Interacting Ghost Dark Energy in Non-Flat Universe

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A new dark energy model called “ghost dark energy” was recently suggested to explain the observed accelerating expansion of the universe. This model originates from the Veneziano ghost of QCD. The dark energy density is proportional to Hubble parameter, \( \rho_D = \alpha H \), where \( \alpha \) is a constant of order \( \Lambda_{QCD}^3 \) and \( \Lambda_{QCD} \sim 100 MeV \) is QCD mass scale. In this paper, we extend the ghost dark energy model to the universe with spatial curvature in the presence of interaction between dark matter and dark energy. We study cosmological implications of this model in detail. In the absence of interaction the equation of state parameter of ghost dark energy is always \( w_D > -1 \) and mimics a cosmological constant in the late time, while it is possible to have \( w_D < -1 \) provided the interaction is taken into account. When \( k = 0 \), all previous results of ghost dark energy in flat universe are recovered. To check the observational consistency, we use Supernova type Ia (SNIa) Gold sample, shift parameter of Cosmic Microwave Background radiation (CMB) and the Baryonic Acoustic Oscillation peak from Sloan Digital Sky Survey (SDSS). The best fit values of free parameter at 1σ confidence interval are: \( \Omega_0^m = 0.35^{+0.02}_{-0.03} \), \( \Omega_0^D = 0.75^{+0.01}_{-0.04} \) and \( b^2 = 0.08^{+0.03}_{-0.03} \). Consequently the total energy density of universe at present time in this model at 68% level equates to \( \Omega_{\text{tot}}^0 = 1.10^{+0.02}_{-0.05} \).

I. INTRODUCTION

The current acceleration of the cosmic expansion has been strongly confirmed by numerous and complementary observational data [1]. In the context of standard cosmology such an expansion requires the existence of an unknown dominant energy component, usually dubbed “dark energy” whose equation of state parameter satisfies \( w_D < -1/3 \). Although we can affirm that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin is still rather uncertain. (for reviews, see e.g. [2] and references therein). Disclosing the nature of dark energy has been one of the most important challenges of the modern cosmology and theoretical physics in the past decade. A great varieties of dark energy models

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have been proposed, to explain the acceleration of the universe expansion within the framework of quantum gravity, by introducing new degree of freedom or by modifying the underlying theory of gravity [3–6].

Recently a very interesting suggestion on the origin of a dark energy is made, without introducing new degrees of freedom beyond what are already known, with the dark energy of just the right magnitude to give the observed expansion [7, 8]. In this proposal, it is claimed that the cosmological constant arises from the contribution of the ghost fields which are supposed to be present in the low-energy effective theory of QCD [9–13]. It was argued that the Veneziano ghost, which is unphysical in the usual Minkowski spacetime QFT, exhibits important physical effects in dynamical spacetime or spacetime with non-trivial topology. The ghosts are required to exist for the resolution of the $U(1)$ problem, but are completely decoupled from the physical sector [13]. The above claim is that the ghosts are decoupled from the physical states and make no contribution in the flat Minkowski space, but once they are in the curved space or time-dependent background, the cancelation of their contribution to the vacuum energy is off-set, leaving a small energy density $\rho \sim H\Lambda^3_{QCD}$, where $H$ is the Hubble parameter and $\Lambda_{QCD}$ is the QCD mass scale of order a $100\,\text{MeV}$. With $H \sim 10^{-33}\,\text{eV}$, this gives the right magnitude $\sim (3 \times 10^{-3}\,\text{eV})^4$ for the observed dark energy density. This numerical coincidence is remarkable and also means that this model gets rid of fine tuning problem [7, 8]. The advantages of this new model compared to other dark energy models is that it is totally embedded in standard model and general relativity, one needs not to introduce any new parameter, new degree of freedom or to modify gravity. The dynamical behavior of the ghost dark energy (GDE) model in flat universe have been studied [14].

In this paper we would like to extend the previous discussion on ghost dark energy [14] to a universe with spatial curvature. There are enough observational evidences, at present time, for taking into account a small but non-negligible spatial curvature [15]. For instance, the tendency of preferring a closed universe appeared in a suite of CMB experiments [16]. The improved precision from WMAP provides further confidence, showing that a closed universe with positively curved space is marginally preferred [17]. In addition to CMB, recently the spatial geometry of the universe was probed by supernova measurements of the cubic correction to the luminosity distance [18], where a closed universe is also marginally favored.

