A Coordination Space Model for Assemblability Analysis and Optimization during Measurement-Assisted Large-Scale Assembly

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Featured Application: For assembly incoordination caused by excessive assembly deviations, the proposed method can predict the assemblability and solve the assembly features that need accuracy compensation, to improve the assembly efficiency.

Abstract: The assembly process is sometimes blocked due to excessive dimension deviations during large-scale assembly. It is inefficient to improve the assembly quality by trial assembly, inspection, and accuracy compensation in the case of excessive deviations. Therefore, assemblability prediction by analyzing the measurement data, assembly accuracy requirements, and the pose of parts is an effective way to discover the assembly deviations in advance for measurement-assisted assembly. In this paper, a coordination space model is constructed based on a small displacement torsor and assembly accuracy requirements. An assemblability analysis method is proposed to check whether the assembly can be executed directly. Aiming at the incoordination problem, an assemblability optimization method based on the union coordination space is proposed. Finally, taking the space manipulator assembly as an example, the result shows that the proposed method can improve assemblability with a better assembly quality and less workload compared to the least-squares method.

Keywords: measurement-assisted assembly; coordination space; assemblability; small displacement torsor

1. Introduction

Large-scale mechanical products like ships, automobiles, aircrafts, etc. are complex in structure, large in size, and accurate in assembly quality. The assembly workload of the manufacturing process is heavy [1]. These products often need accuracy compensation in the assembly process because of the excessive assembly deviations, which lead to inefficiency. The assembly deviations might be caused by the eventual poor machining quality of parts, or excessive tolerances set by designers. Thus, the trial assembly is often used to detect the assembly deviations in advance, and the parts are then separated to make an accuracy compensation on the bad dimensions. The assembly process takes a long time by the following steps: Trial assembly, measurement of deviations, separation of parts, and re-trial assembly. Therefore, an assemblability analysis and optimization method based on the measurement data is necessary to predict the assembly deviation and make the accuracy compensation in advance.

With the development of measurement-assisted assembly (MAA) [2], measurement technology has become a bridge between the real world and the digital world. Marguet et al. [3] introduced a
MAA application in an Airbus assembly line. The least-squares method was used to calculate the optimal pose. Chen et al. [4] proposed a weighted SVD algorithm to obtain the optimal pose of components, which improved the accuracy of pose evaluation. Li et al. [5] proposed a coaxial alignment method using distributed monocular vision. The iterative reweighted particle swarm optimization method was constructed to improve the measurement ability of complicated wearing holes. Wang et al. [6] calculated the assembly clearance of a wing-fuselage assembly based on the optimal pose. The above methods mainly consider the measurement and calculation of the assembly pose, and then realize alignment through pose adjustment tooling. The assembly will be difficult if the quality of the parts is poor.

Assemblability prediction is the first step to judge whether the assembly is qualified in the measurement-assisted assembly. Sukhan et al. [7] evaluated the assemblability based on tolerance propagation. Sanderson et al. [8] assessed the assemblability by the maximum likelihood problem, which was solved by the Kalman filter algorithm. The traditional assemblability evaluation methods are mainly used to find the assembly problem in the design phase, but not in the assembly phase. Cui and Du [9] proposed the concept of pose feasible space to assess the assembly coordination. Yuan et al. [10] proposed an assembly quality assessment method based on weighted geometric constraints to calculate the optimal pose. Wu et al. [11] proposed a constraint coordination index to assess the assembly quality. Ma et al. [12] developed the assembly precision pre-analysis technique in the simulation of virtual assembly. Du et al. [13] proposed a pose decoupling model of the axis tolerance feature to decouple the analysis of any pose within the tolerance domain.

The accuracy compensation methods are used to improve assemblability. The digital compensation method has become a research highlight to improve the assemblability. Davis et al. [14] put forward the method of measuring the assembly clearance and realizing the digital manufacturing of the accuracy compensation gasket. Fabian et al. [15] introduced a shimming method by 3D printing technology, and the assembly clearance was measured by optical measurement. Wang et al. [16] provided a shimming method based on scanned data for a wing box assembly involving non-uniform gaps. In addition, finite element analysis was taken to improve the shimming scheme. Those methods, however, need to be assembled first, followed by measurement of the deviations to be compensated, resulting in a lower efficiency.

Some scholars proposed predictive shimming and predictive fettling methods to improve the assembly efficiency and quality [17]. Cui et al. [18] proposed the oriented points group to calculate the deviation of multiple shaft-and-holes, and the gap was shimmed. Yang et al. [19] analyzed the deviation from the measured point cloud to the model to improve skin finishing. Yu et al. [20] employed a virtual assembly and repair analysis method based on both the geometric design model and object scanning model. Manohar et al. [21] proposed an alternative strategy for predictive shimming, based on machine learning and sparse sensing to first learn gap distributions from historical data. Lei et al. [22] presented an automated and in situ alignment approach with the assistance of computer numerical controlled (CNC) positioners and laser trackers to reduce the finish machining workload. The above studies are aimed at specific cases.

