Metamaterials have been extensively investigated for the development of invisibility cloaks; however, the role of nonequilibrium thermal fluctuations in the operation of metamaterial-based cloaking remains an unexplored avenue. Here, an analytical study on nonequilibrium thermal fluctuations induced charges and currents in metamaterials which generate electromagnetic modes is presented. It is argued that thermally induced charges in metamaterials cannot remain shielded from incoming electromagnetic radiation and excitation of such charges, by incoming electromagnetic waves, can enhance specific modes, whose radiation limits the invisibility effects associated with metamaterial-based cloaking. The analysis leads to the conclusion that thermodynamic equilibrium is an essential condition of the invisibility of metamaterial-based cloaking.

1. Introduction

Metamaterials are artificially engineered structures which can be used to manipulate the propagation of electromagnetic waves in space.[1] They work on the principle of transformation optics[2] and cancellation of scattering.[1] Research in the field of metamaterials has generated prospects of development of invisibility cloaks,[4–6] perfect lensing through negative refraction,[7–9] and novel photonic and microwave devices.[10–12] The basic physical principles are also being applied to other fields like acoustics[13] and thermodynamics.[14–16]

According to our current understanding based on empirical and theoretical investigations, the fundamental limitation of invisibility is associated with limitations imposed by frequency bandwidth of the incoming radiation.[17–19] The invisibility cloaks are made by developing a set of patterns at nanometer scale using nanofabrication processes like focused ion beam, electron-beam lithography, and laser writing.[20–22] Hence, the ability of fabricating metamaterials at macroscopic scale is another related problem besides the issue of achieving perfect impedance matching.[23] There appears to be some consensus toward the possibility that for specific frequency bands and size, invisibility can be achieved using metamaterials.

Although diverse kinds of physical structures have been investigated for different metamaterials,[1] the issue of nonequilibrium thermodynamic excitations under interaction with a thermal source and development of polarization modes in metamaterials has not been described in the literature until now. In the current work, the excitation of thermally induced polarization modes and external electromagnetic waves and metamaterials and related scattering effects have been explored. A special emphasis of the paper is on the fact that, although, metamaterials can shield objects from an incoming radiation, thermally induced charges on those specific objects cannot remain shielded from an incoming radiation and an interaction between a propagating electromagnetic wave and charges on the material’s outer surface can result in radiation from a metamaterial surface, which can lead to its detection.

Coupling between the external field and collective electrical polarization fields induced in a crystalline solid under nonequilibrium thermal excitation and electromagnetic field excitations lead to generation of Raman effect[24] and the electromagnetic emission from the system can be easily detected. As metamaterials comprise an array of artificially engineered patterns, in principle, such an effect should also be present in such systems, where interaction between incoming radiation and the selective modes existing in a metamaterial structure could generate finite radiation at specific frequencies. A closely related physical effect called Mie scattering has been studied in the context of metamaterials,[25] which underscores the need of investigation of Raman-like effects in such a system.

It has been suggested earlier that a charged particle fired on a metamaterial cloak generates synchrotron radiation as a consequence of acceleration under scattering with the metamaterial which leads to the loss of invisibility.[26] Here, we study a similar effect arising as a consequence of the interaction between thermally induced charges in a metamaterial structure and an incoming electromagnetic wave and generation of radiation from the metamaterial structure.

2. Charge Induction under Thermal Fluctuations

A metamaterial comprises an array of inductive and capacitive elements as shown in Figure 1a, which can be assumed to be in a state of electromagnetic equilibrium as the total sum of current and voltages in the system is zero. An electrical model...
for a metamaterial structure can be considered to be comprising of a capacitor (C), resistor (R), and an inductor L (Figure 1b).\cite{27} Hence, its interaction with a heat bath can be modeled in terms of an induction of voltage $V$ and current $I$ under thermal fluctuations. The energy associated with the induced charge is instantaneously stored as capacitive energy, $W = \frac{1}{2} C V^2$ which is transformed into an inductive energy, $W = \frac{1}{2} L I^2$. It is eventually lost as heat in the resistor, which is the dissipative element, leading to the generation of thermal fluctuations which is transferred back to the system through a closed loop path (Figure 1c).

