Hadronic axion window and the big-bang nucleosynthesis

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Abstract

Hadronic axions with the decay constant $f_a \simeq 10^6$ GeV may fulfill all astrophysical and laboratory constraints discussed so far. In this paper, we reexamine the possibility of the hadronic axion window while taking into account the uncertainties of some parameters describing low energy axion dynamics. It is found that $f_a$ in the range from $3 \times 10^5$ GeV to $3 \times 10^6$ can not be excluded by existing arguments. We then examine the implication of this hadronic axion window for the big-bang nucleosynthesis (NS) by evaluating the energy density of thermal axions at the nucleosynthesis epoch. Our analysis yields $(\rho_a/\rho_\nu)_NS = 0.4 \sim 0.5$ which exceeds slightly the current best bound $(\rho_a/\rho_\nu)_NS \leq 0.3$. 
1. Introduction

One of the most attractive solutions to the strong CP problem is to introduce a spontaneously broken Peccei-Quinn symmetry which gives $\theta = 0$ dynamically \[1\]. This solution predicts the existence of a pseudo-Goldstone boson, the axion, whose decay constant $f_a$ is tightly constrained by astrophysical and cosmological considerations \[2\]. One frequently quoted window for the axion decay constant satisfying all phenomenological constraints is $f_a = 10^{10} \sim 10^{12}$ GeV. Besides this, for hadronic axions which have vanishing tree level coupling to the electron, there can be another window (the hadronic axion window) around $f_a \approx 10^6$ GeV. The existence of the hadronic axion window relies upon (i) the axion-electron coupling is radiatively induced and thus highly suppressed \[3\], (ii) axions on the window are trapped inside the supernova core \[4\], and (iii) a significant cancellation can occur in the axion-photon coupling between the short distance contribution (from the electromagnetic anomaly at $f_a$) and the long distance contribution (from the axion-pion mixing at the QCD scale) \[3\]. Based upon these points, $f_a \approx 10^6$ GeV can be consistent with a variety of the strong astrophysical constraints including those from the supernova SN1987A \[2\].

The hadronic axion window can be relevant for some models of dynamical symmetry breaking. Then it would become important to examine the phenomenological viability of the window more carefully. To this end, recently the effect of relic hadronic axions upon the diffuse extragalactic background radiation has been studied extensively \[5\]. By examining the photon flux from decaying axions \[6\], it has been noted that a severe cancellation should occur in axion-photon coupling for $f_a \approx 10^6$ GeV to be consistent with the observed data \[5\]. In fact, there is a possible loophole for this conclusion. Even when $f_a$ and the axion-photon coupling are fixed, there still exists a large uncertainty of the photon flux associated with the uncertainty of the light quark
mass ratio \( z = m_u/m_d \). For a given value of \( f_a \), the axion mass is determined by \( z \) (see eq. (5)). In ref. [4], \( z = 0.56 \) was taken to arrive at the above conclusion. However, it has been argued by several authors [7] that, due to instanton effects, the true value of \( z \) can be significantly smaller than the conventionally used value 0.56. A smaller \( z \) would give a smaller axion mass for which both the position and the intensity of the photon flux become lower. Clearly then the required cancellation of the axion-photon coupling becomes weaken. At any rate, in generic hadronic axion models, one can achieve the required cancellation by appropriately arranging the electromagnetic anomaly at \( f_a \).

In fact, the axion-photon coupling is the least known parameter among those describing the low energy dynamics of hadronic axions. It is thus desirable to consider an effect which is independent of the axion-photon coupling. In this paper, we wish to examine one such effect, that on the big-bang nucleosynthesis\(^1\). The organization of this paper is as follows. In the next section, we briefly review low energy axion couplings [3, 9] which will be used in later discussion. In sec 3 we collect arguments defining the hadronic axion window. Taking into account the uncertainties of the involved parameters, we argue that existing arguments allow the window: \( 3 \times 10^5 \leq f_a \leq 3 \times 10^6 \) (in GeV unit). In sec 4 we evaluate the rate of thermal axion production after the quark-hadron phase transition which was roughly estimated in ref. [10]. This will determine the energy density of thermal axions at the nucleosynthesis epoch. For the window defined in sec 3, we find \( (\rho_a/\rho_\nu)_{NS} = 0.4 \sim 0.5 \) which exceeds slightly the current best bound \( (\rho_a/\rho_\nu) \leq 0.3 \) [11]. Sec 5 is given for the conclusion.

