Sparsest Feedback Selection for Structurally Cyclic Systems with Dedicated Actuators and Sensors in Linear Time

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Abstract—This paper solves the sparsest feedback selection problem for linear time invariant structured systems, a long-standing open problem in structured systems. We consider structurally cyclic systems with dedicated inputs and outputs. We prove that finding a sparsest feedback selection is of linear complexity for the case of structurally cyclic systems with dedicated inputs and outputs. This problem has received attention recently but key errors in the hardness-proofs have resulted in an erroneous conclusion there. This is also elaborated in this brief paper together with a counter-example.

Index Terms—Linear structured systems, Arbitrary pole placement, Linear output feedback, Sparsest feedback selection.

1. INTRODUCTION

Feedback selection for control systems that guarantees desired closed-loop performance is a fundamental design problem in control theory. The challenging part of the design problem is to accomplish an optimal design, for example in the sense of number of connections or cost of connections. We consider feedback selection in large scale linear dynamical systems. However, the analysis done in this paper is based on the zero/non-zero (referred as sparsity) pattern of the system. The rationale behind performing this analysis is, in most large scale systems and real time systems, the numerical values of the non-zero entries in the system description are either not known at all, like social networks, biological systems, or they are not known accurately, like electric networks, power grids, robotics. To this end, various system properties of these systems are studied using the sparsity pattern of the system referred as structural analysis [1].

Structural analysis of control systems, namely structural controllability was introduced by Lin in [2]. Research in this area has become relevant due to applicability in various complex systems: see [3] and references therein for details. This paper discusses sparsest feedback selection that guarantees arbitrary pole placement. Necessary and sufficient graph theoretic condition that guarantee arbitrary pole placement is given in [4] using the concept of fixed modes [5].

Given a large scale dynamical system, our aim is to find a minimum set of feedback edges, i.e., which output to be fed to which input, that arbitrary pole placement of the closed-loop system is possible. In other words, given the digraph representing the state dynamics, the inputs and the outputs of the system, our objective is to find a minimum set of feedback connections that ensure the desired design objective.

Finding sparsest feedback matrix for a given structured system is considered in [6]. The approach proposed requires a minimum input-output set to be found, which in itself is an NP-hard problem [7]. The authors in [8] discuss minimum cost feedback selection, which is a more general problem. The method proposed there requires solving a multi-commodity network flow problem: an NP-hard problem. Thus neither [6] nor [8] can yield a polynomial time algorithm to the feedback selection problem, and hardness of the sparsest feedback selection problem remained unsolved. For a structured state matrix, the problem of finding jointly sparsest input, output and feedback matrices is addressed in [9]. On the contrary, [10] considers the problem when there is no flexibility in choosing the input and output matrices: given structured state, input and output matrices and cost associated with each of them (i.e., each input, output and feedback edge is associated with cost), find the minimum cost input-output set and the feedback matrix. This problem is known to be NP-hard and hence [10] considers a special class of systems where the state matrix is irreducible. For the case when the state, input and output matrices are fixed, the problem of finding a sparsest feedback matrix has been formulated in [11], [12], where the authors claim and ‘prove’ the NP-hardness of the problem. Later below in Section 5 we elaborate about how NP-hardness of finding a particular solution with special properties (namely, when the sparsest solution’s closed-loop system digraph has exactly two SCCs) is not sufficient for a reduction procedure in general. A counter-example is also described there.

In the context of NP-hardness of the sparsest feedback selection problem, we proved the NP-hardness recently in [13] using a reduction of the set cover problem. In this paper we formulate a subclass of systems referred as structurally cyclic systems with dedicated inputs and outputs: for this subclass, we prove that finding a sparsest feedback matrix has linear time complexity. A system is said to be structurally cyclic if the state bipartite graph (see Section 2-B) has a perfect matching and an input (output, resp.) is said to be dedicated if it can actuate (sense, resp.) a single state only. The class of systems with the state bipartite graph having a perfect matching is wide: for example, self-damped systems (see [14]) including consensus dynamics in multi-agent systems and epidemic equations. Further, for systems whose system dynamics are invertible, the state bipartite graph necessarily has a perfect matching.

