Mini-review of rare charmonium decays at BESIII *

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Abstract: Recently, LHCb experiment announced a 3.5σ evidence for direct CP violation in D0 decay by looking at the difference between \( A_{CP}(D^0 \rightarrow K^+ K^-) \) and \( A_{CP}(D^0 \rightarrow \pi^+ \pi^-) \). This is the first evidence of \( CP \) violation in charm system, which may indicate new physics beyond the Standard Model. Motivated by this measurement, we review rare processes in charmonium decay, especially, the weak decay, \( C \) or \( P \) violated decay and lepton flavor violated decays. In case the new physics appears in charm sector, these rare decays of charmonium states will provide opportunity to search for significant contributions from physics beyond the Standard Model. With huge \( J/\psi \) and \( \psi(2S) \) samples in BESIII experiment, the rare decays may be feasible.

Key words: BESIII, charmonium, weak decay, new physics, lepton flavor violation

PACS: 13.25.Gv, 13.20.Gd, 14.40.Pq, 14.65.Dw, 12.15.Mm

1 Introduction

At present, two general trends can be distinguished in accelerator particle physics. On one hand, very high energy accelerators, for example, the LHC, provide the ability to explore physics at high energy frontier. On the other hand, smaller experiments at lower energies but with very high intensities and low backgrounds, for example, the \( B \) factories and BESIII, provide the capabilities for performing precise tests and accurate determinations of many parameters of the Standard Model (SM). Moreover, the close scrutiny of rare processes may illuminate new physics in a complementary fashion to high-energy colliders.

On November 14, 2011, LHCb experiment announced 3.5σ evidence for direct \( CP \) violation in \( D^0 \) decay. They looked at the difference between \( A_{CP}(D^0 \rightarrow K^+ K^-) \) and \( A_{CP}(D^0 \rightarrow \pi^+ \pi^-) \), and found that the \( \Delta A_{CP} = [-0.82 \pm 0.21(\text{stat.}) \pm 0.11(\text{sys.})]\% \). Since the SM predicted that the \( CP \) asymmetry in charm sector should be less that 0.1\% [2], this is the first indication of new physics at LHC. If the new physics could appear in the up quarks (charm and top), it urgently needs confirmation. One possible way is to probe the charmonium rare decays with huge \( J/\psi \) and \( \psi(2S) \) data sample. The BESIII experiment can provide important tests of the SM, with the accompanying possibility for uncovering new physics induced deviations in charm sector, especially, the weak charmonium decay, invisible decay, lepton flavor violated decays and so on. In this paper, we review the possibility of such kinds of rare charmonium processes.

2 Weak decays of charmonium

The low-lying charmonium states, \textit{i.e.} those below the open-charm threshold, usually decay through intermediate photons or gluons produced by the annihilation of the parent \( c\bar{c} \) quark pair. These OZI-violating but flavor-conserving decays result in narrow natural widths of the \( J/\psi \) and \( \psi(2S) \) states. In the SM framework, flavor-changing weak decays of these states are also possible, although they are expected to have rather low branching fractions. The huge \( J/\psi \) data samples at BESIII will provide opportunities to search for such rare decay processes, which in some cases may be detectable, even at SM levels. The observation of an anomalous production rate for single charmed mesons in \( J/\psi \) or \( \psi(2S) \) decays at BESIII would be a hint of possible new physics, either in underlying continuum processes via flavor-changing-neutral-currents [3–9] or in the decays of the \( \psi \) resonances due to unexpected effects of quark dynamics [10,11].
2.1 Semileptonic decays of charmonium

The inclusive branching fraction for $J/\psi$ weak decays via a single quark (either the $c$ or the $\bar{c}$) had been estimated to be $(2 \sim 4) \times 10^{-8}$ by simply using the $D^0$ lifetime \cite{4}. Such a small branching fraction makes the observation of weak decays of the $J/\psi$ or $\psi(2S)$ quite challenging, despite the expected cleanliness of the events. However, BEPC-II, running at designed luminosity, will produce of order $10^{10}$ $J/\psi$ events per year of data taking, leading to $\approx 400$ weak decays for the predicted SM branching fraction. The semi-leptonic decay of a $c\bar{c}$ $(1^-)$ vector charmonium state below the open-charm threshold is induced by the weak quark-level transition $c \to q W^*$, where $W^*$ is a virtual intermediate boson. Hence, the accessible exclusive semi-leptonic channels are:

$$\psi(nS) \to D_q l \nu, \quad (1)$$

$$\psi(nS) \to D^*_q l \nu, \quad (2)$$

where $n = 1$ or 2, and $q$ can be either a $d$- or $s$-quark, which corresponds to a $D^\pm$ (Cabibbo-suppressed mode) or $D_s$ (Cabibbo-allowed mode) meson. Semi-leptonic weak decays of the $J/\psi$ will offer several advantages over the purely hadronic ones from both the experimental and theoretical points of view: the prompt charged lepton $l = e, \mu$ can be used to tag the events, removing a large fraction of conventional $\psi(nS)$ hadronic decays. In addition, the missing energy due to the escaping neutrino can be also exploited to remove backgrounds. The identification of the charm meson in the final state would provide an unambiguous signature of the semi-leptonic weak decays of $\psi(nS)$. Meanwhile, decays of the excited mesons $D^*_s$ and $D^*\pm$ produced in reaction (2) would provide useful additional experimental handles. In the lab system, the detectable photons from the $D^*\pm \to D^0 \pi^\pm$, radiative transition are in the 90~200 MeV energy interval. These, and the soft pion produced from $D^\pm \to D^0 \pi^\pm$ decay, can provide powerful constraints to help identify a $D_s$ or $D^0$ meson produced in the weak decay of a charmonium state.

