Self-consistent model of fermions

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Abstract

A composite model of the fundamental fermions based on colour preons is discussed. It is found that, if endowed with the pairwise (repulsive/attractive) chromoelectric fields, preons would cohere in a series of structures, resembling by their properties the three generations of the fundamental fermions. The model is self-consistent in the sense that it makes no use of free or experimental input parameters.

1 Introduction

The Standard Model of particle physics does not explain the hierarchical pattern observed in the masses of quarks and leptons because it uses these masses as its input parameters and it is not aimed at answering the question as to why the masses are distributed in the particular way they are. At a first glance they seem to be random, except for the fact that the masses generally increase with the generation number.

Numerous attempts to solve this puzzle have been made over the last twenty years, but the problem is still there. The observed pattern of the particle masses insistently points to structures beyond the quark scale. A number of models for these hypothetical structures have been proposed, focusing on the ideas of unification [1], technicolor [2], supersymmetric unification [3], strings and branes [4]. The latter approach gives some encouraging results, such as distributions resembling the observed families of particles, but no more.

There exist also models where the fundamental particles are composite entities [5], but these models are not very popular, being far from reality and facing problems with resolving the mass paradox and meeting the ‘t Hooft anomaly matching constraints. Finally, there are models discussing the possibility of randomness of these masses [6], which could easily be falsified by constructing a counter-example where the fermion masses would be functionally dependent. For example, G.R.Filewood proposed a model [7], regarding particles as almost crystalline symmetric structures, predicting some fermion masses with based on the geometrical approach to the structures a fairly good accuracy. This shows that at least some of the particle masses have a common origin. Here we shall discuss yet another composite model based on sub-quark primitive particles usually called preons. In author’s view, this model is closer to reality – at least it correctly reproduces the masses of all three generations of the fundamental fermions.

2 Simple structures

Let us suppose that the preon $\Pi$ (the basic building block of the composite fermions) could be regarded as a source of a spherically-symmetric pairwise field $F$ with the colour (tripolar) symmetry, $\Pi, \bar{\Pi} \in S_3$. Except for its tripolarity, the field $F$ is analogous to the Lennard-Jones fields used in molecular physics for modelling long-range attractive ($\phi_s$) and short-range repulsive ($\phi_e$) forces. A trade-off between these forces leads to equilibrium configurations of basic particles. Assuming that infinite energies are not accessible in nature we can hypothesise that the energy of both $\phi_s$ and $\phi_e$, after reaching the maximum, decays to zero at the origin. The simplest form for such a pairwise field would be:

$$F = \phi_s + \phi_e,$$

$$\phi_s = s \exp (-\rho^{-1}), \quad \phi_e = -\phi'_s (\rho),$$

(1)
where the signature \( s = \pm 1 \) indicates the sense of the interaction (attraction or repulsion); the derivative of \( \phi \) is taken with respect to the radial coordinate \( \rho \). Far from the source, the second component of the field \( F \) mimics the Coulomb gauge, whereas the first component extends to infinity being almost constant (similarly to the strong field). Let us formally represent the preon by using a set of auxiliary \( 3 \times 3 \) singular matrices \( \Pi \) with the following elements:

\[
\pm \pi_{jk}^{\pm} = \pm \delta_{ij}^{\pm}(-1)^{jk},
\]

(2)

where \( \delta_{ij} \) is the Kronecker delta-function; the \( \pm \) signs correspond to the sign of the charge; and the index \( i \) stands for the colour \( (i = 1, 2, 3 \) or red, green and blue). The diverging components of the field can be represented by reciprocal elements: \( \tilde{\pi}_{jk}^{\pm} = \pi_{jk}^{\pm} \). Then, we can define the preon’s (unit) charges and masses by summation of these matrix elements:

\[
q_{\Pi} = u^{\dagger} \Pi u, \quad \tilde{q}_{\Pi} = u^{\dagger} \tilde{\Pi} u
\]

\[
m_{\Pi} = |u^{\dagger} \Pi u|, \quad \tilde{m}_{\Pi} = |u^{\dagger} \tilde{\Pi} u|
\]

(3)

