A Control-Oriented Dynamic Model for Wakes in Wind Plants

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Abstract. In this paper, we present a novel control-oriented model for predicting wake effects in wind plants, called the FLOw Redirection and Induction Dynamics (FLORIDyn) model. The model predicts the wake locations and the effective flow velocities at each turbine, and the resulting turbine electrical energy productions, as a function of the control degrees of freedom of the turbines (the axial induction and the yaw angle of the different rotors). The model is an extension of a previously presented static model (FLORIS). It includes the dynamic wake propagation effects that cause time delays between control setting changes and the response of downstream turbines. These delays are associated with a mass of air in the wake taking some time to travel from one turbine to the next, and the delays are dependent on the spatially- and time-varying state of the wake. The extended model has a state-space structure combined with a nonlinear feedback term. While including the control-relevant dynamics of the wind plant, it still has a relatively small amount of parameters, and the computational complexity of the model is small enough such that it has the potential to be used for dynamic optimization of the control reference signals for improved wind plant control.

1. Introduction
Each wind turbine has a wake of turbulent flow downstream of its rotor. Because the turbine extracts energy from the flow, the wind velocity in the wake is reduced with respect to the free-stream velocity. Wake expansion and mixing makes that the wake velocity will recover towards the free-stream velocity further downstream. In a cluster of wind turbines (a wind plant), the wake of one turbine can overlap with another turbine rotor, which affects the power production of that turbine. The topology and amount of the wake interaction depends on time-varying atmospheric conditions (e.g., inflow direction, speed and turbulence, and atmospheric stability), and on the control settings of each turbine: the rotor speed and pitch angles of the blades affect the axial induction and thus the wake velocity deficit [1], and the rotor yaw angle affects both the velocity deficit and flow direction in the wake [2, 3]). In [4], we developed the FLOw Redirection and Induction in Steady-state (FLORIS) model, a simplified control-oriented model that predicts the steady-state characteristics of wakes in a wind plant as a function of the axial inductions and yaw angles of the rotors. The complete flow field in a plant does not respond instantaneously to a change in turbine control settings however, since the flow takes some time to move downstream, resulting in a delay of the response of the downstream turbine. The lengths of these delays are dependent on the spatially- and time-varying flow velocity profile in the wake. Therefore, we extend the FLORIS model with a simplified dynamic model for the propagation...
of the effects of control settings changes through the wake. We refer to the model as the FLOw Redirection and Induction Dynamics (FLORIDyn) model.

In this paper we describe the FLORIDyn model (section 2), and show the results of a case study in which we compare the predictions given by the FLORIDyn model with those of a high-fidelity CFD simulation (section 3). Finally, conclusions are given in section 4.

2. Model description

The FLORIDyn model is a combination of static nonlinear mappings describing the wake velocity profile, based on an augmented Jensen model [5, 6], and the wake deflection through yaw (based on [3]), extended with a state-space model describing the propagation of control settings changes through the wake.

2.1. Turbine power

We use the index \( t \) to count the different wind turbines in a wind plant, and \( \mathcal{P} = \{1, 2, \ldots, N_T\} \) to denote the set of indexes of all turbines in the plant, with \( N_T \) denoting the total number of turbines in the plant. We use the index \( k \) to denote the discrete time steps. When the effective wind speed at a time \( k \) at a turbine \( t \in \mathcal{P} \), denoted as \( u_T(t, k) \), is known, the electrical power of each turbine is calculated as:

\[
P_T(t, k) = \frac{1}{2} \rho A_T(t) C_P(a_T(t, k), \gamma_T(t, k)) u_T(t, k)^3 \forall t \in \mathcal{P}
\]

where \( \rho \) is the air density, and \( A_T(t) \) and \( C_P \) are the rotor swept area and the power coefficient of the turbine \( t \). In [4] we derived the following heuristic relationship between the axial induction factor \( a_T \) and the yaw angle \( \gamma_T \) of the rotor and the power coefficient \( C_P \):

\[
C_P(a_T(t, k), \gamma_T(t, k)) = 4a_T(t, k)[1 - a_T(t, k)]^2 \eta \cos(\gamma_T(t, k)) P_P.
\]

Note that \( a_T \) in this relation corresponds to the axial induction factor when the rotor is not yawed, which can be found from the blade pitch angle and rotor speed using knowledge of the turbine characteristics. Scalars \( P_P \) and \( \eta \) are model parameters.

