A Model for Weibull Deteriorate Items with Price Dependent Demand Rate and Inflation

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Abstract

The present paper deal with an inventory model which is developed for most of the physical goods undergoes decay or deterioration over time. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. The deterioration rate follows the Weibull distribution with two parameters. Demand rate is assumed as price dependent in linear form. It is an ordinary fact that unique price of items attracts more customers. The model is solved numerically by when shortages are partially backlogged with inflation. Analytical and numerical solution of the model is obtained to verify the optimal solution. Comprehensive sensitivity analysis has been given for viewing the result of variation in the parameter. The model is solved analytically by maximizing the total profit.

Keywords: Deteriorating Items, EOQ Model, Inventory, Price Dependent Demand Rate

1. Introduction

It has been seen that delivery of the goods is affected by the price dependent demand of the product. The attractive price of the items motivates the customer to buy more goods and that situation creates the greater demand of the goods. This condition motivates the retailer to increase their order quantity and the retailers earn the more profit to increase their revenue. The condition become more complex when the inventory is deteriorates in nature. Deterioration of goods is one of the important factors in any inventory and production system.

The several researchers have worked for inventory with deteriorating items in recent years because most of the physical goods undergo decay or deterioration over time. A model is developed for an exponentially decaying inventory¹. Inventory models with a time dependent rate of deterioration were considered². A deterministic lot-size inventory model for deteriorating with shortages and a declining market was introduced by Wee³. Deterministic models are suggested for perishable inventory with stock dependent rate and non-linear holding cost⁴. Some of the significant recent work has done in this field of Structural properties of an inventory system with deterioration and trended demand⁵. Optimum ordering policy is developed for decaying items with stock dependent demand under inflation in a supply chain⁶. A production-inventory problem of a product is introduced in inventory system with time varying demand as well as production and deterioration rates also⁷. An integrated production inventory model is defined with Expiration date and uncertain lead time in an inventory system⁸. Inventory model for ameliorating items for price dependent demand rate was proposed⁹. A production-inventory model for a deteriorating item is introduced with trended demand and shortages¹⁰. An ordering policy is formulated with stock dependent demand with partial backlogging and inflation¹¹. Shortages were also considered in this model by them. An inventory Model for deteriorating items is proposed with Weibull deterioration with time dependent demand and Shortages¹². Integrated vendor-buyer cooperative model is developed with multivariate demand and

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progressive credit period. Optimal Inventory Model with Weibull deterioration with Trapezoidal demand and shortages was introduced in this system. The holding cost considered as a constant. Optimal ordering policy for deteriorating items is proposed with stock dependent demand under two-warehouse facility. An inventory model for deteriorating items is introduced with shortages and time varying demand. A Multi Item Inventory Model for Deteriorating Items with Expiration Date and Allowable Shortages. Developing an Inventory Mathematical Model with Deterioration Variables for Discounted Stochastic Goods. An inventory model for decaying items, considering multi variate consumption rate with partial backlogging. Multilevel inventory techniques for minimizing cost-a case study.

In most of the research paper demand rate is considered as time dependent in all situations. But in realistic demand is not always dependent on time. A discount price attracts more customers to buy the product in a supermarket.

In this model, we consider here an optimal inventory model for Weibull deteriorating items. Demand rate is price dependent in linear form.

This paper is formulated with Selling Price Dependent Demand with Partially Backlogged under Inflation. We solve the model to optimize the total profit. Model is illustrated with numerical examples and Comprehensive sensitivity analysis.

2. Assumptions and Notations

The proposed inventory model is developed under the following assumptions and notations:

2.1 Assumptions
The following assumptions are made for development of mathematical model:

- Demand rate is considered as selling price dependent of the items in linear form
  \[ D(s) = a - bs \]

  Where, \( a > 0, 0 \leq b \leq 1 \), \( b \) is the selling price dependent demand rate parameter.
- The ordering cost \( A_o \) is constant.
- The inflation is also consider, the inflation rate is assume \( r \).

