Evaluation of Water Resource Carrying Capacity Based on Fuzzy Matter-element Model

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Abstract. Water resources carrying capacity (WRCC) is a significant foundation for scientific management of water resources that connected with development of water resources, population, society, and economy. With the rapid development of economy and society, the shortage of water quantity is becoming one of the most profound global issues. In this paper, to evaluate WRCC scientifically and reasonably, index system of WRCC was established and applied for assessing the WRCC of four cities in China by fuzzy matter element (FME) model comprehensively. The indices were weighted by entropy method. The WRCC of four cities of Beijing, Tianjin, Shanghai, and Chongqing are 2.620, 2.503, 2.590, and 2.457, respectively. The results indicated that the WRCC of four cities decreased in the order of Chongqing > Tianjin > Shanghai > Beijing. The method proposed can be applied to other evaluation issues, and the results can help managers realize the importance of water resources in developing the economy and society.

Keywords: Water resources carrying capacity; fuzzy matter element model; entropy method.

1. Introduction
With rapid population growth and industrialization process, resources become the most profound issues globally, especially for the water resources shortage (Zhou et al., 2017). In order to study the effect extend of water shortage on local/regional economic development, the framework of water resource carrying capacity (WRCC) was applied worldwide (Feng et al., 2008; Liu et al., 2017). The research of WRCC has gradually improved from support scale of water resources to focusing on the harmonious development of water resource, society, economy, and environment. By studying the WRCC of cities, the scale of the water resource sustain and restrain the society and economy can be determined, and the results can benefit the sustainable regulation of the development plans.

Since WRCC is a complicate issue related to water resource, economy, population, technology, planning, and environment, etc. with interactions as well as feedback mechanisms exist among them, it is inherently complex (Zhou et al., 2017). The primary research method of WRCC is indicator evaluation method based on sufficient consideration social development and water resources (Wang, 2017). Various approaches were established to evaluate WRCC comprehensively, such as hierarchical multi-criteria method (Xia, 2013), fuzzy comprehensive evaluation method (Wang et al., 2010), project pursuit method (Wang and al., 2003), press-state-response framework (Wang et al., 2013a), cloud model (Cheng, 2018), and BP neural network (Yang and Tong, 2013). Moreover, in Guanzhong region, Shaanxi, Multi-objective large system decomposition and coordination model was applied to study WRCC (Jiang et al., 2001). Ecological footprint (Li et al., 2018), and ecological pressure index (EPI) was introduced to compare the balance between water supply and consumption in Liao-Hun and Taizi Watersheds (Wang et al., 2013b). The SD method was successfully applied to study WRCC,
which considered the interactions among subsystems of society, economy, and environment (Wang, 2017; Yang et al., 2015; Zhang et al., 2014). The fuzzy matter element (FME) model was widely applied in pattern recognition, scientific decision, and comprehensive evaluation with consideration of fuzziness of variables (Wang, 2019a; Wang, 2019b), which was seldom applied for evaluating WRCC. In this paper, the FME model was applied for evaluating the WRCC of four cities in China based on the proposed index system. The obtained results can order the four cities in view of WRCC, and provide water resources managers with scientific suggestions.

2. Methods

2.1. Study Area

In this paper, four cities in China were selected, which are Beijing, Tianjin, Shanghai, and Chongqing. The four cities play more important roles in national politics, economy, and culture. However, they are facing crisis of water shortage, which seriously restrict the economic and social development. As such, scientific and reasonable evaluation of WECC can help make sustainable water resource planning and social development policy.

2.2. Fuzzy Matter-element Model

① Weights determination

The weights were calculated based on “entropy” technology that originated from modern information theory. The entropy was applied in various uncertain fields. The entropy of the indicators can be determined by Eq. (1):

$$H_i = - \sum_{k=1}^{K} p_i^k \cdot \ln p_i^k$$  \hspace{1cm} (1)

Where the entropy of the $i$th indicator is indicated by $H_i$, $p_i^k$ refers to frequency for the $i$th indicator of the $k$th object (city), $p_i^k = y_i^k / \sum_{k=1}^{K} y_i^k$, where $y_i^k$ is the normalized average value of the $i$th indicator for the $k$th city, and when $p_i^k = 0$, let $\ln p_i^k = 0$. The normalization for cost indicators and efficiency indicators are obtained by Eqs. (2) and (3), respectively.

$$y_i^k = (\max(x_i^k) - x_i^k) / (\max(x_i^k) - \min(x_i^k))$$  \hspace{1cm} (2)

$$y_i^k = (x_i^k - \min(x_i^k)) / (\max(x_i^k) - \min(x_i^k))$$  \hspace{1cm} (3)

Then weights of the $i$th indicators $w_i$ based on entropy can be calculated by Eq. (4) as follows.

$$w_i = (1 - H_i) / \left( m - \sum_{i=1}^{m} H_i \right)$$  \hspace{1cm} (4)

where $m$ is the number of indicator.

