Anomalous dynamic response in the two-dimensional lattice Coulomb gas model: Effects of pinning

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It is demonstrated through Monte Carlo simulations that the one component lattice Coulomb gas model in two dimensions under certain conditions display features of an anomalous dynamic response. We suggest that pinning, which can either be due to the underlying discrete lattice or induced by disorder, is an essential ingredient behind this anomalous behavior. The results are discussed in relation to other situations where this response type appears, in particular the two components neutral Coulomb gas below the Kosterlitz-Thouless transition, as well as in relation to other findings from theory, simulations, and experiments on superconductors.

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I. INTRODUCTION

Two-dimensional (2D) vortex physics is strongly reflected in the properties of systems related to 2D superfluids like Josephson junction arrays, superconducting films, and 4He films, as well as high-Tc superconductors. The vortices are, e.g., responsible for the well-known Kosterlitz-Thouless (KT) transition and there is a fairly good understanding of the thermodynamic properties related to 2D vortex physics. However, the dynamical aspects, which can be probed by the complex impedance and the flux noise measurements in superconductors, is and by the torsional oscillator period shift in 4He-films, are much less well understood. The first attempt to describe the dynamic vortex response phenomenologically was given by Ambegaokar-Halperin-Nelson-Siggia (AHNS) in Ref. 1. However, it was later discovered that the Minnhagen phenomenology (MP) in Ref. 2 based on essentially the same ingredients as AHNS, described experiments and simulations better.

A particularly interesting aspect in the MP is that it reflects an anomaly in the 2D vortex response which, for example, in a superconductor takes the form of a logarithmic divergence of the complex conductivity: \( \sigma(\omega) \propto -\ln \omega \) for small frequency \( \omega \). There still remain open questions on what ingredients can cause this anomaly and give rise to the anomalous response form.

There has been a number of attempts to obtain the MP form with other phenomenological approaches as well as somewhat formally more rigorous ones. One possibility is that it is an intrinsic property of the 2D vortex system which is linked to the long range logarithmic interaction, as presumed by the original motivation for the MP response form. Another possibility is that it is caused by the coupling between the perpendicular currents described by the vortices and the longitudinal currents (often referred to as the spin-wave part). In the present paper we suggest that there is yet another possibility; the MP type response can also arise from pinning influencing the vortex system.

The approach to this problem, which we subscribe to here, is to systematically investigate various models by aid of simulations. Vortices can be described as 2D Coulomb gas particles, in the sense that a vortex with vorticity \( \pm 1 \) corresponds to a Coulomb gas particle with charge \( \pm 1 \) (See, e.g., Ref. 3 for details on the mapping between the two models). Since the dynamics of the vortices can to a good approximation be described by the Langevin dynamics, we can view the 2D Coulomb gas model with Langevin dynamics as the dynamic model for the “pure” 2D vortex response which includes neither any coupling to a spin-wave part nor any pinning. For a superconducting film in the absence of an external perpendicular magnetic field (which corresponds to the absence of a net rotation in case of superfluid 4He film), the numbers of vortices and antivortices (with vorticities 1 and \(-1\), respectively) are always equal, and the system is described by the charge neutral two components 2D Coulomb gas model on which the AHNS, as well as the MP, are based. Although it was clearly shown from simulation of this neutral 2D Coulomb gas model that the low-temperature phase is well described by the MP response form with the logarithmic divergence of the conductivity there still remains the possibility that the coupling to the spin-wave part or pinning could also influence and perhaps lead to the anomalous response in certain cases.

In Ref. 4 it has been found in experiments that the 2D triangular Josephson array in a perpendicular magnetic field is also well described by the MP response form, which has again been confirmed in simulations of the related frustrated 2D XY model with relaxation dynamics. Since in these cases pinning could with large probability be excluded as the cause of the effect, the observed anomalous behavior can in these cases be attributed to either the logarithmic vortex interaction, or to the interplay between the vortex and the spin-wave interactions, or to a combination of both. From the analysis of the experimental results in Ref. 5 for thin MoGe superconducting film in a perpendicular magnetic field it has also been pointed out that the data show MP behavior. Since in Ref. 6 it was argued that for this sample the vortex pinning due to disorder was important, this raises the question of how pinning influences the dynamic behavior. Could the MP response form also arise from an interplay between the vortex system and pinning? In the present work we conclude, from simulations of the lattice Coulomb gas model, that the vortex system, in the absence of couplings to the spin-wave part but in the presence of pinning, indeed in certain parameter regions gives rise to MP features reflecting an anomalous diffusion.

