Josephson junction based on highly disordered superconductor/low-resistive normal metal bilayer

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Abstract. We calculate current-phase relation (CPR) of a SN-S-SN Josephson junction based on a variable thickness SN bilayer composed of highly disordered superconductor (S) and low-resistive normal metal (N) with proximity induced superconductivity. In case when the thickness of S,N layers and length of S constriction is about of superconducting coherence length the CPR is single-valued, could be close to sinusoidal one and the product $I_c R_n$ can reach $\Delta(0)/2|e|$ ($I_c$ is the critical current of the junction, $R_n$ is its normal-state resistance, $\Delta(0)$ is the superconductor gap of single S layer at zero temperature). We argue that the normal layer should provide good heat removal from S constriction and there is range of parameters when current-voltage characteristic is not hysteretic and $I_c R_n$ is relatively large.

Keywords: normal metal-superconductor bilayer, Josephson junction, Joule heating
1. Introduction

Various technological applications of Josephson junctions (AC voltage standard [1], rapid single-quantum logic [2], SQUID-magnetometers [3] and particle detectors [4]) require to have nonhysteretic current-voltage characteristic (IVC). Tunnel superconductor-insulator-superconductor (S-I-S) junctions is characterized by small critical current densities and hysteretic IVC (the latter is related with large capacitance of the insulator layer) which restricts their applicability. S-N-S and S-S’-S junctions (where N is a normal metal and S’ is the geometric constriction or superconductor with smaller critical current) have small capacitance of the weak link but IV curves are hysteretic due to Joule dissipation [3,5–7].

The current-phase relation (CPR) of these types of the Josephson junctions is often different from the sinusoidal form $I = I_c \sin \varphi$ where $I_c$ is the junction critical current and $\varphi$ is the phase difference between electrodes [8]. Specific form of the CPR depends on the junction parameters and temperature [9]. In the case of the weak link made of pure superconductor or normal metal (having mean free path $\ell$ much larger than the coherence length of the electrode $\xi_1$ and the coherence length of the weak link $\xi_2$), the CPR transform from the sinusoidal one at temperature close to critical temperature of electrodes $T_c$ to the saw-toothed shape with the maximum at $\varphi = \pi$ at $T \ll T_c$. In dirty S-S’-S junctions ($\ell \ll \xi_1$) the CPR with decreasing temperature can change from the sinusoidal one to the quite different multi-valued relation. In the latter case the maximum is attained at $\varphi > \pi$ and two values of current correspond to a fixed value of $\varphi$. Similar multi-valued CPR is typical for the weak link in the form of superconducting bridge whose length is much larger than the superconducting coherence length $\xi(T)$. In the case of short bridges (whose length is smaller than the bridge coherence length $\xi_2$) the CPR remains single-valued at all temperatures but it is sinusoidal one only at temperature close to critical temperature and in the case of sufficiently small ratio $\xi_2/\xi_1$.

For technological applications important characteristic is the characteristic voltage $V_c = I_c R_n$ where $R_n$ is the normal-state resistance of the junction. On the one side, to have large $V_c$ one needs S-N-S or S-S’-S junction with high-resistive N or S’ layer. On the other side, in these junctions IVC becomes hysteretic below certain temperature which is associated with Joule heating in the weak link ($\sim I_c V_c \sim I_c^2 R_n$) and the formation at $I > I_c$ of the stable region of suppressed superconductivity (so called 'hot spot') [3,5–7]. Therefore, the eliminating of the thermal hysteresis without sacrificing the voltage $V_c$ is important and nontrivial problem. One solution is a normal metal shunt either on top of the junction [10] or in parallel to it [11]. However, in this case the resistance and the position of the shunt play important role and they can lead to reduction of the junction characteristics because of the proximity effect or very small shunt resistance.