2. Low energy couplings of hadronic axion

In this section, we review the low energy couplings of hadronic axions [3, 8]. We

\(^1\) The constraints imposed by the nucleosynthesis on the interactions of majorons have been discussed by Bertolini and Steigman [8]
will mainly follow ref. [9]. To be definite, let us consider the simplest hadronic axion model [12] which was first considered by Kim. The model contains an isosinglet Dirac quark $Q$ with the electromagnetic charge $Y_Q$ and also a gauge singlet complex scalar field $\sigma$. These exotic fields carry nonzero PQ charges: 1, $-1$, and 2 for $Q_L$, $Q_R$ and $\sigma$ respectively, while all other fields have vanishing PQ charge. The PQ symmetry is spontaneously broken by $\langle \sigma \rangle = f_a/\sqrt{2}$ and then the axion $a$ can be defined by

$$\sigma = \frac{1}{\sqrt{2}}(f_a + \rho) \exp(i a / f_a).$$  \hspace{1cm} (1)$$

Integrating out heavy degrees of freedom, viz $Q$ and $\rho$ whose masses are of the order of $f_a$, we obtain the axion effective lagrangian at the renormalization point $\mu = f_a$:

$$\frac{1}{2} (\partial_\mu a)^2 + \frac{a}{f_a} \left( \frac{g^2}{32\pi^2} G\tilde{G} + \kappa \frac{g_Y^2}{32\pi^2} Y\tilde{Y} \right).$$  \hspace{1cm} (2)$$

Here $G$ and $Y$ denote the field strengths of $SU(3)_c$ and $U(1)_Y$ respectively, while $\tilde{G}$ and $\tilde{Y}$ are their duals, and $\kappa = 6 Y_Q^2$ denotes the electromagnetic anomaly arising from the triangle loop of $Q$. Note that there is no tree level axion-electron coupling, which is the characteristic property of hadronic axions.

Given the axion effective lagrangian at $\mu = f_a$, we can derive a variety of axion properties at low energies by scaling the effective lagrangian down to the QCD scale $\Lambda \approx 1 \text{ GeV}$ [9]. Below $\Lambda$, the QCD anomaly and instanton effects become strong and also the chiral symmetry is spontaneously broken. It is then convenient to eliminate the $aG\tilde{G}$ coupling through an axion-dependent chiral rotation of the light quark fields $q = (u, d, s)$:

$$q_L \rightarrow \exp(i a Q_A / f_a) q_L, \quad q_R \rightarrow \exp(-i a Q_A / f_a) q_R,$$  \hspace{1cm} (3)$$

where

$$Q_A = M^{-1} / 2 \text{tr}(M^{-1}),$$  \hspace{1cm} (4)$$

and $M = \text{diag}(m_u, m_d, m_s)$ denotes the light quark mass matrix. With this chiral
rotation, the axion field $a$ becomes a physical mass eigenstate with

$$m_a = \frac{\sqrt{z} f_\pi m_\pi}{1 + z f_a},$$

(5)

where $f_\pi = 93$ MeV denotes the pion decay constant and $z = m_a/m_d$. Also the axion couplings with the electron and the photon renormalized at $\mu < \Lambda$ are given by

$$C_{a\gamma} \frac{a}{f_a} \frac{e^2}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} + C_{ae} \frac{\partial_\mu a}{f_a} \bar{\epsilon} \gamma^\mu \gamma_5 e,$$

(6)

where

$$C_{a\gamma} = \kappa - \frac{2(4 + z)}{3(1 + z)},$$

$$C_{ae} = \frac{3\alpha^2_{em}}{8\pi^2} [\kappa \ln \left(\frac{f_a}{\mu}\right) - \frac{2(4 + z)}{3(1 + z)} \ln \left(\frac{\Lambda}{\mu}\right)].$$

(7)

There are two kind of axion couplings with hadrons; derivative and nonderivative couplings [9]. In fact, axion phenomenology associated with the hadronic axion window is governed mainly by derivative couplings. At $\mu \simeq \Lambda$, after the chiral rotation (3), axion derivative couplings with the light quarks are given by

$$\frac{\partial_\mu a}{f_a} \left[ \frac{1}{6}(1 + \gamma)\bar{q}\gamma^\mu \gamma_5 q + \frac{1}{2} x_a \bar{q}\gamma^\mu \gamma_5 \lambda^a q \right],$$

