Feedback from Pixels:
Output Regulation via Learning-based Scene View Synthesis

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Abstract
We propose a novel controller synthesis involving feedback from pixels, whereby the measurement is a high dimensional signal representing a pixelated image with Red-Green-Blue (RGB) values. The approach neither requires feature extraction, nor object detection, nor visual correspondence. The control policy does not involve the estimation of states or similar latent representations. Instead, tracking is achieved directly in image space, with a model of the reference signal embedded as required by the internal model principle. The reference signal is generated by a neural network with learning-based scene view synthesis capabilities. Our approach does not require an end-to-end learning of a pixel-to-action control policy. The approach is applied to a motion control problem, namely the longitudinal dynamics of a car-following problem. We show how this approach lend itself to a tractable stability analysis with associated bounds critical to establishing trustworthiness and interpretability of the closed-loop dynamics.

Keywords: Pixels, Feedback Control, View Synthesis, Visual Servoing, Car-Following, Stability.

1. Introduction
Our aim is to investigate the integration of visual signals into feedback loops for the purpose of controller synthesis and analysis, and without requiring a perception module in the loop. We treat the camera as a high-dimensional sensor and propose a principled approach grounded in mathematical control theory to investigate stability and associated theoretical limitations of the closed-loop performance.

In this paper, we consider output regulation class of problems where the output measurement includes a pixelated image. We feel the contribution of this paper is as follows:

- We treat each RGB pixel as a a measurement and do not attempt to grayscale or threshold the image and can handle an arbitrary image size or resolution.
- Compared to visual servoing approaches, our work does not involve hand-crafted geometrical feature extractions, correspondence or matching, pose estimation, or an interaction matrix.
• Unlike most existing approaches, we integrate vision into reactive low-level control without a need for a perception module, end-to-end imitation learning, the estimation of states or similar latent representations.
• Our approach works for moving targets and non-stationary environments.
• Embedded in our controller is an internal model of the tracked visual reference. This is achieved by incorporating a view synthesizer in the loop at inference or execution time.
• We show a systematic way to synthesize static output feedback controllers, such as a proportional controller, via necessary and sufficient conditions in the literature.
• Our approach does not require discretizing the action space or the state space, and works in continuous-time synthesis and analysis.
• Our work is amenable to stability analysis.
• In the car-following example, our approach maintains physically interpretable representations of the underlying dynamics, e.g. state-space variables from first principles.

In Section 1.1, we provide a context to our contribution by reviewing related work. Section 1.2 covers notational remarks. Section 2 introduces the problem statement concisely in the context of an application domain, while Section 3 presents the main result. In Section 4 we provide conclusions and future directions. Appendix B shows simulations using CARLA from Dosovitskiy et al. (2017).

1.1. Related Work

Several recent results for vision-in-the-loop control attempt to leverage learning-based approaches via end-to-end learning, mainly imitation learning, to essentially map pixels to actions via a static map as in Bojarski et al. (2016) and Amini et al. (2018) in the context of driving. Another body of work attempts to first get a latent representation of the underlying dynamics of the process from visual input as in Watter et al. (2015), Banijamali et al. (2018), Hafner et al. (2019) and structured latent representations as in Johnson et al. (2016). In Zhang et al. (2019), such latent representations are used in model-based reinforcement learning in the context of manipulation.

In Collewet and Marchand (2011), geometric feature extraction or matching was alleviated by using the luminance of all pixels in 2D direct visual servoing. However, such methods require computing explicitly an interaction matrix and solving a nonlinear optimization problem resulting in a small region of convergence. Therefore, in Saxena et al. (2017) and Bateux et al. (2018), the relative pose error is learned from a current and reference images for the purpose of posed-based visual servoing. While these methods alleviate the need for camera parameters and scene geometry, servoing is done towards a non-moving target.

In Amini et al. (2020), a data-driven simulator is used to train a policy via reinforcement learning from an initial stable policy provided by a human driver. The simulator generates perturbations along an initial policy by taking a 2D image captured along the initial trajectory, creating a depth map and a 3D point cloud, applying a desired viewpoint transformation on the 3D data, then synthesizing a novel 2D view of the scene. The view synthesizer is not deployed at inference time; only the learned policy is.

A different body of work leverages video prediction in the form of visual foresight and scene view synthesis Hirose et al. (2018) and Hirose et al. (2019a) along with model predictive control as in Hirose et al. (2019b) in the context of robot navigation.

