Efficient polarization entanglement concentration for electrons with charge detection

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We present an entanglement concentration protocol for electrons based on their spins and their charges. The combination of an electronic polarizing beam splitter and a charge detector functions as a parity check device for two electrons, with which the parties can reconstruct maximally entangled electron pairs from those in a less-entanglement state nonlocally. This protocol has a higher efficiency than those based on linear optics and it does not require the parties to know accurately the information about the less-entanglement state, which makes it more convenient in a practical application of solid quantum computation and communication.

**Keywords:** quantum physics, entanglement concentration, electrons, charge detection, quantum computation

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Entanglement plays an important role in quantum information processing and transmission \cite{1,2,3}. For most of the practical quantum computation and communication protocols, the maximally entangled states are usually required. However, an entangled quantum system transmitted in a realistic quantum channel (such as a fiber or a free space) will suffer from noise, which will degrade the quality of its entanglement, and then a maximally entangled state will become a mixed state. Several entanglement purification protocols have been proposed \cite{4,5,6,7,8} since the first one based on controlled-not (CNOT) gates and bi-lateral operations was proposed by Bennett et al. \cite{9} in 1996. For instance, Deutsch et al. \cite{11} improved the efficiency and decreased the difficulty in the first one. Pan et al. \cite{12} proposed an entanglement purification protocol (EPP) with polarizing beam splitters (PBSs) and sophisticated single-photon detectors in 2001. Simon and Pan \cite{13} presented an EPP for a parametric down-conversion (PDC) source in 2003. Recently, we \cite{14} introduced a scheme for polarization-entanglement purification based on PDC sources with cross-Kerr nonlinearity, which is not only suitable for an ideal entanglement source but also for a PDC source.

Different from entanglement purification, entanglement concentration is used to distill a set of less-entanglement pure states for obtaining a subset of maximally entangled states. This topic is interesting as the process for storing quantum systems and even producing entangled states with asymmetrical devices usually makes the entangled quantum systems become less-entanglement ones. The first entanglement concentration protocol (ECP), named Schmidt projection method, was proposed by Bennett et al. \cite{9} in 1996. In their protocol, they need some collective measurements, which are hard to manipulate in experiment at present. They also need to know the accurate information of the less-entanglement state. There is another type of ECP, named entanglement swapping \cite{10,11} in which collective Bell-state measurements are required. Two similar ECPs were proposed independently by Yamamoto et al. \cite{12,13} and Zhao et al. \cite{13}. In their protocol, they use some PBSs to make a parity check for two photons. However, for distinguishing the four-mode instances from others, both the parties should possess some sophisticated single-photon detectors. In 2008, we \cite{14} proposed an ECP based on the cross-Kerr nonlinearities. By iteration of this protocol, the whole efficiency and the yield are higher than those with linear optical elements. However, for getting a higher efficiency and yield, a strong cross-Kerr media or an intense coherent beam is required.

Currently, one hand, most of the entanglement purification and concentration protocols are focused on photons. On the other hand, quantum communication and computation can also be achieved with conduction electrons since Beenakker et al. \cite{15} broke through the obstacle of the no-go theorem \cite{16} in 2004. An electron system has both its spin degree of freedom and its charge degree of freedom. Moreover, Spin and charge commute, so a measurement of the charge leaves the spin qubit unaffected \cite{15}. With the charge detector \cite{17} which can distinguish the occupation number one from the occupation number 0 and 2, but cannot distinguish between 0 and 2, people can construct CNOT gates \cite{15} and charge qubits \cite{15}, entangle spins \cite{15}, and prepare cluster states.

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and a multipartite entanglement analyzer \[20\].

In this Letter, we present an electronic entanglement concentration protocol with the aid of charge detection, following some ideas in Schmidt projection method \[9\] and quantum erasure \[21\]. The combination of a polarizing beam splitter (PBS) and a charge detector functions as a parity check device for electron spins with nondestructive measurements, with which the parties can reconstruct maximally entangled electron pairs from those in a less-entanglement pure state efficiently. This protocol does not require the parties to know accurately the information about the less-entanglement states, i.e., their coefficients. Compared with the protocols based on linear optical elements, it has a higher efficiency as the states in unsuccessful instances in the first entanglement concentration can also be concentrated probabilistically in the next round.

