Working space analysis of one 3-DOF aircraft motion simulator

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Abstract. In this paper, the working space of one 3-DOF aircraft motion simulator is studied. Two types of methods are used to generate searching space, i.e. generate randomly or generate uniformly. Then, the feasible points are selected under the same selection rules, and the performances of these two types of methods are compared. The results show that Monte Carlo simulation based on random method is suitable for understanding the working space of aircraft motion simulator. Whereas the uniform method based on equal step size is more suitable for deep study of the working space because it has high density and uniformly distributed feasible points.

1. Introduction
Aircraft motion simulator is commonly used in training pilots and cabin crews, or testing new facilities[1]. With the help of motion simulator, training time or cost can be significantly reduced. To carry the cabin and load, the drive system of motion simulator is generally designed as parallel mechanism[2]. This system are controlled via applying inverse kinematics on pose of moving platform[3]. Therefore, to simulate a real plane, and analyze position and posture accurately, the working space of aircraft simulator must be studied[4,5].

Monte Carlo method (MCM) is a common method for working space analysis of parallel mechanism[6]. In this method, searching space will be built via generating a large number of original pose points inside a region of pose space. Then the description of working space is obtained by selecting feasible points under certain selection rules.

In this paper the working space of one aircraft motion simulator was studied. The searching space was built in two ways: generate points in space randomly or generate points in space uniformly with equal step. Then two types of searching results were compared. These works make preparation for future error analysis and calibration.

2. Structure of motion simulator
The aircraft motion simulator system is made up of motion simulator and the cabin(Fig.1). In order to ensure the rigidity and stability of the system, the motion platform adopts a redundant parallel mechanism with 3 inputs and 5 branch-chains, which can provide three degrees of freedom of movement in vertical direction, pitch and roll. Due to the existence of redundant structures, the structure of the aircraft motion simulator is more complicated, and the difficulty of kinematics analysis will be increased as well.

The simplified diagram of the aircraft motion simulator is shown in Fig. 2. The aircraft motion simulator is installed on the base and has a motion platform at its end. The base and platform are connected by 5 branch-chains, of which 3 RPS branches are actuators and the other 2 are redundancy.
In order to reduce the inertia effect of components, the hydraulic rod is used as the main structure and driving element in the RPS branch chain of aircraft motion simulator. Each hydraulic rod is connected with the moving platform through a spherical hinge and the bottom base through a hinge. By controlling the hydraulic rods on the three RPS branches, different positions and postures of the platform can be obtained.

![Fig. 1 Structure of aircraft motion simulator](image1)

![Fig. 2 Simplification of flight motion simulator](image2)

3. Inverse position equation of motion simulator

As shown in Fig. 2, point \( A_i (i = 1 \sim 5) \) is the fixed point, which connect between hinge (R) and base. \( B_i (i = 1 \sim 5) \) is the center point of spherical hinge(S). To simplify analysis work, the fixed coordinate system \( O-XYZ \) is set at the center of hinge \( A_5 \), and the kinetic coordinate system \( O_p-X_pY_pZ_p \) is set at the center of spherical hinge \( B_5 \) consolidated with the moving platform. Direction of each axis is shown in Fig. 2.

Let \( (x_{A_i}, y_{A_i}, z_{A_i}) \) be the coordinate of \( A_i (i = 1 \sim 5) \) in fixed coordinate system \( O-XYZ \), and \( (x_{B_i'}, y_{B_i'}, z_{B_i'}) \) be the coordinate of \( B_i (i = 1 \sim 5) \) in kinetic coordinate system \( O_p-X_pY_pZ_p \). As for the pose of moving platform in fixed coordinate system \( O-XYZ \), it can be expressed as \( P = [x, y, z, \alpha, \beta, \gamma]^T \), which is also the pose of kinetic coordinate system \( O_p-X_pY_pZ_p \).

According to simulator structure, the following relationship can be obtained:

\[
\overrightarrow{O}_A i + \overrightarrow{A}_i \overrightarrow{B}_i = \overrightarrow{O}_O p + \overrightarrow{W}_O p B_i
\]

That is

\[
\overrightarrow{A}_i \overrightarrow{B}_i = \overrightarrow{O}_O p + \overrightarrow{W}_O p B_i - \overrightarrow{O}_A i
\]

where

\[
\overrightarrow{O}_O p = [x, y, z]^T
\]
\[ \overline{O_B B_i} = [x_B', y_B', z_B']^T \]  

(4)

Direction cosine matrix \( W \) of kinetic coordinate system \( O_P X_P Y_P Z_P \) is

\[
W = \begin{bmatrix}
W_{11} & W_{12} & W_{13} \\
W_{21} & W_{22} & W_{23} \\
W_{31} & W_{32} & W_{33}
\end{bmatrix}
\]

(5)

Let \( L_i \) be the length of vector \( \overline{A_i B_i} \), then the following equation be obtained

\[
\overline{A_i B_i} = \begin{bmatrix}
L_{xi} \\
L_{yi} \\
L_{zi}
\end{bmatrix} = \begin{bmatrix}
x + W_{11}x_{B_i}' + W_{12}y_{B_i}' + W_{13}z_{B_i}' - x_Ai \\
y + W_{21}x_{B_i}' + W_{22}y_{B_i}' + W_{23}z_{B_i}' - y_Ai \\
z + W_{31}x_{B_i}' + W_{32}y_{B_i}' + W_{33}z_{B_i}' - z_Ai
\end{bmatrix}, (i = 1 \sim 3)
\]

