Finite Temperature and Density Effects in Planar Q.E.D

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Abstract

The behavior of finite temperature planar electrodynamics is investigated. We calculate the static as well as dynamic characteristic functions using real time formalism. The temperature and density dependence of dielectric and permeability functions, plasmon frequencies and their relation to the screening length is determined. The radiative correction to the fermion mass is also calculated. We also calculate the temperature dependence of the electron (anyon) magnetic moment. Our results for the gyromagnetic ratio go smoothly to the known result at zero temperature, $g = 2$, in accordance with the general expectation.

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Introduction

Perturbative quantum field theory at finite temperature and density is of current interest in many areas of elementary particle physics. Apart from its cosmological relevance it is important in determining the properties of systems like the quark-gluon plasma in Q.C.D and the electron-positron plasma in Q.E.D. It is also of some interest in a more exact description of nuclear matter [1]. The recent interest in planar systems motivates the investigation of such systems in 2+1 dimensions [2]; further since by dimensional arguments the behavior of many physical quantities at low temperature is expected to be enhanced in 2+1 dimensions (i.e. \( \sim T \) instead of \( \sim T^2 \) in 3+1 dimensions) it might provide a simpler way of testing Q.E.D in 2+1 accurately.

In this paper we investigate some properties of the two and three point functions in 2+1 dimensional Q.E.D. The Green functions are the basic tools in the study of quantum many body systems. Knowledge of the Green functions allows one to calculate all physically interesting quantities like energy, density, thermodynamic potential etc. Furthermore, the singularities in momentum space of these functions determine the spectrum of excitations of the system. Linear response theory relates the Green functions to the various quantities that characterize the response of the system to small external fields. In the following sections we use the real time formalism [3] to calculate the polarization tensor, the fermion self energy and the vertex function at finite density and temperature. We calculate static properties like the dielectric constant and magnetic permeability as well as the magnetic and electric screening lengths. The magnetic mass vanishes in the leading order as in the 3+1 dimensional case. Dynamical properties such as the transverse and longitudinal plasmon frequency can also be extracted from the photon two point function. We find
that for small momenta \((m_{pl}^{L})^2 = (m_{pl}^{T})^2 = \frac{1}{2}m_{el}^2\) (in the 3+1 dimensional case, for small momenta \((m_{pl}^{L})^2 = (m_{pl}^{T})^2 = \frac{1}{3}m_{el}^2\)).

In 2+1 dimensions there is the possibility of adding a topological gauge invariant mass term, the Chern Simons term (CS), to the action of Q.E.D. The spin and statistics of electrons in this case is altered by this term, \(S = \frac{1}{2} + \frac{1}{\kappa}\) where \(\kappa\) is the CS coefficient. The finite temperature behavior of the CS term has been investigated by several authors \([5],[7],[3]\). Here we calculate its dependence on the density and temperature. The spin of electrons (anyons) is related to the anomalous magnetic moment by the \(g\) factor, \(\mu = g \frac{2}{2m} S\). It has been observed \([17]\) that at zero temperature \(g = 2\) and that this result is an exact one. This was verified to one loop in perturbation theory \([16]\). From the three point function we can directly calculate the anomalous magnetic moment at finite temperature. We verify that to one loop and in the limit \(T, \mu \to 0\), \(g = 2\).

**Finite Temperature Propagators**

There exist two main formulations of field theory at finite temperature. In the imaginary time formalism one replaces the continuous energy variable \(k_0\) by \(2\pi i n T\) for bosons and \(2\pi i (n + \frac{1}{2}) T\) for fermions. and the integration over this variable by a discrete sum \(\int \frac{dk}{2\pi} \to iT \sum_{n=-\infty}^{\infty}\). From this one immediately notices that in the very high temperature case the time direction disappears entirely because the temporal interval shrinks to zero. Also, in this limit only the zero energy processes survive, as nonzero energy necessarily means high energy at high temperatures, owing to discrete nature of the energy variable. Thus at high temperatures all Matsubara modes with \(n \neq 0\) decouple. This feature is certainly attractive in a study of static processes. However, one might want to study processes where the energy is neither of two extremes—large or zero. The imaginary time formalism is clearly not ideally suited for this regime.
Apart from this, if one is interested in addressing non-static problems one has to analytically continue the results in order to recover the real time. This process of analytic continuation can actually become quite arduous in many cases of interest. In the real time formulation, on the other hand the energies remain real but one has to accept a doubling of the number of degrees of freedom. Time in this formalism is treated as a complex quantity and the real time is actually just one branch (A) of a time contour (Fig. 1) in the complex plane, that lies on the real axis. The other branches of the contour however necessitate the introduction of unphysical degrees of freedom which have support on the branch (B) of the contour. Consequently, we are required to deal with a minimum of four different propagators between AA, BB, AB and BA respectively [14].

