Optimized Space Station CV Method and Its Differences from GNSS CV

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Abstract

There will be better atomic clock system and micro-wave time comparison link in the near earth space station, like Chinese Space Station and European ACES (Atomic Clock Ensemble in Space) system, than those in the GNSS (Global Navigation satellite) satellites. Therefore, the space station common-view (CV) will realize more accurate time comparison than GNSS CV in theory. But due to the orbit characteristic of the space station, there are some limitations if traditional GNSS CV time comparison method is applied to the space station. In order to solve these problems, the GNSS CV method is optimized and the method that is appropriate for the space station is proposed. First, the basic CV principle is analyzed, and the delay items which are needed to be considered for GNSS and space station CV are compared and analyzed. Then, the differences between GNSS and space station CV are studied, and the influences of orbit error on these two CV methods are analyzed in detail. The GNSS CV method is optimized to be fit for the space station next. Finally, the performance of the optimized method is validated by simulated experiments. The simulation results show that the space station time comparison accuracy of several tens of picoseconds can be obtained.
The CV technology is wildly applied for time comparison at present. The magnitude of nanosecond time comparison has been achieved by GPS CV. With the development of GNSS systems, the accuracy of GNSS CV is continuously being improved. Its accuracy is about one to five nanoseconds at present (Wang 2017), and it is the main method for high accuracy time comparison over long distances. But with the expanded application, nanosecond time comparison accuracy cannot satisfy many requirements, such as the measurement of VLBI (Very Long Baseline Interferometry), the research of gravitational red shift, and the deep space exploration. Bai (2012) and Miao (2015) all stated the significance of high accurate time comparison technology in their PhD dissertations. Matsakis et al. (2014) analyzed and compared the modern time transfer...
technologies, and revealed that more accurate time comparison techniques are needed.

The stable atomic clock systems with better performance than the clocks on the ground due to the microgravity environment will be carried on Chinese Space Station and European ISS (International Space Station), where the equipment of ACES project will be loaded. Hess M et al. (2012) and Yang et al. (2016) all pointed out this characteristics of the atomic clocks in the space station. Because the main characteristics related to time comparison of the Chinese Space Station and European ACES project are nearly the same as each other, like their atomic clock performances, time compare links and the orbit characteristics, we call them “space stations” collectively for convenience. The analysis results below are all for these two space stations. The clock ensemble carried in the space station will be the most stable time system that will cover almost the whole earth periodically (Hobiger et al. 2013). Besides, the space-to-ground micro-wave time comparison link also has good performance. Duchayne et al. (2008) pointed out that the specification of the micro-wave link which is used to perform space-to-ground time comparisons requested the time comparison stability of 0.3 picoseconds at the interval of 300 seconds and 7 picoseconds at one
day’s interval. Cacciapuoti et al. (2009) and Much et al. (2009) all stated that the uncertainty of space station clock synchronization is in the level of 100 picoseconds, so more accurate time comparison would be realized by the CV method. Delva et al. (2013) and Meynadier et al. (2018) validated the performance of the micro-wave link by simulation experiments, and these analysis results showed that the micro-wave link satisfied the requirement stated above.

Therefore, several tens of picoseconds CV time comparison accuracy will be achieved by the micro-wave link from the space station to the ground. This accuracy is two orders of magnitude higher than what is achieved with current GNSS CV techniques.

The space station orbit height is about 400 kilometers. The orbit of the space station is far lower than that of the GNSS satellite. Therefore, the orbit characteristics of the space station differ greatly from that of the GNSS satellites. The orbit error of space station affects the CV time comparison more than that of the GNSS satellite, which limits the space station time comparison accuracy to several hundreds of picoseconds level. Also, a low orbit causes smaller coverage areas. There are CV blind areas for the
space station. Thus, the GNSS CV method cannot be used to the space station directly.

It is necessary to study the CV time comparison method that is suitable for the space station aiming at several tens of picoseconds accuracy.

The basic CV principle is analyzed, and the delay items which are needed to be considered for GNSS CV and space station CV are compared and analyzed at first.

Then, the differences between GNSS CV and space station CV are studied. The limitations are analyzed according to simulation, which will occur if the GNSS CV method is used to the space station time comparison directly. The GNSS CV method is optimized in order to solve these limitations to obtain better space station CV performance. Finally, the performance of the optimized method is validated by some simulated experiments. Several tens of picoseconds time comparison accuracy is realized in the simulation experiments.

2. Principle of CV time comparison

The target accuracy of GNSS CV is nanosecond, but our target of the space station CV accuracy is several tens of picoseconds. In order to satisfy these two accuracy targets, the mathematical models are established based on the high-accurate general relativity
theory (Liu 2004; Petit et al. 1994). Hobiger et al. (2013) and Duchayne et al. (2008) all introduced the mathematical formulas of the space station based on the general relativity theory. Some errors less than 1 nanosecond must be considered, which are ignored in the traditional GNSS CV method. The GCRS (Geocentric Celestial Reference System) is used as the basic coordinate system of CV time comparison.