Most discussions on dark energy models rely on the fact that its evolution is independent of other matter fields. Given the unknown nature of both dark matter and dark energy there is nothing in principle against their mutual interaction and it seems very special that these two major components in the universe are entirely independent. Indeed, this possibility has got a lot
of attention in the literature in recent years (see \[19–21\] and references therein) and was shown to be compatible with SNIa and CMB data \[22\].

All above reasons, motivate us to study the interacting ghost dark energy model in a nonflat universe. In this paper, we would like to generalize the ghost dark energy model to the universe with spacial curvature in the presence of interaction between the dark matter and dark energy. Taking the interaction between the two different constituents of the universe into account, we study the evolution of the universe, from early deceleration to late time acceleration. In addition, we will show that such an interacting dark energy model can accommodate a transition of the dark energy from a normal state where \( w_D > -1 \) to \( w_D < -1 \) phantom regimes.

This paper is organized as follows. In the next section, we review the ghost dark energy model in a flat universe. In section III, we generalize the study to the universe with spacial curvature in the presence of interaction between dark matter and dark energy. Observational constraints on the free parameters of model will be given in section IV. We summarize our results in section V.

**II. GHOST DARK ENERGY IN FLAT UNIVERSE**

Let us first review the ghost dark energy model in flat Friedman-Robertson-Walker (FRW) universe where first investigated in \[14\]. Although, our approach in dealing with the problem differs to some extent from those of Ref. \[14\].

**A. Noninteracting case**

For the flat FRW universe filled with dark energy and dust (dark matter), the corresponding Friedmann equation takes the form

\[
H^2 = \frac{1}{3M_p^2} \left( \rho_m + \rho_D \right),
\]

(1)

where \( \rho_m \) and \( \rho_D \) are, respectively, the energy densities of pressureless matter and dark energy. The ghost energy density is \[8\]

\[
\rho_D = \alpha H,
\]

(2)

where \( \alpha \) is a constant of order \( \Lambda_{QCD}^3 \) and \( \Lambda_{QCD} \) is QCD mass scale. With \( \Lambda_{QCD} \sim 100 MeV \) and \( H \sim 10^{-33} eV \), \( \Lambda_{QCD}^3 H \) gives the right order of magnitude \( \sim (3 \times 10^{-3} eV)^4 \) for the observed dark energy density \[8\].
We define the dimensionless density parameters as

\[ \Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{\alpha}{3M_p^2H}, \]

(3)

where the critical energy density is \( \rho_{cr} = 3H^2M_p^2 \). Thus, the Friedmann equation can be rewritten as

\[ \Omega_m + \Omega_D = 1. \]

(4)

The conservation equations read

\[ \dot{\rho}_m + 3H\rho_m = 0, \]

(5)

\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = 0. \]

(6)

Taking the time derivative of relation \( (2) \) and using the Friedmann equation we find

\[ \dot{\rho}_D = \rho_D \frac{\dot{H}}{H} = -\frac{\alpha}{2M_p^2}\rho_D(1 + u + w_D). \]

(7)

where \( u = \rho_m/\rho_D \) is the energy density ratio. Inserting this relation in continuity equation \( (6) \) we reach

\[ (1 + w_D)(6M_p^2H - \alpha) = \alpha u. \]

(8)

Substituting ghost energy density \( (2) \) in Friedmann equation \( (1) \) we find

\[ 3M_p^2H = \alpha(1 + u). \]

(9)

Combining Eq. \( (9) \) with \( (8) \) we reach

\[ w_D = -1 + \frac{u}{1 + 2u}. \]

(10)

Using the fact that

\[ u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = \frac{1 - \Omega_D}{\Omega_D}, \]

(11)

we can rewrite Eq. \( (10) \) as

\[ w_D = -\frac{1}{2 - \Omega_D}, \]

(12)