In a metamaterial interacting with a heat reservoir, a specific charge center associated with a particular capacitor absorbs some finite amount of heat from the reservoir which results in an instantaneous interaction force and it is driven out of equilibrium (Figure 1a). The force, which is highly irregular and of fluctuating nature is similar to the interaction force encountered in Brownian motion. The electron also encounters a restoring force tending to bring the system to equilibrium.\cite{28} The instantaneous acceleration of the charge center can be expressed using the Langevin equation

$$\frac{dp_i}{dt} = -\beta v_i - \omega_m^2 x + F(t)$$  \hspace{1cm} (1)$$

where, $p_i$ indicates the momentum of the $i_{th}$ electron, $v_i$ is its velocity, $\beta$ is the drag coefficient which expresses dissipation, $\omega_m$ is resonant frequency of the inductive–capacitive system, and $F(t)$ is the random fluctuating function representing interaction with the thermal source. The correlation between two different instants of the interaction force $F(t)$, at time, $t = 0$ and a given time, $t$, is defined as

$$\langle F(0) F(t) \rangle = 2\beta k_B T \delta(t)$$  \hspace{1cm} (2)$$

Here, $k_B$ is the Boltzmann constant, $\delta(t)$ is Dirac Delta function, and $T$ is temperature. The spectral density can be written as its Fourier Transform

$$S_{\eta}(\omega) = \int_{-\infty}^{\infty} dt e^{j\omega t} \langle F(0) F(t) \rangle$$  \hspace{1cm} (3)$$

where, $j = \sqrt{-1}$ and $\omega$ is angular frequency of oscillation. The spectral density $S_{\eta}(\omega)$ is a stationary function of frequency and is proportional to $(F(\omega))^2$ and we can write, $F(\omega) = \sqrt{2\beta k_B T \Delta f}$, where $\Delta f$ is frequency bandwidth over which noise is measured.

**Figure 1.** a) A typical metamaterial structure having split ring structure comprises an array of capacitive and inductive elements. Under thermal fluctuations, positive and negative charges can get induced leading to a finite amount of instantaneous electric field. The entire structure maintains global charge neutrality, however, charges can be induced under thermal fluctuations. b) An electrical Model of a metamaterial structure in terms of lumped resistive ($R$), inductive ($L$), and capacitive element ($C$) along with a net structural capacitance $C_L$. c) Thermal fluctuations induced in a metamaterial under interaction with a heat bath result in a fluctuating force field $F(t)$ which leads to charge separation and induction of voltage $V$ and current $I$. The injected heat drives the thermal fluctuation which is again fed back into the system through the closed loop path.
On the basis of equipartition theorem\,[28] the thermal energy of the induced charges, $k_B T$ can be equated to the instantaneous energy stored in the capacitive element, $C$, that is, 

$$k_B T = \frac{1}{2} C V^2$$

which corresponds to an instantaneous voltage, $V(t) = \sqrt{2k_B T/C}$. After some time, as the effect of $F(t)$ vanishes, the capacitor is discharged leading to generation of an instantaneous current $i(t)$. For a capacitance of 12 pF, which corresponds to a typical value of capacitance of a metamaterial-based structure of micrometric dimensions,\,[28] at 300K, the noise voltage comes out to be 8.28 nV.

Our analysis implies that the fluctuating force $F(t)$ can be directly mapped to an induced voltage $V_{in}(t)$ which is generated within a given metamaterial structure at time $t$. Making related substitutions in Equation (1), taking the Laplace transform and separating the terms,\,[31,32] we get the expression for induced voltage generated due to the displacement of electron away from its mean position

$$V_n(s) = \frac{V_{in}(s)}{(\omega_m^2 + 2\xi \omega_m s + s^2)}$$

(4)

Here, $V_{in}(s)$ is frequency domain representation of $V_{in}(t)$, $s = \omega \sqrt{1 - \xi^2}$ is the angular frequency of external excitation, $\omega_m$ is the resonant frequency of the system, $\xi$ is damping ratio, which is closely related to the resistance in the material, and $V_{in}(s) = \sqrt{2k_B T/C}$. A graphical illustration of the output voltage denoted by Equation (4) is shown in Figure 2a, where the noise frequency is assumed to vary from 0 to 10 MHz and the system is excited at some specific resonant modes between 6 and 10 MHz. The assumed value of damping ratio is 0.01, which corresponds to a critically damped system. Its inverse Laplace transform gives the value of voltage in time domain

$$V_n(t) = \sqrt{2k_B T/C} \frac{\xi \omega_m}{\omega_m \sqrt{1 - \xi^2}} \sin \omega_m \sqrt{1 - \xi^2} t$$

(5)

The graphical illustration of Equation (5) is shown in Figure 2b, where a resonant frequency is assumed to vary from 5 to 10 MHz in steps of 0.2 MHz and time varies from 1 ns to 10 µs. The above analysis indicates that the resonant modes present in a metamaterial structure enhance specific thermal modes leading to an enhanced voltage at specific frequencies.