The accuracy compensation method is usually applied after assembly. Then, the assembly sometimes needs be separated, which leads to low efficiency. In this paper, an assemblability analysis and optimization method based on the coordination space model is constructed during measurement-assisted large-scale assembly. In Section 2, the coordination space model based on the small displacement torsor is constructed. In Section 3, the assemblability analysis based on the coordination space model is proposed. In addition, the uncoordinated case is further analyzed. In Section 4, the assemblability optimization method based on the union coordination space is proposed for the uncoordinated case. In Section 5, the space manipulator assembly is taken as an example to verify the proposed method. The result shows that the proposed method can optimize the assemblability with less workload and better assembly quality compared to the least-squares method.
2. Coordination Space Model Based on Small Displacement Torsor

Assemblability refers to the ability of parts to satisfy the assembly accuracy requirements in terms of dimensions, which can be expressed by coordination accuracy. Traditionally, coordination accuracy [23] is the difference in the manufacturing dimensions. Figure 1 shows the coordination accuracy of a keyway assembly.

![Figure 1. Coordination accuracy of a keyway assembly.](image1)

The coordination accuracy is

$$\varphi_{AB} = \Delta_1 + \Delta_2 = L_A - L_B,$$

(1)

It can be seen that the coordination accuracy is the amount of the allowance on a certain dimension. In this way, the assembly coordination of a single dimension is well presented by coordination accuracy such as angle, length, etc. However, it is not suitable for complicated assembly. Therefore, the concept should be extended to pose allowance space and the space can be predicted by digital measurement data during large-scale assembly. This space is named the assembly coordination space, which is the ability of pose variation under the condition of assembly accuracy requirements, as Figure 2 shows.

![Figure 2. The pose variations under the condition of assembly accuracy requirements.](image2)

The parts of the assembly are divided into the reference part and the align part. The reference part is the fixed part during assembly and the align part will move to the target pose by the pose adjustment tooling. Assume that the primary measurement data of the two parts are

$$\begin{align*}
\{ \mathbf{P}^R & = [p^R_1 \ p^R_2 \ ... \ p^R_n] \\
\mathbf{P}^A & = [p^A_1 \ p^A_2 \ ... \ p^A_n]'
\end{align*},$$

(2)

where \(\mathbf{P}^R\) and \(\mathbf{P}^A\) are the point sets of the reference part and align part, separately, where \(p^R_i\), etc. and \(p^A_i\), etc. are the points of the sets \(\mathbf{P}^R\) and \(\mathbf{P}^A\), respectively. The two parts are separated first. According to the least-squares method, the optimal assembly pose can be calculated by

$$\begin{align*}
\mathbf{P}^R & \approx R\mathbf{P}^A + T \\
(\varepsilon = \min \{\sum_{i=1}^{n} || R\mathbf{p}^A_i + T - \mathbf{p}^R_i ||^2 \}'),
\end{align*}$$

(3)

where \(R\) is the rotation matrix, \(T\) is the movement matrix, and \(\varepsilon\) is the minimum residual sum of squares. The singular value decomposition method [24] is taken to calculate the parameter \(R\) and \(T\). Then, the optimal pose of the align part based on the least-squares method is
The assembly deviation can be predicted by the pose $\omega_0$ of the align part. The key assembly characteristics (KAC) [25] are the important geometric structures that have key influences on assembly quality. They are described by measurement data and some dimensions that are not necessary to be measured.

$$K = \{P, G\},$$

where $K$ is the parameters of a KAC, $P$ is the measurement data, $G$ is the dimensions that are not necessary to be measured. The KACs have an irregular distribution in space during large-scale assembly. As shown in Figure 3, the wing-fuselage assembly is completed by 4 pairs of joints. There are four assembly accuracy requirements on each pair of joints: Two on coaxialities and two on clearances.

![Figure 3. Wing-fuselage assembly.](image)

The KACs are restrained by assembly accuracy requirements. The assembly accuracy is described as

$$T^i_j = f^i_j(K^R_i, K^A_i),$$

where $K^R_i$ is the parameters of the $i$th KAC on the reference part (fuselage), $K^A_i$ is the parameters of the $i$th KAC on the align part (wing), $T^i_j$ is the $j$th assembly accuracy of the $i$th KAC, and $f^i_j$ is the mapping from parameters to the assembly accuracy. The assembly accuracy should meet the requirements of assembly accuracy, which is formulated in Equation (7)

$$T^i_j \in [T^i_{j-min}, T^i_{j-max}],$$

where $T^i_{j-min}$ and $T^i_{j-max}$ are the ranges of $T^i_j$. Substitute Equation (6) into Equation (7):

$$f^i_j(K^R_i, K^A_i) \in [T^i_{j-min}, T^i_{j-max}],$$

For the $m$ assembly accuracy requirements on $n$ KACs of the assembly, the constraint equations can be expressed as

$$\forall f^i_j(K^R_i, K^A_i) \in [T^i_{j-min}, T^i_{j-max}], i \in [1, n], j \in [1, j_i],$$

where $j_i$ is the assembly accuracy requirement number of the $i$th KAC, $m = \sum_{i=1}^n j_i$. When all KACs satisfy their assembly accuracy requirements, the pose is a valid pose to be aligned.

