Some models of holographic dark energy on the Randall-Sundrum brane and observational data

Artyom V. Astashenok, Alexander S. Tepliakov
Immanuel Kant Baltic Federal University
Department of Physics, Technology and IT
236041 Kaliningrad, Russia, Nevskogo str.14

The some models of holographic dark energy for Randall-Sandrum brane are considered. For first class of dark energy models we take energy density in form $\sim L^{2\gamma -4}$ where $L$ is size of events horizon in Universe and $\gamma$ is parameter (Tsallis holographic energy). Analysis of observational data allows to define upper limit on value of $\delta = \rho_0/2\lambda$ ($\rho_0$ is current energy density in the Universe and $\lambda$ is brane tension). Then we investigate models for which dark energy density has form $\rho_{de} = C^2 L^{-2} - C^4 H^2$ where $H$ is Hubble parameter.

I. INTRODUCTION

From moment of discovery of cosmological acceleration in 1998 [1, 2] explanation of this fact became one of the most puzzles for theoretical physics and cosmology. The main way for resolution of this ambiguous task is postulate the existence of so called dark energy with very unusual properties. The parameter of state $w = p/\rho$ for dark energy is negative. According to most simple and successful model dark energy is nothing else than Einstein cosmological constant or vacuum energy and its density consist of 70% of total energy density in the Universe [3]-[9].

It is assumed that ultimate resolution of dark energy problem will be achieved in frames of quantum gravity. An interesting approach to this task is related to holographic principle. According to holographic principle all physical quantities inside Universe including dark energy density can be described by some values on spacetime boundary [10–12]. There are only two parameters through which one can calculate dark energy density, Planck mass $M_p$ and some characteristic lengthscale $L$ namely

$$\rho_{de} = 3C^2 M_p L^{-2}. \quad (1)$$

For $L$ one can take for example size of events horizon, particles horizon or inverse value of Hubble parameter $H$:

$$L_e = a(t) \left( \int_0^\infty \frac{dt'}{a(t')} \right),$$

$$L_p = a(t) \left( \int_0^t \frac{dt'}{a(t')} \right),$$

$$L_h = \frac{1}{H}. \quad (2)$$

The various aspects of holographic dark energy are investigated in many papers (see [13]-[22] and reference therein). As shown model with Hubble horizon are not suitable for description cosmological evolution of our universe. The main line of investigations is using size of events horizon as infrared cut-off. It is interesting to note that holographic principle can be applied to early universe too [24] with obtaining of inflation scenario. The cosmological bounce from holographic principle was considered in [25].

In 1988 K. Tsallis proposed the generalized expression for black hole entropy $S_{BH}$ [26]:

$$S_{BH} = \mu A^\gamma. \quad (3)$$

Here $\mu$ is unknown constant, $\gamma$ is non-additivity parameter and $A$ is black hole horizon area. It is obviously that well-known Bekenstein entropy

$$S_{BS} = \frac{A}{4}$$

follows from equation (3) if one put $\gamma = 1$ and $\mu = \frac{1}{4}$. If one assume that such approach is suitable for dark energy then its density can be written as [27]:

$$\rho_{de} = C^2 L^{2\gamma -4}. \quad (4)$$
The model of Tsallis holographic dark energy for Friedmann universe was proposed recently \[28\] for describing of late-time acceleration. Authors of \[29\] considered the generalization of Tsallis holographic dark energy model.

In the simplest holographic dark energy model \((\gamma = 1)\) the value of \(C\) is around 0.75 and universe ends its life in big rip singularity. There are several approaches for resolving problem of singularity have been proposed. One of them is brane world scenario \[30\]. According to Randall-Sundrum model brane is our 4-dimensional Universe with infinitely thin wall located in 5-dimensional spacetime \[31\ \[32\]. All fields of Standard Model “lives” only on brane except gravity which can be appear in additional dimensions. The cosmological equation on the brane change their form in comparison with the standard Friedmann cosmology namely:

\[
H^2 = \rho \left( 1 + \frac{\rho}{2\lambda} \right), \quad H^2 \equiv \frac{\dot{a}}{a},
\]

where \(\rho\) is energy density, \(a\) is scale factor and \(\lambda\) is brane tension. Hereinafter we use natural system of units \((8\pi G/3 = c = 1)\). The dependence of energy density \(\rho\) from scale factor can be obtained from the equation:

\[
\dot{\rho} + 3H(\rho + p) = 0.
\]

