Charged Bilepton Pair Production at LHC
Including Exotic Quark Contribution

E. Ramirez Barreto, Y. A. Coutinho
Instituto de Física,
Universidade Federal do Rio de Janeiro,
Rio de Janeiro, RJ, Brazil

J. Sá Borges
Universidade do Estado do Rio de Janeiro,
Rio de Janeiro, RJ, Brazil

Abstract

The production of \( W^+W^- \) pair in hadron colliders was calculated up to loop corrections by some authors in the Electroweak standard model (SM) framework. This production was also calculated, at the tree level, in some extensions of the SM such as the vector singlet, the fermion mirror fermion and the vector doublet models by considering the contributions of new neutral gauge bosons and exotic fermions. The obtained results for \( e^+e^- \) and \( pp \) collisions pointed out that the new physics contributions are quite important. This motivates us to calculate the production of a more massive charged gauge boson predicted by the \( SU(3)_C \times SU(3)_L \times U(1)_X \) model (3-3-1 model). Thus, the aim of the present paper is to analyze the role played by the extra gauge boson \( Z' \) and of the exotic quarks, predicted in the minimal version of the 3-3-1 model, by considering the inclusive production of a pair of bileptons \( (V^\pm) \) in the reaction \( p + p \rightarrow V^+ + V^- + X \), at the Large Hadron Collider (LHC) energies.

Our results show that the correct energy behavior of the elementary cross section follows from the balance between the contributions of the extra neutral gauge boson with those from the exotic quarks. The extra neutral gauge boson induces flavor-changing neutral currents (FCNC) at tree level, and we have introduced the ordinary quark mixing matrices for the model when the first family transforms differently to the other two with respect to \( SU(3)_L \). We obtain a huge number of heavy bilepton pairs produced for two different values of the center of mass energy of the LHC.

PACS: 12.60.Cn, 12.15.Ff, 14.70.Pw
email: elmer@if.ufrj.br, yara@if.ufrj.br, saborges@uerj.br
1 Introduction

The gauge sector of the Standard Model (SM), has been extensively tested by LEP, SLD and Tevatron experiments. Among these tests, the production of $W$ pairs is important because it is quite sensitive to a balance between $s-$ and $t-$channel contributions. The gauge boson pair production can also reveal the nature of the triple gauge coupling, but until now there is no evidence of the existence of anomalous gauge couplings. In fact, the analysis of the $Z$ transverse momentum distribution in the process $p + \bar{p} \rightarrow W + Z + X \rightarrow \ell^+ \nu_\ell + \ell + \bar{\ell}$ ($\ell$ and $\ell'$ are electrons and muons) at Tevatron ($\sqrt{s} = 1.96$ GeV) gives a more restrict limit on the $WWZ$ coupling parameters [1]. They are: $-0.17 \leq \lambda_Z \leq 0.21$ ($\Delta \kappa_Z = 0$) and $-0.12 \leq \Delta \kappa_Z \leq 0.29$ ($\lambda_Z = 0$) assuming that $\Delta g^Z = \Delta \kappa_Z$. The effective Lagrangian with the parametrization of anomalous couplings involving $WW\gamma$ and $WWZ$ is found in [2, 3].

The standard model $W^+W^-$, $Z^0Z^0$ production in $e^+e^-$ and in hadron colliders was studied in [4, 5, 6, 7], for example; the authors have shown that the $s-$ and $t-$channels balance in $W^+W^-$ production is essential for the good behavior of total cross sections. Such behavior must be preserved when the c.m. energy of colliding particles increase ($\sqrt{s} \gg M_Z$) probably producing new particles, which appear in many extensions the SM or alternative models [8, 9, 10, 11, 12]. For example, the $W^+W^-$ production in linear and hadron colliders was analyzed in some extensions of the SM which have the same gauge boson content but where the fundamental matter representation includes new exotic fermions (very massive leptons and heavy quarks) [13]; using the unitarity constraint the authors determined some relations between model parameters [14]. In the same context, the new neutral gauge boson contribution for left-right models in $e^+e^-$ collisions was analyzed [15].

At high energy, the production of ordinary or exotic gauge boson pair can be studied from alternative models with a large particle spectrum. The boson pair production takes place through standard and new gauge bosons $s$-channel contribution and from $t$-channel exchange of ordinary or new fermions. For these models the number of triple gauge couplings increase and high energy processes can also reveal anomalous couplings, excluded in a previous analysis [16].

We are exploring the phenomenological aspects of an alternative to the SM based on the $SU(3)_c \times SU(3)_g \times U(1)_X$ gauge symmetry (3-3-1 model) [17, 18, 19, 20] which predicts new very massive particles mixed to the observed states. Together with the exotic fermion content and an extra neutral gauge boson, the model includes gauge bosons carrying lepton number equal 2, called bilepton [21], that also occur in $SU(15)$ grand unified theories [22]. There exist, in the literature, the supersymmetric extension of the model [23] and several phenomenological consequences of the model are being explored [24].

Let us outline some features of the model considered in this paper. Although at low energies the model coincides with the SM, it offers an explanation for basic questions such as the family replication problem and the observed bound for the Weinberg angle [25]. The family problem is solved by considering the model anomaly cancellation procedure, requiring that the number of fermion families must be a multiple of the quark color number [26]. Considering that QCD asymptotic freedom condition is valid only if the number of families of quarks is less than five, one concludes that there are three generations. On
the other hand, to keep the validity of perturbation calculation, one obtains a bound for the Weinberg angle at each energy scale $\mu$, $(\sin^2 \theta_W(\mu) \leq 1/4)$ this constraint follows from the coupling constants $(g_{U(1)}, g_{SU(3)}^L)$ ratio. The experimental value of $\sin^2 \theta_W(M_Z) \simeq 1/4$ leads to an upper bound associated with the spontaneous $SU(3)_L$ symmetry breaking [27, 28], which implies directly on a restriction on exotic boson masses [29].

