Dynamics of ghost domains in spin-glasses‡.

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Abstract. We revisit the problem of how spin-glasses “heal” after being exposed to
tortuous perturbations by the temperature/bond chaos effects in temperature/bond
cycling protocols. Revised scaling arguments suggest the amplitude of the order
parameter within ghost domains recovers very slowly as compared with the rate it
is reduced by the strong perturbations. The parallel evolution of the order parameter
and the size of the ghost domains can be examined in simulations and experiments by
measurements of a memory auto-correlation function which exhibits a “memory peak”
at the time scale of the age imprinted in the ghost domains. These expectations are
confirmed by Monte Carlo simulations of an Edwards-Anderson Ising spin-glass model.

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1. Introduction

A class of scaling theories [1,2,3] for randomly frustrated glassy systems has pointed out
a striking fragility of their free-energy landscapes. While they realize some glassy order
within a given environment specified for instance by temperature, even an infinitesimal
change of the latter lead to radical reformation of the free-energy landscape to a
globally uncorrelated new one. Such non-perturbative, global shuffling of the free-
energy landscape with infinitesimal changes of control parameters are called as chaos
effects. Indeed theoretical studies of some microscopic models including studies on
Edwards-Anderson (EA) Ising spin-glass models by Migdal-Kadanoff renormalization
group (MKRG) method [4,5,6] and mean-field theory [7] (and references there in) and
directed polymers in random media (DPRM) [8] have partially or almost fully confirmed
such striking effects. Further works may clarify to what extent these unusual phenomena
are universal.

A natural interest is to see how slowly relaxing or aging glassy systems will react
to such tortuous perturbations [9,10,11,12,13,14]. While systems like simple phase
separating systems would either keep aging accumulatively (domain growth) or stop
aging under external driving forces (e.g. stirring oil+ vinegar) [15], spin-glasses exhibit
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rejuvenation-memory effects [9] which are far more puzzling and richer. In [12] a minimal description for such a dynamics was obtained for the case of Ising spin-glasses in terms of ghost domains, which is a direct extension of the concept of the standard scaling theory for domain growth [10] in isothermal aging. In contrast to isothermal aging, the amplitudes of the order parameters or bias within domains become dynamical variables which play a central role: they act as internal driving forces which perturb the trajectory of the domain growth itself. As the result a concrete mechanism of imprinting/retrieving multiple memory under the tortuous chaos effects was found. Recently the MKRG method was applied to the dynamics of the EA model subjected to chaos effects and such a mechanism was demonstrated explicitly. [13]

In the present paper we revise the ghost domain scenario based on the theory by Bray and Kisner [17] on the growth of the bias during domain growth dynamics. We consider a simplest one-step “perturbation-healing” protocol. An example is the one-step temperature-cycling protocol [18, 19] first used in spin-glasses. It proceeds as follows.

(1) Initial aging stage. First a spin-glass is equilibrated at a high enough temperature above the glass transition temperature $T_g$. Then at time $t = 0$ the temperature is quenched down to a temperature say $T_A$ below $T_g$ where the system is aged for some time $t_w$. This stage is just the same as usual isothermal aging.

(2) Perturbation stage. The temperature is changed to $T_B = T_A + \Delta T$ (with $\Delta T$ being either positive or negative) where the system is aged for some time $\tau_p$. Strong restart of aging or rejuvenation is observed, for instance, by measuring the AC magnetic susceptibility in the spin-glasses and ceramic superconductors [20, 21]. Other glassy systems such as super cooled liquids [22], polymer glasses [23] exhibit no or much weaker rejuvenations. It may suggest absence of chaos effects in some classes of glassy systems. One should also keep in mind that large enough length/time scales compared with the overlap length (See Eq. (3)) must be explored to see chaos effects. Failures of some experiments and simulations to detect rejuvenations may be related to this difficulty.

(3) Healing stage. Finally the temperature is put back to $T_A$. In spin-glasses strong restart of aging or rejuvenation is observed again [18, 24, 11]. After some recovery time say $\tau_{rec}$ this restarted process disappears and the rest of the relaxation becomes a continuation of the initial aging stage, which is called the memory effect. We closely discuss the two stage processes in the healing stage based on the ghost domain scenario.

Many systems “heal” by waiting some recovery time $\tau_{rec}$ after being exposed to a perturbation for a certain time $\tau_p$. Simple minded “length scale(s)” (or some equivalent “energy-barrier”) arguments which neglect the internal driving due to the remanent bias may lead to two contradictory possibilities: A) healing is impossible after such strong perturbations due to chaos effects or that B) healing is somehow possible and the recovery time $\tau_{rec}$ is just identical to the time scale at which the length scale $L(\tau_{rec})$
(energy barrier) explored after switching off the perturbation becomes as large as the length scale $L(\tau_p)$ (energy barrier) explored during the perturbation. Furthermore one could argue the “effective age” of the system imprinted in the system would be largely modified once $L(\tau_p)$ becomes larger than the length (energy) scale corresponding to the age. Somewhat surprisingly we find that all these intuitions fail in general for the perturbations operated in the strongly perturbed regime of the chaos effect. In the present paper we also consider dynamics operated in the weakly perturbed regime of the chaos effect. This allows us to take into account effects of slowness of the switching on/off perturbations in realistic circumstances.

After introducing the spin-glass model in the next section, the definition of ghost domains is summarized and the scaling theory by Bray and Kisner is briefly reviewed in section 3. In sections 4 and 5 the revised ghost domain scenario is introduced focusing on the simple one-step perturbation-healing protocol mentioned above and the scenario is examined numerically on the 4 dimensional EA Ising spin-glass model. In section 6 we propose a simple way to take into account the effects of slow switchings such as heating/cooling rate effects. The conclusion of the paper is presented in the last section.

