Discerning Singlet and Triplet scalars at the electroweak phase transition and Gravitational Wave

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ABSTRACT: In this article we examine the prospect of first order phase transition with a Y=0 real $SU(2)$ triplet extension of the Standard Model, which remains odd under $Z_2$, considering the observed Higgs boson mass, perturbative unitarity, dark matter constraints, etc. Especially we investigate the role of Higgs-triplet quartic coupling considering one- and two-loop beta functions and compare the results with the complex singlet extension case. It is observed that at one-loop level, no solution can be found for both, demanding the Planck scale perturbativity. However, for a much lower scale of $10^4$ GeV, the singlet case predicts first order phase transition consistent with the observed Higgs boson mass. On the contrary, for the two-loop beta functions with one-loop potential, both the scenarios foresee strongly first order phase transition consistent with the observed Higgs mass with upper bounds of 310, 909 GeV on the triplet and singlet masses, respectively. This mass bound shifts to 259 GeV in case of triplet with the inclusion of two-loop contributions to the effective potential and the thermal masses with two-loop beta functions, consistent with the Planck scale perturbativity and the observed Higgs boson mass value. This puts the triplet in apparent contradiction with the observed dark matter relic bound and thus requires additional field for that. The preferred regions of the parameter space in both the cases are identified by benchmark points, that predict the Gravitational Waves with detectable frequencies in present and future experiments.

KEYWORDS: Higgs bosons, Beyond Standard Model, Dark matter, Electroweak symmetry breaking, Gravitational Wave
1 Introduction

The discovery of the Higgs boson around 125.5 GeV was the last stepping stone in the Standard Model (SM) [1, 2] and a proof of spontaneous symmetry breaking (SSB) in generating the masses of some of the SM particles. However, the nature of symmetry breaking is far from understood i.e., the role of another scalar, the order of phase transition (PT), etc, are still to be comprehended. It is intriguing to notice that with one Higgs doublet and the Higgs boson mass around 125.5 GeV, one only finds a smooth cross-over [3–7] but not the first order phase transition. The requirement of the first order phase transition is vastly related to the observed baryon number and lepton number in today’s universe [8–10]. This pushes for additional scalar(s) along with the SM Higgs doublet. The requirement of additional scalar can also be justified as in the SM, there is no cold dark matter(DM) candidate and it can also provide the much needed stability of the electroweak vacuum of the SM, and its various seesaw extensions [11–14]. It is also interesting to see, if these additional scalars are consistent with various constraints coming from the collider experiments, dark matter relic abundance along with the requirement of the strongly first order phase transition. The inspections regarding the first order phase transition exist in various possible extensions of the SM viz. in the supersymmetric scenarios [15–28], inert doublet model[29–33], scalar singlet [34–50], 2HDM [51–54], triplet[55–59] and multiple fields [60–64]. Some of these extensions need a revisit considering various recent experimental constraints along with the theoretical perturbative unitarity.

The first order phase transition originates from the bubble nucleation of the true vacuum at the nucleation temperature $T_n$. These bubbles expand due to the pressure difference between the true and the false vacua and the broken phase extends to the unbroken phase outside [65]. During such bubble expansion in the first order phase transition, bubbles collide and
create Gravitational Wave (GW) [65–73]. Different scenarios foretelling the first order phase transition, generate different GW frequencies that can be detected by the various present and future experiments.

It would be interesting to see, if such different extensions can be distinguished either theoretically or experimentally. We are particularly interested in the study of SM extension with a $Y = 0$ $SU(2)$ triplet, which stabilize the electroweak vacuum till the Planck scale [12] and compare the results with with a singlet extension. Such triplet, odd under $Z_2$, provides the much needed dark matter in terms of its neutral component, which should be $\lesssim 1.2$ TeV to satisfy the DM relic abundance [12]. The scenario is especially interesting, as it provides a charged Higgs boson with displaced decays, which can be detected in the LHC and the MATHUSLA [12, 74, 75]. In the context of supersymmetry $Y = 0$ triplet is also motivated for the reappearance of TeV scale supersymmetry consistent with 125.5 GeV Higgs boson [76–78], predicting correct $B \to X_s \gamma$ [76, 79], triplet charged Higgs boson [80–82] and displaced decays of triplinos [83].

In this article, we explore the possibility of such triplet providing the much needed strongly first order phase transition and the corresponding bound on the triplet mass parameter. The compatibility with perturbative unitarity at one- and two-loop level are also studied along with the bounds from the Higgs data and DM relic. The scenario is also compared with the complex singlet extension of the SM, which is also odd under $Z_2$ [84]. Finally, by measuring the bubble nucleation temperature along with other parameters, we estimate the signal frequencies of the GW created by the bubble collisions. Along with this, the sound wave of the plasma and the turbulence contribute substantially, are also considered here. Such frequencies can be detected by various future space interferometer experiments like Big Bang Observer (BBO) [85], Laser Interferometer Space Antenna (LISA) [86] and earth-based detector LIGO (LIGO) [87–89], and such regions are identified.

The article is organised as follows. In section 2 and section 3 we describe the inert singlet and inert triplet model along with the calculation of thermal corrected potential and masses with broadly defining the regions responsible for first order phase transition. The critical temperature and the effect of the quartic couplings are discussed in section 4. The bounds from perturbative unitarity at one- and two- loops, DM relics are discussed in section 5. The frequencies for the Gravitational Waves and their detectability in various experiments for different benchmark points are discussed in section 7. Finally we conclude in section 8.

## 2 Calculation of finite temperature potential for Inert Singlet scenario

The minimal SM is extended with a complex singlet which is considered to be odd under the $Z_2$ symmetry. The SM Higgs doublet $H$ is even under the $Z_2$ symmetry and transforms as $H \to H$, whereas the singlet $S$ goes to $-S$. Being odd under $Z_2$, the neutral component of the singlet becomes the dark matter candidate. The detailed calculation of the tree-level mass spectrum and the vacuum stability analysis at zero temperature are given in [12]. The corresponding tree-level scalar potential is given by

$$V = -\mu^2 H^\dagger H + m_S^2 S^* S + \lambda_1 |H|^2 |S|^2 + \lambda_2 |H|^2 |S|^2 + \lambda_h |H|^4 |S|^4 + \lambda_h |S|^4 |H|^4,$$

(2.1)

where neutral component $\phi$, of the SM Higgs doublet $H$, acts as the background field. However in case of SM, the field dependent masses for Higgs field $h$, Goldstone bosons $G^0$, the gauge bosons ($W^\pm$ and $Z$ boson) and the dominant top quark contributes to the effective potential. The expressions for the field dependent mass contributing to effective potential from SM are given as follows;

$$M^2_h(\phi) = 3\lambda_1 \phi^2 - \mu^2, \quad M^2_{G^0} = \lambda_1 \phi^2 - \mu^2$$

$$M^2_{W^\pm}(\phi) = \frac{g^2}{4} \phi^2, \quad M^2_2(\phi) = \frac{(g^2 + g^2)}{4} \phi^2$$

(2.2)

$$M^2_2(\phi) = \frac{m^2}{2} \phi^2,$$

where $M_t$ is defined as the top-quark mass. As the singlet does not acquire the vacuum expectation value (vev), the field dependent masses for the singlet will be given in terms of SM background field $\phi$ only. The field dependent mass of singlet contributing to the effective potential is calculated as:

$$M^2_h(\phi) = m_S^2 + \frac{\lambda_h s^2}{2}.$$

(2.3)

The one-loop daisy improved finite temperature effective potential can be written as [16, 84]

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi, 0) + \Delta V_1(\phi, T) + \Delta V_{\text{daisy/ring}}(\phi, T),$$

(2.4)

$^1$This is our notation to use Higgs singlet interaction coupling as $\lambda_{hs}$ and this quartic coupling is defined as $\lambda_{hs} = 2t^2$ in [84].
where $V_0(\phi)$ corresponds to the tree-level potential $V_{\text{tree}}(\phi)$,

$$V_0(\phi) = V_{\text{tree}}(\phi) = \frac{-\mu^2}{2} \phi^2 + \frac{\lambda_1}{4} \phi^4.$$ \hspace{1cm} (2.5)

Here, $V_1(\phi, 0)$ is evaluated at one-loop at zero temperature via Coleman-Weinberg prescription [90]. $\Delta V_1(\phi, T)$ presents the one-loop temperature corrected potential. The potential without daisy resummation can be written as

$$V_{\text{tot}} = V_0(\phi) + V_1(\phi, 0) + \Delta V_1(\phi, T).$$ \hspace{1cm} (2.6)

The total one-loop result ($V_{\text{eff}}(\phi, T)$) includes the resummation over a subclass of thermal loops which are defined as ring diagrams or daisy diagrams and the plasma effects are explained by these ring improved one-loop effective potential [91–96]. These ring diagrams mainly amounts to adding thermal corrections to bosons using $\Delta V_B$. But this method of adding thermal corrections or resummation is not uniquely defined and there are two different methods for adding such thermal corrections, one is Parwani method and second one is Arnold-Espinosa method [97]. In Arnold-Espinosa method, $M^2(\phi) \rightarrow M^2(\phi, T) = M^2(\phi)$ is done only for the cubic term as in Eq: Equation 2.6 and not for every term of the effective potential to obtain the ring-improved effective potential

$$V_{\text{daisy/ring}} = V_{\text{tot}}[M^2(\phi)] + \frac{T}{12\pi} \sum_{\text{bosons}} (M_i^2(\phi) - M_i^2(\phi, T)), \quad \text{Arnold – Espinosa method.} \hspace{1cm} (2.7)$$

In case of Parwani method $M^2(\phi) \rightarrow M^2(\phi, T) = M^2(\phi)$ is done for each term in the effective potential as shown below,

$$V_{\text{ring}} = V_{\text{tot}}[M^2(\phi, T)] \quad \text{Parwani method.} \hspace{1cm} (2.8)$$

Therefore, there is a difference of two-loop order terms in these two prescriptions and can give us idea about the uncertainties in our calculations if we neglect the higher-order terms in the perturbation theory. However, for this analysis we consider Arnold-Espinosa prescription via considering thermal replacement of mass for the cubic mass terms only. Since, fermions do not contribute in the cubic term, so such replacement are ignored here.

The effective potential in high-temperature limit includes $\phi$ depending mass contributions from bosons and fermions of SM and singlet can be written as;

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi, 0) + \Delta V_B(\phi, T) + \Delta V_F(\phi, T),$$ \hspace{1cm} (2.9)

where $V_{\text{tree}}(\phi, 0)$ is the tree-level potential and $\Delta V_B(\phi, T)$ is the one-loop contribution including thermal corrections from bosons. These one-loop contributions from bosons are defined as;

$$\Delta V_B = \sum_{i=h,G,W_L,Z_L,\gamma_L,W_T,Z_T,\gamma_T,S} n_i \Delta V_i,$$ \hspace{1cm} (2.10)

where $G \in \{G^a, G^\pm\}$ and $W_L, Z_L, \gamma_L, W_T, Z_T, \gamma_T$ are the longitudinal and transverse components of gauge bosons $W^\pm, Z$ and photon $\gamma$ with $\Delta V_i$ as detailed below

$$\Delta V_i = \frac{m_i^2(\phi) T^2}{24} - \frac{M_i^4(\phi) T}{12 \pi} - \frac{m_i^4(\phi)}{64 \pi^2} \left[ \log \frac{m_i^2(v)}{c_B T^2} - 2 \frac{m_i^2(v)}{m_i^2(\phi)} + \frac{\lambda_2}{2} \log \frac{m_i^2(v)}{m_i^2(\phi)} \right].$$ \hspace{1cm} (2.11)

As mentioned earlier, for fermions only the dominant contribution from top quark is considered in $\Delta V_F(\phi, T)$ and it does not have any cubic term, so no thermal corrections to masses are considered here as shown below,

$$\Delta V_F = n_t \left[ \frac{m_t^2(\phi) T^2}{48} + \frac{M_t^4(\phi)}{64 \pi^2} \left[ \log \frac{m_t^2(v)}{c_F T^2} - 2 \frac{m_t^2(v)}{m_t^2(\phi)} \right] \right].$$ \hspace{1cm} (2.12)

In Equation 2.10 the number of degrees of freedom for SM fields and triplet bosons are given as;

$$n_h = 1, n_G = 3, n_S = 2, n_t = 12$$

$$n_{W_L} = n_{Z_L} = n_{\gamma_L} = 1, n_{W_T} = n_{Z_T} = n_{\gamma_T} = 2,$$ \hspace{1cm} (2.13)

while the coefficients $c_B$ and $c_F$ used in above Equation 2.11 and Equation 2.12 are defined by: $\log c_B = 3.9076$, $\log c_F = 1.1350$. The Debye masses used in Eq: (2.11), $M_i^2(\phi)$ for $i = h, G, T, W_L, W_T, Z_T, \gamma_T$ are as follows;

$$M_i^2 = m_i^2(\phi) + \Pi_i(\phi, T).$$ \hspace{1cm} (2.14)
where $m_t^2(\phi)$ are the field-dependent masses and $\Pi_i(\phi,T)$ are the self-energy contributions given by:

\[
\Pi_h(\phi,T) = \left(\frac{3g_2^2 + g_1^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^2}{4} + \frac{\lambda_{hs}}{12}\right)T^2,
\]

\[
\Pi_G(\phi,T) = \left(\frac{3g_2^2 + g_1^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^2}{4} + \frac{\lambda_{hs}}{12}\right)T^2,
\]

\[
\Pi_T(\phi,T) = \frac{2\lambda_s + \lambda_{hs}T^2}{6},
\]

\[
\Pi_{W_L}(\phi,T) = \frac{11}{6}g_2^2T^2,
\]

\[
\Pi_{W_T}(\phi,T) = \Pi_{Z_T}(\Phi,T) = \Pi_{\gamma_T} = 0.
\]

Here, the self energy contribution to the transverse component of gauge bosons $W_T, Z_T$ and $\gamma_T$ is zero and only the longitudinal components get the self energy contribution. The Debye mass expressions for $Z_L$ and $\gamma_L$ are written as follows:

\[
M_{Z_L}^2 = \frac{1}{2}[m_Z^2(\phi) + \frac{11}{6}g_2^2\cos^2\theta_WT^2 + \Delta(\phi,T)],
\]

\[
M_{\gamma_L}^2 = \frac{1}{2}[m_\gamma^2(\phi) + \frac{11}{6}g_2^2\cos^2\theta_WT^2 - \Delta(\phi,T)],
\]

where $\Delta$ is given as

\[
\Delta^2(\phi,T) = m_Z^2(\phi) + \frac{11}{3}g_2^2\cos^2\theta_W\left(M_Z^2(\phi) + \frac{11}{12}g_2^2\cos^2\theta_WT^2\right)T^2.
\]

