Scheduling Optimization of Joint Fire Strike of Time Series Targets Based on Grey Target Decision

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Abstract. Aiming at targets in joint fire fighting importance measure accuracy and timeliness of the problem, analyzing the characteristics of joint fire operations and the demand of the important target sequence optimization, based on grey target decision is established on the basis of the sequential hierarchical ordering model, is used to solve the problem of timing target importance. Based on the classification of the combat target system, this model reasonably sets the time series target index weight, and combines the main characteristics of each stage of the joint fire attack, proposes the model algorithm of the classification of the target system. Simulation verification was carried out according to the 6 types of targets of joint fire strike. Simulation results show that the scheduling optimization model based on grey target decision can meet the requirements of real-time operations.

Keywords: Grey Target Decision, Temporal Targets, Combined Fire Strike

1. Introduction

Due to the discreteness and randomness of firing targets in time, azimuth and distance, military modeling of such problems can be attributed to time-series target decision problem [1]. In fact, the sequential multi-objective decision making problem is to make decisions and comprehensively sort a finite number of discrete schemes in a three-dimensional space with time, objectives and schemes. Its basic feature is the addition of time space on the basis of decision space and target space, so the decision process and results of such problems reflect the characteristics of time sequence (dynamic). In literature [2], the analytic hierarchy process (ahp) was used to solve the practical multi-objective decision-making problem of specific time series to a certain extent. However, there were still some shortcomings such as large computation and unclear physical meaning, and the application was limited [3]. In literature [4], several kinds of fuzzy theoretical methods for time-series multi-objective decision making were proposed by using fuzzy mathematical tools, but the determination of fuzzy membership functions and the fuzziness of index parameters would add human factors and lose useful information. Correlation analysis of grey system method based on grey correlation as a measure of a kind of decision sorting method, literature [5] using grey relational analysis method for sequential multi-objective decision making problems are studied, but as a result of grey correlation degree is
affected by the huge difference between two levels and tiny difference is bigger, the as decision-making basis will appear the phenomenon of "distortion", the other correlation value tend to be homogenization, resolution is low, not easy decision.

2. Sequential Multi-Objective Decision Modeling
The number of time periods (i.e., time sample points) of the time series multi-objective decision problem is set as \( t = (t = 1,2,\cdots,T) \), the target or indicator set is set as \( P(i = 1,2,\cdots,m) \), and the decision plan set of the system is set as \( S_j(j = 1,2,\cdots,n) \). Accordingly, there are \( t \) number of decision plans to be selected in the time period \( n \) of the system, and each plan has \( m \) number of indicators to form the indicator set for evaluating the merits of the plan. According to the evaluation of \( m \) evaluation index on the alternative decision scheme, the index characteristic matrix of \( t \) scheme in period \( n \) can be obtained [6]:

\[
A_t = (A_{t1}, A_{t2}, \cdots, A_{tn})
\]  

(1)

Where, \( A_{jt} = (a_{1jt}, a_{2jt}, \cdots, a_{mjt})^T \) is the decision plan \( t \) of time period \( j \), and the sign " \( ^T \) " is the transpose sign. The characteristic quantity matrix of different time series indexes in the decision plan \( j \) is:

\[
A_j = (A_{j1}, A_{j2}, \cdots, A_{jn}) = \begin{bmatrix}
a_{11j} & a_{12j} & \cdots & a_{1mj} \\
a_{21j} & a_{22j} & \cdots & a_{2mj} \\
\vdots & \vdots & \ddots & \vdots \\
a_{mj1} & a_{mj2} & \cdots & a_{mjm}
\end{bmatrix}
\]  

(2)

In the actual research, the indexes of the system generally fall into the following four categories: cost type (the smaller the better), efficiency type (the bigger the better), moderate type (the closer to a fixed value the better), and interval type (the best is to fall into a fixed range). According to different classification of indicators, the indicator set \( P \) is divided as follows:

\[
P = \bigcup_{i=1}^{4} U_i, U_i \cap U_j = \emptyset, i \neq j,
\]

\[
i, j \in \{1, 2, 3, 4\}, U_i(i = 1,2,3,4)
\]

Is the index set of cost type, benefit type, moderate type and interval type, and \( \emptyset \) is the empty set.

