Laser cooling of unbound atoms in nondissipative optical lattices

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The semiclassical theory of laser cooling is applied for the analysis of cooling of unbound atoms with the values of the ground and exited state angular moments 1/2 in a one-dimensional nondissipative optical lattice. We show that in the low-saturation limit with respect to the pumping field a qualitative interpretation of the cooling mechanisms can be made by the consideration of effective two-level system of the ground-state sublevels. It is clarified that in the limit of weak Raman transitions the cooling mechanism is similar to the Doppler mechanism, which is known in the theory of two-level atom. In the limit of strong Raman transitions the cooling mechanism is similar to the known Sisyphus mechanism. In the slow atom approximation the analytical expressions for the coefficients of friction, spontaneous and induced diffusion are given, and the kinetic temperature is estimated.

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1 Introduction

Laser cooling of neutral atoms is necessary in various fundamental and applied problems, such as high precision spectroscopy [1], atomic frequency standards [2, 3, 4], Bose-Einstein condensation [5], atomic nanolitography [6, 7] and others. The methods of cooling of neutral atoms in magneto-optical traps and optical molasses that give the temperature of atomic ensemble about µK have been developed for he last 20 years. However, lower tempe-
tures are required for some applications. Particularly, the sub-$\mu$K transversal cooling would allow one to achieve the higher precision and stability in the modern laser-cooled atomic frequency standards (the atomic fountains [3, 4], the atomic clock in the condition of microgravitation [8]). At the present time there exist several methods of laser cooling, which allow one to achieve the temperature of atomic ensemble below $\mu$K: the velocity-selective coherent population trapping [9], the cooling by Raman pulses [10, 11] and the degenerate sideband Raman laser cooling (further DSRLC) [12, 13]. DSRLC appears to be an adaptation of the resolved-sideband laser cooling of ions for neutral atoms [14]. In comparison with the other methods of laser cooling DSRLC has some advantages: high efficiency, relatively short cooling time (about ms), relative simplicity of experimental realization. This method is based on the use of the Raman two-photon transitions between the vibrational levels of the Zeeman substates of atoms, that are trapped in an optical lattice. In the paper [12] the experiments on the two-dimensional cooling of cesium atoms by this method up to the ground state of a far-resonance optical lattice have been reported. In these experiments the DRSLC stage was preceded by the precooling stage in a near-off-resonance optical lattice, that ensured the high efficiency of cooling (95% of atoms that were captured in magneto-optical trap were cooled up to the ground vibrational state of lattice), but it caused some complications of the experimental realization. Chu with co-authors carried out similar experiments on three-dimensional cooling of cesium atoms in optical lattice up to the kinetic temperature 290 nK (after the adiabatic release of atoms from a lattice [13]). The distinction of this experiments from [12] appears to be the absence of the precooling stage. Nevertheless, 80% of atoms, that are transferred in a three-dimensional lattice, are cooled up to their ground vibrational state.

The high cooling efficiency which has been achieved [13], was most likely the evidence of the co-existence of cooling mechanisms of bound and unbound atoms that was shortly discussed in [13]. Later the experiments on 2D laser collimation of a continuous beam of cold cesium atoms by the method of DSRLC [15] were carried out to improve the corresponding frequency standard. In these investigations the cooling scheme similar to that in [15] is employed, but with some distinctions, that, particularly, lie in the use of a two-dimensional optical lattice of the original configuration. However, the efficiency of transversal cooling (collimation) of atomic beam is not high enough and it was essentially lower in comparison with [13]. The reasons of the lower cooling efficiency have not been investigated in [15].
Therefore the necessity of more detailed investigation of cooling in a nondissipative optical lattices arises. Particularly, it is important for the revealing of conditions, when the co-existence of cooling mechanisms of bound and unbound atoms takes place. In the present paper the semiclassical theory of laser cooling is used for the analysis of cooling of unbound atoms in a nondissipative optical lattice. This analysis is made in the framework of the simplest model of atoms with the degenerate ground state and for one-dimensional configuration of the lattice field that reflect essential features of the experimental scheme [15]. We consider the one-dimensional atomic motion, neglecting the recoil in all other directions.