Now, after getting the full one-loop effective potential including thermal corrections we can do the complete numerical analysis. To see the effectiveness of plasma screening, we can first include the dominant contribution from the singlet field only by neglecting the contributions from other bosons in SM. Considering the contribution from singlet only in Equation 2.10-Equation 2.11 and substituting in Equation 2.9, we get the $\phi$ dependent part of one-loop effective potential as follows:

\[
V(\phi) = A(T)\phi^2 + B(T)\phi^4 + C(T)(\phi^2 + K^2(T))^2.
\]

Here the temperature dependent coefficients are given as;

\[
A(T) = -\frac{1}{2}\mu_T^2 + \frac{1}{3}\left(\frac{\lambda_{hs}}{6} + \frac{y_t^2}{2}\right)T^2,
\]

\[
B(T) = \frac{1}{4}\lambda_s,
\]

\[
C(T) = -\left(\frac{\lambda_{hs}}{2}\right)^2\frac{T}{6\pi},
\]

\[
K^2(T) = \frac{(\lambda_{hs} + 2\lambda_s)T^2 + 6m_\gamma^2}{3\lambda_{hs}},
\]

where,

\[
\mu_T^2 = \mu^2 - \frac{\lambda_{hs}}{16\pi^2}\left(M_Z^2(v) + m_Z^2\log\frac{c_BT^2}{m_Z^2(v)}\right) + \frac{3}{8\pi^2}y_t^2m_{\text{top}}^2(v)\log\frac{m_{\text{top}}(v)}{c_FT^2}.
\]

\[
\lambda_T = \lambda_1 + \frac{\lambda_{hs}}{32\pi^2}\log\frac{c_BT^2}{m_Z^2(v)} + \frac{3}{16\pi^2}y_t^2\frac{m_{\text{top}}^2(v)}{c_FT^2}.
\]

It is clear from Equation 2.18 that $\phi = 0$ is the local minima at very earlier epoch, if $A(T) > 0$, which leads to following constraint:

\[
-\frac{1}{2}\mu_T^2 + \frac{1}{3}\left(\frac{\lambda_{hs}}{6} + \frac{y_t^2}{2}\right) > 0.
\]

After EW symmetry breaking $\phi = 0$ is the maxima and we can find an epoch in between, where, a particular temperature $T_{32}$ is defined by demanding $V''(0) = 0$. This will give a constraint as follows:

\[
4A^2 + 9C^2K^2 = 0.
\]

The $\phi = 0$ is still the minimum above this temperature i.e. $T > T_{32}$, but there exist another maximum and minima at $\phi_-(T)$ and $\phi_+(T)$, respectively [84]. This can be calculated by putting $V''(\phi) = 0$ and demanding that $\phi \neq 0$, which leads to,

\[
\phi_{\pm}(T) = \frac{1}{32B^2}\left(9C^2 - 16AB \pm |C|\sqrt{9C^2 + 32(2B^2K^2 - AB)}\right).
\]
These two extrema can merge resulting \( \phi_-(T) = \phi_+(T) \) at a particular temperature \( T_1 \) which is higher than \( T_2 \) but lower than the symmetric temperature \( T \). The \( \phi_-(T) = \phi_+(T) \) condition from Equation 2.24 implies

\[
9C^2 + 32(2B^2K^2 - AB) = 0. \tag{2.25}
\]

Using the set of equations from Equation 2.19-Equation 2.25, \( T_1 \) and \( T_2 \) are determined as

\[
T_1^2 = \frac{2\lambda_{\omega_1}(\lambda_{\omega_2}m^2_{\omega_1} + 2\lambda_{\omega_1}m^2_{\omega_2})}{\lambda_{\omega_1}((\frac{\lambda_{\omega_2}}{2} + \frac{\mu^2}{2})\lambda_{\nu_1} - \frac{\lambda_{\omega_2}}{4\pi^2}(\lambda_{\omega_1} + 2\lambda_{\nu_1}))}, \tag{2.26}
\]

\[
T_2^2 = \frac{1}{2\alpha}(\Lambda^2(T^2) + \sqrt{\Lambda^4(T^2) - 16\alpha^2m^2_{\nu_2}}), \tag{2.27}
\]

where,

\[
\alpha = (\frac{\lambda_{\omega_1}}{6} + \frac{\mu^2}{2})^2 - \frac{1}{24\pi^2}\lambda^2_{\omega_2}\left(\lambda_{\omega_1} + 2\lambda_{\nu_1}\right),
\]

\[
\Lambda^2(T) = \frac{1}{4\pi}\lambda^2_{\omega_1}m^2_{\omega_2} + 4\left(\frac{\lambda_{\omega_1}}{6} + \frac{\mu^2}{2}\right)m^2_{\nu_2}. \tag{2.28}
\]

### 3 Calculation of finite temperature potential for Inert Triplet scenario

We extend the SM with a \( Y=0 \) (hypercharge=0) real \( SU(2) \) triplet which is odd under the \( Z_2 \) symmetry. The SM Higgs doublet \( H \) as given below, transforms under \( Z_2 \) as \( H \rightarrow \pm H \), where as the triplet \( T \) goes to \( -T \). The triplet has one complex charged component \( T^\pm \) and one neutral component \( T^0 \) as shown below. Being \( Z_2 \), the neutral component of the triplet \( T^0 \) becomes the dark matter candidate. The detailed tree-level mass spectrum and zero temperature vacuum stability analysis are given in [12].

The corresponding scalar potential is given by

\[
V = -\mu^2 H^1 H + m^2_T Tr(T^\dagger T) + \lambda_1|H|^4 + \lambda_i(T^i T)|^2 + \lambda_{hi}H^1 H Tr(T^\dagger T), \tag{3.1}
\]

\[
H = \left(\frac{1}{\sqrt{2}}(\phi + h) + g\phi^0\right), \quad T = \frac{i}{\sqrt{2}}\begin{pmatrix} T_0 & \sqrt{2}T^+ \\ \sqrt{2}T^- & -T_0 \end{pmatrix},
\]

where neutral component of SM Higgs doublet \( H \), given by \( \phi \), acts as the background field. However, the field dependent masses, which contribute to the effective potential in the SM includes Higgs field \( h \), Goldstone bosons \( G^0 \), the gauge bosons \( (W^\pm \text{ and } Z \text{ boson}) \) and the dominant top quark. The field dependent mass expressions contributing to effective potential from SM are calculated as follows;

\[
\begin{align*}
M^2_h(\phi) &= 3\lambda_1\phi^2 - \mu^2, & M^2_{h0} &= \lambda_1\phi^2 - \mu^2 \\
M^2_{hW}(\phi) &= \frac{g^2}{4}\phi^2, & M^2_{hT}(\phi) &= \frac{(g^2 + g'^2)}{4}\phi^2 \\
M^2_{hT}(\phi) &= \frac{\mu^2}{2}\phi^2,
\end{align*} \tag{3.2}
\]

where \( M_t \) is the top-quark mass. As the triplet does not get vacuum expectation value (vev), the field dependent masses for triplet will be in terms of SM background field \( \phi \) only. The neutral component \( T_0 \) and charged component \( T^\pm \) both will contribute to the effective potential as we present their field dependent masses:

\[
\begin{align*}
M^2_{T0}(\phi) &= m^2_T + \frac{\lambda_{hT}}{2}\phi^2, \\
M^2_{T\pm}(\phi) &= m^2_T + \frac{\lambda_{hT}}{2}\phi^2. \tag{3.3}
\end{align*}
\]

In this scenario, the one-loop contributions from bosons are given as;

\[
\Delta V_B = \sum_{i=h, G, W, Z, T} n_i \Delta V_i, \tag{3.4}
\]

where \( G \in \{G^0, G^\pm\}, T \in \{T_0, T^\pm\} \) and \( W_L, Z_L, W_T, Z_T, \gamma_T \) are defined as the longitudinal and transverse components for gauge bosons \( W^\pm, Z \) and photon \( \gamma \) and \( \Delta V_i \) is given below

\[
\Delta V_i = \frac{m_i^2(T^2)}{24} - \frac{M^2_i(\phi)T}{12\pi} - \frac{m_i^2(\phi)T}{64\pi^2} \left[ log \left( \frac{m_i^2(v)}{m_i^2(\phi)} \right) - 2 \frac{m_i^2(v)}{m_i^2(\phi)} + \delta_i \log \left( \frac{m_i^2(v)}{m_i^2(\phi)} \right) \right], \tag{3.5}
\]

\[\text{--- 5 ---}\]
In Equation 3.4 the number of degrees of freedom for SM fields and triplet bosons are given as:

\[ n_h = 1, n_G = 3, n_T = 3, n_t = 12 \]
\[ n_WL = n_{Z_L} = n_{\gamma_L} = 1, n_{WT} = n_{Z_T} = n_{\gamma_T} = 2, \]
\[ (3.6) \]

The Debye masses used in Eq: (3.5) for inert Triplet scenario, \( M_i^2(\phi) \) for \( i = h, G, T, W_L, W_T, Z_T, \gamma_T \) are as follows;

\[ M_i^2 = m_i^2(\phi) + \Pi_i(\phi, T), \]
\[ (3.7) \]

where the field-dependent masses, \( m_i^2(\phi) \) and the self-energy contributions, \( \Pi_i(\phi, T) \) are given by;

\[ \Pi_h(\phi, T) = \left( \frac{3g_2^2 + g_t^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^2}{4} + \frac{\lambda_{ht}}{12} \right) T^2, \]
\[ \Pi_G(\phi, T) = \left( \frac{3g_2^2 + g_t^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^2}{4} + \frac{\lambda_{ht}}{12} \right) T^2, \]
\[ \Pi_T(\phi, T) = \frac{2\lambda + \lambda_{ht}}{6} T^2, \]
\[ \Pi_{WL}(\phi, T) = \frac{11}{6} g_t^2 T^2, \]
\[ \Pi_{WT}(\phi, T) = \frac{11}{6} g_t^2 T^2, \]
\[ \Pi_{ZT}(\phi, T) = \frac{11}{6} g_t^2 T^2, \]
\[ (3.8) \]

Similar to the previous scenario, only the longitudinal components get the self energy contribution while the self energy contribution to the transverse component of gauge bosons \( W_T, Z_T \) and \( \gamma_T \) is zero. The Debye mass expressions for \( Z_L \) and \( \gamma_L \) are same as the earlier and are written as follows;

\[ M_i^2 = \frac{1}{2} \left[ m_i^2(\phi) + \frac{11}{6} g_t^2 \cos^2 \theta_W T^2 \right] + \Delta(\phi, T), \]
\[ (3.9) \]

where \( \Delta \) is given as

\[ \Delta^2(\phi, T) = m_i^2(\phi) + \frac{11}{3} g_t^2 \cos^2 \theta_W \left[ M_i^2(\phi) + \frac{11}{12} g_t^2 T^2 \right] T^2. \]
\[ (3.10) \]

Here the temperature dependent coefficients in are now given as;

\[ A(T) = -\frac{1}{2} \mu_T^2 + \frac{1}{4} \left( \frac{\lambda_{ht}}{2} + \frac{y_t^2}{2} \right) T^2, \]
\[ B(T) = \frac{1}{4} \lambda_T, \]
\[ C(T) = \left( \frac{\lambda_{ht}}{2} \right)^2 \frac{\lambda}{4\pi}, \]
\[ K^2(T) = \left( \frac{2\lambda_{ht} + 4\lambda_t}{} \right) T^2 + 6\mu_T^2 \]
\[ (3.11) \]

where,

\[ \mu_T^2 = \mu^2 - \frac{3\lambda_{ht}}{8\pi^2} \left( \sum_{i=0, T} m_i^2(v) + \sum_{i=0, T} \log \frac{c_B T^2}{m_i^2(v)} \right) + \frac{3}{8\pi^2} y_t^2 m_{top}^2(v) \log \frac{m_{top}^2(v)}{c_B T^2}, \]
\[ (3.12) \]

\[ \lambda_T = \lambda_1 + \frac{3\lambda_{ht}}{64\pi^2} \log \frac{c_B T^2}{m_i^2(v)} + \frac{3}{16\pi^2} y_t^2 \log \frac{m_{top}^2(v)}{c_B T^2}. \]
\[ (3.13) \]

It is clear from Equation 3.11 that at very earlier epoch, \( \phi = 0 \) is the local minima if \( A(T) > 0 \), which leads to following condition;

\[ -\frac{1}{2} \mu_T^2 + \frac{1}{4} \left( \frac{\lambda_{ht}}{2} + \frac{y_t^2}{2} \right) > 0. \]
\[ (3.14) \]

After symmetry breaking \( \phi = 0 \) will be maxima and in between we can find a epoch, where, we can define a particular temperature \( T_2 \) by demanding \( V''(0) = 0 \). This will give a condition as follows;

\[ 4A^2 + 9C^2K^2 = 0. \]
\[ (3.15) \]

If we go above this temperature i.e. \( T > T_2 \), then \( \phi = 0 \) is still the minimum but there exist other maximum at \( \phi_-(T) \) and minima at \( \phi_+(T) \), respectively [84]. This can be achieved by putting \( V'(\phi) = 0 \) and demanding \( \phi \neq 0 \), that give to,

\[ \phi_{\pm}(T) = \frac{1}{\sqrt{2}B^2} \left( 9C^2 - 16AB \pm |C| \sqrt{9C^2 + 32(2B^2 K^2 - AB)} \right). \]
\[ (3.16) \]
regions of first and second order phase transition. For a comparison with the complex singlet we consider the potential of a
proceeds by the spontaneous breaking of the minima. There is also two-step phase transition possible in a
this gives second-order phase transition. We considered only the direct one-step transitions from EW symmetric and broken
Here we explore the dependency of the scalar quartic couplings by presenting
quartic coupling and bare masses of the extra scalars in determining the order of phase transition.

3.1 Effect of scalar quartic couplings in phase transition

At temperature higher than \( T_2 \) but lower than the symmetric temperature \( (T) \) these two extrema can merge resulting \( \phi_-(T) = \phi_+(T) \), which is defined as \( T_1 \). The condition for \( \phi_-(T) = \phi_+(T) \) from Equation 3.16 implies

\[
9C^2 + 32(2B^2K^2 - AB) = 0.
\]

Just to remind ourselves, that the temperatures higher than \( T_1 \), i.e. \( T > T_1 \) which designates the symmetric phase, has just one minimum, i.e. \( \phi = 0 \). Figure 1(b) shows the shapes of the potential at different thermal epoch. We shall see that these transitions can lead to the first order phase transitions as compared to the smooth second order phase transition as shown in Figure 1(a).

