Quark Correlations and Single Spin Asymmetries

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We analyze the Sivers asymmetry in light-cone gauge. The average transverse momentum of the quark distribution is related to the correlation between the quark distribution and the transverse component of the gauge field at $x^\pm = \pm \infty$. We then use finiteness conditions for the light-cone Hamiltonian to relate the transverse gauge field at $x^- = \pm \infty$ to the color density integrated over $x^-$. This result allows us to relate the average transverse momentum of the active quark to color charge correlations in the transverse plane.

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I. INTRODUCTION

Many high energy inclusive hadron processes show surprisingly large transverse polarizations or single-spin asymmetries [1]. The most prominent example is inclusive hyperon production, but similar phenomena are observed in many other reactions as well. For example, in the inclusive photo-production of pions on a transversely (relative to the photon momentum) polarized nucleon target, a left-right asymmetry of the produced pions is observed. Theoretically, two mechanisms (which are not exclusive!) have been identified as a potential source of the asymmetry: the Sivers and the Collins mechanisms. In the Collins mechanism [2], the asymmetry arises when a transversely polarized quark fragment into pions with a left-right asymmetry. In contrast, in the Sivers mechanism [3] the asymmetry results from an intrinsic transverse momentum asymmetry of the quarks in the target nucleon. At first, such an intrinsic transverse momentum asymmetry was expected to vanish due to time-reversal invariance of the strong interaction. However, more recently it was realized that, even at high energies, the final state interactions (FSI) of the struck quark can play an important role for the single-spin asymmetry [4]. Formally the FSI can be taken into account by introducing an appropriate Wilson line gauge link along the trajectory of the ejected quark [5]. The gauge invariantly defined transverse momentum distributions with a gauge link along the light-cone to infinity are no longer required to vanish to to time-reversal invariance and a nonzero Sivers asymmetry is possible.

However, while the above reasoning explains the existence of the Sivers asymmetry it leaves many questions unanswered, for example what sign and what magnitude should one expect for the asymmetry, i.e. is it just an obscure small effect or is it large? If there is a large asymmetry, what does any information about the asymmetry teach us about the structure of the nucleon since the asymmetry hinges on the inclusion of FSI. In this paper an attempt will be made to make a step towards answering these questions.

The paper is organized as follows: in Section 2, we review the definitions of gauge invariant transverse momentum distributions and the role of the gauge field at $x^- = \pm \infty$ in the light-cone gauge. In Section 3, we use finiteness constraints for light-cone Hamiltonians to derive operator constraints that allow one to relate the gauge fields at $x^- = \pm \infty$ to degrees of freedom at finite $x^-$. In Sections 4 (QED) and 5 (QCD) we use these operator constraints to relate the average transverse momentum to charge (color charge) correlations in the transverse plane.

II. INITIAL (FINAL) STATE INTERACTIONS AND TRANSVERSE SPIN ASYMMETRIES

Ref. [5] explains how final state interactions (FSI) and initial state interactions (ISI) allow the existence of T-odd parton distribution functions. Formally the FSI (ISI) can be incorporated into $k_\perp$ dependent parton distribution functions (PDFs) by introducing a gauge string from each quark field operator to infinity [5].

$$q(x, k_\perp, s_\perp) = \int \frac{dy^-d^2y_\perp}{16\pi^3} e^{-i p^+y^- + i k_\perp \cdot y_\perp} \langle p | q(y^-, y_\perp) \gamma^+ [y^-, y_\perp; \infty^- , y_\perp] [\infty^- , 0_\perp; 0^- , 0_\perp] q(0) | p \rangle. \quad (1)$$