As a result, the qualitative interpretation of the cooling mechanisms is given, and the analytical expressions for the force acting on an atom, the coefficients of spontaneous and induced diffusion are obtained. This allows the quantitative estimations of the atomic kinetic parameters, particularly, of the temperature.

2 Statement of the problem

Let us consider a two-level atom with the angular momentum of the ground state $J_g = 1/2$ and the momentum of the exited state $J_e = 1/2$, moving in an optical lattice. The lattice field is formed by two counterpropagating (running along $y$ axis) linearly polarized laser beams, their polarization vectors $e_1$ and $e_2$ are directed with angle $\theta$ to each other making $lin-\theta-lin$ configuration (fig.1).

In the spherical basis the lattice field has the following form:

$$\mathbf{E}_L(y) = E_0^L \exp(-i\omega_L t) \sum_{q=0,\pm1} e_L^q(y)e_q,$$  \hspace{1cm} (1)

where $E_0^L$ is the amplitude of single beam, $e_q$ are spherical orts. Let polarization vectors $e_1$ and $e_2$ are directed with the angle $\theta/2$ to the axis of the quantization (axis $z$ on fig.1), then the contravariant components $e_L^q(y)$ are written as:

$$e_L^0 = 2 \cos(\theta/2) \cos(ky),$$
$$e_L^{-1} = \sqrt{2}i \sin(\theta/2) \sin(ky),$$
$$e_L^{+1} = -\sqrt{2}i \sin(\theta/2) \sin(ky).$$  \hspace{1cm} (2)
It is assumed that the lattice field is detuned far enough from the resonance: $|\delta_L| >> \gamma$, ($\delta_L = \omega_L - \omega_0$ is the detuning of the lattice field frequency $\omega_L$ from the frequency of atomic transition $\omega_0$, $\gamma$ is the relaxation rate of the exited state), so that we can neglect the real transitions of atom from the ground state to the exited state under the action of lattice field. Since the spontaneous emission of photons is also negligibly small, such a lattice is nondissipative. So, the lattice action comes to the forming of periodical potential and the inducing of Raman two-photon transitions between Zeeman sublevels of the ground state (on fig.2 these transitions are labeled with the thick double arrow).

The lattice field action on atoms alone is not enough for cooling, because the lattice is nondissipative, that is the atomic motion in such a lattice has the conservative character. It is necessary for the realization of cooling in this system the presence of pumping field, which is tuned to resonance with the atomic transition, and spatially uniform splitting of the ground state Zeeman sublevels. The pumping field represents the circularly polarized beam, which is directed along the axis $z$:

$$E_p = E_0^p \exp(-i\omega_p t) \exp(ikz)e_{+1},$$

where $E_0^p$ is the pumping field amplitude The resonant pumping field induces the one-photon transitions from the ground state sublevel with the projection $-1/2$ to the exited state sublevel with the projection $1/2$ (the thin arrow on fig. 2). Further the spontaneous decay of exited state occurs (the wavy
Figure 2: Scheme of transitions. The thick double arrow labels Raman two-photon transition under the lattice field action, the thin arrow labels transition under the pumping field action, the wavy arrows label the spontaneous decay of exited state.

arrows on fig. 2). We neglect the recoil effect pumping field, because we consider cooling only in the $y$ direction. So, the pumping field action (together with the spontaneous decay from the exited state) leads to the effective relaxation of the ground state sublevels system. The spatially uniform shift of the Zeeman sublevels is produced by a static magnetic field (Zeeman effect), its direction coincides with the direction of pumping field wave vector. We do not take into account the splitting of the exited state Zeeman sublevels, considering it to be much smaller of the exited state natural width. Cooling in this system can be achieved by a proper choice of the ground state Zeeman splitting magnitude.

