A Numerical Investigation to Identify Dimensionless Parameters for Dual-Rotor Horizontal Axis Wind Turbines

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Abstract. A numerical study was carried out to identify non-dimensional parameters for dual-rotor horizontal axis wind turbines (DRWTs). Based on some important DRWT parameters such as the rotor speeds, rotor diameters and the distance between the rotors, three dimensionless parameters were derived from the Buckingham Pi theorem. Hypothetical DRWT models were created using geometrically-scaled National Renewable Energy Laboratory (NREL) Phase VI rotor geometry and operating conditions in order to confirm the validity of these parameters. The performance of each turbine was simulated using DR_HAWT, an in-house prediction tool for single and dual-rotor wind turbines created by the current authors. The variation in normalized output power as a function of the dimensionless parameters suggests that an improved performance of DRWTs can be obtained at lower diameter and gap ratios. The NREL Phase VI rotor equipped with a 5 m geometrically-scaled upwind rotor can generate about 88% of the combined power output of two equivalent single-rotors. In addition, the effect of having an auxiliary upwind rotor reduces the angle of attack along the inboard section of the downwind blade.

1. Introduction

With the increasing demand in renewable energy, research and development in wind energy systems have greatly increased to improve the power extraction from the wind. Greater emphases have been directed to increase the efficiency of single-rotor wind turbines (SRWTs) by increasing their swept area (i.e. the rotor diameter) since the power output of a wind turbine scales with the square of its rotor diameter. Other examples include wind farm layout optimization and the addition of vortex generators on turbine blades.

In more recent years, a growing interest in dual-rotor systems which had been extensively employed in aerospace and marine applications to increase the aerodynamic efficiency of the systems while eliminating the asymmetrical torque experienced by conventional single-rotors, have resurfaced. Although the idea of increasing the amount of rotors per tower is not well accepted within the wind industry, some industrial and academic researchers are exploring alternatives to the conventional SRWTs such as the recent project from Vestas, investigating a multi-rotor concept consisting of four 225 kW turbines with 29 m diameter rotor [1]. With typical wind turbine tower accounting for around 14%-17% of the total cost of the turbine
including operation and management, grid connection, and foundation costs [2, 3], dual-rotor systems have the potential to lower the overall cost by reducing the number of towers, albeit the need for stronger towers and more complicated gearboxes.

Dual-rotor wind turbines (DRWTs) are equipped with two co-axial rotors in a back-to-back configuration or with an upwind and a downwind rotors separated by the nacelle. Since the shape of the inboard sections of SRWT blades extract little energy from the wind, it is hypothesized that by adding a smaller rotor upwind of a SRWT, more energy can be harvested. During the study performed by Jung et al. [4] using a 30 kW counter-rotating DRWT consisting of an 11 m diameter main rotor with a 5.5 m auxiliary upwind rotor, an increase in power output of about 21% was achieved over a conventional 11 m diameter rotor turbine at a rated wind speed of 10.6 m/s. Shen et al. [5] investigated the performance of a counter-rotating DRWT equipped with two Nordtank 500 kW turbines using the Navier-Stokes solver EllipSys3D. The analysis showed that the turbine can produce 43.5% more annual energy than a similar SRWT. Habash et al. [6] carried out a wind tunnel study with a model DRWT system equipped with a pair of three-bladed, 23 cm diameter counter-rotating rotors and a distance between the rotors that varied from 7 cm to 54 cm, which resulted in up to 60% more energy than a SRWT system. More recently, Ozbay et al. [7] conducted a wind tunnel investigation of DRWTs in either co-rotating or counter-rotating configuration using a model DRWT equipped with two 24 cm diameter rotors. It was found that the counter-rotating DRWT model which harvested approximately 7.8% more energy than the co-rotating model, produced up to 60% more wind energy as compared with the SRWT model.