2. Model

Specifically we consider the Edwards-Anderson (EA) Ising spin-glass models described by a Hamiltonian

$$H = -\sum_{i,j} J_{ij} S_i S_j$$

where $S_i$ is an Ising spin at site $i$ located at $\vec{r}_i$ on a $d$ dimensional lattice with $N$ lattice sites and $J_{ij}$ is a random interaction bond which takes $+J$ and $-J$ randomly for each nearest neighbor pair $(i,j)$. Here $J > 0$ is the unit of energy scale. For convenience we denote the scaled thermal energy $k_B T/J$ as temperature $T$ in the following. Here $k_B$ is the Boltzmann’s constant. We consider two kinds of perturbations, (1) temperature changes $T \rightarrow T + \Delta T$; (2) bond changes $\mathcal{J} \rightarrow \mathcal{J}'$. A new set of bonds $\mathcal{J}' = \{J'_{ij}\}$ is created from the original one $\mathcal{J} = \{J_{ij}\}$ as follows. For each pair $(i,j)$ we choose $J'_{ij} = -J_{ij}$ randomly with probability $p$ and $J'_{ij} = J_{ij}$ with probability $1-p$.

In the numerical simulation presented in section 4 we use the $d = 4$ model on the hyper-cubic lattice and the single spin flip heat-bath Monte Carlo method. In simulations we limit ourselves to bond changes since computational power is too limited to study temperature changes efficiently.

3. Ghost domains

Let us introduce basic ingredients of the ghost domain scenario to prepare for the discussion of the simple one-step perturbation-healing protocol (e. g. the one-step temperature-cycling experiments) in the next two sections. For simplicity we assume that an equilibrium states $\Gamma^{T,\mathcal{J}}$ of a spin-glass a system with a set of random interaction
bonds $J$ at temperature $T$ below the spin-glass transition temperature $T_g$ is given by its typical spin configuration. Such a configuration at site $i$ may be described as $\sqrt{q_{EA}(T)}\sigma_i^{T,J}$ where $\sigma_i^{T,J}$ is an Ising variable and $q_{EA}(T)$ is the Edwards-Anderson (EA) order parameter which takes into account the effects of thermal fluctuations.

Furthermore we assume the only possible other phase at same environment $(T,J)$ is $\bar{\Gamma}^{T,J}$ whose configuration is given by $-\sqrt{q_{EA}(T)}\sigma_i^{T,J}$. However extensions to the cases that more phases exist for a given environment may be considered as well.

3.1. Weakly and strongly perturbed regimes of chaos effects

The chaos effects become stronger at larger length scales. Since the distinction between the weakly and strongly perturbed regimes are important in the following here we summarize the picture on the crossover between the two regimes given in \[8, 25, 13\].

Let us consider a generic perturbation which may induce a droplet excitation of size $L$ with respect to the “ground state” $\{\sigma_i^{T,J}\}$. The excited state has a certain free-energy gap $F_L(>0)$ with respect to the ground state. Suppose that we have a perturbation such that the excited state obtains a gain of the free-energy of order $\Delta U_L/L^a$. Here $L_0$ is a microscopic unit length scale. Then a droplet excitation will be induced if $\Delta U_L$ turns out to be greater than the free-energy gap $F_L$. The free-energy gap is expected to have a broad distribution characterized by a distribution function $\rho_L(F)$ with the scaling form $\rho_L(F) \sim dF \rho(F/L^a/dF/L^a).$ Using these properties the probability $p_L(\delta)$ that a perturbation of strength $\delta$ induces a droplet excitation of size $L$ is found as

$$p_L(\delta) \sim \int_0^{\Delta U_L} dF \rho(F) = \int_0^{(L/\xi(\delta))^{\zeta}} dy \tilde{\rho}(y)$$

where

$$\xi(\delta) = L_0^{\zeta-1/\zeta}$$

is the characteristic crossover length, called overlap length, beyond which $p_L(\delta)$ becomes $O(1)$. The exponent $\zeta$, the so called chaos exponent, is given by $\zeta = a - \theta$. One can see that if $\zeta > 0$ ($a > \theta$) the probability $p_L(\delta)$ continuously increases with increasing $L/\xi(\delta)$. In the following we distinguish between the strongly perturbed regime $L/\xi(\delta) > 1$ and the weakly perturbed regime $L/\xi(\delta) < 1$.

In the strongly perturbed regime $L/\xi(\delta) > 1$, the original ground state $\{\sigma_i^{T,J}\}$ is completely unstable with respect to the droplet excitations, i.e. a new equilibrium state must form. The term chaos $[11, 2, 26, 3]$ properly describes the fact that a strongly perturbed regime eventually emerge even for arbitrary small $\delta \ll 1$ at sufficiently large length scales. However, chaos does not set in abruptly at the overlap length $\xi(\delta)$ but, in the weakly perturbed regime $L/\xi(\delta) < 1$ chaos like droplet excitations already occur at length scales smaller than $\xi(\delta)$ with non-zero probability $p_L(\delta)$ $[8, 25, 13].$

In the case of temperature shifts of strength $\Delta T$ the possible free-energy gain of a droplet excitation of size $L$ is the entropy gain ($\Delta T$) which is expected to
scales as $\Delta U_L/J \sim \Delta T (L/L_0)^{d_s/2}$ where $d_s$ is the surface fractal dimension of droplet excitations. (See [3] for a detailed discussion) In the case of bond perturbations, the random gain of energy of a droplet excitation happen at around its surface so that $\Delta U_L/J \sim p (L/L_0)^{d_s/2}$. Thus temperature and bond perturbations should lead to a chaos effect of the same universality class with $\xi = d_s/2 - \theta$.

3.2. Definition of ghost domains

Let us consider a generic protocol such the working environment is changed from time to time among a set of target environments $\{A, B, \ldots\}$ which consists of different temperatures $\{T_A, T_B, \ldots\}$ (all below $T_g$ ) and/or different bonds $\{J_A, J_B, \ldots\}$ whose equilibrium states are represented by $\sqrt{q_{EA}(T_A)}\sigma^A_i, \sqrt{q_{EA}(T_B)}\sigma^B_i, \ldots$.