Working with two versions of the 3-3-1 model, we have analyzed the $e^+ + e^- \rightarrow f + \bar{f}$, (where $f$ denotes ordinary leptons or quarks) for ILC energies in order to establish some signatures of the extra neutral gauge boson $Z'$ existence and to obtain lower bounds on its mass. The obtained bounds were confirmed by extending our analysis to $pp$ and $pp$ collisions [30]. In another publication, we have included the $Z'$ contribution to the production of a pair of double charged bileptons in $e^+e^-$ for ILC energy [31]. The beginning of activities of the LHC, operating at high energy, opens the search for new discoveries. Among these findings one expect the presence of some signatures for new particles as those predicted by the 3-3-1 model, in particular new gauge bosons, bileptons and exotic quarks.

In the present paper we analyze the production of a pair of single charged bilepton ($V^\pm$) at the Large Hadron Collider (LHC) at CERN with $\sqrt{s} > 10$ TeV, through the process $p + p \rightarrow V^+ + V^- + X$, where $s-$ channel contributions come from $\gamma, Z$ and $Z'$ and where $t-$ channel includes only the exotic quarks contributions. Our calculation is performed at the tree level employing parton distribution functions [32] in a Monte Carlo code.

In the section II we review the basic aspects of the minimal version of the 3-3-1 model. In the section III we present the calculation of $q + \bar{q} \rightarrow V^+ + V^-$ cross section as well as the final results for $p + p \rightarrow V^+ + V^- + X$ adding some comments of our results. Finally, in the section IV, we present the conclusions of our work.

2 Model

In the 3-3-1 model the electric charge operator is defined as:

$$Q = T_3 + \beta T_8 + XI$$

where $T_3$ and $T_8$ are two of the eight generators satisfying the $SU(3)$ algebra

$$[T_i, T_j] = if_{i,j,k} T_k \quad i, j, k = 1..8,$$

$I$ is the unit matrix and $X$ denotes the $U(1)$ charge.

The electric charge operator determines how the fields are arranged in each representation and depends on the $\beta$ parameter. Among the possible choices, $\beta = -\sqrt{3}$ [17, 18] corresponds to the minimal version of the model that is used in the present application.

The lepton content of each generation ($a = 1, 2, 3$) is:

$$\psi_{aL} = (\nu_a \; \ell_a \; \ell_a^c)^T_L \sim (1, 3, 0),$$

where $\ell_a^c$ is the charge conjugate of $\ell_a$ ($e, \mu, \tau$) field. Here the values in the parentheses denote quantum numbers relative to $SU(3)_C, SU(3)_L$ and $U(1)_X$ transformations.
In order to cancel anomalies, the first quark family is accommodated in SU(3)$_L$ triplets and the second and third families ($m = 2, 3$) belong to the anti-triplet representation, as follows:

$$Q_{1L} = (u_1, d_1, J_1)^T_L \sim (3, 3, 2/3),$$
$$Q_{mL} = (d_m, u_m, J_m)^T_L \sim (3, 3^*, -1/3).$$

(4)

$$u_{aR} \sim (3, 1, 2/3), \quad d_{aR} \sim (3, 1, -1/3),$$
$$J_{1R} \sim (3, 1, 5/3), \quad J_{mR} \sim (3, 1, -4/3),$$

(5)

where $a = 1, 2, 3$ and $J_1, J_2$ and $J_3$ are exotic quarks with respectively $5/3, -4/3$ and $-4/3$ units of the positron charge ($e$).

This version has five additional gauge bosons beyond the SM ones. They are: a neutral $Z'$ and four heavy charged bileptons, $Y^{\pm,}, V^{\pm}$ with lepton number $L = \mp 2$. In order to avoid model anomalies, only one quark family must be assigned to a different SU(3) representation, but this procedure does not specify what is the family to be elected [33]. We will comment, in the conclusion section, about the consequences of our choice where the first family is treated differently from the other two.

The minimum Higgs structure necessary for symmetry breaking and that gives quark and lepton acceptable masses are composed by three triplets $(\chi, \rho, \eta)$ and one anti-sextet $(S)$. The neutral field of each scalar triplet develops non zero vacuum expectation values ($v_\chi, v_\rho, v_\eta$, and $v_S$) and the breaking of 3-3-1 group to the SM is produced by the following hierarchical pattern:

$$SU_L(3) \otimes U_X(1) \overset{<v_\chi>}{\rightarrow} SU_L(2) \otimes U_Y(1) \overset{<v_\rho, v_\eta, v_S>}{\rightarrow} U_{e.m.}(1).$$

The consistency of the model with the SM phenomenology is imposed by fixing a large scale for $v_\chi$, responsible to give mass to the exotic particles ($v_\chi \gg v_\rho, v_\eta, v_S$), with $v_\rho^2 + v_\eta^2 + v_S^2 = v_W^2 = (246)^2$ GeV$^2$.

In the minimal version, the relation between $Z', V$ and $Y$ masses [34, 29] is:

$$\frac{M_V}{M_{Z'}} \approx \frac{M_Y}{M_{Z'}} \approx \frac{\sqrt{3 - 12 \sin^2 \theta_W}}{2 \cos \theta_W}.$$  

(6)

This special constraint respects the experimental bounds that, even being a consequence of the model, is not often used in the literature. We keep this relation through our calculations. For example, this ratio is $\approx 0.3$ for $\sin^2 \theta_W = 0.23$ [35], so that $Z'$ can decays into a bilepton pair.

The interactions of quarks and neutral gauge bosons are described by the Lagrangian:

$$\mathcal{L}_{NC} = \sum_q e q_i \bar{\Psi}_i \gamma^\mu \Psi_i A_\mu - \frac{g}{2 \cos \theta_W} \left\{ \bar{\Psi}_i \gamma^\mu (g_{V_i} - g_A \gamma^5) \right\} \Psi_i Z_\mu$$
$$+ \bar{\Psi}_i \gamma^\mu (g'_{V_i} - g'_A \gamma^5) \Psi_i Z'_\mu, $$

(7)

where $e q_i$ is the quark electric charge and $g_{V_i}, g_A, g'_{V_i}$ and $g'_A$ are the quark vector and axial-vector couplings with $Z$ and $Z'$ respectively.

