Questions in quantum physics: a personal view

Rudolf Haag

Waldschmidtstraße 4b, D–83727 Schliersee-Neuhaus, Germany

Abstract

An assessment of the present status of the theory, some immediate tasks which are suggested thereby and some questions whose answers may require a longer breath since they relate to significant changes in the conceptual and mathematical structure of the theory.

1 Introduction

Personal views are shaped by past experiences and so it may be worth while pondering a little about accidental circumstances which channelled the course of one’s own thinking. Meeting the right person at the right time, stumbling across a book or article which suddenly opens a new window.

Fifty years ago, as eager students at Munich University just entering the phase of our own scientific research, we were studying the enormous papers by Julian Schwinger on Quantum Electrodynamics, following the arguments line by line but not really grasping the message. I remember the feelings of frustration, realizing that we were far away from the centers of action. But, mixed with this, also some dismay which did not only refer to the enormous arsenal of formalism in the new developments of QED but began with the standard presentation of the interpretation of Quantum Theory. I remember long discussions with my thesis advisor Fritz Bopp, often while circling some blocks of streets ten times late in the evening, where we looked in vain for some reality behind the enigma of wave-particle dualism. Why should physical quantities correspond to operators in Hilbert space? Why should probabilities be described as absolute squares of amplitudes?, etc., etc. Since we did not seem to make much headway by such efforts I decided to postpone philosophy and concentrate on learning what was really done, which aspects were used in an essential way and were responsible for the miraculous success of Quantum Theory. Leave aside for a while questions of interpretation discussed by Bohr and Heisenberg, extrapolations like Dirac’s transformation theory or von Neumann’s theory of measurement and return to the more pragmatic attitude pervading for instance the book by Leonard Schiff on Quantum Mechanics. For this purpose Walter Heitler’s book ‘Quantum Theory of Radiation’ (second part of the first edition) proved immensely helpful. Here I saw what the typical problems were which Quantum Field Theory
tried to address and I also learned to appreciate the progress made meanwhile by
covariant perturbation theory and Feynman diagrams.

The next great piece of luck for me was (indirectly) caused by Niels Bohr. In
the course of the planning of a great joint European laboratory for high energy
physics (now CERN) he saw the need to introduce a young generation of theoretical
physicists in Europe to this area and offered the hospitality of his Institute. He
called an international conference in 1952 which I could attend and a year later I got
a fellowship for spending a year in Copenhagen. This clearly ended the frustration of
being isolated from the great world. The first fringe benefit of the 1952 conference was
a garden party at the residence of Niels Bohr where I met Arthur Wightman who gave
me the invaluable advice to read the 1939 paper by Wigner on the representations
of the inhomogeneous Lorentz group. Returning to Munich I followed the advice
and it was a revelation. Not only by putting an end to our concern about wave
equations for particles with higher spin but because here I recognized a most natural
starting point. The group, nowadays called the Poincaré group, is the symmetry
group of the geometry of space-time according to the theory of special relativity.
What would be more natural than to ask for the irreducible representations of this
group? Equally remarkable was the result that these representations (more precisely
those of positive energy) correspond to the quantum theory of the simplest physical
system, a single particle. It has just two attributes: a mass and a spin. Everything
that can be observed for such a system, including, as far as possible, a position at
given time, could be expressed within the group algebra. No reference to canonical
commutation relations, guessed from classical mechanics, nor to the wave picture.
The wave equation arises from the irreducibility requirement. I could not understand
why this paper had remained almost unnoticed by the physics community for such a
long time. In fact, even in 1955 I was introduced at some conference in Paris as: ‘He
is one who has read the 1939 paper of Wigner’. This was indeed my major claim to
fame.

Probably all the young theorists who had the privilege of spending a year as
members of the CERN theoretical study group in Copenhagen will remember this
as a wonderful time. The atmosphere at the Institute with the *spiritus loci*, ema-
nating from the personality of Niels Bohr, and upheld by Aage Bohr and Christian
Møller who was the official director of the study group, combined in a rare way sci-
entific aspirations on the highest level with a friendliness discouraging any competitive
struggles and allowing everyone to proceed at his own pace. Though there was some
joint topic suggested, at my time it was the Tamm-Dancoff method and alternatives
to perturbation theory, with some emphasis on nuclear physics, everyone was allowed
to work on the subject of his own choice. So, after a short excursion into nuclear
physics I returned to my pet subjects. Prominent among them was collision theory in
quantum mechanics and field theory. The widely used recipe of ‘adiabatic switching
off of the interaction’ appeared to me not only as ugly but also as highly suspect
because the notion of ‘interaction’ was not clear *a priori* and if one switched off
the wrong thing one would decompose a nucleus into its fragments. This led me to
appreciate the physical significance of various topologies for vectors and operators in Hilbert space. Since in high energy physics the experiments were not concerned with fields but with particles there was the idea that the role of a field was just to interpolate between incoming and outgoing free fields which were associated to some specific type of particle. Trying to implement this I unfortunately used the wrong topology. But just at the end of my stay in Copenhagen we received a preprint of the paper by Lehmann, Symanzik and Zimmermann who did things correctly and thereby provided an elegant algorithm relating ‘Green’s functions’ in quantum field theory to scattering amplitudes for particles.

