A QUANTUM ANALYSIS ON RECOMBINATION OF D-BRANES AND ITS IMPLICATIONS FOR AN INFLATION MODEL

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A quantum-mechanical technique is used within the framework of U(2) super-Yang-Mills theory to investigate processes after recombination of two D-p-branes at one angle. Two types of initial conditions are considered, one of which with \( p = 4 \) is a candidate of inflation mechanism. It is shown that the branes’ shapes come to have three extremes due to localization of tachyon condensation. “Pair-creations” of open strings connecting the recombined branes is also observed. The appearance of closed strings is also discussed; the decaying branes are shown to radiate non-vanishing gravitational wave, which may be interpreted as evidence of closed string appearance. A few speculations are also given on implications of the above phenomena for an inflation model.

This work is based on my talk on July 31 in the conference “String Phenomenology 2003”. More detail is presented in ref. [1] (to be revised at that time).

Recombination of D-branes at angles is a process increasing its value from both phenomenological and theoretical point of view, related to the presence of tachyonic modes which appear as modes of open strings between the two D-branes [2]. In the brane inflation models, these are some of the promising candidates of the mechanism of inflation [3, 4, 5, 6]. Intersecting brane models are also, among various string constructions of Standard Model, one class of hopeful candidates [7, 8, 9], in some of which it has been proposed that the recombination process occurs as Higgs mechanism [8, 10]. On the other hand, this system and process can be regarded as a generalized setting of \( D \bar{D} \) system and its annihilation process, which has been studied thoroughly [11, 12, 13]. In this way, this is one of the most important phenomena to explore at present in string theory.

The behavior of the system after the recombination, however, had been almost unexamined until recently. Suppose one consider a recombination of two D-branes at one angle, wrapping around some cycles in a compact space. One might expect that after the recombination, each brane would begin to take a “shorter cut” and then make a damped oscillation around some stable configuration of branes, while radiating RR gauge and gravitational waves (and some others), leading to the stable configuration. However, the case was that only the shape of the recombined branes at the initial stage was merely inferred [19], though the final configuration can be determined in some cases [6].

Recently, K. Hashimoto and Nagaoka [14] proposed that a T-dual of super Yang-Mills theory (SYM) can be an adequate framework to describe the process, and in the case
of two D-strings at one angle and via classical analysis, they presumed how their shapes develop at the very initial stage.

The main purpose of this work is to investigate the process after the recombination via SYM “more rigorously” and to reveal what happens in the process; we will focus on the two points: time-evolution of the D-branes’ shape, and behavior (appearance) of fundamental strings. The former can be of value in that we describe time-dependent, i.e. dynamical behavior of curved D-branes though for rather short time (which enables us to discuss e.g. gravitational waves radiated by the branes). The latter point is what happens in the process itself. What we mean by “more rigorously” is that we set concrete initial conditions and make a quantum analysis: These are necessary steps in order to understand precise behavior of the system, because the system, its tachyon sector in particular, is essentially a collection of inverse harmonic oscillators, and its behavior depends crucially on the initial fluctuations of the system, which can only be evaluated appropriately via a quantum analysis.

The second purpose of this work is to discuss implications of the above two points (phenomena), for the inflation scenario of the setting that two D-4-branes are approaching each other at one angle as in ref. [4], [5], aiming at the understanding of the whole process of the scenario.

For the above two purposes, we consider two types of initial conditions for D-p-branes for each $p \geq 1$. In order to make a quantum analysis, we need the fact that all the modes have positive frequency-squareds at $t = 0$. So, the first one (denoted as case (I)) is that two D-p-branes have been overlapping until $t = 0$ but are put intersected at one angle $\theta$ at the instant $t = 0$. The second one (the case (II)) is that one of parallel two D-p-branes were rotated by a small angle and are approaching each other very slowly. The case (I) is one of the simplest conditions, rather easier to study the process itself. The case (II) is more practical; the case with $p = 4$ is one of the hopeful setting for the brane inflation scenarios [4, 5].