In brane cosmology the size of events horizon tends to \(L_e \rightarrow L_0 \neq 0\) for \(t \rightarrow \infty\). Therefore the density of holographic energy tends to constant value and we have effective ΛCDM model.

In paper we considered the holographic dark energy model in Tsallis form on Randall-Sandrum brane for varios values of \(\gamma\). We take \(1 < \gamma < 2\) and compare results with case of “ordinary” holographic dark energy \((\gamma = 1)\). We investigated the future evolution of the universe. Using observational data allows to define limit on possible relation of current energy density to brane tension \(\rho_0/2\lambda\). We considered the dependence between apparent magnitude and redshift for distant supernovae Ia, Hubble parameter for some redshifts and baryon acoustic oscillations. In wide range of parameters these data are described well but only separately. For relatively small brane tensions there are no common parameters at which all data are satisfied with good accuracy. Finally we studied model in which besides classical holographic contribution to dark energy \(\sim L_c^{-2}\) term \(H^2\) appears.

II. TSALLIS HOLOGRAPHIC DARK ENERGY MODEL ON RANDALL-SANDRUM BRANE

For our analysis we assume that spatially flat Universe is filled by cold matter and dark energy only (contribution of radiation and other components is negligible), i.e.

\[
\rho = \rho_{de} + \rho_m.
\]

For dark energy density we choose following representation:

\[
\rho_{de} = \frac{C^2}{L_c^{2\gamma}}.
\]

Let’s investigate this cosmological model for some values of ratio between current energy density and tension of the brane \(\delta = \rho_0/2\lambda\). For this purpose it is convenient to rewrite energy density \(\rho_0\) via Hubble parameter:

\[
\rho_0 = H_0^2(1 + \delta)^{-1}.
\]

One can introduce dimensionless units for Hubble parameter, density and brane tension by following

\[
H \rightarrow H_0 \tilde{H}, \quad \rho \rightarrow \rho_0 \tilde{\rho}, \quad \lambda \rightarrow \tilde{\lambda} H_0^2.
\]

Then rewrite first Friedmann equation on the brane in the dimensionless form:

\[
H^2 = (1 + \delta)^{-1}(\rho_{de} + \rho_m)(1 + \delta(\rho_{de} + \rho_m)).
\]

Here tildes are omitted. One consider the \(\Omega_{de} = \rho_{de}/\rho_0\) and constant \(C\) as varying parameters. Therefore fraction of matter energy density is equal \(1 - \Omega_{de}\) and dimensionless matter density can be written as

\[
\rho_m = \frac{1 - \Omega_{de}}{a^3}.
\]

For our moment of time one can put \(a(0) = 1\) without loss of generality. Differentiation on time of \(L_c/a\) gives following equation:

\[
\frac{d(L_c/a)}{dt} = -\frac{1}{a(t)}.
\]
Equations (8), (9) consist of first-order system of differential equations for scale factor $a(t)$ and function $L_e(t)/a(t)$. For condition on $L_e(0)$ one need define such value that dimensionless energy density for dark energy is $\Omega_{de}$ therefore

$$L_e(0) = \left(\frac{C^2}{\Omega_{de}}\right)^{\frac{1}{2-\gamma}}$$

Acceptable models should describe astrophysical data with good accuracy. We use standard statistical approach for estimation of likelihood of cosmological model with some parameters namely $\chi^2$-criteria. The following data are included in our consideration.

