Impact of Gas Removal on the Evolution of Embedded Clusters

Christian Boily

_Astronomisches Rechen-Institut, Heidelberg_

Pavel Kroupa

_Institut für Theoretische Physik und Astrophysik, Kiel_

Abstract. We study both analytically and numerically the disruptive effect of instantaneous gas removal from an embedded cluster. We setup a calculation based on the stellar velocity distribution function, to compute the fraction of stars that remain bound once the cluster has ejected the gas and is out of equilibrium. We find tracks of bound mass-fraction vs star formation efficiency similar to those obtained with N-body calculations. We use these to argue that embedded clusters must develop high-binding energy cores if they are to survive as bound clusters despite a star formation rate as low as 20% or lower suggested by observations.

1. Introduction

Most if not all stars are formed in embedded clusters and associations, and therefore this is likely the predominant mode of star formation contributing to the Galactic-field population. To understand how such aggregates form and evolve remains a severe challenge however, since no theory exists yet that accounts in detail for the (presumably) simpler case of the formation of individual stars. On the observational front, surveys of star-forming regions suggest that the mass-fraction of gas used up at birth does not exceed 10 to 20% of the total (Lada 1999). This low star-formation efficiency (sfe) implies that young stars or clusters of stars should be embedded in gas. Yet populous clusters with ages \( \lesssim 1 \) Myr (e.g. the Orion Nebula Cluster, R136 in 30 Doradus) are already void of gas. The fraction of gas left behind after the epoch of star formation must therefore be removed quickly (well before the first supernova explodes) to reveal a bare cluster. By the time this occurs, most of the cluster must be assembled. Hydrodynamic collapse solutions point to the rapid formation of stellar cores followed by time-dependent accretion (Boily & Lynden-Bell 1995). Stellar masses are then accrued over a period \( 10^5 - 10^6 \) years, long before the gas-evacuation timescale. Therefore gas-evacuation itself inhibits further star formation in the cluster. The gas is driven out by OB stars that culminate the star-formation process. Stellar winds from these OB stars yield a momentum flux \( F \sim 8 \times 10^{-3} M_\odot \) km sec\(^{-1}\)yr\(^{-1}\) (Churchwell 1999). This is sufficient to blow out gas from an \( 10^3 M_\odot \), 1 pc radius embedded cluster in \( \approx 10^5 \) years, which is comparable to its crossing time \( t_{cr} = 2R/\sigma \). In addition, the ionising radiation

\[ F \sim 8 \times 10^{-3} M_\odot \] km sec\(^{-1}\)yr\(^{-1}\)
heats the gas to $10^4$ K, causing an overpressure and expansion at the sound velocity $\approx 10$ km/s. We want to establish how significant mass-loss affects the structure of a cluster, in particular what fraction of the stars remain bound.

This contribution marks the start of a research programme aiming at identifying and quantifying the fundamental physical processes responsible for the formation of open clusters. This programme is inspired by the finding from high-precision $N$-body computations that open clusters can form readily despite low star-formation efficiencies, in contradiction to previous results (Kroupa, Aarseth & Hurley 2001). Three hypothesis are raised that may explain these $N$-body results. (i) Either two-body encounters during the expansion phase of the cluster after gas expulsion condense a part of the radial flow to a bound entity, or (ii) encounters between the ubiquitous binary systems during the radial outflow affect the condensation, or (iii) two-body and binary–binary encounters before gas-expulsion (i.e. during the embedded phase) cause the segregation of a tightly bound core that resists expansion when the gas leaves the system.

In order to weigh the relative contributions from each of points (i)-(iii), all drawn from particle-particle (collisional) evolution, we seek first to isolate the salient features of collisionless, smooth open cluster evolution. We take an analytic approach to the problem, based on the velocity distribution function of stars. By applying a new, fast and self-consistent iterative method for computing the fraction of bound stars, we show that to account for observations of low SFE, clusters must develop strongly bound cores to avoid dissolving completely owing to mass loss. In addition to our analytical approach we use numerical calculations to assess its range of applicability.