Our notation and set-ups are; we set $l_s = 1$ (and revive it when needed), and $g_s \ll 1$, and consider D-p-branes in a 10D flat spacetime with coordinates $x_a \ (a = 0, \ldots, 9)$. The world-volumes are parametrized by $x_0, x_1, \ldots, x_p$. We compactify space dimensions $x_p, \ldots, x_9$ on a (10-p)-torus with periods $L_a$ for $a = p, \ldots, 9$. We denote $x_0$ as $t$ and $x_p$ as $x$ below. The simplest setting of the initial condition for each case is represented in Fig.1

![Figure 1: The initial configuration of D-p-branes (z(t) = 0 for case (I))](image)

*Recently, the process is also analyzed via tachyon effective field theory in ref. [15] and very recently, subsequent work of ref. [14] was done by K. Hashimoto and W. Taylor. (Refer to K. Hashimoto’s talk in Strings 2003.)
The framework is a T-dual of SYM, a low energy effective theory of open strings around the D-branes when distances between the branes are small, as in ref.\[14\]. We denote the U(2) gauge field by $A_\mu$ for $\mu = 0, 1, \ldots, p$ and adjoint scalar fields corresponding to $x_i$, transverse to the branes, as $X_i$ for $i = p + 1, \ldots, 9$. We note that it holds $L_p < \frac{1}{\beta}$ to satisfy the condition that the displacements of branes are smaller than $l_s (= 1)$. The “background” D-p-branes for each case are represented by\[17\]:

$$X_{p+1} = \begin{pmatrix} \beta x & 0 \\ 0 & -\beta x \end{pmatrix}, \quad X_9 = \begin{pmatrix} z/2 & 0 \\ 0 & -z/2 \end{pmatrix}, \quad X_{p+2} = \cdots = X_8 = A_\mu = 0. \quad (0.1)$$

where $\beta \equiv \tan(\theta/2)$. For the case (I) ($z(t) = 0$), this is T-dual to the configuration of two D-(p+1)-brane with a constant flux $F_{p,p+1} = \beta$, in which tachyonic modes were shown to appear in off-diagonal ones of $A_p$ and $A_{p+1}\,[16, 17]$. For case (II) ($z(t) \neq 0$), since the force between the branes are so weak in this setting\[18\], we approximate $z = z_0 - vt$ for a constant $v$. In both cases we can show that potentially tachyonic modes appear in the non-diagonal elements of $A_p$ and $X_{p+1}$. So, we denote the fluctuations as

$$A_\mu = \begin{pmatrix} 0 & c_\mu^* \\ c_\mu & 0 \end{pmatrix}, \quad A_p = \begin{pmatrix} 0 & c^* \\ c & 0 \end{pmatrix}, \quad X_{p+1} = \begin{pmatrix} 0 & d^* \\ d & 0 \end{pmatrix} \quad (0.2)$$

where $\mu = 0, 1, \ldots, p - 1$.

We evaluate time-evolution of typical values of the tachyonic fluctuations in the following; we define typical values of the fluctuations at time $t$ using VEV’s of squares of the fluctuations, and make mode-expansion of them to diagonalize their second order terms in SYM action, using $u_k(x_\mu) \equiv e^{i k_\mu x_\mu}$ as for $x_\mu$ ($\mu = 1, \ldots, p - 1$) and $f_n \propto e^{-\beta x^2} H_n(\sqrt{2 \beta} x)$ ($H_n$ is Hermite function) as for $x$. Then, we can show that only the modes of $\tilde{c}(x_\mu) \equiv \frac{c - i d}{\sqrt{2}}$ with $n = 0$ and the momentum-squared $k^2 \leq 2 \beta$ are tachyonic ones. So we obtain the typical value by using time-evolution of the wave function of $\tilde{c}_{0,k}$ and focusing only on the contribution of the tachyonic modes. We note that though we use second order approximation in the fluctuations, it is enough good until the tachyons blow up, because higher order terms are suppressed by a factor $O(g_s)$ since typical values of non-tachyonic modes are proportional to $1/\sqrt{T_{Dp}} = \sqrt{g_s}$. Defining $|c(x_\mu)|_{\text{typical}}^2 = |d(x_\mu)|_{\text{typical}}^2 \equiv (A(t) f_0)^2$, we have for case (I),

$$(A(t))^2 = \frac{g_s \Omega_{p-2}}{2(2\pi)^{p-1}} \int_0^{2\beta} k^{p-2} dk \left[ \frac{1}{k^2} + \frac{2\beta}{2\beta - k^2} \sinh^2(\sqrt{2\beta - k^2}t) \right] \quad (0.3)$$

for D-p-brane for $p \geq 2$ \footnote{$\frac{1}{\beta} \ll L_p$ is needed here to use $f_n$.}, where $\Omega_{p-2}$ is the volume of (p-2)-sphere and $f_0 = \sqrt{2\beta/\pi} e^{-2\beta x^2}$. For the case (II), the square bracket in (0.3) is replaced by a complicated function. We do

