Exergy analysis and the dissipation function of the composting process

Hirakazu SEKI a, †

(†Kanazawa University, Kanazawa, 920–1192 Japan)

Abstract

By regarding exergy as a fundamental driving force of composting, the exergy balance equation for composting is derived and its applicability is discussed. In composting, the available energy (termed exergy) is released from decomposition of the substrate by microbial reactions. Some of the released exergy is consumed by microbial growth and its movement, some is temporarily accumulated in the compost bed as a temperature increase, and the remainder is irreversibly dissipated and changed to energy (non-available energy). There are two types of dissipation functions: bound and external dissipation functions. The external dissipation function governs the composting system. By considering the bound dissipation function as positive, the maximum value of microbial cell yield \( Y \) is theoretically formulated. Using the previously obtained experimental results, the proportion of the dissipation function of the entire amount of released exergy is confirmed to be as large as 90%. It is further shown that the application of the exergy analysis to the functional analysis of the composting process will be promising.

Key words: Composting, Ecosystem, Exergy dissipation, Temperature, Thermodynamics.

1. Introduction

The importance of waste resourcing has been recognized for several decades. Composting is a waste resourcing method using micro-organisms as decomposers and can be regarded as an ecosystem. Composting is a phenomenon involving simultaneous heat and mass transfer with non-linear microbial reactions. The temperature and mass concentration necessarily emerge during the composting process. However, to directly understand the whole map of the process, deriving the balance equation of its fundamental potential would be convenient.

Exergy is an overall physical quantity concerning the change in heat (temperature) and mass (concentration), and is thought to be the most convenient parameter for describing the thermodynamic state variables for ecological systems (Jørgensen, 2001). There are two types of exergies: physical and chemical exergies. The former corresponds to heat and mass transfer, and the latter corresponds to chemical or biochemical reactions. By regarding exergy as a fundamental potential, a new system model applied to the composting process as a biological system is derived in this study. Thus far, this type of biological system has not been fully conceptualized in terms of thermodynamics. This type of modeling will also be important in the field of agricultural engineering for crop production and management.

In this study, the exergy balance equation is derived and the dissipation function for the composting process is formulated. An interpretation of the physical meaning of the dissipation function is provided. Finally, it is reported that the derived dissipation function will be useful in discussing the applicability of the extremal principle to the composting process as an ecosystem.

2. Literature review of the fundamental potential and exergy analysis of composting

Although the concept was not originally defined for composting, Kawashimo (1971) introduced the concept of "entropy functional" and proposed its analytical method by its minimal principle, regarding entropy as the fundamental potential for heat conduction. This concept was proposed as a method for obtaining analytical solutions to non-linear heat conduction problems. In general, these problems are difficult to solve analytically compared to standard linear heat conduction problems, which had already been solved by the 1970s. It is academically interesting, however, it seems somewhat over-technologized because a phenomenological coefficient is introduced, and coldness (the reciprocal of absolute temperature) is used for variation analysis. Because exergy has the same dimension as energy, it should be more available than entropy.

Concerning exergy analysis of composting, Seki and Komori (1992) pointed out that 90% of released exergy irreversibly dissipates during the composting process under conditions of perfect mixing. They also demonstrated that the process of heat extraction from a compost bed has very small exergy extraction efficiency (Seki and Komori, 1993a, b). In those studies, however, the contents of the dissipation function were not discussed, so the means of using the dissipation function as a criterion for system progress has not been explained.

3. Exergy balance in the composting process

Composting obeys thermodynamic principles at the macroscopic scale, because a compost heap is an ecosystem. However, microbial metabolism is dynamic and accompanied by microscopic fluctuation, and microbes are also in motion. Furthermore, heat and mass transfer also occur in compost.

Microscopic treatment of phenomena is dependent on statistical thermodynamics; however, its application to composting is very difficult because composting occurs in a complicated dispersed
media and there are too many unknown parameters. Actual control of composting systems usually focuses on macroscopic variables. Thus, the discussion herein is confined only to the macroscopic model.