(8)

where $x_a = \text{tr}(Q_A \lambda^a)$ and $\gamma(\mu)$ is due to the renormalization of the singlet current occurring between $f_a$ and $\mu$. Here $\text{tr}(\lambda^a \lambda^b) = 2\delta_{ab}$. This then gives

$$\frac{\partial_\mu a}{f_a} \left( \frac{1}{6} J_0^\mu + \frac{1}{2} x_a J_a^\mu \right),$$

(9)

where $J_0^\mu$ and $J_a^\mu$ are the hadronic currents made of the pseudoscalar meson octet $\Sigma = \exp(i\pi^a \lambda^a/f_\pi)$ and the baryon octet $B = B^a \lambda^a/\sqrt{2}$:

$$J_a^\mu = \frac{i}{2} f_\pi^2 \text{tr}(\lambda^a (\Sigma D^{\mu} \Sigma^\dagger - \Sigma^\dagger D^{\mu} \Sigma)) + \text{tr}(\bar{B} \gamma^\mu [\lambda^a_\pi(\xi), B]) + \text{tr}(\bar{B} \gamma^\mu \gamma_5 [\lambda^a_8(\xi), B]),$$

$$J_0^\mu = S \text{tr}(\bar{B} \gamma^\mu \gamma_5 B).$$

(10)

\footnote{Note that our result for $C_{ae}$ is smaller than the Srednicki’s in ref. [3] by the factor $1/2\pi$.}
Here $\lambda_\Sigma^a(\xi) = (\xi \lambda^a \xi^\dagger \xi^\dagger \lambda^a \xi)/2$, $\lambda_\Sigma^a(\xi) = (\xi \lambda^a \xi^\dagger + \xi^\dagger \lambda^a \xi)/2$ for $\xi^2 = \Sigma$, and $F$ and $D$ denote the baryon matrix elements of the $SU(3)$ octet axial vector currents, while $S$ is defined as

$$ (1 + \gamma) < B|\bar{q}\gamma^\mu\gamma_5 q|B > =Sn^\mu, \quad (11) $$

where $n^\mu$ denotes the covariant spin vector of the baryon state $B$. Note that $S$ is independent of the QCD renormalization point $\mu$.

Among axion couplings with hadrons, those of particular interest for us are the axion-nucleon couplings $\mathcal{L}_{aN}$ responsible for the processes $NN \rightarrow NNa$ and $N\pi \rightarrow Na$ and also the axion-pion couplings $\mathcal{L}_{a\pi}$ for $\pi\pi \rightarrow \pi a$. These couplings can be read from eqs. (9) and (10) giving

$$ \mathcal{L}_{aN} = \frac{\partial_{\mu}a}{f_a} \left[ C_{an} \bar{n}\gamma^\mu\gamma_5 n + C_{ap} \bar{p}\gamma^\mu\gamma_5 p + i \frac{C_{aN}}{f_\pi} (\pi^+\bar{p}\gamma^\mu n - \pi^-\bar{n}\gamma^\mu p) \right] $$

$$ \mathcal{L}_{a\pi} = C_{a\pi} \frac{\partial_{\mu}a}{f_a f_\pi} (\pi^0\pi^+\partial_\mu\pi^0 + \pi^0\pi^-\partial_\mu\pi^- - 2\pi^+\pi^-\partial_\mu\pi^0) \quad (12) $$

where

$$ C_{an} = \frac{z}{2(1+z)}F + \frac{z-2}{6(1+z)}D + \frac{1}{6}S, $$

$$ C_{ap} = \frac{1}{2(1+z)}F + \frac{1-2z}{6(1+z)}D + \frac{1}{6}S, $$

$$ C_{aN} = \frac{1-z}{2\sqrt{2}(1+z)} $$

$$ C_{a\pi} = \frac{1-z}{3(1+z)} \quad (13) $$

Several parameters appear in eqs. (7) and (13) to determine the low energy axion couplings for a given value of $f_a$. Let us discuss the values of those parameters. First of all, the electromagnetic anomaly $\kappa$ is due to the physics at the axion scale $f_a$ and thus is totally unknown. In principle, one can arrange the particle content at $f_a$ to have $\kappa$ taking an arbitrary value of order unity. The axial vector coupling constants $F$ and $D$ of baryons can be determined by the nucleon and hyperon beta decays.
as \( F = 0.47 \) and \( D = 0.81 \) \cite{13}. Then recent EMC measurement\cite{13} of the polarized proton structure function gives \( S \simeq 0.13 \pm 0.2 \) \cite{13}. About \( z \), one usually uses the result \( z = 0.56 \) from first order chiral perturbation theory \cite{16}. In second order chiral perturbation theory, \( z \) receives a correction \( \delta z = O(m_s/4\pi f_\pi) \) which is essentially due to instanton effects \cite{7}. A larger value of this instanton-induced correction means a smaller \( z \). It has been argued that \( \delta z \) can be large enough to imply \( z = 0 \) \cite{7}. With this point taken into account, we will allow \( z \) take a value significantly smaller than 0.56.

