Modeling heat conduction in open-cell metal foams by means of the Three-Dimensional Thermal Fin theory

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Abstract. Their relatively high thermal conductivity makes metal open-cell foams promising heat transfer enhancers for lightweight applications. Conduction occurs through both solid and fluid phases, depending on the material and the microstructure. The effective conductivity of a foam is predicted by means of analytical and numerical methods, based on both idealized and realistic foam geometries, as well as by empirical correlations based on experiments. In this paper, a new model for the prediction of the effective thermal conductivity of foams is proposed. The proposed Three-Dimensional Thermal Fin (TTF) theory is based on the Electrochemical Fin (ECF) theory previously applied to the analysis of ion and electron transport through nanoporous materials. It has computational time two – three times shorter than those of current numerical techniques, such as finite-element methods. Starting from a tomography-reconstructed metallic open cell foam, the 3-D structure is modeled as a network. Conventional fin heat transfer scaled equations, applied to each part of the network, allow for the evaluation of the temperature distribution and heat rates. The effective thermal conductivity of the metal foam is then derived. The proposed model is validated by comparison with experimental results as well as with predictive models from the open literature and a finite-element based model.

Nomenclature

\begin{align*}
A & \quad \text{area, m}^2 \\
B, c & \quad \text{coefficients in equation (3.3)} \\
C & \quad \text{coefficient in equation (3.4)} \\
l/h & \quad \text{unit surface thermal resistance, m}^2 \text{K W}^{-1} \\
i & \quad \text{electrical current, A} \\
k & \quad \text{thermal conductivity, W m}^{-1} \text{K}^{-1} \\
l & \quad \text{fin length, m} \\
L & \quad \text{length of the computational domain, m} \\
L^* & \quad \text{sample scaled size} \\
\dot{Q} & \quad \text{heat rate, W} \\
R_{ct} & \quad \text{unit charge transfer resistance, } \Omega \text{ m}^2 \\
SQ & \quad \text{sintering quality} \\
T & \quad \text{temperature, K} \\
V & \quad \text{volume, m}^3 \\
x & \quad \text{strut axial coordinate, m} \\
\varepsilon & \quad \text{porosity} \\
\rho & \quad \text{resistivity, } \Omega \text{ m}^{-1} \\
\sigma_{io} & \quad \text{ionic conductivity, S m}^{-1} \\
\varphi & \quad \text{voltage, V}
\end{align*}


1. Introduction

A foam is a cellular structure made up by a network of solid struts, which are the edges and faces of cells containing a large volume fraction of fluid. The foam structure is generally represented as a three-dimensional polyhedral structure whose pores either are sealed or form an interconnected network. The shape of the cross-section of this finite-dimension network depends on the manufacturing technique, that can also make the cross section of the strut variable.

With reference to their structure, metal foams are classified as open-cell foams or closed-cell foams. Open-cell foams consist of interconnected cells that allow a fluid to pass through, whereas in a closed-cell foam the cells are single enclosures within the material. In other words a cell is the unit that repeats through the space; a pore is the window between adjacent cells. The selection of the foam’s solid material is generally based on the end-application. Because of their mechanical characteristics [1], closed-cell foams are mostly used in structural, load-bearing applications. Open-cell foams have good thermal properties [1, 2] and are currently used in many applications, such as automotive and biomedical applications or compact heat exchangers and solar air receivers [2 - 4].

Heat conduction in foams is currently investigated experimentally, analytically and numerically. The effective thermal conductivity of a foam is defined as the equivalent thermal conductivity of the two-phase porous medium. Different experimental techniques were employed. Bhattacharya et al.[5] carried out experiments by brazing metal foams between two aluminum plates at different uniform temperatures, measured by four thermocouples, attached to each plate. Analytical models were developed, based on idealized foam geometries. Boomsma and Poulikakos [6] and Yang et al. [7] presented analytical predictive models based on a tetrakaidekahedron geometry. Predictions of both studies were in good agreement with experiments for porosities higher than 0.90. The numerical solution of governing equations allows for the prediction of the effective thermal conductivity of more complex geometries [8, 9].

