New physical effects on the decay $B_{s(d)} \rightarrow \gamma\gamma$ in the sequential fourth Generation model

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Abstract

We study the contributions to the branching ratios of $B_{s(d)} \rightarrow \gamma\gamma$ decay in the sequential fourth generation model (SM4). We find that the theoretical values of the branching ratios, $\text{BR}(B_{s(d)} \rightarrow \gamma\gamma)$, including the contributions of $m_{t'}$ and the new $4 \times 4$ CKM (CKM4) matrix factors, $|V_{t's}V_{t'b}|$ and $|V_{t'd}V_{t'b}|$, are much different from the minimal standard model (SM) predictions. The new physics effects, especially contributed from the CKM4 matrix factors, can provide more than one order enhancement to the SM prediction. It is shown that the decay $B_{s(d)} \rightarrow \gamma\gamma$ can test the new physics signals from SM4.

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SM a very successful theory of the electroweak interactions. But it should not be the final theory. Theoretically, it has too many unknown parameters to be put by hand and can not unify the three gauge interactions. Also, SM has been faced to some troubles from the experiments. We need the new physics beyond SM. Many new physics models have been proposed to resolve the difficulties of SM and to explain the experiments. Of course, they have to be tested in many high energy experiments, such as the rare decays of mesons.

The startup of the LHC opens many new frontiers in precision flavour physics. As is well known, the rare radiative decays of B mesons are particularly sensitive to the contributions from new physics. Both inclusive and exclusive processes have been researched in the last 20 years. For example, $B_{s(d)} \to \gamma\gamma$ has been studied extensively in the SM and new physics scenarios. The present experimental limit on the decay $B_{s(d)} \to \gamma\gamma$ is

$$\text{BR}(B_s \to \gamma\gamma) \leq 8.6 \times 10^{-6} \ (90\%C.L.),$$
$$\text{BR}(B_d \to \gamma\gamma) \leq 3.2 \times 10^{-7} \ (90\%C.L.).$$

Within the SM one finds,

$$\text{BR}(B_s \to \gamma\gamma) \simeq 1 \times 10^{-6},$$
$$\text{BR}(B_d \to \gamma\gamma) \simeq 3 \times 10^{-8}.$$  

The upper bound of $B_{s(d)} \to \gamma\gamma$ is about $O(1)$ larger than the SM values. We believe, with the continuous accumulate of the experiment data, especially in the era of LHC and ILC, these branching ratios will be more and more precise. They will leave less room for the new physics. That is to say, $B_{s(d)} \to \gamma\gamma$, is very suitable to test the new physics models.

In ref. we investigated the new physical effects on $B_s \to \gamma\gamma$ in the one generation Technicolor model (OGTM) and got some interesting results. In this note, we consider the sequential fourth generation model to estimate the possible contributions to the decay $B_{s(d)} \to \gamma\gamma$. Recently, SM4 attracts an increasing interest and seems warming up. The electroweak precision data does not exclude completely existence of the fourth family and there are many reasons to introduce an extra generation of heavy particles, (for a recent brief review on the 4th generation, see ). Especially, LHC has the potential to discover or fully exclude existence of a fourth generation of quarks up to 1 TeV, even if they are
too heavy to observe directly they will induce a large signal in $gg \to ZZ$ that will be clearly visible at the LHC\[8\]. Maybe this model will be firstly tested by the early LHC data.

The sequential fourth generation model is a simple and non-supersymmetric extension of the SM, which does not add any new dynamics to the SM, with an additional up-type $t$ and an down-type $d$ quarks, a heavy charged lepton $\tau'$ and a heavy neutrino $\nu'$.

The model retains all the properties of the SM. The $t$ quark like the other up-type quarks contribute to the $b \to s$ transition at the loop level. Due to the additional fourth generation there will be mixing between the $t$ quark the three down-type quarks of the standard model and the resulting mixing matrix will become a $4 \times 4$ matrix,

$$
V_{\text{CKM}4} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} & V_{ub'} \\
V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\
V_{td} & V_{ts} & V_{tb} & V_{tb'} \\
V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'}
\end{pmatrix},
$$

where $V_{qb'}$ and $V_{t'q}$ are the new matrix elements in the SM4. The parametrization of this unitary matrix requires six mixing angles and three phases\[9\].