Closed-loop stability is emphasized in Nagai and Sakai (2013) in the context of sloshing dynamics, where no geometric feature extraction is done. Instead, a single-input multi-output system
identification is used to approximate and map linearly the input to a matrix representing a reduced grayscale image of the liquid surface. The linear time-invariant (LTI) system is then converted to a port-Hamiltonian system where a passivity-based controller is applied. To reduce computational intensity, Sakai and Ando (2014) applies model reduction on the matrix space to reduce output size then performs LQG control, while Sakai and Sato (2014) uses feature extraction to map the liquid surface to polynomial space.

Another recent approach by Dean et al. (2020a) proposes to learn a perception map from high-dimensional data, the image, to a low dimensional latent representation as a state or partial state observation. Robust control is applied on the low dimensional latent representation, resulting in a dynamic output feedback controller and stability is shown under specific conditions, and extended by Dean and Recht (2020) and Dean et al. (2020b) to show safety.

In Suh and Tedrake (2020), Lyapunov stability to a target set is shown for an approach based on image visual foresight using linear models to solve a quasi-static pile manipulation problem. The state represents a grayscale image of the pile and image-to-image transitions are learned via switched-linear models. The action space is discrete and switching among actions corresponds to switching among linear models.

1.2. Notation

\( \mathbb{R} \) denotes the real line. Given multidimensional array \( Y \in \mathbb{R}^{p \times q \times r} \), \( \text{vec}(\cdot) \) orderly stacks the \( q \times r \) columns of \( Y \) one slice at a time until \( r \). An all one-entries \( n \times m \) matrix is denoted by \( \mathbf{1}_{n \times m} \), and by \( \mathbf{1} \) when the size is context-dependent. A continuous vector function \( f \) of dimension \( m \) that is a function of another \( n \) dimensional vector is represented by \( \mathcal{C}(\mathbb{R}^{n \times 1}, \mathbb{R}^{m \times 1}) = \{ f : \mathbb{R}^{n \times 1} \to \mathbb{R}^{m \times 1} | f \in \mathcal{C} \} \). \( I_{\text{Cam}} \in \mathbb{R}^{W \times H \times C} \) denotes an RGB image from a camera, and \( I_{\text{Syn}} \in \mathbb{R}^{W \times H \times C} \) is an RGB image from a synthesizer, of width, height and channel sizes denoted by \( W, H \) and \( C \) respectively.

2. Problem Formulation — Car-Following

![Car-following](Figure 1: Car-following.)

- \( v_1(t) \): leader speed,
- \( v_2(t) \): follower speed,
- \( f_2(t) \): follower force,
- \( s(t) \): spacing,
- \( \bar{s} \): leader desired speed,
- \( \bar{f}_2 \): steady-state force.

error signals:
- \( x_1(t) = \bar{v}_1(t) = \bar{v}(t) - v_1(t), \)
- \( x_3(t) = \bar{v}_2(t) = \bar{v}(t) - v_2(t), \)
- \( x_2(t) = \bar{s}(t) = \bar{s} - s(t), \)
- \( u(t) = \bar{f}_2(t) = \bar{f}_2(t) - f_2(t), \)
- \( m_1, m_2 > 0 \) mass of vehicles,
- \( \alpha_1, \alpha_2 > 0 \) drag coefficients.

We formulate the problem in the context of a concrete example from the application domain of autonomous driving, namely car-following as depicted in Figure 1. In this case, the objective is for the autonomous blue car to follow a leading red car by matching its speed and keeping a desired longitudinal inter-vehicle spacing. The error dynamics can be written as follows:

\[
\begin{align*}
\dot{x}_1(t) &= -\frac{\alpha_1}{m_1} x_1(t), \quad & (1a) \\
\dot{x}_2(t) &= x_1(t) - x_3(t), \quad & (1b) \\
\dot{x}_3(t) &= -\frac{\alpha_2}{m_2} x_3(t) + \frac{1}{m_2} u, \quad & (1c) \\
y(t) &= I_{\text{Cam}}(\bar{s} - x_2, \Theta, \Omega), \quad & (1d) \\
e(t) &= \bar{y} - y = I_{\text{Cam}}(\bar{s}, \Theta, \Omega) - I_{\text{Cam}}(\bar{s} - x_2, \Theta, \Omega). \quad & (1e)
\end{align*}
\]
Equations (1a) to (1c) follows from Levine and Athans (1966). Equation (1d) is a measurement model where $I_{Cam}(s, \Theta, \Omega)$ represents an image captured by a front-facing camera attached to the follower, and where $\Theta$ represents specific parameters of the leader, while $\Omega$ represents specific parameters of the driving environment background. Moreover, $I_{Cam}(\bar{s}, \Theta, \Omega)$ is a reference image for the same $\Theta$ and $\Omega$ had the spacing been the desired spacing $\bar{s}$. In some sense $I_{Cam}(\bar{s}, \Theta, \Omega)$ can be thought of as imagined instead of measured unlike $I_{Cam}(s, \Theta, \Omega)$ which is measured. This builds on neuroscientific concepts of analysis-by-synthesis where it is believed that mental imagery plays a role in human vision Yildirim et al. (2020). We show in Section 3.2 how to obtain $I_{Cam}(\bar{s}, \Theta, \Omega)$.

**Assumption 1**  Background Invariance: Assume that

$$e(t) = I_{Cam}(\bar{s}, \Theta, \Omega) - I_{Cam}(\bar{s} - x_2, \Theta, \Omega) = H(x_2, \bar{s}, \Theta).$$  \hspace{1cm} (2)

This says that $e(t)$ is invariant to background changes $\Omega$.

**Assumption 2**  Null Space: For $H(x_2, \bar{s}, \Theta)$ in (2), assume that $H(x_2, \bar{s}, \Theta) = 0 \iff x_2 = 0$. Then it follows that for a given $\bar{s}, \Theta$

$$\ker(e(x)) = \{x \in \mathbb{R}^{n \times 1} : x_2 = 0\}.$$  \hspace{1cm} (3)

**Assumption 3**  Error Direction: Let $h(x_2, \bar{s}, \Theta) = vec(H(x_2, \bar{s}, \Theta))$. Without loss of generality,

$$\bar{s} - s \geq 0 \iff 1^T h(\bar{s} - s, \bar{s}, \Theta) \geq 0.$$  \hspace{1cm} (4)

**Assumption 4**  Monotonic: For a given $\bar{s}$ and $\Theta$, consider $h(x_2, \bar{s}, \Theta)$ in (4). If $\beta \geq \alpha \geq 0$ or $-\beta \geq -\alpha \geq 0$, then

$$h(\beta, \bar{s}, \Theta)^T h(\bar{s}, \bar{s}, \Theta) \geq h(\alpha, \bar{s}, \Theta)^T h(\alpha, \bar{s}, \Theta).$$  \hspace{1cm} (5)

**Assumption 5**  Locally Quadratic: For a given $\bar{s}$ and $\Theta$, consider $h(x_2, \bar{s}, \Theta)$ in (4). We assume that over a local domain $D \subset \mathbb{R}^{n \times 1}$, where $x = 0 \in D$, that

$$h(x_2, \bar{s}, \Theta)^T h(x_2, \bar{s}, \Theta) \approx c^2(\bar{s}, \Theta)x_2^2,$$  \hspace{1cm} (6)

for some nonzero constant $c(\bar{s}, \Theta) \in \mathbb{R}$.

**Definition 1** Khalil (2002) Uniformly Ultimately Bounded (UUB): A solution of $\dot{x}(t) = f(t, x)$ is said to be UUB with ultimate bound of $\epsilon$ if $\exists \epsilon > 0, \Delta > 0$ such that $\forall \delta \in (0, \Delta)$, $\exists T(\delta, \epsilon) \geq 0$:

$$||x(t_0)|| \leq \delta \implies ||x(t)|| \leq \epsilon, \forall t \geq t_0 + T(\delta, \epsilon).$$

**Problem 1**  Output Regulation Solvability: Consider the car-following dynamics (1). Determine the existence of a static policy

$$u = F(y, \hat{y}),$$  \hspace{1cm} (7)

such that the regulated output $\text{vec}(e(t))$ is asymptotically stable with $x_1$, $x_2$, and $x_3$ bounded.

Note that the static control policy (7) does not require knowledge of $\bar{v}$.

