Optical heterodyne imaging of magnons in 1D magnonic crystals

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Periodic modulation of magnetic permeability by artificial structures called magnonic crystals results in modifying the band structures of magnetostatic modes. Magnonic crystal could be leveraged to manipulate propagation and localization of magnons in those modes. To facilitate further development of magnonic crystals we demonstrate a real-space imaging technique utilizing heterodyne detection of light that is scattered by coherently driven magnons in magnetostatic modes. This method is used to characterize a simple 1D magnonic crystal for a surface magnetostatic mode (Demon-Eshbach mode), which is formed by patterned aluminum strips deposited on the ferromagnetic film. The resultant band structures are deduced from the Fourier transforms of the real-space image of the amplitude and phase.

I. INTRODUCTION

Bloch electrons in crystals exhibit ample phenomena in solid state physics largely due to the associated band structures [1]. Inspired by these electrons in crystals, photonic crystals [2, 3] have been explored to manipulate photons by way of periodic modulation of refractive index. The similar idea can be applied to the other bosonic excitations, such as phonons and magnons.

Among other bosonic excitations magnons in insulating ferromagnets are peculiar in the following three senses; they are (i) breaking time-reversal symmetry (ii) having long coherence time, and (iii) optically detectable. These features offer interesting opportunities to magnons as a carrier of information, entropy, energy, momentum, and angular momentum [4–7]. In this field of magnonics, magnon band engineering with magnonic crystals [8] have been explored to manipulate propagation and localization of magnons [9] for data processing with magnons [10], and to render magnons topologically protected [11, 12].

To facilitate further development of magnonics, imaging of magnetostatic modes in real-space can be a vital approach. The micro-focused Brillouin light scattering (μ-BLS) imaging technique [13] has been developed and widely used in this respect. The μ-BLS imaging is capable of mapping the intensity profile of magnetostatic modes with high spatial resolution and high sensitivity. These are achieved by meticulously filtering the photons from the inelastic Brillouin scattering with Fabry-Pérot cavities and detecting the filtered photons with a photon counter. The photon counting method used in the μ-BLS imaging is intrinsically incapable of detecting the phase of light. The phase information is, however, essential to fully understand the dynamics of magnons and plays a vital role in topological physics [14]. With an additionally introduced local oscillator it is possible to make the μ-BLS imaging phase-sensitive, though [13, 15–18].

Here, we demonstrate an optical heterodyne μ-BLS imaging of magnetostatic modes, where the beat note between the scattered sideband optical field and the input carrier field is detected with a high-speed photodetector. The advantage of the heterodyne imaging is to simultaneously obtain the amplitude and the phase of a microwave-driven magnetostatic mode. We use this method to characterize a simple 1D magnonic crystal for surface magnetostatic modes (Demon-Eshbach modes) [19–22]. Formed by patterned aluminum strips deposited on the ferromagnetic film. From the Fourier transforms of the spatial images the resultant modification of the band structure is verified. This frequency-domain spectroscopic reconstruction of the dispersion of the magnetostatic mode can be viewed as complementary to the all-optical time-domain reconstruction of spin wave dispersion [23].

II. EXPERIMENTAL SETUP

Figure 1 shows the simplified experimental setup for the optical heterodyne imaging. A field from a CW laser of 1550 nm in wavelength is purified in polarization along the z-axis with the first polarizer as shown in Fig. 1(b) and propagates toward the negative x-axis direction as shown in Fig. 1(a). The input field is tightly focused (waist radius ≈ 4.3 μm) onto a sample of a ferromagnetic film by an objective lens. The sample can be moved in 3D with respect to the focus of the input optical field with a 3-axis stage with stepper motor actuators [THORLABS: ZFS13B], of which spatial resolution is 0.5 μm. Although the input field has the relatively long Rayleigh range (∼ 37 μm), the field is mainly scattered by the magnons on the upper side of the film since the antenna mainly excites the magnons on that side. The field then undergoes Brillouin light scattering which creates sideband fields with a different polarization as we will describe below. The scattered sideband fields, as well as the unscattered carrier field, are collected by...
FIG. 1. (a) Experimental setup. A field from a CW laser of 1550 nm in wavelength is sent through polarizer 1 propagates toward the negative x-axis direction. The input field is tightly focused onto a sample by objective lens 1. The sample can be moved with a 3-axis motorized stage. The scattered sideband fields, as well as the unscattered carrier field, are collected by objective lens 2 and coupled to a photodetector through polarizer 2. The heterodyne beat note between the scattered sideband fields and the carrier field is amplified and fed into a vector network analyzer (VNA). (b) Schematic picture for polarization mixing: the polarizer 1 sets the polarization of the input carrier field as z-polarization. The polarization of the scattered sideband field is y polarization. The carrier field and the sideband field are mixed through the polarizer 2, which is rotated by 45° with respect to the polarizer 1.