2.2. Front and downstream turbines

Given a certain inflow direction, we can distinguish some front turbines in the wind plant, for which the rotor is not overlapping with the wake of any other upstream turbines (those turbines are in the set \( \mathcal{F} \subset \mathcal{P} \)). From a controls point of view, the velocity of the inflow to these turbines, is a given input (disturbance) to the wind plant system. From measurements at these turbines (power, yaw and axial induction) we estimate the local free-stream velocities, denoted by \( U_T \), by inverting relation (1) \( U_T(t, k) = u_T(t, k) \forall t \in \mathcal{F} \). Through adjusting the axial induction and yaw of the turbine rotors, we affect the wake effects on the turbines that are standing downstream of those front turbines (the set \( \mathcal{D} \)). In the remainder of this section, it will be described how the effective inflow speeds \( u_T \) at each turbine \( t \in \mathcal{D} \) are estimated by the model, through a dynamic model of the wake characteristics.

2.3. Coordinates, wake zones and observation points

In order to describe the spatial properties of the wakes in the wind plant, we are adopting a Cartesian coordinate framework \((x, y)\) in which the \( x \)-axis is pointing downwind along an estimated mean inflow direction in the plant, and the \( y \)-axis is pointing orthogonal to the \( x \)-axis in the horizontal direction, i.e., along the cross-wind direction (see Fig. 1). In this work, we assume that each turbine has the same hub-height, and the turbine locations in this downwind-crosswind coordinate frame are denoted as \((x_T(t), y_T(t))\).
Following the approach of the FLORIS model, in the lateral (crosswind) direction we divide the wake in different zones (see Fig. 1), each with their own expansion and recovery properties. A difference with the original model from [4], is that the three zones used in the FLORIS model (far wake, near wake and mixing zone) are further divided in a left and right part, and we add a zone to describe the free-stream, resulting in a total of 7 wake zones.

In each wake zone of each turbine, a finite number of so-called Observation Points (OPs) are defined (see Fig. 3) for which we will calculate the local wake characteristics. The first OP is located in the turbine rotor plane. It is assumed that at each discrete timestep, a mass of air will move from one OP to the next downstream OP. The axial distances between the OPs are adjusted accordingly, based on the estimated wind velocity in between the OPs. Turbine and flow variables that are measured at the turbines, are passed on between the OPs, using the time update laws given in the next section, and then these variables are used to calculate the wake characteristics.

2.4. Time update laws for OP downwind positions and delayed turbine measurements
We use the index $z \in \{1, \ldots, 7\}$ to number the zones in a turbine wake. The total number of OPs in one wake zone of a turbine is $N_P$. We use the index $p \in \{1, \ldots, N_P\}$ to number the OPs in a wake zone. We use a notation in which $x_{t,z,p}$ and $u_{t,z,p,k}$ are respectively the downwind position and the velocity of an OP $p$ in a wake zone $z$ of a turbine $t$ at a timestep $k$. For adjusting the downwind positions of the OPs each timestep, we assume that the velocity in between two OPs
is constant over the downwind distance, resulting in the following update law:

\[
\begin{bmatrix}
    x_{t,z,1,k+1} \\
    x_{t,z,2,k+1} \\
    \vdots \\
    x_{t,z,N_p,k+1}
\end{bmatrix}
= \begin{bmatrix} A & 0 \\ 0 & \ddots \\ 0 & \ddots & 0 \end{bmatrix}
\begin{bmatrix}
    x_{t,z,1,k} \\
    x_{t,z,2,k} \\
    \vdots \\
    x_{t,z,N_p,k}
\end{bmatrix}
+ \begin{bmatrix} B & 0 \\ 0 & \ddots \\ 0 & \ddots & 0 \end{bmatrix}
\begin{bmatrix}
    x_T(t) + \Delta T A \\
    u_{t,z,1,k} \\
    u_{t,z,2,k} \\
    \vdots \\
    u_{t,z,N_p,k}
\end{bmatrix} \\
\forall t \in \mathcal{P}, z \in \{1, \ldots, 7\}
\] (3)

Where \(\Delta T\) is the time interval between two discrete timesteps.