The Electrochemical Fin (ECF) theory is an analytical method used in the microstructural analysis of Sr2Fe1.5Mo0.5O6-δ (SFM), a redox stable solid oxide fuel cell (SOFC) electrode [10]. By using digital image processing tools, the microstructure is separated into an underlying network, made up of individual segments, that behave analogously to connected thermal fins. The fins are characterized to determine the parameters to be used in the ECF theory. The potential distribution along the network segments is obtained by the analytical solution of a system of one-dimensional ordinary differential equations developed from charge balances. With ECF theory, the electrode structure is developed as a complex combination of electrical resistances with different geometries. Compared to traditional techniques, such as the finite-element based ones, the above described approach leads to significant savings in computational time and cost, say two or three times [11].

In this work, starting from the ECF theory, a new tool for the prediction of the effective thermal conductivity of metal foams, the Three-Dimensional Thermal Fin (TTF) theory, is first presented. The electrical analogy for heat conduction is employed to extend the ECF theory to conduction heat transfer problems. The geometry of a 0.94 nominal porosity and 40 PPI open-cell aluminum foam is reconstructed by using x-ray computed tomography. The TTF theory is applied on the skeletonized tomographic image. The code is validated comparing its predictions with those given by models and experiments taken from the open literature as well as with predictions from COMSOL Multiphysics.
code. The comparisons validated the reliability of the TTF model and demonstrated it to be a promising tool for the prediction of conduction heat transfer through foam materials.

2. Mathematical model

2.1. Tomographic scans of the sample

The 0.94 nominal porosity and 40 PPI open-cell aluminium foam sample analyzed in the present work was manufactured by ERG Aerospace. The sample is a 10 mm long cylinder with a 10 mm diameter. The sample geometry was described by X-ray tomography carried out with an Xradia MicroXCT-400 scanner. A 17.8 μm (1x lens) resolution was chosen in order to highlight the foam strut shape. Ambrosio et al [12] showed that higher resolutions are not needed since they provide with similar CT-data morphological features. A filtered back-projection software provided by Xradia was employed to perform the 3-D object reconstruction [13].

2.2. The ECF and TTF theories

Under the assumption that the current flow continuously passes through the struts, that allows to neglect the effects of the space charge regions on the electrode performance as well as the assumption of one-dimensional Ohmic conduction in the solid phase (conduction occurs only along the fin axial coordinate, x), the governing equation for the ECF theory is [11]

$$\frac{d^2 \phi}{dx^2} + \frac{1}{A^*_x} \left( \frac{dA^*_x}{dx} \right) \left( \frac{d\phi}{dx} \right) - \frac{1}{\sigma_{\text{io}}} \left( \frac{dA_{\text{surf}}}{dx} \right) \phi = 0 \quad (2.1)$$

The comparison of predictions from the ECF theory with those given by finite element methods showed that the Ohmic conduction could reliably be assumed to be one-dimensional [14].

A key benefit of the above discussed analytical approach is the possibility to scale equations in order to account for the morphology and performance of SOFC electrode microstructures. The interplay between microstructural morphology, material properties and operating conditions is described by a set of basic non-dimensional parameters: a non-dimensional length, x* = x/l, a non-dimensional voltage, φ* = φ/φb, two non-dimensional areas, A* = A/Ac, and A‾surf = Asurf/Ac.

A non-dimensional form of equation (2.1) is now derived

$$\frac{d^2 \phi^*}{dx'^2} + \frac{1}{A^*_x} \left( \frac{dA^*_x}{dx'} \right) \left( \frac{d\phi^*}{dx'} \right) - \frac{1}{\sigma_{\text{io}}} \left( \frac{dA_{\text{surf}}}{dx'} \right) \phi^* = 0 \quad (2.2)$$

The number of variables in equation (2.2) is further reduced with scaling techniques. To this aim, reference is made to some significant parameters describing the electrode morphology and charge transfer characteristics.

The first parameter is the ionic electrical resistivity ratio

$$\frac{\rho_{\text{io}}}{\rho_{\text{ct}}} = \frac{l^2}{R_{\text{ct}}} \left( \frac{1}{A} \right) \left( \frac{dA_{\text{surf}}}{dx} \right) \approx \frac{l^2}{\sigma_{\text{io}} R_{\text{ct}}} \left( \frac{A_{\text{surf}}}{V} \right) \quad (2.3)$$

that is the ratio of the ionic electrical resistivity of the electrode to the charge transfer electrical resistivity. It depends on the electrode thickness, on the variable fin cross-sectional area and fin surface area. As an operator on the normalized potential term, the ionic electrical resistivity ratio represents a non-dimensional surface charge transfer coefficient that quantifies the balance between the rate of conductive charge transfer through the solid structure to the chemically active surface and...
the rate of charge transfer from the chemically active surface. It can be recognized as the coefficient of the third term on the left-hand side of equation (2.2).

