Mathematical model of an intelligent decision support system for road quality management

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Abstract: The experience of operation of decision support systems is analyzed, it is shown that the existing principles of creating these systems do not take into account the failure and change in the performance of individual service element, which depend on the current load on the system. The definition of an adaptive decision support system as an intelligent system that works in an automated mode and is able to function in a changing environment, its internal structure, knowledge base, regulatory framework for management, support tools, system users is given. The tasks of construction a mathematical model of an adaptive decision support system and determining its objective function are set. A distributed decision support system, which receives information containing data from various input sources was studied. For construction a mathematical model of the system under investigation, the procedures for selecting mathematical models and methods of queuing theory were used. To assess the quality and effectiveness of an adaptive decision support system, simulation modeling is proposed.

1. Introduction
High quality decision-making in the analysis of complex problems, characterized by the processing of large volumes of information on the quality of highways, is difficult to achieve without the involvement of computer technology only on the basis of subjective expert assessment of analysts and specialists.

Operating experience of automated decision support systems (DSS) showed [1 ... 3] that the existing principles for their creation do not take into account the change in the speed of action of individual service elements (EO) as the load on the system increases. The situation with the speed of DSSS can be significantly improved if the system is adapted to adapt to changing conditions of the external and internal environment in order to optimize functioning [4].

2. Method
Usually the creation of automated DSS, which uses artificial intelligence methods, is associated with solving many problems. These include information modeling, intelligent design, the generation of
formal models, using methods for representing knowledge. Of no less importance is the development of software with computer support, the improvement of the architecture of the information and communication network to support decision-making, and many other tasks.

The quality of functioning of intelligent DSS significantly increases with the introduction of additional modules that allow taking into account the output, the change in the speed of individual service elements depending on the load on the system. In fact, we are talking about the transformation of DSS into an adaptive system.

In DSS, requests are used as information processing tools. The servicing element (SE) of the requests is the server from which data is collected (on which the multidimensional data warehouse is located). This data is used in the preparation of reports on the results of collection and analysis of system data. If the resources or performance of the server is insufficient, then the management data store can be installed on another computer / computers (second and third SE). To take into account the factors characterizing adaptive DSS, the following enlarged groups of instrumental computer facilities can be distinguished:

− adaptive knowledge base for the rapid accumulation of information about changing conditions of the situation;
− adaptive software and hardware support for choosing the best calculation plan when making a decision;
− an adaptive interface that can adapt to the individual user features and current requirements for the solution.

Selection among the existing models, further development and effective use of the mathematical model of the DSS is carried out in accordance with the chosen optimization criterion that takes into account the operating conditions of the system and the tools with which it is implemented. In [5] various procedures for selecting and evaluating models are described.

Usually the procedure governing the choice of a mathematical model is carried out by means of the formation of a preference function. It is necessary to take into account that in DSS the important tools are the requests for data processing.

The preference function has the form of (linear or nonlinear) convolution of the form [5]:

\[ u_j = \sum k_{j,i} r_i, \quad i = 1 \ldots n \]  

where \( u_j \) – the value of the preference function when choosing the \( j \)-th model, \( r_i \) – (estimate) the quality of the \( i \)-th criterion, \( k_i \) – the weight of the \( i \)-th criterion.

When choosing the \( j \)-th model, the modified convolution can be estimated as follows:

\[ u_j = \left( \frac{r_1 k_1 + r_2 k_2}{r_3 k_3} \right) + r_4 k_4 + r_5 k_5 + r_6 k_6, \]  

where \( u_j \) – modified convolution when choosing the \( j \)-th model, \( r_i \) – performance (the number of requests serviced per unit of time), \( r_i \) – optimization criterion, \( r_i \) – costs (energy, production, etc.), \( r_i \) – rate of DSS adaptation, \( r_i \) – an indicator of the accuracy of processing requests and visualizing the results of processing requests, \( r_i \) – time costs indicator for DSS adaptation, \( k_i \) – weight of the \( i \)-th optimization criterion.

