A CELLULAR AUTOMATON MODEL FOR THE TRAFFIC FLOW IN BOGOTA

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In this work we propose a car cellular automaton model that reproduces the experimental behavior of traffic flows in Bogotá. Our model includes three elements: hysteresis between the acceleration and brake gaps, a delay time in the acceleration, and an instantaneous brake. The parameters of our model were obtained from direct measurements inside a car on motorways in Bogotá. Next, we simulated with this model the flux-density fundamental diagram for a single-lane traffic road and compared it with experimental data. Our simulations are in very good agreement with the experimental measurements, not just in the shape of the fundamental diagram, but also in the numerical values for both the road capacity and the density of maximal flux. Our model reproduces, too, the qualitative behavior of shock waves. In addition, our work identifies the periodic boundary conditions as the source of false peaks in the fundamental diagram, when short roads are simulated that have been also found in previous works. The phase transition between free and congested traffic is also investigated by computing both the relaxation time and the order parameter. Our work shows how different the traffic behavior from one city to another can be, and how important is to determine the model parameters for each city.

Keywords: Cellular automaton models, Fundamental diagrams, Jams, Shock waves, jamming transitions.

1. Introduction

Some years ago, Bogotá was a city with heavy traffic congestion and a chaotic transportation system, just because it has 7 million inhabitants with more than 55,000 taxis, 18,000 buses of different kinds, and a million of private cars roaming the streets. Recent city administrations have tried to solve this problem by introducing transportation strategies such as: a mass transportation system (Transmilenio), almost 250 kilometres of bike paths, pedestrian bridges everywhere and restrictions on the use of private cars at rush hours (Pico y placa). Thanks these efforts, Bogotá
Traffic Flow has reduced the mean travel time in a 40%, accidents in an 80% and pollution in a 50%. However, there are still many improvements to be done in the future.

During the last 10 years, cellular automata models (CA) have been applied with success to traffic simulations. These models are able to reproduce the macroscopic properties of highway traffic from the microscopic behavior of each car. The first model (STCA)\cite{1,2}, proposed by Nagel and Schrekenberg in 1992, is just based in the distance to the next car (\textit{gap}=\Delta x) and the maximal speed, but leads to a quite realistic flow-density relation (fundamental diagram) and reproduces well the spontaneous jam and shock waves formations in highways\cite{2,3}. Later developments add other elements to STCA, like improved gaps including the speed difference to the car ahead\cite{4,5}, the speed at the previous time step\cite{4,5}, and many other parameters. Some theoretical and practical studies have extended these models to two- and three-lane highways\cite{6}.

The drivers’ driving is very different from city to city, and a realistic traffic model should keep in mind the particularities of each place. Simulations and studies of this type have been carried out in cities like Portland, Los Angeles, Tokyo\cite{7,8,9}. However, there are not such studies performed in Colombia, even in Bogotá.

In this work we propose a car cellular automaton model that reproduces the experimental behavior of traffic flows in Bogotá. Our model includes three elements. The first one are the gaps the driver uses to decide to brake (brake gap \textit{gap}_{brake}) or accelerate (acceleration gap \textit{gap}_{accel}). They are, in general, different (hysteresis) and both depend on the speed. The second element is the time it takes the car to reach the next discrete speed value (retarded acceleration, \textit{t}_{up}). The last one is an instantaneous brake reaction (that) we have observed when the car ahead brakes. The parameters of our model were obtained from direct measurements inside a car that was running on Bogotá’s highways. With these parameters, simulations were performed to construct the flow-density fundamental diagram. This result was compared with experimental measures from Bogotá’s highways. The model was also used to compute shock waves, both in the free and congested regimes. Finally, we studied the phase transition between free and congested traffic (jamming transition) by computing both the relaxation time and the order parameter according to their definition in\cite{14,15}. This last study was performed on both the deterministic model proposed above and on the same model plus a probabilistic spontaneous-brake rule, as in STCA.

