ABSTRACT. Purchasing decisions determine the purchasing cost, which is the largest section of the production cost of zinc smelting enterprise (ZSE). An excellent supplier recommendation is significant for ZSE to reduce the cost. However, during the supplier recommendation process, the nonlinear demand feature of purchasing department varies with the production environment, and there are wrong samples that can affect the supplier recommendation effect. To handle these problems, the recommendation strategy based on a multiple-layer perceptron adaptive online transfer learning algorithm (AOTLMLP) are proposed. In this method, the original prediction function is modified based on MLP nonlinear projective function and adaptive loss function, which enables the AOTLMLP algorithm to tackle the nonlinear classification problems and efficiently follow the demand change of purchasing department, thereby improving the result of the recommendation. The performance of the AOTLMO algorithm is evaluated through a common dataset and a purchasing dataset from a zinc smelter that generated by a supplier evaluation model. It can be assumed that AOTLMLP can ignore the influence of wrong samples and provide an effective recommendation confronting the characteristic of zinc ore purchasing.

1. Introduction. The purchasing cost is the largest section for a ZSE. A smart selection strategy of suppliers can improve the ability of sustainable development of enterprises. Zinc ore supplier selection is a multi-attribute decision-making problem. The purchasing department chooses appropriate suppliers for purchasing according to the demand of the ZSE. The suppliers with appropriate features can help the enterprise to save the raw materials and ensure that the enterprise has more liquidity. However, with the development of the ZSE, the number of alternative suppliers increases whilst the demand feature of purchasing department becomes more complex, which may reduce the efficiency of decision-making. This type of problem can be attributed to supplier information overload. On the other hand, during the operation of ZSE, the demand of purchasing department for suppliers may change. Because ZSE needs to maintain continuous production, purchasing demand varies with respect to the change of working conditions and environment.
And this kind of demand change is fast and unavoidable. For example, ZSE has many requirements for raw material suppliers. Under the condition of stable production and inventory, the purchasing department will be more inclined to price factor. When the production situation is unstable, the purchasing department will prefer the supplier with stable supply quality. With the demand change problem, the supplier information overload has a more negative impact on purchasing department decision-making. Thus, we decide to propose a recommendation method that can improve the efficiency of purchasing department. It can adapt to changing needs of the purchasing department and learn the demand of the ZSE to generate appropriate supplier recommendations.

For the supplier selection model, recently there are many research conducting on sustainable supplier selection. Luthra et al.,[18] proposed an integrated framework for sustainable supplier selection. The integrated AHP-VIKOR method are used to organized a serious of criterias and sort the suppliers. AKman[1] proposed a supplier evaluation method that include two steps: normal evaluation and sustainable evaluation. In this works, we focus on the recommendation process after the evaluation of suppliers and the evaluation criterions are set according to these literature and the investigation reusult.

The recommendation method in this works is mainly to solve the demand change problem. There are many proposed recommendation algorithms in terms of interest change. Chen[3] proposed a collaborative filtering recommendation algorithm based on user interest change and trust evaluation. Cheng[4] proposed a recommendation algorithm based on users’ dynamic information to solve the cold start problem of the collaborative filtering algorithm. Gasmì [8] proposed a collaborative filtering algorithm that considers genre information of an item to learn the user’s preference. Kuo [14] proposed a recommendation system based on decay weight and matrix clustering to solve interest change in a book recommendation. These works focus on the collaborative filtering algorithm that aims to solve the interest change for multiple users, which is not suitable for the supplier non-ferrous metallurgy recommendation.

Considering the feature of ZSE, zinc ore supplier recommendation can be regarded as a content-based recommendation problem. Thus, the recommendation can be built with some classical learning algorithms such as SVM, Rocchio algorithm [15, 2], linear classifier, etc. However, to tackle the demand change problem, it is better to use an online learning algorithm to adapt to changing demand feature. The demand change problem can be regarded as a concept drift problem. Online methods such as Incremental support vector machines (ISVM), Online random tree (ORF) are used to solve learning problems for data flow scenarios[6] and concept drift problems[5, 25, 7]. On the other hand, since the online learning algorithm only considers the effect of present samples, the recommendation system based on that may be affected by wrong samples caused by human factors, which will lead to the fluctuation of recommendation accuracy.

To this end, in this paper, we construct a recommendation algorithm based on online transfer learning algorithm (OTL)[24]. OTL algorithm considers both historical data and present data in the online learning process. For the wrong samples in the learning process, OTL reduces its influence by altering the weight of the old feature vector and maintain overall learning performance[10]. Grubinger[9] proposed a residential building climate control method based on OTL. Nonetheless, since the supplier recommendation is a nonlinear classification problem, the original
OTL algorithm cannot tackle it. Recently, many works on the research of OTL algorithm focus on multi-class classification\cite{13, 23} and regression problems\cite{17, 11}. However, there is few research discuss the ability of OTL algorithm on nonlinear problem solving. For nonlinear improvement, it is better to use a nonlinear project function to transfer nonlinear datasets into the linear dataset. Jorge proposed a modified Passive-Aggressive based on the Max-out project function to solve the nonlinear classification problem\cite{12}. On the other hand, OTL has a slow learning speed, when the real demand change happens as it considers the historical feature, which will reduce the recommendation accuracy when demand changes frequently. Thus, it is significant to improve the OTL algorithm in order to provide a better recommendation according to the feature of zinc ore purchasing.

