Schizophrenic Neutrinos and $\nu$-less Double Beta Decay

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We point out a new possibility for neutrinos where all neutrino flavors can be part Dirac and part Majorana. Our primary motivation comes from an attempt to use supersymmetric see-saw models to tie inflation, baryon asymmetry of the Universe and dark matter to the neutrino sector. The idea however could stand on its own, with or without supersymmetry. We present a realization of this possibility within an $S_3$ family symmetry for neutrino masses, where we obtain tri-bi-maximal mixing for neutrinos to the leading order. The model predicts that for the case of inverted hierarchy, the lower limit on the neutrino mass measured in neutrinoless double beta decay experiments is about a factor of two larger than the usual Majorana case.

I. INTRODUCTION

Experiments over the past two decades have conclusively established that neutrinos have mass. The true nature of the neutrino mass however is unknown since available observations based on flavor oscillations do not tell us whether it is its own anti-particle (Majorana type) or not (Dirac type). Unlike the quarks and charged leptons, both these possibilities are allowed for the neutrinos since they are electrically neutral. Numerous neutrinoless double beta decay experiments are under way or in preparation to settle this question.

An intermediate possibility that has been discussed in literature is known as the pseudo-Dirac case\textsuperscript{1} where one includes a very tiny amount of the Majorana mass for each neutrino flavor which has Dirac type mass. The Majorana entry in this case must be very tiny ($\leq 10^{-10}$ eV) in order to be consistent with current solar neutrino observations\textsuperscript{2}. In this note we point out a new class of possibilities where each neutrino flavor can have a large admixture of both Dirac and Majorana masses under certain circumstances. We point out the experimental implications of this possibility as well as its possible theoretical origin.

While discussing the Dirac versus Majorana nature of neutrinos, it is usual to frame the discussion in terms of the neutrino flavor eigenstates that are emitted in beta decay or other weak interaction processes. When the neutrinos travel in free space, however, they do so as mass eigenstates, which are linear superpositions of the flavor eigenstates. This fact is responsible for neutrino oscillation phenomena. In this note we point out that the possibility of one of the neutrino mass eigenstates having a Dirac mass at the tree level with the others having Majorana type mass, appears consistent with all current observations. Since in this case, each flavor eigenstate is a large admixture of both Dirac and Majorana masses, we call this “schizophrenic neutrino” alternative. This is different from the usual pseudo-Dirac cases discussed in literature where the Majorana admixture is tiny compared to the Dirac mass whereas in our case Majorana admixture is as large or larger than the Dirac mass for each flavor. This possibility implies distinct predictions for neutrinoless double beta decay searches compared to the case where the neutrinos are pure Majorana type and could be used to test the schizophrenic hypothesis.

On the theoretical side, the mass eigenstate having the Dirac mass must have a Dirac Yukawa coupling which is extremely tiny ($\sim 10^{-12}$) whereas the other masses could arise from high mass scale physics as in seesaw models\textsuperscript{3} with much larger Yukawa couplings. The Dirac type mass eigenstate would pair up with a right-handed (RH or sterile) neutrino ($\nu_s$) to form the Dirac mass. A priori, we do not know which of the three mass eigenstates has the Dirac nature. In this paper, we consider a specific model where we want to get tri-bi-maximal pattern\textsuperscript{5} for the PMNS matrix. This suggests that the eigenstates be representations of an $S_3$ symmetry group. The model then picks the “middle” eigenstate $\nu_2$ (the one that determines solar neutrino oscillations) as Dirac type since this is an $S_3$ singlet whereas the other two are Majorana. In a generic model, any or in fact any two of the mass eigenstates could have Dirac type mass.

While one would like to understand the small Dirac Yukawa coupling as a consequence of some high scale theory, it is comforting to know that it stable under radiative corrections due to chiral symmetry (or in this case under the symmetry $\nu_s \rightarrow -\nu_s$). There may be other motivations for the existence of such tiny Yukawa couplings. One such motivation in supersymmetric versions of such models comes from attempts to use the RH sneutrino to drive inflation\textsuperscript{4}. In such a scenario, small Dirac coupling is essential to make the inflation predictions for density fluctuations.
consistent with observations. However the hypothesis of schizophrenic neutrinos could be considered independently of this. As indicated earlier, a testable prediction of this model is that in the case of inverted hierarchy, the lower bound on $m_{\beta\beta}$ measured in neutrinoless double beta decay searches is roughly twice that of usual inverted hierarchy models in literature. This model will therefore be easier to rule out by the current generation of $\beta\beta$ experiments if long base line oscillation searches indicate inverted neutrino mass ordering.

