Stability Threshold of Herringbone Grooved Aerodynamic Journal Bearings with External Stiffness and Damping Elements*

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Abstract

Herringbone grooved aerodynamic journal bearings have better stability characteristics than plane aerodynamic journal bearings. In addition, the use of flexibly supported bearings is one of the efficient way to improve the bearing stability. Therefore, the combination bearing systems have prospects of realization of ultra-high-speed operations. The purpose of this work is to evaluate the effect of external stiffness and damping elements on the bearing stability. The Voigt rheological model was used for the external elements, in which the stiffness and the damping were treated as the linear elements. It was clarified that the stability boundary of the herringbone grooved aerodynamic journal bearings supported by suitable stiffness and damping elements can be heightened more than three times compared with the rigidly supported bearings.

Key words: External Stiffness and Damping Elements, Stability Threshold, Herringbone Bearing, Self-Excited Vibration, Lubrication, Tribology

1. Introduction

Recently, ultracompact turbo machineries such as palm-top sized generators are focused. It is necessary for those machineries to develop bearing systems that can stably support a shaft rotating at ultra-high-speed rotational regions. Typical bearings widely discussed in this research field are air bearings with low friction. Ehrich et al. (1) and Hikichi et al. (2) achieved the rotational speed of over 1,000,000 min\(^{-1}\) by using aerostatic air bearings. However, the aerostatic bearings used in those researches must be operated with large ancillary equipment. Therefore, it seems that aerodynamic bearings are better for decreasing sizes due to their simple structures.

Therefore, the authors (3)(4) studied the stability of herringbone grooved aerodynamic journal bearings and indicated that the threshold of instability was significantly increased by installing rubber O-rings. As the results, a rotor was operated over 530,000 min\(^{-1}\) without the unstable phenomena. However, these researches are limited to experimental approaches. To grasp more concrete design guides, it is important to theoretically investigate the advantage of the viscoelastic supports for the bearing stability.

In generally, aerodynamic journal bearings induce self-excited vibrations at high-speed rotational regions. Therefore, to push the stability up to ultra-high-speed rotation, it is important to appropriately design the bearing support elements. Although the stabilization method using such external support elements are widely performed in practical levels, theoretical investigations for the bearing systems are not satisfied (5). Therefore, it is...
necessary for further high speed operation to study both theoretically and experimentally the stability of herringbone grooved aerodynamic journal bearings with the external stiffness and damping elements.

Tatara et al. (6) studied the stability of an aerostatic journal bearing with O-ring supports by the film coefficients under the concentric and nonrotating conditions, and showed some design guides. Kazimierski et al. (7) studied the effect of supply pressure on an aerostatic bearing system with O-ring supports. However, since these researches were only on the aerostatic bearings, their results cannot be directly used for aerodynamic bearing systems.

On the other hand, Lund (8) theoretically investigated the effect of external support elements on the stability of a plane aerodynamic bearing. In this study, he assumed that the mass of the bearing sleeve could be neglected compared with that of the shaft. However, this assumption was not appropriate. Kogure et al. (9) analyzed the stability of hydrodynamic journal bearings with flexible supports and showed the effect of the stiffness ratios of the rotor and the support element on the stability chart, but they limited to the approximations by the short bearing theory.

As far as the authors know, there is no research about herringbone grooved aerodynamic journal bearings with the external stiffness and damping elements. Therefore, the effect of the elements on their bearing stability has not been fully understood. For achieving ultra-high-speed rotations by the bearing system, it is important to investigate what combinations of stiffness and damping coefficients increase the threshold of instability.

In this paper, it is theoretically studied how the stability of the bearing system change with different combinations of the stiffness and damping coefficients, applying a viscoelastic rheological model.

2. Nomenclatures

The nomenclatures used in this paper are as follows. The variables with subscripts ^ mean dimensional quantities while those without ^ mean non-dimensional quantities.

