1. Introduction

Nowadays, a numbers of available oil reserves are becoming lesser and lesser. Therefore, it is difficult to imagine a modern process of oil production without methods of increasing oil recovery in the reservoirs. So far, the most effective method to overcome this problem is the thermal method. However, the possibilities of this method are limited by its high energy consumption and the cost of relevant practice research. Thus, less expensive corresponding methods of mathematical modeling become more important. This investigation uses a combined finite-element-difference method for the non-stationary thermal conductivity problem. Numerical modeling of the temperature distribution around heat injection wells are carried out, taking into account the heterogeneity of the thermal properties of the oil reservoir and the conditions of convective heat exchange at the reservoir’s boundaries. The proposed method, due to its high accuracy and convergence of the solutions, allows to obtain reliable practical results and has a number of advantages in comparison with the same research methods. It is established that the process of heating of oil reservoirs is slow and energy consuming, so to increase profitability, it is obviously necessary to use associated production products, such as associated gas. It is shown that less wet layers heat up better and there is no sense to heat the layer for more than two weeks, because the radius of the effective heating area (with a temperature exceeding 80 °C required for outcome of high-viscosity oil from the rock) in this case is sufficient. It is also found that the operation of heat-injection wells is more profitable with their joint interaction, in that case the effective heating area of the oil reservoir and, accordingly, the number of production wells will be the largest. Another hand, the main factor in the location of heat-injection wells is defined by special characteristics of the oil-bearing section of the reservoir in each case. The configurations of the location of heat-injection wells, which were presented in this paper, cover the most optimal cases of the installations of considered oil-bearing section of the reservoirs and can be used in practice.

Keywords: finite element difference method, computer modeling, heating processes, hard-to-reach oil reservoirs, heat-injection wells.

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MODELING OF THE RESERVOIRS HEATING WITH THE AIM OF OIL RECOVERY INCREASING

The object of the research is optimal installation of the heat injection wells for reservoirs heating in order to increase oil recovery and, accordingly, support oil production in the hard-reaching heterogeneous reservoirs. One of the most problematic areas in modern oil production is the difficulty of extracting high-viscosity oil from the reservoirs. So far, the most effective method to overcome this problem is the thermal method. However, the possibilities of this method are limited by its high energy consumption and the cost of relevant practice research. Thus, less expensive corresponding methods of mathematical modeling become more important. This investigation uses a combined finite-element-difference method for the non-stationary thermal conductivity problem. Numerical modeling of the temperature distribution around heat injection wells are carried out, taking into account the heterogeneity of the thermal properties of the oil reservoir and the conditions of convective heat exchange at the reservoir’s boundaries. The proposed method, due to its high accuracy and convergence of the solutions, allows to obtain reliable practical results and has a number of advantages in comparison with the same research methods. It is established that the process of heating of oil reservoirs is slow and energy consuming, so to increase profitability, it is obviously necessary to use associated production products, such as associated gas. It is shown that less wet layers heat up better and there is no sense to heat the layer for more than two weeks, because the radius of the effective heating area (with a temperature exceeding 80 °C required for outcome of high-viscosity oil from the rock) in this case is sufficient. It is also found that the operation of heat-injection wells is more profitable with their joint interaction, in that case the effective heating area of the oil reservoir and, accordingly, the number of production wells will be the largest. Another hand, the main factor in the location of heat-injection wells is defined by special characteristics of the oil-bearing section of the reservoir in each case. The configurations of the location of heat-injection wells, which were presented in this paper, cover the most optimal cases of the installations of considered oil-bearing section of the reservoirs and can be used in practice.

Keywords: finite element difference method, computer modeling, heating processes, hard-to-reach oil reservoirs, heat-injection wells.
distribution in real complex oil production conditions, so it has a number of advantages among the same methods.

Thus, the object of research is optimal installation of the heat injection wells for reservoirs heating in order to increase oil recovery and, accordingly, support oil production in the hard-reaching heterogeneous reservoirs.

The aim of research is mathematical modeling of the reservoir heating process in order to optimize oil recovery and, respectively, improvement of oil production in the hard-reaching heterogeneous reservoirs.