The paper proceeds as follows. In section 2, we make a detailed description of our model, the values of the measured parameters and the vehicle rules. Section 3 we show the simulation results and compare them with experimental data. Section 4 includes our study on the jamming transition. Section 5 contains the main conclusions and discussions of our work. Finally, appendix A and B describe in detail the experimental methods we used to obtain the model parameters and the experimental flux-density diagrams.
2. Our Model Description

In our model, the highway is represented by a one-dimensional array of length \( L \) with periodic boundary conditions. Each site of the array is a cell of length 2.5\( m \), that is a finer discretization than the used in STCA model. Vehicles can only have integer velocity values, \( v=0,1,...,v_{\text{max}} \). We used \( v_{\text{max}}=7 \) and a speed unity \( v_{\text{unity}}=10\frac{\text{km}}{\text{h}} \). This corresponds to time steps of \( t_{\text{step}}=0.9\text{s} \), that is near to the usual value to driver’s reaction time. These discretizations are usual for any CA traffic model, with the only difference that a vehicle occupies two consecutive cells, the length of a car (4.5\( m \)) plus the distance between cars in a jam (1\( m \)). Then, the maximal number of vehicles in the highway is given by \( N=\frac{L}{2.5} \). At time \( t \) the \( n \)-vehicle is completely defined by: its position \( x_n(t) \), its velocity \( v_n(t) \) and its brake-light status, \( b_n(t) \), which is \( b_n=1(0) \) when the driver brakes (or not) at the previous time step \( (t-1) \), like \(^{12}\). The effective gap is defined as \( \text{gap}=\Delta x(t)+\Delta v(t) \), where \( \Delta x(t)=x_{n+1}(t)-x_n(t)-1 \) is the number of cells empty to the vehicle ahead and \( \Delta v(t)=v_{n+1}(t)-v_n(t) \) is the speed difference to the car ahead.

As already mentioned, our model includes three elements: the hysteresis between brake and accelerate gaps, the retarded acceleration and the instantaneous break. The three parameters \( \text{gap}_{\text{brake}} \), \( \text{gap}_{\text{accel}} \) and \( t_{\text{up}} \), are function of speed and they represent on the whole the drivers’ driving. These parameters were experimentally found from inside a car on Bogotá’s highways (see appendix A) and are summarized in table 1.

| Speed | \( \text{gap}_{\text{brake}} \) | \( \text{gap}_{\text{accel}} \) | \( t_{\text{up}} \) |
|-------|-----------------|-----------------|----------------|
| 0     | 0               | 3               | 1              |
| 1     | 3               | 4               | 1              |
| 2     | 3               | 5               | 1              |
| 3     | 4               | 5               | 1              |
| 4     | 5               | 6               | 2              |
| 5     | 6               | 7               | 2              |
| 6     | 6               | 8               | 2              |
| 7     | 7               | 9               | 2              |

Table 1. Drivers’ driving from Bogotá.

Summarizing, all cars execute in parallel the following set of rules:
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Rules

• Compute its $\text{gap}(t)$.

• Read its parameters $\text{gap}_{\text{accel}}$, $\text{gap}_{\text{brake}}$ and $t_{\text{up}}$ from table 1.
  
  - normal brake: If $\text{gap}(t) \leq \text{gap}_{\text{brake}}$, speed down to the maximal speed $v(t+1)$ such that $\text{gap}_{\text{brake}}' \leq \text{gap}(t) \leq \text{gap}_{\text{accel}}'$, where $\text{gap}_{\text{brake}}'$ and $\text{gap}_{\text{accel}}'$ are the parameters at speed $v(t+1)$. In addition, let $\text{delay}=0$ and turn on brake lights ($b_{n+1}(t+1)=1$).
  
  - If $\text{gap} \geq \text{gap}_{\text{accel}}$, then
    
    * instantaneous brake: If $\text{gap}(t) \leq \text{gap}_{\text{accel}} + 2$ and the brake lights of the car ahead are on ($b_{n+1}(t)=1$), let $v(t+1)=v(t) - 1$ (brake), turn on brake lights ($b_{n}(t+1)=1$) and let $\text{delay}=0$.
    