- \( b_h \) : external damping coefficient = \( \hat{C}_{b/h} \hat{R} \)
- \( b_i \) : damping coefficients of air film = \( \hat{C}_{b/i} \hat{D} \)
- \( \hat{C}_{r/o} \) : radial clearance at nonrotating
- \( \hat{D} \) : shaft diameter = \( 2 \hat{R} \)
- \( \hat{\rho} \) : eccentricity
- \( F \) : Internal force of shaft = \( \hat{F}/(\hat{p}_a \hat{D} \hat{L}) \)
- \( h \) : film thickness of air film = \( \hat{h}/\hat{C}_{r/o} \)
- \( H_{o} \) : initial groove depth ratio = \( \hat{H}_{o}/\hat{C}_{r/o} \)
- \( k_s \) : external stiffness coefficient = \( \hat{C}_{k/s} \hat{R} \)
- \( k_i \) : stiffness coefficients of air film = \( \hat{C}_{k/i} \hat{D} \)
- \( \hat{L} \) : bearing length
- \( \hat{m}_b \) : mass of bearing
- \( \hat{m}_s \) : mass of shaft
- \( m \) : mass ratio = \( \hat{m}_b/\hat{m}_s \)
- \( M \) : dimensionless mass of shaft = \( \hat{m}_s/\hat{C}_{r/o} \)
- \( \hat{p} \) : film pressure
- \( \hat{p}_a \) : atmospheric pressure
- \( \hat{R} \) : radius of shaft
- \( W \) : film force = \( \hat{W}/(\hat{p}_a \hat{D} \hat{L}) \)
- \( X, Y \) : coordinates in rotor = \( \hat{x}/\hat{C}_{r/o}, \hat{y}/\hat{C}_{r/o} \)
- \( X_k, Y_k \) : coordinates in bearing = \( \hat{x}_k/\hat{C}_{r/o}, \hat{y}_k/\hat{C}_{r/o} \)
- \( z \) : coordinate in z-direction = \( 2\hat{z}/\hat{L} \)
3. Theory

3.1 Analytical model

Figure 1 shows the Voigt rheological model \(^{(10)}\) used in this study as spring and damping elements, in which a spring and a dashpot are connected in parallel. Figure 2 shows the analytical model. The spring and damping coefficients are assumed to be linear within the range of a small displacement and same in the horizontal and vertical directions.

The herringbone grooves are in the inner surface of the bearing sleeves and symmetrical to the center of the axial direction. The surface of the rotational shaft is smooth. In addition, the shaft stiffness is expressed as \(k_s\).

\[ \alpha : \text{groove width ratio} = \frac{\hat{a}_s}{(\hat{a}_s + \hat{a}_b)} \]
\[ \beta : \text{groove angle} \]
\[ I' : \text{expansion ratio} = \frac{\Delta R}{\hat{C}_{ro}} \]
\[ \delta : \text{groove depth} \]
\[ \epsilon : \text{eccentricity ratio} = \frac{\hat{e}}{\hat{C}_{ro}} \]
\[ \zeta_b : \text{damping ratio} = \frac{\hat{b}_b}{\sqrt{m_b k_b}} \]
\[ \vartheta : \text{coordinate in circumferential direction} = \frac{\hat{\vartheta}}{\hat{R}} \]
\[ \lambda : \text{length to diameter ratio} = \frac{L}{\hat{D}} \]
\[ \Lambda : \text{bearing number} = \frac{6 \mu \hat{\delta} \hat{R}^2}{(\hat{p}_f \hat{C}_{ro}^{1.64})} \]
\[ \mu : \text{viscosity of air film} \]
\[ \nu : \text{frequency ratio} = \frac{\hat{\nu}}{\hat{\omega}} \]
\[ \xi, \eta : \text{coordinates in journal} = \frac{\hat{x}}{\hat{C}_{ro}}, \frac{\hat{y}}{\hat{C}_{ro}} \]
\[ \tau : \text{dimensionless time} = \frac{\hat{\tau}}{\hat{\omega}} \]
\[ \Psi : \text{expansion coefficient} = (1 - \nu_p) \frac{\hat{\rho}_b}{\hat{C}_{ro}^{0.44}} \frac{\hat{\mu} \hat{\omega} \hat{R}}{144} \]
\[ \Omega : \text{stability parameter} = \frac{\hat{m} \hat{p}_f \hat{C}_{ro}^{1.64}}{\hat{\mu}^2 \hat{R}^5} \equiv \frac{72 M}{N^2} \]

subscripts

\(r\) : ridge part of herringbone grooves
\(g\) : groove part of herringbone grooves