The paper is organized as follows: In Sec. 7 we describe the Coulomb gas model and the correlation functions we study in simulations. Section 8 gives a brief description of the features of the MP response form and make comparisons with...
II. THE LATTICE COULOMB GAS MODEL

In this work, we use the 2D Coulomb gas model on an $L \times L$ triangular lattice, whose Hamiltonian in the absence of disorder is written as:

$$H = \frac{1}{2} \sum_{ij} (n_i - f)V_{ij}(n_j - f),$$

(1)

where $n_i$ is the integer charge on the $i$th site and the frustration $f$ controls the total number of charges $N_c \equiv \sum_i n_i = fN$ ($N \equiv L^2$). The interaction $V_{ij}$ between charges at positions $r_i$ and $r_j$ for a triangular lattice is given by

$$V_{ij} = \frac{1}{N} \sum_{k \neq 0} \frac{3\pi e^{i\mathbf{k} \cdot (r_i - r_j)}}{6 - 2\cos \mathbf{k} \cdot \mathbf{a}_1 - 2\cos \mathbf{k} \cdot \mathbf{a}_2 - 2\cos \mathbf{k} \cdot \mathbf{a}_3},$$

(2)

where $\mathbf{a}_1 = \mathbf{a}$, $\mathbf{a}_2 = \mathbf{b}$, and $\mathbf{a}_3 = \mathbf{a} - \mathbf{b}$ with two primitive translational vectors $\mathbf{a}$ and $\mathbf{b}$ for 2D triangular lattice. (We choose $\mathbf{a} = \hat{x}$ and $\mathbf{b} = \hat{x}/2 + \sqrt{3}\hat{y}/2$ with lattice spacing set to unity.)

We also study the effects of quenched disorder and consider the Hamiltonian

$$H = \frac{1}{2} \sum_{ij} (n_i - f)V_{ij}(n_j - f) - \sum_i U_i n_i,$$

(3)

where the pinning potential $U_i$ at $i$ has value $U^p(>0)$ if the disorder is located at $i$ and $U_i = 0$ otherwise. The randomly distributed disorder realization is parameterized by the pinning strength $U^p$ and the ratio $r_p$ between the number of pinning sites $N_p$ and the total number of charges $N_c$: $r_p \equiv N_p/N_c$. This corresponds to point disorder since we can express it as $U(r) = \sum_{i=1}^{N_p} U^p \delta(r - r_j)$ in continuum limit, where $r_j$ denotes the positions of disordered sites.

We use Monte Carlo (MC) dynamics which can be implemented as follows: Allow two different MC tries where one is the charge hopping to one of its nearest neighbors, i.e., $(1,0) \rightarrow (0,1)$, and the other is the generation of a charge-anticharge dipole on one bond, i.e., $(0,0) \rightarrow (1,-1)$. As was already observed in Ref. [24], for parameters used in this work the acceptance ratio for generation of charge dipole is found to be extremely small even at temperatures much higher than the critical temperatures (referring to the depinning and melting of the solid phase). We are here studying a temperature region where no charge dipoles are in practice created. Consequently, we only include the charge hopping in our MC try. This means that the model we are studying only contain charges of one sign. Our algorithm is thus: At each step, we first pick up one charge at random and try to move it to one of its six nearest neighboring sites which is also randomly chosen. This MC try is accepted or not according to the standard Metropolis algorithm. One can make the MC simulation more efficient by using a nonlocal update (e.g., the randomly chosen charge can be allowed to hop to a distant site). On the other hand, as far as the dynamic behavior is concerned, the local update rule like the one used in the present work is believed to better imitate the relaxation dynamics of the corresponding continuum model (which corresponds to the Coulomb gas model with Langevin dynamics). [24]

