Focused energy delivery with low grating lobes for precision electronic warfare via BCD framework

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This letter presents a new method of focused energy delivery (FED) with low grating lobes for precision electronic warfare (PREW). The transmitted signal model is modified by equipping each platform with a linear uniform antenna array, enabling the same performance with lower power and providing a greater degree of freedom in the design of the transmitted signals than traditional methods. Moreover, the $L_\infty$-norm is considered as a regularization term in the model to simultaneously realize FED and alleviate the grating lobes. To solve the nonconvex NP-hard multisubjective optimization problem (MOP), block coordinate descent (BCD) is employed to separate the problem into two subproblems with respect to the amplitude and phase components of the signals. Then, these two subproblems are solved using Taylor expansion (TE) and the alternating direction penalty method (ADPM), respectively. Simulation results demonstrate that the proposed method exhibits superior performance compared to previous methods.

Introduction: In focused energy delivery (FED), an ultrasparsely array is usually adopted to precisely direct energy to a specific area while ensuring that other areas are not affected. This technology has aroused great interest in many applications, especially in precision electronic warfare (PREW) [1, 2]. However, traditional methods, such as transmit beamforming and spatial power combining, can only focus energy into a specific range of angles, resulting in unexpected electromagnetic injury to collateral devices. Due to its high-precision characteristics, FED plays a key role in PREW.

To our knowledge, related papers in this field are limited, and the existing methods can be divided into two categories. The first category focuses on the design of the covariance matrix of the transmitted signal, as discussed by Song et al. [1]. However, the computational complexity of such methods is high due to the adoption of a semidefinite (SDP) model and the subsequent rank-one approximation of the estimated covariance matrix. Moreover, array sparsity results in the grating lobe effect, which increases the risk of accidental injury to unknown devices and weakens the anti-reconnaissance capability of the interference system. In 2020, Chen et al. [2] proposed an efficient method of directly designing the transmitted signals. A unimodular quadratic programming (UQP) model was established to design the phase of the signals subject to constraints on the maximum power. Moreover, these authors introduced a regularization term based on the $L_1$-norm to mitigate the grating lobes. However, the transmitted signal dimensions are limited by the number of ultrasparsely arrays. The $L_1$-norm cannot encourage the sparsity of the large-scale matrix related to the grating lobe, and this regularization term alleviates only the total grating lobes. Thus, the performance is unsatisfactory in some cases in which the sum of the grating lobes is mitigated but some individual grating lobes are extremely severe.

To tackle these problems, we first equip each ultrasparsely distributed platform with a linear uniform antenna array to implement FED. This approach yields several advantages over previous methods as discussed below. Then, a new regularization term based on the $L_\infty$-norm is proposed and introduced into the established model to simultaneously realize FED and alleviate the grating lobes. As the original problem is difficult to solve, we adopt block coordinate descent (BCD) to divide the problem into two subproblems in terms of the amplitude and phase components of the signals. These two subproblems are then solved using Taylor expansion (TE) and the alternating direction penalty method (ADPM), respectively. Finally, simulation results reveal obvious improvements in the performance of grating lobe alleviation and FED compared to state-of-the-art methods.

System model and problem formulation: Consider an interference system consisting of $M$ platforms, which are distributed nonuniformly and ultrasparsely, as shown in Figure 1. The location of platform $S_m$ is denoted by $\mathbf{y}_m \in \mathbb{R}^2, m = 1, 2, \ldots, M$. When each platform is equipped with $N$ antennas in a linear uniform array, three advantages are gained over the ultrasparsely array adopted by Song et al. [1] and Chen et al. [2]: (1) each platform can achieve better directivity by adjusting the phase components of the signals; (2) the amplitude components of the signals are controllable, with the $N$ antennas sharing a single interference power source; and (3) the interference system has low power requirements, achieving the same effect with only $1/N$ of the maximum interference power through the combination of the signals from the $N$ antennas.

Let the signal transmitted from the $n$th antenna of the $m$th platform be denoted by $s_{mn}(t)$. Then, we express the transmitted signal at the current sample time, $s \in \mathbb{C}^{MN \times 1}$, in vector form as follows:

$$
\mathbf{s} = [s_1, s_2, \ldots, s_{MN}]^T
$$

where $s_{mn} \triangleq \rho_{mn}e^{j\beta_{mn}}$, with $\rho_{mn}$ denoting the amplitude and phase components, respectively, of the signal $s_{mn}$.