We use light-front (LF) coordinates, which are defined as: $y^\mu = (y^+, y^-, y_\perp)$, with $y^\pm = (y^0 \pm y^3)/\sqrt{2}$. In all correlation functions, $y^+ = 0$ and we therefore do not explicitly show the $y^+$ dependence. The path ordered Wilson-line operator from the point $y$ to infinity is defined as

$$[\infty^- , y_\perp; y^-, y_\perp] = P \exp \left( -ig \int_{y^-}^{\infty} dz^- A^+(y^+, z^-, y_\perp) \right). \quad (2)$$
This implies for the average transverse momentum for quark flavor $q$ and interaction provides the asymmetry $A$. A similar result holds for the unintegrated parton density relevant for the Drell-Yan process, where the initial state complex phase in Eq. (1) is reversed under time-reversal and therefore $T$-odd PDFs may exist [5], which is why a non-zero Sivers asymmetry is possible.

Naively, the single spin asymmetry seems to be absent in light-cone gauge $A^+ = 0$, since the Wilson lines in Eq. (1) are in the $x^-$ direction and therefore $\int dz^- A^+ = 0$. Without the phase factor any single spin asymmetry vanishes due to time reversal invariance.

This apparent puzzle has been resolved in Ref. [6], where it has been emphasized that a truly gauge invariant definition for unintegrated parton densities requires closing the gauge link at $x^- = \infty$, i.e. a fully gauge invariant version of Eq. (1) reads

$$q(x, k_\perp, s_\perp) = \int \frac{dy^-d^2y_\perp}{16\pi^3} e^{-ixp^+y^- + ik_\perp y_\perp} \langle p | \bar{q}(y^- y_\perp) \gamma^+ [y^- y_\perp; \infty^-, y_\perp] [\infty^-, y_\perp, \infty^-, 0_\perp] [\infty^-, 0_\perp; 0^-, 0_\perp] q(0) | p \rangle.$$ (3)

In all commonly used gauges, except the light-cone gauge, the gauge link at $x^- = \infty$ is not expected to contribute to the matrix element, since the gauge fields are expected to fall off rapidly enough at $\infty$. However, this is not true in light-cone gauge and therefore it has been suggested in Ref. [6] that, in light-cone gauge, the entire single-spin asymmetry arises from the phase due to the gauge link at $x^- = \infty$

$$q(x, k_\perp, s_\perp) = \int \frac{dy^-d^2y_\perp}{16\pi^3} e^{-ixp^+y^- + ik_\perp y_\perp} \langle p | \bar{q}(y^- y_\perp) [\infty^-, y_\perp, \infty^-, 0_\perp] \gamma^+ q(0) | p \rangle. \quad (A^+ = 0)$$ (4)

This implies for the average transverse momentum for quark flavor $q$

$$\int d^2k_\perp q(x, k_\perp, s_\perp)k_\perp = -g \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \langle p | \bar{q}(y^- 0_\perp) \gamma^+ A_\perp(\infty^-, 0_\perp) q(0) | p \rangle. \quad (A^+ = 0)$$ (5)

A similar result holds for the unintegrated parton density relevant for the Drell-Yan process, where the initial state interaction provides the asymmetry

$$q_{\text{past}}(x, k_\perp, s_\perp) = \int \frac{dy^-d^2y_\perp}{16\pi^3} e^{-ixp^+y^- + ik_\perp y_\perp} \langle p | \bar{q}(y^- y_\perp) [-\infty^-, y_\perp, -\infty^-, 0_\perp] \gamma^+ q(0) | p \rangle. \quad (A^+ = 0)$$ (6)

and

$$\int d^2k_\perp q_{\text{past}}(x, k_\perp, s_\perp)k_\perp = -g \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \langle p | \bar{q}(y^- 0_\perp) \gamma^+ A_\perp(-\infty^-, 0_\perp) q(0) | p \rangle. \quad (A^+ = 0)$$ (7)

