Status on $^{12}$C+$^{12}$C fusion at deep subbarrier energies: impact of resonances on astrophysical $S^*$ factors

C. Beck\textsuperscript{a}, A.M. Mukhamedzanov\textsuperscript{b} and X. Tang\textsuperscript{c}\textsuperscript{1}

\textsuperscript{1a}Département de Recherches Subatomiques, Institut Pluridisciplinaire Hubert Curien, INP-CNRS and Université de Strasbourg - 23, rue du Loess BP 28, F-67037 Strasbourg Cedex 2, France
E-mail: christian.beck@iphc.cnrs.fr

\textsuperscript{b}Cyclotron Institute, Texas A\&M University, College Station, Texas, 77843, USA
E-mail: akram@comp.tamu.edu

\textsuperscript{c}Institute of Modern Physics, CAS, Lanzhou, P.R.China Joint Department for Nuclear Physics, Lanzhou University and Institute of Modern Physics, CAS, China
E-mail: xtang@impcas.ac.cn

Since the discovery of molecular resonances in $^{12}$C+$^{12}$C in the early sixties a great deal of research work has been undertaken to study $\alpha$-clustering and resonant effects of the fusion process at sub-Coulomb barrier energies. The astrophysical $S^*$ factors of $^{12}$C+$^{12}$C fusion have been extracted from several recent direct fusion measurements at deep subbarrier energies near the Gamow window. They were also obtained by the indirect Trojan horse method (THM). A critical comparison of recent direct measurements and the THM, which elucidates problems in the analysis of the THM, is proposed in this Letter to the Editor.

I. INTRODUCTION

In the last decades, one of the greatest challenges in nuclear science is the understanding of the clustered structure of nuclei from both experimental and theoretical perspectives. The role of cluster configurations in stellar He burning is well established. One of the most exciting topics of contemporary nuclear astrophysics is the nature and the role of resonance structures that characterize the low-energy cross section of the $^{12}$C+$^{12}$C fusion process which plays a very important role in a wide variety of stellar burning scenarios such as massive stars, type Ia supernovae and superbursts. One of the possible scenarios of formation of Supernovae Ia is merging of the binary system of two white dwarfs [1]. The outcome of this merging is controlled by the $^{12}$C+$^{12}$C fusion.

The $^{12}$C+$^{12}$C has a complicated resonance feature which continues all the way from the Coulomb barrier energy down to the lowest measured energies. These resonances have a characteristic width of about 100 keV. Since there is no way to model the resonances in the unmeasured energy range, the averaged $S^*$-factors are used for extrapolation while the resonances are treated as the fluctuations. Since the width of the Gamow window is comparable or wider than the $^{12}$C+$^{12}$C resonance width, the averaged modified astrophysical factor $S^*(E) = \sigma(E) E \exp(\frac{2\pi E}{\sqrt{E}}) + 0.46 E$, where $E$ is the $^{12}$C+$^{12}$C relative energy, $\sigma(E)$ is the fusion cross-section, is a reasonable approach for the astrophysical application.

An extensive scientific discussion about this reaction is underway in recent experimental investigations using direct [2-9] and indirect THM [10] measurements, see Fig 1. The standard $^{12}$C+$^{12}$C reaction rate was established by Caughan and Fowler [11]. Their simple extrapolation agrees reasonably with some recent theoretical calculations, such as CC-M3Y+Rep [12], TDWP [13] and barrier penetration model based on the global Sáo Paulo potentials (SPP) [16], see Fig 1, as well with sophisticated coupled-channels calculations [14, 15] and recent Hartree-Fock and time-dependent Hartree-Fock calculations [17]. We note that the predictions based on the CC-M3Y+Rep method are considered to be the upper limit of the $^{12}$C+$^{12}$C fusion cross section [18, 19].

Guided by the experimental astrophysical factors of the medium-heavy systems, a phenomenological hindrance model was developed in [20], which predicts the $^{12}$C+$^{12}$C $S^*$-factor reaches its maximum as energy decreases and then rapidly drops becoming by many orders of magnitude smaller than the standard rates used for astrophysical modeling. However, the new $^{12}$C+$^{12}$C measurement confirms the trends predicted by the TDHF, CC-M3Y+Rep and disapproves the prediction by the hindrance model [21].

