Scaling Behavior of Anomalous Hall Effect and Longitudinal Nonlinear Response in High-$T_C$ Superconductors

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Based on existing theoretical model and by considering our longitudinal nonlinear response function, we derive a nonlinear equation in which the mixed state Hall resistivity can be expressed as an analytical function of magnetic field, temperature and applied current. This equation enables one to compare quantitatively the experimental data with theoretical model. We also find some new scaling relations of the temperature and field dependency of Hall resistivity $\rho_{xy}$. The comparison between our theoretical curves and experimental data shows a fair agreement.

PACS: 74.60.Ge, 72.15.Gd, 74.60.Ec

Introduction. The anomalous Hall effect (AHE) remains one of most intriguing phenomena in the area of high-temperature superconductors (HTS). A sign reversal of the Hall resistivity $\rho_{xy}$ has been observed in the most of HTS just below the transition temperature $T_C$ [5] although the conventional theories of flux motion proposed by Bardeen and Stephen [2] as well as Nozières and Vinen [3] predict that the Hall effect stems from quasineutral-normal core and hence has the same sign as in the normal state. Several attempts at a theoretical understanding of this surprising sign reversal have been undertaken [4].

Taking into account the backflow current due to pinning and thermal fluctuation effect, Wang, Dong and Ting (WDT) [5] developed a unified theory for the anomalous sign reversal and the observed puzzling scaling relation between the Hall and longitudinal resistivities $\rho_{xy} \propto \rho_{xx}^2$ [6]. They obtained a relation:

$$\rho_{xy} = \frac{\beta_0 \rho_{xx}^2}{\Phi_0 B} \left\{ \eta(1 - \gamma) - 2\pi\gamma \Gamma(v_L) \right\}$$  \hspace{1cm} (1)

where $\beta_0 = \mu_m H_{C2}$ with $\mu_m = m^e / m$ the mobility of the charge carrier and $H_{C2} = \Phi_0 / 2\pi\xi^2$ being the usual upper critical field with $\xi$ the superconducting coherence length, and $\eta$ is the viscous coefficient. $\gamma = \gamma(1 - H / H_{C2})$ with $H$ the average magnetic field over core and $\gamma$ the parameter describing contact force on the surface of core. $B$ is the magnetic field. $\Gamma(v_L)$ is a positive scale function between the time average pining force $\langle \mathbf{F}_p \rangle_t$ and flux motion velocity $v_L = \langle \mathbf{v}_{(i)} \rangle_t$ defined as:

$$\langle \mathbf{F}_p \rangle_t = -\Gamma(v_L) \mathbf{v}_L$$ \hspace{1cm} (2)

However, as mentioned by WDT, it is difficult to find analytical expression for $\Gamma(v_L)$ theoretically though two approximate methods were suggested.

Another school of thought considers the intrinsic vortex properties. Vinokur, Geshkenbein, Feigel'man and Blatter (VGBF) [6] explain the observed scaling behavior $\rho_{xy} \propto \rho_{xx}^2$ assuming that the Hall conductivity $\sigma_{xy}$ does not depend on disorder. A microscopic analysis of intrinsic dynamical single vortex properties provides an interpretation of the observed sign change in the Hall effect in the superconductors with mean free path $\ell$ of the order of coherence length $\xi$ in terms of broken particle-hole symmetry, which is related to detail of the microscopic mechanism of superconductivity.

For testing the role of pinning and intrinsic vortex properties on the Hall effect, experiments to measure the $\rho_{xy}$ of $\sigma_{xy}$ before and after heavy-ion irradiation were carried on since heavy-ion track are very effective pinning centers [7]. Another way to vary the effect of pinning is measuring $\rho_{xy}$ at different current density $J$ [8]. In spite of much recent efforts, qualitative comparison between theories and experimental data remains controversial and consensus on the origin of AHE is still lacking.

In present work, we try to make a rather quantitative comparison between experimental data and theoretical models. By using a recently found nonlinear material equation [10,11], we derive a theoretical expression for $\rho_{xy}$ which is a function of temperature, applied magnetic field and applied current. We also find some new scaling relations of the temperature and field dependency of Hall resistivity $\rho_{xy}$. Comparison between the scaled experimental data and our theoretical curves for $\rho_{xy}$ showed fair agreement.

