Abstract

The parameters of the original log-normal mass spectrum of primordial black holes (PBH) are adjusted based on the existing observational data on supermassive black holes in the galactic centers and the mass distribution of the near-solar mass black holes in the Galaxy. Together with the assumption that PBHs make all or a large mass fraction of the cosmological dark matter it allows to fix the parameters of the original mass spectrum. The predicted, in this way, number density of MACHOs is found to be about an order of magnitude below the observed value. Possible resolution of the controversy may be prescribed to non-isotropic and inhomogeneous distribution of MACHOs or to the two-maximum spectra of PBH.

1 Introduction

A quarter of century old idea [1, 2] that primordial black holes (PBHs) are abundant in the present day universe is gaining more and more popularity. According to the mechanism of PBH production proposed in refs. [1, 2] the mass spectrum of PBH at the moment of creation has the simple log-normal form:

$$\frac{dN}{dM} = \mu^2 \exp \left[ -\gamma \ln^2 \left( \frac{M}{M_m} \right) \right]$$

(1)

where $\gamma$ is dimensionless constant and parameters $\mu$ and $M_m$ have dimension of mass or, what is the same, of inverse length (here the natural system of units with $c = k = \hbar = 1$ is used). Probably log-normal spectrum is a general feature of inflationary production of PBH or, to be more precise, is a consequence of the creation of appropriate conditions for the PBH formation at inflationary cosmological stage, while the PBHs themselves might be formed long after inflation. In the considered model they were formed after the QCD phase transition at the temperature about 100 MeV. Some other forms of the spectrum were postulated in the literature, in particular, the delta-function one and a power law spectrum. In this work we confine ourselves to the log-normal spectrum which has rigorous theoretical justification. Such spectrum is an example of the so called extended mass spectrum which came to life recently instead of narrow (monochromatic) mass spectra assumed previously, see e.g. ref. [3].

As it was envisaged in ref. [1], cosmological dark matter could consist entirely of PBHs. It was even claimed recently that practically all black holes in the contemporary universe, with masses starting from a fraction of the solar mass, $M_\odot$, up to supermassive black holes of billions
solar masses, and intermediate mass black holes with $M = (10^3 - 10^6)M_\odot$ are predominantly primordial [4–6].

In this work we will use available observational data to fix the parameters of distribution (1). This task is highly non-trivial because the original mass spectrum of PBHs was surely distorted through the matter accretion by PBHs in the course of cosmological evolution. This problem was addressed in two works of one of us (with collaborators) [7,8]. Here we will use a different set of observational data and somewhat change the assumptions of the evolution of the mass spectrum.

In ref. [8] we assumed that:
1. MACHOs are primordial black holes and their cosmological mass density makes the fraction $f = 0.1$ of the cosmological mass density of dark matter.
2. All primordial black holes constitute the whole cosmological dark matter,
3. The number density of the primordial black holes with masses above $10^3M_\odot$ is equal to the number density of the observed large galaxies.

The basic assumptions which are relied upon in this work are the following:
1. The total cosmological mass density of black holes in the universe makes the fraction $f$ of the dark matter density with $f$ being a free parameter. The most interesting case is $f = 1$ of course. We also considered the case $f = 0.1$.
2. The value of $M_m$, at which the distribution (1) reaches maximum, is taken from the data on mass spectrum of black holes in the Galaxy.
3. The observed number density of large galaxies is equal to the number density of the heavy black holes with masses exceeding some boundary value, $M_b$. While $M_b$ is supposed to be much smaller than the masses of the supermassive black holes (SMBH) observed in the centers of large galaxies, they could serve as appropriate seeds for the SMBH creation not only in the present day universe but also in the young one at the redshifts $z \sim 10$.