As shown in Figure 4, the valid pose may not be the only one that satisfies all assembly accuracy requirements. Therefore, the adjacent poses of the primary pose shown in Figure 4a can be analyzed. A small displacement torsor (SDT) [26] represents a tiny rigid body’s pose variation. It is described as

$$\omega_\Delta = (x_\Delta, y_\Delta, z_\Delta, \alpha_\Delta, \beta_\Delta, \gamma_\Delta),$$
The homogeneous transformation matrix of an SDT is

\[
\omega_{\Delta}^H = \begin{bmatrix}
 C \gamma_{\Delta} C \beta_{\Delta} & -S \gamma_{\Delta} C \beta_{\Delta} & S \beta_{\Delta} & x_{\Delta} \\
 S \gamma_{\Delta} C \alpha_{\Delta} + C \gamma_{\Delta} S \beta_{\Delta} S \alpha_{\Delta} & C \gamma_{\Delta} C \alpha_{\Delta} - S \gamma_{\Delta} S \beta_{\Delta} S \alpha_{\Delta} & -C \beta_{\Delta} S \alpha_{\Delta} & y_{\Delta} \\
 S \gamma_{\Delta} S \alpha_{\Delta} - C \gamma_{\Delta} S \beta_{\Delta} C \alpha_{\Delta} & C \gamma_{\Delta} S \alpha_{\Delta} + S \gamma_{\Delta} S \beta_{\Delta} C \alpha_{\Delta} & C \beta_{\Delta} C \alpha_{\Delta} & z_{\Delta} \\
 0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( S \) is sin, \( C \) is cos, \( \lim_{\alpha_{\Delta} \to 1} C \alpha_{\Delta} = 1 \), \( \lim_{\alpha_{\Delta} \to 1} S \alpha_{\Delta} = a_{\Delta} \), and \( \lim_{\alpha_{\Delta} \to 1} S \alpha_{\Delta} S \beta_{\Delta} = 0 \). The change in point \( p = (x, y, z) \) after a slight change in the rigid body’s pose is

\[
p_{ex}' = [x' \ y' \ z'] = \begin{bmatrix}
 1 & -\gamma_{\Delta} & \beta_{\Delta} & x_{\Delta} \\
 \gamma_{\Delta} & 1 & -\alpha_{\Delta} & y_{\Delta} \\
 -\beta_{\Delta} & \alpha_{\Delta} & 1 & z_{\Delta} \\
 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
 x \\
 y \\
 z \\
 1
\end{bmatrix} = [x + x_{\Delta} + \beta_{\Delta} \cdot z - \gamma_{\Delta} \cdot z \ y + y_{\Delta} - \alpha_{\Delta} \cdot z + \gamma_{\Delta} \cdot x \ x + x_{\Delta} + \beta_{\Delta} \cdot z + \gamma_{\Delta} \cdot z \ 1],
\]

Then, the assembly accuracy would be

\[
T_{i}^{j-\omega_{\Delta}} = f_{j}^{i}(K_{R}^{j}, K_{L}^{i}, \omega_{\Delta}^{H}) = f_{j}^{i}(P_{R}^{j}, G_{R}^{j}, P_{L}^{i}, G_{L}^{i}),
\]

On this pose, if the assembly accuracy requirements are still satisfied as

\[
\forall f_{j}^{i}(K_{R}^{j}, K_{L}^{i}, \omega_{\Delta}^{H}) \in [T_{i}^{j-\min}, T_{i}^{j-\max}], i \in [1, n], j \in [1, j_{i}],
\]

the pose is still a valid pose. The coordination space model can, hence, be expressed as

\[
\Phi_{CS} = \{ \omega_{\Delta} | \forall f_{j}^{i}(K_{R}^{j}, K_{L}^{i}, \omega_{\Delta}^{H}) \in [T_{i}^{j-\min}, T_{i}^{j-\max}], i \in [1, n], j \in [1, j_{i}] \},
\]

where \( \Phi_{CS} \) is the coordination space, which is the whole pose variation space under the condition of assembly accuracy requirements.

3. Assemblability Analysis Based on Coordination Space Model

Assemblability refers to the geometric consistency of the matching geometric structures of the two assembling parts. It can be judged whether the assembly can directly be carried out by assemblability prediction.

The assemblability is good if the coordination space is greater than 0, which means at least one pose conforms to Equation (15). Otherwise, the assemblability is bad. Therefore, the assemblability analysis flow is shown in Figure 5.
Figure 5. Assemblability analysis process.