In the work [12] it was proposed to use as the Josephson junction the variable thickness SN-N-SN bilayer where the superconducting layer was partially (or entirely) etched by a focused ion beam. Sufficiently thick normal-metal layer act as a heat sink which provides nonhysteretic current-voltage characteristic even at low temperatures. But the increase
of the N layer thickness leads to significant decrease of $R_n$ and, hence, smaller $V_c$.

In our work we calculate current phase relation for recently proposed variable thickness SN-S-SN Josephson junction based on thin dirty superconductor with large normal state resistivity $\rho_S \gtrsim 100 \mu \Omega \cdot cm$ and thin normal metal layer with low $\rho_N \gtrsim 2 \mu \Omega \cdot cm$ \cite{13}. In \cite{14} it has been demonstrated theoretically and experimentally that in such a bilayer the superconducting current mainly flows in N layer (due to proximity induced superconductivity and $\rho_S/\rho_N \gg 1$) and the critical current of SN bilayer may exceed the critical current of single S layer if thicknesses of S and N layers are about of superconducting coherence length. Below we show that in comparison with SN-N-SN junction the critical current density could be about of depairing current density of S layer, which makes it possible to have $I_c R_n \sim \Delta(0)/2|e|$. Due to large diffusion coefficient $D_N$ and small minigap in N layer the heat could be effectively removed from the junction area and current-voltage characteristic could be not hysteretic. Besides, because of $D_N \gg D_S$ current-phase relation could be single-valued at all temperatures and close to sinusoidal one at temperature near the critical temperature of bilayer.

2. Model

The model system consists of SN bilayer strip with length $L$ made of superconducting film with thickness $d_S$ and normal-metal film with thickness $d_N$. At the center of bilayer there is a constriction with length $a$ and thickness $d_c$ where N layer and partially S layer are removed (see figure \ref{fig1}). We assume that in our system the current flows in the $x$ direction and in the $y$ direction the system is uniform. To find the current-phase relation of such SN-S-SN Josephson junction at all temperatures below $T_c$ we solve two-dimensional Usadel equation for quasiclassical normal $g$ and anomalous $f$ Green functions. With the angle parametrization $g = \cos \Theta$ and $f = \sin \Theta \exp(i\phi)$ this equation in different layers can be written as

\[
\frac{\hbar D_S}{2} \left( \frac{\partial^2 \Theta_S}{\partial x^2} + \frac{\partial^2 \Theta_S}{\partial z^2} \right) - \left( \hbar \omega_n + \hbar \frac{D_S}{2} q^2 \cos \Theta_S \right) \sin \Theta_S + \Delta \cos \Theta_S = 0, \tag{1}
\]

Figure 1: Sketch of SN-S-SN Josephson junction based on variable thickness SN strip.
\[ \frac{\hbar D_N}{2} \left( \frac{\partial^2 \Theta_N}{\partial x^2} + \frac{\partial^2 \Theta_N}{\partial z^2} \right) - \left( \hbar \omega_n + \hbar D_N \frac{q^2}{2} \cos \Theta_N \right) \sin \Theta_N = 0, \tag{2} \]

where subscripts S and N refer to superconducting and normal layer, respectively. Here \( \hbar \omega_n = \pi k_B T(2n + 1) \) are the Matsubara frequencies (\( n \) is an integer number), \( q = \nabla \phi = (q_x, q_z) \) is the quantity that is proportional to supervelocity \( v_s \), \( \phi \) is the phase of superconducting order parameter. \( \Delta \) is the magnitude of order parameter which should satisfy to the self-consistency equation

\[ \Delta \ln \left( \frac{T}{T_{c0}} \right) = 2\pi k_B T \sum_{\omega_n > 0} \sin \Theta_S - \frac{\Delta}{\hbar \omega_n}, \tag{3} \]

where \( T_{c0} \) is the critical temperature of the single S layer. We assume that \( \Delta \) is nonzero only in the S layer because of absence of attractive phonon mediated electron-electron coupling in the N layer. Equations (1), (2) are supplemented by the Kupriyanov-Lukichev boundary conditions [15] between layers

\[ D_S \frac{d \Theta_S}{dz} \bigg|_{z=d_S-0} = D_N \frac{d \Theta_N}{dz} \bigg|_{z=d_S+0}. \tag{4} \]

In the model we assume transparent interface between N and S layers which leads to continuity of \( \Theta \) on the NS boundary. At boundaries of the system with the vacuum we use \( d \Theta / dn = 0 \).

