Monopole Condensation and Confinement in SU(2) QCD (2)

Hiroshi Shiba	extsuperscript{1} and Tsuneo Suzuki	extsuperscript{2}

Department of Physics, Kanazawa University, Kanazawa 920-11, Japan

(August 8, 2018)

Monopole and photon contributions to Wilson loops are calculated using Monte-Carlo simulations of SU(2) QCD in the maximally abelian gauge. The string tensions of SU(2) QCD are well reproduced by extended monopole contributions alone.

12.38.Aw, 12.38.Gc, 14.80.Hy

To understand color confinement mechanism in QCD is absolutely necessary for us to analytically explain hadron physics starting from QCD \cite{1, 2}. The 'tHooft idea of abelian projection in which a partial gauge-fixing is done keeping the maximal abelian torus group unbroken is very interesting \cite{3}. Then QCD can be regarded as a $U(1) \times U(1)$ abelian gauge theory with magnetic monopoles and electric charges. An interesting gauge has been found among infinite ways of gauge-fixing for the abelian projection. It is called maximally abelian (MA) gauge \cite{4–7} in which link gauge fields are forced to become abelian as much as possible.

In the preceding note, the authors \cite{8} have shown in the MA gauge and in SU(2) QCD that entropy dominance over energy of the monopole loops, i.e., condensation of the monopole loops occurs in the confinement phase if extended monopoles \cite{10} are considered. After the abelian projection in the MA gauge, infrared behaviors of SU(2) QCD seem to be described by a compact-QED like U(1) theory with the running coupling constant instead of the bare one and with the monopole mass on a dual lattice. The confinement in SU(2) QCD may be interpreted as the (dual) Meissner effect due to the abelian monopole condensation.

If the monopoles alone are responsible for the confinement mechanism, the string tension which is a key quantity of confinement must be explained by monopole contributions. This is realized in compact QED as shown recently by Stack and Wensley \cite{11}. The aim of this note is to show that the same thing happens also in SU(2) QCD by means of evaluating monopole and photon contributions to Wilson loops.

After the abelian projection in the MA gauge, a diagonal matrix $u(s, \mu)$ can be extracted uniquely from the original SU(2) link field. The diagonal matrix $u(s, \mu)$ corresponds to a $U(1)$ gauge field written by an angle variable $\theta_\mu(s)$.

Now we show an abelian Wilson loop operator (which we consider after the abelian projection) is rewritten by a product of monopole and photon contributions. Here we take into account only a simple Wilson loop, say, of size $I \times J$. Then such an abelian Wilson loop operator is expressed as

$$W = \exp \{ i \sum J_\mu(s) \theta_\mu(s) \},$$

where $J_\mu(s)$ is an external current taking $\pm 1$ along the
Wilson loop. Since \( J_\mu(s) \) is conserved, it is rewritten for such a simple Wilson loop in terms of an antisymmetric variable \( M_{\mu\nu}(s) \) as \( J_\mu(s) = \partial_\mu M_{\mu\nu}(s) \), where \( \partial_\mu \) is a backward derivative on a lattice. \( M_{\mu\nu}(s) \) takes \( \pm 1 \) on a surface with the Wilson loop boundary. Although we can choose any surface of such a type, we adopt a minimal surface here. We get

\[
W = \exp\{-\frac{i}{2} \sum M_{\mu\nu}(s) f_{\mu\nu}(s) \},
\]

where \( f_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s) \) and \( \partial_\mu \) is a forward derivative on a lattice. The gauge plaquette variable can be decomposed into \( f_{\mu\nu}(s) = f_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s) \) where \( f_{\mu\nu}(s) \in [-\pi, \pi] \) corresponds to a field strength and \( n_{\mu\nu}(s) \) is an integer-valued plaquette variable denoting the Dirac string. Since \( M_{\mu\nu}(s) \) and \( n_{\mu\nu}(s) \) are integers, the latter does not contribute to Eq. (2). Hence \( f_{\mu\nu}(s) \) in Eq. (2) is replaced by \( f_{\mu\nu}(s) \). Using a decomposition rule

\[
M_{\mu\nu}(s) = - \sum D(s - s') \delta(s) [\partial_\nu(\partial_\mu M_{\alpha\nu} - \partial_\nu M_{\alpha\mu})(s')]
+ \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} \epsilon_{\alpha'\beta'\rho\sigma} \partial_\nu \partial_{\sigma} M_{\rho\sigma}(s')]
\]

