Supersymmetry Breakdown
at Distant Branes:
The Super–Higgs Mechanism

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Abstract

A compactification of 11-dimensional supergravity with two (or more) walls is considered. The whole tower of massive Kaluza-Klein modes along the fifth dimension is taken into account. With the sources on the walls, an explicit composition in terms of Kaluza-Klein modes of massless gravitino (in the supersymmetry preserving case) and massive gravitino (in the supersymmetry breaking case) is obtained. The super–Higgs effect is discussed in detail.

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1 Introduction

The M–theoretic extension of the heterotic $E_8 \times E_8$ string leads to a geometric picture of two walls (branes) at the ends of a finite 11-dimensional interval [1]. While the supergravity multiplet can penetrate in the $d = 11$ bulk, the two $E_8$ gauge multiplets are confined to the two walls, respectively. When further six of the dimensions are compactified one can construct models with gauge fields living on the $d = 4$ walls while gravity could, in addition, extend to the higher dimensional interval [2]. This could then be viewed as the M-theoretic generalization of hidden sector supergravity models [3]. These models typically contain two sectors, an observable sector that contains the usual fields like quarks, leptons and gauge bosons as well as their supersymmetric partners and a hidden sector, coupled to the observable sector via interactions of gravitational strength. Here the two sectors can now be identified with the gauge systems that live on the two separated walls. Such a picture is common in modern string-brane theories. The role of the walls is in general played by higher dimensional p-branes that support gauge groups, while gravitational interactions can communicate between spatially separatored branes. In type I theories we have e.g. D-branes with gauge bosons originating from open string whose ends are confined to the (stack of coincident) D-branes, while closed strings (and thus gravitational interactions) can live in the bulk.

The hidden sector of the above mentioned supergravity models was supposed to be responsible for the breakdown of supersymmetry [4]. This breakdown of supersymmetry was transmitted to the observable sector via gravitational interactions. If the breakdown originated through the vacuum expectation value of an auxiliary field of size $F = M_S^2$, the value of the gravitino mass is given by

$$m_{3/2} \sim \frac{M_S^2}{M_{\text{Planck}}},$$

where the Planck mass represents the suppression due to the gravitational interactions, and the size of the soft supersymmetry breaking terms in the observable sector was given by the gravitino mass.

In the modern picture one would now assume that supersymmetry is broken at a hidden wall [5] and the transmission of that breakdown to the observable wall is mediated via bulk fields. The size of supersymmetry breakdown in the observable sector would be suppressed for widely separated walls. Naively one
might have assumed that the new picture would lead to a value of the gravitino mass even more suppressed than in the classical case \([1]\). A closer inspection, however, shows a similar suppression \([6, 7]\)

\[
m_{3/2} \sim \frac{M_S^2}{R M_D^2} \sim \frac{M_S^2}{M_{\text{Planck}}},
\]

(2)

once the distance \(R\) between the branes and the the higher dimensional Planck mass \(M_D\) are adjusted to fit the correct value of the \(d = 4\) Planck mass \(M_{\text{Planck}}\).

The estimate of the gravitino mass in (2) was obtained \([6, 8, 9]\) using a the simplified approximation according to which the higher dimensional bulk fields were integrated out via an averaging procedure\(^1\). In this picture, the goldstino mode was represented by the lowest Kaluza–Klein \(\Psi_0\) mode of a higher dimensional field \(\Psi\). In the super–Higgs mechanism this mode supplies the additional degrees of freedom to render the gravitino massive. Qualitatively this simplified approximation does give a consistent picture, but there remain some open questions and potential problems when one looks into details of the super–Higgs mechanism. In this paper we would like to point out these potential problems and show how they can be resolved. The open questions will be presented in the following section. In section 3 we shall then discuss the gravitino in the case of unbroken supersymmetry in full generality. Broken supersymmetry and the super–Higgs mechanism will be analysed in section 4. In the following section we shall discuss the consequences of our analysis. This will include a discussion of the possible nature of the goldstino (is it a bulk or a wall field), the relation to the Scherk-Schwarz mechanism \([12]\) in that context \([13]\) and an upper limit for the gravitino mass in the present picture. We shall argue that a meaningful realization of the super–Higgs mechanism seems to require some modes in the bulk other than the graviton and the gravitino. Finally we shall comment on the phenomenological consequences of this findings, including a discussion of the nature of the soft breaking terms on both walls.

### 2 Some open questions and puzzles

Specifically we want to address the following two questions:

\(^1\)A corresponding analysis in global supersymmetry has been performed in ref. \([10]\). Related work in the supergravity case has been given in \([11]\).
(i) the nature of the massless gravitino in the presence of several $F$–terms on different walls that cancel and lead to unbroken supersymmetry

(ii) the identification of the goldstino in the case of broken supersymmetry.