If ground stations A and B receive the time comparison signal send by the spatial aircraft, such as the space station or GNSS satellite, at $t_0$ time, the clock biases between the ground station and the aircraft can be expressed as follows (Meynadier et al. 2018; Duchayne et al. 2008):

\[
\begin{align*}
VT_{AS} &= P_A - \left( \frac{\rho_{SA}}{c} + \frac{\nu_{SA}}{c^2} + \frac{\dot{\rho}_{SA}}{2c^3} \left( \nu_A^2 + \frac{\nu_{SA}^2}{\rho_{SA}^2} \right) + \frac{\nu_A}{\rho_{SA}^2} \right) + \frac{2GM_e}{c^3} \ln \frac{r_s + r_A + \rho_{SA}}{r_s + r_A - \rho_{SA} + \nu_{SA}} \\
VT_{BS} &= P_B - \left( \frac{\rho_{SB}}{c} + \frac{\nu_{SB}}{c^2} + \frac{\dot{\rho}_{SB}}{2c^3} \left( \nu_B^2 + \frac{\nu_{SB}^2}{\rho_{SB}^2} \right) + \frac{\nu_B}{\rho_{SB}^2} \right) + \frac{2GM_e}{c^3} \ln \frac{r_s + r_B + \rho_{SB}}{r_s + r_B - \rho_{SB} + \nu_{SB}}
\end{align*}
\]

where $VT_{AS}$ denotes the clock bias between station A and the aircraft, $VT_{BS}$ denotes the clock bias between station B and the spatial object, $P_A$ and $P_B$ denote the pseudo range of station A and B respectively, $\rho_{SA}$ and $\rho_{SB}$ denote the distance vector from the aircraft to ground station A and B respectively, $c$ denotes the light speed, $\nu_A$ and $\dot{\nu}_A$ denote
\[ \begin{align*} &\text{denote the speed and acceleration vector of station A respectively, } \dot{v}_A \text{ and } \ddot{v}_A \text{ denote the speed and acceleration vector of station B respectively, } \\
&\tau_{im}^{SA} \text{ and } \tau_{im}^{SB} \text{ denote the ionospheric delay, } \tau_{tr}^{SA} \text{ and } \tau_{tr}^{SB} \text{ denote the tropospheric delay, and other terms are the delay components related to the relativity theory.} \\
&\text{\( \dot{\rho}_{SA} \dot{v}_A \) and } \text{\( \ddot{\rho}_{SA} \ddot{v}_A \) denote the Sagnac delay, they are in the magnitude of ten nanoseconds for the space station, but they are in the magnitude of hundred nanoseconds for GNSS satellites.} \\
&\text{The last but one on the right of (1) and (2) are the Shapiro delay terms, in which } G \text{ denotes Newton's gravitational constant, } M_E \text{ denotes the earth mass, } r_S \text{ denotes the distance between the center of the earth to the space object, } \\
&r_A \text{ denotes the distance between the center of the earth to ground station A, and } r_B \text{ denotes the distance between the center of the earth to ground station B. The effect of Shapiro delay is about ten picoseconds for the space station and more than one hundred picoseconds for GNSS satellites. The last terms on the right of (1) and (2) are converting delay from coordinate time to} \\
\end{align*} \]
the ground station proper time, which should be considered in general relativity theory.

According to the relationship between time-space interval and metric tensor in general relativity theory, the relationship between the ground station proper time and coordinate time can be expressed as follows:

\[
\frac{d\tau_A}{dt} = 1 - \left( \frac{GM_E}{r_Ac^2} + \frac{v_A^2(t)}{2c^2} \right)
\]

\[
\frac{d\tau_B}{dt} = 1 - \left( \frac{GM_E}{r_Bc^2} + \frac{v_B^2(t)}{2c^2} \right)
\]

where \( \tau_A \) and \( \tau_B \) denote the proper time, \( t \) denotes the coordinate time.

Therefore, \( \tau_{SA}^{\text{trans}} \) and \( \tau_{SB}^{\text{trans}} \) can be expressed as follows:

\[
\tau_{SA}^{\text{trans}} = \left( \frac{GM_E}{r_Ac^2} + \frac{v_A^2(t)}{2c^2} \right)T_{SA}
\]

\[
\tau_{SB}^{\text{trans}} = \left( \frac{GM_E}{r_Bc^2} + \frac{v_B^2(t)}{2c^2} \right)T_{SB}
\]

where \( T_{SA} \) and \( T_{SB} \) denote the signal propagating delay from the aircraft to the two ground stations, which contain the delay items in the bracket on the right side of (1) and (2) except for \( \tau_{SA}^{\text{trans}} \) and \( \tau_{SB}^{\text{trans}} \). Their effects of \( \tau_{SA}^{\text{trans}} \) and \( \tau_{SB}^{\text{trans}} \) are about several picoseconds for the space station and ten picoseconds for GNSS satellites.