The evolution of atomic system is described by the quantum kinetic equation (QKE) for the atomic density matrix. In our case, in the general form, without the concretization of representation, QKE can be written as:

$$\frac{d\hat{\rho}}{dt} = -i\frac{\hbar}{\hbar} \left[ \hat{H}_0 + \hat{\rho} \frac{\hat{p}_y^2}{2M}, \hat{\rho} \right] - i\frac{\hbar}{\hbar} \left[ \hat{V}_L + \hat{V}_P + \hat{V}_B, \hat{\rho} \right] - \hat{\Gamma}\{\hat{\rho}\}. \quad (4)$$

Here $\hat{\rho}_y$ is the operator of atomic momentum projection on the axis $y$, $\hat{H}_0$ is
the Hamiltonian of free atom in the rest:

$$\hat{H}_0 = \hbar \omega_0 \sum_{\mu_e} |J_e\mu_e\rangle \langle J_e\mu_e|,$$  

(5)

where $\omega_0$ is the atomic transition frequency, $J_e$ is the angular momentum of the exited state, $\mu_e$ is its projection on the quantization axis. The operator of interaction of atom with lattice field has the form:

$$\hat{V}_L(y) = \hbar \Omega_L \sum_q \hat{T}_q e^q_L(y) \exp(-i\omega_L t) + h.c.,$$  

(6)

where $\Omega_L = -\frac{\tilde{d}E^L}{\hbar}$ is the Rabi frequency per a single beam of lattice field, $(\tilde{d}$ is the reduced matrix element of the dipole moment operator). According to the Wigner-Eckart [17] theorem the dependence of operator $\hat{V}_L(y)$ on the magnetic quantum numbers is contained in the Wigner operator:

$$\hat{T}_q = \sum_{\mu_e\mu_g} |J_e\mu_e\rangle C_{J_g\mu_g}^{J_e\mu_e} \langle J_g\mu_g|,$$  

(7)

where $J_g$ and $\mu_g$ is the ground-state angular momentum and its projection, $C_{J_g\mu_g}^{J_e\mu_e}$ is the Clebsch-Gordan coefficients. The operator of interaction of atom with pumping field is analogously written as:

$$\hat{V}_p = \hbar \Omega_p \hat{T}_{+1} \exp(ikz) \exp(-i\omega_p t) + h.c.,$$  

(8)

where $\Omega_p$ is the pumping field Rabi frequency. The operator of interaction of atom with magnetic field can be written, taking into account only the linear Zeeman effect in the ground state:

$$\hat{V}_B = -\hbar \omega_z \hat{J}_{gz},$$  

(9)

where $\omega_z$ is the Zeeman splitting of ground state sublevels, $\hat{J}_{gz}$ is the operator of $z$-projection ground-state angular momentum. The action of the atomic radiation relaxation operator $\hat{\Gamma}\{\hat{\rho}\}$ can be presented as:

$$\hat{\Gamma}\{\hat{\rho}\} = \frac{\gamma}{2} \{\hat{P}_c, \hat{\rho}\} - \frac{\gamma^3}{2} \left\langle \sum_{s=1,2} (\hat{T} \cdot \hat{e}_s(k)) \right\rangle \hat{\rho} \exp(-ikg \hat{y}) \exp(ikg \hat{y}) \langle \hat{T} \cdot \hat{e}_s(k) \rangle \Omega_k,$$  

(10)

$$(11)$$
where $\hat{P}_e$ is the projector on the exited state:

$$\hat{P}_e = \sum_{\mu_e} |J_{e\mu_e}\rangle \langle J_{e\mu_e}|,$$

(12)

$k$ is the wave vector of spontaneous photon, $e_s(k)$ is the unit polarization vectors of spontaneous photon, which are orthogonal $k$, $<\ldots>_\Omega_k$ denotes averaging on running direction of spontaneous photons, $k_y = (k \cdot e_y)$.