For \( T < T_2 \), \( \phi = 0 \) is the maximum and there exists minimum at \( \phi \neq 0 \) which evolves towards the zero temperature minimum. Using the set of equations from Equation 3.11-Equation 3.17 we determine \( T_1 \) and \( T_2 \)

\[
T_1^2 = \frac{6144\pi^2 \lambda \mu_t^2 (\lambda_{bt} \mu_t^2 + 2\lambda_{bt} m_t^2)}{\lambda_{bt} (3072\pi^2 (\lambda_{bt} + \frac{y_t^2}{2}) \lambda_{bt} - 27 \lambda_{bt}^2 - \frac{2048\pi^2 \lambda_{bt}^2 (2\lambda_{bt} + 4\lambda_t)}{\lambda_{bt}})},
\]

\[
T_2^2 = \frac{1}{\alpha}(\Lambda^2(T2) + \sqrt{\Lambda^4(T2) - 65536\alpha^2 m_{T2}^2}),
\]

where,

\[
\alpha = \left(\frac{\lambda_{bt}}{4} + \frac{y_t^2}{2}\right) - \frac{3}{128\pi^2} \lambda_{bt}^2 \left(2\lambda_{bt} + 4\lambda_t\right),
\]

\[
\lambda^2(T) = 9\lambda_{bt} m_{T2}^2 + 256 \left(\frac{\lambda_{bt}}{4} + \frac{y_t^2}{2}\right) \mu_{T2}^2.
\]

From temperature \( T_1 \) and \( T_2 \), we can get the idea about the nature of the phase transition. The condition, when \( T_1 = T_2 \) for particular value of parameters \( (\lambda_{bt}, \lambda_t, m_t) \), the nature of phase transition becomes second-order to first-order. The first-order phase transition happens via bubble nucleation, when the bubbles of broken phase starts nucleating in the sea of symmetric phase. This process requires \( T_1 > T_2 \), when at lower temperature \( T_2 \), \( \phi = 0 \) is the maximum and there exists an deeper \( \phi \neq 0 \) minimum. While for \( T_1 < T_2 \), at lower temperature \( T_1 \) there is no second minima deeper than \( \phi = 0 \) and this gives second-order phase transition. We considered only the direct one-step transitions from EW symmetric and broken minima. There is also two-step phase transition possible in a \( Z_2 \) symmetric scenario, where the electroweak phase transition proceeds by the spontaneous breaking of the \( Z_2 \) symmetry. \(^2\) In the following subsection we investigate such effect of Higgs quartic coupling and bare masses of the extra scalars in determining the order of phase transition.

3.1 Effect of scalar quartic couplings in phase transition

Here we explore the dependency of the scalar quartic couplings by presenting \( T_1 = T_2 \) lines in \( \lambda_t - \lambda_{bt} \) plane to segregate regions of first and second order phase transition. For a comparison with the complex singlet we consider the potential of a

\(^2\)The spontaneous breakdown of \( Z_2 \) symmetry gives rise to the domain wall problem and \( Z_2 \) breaking transition is expected to be of second order, but not possible to verify within perturbative effective theory [98].
complex singlet ($S$) extended SM as given in [84], where the Higgs-singlet quartic coupling $\lambda_{hs}$, $\lambda_s$ is the self quartic coupling for the singlet and $m_S = M$ is the bare mass term for the singlet. The nature of phase transition is discussed in Figure 2 by varying the parameters $\lambda_s/\lambda_t$ vs $\lambda_{hs}/\lambda_{ht}$ for singlet and triplet, respectively. The coloured lines correspond to the condition $T_1 = T_2$ for different values of mass parameter, which defines the cross-over from first-order to second-order phase transition. The region above the $T_1 = T_2$ condition is first-order and below one is second-order. For this analysis we considered the current experimental values $m_h = 125.5 \text{ GeV}$, $m_t = 173.2 \text{ GeV}$ respectively [99]. The mass parameters $m_S/m_T$ are varied from 0-300 GeV and 0-100 GeV with a gap of 50 GeV for singlet and triplet, respectively. The lower lines denote $m_S/m_T = 0 \text{ GeV}$ and the uppermost lines correspond to 300 GeV and 100 GeV for singlet and triplet case. It is evident from both Figure 2(a) & (b) that as we enhance the value of the mass parameter $m_S/m_T$, the required Higgs quartic couplings $\lambda_{hs}/\lambda_{ht}$ for $T_1 = T_2$ are also enhanced, i.e. the first order phase transition now needs higher quartic couplings. The effect of self quartic coupling is very minimal and reduces further as we increase the bare mass parameter.

![Figure 2](image.png)

**Figure 2.** Plot for the condition $T_1 = T_2$ by varying parameters $\lambda_s/\lambda_t$ vs $\lambda_{hs}/\lambda_{ht}$ for singlet and triplet, respectively. The mass parameter $m_S/m_T$ is varied from 0-300 GeV and 0-100 GeV with a gap of 50 GeV for singlet and triplet, respectively. We considered the current experimental values $m_h = 125.5 \text{ GeV}$, $m_t = 173.2 \text{ GeV}$. The region above condition $T_1 = T_2$ corresponds to first-order and below region for second-order phase transition. The lower green curve is for $m_S/m_T = 0 \text{ GeV}$ and the upper one is for $m_S/m_T = 300/100 \text{ GeV}$ with a gap of 50 GeV in case of singlet and triplet respectively.

In section 4 we analyse both singlet and triplet scenarios considering all the bosonic degrees of freedom, coupling constants within the perturbativity at two-loop and calculating the exact critical temperature $T_C$.

### 4 Critical Temperature and Electroweak Baryogenesis

In this section, we focus on electroweak baryogenesis and critical temperature during electroweak phase transition caused by strongly first-order phase transition and the out of equilibrium condition. Inside the bubble walls a net baryon number is generated due to the first order phase transition as well as the suppressed sphaleron transition. Such $\text{B}$-violating interactions inside the bubble walls also achieve the out of equilibrium which helps in baryogenesis. The required criteria for the strongly first-order phase transition can be defined as follows [100, 101]:

$$\frac{\phi_+(T_C)}{T_C} \geq 1,$$

where $T_C$ is defined as the critical temperature and $\phi_+(T_C)$ is the parameter which defines the strength of phase transition. At critical temperature, different two minima of the potential are degenerate i.e., the same depth and such condition defines the critical temperature as;

$$V(0; T_C) = V(\Phi_+(T_C); T_C),$$

where $V(0; T_C)$ is the potential at $\phi = 0$ minima and $V(\Phi_+(T_C); T_C)$ is the second minima at $\phi_+$. In order to calculate...
The couplings see in section 5. It is clear from Figure 3 and Figure 4 that generated by SARAH \[102\] as given below;

In this section, we study the RG evolution of the scalar quartic couplings \(\lambda_s/\lambda_t\) are assigned three different values 0, 0.5 and 1.0, which are depicted by blue, orange and green curves, respectively for fixed mass parameter \(m_S/m_T = 50\) GeV with the current experimental values i.e \(m_h = 125.5\) GeV, \(m_t = 173.2\) GeV.

For lower values of \(\lambda_{hs}/\lambda_{ht}\), the dominant contributions are mainly from the SM fields. For the singlet case Figure 3(a) we cross \(\lambda_{hs} \gtrsim 1\) the effect of the singlet filed starts showing up and for \(\lambda_{hs} \gtrsim 1.65\), we attain the regions with \(\frac{\phi_s(T_c)}{T_c} > 1\). On the contrary, due to more degrees of freedom in the case of the triplet, we see such transitions much earlier i.e., \(\lambda_{ht} \gtrsim 1.3\). One interesting point to note that with the increase of the self couplings i.e. \(\lambda_s/\lambda_t\), the \(\frac{\phi_s(T_c)}{T_c}\) requires higher values of the interactive Higgs couplings i.e. \(\lambda_{hs}/\lambda_{ht}\).

In Figure 4 we describe the similar variations with respect to \(\lambda_{hs}/\lambda_{ht}\) for the fixed values of self quartic couplings i.e., \(\lambda_s/\lambda_t = 0\) to maximize \(\frac{\phi_s(T_c)}{T_c}\) for the singlet and the triplet, respectively. We also check the dependency over the soft mass parameter \(m_S/m_T\) by varying them for 0 – 400 GeV with a gap of 50 GeV and 1000 GeV, respectively and are denoted by blue, orange, green, red curves and so on. It can be seen that as we increase the soft mass \(m_S/m_T\) the value of \(\frac{\phi_s(T_c)}{T_c}\) decreases for a fixed value of \(\lambda_{hs}/\lambda_{ht}\). From Figure 4 (a) we see that after \(m_S \geq 350\) GeV getting \(\frac{\phi_s(T_c)}{T_c} > 1\) will require \(\lambda_{hs} > 3.0\). However, for the triplet scenario in Figure 4 (b) \(m_T = 400\) GeV can still give rise to \(\frac{\phi_s(T_c)}{T_c} > 1\) with \(\lambda_{ht} \geq 2.6\). The couplings \(\lambda_{hs}/\lambda_{ht}\) are restricted differently for the singlet and the triplet case from the perturbative unitarity as we will see in section 5. It is clear from Figure 3 and Figure 4 that \(\frac{\phi_s(T_c)}{T_c}\) parameter is maximum for mass parameter \(m_S/m_T = 0\) for fixed value of self quartic coupling of singlet/triplet and is also maximum for \(\lambda_s/\lambda_t = 0\) for fixed value of mass parameter.

5 RG evolution of Scalar Quartic Couplings

The RG evaluation of the scalar quartic couplings can give sufficient constraints to the regions responsible for the first order phase transition from their perturbative unitarity. We explore such possibility via considering both one- and two-loop beta functions as explained in the following subsections.

5.1 Constraints from one-loop perturbativity

In this section, we study the RG evolution of the scalar quartic couplings \(\lambda_s, \lambda_t\) and \(\lambda_{ht}\) with their one-loop \(\beta\)-functions generated by SARAH \[102\] as given below;

\[
\beta_{\lambda_s} = \beta_{\lambda_s}^{SM} + \Delta \beta_{\lambda_s}^{ITM},
\]

\[
(5.1)
\]
Figure 4. Variation of $\phi_+(T_c)$ with quartic coupling $\lambda_{hs}/\lambda_{ht}$ for the fixed values of self quartic couplings $\lambda_s/\lambda_t=0$ for the singlet and the triplet, respectively. The mass parameter $m_S/m_T$ are varied for $0 - 400$ GeV with a gap of 50 GeV and 1000 GeV, respectively.

\[
\begin{align*}
\beta_{\lambda_1}^{\text{SM}} &= \frac{1}{16\pi^2} \left[ \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - \frac{9}{5} g_1^2 \lambda_1 - 9 g_2^2 \lambda_1 + 24 \lambda_1^2 \
&\quad + 12 \lambda_1 \text{Tr} \left( Y_u Y_u^\dagger \right) + 12 \lambda_1 \text{Tr} \left( Y_d Y_d^\dagger \right) + 4 \lambda_1 \text{Tr} \left( Y_e Y_e^\dagger \right) \
&\quad - 6 \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) - 6 \text{Tr} \left( Y_d Y_d^\dagger Y_d Y_d^\dagger \right) - 2 \text{Tr} \left( Y_e Y_e^\dagger Y_e Y_e^\dagger \right) \right], \\
\Delta \beta_{\lambda_1}^{\text{TM}} &= 8 \lambda_{ht}^2, \\
\beta_{\lambda_s} &= \frac{1}{16\pi^2} \left[ -24 g_2^2 \lambda_s + 88 \lambda_s^2 + 8 \lambda_{ht}^2 + \frac{3}{2} g_2^4 \right], \\
\beta_{\lambda_t} &= \frac{1}{16\pi^2} \left[ \frac{3}{4} g_2^4 - \frac{9}{10} g_1^2 \lambda_{ht} - \frac{33}{2} g_2^2 \lambda_{ht} + 12 \lambda_{ht} + 16 \lambda_{ht}^2 + 24 \lambda_{ht} \lambda_1 + 6 g_2^2 \lambda_{ht} \right].
\end{align*}
\]

where $\Delta \beta_{\lambda_1}^{\text{TM}}$ is the additional contribution to SM $\beta_\lambda$ from inert triplet. Since $\phi_+(T_c)$ is maximum, where both mass parameter $m_S/m_T$ and the self quartic coupling $\lambda_s/\lambda_t$ are zero. We have chosen $\lambda_s/\lambda_t=0$ at the EW scale for our analysis and RG evolutions at one- and two-loops govern the couplings at any other scales. Hence, to maximize $\phi_+(T_c)$, we choose $\lambda_s/\lambda_t = 0$ at the EW scale for further analysis. One point to note here is that the mass parameter does not enter in the running of quartic couplings thus the choice of $\lambda_s/\lambda_t=0$ is sufficient for the perturbative unitarity. To keep the SM Higgs mass around 125.5 GeV, we keep the SM quartic coupling $\lambda_1 = 0.13$ at the EW scale. In Table 1, $\Lambda$ designates the perturbative scale where any of the coupling crosses the perturbativity ($4\pi$). We fix quartic coupling $\lambda_{hs}/\lambda_{ht}$ at the EW scale and check the perturbative unitarity till a particular scale $\Lambda$. To show the effect of the top quark mass, we present the maximum values of the quartic couplings at the EW scale allowed for two different top quark masses i.e. 120.0, 173.2 GeV, respectively for the singlet and the triplet scenarios. We see that due to larger scalar degrees of freedom triplet scenario gets more restriction than the singlet one. For example considering Planck scale perturbativity the singlet can have a $\lambda_{hs}^\text{max} = 0.237(0.248)$, whereas the triplet scenario gets $\lambda_{ht}^\text{max} = 0.2180(0.2202)$ for $m_t = 173.2(120.0)$ GeV. For lower top mass the large negative contribution from the top quark slows down the running of scalar quartic coupling towards perturbative limit. It can also be observed that as we demand lower scale for the perturbativity, higher values of $\lambda_{hs}^\text{max}/\lambda_{ht}^\text{max}$ at the EW scale can be attained. In the next subsection we would discuss such effects at the two-loop level.