Which is define as follows:

\[ f(t) = e^{-rt} \quad r > 0 \]

- The cycle length is assumed \( 0 < t < T \).

2.2 Notations

The following notations are made for development of mathematical model:

- \( I(t) \) be the inventory level at time \( t \) (\( 0 \leq t < T \)).
- \( T \) is the cycle length.
- \( q \) is The ordering quantity is.
- \( A_o \) is the ordering cost.
- \( s \) is the selling price per unit item.
- \( h \) is the inventory holding cost per unit item per unit time.
- \( C_1 \) is the shortage cost per unit item per unit time.
- \( C_2 \) is the deterioration cost per unit item per unit time.
- \( C_3 \) is the lost sale cost per unit item per unit time.
- \( D(t) = a - bs \ (a > 0, b > 0) \) is the Demand rate.
- The deterioration of time as follows by Weibull parameter (two) distribution \( \theta (t) = \alpha \beta^t \) where, \( 0 < \alpha < 1 \) is the scale parameter and \( \beta > 0 \) is the shape parameter.
- \( r \) is the inflation rate.
- \( \gamma \) is the partially backlogging rate, where \( 0 < \gamma < 1 \).
- \( Q \) is the optimal inventory level.
- \( S(T, s) \) is expected total profit.

3. Formulation of the Model with Selling Price Dependent Demand with Partially Backlogged Shortages under Inflation

The inventory is depleting during time \( t \). Due to the demand and deterioration of the items. The inventory level becomes zero at the time \( t \) and shortages are occurs in the period \( (t_1, T) \) which are partially backlogged. The length of the cycle is \( T \). Let \( I(t) \) be the inventory level at time \( t \) (\( 0 \leq t < T \)).

Differential equations of the inventory system

\[ I'(t) + a \beta^t \ (\beta-1) \ I(t) = -(a-b) \ I(t) \quad 0 \leq t \leq t_1 \quad (3.1) \]

\[ I'(t) = -(a-b) \ t_1 \leq t \leq T \quad (3.2) \]
With \( I(t) = 0 \) at \( t = t_1 \)
From equation (3.1) we get:
\[
I(t) = \left[ 1 - \alpha t \right] \left[ a(t_1 - t) - bs(t_1 - t) + \frac{aa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) - \frac{bsa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) \right] 0 \leq t \leq t_i
\]
(3.3)

From equation (3.2) we get
\[
I(t) = \left[ a(t_1 - t) - bs(t_1 - t) \right] t_i \leq t \leq T
\]
(3.4)

3.1 Expected Holding Cost under the Inflation Rate \( r \)
The holding cost during the time period 0 to \( t_1 \)
\[
H = \int_0^{t_1} e^{-rt} \cdot I(t) \, dt
\]
The total holding cost during the time period 0 to \( t_1 \)
\[
H = h \int_0^{t_1} e^{-rt} \cdot I(t) \, dt
\]
\[
H = h \int_0^{t_1} e^{-rt} \left[ 1 - \alpha t \right] \left[ a(t_1 - t) - bs(t_1 - t) + \frac{aa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) - \frac{bsa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) \right] \, dt
\]
Now total holding cost will be:
\[
H = h \left[ a(t_1 - t) - bs(t_1 - t) \right] + \frac{aa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) - \frac{bsa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right)
\]
(3.5)

3.2 Expected Deterioration Cost under the Inflation Rate \( r \)
The total deterioration cost for the period of 0 to \( t_1 \) is given by,
\[
D_T = C_2 \int_0^{t_1} e^{-rt} \cdot \alpha \theta \left[ 1 - a(t_1 - t) - bs(t_1 - t) + \frac{aa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) - \frac{bsa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) \right] \, dt
\]
\[
D_T = C_2 \left[ a(t_1 - t) - bs(t_1 - t) + \frac{aa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) - \frac{bsa}{(\beta + 1)} \left( t_1^{(\beta + 1)} - t^{(\beta + 1)} \right) \right] \, dt
\]
(3.6)

