An upper limit on the initial temperature of the radiation-dominated Universe

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Abstract. Gravitational waves (GWs) are produced by colliding particles through the gravitational analogue of electromagnetic bremsstrahlung. We calculate the contribution of free-free emission in the radiation-dominated Universe to the stochastic GW background. We find that the energy density of the resulting GW radiation is heavily dependent on the number of elementary particles, \(N_{\text{tot}}\), and the maximum initial temperature, \(T_{\text{max}}\). We rule out \(N_{\text{tot}} \gtrsim 10^2 \times N_{\text{SM}}\) for \(T_{\text{max}} \sim T_{\text{Planck}} \approx 10^{19}\) GeV and \(N_{\text{tot}} \gtrsim 10^{16} \times N_{\text{SM}}\) for \(T_{\text{max}} \approx 10^{16}\) GeV, where \(N_{\text{SM}}\) is the number of particles in the Standard Model. In the case of inflation, existing cosmological data constrain \(T_{\text{max}} \lesssim 10^{16}\) GeV. However, alternative models to inflation such as bouncing cosmologies allow for \(T_{\text{max}}\) near \(T_{\text{Planck}}\). At the energy scales we are considering, the extra number of particles arise naturally in models of extra dimensions.

Keywords: alternatives to inflation, cosmology of theories beyond the SM, cosmology with extra dimensions, physics of the early universe

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1 Introduction

The stochastic gravitational wave (GW) background originates from independent physical processes throughout the history of the Universe [1]. Potential sources include the coalescence of compact binaries [2–4], quantum fluctuations during inflation [5–7], cosmic strings [8, 9], first order phase transitions in the early Universe [10, 11], and other exotic phenomena [12, 13]. Ref. [1] provides a comprehensive review summarizing the sources, current and planned observation methods, and constraints on the stochastic GW background.

The GW background spans a wide range of frequencies. The cosmic microwave background (CMB) anisotropies probe GWs with frequencies $f \sim 10^{-20} - 10^{-15}$ Hz [14–16]; pulsar timing arrays such as the EPTA, PPTA, and NANOGrav are sensitive at $f \sim 10^{-9} - 10^{-7}$ Hz [17–19]; ground-based detectors such as LIGO and Virgo at $f \sim 10^1 - 10^3$ Hz [1, 20, 21], and space-based detectors such as LISA will be sensitive at $f \sim 10^{-4} - 1$ Hz [22, 23]. Although less often discussed in the literature, there has been growing interest in building detectors sensitive to $f \sim 10^8$ Hz [24–26] and even higher frequencies [27]. As such, it is instructive to study potential sources of higher-frequency GWs.

Bremsstrahlung radiation from the collision of charged particles [28] is analogous to GWs produced through the gravitational scattering of two particles, a process known as free-free emission. Gravitational free-free emission has been studied classically in the context of, for example, high-speed black hole encounters [29], massive objects [30]. More recently, the case of free-free emission from collisions of massless particles has been considered as well [31, 32].

Here, we use the contribution of free-free emission in the radiation-dominated Universe to the stochastic GW background [33] to constrain the number of particles beyond the Standard Model. Conservatively, we ignore other sources of high frequency GWs in the range of $10^{10} - 10^{15}$ Hz [34], such as thermal GWs from stars [35], the amplification of quantum fluctuations of the gravitational field in the early Universe by inflation [36, 37], graviton to photon conversion in the presence of large scale magnetic fields [38], thermalized photons converting into gravitons in the presence of strong primordial magnetic fields [39], primordial black hole evaporation [40, 41], and black hole mergers in the early Universe [42].