Many theoretical calculations have been done based on various QCD frameworks, including, e.g., heavy quark spin symmetry (HQSS) \cite{5}, QCD sum rules \cite{6} and covariant light-front quark model \cite{7}. The predicted branching fractions are in the range of $10^{-5} \sim 10^{-10}$ and $10^{-10} \sim 10^{-11}$ for $J/\psi \to D^*_s l^+ l^- \nu$ and $J/\psi \to D^*(s) l^+ l^- \nu$, respectively. The branching fractions for the sum of dominant semi-leptonic decay modes can arrive at $5 \times 10^{-9} \sim$, which are hopefully to be detected at BESIII with $10 \times 10^{10}$ $J/\psi$ events accumulated each year. The overall theoretical uncertainty is up to $\sim 50\%$ due to the treatment of the nonperturbative QCD dynamics. In Table 1 the theoretical predictions for $J/\psi$ semi-leptonic decays are summarized.

| decay mode | Ref. [4] | Ref. [5] | Ref. [6] | Ref. [7] |
|------------|----------|----------|----------|----------|
| $J/\psi \to D^- l^+ l^- \nu + c.c.$ | 26 | 20 | 3.5 | 11.2 |
| $J/\psi \to D^+_s l^+ l^- \nu + c.c.$ | 42 | 32 | 11 | 34.7 |
| $J/\psi \to D^+ l^+ l^- \nu + c.c.$ | 1.4 | 1.2 | 0.14 | 1.1 |
| $J/\psi \to D^- l^+ l^- \nu + c.c.$ | 2.3 | 2.0 | 0.73 | 3.4 |

It is interesting to note that the ratio of Cabibbo-allowed decay to Cabibbo-suppressed decay can be obtained cleanly since many theoretical uncertainties cancel out in the ratio. The ratios $R_{s/d} = \frac{\text{BR}(J/\psi \to D^+_s \ell^+ \nu)}{\text{BR}(J/\psi \to D^- \ell^+ \nu)}$ and $R^{*}_{s/d} = \frac{\text{BR}(J/\psi \to D^{*}_s \ell^+ \nu)}{\text{BR}(J/\psi \to D^{*} \ell^+ \nu)}$ should be equal to $\frac{V_{cs}}{V_{cd}} \approx 18.4$ under the SU(3) flavor symmetry limit \cite{4}, where $V_{cs} = 0.973$ and $V_{cd} = 0.2271$ denote the relevant Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements. In Ref. [4], the calculations based on the QCD sum rules show $R_{s/d} \cong 24.7$ and $R^{*}_{s/d} \cong 15.1$, which implies a large effect of the SU(3) symmetry breaking. Under the assumption of heavy-quark spin symmetry (HQSS) and the non-recoil approximation \cite{12, 13}, the ratio of $\frac{R_{V/P}}{R} = \frac{\text{BR}(J/\psi \to D^{*}_s \ell^+ \nu)}{\text{BR}(J/\psi \to D^{*} \ell^+ \nu)}$ is predicted to be 1.6 \cite{4}, while it is calculated to be 3.1 based on the QCD.
sum rules in Ref. 6, which indicates that high order corrections are important.

Following the first proposal by Zhang 14, BESII has measured the $J/\psi$ weak decays to be $BR(J/\psi \rightarrow D^-_s e^+\nu + c.c.) < 3.6 \times 10^{-5}$ and $BR(J/\psi \rightarrow D^- e^+\nu + c.c.) < 1.2 \times 10^{-5}$ at 90% confidence level based on 5.8 × 10^6 $J/\psi$ decay events 15. The current experimental limits are much higher than the predicted values. It will be important to search for $J/\psi$ weak decays at the BESIII experiment, which will be discussed in detail in Section 2.3.

The flavor-changing-neutral-current (FCNC) induced semi-leptonic decays of $J/\psi \rightarrow D^{(*)0}l^+l^-$ are studied in Ref. 8 based on the QCD sum rules, and the decay rates for $J/\psi \rightarrow D^0 l^+l^-$ and $J/\psi \rightarrow D^{(*)0} l^+l^-$ are predicted to be $10^{-14}$ and $10^{-13}$, which are much below the BESII upper limit: $BR(J/\psi \rightarrow D^0 e^+e^-) < 1.1 \times 10^{-5}$ 16. But new physics in the loop may enhance the decay rate considerably as discussed in Ref 14, further experimental search at BESIII will be interesting.

2.2 Two-body weak hadronic decays of charmonium

Non-leptonic two-body weak decays of the $J/\psi$ and $\psi(2S)$ are studied based on the covariant light-front quark model 7, the factorization approach 9, and HQSS 5, 16 in the context of the factorization scheme for both the Cabibbo-allowed ($c \rightarrow s$) and Cabibbo-suppressed ($c \rightarrow d$) quark-level transitions. In the literature, the expressions for branching fractions for $J/\psi \rightarrow PP/PV/VV$ decays (where $P$ and $V$ represent pseudoscalar and vector mesons) had been given. For $J/\psi \rightarrow PP$ mode, the dominant decay is $J/\psi \rightarrow D^-_s \pi^+ + c.c.$ (including charge conjugate process), which is Cabibbo and color allowed. Table 2 shows the decay rate for the $J/\psi \rightarrow PP$ modes under various theoretical predictions 5, 7, 9, 16.

