Economic and mathematical analysis of efficiency leasing in the electronic industry

Yu V Kirillov
Novosibirsk State Technical University, 20, K. Marx Ave., Novosibirsk, 630092, Russian Federation

E-mail: kirillov_yu@ngs.ru

Abstract. The article, using the example of electronic industry enterprises, examines the operation of financial leasing of high-tech equipment to upgrade fixed assets in order to produce competitive products. An analysis is made of the effectiveness of the use of equipment necessary for the production of leased (rented) equipment as a special investment project implemented by the lessee, in which rental payments are an investment part of the project, and the profit from the sale of manufactured products makes up its profitable part. Analytical indicators of reduced net income and profitability index are supplemented by a payback period indicator, for the calculation of which a laborious recursive-logical procedure is used in practice. To obtain an analytical expression of the payback period, an original technique was used based on replacing a discrete payment stream with a financially equivalent continuous stream of payments. The obtained analytical formula made it possible to determine the fundamental economic and mathematical conditions for the payback of this operation for the lessee.

1. Introduction
One of the most important trends in economic development at the present stage is innovative activity. The well-known foreign economic restrictions of recent times should also contribute to the “turn” of the economy onto the tracks of innovative development, which will further ensure the achievement of the goals set in the strategic program [1] with the aim of ensuring national economic security and strengthening the competitiveness of our country in foreign markets.

From the point of view of achieving the final result, innovative activity can be considered as the process of implementing a special investment project [2, 3], however, the deterioration of fixed assets of Russian enterprises and organizations impedes its implementation, since the renewal process is constrained by insufficient retained earnings of enterprises or lack of available borrowed capital. This circumstance is of particular importance for high-tech industries and, first of all, for the electronics industry, which is the basis of the entire modern technological base for the implementation of major national projects [4]. Today, depreciation of fixed assets in the domestic electronic industry is estimated at 60-75%, so it lags behind the world technical level and is not yet able to equip domestic equipment with electronic devices of its own production [5]. The structural share of Russia in the export of computers and electronic optical equipment is 10-15 times less than the shares of Germany, France, the USA, Japan and Singapore, and the lag behind China is two orders of magnitude [6].

Naturally, in such conditions, the problem of finding a source of financing that will provide modern enterprises with access to advanced technology and ensure their innovative development comes to the...
Modern technologies for attracting investments and, in particular, such an effective financial instrument as leasing, which, as an alternative financing mechanism, can play an important role in solving the strategic objective mentioned above, can help find a way out of this situation [7].

This type of business activity is extremely widespread in the West (for example, in the United States at least half of loans for the development of the material and technical base of companies are carried out through a system of leasing relations), but in Russia until now it has not yet received widespread application [8].

2. Problem statement
The most common financial leasing in practice implies that the organization of the electronic industry can purchase the necessary equipment not by buying, but by renting it from the lessor’s company, which is either the owner of the equipment, or can purchase it from the manufacturer. In turn, the lessee (tenant) uses the equipment received in the production of its products, and uses the profits from its sale to pay lease (rental) payments. Thus, the dynamics of the operation of using leased equipment can be represented by an investment project in which lease payments are investment payments, and sales proceeds are income payments. The general operation scheme is shown in figure 1.

![Figure 1. The general scheme of financial leasing operations.](image)

From the foregoing, an unambiguous conclusion follows that the effectiveness of such an operation from the point of view of the lessee can be quantified using well-known indicators of investment efficiency: NPV (net present value), DPI (discounted profitability index), DPP (discounted payback period), IRR (internal rate of return).

3. Theoretical part
Financial-time diagram for implementing the operation of using leased equipment is shown in figure 2.

![Figure 2. Financial-time diagram of the operation of leasing.](image)

In figure 2: ST \([R_{\text{prof}}(t)]\) – stream type of investment leasing payments \([R_{\text{prof}}(t)]\) duration \(n_{\text{prof}}\), which is determined by the terms of the contract with the lessor; ST \([R_{\text{prof}}(t)]\) – income stream type, the size of which \(R_{\text{prof}}(t)\) – is determined by the profit from the sale of products, and the duration \(n_{\text{prof}}\).

The indicators of economic efficiency of this operation as an investment project – net present value and
profitability index – are determined by obvious ratios:

\[ NPV = P_0 \left[ R_{\text{prof}}(t) \right] - P_0 \left[ R_{\text{los}}(t) \right], \]

\[ DPI = \frac{P_0 \left[ R_{\text{prof}}(t) \right]}{P_0 \left[ R_{\text{los}}(t) \right]}, \]

\[ P_0 \left[ R_{\text{prof}}(t) \right] = \sum_{i=0}^{n} \frac{R_{\text{prof}}(t)}{(1+i)^i} \] – the cost of income payments reduced to the beginning of the operation \((t = 0)\); \(i\) – rate of return for the lessee; \(P_0 \left[ R_{\text{los}}(t) \right] = \sum_{i=0}^{n} \frac{R_{\text{los}}(t)}{(1+r)^i} \) – the cost of cost of investment (rental) payments reduced to the beginning of the operation \((t = 0)\); \(r\) – lessor rate of return.