It is easy to see that at the early time where \( \Omega_D \ll 1 \) we have \( w_D = -1/2 \), while at the late time where \( \Omega_D \rightarrow 1 \) the ghost dark energy mimics a cosmological constant, namely \( w_D = -1 \). It is worthy to note that in \( w_D \) of this model, there is no free parameter. In the left panel of figure \( \text{[1]} \)
FIG. 1: Left panel shows the evolution of $w_D$ for ghost dark energy. In the right panel the behavior of the deceleration parameter for ghost dark energy is illustrated. Here we have taken $\Omega^0_D = 0.72$.

we plot the evolution of $w_D$ versus scale factor $a$. From this figure we see that $w_D$ of the ghost dark energy model cannot cross the phantom divide and the universe has a de Sitter phase at late time.

We can also calculate the deceleration parameter which is defined as

$$q = -1 - \frac{\dot{H}}{H^2}. \quad (13)$$

When the deceleration parameter is combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. Using Eq. (17) and definition $\Omega_D$ in (3) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \Omega_D (1 + u + w_D). \quad (14)$$

Substituting this relation into (13), after using (12) we find

$$q = \frac{1}{2} - \frac{3}{2} \frac{\Omega_D}{2 - \Omega_D}. \quad (15)$$

At the early time where $\Omega_D \to 0$ the deceleration parameter becomes $q = 1/2$, while at the late time where the dark energy dominates ($\Omega_D \to 1$) we have $q = -1$. This implies that at the early time the universe is in a deceleration phase while at the late time it enters an acceleration phase.

We have plotted the behavior of $q$ in the right panel of figure (1). From this figure we see that the transition from deceleration to acceleration takes place at $a \simeq 0.64$ or equivalently at redshift
$z \simeq 0.56$. Note that $1 + z = a^{-1}$ and we have set $a_0 = 1$ for the present value of scale factor. Besides, taking $\Omega_D^0 = 0.72$ we obtain $q \approx -0.34$ for the present value of the deceleration parameter which is in agreement with recent observational data [23]. Taking the time derivative of Eq. (3) and using relation $\dot{\Omega}_D = H \frac{d\Omega_D}{d\ln a}$ as well as relation (13) we reach

$$\frac{d\Omega_D}{d\ln a} = \Omega_D (1 + q). \quad (16)$$

Using Eq. (15) we get

$$\frac{d\Omega_D}{d\ln a} = 3\Omega_D \frac{1 - \Omega_D}{2 - \Omega_D}. \quad (17)$$

This is the equation governing the evolution of ghost dark energy. The dynamics of ghost dark energy is plotted in figure (2) where we have taken $\Omega_D^0 = 0.72$ as the initial condition. This figure shows that at the late time the dark energy dominates, as expected.

![FIG. 2: The evolution of $\Omega_D$ for ghost dark energy. Here we have taken $\Omega_D^0 = 0.72$.](image)

**B. Interacting case**

Next we extend the discussion to the interacting case and study the dynamics of the ghost dark energy. Although at this point the interaction may look purely phenomenological but different Lagrangians have been proposed in support of it (see [24] and references therein). Besides, in the absence of a symmetry that forbids the interaction there is nothing, in principle, against it. In addition, given the unknown nature of both dark energy and dark matter, which are two major
contents of the universe, one might argue that an entirely independent behavior of dark energy is very special [21, 25]. Further, the interacting dark matter-dark energy (the latter in the form of a quintessence scalar field and the former as fermions whose mass depends on the scalar field) has been investigated at one quantum loop with the result that the coupling leaves the dark energy potential stable if the former is of exponential type but it renders it unstable otherwise [26]. Thus, microphysics seems to allow enough room for the coupling; however, this point is not fully settled and should be further investigated. The difficulty lies, among other things, in that the very nature of both dark energy and dark matter remains unknown whence the detailed form of the coupling cannot be elucidated at this stage. In this case, the energy densities of dark energy and dark matter no longer satisfy independent conservation laws. They obey instead

\[ \dot{\rho}_m + 3H\rho_m = Q, \]  
\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \]

where \( Q \) represents the interaction term and we take it as

\[ Q = 3b^2 H(\rho_m + \rho_D) = 3b^2 H\rho_D(1 + u), \]

with \( b^2 \) being a coupling constant. It is worth noting that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural

FIG. 3: Left panel corresponds to the evolution of \( w_D \) for interacting ghost dark energy and different interacting parameter \( b^2 \) while right panel shows the evolution of the deceleration parameter for interacting ghost dark energy and different interacting parameter \( b^2 \). Here we took \( \Omega^0_D = 0.72. \)
choice can be the Hubble factor $H$) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) $Q \propto H \rho_D$, (ii) $Q \propto H \rho_m$, or (iii) $Q \propto H (\rho_m + \rho_D)$. Thus we can present the above three choices in one expression as $Q = \Gamma \rho_D$, where

$$
\begin{align*}
\Gamma &= 3b^2 H & \text{for } Q \propto H \rho_D, \\
\Gamma &= 3b^2 H u & \text{for } Q \propto H \rho_m, \\
\Gamma &= 3b^2 H (1 + u) & \text{for } Q \propto H (\rho_m + \rho_D),
\end{align*}
$$

(21)

It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction $Q$. In the present work for the sake of generality, we choose the third expression for the interaction term.

Inserting Eqs. (7) and (20) in Eq. (19) and using (11) we find

$$
w_D = -\frac{1}{2 - \Omega_D} \left(1 + \frac{2b^2}{\Omega_D}\right),
$$

(22)

One can easily check that in the late time where $\Omega_D \rightarrow 1$, the equation of state parameter of interacting ghost dark energy necessary crosses the phantom line, namely, $w_D = -(1 + 2b^2) < -1$ independent of the value of coupling constant $b^2$. For present time with taking $\Omega_D^0 = 0.72$, the phantom crossing can be achieved provided $b^2 > 0.1$. This value for coupling constant is consistent with recent observations [21].

In the presence of interaction the deceleration parameter is obtained by substituting (22) in (14) and using (13). The result is

$$
q = \frac{1}{2} - \frac{3}{2} \frac{\Omega_D}{(2 - \Omega_D)} \left(1 + \frac{2b^2}{\Omega_D}\right),
$$

(23)

while the evolution of dark energy follows the following equation

$$
\frac{d\Omega_D}{d \ln a} = \frac{3}{2} \Omega_D \left[1 - \frac{\Omega_D}{2 - \Omega_D} \left(1 + \frac{2b^2}{\Omega_D}\right)\right].
$$

(24)

The evolution of the cosmological parameters $w_D$, $q$ and $\Omega_D$ are shown in figures (3) and (4) for different interacting parameter $b^2$. We have taken $\Omega_D^0 = 0.72$ as the initial condition. We can also obtain the scale factor $a$ as a function of $t$. Integrating the relation $\Omega_D = \alpha/(3M_p^2 H)$, we find

$$
\int \Omega_D \frac{da}{a} = \int_{t_0}^t \frac{\alpha}{3M_p^2} dt = \frac{\alpha}{3M_p^2} (t - t_0),
$$

(25)

where $\Omega_D$ is given by Eq. (24). The behavior of $a(t)$ is shown in the right panel of figure (4).
FIG. 4: The evolution of dark energy density for interacting ghost dark energy is shown in left panel. Right panel corresponds to the evolution of the scale factor for interacting ghost dark energy with different $b^2$. The rest of parameters are the same as for figure [3].

III. INTERACTING GHOST DARK ENERGY IN NON-FLAT UNIVERSE

Next we reach to the main task of the present work, namely studying the dynamic evolution of ghost energy density in a universe with special curvature. As we discussed in the introduction a closed universe is marginally favored. Taking the curvature into account, the Friedmann equation is written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_D),$$

(26)

where $k$ is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. We define the curvature density parameter as $\Omega_k = k/(a^2H^2)$, thus the Friedmann equation takes the form

$$1 + \Omega_k = \Omega_m + \Omega_D,$$

(27)

Using the above equation the energy density ratio becomes

$$u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = \frac{1 + \Omega_k - \Omega_D}{\Omega_D}.$$  

(28)