Suppose that the system is now evolving in a certain working environment, say $W = (T_W, J_W)$ at a certain time $t$. Short time averages may be took to average out short time thermal fluctuations. Then the temporal spin configuration can be represented as $\sqrt{q_{EA}(T_W)}s_i(t)$ where $s_i(t)$ takes Ising values. It can be projected onto the equilibrium states of any environment $R \in \{A, B, \ldots\}$ as

$$\tilde{s}_R^i(t) = \sigma^R_i s_i(t).$$

Then the projected image $\tilde{s}_R^i(t)$ is described in a coarse-grained way by the following two features.

(i) the domain wall configuration: configuration of the spatial pattern of the sign of the projection $\tilde{s}_R^i(t)$.

(ii) the order parameter: the amplitude of the projection $\rho_R(t) = |[\tilde{s}_R^i(t)]_{\text{domain}}|$ where $[\ldots]_{\text{domain}}$ denote the spatial average within a ghost domain.

It is useful to consider decomposition of a ghost domain $\Gamma^R (\bar{\Gamma}^R)$ into “patches”,

- The strength of the bias has the full amplitude 1 within a patch.
- The “signs +/−” of the bias is however different on different patches—the majority has the same sign as that of the ghost domain to which they belong to. Minorities have the opposite sign.

The probability $p_{\text{minor}}(t)$ that a patch belongs to the minority phase $\bar{\Gamma}^R (\Gamma^R)$ in a ghost domain of $\Gamma^R (\Gamma^R)$ is related to the strength of the bias $\rho_R(t)$ as

$$p_{\text{minor}}(t) = (1 - \rho_R(t))/2.$$  \hfill (5)

If one chooses $R = W$, a ghost domain reduces to an ordinary domain which is enough in isothermal aging where the order parameter is a constant. In the cycling protocols, minimal description is to keep track of projections on to the equilibrium states of all target environments. We call such projections as ghost domains. Very important point is that not only (i) the domain wall structures but also (ii) the amplitude of the order parameter within the domains are dynamical variables.
3.3. Physical observable

Let us note here that basic quantities measured in experiments and simulations are essentially gauge invariant (or ghost invariant), i.e. they do not depend on specific choice of projections. An important example is the spin auto-correlation function

\[ C(t, t') = \frac{1}{N} \sum_i <S_i(t)S_i(t')> \tag{6} \]

where \( N \) is the number spins and \(< ... >\) represents taking an averages over different trajectories. The auto-correlation function can be re-expressed in terms of any projection field \( \tilde{s}_R^i(t) \) as

\[ C(t, t') = q_{EA}(T_R)(1/N) \sum_i <\tilde{s}_i^R(t)\tilde{s}_i^R(t')> \]

because \( \sigma_R^i = \pm 1 \). Thus it does not depend explicitly on the specific choice of projections, i.e. gauge invariant except for the prefactor. The auto-correlation function can be measured experimentally by monitoring spontaneous thermal fluctuations of the magnetization \( M(t) \) because the leading \( O(N) \) part of the magnetic auto-correlation function is \( NC(t, t') \) in spin-glasses with no ferromagnetic or anti-ferromagnetic bias in the distribution of the bonds. In experiments linear magnetic susceptibilities to uniform external magnetic field are often measured. By the same token as above it can be seen that the linear magnetic response functions (per spin) of spin-glasses to a uniform external field \( h(t') \),

\[ R(t, t') = \frac{1}{N}\partial <M(t)> / \partial h(t') \]

is essentially gauge-invariant.

3.4. Basic dynamics at a working environment

Suppose that the system is temporally evolving at a certain working environment \( W \) with temperature \( T = T_W \) with a certain set of bonds \( J_W \). Here we summarize basic properties of the dynamics of the (ghost) domains of \( \Gamma_W / \bar{\Gamma}_W \).

At coarse-grained mesoscopic level, the relaxational dynamics is considered as a thermally activated process of a droplet like excitation. The energy barrier associated with a droplet of size \( L \) scales as \( \Delta(T)(L/L_0)^\psi \) with \( \psi > 0 \). Thus at a given logarithmic time scale \( \log(t/\tau_0(T)) \) a droplet as large as \( L_T(t) \sim L_0[(k_B T / \Delta(T)) \ln(t/\tau_0(T))]^{1/\psi} \)

\[ L_T(t) \sim L_0[(k_B T / \Delta(T)) \ln(t/\tau_0(T))]^{1/\psi} \tag{7} \]

can be thermally activated. In the above formula the effects of critical fluctuations can be took into account in a renormalized way in the characteristic energy scale \( \Delta(T) \) for the free-energy barrier and the characteristic time scale \( \tau_0(T) \).

Suppose that the projection of the initial spin configuration onto the equilibrium state \( \Gamma_W^0 \) at time \( t = 0 \) is strongly disordered such that its spatial correlation function decays rapidly beyond some correlation length \( \xi_{ini} \),

\[ [(\tilde{s}_i^W(0) - \rho^W(0))(\tilde{s}_j^W(0) - \rho^W(0))] = F \left( \frac{|r_i - r_j|}{\xi_{ini}} \right) \]

\[ [\tilde{s}_i^W(0)] = \rho^W(0) \tag{8} \]

Here \([...]\) denotes the average over space and \( F(x) \) is a certain rapidly decreasing function. Note that the bias \( \rho^W(0) \) is made homogeneous within the system.
Domain growth without bias- If the initial bias is absent $\rho^W(0) = 0$, the global $Z_2$ symmetry of the system is not broken and the domain growth (aging) never stops. The mean separation between the domain walls at time $t$ is $L_T(t)$ given in Eq. (7) [26]. In such a critical quench the spatial correlation function

$$C^W(r, t, t') = \langle \hat{s}_r^W(t) \hat{s}_j^W(t') \rangle_{|r=|r_i-r_j|}$$

exhibits universal scaling properties [16]. In the so-called aging regime $L_T(t) > L_T(t')$ it scales as

$$C_0^W(r, t, t') \sim \left( \frac{L_T(t)}{L_T(t')} \right)^{-\bar{\lambda}} h \left( r \frac{L_T(t)}{L_T(t')} \right)$$

(10)

The subscript “0” is meant to emphasize that the spin configuration is random at $t = 0$ with respect to the target equilibrium state. Here $h(x)$ is a decreasing function with $h(0) = 1$. The exponent $\bar{\lambda}$ is a non-equilibrium dynamical exponent introduced by Fisher-Huse in [26]. (Note that in some literatures e.g. Refs [26] and also [12, 28] $\bar{\lambda}$ is denoted as $\lambda$. In the present paper we follow [17, 16] and use $\bar{\lambda}$ for the decay of the correlation function and $\lambda$ for the growth of bias discussed below (See Eq. (12)).