(6)

Then the length of each actuator at different pose can be calculated:

\[
L_i = \sqrt{L_{xi}^2 + L_{yi}^2 + L_{zi}^2}, (i = 1 \sim 3)
\]

(7)

4. Analysis of working space

4.1. Generation and selection of original points

Assuming there is no structural error, movement of \( O_P \) is limited in a vertical plane, because of the hinge \( A_5 \). Similarly, movement of \( B_4 \) is also limited in the same vertical plane, because of hinge \( A_4 \) and \( C_4 \).

Let \( L_5 \) be the length of vector \( \overline{A_5 B_5} \), and \( \theta \) be the angle between \( \overline{A_5 B_5} \) and \( \overline{OX} \). Then the pose of motion platform can be expressed as \( P = [0, L_5 \cos \theta, L_5 \sin \theta, \alpha, \beta, 0] \), and a 3D searching space can be created: \( \pi/2 \leq \theta \leq \pi, -\pi/2 \leq \alpha \leq \pi/2, -\pi/2 \leq \beta \leq \pi/2 \).

To study the working space of motion simulator, MATLAB was adopted, and three methods were used to create \( n^3 \) original points in searching space:

1. Randomly generated: use MATLAB function rand() directly, and \( n \) random values in each dimension were generated;
2. Randomly generated: also use MATLAB function rand(), but \( n^3 \) random points were generated directly;
3. Uniformly generated: convert each search range into arithmetic progression with \( n \) items.

To search working space from generated original points, the following principles were proposed:

- Length of each actuators: \( L_{\text{min}i} \leq L_i \leq L_{\text{max}i}, i = 1, 2, 3 \);
- Distance between \( A_4 \) and \( B_4 \): \( L_{AB4} \leq L_{AA4} + L_{C4B4} \);
- Height of each joint on motion platform: \( z_{Bi} > 0, i = 1, 2, 3, 4 \).

One original point was named as feasible point, if all requirements were met.

4.2. Comparison between different generation methods of original points

Firstly, 100 values were generated in each dimension, i.e. \( \theta, \alpha, \beta \). Then 100^3 original points can be obtained. By applying selection rules in 4.1.:

- 3244 feasible points were selected from method 1 generated points;
- 2462 feasible points were selected from method 2 generated points;
- 2455 feasible points were selected from method 3 generated points.

From Fig. 3 it is easy to find that the working space of motion platform in each phase is the best presented by method 2, although the quantity of feasible points in this method is not the largest. For method 1, some area in working space is not well covered. This could be a risk for unknown parallel mechanism, since some important points/areas may not be analyzed during simulation. And for method 3, the projection in each plane is not as density as that of method 2. The main reason is
overlapping in all directions. For example, there are as much as 57 feasible points with the same $\theta = 2.8903$.

![Graphs showing results of generation methods](image)

(a) Results of generation method 1   (b) Results of generation method 2   (c) Results of generation method 3

Fig. 3 Projection of working space in each coordinate plane

4.3. Increased amount of original points

Increasing the amount of original points seems to be a reasonable way to deal with the “un-covered” issue. Therefore, more original points($50^3, 100^3, 150^3, 200^3, 250^3, 300^3$) were generated in three ways, and feasible points were selected according to the mentioned rules. During simulation, method 2 was unable to generate $200^3$ random points or more at one-time, because of PC memory limitation. Let

$$\text{Ratio of feasible points} = \frac{\text{Number of feasible points}}{\text{The amount of original points}} \times 100\%$$

Simulation results show that the ratio of method 1(points in Fig. 4) are not as stable as that of method 2(solid line in Fig.4) and 3(dashed line in Fig. 4). For example, the ratio was as much as 0.34% in one simulation with $2.7*10^7$ random original points. In another simulation with the same amount of random points, this ratio was only 0.13%. This means that the quantity of feasible points in the later simulation is only about one third of the former one. Moreover, the “un-covered” issue still exists(Fig. 5(a)), even $8.3*10^4$ feasible points were selected from $300^3$ original points.

![Graph showing ratio of feasible points](image)

Fig. 4 Ratio of feasible points under different quantities of generated points
As for methods 2 and 3, more original points generally mean more feasible points. For example, there are 66998 feasible points when the amount original point is $2.7 \times 10^7$ in method 3. The density is more than 27 times of that with $1 \times 10^6$ original points. More details of working space can be expected to be discovered with the help of those increased feasible points (Fig. 5(c)).

5. Conclusion

Based on the results and discussions presented above, the conclusions are obtained as below:

1. Studying search space generated randomly is the most efficient way to get brief view of working space of aircraft motion simulator with limited original points. In this space, the original points are generated directly.

2. On the other hand, searching space generated uniformly is the best choice for deep study of working space of aircraft motion simulator, because it has high density and uniformly distributed feasible points.

These results will be the base of future error analysis and motion calibration.

Acknowledgments

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