2. The problem

A cluster of stars forms converting a fraction $\epsilon$ of the gas into stars in the process. Stellar winds or a supernova event blow/s out the remainder on a timescale $\tau \ll t_{\text{cr}}$. What fraction of the stars remain to form a bound cluster? How is the equilibrium profile linked to the initial mass density? Since the star formation epoch will have lasted as long as or longer than a cluster dynamical time $t_{\text{cr}}$, the system as a whole is close to virial equilibrium before gas is expelled.

Hills (1980) argued that equilibrium, self-gravitating stellar systems would expand or even dissolve if half the mass is lost very quickly: stars then preserve their kinetic energy established under a deeper potential well, hence may escape if their binding energy becomes positive. We write the total energy

$$E = -\kappa \frac{GM^2}{R} + \frac{1}{2} M \langle v^2 \rangle < 0 ,$$

where $M, R$ are the mass and radius of a uniform-density spherical distribution of mean square velocity $\langle v^2 \rangle$, and $\kappa = 3/5$ for the particular geometry and density profile considered. Before gas-expulsion the total mass in gas and stars is $M = M_{\text{init}} = M_{\text{gas}} + M_*$, while after the gas is expelled $M = M_*$ but the velocity dispersion, $\langle v^2 \rangle = \kappa GM_{\text{init}}/R$, from the scalar virial theorem. Thus we find a solution for $E \geq 0$ if $M_* \leq M_{\text{init}}/2$, i.e. as the mass is reduced by 50% or more ($\epsilon \leq 1/2$), the remaining system has zero or positive energy globally.
But if the law of averages applies, nothing stops a hard-bound core forming at the expense of an expanding, loose envelope of stars, while \( E \geq 0 \) for the system as a whole. We need to find out under which circumstances this will occur. To do this we introduce an iterative procedure based on the stellar distribution function in order to determine the end-product fraction of bound stars.

3. Distribution function: iterative scheme

As a simplifying assumption we take the sfe to be independent of position in the cluster. Hence \( \epsilon = \text{constant} \) (see Adams 2000 for a different approach). Thus gas and stars are initially mixed in the same proportion throughout, however note that \( \epsilon \) does not fix the profiling of stellar masses with radius, rather the total mass of gas made into stars:

\[
\frac{\sum_i (\text{number of stars in mass bin } i) \times (\text{stellar mass } m_i)}{m_{\text{gas}}} = \text{constant}
\]

at any radius \( r \). Both gas and stars are distributed according to the same distribution function which for a spherical system depends only on energy, \( f(E) \). The total system mass is then

\[
M = M_* + M_{\text{gas}} = \epsilon^{-1} M_* = \int_{E_c}^{0} f(E) dE,
\]

where \( |E_c| \) is the maximum binding energy and with a suitable normalisation of \( f(E) \). The cluster potential \( \phi(r) \) may be written

\[
\phi(r; M, R) = \int_{\infty}^{r} \frac{du}{u^2} \int_{0}^{u} 4\pi G \rho(w) w^2 dw \equiv y(r/R) \frac{GM}{R}, \tag{1}
\]

where \( y(x) \) is a dimensionless function. Keeping \( R \) constant while removing the gas instantly so the potential includes stars only, we have

\[
\phi(r; M, R) \rightarrow \phi_* = \epsilon \phi, \tag{2}
\]

and hence the fraction of stars at radius \( r \) which now have positive energy is

\[
1 - f_e = \frac{\int_{v_{e,*}}^{v_e} f(v, r/R) v^2 dv}{\int_{0}^{v_e} f(v, r/R) v^2 dv}, \tag{3}
\]

with \( v_e \) the (local) escape velocity computed for \( \phi \) (before gas-expulsion), and similarly for \( v_{e,*} \) obtained for \( \phi_* \).

Thus for a given d.f. and \( \epsilon \), we may compute the quantity \( 1 - f_e \) at all radii. Note that should the sfe be a function of radius, the simple renormalisation (2) leading to \( \phi_* \) would not apply, however (3) may still be computed if the stars’ potential is given and \( v_e \) known from the initial gas + stars mixture. The key step is to adjust the potential \( \phi_* \) itself, since potential and velocity field do...
not match any longer. This is normally computed using N-body integration to take into account the full star-star interactions, and the redistribution of energy between them in the time-dependent potential. If we thought that stars acquire only little energy during the time that they escape, then the fraction of stars remaining might be computed as follows.