**Problem 2**  Learning-based Output Regulation: Find a static policy (8) for the dynamics (1), where $\hat{y}$ is learned to approximate $\bar{y}$, such that $\text{vec}(\hat{e}(t)) = \text{vec}(\hat{\bar{y}} - y)$, $x_1$, $x_2$, and $x_3$ are UUBs:

$$u = F(y, \hat{\bar{y}}).$$  \hspace{1cm} (8)
3. Main Result

In Section 3.1 we show the existence of solutions to Problem 1 by casting the problem as a static output feedback problem. In Section 3.2, we show the architecture of a view synthesizer that will be used to provide reference images needed to compute the tracking error and regulated output (1e). In Section 3.3, we show a block diagram of the proposed controller, and discuss how to treat the RGB values so that generality is not lost as stated in Assumption 3. Later, we show the closed-loop stability with the camera and the reference view synthesizer in the loop Section 3.4.

3.1. Existence of Solutions

To address Problem 1, we first note that (7) is a static policy. One direction to follow is therefore to reduce Problem 1 into the following problem.

Problem 3 Static Output Feedback: Consider the car-following dynamics (1). Determine the existence of a static policy (9) such that $x_1$, $x_2$, and $x_3$ are asymptotically stable:

$$u(t) = F(e(t)).$$

(9)

Problem 3 is a state-regulation problem. The next theorem shows the existence of a solution to this state-regulation Problem 3, and thus to the output regulation Problem 1.

Lemma 1 For a fixed $\bar{s}$, $\Theta$, consider writing (1a), (1b) and (1c) in the form $\dot{x} = f(x) + g(x)u(x)$ and $h(x) = h(x_2, \bar{s}, \Theta)$. There exists $V(x) = x^TPx$ with $P = P^T \geq 0$ and $G(x) \in C(\mathbb{R}^{n \times 1}, \mathbb{R}^{m \times 1})$ such that over a domain $D \subset \mathbb{R}^{n \times 1}$, where $x = 0 \in D$:

$$\begin{align*}
0 &= \frac{dV(x)^T}{dx} f(x) - \frac{1}{4} \frac{dV(x)^T}{dx} g(x) g^T(x) \frac{dV(x)}{dx} + h^T(x) h(x) + G^T(x) G(x), \\
0 &= \frac{dV(x)^T}{dx} f(x), \quad \forall x \in \ker(h(x)).
\end{align*}$$

(10a, 10b)

Theorem 1 A static output feedback policy (9) exists that solves Problem 3, and thus Problem 1.

Proof First, if Problem 3 has a solution, this implies that Problem 1 is solvable because $u(t) = F(y, \bar{y}) = F(e(t))$ and $\lim_{t \to \infty} x_2(t) = 0 \implies \lim_{t \to \infty} H(x_2(t), s, \Theta) = 0$ by Assumption 2 and local continuity from Assumption 5. From Lemma 1 there exists a positive semi-definite solution to the HJ equation (10) over a domain $D \subset \mathbb{R}^{n \times 1}$. It follows from Astolfi and Colaneri (2002) and Astolfi and Colaneri (2001) that there exists a stabilizing state-feedback policy

$$u(x) = G(x) - \frac{1}{2} g^T(x) \frac{dV(x)}{dx},$$

(11)

and using the rank theorem, (11) can be written as a static output feedback policy $u(h(x)) = F(e(t))$ over a region around the equilibrium point. Thus Problem 3 has a solution.

3.2. Reference View Synthesis

We show how to synthesize an imagined reference image $\bar{y} = I_{Cam}(\bar{s}, \Theta, \Omega)$ that places the leading car at the desired inter-vehicle spacing $\bar{s}$ as would be viewed by the following car. In doing so, $\bar{y}$ needs to ideally satisfy Assumption 1. To do so, we consider an approach based on appearance flow Zhou et al. (2016) which has been proposed in the context of 3D view transformation Tatarchenko et al. (2016). However, our objective herein is not to transform the entire view, but rather to generate...
a view that corresponds to moving an object in the scene closer or farther from the observer through a frozen background. Moreover, unlike other work subsequent to Zhou et al. (2016), namely Park et al. (2017), we do not worry about occlusion issues that are more relevant in the rotation of 3D objects and the need to inpaint the hidden sides of the object by hallucinating a view completion.