1) **The dependence magnitude - redshift for supernovae Ia.** The theoretical value of apparent magnitude $\mu_t$ for supernova Ia with redshift $z$ can be calculated using formula:

$$\mu_{th} = 5 \log_{10} \left(\frac{d_L(z)}{Mpc}\right) + 25$$

The luminosity distance $d_L$ for spatially flat Universe is

$$d_L = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}$$

where $E(z) = H(z)/H_0$ is dimensionless Hubble parameter. For $\chi^2_{SN}$ we have simple expression:

$$\chi^2_{SN} = \sum_i \frac{(\mu_{obs}(z_i) - \mu_t(z_i))^2}{\sigma_i^2}.$$  

Here $\sigma_i$ is value of error for given measurement. We use data on supernovae Ia given in [27].

2) **Baryon acoustic oscillations.** For these data it is important to calculate so called acoustic parameter $A(z)$. Theoretical value of acoustic parameter is given by

$$A_t(z) = \frac{D_v(z) H_0 \sqrt{\Omega_{mo}}}{z},$$

where $D_v(z)$ is distance parameter defined by relation:

$$D_v(z) = \left\{(1 + z)^2 d_A^2(z) \frac{cz}{H(z)} \right\}^{1/3}.$$ 

Here $d_A(z)$ is angular diameter distance:

$$d_A(z) = \frac{y(z)}{H_0 (1 + z)}, \quad y(z) = \int_0^z \frac{dz}{E(z)}.$$  

The parameter $A_t(z)$ can be evaluated via dimensionless quantities:

$$A_t(z) = \sqrt{\Omega_{mo}} \left(\frac{y^2(z)}{z^2 E(z)}\right)$$

For $\chi^2_A$ we calculate by the following way:

$$\chi^2_A = \Delta A^T (C_A)^{-1} \Delta A$$

where $\Delta A$ is vector with components $\Delta A_t = A_t(z_i) - A_{obs}(z_i)$ and $(C_A)^{-1}$ is inverse matrix to covariance matrix $3 \times 3$, which elements are given in table [II].

3) **The dependence of Hubble parameter from redshift.** The past evolution of Hubble parameter from time is studied sufficiently well now. The Hubble parameter can be defined from relation:

$$dt = -\frac{1}{H} \frac{dz}{1 + z}.$$
Table I: Observed values of acoustic parameter for various redshifts from [34]

| $z$ | $A(z)$ | $\sigma_A$ |
|-----|---------|------------|
| 0.44 | 0.474   | 0.034      |
| 0.60 | 0.442   | 0.020      |
| 0.73 | 0.424   | 0.021      |

Table II: The dependence of Hubble parameter $H$ (km/s/Mpc) from redshift $z$.

| $z$ | $H(z)$ | $\sigma_H$ | $z$ | $H(z)$ | $\sigma_H$ | $z$ | $H(z)$ | $\sigma_H$ |
|-----|---------|------------|-----|---------|------------|-----|---------|------------|
| 0.070 | 69       | 19.6      | 0.270 | 77       | 14         | 0.593 | 104      | 13         | 0.900       | 117         | 23 |
| 0.090 | 69       | 12.0      | 0.280 | 88.8     | 36.6       | 0.600 | 87.9     | 6.1        | 1.037       | 154         | 20 |
| 0.120 | 68.6     | 26.2      | 0.350 | 76.3     | 5.6        | 0.680 | 92       | 8          | 1.300       | 168         | 17 |
| 0.170 | 83       | 8.0       | 0.352 | 83       | 14         | 0.730 | 97.3     | 7          | 1.430       | 177         | 18 |
| 0.179 | 75       | 4.0       | 0.400 | 95       | 17         | 0.781 | 105      | 12         | 1.530       | 140         | 14 |
| 0.199 | 75       | 5.0       | 0.440 | 82.6     | 7.8        | 0.875 | 125      | 17         | 1.750       | 202         | 40 |
| 0.200 | 72.9     | 20.6      | 0.480 | 97       | 62         | 0.880 | 90       | 40         | 2.300       | 224         | 8 |

