A Statistical Stability Analysis of Earth-like Planetary Orbits in Binary Systems

MARCO FATUZZO,1 FRED C. ADAMS,2,3 RICHARD GAUVIN,1 AND EVA M. PROSZKOW2

ABSTRACT. This paper explores the stability of an Earth-like planet orbiting a solar-mass star in the presence of a stellar companion, using \(~400,000\) numerical integrations. Given the chaotic nature of the systems being considered, we perform a statistical analysis of the ensuing dynamics for \(~500\) orbital configurations defined by the following set of orbital parameters: the companion mass \(M_c\), the companion eccentricity \(e\), the companion periapsis \(\rho\), and the planet’s inclination angle \(i\) relative to the stellar binary plane. Specifically, we generate a large sample of survival times \(\langle \tau \rangle\) for each orbital configuration through the numerical integration of \(N \gg 1\) equivalent experiments (e.g., with the same orbital parameters but randomly selected initial orbital phases). We then construct distributions of survival time using the variable \(\mu_s \equiv \log \tau\), (where \(\tau\) is in years) for each orbital configuration. The primary objective of this work is twofold. First, we use the mean of the distributions to gain a better understanding of orbital configurations that, while unstable, have sufficiently long survival times to make them interesting to the study of planet habitability. Second, we calculate the width, skew, and kurtosis of each \(\mu_s\) distribution and look for general features that may aid further understanding and numerical exploration of these chaotic systems. To leading order, most distributions are nearly Gaussian, with a width of \(\sigma \approx 0.5\), although the longest-lived systems display non-Gaussian tails. As a result, many independent realizations of these systems must be considered in order to characterize the survival time. The situation is more complicated for orbital configurations with longer mean survival times, owing in part to the increasing importance of resonances.

1. INTRODUCTION

Recent (and ongoing) discoveries of exoplanetary systems (e.g., Butler et al. 1999; Marcy et al. 2001; Fischer et al. 2002; Tinney et al. 2002; McArthur et al. 2004) have shown that Sun-like stars are orbited by planets with a wide variety of orbital configurations. Of course, current planetary searches are biased toward large bodies with short orbital periods, resulting in the discovery of planets with masses ranging from \(\sim 0.01 M_J\) to \(10 M_J\) and semimajor axes typically ranging from \(\sim 0.04\) to \(4\) AU. It is expected that Earth-like planets will also form alongside their Jovian counterparts (e.g., Ruden 1999; Lissauer 1993), even in binary stellar systems (e.g., Whitmire et al. 1998; Quintana et al. 2002; Lissauer et al. 2004).

The discovery of exoplanetary systems has spurred a renewed interest in planetary dynamics, with a significant amount of attention being given to the stability of specific multiplanet systems (e.g., Laughlin & Adams 1999; Süli et al. 2005) and to the stability of (hypothesized) terrestrial planets located within a specific system’s habitable zone (e.g., Noble et al. 2002; Asgari et al. 2004; Jones et al. 2005; Haghighipour 2006; Ji et al. 2005). In addition, the identification of Jupiter-like planets in orbit around members of multiple-star systems (e.g., Eggenberger et al. 2004 and references therein) has also provided an observational basis for studying planetary stability in binary systems (e.g., Harrington 1977; Pendleton & Black 1983; Rabl & Dvorak 1988; Holman & Wiegert 1999; David et al. 2003; Pilat-Lohinger et al. 2003; Musielak et al. 2005; Mudryk & Wu 2006). Of course, one must differentiate between stability within a habitable zone and the more general study of overall system stability, with the former imposing stricter conditions on a planet’s allowed orbital motion.

Planetary orbits in binary systems can be found in a wide range of configurations. The two most common orbital classes are P-type orbits, in which the planet’s semimajor axis is larger than that of the binary, so the planet orbits about both stars, and S-type orbits, in which the planet revolves around one of the stars, with a semimajor axis smaller than that of the binary (e.g., Szebehely 1967; Dvorak et al. 1989; Pilat-Lohinger & Dvorak 2002). If the semimajor axis of the planetary orbit is sufficiently large (for P-type orbits) or sufficiently small (for S-type orbits), the orbital motion is stable and well ordered. However, in binary configurations for which the stellar bodies come sufficiently close to the planet, the motion can be chaotic and even unstable. One goal of this paper is to determine the regime of binary parameter space for which S-type planetary orbits are stable. In the regime of parameter space near the border between stability and instability, the orbits tend to be chaotic.

A complete analysis of planetary stability in stellar binaries
must reconcile the underlying chaotic nature of the systems being explored. While theoretical conditions can be used to determine whether a planetary system is unstable, it is still possible for such systems to last for vast spans of time. Numerical work must then be used to delineate the boundaries of “effectively” stable parameter space. However, for a given orbital configuration, the survival time \( \tau \) of an unstable system (defined throughout this work as the number of years it takes for the planet to either be ejected or collide with either star) varies widely, depending on the choices of initial orbital phases. Because the systems are chaotic; this variation is not smooth; i.e., small differences in the starting phase angles can lead to large differences in the resulting survival times. As a result, the survival time for any set of binary properties can only be fully characterized in terms of a distribution of output measures. Furthermore, as shown herein, the distribution of survival times has a substantial width. Even though the systems are chaotic in the regime of interest and hence display a wide distribution of survival times for effectively equivalent starting conditions, we stress that the distributions themselves are well defined. As a result, the answer to the question, how long does a planet survive in any given binary system? is a full distribution. One goal of this work is to find such distributions for binary systems with intermediate semimajor axes \( a \approx 1–50 \) AU.