To clarify the macroscopic behavior, it is necessary to express quantitatively the energy dissipation process based on irreversible thermodynamics. First, the dissipation function of composting is considered (Seki and Shijuku, 2013). In this case, the exergy balance in a small volume element between \( z \) and \( z + \Delta z \) within a small time interval \( \Delta t \) is shown in Figure 1.

\( T \) is temperature, \( T_0 \) is ambient temperature, \( K \) is effective thermal conductivity of compost bed, \( z \) is the spatial coordinate (positive downward), \( r \) is the radius of the cylindrical compost vessel, \( J_{\text{(a)}} \) is the mass flux of aeration, \( \sigma \) is the entropy production rate, \( C \) is the specific heat of the compost bed, \( \rho \) is the density of the compost bed, \( \epsilon_0^c \) is the specific chemical exergy of the microbes, \( U \) is the overall heat transfer coefficient for heat loss through the side wall of the fermenter, \( S \) is the mass of substrate per unit volume of the compost bed, \( X \) is the mass of microbes per unit volume of the compost bed, \( \epsilon_{\text{w}} \) is the specific exergy of the water vapor, \( \epsilon_{\text{d}} \) is the specific exergy of the dry air, and \( J_{\text{ma}} \) is the mass flux of water vapor. Of the abovementioned items, specific exergy is the amount of energy per unit mass of a substance.

The symbol (a) represents accumulation, (b) is influx due to heat conduction, (c) is efflux due to heat conduction, (d) is influx of moist air due to convection, (e) is efflux of moist air due to convection, (f) is the part released due to substrate decomposition, (g) is the amount used for microbial growth, (h) is effluent from the side wall of the fermenter, and (i) is the rate of irreversible exergy dissipation.

As the accumulation term is equal to the net influx exergy, it is possible to obtain the exergy balance in a small-volume element \( \Delta z \). Letting \( \Delta z \) and \( \Delta t \) approach zero, the following partial differential equation is obtained:

\[
\frac{\partial}{\partial t} \left[ C_p \rho (T - T_0 + T_0 \ln(T_0 / T)) \right] = \frac{\partial}{\partial z} \left[ \left( 1 - \frac{T_0}{T} \right) K \frac{\partial T}{\partial z} \right] - \frac{\partial}{\partial z} \left( \epsilon_{\text{H}} J_{\text{ma}} + \epsilon_a J_{\text{ma}} \right) + \epsilon_0^c \Delta S
\]

where \( i \) is the enthalpy of air for aeration.

Using the above, Eq. (1) can be rewritten as

\[
\frac{\partial}{\partial t} \left( 1 - \frac{T_0}{T} \right) J_{\text{ma}} \frac{\partial i}{\partial z} - \epsilon_0^c \frac{\partial S}{\partial t}
\]

The enthalpy (\( \approx \) energy \( \approx \) heat) balance in composting is illustrated in Figure 2, where \( \Delta h_i^0 \) is the enthalpy change in the substrate decomposition reaction per unit mass and \( \Delta h_i^0 \) is the enthalpy change in the microbial growth reaction per unit mass. The symbol (a) represents accumulation, (b) is influx due to heat conduction, (c) is efflux due to heat conduction, (d) is influx of moist air, (e) is efflux of moist air, (f) released exergy due to substrate decomposition, (g) exergy used for microbial growth, (h) efflux from the side wall of the fermenter, and (i) irreversible exergy dissipation rate.