To find the phase distribution \( \phi \) the equations (1) – (3) are supplemented by two-dimensional equation

\[ \text{div} j_s = 0, \tag{5} \]

where \( j_s \) is the superconducting current density, which is determined by the following expression

\[ j_s = \frac{2\pi k_B T}{e \rho} q \sum_{\omega_n > 0} \sin^2 \Theta, \tag{6} \]

where \( \rho \) is the residual resistivity of the corresponding layer. At the SN-interface we use the boundary condition similar to (4) and for the interfaces with the vacuum we use \( d \phi / dn = 0 \). At the system ends the rigid boundary conditions are imposed

\[ \phi(0, z) = -\delta \phi / 2, \phi(L, z) = \delta \phi / 2, \tag{7} \]

where \( \delta \phi \) is the fixed phase difference between the system ends. One should differs it with the phase drop near the junction which we define as

\[ \varphi = \delta \phi - kL, \tag{8} \]

where \( k = q_x(x = 0) \) is far from the constriction (in similar way \( \varphi \) is defined in [16,17]). The value of \( k \) is found from self-consisting solution of (1) – (3).

In numerical calculations we use dimensionless units. The magnitude of the order parameter is normalized by \( k_B T_{c0} = \Delta(0) / 1.76 \), length is in units of \( \xi_c = \sqrt{\hbar D_S / k_B T_{c0}} \simeq 1.33 \xi(0) \) (\( \xi(0) = \sqrt{\hbar D_S / \Delta(0)} \) is the superconducting coherence length at \( T = 0 \)) and current is in units of depairing current \( I_{dep} \) of superconductor at \( T = 0 \).
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To calculate the CPR we numerically solve (1) – (3),(5) by iteration procedure with fixed $\delta \phi$. When the self-consistency is achieved (we stop calculations when maximal relative change of $\Delta$ between consequent iterations is less than $10^{-4}$) the Green functions are used to calculate $j_s$ and the supercurrent per unit of width $I_s$

$$I_s = \int_0^{d_s+d_N} j_{sx}(x = 0) dz. \quad (9)$$

We also compare calculated CPR with the current-phase relation for 1D S'-S-S' system with large ratio of diffusion coefficients $D_{S'}/D_S \gg 1$ (length of S superconductor is equal to $a$). To calculate it we use 1D Usadel equation.

3. Current-phase relation of SN-S-SN Josephson junction

The dependence $I_s(q)$ in SN bilayer may have one or two maxima depending on value of $d_S$ (see figure 2) or $d_N$ (see figure 3(a) in [14]). The maxima at small $q$ is connected with suppression of proximity induced superconductivity in N layer at $q > q_{c1} \sim 1/\sqrt{D_N}$ while the second maxima at $q = q_{c2} \sim 1/\sqrt{D_S} \gg q_{c1}$ comes from suppression of superconductivity in S layer when $q > q_{c2}$. Large difference in $q_{c1}$ and $q_{c2}$ leads to larger phase concentration in S constriction (see figure 1) in comparison with the variable thickness strip (or Dayem bridge) made of the same material and having the similar geometrical parameters. Because of that for relatively thin S layers the CPR is single-valued (see figure 3 (a)) which is not easy to achieve for Dayem bridge [18]. For relatively large $d_S$ there is noticeable contribution to total supercurrent from S layer which means smaller current (phase) concentration in constriction like in ordinary Dayem bridge and CPR becomes multi-valued (see figure 3(a) for $d_S = 2, 3\xi_c$).