another lens and coupled to a high-speed photodetector [NEW FOCUS: 1554-B] through a single-mode fiber after the second polarizer tilted 45 degrees from the y-axis as shown in Fig. 1(b). The heterodyne beat note between the scattered sideband fields and the carrier field is then amplified [MINI CIRCUITS: ZX60-83LN-S+] and fed into a vector network analyzer [AGILENT TECHNOLOGIES: N5232A], which demodulates the beat signal with the drive signal used to excite the magnons.

The Brillouin light scattering takes place when the field creates additional magnons or annihilates magnons. Since each magnon possesses a spin angular momentum \( \hbar \), creation or annihilation of magnon means the change of the polarization of a photon in the scattered field. This correspondence between the magnon and the scattered photon in terms of spin angular momentum is due to the conservation of angular momentum. The number of the scattered photons and that of magnons are also correlated because of the conservation of energy, while the relation between the phase of the scattered light and that of driving magnetostatic mode is also secured thanks to the conservation of momentum [24]. Thus by looking at the scattered photons, we could obtain full information regarding the magnons involved in the scattering.

At this juncture, let us discuss the performance of the heterodyne \( \mu \)-BLS imaging. The bandwidth of the heterodyne \( \mu \)-BLS imaging is only limited by the photodetector bandwidth, which can be as large as 10 GHz. The sensitivity of the heterodyne imaging is shot-noise-limited, which differs from the conventional photon-counting-based \( \mu \)-BLS imaging for which the sensitivity is limited by the dark counts. This means that, given that the number of input carrier photons is \( N \), the noise-equivalent scattering rate would be \( \sqrt{N} \) per second.

FIG. 2. Micrographs of samples: (a) plain ferromagnetic film. (b) 1D magnonic crystal with aluminum stripes. Vector network analyzers (VNA) are connected to metallic antennas for excitation. The static magnetic field \( (B_{ext} \sim 106 \text{ mT}) \) is applied for each sample along the z-axis. (c) Unit-cell design for the magnonic crystal.
III. RESULTS

A. Plain ferromagnetic film

1. Sample

The samples are tangentially magnetized under a static magnetic field \((B_{\text{ext}} \sim 106 \text{ mT})\) along positive z-axis in as shown in Fig. 1, which is produced by a pair of permanent magnets with a pure iron magnetic circuit and an additional solenoid winding around the magnetic circuit so as to vary the static magnetic field by the fraction of the one created by the permanent magnets. The samples are thin films made of Yttrium Iron Garnet (YIG) [thickness: \(d = 9.5 \mu\text{m}\)] on Gadolinium Gallium Garnet (GGG) substrate [thickness: 0.5 mm]. Figure 2(a) shows a sample with a microwave antenna made of aluminum, which is deposited on the plain YIG film. The antenna is designed to excite mainly magnons in a surface magnetostatic mode called Demon-Eshbach mode [19, 20]. The magnons in the Demon-Eshbach mode are propagating only in one direction on the upper surface of the film (to the positive y-axis) and doing in the other on the bottom surface (to the negative y-axis).

2. Dispersion of the Demon-Eshbach modes

Figures 3(a) and (b) respectively show the 1D real-space image of the amplitude and phase of the heterodyne signal as a function of drive frequency for the magnons in the plain YIG film shown in Fig. 2(a). Here, the phase is defined with respect to the edge of the antenna as shown by arrows in the figure. Here, the intense signals are found around the dashed lines in Figs. 3(a) and (b). These are attributed to a beam-like mode, called the caustic wave beam [25], which is diagonally propagating from the corner of the antenna. We shall explain how the caustic wave beam emerges from the peculiar hyperbolic-like dispersion of the Demon-Eshbach modes in Appendix A.