As referred before, in the 3-3-1 model, one family must transform with respect to SU(3) rotations differently to the other two. This requirement manifests itself when we collect the quark currents in a part with universal coupling
with $Z'$ similar to the SM and another part corresponding to the non-diagonal $Z'$ couplings. The transformation of these non-diagonal terms, in the mass eigenstates basis, leads to the flavor changing neutral Lagrangian

$$L_{FCNC} = \frac{g \cos \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W} \left( \bar{U}_L \gamma^\mu U_L^T B \bar{U}_L U_L + \bar{D}_L \gamma^\mu V_L^T B \bar{V}_L D_L \right) Z^\prime_{\mu}. \quad (8)$$

where

$$U_L = (u \ c \ t)^T_L, \quad D_L = (d \ s \ b)^T_L \quad \text{and} \quad B = \text{diag} \ (1 \ 0 \ 0).$$

The mixing matrices $U$ (for up-type quark) and $V$ (for down-type quark), that give raise to the quark masses, come from the Yukawa Lagrangian and are related to the Cabibbo-Kobayashi-Maskawa matrix, as

$$U^u_d V = V_{CKM}, \quad (9)$$

By convention, in the SM, it is usual to assume that for up-type quark the gauge interaction eigenstates are the same as the mass eigenstates, which corresponds to $U^u = I$. This assumption is not valid in the 3-3-1 model because, in accordance to the renormalization group equations (RGE), all matrix elements evolve with energy and are unstable against radiative corrections. It turns out that $U^u$ must be $\neq I$. As the Eq. (9) is independent of representation, one is free to choose which quark family representation must be different from the other two. We recall that our choice was for the first family to belong to the triplet $SU(3)$ representation. In the next section we will discuss the consequences of our choice.

All universal neutral couplings diagonal and non-diagonal are presented in the Tables 1 and 2 respectively.

The dominant couplings between ordinary to exotic quarks are driven by single charged bilepton as follows:

$$L_{CC} = -\frac{g}{2\sqrt{2}} \left[ d^\mu(1 - \gamma^5) \left( V_{21} j_2 + V_{31} j_3 \right) + J_1 \gamma^\mu(1 - \gamma^5) U_{11} u \right] V^+_{\mu}. \quad (10)$$

where $V_{21}, V_{31}$ and $U_{11}$ are mixing matrices elements (Eq. (9)).

In addition to the SM gauge boson Lagrangian the trilinear terms used in the present work are:

$$L_{gauge} = -ig \sin \theta_W \left[ A^\nu (V^-_{\mu\nu} - V^+_{\mu\nu} - V^-_{\mu\nu}) + A_{\mu\nu} V^-_{\mu\nu} V^+_{\mu\nu} \right]$$

$$+ \frac{ig}{2} \left( \cos \theta_W + 3 \sin \theta_W \tan \theta_W \right) \left[ Z^\nu (V^-_{\mu\nu} - V^+_{\mu\nu} - V^-_{\mu\nu} V^+_{\mu\nu}) \right]$$

$$+ \frac{ig}{2} \sqrt{3} \left( 1 - 3 \tan^2 \theta_W \right) \left[ Z^\nu (V^-_{\mu\nu} V^+_{\mu\nu} - V^+_{\mu\nu} V^-_{\mu\nu}) + Z'_{\mu\nu} V^-_{\mu\nu} V^+_{\mu\nu} \right], \quad (11)$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, with $B = A, Z, Z'$ and $V^\pm$.

Finally, one of the main features of the model comes from the relation between the $SU_L(3)$ and $U_X(1)$ couplings, expressed as:

$$\frac{g'^2}{g^2} = \frac{\sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}. \quad (12)$$

that fixes $\sin^2 \theta_W < 1/4$, which is a peculiar characteristic of this model, as explained in the Introduction section.
### Table 1: The $Z$ and $Z'$ vector and axial-vector couplings to quarks ($u_1 = u, u_2 = c, u_3 = t, d_1 = d, d_2 = s, d_3 = b$) in the Minimal Model; $\theta_W$ is the Weinberg angle and $U_{ii}$ and $V_{jj}$ are $U$ and $V$ diagonal mixing matrix elements.

| Couplings       | Vector Couplings                                                                 | Axial-Vector Couplings                                                                 |
|-----------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| $Zu_1u_i$       | $\frac{1}{2} - \frac{4 \sin^2 \theta_W}{3}$                                  | $\frac{1}{2}$                                                                      |
| $Zd_jd_j$       | $\frac{1}{2} + \frac{2 \sin^2 \theta_W}{3}$                                  | $\frac{1}{2}$                                                                      |
| $Z'u_1u_i$      | $\frac{1 - 4 \sin^2 \theta_W - U_{u_1}^*U_{u_i} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$ | $\frac{-1 - 4 \sin^2 \theta_W + U_{u_1}^*U_{u_i} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$ |
| $Z'd_jd_j$      | $\frac{1 + 2 \sin^2 \theta_W - V_{jj}^*V_{j_1j_2} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$ | $\frac{-1 + 2 \sin^2 \theta_W + V_{jj}^*V_{j_1j_2} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$ |

3 Results

In this paper we focus on the bilepton ($V^\pm$) pair production in $pp$ collision at LHC. This particle is predicted in many extensions of the SM and in particular in the 3-3-1 model that was used in the present paper. We restrict our calculation to a version of the model where the bilepton mass is related to the mass of the extra neutral gauge boson $Z'$ also predicted in the model, by the Eq. (6).

We fix the exotic quark masses ($j_1, j_2$ and $j_3$) to be 600 GeV and for the $V^\pm$ mass we use a set of values compatible with the findings related to the $Z \rightarrow b\bar{b}$ [30], where the authors obtained the allowed region for exotic quark and bilepton masses, through the deviation between the SM calculation and the experimental data. We adopt the $Z'$ mass in the range from 800 to 1200 GeV, which is in accordance with accepted bounds [33]. All these values are shown in the Table 3.

The group structure of model is such that bileptons couple ordinary to exotic quarks and leptons ($e, \mu$ and $\tau$) with their neutrinos. In the hadronic channel the bilepton can decay in $d$-type quark with $j_{2,3}$ and $u$-type quark with $j_1$. However, for the range of extra neutral gauge boson mass considered here the only decay mode is leptonic because the exotic-quark ordinary-quark channel will only opens when $M_{Z'} = 2 \text{ TeV}$, associated with a bilepton heavier than 600 GeV. In contrast with $W$, which decays into $\bar{\nu}_\ell + \ell$ where $\ell$ is emitted softly, the leptons coming from bilepton carry high transverse momentum. This signature can be used to disentangle the processes of bilepton pair production from $W$ pair production.