Therefore definition of $dz/dt$ allows to measure $H(z)$ directly. These measurements are possible due to data about ages of galaxies determined from star population models. The theoretical dependence of Hubble parameter from redshift can be defined as

$$H(z) = H_0 E(z), \quad E(z) = (\rho(z)/\rho_0)^{1/2}.$$  \hspace{1cm} (19)

For Randall-Sandrum brane one need to slightly modify this relation:

$$H(z) = H_0 E(z)(1 + \delta E(z))^{1/2}(1 + \delta)^{-1/2}.$$  \hspace{1cm} (20)

The value of $\chi^2_H$ is equal to

$$\chi^2_H = \sum_i \frac{(H_{\text{obs}}(z_i) - H_i(z_i))^2}{\sigma_i^2}.$$  \hspace{1cm} (21)

The data about $H(z)$ are taken from [35] and presented in table I.

We defined 1σ and 2σ allowed areas for parameters $C$ and $\Omega_{de}$ at some fixed values of $\delta$ and $\gamma$, $\gamma \neq 2$ (in this case model simply coincides with standad cosmological $\Lambda$CDM model). For two-parametric models 68.3% and 95.4% level of likelihood corresponds to $\chi^2$ for which $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} < 2.3$ $\Delta \chi^2 < 6.17$ correspondingly. Results of our calculations for some $\delta$ and $\gamma = 1.5$ are given on Fig. I. For $\gamma = 1.25$ and 1.75 similar picture was obtained.

One note that for $\delta \approx 0.02$ we have no joint intersection between 1σ and 2σ allowed regions for three types of considered data. Therefore we have upper limit on parameter $\delta \approx 0.02$ for this models and this limit doesn’t depend from value of parameter $\gamma$.

The next question is difference between considered model and $\Lambda$CDM cosmology. For analysis it is useful to investigate behavior of so called deceleration parameter

$$q = -\frac{d^2a}{dt^2} \frac{1}{aH^2}.$$  \hspace{1cm} (22)

and $r$-parameter

$$r = \frac{d^3a}{dt^3} \frac{1}{aH^3}.$$  \hspace{1cm} (23)

Sahni et al. [36] proposed statefinder pair $(r,s)$ for analysis of cosmological evolution where $s$ is

$$s = \frac{r - 1}{3(q - 1/2)}.$$  \hspace{1cm} (24)
For $\Lambda$CDM model $r = 1$ and $s = 0$. For considered models the possible evolution of $s$, $r$ parameters strongly depends from $C$ and $\gamma$ (see Fig. 2, 3). For $C$ and $\Omega_{de}$ we take values for which the concordance with observational data is close to $\Lambda$CDM model.

For brane cosmology the value of $s$ is more close to 0 in present time in comparison with Friedmann cosmology. For $\gamma = 1$ (standard holographic dark energy model) $s \to \text{const}$ for brane and Friedmann cosmology.

The deviation of $s$ from 0 with time grows faster in Friedmann cosmology. The same one can see for parameter $r$.

One need to note that for some values of $\gamma$ and $C$ state-finder parameters are very close to $\Lambda$CDM values in a wide range of time $\sim -0.5 \leq t \sim 1$.

It is interesting to investigate question about future singularities in considered model. The singularity occurs if $1 < \gamma < 2$ both for cosmology on the brane and Friedmann universe (for values of parameters allowed by observational data analysis). For given $C$ and $\Omega_{de}$ the time before singularity increases with increasing of $\delta$ in comparison with Friedmann cosmology (see table III). One also note that for $1.5 \leq \gamma < 2$ life of universe ends in so called “big freeze” singularity (Hubble parameter diverges for finite time but scale factor $a = a_f \neq \infty$). If $1 < \gamma < 1.5$ big freeze singularity occurs only in Friedmann universe while as for brane big rip take place.