![Fig.1 Dynamic model for external spring and dashpot](image1)

![Fig.2 Analysis model](image2)
3.2 Lubrication equation and analytical method

Equation (1) shows the compressible lubrication equation based on the narrow groove theory by Vohr et al. (11). In this equation both the perfect gas and the isothermal variation in the lubrication film are assumed.

\[
\frac{\partial}{\partial \theta} \left[ \left( \frac{E_s}{E_0} \right)^2 \frac{\partial E_s}{\partial \theta} + \frac{1}{\lambda} \left( \frac{E_s}{E_0} \right)^2 \frac{\partial^2 E_s}{\partial \theta^2} - \left( \frac{E_s}{E_0} \right)^2 \frac{\partial p}{\partial \theta} + 2 \frac{AE_s}{p} \right] \\
+ \frac{1}{\lambda} \frac{\partial}{\partial \theta} \left[ \left( \frac{E_s}{E_0} \right)^2 \frac{\partial E_s}{\partial \theta} + \frac{1}{\lambda} \left( \frac{E_s}{E_0} \right)^2 \frac{\partial^2 E_s}{\partial \theta^2} - \left( \frac{E_s}{E_0} \right)^2 \frac{\partial p}{\partial \theta} \right] \\
- 2 \lambda \left( \frac{\partial}{\partial \theta} \frac{\partial \phi}{\partial \theta} + \frac{\partial}{\partial \theta} \frac{\partial \phi}{\partial \theta} \right) p E_s = 0
\]

(1)

where \( E_0 \sim E_s \) are expressed as follows:

\[ E_0 = a h_r^3 + (1-\alpha) h_g^3 \]
\[ E_1 = h_r^3 h_g^3 + \alpha(1-\alpha)(h_g^3 - h_r^3)^2 \cos^2 \beta \]
\[ E_2 = \alpha(1-\alpha)(h_g^3 - h_r^3)^2 \cos \beta \sin \beta \]
\[ E_3 = a h_r^3 (h_g - h_r) + \alpha(1-\alpha)(h_g^3 - h_r^3)(h_g - h_r) \cos^2 \beta \]
\[ E_4 = \alpha(h_g - h_r) \]
\[ E_5 = \alpha(1-\alpha)(h_g^3 - h_r^3)^2 \cos \beta \sin \beta \]
\[ E_6 = h_r^3 h_g^3 + \alpha(1-\alpha)(h_g^3 - h_r^3)^2 \sin^2 \beta \]
\[ E_7 = \alpha(1-\alpha)(h_g^3 - h_r^3)(h_g - h_r) \cos \beta \sin \beta \]
\[ E_8 = a h_g^3 + (1-\alpha) h_r \]

In this study, the reduction of film thickness due to the centrifugal growth of shaft is considered. Defining the expansion ratio of shaft as \( \Gamma = \frac{\Delta \tilde{R}}{\tilde{C}_R} \), the film thickness in the ridge and groove parts are expressed as Eqs. (2) and (3), respectively:

\[ h_r = 1 - \Gamma + \epsilon \cos \theta \]  
(2)
\[ h_g = H_0 - \Gamma + \epsilon \cos \theta \]  
(3)

The expansion ratio of shaft \( \Gamma \) is:

\[ \Gamma = \Psi^2 \]  
(4)

where the expansion coefficient \( \Psi \) is given as follows:

\[ \Psi = (1-v_p) \rho a^2 C_{R}^3 \mu E \mu^2 R \]  
(5)

In Eq.(5), \( E \) is the Young’s modulus of shaft (72 GPa), \( v_p \) is the poison ratio of shaft (0.34), \( \rho \) is the density of shaft (2700 kg/m³), \( \mu \) is the viscosity of air film (1.8 x10⁻⁵ Pa·s), and \( R \) is the radius of bearing (0.003 m).