Speaking of Copenhagen and quantum theory the ‘Copenhagen interpretation’ immediately comes to mind. I prefer to call it the ‘Copenhagen spirit’ or, more specifically, the natural philosophy of Niels Bohr. I did have some opportunities to talk to the great master but, in spite of my great admiration and some efforts, this was not fruitful. It was only in later years that I understood the depth of various parts of his philosophy. But there always remained one disagreement which came from the question: what are we trying to do and what is guiding us? Physics began by the recognition that there are relations between phenomena which are reproducible. These could be studied systematically, isolating simple processes, controlling and refining the conditions under which they occur. The formulation of the regularities found and the unification of the results of many different experiments by one coherent picture was achieved by a mapping into abstract worlds: a world of appropriate concepts and a world of mathematical structures dealing with relations within and between various sets of mental constructs, one of them being the set of complex numbers. This endeavor manifestly led to some level of understanding of ‘the laws of nature’ as evidenced by the development of a technology which provided mankind with enormous powers to serve their conveniences and vices. But what was understood and what was the relative role in this process played by observations, by creation of concepts and by mathematics? When Dirac wrote on the first pages of the 1930 edition of his famous book: ‘The only object of theoretical physics is to calculate results that can be compared with experiment’, this can hardly be taken at face value. As he often testified later, he was searching for beauty and he found it in mathematical structures. So much so that he preferred to look for beautiful mathematics first and consider their possible physical relevance later. Indeed, the road from phenomena to concepts and mathematics is not a one-way street. As the studies shifted from coarser to finer features the theory could not be derived directly from experiments but, as Einstein put it, it had to be freely invented and tested subsequently by suggesting experiments. In this passage back and forth between phenomena and mental structures many aspects entered which cannot be rationalized. The belief in harmony, simplicity, beauty are driving forces and they relate more to musicality than to logic or observations.

There was one further highly significant and somewhat accidental occurrence in shaping my subsequent work and this may also illustrate the above remarks. Professors F. Bopp and W. Maak in Munich decided in 1955 that it was important to
exchange experience between theoretical physicists and mathematicians. This initiative was not rewarded by visible success. The number of participants dwindled quickly and the enterprise ended after a few months. But for me it was of paramount importance. I was introduced there to rather recent work of the Russian mathematicians Gelfand and Naimark on involutive, normed algebras and to the work of von Neumann on operator algebras and reduction theory. I saw that Hilbert space resides in some wider setting which, at least from the mathematical point of view, constitutes a rather canonical structure resulting from a few natural structural relations. Besides the standard algebraic operations it needed a *-operation (involution) and one is led to a natural topology induced by a unique ‘minimal regular norm’. It appeared highly likely that this structure was behind the scenes in the mathematical formalism of quantum theory. The prototype of such an algebra is furnished by group theory. There a most important tool is the consideration of functions of the group with values in the complex numbers. They form obviously a linear space because we can multiply them by complex numbers and add them. If there is a distinguished measure on the group (which is the case for compact groups and, up to a normalization factor, for locally compact ones) the group multiplication defines a product in the space of these functions, the convolution product. The inverse in the group defines a *-operation in this algebra of functions by $f^*(g) = \bar{f}(g^{-1})$. The resulting algebra yields the representation theory of the group. An irreducible representation corresponds to a minimal (left) ideal in the algebra.

If in the following I try to describe things which I believe to have learned concerning the interrelation between observed phenomena, concepts and mathematical structures I must precede this with some apologies. The inadequate handling of references is due to the state of disorder in my notes and lack of time. The abstractions used in describing the procedures of acquiring knowledge may be too schematic. There is a painful gap between their qualitative character and the very precise mathematical structures into which they are mapped.

2 Concepts and Mathematics in Quantum Theory

A paper describing some fascinating recent experiments was entitled ‘Reality or Illusion?’ These experiments (see e.g. refs. [4, 5, 6]) have lent impetus to the long standing discussion about the meaning of reality in quantum theory. Do the discoveries force us to abandon the naive idea of an outside world called nature whose laws we try to find? What is the role of the observer? Do the puzzles relate to the mind-body problem?

