Laboratory measurements of elastic anisotropy parameters for the exposed crustal rocks from the Hidaka Metamorphic Belt, Central Hokkaido, Japan

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SUMMARY
To study crustal rock seismic anisotropy and its effect on seismic wave propagation, we measure the seismic velocity anisotropy of two amphibolites, one biotite gneiss and one biotite schist from the Hidaka metamorphic belt in central Hokkaido, Japan, under confining pressures up to 150 MPa. The rock microstructures show foliation and lineation characterized by lattice preferred orientation (LPO) of hornblende or biotite. P- and S-wave velocities are measured along the direction perpendicular to the foliation plane and two directions in the foliation plane: perpendicular and parallel to the lineation. We assume orthorhombic symmetry based on the rock microstructures and obtain Tsvankin’s anisotropic parameters (an extension of Thomsen’s parameters for orthorhombic symmetry). P- and S-wave phase velocity surfaces are calculated from anisotropy parameters and compared with the measured velocities along particular directions and with the velocity contour maps calculated from the Voigt averages of single-crystal elastic constants based on the orientation of measured LPO data. Qualitatively, the measured velocity anisotropy agrees with the velocity contour calculated from the LPO data, although large quantitative differences exist between them. All anisotropy patterns can be approximated as transverse isotropy or its modification, appearing as orthorhombic symmetry. Biotite schist (containing 30 per cent volume ratio biotite) shows strong S-wave anisotropy, and the phase velocity surfaces of P waves show a large deviation from ellipticity in the plane perpendicular to the foliation and parallel to the lineation. In the same plane, S waves show a singularity due to a large bulge of the SV velocity surface.

Key words: anisotropy, crustal structure, seismic velocities.

1 INTRODUCTION
Seismic velocity anisotropy is commonly observed in the crust, evidenced by azimuthal velocity variation of the longitudinal (P) wave or velocity differences of the two polarized shear (S) waves (reviews are given by Babška & Cara 1991; Leary et al. 1990). We must consider the effects of seismic anisotropy when treating the seismic waves propagating through the crust. The phase velocity surfaces in anisotropic media sometimes deviate considerably from an elliptic shape for both P and S waves. The S wave splits into fast and slow waves depending on the polarization direction. Those characteristics affect conventional processing techniques of seismic reflection surveys such as moveout correction (normal moveout, NMO, and dipping layer moveout, DMO), reflectivity and amplitude variation with offset (AVO), and they should be modified from the isotropic case to the anisotropic case (Thomsen 1986; Thomsen 1988). S-wave splitting provides important information about crustal structure and tectonic activity (Crampin 1987; Kaneshima 1990; Crampin 1990). In order to study the effects of seismic velocity anisotropy on seismic wave propagation in the Earth’s crust, we first need laboratory data of seismic anisotropy in crustal rocks.

Seismic anisotropy in crustal rocks results from the preferred orientation of fractures or cracks (Hudson 1981; Nishizawa 1982; Crampin 1984) or the lattice preferred orientation (LPO) of major rock-forming minerals with strong anisotropy (Christensen 1984; Siegesmund et al. 1989). In the deeper part of the crust, the LPO of anisotropic minerals will be more effective than preferred orientations of cracks and fractures because most cracks and fractures will be closed under high pressures. There have been many studies that have measured the intrinsic seismic
anisotropy of crustal rocks in order to interpret seismic data (Barruol & Kern 1996; Burlini & Fountain 1993; Kern 1988; Kern & Schenk 1985; Siegsmund et al. 1989; Siegsmund & Volbrecht 1991). Most of those studies measured the velocities along the microstructural axes defined by foliation and lineation and compared them with the calculated velocities based on LPO data. To study the details of wave propagation in an anisotropic rock, we need a full set of elastic parameters of the rock or to measure velocities of all directions. Assuming orthorhombic symmetry in some granitic rocks, Sano et al. (1992) determined full sets of elastic constants and the symmetry axes simultaneously. Pros et al. (1998) measured the P velocity for many paths in spherical samples and obtained the P-velocity contour over all directions. However, those measurements are not easy to perform and are not suited for most practical cases. Therefore, most of the measured anisotropic data on crustal rocks are not good enough for an understanding of seismic wave propagation in anisotropic crust.

Since most of the velocity anisotropy in rocks can be approximated by simple anisotropy (Thomsen 1986) or its modification (Tsvankin 1997b), we can use approximate methods to study velocity anisotropy. The most common case is transverse isotropy (Tsvankin 1997b), we can use approximate methods approximated well by calculations based on Thomsen’s anisotropy parameters (c, γ and δ). We can calculate velocities of all directions by three anisotropy parameters, and P and S velocities along the symmetry axis. Thomsen’s anisotropic parameters can be obtained by measuring seismic wave velocities in particular directions. Therefore, it is worthwhile to measure Thomsen’s parameter (or its modification) in natural anisotropic rocks and calculate phase velocities based on the parameters, and then compare them with those inferred from the LPO data or model calculations. If we find that Thomsen’s anisotropy parameter or its modification is useful for describing wave propagation in rocks, we can study wave propagation in the crust more easily, instead of measuring rock velocities in all directions or measuring a full set of elastic constants.

In this paper we focus on the seismic velocity anisotropy of metamorphic rocks that show typical biotite and hornblende LPO. We measured Tsvankin’s anisotropy parameters (Tsvankin 1997b) for those rocks. By using measured anisotropy parameters, we calculated P, SV and SH phase velocity surfaces in particular planes, and evaluated the effect of anisotropy on seismic wave propagation. We discuss the meaning of measured anisotropy.

2 GEOLOGICAL SETTING AND MICROSTRUCTURAL CHARACTERISTICS OF SAMPLES

The Hidaka metamorphic belt consists of the Hidaka Western Zone and the Hidaka Main Zone (Fig. 1). They are bounded by the Hidaka Main Thrust (HMT). The Hidaka Western Zone consists mainly of greenschists and amphibolites, which are metamorphosed oceanic crustal rocks. The Hidaka Main Zone consists of felsic granulites, amphibolites, biotite gneisses, biotite schists and hornfels from west to east; the trend corresponds to metamorphic grade, from higher to lower. They are considered to represent a section of an ancient island arc crust tilted to the east, and the exposed crustal thickness is approximately 23 km (Komatsu et al. 1983). The rocks used in this study are green hornblende amphibolite (SB-1) of the Hidaka Western Zone and brown hornblende amphibolite (SB-5), biotite gneiss (SB-7) and biotite schist (ST-3) of the Hidaka Main Zone (Fig. 1).

The rocks have developed foliations and lineations, which are characterized by aligned hornblende and biotite crystals. The three mutually orthogonal axes x1, x2 and x3 are selected as follows: x1 is parallel to the lineation; x3 is in the foliation plane and perpendicular to the lineation; and x3 is perpendicular to the foliation plane (Fig. 2). The relationship between the axes determined by microstructures and the velocity and polarization directions of P and S waves are shown in Fig. 2. Fig. 2 also shows that the S wave propagating along the axial direction splits into fast and slow waves that are polarized parallel to the other two axes when rock anisotropy is TI or orthorhombic. However, in general directions the polarization directions of the fast and slow S waves are not necessarily perpendicular to the propagation direction.