When the film pressure shown in Eq.(1) can be expressed with the sum of the static pressure \( (p_0) \) and the dynamic pressure, the Eq.(6) is given.

\[ p = p_0 + p_{c0} \Delta \varepsilon + p_{c1} \Delta \dot{\varepsilon} + p_{c2} \Delta \ddot{\varepsilon} + \cdots + p_{p0} \Delta \phi + p_{p1} \Delta \dot{\phi} + p_{p2} \Delta \ddot{\phi} + \cdots \]  
(6)

It is not easy to obtain the pressure \( p \) with calculating all terms of Eq.(6). Therefore, this study supposes that the journal has a small sine vibration on the stability boundary. In this case, the small changes of the eccentricity ratio and the attitude angle are given as follows:

\[ \Delta \varepsilon = \varepsilon e^{i(\nu \tau + \phi_\tau)} \]
\[ \Delta \phi = \phi e^{i(\mu \tau + \phi_\mu)} \]  
(7)

Then,
Substituting Eqs. (7), (8) and (9) into Eq. (6), the film pressure $p$ is expressed as Eq. (10).

$$p = p_0 + p_1 \Delta \varepsilon + p_2 \Delta \varepsilon^2 + p_3 \Delta \phi + p_4 \Delta \phi^2$$

The boundary conditions for the pressure are:

$$p_0(\theta, -1) = 1$$
$$p_{0,-1}(\theta, -1) = 0$$
$$p_{0,-1}(\theta, z) = p_{0,1}(\theta + 2\pi, z)$$
$$\frac{\partial p_{0,-1}}{\partial \theta} = \frac{\partial p_{0,1}}{\partial \theta}$$
$$\alpha(M_z) + (1 - \alpha(M_z)) = 0 \quad \text{at} \quad z = 0$$

where $M_z$ is the mass flow in the axial direction. The load capacity, the attitude angle and the air film coefficients are calculated by integrating the static and dynamic pressures with respect to the bearing surface.

### 3.3 Equations of motion

Let us consider the equations of motion for the bearing system shown in Fig. 2. Considering the force balances at the equilibrium positions of rotor, shaft, journal and bearing, as shown in Fig. 3, the equations of motion are given as follows:

$$M\ddot{X} = -F_x$$
$$M\ddot{Y} = -F_y$$
$$F_x = k_x(X - \xi)$$
$$F_y = k_y(Y - \eta)$$
$$F_{\xi} = W_{\xi}$$
$$F_{\eta} = W_{\eta}$$
$$mM\ddot{\xi}_b + (Ah_b/12)\ddot{X}_b + k_bX_b = W_{\xi}$$
$$mM\ddot{\eta}_b + (Ah_b/12)\ddot{Y}_b + k_bY_b = W_{\eta}$$
$$W_{\xi} = k_1(\xi - X_b) + k_2(\eta - Y_b) + b_1(\ddot{\xi} - \ddot{X}_b) + b_2(\ddot{\eta} - \ddot{Y}_b)$$
$$W_{\eta} = k_3(\xi - X_b) + k_4(\eta - Y_b) + b_3(\ddot{\xi} - \ddot{X}_b) + b_4(\ddot{\eta} - \ddot{Y}_b)$$

\[\text{Fig. 3 Dynamics system of journal bearing}\]
By assuming the solutions for $X, Y, X_b$ and $Y_b$ as shown in Eq.(22) and substituting them into the equations of motion, Eq.(23) can be derived.