\((u\) is the diagonal of a unit matrix; \( \tilde{q}_{\Pi} \) and \( \tilde{m}_{\Pi} \) diverge). By assuming that the field \( F(\rho) \) does not act instantaneously at a distance we can define the mass of a system containing, say, \( N \) preons, as proportional to the number of these particles, wherever their field flow rates are not cancelled. For this purpose, we shall regard the total field flow rate, \( v_N \), of such a system as a superposition of the individual volume flow rates of its \( N \) components. Then, the net mass of the system can be calculated (to a first order of approximation) as the number of particles, \( N \), times the normalised to unity (Lorentz-additive) field flow rate \( v_N \):

\[
m_N = |N v_N|.
\]

(4)

Here \( v_N \) is computed recursively from:

\[
v_i = \frac{q_i + v_{i-1}}{1 + |q_i v_{i-1}|},
\]

(5)

with \( i = 2, \ldots, N \) and putting \( v_1 = q_1 \). The normalisation condition \( (5) \) expresses the common fact that the superposition flow rate of, say, two antiparallel flows \( (\uparrow \downarrow) \) with equal rate magnitudes \( |v_\uparrow| = |v_\downarrow| = v \) vanishes \( (v_\uparrow^\uparrow = 0) \), whereas, in the case of parallel flows \( (\uparrow \uparrow) \) it cannot exceed the magnitudes of the individual flow rates \( (v_\uparrow \leq v) \). Then, when two unlike-charged preons combine (say, red and antigreen), the magnitudes of their oppositely directed flow rates cancel each other (resulting in a neutral system). The corresponding acceleration also vanishes, which is implicit in \( (4) \). This formula implies the complete cancellation of masses in the system with vanishing electric fields (converted into the binding energy of the system), but this is only an approximation because in our case the preons are separated from each other by distance of equilibrium, whereas the complete cancellation of flows is possible only when the flow source centres coincide. Making use of the known pattern of attraction and repulsion between colour charges (two like-charged but unlike-coloured particles are attracted, otherwise they repel) we can write the signature \( s_{ik} \) of the chromoelectric interaction between two preons with colours \( i \) and \( k \) as

\[
s_{ik} = -u^{\dagger} \Pi_{ik} u.
\]

(6)

This pattern implies that the preons of different colours and charge polarities will cohere in structures, the simplest of which will be the charged and neutral colour-doublets (dipoles)

\[
\phi_{ik}^0 = \Pi^i + \Pi^k, \quad g_{ik}^0 = \Pi^i + \Pi^k, \quad i, k = 1, 2, 3.
\]

According to \( (3) \),

\[
q(g_{ik}) = \pm 2, \quad m(g_{ik}) = 2, \quad \tilde{m}(g_{ik}) = \infty,
\]

and

\[
q(g_{ik}) = 0, \quad m(g_{ik}) = 0, \quad \tilde{m}(g_{ik}) = \infty.
\]

If an additional charged preon is added to the neutral doublet, the mass and the charge of the system are restored:

\[
q(g_{ik}) = \pm 1, \quad m(g_{ik}) = 1,
\]

(7)
\[ \tilde{m}(q_{ik}) = \infty. \] (8)

The charged dipoles \( g (2\Pi \text{ and } 2\Pi') \) cannot be free because of their diverging strong fields. Any distant preon of the same charge but with a complementary colour will be attracted to the dipole, thus, forming a (charged) tripole denoted here as \( y \) (and \( \overline{y} \) for the opposite polarity):

\[ y = \sum_{i=1}^{3} \Pi^i \quad \text{or} \quad \overline{y} = \sum_{i=1}^{3} \Pi^i. \]

The tripoles (\( y \)-particle) is colourless at infinity but colour-polarised nearby, which means that tripoles can combine strings (pole-to-pole to each other) due to their residual chromaticism. In order to formalise representation of these structures let us introduce the following matrices:

\[
\begin{align*}
\alpha_0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \overline{\alpha}_0 &= -\frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \\
\alpha_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, & \overline{\alpha}_1 &= -\frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \\
\alpha_2 &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & \overline{\alpha}_2 &= -\frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.
\end{align*}
\]

