Multifractal fluctuations of waiting time sequence of aftershocks

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Abstract. After a large earthquake there will be series of aftershocks. The analyzed regularity of these aftershocks will be conducive to recognize the characteristics of the aftershocks and explore their formation and evolution, and also to provide theoretical support for Earthquake Disaster Mitigation. Detrended fluctuation analysis (DFA) and multifractal methods are applied to the time-scaling properties analysis of the waiting time sequence of aftershocks (WTSAs) of the Wenchuan Ms8.0 earthquake occurred on Longmen mountain tectonic zone of Sichuan Province in China. The results show that the WTSAs is characterized by multifractal scaling.

1. Introduction
The Wenchuan Ms8.0 earthquake occurred on Longmen mountain tectonic zone of Sichuan Province in China. The abundant aftershocks have occurred after the main event [1]. Understanding the physical mechanism of Wenchuan aftershocks is of great practical importance.

In recent years, the researches of waiting time sequence of aftershocks (so-called, as the time interval from its preceding strong aftershock) of devastating earthquake aftershocks have made outstanding achievements. Gu et al. analyzed the relationship between the waiting time \(\Delta t\) and the time of its occurrence (time interval from the moment of onset of the main shock) [2]. Qin et al. analyzed aftershocks of Qiongshan Ms7.5 earthquake (1605), Xinfengjiang Ms6.1 earthquake (1962) and Xunwu Ms5.5 earthquake (1987), whose logarithmic values between waiting time and occurring time have good relationships in linear [3, 4]. Abe and Suzuki studied the properties of the seismic time sequence that correlation between earthquake events both inside and outside of the Omori regime in southern California, and found that inside of the Omori regime correlation exhibits the aging phenomenon, in marked contrast to the fact that no aging is observed outside of the Omori regime [5]. Caccamo et al. and Teleseca et al. calculated the Hurst exponent, \(p\)-value and the spectral index of six group aftershock sequences in Greece during 1980-1997 and found that the temporal fractality of aftershock sequences is primarily due to the power-law decay of the aftershock rate [6, 7]. Sornette and Ouillon carefully tested this, predicted earthquake sequences in the Southern California earthquake catalog by using multifractal stress activation [8]. According to chaos theory, through the WTSAs of Wenchuan earthquake, Liu et al. believed that there was an obvious nonlinear chaotic characteristic which was resulted by the non-linear chaotic dynamic system evolution in the WTSAs.
Based on a new concept of magnitude clusters describing the fluctuation of aftershock energy release of the Wenchuan Earthquake [9], Zhang et al. discovered that different magnitudes of aftershock within specific size range follow a power-law rather than a non-Poisson distribution, and this suggested that the aftershocks with high magnitudes are statistically clustered [10]. Lee et al. confirmed the existence of multifractal characteristics in the examined aftershock sequence of Chi-Chi (Taiwan) main earthquake (ASCME), which occurred in 1999/9/20/17/47 [11].

In this study, firstly, we investigate the temporal scaling and long memory properties for the aftershocks series of the Wenchuan Ms8.0 earthquake by detrended fluctuation analysis (DFA) and multifractal analysis, which are effectively used for detecting complexity of the WTSAs.

2. Observational seismicity data
In this paper, we analyze the aftershock sequence of Wenchuan main earthquake that occurred at 14:28 May 12, 2008 with magnitude 8.0. The monitored aftershock sequences occurred at locations defined at 30.0-33.5°N, 102.7-106.3°E (The earthquake epicenter was located at 31.0°N, 103.4°E) during 14:28 on May 12th 2008 to 16:40 on October 18th, 2008. The minimum magnitude is 2.1 (a total of 21, 7572 minutes and 2595 times). Set the first waiting time between the first and second aftershocks as case No. 1, then set the second waiting time between the second and third aftershocks as No. 2, and accordingly set the 2594 waiting time between the 2594th and 2595th aftershocks as No. 2594 in waiting times earthquake of aftershocks respectively. So the length of the occurrence sequence is 2595 as shown in figure 1.

![Figure 1. The waiting time sequence of aftershocks is used.](image)

3. Methods

3.1. Detrended fluctuation analysis
The detrended fluctuation analysis (DFA) is an advanced method for determining the scaling behavior of data in the presence of possible trends without knowing their origin [12].