The items less than one picosecond can be neglected in (1) and (2) for space station CV. Differencing (1) and (2), the following equation can be obtained:
where \( V_{T_{AB}} \) denotes the time bias between ground station A and B.

The items less than one nanosecond can be neglected in (1) and (2) for GNSS CV, thus \( V_{T_{AB}} \) for GNSS CV can be expressed as

\[
V_{T_{AB}} = (P_A - P_B) - \frac{(\rho_{SA} - \rho_{SB})}{c} - (r_{SA}^{\text{ion}} - r_{SB}^{\text{ion}}) - (r_{SA}^{\text{tro}} - r_{SB}^{\text{tro}}) - \frac{\gamma_{SA} v_A + \gamma_{SB} v_B}{c^2} - \frac{2GM}{c^2} \left( \ln \frac{r_S + r_A - \rho_{SA}}{r_S + r_B - \rho_{SB}} - \ln \frac{r_S + r_A - \rho_{SA}}{r_S + r_B - \rho_{SB}} \right)
\]

Equations (7) and (8) show that differencing operation is the essence of CV principle. The common errors can be canceled by the differencing operation, so the time comparison accuracy is improved by CV method. But the CV time comparison requires that two ground stations observe the spatial aircraft at the same time to compute their clock bias by using the atomic clocks in the aircraft as a medium. By contrasting (7) and (8), we can see that the Shapiro delay and the converting delay from coordinate time to the proper time are computed in space station CV but not in GNSS CV. These two items are greater than one picosecond but less than one nanosecond. So, these delay
items cannot be neglected if we want to get several tens of picoseconds accuracy. But they can be neglected in GNSS CV which aims at nanosecond time comparison accuracy.

3. Comparising space station and GNSS CV

As mentioned above, due to the target accuracy between the space station CV and GNSS CV is different, the delay items to be considered are also different. Furthermore, there are several differences between them, such as the orbit characteristic and effect, space atomic clock performance and down-link signal structure, and so on. These differences limit the application of the traditional GNSS CV method to the space station.

3.1 System design differences between space station and GNSS CV techniques

The system design differences between the space station and GNSS satellite mainly contain three aspects: the orbit characteristics, space atomic clock system and signal structure of the time comparison down-link.

First, the orbit characteristics of these two systems are compared. The space station has low orbit, and the average orbit height is only about 400 kilometers. The speed of
the space station is far faster than that of GNSS satellites. The average orbital period is
about 1.5 hours, and there are about nine visible flight periods in one day. The average
visible time interval per one orbit period for the ground station is also very short and
about 400 seconds. These characteristics of the space station are mentioned by Yang et
al. (2016), Duchayne et al. (2008) and Föckersperger et al. (2004). Thus, the space
station CV time comparison can only be operated in a few minutes per day. When the
ground stations are in the blind areas, the traditional CV operation cannot be carried out.
The space station visibility of one day to several representative cities in China is shown
in Table 1. In this table, the cut-off elevation is ten degree, and the data in the table
denotes the cumulative minutes for one day. The table shows that every visible interval
of each city is very short and its mean value is about four minutes. Mohe is the
northernmost city of China, there is only one valid observation time interval which is
about four minutes. The other observation time for other cities is longer than that of
Mohe. Table 1 also shows that some cities cannot observe the space station at the same
time, thus traditional GNSS CV computation cannot be done. In other words, there are
blind areas if GNSS CV method is applied to the space station. For example, if one
station is located in Kashi and another station is located in Sanya, GNSS CV method is invalid.

But there are many GNSS satellites, which can supply uninterrupted CV time comparison service (Chen et al. 2014; Zhang et al. 2017; Kou 2012). For example, there are more than twenty MEO satellites in GPS, their orbit heights are about 20000 km, and their orbit periods are about 11 hours and 58 minutes. Thus, the coverage capability of GPS satellite is the whole earth. Taking Beidou system for example, there are GEO, IGSO and MEO satellites, and Beidou satellites also can be used as continuous CV objects.