Here we just quote the expressions for the fermion and the photon propagators of the AA kind. The fermion propagator is

\[ S_F(k) = \frac{i}{k - m + i\epsilon} - 2\pi(k + m)\delta(k^2 - m^2)n_F(k_0), \]  

where

\[ n_F(k_0) = \frac{\Theta(k_0)}{e^{\beta(k_0 + \mu)} + 1} + \frac{\Theta(-k_0)}{e^{\beta(k_0 - \mu)} + 1}. \]  

The photon propagator in 2+1 dimensions in the Landau gauge is

\[ D_{\mu\nu}(k) = -i \left[ g_{\mu\nu} - k_\mu k_\nu k^2 + iM\epsilon_{\mu\nu\rho} \frac{k^\rho k^2}{k^2} \left( \frac{1}{k^2 - M^2} - 2\pi i\delta(k^2 - M^2)n_B(k_0) \right) \right], \]  

where

\[ n_B(k_0) = \frac{1}{e^{\beta k_0} - 1}. \]  

In one loop calculations we need only these propagators [3].

**Photonic Two point function**

The polarization tensor at finite temperature and density determines the dielectric constant, magnetic permeability and other physically interesting quan-
tities \[4\]. Here we calculate the vacuum polarization to one loop at finite temperature and density \(\mu\). The usual notion of temperature presupposes the existence of a preferred frame of reference. We choose to stay with this definition and thus lose a manifestly covariant description. We therefore compute \(\Pi_{00}\) and \(\Pi_{ij}\) separately \[5\]. In contrast to similar calculations in 3+1 dimensions it is possible to express all the results in terms of simple analytic functions.

The most general expression for the parity-even part of the photon two point function is given by \[4\]

\[
\Pi_{\mu\nu} \equiv \Pi_T(\omega, \vec{k}) P_{\mu\nu} + \Pi_L(\omega, \vec{k}) Q_{\mu\nu} ,
\]

where

\[
P_{\mu\nu} \equiv g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} - \frac{k_0}{k^2}(k_\mu g_{\nu 0} + k_\nu g_{\mu 0}) + \frac{k^2}{k^2} g_{\mu 0} g_{\nu 0} \]

and

\[
Q_{\mu\nu} \equiv -\frac{1}{k^2 k^2}(k_0 k_\mu - k^2 g_{\mu 0})(k_0 k_\nu - k^2 g_{\nu 0}) .
\]

This satisfies the Ward identity

\[
k^\mu \Pi_{\mu\nu} = 0 .
\]

The functions \(\Pi_T\) and \(\Pi_L\) can be obtained from \(\Pi_{\mu\nu}\) as

\[
\Pi_L(\omega, k) = -\frac{k^2}{|k|^2} \Pi_{00}(\omega, k) ,
\]

\[
\Pi_T(\omega, k) = \frac{1}{2} g^{\beta\gamma} \Pi_{ij}(\omega, k) .
\]

We are mostly concerned with the static limit, where the time component \(k_0\) of the four-vector \(k_\mu\) is set to zero. This is appropriate to the case where the linear response to a static external electromagnetic field is investigated. Here, the Ward identity assumes the form

\[
k^\mu \Pi_{\mu\nu} = 0 .
\]

\(^1\) Of course one could formulate the problem covariantly a la Weldon \[4\].
In this particular situation, where $P_{00} = Q_{00} = Q_{ij} = 0$, this is tantamount to
\[ k^i \Pi_{ij} = 0 \tag{11} \]

Thus
\[ \Pi_{ij} = \Pi_T(0, \vec{k})(g_{ij} + \frac{k_i k_j}{k^2}) . \tag{12} \]

It is easily verified that the trace of the polarization tensor is just $\Pi_T(0, \vec{k})$ in this case.

In the real time formalism one can easily separate the vacuum contribution from the finite density and temperature contribution. In the leading order $\Pi_{\mu\nu}(p)$ can be written as
\[ \Pi_{\mu\nu}(p) = i e^2 \int \frac{d^3 k}{(2\pi)^3} T \gamma_\mu S_F(p + k) \gamma_\nu S_F(k) \] (13)
\[ = \Pi_{\mu\nu}^{(0,0)}(p) + \Pi_{\mu\nu}^{(\beta,0)}(p) , \tag{14} \]

where $\Pi_{\mu\nu}^{(0,0)}(p)$ is the zero temperature and density polarization tensor. The temperature and density dependent terms are given by:
\[ \Pi_{\mu\nu}^{(\mu,\beta)}(p) = i e^2 \int \frac{d^3 k}{(2\pi)^3} L_{\mu\nu}(p, k) 2\pi i \left\{ \frac{n_F(k)\delta(k^2 - m^2)}{(k + p)^2 - m^2 + i\epsilon} + \frac{n_F(p + k)\delta((p + k)^2 - m^2)}{k^2 - m^2 + i\epsilon} \right\} , \tag{14} \]
where
\[ L_{\mu\nu}(p, k) = 2[2k_\mu k_\nu + p_\mu k_\nu + p_\mu k_\nu - g_{\mu\nu}(k^2 - m^2 + p \cdot k) + im\epsilon_{\mu\nu\rho}p^\rho] . \tag{15} \]

From this rather general expression for the one-loop polarization tensor, we extract useful information about the screening length of the electric field at finite temperature and density (from the electric mass $m_{el}$), the static dielectric function and the static magnetic susceptibility.