2. Research methodology

Further let’s consider productive oil reservoirs with a thickness not lesser than a few meters, assuming that the average thickness of the reservoir is much smaller than horizontal dimensions of the considered area. Then let’s assume that the heat does not spread to the roof of the reservoir and use a two-dimensional non-stationary model of thermal conductivity [9, 10]. In this case, the general formulation of the nonstationary thermal conductivity problem, taking into account the heat exchange at the reservoir boundaries, in the Cartesian coordinate system (x, y) associated with the reservoir, has a view [10]:

\[
\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{q}{c\rho},
\]

\[T(t=0) = T_0;\]

\[\lambda \text{grad} T = \alpha (T - T_b).\]

where (1) – the equation of thermal conductivity; (2) – initial condition; (3) – boundary condition of the heat exchange with the environment; \(T(x, y, t)\) – temperature as a function of coordinates and time; \(a\) – thermal conductivity coefficient; \(q\) – power of the heat source; \(c\) – volumetric heat capacity; \(T_0\) – initial reservoir temperature; \(\lambda\) – coefficient of thermal conductivity of the reservoir; \(\alpha\) – heat transfer coefficient at the boundary of the reservoir; \(T_b\) – temperature at the boundary of the reservoir; \(\rho\) – reservoir’s rock density.

For solving the nonstationary problem of thermal conductivity (1)-(3) let’s use the variational finite-element method, which leads to the solution of the variational equation of thermal conductivity:

\[\delta I(T) = 0,\]

where \(I(T)\) – functional of the thermal conductivity problem (1)-(3), which can be presented in the form [10]:

\[I(T) = \frac{1}{2} \int \left[ \lambda \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{1}{c\rho} \int \frac{\partial T}{\partial t} \left( 2T - 2qT \right) dx dy - \frac{1}{2} \int \alpha (T - 2T_b) T dl,
\]

where \(S\) – cross-sectional area of the study area; \(L\) – contour of the area; \(dl\) – contour’s element.

For solving the variational equation (4), let’s use an eight-node isoparametric quadrilateral finite element [10].

As a global coordinate system, where all finite elements, which divide area \(S\), are united, let’s use the Cartesian system (x, y) as a local coordinate system, where the approximation functions \(\Phi_i\) are defined within every finite element and numerical integration takes place, let’s use a normalized coordinate system \((\xi, \eta)\) [10]. In this system, coordinates, temperature, initial temperature, surrounding temperature, heat transfer coefficient of the environment, and derivatives of the temperature over coordinates are approximated on the base of quadratic polynomials in such way:

\[
\frac{\partial T}{\partial x} = \sum_{i=1}^{8} T_i \frac{\partial \Phi_i}{\partial x}; \quad \frac{\partial T}{\partial y} = \sum_{i=1}^{8} T_i \frac{\partial \Phi_i}{\partial y};
\]

\[
\Psi_i = \frac{1}{k} \int \left( \frac{\partial \rho}{\partial T} \frac{\partial \rho}{\partial T} \frac{\partial \phi_i}{\partial T} \frac{\partial \phi_i}{\partial T} \right) dS;
\]

\[
\Phi_i = \frac{1}{k} \int \left( \frac{\partial \rho}{\partial T} \frac{\partial \rho}{\partial T} \frac{\partial \phi_i}{\partial T} \frac{\partial \phi_i}{\partial T} \right) dS.
\]

where

\[
J = \frac{\partial \rho}{\partial T} \frac{\partial \rho}{\partial T} \frac{\partial \phi_i}{\partial T} \frac{\partial \phi_i}{\partial T}.
\]

– Jacobian transition between systems \((x, y)\) and \((\xi, \eta)\).

Using the variational equation (4) and assuming that the nodal values of the temperature derivatives over time \(dT/dt\) are known values and can’t be varied, it is possible to obtain a system of differential equations for the \(k\)-th node of the \(p\)-th finite element in the form:

\[
x = \sum_{i=1}^{8} x_i \phi_i; \quad y = \sum_{i=1}^{8} y_i \phi_i; \quad T = \sum_{i=1}^{8} T_i \phi_i;
\]

\[T_0 = \sum_{i=1}^{8} T_0 i \phi_i; \quad T = \sum_{i=1}^{8} T_i \phi_i; \quad \alpha = \sum_{i=1}^{8} \alpha_i \phi_i;\]

\[
\phi_i = \frac{1}{4} (1 - \xi^2) (1 - \eta^2) (1 - \xi - \eta - 1);
\]

\[
\phi_i = \frac{1}{4} (1 + \xi) (1 - \eta) (1 + \xi - \eta - 1);
\]

\[
\phi_i = \frac{1}{4} (1 + \xi) (1 + \eta) (1 + \xi + \eta - 1);
\]

\[
\phi_i = \frac{1}{4} (1 - \xi) (1 + \eta) (1 - \xi + \eta - 1);
\]

\[
\phi_i = \frac{1}{2} (1 - \xi^2) (1 - \eta); \quad \phi_i = \frac{1}{2} (1 - \eta^2) (1 - \xi);
\]

\[
\phi_i = \frac{1}{2} (1 - \xi^2) (1 + \eta); \quad \phi_i = \frac{1}{2} (1 - \xi) (1 - \eta^2);
\]

\[
\frac{\partial T}{\partial t} = \sum_{i=1}^{8} \left[ H_i^T \frac{dT}{dx} \right] + \left( P_i^T + Q_i^T \right) T - Q_i^T = 0;
\]

\[
H_i^T = \int \left[ c \rho \phi_i \right] [dS];
\]

\[
P_i^T = \int \left[ a \rho \phi_i \right] [dl];
\]

\[
Q_i^T = \int \left[ a \rho \phi_i \right] [dl];
\]

