Interaction-induced topological superconductivity in antiferromagnet-superconductor junctions

Senna S. Luntama,1 Päivi Törmä,1 and Jose L. Lado1,∗
1Department of Applied Physics, Aalto University, 00076 Aalto, Espoo, Finland
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We predict that junctions between an antiferromagnetic insulator and a superconductor provide a robust platform to create a one-dimensional topological superconducting state. Its emergence does not require the presence of intrinsic spin-orbit coupling nor non-collinear magnetism, but arises solely from repulsive electronic interactions on interfacial solitonic states. We demonstrate that a topological superconducting state is generated by repulsive interactions at arbitrarily small coupling strength, and that the size of the topological gap rapidly saturates to the one of the parent trivial superconductor. Our results put forward antiferromagnetic insulators as a new platform for interaction-driven topological superconductivity.

The search for topological superconductors has been one of the most active areas in condensed matter physics in recent years [1–15]. These systems, pursued in particular for the emergence of Majorana zero modes, represent one of the potential solid state platform for the implementation of topological quantum computing [16, 17]. Due to their elusive nature, topological superconductors are often artificially engineered. A variety of platforms have been proposed and demonstrated for this purpose [13, 18, 19], generically relying on a combination of conventional s-wave superconductivity, ferromagnetism and strong spin-orbit coupling [7–11, 20, 21].

While ferromagnets have played a central role for artificial topological superconductivity, antiferromagnetic insulators have been overlooked for this purpose. Recently, antiferromagnets have attracted a great amount of attention due to their unique properties for spintronics [22–26] and for creating novel types of topological matter [27–32]. Interestingly, whereas ferromagnetism is generically incompatible with superconductivity, antiferromagnetism does not have to give rise to such competition [33–37]. This suggests that topological gaps in antiferromagnetically-based topological superconductors can be more robust than their ferromagnetic counterparts.

Here we show that two-dimensional topologically trivial antiferromagnetic insulators provide a platform to design one-dimensional topological superconductivity. In our proposal, spin-orbit coupling effects are not necessary for topological superconductivity to appear, nor a fine-tuning between the different components of the system. In contrast, we show that long-range interactions alone give rise to a non-trivially gapped state hosting Majorana excitations, and that the interaction-induced gap opening is topological irrespective of details. We demonstrate that the robustness of this unique state stems from the solitonic nature of the emergent excitations at the interface, in which interaction-induced gap opening unavoidably gives rise to a topological superconducting state. Our results put forward antiferromagnet-superconductor junctions as a robust platform to engineer interaction-induced topological superconductivity.

![Figure 1](https://example.com/figure1.png)

FIG. 1. (a) A sketch of the two dimensional antiferromagnet (AF) and superconductor (SC) forming a one-dimensional AF-SC interface. The spectral function at the surface of the AF (b), at the surface of the SC (c) and at the interface between AF and SC (d) as given by our model Hamiltonian (1) in a honeycomb lattice. Panel (e) shows the spatial distribution of the interfacial modes. Here we chose $\Delta = 0.3t$, $m_{AF} = 0.5t$, $\mu = t$ and $V_1 = V_2 = 0$. 
Our system consists of a junction between a conventional s-wave superconductor and antiferromagnetic insulator, as shown in Fig. 1a. To model this system, we take a Hamiltonian in the honeycomb lattice of the form

\[ \mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{AF}} + \mathcal{H}_{\text{SC}} + \mathcal{H}_{\text{int}} \]  

where

\[ \mathcal{H}_{\text{kin}} = t \sum_{\langle ij \rangle, s} c_{i,s}^\dagger c_{j,s} + \sum_{i,s} \mu(r_i)c_{i,s}^\dagger c_{i,s} \]  

\[ \mathcal{H}_{\text{AF}} = \sum_{i,s} m_{\text{AF}}(r_i)\tau_{s,i}^x \sigma_{i,s}^z c_{i,s}^\dagger c_{i,s} \]  

\[ \mathcal{H}_{\text{SC}} = \sum_i \Delta(r_i)c_{i,\uparrow}c_{i,\downarrow} + \text{H.c.} \]  

\[ \mathcal{H}_{\text{int}} = V_1 \sum_{\langle ij \rangle} \left( \sum_s c_{i,s}^\dagger c_{i,s} \right) \left( \sum_s c_{j,s}^\dagger c_{j,s} \right) + V_2 \sum_{\langle i,j \rangle} \left( \sum_s c_{i,s}^\dagger c_{j,s} \right) \left( \sum_s c_{j,s}^\dagger c_{i,s} \right) \]  

where \( c_{i,s}^\dagger \) is the fermionic creation operator for site \( i \) and for the sublattice Pauli matrix, \( \tau^z \) the self-consistent mean-field parameters. [39] On-site interactions are incorporated in \( m_{\text{AF}}(r) \) and \( \Delta(r) \) at the mean-field level.