Many different views concerning such questions have been voiced throughout the past seventy years. So if I try to express mine it may be pardonable to proceed in an extremely pedantic fashion.

A single experiment of the type alluded to above combines many individual clicks of some detectors. Though each click is unique and neither repeatable nor predictable even under optimally controlled circumstances, we may regard it as a ‘real fact’
in the sense in which these words are used in any other context. Let us call it an ‘event’. Its existence is not dependent on the state of consciousness of human individuals. In modern times it is usually registered automatically, stored in computer memories and there is no dispute between the members of a group of experimenters about its ‘reality’. The outcome of the experiment refers to the frequency with which a particular configuration of events occurs in many runs and this is reported as a probability of the phenomenon under precisely described circumstances. To be accepted, this result must be reproducible by any other group of scientists who is willing to invest time and resources in repeating the experiment. The events mentioned are coarse. A detector is macroscopic. We regard macroscopic bodies as ‘real objects’ and statements about their placement in space and time as ‘real attributes’. The word ‘real’ just means here that such objects, events and their space-time attributes belong to common experience shared by many persons and do not depend on the state of consciousness of an individual. The observed relations between them constitute the only empirical basis from which the ‘free invention’ of a theory can proceed. With regard to this mental task there is a piece of wisdom which I learned from F. Hund. It might be called Hund’s zeroth rule. He pointed out that the progress of physical theory depended on the lucky circumstance that always some effects were small enough to remain unnoticed or could be disregarded as insignificant at the time a particular piece of the theory was proposed. We cannot take many steps at the same time. We should regard a theory always as preliminary; it will disregard some fine features of which we are luckily ignorant or which we neglect in order to obtain a tractable idealization.

The purpose of these lengthy elaborations is twofold. First, I do not think that physics can make any contribution to the mind-body problem. The attempt to explain some puzzling aspects of quantum physics by invoking subjective impressions and the role of the consciousness of individual human beings is not an appropriate answer. Secondly, the concept of event is necessary in quantum physics. It is an independent concept. The mental picture that it corresponds to an interaction process between an atomic object and a macroscopic one is misleading because experiments tell us that there are no atomic objects in an ontological sense (see below). Of course, from this we may conclude that there are no macroscopic objects either and that their apparent reality results from an asymptotic idealization. This is true (see e.g. the discussion in ref. [4] of the emergence of classical concepts due to large size and decoherence). But the idealization is covered by Hund’s zeroth rule and is essential for the form of the present theory. If we want to avoid it we must take the next step in the development of the theory. Let us address now some specific aspects.

1. In experiments we usually (necessarily?) distinguish two parts: a source which determines the probability assignments (subsumed under the notion of ‘state’), and a set of detectors to whose responses the probabilities apply. Though in the description of the source a variety of considerations enter which will have to be looked at more closely, we shall, for simplicity of language, just idealize it as characterized by some pattern of ‘source events’. The total setting, consisting of source events and target
events, where the former determine a conspicuous probability assignment for the occurrence of the latter, may be called a ‘quantum process’. Bohr emphasized the indivisibility of the process as one of the key lessons of quantum theory. This poses the question of how we can isolate such a process from the rest of the world. In technical terms, what do we have to take into account in ‘preparing a state’ in order to get a reproducible probability assignment for a pattern of target events (defined by some arrangement of detectors). Here we are helped by lucky circumstances. We live in a reasonably steady environment; its influence does not change rapidly in space and time. So, if we are stupid in the state preparation we just get an uninteresting probability assignment, a ‘very impure state’. The art of the experimenter is needed to improve state preparation and render the probability assignment as conspicuous as possible. It appears, however, that there is a limit to such improvements, idealized by the notion of a ‘pure state’.

2. Given some definite process one would like to assign to it a ‘physical system’ as the agent producing the target events or, more carefully, as the messenger between source and target events. This is clearly a mental construct. Can we attach any element of reality to it? If we focus on a single event involving one detector far removed from the source we may think of a single particle as this messenger. But we may also consider patterns of several events, seen in coincidence arrangements of detectors far removed from the source and from each other. Then we sometimes find correlations in their joint probabilities which are of a very peculiar type. If we believe that there is a specific messenger from the source to each target event (for instance a particle) then, whatever notion of state we try to assign to those, we cannot represent the joint probability for the pattern of events as arising from joint probabilities for a corresponding set of individual states of the messengers. This is the conclusion to be drawn from the violation of Bell’s inequality. It is not so easily seen in the first discussions which focused on hidden variables but emerges clearly in ref. [5]. Another equally surprising effect has been demonstrated by Hanbury-Brown and Twiss. They start from two entirely independent source events (for instance photons emitted from two far distant surface regions of a star which happen to arrive almost simultaneously in the observatory). So they can cause a coincidence in two detectors. Each detector responds to one photon but can, of course, not distinguish from which source event it comes. By varying the difference of the optical paths from the telescope exit to the two detectors one finds varying intensity correlations in the coincidence signals. This means that the cause for the response of one detector cannot be attributed to the arrival of either a messenger from the left edge or from the right edge of the star. Both photons work together though there is no phase relationship between the two emission acts. There is a causal relation between the pattern of two source events and the pattern of two target events but it cannot be split into causal ties between single events.