The discounted payback period of the project is the most “inconvenient”, from the point of view of computational complexity, performance indicator, since it is determined not by the analytical formula, in contrast to (1) and (2), but by the ratio \([9]\):

\[ DPP(x_i, y_i, i, t) = n_i + \Delta DPP, \]

\(n_i\) – duration of the investment part of the project, \(x_i\) and \(y_i\) – sizes of investment and profitable payments, respectively, \(i\) – investor rate of return, moreover, \(\Delta DPP\) it is calculated using the recursive-logical procedure [9]:

\[ \Delta DPP = k + \frac{|S_{n_i} - P_{n_i}^{(k)}|}{(P_{n_i}^{(k+1)} - S_{n_i}) \cdot |S_{n_i} - P_{n_i}^{(k)}|} \]

\[ P_{n_i}^{(k)} < S_{n_i} < P_{n_i}^{(k+1)} \]

where \(S_{n_i} = \sum_{i=1}^{n_i} x_i \cdot (1+i)^{n_i-i}\) – reduced to the point the accumulated amount of investment payments of the project; \(P_{n_i}^{(k)} = \sum_{i=n_{i-1}+1}^{n_{i+k}} \frac{y_i}{(1+i)^{n_{i+k}}}\) – the present value of the project revenue reduced to the point \(t = n_i\), duration \(n_i + k (k = 1, 2,...)\); \(P_{n_i}^{(k+1)} = \sum_{i=n_{i+k}}^{n_{i+k+1}} \frac{y_i}{(1+i)^{n_{i+k+1}}}\) – the present value of the project revenue reduced to the point \(t = n_i + k + 1 (k = 1, 2,...)\). From (4) and (5) it follows that the calculation \(\Delta DPP\) is a process of \(k+1\) consecutive steps, in each of which it is necessary to calculate the value \(P_{n_i}^{(k)}\) and compare it with the value \(S_{n_i}\), until relation (5) is satisfied. Only then can it be calculated \(\Delta DPP\) by formula (4), and then \(DPP\) by formula (3).

In order to present the ratio for calculating \(DPP\) in an analytical form, it is necessary to use the method proposed in [9], the idea of which is to replace the discrete stream of income payments with a financially equivalent continuous stream of the income part of the investment project, since the analysis of relations (4) and (5) shows that the smaller the difference \(P_{n_i}^{(k+1)} - P_{n_i}^{(k)}\), the more accurately we can determine \(\Delta DPP\) and, accordingly, \(DPP\) [10].

The present value of the revenue stream of the project with continuous payments \(\bar{R}_{\text{prof}}(t)\) in accordance with the rules of financial mathematics [11] will be determined as
Financial equivalence of such a discrete payment stream $R_{prof}(t)$ will be ensured if equality is satisfied

$$ P_0 \left[ R_{prof}(t) \right] = P_0 \left[ \bar{R}_{prof}(t) \right] , $$

whence it follows

$$ \bar{R}_{prof}(t) = \frac{\ln(1+i) \cdot \sum_{t=0}^{n_{prof}} R_{prof}(t)}{1-(1+i)^{n_{prof}}} : \quad \text{(6)} $$

For the financial scheme under consideration (see figure 3) in the economic sense of the indicator of the discounted payback period, the present value of a continuous flow of DPP duration should be equal to the present value of the entire flow of investment (leasing) payments. Then

$$ P_0 \left[ R_{leas}(t) \right] = P_0 \left[ \bar{R}_{prof}(t) \right] \bigg|_{t=DPP} = \bar{R}_{prof}(t) \cdot \frac{1-(1+i)^{DPP}}{\ln(1+i)} . \quad \text{(7)} $$

Figure 3. The financial equivalent continuous stream.

Considering here (6) and having performed simple algebraic transformations, we obtain

$$ P_0 \left[ \bar{R}_{prof}(t) \right] \bigg|_{t=DPP} = \frac{1-(1+i)^{DPP}}{1-(1+i)^{n_{prof}}} \cdot \sum_{t=0}^{n_{prof}} R_{prof}(t) \cdot \frac{1-(1+i)^{DPP}}{1-(1+i)^{n_{prof}}} \cdot P_0 \left[ R_{prof}(t) \right] . \quad \text{(8)} $$

Using the results of (8) in relation (7), we obtain the equation

$$ P_0 \left[ R_{leas}(t) \right] = \frac{1-(1+i)^{DPP}}{1-(1+i)^{n_{prof}}} \cdot P_0 \left[ R_{prof}(t) \right] , $$

solving the problem with respect to DPP, after appropriate transformations and logarithm, we get the final result

$$ \ln \left\{ 1 - \frac{P_0 \left[ R_{leas}(t) \right]}{P_0 \left[ R_{prof}(t) \right]} \cdot \frac{1-(1+i)^{n_{prof}}}{1-(1+i)^{DPP}} \right\} = \ln(1+i) . \quad \text{(9)} $$