Taking the time derivative of the Friedmann equation (26) we find

$$\frac{\dot{H}}{H^2} = \Omega_k - \frac{3}{2} \Omega_D[1 + u + w_D],$$

(29)
and therefore
\[ \frac{\dot{\rho}_D}{H} = \rho_D \frac{\dot{H}}{H^2} = \rho_D \left( \frac{\Omega_k}{2} - \frac{3}{2} \frac{\Omega_D}{3} [1 + u + w_D] \right). \] (30)

Combining this relation with continuity equation (19), after using (20) and (28), we find the equation of state parameter of interacting ghost dark energy in non-flat universe
\[ w_D = -\frac{1}{2 - \Omega_D} \left( 1 - \frac{\Omega_k}{3} + \frac{2b^2}{\Omega_D} (1 + \Omega_k) \right). \] (31)

The deceleration parameter is obtained as
\[ q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{\Omega_D}{3} [1 + u + w_D] \] (32)

Substituting Eqs. (28) and (31) in (32) we obtain
\[ q = \frac{1}{2} (1 + \Omega_k) - \frac{3\Omega_D}{2(2 - \Omega_D)} \left[ 1 - \frac{\Omega_k}{3} + 2b^2\Omega_D^{-1} (1 + \Omega_k) \right], \] (33)

FIG. 5: The evolution of \( w_D \) for interacting ghost dark energy in nonflat universe. Right panel illustrates the evolution of the deceleration parameter for interacting ghost dark energy in nonflat universe. Here we set \( \Omega^0_D = 0.73 \) and \( \Omega^0_m = 0.28 \).

In a non-flat FRW universe, the equation of motion of interacting ghost dark energy is obtained following the method of the previous section. The result is
\[ \frac{d\Omega_D}{d\ln a} = \frac{3}{2} \frac{\Omega_D}{2 - \Omega_D} \left( 1 + \frac{\Omega_k}{3} - \frac{\Omega_D}{3} \right) \left[ 1 - \frac{\Omega_k}{3} + 2b^2\Omega_D^{-1} (1 + \Omega_k) \right]. \] (34)
FIG. 6: Left panel shows the evolution of dark energy density for interacting ghost dark energy in nonflat universe. Right panel corresponds to the evolution of the scale factor for interacting ghost dark energy in non flat universe. The value of $\Omega_k$ in the present time is 0.01 (closed universe). The rest of parameter are as in figure [5].

The evolution of $\Omega_k$ can be obtained by combining Eq. (3) with definition $\Omega_k = k/(a^2H^2)$. We find

$$\Omega_k = \frac{k}{a^2H^2} = \left(\frac{9M_p^4k}{\alpha^2}\right) \frac{\Omega_D^2}{a^2}. \quad (35)$$

We calculated the evolution of deceleration parameter and $\Omega_D$ and plotted them in figures [5] and [6], respectively. $a(t)$ versus $t$ in the non-flat universe for different values of coupling constant is shown in figure [6]. In the limiting case $\Omega_k = 0$, Eqs. (32)-(35), restore their respective equations in interacting ghost dark energy model in flat universe derived in the previous section (see also [14]).

IV. OBSERVATIONAL CONSTRAINTS

In this section we use the recent observational data sets for supernova type Ia (SNIa) [27, 28], shift parameter of Cosmic Microwave Background Radiation based on WMAP-7 [29, 31] and Baryonic Acoustic Oscillation (BAO) based on Sloan Digital Sky survey (SDSS) [32] to put constraints on the free parameters of our model. To avoid the rewriting unnecessary things we refer the reader to some references such as [33–36] for more details. In Table II we summarize the list of free
TABLE I: Priors on the free parameter space.

| Parameter | Prior       |
|-----------|-------------|
| $\Omega_m$ | $[0.00 - 1.00]$ | Top hat |
| $\Omega_b$ | $[0.00 - 1.00]$ | Top hat |
| $H_0$     | Free [37, 38] |
| $b^2$     | $[0.00 - 0.20]$ | Top hat |

parameters of model as well as priors for using in the likelihood analysis.

