Derivative F-Terms from Heterotic M-Theory
Five-brane Instantons

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Abstract

We study non-perturbative effects due to a heterotic M-theory five-brane wrapped on Calabi-Yau threefold. We show that such instantons contribute to derivative F-terms described recently by Beasley and Witten rather than to the superpotential.
1 Introduction

In \cite{1} and \cite{2} Beasley and Witten described a new class of instanton effects in $N = 1$ supersymmetric field theory and string theory. These effects are generating of derivative, or multi-fermion, F-terms. In superfields, the corresponding contributions to the effective action are $F$-terms containing spinor covariant derivatives. In components, this corresponds to four- and higher-fermionic terms. The simplest non-trivial example of such terms is two-derivative, or four-fermion, F-terms. They correspond to non-perturbative deformation of the classical moduli space. A field theory example where such effects take place is $N = 1$ supersymmetric QCD with $N_f = N_c$ \cite{3,4}. In general, as was shown in \cite{1}, for $N_f \geq N_c$ instantons contribute to interactions of $2(N_f - N_c) + 4$ fermions. In the case of string theory, it was shown in \cite{2} that derivative F-terms arise, in particular, from wrapping heterotic strings on non-isolated genus zero or higher genus curves. It is worth pointing out that, just like superpotential, derivative F-terms have a certain holomorphic signature.

In this note, we present an explicit example of a string instanton that contributes to four-fermion terms in the low-energy effective action. The instanton is a heterotic M-theory five-brane wrapped on Calabi-Yau threefold. Being a bulk instanton, a five-brane wrapping Calabi-Yau threefold has four fermionic zero modes. As the result it cannot contribute to the superpotential. Instead, we show that it contributes to derivative F-terms. This situation is different from the case of string or open membrane instantons \cite{5,6,7,8,9,10,11}. These instantons are located on the boundary of the interval and, thus, have less fermionic zero modes. In fact, they have only two zero modes and contribute to the superpotential for various moduli.

This note is organized as follows. In section 2, we give a brief review of the structure of derivative F-terms. In section 3, we study the action of a bulk five-brane instanton wrapping Calabi-Yau manifold. We show that this instanton contributes to four-fermion F-terms. In section 4, we discuss some additional features of five-brane instantons.

2 Review of Derivative F-terms

In this section, we give a brief review of the derivative F-terms following \cite{1,2}. These terms have the following superfield structure

$$\delta S = \int d^4x \ d^4\theta \ \omega_{ij}(\Phi, \bar{\Phi}) \bar{D}_\alpha \bar{\Phi}^i \bar{D}^\alpha \bar{\Phi}^j,$$ \hspace{1cm} (2.1)
where $\Phi^i$ and $\bar{\Phi}^i$ are chiral and anti-chiral superfields whose lowest components $\phi^i$ and $\bar{\phi}^i$ parametrize the classical moduli space $M_d$. Furthermore, $\omega_{ij}$ is defined as

$$\omega_{ij} = \frac{1}{2} \left( g_{\bar{a}i} \omega_{\bar{j}^i} + g_{ij} \omega_{i} \right),$$

(2.2)

where $g_{\bar{a}i}$ is the Kahler metric on $M_d$ and $\omega_{\bar{j}^i}$ represents an element in the Dolbeault cohomology group $H^1_{\bar{\partial}}(M_d, TM_d)$. Thus, $\omega_{\bar{j}^i}$ is independent of the metric and parametrizes infinitesimal deformations of the complex structure of $M_d$. Let us note that it is possible to choose a coordinate system on $M_d$ in which the antiholomorphic components of the Christoffel symbols, $\Gamma^k_{ij}$, vanish. In this coordinate system, $\omega_{ij}$ becomes holomorphic, that is a function of $\Phi^i$ only. The operator $\omega_{ij} D_{\alpha} \bar{\Phi}^{\bar{j}i} \bar{D}^{\alpha} \bar{\Phi}^{\bar{j}i}$ is not manifestly chiral. Its chirality follows from the equations of motion of the unperturbed sigma-model with the metric $g_{\bar{a}i}$ and holomorphy of $\omega_{\bar{j}^i}$. Being an element of $H^1_{\bar{\partial}}(M_d, TM_d)$ rather than a function, $\omega_{\bar{j}^i}$ is locally exact. Therefore, locally, one can always rewrite $\delta S$ as an integral over the whole superspace. However, if the cohomology class represented by $\omega_{\bar{j}^i}$ is non-trivial, we cannot write $\delta S$ globally as a D-term. In this sense, $\delta S$ represents an F-term. The expression in (2.1) can be generalized to higher derivative F-terms