We hasten to point out that this kind of pattern for neutrino masses is not protected by a symmetry. As a result, when loop corrections are taken into account, tiny corrections of order $\frac{g^2 m^2_\nu}{32\sqrt{6}\pi^2 M_W^2} \sim 4 \times 10^{-7}$ appear giving the Dirac eigenstate a pseudo-Dirac mass splitting of order $10^{-14}$ eV. These corrections have no impact on our prediction for $\beta\beta_{0\nu}$ decay.

II. MOTIVATION FROM COSMOLOGY

In this section, we review the cosmological motivation for small neutrino Dirac coupling in a supersymmetric seesaw model. We consider a supersymmetric extension of MSSM based on the gauge group $SU(2)_L \times U(1)_{B-L}$ which requires that there be three RH neutrinos $N$ ($\equiv \nu^c$ in SUSY language) to cancel anomalies (with the eventual possibility of $SO(10)$ grand unification). A combination of superpartners $\phi \equiv (\tilde{N} + H_u + \tilde{L}/\sqrt{3} )$ in this theory is a $D$-flat direction under the whole gauge symmetry, and is also $F$-flat when neutrino Yukawa couplings $h$ are turned off. As shown in [4], this flat direction can act as the inflaton. The neutrino Yukawa couplings in combination with the soft mass and $A$-term for $\phi$ leads to a potential for the radial component of $\phi$ (denoted as $\varphi$) of the form [4]

$$V(\varphi) = \frac{m^2}{2} \varphi^2 + \frac{h^2}{12} \varphi^4 - \frac{Ah}{6\sqrt{3}} \varphi^3 \quad (1)$$

where $m^2 = \left(m^2_{\tilde{N}} + m^2_{H_u} + m^2_{\tilde{L}}\right)/3$ can lead to inflection point inflation [4] and the amplitude of observationally relevant density perturbations (as measured by COBE and WMAP) matches the observed value $\delta_H \sim 1.9 \times 10^{-5}$ for weak scale masses $m_{\phi} \sim O(100)$ GeV, provided that $h \sim 10^{-12}$ (for details, see [4]). The neutrino mass is intimately connected to $h$. For instance, if neutrinos are Dirac fermions, we have $m_\nu = h \langle H_u \rangle$, where $\langle H_u \rangle = (174$ GeV $) \sin \beta$ and $\tan \beta$ is the ratio of vacuum expectation value (VEV) of Higgs fields of the minimal supersymmetric standard model (MSSM). Then $h \sim 10^{-12}$ would give rise to $m_\nu \sim O(0.1)$ eV, which is precisely in the range of interest for neutrino oscillations. We could take this as a hidden message from cosmology that at least one of the neutrinos can be dominantly of Dirac nature, and study its implications for neutrino masses and mixings.

It is important to emphasize that the inflation model constrains the coupling of only one of the RH neutrinos whose superpartner is responsible for inflation. That RH neutrino could be the Dirac partner of one of the light neutrino combinations making it a Dirac neutrino. The other two RH neutrinos have unconstrained Yukawa couplings that take natural values ($\sim 10^{-5} - 0.1$), and hence their mass must be heavy. Note however that the heavy RH neutrinos must not mix with the RH neutrino whose superpartner is part of the inflaton so as not to spoil the picture of inflation mentioned above. The simplest possibility for neutrino masses in this case would therefore appear to be that one linear combination of the flavor eigenstates is a Dirac fermion whereas the other two will be Majorana and get their mass via the see-saw mechanism. Below we suggest this as new picture for neutrino masses.

III. AN $S_3$ MODEL FOR SCHIZOPHRENIC NEUTRINOS

One of the challenges in neutrino mass physics is to understand the observed near tri-bi-maximal mixing pattern among different flavors. Discrete symmetries have been discussed extensively as a way to address this issue [7] and the group $S_3$ is one of the symmetries that appears promising in this context and we use it in our discussion in this paper. The basic assumptions of our neutrino model can therefore be summarized as follows:

- The extended gauge group responsible for neutrino masses consists of a local $B - L$ symmetry [7], which requires the existence of three RH neutrinos for anomaly cancellation.
- One of the RH neutrinos (whose superpartner is the inflaton field) couples to a linear combination of all neutrino flavors with a Yukawa coupling of order $10^{-12}$ so that it gets a Dirac mass without any need for see-saw, whereas the remaining orthogonal combinations get their masses from the conventional see-saw mechanism.
- The three standard model lepton doublets transform into one another under a flavor $S_3$ discrete symmetry.
The first assumption is quite well motivated and has been widely discussed in literature. It also naturally incorporates $\bar{N}$ along with $H_u$ and $\bar{L}$ into a single $D$-flat direction that can drive inflection point inflation. The second assumption is motivated by the cosmological scenario discussed above.

