Dynamical Spin-Blockade in a quantum dot with paramagnetic leads

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Abstract. – We investigate current fluctuations in a three-terminal quantum dot in the sequential tunneling regime. Dynamical spin blockade can be induced when the spin-degeneracy of the dot states is lifted by a magnetic field. This results in super-Poissonian shot noise and positive zero-frequency cross-correlations. Our proposed setup can be realized with semiconductor quantum dots.

Introduction. – Non-equilibrium current noise in mesoscopic structures is a consequence of the discreteness of the charge carriers (for reviews, see Refs. [1, 2]). For conductors with open channels the fermionic statistics of electrons results in a suppression of shot noise below the classical Schottky limit [3]. This was first noted by Khlus [4] and Lesovik [5] for single channel conductors. Subsequently, Büttiker generalized this suppression for many-channel conductors [6]. Mesoscopic conductors are often probed by two or more leads. The quantum statistics induces cross-correlations between the currents in different terminals. Since these cross-correlations vanish in the classical limit, even their sign is not obvious a priori. Using only the unitarity of the scattering matrix, Büttiker proved that cross-correlations for non-interacting fermions are always negative for circuits with leads maintained at constant potentials [7]. Note that this also holds in the presence of a magnetic field. It has also been found that an interacting paramagnetic dot shows negative cross-correlations in the absence of a magnetic field [8]. Spin-dependent cross-correlations in a non-interacting 4-terminal spin valve were studied [9] and found to be negative. On the experimental side negative cross-correlations were measured by Henny et al. [10, 11] and Oliver et al. [12] in mesoscopic beam splitters.

Several ways to produce positive cross-correlations in fermionic systems have been proposed (see e.g. [13] for a recent review). Among these possibilities are sources which inject correlated electrons [14–28] and finite-frequency voltage noise [13, 29]. The question of the existence of intrinsic mechanisms, i. e. due to interactions occurring in the beam-splitter device itself, has been answered positively by us [30]. Surprisingly, a simple quantum dot connected to ferromagnetic contacts can lead to positive cross-correlations due the so-called dynamical spin-blockade. Simply speaking, up- and down-spins tunnel through the dot with different rates. In the limit where the Coulomb interaction prevents a double occupancy of the dot, the
spins which tunnel with a lower rate modulate the tunneling of the other spin-direction, which leads to an effective bunching of tunneling events. In a three terminal geometry with one input and two outputs, this results in positive cross-correlation between the two output currents. Independently, Sauret and Feinberg proposed a slightly different setup of a ferromagnetic quantum dot, which also produces positive cross-correlations [31].

Experimentally, it is more difficult to fabricate quantum dots with ferromagnetic leads. However, quantum dots with paramagnetic leads have shown to exhibit spin-dependent transport. A magnetic field lifts the spin-degeneracy and a spin-polarized current with nearly 100% efficiency can be created [32, 33]. In this Letter, we will address the current correlations in a few-electron quantum dot connected to three paramagnetic leads. We will show below that positive cross-correlations can be produced in this device simply by applying a magnetic field. Furthermore, this system also shows a super-Poissonian shot noise.

To arrive at these conclusions we consider a quantum dot with one orbital energy level $E_0$ connected to three terminals by tunnel contacts. The junctions are characterized by bare tunneling rates $\gamma_i$ ($i = 1, 2, 3$) and capacitances $C_i$. We assume that a magnetic field $B$ is applied to the dot, which leads to a Zeeman splitting of the level according to $E_i(t) = E_0 + (-g)\mu_B B/2$, where $\mu_B = e\hbar/2m$ is the Bohr magneton. The double occupancy of the dot costs the charging energy $E_c = e^2/2C$, with $C = \sum_i C_i$. The energy spacing to the next orbital is $\Delta$. We will assume

$$k_B T, eV, \mu_B B \ll E_c, \Delta.$$  

According to these inequalities, the dot can be only singly occupied and we have to take into account only one orbital level.