To resolve the system of linear differential equations of the first order (7) with the initial conditions of (6),
let’s use the finite difference method, in which the time derivative is approximated on the basis of an implicit difference scheme:

\[
\frac{dT}{dt} = \frac{T(t + \Delta t) - T(t)}{\Delta t}.
\]  

(8)

Substituting expression (8) into system (7), let’s obtain the following system of linear algebraic equations:

\[
0 = \begin{pmatrix}
s_i & 1 & H_k & P_n & Q_k
\end{pmatrix} \begin{pmatrix}
1 & H_k & T_n(t) - Q_k T_0
\end{pmatrix} + q_t^k = 0 \ (k = 1 - 8),
\]

(9)

By adding equations (9) to all finite elements, let’s obtain a global system of linear algebraic equations that allows to determine the unknown values of temperature at the time \(t + \Delta t\) via their value at the previous moment of time \(t\). The resolving of the global system of equations is based on the numerical Gaussian method without choosing the main element [10]. As a result of the resolving, the temperature can be determined at all nodes of the finite element grid. Due to the found nodal values, the temperature can be determined at any points of the reservoir at any moments of time.

3. Research results and discussion

Let’s consider medium and weakly wet oil-bearing reservoir areas with sizes of 90×90 m². Let’s choose the average thermal parameters for medium and weakly wet oil reservoirs, respectively [1, 2, 10]: thermal conductivity – 3.0, 5.0 W/m·deg; volumetric heat capacity – 2.0, 1.0·10⁶ J/m³·deg. Let’s suggest in modelling the initial temperature in the reservoir equals 50 °C. Let’s also assume that the average power of the injection well is 2 kW, which roughly corresponds to the injection of water with a temperature of 200 °C. To minimize the edge effects in the modelling let’s choose at the boundaries of the considered area the heat transfer coefficients equals 0.01 W/m²·deg. The modelling results show that the process of heating of the oil reservoir is quite slow and energy consuming.

Fig. 1 presents the process of heating of homogeneous weakly wet oil reservoir with a heat source of 2 kW during for 1, 5, 15, 30 days. It is possible to see that over one day the effective heating area (with a temperature exceeding 80 °C) in the vicinity of the heat injection well has a radius of about 2 m, after 5 days – this radius increases to 3 m, after 15 days – this radius is 7 m, after 30 days – it grows up to 15 m. Fig. 2 shows the process of heating a homogeneous medium-humidity oil reservoir with a similar source for the same periods of time. Over day, the effective heating area does not exceed 1 m, after 5 days – 2 m, after 15 days – 5 m, after 30 days – 10 m. So, it is possible to see that reservoir heating process is very energy-consuming and for profitability increasing it is necessary to use concomitant products, for example, concomitant gas. Also, it is possible to see that lesser wet layers are better heating. Another hand, it’s clear from economic reasons that there is no need to heat the reservoir more than two weeks. Further, let’s consider the heating of a homogeneous medium-humidity oil reservoir by systems of heat-injection wells during for 15 days. Fig. 3 presents the heating process in reservoir around several heat-injection wells. Analysis of the Fig. 3 shows that in the case of two heat-injection wells installation at a distance of 20 m (Fig. 3, a) there is a more elongated effective heating area in comparison with Fig. 3, b (wells located at a distance of 10 m), but in the latter case there is a more concentrated-symmetric heating of the local section of the reservoir. Location case (Fig. 3, c) – more effective for heating of compact area, location (Fig. 3, d) – can be useful for heating an elongated area. Obviously, for the largest number of oil production wells installation, the configuration in Fig. 3, c is more favourable. Fig. 4 presents the heating of the reservoir place by the system of heat-injection wells. The least profitable combination here is (Fig. 4, a), because in this case two strips of located heat-injection wells don’t interact one with other and so heat a smaller area of the reservoir. As for the combinations (Fig. 4, b–d), they are all effective for large heating area and, accordingly, for the largest number of located production wells and can be used in a respective practical cases.