Since the fraction (3) of positive-energy stars is known at all \( r \), we recompute the gravitational potential counting only stars with \( E \leq 0 \) at each radius. Neglecting dynamical evolution, the cluster radius \( R \) is unchanged and hence the potential can be recomputed by integration, from \( \infty \), inwards. Once the new potential is known, a fraction of the remaining stars will again be unbound by virtue of the stars lost during the previous iteration. We therefore repeat the procedure until finally the cluster mass converges to a finite quantity, in which case no more stars escape and the original distribution function is depleted from all escapers in a self-consistent manner.

We consider three cases in detail, then turn briefly to numerical calculations.

3.1. Power-law d.f.

The case \( f(v) \propto v^\beta \), where \( \beta \) is constant, provides a useful illustrative starting point. The d.f. is truncated at the local escape velocity. Then the velocity dispersion \( \sigma^2 \propto v_e^2 = 2\phi(r) \) at each radius. We find, on substituting \( \rho(w) \rightarrow \epsilon \rho(w) \), etc, so that after \( n \) iterations the net mass of bound stars, \( M^b_\star \), becomes

\[
M^b_\star = \epsilon \left( \frac{(\beta+1)}{2} \right)^n \cdot \epsilon \left( \frac{(\beta+1)}{2} \right)^{n-1} \cdots \epsilon \left( \frac{(\beta+1)}{2} \right) M \equiv \Pi_{k=1}^n \left( \epsilon \left( \frac{(\beta+1)}{2} \right)^k \right) M_\star,
\]

where we took \( \beta > -1 \). Thus \( f_e = \epsilon^{(\beta+1)/2} \) is independent of radius. Repeating the procedure to take account of the positive-energy stars, we substitute \( \epsilon \rightarrow f_e \cdot \epsilon \), etc, so that after \( n \) iterations the stellar density \( \rho_\star[r] \) wrt \( \rho[r] \). The multiplicative operator \( \Pi \) in (5) leads to a non-zero (positive) value as \( n \rightarrow \infty \) only when \( \beta < 1 \), since \( \epsilon \leq 1 \). All d.f.’s with \( \beta > 1 \) lead to cluster disruption, because the high-velocity range of the d.f. is too densely populated, leading to catastrophic stellar loss after the expulsion of any amount of gas.

3.2. Plummer model

We wish to compare our basic result (5) to a standard fit to globular clusters. We consider the Plummer model, where \( f(E) \propto (-E)^{7/2} \). For this case the total system mass is finite but infinite in extent; the velocity dispersion maximises at the centre, as the density. The velocity d.f., \( f(v) \propto (1 - (v/v_e)^2)^{7/2} \) (see Spitzer 1987); writing \( p(\epsilon) = 105 - 1210\epsilon + 2104\epsilon^2 - 1488\epsilon^3 + 384\epsilon^4 \), we find
\[ 1 - f_e = \int_{1/2}^{1} \frac{(1-u^2)^{7/2} u^2 \, du}{\int_{0}^{1} (1-u^2)^{7/2} u^2 \, du} = 1 + \frac{\epsilon^{1/2} \sqrt{1-\epsilon} p(\epsilon) - 105 \sin^{-1} \epsilon^{1/2}}{105 \epsilon^{1/2}}. \tag{6} \]

We may repeat the procedure until \( 1 - f_e \to 0 \) and no additional stars are lost. We were not able to express the resulting expression in simple form, however we note that, as in the first case, the solution is independent of radius, hence the density profile is simply renormalised at each radius. Therefore the potential \( \phi_* \) and escape velocity follow from (2).