Assuming factorization as suggested by Bjorken 17, it is interesting to construct the ratio of Cabibbo suppressed to Cabibbo allowed decay modes:

$$BR(J/\psi \rightarrow D_s K) \approx \frac{|V_{us}f_K|^2}{|V_{us}f_s|^2} \approx 0.081,$$

where $f_K$ and $f_s$ are the decay constants for $K$ and $\pi$ mesons. The ratio was predicted to be 0.08 based on the factorization approach 9.

For $J/\psi \rightarrow PV$ mode, the Cabibbo favored and color allowed decay $J/\psi \rightarrow D_s^+ \rho^- + c.c.$ is dominant and predicted to be $12 \sim 50 \times 10^{-10}$ as shown in Table 3, which can be accessible in BESIII experiment. Moreover, the ratio, $BR(J/\psi \rightarrow D_s \rho) / BR(J/\psi \rightarrow D_s K)$, was predicted to be $6.3$ in 9 and $4.2$ in 10, which can be tested by BESIII experiment.

For $J/\psi \rightarrow PV$ mode as listed in Table 3, the dominant decay mode is $J/\psi \rightarrow D_s^+ \rho^- + c.c.$, which was predicted to be $5.3 \times 10^{-9}$ based on the factorization approach 9. This decay mode is the most promising mode to be measured at BESIII.

Table 2. The theoretical predicted branching fractions (in unit of $10^{-10}$) for $\psi \rightarrow PP$. The transition mode, $\Delta C = \Delta S = +1$, corresponds to the Cabibbo-allowed decay modes, while $\Delta C = +1, \Delta S = 0$ corresponds to the Cabibbo-suppressed decay modes. Only the central values are cited from the theoretical predictions.

| Transition mode | Decay mode | Ref. [5] | Ref. [7] | Ref. [9] | Ref. [16] |
|-----------------|------------|----------|----------|----------|----------|
| $\Delta C = \Delta S = +1$ | $\psi \rightarrow D^-_s \pi^+ + c.c.$ | 7.36 | 2.5 | 2.0 | 8.74 |
| | $\psi \rightarrow D^0 K^0 + c.c.$ | 1.39 | 0.5 | 0.36 | 2.80 |
| $\Delta C = +1, \Delta S = 0$ | $\psi \rightarrow D_s^+ K^- + c.c.$ | 0.53 | - | 0.16 | 0.55 |
| | $\psi \rightarrow D^+ \pi^- + c.c.$ | 0.29 | - | 0.08 | 0.55 |
| | $\psi \rightarrow D^0 \eta + c.c.$ | 0.070 | - | - | 0.016 |
| | $\psi \rightarrow D^0 \eta' + c.c.$ | 0.004 | - | - | 0.003 |
| | $\psi \rightarrow D^0 \pi^0 + c.c.$ | 0.024 | - | - | 0.055 |

The $J/\psi$ semi-leptonic decay modes can be related to the two-body hadronic decay modes by applying both the spin symmetry and the non-recoil approximation to the semi-leptonic decay rates 4. For $J/\psi \rightarrow D_s^+(D_s^{(*)+})\pi^-$ decay modes, $q^2 = (p_e - p_D)^2 = m_{\pi}^2$ (here $p_e$ and $p_D$ are the four momenta of the initial and final state heavy mesons) and, assuming factorization as suggested by Bjorken 17 for $B$ decays,
and in the non-recoil approximation for the hadronic transition amplitudes, one can give the relation between relative branching ratios:

\[
\frac{r}{\Delta C = \Delta S = +1} = \frac{BR(J/\psi \rightarrow D_s^+ \pi^-)}{BR(J/\psi \rightarrow D_s^+ \pi^-)} = \frac{d\Gamma(J/\psi \rightarrow D_s^+ l^- \nu)}{dq^2} \left[ \frac{d\Gamma(J/\psi \rightarrow D_s^+ l^- \nu)}{dq^2} \right]_{q^2 = m_c^2},
\]  

(4)

Table 3. The theoretical predicted branching fractions (in unit of $10^{-10}$) for $J/\psi \rightarrow PV$. The transition mode, $\Delta C = \Delta S = +1$, corresponds to the Cabibbo-allowed decay modes, while $\Delta C = +1, \Delta S = 0$ corresponds to the Cabibbo-suppressed decay modes. Only the central values are cited from the theoretical predictions.

| Transition mode | Decay mode   | Ref. [5] | Ref. [7] | Ref. [9] | Ref. [10] |
|-----------------|--------------|----------|----------|----------|-----------|
| $\Delta C = \Delta S = +1$ | $\psi \rightarrow D_s^+ \rho^- + c.c.$ | 50.5     | 28.0     | 12.6     | 36.30     |
|                 | $\psi \rightarrow D^0 K^{*0} + c.c.$ | 8.12     | 5.5      | 1.54     | 10.27     |
| $\Delta C = +1, \Delta S = 0$ | $\psi \rightarrow D_s^+ K^{*-} + c.c.$ | 2.79     | -        | 0.82     | 2.12      |
|                 | $\psi \rightarrow D^0 \rho^- + c.c.$ | 2.13     | -        | 0.42     | 2.20      |
|                 | $\psi \rightarrow D^0 \rho^0 + c.c.$ | 0.18     | -        | -        | 0.22      |
|                 | $\psi \rightarrow D^0 \omega + c.c.$ | 0.16     | -        | -        | 0.18      |
|                 | $\psi \rightarrow D^0 \phi + c.c.$ | 0.41     | -        | -        | 0.65      |