A special case of much interest is the auto-correlation function ($r = 0$) which is just the spin auto-correlation function $C(t, t')$ defined in Eq. (6) which is a gauge invariant quantity. It generically follows a scaling of the form $C_0(t, t') = C_0(L_T(t)/L_T(t'))$. The scaling function $C_0(x)$ remains at 1 in the quasi-equilibrium regime $x < 1$. In the aging regime $x > 1$, it approaches 0 asymptotically as $C_0(x) \sim x^{-\bar{\lambda}}$. In Ising spin glasses

$$d/2 \leq \bar{\lambda} < d$$

(11)

is proposed [26]. This very slow relaxation is in sharp contrast to the exponential decay in the paramagnetic phase $C_0(t, t') \propto \exp(-|t-t'|/\tau_{eq}(T))$ where $\tau_{eq}(T)$ is the correlation time in the paramagnetic phase.

Domain growth with bias- Even if the initial bias $\rho^W(0)$ is small, the $Z_2$ symmetry is explicitly broken if it is non-zero. One expects that the strength of the symmetry breaking will increase with time and eventually terminates the aging just as if external symmetry breaking field is applied. This problem was considered theoretically first by Bray and Kisner [17]. They noticed that the non-zero homogeneous bias grows with time $t$ as as

$$\rho^W(t) \sim \rho^W(0) \left( \frac{L_T(t)}{\xi_{ini}} \right)^{\lambda}$$

(12)

and the dynamical exponent $\lambda$ is related to $\bar{\lambda}$ as

$$\bar{\lambda} + \lambda = d$$

(13)

Here let us summarize the derivation [17] within our context. First one can see that the bias is nothing but the $k = 0$ component of the Fourier transform of $\hat{s}_i^W$. Then one assumes that the amplitude of the $k = 0$ component at time $t$ can be computed as a linear-response to the change of its initial value. Second assuming the Gaussian characteristics of the random initial condition Eq. (8) one finds the linear response
function is the same as the $k = 0$ component of the spin correlation function Eq. (9) up to some proportionality constant $c$. As the result one obtains

$$\rho^W(t) = c\rho^W(0)C_{k=0}(t, 0)$$

(14)

Then using Eq. (9) one finds Eq. (12) and Eq. (13). Combining the scaling relation Eq. (13) and the inequality Eq. (11) one finds

$$\bar{\lambda}/\lambda \geq 1.$$  

(15)

As we discuss below this inequality suggests healing of spin-glasses after chaotic perturbations takes an enormously long time. [29]

4. A cycle on a globally symmetry broken state

Let us now begin with the perturbation-healing protocol by considering an idealized limit. This corresponds for example to the one-step temperature-cycling experiments mentioned in the introduction $T_A \rightarrow T_B \rightarrow T_A$ but with the initial aging done for an extremely long time $t_w = \infty$: the spin configuration is globally equilibrated with respect to an equilibrium state $\Gamma_A$ at the beginning. Then perturbation is performed - change temperature or bond and let the system evolve for a certain time $\tau_p$ so that domains of $\Gamma_B/\overline{\Gamma_B}$ grow. Lastly healing is performed - switch off the perturbation and let the system evolve after wards for some time $\tau_h$. Here $A$ and $B$ would stand for (1) $T_A$ and $T_B = T_A + \Delta T$ in the case of temperature-cycling or (2) $J_A$ and $J_B$ (which is created by randomly changing the sign of a fraction $p$ of the bonds in the original set $J_A$) in the case of bond-cycling. For simplicity we assume the switchings are instantaneous. The effects of slow switching times, e.g. finite heating/cooling rates, will be discussed later in section 6.

**Strongly perturbed regime** - If the duration of the perturbation $\tau_p$ is long enough such that $L_B(\tau_p) > \xi(\delta)$, where $\xi(\delta)$ is the overlap length given in Eq. (3) and $\delta$ can be either $\Delta T$ or $p$, the strongly perturbed regime of the chaos effects (see section 3.1) should come into play. For simplicity here we neglect dynamics at short time scales which belong to the weakly perturbed regime. An extreme example of $\xi(\delta) = 1$ is shown in Fig 1 using a Monte Carlo simulation of a 2-dimensional Ising Mattis model.

The initial spin configuration is globally aligned to $\Gamma_A$ so that it is fully biased as $\tilde{s}^A_i = 1$. But simultaneously $\tilde{s}^B_i$ is completely disordered (beyond $\xi(\delta)$) with no bias $[< \tilde{s}^A_i >] = 0$. Thus during the perturbation stage, the domains of both $\Gamma_B/\overline{\Gamma_B}$ grow competing with each other just as the usual domain growth. Their typical size becomes $L_B(\tau_p)$. As can be seen in the Figure 1 this amounts to reduction of the bias (or staggard magnetization) with respect to $\Gamma_A$. The remanent bias $\rho^A_{rem}$ becomes

$$\rho^A_{rem}(\tau_p) = (1/N) \sum_i \tilde{s}^A_i(\tau_p) = (1/N) \sum_i \tilde{s}^A_i(0)\tilde{s}^A_i(\tau_p) = (1/N) \sum_i \tilde{s}^B_i(0)\tilde{s}^B_i(\tau_p)$$

$$= C_0(\tau_p, 0) \sim (L_B(\tau_p)/\xi(\delta))^{-\bar{\lambda}}.$$  

(16)

Here we used the properties of the initial condition and the gauge invariance of the autocorrelation function. The subscript “0” is used in the last equation because the initial
condition is completely random with respect to the relaxational dynamics which is at work during the perturbation. Here it can be seen clearly that the strong perturbation due to the chaos effect is not equivalent to put a system to a paramagnetic phase. In the latter case one would find exponential decay of the bias \( \rho^A_{\text{rem}}(\tau_p) \sim \exp(-\tau_p/\tau_{\text{eq}}(T_B)) \) where \( \tau_{\text{eq}}(T_B) \) is the relaxation time in the paramagnetic phase. Thus chaos effect do not amount pushing the system to the disordered phase contrary to what might have been suspected.