As is well known [18], one of the conditions for quasiclassical atomic translation motion is a smallness of recoil parameter, which is the ratio of photon momentum $\hbar k$ to atomic momentum dispersion $\Delta p$:

$$\frac{\hbar k}{\Delta p} << 1.$$  

(13)

The execution of condition (13) allows one to separate the fast process of the ordering on internal degrees of freedom from slow processes, which are connected with translation motion. At the kinetic evolution stage (in our case at $t >> (\gamma_S p)^{-1}$), when the stationary distribution on internal degrees of freedom has established, the atomic ensemble dynamics is defined by slow processes of change of distribution function on translation degrees of freedom. Usually the Wigner representation is used for the translation degrees of freedom, then the initial QKE (4) is reduced (with account for the terms of second order in the recoil parameter ) to the closed equation of the Fokker-Plank type for the Wigner distribution function $W$ :

$$\left(\frac{\partial}{\partial t} + \frac{p_y}{M} \frac{\partial}{\partial y}\right) W(y, p_y) = \left[-\frac{\partial F(y, p_y)}{\partial p_y} + \frac{\partial^2 D(y, p_y)}{\partial p_y^2}\right] W(y, p_y).$$  

(14)

The function $W$ can be interpreted as the probability density in the phase space in the case $W$ is a positive definite. The coefficients $F(y, p_y)$ and $D(y, p_y)$ have meaning of the force and diffusion in the momentum space, respectively. The coefficient of diffusion $D(y, p_y)$ is presented in the form:

$$D(y, p_y) = D_{sp}(y, p_y) + D_{ind}(y, p_y),$$  

(15)

where $D_{sp}(y, p_y)$ is the coefficient of spontaneous diffusion, $D_{ind}(y, p_y)$ is the coefficient of induced diffusion [19].
3 Effective two-level system

For a qualitative interpretation of cooling in the system under consideration it is appropriately to use the approximation of low saturation by pumping field, that corresponds to the experimental conditions [15]. This condition can be written as:

\[ S_p = \frac{\Omega_p^2}{(\gamma/2)^2 + \delta_p^2} << 1, \]  

(16)

where \( S_p \) is the saturation parameter for pumping field, \( \delta_p \) is the pumping field detuning from resonance. When the condition (16) is satisfied, the atomic model under consideration is equivalent to the two-level ground state substates system. Really, in this case in the equation (4) the standard reduction procedure to the ground state [18] can be made. The obtained equation system for the ground-state density matrix can be compared with the well-known equations for a two-level atom [19]. It is obvious from this comparison, that the ground state sublevels system is equivalent to the effective two-level system, where the ground state sublevels with the momentum projection \( \pm 1/2 \) play a part of the ground and exited state, consequently. At that the effective two-level system parameters are expressed through the initial model parameters as:

\[ \Gamma_1 = \frac{2}{9} \gamma S_p, \]
\[ \Gamma_2 = 3 \Gamma_1, \]
\[ \Delta = -\frac{2}{3} \delta_p S_p - \omega_z, \]
\[ \chi = \frac{2 \Omega_L^2 \sin \theta}{3 \delta_L}. \]  

(17)

Here \( \Gamma_1 \) is the effective relaxation rate of populations, \( \Gamma_2 \) is the effective relaxation rate of coherence, \( \Delta \) is the effective detuning from the two-photon resonance, \( \chi \) is the effective Rabi frequency. As is mentioned above, that in the limit (16) the atomic model under consideration corresponds to the effective two-level system. This fact lies in the base of the qualitative interpretation of cooling mechanisms.

8
4 Qualitative interpretation of cooling mechanisms

Our analysis of cooling mechanisms in the ground state sublevels system is based on the well-known cooling mechanisms in the two-level system. They are the Doppler mechanism in the weak field limit and the Sisyphus mechanism in the strong field limit [20]. At that together with the conservation of basic properties of these mechanisms some specific features appear. They are connected with the two-photon character of excitation and two-step character of relaxation in the effective two-level system.