In the case of cosmological inflation [43–45], data from the Planck satellite [46] and BICEP2/Keck [47] provide constraints on the tensor-to-scalar ratio, $r$, which, together with the amplitude of the power spectrum of the primordial scalar perturbations, $A_s$, can be translated to an upper bound on the energy scale of the inflationary potential when the pivot scale exits.
the Hubble radius, \( V_s < (1.6 \times 10^{16} \text{ GeV})^4 \). The resultant energy density of a thermal bath of massless particles after reheating can be expressed as \( V^* = g_*(T) T^4 \pi^2 / 30 \), where \( g_*(T) \) is the total number of effectively massless degrees of freedom and \( T \) is the temperature of the thermal bath. With \( g_*(T_{\text{max}}) \approx 100 \) \cite{48}, we can constrain the maximum post-reheating temperature of the radiation-dominated Universe, \( T_{\text{max}} \lesssim 10^{16} \text{ GeV} \), or possibly lower from gravitino production \cite{49, 50}.

Here, we derive equivalent constraints on \( T_{\text{max}} \) for alternatives to inflation. As an example, in bouncing cosmologies the universe began in a contracting phase, experienced a bounce, and eventually entered the radiation-dominated phase of Big Bang cosmology \cite{51}. In these models, \( T_{\text{max}} \) during the bounce can be near the Planck scale \( T_{\text{Planck}} \approx 10^{19} \text{ GeV} \).

We assume for simplicity the standard cosmological history of a single matter-dominated phase following the radiation-dominated epoch and ignore more complicated alternatives \cite{53}.

## 2 Free-free GW emission

We define the present-day GW energy density per logarithmic frequency interval relative to the closure density of the universe as

\[
\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d \rho_{\text{GW}}}{d \ln f},
\]

where \( \rho_{\text{GW}} \) is the energy density of the radiation and \( \rho_c \) the critical density,

\[
\rho_c = \frac{3 H_0^2 c^2}{8 \pi G},
\]

where \( H_0 \) is the Hubble constant today, \( G \) Newton’s constant, and \( c \) the speed of light. Throughout our work, we adopt \( h = H_0 / (100 \text{ km s}^{-1} \text{Mpc}^{-1}) = 0.7 \) \cite{15, 54}.

The observed energy density of the GWs per logarithmic frequency interval is,

\[
\frac{d \rho_{\text{GW}}}{d \ln f} = \int \Gamma \frac{d E_{\text{GW}}}{d \ln f} \frac{1}{(1 + z)^4} dt = \int \Gamma \frac{d E_{\text{GW}}}{d \ln f} \frac{1}{H(z)(1 + z)^5} dz,
\]

where \( \Gamma \) is the collision frequency per proper volume, \( E_{\text{GW}} \) is the GW energy, \( t \) is cosmic time, and \( z \) is redshift. The factor \( (1 + z)^{-3} \) normalizes to comoving volume, an additional factor \( (1 + z)^{-1} \) accounts for the redshifting of the energy \cite{55}, and \( H(z) \) is the Hubble parameter. Transforming \( d E_{\text{GW}}/d \ln f \) to the source rest frame with \( f_r = f(1 + z) \),

\[
\frac{d \rho_{\text{GW}}}{d \ln f} = f \int \Gamma \frac{d E_{\text{GW}}}{d f} \frac{1}{H(z)(1 + z)^5} dz = f \int \Gamma (1 + z) \frac{d E_{\text{GW}}}{d f_r} \frac{1}{H(z)(1 + z)^5} dz,
\]

from which we get,

\[
\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \int \Gamma \frac{d E_{\text{GW}}}{d f_r} \frac{1}{H(z)(1 + z)^4} dz.
\]

The collision frequency per proper volume is, \( \Gamma = n_1 n_2 \sigma c \), with \( n_1, n_2 \) being the number densities and \( \sigma \) the interaction cross section. For massless gauge-boson exchanges \cite{48},

\[
\sigma \sim \left( \frac{\alpha k_B T}{k_B T} \right)^2 \ln \Lambda,
\]
with \( \alpha \) being the gauge coupling constant, \( \hbar \) the reduced Planck constant, \( k_B \) the Boltzmann constant, and \( T \) the temperature of the cosmic plasma. Small-angle collisions, which we consider in this work, are more effective than large-angle collisions by a factor of the Coulomb logarithm, \( \ln \Lambda \sim 1 \sim 10 \) for a relativistic plasma [56]; we conservatively take \( \ln \Lambda \sim 1 \). We assume that at the energies considered in our work — at or above the GUT scale — the strong, weak, and electromagnetic interactions are of similar strength and can be described using QED scattering equations, ignoring order unity corrections. We take a constant \( \alpha \sim 1/26 \) from the supersymmetric extension of the Standard Model at the GUT scale [57].