Taken together these experiences imply that the notion of a ‘physical system’ does not have independent reality. What is relevant for the click of a single detector is some notion of ‘partial state’ prevailing in its neighborhood. In both of the above
examples this is described as an impure state of a single particle. In the EPR-example as discussed by Bell it is determined by one source event, the decay of an unstable particle. In the second example it is caused by two source events. The probability for the response of both detectors in coincidence depends on the partial state in the union of the two neighborhoods and this is apparently not determined by the pair of partial states around the individual detectors. Thus quantum physics exemplifies the saying: ‘the whole is more than the sum of its parts’ and it does so in extreme fashion. The Pauli principle claims that all electrons in the universe are correlated. The reality behind the mental picture of a physical system consisting of a certain number of particles refers to a certain set of events with causal connections between them, manifested by the existence of a probability for the total process. In an ideal experiment this is obtained by counting the number of times the pattern of target events is realized, dividing it by the number of times the source events occurred. In the usually prevailing cases where the source is not adequately known we can still determine relative probabilities of different patterns of target events assuming that the source remains constant.

The holistic aspect mentioned above is often called the ‘essential non-locality’ of quantum theory. But this is an unfortunate terminology because the only reason why we can talk about specific processes at all resides in the locality of individual events and the causal structure of space-time.

3. The reader may have wondered why I specialized the usual notion of ‘observable’ to that of a detector and talked about events instead of measuring results indicated by the position of a pointer of some instrument. The spectral resolution of self-adjoint operators which played such an essential role in the development of early quantum mechanics was not even mentioned so far. One reason for this is the problem of how to achieve the mapping from a particular arrangement of instruments to its representative in the mathematical scheme. In early quantum mechanics the idea that we consider a physical system consisting of a definite number of particles seemed to pose no problem (a beautiful illustration of Hund’s zeroth rule). The degrees of freedom were positions and momenta, appearing in the canonical formalism of classical mechanics on equal footing. Though it became clear that these degrees of freedom could not be real attributes of the system one still talked about measuring one of them (or simple functions of them like energy and angular momentum). Bohr emphasized that the full description of the experimental arrangement is needed ‘to tell our friends what we learned’ and that this could only be done in plain language. But, since the classical degrees of freedom persisted, this description of the arrangement could ultimately be summarized by one mathematical object which corresponded in a symbolic way to a classical quantity. How can one proceed in this passage from the description of an arrangement of hardware to a mathematical symbol relating to the system? The primary piece of information is given by the placement of macroscopic bodies in space-time. These bodies perform different functions. Some parts may be considered as ‘state preparation procedures’ representing the source events. Other parts yield the measuring result which is an unresolvable phenomenon, an unpredictable
decision in nature, a coarse event. Its primary attribute is an approximate position in space-time. The representation of the whole arrangement (apart from the primary source) by a self-adjoint operator, interpreted as describing the measurement of a quantity related to some function of the classical degrees of freedom involves the theory (Schrödinger equation) in conjunction with idealizations and approximations which are transparent only in simple cases. The operators corresponding to momentum and energy have clear significance as generators of translations in space and time but are only indirectly related to observations, which in the last resort concern the position of an event in space-time. The position operator of a particle at a prescribed time yields spectral projectors which can approximately characterize an event. But the assumed existence of a family of mutually exclusive events with certainty that one of them must happen is an extrapolation which becomes highly unnatural in relativistic situations. This is mildly indicated already by the ambiguities arising in the attempt to define a position operator in Wigner’s analysis.

The fundamental discovery, that the ‘elementary particles’, formerly believed to be the building stones of matter, are not eternal but can be created or destroyed in processes, forces us to consider states whose particle content is not only varying but undefined in some regions. While the concepts of ‘system’ or ‘particle’ suggest some object existing in an ontological sense, the concept of ‘state’ belongs to the realm of possibilities (potentialities, propensities) for the realization of something coming into existence, an event. This would not be so in a deterministic theory but if we believe that the indeterminacy in the prediction of phenomena, inherent in the formulation of quantum physics, is a feature of the laws of nature and not just due to ignorance which could be lifted by future studies then the distinction between the realm of possibilities and the realm of facts becomes imperative. The ‘state’ belongs to the former. Strictly speaking it provides a quantitative description of a contribution to the probability for the occurrence of events. The other contribution is given by the placement and type of detectors. Thus also the notions of ‘system’ and ‘particle’ belong to the realm of possibilities. But they retain their importance. They allow us to classify (at least under favorable circumstances) the possible partial states in a region by a denumerable set.