In Figure 5 we present the variation of $\phi_+(T_c)$ i.e. the strength of phase transition with SM Higgs boson mass for the singlet and the triplet scenario, where we consider $\lambda_{hs}^\text{max}/\lambda_{ht}^\text{max}$ as given in Table 1 for a given scale $\Lambda$. Figure 5(a) depicts the situation for the complex singlet extension, where it is evident that higher values of $\phi_+(T_c)$ are possible with lower perturbativity scale and lower SM Higgs boson mass. It is interesting to note that $m_h = 125.5$ GeV and $\phi_+(T_c) > 1$ is not possible even for the perturbative scale $\Lambda = 10^9$ GeV and only $\Lambda = 10^6$ GeV can barely satisfy the condition of the first order phase transition. The values of $\lambda_{hs}^\text{max}/\lambda_{ht}^\text{max}$ are similar for $\Lambda = 10^6$ GeV, however due to more degrees of freedom the
Figure 5. Variation of $\frac{\phi_+ (T_c)}{T_c}$ with respect to the Higgs boson mass $m_h$ in GeV for fixed initial values of $\lambda_{hs}^{max}/\lambda_{ht}^{max}$ at different perturbative scales as shown in Table 1. The mass parameter $m_S/m_T$ and self quartic coupling $\lambda_s/\lambda_t$ is chosen to be zero to maximize the strength of phase transition with $m_t = 173.2$ GeV.

The triplet scenario guarantees larger $\frac{\phi_+ (T_c)}{T_c}$ for a given $m_h$. The perturbative scale $\Lambda = 10^6$ GeV allows larger $\lambda_{ht}^{max}$ compared to $\lambda_{hs}^{max}$ resulting an enhancement of $\frac{\phi_+ (T_c)}{T_c}$ in favour of the singlet and it barely makes it for $\frac{\phi_+ (T_c)}{T_c} \simeq 1$ at $m_h = 125.5$ GeV, however the triplet case fails to achieve that at one-loop level.

Figure 6. Variation of $\phi_+ (T_c)/T_c$ with Higgs mass $m_h$ for two different values of top mass $m_t = 120.0, 173.2$ GeV designated by blue and orange curves for the perturbative scale of $10^6$ GeV.

The dependence of the top quark mass is explored in Figure 6 for the variation of $\frac{\phi_+ (T_c)}{T_c}$ with the Higgs boson mass for the choices of the mass parameters $m_S/m_T$ and self quartic coupling $\lambda_s/\lambda_t$ equal to zero for the perturbative scale $\Lambda = 10^6$ GeV. The maximum allowed quartic couplings, $\lambda_{hs}^{max}/\lambda_{ht}^{max}$ are estimated using $m_t = 120.0, 173.2$ GeV and $m_h = 125.5$ GeV at the electroweak scale for the perturbative scale of $\Lambda = 10^6$ GeV and $m_t = 173.2$ values are described in Table 1. The blue and orange curves present $m_t = 120.0, 173.2$ GeV cases, respectively for the singlet (Figure 6(a)) and the triplet scenario (Figure 6(b)). The maximum allowed quartic coupling $\lambda_{ht}^{max}$ is lower for the triplet due to more degrees of freedom which catalyses an early perturbative restriction. Nevertheless, the slight decrement of $\lambda_{ht}^{max}$ compared to $\lambda_{hs}^{max}$ is over powered by more degrees of freedom giving little higher values of the $\frac{\phi_+ (T_c)}{T_c}$ for a given $m_h$. The upper bound on Higgs mass to
avoid Baryon asymmetry washout i.e. \( \frac{\phi_+(T_c)}{T_c} > 1 \) is 91.0 GeV and 93.0 GeV for the singlet and the triplet, respectively. Thus, we can conclude that these upper bounds on Higgs mass from Baryon asymmetry for both cases, considering one-loop perturbativity of the quartic couplings, are not consistent with the current observed experimental Higgs mass of 125.5 GeV.

### Table 1

| \( \Lambda \) (GeV) | \( \lambda_{hs} = \frac{\lambda^\text{max}_{hs}}{m_t} \) (GeV) | \( \lambda_{ht} = \frac{\lambda^\text{max}_{ht}}{m_t} \) (GeV) |
|---------------------|---------------------|---------------------|
| \( 10^4 \)         | 1.6545              | 1.3710              |
| \( 10^6 \)         | 0.7290              | 0.7067              |
| \( 10^8 \)         | 0.5120              | 0.4873              |
| \( 10^{11} \)      | 0.4780              | 0.3477              |
| \( 10^{16} \)      | 0.3090              | 0.2490              |
| \( 10^{19} \)      | 0.2370              | 0.2180              |

Figure 7. Variation of \( \phi_+(T_c)/T_c \) in \( \lambda^\text{max}_{hs}/\lambda^\text{max}_{ht} - m_h \) plane, where \( \lambda^\text{max}_{hs}/\lambda^\text{max}_{ht} \) are the maximum allowed values of quartic coupling at different perturbative scale in GeV for the singlet and the triplet scenarios, respectively. The colour band from deep blue to red regions signify \( \phi_+(T_c)/T_c \) in \( 0 - 3.5 \) for both scenarios.

Before ending the discussion of one-loop perturbativity and move on to two-loop results, we present the results in a 3-dimensional graph in Figure 7, where we study the variation of \( \phi_+(T_c)/T_c \) in \( \lambda^\text{max}_{hs}/\lambda^\text{max}_{ht} - m_h \) plane. The colour band of \( \phi_+(T_c)/T_c \) from deep blue to red regions signify \( \phi_+(T_c)/T_c \) in \( 0 - 3.5 \) for both scenarios. The self couplings for the singlet and the triplet, and their corresponding soft masses are chosen to zero in order to enhance \( \phi_+(T_c)/T_c \). It is very apparent from Figure 7 that a much lower perturbative scale \( \Lambda \) and lighter SM Higgs boson are preferred in order to achieve first order phase transition i.e. \( \phi_+(T_c)/T_c > 1 \). Only for singlet case, \( \Lambda = 10^5 \) GeV scale can have a first order phase transition with SM Higgs boson mass around 125.5 GeV. The choice of zero soft masses in order to have first order phase transition for both scenarios may restrict the physical singlet and triplet scalars. However, as we explore in the following subsection the

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**Figure 7.** Variation of \( \phi_+(T_c)/T_c \) in \( \lambda^\text{max}_{hs}/\lambda^\text{max}_{ht} - m_h \) plane, where \( \lambda^\text{max}_{hs}/\lambda^\text{max}_{ht} \) are the maximum allowed values of quartic coupling at different perturbative scale in GeV for the singlet and the triplet scenarios, respectively. The colour band from deep blue to red regions signify \( \phi_+(T_c)/T_c \) in \( 0 - 3.5 \) for both scenarios.
two-loop perturbativity gives little breather and such upper limits on the physical singlet and triplet masses are enhanced.

5.2 Constraints from two-loop perturbativity

For the given values of quartic couplings at the electroweak scale i.e. $\lambda_1, \lambda_{hs/ht}, \lambda_{ij}$, they hit the Landau pole at the same scale considering one-loop RG-evolution. Depending on the validity scale of perturbativity certain constraints come for the maximum electroweak values of the couplings, as we have seen in Table 1. For example if we choose the perturbativity scale as Planck scale i.e. $\Lambda = 10^{19}$ GeV, $\lambda^{max}_{hs}$ and $\lambda^{max}_{ht}$ are restricted to 0.23 and 0.21 at one-loop level. The slight difference comes due to the variation of $\lambda_1$ affecting $\lambda_i$.

The situation changes a lot as we move to two-loop RG-evolution Appendix A. Contrary to one-loop case, here $\lambda_1$ hits the Landau pole before $\lambda_{hs/ht}, \lambda_{ij/t}$. However, the growth of $\lambda_1$ coupling slows down at two-loop as compared to one-loop, due to the negative contributions i.e. $-312\lambda_1^2, -80\lambda_1\lambda_{ht}, -128\lambda_1^3$ as shown in subsection A.1. Similarly, other quartic couplings i.e. $\lambda_{ht}, \lambda_i$ also slow down due to some extra negative contributions appearing at two-loop (see subsection A.1). In comparison, the singlet also suffers from the negative contributions of $-312\lambda_1^3, -40\lambda_1\lambda_{hs}^2, -32\lambda_{hs}^3$ as can be read from subsection B.1. However, if we look at the maximum allowed value ($\lambda^{max}_{hs}/\lambda^{max}_{ht}$) at the electroweak scale at two-loop level in Table 2 for the Planck scale perturbativity, the singlet can access double the value that of the triplet one. The can be understood as the triplet has more positive contributions in terms of $10g^2\lambda_{ht}, 32g^2\lambda_{hs}^2$ in $\lambda_1$, which are absent in the singlet one. Thus the growth of $\lambda_1$ in the triplet case is faster hitting the Landau pole much earlier, as compared to the singlet one.

Presence of such extra positive contributions at the two-loop level for the triplet, explains the larger difference in $\lambda^{max}_{hs}$ and $\lambda^{max}_{ht}$ (See Table 2) as compared to one-loop (see Table 1).

In order to examine the situation of the possibility of the first order phase transition with the perturbativity at the two-loop level, we calculate the maximum allowed values of the quartic couplings i.e. $\lambda^{max}_{hs}/\lambda^{max}_{ht}$ with the Planck scale perturbativity ($\Lambda = 10^{19}$ GeV) as given in Table 2. The slow-growing quartic coupling at the two-loop compared to one-loop enhanced the allowed couplings for $\lambda^{max}_{hs} = 4.00$ and $\lambda^{max}_{ht} = 1.95$ at the electroweak scale. These are now enormously amplified compared to the corresponding one-loop values $\lambda^{max}_{hs} = 0.25670$ and $\lambda^{max}_{ht} = 0.2180$, which result in higher values of $\phi_+(T_T)/T_T$ strengthening the possibility of first order phase transition for both scenarios.

| $\Lambda$ (GeV) | $\lambda^{max}_{hs}$ | $\lambda^{max}_{ht}$ |
|-----------------|----------------------|----------------------|
| $10^{19}$       | 4.00                 | 1.95                 |

Table 2. Maximum allowed value of quartic couplings i.e. $\lambda^{max}_{hs}/\lambda^{max}_{ht}$ allowed at the electroweak scale for the perturbativity till Planck scale at two-loop, for the singlet and the triplet scenarios.

Equipped with relatively larger $\lambda^{max}_{hs}/\lambda^{max}_{ht}$ for $\Lambda = 10^{19}$ GeV, we now perform the variation of $\phi_+(T_T)/T_T$ with respect to $m_h$ in Figure 8, where the scalar self coupling are chosen to be zero. The mass parameters varied for $m_S(m_T) = 500(100), 840(200), 1000(300)$ are denoted by blue, orange and green curves, respectively. The red star in both the cases denotes $\phi_+(T_T)/T_T = 1$ and $m_h = 125.5$ GeV point. Higher mass values diminish the $\phi_+(T_T)/T_T$ and push for second order phase transition for both scenarios. However, for the singlet scenario we see a maximum of $m_S = 840$ GeV can still be consistent with SM Higgs boson mass as well as first order phase transition, whereas, for the triplet scenario such bounds comes for rather low mass i.e. $m_T \approx 193$ GeV. We see an order of magnitude difference in the upper bound on the soft mass parameter in the singlet and the triplet scenarios.

In Figure 9 we present $\phi_+(T_T)/T_T$ in $m_S/m_T - m_h$ plane for the maximum allowed values of quartic coupling $\lambda^{max}_{hs}/\lambda^{max}_{ht}$ at the electroweak scale for the perturbativity till Planck scale ($10^{19}$) GeV for the singlet and the triplet scenarios, respectively. The colour band from deep blue to red regions signify $\phi_+(T_T)/T_T$ in $0 - 3.5$ for both scenarios. It is evident that for higher mass values $m_T \geq 193$ GeV in the triplet case stay in the deep blue region for $m_h = 125.5$ GeV conferring a second order phase transition. On the contrary, the singlet case one can obtain regions up to $m_S \approx 840$ GeV satisfying first order phase transition at $m_h = 125.5$ GeV.

5.3 Two-loop resummed potential

The field-dependent terms in the effective potential from one-loop daisy resummation is $O(g^3)$ but achieving accuracy of $O(g^4)$ requires two-loop corrections when two-loop $\beta$-functions are analysed. The most efficient two-loop contributions are of the form $\phi^3\log(\phi)$, which are induced by the Standard Model weak gauge boson loops[10]. The diagrams contributing to the two-loop potential for the minimal standard model are given in [97, 103, 104]. In case of inert singlet, there is no additional diagram, which contributes to the two-loop potential. Therefore, the two-loop correction for inert singlet comes

\[ \phi^3\log(\phi) \]
\[ \Lambda = 10^{19} \text{ GeV} \]

\[ m_T = 100 - 300 \text{ GeV} \]

\[ \lambda_t = 0 \]

\[ \lambda_{hs}/\lambda_{ht} \]

\[ \text{with the perturbativity at the Planck scale} \ (10^{19} \text{ GeV}) \]

\[ \text{and the self coupling are chosen to be zero.} \]

\[ V_2 \simeq \log \frac{T \phi^2 T^2}{32 \pi^2} \left[ \frac{51}{16} g_4^4 \right]. \]  

(5.6)

Similarly, in case of inert triplet, there are diagrams which give additional contributions to the two-loop potential along with the SM part. The two-loop resumed potential for the inert triplet scenario is given as

\[ V_2 \simeq \log \frac{T \phi^2 T^2}{32 \pi^2} \left[ \frac{51}{16} g_4^4 + \frac{3}{16} g_4^4 \right], \]  

(5.7)

where the first term comes from SM and the second term comes from the inert triplet.
After adding these two-loop contributions to the full one-loop effective potential, the strength of phase transition enhances\cite{105,106} in both cases, which actually changes the mass bounds. However, for the singlet one-loop maximum mass required for first order phase transition \( \simeq 909 \text{ GeV} \), which already decouples and does not alter the phase transition. With inclusion of two-loop correction, this bound is still consistent with Planck scale perturbativity and satisfies the Higgs boson mass bound within 1\( \sigma \) uncertainty because of increase in the strength of phase transition. Thus, the singlet mass bound still remains same at the two-loop resumed potential. On the contrary, the effect is visible in the case of inert triplet, owning to lower mass bound of \( \simeq 310 \text{ GeV} \) at one-loop potential. Inclusion of the two-loop resumed potential inflate the mass bound slightly to \( \simeq 320 \text{ GeV} \), satisfying the Planck scale perturbativity and the current experimental Higgs boson mass bound.

