The application of a shape synthesis process for electrically-small three-dimensional (3D) conducting surface antennas is described. A script (shaping controller) links a computational electromagnetics engine and genetic optimisation algorithm, and implements a prescribed process that shapes the antenna from any specified starting shape until a self-resonant structure is produced whose performance satisfies requirements incorporated into an objective function defined in the shaping manager. Examples of successfully shape synthesized 3D antennas are discussed.

**Introduction:** There is a need for electrically-small antennas that can fit on/in a restricted physical surface/volume for use in sensors (e.g. Internet-of-Things) needing to talk to some data centre via the wireless network [1]. The design difficulties of electrically-small antennas are well-known [2, Chap. 59]. It would be advantageous to do so in a way that exploits as much of the space allotted to them (e.g. the surface of some box containing the sensor electronics), and the proximity effects of other objects present, to arrive at functioning antennas. This is possible using antenna shape synthesis [3–6] that, in contrast to traditional design approaches, uses computational electromagnetic (CEM) modelling and optimisation algorithms in a way that the shape of the radiating structure itself, and not merely a sized version of a pre-selected shape, is an outcome of the synthesis process. In what follows we assume the reader is familiar with the CEM technique known as the method of moments (MM) [2, Chap. 59], genetic algorithm (GA) optimisation [3, Chap. 9], and characteristic mode (CM) concepts [7].

Seminal work on the shape synthesis of conducting surface antennas is found in [3]. The conductor geometry undergoes shaping by removal or retention of pixels into which some starting conductor shape is divided. These pixels may (as in the present paper) be those of the MM mesh of the CEM model of the structure, but can be defined independent of it. Key to the MM, at least in the present case where the electric current density \( J \) is the only unknown quantity, is the construction of the impedance matrix \( Z \). Use of the direct matrix method (DMM) strategy [3, Chap. 9] reduces the computational burden, because the appropriate \( Z \) need not be recomputed each time during shaping. A genetic optimisation algorithm (GA) [3, Chap. 1] is used in [3, Chap. 9] and the present paper. Each different shape is described by a chromosome, with each bit in the latter representing a pixel, and the removal or retention of a pixel achieved by assigning it the value 0 or 1. The MM is used to find \( J \) on all conducting surfaces present, and from this to evaluate an objective function \( F_{obj} \) defined so that its minimisation produces a shaped antenna of some desired performance. Use of the CM concept allows one, for electrically small antennas where a single CM dominates (“single-mode antennas”), to perform such antenna shape synthesis in a way that the antenna feedpoint need only be selected [4] once the shape synthesis is complete, further unfettering the design process. This shape-first/feed-next approach [4–6] is followed here. Until now such shape synthesis has only been reported for planar (2D) conducting surface (as opposed to wire) antennas. Here we describe the application of a CEM based tool able to do this for three-dimensional (3D) cases. The goal is not to “break performance records” of antennas designed by traditional approaches.

It is rather to extend, to a wider range of problems, those methods that exploit as much of the space allotted to them (e.g. the surface of some box containing the sensor electronics), and the proximity effects of other objects present, to arrive at functioning antennas. This is possible using antenna shape synthesis [3–6] that, in contrast to traditional design approaches, uses computational electromagnetic (CEM) modelling and optimisation algorithms in a way that the shape of the radiating structure itself, and not merely a sized version of a pre-selected shape, is an outcome of the synthesis process. In what follows we assume the reader is familiar with the CEM technique known as the method of moments (MM) [2, Chap. 59], genetic algorithm (GA) optimisation [3, Chap. 9], and characteristic mode (CM) concepts [7].

Customisation of the shaping prescription: The choice of parameters (selection method, crossover, mutation and migration [9]) for successful operation of a GA depends on the particular problem. What we next state applies to the 3D electrically-small conducting surface antenna shaping problem at hand. When the GA is used with the default [9] settings of the above parameters the shaping process either stagnates, or too much conductor is removed too soon in the initial iterations and the GA is unable to put back conducting pixels in an acceptable number of iterations when it ‘realises its mistake’. Heuristic adjustment of these parameters does not consistently improve matters. Thus a set of shaping experiments was carried out using a particular case (to be shown as the first example) as a vehicle. The outcome was a customised approach (described below) that proved effective in all subsequent shape synthesis examples. Such details are often not provided in publications, but are essential if the reader wishes to repeat what has been described.