In the weak Raman transitions limit $|\chi| \ll \sqrt{\Gamma_2^2 + 4\Delta^2}$ in the system under consideration cooling mechanism is similar to the Doppler mechanism. We will discuss it in more detail. The two-photon Raman transition (the thick arrow on fig.2) can occur in two ways: with the virtual absorption of $\pi$ and the emission of $\sigma^+$ lattice field components or with the absorption of $\sigma^-$ and the emission of $\pi$ components. The probability amplitudes of these processes are equal (see (2)) and proportional $\sin(\theta) \sin(2k_y)$. The probability amplitude under consideration contains the contributions from two effective running waves ($K_1$ and $K_2$ on fig. 3) with the wave vectors projections on the axis $y K_1 = 2k$ and $K_2 = -2k$, because $\sin(2k_y)$ is the superposition of two exponents $\exp(\pm 2iky)$. Let the effective detuning is negative. Then as atom moves towards the wave $K_1$, its emission comes near the two-photon resonance due to the Doppler effect, but the emission of the wave $K_2$ comes far the resonance. So, the moving atom more probably interacts with the contrepropagating effective wave, at that it gets the momentum $2\hbar k$. This process is two-photon, that is the specificity of the cooling mechanism under consideration relative to the standard Doppler mechanism. Other distinction is the two-step relaxation of the exited state $\mu_g = -1/2$, which is characterized by the effective relaxation rate $\Gamma_1$.

When the Raman transitions is strong $|\chi| \gg \sqrt{\Gamma_2^2 + 4\Delta^2}$ cooling mechanism is similar to the Sisyphus mechanism [20], but it also has two features. Firstly, the adiabatic potentials has the two times shorter spatial period, secondly, the two-step transitions between the adiabatic states are present, which are caused by the effective relaxation.
5 Kinetic coefficients in slow atom approximation

In the general case of the atomic motion in nonuniform field the kinetic coefficients $F$ and $D$ can be calculated by numerical methods (for example, by the continuous fraction method [19]). In order to obtain the analytical expressions for $F$ and $D$ one should use some approximations. The slow atom approximation has a great importance (in particular, for the temperature estimation). In our case it can be written as:

$$kv << \gamma S_p,$$

where $v$ is the atomic velocity. This condition means that atom shifts over a distance far less than the light wave length during the optical pumping time. In this limit (18), to describe the dissipative processes it is sufficient to consider only the two first terms in the expansion of the force in velocity:

$$F(y, p_y) \simeq F_0(y) + \alpha(y)v + ....$$

(19)

Here $\alpha$ is the friction coefficient, $F_0$ is the force in the in zeroth order in velocity. For the diffusion coefficient we take in to only the zeroth-order terms:

$$D(y, p_y) \simeq D(y) = D_{sp}(y) + D_{ind}(y).$$

(20)
The analytical expressions for the coefficients \( F_0(y), \alpha(y), D(y) \) can be obtained by the method of work [18].

Let us demonstrate the results of analytical calculations for the local magnitudes of the Fokker-Plank equation kinetic coefficients in the slow atom approximation. It is convenient for brevity to use the effective two-level system parameters (17) and introduce the effective saturation parameter

\[
S = \frac{\chi^2 \sin^2(2ky)}{\Gamma_2^2/2 + \Delta^2}.
\]  

(21)

The force in the zeroth order in velocity is

\[
F_0 = 4\hbar k \coth(\theta) \chi \sin(2ky) - \frac{4\hbar k \Gamma_1 \Delta S}{9(\Gamma_1 + 2\Gamma_2 S)}.
\]  

(22)

The friction coefficient is written as:

\[
\alpha = 32\hbar^2 k^2 \Gamma_1 \Delta \left( \frac{\Gamma_1^2 \Gamma_2 S - [\Gamma_2^3 + \Gamma_1 \Gamma_2^2 + 4\Delta^2(\Gamma_2 - \Gamma_1)]S^2}{\Gamma_2^2 + 4\Delta^2} - 2\Gamma_2 S^3 \right) / (\Gamma_1 + 2\Gamma_2 S)^3.
\]  