\footnote{For D-strings, no integration with respect to $k$ is included, and $k^2$ is replaced by some regularization scale $(z^{(0)})^2$.}
not present it here (see ref. [1]), but the important thing is that we have obtained explicit
functions of typical tachyonic fluctuations for each initial condition. We note that in both
cases, the tachyon condensation is localized around $x = 0$ with the width $1/\sqrt{\beta}$ due to $f_0$.

The information of the shape is obtained from a transverse U(2) scalar field by diagonalizing its VEV’s (i.e. choosing a certain gauge) and looking at its diagonal parts [14]. (For case (II), a non-commutativity between VEV’s of scalar fields appears, implying their uncertainty relation. We proceed with discussion by concentrating on a VEV of one field, leaving alone another.) Then, the formula for the shape of one of the recombined branes is

$$y \equiv \sqrt{(\beta x)^2 + |d|^2} = \sqrt{(\beta x)^2 + \frac{A(t)^2}{2} e^{-2\beta x^2}}. \quad (0.4)$$

This is the same form as obtained in ref. [14]. However, we can make a precise analysis on
the shape of the branes at arbitrary $t$ based on (0.4), since we have the explicit function of $A(t)$ for each case including the overall factor and fine coefficients; we can know explicitly when and which applicable condition of the approximations (SYM, WKB and some others) breaks.

Here, we show that a seemingly queer behavior occurs; the shape of each brane deviates
from the approximate hyperbola and comes to have three extremes as in Fig.2. This
happens for the cases of D-p-brane for $p \geq 2$, and is also expected to happen for the case
of D-strings, as we discuss below.\footnote{The appearance of such a shape was already discussed, but was denied in the case of D-strings in ref. [14].}

![Figure 2: Time-evolution of the shape of a D-brane after recombination](image)

The values of $x$ giving extremes of (0.4) are formally given by $x = 0, \pm \sqrt{\ln(A^4/\pi \beta)/4\beta}$, which means that each has three extremes when

$$A > (\pi \beta / 2)^{1/4} \equiv A_{\text{critical}}. \quad (0.5)$$

The most important condition to be checked is that of the WKB approximation: higher
order terms of the fluctuations in the action do not disturb the behavior. The condition gives

$$A(t) \ll \sqrt{(2\pi)^{p-1/2}/\Omega_{p-2}(2\beta)^{(p-2)/2}}. \quad (0.6)$$
We can easily see that for a small $\theta \simeq 2\beta$ and $p \geq 2$, the inequality (0.6) is sufficiently compatible with (0.5). The other conditions, especially that of SYM, that the displacement of branes should be smaller than $l_s$ ($A(t) < l_s$) and D-branes should have some low velocity are also shown easily to be satisfied. Therefore, the shape of the recombined D-p-brane for $p \geq 2$ surely come to have three extremes. This happens essentially due to the localization of tachyons around $x$, since the factor $e^{-2\beta x^2}$ in (0.4) is directly responsible for the multiple extremes. For the case of D-strings, the case is a bit more subtle, but if we solve numerically the equation of motion for $\tilde{c}_0(t)$ we can see that $|\tilde{c}_0(t)|$ really develop to exceed $A_{\text{critical}}$ in (0.5) before any approximations break.

The physical interpretation of this seemingly queer behavior is as follows; the energy released via tachyon condensation pushes the recombined branes away from each other, but it is given only to the local part of the branes around $x = 0$ due to localization of tachyon condensation, so, only the part is much accelerated. Though the D-branes have a large tension, they also have a large inertia, and when the given energy of the local part is large enough, the branes extend, surpassing the tension, to form three extremes. That is, localization of tachyon condensation or that of the released energy, and the (large) inertia causes the shapes of branes to have three extremes. We note that this is the “physical” gauge where the d.o.f. of each open string starting from and ending on certain branes directly corresponds to the d.o.f. of each U(2) matrix element, since only in this gauge each of the diagonal elements represents the position of each brane.