In each timestep, some of the variables that are measured at the turbines or estimated from measurements at the turbines (the turbine yaw angle \(\gamma_T\), and axial induction \(a_T\), and the free-stream velocities \(U_T\)) are ‘passed down’ the stream from one OP to the next downstream OP in a zone (see also Fig. 3). From these quantities, local wake properties (lateral position and velocity) at the OPs are calculated. This makes that at the first OP, the effects on the wake of a change in yaw are observed after one timestep, and in the second OP the effects of the yaw properties two timesteps ago is observed, etc., and likewise for the rotor axial induction and the free-stream velocity. The free-stream velocity, axial induction, and yaw effective at an OP \(p\) in zone \(z\) of turbine \(t\) at timestep \(k\), are denoted respectively as \(U_{t,z,p,k}\), \(a_{t,z,p,k}\) and \(\gamma_{t,z,p,k}\). The yaw and axial induction property is only passed on in this way in zones 1 to 6, since by definition the free-stream flow is not affected by the axial inductions and yaw angles of turbines. Using a similar vector notation as above, the update laws for passing on the measurements between the OPs in each timestep are:

\[
\begin{bmatrix}
    U_{t,z,1,k+1} \\
    U_{t,z,2,k+1} \\
    \vdots \\
    U_{t,z,N_p,k+1}
\end{bmatrix}
= \begin{bmatrix} A & 0 \\ 0 & \ddots \\ 0 & \ddots & 0 \end{bmatrix}
\begin{bmatrix}
    U_{t,z,1,k} \\
    U_{t,z,2,k} \\
    \vdots \\
    U_{t,z,N_p,k}
\end{bmatrix}
+ BU_T (t, k) \quad \forall t \in \mathcal{P}, z \in \{1, \ldots, 7\}
\] (4)

\[
\begin{bmatrix}
    \gamma_{t,z,1,k+1} \\
    \gamma_{t,z,2,k+1} \\
    \vdots \\
    \gamma_{t,z,N_p,k+1}
\end{bmatrix}
= \begin{bmatrix} A & 0 \\ 0 & \ddots \\ 0 & \ddots & 0 \end{bmatrix}
\begin{bmatrix}
    \gamma_{t,z,1,k} \\
    \gamma_{t,z,2,k} \\
    \vdots \\
    \gamma_{t,z,N_p,k}
\end{bmatrix}
+ B\gamma_T (t, k) \quad \forall t \in \mathcal{P}, z \in \{1, \ldots, 6\}
\] (5)

and similarly for the axial induction, with \(A\) and \(B\) as in eq. (3).

2.5. Calculation of wake characteristics

In this section, we will explain how in the FLORIDYN model the wake characteristics at each OP are estimated at each timestep, based on the OP downwind position, and the locally effective delayed turbine measurements of yaw, axial induction, and free-stream speed. From these wake characteristics, the effective flow velocities at the downstream turbines are estimated. These estimations are based on static nonlinear mappings that follow from the FLORIS model. Therefore in the remainder we will omit the time index \(k\) in the notations.

2.5.1. Deflection and expansion of the wake zones

From the locally effective delayed yaw angle \(\gamma_{t,z,p}\) and axial induction \(a_{t,z,p}\) at an OP \(p\) in wake zone \(z\) of turbine \(t\), we calculate the deflection and expansion of that wake zone. The crosswind position of the centerpoint of the total wake at OP \(p\), denoted as \(y_{C,t,z,p}\), is calculated as:

\[
y_{C,t,z,p} = y_T(t) + \Delta y_{\text{rotation}} (\Delta x_{t,z,p}) + \Delta y_{\text{yaw}} (\Delta x_{t,z,p}, \gamma_{t,z,p}, a_{t,z,p}, D_T(t))
\] (6)
Figure 3. Illustration of the state update mechanism in the FLORIDyn model. Since we only consider one turbine and wake, \( t \) and \( z \) indices are omitted in the notation.