we get

\[
W = W_1 \cdot W_2
\]

\[
W_1 = \exp\{-i \sum \partial_\mu f_{\mu\nu}(s) D(s - s') J_\nu(s') \}
\]

\[
W_2 = \exp\{2\pi i \sum k\beta(s) D(s - s') \frac{1}{2} \epsilon_{\alpha\beta\rho\sigma} \partial_\nu M_{\rho\sigma}(s') \}
\]

where a monopole current \( k\mu(s) \) is defined as \( k\mu(s) = (1/4\pi)\epsilon_{\mu\alpha\beta\gamma} \partial_\alpha f_{\beta\gamma}(s) \) following DeGrand-Toussaint [12]. \( D(s) \) is the lattice Coulomb propagator. Since \( f_{\mu\nu}(s) \) corresponds to the field strength of the photon field, \( W_1(W_2) \) is the photon (the monopole) contribution to the Wilson loop.

To study the features of both contributions, we evaluate the expectation values \( \langle W_1 \rangle \) and \( \langle W_2 \rangle \) separately and compare them with those of \( W \).

The Monte-Carlo simulations were done on 24^4 lattice from \( \beta = 2.4 \) to \( \beta = 2.8 \). All measurements were done every 30 sweeps after a thermalization of 1500 sweeps. We took 50 configurations totally for measurements. The gauge-fixing criterion is the same as done in Ref. [13]. Using gauge-fixed configurations, we evaluated monopole currents. As shown in the previous note [8], type-2 extended monopole loops with \( b > b_c \approx 5.2 \times 10^{-3}(\Lambda_L)^{-1} \) condense, where \( b = n\alpha(\beta) \) for \( n^3 \) extended monopoles and \( \alpha(\beta) \) is the lattice constant. So we measured \( 2^4 \) extended monopole with \( b = 2\alpha(\beta) \) of the type-2 [10]. Then the effective (renormalized) lattice volume becomes \( 12^4 \). Since the original lattice is \( 24^4 \), \( 2^4 \) extended monopoles are the largest from which we can get useful data of the static potentials from Wilson loops. For \( \beta = 2.7 \) and 2.8, the value \( b = 2\alpha(\beta) \) becomes less than \( b_c \) and so the monopoles may not reproduce the string tension.
We have evaluated the averages of $W$ using abelian link variables (called abelian), of $W_1 \cdot W_2$ (called total), of $W_1$ (photon part), and $W_2$ (monopole part), separately. Both the first and the second averages are evaluated to check reliability of the data, since both should be equivalent as known from Eq. (3).

The results are summarized as follows.

1. The monopole contributions to Wilson loops are obtained with relatively small errors. Surprisingly enough, the Creutz ratios of the monopole contributions are almost independent of the loop size as shown partially in Table I. This means that the monopole contributions are composed only of an area, a perimeter, and a constant terms.

2. Assuming the static potential is given by a linear + Coulomb + constant terms, we can determine them from the least square fit to the Wilson loops \[\text{[14]}\]. We plot their data in Fig. 1 (at $\beta = 2.5$) and in Fig. 2 (at $\beta = 2.6$). We find the monopole contributions are responsible for the linear-rising behaviors. The photon part contributes only to the short-ranged region. There seems to exist a small discrepancy between the abelian and the monopole + photon parts for $R/a = 12$, but finite-size effects are expected there. Similar data are obtained for $\beta = 2.4$.

3. This is seen more clearly from the data of the string tensions which are determined from the static potentials. They are shown in Fig. 3. Systematic errors coming from various least square fits are not completely certain and are not plotted in the figure, although they are not negligible. The string tensions are well reproduced by the monopoles alone for $\beta \leq 2.6$ and the photon part has almost vanishing string tensions. The string tensions evaluated from the total part (which are not shown here) are consistent with those of abelian and monopoles.

