Two-component model in quantum statistical framework compared with multiplicity distributions in proton-proton collisions at energies up to $\sqrt{s} = 7$ TeV

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Abstract

Proton-proton collisions at new high energies ($\sqrt{s} = 2.36$ and 7 TeV) at LHC resulted into greater mean multiplicities ($\langle n \rangle$) of charged particles in the mid-rapidity region than estimated ones by different models and event generators. Another significant observation in multiplicity data is the change in slope in the distribution of primary charged hadrons in symmetric pseudorapidity interval $|\eta| < 2.4$. The change is most prominent with data at $\sqrt{s} = 7$ TeV. These new observations merit further studies. We consider a two-component model of particle production to analyze multiplicity distributions of charged hadrons from proton-proton collisions at centre-of-mass energies $\sqrt{s} = 0.9, 2.36$ and 7 TeV in symmetric pseudorapidity intervals $|\eta|$ of increasing width around the centre-of-mass pseudorapidity $\eta_{cm} = 0$. The model, based on quantum statistical (QS) formalism, describes multiplicity distribution by convolution of a Negative Binomial Distribution (NBD), representing a chaotic component, and a Poisson Distribution (PD), representing a coherent component of particle productions. The behaviour of characteristic parameters of the model is followed by the LHC data, while a scaling law, involving information entropy in quantum statistical viewpoint and derived as a function of chaotic multiplicity obtained from the two-component model, is not obeyed by the data, satisfactorily. An attempt to match the measured multiplicity distributions and suggested convolutions with values of characteristic parameters extracted from the data confirms disagreement between the data and the model.

Keywords: LHC Energy, proton-proton collisions, multiplicity distributions, two-component model, quantum statistics.

1. Introduction

The probability distribution $P_{s}(n)$ of production of $n$ particles from collisions, has been the premier of basic observables, characterizing the final states of multi-particle production process in high energy physics experiments since the beginning of such studies. In recent times, the Large Hadron Collider (LHC) at CERN has taken high energy collisions in laboratories to a new energy domain, facilitating proton-proton ($pp$) collisions at centre-of-mass energies, $\sqrt{s} = 0.9, 2.36$ and 7 TeV [2, 3, 4, 5]. The Compact Muon Solenoid (CMS) experiment at LHC has measured multiplicity distributions of primary charged hadrons for all the three LHC energies, available so far, in the mid-pseudorapidity ($\eta$) region (where $\eta = -\ln [\tan (\theta/2)]$ and $\theta$ is the polar angle of the particle with respect to the counterclockwise beam direction) in five symmetric overlapping $\eta$-intervals $|\eta|$ or $\eta_{c} < 0.5, 1.0, 1.5, 2.0$ and 2.4 [4] around the centre-of-mass pseudorapidity ($\eta_{cm} = 0$). The measured mean multiplicities at the new LHC energies ($\sqrt{s} = 2.36$ and 7 TeV) in the range of mid-pseudorapidity, have been found to be largely underestimated by existing models (event generators like PYTHIA, PHOJET etc). Other experiments at LHC corroborate the finding, generating strong motivation to look into the LHC data from different approaches. Another significant observation by the CMS experiment is the change in slope in the distribution of primary charged hadrons in pseudorapidity interval $|\eta| < 2.4$ at $\sqrt{s} = 7$ TeV. Appearance of a change in slope in the measured distribution indicates to the possible existence of more-than-one process or source of particle production. In this article, we present results of our analysis of multiplicity distributions data from $pp$ collisions at LHC, in the light of a two-component model [6] formulated in a quantum statistical approach. We extend our study further to check the validity of a scaling law involving information entropy in quantum statistical point of view.