6 Dimensional reduction

The effective potential at finite temperature has residual scale dependence at \( \mathcal{O}(g^4) \). The cancellation of this scale dependence at \( \mathcal{O}(g^4) \) requires the inclusion of two-loop thermal masses to bare masses for Higgs, singlet and the triplet i.e. \( \mu, m_S \) and \( m_T \), respectively. The most common way is to utilise high-temperature dimensional reduction to a three-dimensional effective field theory (3d EFT) in order to derive the full \( \mathcal{O}(g^4) \) thermal effective potential. The dimensional reduction technique is an systematic approach required at high temperature to the resummations done order-by-order in power of couplings \cite{28,29}. The \( \mathcal{O}(g^4) \) result for the \( \mathbb{Z}_2 \)-symmetric real scalar theory with the two-loop results has been derived long ago. The one-loop potential to this order reads as:

\[
V^{\text{thermal}}_{\text{one-loop}}(v) = \frac{1}{(4\pi)^2} \left[ \frac{1}{12} g_2^4 T^2 v^2 \left( \frac{1}{2} \log \left( \frac{M^2(v) + \mu - \Lambda}{T^2} \right) - \frac{3}{8} L_b(\Lambda) - c + \frac{1}{4} \right) \right] + \frac{1}{(4\pi)^2} \left( - \frac{1}{4} g_2^2 \mu^2 v^2 L_b(\Lambda) - \frac{1}{16} g_2^4 M^4 L_b(\Lambda) \right) + \text{const},
\]

where, we have introduced the notation using following Refs.,

\[
c = -\log \left( \frac{3\pi^2}{4\Lambda^2} \right) = -0.348723\ldots,
\]

\[
L_b(\Lambda) = 2 \log \left( \frac{e^2 \Lambda}{4\pi T} \right).
\]

where, \( \Lambda \) is the Glashow-Kinkelin constant and \( \gamma \) is the Euler-Macheroni constant. The full two-loop effective potential expression is as follows:

\[
V^{\text{thermal}} = TV_{\text{eff}}^{3d} = T \left[ \frac{1}{12} g_2^4 T^2 v^2 \left( \frac{1}{2} \log \left( \frac{M^2(v) + \mu - \Lambda}{T^2} \right) - \frac{3}{8} L_b(\Lambda) - c + \frac{1}{4} \right) \right] + \frac{1}{(4\pi)^2} \left( - \frac{1}{8} g_2^2 M^2 - \frac{1}{24} g_2^4 v^2 \left[ 1 + 2 \ln \left( \frac{\mu^2}{3M_\text{teff}} \right) \right] \right),
\]

where, the one-to-one correspondence between Higgs quartic coupling \( \lambda_3 \) and \( g_{2,3} \) is \( \lambda_3 = \frac{1}{4} g_{2,3}^2 \) and the expressions for two-loop thermal masses are as follows:

\[
M_2^2 = \mu^2 + \frac{1}{2} g_{2,3}^2 v_3^2.
\]

The 3d effective parameters to the same \( \mathcal{O}(g^4) \) order are given as:

\[
g_{2,3}^2 = T \left( g_2^2 (\Lambda) - \frac{3}{2(4\pi)^2} g_2^4 L_b(\Lambda) \right),
\]

\[
\mu^2 = \mu^2 (\Lambda) + \frac{1}{24} g_2^2 (\Lambda) T^2 - \frac{1}{16} g_2^4 L_b(\Lambda) + \frac{1}{16} g_2^4 T^2 L_b(\mu) + \frac{1}{6} g_{2,3}^2 \left[ c + \ln \left( \frac{3T}{\Lambda_\text{teff}} \right) \right],
\]

\[
v_3 = \frac{v}{\sqrt{T}}.
\]

In the next section, we present the similar expressions for dimensionally reduced 3d theory for the SM extended with a inert singlet and an inert triplet.

6.1 Singlet extension

The scalar potential given in Equation 2.1 for the inert singlet scenario in the dimensionally reduced 3D effective theories (DR3EFTs) is given as:

\[
V = -\mu_3^2 H^1 H + m_{2,3}^2 S^* S + \lambda_{1,3} |H^1|^2 + \lambda_{s,3} |S^* S|^2 + \lambda_{hs,3} (H^1 H)(S^* S).
\]
The third and the fourth term in the full two-loop potential at (which are the tree-level parameters) are computed as follows \[98, 109\]:

\[
\lambda_{1,3} = T \left[ \lambda_1 (\Lambda) + \frac{1}{4(\pi^2)} \left( \frac{2 - 3L_h}{16} (3g_3^2 + 2g_2^2g_2^1 + g_1^1) + N_c L_f (g_1^1 - 2\lambda_1 g_1^1) + L_b \left( \frac{3}{2} (3g_2^2 + g_1^1) \lambda_1 - 12\lambda_1^2 - \frac{1}{4} \lambda_{hs} \right) \right) \right],
\]

\[
\lambda_{s,3} = T \left[ \lambda_s (\Lambda) - \frac{1}{(4\pi^2)} L_b (\lambda_{hs}^2 + 9\lambda_s^2) \right],
\]

\[
\lambda_{hs,3} = T \left[ \lambda_{hs} (\Lambda) + \frac{\lambda_{hs}}{(4\pi^2)^2} \left( L_b \left( \frac{3}{4} (3g_2^2 + g_1^1) - 6\lambda_1 - 2\lambda_{hs} - 3\lambda_s \right) - N_c L_f y_1^2 \right) \right],
\]

(6.10)

where

\[
L_b = \ln \left( \frac{\Lambda^2}{T^2} \right) - 2[\ln(4\pi) - \gamma],
\]

(6.11)

\[
L_f = L_b + 4\ln 2.
\]

(6.12)

Here, \( L_b \) and \( L_f \) are logarithms that arise frequently from one-loop bosonic and fermionic sum integrals with \( \Lambda \) is the \( \overline{MS} \) scale and \( \gamma \) is the Euler-Mascheroni constant. The expressions for the two-loop mass parameters are computed as follows:

\[
\mu_3^2 = (\mu_3^2)_{SM} + \frac{T^2}{24} \lambda_{hs} (\Lambda) - \frac{L_b}{(4\pi^2)^2} \left( \frac{3}{2} \lambda_{hs} \mu_3^2 (\Lambda) \right) + \frac{1}{(4\pi^2)^2} \left( \frac{3}{2} (3g_2^2 + g_1^1) L_b - N_c g_1^1 L_f \right) \left( \frac{T^2}{24} \lambda_{hs} \right) - \frac{T^2}{(4\pi^2)^2} L_b \lambda_{hs} \left( \frac{1}{4} \lambda_1 + \frac{5}{24} \lambda_{hs} \right)
\]

\[
+ \frac{1}{(4\pi^2)^2} \lambda_{hs,3} \left( c + \ln \left( \frac{3T}{\Lambda_{3d}} \right) \right),
\]

(6.13)

where

\[
(\mu_3^2)_{SM} = -\mu_3^2 (\Lambda) + \frac{T^2}{12} \left( \frac{3}{4} (3g_2^2 (\Lambda) + g_1^1 (\Lambda)) + N_c g_1^1 (\Lambda) + 6\lambda_1 (\Lambda) \right) + \frac{L_b (\Lambda)^2}{(4\pi^2)^2} \left( \frac{3}{4} (3g_2^2 + g_1^1) - 6\lambda_1 \right) L_b - N_c g_1^1 L_f
\]

\[
+ \frac{T^2}{(4\pi^2)^2} \left( \frac{167}{96} g_3^4 + \frac{1}{288} g_1^1 - \frac{3}{16} g_2^2 g_1^1 + \frac{(1 + 3L_h)}{4} \lambda_1 (3g_2^2 + g_1^1) + L_b \left( \frac{17}{16} g_1^1 - \frac{45}{48} g_1^1 - \frac{3}{16} g_2^2 g_1^1 - 6\lambda_1^2 \right) \right)
\]

\[
+ \frac{1}{(4\pi^2)^2} \left( c + \ln \left( \frac{3T}{\Lambda_{3d}} \right) \right) \left( \frac{39}{16} g_3^4 + 12g_2^2 g_3 + 6g_3^2 - 9g_3^2 \lambda_{hs,3} - 12\lambda_1^2 - \frac{5}{16} g_3^4 - \frac{9}{8} g_3^2 g_3^2 \right) - 2h_2^2 - 3h_3^2
\]

\[
+ 3g_3^2 \lambda_{hs,3} - \frac{1}{96} (9L_b - 3L_f - 2) \left( (N_c + 1) g_3^4 + \frac{1}{6} Y_f g_1^1 \right) Y_f + \frac{N_c}{32} \left( 7L_b - L_f - 2 \right) g_1^1 y_i^2
\]

\[
- \frac{N_c}{4} \left( 3L_b + L_f \right) \lambda_{hs} y_i^2 + \frac{N_c}{96} \left( 9(L_b - L_f) + 4 \right) Y_f^2 - 2 (L_b - 4L_f + 3) (Y_f^2 + Y_f^1) g_1^1 y_i^2
\]

\[
- \frac{N_c C_F}{6} \left( \frac{L_b - 4L_f + 3}{2} g_1^1 y_i^2 + \frac{N_c}{24} \left( 3L_b - 2(N_c - 3)L_f \right) g_1^1 \right),
\]

(6.14)

with \( C_F = \frac{N_c^2 - 1}{N_c} = \frac{4}{3} \) and \( c \sim -0.348723 \) is the fundamental quadratic Casimir of \( SU(3) \). And the two-loop mass parameter for singlet is given as:

\[
m_s^2 (\Lambda) = m_s^2 (\Lambda) + T^2 \left( \frac{1}{6} \lambda_{hs} (\Lambda) + \frac{1}{4} \lambda_s (\Lambda) - \frac{L_b}{(4\pi^2)^2} \left( 2\lambda_{hs} \mu^2 + 3\lambda_s m_s^2 \right) + \frac{1}{(4\pi^2)^2} \left( (3g_3^2 + g_1^1) \lambda_{hs,3} - 2\lambda_{hs,3}^2 - 6\lambda_3^2 \right) \right)
\]

\[
\left( c + \ln \left( \frac{3T}{\Lambda_{3d}} \right) \right) + \frac{T^2}{(4\pi^2)^2} \left( \frac{2 + 3L_b}{24} (3g_2^2 + g_1^1) \lambda_{hs} - L_b \left( \lambda_1 + \frac{7}{12} \lambda_{hs} + \frac{1}{3} \lambda_s \right) \lambda_{hs} + \frac{9}{4} \lambda_3^2 \right) - \frac{N_c}{12} \left( 3L_b - L_f \right) y_i^2 \lambda_{hs}.
\]
The other parameters which are used in the above expressions are computed as follows:

\[ g_{2,3}^2 = g_2^2(\Lambda)T \left[1 + \frac{g_2^2}{(4\pi)^2} \left(\frac{43}{6}L_b + \frac{2}{3} - \frac{(N_f + 1)n_f}{3} L_f\right)\right], \]

\[ g_{1,3}^2 = g_1^2(\Lambda)T \left[1 - \frac{g_2^2}{(4\pi)^2} \left(\frac{1}{3}L_bY_0^2 + L_fY_2n_f\right)\right], \]

\[ h_3 = \frac{g_3^2(\Lambda)T}{4} \left[1 + \frac{1}{(4\pi)^2} \left(\frac{43}{6}L_b + \frac{17}{2} - \frac{(N_f + 1)n_f}{3} (L_f - 1)\right)g_3^2 + \frac{g_2^2}{2} - 2N_c y_i^2 + 12\lambda_i\right], \]

\[ h_5 = \frac{g_5^2(\Lambda)T}{4} \left[1 + \frac{1}{(4\pi)^2} \left(3\frac{g_2^2}{2} - \frac{(L_f - 1)Y_0^2 + (L_f - 1)Y_2n_f}{2} g_3^2 - 2(Y_0^2 + Y_2^2)N_c y_i^2 + 12\lambda_i\right)\right], \]

\[ h_{32}^2 = \frac{g_3^2(\Lambda)g_1^2(\Lambda)}{2} \left[1 + \frac{1}{(4\pi)^2} \left(\frac{43}{12}L_b - 1\right)g_2^2 - \frac{Y_0^2}{3} \left(\frac{4}{3}L_b - 1\right)g_2^2 + 4\lambda_i + \frac{2}{3} N_c y_i^2 - (L_f - 1) \left(\frac{N_f + 1}{6} g_2^2 + \frac{Y_2}{12} g_2^2\right) n_f\right]. \]

With the inclusion of two-loop corrections to the thermal masses specially to \( m_B \) and the Higgs, the upper bound on the singlet mass coming from the first-order phase transition and the current Higgs mass bound remains the same. Since, the two-loop corrections are less for the chosen benchmark point from Planck scale perturbativity, the strength of the order of phase transition does not change significantly and the Higgs mass bound is now satisfied in the 1\( \sigma \) limit with this slight change.