$$
X = X_0 e^{r_1 s}, \quad Y = Y_0 e^{r_2 s},
$$

$$
X_b = X_b e^{r_1 s}, \quad Y_b = Y_b e^{r_2 s}, \tag{22}
$$

$$
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
X_b \\
Y_b
\end{bmatrix} = 0 \tag{23}
$$

where $a_{ij} - a_{44}$ are expressed as follows:

$$
a_{11} = k_1 + b_1 s + \left( M + \frac{M k_1}{k s} \right) s^2 + \frac{Mb_1}{k s} s^3
$$

$$
a_{12} = k_2 + b_2 s + \frac{M k_2}{k s} s^2 + \frac{Mb_2}{k s} s^3
$$

$$
a_{13} = -k_1 - b_1 s, \quad a_{14} = -k_2 - b_2 s
$$

$$
a_{21} = k_3 + b_3 s + \frac{M k_3}{k s} s^2 + \frac{Mb_3}{k s} s^3
$$

$$
a_{22} = k_4 + b_4 s + \left( M + \frac{M k_4}{k s} \right) s^2 + \frac{Mb_4}{k s} s^3
$$

$$
a_{23} = -k_3 - b_3 s, \quad a_{24} = -k_4 - b_4 s
$$

$$
a_{31} = Ms^3, \quad a_{32} = 0, \quad a_{33} = k_b + \frac{Ab_b}{12} s + m M s^2
$$

$$
a_{34} = 0, \quad a_{41} = 0, \quad a_{42} = Ms^2
$$

$$
a_{43} = 0, \quad a_{44} = k_b + \frac{Ab_b}{12} s + m M s^2
$$

For the nontrivial solution of Eq.(23), the determinant of the coefficient matrix should be equal to zero. Consequently, the characteristic equation is derived as follows:

$$
\sum_{n=0}^{10} a_n s^n = 0 \tag{24}
$$

At the stability boundary, the solution of the characteristic equation should be pure imaginary. Therefore, substituting $s = i \nu$ into Eq.(24), the characteristic equation is divided into two equations for the real and imaginary parts, as shown in Eqs.(25) and (26):

$$
-a_{10} \nu^{10} + a_8 \nu^8 - a_6 \nu^6 + a_4 \nu^4 - a_2 \nu^2 + a_0 = 0
$$

$$
a_2 \nu^2 - a_4 \nu^4 + a_6 \nu^6 - a_8 \nu^8 + a_{10} = 0 \tag{26}
$$

where the coefficients $a_{10} - a_{10}$ are the function of $A$. When the shaft stiffness is very large, it can be put to be $k_s = \infty$. Then, $a_{10} - a_{10}$ of Eqs.(25) and (26) are given as follows:

$$
a_{10} = 0
$$

$$
a_0 = 0
$$

$$
a_4 = m^2 M^4
$$

$$
a_2 = A_2 (1 + m) m M^3 + \frac{Ab_b}{6} m M^3
$$

$$
a_6 = A_6 (1 + m) m M^3 + A_4 \frac{Ab_b}{12} m M^2 + 2 k_b m M^3
$$

$$
+ A_4 (1 + m) m M^2 + A_4 \frac{Ab_b}{12} (1 + m) M^2 + \left( \frac{Ab_b}{12} \right)^2 M
$$
Figure 4 is the flow chart for the calculation. First of all, the bearing configurations (ε, λ, Ω, \(H_0\), α, β, m) and the bearing support parameters (\(K_b\) and \(b_b\)) are set. Next, the expansion ratio \(\Gamma\), the film thickness and the pressure distribution are calculated by using initial assumed bearing number \(\Lambda\) and whirl ratio \(\nu\). Then, the load capacity \(W\), the attitude angle \(\phi\), and the air film coefficients \(k_1\sim k_4\) and \(b_1\sim b_4\) are calculated. The new values of the bearing number \(\Lambda\) and the whirl ratio \(\nu\) are calculated from Eqs.(25) and (26). The procedure is repeated until the bearing number \(\Lambda\) and the whirl ratio \(\nu\) are converged less than a fixed error limit.

Then, all the solutions of Eq.(24) are calculated with converged \(\Lambda\), \(\nu\) and the air film coefficients by the Hitchcock Bearstow Method, and checked their signs of the real part.

