Thin accretion disk around a Kaluza-Klein black hole with squashed horizons

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Abstract

We study the accretion process in the thin disk around a squashed Kaluza-Klein black hole and probe the effects of the extra dimensional scale $\rho_0$ on the physical properties of the disk. Our results show that with the increase of the parameter $\rho_0$, the energy flux, the conversion efficiency, the radiation temperature, the spectra luminosity and the spectra cut-off frequency of the thin accretion disk decrease, but the inner border of the disk increases. This implies that the extra dimension scale imprints in the mass accretion process in the disk.

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I. INTRODUCTION

String theory is widely believed to be the most promising candidate theory for a unified description of everything in our Universe, which predicts the existence of the extra dimension. Thus, the detection of the extra dimension has attracted a great deal of attention recently since it can present the signature of the string and the modification of string theory. It is shown that the quasinormal modes originated from the perturbation around high dimensional black holes \[1-4\] carry the peculiar information about the extra dimension, which would be tested in the gravitational wave probe in the near future. Moreover, the spectrum of Hawking radiation from the high dimensional black holes could provide another possible way to observe the extra dimension, which is expected to be detected in particle accelerator experiments \[5-13\]. The investigations of the strong gravitational lensing of the brane world black holes \[14\] indicate that the extra dimension imprints in the deflection angle, the angular position and magnification of the relativistic images, which implies that the extra dimension might be observed by measuring these lensing parameters in the astronomical experiments.

Since the accretion processes is a powerful indicator of the physical nature of the central celestial objects, the analysis of the signatures of the accretion disk could provide another possible way to detect the extra dimension. The accretion disk is such a structure formed by the diffuse material in orbital motion around a central compact body, which now is an important research topic in the astrophysics. The simplest theoretical model of the accretion disks is the steady-state thin accretion disk model in which the disk has negligible thickness \[15-18\]. In this model, the disk is in hydrodynamical equilibrium since the heat generated by stress and dynamic friction in the disk can be dispersed through the radiation over its surface. Moreover, the mass accretion rate in the disk maintains a constant and is independent of time variable. The physical properties of matter forming a thin accretion disk in a variety of background spacetimes have been investigated extensively in \[19-25\]. The special signatures appeared in the energy flux and the emission spectrum emitted by the disk can provide us not only the information about black holes in the Universe, but also the profound verification of alternative theories of gravity. Therefore, the study of properties of the thin disk around high dimensional black holes could help us to probe the extra dimension in astronomical observation in the future.

The Kaluza-Klein black hole with squashed horizons is a new kind of interesting Kaluza-Klein type metrics \[26-32\], which is the solution of the Einstein-Maxwell equations in the five-dimensional spacetime. This family of black holes have a structure of the five-dimensional black holes in the vicinity of the black hole horizon, but are locally the direct product of the four-dimensional Minkowski spacetime and the circle in the region far
from the black holes. Recently, the Hawking radiation has been considered in these squashed Kaluza-Klein black holes under the low-energy approximation, which indicates that the luminosity of Hawking radiation can tell us the size of the extra dimension which could open a window to detect extra dimensions \(^{33, 34}\). The quasinormal modes for the scalar and gravitational perturbations in the background of the Kaluza-Klein black hole with squashed horizons have been investigated in \(^{35, 36}\), which implies that the quasinormal frequencies contain the information of the extra dimension. Moreover, the precession of a gyroscope in a circular orbit in the squashed Kaluza-Klein black hole spacetime have been studied in \(^{37}\), which shows that the modification from the extra dimension is proportional to the square of the ratio between the size of extra dimension and the gravitational radius of central object. The study of the strong gravitational lensing of a squashed Kaluza-Klein black hole also manifests that the size of the extra dimension imprints in the observational lensing parameters, such as deflection angle, the angular position and magnification of the relativistic images \(^{38}\).