The first question (i) arises because of a particular nonlocal effect of supersymmetry breakdown first observed by Hořava [5]. A given source of supersymmetry breakdown (parametrized by a vacuum expectation value (vev) of an auxiliary field $F$) on one wall could be compensated by a similar but opposite value ($-F$) on another (separated) wall. Any calculation and approximation of the system thus has to reproduce this behaviour. The previously mentioned averaging procedure over the bulk distance does this in a trivial way, leading to unbroken supersymmetry as expected. A detailed inspection of the gravitino, however, reveals a problem. If we start with the situation $F = 0$ it is easy to define the massless gravitino $\Psi_0$ in the $d = 4$ theory. Switching on a nontrivial $F$ on one brane and ($-F$) on the other still should give a massless gravitino, but $\Psi_0$ turns out to be no longer a mass eigenstate. The resolution of this problem and the correct identification of the gravitino will be given in section 3. It is a particular combination of the possible gravitini that appear when one, for example, reduces a 5-dimensional theory to a theory in $d = 4$ on a finite $d = 5$ interval. The theory on a $d = 5$ circle would lead to $N = 2$ supersymmetry in $d = 4$ and two massless gravitini (zero modes on the circle). The $Z_2$ projection on the interval removes one of the gravitini and is $N = 1$ supersymmetric. A nonvanishing vev of $F$ now interferes with the boundary conditions and the massless gravitino will be a linear combination of the zero mode and all the excited KK modes whose coefficients will depend on $F$ (assuming, of course, unbroken supersymmetry due to a compensating vev $-F$ on another wall).

The second question (ii) deals with the nature of the goldstino (i.e. the longitudinal components of the gravitino) in the case of broken supersymmetry. Remember that the simplified averaging procedure leads to a goldstino that corresponds to the lowest Kaluza–Klein mode $\Psi_0$ of a higher-dimensional bulk field $\Psi$. Inspecting the gravitino mass matrix in this case reveals the fact that this field $\Psi_0$ is not a mass eigenstate, but mixes with infinitely many higher Kaluza–Klein modes.

\footnote{In this paper we generically use the notation $F$–term for the source of supersymmetry breakdown. Depending on the specific situation this could represent a $D$–term or a gaugino condensate as well.}
modes $\Psi_n$. A consistent manifestation of a super–Higgs mechanism would require a diagonalization of this mass matrix and an identification of the goldstino. This problem, that has not yet been addressed in the literature, will be solved in section 4.

This resolution of the puzzles clarifies some of the other questions of the approach.

- The nonlocality of the breakdown shows some resemblance to the breakdown of supersymmetry via the Scherk–Schwarz \cite{12} mechanism. Here, however, the real goldstino of the spontaneous breakdown of supersymmetry can be unambiguously identified.

- The possibility to cancel the supersymmetry breakdown on a distant wall by a vev on the local wall tells us, that the mass splittings of broken supersymmetry have to be of order of the gravitino mass $m_{3/2}$ on both walls.

- In terms of the physical quantities there is no real extra suppression, once we separate the walls by a large distance $R$. In the limit $R \to \infty$ we will have $M_{\text{Planck}} \to \infty$ as well. The suppression of the soft breaking parameters will always be gravitational, as given in (2).

- In general, when we have a system of many separated branes with potential sources of supersymmetry breakdown, the actual breakdown will be obtained by the sum of these contributions. The averaging procedure will be very useful to decide whether supersymmetry is broken or not. The identification of the goldstino, however, is more difficult and requires a careful calculation.

- A successful implementation of the super–Higgs mechanism will require some fields other than gravitino and graviton in the bulk\cite{13}. This implies that in the absence of such fields (as has been considered in \cite{14}) a consistent spontaneous breakdown of supergravity might not be achieved.

In the following sections we will show how the goldstino and gravitino can be defined in the correct way. We shall do the explicit calculations in the framework of the heterotic M-theory, although a similar calculation will apply under more

\footnote{Usually they arise as modes of the higher dimensional supergravity multiplet.}
general circumstances (like the inclusion of 5-branes or the consideration of multi-D-brane systems in Type I theory), which we shall briefly discuss in section 5. As the source of supersymmetry we consider the mechanism of gaugino condensation. Again this just should represent a generic breakdown of supersymmetry in this specific example.

3 Gravitino in the case of unbroken supersymmetry

The low energy limit of the heterotic M–theory is given by the following lagrangian

\[ \mathcal{L} = \frac{1}{\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g} \left[ \frac{-1}{2} R - \frac{1}{2} \bar{\Psi}_l \Gamma^{IJK} D_J \left( \frac{\Omega + \hat{\Omega}}{2} \right) \Psi_K - \frac{1}{48} \hat{G}_{IJKL} \hat{G}^{IJKL} ight. 
\]

\[ \left. - \frac{\sqrt{2}}{384} \left( \bar{\Psi}_I \Gamma^{IJJKLMN} \Psi_N + 12 \bar{\Psi}_I \Gamma^{KL} \Psi^M \right) \left( \hat{G}_{JKLM} + \hat{G}_{JKLM} \right) 
\]

\[ \left. - \frac{\sqrt{2}}{3456} \epsilon_{I_1 I_2 \ldots I_{11}} \hat{C}_{I_1 I_2 I_3} \hat{G}_{I_4 \ldots I_7} \hat{G}_{I_8 \ldots I_{11}} \right] \]

