (The non-relativistic space–time is the best known example of a fiber space. It has the
one-dimensional base, which is interpreted as time, and three-dimensional fiber, which is interpreted as a proper space.) The unitary group $U(1)$ and its action in the space $C_2(\varepsilon = 0)$ do not change and are given by (1).

The space $C_2(\varepsilon)$ can be obtained from $C_2$ by substitution of $z_2$ by $\varepsilon z_2$, which induces the other ones for Lie algebra generators $T_1 \rightarrow \varepsilon T_1, T_2 \rightarrow \varepsilon T_2, T_3 \rightarrow T_3$. These new generators obey the commutation relations

$$[T_1, T_2] = i\varepsilon^2 T_3, \quad [T_3, T_1] = iT_2, \quad [T_2, T_3] = iT_1$$

for Lie algebra $su(2; \varepsilon)$. The structure of the algebra $su(2; \varepsilon = 0)$ is a semi-direct sum of Abelian subalgebra $t_2 = \{T_1, T_2\}$ and subalgebra $u(1) = \{T_3\}$: $su(2; \varepsilon) = t_2 \oplus u(1)$.

As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely:

$$A^1_\mu \rightarrow \varepsilon A^1_\mu, \quad A^2_\mu \rightarrow \varepsilon A^2_\mu, \quad A^3_\mu \rightarrow A^3_\mu, \quad B_\mu \rightarrow B_\mu.$$\hspace{1cm} (24)

From the commutativity and associativity properties of multiplication by the scalar $\varepsilon$, we have

$$su(2; \varepsilon) \ni \{A^1_\mu(\varepsilon T_1) + A^2_\mu(\varepsilon T_2) + A^3_\mu T_3\} =$$

$$= \{(\varepsilon A^1_\mu)T_1 + (\varepsilon A^2_\mu)T_2 + A^3_\mu T_3\}. \hspace{1cm} (25)$$

The substitution $\beta \rightarrow \varepsilon \beta$ induces multiplication of the standard gauge fields

$$W^\pm_\mu \rightarrow \varepsilon W^\pm_\mu, \quad Z_\mu \rightarrow Z_\mu, \quad A_\mu \rightarrow A_\mu.$$\hspace{1cm} (26)

The left lepton $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix}$, and quark $Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix}$ fields are $SU(2)$-doublets, so their components are transformed in the similar way as the components of the vector $z$, namely:

$$e_l \rightarrow \varepsilon e_l, \quad d_l \rightarrow \varepsilon d_l, \quad \nu_l \rightarrow \nu_l, \quad u_l \rightarrow u_l.$$\hspace{1cm} (27)

The right lepton and quark fields are $SU(2)$-singlets and therefore are not changed.

The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, which is used to generate mass of vector bosons and other elementary particles.
of the model. In this mechanism one of the Lagrangian $L_B$ ground states 
\[ \phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad A_k^0 = B_k = 0 \]

is taken as the model vacuum, and then low field excitations $v + \chi(x)$ with respect to the second component of the vacuum vector are considered. Therefore, the Higgs boson field $\chi$, constant $v$, and $v$-dependent particle masses $m_p$ are multiplied by the contraction parameter:

\[ \chi \to \epsilon \chi, \quad v \to \epsilon v, \quad m_p \to \epsilon m_p, \quad p = \chi, W, Z, e, u, d. \] (28)

As a result of transformations (26)–(28) the boson Lagrangian (4), (5) can be written in the form

\[ L_B(\epsilon) = -\frac{1}{4} Z_{\mu\nu}^2 - \frac{1}{4} F_{\mu\nu}^2 + \epsilon^2 L_{B,2} + \epsilon^3 g W^+ \mu W^- \chi + \epsilon^4 L_{B,4}, \] (29)

where

\[ L_{B,4} = m_W^2 W^+ \mu W^- \mu - \frac{1}{2} m_\chi^2 \chi^2 - \lambda v \chi^3 - \frac{\lambda}{4} \chi^4 + \]
\[ \frac{g^2}{4} \left( W^+ \nu W^- \nu - W^- \nu W^+ \nu \right)^2 + \frac{g^2}{4} W^+ \nu W^- \nu \chi^2, \] (30)