The second parameter, the sintering quality, can be derived from the coefficient of the second term on the left-hand side of equation (2.2)

\[ SQ = \frac{1}{\left( \frac{l}{A} \right) \frac{dA}{dx}} = \left( \ln \frac{A_{c,\text{min}}}{A_{c,\text{max}}} \right)^{-1} \]  

that accounts for the variation in the cross-sectional area along the length of an electrode. The partitioning of the solid phase and the consequent necessity of morphological and topological information for the analytical modelling approach imply to make reference to the average values of the sintering quality and resistivity ratio for each segment of the structure. As to the sintering quality, this implies that the profiles which can be rigorously treated are only the exponential variations of the cross-section. The above profiles require to measure only the intra-phase interfacial surfaces \( A_{c,\text{min}} \) and \( A_{c,\text{max}} \). As to the resistivity ratio, the geometric factor in parentheses is approximated using the measured surface area to volume ratio (see the second term in equation (2.3)).

The final form of the scaled equation is

\[ \frac{d^2 \phi^*}{dx^2} + \frac{1}{SQ} \left( \frac{d \phi^*}{dx} \right) - \frac{\rho_w}{\rho_\phi} \phi^* = 0 \]  

It is worth underlining that the assumption of an exponential variation of the strut cross section along its length fits well for open-cell foams, as it is shown in figure 1, where the strut magnification of a 0.94 nominal porosity 40 PPI aluminum-foam is reported and the strut surface is highlighted in yellow.

Starting from the ECF theory, the analogy between thermal and electrical transport allows one to extend the ECF theory to the study of heat conduction in metal foams. The thermoelectric analogy between Fourier’s and Ohm’s laws is summarized in table 1. The non-dimensional governing equation of the ECF theory, equation (2.2), also holds for heat transfer. Continuous temperature and heat flux across each junction are assumed. Temperature differences as small as to make the dependence of the thermal conductivity on the temperature negligible are imposed.

2.3. Foam skeletonization

The generation of the skeleton starting from the tomography of the of 0.94 nominal porosity 40 PPI aluminum-foam.

![Figure 1. Strut magnification of a 0.94 nominal porosity 40 PPI aluminum-foam.](image-url)
Table 1. Thermoelectric analogy.

| Ohm’s law (Electrical) | Fourier’s law (Thermal) |
|------------------------|------------------------|
| \( i = \sigma_{io} A \frac{\Delta \phi}{l} \) | \( \dot{Q} = k A \frac{\Delta T}{l} \) |
| \( i \) | Electrical current, A |
| \( \Delta \phi \) | Voltage difference, V |
| \( \sigma_{io} \) | Ionic conductivity, S m\(^{-1}\) |
| \( R_{ct} \) | Unit charge transfer resistance, \( \Omega \) m\(^2\) |
| \( Q \) | Heat rate, W |
| \( \Delta T \) | Temperature difference, K |
| \( k \) | Thermal conductivity, W m\(^{-1}\) K\(^{-1}\) |
| \( \frac{1}{h} \) | Unit surface thermal resistance, m\(^2\) K W\(^{-1}\) |

metal foam is presented in figure 2. The skeletonization process is performed by means of postprocessing tools like Avizo and MATLAB. The process starts with the reconstruction of the X-ray microtomography data to generate a grayscale tomogram (figure 2a), and adjusted by using a Gaussian blur filter to improve the contrast in the image. A thresholding of a region of interest (ROI) within the grayscale image (figure 2b) is performed using the Otsu method. Erosion/dilation operations are also performed to clean the surface. Finally, a skeletonization algorithm generates a microstructural skeleton (figure 2c), which is further partitioned into a network. The particles in the network are characterized to yield fin morphological parameters and the network performance is subsequently assessed (figure 2d).