Recently the Trojan Horse Method (THM) has been applied to determine the $^{12}$C+$^{12}$C fusion $S^*$-factor at energies below $E = 2.7$ MeV [10]. The THM $S^*$-factor demonstrates a profound rise of the $S^*$-factors in both $\alpha-$ and $p-$ channels as energy $E$ decreases, which is much faster than any data and models presented in Fig. 1.

In this Letter to the Editor, we present a critical comparison of direct and indirect $S^*$ factors for the $^{12}$C+$^{12}$C fusion. We also reveal serious flaws in the THM analysis [10] which call for new indirect measurements.

II. CRITICAL COMPARISON OF DIRECT AND INDIRECT DATA

The resonant structures at very low energies have still been identified as molecular $^{12}$C+$^{12}$C configurations in the $^{24}$Mg compound nucleus [5]. However, the reaction rate of this reaction is calculated using an average cross section integrated over the molecular resonance compo-
ments. Fig. 1 displays the $S^\ast$-factors of the $^{12}\text{C}+^{12}\text{C}$ fusion.

Indications of possible existence of a pronounced single low-energy resonance in the system $^{12}\text{C}+^{12}\text{C}$ at the relative energy of $^{12}\text{C}$ nuclei $E = 2.14$ MeV [5] that can only be explained by a strong $^{12}\text{C}$ cluster configurations that can only be explained by a strong nuclear molecular configuration of the corresponding state in $^{24}\text{Mg}$. Moreover, the recent STELLA $S^\ast$ factors [8, 9] agree with [5] and confirm the occurrence of the isolated resonance at $E \approx 2.14$ MeV. Meantime, the THM data contradict to data from [5] showing a sequence of resonances in the interval $2.1 - 2.67$ MeV.

But the most striking difference between the direct and the THM data can be revealed by comparison of the trend of the direct data versus the THM ones. While all the direct data are more or less flat in the interval $2.1 - 2.67$ MeV with a possible single resonance at $2.14$ MeV the THM $S^\ast$-factor shows completely opposite trend: a sharp rise as energy decreases from $2.67$ to $2.1$ MeV with explicit resonance structure. The difference in the trends, which is the result of using the invalid plane-wave approximation (PWA) analysis [22] of THM data, is so large that renormalization of the THM data to direct ones cannot reconcile indirect and direct $S^\ast$-factors.

![FIG. 1. (Color online) Total $S^\ast$ factors of the $^{12}\text{C}+^{12}\text{C}$ fusion. The $^{12}\text{C}+^{12}\text{C}$ data are from [2-7] shown as red rectangle, green dots, magenta diamonds, blue triangles and brown stars, respectively. Model calculations, CC-M3Y+Rep (thick dark red) [12], TDWP(light blue) [13], SPP(oragen) [16] and hindrance(blue dashed) [20] are also shown, respectively. The recommended averaged $S^\ast$ factor by CF88 [11] is shown as red dashed line. The THM [10] result and the fit are shown as black dashed and solid lines. The uncertainty of THM $S^\ast$ factor is $\pm 20\%$ which is not shown in the figure. This figure with direct, indirect data and model calculations are taken from [23].](image)

Furthermore, the trend of the recent STELLA $S^\ast$-factors [8, 9] (not displayed in Fig. 1), which are in agreement with the Argonne results [7]), appear to be in a qualitative agreement with classical coupled-channel CC-M3Y+Rep calculations in [12], TDWP calculations of [13], the barrier penetration model of [16] and more recent theoretical investigations in [14].

We note also that the direct data from [5] and [8, 9], in which the lowest directly measured energies are achieved, show the single strong resonances at $E \approx 2.14$ MeV, while the THM data revealed resonances below and above these directly measured resonances. It is also observed that the THM $S^\ast$ factor is several orders of magnitude higher than the upper limit based on the CC-M3Y-Rep calculations [21].

In Fig 2 we show the $\chi^2$ test of the THM [10] and [5] data by interpolating THM data for 21 points between $E = 2.1$ and $2.7$ MeV. This comparison is resulted in the reduced $\chi^2 = 4.9$. Note that for $\chi^2 > 2.226$, the possibility of the coincidence of data is only $0.1\%$.