This procedure involves the center piece of the mathematics of quantum theory: the superposition principle and eigenvalue problems; more generally, the determination of invariant subspaces in a complex linear space with respect to the action of the symmetry group of the theory. The intuitive steps leading to the recognition that this mathematical structure (Hilbert space, involutive algebras, representation theory of groups) offers the key to quantum theory appear to me as a striking corroboration of Einstein’s emphasis on free creations of the mind and Dirac’s conviction that beauty and simplicity provide guidance.

Returning to our line of argument: the ordering of states in classes by the concept of a ‘system’ corresponds to the selection of an invariant subspace, under the action of the symmetry group. A minimal invariant subspace, an irreducible representation, may be called an elementary system. Its attributes are group characters. If we
consider only the symmetry group of space-time, the Poincaré group, the irreducible representations give us states of a stable system, a system which could persist eternally if it were alone in the world and no events could occur. This simple system is a single particle. Its attributes are a value of the mass and the spin, which define a group character. The reason why the simplest systems play such an important role for observations is due to the circumstance that in many experiments the partial state pertaining to a large but limited region of space-time can be very closely approximated by the restriction of a global single particle state to the region. This will, in fact be the standard situation in the overwhelming part of space-time if the mean density of matter is small. To obtain a basis in the space of single particle states we must choose some maximal set of commuting generators. In this choice the generators of space-time translations, whose spectral values are energy-momentum 4-vectors, play a preferred role in the following respect. If we look in a region whose extension is small compared to its mean distance from source events then the partial state there is well approximated by a mixture of parts of plane waves, (improper) eigenstates of the generators of translations, each belonging to a specific energy-momentum vector.

We confined attention so far to the Poincaré group describing the space-time symmetry. The full symmetry group includes ‘gauge symmetries’ whose characters are charge quantum numbers. The first example was the electric charge with its description as a character of U(1). The generalizations of this in high energy physics led to flavor and color multiplets associated with the groups SU(2), SU(3). To avoid misunderstandings it must be stressed that we talk here about a global gauge group. The significance of local gauge invariance will be addressed later.

2.1 Conclusions

Position and momentum belong to different parts of the scheme. Position is an (approximate) attribute of an event, not of a particle, and the event marks a position in space-time not a position in space at an arbitrarily assumed time (as the picture of a world line for a particle would suggest). In simple cases the event may be regarded as the interaction process between a particle and a detector. But the notion of ‘particle’ does not correspond to that of an object existing in any ontological sense. It relates to the simplest type of global state and describes possibilities, not facts. The notion of ‘partial state’ demands in addition that we ignore all possible events outside some chosen region and thus ignore possible correlations with outside events. The concepts of ‘particle’ and ‘physical system’ arise from the possibility of ordering global states into distinct classes defined by the symmetry group of the theory. A particle corresponds to an irreducible representation of this group. Its attributes are group characters. A system corresponds to some subrepresentation of the tensor product of irreducible representations. Experience tells us that only a countable set of irreducible representations (particle types) appears in nature. The determination of these (the masses, spins, charge quantum numbers of physical particles) is one of the tasks of the theory.

In observations we are concerned with partial states which result by the restriction
of global states to some regions in which we choose to place detectors. For a fixed
global state the partial state in a region can be approximately described by the
restriction of a global state which belongs to the class of some specific system. In
other words: if we focus attention on some particular region then the global state
may tell us for instance that in there the probability for events is almost the same as
that predicted from the restriction of some single particle state. Indeed, if we choose
the region sufficiently small then it will usually suffice to consider only mixtures of
single particle states with definite momenta. The existence of zero mass particles
complicates this picture somewhat, as evidenced by laser beams and by infrared
problems where the number of particles is no longer useful for the description of a
partial state.

The analysis of global states in terms of various systems approximating the partial
states in various regions of space and time is the other task of the theory (the theory
of collision processes).