3. Results and discussion

3.1. Data reduction
Under the previously described appropriate boundary conditions, a COMSOL finite element code generates output data like heat flux and heat rate. Based on the boundary conditions, the effective

![Figure 2. Skeleton work-flow: a) grey-scale image; b) thresholded image; c) reconstructed skeleton; d) network.](image_url)
thermal conductivity can be determined by assuming heat transfer along each of the three principal directions in the foam domain. Convection heat transfer is neglected, thus only conduction through the foam is accounted for. Under the above assumptions, the Fourier law allows to derive the effective thermal conductivity of each phase

\[ k_{\text{eff}} = k_{\text{eff},s} + k_{\text{eff},f} = \frac{QL}{A_\Delta T} + \varepsilon k_f = \frac{QL}{(1-\varepsilon)A_\Delta T} + \varepsilon k_f \]  

(3.1)

where the heat rate is given as a code output and the porosity, as well as the cross-section area, are taken from imaging tools. With reference to equations (2.3) and (2.5), it is worth remarking that, the effective thermal conductivity obtained with the proposed Three-Dimensional Thermal Fin theory accounts also for convective heat transfer effects, since the model includes the unit charge transfer resistance, \( R_{ct} \). Convection heat transfer was assumed to be negligible, in order to predict overall heat transfer in the struts without carrying out the expensive simulation of the foam fluid phase.

3.2. Comparisons with literature data

3.2.1. Experiments. The average values of the effective thermal conductivities in the three directions from the TTF theory and the experimental data presented in [5, 15], for a 0.94 porosity and 40 PPI foam, are reported in table 2. Since few experimental data for 0.94 porosity foams were found in the literature, data for 0.94 ± 1% porosity have been reported in the table. It is important to remark that experimental data do not provide information on the anisotropy of the analyzed samples. The importance of anisotropy will be pointed out in Subsection 3.3. The good agreement with experimental data validates the feasibility of the herein proposed TTF theory.

3.2.2. Predictive models. Predictions from the TTF theory are now compared with those from models available in the open literature and a finite element code. Lemlich [16] derived analytically the correlation

\[ k_{\text{eff}} = \left( \frac{1-\varepsilon}{3} \right) k_s + \varepsilon k_f \]  

(3.2)

Calmidi and Mahajan [17], on the basis of experimental results, derived the correlation

\[ k_{\text{eff}} = k_f \left[ \varepsilon + B (1-\varepsilon) k_s/k_f \right] \]  

(3.3)

with the coefficients \( B = 0.181 \) and \( c = 0.763 \) for air, with 6.9% maximum and 3.7% average deviations.

Bhattacharya et al. [5] presented the empirical correlation

\[ k_{\text{eff}} = C \left[ \varepsilon k_f + (1-\varepsilon) k_s \right] + \frac{1-C}{\varepsilon/k_f + [(1-\varepsilon)/k_s] \Delta T} \]  

(3.4)

with the best fit for \( C = 0.35 \), with \( R^2 = 0.97 \).

Table 2. Average value of the effective thermal conductivities in the three directions from the TTF theory and experimental data presented in [5, 15] for a 0.94 porosity and 40 PPI foam.

| Average value from the TTF theory | Experimental data [5] | Experimental data [15] |
|----------------------------------|-----------------------|-----------------------|
| \( k_{\text{eff}} \) (W m\(^{-1}\) K\(^{-1}\)) | \( k_{\text{eff}} \) (W m\(^{-1}\) K\(^{-1}\)) | \( k_{\text{eff}} \) (W m\(^{-1}\) K\(^{-1}\)) |
| 3.9                              | 4.5                   | 3.8                   |
Numerical simulations on the same scanned foam were also carried out. The binary data, sketched in figure 2b, was imported into MATLAB and a tetrahedral mesh for the solid phase was built up by using the open-source tool iso2mesh. The heat conduction equation was solved with the finite-element commercial code COMSOL Multiphysics, by assuming the solid/fluid internal walls and four boundaries to be adiabatic; the remaining two boundaries were assumed to be at uniform temperatures, 20.5 °C and 20.0 °C, respectively. The above boundary conditions make the problem one-dimensional and the effective thermal conductivity is obtained from the calculated heat rate. The reason of the adiabatic solid/fluid internal walls assumption is the four order of magnitude difference between aluminum and air thermal conductivities, which makes negligible the heat transferred through the fluid phase.