According to the problems in zinc ore supplier recommendation, we first establish a zinc ore supplier evaluation model to generate the purchasing dataset. Then, we proposed a nonlinear OTL algorithm with MLP nonlinear projection function to solve recommendation problems (nonlinear binary classification problem). Besides, we add the adaptive loss function into the new nonlinear prediction function to enhance the learning speed during the demand change. The main contribution of this study are as follow:(1) We build a zinc ore supplier evaluation model based on AHP-TOPSIS method.(2) We proposed a nonlinear Online transfer learning algorithm to solve nonlinear binary classification problems.(3) To address the wrong samples in the learning process, we introduce an adaptive loss function into the algorithm above, which improves the transfer speed confronting real demand change. It can be assumed that the zinc ore supplier evaluation model and the new recommendation method can be used in non-ferrous metallurgy supplier recommendation or other similar recommendation problems involving demand change such as the recommendation of music\cite{20}, movie\cite{22}, email, etc.

The rest of this paper is organized as follows. Section 2 provides the zinc ore supplier evaluation model, the details of non-ferrous metallurgy supplier recommendation problems and the structure of the AOTLMLP algorithm. The numerical and case study is illustrated and discussed in section 3. Section 4 concludes the study.

2. The zinc ore suppliers evaluation model and problems statement. According to the investigation result, we propose a supplier evaluation method contains 10 evaluation criterias. We classify these criterias into two categories: quantifiable criteria and fuzzy criteria, and determine the weight of each criteria by AHP method. Fig 1 shows the AHP evaluation model of zinc ore suppliers.

AHP method is proposed by Saaty\cite{21} in 1980. It helps researchers determine the weight of each evaluation criterions. Fig shows, the evaluation problem is divided into two layers. The each criterion in layer A contains some sub-criterions in layer B. The importance of one criterion relative to other criterions are obtain through investigation. For the fuzzy criterions, we define the value of criterias according to the evaluation of experts (Good, Average and Bad). Therefore, we can get the weight vector $\mathbf{w}$ of supplier evaluation model. With the supplier feature vector $\mathbf{x}$, we can obtain the score of each supplier $S$ by equation 1.

$$S = \mathbf{x} \cdot \mathbf{w}$$  \hspace{1cm} (1)

By using a threshold, we can classify recommended suppliers and unrecommended suppliers. Establishing the supplier evaluation is the first step of supplier recommendation. Determining the purchasing plan through AHP supplier evaluation is
theoretically feasible, but it is not practical confronting numerous suppliers information and changing purchasing demand. Therefore, we propose a supplier recom-

![Diagram](image)

**Figure 1.** The framework of evaluation criteria for zinc ore suppliers

![Diagram](image)

**Figure 2.** The requirement change problem in supplier recommendation.

The process of zinc ore suppliers recommendation is shown in Fig 2. The supplier recommendation can be regarded as a binary classification problem using a sample which is the combination of supplier feature $\mathbf{x}_t$ and purchase activity $y_t$, where $\mathbf{x}_t \in \mathbb{R}^d$ and $y_t \in \{-1, 1\}$. With the increasing number of samples, it can learn the present demand feature of purchasing department and provide an effective recommendation for next purchasing.

Nonetheless, there are some difficulties in this recommendation problem. First, the demand feature can be used to get predicted label $\hat{y}_t$ of supplier by the prediction function $\hat{y}_t = \text{sign}(\mathbf{w}^\top \mathbf{x}_t)$. However, because of the complexity of demand
feature, recommended and unrecommended supplier cannot be classified by a single hyperplane, which means the prediction function $f(x)$ should be a nonlinear mapping function. The nonlinear feature of the supplier recommendation problem is caused by the coupling between evaluation criteria\cite{16}. Since the evaluation of Zinc ore suppliers involves many criteria, they all affect the selection of suppliers and influence each other either\cite{19}. This limits the selection of learning algorithm. Second, the demand feature may change during the training process, which can be regarded as a concept drift problem. Before the demand change, it can be assumed that all the sample are extracted from source domain $D_s$. And after the requirement change, the sample are derived from target domain $D_t$. These two domains represent different demand feature. The supplier recommendation is supposed to capture the changing demand and maintains the accuracy of recommendation. Besides, considering the frequency of demand change, the transfer speed from $D_s$ to $D_t$ should be fast to maintain the effectiveness of recommendation. Moreover, wrong samples caused by human factors may affect the accuracy of recommendation. Wrong samples which are not belong to the same domain with normal samples will make the recommendation accuracy fluctuate. It is essential for supplier recommendation to distinguish wrong sample and maintain the stability of recommendation.