The motivation behind the current study was initiated during the collaboration with ZEC Wind Power [8], an industry partner in Ottawa which envisioned dual-rotor turbines as a promising alternative to diesel-fueled generators in remote areas or areas with limited access to the electric utility’s grid. Since the DRWT is a fairly new concept in the wind industry, non-dimensional properties which allow for comparison between different DRWTs, are not readily available in the literature as per the authors knowledge. For this reason, this study aims at evaluating potential parameters for the characterization of an optimal dual-rotor turbine design using DR_HAWT [9], an in-house vortex filament code implemented by the present authors. The study also includes the verification and investigation of dimensionless parameters to non-dimensionalize the performance for DRWTs.

2. Performance Prediction Method

The performance of wind turbines studied in this paper were computed using DR_HAWT [9], an in-house code implemented by the current authors for single and dual-rotor horizontal axis wind turbine. This gridless, free-vortex wake model which uses a vortex filament method, was adapted from Strickland et al. [10] and validated using the Sequence S of the NREL (National Renewable Energy Laboratory) Phase VI experiment [11] as well as the MEXICO (Model rotor EXperiments In CONtrolled conditions) experiment [12]. These validation cases are briefly described in Section 2.3. Although vortex filament method provides a good spatial and temporal representation of the wake, its accuracy is dependent on the azimuthal discretization and core regularization. A comprehensive calibration is therefore required to obtain accurate solutions.

The rotor blades modeled in DR_HAWT, are discretized into elements where the vortex strength is determined based on the Lifting Line Theory as a function of the local relative velocity. At any radial location along the blade, the local relative velocity can be evaluated as the sum of the oncoming wind velocity, the local blade rotational velocity, and the total velocity induced by the vortex filaments in the wake. Figure 1 illustrates a schematic of the wake formed behind a rotor blade. As the blade rotates, vortex filaments comprising of trailing and spanwise vortices are shed from the trailing edge of the blade. The trailing and spanwise vortices are generated by spanwise variations of the bound vorticity along the blade and by the temporal
2.1. Biot-Savart Law
The induced velocity $\vec{V}_P$ at a point $P$, located at a distance $h$ perpendicular to the filament vortex of length $r_0$ and of strength $\Gamma_v$, can be evaluated using the well-known Biot-Savart Law. For a straight filament vortex as illustrated in Figure 2, the Biot-Savart equation can be simplified to give Equation 1, where $\vec{r}_1$ and $\vec{r}_2$ represent the direction vectors AP and BP, respectively.

$$\vec{V}_P = \frac{\Gamma_v}{4\pi} \frac{(|\vec{r}_1| + |\vec{r}_2|) (\vec{r}_1 \times \vec{r}_2)}{|\vec{r}_1||\vec{r}_2| (|\vec{r}_1||\vec{r}_2| + \vec{r}_1 \cdot \vec{r}_2)}$$  (1)

As $\vec{r}_1 \cdot \vec{r}_2$ approaches zero, Equation 1 shows singular behaviour. To remove this singularity, a vortex core model that mimics the viscous behaviour of a real vortex, is introduced in the Biot-Savart Law. A linear cut-off method and smoothing method are the two methods commonly used in the literature (see Ref. [13]) to desingularise the Biot-Savart equation. In this study, the latter is employed where Equation 1 is multiplied by a viscous effect factor $K$ given by Equation 2 where $n = 2$ according to the Vatistas vortex model [14].

$$K = \frac{h^2}{(r_c^{2n} + h^{2n})^{1/n}}$$  (2)

The core growth model [15] employed in this study to account for viscous and turbulent diffusion, defines the vortex core radius $r_c$ as a function of the vortex core radius at zero wake age ($r_{co}$, usually a percentage of the local chord [16]), wake age $(t)$, kinematic viscosity $(\nu)$, eddy viscosity coefficient $(\delta = 1 + a_1 \Gamma_v/\nu$ where $a_1 = 2 \times 10^{-4}$), and $\alpha = 1.25643$, given by the following equation.

