Cusp Annihilation on Ordinary Cosmic Strings

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Abstract

The order of magnitude of energy emission from cusps to light bosons on ordinary cosmic strings is calculated perturbatively. The analysis is applicable to both closed string loops and long cosmic strings. The perturbative result obtained here is much less than what is found by non-perturbative approximations.

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1. Introduction

Cosmic strings are linear topological defects formed during a phase transition in the early universe [1]. They may have an important role in structure formation if the mass per length parameter, $\mu$, satisfies $G\mu \simeq 10^{-6}$ [2]. This value is consistent with the scale of the symmetry breaking in grand unified theories (GUT).

Cosmic strings emit gravitational [3] and non-gravitational [4,5,6] radiation. It has been shown, however, that gravitational radiation dominates [4,5,6].

The non-gravitational radiation of cosmic strings can be induced by oscillations of ordinary cosmic string [4,5,6] or superconducting cosmic string [7] and by the phenomenon of cusp annihilation [6,8].

An important contribution to the non-gravitational radiation is cusp annihilation. Cusps are region of cosmic strings where the string doubles back onto itself, the tangent angle to string changes by 180 degrees and its velocity is near that of light. Cusps can be formed on long strings [9], as well as on loops[13].

Here we calculate the order of magnitude of particle production at cusps on ordinary cosmic strings by perturbative field theoretical methods and compare the result with the naive non-perturbative assumptions[6]. In this work we generalize the field theoretical calculations of [4,6] to more general string configurations (closed and long string), focusing on the cusp region. We will see that the amount of energy predicted by this work is much smaller than what was obtained by previous non-perturbative approximations for cusp annihilation. As a result, the background gamma-ray attributed to cosmic strings would be much smaller than what was predicted by using the other approximations [9,14].

For a global $U(1)$ string the action, in terms of its constituting field $\phi$, is

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - f(|\phi|^2 - \sigma^2)^2 \right],$$

(1.1)

where $\phi$ is a complex scalar field and $f$ is the self coupling constant. Assuming
that the field configuration corresponds to a string with world sheet $\chi^\mu(s, \tau)$ in four dimensional space-time, where $s, \tau$ and $h_{ij}$ are its world sheet coordinates and metric, the above field theory action reduces [11] to the Nambu action for the world sheet:

$$S = \mu \int d^2 s \sqrt{h} \partial_i \chi^\mu(s, \tau) \partial_j \chi^\mu(s, \tau) h^{ij}$$

(1.2)

where $i$ and $j$ are world sheet indices. This action to a good accuracy describes the motion of the string world sheet provided

$$\frac{w}{R} \ll 1$$

(1.3)

where $w$ and $R$ are the width and curvature radius of the string, respectively. The width $w$ of the string is given in terms of $\mu$ by $w \sim \mu^{-1/2}$. In case the equation (1.3) is not valid the action (1.2) should be corrected by higher order terms [12].

By the equations of motion derived from the action (1.2) it can be shown that cusps, or the points where $\dot{\chi} = 1$ and $\chi' = 0$ can be formed on loops [13] or long strings[9].

2. The field theoretical calculation of cusp annihilation

Cusps are regions where the cosmic strings double back onto themselves. Hence, there is no topological constraint which prevents the strongly correlated string field configuration from decaying into outgoing jets of particles which can be fermionic or bosonic. As a toy model, inspired by the field potential of the cosmic string we consider the interaction lagrangian

$$\mathcal{L}_I = f \tilde{\phi}^2 \Psi^2$$

(2.1)

where $\Psi$ is the bosonic field of the outgoing pair of particles, $\tilde{\phi}$ is the real part of the higgs field of string expanded around the real vacuum $|\langle \phi \rangle| = \sigma$, and $f$ is the coupling constant.
For a long string, we consider a line of string in spatial $z$ direction with small wiggles on it and assign a quantum state $|S\rangle$ to its configuration. The state could be interpreted as a coherent state of higgs field configuration.

The initial state can be $|S\rangle$ and the final state $|S\rangle|\psi(k_1)\psi(k_2)\rangle$, where $|\psi(k)\rangle$ represents the bosonic outgoing state with momentum $k$.

The $S$-matrix element

$$S_{fi} = \langle S, \psi(k_2)\psi(k_1)|S\rangle$$

(2.2)

can be written using the LSZ construction as

$$S_{fi} = f \int d^4x e^{i(k_1+k_2)\cdot x} \langle S| : \tilde{\phi}^2 : |S\rangle$$

(2.3)

where $::$ denotes normal ordering.