As already mentioned, our neutrino model is based on the idea that only one of the neutrino flavor combinations corresponding to a mass eigenstate has a small Yukawa coupling to one RH neutrino whereas the other two combinations get their masses from the seesaw mechanism. If we take the tri-bi-maximal matrix as the leading order PMNS matrix, then one might start thinking of a discrete symmetry group which has one singlet and one doublet as part of its irreducible representations and the singlet one being the Dirac neutrino whereas the doublet combinations becoming Majorana. One such example used in literature is the $S_3$ group which proves convenient for our discussion.

We assume the $S_3$ to permute the three families of leptons $(L_e, L_\mu, L_\tau)$ among themselves. Of course, it is well known that this is a reducible representation of $S_3$ group but the following linear combinations of these fields transform as singlet and two dimensional representations of $S_3$:

$$L_2 = \frac{1}{\sqrt{3}}(L_e + L_\mu + L_\tau) \quad \text{(Singlet)}$$
$$L_1 = \frac{1}{\sqrt{6}}(2L_e - L_\mu - L_\tau) \quad \text{(Doublet)}$$
$$L_3 = \frac{1}{\sqrt{2}}(L_\mu - L_\tau) \quad \text{(Doublet)}. \quad (2)$$

We assume that muon type RH neutrino is the $S_3$ singlet whereas $(N_e, N_\tau)$ form a doublet. The masses of these doublet RH neutrinos can be chosen different by appropriate choice of symmetry breaking (see below). The effective lepton Yukawa coupling after integrating out $N_e$ and $N_\tau$ can then be written as

$$\mathcal{L}_\nu = h L_2 H_u N_\mu + \frac{h_2^2}{M_{N_e}} (L_1 H_u)^2 + \frac{h_3^2}{M_{N_\tau}} (L_3 H_u)^2 + \text{h.c.} \quad (3)$$

After the electroweak symmetry breaking, this gives rise to one Dirac neutrino corresponding to the mass eigenstate $\nu_2$ and two Majorana eigenstates $\nu_1, \nu_3$ and clearly leads to tri-bi-maximal form for the PMNS matrix provided the charged lepton mass matrix is diagonal.

The effective Lagrangian in (3) could for instance arise in an $SU(2)_L \times U(1)_{B-L}$ theory supplemented by a global discrete symmetry $S_3 \times D$, (where $D$ is a product of extra $Z_n$ symmetries) if we take an $S_3$ doublet Higgs fields $(\Delta^1, \Delta^2)$ and a singlet field $\Delta^3$ (all with $B - L$ charge $-2$ and $I_{3R}$ charge $+1$) with VEVs $\langle \Delta^1 \rangle = 0$ and others with non-zero VEV. This will generate different Majorana masses $M_{N_e}$ and $M_{N_\tau}$ for the $S_3$ doublet RH neutrinos.

In this model, inflation occurs along the flat direction corresponding to the first superpotential term in Eq. (3). The coupling between $N_\mu$ and $(N_e, N_\tau)$ can be forbidden by e.g., a $Z_8$ symmetry contained in $D$ under which $N_\mu \rightarrow -i N_\mu$ and a gauge singlet field $X$ with $X \rightarrow e^{i\pi/4}X$ with all other fields invariant. The Dirac coupling of $N_\mu$ can be obtained from a higher dimensional coupling $(\lambda L_2 H_u N_\mu X^2/M_{P}^2)$, where $\langle X \rangle \sim 10^{12} \text{GeV or so. At the inflection point VEV}$ ($\sim 10^{12} \text{GeV}$ []), this interaction then yields the effective Dirac coupling of $N_\mu$ in Eq. (3). An additional RH neutrino mixing term $(N_\mu N_e \tau \Delta^1 H_u X^2/M_{P}^2)$ is allowed by the $Z_8$ symmetry, but has negligible contribution to masses and mixings and can be ignored. We need to add the fields $(\Sigma_1, \Sigma_2)$ and a singlet field $\Sigma_3$ to preserve supersymmetry below the $B - L$ scale as well as to cancel anomalies.

