Λ and ¯Λ spin interaction with meson fields generated by the baryon current in high energy nuclear collisions

L. P. Csernai¹, J. I. Kapusta² and T. Welle²

¹Institute of Physics and Technology, University of Bergen, Allegaten 55, 5007 Bergen, Norway
²School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455 USA

(Dated: August 1, 2018)

We propose a dynamical mechanism which provides an interaction between the spins of hyperons and anti-hyperons and the vorticity of the baryon current in non-central high energy nuclear collisions. The interaction is mediated by massive vector and scalar bosons which is well-known to describe the nuclear spin-orbit force. It follows from the Foldy-Wouthuysen transformation and leads to a strong-interaction Zeeman effect. The interaction may explain the puzzle of the difference in polarizations of Λ and ¯Λ hyperons as measured by the STAR collaboration at RHIC.

Experiments at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC) have provided a wealth of data on the hot and dense matter created in collisions between heavy ions [11]. Among these data are the coefficients of a Fourier expansion in the azimuthal angle for various physical observables. They provide strong evidence for collective expansion of the hot and dense matter and provide information on transport coefficients such as the shear viscosity [2]. Measurements of the polarizations have been made by the STAR collaboration from the lowest to the highest beam energies at RHIC [5–7], noting that RHIC produces matter with the polarization of Λ and ¯Λ hyperons as measured by the STAR collaboration at RHIC.

The standard picture of Λ and ¯Λ polarization in non-central heavy ion collisions assumes equipartition of energy [5–9]. This requires an interaction between the spin of the hyperon and the vorticity of the matter which heretofore has not been identified. There is also a puzzle presented by the experimental data: The Λ polarization is greater than the Λ polarization by about a factor of 4 at √s_{NN} = 7.7 GeV for Au+Au collisions. Both the difference between the two and their absolute values decrease with increasing beam energy until they are approximately equal at √s_{NN} = 200 GeV, whereas equipartition would suggest no difference. The interaction that we propose addresses the issue of the polarization difference.

It has been known since the early days of the nuclear shell model that a spin-orbit interaction is required to explain the single particle energy levels [10]. It was subsequently shown that attractive scalar and repulsive vector meson exchanges naturally lead to such spin-orbit interactions via a non-relativistic reduction of the Dirac equation [11]. Starting with the so-called Walecka model [12] much success has been achieved in describing nuclear structure, proton-nucleus scattering, and high density matter using various versions of these relativistic Lagrangians incorporating baryons and mesons [13] [14]. The fact that they include the strong interaction equivalent of the magnetic force and the spin-orbit force, including hyperons [15], suggest that this approach provides a natural explanation for the interaction between spin and vorticity and for the difference between Λ and Λ polarizations.

Suppose that the strong interaction among baryons is mediated by a scalar field σ and a vector field ωμ. The effective Lagrangian is

\[
\mathcal{L}_{\text{eff}} = \sum_j \bar{\psi_j} (i \gamma \cdot \partial - m_j + g_{\sigma j} \sigma - g_{\omega j} \cdot \omega) \psi_j + \frac{1}{2} (\partial_\mu \sigma \partial_\nu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{2} \omega_{\mu \nu} \omega_{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_{\mu \nu} \omega^{\mu \nu}. \tag{1}
\]

Here \( j \) represents one of the spin-1/2 baryons in the octet, the coupling constants are all positive, and the field strength tensor for the vector field is

\[
\omega_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu. \tag{2}
\]

In general there may be a potential \( V(\sigma) \) which has terms cubic and quartic in \( \sigma \) but its exact form will not be needed here.

One may perform a Foldy-Wouthuysen transformation \( [16] [18] \) (an expansion in powers of the inverse of the baryon mass, higher order corrections may be found in \( [19] \)) to obtain the nonrelativistic interaction between the \( \omega \) field and the spin operator \( \mathbf{S} \) of the Λ and ¯Λ. (We set \( \hbar = c = 1 \).) The interaction of the spin with the \( \omega \) vector meson is

\[
H_{\text{spin}}^\omega = -i \frac{g_\omega}{m_\Lambda} \beta \cdot \mathbf{B}_\omega - i \frac{g_\omega}{4m_\Lambda^2} \mathbf{S} \cdot \mathbf{E}_\omega - \frac{g_\omega}{2m_\Lambda^2} \mathbf{S} \cdot \mathbf{E}_\omega \times \mathbf{p} \tag{3}
\]