Figure 4. Upstream and downstream OP in an overlapping wake (see Section 2.5.2)
with $\Delta x_{t,z,p} = x_{t,z,p} - x_T(t)$, $D_T(t)$ the rotor diameter of turbine $t$, and with functions:

$$
\Delta y_{w,\text{rotation}}(\Delta x) = \alpha_d + \beta_d \Delta x
$$

$$
\Delta y_{w,yaw}(\Delta x, \gamma, a, D_T) = \frac{\tilde{C}_T(\alpha, \gamma)}{2\eta_d} \left[ \frac{\eta_d^2}{D_T^2} + 1 \right]^{3/2} \left[ \frac{2k_d \Delta x}{D_T^2} + 1 \right]^{1/2} - \frac{\bar{C}_T(\alpha, \gamma) D_T [1 + \bar{C}_T(\alpha, \gamma)]}{30k_d}
$$

where $\bar{C}_T(\alpha, \gamma) = \frac{1}{2} \cos^2(\gamma) \sin(\gamma) [4\alpha [1 - a]$

with coefficients $\alpha_d$, $\beta_d$ and $k_d$ as model parameters. We refer to [4] for the derivation of the above functions. In the FLORIS model, only three wake zones were defined: near wake, far wake and the mixing zone (see Fig. 1). At each OP, the diameters of these zones, respectively $D_{w,n}$, $D_{w,f}$, $D_{w,m}$, are given by:

$$
D_w(\bullet) (\Delta x_{t,z,p}) = \max (D_T(t) + 2k_e m_{\bullet,w} \Delta x_{t,z,p}, 0) \text{ with } \bullet = n, f \text{ or } m
$$

and with parameters $m_{e,n}, m_{e,f}, m_{e,m}$, $k_e$ being coefficients defining the expansion of the zones. Using these diameters, we define the position of an inner boundary point, with coordinates $x_{t,z,p}$, $y_{t,z,p}$, $z_{t,z,p}$ and an outer boundary point $(x_{t,z,p}, y_{O,t,z,p})$ of the wake zones 1 to 6 (see Fig. 3), as follows:

$$
y_{t,1,p} = y_{C,t,1,p}, \quad y_{O,t,1,p} = y_{C,t,1,p} + \frac{1}{2} D_{w,n} (\Delta x_{t,z,p});$$
$$y_{t,2,p} = y_{C,t,2,p}, \quad y_{O,t,2,p} = y_{C,t,2,p} - \frac{1}{2} D_{w,n} (\Delta x_{t,z,p});$$
$$y_{t,3,p} = y_{C,t,3,p} + \frac{1}{2} D_{w,n} (\Delta x_{t,z,p}); \quad y_{O,t,3,p} = y_{C,t,3,p} + \frac{1}{2} D_{w,f} (\Delta x_{t,z,p});$$
$$y_{t,4,p} = y_{C,t,4,p} - \frac{1}{2} D_{w,n} (\Delta x_{t,z,p}); \quad y_{O,t,4,p} = y_{C,t,4,p} - \frac{1}{2} D_{w,f} (\Delta x_{t,z,p});$$
$$y_{t,5,p} = y_{C,t,5,p} + \frac{1}{2} D_{w,f} (\Delta x_{t,z,p}); \quad y_{O,t,5,p} = y_{C,t,5,p} + \frac{1}{2} D_{w,m} (\Delta x_{t,z,p});$$
$$y_{t,6,p} = y_{C,t,6,p} - \frac{1}{2} D_{w,f} (\Delta x_{t,z,p}); \quad y_{O,t,6,p} = y_{C,t,6,p} - \frac{1}{2} D_{w,m} (\Delta x_{t,z,p});$$
$$y_{M,t,z,p} = \frac{1}{2} (y_{t,z,p} + y_{O,t,z,p}) \text{ for all } z \in \{1, \ldots, 6\}
$$

2.5.2. Wake velocity profile Interaction between the turbines and their wakes is modelled by correcting the velocities in the wake by means of a method based on [6]. If an OP is situated in the wake zone of a turbine, the velocity at that OP is reduced by a factor dependent on the ‘delayed’ turbine measurements and the downwind distance to the turbine. First we consider a single turbine influencing its own wake only, then we combine the wake effects of several turbines.