**Remark 1.** We considered model of Tsallis dark energy on brane but the similar cosmological evolution can be obtained in frames of generalized holographic dark energy proposed in [23]. For universe filled of dark energy only we can put

$$\rho_{\Lambda} = \frac{\dot{C}^2}{L_{\Lambda}^4} = \frac{C^2}{L_{\Lambda}^{4-2\gamma}} \left( 1 + \frac{C^2}{2M_{\Lambda}^{4-2\gamma}} \right).$$

Here $L_{\Lambda}$ is function of $L_{\Lambda}$ and parameters $C$ and $\lambda$. In this case Friedmann equation on the brane take the form of usual cosmological equation:

$$H^2 = \frac{\dot{C}^2}{L_{\Lambda}^4}$$

(25)
Figure 2: Time evolution of state-finder parameters $r$ and $s$ and corresponding diagram on $(r,s)$-plane for holographic dark energy model (7) in a case of Friedmann cosmology. Negative values of $t$ corresponds to past, time is given in units of inverse Hubble parameter. Solid and dashed lines corresponds to $C = 0.7$ and $C = 0.8$. The value of $\Omega_{de}$ is assumed 0.72.

Table III: Time before singularity $t_f$ in model (7) for brane ($\delta = 0.015$) and Friedmann cosmology ($\delta = 0$) for $1 \leq \gamma < 2$. In brackets the type of singularity is given according to classification offered in [37].

| Parameters       | $t_f, H_0^{-1}$ |
|------------------|-----------------|
| $\delta = 0, \gamma = 1, C = 0.7, \Omega_{de} = 0.72$ | 2.922 (II)      |
| $\delta = 0, \gamma = 1.25, C = 0.7, \Omega_{de} = 0.72$ | 1.488 (III)     |
| $\delta = 0, \gamma = 1.5, C = 0.7, \Omega_{de} = 0.72$ | 1.215 (III)     |
| $\delta = 0, \gamma = 1.75, C = 0.7, \Omega_{de} = 0.72$ | 0.628 (III)     |
| $\delta = 0.015, \gamma = 1.0, C = 0.8, \Omega_{de} = 0.72$ | $\infty$       |
| $\delta = 0.015, \gamma = 1.25, C = 0.8, \Omega_{de} = 0.72$ | 2.652 (II)      |
| $\delta = 0.015, \gamma = 1.5, C = 0.8, \Omega_{de} = 0.72$ | 2.287 (III)     |
| $\delta = 0.015, \gamma = 1.75, C = 0.8, \Omega_{de} = 0.72$ | 1.594 (III)     |

Therefore required scale of cut-off is equal

$$L_\Lambda = \alpha L_e^{4-2\gamma} (L_e^{4-2\gamma} + \beta)^{-1/2}, \quad \alpha = \ddot{C}/C, \quad \beta = C^2/2\lambda.$$  

One can consider fluid with specific equation of state instead cosmological model on the brane (see for example [38]).

The energy density $\tilde{\rho}$ and pressure $\tilde{p}$ for equivalent one-fluid Friedmann model mimicking brane cosmology are

$$\tilde{\rho} = \rho \left(1 + \frac{\rho}{2\lambda}\right),$$

$$\tilde{p} = \rho \left(1 + \frac{\rho}{3\lambda}\right).$$
Figure 3: Time evolution of state-finder parameters $r$ and $s$ and corresponding diagram on $(r,s)$-plane for holographic dark energy model \((7)\) in a case of Randall-Sandrum brane for $\delta = 0.015$. Solid and dashed lines corresponds to $C = 0.8$ and $C = 0.85$ on brane. The value of $\Omega_{de}$ is assumed 0.72.

\[
\tilde{\rho} = p + \frac{\rho}{2\lambda} (2p + \rho).
\]

One can find that

\[
\frac{d\tilde{\rho}}{d\tilde{\rho}} = \frac{dp}{d\rho} + \frac{\rho + p}{\rho + \lambda}.
\]

Realistic model requires to account the existence of matter also. In frames of equivalent Friedmann cosmological model this leads to the interaction between dark energy and matter. One can obtain the following expressions for dark energy density and pressure in Friedmann cosmology in terms of $\rho_{de}$, $\rho_m$ and $p_{de}$ on the brane:

\[
\tilde{\rho}_{de} = \rho_{de} \left(1 + \frac{\rho_{de}}{2\lambda}\right) + \rho_m \left(\rho_m + 2\rho_{de}\right),
\]

\[
\tilde{p}_{de} = p_{de} \left(1 + \frac{\rho_{de}}{\lambda}\right) + \rho_{de}^2 \frac{\rho_m}{2\lambda} + \rho_m \left(\rho_m + 2\rho_{de} + 2p_{de}\right).
\]