\[ + \frac{1}{4\pi (4\pi \kappa^2)^{2/3}} \sum_{i=1}^2 \int_{M^{10}} d^{10}x \sqrt{g} \left[ -\frac{1}{4} F_{iAB}^a F_i^{a,AB} - \frac{1}{2} \hat{\chi}_i^a \Gamma^A D_A (\hat{\Omega}) \hat{\chi}_i^a 
\]

\[ - \frac{1}{8} \bar{\Psi}_A \Gamma^{BC} \Gamma^A \left( F_{iBC}^a + \hat{F}_{iBC}^a \right) \chi_i^a + \frac{\sqrt{2}}{48} \left( \bar{\chi}_i^a \Gamma^{ABC} \chi_i^a \right) \hat{G}^{ABC \ldots 11} \right]. \]

where \( I, J, K, \ldots = 1, 2, \ldots, 11; \ A, B, C, \ldots = 1, 2, \ldots, 10; \) and \( i = 1, 2 \) counts the 10–dimensional boundaries (walls) of the space. The first integral describes the supergravity in the 11–dimensional bulk while the second one describes interactions with the super Yang–Mills fields living on two 10–dimensional walls. Our signature is \((-\ldots, +, \ldots, +)\). In the above lagrangian only the two first terms in the long wavelength expansion are kept. They are of relative order \( \kappa^{2/3} \). All higher order terms (order \( \kappa^{4/3} \) or higher) will be consistently dropped in this paper.

We work in the upstairs approach in which the 11–dimensional integrals are defined as

\[ \int d^{11}x = \frac{1}{2} \int_{-\pi \rho}^{\pi \rho} dx^{11} \int d^{10}x. \]
and we use the $Z_2$ symmetry conditions:

$$\Psi_A(-x^{11}) = +\Gamma^{11} \Psi_A(x^{11})$$
$$\Psi_{11}(-x^{11}) = -\Gamma^{11} \Psi_{11}(x^{11})$$
$$\tilde{G}_{ABCD}(-x^{11}) = -\tilde{G}_{ABCD}(x^{11})$$
$$\tilde{G}_{BCD11}(-x^{11}) = +\tilde{G}_{BCD11}(x^{11})$$  \hspace{1cm} (5)

The Bianchi identity in this approach is modified on both walls:

$$d\tilde{G}_{11ABCD} = -\frac{3\sqrt{2}}{2\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \sum_{i=1}^{2} \delta(x^{11} - x^{11}_i) \left( \text{tr} F_{i[AB} F_{iCD]} - \frac{1}{2} R_{[AB} R_{CD]} - \frac{1}{2} R_{[AB} R_{CD]} \right). \hspace{1cm} (6)$$

where $x^{11}_i$ is a position of the $i$–th wall.

The fields $\Psi_\mu$ and $\Psi_{11}$ as 11–dimensional Majorana spinors have 32 components. Imposing $SU(3)$ invariance on Calabi–Yau reduces this number to 8 components – they can be assembled into two sets with 4 components each distinguished by 10-dimensional chirality. Some of the formulae given below are valid only after imposing $SU(3)$ invariance but we treat all spinors as 11–dimensional along the way. Only at the very end, after compactification to 4 dimensions we assemble each set into a 4–dimensional Majorana spinor to give the final formula for the effective 4–dimensional action for spinors.

This theory is defined in 11 dimensions so 7 dimensions must be compact. The spacetime is given (in the lowest approximation) by the product: $M^4 \times X^6 \times S^1/Z_2$ where $X^6$ is a Calabi–Yau manifold and $S^1/Z_2$ is the interval between the two walls. For simplicity we will use a truncation and reduction method instead of compatifying on a Calabi–Yau manifold. The 11–dimensional metric is given in this case by

$$g^{(11)}_{MN} = \begin{pmatrix} e^{-\gamma} e^{-2\sigma} g_{\mu\nu} & e^\sigma g_{mn} \\ e^\sigma g_{mn} & e^{2\gamma} e^{-2\sigma} \end{pmatrix} \hspace{1cm} (7)$$

At order $\kappa^{2/3}$ the moduli $\gamma$ and $\sigma$ depend linearly on $x^{11}$ and the 11–dimensional spacetime is no longer a direct product of the three factors.

\footnote{In the downstairs approach we would use

$$\int d^{11}x = \int_0^{\pi} d\rho \int d^{10}x$$

and the appropriate boundary conditions instead of $Z_2$ symmetry.}
Let us now describe a process of reduction of the theory to 4 dimensions. We will be mainly interested in the fermion fields since our main goal is to identify the 4–dimensional (massless or massive, depending on supersymmetry breaking) gravitino. It turns out that in the resulting lagrangian all the fermion fields are mixed. To diagonalize that lagrangian one has to make a number of field redefinitions. In order to make the result of the calculation more transparent we will perform the appropriate redefinitions (necessary to get the final result in a diagonal form) step by step.