On the basis of relations (1) and (2), an algorithm for selecting mathematical models is constructed. To minimize the development costs of these models, it is advisable to apply linear programming methods. From the point of view of the result, models should be described by similar characteristics:

\[ F = \sum_{j=0}^{n} c_j x_j \rightarrow \min \]  

\[ \sum_{j=0}^{n} h_{i,j} x_j \leq b_i, \quad d_j \leq x_j \leq D_j, \quad i = 1 \ldots m, j = 1 \ldots n \]  

where \( F = f(x_j) \) – objective function, \( g_i(x_j) \leq b_i \) – confines, \( d_j \leq x_j \leq D_j \) – border conditions, \( x_j \) – group of \( j \)-type models, \( h_{i,j} \) – Required resource of the \( i \)-th type type when developing \( j \)-th model, \( b_i \) – limitations of the \( i \)-th resource (value), \( c_j \) – cost of developing the \( j \)-th group of models.

In determining the objective function, the (total) cost of model development, the time of solving the
problem (scenario), and the indicators of financial, time or personnel (professional) resources are often taken into account [6]. Obviously, the lower cost of developing a model and the shorter time of solving the problem do not always lead to effective results.

It is advisable to take the number of requests processed per unit time as the objective function of adaptive DSS, which is a measure of the efficiency of using an information resource. DSS with SE is presented as a discrete event queuing system. The number of requests processed per unit time tends to the maximum and depends on the following variables: the probability of using EO at different loads, the coefficient of use of EO at different loads, the probability of refusal to service EO, the probability of several service elements working simultaneously, and the average number of requests in the system.

Restrictions on the modes of operation of EA are associated with a restriction on the number of functioning and backup EA, serviced requests and denials of service. More details are considered in [7]. This choice is due to the fact that SE and operations with them are typical functions realized by discrete processes, and computer systems and communication channels for transferring the results of computation in infocommunication interaction with the support of decision making and remote control can be represented as networks of single- and multichannel queuing systems (QS) with failures and various maintenance disciplines [8].

It is assumed that the investigated DSS is distributed, and it receives information containing data from various input sources that is written to databases (data warehouses).

Under such initial assumptions, the problem to be solved is reduced to finding the maximum efficiency of using the information resource of adaptive DSS in the conditions of its normal functioning. A means of achieving this goal is the development of algorithms, methods and means of solving the problems of integration and data quality of various suppliers of information resources.

Based on the selected objective function, a simulation model of adaptive DSS can be constructed. In the process of conducting theoretical and experimental studies, this model allows us to determine the change in the parameters of the change in the number of service elements depending on the intensity of a given flow. The efficiency of using an information resource is increased by reducing the cost of maintaining the data of the EA and reducing the proportion of lost requests due to large queues.

3. Mathematical model

The formula for calculating the probability of the initial state of the system is obtained by setting a limit on the maximum possible number of SE in DSS

\[ p_0 = \left( 1 + \sum_{S=1}^{E_{max}} \frac{\psi^{S-1}}{(S-1)!} \right)^{-1}, \]

where \( S \) – the number of SE; \( E_{max} \) – the maximum number of SE elements; \( \psi \) – load on the system; \( K \) – the number of system states, which depends on the length of the queue.

The obtained result \( p_0 \) corresponds to the state of the system when there is no service request in it, i.e. the system is completely idle.

We determine the probability of the presence of at least one request in the system

\[ P = 1 - p_0. \]

We calculate the probability of several \( S \) (in the presented example, one, two, and all three) SE

\[ P_S = p_0 \left( \frac{\psi^{S-1}}{(S-1)!} \right)^{K} \left( \frac{\psi}{S} \right)^{K} \left( \frac{\psi}{S} - 1 \right) \left( \frac{\psi}{S} - 1 \right). \]

The values of the SE utilization coefficients were obtained with the addition of a restriction on the maximum possible number of SE [7]
\[ a_n = \sum_{S=n}^{E_{\text{max}}} P_S = p_0 \sum_{S=n}^{E_{\text{max}}} \left( \frac{\psi^{S-1}}{(S-1)!} \right)^k \left( \frac{\psi}{S} \right) \left[ \frac{(\psi/S)^k - 1}{\psi/S - 1} \right] = 1 - p_0 - \sum_{S=1}^{n-1} P_S. \]  

(7)

where \( n \) – the number of requests.