In this letter, we adopt several assumptions regarding the signal model, which are typical of studies of distributed arrays [3, 4]: the antennas exhibit no coupling effect and the signals propagate in free space with no reflection or scattering effects, such as multipath fading or shadowing. Therefore, we represent the steering vector directed towards point $\sigma$ as:

$$
\mathbf{a}(\sigma) = [a_1, a_2, \ldots, a_{21}, a_{22}, \ldots, a_{2N}, a_{2N+1}, \ldots, a_{MN}]^T
$$

(2)

where $\rho_{mn} \triangleq \beta_{mn}e^{j\gamma_{mn}}$, with $\beta_{mn}$ denoting the attenuation suffered during propagation in space, $\gamma_{mn}$ denoting the carrier frequency of the transmitted signal, and $\tau_{mn}(\sigma)$ denoting the propagation time that the transmitted signal takes from the $n$th antenna of the $m$th platform to $\sigma$.

Thus, the combined power at point $\sigma$ is given by:

$$
P(\sigma, s) = |\mathbf{a}(\sigma)^Hs|^2
$$

(3)

where $(\cdot)^H$ denotes the conjugate transpose operator. Then, the transmitted power in area $\Omega_3$ is represented as:

$$
E_{\Omega_3} = \int_{\Omega_3} P(\sigma, s)\,d\sigma = \int_{\Omega_3} \mathbf{s}^*H(\sigma)\mathbf{a}(\sigma)^H\mathbf{s}\,d\sigma
$$

(4)

The closed-form expression for $E_{\Omega_3}$ is approximately obtained by discretising $\Omega_3$ into $K$ uniform square grid cells, and we represent it as:

$$
E_{\Omega_3} = \mathbf{s}^*Q_{\Omega}\mathbf{s}
$$

where $K \rightarrow \infty, \Delta\sigma$ denotes the area of each grid cell, and $Q_{\Omega} = \sum_{k=1}^{K} \Delta\sigma(\sigma_k)a(\sigma_k)^H \in \mathbb{C}^{MN \times MN}$ is a Hermitian matrix, with $\sigma_k$ denoting the $k$th point in $\Omega_3$. Based on (5), a universal metric for FED is formulated as discussed by Song et al. [1] and Chen et al. [2]:

$$
\Omega_{FED} = \mathbf{s}^*Q_{\Omega}\mathbf{s} - \mathbf{s}^*Q_{\Omega}\mathbf{s} = \mathbf{s}^*Q_{\Omega}\mathbf{s}
$$

(6)
where $Q_T$ is the Hermitian matrix of the target regions, $Q_P$ is the Hermitian matrix of the regions to be protected, and the expression is simplified by introducing $Q = Q_T - Q_P$. When the numbers of target regions and protected regions are greater than one, $Q_T$ is given by $\sum_1^T Q_k$ and $Q_P$ is given by $\sum_1^P Q_k$, respectively, where $Q_{1:1}$ and $Q_{1:2}$ denote the $T$-th target region and the $P$-th protected region, respectively.

Regarding the grating lobes in FED, the $\infty$-norm is adopted to measure them. We eliminate all target regions and protected regions from the whole area of interest $\Omega$ and denote the remaining regions by $\Omega_G$. Then, the maximum modulus of the combined signal in $\Omega_G$ is given by:

$$O_{\Omega_G} = \|G^{1/2} x\|_{\infty}$$

where $G$ is obtained by combining all steering vectors of the points in $\Omega_G$. In letter, $O_{\Omega_G}$ is adopted to evaluate the grating lobes in PREW.

