Are photon momenta in left-handed materials reversed?

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(Dated: September 20, 2008)

We develop a semiclassical theory to describe the photon momenta in left-handed materials (LHMs). A single two-level atom is introduced as an “explorer” to probe the momenta of photons. We demonstrate that the linear momentum of the photons reverses its direction in LHMs. However the orbital angular momentum is remains unreversed, although the wave-fronts reversed their screwing fashion. We theoretically predict that the spin angular momentum is also unreversed. The investigation of photon momenta will provide insights into the fundamental properties of LHMs.

PACS numbers: 42.25.-p; 42.79.-e; 41.20.Jb; 78.20.Ci

Left-handed materials (LHMs) would reverse many known optical properties [1], such as negative refraction [2], reversed Doppler effect [3], negative Goos–Hänchen shift [4], and reversed Cherenkov effect [5]. Vesalago theoretically predicted that the electromagnetic momentum should reverse its direction in LHMs [1]. However, there exist different arguments on the direction of linear momentum in LHMs [6, 7, 8, 9]. The question naturally arises: Whether is the linear momentum of photons in LHMs reversed? In this case, E, H, and k form a left-handed triplet. Under the paraxial approximation |ΔE/Δz| ≪ k|E|, we obtain the wave equation

\[
\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \varphi^2} + 2ik \frac{\partial}{\partial z} \right) E(\rho, \varphi, z) = 0,
\]

where \( \mathbf{r} = (\rho, \varphi, z) \) is cylinder polar coordinates and \( k = \omega / c \). A particularly important solutions of the paraxial wave equation are given by LG set of modes. In general, the LG field in LHMs can be written as [12]

\[
E_{pl} = \frac{C_{pl}}{w(z)} \left[ \sqrt{2} \rho \right]^{||} L_{l}^{||} n (2 \rho^2 / w^2(z)) \exp \left[ -\rho^2 / w^2(z) \right] \right. \times \exp \left[ i k_0 z \right] \exp \left[ -i k_0 \rho^2 z / R(z) \right] \exp [il \varphi] \times \exp \left[ -i(2p + \left| l \right| + 1) \arctan z / z_R \right],
\]

\[
w(z) = w_0 \sqrt{1 + (z / z_R)^2}, \quad R(z) = z + \frac{z_R^2}{z}.
\]

Here \( C_{pl} \) is the normalization constant, \( L_{l}^{||} \) is a generalized Laguerre polynomial, \( k_0 = \omega / c \) is the wave number in vacuum, \( z_R = n k_0 w_0^2 / 2 \) is the Rayleigh length, \( w(z) \) is the beam size, and \( R(z) \) the radius of curvature of the wave front. The last term denotes the Gouy phase which is given by \( \Phi = -(2p + \left| l \right| + 1) \arctan z / z_R \).

Laguerre-Gaussian beams with helical phase fronts, characterized by an \( \exp [il \varphi] \) azimuthal phase dependence, the angular orbital momentum in the propagation

\[\omega \] propagating in a homogeneous medium whose permittivity \( \varepsilon \) and permeability \( \mu \) are negative simultaneously.

The field can be described by the Maxwell’s equations and the constitutive relations

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\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \mathbf{B} = \mu_0 \mathbf{H},
\]

\[
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \quad \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}.
\]

From the Maxwell’s equations, we can easily find that the wave propagation is permitted in the medium with \( \varepsilon, \mu < 0 \). In this case, E, H, and k form a left-handed triplet.

In order to explore the momenta in LHMs, we develop a semiclassical theory to describe the photon momentum in LHMs. Here we do not want to involve in the famous Abraham-Minkowski controversy. The accepted convention associates the term Abraham momentum with the quantity \( \mathbf{D} \times \mathbf{B} \) and the term Minkowski momentum with the quantity \( \mathbf{E} \times \mathbf{H} / c^2 \) [10]. In our theory model, we introduce a two-level atom as an “explorer” to probe the photon momenta. The interaction of atom with a single mode of laser beam in which the atom is treated as a quantum two-level system and laser field is treated as classically. It is well known that Laguerre-Gaussian (LG) beam can carry two kinds angular momentum: spin angular momentum of magnitude \( \hbar / 2 \) per photon due to its polarization state and orbital angular momentum of \( \hbar \) per photon due to an azimuthal phase term \( \exp [i l \varphi] \) [11]. Thus, we introduce the LG beam to describe the orbital angular momentum.

We consider a monochromatic electromagnetic field \( \mathbf{E}(r,t) = \text{Re} \left[ \mathbf{E}(r) \exp(-i \omega t) \right] \) and \( \mathbf{H}(r,t) = \text{Re} \left[ \mathbf{H}(r) \exp(-i \omega t) \right] \) of angular frequency \( \omega \) propagating in a homogeneous medium whose permittivity \( \varepsilon \) and permeability \( \mu \) are negative simultaneously.

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Laguerre-Gaussian beams with helical phase fronts, characterized by an \( \exp [il \varphi] \) azimuthal phase dependence, the angular orbital momentum in the propagation

\[\omega \] propagating in a homogeneous medium whose permittivity \( \varepsilon \) and permeability \( \mu \) are negative simultaneously.
This is a time-dependent as well as spatially dependent force and it turns out that it is divisible into two types of force, namely, the dissipative force and a dipole force. The dissipative force represents the force due to the absorption and reemission of the light by the atom [13]. To explore the linear momentum and orbital angular momentum of photons, we want to investigate the dissipative force, which is given by

$$\langle F_{\text{diss}}(R, V) \rangle = \frac{2\hbar \Gamma \Omega_{lp}^2(R) \nabla \Theta_{lp}(R)}{\Delta_{lp}(R)^2 + 2\Omega_{lp}(R)^2 + \Gamma^2},$$