    * accelerate: Else, turn off brake lights ($b_{n}(t+1)=0$) and
      
      - If $\text{delay}=t_{\text{up}}$, let $v(t+1)=v(t) + 1$ (accelerate) and let $\text{delay}=0$.
      
      - Else, let $\text{delay}=\text{delay} + 1$ and preserve $v(t+1)=v(t)$.
  
  - Otherwise, let $\text{delay}=0$, turn off brake lights ($b_{n}(t+1)=0$) and preserve $v(t+1)=v(t)$.

• Finally, move $v$ cells ahead,

$$x(t+1) = x(t) + v(t+1) .$$

The counter $\text{delay}$ defines if $t_{\text{up}}$ has been completed. The variable $b_{n+1}(t)$ defines the brake light status of the car ahead. The instantaneous brake rule represents the braking reaction we have observed when the car ahead also brakes. This reaction is observed for all distances but is just included in the gaps when $\text{gap} \leq \text{gap}_{\text{accel}}$. Thus, we have included it as an additional rule only if $\text{gap}_{\text{accel}} \leq \text{gap}(t) \leq \text{gap}_{\text{accel}} + 2$ through a brake light on each car.

3. Results

The system starts with an initial configuration of $N$ cars, with random distributions of speeds and positions and $b_{j}(0)=0$ for all $j$. In order to prevent traffic accidents, as a previous step, we limit the speed values to the headway ($v_{\text{ini}} \leq \Delta x$). Starting from this initial configuration, we measure the average velocity $\bar{v}(t)$ over all cars at each time step, $t$. After many time steps ($t \to \infty$), when the system reaches a stationary velocity state $\bar{v}(\infty)=v(\infty)$, the flow is computed by the relation $q=v(\infty) \cdot \rho=v(\infty) \cdot \frac{N}{L}$. The whole process is repeated 100 times for each density value, just to make statistics, and, so, the fundamental diagram is obtained.

Our model is able to reproduces the phases observed in real traffic: free-flow, synchronized, and stop-and-go. The shock waves and jams formations for these flow regimes can be observed in space-time plots (figure 1).
Before comparing the simulation results with the experimental data, we study in detail the finite size effects. For small systems, appears in the fundamental diagram a false peak of maximal flow, which disappears when the system is sufficiently large. Figure 2 shows the fundamental diagram for different system sizes. One can observe that the false peak appears around $\rho=0.12$ (in the synchronized traffic phase), and it has completely disappeared for $L=2000$. These false peaks have also been observed in many other models. In contrast, we have found that these false peaks are due to extraordinary configurations that exists only in small systems. Due to the periodic-boundary conditions, it is possible to think a configuration where all cars have the maximal speed and all have the same gap, equals to that maximal speed.

In these configurations the system does not relax, but remains forever, with a mean velocity that is larger than the average velocity for the relaxed system. Thus, they push up the values of $\langle v(\infty) \rangle$, where the $\langle \rangle$ denotes the average on many realizations, generating the false peak. In conclusion, the peak should be located at densities $\rho=1/v_{\text{max}}=0.14$, in good agreement with the computational results.

Since our model takes into account three elements, we were interested in looking at the effect of including each one of them, one by one, namely: the hysteresis between brake and acceleration gaps, the retarded acceleration and the instantaneous brake.

Figure 3 shows the fundamental diagrams obtained from our model with (a) brake and acceleration gaps only, (b) gaps plus retarded acceleration and (c) all three elements. They are compared with (d) the fundamental diagram from the STCA model with our time and space discretization. One notes small differences between the fundamental diagrams corresponding to STCA and the most elementary model. This means that the hysteresis doesn’t have any relevant effect on the fundamental
diagram. Its consequences, however, should be seen in the behavior of jams and shock waves, but they will be not investigated here and they would be theme of future work.

When the retarded acceleration is included, two effects emerge, namely: a decrease of the maximum flow and a its shift towards larger densities. Finally, the instantaneous brake effect shows up slightly into the congestion region.

