Entanglement conditions for tripartite systems via indeterminacy relations

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Abstract

Based on the Schrödinger–Robertson indeterminacy relations in conjunction with the partial transposition, we derive a class of inequalities for detecting entanglement in several tripartite systems, including bosonic, SU(2) and SU(1, 1) systems. These inequalities are in general stronger than those based on the usual Heisenberg relations for detecting entanglement. We also discuss the reduction from SU(2) and SU(1, 1) to bosonic systems and the generalization to a multipartite case.

1. Introduction

The Heisenberg uncertainty relation (HUR) plays a fundamental role in quantum mechanics, and recent developments in quantum information theory display that it is useful for deriving some entanglement criteria [1–4]. Given two noncommuting observables \{A, B\} satisfying \[\{A, B\} = C\], the HUR is given by [5]

\[\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \frac{1}{4} |\langle C \rangle|^2, \tag{1}\]

where \(\text{Var}(A) \equiv \langle (\Delta A)^2 \rangle = \langle A^2 \rangle - \langle A \rangle^2\) denotes the variance or the uncertainty of the observable \(A\). It is evident that the product of two uncertainties is bounded below by \(|\langle C \rangle|^2 / 4\).

Actually, there exists a stronger bound \(|\langle C \rangle|^2 / 4 + \text{Cov}(A, B)^2\), where the covariance \(\text{Cov}(A, B) = \langle (A B + B A) / 2 \rangle - \langle A \rangle \langle B \rangle\). The corresponding uncertainty relation is the Schrödinger–Robertson indeterminacy relation (SRIR) given by [6, 7]

\[\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \frac{1}{4} |\langle C \rangle|^2 + \text{Cov}(A, B)^2. \tag{2}\]

Very recently, the SRIR was also used by Nha [8] and Yu and Liu [9] to obtain entanglement conditions. In general, the entanglement criteria based on SRIRs are stronger than those via HURs.

Many methods are developed to obtain entanglement conditions in the literature [10–21]. The method based on the uncertainty relations has its own advantages for deriving entanglement criteria. It can apply not only to continuous-variable but also discrete-variable systems, or even hybrid systems. Another advantage is that it is easier to use to derive entanglement criteria compared with several other approaches. Finally and importantly, the entanglement criteria based on this method often provide strong detection of the separability. For instance, the entanglement inequality based on the SRIR for two qubits gives a necessary and sufficient condition for separability [9].

In this paper, we consider tripartite states and study their separability problem via indeterminacy relations. Some separability inequalities have been obtained previously in [22] from a different approach. It will be seen that the inequalities obtained here are more general and stronger. We consider not only continuous-variable systems, but also SU(2) and SU(1, 1) systems.

2. Method based on indeterminacy relations

First, we introduce the method and demonstrate its usefulness by rederiving the inequality given by Duan et al [23]. Consider the SRIRs for operators \(A, B, C\) acting on a composite multipartite system. The SRIR, of course, holds for a separable state represented by the density operator \(\rho\). The separable state is still separable after partial transposition with respect to any
subsystems, namely the partially transposed density operator $\rho^{TP}$ is still physical. Thus, the SRIR also holds for state

$$\langle (\Delta A)^2 \rangle_{\rho^{TP}} \langle (\Delta B)^2 \rangle_{\rho^{TP}} \geq \frac{1}{2} \langle |C|^2 \rangle_{\rho} + \text{Cov}(A, B)^2_{\rho^{TP}}. \quad (3)$$

This is of the form of product of two uncertainties. By using $a^2 + b^2 \geq 2ab$, one can also achieve the following:

$$\alpha \langle (\Delta A)^2 \rangle_{\rho^{TP}} + \beta \langle (\Delta B)^2 \rangle_{\rho^{TP}} \geq \sqrt{\alpha \beta} \sqrt{\langle |C|^2 \rangle_{\rho} + 4 \text{Cov}(A, B)^2_{\rho^{TP}}}, \quad (4)$$

which is of the form of an arbitrary sum of two uncertainties. Here, $\alpha, \beta$ are real. By defining positive $c = \sqrt{\beta/\alpha}$, the above equation can be written as

$$\langle (\Delta A)^2 \rangle_{\rho^{TP}} + c^2 \langle (\Delta B)^2 \rangle_{\rho^{TP}} \geq c \sqrt{\langle |C|^2 \rangle_{\rho} + 4 \text{Cov}(A, B)^2_{\rho^{TP}}}. \quad (5)$$