For collisions of photons, the number densities are

\[
n_1 = n_2 = g \int \frac{d^3p}{(2\pi)^3} f(p) \approx \frac{g(k_BT)^3}{\pi^2(\hbar c)^3}, \tag{2.7}
\]

allowing us to write the collision frequency per proper volume as,

\[
\Gamma \approx \left[ \frac{g(k_BT)^3}{\pi^2(\hbar c)^3} \right]^2 \left( \frac{\alpha \hbar c}{k_BT} \right)^2 c \ln \Lambda. \tag{2.8}
\]

Ref. [31] calculated classical gravitational free-free emission from the gravitational scattering of two massless particles at leading order in the center of mass deflection angle \( \theta \ll 1 \).

Under the approximation that the characteristic scale of the angular frequency \( \omega \) is \( c^5/GE \),

\[
\frac{dE_{GW}}{d\omega_r} = \frac{4}{\pi} \frac{\theta^2 E^2 G}{c^5} \ln \left( \frac{c^5}{E\omega_r G} \right), \tag{2.9}
\]

with \( \omega_r = 2\pi f_r = 2\pi f(1 + z) \) being the source frame angular frequency. This result holds in the range \( c/b < \omega_r < c^5/GE \), with \( b \) the impact parameter. However, energy conservation imposes a stricter upper bound of \( \omega_r \lesssim 3k_BT/\hbar \), the average total energy of an extreme relativistic gas. We note that the aforementioned assumption that \( \omega \sim c^5/GE \) holds only for \( T_{\text{max}} \lesssim 0.1 \sqrt{3k_BT_0} \).

The center of mass deflection angle, \( \theta \), is a function of the impact parameter \( b \). In the relativistic regime, \( b_{\text{min}} \approx h/p = hc/E \), and we approximate \( b \sim b_{\text{min}} \). The classical deflection angle is \( \theta = 8EG/\hbar c^4 \) [32], where \( E = k_BT = k_BT_0(1 + z) \), with \( T_0 = 2.725 \text{K} \) being the present-day CMB temperature. With these parameters, we can use eqs. (2.8) and (2.9) to integrate eq. (2.5) from present time, \( z = 0 \), to the end of inflation, \( z_{\text{max}} \sim T_{\text{max}}/T_0 \). This ignores the modest (order unity) heating of the CMB but not of the GW background by annihilations (similar to \( e^+e^- \) heating of the CMB relative to the neutrino background).

We only calculated \( \Omega_{GW}(f) \) for the case of photon collisions. However, near the Planck scale we ignore order unity corrections that distinguish other types of particles from photons. Instead, we multiply the expression for \( \Omega_{GW}(f) \) in eq. (2.5) by the number of elementary particles. Most of the contribution to \( \Omega_{GW}(f) \) comes from \( z \sim z_{\text{max}} \sim T_{\text{max}}/T_0 \).

In deriving our results we focus on two ways that the contribution of free-free emission to the stochastic GW background can be enhanced: (i) through the number of elementary particles, which can constrain physics beyond the Standard Model, and (ii) through the proximity of \( T_{\text{max}} \) to \( T_{\text{Planck}} \). In our results, we express the temperature relative to the Planck scale, \( T_{\text{Planck}} = \sqrt{\hbar c^3/Gk_B^2} \approx 1.2 \times 10^{19} \text{GeV} \).
3 Constraints on free-free GW emission

Big Bang Nucleosynthesis (BBN) and the CMB can be used to set upper bounds on the total energy density of a cosmological GW background [58],

$$\Omega_{GW} = \int \Omega_{GW}(f) d\ln f = \frac{\rho_{GW}}{\rho_c}. \quad (3.1)$$