Our reference view synthesizer is shown in Figure 2 which takes as input a raw camera RGB image and the desired inter-vehicle spacing, and generates as output a view placing the leading vehicle at the desired spacing away from the following vehicle. The raw camera image is an input to an autoencoder that is trained to generate an appearance flow as its output based on \( \bar{s} \), where this appearance flow determines which pixels from the camera raw image to copy from as opposed to generating pixels from scratch. The generated appearance flow and the raw camera image are both fed to a bilinear sampler and the output is an RGB image representing the synthesized view. Note that the camera raw image is an input to both the autoencoder, and the bilinear sampler. The bilinear sampler is differentiable for backpropagation purposes as shown in Jaderberg et al. (2015).

The encoder is constructed from 8 convolutional neural networks (CNNs) each followed by a rectified linear unit (RELU) and with the last layer flattened. All have a stride of 2, padding of 1 and kernel of 4 except for the first layer which has a kernel size of 3, stride of 1 and padding of 1.

The decoder is constructed from 7 convolutional transpose neural networks with stride 2, padding 1 and kernel of 4, each followed by a RELU and a CNN with kernel 3, stride 1 and padding 1 followed by a tangent hyperbolic function. The last layer clearly outputs values between -1 and 1, and represent the appearance flow. The input to the decoder is the flattened output of the encoder in addition to the desired spacing \( \bar{s} \).

The bilinear sampler takes as input the raw camera RGB tensor and the appearance flow-field tensor which acts on an identity sampling grid to form a modified sampling grid. The modified sampling grid determines, for each output pixel, the location of the input pixels to copy from. Almost all background pixels are copied from their original locations as is to ensure background invariance, while the pixels representing the current location of the leading car and the desired location are impacted. The reference image can therefore be represented as

\[
\hat{y} = I_{\text{Syn}}(\bar{s}, I_{\text{Cam}}(\bar{s} - x_2, \Theta, \Omega)),
\]

which will be assumed to satisfy Assumptions 1 to 5.

**Assumption 6  View Synthesis Error:** For a given \( \Theta \) and \( \bar{s} \), \( \exists \epsilon_1 > 0 \) such that

\[
\|\text{vec}(\begin{bmatrix}
I_{\text{Cam}}(\bar{s}, \Theta, \Omega) \\
I_{\text{Syn}}(\bar{s}, I_{\text{Cam}}(s, \Theta, \Omega))
\end{bmatrix} - \hat{y})\|_2 \leq \epsilon_1.
\]

3.3. Block Diagram of the Feedback Loop

We start by showing a block diagram of the proposed controller with a camera and a reference view synthesizer in the loop as shown in Figure 3.
As straightforward as this may seem, the block diagram of Figure 3 may lose the generality Assumption 3 states. To see this, consider a case where \( \bar{s} - s \geq 0 \) and the following two 3-by-3 pixel images where we show a single color channel only, e.g. Green:

\[
\bar{z} = \begin{bmatrix} B & O & B \\ B & B & B \\ B & B & B \end{bmatrix}, \quad z = \begin{bmatrix} B & B & B \\ B & B & B \\ O & O & O \end{bmatrix}.
\] (14a) \hspace{1cm} (14b)

The reference image \( \bar{z} \) has 1 pixel in the first row denoted by the letter \( O \) representing the color of an object traversing a background of color denoted by \( B \). The image \( z \) has 3 pixels in the last row representing the same observed object at a closer distance to the observer thus occupying more pixels. From (4), we get

\[
1^T \cdot \text{vec}(\bar{z} - z) = (O - B) + 3(B - O) = 2(B - O).
\] (15)

If the object is black moving in a green background, then we have \( O = 0 \) and \( B = 1 \) and thus \( 1^T \cdot \text{vec}(\bar{z} - z) \geq 0 \), otherwise if the object is green and moving through a black background, then \( O = 1 \) and \( B = 0 \) and thus \( 1^T \cdot \text{vec}(\bar{z} - z) \leq 0 \).

To enforce the generality of Assumption 3, we need an expression that is invariant to the polarity of \( (B - O) \), in other words a function \( |B - O| \). Consider a 3-by-3 pixel image \( z_0 \) representing the background only whose elements are all \( B \) values. By adding and subtracting \( z_0 \) to (15) and the taking of absolute values, we get the following

\[
1^T \cdot \text{vec}(-|z_0 - \bar{z}| + |z_0 - z|) = -|B - O| + 3|B - O| = 2|B - O|,
\] (16)
which is the desired expression. Equation (16) provides a clear breakdown to how the error signal can achieve the desired error directionality and magnitude. We therefore reorganize the block diagram in Figure 3 as shown in Figure 4 to ensure the generality of Assumption 3 is not lost.