(Here and in the rest of this paper we will work in light-cone gauge and therefore no longer emphasize explicitly that $A^+ = 0$). The fact that these asymmetries hinge on the value of the transverse gauge field at $x^- = \pm \infty$ makes the evaluation of these matrix elements rather tricky. Only a careful regularization prescription for the $k^+ = 0$ singularity of the gauge field propagator is capable to generate the complex phase that is necessary for a non-vanishing SSA. Therefore, the question arises whether knowledge of the light-cone wave function for the nucleon would (in principle)
Therefore

\[ \int d^2k_\perp q(x, k_\perp, s_\perp)k_\perp = -\int d^2k_\perp q_{\text{past}}(x, k_\perp, s_\perp)k_\perp. \]

Therefore

\[ \tilde{k}_\perp q(x) = \frac{1}{2} \left[ \int d^2k_\perp q(x, k_\perp, s_\perp)k_\perp - \int d^2k_\perp q_{\text{past}}(x, k_\perp, s_\perp)k_\perp \right] \]

\[ = -\frac{g}{2} \int dy^- e^{-ixp^+} \left\langle p \mid \tilde{q}(y^-) A_\perp(\infty^-, 0_\perp) - A_\perp(-\infty^-, 0_\perp) \right\rangle \gamma^+ q(0) \mid p \rangle. \]

For later applications, we also rewrite Eq. (9) in the more familiar color component notation

\[ \tilde{k}_\perp q(x) = -\frac{g}{2} \int dy^- e^{-ixp^+} \left\langle p \mid \tilde{q}(y^-) \gamma^+ \frac{\lambda_a}{2} q(0) \alpha^a_\perp(0_\perp) \mid p \rangle \right. \]

where \( \lambda_a \) are the Gell-Mann matrices and

\[ \alpha^a_\perp(b_\perp) \equiv A^a_\perp(\infty^-, b_\perp) - A^a_\perp(-\infty^-, b_\perp). \]

Eqs. (10) and (11) still involve the gauge field at \( x^- = \pm \infty \). In the following section, we will derive an operator relation that relates \( A_\perp(\pm \infty, y_\perp) \) to degrees of freedom at \(-\infty < x^- < \infty\).

### III. FINITENESS CONDITIONS

In a light-cone formulation of QCD (\( x^+ \) is time), not all components of the quark and gluon field are dynamical, since they satisfy certain constraint equations (Lagrangean contains no \( x^+ \) derivative for these degrees of freedom). In particular, \( A^- \) as well as the “bad” component of the quark field \( q_{\text{bad}} \equiv \frac{1}{2} \gamma^+ \gamma^- q \) are such constrained degrees of freedom in QCD. Upon eliminating both \( A^- \) and \( q_{\text{bad}} \) using their constraint equations, one arrives at the canonical light-cone Hamiltonian for QCD in light-cone gauge \( A^+ = 0 \), which governs the \( x^+ \) evolution

\[ P^- = \int dx^- d^2x_\perp \left\{ F^+_i \frac{1}{i\partial_-} F_i + F^-_i \frac{1}{i\partial_-} F_i + \frac{1}{2} \text{tr} \left[ -\tilde{J} \frac{1}{\partial_-^\perp} J + (F^+_{12})^2 \right] \right\} \]

where

\[ F^+_{12} = \partial_+ A_2 - \partial_2 A_1 + ig [A_1, A_2] \]

\[ F^-_i = i [\partial_i + i\partial_+ + ig (A_x + iA_y)] u_- + \frac{m}{\sqrt{2}} u_+ \]

\[ \tilde{J} = \partial_- \partial_+ A_i + J \]

\[ = \partial_- \partial_+ A_i + g [A_i, \partial_+ A_1] + g \left( q_+ q^i_+ + q_- q^i_- \right) \]

In the above expression \( q^i_\pm \) form respectively the positive and negative chirality components of the ‘good’ component \( q_{\text{good}} \equiv \frac{1}{2} \gamma^+ \gamma^- q \) of the quark field. A summation over quark flavors is implicit.