### 6.2 Triplet extension

In the similar way, the scalar potential given in Equation 3.1 for the inert triplet in the dimensionally reduced 3D effective theories (DR3EFTs) is given as:

\[ V = -\mu_3^2 H^H H + m_{3,T}^2 Tr(T^T T) + \lambda_{1,3} |H|^2 + \lambda_{3,3} (Tr(T^T T))^2 + \lambda_{3,3} H^H H Tr(T^T T), \]

(6.16)

The matching relations for the corresponding quartic couplings are given as:

\[ \lambda_{1,3} = T \left[\frac{1}{(4\pi)^2} \left(\frac{3}{2}g_2^2 + g_1^2 + 2g_1^2g_2^2\right) + 3L_f \left(y_1^2 - 2\lambda_1 y_i^2\right) - L_b \left(\frac{3}{16} \left(3g_2^2 + g_1^2 + 2g_1^2g_2^2\right)\right)\right], \]

\[ \lambda_{3,3} = T \left[\frac{1}{(4\pi)^2} \left[2g_2^2 - 3\lambda_3 y_1^2 L_f - L_b (2\lambda_3^2 + 5\lambda_3 \lambda_4 + 3g_2^2 + 6\lambda_3 \lambda_4 - \frac{3}{4} \lambda_4 (g_1^2 + 11g_2^2)\right)\right], \]

\[ \lambda_{3,3} = T \left[\frac{1}{(4\pi)^2} \left(4g_2^2 - L_b \left(\lambda_3^2 + 11\lambda_3^2 - 12g_2^4 + 6g_2^4\right)\right)\right], \]

(6.17)

\( y_i \) \( L_b = \ln \left(\frac{\Lambda^2}{2\pi}\right) - 2[\ln(4\pi) - \gamma], \)

\( L_f = L_b + 4\ln 2. \)

(6.20)

The matching relations for the corresponding bare mass parameters are given as [110]:

\[ \mu_3^2 = (\mu_3^2)_{SM} + \frac{T^2}{8} \lambda_{3,3} (\Lambda^2) + \frac{1}{16\pi^2} \left[ + \frac{\lambda_{3,3} m_T^2 L_b + T^2 \left(\frac{5}{24} g_2^2 + \frac{1}{2} \lambda_{3,3} g_2^2 - \frac{3}{8} \lambda_{3,3} y_1^2 L_f + L_b \left(\frac{7}{16} g_2^2 - \frac{5}{8} \lambda_{3,3}\right)\right)\right], \]

\[ \lambda_{3,3}^2 = \left(\mu_3^2\right)_{SM} + \frac{T^2}{8} \lambda_{3,3} (\Lambda^2) + \frac{1}{16\pi^2} \left[ - \mu_3^2 \left(\frac{3}{4} \left(3g_2^2 + g_1^2\right) - 6\lambda_1 \right) L_b - 3g_2^2 L_f\right], \]

(6.22)

\[ + \left(\frac{167}{96} g_2^2 + \frac{1}{258} g_1^2 + \frac{3}{16} \lambda_1 \left(3g_2^2 + g_1^2\right) + L_b \left(\frac{17}{16} g_2^2 - \frac{3}{8} \lambda_1 \left(3g_2^2 + g_1^2\right) - 6\lambda_1^2\right)\right)\]

\[ + (c + \ln(3T/\Lambda_{3d})) \left(\frac{39}{16} g_2^2 + \frac{5}{16} \lambda_3 g_1^2 + 12g_2^2 + 6\lambda_1^2 - 2h_3^2 - 3h_3^2 + 3\lambda_3 \left(3g_2^2 + g_1^2\right) - 12\lambda_1^2\right)\]

\[ - \lambda_1 \left(\frac{3}{16} g_2^2 + \frac{11}{48} g_1^2 + 2g_1^2\right) + \left(\frac{5}{12} g_2^2 + \frac{5}{108} g_1^2\right) N_f + L_f \left(\frac{9}{16} g_2^2 + \frac{17}{48} g_2^2 + 2g_2^2 - 3\lambda_1\right) + \frac{3}{8} g_1^2 + \left(\frac{1}{4} g_2^2 + \frac{3}{36} g_1^2\right) N_f\right)\]

\[ + \ln(2) \left(\frac{9}{16} g_2^2 - \frac{47}{72} g_2^2 + \frac{8}{3} g_2^2 + 9\lambda_1\right) - \frac{3}{2} g_1^2 + \left(\frac{3}{4} g_2^2 + \frac{5}{6} g_1^2\right) N_f\right)\]
and

\[ m_{T,3}^2 = -m_T^2 + T^2 \left( \frac{1}{6} \lambda_{bt} \lambda_t + \frac{5}{12} \lambda_t \lambda_t + \frac{1}{2} g_2^2 (\Lambda) \right) + \frac{1}{16} \pi^2 \left[ - (6 g_2^2 - 3 \lambda_t) m_T^2 L_b + 2 \mu^2 \lambda_{bt} L_b + T^2 \left( \frac{71}{18} + \frac{2}{9} N_f \right) g_1^2 + \frac{5}{3} \lambda_1 g_2^2 + \frac{1}{4} \lambda_{bt} g_2^2 + \frac{1}{12} \lambda_t \lambda_t g_1^2 + L_b \left( \frac{5}{12} g_1^2 - \frac{3}{4} \lambda_t - \frac{55}{12} \lambda_t^2 + \frac{11}{8} \lambda_{bt} g_2^2 + \frac{1}{8} \lambda_t \lambda_t g_1^2 + 5 \lambda_t g_2^2 - \frac{5}{6} \lambda_t \lambda_t - \lambda_t \lambda_t \lambda_t \right) + \left( c + \ln \left( \frac{3 T}{\Lambda_{std}} \right) \right) \left( -2 \lambda_{bt} A - 10 \lambda_t^2 + \lambda_t \lambda_t (3 g_2^2 + g_1^2) + 20 \lambda_t g_2^3 - 3 g_2^2 + 24 g_2^2 \delta_3 - 24 \delta_3^2 + 8 g_2^2 \delta_3 - 16 \delta_3^2 \right) \right] - 16 \delta_3^2 - L_f \left( \frac{1}{2} \lambda_{bt} g_2^2 + \frac{1}{2} g_1^2 N_f \right) + \ln(2) \left( 3 \lambda_t g_2^2 + 4 g_2^2 N_f \right) \right], \]

(6.24)

Below are the expressions for the quantities which are used above:

\[ g_{2,3}^2 = g_2^2 (\Lambda) T \left[ 1 + \frac{g_2^2}{4 \pi^2} \left( \frac{44 - N_d - 2 N_t}{6} L_b + \frac{2 + 4 N_t}{3} L_f \right) \right], \]

(6.25)

\[ g_{2,3}^2 = g_2^2 (\Lambda) T \left[ 1 + \frac{g_2^2}{4 \pi^2} \left( - \frac{N_d}{6} L_b - \frac{20 N_t}{9} L_f \right) \right], \]

(6.26)

\[ h_3 = \frac{g_2^2 (\Lambda) T}{3} \left[ 1 + \frac{1}{4 \pi^2} \left( \frac{44 - N_d - 2 N_t}{6} L_b + \frac{53}{6} - \frac{N_d}{3} - \frac{2 N_t}{3} - \frac{4 N_f}{3} (L_f - 1) \right) g_2^2 + \frac{g_1^2}{2} - 6 g_1^2 + 12 \lambda_t + 8 \lambda_{bt} \right], \]

(6.27)

\[ h_5 = \frac{g_2^2 (\Lambda) L_b}{4} \left[ 1 + \frac{1}{4 \pi^2} \left( \frac{3 g_2^2}{2} + \frac{1}{3} - \frac{N_d}{6} (2 + L_b) + \frac{20 N_t}{9} (L_f - 1) \right) g_2^2 - \frac{34}{3} g_1^2 + 12 \lambda_1 \right], \]

(6.28)

\[ \delta_3 = \frac{1}{2} \frac{g_2^2 (\Lambda) T}{1 + \frac{1}{4 \pi^2} \left[ \lambda_{bt} + 8 \lambda_t + g_2^2 \left( \frac{16 - N_d - 2 N_t}{3} - \frac{4}{3} N_f (L_f - 1) + L_b \frac{44 - N_d - 2 N_t}{6} \right) \right], \]

(6.29)

\[ \delta_5 = - \frac{1}{2} \frac{g_2^2 (\Lambda) T}{1 + \frac{1}{4 \pi^2} \left[ 4 \lambda_t + g_2^2 \left( - \frac{20 + N_d + 2 N_t}{3} - \frac{4}{3} N_f (L_f - 1) + L_b \frac{44 - N_d - 2 N_t}{6} \right) \right]}. \]

(6.30)

where, \( N_d = 1 \), \( N_t = 1 \) and \( N_f = 3 \) to identify the contributions from the SM Higgs doublet, the real triplet and the fermions, respectively.

In case of triplet, there is significant change after the two-loop corrections to the thermal masses are added. The upper mass bound which was previously 310 GeV is now constrained more and reduced to 259 GeV.

### 6.3 Constraints from DM relic

Both complex singlet and the inert triplet scenarios considered here offer a dark matter candidate being odd under \( Z_2 \). In order to fulfill the criteria of only dark matter candidate, the neutral component in both scenarios independently should satisfy the observed dark matter relic by the Planck experiments [111]

\[ \Omega_{DM} h^2 = 0.1199 \pm 0.0027. \]

(6.31)

The interactions of the \( Z_2 \) odd particles with the particles in the thermal bath are the gauge couplings and the quartic couplings and the values of these couplings are quite large. Hence, the DM particle is considered to be in equilibrium with the thermal bath initially. As the Universe expands, the interaction rates of the DM falls short to maintain the equilibrium number density and freezes out. After freeze out, the number density of the DM remains constant in the comoving frame which gives the DM relic abundance in the current epoch. So, we constrain our parameter space to satisfy the thermal relic abundance as given in Equation 6.31. In the case of singlet the main annihilation comes via s-channel Higgs boson on- or off-shell. It is noticed that for maximum region of parameter space the singlet dark matter matter can satisfy the required observed dark matter relic [75, 112, 113], thus seems phenomenologically much more viable. Contrastingly, the inert triplet scenario, the neutral part \( T^0 \) annihilates mainly \( W^+ W^- \) and co-annihilates via \( t^\pm T^0 \to W^\pm Z \) and thus demands \( m_{T^0} \geq 1176 \text{ GeV} \) [12] to satisfy the required dark matter relic in Equation 6.31. This is incompatible with the demand of first order phase transition at \( m_h = 125.5 \text{ GeV} \) that we just observed in the previous section which states \( m_T \geq 193 \text{ GeV} \) and \( m_{T^0} < 310.24 \text{ GeV} \). For the first-order phase transition occurring at temperatures after the freeze-out of species, the entropy injection during the first-order phase transition can lead to dilution of the relic species that has decoupled from the thermal bath in the early universe. This dilution factor can only reach on order of 10, in case of purely bosonic models which still does not make inert triplet model relic mass bound of TeV order viable [114]. Certainly, the inert triplet scenario cannot satisfy both demands: of obtaining the first order phase transition consistent with current experimental Higgs boson mass bound and satisfying the dark matter relic. A simple gateway would be one more contributor viz. singlet, in order to satisfy the dark matter relic which would also enhance the possibility of the first order phase transition even further [60–64].
7 Calculating frequency detectable by LISA, LIGO and BBO

The phase transition from symmetric phase to broken phase proceeds via bubble nucleation when bubbles of the false vacua nucleate in the sea of symmetric phase and then keep on expanding. These expanding bubbles collide and gives rise to Gravitational waves (GW) which is described below. The frequencies of such gravitational waves can be estimated via thermal parameters which are described in the next subsections. Before we move on to the calculation of the frequencies of the gravitational waves, let us revisit the effective potential in order to implement in the CosmoTransition [115]. The effective potential at finite temperature which can be written as;

\[ V_{\text{eff}} = V_{\text{tree}} + V_1(\phi, 0) + V_1(\phi, T), \]  

where \( V_{\text{tree}} \) is the tree-level potential, \( V_1(\phi, 0) \) is the quantum correction at the zero temperature and \( V_1(\phi, T) = \Delta V_1(\phi, T) = \Delta V_{\text{daisy/ri}}(\phi, T) \) as shown in Equation 2.4. The one-loop quantum correction at zero-temperature is estimated via Coleman-Weinberg method [90] working in the Feynman gauge and also implemented in CosmoTransition [115];

\[ V_1(\phi, T) = \frac{T^2}{2\pi^2} \sum_i n_i m_i^2 \left[ \log \frac{m_i^2}{T^2} - c_i \right]. \]  

where \( n_i \) and \( m_i \) are the degrees of freedom and field-dependent masses as described in Equation 3.2, Equation 3.3 and Equation 3.6, respectively. Here \(+(-)\) signs come for bosonic(fermionic) degrees of freedom. The expression for the potential coming from non-zero temperature including the daisy/ring resummation (also in the Feynman gauge) are expressed as [115];

\[ V_1(\phi, T) = \frac{T^2}{2\pi^2} \sum_i n_i J_\pm \left[ \frac{m_i^2}{T^2} \right], \]  

where \( J_\pm \) are spline functions with \(+(-)\) for bosons(fermions), respectively and are defined as;

\[ J_\pm \left( \frac{m_i^2}{T^2} \right) = \int_0^\infty dy y^2 \log \left( 1 \mp e^{-y^2 + m_i^2/T^2} \right). \]  

Next we discuss the relevant parameters needed to calculate the frequencies of the Gravitational Waves(GW) using CosmoTransition [115] and Bubbleprofiler[116].

7.1 Thermal parameters

The Gravitational Waves(GW) are created when bubble collision occurs and thus depends on the bubble nucleation rate as given below [65]

\[ \Gamma(t) = A(t) e^{-S_3(t)}, \]  

where \( S_3 \) is the Euclidean action of the background field \( \phi \) written in spherical polar coordinate, of the critical bubble as follows[65]:

\[ S_3 = 4\pi \int drr^2 \left[ \frac{1}{2} (\partial_\phi)^2 + V_{\text{eff}} \right]. \]  

Here, \( V_{\text{eff}} \) is the total potential as given in Equation 7.1.

The temperature of the thermal bath at time \( t_* \) is defined as \( T_* \) and without significant reheating effect, \( T_* \approx T_n, \) the nucleation temperature. At the nucleation temperature \( T_n, \) the bubble nucleation starts and the bubble nucleation rate, \( \Gamma, \) should be large enough that a bubble is nucleated per horizon volume with probability of order 1[65]. In terms of bubble nucleation rate, inverse time duration of the phase transition, \( \beta \) is given as

\[ \beta = -\frac{dS}{dt} \bigg|_{t=t_*} \approx \frac{\Gamma}{\Gamma}. \]  

\( t_* \) being the instant of time where the first order phase transition completes. The parameter \( \beta \) defines the time variation of the bubble nucleation rate and therefore describe the length of the time in which the phase transition occurs. There are two relevant parameters which control the Gravitational Wave (GW) signal, one of them is the fraction \( \frac{\beta}{H_*} \), where \( H_* \) is the Hubble parameter at temperature \( T_* \). To achieve large Gravitational wave (GW) signal, relatively slow phase transition is required and hence the fraction, \( \frac{\beta}{H_*} \) should be small for stronger signals. This ratio \( \frac{\beta}{H_*} \) instrumental for this is defined as

\[ \frac{\beta}{H_*} = T_* \frac{dS}{dt} \bigg|_{t=t_*}, \]  

where \( T_* \) is the temperature at time \( t_* \), i.e. \( T_* = T_{t_*} \), and it becomes \( T_* \approx T_n \) with negligible reheating effect. The ratio \( \frac{\beta}{H_*} \) required for the visible signal in LISA is \( \frac{\beta}{H_*} \lesssim 10^4 \) [117]. This is a dimensionless quantity and it mainly depends on
the effective potential size at the nucleation temperature. The another essential parameter is $\alpha$, defined as the ratio of the vacuum energy density which is released during the phase transition to that of radiation bath and it is defined as below;

$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}}$$  \hspace{1cm} (7.9)

where $\rho_{\text{rad}}^* = g_* \pi^2 T_*^4/30$, and $g_*$ is the number of relativistic degrees of freedom at temperature $T_*$. Other relevant parameters for the appraisal of the GW frequencies are

$$\kappa_v = \frac{\rho_v}{\rho_{\text{vac}}} \quad \kappa_\phi = \frac{\rho_\phi}{\rho_{\text{vac}}}$$  \hspace{1cm} (7.10)

where $\kappa_v$ is the fraction of vacuum energy that is converted into bulk motion of the fluid and $\kappa_\phi$ is the fraction of vacuum energy converted into gradient energy of the Higgs-like field. And $v_w$ is defined as the fluid bubble wall velocity.