where \( \mathbf{E}_\omega \) and \( \mathbf{B}_\omega \) are the vector meson electric and magnetic fields corresponding to Eq. (2). \( \mathbf{S} \) is the momentum of the Λ or ¯Λ, and

\[
\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}
\]

is the usual Dirac 4x4 \( \beta \) matrix. When acting on the spinors of Λ and ¯Λ they result in opposite signs whereas the second and third terms have the same sign. In fact the second and third terms contribute to the usual nuclear spin-orbit energy. Only their sum is Hermitian, not
the individual terms. (According to the Bianchi identity we can replace $\nabla \times E_\omega$ with $-\partial B_\omega / \partial t$.) For a spherically symmetric static potential only the third term remains, which becomes

$$H^\sigma_{\text{spin-orbit}} = \frac{g_{\sigma N}}{2m^2} \frac{1}{r} \frac{\partial \omega_0}{\partial r} S \cdot L \quad (5)$$

where $L = r \times p$ is the orbital angular momentum.

For the scalar field the spin-orbit interaction is

$$H^\sigma_{\text{spin-orbit}} = \frac{g_{\sigma N}}{2m^2} S \cdot \nabla \sigma \times p \quad (6)$$

while there is no “magnetic” interaction. For central potentials this becomes

$$H^\sigma_{\text{spin-orbit}} = \frac{g_{\sigma N}}{2m^2} \frac{1}{r} \frac{\partial \sigma}{\partial r} S \cdot L \quad (7)$$

With the sign convention in Eq. (1) $\sigma > 0$ represents an attraction interaction and $\omega_0 > 0$ represents a repulsive interaction. They contribute with the same sign to the spin-orbit interaction with approximately equal strengths, whereas their contributions to the total binding energy approximately cancel.

In the mean field approximation the vector field is calculated as follows [12,14].

$$\partial^2 \psi + m^2 \psi + \sum_j g_{ij} J^\nu_j \quad (8)$$

Here $J^\nu_j$ is the baryon current $\langle \bar{\psi} \gamma^\nu \psi \rangle$ contributed by species $j$, such that protons and anti-protons contribute with opposite signs, for example. A survey of results in the literature leads to $g_{\sigma N} \approx 8.65$, $g_{\sigma \Lambda} = g_{\sigma \Sigma} \approx 0.55g_{\sigma N}$, and $g_{\sigma \Xi} \approx \frac{1}{2}g_{\sigma N}$ [14]. The mean scalar field is determined by

$$\partial^2 \sigma + m^2 \sigma + \frac{dV}{ds} = \sum_j g_{s_j n_{s_j}} \quad (9)$$

where $n_{s_j}$ is the scalar density $\langle \bar{\psi} \psi \rangle$ contributed by species $j$, such that protons and anti-protons contribute with the same sign, for example. The same survey leads to $g_{\sigma N} \approx 8.65$ and $g_{\sigma \Lambda} = g_{\sigma \Sigma} = g_{\sigma \Xi} \approx \frac{1}{2}g_{\sigma N}$ [14]. These interactions are anticipated to become relevant around the time of hadronization of the hot and dense matter created in the collisions which is generally accepted to be on the order of 3 to 5 fm/c or longer. The corresponding energy scale is much less than $m_\omega \approx 780$ MeV and $m_\sigma \approx 600$ MeV so that the derivatives in Eqs. (8,9) can be neglected.

For non-central potentials, $\nabla \times E_\omega = -\partial B_\omega / \partial t \neq 0$, $\nabla \sigma \neq (r/r^2) \partial \sigma / \partial r$, the spin-orbit terms represent an exchange of energy and angular momentum with the fields. For some systems in the condensed matter with angular momentum, the electromagnetic spin-orbit interaction has been used to derive the Gilbert term which describes Gilbert damping, the rate at which magnetization relaxes to equilibrium, in Refs. [20,21]. The damping of magnetization is commensurate with the emission of electromagnetic radiation. Assuming that the baryons in high energy nuclear collisions have a vortical flow motion, the scalar and vector meson interactions given above can provide a mechanism for hyperon polarization. In addition, note that the “magnetic” interaction is opposite in sign for hyperons and anti-hyperons due to the factor of $\beta$.

