Early Shape Morphing: the Metamorphosis of Polygons in Antoni Gaudi’s Sagrada Familia Cathedral and Le Corbusier’s Firminy Chapel

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Abstract

This paper first discusses the fundamental notion of shape morphing and morphing techniques. Then it sets out to introduce early applications of the notion in architecture. Antoni Gaudi’s Sagrada Familia Cathedral in Spain and Le Corbusier’s Firminy Chapel in France are examined with regard to shape morphing. The examination seeks to cast new light on the significance of the two designs whose morphing method has become the legacy of an innovative characteristic of considerable contemporary architecture.

Keywords: shape morphing; polygons; gaudi; le corbusier

Introduction

In recent years shape morphing has become popular as a form-making process, enabled by the computational power of modern computer technology. Some contemporary architects have quickly and cleverly adopted the method, albeit claiming it as new technique in architecture (Rahim, 2002, Lynn, 1998). The potency of the method is that during the morphing process, a number of in-between evolutionary forms are algorithmically generated in the computer, after which an architect selects one of many and converts into an architectural design. The method is based on the sequential evolution of a mapping from one image to the other.

Although this sort of approach may lead to an interesting visual experimentation, it sometimes provokes an extensive resistance. Some critics consider the process to produce “merely unbuildable utopias” which “lock the discipline into endless loops of aesthetic experimentation” (Andia, 2002). One of the more serious criticisms is that the shape-morphing approach is a way of determining form without engaging cultural connotations, human activities, or the philosophical insights of building concepts. The criticism comes from a belief that the process merely derives from visual pursuits of architectural forms, thus providing illusive and distorted forms which are then described with honeyed words. It is true that the recent trend of the morphing paradigm tends to rely heavily on the form making mechanism within the computer software itself.

The notion of shape morphing and its application in architecture is not new at all. It has been employed as an architectural form-making process since long before computer technology emerged. Architects have profiled morphed spatial forms while having structural modus operandi in mind. Here, geometric laws and mathematical precisions were the basis of formal complexity. Two early modernists who experimented with unique shape morphing techniques were Antoni Gaudi and Le Corbusier. In this paper, Gaudi’s Sagrada Familia Cathedral and Le Corbusier’s Firminy Chapel are chosen to explain the application of the technique. The argument is put forward that the use of shape morphing was one of the early modernists’ ventures in their attempt to create architectural forms. Prior to this, this paper introduces a basic operative notion of shape morphing.

Metamorphosis techniques

Simply put, shapemorphing is the metamorphosis of one shape (source) into another (target), creating a variety of in-between shapes. The underlying premise of the approach is based on a basic algorithm that ensures prospective generations of the form learn from the previous ones during the morphing process. Hence, as the process proceeds, the first shape is gradually distorted to the second shape, and so on.

Several theoretical and computational techniques have been developed to aid the shape morphing process. The simplest approach involves the representation of a pair of polygons. The vertices of the first object are then displaced over time to coincide with the position of corresponding vertices of the second object. The major problem with this technique is the difficulty of establishing a desirable vertex correspondence. The following figure illustrates morphing of a rectangle (4 points) onto a triangle (3 points) by equalizing the number of vertices. Here two points meet into one point. Then straight-line interpolation is used to map from each point in the first polygon (formerly the rectangle) onto
a corresponding point in the second polygon (the triangle). In the process, a number of intermediate shapes are emerged in relational changes of the two shapes. Terzidis (2003) provides an excellent presentation of the notion.

![Diagram of Shape Morphing between a Square (A) and a Triangle (B)](image)

Fig.1. Diagram of Shape Morphing between a Square (A) and a Triangle (B)

The concept of morphing transformation has been implemented incrementally in computer-aided design. An interesting early exploration of shape morphing involved changing one digital image into another by a simple cross-dissolve between the two images in two dimensions (Wolberg, 1990). Recently, the method has been popularly implemented for the interpolation of three-dimensional objects.

The basic notion of shape morphing is clearer in algebraic structure of numbers (March and Steadman, 1971). There is an equivalent relation in number theory whose importance has been wisely recognized because of its many applications in computing. For example, following one to one mapping is the simplest case.

\[
0 \rightarrow 1, 1 \rightarrow 2, 2 \rightarrow 3, \ldots, n \rightarrow n+1
\]

It is a case of “a” mapping onto “b”. There is also many to one mapping in arithmetic. “Time-of-Day” arithmetic may be a good example of such case. For example, one asks what time will it be in 7 hours if it now 8:00 A. M.? You can get the answer 3:00 pm by adding 7 to 8, and then, subtracting 12 to get 3. This is a particular case of an equivalent relation. Similar techniques are useful in many problems involving repeating patterns. The technique can be generalized that if a is equivalent to b modulo m written a ≡ b (mod m), if b-a is a multiple of m. The number m is called the modulus of congruence. For example, mod 3 of the numbers 0 to 10 maps into \{0,1, 2\}. The equivalent relation of the number connectivity is shown with the graph. Consequently, large integers can be converted to corresponding problems with small integers.