It is instructive to examine the electronic bandstructure in the presence of interactions and in the absence of an interface. For this sake, let us consider a semi-infinite slab in the \( y \)-direction, having translational symmetry in the \( x \)-direction as depicted in Fig. 1a. For that geometry, we compute the momentum-resolved spectral function at the edge \( A(k_y, \omega) = -\frac{1}{2} \text{Im} \left( \omega - \mathcal{H}(k_y) + i0^+ \right)^{-1} \) using the Dyson formalism. [40] For both isolated superconductor and antiferromagnet, the surface spectral function presents a gap, as shown in Fig. 1b, that simply stems from the gapped topologically trivial band structure. In the case of the superconductor the gap is controlled by \( \Delta \), whereas in the antiferromagnet, the gap is determined by \( m_{\text{AF}} \). In stark contrast, when the antiferromagnet and superconductor are joined together, a new branch of interfacial modes appear as shown in Fig. 1d. By computing the spectral function in real space at zero energy \( A(r, \omega = 0) \) it is clearly seen that the new branch is heavily localized at the junction between the superconductor and the antiferromagnet. We have verified, that for different values of the superconducting and antiferromagnet order parameters, zero modes emerge as long as the order parameters are not substantially bigger than the typical bandwidth. This robustness can also be rationalized analytically as explained below.

The emergence of the interfacial zero modes can be rationalized from a low energy model. [41–45] For the following analytic derivation, it is convenient to take \( \mu = 0 \) so that the full antiferromagnet-superconductor can be described with a generalized Dirac equation at the \( K \) point of the honeycomb lattice. The low energy excitations can be captured by an effective model around the valleys \( \kappa^2 = \pm 1 \), and we will focus first on taking the momentum parallel to the interface \( p_x = 0 \). By defining the Nambu spinor \( \Psi = (c_{A,\uparrow}, k^*, c_{B,\uparrow}, k^*, c_{A,\downarrow}, -k, c_{B,\downarrow}, -k) \), the Hamiltonian in the electron-up/hole-down sector (\( \uparrow \)) can be written as \( \mathcal{H}(p_x = 0, p_y) = \frac{1}{2} \Psi^\dagger \mathcal{H}_\kappa \Psi \) with

\[ \mathcal{H}_\kappa = \begin{pmatrix} m_{\text{AF}}(r) & -p_x & \Delta(r) & 0 \\ p_x & m_{\text{AF}}(r) & 0 & -\Delta(r) \\ \Delta(r) & 0 & -p_x & m_{\text{AF}}(r) \\ -\Delta(r) & \Delta(r) & 0 & -p_x & m_{\text{AF}}(r) \end{pmatrix} \]  

FIG. 2. (a) Non-interacting bands in a ribbon geometry. First neighbor interactions do not lead to a gap (b), whereas second neighbor interactions drive a gap opening (c). When both first and second neighbor interactions are present the gap remains. The parameters are \( V_1 = t \) in (b), \( V_2 = 1.7t \) in (c) \( V_1 = t \) \( V_2 = 2t \) in (d) and \( m_{\text{AF}} = 0.8t \Delta = 0.4t \) in (a-d).
The spectrum of this effective model is gapped at \( y = \pm \infty \), as expected from its asymptotic antiferromagnet/superconductor gap. However, a zero energy mode \( H|\psi_0\rangle = 0 \) at the interface can be always built, taking the functional form \( \psi_0^\dagger = e^{-\int_y^\infty [\Delta(y') - m_{AF}(y')] dy'} (c_{A,1}^\dagger + i c_{B,1}^\dagger - i c_{A,4} - i c_{B,4}) \). The nature of this zero mode is analogous to the Jackiw-Rebbi soliton[41], and therefore can be understood as an antiferromagnet-superconducting soliton.

The complementary electron-down/hole-up (↓) sector of the Hamiltonian will therefore also host a zero mode, that we label as \( \psi_\uparrow \). Away from the point \( p_x = 0 \), the previous state acquires a finite dispersion given by first order perturbation theory \( v_F p_x = (\psi_\uparrow|H|\psi_\uparrow) \). As a result, close to the \( K \)-points two branches of zero modes appear, giving rise to the effective low energy Hamiltonian

\[
H(p_x) = \sum_{\kappa} v_F p_x v^z_{\kappa,\kappa}[\psi_{\uparrow,\kappa,p_x} \psi_{\uparrow,\kappa,p_x} - \psi_{\downarrow,\kappa,p_x} \psi_{\downarrow,\kappa,p_x}]
\]

where \( \kappa \) runs over the two valleys. It is interesting to note that the four modes are not independent, but are related by electron-hole symmetry operator \( \Xi = \theta^\dagger_\kappa \theta^\dagger \) with \( \theta^\dagger \) the Nambu Pauli matrix and \( \mathcal{C} \) complex conjugation as \( \Xi^{-1} \psi_{\uparrow,+1,p_x} \Xi = \psi_{\downarrow,-1,-p_x} \) due to the built-in Nambu electron-hole symmetry of the Hamiltonian.