We can make a simple estimate of the magnitudes and signs of the effects. We work in the center-of-momentum frame of the colliding nuclei at mid-rapidity and neglect Lorentz $\gamma$ factors. The $x$-$z$ plane is taken as the reaction plane with the projectile nucleus moving along the $+z$ direction at $x = b/2$ and the target nucleus moving along the $-z$ direction with $x = -b/2$. Then the angular momentum of the produced matter is oriented in the $-y$ direction. The baryon species all couple to the vector meson with coefficients that differ by not more than factors of 2. Therefore we approximate

$$m^2 \omega^\nu = \bar{g}_\omega J^\nu_B \quad (10)$$

with an effective coupling $\bar{g}_\omega \approx 5$. Write the baryon current as $J^\nu_B = n_B(t)$ and $J_B = n_B(t) \psi(x,t)$ with the velocity parameterized by

$$\psi = (\psi_x(t)x + c_1 z/t, \psi_y(t)y, z/t + c_3 z/t) \quad (11)$$

The third component with $z/t = \tan \eta$, where $\eta$ is space-time rapidity, is the usual longitudinal expansion in the Bjorken model [22]. The $\psi_3(t)x$ and $\psi_3(t)y$ terms represent transverse expansion and when they are different they reflect elliptic flow. The $c_1$ term represents directed flow of the baryons as they are deflected away from the beam axis. The $c_3$ term represents shear flow along the beam axis. The $c_1$ terms represent contributions to vorticity since $\nabla \times \psi = 0$, which can be positive or negative. Baryon conservation leads to $\dot{n}_B(t) + (\psi_x(t) + \psi_y(t) + t^{-1})n_B(t) = 0$. In general, for fixed transverse coordinate one expects the $\psi$ to start near zero, rise with time and then fall to zero. Since we are interested in the time around hadronization we take $\psi_3(t) = a_x/t$ and $\psi_y(t) = a_y/t$ with $a_x$ and $a_y$ constants. Then

$$n_B(t) = n_B(t_{ch}) \left( \frac{t_{ch}}{t} \right)^{a_x + a_y + 1} \quad (12)$$

where $t_{ch}$ is the time of hadronization. The limit $a_x = a_y = 0$ corresponds to longitudinal expansion only, while the limit $a_x = a_y = 1$ corresponds to homologous spherical expansion. Consistent with this is the approximation that the scalar density $n_\sigma$ is a function of $t$ only so that the scalar field does not contribute to the polarization, at least in this simple model.
This is basically a blast wave model [23]. At some time $t_f > t_{ch}$, hydrodynamic flow ceases and free-streaming begins. At that time $x^2 + y^2 \lesssim R^2$, where $R$ is a cutoff on the transverse extent of the matter. The resulting fields are

$$B_\omega = \frac{\bar{\gamma}_\omega}{n_0} \left( \frac{\Delta c}{t} \right) n_B(t) \hat{y}$$

$$E_\omega = \frac{2\bar{\gamma}_\omega}{m_\omega} \left( 1 + a_{av} \right) n_B(t) \mathbf{v}$$

with

$$\nabla \times E_\omega = \frac{\partial B_\omega}{\partial t} = \frac{2\bar{\gamma}_\omega}{m_\omega} \left( 1 + a_{av} \right) n_B(t) \Delta c \hat{y}$$

where $a_{av} \equiv \frac{1}{2}(a_s + a_y)$.

We can estimate the magnitudes by using as reference values $n_B(t_{ch}) = n_0 = 0.15/fm^3$ (density of normal nuclear matter) and $t_{ch} = 3$ fm/c.

$$B_\omega \approx (25 \text{ MeV})^2 \Delta c \left( \frac{n_B(t_{ch})}{n_0} \right) \left( \frac{t_{ch}}{t} \right)^{2(1+a_{av})} \hat{y}$$

$$\frac{\partial B_\omega}{\partial t} \approx -(132 \text{ MeV}) (1 + a_{av}) \left( \frac{t_{ch}}{t} \right) B_\omega$$

$$E_\omega \approx (50 \text{ MeV})^2 (1 + a_{av}) \left( \frac{n_B(t_{ch})}{n_0} \right) \left( \frac{t_{ch}}{t} \right)^{2(1+a_{av})} \mathbf{v}$$

Then the ratio of the second to the first coefficient in Eq. (18) is $\left( \frac{1 + a_{av}}{3t_{ch}} \right) \left( \frac{t_{ch}}{t} \right)$ which is 9% or less. Due to the symmetries of our simple model $\langle E_\omega \times p \rangle = 0$ when averaging is done with a Boltzmann distribution boosted by the flow velocity $\mathbf{v}$.