Hall resistivity equation. In the mixed state of type-II superconductors, flux enters the superconductors in form of quantized flux lines or vortices according to the theory of Abrikosov. In the case of nonideal type-II superconductors the flux lines are pinned by imperfections, small current flow usually results in vanishing resistance. However, with a sufficiently large driving force produced by a significant transport current, flux lines can be depinned. Their motion is subject to a viscous drag, giving rise to
dissipation [2]. Therefore, for the steady state of flux motion in nonideal type-II superconductors, the mean current transport density \( J \) can be phenomenologically expressed as a sum

\[
J = J_p + J_f
\]

(3)

with \( J_f = E(J)/\rho_f \) the component due to the moving vortices of uniform density and \( J_p \) denotes contribution from the pinned vortices with nonuniform distribution. Therefore, the pinning force \( (\mathbf{F}_p)_t \) can be expressed as

\[
(\mathbf{F}_p)_t = -\mathbf{J}_p \times \mathbf{B}
\]

(4)

and we have

\[
\Gamma(v_L) = \frac{(J - J_f)B^2}{\rho_x}
\]

(5)

In writing this, we have used Maxwell equation \( \mathbf{E} = \mathbf{B} \times \mathbf{v}_L \) with \( \mathbf{v}_L \) flux motion velocity which induces an electric field \( \mathbf{E} \). Substituting the right hand side of Eq.(5) into the term \( \Gamma(v_L) \) in Eq.(1) and following the estimation in Ref.[2] as \( \eta = B^2/\rho_f \) and the resistivity due to flux flow \( \rho_f = E/J_f = \rho_n B/B_{c2} \), we get

\[
\rho_{xy} = \frac{\beta_0 B^2 \rho_n}{\Phi_0 B_{c2}} \left\{ (1 + \gamma) \frac{\rho_{xx}}{\rho_f} - 2\gamma \frac{\rho_{xx}}{\rho_f} \right\}
\]

(6)

where \( \rho_n \) and \( B_{c2} \) are the resistivity of normal state and the upper critical magnetic field, respectively.

The form of Eq.(6) implies that the behavior of \( \rho_{xy} \) strongly depends on the longitudinal \( \rho_{xx} \), i.e., the longitudinal nonlinear response of HTS, which will lead to nonlinear Hall resistivity and even sign reversal. P.J.M.Wöltgens et al. [2] measured the nonlinear Hall resistivity in \( Y\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta} \) film near the vortex-glass transition. Their results firmly indicated that the Hall resistivity has the similar scaling relation as for longitudinal resistivity which has been discussed extensively by M.P.A.Fish et al. [3]. Therefore, the main problem for

the Hall resistivity equation is to find out a general analytical expression of the nonlinear longitudinal \( \rho_{xx} \) usually formulated in the form

\[
\rho_{xx} = E(J)/J = \rho_f e^{-U(T,B,J)/kT}
\]

(7)

in which different types of the current dependency \( U(J) \) have been suggested to approximate the real barrier, for instance, the Anderson-Kim model [4] with \( U(J) = U_C(1 - J/J_{C0}) \), the logarithmic barrier \( U(J) = U_C \ln(J/J_{C0}) \) [5] and the inverse power-law with \( U(J) = u_C(1/J_{C0}/J)^\beta - 1 \) [6].

We find, if one makes a common modification to the different model barriers \( U(J) \) as

\[
U(J) \rightarrow U(J_p = J - E/\rho_f)
\]

(8)

the corresponding modified material equation

\[
E(J) = J \rho_f e^{-U(J_p)/kT}
\]

(9)

leads to a common form of longitudinal resistivity [10,11]

\[
\rho_{xx} = \rho_f \exp\left[-a \frac{U_C}{kT} \cdot \frac{J_p}{J} \left( 1 + \frac{\rho_{xx}/\rho_f}{J/C_0} - \frac{1}{J/C_0} \right)^{y} \right]
\]