Let us start with the following redefinition of the gravitino fields

\[ \Psi_\mu = e^{-\gamma/4} e^{-\sigma/2} \left( \psi_\mu + \frac{1}{\sqrt{6}} \Gamma_\mu \psi_{11} \right) \]
\[ \Psi_{11} = -\frac{2}{\sqrt{6}} e^{5\gamma/4} e^{-\sigma/2} \Gamma^{11} \psi_{11} . \]  

As a result the kinetic term of the lagrangian (3) reads:

\[ -\frac{1}{2} \varepsilon_{11} \nabla_I \Gamma^{IJK} D_J \left( \frac{\Omega + \hat{\Omega}}{2} \right) \Psi_K = \]
\[ -\frac{1}{2} \varepsilon_4 \bar{\psi}_\mu \Gamma^{\mu\rho} \psi_{\rho} - \frac{1}{2} \varepsilon_4 \bar{\psi}_{11} \Gamma^{\mu} \psi_{11} \]
\[ + \frac{1}{2} e_4 e^{-3\gamma/2} \bar{\psi}_\mu \Gamma^{\mu} \Gamma^{11} \partial_{11} \psi_{11} - \frac{\sqrt{6}}{4} e_4 e^{-3\gamma/2} \psi_{11} \Gamma^{11} \partial_{11} \psi_{11} \]
\[ - \frac{\sqrt{6}}{4} e_4 e^{-3\gamma/2} \psi_\mu \Gamma^{\mu} \Gamma^{11} \partial_{11} \psi_{11} + e_4 e^{-3\gamma/2} \psi_{11} \Gamma^{11} \partial_{11} \psi_{11} \]
\[ - \frac{\sqrt{6}}{4} e_4 e^{-3\gamma/2} \psi_{11} \Gamma^{\mu} \psi_{11} \partial_{\mu} \gamma + \frac{\sqrt{6}}{4} e_4 \psi_{11} \Gamma^{11} \partial_{11} \gamma \]  

Let us introduce

\[ \psi_\mu = \psi_\mu^+(x^{11}) + \psi_\mu^-(x^{11}) \]
\[ \psi_{11} = \psi_{11}^-(x^{11}) + \psi_{11}^+(x^{11}) \]  

(\text{the signs } "^+" \text{ and } "^-" \text{ denote the chirality with respect to } \Gamma^{11}). \text{ The relations } (3) \text{ show that}

\[ \psi_\mu^+(-x^{11}) = +\psi_\mu^+(x^{11}) \]
\[ \psi_\mu^-(-x^{11}) = -\psi_\mu^-(x^{11}) \]
\[ \psi_{11}^+(-x^{11}) = -\psi_{11}^+(x^{11}) \]
\[ \psi_{11}^-(-x^{11}) = +\psi_{11}^-(x^{11}) \]  

(11)
Therefore the zero modes are possible only for $\psi_\mu^+$ and $\psi_{11}^-$. All fields have an implicit dependence on $x^\mu$ and we will often omit $x^{11}$ dependence of the fields.

Let us now consider switching on vev of the tensor field $\tilde{G}$. As explained in [2] due to the nonzero r.h.s. of eq. (6) fields $\tilde{G}_{a\bar{a}b\bar{c}d}$ (where $a,\ldots$ are holomorphic and $\bar{a},\ldots$ are antiholomorphic indices on $X^6$) acquire nonzero vev satisfying

$$\langle \Gamma^{a\bar{a}b\bar{c}d} \tilde{G}_{a\bar{a}b\bar{c}d} \rangle \psi = -\frac{\alpha}{2} \epsilon(x^{11}) \psi$$

on any $SU(3)$ invariant spinor $\psi$ (with arbitrary chirality). The parameter $\alpha$ is given by

$$\omega^{AB} \omega^{CD} \tilde{G}_{ABCD} = 4 \omega^{a\bar{a}} \omega^{\bar{a}\bar{b}} \tilde{G}_{a\bar{a}b\bar{b}} = \alpha.$$  (13)

In the case of unbroken supersymmetry there is a relation between $\alpha$ and the slopes of $\gamma(x^{11})$ and $\sigma(x^{11})$ [2, 7]:

$$\partial_{11} \gamma = -\partial_{11} \sigma = \frac{\sqrt{2}}{24} \alpha \epsilon(x^{11})$$  (14)

In order to get the canonical kinetic terms for $G$ we make the following redefinitions:

$$\tilde{G}_{a\bar{a}b\bar{c}} = e^{3/2} e^{3\gamma/2} \left(G_{a\bar{a}b\bar{c}} - \frac{\sqrt{3}}{3} \bar{\psi}_{11} \Gamma^\mu \Gamma_{a\bar{a}b\bar{c}} \psi^-_\mu - \frac{2}{3} \bar{\psi}_{11} \Gamma_{a\bar{a}b\bar{c}} \psi^-_{11}\right)$$

$$\tilde{G}_{11a\bar{b}c} = e^{3\gamma/2} e^{3\sigma/2} G_{11a\bar{b}c}$$

$$\tilde{G}_{11\bar{a}\bar{b}c} = e^{3\gamma/2} e^{3\sigma/2} G_{11\bar{a}\bar{b}c}$$

(15)

The unusual additional terms in the redefinition of $\tilde{G}_{a\bar{a}b\bar{c}}$ are necessary to cancel some of the fermion nondiagonal terms (as will be shown after eq. (17)).