For the characterizations of the thermodynamic properties, we measure (i) the dielectric constant $1/\epsilon(0)$, which corresponds to the helicity modulus in the vortex systems and detects the superconducting to normal transition, (ii) the orientational order parameter $\phi_6$, which probes the melting of the solid-like structure, and (iii) the specific heat $C_v$ (see Ref. [24]):

$$\frac{1}{\epsilon(0)} \equiv \lim_{k \rightarrow 0} \frac{1}{TNk^2} \left\langle n_k n_{-k} \right\rangle,$$

(4)

$$\phi_6 \equiv \frac{1}{N_c} \sum_{ij}(e^{i\phi(r_i - r_j)}),$$

(5)

$$C_v \equiv \frac{\langle H^2 \rangle - \langle H \rangle^2}{T^2 N},$$

(6)

where $\langle \cdots \rangle$ is the thermal average, $n_k \equiv \sum_i n_i e^{-i\mathbf{k} \cdot \mathbf{r}_i}$ is the Fourier transformation of the charge distribution, $\theta_i$ is the angle between a reference direction, say $x$, and the line connecting the $i$th particle and its closest neighbor. Since the smallest wavevector is limited by the system size, we choose $\mathbf{k} = A/L, B/L$, and $-(A + B)/L$ with reciprocal lattice vectors $A = 4\pi \hat{y}/\sqrt{3}$ and $B = 2\pi(\hat{x} - \hat{y}/\sqrt{3})$, and take the average over these three smallest $k$’s (with magnitude $k = \sqrt{1/3}/3\sqrt{N}$) to obtain the static dielectric function $1/\epsilon(0)$ in Eq. [4].

The dynamic behaviors are described by the dynamic dielectric function $1/\epsilon(\omega)$ (see Refs. [12 and 13] for details):}

$$\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)} = -i\omega \int_0^\infty dt e^{i\omega t} G(t),$$

(7)

$$G(t) \equiv \lim_{k \rightarrow 0} \frac{2\pi}{NK^2} \left\langle n_k(t) n_{-k}(0) \right\rangle,$$

(8)

where $\langle n_k(t) n_{-k}(0) \rangle$ is the charge correlation function, and $1/\epsilon(0)$ is the static dielectric constant given above. Just as for $1/\epsilon(0)$ in Eq. [4], $G(t)$ is obtained as the average over the three smallest wavevectors. We measure time $t$ in units of the MC step so that one time unit corresponds to $N_c$ MC tries to move the randomly chosen particle to one of its nearest neighbors.

III. DYNAMIC RESPONSE

The dynamic response function we focus on is the complex dielectric function $1/\epsilon(\omega)$ given by Eq. [7]. The anomalous vortex diffusion is reflected in the fact $G(t) \propto 1/t$ for large $t$ [24]. From this one directly infers that to leading order in small $\omega$ (Ref. [16]
\[
\text{Re}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = \frac{1}{\epsilon} \frac{\omega}{\omega^2 + \tau^2}, \\
\text{Im}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = -\frac{1}{\epsilon} \frac{\omega \tau^{-1}}{\omega^2 + \tau^2},
\]

where \(\frac{1}{\epsilon}\) is a constant and the first equality in Eq. (10) follows because \(1/\epsilon(0)\) is always a real quantity. The complex conductivity for a superconductor \(\sigma(\omega)\) corresponds to 
\[ -1/\omega\epsilon(\omega) \]
which means that \(\text{Re}\sigma(\omega) \propto -\ln \omega\) for small \(\omega\).

This logarithmic divergence is another characteristic of the anomalous vortex diffusion.

The MP form is given by
\[
\text{Re}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = \frac{1}{\epsilon} \frac{\omega}{\omega + \omega_0}, \\
\text{Im}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = -\frac{2}{\epsilon} \frac{\omega \omega_0 \ln \omega/\omega_0}{\omega^2 - \omega_0^2},
\]