**Velocities in a single wake** Following the Jensen model [5], the velocity deficit in the far wake decays quadratically with the expansion of the wake. In the near wake, we use an arctangent function as a velocity correction factor (based on the method in [7] and the velocity profiles found in [8]). This leads to the following formulation for the velocity at an OP $p$ in the wake zone $z$ of a turbine $t$:

$$
r_{t,z,p} = 2a_{t,z,p} \left[ \frac{1}{2} + \tan^{-1} \left( \frac{2 \Delta x_{t,z,p}}{\pi D_T(t)} \right) \right] \left[ \frac{D_T(t)}{D_T(t) + 2k_e m_{U,z}(\gamma_{t,z,p}) \Delta x_{t,z,p}} \right]^2
$$

$$
u_{t,z,p} = U_{t,z,p} (1 - r_{t,z,p})
$$

The coefficients $m_{U,z}$ define how quickly the velocities recover to the free-stream velocity $U_{t,z,p}$ in different zones, as the distance to the rotor $\Delta x_{t,z,p}$ increases. These coefficients are adjusted for the rotor yaw angle as follows (we refer to [4] for a more detailed explanation):

$$
m_{U,z}(\gamma_{t,z,p}) = \frac{M_{U,z}}{\cos(\alpha_U + \beta_U \gamma_{t,z,p})}
$$

where $M_{U,1} = M_{U,2} = M_{U,n}$, $M_{U,3} = M_{U,4}$ and $M_{U,5} = M_{U,6} = M_{U,m}$, and $\alpha_U$ and $\beta_U$ are model parameters defining how quickly the velocity in the different wake zones recover to the free-stream velocity.
Combining wakes to find velocities in overlapping wakes For all the OPs \( p \) in a wake zone \( z \) of a certain turbine \( t \), we calculate the reduction factors induced by that turbine \( t \), \( r_{t,z,p} \). To consider the case in which wakes of multiple turbines overlap with a certain OP, we have to combine the reduction factors of several turbines. Because the OPs in one wake are located in different downwind positions \( x \) than in other wakes, we apply an interpolation to find all the effective reduction factors of the different turbine wakes overlapping with a certain OP. The effective (interpolated) reduction factor at an OP \( p \) in a wake zone \( z \) of a different turbine \( t \), is denoted as \( r_{(i,\tilde{z} \rightarrow t,z,p)} \). We find \( r_{(i,\tilde{z} \rightarrow t,z,p)} \) by interpolating the reduction factor of the two nearest OPs (in the upwind and downwind direction) in the wake zone \( \tilde{z} \) of turbine \( \tilde{t} \) (See Fig. 4). We use the notation \( upstr (\tilde{t}, \tilde{z} \rightarrow t, z, p) \), for the index of the nearest upwind OP belonging to the wake zone \( \tilde{z} \) of turbine \( \tilde{t} \). The nearest downstream OP in zone \( \tilde{z} \) of turbine \( t \) then is \( upstr (\tilde{t}, \tilde{z} \rightarrow t, z, p) + 1 \). This results in the following interpolation:

\[
r_{(i,\tilde{z} \rightarrow t,z,p)} = f_{\text{int}} \left(r; i, \tilde{z} \rightarrow t, z, p \right)
\]

where \( f_{\text{int}} \) is a linear interpolation operator, which for some variable \( \xi \) defined at each OP is given by:

\[
f_{\text{int}} \left(\xi; i, \tilde{z} \rightarrow t, z, p \right) = \frac{x_{i,\tilde{z},upstr(t,\tilde{z} \rightarrow t,z,p)}-x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}+1}{x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}-x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}+1} \xi_{i,\tilde{z},upstr(t,\tilde{z} \rightarrow t,z,p)} + \cdots \frac{x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}-x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}+1}{x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}-x_{i,\tilde{z},upstr(\tilde{t},\tilde{z} \rightarrow t,z,p)}+1} \xi_{i,\tilde{z},upstr(t,\tilde{z} \rightarrow t,z,p)}+1
\]

In a similar way, we interpolate the crosswind positions of the wake boundaries to establish for each OP which wake zones of which turbines are overlapping with it. For brevity, here we omit the exact conditions which follow from simple geometry, and state that if an OP \( p \) belonging to the wake zone \( z \) of turbine \( t \) is in the wake zone \( \tilde{z} \) of a turbine \( \tilde{t} \), then the pair \((i,\tilde{z})\) belongs to the set \( O_{t,z,p} \). We combine the effective reduction factors of each wake to find the velocity at an OP, through:

\[
u_{t,z,p} = U_{t,z,p} \prod_{(i,\tilde{z}) \in O_{t,z,p}} \left(1 - r_{(i,\tilde{z} \rightarrow t,z,p)} \right)
\]