In order to eliminate the influence of different indexes and different dimensions, the index characteristic matrix \( A_j \) of each time series decision scheme was normalized and transformed into \( S_j \). Different normative methods are adopted for different types of indicators. Element \( S_{jt} \) in matrix \( S_{jt} \) adopts the following normalized formula:

For the cost indicator,

\[
S_{jt} = \frac{\text{max}_i \text{max}_j a_{jt} - a_{jt}}{\text{max}_j \text{max}_i a_{jt} - \text{min}_j \text{min}_i a_{jt}} \quad i \in U_i
\]
For the efficiency indicator, let

$$S_i = \frac{a_i - \min_{j} \min_{i} a_{ij}}{\max_{j} \max_{i} a_{ij} - \min_{j} \min_{i} a_{ij}} \quad i \in U_2$$

For moderate goals, let

$$S_i = \left\{ \begin{array}{ll}
1 & a_i = a_i^* \\
\frac{1}{m} & a_i = a_i^* \quad i \in U_3
\end{array} \right.$$  

Where, $a_i^*$ is the best fixed value of indicator $P_i$ for time period $t$.

For the interval type index, let

$$S_i = \left\{ \begin{array}{ll}
\frac{a_i^1 - a_i}{\max\{a_i^1 - \min_{j} a_{ij}, \max_{j} a_{ij} - a_i^2\}} & a_i = a_i^1 \\
1 & a_i = a_i^2 \quad i \in U_3
\end{array} \right.$$  

Where, $[a_i^1, a_i^2]$ is the optimal stable interval of indicator needle $P_i$ to time $t$.

Thus, equation (1) can be transformed into a normalized decision matrix:

$$S_i = (S_{i1}, S_{i2}, \ldots, S_{in}) = \left[ \begin{array}{cccc}
s_{i11} & s_{i12} & \cdots & s_{i1n} \\
s_{i21} & s_{i22} & \cdots & s_{i2n} \\
\vdots & \vdots & \ddots & \vdots \\
s_{in1} & s_{in2} & \cdots & s_{inn}
\end{array} \right]$$

Where, $S_j = (S_{j1}, S_{j2}, \ldots, S_{jn})$ is the $j$th timing decision scheme of time period $t$.

$$S_j = (s_{j11}, s_{j12}, \ldots, s_{j1n}) = \left[ \begin{array}{cccc}
s_{j11} & s_{j12} & \cdots & s_{j1n} \\
s_{j21} & s_{j22} & \cdots & s_{j2n} \\
\vdots & \vdots & \ddots & \vdots \\
s_{jn1} & s_{jn2} & \cdots & s_{jnn}
\end{array} \right]_{m \times n}$$

The above formula is the normalization matrix of the fourth sequential multi-objective decision scheme.

### 2.1 Establishment of Index Weight

In the multi-objective decision making optimization of time series, due to the difference of importance between time periods and indexes, this model starts from the grey relational degree and determines the weight through the grey relational degree.

Let the index weight vector be: $W = (w_1, w_2, \ldots, w_m), 0 < w_i < 1, \sum_{i=1}^{m} w_i = 1$. The period weight vector is $\Gamma = (\lambda_1, \lambda_2, \cdots, \lambda_T), 0 < \lambda_T < 1, \sum_{i=1}^{T} \lambda_i = 1$. In the actual evaluation, the period weight is generally averaged, that is, $\lambda_1 = \lambda_2 = \cdots = \lambda_T = \frac{1}{T}$. Since the index weight of each period is different, the establishment of the index weight should be analyzed according to the specific period.

First, the evaluation index is dimensionless, and then the initial value is processed. The index characteristic quantity matrix $A_i (t = 1, 2, \cdots, T)$ of $n$ schemes in period $t$ in (1) is initialized. The method of removing all the components of a vector by its first component to get a new vector and
normalizing it is called initialization.