Suppose that the spins were in equilibrium at temperature $T(t_{ch})$ at time $t_{ch}$. With the quantization axis in the $y$ direction the average polarization in the high temperature/weak field limit would be

$$P_\beta = \beta \frac{\bar{\gamma}_\omega}{m_\omega} \left| \frac{B_\omega}{2T} \right| \approx 1.4\% \left( \frac{n_B(t_{ch})}{n_0} \right) \left( \frac{140 \text{ MeV}}{T(t_{ch})} \right) \Delta c$$

meaning that $\Lambda$’s are polarized in the $+y$ direction while $\bar{\Lambda}$’s are polarized in the $-y$ direction if $\Delta c > 0$, and the opposite if $\Delta c < 0$.

For comparison the true magnetic field produced in high energy heavy ion collisions points in the $-y$ direction. The equilibrium $\Lambda$ polarization due to that field is $P_\beta = -\mu_B B/T$ which orients the spin in the $+y$ direction because the magnetic moment is negative: $\mu_B = -0.61 \mu_N$ where $\mu_N$ is the nuclear Bohr magneton. Being its anti-particle, the $\Lambda$ would be polarized in the $-y$ direction. The magnetic field has been calculated with the inclusion of the electrical conductivity $\sigma_E$ of the produced matter; in its absence the magnetic field at the time of hadronization is orders of magnitude smaller [24]. At time $t$ at $z = 0$ its value is

$$B = \frac{eb\sigma_E}{8\pi^2} \exp(-b^2 \sigma_E/4t)$$

where $b$ is the impact parameter. Evaluated at $t = t_{ch} = 3$ fm/c, $b = 7$ fm, $T = T(t_{ch}) = 140 \text{ MeV}$, and $\sigma_E = 6 \text{ MeV}$ the magnitude of the polarization is $|P_\beta| = 7.4 \times 10^{-6}$, totally irrelevant compared to the strong interaction induced polarization. Note also that as long as the condition $\gamma_{beam} b \sigma_E > 1$ is satisfied there is no beam energy dependence to the magnetic field. Realistic transport model calculations show that the time extent of the magnetic field is on the order of 0.2 fm/c, which is too short to build up observable polarization [25].

There are many complications before one is able to make precision comparisons to data. These include, but are not limited to: feed down from decays of heavier hyperons, such as $\Sigma$, $\Xi$, and $\Omega$; contributions from other vector mesons, such as $\rho$ (coupled to isospin or electric charge) and $K^*$, while the mean value of the $\phi$ meson field should be zero because the net strangeness is zero; feedback of the polarized spins to produce an effective vector meson magnetic field via susceptibility; and a more realistic, relativistic space-time evolution of the baryon current. Nevertheless, we make some preliminary comparisons here. The difference in polarization in the $-y$ direction according to Eq. (18) has the form

$$P_\Lambda - P_{\bar{\Lambda}} = C \left( \frac{n_B(t_{ch})}{n_0} \right) \left( \frac{140 \text{ MeV}}{T(t_{ch})} \right).$$

For the chemical potential and temperature at $t_{ch}$ as functions of $\sqrt{s_{NN}}$ we use the parametrization given in [26]. We then use a crossover equation of state from [27] to determine the baryon density. For illustration, since the precise magnitude is rather uncertain for the reasons given above, we consider two cases. In case I $C$ is independent of beam energy. In case II $C \sim 1/\sqrt{s_{NN}}$ because generally the directed flow and the shear flow of net baryons is expected to decrease with increasing energy. We take $C = 0.03$ for case I and $C = 0.45 \text{ GeV}/\sqrt{s_{NN}}$ for case II; both assume that $\Delta c > 0$. The coefficients are chosen to give a reasonable visual fit to the polarization data as shown in Fig. 1. The difference in polarizations rises with decreasing energy because the net baryon density increases, the temperature decreases, and in case II the factor $C$ rises with decreasing energy. It is interesting to note that the directed flow of both net protons [28] and net $\Lambda$’s [29] is actually negative in the range $10 < \sqrt{s_{NN}} < 30 \text{ GeV}$. This may reflect a change in the equation of state of the produced matter [30]. Because the polarization difference is sensitive to the baryon current it is a probe of the reaction dynamics.

Now let us turn to the problem of relaxation of a small departure from equilibrium. For this we turn to studies in the area of spintronics. A solution to the Bloch
spin-orbit interaction we estimate that

$$\langle \Omega_y^2 \rangle + \langle \Omega_z^2 \rangle \approx \left( \frac{2g_\omega g_\omega \Lambda}{m_\omega^2 m_\Lambda^2} \right)^2 \frac{v^2}{t^2} \int d^4k \frac{f(k)}{(p^2)}.$$  

One finds that \(\Gamma_s \ll \Gamma_c\) and therefore polarizations are established at the earliest times and thereafter do not change to any noticeable degree.

