On a first order transition in QCD with up, down and strange quarks

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Abstract We consider the quark-mass dependence of the baryon octet and decuplet ground state masses. It is predicted that QCD dynamics implies a first order transition when increasing the strange quark mass from its chiral limit towards its physical value. Our claim relies on a global fit to the available QCD lattice data on such baryon masses. Quantitative results based on an application of the chiral SU(3) Lagrangian at N\textsuperscript{3}LO are discussed. We predict an anomalous sector of QCD where stable baryonic matter would be composed of \( \Lambda \) or \( \bar{\Lambda} \) particles rather than nucleons and anti-nucleons

Keywords chiral symmetry · flavor SU(3) · lattice QCD

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1 Introduction

A most fundamental question of modern theoretical physics is to explain how small-scale structures are formed in terms of local quantum field theories. Nature shows a plethora of phenomena, like finite nuclei, conventional or exotic hadrons, which are thought to be a consequence of the strong interaction as encoded in quantum chromodynamics (QCD). The non-perturbative nature of the latter makes it quite difficult to arrive at a quantitative link from QCD to observable quantities as measured in the laboratory (see e.g. \textsuperscript{[8,11]}). While QCD lattice simulations have seemingly converged conclusions on a smooth crossover behaviour of the chiral transition at finite temperature, \( T \), similar studies at finite baryon chemical potential, \( \mu \), are still not possible (see e.g. \textsuperscript{[12,13]}). In this Letter we wish to point at another sector of QCD, which is within reach of current lattice QCD simulation technology. While QCD is expected to show a rich phase structure on the external parameters \( T \) and \( \mu \), possible parametric phase transitions in the up, down and strange quark masses did not receive much attention so far. The possibility of a discontinuous quark-mass dependence in the baryon masses was discussed in \textsuperscript{[14]}. The purpose of this Letter is to take up this issue, given the rather large data set generated on various QCD lattice ensembles over the last 15 years. We argue that by now it is possible to arrive at more definite conclusions.

QCD lattice simulations with three light flavors at pion and kaon masses smaller than 600 MeV are considered that are available publicly. That leaves data sets from PACS-CS, LHPC, HSC, NPLQCD, QCDSFUKQCD and ETMC \textsuperscript{[15,16,17,18,19,20]}. We are aware of the recent lattice ensembles of the CLS group with 2+1 flavors based on nonperturbatively improved Wilson fermions \textsuperscript{[21,22,23]}. Results for baryon masses are not available yet.

2 An effective field theory approach

We study QCD in terms of its chiral SU(3) Lagrangian. The strong interaction of hadrons can be described efficiently in terms of effective degrees of freedom at least
at low enough energies [24]. The most prominent effective fields interpolate the pseudo Goldstone bosons, the pion, kaon and eta mesons. This sector [25] of the chiral Lagrangian is well established

\[
\mathcal{L} = -f^2 \left( \{ U_\mu, U_\nu \} + \frac{1}{2} f^2 \left\{ \chi_+ \right\} \right) - 8 L_4 \left( \{ U_\mu U^\mu \} \right) \left\{ \chi_+ \right\} + 8 L_6 \left( \{ U_\mu U^\mu \} \right) \left\{ \chi_+ \right\} + 4 L_7 \left( \{ U_\mu U^\mu \} \right) \left\{ \chi_- \right\} + 4 L_7 \left( \{ U_\mu U^\mu \} \right) \left\{ \chi_- \right\}
\]

\[
+ 2 L_8 \left( \{ \chi_+ \chi_+ + \chi_- \chi_- \} \right), \quad u = e^{\frac{2i}{f}},
\]

\[
U_\mu = \frac{1}{2} u^\dagger \left( \partial_\mu e^{\frac{2i}{f}} \right) u^\dagger, \quad \chi_{\pm} = \frac{1}{2} \left( \chi_0 u \pm u^\dagger \chi_0 u^\dagger \right),
\]

where we display terms only that are relevant for our current work. While the matrix field \( \Phi \) in [1] combines the pion, kaon and eta meson fields into a suitable 3\times3 matrix, the symmetry breaking field \( \chi_0 \sim \text{diag}(m_u, m_d, m_s) \) is proportional to the current quark masses of QCD. By means of [1] the quark-mass dependence of the pion, kaon and eta meson masses can be computed in QCD. At the one-loop level besides the quark masses the low-energy constants \( f, L_n \) are involved only. Here we use resummed expressions where the tadpole terms involve the on-shell meson masses [27,28].