Turning to the charged lepton mass matrix, neutrino oscillation data require that it be nearly diagonal. We employ the technique used in the second reference in [5]. We add three gauge singlet superfields $(\sigma_e, \sigma_\mu, \sigma_\tau)$ and three extra $Z_n$ symmetries, i.e. $Z_{n,e} \times Z_{n,\mu} \times Z_{n,\tau}$, with RH lepton fields $e^c, \mu^c, \tau^c$ transforming as $\omega^e_{n,\mu,\tau}$ and singlet fields transforming as $\omega_{n,\mu,\tau}$. Both sets of three fields also transform under $S_3$ exactly like the lepton doublet fields. We can write down the corresponding Yukawa superpotential as

$$W_{l,Y} = \frac{1}{M} h_{e} H_d (L_e \sigma_e e^c + L_\mu \sigma_\mu \mu^c + L_\tau \sigma_\tau \tau^c). \quad (4)$$

There can be another term where the $L_e, L_\mu, L_\tau$ are permuted among themselves. This will contribute to the off-diagonal elements of the charged lepton mass matrix after symmetry breaking. We set this coupling to zero. Now by adjusting the VEVs of the singlet fields, we can get diagonal mass matrix for the charged leptons. On the other hand, if the small contributions to the mass matrix coming from the permuted terms are kept, there will corrections to the tri-bi-maximal form e.g. it will lead to non-zero $\theta_{13}$.
This neutrino mass model has an interesting implication for neutrinoless double beta decay. Recall that in the conventional all Majorana neutrino case, the light neutrino contribution to $\beta\beta_0\nu$ decay takes the form $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$, ($i = 1, 2, 3$), where $U_{ei}$ are entries of the PMNS matrix. In the inverted hierarchy scenario, this leads to the following lower bound for the conventional three Majorana neutrino case \[ |m_{\beta\beta}| \simeq |(\cos^2 \theta_\odot + e^{i\alpha} \sin^2 \theta_\odot) m_{\text{atm}}| \geq \frac{m_{\text{atm}}}{3} \approx 17 \text{ meV} \] (Conventional). \[(5)\]

In our model, however, the second neutrino mass eigenstate is a Dirac type state and therefore has no contribution to $\beta\beta_0\nu$ decay. This leads to the following lower bound for inverted case: \[ |m_{\beta\beta}| \simeq \cos^2 \theta_\odot m_{\text{atm}} \geq \frac{2m_{\text{atm}}}{3} \approx 34 \text{ meV} \] (Dual), \[(6)\]

which is roughly twice the value of the conventional case \[\text{(5)}.\] This makes it easier to rule out our model in the current generation of neutrinoless double beta decay searches, provided we have independent evidence, e.g. from long baseline neutrino experiments for inverted mass hierarchy.

In the normal hierarchy scenario, the corresponding formula becomes $m_{\beta\beta} \simeq (m_1 \cos^2 \theta_\odot + e^{i\alpha'} \sin^2 \theta_{13} m_{\text{atm}})$, which is different from the conventional three Majorana case. The precise value in this case depends on the unknown Majorana mass of $\nu_1$ as well as the value of $\theta_{13}$.

In Fig. 1 we plot $|m_{\beta\beta}|$ as a function of the lightest neutrino mass $m_{min}(m_j) = m_{min}$ (which sets the absolute neutrino mass scale in the case of inverted hierarchy). The red (dark) band shows the prediction of our scenario and the gray shaded region shows the usual three Majorana neutrino scenario in the case of inverted hierarchy. The masses and mixing angles used for the figure are as follows \[\text{[10]}: \] $\Delta m^2_\odot = (7.59 \pm 0.20^{+0.14}_{-0.09}) \times 10^{-5} \text{eV}^2$, $\Delta m^2_{\text{atm}} = (2.46 \pm 0.12 \pm 0.07) \times 10^{-3} \text{eV}^2$, $\theta_{13} < 12.5^\circ$, $\theta_\odot = 34.4^\circ \pm 1^{+3.2}_{-2.9}$ and $\theta_{\text{atm}} = 42.8^\circ ^{+4.7}_{-7.3}$. We can see that in the case of inverted hierarchy (corresponding to $m_{\text{min}} < 0.05$ eV) the lower limit on $|m_{\beta\beta}|$ measured in neutrinoless double beta decay experiments is about a factor of two larger than the conventional case.

V. COMMENTS

We now make some comments on the model described here.