The average of the effective thermal conductivities in the three directions from the TTF theory and the percent differences among it and those predicted by equations (3.2 - 3.4) for a 0.94 porosity 40 PPI foam are reported in table 3. The higher percent differences between TTF theory predictions and predictions from [5] and [16] is likely due to different values of the effective thermal conductivity when evaluated along the stretch direction, due to strut elongation during the manufacturing process. This anisotropic effect will be further investigated in the next subsection.

### 3.3. Anisotropy analysis

The effective thermal conductivity was observed to depend on the directions on which it is computed [8]. The effective thermal conductivities in the three directions, their average value and the value predicted by the analytical model [16] as a function of the scaled sizes of cubic samples, are presented in figure 3. The scaled size of a cubic sample, $L^*$, is the ratio of the sample size to the pore size taken from [12]. Figure 3 shows that, above a certain cubic sample scaled size, the effective thermal conductivity can be considered to be nearly independent of the scaled size of the sample.

A value of $L^* = 4.8$ can, therefore, be assumed as the scaled size of cubic sample that guarantees the invariance of the foam effective thermal conductivity [18]; the maximum difference among the values predicted in the three directions is 33%. The figure also exhibits a more marked anisotropy along the direction 3, which, as it is shown in figure 4, is the foam stretching direction.

The comparison of predictions from TTF theory along the three directions with values predicted by [16] shows that direction 3 provides values closer to those given by [16]. The effective thermal conductivity in the direction 3 is affected by the viscous and gravity forces during manufacture. One can remark that, for $L^* \leq 4.8$, the minimum differences are those between predictions from [16] and those along direction 3; differences with average TTF theory predictions are larger while the maximum differences are exhibited in comparison with directions 1 and 2. This means that the average value might be a good compromise in predicting the effective thermal conductivity.

Visual evidence of foam stretch is shown by the tomographic scan of a 0.94 porosity and 40 PPI aluminum foam cell reported in figure 4. The yellow-highlighted cells show that the foam is stretched along the direction 3. The cell elongation in direction 3 is suspected to be due to viscous and gravity forces during the foaming process.

**Table 3.** Averaged effective thermal conductivities in the three directions from the TTF theory and percent differences among it and those from other models for a 0.94 porosity 40 PPI foam.

| Averaged effective thermal conductivities in three directions from TTF theory $k_{eff} \,(W \, m^{-1} \, K^{-1})$ | Percent differences in the average effective thermal conductivity from TTF theory and those from other models |
|---|---|---|---|
| 3.7 | 23.5 | 19.1 | 4.4 |
4. Conclusions

A new tool for the prediction of the effective thermal conductivity of metal foams, the Three-Dimensional Thermal Fin (TTF) theory has been presented. It extends the Electrochemical Fin (ECF) theory to conduction heat transfer problems. Its computational efficiency is attractive when the effective thermal conductivity of foams must be iteratively evaluated.

The temperature distribution and heat rates in a 0.94 nominal porosity 40 PPI open-cell aluminium foam sample have been predicted employing the TTF theory. Then the effective thermal conductivity of the foam has been predicted. The TTF theory has been validated by comparison to published experimental data and predictions from models taken from the open literature as well as with predictions from COMSOL Multiphysics finite element simulations.

Differences between the average value of the effective thermal conductivities along the three orthogonal directions predicted by TTF theory and experimental data were about 20%, similar to those compared with predictions from models taken from the literature. The differences could likely depend on the foam anisotropy, caused by viscous and gravity forces during the foaming process. Far smaller differences were found in the comparison of TTF with COMSOL predictions.

Figure 3. The effective thermal conductivities predicted by TTF in the three Cartesian directions, their average value, and values predicted by [16] vs. the scaled sizes of cubic samples for a 0.94 and 40 PPI porosity foam.

Figure 4. Tomographic scan of a 0.94 porosity aluminum foam cell in the 1 - 3 plane (2 is orthogonal to the 1 - 3 plane).
The foam anisotropy has also been analysed and it is found to be larger along the foam stretch direction. The best agreement was found between the TTF theory predictions along the stretching direction and predictions from [16]. Since the agreement between the average value of TTF theory predictions along three orthogonal directions is also rather satisfactory, we can conclude that the average value might be a good compromise in predicting the effective thermal conductivity of the foam.

In the future, authors will also perform simulations both at equal computational cost and equal time, in order to quantify savings, when TTF is used instead of a finite element method.

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