The Thales experiment

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1. Introduction: Thales and the Great Pyramid

Thales of Miletus (Θαλες, 625 ∼ 547 B.C.) was considered by ancient
Greeks as one of their Seven Sages, as the father of scientific approach to the
description of natural phenomena, and perhaps as the first person deserving
the title of mathematician.

Thales became famous for his prediction of solar eclipse of 585 B.C., and
for his ability to evaluate dimensions of objects at a distance, by comparing
their shadows with the shadow of a stick of known dimension.

The relationship of proportionality used by Thales to determine the height
of the Great Pyramid is also an introduction of linear dependence, the essence
of linear algebra.

It has become such a commonplace, that the physical aspects of this
fundamental experience are rarely considered in a more detailed manner.

In fact, Thales has performed an important physical experiment relating
different definitions of geometry; to put it more precisely, the notions of
straight lines and right angles. It turns out that the phenomena involved
in this experiment belong to quite different domains of physics: gravitation,
quantum mechanics and electromagnetism. The fact that they lead to three
different, but compatible definitions of geometry, suggests that these distinct
aspects of physical reality are apparently related. This fact gives rise to one
of the most important and fundamental questions concerning physics and our
perception of physical world, still open after more than twenty-five centuries.
2. The three definitions of geometry

Let us analyze the premises and hypotheses that enabled Thales to draw his conclusions and to state the theorem of parallel lines cutting the angle formed by two intersecting straight lines.

The first two assumptions are that the segment \( OQ' \) on the ground is indeed a straight line, and that the two segments, the height of the pyramid \( QQ' \) and the stick \( MM' \) are also straight, and form the same angle with the line of the ground \( OM'Q' \) (in this case, the right angle of 90°).

This is a physical statement, and the fact that the two objects are straight and vertical could be checked using of the well known instruments based on the exploitation of gravity.

Both instruments shown in Fig. 2 are based on the use of the gravity field of Earth, defining local vertical directions and horizontal planes (equipotential surfaces).
The fact that the two segments are vertical and straight is based on the assumption that the string sustaining a heavy object in the gravitational field on the surface of Earth may serve as a definition of a *vertical straight line*. Checking the horizontality of the ground is performed using the same principle. To be more precise, the fact that the string supporting the heavy object takes on the straight shape is due to the tension to which it is subjected due to the gravitational force.

A straight line can be obtained in this manner even inside an artificial satellite orbiting around Earth, in absence of gravitational forces, in total weightlessness. Any elastic string subjected to tension will take on the form of a straight line. The tension can be caused by forces having nothing to do with gravitation - e.g. the force of our muscles, or some mechanical or electrical device.

However, Earth’s gravitation is crucial in defining the *right angle* between the horizontal ground surface and the two *distant* straight lines, the height of the pyramid $QQ'$ and the stick $MM'$, thus determining what is often called *distant parallelism*.

Although the Thales theorem seems to concern exclusively spatial relationships between straight lines of certain type, idealizing spatial interplay between physical objects, *time* is implicitly involved in physical hypotheses necessary to justify the result of Thales’ measurements.

When Thales was performing his experiment, the Great Pyramid was more than 2000 years old, which by the way explains why its exact dimensions have been since long forgotten. A tacit assumption was that it kept its initial form, including all angles and dimensions. Even if we exclude the occurrence of seismic events, he had still to admit that the stones forming the pyramid kept their shape unchanged during very long periods of time.

The fact that the stick also remains straight and stiff is due to the similar assumption, namely, that it is made of a material whose cohesion is sufficient to keep its shape unchanged (a common definition of a solid body). As seen from our present perspective, this hypothesis is based on the assumption that atoms can form stable structures able to keep unchanged under reasonable conditions (e.g. the ambient temperature not exceeding certain values). From the four-dimensional point of view, this means that atoms and molecules can follow parallel timelike geodesics, with null geodesic deviation.

Incidentally, the ability of atoms and molecules to form stable periodic structures makes possible an alternative definition of straight lines and right (and not only right) angles. Crystals represented in Fig. 3 show remarkable
linear structure as well as apparently perfect angles, 90° in the case of cubic lattice of NaCl, and 60° and 120° in the case of quartz (SiO₂).

Figure 3: Crystals of ordinary salt NaCl, of quartz SiO₂, and an example of crystalline lattice (SiO₂ - wurtzite). The interatomic forces impose the shapes and the geometry of solid bodies.

The straight lines and right angles obtained in the traditional way, by using compass, ruler and a sheet of paper, are based on the same physical principle, which is the existence of solid bodies serving as standards of length. The geometry based on solid bodies’ shapes is independent of gravitational field that determines parallel vertical lines and the horizontal plane in Thales’ experiment. From the present point of view, the existence of stable configurations of atoms, as well as that of atoms themselves, can be understood only using the principles of quantum mechanics, until now seemingly independent of gravitational phenomena.