The GW background acts as an additional component of the radiation field in the Hubble expansion rate,

$$H(z) = H_0 \left[ (\Omega_{\text{rad}} + \Omega_{GW}) (1 + z)^4 + \Omega_{m} (1 + z)^3 + \Omega_{\Lambda} \right]^{1/2}, \quad (3.2)$$

where $\Omega_{\text{rad}}$, $\Omega_{m}$, and $\Omega_{\Lambda}$ are contributions of the standard radiation (CMB + neutrino background), matter, and the cosmological constant, respectively. BBN and the CMB probe the cosmic energy budget and constrain $\Omega_{GW}$. The change in the radiation energy density, $\Delta \rho_{\text{rad}}$, is then $\Delta \rho_{\text{rad}} \equiv (\rho_{\text{rad}} - \rho_c \Omega_{\text{rad}})$. This can be expressed in terms of $\Delta N_{\nu}$ extra neutrino species,

$$\Delta \rho_{\text{rad}} = \frac{\pi^2}{304} \Delta N_{\nu} T^4. \quad (3.3)$$

By requiring $\rho_{GW} \leq \Delta \rho_{\text{rad}}$, we get,

$$h^2 \Omega_{GW} \leq 5.6 \times 10^{-6} \Delta N_{\nu} = 5.6 \times 10^{-6} \Delta N_{\text{eff}}, \quad (3.4)$$

with $N_{\text{eff}}$ the effective number of neutrino species present in the thermal bath after $e^+e^-$ annihilation.

Assuming GWs with homogeneous initial conditions, the Planck satellite and other cosmological data constrain $\Omega_{GW} \lesssim 4 \times 10^{-7}$ [58].

4 Results

To gauge the contribution to $\Omega_{GW}$ per logarithmic redshift interval, we examine

$$\frac{d\Omega_{GW}(f)}{d\ln(1+z)} = \frac{d^2\Omega_{GW}}{d\ln(1+z)d\ln f} = \frac{f}{\rho_c} \frac{dE_{GW}}{df} = \frac{1}{H(z)(1+z)^3}. \quad (4.1)$$

Assuming $H(z) \approx H_0 \sqrt{\Omega_{\text{rad}}(1+z)^2}$ in the radiation dominated era and defining the following constants,

$$\Gamma_0 \equiv \left[ \frac{g (k_B T_0)^3}{\pi^2 (hc)^3} \right]^2 \left( \frac{\alpha hc}{k_B T_0} \right)^2 c \ln \Lambda,$$

$$\theta_0 \equiv \frac{8G(k_B T_0)^2}{hc^5},$$

$$\varepsilon_0 \equiv 2\pi \left( \frac{4}{\pi} \right) \frac{\theta_0^2 (k_B T_0)^2 G}{c^5},$$

$$\phi_0(f) \equiv \frac{c^5}{k_B T_0 2\pi G f^2}. \quad (4.2)$$
Figure 1. \( \log_{10} \Omega_{\text{GW}} \) as a function of \( N_{\text{tot}}/N_{\text{SM}} \) and of \( T_{\text{max}}/T_{\text{Planck}} \). In general, as either \( N_{\text{tot}} \) or \( T_{\text{max}} \) increases, the energy density of the radiation increases. The solid purple line indicates the upper bound given by eq. (3.4), with the region above and to its right ruled out based on existing cosmological data. The dotted line is obtained from extrapolating the solid line, and lies in the region where eq. (2.9) loses its validity.

we can write eq. (4.1) as

\[
\frac{d^2 \Omega_{\text{GW}}}{d \ln(1+z) d \ln f} = \frac{f \varepsilon_0 \Gamma_0}{\rho_c H_0 \sqrt{\Omega_{\text{rad}}}} \ln \left[ \phi_0(f) \right] (1 + z)^5
\]

\[
\propto \ln \left[ \frac{\phi_0(f)}{(1 + z)^2} \right] (1 + z)^5.
\]

(4.3)

Numerically, we find that eq. (4.1) is well-described by a power law,

\[
\frac{d^2 \Omega_{\text{GW}}}{d \ln(1+z) d \ln f} \propto z^\beta,
\]

(4.4)

with \( \beta \approx 4.77 \) over the observed frequency range. It is evident that the highest values of \( z \) dominate the contribution to \( \Omega_{\text{GW}}(f) \). Hence, one can simply examine the dependence of \( \Omega_{\text{GW}} \) on \( T_{\text{max}} \).