\[ L_{B,2} = \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} m_Z^2 (Z_\mu)^2 - \frac{1}{2} W^+ \mu W^- \mu + \]
\[ + \frac{g m_z}{2 \cos \theta_W} (Z_\mu)^2 \chi + \frac{g^2}{8 \cos^2 \theta_W} (Z_\mu)^2 \chi^2 - \]
\[ - 2ig \left( W^+ \nu W^- \nu - W^- \nu W^+ \nu \right) \left( F_{\mu\nu} \sin \theta_W + Z_{\mu\nu} \cos \theta_W \right) - \]
\[ - \frac{i}{2} e \left[ A_\mu \left( W^+ \nu W^- \nu - W^- \nu W^+ \nu \right) + A_\nu \left( W^+ \mu W^- \mu - W^- \mu W^+ \mu \right) \right] - \]
\[ - \frac{i}{2} g \cos \theta_W \left[ Z_\mu \left( W^+ \mu W^- \nu - W^- \mu W^+ \nu \right) - \]
\[ - Z_\nu \left( W^+ \nu W^- \mu - W^- \nu W^+ \mu \right) \right] - \frac{e^2}{4} \left\{ \left( W^+ \nu \right)^2 + \left( W^- \nu \right)^2 \right\} (A_\nu)^2 - \]
\[ - 2 \left( W^+ \nu W^+ \nu + W^- \nu W^- \nu \right) A_\mu A_\nu + \left[ \left( W^+ \nu \right)^2 + \left( W^- \nu \right)^2 \right] (A_\mu)^2 \right\} - \]
\[ - \frac{g^2}{4} \cos \theta_W \left\{ \left( W^+ \mu \right)^2 + \left( W^- \mu \right)^2 \right\} (Z_\mu)^2 - \]
\[ - 2 \left( W^+ \mu W^+ \mu + W^- \mu W^- \mu \right) Z_\mu Z_\nu + \left[ \left( W^+ \nu \right)^2 + \left( W^- \nu \right)^2 \right] (Z_\mu)^2 \right\} - \]
\[
-eg \cos \theta_W \left[ W^+_\mu W^-_\nu A_\mu Z_\nu + W^+_\nu W^-_\mu A_\mu Z_\nu \right] - \\
-\frac{1}{2} \left( W^+_\mu W^-_\nu + W^+_\nu W^-_\mu \right) (A_\mu Z_\nu + A_\nu Z_\mu).
\]

The lepton Lagrangian (13), (17) in terms of electron and neutrino fields takes the form
\[
L_L(\epsilon) = L_{L,0} + \epsilon^2 L_{L,2} = \\
= \nu^\dagger_l i \tau_\mu \partial_\mu \nu_l + e^\dagger_l i \tau_\mu \partial_\mu e_r + g' \sin \theta_w e^\dagger_l \tau_\mu Z_\mu e_r - \\
- g' \cos \theta_w e^\dagger_l \tau_\mu A_\mu e_r + \frac{g}{2 \cos \theta_w} e^\dagger_l \tau_\mu Z_\mu e_l + \\
+ \epsilon^2 \left\{ e^\dagger_l i \tau_\mu \partial_\mu e_l - m_e (e^\dagger_l e_l + e^\dagger_r e_r) + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e^\dagger_l \tau_\mu Z_\mu e_l - \\
- e e^\dagger_l \tau_\mu A_\mu e_l + \frac{g}{\sqrt{2}} (\nu^\dagger_l \tau_\mu W^+_{\mu} e_l + e^\dagger_l \tau_\mu W^-_{\mu} \nu_l) \right\}.
\]

The quark Lagrangian (18), (20) is written as
\[
L_Q(\epsilon) = L_{Q,0} - \epsilon m_u (u^\dagger_r u_l + u^\dagger_l u_r) + \epsilon^2 L_{Q,2},
\]
where
\[
L_{Q,0} = d^\dagger_l i \tau_\mu \partial_\mu d_l + u^\dagger_l i \tau_\mu \partial_\mu u_l + u^\dagger_r i \tau_\mu \partial_\mu u_r - \\
- \frac{1}{3} g' \cos \theta_w d^\dagger_l \tau_\mu A_\mu d_l + \frac{1}{3} g' \sin \theta_w d^\dagger_l \tau_\mu Z_\mu d_l + \frac{2e}{3} u^\dagger_l \tau_\mu A_\mu u_l + \\
+ \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u^\dagger_l \tau_\mu Z_\mu u_l + \\
+ \frac{2g'}{3} \cos \theta_w u^\dagger_r \tau_\mu A_\mu u_r - \frac{2g'}{3} g' \sin \theta_w u^\dagger_r \tau_\mu Z_\mu u_r,
\]

\[
L_{Q,2} = d^\dagger_l i \tau_\mu \partial_\mu d_l - m_d (d^\dagger_l d_l + d^\dagger_r d_r) - \frac{e}{3} d^\dagger_l \tau_\mu A_\mu d_l - \\
- \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d^\dagger_l \tau_\mu Z_\mu d_l + \\
+ \frac{g}{\sqrt{2}} \left[ u^\dagger_l \tau_\mu W^+_{\mu} d_l + d^\dagger_l \tau_\mu W^-_{\mu} u_l \right].
\]

The complete Lagrangian of the Electroweak Model with contracted gauge group is given by the sum
\[
L_{EW,M}(\epsilon) = L_B(\epsilon) + L_L(\epsilon) + L_Q(\epsilon)
\]
and takes the form
\[
L_{EW,M}(\epsilon) = L_0 + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3 + \epsilon^4 L_4.
\]
where
\[ L_0 = L_{B,0} + L_{L,0} + L_{Q,0}, \quad L_1 = L_{Q,1} = -\epsilon m_u (u_r^1 u_l + u_l^1 u_r), \]
\[ L_2 = L_{B,2} + L_{L,2} + L_{Q,2}, \quad L_3 = L_{B,3}, \quad L_4 = L_{B,4}. \] (37)