Then, the atomic clock carried by the space station is compared with that carried by the GNSS satellite. The space station atomic clock has better performance. First, the laser cooling techniques and the space microgravity environment will be combined to improve the stability of the atomic clock in the space station. Although the GNSS satellites are also in the space microgravity environment, there are no cooled atomic clocks carried on the GNSS satellites. Second, the performance level of the space station atomic clock is higher than that of the GNSS satellite clock. There will be two
atomic clocks carried in the ISS for the ACES project. One is a hydrogen maser whose
frequency stability is better than 1.5E-13 when the averaging time interval $\tau$ is equal
to 1 second and is better than 1.5E-15 when $\tau$ is equal to 86400 seconds. The other is a
cold atom cesium clock, whose frequency stability should be better than $1E^{-13}/\sqrt{\tau}$.
These two clocks will be integrated to output better time and frequency signals, so the
frequency stability of ACES integrated signal is better than 1E-13 when $\tau$ is equal to 1
second and is better than 1E-15 when $\tau$ is equal to 86400 seconds. The payload of
 Chinese Space Station not only contains ordinary atomic clocks, but also contains a
strontium atom light clock with the performance at least an order of magnitude higher
than that of ACES clocks. Most of clocks carried by GNSS satellites are rubidium or
cesium atomic clocks. Although there are some passive hydrogen atomic clocks in
Galileo satellites, the frequency stability of integrated signal is an order of magnitude
worse than the ACES signal. The atomic clocks of the space station and GNSS satellites
are the CV references. Although the clock errors have been canceled by the CV
algorithm, excellent clock reference is helpful to optimize the space station CV method
that will be mentioned latter.
Finally, the signal structure differences between these two systems are explained. The differences related to time comparison performance consist of the code chip rate and the signal frequency. The down link code chip rate of the space station is 100 Mchip/s, which is nearly ten times the GPS P-code. Thus, the downlink signal pseudo code ranging accuracy of the space station is higher than that of the GNSS satellite. The signal frequency of GNSS is commonly in L band, but that of ACES is in the Ku band, and that of the Chinese Space Station is higher and in Ku and Ka bands. The ionospheric delay is inversely proportional to the square of the signal frequency. The ionospheric delay of the space station is far less than that of GNSS. For GNSS CV, the ionospheric delay is the main error source. But for space station CV, the ionospheric delay can be calculated by the pseudorange combination of two or three carriers in different frequencies with the accuracy of picosecond magnitude. Therefore, the time comparison performance of the space station down link is better than that of GNSS, and higher CV time comparison accuracy can be achieved by this link.

3.2 Effect differences of orbit error on space station and GNSS CV

Because the target accuracy of the space station CV is two orders magnitude higher than
that of GNSS CV, their error correcting methods are different. The atmospheric delay, the earth rotation, and the gravity delay are not explained here, and only the influence of orbital error on the space station CV and GNSS CV is analyzed in detail.

From the equations (7) and (8), the differential coefficient of the aircraft position error can be gotten by the following equation:

\[ dV_{AB} = \frac{1}{c} \left( \frac{\dot{\rho}_{SB} - \dot{\rho}_{SA}}{\rho_{SB}} \right) d\dot{X}_S \tag{9} \]

where \( \dot{X}_S \) denotes the position error of the aircraft. Equation (9) can be rewritten as:

\[ dV_{AB} = \frac{1}{c} \left( \frac{\rho_{SA} \dot{\rho}_{SB} - \rho_{SB} \dot{\rho}_{SA}}{\rho_{SB} \rho_{SA}} \right) d\dot{X}_S \tag{10} \]

Assuming that \( \rho_{SA} = \rho_{SB} + V \rho \), we can get the following equation:

\[ dV_{AB} = \frac{1}{c} \left( \frac{\rho_{SB} \dot{\rho}_{BA} + \Delta \rho \dot{\rho}_{SB}}{\rho_{SB} \rho_{SA}} \right) d\dot{X}_S \tag{11} \]

where \( \dot{\rho}_{BA} \) denotes the distance vector from station B to A. After some simple transformation of (11), the following inequality is obtained:

\[ |dV_{AB}| \leq \frac{1}{c} \left( \frac{|\dot{\rho}_{BA}| + |\Delta \rho|}{\rho_{SA}} \right) |d\dot{X}_S| \tag{12} \]

which shows that the effect of orbit error on CV time comparison is related to the distance between the aircraft and the ground station, the length of the CV baseline and the orbit error itself.
Generally, GNSS satellite orbit height is usually about 20000 kilometers, so the following relation expression can be obtained for GNSS:

\[
\frac{|\Delta \rho|}{\rho_{SA}} = 1
\]  
(13)

Equation (12) can be rewritten approximately as for GNSS CV:

\[
|dV_{TA}| \leq \frac{1}{c} \frac{\rho_{rel}}{\rho_{SA}} \left| \frac{dX_{S}}{c} \right|
\]  
(14)

GNSS CV baseline is up to thousand kilometers, which is much less than \( \rho_{SA} \). So the effect of GNSS satellite position error is reduced. Taking the CV baseline as 2000 kilometers and GNSS orbit error as one meter for example, the maximum influence magnitude of the orbit error to GNSS CV is about 300 picoseconds according to (14).

This effect can be ignored for a 3 to 5 nanoseconds time comparison accuracy. Additional orbital improvements need not to be made for GNSS CV, and the CV algorithm itself is enough.