To solve the concept drift problem in demand feature, recommendation system based on online learning algorithm is preferable. The online learning algorithm only considers the influence of present sample, and it can follow the variation of demand feature and guarantee the recommendation accuracy after concept drift. However, the online learning algorithms are sensitive to wrong samples, which will reduce the recommendation accuracy. In order to tackle this point, we establish a recommendation algorithm based on OTL to combine the historical data with online learning. In addition, to tackle the nonlinear classification problem and the wrong samples issue, we modify the prediction function based on MLP and add adaptive loss function to perform better recommendation.

3. **Supplier recommendation based on nonlinear adaptive online transfer learning.** To illustrate the method better, the symbol reference table is shown in 3 to state the meaning of symbol.

| Symbol | Paraphrase |
|--------|------------|
| $x_t$  | Supplier feature vector |
| $y_t$  | Recommendation outcome |
| $v$    | Linear previous demand feature vector |
| $w_t$  | Linear present demand feature vector(time-varying) |
| $v_\phi$ | Nonlinear previous demand feature matrix |
| $w_\phi t$ | Nonlinear present demand feature matrix(time-varying) |
| $z_t$  | The hidden layer node vector(time-varying) |
| $z_{(j)t}$ | The jth hidden layer node vector(time-varying) |
| $r_t$  | The weight vector for ReLU units(time-varying) |
| $r_{(j)t}$ | The jth layer weight vector for ReLU units(time-varying) |
| $\beta$ | The restriction parameter |
| $\varphi$ | The preference parameter |
| $\eta$ | The transfer speed rate |
3.1. **Online transfer learning.** Original online transfer learning follows ensemble learning strategy, and the prediction function [24] is shown in equation 2:

\[ \hat{y}_t = \text{sign}(\alpha_{1,t}\Pi(v^\top x_t) + \alpha_{2,t}\Pi(w_t^\top x_t) - \frac{1}{2}) \] (2)

where \( v \in \mathbb{R}^d \) represents old prediction function, and \( \Pi(z) = \max(0, \min(1, \frac{z+1}{2})) \) is a projection function. \( w_t \in \mathbb{R}^d \) represents present prediction function updated by Passive-aggressive algorithm. Both prediction functions are combined by weighting parameters \( \alpha_{1,t} \) and \( \alpha_{2,t} \) [24], which is updated by the equation 3:

\[
\begin{align*}
\alpha_{1,t+1} &= \frac{\alpha_{1,t} F_t(v)}{\alpha_{2,t} F_t(w_t) + \alpha_{1,t} F_t(v)}; \\
\alpha_{2,t+1} &= \frac{\alpha_{2,t} F_t(w_t)}{\alpha_{2,t} F_t(w_t) + \alpha_{1,t} F_t(v)}
\end{align*}
\] (3)

where \( F_t(u) = \exp\{-\eta \ell(\Pi(u^\top f(x_t)), \Pi(y_t))\} \), \( u \in \mathbb{R}^d \) and \( u \in \{w_t, v\} \). \( \ell(z, y) = (z - y)^2 \) is a loss function, and the \( \eta \) is the transfer speed rate parameter that should be set according to the feature of recommendation problem. In the purchasing process, the recommendation system learning the demand feature through the sequence of purchasing department operation. After running for some time, it records a stable demand feature denoted as \( v \) in the previous period, and keep update the present demand feature denoted as \( w_t \) to follow the demand change. With the weighting parameters, it can focus on the online learning part to follow the changing demand, or it will emphasize the historical demand feature to alleviate the fluctuation of recommendation accuracy confronting wrong sample. However, the original online transfer learning cannot handle the nonlinear classification problem. To solve this point, inspired by MLP, we map the nonlinear dataset into a new space by seperating the nonlinear space into multiple linear space. The new prediction function can use the update strategy of original OTL.

![Figure 3. The structure of nonlinear prediction function based on MLP.](image)

3.2. **Define the new prediction function based on MLP network.** To cope with nonlinear classification, we decide to use the output layer to roughly classify the
samples, and using the hidden layer to finish the classification of the rest samples. The structure of nonlinear prediction function based on MLP network is shown in Fig 3. The label prediction function will be provided as equation 4:

$$\hat{y}_t = \text{sign}(\alpha_{1,t} f_v (x_t)) + \alpha_{2,t} f_{w_{\phi}} (x_t) - \frac{1}{2}$$

where $v \in \mathbb{R}^d$ and $w_t \in \mathbb{R}^d$ represent the weight vector of output layer for the old prediction function and the online prediction functions respectively. $f_{\phi}(x)$ is the nonlinear projection function consists of nonlinear activation function and hidden layers, and $w_{\phi t}$ and $v_{\phi}$ are the nonlinear present and previous demand feature matrix, which is the combination of weight vector of ReLU units of AOTLMLP network. According to the HomOTL-I, we set the updating equation for weight parameters $\alpha_{1,t}$ and $\alpha_{2,t}$ as equation 5 and 6:

$$\alpha_{1,t+1} = \frac{\alpha_{1,t} F_t(v, v_{\phi})}{\alpha_{2,t} F_t(w_t, w_{\phi t}) + \alpha_{1,t} F_t(v, v_{\phi})}$$

$$\alpha_{2,t+1} = \frac{\alpha_{2,t} F_t(w_t, w_{\phi t}) + \alpha_{1,t} F_t(v, v_{\phi})}{\alpha_{2,t} F_t(w_t, w_{\phi t}) + \alpha_{1,t} F_t(v, v_{\phi})}$$

where $F_t(u, u_{\phi}) = \exp(-\eta \ell_t(u, u_{\phi}))$ is a loss function. According to PA algorithm, the structure output layer can be regarded as a linear binary classification problem, thus it can be solved by the PA formulation. The output layer prediction function roughly classify the nonlinear dataset by restricting the total loss with $\beta$ proportion. The $\xi$ parameter is also selected according to the feature of recommendation problem, which has the same function with beta parameter. Changing the value of sigma is the same with changing the beta value. Thus, during the modeling process, we generally set sigma at 1.

$$w_{t+1} = \arg \min_w \frac{1}{2} \| w - w_t \|^2$$

s.t

$$\beta \ell_t \leq \xi$$

$$\xi \geq 0$$

$$\ell_t = [1 - y_t w_t^T f_{\phi}(x_t)]_+$$

Solving this constrained optimization 7 and 8, we can get the updating equation 9 for $w_t$

$$w_{t+1} = w_t + \tau_{w_t} y_t f_{\phi}(x_t)$$

$$\tau_{w_t} = \frac{(1 - \beta) \ell_t}{\| f_{\phi}(x_t) \|^2}$$

Moreover, the rest loss is reduced by the nonlinear part. $f_{\phi}(x_t)$ is denoted as $z_{t+1}$ the hidden layer node vector. It can be assumed that the updating of $z_{t+1}$ can be solved by the PA formulation as well.

$$z_{t+1} = \arg \min_z \frac{1}{2} \| z - z_{t+1} \|^2$$

s.t

$$\ell_t = 0$$

We set the lowest lost to finish the nonlinear classification by equation 11, and the updating rules is shown in equation 12:

$$z_{t+1} = z_t + \tau_{z_t} y_t w_t$$
\[ \tau_{z_t} = \frac{\ell_t}{\|z_t\|^2} \]  

From the above procedure, the variation between \( z_t \) to \( z_{t+1} \) is obtained. This can be used to update the nonlinear parameter group \( w_{\phi_t} \), which provides a projection of \( z_t \) with respect to the current sample.

### 3.3. The implementation of MLP nonlinear projection for single hidden layer

Considering online learning strategy, it is better to use linear components to build the nonlinear projection function. Thus, ReLU activation function is used. The nonlinear projection function for \( z \in \mathbb{R}^h \) is shown in equation 14 and 15:

\[ z_t = [z_t^1, z_t^2, z_t^3, ..., z_t^h] \]  
\[ z_t^i = \max(0, r_t^i \cdot x_t), i \in [1, ..., h] \]  

Although the ReLU activation function is nonlinear, it still can be divided into two linear sections. Thus, we define the parameter vector for nonlinear projection as equation 16:

\[ r_t = [r_t^1, r_t^2, ..., r_t^h] \]  

The parameter vector of \( r_t \) are the combination of \( h \) Rectified Linear Units \( r_{t+1} \in \mathbb{R}^d \). And the regression problem for each \( r_{t+1} \) can be summarized as equation 17:

\[ r_{t+1}^i = \arg \min_{r^i} \frac{1}{2} \| r^i - r_t^i \|^2 \]  
\[ \text{s.t} \]  
\[ \ell_z(r_t^i; (x_t, z_t^{i+1})) = 0 \]  
\[ \ell_z(r_t^i; (x_t, z_t^{i+1})) = \| (r_t^i \cdot x_t) - z_{t+1}^i \|_+ \]  

The updating equation of weight for each ReLU are provide by PA formulation, which is shown in equation 19.

\[ r_{t+1}^i = r_t^i + \text{sign}(z_{t+1}^i - r_t^i \cdot x_t) \tau_{r_t^i} x_t \]  
\[ \tau_{r_t^i} = \ell_z(r_t^i; (x_t, z_t^{i+1})) \| x_t \|^2 \]  

### 3.4. The implementation of MLP nonlinear projection for multiple hidden layers

In some situations, nonlinear classification problems may be complex, which means they cannot be expressed by a single hidden layer network. Therefore, the implementation of MLP nonlinear projection function is proposed. According to the work above, we denote the jth hidden layer nodes as \( z_{(j)}^t \) (equation 21):

\[ z_{(j)}^t = [z_{(j)}^1, z_{(j)}^2, ..., z_{(j)}^h] \]  