### 7.2 Production of the Gravitational Wave signal

The first order phase transition happen via bubble nucleation and because of the pressure difference between the false and true vacua these bubbles start expanding. The collision of these bubbles then break the spherical symmetry of each bubble and Gravitational waves (GW) are produced while for uncollided bubbles, the spherical symmetry remains preserved and no Gravitational waves (GW) are produced. The Gravitational wave background spectrum arising from cosmological phase transition depends on various sources. The sources which are most relevant for the GW, depend on the dynamics of bubble expansion an the plasma as we discuss below.

### 7.3 Relevant contributions to the Gravitational Wave spectrum

The following processes are involved in first-order phase transition for the production of Gravitational Waves:

- **Bubble wall collision** [118–123] and shocks in the plasma. The technique referred as ‘envelope approximation’ is used in this scenario. In this approximation, the contribution of scalar field, $\phi$, is considered in computing the GW spectrum.

- **Sound waves in the plasma**: when a part of energy released in the transition is dissipated as kinetic energy resulting in the bulk motion of fluid in plasma. [124–128].

- **Bubble collision** leads to the formation of Magnetohydrodynamic turbulence in the plasma [129–133].

These three processes generally coexist and linearly combine to give the contribution to the GW background as follows [134]:

$$h^2 \Omega_{\text{GW}} \simeq h^2 \Omega_{\phi} + h^2 \Omega_{vw} + h^2 \Omega_{\text{turb}}.$$  \hspace{1cm} (7.11)

The detailed forms of each contributions are discussed successively.

**Bubble Collision**: The scalar field contribution to the Gravitational Wave (GW), involved in the phase transition can be treated by envelope approximation [119, 121]. In "envelope approximation", the expanding bubbles are configured with the overlapping of corresponding set of infinitely thin shells. Once the phase transition is completed, the envelope disappears and the production of Gravitational waves (GW) stops. It has been found that the peak frequency for the Gravitational wave (GW) signal is determined by the average size of the bubble at collision. The GW contribution to the spectrum using the envelope approximation via numerical simulations can be written as,

$$h^2 \Omega_{\text{env}}(f) = 1.67 \times 10^{-5} \left(\frac{\beta}{\Pi} \right)^{-2} \left( \frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} \left( \frac{0.11 v_w^3}{0.42 + v_w^3} \right) \frac{3.8(f/f_{\text{env}})^{2.8}}{1 + 2.8(f/f_{\text{env}})^{4.8}},$$  \hspace{1cm} (7.12)

with

$$\beta = \left[ HT \frac{d}{dT} \left( \frac{S_3}{T} \right) \right]_{T_n}.$$  \hspace{1cm} (7.13)

where $T_n$ is defined as the nucleation temperature and $H_0$ is the Hubble parameter at temperature $T_n$. The estimation of the bubble wall velocity $v_w$ used in the above equation is given as [122, 135–137];

$$v_w = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha}. \hspace{1cm} (7.14)$$

The $\kappa_\phi$ parameter used in the calculation is defined as the fraction of latent heat deposited in a thin shell and is expressed as,

$$\kappa_\phi = 1 - \frac{\alpha_{\infty}}{\alpha}.$$  \hspace{1cm} (7.15)
The collision of bubbles can also induce turbulent motion of fluid [139]. This can give rise to Gravitational waves (GW). Therefore, the contribution to the Gravitational wave from sound wave (SW) can be estimated as follows.

\[
\rho_{\text{vac}} = (V_{\text{eff}}^{\text{high}} - T \frac{dV_{\text{eff}}^{\text{high}}}{dT}) - (V_{\text{eff}}^{\text{low}} - T \frac{dV_{\text{eff}}^{\text{low}}}{dT})
\]

where \( \kappa \) is defined in Equation 7.10 as the fraction of latent heat which is transferred to the bulk motion of the fluid, can be rewritten as

\[
\kappa_v = \frac{\alpha_{\infty}}{\alpha_{\infty}} [\frac{0.73 + 0.083\sqrt{\alpha_{\infty}}}{\alpha_{\infty} + \alpha_{\infty}}].
\]

The peak frequency contribution \( f_{SW} \) to the GW spectrum produced by sound wave mechanisms is

\[
f_{SW} = 1.9 \times 10^{-5} Hz \left( \frac{\beta}{M} \right) \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_{*}}{100} \right)^{\frac{1}{2}}.
\]

Turbulence: The collision of bubbles can also induce turbulent motion of fluid [139]. This can give rise to Gravitational waves (GW) even after the transition is finished. Lastly, the contribution to GW from the Magnetohydrodynamic turbulence can be evaluated as

\[
h^2 \Omega_{\text{turb}} = 3.35 \times 10^{-4} \left( \frac{\beta}{M} \right)^{-1} v_w \left( \frac{\epsilon \kappa_v \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left( \frac{g_{*}}{100} \right)^{-\frac{1}{2}} \left( \frac{f_{\text{turb}}}{f_{SW}} \right)^{11} \left( \frac{1 + \frac{f_{\text{turb}}}{f_{SW}}}{1 + \frac{g_{*}}{100}} \right),
\]

where \( \epsilon = 0.1 \) and \( f_{\text{turb}} \) is again the peak frequency contribution to the GW spectrum produced by the turbulence mechanism

\[
f_{\text{turb}} = 2.7 \times 10^{-5} Hz \left( \frac{\beta}{M} \right) \left( \frac{T_n}{100 \text{GeV}} \right) \left( \frac{g_{*}}{100} \right)^{\frac{1}{2}}.
\]

The updated expression for \( \kappa_v \), given in Eq. (7.21) which is used in this analysis is as follows:

\[
\kappa_v \approx \frac{\alpha_{\infty}}{0.73 + 0.083\sqrt{\alpha_{\infty} + \alpha_{\infty}}}
\]

\[7.26\]
7.4 Benchmark points

In this section we compare the triplet and the singlet scenarios with their gravitational wave frequencies detectable by LISA, LIGO and BBO experiments [85–87]. For this purpose we choose the benchmark points in the singlet and the triplet cases as given in Table 3.

The thermal parameters required for the calculation of GW spectrum are mainly the nucleation temperature \( T_n \), the strength of phase transition \( \alpha \), length of the time of phase transition \( \beta \), Higgs vev at the nucleation temperature \( v_n \) and the bubble wall velocity \( v_w \). The calculation of the Gravitational Wave(GW) intensity requires the phase transition temperature. Hence, the finite temperature effective potential is computed for the calculation of transition temperature. These calculations are performed using a publicly available package \texttt{CosmoTransition}[115]. The tree-level potential is given as an input to this package and it provides the thermal parameters required for the calculation of Gravitational Wave(GW) intensity. These thermal parameters corresponding to the benchmark points in Table 3, predicting strongly first order phase transition and allowed by 125.5 GeV Higgs boson are shown in Table 4–Table 5 for the singlet and the triplet scenarios, respectively.

### Table 3. BPs for frequency analysis for singlet and triplet scenario.

|      | \( m_S/m_T \) | \( \lambda_s/\lambda_t \) | \( \lambda_{hs}/\lambda_{ht} \) |
|------|--------------|-----------------|-----------------|
| BP1  | 150.23       | 0.10            | 0.10            |
| BP2  | 120.23       | 0.01            | 0.01            |

### Table 4. Thermal parameters required for frequency analysis of the singlet for the chosen benchmark points, where \( T_n \) is the nucleation temperature, \( \alpha \) is the strength of transition, \( \beta \) is the length of the time of phase transition and \( v_n \) is the Higgs vev at the nucleation temperature.

|      | BP1       | BP2       |
|------|-----------|-----------|
| \( T_n \)[GeV] | 121.03    | 119.25    |
| \( \alpha \)     | 0.17      | 0.18      |
| \( \beta/H \)    | 332.83    | 327.94    |
| \( v_n/T_n \)    | 1.10      | 1.16      |

### Table 5. Thermal parameters required for frequency analysis of the inert triplet for the chosen benchmark points where \( T_n \) is the nucleation temperature, \( \alpha \) is the strength of transition, \( \beta \) is the length of the time of phase transition and \( v_n \) is the Higgs vev at the nucleation temperature.

|      | BP1       | BP2       |
|------|-----------|-----------|
| \( T_n \)[GeV] | 115.07    | 113.55    |
| \( \alpha \)     | 0.86      | 0.89      |
| \( \beta/H \)    | 284.22    | 278.87    |
| \( v_n/T_n \)    | 1.16      | 1.22      |

The Gravitational wave(GW) spectrum arising from the first-order phase transition for the benchmark points are given in Figure 10. The constraints for different experiments are drawn by the respective sensitivity curves for the different GW detectors viz. LISA, LIGO and BBO. The higher value of \( \alpha \) and lower value of \( \beta \) actually provides stronger GW signals. It is clear from Table 4 and Table 5 that the nucleation temperature \( T_n \) is lower than the critical temperature \( T_c \) for all benchmark points in singlet and triplet and the value of ratio \( v_n/T_n \) is \( \gtrsim 1 \), giving strongly first-order phase transition. The values of nucleation temperature for inert triplet, \( T_n = 115.07 (113.55) \) GeV are lower compared to singlet ones, \( T_n = 121.03 (119.25) \) GeV, that ensure stronger signals detectable by various experiments. For both the benchmark points, the GW intensity lie within the sensitivity curves of LISA and BBO in the singlet and the triplet scenarios, respectively. The detectable frequencies for singlet lie between \( \sim 1.15 \times 10^{-3} - 1.06 \times 10^{-2} \) Hz, while for the triplet, the allowed ranges enhance to range \( \sim 4.18 \times 10^{-4} - 1.99 \times 10^{-2} \) Hz, for the LISA experiment as can be seen from Figure 10. It is also inferred from Figure 10 that
the Gravitational Wave (GW) intensity mainly depends on the parameter $\beta$. The smallest value of parameter $\beta$ is attained for BP2 of the inert triplet scenario, which leads to highest Gravitational wave (GW) intensity. For LIGO, the Gravitational Wave (GW) intensities lie outside the detectable region in both the singlet and the triplet scenarios. In comparison BBO has more region of parameter space that can be detected for both, with triplet having larger spectrum with slight larger frequency range compared to the singlet case. Also the signal to noise ratio (SNR) for a particular detector, which is given as

$$\text{SNR} = \sqrt{\frac{2t_{\text{obs}}}{t_{\text{min}}} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\Omega_{\text{GW}}(f) h^2}{\Omega_{\text{noise}}(f) h^2}}$$

where $t_{\text{obs}}$ is defined as the duration of the observation in unit of seconds and $\Omega_{\text{noise}}(f) h^2$ is defined as the effective strain noise power spectral density for the considered detector. $\Omega_{\text{GW}}(f)$ is detectable for signal to noise ratio (SNR) $\text{SNR} > 1$, which is possible for $\Omega_{\text{noise}}(f) < \Omega_{\text{GW}}$. Therefore, there is a finite chance that the frequency range in Figure 10 covered by BBO experiment is detectable. [142–147]

However, the future advanced Gravitational wave (GW) detectors such as eLISA and BBO are expected to explore millihertz to decihertz of frequency ranges in future. Similarly the ground based detector like aLIGO can explore the lower frequency range with much higher sensitivity. There can be two to three orders-of-magnitude theoretical uncertainty in the peak GW amplitude using daisy-resummation approach due to renormalization scale dependence. Using higher order terms in the perturbative calculations i.e. dimensional reduction approach, the scale dependence can be reduced and the theoretical uncertainty can be reduced to $\mathcal{O}(10^0 - 10^1)$ [107, 148].

In order to ensure that the physical quantities in any field theory are independent of the particular renormalization scheme (RS), if the true result is exactly RS independent then the best approximation should be least sensitive to the small changes in RS. This is known as principle of minimal sensitivity [149]. This principle states that for unphysical parameters, the exact result is a constant. Hence, the calculated result cannot be a successful approximation where it is varying rapidly. If the variation is considered with the renomalization scale then the extrapolation from $10^2$ GeV can be judged by observing how flat the result is at higher scales. The variation will not be flat everywhere, so one can always choose the scale to lie in the middle of the flat portion of the variation. The variation of all the quartic couplings become almost constant after $10^6$ GeV, and further higher scales. Therefore, we consider the variation of the quartic couplings using the two-loop $\beta$-functions in the Daisy resummation approach including the two-loop potential from Equation 5.6-Equation 5.7 at three different scales i.e. $10^2$ GeV, $10^3$ GeV and $10^6$ GeV, respectively. The benchmark points and the thermal parameters for singlet and triplet scenario are given in Table 6 to 8.