Proposed method: As our goal is to simultaneously realize FED and alleviate the grating lobes, we establish a multiobjective optimization problem (MOP) to maximize $O_{\Omega_G}$ and minimize $O_{\Omega_G}$. When $O_{\Omega_G}$ is considered as a regularization term, the optimization problem is established as follows:

$$\begin{align*}
\text{max} & \quad s^T Q s - \gamma \|G^{1/2} s\|_{\infty} \\
\text{s.t.} & \quad |s(m1)|^2 + |s(m2)|^2 + \cdots + |s(mN)|^2 = 1 \\
& \quad m = 1, 2, \ldots, M
\end{align*}$$

where $s(m)$ represents the nth element of the vector $s$, $\gamma$ is a regularization parameter determined according to practical requirements, and the constraints ensure that the total power of the $N$ antennas on the nth platform is maximum and normalized by $1/N$, as discussed regarding the third advantage identified in the section above. Then, we transform the problem by introducing $Q = \lambda I_{MN} - Q$, which has no effect on the solutions to (8), where $\lambda$ denotes the maximum eigenvalue of the known matrix $Q$ and $I_{MN}$ denotes an $MN$-order identity matrix. Thus, $Q$ is an $MN$-positive semidefinite Hermitian matrix. Furthermore, problem (8) is converted into an equivalent form:

$$\begin{align*}
\text{min} & \quad s^T Q s + \gamma \|G^{1/2} s\|_{\infty} \\
\text{s.t.} & \quad |s(m1)|^2 + |s(m2)|^2 + \cdots + |s(mN)|^2 = 1 \\
& \quad m = 1, 2, \ldots, M
\end{align*}$$

Because (9) is a nonconvex NP-hard MOP that is difficult to solve, we adopt the BCD framework to separate the problem into two subproblems in each iteration, the purpose of which is to individually update the amplitude and phase components of the transmitted signals.

Update of the amplitude components of the signals based on TE: In the iterative process of BCD, the phase components of the signals are known from the previous step. Thus, we rewrite the transmitted signals $s$ as:

$$\begin{align*}
\text{s} &= Xp \\
X &= \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_1}, e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_2}) \\
p &= [p_{1:1}; p_{1:2}; \ldots; p_{1:N}; p_{2:1}; p_{2:2}; \ldots; p_{2:N}; p_{M:1}; p_{M:2}; \ldots; p_{MN}]^T
\end{align*}$$

where $X$ is a known diagonal matrix consisting of the phase components of the transmitted signals and $p$ represents the amplitude components of the transmitted signals to be updated. Substituting (10a), (10b), and (10c) into (9), we present the subproblem to update the amplitude components of the transmitted signals as follows.

Subproblem I:

$$\begin{align*}
\text{min} & \quad p^T XQX p + \gamma \|G^{1/2} X p\|_{\infty} \\
\text{s.t.} & \quad \sum m2 + p(m2)^2 + \cdots + p(mN)^2 = \frac{1}{N} \\
& \quad m = 1, 2, \ldots, M
\end{align*}$$

By introducing $Q_{1} = XQX$ and $G_{1/2} = \sqrt{G} X$, subproblem I is simplified as:

$$\begin{align*}
\text{min} & \quad p^T Q_1 p + \gamma \|G_1^{1/2} p\|_{\infty} \\
\text{s.t.} & \quad p(m1)^2 + p(m2)^2 + \cdots + p(mN)^2 = 1 \\
& \quad m = 1, 2, \ldots, M
\end{align*}$$

Then, this problem is converted into a sequential quadratic programming (SQP) problem by performing TE on $p$. By ignoring the constant terms in the objective function, the problem is transformed into:

$$\begin{align*}
\text{min} & \quad 2\text{Re}[p_1^T Q_1 \Delta p + \Delta p^T Q_1 \Delta p + \gamma \|G_{1/2}^1 (p_1 + \Delta p)\|_{\infty}] \\
\text{s.t.} & \quad \Delta p(m1)^2 + \Delta p(m2)^2 + \cdots + \Delta p(mN)^2 + 2\text{Re}\{p_1\} = \frac{1}{N} - (p_1(m1)^2 + p_1(m2)^2 + \cdots + p_1(mN)^2) m = 1, 2, \ldots, M
\end{align*}$$

where $\text{Re}[z]$ denotes the real part of complex number $z$. By finding the optimal solution to (13) and updating $p_{1:+1} = p_1 + \Delta p$ until convergence, problem (12) is solved through iteration.