The measured 1D real-space image of the amplitude and the phase, shown in Figs. 3(a) and (b), respectively, can be used to reconstruct the dispersion relations by performing the discrete Fourier transformation of the real-space image. The discrete Fourier transformation is given by

\[
[F(s)]_m = \frac{1}{N} \sum_{n=0}^{N-1} s_n \exp \left( 2\pi j \frac{mn}{N} \right),
\]

(1)

where the complex transmission amplitude \(S_{21}\) at \(x = x_j = j\Delta x\) \((j = 0, 1, \cdots, N - 1)\) is denoted by \(s_j\), and \([F(s)]_m\) with \(s = [s_0, s_1, \cdots, s_{N-1}]^T\) is the complex amplitude at the angular wavenumber \(k_m = m\Delta k\), the step of which is \(\Delta k = 2\pi/(N\Delta x)\) with the length interval \(\Delta x\) of data points. Figure 3(c) shows the resultant Fourier transform of the 1D real-space image.

FIG. 3. 1D real-space images of (a) intensity and (b) phase of the heterodyne signal from the Brillouin light scattering by magnons in the plain ferromagnetic film as a function of the microwave frequency (vertical axis) at which magnons are excited. The dashed lines in (a) and (b) represent the position where the caustic wave beam coming from the antenna corner hits the 1D scanning line (see Appendix A). The phase reference point with zero phases is represented by arrows in (b). (c) Dispersion reconstructed from the real-space image shown in (a) and (b) by Fourier transformation. The dashed line indicates the theoretically evaluated dispersion with \(M_s = 166.5 \text{ kA/m}\) and \(B_{\text{ext}} = \mu_0 H_{\text{ext}} = 108.9 \text{ mT}\).
In the magnetostatic region, the Demon-Eshbach mode obeys the following dispersion relation, [20–22]:

\[ k = -\frac{1}{2d} \ln \left[ 1 + \frac{4}{\gamma M_B^2} \left( \omega_0^2 + \omega_M^2 \right) - \omega^2 \right], \]  

(2)

where \( \omega_M = -\gamma \mu_0 M_s \) and \( \omega_0 = -\gamma B_{\text{ext}} \) with \( d \) being thickness of the film, \( \mu_0 \) being the vacuum permeability, \( M_s \) being the saturation magnetization, and \( \gamma \) \((<0)\) being the gyromagnetic ratio. For YIG, we have \( \gamma \approx -1.761 \times 10^7 \text{ rad GHz/T} \). The blue dashed line in Fig. 3(c) corresponds to the theoretical dispersion of the Demon-Eshbach mode \([\text{the inverse version of Eq. (2)}]\):

\[ \omega = \sqrt{\omega_0^2 + \omega_M^2} \left[ 1 - \exp(-2kd) \right], \]  

(3)

where \( M_s \) and \( B_{\text{ext}} \) are used as the fitting parameters. From the fitting, we obtain \( M_s = 166.5 \pm 0.4 \text{ kA/m} \) and \( B_{\text{ext}} = \mu_0 H_{\text{ext}} = 108.9 \pm 0.2 \text{ mT} \), which are reasonable agreements with the expected value of \( M_s \) at room temperature, 140 kA/m \([26]\), and the measured value of \( B_{\text{ext}} \approx 106 \text{ mT} \), respectively. This suggests that the method to reconstruct the dispersion relation from the real-space imaging works pretty well. It is emphasized that to reconstruct the dispersion from the real-space imaging it is vital to have both of the amplitude and the phase of the optical signal. We envision that not only the magnitude of the dispersion shown in Fig. 3(c) but also the phase in the reciprocal space can, in principle, be reconstructed. The phase information in the reciprocal space could be useful when the topological aspects of the dispersion are of interest.

**B. 1D magnonic crystal**

1. Sample

Figure 2(b) shows a sample with 4 microwave antennas embedded in the 1D magnonic crystal \([27–30]\). The unit cell for the magnonic crystal is defined in the 1 mm \( \times \) 120 \( \mu \)m as shown in Fig. 2(c). The aluminum region imposes an additional boundary condition on the tangential electric field, and thus modifies the magnetic permeability.