The main purpose of the present Letter is to study the properties of the thin accretion disk in the squashed Kaluza-Klein black hole spacetime and see whether it can leave us the signature of the extra dimension in the energy flux and the emission spectrum emitted in the mass accretion process.

The Letter is organized as follows: in the following section we will present the geodesic equations for the timelike particles moving in the equatorial plane in the squashed Kaluza-Klein black hole spacetime. In Sec.III, we study the physical properties of the thin accretion disk around the squashed Kaluza-Klein black hole and probe the effects of the extra dimension on the energy flux, temperature, emission spectrum and efficiency of the thin accretion disks onto this black hole. We end the paper with a summary.

### II. THE GEODESIC EQUATIONS IN THE SQUASHED KALUZA-KLEIN BLACK HOLE SPACETIME

The five-dimensional neutral static Kaluza-Klein black hole with squashed horizons is described by \(^{26, 29}\)

\[
d s^2 = -f(\rho) \, dt^2 + \frac{K}{f(\rho)} \, d\rho^2 + K \rho^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) + \frac{r_{\infty}^2}{4K} \, (d\psi + \cos^2 \theta \, d\phi)^2,
\]

with

\[
f(\rho) = 1 - \frac{\rho_H}{\rho}, \quad K = 1 + \frac{\rho_0}{\rho},
\]

where the angular coordinates \(\theta \in [0, \pi]\), \(\phi \in [0, 2\pi]\), and \(\psi \in [0, 4\pi]\). \(\rho_H\) is the radius of the black hole event horizon. \(r_{\infty}\) corresponds to the spatial infinity. The quantity \(\rho_0\) is related to the parameters \(r_{\infty}\) and \(M\) by

\[
\rho_0^2 = \frac{r^2_{\infty} - M}{r^2_{\infty} - M}.
\]

The Komar mass of the squashed Kaluza-Klein black hole can be described by

\[
M = \pi r_{\infty} \rho_H / G_5
\]
where $G_5$ is the five-dimensional gravitational constant. Moreover, one can find that the event horizon radius $\rho_H$ can be written further as $\rho_H = 2G_4 M$ since the relationship between $G_5$ and $G_4$ (the four-dimensional gravitational constant) can be expressed as $G_5 = 2\pi r_\infty G_4$ in this black hole spacetime. Since the metric (1) reduces to a four-dimensional Schwarzschild black hole with a constant twisted $S_1$ fiber as $\rho_0 \rightarrow 0$ and tends to a spherical symmetrical five-dimensional Schwarzschild black hole as $\rho_H \ll \rho_0$, $\rho_0$ can be regarded as a scale of transition from five-dimensional spacetime to an effective four-dimensional one.

In order to probe the properties of the thin accretion disk in squashed Kaluza-Klein black hole spacetime (1), we must study the geodesics equations of motion for the particle moving in this neutral static background. As in Refs. [17–25, 37, 38], we here consider only the orbits in the equatorial plane. The $\theta$-component of the geodesics for the particle moving in the orbits with the condition $\theta = \pi/2$ tells us

$$\frac{d\phi}{d\lambda} \frac{d\psi}{d\lambda} = 0,$$

(3)

where $\lambda$ is an affine parameter along the geodesics. This means that either $\frac{d\phi}{d\lambda} = 0$ or $\frac{d\psi}{d\lambda} = 0$. Here we set $\frac{d\psi}{d\lambda} = 0$, so that we can compare with the results obtained in the four-dimensional black hole spacetime. In doing so, one can find that the timelike geodesics equations can be simplified as

$$\frac{dt}{d\lambda} = \hat{E} \frac{g_{tt}}{f(\rho)},$$
$$\frac{d\phi}{d\lambda} = \hat{L} \frac{g_{\phi\phi}}{K \rho^2},$$

$$g_{tt} g_{\rho\rho} \left( \frac{d\rho}{d\lambda} \right)^2 + V_{eff}(\rho) = \hat{E}^2,$$