During the healing stage, the domains of both \( \Gamma^A/\bar{\Gamma}^A \) grow competing with each other starting from the minimum length scale \( \xi(\delta) \). Their typical size becomes \( L_A(\tau_h) \). However this is a domain growth with biased initial condition as Eq. (8). From Eq. (12)

![Figure 1](image_url)

**Figure 1.** Evolution of ghost domains in a simple “perturbation-healing” protocol on a globally symmetry broken state. This is a demonstration using a heat-bath Monte Carlo simulation of 2-dim Ising Mattis model \((N = 400 \times 400)\) in which the interaction bonds in Eq. (1) are given as \( J_{ij} = J_{\sigma_i\sigma_j} \) where \( \sigma_i \) is a random Ising (gauge) variables given at each site. One immediately finds the equilibrium state (ground state) for each \( \mathcal{J} \) is simply given by the set \( \{\sigma_i\} \). The initial spin configuration is chosen to be identical to a random ground state \( \sigma^A \). **Perturbation:** For time \( \tau_p = 100 \) (MCS) the system is strongly perturbed by using the Hamiltonian of a different ground state \( \sigma^B \) which is completely uncorrelated with \( \sigma^A \): \( \xi(\delta) = 1 \) in the unit lattice. **Healing:** Then the Hamiltonian is put back to the original one which is used for additional 1000 (MCS). In the present examples temperature is set to \( T = 2.0 \). The different colors (greyscale) represent the sign + and − of the projections on to the ground states. The 3 columns on the left sides are snapshots at time 0, 10, 100 (MCS) during the perturbation. Domains of \( \Gamma^B/\bar{\Gamma}^B \) grow while the bias \( \rho^A \) decreases. In this example \( \rho^A \) has become 0.03 which is too small to distinguish by eye. In the 3 columns on the right are snapshots at time 10, 100, 1000 (MCS) during the healing. Here domains of \( \Gamma^A/\bar{\Gamma}^A \) grow but the minority phase \( \bar{\Gamma}^A \) slowly disappears and \( \rho^A \) increases. The recovery of the \( \rho^A \) turns out to be much longer time than \( \tau_p \). In this model the growth law Eq. (7) should be replaced by \( L(t)/L_0 \sim \sqrt{t/t_0} \) since it is equivalent to the ferromagnetic Ising model [10]. We found numerically \( \lambda \sim 0.8 \) and \( \bar{\lambda} \sim 1.2 \) in this model being consistent with Eq. (13).
the strength of the global bias is found to grows as

$$\rho_{\text{rec}}^A(\tau_n) = \rho_{\text{rem}}^A(\frac{L_A(\tau_n)}{\xi(\delta)})^\lambda$$  \hspace{1cm} (17)

where $\rho_{\text{rem}}^A$ is the remanent bias at the end of the perturbation stage (or the beginning of the healing stage). Note that $\rho_{\text{rec}}^A(\tau_n)$ is proportional to the initial remanent bias $\rho_{\text{rem}}^A$ which is the direct consequence of the "linear-response" of the temporal bias with respect to the change of the initial bias as noticed by Bray and Kisner.

The above situations may be rephrased as the following. During the perturbation stage the system may be decomposed into patches whose size is given by the overlap length $\xi(\delta)$. The probability $p_{\text{minor}}$ that a patch belong to the minority phase $\bar{\Gamma}^A$ is related to the bias $\rho^A$ as given in Eq. (5) so that it increases with $\tau_p$ during the perturbation stage. Next in the healing stage the system may be decomposed into patches of size $L_A(\tau_n)$, which now grows with $\tau_n$. The probability $p_{\text{minor}}$ now decreases because $\rho^A$ increases. Consequently the minority phase eventually disappears and the domain growth stops at the recovery time $\tau_{\text{rec}}$.

Combining Eq. (16) and Eq. (17) one finds the recovery time $\tau_{\text{rec}}^{\text{strong}}$ of the strength of the bias as

$$\frac{L_A(\tau_{\text{rec}}^{\text{strong}})}{\xi(\delta)} = \left(\frac{L_B(\tau_p)}{\xi(\delta)}\right)^{\bar{\lambda}/\lambda}$$ \hspace{1cm} (18)

Here the super-script "strong" is mean to emphasize that it is a formula valid in the strongly perturbed regime. Since $\bar{\lambda}/\lambda \geq 1$ as given in Eq. (15), we conclude that the recovery time can be significantly large. [29]

The above considerations can be extended to multi-step cycling. Very counter-intuitive consequences follow due to multiplicative nature of the effect of multiple perturbations [12]. For example another perturbation stage to grow $\Gamma^C/\bar{\Gamma}^C$ for some time $\tau_p'$ can be added before the healing stage in the perturbation-healing protocol discussed above. Here $\Gamma^C$ is assumed to be decorrelated with respect to both $\Gamma^A$ and $\Gamma^B$ beyond the overlap length $\xi(\delta)$. Then the recovery time $\tau_{\text{rec}}^{\text{strong}}$ of the order parameter of $A$ becomes,

$$\frac{L_A(\tau_{\text{rec}}^{\text{strong}})}{\xi(\delta)} = \left(\frac{L_B(\tau_p) L_C(\tau_p')}{\xi(\delta)}\right)^{\bar{\lambda}/\lambda}$$ \hspace{1cm} (19)

which can yield huge recovery time.