$$\delta S = \int d^4x \, d^4\theta \, \omega_{i_1 \ldots i_p} \omega_{\bar{j}_1 \ldots \bar{j}_p} \left( D_{\alpha} \bar{\Phi}^{\bar{j}_1} \bar{D}^{\alpha} \bar{\Phi}^{\bar{j}_1} \right) \ldots \left( D_{\alpha} \bar{\Phi}^{\bar{j}_p} \bar{D}^{\alpha} \bar{\Phi}^{\bar{j}_p} \right).$$

(2.3)

See [1, 2] for details.

In components, the action (2.1) becomes

$$\delta S = -4 \int d^4x \, \omega_{ij} \frac{1}{4} \int d^4x \, \nabla_k \nabla_l \omega_{ij} \psi^{i\alpha} \bar{\psi}_{\alpha} \bar{\psi}_{\bar{j}} \bar{\psi}_{\bar{\alpha}} + \ldots ,$$

(2.4)

where $\psi_{\alpha}^i$ is the fermionic superpartner of $\phi^i$.

The most convenient way to see if an instanton contributes to $\delta S$ is to look at the correlation function of four fermions

$$\langle \psi^{i\alpha}(x_1) \psi_{\alpha}^j(x_2) \bar{\psi}^{i\bar{\alpha}}(x_3) \bar{\psi}^{j\bar{\alpha}}(x_4) \rangle .$$

(2.5)

In order for this correlation function to be non-zero, the action of the instanton has to be multiplied by four fermions. This implies that the instanton has to have four fermionic zero modes. In the next section, we will present an example of such an instanton, a heterotic M-theory five-brane wrapped on a Calabi-Yau manifold. The general expression for the correlation function that we want to consider is

$$\int \mathcal{D} \Phi \mathcal{D} \bar{\Phi} e^{-S_{4D}} \psi^{i\alpha}(x_1) \bar{\psi}_{\alpha}^j(x_2) \bar{\psi}_{\alpha}^{i\bar{\alpha}}(x_3) \bar{\psi}^{j\bar{\alpha}}(x_4) \int \mathcal{D} \mathcal{Z} e^{-S_{5}},$$

(2.6)
where by $S_{4D}$ we denote the four-dimensional action and $\int \mathcal{D}Z e^{-S_5}$ is the partition function of the five-brane wrapped on Calabi-Yau manifold.

## 3 Five-brane Instantons

We consider a (Euclidean) five-brane wrapped on a Calabi-Yau manifold and located at an arbitrary point in the interval away from any of the orbifold fixed planes. The relevant part of the five-brane action [12] is

$$S_5 = T \int d^6\sigma \sqrt{\det g_{ij}} + i T \int \mathcal{C}^{(6)}.$$  (3.1)

Here $\sigma^i, i = 1, \ldots, 6$ are the coordinates along the five-brane worldvolume, $T$ is the five-brane tension, $g_{ij}$ is the pullback of the metric superfield to the worldvolume and $\mathcal{C}^{(6)}$ is the superfield whose lowest component is the dual six-form potential $C_{M_1 \ldots M_6}$. More precisely, $g_{ij}$ is defined as

$$g_{ij} = \eta_{AB} E^A_M E^B_N \partial_i Z^M \partial_j Z^N = g_{MN} \partial_i Z^M \partial_j Z^N. \quad (3.2)$$

Our index convention is the following. The indices $A, B$ are flat and run from 0 to 10. The superindex $\mathbb{M}$ is split into the space-time index $M, M = 0, \ldots, 10$ and the spinor index $m, m = 1, \ldots, 32$. The index $M$ itself splits into the four-dimensional index $\mu, \mu = 0, 1, 2, 3$, the indices along the Calabi-Yau threefold $U, U = 4, \ldots, 10$ and the eleventh direction. The supercoordinates $Z^\mathbb{M}$ split into the space-time coordinates $X^M$ and fermionic coordinates $\Theta^m$. To proceed we have to expand the superfields $E^A_M$ and $\mathcal{C}^{(6)}$ in powers in the fermionic coordinates $\Theta$. For our purposes it is enough to expand to the linear order in gravitino $\Psi_M$ similarly to the case of membrane instantons in [8, 9]. We have [13]

$$E^A_M = E^A_M + 2 \bar{\Theta} \Gamma^A \Psi_M + \ldots,$$

$$E^A_m = 0,$$

$$\mathcal{C}^{(6)}_{M_1 M_2 M_3 M_4 M_5 M_6} = C_{M_1 M_2 M_3 M_4 M_5 M_6} - \bar{\Theta} \Gamma_{[M_1 M_2 M_3 M_4 M_5} \Psi_{M_6]} + \ldots. \quad (3.3)$$