(4)

where $\hat{E}$ and $\hat{L}$ are the specific energy and the specific angular momentum of the particle, respectively. The effective potential $V_{eff}(\rho)$ has the form

$$V_{eff}(\rho) = g_{tt} \left( 1 + \frac{\hat{L}^2}{g_{\phi\phi}} \right) = \left( 1 - \frac{\rho_H}{\rho} \right) \left[ 1 + \frac{\hat{L}^2}{\rho(\rho + \rho_0)} \right].$$

(5)

For stable circular orbits in the equatorial plane, we have $V_{eff}(\rho) = \hat{E}^2$ and $V_{eff}(\rho)_{,\rho} = 0$. Making use of these conditions, one can get the specific energy $\hat{E}$, the specific angular momentum $\hat{L}$, and the angular velocity $\Omega$ of the particle moving in circular orbit in the squashed Kaluza-Klein black hole spacetime

$$\hat{E} = \frac{g_{tt}}{\sqrt{g_{tt} - g_{\phi\phi} \Omega^2}} = \sqrt{\frac{(2\rho + \rho_0)(\rho - \rho_H)^2}{\rho^2(2\rho - 3\rho_H) + \rho \rho_0 (\rho - \rho_H)}},$$

(6)

$$\hat{L} = \frac{g_{\phi\phi} \Omega}{\sqrt{g_{tt} - g_{\phi\phi} \Omega^2}} = \sqrt{\frac{\rho(\rho + \rho_0)^2}{\rho(2\rho - 3\rho_H) + \rho \rho_0 (\rho - \rho_H)}},$$

(7)

$$\Omega = \frac{d\phi}{dt} = \sqrt{\frac{g_{tt}}{g_{\phi\phi,\rho}}} = \sqrt{\frac{\rho_H}{\rho^2(2\rho + \rho_0)}},$$

(8)
The marginally stable orbit of the particle around the black hole is given by the condition $V_{\text{eff}}(\rho),_{\rho\rho} = 0$. Combining Eqs. (6), (7) with $V_{\text{eff}}(\rho),_{\rho\rho} = 0$, one can find that the radius $\rho_{\text{ms}}$ of the marginally stable orbit for the particle moving in the squashed Kaluza-Klein black hole spacetime can be expressed as

$$\rho_{\text{ms}} = \rho_H \left[ 1 + \left( 1 + \frac{\rho_0}{\rho_H} \right)^{\frac{1}{3}} + \left( 1 + \frac{\rho_0}{\rho_H} \right)^{\frac{2}{3}} \right].$$

(9)

With the increase of $\rho_0$, one can find that for the particle moving in the circular orbit, the specific energy $\tilde{E}$, the specific angular momentum $\tilde{L}$, and the marginally stable orbit radius $\rho_{\text{ms}}$ increase, but the angular velocity $\Omega$ decreases. As the parameter $\rho_0 \to 0$, the above quantities $\tilde{E}$, $\tilde{L}$, $\Omega$, and $\rho_{\text{ms}}$ are reduced to those in the four-dimensional Schwarzschild black hole spacetime.

III. THE PROPERTIES OF THIN ACCRETION DISKS IN THE SQUASHED KALUZA-KLEIN BLACK HOLE SPACETIME

In this section, we will investigate the accretion process in the thin disk around the squashed Kaluza-Klein black hole and probe the effects of the extra dimensional scale $\rho_0$ on the physical properties of the thin accretion disk. For the thin accretion disk model in the squashed Kaluza-Klein black hole spacetime, we can assume similarly that the central plane of the disk is located in the equatorial plane of the black hole and the disk height $H$ (the maximum half thickness of the disk) is much smaller than the characteristic radius $\rho$ of the disk, i.e., $H \ll \rho$. Moreover, the thin disk can stay in hydrodynamical equilibrium since the heat generated by stress and dynamic friction in the disk can be dispersed through the radiation over its surface, which makes the disk stabilize its thin vertical size. For simplification, we here assume that the thin disk around the squashed Kaluza-Klein black hole is modeled by a steady state accretion disk [17, 18] in which the mass accretion rate $\dot{M}_0$ stays a constant. As in the disk around the usual black holes [17–25], we can also measure the physical quantities describing the accreting matter by averaging over a characteristic time scale $\Delta t$, the azimuthal angle $\Delta \phi = 2\pi$, and the height $H$ of the disk.