Firstly, the KACs are measured by a laser tracker or other digital measurement devices. Then, the coordination space model is constructed based on the assembly accuracy requirements. The volume of the coordination space is solved to judge whether it is assemblable. It will be assemblable when $\emptyset_{CS}$ is greater than 0. The assembly can be executed by calculating the optimal pose and aligning the parts. It will be uncoordinated when $\emptyset_{CS}$ is 0. Then, the assembly deviation should be analyzed and compensated to make it assemblable.

The solution process of the coordination space is based on the Monte Carlo method:

1. Calculate the optimal pose based on the least-squares method.
2. According to the dimensions and assembly accuracy requirements, a maximum pose space is assumed, as Equation (16) shows. All poses out of the space are not valid for any assembly accuracy requirements.

\[
\omega_d: (-x_d, -y_d, -z_d, -\alpha_d, -\beta_d, -\gamma_d) \rightarrow (x_d, y_d, z_d, \alpha_d, \beta_d, \gamma_d),
\]

(16)

3. Generate a random SDT uniformly for $n_T$ times and check the SDTs by Equation (15).
4. If $n_T$ of $n_T$ SDTs are valid, the coordination space is

\[
\emptyset_{CS} = \frac{n_T}{n_T} 64x_d y_d z_d \alpha_d \beta_d \gamma_d
\]

(17)

In the case of incoordination, the coordination space should be further analyzed. According to Equation (15), the coordination space is the intersection of KAC’s constraint equations. All constraints are divided by KACs. Equation (15) will be translated to

\[
\left\{ \begin{array}{l}
\forall f_j^i(K^R, K^A, \omega_d^j) \in [T^1_{j-min}, T^1_{j-max}], j \in [1, j_1], \\
\forall f_j^i(K^R, K^A, \omega_d^j) \in [T^2_{j-min}, T^2_{j-max}], j \in [1, j_2], \\
\forall f_j^i(K^R, K^A, \omega_d^j) \in [T^n_{j-min}, T^n_{j-max}], j \in [1, j_n]
\end{array} \right.
\]

(18)

Let

\[
\emptyset^{KAC}_i = \{ \omega_d^{i-KAC} | \forall f_j^i(K^R, K^A, \omega_d^j) \in [T^1_{j-min}, T^1_{j-max}], j \in [1, j_1] \}
\]

(19)

where $\emptyset^{KAC}_i$ is the KAC coordination space formed by the assembly accuracy requirements of a KAC, and $\omega_d^{i-KAC}$ is an SDT in the KAC coordination space. The $\emptyset_{CS}$ would be

\[
\emptyset_{CS} = \cap_{i=1}^{n} \emptyset^{KAC}_i
\]

(20)

The relationship between the KAC coordination space and the assembly coordination space is shown in Figure 6a.
Figure 6. The relationship between the part and feature PFS.

Figure 6b shows the status of the KAC coordination space when the assembly is uncoordinated. Each area of the same color represents a KAC coordination space. The divided zone is named the coordination zone. The accuracy compensation method is needed to improve the assemblability.

According to Figure 6b, set the union of KAC coordination space as a union coordination space. It is formulated as

$$\emptyset_{UCS} = \bigcup_{i=1}^{n} \emptyset_{i}^{KAC},$$  \hspace{1cm} (21)$$

where $\emptyset_{UCS}$ is the union coordination space. In the union coordination space, all poses are valid for some KACs but not valid for all. Some divided zones are valid for more KACs than others, e.g., the two zones marked with 3 are better than those marked with 1 or 2. The marked number is named the coordination zone index, which is the valid KACs’ number in the coordination zone. If a pose in the zone marked with 3 is selected, only one KAC needs to be compensated. In this way, an assemblability optimization method is put forward by selecting a coordination zone with larger volume and KAC number. The larger volume means a better geometric consistency, and the larger KAC number means fewer KACs need to be compensated.

4. Assemblability Optimization Based on the Union Coordination Space

The accuracy compensation process is time- and effort-consuming [22] when the assemblability is poor. For example, it needs programming, clamping, tool setting, machining, loosen clamping, and other steps when finishing a KAC with cutting. Therefore, reducing the number of KACs to be processed is an effective means to improve the assembly efficiency in many cases. The optimal pose is usually obtained under the condition of optimal assembly accuracy. If each unqualified KAC is compensated one by one under the optimal pose, more work may be needed and the assembly quality might not be good, due to the unknown assembly quality after accuracy compensation. If the assembly quality is bad after compensation, there are no alternative compensation schemes based on the least-squares method. Therefore, the assemblability optimization method is proposed to solve the incoordination problem. The key to optimize the assemblability is whether there is one or more coordination zones that can satisfy assembly accuracy requirements with fewer KACs to be compensated and a better or approximate volume of coordination space.