A statistical analysis of the stability of an Earth-like planet orbiting a one solar-mass star in the presence of an outlying companion was recently performed by David et al. (2003; hereafter D03), leading to an estimate of the fraction \( F_b \) of binary star systems that allow Earth-like planets to remain in the system over a timescale of 4.6 Gyr. Specifically, D03 found that \( F_b \approx 0.5 \) by calculating the survival time of an Earth-like planet (with an initial circular orbit of \( R = 1 \) AU) for a range of companion masses \( M_c = 0.001–0.5 \) \( M_\odot \), initial eccentricities \( e \), and semimajor axis \( a \). For the parameter space explored by D03, the planet’s survival time (for a specified companion mass) depends most strongly on the companion’s initial periastron distance \( p = a(1 - e) \) but spans over two decades when sampled over a range of semimajor axis values (with \( e \) then set to give the desired value of \( p \)). Nevertheless, the mean value of \( \log \tau \) exhibits a clear exponential dependence on the periastron. It is important to note that the range of survival times versus periastron presented in D03 results from both the chaotic nature of the three-body problem and the sampling over different \((a, e)\) pairs for a given periastron value (see \$5.2 for a more complete discussion).

This paper extends previous work on the stability of an Earth-like planet orbiting a Sun-like star in the presence of a stellar companion. Although much of the previous work focused on coplanar systems (e.g., D03), this work explores the full range of inclination angles. Specifically, we explore \( \sim 500 \) different orbital configurations (defined through the choice of the companion mass \( M_c \), eccentricity \( e \), and periastron \( p \), and the planet’s inclination angle \( i \) relative to the stellar orbital plane), and we broadly sample the orbital parameter space inhabited by most observed binary systems. Our main body of work explores 376 different orbital configurations organized in four series (1–4). An additional 136 orbital configurations with low inclinations, organized in three series (5–7), are performed to further explore some of the detailed structure exhibited by the output measures of the first four series. We generate a large sample of survival times \( (\tau) \) for each orbital configuration through the numerical integration of \( N \gg 1 \) equivalent experiments (with the same basic orbital parameters but randomly selected initial orbital phases), performing a total of \( \sim 390,000 \) separate orbital simulations; this sample of orbital simulations is thus an order of magnitude larger than our previous study (D03). This broad survey of parameter space provides a cleaner delineation of the orbital configurations that (while potentially unstable) have sufficiently long survival times to be interesting for planetary habitability (as noted above, however, stability within a planet’s habitable zone imposes additional constraints on the planet’s orbital motion).

Another focus of this new work is the characterization of the distributions of survival times. As discussed above, the regime of parameter space near the stability/instability border is chaotic, and the results must be described statistically. Performing multiple realizations for each orbital configuration is a necessity. This paper constructs distributions of survival times—using the variable \( \mu_s \equiv \log \tau, \) with \( \tau \) defined in years—for each orbital configuration under consideration. The resulting distributions are then characterized by their mean, width, skew, and kurtosis. We also calculate the fraction of runs that lead to the planet’s ejection from the system (and the fraction accreted by one of the stars). With the construction of these distributions, we can study how they vary over the regime of binary parameter space; these distributions also provide important guidance for future work (e.g., the width of the distribution determines how many independent realizations of the numerical problem are necessary to achieve a desired accuracy in estimating the mean value).

The paper is outlined as follows: We discuss the numerical method used in our work in \$2. We present the results of our numerical work for series 1–4 in \$3 and characterize the resulting distributions for these series in \$4. We present the results of series 5–7 in \$5 and use the results of these runs to explore certain aspects of the rich structure exhibited by these three-body systems. Specifically, we (1) consider the effect of integer ratios of initial planet/companion orbital periods—a necessary but not sufficient condition for resonance (see, e.g., Murray & Dermott 2000), (2) explore more fully the dependence of ejection time on eccentricity and on periastron, (3) characterize the fraction of ejection events, and (4) consider the stability exhibited by high inclination, low eccentricity orbits. We present our conclusions in \$6.

2. NUMERICAL SCHEME

We present here the numerical method by which we calculate the survival time of an Earth-like planet orbiting a Sun-like star in the presence of a stellar outer companion. Through long-
term dynamical interactions with the outer companion, the orbital elements of the Earth-like planet evolve, generally in chaotic fashion, until the planet is either ejected from the system or collides with one of the two stars. In order to explore this stability issue on intermediate timescales, Newton’s equations of motion are integrated directly, using a Bulirsh-Stoer (B-S) scheme (Press et al. 1992). Although direct integration is computationally more expensive (e.g., compared to symplectic integration), it is accurate and explicit. For the systems at hand, our B-S scheme incurs errors in relative accuracy of an order of 1 part in $10^{11}$ per total time step, where each time step in the three-body problem is variable but has a typical value of about 10 days. The accumulated error for a given integration is variable but has a typical value of $\sim 10^{-5}$ for the systems that yield the longest survival times in our study (where $a_p$ is the Earth’s semimajor axis), we check a posteriori that $\Delta E/E$ remains less than $\sim 10^{-8}$ to ensure that accumulated errors do not affect our results.

The planet’s initial orbit is always set to be circular ($e_p = 0$), with radius $R = a_p = 1$ AU. The companion mass $M_c$, eccentricity $e$, and periastron $p$, as well as the planet’s initial inclination angle $i$ (relative to the stellar binary plane), are then specified for each run, and the system is integrated forward in time. For the sake of definiteness and in order to cover a large range of parameter space, we use an upper limit integration time of $\tau_{\text{run}} = 10^5$ yr for the runs presented in § 3 (series 1–4) and $\tau_{\text{run}} = 10^6$ yr for the runs presented in § 5 (series 5–7). Our experiments therefore give us either a timescale for survival or a lower limit of $\tau_{\text{run}}$ on the possible survival time. The planet is considered to be ejected if any of the following conditions are met: the energy of the planet becomes positive, the eccentricity of the planet exceeds unity, or the semimajor axis of the planet exceeds a maximum value (taken here to be 100 AU). The planet is considered to collide with the solar-mass star if the periastron of the planet becomes smaller than the stellar radius (assumed to be $1 R_\odot$) so that the planet is accreted, and to collide with the companion if its orbit crosses within one radius of the companion’s center-of-mass position (although for the orbital configurations considered here, the latter result is effectively ruled out). The survival time for a given orbital configuration is explored statistically by sampling over $N$ equivalent realizations set through the random assignment of the initial orbital phase angles, where $N = 10^3$ for series 1–4 (presented in § 3) and $N = 10^2$ for series 5–7 (presented in § 5).