In this paper we keep track of only $(2,2,0)$, $(3,0,1)$ and $(0,3,1)$ components of $<G>$. It was shown by Witten [2] that the presence of the vacuum expectation value for the $(2,2,0)$ component of $G$ does not break supersymmetry when the functions $\gamma$ and $\sigma$ have definite dependence on $x^{11}$ (see eq. (14)). The presence of the $(3,0,1)$ and $(0,3,1)$ components of $<G>$ located on the walls generically breaks supersymmetry and in the next section we will provide the explicit formula for the mass of the gravitino, its expansion in the Kaluza–Klein modes and the disappearance of one spin 1/2 state (super–Higgs mechanism). In this section we will consider the case of unbroken supersymmetry discussed by Hořava [5]. We now need the couplings of $<G>$ to the fermion fields

$$-\frac{\sqrt{2}}{192} e_{11} \bar{\Psi}_J \Gamma^{JKLNM} \Psi_J \tilde{G}_{KLNM} =$$
Using the redefinition (13) and the vacuum expectation value for $G^{\alpha\bar{\beta}\bar{\beta}}$ (12) we rewrite the kinetic term for $\tilde{\psi}$:

$$-\frac{\sqrt{2}}{96} e_4 \left( 3\bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta\gamma\delta} \psi_\nu + \sqrt{6} \bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta\gamma\delta} \psi_{11x} + 2\bar{\psi}_{11\mu} \Gamma^{\alpha\beta\gamma\delta} \psi_{11\nu} \right) <G_{\alpha\beta\gamma\delta}>$$

$$-\frac{2\sqrt{2}}{96} e_4 \left( \bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_\nu + \sqrt{6} \bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_{11x} - 2\bar{\psi}_{11\mu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_{11\nu} \right) <G_{\alpha\beta11x}>$$

$$-\frac{2\sqrt{2}}{96} e_4 \left( \bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_\nu + \sqrt{6} \bar{\psi}_\mu \Gamma^{\mu\nu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_{11x} - 2\bar{\psi}_{11\mu} \Gamma^{\alpha\beta} \Gamma^{11x} \psi_{11\nu} \right) <G_{\alpha\beta11x}>$$

$$+ \ldots \quad (16)$$

Terms with the fermion fields in the above formula are necessary to cancel some of the nondiagonal terms coming from (9) and (17). With all these redefinitions we are now ready to evaluate the sum of (9), (16) and (17) with the vacuum expectation value for $G_{\alpha\beta\gamma\delta}^{11x}$ (12) – it is the final result for the case considered by Witten. Using also (14) and (15) we get the following terms bilinear in the fermionic fields

$$\mathcal{L} = -\frac{1}{2} e_4 \bar{\psi}_\mu \Gamma^{\mu\nu} D_\nu \psi_\rho - \frac{1}{2} e_4 \bar{\psi}_\mu \Gamma^{\mu\nu} D_\nu \psi_\rho - \frac{1}{2} e_4 \bar{\psi}_{11\mu} \Gamma^{\mu\nu} D_\nu \psi_{11\rho} - \frac{1}{2} e_4 \bar{\psi}_{11\mu} \Gamma^{\mu\nu} D_\nu \psi_{11\rho} + e_4 e^{-3\gamma/2} \bar{\psi}_\mu \Gamma^{\mu\nu} \partial_\nu \psi_\rho - \frac{\sqrt{6}}{2} e_4 e^{-3\gamma/2} \bar{\psi}_\mu \Gamma^{\mu\nu} \partial_\nu \psi_\rho + \frac{\sqrt{6}}{2} e_4 e^{-3\gamma/2} \bar{\psi}_{11\mu} \Gamma^{\mu\nu} \partial_\nu \psi_{11\rho} - 2 e_4 e^{-3\gamma/2} \bar{\psi}_{11\mu} \Gamma^{\mu\nu} \partial_\nu \psi_{11\rho} + \ldots \quad (18)$$

Since the eleventh dimension is compact we can make the Fourier expansion of the fields:

$$\psi_\mu(x^\mu, x^{11}) = \psi_\mu^{0+} + \sqrt{2} \sum_{n=1}^\infty \psi_\mu^{n+} \cos(nx^{11}/\rho) + \sqrt{2} \sum_{n=1}^\infty \psi_\mu^{n-} \sin(nx^{11}/\rho),$$

$$\psi_{11}(x^\mu, x^{11}) = \psi_{11}^{0+} + \sqrt{2} \sum_{n=1}^\infty \psi_{11}^{n+} \cos(nx^{11}/\rho) + \sqrt{2} \sum_{n=1}^\infty \psi_{11}^{n-} \sin(nx^{11}/\rho), \quad (19)$$

where the coefficients of the expansion depend only on $x^\mu$ (so they correspond to 4–dimensional spinor fields).
Substituting this into eq. (18) and integrating over $x^{11}$ we can see that $\psi_{\mu}^{0+}$ and $\psi_{11}^{0-}$ are massless – therefore they correspond to the 4-dimensional gravitino and the massless spin 1/2 fermion fields. The remaining fields form the usual infinite tower of KK modes with masses equal to $n/\rho$. This is very similar to the standard Kaluza–Klein reduction. The only difference is that we had to redefine the tensor field $\tilde{G}$ (15) in order to remove some nondiagonal kinetic terms from the lagrangian.