The modal compositions and outline of LPO for the measured rock samples are described in Table 1. Photomicrographs of SB-1 and ST-3 are shown in Figs 3 and 4, respectively, for (a) the x1-x2-planes and (b) the x2-x3-planes. In these photomicrographs, we see crystal alignments and hornblende and biotite crystal shapes.

Porosities of rock samples were measured with a helium porosimeter. Table 2 shows densities and porosities of rock samples. Except for SB-1, the porosity values are large compared to those of common metamorphic rocks. The high porosity may affect velocity measurements, but the change of anisotropy parameters under pressure indicates that anisotropy parameters become almost constant over 100 MPa, as is shown in Fig. 5.

3 ANISOTROPY PARAMETERS

3.1 Symbols and notation

To obtain anisotropic parameters from the measured velocity data, we first assume that the rock symmetry is TI, and then assume orthorhombic symmetry with axes along x1, x2 and x3, which is determined by the microstructure of the rocks. We denote P- and S-wave velocities by Vp and Vs, respectively. For describing the propagation and polarization directions of the S wave along the axis, we use subscripts 1, 2 and 3 corresponding to the axes x1, x2 and x3, respectively. The first subscript i of Vpi indicates the propagation direction, and the second subscript j of Vsi indicates the polarization direction of the shear wave in the x3-xi-plane. The velocities propagating along the three orthogonal axes are calculated by the diagonal elements of the elastic constants (Cij, i = 1, 2, . . . , 6, in Voigt notation). The velocity in an arbitrary direction is a function that includes the off-diagonal elements of elastic constants. The off-diagonal

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elastic constants are calculated from velocities of more than two directions. In general directions, the polarization directions of P and S waves do not necessarily correspond to directions parallel or perpendicular to the propagation direction. These waves are called quasi-longitudinal and quasi-shear waves and are denoted by $q_P$, $q_{S_1}$ and $q_{S_2}$ (Winterstein 1990). In this paper we mostly use the notation of $P$, $SV$ and $SH$ waves because our major concern is the velocity and its associated properties in TI or particular directions of orthorhombic symmetry. The simple terminology, therefore, will not cause misunderstandings. General anisotropy appears only in LPO-based velocities, where we use the notation $P$, $S_1$ and $S_2$.

### Table 1. Mode and characterization of rock samples.

| Sample | Mode | Remarks (lattice preferred orientation: LPO) |
|--------|------|----------------------------------|
| SB-1   | 70% green Hb | Hb $c$-axis: strong single maximum in $x_1$ |
|        | 18% Plg | $b$-axis: a girdle to two maxima in $x_2x_3$-plane |
|        | 9% sphene | |
| SB-5   | 55% brown Hb | Hb $c$-axis: two-maxima in $x_1x_2$-plane, no strong LPO |
|        | 40% Plg | $b$-axis: single maximum in $x_2$ |
| Brown hornblende amphibolite, lower part of Hidaka Main Zone | |
| SB-7   | 39% Plg | Bt $c$-axis: girdle pattern in $x_2x_3$-plane |
|        | 34% Bt | with maximum in $x_3$ |
|        | 27% Qz | |
| ST-3   | 43% Qz | Bt $c$-axis: single maximum in $x_3$ |
| Biotite schist | 36% Bt | elongated in $x_2x_3$-plane |
|        | 20% Plg | |

**Figure 1.** Geological map of the central part of the Hidaka metamorphic belt. MZ: Hidaka Main Zone; WZ: Hidaka Western Zone; HMT: Hidaka Main Thrust; WBT: Western Boundary Thrust. Sample locations are shown in the figure.

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3.2 Transverse isotropy (TI)

In this study, we start from TI and then move to orthorhombic symmetry as a modification of TI; this approach seems reasonable when we consider the microstructure of rocks, because rock microstructures are described by using three orthogonal axes. TI is described in terms of the symmetry axes; for example, TI(3) is the transverse isotropy with symmetry axis parallel to the \(x_3\) direction. By using Thomsen’s parameters \(\varepsilon\), \(\gamma\), and \(\delta\) (Thomsen 1986), we can express \(P\)- and \(S\)-wave phase velocities as simple functions of the polar angle \(\theta\) measured from the symmetry axis.

Assuming TI(3), Thomsen’s anisotropic parameters \(\varepsilon\) and \(\gamma\) are given by

\[
\varepsilon = \frac{C_{11} - C_{33}}{2C_{33}} = \frac{V_{p1}^2 - V_{p3}^2}{2V_{p3}^2} \geq \frac{V_{p1} - V_{p3}}{V_{p3}} = \frac{V_{p2} - V_{p3}}{V_{p3}},
\]

\[
\gamma = \frac{C_{66} - C_{44}}{2C_{44}} = \frac{V_{S12}^2 - V_{S31}^2}{2V_{S11}^2} \geq \frac{V_{S12} - V_{S31}}{V_{S31}} = \frac{V_{S21} - V_{S23}}{V_{S23}}.
\]

Note that \(V_{p1} = V_{p2}\) (the \(P\)-wave velocities in the isotropic plane are all equal), \(V_{S31} = V_{S32}\) (the \(S\) wave along the symmetry axis) and \(V_{S12} = V_{S21}\) (the in-plane polarized \(S\) wave propagating in the isotropic plane). \(\varepsilon\) and \(\gamma\) are the parameters derived directly from the diagonal elements of the elastic constants and calculated from the \(P\)- and \(S\)-wave velocities in the \(x_1, x_3\)-plane and along the \(x_3\)-axis. The third parameter, \(\delta\), is given by

\[
\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}.
\]

\(\delta\) contains an off-diagonal element of elastic constants, \(C_{13}\). \(P\)-wave phase velocity along the polar angle \(\theta\) is approximately given by

\[
V_p(\theta) \approx V_{p3}(1 + \delta \sin^2 \theta \cos^2 \theta + \varepsilon \sin^4 \theta).
\]

If \(\varepsilon\) and \(V_{p3}\) are known, \(\delta\) is calculated from the \(P\)-wave velocity propagating in the direction \(\theta\) (\(\theta \neq 0, \pi/2\)); for example, the
P velocity in the direction $\theta = \pi/4$ gives $\delta$ as

$$\delta \approx 4 \left( \frac{V_P(\pi/4)}{V_{P1}} - 1 \right) \frac{V_{P1}}{V_{P3}} - 1 \right). \tag{5}$$

If the rock anisotropy is TI$^{(1)}$ (or TI$^{(2)}$), $x_3$ and $x_1$ (or $x_2$) should be swapped.