II. BRANCHING RATIOS OF $B_s(d) \to \gamma\gamma$

At quark level, $b \to s\gamma$, $b \to s\gamma\gamma$ and the exclusive decays $B_s \to \gamma\gamma$ have a close relation. Up to the corrections of order $1/m_W^2$, the effective Hamiltonian for $b \to s\gamma\gamma$ at scales $\mu_b = O(m_b)$ is identical to the one for $B \to X_s\gamma$ transition [1] and takes the form

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} \left[ \sum_{i=1}^{6} C_i(\mu_b)Q_i + C_{7\gamma}(\mu_b)Q_{7\gamma} + C_{8G}(\mu_b)Q_{8G} \right],
$$

here $Q_1 \ldots Q_6$ are the usual four-fermion operators whose explicit form is given below. The last two operators in the Eq.\[6\], characteristic for this decay, are the magnetic–penguin operators. The complete list of operators is given as follows

$$
Q_1 = \langle \bar{\ell}_{L\beta}\gamma'^\mu b_{L\alpha} \rangle (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}),
$$

$$
Q_2 = \langle \bar{\ell}_{L\alpha}\gamma'^\mu b_{L\alpha} \rangle (\bar{s}_{L\beta}\gamma_\mu c_{L\beta}),
$$

$$
Q_3 = \langle \bar{s}_{L\alpha}\gamma'^\mu b_{L\alpha} \rangle \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma_\mu q_{L\beta}),
$$
\[ Q_4 = (\bar{s}_{L\alpha} \gamma^{\mu} b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta} \gamma_\mu q_{L\alpha}), \]  
\[ Q_5 = (\bar{s}_{L\alpha} \gamma^{\mu} b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\beta}), \]  
\[ Q_6 = (\bar{s}_{L\alpha} \gamma^{\mu} b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\alpha}), \]  
\[ Q_7 = (e/16\pi^2) m_b \bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\beta} F_{\mu\nu}, \]  
\[ Q_8 = (g/16\pi^2) m_b \bar{s}_{L\alpha} \sigma^{\mu\nu} T^a_{R\beta} G^a_{\mu\nu}. \]

where \( \alpha \) and \( \beta \) are color indices, \( \alpha = 1, \ldots, 8 \) labels \( SU(3)_C \) generators, \( e \) and \( g \) refer to the electromagnetic and strong coupling constants, while \( F_{\mu\nu} \) and \( G^a_{\mu\nu} \) denote the QED and QCD field strength tensors, respectively. It is the magnetic \( \gamma \)-penguin operator \( Q_7 \), which plays the crucial role in this decay. The effective Hamiltonian for \( b \to d\gamma\gamma \) is obtained from Eqs.(6-14) by the replacement \( s \to d \).

The Feynman diagrams that contribute to the matrix element as the following, see Fig. 1.

![Feynman Diagrams](image)

**FIG. 1:** Examples of Feynman diagrams that contribute to the matrix element.

Within the SM, at scale \( m_W \), the Wilson coefficients \( C_i(m_W) \) at the leading order (LO) approximation have been given for example in [10],

\[ C_i(m_W) = \begin{cases} 0 & (i = 1, 3, 4, 5, 6), \\ 1 & (i = 2), \end{cases} \]

\[ C_7(m_W) = \frac{8x_t^3 + 5x_t^2 - 7x_t}{24(1 - x_t)^3} - \frac{2x_t^3 - 3x_t^2}{4(1 - x_t)^4} \log[x_t], \]

\[ C_8(m_W) = \frac{x_t^3 - 5x_t^2 - 2x_t}{8(1 - x_t)^3} - \frac{3x_t^3}{4(1 - x_t)^4} \log[x_t], \]

where \( x_t = m_t^2/m_W^2 \).

By using QCD renormalization group equations [10], it is straightforward to run Wilson coefficients \( C_i(m_W) \) from the scale \( \mu = O(m_w) \) down to the lower scale \( \mu = O(m_b) \). The
leading order results for the Wilson coefficients $C_7(\mu)$ with $\mu \approx m_b$ are of the form

$$C_7(\mu) = \eta^{16/23}C_7(m_W) + \frac{8}{3} \left(\eta^{14/23} - \eta^{16/23}\right) C_8(m_W) + \sum_{i=1}^{8} h_i \eta^{a_i},$$  \hspace{1cm} (18)$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$,

$$a_i = (14/23, 16/23, 6/23, -12/23, 0.4086, -0.4230, -0.8994, 0.1456),$$

$$h_i = (2.2996, -1.0880, -3/7, -1/14, -0.6494, -0.0380, -0.0185, -0.0057).$$

In the sequential 4th generation model, there exists an additional contribution to $b \to s\gamma$ induced by the 4th generation up quark $t'$, which produce the penguin diagrams, see Fig. 2.

\[\text{FIG. 2: Magnetic photon and gluon penguin diagrams with the fourth generation } t' \text{ quark.}\]

The new Wilson coefficients contributed by $t'$ are same as their counterparts in Eq. (16) and (17) except for exchanging $t'$ quark not $t$ quark.