The orbit height of space station is about 400 kilometers. But the distance between the space station and the ground station ranges from several hundred to several thousand of kilometers. Thus, the following express is sometimes correct for the space station:

\[
\frac{|\Delta \rho|}{\rho_{SA}} > 1
\]  
(15)
thus the effect of space station orbit error may be magnified.

On the other hand, the following express is also correct for the space station at most times:

$$\frac{\hat{\rho}_{BA}}{\rho_{SA}} > 1$$  \hspace{1cm} (16)

So, if the CV baseline length is longer than the distance between the space station and the ground station, the effect of orbit error is probably enlarged. But the CV baseline length is often longer than several hundred kilometers. The space station position data is provided by a GNSS receiver. With the development of GNSS technology, the positioning accuracy of GNSS receiver reaches decimeter or even centimeter at present (Martin et al. 2012; Cerri et al. 2010; Lemoine et al. 2010). If the baseline is 2000 kilometers and the orbit error is 10 centimeters, the effect of orbit error on the space station CV is up to one nanosecond. This impact is significant for the target accuracy of several tens of picoseconds. Thus, GNSS CV method cannot be applied to the space station CV time comparison directly. This method must be optimized to meet the high accurate requirement.

3.3 Application limitations when using GNSS CV method to the space station
From the analysis results above, it is known that the traditional GNSS CV method requires two ground stations observing the space station simultaneously. Due to the low orbit and fast orbital speed of the space station, the ground stations in some areas cannot observe the space station simultaneously all the time. Therefore, GNSS CV method cannot be used for this case. This is the first application limitation.

The other limitation relates to CV time comparison accuracy. As mentioned above, the GNSS CV method will result in great time comparison errors for the space station CV. This limitation is validated by simulation experiments in which the GNSS CV method is applied to the space station.

In the simulation, the orbit error is divided into three components, the radial (R), tangential (T) and the normal (N) vector components. Each component is set to 0.1 meters in R, T and N. The mean value of observation noise is set to zero, and its standard deviation is set to 1 picosecond. The ionospheric delay data is generated based on the VTEC (Vertical Total Electronic Content) data provided by IGS. The tropospheric delay is modeled by the Saastamoinen model. Minimum elevation of the space station is taken as 10°. The temperature is set to 298 K, the pressure is set to 1 bar,
and the water vapor pressure is set to 0.5 bar. The two ground stations are located in Xi’an and Sanya respectively, so the baseline length is about 2000 kilometers. The frequency stability of the Xi’an atomic clock when $\tau$ is equal to 1 second is set to $1\times10^{-13}$, and the frequency stability when $\tau$ is equal to 86400 seconds is set to $1\times10^{-15}$. The frequency stability of the Sanya atomic clock when $\tau$ is equal to 1 second is set to $5\times10^{-13}$, and the frequency stability when $\tau$ is equal to 86400 seconds is set to $1\times10^{-14}$.

Xi’an and Sanya space station CV time comparison errors are shown in Figure 1. The traditional GNSS CV method is applied in this figure. The data displayed in the abscissa indicates the cumulative seconds of one day, and the data displayed in the ordinate indicates the value of time comparison error. It can be seen from the figure that there are two CV intervals for Xi’an and Sanya to do space station CV time comparison for one day, and each interval lasts for about three minutes. It also can be seen that the maximum time comparison error is close to 800 picoseconds, and the error fluctuation range is about 600 picoseconds. Thus, GNSS CV method cannot be directly used to the space station CV time comparison, when the time comparison target accuracy is better than several tens of picoseconds.
Figure 2 shows the effect of the space station orbit error on time comparison when GNSS CV time comparison method is applied. The data of Figure 2 is obtained by the method of geometrical computation, and the computation formula will be introduced in the next section in detail. Comparing Figures 1 and 2, we find that the error distributions of these two figures are virtually identical. Therefore, we know that the orbit error is the main reason that results in the low accuracy if GNSS CV method is used to the space station CV time comparison.

In order to verify the correctness of the above analysis result, other simulation experiments also have been done, in which the ground stations are located in Shanghai and Kunming or Xi’an and Changchun. Consistent conclusions have been obtained. So, in order to realize the CV accuracy of several tens of picoseconds, the traditional GNSS CV method must be optimized.

4. Optimization of space station CV method

It is known that the traditional GNSS CV method cannot be used to the space station directly for two aspects of reasons. One is the existence of the CV blind area. The other is the time comparison accuracy is limited to hundreds of picoseconds. In order to solve
these problems, the GNSS CV method is optimized.

The effect of orbit error on time comparison can also be expressed by the following equation:

\[ c[dY_{AB}] = dX_S^R (\cos \alpha_{SA} - \cos \alpha_{SB}) + dX_S^T (\cos \beta_{SA} - \cos \beta_{SB}) + dX_S^N (\cos \gamma_{SA} - \cos \gamma_{SB}) \]

\[ (17) \]

where \( dX_S^R \), \( dX_S^T \) and \( dX_S^N \) denote the R, T and N orbit error component respectively, \( \alpha_{SA} \), \( \beta_{SA} \) and \( \gamma_{SA} \) denotes the angle between the vector from the space station to ground station A and the R, T and N vector respectively, \( \alpha_{SB} \), \( \beta_{SB} \) and \( \gamma_{SB} \) denotes the angle between the vector from the space station to ground station B and the R, T and N vector respectively. The data of Figure 2 is computed by (17).