3.3 Expected Shortage Cost under the Inflation Rate \( r \)
The total shortage cost for the period \( t_i \) to \( T \) is given by,
\[
sh = -C_3 \int_{t_i}^{T} e^{-rt} \cdot I(t) \, dt
\]
\[
sh = -C_3 \left[ a(t_1 - t) - bs(t_1 - t) \right] \, dt
\]
(3.7)

3.4 Expected Lost Sale Cost under the Inflation Rate \( r \)
The lost sale cost is given by,
\[
\text{Lost sale cost} = -C_4 \int_{t_1}^{T} e^{-rt} \cdot (1 - \gamma)(a - bs) \, dt
\]
3.5 Expected Total Profit

From the above equations the total profit per unit time can describe:

\[ S(T, s) = s(a - bs) - \frac{1}{T} [\text{Ordering cost + Expected holding cost} + \text{Expected deterioration cost + Expected shortage cost} + \text{Lost sale cost}] \]

\[
S(T, s) = s(a - bs) - \frac{1}{T} \left[ A_T + h(a - bs) \left( \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right) - \frac{a}{(\beta + 1)(\beta + 2)} t_1^{(\beta + 2)} + \frac{\alpha r}{(\beta + 1)} t_1^{(\beta + 3)} \right] \]

\[
+ C_a \beta (a - bs) \left[ \frac{1}{\beta + 1} t_1^{(\beta + 1)} + \frac{\alpha r}{(\beta + 1)^2} t_1^{(\beta + 2)} \right] \]

\[
- \frac{\alpha}{(\beta + 2)} t_1^{(\beta + 2)} + \frac{\alpha r}{(\beta + 1)(\beta + 2)} t_1^{(\beta + 3)} \right] \]

\[
+ C_b \left( a - bs \right) \left[ \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right] \left[ t_1 - \frac{T}{2} \right] \]

\[
+ C_b (1 - \gamma)(a - bs) \left[ \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right] \left[ t_1 - \frac{T}{2} \right] \]

\[
(3.10)
\]

\[
\text{And} \quad \frac{\partial S(T, s)}{\partial s} = 0 \quad (3.11)
\]

The optimal value of \( T^* \) and \( s^* \) can be calculated simultaneously by equation (3.10) and equation (3.11) by using the Mathematica 8.0. The optimal values of profit function \( S(T, s) \) under inflation also determined by equation (3.9). Sufficient conditions for maximizing profit function \( S(T, s) \) are satisfied by the optimal value of \( T^* \) and \( s^* \).

\[
\frac{\partial^2 S(T, s)}{\partial T^2} < 0, \quad \frac{\partial^2 S(T, s)}{\partial s^2} < 0 \quad (3.12)
\]

And \( \frac{\partial^2 S(T, t_1)}{\partial T^2} \cdot \frac{\partial^2 S(T, t_1)}{\partial s^2} - \frac{\partial^2 S(T, t_1)}{\partial T \partial s} > 0 \) And at \( T = T^* \) optimal value \( s = s^* \) \( (3.13) \)

3.6 Mathematical Formulation of the Model

Our key purpose to maximize the profit function \( S(T, t_1) \) the necessary condition for maximizing the profit are

\[
\frac{\partial S(T, t_1)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial S(T, t_1)}{\partial s} = 0
\]

\[
\frac{\partial S(T, s)}{\partial t_1} = 0
\]

\[
+ \frac{1}{T^2} \left[ A_T + h(a - bs) \left( \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right) - \frac{\alpha}{(\beta + 1)(\beta + 2)} t_1^{(\beta + 2)} \right] \]

\[
+ \frac{\alpha r}{(\beta + 1)(\beta + 2)} t_1^{(\beta + 3)} \right] \]

\[
+ C_b \left( a - bs \right) \left[ \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right] \left[ t_1 - \frac{T}{2} \right] \]

\[
+ C_b (1 - \gamma)(a - bs) \left[ \frac{1}{2} t_1^3 + \frac{r}{6} t_1^3 \right] \left[ t_1 - \frac{T}{2} \right] \]

\[
(3.9)
\]

3.7 Numerical Illustration

The model has been explored numerically as well. We have considered the following data for the study which based on previous study.