3. Electromagnetic Radiation under Thermal Fluctuations

The current density influenced by thermal fluctuations, should have a similar form factor as defined by Equation (5). However, a combination of all the existing modes at different frequencies along the structure can be modeled using a Gaussian function. This is a reasonable assumption considering the fact that thermal fluctuations have normal distribution, which lead to a Gaussian distribution of charge density. The current density associated with each of the sinusoidal modes can be expressed as

$$J(r, t) = J_0 \delta(x - x_0) \delta(y - y_0) \delta(z - z_0) \frac{1}{\xi \sqrt{2\pi}} e^{-\frac{[x-x_0]^2}{2\xi^2}}$$

(6)

where $J_0$ represents the maximum value of instantaneous current density induced under thermal fluctuation, $\xi$ is the spectral width of the current distribution, and $v$ is the velocity of charge travelling under the effect of thermal fluctuation. Assuming that a metamaterial structure has around 1000 such capacitors in parallel, each having a capacitance of 1 pF, we can assume the total capacitance to be 1 nF. Equating the Boltzmann energy at room temperature to capacitive energy leads to a maximum voltage generation of 2.03 µV. As metamaterials are comprised of inductors, which are metallic and can have finite resistance, the discharge can lead to an initial current of $V/ \sqrt{R}$. Assuming a resistance of 1Ω, which corresponds to the Lorentz model of electrical parameters of a metamaterial, in terms of orders of magnitude,\,[27] it leads to a current of 2.03 µA. If the area is 1 mm², the current density can be assumed to be 2.03 A m⁻².

Incorporation of the current density defined by Equation (6) in the Helmholtz equation leads to an expression, which cannot be solved analytically due to the presence of current in Gaussian form, which is nonlinear. However, the equation can be solved numerically. For simplicity, we solve the equation within a 2D space of dimensions 4 x 4 cm², assuming a unity value of current density and phonon velocity. The geometric form of magnetic vector potential, calculated numerically, is illustrated...
in Figure 3a. Its magnitude undergoes a change, as illustrated in Figure 3b, when the phonon velocity is assumed to be in the range of 8433.2 m s\(^{-1}\), which is the acoustic speed of sound in silicon along the longitudinal direction.\(^{[33]}\)

The thermally induced charges in a given volume are schematically illustrated in Figure 4. A metamaterial structure remains invisible as the incoming radiation grazes through the surface, but charges induced on the surface under thermal fluctuations cannot be shielded from an external electromagnetic wave and can induce finite amount of current on the outer surface of the metamaterial resulting in electromagnetic radiation.

4. Loss of Invisibility in Metamaterials

An incident electromagnetic wave with an electric field \(\mathbf{E}_IN\) impinging on the metamaterial surface interacts with the surface charge inducing a current density \(\mathbf{J}_S.\)\(^{[34]}\) It generates a radiation which can be called scattered electric field \(\mathbf{E}'\). At a given point in space, the total field is expressed as, \(\mathbf{E} = \mathbf{E}' + \mathbf{E}^s\), where \(\mathbf{E}^s\) is the incident field and \(\mathbf{E}'\) is the scattered field. If the metamaterial surface is perfectly conducting, the total tangential field vanishes on the surface. The radiated field for a cylindrical structure on the surface at a distance, \(r = r_e\), has a tangential component \(\mathbf{E}_T\) and a radial component \(\mathbf{E}_r\). Thus, on the surface of the wire, the tangential component is expressed as, \(\mathbf{E}_T^s(r = r_e) = \mathbf{E}_T(r = r_e) + \mathbf{E}_T^s(r = r_e) = 0\), which leads to, \(\mathbf{E}_T^s(r = r_e) = -\mathbf{E}_T^s(r = r_e)\). The scattered field is expressed as

\[
\mathbf{E}'(r) = j\omega \mathbf{A} - j \frac{c^2}{\omega} \left[ k^2 \mathbf{A} + \mathbf{\nabla} \mathbf{\nabla} \cdot \mathbf{A} \right] \tag{7}
\]