In the whole scheme we still need an observer. No facts are created if no detectors
are around anywhere. Though the consciousness of an individual plays no role (it was
eliminated by the assumed ‘as if’ reality of macroscopic bodies and coarse events)
the scheme still appears somewhat artificial. It is a description of what we may learn
by experiments. But looking at the detailed mathematical structure, developed to
cope with the above mentioned tasks of the theory, it seems clear that the notions
of macroscopic bodies and coarse events are asymptotic concepts. If, on the other
hand, we wish to replace them by finer ones we encounter difficulties. They can,
I believe, not be overcome without a radical change of the formalism involving our
understanding of space and time. As long as there is an enormous disparity between
collision partners, one being a macroscopic body the other an atomic object, we
can talk about an approximate position of the event and give upper bounds for its
uncertainty, relating to the size and the time of sensitivity of the (effective part of the)
detector, and we can give lower bounds for the energy- momentum transfer needed
to overcome the barriers against the appearance of a significant change. This suffices
for practical purposes but does not seem to be the ultimate answer if we look at the
vertex of a high energy event in a storage ring.

3 The mathematical structure in relativistic quant-
tum physics and its interpretation

In quantum field theory the basic mathematical objects, the fields, are functions of
points in space-time. These are singular objects which have to be smeared out over
some finite regions to yield observables which can be represented by operators in a
Hilbert space. There are problems. Some serious ones are related to gauge invariance,
specifically to the local gauge principle first encountered in quantum electrodynamics
(QED). If one wants to avoid ‘unphysical states’ one has to restrict attention to gauge
invariant quantities. From these one may hope to construct algebras of observables.
To be precise: we abstract from this heuristic consideration that we can obtain a normed, involutive algebra (for short, a $C^*$-algebra) for each bounded, open region $\mathcal{O}$ of space-time. The correspondence
\[ \mathcal{O} \to \mathcal{A}(\mathcal{O}) \tag{3.1} \]
between regions and algebras yields one essential piece of information for the analysis of the consequences of the theory. We call $\mathcal{A}(\mathcal{O})$ the algebra of observables of the region $\mathcal{O}$. There are some natural relations between these local algebras. Obvious is the inclusion relation:

(i) $\mathcal{O}_1 \subset \mathcal{O}_2$ implies $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$.

This allows the definition of a global $C^*$-algebra $\mathfrak{A}$ as the ‘inductive limit’, the completion of the union of all local algebras in the norm topology.

The second important relation reflects the causal structure of space-time:

(ii) If $\mathcal{O}_1$ is space-like to $\mathcal{O}_2$ then $\mathcal{A}(\mathcal{O}_1)$ and $\mathcal{A}(\mathcal{O}_2)$ commute.

The third basic ingredient is covariance with respect to the Poincaré group $\mathcal{P}$.

We need a realization of $\mathcal{P}$ by automorphisms of $\mathfrak{A}$; to each element $g \in \mathcal{P}$ there is an automorphism of $\mathfrak{A}$ denoted by $\alpha_g$ which should have the obvious geometric significance:

(iii) If $A \in \mathcal{A}(\mathcal{O})$, then $\alpha_gA \in \mathcal{A}(g\mathcal{O})$, where $g\mathcal{O}$ denotes the region resulting from shifting $\mathcal{O}$ by $g$. It is convenient to assume that these algebras have a common unit element.

We call the structure defined by the correspondence (3.1) with the properties mentioned a (covariant) net of local algebras. A general state $\omega$ corresponds to a normalized, positive linear form i.e. a linear function $A \to \omega(A)$ from the algebra $\mathfrak{A}$ to the complex numbers which takes real, non-negative values on the positive elements of the algebra:
\[ \omega(A^*A) \geq 0 \text{ for any } A \in \mathfrak{A}; \quad \omega(1) = 1. \tag{3.2} \]

A partial state in some region $\mathcal{O}$ is defined in the same way with $\mathfrak{A}$ replaced by $\mathcal{A}(\mathcal{O})$. It corresponds to the restriction of a class of global states to the subalgebra considered.

From section 2 we see that the physical interpretation requires a characterization of those elements of $\mathfrak{A}$ which represent detectors for an event in a region $\mathcal{O}$. The first guess might be to identify the projectors in $\mathcal{A}(\mathcal{O})$ with such detectors. This is, however, not sufficient. A detector, in contrast to a source, must be passive; it should not click in the vacuum situation. We must control the energy-momentum transfer. Starting from any element $A \in \mathfrak{A}$ we can construct elements
\[ A(f) = \int (\alpha_x A)f(x) d^4x, \tag{3.3} \]
where $x$ refers to a translation in space-time. If the Fourier transform of the function $f$ has support in a region $\Delta$ in $p$-space then the energy-momentum transfer of $A(f)$ is limited to $\Delta$. Therefore we add to the structure described so far the (somewhat
over-idealized) assumption that there exists a ground state \( \omega_0 \), the vacuum, which is invariant with respect to the Poincaré group and is annihilated by any \( L \in \mathfrak{A} \) which is of the form (3.3) with support \( \Delta \) outside of the closed forward cone \( V^+ \) (positive time-like vectors in \( p \)-space including 0). This assumption, called the spectrum condition, allows us to define detectors which are approximately associated to a region \( \mathcal{O} \) in position space and to some window \( \Delta \) in \( p \)-space indicating the minimal energy-momentum of the ‘atomic object’ needed for the response of the detector. Any element

\[
P = L^* L \quad \text{with} \quad \|P\| = 1, \quad L = A(f)
\]  