The requirement that \( P^- \) acting on a hadron state is free of infrared \( x^- = \pm \infty \) divergences implies that each of the four terms in Eq. (12) are free of such divergences. This has a number of consequences. In Ref. [3], it was investigated what conditions on the hadron Fock state result from the requirement that such divergences are absent in the first two terms. In this work we will focus on the third term. The condition

\[ \int dx^- Tr \left[ -\tilde{J} \frac{1}{\partial_-^\perp} J \right] = \text{finite} \]
implies that

$$\int_{-\infty}^{\infty} dx^- \tilde{J}(x^-, x_\perp) = 0 \quad \forall x_\perp$$  \hspace{1cm} (16)

This (weak) condition, which should hold as a condition on all physical states, forms one of the crucial ingredients of this investigation. Since the first term in \( J \) is a total derivative, Eq. (16) implies (now expressed in terms of the more familiar cartesian components \( i = 1, 2 \), which are implicitly summed when they appear in pairs)

$$\partial^i \alpha^i(x_\perp) = -\rho(x_\perp) \equiv - \int dx^- J(x^-, x_\perp),$$  \hspace{1cm} (17)

where

$$\alpha^i(x_\perp) \equiv A^i(\infty^-, x_\perp) - A^i(-\infty^-, x_\perp)$$

$$J(x^-, x_\perp) = ig [A^i, \partial_+ A^j] + g \left( q_+ q_+^\dagger + q_- q_-^\dagger \right).$$  \hspace{1cm} (18)

The physical meaning of \( \rho(x_\perp) \) is the total charge (quarks plus gluons) along a line with fixed \( x_\perp \). For later use, we also express the above results using color components (instead of matrix notation)

$$\partial^i \alpha^i_0(x_\perp) = -\rho_a(x_\perp) \equiv - \int dx^- J_a(x^-, x_\perp)$$  \hspace{1cm} (19)

$$J_a(x^-, x_\perp) = -gf_{abc} A_b^i \partial_+ A_c^i + g \sum_q \tilde{q} \gamma^+ \lambda_a \frac{\gamma}{2} q,$$  \hspace{1cm} (20)

where \( \lambda_a \) are the Gell-Mann matrices. In QED, the analogous conditions read and

$$\partial^i \alpha^i(x_\perp) = -\rho(x_\perp) \equiv - \int dx^- J(x^-, x_\perp)$$

$$J(x^-, x_\perp) = \sum_q e_q \tilde{q} \gamma^+ q.$$  \hspace{1cm} (21, 22)

An additional condition arises from the requirement that the tr \([\mathcal{F}_{12}]^2\) term in the Hamiltonian is convergent at \( x^- = \pm \infty \): the field \( \perp \) strength tensor itself must vanish at \( x^- = \pm \infty \)

$$\mathcal{F}_{12}(\pm \infty^-, x_\perp) = 0,$$  \hspace{1cm} (23)

i.e. \( A^I(\pm \infty^-, x_\perp) \) must be pure gauge. \( \bar{a} \)

$$A^I(\pm \infty^-, x_\perp) = \left\{ \begin{array}{ll}
-i g V_+(x_\perp) \partial_+ V_+^\dagger(x_\perp) & \text{if } x^- = -\infty \\
-i g V_- (x_\perp) \partial_+ V_-^\dagger(x_\perp) & \text{if } x^- = +\infty
\end{array} \right.$$

$$A^I(-\infty^-, x_\perp) = \left\{ \begin{array}{ll}
-i g V_+(x_\perp) \partial_+ V_+^\dagger(x_\perp) & \text{if } x^- = -\infty \\
-i g V_- (x_\perp) \partial_+ V_-^\dagger(x_\perp) & \text{if } x^- = +\infty
\end{array} \right.$$