Example: Let us consider, \( A = 750, a = 200, b = 0.25, \)
\(
\alpha = 0.1, \beta = 1.3, h = 1.2, C_1 = 1.1, C_2 = 0.02, t_1 = 1.25, \)
\( r = 0.5, \gamma = 0.2 \)
We solve the problem numerically with the help of computer software Mathematica-8.0, and calculate the optimal value of \( S(T,s) \), \( T^* \) and \( s^* \) simultaneously by equation (3.9), equation (3.10) and equation (3.11).

\[
S(T,s) = 1582.63, \quad T^* = 3.24169, \quad s^* = 9.8325.
\]

3.8 Sensitivity Analysis and Observations

We have study the effects of changes of the parameters on the optimal values of \( S(T,s) \), \( T^* \) and \( s^* \) derived by proposed method. The sensitivity analysis is performed in view of the numerical example given above. We have executed sensitivity analysis by changing the parameters \( \alpha \), \( \alpha \), \( \beta \) and \( b \) as +20%, +50%, −20% and −50%. All remaining parameters have original values with respect to these changes. The corresponding changes in \( S(T,s) \), \( T^* \) and \( s^* \) are showed in below (Table 1).

| parameters | % change | \( T^* \) | \( s^* \) | \( S(T,s) \) |
|-----------|---------|---------|---------|---------|
| \( \alpha \) | −50 | 4.63793 | 10.60215 | 79.1315 |
| | −20 | 3.80958 | 10.05240 | 791.315 |
| | +20 | 2.59218 | 9.66750 | 2579.6869 |
| | +50 | 1.80571 | 9.79768 | 4510.4955 |
| | −50 | 3.28645 | 10.84990 | 1609.5347 |
| | −20 | 3.25390 | 10.43818 | 1593.8666 |
| | +20 | 3.15173 | 9.22810 | 1571.3933 |
| | +50 | 3.17248 | 8.72388 | 1555.7257 |
| \( \alpha \) | −50 | 3.39729 | 11.92046 | 1584.6804 |
| | −20 | 3.30004 | 10.45436 | 1583.3134 |
| | +20 | 3.20408 | 9.65643 | 1582.7047 |
| | +50 | 3.10878 | 9.56719 | 1582.7595 |
| | −50 | 3.57634 | 10.47387 | 332.3532 |
| | −20 | 3.41023 | 10.97911 | 923.1480 |
| \( \beta \) | −50 | 3.14012 | 9.69506 | 2019.8802 |
| \( b \) | −50 | 2.97876 | 9.80501 | 2405.5976 |

We study above (Table 1.) brings out the following.

We observed that as parameter \( a \) and \( b \) increase, optimal value of \( T^* \) and \( s^* \) decrease while the average total profit \( S(T,s) \) of an inventory system increases, whereas parameter \( \alpha \) and \( \beta \) decrease, optimal value of \( T^* \) and \( s^* \) increase while the average total profit \( S(T,s) \) of an inventory system decreases. It is interesting to observe that deterioration parameter \( \alpha \) increase, optimal value of \( T^* \) and \( s^* \) decrease while the average total profit \( S(T,s) \) of an inventory system decreases. If deterioration parameter \( \beta \) decrease, optimal value of \( T^* \) and \( s^* \) increase while the average total profit \( S(T,s) \) of an inventory system increases.
shortage cost are considered with inflation in this model. Manager of the industry always take care of selling price parameters which affect the profit quickly. The numerical examples are given to illustrate the development of model. Comprehensive sensitivity analysis (Table 1.) has been carried out for showing the effect of variation in the parameter. The model is solved analytically by maximizing the total profit. In the numerical examples we found the maximum value of profit.

5. References

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