If the electric field due to a charge placed in a medium picks up a mass term as a result of interactions with the medium, it becomes short ranged and
the charge is screened. Screening occurs naturally in a plasma as the charge attracts a halo of charges of the opposite signature around it, polarizing the plasma and thereby effectively reducing its own strength. The starting point for a calculation of the screening length is the vacuum polarization (photonic two point) function in the plasma in the static limit. In fact, the potential energy of a static $e^+ - e^-$ pair, \( V(R, \mu, T) \) is given by

\[
V(R, \mu, T) = \int \frac{d^2 k}{(2\pi)^2} e^{ik \cdot R} V(k, \mu, T) = -\int \frac{d^2 k}{(2\pi)^2} e^{ik \cdot R} \frac{e^2}{k^2} \left[ 1 - \Pi_{00}(0, k)/k^2 \right].
\]  

For large \( R \) this potential becomes \( V(R, \mu, T) \to \frac{e^2}{4\pi} \frac{e^{-m_{el} R}}{\sqrt{m_{el} R}} \), so that it is actually screened with screening length \( \sim \frac{1}{m_{el}} \). In order to calculate the screening length of static external electric fields we compute \( \Pi_{00} \) in the static infrared limit \( \lim_{p \to 0} p^\mu = (0, p) \):

\[
\text{Re}(\Pi_{00}(p)) = m^2_{el} + \tilde{\Pi}_{00}(0)p^2 + \ldots
\]  

We thus first restrict our attention to the real part of \( \Pi_{00} \). From eq.(14) it follows that the terms that contribute to this part are linear in \( n_F \).

\[
\text{Re}(\Pi^{(\mu, \beta)}_{00}(p)) = -e^2 \int \frac{d^3 k}{2\pi^2} (k_0^2 + w^2 + p \cdot k) \left[ \frac{\delta(k_0^2 - w^2)}{k_0^2 - p^2 - 2p \cdot k - w^2} + \frac{\delta(k_0^2 - w'^2)}{k_0^2 - w'^2} \right] n_f(k)
\]

where \( w = \sqrt{k^2 + m^2} \) and \( w' = \sqrt{w^2 + p^2 + 2p \cdot k} \). Expanding the second term in eq.(18) around \( p = 0 \) we find that it exactly cancels the contribution of the first term, with the remaining contribution being easy to evaluate through integration by parts. Thus:

\[
m^2_{el} = \frac{e^2}{2\pi \beta} \log[1 + 2e^{-\beta m} \cosh(\mu \beta) + e^{-2\beta m}] + \frac{e^2 m}{2\pi} \left[ \frac{1}{e^{\beta(m-\mu)} + 1} + \frac{1}{e^{\beta(m+\mu)} + 1} \right],
\]
and

\[ \tilde{\Pi}_{00}(0) = -\frac{e^2}{2\pi m} \left[ \frac{1}{e^{\beta(m+\mu)} + 1} + \frac{1}{e^{\beta(m-\mu)} + 1} \right] + \frac{\beta e^2}{8\pi} \left[ \frac{e^{\beta(m+\mu)}}{(e^{\beta(m+\mu)} + 1)^2} + \frac{e^{\beta(m-\mu)}}{(e^{\beta(m-\mu)} + 1)^2} \right]. \]  

\[ \text{(20)} \]

In the limit \( T > \mu > m \), \( m^2_{el} \) becomes

\[ m^2_{el} \simeq e^2 4 \pi \log(2). \]

For \( \mu > T > m \) we find

\[ m^2_{el} \simeq \frac{e^2 m}{2\pi} [1 + e^{-\beta(m+\mu)}] \] (see fig. 2, 3).

The dielectric function which describes the response of the system to small constant external electric fields is given by

\[ \text{Re} \ \epsilon(p) = 1 + \frac{\Pi_{00}(0, p)}{p^2} = 1 + \tilde{\Pi}_{00}(0, 0) + \frac{m^2_{el}}{p^2}. \]  

\[ \text{(21)} \]

The spatial part of the polarization tensor has the following form in the static limit

\[ \Pi_{ij} = \Pi^T (g_{ij} + \frac{p_ip_j}{p^2}) \]  

\[ \text{(22)} \]

Taking the trace we get \( \Pi^T_0(0, p) = p^2 \Pi^T(p^2) \) or

\[ \text{Re}(\Pi^T_0(p)) = -e^2 \int \frac{d^3k}{\pi^2} (w^2 - k_0^2 - k^2) \left\{ \frac{\delta(k_0^2 - w^2)}{k_0^2 - w^2} + \frac{\delta(k_0^2 - w'^2)}{k_0^2 - w'^2} \right\} n_F(k). \]  

\[ \text{(23)} \]