One can find the equation-of-state parameter $w = p_{de}/\rho_{de}$ for considered model on brane and parameter $\tilde{w}$ equivalent model in Friedmann cosmology. From equation of energy conservation

\[
\dot{\rho}_{de} + 3H (\rho_{de} + p_{de}) = 0
\]

follows that

\[
w = -1 - \frac{1}{3} (\ln \rho_{de})'.
\] (26)
Here prime denotes differentiation on ln $a$. From (7) we have

$$w = -1 + \frac{1}{3} (4 - 2\gamma) (\ln L_e)'.$$

One can rewrite equation for $L_e$ as

$$L_e = a \int_a^\infty \frac{da}{Ha^2}$$

and taking derivative obtain for $w$:

$$w = \frac{1}{3} - \frac{2\gamma}{3} - \frac{1}{3C} (4 - 2\gamma) \sqrt{\frac{\rho_{de}}{H^2}}$$

(28)

Or via quantities $\Omega_{de}$ and energy density $\rho = \rho_{de} + \rho_m$:

$$w = \frac{1}{3} - \frac{2\gamma}{3} - \frac{1}{3C} (4 - 2\gamma) \sqrt{\frac{\Omega_{de}}{1 + \delta x}} \quad x = \rho/\rho_0.$$ 

After calculations using equations for $\dot{\rho}_{de}$ and $\dot{p}_{de}$ one obtain for equivalent Friedmann model:

$$\dot{w} = \frac{w(1 + 2\delta x \Omega_{de}) + \delta x}{\Omega_{de} + \delta x}.$$ 

(29)

If $L_e$ decreases and $L_e \to 0$ we have that $x = \rho/\rho_0 >> 1$ and therefore

$$w \to \frac{1}{3} - \frac{2\gamma}{3}$$

It is interesting to note that parameter $\dot{w}$ for this case tends to

$$\dot{w} \to 2w.$$ 

In particular for $\gamma \geq 1.25$ $\ddot{w} \to \ddot{w}_f < -1$ and singularity occurs (see Table III).

Of course one can ask, how we can see that considered model is from braneworld, not just from specific fluid? Of course this question arises for any cosmological model on the brane. Maybe one of the arguments in favor of brane cosmology is that this model consist of relatively simple ingredients in comparison with complicated form of equation-of-state for cosmological fluid in Friedmann universe: simple model of holographic dark energy and simple multidimensional model proposed by Randall and Sundrum.

**Remark 2.** Another interesting question concerns possibility of complete description of universe history in considered model including early inflation. For brane we can apply the approach proposed recently in [24] for Friedmann cosmology. Namely one put

$$L = \sqrt{L_e + \Lambda_{UV}^{-2}},$$

(30)

where $\Lambda_{UV}$ is some correction due to the ultraviolet cutoff. For early times the term $\rho/2\lambda >> 1$ and

$$H^2 \approx \frac{\rho_0}{2\lambda}.$$ 

Neglecting contribution of matter and radiation we assume that $\rho = \rho_{de}$ and

$$H \approx \frac{C^2}{\sqrt{2\lambda}} \frac{1}{L^{3-2\gamma}}.$$ 

(31)

We see for example that for $\gamma = 1.5$ this model coincides with model considered in [24]. Value $C^2/\sqrt{2\lambda}$ plays role of parameter $C$ in [24]. For arbitrary $0 < \gamma < 2$ we have similar result i.e. the ultraviolet cutoff causes exponential expansion at early times.
III. ANOTHER MODEL OF HOLOGRAPHIC DARK ENERGY ON THE RANDALL-SUNDRUM BRANE