Note that we may use the view synthesizer to generate a background by choosing \( s_0 \) to be a large value, thus the leading car essentially is vanishing from the view.

### 3.4. Stability Analysis of Learning-based Controller

The stability analysis will be discussed for the block diagram of Figure 3. We treat the following nonlinear controller which has a proportional gain acting on a neural network based error signal

\[
u = \text{vec}(K)^T \cdot \text{vec}(\hat{y} - y) = \text{vec}(K)^T \cdot \text{vec}(\hat{e}),
\]

(17)

which relates to (7) and mainly (9); and where \( \hat{y} - y = I_{Syn}(\bar{s}, I_{Cam}(s, \Theta, \Omega)) - I_{Cam}(s, \Theta, \Omega) \). Note that \( \hat{y} \) reflects an internal model principle. \( \hat{e} \) enables background invariance, thus generalization to backgrounds. Generalization and sample efficiency are key performance issues Chen et al. (2020) and Sax et al. (2019).

We first note that the dynamical system (1) can be decomposed into two subsystems, a stable uncontrollable subsystem governing the dynamics of \( x_1(t) \) and a controllable subsystem governing the dynamics of \( x_2(t) \) and \( x_3(t) \). By decoupling the stable uncontrollable subsystem, we have:

\[
\dot{x}_2(t) = -x_3(t),
\]

(18a)

\[
\dot{x}_3(t) = -\frac{\alpha_2}{m_2}x_3(t) + \frac{1}{m_2}u,
\]

(18b)

\[
y(t) = I_{Cam}(\bar{s} - x_2, \Theta, \Omega),
\]

(18c)

\[
\dot{\bar{e}}(t) = \hat{y} - y = I_{Syn}(\bar{s}, I_{Cam}(s, \Theta, \Omega)) - I_{Cam}(\bar{s} - x_2, \Theta, \Omega).
\]

(18d)

The following theorem demonstrates stability and thus addresses Problem 2.

**Theorem 2** Consider controller (17) and let \( K = 1 \). The dynamics (18) for a fixed \( \Theta \) is UUB.

**Proof** We first construct an appropriate Lyapunov function candidate. Let \( u^*(x_2) = \text{vec}(K)^T \cdot \text{vec}(H(x_2, \bar{s}, \Theta)) \). Consider the following positive definite function for subsystem (18)

\[
V(x_2, x_3) = w_1x_2^2 + w_2x_2x_3 + w_3x_3^2 + w_4 \int_0^{x_2} u^*(z)dz,
\]

(19)

where \( w_1 > 0 \) is arbitrary, and \( w_3 > 0 \), and \( w_2 \) are chosen appropriately and such that \( w_1x_1^2 + w_2x_2x_3 + w_3x_3^2 \) is positive definite in \( x_2 \) and \( x_3 \). Moreover \( w_4 > 0 \) will be chosen appropriately noting that the integral term is nonnegative due to Assumption 3 and \( K = 1 \).

From Assumption 5, \( u^*(z) \) is locally continuous in \( z \). Differentiating \( V(x_2, x_3) \) along the trajectories of (18), we get

\[
\dot{V}(x_2, x_3) = 2w_1x_2\ddot{x}_2 + w_2\ddot{x}_2x_3 + w_2x_2\ddot{x}_3 + 2w_3x_3\ddot{x}_3 + w_4\ddot{x}_2u^*(x_2), = \left(-2w_1 - \frac{\alpha_2}{m_2}w_2\right)x_2x_3 + \left(-w_2 - 2\frac{\alpha_2}{m_2}w_3\right)x_3^2 + \frac{w_2}{m_2}x_2u + \left(\frac{w_3}{m_2}u - w_4u^*\right)x_3.
\]

(20)
We demonstrated that stable feedback control directly from raw pixels is plausible and promising, automatic control applications where a cheap camera sensor can be deployed for motion control. The approach can extend to various control theory directly to pixels and establish safe and trustworthy dynamical systems that are more reasonably invariant to background changes. The approach provides a more clear path to apply driving backgrounds that have not been seen before due to the ability of the synthesizer to be different types of motions and tracked objects, and to further provide quantitative and qualitative to relax strong assumptions and have the theory encompassing of more practical scenarios and simulator environment. For further improvements and scalability, we need to investigate approaches and that introduced assumptions hold reasonably well for the application domain considered within a