![FIG. 2. (Color online) $\chi^2$ test of the [10] and [5] data in the interval $2.1 - 2.7$ MeV.](image)

There have been predictions based on phenomenological considerations of explosive stellar events such as superbursts that suggest a strong $^{12}\text{C}+^{12}\text{C}$ cluster resonance around $E = 1.5$ MeV in $^{24}\text{Mg}$ that would drastically enhance the energy production and may provide a direct nuclear driver for the superburst phenomenon. However, no indication for such a state has not yet been reported in direct measurements in which the minimal measured energy is $E \approx 2.1$ MeV.

From Figs. 1 and 2 we conclude that direct data and model calculations are antagonistic to the THM ones.

III. CRITICAL REVIEW OF ANALYSIS USED TO EXTRACT THM $S^\ast$ FACTORS FOR $^{12}\text{C}+^{12}\text{C}$ FUSION

The THM is a powerful and unique indirect technique that allows one to measure the astrophysical factors of the resonant reactions at low energies, where direct methods are not able to obtain data due to very small cross sections. The method was originally suggested by Baur [24] but became well known and one of the powerful indirect methods due to the leadership of Prof. Claudio Spitaleri [25-27]. The criticism of the THM in this Letter is not aimed to taint the whole method which demonstrated its power in more than hundred publications. We critically review only the analysis of the data in [10].

The THM resonant reaction $a+A \rightarrow s+F^\ast \rightarrow s+b+B$
where \( a = (xA) \) is the Trojan horse (TH) particle, is described by the two-step mechanism in which the first step is transfer reaction \( a + A \rightarrow s + F^* \), populating the resonance state \( F^* = x + A \), and the second step is decay of the resonance \( F^* \rightarrow b + B \). Thus, the THM reaction is the process leading to three particles in the final state making analysis of such a reaction quite complicated. Special kinematical conditions should be fulfilled to allow one to extract from the THM reaction an information about resonant binary sub-reaction \( x + A \rightarrow F^* \rightarrow b + B \). One of the major requirements in the THM is that the nucleus \( s \) is a spectator, the presence of which does not affect the binary sub-reaction.

To analyze the measured data Tumino et al. [10] used a simple plane-wave approximation (PWA). This approximation neglects the Coulomb interactions between the fragments. In Refs. [28, 29], a generalized R-matrix approach was developed within the surface integral formalism. This approach uses distorted waves in both initial and final states of the transfer reaction (see Eq. (117) of Ref. [22]). The PWA follows from this more general approach when the distorted waves are replaced by the plane waves.

In [10] was reported a sharp rise of the astrophysical \( S \)-factor for carbon-carbon fusion determined using the indirect THM. However, in [22] it was demonstrated that the rise at low energies seen in the aforementioned work was an artifact of using an invalid PWA that neglects the Coulomb interactions. It was shown that such a rise disappears if the Coulomb (or Coulomb-nuclear) interactions in the initial and final states are included.

Here are outlined the most striking inconsistencies in the THM data from [10]. All the notations are given in [22].

1. The THM double differential cross section (DCS) in [10] does not correspond to the one described by the two-step THM mechanism for the THM reaction \( ^{14}N + ^{12}C \rightarrow d + ^{24}Mg^* \rightarrow \alpha(p) + ^{20}Ne(^{23}Na) \). The double DCS for the THM mechanism is given by Eq. (39) from [22]. Specifically, for the reaction under consideration described by the THM mechanism, the energy of the outgoing deuterons corresponding to the \( ^{24}Mg \) resonance energy at \( E = 2.1 \) MeV is \( E_{\text{res}}^{^{24}Mg} = 1.47 \) MeV while for \( E = 2.6 \) MeV \( E_{\text{res}}^{^{24}Mg} = 0.97 \) MeV. Thus, the deuterons are well below the Coulomb barrier of 3 MeV in the system \( d + ^{24}Mg \). Hence, the DWBA DCS \( \frac{d^2\sigma}{d\Omega_{\text{res}}^{E}} \) of the transfer reaction \( ^{14}N + ^{12}C \rightarrow d + ^{24}Mg^* \), which is the first step of the THM reaction, drops by two orders of magnitude on the interval \( 2.1 - 2.64 \) MeV interval what should lead to the decrease of the THM double DCS. Hence, one can expect that as energy \( E \) increases the non-THM mechanisms, which are background, should dominate.