Similarly, we suppose that the accreting matter in the disk can be described by an anisotropic fluid with the energy-momentum tensor [17, 18]

$$T^{\mu\nu} = \varepsilon_0 u^\mu u^\nu + 2u^{(\mu} q^{\nu)} + t^{\mu\nu},$$

(10)

where the rest mass density $\varepsilon_0$, the energy flow vector $q^\mu$ and the stress tensor $t^{\mu\nu}$ of the accreting matter are defined in the averaged rest-frame of the orbiting particle with four-velocity $u^\mu$. In the averaged rest-frame, we have $u_\mu q^\mu = 0$ and $u_\mu t^{\mu\nu} = 0$ since both $q^\mu$ and $t^{\mu\nu}$ is orthogonal to $u^\mu$. With the laws of conservation
of the rest mass, the energy and the angular momentum, one can obtain three time-averaged radial structure
equations of the thin disk around the squashed Kaluza-Klein black hole

$$\dot{M}_0 = -2\pi(\rho + \rho_0)\Sigma(\rho)u^\rho = \text{Const},$$  \hspace{1cm} (11)

$$[\dot{M}_0\dot{E} - 2\pi(\rho + \rho_0)W^\rho_{\phi}]_{,\rho} = 2\pi(\rho + \rho_0)F(\rho)\dot{E},$$  \hspace{1cm} (12)

$$[\dot{M}_0\dot{L} - 2\pi(\rho + \rho_0)W^\rho_{\phi}]_{,\rho} = 2\pi(\rho + \rho_0)F(\rho)\dot{L},$$  \hspace{1cm} (13)

where a dot represents the derivative with respect to the time coordinate \(t\). The averaged rest mass
density \(\Sigma(\rho)\) and the averaged torque \(W^\rho_{\phi}\) are given by

$$\Sigma(\rho) = \int_{-H}^{H} (\langle \varepsilon \rangle_{0}) \, dz,$$

$$W^\rho_{\phi} = \int_{-H}^{H} (\langle t^\rho_{\phi} \rangle) \, dz,$$  \hspace{1cm} (14)

respectively. The quantity \(\langle t^\rho_{\phi} \rangle\) is the average value of the \(\phi - \rho\) component of the stress tensor over a
characteristic time scale \(\Delta t\) and the azimuthal angle \(\Delta \phi = 2\pi\). Combining Eqs. (12) and (13) with the energy-
angular momentum relation for circular geodesic orbits \(\tilde{E}_{,\rho} = \Omega \tilde{L}_{,\rho}\), one can eliminate \(W^\rho_{\phi}\) and obtain the
expression of the energy flux

$$F(\rho) = -\frac{\dot{M}_0}{4\pi(\rho + \rho_0)} \frac{\Omega^\rho}{(\tilde{E} - \Omega\tilde{L})^2} \int_{\rho_{ms}}^{\rho} (\tilde{E} - \Omega\tilde{L})\Omega_{,\rho} d\rho.$$  \hspace{1cm} (15)

As in Refs. [19], we here consider the mass accretion driven by black holes with a total mass of \(M = 10^6M_\odot\),
and with a mass accretion rate of \(\dot{M}_0 = 10^{-12}M_\odot \, yr^{-1}\). In Fig. (1), we present the total energy flux \(F(\rho)\)
radiated by a thin disk around the squashed Kaluza-Klein black hole for different \(\rho_0\). It is shown that the
energy flux \(F(\rho)\) decreases with the extra dimensional scale \(\rho_0\). The main mathematical reason is that the
presence of \(\rho_0\) increases the marginally stable orbit radius \(\rho_{ms}\) and enhances the lower limit of integral in the
energy flux (15). The position of the peak value of \(F(\rho_0)\) moves along the right with the increase of \(\rho_0\).