### 3.3. Hernquist profile

To contrast with the smooth-density Plummer solution, we consider a peaked Hernquist (1990) profile. The density \( \rho \) and one-dimensional velocity dispersion \( \sigma \) vary radially according to

\[ \rho(r \text{ or } x) \equiv \frac{M}{2\pi r^3} \frac{1}{(x + 1)^3} = \frac{1}{2\pi G r_c^2} \frac{\phi(x)}{x(x+1)^2}, \]

\[ \sigma^2(x) = \phi(x) x (1+x)^4 \left\{ \ln \frac{1+x}{x} - \frac{1/4}{(1+x)^3} - \frac{1/3}{(1+x)^3} - \frac{1/2}{(1+x)^3} - \frac{1}{1+x} \right\}, \]

with \( r_c \) a free length fixing the point of the power-law turnover, and \( x \equiv r/r_c \). The velocity d.f., \( f(v) \), is only constrained locally by \( \sigma(r) \); we thus have the freedom to choose any profile satisfying \( \sigma(r) \). We set \( f(v) \propto v^2 \exp\left(-v^2/2\sigma^2\right) \), a Maxwellian profile. This is found to give stable equilibria in N-body calculations (Hernquist 1993). Inserting this in (7) yields

\[ 1 - f_e(\Psi_*) = 1 - \frac{\sqrt{\Psi_* e^{-\Psi_*}} - \text{erf}(\sqrt{2\Psi_*})}{\sqrt{\Psi e^{-\Psi}} - \text{erf}(\sqrt{2\Psi})}, \tag{7} \]

where the dimensionless potential \( \Psi \equiv \phi/\sigma^2 \) and \( \text{erf}(x) \) is the error function. Note that although the dispersion \( \sigma^2 \propto \phi \) as before, here the fraction of positive-energy stars depends on the local potential, and hence it is a function of radius. To compute the net fraction of bound stars we must therefore recompute the potential numerically for each evaluation of \( f_e \) in (7). This poses no problem since \( \phi(r \to \infty) \to 0 \), and the density is known at each step (though it is no longer a Hernquist profile).

### 4. Results for three illustrative cases

Our results are shown in Fig. 1 for the three cases discussed above. The solid lines show the solutions for the power-law d.f. with \( \beta = -3/4 \) and 0. Note that in either case the fraction \( M_b^*/M_* \) of stars that remain bound is not dropping to zero until \( \epsilon \) itself is zero. The case \( \beta = 0 \) corresponds to a flat distribution, \( f(v)v^2 dv = \text{constant} \times dv \). Assuming only that no star has velocity greater than the local escape velocity, the solution (6) decreases only marginally faster than
as $\epsilon \to 0$. By contrast, a Plummer or Maxwellian-Hernquist model shows a sudden drop to a null fraction of bound stars for finite sfe: for Plummer models we obtain a critical value $\epsilon \approx 0.445$, while for the Hernquist model the fraction of bound stars exceeds 5% or so until $\epsilon \approx 0.28$. In both cases the iterative scheme converged to machine accuracy.

Note that for the two self-consistent d.f.’s discussed here, the power-law solutions provide an illustrative description of the survival rate as $\epsilon$ approaches a critical value. The more robust Hernquist model favours low-velocity stars (near the centre $\sigma \to 0$) and hence is better fitted with the solution $\beta = -3/4$ in the range $0.5 < \epsilon \leq 1$. Figure 2 shows for this case that the fraction of bound stars $f_\epsilon$ remains larger near the core (see Fig. 2b). As a result, the initial density profile becomes steeper with radius. Since we have only made a selection by energy, the expectation is that the bound system indeed should be more peaked than initially.

So far we only considered a mapping of static configurations under the selection of particles according to (3). In reality stars must orbit in space before leaving the system, and they may exchange gravitating energy with the background system in the process. N-body calculations are of some help here. N-body studies with time-dependent potential (Lada, Margulis & Deardorn 1984) or a star-gas mixture (Geyer & Burkert 2000) also find a critical sfe below which clusters dissolve. We decided to conduct our own N-body calculations with a collisionless grid code (Fellhauer et al. 2000) and 100,000 particle equilibrium Plummer models. Our analytic approach gives a critical value for survival of $\epsilon = 0.4448$. We therefore setup two N-body calculations with sfe = 50% and
40%. The fraction of stars which remain bound in each case brackets the results derived from (6) (see Fig. 1, large open circles). Notably we find no indication that any stars remain for the case where $\epsilon = 0.40$, in agreement with the findings by others (Lada, Margulis & Deardorn 1984; Geyer & Burkert 2000) for similar setups.