4. Analysis of results

Analysis of the performance indicators of the operation under consideration for the use of leased equipment on the basis of the obtained analytical expressions (1), (2) and (9) allows us to conclude that such an operation will be effective for the lessee if the main condition is met:

$$ P_0 \left[ R_{leas}(t) \right] < P_0 \left[ R_{prof}(t) \right] . \quad \text{(10)} $$

The critical condition for the principal payback of this operation is determined by the ratio:
\[
\frac{P_0[R_{\text{leasing}}(t)]}{P_0[R_{\text{profit}}(t)]} = 1 - (1 + i)^{-n_{\text{profit}}},
\]

where \( P_0 \) is the present value of the lease payments, \( P_0[R_{\text{leasing}}(t)] \), and \( P_0[R_{\text{profit}}(t)] \) is the present value of the profit payments from sales of products, \( R_{\text{leasing}} \) and \( R_{\text{profit}} \)

at which profitable payments from the sale of products will be able to pay off only the interest on the lease, but the main debt will never be paid off (perpetual rent). If

\[
\frac{P_0[R_{\text{leasing}}(t)]}{P_0[R_{\text{profit}}(t)]} < 1 - (1 + i)^{-n_{\text{profit}}}
\]

negative outflows of rental payments will be exceeded by positive inflows of profits from sales of products. If

\[
\frac{P_0[R_{\text{leasing}}(t)]}{P_0[R_{\text{profit}}(t)]} > 1 - (1 + i)^{-n_{\text{profit}}}
\]

the main debt with interest on lease will not only not be repaid, but will also increase.

As an illustration of the results in figure 4 shows a graph of dependence \( DPP = f(R_{\text{profit}}) \) according to formula (9) based on the following data: types of lease and income payment stream – annual annuities postnumerando, \( R_{\text{leasing}} = 10 \) thousand roubles, \( n_{\text{leasing}} = 6 \) years, \( r = 10\% \), \( n_{\text{profit}} = 7 \) years, \( i = 10\% \).

If \( 9 < R_{\text{profit}}(t) < 15 \) the project pays off and makes a profit, because condition (12) is fulfilled and the value \( DPP \) remains less than the critical value in the given conditions \( DPP_{\text{crit}} = n_{\text{profit}} = 7 \) years. At \( R_{\text{profit}} \approx 9 \) condition (11) is satisfied and \( DPP = DPP_{\text{crit}} \).

At \( R_{\text{profit}}(t) < 9 \) the main condition (10) is not satisfied and the project becomes unprofitable, and when \( R_{\text{profit}}(t) < 5 \) condition (13) is fulfilled, under which it becomes impossible to calculate the payback period, which is reflected by a “failure” in the graph in figure 4.

![Figure 4](image_url)

**Figure 4.** Dependence of the \( DPP \) on the value \( R_{\text{profit}}(t) \).

5. Conclusion

Obviously, the results obtained will make it possible to analyze the effectiveness of such operations with greater efficiency, which is especially important in modern business conditions, when the time for
making managerial decisions is rapidly decreasing. In addition, the obtained analytical expressions for performance indicators are functions of the same variables, which, undoubtedly, will make it possible to use them to solve the problem of finding not only an operational, but also an optimal solution under given conditions from the point of view of the chosen criterion.

References

[1] 2020 The strategy of innovative development of the Russian Federation for the period until 2020. The official website of the Government of Russia. Retrieved from http://government.ru/docs/9282

[2] Ilyenkova S 2012 *Innovation Management: A Textbook for High Schools* (Moscow, Russia: UNITY-DANA) p 335

[3] Barysheva A 2015 *Innovation Management: A Study Guide* (Moscow, Russia: Publishing and Trading Corporation "Dashkov and Co.") p 384

[4] The development strategy of the electronic industry of the Russian Federation for the period until 2030. Official website of the Government of Russia. Retrieved from http://government.ru/docs/38795/

[5] Khrustalyov E and Slavyanov A 2018 *Economic analysis: theory and practice* 17(6) 1000-13

[6] Obolenskaya L and Moreva E 2019 *Drucker Herald* 5 (31) 63-74

[7] Fedorova A, Dorozhkina N and Chernyshova O 2017 *Socio-economic phenomena and processes* 12(1) 100-6

[8] Kovalev V 2015 *Leasing: Financial, Accounting, Analytical and Legal aspects: a Training Manual* (Moscow, Russia: Prospekt) p 448

[9] Kirillov Yu and Nazimko E 2012 *Economic analysis: theory and practice* 45(300) 49-54

[10] Kirillov Yu and Nazimko E 2016 *Financial analytics: problems and solutions* 16(298) 20-9

[11] Chetyrkin E 2010 *Financial Mathematics: A Textbook for High Schools* (Moscow, Russia: Delo) p 400