In order to calculate the total cross section for bilepton pair production we start by considering the elementary process, $q_i + \bar{q}_i \rightarrow V^+ + V^-$ ($q_i = u, d$), taking into account all contributions: $\gamma, Z$ and a new neutral gauge boson $Z'$. 
Table 2: The flavor changing vector and axial-vector couplings to quarks (u- and d-type) induced by $Z'$ in the Minimal Model.

| Couplings       | Vector Couplings                                                                 | Axial-Vector Couplings                                                                 |
|-----------------|----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| $Z'\bar{c}u$    | $\frac{U_{12}^* U_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{U_{12}^* U_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |
| $Z'\bar{t}u$    | $\frac{U_{13}^* U_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{U_{13}^* U_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |
| $Z'\bar{t}c$    | $\frac{U_{13}^* U_{12} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{U_{13}^* U_{12} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |
| $Z'\bar{d}s$    | $\frac{Y_{12}^* V_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{Y_{12}^* V_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |
| $Z'\bar{b}d$    | $\frac{Y_{13}^* V_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{Y_{13}^* V_{11} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |
| $Z'\bar{b}s$    | $\frac{Y_{13}^* V_{12} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$        | $\frac{Y_{13}^* V_{12} \cos^2 \theta_W}{\sqrt{3} - 12 \sin^2 \theta_W}$            |

in the $s-$ channel and exotic heavy quarks $Q_j$ ($J_1, j_2$ and $j_3$) in the $t-$ channel, as displayed in the Figure 1. At the beginning of our calculation, we have taken into account the heavy quark and $Z'$ widths, however we have verified that our results do not depend on the heavy quarks width then we keep only $Z'$ width in all calculations. We perform the amplitude algebraic calculation with FORM [37].

The elementary differential cross section obtained, as a function of kinematical invariants ($\hat{s}, \hat{t}, \hat{u}$), can be computed, for $k, l = \gamma, Z, Z'$ and $Q_j = J_1, j_2$ and $j_3$, as:

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi\alpha^2}{\hat{s}^2} \sum_{kl} B_{kl}$$ (13)

We present below the amplitudes ($B_{kl}$) corresponding to the diagrams shown.
where \( M \) in the Figure 1:

\[
B_{\gamma} = (eq_i)^2 A(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{ZZ} = (\frac{cz}{e^2})^2 \left[ g_V^2 + g_A^2 \right] \Delta_Z^2 A(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{\gamma} = -2eq_i (\frac{cz}{e^2}) g_V \Delta_Z A(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{Z,Z'} = (\frac{cz}{e^2})^2 (g_V^2 + g_A^2) \Delta_Z^2 A(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{Z,Q_j} = 2sgn(eq_i) \frac{g_V^2 A \frac{cz}{e^2}}{e^2} \left[ g_V^2 (a_j^2 + b_j^2) - 2a_jb_jg_A \right] \Delta Z I_1(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{ZZ'} = -\left(\frac{cz}{e^2}\right) \left(\frac{cz}{e^2}\right) (g_A g_A^2 + g_V g_V^2) A(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{\gamma,Q_j} = -2eq_i \frac{sgn(eq_i)}{G_{V,A}^2} \left[ a_j^2 + b_j^2 \right] I_1(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{Q_j,Q_j} = \frac{G_{V,A}^2}{e^2} \left[ a_j^2 + b_j^2 + 6(a_j b_j)^2 \right] E_2(\dot{s}, \dot{\ell}, \dot{u}) + M_{Q_j}^2 (a_j^2 - b_j^2)^2 E_H(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{ZQ_j} = 2sgn(eq_i) \frac{g_V^2 A \frac{cz}{e^2}}{e^2} \left[ g_V^2 (a_j^2 + b_j^2) - 2a_jb_jg_A \right] \Delta Z I_1(\dot{s}, \dot{\ell}, \dot{u}) \\
B_{\gamma,Z'} = -2eq_i \left(\frac{cz}{e^2}\right) g_V^2 \Delta Z' A(\dot{s}, \dot{\ell}, \dot{u})
\]

(14)

where \( M_Z, M_Z' \) are the neutral gauge boson masses and \( \Gamma_{Z'} \) is the \( Z' \) width; \( a_j \) and \( b_j \) are the ordinary quark-exotic quark-bilepton couplings \( (a_j = b_j = 1) \),

\[
G_{V,A} = \frac{g}{2\sqrt{2}} \gamma_{ij}^\nu, \Delta_Z = \frac{\dot{s}}{s - M_Z^2} \quad \text{and} \quad \Delta Z' = \frac{\dot{s}}{s - M_Z^2 + M_Z \Gamma_{Z'}}.
\]

The trilinear coupling constants \( e_Z \) and \( e_Z' \) are:

\[
e_Z = \frac{e}{2} \frac{1 + 2 \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} \quad \text{and} \quad e_Z' = \frac{e}{2} \frac{\sqrt{3} - 12 \sin^2 \theta_W}{\sin \theta_W \cos \theta_W}.
\]

The functions \( A(\dot{s}, \dot{\ell}, \dot{u}) \), \( E(\dot{s}, \dot{\ell}, \dot{u}) \) and \( I(\dot{s}, \dot{\ell}, \dot{u}) \) are:

\[
A(\dot{s}, \dot{\ell}, \dot{u}) = \left(\frac{\dot{u} \dot{\ell}}{M_V} - 1\right) \left(\frac{1}{4} \frac{M_V^2}{s} + \frac{3M_V^4}{s^2}\right) + \frac{\dot{s}}{M_V} - 4
\]

\[
I(\dot{s}, \dot{\ell}, \dot{u}) = \left(\frac{\dot{u} \dot{\ell}}{M_V} - 1\right) \left(\frac{1}{4} \frac{M_V^2}{2s} - \frac{M_V^4}{s \dot{t}}\right) + \frac{\dot{s}}{M_V} + \frac{2M_V^2}{\dot{t}} - 2
\]

\[
E(\dot{s}, \dot{\ell}, \dot{u}) = \left(\frac{\dot{u} \dot{\ell}}{M_V^4} - 1\right) \left(\frac{1}{4} + \frac{M_V^4}{t^2}\right) + \frac{\dot{s}}{M_V^2}.
\]