We assume that the contraction parameter is monotonous function \( \epsilon(T) \) of the temperature (or average energy) of the Universe with the property \( \epsilon(T) \to 0 \) for \( T \to \infty \). When \( \epsilon \to 0 \), the terms with higher powers of \( \epsilon \) make a smaller contribution as compared to the terms with lower powers. Thus, the Electroweak Model demonstrates five stages of the behavior as \( \epsilon \to 0 \), which differ by the powers of the contraction parameter. This offers an opportunity for construction of intermediate limit models. One can take the Lagrangian \( L_0 \) for the initial limit system, then add \( L_1 \) and obtain the second limit model with the Lagrangian \( L_1 = L_0 + L_1 \). After that one can add \( L_2 \) and obtain the third limit model \( L_2 = L_0 + L_1 + L_2 \). The last limit model has the Lagrangian \( L_3 = L_0 + L_1 + L_2 + L_3 \).

From the contraction of the Electroweak Model we can classify events in time as earlier-later, but we can not determine their absolute time without additional assumptions. At the level of classical gauge fields we can already conclude that the \( u \) quark first restores its mass in the evolution of the Universe. Indeed the mass term of the \( u \) quark in the Lagrangian \( L_1 = -m_u (u_r^1 u_l + u_l^1 u_r) \) is proportional to the first power \( \epsilon \), whereas the mass terms of \( Z \) boson, electron and \( d \) quark are multiplied by the second power of the contraction parameter \( \epsilon^2 \).

Massless charged \( W \) bosons and Higgs boson \( \chi \) appear at the same time. These particles restore their masses after all other particles of the Electroweak Model because their mass terms are proportional to the fourth power \( \epsilon^4 \).

The main part of the electroweak interactions are restored in the epoch which corresponds to the second order of the contraction parameter. There is one term in Lagrangian \( L_3 = g W_\mu^+ W_\mu^- \chi \) proportionate to \( \epsilon^3 \). The final reconstruction of the electroweak interactions and the restoration of mass of all particles takes place at the last stage \( (\approx \epsilon^4) \).

Two other generations of leptons \( (15) \) and quarks \( (16) \) have similar properties: for the ”infinite” energy there are only massless right \( \mu \) and \( \tau \) muons, left \( \mu \) and \( \tau \) neutrinos, as well as massless left and right quarks \( c_l, c_r, s_r, t_l, t_r, b_r \).
c- and t quarks first acquire their mass and after that μ, τ muons, s, b quarks become massive.

4 Quantum Chromodynamics

Strong interactions of quarks are described by the Quantum Chromodynamics (QCD). QCD is a gauge theory based on the local color degrees of freedom [10]. The QCD gauge group is SU(3), acting in three dimensional complex space $C_3$ of color quark states $q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \equiv \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix} \in C_3$, where $q(x)$ are quark fields $q = u, d, s, c, b, t$ and $R$ (red), $G$ (green), $B$ (blue) are color degrees of freedom. The SU(3) gauge bosons are called gluons. There are eight gluons in total, which are the force carrier between quarks. The QCD Lagrangian is taken in the form

$$\mathcal{L} = \sum_q (\bar{q}^i (i\gamma^\mu) (D_\mu)_{ij} q^j - m_q q^i q_i) - \frac{1}{4} \sum_{\alpha=1}^{8} F_{\mu\nu}^\alpha F^{\mu\nu \alpha},$$

(39)

where $D_\mu q$ are covariant derivatives of quark fields

$$D_\mu q = \left( \partial_\mu - ig_s \left( \frac{\lambda^\alpha}{2} A_\mu^\alpha \right) \right) q,$$

(40)

g_s is the strong coupling constant, $t^a = \lambda^a/2$ are the generators of SU(3), $\lambda^a$ are Gell-Mann matrices in the form

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (41)$$

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Gluon strength tensor is defined by the equation

\[ F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g_s f^{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu. \]  \hspace{1cm} (42)

Here \( f^{\alpha\beta\gamma} \) are the structure constant of the algebra \( su(3) \): \([t^\alpha, t^\beta] = if^{\alpha\beta\gamma} t^\gamma\), \( \alpha, \beta, \gamma = 1, \ldots, 8 \). They are antisymmetric on all indices and its nonzero values are as follows:

\[ f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2}, \]

\[ f^{156} = f^{367} = -\frac{1}{2}, \quad f^{458} = f^{678} = \sqrt{3} \frac{1}{2}. \]  \hspace{1cm} (43)

