The research of optimal selection method for wavelet packet basis in compressing the vibration signal of a rolling bearing in fans and pumps

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Abstract. Compressing the vibration signal of a rolling bearing has important significance to wireless monitoring and remote diagnosis of fans and pumps which is widely used in the petrochemical industry. In this paper, according to the characteristics of the vibration signal in a rolling bearing, a compression method based on the optimal selection of wavelet packet basis is proposed. We analyze several main attributes of wavelet packet basis and the effect to the compression of the vibration signal in a rolling bearing using wavelet packet transform in various compression ratios, and proposed a method to precisely select a wavelet packet basis. Through an actual signal, we come to the conclusion that an orthogonal wavelet packet basis with low vanishing moment should be used to compress the vibration signal of a rolling bearing to get an accurate energy proportion between the feature bands in the spectrum of reconstructing the signal. Within these low vanishing moments, orthogonal wavelet packet basis, and ‘coif’ wavelet packet basis can obtain the best signal-to-noise ratio in the same compression ratio for its best symmetry.

1. Introduction

A rolling bearing is a general mechanical component which has been widely used in various rotating machineries. Its performance and reliability has great effect on the rotating machinery. As a result, the research about vibration monitoring of rolling bearings have been developed widely[1][2][3]. A Rolling bearing belongs to large-scale device which needs to be replaced frequently and the working environment of it is quite harsh. These conditions could bring a lot of limitations to traditional wired supervisory systems. Thus, wireless monitoring has become a hotspot of recent research[4][5][6]. However, the amount of data contained within a vibration signal is generally very large, and could bring great pressure to wireless transmission. So the data compression in a vibration signal has significance in terms of decreasing transmission bandwidth and increasing storage capacity.
Presently, common methods which are used for compressing a vibration signal mainly include
data sparseness, orthogonal transformation and predictive coding, etc. Among them, wavelet
transform is especially useful since it has the capability to depict the partial characters in time
domain and frequency domain. However, there are lots of wavelet bases that could be used in wavelet
transform and the vibration signal of a rolling bearing has its own characteristics such as complicated
transient signals and abundant high frequency components. So, there exists two questions: one is how
to choose an appropriate wavelet basis, the other is how to preserve the characters of the vibration
signal which is needed to be compressed in a required compression ratio. In this paper, we first
propose the general compression method which uses wavelet transform. Then we analyse the
characteristics of the vibration signal of a rolling bearing and analyse several important properties of
wavelet bases including: compactly supported length, vanishing moment, regularity, symmetry, and
orthogonality. Based on the analysis, a method to choose the appropriate wavelet basis is proposed.
This method and conclusion will have great reference significance towards further research in this
field.

2. Characteristics of the vibration signal of rolling bearings
The frequency range of the vibration signal of a rolling bearing has a great relationship with the type
of fault. Acceleration signal is often selected as the measurement parameter when making fault
diagnosis toward a rolling bearing. Besides trend terms, there are a lot of high frequency noise in
acceleration signal.

Suppose that \( F(n\Delta t) \) stands for the discrete time series of the vibration signal formed by
sampling, \( \Delta t \) stands for the sampling period. Consider that, the vibration signal of a rolling bearing
can always be represented as a combination composed of harmonics which the characteristic
frequency is the fundamental frequency \( \omega_1 \), and each harmonic has its amplitude \( A_i \) and phase \( \theta_i \).
The vibration signal can be represented as follows:

\[
F(n\Delta t) = \sum_{i=1}^{N} A_i \cos(\omega_i n\Delta t + \theta_i)
\]  

Moreover, in some faults (such as local defects on rolling elements and race way), most energy will
be concentrated to high frequencies. These faults can produce transient signals \( F_{t}(n\Delta t) \) in time
domain signal. In addition, consider noise signal \( F_{s}(n\Delta t) \), a complete representation of the vibration
signal can be represented as follows:

\[
F(n\Delta t) = \sum_{i=1}^{N} A_i \cos(\omega_i n\Delta t + \theta_i) + F_{t}(n\Delta t) + F_{s}(n\Delta t)
\]  

3. Compression method based on wavelet packet transform(WPT) and threshold function
The process of compression coding is usually divided into three steps: sampling, quantization and
coding. The number of sampling is determined by Nyquist theorem, So compression ratio(CR) and
signal-to-noise ratio(SNR) are mainly determined by quantization. The relativity of time domain can be
removed maximally through wavelet transform. Moreover, the total transmit number of bits can be compressed by allocating a different number of bits to each sub-band based on its energy proportion.