2. Background

In the last few decades, with the advent of accelerator technology, collider facilities, capable of delivering higher and higher centre-of-mass energy ($\sqrt{s}$), could be made available for proton-proton ($pp$) and proton-antiproton ($p\bar{p}$) collisions. While the Intersecting Storage Ring (ISR), CERN facilitated $pp$ collisions at $\sqrt{s}$ range of 23.9 to 62.2 GeV [7], the Super Proton Synchrotron (SPS), CERN had $pp$ collisions at $\sqrt{s} = 200$ to $\sqrt{s} = 900$ GeV [8]. In the Tevatron at Fermilab, the energies of $p\bar{p}$ collisions were $\sqrt{s} = 540$ GeV to $\sqrt{s} = 1.8$ TeV [9] and finally at the Large Hadron Collider (LHC), CERN, energy of collisions of protons have reached $\sqrt{s}$ as high as 7 TeV [3, 4, 5]. With all these data along with data at the fixed-target (non-collider) experiments at pre-ISR period, a rich set of experimental data on multiplicity distributions from $pp$ ($p\bar{p}$) collisions is

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now available for a wide range of $\sqrt{s}$ for a comprehensive and systematic study in different theoretical and phenomenological formalisms for better understanding of multiparticle production mechanisms. The two-component model [6] of particle productions is one such formalism which has been thoroughly used in analyzing experimental data of $pp$ ($p\bar{p}$) collisions at energies available up to the SPS.

Of the statistical distribution functions, the Negative Binomial Distribution (NBD),

$$P(n, \langle n \rangle, k) = \frac{(n + k - 1)!}{n!(k + 1)!} \left[ \langle n \rangle \right]^n \frac{k}{k + \langle n \rangle}$$

(1)

has been the most successful one in describing the probability distributions of final state charged particles from $pp$ ($p\bar{p}$) collisions in the discussed energy domain. At the lower part of the energy range, the multiplicity distributions of final state charged particles in the full phase space could be described by binomial distribution (when the parameter $k$ in equation-1 is negative and an integer). At around $\sqrt{s} = 5$ GeV, the distribution of produced charged particles turned broader and started following Poisson distribution ($k$ is infinite). Above $\sqrt{s} = 30$ GeV, the NBD matched fairly well with even broader distributions in the full phase space (pseudorapidity space) data up to $\sqrt{s} = 540$ GeV at SPS. At $\sqrt{s} = 900$ GeV at SPS, when a shoulder-like structure appeared in the tail of the multiplicity distributions, a single NBD failed in matching with the data at large pseudorapidity intervals. At 900 GeV and also at 1.8 TeV at Tevatron, the NBD function was successful only in the restricted mid-pseudorapidity ($|\eta| < 0.5$) region. The distribution in large $|\eta|$-interval could be reasonably described with the sum of two NBDs.

3. Motivation

At the LHC energies, multiplicity distributions in non-single diffractive (NSD) inelastic proton-proton collisions have been measured and reported by different experiments, in different kinematic ranges, depending on capability of respective detector setup in terms of geometrical acceptance, detection efficiencies etc. A Large Ion Collider Experiment (ALICE) at LHC has measured primary charged particles at $\sqrt{s} = 0.9$ and 2.36 TeV in the mid-$\eta$ region in three overlapping $\eta$-intervals $|\eta| < 0.5$, 1.0 and 1.3 [2]. At $\sqrt{s} = 7$ TeV, instead of NSD inelastic events, ALICE analyzed [3] an event class requiring at least one charged particle in $|\eta| < 1$ and measured multiplicity distribution in that $\eta$-interval only. The ATLAS experiment at LHC has measured charged particle multiplicities for different event classes characterized by different lower cuts on the number of charged particles ($n_{ch} < 1, 2$ and 6) in different kinematic ranges ($p_T > 100$ MeV, 500 MeV in $|\eta| < 2.5$). The distributions measured by ALICE in the three $|\eta|$-intervals, $|\eta| < 0.5$, 1.0 and 1.3, at the two energies, $\sqrt{s} = 0.9$ TeV, $\sqrt{s} = 2.36$ TeV have been reported to match fairly well with NBD. The NBD fit to the distribution at $\sqrt{s} = 7$ TeV, measured by ALICE, has been reported to be slightly underestimating the data at low multiplicity ($n < 5$) and slightly overestimating the data at high multiplicity ($n > 55$). Measurements of multiplicity distributions in a wider phase space and in all the three energies by CMS experiment, reveal interesting features. Presenting [4] multiplicity distributions, without fitting to any distribution function, CMS reported a change of slope in $P_n$ for $n > 20$ in its largest $|\eta|$-interval of $|\eta| < 2.4$. This feature becomes more pronounced with increasing $\sqrt{s}$.