Let’s investigate another model of holographic dark energy model on brane. One take dark energy density in the following form:

\[ \rho_{de} = \frac{C^2}{L_e^2} - C_1^2 H^2; \]  

(32)

Here \( C_1 \) is some constant. Let’s consider quantity (energy density for \( t = 0 \) without \( \sim H^2 \) term):

\[ \rho_0 = \frac{C^2}{L_e(0)^2} + \rho_{m0}. \]

The value of \( \rho_0 \) can be found from equation:

\[ (\rho_0 - C_1^2 H_0^2) (1 + \delta) = H_0^2. \]

One use \( \Omega_{de} = \rho_{de}/\rho_0 \) and \( C \) as varying parameters for fixed \( C_1 \) and \( \delta = (\rho_0 - C_1^2 H^2)/2\lambda \). The system of cosmological equations for this model can be written in form (we again use dimensionless quantities \( H/H_0 \rightarrow H, \ L_e H \rightarrow L_e, \ \rho_0/H_0^2 \rightarrow \rho_0 \)):

\[ H = \sqrt{\rho - C_1^2 \frac{\dot{a}^2}{a^2} \sqrt{1 + \delta - \frac{\rho}{\rho_0 - C_1^2}}}, \rho = \frac{C^2}{L_e^2} + \frac{1 - \Omega_{de}}{a_0^3} \rho_0, \]

\[ \frac{d(L_e/a)}{dt} = -\frac{1}{a(t)}, \]

\[ v = \frac{da}{dt}. \]

(33)

(34)

Initial conditions are \( a(0) = 1 \) and \( v(0) = 1 \) (latter corresponds to that the present dimensionless value of Hubble parameter is simply 1). For initial value of \( L_e \) we have via the value of \( \Omega_{de} \):

\[ L_e(0) = \left( \frac{C^2}{\Omega_{de} \rho_0} \right)^{1/2}. \]

This model is studied as previous. The allowed 1σ and 2σ regions for parameters \( C \) and \( \Omega_{de} \) for fixed values of \( \delta \) and \( C_1 \) are given on Fig.4.

The dependence of state-finder parameters from time are given on 5.

One can see as in previous case that at \( \delta = 0 \) (Friedmann cosmology) observational data are described better in comparison with model on the brane. For \( \delta \approx 0.02 \) and relatively small values of \( C_1 \) there are no intersections between allowed areas for various observational data. Therefore we have again some limit on parameter \( \delta \) as in case of Tsallis model. One note also that in this model there is no final big rip singularity.

IV. CONCLUSION

Two classes of holographic dark energy models on Randall-Sandrum brane are investigated in comparison with Friedmann cosmology and ΛCDM model. For first type of models the dark energy density is assumed to proportional some degree of events horizon length \( \sim L_e^{2\gamma - 4} \). For second class we presented dark energy density as sum of two contributions, classical \( \sim L_e^{-2} \) and \( \sim H^2 \). Using observational data such as dependence “magnitude-redshift” for SN Ia, dependence of Hubble parameter from redshift and values of acoustic parameter at some redshifts one can give allowable areas for parameters of the models (\( \Omega_{de} \) and \( C \)).

For Tsallis holographic energy model on the brane if \( 1 < \gamma < 2 \) one cannot avoid singularities in future although time for singularity increases or singularity became in some sense more soft (for \( \gamma > 1.5 \) big rip occurs instead singularity of type III in Friedmann universe).
For second case we there are no singularities in future because size of events horizon tends to constant value. Such model can be considered as effective ΛCDM model in future although value of “cosmological contant” can differs considerably.

From analysis of observational data we also obtained the limit on ratio between current energy density in universe and brane tension. As follows from our calculations this limit weakly depends from γ.

In conclusion one mention about possible perspectives for future work. One of the important task for example is the reconstruction of scalar field potential for Tsallis holographic energy on the brane. Another question is construction of equivalent modified gravity theory. It is interesting to consider this model in early universe for loop quantum gravity also. We are going to consider these question in the future papers.
Figure 5: Time evolution of state-finder parameters $r$ and $s$ and corresponding diagram on $(r, s)$-plane for holographic dark energy model [32] in a case of Friedmann cosmology ($\delta = 0$) and Randall-Sundrum brane for some $C_1$.