and the corresponding explicit form of \(G(t)\), which we denote by \(G_{\text{MP}}\), hence incorporates the features associated with \(G(t) \propto 1/t\). The MP form is associated with a single characteristic frequency \(\omega_0\). In the 2D neutral Coulomb gas all vortices are bound in vortex-antivortex pairs in the low-temperature phase. This implies an infinite correlation length which for a superconductor means a vanishing resistance \(R\). Since the logarithmic divergence of the conductivity means that \(R = 0\), a possible scenario is that the MP response form describes the response of the vortex-antivortex bound pairs and hence should apply to the low-temperature phase. It has been verified, from simulations of the 2D neutral Coulomb gas with Langevin dynamics, that the MP form indeed gives a very good description in this case. Above the KT transition there are both free vortices and bound vortex-antivortex pairs present and hence there is a finite correlation length. This means that \(G(t)\) has an exponential decay of the form \(e^{-t/\tau}\), where \(\tau\) is a relaxation time and \(R \propto 1/\tau\). Thus a finite resistance is associated with an exponential decay of \(G(t)\). If there were only free vortices then \(G(t)\) would be dominated by the exponential factor and the response should to good approximation be described by the standard Drude form \(G_D \propto e^{-t/\tau}\), i.e.,

\[
\text{Re}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = \frac{1}{\epsilon} \frac{\omega^2}{\omega^2 + \tau^{-2}}, \\
\text{Im}\left(\frac{1}{\epsilon(\omega)} - \frac{1}{\epsilon(0)}\right) = -\frac{1}{\epsilon} \frac{\omega \tau^{-1}}{\omega^2 + \tau^{-2}}.
\]

More generally, when free vortices are present the response form \(G_{\text{MPD}} = G_{\text{MP}}(t)G_D(t)\) has from simulations been shown to give a good parameterization of the data.

One difference between the MP response and the Drude response, which is easy to gauge in experiments and simulations, is the peak ratio: The peak ratio (PR) is defined as the ratio between the real and imaginary part of the complex dielectric function at the frequency where \(\text{Im}[1/\epsilon(\omega)]\) has its maximum, i.e., at the dissipation peak (see Fig. 6). For the standard Drude response this ratio is 1 whereas it is \(2/\pi \approx 0.64\) for the MP response. In general when the resistance is finite the peak ratio should be between these two values, \(2/\pi \leq \text{PR} \leq 1\).

Our strategy is to calculate \(G(t)\) as described above and then to gauge the response by studying the peak ratio, \(G(t)\) for large \(t\), and to what extent the response is parameterized by the MP form.

IV. SIMULATION RESULTS

A. Lattice Coulomb gas without disorder

In this section, we first present the results of the MC simulations of the lattice Coulomb gas model given by the Hamiltonian in Eq. (1) for the case without random pinning sites, i.e., \(U_i = 0\) in Eq. (3). It is known that the system can undergo two phase transitions in this case. One is the depinning of the solid at \(T_s\), corresponding to the superconducting to normal transition, and the other is the solid to liquid transition at \(T_m\) and \(T_c\) (\(T_c \leq T_m\)). These two transitions can be identified by a jump in the quantities \(1/\epsilon(0)\) and \(\phi_0\), respectively. Since the continuum system corresponds to an infinitesimally small lattice spacing and the frustration \(f\) is proportional to the area of the elementary plaquette, the continuum limit of the lattice Coulomb gas is given by \(f \rightarrow 0, \text{ but with the number of particles } N_c \text{ still finite: } N_c = Nf = \text{const} \). In Ref. [24] it is shown that \(T_c\) becomes smaller as \(f\) is decreased indicates that \(T_c \rightarrow 0\) in the continuum system so that the system only has one transition at a nonzero temperature \(T_m\).