The value $M_m$, at which the distribution (1) reaches maximum according to the papers [7,8], was taken to be equal to one solar mass. However, in this work we assume that $M_b$ is in the interval $(6 - 8)M_\odot$ as dictated by the observations of the mass spectrum of the black holes in the Galaxy. With this choice of the three basic sets of the observational data the mass density of MACHOs derived here is about $f \lesssim 10^{-3}$. The apparent contradiction of the observations can be resolved if the MACHOs are non-homogeneously distributed in space, see discussion below, or the mass spectrum (1) is generalized to a more complicated form having two or several maxima, as is envisaged in ref. [2].

2 Total mass density of black holes

The total cosmological mass density of the primordial black holes at the present time is given by the integral

$$\rho_{BH} = \mu^2 \int_0^{M_{max}} dMM \exp \left[ -\gamma \ln^2 \left( \frac{M}{M_m} \right) \right]$$

under assumption that the spectrum (1) is weakly distorted by accretion in the essential mass range where $M$ is close to $M_m$. As shown in ref. [7], the spectrum has a cutoff at large mass, $M_{max}$. The maximum value of PBH mass is estimated in ref. [7] a function of the model parameters. According to this work a reasonable value of $M_{max}$ may lay in the range $M_{max} = (10^5 - 10^6)M_\odot$. Since $M_m$ is below $10M_\odot$, see the next section, integral (2) can be safely extended to infinity.

Assuming that $\rho_{BH}$ makes a fraction $f$ of the mass density of dark matter, $\rho_{BH}/\rho_{DM} = f$, where

$$\rho_{DM} \approx 2.5 \cdot 10^{-30} \text{g/cm}^3 \approx 3.7 \cdot 10^{10} M_\odot/\text{Mpc}^3$$

(3)
we find the first equation for fixation of the parameters of the distribution:

\[
\mu^2 \int_{0}^{M_{\text{max}}} dM M \exp \left[ -\gamma \ln^2 \left( \frac{M}{M_m} \right) \right] = f \varrho_{DM}.
\]  

(4)

For the numerical estimates it is convenient to present the solar mass in different units not only in gramma but in inverse megaparsec as well:

\[
M_\odot = 2 \cdot 10^{33} \text{ g} = 1.75 \cdot 10^{95} / \text{Mpc}.
\]  

(5)

There is no agreement on the value of \( f \) in the literature. According to the recent work [45], the mass fraction of black holes should be rather small, \( f < 0.1 \). However, this result is valid for a high value of the median mass \( M_m \geq 20M_\odot \). On the other hand, the data on the mass spectrum of galactic black holes indicate that \( M_m = (6 - 9)M_\odot \). For \( M_m \) in this interval the limits are much weaker. In what follows we assume the extreme case \( f = 1 \), which might be not excluded.

3 Mass spectrum of black holes in the Galaxy

The mass spectrum of black holes in the Galaxy shows striking features unexpected in the standard picture of stellar mass BH formation through stellar collapse after a star exhausted its nuclear fuel and if it has a sufficiently large mass. The observed picture strongly disagrees with natural expectation from this scenario. According to ref. [10] the masses of the observed black holes are surprisingly high and are concentrated in a narrow interval \( (7.8 \pm 1.2)M_\odot \). This result is supported by another work [11], according to which the spectrum maximum is situated at \( M \sim 8M_\odot \) and sharply drops above \( M \sim 10M_\odot \) and below 5\( M_\odot \).

It is also observed [12] that black holes in the Galaxy have two-peak mass distribution with the second peak situated above the maximum mass of neutron star but below the lower limit of the BH masses found in the quoted above papers [10,11]. The lower mass BHs are presumably produced by the usual mechanism of stellar collapse. So we expect that galactic black holes have log-normal distribution of heavier BHs, but lower mass BHs have a replica of stellar mass distribution of of stars exceeding the Chandrasekhar limit.

Matter accretion in the course of galactic evolution may lead to some increase of the galactic black hole masses. Bearing this in mind, we take as the test values \( M_m/M_\odot = 6, 7, \text{ and } 8 \).