2. 1D magnonic crystal and bandgap formation

Figures 4(a) and (b) respectively show the results of the real-space imaging of the amplitude and the phase of the heterodyne signal obtained with the 1D magnonic crystal. With these data the discrete Fourier transformation, Eq. (1), is carried out. Figure 4(c) shows the resultant Fourier transform. We can clearly see the bandgaps around 5.3 GHz and 5.5 GHz. Note that in the results shown in Fig. 4(c) the portion of the dispersion curves where the magnon has a negative group velocity is absent. This can be understood by the fact that the magnons are excited by the antenna, which mainly excites the magnons on the upper side of the film, and thus flow from left to right on that side of the film.

Now let us analyze the observed bandgaps. The magnonic crystal shown in Fig. 2(b) has metallic stripes. The region with a top metal is denoted by M, while that without the metal is written as A. If the metal thickness is larger than the microwave penetration depth \((\approx 1.2 \mu \text{m} \text{ at } 5 \text{ GHz for aluminum with the conductivity } \sigma_{\text{dc}} = 27 \text{ nS/m})\), the metal can be safely considered perfectly conducting, and the dispersion relation in the region \( M \) can be given by \([22, 31]\):

\[ k_{\pm} = -\frac{1}{2d} \ln \left[ \left( 1 + \frac{2\omega_0^2}{\omega_M^2} \pm \frac{2\omega}{\omega_M} \right) \frac{\omega_0 + \omega_M \mp \omega}{\omega_0 + \omega_M \pm \omega} \right], \]  

(4)

for propagation toward the positive (negative) \( y \)-direction. This modification of the dispersion compared to that in the region \( M \) \([\text{given in Eq. (2)}]\) is the key to realize the magnonic crystal.

Bandgap formation due to the magnonic crystal can be captured by a simple 1-dimensional transfer matrix model. Consider a \textit{magnetostatic potential} \([22]\), \( \psi(y, t) = \tilde{\psi}(y) \exp(j\omega t) + c.c. \), inside the magnonic crystal with the complex amplitude \( \tilde{\psi}(y) \), which is defined in terms of the induced \( \text{H} \)-field along the \( y \)-axis, \( h(y, t) \), by

\[ h(y, t) = -\frac{\partial}{\partial y} \tilde{\psi}(y, t). \]  

(5)

Here, for simplicity, we suppress \( z \) and \( x \) dependencies in \( \psi \). We can decompose \( \tilde{\psi}(y) \) into the counter propagating components \( \tilde{\psi}_+(y) \) and \( \tilde{\psi}_-(y) \) as \( \tilde{\psi}(y) = \tilde{\psi}_+(y) + \tilde{\psi}_-(y) \), where \( + \) and \( - \) represent positive and negative propagation along the \( y \)-axis, respectively. These components are combined and denoted as \( \tilde{\psi}(y) = \left[ \begin{array}{c} \tilde{\psi}_+(y) \\ \tilde{\psi}_-(y) \end{array} \right] \). To calculate the dispersion relation of the magnonic crystal, we define a transfer matrix of the unit cell shown in Fig. 5 as

\[ T_{\text{unit}} = T_{tA}^A T_{sAM}^A T_{tM}^M T_{sMA}^M. \]  

(6)

Here, \( T_{tA} \) \((T_{tM})\) is a free propagation transfer matrix in the region \( A \) \((M)\). At the interface between the region \( A \) \((M)\) and \( M \) \((A)\) a distinct transfer matrix \( T_{sAM}^A \) \((T_{sMA}^M)\) can be defined. For more details regarding each transfer matrix we shall explain later on. For a Bloch state \( \psi \) with \( y \)-component \( k_c \) of the crystal angular wave vector, we have

\[ T_{\text{unit}} \tilde{\psi} = \exp(jk_c a) \tilde{\psi} \]  

(7)

with the unit cell length \( a \). Therefore, by diagonalizing \( T_{\text{unit}} \) for a given \( \omega \), we obtain \( k_c \) as a function of \( \omega \).