![Fig. 1. Exergy balance in a small volume element. All terms concerning exergy balance in a fixed-volume element between \( z \) and \( z + \Delta z \) for one-dimensional distributing composting system are presented, where (a) accumulation, (b) influx due to heat conduction, (c) efflux due to heat conduction, (d) influx of moist air exergy, (e) efflux of moist air exergy, (f) released exergy due to substrate decomposition, (g) exergy used for microbial growth, (h) efflux from the side wall of the fermenter, and (i) irreversible exergy dissipation rate.](image-url)
4. Interpretation of the dissipation function

Eq. (5) can be rewritten as follows:

\[
T_{d\sigma} = -e_X^0 \frac{\partial X}{\partial t} + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial t} \right) \left( C_p \rho \Delta (T - T_0) \right) + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial t} \right) \left( \Delta h_X^0 \right) \frac{\partial X}{\partial t} + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial t} \right) \left( \Delta h_S^0 \right) \frac{\partial S}{\partial t}
\]

\[
+ K \frac{T_0}{T^2} \left( \frac{\partial T}{\partial z} \right)^2
\]

(6)

Figure 3 shows the relation between the individual terms in Eq.
(6). In this figure, (a) \( \Psi_0 \) is the released exergy from substrate decomposition, (b) \( \psi_t \frac{\partial \gamma}{\partial t} \) is the accumulated exergy from microbial growth, (c) \( (1 - \frac{\partial}{\partial t}) \sigma \frac{\partial S}{\partial t} - \omega \frac{\partial X}{\partial t} \) is the part of released exergy that is transformed into sensible heat exergy, and (d) \( \psi_t \frac{\partial (\frac{\partial \gamma}{\partial t})}{\partial t} \) is the part of released exergy that is dissipated together with heat conduction.

To summarize the figure, the exergy remaining after subtraction from the released exergy because of the substrate decomposition of both the accumulated exergy resulting from microbial growth and the reversibly changed portion of the sensible heat exergy accompanied by temperature rise is dissipated.

The dissipation function of the composting process \( (T_0 \sigma; \text{Eq. (5)}) \), can be interpreted using the theory of Lamprecht and Zotin (1978) regarding the dissipation function for biological phenomena as follows:

\[
T_0 \sigma = \Psi_u + \Psi_d \tag{7}
\]

where \( \Psi_u \) is the bound dissipation function used for irreversibly progressed microbial growth within the system and \( \Psi_d \) is the external dissipation function reversibly converted to energy as a result of heat loss from the system. These quantities are expressed as

\[
\Psi_u = -e_0^0 \frac{\partial S}{\partial t} + e_X^0 \frac{\partial X}{\partial t} - \left( \Delta h_S^0 \frac{\partial S}{\partial t} - \Delta h_X^0 \frac{\partial X}{\partial t} \right) \tag{8}
\]

\[
\Psi_d = \frac{T_0}{T}\left( \Delta h_S^0 \frac{\partial S}{\partial t} - \Delta h_X^0 \frac{\partial X}{\partial t} \right) + K T_0 \left( \frac{\partial T}{\partial t} \right)^2 \tag{9}
\]

Lamprecht and Zotin (1978) set the external dissipation function as only the first term of the right-hand side of Eq. (9); however, the second term is additionally included because this term, which models the dissipation due to heat conduction, has no relation to microbial growth and should be included in \( \Psi_d \).

Eq. (8) can be rewritten as

\[
\Psi_u = (-e_0^0 - \Delta h_S^0) \frac{\partial S}{\partial t} + (-e_X^0 + \Delta h_X^0) \frac{\partial X}{\partial t} \tag{10}
\]

If the substrate and the microbes are physically in equilibrium with their surroundings, their specific enthalpies are written as \( h_{S0} \) and \( h_{X0} \), respectively. If the substrate and the microbes are in equilibrium both physically and chemically, their specific enthalpies are written as \( h_{S00} \) and \( h_{X00} \), respectively. The subscripts 0 and 00 indicate physical and chemical equilibrium states, respectively. Although the values of the four specific enthalpies are unknown, the differences \( h_{S0} - h_{S00} \) and \( h_{X0} - h_{X00} \) can be obtained from the enthalpy change when they are chemically composed.