The paper is organized as follows. In Section 2 we formally define the problem that is considered here. Here, we also provide some required preliminaries and state some known results that we use subsequently. In Section 3 we elaborate on the fallacies in the proof of NP-hardness result of the proposed problem given in [11]. In Section 4 we provide a

1A graph is said to be irreducible if there exists a directed path between any two vertices in the graph, i.e., strongly connected.
linear complexity algorithm for solving the proposed problem. Finally, we conclude in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this section we formulate the problem considered in this paper and then give few preliminaries used in the sequel.

A. Problem Formulation

Consider a linear time-invariant system \( \dot{x} = Ax + Bu, y = Cx \), where \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m} \) and \( C \in \mathbb{R}^{p \times n} \). Here \( \mathbb{R} \) denotes the set of real numbers. The structural representation of this system referred as structured system is denoted by \((A, B, C)\), where \(\bar{A}, \bar{B}\) and \(\bar{C}\) has the same structure as that of \(A, B\) and \(C\) respectively.

\[
\begin{align*}
A_{ij} &= 0 \text{ whenever } \bar{A}_{ij} = 0, \\
B_{ij} &= 0 \text{ whenever } \bar{B}_{ij} = 0, \\
C_{ij} &= 0 \text{ whenever } \bar{C}_{ij} = 0.
\end{align*}
\] (1)

Given \((\bar{A}, \bar{B}, \bar{C})\), any tuple \((A, B, C)\) that satisfies (1) is referred as a numerical realization of the structured system. Let \( K \in \{0, *\}^{m \times p} \) denote a feedback matrix, where \( \bar{K}_{ij} = * \) if \( j \)th output is fed to \( i \)th input. We define, \( |K| := \{ K : \bar{K}_{ij} = 0, \text{ if } \bar{K}_{ij} = 0 \} \).

**Definition 1.** The structured system \((\bar{A}, \bar{B}, \bar{C})\) and the feedback matrix \(\bar{K}\) is said not to have structurally fixed modes (SFM) if there exists a numerical realization \((A, B, C)\) of \((\bar{A}, \bar{B}, \bar{C})\) such that \( \cap_{K \in |K|} \sigma(A + BKC) = \phi \), where \( \sigma(T) \) denotes the set of eigenvalues of any square matrix \( T \).

Given a structured system \((\bar{A}, \bar{B}, \bar{C})\), our aim is to find a minimum set of feedback edges (i.e., sparsest \( \bar{K} \)) such that the closed-loop structured system \((\bar{A}, \bar{B}, \bar{C}, \bar{K})\) has no SFMs. Let \( \mathcal{K}_s := \{ K \in \{0, *\}^{m \times p} : (\bar{A}, \bar{B}, \bar{C}, \bar{K}) \) has no SFMs \} \). For a structured system \((\bar{A}, \bar{B}, \bar{C})\), without loss of generality, we assume that \( \mathcal{K}_s \) is non-empty. Specifically, \( \bar{K} \in \mathcal{K}_s \), where \( \bar{K}_{ij} = * \) for all \( i, j \). Next we describe the problem addressed in this paper.

**Problem 1.** Given a structured system \((\bar{A}, \bar{B}, \bar{C})\), find

\[
\bar{K}^* \in \arg\min_{K \in \mathcal{K}_s} \|K\|_0.
\]

Here \( \| \cdot \|_0 \) denotes the zero matrix norm\(^2\). We refer to Problem 1 as sparsest feedback selection problem.

B. Preliminaries

Graph theory is a key tool in the analysis of structured systems since a structured system can be represented as a digraph and there exists necessary and sufficient graph theoretic conditions for various structural properties of the system. \(^1\) Given a structured system \((\bar{A}, \bar{B}, \bar{C})\) we first construct the system digraph denoted as \(D(\bar{A}, \bar{B}, \bar{C})\) which is constructed as follows: we define the state digraph \(D(\bar{A}) := D(V_X, E_X)\) where \( V_X = \{x_1, \ldots, x_n\} \) and an edge \((x_j, x_i) \in E_X\) if \( \bar{A}_{ij} \neq 0 \). Thus a directed edge \((x_j, x_i) \) exists if state \( x_j \) can influence state \( x_i \).