**Weakly perturbed regime**- If the duration of the perturbation $\tau_p$ is small such that $L_B < \xi(\delta)$, the effect of the perturbation should remain mild as explained in section 3.1 Here the mutual interferences between ghost domains just amount to induce rare droplet excitations of various size $L$ with probability $p_L(\delta) \ll 1$ (see Eq. 2) on top of each other. They are just independent islands of the minority phase which rarely overlap with each other. Thus one only needs to keep track of switch on/off of such isolated objects during perturbation and healing stages. This means a naive “length scale(s)” argument to estimate the recovery time of bias fortunately do not fail in this regime.
More precisely the result of [13] implies the remanent bias decreases due to the increase of rare islands of the minority phase as
\[
\rho_{\text{rem}}(\tau_p) = 1 - c p L_B(\tau_p)(\delta) + O(p^2) \simeq 1 - c(L_B/\xi(\delta))^\zeta
\]
in the perturbation stage and increases as
\[
\rho_{\text{rec}}(\tau_h) = \rho_{\text{rem}}(\tau_p) + c(L_A/\xi(\delta))^\zeta
\]
by removing islands one by one in the healing stage. Here \( c \) is some numerical constant.

We have neglected higher order terms of \( O(p^2) \). In the MKRG analysis [13], it has been shown that both Eq. (20) and Eq. (16) are limiting behaviors of a universal scaling function of \( L_B/\xi(\delta) \). Note that the bias recovers in additive fashion in Eq. (21) which is markedly different from the multiplicative fashion found in the strongly perturbed regime Eq. (17).

One finds the recovery time in the weakly perturbed regime is simply given as
\[
L_A(\tau_{\text{weak}}) = L_B(\tau_p) \quad \text{or} \quad \tau_{\text{rec}}^{\text{weak}}/\tau_0(T_A) = (t/\tau_0(T_B))^{\Delta(T_A)/\Delta(T_B)}(T_B/T_A)^\eta(22)
\]
Here the super-script “weak” is mean to emphasize that it is a formula valid only in the weakly perturbed regime. The 2nd equation holds in the case of activated dynamics Eq. (7) which simplifies further at low enough temperature as \( \tau_{\text{rec}}^{\text{weak}}/\tau_0 = (t/\tau_0)^{(T_B/T_A)} \) where temperature dependence of the unit time \( \tau_0(T) \) and the energy scale \( \Delta(T) \) can be neglected. There one only needs to know the microscopic time scale \( \tau_0 \), which is known to be around \( 10^{-12} - 10^{-13} \) (sec) in real spin-glass materials, to estimate the recovery time \( \tau_{\text{rec}}^{\text{weak}} \).

Moreover it is easy to see that non-overlapping islands of the minority phase cannot cause any non-trivial effect of multiple perturbations (see Eq. (19)). This point becomes important when we consider the effects of slow switching, e.g. heating/cooling rate effects in section 6.

Other non-chaotic, mild effects of perturbations can be considered in similar ways. For instance change of thermally active droplets can be took into account by changing \( p_L(\delta) \) above by \( \Delta T/(L/L_0)^\theta \). The latter amounts to an even weaker effect at larger time scales but may be dominant at short time scales.

5. Parallel evolution of domain sizes and order parameter

Let us complete the scenario for the one-step perturbation-healing protocol by now allowing the waiting time \( t_w \) in the initial aging to be finite. Suppose the system is completely disordered with respect to both \( \Gamma^A \) and \( \Gamma^B \) at the beginning and aged for some waiting time \( t_w \) at \( A \). Then instead of an infinitely large domain of \( \Gamma^A \), there will be domains of \( \Gamma^A \) and \( \bar{\Gamma}^A \) of size \( L_A(t_w) \). In the following we consider the perturbation-healing protocol exerted onto this initial state.

The time evolution of the system in a cycling protocol can be concisely described by the time evolution of the ghost domains of \( \Gamma^A/\bar{\Gamma}^A \) and \( \Gamma^B/\bar{\Gamma}^B \). The situation may be visualized again simply by patches. A ghost domain \( \Gamma^A \) of size \( L_A(t_w) \) may
be decomposed into patches of size $\xi(\delta)$ during perturbation stage and patches of size $L_A(\tau_h)$ during healing stage. Within a patch the bias is always homogeneous and has the full amplitude 1. But the “signs” of the bias is different on different patches. The probability $p_{\text{minor}}$ (Eq. (11)) that a patch belong to the minority phase $\Gamma^A$ increases with time as Eq. (16) in the perturbation stage and decreases as Eq. (17) in the healing stage. Since the size of the ghost domain itself is finite, it also continues to grow during the healing stage. Following [12] we call the new domain growth under the background bias field during the healing as inner-coarsening and the further growth of the size of the ghost domain itself as outer-coarsening.

The crucial point is that the projection $\tilde{s}^A$ keeps the same long wavelength spatial structure of sign of the bias as the original domain structure just before the perturbation throughout the perturbation-healing stages. In the absence of such an explicit mechanism of a sort of symmetry breaking, the new domains grown during the healing would have completely wrong signs of bias and could lead to total erasure of the memory. (The scenario (A) mentioned in the introduction.) The latter was the main problem in the previous attempt to model multiple domains by Koper and Hilhorst [30] and many other popular “length scale(s)” arguments which neglect the role of the internal driving by the remanent bias.

5.1. Memory correlation function

The memory of the “state” of the system just before the perturbation (or the end of the initial aging stagey) can be directly quantified by the spin auto-correlation function as,

$$C_{\text{mem}}(\tau + t_w, t_w) = C(\tau + t_w, t_w).$$ (23)

which we call as memory correlation function. The hamming distance $d = 1 - C_{\text{mem}}$ gives a measure of the closeness in the phase space between the phase points at the two times. Note that in the limit $t_w \to \infty$ it reduces to the global bias $\rho^A$ discussed in section 4. It is useful to recall that the auto-correlation function is gauge invariant so that it is suitable for experiments/simulations of spin glass systems where one does not know a priori any equilibrium states below $T_g$.