Suppose that the attribute value of ideal decision plan \( S_0 \) to indicator \( P_i \) is \( Q_{0i} \), and it satisfies: when the indicator is \( P_i \) benefit indicator, \( Q_{0i} = \max(Q_{1i}, Q_{2i}, \ldots, Q_{ni}) \); When the index is \( P_i \) cost-type index, \( Q_{0i} = \min(Q_{1i}, Q_{2i}, \ldots, Q_{ni}) \). Then matrix \( Q=(Q_{ji})_{(n+1)\times m} (j=0,1,2,\ldots,n; i=1,2,\ldots,m) \) is called the decision matrix of scheme set \( S_j \) versus index set \( P_i \). If \( Q'=(Q'_{ji})_{0i} (j=0,1,2,\ldots,n; i=1,2,\ldots,m) \), then matrix \( Q'=(Q'_{ji})_{(n+1)\times m} \) is called the initialization matrix of matrix \( Q=(Q_{ji})_{(n+1)\times m} \). Obviously, \( Q'_{0i} = 1(i=1,2,\ldots,m) \), ideal plan \( S_0 = \{1,1,\ldots,1\}_m \), with \( Q'_{0i} \) as the parent factor and \( Q'_{ji} \) as the sub-factor, shows the grey correlation degree of other plans and ideal plan \( \gamma_{ji} (j=0,1,2,\ldots,n; i=1,2,\ldots,m) \).

\[
\gamma_{ji} = \frac{\min \max \{Q_{ji}-Q_{0i}+\rho \max \min \{Q_{ji}-Q_{0i}'\} \}}{\max \min \{Q_{ji}-Q_{0i}+\rho \max \min \{Q_{ji}-Q_{0i}'\} \}} \\
(5)
\]

Where, \( \rho \) is the discrimination coefficient, the value range is \( 0 < \rho < 1 \), usually \( \rho = 0.5 \).

The matrix \( F(\gamma_{ji})_{n\times m} \) composed of \( \gamma_{ji} (j=1,2,\ldots,n; i=1,2,\ldots,m) \) grey relational degree \( n \times m \) is defined as the judgment matrix of multi-objective grey relational degree.

\[
F = \begin{bmatrix}
\gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} \\
\gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nm}
\end{bmatrix}
\]

Consider that \( (r_{1i}, r_{2i}, \cdots, r_{ni}) \) is the grey correlation degree of \( Q'_{0i} \) for the indicator \( i \) of each plan as the parent factor of \( n \), with \( Q'_{ji} (j=1,2,\ldots,n; i=1,2,\ldots,m) \) as the sub-factor. In fact, they reflect the correlation degree between the actual factor value \( A \) and the ideal value of each plan. The average value is:

\[
\overline{\omega}_i = \frac{1}{n} \sum_{j=1}^{n} \gamma_{ji} \quad (i=1,2,\ldots,m) \\
(6)
\]

According to the analysis, \( \overline{\omega}_i \) reflects the proportion of index \( i \) in the whole index space \( P \).

Normalize \( \overline{\omega}_i (i=1,2,\ldots,m) \), \( W_i = \frac{\overline{\omega}_i}{\sum_{i=1}^{m} \overline{\omega}_i} (i=1,2,\ldots,m) \), so \( W = (w_1, w_2, \cdots, w_m) \) can be used as the weight of the index.

### 2.2 Target Hierarchical Sorting Model

It is of relative significance to optimize or sort among \( n \) schemes, and the matrices of time-series ideal decision scheme and negative ideal decision scheme are defined as:
Obviously, the degree of correlation is the physical meaning of the corresponding decision scheme. According to the grey system theory correlation analysis method and the extension of time series, the plan $S_{ij}$ in the scheme $S_{j}$ of the scheme to be decided is respectively associated with element $A$ in the time-series ideal scheme matrix and time-series negative ideal scheme matrix. The correlation coefficients of $g_{it}$, $b_{it}$ are as follows:

$$
\xi_{it}^+ = \frac{\min_{i,j} \min_{j} g_{ij} - S_{ij} + \rho \max_{i,j} \max_{j} g_{ij} - S_{ij}}{g_{ij} - S_{ij} + \rho \max_{i,j} \max_{j} g_{ij} - S_{ij}}
$$

$$
\xi_{it}^- = \frac{\min_{i,j} \min_{j} b_{ij} - S_{ij} + \rho \max_{i,j} \max_{j} b_{ij} - S_{ij}}{b_{ij} - S_{ij} + \rho \max_{i,j} \max_{j} b_{ij} - S_{ij}}
$$