But this is not the end of the story. A third type of straight line is involved in the experiment, the light ray along the line QPNMO. The character of this line is due to the properties of electromagnetic waves’ propagation in vacuo (as far as the influence of air can be neglected), which a priori is independent of gravitational phenomena as well as of the forces predominant on the atomic level. Light wavefronts and rays set forth an alternative notion of straight lines and angles, resulting in conformal geometry, which preserves the notions of straight lines and angles, but ignores the notions of length and distance.

\[1\] Thales made also an extra tacit assumption, namely, that the properties used for the definition of straight lines, parallelism and right angles were scale independent, i.e. they were the same for the small stick and for the Great Pyramid. The extension towards even greater dimensions, including the Skies, seemed also obvious.
3. The three realms of physical world

The results of Thales’ experiment can be interpreted in two ways. In fact, he established the coincidence of three completely different definitions of a straight line. The first came from the natural shape a string with a heavy body attached to its end takes under the influence of the gravitational field of the Earth.

The gravitational field defines also the right angle between the horizontal ground and two distant versicals, the height of the pyramid and the stick. The mathematical expression of this assertion is given by the potential function \( U(x, y, z) \) defining the equipotential surfaces \( U = \text{Const} \). On the surface of Earth this equation defines the horizontal plane and the vertical direction, since we have

\[
dU = \nabla U \cdot dr = 0,
\]

with the vector

\[
\nabla U = \left[ \frac{\partial U}{\partial x}, \frac{\partial U}{\partial x}, \frac{\partial U}{\partial x} \right]
\]

defining the local vertical direction, while all displacements orthogonal to it define (locally) a horizontal plane.

The second definition comes from the material shape of the stick. The existence of solid bodies which can be used as standards of lengths and angles results from symmetry properties of interatomic forces, which in turn can be derived \textit{ab initio} according to the rules of quantum mechanics, valid on the atomic scale.

The third straight line is given by the light ray, which comes from an idealization of electromagnetic wave propagation from a very distant source. The sunlight illuminating the Earth is well described by a plane wave:

\[
A \cos(\omega t - k \cdot r),
\]

with planar wavefronts given by the implicit equation \( \omega t - k \cdot r = \text{Const} \). The rays are parallel to the wave vector \( k \), everywhere perpendicular to planar wavefronts.

The first interpretation of the experiment coinciding with what Thales was interested in, is based on the supposition that all definitions of straight lines and angles do coincide, which enabled him to evaluate the height of the Great Pyramid.

The second interpretation would be, with the height of the pyramid considered as a known quantity, as well as the height of the stick, to see the
result of Thales experiment as a proof that the light rays follow straight lines compatible with the two definitions involving gravitation and interatomic forces. Or else, that the straight lines and right angles defined by means of gravitational field coincide with those defined by light rays and solid rods.

The three alternative definitions of geometry involved in Thales’ experiment are directly related to three different aspects of our perception of nature. Since the advent of modern physics, the description of the world surrounding us is based on three essential realms, already present in the Thales experiment, which are

- **Space and time**
- **Material bodies**
- **Forces acting between them**

The three main aspects of our perception of physical reality can be distinctly seen in the fundamental equation expressing Newton’s third law of dynamics:

\[ a = \frac{1}{m} F \]  

shows the relation between three different realms which are dominant in our perception and description of physical world: massive bodies (“m”), force fields responsible for interactions between the bodies (“F”) and space-time relations defining the acceleration (“a”).

The same three ingredients are found in physics of fundamental interactions: we speak of elementary particles and fields evolving in space and time we deliberately formulated Newton’s law of dynamics in a slightly unusual way, \( a = \frac{1}{m} F \), in order to separate the directly observable entity ( \( a \) ) from the product of two entities whose definition is much less direct and clear.

Also, by putting the acceleration alone on the left-hand side, we underline the causal relationship between the phenomena: the force is the cause of acceleration of mass under its influence, and not vice versa.

*In modern language, the notion of force is generally replaced by the new concept, the fields of various types.*

The fact that the three ingredients are related by the equation \([\text{1}]\) may suggest that perhaps only two of them are fundamentally independent, the third one being the consequence of the remaining two.

Let us represent the three aspects of theories of fundamental interactions by three orthogonal axes, as shown in the following figure, which displays
also three possible choices of two independent aspects of physical reality from which we are supposed to be able to derive the third one.

![Diagram of three realms of physics]

Figure 4: The three realms of physics.