Figure 1 shows \( \log_{10} \Omega_{\text{GW}} \) as a function of the total number of elementary particles divided by the number of elementary particles in the Standard Model, \( N_{\text{tot}}/N_{\text{SM}} \), where \( N_{\text{SM}} = 17 \), and as a function of the maximum temperature of the Universe normalized by the Planck temperature, \( T_{\text{max}}/T_{\text{Planck}} \). The solid diagonal line is the upper limit given by eq. (3.4); the region above and to its right is ruled out from cosmological data. The dashed diagonal line in figure 1 is the solid line extrapolated to higher temperatures, where the assumption that \( \omega \sim c^5/GE \), used in deriving eq. (2.9), is no longer valid.

As either \( N_{\text{tot}} \) or \( T_{\text{max}} \) increases, the energy density of the radiation increases. For \( T_{\text{max}} \) near \( T_{\text{Planck}} \sim 10^{19} \text{ GeV} \), the cosmological constraint rules out particle physics models that predict \( N_{\text{tot}} \sim 10^2 \times N_{\text{SM}} \).

Figure 2 shows the ratio of GW to the radiation energy density, \( \Omega_{\text{GW}}(f)/\Omega_{\text{rad}} \), versus observed GW frequency. Each band in figure 2 represents a different value of \( T_{\text{max}} \), which
we vary from $10^{-4} \times T_{\text{Planck}} \sim 10^{15} \text{ GeV}$ to $T_{\text{Planck}} \sim 10^{19} \text{ GeV}$. The lower boundary of each band is for $N_{\text{tot}} = N_{\text{SM}}$ and the upper boundary is for $N_{\text{tot}} = 10 \times N_{\text{SM}}$.

There is a thermal cutoff near an observed frequency of $f \sim 10^{12} \text{ Hz}$, which is similar to the frequency cutoff of the CMB. The lower boundary on $f$ originates from the smallest possible value of the impact parameter $b_{\text{min}}$ for classical free-free emission. Over this frequency range, $\Omega_{\text{GW}}(f)$ varies weakly as a function of $f$, but varies strongly as a function of $N_{\text{tot}}$ or $T_{\text{max}}$.

5 Discussion

We find that the energy density of the GW background from free-free emission in the early Universe can be used to rule out $N_{\text{tot}} \gtrsim 10^2 \times N_{\text{SM}}$ for $T_{\text{max}} \sim 10^{19} \text{ GeV}$. If we assume a higher-dimensional Planck scale such as $M_{4+n} \sim 10^3 \text{ GeV}$, near the lower bound imposed from astrophysics and cosmology [59–62], about $10^{10}$ Kaluza-Klein (KK) states could easily be accessed even for $n$ as low as one [63, 64]. From a four-dimensional point of view, these KK excitations are distinct particles, meaning this extra number of particles could easily arise from theories that predict ten, eleven, or more dimensions [65–67].

However, we caution that our work concerns energy scales at which four-dimensional effective field theories may no longer be accurate [68]. An intrinsically higher-dimensional description of free-free GW emission might be necessary. We also note that in theories with a large number $N$ of particle species, it has been pointed out that black hole physics imposes an upper bound on the energy at which these theories are valid, $T \sim M_{\text{Planck}}/\sqrt{N}$ [69]. The upper bounds we place on $N_{\text{tot}}$ as a function of $T_{\text{max}}$ lie beyond this energy limit for $T_{\text{max}} \lesssim 10^{18} \text{ GeV}$.

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References

[1] N. Christensen, Stochastic gravitational wave backgrounds, Rept. Prog. Phys. 82 (2019) 016903 [arXiv:1811.08797] [inSPIRE].