The transfer matrix of free propagation along length \( l \) in the region \( A \) is reciprocal and written as

\[ T_{lA}^A = \left[ \begin{array}{cc} \exp(jkl) & 0 \\ 0 & \exp(-jkl) \end{array} \right]. \]  

(8)
where, for a given $\omega, k$ is given by Eq. (2). On the other hand, that in the region M is nonreciprocal and written as

$$T_i^M = \begin{bmatrix} \exp(jk_+ l) & 0 \\ 0 & \exp(-jk_- l) \end{bmatrix},$$

where, for a given $\omega$, we obtain $k_\pm$ through Eq. (4).

At the interface $y = \xi$ between the region M and the region A, for instance, the scattering occurs and mixes up $\psi_+$ and $\psi_-$. This mixing is denoted with the transfer matrix $T_{sAM}$ by $\tilde{\psi}^A = T_{sAM}^A \tilde{\psi}^M (\xi)$. Here, $T_{sAM}$ can be determined from the following boundary conditions: At $y = \xi$, $\tilde{\psi}^A$ and $B_y^A \approx -\mu_{yy} \partial_y \tilde{\psi}^A$ must be equal to $\tilde{\psi}^M$ and $B_y^M \approx -\mu_{yy} \partial_y \tilde{\psi}^M$, where $\mu_{yy}$ represents $yy$-component of the permeability tensor. These conditions amount to

$$M_A \tilde{\psi}^A (\xi) = M_M \tilde{\psi}^M (\xi),$$

where $M_A = \begin{bmatrix} 1 & 1 \\ k & -k \end{bmatrix}$ and $M_M = \begin{bmatrix} 1 & 1 \\ k_+ & -k_- \end{bmatrix}$. The transfer matrix $T_{sAM}$ at the interface $y = \xi$ is thus given by

$$T_{sAM} = M_A^{-1} M_M.$$

The transfer matrix $T_{sMA}$ is similarly obtained as

$$T_{sMA} = M_M^{-1} M_A.$$

For a given angular frequency $\omega$, we have a $2 \times 2$ transfer matrix $T_{uni}$ for the unit cell of the magnonic crystal given by Eq. (7). Form this characteristic equation we can obtain $k_\xi$ as a function of $\omega$, which is shown as the blue dashed lines in Fig. 4(c). Here, the fitting parameters are $B_{ext}$ and $M_s$. These values are found as $B_{ext} = 99.55 \pm 0.03$ mT, $M_s = 164.91 \pm 0.06$ kA/m, which are similar to those obtained from the plain film above. This agreement indicates that the model we developed captures the physics behind the bandgap formation. The optical heterodyne imaging thus provides a powerful mean to diagnose magnon propagation in magnonic crystals and related artificial magnetic structures.
IV. SUMMARY

The heterodyne μ-BLS imaging is demonstrated to study a simple 1D magnonic crystal. In this method, both the amplitude and phase of the optical heterodyne signal, which stems from the scattering by the microwave-driven magnons, are simultaneously obtained. The real-space image of the information can be acquired by scanning the sample with respect to the optical path. The Fourier transforms of the real-space image can be used to obtain dispersion relations of the magnetostatic modes and to verify the opening of the magnonic bandgap caused by the 1D magnonic crystal. Our results show that the heterodyne μ-BLS imaging could be a powerful and simple way to probe magnons in ferromagnetic films, paving a way to investigate more complex phenomena, such as Anderson localization and topological phenomena with magnons.

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Appendix A: Effect of magnon caustics

Here, we discuss the physical origin of the intense signals around the dashed lines in Figs. 3(a) and (b). We shall see that the signal is due to a non-diffracting beam-like mode (called caustic wave beam [25]), which is generated around the antenna corner.

To this end, let us consider the dispersion of the Demon-Eshbach mode in 2D. Since the Demon-Eshbach mode is the surface mode, the dispersion is peculiar, whose isofrequency contour is hyperbolic-like shape as opposed to the typical elliptical shape. The isofrequency contour is given by \[ \Omega = \omega/\omega_M \]
with \( \kappa = \omega_M\omega_0/(\omega_0^2 - \omega^2) \), \( \nu = \omega_M\omega/(\omega_0^2 - \omega^2) \). Here, \( d \) is the thickness of the film. Figure 6 depicts such isofrequency contours for \( \Omega = \omega/\omega_M \) from 0.875 to 0.995 with \( \Omega_0 = \omega_0/\omega_M = 0.5 \).

