Hadron-quark mixed phase in neutron stars

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Possibility of structured mixed phases at first order phase transitions is examined by taking into account of charge screening and surface effects. Hadron-quark phase transition in dense neutron star interiors is considered as a concrete example.

1. INTRODUCTION

Recently many efforts have been made for the derivation of the equation of state (EOS) of the neutron star matter exhibiting possibilities of various phase transitions in neutron star interiors. Some of these possible phase transitions are the first order phase transitions (FOPT) essentially affecting the EOS. We will discuss this issue by considering the hadron-quark (H-Q) matter phase transition\cite{1}.

In Ref. \cite{2} Glendenning has pointed out that the usual Maxwell construction is applicable only for systems with one particle species and respectively one chemical potential, whereas in neutron stars there are two relevant quantities, the charge and baryon number chemical potentials. He has advocated to use the Gibbs conditions and demonstrated the appearance of a wide region of the structured mixed phase, consisting of droplets, rods or slabs of one phase embedded into the other phase, instead of the Maxwell construction. Finite size effects, the Coulomb interaction and the surface tension, in the treatment of the mixed phase were disregarded. His idea was further exploited by many authors. It was concluded that a wide region of the mixed phase is inevitable and there should be no region of constant pressure, opposite to that follows from the Maxwell construction.

On the other hand, Heiselberg et al. pointed out the importance of the inclusion into consideration of the finite size effects to realize the structured mixed phase \cite{4}. They demonstrated that for rather large values of the surface tension the mixed phase constructed of rather small size droplets becomes energetically unfavorable compared to the two bulk-phase structure given by the Maxwell construction. Recently it was argued that the Gibbs condition of the equality of the electron chemical potentials is

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not satisfied at the interface between the color-flavor-locked phase (no electrons) and the nucleon matter phase and it is described by the Maxwell construction \[3\]. In the above treatments the charged particle density profiles are uniform in each phase and furthermore corrections due to charge screening effects are disregarded. In spite of the question on the role of the screening has been raised long ago, up to now there was no consistent treatment of these inhomogeneity effects. Therefore, a further study of the screening and surface effects seems to be of prime importance.

2. DENSITY FUNCTIONAL METHOD

Consider a quark droplet (I) immersed in the \((n,p,e)\) nuclear matter (II) within the Wigner-Seitz cell. The narrow boundary region between them is approximated by the sharp boundary \(\partial D\). Then the total energy functional \(E\) is given by a functional of particle densities \(\rho_i[5]\):

\[
E[\rho(r)] = \int_{D_I} d^3r \epsilon_{\text{kin}}^I(\rho^I_i(r)) + \int_{D_{II}} d^3r \epsilon_{\text{kin}}^{II}(\rho^{II}_i(r)) + \int_{\partial D} d^2r \epsilon_{S} + E_V,
\]

where the first and second terms are the kinetic plus strong interaction contributions in both phases, the third term is the surface energy contribution. The last term is the usual Coulomb interaction energy,

\[
E_V = \frac{1}{2} \int \int d^3r d^3r' \sum_{i,j} \frac{Q_i \rho_i(r) Q_j \rho_j(r')}{|r - r'|},
\]

with \(Q_i\) being the particle electric charge. Then the generating functional under the constraint of the particle number conservation is introduced via the Legendre transformation,

\[
\Omega[\rho(r)] = E[\rho(r)] - \sum_i \mu^I_i \int_{D_I} d^3r \rho^I_i(r) - \sum_i \mu^{II}_i \int_{D_{II}} d^3r \rho^{II}_i(r).
\]

Note that \(\Omega[\rho(r)]\) is the analog of the thermodynamic potential and the parameters \(\mu^\alpha_i\) are the chemical potentials. When we consider two conservation laws relevant in the mixed phase: baryon number and charge conservation, these quantities are well defined over the Wigner-Seitz cell, not restricted to each domain. Accordingly the baryon number and charge chemical potentials (\(\mu_B\) and \(\mu_Q\)), being linear combinations of \(\mu^\alpha_i\), become constants over the whole space,

\[
\mu^I_B = \mu^{II}_B \equiv \mu_B, \quad \mu^I_Q = \mu^{II}_Q \equiv \mu_Q;
\]

e.g. \(\mu_Q = \mu_e^\alpha\) and \(\mu_B = \mu_n = 2\mu_d + \mu_u\).