In some respect studied Josephson junction resembles Josephson junction based on S'-S-S'system composed of two superconductors S and S' having $D_{S'} \gg D_S$ and the same thicknesses $d_S = d_{S'}$ [16][19][20]. Josephson junction based on this quasi 1D system has single-valued CPR which tends to the sinusoidal shape with increasing temperature. In figure 3 (b) we compare CPR calculated for 1D S'-S-S' and 2D SN-S-SN systems. Since in 1D model there is no suppression of $T_c$ by N layer, in calculations we use ratio $T/T_{c0}$ which corresponds to ratio $T/T_c$ of 2D SN structure. Visible differences between CPRs calculated using different models could be related with transversal inhomogeneity near the S constriction in the 2D case.

We have studied evolution of CPR of SN-S-SN Josephson junction by varying different parameters. In figure 4(a) we demonstrate that with increase of the temperature the current phase relation becomes closer to sinusoidal one which is typical for S'-S-S' junctions [20] and it is related with increase of the temperature-dependent coherence length $\xi(T)$. Effect of different $d_N$ is shown in figure 4(b). An increase in $d_N$ leads to slight shift of maximum of $I_s(\varphi)$ to the left and decrease of $I_c$ which are explained by lowering of $T_c$ of SN bilayer for thicker N layers. Lower $I_c$ means smaller
Figure 2: Dependence of the superconducting current $I_s$ flowing along SN bilayer on $q$ for different $d_S$. Solid line shows the dependence $I_s$ on $q$ for the single S strip. Dashed lines show the critical values of $q$. Current is normalized by the depairing current $I_{dep}$ of the single S strip with thickness $d_S$ at $T = 0$.

$I_c R_n$ but how we discuss below large $d_N$ provides better cooling of S constriction and nonhysteretic IV curves.

An increase of the weak-link length $a$ leads to the shift of the maximum of $I_s(\varphi)$ to the right (see figure 4(c)) as it is typical for ordinary variable thickness Josephson junctions. Interestingly, that contrary to that junctions the $I_c$ increases in SN-S-SN system. This result is explained by lower value of superconducting order parameter in SN banks in comparison with $\Delta$ in S constriction at $I_s = 0$. With increasing $a$ the superconducting order parameter in constriction increases and $I_c$ increases too.

And finally figure 4(c) illustrates that three-fold decrease of ratio $\rho_S/\rho_N$ does not change current-phase relation drastically. Both the critical current and shape of CPR vary a little.

4. Effect of Joule heating in SN-S-SN junctions

The absence of hysteresis in current-voltage characteristic is important for devices based on Josephson junctions. The hysteresis in Dayem, variable thickness, S'-S-S’ or S-N-S junctions is mainly caused by the temperature rise in the weak-link region in the
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Figure 3: (a) Current-phase relation of SN-S-SN Josephson junction at different $d_S$. Current is normalized by the depairing current $I_{dep}$ of the single S strip with thickness $d_c$ at $T = 0$. The junction parameters are shown in the figure. (b) Comparison of current-phase relations calculated on the basis of 1D and 2D models. For 2D case the parameters are following: $d_S = d_N = \xi_c$, $d_c = 0.5\xi_c$, $T = 0.2T_c$. In the 1D case temperature $T = 0.6T_c$ corresponds to $T = 0.6T_c$, where $T_c = 0.32T_c$ is critical temperature of SN bilayer with chosen parameters. The superconducting current is normalized by critical current of Josephson junction.

resistive state due to Joule heating and the formation of hot spot [3, 6, 7]. Local heat production should be large in SN-S-SN junction due to large critical current density which is about of the depairing current density of the superconductor. But as we show below the presence of relatively thick N layer with large diffusion coefficient provides efficient cooling of constriction.