This allows us to gauge transform \( A_I(-\infty^-, x_\perp) \) to zero while preserving \( A^+ = 0 \). Since in this gauge \( A^I(+\infty^-, x_\perp) = 0 \) is still pure gauge, i.e. \( A^I(\infty^-, x_\perp) = -\frac{i g}{2} U^I(x_\perp) \partial_+ U(x_\perp) \) with \( U(x_\perp) = V_+(x_\perp) V_-(x_\perp) \) we thus conclude that

$$\alpha^i(x_\perp) = A^I(\infty^-, x_\perp) - A^I(-\infty^-, x_\perp)$$

must be (in this gauge) of the form

$$\alpha^i(x_\perp) = -\frac{i g}{2} U^I(x_\perp) \partial_+ U(x_\perp).$$

Eq. (26) together with Eq. (17) thus determine \( \alpha^i(x_\perp) \) uniquely (up to some trivial constants).

The above results have a number of applications as we will discuss below. First of all, Eqs. (19, 21) alert us again that in light-cone gauge one must not assume a vanishing of the gauge fields at \( x^- = \pm \infty \). However, the most important application of Eqs. (19, 21) lies in the fact that it allows us to reexpress \( \alpha^i(x_\perp) = A^I(\infty^-, x_\perp) - A^I(-\infty^-, x_\perp) \) in terms of other degrees of freedom. The interesting aspect about this observations is the fact that \( \alpha^i(x_\perp) \) also appears in the correlation function \( \bar{a} \) for the average transverse momentum. In the rest of this paper we will discuss the implication of this fundamental result.
IV. AVERAGE $\perp$ MOMENTUM IN THE ABELIAN CASE

Before proceeding to QCD, we will first discuss an abelian theory, where the nucleon contains quarks of different flavor $q$ with charges $e_q$ respectively as well as photons. For simplicity, we will consider here only the Sivers asymmetry averaged over all $x$

$$\langle k_{\perp q} \rangle = \int dx \int d^2k_{\perp} q(x, k_{\perp}, s_{\perp}) k_{\perp} = \int dx \bar{k}_{\perp q}(x).$$

(27)

Considering only the $x$-averaged asymmetry not only helps keep the resulting expressions simpler, but may also help to cancel possible divergences from endpoint singularities [12].

In QED, the requirement that $\alpha^i(x_{\perp})$ is pure gauge can be rewritten as

$$\alpha^i(x_{\perp}) = -\partial^i \phi(x_{\perp}),$$

(28)

where $\phi(x_{\perp})$ is some scalar function, which can be determined by solving the 2-dimensional Poisson equation. This yields

$$\alpha^i(x_{\perp}) = -\int d^2y_{\perp} \frac{x^i - y^i}{|x_{\perp} - y_{\perp}|} \rho(y_{\perp}).$$

(29)

What we have found is an operator condition (29) that determines the transverse gauge field at $x^- = \pm \infty$ in terms of the charge density at $-\infty < x^- < \infty$. This condition needs to be satisfied for states in order to have an infrared finite energy. Upon inserting this result into the expression for $\langle k_{\perp q} \rangle$ we find

$$\langle k_{\perp q} \rangle = \frac{e_q}{4p^+-} \int \frac{d^2y_{\perp}}{2\pi} \frac{y_{\perp}}{|y_{\perp}|} \langle p | \bar{q}(0) \gamma^+ q(0) \rho(y_{\perp}) | p \rangle$$

(30)

where

$$\rho(y_{\perp}) = \sum_{q'} e_{q'} \int dy^- \bar{q}'(y) \gamma^+ q'(y)$$

(31)

is the charge density (integrated over $x^-$) from all flavors (actually, for symmetry reasons, $q' = q$ does not contribute in Eq. (30).

Eq. (30) is gauge invariant and provides a regularized expression for the Sivers asymmetry that depends on the light-cone wave function of the target only. Although this should be clear from our derivation, it should be emphasized that the final state interactions are included in Eq. (30), but the gauge fields that give rise to the final state interactions have been reexpressed in terms of charge density correlations inside the target. The relevant density-density correlations are correlations in the transverse plane of the charge density integrated along $x^-$. The kernel in the correlation function is the Lorentz-boosted Coulomb force integrated along $x^-$ as well. Such a result should not be surprising since this is simply the Coulomb force from the spectators acting on the escaping quark. In fact, for the specific example of the scalar diquark model this result was already obtained in Ref. [13].