However, localities of time domain and frequency domain can go better and worse respectively, along with the decomposition level of wavelet transform increasing. Thus, distinguishing signal and noise in high frequencies can be difficult. Based on the characters of the vibration signal of a rolling bearing, a compression method using WPT is proposed in this paper. The basic idea is to ulteriorly decompose the wavelet subspace so that the frequency window, which goes wider along with the decomposition level, can be decomposed to a series of smaller windows. Thus, the resolution in high frequencies increases and the SNR of reconstructing the signal can be improved\(^{[14]}\).

The WPT has the following definition: supposing \(\{h_k\}_{k \in Z}\) and \(\{g_k\}_{k \in Z}\) are Quadrature Mirror Filter(QMF), \(\varphi(t)\) is scaling function, \(\psi(t)\) is wavelet function. Assuming that \(\mu_0(t) = \varphi(t)\), \(\mu_1(t) = \psi(t)\), the two-scale equation can be shown as follows:

\[
\mu_0(t) = \sqrt{2} \sum_{k \in Z} h_k \mu_0(2t - k) \tag{3}
\]

\[
\mu_1(t) = \sqrt{2} \sum_{k \in Z} g_k \mu_0(2t - k) \tag{4}
\]

The forms in frequency domain are:

\[
\hat{\mu}_0(\omega) = H\left(\frac{\omega}{2}\right) \hat{\mu}_0\left(\frac{\omega}{2}\right) \tag{5}
\]

\[
\hat{\mu}_1(\omega) = G\left(\frac{\omega}{2}\right) \hat{\mu}_0\left(\frac{\omega}{2}\right) \tag{6}
\]

From the point of view in filtering signal \(f\), the process of WPT is to filter the signal using a low-pass filter and a high-pass filter respectively. Thus, a low frequency signal \(Hf\) and a high frequency signal \(Gf\) can be obtained. Then, execute the foregoing process iteratively. The whole process is shown in table 1:

| F   |             |             |
|-----|-------------|-------------|
| Hf  | Gf          |             |
| HHf | GHf         | HGf         |
| GGF |             |             |
| ...... | ...... | ...... |

Table 1. the structure of wavelet packet decomposition.

Compression method using the threshold of wavelet was proposed firstly by Donoho in the 1990s\(^{[15]}\). The main idea is to dispose wavelet coefficients which are more and less than certain thresholds respectively after WPT. Then reconstruct the signal using the wavelet coefficients after disposed.
Threshold functions mainly include hard threshold and soft threshold, it is very important to choose an appropriate threshold function among them. Massive literatures gives the method of choosing threshold function\textsuperscript{[16][17][18]}, and it can mainly classified into four kinds\textsuperscript{[9]}:

- Fixed Threshold.
- Unbiased Likelihood Estimating Threshold.
- Heuristic Threshold.
- Minimax Threshold.

In recent years, many compression methods using innovative threshold functions were proposed, such as Birge-Massart strategy\textsuperscript{[20]}, sparse algorithm\textsuperscript{[21]}, removing zero value\textsuperscript{[22]}, specific energy threshold\textsuperscript{[23]}, energy relativities threshold\textsuperscript{[24]}, multilevel threshold\textsuperscript{[25]}, and gradient threshold\textsuperscript{[26]}, etc. These methods enrich the theory of threshold compression maximally, and makes it to become the basic method of wavelet compression.

4. Optimal selection of wavelet basis

Based on the whole analysis above, general requirements of compressing the vibration signal can be shown as follows:

- Decreasing relativity in time domain as much as possible.
- After transforming, making the number of wavelet coefficients as few as possible and the amplitudes of wavelet coefficients as small as possible.
- Paying special attention to reserve information which has relation to various faults at the condition of certain CR.