![An Equivalent Relation where Integers between 0 and 10 Congruent Modulo 3](image)

Fig.2. An Equivalent Relation where Integers between 0 and 10 Congruent Modulo 3

Very early morphing is represented in the work of Albrecht Dürer (1471-1528). During the Renaissance period, Dürer employed various transformations including stretching, shearing, and perspective, to generate wide ranges of variants on the forms of the human face based on grid lines (Strauss, 1972). This study provides a basis for the scientific morphology, in particular, in the work of D’Arcy Thompson (1860-1948), growth and form in nature. In his work, fish species are mapped onto others using non-linear transformations based on warping a coordinate system. Morphing has much in common with early film-making techniques such as optical cross-dissolve and clever cuts. Another technique similar to morphing is stop-motion animation where the subject (such as a malleable clay figure) is progressively photographed through various transformations, one frame at a time. These are assembled together to provide the illusive metamorphosis. Recently, the idea has been introduced to architecture with the term “phenomenal transparency” by Rowe and Slutzky (1997) to read various underlying layers in a deeper formal space on a surface.

![Albrecht Dürer’s Use of Various Two-dimensional Transformations to Generate Different Human Faces](image)

Fig.3. Albrecht Dürer’s Use of Various Two-dimensional Transformations to Generate Different Human Faces (after Strauss, from The Human Figure by Albrecht Dürer-The complete Dresden Sketchbook, 1972)

The application of the shape morphing process has appeared in various architectural designs from detailed elements to overall building forms. Particularly, the morphing method stands out in the work of the two early modernist architects, Antoni Gaudi and Le Corbusier, whose careers marked an intricate interplay between knowledge of geometry and construction technique. In Gaudi’s Sagrada Familia cathedral columns and Le Corbusier’s Firminy chapel, both architects utilize polygonal morphing to shape the geometric structure of architectonic forms. Their use of the method involves not only creating structural and tectonic forms but also explicit conceptual and philosophical intention.
Antoni Gaudi’s the Sagrada Familia Cathedral columns

Antoni Gaudi (1852-1926) is known for the daring and idiosyncratic originality of his architectural forms and structures. His work presents much beyond the conventional styles of the nineteenth-century eclecticism of the period, thus making it almost impossible to classify his work in contemporary architectural historiography.

Behind the audacity and precision of his architectural forms, there are two significant principles observed in Gaudi’s work: nature and geometry. Organic form from nature is Gaudi’s inspirational model. He writes, “Everything comes from nature’s great book” (Bergos, 1954). Organic forms became essential structural elements in his architecture, along with geometry. The knowledge of geometry helps him to calculate the assemblage of complex forms, and then, to translate the abstract forms into a pragmatic construction technique. He writes, “my structural and aesthetic ideas have an ‘indisputable’ logic.” (Martinell, 1951)

The Sagrada Familia Cathedral stands as Gaudi’s magnum opus. Gaudi started working on the cathedral from 1883, and dedicated the rest of his career to the project until his sudden death in 1926. It has been known that Gaudi translated the geometry of natural forms like tree trunks and branches into the cathedral design, reinforcing his belief that the natural forms encouraged him for inspiration. There exist a number of geometric analyses of the cathedral (Sweeney and Sert, 1960; Martinell, 1967). In particular, the use of hyperbolic paraboloid in the cathedral has been commonly known. Recent computational analysis of the cathedral columns by Bonet (2000) turns out the enigma of the ruled surface of the columns. Various polygons are the entity in two-dimensional cross-sections of columns, when the columns are cut in various heights.

According to Bonet, multiple layers of polygons are interpolated and extrapolated in different elevations to form the structure of the columns. This results in a geometrically varied profile of the columns. Each column differs in design according to its purposes such as aisles, nave, transept, and crossing. They also use different polygons. An examination of a column over the choir demonstrates how polygons are used. Six different polygons are found in generating the column, including circle, hexadecagon, octagon, square, rhomboid, and rectangle. Basically, this column derives from the tetragon.

Then, each polygon is continuously morphed to form the column. Some other columns are based on pentagon, or two squares forming a star-shaped octagon. When pentagon is used as basic shape, then decagon and icosagon, are used for the column. These polygons are set according to different heights. Then, these are morphed in-between shapes to form the profile of the column. An example of the hierarchical juxtaposition of the polygons, their heights, morphed polygons are shown below.

The basic morphing process of Gaudi’s column relies on a simple vertex correspondence in three-dimensional space, where a vertex of n-gon matches to two vertexes of 2n-gon. The matching process simulates an equivalent relation in mathematics. For example, if one morphs a triangle (3 points) into a hexagon (6 points), one needs to match one vertex point of the triangle to two vertex points of the hexagon. Then, straight-line interpolation is used to map from each point in the first polygon (the triangle) onto two corresponding points in the second polygon (the hexagon). The resulting form is like an elementary tree structure of correspondence.

The use of these polygons in Gaudi’s work is not surprising. In *The Geometrical Foundation of Natural Structure* (1979), Williams observed that the metamorphosis of natural forms is found in the geometry. According to Williams (1979), circles and polygons are the primary entities in two-dimensional cross-sections.
of natural structures, Figure 6. Williams asserts that aggregated entities in nature occupy positions in highly organized and ordered spaces. In particular, hexagon in aggregated structures and pentagon in plant and animal growth systems predominate, noting to D’Arcy Thompson.