Now, we detail how our ECP works. Let us consider two pairs of entangled electrons in the following unknown polarization states:

\[
|\Phi_a\rangle_{a_1b_1} = \alpha|\uparrow\rangle_{a_1}|\downarrow\rangle_{b_1} + \beta|\downarrow\rangle_{a_1}|\uparrow\rangle_{b_1},
\]

\[
|\Phi_a\rangle_{a_2b_2} = \alpha|\uparrow\rangle_{a_2}|\downarrow\rangle_{b_2} + \beta|\downarrow\rangle_{a_2}|\uparrow\rangle_{b_2},
\]

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the spin up state and the spin down state, respectively, and $|\alpha|^2 + |\beta|^2 = 1$. Alice owns the electrons $a_1$ and $a_2$, and Bob owns the electrons $b_1$ and $b_2$. Here we call the mode $a_1b_1$ the upper mode, and $a_2b_2$ the lower mode, shown in Fig. 1. The original nonmaximally entangled state of the four electrons can be written as:

\[
|\Psi\rangle = |\Phi_a\rangle_{a_1b_1} \otimes |\Phi_a\rangle_{a_2b_2} = |\uparrow\rangle_{a_1}|\downarrow\rangle_{b_1}|\uparrow\rangle_{a_2}|\downarrow\rangle_{b_2} + \alpha\beta|\downarrow\rangle_{a_1}|\uparrow\rangle_{b_1}|\downarrow\rangle_{a_2}|\uparrow\rangle_{b_2} + \beta^2|\downarrow\rangle_{a_1}|\downarrow\rangle_{b_1}|\uparrow\rangle_{a_2}|\downarrow\rangle_{b_2} + \alpha\beta|\downarrow\rangle_{a_1}|\downarrow\rangle_{b_1}|\downarrow\rangle_{a_2}|\uparrow\rangle_{b_2}.
\]

(1)

Before the two electrons $a_2$ and $b_2$ are transmitted to Alice and Bob, respectively, two electronic half-wave plates are used to transfer $|\uparrow\rangle$ to $|\downarrow\rangle$ or vice versa. The whole state of the two electron pairs becomes:

\[
|\Psi\rangle' = |\uparrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{a_3}|\uparrow\rangle_{a_4} + \alpha\beta|\downarrow\rangle_{a_1}|\uparrow\rangle_{a_2}|\uparrow\rangle_{a_3} + \beta^2|\downarrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{a_3} + \alpha\beta|\downarrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\uparrow\rangle_{a_3} + \beta^2|\downarrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{a_3} + \alpha\beta|\downarrow\rangle_{a_1}|\uparrow\rangle_{a_2}|\downarrow\rangle_{a_3}.
\]

(2)

We use $a_3(b_3)$ to substitute $a_2(b_2)$ after the 90° rotation $R_{90}$ which transfers $|\uparrow\rangle$ to $|\downarrow\rangle$ and vice versa. It is obvious that the two terms $|\uparrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{a_3}$ and $|\downarrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{a_3}$ have the same coefficient of $\alpha\beta$, and the other two terms are different ($\alpha^2$ or $\beta^2$). Bob lets the two electrons $b_1$ and $b_3$ pass through a PBS which fully transmits $|\uparrow\rangle$ polarization electrons and fully reflects $|\downarrow\rangle$ electrons.

After the PBS, one can see that in Bob’s laboratory, the states $|\uparrow\rangle_{b_1}|\uparrow\rangle_{b_3}$ and $|\downarrow\rangle_{b_1}|\downarrow\rangle_{b_3}$ will make each of the two spatial modes $c_1$ and $c_2$ contains one electron. The charge detector $P$ will detect only one electron with a nondestructive measurement. However, the state $|\uparrow\rangle_{b_1}|\downarrow\rangle_{b_3}$ will make two electrons in the lower mode $c_2$ and $|\downarrow\rangle_{b_1}|\uparrow\rangle_{b_3}$ will make the two electrons in the upper mode $c_1$, respectively, which means that the charge detector will detect two electrons or no electrons.

If the charge detector detects only one electron, the four electrons will collapse to the state as:

\[
|\Psi\rangle'' = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{a_1}|\uparrow\rangle_{a_2}|\uparrow\rangle_{c_1}|\downarrow\rangle_{c_2} + |\downarrow\rangle_{a_1}|\downarrow\rangle_{a_2}|\downarrow\rangle_{c_1}|\uparrow\rangle_{c_2}).
\]

(4)

The probability for the charge detector to detect one electron in each mode is $2|\alpha\beta|^2$.

The state described with Eq. (4) is a maximally entangled state for four electrons. It is easy to get a maximally entangled two-electron state. We only need to perform a Hadamard operation on each of the two electrons $a_3$ and $c_2$, and then measure them with the basis $Z = \{|\uparrow\rangle, |\downarrow\rangle\}$, shown in Fig. 1. In detail, after the two Hadamard (H) operations on $a_3$ and $c_2$ (the H operation completes the transformations $|\uparrow\rangle \rightarrow (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ and $|\downarrow\rangle \rightarrow (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$), the whole state of the four electrons becomes:

\[
|\Psi\rangle''' = \frac{1}{2\sqrt{2}}(|\uparrow\rangle_{a_1}|\uparrow\rangle_{c_1} + |\downarrow\rangle_{a_1}|\downarrow\rangle_{c_1})(|\uparrow\rangle_{a_2}|\uparrow\rangle_{c_3} + |\downarrow\rangle_{a_2}|\downarrow\rangle_{c_3}).
\]