$$r_c(t) = \sqrt{r_{co}^2 + 4\alpha \delta \nu t}$$  (3)

2.2. Stall Delay Model
The effect of rotation affects the boundary layer on rotor blades at the onset of stall. These rotational effects are induced by a phenomenon referred to as rotational augmentation which delays the separation point to a higher angle of attack as compared to two-dimensional (2D) case. In the 1940’s, Himmelskamp [17] was the first to quantitatively analyze the three-dimensional variations as the blade rotates, respectively. Lagrangian markers are used to keep track of the shed vortices where any two successive markers form a vortex filament in the rotor wake. The Lagrangian markers are allowed to convect freely based on the second-order Adams-Bashford explicit integration formula.
(3D) effect on propeller blades and confirmed that rotation does delay the laminar separation point. Since then, various rotational effects or 'stall delay' models have been formulated by Snel et al. [18], Du & Selig [19], Chaviaropoulos & Hansen [20], Lindenburg [21], and Bak et al. [22], to name a few. These models correct the non-rotating lift \( (C_{L,2D}) \) and drag \( (C_{D,2D}) \) coefficients with an increment in lift \( (\Delta C_L) \) and a decrement in drag \( (\Delta C_D) \) to simulate rotational effect at each radial location along the blade [19]. Equations 4 and 5 describe the methodology used in the stall delay models

\[
C_{L,3D} = C_{L,2D} + f_L \Delta C_L
\]

\[
C_{D,3D} = C_{D,2D} + f_D \Delta C_D
\]

where the subscripts \( 2D \) and \( 3D \) refer to non-rotating and rotating characteristics, respectively. \( \Delta C_L \) and \( \Delta C_D \) are the difference between the \( C_{L,2D} \) and \( C_{D,2D} \) wind tunnel measurements and the \( 2D \) \( C_L \) and \( C_D \) if hypothetically the flow would remained attached at all angle of attacks [22]. In other words, \( \Delta C_L \) is the difference between \( C_{L,2D} \) and the inviscid \( C_L \) \( (C_{L,inv} = 2\pi(\alpha - \alpha_0)) \) where \( \alpha_0 \) is the zero-lift angle) and \( \Delta C_D \) is the difference between the \( C_{D,2D} \) and the drag coefficient at zero angle of attack, \( C_{D,0} \) [19–21, 23]. Sometimes, the extended linear lift \( (C_{L,lin}) \) is used in place of the \( C_{L,inv} \) [24]. \( f_L \) and \( f_D \) are functions used in the specific stall delay models. It should be noted that some models contain solely a correction for \( C_{L,2D} \) in which case, \( f_D \) is zero. The Snel et al. correction which was found to provide better agreement for both validation cases, is employed in this study unless otherwise stated, where \( f_L = 3 \left( \frac{c}{r} \right)^2 \) and the effect on the drag coefficient (i.e., \( \Delta C_D \)) is not accounted for.

2.3. Validation Experiments

After a thorough calibration, a 10° azimuth angle increment with 10% and 5% vortex core radii were employed for the NREL Phase VI and the MEXICO wind turbines, respectively. Additionally, a wake length of 4\( D \) was found to be sufficient to obtain converged solutions, with a maximum of 0.7% difference between the solution at 4\( D \) and 10\( D \). Figure 3 shows the measured and computed torque for the NREL Phase VI and MEXICO validation cases in the axial flow condition. Both cases showed good agreement with their respective torque measurements. The axial thrust (not shown here) displayed a similar comparison with the experimental results.