The choice of Gell-Mann matrices in the form (41) fix the basis in \( SU(3) \). This enables us to write out the covariant derivatives (40) in the explicit form

\[ D_\mu = \mathbf{1} \partial_\mu - i \frac{g_s}{2} \begin{pmatrix} A^3_\mu + \frac{1}{\sqrt{3}} A^8_\mu & A^1_\mu - i A^2_\mu & A^4_\mu - i A^5_\mu \\ A^1_\mu + i A^2_\mu & \frac{1}{\sqrt{3}} A^8_\mu - A^3_\mu & A^6_\mu - i A^7_\mu \\ A^4_\mu + i A^5_\mu & A^6_\mu + i A^7_\mu & -2 \frac{1}{\sqrt{3}} A^8_\mu \end{pmatrix} \]

\[ = \mathbf{1} \partial_\mu - i \frac{g_s}{2} \begin{pmatrix} A^R_\mu & A^G_\mu & A^B_\mu \\ A^G_\mu & A^G_\mu & A^B_\mu \\ A^B_\mu & A^B_\mu & A^B_\mu \end{pmatrix}, \]  \hspace{1cm} (44)

where

\[ A^R_\mu = \frac{1}{\sqrt{3}} A^8_\mu + A^3_\mu, \quad A^G_\mu = \frac{1}{\sqrt{3}} A^8_\mu - A^3_\mu, \quad A^B_\mu = -2 \frac{1}{\sqrt{3}} A^8_\mu, \]

\[ A^R_\mu + A^G_\mu + A^B_\mu = 0, \quad A^G_\mu = A^1_\mu + i A^2_\mu = \bar{A}^R_\mu, \]

\[ A^B_\mu = A^4_\mu + i A^5_\mu = \bar{A}^R_\mu, \quad A^B_\mu = A^6_\mu + i A^7_\mu = \bar{A}^G_\mu, \]  \hspace{1cm} (45)

and Lagrangian (39)

\[ \mathcal{L} = \bar{u}_i (i \gamma^\mu) (D_\mu)^{ij} u_j - m_u \bar{u}_i u_i + \ldots - \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} \equiv \]

\[ \equiv L_u + \ldots - \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha}, \]  \hspace{1cm} (46)

where only the \( u \) quark part is given. Let us note, that in QCD the special mechanism of spontaneous symmetry breaking is absent. Therefore, gluons are massless particles.

The QCD Lagrangian has rich dynamical content. It describes complicated hadron spectrum, color confinement, asymptotic freedom and many other effects.
5 QCD at High Energies

The contracted special unitary group $SU(3; \epsilon)$ is defined by the action

$$q'(\epsilon) = \begin{pmatrix} q'_1 \\ \epsilon q'_2 \\ \epsilon^2 q'_3 \end{pmatrix} = \begin{pmatrix} q_1 \\ \epsilon q_2 \\ \epsilon^2 q_3 \end{pmatrix} = U(\epsilon)q(\epsilon)$$

(47)

in the color space $C_3(\epsilon)$ as $\epsilon \to 0$. Under this action the hermitian form

$$q^\dagger(\epsilon)q(\epsilon) = |q_1|^2 + \epsilon^2 |q_2|^2 + \epsilon^4 |q_3|^2$$

(48)

remains invariant. Transition from the classical group $SU(3)$ and space $C_3$ to the group $SU(3; \epsilon)$ and space $C_3(\epsilon)$ is given by the substitution

$$q_1 \to q_1, \quad q_2 \to \epsilon q_2, \quad q_3 \to \epsilon^2 q_3,$$

$$A^{GR}_\mu \to \epsilon A^{GR}_\mu, \quad A^{BG}_\mu \to \epsilon A^{BG}_\mu, \quad A^{BR}_\mu \to \epsilon^2 A^{BR}_\mu,$$

(49)

and diagonal gauge fields $A^{RR}_\mu, A^{GG}_\mu, A^{BB}_\mu$ remain unchanged. Substitutions (28) and (49) lead to the quark part of the QCD Lagrangian in the form

$$L_q(\epsilon) = L^0_q + \epsilon L^{(1)}_q + \epsilon^2 L^{(2)}_q + \epsilon^3 L^{(3)}_q + \epsilon^4 L^{(4)}_q + \epsilon^5 L^{(5)}_q,$$

(50)

where

$$L^0_q = \sum_q \left\{ i\bar{q}_1 \gamma^\mu \partial_\mu q_1 + \frac{g_s}{2} |q_1|^2 \gamma^\mu \left( \frac{1}{\sqrt{3}} A^8_\mu + A^3_\mu \right) \right\},$$

(51)

$$L^{(1)}_q = -\sum_q m_q |q_1|^2, \quad L^{(3)}_q = -\sum_q m_q |q_2|^2, \quad L^{(5)}_q = -\sum_q m_q |q_3|^2,$$

(52)

$$L^{(2)}_q = \sum_q \left\{ i\bar{q}_2 \gamma^\mu \partial_\mu q_2 + \frac{g_s}{2} |q_2|^2 \gamma^\mu \left( \frac{1}{\sqrt{3}} A^8_\mu - A^3_\mu \right) + \right.$$  \left. +q_1 \bar{q}_2 \gamma^\mu (A^1_\mu + iA^2_\mu) + \bar{q}_1 q_2 \gamma^\mu (A^1_\mu - iA^2_\mu) \right\},$$