Expanding \( w' \) and collecting all terms we find

\[ \text{Re}(\Pi^T(0, p)) = \frac{3e^2}{4\pi m} \left[ \frac{1}{e^{\beta(m+\mu)} + 1} + \frac{1}{e^{\beta(m-\mu)} + 1} \right] p^2 + O(p^4). \]  

\[ \text{(24)} \]

This can now be used to determine the magnetic permeability function \( \mu(0, p) \) which is defined by (see Fig. 4)

\[ \frac{1}{\mu(0, p)} = 1 + \frac{\text{Re}\Pi^T(0, p)}{p^2} \left[ \frac{1}{e^{\beta(m+\mu)} + 1} + \frac{1}{e^{\beta(m-\mu)} + 1} \right] + O(p^2). \]  

\[ \text{(25)} \]

We note that as expected \( \epsilon(0, p)\mu(0, p) = 1 \) for \( T \ll m \).

The parity odd part of the polarization tensor is especially interesting in 2+1 dimensions. In particular the parity odd part of \( \Pi_{\mu\nu} \) is known to generate
a topological gauge invariant mass \[11\]. Following \[3\] we write the full real time fermion propagator as

\[
S_{F}^{\mu,\beta}(p) = U_{F}(p_{0})S_{F}^{\beta,0}(p)U_{F}(p_{0})^{\dagger},
\]

where

\[
U_{F}(p_{0}) = \begin{bmatrix}
\cos\Theta(p, \beta \mu) & -e^{\beta \mu/2} \sin\Theta(p, \beta \mu) \\
e^{-\beta \mu/2} \sin\Theta(p, \beta \mu) & \cos\Theta(p, \beta \mu)
\end{bmatrix}
\]

and

\[
\cos\Theta(p, \beta \mu) = \frac{\Theta(p_{0})e^{\beta(p_{0}-\mu)/4} + \Theta(-p_{0})e^{-\beta(p_{0}-\mu)/4}}{\sqrt{e^{\beta(p_{0}-\mu)^{2}} + e^{-\beta(p_{0}-\mu)^{2}}}} \quad \sin\Theta(p, \beta \mu) = \frac{\Theta(p_{0})e^{\beta(p_{0}-\mu)/4} - \Theta(-p_{0})e^{-\beta(p_{0}-\mu)/4}}{\sqrt{e^{\beta(p_{0}-\mu)^{2}} + e^{-\beta(p_{0}-\mu)^{2}}}}.
\]

In this notation the parity odd part of \(\Pi^{\mu\nu}\) becomes

\[
\Pi_{\mu\nu}^{\text{odd}}(p) = -ie^{2} \int \frac{d^{3}k}{(2\pi)^{3}} \text{Tr} [\gamma_{\mu}U_{F}(p+k)\frac{p + k + m}{(p + k)^{2} - m^{2}}U_{F}(p + k)^{\dagger} \gamma_{\nu}U_{F}(k)\frac{(k + m)}{k^{2} - m^{2}}U_{F}(k)]
\]

\[
\simeq -2me^{2}\epsilon_{\mu\rho\nu}p^{\rho} \frac{\partial}{\partial m^{2}} \int \frac{d^{3}k}{(2\pi)^{3}} U_{F}(k) \frac{1}{k^{2} - m^{2}} U_{F}(k)^{\dagger}
\]

\[
\simeq -2me^{2}\epsilon_{\mu\rho\nu}p^{\rho} \frac{\partial}{\partial m^{2}} \int \frac{d^{3}k}{(2\pi)^{3}} \left[ \frac{1}{k^{2} - m^{2}} + 2\pi i\delta(k^{2} - m^{2})m_{F}(k) \right]
\]

where the \(\simeq\) means that we have expanded the integrand around \(p_{\mu} = 0\). Collecting all terms we arrive at the following expression

\[
\Pi_{\mu\nu}(p) = -\frac{ie^{2} m}{4\pi |m|} \left[ \tanh(\beta(m + \mu)) + \tanh(\beta(m - \mu)) \right] + O(p^{2}).
\]

In the limit of zero temperature and zero chemical potential eq.(30) reduces to the result of \[3\], \[5\], \[7\].

**Plasma oscillations**

The application of a time dependent perturbation on a plasma sets up oscillations in the system, which are of the form of longitudinal and transverse traveling waves. This is to be contrasted with the static case of screening discussed earlier, where the introduction of a static charge in the plasma polarized the plasma and screened the electric field.
The longitudinal wave is associated with compression and relaxation of the plasma and its dispersion relation is naturally obtained from the density-density correlation function, namely $\Pi_{00}$. The value of the frequency as the momentum is taken to zero in the dispersion relation yields what may be called the longitudinal plasmon mass. Thus the plasmon is a long-wavelength collective effect in the plasma, embodying the contributions due to the many-body effect. The oscillatory electric and magnetic fields accelerate the electrons and positrons in the plasma and this in turn means that it takes a finite amount of energy to excite an oscillation with vanishing momentum. This is what renders the plasmon massive.