The methodology operates on the WTSAs, \( \{ x(t), t=1,2,…, N \} \), where \( N \) is the length of the series. \( \bar{x} \) is the average value of the original time series.

The series is first integrated as follows:

\[
y(k) = \sum_{t=1}^{k} (x(t) - \bar{x}) \quad (k = 1, 2, L, N) \tag{1}
\]

Next, one measures the vertical characteristic scale of the accumulated time series by dividing the latter into boxes of equal length, \( n \). In each box, a least-squares line is fit to the data, representing the trend in that box. The ordinate of the straight-line segments is denoted by \( y_n(k) \). Next, the accumulated series, \( y(k) \) for observation \( k \), is detrended by subtracting the local trend, \( y_n(k) \), in each box. The root-mean-square fluctuation of this integrated and detrended time series is calculated using
\[
F(n) = \sqrt[2]{\frac{1}{n} \sum_{k=1}^{n} [y(k) - y_n(k)]^2}
\] (2)

If \(F(n)\) behaves as a power-law function of box sizes \(n\) then the data present scaling: \(F(n) \propto n^d\). The DFA exponent \((d)\) is defined as the slope of the regression line for all points \([\lg(n), \lg(F(n))]\).

For white noise, where the value at one instant is completely uncorrelated with any previous values, the integrated value \(y(k)\) corresponds to a random walk and \(d=0.5\); \(0.5 \leq d \leq 1\), indicating persistent long-range power-law correlations; \(0 < d \leq 0.5\), power-law anti-correlations are present; \(d > 1\), the correlation exists but cease to be of the power-law form; \(d = 1.5\) indicates a brown noise, namely the integration of white noise.

3.2. Multifractal analysis

Multifractal analysis was first introduced for the study of turbulence by Mandelbrot and Halsey et al. and was then much developed in a mathematical framework, for instance, and many authors extended this analysis to point functions \([13, 14]\), obtaining quite complete descriptions. A multifractal analysis is defined for sequences of Choquet capacities with respect to a general class of measures, and some preliminary results are presented concerning the usual spectra.

We further investigate the possibility that time series generated by certain climatic systems may be members of a special class of complex processes, termed multifractal, which require a large number of exponents to characterize their long memory properties. Further detail computation can be seen the related literatures \([11]\).

The calculation steps of the statistical method are briefly described as follows Halsey et al. and Chen et al.. The multifractal analysis is applied to operate on the WTSAs \(\{X_t\}\), where \(t = 1, 2, \ldots, N\) and \(N\) is the length of the series.

Step 1: The normalized the WTSAs is determined by

\[
M_n = X_t \sum_{i=1}^{n} X_i
\] (3)

Step 2: \(M_n\) is divided into \(v\) non-overlapping intervals of a certain time resolution, \(s\). Each interval is characterized by a time resolutions \(s\). Then for each \(s\), we compute the sum of normalized the series \(M_n\) in each interval. And we obtain the box probability of each interval

\[
P_s(v) = \sum_{i(v-1)+1}^{i(v)} M_i, v = 1, 2, \ldots, 2Ns
\] (4)

Where \(Nv = [N/s]\) [\(\:\\) represents the integral zed Gauss mark.].

Step 3: The partition function \(Z(q, s)\) \([15, 16]\) has to be defined by

\[
Z(q, s) = \sum_{v=1}^{2N} \left| P_s(v) \right|^q
\] (5)

Where \(q\) is a real number ranging from \(-\infty\) to \(\infty\). For multifractal distributed measures, the partition function scales with the resolution as

\[
Z(q, s) \propto s^{\tau(q)}
\] (6)

where \(\tau(q)\) is the mass exponent of order \(q\) (namely \(q\)th-order moment structure partition function) \([17]\). The mass exponent for each \(q\)-value can be obtained by plotting \(\log Z(q, s)\) vs. \(\log s\). The obtained \(\tau(q)\) may be regarded as a characteristic function of the fractal behavior. If \(\tau(q)\) against \(q\) is a straight line (convex function), the data set is monofractal (multifractal) \([18]\).