For TI$^{(3)}$, S-wave velocities are approximated as

$$V_{SV} = V_{S3}(1 + \sigma \sin^2 \theta \cos^2 \theta), \tag{6}$$

$$V_{SH} = V_{S3}(1 + \gamma \sin^2 \theta), \tag{7}$$

where $V_{SV}$ and $V_{SH}$ denote the S waves polarized in the plane including the $x_3$-axis and in the plane perpendicular to the $x_3$-axis, respectively, and $V_{S3}$ is the S velocity along the $x_3$-axis (no S-wave splitting in $x_3$ of TI$^{(3)}$). $\sigma$ is called the SV-wave anisotropy parameter (Banik 1987), which is defined by $V_{P3}$ and $V_{S3}$ and the difference between $\varepsilon$ and $\delta$:

$$\sigma = \frac{(V_{P3})^2}{V_{S3}}(\varepsilon - \delta). \tag{8}$$

The $P$- to S-wave velocity ratio along the symmetry axis and the difference between $\varepsilon$ and $\delta$ control the phase velocity of the $SV'$ wave. The $SV'$ velocity is given by a linear combination of 20 and 40 terms of sine or cosine functions, and deviates from a circular shape when $\varepsilon \neq \delta$. $\sigma$ is always zero or positive, and the phase velocity surface of the $SV'$ wave bulges with the maximum at $\theta = \pi/4$, while the $SH$ wave always shows an elliptical phase velocity surface. The large bulge of the $SV'$-wave phase velocity surface causes a singularity, where the $SV'$- and $SH$-wave phase velocity surfaces cross each other, and the fast and slow S waves change their polarities.

### 3.3 Orthorhombic case

When rocks show deviations from TI and are approximated as orthorhombic symmetry, Thomsen’s parameters are extended to the orthorhombic case (Tsvankin 1997b). Parameters are defined similar to the TI case, but they contain superscripts indicating the axis perpendicular to the mirror plane of the orthorhombic symmetry:

$$\varepsilon^{(1)} = \frac{V_{P2}^2 - V_{P3}^2}{2V_{P3}^2} \approx \frac{V_{P2} - V_{P3}}{V_{P3}} \tag{9}$$

$$\varepsilon^{(2)} = \frac{V_{P1}^2 - V_{P2}^2}{2V_{P2}^2} \approx \frac{V_{P1} - V_{P2}}{V_{P2}} \tag{10}$$

$$\gamma^{(1)} = \frac{V_{S21}^2 - V_{S31}^2}{2V_{S31}^2} \approx \frac{V_{S21} - V_{S31}}{V_{S31}} \tag{11}$$

$$\gamma^{(2)} = \frac{V_{S32}^2 - V_{S31}^2}{2V_{S32}^2} \approx \frac{V_{S32} - V_{S31}}{V_{S32}}, \tag{12}$$

where the superscripts (1) and (2) indicate the normals of the $x_2x_3$ and $x_1x_3$ mirror planes, respectively. TI-type anisotropy is assumed on each mirror symmetry plane. $\varepsilon^{(0)}$ and $\gamma^{(0)}$ are obtained from $P$- and two polarized S-wave velocities along the axes of orthorhombic symmetry. The parameters $\varepsilon^{(3)}$ and $\gamma^{(3)}$ are omitted because they are calculated from other parameters. Normally the $x_3$-axis is assumed as the symmetry axis for expressing $\varepsilon^{(3)}$ and $\gamma^{(3)}$. 

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**Figure 4.** Optical micrographs of (a) the $x_1x_3$ and (b) the $x_2x_3$ sections of ST-3. Bt = biotite. Plane-polarized light. Biotite crystals in the $x_1x_3$ section (a) are rather thin in shape and strongly oriented parallel to the $x_1$-direction, while they are relatively thick and poorly oriented in the $x_2x_3$-section (b).

**Figure 5.** Change of anisotropy parameters $\varepsilon^{(2)}$, $\gamma^{(2)}$, $\delta^{(2)}$ in ST-3 as a function of confining pressure. Anisotropy parameters show changes in pressures below 100 MPa. For pressures above 100 MPa, the parameters are almost constant, showing that most of the thin cracks affecting rock anisotropy are closed.
The anisotropy parameters $\delta$ are

\[
\delta^{(1)} = \frac{(C_{33} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})},
\]

\[
\delta^{(2)} = \frac{(C_{13} + C_{55})^2 - (C_{13} - C_{55})^2}{2C_{13}(C_{13} - C_{55})},
\]

\[
\delta^{(3)} = \frac{(C_{12} + C_{66})^2 - (C_{11} - C_{66})^2}{2C_{11}(C_{11} - C_{66})}.
\]

The phase velocity of the $P$ wave in an orthorhombic medium is given by eq. (49) of Tsvankin (1997a),

\[
V_P(\theta, \phi) = V_{P3}[1 + \delta(\phi) \sin^2 \theta \cos^2 \theta + \epsilon(\phi) \sin^4 \theta],
\]

where $\theta$ and $\phi$ are the polar and azimuthal angles measured from $x_3$ and $x_1$, respectively, and $\delta(\phi)$ and $\epsilon(\phi)$ are given by

\[
\delta(\phi) = \delta^{(1)} \sin^2 \phi + \delta^{(2)} \cos^2 \phi,
\]

\[
\epsilon(\phi) = \epsilon^{(1)} \sin^4 \phi + \epsilon^{(2)} \cos^2 \phi + (2\epsilon^{(2)} + \delta^{(3)}) \sin^2 \phi \cos^2 \phi. \tag{17}
\]

In the $x_1x_3$- and $x_2x_3$-planes, $\phi$ equals 0 and $\pi/2$, respectively, and the $P$ velocities are described by Tsvankin’s parameters with the superscripts (2) and (1), respectively.

$S$-wave velocities in the mirror symmetry plane can also be given in a manner analogous to $Tl$. Phase velocities of $SV$ and $SH$ waves in the $x_1x_3$-plane ($V_{SV}^{(2)}$, $V_{SH}^{(2)}$) are given by Tsvankin (1997b),

\[
V_{SV}^{(2)}(\theta) = V_{SV}^{(0)}(1 + \sigma^{(2)} \sin^2 \theta \cos^2 \theta),
\]

\[
V_{SH}^{(2)}(\theta) = V_{SH}^{(0)}(1 + \gamma^{(2)} \sin^2 \theta),
\]

where $V_{SV}^{(0)}$ and $V_{SH}^{(0)}$ are equal to $V_{C1}$ and $V_{C2}$, respectively, and $\sigma^{(2)}$ is defined by $V_P^{(2)}(\theta) = V_P^{(0)}$ and $V_{SV}^{(2)}(\theta)$ and the difference between $\sigma^{(2)}$ and $\delta^{(2)}$,

\[
\sigma^{(2)} = \frac{V_{SV}^{(2)}(\theta)^2}{V_{SH}^{(2)}(\theta)} ((\sigma^{(2)} - \delta^{(2)}). \tag{19, 20}
\]

$SV$ and $SH$ velocities in other planes are calculated in the same manner.