\(^2\)Although \( \| \cdot \|_0 \) does not satisfy all the norm axioms, the number of non-zero entries in a matrix is conventionally referred to as the zero norm.

Define the system digraph \(D(\tilde{A}, \tilde{B}, \tilde{C}) := D(V_X \cup V_U \cup V_Y, E_X \cup E_U \cup E_Y)\), where \( V_U = \{u_1, \ldots, u_m\} \) and \( V_Y = \{y_1, \ldots, y_p\} \). An edge \((u_j, x_i) \in E_U\) if \( \tilde{B}_{ij} \neq 0 \) and an edge \((x_j, y_i) \in E_Y\) if \( \tilde{C}_{ij} \neq 0 \). Thus a directed edge \((u_j, x_i) \) exists if input \( u_j \) can actuate state \( x_i \) and a directed edge \((x_j, y_i) \) exists if output \( y_i \) can sense state \( x_j \) and this completes the construction of the system digraph.

Given a structured system \((\bar{A}, \bar{B}, \bar{C})\) and a feedback matrix \(\bar{K}\), we define the closed-loop system digraph \(D(\bar{A}, \bar{B}, \bar{C}, \bar{K}) := D(V_X \cup V_U \cup V_Y, E_X \cup E_U \cup E_Y \cup E_K)\), where \( \{y_j, u_i\} \in E_K \) if \( \bar{K}_{ij} \neq 0 \). Here a directed edge \((y_j, u_i) \) exists if output \( y_j \) can be fed to input \( u_i \).

A digraph is said to be strongly connected if for each ordered pair of vertices \((v_1, v_k)\) there exists an elementary path from \( v_1 \) to \( v_k \). A strongly connected component (SCC) is a subgraph that consists of a maximal set of strongly connected vertices. Now, using the closed-loop system digraph \(D(\bar{A}, \bar{B}, \bar{C}, \bar{K})\) the following result has been shown \(^4\).

**Proposition 1** (4, Theorem 4). A structured system \((\bar{A}, \bar{B}, \bar{C})\) have no structurally fixed modes with respect to an information pattern \(\bar{K}\) if and only if the following conditions hold:

a) in the digraph \(D(\bar{A}, \bar{B}, \bar{C}, \bar{K})\), each state node \( x_i \) is contained in an SCC which includes an edge from \( E_K \), and

b) there exists a finite node disjoint union of cycles \(\bar{C}_g = (V_g, E_g)\) in \(D(\bar{A}, \bar{B}, \bar{C}, \bar{K})\) where \( g \) belongs to the set of natural numbers such that \( V_X \subseteq \cup_{g} V_g \).

Given a closed-loop structured system \((\bar{A}, \bar{B}, \tilde{C}, \tilde{K})\) we can check condition a) in \(O(n^2)\) computations and condition b) in \(O(n^{2.5})\) computations \(^15\). Thus checking SFMs in a structured system has complexity \(O(n^{2.5})\). The objective here is to find a sparsest feedback matrix such that the resulting closed-loop system has no SFMs. We consider structurally cyclic systems with dedicated inputs and outputs. A structurally cyclic system is defined as follows: see \(^16\).

**Definition 2.** A structured system \(\bar{A}\) is said to be structurally cyclic if the state bipartite graph \(B(\bar{A})\) has a perfect matching.

Thus in a structurally cyclic system all state vertices lie in disjoint union of cycles which consists of only \( x_i \)'s and thus condition b) in Proposition 1 is satisfied. Thus the feedback selection problem needs to satisfy only condition a) in Proposition 1. Henceforth we consider structurally cyclic systems with dedicated inputs and outputs. Thus the following assumption holds.

**Assumption 1.** The structured system \((\bar{A}, \bar{B}, \bar{C})\) satisfies \(\bar{B} = \bar{I}_n\), \(\bar{C} = \bar{I}_p\) and \(B(\bar{A})\) has a perfect matching.