4 Supermassive PBH in the centers of large galaxies

Astronomical observations strongly indicate that in each large galaxy resides a supermassive black hole (SMBH) [13]. Moreover, SMBGHs are also observed in some small galaxies and even in practically empty space, for a review see [5,6].

Origin of such black holes is mysterious. According to conventional understanding SMBHs in galactic centers appeared as a result of matter accretion on a massive seed. However, the estimates of the necessary accretion rate to create such giants demand it be much larger than any reasonable value. These facts create serious doubts about the traditional picture of of the galaxy and SMBH formation, according to which the galaxy was created first and later a SMBH was formed in the center by accumulation of the galactic matter. The data certainly indicates to inverted picture that SMBHs were formed first and they served as a seed for the galaxy formation [11,12,13]. Recent observations of high red-shift, \( z \sim 10 \) SMBHs [5,6], surely support this assertion.

Accordingly we assume that the density of supermassive primordial black holes is equal to the density of galaxies. As is assumed in ref. [7], the initially formed superheavy PBH might have much smaller masses, roughly speaking in the range \((10^3 - 10^5)M_\odot \) which could subsequently
grow up to $10^9 M_\odot$ because of an efficient accretion of matter on the preexisting very massive seeds and mergings. A similar statement is done in ref. [15], namely that the PBHs with masses around $(10^4 - 10^5) M_\odot$ may subsequently grow to $10^9 M_\odot$

This mass enhancement factor is much stronger for heavier BH and thus their mass distribution may be different from (1). We assume the simplified picture that the original PBHs were created with the distribution (1) but a PBH with mass larger than a certain boundary value $M_b$ became a supermassive seed of galaxy formation. Correspondingly the number density of PBH with masses larger than $M_b$ should be equal to the present day number density of (large) galaxies:

$$N_b = \mu^2 \int_{M_b}^{M_{\text{max}}} dM \exp \left[ -\gamma \ln^2 \left( \frac{M}{M_m} \right) \right] = N_{\text{gal}}$$

In what follows we take the following two sampling values

$$M_b = [10^4, 10^5] M_\odot.$$  

Evidently we must choose $M_{\text{max}} > M_b$. If $M_{\text{max}} \gg M_b$, the upper limit in eq. (6) may be extended to infinity. If accidentally $M_{\text{max}}$ is close by magnitude to $M_b$, the integral in Eq. (6) would be strongly diminished.

The number density of galaxies is not well known. We take it as

$$N_{\text{gal}} = K/\text{Mpc}^3.$$  

with $K$ presumably in the generous interval $K = (0.1 - 0.001)$. This estimate is in a reasonable agreement with those presented in refs. [16, 17].

This relations give the third and last necessary condition for determination of the parameters of distribution (1).

5 Determination of the parameters

Using the presented above conditions we can determine the parameters: $\gamma$ and $\mu$. The value of median mass $M_m$ is fixed in the interval $6 M_\odot < M_m < 8 M_\odot$ by the mass spectrum of the Galactic black holes, see Sec. 3.

From equations (4,6,8) we find:

$$\frac{\rho_{\text{DM}}}{\dot{\rho}_{\text{BDM}}} = 3.7 \times 10^{10} f/K = \frac{I_1(0, x_{\text{max}}, x_m, \gamma)}{I_0(x_b, x_{\text{max}}, x_m, \gamma)},$$  

where

$$I_n(x_{\text{min}}, x_{\text{max}}, x_m, \gamma) = \int_{x_{\text{min}}}^{x_{\text{max}}} dxx^n \exp \left[ -\gamma \ln^2 \left( \frac{x}{x_m} \right) \right]$$

with $x_{\text{min}} = M_{\text{min}}/M_\odot$, $x_{\text{max}} = M_{\text{max}}/M_\odot$, $x_b = M_b/M_\odot$, and $x_m = M_m/M_\odot$.