If the cosine of \( \alpha_{SA} \), \( \beta_{SA} \) and \( \gamma_{SA} \) is approximately equal to the cosine of \( \alpha_{SB} \), \( \beta_{SB} \) and \( \gamma_{SB} \) respectively, the calculation result of the right side of (17) is close to zero. The orbit error has little effect on time comparison at this time. Therefore, the relative position relationship between the two ground stations and the space station is very important for cancelling the effect of the orbit error. The relative position relationship satisfying the condition that the cosine of \( \alpha_{SA} \), \( \beta_{SA} \) and \( \gamma_{SA} \) is close to the cosine of \( \alpha_{SB} \),
\[ \beta_{SB} \text{ and } \gamma_{SB} \] respectively need to be found. The CV time comparison computation is done based on this position relationship and most influence of the orbit error can be removed. The following equation is used as a judgment condition to find this position relationship:

\[
\text{flag} = |\cos \alpha_{SA} - \cos \alpha_{SB}| + |\cos \beta_{SA} - \cos \beta_{SB}| + |\cos \gamma_{SA} - \cos \gamma_{SB}| \leq T_{hod} \quad (18)
\]

where \( \text{flag} \) denotes the decision factor, and \( T_{hod} \) denotes the decision threshold.

Combining equation (17) and (18), the relationship between \( T_{hod} \) and \( |dVT_{AB}| \) can be expressed as:

\[
c|dVT_{AB}| \leq \text{flag} \times \left( |dX^R_S|, |dX^T_S|, |dX^N_S| \right)_{\max} \leq T_{hod} \times \left( |dX^R_S|, |dX^T_S|, |dX^N_S| \right)_{\max} \quad (19)
\]

where \( \left( |dX^R_S|, |dX^T_S|, |dX^N_S| \right)_{\max} \) denotes the maximum of \( |dX^R_S|, |dX^T_S| \) and \( |dX^N_S| \).

Therefore, the influence of orbit error on CV time comparison can be adjusted by the decision threshold \( T_{hod} \). For example, if the orbit errors in R, T and N are all less than 0.1 meters, and the decision threshold \( T_{hod} \) is set to be 0.03, the time comparison error caused by the orbit error will be less than ten picoseconds. If \( T_{hod} \) is set to be 0.06, the time comparison error caused by the orbit error will be up to twenty picoseconds. But it is important to note that the decision threshold does not follow the rule less the better.
The less the decision threshold \( Thod \) is, the less position combination that meet the judgment condition (18) will be found. In a general way, \( Thod \) in the range from 0.03 to 0.05 is ok. But for some long baseline CV, \( Thod \) need to be magnified properly. For example, the baseline length of Changchun and Kunming is about 3500 km, \( Thod \) need to be set as 0.06. If \( Thod \) is set to be 0.05, the condition (18) can not be satisfied.

Furthermore, two ground stations that meet the judgment condition of (18) do not need to observe the space station simultaneously. For example, ground station A collects observation data at \( t_1 \), ground station B collects observation data at \( t_2 \), the relative position of ground station A and the space station at \( t_1 \) and the relative position of ground station B and the space station at \( t_2 \) satisfy the judgment condition of (18). Thus, the orbit error component in the calculated clock bias between station A and the space station at \( t_1 \), \( \nu_{AS}(t_1) \), is close to the orbit error component in the calculated clock bias between station B and the space station at \( t_2 \), \( \nu_{BS}(t_2) \). \( \nu_{AS}(t_1) \) and \( \nu_{BS}(t_2) \) is computed by the one-way time comparison method. After the differencing operation of \( \nu_{AS}(t_1) \) and \( \nu_{BS}(t_2) \), the remnants of orbit error are almost removed.

Because the relative frequency bias between two atomic clocks makes the phase bias
change with time. $V_{T_{\text{AS}}}(t_1)$ cannot be minus $V_{T_{\text{BS}}}(t_2)$ directly to calculate the clock bias of stations A and B. It is necessary to get the clock biases $V_{T_{\text{AS}}}(t)$ and $V_{T_{\text{BS}}}(t)$ at the same time to calculate the clock bias between two ground stations. This is the fundamental idea of CV time comparison method.