For any operators $A, B$, acting on a state $\rho$, we have

$$\langle (A) \rangle_{\rho} = \langle (A^T) \rho \rangle. \quad (6)$$

Then, using this fact, inequalities (3) and (4) can be written in the form of partial transposition of operators other than states. They are given by

$$\left[ \langle (A^2)angle_{\rho} - \langle (A^T)^2 \rho \rangle \right] \times \left[ \langle (B^2)angle_{\rho} - \langle (B^T)^2 \rho \rangle \right]$$

$$\geq \frac{1}{4} \langle |C|^2 \rangle_{\rho} + \left[ \langle [A, B]^2 \rangle_{\rho} / 2 \right] - \langle (A^T)^2 \rho \rangle \langle (B^T)^2 \rho \rangle, \quad (7)$$

and

$$\left[ \langle (A^2)angle_{\rho} - \langle (A^T)^2 \rho \rangle \right] + c^2 \left[ \langle (B^2)angle_{\rho} - \langle (B^T)^2 \rho \rangle \right]$$

$$\geq c \sqrt{\langle |C|^2 \rangle_{\rho} + 4 \left[ \langle [A, B]^2 \rangle_{\rho} / 2 \right] - \langle (A^T)^2 \rho \rangle \langle (B^T)^2 \rho \rangle}, \quad (8)$$

respectively. Here, $[A, B]_\rho = AB - BA$. Note that in general $(A^2)^T \neq (A^T)^2$, $(AB)^T \neq B^TA^T$. The inequalities hold for any separable states, and conversely any state violating this inequality must be entangled.

Now, we rederive the inequality for a two-mode system given by Duan et al [23] using the present approach. Consider the following operators:

$$u' = |a|x_1 + \frac{1}{a}x_2, \quad v' = |a|p_1 + \frac{1}{a}p_2, \quad (9)$$

where $x_i$ and $p_i$ are position and momentum operators for mode $i$, respectively. It is easy to check that for any state, the two operators satisfy the HUR:

$$\langle (\Delta u')^2 \rangle \langle (\Delta v')^2 \rangle \geq \frac{1}{4} \left( a^2 + \frac{1}{a^2} \right)^2, \quad (10)$$

Therefore, we have

$$\langle (\Delta u')^2 \rangle + \langle (\Delta v')^2 \rangle \geq a^2 + \frac{1}{a^2}, \quad (11)$$

hailing for any state. For a separable state $\rho$, we have

$$\langle (\Delta u')^2 \rangle_{\rho_1} + \langle (\Delta v')^2 \rangle_{\rho_2} \geq a^2 + \frac{1}{a^2}, \quad (12)$$

for any separable states. Here, $u = u'$, $v = |a|p_1 - \frac{1}{a}p_2$. We see that from the uncertainty relation in conjugation with the partial transposition, the inequality by Duan et al is neatly obtained, indicating the effectiveness of the approach.

### 3. Entanglement conditions for tripartite systems

We consider the entanglement of tripartite systems and begin our discussions on the case of three bosonic modes.

#### 3.1. Continuous-variable systems

Let operators $a$, $b$ and $c$ be the annihilation operators of the first (A), second (B) and third (C) modes respectively. We define a set of operators $L_x, L_y$ and $L_z$ which obey the commutation relations $[L_x, L_y] = iL_z$. Note that these three operators do not need to form an algebra. It can be realized in optics using three-mode fields represented by the annihilation operators, $L_x = \frac{1}{2}(a^\dagger b^\dagger c + abc^\dagger)$, $L_y = \frac{1}{2i}(ab^\dagger c - abc^\dagger)$, $L_z = \frac{1}{2}[(N_aN_b(N_c + 1) - (N_a + 1)(N_b + 1)N_c)],$ where $N_a = a^\dagger a, N_b = b^\dagger b$ and $N_c = c^\dagger c.$ We further define another set of operators $H_x, H_y$ and $H_z$ that satisfy $[H_x, H_y] = iH_z$. The operators can be given by