### 3. RESULTS OF PARAMETER SPACE SURVEY

This section presents the results of 376,000 numerical experiments, with orbital parameters organized into four series with the following values for $M_c$ and $e$, respectively: (1) 0.1 $M_\odot$, 0.5; (2) 0.5 $M_\odot$, 0.5; (3) 0.5 $M_\odot$, 0.75; and (4) 0.5 $M_\odot$, 0.25. Values for the companion periastron and the planet’s inclination angle relative to the Sun-companion orbital plane range from $p = 2$ to 60 AU and $i = 0^\circ$ to 90$^\circ$, respectively, for each series.\(^3\) A total of $N = 10^4$ equivalent realizations (with initial phase angles sampled randomly) were performed for each of the 376 different orbital configurations explored (defined by the parameter space four-vector $\{M_c, e, p, i\}$).

For the chaotic systems being explored herein, different choices of initial phase angles can lead to widely different dynamical behavior, and hence the values of survival time $\tau$, can differ by orders of magnitude for effectively equivalent starting states. The survival time for a given orbital configuration is best characterized in terms of a distribution in $\mu_s = \log \tau$, (where the survival time is expressed in years). Each orbital configuration is therefore characterized by the mean or expectation value $\langle \mu_s \rangle$, the width

$$\sigma = \left[ \frac{\sum (\mu_s - \langle \mu_s \rangle)^2}{N} \right]^{1/2},$$

the skew

$$sk = \left[ \frac{\sum (\mu_s - \langle \mu_s \rangle)^3}{N\sigma^3} \right],$$

and the kurtosis

$$ku = \left[ \frac{\sum (\mu_s - \langle \mu_s \rangle)^4}{N\sigma^4} \right] - 3$$

of its corresponding distribution. A secondary output measure also explored here is the fraction $f_e$ of ejection events, defined as the number of runs that result in the planet being ejected from the system, divided by the total number of ejection/accretion events (see § 5 for further discussion).

The calculated values of $\langle \mu_s \rangle$ are plotted versus inclination angle $i$ for several periastron tracks in Figure 1, with series 1–4 separated into Figures 1a–1d. For all cases, a clear transition between longer- and shorter-lived systems occurs at $i \approx 40^\circ$, in agreement with results of previous studies (e.g., Harrington 1977; Pendleton & Black 1983; Innanen et al. 1997; Haghighipour 2006). In addition, a clear region of stability is found at $i \approx 60^\circ$ for series 4. Note that Kozai resonances occur for\(^5\) Although high-inclination configurations have been included for completeness, observations suggest that planetary systems are likely to form in binaries with relatively small ($i \approx 20^\circ$) inclination angles. For example, disks in binary systems are close to coplanar (e.g., Mathieu et al. 1991).

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\(^4\) The error accumulation involves a random walk process so that high accuracy can (usually) be maintained even when the product of the error per time step and the number of time steps exceeds unity.

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large inclination angles (see § 5 for further analysis and discussion).

Clear trends are exhibited by our results. As expected, survival time depends strongly on the companion mass and periastron, and somewhat weakly on inclination angle for $i \leq 40^\circ$. In addition, Figures 1b–1d, corresponding to series 2–4 ($M_c = 0.5 M_\oplus$; $e = 0.5$, 0.75, and 0.25), indicate that while survival times increase with eccentricity for periastron values below $p \sim 3$ AU (as one would naively expect, given the longer orbital period of the companion orbits), the opposite trend is suggested for periastron values greater than 3 AU. The dependence of survival time on eccentricity is further explored in § 5. The robust structure of orbital systems (being chaotic in nature) is also clearly illustrated by the interweaving of the $p = 2.5$ AU and $p = 3$ AU tracks in Figure 1a. Naively, one expects the survival time to increase with increasing periastron, and this general trend is clearly seen in all four panels of Figure 1. The enigmatic behavior of the $p = 3$ AU case in series 1 thus suggests some type of resonance behavior. Indeed, this set of orbital parameters corresponds to an initial companion orbital period exactly 14 times larger than that of the planet, a necessary but not sufficient condition for resonance to occur.
One of the most important aspects of this work is the characterization of the \( \mu_i \) distribution widths. These values quantify the degree of chaos in these three-body systems and allow one to determine the precision of the mean value (\( \langle \mu_i \rangle \)) calculated from \( N \) equivalent numerical experiments. For example, if the parent distribution is nearly Gaussian, then the mean value calculated from \( N \) equivalent numerical experiments has a 68%, 95%, and 99% probability of being within 1, 2, and 3 \( \sigma/\sqrt{N} \), respectively, of the true mean.

The dependence of the distribution width on inclination angle and mean survival time (characterized by \( \langle \mu_i \rangle \)) for the distributions of the 376 orbital configurations in series 1–4 is illustrated by the scatter plots presented in Figures 3a and 3b. The solid line in Figure 3a connects the mean widths calculated at each inclination angle, determined by combining all of the results from orbital configurations with the same inclination angle (but offset by the mean so as to center each distribution about zero) to form a single distribution, and then calculating the ensuing width as per equation (1). Most of the distributions for the orbital configurations considered in this section have widths in the range \( \sigma \approx 0.2–0.6 \). In addition, the distribution width has only a weak dependence on inclination angle and no clear trend that differentiates the four series from each other. The distribution width also seems to be independent of the mean survival time for orbital configurations with \( \langle \mu_i \rangle \approx 4 \), although wider distributions appear increasingly likely when \( \langle \mu_i \rangle \approx 4 \). This result is further explored in \( \S \) 5.