In the sequential-tunneling limit $\hbar \gamma_j \ll k_B T$, the time evolution of the occupation probabilities $p_\psi(t)$ of states $\psi \in \{\uparrow, \downarrow, 0\}$ is described by the master equation:

$$\frac{d}{dt} p_\psi = M_{\psi \sigma} p_{\sigma},$$  

where

$$\dot{M} = \begin{bmatrix}
-\Gamma_\uparrow - \Gamma_{\downarrow \uparrow} & \Gamma_{\uparrow \downarrow} & \Gamma_{\uparrow \uparrow} \\
\Gamma_{\downarrow \uparrow} & -\Gamma_\downarrow - \Gamma_{\downarrow \downarrow} & \Gamma_{\downarrow \downarrow} \\
\Gamma_{\downarrow \downarrow} & \Gamma_{\uparrow \downarrow} & -\Gamma_\downarrow - \Gamma_{\downarrow \downarrow}
\end{bmatrix}.  \tag{3}$$

The rate for an electron to tunnel on/off the dot ($\epsilon = +/-$) through junction $j$ is given by $\Gamma_j^{\epsilon} = \gamma_j / (1 + \exp(e(E_\sigma - eV_j)/k_B T))$, where $V_1 = V_3 = -C_2 V/C$ and $V_2 = (C_1 + C_3) V/C$. Here, we took the Fermi energy $E_F = 0$ for lead 2 as a reference. The total tunneling rates are $\Gamma_\sigma = \sum_j \Gamma_j^{\epsilon}$ and $\gamma = \sum_j \gamma_j$. Spin flips on the dot are described by rates $\Gamma_{\downarrow \uparrow}$, which obey the detailed balance rule $\Gamma_{\uparrow \downarrow} / \Gamma_{\downarrow \uparrow} = \exp(g\mu_B B / k_B T)$. From Eq. (2) the stationary occupation probabilities $\bar{p}_\sigma$ are

$$\bar{p}_\sigma = \frac{\frac{\Gamma_{\uparrow \sigma} + \Gamma_{\downarrow \sigma}}{\gamma^2 - \Gamma_{\uparrow \sigma} \Gamma_{\downarrow \sigma} + (\gamma + \Gamma_{\uparrow \sigma}) \Gamma_{\downarrow \uparrow} + (\gamma + \Gamma_{\downarrow \sigma}) \Gamma_{\uparrow \uparrow}}}{\sum_{\sigma'} \frac{\Gamma_{\epsilon, \sigma'} + \Gamma_{\sigma', \sigma} (\Gamma_{\epsilon \uparrow} + \Gamma_{\epsilon \downarrow})}{\gamma^2 - \Gamma_{\epsilon \sigma} \Gamma_{\epsilon \sigma} + (\gamma + \Gamma_{\epsilon \sigma}) \Gamma_{\epsilon \epsilon} + (\gamma + \Gamma_{\epsilon \sigma}) \Gamma_{\epsilon \epsilon}}},$$  

and $\bar{p}_0 = 1 - \bar{p}_\uparrow - \bar{p}_\downarrow$. These probabilities can be used to calculate the average value $\langle I_j \rangle$ of the tunneling current $I_j(t)$ through junction $j$ as

$$\langle I_j \rangle = e \sum_{\sigma, \epsilon} \epsilon \Gamma_{\epsilon, \sigma} \bar{p}_A(\sigma, -\epsilon),$$  

where $A(\sigma, \epsilon)$ is the state of the dot after the tunneling of an electron with spin $\sigma$ in the direction $\epsilon$, i.e., $A(\sigma, -1) = 0$ and $A(\sigma, +1) = \sigma$. The frequency spectrum of the noise
correlations can be defined as

$$S_{ij}(\omega) = 2 \int_{-\infty}^{+\infty} dt \exp(\text{i}\omega t) \langle \Delta I_i(t) \Delta I_j(0) \rangle,$$  

(6)

where $\Delta I_i(t) = I_i(t) - \langle I_i \rangle$ is the deviation from the average current in terminal $i$. It can be calculated using the method developed in Refs. [34–36] as:

$$\frac{S_{ij}(\omega)}{2e^2} = \sum_{\epsilon,\sigma} \Gamma_{j\sigma}^e \bar{p}_A( - \epsilon, \sigma) \delta_{ij} + \sum_{\sigma'\epsilon'} \epsilon' S'_{i\sigma'j\sigma'}(\omega),$$  

(7)

where the first term is the Schottky noise produced by tunneling through junction $j$, and

$$S'_{i\sigma'j\sigma}(\omega) = \Gamma_{i\sigma}^e G_{A(\sigma, - \epsilon'), A(\sigma', \epsilon)}(\omega) \Gamma_{j\sigma'}^e \bar{p}_A( - \sigma, - \epsilon) + \Gamma_{j\sigma}^e G_{A(\sigma', - \epsilon), A(\sigma, \epsilon)}( - \omega) \Gamma_{i\sigma}^e \bar{p}_A( - \sigma, - \epsilon),$$  

(8)

with $G_{\psi, \omega}(\omega) = \bar{p}_\psi / i\omega - (i\omega + M)^{-1}_{\psi, \omega}$.