To estimate the increase of temperature in the resistive state we use two temperature (2T) model [21, 22] for SN-S-SN junction. We suppose that electron $T_e = T + \delta T_e$ and phonon $T_p = T + \delta T_p$ temperatures are near the substrate temperature $\delta T_c$, $\delta T_e \ll T$ and do not vary along the thickness. Because of inverse proximity effect the gap in relatively thin S layer ($d_S \lesssim 1.5\xi_c$) is suppressed in comparison with single S layer, which permits heat diffusion from N to S layer in SN banks. In S constriction being in the resistive state at $I > I_c$ the superconducting order parameter is also suppressed.
Figure 4: Variation of current-phase relation of SN-S-SN junction with change of:
(a) temperature; (b) thickness of N layer \( d_N \); (c) length of constriction \( a \); (d) ratio of resistivities. Current is normalized by the depairing current \( I_{\text{dep}} \) of the superconducting strip with thickness \( d_c \) at \( T = 0 \).

It allows us to use normal state heat conductivity both in SN and S regions in heat conductance equation for calculation of \( \delta T_e \). In our model Joule dissipation is taken into account only in S constriction, because in SN bilayer it is considerably lower due to much lower resistivity and lower current density. Because of small length of constriction and large difference in diffusion coefficients and thicknesses in constriction and banks we can neglect heat flow to the phonons and substrate in constriction (main cooling of junction comes from diffusion of hot electrons to SN banks). In SN bilayer \( D_N \gg D_S \) and heat diffusion occurs mainly along N layer. With above assumptions we have following equation for \( \delta T_e \)

\[
\frac{d^2 \delta T_e}{dx^2} + \frac{\rho_S (j_c)^2}{\kappa_S} = 0, \quad |x| \leq \frac{a}{2},
\]

\[
\frac{d^2 \delta T_e}{dx^2} - \frac{\delta T_e}{\lambda_T^2} = 0, \quad |x| \geq \frac{a}{2},
\]

where \( \kappa_S = 2D_S N(0) k_B T / 3 \) is the electron heat conductivity of S layer in the normal
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state, \( N(0) \) is the one spin density of states on the Fermi level,

\[
\lambda_T = \sqrt{D_N \tau_0} \left( \frac{T_{c0}}{T} \right)^{3/2} \sqrt{\frac{\pi^2 (1 + \beta)}{720 \zeta(5)}}
\]

(11)
is the healing length, \( \beta = [\gamma \tau_{\text{esc}} 450 \zeta(5) T / [\tau_0 \pi^4 T_{c0}]] \), \( \zeta(5) \approx 1.03 \), \( \tau_{\text{esc}} \) is the escape time of nonequilibrium phonons to substrate, \( \gamma = 8\pi^2 C_e(T_{c0})/C_p(T_{c0}) \) is the ratio of electron and phonon heat capacities at \( T = T_{c0} \) and \( \tau_0 \) determines the strength of electron-phonon inelastic scattering in S and N layers (see equations (4,6) in [22]). For \( \tau_0 \) we use the smallest time for S and N materials due to assumed good transfer of electrons between S and N layers and their small thickness. On the boundary between S and SN regions we use continuity of the electron temperature (\( \delta T_e|_{a/2-0} = \delta T_e|_{a/2+0} \)) and heat flux (\( dS D_S \delta T_e|_{a/2-0} = dN D_N \delta T_e|_{a/2+0} \)).

Using (10) and above boundary conditions we find maximal temperature increase in the constriction

\[
\frac{\delta T_e^{\text{max}}}{T} = 0.23 \left( \frac{a}{\xi_c} \right)^2 \left( \frac{T_{c0}}{T} \right)^2 \left( \frac{I_c}{I_{\text{dep}}(0)} \right)^2 \left( \frac{D_S d_S}{D_N d_N} \frac{4\lambda_T}{a} + 1 \right)
\]

(12)