**Strongly perturbed regime**- Let us first consider a cycling operated in the strongly perturbed regime. During the perturbation stage one can easily see $C_{\text{mem}}$ is identical to $\sqrt{q_{EA}(T_A)}\sqrt{q_{EA}(T_B)}C_0(\tau_p, 0)$ where $C_0(\tau_p, 0) = \rho^A_{\text{rem}}(\tau_p) = (LB(\tau_p)/\xi(\delta))^{-\lambda}$ (See Eq. (10)) During the healing stage the analytical result of a spherical Mattis model suggest the following factorization (See Eq. (109) of [12])

$$C_{\text{mem}})(\tau_h + \tau_p + t_w, t_w) = q_{EA}(T_A)\rho_{\text{rec}}^A(\tau_h; \rho_{\text{rem}}^A)C_0(\tau_h + t_w, t_w).$$ (24)

Here the factor $\rho_{\text{rec}}^A(\tau_h; \rho_{\text{rem}}^A)$ represents the growth of the bias within the ghost domains by the inner-coarsening (See Eq. (17)) and the factor $C_0(\tau_h + t_w, t_w)$ represents the outer-coarsening which is the auto-correlation function in isothermal aging (without perturbation $\tau_p = 0$). Thus the memory correlation function Eq. (24) behaves non-monotonically with time in the healing stage and exhibits a peculiar “memory peak”
because of the two competing factors: $\rho_{\text{rec}}(\tau_h)$ increases while $C_0(\tau_h + t_w, t_w)$ decreases with $\tau_h$.

In Figure 2 we show the memory auto-correlation function of the 4 dim EA model after a bond perturbation of strength $p = 0.2$. By an independent numerical study of a bond-shift simulation done in the same way as in a recent temperature-shift experiment [10, 25], we checked our time window lies almost entirely in the strongly perturbed regime with $p = 0.2$ as reported elsewhere [11]. In the scaling plot (b), the expected multiplicative recovery of bias (memory) (See Eq. (17)) is demonstrated.

In a previous study of this model [28] $\lambda \sim 3.0 - 3.5$ was found by analyzing the relaxation of thermo remanent magnetization (TRM). As shown in the scaling plot (b) the present result appear to be consistent with $\lambda \sim 0.8$ and $\lambda \sim 3.2$ (thus $\lambda/\lambda \sim 4$) being consistent with the scaling relation Eq. (13) and the inequality $\lambda/\lambda \geq 1$ Eq. (15). Indeed it can be seen that the recovery time at which the data merges with the reference data of $C_0(\tau_h + t_w, t_w)$ (thus $\rho^A \to 1$) is already as large as $O(10^4)$ (MCS) even with very short perturbation $\tau_p = 10$ (MCS).

The factorization in Eq. (24) strongly suggests independence of the evolution of the amplitude of the bias or the order parameter and size of the domain. Consequently somewhat surprisingly the above result suggest the memory peaks can be always identified no matter how long the perturbation is kept on. Note that nothing special happens when $L(\tau_p)$ exceeds $L(t_w)$. Only the amplitude of the signal will be smaller for longer perturbation so that higher resolution is required.

Rather amusingly the factorization Eq. (24) allows one to extract the growth of the amplitude of the bias $\rho$ even with no knowledge of the underlying equilibrium state $\Gamma/\bar{\Gamma}$ thanks to the gauge (ghost) invariance of the auto-correlation functions. Probably it is interesting to apply the same trick to other spin-glass models. In $d = 3$ Ising EA model the data reported in [32] on the relaxation of the auto-correlation function in isothermal aging suggest roughly $\lambda \sim 2$ and hence $\lambda/\lambda \sim 2$ assuming the scaling relation Eq. (13).

Weakly perturbed regime- Naturally the factorization of the time evolution of the bias and the size of the domains Eq. (24) should also hold in the weakly perturbed regime. In the 4 dim EA model we also performed bond cycling simulations operated in the weakly perturbed regime with very small $p$ such as $p = 0.02$. There we found the recovery of the bias is additive as suggested by Eq. (21) and the recovery time was found to be the trivial one $\tau_{\text{rec}} \sim \tau_p$ being consistent with Eq. (22).

5.2. Magnetic susceptibilities

In experiments AC/DC magnetic susceptibilities are often used to study dynamics of spin-glass materials. As noted in section 3.3 these are also essentially gauge invariant quantities. Here we discuss possible scaling properties of those in the healing stage.

The relaxation of the AC susceptibility of frequency $\omega$ can be considered as a probe of the increase of the effective stiffness of a droplet excitation of size $L_T(1/\omega)$ due to decrease of domain wall density [26]. The scaling ansatz which follows this picture has
been supported by recent numerical and experimental studies \[33, 34, 35, 28\]. In the healing stage, there must be excess contributions from the domain walls around the islands of the minority phase. A given spin can be surrounded by such a wall with probability \( p_{\text{minor}}(\tau_h, \tau_p) = (1 - \rho_{\text{rec}}(\tau_h, \rho_{\text{rem}}(\tau_p))) \) (See Eq. (5)) which increases with \( \tau_p \) and decreases with \( \tau_h \). Then a natural scaling is

\[
\chi''(\omega, \tau_h + \tau_p + t_w) = p_{\text{minor}}(\tau_h; \tau_p)\chi''(\omega, \tau_h) + \chi''(\omega, \tau_h + t_w) \quad \text{for} \quad p_{\text{minor}} \ll 1 \tag{25}
\]

Here \( \chi''(\omega, t) \) is the AC susceptibility of isothermal aging starting from random initial condition at \( t = 0 \). The 1st term is the excess response due to the minority phase. Since \( p_{\text{minor}}(\tau_h, \tau_p) \) decreases with time \( \tau_h \), the excess part slowly fades away. The 2nd term in the r.h.s. is due to the outer-coarsening which is just the AC susceptibility without perturbation. If the cycling is operated in the weakly perturbed regime, the excess part will fade away at the time scale \( \tau_{\text{weak}} \) given in Eq. (22) while it will take an extremely long time \( \tau_{\text{rec}} \) in the strongly perturbed regime. It is interesting to note that anomalously large recovery time which apparently exceeds the simple estimate Eq. (22), which is valid only in the perturbative regime, has been found in recent measurements of the AC susceptibility \[24, 11\].