Equations of motion are given by the variational principle:

\[
\frac{\delta \Omega[\rho(r)]}{\delta \rho^\alpha_i(r)} = 0 \quad \text{or} \quad \mu^\alpha_i = \frac{\partial \epsilon_{\text{kin}}^\alpha \rho^\alpha_i}{\partial \rho^\alpha_i} - (Q_i^\alpha/e)V^\alpha(r), \quad \alpha = \{I, II\},
\]

where \(V^\alpha(r)\) is the electric potential,

\[
V = - \int d^3r' \sum_i \frac{e Q_i \rho_i(r)}{|r - r'|} = \begin{cases} V^I(r) \quad r \in D^I, \\ V^{II}(r) \quad r \in D^{II}, \end{cases}
\]
and satisfies the Poisson equation,
\[ \Delta V^\alpha = 4\pi e \sum_i Q_i \rho^\alpha_i. \]  
(7)

Note here that the electric potential can be shifted by an arbitrary constant, \( V^0 \), and theory should be invariant under the redefinition of chemical potentials in a proper way; \( V \rightarrow V - V^0, \mu_i^\alpha \rightarrow \mu_i^\alpha + (Q_i^\alpha/e)V^0 \). One convenient choice is \( V^0 = 0 \) that means that the electron density can be expressed as \( \rho_e = (\mu_e - V)^3/(3\pi^2) \). Another convenient choice is \( V^0 = -\mu_e \) corresponding to \( \rho_e = -V^3/(3\pi^2) \), see [1].

If we expand the charge density in the r.h.s. of the Poisson equation with respect to \( V^\alpha \) around a reference value \( V^\alpha_r \) up to the second order, we arrive at the equation
\[ \Delta \tilde{V}^\alpha = 4\pi \sum_j Q_j^\alpha \rho_j^\alpha (V^\alpha = V^\alpha_r) + \kappa_\alpha^2 \tilde{V}^\alpha \]  
(8)

with \( \tilde{V}^\alpha = V^\alpha - V^\alpha_r \). The coefficient \( \kappa_\alpha \) is the inverse of the Debye screening length,
\[ \kappa_\alpha^2 \equiv 1/(\lambda_D^\alpha)^2 = 4\pi \sum_{i,j} Q_j^\alpha \rho_i^\alpha \frac{\partial \mu_i^\alpha}{\partial \mu_j^\alpha} \bigg|_{V^\alpha = V^\alpha_r}. \]  
(9)

Electrons (quarks) can be treated as having the uniform charge density profile only if the droplet size and the inter-droplet distance are essentially smaller than the screening length \( \lambda_e, D (\lambda_q, D) \). Otherwise, as we shall demonstrate it below, the screening effects become essential and the charged density profiles are not uniform. Estimates show that the screening length for electrons \( \lambda_e, D \geq 13 \text{ fm} \) is longer than that for the quarks, \( \lambda_q, D \simeq 5 \text{ fm} \).

Pressure in each phase is constant owing to the equation of motion (5), and the pressure balance condition at the boundary \( \partial D \) reads
\[ P^I = P^{II} + P_S, \]  
(10)

where \( P_S \) denotes the surface contribution. In the following we assume the spherical geometry and presence of the sharp boundary parameterized by the constant surface tension \( \sigma \). Then Eq. (10) is equivalent to the extremum condition, \( \partial \Omega/\partial R = 0 \), where \( R \) is the droplet radius. Eqs. (4), (10) are manifestation of the Gibbs conditions.