$$H_x = \frac{1}{2}(a^\dagger b^\dagger c + abc^\dagger), \quad (13)$$

$$H_y = \frac{1}{2}(ab^\dagger c - abc^\dagger), \quad (14)$$

$$H_z = \frac{1}{2}[(N_aN_bN_c - (N_a + 1)(N_b + 1)N_c)].$$

It is easy to see that the two sets of operators are connected by partial transposition with respect to the third mode as follows:

$$H_x^{T_3} = L_x, \quad H_y^{T_3} = L_y, \quad (15)$$

$$H_z^{T_3} = H_z, \quad L_z^{T_3} = L_z.$$

The partial transposition with respect to the third mode means that we are considering the entanglement between systems $AB$ and $C$. From the discussions in the above section, in order to get entanglement conditions, we need to know the partial transposition of product of two operators. For our case, after some algebra, we obtain

$$H_x^{T_3} = L_x^2 + \frac{1}{2}(N_a + N_b + 1), \quad (16)$$

$$(H_x^T)^{T_3} = L_x^2 + \frac{1}{4}(N_a + N_b + 1), \quad (17)$$

Now by replacing $A, B$ and $C$ in equation (7) with $H_x, H_y$ and $H_z$, respectively, and using equations (15) and (16), we obtain the following inequality:

$$\left[ \langle (\Delta L_x)^2 \rangle_{\rho} + \frac{1}{2} \langle (N_a + N_b + 1) \rangle \right] \left[ \langle (\Delta L_y)^2 \rangle_{\rho} + \frac{1}{2} \langle (N_a + N_b + 1) \rangle \right]$$

$$\geq \frac{1}{4} \langle |H_z|^2 \rangle_{\rho} + \text{Cov}(L_x, L_y)^2,$$
three-mode case and then obtain
\[
\langle \Delta L_y \rangle^2_p + \langle \Delta L_y \rangle^2_p + \frac{(1 + c^2)}{4} (N_a + N_b + 1) \geq \sqrt{\frac{1}{4} (M_x + (N_a + 1))^2 + 4 \text{Cov}(L_x, L_y)^2}. 
\]
(18)
For \( c = 1 \), the above equation reduces to
\[
\langle \Delta L_y \rangle^2_p + \langle \Delta L_y \rangle^2_p \geq \frac{1}{2} (M_x + N_c), 
\]
(19)
by letting \( \text{Cov}(L_x, L_y) = 0 \). This inequality is just the one obtained from a different procedure [22]. Inequality (19) is a special case of inequality (18). Having studied three-mode systems, we next consider the SU(2) spin systems and SU(1, 1) systems.

3.2. SU(2) spin and SU(1, 1) systems

3.2.1. SU(2) spin systems. A spin is described by the operators \( J_+ \) and \( J_- \), which obey the following commutation relations:
\[
[J_+, J_-] = 2J_z, \quad [J_z, J_\pm] = \pm J_\pm. 
\]
(21)
In the spin system, we can define the ‘number’ operator \( \mathcal{N} = J_+ J_- \). For tripartite systems, we define
\[
A_x = \frac{1}{2} (J_a + J_b - J_c - J_a - J_b - J_c), \\
A_y = \frac{1}{2i} (J_a + J_b - J_c - J_a - J_b - J_c), \\
A_z = \frac{1}{2} [J_a + J_b - J_c - J_a - J_b - J_c - J_a - J_b - J_c], 
\]
(22)
satisfying \( [A_x, A_y] = iA_z \). By using
\[
J_+ J_- = \mathcal{N}(2J_+ J_- + 1), \quad J_+ J_+ = (\mathcal{N} + 1)(2J_+ J_- + 1), 
\]
(23)
operator \( A_z \) can be written as
\[
A_z = \frac{1}{2} [\mathcal{N} - (J_a + J_b + J_c) - (J_a + J_b + J_c)] - (J_a + J_b + J_c), \\
\times (2J_+ J_- + 1)(J_+ J_- + 1). 
\]
(24)
Another set of operators satisfying \( [B_x, B_y] = iB_z \) are given by
\[
B_x = \frac{1}{2} (J_a + J_b - J_c + J_a - J_b - J_c), \\
B_y = \frac{1}{2i} (J_a + J_b - J_c + J_a - J_b - J_c), \\
B_z = \frac{1}{2} [J_a + J_b - J_c + J_a - J_b - J_c - J_a - J_b - J_c]. 
\]
(25)
By using equation (23), operator \( B_z \) can be written as
\[
B_z = \frac{1}{2} [\mathcal{N} + (J_a + J_b - J_c) - (J_a + J_b + J_c)] - (J_a + J_b + J_c), \\
\times (2J_+ J_- + 1)(J_+ J_- + 1). 
\]
(26)
From the definitions of the above operators, one finds
\[
B_{xT}^T = A_x, \quad B_{yT}^T = A_y, \quad B_{zT}^T = A_z, \\
(A_x^T)^T = A_x^2 + \frac{1}{4} E, \quad (A_y^T)^T = A_y^2 + \frac{1}{4} E, \\
\]
(27)
where
\[
E = 2(N_c - j) [N_a N_b (2j_a - N_a + 1)(2j_b - N_b + 1) - (N_a + 1)(N_b + 1)(2j_a - N_a)(2j_b - N_b)]. 
\]
(28)
Then, from equation (7), we obtain
\[
\left[ \frac{1}{2} \langle \Delta L_x \rangle^2 + \frac{1}{2} \langle \Delta E \rangle \right]\left[ \frac{1}{2} \langle \Delta L_x \rangle^2 + \frac{1}{2} \langle E \rangle \right] \geq \frac{1}{4} (B_x)^2 + \text{Cov}(A_x, A_y)^2. 
\]
(29)
This is the entanglement condition for tripartite SU(2) systems and can be used to detect entanglement between \( AB \) and \( C \).