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**Fig. 2.** Effects of small system sizes ($L$) into the simulated fundamental diagram. One observes the false peak of maximal flux due to the finite boundary conditions and how it disappears when $L=2000$.
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To validate our model we performed a comparison with experimental measurements. Figure 4 compares the flow-density diagram from our model with measurements over Bogotá’s highways on broad density ranges, both as a) dispersed data and b) averaged data. A second alternative, consisting of a STCA model plus retarded acceleration, is also included.

On one hand, one observes that both simulations are in good agreement with the experimental data, especially if we compare the numerical values, simulated and measured, of the road capacity and the density of maximal flux.

\[ \rho(q_{\text{max}})_{\text{exp}} = 0.33(3) \quad q_{\text{max}}_{\text{exp}} = 1.43(6), \]
\[ \rho(q_{\text{max}})_{\text{sim}} = 0.33(4) \quad q_{\text{max}}_{\text{sim}} = 1.320(4). \]

The difference in the scatter between simulated and experimental data shows up (at least in part) because the first ones are averages over much more cells than the second ones. The two models have different behaviours just in the zone of high congestion, were our model predicts lower fluxes. This is the effect of hysteresis that is expected to play a role in jam formations. On the other hand, these values of road capacity are slightly larger than those measured in other countries. This suggests that the traditional Bogotá’s aggressive driving makes the traffic flux more efficient. The price is, however, one of the most high rates of fatal victims on the world (in fact, one out of six victims of violent causes in Colombia dies in a car accident). For a recent work including aggressive drivers see.

Cellular automata models for traffic flow exhibits sometimes a phase transition from a free-flow phase to a congested phase. In most cases this transition is first order, but some models, like STCA, shows a second-order phase transition at a single point in the phase space. The relaxation time and the order parameter are the quantities which characterize this transition.

To compute the relaxation time we employed the definition of Csányi and Kertész. As they do, we start from a configuration random initial positions and zero speed for all cars. Then, the relaxation time is computed as

\[ \tau = \int_0^\infty [\min\{v^*(t), \langle \bar{v}(\infty) \rangle\} - \langle \bar{v}(t) \rangle] dt, \]

where \( v^*(t) = t \) denotes the speed a car obtains at time \( t \) when there are no cars ahead (free acceleration).

As we already know, a characteristic feature of a second order phase transition is the divergence of the relaxation time at the transition point. Figure 5 shows how the relaxation time has a maximum at a pseudo-critical density \( \rho_c = 0.33 \), which is the same value of maximal-flow density one can read from the flux-density fundamental diagram (figure 4), as expected for a deterministic model.

The typical order parameter for traffic cellular automata is

\[ m = \frac{1}{L} \sum_{i=1}^{L} n_i n_{i+1}, \]

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Fig. 4. Comparison of the flow-density diagram for the simulated models with the experimental measures. Both simulated models, our model and the STCA model plus retarded acceleration, are in good agreement with the actual data. Above: dispersed data. Below: averaged data.
Since a car in our model occupies two consecutive cells, we redefine the order parameter as
\[ m = \frac{1}{L} \sum_{i=1}^{L} [n_i n_{i+2} + n_i (1 - n_{i+2}) n_{i+3}] \] . (6)

Like 14, figure 6 shows that the order parameter exhibits a continuous transition. The situation is quite similar to the behavior of the order parameter in finite systems. That is, the order parameter have small values for small density, and around the \( \rho_c \) it begins to have a non-zero value.

Fig. 5. Relaxation time \( \tau \) near the transition density \( \rho_c \) for different system sizes.

Fig. 6. Order parameter for different system sizes. Below the transition density \( m(\rho) \) decreases to zero.
Finally, we wanted to explore what occurs when we add the randomization rule of the STCA model. This is,

- Randomization: If after the above steps the velocity is larger than zero \(v \geq 0\), then, with probability \(p\), let \(v(t) = v(t) - 1\).

Figure 7 shows the fundamental diagrams for different values of \(p\). One observes that when the noise \(p\) increases, the flow decreases and the maximum towards smaller densities. In addition, for larger densities, the randomization rule causes an early collapse in the system.

![Fundamental diagram](image1.png)

Fig. 7. Fundamental diagram for different values of \(p\). This figure corresponds to our model plus randomization rule of the STCA model.