## 4 VELOCITY MEASUREMENTS AND CHANGE OF ANISOTROPIC PARAMETERS UNDER PRESSURE

We cored rock samples (25 mm in diameter and 50–70 mm in length) parallel to the three orthogonal axes, and mounted piezoelectric transducers on each core end. Rectangular parallelepiped samples of similar dimension were also used for the measurements when core samples were not available. Samples were dried and covered with silicon rubber to prevent immersion of the pressure medium (oil). They were then installed in a hydrostatic pressure vessel. $P$ and $S$ waves were measured under the confining pressures up to 150 MPa using a pulse transmission method. Confining pressure was measured using a calibrated Heise gauge. Coaxial-type feedthroughs were installed in the vessel (Nishizawa 1997), which enabled us to obtain low-noise transmitted waveforms showing clear first arrivals. We used a pair of $P$- or $S$-wave piezoelectric transducers as the source and receiver for $P$ and $S$ waves. The $P$-wave transducer is a 5 mm diameter disc with a dilation mode of 2 MHz characteristic frequency. The $S$-wave transducer is a $3 \times 6$ mm rectangular plate that produces a shear wave polarized along the long direction of the rectangle with 1 MHz characteristic frequency. Both transducers radiate and receive most of the $P$- and $S$-wave energy along the normal of the transducer surface. We measured the phase velocity of $P$ and polarized $S$ waves along the direction of the source and receiver transducers, as in the experiments of Johnston & Christensen (1994). The waveform was digitized by a fast A/D converter with 20 MHz sampling rate and 10 bit full-scale resolution and then it was transferred and stored on a computer hard disk. To pick up the first $S$-wave arrival, the signal waveforms of different pressures were gathered. By comparing the gathered waveforms, we can pick up the $S$-wave arrivals even for data containing scattered $P$ waves before the $S$ phase. For traveltime calibration, we measured the first arrivals of $P$ waves in brass rods with different lengths and plotted arrival times against the rods’ lengths (travel distances). Extrapolating the plotted line to zero length, we obtained the systematic delay (or advance) time of our measurement. The technical details of our measurements are described in Matsuzawa et al. (1995). The accuracy of the present experiments is better or at least not worse than other methods reported so far.

The anisotropic parameters $\epsilon^{(0)}$ and $\gamma^{(0)}$ are obtained from the $P$ and $S$ velocities in the axial directions (eqs. 9–12). Exact measurements of $\delta^{(0)}$ are not easy because $\delta^{(0)}$ is calculated from the $P$ velocity in the $\theta$ direction and the two axial velocities in the corresponding symmetry plane. The accuracy of $\delta^{(0)}$ depends on the incident angle of the velocity because the coefficients of $\delta^{(0)}$ and $\epsilon^{(0)}$ in eq. (16) change with $\theta$. If we measure the velocity close to the symmetry axis of $Tl$, the coefficient of $\delta^{(0)}$ in eq. (4) is larger than that of $\epsilon^{(0)}$, but the velocity change relative to $V_P^{(2)}$ is very small. A small error in the velocity measurements strongly affects the value of $\delta^{(0)}$. We therefore use the $P$-velocity data measured at the polar angle $\pi/4$ from the symmetry axis and calculate $\delta^{(0)}$ using eq. (5), because the velocity change from the axial direction is expected to be large and the errors from the velocities at $\pi/4$ are expected to be of the same magnitude as the other two axial velocities.

To study the effects of cracks on velocity, we measured the velocity change with increasing confining pressure. Thin cracks strongly affect velocity anisotropy when they are aligned in parallel (Anderson et al. 1974; Nishizawa 1982; Douma 1988). However, most thin cracks close at pressures of less than 100 MPa (Walsh 1965; Simmons et al. 1974). The effects of cracks on velocity anisotropy become weak above 100 MPa. Fig. 5 shows the change in anisotropy parameters under pressure for sample ST-3 in the $x_1x_3$-section. Anisotropy parameters change at pressures below 50 MPa, but they are almost constant above 100 MPa, except $\delta^{(2)}$, which is affected by small changes of the $P$ velocities in the three directions and is not accurate compared to the other parameters. We thus consider that the velocities at 150 MPa show intrinsic anisotropy.

## 5 RESULTS

### 5.1 $P$- and $S$-wave velocities and anisotropy

$P$- and $S$-wave velocities under 150 MPa are shown in Fig. 6 and the values are given in Table 3. In Table 3, the relationships between the axial $S$ velocities and the orthorhombic

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elastic constants are included for comparing the axial velocities with the velocities of orthorhombic media. Anisotropy patterns estimated from \(P\)- and \(S\)-wave velocities are indicated in the last column of Table 3(a).

### 5.1.1 SB-1

A significant velocity difference between \(V_{P1}\) and the other two velocities indicates that the rock anisotropy is close to TI\(^{(1)}\), but the symmetry slightly shifts to orthorhombic. The small \(S\)-wave splitting (\(D_{VS} = 1.3\) per cent) in the \(x_1\) direction also supports the idea that the symmetry of SB-1 is close to TI\(^{(1)}\). The difference in \(S\)-wave splitting between the \(x_2\)- and \(x_3\)-directions seems to indicate an orthorhombic perturbation. If we approximate the rock as having orthorhombic symmetry, the following relations should hold:

\[
V_{S21} = V_{S31}, \quad V_{S23} = V_{S32}, \quad V_{S13} = V_{S12}.
\]

However, the difference between \(V_{S13}\) and \(V_{S12}\) is significant. It may be more realistic to assume that the symmetry of SB-1 is basically a TI\(^{(1)}\) with some perturbation, rather than to approximate it by orthorhombic symmetry. The velocity calculation based on LPO data also supports a TI\(^{(1)}\) symmetry, as we show later.

### 5.1.2 SB-5

Large velocity differences between \(V_{P3}\) and the other two \(P\)-wave velocities indicate that the rock anisotropy is close to TI\(^{(3)}\). The assumption of orthorhombic symmetry does not hold for the \(S\) wave because of the large difference between \(V_{S12}\) and \(V_{S23}\). It may be better to assume that the basic anisotropy of SB-5 is TI\(^{(3)}\) with a strong perturbation, which produces an orthorhombic symmetry. The velocity contour of the \(S\) wave based on LPO data suggests a weak TI\(^{(3)}\) symmetry.
5.1.3 SB-7

P-wave velocities show small differences between $V_{P1}$ and the other two velocities, suggesting a weak TI$^{(1)}$-type anisotropy. However, there is a significant difference between $V_{S12}$ and $V_{S13}$. The S-wave anisotropy suggests TI$^{(3)}$-type anisotropy because $\Delta V_S$ in the $x_3$-direction is significantly smaller than in the other two directions. The orthorhombic assumption seems to hold for S-wave velocities in axial directions within the error of 0.1 km s$^{-1}$ ($\approx 3$ per cent). This anisotropy can be interpreted as TI$^{(1)}$-type with an orthorhombic-type perturbation, which is reasonably well supported by biotite LPO data.

5.1.4 ST-3

The rock shows different P-wave velocities in the three directions, indicating significant deviation from transverse isotropy and suggesting an orthorhombic symmetry. S-wave splitting in the $x_1$- and $x_2$-directions shows large values, 0.80 and 0.73 km s$^{-1}$, corresponding to 24.8 and 19.8 per cent, respectively. The orthorhombic assumption of axial S-wave velocities holds within 1 per cent except for $V_{S12}$ and $V_{S21}$, which show 3.5 per cent difference. We assume that ST-3 is a typical TI$^{(3)}$ anisotropy with an orthorhombic perturbation. We will show later that this assumption is reasonably well supported by LPO data and velocity contour maps based on LPO data.