The dispersion relation for the transverse oscillations is obtained, correspondingly from the transverse part of the photon two point function. The frequency at vanishing momentum is the transverse plasmon mass. However, rotational invariance requires that at zero momentum the two plasmon masses should be equal. This has been explicitly seen in 3+1 dimensions [4], [15]. In what follows, we verify this equality for a planar plasma and find an explicit expression for the plasmon mass.

The dispersion relations for the longitudinal and the transverse plasmons are obtained respectively from

$$\omega_{L,T}^2 = k^2 + \Pi_{L,T}(\omega, k)$$  \hspace{1cm} (31)

in the limit where $|k| \ll \omega$. $\Pi_{L,T}$ have already been defined in earlier sections eq.(9). In terms of the components of $\Pi_{\mu\nu}$, they are given by

$$\Pi_L(\omega, k) = -\frac{k^2}{k^2} \Pi_{00}$$  \hspace{1cm} (32)

and

$$\Pi_T(\omega, k) = \frac{1}{2} \left( g^{ij} + \frac{k^i k^j}{k^2} \right) \Pi_{ij}(\omega, k).$$  \hspace{1cm} (33)
Here, we choose to work with a neutral plasma (i.e., we set the chemical potential $\mu = 0$). We expand $\Pi_L$ and $\Pi_T$ in powers of $|p|/p_0$ and in the high temperature limit where we can drop the electron mass in comparison with the temperature, we obtain

$$\Pi_L(\omega, 0) \simeq \frac{e^2}{2\pi} \int_m^\infty dx \, n_F(x)$$

(34)

and

$$\Pi_T(\omega, 0) \simeq \frac{e^2}{2\pi} \int_m^\infty dx \, n_F(x),$$

(35)

where $n_F(x) \equiv \frac{1}{e^{x/T} + 1}$. The integral can be performed trivially and in the high temperature limit, yields

$$m_T^2 = m_L^2 \simeq \frac{e^2}{2\pi} T (\log 2).$$

(36)

Comparing with what we had for the electric mass in the case of screening in an earlier section eq.(19), we see that

$$m_T^2 = m_L^2 = \frac{1}{2} m_{el}^2.$$  

(37)

It is instructive to compare this with what is obtained in 3+1 dimensions. There,

$$m_T^2 = m_L^2 = \frac{1}{3} m_{el}^2.$$  

(38)

Thus we have, in this section explicitly shown the equality of the masses of the two plasmons and have related this mass to the inverse of the Debye screening length obtained earlier.

**Radiative Fermion Mass**

It is well known that a fermion mass term in 2+1 dimensional QED (with an odd number of flavors) breaks invariance under parity explicitly. This results in the radiative generation of a C-S term for the gauge field. This in turn provides a gauge-invariant mass for the gauge field. Here, we are interested
in the converse effect, namely, the radiative generation of fermion mass. This already occurs at zero temperature and density. In what follows, we investigate the effect of finite temperature and density on this radiative mass.

For simplicity we assume a zero bare fermion mass. The radiative mass is computed from the self-energy correction graph using a standard procedure. At zero temperature and density the leading order correction to the fermion mass is obtained from:

\[ \Sigma(p) = ie^2 \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S_F(p-k) \gamma_\mu D^{\mu\nu}(k). \] (39)

This corresponds to:

\[ \Sigma(p) = ie^2 \int \frac{d^3 k}{(2\pi)^3} \frac{L_{\mu\nu}(p,k)}{(p-k)^2(k^2-M^2)} \left[ P^{\mu\nu}(k) + iM \epsilon^{\mu\nu\lambda\rho} \frac{k_\lambda k_\rho}{k^2} \right], \] (40)

where,

\[ L_{\mu\nu}(p,k) = \gamma_\mu (p-k)_\nu + \gamma_\nu (p-k)_\mu - g_{\mu\nu} (p-k)^2 + i\epsilon_{\mu\nu\rho}(p-k)^2. \] (41)

and \( M \) is the topological photon mass. The physical fermion mass is defined as the location of the pole in the propagator. Expanding eq.(41) around \( p = 0 \) we find

\[ m_{\text{phys}} = \frac{e^2}{2\pi} \frac{M}{|M|}. \] (42)