(53)

$$L^{(4)}_q = \sum_q \left\{ i\bar{q}_3 \gamma^\mu \partial_\mu q_3 + \frac{g_s}{2} \left( -\frac{2}{\sqrt{3}} A^8_\mu |q_3|^2 \gamma^\mu A^{BB}_\mu + \right. \right.$$  \left. +q_1 \bar{q}_3 \gamma^\mu (A^4_\mu + iA^5_\mu) + \bar{q}_1 q_3 \gamma^\mu (A^4_\mu - iA^5_\mu) + \right.$$  \left. +q_2 \bar{q}_3 \gamma^\mu (A^6_\mu + iA^7_\mu) + \bar{q}_2 q_3 \gamma^\mu (A^6_\mu - iA^7_\mu) \right\}.$$

(54)
The gluon part \( L_{gl} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} \) of the Lagrangian has the form

\[
\mathcal{L}_{gl}(\epsilon) = L_{gl}^0 + \epsilon^2 L_{gl}^{(2)} + \epsilon^4 L_{gl}^{(4)} + \epsilon^6 L_{gl}^{(6)} + \epsilon^8 L_{gl}^{(8)},
\]

(55)

where

\[
L_{gl}^0 = -\frac{1}{4} \left\{ \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 + \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right\},
\]

(56)

\[
L_{gl}^{(2)} = -\frac{1}{4} \left\{ \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 + \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right\} + \frac{g_s}{2} \left[ \left( A_\mu^3 A_\nu^7 - A_\mu^7 A_\nu^3 \right) + \sqrt{3} \left( A_\mu^5 A_\nu^8 - A_\mu^8 A_\nu^5 \right) \right]^2 + \left( \partial_\mu A_\nu^5 - \partial_\nu A_\mu^5 \right)^2 + \left( \partial_\mu A_\nu^7 - \partial_\nu A_\mu^7 \right)^2 + \frac{g_s}{2} \left[ A_\mu^3 A_\nu^6 - A_\mu^6 A_\nu^3 \right] + \sqrt{3} \left( A_\mu^6 A_\nu^8 - A_\mu^8 A_\nu^6 \right) \right]^2 + g_s \left[ \left( 2 A_\mu^1 A_\nu^2 - A_\mu^2 A_\nu^1 \right) - A_\mu^3 A_\nu^7 + A_\mu^7 A_\nu^3 \right] \left( \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 \right) + \left( \partial_\mu A_\nu^1 A_\nu^3 - \partial_\nu A_\mu^3 A_\nu^1 \right) + \left( \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 \right),
\]

(57)

\[
L_{gl}^{(4)} = -\frac{1}{4} \left\{ \left( \partial_\mu A_\nu^4 - \partial_\nu A_\mu^4 \right)^2 + \left( \partial_\mu A_\nu^5 - \partial_\nu A_\mu^5 \right)^2 + \frac{g_s}{2} \left[ \left( A_\mu^4 A_\nu^7 - A_\mu^7 A_\nu^4 \right) + \left( A_\mu^5 A_\nu^9 - A_\mu^9 A_\nu^5 \right) \right] \right\} + \frac{g_s}{2} \left[ \left( A_\mu^4 A_\nu^5 - A_\mu^5 A_\nu^4 \right) + \left( A_\mu^4 A_\nu^6 - A_\mu^6 A_\nu^4 \right) + \left( A_\mu^5 A_\nu^7 - A_\mu^7 A_\nu^5 \right) \right] \left( \partial_\mu A_\nu^5 - \partial_\nu A_\mu^5 \right) + \left( \partial_\mu A_\nu^4 - \partial_\nu A_\mu^4 \right)^2 + \left( \partial_\mu A_\nu^1 - \partial_\nu A_\mu^1 \right)^2 + \left( \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 \right)^2 + \left( \partial_\mu A_\nu^4 - \partial_\nu A_\mu^4 \right)^2 + \left( \partial_\mu A_\nu^5 - \partial_\nu A_\mu^5 \right)^2 + \left( \partial_\mu A_\nu^6 - \partial_\nu A_\mu^6 \right)^2 + \left( \partial_\mu A_\nu^7 - \partial_\nu A_\mu^7 \right)^2 + \left( \partial_\mu A_\nu^8 - \partial_\nu A_\mu^8 \right)^2 + \left( \partial_\mu A_\nu^9 - \partial_\nu A_\mu^9 \right)^2,
\]