Therefore, the Hamiltonian Eq. 7 hosts only two physical degrees of freedom, each one propagating in opposite directions, realizing an effective spinless one-dimensional model. These singly-degenerate channels are analogous to quantum Hall edge states,[11] and helical channels in topological insulators,[7] states that provide a starting point for engineering a topological superconducting gap. Remarkably in our case, as will be shown below, the solitonic gapless channels will open up a topological superconducting gap once electron-electron interaction effects are included.

Let us now move on to consider the impact of long-range electronic interactions in the solitonic modes. For computational convenience, we now perform our calculations in ribbon of finite width in the \( x \)-direction, in which we take the transverse direction wide enough to avoid finite-size effects. The previous gapless interface modes of Fig. 1d and derived in Eq. 7 appear in this ribbon geometry as shown in Fig. 2a, where \( S_z = \frac{1}{2} \sum_{n,s} \sigma^z_n c^\dagger_{n,s} c_{n,s} \psi_k \) with \( \Psi_k \) the eigenstate. It is shown that in the absence of interactions, the sectors \( S_z = \pm 1/2 \) are fully decoupled, stemming from the \( U(1) \)-spin symmetry of the Hamiltonian. With this lattice model, we now explore the impact of electronic interactions by solving self-consistently Eq. 1. Note that the interactions apply both along the interface and across it. We start by considering only first neighbor interactions, taking \( V_2 = 0 \). In this situation, a gap does not open even when \( V_1 \) is increased, as shown in Fig. 2b. We now move on to the case of \( V_2 \), taking first \( V_1 = 0 \). As observed in Fig. 2c, it is clearly seen that now a gap opens up. This behavior also takes place when \( V_1 \) is taken to be non-zero, see Fig. 2d. As a result, second neighbor interactions are the only interaction capable of opening up a gap on the topological interface modes, whose magnitude is marginally affected by the first neighbor interactions.

The emergence of a gap opening driven by electronic interactions raises the question of potential non-trivial topological properties. From the point of view of the effective low energy model, interactions create an effective term in Eq. 7 of the form \( H^{\text{MF}} \sim (\Psi_{\uparrow} \Psi_{\downarrow}^\dagger) \Psi_{\uparrow} \Psi_{\downarrow} + \text{H.c.} \). It is interesting to note that due to the solitonic functional form of \( \Psi_{\uparrow} \) and \( \Psi_{\downarrow} \) and their relation via electron-hole symmetry, the gap \( (\Psi_{\uparrow} \Psi_{\downarrow}^\dagger) \) created is odd with respect to \( \kappa \), the valley index, suggesting the emergence of an effective topological superconducting state. To verify the non-trivial topological nature of the interaction-driven gapped state, we compute both its \( Z_2 \) topological invariant[1, 46] and surface spectral function. We revealed that the gapped system has a topologically non-trivial \( Z_2 \) invariant, signaling the existence of a topological superconducting state. This is further verified when computing the density of states at the edge of the interface in a ribbon that spans from \( x = 0 \) to \( x = \infty \), as shown in Fig. 3a. It is observed that the edge of the system hosts a zero-mode resonance associated with the unpaired Majorana stemming from the non-trivial elec-
tronic structure. This is contrasted with the finite gap present in the bulk of the system shown in Fig. 3b. The localization of the zero-mode can also be seen when computing the spectral function for \( \omega = 0 \), Fig. 3c.

Let us now move on to look at the impact of long-range interactions, and in particular, at the interplay between the first and second neighbor interactions at the mean field level. For the sake of simplicity in the following discussion we will only consider effects that appear by means of a mean field decoupling of Eq. 5, without considering beyond mean-field effects or additional \( t - J \) contributions. At the mean-field level, the interaction term of Eq. 5 can give rise to two potential effects: first, interaction induced hoppings and second, symmetry broken states such as charge density waves. In particular, in the weak coupling regime considered here, only interaction-induced hopping terms arise. In the particular case of graphene, it is known that interactions give rise to a Fermi renormalization effect, that is rationalized as an interaction-induced hopping. Nevertheless, interaction-induced hopping can be of spin-dependent nature, therefore artificial spin-orbit coupling effects can appear due to interactions. This is the case in our present system, as gaps in the solitonic interface modes can only appear due to interaction-induced spin-mixing effects.