The observed change in slope in the distributions as measured by CMS could be attributed to the existence of more than one kind of source or process of particle production. There have been several proposals of models [6, 10, 11, 12, 13, 16] involving multi-particle production from more than one process / source. In Ref. [10], it is the weighted superposition of two multiplicity distributions (each assumed to follow NBD); one due to soft events (without mini-jets) and the other representing semi-hard events (with mini-jets). In the multiparton interactions model described in Refs. [11, 12], the soft component, which constitute the bulk of events in the final state, corresponds to single parton-parton collisions in the framework of dual parton model and produces KNO [14] distribution. The other process, which seems to be superimposed on top of the KNO producing process, involves two or more independent parton-parton collisions. Ref [13] is a two-component dual parton model (DPM), the soft process is described by the supercritical Pomeron and the hard component is described by perturbative QCD. This two-component DPM includes diffractive processes also. In spite of all these efforts, the change in slope in distributions is not yet a fully understood phenomenon.

We recollect that the distribution at $\sqrt{s} = 900$ GeV in broad phase space has been explained by sum of two NBDs. Also, there has been agreement [12] on the soft parts (as measured in UA5 and E735 experiments) obtained by the models described in Ref. [10] and Ref. [11]. But, at this point, it is important to consider relative position of the change in slope in the distributions of data at $\sqrt{s} = 900$ GeV at SPS and $\sqrt{s} = 7$ TeV at LHC. The change in slope appeared in SPS data at a higher multiplicity ($n$) and a sum of two NBDs (representing two broad distributions) could describe the data satisfactorily. For the LHC data, the most prominent change in slope appears in the multiplicity distribution in the symmetric pseudorapidity interval, $\eta_s < 2.4$ at $\sqrt{s} = 7$ TeV at much lower multiplicity ($n < 20$), indicating that the measured distribution could be due to contributions from a broad distribution and a narrow distribution.

Such a scenario is provided by a two-component model [6, 15, 16, 17] in quantum statistical approach, where a weighted average of a NBD (representing a broad distribution) with $k = 1$ due to a chaotic source and a Poisson distribution (representing a narrow distribution) due to a coherent source results into the final distribution. We follow this two-component model in QS viewpoint to analyze the LHC data.

4. Methodology

In the two-component model [6] in QS approach, the total mean multiplicity is given by:

$$\langle n \rangle = \langle n_{ch} \rangle + \langle n_{co} \rangle$$

(2)
(n_{co}) is the coherent component and (n_{ch}) is the chaotic component of the mean of total multiplicity.

In quantum statistics, a completely chaotic source produces particles following a negative binomial distribution given by equation-1 (\( n \) in equation-1 is to be read as \( n_{ch} \) in the present formalism), where \( k \) is the number of cells in the phase space or the number of independent quantum states. On the other hand, a completely coherent source gives rise to a Poisson distribution:

\[
P((n_{co}), k_p) = \frac{(n_{co})^{k_p} e^{-(n_{co})}}{k_p!}
\]

The mean of the chaotic component of total multiplicity, \( \langle n_{ch} \rangle \) is obtained from the measured mean of total multiplicity and the second moment of the distribution by:

\[
\langle n_{ch} \rangle = \bar{\rho}(n)
\]

where

\[
\bar{\rho} = [K(C_2 - (1 + 1/(n^2)))]^{1/2}
\]

is the chaoticity parameter, \( C_2 = \langle n^2 \rangle/\langle n \rangle^2 \) is the second moment of multiplicity distribution and \( K \) can have value either 1 or 2. However, in narrow symmetrical \( \eta \) intervals in data, only \( K=1 \) leads to physical solutions, in most of the cases. The chaoticity, \( \bar{\rho} (0 < \bar{\rho} < 1) \) indicates measure of fraction of chaoticity involved in the source. For a completely chaotic source, \( \bar{\rho} = 1 \) and for a completely coherent source, \( \bar{\rho} = 0 \). Any value, in between, represents a convoluted distribution of the two.