Contemporary computer technology opens up the vehicle to generate a multitude of morphed designs. It can easily generate almost countless spatial forms of morphing permutations where one shape mutates into another. Figure 8 provides some examples of two different polygons where each polygon is placed in parallel in a certain height along the perpendicular axis on center. Two polygons can also be set and morphed in inclined angles off the central axis. Various polygons can be consecutively morphed in three-dimensional space to form amorphous vertical structures like tree branches as Gaudi envisioned.

**Le Corbusier’s Firminy Chapel**

Le Corbusier (1887-1965), who expressed a great respect for Gaudi (Collins, 1960), also experimented with a similar method to form building structure. After Gaudi, Le Corbusier cryptically referred the geometry of the design to a hyperbolic-paraboloid. This problem was of interest much earlier to mathematicians, like Theodore Olivier (1793-1853). A hyperbolic-paraboloid is an infinite surface in three dimensions with hyperbolic and parabolic cross-sections. Simply, it is a saddle shape. It is well known that the use of geometry and platonic primary solids is an embryonic notion in Le Corbusier’s architecture (Le Corbusier, 1999).

Le Corbusier used the method in the late work of his design. Particularly, the hyperboloid of one sheet that is a ruled surface is used in the Parliament building at Chandigarh in 1951-63. The hyperbolic-paraboloid with deformable wire and thread model is used in the Philips Pavilion in 1956-1958. The use of the wire and thread echoes Gaudi’s earlier structural model loaded with proportional weights for the chapel at Colonia Güell in 1900. The choice of the geometry was specifically intended to solve his design problem inspired by “the majestically evocative forms of machine-age Ahmedabad’s cooling tower” (Frampton and Kolnowski, 1981). Le Corbusier experimented designs with wire and thread models to explain the exact determination of the complex geometry. The model also exposes the inside-out ribs of structural framework to show the structural and construction rationale. He also used tensile membrane skin to illustrate the principle of the warped cladding as building envelope.
Le Corbusier seemed to adopt a similar method to his unfinished Firminy Chapel. The chapel has been known as Le Corbusier’s last project commissioned in 1960 but incomplete. The chapel “represents a new type of church” after Ronchamp and La Tourette (Frampton and Kolnowski, 1981). Le Corbusier developed numerous study sketches and models with the wire and thread to present his geometric idea and construction.

The plan parti of the Firminy Chapel is derived from a square. All dimensions in the chapel derive from Le Corbusier’s Modulor system, clearly expressed on the drawings that Jose Oubrerie (Le Corbusier’s assistant for this project) developed. Although there had been subsequent changes of the design, the roof is constructed on the radius of a circle not found in the plan. The center of the circle is carefully shifted and set back from the center of the square. This means the bottom square and the roof circle are not concentric. In three dimensions of the form, from the bottom, as it moves up in height, the volume is progressively reduced and morphed into a truncated conic pyramid. The result form is that one side of the bottom square is obviously perpendicular to the line tangent to the circular roof, and another side forms obliquely angled, thus creating inclined and truncated spatial form in-between two shapes.

Fig.10. Earlier Design of the Firminy Chapel, 1960, First Floor Plan and Section

Nevertheless, in a strict sense, the geometry of the chapel was not hyperboloid. Whereas the Parliament building of the Chandigarh used hyperboloid of one sheet with the twisted spiral, the design in the chapel was a way of shape morphing between a square and a circle in a certain distance. Le Corbusier’s sketches clearly show his early intention of the roof of the chapel.

For Le Corbusier, the technique was conceived through the conceptualization process rather than an architects to explore new possibilities for the development and transformation of complex spatial forms. Traditional Cartesian static space seems no longer adequate to describe the geometric form of design. Besides, spaces created with the technique are continuously read through a constant comparison between the two shapes in three-dimensions. The technique can be a unique tactical device in creating architectural forms and in exploring numerous new formal typologies with the help of computer technology.

Gaudi’s Sagrada Familia Cathedral and Le Corbusier’s Firminy Chapel have been analyzed and interpreted in terms of their structural geometry. In their designs, Le Corbusier sought to achieve a metaphysical or platonic form, while Gaudi searched for the organic form. Interestingly enough, both projects were the final projects for their respective architectural journeys. And both were churches. Although they rely on different philosophical grounds, both architects use the shape morphing method in an attempt to create a new prototype for the modern church. Their approaches in the architectural discourse may have been surely radical and inventive at that time; yet their method became a stepping-stone for the development of contemporary architectural design. The legacy of the method is continuously found in the sheer number of contemporary designs.

All in all, it should be noted that designing is a process of conceptualizing spatial form and realizing it in material form from inception to completion. Perhaps, the shape morphing method itself may not be a driving force for architects’ spatial conceptions. However, when the device is used with philosophical grounds as well as, structural and construction reasoning, it can be promising to be continuously enriched with advances in computer technology as a form-making agent.
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