4. Results and discussion

The analytical results are shown in this section. The configurations of the herringbone grooves are as follows: the groove depth ratio \(H_0\) is 3.0, the groove width ratio \(\alpha\) is 0.44 and the groove angle \(\beta\) is 20 degree. In this study, the calculation results are graphed with the damping ratio \(\zeta_b = \frac{b_b}{2\sqrt{m_bk_b}}\).
4.1 Effect of damping ratio

4.1.1 Results for $\zeta_b = 0$

Figure 5 shows the stability chart for the case that the bearing is supported by only stiffness elements, i.e. $\zeta_b = 0$. In Fig. 5, the solid lines are for $k_b=0.01$, 0.1 and 1, while the broken line are for the rigid support ($k_b=\infty$). The stable region decreases as the stiffness coefficient decreases, and there is no case that the stability region increases than that of the rigid support.

Figure 6 shows the stability threshold for some damping ratio $\zeta_b$ with respect to $k_b=0.01 \sim 10$. Figure 6(a) is for the case of $\zeta_b < 0.1$, while Fig. 6(b) is for the case of $0.1 \leq \zeta_b \leq 1$. Figure 7 shows the air film coefficients at the stability threshold with $\varepsilon=0.1$ and $\zeta_b=0.3$. It can be seen that in these cases, the stiffness coefficient $k_b$ and the damping ratio $\zeta_b$ have great effects on the stability chart.

When the stiffness coefficient $k_b$ is fully large compared with the stiffness of air film, the stability thresholds are almost equal regardless of the damping ratios $\zeta_b$. Since these values are near enough that of the rigid support, i.e. $\lambda=11.869$, it is stated that in this condition, there is few effect of the external support elements on the bearing stability. This...
is because that the large support stiffness compared with that of air film yields the situation like the rigid support.

On the other hand, as the stiffness of the external support element and the lubrication film are close in value, the stability threshold intricately varies with the magnitude of the damping ratio $\zeta_b$. When $\zeta_b=0.3$, the stability region is the largest and the stability threshold can be increased more than three times compared with that of the rigid support. However, as the damping ratio decreases, the unstable region increases and there are even the cases that the stability threshold decreases less than that of the rigid support. As shown in Fig.6(a), this trend is clearer when $\zeta_b<0.1$. This is considered that the lower damping leads to the similar effect for the case of $\zeta_b=0$ described in 4.1.1. As the results, it is clarified that for $0.2 \leq \zeta_b \leq 0.6$, the bearing system can be stabilized effectively by the external stiffness and damping elements. On the other hand, if the damping is too large, the external support elements behave like a rigid constraint as the whirl frequency increases. Therefore, when the damping ratio is too large or too small, the bearing system cannot offer high stability.

As the stiffness coefficient of the external support element is smaller compared with that of the air film, the stability threshold becomes about twice larger than that of the rigid support. In addition, it increases slightly as the damping ratio increases. In this condition, the external support elements with larger damping ratio have potential for the stabilization.

For $\zeta_b \leq 0.5$, there are a number of the stability regions for certain stiffness coefficients $k_b$. This means that the unstable bearing system can turn stable again with further increase in the rotating speed. This result is different from those for the aerostatic bearings, shown by Tatara et al. (5).

![Stability chart](image-url)

Fig.6 Effect of the external springs and damping on stability chart
4.2 Effect of the eccentricity ratio

Figure 8 shows the relationship between the eccentricity ratio $\varepsilon$ and the bearing number $A$ for $k_b=0.5$ and 1.5 with $\zeta_b=0.2$. In addition, the operation curves for $W=0.003$, 0.1 and 1.0, meaning the relation between $\varepsilon$ and $A$, are also shown. As shown in Fig.8(a), for $k_b=0.5$ the unstable region is appeared in the stable region of the high eccentricity ratio. Therefore, when $W=1.0$, the bearing system turns unstable at A shown in Fig.8(a) while turns stable at B as the shaft speed increases. And then, the bearing system turns unstable again at C. On the other hand, as shown in Fig.8(b), for $k_b=1.5$ the bearing system turns unstable around the bearing number of 10. However, at the high bearing number, the stable region appeared in the unstable region of the low eccentricity ratio.
5. Conclusions

In this study, it is theoretically studied how the stability of the herringbone grooved aerodynamic journal bearing changes with different external stiffness and damping coefficients. The results obtained are as follows.