3. The window

In this section, we will collect arguments which define the hadronic axion window. Taking into account the uncertainties of \( \kappa \) and \( z \), we will argue that existing arguments do not exclude the window

\[
3 \times 10^5 \text{GeV} \leq f_a \leq 3 \times 10^6 \text{GeV}.
\] (14)

A key property of hadronic axion is that its coupling to the electron is radiatively induced and thus is highly suppressed. As is well known, helium ignition of red giants provides a strong constraint on the axion-electron coupling \cite{17}, \( C_{ae} m_e / f_a \leq 1.5 \times 10^{-13} \) in our notation. This gives\cite{17} \( f_a \geq \kappa \times 10^5 \) GeV which is satisfied by the window (14) for \( \kappa \) of order unity.

The list of other arguments defining the hadronic axion window is as follows:

\footnote{In fact, what the EMC measurement gives is the value of \( S/(1 + \gamma) \) at \( \mu \simeq 3 \) GeV where \( \gamma \) is negligibly small. Note that at leading order the evolution of \( \gamma \) is given by \( \mu d\gamma/d\mu = -3(\alpha_e/\pi)^2 \gamma \) \cite{15} with \( \gamma(\mu = f_a) = 0 \). It has been pointed out \cite{14} that elastic neutral current experiment gives a similar result \( S \simeq 0.09 \pm 0.24 \).}

\footnote{Since our result for the radiatively induced axion-electron coupling \( C_{ae} \) is smaller than the Srednicki’s, the lower bound on \( f_a \) is relaxed by the factor \( 1/2\pi \). Note that the hadronic axion window becomes widen as a result of this point.}
(A) the axion’s effect on the neutrino burst from SN1987A [4, 18], (B) the effect of axions emitted from SN1987A on the Kamiokande II detector [19], (C) the effects of axion emission on the evolution of helium burning low-mass stars [20], (D) the effect of decaying relic axions on the diffuse extragalactic background radiation [5].

In regard to the argument (A), axions on the window (14) would be trapped inside the supernova core [4]. A detailed numerical analysis for this “trapping regime” has been performed by Burrows et al. [18] who have concluded that if \( f_a \leq 8C_aN \times 10^6 \) GeV (here \( C_aN \simeq (C_{an}^2 + C_{ap}^2)^{1/2}/\sqrt{2} \)) axions would be so strongly trapped that axion emission would not have a significant effect on the neutrino burst [5]. Also by considering (B), it has been pointed out that \( f_a \leq 2C_aN \times 10^6 \) GeV would have produced an unacceptably large signal at the Kamiokande detector [19]. These then give the allowed window

\[
2C_aN \times 10^6 \text{GeV} \leq f_a \leq 8C_aN \times 10^6 \text{GeV}.
\]

Using \( F = 0.47, D = 0.81, \) and \( S = 0.13 \pm 0.2, \) and allowing \( z \) vary between zero and 0.56, we find \( C_aN = 0.15 \sim 0.4 \) for which the above window goes into that of eq. (14).

Can the window (14) be compatible with the constraints from (C) and (D)? The arguments (C) and (D) provide limits to the axion-photon coupling. From (C), one has derived \( C_{a\gamma} \leq (f_a/10^7 \text{GeV}) \). It has been pointed out that, for \( z = m_u/m_d = 0.56 \) and \( f_a \approx 10^6 \) GeV, the argument (D) can lead to a more stringent limit [5], say \( C_{a\gamma} \leq 0.02 \). However the validity of the latter limit strongly relies upon the used value of \( z \). Since \( m_a \propto \sqrt{z} \), if \( |z| \ll 0.56 \) which is an open possibility [7], both the position and the intensity of the photon-line from decaying axions would

\[\text{Although less complete, axion trapping was discussed also by Ishizuka and Yoshimura [21] and Engel et al. [13].}\]

\[\text{Here we ignore possible uncertainties of the astrophysical arguments leading to this range of } f_a.\]
be significantly lowered. Clearly then the limit from (D) becomes invalid or at least weaken.