A proper interpretation of the meaning of a width calculated from a sample distribution requires an underlying assumption about the parent distribution from which the sample was drawn. To assess the meaning of the values of \( \sigma \) calculated for the orbital configurations in series 1–4, we need to gauge whether a Gaussian approximation for the calculated \( \mu_i \) distributions is warranted. To this end, we present scatter plots of skew and kurtosis versus width in Figures 3c and 3d for the distributions of the 376 orbital configurations of series 1–4. The dotted lines in these figures represent the 3 \( \sigma \) values (for the parameter relevant to the given panel) calculated through random sampling (with \( N = 10^3 \)) of a Gaussian parent distribution. For example, the skew of a sample distribution built up by randomly sampling a Gaussian parent distribution 10\(^7\) times would have a 99% probability of falling within the dotted lines of Figure 3c. Clearly, the overwhelming number of orbital configurations leads to distributions with positive skew and kurtosis, with most skew values ranging between 0 and 2, and most kurtosis values ranging from 0 to 5. No obvious trends differentiate series 1–4 in this respect. As a point of reference, we note that the distributions shown in Figure 5 (plotted in terms of \( \ln P \)) have values of skew between 1 and 1.2, and values of kurtosis between 5 and 8. Clearly, while the distributions explored in this section are not perfectly Gaussian, they

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The characterization of the survival times through the output measure \( \mu_i = \log \tau \) was motivated by the large dynamic range of survival times exhibited by different realizations of the same orbital configuration. Although the use of a logarithmic time unit is arbitrary, it appears to be robust for the cases explored in series 1–4. Specifically, the distributions of \( \mu_i \) for most of the orbital configurations presented in \( \S \) 3 are nearly Gaussian to leading order (i.e., \( \tau_i \) is log-normally distributed); furthermore, the distributions have roughly the same widths, of \( \sigma \approx 0.5 \). To further illustrate this point, Figures 2a–2c show the distributions of \( \mu_i \) values obtained for orbital configurations with \( M_c = 0.1 \, M_\odot \), \( p = 2.5 \, \text{AU} \), \( e = 0.50 \), and \( i = 0^\circ, 25^\circ \), and \( 50^\circ \), along with corresponding Gaussian distributions (solid line) with the same computed mean, width, and normalization.\(^6\)

Interestingly, distributions with \( \sigma \approx 0.7 \) in our survey also appear to have Gaussian peaks with widths of \( \sim 0.5 \), where the presence of tails leads to the higher computed values of the total widths. As an example, the distribution for the orbital configuration \( M_c = 0.5 \, M_\odot \), \( p = 3.5 \, \text{AU} \), \( e = 0.50 \), and \( i = 35^\circ \) is shown in Figure 2d. Again, the solid line represents a Gaussian profile with the same width, mean, and normalization as the computed distribution. Clearly, the presence of a

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\( \S \) 3

The degree to which a given distribution is non-Gaussian is measured by its skew and kurtosis. The skew reflects how symmetric about the mean value a distribution is, with “right heavy” distributions having positive skews. The kurtosis reflects how peaked or flattened a distribution is, with flatter than normal distributions having a positive kurtosis, and pointier than normal distributions having a negative kurtosis (e.g., Press et al. 1992).
Fig. 2.—Distributions of $N = 10^3$ equivalent values of $\mu_p = \log \tau_s$ (i.e., set through a random sampling of the initial phase angles) for the following four orbital configurations: (a) $M_C = 0.1 M_\odot$, $p = 2.5$ AU, $e = 0.5$, and $i = 0^\circ$; (b) same as (a), but with $i = 25^\circ$; (c) same as (a), but with $i = 50^\circ$; (d) $M_C = 0.5 M_\odot$, $p = 3.5$ AU, $e = 0.5$, and $i = 35^\circ$. The calculated distribution width, skew, and kurtosis for each distribution is shown in each panel. The solid curve shows a normal distribution with the same mean and width as the computed distributions, normalized to the sample size of $N = 10^3$.

evertheless have Gaussian-like peaks and, to first order, can be reasonably well approximated by Gaussian profiles. Furthermore, the departure of the distributions from a Gaussian form occurs through a tail at high values of $\mu_p$.

To gain further insight into the distribution widths expected from this class of systems, we produce histograms of survival times for subsets of the orbital configurations in series 1–4, with each survival time normalized about the mean of its own distribution (i.e., all of the distributions are normalized so that their mean is zero, and then combined to form new distributions for specific subsets of the data). Figure 4 shows the resulting histograms for each series, plotted in terms of the natural logarithm of the probability $P$ and normalized values of $\mu_p$. Figure 5 presents the resulting histograms for series 2–4 (Fig. 5a) and for various angle subsets of series 2–4 (Figs. 5b–5d). The width of each distribution is included in the figures; the skew and kurtosis for each of the histograms range from 1 to 1.6 and from 3 to 8, respectively. In all cases, power exists in the wings, but a clear peak is always observed. Figure 6 presents probability histograms of $\sigma$ for all of the distributions within each of the four series (these histograms thus represent the distribution of distribution parameters). The calculated mean width $\langle \sigma \rangle$ and distribution width $\sigma_0$ are denoted in each panel. We note that the calculated values of $\langle \sigma \rangle \sim 0.35$ are slightly lower than the mean.
of the distributions shown in Figures 4 and 5. Similar histograms for all of the distributions in series 2–4, as well as the low-angle \( (i \leq 30^\circ) \) subset, are presented in Figure 7.