2. The extraction of the astrophysical factor from the THM double DCS is not straightforward and requires an advanced theory. In the analysis in [10] was used the PWA which was developed by one of us (A. M. M.) for the cases when the spectator is a neutron or when the Coulomb interaction of the spectator with other participating nuclei can be neglected.

The PWA is an approximation to the DWBA and can be used only if the PWA calculations are close to the DWBA ones. In the case under consideration the DWBA and PWA give completely different results [22] what makes application of the PWA invalid. Using the DWBA we can renormalize the PWA based astrophysical factors extracted in [10]. The comparison of the total \( S^* \)-factors obtained from the THM data using the PWA [10] and the DWBA is shown in Fig. 3.

FIG. 3. Total THM astrophysical factors for \(^{12}C + ^{12}C\) fusion. Black solid line is the \( S^*(E) \) factor from [10]. The red dashed line is the renormalized \( R(E)S^*(E) \) factor calculated using the pure Coulomb distortions. The blue dotted line is the renormalized \( R(E)S^*(E) \) factor calculated using the Coulomb plus nuclear distortions. The renormalization factor is given by Eq. (43) from [22].

In the standard DWBA approach the THM reaction amplitude is diverging because it contains the resonant wave function of the system \( ^{12}C + ^{12}C \). To make the DWBA matrix element converging over the variable \( r_{x,A} \) in [22] the binned resonant wave functions were used which have highly oscillatory behavior. That is the reason why in the DWBA approach the internal matrix element becomes negligible and the DWBA reaction amplitude can be approximated by the surface term [28] plus the converging external term.

As we have underscored, the PWA is invalid in the case under consideration. But even if we disregard this fact, the PWA used in [10] is not correct. In the PWA approach used in [10] the matrix element is given by

\[
M^{\text{PWA}} = M^{\text{PWA}}_{\text{int}} + M^{\text{PWA}}_{\text{s}} + M^{\text{PWA}}_{\text{ext}},
\]

where \( M^{\text{PWA}}_{\text{int}} \), \( M^{\text{PWA}}_{\text{s}} \) and \( M^{\text{PWA}}_{\text{ext}} \) are the internal, surface and external amplitudes. The internal matrix element containing the Fourier transform of the resonant wave functions in the internal region over the variable \( r_{x,A} \leq R_{xA} \) describes the non-THM mechanism, and in [10] it was neglected without any justification. But in
the case under consideration even a small correction to the THM reaction amplitude due to the contribution from the internal matrix element will change the channel radius what can significantly affect the off-shell factors $|\mathcal{W}_{l,A}|^2$ and modify the extracted THM $S$-factors. The external matrix element is not converging and requires regularization. It has not been done in [10] and the external integral simply was neglected.

3. The most instructive results of the THM experiment are presented by the double DCSs in four different channels. The presence in the right-hand-side of Eq. (39) [22] of the off-shell factor $|\mathcal{W}_{l,A}|^2$, where $x = A = ^{12}\text{C}$, which appears because the transferred particle $x$ in the THM reaction is off-the-energy shell, changes the spectrum of the resonances in the system $^{12}\text{C} - ^{12}\text{C}$ excited in the THM reaction compared to the one in direct measurements. It makes the relative weights of the resonances observed in the THM double DCS different from the ones in direct measurement. To determine correct strengths of the resonances from the THM double DCS one needs to divide it by $|\mathcal{W}_{l,A}|^2$, which is extremely sensitive to the orbital angular momentum $l_{x,A}$ of the resonance in the system $^{12}\text{C} - ^{12}\text{C}$ and to the channel radius $R_{ch}$ determining the surface term contribution to the THM reaction amplitude. The value of $l_{x,A}$ can be determined by measuring the angular distribution of the products of the $^{24}\text{Mg}^+$ resonance decay. In [10] these angular distributions are not presented and the values of $l_{x,A}$ were determined by fitting of the experimental double DCS in the $R$-matrix approach. Such a fitting is ambiguous because the unknown parameters of the resonances are also fitting parameters. Spin assignments of the resonances in [10] contradict to the results from [34].