(15)
and the functions originated by exotic quark contributions are:

\[
E_1(\hat{s}, \hat{t}, \hat{u}) = \frac{i}{\hat{t} - M_Q^2} E(\hat{s}, \hat{t}, \hat{u})
\]

\[
I_1(\hat{s}, \hat{t}, \hat{u}) = \frac{i}{\hat{t} - M_Q^2} I(\hat{s}, \hat{t}, \hat{u})
\]

\[
E_2(\hat{s}, \hat{t}, \hat{u}) = \left( \frac{i}{\hat{t} - M_Q^2} \right)^2 E(\hat{s}, \hat{t}, \hat{u})
\]

\[
E_H(\hat{s}, \hat{t}, \hat{u}) = \left( \frac{i}{\hat{t} - M_Q^2} \right)^2 \left[ \frac{M_Q^2}{\hat{t}^2} \left( \frac{\hat{s} \hat{t}}{M_Q^2} - 1 \right) + \frac{\hat{s}}{M_Q^2} \left( \frac{1}{4} + \frac{M_Q^4}{\hat{t}^2} \right) \right].
\]

We have performed the \( \hat{t} \) integration within the limits: \( \hat{t}_{min} = M_V^2 + M_q^2 - \hat{s}/2(1 - \beta(-1 + \cos \theta)) \) and \( \hat{t}_{max} = M_V^2 + M_q^2 - \hat{s}/2(1 - \beta(1 - \cos \theta)) \), where \( M_V \), \( M_q \) and \( M_Q \) are the bilepton, ordinary and exotic quark masses and the definition

\[
\beta = \sqrt{\frac{(\hat{s} - 4 M_V^2)(\hat{s} - 4 M_q^2)}{\hat{s}}}
\]

For short we call \( X = \int X(\hat{s}, \hat{t}, \hat{u}) \, dt \), where \( X = A, E, I, I_1, E_2, E_H \) and \( X = A, E, I, I_1, E_1, E_2, E_H \):

\[
A = \beta \left( s - 4 M_V^2 \right) \left( \frac{1}{2} + \frac{5 s}{6 M_V^2} + \frac{s^2}{24 M_V^4} \right)
\]

\[
E = \beta s \left( -2 + \frac{5 s}{6 M_V^2} + \frac{s^2}{24 M_V^4} \right) + (s - 2 M_V^2) \, L_x
\]

\[
I = \beta \left( s - 2 M_V^2 \right) \left( \frac{1}{2} + \frac{5 s}{6 M_V^2} + \frac{s^2}{24 M_V^4} \right) - \frac{(2 s + M_V^2) M_V^2}{s} L_x
\]

(17)

and

\[
E_1 = -\frac{\beta s}{4 M_V^2} \left[ M_Q^6 + \left( s - 2 M_V^2 \right) M_Q^4 + \left( 4 M_V^4 - 3 s M_V^2 + \frac{s^2}{12} \beta^2 - 3 \right) \right] - \frac{M_Q^4}{M_Q^2} L_x
\]

\[
+ \frac{1}{4 M_V^2} \left[ M_Q^6 + (s - 2 M_V^2) M_Q^4 + (5 M_V^4 - 4 M_V^2 s) M_Q^2 - 4(2 M_V^2 - s) M_V^4 + \frac{M_Q^2}{M_Q^2} \right] L_x
\]

\[
E_2 = \left[ \frac{M_Q^6}{M_V^2} + \left( -\frac{3}{2 M_V^2} + \frac{3 s}{4 M_V^4} \right) M_Q^4 + \left( -\frac{2 s}{M_V^2} + \frac{5}{2} \right) M_Q^2 + s - 2 M_V^2 \right] L_x
\]

\[
- \frac{\Delta}{3} \left[ 24 M_Q^6 + 30 (s - 2 M_V^2) M_Q^4 + (96 M_V^4 - 68 M_V^2 s + 5 s^2) M_Q^2
\]

\[
- (s^3 - 94 M_V^2 s + 18 M_V^4 s^2 + 108 M_V^6 s) M_Q^2 - M_V^2 s^2 + 48 M_V^6 - 20 M_V^6 s \right] L_x
\]

\[
E_H = \frac{\Delta}{2} \left[ 8 (s - 4 M_V^2) M_Q^4 + 4 (16 M_V^4 - 10 M_V^2 s + s^2) M_Q^2 - 32 M_V^6 - 8 M_V^2 s^2 \right]
\]

9
\[+36 M_V^4 s + s^3 - s^3 \beta^2 + 4 M_V^2 s^2 \beta^2 - \frac{1}{2 M_V^4} \left(4 M_V^4 - 4 M_Q^2 M_V^2 + M_Q^2 s - 2 M_V^2 s \right) L_\psi\]

\[I_1 = \frac{(s - 2 M_V^2) M_Q^2}{4 M_V^4} L_\psi - \left[\beta (s - 2 M_V^2) - (s - 4 M_V^2) L_\psi \right] \frac{M_Q^4}{4 M_V^4} + \frac{2 \beta \left(M_V^4 - \frac{s^2}{4} - M_V^2 s \right)}{4 M_V^2} \left(4 s^2 - 6 M_V^4 - 5 M_V^2 s \right) \frac{M_Q^2}{4 M_V^4} \]

\[\sum \beta (s - 4 M_V^2) \left(s^2 + 3 M_V^2 s^2 - \frac{14 M_V^4 s}{3} - 4 M_V^4 \right) \frac{48 M_V^2 (2 s + M_V^2)}{s} L_\psi \]

where:

\[L_x = 2 \ln \left(\frac{1 + \beta}{1 - \beta} \right), \quad L_\psi = \ln \left(\frac{2 M_Q^2 - 2 M_V^2 + s + s \beta}{2 M_Q^2 - 2 M_V^2 + s - s \beta} \right)\]

and

\[\Delta = \frac{s \beta}{8 M_V^4 \left(M_Q^4 + (s - 2 M_V^2) M_Q^2 + M_V^4 \right)}\]