At \( T \neq 0 \) and \( \mu \neq 0 \) however the situation is different. Since charge conjugation invariance is explicitly broken, one expects that in addition to the above diagram there is a contributing “tadpole” diagram. To ensure the overall neutrality of the plasma, an uniform background is introduced, described by a classical current \( J_\mu \), which contributes a term \( J_\mu A^\mu \) to the Lagrangian. In 3+1 dimensions, the contribution of the “tadpole” is infrared singular, as it involves the zero momentum limit of the massless photon propagator. This singular contribution
is seen to be canceled by the background charge distribution chosen to maintain the neutrality of the plasma. In 2+1 dimensions, the contribution of the “tadpole” is not entirely infrared singular. The photon is now massive and the contribution has a finite part in addition to the singular parts. However, the background distribution that cancels these infinities also cancels this finite part, as we show below. Thus, even in 2+1 dimensions where the photon acquires a topological mass, the “tadpole” does not contribute to the fermionic two point function. The second order contribution of the neutralizing external current to the fermion propagator is given by

\[
(-i)^2 \int d^d x d^d y J^\mu(x) (T \bar{\psi}(y) \gamma^\nu \psi(y) A_\mu(x) A_\nu(y)) .
\]

(43)

If we choose the background as uniform, we may represent it as \( J^\mu(x) = e a^\mu \), where \( a^\mu \) is a constant. Thus, the above expression reduces to

\[
- e a^\mu S_F(p) \gamma^\nu S_F(p) D_{\mu\nu}(0) ,
\]

(44)

which after truncating the external fermionic lines yields

\[
- e a^\mu \gamma^\nu D_{\mu\nu}(0) .
\]

(45)

The “tadpole” diagram, on the other hand, gives a contribution of

\[
- ie^2 \int \frac{d^dk}{(2\pi)^d} tr[\gamma^\mu S_F(k)] \gamma^\nu D_{\mu\nu}(0) .
\]

(46)

So, the background field required to nullify the “tadpole” contribution can be read off to be

\[
a^\mu = - ie \int \frac{d^dk}{(2\pi)^d} tr[\gamma^\mu S_F(k)] .
\]

(47)

This can be easily seen to be proportional to the density of the plasma, as it should be in order to render it neutral. Thus,

\[
a^\mu = e g^\mu_0 \rho .
\]

(48)
The neutralizing background obviously vanishes when \( \rho = 0 \), as the plasma in that case contains equal numbers of positrons and electrons and is neutral by itself. This background current gets renormalized by radiative corrections to the “tadpole”, which however do not contribute to the radiative fermionic mass.

Let us now return to (40). At finite temperature and density, the self-energy it is given by

\[
\Sigma(p) = \frac{Me^2}{2\pi^2} \int \frac{d^3k}{(2\pi)^3} (p \cdot k - k^2) \left[ \frac{n_F(p_0 - k_0)\delta((p - k)^2)}{k^2(k^2 - M^2)} - \frac{n_B(k_0)\delta(k^2 - M^2)}{k^2(k - p)^2} \right].
\]

(49)

We evaluate this on the mass-shell, \( p = 0 \) and \( p^2 = 0 \). A rather straightforward evaluation yields

\[
m(T, \mu, e^2, M) = \frac{e^2}{2\pi} \frac{M}{|M|} \left[ 1 + \frac{2T(\log 2)}{|M|} + \frac{2T}{|M|} \log(\cosh(\frac{\mu}{2T})) - \frac{2T}{|M|} \ln(1 - e^{-|M|/T}) \right].
\]

(50)

This result disagrees with that of ref.\[12\] in the overall sign of the finite temperature corrections. For the low temperature limit, we get

\[
m \simeq \frac{e^2}{2\pi} \frac{M}{|M|} \left[ 1 + \frac{T(\log 2)}{|M|} \right].
\]

(51)

On the other hand, the result should not be extrapolated naively to high temperatures. In this limit, the radiative mass grows uncontrollably large and one is called upon to invoke self-consistently arguments.

**Anomalous Magnetic Moment**

The anomalous magnetic moment of fermions is defined by using the Gordon decomposition for the fermionic vector current. In 2+1 dimensions it has the following form:

\[
\bar{u}(p + q)\gamma_\mu u(p) = \frac{(p + q)\mu}{2m} \bar{u}(p + q)u(p) + i\epsilon_{\mu\nu\lambda} q^\lambda \frac{q^\nu}{2m} \bar{u}(p + q)\gamma^\nu u(p).
\]

(52)

It is easy to see from this expression that if we couple the fermions to an external electromagnetic field the coupling of the second term describes the coupling to
the magnetic field. Thus the magnetic moment is the coefficient of $i\epsilon_{\mu\nu\lambda}\frac{\alpha^\nu q^\lambda}{m}$.