We start by presenting some static results for \(f = 1/16\) on a \(64 \times 64\) triangular grid. In the simulations, we start from high enough temperatures and anneal the system by decreasing the temperature slowly, to avoid being captured by metastable states at low temperatures. However, the local MC scheme used in this work (a particle is only allowed to hop to its nearest neighbor sites) makes it difficult to achieve equilibrium near and below the transition. Thus very many MC updates are required in this region (our longest runs consists of \(2 \times 10^6\) MC steps). As \(T\) is decreased \(1/\epsilon(0)\) abruptly changes to unity near \(T \approx 0.01\) (see Fig. 1). This abrupt change is associated with the normal (high \(T\)) to superconducting (low \(T\)) transition in the vortex system since \(1/\epsilon(0)\) corresponds to the superfluid density. In the Coulomb gas model it corresponds to a transition where the Coulomb gas systems becomes pinned to the underlying grid. The six-fold orientational order parameter \(\phi_0\) has the value unity for a perfect triangular Abrikosov vortex lattice. Consequently the abrupt drop of \(\phi_0\) seen in Fig. 1 indicates that the vortex solid melts into a vortex liquid near \(T \approx 0.01\). The specific heat \(C_v\) also shows a sharp maximum near this temperature \(T \approx 0.01\). Consequently the data in Fig. 1 for the lattice Coulomb gas model with \(f = 1/16\) are consistent with a single phase transition corresponding to a transition where the superconductor becomes normal and the Abrikosov vortex structure melts, i.e., \(T_c \approx T_m \approx 0.01\).
We next turn to the dynamic results for \( f = 1/16 \). These are shown in Fig. 2. The characteristic frequency \( \omega_0 \) in Fig. 2(a), determined from the peak position of \( |\text{Im}[1/\epsilon(\omega)]| \), shows a dip-like feature near \( T = 0.01 \), reflecting a critical slowing down close to the phase transition (compare the static results in Fig. 1). In general, a system composed of particles is expected to be well described by a Drude approximation at sufficiently high temperatures because the interaction between the particles becomes negligible in comparison with the strong thermal fluctuations. The dynamic response function in such a temperature regime should be well described by the Drude response form Eqs. (3) and (10) and should consequently be well characterized by the peak ratio (PR) equal to one. As is shown in Fig. 1(b), the dynamic dielectric function indeed approaches the Drude PR value one at higher temperatures. However, the PR crosses over to a value close to PR=0.64 which characterizes the anomalous behavior (see the previous section) as we approach \( T \approx 0.01 \) from above.\[ \]

The question is then what causes this crossover behavior. Is it associated with the depinning transition at \( T_c \), or with the melting transition at \( T_m \)? In order to address this question one needs a temperature separation between the two transitions. In Ref. 24, it was shown that as \( f \) is decreased, the two transitions present become well separated. With this in mind we, as a next step, repeat the simulations for a much smaller value of \( f \).

Figure 3 shows the static results \( 1/\epsilon(0), \phi_0, \) and \( C_v \) for the system with \( f = 1/16 \) on a 96 \times 96 lattice. As seen from the figure, the superconducting to normal transition is now at \( T_c \approx 0.0035 \) and is well separated from the vortex solid melting transition at \( T_m \approx 0.007 \), as was found in Ref. 24. As also seen in Fig. 3, the existence of two transitions are also reflected by two peaks in \( C_v \): one near \( T_c \) and another broader one near \( T_m \). The intermediate phase existing between \( T_c \) and \( T_m \) is characterized by a vanishing \( 1/\epsilon(0) \) together with a nonzero \( \phi_0 \) and is interpreted as a floating solid phase.\[ \]

The vortex solid structure is depinned from the underlying discrete grid and floats around, which leads to dissipation of energy.

Next we analyze the dynamic response function for \( f = 1/16 \) in terms of the characteristic frequency \( \omega_0 \) and the peak ratio PR. In Fig. 4 it is shown that \( \omega_0 \) displays a dip-like structure reflecting a critical slowing down near \( T_c \) rather than at \( T_m \). Similarly, the correspondence of the crossover behavior for \( f = 1/16 \) in PR from Drude to anomalous response form shown in Fig. 2(b), is in Fig. 4(b) found to be near \( T_c \) and not near \( T_m \). From this we conclude that the crossover behavior in the dynamic response is associated with \( T_c \) and hence with the pinning of the Coulomb gas to the underlying lattice. Thus for the pure one component lattice Coulomb gas with MC dynamics we infer that there is a crossover behavior from a normal Drude like response to an anomalous response. An essential ingredient for the appearance of the anomalous response is associated with the pinning between the vortex system and the underlying lattice.

B. Lattice Coulomb gas with disorder

In the previous section we inferred that the pinning, caused by the underlying lattice, leads to a crossover from a Drude to an anomalous dynamic response. In the present section we study the role of pinning further by introducing extrinsic vortex pinning caused by disorder into the model. The general idea is that the anomaly in the dynamics is caused by a sluggish motion of the vortex system and that pinning hindering the vortex motion can cause such a behavior. To this end we introduce point impurities into the lattice Coulomb gas model as described by the Hamiltonian in Eq. (3). For a superconductor this corresponds to magnetic point impurities. In this section, we perform MC dynamic simulations for 10 different disorder realizations in order to get disorder averaged quantities.