[2] LIGO Scientific and Virgo collaborations, Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. 116 (2016) 061102 [arXiv:1602.03837] [inSPIRE].

[3] LIGO Scientific and Virgo collaborations, Gw150914: Implications for the stochastic gravitational-wave background from binary black holes, Phys. Rev. Lett. 116 (2016) 131102 [arXiv:1602.03847] [inSPIRE].

[4] LIGO Scientific and Virgo collaborations, Gw170817: Implications for the stochastic gravitational-wave background from compact binary coalescences, Phys. Rev. Lett. 120 (2018) 091101 [arXiv:1710.05837] [inSPIRE].

[5] R. Easther and E.A. Lim, Stochastic gravitational wave production after inflation, JCAP 04 (2006) 010 [astro-ph/0601617] [inSPIRE].

[6] R. Easther, J. Giblin, John T. and E.A. Lim, Gravitational wave production at the end of inflation, Phys. Rev. Lett. 99 (2007) 221301 [astro-ph/0612294] [inSPIRE].

[7] J.L. Cook and L. Sorbo, Particle production during inflation and gravitational waves detectable by ground-based interferometers, Phys. Rev. D 85 (2012) 023534 [Erratum ibid. 86 (2012) 069901] [arXiv:1109.0022] [inSPIRE].

[8] X. Siemens, V. Mandic and J. Creighton, Gravitational-Wave Stochastic Background from Cosmic Strings, Phys. Rev. Lett. 98 (2007) 111101 [astro-ph/0610920] [inSPIRE].

[9] M.R. DePies and C.J. Hogan, Stochastic gravitational wave background from light cosmic strings, Phys. Rev. D 75 (2007) 125006 [astro-ph/0702335] [inSPIRE].

[10] A. Kosowsky, M.S. Turner and R. Watkins, Gravitational waves from first-order cosmological phase transitions, Phys. Rev. Lett. 69 (1992) 2026 [inSPIRE].

[11] M. Kamionkowski, A. Kosowsky and M.S. Turner, Gravitational radiation from first-order phase transitions, Phys. Rev. D 49 (1994) 2837 [astro-ph/9310044] [inSPIRE].

[12] A. Buonanno, G. Sigl, G.G. Raffelt, H.-T. Janka and E. Muller, Stochastic gravitational-wave background from cosmological supernovae, Phys. Rev. D 72 (2005) 084001 [astro-ph/0412277] [inSPIRE].

[13] S. Marassi, R. Schneider and V. Ferrari, Gravitational wave backgrounds and the cosmic transition from Population III to Population II stars, Mon. Not. Roy. Astron. Soc. 398 (2009) 293 [arXiv:0906.0461] [inSPIRE].

[14] P.D. Lasky et al., Gravitational-wave cosmology across 29 decades in frequency, Phys. Rev. X 6 (2016) 011035 [arXiv:1511.05994] [inSPIRE].

[15] PLANCK collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6 [arXiv:1807.06209] [inSPIRE].

[16] M. Kamionkowski, A. Kosowsky and A. Stebbins, Statistics of cosmic microwave background polarization, Phys. Rev. D 55 (1997) 7368 [astro-ph/9611125] [inSPIRE].

[17] M. Krämer and D.J. Champion, The European Pulsar Timing Array and the Large European Array for Pulsars, Class. Quant. Grav. 30 (2013) 224009 [inSPIRE].

[18] D.R.B. Yardley et al., The Sensitivity of the Parkes Pulsar Timing Array to Individual Sources of Gravitational Waves, Mon. Not. Roy. Astron. Soc. 407 (2010) 669 [arXiv:1005.1667] [inSPIRE].

[19] M.A. McLaughlin, The North American Nanohertz Observatory for Gravitational Waves, Class. Quant. Grav. 30 (2013) 224008 [arXiv:1310.0758] [inSPIRE].
[20] M. Pitkin, S. Reid, S. Rowan and J. Hough, *Gravitational Wave Detection by Interferometry (Ground and Space)*, *Living Rev. Rel.* **14** (2011) 5 [arXiv:1102.3355] [inSPIRE].