3.2.2. SU(1, 1) systems. The SU(1, 1) systems are described by su(1, 1) Lie algebra. The generators of su(1, 1) Lie algebra, \( K_z \) and \( K_\pm \), satisfy the commutation relations
\[
[K_+, K_-] = -2K_z, \quad [K_z, K_\pm] = \pm K_\pm. 
\]
(30)
Its discrete representation is
\[
K_+ |m, k \rangle = \sqrt{(m + 1)(2m + 1)} |m + 1, k \rangle, \\
K_- |m, k \rangle = \sqrt{m(2m - 1)} |m - 1, k \rangle, \\
K_z |m, k \rangle = (m + k) |m, k \rangle. 
\]
(31)
Here \( |m, k \rangle \) \((m = 0, 1, 2, \ldots)\) is the complete orthonormal basis and \( k = 1/2, 1, 3/2, 2, \ldots \) is the Bargmann index labelling the irreducible representation \( (k - 1) \) is the value of the Casimir operator). We introduce the ‘number’ operator \( \mathcal{M} \) by
\[
\mathcal{M} = K_z - k, \quad \mathcal{M} |m, k \rangle = m |m, k \rangle. 
\]
(32)
From equation (31), one may find
\[
K_+ K_- = \mathcal{M}(2k + \mathcal{M} - 1), \\
K_+ K_+ = (\mathcal{M} + 1)(2k + \mathcal{M}). 
\]
(33)
Similar to the discussions of the SU(2) case, we consider the \( AB|C \) entanglement conditions for three SU(1, 1) systems. We define
\[
C_x = \frac{1}{2} (K_{xa} K_{xb} K_{xc} - K_{xa} K_{xb} K_{xc}), \\
C_y = \frac{1}{2i} (K_{xa} K_{xb} K_{xc} - K_{xa} K_{xb} K_{xc}), \\
C_z = \frac{1}{2} [K_{xa} K_{xa} K_{xb} K_{xb} K_{xc} - K_{xa} K_{xa} K_{xb} K_{xb} K_{xc}], \\
\]
(34)
satisfying \( [C_x, C_y] = iC_z \). By using equation (33), operator \( C_z \) can be written as
\[
C_z = \frac{1}{2} [M_a M_b |(M_c + 1)(2k_a + M_a - 1) - (M_a + 1)(M_c + 1)(2k_a + M_a)] \\
\times (2k_b + M_b - 1)(2k_b + M_b) \\
[2k_c + M_c - 1]. 
\]
(35)
Another set of operators satisfying \([D_x, D_y] = i D_z\) are given by

\[
D_x = \frac{1}{2} (K_{a+} K_{b+} K_{c+} + K_{a-} K_{b-} K_{c-}),
\]

\[
D_y = \frac{1}{2i} (K_{a+} K_{b+} K_{c+} - K_{a-} K_{b-} K_{c-}),
\]

\[
D_z = \frac{1}{2} \left[ K_{a+} K_{b+} K_{c+} K_{c+} - K_{a-} K_{c-} K_{b-} K_{c-} \right].
\]