In Fig. 5 the PR for the system with \( f = 1/64 \) and \( L = 96 \) at \( T = 0.01 \) is displayed as a function of the pinning strength \( U^p \) (the number of pinning sites \( N_p \) is fixed to 14, which approximately corresponds to 10% of the total number of vortices \( N_v \), i.e., \( N_p \approx 0.1 N_v \)). As seen in Fig. 6 the dynamic response for the pure case \( (U^p = 0) \) has PR close to one at this higher \( T \) [compare Fig. 6(b)]. This means that the pinning to the underlying lattice at this \( T \) is too weak to influence the behavior so that the vortices should obey a Drude response. However, if we introduce disorder then Fig. 6 shows a decreasing PR with increasing pinning strength again suggesting a crossover towards an anomalous dynamic response [compare Fig. 6(b)]. This is consistent with the idea that when the pinning strength becomes stronger, the motion of the vortices becomes more sluggish, resulting in a decrease of the PR and a crossover to an anomalous response.

This scenario implies that if we introduce enough pinning then the crossover to the anomalous response should become complete. To achieve this we investigate a stronger pinning case where \( N_p = N_v \) and the pinning strength \( U^p = 0.1 \). Figure 7 shows the obtained dielectric function \( 1/\epsilon(\omega) \) for this case [see Eqs. (7) and (8)] at \( T = 0.02 \) which is much higher than \( T_c \), as well as \( T_m \), for the pure case \( (T_c \approx 0.0035 \) and \( T_m \approx 0.007 \), respectively). As illustrated in Fig. 8, the PR is indeed close to the anomalous value \( PR = 2/\pi \) in this case. In the MP phenomenology the anomalous dynamics is linked to a \( 1/\tau \) decay of the correlation function \( G(t) \) [compare Eqs. (8) and (12)]. The time-correlation function \( G(t) \) corresponding to \( 1/\epsilon(\omega) \) in Fig. 8 is shown in Fig. 9 and indeed has a \( 1/\tau \) tail as is manifested by the horizontal plateau of \( tG(t) \) for larger \( t \). This suggests that there exists a link between a \( 1/\tau \)-tail in \( G(t) \) and the PR = \( 2/\pi \). Such a link is incorporated into the MP response form Eqs. (11) and (12). The inset in Fig. 9 shows what happens for the same case at the higher \( T = 0.03 \). In this case \( G(t) \) suggests the presence of an exponential decay. This is very similar to previous simulations where \( G_{\text{MPD}} \equiv G_{\text{MP}}(t)G_D(t) \) has been shown to give a good parameterization [see Sec. II below Eq. (14) and Refs. 13, 14, and 17]. Figure 8(a) shows that a reasonable fit of \( 1/\epsilon(\omega) \) to the MP form is obtained in the anomalous case \( T = 0.02 \). This suggests that the basic link between
the $G(t) \propto 1/t$ and the PR=2/π is reasonably well captured by the MP form. Figure [8] shows a fit to the exponentially decaying case at $T = 0.03$ using the parameterization $G_{\text{MPD}} \equiv G_{\text{MP}}(t)G_{\text{D}}(t)$. Again a reasonable fit is obtained with PR≈ 0.9. This suggests that, although it is close to the Drude form, there is a small crossover towards the anomalous response (see Fig. 5).

In this section we have thus shown that for the one component lattice Coulomb gas, at a $T$ so high that the pinning to the underlying lattice plays no role, it is still possible to obtain a crossover to the anomalous response by introducing pinning through randomly distributed pinning sites.

**V. COMPARISON AND DISCUSSIONS**

We have found from simulations that the 2D one component lattice Coulomb gas with Monte Carlo dynamics under certain circumstances displays an anomalous dynamic response. An essential ingredient for the appearance of this anomalous response is found to be pinning; either pinning due to the underlying lattice or external pinning induced by disorder. The anomalous response vanishes gradually as the pinning effects become weaker and from this we conclude that there is no anomaly in the absence of pinning.