The attempts to understand physics with only two realms out of three represented in [4] have a very long history. They may be divided in three categories, labeled \( I, II \) and \( III \).

In the category \( I \) we can easily recognize Newtonian physics, presenting the physical world as a collection of material bodies (particles) evolving in absolute space and time, interacting at a distance. Newton considered light being made of tiny elastic particles obeying the same rules of mechanics as all material bodies. The notion of fields transmitting forces from one body to another was totally absent.

The controversy concerning the nature of light led to deep differences in the interpretation of space. For Huygens, who proved the wave-like propagation of light, space must be filled with some medium enabling the propagation. Two diametrically opposite views on the status of space and motion prevailed since then. The Newtonian view was reinforced by Immanuel Kant, who raised the status of space to the independent and absolute category, existing independently of observers, like the starry sky and the “moral imperative”.

Theories belonging to the category \( II \) assume that physical world can be described uniquely as a collection of fields evolving in space-time manifold. This approach was advocated by Lord Kelvin, A. Einstein, and later on by J.A. Wheeler. The initial impulse was given by M. Faraday and J.C. Maxwell, who introduced a revolutionary, anti-Aristotelean and anti-Newtonian point of view according to which no interaction at a distance is possible. All forces are transmitted by a medium; the space is filled with it. It can be called
“aether”, and the fields of forces become a new physical realm, identified with tensions inside the aether, which in a sense is the space. In a sense, “space” becomes the synonym of “material continuum”, just like from the point of view of a fish, its spatial separation from another fish can be defined as the amount of water contained between the two.

As a follower of Maxwell and Faraday, Einstein believed in the primordial role of fields and tried to derive the equations of motion as characteristic behavior of singularities of the fields, or of the space-time curvature. One can say that in Einstein’s vision, fields replaced the aether. (3)

In the spirit of F. Klein’s programme, H. Minkowski defined the hyperbolic geometry of space united with time in a single entity named “space-time manifold”. Its geometry was defined by the action of the Lorentz-Poincaré group. However, at a closer look, physically measurable entities that are subjected to Lorentz transformations (the “four-vectors”) are not at all the time and space coordinates, but the conserved physical quantities, such as energy $E$ and momentum $p$, or the frequency $\omega$ and the wave vector $k$ of an electromagnetic wave (or more precisely, of a photon).

The Minkowskian spacetime inherits the Lorentz-Poincaré symmetry because it is defined via measurements based on photons and their interaction with electrons, whose energy, momentum and spin are Lorentz-covariant quantities and span representation spaces of the Lorentz-Poincaré group.

The category III represents an alternative point of view supposing that the existence of matter is primary with respect to that of the space-time, which becomes an “emergent” realm - an euphemism for “illusion”. Such an approach was advocated recently by N. Seiberg and E. Verlinde. (9)

It is true that space-time coordinates cannot be treated on the same footing as conserved quantities such as energy and momentum; we often forget that they exist rather as bookkeeping devices, and treating them as real objects is a “bad habit”, as pointed out by D. Mermin (4).

Seen under this angle, the idea to derive the geometric properties of space-time, and perhaps its very existence, from fundamental symmetries and interactions proper to matter’s most elementary building blocks seems quite natural. Many of those properties do not require any mention of space and time on the quantum mechanical level, as was demonstrated by M. Born and W. Heisenberg (5, 6) in their version of matrix mechanics, or by J. von Neumann’s formulation of quantum theory in terms of the $C^*$ algebras (7). The non-commutative geometry is another example of formulation of space-time relationships in purely algebraic terms (8).
Considering quantum physics as the primary underlying reality of which classical objects are an averaged version, one is led to conclude that quantum properties of physical objects must be intimately related to the definition of geometry in the first place.

4. The Thales experiment from todays’ perspective

Let us come back to the experiment carried out by Thales more than twenty-five centuries ago. According to our analysis, we can recognize to which physical realm belongs each of three definitions of straight line. The two parallel vertical lines, the pyramid’s height and the stick, are made of wood and stone, which keep their form due to their solid state. Being made of atoms, the existence and properties of such solids can be derived from rules of quantum mechanics. This is the realm of particles with mass: nucleons and electrons, which form atoms, then molecules, and finally stable crystalline or amorphous solids. The electromagnetic forces play also an important role, keeping the electrons around the nuclei, and creating the residual Lennard-Jones potentials outside the atoms, giving rise to the Van der Waals forces.

The light rays which created also shadows of the pyramid and stick alike are, as we know now, the innumerable swarm of photons creating a common planar wavefront. They are identified with a massless gauge field, thus belonging to the realm of forces making possible the interaction between massive charged particles. The interaction between the photons and electrons of atomic outer shells is described most adequately with the rules of quantum physics.