(10)

where \( U_C \) and \( J_{C0} \) are only dependent on the temperature and magnetic field. Following the discussion in the review article of Cohen et al. [7], we can assume that the temperature and field are separated variables in the \( U_C \) and \( J_{C0} \) and write

\[
U_C \propto B^m(1 - B/B_{c2})^\delta(1 - T/T^*)^\alpha
\]

(11)

\[
J_{C0} \propto (1 - T/T^*)^\beta
\]

(12)

with \( T^* \) the irreversibility temperature and \( m, \delta, \alpha \) and \( \beta \) are exponents. Combining Eq.(6) with Eq.(10-12), we finally get the analytical expression for Hall resistivity

\[
\rho_{xy} = \beta_0 AB^2 \left\{ (1 + \gamma) \exp\left[-a \frac{B^m(1 - B/B_{c2})^\delta(1 - T/T^*)^\alpha}{kT} J_p \left( 1 + \frac{\rho_{xx}/\rho_f}{b(1 - T/T^*)^\beta} - \frac{1}{b(1 - T/T^*)^\beta} \right)^{y} \right] \right.
\]

\[
-2\gamma \exp\left[-a \frac{B^m(1 - B/B_{c2})^\delta(1 - T/T^*)^\alpha}{kT} J_p \left( 1 + \frac{\rho_{xx}/\rho_f}{b(1 - T/T^*)^\beta} - \frac{1}{b(1 - T/T^*)^\beta} \right)^{y} \right] \right\}
\]

(13)

where we have used the approximate resistivity relation \( \rho_n = AT \) for HTS with \( A \) as a constant. In Fig.1, we plot a series of \( \rho_{xy} \) vs. \( B \) curves as the interesting numerical solutions of Eq.(13) with the values of exponents of \( m, \delta, \alpha, \beta \) similar to that of Ref. [7] and different values of \( \gamma \) which describes contact force on the surface of core. Negative Hall resistivity appears between two characteristic fields, \( B_r \) (where sign reversal occurs) and \( B_0 \) (where \( \rho_{xy} \) disappears), or between two similar characteristic temperature \( T_c \) and \( T_0 \) in \( \rho_{xy} \sim T \). It is also clearly seen that the magnitude of negative Hall resistivity decreases as the value of \( \gamma \) decreasing and vanishes at
a definite value. This result coincides with the conclusion of WDT [5]. Moreover, it is worthy to note that, some times the numerical solution of \( \rho_{xy} \sim B \) (see Fig.1) manifests two sign reversals of Hall resistivity \( \rho_{xy} \), i.e., the Hall resistivity changes sign from negative to positive at a lower magnetic field besides the first change from positive to negative at higher field. This behavior is very similar to that experimentally observed by J.Schoenes et al. [8].

Scaling behavior of AHE.—Besides the well known scaling relations between Hall and longitudinal resistivities \( \rho_{xy} \propto (\rho_{xx})^\beta \), we find two further striking scaling relations of AHE as

\[
\frac{\rho_{xy}}{\rho_m} \approx f\left(\frac{B - B_0}{B_0 - B_0}\right)
\]

(14)

\[
\frac{\rho_{xy}}{\rho_m} \approx f'\left(\frac{T - T_0}{T_r - T_0}\right)
\]

(15)

where \( \rho_m \) is the maximal negative Hall resistivity, \( f \) and \( f' \) are scaling functions which in general can be dependent on the specific samples measured in the experiments. However, to our astonishment, we find that the scaled experimental data obtained from different laboratories (Ref. [4], Ref. [6] and Ref. [21]) also fall onto a single scaling curve (see Fig.2). With the similar scaling Eq.(15), the negative Hall resistivity data from S.J.Hagen et al. [21] and Y.Matsuda et al. [22], measured at many different magnetic fields can also collapse onto a single universal functional dependence on the scaled temperature as shown in Fig.3. For making a comparison between the experimental data and theoretical predication, it is convenient to rescale the curves obtained from theoretical expression of \( \rho_{xy} \) according to Eq.(14) and Eq.(15). Fig.2 and Fig.3 show comparison between the scaled experimental data and the scaled theoretical curves with the values of exponents of \( m = 2.0, \delta = 4.2, \alpha = 1.8, \beta = 1.8, p = 1 \) as used in Ref. [17]. We see fair agreements for both the scaled field and temperature dependence of \( \rho_{xy}/\rho_m \).