In this case, we relax problem (13) by abandoning the second-order terms with respect to $\Delta p$ in the constraints, which have a minimal effect on the solution. Thus, problem (13) is converted into:

$$\begin{align*}
\text{min} & \quad 2\text{Re}[p_1^T Q_1 \Delta p + \Delta p^T Q_1 \Delta p + \gamma \|G_{1/2}^1 (p_1 + \Delta p)\|_{\infty}] \\
\text{s.t.} & \quad (I_{1:1} \otimes I_{1:2}) p_1 = \frac{1}{N} - (I_{1:1} \otimes I_{1:2}) p_1
\end{align*}$$

where $P = \text{diag}(p_1)$, denotes the Kronecker product, and $I_{1:1}$ and $I_{1:2}$ denote an $L$-order row vector and an $M$-order column vector, respectively, with all elements equaling 1. As problem (14) is convex, we can efficiently solve for $\Delta p$ in polynomial time by directly using standard tools, such as CVX [5].

Update of the phase components of the signals based on the ADPM: The amplitude components of the signals are first determined by solving subproblem I in the current iteration of the BCD process. Then, we update the phase components of the transmitted signals. To this end, we rewrite the transmitted signals $s$ as:

$$s = P x$$

$$p = [p_{1:1}; p_{1:2}; \ldots; p_{1:N}; p_{2:1}; p_{2:2}; \ldots; p_{2:N}; p_{M:1}; p_{M:2}; \ldots; p_{MN}]^T$$

where $X$ is a known diagonal matrix consisting of the amplitude components of the transmitted signals and $x$ represents the phase components of the transmitted signals. Substituting (15a), (15b), and (15c) into (9), we present the subproblem to be solved to update the phase components of the transmitted signal as follows.

Subproblem II:

$$\begin{align*}
\text{min} & \quad x^T X^2 P^2 P x + \gamma \|G^{1/2} P x\|^2 \\
\text{s.t.} & \quad |x(mn)|^2 = 1 \\
& \quad m = 1, 2, \ldots, M, n = 1, 2, \ldots, N
\end{align*}$$

Similar to (11), we simplify the problem by introducing $Q_x = P^2 P$ and $G_{1/2}^2 = G^{1/2} P$. As the modulus of $G_{1/2}^2$ is equal to 1, subproblem II is further converted into a unimodal optimization problem as:

$$\begin{align*}
\text{min} & \quad x^T Q_x x + \gamma \|G_{1/2}^2 x\|^2 \\
\text{s.t.} & \quad |x(mn)|^2 = 1 \\
& \quad m = 1, 2, \ldots, M, n = 1, 2, \ldots, N
\end{align*}$$

where $x(mn) \in \mathbb{C}$. By introducing an auxiliary variable $y$, the ADPM is adopted to solve problem (17). This approach guarantees both efficiency and convergence [6]. We recast the problem as follows:

$$\begin{align*}
\text{min} & \quad x^T Q_x x + \gamma \|G_{1/2}^2 x\|^2 \\
\text{s.t.} & \quad x = y \\
|y(mn)|^2 = 1 & \quad m = 1, 2, \ldots, M, n = 1, 2, \ldots, N
\end{align*}$$

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The augmented Lagrangian of (18), as required for ADPM, is given by:
\[
L(x, y, \lambda, \rho) = \lambda^T Q x + y \left[ \left\| G^T (y - x) \right\|_2 + \frac{\rho}{2} \left\| y - x \right\|_2^2 \right]
\]
(19)
where \( \lambda \in \mathbb{C}^{MN \times 1} \) and \( \rho > 0 \) are the multiplier vector and the penalty parameter, respectively. In particular, we use \( x_i, y_i, \lambda_i \) and \( \rho_i \) to denote the estimates of \( x, y, \lambda \) and \( \rho \) in the 4th iteration, respectively.