In addition, three parameters of multifractal spectrum are very important to describe the strong or weak characteristics of the degree of multifractality \([19, 20]\).

Another way to characterize a multifractal series is the singularity spectrum \(f(\alpha)\), that is related to \(\alpha(q)\) via a Legendre transform,

\[
\alpha(q) = \frac{d\tau(q)}{dq} \text{ and } f(\alpha) = q\alpha(q) - \tau(q)
\] (7)
where $\alpha$ called Holder singular index, which is also called singular strength or the singularity of the subset of probabilities, reflecting the degree of singular interval measures. And $f(\alpha)$ is the multifractal spectrum, denoting the dimension of the subset of the series that is characterized by $\alpha$. The curve $\alpha$--$f(\alpha)$ is a single-humped function for multifractal, which reduces to a point for monofractal. The shape and the extension of $f(\alpha)$ curve contain significant information about the distribution characteristics of the examined data set.

Two conventional multifractal parameters of them are used to be deduced from the approximated $f(\alpha)$. The spectrum width is defined as $\Delta \alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$, where $\alpha_{\text{max}}$ and $\alpha_{\text{min}}$ are obtained from the relation $f(\alpha) = 0$. The parameter $\Delta \alpha$ describes the inhomogeneity of the distribution of probability measured on the overall fractal structure, which has been identified as the degree of multifractality and intermittency. $f(\alpha)$ takes a maximum value $f_{\text{max}} (f_{\text{max}} = f(\alpha_0))$ at a specific $\alpha_0$, which corresponds to the peak of the multifractal spectrum [7]. To be specific, the bigger the $\Delta \alpha$ and the larger $f_{\text{max}}$, the stronger is the degree of multifractality. Moreover, the difference of the fractal dimensions between the minimum and maximum probability subsets $\Delta f (\Delta f = f(\alpha_{\text{min}}) - f(\alpha_{\text{max}}))$ describes the proportion of the number of elements at the maximum and minimum in the subset, which refers to the proportion of the large and small peaks of vibration signals.

We can make a quantitative characterization of the spectra by least square, fitting it to a quadratic function around the position of maximum $\alpha_0$

$$f(\alpha) = \theta_1 (\alpha - \alpha_0)^2 + \theta_2$$

where $\theta_2$ is an additive constant $\theta_2 = f(\alpha_0) = 1$ and $\theta$ indicates the asymmetry of the spectrum. It is zero for a symmetric spectrum. The better symmetry (namely the closer to 0), the stronger is the degree of multifractality. A larger $\theta$ value (positive) for a process indicates a left-skewed shape of multifractal spectrum and a relative dominance of lower fractal exponents corresponding to more smooth-looking structures, which is ascribed to the time series with long memory [21].

4. Results and analysis

4.1. DFA method

DFA results exhibit a clearly power-law scaling relationship at the time scale of 229,093 minutes (figure 2). In the original WTSAs, $d = 0.67$ with $r^2 = 0.998$. In order to verify that the $d$ value indeed reflects some information of the temporal variation of the WTSAs, we performed the same analysis on randomly shuffled version of the original WTSAs. We calculated the values $d$ for the shuffled sequence which are shown in figure 2. In the randomly shuffled WTSAs, $d = 0.48$ with $r^2 = 0.995$. The randomly shuffled series indicate the obvious randomness, which is significantly different from that of calculated for the original sequence. The original WTSAs indicates high persistence or long memory. The high persistence signifies that the WTSAs fluctuations in aftershocks, from small time intervals to larger ones (up to 229,093 minutes at least), are positively correlated in a power-law fashion. For example, there is a tendency for increase in waiting time to be followed by another increase in waiting time at a different time in a power-law fashion. This suggests that the correlations between the waiting time fluctuations in aftershocks do not obey the classical Markov-type stochastic behavior (exponential decrease with time), but display more slowly decaying correlations. This implies that the long memory may be an important factor in the trend prediction of the waiting time. However, the data set is only 229,093 minutes long. It needs longer series to confirm the critical correlated time scale, where power-law scaling is varied.
4.2. Multifractal analysis
In this section, multifractal method is used to analyze the WTSAs of Wenchuan earthquake in order to make a comprehensive study in its multifractal nature in detail.