To apply the observations from SNIa we calculate the distance modulus as

$$\mu \equiv m - M = 5 \log D_L(z; \Omega_m^0, \Omega_D^0, b^2) + 5 \log \left( \frac{c/H_0}{1 \text{ Mpc}} \right) + 25,$$

(36)

in the above equation

$$D_L(z; \Omega_m^0, \Omega_D^0, b^2) = \frac{(1 + z)}{\sqrt{|\Omega_k|}} \mathcal{F} \left( \sqrt{|\Omega_k^0|} \int_0^z \frac{dz' H_0}{H(z', \Omega_m^0, \Omega_D^0, b^2)} \right),$$

(37)

where

$$\mathcal{F}(x) \equiv (x, \sin(x), \sinh(x)) \quad \text{for} \quad (\Omega_k^0 = 0, \Omega_k^0 > 0, \Omega_k^0 < 0)$$

and $H(z; \Omega_m^0, \Omega_D^0, b^2)$ is computed numerically from Eqs. (1), (2), (3) and (34).

Finally the $\chi^2_{\text{SNIa}}$ is defined by:

$$\chi^2_{\text{SNIa}}(\Omega_m^0, \Omega_D^0, b^2) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{the}}(z_i; \Omega_m^0, \Omega_D^0, b^2)]^2}{\sigma_i^2}$$

(38)

Usually, beside using the peak locations of the CMB power spectrum, one can use the so-called shift parameter $R$, as [39]

$$R = \sqrt{\Omega_m^0} \frac{D_L(z_{\text{dec}}, \Omega_m^0, \Omega_D^0, b^2)}{(1 + z_{\text{dec}})}$$

(39)

here $z_{\text{dec}}$ is the redshift of the last scattering surface [40]. Subsequently the $\chi^2_{\text{CMB}}$ can be written as

$$\chi^2_{\text{CMB}}(\Omega_m^0, \Omega_D^0, b^2) = \frac{[R_{\text{obs}} - R_{\text{the}}(\Omega_m^0, \Omega_D^0, b^2)]^2}{\sigma^2_{\text{CMB}}}$$

(40)

For the last observational constraint, we rely on the large-scale correlation function measured from the sample of SDSS including a clear peak at $100 \text{ Mpc}^{-1}$ [32]. A dimensionless and $H_0$
TABLE II: The best fit values for the free parameters from fitting with SNIa from new Gold sample, SNIa+CMB, SNIa+CMB+BAO experiments at one and two $\sigma$ confidence level.

| Observation       | $\Omega_0^m$ | $\Omega_0^D$ | $b^2$  |
|-------------------|--------------|--------------|--------|
| SNIa              | $0.95^{-0.56}_{+0.99}$ | $0.99^{-0.25}_{+0.44}$ | $0.16^{-0.11}_{+0.16}$ |
| SNIa+CMB          | $0.39^{+0.22}_{-0.16}$ | $0.73^{+0.04}_{-0.17}$ | $0.09^{+0.10}_{-0.05}$ |
| SNIa+CMB+BAO      | $0.35^{+0.02}_{-0.03}$ | $0.75^{+0.01}_{-0.04}$ | $0.08^{+0.03}_{-0.03}$ |

An independent parameter for constraining the cosmological models has been proposed in literatures [32] as follows:

$$A = \sqrt{\Omega_0^m} \left[ \frac{H_0 D_L^2(z_{sdss}; \Omega_0^m, \Omega_0^D, b^2)}{H(z_{sdss}; \Omega_0^m, \Omega_0^D, b^2) z_{sdss}^2 (1 + z_{sdss})^2} \right]^{1/3}$$  (41)

where $z_{sdss} = 0.35$ [32]. So the $\chi^2_{BAO}$ is expressed as:

$$\chi^2_{BAO}(\Omega_0^m, \Omega_0^D, b^2) = \frac{[A_{obs} - A_{the}(\Omega_0^m, \Omega_0^D, b^2)]^2}{\sigma_{BAO}^2}$$  (42)

Figure (7) represent the marginalized likelihood function for model free parameters. In addition joint contour plot for parameters have been illustrated in figures (8) and (9).

The best values and the confidence interval for free parameter at 1$\sigma$ and 2$\sigma$ have been reported in Table (II).

