Time Domain Prediction of Side and Plunge Milling Stability Considering Edge Radius Effect

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This article introduces a time domain model of side and plunge milling stability by considering cutting edge radius as well as dynamic motion of cutter center. Dynamic uncut chip thickness generated by side, plunge and ultrasonic milling can be simulated by tracking the cutter center positions at the present and previous vibrations with phase difference. Finally, the dynamic uncut chip thickness models as well as the mechanistic cutting force coefficients considering cutting edge radius are integrated into the time domain model. Especially, cross edge radius varies due to tool wear which affects cutting force coefficients and dynamics as well. Finally, the machining stability and vibrations are estimated using an identified transfer function and predicted cutting forces through the time domain solution. Experimental tests are compared against predicted results for validation of the proposed model.

Keywords: time domain chatter model, side and plunge milling, ultrasonic vibration, cutting edge radius

1. Introduction

Machining chatter often becomes a big hindrance against high productivity and machining quality. There have been research works on the mechanics and dynamics of milling processes. Tobias and Tlusty introduced first chatter stability laws in frequency domain [1,2]. Sridhar presented the time domain solution with two coupled, delayed differential
equations with time varying coefficients [3]. Minis and Yanushevsky proposed the first analytical solution of milling stability using Floquet's theory by advancing from Sridhar's formulation [4]. Altintas and Budak developed a general and closed solution of the milling stability in frequency domain[5,6]. Some articles proposed added stability lobes to simulate low radial immersion cutting during high speed machining [7,8]. Olgac et. al. included both single and multiple time delays and validated their stability law on uniform and variable pitch cutter results [9]. Ko and Altintas proposed mechanistic and dynamic models in time and frequency domain for plunge milling processes [10,11]. The developed stability model can predict a torsional-axial vibration as well as lateral vibrations. Eksioglu and Altintas [12] proposed a general formulation of flexible cutters and workpiece at high axial and small radial depths of cuts. Recently, process modeling for milling process has been advanced by a few researchers [13-15]. In addition, ultrasonic vibration assisted milling has been also simulated using time integration models [16, 17].

This article introduces time domain model, which can simulate side and plunge milling by tracking dynamic motion of cutter center position. The modeling of dynamic uncut chip thickness reflects regenerative effect with phase difference. In case of ultrasonic vibration assisted milling process, the ultrasonic vibration displacement can be added to cutter center position as well.

The dynamic uncut chip thickness models as well as the mechanistic cutting force coefficients considering cutting edge radius are integrated into time domain solutions. Cutting edge radius according to tool wear varies which is considered into cutting force coefficients and dynamics as well. Finally, the machining stability and vibrations can be estimated using the identified transfer function and the predicted cutting forces from time domain solution. Experimental validations as well as industrial applications are presented with conclusion.

2. Dynamic chip thickness estimation

For the time domain modeling, dynamic chip thickness is first formulated considering dynamic response of a cutter. In order to calculate the dynamic chip thickness shown in Fig. 1, the cutter center position needs to be tracked. So, in time domain solution, the cutter center position is formulated using rigid body motion and vibrations as defined in Eq. (1). The process induced vibration is calculated using transfer function and predicted forces. Ultrasonic vibration motion can be added to cutter center position in order to simulate its effect on machining stability.

\[
\begin{align*}
X_c(t) &= \frac{\text{feed}_x \cdot t + R_c \cdot \sin \phi}{\Omega} + \frac{z(t)}{\Omega} + \frac{U_v(t)}{\Omega} \\
Y_c(t) &= \frac{\text{feed}_y \cdot t + R_c \cdot \cos \phi}{\Omega} + \frac{z(t)}{\Omega} + \frac{U_v(t)}{\Omega} \\
Z_c(t) &= \frac{\text{feed}_z \cdot t}{\Omega} + \frac{z(t)}{\Omega} + \frac{U_v(t)}{\Omega} \\
\theta_c(t) &= \frac{\text{deg}_t}{\Omega} + \frac{\phi(t)}{\Omega}
\end{align*}
\]

where \( R_c \) represents the radial runout of the cutter center.