![Figure 3](image-url)
For the NREL case, 2D wind tunnel aerodynamic coefficients of the S809 airfoil provided in Ref. [11] were used with various 3D corrections to evaluate the performance of this two-bladed 10 m diameter rotor with 3° tip pitch angle; the Snel et al. correction resulted in the best agreement. Several simulations with wind velocities ranging from 5 m/s to 15 m/s at 72 rpm were computed with reference to the Sequence S. Similarly, the aerodynamic polars for the three different airfoil profiles (DU91-W2-250, RISOE-A1-21 and NACA 64-418) along the MEXICO blade were obtained from Ref. [26] and corrected using the Snel et al. 3D correction. This three-bladed 4.5 m diameter rotor has −2.3° pitch angle and 0.25 mm thick zig-zag tape placed at 5% of the chord on both pressure and suction side of the turbine blades to trigger the laminar to turbulent flow transition and to avoid laminar separation. The rotor performance with a rotational velocity of 424 rpm was simulated for wind velocities of 10 m/s, 15 m/s, and 24 m/s.

### 3. Methodology

#### 3.1. Dimensionless Parameters

The performance of dual-rotor wind turbines can be characterized as a function of the oncoming wind velocity ($V_o$), the rotor angular velocity ($\omega_1$ and $\omega_2$), the rotor diameter ($D_1$ and $D_2$), and the distance separating the upwind and downwind rotor ($L$). The fluid density and viscosity are other flow conditions considered. Figure 4 illustrates these characteristic parameters as defined in the vortex model utilized in this study. Although the effect of tower shadow on blade loading is significant, the tower and nacelle are not accounted for in this study due to the optimal placement of the tower still subjected to further study (i.e., having the tower located downwind both rotors or between the rotors). Furthermore, since the upwind rotor is considered as an auxiliary rotor, the analysis was performed with reference to the downwind rotor (main rotor). As a result of employing the Buckingham Pi Theorem, a set of dimensionless numbers (Equations 6 - 8) were derived to compare the performance of DRWTs. These dimensionless numbers include the ratio of the rotor diameters (diameter ratio), the ratio of the distance between the rotors and the diameter of the downwind rotor (gap ratio), and the combined tip-speed ratio (CTSR) of both rotors, respectively. Other pre-existing dimensionless numbers used were the power coefficient, Reynolds number and tip-speed ratio of the downwind rotor.

$$ \text{Diameter Ratio} = \frac{D_1}{D_2} \quad (6) $$

$$ \text{Gap Ratio} = \frac{L}{D_2} \quad (7) $$

$$ \text{CTSR} = \frac{\omega_1 R_2}{V_o} \quad (8) $$

where the subscripts 1 and 2 represent the upwind and downwind rotor, respectively. The downwind rotor radius, $R_2$ was used as the length scale for CTSR to be consistent with the tip-speed ratio ($\lambda = \omega R/V_o$) of a SRWT. Furthermore, unlike other dual-rotor studies found in the literature [4–7], where the power output produced by DRWTs are compared with the main rotor (usually the downwind rotor) in single-rotor configuration, the power output of a DRWT is compared with the power generated by individual SRWTs equipped with the equivalent rotors. In other words, the normalized power is given by Equation 9 as the ratio of the power output generated by a DRWT ($P_{\text{dual}}$) to the sum of the power output generated by the upwind and downwind rotor in single-rotor configurations ($P_{\text{singles}}$). This method is believed to give a better value to the potential that dual-rotor systems can provide.

$$ \text{Normalized Power} = \frac{P_{\text{dual}}}{P_{\text{singles}}} \quad (9) $$
3.2. Wind Turbine Models
Various hypothetical DRWT models were created based on the geometry and operating condition of the NREL Phase VI wind turbine provided in Ref. [11]. The non-dimensional chord and twist distributions of the reference rotor are shown in Figure 5 where the blade chord (c) and radial location (r) are non-dimensionalized with respect to the rotor tip radius (R). Each two-bladed rotor model was geometrically-scaled from the reference rotor following the assumption that the rotor tip-speed remains constant. For all test cases unless otherwise stated, the blade was modeled in DR_HAWT with 26 discretized section, as provided in Ref. [11] with the appropriate scaling factor. A 10° azimuth angle increment with 10% vortex core radius ($r_{co}$) was found to provide adequate solutions for the NREL Phase VI validation case, without greatly affecting the computational cost and were therefore used during the dual-rotor simulations. Similarly, the numerical solution is assumed to be converged when the wake traveled 4$D_2$ downstream of the downwind rotor. Since wind tunnel aerodynamic coefficients are not readily available for a wide range of Reynolds numbers, XFOIL [27] which employs a viscid/inviscid panel method, is used to compute the lift and drag coefficients. Although the latter method over-predicts lift and under-predicts drag, especially under post-stall conditions [28–30], its low computational runtime, allows new designs to be tested quickly.

