Mass and flavor from strong interactions

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Abstract

We provide an economical description of mass and flavor based on strong interactions and some dynamical assumptions. We include a discussion of CP violation in the quark sector and its relation to neutrino masses.

1 Introduction

Over the years there have two basic approaches to finding the theory of flavor and mass. One approach is to consider only those theories we fully understand. That is, one constructs theories which either are perturbative, or are based on some rigorous results of strong interactions, such as those emerging in the study of strongly interacting supersymmetric theories. But so far at least, this approach has led to models of mass and flavor which appear overly complicated, with numerous new interactions and/or matter multiplets. The other approach is to consider only those theories with an economical structure. But so far at least, this approach has forced the model builder to make dynamical assumptions about the behavior of strong interactions. In other words, it is not known that the models being proposed actually work as advertised.

It is not surprising that the first approach has proven much more popular over the years; it is preferable to know what one is talking about! On the other hand, the theory of hadronic interactions, QCD, has more in common with the second approach.

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1Theories which simply parametrize fermion masses, such as the standard model, do not qualify as theories of flavor and mass.
QCD has a simple and economical structure, and yet it is often difficult to extract physical results. QCD is the theory of the hadronic mass spectrum, but we have yet to see this spectrum fully emerge from a theoretical calculation. Even the concept of confinement is still closer to a dynamical assumption than a rigorously derived result of the theory. But none of this shakes our acceptance of QCD, since there have been other ways to get a handle on QCD which have allowed for experimental checks and confirmation of the theory.

Our experience with QCD thus suggests that it is not necessarily fatal for a theory to rely on plausible dynamical assumptions, as long as the structure of the theory is rigid enough to lead to testable consequences. The correct theory of flavor and mass may be such as to not allow for a calculation of the fermion mass spectrum with current tools. But even without being able to provide precise numbers, the following are examples of what we may hope to glean from the correct theory.

- patterns in new flavor dependent effects
- patterns in CP violation
- patterns in neutrino masses
- predictions for the lightest of the new particles

Another common theme in the search for the theory of mass and flavor is to first deal with the question of electroweak symmetry breaking. There are two widely reported approaches to that question, supersymmetry and technicolor, which both provide attractive answers to that single question. These are then taken as the two possible starting points in the search for the theory of mass and flavor. But as indicated in Fig. 1, many obstacles must be overcome in each case before one can approach a comprehensive theory of mass and flavor. In both cases, after the various hurdles are passed, the resulting proposed theories are looking quite complicated and convoluted.

Here we shall consider the possibility that the key to electroweak symmetry breaking is neither supersymmetry or conventional technicolor. We will hope to identify an alternative which leads more simply and naturally to a theory of flavor. The price we will pay is to have electroweak symmetry breaking associated with some aspect
of strong interactions which is less familiar to us, i.e. associated with a dynamical assumption.

Our basic picture is as follows.

- A new strong interaction breaks close to a TeV, unlike technicolor which remains unbroken.

- Associated with this symmetry breaking are the dynamically generated masses for a fourth family of quarks and leptons, which in turn is responsible for electroweak symmetry breaking.

- The new strong interaction is a remnant flavor interaction, and it only acts on the third and fourth families.

- At a higher “flavor scale”, say 100–1000 TeV, the remnant flavor interaction merges with the full flavor interaction, which involves all quarks and leptons.

The full flavor interaction is some strong, chiral gauge interactions which partially breaks itself. The important point is that this symmetry breaking does not include the breakdown of $SU(2)_L \times U(1)_Y$, and thus the known quarks and leptons receive no mass at the flavor scale. The exception are the right-handed neutrinos, which can serve as bilinear order parameters for the flavor breaking. The theory above the flavor scale may also be left-right symmetric, in which case the right-handed neutrino condensates also serve to break $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ down to $SU(2)_L \times U(1)_Y$.