![Uncut chip thickness model of side milling](Image)

![Uncut chip thickness model of plunge milling](Image)

Fig. 1 Dynamic chip thickness calculation in side and plunge milling process

The coordinates of the points along the cutting edge are evaluated by using the cutter geometry information. The coordinates \((X_c, Y_c, Z_c)\) of a point along the cutting edge of tooth \( j \) are expressed as:

\[
X_{c,j}(t) = X_c(t) + r_k \cdot \sin(\phi_j), \quad Y_{c,j}(t) = Y_c(t) + r_k \cdot \cos(\phi_j), \quad Z_{c,j}(t) = Z_c(t) + r_k + (r_k - r) \cdot \tan \psi_r.
\]

Where \( r_k = \sqrt{(X_c-X_e)^2 + (Y_c-Y_e)^2} \) \( \phi_j = \Omega t + (j-1)\phi_p \)

For plane milling processes, dynamic chip thickness is evaluated by subtracting the radial distance of the present cutting edge from previous one with reference to cutter center as follows.

\[
h_{\text{lim}}(\phi_j) = \sqrt{(X_c-X_e)^2 + (Y_c-Y_e)^2} - \sqrt{(X_{\text{lim}}-X_e)^2 + (Y_{\text{lim}}-Y_e)^2}
\]

In plunge milling, the uncut chip thickness is generated by difference between z position of the present edge and that of the previous one at the same edge location angle. For the present location angle of the \( k^{th} \) edge disk element of cutter edge, the possible uncut chip thickness generated in the disk element is calculated as:

\[
h_{\text{lim}}(i, j, k, m) = z_m - z
\]

For side or plunge milling process, final dynamic chip thickness is calculated as follows:

\[
h_{\text{lim}}(i, j, k) = \max[0, h_{\text{lim}}(i, j, k, m)]
\]

where \( h_{\text{lim}}(i, j, k, m) \) represents the possible uncut chip thickness, \( m \) is previously passed teeth \((m = 1, \ldots, N_j)\) at
the same edge location angle as the present edge location
angle \( \phi \), and \( h \) is the real uncut chip thickness.

3. Dynamic cutting force formulation

The cutting force acting on the rake face of a disk element
is divided into two orthogonal components: the normal pressure force, \( dF_n(i,j,k) \), and the frictional force, \( dF_f(i,j,k) \).
It can be obtained from

\[
\begin{align*}
\frac{dh}{dt} &= K_f(T_c)\cos(\theta)h \sin(h) = K_f(T_c)\sin(\theta) \cos(h) \\
\frac{dh}{dt} &= K_f(T_c)\sin(\theta) \cos(h) \sin(h) = K_f(T_c)\cos(\theta)h
\end{align*}
\]

where \( dh = h(\theta) \cos(\alpha) \Delta \theta / \cos(\theta) \), and \( n \) and \( T_c \) are the unit vector shown in Fig. 2, respectively [18].

The relationship between the rescaled uncut chip thickness
\( h(i,j,k) \) and \( \ln(K_f) \) can be obtained using the Weibull function.

\[
\ln(K_f) = -\left( K_f - A_2 \right) \frac{(h(i,j,k))}{h(i,j,k)}
\]

Here, the set of coefficients \( (A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4) \) can be obtained through measured forces and cutting force equation using a few calibration cuts [18]. For considering size effect of cross edge radius \( er(k) \), the rescaled uncut chip thickness \( h(i,j,k) \) is defined to consider its effect on cutting force coefficients.

\[
h(i,j,k) = \frac{h(i,j,k)}{er(k) + \cos(h(i,j,k)} \tag{10}
\]

Cross edge radius \( er(k) \) varies according to tool wear or insert type which affects the size effect of cutting force coefficients. If \( er(k) \) increases, the size effect of the coefficients become significant, resulting in larger cutting force coefficients.

\[
T(\phi, \theta) = \begin{bmatrix}
\cos(\phi) \cos(\theta) & -\sin(\phi) & \cos(\phi) \sin(\theta) \\
\sin(\phi) \cos(\theta) & \cos(\phi) & \sin(\phi) \sin(\theta) \\
-\sin(\phi) & \cos(\phi) & 0
\end{bmatrix}
\]