In the following we study the dot in a beam-splitter configuration, in which a bias voltage $V$ is applied between terminal 2 and terminals 1 and 3. We consider the case $V > 0$, so that it is energetically more favorable for electrons to go from lead 2 to leads 1 and 3. We will limit our discussion to the case in which the two Zeeman sublevels are below the Fermi energy at equilibrium (i.e. $E_0 \pm g\mu_B B/2 < 0$). The opposite case was discussed in Ref. [37] for a two-terminal dot. We are mostly interested in the total zero-frequency current noise $S_{22} = S_{22}(0)$ and the cross-correlations $S_{13} = S_{13}(0)$ between the two output leads. It is useful to define the Fano factor $F_2 = S_{22} / 2e \langle I_2 \rangle$ and, correspondingly, $F_{13} = S_{13} / 2e \langle I_2 \rangle$.

In the following, we will first assume that $k_B T \ll g\mu_B B$. Transport through the down level is energetically allowed for $V \gtrsim V_- = ( - E_0 - g\mu_B B/2 )C/eC_2$. However, for $V \lesssim V_+ = ( - E_0 + g\mu_B B/2 )C/eC_2$, the dot is blocked by an up spin, thus down spins cannot cross the
dot. Around $V \simeq V_+$, the lower Zeeman level is close to the Fermi level of leads 1 and 3, as represented by the level diagram in the lower right inset of Fig. 1. The blockade of the dot is then partially lifted and transport through both levels is allowed. In this regime, we can write the tunneling rates as $\Gamma_{2\sigma} = \gamma_2$, $\Gamma_{2\sigma} = 0$, $\Gamma_{1\sigma}^- = \gamma_1(3)$, $\Gamma_{1\sigma}^+ = (1 - x)\gamma_1(3)$, $\Gamma_{1\sigma}^+ = 0$, and $\Gamma_{1\sigma}^- = \gamma_1(3)$, where $x = 1/(1 + \exp[-(E_0 - g\mu_B B - eV_1)/k_BT])$ ranges from 0 to 1 with increasing voltage. Furthermore, taking $\gamma_{sf} = 0$ and $\gamma = \gamma_1 + \gamma_3$, we find for the current

$$
\langle I_2 \rangle = \frac{2ex\gamma_2\bar{\gamma}}{\gamma + \gamma_2(1 + x)},
$$

for the Fano factor

$$
F_2 = 1 + \frac{2\gamma_2 (\bar{\gamma}(1 - 3x) + (1 - x)^2\gamma_2)}{(\bar{\gamma} + \gamma_2(1 + x))^2},
$$

and for the cross correlations

$$
F_{13} = \frac{\gamma_1\gamma_3}{\bar{\gamma}^2} \frac{2(1 - x)\gamma_2^2 + (1 - 7x + x^2 + x^3)\gamma_2^2\bar{\gamma} - 2(1 - x^2)\gamma_2\bar{\gamma}^2 - (1 - x)\bar{\gamma}^3}{\gamma_2(\bar{\gamma} + (1 + x)\gamma_2)^2}.
$$

We observe that the current increases with voltage (i.e. with $x$) around the voltage step $V_+$. Note that this current is not spin-polarized because up and down spin have the same probability to enter the dot, regardless of what happens at the output. The Fano factor $F_2$ and the cross-correlations $F_{13}$ deviate from their Poissonian values depending on the applied voltage. Our main results are that $F_2$ can be super-Poissonian and $F_{13}$ positive for $x < 1$, as can be clearly seen from (10) or (11) in the limit $\gamma_2 \gg \bar{\gamma}$. These features are a consequence of dynamical spin blockade: up spins leave the dot with a rate smaller than down spins, leading to a bunching of tunneling events [30]. In the limit $x \rightarrow 1$, the Fano factor is always sub-Poissonian and the cross-correlations always negative. This is due to the fact that the tunnel rates of up and down spins are equal, thus the Zeeman splitting plays no role and the dot is equivalent to a simple quantum dot with a spin-degenerate level. In the limit $x \rightarrow 0$, one could also expect the super-Poissonian nature of $F_2$ and the positivity of $F_{13}$ to be lost since the transport is enabled only by thermally activated processes. However, below the voltage threshold $V_+$, the Fano factor tends to