Requests are refused only if two conditions are met at the same time: all available EOs are occupied and all places in the queues for EO are occupied. The probability of this event [9]

\[ p_r = \frac{\psi^{max \cdot k}}{(max)^k} p_0. \]  

(8)

4. Results of calculations and simulation

The effectiveness of introducing the adaptability property into DSS was checked by comparing the results of its functioning with two other common systems. As such, two systems of the limiting type are chosen

• M/M/1/100 system based on one SE (1), in which the incoming flow of requirements is Poisson's (M), the distribution of time for servicing obeys the exponential law (M), an almost infinite queue is allowed (100)

• the M/M/3 system is built on the basis of three SE and assumes the complete absence of queues.

Table 1 presents the results of calculations of the probability of the presence of at least one query in the system. In graphical form, these results are shown in Figure 1. In addition to the calculations, simulation was carried out in the GPSS environment, which showed similar results.

| System in question | 0   | 0.5 | 1   | 1.5  | 2   | 2.5  | 3   |
|--------------------|-----|-----|-----|------|-----|------|-----|
| M/M/3              | 0,334 | 0,500 | 0,600 | 0,666 | 0,716 | 0,750 |
| Adaptive DSS       | 0 | 0,469 | 0,752 | 0,877 | 0,932 | 0,961 | 0,974 |
| M/M/1/100          | 0,503 | 0,996 | 1   |

Table 1. Probability values for the presence of at least one request in the system.

Figure 1. Dependencies of the probability of using the first element of service at different loads for decision support systems: 1 – M/M/3; 2 – adaptive; 3 – M/M/1/100.
Obviously, adaptive DSS (points 2) occupies an intermediate position between a system without $M/M/3$ queues (points 1) and a system with an almost infinite $M/M/1/100$ queue (points 3), confirming the conclusions of theoretical calculations. The system operates in the optimum mode, preventing neither excessive downtime in operation (as in the first case), nor operation in emergency mode (as in the case of the $M/M/1/100$ system).

By determining the percentage difference between the two numbers, it is estimated that on average, an adaptive system provides 28% more time for processing requests than a system without queues, and 11% less downtime than a system with an infinite queue.

You need to know the percentage of denial of service when all EO and all the places in the queue for them are occupied. The results of the calculation of the probability of denial of service according to the formula (8) are presented in table 2.

Table 2. Denial of service probability values

| System in question | System load, $\Psi$ | 0 | 0,5 | 1 | 1,5 | 2 | 2,5 | 3 | 3,5 | 4 | 4,5 | 5 |
|--------------------|----------------------|----|-----|---|-----|---|-----|---|-----|---|-----|---|
| $M/M/3$            |                      | 0,013 | 0,061 | 0,137 | 0,210 | 0,280 | 0,348 | 0,402 | 0,449 | 0,49 | 0,529 |
| Adaptive DSS       |                      | 0   | 0,009 | 0,037 | 0,090 | 0,160 | 0,230 | 0,305 | 0,363 | 0,418 |
| $M/M/1/100$        |                      | 0   | 0    | 0,012 | 0,146 | 0,252 | 0,337 | 0,399 |

The dependencies of the probability of denial of service at different load on the DSS are presented in Figure 2. It is obvious that the adaptive system (points 2) provides a lower probability of denial of service than a system without queues $M/M/3$ (points 1) but greater than a system with almost endless queue of $M/M/1/100$ (points 3).

![Figure 2. Dependence of the probability of a denial of service failure on various load on the system: 1 – $M/M/3$; 2 – adaptive; 3 – $M/M/1/100$.](image-url)
The implementation of tools and management mechanisms of DSS in a single information and analytical complex that includes information-modeling systems provides expansion of functionality, additional analytical support for DSS and performance of information processing.

5. Conclusions
1. Introduction to the DSS adaptation options successfully solves the problem of maximizing the number of requests processed per unit time.
2. The adaptive system on average provides a 28 percent increase in employment by processing requests compared to a queue-free system, combined with an 11% reduction in downtime compared to a system that allows an infinite queue.
3. The probability of failure of the adaptive system compared to the system without queues on average does not exceed 5.8%.

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