The authors in \(^11\) claim that Problem 1 is NP-hard. However, the proof provided is not complete. The instance of Problem 1 constructed in the NP-hardness proof given in \(^11\) satisfies Assumption 1. We show that for this case, the problem can be solved in linear time complexity. We mention briefly the error in the proof and the result given in \(^11\) in detail in the next section. We also give our algorithm of linear complexity for solving Problem 1 when Assumption 1 holds.
3. GRAPH DECOMPOSITION PROBLEM AND THE SPARSEST FEEDBACK SELECTION PROBLEM

In this section we focus on the subtle difference between the approach followed in [11] for solving the sparsest feedback selection problem, in which the authors link this problem with the graph decomposition problem. In this section we elaborate on a key error in their reduction procedure. We later consider a counter-example that helps understand the error.

Recall that Problem 1 aims at finding a sparsest feedback matrix $\bar{K}$ such that the closed-loop system has no SFMs. This problem has been shown to be NP-hard in [11] using reduction from a known NP-complete problem, the graph decomposition problem [17], described below.

Problem 2 (Graph Decomposition Problem [17]). Given a directed acyclic graph $D(V,E)$, find a partition of $V$ into two non-empty sets $\Gamma_1$ and $\Gamma_2$ such that

- no edge in $E$ connects any vertex in $\Gamma_1$ to a vertex in $\Gamma_2$;
- for every vertex $v \in \Gamma_i$, $i = 1, 2$, there exists a source-sink pair and a path between them that contains $v$ and passes only through the nodes in $\Gamma_i$. Here, source (sink, resp.) refers to a node that does not have any incoming (outgoing, resp.) edge.

The authors of [11] have taken a general instance of Problem 2 and constructed an instance of Problem 1 Next, in order to claim NP-hardness of Problem 1 it is shown that if an optimal solution $\bar{K}^*$ to Problem 1 results in two SCCs in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K}^*)$, then the original graph can be decomposed as required. However, this result is not enough. For proving the NP-hardness of a given problem using reduction, one must reduce an arbitrary instance of a known NP-complete or NP-hard problem to an instance of the given problem with polynomial complexity. Further, the reduction must also be such that any optimal solution to the given problem must give an optimal solution to the NP-complete or NP-hard problem chosen and vice versa [18].

However, [11] only shows that an optimal solution $\bar{K}^*$ to Problem 1 that has two SCCs in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K}^*)$ gives a solution to the decomposition problem. This does not answer the case where an optimal solution $\bar{K}^*$ results in a different number of SCCs in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K}^*)$. Importantly, one must also address the case when an optimal solution results in a single SCC and specify what happens in the graph decomposition problem then. More precisely, for the reduction to be complete, one should show that any optimal solution to Problem 1 gives an optimal solution to the graph decomposition problem and vice-versa. Thus, if an optimal solution $\bar{K}^*$ to Problem 1 exists that results in a single SCC in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K}^*)$, then it is not clear about the corresponding solution of the graph decomposition problem.

We demonstrate the ambiguity in the proof using the illustrative examples given in Figure 1. Note that in Figure 1a, the optimal solution $\bar{K}^*$ results in three SCCs. However, there are three possible partitioning of the original graph that can be obtained from this. These partitions are: $\Gamma_1 = \{x_1, \ldots, x_6\}$ and $\Gamma_2 = \{x_7, x_8, x_9\}$, $\Gamma_1 = \{x_1, x_2, x_3\}$ and $\Gamma_2 = \{x_4, \ldots, x_9\}$ and $\Gamma_1 = \{x_1, x_2, x_3, x_7, x_8, x_9\}$ and $\Gamma_2 = \{x_4, x_5, x_6\}$. Now another optimal solution given in Figure 1b results in two SCCs. Corresponding to this solution, there is a partitioning of the original graph, $\Gamma_1 = \{x_1, \ldots, x_6\}$ and $\Gamma_2 = \{x_7, x_8, x_9\}$. The optimal solution given in Figure 1c results in a single SCC. In this case, according to [11], from $\bar{K}^*$ it is not possible to say whether there exists a partitioning of the original problem. Note that in this example, many optimal $\bar{K}^*$ are possible, and the original graph can be decomposed irrespective of the solution chosen. On the contrary, consider the system given in Figure 1d and the corresponding optimal solution. Notice that for this structured system, given $\bar{K}^*$ is the only optimal
solution to Problem \[1\] and it results in a single SCC. Moreover, the original graph can not be decomposed. Thus, the case where optimal solution results in a single SCC, we can not conclude either way for the graph decomposition problem. Thus, the reduction in \([1]\) is inconclusive.