5.2 Anisotropy parameters

If we assume orthorhombic symmetry, the S-wave velocities in axial directions are determined by the diagonal elastic constants $C_{44}$, $C_{55}$ and $C_{66}$. Therefore, three pairs of S-wave velocities in the axial directions should be equal: $V_{S13}=V_{S31}$, $V_{S23}=V_{S32}$ and $V_{S12}=V_{S21}$. In fact, there are some differences in these pairs. For calculating $\gamma^{(i)}$, we use average values of the above velocity pairs in the axial directions. $\varepsilon^{(i)}$ and $\delta^{(i)}$ ($i=1, 2$) are calculated from eqs (9)-(12). $\gamma^{(1)}$ and $\gamma^{(3)}$ are calculated by assuming $x_3$ as a symmetry axis. When the TI approximation is reasonable, one of the $\varepsilon^{(i)}$ values is close to zero; for example, $\varepsilon^{(1)}$ or $\varepsilon^{(3)}$ is expected to be very small, corresponding to TI$^{(1)}$ or TI$^{(3)}$, respectively. The values of $\gamma^{(i)}$ are shown in two directions, (1) and (2), and $\gamma^{(3)}$ is calculated when the anisotropy is close to TI$^{(1)}$.

Anisotropy parameters are shown in Table 4 and Fig. 7. In Fig. 7, the splitting pattern of $\varepsilon^{(i)}$ indicates the anisotropy type. If one $\varepsilon^{(i)}$ is close to zero and splits from the other two $\varepsilon^{(i)}$, which are close to each other, the symmetry is close to TI$^{(1)}$. If every $\varepsilon^{(i)}$ separated, the symmetry would be close to orthorhombic. $\varepsilon^{(1)}$ is close to zero in SB-1 and SB-7 and splits from $\varepsilon^{(2)}$ and $\varepsilon^{(3)}$, suggesting that SB-1 and SB-7 have TI$^{(1)}$-type anisotropy. On the other hand, SB-5 and ST-3 are close to TI$^{(3)}$.

Table 4. Anisotropic parameters of the rock sample.

| Sample | $\varepsilon^{(1)}$ | $\varepsilon^{(2)}$ | $\varepsilon^{(3)}$ | $\delta^{(1)}$ | $\delta^{(2)}$ | $\delta^{(3)}$ | $\gamma^{(1)}$ | $\gamma^{(2)}$ | $\gamma^{(3)}$ |
|--------|------------------|------------------|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| SB-1   | 0.030            | 0.144            | 0.117            | 0.146          | 0.195          | 0.004          | 0.004          | 0.043          | 0.052          |
| SB-5   | 0.090            | 0.123            | 0.030            | 0.062          | 0.043          | 0.062          | 0.062          |                |                |
| SB-7   | 0.019            | 0.081            | 0.064            | 0.063          | 0.089          | 0.063          | 0.089          | 0.022          |                |
| ST-3   | 0.119            | 0.189            | 0.059            | 0.013          | 0.106          | 0.283          | 0.283          |                |                |

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The anisotropy parameter $\delta^{(1)}$ was measured for SB-1 and ST-3, which show strong anisotropy. We measured $\delta^{(2)}$ and $\delta^{(3)}$ for SB-1, and $\delta^{(1)}$ and $\delta^{(2)}$ for ST-3. In Fig. 7, $\delta^{(0)}$ is shown by an open symbol. The distance between $\delta^{(0)}$ and $\delta^{(0)}$ (shown by open and closed symbols, respectively) indicates non-ellipticity of the $P$-wave phase velocity surface because this value controls the magnitude of the $4\theta$ term of sine or cosine functions in eqs (4) and (16).

In SB-1, $\delta^{(0)}$ is very close to $\delta^{(1)}$ but $\delta^{(2)}$ differs from $\delta^{(2)}$, suggesting an elliptic anisotropy in the $x_1x_3$-plane and slightly non-elliptic anisotropy in the $x_1x_2$-plane. The difference between $\delta^{(2)}$ and $\delta^{(2)}$ in ST-3 is quite large—0.295. This indicates a considerable deviation from ellipticity in the phase velocity surface of the $P$ wave in the $x_1x_3$-plane. The large difference between $\delta^{(0)}$ and $\delta^{(0)}$ contributes to the bulge of the $SV$ wave, and the $S$-wave singularity may appear. The shape of the $P$-wave phase velocity surface is discussed below.

Fig. 8 shows the $\lambda_1$ and $\lambda_3$ preferred orientations of hornblende in amphibolites SB-1 and SB-5. Lower-hemisphere, equal-area projections using Neil Mancktelow's StereoPlot 2.0. Contoured at multiples of uniform distribution. Max: maximum densities normalized by uniform distribution; $\lambda_1$, $\lambda_2$, $\lambda_3$: eigenvectors of orientation tensor. Number of measurements is 50 for each projection.

$\delta^{(2)}$ and $\delta^{(3)}$ are shown in Fig. 7(b). $\delta^{(2)}$ and $\delta^{(3)}$ are quite large in ST-3, showing strong $S$-wave anisotropy in the $x_1x_3$- and $x_2x_3$-planes. The $S$-wave splitting in the mirror symmetry plane of orthorhombic symmetry is given by eqs (19) and (20). The phase velocity and splitting of the $S$ wave are discussed below.

5.3 Lattice preferred orientation (LPO)

We consider that the origin of anisotropy is the preferred orientation of hornblende in amphibolite (SB-1 and SB-5) and biotite in gneiss and schist (SB-7 and ST-3). We measured the LPO of hornblende and biotite using a microscope equipped with a universal stage. Other major rock-forming minerals—plagioclase and quartz—were not measured because they did not have such a strong LPO.

The $b$- and $c$-axis orientations of hornblende in two amphibolite rocks (SB-1 and SB-5) and the $c$-axis orientations of biotite in gneiss (SB-7) and schist (ST-3) were measured. Fig. 8 shows the $b$- and $c$-axis pole figures of hornblende in SB-1 and SB-5 and Fig. 9 shows the $c$-axis pole figure of biotite in SB-1 and ST-3.

The $c$-axes of hornblende tend to align parallel to the lineation (the $x_1$-direction) as observed in SB-1, or show a two-maxima distribution with a subsidiary maximum in the $x_3$-direction as observed in SB-5. The $b$-axes of hornblende in SB-1 form a girdle distribution around the $x_1$-axis, while in SB-5 they show a concentration in the $x_2$-direction. A section of SB-1 in the $x_1x_3$-plane (Fig. 3a) shows that hornblende grains are elongated parallel to their $c$-axes and are aligned parallel to the $x_1$ direction, whereas their shapes are rather round in the section perpendicular to the $x_1$-direction ($x_2x_3$-plane, Fig. 3b).

The $c$-axes of biotite in SB-7 form a two-maxima to girdle pattern in the $x_2x_3$-plane, with a maximum concentration in the $x_3$-direction. In ST-3, the $c$-axes are more strongly concentrated in the $x_1$-direction, and form a weak girdle in the $x_2x_3$-plane. In the $x_1x_3$-section (Fig. 4), biotite grains are thin and strongly elongated in the $x_1$-direction. However, in the $x_2x_3$-section, biotite crystals are rather thick compared with those in the $x_1x_3$-section, and are slightly scattered in orientation, affected by gentle microfolding. The $c$-axis distribution of biotite produces a weak girdle in the $x_2x_3$-plane due to this microfolding.