In following estimations we use parameters of NbN (S layer) and Cu (N layer): \( T_{c0} = 10 \) K, \( D_S = 0.5 \) cm\(^2\)/s, \( \rho_S = 200 \) \( \mu \Omega \)-cm, \( D_N = 40 \) cm\(^2\)/s, \( \rho_N = 2 \) \( \mu \Omega \)-cm, \( \tau_0 = 1 \) ns (theoretical estimation for NbN is taken from [22]), \( \xi_c = 6.4 \) nm, \( \gamma = 9 \), \( d_S = 1.25 \xi_c \), \( d_N = 2 \xi_c \), \( \tau_{\text{esc}} = 4(d_N + d_S)/u \approx 41 \) ps (\( u = 2 \cdot 10^5 \) cm\(^2\)/s is a mean speed of sound), \( T/T_{c0} = 0.3 \), \( T_c/T_{c0} = 0.43 \), \( a = 0.5 \xi_c \), \( d_c = 0.5 \xi_c \). With these parameters \( \beta \approx 0.53 \), \( I_c \approx 0.22 I_{\text{dep}}(0) \) (see figure 4(b)) and \( \delta T_e^{\text{max}}/T \approx 0.24 \) is small, thanks to \( D_N \gg D_S \) and \( d_N \gg d_c \).

5. Discussion

We use Usadel model to calculate current-phase relation of SN-S-SN Josephson junction based on high-resistive superconductor and low-resistive normal metal. In [14] from comparison of the experiment and theory it was concluded that Usadel model underestimates proximity induced superconductivity in N layer and overestimates inverse proximity effect in S layer in NbN/Al, NbN/Ag and MoN/Ag bilayers. Namely, the suppression of critical temperature of SN bilayer is smaller while change in magnetic field penetration depth of SN bilayer is larger than Usadel model predicts. Therefore, present results should be considered only as a route for possible experimental realization of SN-S-SN Josephson junction. They demonstrate that the thickness of S layer should not exceed \( \sim 1.5 \xi_c \), otherwise current-phase relation is not single-valued for reasonable length and thickness of S constriction. The thickness of N layer should not be too small (small \( d_N \) leads to large overheating) and not too large (the larger \( d_N \) leads to lower \( T_c \) and smaller \( I_c \) at fixed substrate temperature).
Our results show that SN-S-SN Josephson junction in many respects resembles Dayem, variable thickness bridge, S’-S-S’ or S-N-S junctions. Product

$$V_c = I_c R_n = \frac{\Delta(0)}{|e|} \frac{a}{\xi_c} \frac{I_c}{I_{dep}(0)},$$

(13)
can reach $0.5\Delta(0)/|e|$ at low temperature ($T = 0.1T_\text{c}$) and $a = \xi_c$ (see figure 4(c)) due to use of superconductor in constriction area, instead of normal metal as in [12]. In case of NbN with $T_\text{c} = 10K$ one may have $V_c = 0.75\text{ mV}$ but according to (12) $\delta T_{\text{c}}^{\text{max}}$ will be larger than $T$ at these parameters. However there is a hope, that critical temperature of real SN bilayer is higher than Usadel model predicts (see discussion above) and therefore large $I_c$ could be reached at higher operating temperature $T/T_\text{c}$, leading to drastic reduction of $\delta T_{\text{c}}^{\text{max}}$ (see (12)).

The SN-S-SN junctions made of NbN/Al bilayer have been fabricated recently [13] and indications of Josephson effect (the presence of Shapiro steps and Fraunhofer like dependence of critical current on the magnetic field) have been observed. But due to not optimized parameters ($d_S = d_c \sim 15\text{ nm} \sim 2.3\xi_c$, $d_N \sim 29\text{ nm} \sim 4.5\xi_c$, $a = 20\text{ nm} \sim 3.1\xi_c$) the IV curves were hysteretic already at temperature close to critical one and width of Shapiro steps did not follow the theoretical expectations [13]. Modern technology allows to make constriction with length about 5 nm with help of helium beam, which is smaller than $\xi_c$ in NbN. Successful implementation of this method could lead to creation of low temperature nano-scale Josephson junction or their arrays. For example SN-S-SN junctions can be promising to use in programmable voltage standards [1] where large value of $V_c$ allows to reduce the number of junctions and to use Shapiro steps of order higher than one. Nonhysteretic current-voltage characteristics with large $V_c$ at low temperatures enables to use these structures for various low-temperature applications, e.g., particle detectors [4].