For the orbital configurations explored in this section, distributions have typical widths in the range \( \sigma \sim 0.2–0.6 \). Taken at face value, a “typical” distribution of \( \mu_s \) for the most likely orbital configurations found in our galaxy (i.e., \( i \lesssim 20^\circ \)) is expected to have a width of \( \sigma \sim 0.5 \) (see § 5.3 for a discussion of further complications). As a result, the value of \( \langle \mu_s \rangle \) calculated through a random sampling with a sample size of \( N \) would have a 68% probability of being within \( \sim 0.05(100/N)^{1/2} \) of the true value, and a 99% probability of being within \( \sim 0.15(100/N)^{1/2} \) of the true value, assuming that the parent distribution is nearly Gaussian. This result provides important guidance on the survival time of planetary systems for future studies.
5. STRUCTURE IN THE OUTPUT VARIABLES

The chaotic nature of three-body problems can lead to a rich amount of structure in the output variables that characterize the underlying dynamics. Indeed, a fair amount of structure is evident in the output measures presented in § 3. We explore certain aspects of that structure here through three additional series of runs. Specifically, we consider (1) the structure resulting from integer ratios of the initial companion/planetary orbital periods, (2) the dependence of survival time on periastron and eccentricity, (3) structure in the output measure \( f \) (the fraction of simulations for a given orbital configuration that lead to the planet’s ejection), and (4) the structure at \( \sim 60^\circ \) evident in Figure 1d. The numerical experiments presented in this section (series 5–7) were integrated to a maximum run time of \( \tau_{\text{run}} = 10^5 \) yr. To compensate for this increase, the number of equivalent experiments performed for each orbital configuration was reduced to \( N = 10^3 \). Based on the results of § 4, we still expect good statistical results. The overall goal of this section is to explore further the aforementioned observed structure in the output.
variables, to provide some insight into the underlying mechanisms, and to relate this work to results of previous studies of orbital dynamics.

5.1. Mean Motion Orbital Resonances

As noted in § 3, the mean value of $\mu_s \equiv \log \tau_s$ generally increases with increasing periastron. This result is expected, given that an increase in periastron corresponds to both an increase in distance of closest approach between the planet and companion and an increase in the companion’s orbital period $P_c$ (for a constant eccentricity). However, orbital configurations in series 1 with $p = 3.0$ AU and inclination angles in the range $25^\circ \leq i \leq 60^\circ$ have $\mu_s$ distributions with means that are smaller than all of their $p = 2.5$ AU counterparts, and smaller than their $p = 2.9$ AU counterparts at all but the largest inclination angles. As noted in § 3, this result suggests the presence of a resonance condition at $p = 3.0$ AU, and indeed, the companion and planet’s orbital periods are in a ratio of $n_c : n_p = 14 : 1$.

The presence of resonance conditions can play an important role in the dynamics of a planetary system—and they certainly do so in our own solar system (see, e.g., Murray & Dermott 2000 for a thorough discussion). While an extensive review of resonances is beyond the scope of this paper, we note that an integer ratio of initial orbital periods of the planet and com-
Companion star can have either a stabilizing or destabilizing effect on the system, depending on the initial positions of the two objects. For example, for a 2:1 orbital configuration, if the planet and the companion start at conjunction at the companion’s aphelion, their closest approach distance attains its largest possible value. One would therefore expect such an initial condition to lead to an increase in the planet’s survival time. In contrast, if the planet and companion start at conjunction at the companion’s perihelion, their closest approach distance attains its smallest possible value. This initial condition therefore has a destabilizing effect on the planet’s orbital motion. The goal of this section is to explore the effect that the presence of mean motion orbital resonance conditions has on the survival time distribution of randomly chosen initial orbital phases, which in turn affects the structure of output variables generated in this type of analysis.

To this end, we first consider a series of low-inclination runs with \( M_{\text{c}} = 0.1\ M_{\odot} \) and \( e = 0.5 \), for which the ratio of initial orbital periods is either of the form \( n_{\text{c}} : 1 \) or \( n_{\text{c}} : 2 \), ranging from 7:1 to 14:1 (series 5). The results of these experiments are presented in Figure 8, which plots the mean value of \( \mu \), for a given orbital configuration as a function of periastron. Values from \( n_{\text{c}} : 1 \) orbital configurations are represented with filled symbols, and the values of \( n_{\text{c}} \) are marked below the corresponding data. Values from \( n_{\text{c}} : 2 \) orbital configurations are represented with open symbols. As expected, the mean \( \langle \mu \rangle \) exhibits an overall increase with increasing periastron, but for orbital configurations with \( \langle \mu \rangle \gtrapprox 4 \), the survival times for

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**Fig. 6.**—Distribution of widths calculated from the \( \mu \) distributions for the orbital configurations in (a) series 1, (b) series 2, (c) series 3, (d) series 4. The calculated mean \( \langle \sigma \rangle \) and width \( \sigma \), of the distributions shown in each panel are also presented.
cases with $n_c : 1$ ratios are clearly shorter than those of their $n_c : 2$ counterparts. Interestingly, both cases correspond to the necessary (but not sufficient) condition for resonance (e.g., Murray & Dermott 2000). Our results thus indicate that stabilization is more likely to occur for integer ratios of the form $n_c : 2$ compared to the $n_c : 1$ case. Indeed, of the 4200 runs performed in series 5 (100 equivalent realizations for each of the 42 orbital configurations explored), 38 of the 42 experiments that remained bound over the $\tau_{max} = 10^9$ yr span had $n_c : 2$ orbital configurations. Further evidence of enhanced stabilization in the $n_c : 2$ cases is provided in Figure 9, which shows the $\mu_i$ distributions for the $11 : 1$, $23 : 2$, and $12 : 1$ orbital configurations ($i = 0^\circ$) from series 5 (we note that the last bin for the $23 : 2$ distribution contains seven numerical experiments that survived out to the maximum integration time of $10^9$ yr). While all three distributions show peaks with widths of $\sim 0.5$, the $23 : 2$ distribution (whose orbital configuration has a periastron that falls between those of the $11 : 1$ and $12 : 1$ cases) has its peak centered at a considerably larger value of $\mu_i$ than the others, and has a significant, almost flat tail above the peak.

While a full exploration of the effects of mean motion resonances in these three-body systems is not computationally feasible, we perform a series of runs (series 6) focusing on the $11 : 1$ to $12 : 1$ region of series 5. The results of these experiments are presented in Figure 10, where the mean value of the $\mu_i$ distribution for a given orbital configuration is plotted as a function of periastron at $i = 0^\circ$, $10^\circ$, and $20^\circ$. Figure 10 suggests that a significant amount of structure can be attributed to $n_c : n_p$–type resonances, and that the survival time is longest

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**Fig. 7.**—Same as Fig. 6, but for all of the orbital configurations in series 2–4 (solid line) as well as for the $0^\circ \leq i \leq 30^\circ$ subset (dotted line).