To lowest order in perturbation theory the only diagram that contributes is the vertex correction diagram:

\[
\Lambda_\mu(p, q) = (\mp)^2 \int \frac{d^2 k}{(2\pi)^2} \tilde{u}(p - q) \gamma_\nu S_F(p - k - q) \gamma_\mu S_F(p - k) \gamma_\lambda u(p) D^{\nu\lambda}
\]

\[
= \Lambda_\mu^0(p, q) + \Lambda_\mu^\beta(p, q), \quad (53)
\]

where $\Lambda_\mu^0(p, q), \Lambda_\mu^\beta(p, q)$ are the zero and finite temperature vertex functions respectively. The zero temperature calculation has been done in [16], $\mu^0 = \frac{1}{m} \left[ \frac{1}{2} + \frac{1}{\kappa} \right]$, where $\kappa = 4\pi M$. The anomalous contribution to the magnetic moment is extracted from the real part of the three point function with on-shell electrons and vanishing external spatial momenta:

\[
Re \Lambda_i^\beta = \int \frac{d^2 k}{(2\pi)^2} \frac{L_i(k, p, q) \delta(k^2 - M^2)}{[(k - p)^2 - m^2][(k - q)^2 - m^2]n_b(k) + \delta(k^2 - m^2)n_F(k)} \n \left[ \frac{L_i(k + p, p, q)}{[(k - q + p)^2 - m^2][(k + p)^2 - M^2]} + \frac{L_i(k + p - q, p, q)}{[(k - q - p)^2 - m^2][(k + p - q)^2 - M^2]} \right] \n - [\text{same with } M = 0], \quad (54)
\]

where

\[
L_i(k, p, q) = 2[a_i k_j k_i q_j^m + 2i k \cdot p k_i - i q^i k_i - i(2p - q) k^i - 2p_0 q_i k_m \epsilon^{0jm} \gamma_j].
\]

Using $q^2 \ll m, M$ we find

\[
Re \Lambda_i^\beta(p, q) = \frac{p_0 q_j \epsilon_i^0}{m} \int_M^\infty \frac{d w}{2\pi} \frac{w^2}{[M^4 - 4m^2 w^2]^2} n_b(w) + \int \frac{d w}{2\pi} \frac{w^2}{[M^4 - 4m^2 w^2]^2} n_F(w), \quad (56)
\]

A closed form expression for these integrals cannot be given in the general case. However, in the limits of high and low temperature these integrals can

\footnote{the alternate limit i.e. taking $p_\mu = (0, \vec{p})$ and letting $\vec{p} \to 0$ leads to different results, however this was shown recently [4] to be an artifact of the perturbative expansion.}
be reduced to simple analytic functions. In the low temperature limit one can replace \( n_B(w) \sim e^{-\beta w} \) and \( n_F(w) \sim e^{-\beta w} \). We then find

\[
\Lambda^\beta(p,q)_i = q_j p_0 e^{j_0} \mu(T) = p_0 q_j e^{j_0} \left[ \frac{1}{32 \pi} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{m^4} + \frac{1}{32 \pi} \right] \left[ \left( \frac{\beta M^2}{2m} \right) e^{-\frac{\beta M^2}{2m}} + E_1\left( \frac{\beta M^2}{2m} \right) e^{-\frac{\beta M^2}{2m}} \right] - 8 \beta m^4 E_1(\beta m) + \frac{16 M^2 m^3}{2 m^2 + M^2} e^{-\beta m} + \beta (M^2 - 2m^2)^2 \left( E_1\left( \frac{\beta (2m^2 + M^2)}{2m} \right) e^{\frac{\beta M^2}{2m}} + E_1\left( \frac{\beta (2m^2 + M^2)}{2m} \right) e^{-\frac{\beta M^2}{2m}} \right) \left( -8 \beta m^4 E_1(\beta m) + \frac{16 M^2 m^3}{2 m^2 + M^2} e^{-\beta m} \right) + \beta (M^2 - 2m^2)^2 \left( E_1\left( \frac{\beta (2m^2 + M^2)}{2m} \right) e^{\frac{\beta M^2}{2m}} + E_1\left( \frac{\beta (2m^2 + M^2)}{2m} \right) e^{-\frac{\beta M^2}{2m}} \right) \left( -8 \beta m^4 E_1(\beta m) + \frac{16 M^2 m^3}{2 m^2 + M^2} e^{-\beta m} \right) \right] \frac{1}{m^4},
\]

where \( E_1(x) = \int_x^{-\infty} \frac{e^t}{t} \) is the exponential integral. Fig. 5 shows the behavior of this function for small \( T \). This result together with the calculation of the parity odd part of the photon propagator can be used to calculate the gyromagnetic ratio \( g \) at finite temperature. At zero temperature \( g \) is defined by

\[
\mu = g \frac{2mS}{2} = g \left( \frac{1}{2} + \frac{1}{\kappa} \right).
\]

For anyons, i.e. arbitrary \( \kappa, S \), it has been observed [17] that \( g \) is also 2 and that this is an exact result. To one loop this has been verified in perturbation theory. Here we confirm this result by taking the limit \( \mu, T \to 0 \), when \( Re(\Lambda^\beta) \to 0 \) exponentially.