\[
-\Delta h_S^0 = h_{S0} - h_{S00} \tag{11}
\]

\[
\Delta h_X^0 = h_{X0} - h_{X00} \tag{12}
\]

The specific chemical exergy of the substrate \( (e_S^0) \) and that of microbes \( (e_X^0) \) for an open system are given by (Kojima, 2004)

\[
e_0^0 = h_{S0} - h_{S00} - T_0 \left( \overline{S}_{S0} - \overline{S}_{S00} \right) \tag{13}
\]

\[
e_X^0 = h_{X0} - h_{X00} - T_0 \left( \overline{S}_{X0} - \overline{S}_{X00} \right) \tag{14}
\]

where \( \overline{S}_S \) and \( \overline{S}_X \) are the specific entropy of the substrate and the microbes, respectively. Taking into account Eq. (11) to (14), Eq. (10) can be rewritten as

\[
\Psi_u = T_0 \left( \overline{S}_{S0} - \overline{S}_{S00} \right) \frac{\partial S}{\partial t} + T_0 \left( \overline{S}_{X0} - \overline{S}_{X00} \right) \frac{\partial X}{\partial t} \tag{15}
\]

Introducing the microbial growth yield \( Y \), it is possible to obtain the relation between \( X \) and \( S \) as follows:

\[
\frac{\partial X}{\partial t} = -Y \frac{\partial S}{\partial t} \tag{16}
\]

Eq. (15) can be rewritten as

\[
\Psi_u = T_0 \left[ (\overline{S}_{X0} - \overline{S}_{S00}) - Y(\overline{S}_{X0} - \overline{S}_{X00}) \right] \frac{\partial S}{\partial t} \tag{17}
\]

The substrate and the microbes, because they are organic materials, have a more complicated structure and much smaller specific entropy values \( \overline{S}_{S0} \) and \( \overline{S}_{X0} \) than those of materials in an inorganic soup (Jorgensen, 2001), \( \overline{S}_{S00} \) and \( \overline{S}_{X00} \). Obtaining the absolute value of \( \overline{S}_{X0} \) is impossible because microbes are living organisms (Aoki, 2001). \( \overline{S}_{X0} \) has a negative value, however, because it contains a “value-added entropy” (Kakitani, 2000). Nevertheless, the values of the entropy differences \( \overline{S}_{S0} - \overline{S}_{S00} \) and \( \overline{S}_{X0} - \overline{S}_{X00} \) can be estimated.

The term \( T_0((\overline{S}_{S0} - \overline{S}_{S00}) - Y(\overline{S}_{X0} - \overline{S}_{X00})) \) is the exergy drop for structure formation and has a negative value. As the value of \( (\overline{S}_{S00} - \overline{S}_{S0}) \) is smaller than the value of \( (\overline{S}_{X00} - \overline{S}_{X0}) \), \( Y \) must satisfy the following equation in order for the exergy drop term to be negative.

\[
y < \frac{\overline{S}_{S0} - \overline{S}_{S00}}{\overline{S}_{X00} - \overline{S}_{X0}} \tag{18}
\]

According to the estimation of Haug (1993), the value of \( Y \) is between 0.1 and 0.2 for composting of glucose, a representative substrate. Because substrates are usually consumed, \( \overline{S}\partial t\overline{t}r≈0 \). Therefore, \( \Psi_u \) corresponds to the part of exergy dissipation used up in acquiring structural complexity due to microbial growth.