Let us now include a nonvanishing vev of $G_{abc11}$ and $G_{\bar{a}\bar{b}\bar{c}11}$ fields. Such vevs can be generated for example by condensation of gaugino fields living on the walls. In such a case the vev of $G_{abc11}$ on the wall at $x^{11} = 0$ is given by:

$$\langle G_{abc11} \rangle = \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta \left( x^{11} \right) \langle \chi \Gamma_{abc} \chi \rangle$$

An analogous formula holds for $G_{\bar{a}\bar{b}\bar{c}11}$ and other walls.

In the case of two walls the most general formula reads:

$$\langle G_{abc11} \rangle = 6 \varepsilon_{abc} \left[ G_{+} \left( \delta \left( x^{11} \right) + \delta \left( x^{11} - \pi \rho \right) \right) + G_{-} \left( \delta \left( x^{11} \right) - \delta \left( x^{11} - \pi \rho \right) \right) \right]$$

Hořava [5] discussed the case when the supersymmetry remains unbroken i.e. when $G_{+} = 0$. Let us try to see one manifestation of the unbroken supersymmetry i.e. the massless gravitino in terms of the usual KK modes. Using the Fourier expansion (19) and putting the nonzero vev $G_{-}$ into eqs. (16) and (17) we get terms which mix different modes

$$G_{-} \left[ -\psi_{\mu}^{0+} \sum_{k=1} \left( \frac{1}{2} \Gamma_{\mu} \Gamma_{Y} \psi_{\nu}^{(2k-1)+} + \frac{\sqrt{6}}{4} \Gamma_{\mu} \Gamma_{Y} \psi_{11}^{(2k-1)-} \right) \right]$$

and similarly for $G_{+}$. One can obtain the fermions with definite masses by diagonalization of the mass terms. This, however, requires a very tedious calculation.
It turns out that it is possible to identify the massless states in a much simpler way. Before calculating the mass terms for fermions we perform a rotation of the fields $\psi_\mu$ and $\psi_{11}$:

$$
\psi_\mu \rightarrow (1 + f(x^{11}) \Gamma_Y) \psi_\mu
$$

$$
\psi_{11} \rightarrow (1 - f(x^{11}) \Gamma_Y) \psi_{11}
$$

where

$$
f(x^{11}) = -\sqrt{2} \frac{G_-\epsilon(x^{11})}{8}
$$

Such a rotation makes the new zero mode a combination of the old zero mode and infinitely many excited KK modes. Evaluating the effect of (21) on (16) and taking into account the redefinition (24) we recover the lagrangian (18) but now in terms of the new fields. Hence the lagrangian (18) is the final result in the case considered by Hořava (but the fields are those obtained after the rotation (24)). Therefore, the zero modes of the rotated fields $\psi_\mu^+$ and $\psi_{11}^-$ are now the massless gravitino and the massless spin 1/2 fields.

It is easy now to find an explicit form of the massless fermions in terms of the old KK modes

$$
\psi_{gravitino}^\mu = \psi_{0+}^\mu - \sum_{k=1}^{\infty} \frac{G_-}{2(2k-1)\pi} \Gamma_Y \psi_{0+}^{(2k-1)-}
$$

$$
\psi_{(1/2)} = \psi_{0-}^{11} + \sum_{k=1}^{\infty} \frac{G_-}{2(2k-1)\pi} \Gamma_Y \psi_{0+}^{(2k-1)+}
$$

These fields are different from the constant modes which were massless in the case with vanishing $G_-$. This change of the massless fermions reflects the fact that the unbroken supersymmetry changes when we change the value of $G_-$. 

4 The super–Higgs mechanism

In order to break the supersymmetry in this scenario we have to assume that the condensates on opposite walls do not cancel ($G_+ \neq 0$ in the formula (21)). In this case the fermion mass matrix is even more complicated than in the case with only $G_-$ nonzero. The procedure of diagonalization is very tedious but it turns
out that, as before, it is much simpler to work in the 5–dimensional language. We can rotate the fields in the similar way as in (24):

\[
\psi_\mu \to \left(1 + f(x^{11}) \Gamma_Y \right) \psi_\mu
\]

\[
\psi_{11} \to \left(1 - f(x^{11}) \Gamma_Y \right) \psi_{11}
\]  

(27)

but now with

\[
f(x^{11}) = -\frac{\sqrt{2}}{8} (G_- + G_+) \epsilon(x^{11}) + \frac{\sqrt{2}}{4\pi \rho} G_+ x^{11}
\]  