From this figure, we can recognize that with given angle \( \theta \) defined by

\[ \tan \theta \equiv \eta = \frac{k_z}{k_y} = \frac{\varphi_z}{\varphi_y}, \]
there exists the isofrequency contour whose asymptote lays on a line. This indicates that magnons originate from this asymptote propagate the same direction, which is perpendicular to the asymptote as shown in the left panel of Fig. 7. This leads to beam-like magnon propagations, that is, so-called caustic wave beams [25], as shown in the right panel of Fig. 7. Note here that, around the antenna corner, magnons with various wavenumbers and propagation directions can be generated.

To experimentally address caustic wave beam, we perform the 2D optical heterodyne imaging. Figures 8(a) and (b) show the amplitude and phase of the optical heterodyne signal for magnons propagating around the corner. In Fig. 8(a), caustic wave beams is indeed visible. The 1D dispersions are reconstructed from the Fourier transforms of the experimental data along the arrow labeled by X in Fig. 8(a) for several microwave frequencies, obtaining \( M_s = 151 \text{ kA/m} \) and \( B_{ext} = 104 \text{ mT} \) as the fitting parameters.

Next, we apply the 2D Fourier transformation with hamm windows for the 2D data and obtain 2D dispersion relations for each frequency as shown in Fig. 9. These results clearly indicate the hyperbolic-like nature of the

![ Isofrequency contours given by Eq. (A1) of the Demon-Eshbach modes for \( \Omega \) ranging from 0.875 to 0.995. ]
isofrequency contours of Demon-Eshbach modes. The theoretical isofrequency contours are also plotted. The experimentally reconstructed isofrequency contours agree well with the theoretical contours with the fitting parameters $M_s = 151 \text{ kA/m}$ and $B_{ext} = 104 \text{ mT}$.

With these preparations, let us now analyze the intense signals around the dashed lines in Figs. 3(a) and (b). We claim that the intense signal coming from the caustic wave beam originated from the antenna corner as shown in the right panel of Fig. 7. Only considering asymptotes for Eq. (A1), we can put $\cot(\varphi_x) \to -j$. Then, Eq. (A1) becomes

$$(1 + \eta^2) + 2(1 + \kappa)\sqrt{1 + \eta^2}\sqrt{1 + \frac{\eta^2}{1 + \kappa}} + (1 + \kappa)^2\left(1 + \frac{\eta^2}{1 + \kappa}\right) - \nu^2 = 0, \quad (A7)$$

with $\eta = k_z/k_y$. In the right panel of Fig. 7, we show how to calculate the position $y = l\eta$ along the 1D scanning line for obtaining the result shown in Figs. 3(a) and (b). The caustic wave beams come from the corner of the antenna hits. Using Eq. (A7) with $l = 0.5 \text{ mm}$, $M_s = 166.5 \text{ kA/m}$ and $B_{ext} = \mu_0H_{ext} = 108.9 \text{ mT}$, $d = 9.5 \mu\text{m}$, and $\Omega = \frac{2\pi f}{\omega_\nu}$, the positions $y = l\eta$ as a function of frequency are depicted as the dashed blue line in Figs. 3(a) and (b). These theoretically estimated positions are nicely matched to the intense signals we observed.

[1] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, 1976).

[2] E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987).

[3] S. John, Phys. Rev. Lett. 58, 2486 (1987).

[4] V. V. Kruglyak, S. O. Demokritov, and D. Grundler, J. Phys. D: Appl. Phys. 43, 264001 (2010).

[5] A. A. Serga, A. V. Chumak, and B. Hillebrands, J. Phys. D: Appl. Phys. 43, 264002 (2010).

[6] S. A. Nikitov, D. V. Kalyabin, I. V. Lisenkov, A. Slavin, Y. N. Barabanenkov, S. A. Osokin, A. V. Sadovnikov, E. N. Beginin, M. A. Morozova, Y. A. Filimonov, Y. V. Khvintsev, S. L. Vysotsky, V. K. Sakharov, and E. S. Pavlov, Phys. Usp. 58, 1002 (2015).

[7] A. V. Chumak, V. I. Vasyukha, A. A. Serga, and B. Hillebrands, Nat. Phys. 11, 453 (2015).

[8] M. Krawczyk and D. Grundler, J. Phys.: Condens. Matter 26, 123202 (2014).