The basic two-scale structure of the model is shown in Fig. 2. The physics at the flavor scale shows up on lower scales through 4-fermion operators and other nonrenormalizable operators. These effects, combined with the mass generation at a TeV, feed down masses from the fourth family to the lighter families. The following are some key ingredients for understanding the origin of a complicated fermion mass spectrum.

- There are a wide variety of possible 4-fermion operators, due to strong-coupled flavor physics, and different operators can contribute to different elements of the mass matrices.

- Operators have various transformation properties under the remnant flavor interaction, and in particular, operators have various numbers of fermions coupling to this interaction.
• Since the remnant flavor interaction is strong, we can expect that anomalous scaling gives large relative enhancement of operators. This will be one of the sources of quark and lepton mass hierarchies.

One problem which has plagued the technicolor approach has been the difficulty in understanding the origin of the large isospin breaking inherent in the top mass, in a way compatible with the electroweak correction parameter $\delta \rho$. In our approach, isospin breaking originates at the flavor scale, for example through a dynamical breakdown of $SU(2)_R$. The remnant flavor interaction remaining down to a TeV is isospin preserving and it is this interaction which is responsible for electroweak symmetry breaking. This in itself produces no contribution to $\delta \rho$. Isospin breaking is communicated to the TeV scale via 4-fermion operators, and an operator in particular which must be present is the $t$-mass operator $\overline{t} t \overline{t} t$, where primes denote fourth family members. (It may be that the corresponding $b$-mass operator, $\overline{b} b \overline{b} b$, is generated as a weak radiative $SU(2)_R$ correction to the $t$-mass operator.) It turns out that the contribution of the $\overline{t} t \overline{t} t$ operator to $\delta \rho$ is suppressed by $(m_t/m_{t'})^4$ where $m_{t'} \approx 1$ TeV, and thus the $t$-mass does not directly imply a significant problem for $\delta \rho$. Indeed we are relying on how small the $t$ mass is small relative to the fundamental TeV scale, which differs from the usual emphasis on how large the $t$ mass is. We shall find a dynamical reason as to why the $t$-mass operator is the largest isospin violating operator, due to the anomalous scaling mentioned above.

2 A minimal model

We will now specify the model in more detail [1]. The main object here is to show how a complicated fermion mass spectrum can arise from a simple underlying structure. It is sufficient for us to present the minimal model, since we do not have adequate understanding of the strong dynamics to judge which variation of the model will produce the assumed symmetry breaking pattern. We will consider a 4 family model where the flavor gauge symmetry is $U(2)_V$. Two pairs of families transform as $(2, +)$
and \((\bar{q}, -)\) under \(SU(2) \times U(1)\); we label these two pairs of families as \([Q, L]\) and \([\bar{Q}, \bar{L}]\). The basic structure of the model, including the right-handed neutrino masses which are assumed to occur at the flavor scale and the resulting breakdown of \(U(2)\) to \(U(1)_X\), are depicted in Fig. 3. Notice that \(U(1)_X\) couples only to the two heavy families, and that the fermion basis depicted in the figure is not the mass eigenstate basis.

The main dynamical assumption we make is in the form of the fourth family masses. These masses must be generated by the strong, and broken, \(U(1)_X\) interaction along with possible 4-fermion interactions.

- \(t'\) and \(b'\) quark masses: \(\overline{Q}_{L1} Q_{R1}\)
- \(\tau'\) mass: \(\overline{E}_{L1} E_{R1}\)
- \(\nu_{\tau'}\) mass: \(N_{L1}^2\)

Now consider the 4-fermion operators which feed these masses to the lighter quark and leptons. We find that interesting results follow from the following subset of operators. Other operators may also contribute, but we assume that this subset dominates. The unique characteristics of these operators can of course provide a dynamical reason for their dominance.

- They have the chiral structure \(\overline{\psi}_L \psi_R \overline{\psi}_L \psi_R\), and hence must be generated dynamically by the strong flavor interactions.
- They preserve \(SU(2)_L \times U(1)_Y\) but display maximal \(SU(2)_R\) breaking.
- They preserve the strong \(SU(2)_V\). They may thus be composed of the \(SU(2)_V\) singlet bilinears: \(\overline{Q}_{L1} Q_{R1}\), \(\overline{Q}_{L1} Q_{Rj} \varepsilon_{ij}\), etc.