Substituting eq. (3.3) back in the action and expanding to the linear order in gravitino yields

$$S_5 = T \int d^6\sigma \sqrt{\det g_{MN}} \partial_i X^M \partial_j X^N + i T \int d^6\sigma \epsilon^{i_1 \ldots i_6} \partial_{i_1} X^{M_1} \ldots \partial_{i_6} X^{M_6} C_{M_1 \ldots M_6}$$

$$+ T \int d^6\sigma \sqrt{\det g_{MN}} \partial_i X^M \partial_j X^N \mathcal{V}^N \Psi_N, \quad (3.4)$$

where the vertex operator for the gravitino is given by

$$\mathcal{V}^N = g^{ij} \partial_i X^M \partial_j X^N (\bar{\Theta} \Gamma_M) - \epsilon^{i_1 \ldots i_6} \partial_{i_1} X^{M_1} \ldots \partial_{i_6} X^{M_6} (\bar{\Theta} \Gamma_{M_1 \ldots M_6}). \quad (3.5)$$
Now let us recall that we are considering a five-brane wrapped on a Calabi-Yau threefold. From the four-dimensional point of view this configuration looks like an instanton. We can choose the static gauge where the Calabi-Yau coordinates $X^U$ are identified with the world-volume coordinates $\sigma^i$. The action $S_5$ in this gauge simplifies and becomes

$$S_5 = T \int d^6 \sigma (\sqrt{\det g_{ij}} + i C_{123456}) + T \int d^6 \sigma \sqrt{\det g_{ij}} V^i \Psi_i,$$

where $g_{ij}$ is now the Calabi-Yau metric and the vertex operator $V^i$ is given by

$$V^i = g^{ij} \bar{\Theta} \Gamma_j - \frac{\epsilon^{i_1 ... i_6}}{6! \sqrt{g_{ij}}} \bar{\Theta} \Gamma_{i_1 ... i_5} \cdot$$

Now let us discuss the zero modes. This instanton has five bosonic zero modes, four along four-dimensional Minkowski space and one additional zero mode along the interval. In the low-energy limit, the fields $g_{ij}$ and $\Psi_i$ do not depend on the interval and this additional zero mode can be integrated out to produce an irrelevant constant. The integral over the remaining bosonic zero modes is just the integral over four-dimensional Minkowski space.

Now let us move on to the fermionic zero modes. We start by recalling that $\Theta$ is a Majorana spinor in eleven dimensions. Thus, it has thirty-two real components. The Calabi-Yau background breaks a quarter of supersymmetry. A five-brane reduces the number of surviving supersymmetries by a factor of two. Thus, we find that $\Theta$ has four zero modes in the background of the five-brane instanton. The equation that these zero modes is the equation for the supersymmetries preserved by a five-brane. It reads

$$\frac{1}{6!} \Gamma_{i_1 i_2 i_3 i_4 i_5} \epsilon_{i_1 i_2 i_3 i_4 i_5 i_6} \bar{\Theta} \theta^a \otimes \xi_+ = \Theta.$$

Equivalently, this equation can be understood as the kappa supersymmetry fixing condition in the world-volume action (3.1). Equation (3.8) is just the condition that $\Theta$ has to be a chiral spinor on Calabi-Yau manifold. This means that $\Theta$ can be written as

$$\Theta = \theta^a \otimes \xi_+ \oplus \bar{\theta}_\dot{a} \otimes \xi_+,$$

where $\xi_+$ is the covariantly constant spinor of positive chirality on Calabi-Yau manifold. The spinors $\theta^a$ and $\bar{\theta}_\dot{a}$ are the four-dimensional spinors representing the fermionic zero modes of the five-brane instanton. Of course, one can come to the same conclusion analyzing the fermionic equations of motion derived from the action (3.1). The equation of motion for $\Theta$ is the Dirac equation (14)