Finally, \( dF_n \) and \( dF_f \) can be decomposed into three orthogonal force components in Cartesian coordinates using transformation matrix Eq. (11) as follows:

\[
F_x(i,j,k) = \begin{bmatrix}
K_n \sin(\theta) \sin(h(i,j,k)) - \sin(\alpha_i) \cos(h(i,j,k)) \\
K_f \sin(\theta) \sin(h(i,j,k)) (\cos(\alpha_i) - \cos(h(i,j,k))) \\
K_f \sin(\theta) \sin(h(i,j,k)) (\cos(\alpha_i) - \cos(h(i,j,k)))
\end{bmatrix}
\]

4. Experimental validations

In this section, predicted and measured cutting forces are compared in side and plunge millings and the effect of ultrasonic vibration is addressed in terms of stability and its effect on the surface. The end milling's cross edge radius effect on process damping is discussed considering surface quality as well as vibration phenomenon.
By inputting geometric values, cutting condition, transfer function and material information into the time domain model, the dynamic uncut chip can be calculated and cutting forces can be predicted considering the cutting force coefficients of the workpiece materials. For example, the unsymmetrical plunge mill with the given variables shown in Table 1 was used for machining AL6061 and the cutting forces predicted through the model are well matched with measured ones as in Fig. 3(a). Considering plunge motion in tool axial (z) direction, $F_z$ is much higher than $F_x$ and $F_y$. The lateral forces $F_x$ and $F_y$ are generated mainly by tool unsymmetrical geometry and runout.

Table 1. The geometric values of the tested plunge mill

| $l$  | 12.700 mm | $d$ | 5 deg. |
|-----|-----------|-----|--------|
| $R$ | 15.875 mm | $f_l$ | 4 deg. |
| $d_x$ | 1.659 mm | $f_d$ | 2 deg. |
| $d_y$ | 3.175 | $q$ | 6 deg. |
| $d_z$ | 0.270 mm |

For ball-end side milling, the cutting forces are predicted for S45C material using the presented model in section 3 and compared as shown in Fig. 3(b). In terms of force prediction, the model can predict the magnitude and phases with less than 10 % error.

For tooling axial (z-axis) ultrasonic vibration assisted milling, the simulation considers the ultrasonic motion of the frequency of around 39 kHz and amplitude of 1.6 μm into the dynamic center motion in Eq. (1). The displacement of the tool center in the presence of axial ultrasonic vibration assistance is simulated to be reduced compared to the machining without ultrasonic vibration assistance as illustrated in Fig. 4. So it is expected to improve surface quality with ultrasonic vibration as shown in the milled surfaces of Fig. 5, where the forced and chatter marks were suppressed with ultrasonic vibration assistance. The simulation and experiment results validate that ultrasonic vibration assistance increases machining stability which leads to the reduction of tool vibration and surface quality improvement.

In addition to the above addressed results regarding the simulations on cutting forces and cutter’s dynamic motion, cutter edge radius’s effect on process damping is investigated in the following. Depending on tool wear or tool edge preparation, cutting edge radius varies which affects the process damping and surface quality. So it is important to consider the cutting edge radius of the tool as one of the variables for predicting machining stability. In this article, a worn out blunt-edge is compared against a new sharp-edge.

For example, optical 3D non-contact metrology system has been used to measure edge radius of a worn-out end mill as in Fig. 6.

The measured cross edge radiuses of new tool and worn-out tool are 5.28 μm and 42.86 μm, respectively. A series of end milling of Al6061-T6 were performed with $AE = 2$ mm, $AP = 3$ mm, feed per tooth = 0.05 mm and various RPMs. The
surface roughness according to each condition was measured and compared according to new and worn tools as shown in Table 2.