where \(\mathbf{A}\) is magnetic vector potential, \(c\) is the speed of light in the given medium and \(k\) is the wave number. As a result, the total induced current density, \(\mathbf{J}(t)\) at a given time \(t\), generated as a consequence of the scattered electric field \(\mathbf{E}'\), from a pulse of electromagnetic wave, is \(\mathbf{J}(t) = \mu n(t)\mathbf{E}'\), where \(\mu\) is mobility of the induced charge, \(n(t)\) is electron charge density generated as a consequence of thermal fluctuations, and \(e\) is the charge of an electron.\(^{[34]}\) The current density, arising as a consequence of the interaction between the thermally induced charge density, having a Gaussian profile, at a given position along the metamaterial structure, excited by an incoming radiation of frequency \(\omega_0\), is expressed as

\[
J(r, t) = \mu e \delta(x - x_0) \delta(y - y_0) \delta(z - z_0) n_0(x, t) \left[ \frac{j\omega_0}{\xi \sqrt{2\pi}} e^{-\frac{-\xi^2}{2\pi}} \right] \times \sin(\omega_0 t - kx) \tag{8}
\]

Here, \(n_0(x, t)\) represents the maximum value of thermally induced charge density. The interaction between external electromagnetic excitation and thermally induced charges leads to a new value of magnetic vector potential, which can be associated with propagating modes. Incorporation of the current density defined by Equation (8), where, a Gaussian pulse is modulated sinusoidally, in the Helmholtz equation leads to an expression, which does not have an analytical solution. We present a graphical illustration of its numerical solution within a 2D cross section of area 4 cm\(^2\). We consider the current as well as the velocity of charge flow to have unity values and the geometric form of magnetic vector potential, calculated numerically, is illustrated in
A given metamaterials structure develops a collective set of electromagnetic modes depending on the resonant frequency of its components. If the current density \( J \) induced in the metamaterial is assumed to be linear, we can use the relation, \( j dV = l dl \), where \( dV \) is an infinitesimally small element of volume, \( l \) is current over a small segment \( dl \). The magnetic vector potential \( A \) for a current density \( J \) along the \( z \) direction, is expressed as

\[
A_z = \frac{\mu_0}{4\pi} \int \int \int J_z \frac{e^{-jkr}}{r} dV
\]  

The electric field is expressed as

\[
E_z = -j \frac{1}{\omega \epsilon} \frac{I_z(z')}{2\pi r^2} \left[ k^2 + \frac{d^2}{dz^2} \right] G(z, z') dz'
\]

where, \( G(z, z') = e^{-jkr/(4\pi r)} \) for a thin wire antenna carrying a current \( I_z(z') \) along the \( z \) direction. The electric field along the \( r \) and \( \theta \) directions in a spherical coordinate system \((r, \theta, \phi)\) are given by

\[
E_r(t) = \eta I_0 l \cos \theta \frac{1}{2\pi r^2} \left[ 1 + \frac{1}{jkr} \right] e^{-jkr}
\]

\[
E_\theta(t) = j\eta k I_0 l \sin \theta \frac{1}{2\pi r} \left[ 1 + \frac{1}{jkr} - \frac{1}{(kr)}^2 \right] e^{-jkr}
\]

Here, \( I_0 \) is the current flowing in the antenna of length \( l \) and \( \eta \) is the intrinsic impedance of free space which is 377 \( \Omega \). The electric field along the \( \phi \) direction is \( E_\phi = 0 \). In the above equations, the \( 1/r^3 \) term is the electric field generated by the polarization which is strong in the near field region. The \( 1/r \) term is the radiation term which propagates out of the metamaterial.

Interaction between an incoming electromagnetic signal on a metamaterial and existing modes developed by the thermally induced charges can lead to the generation of additional frequency components. For example, if an incoming wave defined by \( E_0 \cos \omega_0 t \) interacts with an existing mode expressed by, \( E_k \cos \omega_k t \), the current in Equations (9) and (10) will have two different frequency components and resultant electric field can be represented mathematically as

\[
E_T = E_0 E_k [\sin(\omega_0 t) \sin(\omega_k t)]
\]

\[
= \frac{1}{2} E_0 E_k [\cos(\omega_0 - \omega_k) t - \cos(\omega_0 + \omega_k) t]
\]

Thus, as a consequence of the interaction, the emitted radiation has some new frequency components, similar to Raman effect. These conclusions are based on a simplified form of a complex process, which has been highlighted in the numerical forms of solutions described in Figures 3 and 5.