As a corollary, we should also note that if one sums the mean transverse momentum over all quark flavors one gets zero

$$\sum_q \langle k_{\perp q} \rangle = 0.$$ 

(32)

This result follows from the fact that the integration kernel in Eq. (29) is odd under $x_{\perp} \leftrightarrow y_{\perp}$.

To summarize this section, what we have found is that the mean transverse momentum for quarks of flavor $q$ can be related to the correlations between quarks of flavor $q$ and all other quarks in the transverse plane. This is not surprising since the final state interaction is the Lorentz boosted Coulomb interaction and the correlations describe the Coulomb field from the spectators acting on the escaping quark.

V. AVERAGE $\perp$ MOMENTUM IN QCD

In QCD, the condition that the gauge field at $\pm \infty$ is pure gauge is nonlinear, which prevents us from writing down closed form solutions to the finiteness conditions. This can be seen as follows. If one writes

$$U(x_{\perp}) = e^{-ig\phi(x_{\perp})}$$

(33)
then, to lowest order in \( \phi_a \) (see also Appendix B) one finds the QED-like condition

\[
\Delta \phi_a(x_\perp) = -\rho_a(x_\perp),
\]

yielding

\[
\alpha^i_a(x_\perp) = -\partial^i \phi_a = -\int \frac{d^2 y_\perp}{2\pi} \frac{x^i - y^i}{|x_\perp - y_\perp|^2} \rho_a(y_\perp).
\]

Of course, there are nonabelian corrections to Eq. (34) and therefore, unlike in QED, Eq. (35) is not an exact solution to the finiteness conditions. However, since we were unable to find an exact operator solution, we will proceed using Eq. (35).

Upon inserting (29) into Eq. (10) one obtains the nonabelian version of Eq. (30)

\[
\langle k_\perp^i \rangle = -\frac{g}{4p} \int \frac{d^2 y_\perp}{2\pi} \frac{y^i}{|y_\perp|^2} \left< p \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(y_\perp) \right| p \right>.
\]

The physical interpretation of this result is that the average transverse momentum of quarks of flavor \( q \) can be related to correlations on the transverse plane. The specific correlations that appear in Eq. (36) reflect a Coulomb interaction between the active quark and the spectators. Here a Coulomb interaction appears because we have solved the finiteness constraints in QCD only to first order. Eq. (36) is thus equivalent to treating the FSI in lowest order in perturbation theory [4, 11, 13, 14]. Note also that the resulting correlation functions are very similar to the correlation functions that have been used to describe the small-\( x \) gluon distributions in nuclei [16].

To first order, what we also find is that the average transverse momentum due to the FSI of all constituents (quarks + gluons) added together vanishes for symmetry reasons. It is not clear if this happens beyond lowest order.

Eq. (36) may be useful for several reasons. While the original expression for the Sivers asymmetry involved a gauge link, which made a parton model interpretation difficult, Eq. (36) does have an immediate parton model interpretation in terms of color-flavor correlations in the transverse plane. This may be useful in correlating experimental data with our understanding of the nucleon structure. Another use of Eq. (36) is that it can be directly calculated from the light-cone wave functions of the nucleon. Of course, we need to keep in mind that (unlike the QED case) Eq. (36) is only approximation, but we still believe that this result provides a step towards linking the Sivers asymmetry with other features of hadron structure. Finally, we would like to emphasize that Eq. (36) suggests interesting connections between the distribution of partons in impact parameter \( (x_\perp) \) and the sign of the transverse SSA [15]. For example, in a simple quark model, such as the bag model [11], the color part of the matrix element in Eq. (36) would be negative (attraction). If the transverse distribution of \( j_\perp^+ \) is transversely shifted relative to the spectators [17] then the resulting average transverse momentum has the opposite sign of the sign of the transverse distortion in impact parameter space.