4. Results and Discussion
4.1. Effects of Variation in Dimensionless Numbers
Three test cases are performed where the variation in gap ratio as a function of the combined tip-speed ratio (D01), the variation in diameter ratio as a function of the combined tip-speed ratio (D02), and the variation in the gap ratio as function of the diameter ratio (D03) are examined. Since counter-rotating rotors were more popular in the past and proved to be more efficient as compared to co-rotating rotors, all the simulations performed during this study employed...
counter-rotating configuration unless otherwise stated. The results of these three cases (D01, D02 and D03) are shown in Figures 6, 7 and 8, respectively.

The wind turbine model employed in Case D01 is equipped with a 5.0 m diameter upwind rotor and a 10.0 m diameter downwind rotor which results in a 0.5 diameter ratio. For the range of gap ratios (0.25 - 1.00) investigated, similar results can be observed in Figure 6, with more obvious differences at high CTSR. At a CTSR of 10.8, the highest power output generated was obtained at 0.5 gap ratio. As a result, a 0.5 gap ratio was kept constant in Case D02, to obtain more distinct differences while varying the diameter ratio as a function of CTSR. A 10 m diameter downwind rotor was used for all the wind turbine models studied in Case D02 with the upwind rotor varying from 2.5 m to 10.0 m diameter. It can be seen in Figure 7 that the normalized power output is inversely proportional to diameter ratio for all CTSR simulated with the exception of 10.0 and 12.0 CTSR. The results further showed that the downwind rotor for the 10.0 and 12.0 CTSR simulations, appeared to be stationary as a result of the effect of the upwind rotor’s wake and the fact that the oncoming wind speed was below the cut-in wind speed of the reference rotor.

To avoid simulating the downwind rotor outside its design operating range, Case D03 which investigates the effect of varying diameter ratio as a function of gap ratio, was performed at a constant CTSR of 3.8. The results shown in Figure 8, revealed that the dual-rotor system with diameter ratio of 0.5 produced the best overall performance. The results further show that the normalized power for the 1.00 diameter ratio model decreases with increasing gap ratio. It is expected to produce similar performance to SRWTs in a wind farm configuration as the distance between the rotors reaches 7D to 10D (where D is the diameter of the rotors) which are typical wind farm longitudinal spacing with the second turbine producing 35%-40% less power compared to the first turbine [31]. Based on the Case D03 results, as the gap ratio increases, the power deficit obtained ranged from 18% to 28% with the 1.00 diameter ratio turbine. This trend suggests that a gap ratio below 0.25 should provide an optimal distance, however, the distance between the rotors is also dependent on the space required for the turbine’s mechanical system. On the other hand, as the diameter ratio decreases, almost no change in power output can be observed as gap ratio is increased (see Figure 8). Although this case was simulated at a constant CTSR of 3.8, the results exhibit the potential of dual-rotor systems which could be exploited more efficiently after a comprehensive optimization process.
Figure 6. Power output comparison between four DRWT having diameter ratio of 0.50 at various gap ratios (GR = 0.25, 0.50, 0.75, and 1.00)

Figure 7. Power output comparison between four DRWT having various diameter ratios (DR = 0.25, 0.50, 0.75, and 1.00) at a gap ratio of 0.50