(36)

Operator \(D_z\) can be written in the form

\[
D_z = \frac{1}{2} \{M_a M_b M_c (2 k_a + M_a - 1) \times (2 k_b + M_b - 1)(2 k_c + M_c - 1) - (M_a + 1)(M_b + 1)(M_c + 1) \times (2 k_a + M_a)(2 k_b + M_b)(2 k_c + M_c) \}. \tag{37}
\]

From the definitions of the above operators, one finds

\[
D_x^2 = C_x, \quad D_y^2 = C_y, \quad D_z^2 = D_z, \quad C_x C_y = C_z, \quad (D_x^2)^{1/2} = C_x^2 + \frac{1}{4} F, \quad (D_y^2)^{1/2} = C_y^2 + \frac{1}{4} F.
\]

(38)

\[
\left[ (D_x, D_y) a \right] = \left[ C_x, C_y \right],
\]

where

\[
F = 2(M_a + k_a)(M_a + 1)(M_b + k_b) \times (2 k_a + M_a)(2 k_b + M_b) - M_a M_b (2 k_a + M_a - 1)(2 k_b + M_b - 1) \].
\]

(39)

Then, from equation (7), we obtain

\[
\left( \langle \Delta C_x \rangle^2 + \frac{1}{4} \langle F \rangle \right) \left( \langle \Delta C_y \rangle^2 + \frac{1}{4} \langle F \rangle \right) \geq \frac{1}{4} \langle (D_x^2) \rangle^2 + \text{Cov}(C_x, C_y)^2.
\]

(40)

It is known that su(2) and su(1,1) algebras connect with the Heisenberg–Weyl algebra, and thus we expect that the inequalities for SU(2) and SU(1,1) systems also relate to the corresponding inequality for bosonic systems.

3.3. Reduction from SU(2) and SU(1,1) to Bosons

We use the usual Holstein–Primakoff realization of the su(2) algebra [24]:

\[
J_+ = a^\dagger \sqrt{2j} - a^\dagger a, \quad J_- = \sqrt{2j} - a^\dagger a, \quad J_z = a^\dagger a - j.
\]

In the limit of \( j \to \infty \), we have

\[
\frac{J_+}{\sqrt{2j}} \to a^\dagger, \quad \frac{J_-}{\sqrt{2j}} \to a, \quad -\frac{J_z}{j} \to 1,
\]

by expanding the square root and neglecting the terms of \( O(1/j) \). The Holstein–Primakoff transformation [24] representation for the su(1,1) algebra is given by

\[
K_+ = a^\dagger \sqrt{2k} + a^\dagger a, \quad K_- = \sqrt{2k} + a^\dagger a a, \quad K_z = a^\dagger a + k.
\]

In the limit of \( k \to \infty \), we have

\[
\frac{K_+}{\sqrt{2k}} \to a^\dagger, \quad \frac{K_-}{\sqrt{2k}} \to a, \quad \frac{K_z}{k} \to 1,
\]

by expanding the square root and neglecting the terms of \( O(1/k) \). We see that both the su(2) and su(1,1) algebras reduce to the Heisenberg–Weyl algebra in the large \( j \) or \( k \) limit.

Multiplying (29) with \( 1/(2 j_1^2 j_2^2 j_3^2) \), and letting \( j_1, j_2, j_3 \to \infty \), we can see that

\[
\langle \Delta A_x \rangle^2 \to \langle \Delta L_x \rangle^2, \quad \langle \Delta A_y \rangle^2 \to \langle \Delta L_y \rangle^2, \quad \text{Cov}(A_x, A_y) \to \text{Cov}(L_x, L_y).
\]

(41)

From equation (28), in this limit, we find that the operator \( E \to N_a + N_b + 1 \). Thus, inequality (29) for the SU(2) system reduces to inequality (17) for the bosonic system. Similarly, in the limit of \( k_1, k_2, k_3 \to \infty \), inequality (40) for the SU(1,1) system reduces to inequality (17).