To characterize the LPO, we calculated the polar density eigenvalues $\lambda_1$, $\lambda_2$ and $\lambda_3$. From those eigenvalues we calculated the following parameters for describing the preferred orientation pattern (Woodcock 1977; Gapais & Brun 1981):

$$K_f = \frac{\ln(\lambda_3/\lambda_1)}{\ln(\lambda_1/\lambda_2)}$$

(22)

$$\eta_f = \frac{1}{\sqrt{3}} \left[ \left( \ln(\lambda_2/\lambda_1) \right)^2 + \left( \ln(\lambda_2/\lambda_3) \right)^2 + \left( \ln(\lambda_1/\lambda_3) \right)^2 \right]^{1/2}.$$

(23)

$K_f$ indicates the distribution pattern of the crystal axis: a uniaxial cluster when $K_f=0$, a transition from cluster to girdle when $K_f=1$, and a uniaxial girdle when $K_f \to \infty$. $\eta_f$ indicates the randomness of axis orientation. $\eta_f=0$ corresponds to a random orientation and the increase of $\eta_f$ shows a stronger preferred orientation of the measured crystal axis. Table 3 shows the measurements of elastic anisotropy parameters 41

(a) SB-1: Hb $b$-axis  
(b) SB-1: Hb $c$-axis

(c) SB-5: Hb $b$-axis  
(d) SB-5: Hb $c$-axis

Figure 8. $b$- and $c$-axis preferred orientations of hornblende in amphibolites SB-1 and SB-5. Lower-hemisphere, equal-area projections using Neil Mancktelow’s StereoPlot 2.0. Contoured at multiples of uniform distribution. Max: maximum densities normalized by uniform distribution; $\lambda_1$, $\lambda_2$, $\lambda_3$: eigenvectors of orientation tensor. Number of measurements is 50 for each projection.

(a) SB-7  
(b) ST-3

Figure 9. $c$-axis preferred orientation of biotite in (a) biotite gneiss SB-7 and (b) biotite schist ST-3. Lower-hemisphere, equal-area projections using Neil Mancktelow’s StereoPlot 2.0. Contoured at multiples of uniform distribution. Max: maximum densities normalized by uniform distribution; $\lambda_1$, $\lambda_2$, $\lambda_3$: eigenvectors of orientation tensor. Number of measurements is 100 for both projections.
Fig. 10. P-wave phase velocity in the $x_1x_2$-plane of SB-1. The thin curve is an elliptical phase velocity surface calculated using $\delta^{(1)}=\delta^{(2)}$.

Fig. 14(a) shows S-wave phase velocity surfaces in the $x_1x_3$-plane of ST-3. The $ST$-wave phase velocity surface deviates from a circular shape and crosses the $SH$-wave phase velocity surface at a polar angle of about 60°. Crossing of the $SV$ and $SH$ phase velocity surface is called an $S$-wave singularity (Crampin & Yedlin 1981). Velocity measurements at a polar angle of 45° show that measured velocities agree well with the expected velocities obtained from the anisotropy parameters. The deviation from a circular shape of the $SV$-wave phase velocity surface results from the large difference between $\delta^{(1)}$ and $\delta^{(2)}$ (Banik 1987) and the large value of $V_p(0)/V_p(0)$, the $P$- and $S$-wave velocity ratio along the $x_3$-axis. $\delta^{(2)}$ in eq. (21)

6 DISCUSSION

6.1 Phase velocity surface

The purpose of the present study is to obtain anisotropy parameters and then calculate velocities in any direction using the anisotropy parameters. $P$- and $S$-wave velocities in the symmetry planes are shown for SB-1 and ST-3 (Figs 10–14).

The $P$-wave phase velocity surface in the $x_1x_2$-plane of SB-1 is shown in Fig. 10. The velocity surface is slightly non-elliptic because of a small difference between $\delta^{(2)}$ and $\delta^{(3)}$. The $P$-wave phase velocity surface in the $x_1x_3$-plane of ST-3 is shown in Fig. 11. The velocity surface calculated by $\delta^{(2)}$ and $\delta^{(3)}$ deviates considerably (0.4 km s$^{-1}$, ca. 7 per cent) from the elliptic wave surface ($\delta^{(2)}=\delta^{(3)}$). We measured the velocity in the $x_1x_3$-plane for every 15 degrees of $\theta$. These values are shown together with the calculated velocity. The measured velocity values are close to the calculated velocity values, suggesting that the measured anisotropic parameters give consistent results. The phase velocity surface of the $P$-wave in the $x_1x_3$-plane of ST-3 is shown in Fig. 12. Deviation from the elliptic wave surface is not large because the difference between $\delta^{(1)}$ and $\delta^{(1)}$ is not as large as that between $\delta^{(2)}$ and $\delta^{(2)}$.

The $S$ wave in the $x_1x_2$-plane of ST-3 splits into two velocities depending on their polarization directions: an $SV$ wave polarized in the $x_1x_3$-plane and an $SH$ wave polarized parallel to the $x_1x_3$-plane. Fig. 13 shows the phase velocity surfaces of the $SH$ and $SV$ waves in the $x_1x_3$-plane of SB-1. Deviation from a circular shape in the $SV$ wave is small in SB-1, and the $SH$ and $SV$ phase velocity surfaces never intersect. Hence, the fast $S$ wave never changes its polarization direction.

Table 5. Eigenvalues of the crystal lattice preferred orientation intensity ($\lambda_1$, $\lambda_2$ and $\lambda_3$). $K_P$ and $\epsilon_P$ are given by eqs (22) and (23). $K_P$ is the anisotropic intensity.

| Sample | Axis | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $K_P$ | $\epsilon_P$ |
|--------|------|-------------|-------------|------------|-------|-------------|
| SB-1   | $c$  | 0.76        | 0.13        | 0.11       | 3.97  | 15.8        |
|        | $b$  | 0.42        | 0.40        | 0.18       | 16.37 | 0.78        |
| SB-5   | $c$  | 0.65        | 0.24        | 0.11       | 1.26  | 11.6        |
|        | $b$  | 0.67        | 0.18        | 0.15       | 0.15  | 11.5        |
| SB-7   | $c$  | 0.66        | 0.28        | 0.07       | 1.65  | 8.5         |
|        | $b$  | 0.87        | 0.10        | 0.03       | 0.66  | 17.2        |
| ST-3   | $c$  | 0.87        | 0.10        | 0.03       | 0.66  | 2.53        |

The $\lambda_1$, $\lambda_2$ and $\lambda_3$ and the parameters $K_P$ and $\epsilon_P$ together with the $P$-wave anisotropy value $K_P$, which is determined by $(V_{P\text{max}}-V_{P\text{min}})/V_P$, where $V_{P\text{max}}$, $V_{P\text{min}}$ and $V_P$ are the maximum, minimum and average $P$-wave velocities along the three axes.

The $K_P$ and $\epsilon_P$ values of SB-1 indicate uniaxially clustered $c$-axes and girdle-forming $b$-axes of hornblende. $K_P$ and $\epsilon_P$ of SB-5 indicate that the hornblende $c$-axis shows a relatively weak preferred orientation of cluster–girdle transition, while the hornblende $b$-axis forms a strong uniaxial cluster. $K_P$ and $\epsilon_P$ of SB-7 show that the $c$-axis of biotite forms a girdle along the $x_2x_3$-plane. $K_P$ and $\epsilon_P$ of ST-3 indicate a cluster distribution of the biotite $c$-axis around the $x_3$-axis of the rock, but the $K_P$ value of 0.66 also suggests a weak girdle in the $x_2x_3$-plane.

