PHENOMENOLOGY OF $\varepsilon_K$ BEYOND LEADING LOGARITHMS*

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I present the QCD short distance coefficient $\eta_3$ of the $\Delta S = 2$ hamiltonian in the next-to-leading order (NLO) of renormalization group improved perturbation theory. Since now all QCD-factors $\eta_1$, $\eta_2$ and $\eta_3$ are known with NLO accuracy, a much higher precision in the analysis of $\varepsilon_K$ can be achieved. The CKM phase $\delta$, $|V_{td}|$ and the mass difference $\Delta m_{B_d}$ in the $B^0_d - \overline{B^0_d}$-system are predicted from the measured values of $\varepsilon_K$ and $\Delta m_{B_d}$. Finally I briefly look at the $K_L - K_S$-mass difference. This work has been done in collaboration with Stefan Herrlich.

1. The $|\Delta S|= 2$-hamiltonian in the NLO

The low-energy hamiltonian inducing $K^0 - \overline{K^0}$-mixing reads:

$$H^{\Delta S = 2} = \frac{G_F^2}{16 \pi^2} M_W^2 \left[ \lambda_1^2 \eta_1^2 \frac{m_t^2}{M_W^2} + \lambda_t^2 \eta_t^2 S\left( \frac{m_t^2}{M_W^2} \right) \right]$$

$$+ 2 \lambda_c \lambda_t \eta_t^2 S\left( \frac{m_c^2}{M_W^2} \right) \frac{m_t^2}{M_W^2} b(\mu) Q S_2(\mu) + \text{h.c.} \tag{1}$$

Here $G_F$ is the Fermi constant, $M_W$ is the W boson mass, $\lambda_j = V_{jd} V^*_{js}$ comprises the CKM-factors and $Q S_2$ is the local $|\Delta S|= 2$ four-quark operator

$$Q S_2 = (\overline{\pi} \gamma \mu (1 - \gamma_5) d_j) (\overline{\pi} \gamma \mu (1 - \gamma_5) d_k) \tag{2}$$

with $j$ and $k$ being colour indices. $m_\pi^* = m_\pi (m_q)$, $q = c, t$, are running quark masses in the MS scheme. The Inami-Lim functions $S(x)$, $S(x, y)$ describe the $|\Delta S|= 2$-transition amplitude in the absence of strong interaction. The short distance QCD corrections are comprised in the coefficients $\eta_1$, $\eta_2$ and $\eta_3$ with a common factor $b(\mu)$ split off. They are functions of the charm and top quark masses and of the QCD scale parameter $\Lambda_{QCD}$. Further they depend on the definition of the quark masses used in the Inami-Lim functions: In $|\Delta S|= 2$ the $\eta_i$’s are defined with respect to MS masses $m_\pi^*$ and are therefore marked with a star.

With actual values of the input data the results of the old leading log approximation read

$$\eta_1^{\text{LO}} \approx 0.80, \quad \eta_2^{\text{LO}} \approx 0.62, \quad \eta_3^{\text{LO}} \approx 0.36. \tag{3}$$

Now the NLO values read:

$$\eta_1^* = 1.32_{+0.21}^{0.21}, \quad \eta_2^* = 0.57_{-0.01}^{+0.00}, \quad \eta_3^* = 0.47_{-0.04}^{+0.03}. \tag{4}$$

where $m_t^* = 1.3$ GeV and $\Lambda_{\overline{\text{MS}}}^{\text{NLO}} = 0.310$ GeV has been used. $\eta_2$ and $\eta_3$ are almost independent of the input parameters.

The NLO calculation of $\eta_2$ has been performed in $[1]$ and the NLO result for $\eta_1$ can be found in $[2]$. $\eta_3$ in $[3]$ is new. Details of the calculation are presented in $[4]$. A phenomenological analysis using the NLO $\eta_i$’s has been published in $[5]$.

$[3]$ and $[4]$ clearly show the sizeable numerical effect of the NLO correction. Further there are conceptual reasons for going to the NLO:

i) The fundamental QCD scale parameter $\Lambda_{\overline{\text{MS}}}^{\text{MS}}$ cannot be used in LO calculations.

ii) The quark mass dependence of the $\eta_i$’s is not accurately reproduced by the LO expressions. Especially the $m_c$-dependent terms in $\eta_3 \cdot S(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2})$ belong to the NLO.

iii) Likewise the proper definition of the quark masses is a NLO issue: One must go to the NLO to learn how to use $m_t$ measured at FERMILAB in low energy expressions such as $[5]$. In NLO the MS mass $m_t^*$ is smaller than the pole mass $m_t^{\text{pole}}$ by 8 GeV. It has been discussed at this conference to which definition of $m_t$ the quoted CDF and D0 results for $m_t$ refer. Presumably the measured quantity is $m_t^{\text{pole}}$.

iv) In the NLO the large LO error bars caused by renormalization scale dependences are reduced.