Discussion.—Though the comparisons of our Hall resistivity equation (see Eq.(13)) with the pertinent experimental results shown in Fig.2 and Fig.3 convincingly indicate the pinning dependence of \( \rho_{xy} \), it by no means rules out the possible role of particle-hole asymmetry in AHE [1]. Actually, Eq.(13) is not incompatible with a possible broken particle-hole symmetry in the electronic band structure of superconductor. However, we believe the rather quantitative fit of experimental data of different origins with our Hall resistivity equation in the scaling form Eq.(14)(15) may imply that pinning is the key factor in AHE of high-\( T_C \) cuprates. It would be very interesting to see whether the analogous Hall effect is still observed in the recently discovered high-\( T_C \) (\( \sim 40K \)) \( MgB_2 \) [23] which in contrast to the high-\( T_C \) cuprates manifests typical electron-phonon superconductivity. It is also useful to check the \( \rho_{xy} \) scaling behavior in this conventional s-p band metallic compound and compare it with those for the high-\( T_C \) cuprates as strongly correlated electronic systems.

Summary.—Based on the WDT’s model and combining our nonlinear response equation, we derived a theoretical equation for mixed state Hall resistivity in type-II superconductors. This equation predicts the sign reversal of Hall resistivity and has a quantitative agreement with experimental data.

Note added in proof: After preparation of this paper we became aware of the interesting work of R.Jin et al. on the Hall effect in \( MgB_2 \) film [24]. The experimentally observed \( R_H \sim B \) and \( R_H \sim T \) data clearly indicated that there are sign reversals of Hall resistivity in \( MgB_2 \) very similar to our Eq.(14) and Eq.(15) and the behaviors shown in Fig.2 and Fig.3.
[17] Daole Yin, Chuanyi Li and Weiping Bai, Applied Superconductivity Vol. 5, Nos 1-6, 147 (1998).
[18] J.Schoenes, E.Kaldis, and J.Karpinski, Phys.Rev.B 48, 16869 (1993).
[19] J.Luo, T.P.Orlando, and J.M.Graybeal, et al., Phys.Rev.Lett, 68, 690 (1992).
[20] M.R.Cimberle, C.Ferdeghini, G.Grassano, D.Marrè, M.Putti, A.S.Siri, Physica C, 282, (1997).
[21] S.J.Hagen, A.W.Smith, M.Rajeswari, et al., Phys.Rev.B 47, 1064 (1993).
[22] Y.Matsuda and S.Komiyama et al., Phys.Rev.Lett, 69, 3228 (1992).
[23] J.Nagamatsu, N.Nakagawa, T.Muranaka, Y.Zenitani, and J.Akimitsu, Nature 410, 36 (2001).
[24] R.Jin, M.Paranthaman, H.Y.Zhai, H.M.Christen, D.K.Christen, and D.Mandrus (unpublished).

FIG. 1. The theoretical curves of $\rho_{xy}$ as a function of reduced field $B/B_{c2}$ for several value of the parameter $\gamma$ ($0 < \gamma < 1$). The values of $\gamma$ from bottom to top are 0.8, 0.7, 0.6, 0.5, and 0.4.

FIG. 2. Empirical scaling functions for $\rho_{xy}$ vs. scaled magnetic field for various values of temperature. The experimental data (a)-(d) are from A.W.Smith et al. (Ref.[9] Fig.1) and (e), (f) are from J.Luo et al. (Ref.[19] Fig.1) and M.R.Cimberle et al. (Ref.[20]), respectively. The solid curve (g) is theoretical result obtained from Eq.(13). The comparison between the theoretical curve and experimental data shows a fair agreement.

FIG. 3. Empirical scaling functions for $\rho_{xy}$ vs scaled temperature for various values of magnetic fields. The experimental data (a)-(d) are from S.J.Hagen et al. (Ref.[21] Fig.1) and (e)-(j) are from Y.Matsuda et al. (Ref.[22] Fig.2 and Fig.3). The solid curve (k) is theoretical result obtained from Eq.(13). The comparison between the theoretical curve and experimental data shows a fair agreement.