The coordination zone index shows the valid KACs in the certain coordination zone. The total number of all KACs is $n_{KAC}$. The incoordination zone index shows the number of uncoordinated KACs in the coordination zone. Their relationship is

$$n_{IZI} = n_{KAC} - n_{CZI},$$  \hspace{1cm} (22)$$

where $n_{IZI}$ is the incoordination zone index and $n_{CZI}$ is the coordination zone index. If the accuracy of uncoordinated KACs is compensated well in the coordination zone, this coordination zone will change to the assembly coordination space, as Figure 7 shows.
In this way, each coordination zone can be analyzed to check whether it is good to be compensated or not. Two indicators of the coordination zone should be analyzed, one is the incoordination zone index, and the other is the volume of the coordination zone. The Monte Carlo method of Section 3 is improved to judge the state of each coordination zone one by one, and the optimal assemblability optimization schemes of the coordination zone are selected for recording.

The solution process based on the Monte Carlo method is as follows:
1. Solve the optimal pose of the align part;
2. Set a pose space as the pose boundary as shown by the square box of Figure 8;
3. Generate a random SDT in the pose space;
4. According to Equation (19), judge which KAC equations are satisfied (coordination zone index) and which are not (incoordination zone index);
5. Cluster the analysis results of each SDT. The SDTs in the same coordination zone are clustered together;
6. Put the clustered results into the data structure of Equation (23). The KAC number to be compensated is the incoordination zone index. Select the scheme with a better KAC number and space volume of the coordination zone.

\[ \{ \Gamma \} | \Gamma_i = (n_{IZi}, V_{CZ}, b_f, s_w), i < \Gamma_{num} \}, \]  \hspace{1cm} (23)

where \( \Gamma_i \) is the \( i \)th scheme, \( n_{IZi} \) is the incoordination zone index, \( V_{CZ} \) is the space amount of the coordination zone, \( b_f \) is the information of uncoordinated KACs, \( s_w \) is the SDT set, and \( \Gamma_{num} \) is the max number of the schemes.
7. Calculate the center SDT of the SDTs in the selected scheme. The assembly deviation of target features under the SDT is analyzed and the accuracy compensation is carried out.

\[ \omega_{\Delta} = \frac{1}{n_\omega} \sum_{i=1}^{n_\omega} \omega_{\Delta i} \]  \hspace{1cm} (24)

where \( \omega_{\Delta} \) is the center SDT, \( n_\omega \) is the SDT number of \( s_w \), and \( \omega_{\Delta i} \) is an SDT of \( s_w \). All assembly accuracies on \( \omega_{\Delta} \) are calculated. Then, the deviations on the excessive KACs will be compensated.

According to Equations (13) and (24), the compensation amount would be

\[ C_j^i = T_j^{i-\omega_{\Delta}} - T_j^{i-opt} = f_j^i(K_j^{\omega_{\Delta}}, K_j^{\omega_{\Delta}}, \omega_{\Delta}^i) - T_j^{i-opt}, \]  \hspace{1cm} (25)

where \( C_j^i \) is the compensation amount of the \( j \)th assembly accuracy requirement of the \( i \)th KAC, \( T_j^{i-\omega_{\Delta}} \) is the assembly accuracy on the SDT \( \omega_{\Delta} \), and \( T_j^{i-opt} \) is the optimal value of the assembly accuracy.
5. Case Study

5.1. Space Manipulator Assembly

The space manipulator is fixed on the spacecraft, which needs a high assembly accuracy to guarantee the stability when the spacecraft is flying. The assembly is executed by shaft and hole connectors, which are shown in Figure 9a. The connector is shown in Figure 9b. The manipulator is the align part and the spacecraft is the reference part.

The upper connector is fixed on the manipulator, and the bottom connector is fixed on the spacecraft. The KACs are the assembly of the connectors. Due to the slight deformation of the spacecraft and the installation error of the bottom connectors, it is difficult for the connectors to accurately assemble at one time during the assembly of the spacecraft and the manipulator. In the original assembly process, it is necessary to try the assembly first, measure the assembly deviation of the clearance and coaxiality of each pair of connectors, make the accuracy compensation, and retry the assembly to ensure the assembly quality. The assembly takes a long time and the connectors are not convenient to be operated on the spacecraft. Therefore, the laser tracker is used to measure the connectors between the spacecraft and the manipulator. The methods in Section 2 and Section 3 are taken to evaluate the assemblability based on the measurement data. The method in Section 4 is used to find the key connectors to make the accuracy compensation. The assembly is carried out after the accuracy compensation. In this way, the assembly quality is better guaranteed and the assembly efficiency is improved. The flow of the proposed method and the comparison with the original method are shown in Figure 10.
Figure 10. The comparison of the original and proposed assembly processes.

As shown in Figure 10, the assembly process is developed toward digital measurement and analysis. The results obtained in the actual assembly and inspection are replaced by the analysis of the measurement data. Therefore, some unnecessary assembly processes are eliminated and the possibility of repeated trial assembly is greatly reduced.