The crossing relation for the functions coming from \( \hat{t} \) and \( \hat{u} \) channels gives,

\[\int E(\hat{s}, \hat{t}, \hat{u}) d\hat{t} = \int E(\hat{s}, \hat{u}, \hat{t}) d\hat{u}\]

\[\int I(\hat{s}, \hat{t}, \hat{u}) d\hat{t} = \int I(\hat{s}, \hat{u}, \hat{t}) d\hat{u}\]

In order to obtain the elementary total cross section for each parton in the initial proton, we sum the contributions \( B_{lm} \) integrated over \( t \),

\[\hat{\sigma}_{qq} = \int \frac{d\hat{\sigma}}{d\hat{t}} \frac{2\pi \alpha^2}{s^2} \sum \int B_{lm} d\hat{t}\]

It is well known that in the SM the correct behavior of the total cross section at high energy for charged gauge boson pair production is extremely dependent on the balance between \( s \)- and \( t \)-channel contributions \([38]\), as can be seen in the Figure 2, which shows the elementary cross sections \((u\bar{u} \text{ and } d\bar{d})\) for \( W \) pair production. In fact, the renormalizability of the theory is translated into the cancellation between these contributions at high energy. For some models the new neutral boson and/or exotic fermion contributions can guarantee this delicate cancellation \([38]\).

Let us discuss how this comes about for the present model where there are two non standard contributions: \( Z' \) and exotic quarks. We present in Figures 3 and 4 some individual amplitudes \( B_{ij} \) (Eq. 14) that correspond to the elementary sub-processes, \( u\bar{u} \) and \( d\bar{d} \) for \( M_{Z'} = 800 \text{ GeV} \). In the Figure 3 we plot the main contributions \((B_{Z'Z'} , B_{ZZ} , B_{\gamma\gamma} , B_{ZZ' , B_{\gamma} , B_{Q} , B_{Q'}})\). We do not display the remaining interferences \((B_{Z'Z'} , B_{ZQ} , B_{\gamma Q} , B_{Z'Q} , B_{Z'Q'})\), that get values between \( B_{Z'Z'} \) and \( B_{Q} , B_{Q'} \). In the Figure 4, we show the similar relevant contributions for \( d\bar{d} \) with one additional exotic quark. We omit the curves corresponding to
$B_{\gamma Z'} \simeq B_{\gamma Z}, B_{ZQ_1} + B_{ZQ_2} \simeq B_{Q_1 Q_3} + B_{Q_1 Q_2} + B_{Q_2 Q_2}, B_{\gamma Q_1} + B_{\gamma Q_2} \simeq B_{\gamma Z}, B_{Z' Q_1} + B_{Z' Q_2} < B_{ZZ'}$. One can observe again the large extra gauge boson ($B_{Z'Z'}$) component and the tiny exotic quark and interference contributions.

Let us show in the Figure 5 the bad behavior in energy for the elementary cross section when we consider only the neutral boson contributions ($\gamma, Z$ and $Z'$). One can see clearly that, when the energy increases, the $uu$ sub-process violates softly the unitarity behavior (only visible for $\sqrt{s} > 4$ TeV) and, on the other hand, $dd$ leads to a more drastic violation. As expected, the $dd$ process is more sensitive than $uu$, because $dd$ channel receives more exotic quark contribution. This behavior imposes to add the exotic quark contributions ($via\ t$ involving the charged current. The amplitudes balance has to occur between the exotic quark ($via\ t$) and the $s$-channel.

Besides, this cancellation requires to take into account the mixing between the quark eigenstates respecting the constraint given by Eq. (9). The quark mixing depends on what family must belong to a different $SU(3)_L$ representation. Working in the minimal version $[17]$, where the first family is in the $SU(3)$ triplet representation, it is possible to obtain mixing parameters compatible with Eq. (9) and restoring the correct high energy behavior. There is no parametrization, in the literature, for $U$ matrix elements, however some limits for $V$ elements have been obtained from $Z'$ rare decay bounds in $[33, 40, 41]$. We cannot exclude the possibility that another choice of mixing parameters would provide a correct energy behavior, when the third quark family is treated differently.

The Figures 6 and 7 show the elementary $q + \bar{q} \rightarrow V^+ + V^-$ total cross section for the set of quark mixing parameters: $U_{11} = 0.1349989, V_{11} = 0.900542, V_{12} = 0.1009984$, and $V_{31} = V_{11}$. In these figures we have used three values for $M_{Z'} = 800$ GeV, $1000$ GeV and $1200$ GeV for $M_Q = 600$ GeV. It can be noted the good behavior of the elementary cross sections presenting a peak around the $Z'$ mass and becoming broader and smaller as $Z'$ mass increases. This range of values relies on our previous work $[30]$ where we have establish bounds on $Z'$ mass in two versions of 3-3-1 models for $e^+e^-$ and hadron colliders, obtaining results that are compatible with experimental bounds. Here we consider the constraints given by Eq. (6). We display in the Table I some values for $M_{Z'}$, $\Gamma_{Z'}$, $M_V$ and $\Gamma_V$.

The total cross section for $p + p \rightarrow V^+ + V^- + X$ is obtained integrating the elementary total cross section weighted by the distribution function for partons in hadron (proton) $[32]$.

$$\sigma(p + p \rightarrow V^+ + V^- + X) = \sum_{i,j} \int_{thr}^{1} \int_{thr}^{1} dx_1 dx_2 f_i(x_1, Q^2)f_j(x_2, Q^2) \hat{\sigma}_{qq}.$$  

To obtain more realistic results, we have applied an angular cut on the angle between the final bileptons with respect to the initial beam direction, $|\eta| \leq 2.5$.