(23)

the induced diffusion coefficient has following form:

\[
D_{ind} = 2\hbar^2 k^2 S \left( \Gamma_1^3 \Gamma_2^3 / 2 - \frac{\Gamma_1^2 \Gamma_2 S}[\Gamma_2^3 + 4\Delta^2] - 4\Gamma_1 \left[ -3\Gamma_2^3 + 8\Delta^2(\Gamma_1 - \Gamma_2) \right] S^2 + 8(\Gamma_2^3 + 4\Delta^2) \Gamma_2^2 S^3 \right) / (\Gamma_1 + 2\Gamma_2 S)^3
\]  

(24)

the spontaneous diffusion coefficient is

\[
D_{sp} = \frac{\hbar^2 k^2 \Gamma_1 \Gamma_2 S}{2(\Gamma_1 + 2\Gamma_2 S)}
\]  

(25)

We represent the saturation effective parameter \( S \) as \( S = S_0 \sin^2(2ky) \). In this case after the averaging on lattice period the analytical expressions for kinetic coefficients have the following form. The friction coefficient is written as:

\[
\langle \alpha \rangle = 4\hbar^2 k^2 \Delta \left( \frac{\Gamma_1 \Gamma_2^3 S_0^2 (\Gamma_2^2 - 4\Delta^2)}{\Gamma_2^2} - 2\Gamma_1^3 (\Gamma_2^2 + 4\Delta^2) \right)
\]
\[-\Gamma_2^3 S_0^2 \left( \Gamma_2^2 + 4 \Delta^2 \right) + 2 \Gamma_1^{5/2} \sqrt{\Gamma_1^2 + 2 \Gamma_2 S_0 (\Gamma_2^2 + 4 \Delta^2)} +
+ 4 \Gamma_1^{3/2} \Gamma_2 S_0 \sqrt{\Gamma_1^2 + 2 \Gamma_2 S_0 (\Gamma_2^2 + 4 \Delta^2)} - 2 \Gamma_1^2 \Gamma_2 S_0 (\Gamma_2^2 + 12 \Delta^2) \bigg) / \bigg( \sqrt{\Gamma_1 \Gamma_2} (\Gamma_1 + 2 \Gamma_2 S_0)^{3/2} (\Gamma_2^2 + 4 \Delta^2) \bigg). \tag{26}\]

The induced diffusion coefficient is
\[ < D_{\text{ind}} > = h^2 k^2 \left( -\sqrt{\Gamma_1} \Gamma_2 (6 \Gamma_1^2 + 20 \Gamma_1 \Gamma_2 S_0 +
+ 15 \Gamma_2^2 S_0^2) (\Gamma_2^2 + 4 \Delta^2) + 2 (\Gamma_1 + 2 \Gamma_2 S_0)^{3/2}
(\Gamma_2^4 S_0 + 8 \Gamma_2^2 \Delta^2 + 4 \Gamma_1 \Gamma_2 \Delta^2 + 4 \Gamma_2^2 S_0 \Delta^2) +
+ \sqrt{\Gamma_1} \left\{ -(2 \Gamma_1^2 + 6 \Gamma_1 \Gamma_2 S_0 + 3 \Gamma_2^2 S_0^2)
(-3 \Gamma_2^3 + 8 (\Gamma_1 - \Gamma_2) \Delta^2) + \left[ 2 \Gamma_2^4 S_0 \left( 3 \Gamma_2 S_0 (\Gamma_2^2 + 4 \Delta^2) +
\Gamma_1 (\Gamma_2^3 + 4 (1 - 2 S_0) \Delta^2) \right) \bigg] / (\Gamma_2^2 + 4 \Delta^2) \right\} / \left( 2 \Gamma_2^3 (\Gamma_1 + 2 \Gamma_2 S_0)^{3/2} \right) \bigg) \bigg) / \bigg( \Gamma_1 \Gamma_2 \Gamma_2^4 S_0 \left( \Gamma_1 + 2 \Gamma_2 S_0 \right)^{3/2} \bigg). \tag{27}\]

The spontaneous diffusion coefficient has following form:
\[ < D_{\text{sp}} > = \frac{h^2 k^2 \Gamma_1}{4} \left( -1 + \sqrt{\frac{\Gamma_1 + 2 \Gamma_2 S_0}{\Gamma_1}} \right). \tag{28}\]

Note that at \( \Gamma_1 = \Gamma_2 \), our expressions formally coincide (with an accuracy of constant factors) with the corresponding formulas for two-level atom in a standing wave field [21, 20].