VI. SUMMARY

We have studied the average transverse momentum of gauge invariant Quark distributions for a transversely polarized target in light-cone gauge. The Wilson line is along the light-cone to infinity to incorporate the Final state interactions in semi-inclusive DIS. In light-cone gauge, the Wilson-line phase factor receives its only nonzero contribution from the gauge field at \( x^- = \pm \infty \). In a naive Fock space expansion the gauge field at \( x^- = \pm \infty \) is usually implicitly set to zero, thus making a correct treatment of single-spin asymmetries rather difficult (except in perturbation theory, where one can carefully regularize the fields at \( x^- = \pm \infty \) “by hand”).

We have also studied conditions for the infrared \( (x^- = \pm \infty) \) convergence of the light-cone Hamiltonian for gauge theories and derived operator conditions that need to be satisfied in order for the light-cone to be free of infrared divergences arising from otherwise ill-defined operators \( \frac{1}{x^-} \). This operator condition relates the transverse component of the gauge field \( A^i_a(\pm \infty, x_\perp) \) to the color density \( \rho_a(x_\perp) \) integrated over all \( x^- \).

Fortunately, the same kind of operators that governs the average transverse momentum in gauge invariant quark distributions also appears in the finiteness conditions for the light-cone Hamiltonian. We are thus able to eliminate \( A^i_a(\pm \infty, x_\perp) \) in the average transverse momentum in favor of other, less infrared singular, degrees of freedom. In QED we can solve the operator condition arising from finiteness conditions exactly and we are able to express the average transverse momentum in terms of charge density correlations in the transverse plane. In QCD we were only able to solve the operator condition to first order in the color charge density and there we find a similar result as in QED, namely that the average transverse momentum can be related to transverse correlations between the active quark and the spectators.
Single spin asymmetries do not have a simple parton model (or light-cone Fock space) interpretation. The main significance of our results is that we have found relations that allow to relate the average transverse momentum to operators that do have a parton interpretation (in QED exactly, in QCD approximately). One immediate application of these results is that it allows to evaluate the average transverse momentum of the quarks directly from the nucleon wave function in light-cone quark models.

Several extensions of this work are conceivable. First it would be desirable to derive an exact solution (at least in terms of an expansion) for the finiteness conditions in QCD, so that one can study the effects of higher order terms that we have omitted. Secondly, it would be interesting to see if one can translate the results from the work into lattice language (Euclidean as well as transverse lattice) with the goal of being able to compute the average transverse momentum nonperturbatively within these frameworks.

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**APPENDIX A: ALTERNATIVE DERIVATION FOR THE FINITENESS CONDITION**

In this appendix we would like to present an alternative derivation of Eq. (20) that does not require a light-cone Hamiltonian. We start from the + component of the QCD equations of motion in light-cone gauge

$$D_\mu F_\mu^+ = \partial_- F_a^{-+} + \partial^i F_a^{i+} - gf_{abc} A_b^i F_c^{i+} = j_a^+$$

(A1)

where

$$F_a^{-+} = \partial^- A_a^+ - \partial^+ A_a^- - gf_{abc} A_b^- A_c^+ = -\partial_- A_a^-$$

(A2)

$$F_a^{i+} = \partial^i A_a^+ - \partial^+ A_a^i - gf_{abc} A_b^i A_c^+ = -\partial_- A_a^i$$

and the fermion color density is given

$$j_a^+ = g \sum_q \bar{q} \gamma^+ \frac{\lambda_a}{2} q$$

(A3)

is the fermion color density. Written out in components the Field equations (A1) thus read

$$- \partial_- A_a^- - \partial_- \partial^i A_a^i - gf_{abc} A_b^i F_c^{i+} = j_a^+$$