The eigenvalues of $\lambda_1$, $\lambda_2$ and $\lambda_3$ and the parameters $K_P$ and $\epsilon_P$ are given by eqs (22) and (23). The $P$-wave anisotropy value $K_P$, which is determined by $(V_{P\text{max}}-V_{P\text{min}})/V_P$, is shown in Fig. 10. The velocity surface is slightly non-elliptic because of a small difference between $\delta^{(2)}$ and $\delta^{(3)}$. The $P$-wave phase velocity surface in the $x_1x_3$-plane of ST-3 is shown in Fig. 11. The velocity surface calculated by $\delta^{(2)}$ and $\delta^{(3)}$ deviates considerably (0.4 km s$^{-1}$, ca. 7 per cent) from the elliptic wave surface ($\delta^{(2)}=\delta^{(3)}$). We measured the velocity in the $x_1x_3$-plane for every 15 degrees of $\theta$. These values are shown together with the calculated velocity. The measured velocity values are close to the calculated velocity values, suggesting that the measured anisotropic parameters give consistent results. The phase velocity surface of the $P$-wave in the $x_1x_3$-plane of ST-3 is shown in Fig. 12. Deviation from the elliptic wave surface is not large because the difference between $\delta^{(1)}$ and $\delta^{(1)}$ is not as large as that between $\delta^{(2)}$ and $\delta^{(2)}$.

The $S$ wave in the $x_1x_3$-plane of ST-3 splits into two velocities depending on their polarization directions: an $SV$ wave polarized in the $x_1x_3$-plane and an $SH$ wave polarized parallel to the $x_1x_3$-plane. Fig. 13 shows the phase velocity surfaces of the $SH$ and $SV$ waves in the $x_1x_3$-plane of SB-1. Deviation from a circular shape in the $SV$ wave is small in SB-1, and the $SH$ and $SV$ phase velocity surfaces never intersect. Hence, the fast $S$ wave never changes its polarization direction.
becomes 1.00, which gives a 25 per cent excess bulge from a circular shape at the polar angle $p/4$. Fig. 14(b) shows the $S$ wave in the $x_2x_3$-plane of ST-3. The deviation from a circular shape in the $SV$ wave is not as large as that in the $x_1x_3$-plane, and $S$-wave splitting is small at small polar angles, from zero to about 45°. Large $S$-wave splitting is expected only for rays close to the horizontal direction.

We see that the $S$-wave singularity is caused by a large bulge of the $SV$ wave. Table 6 shows the parameters that control the $SV$-wave bulge: the difference between $e^{(i)}$ and $d^{(i)}$, the axial $VP$ to $V_{SV}$ ratio, and $s^{(i)}$. $s^{(i)}$ is calculated from the previous two parameters using eq. (6). We can thus predict the interesting behaviour of the $S$ wave from anisotropy parameters.

| Sample | Plane     | $\delta^{(i)} - \delta^{(0)}$ | $V_{P}/V_{SV}$ | $\sigma^{(i)}$ |
|--------|-----------|-------------------------------|----------------|---------------|
| SB-1   | $x_1x_3$  | 0.002                         | 1.90           | 0.01          |
|        | $x_1x_2$  | 0.078                         | 1.91           | 0.28          |
| ST-3   | $x_2x_3$  | 0.132                         | 1.84           | 0.45          |
|        | $x_1x_3$  | 0.295                         | 1.84           | 1.00          |

Figure 12. $P$-wave phase velocity in the $x_1x_2$-plane of ST-3. The thin curve is an elliptical velocity calculated using $\delta = \epsilon$.

Figure 13. $S$-wave phase velocity in the $x_1x_2$-plane of SB-1.

Figure 14. $S$-wave phase velocity in ST-3 (a) in the $x_1x_3$-plane and (b) in the $x_2x_3$-plane.

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6.2 Comparison of measured and calculated velocities based on LPO data

We found a close correlation between the velocity anisotropy and the LPO patterns of hornblende and biotite, which are characterized by $K_r$ and $\varepsilon_0$. We can obtain overall elastic constants of the rock as the Voigt average of single-crystal elastic constants of constituent minerals with crystal orientations given by measured LPO data (Mainprice 1990; Mainprice & Humbert 1994). The Voigt average generally has the lowest symmetry for 21 elastic constants, and the velocities are calculated by solving Christoffel’s equation. Amphibolites (SB-1 and SB-5) are assumed to be mixtures of plagioclase (An53) and hornblende, where plagioclase has a random orientation but hornblende has the measured LPO. Gneiss (SB-7) and schist (ST-3) are assumed to be mixtures of randomly oriented quartz and plagioclase (An24) crystals and aligned biotite crystals with the measured LPO. We used published elastic constants compiled by Simmons & Wang (1997); the original data are from Ryzhova (1964) for plagioclase, Aleksandrov & Ryzhova (1961a) for hornblende, Hearmon (1956) for quartz and Aleksandrov & Ryzhova (1961b) for biotite. The recent elastic constants of muscovite reported by Vaughan & Guggenheim (1986) were not used, because there is a significant difference in $C_{44}$ between muscovite and biotite (or phlogopite). We think this difference should not be ignored, because $C_{44}$ strongly affects elastic anisotropy of biotite-rich rocks (Nishizawa & Yoshino 2001).

The contour maps of Fig. 15 show distributions of the $P$ velocity and the fast ($V_{S1}$) and slow ($V_{S2}$) velocities of $S$ waves calculated from the Voigt average. The notations $SH$ and $SV$ are not used because the polarization directions of the two splitting waves do not exactly correspond to the $SH$- and $SV$-wave polarization directions of $TI$ or orthorhombic anisotropy. The measured $P$ and $S$ velocities are also shown in rectangular boxes close to each axis. The calculated velocities are all faster than the measured velocities because the Voigt average always gives the upper limit of elastic constants. The distribution patterns of the $P$- and polarized $S$-wave velocities mostly agree with the measured axial velocities; for example, the maximum and minimum values of measured axial velocities appear in the highest and lowest zones of the velocity distribution.

Hornblende is a monoclinic mineral and the velocities along the three crystallographic $a$-, $b$- and $c$-axes are 6.10, 7.16 and 7.85 km s$^{-1}$, respectively. The calculated $P$-velocity anisotropy of SB-1 is close to $TI^{(1)}$ due to the strong concentration of the hornblende’s $c$-axes in the $x_1$-direction and the girdle pattern of the $b$-axes in the $x_2$-$x_3$-plane. The calculated $S$-velocity anisotropy shows a weak $TI^{(0)}$, but a significant discrepancy appears between measured velocity and calculated velocity: the measured maximum axial velocity appears in the minimum zone of the $V_{S2}$ distribution. The calculated $P$- and $S$-velocity distributions of SB-5 show $TI^{(0)}$ with slightly orthorhombic anisotropy due to the high-velocity zone close to the $x_1$-direction, and they agree well with the measured anisotropy except for the absolute values. The calculated $S_2$ velocity shows a slight bulge in both the $x_1$-$x_2$- and the $x_1$-$x_3$-planes.