The assembly accuracy requirements of the connector are coaxiality $dr$ and clearance $dc$ on the matching surface, as shown in Figure 9b. The coaxiality requirement is 0.2 mm, and the clearance requirement is 0.1 mm. Assembly accuracy is compensated by gasket compensation, finishing, or position movement according to the deviation.

5.2. Coordination Space Model

The measurement of the connector is based on the measurement auxiliary tool, which is shown in Figure 11.

Figure 11. Measurement auxiliary tool.

After inserting the shaft into the corresponding hole, measure the four holes of the measurement auxiliary tool. The measurement data are processed as the position $p$ and orientation $\overrightarrow{po}$.

\[
\begin{align*}
\overrightarrow{po} &= \frac{(c_4 - c_1) \times (c_4 - c_2)}{|(c_3 - c_1) \times (c_4 - c_2)|} \\
\overrightarrow{p} &= \frac{1}{4} \sum c_i - l \cdot \overrightarrow{po}
\end{align*}
\]

(26)

where $c_1$, $c_2$, $c_3$, and $c_4$ are the points measured by the laser tracker.
As shown in Figure 12, the clearance \( dc \) and the coaxiality \( dr \) of a connector are

\[
\begin{align*}
\text{dr} &= |\overrightarrow{p_2p_1} \cdot \sin \theta_2| \\
\text{dc} &= |\overrightarrow{p_2p_1} \cdot \cos \theta_2 + r \cdot \sin(\theta_2 - \theta_1)|
\end{align*}
\]  

(27)

where \( \theta_1 \) and \( \theta_2 \) are the angles between \( \overrightarrow{p_2p_1} \) and \( \overrightarrow{p_1o_1} \) or \( \overrightarrow{p_2o_2} \); \( \theta_1 \) can be calculated by \( \theta_1 = \arccos(\overrightarrow{p_1p_2} \cdot \overrightarrow{p_1o_1}/|\overrightarrow{p_1p_2}| \cdot |\overrightarrow{p_1o_1}|) \) and, similarly, \( \theta_2 \) can be calculated by the same way; and \( r \) is the radius of the matching surface, which is 15 mm. There are 20 connectors to be guaranteed at the same time.

Therefore, the coordination space model is

\[
\begin{align*}
\omega_{\Delta} &\geq \begin{bmatrix} 0 < |p_2^{i-o_1} \cdot \omega_{\Delta}^y \cdot \sin \theta_2^{i-o_1}| < 0.2, \\
|p_2^{i-o_1} \cdot \omega_{\Delta}^y \cdot \sin \theta_2^{i-o_1} + r \cdot \sin(\theta_2^{i-o_1} - \theta_1^{i-o_1})| < 0.1 \end{bmatrix} \ 
\end{align*}
\]  

(28)

where \( \omega_{\Delta} \) is the random SDT based on the optimal pose derived from the least-squares method, and \( p_2^{i-o_1} \) is the parameters of Equation (27) changed by \( \omega_{\Delta} \) according to Equation (12), which are listed in Equation (29):

\[
\begin{align*}
p_2^{i-o_1} &= p_2^{i-o_1} \omega_{\Delta}^y \\
p_2^{i-o_1} \cdot \omega_{\Delta}^y &= \frac{(c_1-c_3) \times (c_1-c_2)}{|c_1-c_3| \times |c_1-c_2|} \\
\theta_1^{i-o_1} &= \arccos\left(p_2^{i-o_1} \cdot \overrightarrow{p_1o_1}/|p_2^{i-o_1}| \cdot |\overrightarrow{p_1o_1}|\right) \\
\theta_2^{i-o_1} &= \arccos\left(p_2^{i-o_1} \cdot \overrightarrow{p_2o_2}/|p_2^{i-o_1}| \cdot |\overrightarrow{p_2o_2}|\right)
\end{align*}
\]  

(29)

5.3. Assemblability Analysis

Part of the raw data is listed in Table 1. All the measurement data are listed in Appendix A, Table A1.

| Spacecraft | Manipulator |
|------------|-------------|
| x/mm       | y/mm        | z/mm       | x/mm       | y/mm        | z/mm       |
| 26.060     | 26.037      | -192.674   | 222.509    | 2274.256    | 403.730    |
| 25.941     | -25.881     | -192.645   | 222.471    | 2222.338    | 403.647    |
| -25.972    | -25.945     | -192.558   | 170.389    | 2222.349    | 403.666    |
| -25.973    | 26.032      | -192.581   | 170.632    | 2274.362    | 403.722    |

The least-squares method is taken to calculate the optimal pose and the deviations on the optimal pose. The deviations of the connectors are listed in Table 2 calculated by Equation (27).
It can be seen that 10 connectors need to be adjusted or repaired based on the least-squares method.