Table 4. The theoretical predicted branching fractions (in unit of $10^{-10}$) for $J/\psi \rightarrow VV$. Only the central values are cited from the theoretical predictions.

| Decay mode | Ref. [9] |
|-----------|----------|
| $J/\psi \rightarrow D_s^+ \rho^- + c.c.$ | 52.6     |
| $J/\psi \rightarrow D_s^+ K^{*-} + c.c.$ | 2.6      |
| $J/\psi \rightarrow D^0 \rho^- + c.c.$ | 2.8      |
| $J/\psi \rightarrow D^0 \phi + c.c.$ | 9.6      |

All of the above estimates show an overall enhancement of the final-state vector charm mesons with respect to the pseudoscalar ones. This suggests the use of $D_s^+$ or $D_s^{*+}$ as signals in searches for weak decays of the $J/\psi$ in non-leptonic decay channels as well. According to the predicted results in Ref. [9], the branching fraction for inclusive weak hadronic decays of $J/\psi$ can be as large as $1.3 \times 10^{-4}$, which is in remarkable agreement with the naive estimation in Ref. [9].

In the SM, FCNC induced $J/\psi$ weak hadronic decays are predicted to be unobservably small, and, thus, any observation of such decay would provide a signal for new physics. In Ref. [10], the predictions of various models, such as TopColor models, minimal supersymmetric standard model (MSSM) with R-parity violation and a general two-Higgs-doublet model, are discussed. These authors found that the branching fraction for $J/\psi \rightarrow D/\bar{D} X_u$, which is mediated by the weak $c \rightarrow u$ transition, could be as large as $10^{-9}$ to $10^{-5}$ in some new physics scenarios.

At BESIII it will be difficult to isolate pure, $c \rightarrow u$ mediated, hadronic $J/\psi \rightarrow D^{(*)}/\bar{D}^{(*)} X_u$ decays. On the other hand the decays $J/\psi \rightarrow D^{(*)}/\bar{D}^{(*)} l^+ l^-$ ($l = e, \mu$) and $J/\psi \rightarrow D^{(*)}/\bar{D}^{(*)} \gamma$ decays, which are also dominated by FCNC processes, would be quite distinct.

In addition to the $J/\psi$ and $\psi(2S)$ decays into charm meson, the weak decays of $\psi(2S)$ into final states involved $\Lambda_b$ baryon can also be searched for at BESIII. These decay modes include $\psi(2S) \rightarrow \Lambda^0_b \bar{\Sigma}^- + c.c., \Lambda^0_b \Sigma^0 \pi^- + c.c., \Lambda^+ \bar{\Sigma}^- e^- \bar{\nu} + c.c.$ and $\Lambda_b^0 \bar{\Lambda}^- + c.c.$. In Ref. [10], the decay of $\psi(2S) \rightarrow \Lambda^0_b \bar{\Sigma}^- + c.c.$ are estimated in the framework of the SM, and they found that the decay rate could reach $10^{-10}$.

### 2.3 Searches at BESIII

At BESIII, assuming a $10^{6}$ $J/\psi$ event sample, the total semi-leptonic decay rate could be $5 \times 10^{-9}$, so about 50 semi-leptonic decay events of the type $J/\psi \rightarrow D_s(D^*_s) l \nu$ can be accumulated by BESIII per year. The following event selection criteria would be useful for searching such exclusive semi-leptonic channels.
nels:

- The prompt charged lepton can be used to tag the weak decay: in order to suppress cascade decay backgrounds from \( J/\psi \) strong decays, the tagging lepton momentum could be required to be between 0.5 GeV and 1.0 GeV, close to the upper kinematic limit for the decay under consideration. High quality lepton discrimination from charged pions or kaons is needed for the measurement.

- The missing mass of the reconstructed candidates must be consistent with the (nearly) zero mass of the undetected neutrino.

- The reconstruction of a \( D_s \) or \( D^\pm \) meson would provide an unambiguous signature for a weak decay of a below-open-charm threshold \( \psi(nS) \). Good invariant mass resolution of the \( D_s \) decay products will be important for reducing combinatorial backgrounds.

- Soft photons in the energy interval (90–200) MeV from the \( D^*_s \to \gamma D_s \) transition and soft charged pions from \( D^{\pm} \to \pi^{\pm} D \) decay can provide further suppression of combinatorial backgrounds: the additional constraint of an intermediate \( D^*_s \) state would reconfirm the \( D_s \) signal.

In general, exclusive hadronic decays are probably too tiny to look for in any specific fully reconstructed decay channel. Therefore, it seems that an inclusive search for \( J/\psi \to D^*_s + X \) at BESIII may be more fruitful. The \( \gamma \) from the decay of a \( D^*_s \) meson should be useful as a kinematic constraint to clean up any \( D_s \) meson signal, as discussed in Ref.\(^21\).

### 3 Search for the invisible decays of quarkonium

Invisible decays of quarkonium states offer a window to look for possible new physics beyond the SM. The reason is that other than neutrinos, the SM includes no other invisible final particles that these states can decay into. BESII explored such a window by establishing the first experimental limits on invisible decays of the \( \eta \) and \( \eta' \), which complemented the limit of \( 2.7 \times 10^{-7} \) that was previously established for invisible decays of the \( \pi^0 \)\(^22\).