(i) Since the Dirac nature of the second neutrino mass eigenstate is not protected by any symmetry, radiative corrections will induce Majorana component to its mass. The self-energy corrections to $\nu_1$ masses due to $W^+\ell^-$
intermediate states will lead to kinetic mixings between the different mass eigenstates that depends on the charged lepton masses: \( \epsilon_{ij} \sim (\sum_k U_{ik} U_{jk} g^2 m_k^2 / 32 \pi^2 M_W^2) \). This mixing is of order \( 10^{-7} \). When the kinetic energy term in the Lagrangian is diagonalized, this leads to mixing terms (for the normal hierarchy case), e.g. \( m_4 \nu_3 \nu_2 + \ldots \), where \( m_4 \sim m_{\nu_3} f_{23} + \ldots \). This introduces a Majorana mass term \( \delta m_{23} \nu_2 \nu_2 \) with \( \delta m_2 \sim 10^{-14} \text{eV} \). It, being very small, keeps the Dirac nature of \( \nu_2 \) to very high precision. This is also consistent with a bound \( 10^{-9} \text{eV} \) on this parameter from solar neutrino observations \( [2] \). The same result holds for the inverted hierarchy case with \( \nu_1 \) and \( \nu_3 \) interchanged. In the SUSY version, quantum corrections that mix the slepton states introduce a Majorana component for the Dirac neutrino. At one loop this effect results in \( \delta m_{ij}^2 \sim \left[ (Y_{l}^T Y_{l})_{ij} / 16\pi^2 \right] m_0^2 \ln (M^2 / M_Z^2) \), which is of the same order as that mentioned before.

(ii) We wish to emphasize that our scenario is different from the usual pseudo-Dirac scenario \([1]\) discussed in the literature. Our light neutrino mass matrix is a \( 4 \times 4 \) matrix such that one of its eigenstates forms a Dirac pair with the sterile neutrino and the other two eigenstates are Majorana. The Dirac eigenstate gives rise to a pseudo-Dirac pair only at the one-loop level.

(iii) The masses of the two heavy RH neutrinos depend on the scale at which \( B - L \) is broken, and can be as low as \( O(1) \, \text{TeV} \). The decay of heavy Majorana neutrinos, and their SUSY partners, can generate baryon asymmetry of the Universe. If \( M_{N_2}, M_{N_1} \) are of order TeV, resonant leptogenesis will be a relevant solution. However, in the \( S_3 \) symmetric model, this does not work since it will require the first and the third neutrino masses be almost equal. The oscillation data will be hard to fit with this pattern. However, soft leptogenesis \([11]\) can work well in the model for a wide range of Majorana masses.

(iv) In this model, either the MSSM neutralino or the superpartner of the RH component of the Dirac neutrino can play the role of dark matter. The latter is naturally the lightest of the RH sneutrinos since its mass receives contribution from SUSY breaking alone. If the \( B - L \) is broken around TeV, it can obtain the correct relic density via thermal freeze out \([4]\). This also makes the corresponding \( Z' \) accessible at the LHC. On the other hand, for a high scale \( B - L \) the usual MSSM neutralino is a good dark matter candidate. The role of \( B-L \) in this case is to provide the \( R \)-parity symmetry naturally.

(v) The impact of the RH neutrino, which is responsible for the Dirac mass, on Big Bang Nucleosynthesis also depends on the scale at which \( B - L \) is broken. For example, for \( M_{Z'} \sim 10 \, \text{TeV} \), the RH neutrinos decouple at \( T_D \sim 1 \, \text{GeV} \), while for \( M_{Z'} \sim 1 \, \text{TeV} \) we have \( T_D \sim 100 \, \text{MeV} \). In the latter case this amounts to \( N_{\nu}^{\text{eff}} \simeq 4 \), whereas \( N_{\nu}^{\text{eff}} \simeq 3.1 \) in the former case. For a high scale \( B - L \) the RH neutrinos decouple much earlier, and hence \( N_{\nu}^{\text{eff}} \simeq 3 \).

In summary, motivated by cosmology, we have pointed out a new picture for neutrino masses with the novel feature that one of the mass eigenstates is a Dirac fermion (at the tree-level) whereas the other two are Majorana type. We presented an \( S_3 \) realization of this idea that leads to tri-bi-maximal mixing for leptons in the leading order. This model can be ruled out by the current generation of neutrinoless double beta decay searches because inverted mass hierarchy cannot make the production of long base line neutrino oscillation experiments and neutrinoless double beta decay searches give \( |m_{3\beta\beta}| \lesssim 34 \, \text{meV} \).

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