[9] S. O. Demokritov, ed., *Spin Wave Confinement: Propagating Waves*, second edition ed. (Pan Stanford Publishing, Singapur, 2017).

[10] A. V. Chumak, A. A. Serga, and B. Hillebrands, J. Phys. D: Appl. Phys. 50, 244001 (2017).

[11] R. Shindou, R. Matsumoto, S. Murakami, and J.-i. Ohe, Phys. Rev. B 87, 174427 (2013).

[12] Y.-M. Li, J. Xiao, and K. Chang, Nano Lett. 18, 3032 (2018).

[13] T. Sebastian, K. Schultheiss, B. Obry, B. Hillebrands, and H. Schultheiss, Front. Phys. 3, 35 (2015).

[14] D. Vanderbilt, *Berry Phase in Electronic Structure Theory* (Cambridge University Press, Cambridge, 2018).

[15] A. A. Serga, T. Schneider, B. Hillebrands, S. O. Demokritov, and M. P. Kostylev, Appl. Phys. Lett. 89, 063506 (2006).

[16] F. Föhr, A. A. Serga, T. Schneider, J. Hamrle, and B. Hillebrands, Rev. Sci. Instrum. 80, 043903 (2009).

[17] K. Vogt, H. Schultheiss, S. J. Hermsoofer, P. Pirro, A. A. Serga, and B. Hillebrands, Appl. Phys. Lett. 95, 182508 (2009).

[18] V. E. Demidov, S. Urazhdin, and S. O. Demokritov, Appl. Phys. Lett. 95, 262509 (2009).

[19] J. R. Eshbach and R. W. Damon, Phys. Rev. 118, 1208 (1960).

[20] R. Damon and J. Eshbach, J. Phys. Chem. Solids 19, 308 (1961).

[21] D. D. Stancil and A. Prabhakar, *Spin Waves: Theory and Applications* (Springer Science & Business Media, 2009).

[22] A. G. Gurevich and G. A. Melkov, *Magnetization Oscillations and Waves* (CRC Press, 1996).

[23] Y. Hashimoto, D. Shunsuke, R. Iguchi, Y. Oikawa, K. Shen, K. Sato, D. Bossini, Y. Tabuchi, T. Satoh, B. Hillebrands, G. E. W. Bauer, T. H. Johansen, A. Kirilyuk, T. Rasing, and E. Saitoh, Nature Commun. 8, 15859 (2017).

[24] R. Hisatomi, A. Osada, Y. Tabuchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Phys. Rev. B 93, 174427 (2016).

[25] T. Schneider, A. A. Serga, A. V. Chumak, C. W. Sandweg, S. Trudel, S. Wolf, M. P. Kostylev, V. S. Tiberkevich, A. N. Slavin, and B. Hillebrands, Phys. Rev. Lett. 104, 197203 (2010).

[26] D. D. Stancil, *Theory of Magnetostatic Waves* (Springer-Verlag, 1993).

[27] N. Kanazawa, T. Goto, and M. Inoue, J. Appl. Phys. 116, 083903 (2014).
FIG. 8. 2D real-space images of (a) intensity and (b) phase of the heterodyne signal from the Brillouin light scattering by magnons in the plain ferromagnetic film at the microwave frequency, $\Omega = 1.01$, at which magnons are excited.

[28] M. Mruczkiewicz, E. S. Pavlov, S. L. Vysotsky, M. Krawczyk, Y. A. Filimonov, and S. A. Nikitov, Phys. Rev. B 90, 174416 (2014).
[29] V. D. Bessonov, M. Mruczkiewicz, R. Gieniusz, U. Guzowska, A. Maziewski, A. I. Stognij, and M. Krawczyk, Phys. Rev. B 91, 104421 (2015).
[30] N. Kanazawa, T. Goto, J. W. Hoong, A. Buyandalai, H. Takagi, and M. Inoue, J. Appl. Phys. 117, 17E510 (2015).
[31] S. Seshadri, Proc. IEEE 58, 506 (1970).
FIG. 9. Experimentally reconstructed isofrequency contours in $\Omega = 0.02$ intervals. These contours are reconstructed from the real-space images. The dashed lines and the curved surface indicate the theoretically evaluated contours [Eq. (A1)] with $M_s = 151$ kA/m, $B_{ext} = 104$ mT.