Accurate results at the 10 MeV uncertainty level for the baryon masses can be obtained at next-to-next-to-leading order (N3LO). This involves a large set of low-energy constants (LEC). Here sum rules for the LEC as derived from QCD with a large number of colors \( N_c \) are instrumental and reduce the number of fit parameters significantly [29,30,27]. It is important to consider the baryon octet and baryon decuplet fields on an equal footing. After all in the large-\( N_c \) limit of QCD the two flavor multiplets turn degenerate. Here we recall a selection of terms which involve the baryon octet fields only. Analogous terms with the decuplet fields can be selected of terms which involve the baryon octet fields the two flavor multiplets turn degenerate. Here we recall a selection of terms which involve the baryon octet fields only. Analogous terms with the decuplet fields can be taken from [30,31,27]. The chiral Lagrangian is

\[
\mathcal{L} = \text{tr} \left\{ B \left( \partial^\mu - M \right) B + F \left\{ B i \gamma^\mu \gamma_5 \left\{ U_\mu, U_\nu \right\}, B \right\} \right\} + D^\mu \left\{ B \gamma^\mu \gamma_5 \left\{ i U_\mu, B \right\} \right\} + 2 b_0 \left\{ B \right\} \left\{ \chi_+ \right\} + 2 b_D \left\{ B \right\} \left\{ \chi_- \right\} - \frac{1}{2} g_0^{(S)} \left\{ B \right\} \left\{ U_\mu U^\mu \right\} - \frac{1}{2} g_1^{(S)} \left\{ B U^\mu U_\mu \right\} - \frac{1}{4} g_0^{(D)} \left\{ B \right\} \left\{ U_\mu U_\nu \right\} \left\{ U_\mu U_\nu \right\} \right\} - \frac{1}{4} g_0^{(G)} \left\{ B \right\} \left\{ U_\mu U_\nu \right\} \left\{ U_\mu U_\nu \right\} \right\} - \frac{1}{8} g_1^{(V)} \left\{ B U_\mu \right\} \left\{ U_\nu \right\} \left\{ D^\nu B \right\} + \left\{ B \right\} \left\{ U_\mu \right\} \left\{ U_\nu \right\} \left\{ D^\nu B \right\} - \frac{1}{8} g_0^{(G)} \left\{ B i \gamma^\mu \left\{ U_\mu, U_\nu \right\}, D^\nu B \right\} \right\} - \frac{1}{8} g_0^{(G)} \left\{ B i \gamma^\mu \left\{ U_\mu, U_\nu \right\}, D^\nu B \right\} \right\}.
\]

\[
\Gamma_\mu = \frac{1}{2} u^\dagger \partial_\mu u + \frac{1}{2} u \partial_\mu u^\dagger,
\]

\[
D_\mu B = \partial_\mu B + \Gamma_\mu B - B \Gamma_\mu.
\]
Lattice data sets even more compatible. The only exception being the data on the LHPC ensembles, which appear to set some tension. Like in our previous study such data are reproduced with a somewhat larger uncertainty of 20 MeV only. Therefore, in our chi-square function such data points are assigned a systematic uncertainty of 20 MeV. We checked that our results are stable with respect to a complete omission of such data points.

As pointed out already in our previous works the low-energy constants, $2L_6 - L_4$, $2L_8 - L_5$ and $L_8 + 3L_7$ can be determined quite accurately from a global fit to the baryon masses as measured on various QCD lattice ensembles. This is so since the baryon masses
depend rather sensitively not only on the quark masses but also on the meson masses.