(28)

Evaluating all the terms after the rotation (27) we get

\[
\mathcal{L} = -\frac{1}{2} e_4 \overline{\psi}_\mu^+ \Gamma^{\mu\nu} \psi_\nu^+ - \frac{1}{2} e_4 \overline{\psi}_\mu^- \Gamma^{\mu\nu} \psi_\nu^-
\]

\[
- \frac{1}{2} e_4 \overline{\psi}_{11}^+ \Gamma^\mu D_\mu \psi_{11}^+ - \frac{1}{2} e_4 \overline{\psi}_{11}^- \Gamma^\mu D_\mu \psi_{11}^-
\]

\[
+ \frac{\sqrt{6}}{2} e_4 e^{-3\gamma/2} \overline{\psi}_\mu^+ \Gamma^{\mu\nu} \partial_\nu \psi_\nu^+ - \frac{\sqrt{6}}{2} e_4 e^{-3\gamma/2} \overline{\psi}_{11}^+ \Gamma^\mu \partial_\nu \psi_\nu^+
\]

\[
+ \frac{\sqrt{6}}{2} e_4 e^{-3\gamma/2} \overline{\psi}_\mu^- \Gamma^{\mu\nu} \partial_\nu \psi_\nu^- - 2 e_4 e^{-3\gamma/2} \overline{\psi}_{11}^- \Gamma^\mu \partial_\nu \psi_\nu^-
\]

\[
- \frac{m}{2} e_4 \overline{\psi}_\mu^+ \Gamma^\mu \Gamma_Y \psi_{11}^+ + \frac{m}{2} e_4 \overline{\psi}_\mu^- \Gamma^\mu \Gamma_Y \psi_{11}^-- \frac{m\sqrt{6}}{2} e_4 \overline{\psi}_\mu^- \Gamma^\mu \Gamma_Y \psi_{11}^+
\]

\[
- \frac{m\sqrt{6}}{2} e_4 \overline{\psi}_{11}^+ \Gamma^\mu \Gamma_Y \psi_{11}^- + m e_4 \overline{\psi}_{11}^- \Gamma_Y \psi_{11}^+ - m e_4 \overline{\psi}_{11}^+ \Gamma_Y \psi_{11}^+ + \ldots
\]  

(29)

where

\[
m = \frac{\sqrt{2}}{4\pi \rho} G_+
\]  

(30)

is the mass of the lightest spin 3/2 state – the gravitino. Zero modes of the rotated fields are now the lowest–lying states and from the rotation (27) we can read off their composition in terms of the standard KK modes:

\[
\psi_{\mu}^{\text{gravitino}} = \psi^{0+}_\mu - \sum_{k=1}^\infty \frac{G_- + G_+}{4\pi k} \left[1 - (-1)^k - \frac{G_+}{2\pi k} (-1)^{k+1} \psi^{k+}_\mu
\]

\[
\psi_{11}^{\text{gravitino}} = \psi^{0-}_{11} + \sum_{k=1}^\infty \frac{G_- + G_+}{4\pi k} \left[1 - (-1)^k + \frac{G_+}{2\pi k} (-1)^{k+1} \psi^{k+}_{11}
\]

\[
\psi_{11}^{\text{goldstino}} = \psi^{0-}_{11} + \sum_{k=1}^\infty \frac{G_- + G_+}{4\pi k} \left[1 - (-1)^k + \frac{G_+}{2\pi k} (-1)^{k+1} \psi^{k+}_{11}
\]

A remark is in order here: the formula for the gravitino mass (30) is the same as in the naive approach (just taking the zero mode and not performing the rotation (27)). This mass is already a \(\kappa^{2/3}\) effect, so corrections of the next order in \(\kappa^{2/3}\) (or inversely proportional to \(M_{11}\)) must be dropped. In general we
could expect other corrections which should be kept e.g. proportional to \( \pi \rho \gg M^{-1} \). However, it turns out that no such corrections come from the proper diagonalization of the mass matrix. On the other hand, we find corrections to the composition of the mass eigenstates (in perturbation theory corrections to the eigenfunctions are usually more difficult to obtain than corrections to the eigenvalues).

Let us now discuss the super–Higgs effect which should take place when supersymmetry is spontaneously broken as in the present case. In the Lagrangian (29) there are many terms containing both the gravitino and the goldstino fields. There should exist a way of “eating” the massless goldstino and leaving only the massive gravitino. And indeed let us define

\[
\psi_{\mu}^{(3/2)} = \psi_{\mu}^{\text{gravitino}} + \frac{2}{\sqrt{6} m} \Gamma_Y D_\mu \psi_{11}^{\text{goldstino}} + \frac{1}{\sqrt{6}} \Gamma_\mu \psi_{11}^{\text{goldstino}}. \tag{32}
\]

Then as a part of (29) we obtain a lagrangian for the massive gravitino

\[
-\frac{1}{2} e_4 \psi_{\mu}^{(3/2)} \Gamma^{\mu
u\rho} D_\nu \psi_{\rho}^{(3/2)} - \frac{m}{2} e_4 \psi_{\mu}^{(3/2)} \Gamma^{\mu\nu} \psi_{\nu}^{(3/2)} \tag{33}
\]

and the field \( \psi_{11}^{\text{goldstino}} \) completely disappears from (29). This is precisely the super–Higgs effect since the goldstino provided the degrees of freedom needed for the massless gravitino to become massive.