Some theories of beyond the SM physics predict new particles with masses that are accessible at BESIII, such as the light dark matter (LDM) particles discussed in Ref.\(^23\). These can have the right relic abundance to constitute the nonbaryonic dark matter of the universe, if they are coupled to the SM via a new light gauge boson \( U \)\(^24\), or exchanges of heavy fermions. A light neutralino with a coupling to the SM that is mediated by a light scalar singlet in the next-to-minimal supersymmetric standard model has also been considered\(^25\).

These considerations have received a boost in interest by the recent observation of a bright 511 keV \( \gamma \)-ray line from the galactic bulge reported by the SPI spectrometer on the INTEGRAL satellite\(^26\). The corresponding galactic positron flux, as well as the smooth symmetric morphology of the 511 keV \( \gamma \) emission, could be interpreted as originating from the annihilation of LDM particles into \( e^+e^- \) pairs\(^23\,27\). It would be very interesting to see evidence for such light invisible particles in collider experiments. CLEO gave an upper bound on \( \Upsilon(1S) \to \gamma + \text{invisible}, \) which is sensitive to dark matter candidates lighter than about 3 GeV/c\(^2\)\(^28\), and also provided an upper limit on the axial coupling of any new \( U \) boson to the \( b \) quark. It is important, in addition, to search for the invisible decays of other light quarkonium states \( (qq', q = u,d, \text{ or } s \text{ quark}), \) since these can be used to constrain the masses of LDM particles and the couplings of a \( U \) boson to the light quarks\(^21\).

It has been shown that measurements of the \( J/\psi \) invisible decay widths can be used to constrain new physics models\(^29\). It is straightforward for one to calculate the SM ratio of branching fractions for \( J/\psi \) invisible decays and its measured branching fraction for decays into electron-positron pairs\(^29\). Within the SM, the invisible mode consists solely of decays into the three types of neutrino-antineutrino pairs. Neglecting polarization effects and taking into account \( e^+e^- \) production through a photon only, one gets\(^29\):

\[
\frac{\Gamma(\psi \to \nu \bar{\nu})}{\Gamma(\psi \to e^+e^-)} = \frac{27G^2M_{J/\psi}^4}{256\pi^2\alpha^2} \left( 1 - \frac{8}{3} \sin^2 \theta_W \right)^2 = 4.5 \times 10^{-7},
\]

where \( G \) and \( \alpha \) are the Fermi and fine-structure constants, respectively, and \( M_{J/\psi} \) is the \( J/\psi \) mass. The uncertainty of the above relation is about 2–3% and comes mainly from the corrections to the \( J/\psi \) wave function, \( e^+e^- \) production via the \( Z \) boson and electroweak radiative corrections\(^29\).

In BES experiment, one can tag the charmonium states that decay invisibly by looking for a particular cascade transition, such as \( \psi(2S) \to \pi^+\pi^- J/\psi, \).
$\psi(2S) \rightarrow \gamma \chi_c$ and so on, where the soft $\pi^+ \pi^-$ pair or the monoenergetic radiative $\gamma$ serves as a tag for the invisibly decaying $J/\psi$ or $\chi_c$ state. The BESIII experiment performed the first search for invisible decays of the $J/\psi$ using $\psi' \rightarrow \pi^+ \pi^- J/\psi$ events detected in a sample of 14.0 million $\psi'$ decays. The upper limit on the ratio $\frac{B(\psi' \rightarrow \text{invisible})}{B(\psi' \rightarrow \pi^+ \pi^- J/\psi)}$ at the 90% confidence level is $1.2 \times 10^{-2}$ [30]. This measurement improves by a factor of 3.5 the bound on the product of the coupling $\Gamma(\psi' \rightarrow \text{invisible})$ of Ref. [31], a limit of $|c_\chi f_{\psi'\gamma}| < 8.5 \times 10^{-3}$, which is almost a factor of 2 stronger than the corresponding limit $|c_\chi f_{\psi\gamma}| < 1.4 \times 10^{-2}$ derived from the invisible decays of the $\Upsilon(1S)$ as described in Eq. (106) in Ref. [32], where $c_\chi$ and $f_{\psi'\gamma}$ ($f_{\psi\gamma}$) denote the $U$ boson couplings to the LDM particle $\chi$ and $c$ ($b$) quark. We expect a more precise measurement can be obtained in the future BES-III experiment. A list of potentially useful decay chains is provided in Table 5 in the BESIII experiment.

Table 5. $\psi(2S)$ and $J/\psi$ decay modes that can be used to search for invisible decays of the $J/\psi$, $\chi_{c0}$, $\chi_{c1}$, $\chi_{c2}$, $\eta_c(2S)$ and $\eta_c(2S)$. The branching fractions are taken from the PDG [33]. For each mode, a “tagging topology” is given, which is the set of visible particles that are seen within the detector’s acceptance. In each case the tagging topology has well defined kinematics. The number of events are the expected event yield in 3 billion $\psi(2S)$ (10 billion $J/\psi$) data set, in which we did not consider the decay probabilities of the tagging particles.