6. Conclusion

In conclusion, we have calculated current phase relation of Josephson junction based on variable thickness SN-S-SN strip, where S is dirty superconductor with large normal state resistivity and N is low resistive normal metal. We find the range of parameters when CPR is single-valued, close to sinusoidal one and product $I_c R_n \lesssim \Delta(0)/2|e|$. Our estimations demonstrate that relatively thick N layer serves as effective heat-conductor providing weak overheating and nonhysteretic current-voltage characteristic of SN-S-SN Josephson junction.

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References

[1] Benz S P 1995 Appl. Phys. Lett. 67 2714
[2] Likharev K K and Semenov V K 1991 IEEE Trans.Appl. Supercond. 1 328
[3] Hazra D, Pascal L M A, Courtois H and Gupta A K 2010 Phys.Rev.B 82 184530
[4] Tarte E J, Moseley R W, Kolbl M R, Booij W E, Burnell G and Blamire M G 2000 Supercond. Sci. Technol. 13 983
[5] Skocpol W J, Beasley M R and Tinkham M 1974 J. Appl. Phys. 45 405466
[6] Courtois H, Meschke M, Peltonen J T and Pekola J P 2008 Phys. Rev. Lett. 101 067002
[7] Biswas S, Winkelmann C B, Courtois H and Gupta A K 2018 Phys. Rev. B 98 174514
[8] Barone A and Paterno G 1982 Physics and Applications of the Josephson Effect (New York: Wiley)
[9] Golubov A A, Kupriyanov M Yu and Ilichev E 2004 Rev. Mod. Phys. 76, 411
[10] Lam S K H and Tilbrook D L 2003 Appl. Phys. Lett. 82 107880
[11] Mck M, Rogalla H and Heiden C 1988 Appl. Phys.A 47 2859
[12] Hadfield R H, Burnell G, Booij W E, Lloyd S J, Moseley R Wand Blamire M G 2001 IEEE Trans. Appl.Supercond. 11 11269
[13] Levichev M Yu, El'kina A I, Bukharov N N, Petrov Yu V, Aladyshkin A Yu, Vodolazov D Yu and Klushin A M 2019 Phys. Solid State 61 15441548
[14] Vodolazov D Yu, Aladyshkin A Yu, Pestov E E, Vdovichev S N, Ustavshikov S S, Levichev M Yu, Putilov A V, Yumin P A, El'kina A I, Bukharov N N and Klushin A M 2018 Supercond. Sci. Technol. 31 115004
[15] Kupriyanov M Yu and Lukichev V F 1988 Sov. Phys. JETP 67 1163
[16] Baratoff A, Blackburn J A and Schwartz B B 1970 Phys. Rev. Lett. 25 1096
[17] Zubkov A A and Kupriyanov M Yu 1983 Fiz. Nizk. Temp. 9 548
  Zubkov A A and Kupriyanov M Yu 1983 Sov. J. Low Temp.Phys. 9 279 (Engl. transl.)
[18] Vijay R, Sau J, Cohen M and Siddiqi I 2009 Phys. Rev. Lett. 103 087003
[19] Blackburn J A, Schwartz B B and Baratoff A 1975 J. Low Temp. Phys. 20, 523
[20] Kupriyanov M Yu and Lukichev V F 1981 Fiz. Nizk. Temp. 7 281
[21] Perrin N and Vanneste C 1983 Phys. Rev.B 28 5150
[22] Vodolazov D Yu 2017 Phys. Rev. Appl. 7 034014