$$\Gamma^i (\partial_i + \omega_i^{AB} \Gamma_{AB}) \Theta = 0,$$
where $\omega_i^{AB}$ is the spin connection. Since a Calabi-Yau background breaks a quarter of supersymmetry, a solution to (3.10) has eight independent components. This solution involves fermions of both chirality. A supersymmetric five-brane instanton corresponds to a solution of positive chirality (similarly an anti-five-brane instanton corresponds to a solution of negative chirality) and we obtain a solution for $\Theta$ given by equation (3.9). The five-brane partition function can now be written as

$$\int d^4x d^2\theta d^2\bar{\theta} e^{-S_5},$$

where $S_5$ is given by (3.6) and $\Theta$ is given by eq. (3.9). The existence of four fermionic zero modes makes a five-brane instanton different from string or open membrane instantons studied in [7, 8, 9]. String and open membrane instantons are boundary instantons. They have only two fermionic zero modes because the boundary in heterotic M-theory reduces the number of preserved supercharges by two.

The next step is to evaluate the action $S_5$. Let us start with the first term which is purely bosonic. Upon the dimensional reduction, the Calabi-Yau metric $g_{ij}$ is written as follows [15]

$$g_{ij} = V^{1/3} \Omega_{ij},$$

where $\Omega_{ij}$ is the reference metric and $V$ is the volume modulus. The field $C_{123456}$ is related to the axion. This can be shown as follows. Take the three-form potential $C_{MNP}$ and consider the components $C_{\mu\nu11}$. To obtain the axion $\sigma$, we dualize the field strength $G_{\mu\nu\lambda1}$

$$G_{\mu\nu\lambda1} = \epsilon_{\mu\nu\lambda\rho1...i6} \partial^\rho \sigma.$$

On the other hand, by definition of $C_{123456}$, we have

$$G_{\mu\nu\lambda1} = \frac{1}{6!} \epsilon_{\mu\nu\lambda\rho i1...i6} \partial^\rho C^{i1...i6} = \epsilon_{\mu\nu\lambda\rho} \partial^\rho C_{123456}.$$  \hspace{1cm} (3.14)

Comparing (3.13) and (3.14), we obtain

$$C_{123456} = \sigma.$$  \hspace{1cm} (3.15)

Thus, the first term in (3.6) gives

$$TvS,$$

where

$$S = V + i\sigma$$

(3.17)
and $v$ is the Calabi-Yau reference volume. Now let us consider the term $V^i \Psi_i$ in eq (3.6). We split the index $i$ into holomorphic and antiholomorphic indices $u, \bar{u}$. Then the dimensional reduction of the gravitino $\Psi_u$ is

$$\Psi_u = \psi_\alpha \otimes \Gamma_u \xi_+ \oplus \bar{\psi}_\dot{\alpha} \otimes \Gamma_{\bar{u}} \xi_+. \quad (3.18)$$

Substituting (3.9) and (3.18) in (3.6) and using the following relations for the covariantly constant spinors

$$\Gamma_{\bar{u}} \xi_+ = 0, \quad \xi_+^\dagger \xi_+ = 1, \quad (3.19)$$

we find that the five-brane partition function is

$$\int d^4 x d^2 \theta d^2 \bar{\theta} e^{-TV} e^{-\theta \bar{\psi}_\alpha - \bar{\theta} \bar{\psi}_{\dot{\alpha}}}. \quad (3.20)$$

To perform the integral over the fermionic zero modes, we expand the second factor in (3.20) to the fourth order. This gives the following expression for the partition function

$$\int d^4 x e^{-TV} (\psi_\alpha \bar{\psi}_{\dot{\alpha}})(\bar{\psi}_{\bar{\alpha}} \bar{\psi}_{\dot{\alpha}}). \quad (3.21)$$

This partition function is interpreted as the four-fermionic term in the action $\delta S$ given by eq. (2.4) (or, equivalently, (2.1)) with the holomorphic section $\omega$ given by

$$S = e^{-TV}. \quad (3.22)$$

Of course, this function $\omega$ has to be multiplied by the complex structure dependent bosonic and fermionic one-loop determinants. Analysis of fluctuations around the zero modes and their one-loop determinants is rather complicated. In the static gauge, the fluctuations involve five scalars $\delta X^\mu, \mu = 0, \ldots, 3, \delta X^{11}$, four chiral fermions $\delta \Theta_+$ (the fermions of opposite chirality are projected out by the condition (3.8), or, equivalently, by fixing the kappa supersymmetry) and also the anti-self-dual two-form, whose coupling to the world-volume we have been ignoring. Note that the fluctuations form the tensor multiplet propagating on the five-brane worldvolume. Various issues about the partition function of the five-brane world-volume fields, including anomaly cancellations, were studied by Witten in [16]. We will not discuss them in this paper. Because of the one-loop determinants, there is a non-trivial non-perturbative mixing of the moduli spaces of the volume and complex structure multiplets. So, to be precise, eq. (3.22) defines only the $S\bar{S}$ component of $\omega$. 