The coordination space is 0 at the optimal pose based on the method in Section 3, which means it cannot be assembled directly. Therefore, the assemblability should be optimized.

Table 2. Assembly deviation prediction by least-squares method.

| No. | \(dr/\text{mm}\) | \(dc/\text{mm}\) | No. | \(dr/\text{mm}\) | \(dc/\text{mm}\) |
|-----|-----------------|-----------------|-----|-----------------|-----------------|
| 1   | 0.154           | 0.043           | 11  | 0.171           | -0.012          |
| 2   | 0.150           | -0.027          | 12  | 0.338           | 0.002           |
| 3   | 0.205           | 0.072           | 13  | 0.286           | -0.010          |
| 4   | 0.196           | 0.042           | 14  | 0.530           | -0.274          |
| 5   | 0.191           | 0.012           | 15  | 0.247           | 0.033           |
| 6   | 0.209           | 0.083           | 16  | 0.165           | -0.029          |
| 7   | 0.066           | 0.045           | 17  | 0.188           | 0.033           |
| 8   | 0.195           | -0.025          | 18  | 0.402           | -0.154          |
| 9   | 0.118           | -0.006          | 19  | 0.086           | 0.183           |
| 10  | 0.298           | -0.010          | 20  | 0.386           | -0.028          |

5.4. Assemblability Optimization

The proposed method in Section 4 is taken to find the accuracy compensation schemes. The results are shown in Figure 13.

Figure 13 shows the accuracy compensation schemes. The first point on the X axis is the KAC quantity to be compensated. The second point is the volume of the coordination zone of the scheme. The latter ones are the number of KACs. The Y axis is the scheme number. The Z axis is the value of the X axis. Seven connectors need to be adjusted to complete assembly in scheme 1. Finally, scheme 18, which needs nine connectors to be compensated, is taken by considering the assembly quality. The coordination space of the scheme is 56d. d is the volume of the maximum pose space divided by random times. In this case, d is \(2.46 \times 10^{-14} \text{ mm}^3\text{rad}^3\). The KAC number to be compensated is 5, 6, 8, 10, 12, 14, 18, 19, and 20. The center SDT of the coordination zone in scheme 18 is \((-0.0363 \text{ mm}, 0.0210 \text{ mm}, 0.0098 \text{ mm})\), \(2.52 \times 10^{-6} \text{ rad}, 1.64 \times 10^{-6} \text{ rad}, -2.35 \times 10^{-6} \text{ rad}\).

The deviation is calculated under the center SDT listed in Table 3.
Table 3. The deviations of the connectors.

| No. | dr/mm | dc/mm | No. | dr/mm | dc/mm |
|-----|-------|-------|-----|-------|-------|
| 1   | 0.145 | 0.034 | 10  | 0.150 | −0.036|
| 2   | 0.188 | −0.031| 11  | 0.351 | −0.013|
| 3   | 0.169 | 0.064 | 12  | 0.142 | −0.022|
| 4   | 0.151 | 0.031 | 13  | 0.391 | −0.286|
| 5   | 0.230 | 0.004 | 14  | 0.133 | 0.016 |
| 6   | 0.249 | 0.070 | 15  | 0.031 | −0.039|
| 7   | 0.025 | 0.037 | 16  | 0.056 | 0.032 |
| 8   | 0.222 | −0.034| 17  | 0.204 | −0.145|
| 9   | 0.162 | −0.021| 18  | 0.204 | 0.179 |
| 10  | 0.255 | −0.029| 19  | 0.233 | −0.027|

After simulation accuracy compensation for the above nine connectors, which is in bold and italics in Table 3 (the proposed method), the coordination space is 101d, which is greater than 0. The average coaxiality is 0.068 mm and the average clearance is 0.014 mm. The assemblability is good and the assembly can be executed directly.

After simulation accuracy compensation for the above 10 connectors, which is in bold and italics in Table 2 (the least square method), the coordination space is 9d. The average coaxiality is 0.084 mm and the average clearance is 0.014 mm. The assemblability is good but the assembly quality on coaxiality is worse.

The result shows that the proposed method will generate a better accuracy compensation scheme with less workload and better assembly quality, which improves the assemblability.

The measurement and connector adjustment process took about 8 h during the assembly. The pose adjustment process took about 2 h. Therefore, it took about 10 h in total based on the proposed method. The original assembly process took more than 20 h because the first trial assembly and accuracy compensation process cannot realize the re-trial assembly smoothly. Three or four times the assembly are needed to guarantee the assembly quality.

6. Discussion

Compared to the previous research, the major contributions in this paper are listed as follows:
(1) The concept of assemblability and coordination accuracy in the design/drawing stage are extended into the measurement-assisted assembly.
(2) An assemblability analysis method based on the measurement data and the coordination space model is proposed for predicting the key assembly deviations.
(3) The accuracy compensation methods based on the optimal pose might lead to more workload and worse assemblability. Therefore, an assemblability optimization method is proposed for less workload and better assembly quality. In addition, the space manipulator assembly is taken as an example. The result shows that the proposed method can optimize the assemblability with less workload and better assembly quality compared to the accuracy compensation method based on the optimal pose.