The correlation degree between the correspondences of each decision plan and the time-series ideal plan $G$ and the time-series negative ideal plan $B$ is as follows:

$$
\gamma_j^+ = \sum_{t=1}^{T} \sum_{i=1}^{m} \lambda_i w_j \xi_{it}^+
$$

$$
\gamma_j^- = \sum_{t=1}^{T} \sum_{i=1}^{m} \lambda_i w_j \xi_{it}^-
$$

Obviously, the relation degree of the above extension consideration satisfies the four axioms of grey relation degree.

Correlation as a measure (factors) sequence similarity measure, for range the amount of change, and the correlation is more close to 1, the sequence similarity degree, the greater the correlation is more close to 0, the smaller the sequence similarity degree, in order to measure the degree of sequence differences, the correlation difference degree as the degree of difference between sequence metrics.

Definition: correlation difference degree $= 1 - \gamma_j^+$ correlation degree. It can be seen that the physical meaning of correlation difference degree is opposite to that of correlation degree. Then the difference degree of time-series ideal correlation and the difference degree of time-series negative ideal correlation are as follows:

$$
\gamma_j^- = 1 - \gamma_j^+ = 1 - \sum_{t=1}^{T} \sum_{i=1}^{m} \lambda_i w_j \xi_{it}^+
$$
\[
\bar{\gamma}_j = 1 - \gamma_j = 1 - \sum_{i=1}^{m} \sum_{t=1}^{T} \lambda_i z_{it} g_{it} \\
(14)
\]

Due to the disadvantages of the correlation degree, which tends to be homogenized and the resolution is low, it is not easy to make the decision between the schemes with approximate index eigenvalues. For measuring scheme is \( j \) complete and ideal scheme, the introduction of the concept of subordinate degree of fuzzy mathematics, such as the first \( j \) solution to \( u_j \), subordinate to timing plan \( G \) ideal, then it again at the same time with \((1 - u_j)\) from the negative ideal solution \( B \), belong to the temporal associate \( u_j \) as weight is introduced into the difference degree, is \( u_j \cdot \bar{\gamma}_j = u_j(1 - \gamma_j) \) right timing for ideal correlation difference degree, by the same token, the negative ideal correlation difference degree is \((1 - u_j) \cdot \bar{\gamma}_j = (1 - u_j)(1 - \gamma_j) \) right timing.

To determine the ideal degree of dependency \( u_j \), the following objective function is established:

\[
\min \left\{ \sum_{j=1}^{n} \left[ u_j(1 - \gamma_j)^2 + (1 - u_j) \gamma_j^2 \right] \right\} \\
(15)
\]

The least square criterion of "distance squared sum minimum" is extended, that is, the sum of the squares of ideal correlation difference degree of weighting time series and the negative ideal correlation difference degree of weighting time series of all \( n \) timing decision schemes is minimized [7].

Take the partial derivative of the multivariate function \( F(u_1, u_2, \ldots, u_n) \) and set it to zero, namely

\[
\frac{\partial F}{\partial u_j} = 2u_j(1 - \gamma_j)^2 - 2(1 - u_j)(1 - \gamma_j)^2 = 0
\]

\[
u_j = \frac{(1 - \gamma_j)^2}{(1 - \gamma_j)^2 + (1 - \gamma_j)^2} = \frac{(1 - \sum_{i=1}^{m} \lambda_i z_{it} g_{it})^2}{(1 - \sum_{i=1}^{m} \lambda_i z_{it} g_{it})^2 + (1 - \sum_{i=1}^{m} \lambda_i z_{it} g_{it})^2}
\]

(16)

It can be seen from above that \( F(u_1, u_2, \ldots, u_n) \) is A convex function, so equation (15) is A convex programming, and equation (16) is the optimal solution of equation (15).