Pairs of like-charged tripoles \( y \) would combine in short strings with their components 180°-rotated with respect to each other. The corresponding charged structure \( \delta^\pm \) can be written as

\[ \delta^\pm = \alpha_i y + \overline{\alpha}_i \overline{y}. \]

with \( q_\delta = \pm 6, \ m_\delta = \tilde{m}_\delta = 6 \). The states corresponding to different \( i \) are equivalent. The pairs of unlike-charged \( y \)-particles would also combine (rotated by \( \pm 120^\circ \) with respect to each other):

\[
\overline{\gamma}^0 = \alpha_i y + \alpha_k \overline{y} \quad \text{(9)}
\]

with \( q_\gamma = 0, \ m_\gamma = \tilde{m}_\gamma = 0, \ i \neq k, \ \gamma^0 \equiv 3(q \overline{y}). \)

The \( Z_3 \)-symmetry of the tripoles \( y \) implies that a string of three like-charged tripoles (triplet) would close in a loop \( 3y \) (or \( 3\overline{y} \)) denoted here as \( e \) because, as we shall see, this structure by its properties can be identified with the electron. The triplet \( e \) is charged, with its charge \( q_e = \pm 9 \) and mass \( m_e = 9 \) (expressed in units of preon’s charge and mass). The tripoles in this structure can be directed with their vertices towards or away from the centre of the loop. However, these configurations correspond to two different phases of the same structure, since the tripoles here have a rotational degree of freedom (around their common ring-closed axis). At the same time, the tripoles will orbit the centre of the structure moving along the ring-closed axis. The resulting currents have helical shapes with two possible helicity signs (clockwise or anticlockwise). These different helicities can be identified with two spins of the structure \( (e_\uparrow \text{ and } e_\downarrow) \). The pairs of unlike-charged tripoles can form longer strings:

\[
\nu_{e\uparrow} = \begin{pmatrix} a_0 & a_1 & \overline{a}_1 & \overline{a}_2 \\ a_2 & a_0 & \overline{a}_0 & \overline{a}_1 \\ a_1 & a_2 & \overline{a}_2 & \overline{a}_0 \end{pmatrix} \begin{pmatrix} y \\ \overline{y} \\ \overline{y} \\ y \end{pmatrix},
\]

or

\[
\nu_{e\downarrow} = \begin{pmatrix} a_0 & a_2 & \overline{a}_2 & \overline{a}_1 \\ a_1 & a_0 & \overline{a}_0 & \overline{a}_2 \\ a_2 & a_1 & \overline{a}_1 & \overline{a}_0 \end{pmatrix} \begin{pmatrix} y \\ \overline{y} \\ \overline{y} \\ y \end{pmatrix}.
\]

with the pattern of colour charges repeating after each six consecutive \( y \overline{y} \)-pairs, which allows the closure of such a string in a loop (denoted here as \( \nu_e = 6y \overline{y} \)). The structure \( \nu_e \) (formed of 36 preons) is electrically neutral and has a vanishing mass, according to (4), unless combined with a charged particle, say \( y \) or \( 3y \), which would restore the entire mass of the system.
3 Combining $y$, $e$, and $\nu_e$

Since the strong fields of $3g$ ($e$) and $6g\bar{y}$ ($\nu_e$) are closed, these particles can be found in free states. The particles $y$, $e$, and $\nu_e$ can combine with each other because of their residual chromaticism. The structure $y_1 = y + \nu_e$ will have a mass of 39 preon units ($m_{y_1} = n_y + m_y = 36 + 9$) and be charged, with its charge $q_{y_1} = \pm 3$, corresponding to the charge of the $y$-particle. The charge of the configuration $e + \nu_e$ will correspond to the charge of the triplet $e$ ($q_{\nu_e} = \pm 9$). Its mass will be 45 units as expressed in preon’s units of mass ($n_e + m_e = 36 + 9$).

Like-charged particles $y_1$ of the same helicity signs would further combine (through an intermediate $\nu_e$-particle with the opposite helicity) forming three-component strings. The string $y_1 e y_1$ can be identified with the $u$-quark. Its charge will correspond to the charge of two $y$-particles ($q_u = \pm 6$) and its mass will roughly be the sum of the masses of its two $y_1$-components: $m_u = 2 \times 39 = 78$ (preon mass units). The positively charged $u$-quark (78 preon mass units) would be able to combine with the negatively charged particle $e - \nu_e$ (45 preon mass units) mass, forming the $d$-quark with a mass of 123 preon mass units, $m_d = m_u + m_{e\nu_e} = 78 + 45 = 123$, and with the charge derived from the charges of its constituents $q_d = q_u + q_e = +6 - 9 = -3$ units.