$$
F_2 = 1 + \frac{2\gamma_2}{\gamma_2 + \bar{\gamma}}.
$$

which is always super-Poissonian. If the coupling to terminal 2 dominates, i.e. $\gamma_2 \gg \bar{\gamma}$, the Fano factor takes a maximal value of 3. In the opposite limit $\bar{\gamma} \gg \gamma_2$, $F_2$ approaches the Poisson limit of uncorrelated single charge transfer. It is interesting to note that a symmetric junction $\gamma = \gamma_2$ produces twice the Poisson noise level. The cross-correlations in the same limit have the form

$$
F_{13} = \frac{\gamma_1\gamma_3}{\bar{\gamma}^2} \frac{(2\gamma_2 + \bar{\gamma})(\gamma_2 - \bar{\gamma})}{\gamma_2(\gamma_2 + \bar{\gamma})}.
$$

In the three cases discussed above, the cross-correlations thus take the limiting values $F_{13} = -\gamma_1\gamma_3/\gamma_2$ for $\bar{\gamma} \gg \gamma_2$, $F_{13} = 0$ for $\bar{\gamma} = \gamma_2$ and $F_{13} = 2\gamma_1\gamma_3/\bar{\gamma}^2$ for $\gamma_2 \gg \bar{\gamma}$. Hence, both the super-Poissonian nature of $F_2$ and the positivity of $F_{13}$ can persist for $x \rightarrow 0$. Even if the transport is enabled only by thermally activated processes, dynamical spin blockade already results in a correlated transfer of electrons.

We now turn to the discussion of the general results displayed in Figs. 1-3, obtained from an exact treatment of the full Master equation. Fig. 4 shows the full voltage dependence...
of the average current. As expected, the current shows a single step at \( V \approx V_+ \) [38–42]. The step width is about \( 10k_BT/eC_2 \), whereas its position varies only slightly with the asymmetry of the junctions (the maximal variation is about \( 0.27k_BT/eC_2 \)).

The left panel of Figure 2 shows the voltage dependence of the Fano factor in the absence of spin-flip scattering, for some values of \( \gamma_2/\tilde{\gamma} \). The divergence of the Fano factor at zero voltage is simply a result of the dominating thermal noise in the limit \( k_B T > eV \).

Note that similarly to \( \langle I^2 \rangle \), the Fano factor \( F_2 \) shows one single step at \( V \approx V_+ \). The right panel of Fig. 2 shows the effect of spin-flip scattering on \( F_2 \), for the case \( \gamma_2 = 5\tilde{\gamma} \). For \( V_- < V < V_+ \) spin flips become effective when \( \Gamma_{\uparrow\downarrow} = \gamma_{sf} \exp(g\mu_B B/2k_BT) \approx \gamma_t \), see Eq. (3).

The sensitivity to \( \gamma_{sf} \) thus increases with \( B \). Below \( V_- \), even smaller spin-flip rates suppress the super-Poissonian noise because the dwell time of electrons on the dot is very long. Far above \( V_+ \), spin-flip scattering has no effect on the sub-Poissonian noise.

The left panel of Fig. 3 shows the voltage dependence of the cross-correlations factor \( F_{13} \) between the two output terminals, for the same parameters as in Fig. 2. First, around the voltage threshold \( V_+ \), we observe the features discussed above. The cross-correlations develop from a positive or negative level below \( V_+ \) depending on the ratio \( \gamma_2/\tilde{\gamma} \) to the usual negative cross-correlations above \( V_+ \), where the spin-splitting plays no role anymore. Remarkably, in contrast to \( F_2 \), the cross-correlations also show a step around the lower voltage threshold \( V_- \). This illustrates clearly that \( F_{13} \) and \( F_2 \) are qualitatively different. The absence of the lower step for \( F_2 \) can be interpreted as a consequence of the unidirectionality of tunneling through junction 2. Indeed, \( \Gamma_{\sigma} \rightarrow 0 \) means that \( F_2 \) depends only on \( \tilde{p}_0 \) and \( G_{0,\upsigma} \) see Eq. (4) and (5).