![Order parameter behavior](image2.png)

Fig. 8. Behavior of the order parameter for our model with a randomization rule. a) Order parameter for different values of \(p\) and b) Scaling plot for the order parameter (excluding \(p=0\)).
Figure 8 shows the order parameter behavior for different values of $p$. It converges smoothly to zero for densities smaller to $\rho_c$. To the right hand, we performed the respective scaling. The following form summarize the simple scaling:

$$M(\rho) = m(\rho + \frac{\Delta}{2})$$ \hspace{1cm} (7)

Herein $\Delta$ is the shift of the transition density compared to the same value for $p=0.125$, because it was impossible to make the scaling for the deterministic case.

4. Conclusions

In this work, we present one of the first cellular automaton models that applies to the reality of Bogotá. Their objective is not just to propose a first model, but also, to investigate which models apply to our city. The work includes also measures of the driving parameters of a car in Bogotá and of the fundamental diagram of a highway.

Our model includes three elements, to summarize are the drivers’ driving: hysteresis between brake and acceleration gaps, retarded acceleration and instantaneous brake.

The simulation results are in a good agreement with the experimental measurements. That is, the simulated fundamental diagram reproduces successfully the shape of the fundamental diagram and the numerical values for both the road capacity ($q_{\text{max}}$) and the density of maximal flux ($\rho(q_{\text{max}})$). The obtained values are:

$$\rho(q_{\text{max}})_{\text{exp}} = 0.33(3) \hspace{1cm} q_{\text{max,exp}} = 1.43(6)$$ \hspace{1cm} (8)

$$\rho(q_{\text{max}})_{\text{sim}} = 0.33(4) \hspace{1cm} q_{\text{max,sim}} = 1.32(4).$$ \hspace{1cm} (9)

For small system sizes we have found that the false peak of maximal flux in the fundamental diagram is due to some extraordinary configurations. In these configurations the system remains in a not-relaxation state, with a mean velocity larger than $\langle v(\infty) \rangle$. Finally, these extraordinary configurations only exist for systems with $L \leq 2000$.

The hysteresis element in our model doesn’t have any relevant effect on the fundamental diagram. If we compares it with the STCA model, the differences appears only into the congestion region. However, we believe that the spontaneous formations of jams and shock waves should be affected by this elements, and this is an interesting area of future work. In contrast, the retarded acceleration is much more important. It fixes the real shape of the fundamental diagram and the characteristic values of $q_{\text{max}}$ and $\rho(q_{\text{max}})$ . Moreover, an STCA model plus the retarded acceleration is enough to reproduce the fundamental diagram we found in Bogotá’s highways.

By looking at the phase-transition, we found qualitatively the same behavior of both relaxation time and order parameter as shown in $^{14,15}$. One observes a
maximum of the relaxation time \( \tau \) near the density transition \( \rho_c \), specially for the deterministic case \( \rho_c = \rho(q_{\text{max}}) \). The divergence is more marked when the system size increases. The behavior of the order parameter shows a smooth convergence to zero for small densities.

Summarizing, both our model an the STCA model plus retarded acceleration reproduces the fundamental diagram for Bogotá. They could be used to perform more complicated simulations in future works, like semaphorized, intersections and even the Bogotá’s net of main highways. It also remains to study the jam formation in both models.

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Appendix A : Our Model Parameters

By installing a videocamera inside a car, we taped the driving of a driver on the highways in Bogotá. The videocamera was calibrated with the plates of the other cars, therefore we made a chart, which relates the longitude of the plates with the distance to the car in front, \( \Delta x \). During the trips, the copilot registers the car speed the velocity and if the driver brakes, accelerates or preserves this speed. With the films on hand, one can know at any time the vehicle speed, its state (braking, accelerating or preserving) and the distance to the car in front \( (\Delta x) \). The relative velocity \( \Delta v \) is calculated as

\[
\Delta v = \frac{\Delta x_2 - \Delta x_1}{2},
\]

where \( \Delta x_1 \) is the usual \( \Delta x \), and \( \Delta x_2 \) is the distance to the car in front measure after two seconds.