Let us now to probe the effect of \(\rho_0\) on the conversion efficiency \(\epsilon\) for the thin accretion disk around the
squashed Kaluza-Klein black hole. The conversion efficiency is another important characteristics of the mass
accretion process, which describes the capability of the central object converting rest mass into outgoing
radiation. In general, the conversion efficiency can be given by the ratio of two rates measured at infinity
[16, 17]: the rate of the radiation energy of photons escaping from the disk surface to infinity and the mass-
energy transfer rate of the central compact object in the mass accretion. If all the emitted photons can escape
to infinity, one can find that the efficiency \(\epsilon\) is related to the specific energy measured at the marginally stable
orbit \(\rho_{ms}\) by

$$\epsilon = 1 - \tilde{E}_{ms}. $$  \hspace{1cm} (16)
Substituting Eqs. (6) and (9) into Eq. (16), we can probe the effects of $\rho_0$ on the conversion efficiency $\epsilon$ for the mass accretion process occurred in the squashed Kaluza-Klein black hole spacetime. The dependence of $\epsilon$ on $\rho_0$ is plotted in Fig.(2), which shows that the larger values of $\rho_0$ leads to a much smaller efficiency $\epsilon$. This means that the conversion efficiency of the thin accretion disk in the squashed Kaluza-Klein black hole spacetime is less than that in the four-dimensional Schwarzschild one.

In the steady-state thin disk model [17, 18], the accreting matter is generally assumed to be in thermodynamical equilibrium. This means that the radiation emitted by the disk surface can be considered as a perfect black body radiation. The radiation temperature $T(\rho)$ of the disk is related to the energy flux $F(\rho)$ through
the expression $T(\rho) = [F(\rho)/\sigma]^{1/4}$, where $\sigma$ is the Stefan-Boltzmann constant, respectively. Thus, one can find that the dependence of $T(\rho)$ on $\rho_0$ is similar to that of the energy flux $F(\rho)$ on $\rho_0$, which is also shown in Fig. (3). As in [22], the observed luminosity $L(\nu)$ for the thin accretion disk around the squashed Kaluza-Klein black hole can be expressed as

$$L(\nu) = 4\pi d^2 I(\nu) = \frac{16\pi^2 h \cos \gamma}{c^2} \int_{\rho_i}^{\rho_f} \frac{\nu^3 \rho d\rho}{e^{\hbar \nu/K T(\rho)} - 1},$$

(17)

where $d$ is the distance to the source, $I(\nu)$ is the thermal energy flux radiated by the disk, and $\gamma$ is the disk inclination angle. The quantities $\rho_f$ and $\rho_i$ are the outer and inner border of the disk, respectively. In order to calculate the luminosity $L(\nu)$ of the disk, we choose $\rho_i = \rho_{ms}$ and $\rho_f \to \infty$ since the flux over the disk surface vanishes at $\rho_f \to \infty$ in the squashed Kaluza-Klein black hole spacetime. Applying the formulas (17), we present the spectral energy distribution of the disk radiation in Fig. (4). It is shown that the observed luminosity of the disk decreases with the extra dimensional scale $\rho_0$ in the total frequency domain ranges. The effect of $\rho_0$ becomes more distinct as $\nu > 5 \times 10^{13}$ Hz for the chosen values of $\dot{M}_0$, $M$ and $\gamma$. Moreover, we also find that the larger value of $\rho_0$ leads to the lower cut-off frequencies, which means that the spectra of the disk becomes softer in the squashed Kaluza-Klein black hole background.