Summarizing the above, the dissipation function in composting is

\[
T_0 \sigma = \Psi_d = \frac{T_0}{T} \left[ \frac{\partial S}{\partial t} + T \left( \Delta h_S^0 \frac{\partial S}{\partial t} - \Delta h_X^0 \frac{\partial X}{\partial t} \right) + K T_0 \left( \frac{\partial T}{\partial t} \right)^2 \right] \tag{19}
\]

According to Lamprecht and Zotin (1978), fermentation proceeds very effectively, as observed in continuous culture of yeast, and the bound dissipation function becomes negligibly small during the stable growth period immediately after the logarithmic growth stage, though the function is large in the initial stage when metabolic action is active (Maeda and Fujita, 1983). Assuming that this relation is also applicable to the composting process, the dissipation function can be approximated by the external dissipation function.

\[
T_0 \sigma = \Psi_d = \frac{T_0}{T} \left( \Delta h_S^0 \frac{\partial S}{\partial t} - \Delta h_X^0 \frac{\partial X}{\partial t} \right) + K \frac{T_0}{T^2} \left( \frac{\partial T}{\partial t} \right)^2 \tag{20}
\]

This equation is consistent with the statement that the dissipation...
function in a chemical reaction system is almost entirely governed by the heat generation term, as pointed out by Oshida (1986).

5. Proportion of the dissipation function to the total released exergy

The proportion of the dissipation function to the total released exergy in the entire fermenter is given by

$$\varepsilon = \frac{\pi r^2 \int_0^l 0 T_0 dz}{\pi r^2 \int_0^l -e^S_0 \frac{\partial S}{\partial t} dz}$$

Using the experimental results of Shishido and Seki (2015), we calculated this value $\varepsilon$ and plotted the results against time (Figure 4). In the calculation, Eq. (20) was used for $T_0$ approximately, and enthalpy change in the substrate decomposition process $-\Delta h_s$ ($=18000 \text{ kJ kg}^{-1}$) was used for $e^S_0$ because $e^S_0$ had been confirmed to be almost equal to the standard heat of the combustion of the substrate (Seki and Komori, 1992).

This experiment was conducted using mixed compost material of rice bran, chicken manure, and sawdust, of which the proportional ratio (dry mass) was 27.3, 25.4, and 47.3 wt%, respectively. The composting vessel was an acrylic resin cylindrical container with an inner radius of 0.2 m and a height of 0.69 m. The density of the packed materials was 550 kg m$^{-3}$ and the moisture content was 0.571 kg-water kg$^{-1}$-total mass. Air was supplied from the top of the container at a constant flow rate of 1.0 l min$^{-1}$, and the duration of the experiment was approximately 300 h. Because of the microbial shift from mesophiles to thermophiles in 20–30 h, the rate of exergy release drastically changed from the mesophilic action phase around 10 h to the thermophilic action phase around 40 h, and this is a general feature of the composting process.

As pointed out by Seki and Komori (1992), the value of $\varepsilon$ is around 90% for $t = 50–200$ h, so $\varepsilon$ has a very large value.

The changes in the characteristic temperature and substrate profiles with time form in composting process, one of eco-systems, as correspond to the emergence of a dissipated structure. We can interpret that this structure is formed by utilizing about 90% of the released exergy during the process. In other words, this proportion corresponds to the degree of the attainment of dissipated structure formation, and this degree is shown to be very high in the composting process.

6. Applicability of the exergy dissipation function to prediction of the composting process

Generally, it is known that the Lyapunov function is an excess entropy and a system is stable when the function has a negative value and unstable when the function becomes positive by certain temporal fluctuations of the system. This instability is a driving force for new dissipative structure formation (Nicolis and Prigogine, 1977). From the viewpoint of exergy, excess exergy is the Lyapunov function, and the system is stable when it is positive and unstable when it is negative. Especially, Svirezhev (2001) reported that exergy itself is the Lyapunov function. As described above, the value of exergy relates to the stability criterion of the system.

The general principle of minimum entropy being produced (Nicolis and Prigogine, 1977) does not hold for phenomena that fall into the category of non-linear thermodynamics, such as the composting process, which shows bifurcation caused by the fluctuation of state variables.

On the other hand, Jorgensen (2001) proposed a tentative
fouth law of thermodynamics, which states that exergy maximization is the process direction of an ecosystem, and reported the applicability of the extremal principle to the ecosystems’ progress prediction.