4. Another important issue to be discussed is the extrapolation of the renormalized in [22] astrophysical factor $R(E)S^*(E)$ to higher energies. One can argue that the naive extrapolation of the renormalized $R(E)S^*(E)$ factor to energies $E > 2.7$ MeV, where THM data are not available, will go higher than the directly measured astrophysical factors. This seemingly disagreement is, as a matter of fact, another evidence that the THM double DCS at energies $E > 2.1$ MeV is contributed by the non-THM mechanisms which provide practically flat energy dependence of the double DCS rather then the drop which is expected for the THM mechanism as the energy $E$ increases. Note also that in [22] were used the channel radius and the off-shell factors $|\mathcal{W}_{l,A}|^2$ from [10] and the DWBA was applied only to renormalize the $S^*$ factors. However, if one uses the DWBA from the beginning, the channel radius will be different from $R_{ch} = 7.25$ [10] in what can change the off-shell factors, extracted astrophysical factors and their extrapolation to higher energies.

5. Despite of the criticism of the analysis in [10] the experimental THM double DCS reveals two strong resonances at 0.88 and 1.5 MeV what is an undeniable achievement of the THM because such low energies are not yet reachable in direct measurements. A strong resonance at 1.5 MeV, which is inside the Gamow window, is of a crucial importance for the $^{12}\text{C} - ^{12}\text{C}$ fusion rates and will play the same role as the "Hoyle" state [30] in the triple-α process of synthesis of $^{12}\text{C}$ [31].

Note that the models using global potentials in [12, 13, 32, 33] can predict resonances only above $E = 2.5$ MeV (the TDHF calculations of [17] do not take into account any possible resonant behavior in the fusion process). The THM data at low energies call for an improvement of the theoretical models.

There have been predictions based on phenomenological considerations of explosive stellar events such as supernovae that suggest a strong $^{12}\text{C} + ^{12}\text{C}$ cluster resonance around $E = 1.5$ MeV in $^{24}\text{Mg}$ that will drastically enhance the energy production and may provide a direct nuclear driver for the supernova phenomenon. However, no indication for such a state has not yet been reported in direct measurements in which the minimal measured energy is $E \approx 2.1$ MeV. Hence, the discovery of two strong resonances at 0.88 and 1.5 MeV in the THM data is an important contribution of this method in the $^{12}\text{C} - ^{12}\text{C}$ fusion research.

However, the assigned in [10] spins of the resonances are very questionable and contradict to the CC-M3 Y-Rep calculations [12]. Even more surprising are the spin $4^+$ of the resonance at 0.805 MeV because of the very small penetrability factor for such a high spin. The assigned spin $1^-$ for the resonance at 0.877 MeV is a mistake because a resonance with negative parity cannot be populated in the collision of two identical bosons such as $^{12}\text{C}$ nuclei. The assigned resonance spins for the $^{12}\text{C} - ^{12}\text{C}$ system in [10] also contradict to the estimations in [34].

IV. CONCLUSION

A critical comparison of the direct measurements of $^{12}\text{C} - ^{12}\text{C}$ fusion and the indirect THM data [10] is presented. We demonstrated serious disagreements between the direct and THM $S$ factors. We also outlined serious shortcomings in the THM analysis. Moreover, the very recent direct data by Notre Dame University [35] show even more profound failure of the THM $S$ factors from [10]. Taking into account the critical importance of the $^{12}\text{C} + ^{12}\text{C}$ fusion in many astrophysical scenarios we can outline three major problems to be solved in the near future:

1. Extending direct measurements below 2.1 MeV with the final goal to reach 1.5 MeV. It would help us to better understand the trend of the $S^*$ factors at energies $E < 2.1$ MeV and resolve the question about the validity of the hindrance model.

2. Perform new indirect measurements. One such preparation is underway at the Cyclotron Institute, Texas A&M University [36].