6 Discussion of the results

We estimate the kinetic temperature by the standard way [20], neglecting the spatial localization:
\[ k_B T = - \frac{\langle D_{\text{ind}} \rangle + \langle D_{\text{sp}} \rangle}{\langle \alpha \rangle}, \tag{29}\]
where \( k_B \) is the Boltcman constant. In the weak Raman transitions limit expression (29) is written as:
\[ k_B T = - \frac{5 \hbar \left( \Gamma_1^2 \Delta^2 + \Delta^2 \right)}{16 \Delta}. \]
In this limit the minimal temperature is achieved at the effective detuning \( \Delta = -\frac{\Gamma}{2} \), and it is equal \( k_B T = \frac{5}{16} \hbar \Gamma \). In our case it is possible to change the minimal temperatures by variation the effective relaxation constant \( \Gamma_2 \) (for this purpose, it is necessary to change the parameters of \( \Gamma_2 \), for example, the pumping field intensity). This feature presents the important difference from the usual Doppler cooling in two-level system.

Further, let us consider the dependence of averaged friction coefficient and the atomic temperature on the effective Rabi frequency. In fig. 4 the dependence of averaged friction coefficient on the effective Rabi frequency is presented (the effective detuning \( \Delta = -0.1\gamma \)). It is clearly from fig. 4 that in the weak Raman transition limit (when the effective Rabi frequency is small) \( \langle \alpha \rangle < 0 \), that is the cooling of atoms occurs. In the strong Raman transition limit (when \( \chi \) is large) \( \langle \alpha \rangle > 0 \), that is the heating of atoms occurs. This dependence of kinetic processes direction on the effective Rabi frequency qualitatively coincide with the form of analogically dependence in two-level system.

On fig. 5 the dependence of kinetic temperature (29) on \( \chi \) is presented (when \( \Delta = -0.1\gamma \)). This figure illustrates the cooling mechanism in the weak Raman transitions limit. The atomic temperature decrease is observed as \( \chi \) decreases, that corresponds to the Doppler cooling limit in two-level system. When the effective detuning is positive cooling is observed in the strong
Raman transitions limit (fig. 6). In the two-level system it corresponds to Sisyphus cooling mechanism. It is obviously from fig.6 that the temperature increase without limit as $\chi$ increase.

So, the demonstrated dependencies $\langle \alpha \rangle$ and $\langle kT \rangle$ on $\chi$ confirm the qualitative interpretation of cooling mechanisms given above.

7 Comparison with experiment

The previously derived theoretical results can be compared to the experimental data [15]. For this purpose let as calculate the model parameters that are correspond to the experimental conditions: the pumping field detuning $+2.3$ MHz, the pumping field intensity $I_p = 0.24$ mW cm$^{-2}$; the lattice field detuning $-9$ GHz, the single-beam lattice intensity $I_L = 75$ mW cm$^{-2}$, the angle between lattice beams polarization vectors $45^\circ$; the magnetic field changed
in the range from 0 to 200 mG. The pumping and lattice field Rabi frequencies are calculated from the formula \( \Omega_{p,L} = \gamma \sqrt{I_{p,L}/(8I_s)} \), where \( I_s = 1.1 \text{ mWt cm}^{-2} \) is saturation intensity for the \( D_2 \) line \(^{133}\text{Cs} \), \( \gamma = 2\pi \times 5.3 \text{ MHz} \).