4.1. Horizontal comparison of different kinds of wavelet bases at same order vanishing moment

4.1.1. Orthogonal wavelet. An orthogonal wavelet can remove the relativity between various sub-bands maximally. However, it can also decrease signal-to-noise ratio of reconstructing the signal for its nonlinear phase. So an orthogonal wavelet basis with approximately linear phase can be an optimal selection.

A "Daubechie" wavelet is the most classical orthogonal wavelet, it has the following constructing formula:

\[
|H(\omega)|^2 = \left(\cos \frac{\omega}{2}\right)^{2N} P_N (1 - \cos \frac{\omega}{2})
\] (7)

Among them, \(H(\omega)\) is the conjugate mirror filter of scaling function and \(P_N\) is a polynomial which meets:

\[
P_N(y) = \sum_{k=0}^{N-1} \binom{N-1+k}{k} y^k
\]

The filter of \(H(\omega)\) with the length of \(n\) can be constructed using real root(complex root) pairs of \(P_N\) shown as follows:

\[
|H(\omega)|^2 = N U(z)_N U(1/z)
\] (8)
Where these pairs are symmetric with unit circle and \( U(z) \) is the lowest degree polynomial root which meets: 
\[
|U(z)|^2 = P_n(\sin^2(\frac{\omega}{2})).
\]
Moreover, if the modular square of \( U(z) \) is less than 1, \( H(\omega) \) is a "daubechies" wavelet. While if it is more than 1, \( H(\omega) \) is a "Symlets" wavelet. In addition, based on the demands made by R. Coifman, a "Coiflets" wavelet, which has better symmetry, had been constructed by Ingrid Daubechie.

### 4.1.2. Biorthogonal wavelet

A Biorthogonal wavelet can be constructed more flexibly since the conditions for biorthogonality is looser than orthogonality\(^{[27]}\). A Biorthogonal wavelet, which has the best performance, is a CDF biorthogonal wavelet which was made by Cohen, Daubechies and Feauveau using a spline function. This biorthogonal wavelet has four characteristics as shown below\(^{[28]}\):

- finite number of filter banks.
- all wavelet coefficients are rational numbers.
- Symmetry.
- linear phase.

However, after transformed, there are still related redundancies that exist between various sub-bands since the decomposition using a CDF biorthogonal wavelet is not an orthogonal decomposition. Moreover, the length \( L \) of the analysis filter is determined by the order \( N \) of the vanishing moment of analyzing the wavelet and the order \( \tilde{N} \) of regularity of reconstructing the wavelet is shown as follows:

\[
L = 2N + \tilde{N} - 1
\]

It can be found that the compactly supported length of a CDF biorthogonal wavelet is at least two times of an orthogonal wavelet. The waves of typical orthogonal and biorthogonal wavelet bases in time domain are shown in figure 1:

![Figure 1. The time-domain waveform and spectrum graph of vibration signal.](image)

### 4.2. Tradeoff between compactly supported length and vanishing moment

Based on the analysis in section 2, vibration signals of rolling bearings are composed of harmonics, transient signals and noise. Wavelet basis function with higher order of vanishing moment
can decrease amplitudes of wavelet coefficients effectively in high scale, since harmonics are infinitely differentiable. Moreover, in literature [27], it was proven that the reconstructing performance will become better as the order of regularity goes higher. However, the improved velocity of the performance also has relationship with the length of the analysis filter \( h(n) \), that is, the improved performance becomes not obvious as the length of the analysis filter goes longer.

5. Experiment

5.1. Experiment 1: choosing wavelet basis with appropriate vanishing moment

5.1.1. Experimental design. Using an instance of a fault signal of a rolling bearing, we will compare the compressing performances using various wavelet bases in different orders of vanishing moment. On 2008.12.25, there was a multistage centrifugal pump in China National Petroleum Corporation that was working in an unusual service conditions and was diagnosed as wear failure of bearings. The waveform and spectrum graph of the vibration signal are shown in figure 2, the sampling frequency is 20480Hz, the sampling number is 1024.