3. Improving of the theoretical models, which will be
able to predict the low-energy resonances detected in the THM.

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[1] K. Mori et al., MNRAS: Letters 482, L70 (2019); arXiv: 1810.01025 (2018).
[2] H. W. Becker et al., Z. Phys. A 303, 305 (1981).
[3] K.U.Kettner, H.Lorenz-Wirzba, C.Rolfs, Z.Phys. A, 298, 65 (1980).
[4] E. F. Aguilera et al., Phys.Rev. C 73, 064601 (2006).
[5] T.Spillane et al., Phys.Rev.Lett. 98, 122501 (2007).
[6] J. R. Patterson, H. Winkler, and C. S. Zaidins, Astrophys. J. 157, 367 (1969).
[7] C.L. Jiang et al., Phys. Rev. C 97, 012801(R) (2018).
[8] G. Fruet, Ph. D Thesis, Universit´ e de Strasbourg, https://tel.archives-ouvertes.fr/tel-02107569.
[9] C. Beck, Proceedings of the XI International Symposium on Exotic Nuclei EXON2018, Petrozavodsk (Russia), Eds. Yu. Penionzkevich and Y. Sobelev, 20, (2019).
[10] A. Tumino et al., Nature, 557, 687 (2018).
[11] G.R.Caughlan and W.A.Fowler, At.Data Nucl.Data Tables 40, 283 (1988).
[12] H. Esbensen, X. Tang, and C. L. Jiang, Phys. Rev. C 84, 064613 (2011).
[13] A.Diaz-Torres and M.Wiescher, Phys.Rev. C 97, 055802 (2018).
[14] Le Hoang Chien et al., Phys. Rev. C, 98, 064604 (2018); Le Hoang Chien, Dao T. Khoa, Do Cong Cuong, and Nguyen Hoang Phuc, arXiv:1810.07887 (2018).
[15] Dao T. Khoa et al., Nucl. Sci. Tech., 29, 182 (2018).
[16] L. R. Gasques et al., Phys. Rev. C 76, 035802 (2007).
[17] K. Godbey, C. Simenel, and A. S. Umar, Phys. Rev. C 100, 024619 (2019).
[18] M. Notani et al., Phys. Rev. C 85, 014607 (2012).
[19] C. L. Jiang et al., Phys. Rev. Lett. 110, 072701 (2013).
[20] C. L. Jiang et al., Phys. Rev. C 75, 015803 (2007).
[21] N. T. Zhang et al., submitted to Phys. Lett. B (2019).
[22] A.M. Mukhamedzhanov, D. Y. Pang and A.S. Kadyrov, Phys. Rev. C 99, 064618 (2019).
[23] X. Tang, The $^{12}\text{C} - ^{12}\text{C}$ Fusion Reaction at STELLAr Energies. In: Formicola A., Junker M., Gialanella L., Imbriani G. (eds),Proceeding of 15th International Symposium on Nuclei in the Cosmos, Springer Proceedings in Physics, 13 219 (2019).
[24] G. Baur, Phys. Lett. B 178, 135 (1986).
[25] C. Spitaleri et al., Phys. Rev. C 60, 055802 (1999).
[26] C. Spitaleri et al., Eur. Phys. J. A, review (to be published) (2019).
[27] R. E. Tribele et al., Rep. Prog. Phys., 77, 106901 (2014).
[28] A. M. Mukhamedzhanov, Phys. Rev. C 84, 044616 (2011).
[29] A.M.Mukhamedzhanov, D. Y. Pang, C. A. Bertulani, and A. S. Kadyrov, Phys. Rev. C 90, 034604 (2014).
[30] F. Hoyle, Astrophysical Journal Supplement 1, 121 (1954).
[31] M. Freer et al., Rev. Mod. Phys., 90, 035004 (2018).
[32] N. Rowley and K. Hagino, Phys. Rev. C 91, 044617 (2015).
[33] M. Assuncao and P. Descouvemont, Phys. Lett. B 723, 355 (2013).
[34] K. P. Erb and D. A. Bromley, Phys. Rev. C 23, 2781 (1981).
[35] M. Wiescher, private communication (2019).
[36] G. V. Rogachev, private communication (2019).