In Fig. 1 we confront our results with the lattice data set for the baryon octet masses along the trajectory $m/m_{\text{phys}}$. Finite volume effects are taken into account. Here we use the isospin averaged quark mass $m = m_u = m_d$. While the lattice data are shown by filled colored symbols, the chiral extrapolation result by corresponding open symbols displayed on top of the lattice points. Fit 1 results are systematically shown on top of Fit 2 results. The fact that almost always an open symbol covers its corresponding colored symbol confirms that the QCD lattice data are very well reproduced by our two fits. It is important to note that the lattice data set provides the baryon masses at various values of the strange quark mass. This has decisive impact on the determination of the set of low-energy parameters.

Fig. 2 shows the baryon masses in the infinite volume limit as a function of $m/s/m_{\text{phys}}$ based on three distinct scenarios. The first two are implied by $m = m_u = m_d = m_{\text{phys}}$ (solid lines) or $m_u = m_d = m_s$ (dotted lines). The last one is implied by $m + m_s = (m + m_{\text{phys}})$ (dashed lines). The differences of Fit 1 and Fit 2 are largest along the trajectory, for which we show the results of Fit 2 in terms of dot-dashed line. All other lines are with respect to Fit 1. For all trajectories we find a smooth dashed line. All other lines are with respect to Fit 1.

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A first order transition is implied also in the standard model scenario where the ratio $2 m_s/(m_u + m_d) \simeq 26$ takes its physical value. Along such a trajectory the light quark masses $m_u$ and $m_d$ are smaller than their physical values. Our findings are stable against one-sigma variations in the LEC.

| $10^3 (2 L_d - L_A)$ | $10^3 (2 L_s - L_d)$ | $10^3 (L_s + 3 L_T)$ |
|---------------------|---------------------|---------------------|
| $m/s/m_{\text{phys}}$ | $b_0 [\text{GeV}^{-1}]$ | $b_0 [\text{GeV}^{-1}]$ |
| $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ |
| $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ |
| $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ |
| $M [\text{GeV}]$ | $M [\text{GeV}]$ | $M [\text{GeV}]$ |
| $b_0 [\text{GeV}^{-1}]$ | $b_0 [\text{GeV}^{-1}]$ | $b_0 [\text{GeV}^{-1}]$ |
| $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ |
| $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ |
| $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ |
| $M [\text{GeV}]$ | $M [\text{GeV}]$ | $M [\text{GeV}]$ |
| $b_0 [\text{GeV}^{-1}]$ | $b_0 [\text{GeV}^{-1}]$ | $b_0 [\text{GeV}^{-1}]$ |
| $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ | $g_s \frac{S}{\text{GeV}^{-1}}$ |
| $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ | $g_s \frac{V}{\text{GeV}^{-2}}$ |
| $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ | $g_s \frac{D}{\text{GeV}^{-2}}$ |

Table 1 Results for LEC in $\mu$ based on two fit scenarios. The low-energy constants $L_d$ are at the renormalization scale $\mu = 0.77$ GeV. We use $f \approx 92.4$ MeV and $D \approx 0.75$ throughout this work.

3 Summary and conclusions

We have shown that QCD lattice data on the baryon masses are compatible with a first order transition along the trajectory where the strange quark mass goes from its chiral limit to its physical value. Our global fits predict an anomalous sector of QCD where stable baryonic matter would be composed of strange particles, the $\Lambda$, rather than nucleons. In the region

$$9.7 < m_s/m_{\text{phys}} < 14.8 \Leftrightarrow M_A < M_N, \quad (3)$$

normal baryonic matter does not exist in our preferred scenario. The figures are supplemented with a compilation of baryon masses as they are implied by our Fit 1 of the lattice data set, where now the masses are given in the finite box as set up by the lattice groups. Such points illustrate that so far lattice data provide very few direct constraints on hadron masses close to the $m = m_{\text{phys}}$ trajectory. We emphasize that all theory points shown in Fig. 2 correspond to the open-symbol points of Fig. 1 and therefore are as close to their lattice points as shown already in Fig. 1. The theory points are just displayed in a different manner.

A first order transition is implied also in the standard model scenario where the ratio $2 m_s/(m_u + m_d) \simeq 26$ takes its physical value. Along such a trajectory the light quark masses $m_u$ and $m_d$ are smaller than their physical values. Our findings are stable against one-sigma variations in the LEC.

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