2.5.3. Calculation of effective velocities at downstream turbines In this model we combine the effects of the wake zones of different turbines, in order to estimate the effective inflow velocity at the turbines \( t \in \mathcal{D} \). The wind speeds at the turbines are estimated by combining the effect of the wakes on the free-stream velocities. First the interpolation function \( f_{\text{int}} \) is used to find the set of delayed free-stream velocities \( U \) for each wake zone \( \tilde{z} \) of the turbines \( \tilde{t} \) upstream of turbine \( t \), at the location of the rotor plane of \( t \). Since the OPs with index \( p = 1 \) are always located in the rotor plane, we find these interpolated velocities, denoted as \( U_{(i,\tilde{z} \rightarrow t)} \), as follows:

\[
U_{(i,\tilde{z} \rightarrow t)} = f_{\text{int}} \left(U; i, \tilde{z} \rightarrow t, 1, 1 \right)
\]

To find the effective free-stream velocity for each turbine \( t \in \mathcal{D} \) we weight each of the delayed free-stream velocities by the overlap area of the corresponding wake zones \( \tilde{z} \) of other turbines \( \tilde{t} \) with the rotor of turbine \( t \), denoted by \( A_{\tilde{t},\tilde{z} \rightarrow t}^{\text{overlap}} \) (see Fig. 2). For the part of the rotor that is not overlapping with any wake (with area \( A_{t}^{\text{noOverlap}} \)), we use the delayed free-stream velocity in zone 7 of the upstream turbine that is closest to turbine \( t \) in terms of its \( y \) location:

\[
\text{closest} \ (t) = \arg \min_{t \in \mathcal{P}, x_{T}(\tilde{t}) < x_{T}(t)} (|y_{T}(t) - y_{T}(\tilde{t})|)
\]
measurements (see section 2.2), while the free-stream speeds for turbines we also find the effective velocity reduction factor $r$ of the different turbines. Using interpolation and root-sum-square weighting by overlap area, wind speed measurements are essentially passed on from turbine to turbine through the wakes through eq. (18) are fed back to the state-space delay model (eq. 4), such that the free-stream

\[ U(t) = \sum_{i \in P: x_T(t) < x_T(t)} \left( A_{\text{overlap}}^{i \rightarrow t} A_T(t) U(i, \hat{z} \rightarrow t) \right) + A_{\text{noOverlap}}^{i \rightarrow t} U(\text{closest}(t), \hat{z} \rightarrow t) \forall t \in D \]  

(18)

Note that the free-stream speeds $U$ for front turbines $t \in F$ are estimated based on their own measurements (see section 2.2), while the free-stream speeds for turbines $t \forall t \in D$ estimated through eq. (18) are fed back to the state-space delay model (eq. 4), such that the free-stream wind speed measurements are essentially passed on from turbine to turbine through the wakes of the different turbines. Using interpolation and root-sum-square weighting by overlap area, we also find the effective velocity reduction factor $r_T(t)$ for each turbine rotor (similar to [6]):

\[ r_T(t) = \left( \sum_{i \in P: x_T(t) < x_T(t)} \sum_{\hat{z}=1}^{6} \left( A_{\text{overlap}}^{i \rightarrow t} A_T(t) U(i, \hat{z} \rightarrow t) \right) \right) \left( \sum_{\hat{z}=1}^{6} A_{\text{overlap}}^{i \rightarrow t} A_T(t) U(i, \hat{z} \rightarrow t) \right) \]  

(19)

with $r(i, \hat{z} \rightarrow t) = f_{\text{int}}(r; i, \hat{z} \rightarrow t, 1, 1)$. We then correct the free-stream velocity to find the effective velocity for the turbine:

\[ u_T(t) = U_T(t) \left[ 1 - r_T(t) \right] \forall t \in D \]  

(20)

The effective velocities $u_T(t)$ are then used to calculate the powers of through eq. (1).