Now, for \( V \sim V_- \), \( \Gamma_{\uparrow\downarrow} \rightarrow 0 \) implies that the contribution of these terms is independent of \( V \). On the contrary, \( F_{13} \) also depends on \( \tilde{p}_{\upsigma} \) and \( G_{\upsigma,0} \) with \( \upsigma \in \{\uparrow, \downarrow, 0\} \). For \( \Gamma_{\uparrow\downarrow} \rightarrow 0 \), these last terms depend strongly on \( \Gamma_{\uparrow\downarrow} \) which varies itself significantly with \( V \) around \( V_- \). Note that the absence of a step in \( F_2 \) implies a redistribution of the noise between \( S_{11}, S_{13} \) and \( S_{13} \) when the threshold \( V_- \) is crossed (due to charge conservation, \( S_{22} = S_{11} + S_{33} + 2S_{13} \)).
Fig. 3 – Left: Cross-Fano factor $F_{13} = S_{13}/2\langle I_2 \rangle$ between leads 1 and 3 as a function of voltage, for the same circuit parameters as in Fig. 1 and different values of $\gamma_2/\bar{\gamma}$. In all curves of the main frame $\gamma_{sf} = 0$. The inset shows the effect of spin flip scattering for $\gamma_2/\bar{\gamma} = 5$ and different values of $\gamma_{sf}$. Right: $F_{13}$ as a function of voltage for different values of $B$. The curves are shown for $\gamma_2/\bar{\gamma} = 5$. A cross indicates the position of $V_-$ for each case.

The extra step of $F_{13}$ disappears for $\gamma_2 \gg \bar{\gamma}$. In this limit, the cross-correlations display a single low voltage plateau $F_{13} = 2\gamma_1\gamma_3/\bar{\gamma}^2$, which is an upper bound for the two low voltages plateaux found in the general case. The inset in the left panel of Fig. 3 shows the effect of spin-flip scattering on the cross-correlations. As expected, they suppress all spin-effects and the positive cross-correlations become finally negative. Like for the Fano factor, very small spin-flip scattering rates $\gamma_{sf}$ are already sufficient to modify $F_{13}$ for $V < V_+$.

Since the positive cross-correlations found in this work are intimately related to the dynamical spin-blockade, we expect a strong dependence on the magnetic field. The right panel of Fig. 3 shows the voltage dependence of $F_{13}$ around the step $V_+$, for a fixed temperature and various magnetic fields. Just below $V_+$ the limiting value of $F_{13}$ is determined by formula (13). Thus, for a constant voltage $V \leq V_+$ we predict a cross-over from negative to positive cross-correlations with increasing magnetic field. One can see a qualitative change in the curves, which can be understood by a gradual splitting of the voltage steps $V_-$ and $V_+$. The lower step is at $V_- - V_+ = -g\mu_B B C_2 e$. As long as $g\mu_B B \lesssim 10k_B T$, the two voltage steps are indistinguishable. However, positive cross-correlations are already expected for $g\mu_B B \gtrsim 2k_B T$. For $g\mu_B B = 6k_B T$ the two steps still overlap, resulting in a broad peak, whereas for the higher magnetic field $g\mu_B B = 20k_B T$ the lower threshold at $V_-$ is outside the plotting region.

The regime $V \sim V_+$ has the advantage that the current is not exponentially small (c.f. Fig. 1) and thus observable more easily in an experiment. For $\gamma_1 = \gamma_3/5 = \gamma_3$, the maximum value obtained for the cross-correlations is $S_{13} \approx 0.09e^2\gamma_p$ at $x \approx 0.17$, i.e. $V \approx 4.26E_0$ in Fig. 3. With $\gamma_p \approx 5$ GHz, this corresponds to $10^{-29}$A$^2$s, a noise level accessible experimentally [43].

In conclusion we have studied current correlations for a three terminal quantum dot with unpolarized leads, placed in a magnetic field. Below the voltage threshold $V_+$, as a result of dynamical spin-blockade, the Fano factor of the input current shows an interesting super-Poissonian behavior and the cross-correlations in the two output leads can be positive. At higher voltages the Fano factor becomes sub-Poissonian and the cross-correlations negative, as
usual. The effect we predict should be observable in semiconductor quantum dots of Ref. [42].

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