As a future research strategy, we will propose the certification of the validity of Jorgensen’s tentative theory through elucidating the applicability of the extremal principle, i.e., functional analysis, to the composting process as an example. We can use the dissipation function derived here as a powerful tool to discuss the abovementioned subject.

7. Conclusions
The exergy balance equation of composting was obtained for thermodynamic discussion. This equation is useful for consideration of effective utilization of the exergy released during composting. Then, based on the equation, the exergy dissipation function relating to the direction of the composting process was derived.

1) It is clarified that the remainder of the released exergy after subtraction of exergy used for microbial growth and the reversibly changed part of the sensible heat exergy accompanying a temperature rise is dissipated; this exergy corresponds to the dissipation function.

2) Following the idea of Lamprecht and Zotin (1978) concerning the dissipation function for biological phenomena, the obtained dissipation function was divided into two parts, the bound dissipation function and the external dissipation function. Using the condition that the bound dissipation function should be positive, it was shown that the maximum value of the microbial cell yield ($Y$) can be determined. Almost all the external dissipation function consists of the chemically dissipated part, in agreement with the conclusion of Oshida (1986).

3) The applicability of the dissipation function to functional analysis of the composting process was highlighted as a future research strategy.

References
Aoki, I., 2001: Entropy and the exergy principles in living systems. Chap.8. In Thermodynamics and ecological modeling, Lewis publishers, Boca Raton, pp.165–190.

Haug, R. T., 1993: The practical handbook of compost engineering. Lewis publishers, Boca Raton, pp.717.

Jorgensen, S. E., 2001: Thermodynamics and ecological modeling, Lewis publishers, Boca Raton, pp.373.

Kakitani, T., 2000: Strategy for biological energy transformation. Chap.5. In Physics of living materials and energetics—source of vitality—, The physical society of Japan, Shokabo, Tokyo, pp.88–111. (in Japanese).

Kawashimo, K., 1971: Analysis of heat conduction by entropy minimum principle. Chap.15. In Theory of heat conduction (2nd edition). Ohmusha, Tokyo, pp.415–432. (in Japanese).

Kojima, K., 2004: Principle of effective energy utilization. Baifukan, Tokyo, pp.153. (in Japanese).

Lamprecht, I., and Zotin, A. I., 1978: Thermodynamics of Biological Processes. De Gruyter, Berlin, pp.428.

Maeda, Y., and Fujita, T., 1983: Fermentation heat. Chap.3. In Optimum measurement and control of fermentation process, Science forum, Tokyo, pp.282–293. (in Japanese).

Nicolis, G., and Prigogine, I., 1977: Self-organization in nonequilibrium systems. John Wiley & Sons, New York, pp.491.

Oshida, I., 1986: Lecture on exergy. Kyoritsu Shuppan, Tokyo, pp.194. (in Japanese).

Seki, H., and Komori, T., 1992: Structure of energy conversion in composting process. Journal of Agricultural Meteorology, 48, 225–235 (in Japanese with English summary).

Seki, H., and Komori, T., 1993a: Mass, energy and exergy balances for heat recovery operation from compost, Part.1 derivation of microscopic balance equations in composting process. Environmental control in biology, 31, 197–203 (in Japanese with English summary).

Seki, H., and Komori, T., 1993b: Mass, energy and exergy balances for heat recovery operation from compost, Part.2 validity of microscopic balance equations. Environmental control in biology, 31, 205–215 (in Japanese with English summary).

Seki, H., and Shijuku, T., 2013: Estimating the heat generation rate in a forced-aeration composting process by measuring temperature changes. Journal of Agricultural Meteorology, 68, 107–120.

Shishido, T., and Seki, H., 2015: Laboratory-scale experiment for an active-stage composting process under the same material and operating conditions. Journal of Agricultural Meteorology, 71, 111–123.

Svirezhev, Y. M., 2001: Thermodynamics and theory of stability. Chap.5. In Thermodynamics and ecological modeling, Lewis publishers, Boca Raton, pp.117–132.