At the mass scale of $\mu_b$, the Wilson coefficients of the dipole operators are given by

$$C_{7,8}^{\text{eff}}(\mu_b) = C_{7,8}^{(SM)\text{eff}}(\mu_b) + \frac{V_{ts}^* V_{t'b}}{V_{ts} V_{tb}} C_{7,8}^{(4)\text{eff}}(\mu_b),$$

where $V_{ts}^*$ and $V_{t'b}$ are two elements of the $4 \times 4$ CKM matrix. We recall here that the CKM coefficient corresponding to the $t$ quark contribution, i.e., $V_{ts}^* V_{tb}$, is factorized in the effective Hamiltonian as given in Eq. (6).

To calculate $B_{s(d)} \to \gamma\gamma$, one may follow a perturbative QCD approach which includes a proof of factorization, showing that soft gluon effects can be factorized into $B_{s(d)}$ meson wave function; and a systematic way of resuming large logarithms due to hard gluons with
energies between 1Gev and \( m_b \). In order to calculate the matrix element of Eq(1) for the \( B_{s(d)} \to \gamma \gamma \), we can work in the weak binding approximation and assume that both the \( b \) and the \( s(d) \) quarks are at rest in the \( B_{s(d)} \) meson, and the \( b \) quarks carries most of the meson energy, and its four velocity can be treated as equal to that of \( B_{s(d)} \). Hence one may write \( b \) quark momentum as \( p_b = m_b v \) where is the common four velocity of \( b \) and \( B_{s(d)} \). We have

\[
\begin{align*}
p_b \cdot k_1 &= m_b v \cdot k_1 = \frac{1}{2} m_b m_{B_{s(d)}} = p_b \cdot k_2, \\
p_{s(d)} \cdot k_1 &= (p - k_1 - k_2) \cdot k_1 \\
&= \frac{1}{2} m_{B_{s(d)}} (m_{B_{s(d)}} - m_b) \\
&= p_{s(d)} \cdot k_2.
\end{align*}
\]  

(22)

We compute the amplitude of \( B_{s(d)} \to \gamma \gamma \) using the following relations

\[
\begin{align*}
\langle 0 | \bar{s}(\bar{d}) \gamma_\mu \gamma_5 b | B_{s(d)}(P) \rangle &= -i f_{B_{s(d)}} P_\mu, \\
\langle 0 | \bar{s}(\bar{d}) \gamma_5 b | B_{s(d)}(P) \rangle &= i f_{B_{s(d)}} M_B,
\end{align*}
\]  

(23)

where \( f_{B_{s,d}} \) is the \( B_{s(d)} \) meson decay constant.

The total amplitude is now separated into a CP-even and a CP-odd part

\[
T(B_{s(d)} \to \gamma \gamma) = M^+ F_{\mu \nu} F^{\mu \nu} + i M^- F_{\mu \nu} \tilde{F}^{\mu \nu}.
\]  

(24)

We find that

\[
M^+ = -\frac{4\sqrt{2} \alpha G_F}{9\pi} f_{B_{s(d)}} m_{B_{s(d)}} V_{ts(d)}^* V_{tb} \left( B m_b K(m_b^2) + \frac{3 C_7}{8 \Lambda} \right),
\]  

(25)

\[
M^- = \frac{4\sqrt{2} \alpha G_F}{9\pi} f_{B_{s(d)}} m_{B_{s(d)}} V_{ts(d)}^* V_{tb} \left( \sum_q m_{B_{s(d)}} A_q J(m_q^2) + m_b B L(m_b^2) + \frac{3 C_7}{8 \Lambda} \right),
\]  

(26)

with \( B = -(3C_6 + C_5)/4, \bar{\Lambda} = m_{B_{s(d)}} - m_b, \) and

\[
\begin{align*}
A_u &= (C_3 - C_5) N_c + (C_4 - C_6), \\
A_d &= \frac{1}{4} [(C_3 - C_5) N_c + (C_4 - C_6)], \\
A_c &= (C_1 + C_3 - C_5) N_c + (C_2 + C_4 - C_6), \\
A_s &= \frac{1}{4} [(C_3 + C_4 - C_5) N_c + (C_3 + C_4 - C_6)], \\
A_b &= A_s.
\end{align*}
\]  