Thus, each node in the jth hidden layer can be calculated by equation 22.

\[ z_{(j)}^i = \max(0, r_{(j)}^i \cdot x) \]  

Moreover, the weight vector for jth hidden layer is denoted in equation 23:

\[ r_{(j)}^t = [r_{(j)}^1, r_{(j)}^2, ..., r_{(j)}^h] \]  

The updating rules for each node weight \( r_{(j)}^i \) in hidden is similar to the method above. It leads to the following optimization problem 24, where \( \beta \) is loss restriction parameter.

\[ r_{(j)}^i_{t+1} = \arg \min_{r_{(j)}^i} \frac{1}{2} \| r_{(j)}^i - r_{(j)}^i_t \|^2 \]
\[\begin{align*}
\beta \ell_h & \leq \xi \\
\xi & \geq 0 \quad (24)
\end{align*}\]

Solving this optimization problem, we can get the updating rules \( r_i^t(\cdot) \):

\[
r_i^t(j+1)_{t+1} = r_i^t(j)_{t} + \text{sign}(z_i^t(j+1)_{t+1} - r_i^t(j)_{t} \cdot z_i^t(j+1)_{t}) \tau r_i^t(j)_{t} \\
\tau r_i^t(j)_{t} = \frac{(1 - \beta) \ell_h}{\|z_i(j+1)_{t}\|^2} \quad (27)
\]

Similarly, the remaining loss is reduced by the following hidden layer, and we can obtain the updating rules \( z_i^t(j+1)_{t} \):

\[
z_i(j+1)_{t+1} = \arg \min_{z_i(j+1)_{t}} \frac{1}{2} \|z_i(j+1)_{t} - z_i(j+1)_{t}\|^2 \\
\text{s.t} \quad \|r_i^t(j)_{t} \cdot z_i(j+1)_{t} - z_i^t(j+1)_{t+1}\| = 0 \quad (28)
\]

Solving this optimization problem, we can obtain the updating rules \( z_i^t(j+1)_{t} \) for each \( j+1 \)th hidden layer node:

\[
z_i^t(j+1)_{t+1} = z_i^t(j+1)_{t} + \text{sign}(r_i^t(j)_{t} \cdot z_i(j+1)_{t} - z_i^t(j+1)_{t+1}) \tau z_i(j+1)_{t} r_i^t(j)_{t} \\
\tau z_i(j+1)_{t} = \frac{\ell_h}{\|z_i(j+1)_{t}\|^2} \quad (30)
\]

With the variation from \( z_i^t(j+1)_{t} \) to \( z_i^t(j+1)_{t+1} \), the updating rules for the next hidden layer can be obtained.

### 3.5. Self-adaptation for MLP nonlinear online transfer learning.

The recommendation system based on MLP nonlinear online transfer learning considers both the historical feature \( \Theta(v_t, v_{\Phi_t}) \) and present feature \( \Theta(w_t, w_{\Phi_t}) \) of purchasing department. If the demand is stable over a period, the prediction function will prefer the historical feature. And if the demand is changing, the prediction function will start to emphasize the present feature. Nonetheless, because of the transfer speed from historical feature to present feature, the recommendation accuracy will maintain a low level for a period. And if this period is too long, the recommendation result will harm the decision of purchasing department. Thus, to speed up the transfer speed after the demand changing, we proposed an adaptive MLP nonlinear online transfer learning method.

To implement the high transfer speed, one way is to enable the algorithm to recognize the demand changing by detecting misclassified cases. However, the wrong sample caused by human factors may have the same effect with demand changing that will also reduce the recommendation accuracy. If the transfer speed increases when the demand changing happen, it will also change when the wrong samples emerge. Therefore, we introduce an adaptive loss function \( l_{a} \) to distinguish both situations.

\[
l_{a} = e^{f(MC)} \\
f(MC) = \varphi \frac{MC - \min(MC)}{\max(MC) - \min(MC)} \quad (32)
\]
\( l_\alpha \) is the adaptive parameter, and \( MC \) is the prediction error rate for a single batch. \( \varphi \) is the preference parameter. \( \max(MC) \) and \( \min(MC) \) are the maximum and minimum error rates for a single batch respectively. And the modified prediction function is showed in equation 33:

\[
y'_t = \text{sign}(l_\alpha[\alpha_{1,t} \Pi(v^\top f_{v,t}(x_t)) + \alpha_{2,t} \Pi(w_t^\top f_{w,t}(x_t))] - \frac{1}{2})
\]  

(33)

When the single batch prediction error rate is low (wrong samples), \( l_\alpha \) will not affect the original transfer speed. And when the single batch prediction error rate is high (demand changing), the transfer speed will be improved, thereby increasing the recovery speed of recommendation accuracy.