We calculate the ratio in the r.h.s. of eq. (9) as a function of $\gamma$ for $f = 1, 0.1$; $K = 0.1$; $x_b = 10^4, 10^5$ and $x_{\text{max}} = 10^5, 10^6$. According to the definition $M_b$ should be smaller than $M_{\text{max}}$ in each sample of the parameters. The results are not significantly different except for the case when $M_b$ closely approaches $M_{\text{max}}$ from below.

The values of the parameters $\gamma$ and $\mu$ have been calculated in the appendix for $M_b = 10^4, 10^5$. In that table $\mu_1$ is the value of parameter $\mu$ calculated from the condition $N_{\text{gal}} = 0.1/\text{Mpc}^3$ and $\mu_2$ is the value of the same parameter calculated from the condition $\rho_{\text{BDM}} = 2.5 \times 10^{-30} \text{f g/cm}^3$. As mentioned in introduction, we have taken two sample values of $f$, 1 and 0.1.
According to ref. [8] fitting the PBH mass function normalization in the $10 - 100 M_\odot$ range to the BH+BH merging rate derived from the LIGO BH+BH detections ($9 - 240$ events a year per cubic Gpc), we should only take care that the mass density of primordial SMBHs does not contradict the existing SMBH mass function as inferred from observations of galaxies, $dN/(d\log M)dV \sim 10^{-2} - 10^{-3}$ Mpc$^{-3}$.

The initially formed superheavy AD PBH might have much smaller masses (around $(10^4 - 10^5)M_\odot$) to subsequently grow to $10^9 M_\odot$ because of an efficient accretion of matter and mergings, see the state-of-the-art SMBH growth calculations in [14].

6 Problems with MACHOs

As we have found in the previous section, $\gamma$ is typically about 0.5. If we choose $M_m = (6-8) M_\odot$, then the calculated mass density of MACHOs would be several orders of magnitude lower than most results on the measured MACHO density for all reasonable values of $\gamma$.

The data presented by different groups are rather controversial. The state of the art is reviewed and summarized in refs. [7,18-20]. Briefly the situation is the following.

MACHO group [21] reported registration of 13 - 17 microlensing events in the Large Magellanic Cloud (LMC), which is significantly higher than the number which could originate from the known low luminosity stars. On the other hand it is not sufficient to explain all dark matter in the halo. The fraction of the mass density of the observed objects, which created the microlensing effects, with respect to the energy density of the dark matter in the galactic halo, $f$, according to the observations [21] is in the interval:

$$0.08 < f < 0.50,$$

(11)

95% CL for the mass range $0.15 M_\odot < M < 0.9 M_\odot$.

EROS collaboration [22] has placed the upper limit on the halo fraction, $f < 0.2$ (95% CL) for the objects in the specified above MACHO mass range, while EROS-2 [23] gives $f < 0.1$ for $0.6 \times 10^{-7} M_\odot < M < 15 M_\odot$ for the survey of Large Magellanic Clouds. It is considerably less than that measured by the MACHO collaboration in the central region of the LMC.

The new measurements of 2013 by EROS-2, OGLE-II, and OGLE-III collaborations [26] towards the Small Magellanic Cloud (SMC) revealed five microlensing events towards the SMC (one by EROS and four by OGLE), which lead to the upper limits $f < 0.1$ obtained at 95% confidence level for MACHO’s with the mass $10^{-2} M_\odot$ and $f < 0.2$ for Machos with the mass $0.5 M_\odot$.

Search for microlensing in the direction of Andromeda galaxy (M31) demonstrateded some contradicting results [18,19] with an uncertain conclusion. E.g. AGAPE collaboration [24], finds the halo Macho fraction in the range $0.2 < f < 0.9$, while MEGA group presented the upper limit $f < 0.3$ [25]. On the other hand, the recent discovery of 10 new microlensing events [36] is very much in favor of MACHO existence. The authors conclude: “statistical studies and individual microlensing events point to a non-negligible MACHO population, though the fraction in the halo mass remains uncertain”.

Some more recent observational data and the other aspects of the microlensing are discussed in ref. [27].