We gauge the degree of anomaly in the response by the behavior of the correlation function $G(t)$ for large $t$, the peak ratio, and to what extent the response can be parameterized by the MP form. We find that the full anomaly is characterized by $G(t) \propto 1/t$ for large $t$, a peak ratio consistent with 2/π and note that these two features are also incorporated into the phenomenological MP form. For the one component lattice Coulomb gas this means that an anomalous response in the absence of externally introduced pinning only appears very close to the depinning of the vortex system from the underlying lattice structure. From this we further infer that the pure continuum one component Coulomb gas has no anomalous response and that in this case external pinning is required to change the response from normal to anomalous.

It is interesting to compare this with the 2D neutral (two components) continuum Coulomb gas with Langevin dynamics which is the generic model for the vortex physics of a 2D superconductor(superfluid) in the absence of an external perpendicular magnetic field (net rotation). This model undergoes a KT transition and below this transition the dynamic response has from simulations been shown to be anomalous characterized by $G(t) \propto 1/t$ for large $t$, a peak ratio consistent with 2/π, and to be well described by the MP form. Since the same response form is found for the one component Coulomb gas in the presence of external pinning, as for the 2D neutral (two component) Coulomb gas below the KT transition, one might speculate about a common physical feature. We here suggest that the common physical feature is pinning; in the one case externally introduced and in the other intrinsic in the sense that negative vortices act as (moving) pinning centers for positive ones (and vice versa). In both cases the pinning mechanism suppresses the free vortex diffusion. When this suppression is complete the response becomes anomalous as for the one component Coulomb gas with enough external pinning and for the neutral Coulomb gas below the KT transition where all positive and negative vortices are bound together in pairs.

Thus our conclusion based on simulations is that the 2D one component continuum Coulomb gas cannot have an anomalous response by itself. This is in contrast to the theoretical considerations in Ref. [19], based on a Mori approximation scheme, which suggest that the one component model in certain parameter regions could have an anomalous response. We have not been able to find any support for this suggestion in our simulations.

The MP form was originally motivated for the 2D neutral (two components) Coulomb gas as an extension of the AHNS phenomenology and was assumed to describe the response of bound vortex pairs. Other recent improvements of the AHNS phenomenology also gives a peak ratio close to 2/π. However, the MP form is to our knowledge the only form that incorporates both the large $t$ behavior $G(t) \propto 1/t$ and the peak ratio $2/\pi$, both of which seems to be characteristics of the anomalous response. In the one component Coulomb gas case with external pinning there is at present really no motivation for the MP form; it is just a simple form which simultaneously incorporates two features which seem to be characteristics of the anomalous response.

Our basic conclusion is that the anomalous response form can indeed arise from an interplay between pinning and a vortex system associated with an external perpendicular magnetic field. In this context one may note that a $2/\pi$ peak ratio was found for the experimental data in Ref. [23]. This was a MoGe superconducting film in a perpendicular magnetic field where pinning was expected to be important. We suggest that this might be an experimental example of our present findings from simulations.

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28 In connection with Fig 3 for the disordered case the possibility of a link between PR=2/π and G(t) ~ 1/t is discussed. This link also seems to be borne out in the present case without disorder although the convergence of the data is not as impressive as in Fig 3.
29 The fact that ω0 does not reach zero in this case is due to the finite lattice size. We believe that this is also the reason why the PR does not decrease all the way to 2/π.
30 The fact that disorder may cause a slow decay of the correlations is reminiscent of glassy states. However, the special feature of the anomalous response discussed here is that the correlations decay precisely as 1/t.
31 The cause of the increase of PR for decreasing T at the lowest temperatures is not clear for us. It cannot be ruled out that it may be an artifact of the finite-size lattice.
FIG. 1. Static dielectric function $1/\varepsilon(\omega = 0)$, the orientational order parameter $\phi_6$, and the specific heat $C_v$ vs temperature $T$ for the 2D lattice Coulomb gas model on a $64 \times 64$ triangular grid with the frustration $f = 1/16$. The depinning transition at $T_c$, corresponding to the superconducting to normal transition, is signaled by an abrupt change of $1/\varepsilon(0)$. The solid to liquid transition at $T_m$ probed by $\phi_6$ occurs at approximately the same temperature, i.e., $T_c \approx T_m \approx 0.01$. $C_v$ also shows a sharp maximum near the same temperature.