The assemblability optimization method based on accuracy compensation improves the ability to detect assembly problems in advance, which will benefit the automation assembly. Further, the coordination space model and the small displacement torsor are useful for analyzing the assemblability and optimizing the tolerances in the design/drawings phase, but the assemblability optimization method is not useful. In the implementation of the method, high-precision digital measurement equipment are needed. Measurement uncertainty will affect the reliability of the final results.

Future research include evaluating the influence of the measurement uncertainty on the coordination space model. Then, the uncertainty of pose adjustment should be taken into consideration compared to the volume of the coordination space to judge the feasibility of automatic pose adjustment.
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Appendix A

Table A1. Raw measurement data.

| Manipulator | Spacecraft | x/mm | y/mm | z/mm |
|-------------|------------|------|------|------|
| 2.061       | -26.037    | -192.674 | 2525.979 | 25.966 | -192.995 |
| 25.941      | -25.881    | -192.645 | 2525.879 | -26.066 | -192.988 |
| -25.972     | -25.945    | -192.588 | 2473.905 | -25.939 | -192.634 |
| -25.973     | 26.032     | -192.581 | 2473.927 | 26.032 | -192.648 |
| 25.964      | -1103.901  | -192.620 | 2525.886 | -1103.863 | -192.642 |
| 25.995      | -1103.853  | -192.533 | 2525.963 | -1155.983 | -192.687 |
| -26.108     | -1103.891  | -192.633 | 2473.910 | -1155.938 | -192.617 |
| -26.138     | -1103.914  | -192.600 | 2473.997 | -1155.936 | -192.532 |
| 526.046     | 26.182     | -192.691 | 25.984 | 371.512 | -292.968 |
| 526.032     | -25.859    | -192.578 | 26.082 | 328.937 | -322.879 |
| 474.044     | -25.874    | -192.632 | -25.948 | 328.937 | -322.879 |
| 473.999     | 26.048     | -192.660 | -25.989 | 371.392 | -292.860 |
| 525.926     | -1103.994  | -192.665 | 26.306 | 779.693 | -581.183 |
| 525.941     | -1156.026  | -192.670 | 26.326 | 737.195 | -611.193 |
| 473.906     | -1155.956  | -192.534 | -25.677 | 737.131 | -612.127 |
| 474.001     | -1103.936  | -192.549 | -25.718 | 779.531 | -581.201 |
| 1025.900    | 25.892     | -192.696 | 26.057 | 371.467 | -292.910 |
| 1025.991    | -26.042    | -192.675 | 525.980 | 329.046 | -322.917 |
| 973.956     | -26.131    | -192.596 | 473.953 | 329.017 | -322.836 |
| 973.988     | 25.990     | -192.571 | 473.989 | 371.499 | -292.943 |
| 1026.139    | -1104.097  | -192.694 | 526.004 | 779.759 | -581.219 |
| 1026.085    | -1156.050  | -192.740 | 525.882 | 737.293 | -611.181 |
| 974.124     | -1156.044  | -192.605 | 474.011 | 737.365 | -611.149 |
| 974.105     | -1104.011  | -192.634 | 474.022 | 779.818 | -581.198 |
| 1525.925    | 26.129     | -192.717 | 25.899 | -1458.921 | -322.909 |
| 1525.908    | -25.848    | -192.628 | 26.022 | -1501.395 | -292.846 |
| 1473.914    | -25.921    | -192.601 | -26.061 | -1501.356 | -292.887 |
| 1473.942    | 26.171     | -192.663 | -29.580 | 1458.862 | -322.971 |
| 1526.063    | -1104.006  | -192.639 | 26.112 | -1687.403 | -611.275 |
| 1525.951    | -1155.886  | -192.590 | 25.999 | -1909.862 | -581.157 |
| 1473.970    | -1155.991  | -192.610 | -25.945 | -1909.883 | -581.133 |
| 1474.029    | -1103.917  | -192.536 | -25.952 | -1687.415 | -611.214 |
| 2026.006    | 25.807     | -192.602 | 525.901 | -1501.548 | -322.918 |
| 1974.072    | -26.202    | -192.597 | 474.025 | -1501.571 | -292.828 |
| 1974.950    | 25.758     | -192.615 | 473.983 | -1459.106 | -322.849 |
| 2025.839    | -1103.981  | -192.592 | 525.886 | -1687.434 | -611.169 |
| 2025.808    | -1155.974  | -192.526 | 525.806 | -1909.947 | -581.114 |
| 1973.868    | -1155.897  | -192.571 | 473.872 | -1909.917 | -581.157 |
| 1973.894    | -1103.986  | -192.651 | 473.840 | -1687.343 | -611.211 |

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