(3.4)

represents such a detector if we start with \( A \in \mathcal{A} (\mathcal{O}) \) and choose the function \( f \) so that (apart from small negligible tails) the support in \( x \)-space is a small region around the origin and its Fourier transform is practically zero outside of the region \( \Delta \). Of course this does not yield a precise localization or momentum transfer but this is not relevant if we think of a detector as a macroscopic body. Let us note that \( P \) will in general not be a projector but this is not necessary either, because we need not consider the negation, an instrument which indicates with certainty that no event has happened.

The above characterization of a detector does not tell us what the detector detects. But we discover that such additional information is not needed \textit{a priori}. If the net is given and a vacuum state exists (the spectral condition), then we have the tools to analyze the physical content of the theory by studying the response of coincidence arrangements, represented by products of \( P_k \) belonging to mutually space-like situated localization regions, in any state. A single particle state, for instance, can be defined as a state which is ‘simply localized at all times’, i.e. never capable of producing a coincidence of two (space-like separated) detectors:

\[
\omega(P_1 P_2) = 0
\]  

(3.5)

for any such choice of the \( P_k \) \((k = 1, 2)\), but \( \omega(P) \neq 0 \) for some \( P \). For further elaborations see [6].

A net satisfying only the requirements mentioned so far need not yield a physically reasonable theory. It may, for instance, describe no particles at all or a non-denumerable number of different types. Further properties are needed. Some necessary conditions are known which concretize the structure considerably and relate to various physical aspects ranging from the appearance of charge quantum numbers in particle physics to properties of thermodynamic equilibrium states [6, 7]. But we do not know yet how to formulate restrictive conditions powerful enough to define a specific net, let alone the ambitious aim of constructing a net whose physical content is corroborated by experiments.

It is my personal conviction that in this step the local gauge principle plays a crucial role. This assessment stems partly from progress in theoretical high energy physics in the past decades and partly from my belief in simplicity and naturalness of fruitful basic concepts. The principle mentioned tells us that we should not try to focus on global symmetries. In a local theory the symmetries should only govern the
structure in the small and the comparison of their action in different regions needs additional information which is called a ‘connection’ because the comparison depends on the way we pass from one region to the other. In the two important classical field theories which have proved their worth for physics, Maxwell’s electrodynamics and Einstein’s general relativity, this principle is encoded. In the former it was recognized rather late and refers to the gauge symmetry related to electric charge; in the latter it was one of the guiding principles and refers to the Poincaré symmetry of space-time. The Lorentz part, which keeps one point in space-time fixed, is reduced to a local symmetry for the tangent space at this point; the translations are replaced by the connection. Quantum physics as we know and use it is anchored on the uncritical acceptance of space-time as an arena in which we can place instruments, an arena with known geometry including a causal structure. Some aspects of the theory depend also on the existence of a global symmetry for this geometry. If we loose this anchor completely we enter an area in which the conceptual structure and mathematical formalism of quantum physics cannot persist. In this area the problem mentioned at the end of section 2 and some of the questions addressed in the next section may become imperative. So, to stay on the present level, we wish to keep global Poincaré symmetry and only reduce the internal symmetries, relating to the charge structure, to local significance needing the definition of a connection. In a classical field theory the formalism of Yang-Mills theories, generalizing electrodynamics to non-Abelian local internal symmetry groups, is well understood, using the notions of sections and connections in a fiber bundle. The transfer of this formalism to quantum theory is highly nontrivial and, in my opinion, not yet adequately understood. If we use the approach via algebras of observables sketched above then the incorporation of the additional structure due to (not directly observable) local internal symmetries is obscured by the singular nature of points and lines used in the classical case. To handle this we need knowledge about the short distance behavior (ultraviolet limit) of the theory. A few tentative suggestions concerning the notion of a quantum connection are given in ref. I consider the clear understanding of how local internal symmetries can be incorporated in a well defined mathematical structure as one of the most important immediate aims on which many subsequent developments may hinge. It constitutes, of course, a hybrid theory since the global nature of the geometric symmetries is kept. So it may not be of primary importance to clarify whether the continuum limit really exists.