Calculating (3) requires some knowledge of $\tilde{\phi}$. There is, however, no solution in closed form for the cosmic string field configuration. Here, we use the following approximation [4,6]

$$\langle S| : \tilde{\phi}^2(\chi) : |S\rangle \simeq \sigma^2w^2 \int ds |\chi'|^2 \delta^3(x - \chi(t,s))$$

(2.4)

where $\sigma$ and $w$ are the value of $\phi$ at the center of the string and its thickness respectively. $\chi(t,s)$ represents the world sheet of the traveling wave [10] along the string.

Now the integral (2.3) can be written as

$$S_{fi} = f \int d^4x \int ds |\chi'|^2 e^{ikx} \sigma^2w^2 \delta(x - \chi(t,s))$$

$$= f \sigma^2w^2 \int dt ds |\chi'|^2 e^{i(Et-k\cdot \chi(t,s))}$$

(2.5)

The main contribution to (2.5) is from the cusp vicinity where the phase $\Phi$ is
stationary, namely

$$\dot{\Phi}(t, s) = 0$$
$$\Phi'(t, s) = 0$$

(2.6)

where \( \cdot \) and \( ' \) mean the derivatives with respect to time and \( s \), respectively.

We use the SPG[6] result to expand the determinant and phase of the integrand (2.5) around the cusp site:

$$\chi = \begin{pmatrix}
t \\
\frac{1}{2}(\alpha^2 + \beta^2 + \gamma^2)((\frac{1}{3}t^3 + s^2t)) - \alpha\beta(t^2s + \frac{1}{3}s^3) \\
\alpha\left(\frac{1}{2}(t^2 + s^2)\right) + \beta st + \delta\left(\frac{1}{3}t^3 + s^2t\right) + \zeta(t^2 + \frac{1}{3}s^3) \\
\gamma st + \epsilon\left(\frac{1}{3}t^3 + s^2t\right) + \xi(t^2s + \frac{1}{3}s^3)
\end{pmatrix}$$

(2.7)

where \( \alpha, \beta, \gamma, \delta, \epsilon, \xi \) and \( \zeta \) are some constants of dimension \( \frac{1}{\text{length}} \) or \( \frac{1}{(\text{length})^2} \) and for closed string with size \( R \), approximately \( \frac{1}{R} \). Here, we assume \( \alpha \approx \beta \approx \gamma \). Note that the expansion (2.7) is approximately valid while \( |t| \& |s| \ll \alpha \).

The parameter \( \alpha \), with dimension \( \frac{1}{\text{length}} \), is proportional to the size of the incoming traveling wave [10] on strings. For closed strings with typical length \( R \) we have [6, 8] \( \alpha^{-1} \sim R \) and for long string with incoming traveling wave size \( l \), we have \( \alpha^{-1} \sim l \).

Plugging the expansion (2.7) into the stationary phase condition (2.6) it can easily be seen that \( k_x \sim E \). Hence, by the mass shell condition \( k_y \& k_z \approx 0 \) near the cusp. More precisely the condition

$$k_z, k_y \ll E^{\frac{2}{3}}\alpha^{\frac{1}{2}}$$

(2.8)

can be obtained from (2.6) and the relation (2.12) and therefore the maximum angle between the two vectors \( k_x \) and \( k_y \) can be approximated as

$$\theta_{\text{max}} \sim \frac{k_y}{k_x} \sim E^{-\frac{1}{2}}\alpha^{\frac{1}{4}}.$$
By substituting \( z = s \), the phase in the integrand (2.5) will take the form

\[
\Phi \simeq \frac{1}{2} \alpha^2 \left( \frac{1}{3} t^3 + z^2 t \right) - \alpha^2 \left( t^2 z + \frac{1}{3} z^3 \right) E. \tag{2.10}
\]

The considerable contribution to the integral (2.5) comes from the region where

\[
\Phi \ll 1 \tag{2.11}
\]

or

\[
t_{\text{max}}, z_{\text{max}} \leq \left( \frac{1}{\alpha^2 E} \right) \frac{1}{4}. \tag{2.12}
\]

Thus, \( t_{\text{max}} \) and \( z_{\text{max}} \) will be the upper bounds for the integration.