4. Generalization to multipartite systems

The methods employed above for tripartite states can be extended to \( n \)-partite states. For the sake of illustration, we consider \( n \) modes whose annihilation operators are given by \( a_1, a_2, \ldots, a_n \), and study the entanglement between the \( n \)th mode and the rest. We have two set of operators:

\[
L_x = \frac{1}{2} \left[ a_1 a_2^\dagger \cdots a_{n-1}^\dagger a_n + a_1 a_2 \cdots a_{n-1} a_n \right],
\]

\[
L_y = \frac{1}{2i} \left[ a_1 a_2^\dagger \cdots a_{n-1}^\dagger a_n - a_1 a_2 \cdots a_{n-1} a_n \right],
\]

\[
L_z = \frac{1}{2} \left[ (N_{a_1} + 1) \prod_{i=1}^{n-1} N_i - N_{a_1} \prod_{i=1}^{n-1} (N_i + 1) \right],
\]

and

\[
H_x = \frac{1}{2} \left[ a_1 a_2^\dagger \cdots a_n^\dagger a_1 a_2 \cdots a_n \right],
\]

\[
H_y = \frac{1}{2i} \left[ a_1 a_2^\dagger \cdots a_n^\dagger a_1 a_2 \cdots a_n \right],
\]

\[
H_z = \frac{1}{2} \left[ \prod_{i=1}^{n-1} N_i - \prod_{i=1}^{n-1} (N_i + 1) \right],
\]

(42)

(43)

(44)

(45)

satisfying \([L_x, L_y] = i L_z, [H_x, H_y] = i H_z\).

From the definitions of the above operators, one finds

\[
(\hat{H}_x^2)^{1/2} = L_x, \quad (\hat{H}_y^2)^{1/2} = L_y, \quad (\hat{H}_z^2)^{1/2} = L_z,
\]

(46)

\[
(\hat{H}_x^2)^{1/2} = L_x^2 + \frac{1}{4} \left( \prod_{i=1}^{n-1} (N_i + 1) - \prod_{i=1}^{n-1} N_i \right),
\]

\[
(\hat{H}_y^2)^{1/2} = L_y^2 + \frac{1}{4} \left( \prod_{i=1}^{n-1} (N_i + 1) - \prod_{i=1}^{n-1} N_i \right),
\]

(47)

\[
[H_x, H_y] = [L_x, L_y] + [L^2, L^2] + \text{Cov}(L_x, L_y).
\]

Then, from equation (7), we obtain

\[
\left[ \langle \Delta L_x \rangle^2 + \frac{1}{4} \left( \prod_{i=1}^{n-1} (N_i + 1) - \prod_{i=1}^{n-1} N_i \right) \right] \times \left[ \langle \Delta L_y \rangle^2 + \frac{1}{4} \left( \prod_{i=1}^{n-1} (N_i + 1) - \prod_{i=1}^{n-1} N_i \right) \right] \geq \frac{1}{16} \left[ \prod_{i=1}^{n-1} (N_i + 1) - \prod_{i=1}^{n-1} N_i \right]^2 + \text{Cov}(L_x, L_y)^2.
\]

This inequality is applicable to studies of entanglement properties between the \( n \)th mode and the rest. It is straightforward to obtain relevant inequalities for entanglement between a finite selected mode and the rest.
5. Conclusions

In summary, we have presented a family of entanglement criteria which are able to detect entanglement in tripartite systems. The method is based on the indeterminacy relations in conjugation with the partial transposition. To detect entanglement, one needs to appropriately define two sets of operators and write out the indeterminacy relation in terms of the variances, covariances and expectation values. Then, after partial transposition on operators other than states, we can obtain the entanglement criteria. One merit of this method is that it is efficient to get useful strong entanglement criteria.

We have considered three typical systems: bosonic, SU(2) and SU(1, 1) systems. We also discussed the reduction from SU(2) and SU(1, 1) to bosonic systems and the generalization to a multipartite case. We highlight the importance of uncertainty relations and the indeterminacy relations. They are not only important in the understanding of fundamental problems such as measurement problem in quantum mechanics, but also provide a convenient way to detect entanglement together with the partial transposition. We hope that this work will stimulate more discussions on applications of the indeterminacy relations in entanglement detection problems.

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