1Quantum physics in ‘curved space-time’ (representing a given, external gravitational field) retains the first part of these requirements. This means that the net structure of local algebras with the properties (i), (ii) persists. The loss of (iii) implies that the spectrum condition has to be replaced. A considerable amount of work has been devoted to this problem but we shall not discuss it here; it would be beyond the scope of this paper.
4 Retrospective and Perspectives

Comparing the picture sketched so far with the discussions on the interpretation of quantum theory seventy years ago we may note:

1. The ‘language of classical physics’ stressed so much by Bohr as indispensable for the observer (to enable him to tell what was done and learned) remains an essential ingredient but, if we disregard questions of convenience, it may be reduced to the description of geometric relations in the placement of various macroscopic bodies and the coarse events observed in space and time. All further information is contained in the mathematical structure. The correspondence principle needed to map the description into the realm of mathematical symbols is provided by the reference to classical space-time and its geometric symmetry on both sides. It is the correspondence \((3.1)\) together with the action of the translation group, needed to characterize the passive nature of detectors by \((3.4)\). Apart from this the global symmetry of space-time is needed in two respects. In a single experiment because it studies the statistical relations in an ensemble of many individual event patterns which occur at different times and in the communication with other observers who would like to test the results in a different region of space-time.

2. The indivisibility of a process emphasized by Bohr leads to the concept of an event as an irreducible unit and it manifests itself also in the holistic aspect of the causal relations between events. The isolation of an individual process as a distinguishable, coherent part in the history of the universe, a pattern of events which can be considered by itself without mentioning its ties with other parts, depends to some extent on the choice of the observer of how much he wants to consider but this choice is limited by the requirement that it must lead to a well defined conspicuous probability assignment for the total process, a requirement which can be precisely fulfilled only in a steady environment.

3. The distinction between possibilities and facts which appears to be unavoidable in a formulation of indeterministic laws implies a distinction between future and past. Bohr mentions the ‘essential irreversibility inherent in the very concept of observation’. If the term observation does not mean that the ultimate responsibility for deciding what constitutes a fact is delegated to the consciousness of an individual human being then we must accept the essential irreversibility inherent in the concept of an event. This endows the ‘arrow of time’ with an intrinsic significance in the physical theory and corresponds to a picture of reality as evolving in successive steps of a process with a moving boundary, separating past facts from future possibilities. It corresponds to the picture drawn by the philosopher A.N. Whitehead. This does not conflict with the existence of a time reversal symmetry of the theory which describes a symmetry in the probability assignments for processes. The significance of the arrow of time is encoded in the existing theory by the spectrum condition for the energy-momentum of states entering in the characterization of detectors (which provide one contribution for the probability of an event). The time reversal operator,

\[ T \]

\[ I \text{ think this would be too unreliable to be useful for the purposes of physics.} \]
being anti-unitary, does not change the sign of the energy.

Turning now to perspectives for future development of the theory, we might take a few hints from the preceding discussion. First, that all symmetries should be considered as local but that we should not associate the meaning of local with a point in a space–time continuum but with a possible event. A pattern of events with a web of causal ties between them bears some analogy to a section in a fiber bundle whose base space is the set of events and the typical fiber is a direct sum of representations of the symmetry group. The causal ties provide the connection. The dynamical law must then describe the probability assignment for different possibilities of growth of such a pattern in the evolution process in which possibilities turn into facts and the boundary between past and future changes. Included in this task is the determination of the subset of representations in the fiber of an event, the generalized eigenvalue problem yielding the relation between masses, spins and charge quantum numbers. I shall not try to elaborate on the many questions connected with such a picture and its relation to existing formalism. This is beyond the scope of this paper and the capabilities of its author.

References

[1] Ph. Blanchard and A. Jadczyk (eds.) Quantum Future, Proc. Przesieka Conf. 1997, Springer Verlag, Heidelberg, 1999.

[2] R. Bonifacio (ed.) Mysteries, Puzzles and Paradoxes in Quantum Mechanics, Proc. Lake Garda Conf. 1998, AIP Conf. Proc. 461.

[3] Proc. Lake Garda Conf. 1999, to appear.

[4] R. Omnès, The Interpretation of Quantum Mechanics, Princeton University Press, 1994.

[5] J.F. Clauser and M.A. Horne, Phys. Rev. D 105261974.

[6] R. Haag, Local Quantum Physics, second edition. Springer Verlag, Heidelberg, 1996.

[7] D. Buchholz and R. Haag, The quest for understanding in relativistic quantum physics, preprint hep-th/9910243, to appear in J. Math. Phys., special issue.

[8] A.N. Whitehead, Process and Reality, Macmillan Publishing Co., 1927.