V. CONCLUSION

It is a general belief that our universe is currently undergoing a phase of accelerated expansion likely driven by dark energy. Unfortunately, until now, the nature and the origin of such dark
FIG. 7: Marginalized likelihood functions of model free parameters. The solid, dash and dashdot lines correspond to fitting the model with SNIa data new gold sample, SNIa+CMB and SNIa+CMB+BAO, respectively. The horizontal solid and dashed lines represent the bounds with $1\sigma$ and $2\sigma$ level of confidence, respectively.

energy is still the source of much debate and we don’t know what might be the best candidate for dark energy to explain the accelerated expansion. Thus, various models of dark energy have been proposed, to explain the accelerated expansion by introducing new degree of freedom or by modifying the standard model of cosmology. In this regard, a so called “ghost dark energy” was recently proposed [7, 8] which originates from the Veneziano ghost of QCD. The QCD ghost has no contribution to the vacuum energy density in Minkowski spacetime, but in curved spacetime it gives rise to a small vacuum energy density [8]. The dark energy density is proportional to Hubble
FIG. 8: Joint likelihood function of $(\Omega_D^0, \Omega_m^0)$. Left panel corresponds to SNIa observation while right panel shows SNIa+CMB+BAO data sets.

FIG. 9: Joint likelihood function of free parameters. left panel corresponds to $(b^2, \Omega_m^0)$ while right panel shows for $(b^2, \Omega_D^0)$. Here SNIa+CMB+BAO observations used to confine the values of free parameters.

parameter, $\rho_D = \alpha H$, where $\alpha$ is a constant of order $\Lambda_{QCD}^3$ and $\Lambda_{QCD}$ is QCD mass scale. With $\Lambda_{QCD} \sim 100 MeV$ and $H \sim 10^{-33} eV$, $\Lambda_{QCD}^3 H$ gives the right order of magnitude $\sim (3 \times 10^{-3} eV)^4$ for the observed dark energy density [8]. The advantages of this new proposal compared to the previous dark energy models is that it totally embedded in standard model so that one needs not
to introduce any new parameter, new degree of freedom or to modify general relativity [14].

In this paper, we generalized the ghost dark energy model, in the presence of interaction between dark energy and dark matter, to the universe with spatial curvature. Although it is believed that our universe is spatially flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-folding is not very large [41]. Besides, some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature [15]. With the interaction between the two different dark components of the universe, we studied the evolution of the universe, from early deceleration to late time acceleration. We found that in the absence of interaction the equation of state parameter of ghost dark energy is always larger than $-1$ and mimics a cosmological constant in the late time. We also found that the transition from deceleration to acceleration take places at $a \simeq 0.64$ or equivalently at redshift $z \simeq 0.56$. We observed that, in the presence of interaction, the equation of state parameter can cross $-1$ at the present time provided the interacting parameter satisfy $b^2 > 0.1$.

To check the observational consistency of interacting Ghost Dark Energy model, we used Supernova type Ia (SNIa), CMB shift parameter and Baryonic Acoustic Oscillation (BAO). Our results demonstrated that the best values of free parameters when we combine all observational data are: $\Omega_m^0 = 0.35^{+0.02}_{-0.03}, \Omega_D^0 = 0.75^{+0.01}_{-0.04}$ and $b^2 = 0.08^{+0.03}_{-0.03}$ at $1\sigma$ confidence interval. Our analysis shows that at $1\sigma$ level of confidence the value of so-called interacting parameter does not cross zero. Also the total value of energy density of universe at present time is $\Omega_{tot}^0 = \Omega_m^0 + \Omega_D^0 = 1.10^{+0.02}_{-0.05}$ at $68\%$ level.

Finally, we would like to mention that if there is any kind of ghost field which gives rise to an energy density $\rho \propto H$, its cosmological implications is exactly similar to the present work independent of its origin. Although the existence of a well-motivated physical model where this energy density is obtained is of course a valid starting point, however, the details of this study can be found in the previous works such as [7, 8] and we have not repeated them here. In this work our main task was to study the cosmological implications of this new ghost dark energy model proposed in [7, 8] without referring to its origin. In particular we generalized the study to the universe with spatial curvature and put some observational constraints on the model parameters.

Acknowledgments

A. Sheykhi thanks Professor Rong Gen Cai for valuable comments and useful discussion. This
work has been supported by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

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