FIG. 2. Characteristic features of the dynamic dielectric function for the system with the frustration $f = 1/16$ and size $L = 64$. (a) The characteristic frequency scale $\omega_0$ as a function of $T$ obtained from the peak position of $|\text{Im}[1/\varepsilon(\omega)]|$: $\omega_0$ decreases near $T \approx 0.01$, reflecting a critical slowing down, in accordance with Fig. 1. (b) The peak ratio PR defined by the ratio between real and imaginary part of $1/\varepsilon(\omega)$: The response function is described by the simple Drude response form with PR= 1 at high enough temperatures and crosses over to the anomalous behavior with PR= 2/π as $T$ is decreased.

FIG. 3. Static dielectric function $1/\varepsilon(\omega = 0)$, the orientational order parameter $\phi_6$, and the specific heat $C_v$ vs temperature $T$ for the system with $f = 1/64$ and $L = 96$. The depinning transition is at $T_c \approx 0.0035$ and melting transition is at $T_m \approx 0.007$. Also note that $C_v$ displays two peaks: one near $T_c$ and the other one near $T_m$. The intermediate phase existing between $T_c$ and $T_m$ has a nonzero orientational order and corresponds to a floating solid phase.

FIG. 4. Characterization of the dynamic dielectric function for the system with the frustration $f = 1/64$ and size $L = 96$. (a) The characteristic frequency $\omega_0$ vs temperature $T$ shows a dip-like structure near $T \approx T_c \approx 0.0035$ (see Fig. 3 for the static results). (b) The peak ratio PR vs $T$ reflects a crossover behavior from Drude response form in high temperatures to the anomalous one near $T_c$. 

FIG. 5. The peak ratio PR vs the pinning strength $U_p$ for the system with $f = 1/64$ and $L = 96$ at $T = 0.01$. The number of pinning sites was fixed to 14 (which is about 10% of the total number of vortices and the results correspond to an average over 10 random pinning realizations). The PR starts from unity for the pure case ($U_p = 0$), which is the value for the Drude response, and decreases as $U_p$ is increased, reflecting that the vortex motion becomes more sluggish.

FIG. 6. The dynamic dielectric function obtained for the system with $f = 1/64$ and $L = 96$ in the presence of extrinsic pinning with $U_p = 0.1$ and $N_p = N_c$ for $T = 0.02$ (10 random pinning configurations were used). Filled circles and open circles correspond to $\text{Re}[1/\epsilon(\omega)]$ and $|\text{Im}[1/\epsilon(\omega)]|$. The PR (the ratio between the real and imaginary part at the maximum of the imaginary part) is found to be close to the MP value $2/\pi$.

FIG. 7. The time-correlation function $G(t)$ for the same case as in Fig. 6 plotted as $\ln G(t)$ vs $t$. The horizontal plateau for larger $t$ shows that the long-time behavior of $G(t)$ has a $1/t$ decay. The inset shows the same case at $T = 0.03$ and at this higher temperature $G(t)$ appears to have an exponential decay.

FIG. 8. Fits of the dynamic dielectric function to the MP form. (a) $1/\epsilon(\omega)$ corresponding to the anomalous $G(t)$ for $T = 0.02$ in Fig. 7 is fitted to Eqs. (11) and (12). Filled circles, open circles and full drawn curves correspond to $\text{Re}[1/\epsilon(\omega)]$, $|\text{Im}[1/\epsilon(\omega)]|$, and the MP form, respectively. A reasonably good fit is obtained. (b) $1/\epsilon(\omega)$, corresponding to the exponentially decaying $G(t)$ for $T = 0.03$ in the inset in Fig. 7, is fitted to the parameterization obtained from $G_{\text{MPD}}(t) \equiv G_{\text{MP}}(t)G_D(t)$ (see the text). Filled circles, open circles and full drawn curves correspond to $\text{Re}[1/\epsilon(\omega)]$, $|\text{Im}[1/\epsilon(\omega)]|$, and the $1/\epsilon(\omega)$ from $G_{\text{MP}}$, respectively. A reasonably good fit is found for $\text{PR} \approx 0.9$ which is slightly smaller than $\text{PR} = 1$ corresponding to the pure Drude value.