To include the retarded acceleration it is necessary to measure the parameter \( t_{\text{up}} \). For this purpose, the car was accelerated from 0 \( \frac{\text{km}}{\text{h}} \) to 100 \( \frac{\text{km}}{\text{h}} \) and we measure the time, it takes to reach the next discrete speed value, in steps of \( (10\frac{\text{km}}{\text{h}}) \).

Appendix B : Experimental Measurements

The experimental data were obtained by capturating on tape (from a pedestrian bridge), the traffic flow in a highway segment. The 30\text{th} avenue is perhaps the most important avenue of Bogotá, because it communicates directly the south with the north of the city. At daily rush hours the 30\text{th} avenue is not able to cope with the demand and bored traffic jams are generated. We chose for our measurements the south-north high-speed lane of the 30\text{th} avenue between 53\text{th} street and the Campín Football Stadium.

This is a two-lane sector and have a length of \( L = 169(5) \text{m} \approx 62(2) \) cells, i.e, a maximal number of cars \( N_{\text{max}} = 31(1) \).
We carried out the following process:

- At each time, the density of the system is computed by $\rho = \frac{N(t)}{N_{\text{max}}}$, where $N(t)$ is the number of cars over the highway sector.

- The velocity of the system is the average velocity over the $N$ cars. It is calculated as an arithmetic mean, $\bar{v}(t) = \frac{1}{N} \sum_{j=1}^{N} \bar{v}_j(t)$, where $\bar{v}_j(t)$ is the average velocity of each car over the highway sector.

- Finally the flow is calculated as $q = \rho \cdot \bar{v}(t)$.

With this process we obtained the experimental data to perform the flux-density fundamental diagram.
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1. K. Nagel and M. Schreckenberg, J. Phys. I(France) 12, 2221 (1992).
2. K. Nagel, Phys. Rev. E 53, 4655 (1996).
3. A. Schadschneider and M. Schreckenberg, J. Phys. A 26, L679 (1997).
4. D.E. Wolf, Physica A 263, 438 (1999).
5. C.L. Barrett, M. Wolinsky, and M.W. Olesen. "Emergent local Control in particle hopping traffic simulations". In 9, pp. 169-173.
6. R. Chrobok, S.F. Hafstein, A. Pottmeier. OLSIM: A New Generation of Traffic Information Systems. University of Duisburg-Essen.www.autobahn.nrw.de.
7. K. Nagel, Dietrich E. Wolf, Peter Wagner and Patrice Simon. Phys. Rev. E 58, 1425 (1998).
8. P. Wagner. "Traffic simulations using cellular automata: Comparison with reality". In 9, pp. 199-203.
9. Traffic and Granular Flow, edited by D.E. Wolf, M. Schreckenberg and A. Bachem (World Scientific, Singapore, 1995).
10. S. Krauss, P. Wagner and C. Gawron. Phys. Rev. E 54, 3707 (1996).
11. S. Krauss, P. Wagner and C. Gawron. Phys. Rev. E 55, 5597 (1997).
12. Knospe et al, J. Phys. A 33, L477 (2000).
13. K. Nagel. Multi-agent transportation simulation, 2004; http://www.sim.inf.ethz.ch/.
14. B. Eisenblätter, L. Santen, A. Schadschneider and M. Schreckenberg, Phys. Rev. E 57, 1309 (1998).
15. G. Csányi and J. Kertész, J. Phys. A 28, L427 (1995).
16. M. Sasvári and J. Kertész, Phys. Rev. E 56, 4104 (1997).
17. D. Jost and K. Nagel, Probabilistic Traffic Flow Breakdown In Stochastic Car Following Models; to be published in Traffic and granular flow ’03, P. Bovy, S. Hoogendoorn, M. Schreckenberg, and D.E. Wolf editors; http://www.vsp.tu-berlin.de/.
18. Data from Organización Mundial de la Salud, see in http://www.col.ops-oms.org/.
19. Data given by Instituto Nacional de Medicina Legal y Ciencias Forenses. See in http://www.saludcolombia.com.
20. Lee et al, PRL 92, 238702, (2004).