### IV. SUMMARY

In summary, we have studied the properties of the thin accretion disk around the squashed Kaluza-Klein black hole and found that the size of the extra dimension imprints in the energy flux, temperature distribution and luminosity of the disk.
and emission spectra of the disk. With the increase of the parameter $\rho_0$, except the inner border of disk, the energy flux, the conversion efficiency, the radiation temperature, the spectra luminosity and cut-off frequency of the thin accretion disk decrease. The presence of the lower cut-off frequencies means that the spectra of the disk becomes softer in the squashed Kaluza-Klein black hole background. Theoretically, we could detect the effects of the extra dimension on the accretion around the black hole by the astronomical observations and then make a constraint on the parameter $\rho_0$. However, the sensitivity of the current observations is far from being sufficient to detect these effects. Perhaps with the development of technology, the effects of the extra dimension on the accretion may be detected in the future.

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[1] J. Y. Shen, B. Wang and R. K. Su, Phys. Rev. D 74 (2006) 044036.
[2] E. Abdalla, B. Cuadros-Melgar, A. B. Pavan and C. Molina, Nucl. Phys. B 752 (2006) 40.
[3] S. B. Chen, B. Wang, R. K. Su, Phys. Lett. B 647 (2007) 282, arXiv:hep-th/0701209.
[4] P. Kanti, R. A. Konoplya, A. Zhidenko, Phys. Rev. D 74 (2006) 064008; P. Kanti, R. A. Konoplya, Phys. Rev. D 73 (2006) 044002; D. K. Park, Phys. Lett. B 633 (2006) 613.
[5] P. Kanti, hep-ph/0310162.
[6] C. M. Harris and P. Kanti, JHEP 0310 (2003) 014; P. Kanti, Int. J. Mod. Phys. A 19 (2004) 4899; P. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B441 (1998) 96; T. Banks and W. Fischler, hep-th/9906038; R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85 (2000) 499.
[7] E. Jung and D. K. Park, Nucl. Phys. B 717 (2005) 272; N. Sanchez, Phys. Rev. D 18 (1978) 1030; E. Jung and D. K. Park, Class. Quant. Grav. 21 (2004) 3717.
[8] A. S. Majumdar, N. Mukherjee, Int. J. Mod. Phys. D 14 (2005) 1095; G. Kofinas, E. Papantonopoulos and V. Zamarias, Phys. Rev. D 66 (2002) 104028; G. Kofinas, E. Papantonopoulos and V. Zamarias, Astrophys. Space Sci. 283 (2003) 685; A. N. Aliev, A. E. Gumrukcuoglu, Phys. Rev. D 66 (2002) 104028; V. P. Frolov, D. Stojkovic, Astrophys. Space Sci. 283 (2003) 685; A. N. Aliev, A. E. Gumrukcuoglu, Phys. Rev. D 66 (2002) 104028; S. Kar, S. Majumdar, Int. J. Mod. Phys. A 21 (2006) 6087; S. Kar, S. Majumdar, Phys. Rev. D 74 (2006) 066003; S. Kar, Phys. Rev. D 74 (2006) 126002.