The simulation results show that the process of heating of the oil reservoir is quite slow and energy-consuming, so it requires careful study. In Fig. 1 it is possible to detect the degree of intensity of the thermal process around the heat-injection well in a weakly wet oil reservoir during for 1, 5, 15, 30 days, respectively. In Fig. 2 presented the same thermal processes in a moderately wet oil reservoir. As it is possible to see, the heating of the oil reservoir is quite expensive and for increasing profitability, it’s necessary to use associated oil production products, for example, such as associated gas.

**Fig. 1.** Temperature distribution near heat-injection well with power of 2 kW in the weak wet homogeneous oil reservoir over different time intervals: a – 1; b – 5; c – 15; d – 30 days
Fig. 2. Temperature distribution near heat-injection well with power of 2 kW in the middling wet homogeneous oil reservoir over different time intervals: a – 1; b – 5; c – 15; d – 30 days

Fig. 3. Temperature distribution near a few heat-injection wells with power of 2 kW installed in the middling wet homogeneous oil reservoir over 15 days: a – 2 wells (w) over 10 m; b – 2 w – 20 m; c – 4 w – 20 m compact; d – 4 w – 20 m strip
It is also seen that less wet reservoir layers are better heated. Another hand, it’s clear that no sense to heat the layer for more than two weeks, because increasing of the effective radius in this case is not essential. Fig. 3 presents the heating process in reservoir around several heat-injection wells. Analysis of the Fig. 3 shows that in the case of two heat-injection wells placing at a distance of 20 m (Fig. 3, a) there is a more elongated effective heating area in comparison with Fig. 3, b (wells located at a distance of 10 m), but in the latter case there is a more concentrated-symmetric heating of the local section of the reservoir. Location case (Fig. 3, c) – more effective for heating of compact area, location (Fig. 3, d) – can be useful for heating an elongated area. Obviously, for the largest number of oil production wells installation, the configuration (Fig. 3, c) is more favourable. Fig. 4 presents the heating of the reservoir place by the system of heat-injection wells. The least profitable combination here is (Fig. 4, a), because in this case two strips of located heat-injection wells don’t interact one with other and so they heat a smallest area of the reservoir. As for the combinations (Fig. 4, b–d), they are all effective for large heating area and, respectively, for largest number of located production wells and can be used in a respective practical cases.

Fig. 4. Temperature distribution near the systems of heat-injection wells of power 2 kW installed in the middling wet homogeneous oil reservoir over 15 days: a – two strips; b – two close strips; c – star; d – two triangles system.
Thus, it is possible to make the general conclusions. The operation of heat-injection wells is more profitable with their joint interaction, in this case the effective heating area of the oil reservoir and, accordingly, the number of production wells will be the largest. Another hand, the main factor for optimal location of the heat-injection wells is the calculation of specific characteristics of the oil reservoir in each practical case. The configurations of the location of heat-injection wells presented in this paper demonstrate the most effective cases of the hard-reaching oil reservoirs heating for improvement of the oil recovery and can be used in oil production practice.

The limitations of this study are productive oil reservoirs with a thickness not less than a few meters are considered, assuming that the average thickness of the reservoir is much smaller than horizontal dimensions of the considered area. And then it is assumed that the heat does not spread to the roof of the reservoir and used a two-dimensional non-stationary model of thermal conductivity.

In further it is possible to develop this method for investigation of heating processes near heat-injection wells in the hard-reaching oil reservoirs with heterogeneous and anisotropic heating properties.

4. Conclusions

The developed finite-element-difference method for solving the nonstationary problem of thermal conductivity in heterogeneous oil reservoirs allows adequately on a quantitative level to describe the temperature distribution around heat-injection wells in real oil production conditions for improvement of the oil recovery. The simulation results show that process of heating of the oil reservoirs is quite slow and energy-consuming and requires careful study. Thus, to increase profitability, it is necessary to use associated oil production products, such as associated gas. Also, it is possible to see that less wet reservoir layers are better heated. Another hand, it’s clear that no sense to heat the reservoir for more than two weeks, because increasing of the effective region radius (with a temperature above 80 °C required for exit of high-viscosity oil from the rock) in this case is not essential. Therefore, it is possible to with their joint interaction, in this case the effective heating area of the oil reservoir and, accordingly, the number of production wells will be the largest. Another hand, for optimal location of the heat-injection wells it’s necessary to calculate the specific characteristics of the oil reservoir in each practical case. The configurations of the location of heat-injection wells, which presented in this paper, demonstrate the effective cases of the hard-reaching oil reservoirs heating for improvement of the oil recovery. So, this information can be used in hard reaching oil reservoir production practice with the purpose of increasing of the oil recovery.

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