**Evaluating the chemical potential**

In order to write our results in terms of physical quantities one must replace the lagrange multiplier \( \mu \) in all the physical quantities that we have computed by the temperature dependent particle density \( \rho/e \). To do this we calculate \( \rho \) to leading order in \( e^2 \).

\[
\frac{\rho}{e} = \langle \bar{\psi} \gamma_0 \psi \rangle = \int \frac{d^2 k}{(2\pi)^2} Tr(\gamma_0(k + m)) 2\pi \delta(k^2 - m^2) n_F(k_0)
\]

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\[
\int_{m}^{\infty} \frac{dw}{2\pi} \frac{1}{\beta} \left\{ \log \left[ \frac{1 + e^{-\beta(w+\mu)}}{1 + e^{-\beta(w-\mu)}} \right] \right\} = \frac{1}{2\pi} \left\{ \frac{\text{dilog}(1 + e^{-\beta(m+\mu)})}{m + \mu} - \frac{\text{dilog}(1 + e^{-\beta(m-\mu)})}{m - \mu} \right\}
\] (58)

**Conclusions**

In this paper we have discussed various properties of a planar electron positron plasma. Static properties like the screening length were calculated using a real time formalism. Dynamic properties such as the plasmon frequency were also calculated; We find that \((m_{pl}^L)^2 = (m_{pl}^T)^2 = \frac{1}{2} m_{el}^2\). The coefficient of the Chern-Simons term which is unique to 2+1 dimensional gauge theories with explicitly broken parity, was calculated for finite temperature and density. In the limit of zero density the results agree with those of [3], [5]. In this limit we have also computed the correction to the electron magnetic moment to one loop and extracted the value of the gyromagnetic ratio. The value of \(g\) goes smoothly to 2 as \(T \to 0\), thus agreeing with the zero temperature result of [16], [17].

Even though we have discussed relativistic systems one could also use the results for a condensed matter system where the quasi-particles have linear dispersion relation. The extension of to the nonabelian case might be interesting especially since some condensed matter systems can be represented by quasi-particles with non-abelian interactions.

A further interesting feature of this system that one can address fruitfully is the effect of dissipative processes. One can discuss the viscosity of the plasma as well as the damping of the plasmon modes due to dissipation. Work in this direction is in progress.

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References

[1] W. D. Brown, R. D. Puff and L. Willets, Phys. Rev. C 2, 331, (1970)

[2] B. Rosenstein, B. J. Warr and S. H. Park, Phys. Rep. 205, 59, (1991)

[3] K. S. Babu, Ashok Das and Prasanta Panigrahi, Phys. Rev. D 36, 3725 (1987)

[4] H. A. Weldon, Phys. Rev. D 26, 1394, (1982)

[5] I. J. R. Aitchison, C. D. Fosco and J. A. Zuk, Phys. Rev. D 48, 5895, (1993)

[6] José F. Nieves and Palash B. Pal, “The zero-momentum limit of thermal Green functions” [hep-ph 9402290] 1994

[7] Ashok Das and Sudhakar Panda, “Temperature dependent Anomalous Statistics” J. Phys. A 25, L 245, (1992)

[8] S. Midorikawa, Prog. Theor. Phys. 67, 661, (1982)

[9] A. I. E. Johansson, G. Peressutti and B. S. Skagerstam, Nuc. Phys. B 278, 324, (1986)

[10] L. Dolan and R. Jackiw, Phys. Rev. D9, 3320, (1974)

[11] S. Deser, R. Jackiw and S. Templeton, Ann. Phys. 140, 372, (1982)

[12] Y-C Kao, Mod. Phys. Lett. A6, 3261, (1991)

[13] N. Redlich, Phys. Rev. D 29, 2366, (1984); Phys. Rev. Lett. 52, 18, (1984)
[14] See for example N. P. Landsman & Ch. G. van Weert, *Phys. Rep.* 145, 141, (1987) and the extensive list of references therein

[15] J.I. Kapusta, *Finite Temperature Field Theory*, Cambridge University Press, (1989)

[16] I. I. Kogan and G. W. Semenoff *Nuc. Phys. B* 368, 718 (1992)

[17] Chi-hong Chou, V. P. Nair and A. P. Polychronakos, *Phys. Lett* B 304, 105 (1993)

[18] R. Efraty and V. P. Nair, *Phys. Rev* D 47, 5601 (1993)
Figure 1: The Contour of integration in the Complex Time Plane
Figure 2: \( m_{\text{cl}}^2 \) as a function of \( T \) for \( \mu = 0.5 \).
Figure 3: $\frac{\Omega(0,0)}{e^2}$ as a function of $T$ for $\mu = 0.5$. 
Figure 4: Leading term in the expansion of $\frac{1}{e^{\mu(0,p)} - 1}$ in $p$ as a function of $T$ for $\mu = 0.5$. 
Figure 5: The temperature dependent part of the leading term of the anomalous magnetic moment of electrons ($\mu = 0$). Here we take $m = 1$ and $M = 10$. 

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