3.6. The strategy of parameter adjustment. In the AOTLMLP algorithm, the restriction parameter \( \beta \), transfer speed rate \( \eta \), and preference parameter \( \varphi \) need to be fixed before training. Initially, the selection of \( \beta \) depends on the complexity of the nonlinear dataset. The more complicated the nonlinear dataset requires smaller \( \beta \) in order to allocate more classification tasks to the nonlinear part, and the selection of \( \beta \) may also affect the training speed of the algorithm. For the transfer speed rate \( \eta \) adjusting, the complexity of the supplier feature (dimensionality) needs to be considered. During the training process, appropriate \( \eta \) will enhance the recovery speed of recommendation accuracy. Setting a too small \( \eta \) may ignore the effect of historical feature, and setting a too large \( \eta \) may cause the divergence of the system. The preference parameter \( \varphi \) determines whether the algorithm favors stability or mobility. If the system emphasizes the recommendation stability when the demand is stable, \( \varphi \) should be reduced. However, if the system emphasizes the response speed for demand changing, \( \varphi \) should be increased.

On the other hand, the number of hidden layers nodes \( h \) hand the number of hidden layers \( h_j \) should be decided with respect to the complexity of the nonlinear dataset, and the strategy is similar to the neural network. The more layers the algorithm has, the more complicated dataset it can solve, and the more training time it will need.

3.7. The structure of AOTLMLP. To sum up, the recommendation system based on AOTLMLP algorithm has the following parameters: the restriction parameter \( \beta \); the preference parameter \( \varphi \); the transfer speed rate \( \eta \); the number of hidden layers and the number of nodes \( j, h \); The old feature is obtained from the previous batch training. And the update steps of algorithm is shown in Fig 4. After initializing the weight parameter \( \alpha_{2,t}, \alpha_{1,t} \) and the weight vector of each layer, the recommendation system start training. The online demand feature of purchasing department will be updated only when the prediction is wrong. During the updating process, the weight vector \( r_{(j)} \) is updated layer by layer. Finally, the new adaptive parameter is calculated and applied to the next batch training.

For the computational complexity, the prediction function of AOTLMO includes the old prediction function and present prediction function. Both of them include the linear part and the nonlinear part. The nonlinear projection based on MLP is the combination of vector/matrix multiplications for each layer. After the performing of projection, the label is provided by the multiplication between \( z_t \) and \( w \). Thus, the complexity is \( O(2(d \cdot h^l + h)) \).

4. Results. In this section, we evaluate the performance of AOTLMLP recommendation algorithm through a Common dataset in UCI Benchmark Repository.
which is a case of binary classification problem, and a purchasing dataset of a zinc smelter. First, we evaluate the performance AOTLMLP recommendation algorithm for different values of $\beta$ to discuss the effect of $\beta$ on nonlinear problem learning. Then, we set different MLP parameters $j, h$ for both datasets to find the strategy for MLP parameter adjustment.

The image dataset in UCI Benchmark Repository has 1300 training patterns and 1010 test patterns, and the input dimension is 18. The purchasing dataset from zinc smelter has 11 inputs, 2000 training patterns, and 1000 test patterns. To mimic the recommendation in the purchasing process, we set the batch size at 10 to see the online learning performance of the AOTLMLP algorithm.

4.1. The influence of $\beta$ and MLP parameters on the nonlinear binary classification problem. To test the performance of AOTLMLP recommendation algorithm on nonlinear classification problems, the datasets without concept drift are used. In this situation, the concept drift for supplier recommendation problem is the changing demand of purchasing department, which means the demand model of the purchasing department varies with the time. Fig 5 and Fig 6 show the cumulative error rate during training process considering different value for $\beta$. According to the parameter selection strategy for MLP, the network structure parameters for nonlinear projection are set, where the number of hidden layers and the number of nodes $j, h$ for the common dataset are 2 and 20 respectively and 2 and 15 for the purchasing dataset respectively. Moreover, the preference parameter $\varphi$ for both training process is set at 0.5, and the transfer speed rate $\eta$ are set at 0.005.

For the common dataset, we can see that the recommendation algorithm based on AOTLMLP has the best training result when beta=0.85. In terms of purchasing dataset, the recommendation has the best training result when beta=0.9. With this result, it can be assumed that the selection of beta is related to the complexity of the nonlinear problem. The image dataset has 18 inputs and purchasing dataset has 11 inputs, where the image dataset is more complex than the purchasing dataset. The

![Figure 4. The update steps of algorithm.](image-url)
Algorithm 1 Adaptive nonlinear Online transfer learning based on Multiple layer perceptron

**Input:**
The restriction parameter, $\beta$;
The preference parameter, $\varphi$;
The transfer speed rate, $\eta$;
The number of hidden layers and the number of nodes, $j,h$;
The old feature, $\Theta(v,v_{\Phi})$;
The adaptive parameter, $l_a$