[9] E. Jung, S. H. Kim and D. K. Park, Phys. Lett. B 615 (2005) 273; E. Jung, S. H. Kim and D. K. Park, Phys. Lett. B 619 (2005) 347; D. Ida, K. Oda and S. C. Park, Phys. Rev. D 67 (2003) 064025; G. Duffy, C. Harris, P. Kanti and E. Winstanley, JHEP 0509 (2005) 049; M. Casals, P. Kanti and E. Winstanley, JHEP 0602 (2006) 051; E. Jung and D. K. Park, Nucl. Phys. B 731 (2005) 171; A. S. Cornell, W. Naylor and M. Sasaki, JHEP 0602 (2006) 012; V. P. Frolov, D. Stojkovic, Phys. Rev. Lett. 89 (2002) 151302; Valeri P. Frolov, Dejan Stojkovic, Phys. Rev. D 66 (2002) 084002; D. Stojkovic, Phys. Rev. Lett. 94 (2005) 011603.
[10] D.K. Park, Class. Quant. Grav. 23 (2006) 4101.
[11] Eylee Jung and D. K. Park, hep-th/0612043; V. Cardoso, M. Cavaglia, L. Gualtieri, Phys. Rev. Lett. 96 (2006) 071301; V. Cardoso, M. Cavaglia, L. Gualtieri, JHEP 0602 (2006) 021.
[12] D. Dai, N. Kaloper, G. Starkman and D. Stojkovic, Phys. Rev. D 75 (2007) 024043.
[13] L. H. Liu, B. Wang, G. H. Yang, Phys. Rev. D 76 (2007) 064014.
[14] R. Whisker, Phys. Rev. D 71 (2005) 064004.
[15] N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. 24 (1973) 33.
[16] I. D. Novikov and K. S. Thorne, in Black Holes, ed. C. DeWitt and B. DeWitt, New York: Gordon and Breach (1973).
[17] D. N. Page and K. S. Thorne, Astrophys. J. 191 (1974) 499.
[18] K. S. Thorne, Astrophys. J. 191 (1974) 507.
[19] T. Harko, Z. Kovacs and F. S. N. Lobo, Class. Quant. Grav. 27 (2010) 105010; Phys. Rev. D 80 (2009) 044021; Phys. Rev. D 78 (2008) 084005; Phys. Rev. D 79 (2009) 064001; Class. Quant. Grav. 26 (2009) 215006.
[20] S. Bhattacharyya, A. V. Thampan and I. Bombaci, Astron. Astrophys. 372 (2001) 925.
[21] Z. Kovacs, K. S. Cheng and T. Harko, Astron. Astrophys. 500 (2009) 621.
[22] D. Torres, Nucl. Phys. B 626 (2002) 377.
[23] Y. F. Yuan, R. Narayan and M. J. Rees, Astrophys. J. 606 (2004) 1112.
[24] F. S. Guzman, Phys. Rev. D 73 (2006) 021501.
[25] C. S. J. Pun, Z. Kovacs and T. Harko, Phys. Rev. D 78, 084015 (2008); Phys. Rev. D 78 (2008) 024043.
[26] H. Ishihara and K. Matsuno, Prog. Theor. Phys. 116 (2006) 417.
[27] S. S. Yazadjiev, Phys. Rev. D 74 (2006) 024022.
[28] Y. Brihaye and E. Radu, Phys. Lett. B 641 (2006) 212.
[29] H. Ishihara, M. Kimura, K. Matsuno, and S. Tomizawa, Phys. Rev. D 74 (2006) 047501; Class. Quantum Grav. 23 (2006) 6919.
[30] T. Harmark, V. Niarchos and N. A. Obers, Class. Quant. Grav. 24 (2007) R1-R90.
[31] T. Wang, Nucl. Phys. B 756 (2006) 86.
[32] V. Frolov and D. Stojkovic, Phys. Rev. D 67 (2003) 084004; H. Nomura, S. Yoshida, M. Tanabe and K. Maeda, Prog. Theor. Phys. 114 (2005) 707-712.
[33] H. Ishihara and J. Soda, Phys. Rev. D 76 (2007) 064022.
[34] S. Chen, B. Wang and R. Su, Phys. Rev. D 77 (2008) 024039.
[35] X. He, B. Wang and S. Chen, Phys. Rev. D 79 (2009) 084005; X. He, S. Chen, B. Wang, R. G. Cai, C. Lin, Phys.Lett.B 665 (2008) 392.
[36] H. Ishihara, M. Kimura, R. A. Konoplya, K. Murata, J. Soda and A. Zhidenko, Phys.Rev.D 77 (2008) 084019.
[37] K. Matsuno and H. Ishihara, Phys. Rev. D 80 (2009) 104037.
[38] Y. Liu, S. Chen and J. Jing, Phys. Rev. D 81 (2010) 124017.
[39] Y. Kurita and H. Ishihara, Class. Quant. Grav. 24 (2007) 4525; Y. Kurita and H. Ishihara, Class. Quant. Grav. 25 (2008) 085006.