Only the third side of each of the two triangles appearing in Thales’ experiment, the parallel vertical lines, seem to have nothing to do with quantum physics, their directions being defined by the gravitational field of Earth. But after closer scrutiny we can conclude that even in this case the devices made of solids are necessary to detect the presence of gravitation, and the information about their behavior is carried forth by photons.

At this point we can ask whether the Thales experiment could be performed without gravity - and the answer is ”yes”. To construct a plane and two vertical parallel lines the solid standards of length and right angle would suffice, it can be done with standard compass and ruler. Therefore, the experience following Thales’ scheme, can be viewed as checking whether the laws of gravity are compatible with the geometry defined by solid bodies and light, i.e. by the classical limit of quantum mechanics and quantum field theory.
By the way, with very precise measurements of angles we would be able to find out the actual curvature of Earth surface, because the verticals defined by its gravitational field are not parallel in fact: the distance of about 31 metres corresponds to one second of arc between the local vertical directions defined by Earth’s gravitation.

The present analysis of Thales’ experiment suggests that among the three realms of physics represented in (4), particles and fields (quantum physics) define the geometry when they constitute classical objects like solid bodies and wavefronts, while the presence or absence of gravitation is checked with the help of other classical objects. To put it in a very rough manner, solid bodies made of atoms and wavefronts made of photons are there no matter whether gravity exists or not; on the contrary, gravity, as well as the geometry of space-time itself, is defined through the properties of solid bodies and light rays. The very detection of gravitational effects cannot be performed without extended massive bodies, behaving like classical objects. Even the famous experiment confirming the variation of proper time under the influence of gravity, performed by Pound and Rebka ([10]) in 1959, uses the Mössbauer effect based on the collective behavior of crystalline lattice which cancels the recoil effect during photon absorption.

Thales’ theorem led the way to all subsequent measurements of great distances, first on land and sea, then applied to the measurement of radius of the Earth by Eratosthenes, then for determining astronomical distances by Aristarchos of Samos. Later on the measurement of distance to the closest stars due to the observed annual parallax is just another application of the Thales theorem. The determination of shapes of planetary orbits by Kepler was based on triangulation, which is also a variety of the same theorem. The subsequent determination of the true dimensions of Solar System was made only in 1769 due to the observation of Venus’ transit and the exact knowledge of longitude by Captain Cook who performed the observations on the island of Tahiti. The crucial measurement concerned the exact time of the phenomenon as observed from distant places on Earth. The longitude could be determined also due to the invention of chronometre by Huyghens.

The speed of light in the vacuum being constant for all Galilean observers, nowadays the measurements of distances in space can be replaced by precise measurements of time delays, like with the Global Positioning System (GPS). And it is not accidental that very large distances are measured in time equivalents, the light-years. The exact measurements of time, which nowadays attains the precision of $10^{-12}$ second, enable us to determine dis-
stances with similar degree of precision - less than 1 cm on the surface of our globe. Such time measurements are possible due to atomic clocks obeying quantum mechanical rules.

All the information we receive from the surrounding world is carried by photons, leptons and baryons, elementary particles whose properties and behavior are extremely well described by quantum physics. However, we can perceive and analyze them only through devices representing the classical limit of quantum mechanics. No wonder that the geometry built on the base of the obtained data reflects the symmetry group acting in the space of states of elementary particles - the Lorentz-Poincaré group. The transformation properties of conserved physical entities such as the four-vectors $k^\mu = \begin{pmatrix} \omega \\ c \end{pmatrix}$ or $P^\mu = \begin{pmatrix} E \\ c \end{pmatrix}$ are extended to the dual space of differential forms $dx^\mu = [cdt, dx]$. These, in turn, are defined experimentally using classical objects, whose very existence (rigid bodies made up from atoms, light wavefronts made out of photons) is explained by quantum theory.

Thus the conclusion in the case of Thales experiment is that in order to construct the Euclidean geometry of space, only these two physical phenomena were needed, the light playing the role of the ruler (defining the straight lines), and the rigid bodies playing the role of compass (defining distances). Gravity was used to define parallel straight lines and right angles, but its use was not necessary. On the contrary, its influence can be measured using exclusively classical objects.

Apparently, the gravitation can be perceived only in the classical limit, and not on the quantum level. In spite of numerous attempts, there is no quantum limit of classical physics. This suggests two conclusions:

first, that space and its geometry are defined only in the classical limit of quantum theory;

second, that gravity is also a classical phenomenon, appearing only when the collective effects can be perceived, just like classical thermodynamics can be defined only as a limit of statistical physics

In this case, quantizing gravitational waves is as hazardous an enterprise as an attempt to quantize the waves on the surface of water.
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