In summary, in our opinion, the problem that is shown to be NP-hard in \([1]\) can be stated as: given a structured system \((\tilde{A}, \tilde{B}, \tilde{C})\), find \(\tilde{K}\) such that \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K})\) has two or more SCCs. Note that this is slightly different than Problem \([1]\). In the next section, we provide a linear time algorithm for solving Problem \([1]\).

### 4. Linear Complexity Algorithm for Solving Problem \([1]\)

The structured system \((\tilde{A}, \tilde{B}, \tilde{C})\) considered in the NP-hard proof given in \([1]\) satisfies Assumption \([1]\). In this section we show that if the structured system satisfies Assumption \([1]\) then Problem \([1]\) can be solved in linear time. The proposed solution is the consequence of the following important observation.

**Lemma 1.** Consider a structured system \((\tilde{A}, \tilde{B}, \tilde{C})\). Let \(\tilde{K}^*\) be an optimal solution to Problem \([1]\) such that all state nodes lie in \(B\) number of SCCs in \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K}^*)\), where \(B > 1\). Then, there exists another optimal solution \(\tilde{K}_{\text{new}}^*\) such that \(\|\tilde{K}_{\text{new}}^*\|_0 = \|\tilde{K}^*\|_0\) and all state nodes lie in \(B - 1\) number of SCCs in \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K}_{\text{new}}^*)\).

**Proof.** Given \(\tilde{K}^*\) is an optimal solution to Problem \([1]\) and all state nodes lie in \(B\) number of SCCs in \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K}^*)\), say \(\tilde{C}_1, \ldots, \tilde{C}_B\). Pick two SCCs, say \(\tilde{C}_i, \tilde{C}_j\). Since \(\tilde{K}^*\) satisfies condition a) both \(\tilde{C}_i\) and \(\tilde{C}_j\) has a feedback edge in it. Let \((y_a, u_b) \in \tilde{C}_i\) and \((y_c, u_d) \in \tilde{C}_j\). Now, break the edges \((y_a, u_b), (y_c, u_d)\) and make the edges \((y_b, u_a), (y_d, u_c)\). We claim that now all the nodes in \(\tilde{C}_i\) and \(\tilde{C}_j\) lie in a single SCC with feedback edges \((y_a, u_b), (y_c, u_d)\). To prove this we need to show that there exists a directed path between two arbitrary vertices in them. Consider any four arbitrary vertices \(v_p, v_q, v_r, v_s\) in \(\tilde{C}_i\) and \(\tilde{C}_j\). Since \(\tilde{C}_i\) is an SCC, notice that there exists a directed path \(v_p, v_q, v_r, v_s\). Similarly, there exists a directed path \(v_p, v_{\tilde{C}_j}, v_s\). Thus there exists a directed path \(v_p, v_{\tilde{C}_j}, v_s\). Further, using the same argument on \(\tilde{C}_j\) we can show that there exists a directed path from \(v_{\tilde{C}_j}\) to \(v_s\) passing through \(v_{\tilde{C}_j}\). This completes the proof.

As a consequence of Lemma \([1]\) we have the following corollary.

**Corollary 1.** Consider a structured system \((\tilde{A}, \tilde{B}, \tilde{C})\). Then, there exists an optimal solution \(\tilde{K}^*\) to Problem \([1]\) such that all state nodes lie in a single SCC in \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K}^*)\).

The above corollary is true, since given any optimal solution \(\tilde{K}^*\) to Problem \([1]\) we can apply Lemma \([1]\) recursively such that we arrive at an optimal solution \(\tilde{K}^*\) such that all state nodes lie in a single SCC in \(D(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{K}^*)\). Thus solving Problem \([1]\) on a structured system \((\tilde{A}, \tilde{B}, \tilde{C})\) is same as finding the minimum number of feedback edges to add in the digraph \(D(\tilde{A}, \tilde{B}, \tilde{C})\) such that in the resulting digraph all state nodes lie in an SCC.