In isothermal aging which start from a random initial condition at \( t = 0 \) the ZFC susceptibility \( M_{\text{ZFC}}(\tau = t - t_w) \) measured under a probing field switched on after waiting time \( t_w \) exhibits a rapid increase at around \( \tau \sim t_w \). The latter is reflected as a peak of the relaxation rate \( S(\tau) = dS(\tau)/d\log(\tau) \) at around \( \tau \sim t_w \). In the cycling operated in the strongly perturbed regime, it is very likely to happen that the population of the minority phase within the ghost domains \( p_{\text{minor}}(\tau_h) \) remains non-zero at the time scale \( \tau_h \sim t_w \). This may explain the substantial reduction of the amplitude of the memory peak of \( S(\tau_h) \) at around \( \tau_h \sim t_w \) in one-step temperature-cycling experiments operated in the strongly perturbed regime \[37, 11\].

6. Renormzalization of slow switching effects

So far we considered idealized situations that perturbations are switched on/off instantaneously which is not possible in reality. For example typically heating/cooling rates are \( v_T = 1 \) Kelvin/second in “quench”experiments \[9\] which is equivalent to \( v_T = 10^{-15} \) J/MCS in simulations (assuming \( T_g = 10 \) K and the microscopic time scale \( \tau_0 = 10^{-13} \) sec). The surprising weakness of heating/cooling rate effect in spin-glasses \[9\] already suggests relevance of the chaos effects.

Let us illustrate here some important consequences of such a slow switching by considering a continuous bond change protocol as an example. Suppose that the signs of a fraction \( p \) of \( \pm J \) bonds in a temporally set \( J(t) \) are changed randomly in a unit time \( \tau_0 \). After some transients the system should become stationary by the chaos effect such that the size of the ghost domains of \( J(t) \) becomes constant in time \( L_{v_J} \) which decreases by increasing the bond change rate \( v_J = pJ/\tau_0 \). Then we can consider for example a one-step bond cycling \( J_A(t_w) \to J_B(\tau_p) \to J_A(\tau_h) \) with such a gradual bond changes. The
point is that at length scales smaller than $L_{\nu_J}$ the whole cycling process just amounts to successive operations in the weakly perturbed regime. There the multiplicative effects Eq. (19) are avoided as explained in section 3.1. Then the cycling can be coarse-grained by taking $L_{\nu_J}$ as the new microscopic length scale instead of the overlap length $\xi(\delta)$ between $A$ and $B$ which yields a coarse-grained cycling $J_A \rightarrow J_B \rightarrow J_A$ operated in a strongly perturbed regime with instantaneous bond changes. The scaling properties of the strongly perturbed regime will hold for the latter but the original overlap length $\xi(\delta)$ should be replaced by the renormalized overlap length $L_{\nu_J}$, for example in Eq. (18), which leads to a certain “rounding” of the strong chaos effects in realistic circumstances.

Although the temperature dependence of the growth law Eq. (17) induce some obvious complications, essentially the same argument for the case of continuous temperature changes leads to a corresponding renormalized overlap length $L_{\nu_T}$ which decreases with increasing heating/cooling rate $v_T$.

7. Conclusion

To summarize we studied how spin-glasses heal after being exposed to strong perturbations which induce the chaos effects in simple perturbation-healing protocols (e.g. one-step temperature-cycling). The bias or the order parameter within the ghost domains decays as $L^{-\bar{\lambda}}$ in the perturbation stage and increases as $L^{\lambda}$ in the healing stage with increasing dynamical length scales $L$. The inequality of the exponents $\bar{\lambda} \geq \lambda$ immediately suggests anomalously large recovery times of the order parameter. The memory auto-correlation function is suited for direct examination of the time evolution of the order parameter. It should exhibit the memory peak in the healing stage at the time scale of the “age” imprinted in the ghost domains due to the parallel evolution of the order parameter (inner-coarsening) and the size of the ghost domains (outer-coarsening). The predictions were checked quantitatively by numerical simulations in the 4 dim EA model. It should be very interesting to measure experimentally the memory auto-correlation function by the noise-measurement technique [27] for example in the standard one-step temperature-cycling protocol. Extensive experimental and numerical investigations which examine the ghost domain scenario in other observables such as the AC/DC magnetic susceptibilities will be reported elsewhere [11]. Important features of the weakly perturbed regime of the chaos effect were also discussed which leads to a proposal to take into account the effect of finiteness of switching on/off the perturbations in experimental circumstances by the renormalized overlap length.

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Figure 2. Memory correlation function of the 4 dim EA model. The perturbation is a bond perturbation of strength \( p = 0.2 \). The temperature is \( T = 1.2 \) (\( T_g = 2.0 \)). The system size is \( N = 24^4 \) which is large enough to avoid finite size effects within the present time window [28]. The initial waiting time is fixed as \( t_w = 10^4 \) (MCS). (a) The data points labeled “perturbation stage” is \( C_{\text{mem}}(\tau_p + t_w, t_w) \). Other data points are those in the healing stage \( C_{\text{mem}}(\tau_h + \tau_p + t_w, t_w) \) after various duration of the perturbation \( \tau_p = 10, 10^2, 10^3, 10^4, 10^5 \) (MCS) from the top to the bottom. Note that the last one is even larger than the initial waiting time \( t_w \). The solid line is the reference curve of \( C_0(\tau_h + t_w, t_w) \) obtained by a simulation of isothermal aging with \( t_w = 10^4 \) (MCS). (b) The memory correlation functions scaled by the remanent bias \( \rho_{\text{rem}}(\tau_p) \) are shown. The latter is directly read off from the data in the perturbation stage (shown in (a)) as \( \rho_{\text{rem}}(\tau_p) = C_{\text{mem}}(\tau_p + t_w, t_w) \). Here the dynamical length \( L_{T=1.2}(t) \) is used which has been obtained in a previous study of the same model [31][28]. The straight lines are the power laws \( x^\lambda \) and \( x^{-\bar{\lambda}} \) with \( \lambda = 0.8 \) and \( \bar{\lambda} = 3.2 \).