Accordingly, the iterative process of the ADPM for (18) is expressed as follows:

\[
y_{k+1} = \arg \min_{y_i} \{ L(x_i, y_i, \lambda_i, \rho_i) \} \quad (20a)
\]

\[
x_{k+1} = \arg \min_{x_i} \{ L(x_i, y_{k+1}, \lambda_i, \rho_i) \} \quad (20b)
\]

\[
\rho_{k+1} = \begin{cases} 
\rho_1, & \left\| x_{k+1} - y_{k+1} \right\|_2 \leq \delta_1 \left\| x_k - y_k \right\|_2 \\
\rho_2, & \text{else}
\end{cases}
\] (20c)

\[
\lambda_{k+1} = \begin{cases} 
\lambda_1, & \left\| \hat{y}_{k+1} - \hat{x}_{k+1} \right\|_2 \leq \delta_1 \\
\lambda_2, & \left\| \hat{y}_{k+1} - \hat{x}_{k+1} \right\|_2 \geq \delta_1
\end{cases}
\] (20d)

where \( \lambda_{k+1} = \lambda_k + \rho_i (x_i - y_i) \), \( \delta_1 < 1 \) and \( \delta_2 > 1 \) are constants close to 1, and \( \delta_1 \) is a sufficiently large positive constant. By neglecting the constant terms, \( y_{k+1} (mn) \) is obtained as follows:

\[
y_{k+1} (mn) = \arg \{ \rho (x_i (mn) - \lambda_i (mn)) \} \quad (21)
\]
where \( \arg(z) \) denotes the argument of the complex number \( z \). As problem (20b) is convex and unconstrained, we employ CVX in Matlab to solve it directly [5]. Therefore, the BCD framework applied to problem (9) is given based on the update methods described above.

Simulation results: In this section, the performance of the proposed method is discussed and compared with that of other state-of-the-art methods. In the experiment, the area of interest is (-50 m, 50 m) x (-50 m, 50 m) in the XOY plane, and the size of each discrete grid cell is 0.25 m x 0.25 m. The interference system consists of \( M = 10 \) platforms, which are located in a horizontal plane at a distance of \( d = 1000 \) m. The carrier frequency of the transmitted signals is \( f_0 = 1 \) GHz. Considering the size of the platforms, we set \( N = 5 \). For convenience, we abbreviate the first and the second method proposed by Chen et al. [2] as Chen et al. [2] (first) and Chen et al. [2] (second), respectively, and our proposed method is abbreviated as FED-LGL (ours).

In Figure 2, the target region is marked with a black circle, and four protected regions are marked with white circles. The energy levels are shown by different colours, as indicated by the colour bar. Figures 2a and 2b display the energy distribution generated by Song et al. [1] and Chen et al. [2] (first) respectively. The grating lobe effect is extremely high, though the goal of PREW is achieved. The reason is that the grating lobes are mitigated in the corresponding models. The energy distribution shown in Figure 2c is generated by Chen et al. [2] (second). The grating lobes are mitigated to some extent due to the L1-norm regularization term, but some remain severe because the limited signal dimensions cannot encourage the L1-norm to be sufficiently sparse; thus, only the total grating lobe effect is mitigated. Figure 2d shows that FED-LGL (ours) achieves the expected performance. The grating lobes are obviously lower, while interestingly, FED is also better achieved.

In conclusion, we analyse the indicators of different FED methods, where \( O_{EVD} \) measures the performance of FED, \( \text{Max}_{E} \) is used to evaluate the maximum energy level among all grating lobes, and \( P_{E} \) is the average energy level in \( \Omega_2 \). The experiments are implemented under the given array by repeating 50 times with different methods. It is worth noting that the ideal expected values of these indicators highly depend on the specific type of the interference platforms, the target devices, and the collateral devices adopted in PREW [1, 2]. Thus, we neglect the ideal expected values in the experiments and demonstrate the superiority of our method by comparing these indicators. In Table 1, we find that all methods achieve the goal of FED, but the grating lobes performance varies a lot.

Table 1. The indicators of different FED methods

| Indicators (dB) | Song et al. [1] | Chen et al. [2] (first) | Chen et al. [2] (second) | FED-LGL (ours) |
|----------------|----------------|------------------------|------------------------|----------------|
| \( O_{EVD} \)  | 10.96          | 11.93                  | 11.77                  | 12.11          |
| \( \text{Max}_{E} \) | 21.02          | 22.90                  | 21.75                  | 15.83          |
| \( P_{E} \)    | 13.47          | 12.23                  | 10.85                  | 9.64           |

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