**Table 2. Comparisons of measured roughness (Ra)**

| Test no. | RPM | 1      | 2      | 3      |
|----------|-----|--------|--------|--------|
|          |     | 5000   | 6500   | 8000   |
| Surface roughness (Ra) | μm with a new tool | 0.1064 | 0.2823 | 1.2559 |
| Surface roughness (Ra) | μm with a worn-out tool | 0.1414 | 0.1601 | 0.1731 |

As shown in Table 2, it is observed that the roughness is reduced at lower RPM due to process damping. At RPM 5000, vibration was damped for both new tool and worn-out tools due to stronger process damping induced by its high ratio between vibration frequency and spindle rotation one. At RPM 5000, the surface roughness with the new tool is lower than the roughness with the worn-out tool since the new tool’s cutting edge radius is sharper than the worn-out tool’s one. At unstable conditions with RPM 6500 and 8000, worn-out tool’s Ra becomes lower than new tool’s one because worn-out tool’s edge radius generates more damping than the new tool’s edge radius.

The worn and new end mills were used to validate the time domain simulation of side milling process. The modal parameters are taken for x and y directions considering side milling operation as shown in Table 3. For example, the cutter center position’s movement is simulated at RPM 8000 using the proposed time domain solution and FFTs are analysed.

**Table 3. Measured modal parameters of the end mill attached to Roeders 760**

| Mode no. | Natural Frequency (Hz) | Damping Ratio (ζ) | Modal Stiffness (k) (N/m) |
|----------|------------------------|------------------|--------------------------|
| x        | 1                      | 4843              | 0.019                    | 14020267 |
|          | 2                      | 1068              | 0.012                    | 57030270 |
|          | 3                      | 2100              | 0.034                    | 39301050 |
|          | 4                      | 6706              | 0.020                    | 110470627 |
| y        | 1                      | 4843              | 0.021                    | 13474275 |
|          | 2                      | 1068              | 0.012                    | 46792539 |
|          | 3                      | 4431              | 0.018                    | 32886777 |
|          | 4                      | 1950              | 0.026                    | 39984415 |

As shown in Fig. 7, the sound signal’s FFT shows that the chatter frequency around 4.8 kHz is suppressed with the damping effect of the increased edge radius of the worn tool. In the simulation of cutter center displacement, when the edge radius is input into e(k) of the proposed time domain solution, vibration with around 4.8 kHz is damped with the worn tool as shown in Fig. 8. As cross edge radius increases due to tool wear, the edge and flank surface of the worn tool has more contact with workpiece compared to the one of the new tool. In the simulation, the contact area is simulated by calculating intersection area between cutter flank and workpiece and its effect on the process damping is estimated. The damping effect can be added into Eq. (14) or damping force can be calculated and integrated into cutting force model as well. In this article, empirical approach has been used to model process damping phenomenon due to its complex mechanism and further details on analytical approach will be addressed in next publications.

In the both simulation and experiment, it is obvious that worn edge radius 42.86 μm generate more process damping at high frequency around 4.8 kHz than sharp edge radius 5.28 μm. If RPM increases further than RPM 8000, its process damping effect is expected to be reduced since the vibration wave length on the machined surface increases at higher RPM and the contact area between tool flank and vibration wave of the machined surface becomes less.

**Fig. 8 FFTs of displacements of new and worn-out tools (RPM = 8000)**

5. Conclusion

This article proposes the dynamic chip thickness formulation based on cutter center tracking method which can be applied to side, plunge and ultrasonic vibration assisted milling. Since the effect of the cross edge radius is significant for process damping, the process model integrates the cross edge radius and its effect on process damping. The experimental results have been in good agreement with the predicted ones from the proposed time domain solution. The high frequency component around 4.8 kHz of the milling process has been found to be damped with the worn edge radius 42.86 μm of the tested end mill for RPM 8000 as demonstrated in both experimental and simulation results. The prediction model is able to replace trial and error approach in determining cutting condition in milling process considering dynamic response and process damping phenomenon. It has been applied to industries for stabilizing process and increasing material removal rate. So far die/mold, oil/gas, aerospace, and component machining companies have utilized the proposed models to find optimal cutting condition for side and plunge milling process.

In near future, the process damping model will be further refined to improve prediction accuracy considering different type of tool wear such as flank wear. Further details on the
development and industry applications will be shared in next articles.

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