Some of these operators break \(U(1)_X\), and we will assume that this generates an \(X\) mass of order a TeV, or somewhat higher.

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4 The fact that the 4th family and not the 3rd family masses form must be due to a cross-channel coupling, which should be familiar to builders of multi-Higgs potentials. Note that the \(\tau'\) mass forms in the \(U(1)_X\)-singlet channel, unlike the \(q'\) masses, which may be due to flavor induced 4-fermion operators which distinguish quarks and leptons and which are enhanced by \(U(1)_X\) anomalous scaling, e.g. \(\tau_L \tau_R \overline{\tau}_R \tau_{L'}\).
3 Quark masses

We now briefly describe the various operators which are responsible for quark and lepton masses. We first discuss the quark sector. The following operators feed mass down from $t'$ and $b'$ (and in the last case from $t$):

$$
\begin{bmatrix}
U_{L1}D_{R1} & \overline{D}_{L1}U_{R1} & B \\
\overline{D}_{L1}U_{R1} & U_{L1}D_{R1} & \bar{B} \\
U_{L1}D_{R1} & \overline{D}_{L1}U_{R2} & C \\
\overline{D}_{L1}U_{R1} & U_{L1}D_{R2} & \bar{C} \\
\overline{U}_{L2}D_{R1} & \overline{D}_{L1}U_{R1} & \bar{D} \\
\overline{D}_{L2}U_{R1} & \overline{U}_{L1}D_{R1} & \bar{D} \\
Q_{Li}U_{Rj}\varepsilon_{ij}Q_{Lk}D_{Rl}\varepsilon_{kl} & Q_{Li}U_{Rj}\varepsilon_{ij}Q_{Lk}D_{Rl}\varepsilon_{kl} & \mathcal{E}
\end{bmatrix}
$$

while the following operators feed mass down from $\tau'$:

$$
\begin{bmatrix}
\overline{E}_{L1}E_{R1}U_{L1}U_{R1} & \mathcal{G}_1 \\
\overline{E}_{L1}E_{R1}U_{L2}U_{R2} & \mathcal{G}_2 \\
\overline{E}_{L1}E_{R1}U_{L1}U_{R1} & \mathcal{H}_1 \\
\overline{E}_{L1}E_{R1}U_{L2}U_{R2} & \mathcal{H}_2 \\
\overline{E}_{L1}E_{R1}U_{L1}U_{Rj}\varepsilon_{ij} & \mathcal{T} \\
\overline{E}_{L1}E_{R1}U_{L1}U_{Rj}\varepsilon_{ij} & \mathcal{J}
\end{bmatrix}
$$

The main point is that each operator contributes to a different mass element.

$$
M_u = \begin{bmatrix}
0 & \mathcal{G}_2 & \mathcal{T} & 0 \\
\mathcal{H}_2 & \mathcal{E} & \mathcal{D} & \mathcal{J} \\
\mathcal{T} & \mathcal{C} & \mathcal{B} & \mathcal{G}_1 \\
0 & \mathcal{J} & \mathcal{H}_1 & \mathcal{A}
\end{bmatrix}
$$

$$
M_d = \begin{bmatrix}
\mathcal{F} & 0 & 0 & 0 \\
0 & \mathcal{E} & \bar{D} & 0 \\
0 & \bar{C} & \bar{B} & 0 \\
0 & 0 & 0 & \mathcal{A}
\end{bmatrix}
$$

Note that essentially all of the CKM mixing arises in the up sector, and that the mass matrices are not symmetric. Various mass hierarchies arise for the following reasons.