(58)
\[+(A_\mu^2 A_\nu^4 - A_\mu^1 A_\nu^2 - A_\mu^A_\nu^8 + A_\mu^5 A_\nu^1) \left( \partial_\mu A_\nu^6 - \partial_\nu A_\mu^6 \right) +
+(A_\mu^1 A_\nu^4 - A_\mu^3 A_\nu^8 + A_\mu^5 A_\nu^2) \left( \partial_\mu A_\nu^7 - \partial_\nu A_\mu^7 \right) +
+\sqrt{3} \left( A_\mu^4 A_\nu^5 - A_\mu^5 A_\nu^4 \right) \left( \partial_\mu A_\nu^6 - \partial_\nu A_\mu^6 \right) +
+g_s^2 \left[ (A_\mu^1 A_\nu^2 - A_\mu^2 A_\nu^1)^2 + (A_\mu^6 A_\nu^7 - A_\mu^7 A_\nu^6)^2 -
-(A_\mu^1 A_\nu^2 - A_\mu^2 A_\nu^1) (A_\mu^6 A_\nu^7 - A_\mu^7 A_\nu^6) -
-(A_\mu^1 A_\nu^3 - A_\mu^3 A_\nu^1) (A_\mu^4 A_\nu^6 - A_\mu^6 A_\nu^4 + A_\mu^5 A_\nu^7 - A_\mu^7 A_\nu^5) +
+(A_\mu^2 A_\nu^3 - A_\mu^3 A_\nu^2) (A_\mu^4 A_\nu^7 - A_\mu^7 A_\nu^4 - A_\mu^5 A_\nu^6 + A_\mu^6 A_\nu^5) +
+\frac{1}{2} \left( A_\mu^2 A_\nu^7 - A_\mu^7 A_\nu^2 + \sqrt{3} \left( A_\mu^4 A_\nu^6 - A_\mu^6 A_\nu^4 \right) \right) \times
\left( A_\mu^1 A_\nu^4 - A_\mu^4 A_\nu^1 + A_\mu^5 A_\nu^6 - A_\mu^6 A_\nu^5 \right) -
-\frac{1}{2} \left( A_\mu^3 A_\nu^6 - A_\mu^6 A_\nu^3 + \sqrt{3} \left( A_\mu^4 A_\nu^8 - A_\mu^8 A_\nu^4 \right) \right) \times
\left( A_\mu^1 A_\nu^4 - A_\mu^4 A_\nu^1 + A_\mu^2 A_\nu^5 - A_\mu^5 A_\nu^2 \right) +
+\frac{1}{2} \left( A_\mu^1 A_\nu^7 - A_\mu^7 A_\nu^1 + A_\mu^2 A_\nu^6 - A_\mu^6 A_\nu^2 +
+A_\mu^3 A_\nu^5 - A_\mu^5 A_\nu^3 - \sqrt{3} \left( A_\mu^4 A_\nu^8 - A_\mu^8 A_\nu^4 \right)^2 +
+\frac{1}{2} \left( A_\mu^1 A_\nu^6 - A_\mu^6 A_\nu^1 - A_\mu^2 A_\nu^7 + A_\mu^7 A_\nu^2 +
+A_\mu^3 A_\nu^4 - A_\mu^4 A_\nu^3 - \sqrt{3} \left( A_\mu^4 A_\nu^8 - A_\mu^8 A_\nu^4 \right)^2 \right) \right] \right) \right) \right) \right). \tag{58} \]

\[L_{gl}^{(6)} = -\frac{g_s^2}{16} \left\{ \left( A_\mu^4 A_\nu^7 - A_\mu^7 A_\nu^4 - A_\mu^5 A_\nu^6 + A_\mu^6 A_\nu^5 \right)^2 +
\left( A_\mu^4 A_\nu^6 - A_\mu^6 A_\nu^4 + A_\mu^5 A_\nu^7 - A_\mu^7 A_\nu^5 \right)^2 +
\left( A_\mu^2 A_\nu^4 - A_\mu^4 A_\nu^2 - A_\mu^1 A_\nu^5 + A_\mu^5 A_\nu^1 \right)^2 +
\left( A_\mu^1 A_\nu^4 - A_\mu^4 A_\nu^1 + A_\mu^2 A_\nu^5 - A_\mu^5 A_\nu^2 \right)^2 + \right\} \]
Thus, the Lagrangian of the modified QCD can be represented as an expansion in powers of the contraction parameter

\[ \mathcal{L}_{\text{QCD}}(\epsilon) = L^0 + \epsilon L^{(1)} + \epsilon^2 L^{(2)} + \epsilon^3 L^{(3)} + \epsilon^4 L^{(4)} + \epsilon^5 L^{(5)} + \epsilon^6 L^{(6)} + \epsilon^8 L^{(8)}, \]

where

\[ L^0 = L^0_q + L^0_{gl}, \quad L^{(1)} = L^{(1)}_q, \quad L^{(2)} = L^{(2)}_q + L^{(2)}_{gl}, \quad L^{(3)} = L^{(3)}_q, \]

\[ L^{(4)} = L^{(4)}_q + L^{(4)}_{gl}, \quad L^{(5)} = L^{(5)}_q, \quad L^{(6)} = L^{(6)}_{gl}, \quad L^{(8)} = L^{(8)}_{gl}. \]

According to our hypothesis, the contraction parameter is a monotonic function of temperature \( \epsilon \rightarrow 0 \) as \( T \rightarrow \infty \). Very high (“infinite”) temperatures can exist at the first stages of the Big Bang immediately after inflation in the pre-electroweak epoch \([5, 6]\).

### 6 Estimation of boundary values in the evolution of the Universe

As noted, the contraction of the gauge group of the Standard Model makes it possible to order in time different stages of its development, but does not allow determining their absolute dates. For this additional assumptions are needed. We assume that the contraction parameters for QCD and the Electroweak Model are the same.