If \( S_j \) is not A negative ideal scheme, equation (16) can be converted to:

\[
u_j = \frac{1}{1 + \left( \frac{(1 - \gamma_j)^2}{(1 - \gamma_j)^2} \right)^2} = \frac{1}{1 + \left[ \frac{(1 - \sum_{i=1}^{m} \lambda_i z_{it} g_{it})^2}{(1 - \sum_{i=1}^{m} \lambda_i z_{it} g_{it})^2} \right]^2} \]

\[
(1 - u_j) = j = 1, 2, \ldots, n
\]

In summary, the calculation steps of this model are as follows:

Step 1: determine and normalize the characteristic matrix of the time series multi-objective decision making problem.

Step 2: determine the time sequence ideal decision scheme and the negative ideal decision scheme matrix.

Step 3: calculate the grey correlation degree and correlation difference degree of each decision
scheme;
Step 4: solve the grey dependency degree \( u_j \) of each decision plan;
Step 5 sort the evaluation by the size of \( u_j \).

3. Hierarchical Sorting Of the Target System

3.1 Determination of Index Data
- Joint fire early strike phase
  Leaders and command organs are the first level targets in all stages of joint fire attack, especially leaders are the most important; Highway hub and naval forces belong to the second level of targets, naval forces because of the direct impact of our army to seize the sea is more important; Ocean-going transportation lines and media organizations belong to the third level of targets.
- Combined fire in the middle attack phase
  At this stage, the joint fire strike action reflects the "shock and awe" effect. The media were upgraded to the second tier and more important; Highway hubs and naval forces were downgraded to tier 3, with the same degree of mutual importance, both more important than ocean shipping lines.
- Joint fire follow-up attack phase
  At this stage, the joint fire strike action reflects the idea of "paralysis". The navy rose to the first rank and assumed a more important position; Media organizations were downgraded to tier 3, with their mutual importance unchanged.

3.2 Model Solution
The normalization matrix can be obtained by normalizing the six index characteristic quantity matrices according to different normalization formulas:

\[
S_1 = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.33 & 0.33 & 0.33 \\ 1.00 & 1.00 & 1.00 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.88 & 0.88 & 0.88 \\ 0 & 0 & 0 \\ 0.43 & 0.43 & 0.43 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.50 & 0.50 & 0.50 \\ 0 & 0 & 0 \end{bmatrix}, \quad S_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.00 & 1.00 & 1.00 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 0.25 & 0.63 & 0.25 \\ 0 & 0 & 0 \\ 0.57 & 0.57 & 0.57 \end{bmatrix}, \quad S_6 = \begin{bmatrix} 0.50 & 0.25 & 1.00 \\ 0 & 0 & 0 \\ 0.85 & 0.85 & 0.85 \end{bmatrix}
\]

The optimal scheme of time series and the negative ideal scheme of time series can be obtained as follows:

\[
G = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 1.00 & 1.00 & 1.00 \\ 1.00 & 1.00 & 1.00 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

The correlation coefficients \( \xi_j^+ \) and \( \xi_j^- \) are solved according to formula (9), and then A and A are solved according to formula (12)

\[
\gamma_j^+ = (0.816, 0.544, 0.456, 0.560, 0.585, 0.659) \quad \gamma_j^- = (0.419, 0.626, 0.706, 0.712, 0.617, 0.469)
\]

According to the model (16) or (17), the final evaluation result A of six types of objectives can be obtained.

\[
u_j = (0.909, 0.402, 0.226, 0.299, 0.460, 0.708)
\]

From the above data, it can be seen that the ranking results of six categories of targets are as follows:
Level 1 goal: leader
Level 2 target: media organizations
Level 3 targets: naval forces, command structures, ocean shipping lines, highway hubs

According to the analysis, in the future joint fire strike, the leader because of its special role will be listed as the preferred target. At the same time, the proportion of attacking psychological targets will be increased, which also reflects the basic content of the idea of "shock and awe" in the information age. The multi-functional development of combat forces and command structures and the improvement of regeneration capability will reduce the degree of hard strikes against these two types of targets. In addition, the attack on the cyclic subsystem will help achieve the overall combat objective, so our army will adopt the campaign style of joint landing and joint blockade.

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