Now the S-matrix (2.5) can be written as

\[
S_{fi} = f \sigma^2 w^2 \int_{-t_{\text{max}}}^{t_{\text{max}}} dt \int_{-z_{\text{max}}}^{z_{\text{max}}} dz \Delta^2 e^{i\Phi} \tag{2.13}
\]

where \( \Delta^2 \) is approximately

\[
\Delta^2 \simeq \alpha^2 (z^2 + t^2) + \alpha^4 (z^4 t^2 + z^4 + t^4) \tag{2.14}
\]

where we have considered the terms in the integral that do not vanish and absorbed all the constants in \( \alpha \). The total energy emitted can be found by working out the following integral

\[
\tilde{E} = \int \frac{d^3 k_1}{(2\pi)^3 k_1^0} \frac{d^3 k_2}{(2\pi)^3 k_2^0} |S_{fi}|^2 E
\]

\[
= f^2 \sigma^4 w^4 \int \frac{d^3 k_1}{(2\pi)^3 k_1^0} \frac{d^3 k_2}{(2\pi)^3 k_2^0} (k_1^0 + k_2^0) \left| \int dt \int dz \Delta^2 e^{i\Phi} \right|^2 \tag{2.15}
\]

The upper limit for \((k_1^0 + k_2^0)\) is the scale of the energy of the cosmic string i.e \( \sigma \). More precisely this cutoff can be figured out from the action (1.1), by considering
the mass of particles i.e. \( f \frac{\sigma}{f} \) \[6,15\]. Therefore the compton wavelength will be \( d_c \sim \frac{1}{f \frac{\sigma}{f}} \), where \( f \) is the coupling constant and \( f \lesssim 1 \). The energy cutoff will, consequently, be the amount of energy in the string within a distance \( d_c \), i.e.

\[
E_{\text{cut-off}} \sim \frac{1}{d_c} \sim f \frac{\sigma}{f}
\]  

(2.16)

An upper bound to (2.15) can be founded by approximating \( d\Omega \) using (2.9)

\[
d\Omega \sim \theta_{\max}^2 \sim E^{-2/3} \alpha^\frac{2}{3}
\]  

(2.17)

Therefore, the upper limit to the energy of ejected particles from a cusp, \( \tilde{E}_{\text{max}} \), will be

\[
\tilde{E}_{\text{max}} = \frac{1}{(2\pi)^6} \frac{\sigma^4 w^4 f^2}{\alpha^\frac{2}{3}} \int_{k_1 = E_{\text{min}}}^{E_{\text{min}} + k_0} \int_{k_2 = E_{\text{min}}}^{E_{\text{min}} + k_0} \frac{f \frac{\sigma}{f} - k_0}{k_1^0} \frac{f \frac{\sigma}{f} - k_0}{k_2^0} (k_1^0 + k_2^0) d\Omega_1 d\Omega_2 (k_1^0 + k_2^0)^{-8/3}
\]

(2.18)

For \( E_{\text{min}} \ll \sigma \), eq.(2.17) can be written as

\[
\tilde{E}_{\text{max}} \sim f^2 w^4 \sigma \frac{1}{2} \left( f \frac{\sigma}{f} - \frac{3}{2} E_{\text{min}} \right)
\]  

(2.19)

Therefore the total energy of the emitted pairs can be easily found by substituting \( \sigma w = 1 \) and neglecting \( E_{\text{min}} \), namely

\[
E_{\text{tot}} = \frac{1}{2} f \frac{\sigma}{f}
\]  

(2.20)

As a result the predicted amount of energy emitted by cusp annihilation, in this perturbative calculation, is proportional to the energy scale of symmetry breaking of the cosmic strings.
In conclusion, considering the absence of back reactions due to gravitational radiation, we have calculated to an order of magnitude, the maximum amount of energy that the light bosons can take when cusps on cosmic strings annihilate. These bosons can further decay to bursts of gamma-rays and contribute to the gamma-ray background [9,14].

The maximum energy is proportional to $\sigma$, the scale of symmetry breaking of the cosmic strings. Using equation (2.8) and the fact that $\alpha \ll \sigma$ (i.e. the size of the incoming traveling wave is much bigger than the thickness of the cosmic string), it is easy to show that

$$\sigma \ll E_{\text{cusp}} \quad (2.21)$$

where $E_{\text{cusp}}$ is the energy in the cusp region, namely

$$E_{\text{cusp}} = \mu l_{\text{cusp}} = \mu \sigma^{-\frac{1}{3}} \alpha^{-\frac{2}{3}}$$

where $l_{\text{cusp}}$ is the length of the cusp (overlap) region. Therefore the maximum amount of energy predicted by the perturbative calculations is much less than the results obtained by non-perturbative approximations [6]. As a result the background gamma ray predicted by this work is much smaller than what was determined in the previous calculations [9,14]

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