The atomic clock of the space station has the stability higher than that of the ground atomic clock. The frequency stability of the space clock when $\tau$ is equal to 1 second is better than 1E-13, and the frequency stability when $\tau$ is equal to 86400 seconds is better than 1E-15. The ground clocks that need several tens of picoseconds time comparison accuracy also have good stability and little frequency drift. Thus, we can use the linear polynomial to model the clock bias between the ground station and the space station. The modeling target is to compute the relative frequency deviation of these two clocks. The clock bias model is shown as follows:

$$V_{T_{\text{AS}}}(t) = a + b \times (t - t_0)$$

(20)

where $a$ denotes the constant term, $b$ denotes the first-order coefficient of the polynomial, and $t_0$ denotes the model starting time.

The following equation will be used to obtain the clock bias between station A and
the space station at \( t_2 \) based on the extrapolation principle.

\[
 VT_{AS}^\prime (t_2) = VT_{AS}(t_1) + b \times (t_2 - t_1) \quad (21)
\]

where \( VT_{AS}(t_1) \) denotes the clock bias between station A and the space station. Then, we can compute the clock bias of station A and B by the following equation:

\[
 VT_{AB}(t_2) = VT_{AS}^\prime (t_2) - VT_{BS}(t_2) = (VT_{AS}(t_1) - VT_{BS}(t_2)) + b \times (t_2 - t_1) \quad (22)
\]

because the orbit error component in \( VT_{AS}(t_1) \) and \( VT_{BS}(t_2) \) is nearly equal to each other, the orbit error is almost removed in (22). So, the GNSS CV method has been optimized and new space station CV method has been proposed.

It can be seen from (20) that the modeling data source is the clock bias that is calculated based on one-way time comparison method. So, the data source contains the orbit error, and some modeling error must be applied to (22).

The error of \( VT_{AS}(t_2) \) can be expressed by the following equation:

\[
 \epsilon (VT_{AB}(t_2)) = (\epsilon_{\text{orbit}}(VT_{AS}(t_1)) - \epsilon_{\text{else}}(VT_{BS}(t_2))) + (\epsilon_{\text{orbit}}(VT_{BS}(t_2)) - \epsilon_{\text{else}}(VT_{BS}(t_2))) + \epsilon_{\text{mod}} \quad (23)
\]

where \( \epsilon_{\text{orbit}}(VT_{AS}(t_1)) \) and \( \epsilon_{\text{orbit}}(VT_{BS}(t_2)) \) denote the orbit error in \( VT_{AS}(t_1) \) and \( VT_{BS}(t_2) \) respectively, \( \epsilon_{\text{else}} \) denotes other errors except the orbit error, \( \epsilon_{\text{mod}} \) denotes the modeling
error of (20). Because the relative position of ground station A and the space station at $t_1$
and the relative position of ground station B and the space station at $t_2$ satisfy the
judgment condition of (18), the first error term of the right side of (23) is cancelled. The
orbit error is canceled by CV principle, but the modeling error is introduced. This
method need to obtain the linearized parameter $b$ of clock bias through modeling.
Although the orbit error in the modeling data source is large, the time comparison error
caused by modeling is much less than the original orbit error. The simulation
experiments will be used to validate this method later.

From above optimizing procedure, it is known that (18) presents the basic principle
of the optimized method. Two ground stations must obtain the space-ground clock bias
of the same time to calculate the clock bias between these two ground stations based on
CV method. But the synchronous space-ground clock biases of the two ground stations
are not all calculated by the observing data. One is calculated by the observing data. The
other is obtained by the method of modeling and extrapolating. The optimized method
makes use of the high stability of the space and ground atomic clocks to model the
space-ground clock bias. By modeling and extrapolating, the space-ground clock biases
at the same time are obtained, and the orbit errors are cancelled at that time. After calibrating other errors of (7), several tens of picoseconds CV time comparison accuracy can be achieved.

Moreover, the optimized method does not require that two ground stations observe the space station at the same time, so this method can be applied to the blind areas where GNSS CV method is invalid.

5. Simulation verification of optimized method

In order to verify the validity of the optimized space station CV time comparison method mentioned above, we selected two ground station groups as examples to do the simulation experiments. Xi’an and Sanya stations form the first group and Kashi and Sanya stations form the second group which faces the problem of GNSS CV blind area.

5.1 Analysis of Xi’an and Sanya Simulation

The simulation condition of the optimized method is almost the same as that of GNSS CV method. The decision threshold of (12) is set to be 0.03. The optimized method is used to realize the CV time comparison of Xi’an and Sanya stations, the decision factor is shown in Figure 3, and time comparison error is shown in Figure 4. These two figures
are all in three-dimensional coordinate format. The data of X and Y axis indicates the cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis of Figure 3 indicates the decision factor, and that of Figure 4 indicates the time comparison error.

It can be seen from Figure 3 that the decision factor is calculated based on un-simultaneous space station position for Xi’an and Sanya. Thus, the space-ground clock biases of these two stations are not derived from the synchronous observation data.

Figure 4 shows that the absolute time comparison errors are less than 30 picoseconds based on the optimized space station CV method.