For hadronic axions, $C_{a\gamma}$ can be divided into two pieces, the short distance contribution $\kappa$ denoting the electromagnetic anomaly at the axion scale $f_a$ and the long distance part $-2(4+z)/3(1+z)$ arising from the axion-pion mixing. Then an interesting point is there can be a severe cancellation between the short and long distance contributions. In order to see this, let us consider two examples: (e.1) $z = 0.56, \ Y_Q = 4/7$; (e.2) $z = 0.05, \ Y_Q = 2/3$. Here $Y_Q$ denotes the electromagnetic charge of the heavy Dirac quark $Q$ and thus $\kappa = 6Y_Q^2$. We then have (e.1) $C_{a\gamma} = O(10^{-2})$; (e.2) $C_{a\gamma} \simeq 0.1$. These examples simply show that a significant cancellation can occur rather naturally, but the degree of cancellation is highly model-dependent. At any rate, it is true that, for $f_a$ on the window (14), one can always arrange the model to have $\kappa$ for which $C_{a\gamma}$ becomes small enough to satisfy the constraints from (C) and (D). We thus conclude that, due to our ignorance of $\kappa$ (and also of $z$), the hadronic axion window (14) is allowed also by (C) and (D).

4. Thermal hadronic axions and the nucleosynthesis

In this section, we consider the implication of the hadronic axion window (14) for the big-bang nucleosynthesis (NS). As is well known, in the standard model of NS, the energy density of exotic particles at the NS epoch is severely constrained. The primordial yields of $^4\text{He}$ is sensitive to the universal expansion rate $H \propto \rho^{1/2}$ at $T \simeq 1$ MeV and, thereby, leads to a bound on any exotic contribution to the energy density $\rho$. The recent work of Walker et al. [11] provides the most stringent bound on the axion energy density normalized to the energy density of the electron-neutrino:

$$\delta N_\nu \equiv \left( \frac{\rho_a}{\rho_\nu} \right)_{NS} \leq 0.3.$$  \hspace{1cm} (16)

Since both axions and neutrinos are relativistic at $T \simeq 1$ MeV, one simply has...
$\delta N_\nu = (4/7) (T_a / T_\nu)^4_{NS}$. Suppose axions were initially in equilibrium with the thermal bath of normal particles ($\gamma, e^\pm, \nu, \ldots$) at high temperature, but later become decouple at temperature $T_D > 1$ MeV. Entropy conservation then leads to the relation

$$\delta N_\nu = \frac{4}{7} \left( \frac{43/4}{g_* (T_D)} \right)^{4/3}, \quad (17)$$

where $g_*$ counts interacting degrees of freedom whose entropy density is given by $s = 2\pi^2 g_* T^3 / 45$. Here the axion decoupling temperature $T_D$ can be found by comparing the expansion rate $H$ with the axion production rate

$$\Gamma = n_a^{-1} \sum_{i,j} n_i n_j \langle \sigma_{ij} v \rangle, \quad (18)$$

where $n_a$ denotes the axion number density and the sum extends to all production processes involving as initial states the particles $i$ and $j$ which are still in equilibrium at $T_D > 1$ MeV. As was pointed out by Bertolini and Steigman \[8\], for the NS constraint (16) to be satisfied, axions (or any light scalar with $m \ll 1$ MeV) must decouple before $T = 100$ MeV.

Clearly $\delta N_\nu \leq 4/7$. If axions decouple before the quark-hadron phase transition, we would have $\delta N_\nu \leq 0.06$ because of the relative “heating” of neutrinos during the quark-hadron phase transition. We are thus led to consider the possibility for axions in thermal equilibrium after the quark-hadron phase transition. Then there are two types of processes dominantly producing thermal axions: (a) $\pi^0 \pi^\pm \to a \pi^\pm$, $\pi^+ \pi^- \to a \pi^0$, and (b) $\pi^0 N \to aN$ ($N = n$ or $p$), $\pi^+ n \to ap$, $\pi^- p \to an$ and their CP conjugates. In fact, the rate of the process (b) was roughly estimated in ref. \[4\]. Here we will carefully evaluate the rates of both (a) and (b).