(5)

The last step is to measure the spins of the electrons $a_4$ and $c_3$ with the basis $Z$. If the two detectors $D_1$ and $D_2$ have the same results, the electron pair $a_1c_1$ will collapse to the state:

\[
|\phi^+\rangle_{a_1c_1} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{a_1}|\uparrow\rangle_{c_1} + |\downarrow\rangle_{a_1}|\downarrow\rangle_{c_1}).
\]

(6)
Otherwise, we will get
\[
|\phi^-\rangle_{a_1c_1} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{a_1}|\uparrow\rangle_{c_1} - |\downarrow\rangle_{a_1}|\downarrow\rangle_{c_1}). \tag{7}
\]

Alice or Bob performs a phase-flip operation on her or his electron to get $|\phi^+\rangle_{a_1c_1}$. That is, a maximally entangled two-electron state $|\phi^+\rangle$ can be generated with the steps described above.

![Diagram of entanglement concentration process](image)

FIG. 2: The principle for obtaining some less-entanglement states from the fail instances. Another PBS is used to make each spatial mode contain only one electron.

In our protocol, the charge detector $P$ is used to detect the parity of two electrons. If the two electrons are in an even parity (both spin up or down), the charge detector will detect only one electron, and the less-entanglement state $|\Phi\rangle$ can be concentrated to the maximally entangled state. Otherwise, the charge detector will detect 0 or 2 electrons, which means the entanglement concentration process fails. In this time, the four-electron system collapses to another less-entanglement state (without being normalized):
\[
|\Phi_1\rangle = \alpha^2|\uparrow\rangle_{a_1}|\downarrow\rangle_{a_3}|\uparrow\rangle_{c_1}|\downarrow\rangle_{c_2} + \beta^2|\downarrow\rangle_{a_1}|\uparrow\rangle_{a_3}|\downarrow\rangle_{c_1}|\uparrow\rangle_{c_2}. \tag{8}
\]

Different from the ECPs with linear optics $[12, 13]$, the states in the unsuccessful instances in this protocol can also be concentrated to the maximally entangled state in the next round. We show the principle in Fig.2. Another PBS is added to divide the two electrons into two different spatial modes $c_3$ and $c_4$. After the measurements on the electrons $a_4$ and $c_5$, Eq. (8) become:
\[
|\Phi_1\rangle' = \alpha^2|\uparrow\rangle_{a_1}|\downarrow\rangle_{a_3} \pm \beta^2|\downarrow\rangle_{a_1}|\downarrow\rangle_{c_3}. \tag{9}
\]

The $'+$ or $'-'$ depend on the facts that the results of $D_1$ and $D_2$ are the same one or different ones, respectively. It is obvious that Eq. (9) has the same form as Eq. (11). We can pick up two pairs of electrons in the less-entanglement state shown in Eq. (9) and perform the similar concentration process as our ECP does not require the two parties to know accurately the information about the coefficients $\alpha$ and $\beta$. With the iteration of the entanglement concentration process above, the efficiency of our protocol is higher than the protocols based on linear optics $[12, 13]$.

It is straightforward to generalize this entanglement concentration protocol to the case for multipartite pure entangled states. Let us suppose that the pure entanglement states of $n$-electron quantum systems are
\[
|\Phi_2\rangle = \alpha|u\rangle|\uparrow\rangle_{a_1}|\uparrow\rangle_{b_1} + \beta|v\rangle|\downarrow\rangle_{a_1}|\downarrow\rangle_{b_1}, \tag{10}
\]

where $|u\rangle$ and $|v\rangle$ are states of $n-2$ electrons, which are owned by the other $n-2$ parties (not Alice and Bob). The $n$ parties can accomplish the entanglement concentration as the same as that discussed above by replacing $\alpha$ and $\beta$ in Eq. (11) with $\alpha|u\rangle$ and $\beta|v\rangle$, respectively. With the similar operations, the $n$ parties can reconstruct maximally entangled $n$-electron Greenberger-Horne-Zeilinger (GHZ) states $|\phi^+\rangle_n = \frac{1}{\sqrt{2}}(|u\rangle|\uparrow\rangle_{a_1}|\uparrow\rangle_{b_1} + |v\rangle|\downarrow\rangle_{a_1}|\downarrow\rangle_{b_1})$ from partially entangled GHZ-class states $|\Phi_2\rangle$.

In conclusion, we propose an electronic entanglement concentration protocol based on charge detection. We exploit the combination of an electronic polarizing beam splitter and a charge detector to distinguish the parity of two electrons. Compared with other ECPs, this protocol is simpler and convenient as it does not require collective measurements and sophisticated detectors. Moreover, it does not require the parties to know accurately the information about the less-entanglement state. By iterating the entanglement concentration processes, this protocol has a higher efficiency than those based on linear optics. These advantages make our scheme have a good application in solid quantum computation and communication.

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