The work is supported by project 1.4539.2017/8.9 (MES, Russia).

[1] A. G. Riess, et al., Astron. J. 116 (1998) 1009.
[2] S. Perlmutter, et al., Astrophys. J. 517 (1999) 565.
[3] P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 559.
[4] T. Padmanabhan, Phys. Rep. 380 (2003) 235.
[5] E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753.
[6] J. Frieman, M. Turner, D. Huterer, Ann. Rev. Astron. Astrophys. 46 (2008) 385.
[7] R. R. Caldwell, M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. 59 (2009) 397.
[8] A. Silvestri, M. Trodden, Rept. Prog. Phys. 72 (2009) 096901.
[9] M. Li, X.-D. Li, S. Wang, Y. Wang, Frontiers of Physics 8 (2013) 828.
[10] J. D. Bekenstein, Phys. Rev. D7 (1973) 2333.
[11] S. W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[12] G.t Hooft, arXiv:gr-qc/9310026
[13] A. G. Cohen, D. B. Kaplan, A. E. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
[14] P. Horava, D. Minic, Phys. Rev. Lett. 85 (2000) 1610.
[15] S. D. H. Hsu, Phys. Lett. B 594 (2004) 13.
[16] M. Li, Phys. Lett. B 603 (2004) 1.
[17] J. Shen, B. Wang, E. Abdalla, R. K. Su, Phys. Lett. B 609 (2005) 200.
[18] A. Sheykhi, Phys. Lett. B 680 (2009) 113.
[19] A. Sheykhi et al., Gen. Relativ. Gravit. 44 (2012) 623.
[20] B. Wang, E. Abdalla, F. Atrio-Barandela, D. Pavon, Rep. Prog. Phys. 79 (2016) 096901.
[21] Y. S. Myung, Phys. Lett. B 652 (2007) 223.
[22] S. Wang, Y. Wang, M. Li, Physics Reports 696 (2017) 1.
[23] Shin’ichi Nojiri, Sergei D. Odintsov, Gen. Rel. Grav. 38 (2006) 1285.
[24] Shin’ichi Nojiri, Sergei D. Odintsov, Emmanuel N. Saridakis, Phys. Lett. B797 (2019) 134829.
[25] Shin’ichi Nojiri, Sergei D. Odintsov, Emmanuel N. Saridakis, [arXiv:1908.00389] [gr-qc].
[26] C. Tsallis, J. Stat. Phys. 52, 479 (1988).
[27] C. Tsallis, L. J. L. Cirto, Eur. Phys. J. C73 (2013) 2487.
[28] M. Tavayef, A. Sheykhi, Kazuharu Bamba, H. Moradpour, Phys. Lett. B781 (2018) 195.
[29] Shin’ichi Nojiri, Sergei D. Odintsov, Emmanuel N. Saridakis, Eur. Phys. J. C79 (2019) 242.
[30] X. Zhang, Phys. Lett. B 683 (2010) 81.
[31] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370.
[32] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.
[33] R. Amanullah et al., Astrophys. J. 716 (2010) 712.
[34] C. Blake et al., MNRAS 418 (2011) 1707.
[35] Y. Chen, S. Kumar, B. Ratra, Astrophys. J. 835 (2017) 868.
[36] V. Sahni, T.D. Saini, A.A. Starobinsky and U. Alam, JETP Lett. 77 (2003) 201.
[37] Shin’ichi Nojiri, Sergei D. Odintsov, Shinji Tsujikawa, Phys. Rev. D71 (2005) 063004.
[38] I. Brevik, V. V. Obukhov, A. V. Timoshkin, Y. Rabochaya, Astrophys. Space Sci. 346 (2013) 267.
[39] J.-F. Zhang, X.Zhang, H.-y. Liu, Eur. Phys. J. C52 (2007) 693.