(1) When the bearing sleeve is supported by only stiffness elements, the threshold of instability decreases as the stiffness coefficient decreases. In this condition, there is no case that the stability increases than that of the rigid support.

(2) When the stiffness coefficient of the external support element is fully large compared with the stiffness of air film, there is few effect of the external support element on the bearing stability.

(3) As the stiffness of the external support element is close to that of air film, the effect strongly appears. When $0.2 \leq \zeta_b \leq 0.6$, the bearing system can be stabilized effectively and it has large stable region. Especially for $\zeta_b = 0.3$, the stability threshold is increased more than three times compared with that of the rigid support.

(4) As the stiffness of external support element is smaller compared with that of air film, the stability can be increased about twice larger than that of the rigid support. In this condition, the threshold of instability increases as the damping ratio increases.

(5) In the herringbone grooved aerodynamic journal bearings with the external stiffness and damping elements, whirl occurred can be disappeared with further increases in the rotating speed and the bearing system can be operated stably again.

References

(1) Ehrich, F.F. and Jacobson, S.A., Development of High-Speed Gas Bearing for High-Power Density Microdevices, *Transaction of the ASME, Journal of Engineering for Gas Turbine and Power*, Vol.125, No.1 (2003), pp. 141-148.

(2) Hikichi, K. et al., Hydroinertia Gas Bearings and their Application to High Speed Micro Spinners (in Japanese), *Journal of Japanese Society of Tribologists*, Vol.50, No.6 (2005), pp.39-44.

(3) Tomioka, J., Miyanaga, N., Outa, E., Ogimoto, K., Matuei, S., Mori, T. and Kagami, F., Study of the Herringbone Grooved Aerodynamic Journal Bearings for the Support of an Ultra-High-Speed Rotor, *Proceedings of 3rd Asia International Conference on Tribology* (2006-10), pp.215-216.

(4) Tomioka, J., Miyanaga, N., Outa, E., Takahashi, T., Ogimoto, K., Mori, T. and Kagami, F., Development of Herringbone Grooved Aerodynamic Journal Bearings for the Support of Ultra-High-Speed Rotors (in Japanese), *Transactions of the Japan Society of Mechanical*
(5) Tatara, A., Dynamic Characteristic of Flexibly Supported Sleeve Bearings (in Japanese), *Journal of Japanese Society of Lubrication Engineers*, Vol.19, No.12 (1974), pp.883-890.

(6) Tatara, A., Koike, H. and Iwasaki, A., Stability Pressurized Gas Bearing with Flexibly Support, *Transactions of the Japan Society of Mechanical Engineers*, Vol.39, No.318 (1973), pp.643-649.

(7) Kazamierski, Z and Jarzecki, K., Stability Threshold of Flexibly Supported Hybrid Gas Journal Bearings, *Transactions of the ASME, Journal of Lubrication Technology*, Vol.101, No.4 (1979), pp.451-457.

(8) Lund, J. W., The Stability of an Elastic Rotor in Journal Bearings with Flexible, Damped Supports, *Transactions of the ASME, Journal of Applied Mechanics*, Vol.87, No.4 (1965), pp.911-920.

(9) Kogure, K. and Tamura, A., Stability of Journal Bearing and Elastic Rotor system with Flexibly Support, *Transactions of the Japan Society of Mechanical Engineers*, Vol.43, No.367 (1977), pp.920-927.

(10) The Japan Society of Mechanical Engineers ed., *JSME Date Handbook : A4*, (1984), pp.18.

(11) Vohr, J.H. and Chow, C.Y., Characteristics of Herringbone-Grooved, Gas-Lubricated Journal Bearings, *Transaction of the ASME, Journal of Basic Engineering*, Vol. 87, No.3 (1965), pp. 568-578.