2. Phenomenology of $\varepsilon_K$

Let us first recall our present knowledge about the CKM matrix $V_{CKM}$: The precise measurements of $|V_{ud}|$ and $|V_{us}|$ also constrain $|V_{ud}|$, $|V_{cs}|$ and $|V_{tb}|$ via the unitarity...
of $V_{CKM}$. $|V_{cb}|$ is expected to lie in the range

$$0.037 \leq |V_{cb}| \leq 0.043$$

(5)

After fixing $|V_{cb}|$ unitarity likewise pins down $|V_{ts}|$. We will further need

$$0.06 \leq \frac{|V_{ub}|}{|V_{cb}|} \leq 0.10.$$  

(6)

Yet even for fixed $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ the magnitude of the remaining CKM element $|V_{td}|$ is a sensitive function of the phase $\delta$. Hence $\Delta m_{B_d}$ and $|\varepsilon_K|$ provide complementary information on $|V_{td}|$: The former determines this element directly and the latter indirectly through $\delta$.

Apart from $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ two other key parameters are involved: The actual value $m_t^{pole}$ is $(180 \pm 12)$ GeV for the top quark pole mass.

$$160 \text{ GeV} \leq m_t^p \leq 184 \text{ GeV}.$$  

(7)

Finally the hadronic matrix element of $Q_{S2}$ in (2) is parametrized as

$$\langle |K| Q_{S2}(\mu) |K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K/b(\mu).$$

We take

$$0.65 \leq B_K \leq 0.85,$$

(8)

which reflects the $1/N_c$ result as well as the ballpark of the lattice determinations.

The uncertainties in the input parameters other than $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $m_t$, and $B_K$ do not significantly affect the analysis (see (4)). Using the LO $\eta_i$’s, however, imposes an error onto $\varepsilon_K$, which is roughly of the same order as the one due to (5-8). The NLO shift in $\eta_i$ affects $\varepsilon_K$ as much as pushing $B_K$ from 0.75 to 0.82. This uncertainty has been neglected in most phenomenological analyses.

After fixing three of the key parameters the measured value for $\varepsilon_K$ yields a lower bound at the fourth one. This feature has been used in the pre-top era to constrain $m_t$. Yet today one should focus on the CKM elements instead. The allowed range for $(|V_{cb}|, |V_{ub}/V_{cb}|)$ is shown in fig. 1.

Next $\delta$ is obtained from a simultaneous analysis of $\varepsilon_K$ and $\Delta m_{B_d}$: For $|V_{ub}/V_{cb}| = 0.08$, $|V_{cb}| = 0.04$, $m_t^p = 172 \text{ GeV}$, $B_K = 0.75$ one finds the two solutions:

$$\delta_{\text{low}} = 85^\circ, \quad \delta_{\text{high}} = 121^\circ.$$  

(9)

They correspond to

$$|V_{td}|_{\text{low}} = 9.1 \cdot 10^{-3}, \quad |V_{td}|_{\text{high}} = 10.7 \cdot 10^{-3}.$$  

(10)

Accounting for the errors (3-5) leads to

$$47^\circ \leq \delta \leq 135^\circ, \quad 7.3 \leq |V_{td}| \cdot 10^{-3} \leq 11.9.$$  

(11)

Figure 1: For each pair $(m_t^p, B_K)$ the measured value for $\varepsilon_K$ defines a curve. The points below the curve are excluded. The rectangle limits the allowed range for $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ obtained from tree-level b-decays according to (2) and (3).

where the upper bounds stem from $\Delta m_{B_d} = (0.470 \pm 0.0025) \text{ ps}^{-1}$. Here $F_{B_d} \sqrt{B_{B_d}} = (195 \pm 45) \text{ MeV}$ has been used.

The ratio $\Delta m_{B_d}/\Delta m_{B_s}$ is theoretically much better understood than the mass differences separately. We can use the result (11) for $|V_{td}|$ to predict $\Delta m_{B_s}$:

$$6.2 \text{ ps}^{-1} \leq \Delta m_{B_s} \leq 26 \text{ ps}^{-1}.$$  

(12)

More details can be found in [4], where slightly different values for $m_t$ and $\Delta m_{B_d}$ have been used.

3. The $K_L - K_S$ mass difference

In the 1980s it was generally believed that long distance interactions make up more than half of the observed $K_L - K_S$ mass difference. Today the picture has changed due to the NLO enhancement of $\eta_1$ and $\eta_3$ and the larger value for $\Lambda_{\text{had}}$, which pushes $\eta_1$ further up. One finds

$$\frac{\langle \Delta m_K \rangle_{\text{SD}}}{\langle \Delta m_K \rangle_{\text{exp}}} = 0.7 \pm 0.2$$

(13)

exhibiting a short distance dominance. This is in agreement with naive power counting: One expects the long distance part to be suppressed with a factor of $(\Lambda_{\text{had}}/m_c)^2$ compared to (13). Here $\Lambda_{\text{had}}$ is a hadronic scale.

References

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