② Fuzzy matter-element model

This model is composed of a triple ordered matrix of objects, characteristics, and fuzzy values, termed as $U = (C, G, \mu)$, and expressed by Eq. (5).

$$U_{mn} = \begin{bmatrix} G_1 & G_2 & L & G_n \\ C_1 & \mu_{11}^k & \mu_{12}^k & L & \mu_{1n}^k \\ C_2 & \mu_{21}^k & \mu_{22}^k & L & \mu_{2n}^k \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_m & \mu_{m1}^k & \mu_{m2}^k & L & \mu_{mn}^k \end{bmatrix}$$  \hspace{1cm} (5)

where $U_{mn}^k$ is fuzzy matter element matrix of the $k$th study object (city), $C_i$ is the $i$th indicator, $i =1, 2, \ldots, m$; $G_j$ is the $j$th grade, $j =1, 2, \ldots, n$; $\mu_{ij}^k$ is the fuzzy membership degree of the $i$th indicator to the $j$th grade, which is defined according to fuzzy membership functions and classifications of WRCC.
Since the normalized membership function is widely applied (Liu et al., 2012), normal distributions of membership functions were adopted, and expressed by Eq. (6) as follows:

$$\mu^j_k = \exp \left[ -\frac{(x^j_k - a^j_k)^2}{2b^j_k} \right]$$

where \( \mu^j_k \) is the membership function of the \( i \)th indicator to the \( j \)th classification criterion for the \( k \)th city, \( x^j_k \) is value of the \( i \)th indicator in the \( k \)th city, \( a^j_k \) and \( b^j_k \) are the constants with \( a^j_k > 0 \) and \( b^j_k > 0 \). The parameters \( a^j_k \) and \( b^j_k \) were defined and expressed by Eqs. (7) – (8), respectively, shown as follows.

$$a^j_k = \begin{cases} x^j_k, & j = 1 \\ \frac{x^j_k + x^j_u}{2}, & j = 2, 3 \\ x^j_u/x^j_k, & j = 4 \end{cases}$$

$$b^j_k = \begin{cases} x^j_u - x^j_k, & j = 1 \\ \frac{x^j_u - x^j_k}{2^{\frac{1}{2}}} & j = 2, 3 \\ \frac{x^j_u - x^j_k}{2^{\frac{1}{2}}} & j = 4 \end{cases}$$

where \( x^j_k \) and \( x^j_u \) refer to the lower and upper boundaries of the \( j \)th classification grade of the \( i \)th indicator, respectively. In Eq. (7), the parentheses are for the efficiency indicators.

where \( r^j_k \) is the fuzzy membership degree of the \( i \)th indicator to the \( j \)th grade for the \( k \)th city after normalization, and termed as \( R_{mn}^k \), shown in Eq. (9) as follows,

$$R_{mn}^k = \begin{bmatrix} G_1 & G_2 & \cdots & G_n \\ C_1 & r_{11} & \cdots & r_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ C_m & r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

where \( R_{mn}^k \) is the fuzzy matter element matrix for the \( k \)th city after normalization.

3 Fuzzy nearitude calculation

The fuzzy nearitude of the \( k \)th object to the \( j \)th WRCC \( \rho H^j_k \) is calculated by Hamming nearitude (\( \rho H \)) shown by Eq. (10).

$$\rho H^j_k = 1 - \frac{1}{n} \sum_{i=1}^{n} w_i |r^j_k - r_{1i}|$$

Where \( r^j_k \) is the element of fuzzy matter element matrix after normalization, and \( r_{1i} \) is the element of the ideal fuzzy matter element matrix after normalization.