Most of the mica-group minerals are monoclinic, so the $c$-axis is not perpendicular to the banded plane of the crystal. However, elastic constants of biotite can be approximated as hexagonal symmetry by choosing the symmetry axis perpendicular to the sheet (Aleksandrov & Ryzhova 1961b). Under the approximation of hexagonal symmetry, the two pairs of diagonal elastic constants satisfy the relations $C_{11} \cong C_{22}$ and $C_{44} \cong C_{55}$. The off-diagonal elastic constants $C_{12}$, $C_{23}$, $C_{35}$ and $C_{46}$ are close to zero (Huntington 1958; Aleksandrov & Ryzhova 1961b; Vaughan & Guggenheim 1986). The hexagonal assumption makes the biotite anisotropy simple.

Nishizawa & Yoshino (2001) calculated the velocity anisotropy of biotite-rich rocks by applying an inclusion model, where mica crystals are embedded in an isotropic matrix as inclusions. They revealed that the $P$-wave phase velocity surface deviates considerably from ellipsoidal and the $SV$-wave phase velocity surface shows a large bulge when the volume ratio of biotite increases. An $S$-wave singularity appears in the plane including the symmetry axis. The $S$-wave anisotropy becomes large when the biotite crystals become thin.

The measured axial $P$ velocities of SB-7 indicate anisotropy close to $TI^{(0)}$. However, the velocity contours of the $P$ wave extend along the $x_1$-$x_2$-plane, which suggests that the anisotropy is basically $TI^{(0)}$. The calculated $S$-velocity contour and the measured $S$ velocities also suggest a $TI^{(3)}$-type anisotropy. The $P$-wave velocity difference between the $x_1$- and $x_2$-directions is probably a result of a weak girdle-type distribution of the $c$-axis of biotite in that plane. If biotite crystals form a girdle in the $x_2$-$x_3$-plane, the symmetry axis of $TI$ shifts from $x_3$ to $x_1$ and the rock anisotropy changes from $TI^{(0)}$ to $TI^{(3)}$.

The calculated $P$- and $S$-velocity contours of ST-3 indicate that the anisotropy is close to $TI^{(1)}$. The bulge of the slow $S$ wave, $V_{S2}$, agrees with the anisotropy calculated from Tsvankin’s anisotropic parameter, which indicates a large bulge of $SV$-wave and singularity of the $S$ wave in the $x_1$-$x_3$-plane. The apparent orthorhombic anisotropy will be produced by weak extension of the biotite $c$-axis maximum in the $x_1$-direction towards the $x_2$-direction due to microfolding (crenulation) (Passchier & Trouw 1996). Crystal shape may also affect the anisotropy (Nishizawa & Yoshino 2001). Both elongated biotite crystals in the $x_1$-$x_3$-plane and weak crenulation in the $x_2$-$x_3$-planes can be seen in Fig. 4.

Fig. 16 shows calculated polarization vectors of the fast and slow $S$ waves on a lower-hemisphere equal-area projection for SB-1 and ST-3. Solid and dashed lines correspond to the projected polarization vectors of the fast and slow $S$ waves, respectively. The length of a vector indicates the plunge of the polarization vector from the projection plane. In SB-1 the directions of the fast $S$-wave polarization are almost in the plane including the $x_1$-axis, and they are continuous on the whole hemisphere, showing no $S$-wave singularity. However, ST-3 shows discontinuities of the polarization vectors. In the area close to the $x_1$-$x_3$-plane, the polarization vectors of the fast $S$ wave are directed almost parallel to the $x_1$-$x_2$-plane. However, around the $x_3$-axis the fast $S$ wave changes its polarization direction. In the $x_1$-$x_3$-plane, the fast $S$ wave changes its polarization direction at around $30^\circ$ from the $x_3$-axis; from the $x_2$- to the $x_3$-axis, the fast $S$ wave first polarizes in the $x_1$-$x_2$-plane and then it polarizes in the $x_1$-$x_3$-plane. The anisotropy pattern agrees well with the estimated phase velocity surface of the $S$ wave calculated from the measured anisotropic parameter.

6.3 Non-ellipticity of the $P$-wave phase velocity surface and the $S$-wave singularity: meaning in seismic exploration

Many authors have studied the effects of anisotropy on seismic reflection: Thomsen (1988) and Lynn & Thomsen (1990) for $TI$ media, Rüger (1998) for the boundary between isotropic and
orthorhombic media, and Vavryčuk & Pšencík (1998) for P–P reflection at the boundary of two anisotropic media. The effect of non-elliptic velocity surfaces on the moveout correction has been discussed (Banik 1987; Thomsen 1986; Tsvankin 1995) and extended to the orthorhombic case (Grechka & Tsvankin 1999). Non-elliptic P-wave velocity surfaces play an important role in moveout correction and pre- and post-stack migrations (Grechka & Tsvankin 1999).

According to our study, the two anisotropic parameters \(\delta^0\) and \(\delta^0\) are almost equal in amphibolite, whereas the difference between \(\delta^0\) and \(\delta^0\) may become significant in biotite-rich rocks (Table 6). Single-crystal biotite has unique elastic properties: the P- to S-velocity ratio is very large (= 3.14) in the c-axis direction and there is strong anisotropy in P and S waves. Nishizawa & Yoshino (2001) showed that non-ellipticity and S-wave singularities are enhanced in biotite-rich rocks compared to cracked rocks (Douma 1988). Cracks affect mainly P-velocity anisotropy whereas biotite minerals affect mainly S-velocity anisotropy. The S-wave splitting in biotite-rich rocks is quite large compared to the rock containing thin cracks (Douma 1988). When the rock formation contains much biotite, formation analysis based on S waves will show different

\[ \begin{align*}
\text{Figure 15. } & P- \text{ and two } S- \text{wave velocities, fast and slow } S- \text{wave velocities } V_{S1} \text{ and } V_{S2}, \text{ in an arbitrary direction. The velocities are calculated from the Voigt average of constituent minerals.}
\end{align*} \]
7 CONCLUSIONS

The phase velocity surface calculated from measured anisotropic parameters ($\epsilon^0$, $\delta^0$, and $\gamma^0$) in metamorphic rocks can describe the rock anisotropy quite well. Since velocity anisotropy in rocks is basically simple, orthorhombic anisotropy can describe the rock anisotropy quite well. Since velocity anisotropy in rocks is basically simple, orthorhombic anisotropy can describe the rock anisotropy quite well. Since velocity anisotropy in rocks is basically simple, orthorhombic anisotropy can describe the rock anisotropy quite well.

The axial anisotropy of ST-3 is not surprisingly large compared with the typical velocity anisotropy of schist compiled by Douma & Crampin (1990). The large bulge of the SV wave in biotite also affects group velocity. The group velocity surface of the SV wave forms a cusp and produces complex waves. Since most seismic exploration uses point sources and obtains group velocities rather than phase velocities, the S-wave singularity will add much complication to seismic exploration using S waves. Strong anisotropy produces strong velocity contrasts and causes strong scattering of seismic waves. The strong anisotropy observed in ST-3 can be a strong scattering source when this rock is embedded underground because of the large velocity contrasts with surrounding rocks that will be produced. If the velocity fluctuation is randomly distributed in space, scattered waves become random and may mask the seismic signal that is essential in determining underground structure. Therefore, the analysis of seismic waves may not be simple.

In order to study crustal anisotropy, laboratory measurements of elastic anisotropy in crustal rocks are of basic importance. Wave propagation in actual crustal material can be studied by measuring the anisotropy parameters of rocks.

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