It would be exciting if all DM were constituted by old stars and black holes made from the high density baryon bubbles as suggested in refs. [11] with masses in still allowed intervals, but more detailed analysis of this possibility has to be done.

There is a series of papers claiming the end of MACHO era. For example in ref. [28] the authors stated ”we exclude MACHOs with masses $M > 43 M_\odot$ at the standard local halo density This all but removes the last permitted window for a full MACHO halo for masses $M > 10^{-7.5} M_\odot$.
In addition to the criticism raised in the paper [28], some more arguments against an abundant galactic population of MACHOs are also presented in ref. [29, 30]. However, according e.g. to the paper [31], the approach of the mentioned works have serious flaws and so their results are questionable. A reply to this criticism is presented in the subsequent paper [32].

The data in support of smaller density of MACHOs in the direction to SMC is presented in ref. [33].

Later, however, another paper of the Cambridge group [34] was published where, on the basis of studies of binary stars, arguments in favor of real existence of Machos and against the pessimistic conclusions of ref. [28] were presented.

The latest investigation on the "end of MACHO era" was presented in ref. [35], where it is concluded that "the upper bound of the MACHO mass tends to less than $5M_\odot$ does not differ much from the previous one. Together with microlensing studies that provide lower limits on the MACHO mass, our results essentially exclude the existence of such objects in the galactic halo".

A nice review of the state of the art and some new data are presented in ref. [36] with the conclusion that some statistical studies and individual microlensing events point to a non-negligible MACHO population, though the fraction in the halo mass remains uncertain.

According to the results of different groups the fraction of MACHO mass density with respect to the total mass density of dark matter varies in rather wide range:

$$f_{MACHO} = \frac{\rho_{MACHO}}{\rho_{DM}} \sim (0.01 - 0.1)$$

(12)

Notice a large variance of the results by different groups. Reasonable agreement between the data and the considered here model can be achieved only if $M_m \sim M_\odot$ [8, 20]. So we either have to reject the possibility that practically all galactic black holes are primordial with masses around $(6 - 8)M_\odot$ or to search for another explanation of the discrepancy between the observed and the predicted density of MACHOs with log-normal mass spectrum of PBHs.

An interesting option is that the spatial distribution of MACHOs may be very inhomogeneous and non-isotropic. Due to selection effect MACHOs are observed only in over-dense clumps where their density is much higher than the average one. For a review and the list of references on dark matter clumping see e.g. [37]. Clumping of primordial back holes, due to dynamical friction, may be much stronger than the clumping of dark matter consisting from elementary particles. This hypothesis would allow to avoid contradiction between the observed high density of MACHOs and the predicted by the log-normal mass spectrum much smaller density of them if $M_m = (7 - 9)M_\odot$.

Another possibility to adjust theory to the observations is to assume multi-maximum log-normal spectrum having, i.e. the superposition of the log-normal spectra having maxima at several different values of $M_m$:

$$\frac{dN}{dM} = \sum_j \mu_j^2 \exp \left[ -\gamma_j \ln^2 \left( \frac{M}{M_m} \right) \right]$$

(13)

Such spectrum may originate from inflationary stage if the coupling of the inflaton field $\chi$ to the scalar with non-zero baryonic number has more complicated polynomial form [2], than that postulated in the original paper [1]:

$$U_{int} = |\chi|^2 \prod_j \lambda_j (\Phi - \Phi_j)^2 / m_{Pl}^{2j-2}.$$  

(14)

In our case the two-maxima mass spectrum, with $j$ running from 1 to 2, is sufficient to describe all observational data with reasonable accuracy. It allows also to avoid many existing bounds on primordial black holes [38 - 42]. Such two-maximum log-normal spectrum is introduced ad hoc in ref. [43] with the same purpose to satisfy the demands of astronomical observations.
7 Black holes with intermediate mass

Black holes with masses from $10^3 M_\odot$ up to $10^6 M_\odot$ are rather arbitrarily called Intermediate Mass Black Holes (IMBH). They were observed during recent few years and now about $10^3$ of them are known [44]. It remains unclear if they can be created by the conventional astrophysical processes, such as stellar collapse or matter accretion to some massive seeds. The hypothesis that they are all primordial looks much more natural.