| $\psi(2S)$ decay mode | Branching fraction (10^{-2}) | Number of events | Invisible decay mode | Tagging topology |
|------------------------|-----------------------------|------------------|---------------------|-----------------|
| $\psi(2S) \rightarrow \pi^+ \pi^- J/\psi$ | 31.7±1.1 | 9.3×10^6 | $J/\psi \rightarrow \text{invisible}$ | $\pi^+ \pi^-$ |
| $\psi(2S) \rightarrow \eta J/\psi$ | 18.6±0.8 | 5.6×10^6 | $\eta$ | $\eta$ |
| $\psi(2S) \rightarrow \chi_{c0}$ | 9.0±0.4 | 2.7×10^6 | $\chi_{c0}$ | $\chi_{c0}$ |
| $\psi(2S) \rightarrow \chi_{c1}$ | 8.7±0.5 | 2.6×10^6 | $\chi_{c1}$ | $\chi_{c1}$ |
| $\psi(2S) \rightarrow \chi_{c2}$ | 8.2±0.3 | 2.5×10^6 | $\chi_{c2}$ | $\chi_{c2}$ |
| $\psi(2S) \rightarrow \eta_c(1S)$ | 0.26±0.04 | 7.8×10^6 | $\eta_c(1S)$ | $\eta_c(1S)$ |
| $J/\psi \rightarrow \gamma$ | 1.3±0.4 | 1.3×10^6 | $\gamma$ | $\gamma$ |

It is also interesting to search for invisible decays of the $\eta$, $\eta'$, $\rho$, $\omega$ and $\phi$ light mesons. Within the SM, the decays of $\eta(\eta') \rightarrow \nu \bar{\nu}$ are tiny due to the helicity suppression. If the $Z^0$ couples to a massive neutrino with the standard weak interaction strength, the branching ratios (BR) for $\pi^0 \rightarrow \nu \bar{\nu}$ and $\eta \rightarrow \nu \bar{\nu}$ have maximum values of $5.0 \times 10^{-10}$ and $1.3 \times 10^{-11}$ [34, 35], respectively, at the $\nu_\tau$ mass upper limit of $m_{\nu_\tau} = 18.2$ MeV/c^2 [36]. Any enhanced signal of invisible decay may indicate New Physics. Possible $\eta/\eta' \rightarrow $ invisible decay products could be LDM particles or light neutralinos. These LDM particles may have an adequate relic density to account for the nonbaryonic mass of the universe.

In the BES experiment, the two-body decay modes $J/\psi \rightarrow \phi \eta$ or $\phi \eta'$ can be selected using only the very clean and distinct $\phi \rightarrow K^+ K^-$ decays, which then tag the presence of an $\eta$ or $\eta'$ meson that has decayed into an invisible final state. For $J/\psi \rightarrow \phi \eta'$, the missing momentum, $P_{\text{miss}} = |\vec{P}_{\text{miss}}|$, is a powerful discriminating variable to separate signal events from possible backgrounds in which the missing side is not from an $\eta (\eta')$. Here, $\vec{P}_{\text{miss}} = -\vec{P}_\phi$. In addition, the regions of the detector where the $\eta$ and $\eta'$ decay products are expected to go are easily defined thanks to the strong boost from the $J/\psi$ decay.

The branching fraction of $\eta(\eta') \rightarrow \gamma \gamma$ was also determined in $J/\psi \rightarrow \phi \phi(\eta')$ decays, in order to provide the ratio $\mathcal{B}(\eta(\eta') \rightarrow \text{invisible})$ to $\mathcal{B}(\eta(\eta') \rightarrow \gamma \gamma)$. The advantage of measuring $\frac{\mathcal{B}(\eta(\eta') \rightarrow \gamma \gamma)}{\mathcal{B}(\eta(\eta') \rightarrow \text{invisible})}$ is that the uncertainties due to the total number of $J/\psi$ events, tracking efficiency, PID, the number of the charged tracks, the cut on $M(KK)$, and residual noise in the BSC all cancel.

Based on $58 \times 10^6 J/\psi$ events in the BESIII experiment, the upper limit on the ratio of the $\mathcal{B}(\eta \rightarrow \text{invisible})$ to $\mathcal{B}(\eta \rightarrow \gamma \gamma)$ was determined to be [37]:

$$\frac{\mathcal{B}(\eta(\eta') \rightarrow \text{invisible})}{\mathcal{B}(\eta(\eta') \rightarrow \gamma \gamma)} < 1.65 \times 10^{-3} (6.69 \times 10^{-2}).$$

Table 6 lists the possible two-body $J/\psi$ decay modes that can be used to study the invisible decays of the $\eta$, $\eta'$, $\rho$, $\omega$ and $\phi$ mesons at BESIII.
Table 6. $J/\psi$ decay modes that can be used to study invisible decays of $\eta$, $\eta'$, $\rho$, $\omega$ and $\phi$ mesons. The branching fractions are from the PDG [32]. For each mode, a “tagging topology” is given, which is the set of visible tracks in the detector’s acceptance. In each case the tagging topology has well defined kinematics.

The produced number of events are the expected events in 10 billion $J/\psi$ event data set at BESIII, with the decay probabilities of the tagging particles included.