5. Discussion

In the absence of thermal fluctuations, there would not be any induced charge on the surface of metamaterial structures and the
incoming electromagnetic wave will follow the curved trajectory illustrated in Figure 4. Hence, there would not be any kind of interaction between incoming electromagnetic radiation and the given structure leading to a cloaking effect defined by the constraints of the structure. The physics behind this phenomenon has been addressed in detail in the existing literature, which neglects the possible effects of thermal fluctuations. The frequency and time domain representation of the noise voltage arising as a consequence of thermal fluctuations are expressed in Equations (4) and (5), which also gives the value of the induced charge for a given capacitance.

In order to achieve the condition of non-invisibility of a metamaterial structure, the emitted radiation should be such that its value is above the ambient noise floor of the receiving antennas. Such systems tend to be influenced by different forms of noise like Johnson noise, shot noise, and flicker noise, whose levels can be calculated by identifying the related variables. The most common among these is Johnson noise, where the noise voltage is expressed as, \( V_n = \sqrt{4k_B RT} \). Its value at a temperature of 300 K, a bandwidth of 1 MHz in a resistor of 50 Ω is 0.91 µV and the noise power, \( P_n \) corresponding to this noise level generated in a resistor of 50 Ω is 16.56 fW. Thus, thermal fluctuations which generate an electromagnetic signal with an electric field defined by Equations (11) and (12), which propagates up to a receiving antenna inducing a voltage above \( V_n \), can be easily detected. The energy flux or the Poynting vector of an electromagnetic wave with an electric field \( E \) is expressed as, \( P = \frac{E^2}{2\epsilon_0} \). If the electric field at the receiving antenna is \( E > \sqrt{2P_n/T} \), it can be detected. Under the current conditions of noise level of 16.56 fW over a bandwidth of 1 MHz, the electric field at the receiving antenna should be greater than 0.53 µV m⁻¹ for viable detection. The exact spacing between the receiving antenna and the metamaterial structure radiating electromagnetic wave under thermal fluctuation can be found by the Friis transmission formula while incorporating the gain of the two systems.

An important question which could be raised in this context is whether thermal fluctuations generated within the cloaked region of a metamaterial structure could reduce its invisibility. The fact is that inner structure of a metamaterial can also be a source of induced charges. According to Gauss’ law of electromagnetism, any charge present within a given closed surface has flux lines which spread out from the center of the charge. Thus, it is impossible to shield such charges from external electromagnetic fields. Assuming that the induced charge accelerates due to time varying current associated with thermal fluctuations, the time varying field will appear along the outer structure and we can associate a radiation field with the system, which cannot be shielded by the metamaterial structure. Although, in the ideal case, a specially designed metamaterial structure, can be shielded from radiation, however, when the metamaterial structure, itself, is the source of radiation, its shielding is impossible unless there is a mechanism to suppress the induced charges. Under such a case, providing a perfect ground plane is not the solution as from the perspective of thermodynamics, the ground plane can also act as an additional source of heat bath inducing thermal fluctuations and increasing the value of the existing charge and current density.

6. Conclusion

We have presented a detailed study on the role of radiation from metamaterials under thermal fluctuations. At an analytical level, it has been illustrated how, the fluctuations of charges in a metamaterial induced under thermal interactions with a thermal source generates a broad range of standing electromagnetic modes which are selectively enhanced as a consequence of the resonant modes of the metamaterials. The interaction between an external electromagnetic wave and thermally induced charges of a metamaterial and generation of new modes has been further analyzed and illustrated through a numerical solution of the corresponding Helmholtz equation. The study indicates that a metamaterial-based structure loses the condition of invisibility under the presence of thermal fluctuations and becomes a source of radio waves. Thus, a complete invisibility can be achieved only if the issue of thermally induced charges, selective dominance of resonant modes, and conductivity are fully addressed. Considering these facts, metamaterial-based cloaks can remain completely invisible only at ultra low temperatures under the complete absence of thermal fluctuations.

Acknowledgements

The author acknowledges financial support from SUTD-MIT Fellowship. [Correction added on March 27, 2019, after first online publication: In Equation (9), \( dV \) was corrected to \( dV \) and the number of integral signs corrected to 3].