Next, we examine the behavior of open strings based the typical value of the electric flux since electric charges on D-branes correspond to ends of open strings attached to the D-branes[20]. We note that we have to study the flux in the physical gauge. An important thing is that even after choosing the “physical gauge” (in which $X_{p+1}$ is diagonalized), only the off-diagonal elements of the typical flux $F_{0p} = \partial_t A_p$ blow up, since it holds

\[
F_{0p}^{(\text{phys})} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & |\partial_t \tilde{c}|_{\text{typical}} e^{-i\epsilon'} \\ |\partial_t \tilde{c}|_{\text{typical}} e^{-i\epsilon'} & 0 \end{pmatrix}
\]  

(0.7)

where $e^{i\epsilon'}$ is a phase factor. The blow-up of the off-diagonal elements in this gauge means that fundamental strings connecting the two D-branes are vastly created.\footnote{This point was also discussed very recently by K. Hashimoto and W. Taylor in Hashimoto’s talk in “Strings 2003”} In addition, since the true VEV of (linear) $F_{0p}$ vanishes, so does the total electric charge. That is, these are effectively “pair-creations” of the strings. Thus, we conclude that vast number of open string pairs connecting the two D-branes are created after the recombination of D-branes. (For estimation of their typical number, see ref.[1].) We note that (at least at the early stage of the decay) about half the released energy is used to create the open string pairs, so its energy turns into that of the open strings rather rapidly.

Furthermore, we discuss (expect) the behavior of the system, in particular fundamental strings, beyond our approximations and SYM. If the distance between the D-branes becomes larger than $l_s$, it is difficult to imagine that the created open strings extend...
unlimitedly, because the string has its tension. It also seems unnatural if one take into account increase of entropy. Then, what will happen? Since open strings are created in pairs with no net NSNS gauge charges, it is expected that each pair of open strings connecting the branes are cut to pieces to form some closed strings and two open strings, each of the latter of which has its both ends on each of the two D-branes. That is, such a picture seems to arise that the decaying (annihilating) D-branes leave vast number of closed strings behind (and radiate some of them) while producing many open strings which start from and end on each brane. This may be regarded as a generalization of the picture for $D \bar{D}$ annihilation that only closed strings are left after the annihilation\[12].

Then, what evidence can we present to support the expectation? We can show that the recombined D-branes surely radiate gravitational wave; The angular distribution of the energy flux due to the radiation $d^2E/dtd\Omega = < (\frac{d^2I_{ij}}{dt^2})^2 - 2(n_i \frac{d^3x}{dt^3})^2 + \frac{2}{8} (n_i n_j \frac{d^3x}{dt^3})^2 >$ in the direction $n_i$ is shown to be non-vanishing when $n_i$ is perpendicular to the $x_px_{p+1}$-plane. $I_{ij}$ is the mass quadrupole moment of the D-branes

$$I_{ij} = T_p \sum_{A=1}^{2} \int d^p x \sqrt{1 + (\partial_x x^A)^2} [x^A_i x^A_j - \frac{1}{9} \delta_{ij} (x^A_k)^2]$$  \hspace{1cm} (0.8)

where $A$ denotes each of the branes and $x_i$ in $I_{ij}$ is to be evaluated at the retarded time. Gravitational waves should be represented by closed strings in string theory, so this may be regarded as evidence of appearance of closed strings. Since we know time-evolution behavior of the D-branes, we can compute it within some approximations. (See revised version of ref.[1] more detail.)

Finally, we speculate on implications of the above results for a brane inflation scenario. Let us suppose that either of the following properties is (would be) a generic one (though it might be a too optimistic extrapolation): (i) Decaying D-branes continue to create vast number of open string pairs, or (ii) they would keep leaving vast number of closed strings behind while producing open strings on each brane, in both cases using some amount of the released energy. Then, it might be expected that the dissipation of the energy released via tachyon condensation would be rather large, i.e. damping factor would be large and only a few times of, or no oscillations would occur. Thus, if this setting is applied to the inflation scenario, reheating might be efficient. Furthermore, in the inflation model, (reheated) fermion and gauge fields have to be produced, but the mechanism how they are produced in this setting of the inflation model has not been clarified. We speculate about a scenario of the mechanism based on our analysis: vast number of creations of the open string pairs connecting the branes and the open strings on each of the branes might directly correspond to the fields. It would be interesting to find a framework describing the process in the region of distance other than SYM or beyond SYM, and discuss the above possibilities.
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