| $J/\psi$ decay mode | Branching fraction ($10^{-4}$) | Invisible decay mode | Tagging topology | Number of events /10 billion $J/\psi$ decays |
|---------------------|-------------------------------|----------------------|----------------|-----------------------------------------------|
| $J/\psi \to \eta$   | 6.5 ± 0.7                    | $\eta \to \text{invisible}$ | $\phi \to K^+K^-$ | (31.4 ± 3.4) × 10^5 |
|                     | 6.5 ± 0.7                    | $\phi \to \text{invisible}$ | $\eta \to \gamma\gamma$ | (25.7 ± 2.8) × 10^5 |
| $J/\psi \to \eta'$  | 3.3 ± 0.4                    | $\eta' \to \text{invisible}$ | $\phi \to K^+K^-$ | (16.2 ± 1.9) × 10^5 |
|                     | 3.3 ± 0.4                    | $\phi \to \text{invisible}$ | $\eta' \to \gamma\rho^0$ | (9.0 ± 1.2) × 10^5 |
| $J/\psi \to \omega\eta$ | 15.8 ± 1.6                | $\omega \to \pi^+\pi^0\pi^0\pi^0$ | $\eta \to \gamma\gamma$ | (13.9 ± 1.4) × 10^6 |
|                     | 15.8 ± 1.6                | $\omega \to \text{invisible}$ | $\eta \to \gamma\gamma$ | (6.2 ± 0.6) × 10^6 |
| $J/\psi \to \omega'\eta'$ | 1.67 ± 0.25               | $\omega' \to \text{invisible}$ | $\omega \to \pi^+\pi^0\pi^0\pi^0$ | (1.5 ± 0.2) × 10^6 |
|                     | 1.67 ± 0.25               | $\omega \to \text{invisible}$ | $\eta' \to \gamma\rho^0$ | (0.7 ± 0.1) × 10^6 |
| $J/\psi \to \rho^0\eta$ | 1.93 ± 0.23               | $\rho^0 \to \pi^+\pi^0\pi^0\pi^0$ | $\eta \to \gamma\gamma$ | (1.9 ± 0.2) × 10^6 |
|                     | 1.93 ± 0.23               | $\rho^0 \to \text{invisible}$ | $\eta \to \gamma\gamma$ | (0.8 ± 0.09) × 10^6 |
| $J/\psi \to \rho^0\pi^0$ | 56 ± 7                    | $\rho^0 \to \text{invisible}$ | $\eta \to \gamma\gamma$ | (55.3 ± 5.8) × 10^6 |

4 Search for $C$ or $P$ violating processes in $J/\psi$ decays

With its huge $J/\psi$ and $\psi(2S)$ data samples, the BESIII experiment will be approaching the statistics regime where studies of rare $\psi$ decays can provide important tests of the SM and possibly uncover deviations. Among the interesting examples are $C$, $P$ or $CP$ violating processes in $J/\psi$ decays. An example of such modes would be $\psi(nS) \to V^0V^0$, where $V^0$ is used to denote $J^{PC} = 1^-\text{vector mesons}$ ($\phi$, $\omega$, $\rho^0$ and $\gamma$). A distinct signal for this class of event would be $\psi(nS) \to \phi\phi$ detected in $\psi(nS) \to K^+K^-K^+K^-$ final states. Because of the $C$ violation, $\psi(nS) \to V^0V^0$ decays can only occur in the SM via $\bar{c}c$ annihilation via a $Z^0$ or $W$-exchange decays as discussed in Ref. [37]. The rate for this type of weak decay can provide a measurement of the charmonium wave function at the origin [37].

In order to make a rough estimate for the rate, we first consider just the rate due to the $W$-exchange contribution, which is straightforward to compute [38]

$$\Gamma(J/\psi \to s\bar{s})^{\text{weak}} \approx \frac{1}{2} \left( \frac{m_{J/\psi}}{m_W} \right)^4,$$

(7)

where $m_{J/\psi}$ and $m_W$ are the masses of $J/\psi$ and $W$ boson, respectively. This leads to $BR(J/\psi \to s\bar{s})^{\text{weak}} \approx 10^{-7}$ for this weak contribution. To form the $\phi\phi$ final state, another $s\bar{s}$ pair must be produced from the vacuum and these $s$-quarks have to bind with the outgoing $s\bar{s}$ from the $\bar{c}c$ decay to produce the $\phi\phi$ final state [37]. When this is considered, it seems that one can expect that the SM exclusive $BR(\psi(nS) \to \phi\phi)$ rate should be below the level of $10^{-8}$ and probably out of reach of the BESIII experiment.

Experimentally, there are some possible backgrounds that will dilute the signal for $J/\psi \to \phi\phi$ decays. One major background is $J/\psi \to \gamma\phi\phi$, which is mainly from $J/\psi \to \gamma\eta_c(1S)$, $\eta_c(1S) \to \phi\phi$. This background can be removed by doing a constrained kinematic fit. A detailed calculation had been done to estimate the background from $J/\psi \to \gamma\phi\phi$ [37]. Another background appears if one studies only $2(K\bar{K})$ invariant pair mass distributions. It arises from the $C$ and $P$-conserving reaction $J/\psi \to \phi(K\bar{K})_{S=\text{wave}}$, due to the fact that the $\phi$ mass is only two $S$-wave-widths away from the $K\bar{K}$ $S$-wave resonance mass, for example, $f_0(980) \to K\bar{K}$. Although it may be difficult to subtract in a small statistical sample, one can, in principle, remove this kind of background by either a spin-parity analysis of the $K\bar{K}$ pairs in a narrow window about $\phi$ mass, or by a subtraction normalized to an observed $S$-wave mass peak. To avoid the $S$-wave contribution, one can reconstruct one $\phi$ from $K^+K^-$ and another $\phi$ from the $K_SK_L$ mode, which is not allowed to form an $S$-wave. It will be easy to look for the missing mass of one $\phi$ reconstructed from $K^+K^-$, to see if there is any peak under the $\phi$ mass region by also requiring $K_S$ and $K_L$ information in the final states.