For our present study, we use the published multiplicity distribution data from CMS experiments in five overlapping symmetric pseudorapidity intervals \(|\eta|\) around the centre-of-mass pseudorapidity with width of the interval extended up to \(|\eta| < 2.4 \) for the three LHC energies, \( \sqrt{s} = 0.9, 2.36 \) and 7 TeV. In some cases, we use published ALICE data in \(|\eta| < 0.5, 1.0 \) and 1.3 for \( \sqrt{s} = 0.9 \) and 2.36 TeV for which the multiplicity distributions in non-single diffractive (NSD) inelastic proton-proton collisions have been measured and published by the experiment.

5. Results and Discussions

5.1. Behaviour of characteristic parameters of the model

Following Fowler et al., using equations-5 and 4 given in section-4 above, we calculate chaoticity \( \bar{\rho} \) and mean of the chaotic multiplicity \( \langle n_{ch} \rangle \) for all the multiplicity distributions in NSD pp collisions as reported by the two LHC experiments, CMS and ALICE. We plot the parameters in Fig. 1 and Fig. 2 respectively. In these plots, we include similar data for \( \sqrt{s} = 540 \) GeV of SPS also for comparison. The chaoticity depicts fractional contribution of chaotic source in the total multiplicity. As could be seen in Fig. 1, the chaoticity decreases with increasing width of the \( \eta \)-interval, \( \eta_c \) for a given \( \sqrt{s} \). Also, for a given width of \( \eta_c \), the chaoticity increases with \( \sqrt{s} \).
Comparing observed dependences of the parameters of the model for the LHC data on $\eta_c$ and $\sqrt{s}$, the particle production scenario at the considered energy range by the present approach could be summarized as follows:

a) The chaotic source dominates mainly the mid-pseudorapidity region and the fractional contribution of the chaotic source as compared to the coherent source decreases as one goes away from the centre-of-mass pseudorapidity ($\eta_{cm} = 0$).

b) The span of pseudorapidity of dominance of chaotic source increases with increase in $\sqrt{s}$.

c) The increase in the total mean multiplicity $\langle n \rangle$ with $\sqrt{s}$ in the mid-pseudorapidity region, in the considered energy range, is dominantly due to the increase in chaotic multiplicity $\langle n_{ch} \rangle$ with $\sqrt{s}$.

In general, the behaviour of the parameters extracted from the LHC data match with characteristic features of the model in a similar way as has been observed in case of SPS energy data. It is naturally logical to analyze the LHC data in terms of a scaling law involving chaotic multiplicity, which has been reported to hold for $pp$ or $p\bar{p}$ data for a wide range of energies at ISR and SPS.

5.2. Scaling of information entropy

We analyze the LHC data of NSD $pp$ collisions up to $\sqrt{s} = 7$ TeV, as measured by the CMS experiment to test the validity of a proposed scaling law namely the scaling of information entropy [18] involving application of the discussed two-component model. The information entropy is a function of the chaotic multiplicity $\langle n_{ch} \rangle$, which is a function of symmetric pseudorapidity interval, $\eta_c$, and the centre-of-energy, $\sqrt{s}$. The entropy for chaotic multiplicities in symmetric pseudorapidity intervals $\eta_c$ is given by:

$$S(\eta_c, \sqrt{s}) = (\langle n_{ch} \rangle + 1) \ln(\langle n_{ch} \rangle + 1)$$

$$- (\langle n_{ch} \rangle \ln \langle n_{ch} \rangle)$$

In Fig. 4 we plot $S/\eta_{max}$ as a function of $\xi = \eta_c/\eta_{max}$ where [18].

$$\eta_{max} = \ln[(\sqrt{s} - 2m_p)/m_T]$$

(7)

It may be noted that the discussed scaling law is not a characteristic feature of the two-component model. The scaling law was developed for information entropy in quantum statistical point of view, modifying the entropy scaling [19] found to obey experimentally measured multiplicity distribution in hadronic interactions for wide range of $\sqrt{s}$, including ISR and SPS data. The response of the LHC-data to the scaling law is worth observing. We study the scaling law with the CMS data of $pp$ collisions at $\sqrt{s} = 0.9, 2.36$ and 7 TeV along with the SPS data of $p\bar{p}$ at $\sqrt{s} = 540$ GeV. The data points along with respective experimental uncertainties are plotted and are joined with straight lines for a given $\sqrt{s}$ to guide the eye. As it is clear from the Fig. 4 the data points of different $\sqrt{s}$ do not follow a common single curve. Moreover, data-points of different energies follow distinctly separate lines. This observation indicates to the deviation of the entropy-scaling at LHC energies.