It is noteworthy that the stability of Xi’an atomic clock is better than that of Sanya, so Xi’an space-ground clock bias is modeled and extrapolated. By this means, less modeling error is introduced, and higher accuracy is obtained.

### 5.2 Analysis of Kashi and Sanya Simulation

It can be seen from Table 1 that traditional GNSS CV method is invalid if one ground station is in Kashi, and the other station is in Sanya. Because these two ground stations
cannot observe the space station simultaneously. But the optimized method is valid to
implement the CV time comparison of these ground stations.

The baseline length of Kashi and Sanya is longer than 5000 kilometers, and the
simulation condition is essentially the same as that of Xi’an and Sanya simulation
except for the atomic clock parameters of Kashi. The frequency stability of the Kashi
clock when $\tau$ is equal to 1 second is set to $5 \times 10^{-13}$, and the frequency stability when $\tau$ is
equal to 86400 seconds is set to $1 \times 10^{-14}$. The decision factor of Kashi and Sanya stations
is shown in Figure 5, and the time comparison error is shown in Figure 6.

Although the baseline length of Kashi-Sanya is almost two times that of Xi’an-Sanya,
and the atomic stability of Kashi is worse than that of Xi’an, but it can be seen from
Figure 6 that the equivalent CV time comparison accuracy is obtained. Figure 6 shows
that the absolute time comparison errors of Kashi and Sanya are less than 35
picoseconds. It is known from this simulation experiment that the optimized method
solves the problem of the blind area and also is effective to realize super high accurate
time comparison.

It can be seen from Figure 5 that there are not many observation time groups of
Sanya and Kashi when the decision threshold is set to 0.03. But the time comparison error of Figure 6 is little. Therefore, the threshold can be appropriately magnified to obtain more times of CV time comparison.

6. Conclusion

The traditional GNSS CV method is optimized to be more effective for the space station. First, the CV time comparison theory has been introduced. The difference between the space station CV and GNSS CV were pointed out. Then, the application limitations of GNSS CV method for the space station were analyzed based on simulation. After that, the traditional CV method was optimized, and the limitations were improved. Finally, the performance of the optimized method was validated by simulation experiments. The following conclusions are drawn:

(1) There are three differences between the space station and the GNSS satellite, the orbit characteristics, downlink signal structure and the atomic clock performance. The space station advantages regarding downlink structure and clock performance are the premise for achieving high accurate CV time comparison. But the space station orbit error cannot be canceled substantially by the traditional CV principle, and additional
orbit error calibrating method must be designed and used.

(2) There are two limitations for traditional GNSS CV method to be applied to the space station. On the one hand, there are GNSS CV blind areas. On the other hand, GNSS CV method is not good in dealing with the orbit error and it is only useful for several hundreds of picoseconds space station CV accuracy.

(3) The optimized method has excellent performance. First, several tens of picoseconds CV time comparison accuracy can be realized by the optimized method, and the CV baseline length can be up to several thousands of kilometers. Second, the optimized method does not need the ground stations observe the space station simultaneously, so it solves the problem of CV blind area handily.

Abbreviations

CV: Common-view; GNSS: Global Navigation satellite System; ACES: Atomic Clock Ensemble in Space; VLBI: Very Long Baseline Interferometry; ISS: International Space Station; GCRS: Geocentric Celestial Reference System; RTN: the radial (R), tangential (T) and the normal (N) vector; VTEC: Vertical Total Electronic Content.

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Authors’ contributions

Doctor Liu Yinhua designed the research, implemented the algorithm and simulated software of the optimized CV method and did the simulation experiments. Doctor Li Xiaohui proposed the original thought of the space station optimized CV method. All authors read and approved the final manuscript.

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Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors declare that they have no competing interests.

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**Figure Legend**

Figure 1: Errors of Xi’an and Sanya traditional CV time comparison. The data displayed in the abscissa indicates the cumulative seconds of one day, and the data displayed in the ordinate indicates the value of time comparison error between Xi’an and Sanya using traditional CV method.

Figure 2: Xi’an and Sanya CV time comparison error caused by orbit error. The data displayed in the ordinate indicates the orbit error contained in time comparison error between Xi’an and Sanya using traditional CV method.

Figure 3: Flag of Xi’an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi’an and Sanya, respectively. The data of Z axis indicates the decision factor.

Figure 4: Errors of Xi’an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi’an and Sanya, respectively. The data of Z axis indicates the value of time comparison error.
between Xi’an and Sanya using optimized CV method.

Figure 5: Flag of Kashi and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Kashi and Sanya, respectively. The data of Z axis indicates the decision factor.

Figure 6: Errors of Kashi and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Kashi and Sanya, respectively. The data of Z axis indicates the value of time comparison error between Kashi and Sanya using optimized CV method.

Table List

Table 1: Space station visibility to several representative cities of China. The data in the table indicates the cumulative minutes of one day, the unit of that is min.