The final results are displayed in the Figure 8, where the total cross section is plotted as a function of the bilepton mass. We consider two energy regimes ($\sqrt{s} = 10$ TeV and $\sqrt{s} = 14$ TeV) to compare with the $W^+ W^-$ production. It is clear from the plot that, even for $\sqrt{s} = 10$ TeV, the production of bileptons pairs with $M_V \leq 300$ GeV is larger than $W$ pair production, allowing for a large number of events originated from bilepton decay. For a low LHC luminosity around $1 \text{fb}^{-1}$ and c.m. energy of $10$ TeV it can be produced a thousand of
Figure 1: The Feynman diagrams for $q + ar{q} \rightarrow V^+ + V^-$ process with s-channel and $t$ (u)-channel contributions.

Figure 1: The Feynman diagrams for $q + ar{q} \rightarrow V^+ + V^-$ process with s-channel and $t$ (u)-channel contributions.

$M_V \simeq 300$ GeV pairs. For $\sqrt{s} = 14$ TeV the same number of pair can be produced for $M_V \simeq 450$ GeV. This scenario is related to a very massive extra neutral gauge boson existence.

4 Conclusions

In this paper we focus on the bilepton ($V^\pm$) pair production in $pp$ collision at LHC. This particle is predicted in many extensions of the SM and in particular in the 3-3-1 model used in the present paper. We restrict our calculation to a version of the model where the bilepton mass is related to the mass of the extra neutral gauge boson $Z'$, also predicted in the model. For the range of the extra neutral gauge boson mass considered here, the dominant $V^\pm$ decay is leptonic ($\nu_\ell + \ell$). The hadronic channel ($J_i + q_i$) will open when $M_{Z'} = 2$ TeV, associated with a bilepton heavier than 600 GeV. For the elementary Drell-Yan process, there are the contributions of $\gamma$, $Z$, and $Z'$ in the s-channel and the exotic quark in the t-channel. $J_1$ is exchanged when the initial quark of colliding proton is an up-type quark, but when the down-type quark is participating, $j_2$ and $j_3$ are exchanged. This is a consequence of our choice for family quark representation.
Figure 2: The elementary total cross section for the process \( q + \bar{q} \rightarrow W^+ + W^- \) for the SM.
Figure 3: The partial amplitudes the process $u + \bar{u} \rightarrow V^+ + V^-$ for the 3-3-1 model considering $M_{Z'} = 800$ GeV and $M_{J_1} = 600$ GeV.
Figure 4: The partial amplitudes the process $d + \bar{d} \rightarrow V^+ + V^-$ for the 3-3-1 model considering $M_{Z'} = 800$ GeV and $M_{j_2} = M_{j_3} = 600$ GeV.
Figure 5: The elementary cross section for $u \bar{u}$ and $d \bar{d}$ sub-process for $M_{Z'} = 800$ considering only the $s$–channel contributions.
Figure 6: The elementary total cross section for the process $u + \bar{u} \rightarrow V^+ + V^-$ for the 3-3-1 model considering $M_{Z'} = 800$ GeV, 1000 GeV and 1200 GeV and $M_{J_1} = 600$ GeV.
Figure 7: The elementary total cross section for the process $d + \bar{d} \rightarrow V^+ + V^-$ for the 3-3-1 model considering $M_{Z'} = 800$ GeV, 1000 GeV and 1200 GeV and $M_{j_2} = M_{j_3} = 600$ GeV.
Figure 8: The total cross section for the process $p + p \rightarrow V^+ + V^- + X$ against $M_V$ for the 3-3-1 model considering $\sqrt{s} = 10$ TeV and $\sqrt{s} = 14$ TeV. The horizontal line is the SM.
The correct high energy behavior of the elementary cross section follows from the balance between the individual contributions. In order to emphasize the role of the exotic quark contribution, we present in the Figure 5 the "bad" behavior of the elementary cross section in the absence of the t-channel contribution for a fixed $Z'$ mass, we see clearly that the $u\bar{u}$ sub-process violates "softly" the unitarity bound and $d\bar{d}$ leads to a more severe violation. As expected, the $d\bar{d}$ process is more sensitive than $u\bar{u}$, because this channel receives additional exotic quark contribution.

When considering the t-channel contribution we take into account the mixing of quark mass eigenstates originated from the Yukawa coupling. In this work we have obtained a set of mixing parameters allowing to a good behavior for the elementary cross section, for different $M_{Z'}$. These parameters are related to our particular choice for $SU(3)_L$ family representation. This result does not exclude any other choice for quark representation.

In the 3-3-1 model, $Z'$ couples to the quarks in a non universal way leading to the existence of flavor changing vertices. As a consequence, the quark mixing parameters are also present in $Z'$ quark vertices. We display in the Tables I and II our results for the flavor changing couplings.

In order to obtain the total cross section for the production of bilepton pairs we employed the cut on the final particle pseudo-rapidity. Considering a conservative integrated luminosity value and $\sqrt{s} = 10$ TeV we predict the production of a thousand of $M_V \simeq 300$ GeV pairs mainly due to the contribution from $M_{Z'} \simeq 1$ TeV. For $\sqrt{s} = 14$ TeV the same number of events is obtained for $M_V \simeq 450$ GeV pairs, associated to a $M_{Z'} \simeq 1.4$ TeV. One can ask about the possibility of Tevatron to find a large amount of bileptons. In fact, as our prediction lies on a large $M_{Z'}$ (greater than 800 GeV), the required energy per quark for an individual sub-process $q\bar{q}$ would be larger than 500 GeV not available at Tevatron, where the energy beam is about 900 GeV. For this reason the Tevatron gives $M_{Z'} > 600$ GeV.

Finally, we observe that it is possible to distinguish the leptons coming from bileptons with those from the background of $W$ decay. In contrast with $W^\pm$.

| $M_{Z'}$ (GeV) | $\Gamma_{Z'}$ (GeV) | $M_{V^\pm}$ (GeV) | $\Gamma_{V^\pm}$ (GeV) |
|---------------|-----------------|-----------------|-----------------|
| 800           | 117             | 217             | 1.84            |
| 1000          | 149             | 271             | 2.28            |
| 1200          | 181             | 407             | 3.44            |

Table 3: Some widths for new gauge bosons $Z'$ and $V^\pm$ in the minimal 3-3-1 model.
which decays into \( \bar{\nu}_\ell \ell \), the charged lepton coming from the bilepton decay has a large transverse momentum. A useful \( p_T \) cut can eliminate this SM background.