(27)
The functions $J(m^2)$, $K(m^2)$ and $L(m^2)$ are defined by

$$ J(m^2) = I_{11}(m^2), $$

$$ K(m^2) = 4I_{11}(m^2) - I_{00}(m^2), $$

$$ L(m^2) = I_{00}(m^2). $$

(28)

with

$$ I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \frac{x^p y^q}{m^2 - 2xyk_1 \cdot k_2 - i\varepsilon}. $$

(29)

The decay width for $B_s(d) \to \gamma\gamma$ is simply

$$ \Gamma(B_s(d) \to \gamma\gamma) = \frac{m^3_{B_s(d)}}{16\pi} (|M^+|^2 + |M^-|^2). $$

(30)

### III. NUMERICAL ANALYSIS AND SUMMARY

In the numerical calculations we use as input parameters

$\alpha_s(m_Z) = 0.118$, $\alpha_s(m_b) = 0.223$, $m_W = 80.22$GeV, $m_c = 1.27$GeV, $m_b = 4.19$GeV, $m_t = 172$GeV, $\tau_{B_s} = 1.49$ps, $f_{B_s} = 230$MeV, $\lambda_{B_s} = \lambda_{B_d} = 350$MeV, $m_{B_s} = 5.37$GeV, $\tau_{B_d} = 1.55$ps, $f_{B_d} = 200$MeV and $m_{B_d} = 5.28$GeV, respectively.

For the mass limit of $t'$, CDF gives $m_{t'} > 256$GeV for the $t' \to qW$ final state [11].

The experimental upper bounds for the fourth family quark CKM matrix elements are $|V_{t'd}| < 0.063$, $|V_{t's}| < 0.46$, $|V_{t'b}| < 0.47$ [12]. By taking the CKM unitarity conditions, $\sum_i V_{is(d)}^* V_{ib} = 0$, ($i = u, c, t, t'$), and the present measurement of $3 \times 3$ CKM matrix [13], We obtain the bounds for the CKM4 matrix elements in SM4,

$$ |V_{t'd}^* V_{t'b}| < (1.83 - 2.03) \times 10^{-2}, $$

(31)

$$ |V_{t's}^* V_{t'b}| < (6.97 - 7.75) \times 10^{-2}. $$

(32)

Fig. 3a shows the dependence of $\text{BR}(B_d \to \gamma\gamma)$ with the CKM4 matrix factor $|V_{t'd}^* V_{t'b}|$ for different values of $m_{t'}$. We can see that the new physics contributions can lead to appreciable changes of the SM predictions which may be enhanced by about more than one orders of magnitude in a reason- able mass range for $t'$. The new physics effects is very sensitive to the value of $|V_{t'd}^* V_{t'b}|$ and becomes tiny as $|V_{t'd}^* V_{t'b}| < 0.5 \times 10^{-2}$. It mains that $B_s \to \gamma\gamma$ can give the great limit room for the CKM4 matrix elements and give the strong constrict for the contributions to CP violation in SM4. But from Fig. 3a, the new physics effects is not
sensitive to the mass of $t'$. This can be seen more clearly in Fig. 3b, which shows the mass dependence of $\text{BR}(B_d \to \gamma\gamma)$ with $t'$ for different values of $m_{t'}$. From Fig. 3b, we can see that the new physics effects become bigger with increasing mass of the $t'$. This is similar to the case of top quark to the rare $B$ meson decays in SM, which the main contributions come from the heavy quark.

Figs. 4 and show the dependence of the decay $\text{BR}(B_s \to \gamma\gamma)$ with the CKM4 matrix factor $|V_{td}^*V_{tb}'|$ for different values of $m_{t'}$; (b) the mass of $t'$ for different values of $|V_{td}^*V_{tb}'|$.

Figs. 4 and show the dependence of the decay $\text{BR}(B_s \to \gamma\gamma)$ with the CKM4 matrix
FIG. 4: The Branching ratio of $B_s \rightarrow \gamma\gamma$ versus (a) CKM4 matrix factor $|V_{t's}^* V_{t'b}|$ for different values of $m_{t'}$; (b) the mass of $t'$ for different values of $|V_{t's}^* V_{t'b}|$.

factor $|V_{t's}^* V_{t'b}|$ for different values of $m_{t'}$. We can get the similar analysis but the new physics effects is much more sensitive to the value of the CKM4 matrix factor, $|V_{t's}^* V_{t'b}|$. For both of these decays, the CKM4 matrix elements provides the dominant new physics contribution.

As a conclusion, the new physics contribution to the rare decay of $B_{s(d)} \rightarrow \gamma\gamma$ in the sequential fourth can enhance rather large in magnitude, and may be detected in the near future precision experiments.
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