4. Conclusion

We demonstrated that stable feedback control directly from raw pixels is plausible and promising, and that introduced assumptions hold reasonably well for the application domain considered within a simulator environment. For further improvements and scalability, we need to investigate approaches to relax strong assumptions and have the theory encompassing of more practical scenarios and different types of motions and tracked objects, and to further provide quantitative and qualitative assessments on generalization and sample complexity. The method generalized well to different driving backgrounds that have not been seen before due to the ability of the synthesizer to be reasonably invariant to background changes. The approach provides a more clear path to apply control theory directly to pixels and establish safe and trustworthy dynamical systems that are more interpretable compared to purely end-to-end learning approaches. The approach can extend to various automatic control applications where a cheap camera sensor can be deployed for motion control.1

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1. Code is available at https://github.com/abukhalaf/FeedbackFromPixels_L4DC2021
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Appendix A. Proof of Lemma 1

Proof Note that \( h(x) = h(x_2, \bar{s}, \Theta) \), and therefore from Assumption 2, it follows that \( \ker(h(x)) = \{ x \in \mathbb{R}^{n \times 1} : x_2 = 0 \} \). Moreover, by writing \( f(x) = Ax, g(x) = B \), and locally \( h^T(x)h(x) = c^2x_2^2 \) from Assumption 5, and \( G = [G_1, G_2, G_3] \),

\[
A = \begin{bmatrix} -\frac{\alpha_1}{m_1} & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -\frac{\alpha_2}{m_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & c & 0 \end{bmatrix},
\]

we can replace the Hamilton-Jacobi (HJ) equation (10a) and (10b) over domain \( D \subset \mathbb{R}^{n \times 1} \) with

\[
0 = A^T P + PA - PBB^T P + C^T C + G^T G, \quad (26a)
\]

\[
0 = N(A^T P + PA) N, \quad N = I - C^T (CC^T)^{-1} C. \quad (26b)
\]

From the kernel condition (26b), we have

\[
P = \begin{bmatrix} p_{11} & \frac{\alpha_1}{m_1} p_{11} & -\frac{\alpha_1 p_{11} + \alpha_2 p_{33}}{m_1} \\ \frac{\alpha_1}{m_1} p_{11} & p_{22} & -\frac{\alpha_2}{m_2} p_{33} \\ -\frac{\alpha_1 p_{11} + \alpha_2 p_{33}}{m_1} & -\frac{\alpha_2}{m_2} p_{33} & p_{33} \end{bmatrix}. \quad (27)
\]

From the algebraic Riccati equation (26a), we obtain the following for \( p_{11}, p_{22}, p_{33} \) and \( G \):

\[
p_{11} = \frac{|c| \alpha_2 + |c|^2 \frac{m_2}{\alpha_2} - |c|^2 \frac{m_1}{\alpha_1} \frac{m_2}{\alpha_2} + \frac{m_1 m_2}{\alpha_1 + \alpha_2}}{|c| \frac{\alpha_1}{\alpha_1 m_2 + \alpha_2 m_1} + \frac{\alpha_2}{m_1}}, \quad (28a)
\]

\[
G_1 = -\frac{\alpha_1 m_2 p_{11}}{\alpha_1 m_2^2 + \alpha_2 m_1 m_2}, \quad (28b)
\]

\[
p_{22} = |c| \alpha_2 + |c|^2 \frac{m_2}{\alpha_2}, \quad (28c)
\]

\[
G_2 = 0, \quad (28d)
\]

\[
p_{33} = |c| \frac{m_2}{\alpha_2}, \quad (28e)
\]

\[
G_3 = |c| \frac{m_2}{\alpha_2}. \quad (28f)
\]

Substitute (28a), (28c) and (28e) in (27), it follows that the principal minors of (27) are nonnegative; hence \( P \geq 0 \).