Further, we assume that the Electroweak Model is fully restored at its characteristic temperature \( E_4 = 100 \text{ GeV} \), and the complete reconstruction of the QCD occurs at the temperature \( E_8 = 0.2 \text{ GeV} \). Let \( \Delta \) be the cutoff level for \( \epsilon^k, \ k = 1, 2, 4, 5, 6, 8 \), i.e. when \( \epsilon^k < \Delta \) all Lagrangians terms proportional to \( \epsilon^k \) are negligibly small. Also we will assume that the contraction parameter is proportional to the inverse temperature

\[ \epsilon(T) = \frac{A}{T}, \]

where

\[ A \quad \text{and} \quad \Delta \quad \text{are constants.} \]
where $A = \text{const}$. From the QCD equation $\epsilon^8(T_8) = A^8 T_8^{-8} = \Delta$ we obtain $A = T_8 \Delta^{1/8} = 0, 2\Delta^{1/8}$ GeV. Using the equation for the $k$th power $\epsilon^k(T_k) = A^k T_k^{-k} = \Delta$, we have

$$T_k = T_8 \Delta^{k-8} \approx 10^{\frac{88-15k}{2}} \text{ GeV}$$

and after simple calculations one easily obtains the boundary values (in GeV)

$$T_1 = 10^{18}, \ T_2 = 10^7, \ T_3 = 10^3, \ T_4 = 10^2, \ T_6 = 1, \ T_8 = 2 \cdot 10^{-1}. \quad (65)$$

The estimate of the "infinity" energy $T_1 \approx 10^{18}$ GeV is comparable with the Planck energy $\approx 10^{19}$ GeV, at which it is necessary to take into account the gravitational effects. Thus, the resulting evolution of elementary particles does not go beyond the problems described by electroweak and strong interactions.

### 7 Evolution of Particles

Combining the modified Lagrangians of the electroweak model (36) and quantum chromodynamics (61), we arrive at the Lagrangian of the Standard Model, represented as an expansion in powers of the contraction parameter

$$\mathcal{L}_{SM}(\epsilon) = \mathcal{L}_0 + \epsilon \mathcal{L}_1 + \epsilon^2 \mathcal{L}_2 + \epsilon^3 \mathcal{L}_3 + \epsilon^4 \mathcal{L}_4 + \epsilon^5 \mathcal{L}_5 + \epsilon^6 \mathcal{L}_6 + \epsilon^8 \mathcal{L}_8, \quad (66)$$

where

$$\mathcal{L}_0 = L_0 + L_q^0 + L_{gl}^0, \quad \mathcal{L}_1 = L_1 + \mathcal{L}_q^{(1)} = L_{q,1} + L_q^{(1)}, \quad \mathcal{L}_2 = L_2 + L_q^{(2)} + L_{gl}^{(2)},$$

$$\mathcal{L}_3 = L_q^{(3)}, \quad \mathcal{L}_4 = L_4 + L_q^{(4)} + L_{gl}^{(4)}, \quad \mathcal{L}_5 = L_q^{(5)}, \quad \mathcal{L}_6 = L_{gl}^{(6)}, \quad \mathcal{L}_8 = L_{gl}^{(8)}. \quad (67)$$

The properties of particles change from epoch to epoch and are described by terms in the corresponding Lagrangians. We may draw some conclusions about the properties of particles and their interactions in different epochs even at the level of the classical fields.

In the "infinite" temperature limit ($\epsilon = 0, T > 10^{18}$ GeV) Lagrangian of the Electroweak Model (36) is written as

$$L_0 = -\frac{1}{4} \mathcal{F}^2_{\mu\nu} - \frac{1}{4} \mathcal{F}^2_{\mu\nu} + \nu_i^{\dagger} i \tau^\mu \partial_\mu \nu_i + u_i^{\dagger} i \tau^\mu \partial_\mu u_i +$$

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i.e., the limit model contains only \textit{massless particles}: photons $A_\mu$ and neutral bosons $Z_\mu$, left quarks $u_l$ and neutrinos $\nu_l$, right electrons $e_r$ and quarks $u_r, d_r$.

This phenomenon has a simple physical explanation: the temperature is so high, that the particle mass is a negligibly small quantity in comparison to the kinetic energy. The electroweak interactions become long-range ones, since they are transferred by massless $Z$ bosons and photons. The charged fields corresponding to the translation subgroup are not included in the limit Lagrangians. The charged boson fields $W^\pm_\mu$ corresponding to the translation subgroup are not included in the limit Lagrangians (68) and (69).

We see from the interaction Lagrangian

\begin{equation}
L^\text{int}_0(A_\mu, Z_\mu) = \left( \sum_q \left\{ i\bar{q}_R \gamma^\mu \partial_\mu q_R + \frac{g_s}{2} |q_R|^2 \gamma^\mu A^\mu_R \right\} -
\end{equation}

that particles of different kind do not interact with one another. For example, neutrinos interact only with each other through neutral currents. All other particles are charged and interact with particles of the same sort by massless $Z_\mu$ bosons and photons. Particles of different kind do not interact. This looks like a sort of stratification of the Electroweak Model with particles of the same kind in each layer.