![Flowchart](image)

**Fig. 1 Flowchart1—The shaping manager/controller**

We have found it necessary to divide each shaping exercise into two parts, a trial run and only then the actual run, as summarized in the flowcharts Figures 2 and 3, respectively. The trial run effectively provides a physically feasible “warm start” for the actual shaping run, with the latter eventually producing the shaped antenna. It is advantageous if both forward and backward migration is allowed. If one has fixed regions on the starting geometry (that may not be shaped, as in the first example) the GA takes longer to reduce \( F_{obj} \) because one is interfering in the route the algorithm wishes to follow. In such cases stagnation can occur unless the crossover values and mutation rate are adjusted slightly at different stages of the shaping process. This is one aspect falling within the user supervision block in Figure 3. In all the shape synthesis examples completed we have found that mutation rates between 0.3% and 0.5% are satisfactory, and the best crossover values between 0.5 and 0.8. The reason is that crossover values that are too low result in too many shapes ‘proposed’ by the GA having far too little conductor, known for physical reasons to not give us what we need. In all the applications shown uniform crossover and tournament selection was employed satisfactorily. We have observed (for 3D single-mode antenna shaping problems) that it is satisfactory to keep the population size at 20 for all trial runs; although its value was experimented with in some actual runs this was not really necessary.
Fig. 2 Flowchart#2—The trial run of the shape synthesis process

Fig. 3 Flowchart#3—The actual run of the shape synthesis process. The bit-sum (Σ) is the ordinary sum of all the bits in a single chromosome

Shape synthesis examples: Symbol \( a \) will here always represent the radius of a sphere that just encloses the starting shape (and hence the shaped antenna). The MM mesh is always sufficiently fine that both accuracy of the CEM model and good geometrical resolution is assured at the operating frequency. The starting shape of the first example is a cuboid with sides of length \( d \), and the lower face removed. There is no ground plane. At the centre-frequency (1 GHz) the dimension \( d = 60 \text{mm} = 0.2 \lambda \), and since \( a = \sqrt{3}d/2 \) we then have \( ka = 1.09 \), where \( k \) is the free space wavenumber. As an indication of the number of degrees of freedom we note that the starting shape was meshed into 860 triangular pixels with which are associated 1023 expansion functions [2, Chap. 59]. The starting shape is open at the bottom to emphasize that it can in fact be any shape whatsoever; the implication in this case is that such a starting shape would ensure the synthesized antenna can be assembled by pushing it over a package. Furthermore, we envisaged that the intention is to fabricate the antenna from a flat sheet of copper and then fold it into a structurally rigid form. We therefore instructed the shaping manager to disallow the removal of those pixels on either side of all cuboid edges during the shaping process. The objective function used was simply

\[
F_{\text{obj}}(f_0) = |\lambda_1(f_0)|,
\]

namely the magnitude of the CM with the lowest eigenvalue at centre-frequency \( f_0 \). The reasoning is that, as long as the structure is indeed a single-mode one, a low \( |\lambda_1(f_0)| \) value means that the shaped structure is resonant in a CM-sense, and it will then be possible to find a feed point with respect to which, as an antenna, it will be self-resonant [4, 5].

The shaped antenna is shown on the left in Figure 4, depicting the feeding system used. The fabricated antenna (on the right in Figure 4) was constructed by downloading its computer-aided-drawing (CAD) file from the CEM engine, using it to create a flattened version, and laser cutting the flat shape from copper sheeting using the resulting CAD file, followed by folding and soldering the antenna. Figure 5 shows the progress of the shaping process; the final shape is reached after 250 iterations. Table 1 confirms the shaped structure is indeed a single-mode one; only the lowest order CM has a significant excitation value when the antenna is fed at the location shown. Figure 6 compares the computed (driven model) and measured input reflection coefficients of the shaped antenna. The discrepancy is due to fabrication imperfections; in actual sensor packages the internal electronics would connect directly to the feedpoint in a manner closer to the actual CEM feed model used. The far-zone pattern prediction capability of CEM models is good, and so patterns need not be shown. As an exception, Figure 7 purposefully shows the computed and measured patterns of the antenna in the \( xy \)-plane; the disagreement in the vicinity of \( \phi = 180^\circ \) is due to the location of the coaxial cable emerging from the test chamber pedestal holding the antenna. The remaining examples intend to demonstrate other factors that can be taken into account in the shape synthesis process, and how insight into what the shape synthesis is doing can be gleaned. Only computed results that facilitate this need be shown for these examples.