The interplay of first and second neighbor interactions can be easily rationalized within this language. From the mean-field point of view, first neighbor interactions can give rise to interaction induced Rashba spin-orbit coupling terms,[47] whereas second neighbor interactions can give rise to interaction-induced Kane-Mele spin-orbit coupling.[48] However, due to valley polarized nature of the solitonic modes, interaction induced Rashba-spin-orbit coupling does not open up a gap in them,[47] whereas Kane-Mele like spin-orbit[48] can create a gap. As a result, second neighbor interactions are the only ones capable of interaction-induced gap opening in the system. In contrast, the effect of the first neighbor interactions is to simply create a Fermi velocity renormalization[49, 50] increasing the kinetic energy of the solitonic modes, yet without any competing mechanism for gap opening.

It is crucial to understand whether the gap opening requires a finite minimum value of interaction strength. We investigate this by taking the first neighbor interaction \( V_1 = 0 \), and looking at the topological gap as a function of the repulsive second neighbor interaction \( V_2 \). It is clearly observed that the topological gap becomes stronger as \( V_2 \) is increased, without the existence of a critical value for the transition (Fig. 4a). In particular, a logarithmic plot of the gap (inset of Fig. 4a) at small coupling strength reveals that the topological gap \( \delta \) follows an exponential dependence \( \delta \sim e^{-\frac{1}{2}V_2^2} \). Interestingly, whereas exponential dependences of that form are typical for superconducting instabilities driven by attractive interactions,[51] in our present case interactions are actually repulsive. This behavior stems from the projection of the interactions in the low energy solitonic model of Eq. 7, driving a topological phase transition at arbitrarily small couplings. At large coupling strengths \( V_2 \), the topological gap saturates to the gap of the superconductor. This behavior should be contrasted with the other schemes proposed for topological superconductivity, in which the topological gap is usually substantially smaller than the original superconductor gap. This saturation of the topological gap can be ascribed to the absence of competition between the superconductor and the antiferromagnet. Including finite first neighbor interactions \( V_1 \) keeps the picture qualitatively unchanged, yet with a slightly renormalized topological gap (Fig. 4b). The interplay between \( V_1 \) and \( V_2 \) shown in Fig. 4c shows that whereas \( V_2 \) opens the topological gap, \( V_1 \) leaves the system gapless or slightly renormalizes the topological gap.

Finally, we address the potential experimental realization of our proposal. For a solid-state realization, no specific requirements are necessary for the superconductor besides conventional s-wave pairing, as realized in NbSe2. The fundamental requirement is having a two-dimensional honeycomb antiferromagnetic insulator, as its electronic structure is expected to have the gapped Dirac points required for the emergence of the topological solitonic modes. Within van der Waals materials, trihalides host a magnetic honeycomb lattice,[52] and in particular antiferromagnetic strained
trihalides \[53, 54\] would be suitable for our proposal. Within oxides, thin films of $\text{InCu}_2\text{V}_3\text{O}_7$ \[55\] or $\beta\text{-Cu}_2\text{V}_2\text{O}_7$ \[56\] has the required antiferromagnetic honeycomb lattice. Generic two-dimensional antiferromagnetic insulators hosting Dirac points in their normal state \[57\] would be suitable materials for our proposal. Finally, future ultracold atom setups \[58\] are potential platforms for the realization of our model, as honeycomb structures \[59\], antiferromagnetic correlations \[60\], long-range interactions\[61–63\] and s-wave correlations \[64\] in the normal state have been separately demonstrated. Interactions can be tuned from attractive to repulsive by magnetic fields; spatially dependent fields could be one way of creating the AF-SC interface, once superfluid correlations in a lattice have been reached.

To summarize, we have shown that an interface between a topologically trivial two-dimensional superconductor and antiferromagnetic insulator gives rise to a one-dimensional solitonic gas. Upon introduction of repulsive long-range interactions, we have demonstrated that a topological gap gets generated, giving rise to Majorana zero energy modes. The emergence of topological superconductivity appears in the absence of intrinsic spin-orbit coupling and is driven by repulsive Coulomb interactions. We showed that the topological gap appears at arbitrarily small interactions, and rapidly saturates to the gap of the parent superconductor, in stark contrast with conventional proposals involving competition between ferromagnetism and superconductivity. Our results propose a new mechanism to generate topological superconductivity based on interacting solitons, putting forward antiferromagnetic insulators as a potential materials platform for Majorana physics.

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