**Fig. 8.**—Results of numerical simulations for series 5 ($M_c = 0.1 M_\odot$ and $e = 0.5$). The mean of the $\mu_i$ distribution generated for each orbital configuration is plotted vs. periastron for the following inclination angles: $0^\circ$ (triangles), $10^\circ$ (squares), and $20^\circ$ (circles). Values from orbital configurations are represented with filled symbols, and the values of $n_c$ are marked below the corresponding data. Values from orbital configurations are represented by open points, with the value of $n_c$ implied by their location.

**Fig. 9.**—Distribution of $\mu_i$ values for three orbital configurations from series 5: $p = 2.553$ AU ($n_c : n_p = 11 : 1$) (dashed line), $p = 2.630$ AU ($n_c : n_p = 23 : 2$) (solid line), and $p = 2.705$ AU ($n_c : n_p = 12 : 1$) (dotted line).
for larger values of $n_c$. This latter result is further suggested by the plot of the width of the $\mu_i$ distributions as a function of periastron, as shown in Figure 11. Clearly, the distributions with the greatest widths are associated with large $n_c$-type orbits. Nevertheless, Figure 10 suggests that the dominant features present in the $(\mu_i)\cdot p$ curve are broad “depressions” that occur around the $n_c: 1$ orbits. Figure 8 further suggests that these “depressions” become more pronounced with increasing periastron (and hence increasing $(\mu_i)$). This issue is complicated and could be the subject of a considerably broader investigation.

An important consequence of these results is the implied additional computational cost in numerically determining the survival times for long-lived systems. Specifically, it appears that $\mu_i$ distributions can become significantly wider than $\sim 0.5$ for orbital configurations with longer survival times, e.g., as shown in Figure 12. This result thus presents an added challenge to the numerical exploration of long-lived planetary systems. In addition to the increased computational time required to perform a given numerical experiment, the number of equivalent runs that must be performed to acquire a valid statistical result for each orbital configuration also increases.

5.2. Ejection Time Versus Periastron and Eccentricity

Previous work on the dynamical stability of Earth-like planetary orbits in binary systems (e.g., D03) suggests that the most important variables affecting the system’s survival time are the companion mass and periastron distance. In the work of D03,
two surveys of the $a$–$e$ plane were performed using two different numerical methods (both B-S and a symplectic integration scheme). Although the survey of parameter space was not as extensive as that of the present paper, the results are in good agreement with those found herein (see below; see also Holman & Wiegert 1999). The mean survival time of a planet in a binary system can be described by a function of periastron, of the form

$$\tau_s = \tau_{s0} \exp[\alpha(p - 1)],$$

where periastron $p$ is in AU and the values of $\alpha$ and $\tau_{s0}$ depend on the companion mass (e.g., see the bottom panels of Figs. 3–6 and Table 1 in D03). For a given periastron value, the survival time has a relatively wide distribution, and equation (4) provides a fit to the mean values.

We note that the width of the distribution of survival times (at constant periastron) arises from two sources: (1) the chaotic nature of the system, which leads to an intrinsic width for any unstable orbital configuration (i.e., for given values of $[a, e]$); and (2) the sampling over $(a, e)$ pairs at constant periastron $p$, which provides an additional systematic width to the distribution. As discussed in § 3, the intrinsic width typically has a value of $\sigma \sim 0.5$ (for the distribution of $\mu_s = \log \tau_s$), although the tails at long survival times can make the effective total width larger. The range of survival times for a given periastron is wider still, where the additional variation is due to the systematic width defined above.

Figure 13 presents an analogous plot of survival time as a function of periastron $p$ for the simulations in this paper. Specifically, we plot the mean values of $\mu_s$ versus periastron for our numerical simulations of the low-inclination ($i \leq 20^\circ$) configurations with $M_c = 0.5 M_\odot$ (series 2–4). The figure also shows the value of $\tau_s$ calculated from equation (4) using the values of $\alpha = 4.7$ and $\tau_{s0} = 0.64$ given in Table 1 of D03 (appropriate for $M_c = 0.5 M_\odot$). The results of this present work are thus in good agreement with those of D03.

We explore more carefully the structure in our output measures that arises from different eccentricities by performing a series of runs with $M_c = 0.5 M_\odot$ and $i = 0^\circ$, sampling over eccentricity for values of $p$ ranging between 2.5 and 4.5 AU (series 7). As with series 5 and 6, $\tau_{s0} = 10^9$ yr and $N = 10^3$. The results of these experiments are presented in Figure 14, which shows the mean of the $\mu_s$ distributions versus eccentricity for different periastron values. The shapes of the $\mu_s$–$e$ curves (for a given periastron value) indicate the presence of competing effects on the stability of the orbital systems being explored. Specifically, for a given periastron, an increased value of eccentricity leads to a longer orbital time, and hence a longer time between closest approaches. As a result, one expects an increase in survival time with increasing eccentricity; a clear trend in the high-$e$ part of the periastron tracks shown in the figure.
Figure 14. However, lower eccentricity orbits also become more stable, in spite of the reduced orbital times. This result points to the importance that the tangential component of the impulse imparted on the planet at closest approach has on destabilizing the system. As a result, low-eccentricity orbits (for which these tangential components are small) can have very long survival times. Indeed, a comparison between \( e = 0.2 \) and 0.8 orbits (with all other orbital parameters being the same), as shown by the \( \mu \) distributions in Figure 15, illustrates how dramatic this effect can be.

As with the widening of distributions due to the presence of integer ratios of orbital periods, the stabilization of low-eccentricity orbits poses a twofold challenge for the numerical analysis of such systems. An increasing survival time requires longer integration times, and an increasing width of the distribution requires an increase in the number of equivalent experiments that must be performed in order to gain good statistical output measures. On the other hand, binary systems are likely to have large eccentricities. Specifically, the eccentricity distribution for binaries with orbital periods \( P_2 \geq 1000 \) days (e.g., \( a_c \geq 2 \) AU) has the form \( f(e) = 2e \) (Duquennoy & Mayor 1991), so that low-eccentricity orbits are considerably less common than their high-eccentricity counterparts.