Initialize: $\alpha_{1,t} = 0.5$, $\phi_{(j)\ell} = [0.5,0.5,...,0.5]$, $w_1 = [0,0,...,0]$, $M = 0$

for each $t \in 1,2,...,T$ do

Receive sample, $x_t$

Predict $\hat{y}_t = \text{sign}(\alpha_{1,t}\Pi(v^T f_{v_{\Phi}}(x_t)) + \alpha_{2,t}\Pi(w^T f_{w_{\Phi}}(x_t)) - \frac{1}{2})$

Correct label: $y_t$

Calculate $\alpha_{1,t+1}, \alpha_{2,t+1}$

Suffer loss: $\ell_t = |1 - y_t w^T f_{\Phi}(x_t)|$

if $\ell_t > 0$ then

Update: $w_{t+1} = w_t + \tau w_t y_t f_{\Phi}(x_t)$

Update: $z_{(1)\ell+1} = z_{(1)\ell} + \tau z_{(1)\ell} y_t w_t$

for each $j \in 1,...,J$ do

for each $i \in 1,...,h$ do

Update: $r^{i}_{(j)\ell+1} = r^{i}_{(j)\ell} + \text{sign}(z_{(j)\ell+1}^i - r^{i}_{(j)\ell}) \cdot z_{(j+1)\ell} - z_{(j+1)\ell}^i \cdot r^{i}_{(j)\ell}$

Update: $z_{(j+1)\ell+1} = z_{(j+1)\ell} + \text{sign}(r^{i}_{(j)\ell} \cdot z_{(j+1)\ell} - z_{(j+1)\ell}^i \cdot r^{i}_{(j)\ell})$

end for

Normalize $z_{(j)\ell+1}$

end for

$M = M + 1$

end if

end for

$MC = \frac{M}{T}$

Calculate the new adaptive parameter, $l_a^*$

**Figure 5.** Cumulative training error rate for different $\beta$, considering IMAGE dataset.
restriction parameter should be reduced when the complexity of the classification problem goes up.

Fig 7 and Fig 8 show the learning performance of AOTLMLP for the common dataset and purchasing dataset considering different values of the number of hidden layers and the number of nodes \( j, h \). For the common dataset, we set \( \beta \) at 0.85. And, for the purchasing dataset, the \( \beta \) is set at 0.9. Moreover, for both datasets, the preference parameter \( \varphi \) and the transfer speed rate \( \eta \) are set at 0.5 and 0.005 respectively.

For the common dataset, we can see that when \( j=2 \) and \( h=20 \), the AOTLMLP recommendation algorithm has the best classification performance. And, in terms of the purchasing dataset, the classification performance is the best when \( j=2 \) and \( h =15 \). It can be assumed that the strategy for adjusting the MLP parameter for the nonlinear projection is similar to the strategy for the neural network. With the enhancement of complexity of the nonlinear problem, more hidden layers and nodes are required.

4.2. The influence of \( \varphi \) and \( \eta \) on the transfer speed. The transfer speed is the speed that the classifier for the source domain transferring to the classifier for the target domain. In supplier recommendation problems, the transfer speed is the recovery speed of recommendation accuracy after demand changing. Thus, the purchasing data with demand changing and wrong samples are used in the following evaluation.
Fig 9 and 10 show the convergence behaviors of AOTLMLP algorithm for training dataset and test dataset considering different value of $\eta$. As the transfer speed rate mainly affect the learning speed when demand changing happen, the convergence behaviour for both training and testing dataset for different value of $\eta$ is not much different.

Fig 11 shows the variation of recommendation accuracy considering different values of $\eta$. According to the result above, $\beta$ is set at 0.85 and the MLP parameter
\( j, h \) are set at 2 and 15 respectively. Moreover, the preference parameter \( \phi \) is set at 0.5. The result shows that when \( \eta(\text{eta})=0.06 \), the recommendation accuracy has the fastest recovery rate, which means the AOTLMLP recommendation algorithm can handle the recommendation more effectively. If we reduce the value of \( \eta \), more samples from the target domain are needed for the recommendation system to learn the new demand feature. However, the oscillation will emerge after the demand change, if the \( \eta \) value is too high.

**Figure 11.** Recommendation accuracy for different \( \eta \), for purchasing demand change.

**Figure 12.** Cumulative training error rate for different \( \phi \), considering training purchasing dataset.

**Figure 13.** Cumulative training error rate for different \( \phi \), considering testing purchasing dataset.
Fig 12 and 13 show the convergence behaviors of AOTLMLP algorithm for training dataset and test dataset considering different value of $\varphi$. Similarly, different value of $\varphi$ does not affect the convergence of AOTLMLP algorithm.