Figure 8. Power output comparison between four DRWT having various diameter ratios (DR = 0.25, 0.50, 0.75, and 1.00) at a CTSR of 3.8
4.2. Scaling Effect
In addition to these three cases, the scaling effect (D04) is investigated by comparing three geometrically-scaled dual-rotor models with diameter and gap ratios of 0.50 as shown in Figure 9. The models, which will be referred to based on the downwind diameter, are equipped with upwind rotors with diameter of 2.5 m, 5.0 m, and 10.0 m and downwind rotor diameters of 5.0 m, 10.0 m, and 20.0 m, respectively. Large discrepancy can be observed at high CTSR which can be justified by the fact that Reynolds number was not constant between the three models. For the purpose of this study, Case D04 was repeated with the Reynolds number artificially kept constant by varying the fluid viscosity between the models. The results confirmed that the differences in power output between the models are solely due to Reynolds number effect. In addition, the results show that the power output of the 5 m turbine is consistent over the range of CTSR simulated and is yet the most efficient of the three models with a 95% average normalized power generated.

![Figure 9. Power output comparison between three geometrically-scaled DRWT with diameter and gap ratios of 0.5](image)

The results from Case D04 further showed that a greater portion of the flow over the downwind rotor in DRWT configuration remained attached compared to the same rotor in SRWT configuration due to a difference in oncoming flow which each rotor sees. In other words, the local angle of attack (AOA) is reduced as a result of the deficit in the wind velocity due to the upwind rotor. Figure 10 illustrates the percentage difference in AOA along the blade span of the downwind rotor in DRWT configuration as compared to SRWT configuration. It can be observed that the reduction in AOA is more pronounced in the inboard section between 25% to 50% span. The effect is not as apparent near the root since this region consists mostly of cylindrical sections where vortex filaments of lower strength are shed. Although to a lesser extent, the wake of the upwind rotor still affects the downwind rotor outer 50% span due to wake expansion which gradually dissipates nearing the blade tip.

4.3. Summary and Future Work
In summary, this study confirmed that a smaller auxiliary rotor placed upstream of the main rotor allow for further kinetic energy extraction from the wind. The results also showed that better performance can be achieved by a dual-rotor system with low diameter and gap ratio. Although more in-depth optimization of the dimensionless parameters and blade geometry are required, this study suggests that having a counter-rotating 5.0 m diameter auxiliary rotor located 5.0 m upstream of the 10 m diameter NREL Phase VI rotor will generate in average
Figure 10. Angle of attack differences between the downwind rotor of the three DRWT models in single and dual-rotor configuration over the inboard section of the blade at CTSR = 10.8.

88% of the power output from two equivalent SRWTs. To better understand the effect of varying the dimensionless parameters described in this study on dual-rotor systems, other wind turbine blade geometry as well as three-bladed turbine case such as the MEXICO project should be investigated. Further analyses should be performed under yaw conditions and flow misalignment as well as incorporating the effect of the tower and nacelle. In addition to using hypothetical dual-rotor models, wind tunnel experiments of dual-rotor wind turbines in various geometric and flow conditions should be performed and compared with computational tools such as DR_HAWT.

5. Conclusion
In the present study, three non-dimensional parameters, namely diameter ratio, gap ratio, and combined tip-speed ratio, were derived for the characterization of dual-rotor horizontal axis wind turbines (DRWTs). With the assumption that tip-speed ratio is conserved, hypothetical DRWT models were created based on blade geometry and operating conditions of the NREL Phase VI wind turbine and used in validation test cases performed using DR_HAWT, an in-house vortex filament code implemented by the current authors. The simulation results revealed that a 0.5 diameter and gap ratios DRWT can generated to about 88% of the combined power of two equivalent turbines in single-rotor configuration. The trends further suggest that the performance of DRWTs is enhanced at low diameter and gap ratios, keeping in mind that the minimum gap ratio allowed by a specific turbine is dependent on the mechanical system used. Furthermore, the angle of attack along the inboard region of the downwind rotor is reduced with addition of an auxiliary upwind rotor.

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