![Figure 2. The time-domain waveform and spectrum graph of vibration signal.](image)

The process of wavelet compression coding has the following steps:
- Choosing wavelet basis.
  Since only finite supported wavelet basis can be used in discrete wavelet transform, we only consider four kinds of orthogonal wavelet: haar, db, sym, coif, and a biorthogonal wavelet: bioNr.Nd and a truncated wavelet: dmey.
- Wavelet packet decomposition.
  Using the wavelet basis above, the wavelet packet decomposition will be done to obtain a whole wavelet packet tree composed of 16 sub-bands.
- Compressing using various thresholds.
  First, we calculate the energy of each sub-band and schedule a sequencing based on each energy. And then, based on the compression ratio (for instance, 30% and 70%), the compressing threshold of each sub-band can be calculated by the energy sequence. Each wavelet coefficient whose amplitude is under their threshold in each sub-band will be set to zero.
- Encoding wavelet coefficients.
Encode residual wavelet coefficients using Huffman coding.

- Reconstructing signal.

The process of reconstructing is the inverse process of compressing, and the spectrum graph of each reconstructing signal using different wavelet basis will be shown.

5.1.2. Experimental analysis. In compressing the signal, we want to get a higher CR on the condition of certain SNR. However, considering this real vibration signal, there are abundant mutational components in high frequency which contain the information of faults. So the main question we consider is how to reserve these mutational components in a certain CR by choosing an appropriate wavelet basis.

Reconstructing SNRs using six foresaid wavelet basis at the CR of 30% and 70% that are shown in table 2 and table 3. The SNR has the following formula:

\[ SNR = 10 \times \log_{10} \frac{\sigma^2}{D} \]  

(10)

Where, \( \sigma^2 = \frac{1}{N} \sum_{i=1}^{N-1} (x_i - \bar{x})^2 \), \( D = \frac{1}{N} \sum_{i=1}^{N-1} (x_i - \bar{x})^2 \), \( \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \). \( x_i \) stands for the discrete component of the original signal; \( \bar{x} \) stands for the discrete component of reconstructing signal; \( \bar{x} \) stands for the average of \( x_i \). The spectrum graph of each reconstructing signal is shown in figure 3 and figure 4.

| Wavelet basis | Vanishing moment | Vanishing moment |
|---------------|------------------|------------------|
| haar          | 9.044            | 1.1              |
| db            | 9.451            | 8.621            |
| sym           | 9.366            | 6.8              |
| coif          | 9.366            | 5.728            |
| dmem          | 8.358            | 5.5              |
| biorNr.Nd     | 8.016            | 7.852            |

Table 2. SNRs of each wavelet basis in the condition of 30% CR.
Figure 3. Reconstructing spectrum of each wavelet basis in the condition of 30% CR.

Table 3. SNRs of each wavelet basis in the condition of 70% CR.

| Wavelet basis | haarr | db1 | sym | coif | dmez | bior | Nr | Nd |
|---------------|-------|-----|-----|------|------|------|----|----|
| Vanishing moment | 3.244 | 3.363 | 1.1 | 3.021 |
| 2 | 3.332 | 3.259 | 3.273 | 1.3 | 3.192 |
| 3 | 3.289 | 3.085 | 3.252 | 1.5 | 3.227 |
| 4 | 3.104 | 3.051 | 3.078 | 2.2 | 2.870 |
| 5 | 3.047 | 3.000 | 2.954 | 2.4 | 2.981 |
| 6 | 2.913 | 2.989 | 4.4 | 2.724 |
| 7 | 2.650 | 2.650 | 5.5 | 2.752 |
| M | 2.474 | 6.8 | 2.428 |
By the upper experiments of comparing six kinds of wavelet basis, it can be found that the frequency spectrum of reconstructing signals are approximately same in high frequency. However, in each reconstructing signal, there is big difference in low frequency. The spectrum of each low frequency sub-band can be exactly reconstructed by wavelet basis with low order of vanishing moment. That is, the energy of fundamental frequency component is greater than each harmonic. However, with the increasing of the order of vanishing moment, the energy of fundamental frequency component decreases and the performance of compression system become worse since the SNR was reduced.