Having the parameters fixed we can calculate the number density of the intermediate black holes $N_{IMBH}$. We find that for each large galaxy there are $\sim 10^3 - 10^4$ number of IMBHs (see appendix). According the ref. [8] such IMBH can seed globular cluster formation dwarf galaxies. At the moment only in one globular cluster a black hole with mass about $2000 M_\odot$ is observed. It is predicted [8] that in every globular cluster there must be an intermediate mass primordial black hole.

8 Conclusion

Massive primordial black holes with extended mass spectrum became viable candidates for the constituents of the cosmological dark matter. Formation of such PBHs is possible due to inflationary expansion of the very early universe because inflation could make physically connected super-horizon scales. In this sense existence of supermassive PBHs can be considered as extra proof of inflation.

Recent observations of abundant supermassive black holes in the early universe leads to a natural conclusion that they are primordial, see e.g. [5]. If they indeed have log-normal or some other extended mass distribution, then it is tempting to conclude that the contribution of PBH to the cosmological dark matter is at least non-negligible.

In principle there could be two, or even several, comparable forms of dark matter: PBHs and different elementary particles species, though such a conspiracy is surely at odds with the Occam razor. On the other hand, there are impressive examples of similar cosmic conspiracies in near equality of energy densities of baryons, dark matter, and dark energy.

Detailed comparison of the observational data with the predicted mass spectrum of black holes at different redshifts could help to solve this deep mystery.

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### Appendix:

| $M_m = 8 \text{M}_\odot$ | $M_b = 10^4 \text{M}_\odot$ | $M_b = 10^5 \text{M}_\odot$ |
|--------------------------|-------------------------------|-------------------------------|
| $f = 1$                  | $\gamma = 0.53$              | $\gamma = 0.53$              |
|                          | $\mu_1 = 2.4 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 8.98 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 4.4 \times 10^{-69}\text{cm}^{-1}$ | $= 1.6 \times 10^{-69}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 3.4 \times 10^5$ | $N_{\text{IMBH}} = 1.9 \times 10^4$ |
|                          | $\mu_2 = 2.5 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 1.1 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 4.6 \times 10^{-69}\text{cm}^{-1}$ | $= 2 \times 10^{-69}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 3.6 \times 10^5$ | $N_{\text{IMBH}} = 2.8 \times 10^4$ |
| $f = 0.1$                | $\gamma = 0.48$              | $\gamma = 0.48$              |
|                          | $\mu_1 = 6.4 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 3.5 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 1.2 \times 10^{-69}\text{cm}^{-1}$ | $= 6.4 \times 10^{-70}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 8.6 \times 10^4$ | $N_{\text{IMBH}} = 8.7 \times 10^4$ |
|                          | $\mu_2 = 6.9 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 3.1 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 1.3 \times 10^{-69}\text{cm}^{-1}$ | $= 5.7 \times 10^{-70}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 10^5$ | $N_{\text{IMBH}} = 6.8 \times 10^4$ |