The interactions between the particles $e$ and $\nu_e$ is be helicity-dependent. The configuration of colour charges of the structure $\nu_e\bar{y}$ does not match that of the structure $e\bar{y}$, which would lead to the mutual repulsion of these particles. Only the structures $e\bar{y} + \nu_e\bar{y}$ or $e\bar{y} + \nu_e\bar{y}$ can be formed since the combined potential of these structures implies attraction between their components. By contrast, if two structures of the same kind combine (e.g., $e$ with $e$ or $\nu_e$ with $\nu_e$), their helicity signs must be opposite in order to create an attractive force between the components of the pair. This coheres with and probably explains the Pauli exclusion principle, which suggests identifying the helicities of the structures in question with their spins.

Normalised to the number of its constituents (divided by nine) the charge of the $3g$-particle, gives us the conventional unit charge of the electron. Then, the charges of the $y$ and $\delta$-particles (fractions of the nine-unit electron charge) would correspond to the quark fractional charges.

It is not unnatural to suppose that the particles of the second and third generations of the fundamental fermions are formed of simpler structures belonging to the first generation. For example, the muon-neutrino can be a bound state of the positively and negatively charged particles $y_1$ and $\bar{y}_1$:

$$\nu_{\mu} = y\nu_{\epsilon\bar{y}}\nu_{\epsilon\bar{y}}\bar{y} = y_1\nu_{\epsilon\bar{y}}\bar{y}_1,$$

whereas the muon’s structure can be written as

$$\mu = (\nu_{\epsilon\bar{y}}\bar{y}_1)(\nu_{\epsilon\bar{y}}\nu_{\epsilon\bar{y}}y) = \bar{y}_1y\nu_{\epsilon\bar{y}}.$$  

(11)

Similarly, the structures of the other higher-generation fermions can be found. It is difficult to regard these structures as rigid bodies: they are rather oscillating clusters with multiple resonances, whose oscillatory energies are likely to contribute to the masses of these systems. In (11) we have enclosed the supposedly clustered components in parenthesis. Obtaining the masses of these systems is not a straightforward task, but one can show that, in principle, they are computable by using a simple empirical formula, which relates the oscillatory energies of the components to the sum $M$ of their masses $m_k$:

$$M = \sum_k m_k,$$

(12)

multiplied by the sum $\tilde{M}$ of their reciprocal masses $1/\tilde{m}_k$:

$$\frac{1}{\tilde{M}} = \sum_k \frac{1}{\tilde{m}_k}.$$  

Then, by setting for simplicity the unit-conversion coefficient to unity, we can compute the masses of the combined structures as

$$M_{\text{total}} = M\tilde{M}$$

We shall abbreviate this summation rule by using the overlined notation:
\[
M_{\text{total}} = m_1 + m_2 + \cdots + m_N = \frac{m_1 + m_2 + \cdots + m_N}{\frac{1}{m_1} + \frac{1}{m_2} + \cdots + \frac{1}{m_N}}. \quad (13)
\]

As an example, let us compute the muon’s mass. The masses of its components, according to its structure, are: \(m_1 = \bar{m}_1 = 48, m_2 = \bar{m}_2 = 39\) (in preon mass units). Then, the mass of the muon will be

\[
m_\mu = \frac{m_1 + m_2}{\frac{1}{m_1} + \frac{1}{m_2}} = 1872 \text{ (preon mass units).}
\]

For the \(\tau\)-lepton, according to its structure, \(m_1 = \bar{m}_1 = 156, m_2 = \bar{m}_2 = 201\), so that

\[
m_\tau = \frac{m_1 + m_2}{\frac{1}{m_1} + \frac{1}{m_2}} = 31356 \text{ (preon mass units).}
\]

For the proton, positively charged particle consisting of two \(u\)- and one \(d\) quarks submerged in a cloud of gluons \(g_{ij}\), the masses of its components are \(m_u = \bar{m}_u = 78, m_d = \bar{m}_d = 123, m_g = 2m_u + m_d = 279, \bar{m}_g = \infty\). The resulting proton mass is

\[
m_p = m_u + m_u + m_d + m_g = 16523 \text{ (preon mass units).} \quad (14)
\]