For specific numerical values of \( \alpha_1, \alpha_2, m_1 \) and \( m_2 \), a numerical procedure shown in Kučera and Souza (1995) and Gadewadikar et al. (2006) can be used to numerically solve (26).
Appendix B. Simulation Results

B.1. Training and Data Sets

Figure 5: Observed Camera Views from three CARLA Towns

We use release 0.9.9 of CARLA Dosovitskiy et al. (2017), a photorealistic urban driving simulator, in this study to both create a dataset to use for training the view synthesizer proposed in Section 3.2, and to demonstrate our proposed feedback control strategy for the car-following scenario introduced in Section 2. Our dataset is a set of raw images for observation views along with associated spacing distances collected at four different streets in three different CARLA towns or maps. Namely, one street in Town 3, two streets in Town 4, and one street in Town 5, all representing different urban environments. At each street, our data collection methodology is as follows:

- Spawn a leader and a follower cars.
- Place a front-facing camera on the follower car that faces the back of the leading car.
- Freeze the background, namely weather conditions, sun movement, cloud motion, wind or trees movements, traffic lights, and any other agents if any. The only thing allowed to move in the frame is the leading car.
- With the follower vehicle fully stopped at speed \(0 \text{ m s}^{-1}\), record the leading car driving away at an arbitrary slow speed.
- Capture images continuously, along with the associated distance, covering an inter-vehicle distance of \(5 \text{ m to } 50 \text{ m}\), with one or two samples or images for each driven meter.
- Repeat the experiment for the same location but with a different color for the leading car. Data for both blue and red colors is gathered.

The neural network is trained in a supervised manner. The network takes two inputs — a scalar and an RGB image, and it generates one output — an RGB image. The inputs are the reference distance \(\bar{s}\) and an RGB image from the front-facing camera showing the leading vehicle at some distance. The output is trained to generated a synthesized view RGB image that shows the leading vehicle at the desired \(\bar{s}\) in the same background of the input RGB image.
The training set is organized in the form of 3-tuples. For each desired spacing $\bar{s}$, there are eight groups with each group corresponding to one of the streets and one of the leading vehicle colors shown in Figure 5. Within each group, a 3-tuple has the following items:

1. A desired spacing $\bar{s}$.
2. A captured RGB image showing the leading car at the desired $\bar{s}$ in the street.
3. A captured RGB image for the same leading car of the same color at a distance between 5.5 m and 40 m in the same street, and therefore the same background.

Within each group, enough 3-tuples are created such that the third item covers the distance 5.5 m to 40 m at a 1 m increment. Our training set considers three different values for $\bar{s}$, namely $\bar{s} = 10$, $\bar{s} = 20$ and $\bar{s} = 30$. Therefore, we have a total of 24 groups of 3-tuples — 8 groups per each value of $\bar{s}$ — for a total of 774 different 3-tuples.

B.2. Open-Loop Results

Figure 6: Synthesized Views for a Fixed $\bar{s} = 10$ m and Varying Camera Views from the Training Set.

In Figure 6, we show synthesized views placing the leading car at the same desired reference spacing of $\bar{s} = 10$ for different camera views. The camera views are drawn from the training set. To show generalization, Figure 7 shows synthesized views placing the leading car at the same desired reference spacing of $\bar{s} = 20$ for different camera views. The camera views are drawn from outside the training set.

In Figure 8, we show synthesized views placing the leading car at different desired reference spacings — namely $\bar{s} = 10$, $\bar{s} = 20$ and $\bar{s} = 30$ — for the same camera view. The camera view is drawn from the training set. To show generalization, Figure 9 shows synthesized views placing the leading car at different desired reference spacings — namely $\bar{s} = 10$, $\bar{s} = 20$ and $\bar{s} = 30$ — for the same camera view. The camera view is drawn from outside the training set.
Figure 7: Synthesized Views for a Fixed $\bar{s} = 20m$ and Varying Camera Views from Outside the Training Set.

Figure 8: Synthesized Views for a Fixed Camera View from the Training Set and Varying Distance $\bar{s}$.

B.3. Closed-Loop Results

Figure 10 shows closed-loop responses for the control policy (17) with $K = \frac{1}{50}I$ using block diagram Figure 3. The data shows that cars drive within the desired spacing $\bar{s}$. In both cases, the cars where initially spawn at an initial spacing distance of $s = 10$. We have allowed the car to break and reverse direction in response to negative values of $u(t)$. Even for such a simple proportional controller, the performance can be improved if the value for $K$ is tuned and optimized. Further, if we additionally close the loop on the speed, we can achieve tighter control.
Figure 9: Synthesized Reference Views for a Fixed Camera View from Outside Training Set and Varying Distance $\bar{s}$.

Figure 10: Closed-loop Responses.
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