5 Discussion

In the case with the gaugino condensate present only at the hidden wall the gravitino mass is given by

\[
m_{3/2} = \frac{1}{16(4\pi)^{5/3}} \frac{\Lambda^3}{M_{Pl}^2} < 10^{-3} \frac{\Lambda^3}{M_{Pl}^2}. \tag{34}
\]

We need the scale of the condensate \( \Lambda \) to be of the order \( 10^{14} \) GeV to get the gravitino mass of about 1 TeV. If we assume that \( \Lambda \) is not bigger than the GUT scale (\( 10^{16} \) GeV) then we obtain the upper bound on the gravitino mass of the order of \( 10^5 \) TeV. A value of \( \Lambda \) much larger than the GUT scale (which is comparable to the 11-dimensional Planck scale \( M_{11} \)) will not be meaningful in this framework.

In the above we have shown how to identify the gravitino and goldstino fields in the case with arbitrary gaugino condensates at two 10–dimensional walls of
the 11–dimensional spacetime. This procedure can be easily generalized to more complicated situations. We can analyse, for example, a model in which supersymmetry breaking sources are present not only at the walls but also at some branes located along the eleventh dimension interval. In this case we have to perform a field redefinition similar to the rotation (27) but with modified function \( f(x^{11}) \) and/or matrix \( \Gamma_Y \). Those modifications should be chosen in such a way that the terms containing \( \partial_{11} \) present in the lagrangian (18) exactly cancel (locally) all \( \delta \)–like sources of supersymmetry breaking. An additional constraint on the function \( f \) comes from the \( Z_2 \) symmetry and relates its values at \( x^{11} = -\pi \rho \) and \( x^{11} = +\pi \rho \). To fulfill such a constraint we generally have to add a linear part to \( f \) (like in (28)). The compactification to 4 dimensions is quite straightforward in terms of the rotated fields because the KK modes of those fields are the mass eigenstates. The linear term in \( f \) gives rise to the gravitino mass after compactification to 4 dimensions. This mass is zero (and supersymmetry is unbroken) only if all the sources add up to zero.

Let us now compare the above discussed mechanism of supersymmetry breaking to the Scherk–Schwarz mechanism [12]. There is one similarity: in both cases the 4–dimensional fields (for example the gravitino) are obtained from higher dimensional fields with nontrivial dependence on the compact coordinate(s). But the origin of this dependence is very different. In the Scherk–Schwarz mechanism we just assume some specific dependence or, in other words, we keep only one (nonconstant) KK mode and drop all the other KK modes (also the constant one). The mass of the gravitino is equal to this KK mass and as a result supersymmetry is explicitly broken. In this paper, on the contrary, we keep all the KK modes. They mix due to the supersymmetry breaking sources (like a vev of \( G \)) and we identify the gravitino as the lightest mass eigenstate with spin 3/2. Supersymmetry is broken spontaneously and the goldstino is “eaten” by the super–Higgs mechanism as was shown explicitly in the previous section. It is thus possible to take into account effects of other (heavier) spin 3/2 states. The mechanism is motivated by the dynamics of the higher dimensional theory. Modified Bianchi identities and the perfect square structure in the lagrangian provide justification for the nontrivial background of the \( G \) field. In this background we are able to perform explicit calculations identifying the lowest–lying gravitino and all the heavier states.
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References

[1] P. Hořava and E. Witten, Nucl. Phys. B460(1996)506; B475(1996)94
[2] E. Witten, Nucl. Phys. B471(1996)135
[3] H. P. Nilles, Phys. Reports 110(1984)1
[4] H. P. Nilles, Phys. Lett. B115(1982)193; Nucl. Phys. B217(1983)366
[5] P. Hořava, Phys. Rev. D54(1996)7561
[6] H. P. Nilles, M. Olechowski and M. Yamaguchi, Phys. Lett. B415(1997)24
[7] H. P. Nilles, M. Olechowski and M. Yamaguchi, Nucl. Phys. B530(1998)43
[8] Z. Lalak and S. Thomas, Nucl. Phys. B515(1998)55
[9] A. Lukas, B. A. Ovrut and D. Waldram, Phys. Rev. D57(1998)7529
[10] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D58(1998)065002
[11] J. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, Nucl. Phys. B540(1999)149; A. Lukas, B.A. Ovrut and D. Waldram, hep-th/9901017, JHEP 9904:009, 1999
[12] J. Scherk and J. H. Schwarz, Nucl. Phys. B153(1979)61
[13] I. Antoniadis and M. Quiros, Nucl. Phys. B505(1997)109
[14] L. Randall and R. Sundrum, hep-th/9810153
[15] E. Witten, Phys. Lett. B155(1985)151