5.3. Test of agreement between the model and the data.

At this stage, one might be interested to know how well the discussed two-component model describes the measured multiplicity distributions. To check the agreement between the model and the data, we chose multiplicity distributions in the pseudorapidity interval, $|\eta| < 2.4$ for the fact that the change in slopes in distributions have been observed by the CMS collaboration for in the same $\eta$-interval with most pronounced structure appearing at $\sqrt{s} = 7$ TeV and that the motivation of this work has been the understanding of change in slope of the multiplicity distribution by the two-component model.

We fit the multiplicity distributions of primary charged hadrons, as measured by the CMS experiment, at energies $\sqrt{s} = 0.9, 2.36$ and 7 TeV in the pseudorapidity interval ($\eta_c < 2.4$) around the mid-$\eta$ with NBD. For clarity in presentation, plots of three energies are given separately in Fig. 5. For. The lower panel of
Figure 5: Primary charged hadron multiplicity distributions for $|\eta| < 2.4$ for $\sqrt{s} = 7.0$ TeV. The continuous line corresponds to a fit to a NBD function and the dotted line corresponds to the convolution of a NBD and a PD with parameters derived from the two component model as described in Ref.[6]. The lower part of the panel contains $\chi^2$/dof as well as the plots of residual analysis to test goodness of the fits. The error-bars shown with the data-points include both the statistical and the systematic uncertainties.

Figure 6: Same as Figure 5 for $\sqrt{s} = 2.36$ TeV. Lines of different styles correspond to different fits similar to those as described in the caption of Figure 5. The lower part of the panel contains $\chi^2$/dof as well as the plots of residual analysis to test goodness of the fits. The error-bars shown with the data-points include both the statistical and the systematic uncertainties.
each of these figures contains information on goodness of respective fits. Interestingly, all three distributions, including the one for $\eta_c < 2.4$ at $\sqrt{s} = 7$ TeV, where a prominent change in slope appears, match well with NBD as it is evident from the values of the $\chi^2$/dof, quoted in respective figures. In the same figures the dotted lines correspond to the fit with the convolution of a NBD (with $k=1$) and a PD with parameters extracted from the data by the discussed two component model.

![Figure 7: Same as Figure - 4 for $\sqrt{s} = 0.9$ TeV. Lines of different styles correspond to different fits similar to those as described in the caption of Figure - 4. The lower part of the panel contains $\chi^2$/dof as well as the plots of residual analysis to test goodness of the fits. The error-bars shown with the data-points include both the statistical and the systematic uncertainties.](image)

As can be seen by visual inspection of the plots (with zoomed insets) as well as from the corresponding plots of residual analysis and the values of $\chi^2$/dof, the multiplicity distributions fit better with single NBD than with the convolution of a NBD and a PD in terms of the discussed two component model, indicating poor agreement between the model and the LHC data of multiplicity distribution. The residual is defined as the difference between a data-point and corresponding fit-value.

6. Summary and Remarks

We have studied multiplicity distributions of primary charged hadrons produced in proton-proton collisions of non-single diffractive class of events at LHC energies, in terms of a two-component model of particle production based on quantum statistical formalism. Though the behaviour of characteristic parameters of the formalism is found to be consistent with the LHC data, a scaling law of quantum statistical information entropy, calculated from chaotic multiplicity, extracted from the LHC data by the model, is violated.

A test of agreement between the model and the data by fitting the multiplicity distributions with convolutions of functions as suggested by the model and using values of the characteristic parameters evaluated from the data reveals that the model indeed fails to describe the data satisfactorily. On the other hand, the commonly used single NBD function fits better than the convolution suggested by the model. Our analysis thus shows disagreement between the LHC data and the discussed two-component quantum statistical model in its present form.

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