We conclude that a large number of single charged bilepton pairs can be produced in the early stage of the LHC.

Acknowledgments: We thank Prof. V. Pleitez and Prof. F. Schwab for useful discussions. E. Ramirez Barreto thanks Capes and Y. A. Coutinho thanks FAPERJ for financial support.

References

[1] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. D 76, 111104R (2007).
[2] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. B 282, 253 (1986).
[3] K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, Phys. Rev. D 48, 2182 (1993).
[4] Ronald F. Peierls, T. L. Trueman, and Ling-Lie Wang, Phys. Rev. D 16, 1397 (1977).
[5] R. W. Brown, K. O. Mikaelian, Phys. Rev. D 19, 922 (1978).
[6] R. Philippe, Phys. Rev. D 26, 1588 (1982).
[7] E. Eichten, H. Hinchiliffe, K. D. Lane, and C. Quigg, Rev. Mod. Phys. 56, 579 (1984); 58, 1065(E) (1986).
[8] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001); N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002).
[9] An extensive list of references can be found in R. N. Mohapatra and P. B. Pal, ”Massive Neutrinos in Physics and Astrophysics”, World Scientific, Singapore, 1998.
[10] For a review see e.g. J. L. Hewett, T. G. Rizzo, Phys. Rep. 183, 193 (1989).
[11] For a pedagogical review see e.g. T. G. Rizzo, hep-ph/0409309.
[12] Howard Georgi, Phys. Rev. Lett. 98, 221601 (2007).
[13] M. C. Gonzalez-Garcia, A. Santamaria, and J. W. F. Valle, Nucl. Phys. B 342, 108 (1990); J. Maalampi, and M. Roos, Phys. Rep. 186, 53 (1990); T. G. Rizzo, Phys. Rev. D 34, 2076 (1986).
[14] Y. A. Coutinho, J. A. Martins Simões, and P. P. Queiroz Filho, Phys. Rev. D 54, 3497 (1996); Y. A. Coutinho, J. A. Martins Simões, and M. C. Pommot Maia, Phys. Rev. D 45, 771 (1992).
[15] J. Maalampi, A. Pietilä, and J. Vuori, Nuc. Phys. B 381, 544 (1992).
[16] Y. A. Coutinho, A. J. Ramalho, R. Walsh, and S. Wilck, Phys. Rev. D 64, 115008 (2001); R. Walsh, A. J. Ramalho, Phys. Rev. D 65, 055011 (2002).

[17] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992).

[18] P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992).

[19] J. C. Montero, F. Pisano, and V. Pleitez, Phys. Rev. D 47, 2918 (1993); R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50, R 34 (1994); Hoang Ngoc Long, Phys. Rev. D 53, 437 (1996); ibid 54, 4691 (1996); V. Pleitez, Phys. Rev. D 53, 514 (1996).

[20] V. Pleitez and M. D. Tonasse, Phys. Rev. D 48, 2353 (1993).

[21] For a review of bileptons see F. Cuypers and S. Davidson, Eur. Phys. J. C 2, 503 (1998).

[22] P. H. Frampton and B.-H. Lee, Phys. Rev. Lett. 64, 619 (1990).

[23] T. V. Duong and E. Ma, Phys. Lett. B 316, 307 (1993); H. N. Long and P. B. Pal, Mod. Phys. Lett. A 13, 2355 (1998); M. Capdequi-Peyranere and M. C. Rodriguez, Phys. Rev. D 65, 035001 (2002); J. C. Montero, V. Pleitez, and M. C. Rodriguez, Phys. Rev. D 65, 095008 (2002); Rodolfo A. Diaz, R. Martinez, J. Mira, and J.- Alexis Rodriguez, Phys. Lett. B 552, 287 (2003); R. Martinez, N. Poveda, and J.- Alexis Rodriguez, Phys. Rev. D 69, 075013 (2004).

[24] Adrian Palcu, Mod. Phys. Lett. A 21, 2591 (2006); J. C. Montero, C. A. De S. Pires and V. Pleitez, Phys. Rev. D 65, 095001 (2002); P. V. Dong, H. N. Long, D. T. Nhung, and D. V. Soa, Phys. Rev. D 73, 035004 (2006).

[25] V. Pleitez, Phys. Rev. D 53, 514 (1996).

[26] Felice Pisano, Mod. Phys. Lett. A 11, 2639 (1996).

[27] A. G. Dias, R. Martinez, V. Pleitez, Eur. Phys. J. C 39, 101 (2005).

[28] A. G. Dias, Phys. Rev. D 71, 015009 (2005).

[29] Daniel Ng, Phys. Rev. D 49, 4805 (1994).

[30] E. Ramirez-Barreto, Y. A. Coutinho, J. Sá Borges, Eur. Phys. J. C 50, 909 (2007).

[31] E. Ramirez-Barreto, Y. A. Coutinho, J. Sá Borges, Phys. Lett. B 632, 675 (2006).

[32] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky, and W. K. Tung, JHEP 207, 12 (2002).

[33] A. Carcamo, R. Martinez, and F. Ochoa, Phys. Rev. D 73, 035007 (2006).

[34] B. Dion, T. Grégoire, D. London, L. Marleau, H. Nadeau, Phys. Rev. D 59, 075006 (1999).
[35] C. Amsler et al., Physics Letters B 667, 1 (2008).

[36] G. A. González-Sprinberg, R. Martínez, and O. Sampayo, Phys. Rev. D71, 115003 (2005).

[37] J. A. M. Vermaseren, mat-ph/0010025

[38] John Ellis, Mary K. Gaillard, Georges Girardi, and Paul Sorba, Ann. Rev. Nuc. Part. Sci. 32, 443 (1982).

[39] Christoph Promberger, Sebastian Schatt, and Felix Schwab, Phys. Rev. D 75, 115007 (2007).

[40] J.-Alexis Rodriguez, Marc Sher, Phys. Rev. D 70, 117702 (2004).

[41] D. Gomez Dumm, F. Pisano, V. Pleitez, Mod. Phys. Lett. A 9, 1609 (1994).