4. At $\beta = 2.7$ and 2.8, the monopoles which have $b < b_c$ do not seem to reproduce the abelian string tensions as shown in Table I. However the string tensions from the total part which should be equal to the abelian ones are also smaller. The origin of the difference resides in the smallness of the renormalized lattice volume. To check it, we have adopted only even-sized abelian Wilson loops which correspond to the total ones with the lattice spacing $2a(\beta)$ and made the least-square fit. The string tensions are almost unchanged for $\beta = 2.4, 2.5$, and 2.6 (see Fig. 3), but they become smaller for $\beta = 2.7$ and 2.8 with large errors. The difference between the abelian (even only) and the total becomes smaller. In conclusion, to get definite results for $\beta \geq 2.7$ in this framework of the analysis, we have to adopt much larger lattices like $48^4$. We can not conclude at present that the monopoles with
\( b > b_c \) alone can reproduce the string tensions as is expected.

5. We have derived also Coulomb coefficients from the static potentials as shown in Fig. 4. The monopole part has almost vanishing Coulomb coefficients which is in agreement with the constant behaviors of the Creutz ratios of the monopole part as shown above. The photon part has large coefficients, but they do not reproduce the coefficients of the abelian static potentials. But again they are not far from the Coulomb coefficients from Eq. (3) (called total in Fig. 4) and those from even-sized abelian Wilson loops. The following may be interesting. The photon parts are evaluated on an effective lattice with \( b = 2a(\beta) \). Hence they have different values of \( b \) for different \( \beta \). The Coulomb coefficients of the photon parts are well reproduced by the \( SU(2) \) running coupling constants \( g(b) \) with \( b = 2a(\beta) \), i.e., \( -g(b)^2/16\pi \), where

\[
\begin{align*}
g^{-2}(b) &= \frac{11}{24\pi^2} \ln\left(\frac{1}{b^2\Lambda^2}\right) + \frac{17}{44\pi^2} \ln\ln\left(\frac{1}{b^2\Lambda^2}\right). \\
&= \frac{11}{24\pi^2}\ln\left(\frac{1}{b^2\Lambda^2}\right) + \frac{17}{44\pi^2}\ln\ln\left(\frac{1}{b^2\Lambda^2}\right). \quad (4)
\end{align*}
\]

The scale parameter \( \Lambda \) determined is \( \Lambda \sim 46\Lambda_L \) which is quite near the value \( \Lambda \sim 42\Lambda_L \) fixed from the monopole action.

Our analyses in this note as well as in the previous one \[3\] show that abelian monopoles are responsible for confinement in \( SU(2) \) QCD and condensation of the monopoles is the confinement mechanism if the abelian projection is done in the MA gauge.

Finally we make some comments on the results of both notes.

1. Why is the MA gauge so nice? Note that an abelian projection reduces QCD into an abelian theory with diagonal gluons as a photon-like particle and off-diagonal gluons as charged particles. The characteristic features of the MA gauge among many abelian projections are in the following. The MA gauge is defined in such a way that a quantity

\[
R = \sum_{s, \mu} \{U_1(s, \mu)^2 + U_2(s, \mu)^2\}
\]

is minimized, where \( U_1(s, \mu) \) and \( U_2(s, \mu) \) are components of a \( SU(2) \) link field \( U(s, \mu) = U_0(s, \mu) + \hat{U}(s, \mu) \cdot \hat{\sigma} \). Hence the gauge condition forces as many link fields as possible to become abelian, i.e., diagonal. However one can not let all link fields diagonal, i.e., \( R = 0 \) as seen from a histogram analysis shown in Ref. \[3\] and from the correlation of \( R \) with the monopole density \[3\]. If we see only long-ranged physics, \( SU(2) \) QCD in the MA gauge may be regarded as a \( U(1) \) theory with abelian link photon fields and abelian monopoles.
Is there any other gauge showing similar behaviors? There seem to exist many candidates. For example, the following gauge may be interesting in which

$$R = \sum_{s, \mu} \{U_1(s, \mu)^2 + U_2(s, \mu)^2\}^n \quad (n > 0)$$

is minimized. The condition also forces link fields to be diagonal as much as possible. The case $n = 2$ leads us to $\sum_{s, \mu} A^+_\mu(s) A^-_{\mu}(s)(\partial_{\mu} \pm ig A^3_{\mu}(s))A^A_{\mu}(s) = 0$ in the continuum limit. The study in the gauge is in progress.