5.3. Ejection Versus Accretion Events

In this section we consider the presence of structure in the output measure \( f_e \), the fraction of events that are ejected from the system (excluding events that remain bound for the duration of the numerical run time \( \tau_{run} \)). In order for ejection to occur, the gravitational energy between the planet and companion at closest approach must exceed the corresponding gravitational energy between planet and Sun. Since the nearest possible approach \( d \) between planet and companion is \( d \sim p - 2 \) AU (when the planet’s eccentricity is near unity), we find that ejection is not energetically possible when \( p \geq 2 + 2M_p/M_\odot \), or \( p = 2.2 \) AU, for series 1, 5, and 6, and \( p = 3 \) AU for series 2–4 and 7. This result agrees relatively well with the results of our numerical experiments, especially for low inclination angles. Our results also indicate a clear dependence of \( f_e \) on inclination angle, as can be seen from Figure 16, which plots \( f_e \) versus inclination angle at different periastron values for series 1–4. As a general trend, ejection fraction values peak at inclination angles ranging between 30° and 60°, but additional structure is seen in the \( f_e-i \) curves.

5.4. Kozai Resonance

Figure 1d shows a clear peak at \( \sim 60^\circ \), a feature that can be attributed to the presence of a Kozai resonance (e.g., Murray & Dermott 2000). This type of resonance exists for low-mass objects in highly inclined orbits perturbed by a larger mass object with an outer, nearly circular orbit (which explains why the feature is not observed at higher companion eccentricities). At a Kozai resonance, the eccentricity and inclination angle of the planet are coupled such that when one is at its maximum value, the other is at its minimum (and vice versa), and the quantity

\[
H_k = (a_c\sqrt{1-e^2})^{1/2}\cos i
\]

remains constant (Kozai 1962).

To confirm the presence of Kozai resonances in the \( \sim 60^\circ \) runs of series 4, we numerically determine both the planet’s eccentricity and inclination angle as a function of time for three different test runs, defined by the following orbital parameters:

- run 1: \( M_c = 0.5\, M_\odot \), \( p = 4 \) AU, \( e = 0.75 \), and \( i = 0^\circ \).
- run 2: \( M_c = 0.5\, M_\odot \), \( p = 4 \) AU, \( e = 0.75 \), and \( i = 60^\circ \).
- run 3: \( M_c = 0.5\, M_\odot \), \( p = 4 \) AU, \( e = 0.25 \), and \( i = 60^\circ \).

The first two runs have orbital configurations that should not lead to the presence of Kozai resonances, owing to a small inclination angle and/or a large eccentricity, whereas run 3 meets the required conditions for Kozai resonances to occur. The results of the three simulations are shown in Figure 17, which plots the eccentricity for the planet as a function of time (over the planet’s survival time) in Figures 17a–17c, and the eccentricity (solid line) and inclination angle (dotted line) for a short time interval of run 3 in Figure 17d. The latter panel shows a clear periodicity in eccentricity and inclination angle, as expected during a Kozai resonance. We also plot the value of \( H_k \) as a function of time for each run,
in Figure 18. The value of $H_K$ clearly oscillates between 0.23 and 0.27 for most of the duration of run 3, indicating that while the system is not perfectly in resonance (since $H_K$ is not exactly constant), it is close to it. In contrast, the values of $H_K$ for runs 1 and 2 show considerably more variability.

**6. CONCLUSION**

This paper extends previous work on numerical simulations of Earth-like planets in binary systems. Specifically, the survival time of an Earth-like planet orbiting a Sun-like star in the presence of an outer companion is determined for different orbital configurations defined by the companion’s mass, eccentricity, periastron, and the planet’s inclination angle relative to the binary orbital plane. Due to the chaotic nature of the systems being explored, the survival time for each orbital configuration was determined for $N$ equivalent realizations with randomly selected initial phase angles, allowing for a statistical analysis of the survival time. We used $N = 10^3$ for the bulk of our exploration of parameter space, which focuses on four pairs of companion masses and eccentricities (series 1–4), and $N = 10^2$ for three additional series of runs (5–7). In all, we performed $\sim 400,000$ numerical integrations and explored $\sim 500$ different orbital configurations. Our results indicate that the values of survival time obtained for a given orbital configuration are log-normally distributed (to leading order). We therefore use the logarithm of the survival time
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Fig. 17.—Time evolution of the planet’s eccentricity for three numerical experiments with different orbital configurations. (a) run 1: \( M_C = 0.5 \, M_\odot, \, p = 4 \, \text{AU}, \, e = 0.75, \) and \( i = 0^\circ; \) (b) run 2: \( M_C = 0.5 \, M_\odot, \, p = 4 \, \text{AU}, \, e = 0.75, \) and \( i = 60^\circ; \) (c) run 3: \( M_C = 0.5 \, M_\odot, \, p = 4 \, \text{AU}, \, e = 0.25, \) and \( i = 60^\circ; \) and (d) same as (c), but for a small time interval of the planet's evolution. This panel also shows the planet’s inclination angle (dotted line) as a fraction of \( \pi/2. \)

\( \mu_s \equiv \log \tau_s \) (where \( \tau_s \) is given in years) as our primary output measure.

We plot the dependence of \( \langle \mu_s \rangle \) on inclination angle and periastron value for each distribution in series 1–4. These results confirm a weak dependence of survival time on inclination angle for cases with \( i \leq 40^\circ, \) and the well-known decrease in survival time at high \( (i \geq 40^\circ) \) inclination angles. A simple extrapolation of our results for low inclination angles indicates that while potentially Hill unstable, orbital configurations with companion masses of \( M_C = 0.1 \, M_\odot \) and \( M_C = 0.5 \, M_\odot \) can nonetheless remain stable for \( \sim 5 \, \text{Gyr} \) when \( p \geq 4 \, \text{AU} \) and \( p \geq 6 \, \text{AU}, \) respectively. Note that these estimates are conservative in that the extrapolation is carried out along the lower envelope of the (wide) range of survival times for a given periastron. These results are consistent with those of D03 and represent lower periastron values than those obtained through a Hill stability analysis (e.g., Gladman 1993; see also Barnes & Greenberg 2006); i.e., systems that are ultimately "unstable" according to analytic criteria can survive over the 4.6 Gyr age of the solar system.