Printing can be done over 3D surfaces, but cost favours its use on a flat surface that is then folded. In such cases the desire is to simply fold and bond the printed shape onto some package without any need for conductive bonding at the edges. Thus in the second example (with centre frequency \( f_0 = 2 \text{GHz} \) specified) the starting shape is the cuboid-like structure shown on the left in Figure 8. The corners are
of the far-zone fields mode’ antenna, it is possible to show the MED can be written in terms of mean effective directivity (MED) concept [2, Chap. 58]. For a ‘single-environment in which link budgets are determined using the antenna location.

The starting shape of the second example (left), and the shape synthesized antenna (right). At centre-frequency 2 GHz the dimensions are \( d = 0.1 \lambda, s = 0.02 \lambda, \) and \( w = h = 0.06 \lambda, \) so that \( \theta_{a} = 0.4826 \)

purposefully missing so that the final synthesized shape satisfies the assembly requirements just noted. The resulting self-resonant shaped antenna is that shown on the right in Figure 8, along with its feedpoint location.

Sensors, like mobile devices, communicate in a wireless propagation environment in which link budgets are determined using the antenna mean effective directivity (MED) concept [2, Chap. 58]. For a ‘single-mode’ antenna, it is possible to show the MED can be written in terms of the far-zone fields \( E_\text{z} (\theta, \phi) \) and \( E_\text{z} (\theta, \phi) \) of this single CM as:

\[
\text{MED} = \frac{2\pi}{\sqrt{\mu_0/\varepsilon_0}} \int_{-\pi}^{\pi} \int_{0}^{\pi} \frac{\left| XPR \left[ E_\text{z} (\theta, \phi) \right] \right|^2 P_\theta (\theta, \phi)}{\left| P_\theta (\theta, \phi) \right|} \sin \theta \, d\theta d\phi
\]

\( P_\theta (\theta, \phi) \) and \( P_\theta (\theta, \phi) \) are distributions describing the incident signal direction statistics, and XPR is the experimentally determined cross-polarisation level of the environment [10]. Equation (1) allows us to determine the MED during the shaping process without the feedpoint location being known. In the third shape synthesis example (using the same starting shape as in Figure 8) we therefore use an objective function:

\[
F_\text{obj} = |\beta| (f_\text{c}) + \left[ 1 - \frac{\text{MED}(f_\text{c})}{4\pi} \right]
\]

The goal is to demonstrate the utility of being able to include MED in the objective function. We do not have a specific propagation environment in mind, and so use XPR = 1, and Gaussian incident signal distributions [10] with \( n_{\text{inc}} = m_{\text{inc}} = 0^\circ, \) and \( \sigma_{\theta} = \sigma_{\phi} = 40^\circ. \) This should result in the shaping process achieving both self-resonance for the shaped antenna at 2 GHz, and making the azimuth radiation pattern more omni-directional. Figure 9 shows this indeed was the outcome.

Sensor packages often have batteries within them. To show that the shaping procedure can account for such things, we consider for the fourth example the starting shape shown on the left in Figure 10. The inner cuboid representing a battery is present in the CEM model, but is constrained from participating in the shaping. The shaped antenna on the right in Figure 10 is self-resonant with the battery present; removal of the battery from the shaped antenna in Figure 10 destroys the self-resonance at \( f_c. \)

Self-resonant electrically-small antennas usually have a dipole-like or loop-like input impedance behaviour [2]. As expected, one finds for various 3D shaped antennas that the different current paths emerging from the shaping procedure govern the resulting input impedance behaviour (over frequency); some quite different from the behaviour just mentioned. The current paths depend on the starting shape (e.g. whether the bottom face of the starting cuboid is present or not, or there is a battery present), since it dictates the material available for shaping.

Conclusion: The application of a shape-first/feed-next approach to the shape synthesis of electrically-small 3D conducting surface antennas has been described, and demonstrated for specific examples.

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