4 Determine the WRCC grade of cities

The WRCC levels of cities can be determined according to the non-integral feature value generated by Eq. (11).

$$J^k = \frac{\sum_{j=1}^{J} \rho H_j}{\sum_{j=1}^{J} \rho H_j}$$

Where \( J^k \) is the WRCC feature value of the \( k \)th city. The lower \( J^k \) value indicates the better WRCC grade and vice versa. The WRCC grade of the \( k \)th city is defined according to classification in Table 1.

| \( J \) | \((1,1.5]\) | \((1.5, 2.5]\) | \((2.5, 3.5]\) | \((3.5, 4]\) |
|---|---|---|---|---|
| Grade | I | II | III | IV |
3. Results

The indicators related closely to WRCC were selected to establish index system, shown in Table 2. The evaluation levels of WRCC were based on the index systems for the study area. The entropy weights of indicators were determined by Eqs. (1) - (4). The WECC evaluations for the four cities were obtained by Eq. (5) – (13), shown in Table 3.

Table 2. Index system of the WRCC.

| Evaluation index | Equation (units) |
|------------------|------------------|
| Population density (C1) | Population / area (p/km²) |
| GDP per capita (C2) | GDP/total population (10⁴ USD/p) |
| Proportion of second industrial increase to GDP (C3) | Second industrial increase / GDP (%) |
| Water consumption per 104 USD GDP (C4) | Total water consumption / total GDP (m³/10⁴ USD) |
| Forest coverage rate (C5) | Total forest area / total area (%) |
| Irrigation coverage (C6) | Effective irrigation area / farmland area (%) |
| Water resources per capita (C7) | Total water resources / total population (m³/p) |
| Modulus of water supply (C8) | Total water supply / area (10⁴m³/km²) |
| Groundwater exploitation rate (C9) | Quantity of groundwater exploitation / total groundwater resource (%) |
| Proportion of lengths of rivers satisfying water quality standards (C10) | River length with water quality above class V / total river length (%) |
| Daily of water consumption per capita (C11) | Yearbook statistical data (L) |
| Underground water consumption rate (C12) | Groundwater consumption/total water consumption (%) |
| Ecological water consumption per capita (C13) | Ecological water consumption / total population (m³/p) |
| Reutilization rate of industrial water (C14) | Reutilized water consumption / total industrial water (%) |
| Fresh water per 10⁴ USD industrial production value (C15) | Industrial water consumption / increased industrial production value (m³/10⁴ USD) |
| Proportion of recycled water (C16) | Quantity of recycled water / total water consumption (%) |

Table 3. Water resources carrying capacity in four cities

| City      | $\rho_{HJ}$ | J   | Grade |
|-----------|-------------|-----|-------|
|           | I | II | III | IV |     |
| Beijing   | 0.058 | 0.065 | 0.059 | 0.081 | 2.620 | III |
| Tianjin   | 0.059 | 0.072 | 0.049 | 0.067 | 2.503 | III |
| Shanghai  | 0.059 | 0.049 | 0.059 | 0.070 | 2.590 | III |
| Chongqing | 0.065 | 0.069 | 0.076 | 0.055 | 2.457 | II |

4. Discussion

The WRCC of four cities decreased in the order of Chongqing > Tianjin > Shanghai > Beijing, which indicated that the WRCC is not consistent with the economic development. To maintain the sustainable economic development, water quantity shortage should be solved effectively, especially for city with lower WRCC. In addition, the WRCC of cities can be definitely compared by means of fuzzy matter element method. Furthermore, in other fields concerning assessment issues, this method can also be of significant value.

5. Conclusions

Base on the index system proposed consisted of 16 indicators consisted of society, economy, water resources, and water environment, the fuzzy matter element model was proposed to evaluate the water resources carrying capacity of four cities in China, which are Beijing, Tianjin, Shanghai, and Chongqing. The weights of 16 indicators were defined by entropy method. The WRCC of Beijing, Tianjin, Shanghai, and Chongqing were calculated as 2.620, 2.503, 2.590, and 2.457, respectively. The results indicated that the WRCC of four cities decreased in the order of Chongqing > Tianjin > Shanghai > Beijing. With the consideration of water resources, the order of the four cities is different from the order of economic scale and social development, which was primarily considered in the past
decades. As such, the city with less WRCC should take measures to increase efficient indicators (such as water resources per capita) and decrease the cost indicators (such as fresh water per $10^4$ USD industrial production value) such as related to water resources. The results can help managers make sustainable planning with consideration of both social development and water resources protection.

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