2. Using the method developed in Refs. [16,17], the abelian monopole action derived in [8] may be mapped on to a field theoretical model of an abelian (dual) Higgs system. The model is just equal to a Ginzburg-Landau type theory which one of the authors (T.S.) derived earlier assuming abelian dominance and monopole condensation [18].

3. To extend our method to a $T \neq 0$ system and also to $SU(3)$ with or without dynamical quarks is very interesting. These studies are also in progress.

We wish to acknowledge Yoshimi Matsubara for useful discussions especially on the entropy decreasing effects of the monopole loops. This work is financially supported by JSPS Grant-in Aid for Scientific Research (c)(No.04640289).

---

* e-mail address:shiba@hep.s.kanazawa-u.ac.jp
** e-mail address:suzuki@hep.s.kanazawa-u.ac.jp
[1] G.'tHooft, in High Energy Physics, edited by A.Zichichi (Editorice Compositori, Bologna, 1975).
[2] S. Mandelstam, Phys. Rep. 23C, 245 (1976).
[3] G. ’tHooft, Nucl. Phys. B190, 455 (1981).
[4] A.S. Kronfeld et al., Phys. Lett. 198B, 516 (1987); A.S. Kronfeld et al., Nucl.Phys. B293, 461 (1987).
[5] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42, 4257 (1990).
[6] S. Hioki et al., Phys. Lett. 285B, 343 (1992).
[7] T. Suzuki, Nucl. Phys. B(Proc. Suppl.) 30, 176 (1993) and see also references therein.
[8] H.Shiba and T.Suzuki, Kanazawa University, Report No. Kanazawa 93-09, 1993.
[9] This condition $\tilde{f}_{\mu\nu}(s) \in [-\pi, \pi]$ is applicable only in the case of defining a smallest 1$^3$ monopole.
[10] T.L. Ivanenko et al., Phys. Lett. 252B, 631 (1990).
[11] J.D. Stack and R.J. Wensley, Nucl. Phys. B371, 597 (1992).
[12] T.A. DeGrand and D. Toussaint, Phys. Rev. D22, 2478 (1980).
[13] S.Hioki et al.,Phys.Lett. 271B, 201 (1991).
[14] S.Itoh et al., Phys. Rev. D33, 1806 (1986).
TABLE I. Creutz ratios from abelian and monopole Wilson loops at $\beta = 2.6$. The Monopole Creutz ratio values are divided by 4, being adjusted to those in unit $a(\beta)$.

| Creutz ratios | abelian | monopole |
|---------------|---------|----------|
| $\chi(2,2)$   | 0.0872(2) |          |
| $\chi(3,3)$   | 0.0460(3) |          |
| $\chi(4,4)$   | 0.0323(4) | 0.0173(2) |
| $\chi(5,5)$   | 0.0270(7) |          |
| $\chi(6,6)$   | 0.0248(9) | 0.0169(3) |
| $\chi(7,7)$   | 0.0255(13) |        |
| $\chi(8,8)$   | 0.0196(25) | 0.0172(4) |
| $\chi(9,9)$   | 0.0147(66) |        |
| $\chi(10,10)$ | 0.0307(160) | 0.0178(6) |
| $\chi(11,11)$ | undeterminable |         |
| $\chi(12,12)$ | undeterminable | 0.0178(9) |

TABLE II. String tensions $\sigma/\Lambda_L^2$ at $\beta = 2.7$ and 2.8. Abelian and total mean those from Wilson loops evaluated using usual link fields and Eq. (3) respectively. Abelian (even) means the least-square fit using only even-sized abelian Wilson loops.

| $\beta$ | abelian | abelian (even) | total | monopole | photon |
|---------|---------|----------------|-------|----------|--------|
| 2.7     | 1962(37)| 1467(220)      | 1238(516) | 1087(394) | -36(62) |
| 2.8     | 2130(32)| 1621(187)      | 1269(326) | 1025(261) | -21(134) |

FIG. 1. Static potentials $aV(R)$ versus $R/a$ at $\beta = 2.5$. The values are shifted by a constant.

FIG. 2. Static potentials $aV(R)$ versus $R/a$ at $\beta = 2.6$. The values are shifted by a constant.

FIG. 3. String tensions at $\beta = 2.4, 2.5,$ and 2.6.

FIG. 4. Coulomb coefficients.