3. Simulation case study

Fig. 5 shows the results of a case study in which we simulate a small wind plant in three ways:

- a high-fidelity 3D large-eddy simulation with the Simulator for On/Offshore Wind Farm Applications (SOWFA) [9, 10]
- a simulation of the same wind plant with the FLORIDYN model, with the model parameters tuned to provide a good match with the SOWFA data (the parameters are listed in Tab. 1)
- calculation of steady-state (SS) solution of the FLORIDYN model, which would be comparable with the predictions that the steady-state FLORIS model would provide

The simulated wind plant consists of 2 rows with 3 NREL 5-MW baseline turbines [11] each, with a 5 rotor diameter (5D) spacing in the row direction, and 3D in the column direction. The turbine rows are rotated 5 degrees with respect to the wind direction. In the SOWFA simulations, an inflow with a 6% turbulence intensity and an 8 m/s mean velocity is used (see the case study in [4] for more details on the setup). After 400 s, the yaw angles of upstream turbines are misaligned with the wind flow to redirect the wakes away from the downstream turbines. 1 In Fig. 5(a), it can be seen that:

- The FLORIDYN model gives an overall better fit to the SOWFA-predicted electrical power production data of the downstream turbines than the SS solution, with the exception of turbine 6. Specifically, in the 400-600 s time range, when the effects of the yaw angle changes propagate through the wind field, the FLORIDYN model performs better. In Fig. 5(b), it

\[ \begin{array}{cccccccc}
\rho & p_p & \eta & N_p & \Delta T & \alpha_d & \beta_d & k_d & k_e \\
1.172 & 1.88 & 0.768 & 80 & 5 s & -4.5 & 0.01 & 0.15 & 0.0963 \\
m_{e,n} & m_{e,f} & m_{e,m} & M_{U,n} & M_{U,f} & M_{U,m} & \alpha_U & \beta_U & \\
-0.5 & 1 & 2.2 & 0.3750 & 1 & 5.1250 & 5 & 1.66 & \\
\end{array} 
\]

Table 1. FLORIDYN model parameter values used in the case study of Section 3

\[ f_{\text{int}}(r; i, \hat{z} \rightarrow t, 1, 1) \]

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1 The first 300 s of the simulation results are not shown, since in the time interval 0-300 s in the SOWFA simulation the wakes are developing, which is not representative of normal operation in a wind plant.
Figure 5. Result of the case study described in Section 3
is shown that both in the SOWFA- and in the FLORIDYN-predicted flow-fields, first the part of the wake closer to the rotor are redirected in response to the yaw change, and later the part further downstream. The SS solution does not include these transient effects.

- There is still some error between the FLORIDYN-predicted power output and the SOWFA-predicted output, which can at least partly be attributed to turbulence, which is not fully represented by the FLORIDYN model.

These notions are also shown in the normalized root-mean square errors (RMSE) listed for each turbine in Fig. 5(a). The FLORIDYN model has a relatively low computational cost: it takes 2 s to evaluate the complete simulation in a MATLAB implementation on a 1.6 GHz PC.

4. Discussion, conclusions and future work

The results from the case study are promising: with a relatively simple extension of the steady-state FLORIS model to include the dynamics of the wake propagation, we were able to provide a prediction of a wind farm response with reasonable accuracy, while still keeping the computational complexity of the model small. In addition to further validation of the model, future work consists of using the control-oriented FLORIDYN model for:

- designing an observer to estimate the velocity field in the wind plant from turbine power measurements, under the influence of the random variations due to turbulence and model inaccuracies. Also an extended observer that adapts the model parameters to changing inflow conditions (turbulence, atmospheric stability), based on power measurements, should improve the performance of the predictions model.

- designing a controller adjusting the yaw angles for improved wind plant power production while taking into account transients effects caused by the wake taking time to propagate through the wind field. While the specific structure of the model (a linear state-space representation for the delay dynamics, combined with a nonlinear static feedback) may be exploited to come to a design of a feedback controller, another possibility is to use the predictions of the low-complexity model for online nonlinear model-predictive control.

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2 The normalized RSME is defined as the root-mean square error between SOWFA-predicted power output and FLORIDYN-predicted power output, divided by the mean SOWFA-predicted power output.