At first, we examine the relationship between $\log Zq$ ($s$) and $\log(s)$, figure 3 is a double-logarithmic plot of $\log Zq$ ($s$) and $\log(s)$ with different values of $q$, where $q$ only takes -30 to 30, 61 values in total. It can be seen from figure 3, whichever value $q$ takes, the data points approximate a straight line, indicating that the fixed value of $q$, the time sequence has fractal scaling characteristics. In addition, for different values of $q$, the slopes of the data points, are both different, which suggests that the fractal time sequence also has a multi-scale features, that is to say the WTSAs displays multifractal characteristics. Secondly, we examine $\tau(q)$--$q$ curves, the nonlinear strength of the degree of $\tau(q)$ is a direct reflection of the degree of the multifractal strength. It can be seen from figure 4, severe dependence of $\tau(q)$ for $q$: $q < 0$, the slope of $q > 0$, with the obvious nonlinear behavior, and the WTSAs with further multifractal characteristics.

Furthermore, we investigate the multifractal spectrum of the WTSAs as shown in figure 5. We find that $f(\alpha)$ is located in a wide interval ($\Delta \alpha = 0.5853$), and distribution curve is concentrated on the larger context in the coordinate system, indicating fluctuations of the waiting time are quite instable, the index distribution is very uneven and reflecting that the multifractal characteristics of the waiting time are very obvious.

At the same time, the $f(\alpha)$ curve obviously likes a hook to the right with $\Delta f = -0.0722 < 0$, which indicates that the frequency of the waiting time values at the lower values is more than higher values in the whole 229, 093 minutes. The larger $\theta$ value (positive) of the process of WTSAs fluctuations indicates that its multifractal spectrum is a relative dominance of lower fractal exponents corresponding more smooth-looking structures. It indicates that small fluctuations of time series play a leading role in the WTSAs, which is mostly ascribed to the aftershock process with long memory and reflects the complexity of the changes in the spatial and temporal distribution of abnormal seismic activity, which may be related to the enhanced degree of the distribution of non-uniform stress field within the structural system of Wenchuan region [22].

![Figure 2. DFA performed on the original WTSTs and randomly shuffled WTSTs.](image)
5. Discussions and conclusions
Through the $Z_q(s)$ and the double logarithm of partitions subinterval $s$, mass index $\tau(q)$ and $q$ relations, and its multifractal spectrum $f(\alpha)$ study of the WTSAs, we found that the waiting time displays multifractal behavior and long-range correlation.

Aftershock system is a subsystem of rock system and the rock system is an open system which exists energy and material exchange with others systems and its subsystems. The system is in a non-equilibrium state, and is layered, which complies with the main features of the dissipation structure. The rock layer system is a dissipative system with dissipative property; secondly. As for the movement of rock layers, it is not possible to use linear kinetic equation to make description, because the rock layer system is nonlinear, dominated by the deterministic laws, and it shows some random phenomena, which means the WTSAs of Wenchuan earthquake has features such as variability, randomness and fuzziness; finally, in the changes of layer system, there are many random factors, with randomness, dissipation, nonlinearity and randomness consists the conditions of forming the fractal structure [9] Therefore, the dissipation nonlinearity randomness of the rock layer system shall be the root causes of the WTSAs being multifractal and with long-range correlation.

According to the study of Bak et al., the crust of the earth had been set up in a highly complex self-organized critical state in which the criticality manifests itself in many different geological phenomena with power-law fractal distribution and dynamics [23]. As a specific example, seismicity patterns appear to be complex and chaotic, yet there is sequence in the complexity. The WTSAs exhibits multifractal behaviors, and aftershocks it is a nondeterministic system and has not exact solution,
which makes it is impossible to give a fairly accurate forecast for aftershocks, especially low aftershocks.

The above-mentioned results are only based on the aftershock sequence of the five-month period after the Wenchuan earthquake. However, the whole period of aftershock may last much longer. Therefore, further studies will be required till the aftershock ceases. Although the existing aftershock sequence is relatively short, it has displayed various apparent multifractal behavior and stationary long memory property. This is valuable for the risk evaluation of post-disaster reconstruction.

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