Let $\Gamma_{(a)}$ and $\Gamma_{(b)}$ denote the axion production rate of (a) and (b) respectively. Then a straightforward computation yields

$$\Gamma_{(a)} = \frac{3}{1024 \pi^5} \frac{1}{f_a^2 f_\pi^2} C_{\pi a}^2 I_{(a)},$$

10
\[
\Gamma_{(b)} = \frac{1}{6\pi^3 f_a^2 f_\pi^2} \left[ g_A^2 (5C_{an}^2 + 5C_{ap}^2 + 2C_{an}C_{ap}) + 6C_{an}^2 \right] I_{(b)},
\]

where

\[
I_{(a)} = n_a^{-1} T^8 \int dx_1 dx_2 \frac{x_1^2 x_2^2}{y_1 y_2} f(y_1) f(y_2) \int_{-1}^{1} d\omega \frac{(s - m_\pi^2)^3 (5s - 2m_\pi^2)}{s^2 T^4},
\]

\[
I_{(b)} = n_a^{-1} n_N T^5 \int dx_1 x_1 y_1^3 f(y_1).
\]

Here \( f(y) = 1/(e^y - 1) \) denotes the pion distribution function, \( n_a \) and \( n_N \) are the axion and nucleon number densities at equilibrium, \( x_i = |\vec{p}_i|/T, \ y_i = E_i/T \ (i = 1, 2) \), and \( s = 2(m_\pi^2 + T^2(y_1 y_2 - x_1 x_2 \omega)) \). To derive \( \Gamma_{(a)} \), we have used the pion-nucleon coupling \( g_A \bar{N} \gamma^\mu \gamma_5 \partial_\mu \bar{\pi} N \) where \( g_A = F + D \) and \( \bar{\pi} = \pi^i \tau^i/2 \) (\( \tau^i \) = the Pauli matrices).

By performing numerical analysis for \( I_{(a)} \) and \( I_{(b)} \), one can see that \( \Gamma_{(a)} \) dominates over \( \Gamma_{(b)} \) when \( T \leq 150 \) MeV. Here to be definite, we have used \( z = 0.56, \ F = 0.47, \ D = 0.81, \) and \( S = 0.13 \), but the result is quite insensitive to the allowed variations of these parameters. Using \( \Gamma/H \simeq 1 \) at \( T = T_D \), we find \( T_D = 30 \sim 50 \) MeV for the range of \( f_a \) from \( 3 \times 10^5 \) GeV to \( 3 \times 10^6 \) GeV. With the relation (17), this gives

\[
\delta N_\nu = 0.4 \sim 0.5,
\]

which is in slight contradiction to the NS bound (16).

5. Conclusion

Hadronic axions with the decay constant around \( 10^6 \) GeV can escape from all astrophysical and laboratory constraints discussed so far. In this paper, we have collected arguments which define the hadronic axion window and found that existing constraints are satisfied. Using the rough estimate of \( \Gamma_{(b)} \) given in ref. [10], \( T_D \simeq 50 \) MeV was found in ref. [3]. Our careful evaluation yields \( \Gamma_{(b)} \) which is significantly smaller than that of ref. [10]. This is mainly due to the suppression caused by the axion-nucleon coupling constants. As a result, if one uses our \( \Gamma_{(b)} \) while ignoring \( \Gamma_{(a)} \), one would get \( T_D = 70 \sim 100 \) MeV for \( f_a = 3 \times 10^5 \sim 3 \times 10^6 \) GeV. However as we have mentioned, around \( T_D \), the pion process (a) largely dominates over the nucleon process (b), leading to a lower \( T_D \).
arguments allow the window $3 \times 10^5 \leq f_a \leq 3 \times 10^6$ (in GeV unit). The phenomenological viability of this window relies strongly upon the model-dependent cancellation that occurs in the axion-photon coupling. As we have noted, in generic hadronic axion models, it is not so unnatural to achieve the required cancellation. Since the axion-photon coupling is quite model-dependent and is totally unknown, it is desirable to consider an effect which is independent of the axion-photon coupling. We thus have considered the effect on the nucleosynthesis which is determined by the axion-hadron couplings. By evaluating the axion production rate after the quark-hadron phase transition, we could determine the energy density of thermal axions at the nucleosynthesis epoch. It is quite certain that $T_D < 100$ MeV and thus the resulting $\delta N_\nu$ exceeds the lower bound 0.3 [1]. However since the deviation is not so significant, $\delta N_\nu = 0.4 \sim 0.5$, it would still be difficult to conclude that the hadronic axion window is excluded by the primordial abundance of $^4$He.

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