In the "infinite" temperature limit only two components of the gluon strength tensor are nonzero

\begin{align*}
F^3_{\mu\nu} &= \partial_\mu A^3_\nu - \partial_\nu A^3_\mu = \frac{1}{2} \left( F^{RR}_{\mu\nu} - F^{GG}_{\mu\nu} \right), \\
F^8_{\mu\nu} &= \partial_\mu A^8_\nu - \partial_\nu A^8_\mu = \frac{\sqrt{3}}{2} \left( F^{RR}_{\mu\nu} + F^{GG}_{\mu\nu} \right),
\end{align*}

so we can write out the limiting QCD Lagrangian explicitly

\begin{equation}
\mathcal{L}^0 = L_q^0 + L_{gl}^0 = \sum_q \left\{ i\bar{q}_R \gamma^\mu \partial_\mu q_R + \frac{g_s}{2} |q_R|^2 \gamma^\mu A^\mu_R \right\} -
\end{equation}
From $\mathcal{L}_0$ we conclude that in this limit only dynamic terms for one color component of massless quarks survive, i.e., quarks become monochromatic. Terms describing interaction of these components with $R$ gluons also persist. Besides $R$ gluons there are also $G$ gluons, which do not interact with the quarks. Thus, there is stratification in the QCD sector as well.

The limit Lagrangian $\mathcal{L}_0$ can be considered as a good approximation after the Big Bang, just as the nonrelativistic mechanics is a good approximation of the relativistic one at low velocities.

In the temperature interval $10^{18}$ GeV $\geq T > 10^7$ GeV the terms

$$\mathcal{L}_1 = -\sum_q m_q |q_1|^2 - \sum_{\bar{q}} m_{\bar{q}} (\bar{q}\Gamma \bar{q} + \bar{q}^\dagger \bar{q}) \quad q = u, d, s, t, b, \quad \bar{q} = u, c, t$$

(72)

are added to the Lagrangian $\mathcal{L}_0$. The mass terms of quarks appear in the Lagrangian. Therefore, all quarks restore their masses in the process of evolution of the Universe in a given interval.

In the interval $10^7$ GeV $\geq T > 10^3$ GeV the electron mass terms $\epsilon^2 \left[ m_e (e^+_l e^+_r + e^+_r e^+_l) \right]$ appear in the Lagrangian. The same is true for the $\mu$ and $\tau$ leptons. All these particles become massive at this epoch.

Quarks acquire the second color degree of freedom and scalar Higgs boson began to interact with charged $W$ bosons between $10^3$ GeV and $10^2$ GeV due to Lagrangian $\mathcal{L}_3 = -\sum_q m_q |q_2|^2 + g W^+ W^- \chi$. The main part of the electroweak and color interactions is restored in this two last stages.

In the next interval $10^2$ GeV $\geq T > 4$ GeV charged $W$ bosons and the Higgs boson $\chi$ are the last to restore mass. More complex interactions arise, such as self-interactions of the Higgs boson, interactions of two Higgs bosons with two charged bosons, and four charged bosons. The Electroweak Model is ultimately restored. Quarks acquire the third color degree of freedom between 4 GeV and 1 GeV due to the terms $\mathcal{L}_5 = -\sum_q m_q |q_3|^2$.

At energies 1 GeV $\geq T > 0.2$ GeV there exist all color interactions except $L_{gl}^{(8)} = -\frac{g_s^2}{4} (A^4_\mu A^5_\nu - A^5_\mu A^4_\nu)^2$. Finally, at $T \leq 0.2$ GeV the Standard Model is entirely restored.

8 Conclusions

We have investigated the high-temperature limit of the Standard Model which was obtained from the first principles of the gauge theory as the con-
traction of its gauge group. It was shown that if the mathematical contraction parameter is taken to be inversely proportional to the temperature of the Universe, then its zero limit corresponds to the "infinite" temperature of the order of the Planck energy of $10^{19}$ GeV. In the process of the evolution of the Universe, the Standard Model passes through several stages, which are distinguished by the powers of the contraction parameter, what gives the opportunity to classify them in time as earlier-later. To determine the absolute dates of these stages the additional assumptions were used, namely: the inverse temperature dependence of contraction parameter $\epsilon$ and the cutoff level $\Delta$ for $\epsilon^k$. The unknown parameters are determined with consideration of typical QCD and Electroweak Model energies.

The exact expressions for the respective Lagrangians for any stage in the Standard Model evolution are obtained. On the base of decompositions (36), (61), and (66), the intermediate Lagrangians $L_k$ for any temperature scale are constructed. It gives an opportunity to draw conclusions on the interactions and properties of the elementary particles in each of the considered epochs.

The evolution of the elementary particles and their interactions in the early Universe obtained with the help of the contractions of the gauge groups of the Standard Model does not contradict the canonical one [10], according to which the QCD phase transitions take place later than the electroweak phase transitions. The developed evolution of the Standard Model present the basis for a more detailed analysis of different phases in the formation of leptons and quark-gluon plasma, in view of the fact that the terms $L_{gl}^{(6)}$ and $L_{gl}^{(8)}$ in the gluon Lagrangian (55) become negligible small at temperatures from 0.2 GeV to 100 GeV and in the temperature interval of 100 GeV to 1000 GeV only the interaction of the Higgs boson with charged $W$ bosons is restored.

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