The problem of finding minimum number of edges to add in a digraph that the resulting graph is strongly connected is referred as strong connectivity augmentation problem \([19]\). We briefly explain the strong connectivity augmentation problem here for the sake of completeness. Given a directed graph \(D(V, E)\), the strong connectivity augmentation problem aims at finding the minimum cardinality set of edges \(E'\) such that \(D(V, E \cup E')\) is strongly connected. First note that if \(D(\tilde{A})\) is irreducible, then \(E' = \emptyset\) and any \(\tilde{K}\) that has a single non-zero entry is optimal. Hence, from now on we only focus on the non-trivial cases such that \(D(A)\) has at least two SCCs. There exists a linear time algorithm for solving the above problem optimally \([20]\). Given a directed graph the algorithm given in \([20]\) gives the minimum set of edges that when added to the graph results in a single SCC. Using this result now we give the linear time algorithm to solve Problem \([1]\) on structured systems that satisfy Assumption \([1]\).

**Theorem 1.** Consider a structured system \((\tilde{A}, \tilde{B}, \tilde{C})\) such that Assumption \([1]\) holds. Then, Problem \([1]\) can be solved in \(O(n)\) complexity, where \(n\) denotes the number of states.

**Proof.** Here we prove that solving strong connectivity augmentation problem on \(D(\tilde{A})\) gives an optimal solution to
Problem II. The structured system given has to satisfy one of the following cases: i) $D(\bar{A})$ is irreducible; ii) $D(\bar{A})$ is reducible. In case i), solution to the strong connectivity augmentation problem, $E_X = \phi$. Then, an optimal solution to Problem II is given by $\{K : K_{11} = \ast \text{ and } 0 \text{ otherwise}\}$. In case ii), we prove that $E_X$ is an optimal solution to Problem II. We first show that $\bar{K}$ is a feasible solution, i.e., $\bar{K} \in K_x$. By the construction of $\bar{K}$ given in Step 2 notice that all state nodes lie in a single SCC in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K})$. Also, $D(\bar{A})$ is not irreducible. Thus condition a) is satisfied for all states. Thus $\bar{K} \in K_x$. Now we prove that $\bar{K}$ is an optimal solution to Problem II i.e., $\|\bar{K}\|_0 = \|\bar{K}'\|_0$. Suppose not. Then there exists $\bar{K}' \in K_x$ such that $\|\bar{K}'\|_0 < \|\bar{K}\|_0$ and by Corollary I all state nodes lie in a single SCC in $D(\bar{A}, \bar{B}, \bar{C}, \bar{K})'$. Consider edges $E_X'$ where $(j, x_i) \in E_X'$ if $\bar{K}''_{ij} = \ast$. Notice that $|E_X'| < |E_X|$ and $D(V_X, E_X \cup E_X')$ is an SCC. This contradicts the assumption that $E_X$ is an optimal solution to the strong connectivity augmentation problem. This proves that the feedback matrix obtained by solving the strong connectivity augmentation problem on $D(\bar{A}), \bar{K} = \{K_{ij} = \ast : (j, x_i) \in E_X\}$, is an optimal solution to Problem II.

Now the complexity of the strong connectivity augmentation algorithm is linear in the number of nodes in the digraph. Since $|V_X| = n$, the result follows.

Though we show that when $B = \bar{C} = I_n$ and all feedback links are feasible, this algorithm gives an optimal solution to Problem II these results do not immediately extend to the cases where some feedback links are not feasible or feedback links are associated with costs. We believe that these problems are NP-hard and approximation algorithms for these problems will be subject of future work.

5. Conclusion

This paper deals with optimal feedback selection of structured systems. The objective here is to obtain a sparsest feedback matrix such that the resulting closed-loop system has no structurally fixed modes. This problem was considered in Carvalho et al. in [11], [12] though we elaborated in this paper on an error in their proof of NP-hardness. We have shown recently in [13] that this problem is NP-hard. Further, in this paper, we also proved that solving this problem is, in fact, not NP-hard on the subclass of systems considered in [11], i.e., structurally cyclic with dedicated inputs and outputs. Finally, we provided an algorithm for this subclass of systems that has linear complexity in the number of states of the system.

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