(9)

where \(\Gamma\) is the half-width of the upper quantum atomic state, and \(\nabla \Theta(R)\) is the spatial gradient of the phase of LG beam:

$$\nabla \Theta(R) = \left[-nk_0 + \frac{nk_0 \rho^2}{2(z^2 + z_R^2)} \left(\frac{2z^2}{z^2 + z_R^2} - 1\right) - \frac{(2p + |l| + 1)z_R}{z^2 + z_R^2}\right] e_z + \frac{nk_0 \rho^2}{R(z)} e_\rho + \frac{l}{\rho} e_l.$$

(10)

The plane-wave phase emerges directly from by setting \(l = 0, p = 0,\) and \(z_R \to \infty.\) The function \(\Omega_{lp}(R)\) is identified as the position-dependent Rabi frequency:

$$\Omega_{lp}(R) = \frac{\Omega_{00} C_{pl}}{1 + z^2/z_R^2} \left[\frac{\sqrt{2} \rho}{w(z)}\right]^{|l|} L_p^{|l|} \left[\frac{2\rho^2}{w^2(z)}\right] \exp \left[-\frac{\rho^2}{w^2(z)}\right].$$

(11)

which is well defined for a given plane-wave Rabi frequency \(\Omega_{00}\) and LG field. The dynamic detuning is defined as

$$\Delta_{pl}(R) = \Delta_0 - \nabla \Theta_{lp}(R) \cdot V,$$

(12)

where \(\Delta_0 = \omega - \omega_0\) is the static detuning, with \(\hbar \omega_0\) the level energy separation of the two-level atom and \(\omega\) the frequency of the light. Laguerre-Gaussian field exerts momenta to the atom is given by \(F_{\text{diss}} = dP/dt.\) To explore whether the photon momentum is reversed, we need to judge the direction of dissipative force. The force of the atom experiencing in a LG field is given by

$$\langle F_{\text{diss}}(R) \rangle = \frac{\hbar \Gamma \mathcal{J}}{1 + \mathcal{J} + \Delta^2/\Gamma^2} \left\{-nk_0 \rho^2 R e_\rho - \frac{l}{\rho} e_\phi \right\} + \left[-nk_0 + \frac{nk_0 \rho^2}{2(z^2 + z_R^2)} \left(\frac{2z^2}{z^2 + z_R^2} - 1\right) - \frac{(2p + |l| + 1)z_R}{z^2 + z_R^2}\right] e_z,$$

(13)

where \(\mathcal{J} = 2G^2(R)/\Gamma^2\) is the position-dependent saturation parameter. The motion of the atom governed by \(F_{\text{diss}} = d^2R(t)/dt^2,\) and the trajectory is depicted in Fig. 2. The dominant component of the dissipative force is the axial term:

$$\langle F_{\text{axial}} \rangle = \frac{nk_0 \hbar \Gamma \mathcal{J}}{1 + \mathcal{J} + \Delta^2/\Gamma^2} e_z.$$
The linear momentum of photon $\mathbf{p}_z = \hbar \mathbf{k}$ is reversed, which demonstrates Veselago’s early prediction [1].

The azimuthal is the only force that is response for a torque on the atom about the propagating axis of LG beam,

$$F_{\text{azimuth}} = \frac{\hbar \Gamma J}{\Delta^2 + \Gamma^2 + 2\Omega^2(R)} \mathbf{p} \times \mathbf{e}_\phi. \quad (15)$$

The torque is given by $\langle \mathbf{T} \rangle = \langle \mathbf{p} \times F_{\text{azimuth}} \rangle$. Clearly, the atom will rotate anticlockwise ($l > 0$) or clockwise ($l < 0$) about the beam axis with a angular velocity. This can be easily understood if we consider the torques under the condition of saturation limit $J \rightarrow \infty$, the torque deduces to the simple form

$$\langle \mathbf{T} \rangle \approx i\hbar \Gamma \mathbf{e}_z. \quad (16)$$

The is proportional to the orbital angular momentum of photon $i\hbar$, which is independent of the refractive index.

The atom still remains its rotation fashion in the LHM (see Fig. 2). Obviously, the orbital angular momentum of photon in LHMs is unreversed. This interesting feature is consistent with the appentence of the unreversed rotational Doppler effect [12].

Note that the light remains linearly polarized and the intrinsic spin plays no role in our theory model. When a birefringent particle such as a calcite fragment is introduced, and circularly polarized light would be converted to linear. In principle, the spin angular momentum should cause a particle to spin about its own axis. The sense of rotation is governed by the spin angular momentum of photons [16, 17]. In our opinion, spin and orbital angular momenta do not depend upon the refractive index and so is said to be intrinsic. Hence we predict that the spin angular momentum should remain its direction unchanged. Further research is needed to demonstrate whether the spin angular momentum is reversed. It is possible that the study of photon momenta in LHMs may make a useful contribution to long established Abraham-Minkowski dilemma.

In summary, we have developed a semiclassical theory to describe the photon momenta in LHMs. A single two-level atom has been introduced to probe the momenta of photons. We have demonstrated that the linear momentum in LHMs should be reversed. However the orbital angular momentum of photons remains unreversed, although the wave-fronts reversed their screwing fashion. We predict that not all optical effects would reverse their properties in LHMs. Some intrinsic optical effect, such as spin and orbital angular momenta of photon, should unreverse its direction. The semiclassical theory can be applied to explore other intriguing phenomena in LHMs, such as photon recoil, Doppler effect, and photon drag effect. To explore whether these effect are reversed would allow us to better understand the interaction of light with LHMs.

We wish to acknowledge the support of projects of the National Natural Science Foundation of China (Grants Nos. 10674045, 10804029, 50802027, and 60538010).

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