The Gravitational Wave (GW) spectrum for the singlet and the triplet scenarios are given in Figure 11. The blue and the green intensity curves correspond to the singlet and the triplet scenarios and the dotted purple, orange and cyan curves
Table 6. Allowed benchmark point for the frequency analysis in singlet and triplet scenario at three different renormalization scales i.e $10^2$ GeV, $10^3$ GeV and $10^6$ GeV using Daisy resummation method with two-loop potential and two-loop $\beta$-functions. The quartic couplings are given for three different renormalization scale variation followed by the running of the two-loop $\beta$-functions.

| $\mu$ (GeV) | $m_S/m_T$ (GeV) | $\lambda_s/\lambda_t$ | $\lambda_{hs}/\lambda_{ht}$ | $\lambda_h$ |
|-------------|-----------------|-----------------------|-----------------------------|------------|
| $10^2$      | 190.23          | 0.10                  | 0.33                        | 0.1264     |
| $10^3$      | 190.23          | 0.11/0.11             | 0.36/0.36                   | 0.1040/0.1040 |
| $10^6$      | 190.23          | 0.19/0.22             | 0.48/0.53                   | 0.1198/0.1253 |

Table 7. Thermal parameters required for the frequency analysis in case of singlet for chosen benchmark points where $T_n$ is the nucleation temperature, $\alpha$ is the strength of transition, $\beta$ is the length of the time of phase transition and $v_n$ is the Higgs vev at the nucleation temperature.

|                  | $10^2$ | $10^3$ | $10^6$ |
|------------------|--------|--------|--------|
| $T_n$ [GeV]      | 130.73 | 131.69 | 185.91 |
| $\alpha$         | 0.15   | 0.15   | 0.10   |
| $\beta/H$        | 292.83 | 295.94 | 327.68 |
| $v_n/T_n$        | 0.97   | 0.97   | 0.83   |

Table 8. Thermal parameters required for the frequency analysis in case of the inert triplet for chosen benchmark points where $T_n$ is the nucleation temperature, $\alpha$ is the strength of transition, $\beta$ is the length of the time of phase transition and $v_n$ is the Higgs vev at the nucleation temperature.

|                  | $10^2$ | $10^3$ | $10^6$ |
|------------------|--------|--------|--------|
| $T_n$ [GeV]      | 128.50 | 129.60 | 181.07 |
| $\alpha$         | 0.16   | 0.16   | 0.11   |
| $\beta/H$        | 291.56 | 294.23 | 320.68 |
| $v_n/T_n$        | 0.98   | 0.98   | 0.84   |
Figure 11. Gravitational Wave (GW) spectrum for the BP allowed by strongly first-order phase transition and perturbative unitarity in comparison with the sensitivity curves based on noise curves of experiments i.e. LISA, LIGO and BBO for three different renormalization scales i.e. $10^2$ GeV, $10^3$ GeV and $10^6$ GeV using Daisy resummation method including the two-loop potential and two-loop $\beta$-functions. The blue and green curve corresponds to the Singlet and the triplet scenarios. The dotted purple, orange and cyan curves denotes the sensitivity curves LISA, BBO and LIGO experiments.

to the compressed spectrum [12, 74, 75, 83]. $Y = 0$, triplet unlike the SM doublet does not couple to fermions which alters the bounds on rare $B$-decays [76, 79] and also difficult to produced at the collider. However, vector boson fusion to charged Higgs and other associate production can be analysed in the $ZW$ decay mode of the charged Higgs via multi-lepton final states, where the triplet takes vev [80–82]. On the contrary, the singlet does not have any charged Higgs bosons and being gauge singlet, it cannot be produced via gauge bosons. The productions are mainly come via the mixing with the SM Higgs bosons or SM Higgs boson decay to singlet pair [35, 118] and for inert singlet it is bounded by Higgs to invisible decay width [40, 119]. Lastly, inert singlet model satisfies the DM relic density bound even with the very small singlet mass along with the first-order phase transition [115–117], but for the triplet it shows under abundance demanding such low triplet mass required for the first-order phase transition [75].

8 Conclusion

In this article we study the $Y = 0$ SU(2) inert triplet which successfully stabilises the electroweak vacuum at the zero temperature and also provide the DM candidate[12], at the finite temperature. The regions of parameter space suitable for the first order phase transitions are designated considering perturbative unitarity at one- and two-loop level along with the demand of a SM-like Higgs boson around 125.5 GeV. It has been noticed that no consistent solutions have been found at
one-loop perturbativity till Planck scale consistent with first order phase transition, and current Higgs boson and top quark masses. Considering the two-loop beta functions with the one-loop resummed potential, one can find the maximum mass values for the singlet and the triplet field as 909, 310 GeV, respectively predicting the first order phase transition which are also consistent with the currently measured Higgs boson and top quark masses. Including the two-loop contributions coming from the effective potential as well as the thermal masses, the mass bound for the singlet remains the same, while satisfying the current Higgs mass within the uncertainty of 1σ. On the other hand, the mass bound for the inert triplet is further constrained to $\lesssim 259$ GeV with these corrections. However, these maximum allowed values of mass correspond to relatively larger values of $\lambda_{ts} (\lambda_{tb}) = 4.00 (1.95)$, respectively. For lower values of these masses correspond to the regions with higher $\phi_s (T_c)$ i.e., more strongly first order phase transition. The self couplings of the singlet and the triplet are considered to be zero to maximize the $\phi_s (T_c)$.

It is interesting to note here that for the singlet mass $\lesssim 909$ GeV one not only realises first order phase transition along with a Higgs boson mass around 125.5 GeV, but also find the parameter space consistent with DM relic [75, 112, 113]. On the contrary, the situation looks grim for the triplet scenario as the correct DM relic abundance demands the triplet scalar mass $\lesssim 1.2 - 1.8$ TeV. Thus with only triplet extension of the SM, we cannot have the first order phase transition along with the correct DM relic. Triplet DM mass $< 320$ GeV gives rise to under abundance for the DM and we need additional fields to satisfy the correct relic [75].

First order phase transition in both cases can give rise to gravitational wave coming from the bubble collision, sound wave of the plasma and the turbulence. These add up to the frequencies that can be observed via the space and earth bases experiments like LISA [86], BBO [85] and LIGO [87]. To observe and distinguish the singlet and the triplet scenarios we benchmark both singlet and triplet scenarios and predict their frequencies observed by various different detectors. The detectable frequency range by LISA is more for the triplet i.e. $\sim 4.18 \times 10^{-4} - 1.99 \times 10^{-2}$ Hz, in comparison to the singlet i.e. $\sim 1.15 \times 10^{-3} - 1.06 \times 10^{-2}$ Hz. For all the benchmark points, the Gravitational wave(GW) intensities lie within the detectable range of LISA and BBO in both singlet and triplet scenarios. With the increase of the renormalization scale using the Daisy resummation method with two-loop $\beta$-functions and two-loop potential, the Gravitational Wave (GW) intensity and also the detectable frequency drop. Thus, the singlet model, constrained from perturbative unitarity and DM relic, is in agreement with the sensitivity curves of Gravitational wave(GW) detectors. However, for the triplet case, the strongly first order phase transition predicts relatively lower mass for the triplet ($\lesssim 320$ GeV), demanding additional multiplets to satisfy the DM relic.

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A Two-loop $\beta$-functions for ITM

A.1 Scalar Quartic Couplings

$$
\beta_{\lambda=\lambda_1} = \frac{1}{16\pi^2} \left[ \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - \frac{9}{5} g_1^4 \lambda_1 - 9 g_2^2 \lambda_1 + 24 \lambda_1^2 + 8 \lambda_{ht}^2 + 12 \lambda_1 \text{Tr} \left( Y_d Y_d^t \right) + 4 \lambda_1 \text{Tr} \left( Y_u Y_u^t \right) \\
+ 12 \lambda_1 \text{Tr} \left( Y_d Y_d^t Y_d Y_d^t \right) - 6 \text{Tr} \left( Y_d Y_d^t Y_u Y_u^t \right) - 2 \text{Tr} \left( Y_d Y_d^t Y_u Y_u^t \right) - 6 \text{Tr} \left( Y_u Y_u^t Y_d Y_d^t \right) \right]
$$

$$
+ \frac{1}{(16\pi^2)^2} \left[ \frac{3411}{2000} g_1^8 - \frac{1677}{400} g_1^6 g_2^2 - \frac{317}{80} g_1^4 g_2^4 + \frac{277}{16} g_2^6 + \frac{1887}{200} g_1^2 \lambda_1 + \frac{117}{20} g_1^2 g_2^2 \lambda_1 - \frac{29}{8} g_2^4 \lambda_1 \\
+ \frac{108}{5} g_1^4 \lambda_1^2 + 10 g_2^4 \lambda_1^2 - 312 \lambda_1^2 + 10 g_2^2 \lambda_{ht} + 32 g_2^2 \lambda_{ht}^2 - 80 \lambda_1 \lambda_{ht} - 128 \lambda_{ht}^2 \\
+ \frac{1}{20} \left( 5 \left( \frac{64\lambda_1}{1} - 5 \lambda_1^2 + 9 \lambda_1 \right) - 90 g_2^2 \lambda_1 + 9 g_1^4 \right) + g_1^4 \left( 50 \lambda_1 + 54 g_2^2 \right) \right] \text{Tr} \left( Y_d Y_d^t \right) \\
- \frac{3}{20} \left( 15 g_1^4 - 2 g_1^2 \left( 11 g_1^2 + 25 \lambda_1 \right) + 5 \left( - 10 g_2^2 \lambda_1 + 64 \lambda_1^2 + g_1^4 \right) \right) \text{Tr} \left( Y_u Y_u^t \right) - \frac{171}{100} g_1^4 \text{Tr} \left( Y_d Y_d^t Y_u Y_u^t \right) \\
+ \frac{63}{10} g_1^2 g_2^2 \text{Tr} \left( Y_u Y_u^t \right) - \frac{9}{4} g_2^4 \text{Tr} \left( Y_u Y_u^t \right) + \frac{17}{2} g_1^2 \lambda_1 \text{Tr} \left( Y_u Y_u^t \right) + \frac{45}{2} g_2^2 \lambda_1 \text{Tr} \left( Y_u Y_u^t \right) \\
+ 80 g_2^2 \lambda_1 \text{Tr} \left( Y_u Y_u^t \right) - 144 \lambda_1^2 \text{Tr} \left( Y_u Y_u^t \right) - \frac{4}{5} g_2^4 \text{Tr} \left( Y_u Y_u^t \right) - 32 g_2^2 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) \\
- 3 \lambda_1 \text{Tr} \left( Y_u Y_u^t Y_d Y_d^t \right) - 42 \lambda_1 \text{Tr} \left( Y_u Y_u^t Y_d Y_d^t \right) - \frac{12}{5} g_1^4 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) \lambda_1 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) \\
- \frac{8}{5} g_1^2 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) - 32 g_2^2 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) - 3 \lambda_1 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) + 30 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) Y_u Y_u^t Y_u Y_u^t \\
- 12 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) Y_u Y_u^t Y_u Y_u^t \right) + 6 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) Y_u Y_u^t Y_u Y_u^t \\
+ 10 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) Y_u Y_u^t Y_u Y_u^t \right) + 30 \text{Tr} \left( Y_u Y_u^t Y_u Y_u^t \right) Y_u Y_u^t Y_u Y_u^t \right)
$$

$$
\beta_{\lambda_t} = \frac{1}{16\pi^2} \left[ -24 g_2^2 \lambda_t + 88 \lambda_t^2 + 8 \lambda_{ht}^2 + \frac{3}{2} g_2^2 \lambda_t \right] \\
+ \frac{1}{(16\pi^2)^2} \left[ -\frac{68}{3} g_1^6 + 10 g_2^2 \lambda_t + \frac{48}{5} g_1^2 \lambda_{ht}^2 + 48 g_2^2 \lambda_{ht}^2 - 128 \lambda_{ht}^4 + \frac{94}{3} g_2^4 \lambda_t - 320 \lambda_t \lambda_{ht} + 64 g_2^2 \lambda_t^2 \\
- 4416 \lambda_t^3 - 48 \lambda_{ht}^2 \text{Tr} \left( Y_u Y_u^t \right) - 16 \lambda_{ht}^2 \text{Tr} \left( Y_u Y_u^t \right) - 48 \lambda_{ht}^2 \text{Tr} \left( Y_u Y_u^t \right) \right]
$$

$$
\beta_{\lambda_{ht}} = \frac{1}{16\pi^2} \left[ \frac{3}{4} g_2^2 - \frac{9}{10} g_1^2 \lambda_{ht} + \frac{33}{2} g_2^2 \lambda_{ht} + 12 \lambda_1 \lambda_{ht} + 16 \lambda_1^2 + 24 \lambda_1 \lambda_t + 6 \lambda_{ht} \text{Tr} \left( Y_d Y_d^t \right) + 2 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) \\
+ 6 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) \right]
$$

$$
+ \frac{1}{(16\pi^2)^2} \left[ -\frac{9}{10} g_1^4 \lambda_{ht} + \frac{329}{48} g_6 + \frac{15}{2} g_2^4 \lambda_t + \frac{1671}{400} g_1^4 \lambda_{ht} + \frac{9}{8} g_1^2 g_2^2 \lambda_{ht} - \frac{1087}{48} g_1^2 \lambda_{ht} + \frac{72}{5} g_1^4 \lambda_{ht} \\
+ 72 g_2^2 \lambda_1 \lambda_{ht} - 60 g_1^2 \lambda_{ht} + \frac{12}{5} g_1^2 \lambda_{ht}^2 + 44 g_2^2 \lambda_{ht}^2 - 288 \lambda_1 \lambda_{ht}^2 - 168 \lambda_{ht}^3 + 20 g_2^4 \lambda_t + 144 g_2^2 \lambda_{ht} \lambda_t \\
- 576 \lambda_{ht}^3 - 544 \lambda_1 \lambda_{ht}^2 - \frac{1}{4} \left( 3 g_1^4 - 45 g_2^2 \lambda_{ht} + \lambda_{ht} \left( -160 g_2^2 + 192 \lambda_{ht} + 288 \lambda - 5 g_2^4 \right) \right) \text{Tr} \left( Y_d Y_d^t \right) \\
- \frac{1}{4} \left( -15 g_2^2 \lambda_{ht} + \lambda_{ht} \left( -15 g_2^2 + 64 \lambda_{ht} + 96 \lambda_1 \right) + g_2^2 \lambda_{ht} \right) \text{Tr} \left( Y_u Y_u^t \right) - \frac{3}{4} g_2^4 \text{Tr} \left( Y_u Y_u^t \right) \\
+ \frac{17}{4} g_1^2 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) + \frac{45}{4} g_2^2 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) + 40 g_2^2 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) - 72 \lambda_1 \lambda_{ht} \text{Tr} \left( Y_u Y_u^t \right) \\
- 27 -
$$
B Two-loop $\beta$-functions for Singlet

B.1 Scalar Quartic Couplings

$$\beta_{\lambda_s}^{(1)} = \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - \frac{9}{5} g_1^2 \lambda_1 - 9 g_2^2 \lambda_1 + 24 \lambda_s^2 + 4 \lambda_s^4 + 12 \lambda_1 \text{Tr}(Y_d Y_d') + 4 \lambda_1 \text{Tr}(Y_e Y_e')$$

$$\beta_{\lambda_s}^{(1)} = \frac{27}{2} \lambda_1 \text{Tr}(Y_d Y_d') - \frac{27}{2} \lambda_1 \text{Tr}(Y_e Y_e')$$

$$- 48 \lambda_s^2 \text{Tr}(Y_d Y_d') - \frac{27}{2} \lambda_1 \text{Tr}(Y_d Y_d') + 21 \lambda_1 \text{Tr}(Y_d Y_d') - \frac{9}{2} \lambda_1 \text{Tr}(Y_e Y_e') - \frac{27}{2} \lambda_1 \text{Tr}(Y_e Y_e') \right].$$

\[ - 48 \lambda_s^2 \text{Tr}(Y_d Y_d') - \frac{27}{2} \lambda_1 \text{Tr}(Y_d Y_d') - 21 \lambda_1 \text{Tr}(Y_d Y_d') - \frac{9}{2} \lambda_1 \text{Tr}(Y_e Y_e') - \frac{27}{2} \lambda_1 \text{Tr}(Y_e Y_e') \]
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