![Graph showing convergence behaviors](image)

**Figure 14.** Recommendation accuracy for different $\varphi$, for purchasing demand change.

Fig 14 shows the variation of recommendation accuracy considering different values of $\varphi$, where the $\beta$ is set at 0.85 and the MLP parameter $j, h$ are set at 2 and 15 respectively. And the transfer speed rate is set at 0.5. We can see that with the increase of $\varphi$ the recovery speed of recommendation accuracy enhance, and it has the best performance when $\varphi(\varphi)=0.5$. However, if we keep improving $\varphi$, there will be oscillation caused by the wrong samples before the demand change. Thus, it can be assumed that the preference parameter determines whether the system prefers the ability of online learning or the robustness to wrong samples.

As a result, the transfer speed rate $\eta$ base of transferability for the AOTLMLP algorithm should be adjusted with respect to the feature of the classification problem. Based on $\eta$, the preference parameter $\varphi$ decides the preference of the system. If $\varphi$ is small, the algorithm is insensitive to a small number of wrong samples and prefers the historical feature of the user. On the other hand, the system will have a faster learning speed if $\varphi$ is increased, and it will increase the sensitivity to wrong samples. Thus, it also should be adjusted according to the feature of classification problems.

4.3. The overall effect of the AOTLMLP. To test the learning speed of AOTLMLP, the AOTLMLP algorithm is compared with some standard nonlinear optimization method include SGD, ADAM and L-BFGS to show the convergence behaviors. Based on the theory of perceptron and stochastic gradient descent, the AOLTMLP algorithm can be regarded as mini-batch GD, and we set batch size at 1 to realized SGD. Fig 15 shows the convergence behaviors for different learning strategy, considering purchasing dataset.

![Graph showing learning speed comparison](image)

We can see the learning speed of SGD is slower than AOTLMLP. However, the recommendation system with learning strategy ADAM and L-BFGS has the same convergence behaviors with AOTLMLP algorithm, which means they have almost the same learning speed performance on purchasing dataset.

Fig 16 shows the performance of four algorithms, including PAMO, OTL, OTL MLP, AOTLMLP. Similarly, the purchasing dataset with demand change and wrong samples are used in this test. The original OTL algorithm cannot handle nonlinear classification problem, thus the recommendation accuracy for OTL are not able to improve. For the PAMO algorithm, the wrong samples will cause the fluctuation...
of the recommendation accuracy, as PAMO only considers the present data. Compared with OTL, the OTLMLP algorithm can tackle the nonlinear classification problem, but it requires 50 samples to bring the recommendation accuracy to normal. AOTLMLP algorithm can follow the demand feature as quickly as PAMO and it can ignore the influence of wrong samples when the demand is stable.

Nonetheless, the nonlinear improvement based on MLP also bring the AOTLMLP algorithm some problems. First, the initialization of algorithm become more complex. In above experiments, the initial value of weight vector for each layer is 0.5. However, for more complex problem, the initial value of weight vector is hard to decide, and wrong selection of initial value may decrease the validation accuracy of algorithm. Second, compared to the original algorithm, new algorithm need to find the appropriate number of hidden layers, hidden neurons, and iterations, which means the frame of algoritm requires more testing to determine. Third, because the MLP is sensitive to feature scaling, the input feature will need processsing such as normalization or standardization.

5. Conclusion. In this paper, we proposed a recommendation method (AOTLMLP) based on an online transfer learning method that was modified with MLP nonlinear projection function and adaptive loss function to provide a better recommendation. For the nonlinear projection, we use MLP to build nonlinear projection, where the original OTL parameter update method can be used in each hidden layer. Moreover, to improve the performance during the demand change, we add the adaptive loss function into the prediction function, where the new algorithm can
distinguish the wrong samples and real demand change and improve the transfer speed when the demand change happened.

For the parameter adjustment, we evaluate the influence of beta $\eta$, $\varphi$ and the number of hidden layers and the number of nodes, $h, j$ on the algorithm. The result shows that the selection of these parameters is related to the complexity of the dataset. For a certain recommendation problem, there should be a group of parameters that is the most appropriate.

In the case study, we evaluate the performance of the AOTLMLP algorithm for nonlinear binary classification problems with common dataset and purchasing dataset from a zinc smelter. The result shows that the AOTLMLP algorithm can address a nonlinear classification problem. Compare to the original OTL algorithm, it can tackle the nonlinear dataset and provide a better learning performance when the demand change happens. Besides, it has the same online learning speed as the PAMO algorithm when confronting the demand change. The recommendation algorithm based on AOTLMLP provides an excellent result with respect to purchasing dataset. Moreover, according to the result for different dataset, the AOTLMLP algorithm can tackle similar nonlinear recommendation problems that such as the recommendation of music, movie, which also has demand change during the recommendation process, merchandise. And the parameter of algorithm should be fixed in the light of the complexity of problem. Further research will be conducted to provide more results for other datasets, and more analyze of application on other recommendation problems.

Acknowledgments. This work was supported by the National key research and development program (2020YFB1713700), Excellent Youth Natural Science Foundation of Hunan Province (2019JJ30032), Natural Science Foundation of Hunan Province(2019JJ50823).

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Received March 2021; revised August 2021; early access November 2021.

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