This work has important implications for searches for Earth-like planets (e.g., Terrestrial Planet Finder) in extrasolar systems, since a large fraction of stars are found in binaries. As summarized above, our numerical results show that all binaries with periastron values greater than 6–7 AU allow Earth-like planets to be stable over the age of our solar system. This finding, in conjunction with the observed distributions of binary parameters for solar-type primaries (see...
future work, a rough criterion can be formulated using our
survive in a binary system that also contains a Jovian planet.
A related question is whether or not an Earth-like planet can
with detected planets, the periastron satisfies this constraint.
The Jovian planet (i.e., ). In all of the binary systems
must be wider than about

gas can inhibit planetesimal formation (Thebault et al. 2006)
does not include gas; in the early stages, interactions with
However, note that the aforementioned planet formation stud-
systems are habitable, according to dynamical considerations.
2004; Marzari & Scholl 2000). Thus, at least half of the binary
numbers of variables, the central limit theorem indicates that
play a role in determining a distribution, the result tends to


Fig. 18.—Time evolution of for the three nu-
meric experiments presented in Fig. 17. The value for run 1 is offset upward by 0.5.
Duquennoy & Mayor 1991 and the compilation of D03), in-
dicates that roughly half of all binaries are wide enough to
contain stable Earths. In addition, binaries that are wide
(with sufficiently large periastron) to allow for orbital
stability of Earth-like planets are also wide enough to allow
for the formation of terrestrial planets through the accumu-
lation of planetesimals (e.g., Quintana et al. 2002; Quintana
2004; Marzari & Scholl 2000). Thus, at least half of the binary
systems are habitable, according to dynamical considerations.
However, note that the aforementioned planet formation stud-
ies do not include gas; in the early stages, interactions with
gas can inhibit planetesimal formation (Thebault et al. 2006)
and lead to tighter constraints on habitability.

Some of the extrasolar planets detected to date are found
in binary systems, although these planets have masses com-
parable to that of Jupiter. In the simulations performed here,
the Earth-like planet acts as a test particle, but even a Jovian-
mass planet will act as a test particle in the potential of stellar
bodies. As a result, one can directly apply our stability criteria
to these systems: in order for a Jovian planet to remain stable
over typical stellar ages of ~5 Gyr, the binary periastron must
be wider than about ~7 times the semimajor axis of the
Jovian planet (i.e., ). In all of the binary systems
with detected planets, the periastron satisfies this constraint.
A related question is whether or not an Earth-like planet can
survive in a binary system that also contains a Jovian planet.
Although the relevant four-body simulations must be left for
future work, a rough criterion can be formulated using our
results to date: the Jovian planet must have a sufficiently large
periastron to allow the Earth-like planet to remain stable (and
this constraint is roughly periastron . and the
binary periastron must be large enough to allow the Jovian
planet to be stable (roughly periastron ).
The next major result of this work concerns the width of
the distributions of survival time. We have calculated the
 distributions over a broad range of orbital parameter space.
It is well known that orbital systems can exhibit chaotic be-
havior in or near the unstable regime. As a result, it is not
possible to define a unique value of survival time for a given
orbital configuration. Instead, the survival time can only be
defined in terms of a distribution, the mean and width of
which then characterizes the underlying dynamics of the sys-
tem. For the orbital configurations explored in series 1–4, the
resulting distributions exhibit Gaussian peaks with widths of
and generally display high-end tails when . In this regime, the value of calculated via equivalent realizations of a given orbital configuration has a
68% probability of being within of the true value, and a 99% probability of being within of the true value. The situation is more complex for orbital
configurations whose distributions have means in excess of . While exhibiting peaks with widths of ~0.5, these distributions may also have significant tails, thereby leading
to larger calculated total width values (~0.7–2). Part of this
increased complexity can be traced to the increasing import-
ance of mean motion orbital resonance effects. In addition,
the loss of a tangential component in the impulse imparted
as the planet moves through closest approach for low-eccen-
ctricity cases can also lead to fairly broad distributions, al-
though the paucity of observed low-eccentricity binary sys-
tems somewhat limits the importance of this latter effect.
Nevertheless, our results indicate that the determination of
survival times for long-lived systems poses a serious nu-
merical challenge in that both the integration times must be
considerably larger than the distribution over a broad range of orbital parameter space.

The form of the distributions of survival times is not un-
expected. Whenever a large number of independent variables
play a role in determining a distribution, the result tends to
exhibit a nearly Gaussian form. In the limit of an infinite
number of variables, the central limit theorem indicates that
the resulting distribution will be Gaussian for any type of
distribution for the input variables (e.g., Richtmyer 1978).
Binary systems with planets contain an intermediate number
of variables. Although the systems are sufficiently compli-
cated to display interesting and complex behavior (Figs. 1–
18), the number of independent variables is not infinite. As
a result, one expects the distribution of any composite variable
(here the survival time for a given orbital configuration) to
approach a Gaussian form but retain non-Gaussian tails (since
convergence is slowest in the tails). The result—a Gaussian
distribution with tails—is in fact what we find here.

As expected from the chaotic nature of the systems being
explored, our output measures exhibit a rich amount of structure. Specifically, we find that the presence of Kozai resonances result in a region of stability at $i \sim 60^\circ$ when the companion orbit has a low eccentricity. In addition, we find a significant amount of structure in the output measure $f_e$, the fraction of events that lead to ejection for a given orbital configuration. To our knowledge, this latter result has not received significant attention in the literature and may warrant further study.

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