In conclusion, we have argued that well-known interactions of baryons with mesons can result in a splitting of the polarizations of \(\Lambda\) and \(\bar{\Lambda}\) hyperons in high energy heavy ion collisions. The results are sensitive to the space-time evolution of the baryon current. In order to obtain the experimentally observed sign of the splitting relative to the reaction plane it must be that the vorticity of the baryon current be opposite to the overall vorticity resulting from global angular momentum conservation. It implies that the flow velocity defined by the baryon current (Eckhart definition) differs from that defined by the energy flow (Landau-Lifshitz definition) and that therefore baryon diffusion may play a significant role in the dynamics. Clearly there is much work to be done before a comprehensive comparison between theory and experiment can be done.

**ACKNOWLEDGMENTS**

We thank J. W. Halley, A. Kamenev, M. Li, D. J. Millener, and C. Plumberg for discussions. The work of L. Cs. was supported by the Research Council of Norway Grant No. 255253/F50 and the work of J. K. and T. W. was supported by the U.S. DOE Grant No. DE-FG02-87ER40328.

[1] See the proceedings of the Quark Matter Conference series, the most recently available being: Nucl. Phys. A 967, (2017) ed. U. Heinz, O. Evdokimov, and P. Jacobs.
[2] First Three Years of Operation of RHIC, Nucl. Phys. A 757, Issues 1-2 (2005).
[3] Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005), [Erratum: Phys. Rev. Lett. 96, 039901 (2006)].
[4] F. Becattini, F. Piccinini, and J. Rizzo, Phys. Rev. C 77, 024906 (2008).
[5] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 76, 024915 (2007), [Erratum: Phys. Rev. C 95, 039906 (2017)].
[6] L. Adamczyk et al. (The STAR Collaboration), Nature 548, 62 (2017).
[7] J. Adam et al. (STAR Collaboration), Phys. Rev. C 98, 014910 (2018).
[8] F. Becattini, L. P. Csernai, and D. J. Wang, Phys. Rev. C 88, 034905 (2013).
[9] F. Becattini, I. Karpenko, M. Lisa, I. Upsilon, and S. Voloshin, Phys. Rev. C 95, 054902 (2017).
[10] M. Goeppert-Mayer, Phys. Rev. 75, 1969 (1949); Haxel, Jensen, and Suess, Phys. Rev. 75, 1766 (1949).
[11] H.-P. Duerr, Phys. Rev. 103, 469 (1956).
[12] J. D. Walecka, Ann. Phys. (NY) 83, 491 (1974).
[13] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986); B. D. Serot, Rep. Prog. Phys. 55, 1855 (1992); B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997).
[14] J. I. Kapusta and C. Gale, Finite Temperature Field Theory (Cambridge University Press, 2006).
[15] C. B. Dover and A. Gal, Prog. Part. Nucl. Phys. 12, 171 (1984).
[16] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).
[17] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, 1964).
[18] W. Greiner, Relativistic Quantum Mechanics (Springer-Verlag, Berlin, Germany, 1987).
[19] Y. Hinschberger and P.-A. Hervieux, Phys. Lett. A 376, 813 (2012).
[20] M. C. Hickey and J. S. Moodera, Phys. Rev. Lett. 102, 137601 (2009).
[21] R. Mondal, M. Berritts, and P. M. Oppeneer, Phys. Rev. B 94, 144419 (2016); R. Mondal, M. Berritts, and P. M. Oppeneer, Phys. Rev. B 96, 024425 (2017).
[22] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
[23] One may change variables so that $x = \tau \sinh \rho \cos \phi$ and $x/t = (\sinh \rho / \cosh \eta) \cos \phi$ where $\rho \geq 0$ is the transverse rapidity, and similarly for $y$.
[24] K. Tuchin, Phys. Rev. C 88, 024911 (2013).
[25] V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, Phys. Rev. C 83, 054911 (2011).
[26] J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C 73, 034905 (2006).
[27] M. Albright, J. Kapusta, and C. Young, Phys. Rev. C 90, 024915 (2014); 92, 044904 (2015).
[28] L. Adamczyk et al. (STAR Collaboration), Phys. Rev. Lett. 112, 162301 (2014).
[29] L. Adamczyk et al. (STAR Collaboration), Phys. Rev. Lett. 120, 62301 (2018).
[30] Y. Nara, H. Niemi, A. Ohnishi, J. Steinheimer, X. Luo, and H. Stöcker, Eur. Phys. J. A 57, 565 (2007). See esp. Eq. (IV.36).