At the calculation of Zeeman shift we use the \( g \)-factor value for lowest hyperfine level of \(^{133}\text{Cs} \) ground state: \( g = -1/4 \) that gives \( \omega_z = 2\pi \times 350 \text{ kHz G}^{-1}B \). Under this conditions the effective Rabi frequency is \( \chi = 2\pi \times 11 \text{ kHz} \), and the effective relaxation rates of two-level system are \( \Gamma_1 = 2\pi \times 76 \text{ kHz} \) and \( \Gamma_2 = 2\pi \times 222 \text{ kHz} \). It is necessary to compare the magnitudes \(|\chi|\) and \( \sqrt{\Gamma_2^2 + 4\Delta^2} \) for definition dominating cooling mechanism under the given conditions. As the the magnetic field changes in the range from 0 to 200 mG (that corresponds to the experimental conditions) \( \sqrt{\Gamma_2^2 + 4\Delta^2} \) changes in the range from 0.29 MHz to 0.4 MHz. As this take place, \(|\chi|\) remains 26 - 36 times smaller than this magnitude. Consequently, the domain with mainly weak Raman connection and the Doppler-like cooling mechanism correspond to the given conditions. According to (30) at the weak Raman coupling the minimal temperature is achieved under the condition \( \Delta = -\Gamma_2/2 \) (that corresponds to \( B_{\text{min}} = 50 \text{ mG} \)). This minimal temperature can be estimated as \( T_{\text{min}} = 0.3 \hbar \Gamma_2/k_B \approx 3.3 \mu\text{K} \). These values are close to the experimentally observed \([15]\) \( (B_{\text{min}} = 45 \text{ G} \ T_{\text{min}} = 1.5 \mu\text{K}) \). The experimentally obtained temperature is more than 2 times smaller than the theoretical limit. This discrepancy is most likely due to the disregarding of the contribution of atoms, confined to the optical potential minima. It is necessary for the more detailed analysis to consider simultaneously the cooling of unbound and bound atoms. This consideration is beyond the purpose of the present work and will appear the subject of further investigations.

Moreover, we express the dependence of the kinetic temperature on the magnetic field magnitude (fig.7) (It was calculated from the formula (29) that is with the taking into account all orders on \(|\chi|\)). It is clear from the comparison fig.7 and the experimental dependence of atomic temperature on magnetic field (work [15], fig. 6) that there is a satisfactory qualitative agreement between the results of our theoretical model and the experimental data.
Figure 7: Temperature dependence on magnetic field under $\delta_p = 2\pi \times 2.3$ MHz, $\delta_L = -2\pi \times 9$ MHz, $I_p = 0.24$ mWt cm$^{-2}$, $I_L = 75$ mWt cm$^{-2}$, $\theta = 45^\circ$

8 Conclusion

Let us summarize some results. We considered the laser cooling of the unbound atoms with the exited state and ground state momentum $J_e$ and $J_g$ that were equal $1/2$ in one-dimensional $\text{lin-}\theta\text{-lin}$ lattice field configuration. It was showed that in the low saturation limit in pumping field (16) the qualitative interpretation of cooling mechanisms could be made in the framework of the consideration of the effective two-level system which was formed by the ground-state sublevels. We compared the equations, that described the effective system with the known equations for two-level system. The dependence of effective parameters on model parameters was found from this comparison. The qualitative interpretation of cooling mechanisms was given. It was showed that in the weak Raman transitions limit the Doppler-like mechanism was observed, and in the strong Raman transitions limit the similar Sisyphus mechanism was demonstrated. The analytical expressions for the force acting on atom, spontaneous and induced diffusion coefficients were obtained. The quantitative estimate of atomic kinetic temperature was made. It was demonstrated that the dependence of friction coefficient and temperature on the effective Rabi frequency confirmed our qualitative interpretation of cooling mechanisms. The comparison of theoretical calculations of temperature and experimental data of work [15] was made and a satisfactory qualitative agreement was revealed. The results of this work can be used for analysis of laser cooling of atoms in nondissipative optical lattices.

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