[21] KAGRA, LIGO Scientific and VIRGO collaborations, *Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA*, *Living Rev. Rel.* **21** (2018) 3 [arXiv:1304.0670] [inSPIRE].

[22] N. Bartolo et al., *Science with the space-based interferometer LISA. IV: Probing inflation with gravitational waves*, *JCAP* **12** (2016) 026 [arXiv:1610.06481] [inSPIRE].

[23] P. Amaro-Seoane et al., *eLISA/NGO: Astrophysics and cosmology in the gravitational-wave millihertz regime*, *GW Notes* **6** (2013) 4 [arXiv:1201.3621] [inSPIRE].

[24] A. Nishizawa et al., *Laser-interferometric Detectors for Gravitational Wave Background at 100 MHz: Detector Design and Sensitivity*, *Phys. Rev. D* **77** (2008) 022002 [arXiv:0710.1944] [inSPIRE].

[25] A.M. Cruise and R.M.J. Ingley, *A prototype gravitational wave detector for 100-MHz*, *Class. Quant. Grav.* **23** (2006) 6185 [inSPIRE].

[26] T. Akutsu et al., *Search for a stochastic background of 100-MHz gravitational waves with laser interferometers*, *Phys. Rev. Lett.* **101** (2008) 101101 [arXiv:0803.4094] [inSPIRE].

[27] L.P. Grishchuk, *Electromagnetic generators and detectors of gravitational waves*, in *1st Conference on High Frequency Gravitational Waves* **6**, 2003 [gr-qc/0306013] [inSPIRE].

[28] J.D. Jackson, *Classical electrodynamics*, 3rd ed., Wiley, New York, NY, U.S.A. (1999).

[29] B. Allen, *The stochastic gravity wave background: Sources and detection*, in *Les Houches School of Physics: Astrophysical Sources of Gravitational Radiation*, pp. 373–417, (1996) [gr-qc/9604033] [inSPIRE].

[30] L.P. Grishchuk, *Discovering Relic Gravitational Waves in Cosmic Microwave Background Radiation*, arXiv:0707.3319 [inSPIRE].
[41] K. Inomata, M. Kawasaki, K. Mukaida, T. Terada and T.T. Yanagida, Gravitational Wave Production right after a Primordial Black Hole Evaporation, Phys. Rev. D 101 (2020) 123533 [arXiv:2003.10485] [inSPIRE].

[42] D. Hooper, G. Krnjaic, J. March-Russell, S.D. McDermott and R. Petrossian-Byrne, Hot Gravitons and Gravitational Waves From Kerr Black Holes in the Early Universe, arXiv:2004.00618 [inSPIRE].

[43] A.H. Guth, The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems, Phys.Rev.D 23 (1981) 347 [inSPIRE].

[44] K. Sato, First Order Phase Transition of a Vacuum and Expansion of the Universe, Mon. Not. Roy. Astron. Soc. 195 (1981) 467 [inSPIRE].

[45] R. Brout, F. Englert and E. Gunzig, The Creation of the Universe as a Quantum Phenomenon, Annals Phys. 115 (1978) 78 [inSPIRE].

[46] Planck collaboration, Planck 2018 results. X. Constraints on inflation, Astron. Astrophys. 641 (2020) A10 [arXiv:1807.06211] [inSPIRE].

[47] BICEP2 and Keck Array collaborations, BICEP2/Keck Array x: Constraints on Primordial Gravitational Waves using Planck, WMAP, and New BICEP2/Keck Observations through the 2015 Season, Phys. Rev. Lett. 121 (2018) 221301 [arXiv:1810.05216] [inSPIRE].

[48] E.W. Kolb and M.S. Turner, The Early Universe, Frontiers in Physics, Westview Press, Boulder, CO, U.S.A. (1990), [DOI].

[49] M. Khlopov and A.D. Linde, Is It Easy to Save the Gravitino?, Phys. Lett. B 138 (1984) 265 [inSPIRE].