3. RESULTS AND DISCUSSION

In Fig. 1 we show the effective energy of the Wigner-Seitz cell per droplet volume associated with the inhogeneity of the electric field profile, \( \delta \tilde{\omega} = 3\Omega[\rho(\mathbf{r})]/(4\pi R^3) \), up to the second order in \( \tilde{V}^\alpha \) as a function of the scaled droplet radius \( \xi = R/\lambda_D^1 \). The curves are presented for two values of the concentration of phase I, \( f = (R/R_W)^3 \), \( R_W \) is the radius of the Wigner-Seitz cell. Solid curves are for \( f = 1/2 \) and dashed curves are for a tiny value \( f = 0.01 \). The “e.m.” curves show the partial contribution of the (screened) electric field to the energy \( (\propto \int (\nabla V)^2 d^3r) \) plus the surface energy. The difference between the non-labeled curves and the “e.m.” curves shows the important role of the secondary (correlation) effects coming from the dependence of the bulk energies \( (\epsilon_{\text{kin}+\text{str}}) \) on the inhomogeneous distribution of the electric field. The ratio of the screening lengths \( \alpha_0 = \lambda_D^1/\lambda_D^{II} \) is assumed to be 1.
We vary the value of the surface tension, $\beta_1 = 3\sigma/(\lambda^4\beta_0)$ with the energy scale parameter $\beta_0$, admitting its uncertainties. E.g., for typical values $\mu_e \simeq 170$ MeV for the electron chemical potential, $\mu_n \simeq 1020$ MeV for the baryon (neutron) chemical potential, $\alpha_c \simeq 0.4$ for the strong coupling constant, and $m_s \simeq 120 \div 150$ MeV for the strange quark mass, we estimate $\beta_0 \simeq 1.6m_s^4$. Then, with the value $\sigma \simeq 1.3m_s^2$ we obtain $\beta_1 \simeq 0.7$, whereas with $\sigma \simeq 10$ MeV/fm$^2 \simeq 0.14m_s^2$ we would get $\beta_1 \simeq 0.08$. The Coulomb curves (labeled by “C”) show the corresponding results obtained neglecting screening effects. We see that the “C” curves demonstrate pronounced minima at $\xi = \xi_C \propto \beta_1^{1/3}$, while there is no minimum for $\beta_1 > \beta_{1c}$ if one includes screening effects. According to our figure $\beta_{1c} \simeq 0.6$ (would be $\beta_{1c} \simeq 0.03$ for ”e.m.” curves, when correlation energy effects are disregarded). Only for $\beta_1 \leq 0.01$ the minima at the ”C” curves differ not essentially from the minima at other curves. Hence our results clearly show that the screening effects are important for all realistic values of $\sigma$ and that the structured mixed phase is proved to be prohibited due to the kinetic instability of the droplets induced by the screening effects provided the surface tension is larger than a critical value. In absence of the mixed phase our charged distributions describe the boundary layer between two separated phases existing within the double-tangent (Maxwell) construction. Consideration of non-spherical droplets (rods and slabs) does not change our conclusions[1].

REFERENCES

1. D.N. Voskresensky, M. Yasuhira and T. Tatsumi, Phys. Lett. B541 (2002) 93; nucl-th/0208067, submitted to Nucl. Phys. A.
2. N.K. Glendenning, Phys. Rev. D46 (1992) 1274.
3. M. Alford, K. Rajagopal, S. Reddy and F. Wilczek, Phys. Rev. D64 (2001)074017.
4. H. Heiselberg, C.J. Pethick and E.F. Staubo, Phys. Rev. Lett. 70 (1993) 1355.
5. R.G. Parr and W. Yang, Density-Functional Theory of Atoms and Molecules, (Oxford U. Press, 1989).
Figure 1. Contribution to the effective energy per droplet volume due to inhomogeneous charge distributions versus scaled droplet radius for the droplet concentration $f = 1/2$ (solid curves) and a tiny concentration $f = 0.01$ (dashed curves), $\beta_0$ is the scale-parameter for the energy, and $\beta_1$ is the parameter of surface tension (see the text). The "C" curves are calculated ignoring screening effects, the "e.m." curves include the electric field energy (with screening effect included) plus surface energy, ignoring however the correlation effects.