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4 Additional Remarks

In this section, we will discuss some additional features of five-brane instantons. First, a generic supersymmetric heterotic M-theory background contains some amount of the $G$-flux. The non-vanishing components are $G_{(2,1,1)}, G_{(1,2,1)}$ and $G_{(2,2,0)}$. Here we are using notation from [17]. The first two indices represent the holomorphic and anti-holomorphic directions along the Calabi-Yau manifold and the last index is the index along the interval. The fact that these components of the fluxes are consistent with supersymmetry means that the Dirac operator on the deformed compactification manifold still has solutions. To be consistent with Poincare symmetry in five dimensions, the variation of the eleven-dimensional gravitino has to have eight zero modes as in the absence of fluxes. This immediately implies that a five-brane wrapped on the deformed manifold will have four fermionic zero modes since it breaks one half of supersymmetry. Thus a five-brane instanton still has four zero modes and contributes to the function $\omega$ as $e^{-T v(V+i \sigma)}$ times the one-loop determinants. However, the flux $G_{(2,2,0)}$ is more subtle. It provides a deformation of the Calabi-Yau metric along the interval [17, 18, 19]

$$ds^2 = e^{-f(x^{11})} \eta_{\mu\nu} dx^\mu dx^\nu + e^{f(x^{11})} (g_{ij} dX^i dX^j + dx^{11} dx^{11}),$$

(4.1)

where the warp factor $e^{f(x^{11})}$ is given by

$$e^{f(x^{11})} = (1 + x^{11} Q)^{2/3}$$

(4.2)

and $Q$ is the amount of the flux. For example, in the absence of five-brane wrapping holomorphic curves, $Q$ is given by [17, 18, 19]

$$Q = \frac{\ell_{11}^3}{32 \pi^2 v} \int \omega \wedge \left( \text{tr} F \wedge F - \frac{1}{2} \text{tr} R \wedge R \right),$$

(4.3)

where $\ell_{11}$ is the eleven-dimensional Planck length and $\omega$ is the Kahler form of the undeformed Calabi-Yau metric. Then it follows from eq. (4.1) that the tension of the five-brane wrapping the Calabi-Yau manifold is $x^{11}$ dependent and, thus, such an instanton is not BPS. The wrapped five-brane has to move along the interval to minimize its tension until it collides with the boundary. Therefore, strictly speaking, the analysis in the previous section is valid only if a five-brane instanton is placed in the region where the flux is zero. The existence of such regions was discussed in detail in [20] and we will not repeat it here.

As the second remark, let us discuss what happens when the five-brane is placed on top of one of the orbifold fixed planes. This system is different than a five-brane in the bulk.
The boundary breaks one half of the bulk supersymmetry. The supercharges preserved by the boundary are given by

\[ Q = Q_\alpha \otimes \xi_+ \oplus \bar{Q}_\dot{\alpha} \otimes \xi_- . \]  

(4.4)

Comparing this with (3.9) we find that the zero mode of the boundary five-brane are given by

\[ \Theta = \theta_\alpha \otimes \xi_+ . \]  

(4.5)

This means that a five-brane instanton will now have only two zero modes, \( \theta_\alpha \) just like an open membrane [8, 9]. Then, one can imagine that such a configuration might contribute to two-fermion terms, that is to the superpotential for the Calabi-Yau volume multiplet. This would have have interesting effects on moduli stabilization [21]. Unfortunately, it is not clear how to prove (or disprove) it quantitatively. The world-volume of a five-brane instanton coincident with an orbifold fixed plane interacts with a tensionless string propagating on the Calabi-Yau manifold. It is not known how to describe this interaction.

As the last remark, we will recall that a five-brane can dissolve in the orbifold plane through a small instanton transition [22, 23, 24]. The resulting configuration is a gauge instanton. If the five-brane is wrapping the entire Calabi-Yau manifold, the dissolved instanton configuration is an instanton on \( R^4 \). Thus, the effects of this dissolved five-brane are the standard gauge instanton effects in field theory.

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