The reason the results in this difference can be composed of the following two parts:

• The wavelet basis with higher order of vanishing moment is corresponding to longer compactly supported length. So, more wavelet coefficients will be produced and removed at the condition of the same CR.

• The wavelet basis with a higher order of vanishing moment makes more energy concentrated at high frequency. So more wavelet coefficients in low frequency will be removed at the condition of same CR.

Thus, in the condition of high CR, more energy will be concentrated to the middle and low frequency sub-band(800Hz~2000Hz), and the middle and high frequency sub-band (5500Hz~7000Hz),and this will make different degrees of distortion (even loss) in the low frequency
sub-band(0Hz~800Hz), the middle frequency sub-band (2000Hz~5500Hz) and the high frequency sub-band(7000Hz~10240Hz).

5.2. Experiment 2: choosing appropriate wavelet basis

It can be known from experiment [1], that the wavelet basis with low order of vanishing moment is more suitable to compress the vibration signal of a rolling bearings. Now, we will construct six compression system using haar, db2, sym2, coif1, bior1.1, bior1.3 and bior1.5, six kinds of wavelet basis. Through different CR, we can get the relationships between CR and SNR of each wavelet basis and compare the performance of each compression system.

5.2.1. Experimental design. We still use the vibration signal in experiment [1]. Through various CR, the relation curves between CR and SNR of different wavelet bases are drawn.

CR and SNR are defined as follows:

- CR is the ratio of bits after compressing and bits of original signal.
- SNR has the following formula.

\[
SNR = 10 \times \log_{10} \frac{\sigma^2}{D}
\]  

The spectrum graph of each reconstructing signal is shown in figure 3 and figure 4.

5.2.2. Experimental analysis. The relation curves between CR and SNR of different wavelet bases are shown in figure 5:

From figure 5, it can be found that, at the condition of low CR, the reconstruction SNRs of coif1 and bior1.5 are obviously better than other wavelets. The main reason causing this result is that, coif1 has the best symmetry among all orthogonal wavelet and biorthogonal wavelet and bior1.5 has
complete symmetry. This symmetry can smooth transient components of vibration signal effectively. Moreover, in biorthogonal wavelet basis, bior1.5 has the highest order of regularity. It can obtain a relatively high reconstruction precision. However, the compactly supported length of biorthogonal wavelet basis is longer than orthogonal wavelet basis, so the SNR of bior1.5 is worse than coif1. In addition to both wavelet bases, the performance using haar wavelet basis is also good for its symmetry. However, because the waveform in time domain of haar wavelet basis is discontinuous, it only can be used in theoretical analysis. In addition, as the CR increases, the SNR of each wavelet basis does the same. However, coif1 is still better than the others.

6. Conclusion
Based on all the analysis above, it can be found that felicitous wavelet basis can help to improve the performance in compressing the vibration signal. Moreover, there are two parts that need to be considered: one is the attribute of wavelet basis, and the other is the character of the vibration signal. The whole attribute of wavelet basis includes compactly supported length, vanishing moment, regularity, symmetry, orthogonality, etc. Only the wavelet basis which is compactly supported can be used in discrete wavelet transform. Furthermore, the wavelet basis with low order of vanishing moment, high order of regularity and symmetry, can help to improve the reconstructing SNR. The orthogonal wavelet basis can help to improve CR. From the point of view in the character of the vibration signal of a rolling bearing, there are mainly periodic harmonics, but also many mutational components and noise. Thus, when compressing, the wavelet basis with lower order of vanishing moment and higher order of regularity should be chosen to improve the reconstruction SNR and preserve complete information of faults. In addition, the wavelet basis of “coif” can obtain a relatively higher reconstruction SNR in the same condition of vanishing moment for its orthogonality and approximate symmetry.

Acknowledgments
I wishing to acknowledge assistance or encouragement from G Jinji, special work by financial support from National Natural Science Foundation of China (Grant No.51135001) and the Major State Basic Research Development Program of China (973 Program) (Grant No. 2012CB026000).

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