Conflict of Interest

The authors declare no conflict of interest.

Keywords

invisibility, limits, metamaterial, thermal fluctuations

Received: September 13, 2018
Revised: December 18, 2018
Published online: February 21, 2019

[1] D. H. Werner, D. H. Kwon, Transformation Electromagnetics and Metamaterials, Springer, Amsterdam 2015.
[2] U. Leonhardt, Science 2006, 1777, 312.
[3] A. Alu, N. Engheta, Phys. Rev. E 2005, 016623, 72.
[4] X. Chen, Y. Luo, J. Zhang, K. Jiang, J. B. Pendry, S. Zhang, Nature Comm. 2011, 176, 2.
[5] T. Ergin, N. Stenger, P. Brenner, J. B. Pendry, M. Wegener, Science 2010, 328, 1186351.
[6] W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, Nat. Photonics 2007, 1, 224.
[7] D. T. Smith, J. B. Pendry, M. C. K. Wiltshire, Science 2004, 305, 788.
[8] G. Rosenblatt, M. Orenstein, Phys. Rev. Lett. 2015, 115, 195504.
[9] T. Xu, A. Agrawal, M. Abashin, K. J. Chau, H. J. Lezec, Nat. Science 2013, 497, 7450.
[10] C. Jouvaud, J. D. Rosny, A. Ourir, Electron. Lett. 2013, 49, 518.
[11] C. M. Soukoulis, M. Wegener, Nat. Photonics 2011, 5, 523.
[12] C. Caloz, T. Itoh, Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications, John Wiley & Sons, Hoboken 2005.
[13] P. A. Deymier, *Acoustic Metamaterials and Phononic Crystals*, Springer Science & Business Media, Berlin 2013.

[14] M. Wegener, *Science* 2013, 342, 939.

[15] R. Hu, S. Zhou, Y. Li, D. Y. Lei, X. Luo, C. W. Qiu, *Adv. Mater.* 2018, 30, 1707237.

[16] S. Zhou, R. Hu, X. Luo, *Int. J. Heat Mass Transf.* 2018, 127, 607.

[17] D. A. B. Miller, *Opt. Express* 2006, 14, 12457.

[18] H. Chen, Z. Liang, P. Yao, X. Jiang, H. Ma, C. T. Chan, *Phys. Rev. B* 2007, 76, 241104.

[19] F. Monticone, A. Alu, *Optica* 2016, 3, 718.

[20] J. Valentine, J. Li, T. Zentgraf, G. Bartal, X. Zhang, *Nat. Mater.* 2009, 8, 568.

[21] L. H. Gabrielli, J. Cardenas, C. B. Poitras, M. Lipson, *Nat. Photon.* 2009, 3, 461.

[22] T. Ergin, N. Stenger, P. Brenner, J. B. Pendry, M. Wegener, *Science* 2010, 328, 337.

[23] W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, *Nat. Photonics* 2007, 1, 224.

[24] R. Loudon, *Adv. Phys.* 1964, 13, 423.

[25] H. Chen, B. I. Wu, B. Zhang, J. A. Kong, *Phys. Rev. Lett.* 2007, 99, 063903.

[26] B. Zhang, B. I. Wu, *Phys. Rev. Lett.* 2009, 103, 243901.

[27] T. P. Meyrath, T. Zentgraf, H. Giessen, *Phys. Rev. B* 2007, 75, 205102.

[28] P. Langevin, *C. R. Acad. Sci.* 1908, 146, 530.

[29] F. Reif, *Fundamentals of Statistical and Thermal Physics*, Waveland Press, Long Grove, IL 2009.

[30] F. Capolino, *Applications of Metamaterials*, CRC press, Boca Raton, FL 2009.

[31] J. Bechhoefer, *Rev. Mod. Phys.* 2005, 77, 783.

[32] K. Ogata, *State Space Analysis of Control Systems*, Prentice-Hall, NJ 1967.

[33] O. Madelung, M. Schulz, H. Weiss, *Physics of Group IV Elements and III-V Compounds*, Landolt-Börnstein: Numerical Data and Functional Relationships in Science and Technology – New Series, Group III, Vol. 17, Springer, Berlin 1982.

[34] M. N. Sadiku, *Elements of Electromagnetics*, Oxford University Press, Oxford 2014.

[35] H. W. Ott, *Noise Reduction Techniques in Electronic Systems*, Wiley, New York 1988.