(A4)

Integrating Eq. (A4) over $x^-$, while making use of the condition that the field strength tensor $F_a^{-+}$ at $x^- = \pm \infty$ vanishes, thus yields

$$\partial^i A_a^i(\infty^-, \vec{x}_\perp) - \partial^i A_a^i(-\infty^-, \vec{x}_\perp) = -\int_{-\infty}^{\infty} dx^- [j_a^+ - gf_{abc} A_b^i \partial_- A_c^i]$$

(A5)

$$= -\int_{-\infty}^{\infty} dx^- J_a(x^-, \vec{x}_\perp) = -\rho_a(\vec{x}_\perp) \equiv \rho(\vec{x}_\perp).$$

**APPENDIX B: HIGHER ORDER COLOR CORRELATIONS**

We want to solve the finiteness condition

$$\partial^j a_a^j(\vec{x}_\perp) = -\rho_a(\vec{x}_\perp)$$

(B1)

subject to the constraint that $a_a^j(\vec{x}_\perp)$ is pure gauge, i.e.

$$a^j(\vec{x}_\perp) = -\frac{i}{g} U^\dagger \partial^j U.$$  

(B2)

In order to satisfy the second condition, one can make the ansatz

$$U(\vec{x}_\perp) = \exp \left( -i g \phi_a(\vec{x}_\perp) \frac{\lambda_a}{2} \right).$$  

(B3)
Upon inserting this ansatz into Eq. (A5) one finds

$$\alpha_i(x_\perp) = -\partial^i \phi(x_\perp) - \frac{ig}{2} [\phi(x_\perp), \partial^i \phi(x_\perp)] + \frac{g^2}{12} [\phi(x_\perp), [\phi(x_\perp), \partial^i \phi(x_\perp)]] + \ldots$$

(B4)

i.e. in component notation

$$\alpha^i_a(x_\perp) = -\partial^i \phi_a(x_\perp) + \frac{g}{2} f_{abc} \phi_b(x_\perp) \partial^i \phi_c(x_\perp) - \frac{g^2}{12} f_{abc} \phi_b(x_\perp) f_{cde} \phi_d(x_\perp) \partial^i \phi_e(x_\perp) + \ldots$$

(B5)

and therefore

$$\partial^i \alpha^i_a(x_\perp) = -\partial^i \partial^j \phi_a(x_\perp) + \frac{g}{2} f_{abc} \phi_b(x_\perp) \partial^i \partial^j \phi_c(x_\perp) - \frac{g^2}{12} f_{abc} f_{cde} \left[ \phi_b(x_\perp) \phi_d(x_\perp) \partial^i \partial^j \phi_e(x_\perp) + \partial^i \phi_b(x_\perp) \phi_d(x_\perp) \partial^j \phi_e(x_\perp) \right] + \ldots$$

$$\partial^i \alpha^i_a(x_\perp) = -\rho_a(x_\perp)$$

(B6)

One may attempt to solve Eq. (B6) by making a formal expansion in powers of \(\rho_a(x_\perp)\)

$$\phi_a(x_\perp) = \phi^{(0)}_a(x_\perp) + \phi^{(1)}_a(x_\perp) + \ldots,$$

(B7)

yielding

$$\partial^i \partial^j \phi^{(0)}_a(x_\perp) = \rho_a(x_\perp)$$

$$\partial^i \partial^j \phi^{(1)}_a(x_\perp) = \frac{g}{2} f_{abc} \phi^{(0)}_b(x_\perp) \partial^i \partial^j \phi^{(0)}_c(x_\perp) = \frac{g}{2} f_{abc} \phi^{(0)}_b(x_\perp) \phi^{(0)}_d(x_\perp) \partial^i \partial^j \phi^{(0)}_e(x_\perp) + \ldots$$

(B8)

e.t.c.

Eq. (35) is then obtained by keeping only the lowest order term in Eqs. (B5) and (B8).

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