| $M_m = 7 \text{M}_\odot$ | $M_b = 10^4 \text{M}_\odot$ | $M_b = 10^5 \text{M}_\odot$ |
|--------------------------|-------------------------------|-------------------------------|
| $f = 1$                  | $\gamma = 0.52$              | $\gamma = 0.31$              |
|                          | $\mu_1 = 3.1 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 1.3 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 5.7 \times 10^{-69}\text{cm}^{-1}$ | $= 2.4 \times 10^{-69}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 3.6 \times 10^5$ | $N_{\text{IMBH}} = 2.1 \times 10^4$ |
|                          | $\mu_2 = 2.8 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 1.3 \times 10^{-50}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 5.1 \times 10^{-69}\text{cm}^{-1}$ | $= 2.4 \times 10^{-69}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 2.9 \times 10^5$ | $N_{\text{IMBH}} = 2.1 \times 10^4$ |
| $f = 0.1$                | $\gamma = 0.47$              | $\gamma = 0.28$              |
|                          | $\mu_1 = 7.7 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 3.2 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 1.4 \times 10^{-69}\text{cm}^{-1}$ | $= 5.8 \times 10^{-70}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 8.5 \times 10^4$ | $N_{\text{IMBH}} = 7.1 \times 10^4$ |
|                          | $\mu_2 = 7.7 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ | $\mu_1 = 3.3 \times 10^{-51}(\text{gm cm}^3)^{-1/2}$ |
|                          | $= 1.4 \times 10^{-69}\text{cm}^{-1}$ | $= 6.0 \times 10^{-70}\text{cm}^{-1}$ |
|                          | $N_{\text{IMBH}} = 8.5 \times 10^4$ | $N_{\text{IMBH}} = 7.6 \times 10^4$ |
| $M_m = 7 M_\odot$ | \( M_b = 10^4 M_\odot \) | \( M_b = 10^5 M_\odot \) |
|---|---|---|
| $f = 1$ | \( \gamma = 0.5 \) | \( \gamma = 0.3 \) |
| \( \mu_1 = 3.2 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_1 = 1.3 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_1 = 1.3 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) |
| = \( 5.8 \times 10^{-69} \text{cm}^{-1} \) | = \( 2.4 \times 10^{-69} \text{cm}^{-1} \) | = \( 2.4 \times 10^{-69} \text{cm}^{-1} \) |
| \( N_{\text{IMBH}} = 2.9 \times 10^5 \) | \( N_{\text{IMBH}} = 1.9 \times 10^4 \) | \( N_{\text{IMBH}} = 1.9 \times 10^4 \) |
| \( \mu_2 = 3.1 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_2 = 1.4 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_2 = 1.4 \times 10^{-50} \text{(gm cm}^3\text{)}^{-1/2} \) |
| = \( 5.7 \times 10^{-69} \text{cm}^{-1} \) | = \( 2.6 \times 10^{-69} \text{cm}^{-1} \) | = \( 2.6 \times 10^{-69} \text{cm}^{-1} \) |
| \( N_{\text{IMBH}} = 2.7 \times 10^5 \) | \( N_{\text{IMBH}} = 2.2 \times 10^4 \) | \( N_{\text{IMBH}} = 2.2 \times 10^4 \) |
| $\gamma = 0.45$ | \( \gamma = 0.45 \) | \( \gamma = 0.27 \) |
| $f = 0.1$ | \( \mu_1 = 7.5 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_1 = 3.0 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_1 = 3.0 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) |
| = \( 1.4 \times 10^{-69} \text{cm}^{-1} \) | = \( 5.5 \times 10^{-70} \text{cm}^{-1} \) | = \( 5.5 \times 10^{-70} \text{cm}^{-1} \) |
| \( N_{\text{IMBH}} = 6.7 \times 10^4 \) | \( N_{\text{IMBH}} = 5.9 \times 10^3 \) | \( N_{\text{IMBH}} = 5.9 \times 10^3 \) |
| \( \mu_2 = 8.4 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_2 = 3.5 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) | \( \mu_2 = 3.5 \times 10^{-51} \text{(gm cm}^3\text{)}^{-1/2} \) |
| = \( 1.5 \times 10^{-69} \text{cm}^{-1} \) | = \( 6.4 \times 10^{-70} \text{cm}^{-1} \) | = \( 6.4 \times 10^{-70} \text{cm}^{-1} \) |
| \( N_{\text{IMBH}} = 8.3 \times 10^4 \) | \( N_{\text{IMBH}} = 8 \times 10^3 \) | \( N_{\text{IMBH}} = 8 \times 10^3 \) |