We can convert \(m_\mu\) and other particle masses from preon mass units into proton mass units, \(m_p\), by dividing these masses by the quantity \((14)\). The computed masses of the composite fermions [converted into the \(m_p\)-units with the use of \((13)\)] are presented in Table 1, where the experimental fermion masses \(\bar{m}\) are listed in the last column for comparison (expressed also in units of the proton mass). This Table illustrates the family-to-family similarities in the particle structures. For instance, in each family, the \(d\)-like quark is a combination of the \(u\)-like quark, with a charged lepton belonging to the lighter family. Charged leptons appear as combinations of the neutrino from the same family with the neutrinos and charged leptons from the lighter family. It is conceivable that ring-closed strings, similar to that of the electron neutrino, may appear on higher structural levels of the hierarchy, which could be regarded as “heavy neutrinos”, \(\nu_h = 6y_1\nu_1\). They can further form “ultra-heavy” neutrinos \(\nu_{uh} = 3(\nu_1\nu_2u)\bar{\nu}_\mu\), and so on. With these neutral structures the components \(y_2\) and \(y_3\) of the \(c\)- and \(t\)-quarks can be written as: \(y_2 = \nu_\mu \bar{\nu}_\mu e^-\) and \(y_3 = \nu_\mu \bar{\nu}_\mu \nu_\mu\).

The experimental masses of the charged leptons are known with high accuracy. They are given in the last column of Table 1 for comparison, so that we can estimate the accuracy of our model to be of about 0.5% (only the experimental masses of the charged leptons are used for the comparison). A higher accuracy could be achieved by taking into account some extra factors contributing to the masses, such as binding energies and/or the (non-vanishing) neutrino masses.

| Particle and its structure | Number of preons in each component |Computed mass \(M_{\text{total}}\) (preon units) | Mass converted into \(m_p\) \((\text{Unit mass})\) | Experimental mass \(\bar{m}\) \((\text{Unit mass})\) |
|---------------------------|-----------------------------------|------------------------------------------|---------------------------------|---------------------------------|
| First generation          |                                   |                                         |                                 |                                 |
| \(e^-\)                   | \(3y\)                            | 9                                       | 0.0005447                       | 0.0005446                       |
| \(\nu_e\)                 | \(6y1\)                           | 36                                      | 0                               | -                               |
| \(u\)                     | \(y_1 + y_1\)                     | 78                                      | 0.00472                         | 0.001 to 0.005                  |
| \(d\)                     | \(u + \nu_e e^-\)                 | 123                                     | 123                             | 0.007443                        |
| Second generation         |                                   |                                         |                                 |                                 |
| \(\mu^-\)                 | \(\nu_e e^-\) \nu_\mu\)          | \(48 + 39\)                             | 1872.6778                       | 0.1133                          |
| \(\nu_\mu\)               | \(y_1 \nu_1 \nu_1\)              | 114                                     | 0                               | 0.1126                          |
| \(c\)                     | \(y_2 + y_2\)                     | \(163 + 163\)                          | 22725                          | 1.6477                          |
| \(s\)                     | \(c + e^-\)                      | \(163 + 163 + 9\)                      | 2751                           | 0.1665                          |
| Third generation          |                                   |                                         |                                 |                                 |
| \(\tau^-\)                | \(\nu_\mu \mu^- \nu_\mu\)        | \(156 + 201\)                          | 31356                          | 1.8977                          |
| \(\nu_\tau\)              | \(u_3 \overline{\nu}_3\)         | 192                                     | 0                               | 1.8939                          |
| \(t\)                     | \(y_3 + y_3\)                     | \(1767 + 1767\)                        | 3122289                        | 188.96                          |
| \(b\)                     | \(t + \mu^-\)                     | \(1767 + 1767 + 48 + 39\)              | 76061.5                        | 4.60                            | 4.2 to 4.7                       |
4 Conclusions

Our model explains the origin of the observed variety of elementary particles and reproduces the masses of three generations of the fundamental fermions without using input parameters. The accuracy of this model is estimated to be of about 0.5%.

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