It is noted that there is possible continuum background produced via a two-photon annihilation process. It is a peaking background that cannot be removed without considering detailed angular distributions in a high statistics sample. It is very hard to
deal with this kind of peaking source with a small sample of signal events. One way is to use off-peak data which are taken below the $J/\psi$ mass peak. The $e^+e^- \rightarrow \gamma\gamma$ process has been investigated before [39], and it has a unique production angle ($\theta^*$) distribution, which is defined as the angle between $\phi$ and $e^-$ beam direction in the Center-of-Mass (CM) frame. The production angle distribution for the two real photon annihilation process has the form

$$\sigma(\cos \theta^*)_{e^+e^+\rightarrow\gamma\gamma} = \frac{1 + \cos \theta^*}{1 - \cos \theta^*}^2,$$  \hspace{1cm} (8)$$
while, in the process of two virtual photon into $V^0V^0$ pairs, the distribution is (to first order) [40]:

$$\sigma(\cos \theta^*)_{e^+e^-\rightarrow\gamma\gamma\rightarrow V^0V^0} = \frac{1 + \cos \theta^*}{k^2 - \cos \theta^*}^2,$$  \hspace{1cm} (9)$$
where factor $k$ is:

$$k = \frac{2m_{\gamma\gamma}^2 - S}{\sqrt{S^2 - 4Sm_{\gamma\gamma}^2}}.$$  \hspace{1cm} (10)$$
where $S$ is the square of CM energy. In principle, by using an angular analysis, one can remove the peaking background with high statistic data sample. To avoid the peaking background from the continuum, $\psi(2S) \rightarrow \pi\pi J/\psi$ could be used to study this kind of rare $J/\psi$ decays with 3 billion $\psi(2S)$ sample, but the statistics will be substantially reduced.

5 Charged-lepton flavor violating processes in decays of $J/\psi$

In the framework of the SM, lepton flavor is an accidentally conserved quantum number due to the massless neutrinos. The new physics beyond the SM introduces oscillations between the neutrino flavors by including the neutrino masses, which violate lepton flavor. However, SM processes involving charged-lepton flavor violation (CLFV) with massive neutrinos are too tiny to be observed because they are suppressed by the quantity $(\Delta M^2_{\nu}/M^2_W)^2 < 10^{-48}$ [41]. Here $\Delta M_{\nu}$ is the difference between the masses of neutrinos of different flavor and $M_W$ is the mass of the charged weak vector boson. Hence, CLFV offers an opportunity to probe signature of new physics [42-46]. The CLFV charmonium decays $J/\psi \rightarrow ll'$ ( $l$ and $l' = \tau, \mu, e$, $l \neq l'$) are predicted by various theoretical models that allow tree-level FCNC, including, e.g., unparticle theory [46, 47], R-parity violating and large tan$\beta$ SUSY scenarios, leptoquarks, and other models inspired by the idea of grand unification [44, 45].

By using unitarity considerations, limits on CLFV $\mu$ and $\tau$ branching fractions [33] have been used to place indirect limits on CLFV $J/\psi$ branching fractions [48-50]. In Ref. [48], unitarity relations between the CLFV $J/\psi$ decay and the CLFV $\mu$ and $\tau$ decays have been exploited. From the existing experimental bounds on the latter process [33], the stringent indirect limits can be obtained to be $4 \times 10^{-13}$ and $6 \times 10^{-9}$ for $J/\psi \rightarrow \mu e$ and $\tau \mu/\tau e$, respectively. In Ref. [50], the experimental bounds on nuclear $\mu^- - e^-$ conversion are used to place indirect limits on the CLFV $J/\psi$ decays, which are at the same level as those in Ref. [48]. As suggested by Zhang in Ref. [49], searching for the CLFV $J/\psi$ decays with a huge sample at the BESIII remains a worthwhile experimental challenge. With a 58 M $J/\psi$ event sample at BESIII, the following upper limits have been established [51]:

$$BR(J/\psi \rightarrow \tau^\pm e^{\mp}) < 8.3 \times 10^{-6};$$  \hspace{1cm} (11)$$
$$BR(J/\psi \rightarrow \tau^\pm \mu^{\mp}) < 2.0 \times 10^{-6};$$  \hspace{1cm} (12)$$
$$BR(J/\psi \rightarrow \mu^\pm e^{\mp}) < 1.1 \times 10^{-6}.$$  \hspace{1cm} (13)$$

The limits on the two-body lepton flavor violating decays of the $J/\psi$ could be reduced to the $10^{-8} \sim 10^{-9}$ level at BESIII with one year full-luminosity run at the $J/\psi$ peak. This would be a significant improvement.

We should note that the indirect limits on CLFV $J/\psi$ decays are significantly more stringent than the expected bounds from the BESIII experiment. However, searches for the CLFV decays of vector mesons remain an important experimental effort since their observation at the rates above the indirect limits would be a manifestation of new CLFV physics, which does not fit into these theoretical analyses. In particular, it may imply a nontrivial mechanism of self cancellation as discussed in Refs. [48-50], which are considered as unnatural.

6 Summary

A mini-review of $J/\psi$ and $\psi(2S)$ rare decays in the BES-III experiment has been done. With one year’s luminosity data, about 10 billion $J/\psi$ and 3 billion $\psi(2S)$ decays can be collected at BESIII experiment. The searches of weak decay, LFV decay, $C$ or $P$ violated decay and invisible decays of charmonium states will be feasible. These rare decay processes will provide opportunity for new physics space, especially, in charm sector. It should be noted that the recent measurement of 3.5 $\sigma$ evidence for direct $CP$ violation in $D^0$ decay at LHCb may indicate possible contributions to charm decays from physics beyond SM. We expect to see surprises from BESIII experiment in rare charmonium decays.

The authors would like to thank X. M. Zhang and D. T. Zhu for further discussions.
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