![Figure 2](image)

Figure 2. Initial and final density profiles for (a) Plummer and (b) Hernquist models for two values of the sfe $\epsilon$. The run of $f_e$, i.e. the ratio of density to the initial density, is also shown as a function of radius. Note how the Hernquist profile becomes steeper for small $\epsilon$.

5. Time-evolution and other considerations

To progress further, we note that only two timescales are important to the problem, namely the cluster crossing time, $t_{cr}$, and the gas removal timescale, $\tau$. We may understand the dynamics by comparing $\tau$ to $t_{cr}$.

5.1. Collisional evolution, but short $\tau$

The key lies with the redistribution of kinetic energy between the stars. This can be achieved on a short timescale by direct collisions or close encounters, when the Safronov cross section $d^2 \left(1 + \frac{1}{N^2 \beta}\right)$ is large (here $d^2$ is a star’s geometric cross section and $N$ the number of stars). This is especially true for small-N open clusters with binaries and multiple stars, in which case $d$ is set equal to their semi-major axis. In this situation collisional effects are never negligible and hence when $\tau \simeq t_{cr}$ or longer, the situation is not one of equilibrium, and evolution must be tackled numerically. Kroupa, Aarseth & Hurley (2001)
Boily & Kroupa evolved an embedded Plummer model with a high-precision direct-summation
$N$-body code and delayed, but near-to instantaneous, gas-removal. They find
a fraction $\simeq 30\%$ of stars remain despite a low sfe of $\epsilon \approx 0.3$. The results are
similar to the analytical results for the Hernquist model (Fig. 1, black square),
while we would have expected complete dissolution for collisionless evolution of
a Plummer model. We are lead to the conclusion that it is the compact core that
develops as a result of two-body relaxation during the embedded phase before gas
expulsion that leads to more robust clusters (see also Section 1).

5.2. Collisionless evolution, but long $\tau$

Large-$N$ systems possess a long two-body relaxation time $t_{\text{col}} \simeq \frac{N}{10\ln\gamma N} t_{\text{cr}}$. When
$t_{\text{cr}} \ll \tau \ll t_{\text{col}}$, two-body effects may be neglected. In this case the age of the
cluster may not have allowed for global two-body relaxation, yet significant mass
removal will have occurred over several stellar orbits concentrated around the
centre and these orbits evolve adiabatically. Since adiabatic evolution is a re-
mapping of an orbit to itself, no stars on such orbits are lost. Thus for finite
or large $\tau$, we anticipate the survival rate of clusters to be intimately linked to
their properties at birth, such as what family of orbits are present initially.

The future of this programme will see additional high-accuracy $N$-body
computations being performed to address how relatively important the three
hypothesis raised in Section 1 are for the formation of bound clusters. Specif-
ically, the formation of sub-condensations as a result of encounters during the
radially expanding flow will be addressed in detail.

References

Adams, F.C. 2000, ApJ, 542, 964
Boily, C.M., & Lynden-Bell, D. 1995, MNRAS, 276, 133
Churchwell, E. 1999, in NATO Science Series C Vol. 450, The Origins of
Stars and Planetary Systems, ed. C.J. Lada & N.D. Kylafis (Dordrecht: Kluwer), 515
Fellhauer, M., Kroupa, P., Baumgardt, H., et al. 2000, New Astronomy, 5, 305
Geyer, M.P., & Burkert, A. 2000, MNRAS, submitted (astro-ph/0007413)
Hernquist, L. 1990, ApJ, 356, 359
Hernquist, L. 1993, ApJS, 86, 389
Hills, J. 1980, ApJ, 225, 986
Kroupa, P., Aarseth, S.J., & Hurley, J. 2001, MNRAS, in press
Lada, C.J., Margulis, M., & Deardorn, D. 1984, ApJ, 285, 141
Lada, E. A. 1999, in NATO Science Series C Vol. 450, The Origins of Stars and
Planetary Systems, ed. C.J. Lada & N.D. Kylafis (Dordrecht: Kluwer), 441
Spitzer, L. 1987, Dynamical Evolution of Star clusters, Princeton: Princeton
University Press, pp. 232