[50] M. Khlopov, Y. Levitan, E.V. Sedelnikov and I.M. Sobol, Nonequilibrium cosmological nucleosynthesis of light elements: Calculations by the Monte Carlo method, Phys. Atom. Nucl. 57 (1994) 1393 [inSPIRE].

[51] R. Brandenberger and P. Peter, Bouncing Cosmologies: Progress and Problems, Found. Phys. 47 (2017) 797 [arXiv:1603.05834] [inSPIRE].

[52] R. Brandenberger and Z. Wang, Nonsingular Ekpyrotic Cosmology with a Nearly Scale-Invariant Spectrum of Cosmological Perturbations and Gravitational Waves, Phys. Rev. D 101 (2020) 063522 [arXiv:2001.00638] [inSPIRE].

[53] X. Chen and S.-H. Tye, Heating in brane inflation and hidden dark matter, JCAP 06 (2006) 011 [hep-th/0602136] [inSPIRE].

[54] A.G. Riess et al., New Parallaxes of Galactic Cepheids from Spatially Scanning the Hubble Space Telescope: Implications for the Hubble Constant, Astrophys. J. 855 (2018) 136 [arXiv:1801.00638] [inSPIRE].

[55] E.S. Phinney, A practical theorem on gravitational wave backgrounds, astro-ph/0108028 [inSPIRE].

[56] R.P. Pilla and J. Shaham, Kinetics of electron-positron pair plasmas using an adaptive Monte Carlo method, Astrophys. J. 486 (1997) 903 [astro-ph/9702187] [inSPIRE].

[57] W. de Boer, Grand unified theories and supersymmetry in particle physics and cosmology, Prog. Part. Nucl. Phys. 33 (1994) 201 [hep-ph/9402266] [inSPIRE].

[58] C. Caprini and D.G. Figueroa, Cosmological Backgrounds of Gravitational Waves, Class. Quant. Grav. 35 (2018) 163001 [arXiv:1801.04268] [inSPIRE].

[59] S. Hannestad and G.G. Raffelt, Stringent neutron star limits on large extra dimensions, Phys. Rev. Lett. 88 (2002) 071301 [hep-ph/0110067] [inSPIRE].

[60] S. Cullen and M. Perelstein, SN1987A constraints on large compact dimensions, Phys. Rev. Lett. 83 (1999) 268 [hep-ph/9903422] [inSPIRE].
[61] V.D. Barger, T. Han, C. Kao and R.-J. Zhang, *Astrophysical constraints on large extra dimensions*, *Phys. Lett. B* **461** (1999) 34 [hep-ph/9905474] [INSPIRE].

[62] C. Hanhart, J.A. Pons, D.R. Phillips and S. Reddy, *The likelihood of GODs’ existence: improving the SN1987a constraint on the size of large compact dimensions*, *Phys. Lett. B* **509** (2001) 1 [astro-ph/0102063] [INSPIRE].

[63] H.-C. Cheng, *Introduction to Extra Dimensions*, arXiv:1003.1162 [INSPIRE].

[64] V.H. Satheesh Kumar and P.K. Suresh, *Gravitons in Kaluza-Klein theory*, gr-qc/0605016 [INSPIRE].

[65] K. Becker, M. Becker and J.H. Schwarz, *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press (2006).

[66] K.R. Dienes, *String theory and the path to unification: A review of recent developments*, *Phys. Rept.* **287** (1997) 447 [hep-th/9602045] [INSPIRE].

[67] A. Maharana and E. Palti, *Models of Particle Physics from Type IIB String Theory and F-theory: A Review*, *Int. J. Mod. Phys. A* **28** (2013) 1330005 [arXiv:1212.0555] [INSPIRE].

[68] H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin-Cummings Pub Co (1984).

[69] G. Dvali and M. Redi, *Black Hole Bound on the Number of Species and Quantum Gravity at LHC*, *Phys. Rev. D* **77** (2008) 045027 [arXiv:0710.4344] [INSPIRE].