- Various operators experience different power-law scaling enhancements from the strong $U(1)_X$. Basically, operators containing heavy fermions in both Lorentz
and $U(1)_X$ singlet combinations are expected to be enhanced the most. The $B$ operator, which is the $t$-mass operator, is expected to be the largest.

$$B > C, D > E \quad (5)$$

$$G_1, H_1 > I, J > G_2, H_2 \quad (6)$$

- There are different heavy masses, $m_{t',v'} > m_{t'} > m_t$, being fed down.

$$E > F \quad (7)$$

$$B > G_1, H_1 \quad (8)$$

$$C, D > I, J \quad (9)$$

- $\bar{B}, \bar{C}$ and $\bar{D}$ can arise from weak radiative corrections (from $SU(2)_R$).

$$B, C, D > \bar{B}, \bar{C}, \bar{D} \quad (10)$$

- Operators are affected differently by the axial interaction mentioned in footnote 3.

$$G > H \quad (11)$$

We get one approximate relation due to the similarity of the $E$ and $F$ operators.

$$\frac{m_d}{m_s} \approx \frac{m_t}{m_{t'}} \quad (12)$$

## 4 Lepton masses

We now turn to the charged lepton mass matrices where we find that the mixed quark-lepton operators play a crucial role. The following operators feed mass down from $t'$,

$$E_{L1} U_{R1} U_{L1} E_{R1} \quad B_t$$

$$E_{L1} U_{R1} U_{L1} E_{R2} \quad C_{\ell}$$

$$E_{L2} U_{R1} U_{L1} E_{R1} \quad D_{\ell}$$

$$E_{L2} U_{R1} U_{L1} E_{R2} \quad E_{\ell} \quad (13)$$

while the following operators feed mass down from $t$.

$$E_{L1} U_{R1} U_{L1} E_{R1} \quad F_{\ell}$$

$$E_{L2} U_{R1} U_{L1} E_{R1} \quad G_{\ell}$$

$$E_{L1} U_{R1} U_{L1} E_{R2} \quad H_{\ell}$$

$$E_{L2} U_{R1} U_{L1} E_{R2} \quad I_{\ell} \quad (14)$$
The following operators are the only ones we mention which are generated by $SU(2)_V$ exchange, and they feed mass down from the $\tau'$ and $\tau$.

\[
\begin{bmatrix}
\mathcal{J}_\ell \\
\mathcal{K}_\ell
\end{bmatrix}
\]

Here is the resulting matrix.

\[
M_\ell = \begin{bmatrix}
\mathcal{K}_\ell & \mathcal{J}_\ell & \mathcal{G}_\ell & 0 \\
\mathcal{E}_\ell & \mathcal{J}_\ell & 0 & \mathcal{D}_\ell \\
\mathcal{C}_\ell & 0 & 0 & \mathcal{B}_\ell \\
0 & \mathcal{H}_\ell & \mathcal{F}_\ell & \mathcal{A}_\ell
\end{bmatrix}
\]

The $\mu$ mass is reasonable,

\[
m_\mu \approx \frac{(1\text{TeV})^3}{(100\text{TeV})^2}
\]

and there is a relation due to the similarity of the $\mathcal{J}_\ell$ and $\mathcal{K}_\ell$ operators:

\[
\frac{m_\mu}{m_\tau} \approx \frac{m_\tau'}{m_\tau'}
\]

We now turn to neutrinos. We have already mentioned that the RH neutrinos have mass at the flavor scale, and that the 4th LH neutrino has a dynamical mass in the 100 GeV to 1 TeV range. The remaining 3 LH neutrino masses can only come from 6-fermion operators. For example, the operator

\[
\mathcal{L}_{\ell_1} \mathcal{L}_{R_1} \mathcal{N}_{L_2} \mathcal{N}_{L_2}
\]

is generated by two $\mathcal{E}_{\ell_1} \mathcal{E}_{R_1} \mathcal{N}_{L_2} \mathcal{N}_{R_2}$ operators after integrating out the large $N_{R_2}$ mass. The result is that the $\tau'$ mass feeds down to produce a small $N_{L_2}$ (i.e. $\nu_e$) mass.\footnote{This is very similar to the standard see-saw mechanism involving scalar fields, except that the dimensions of the operators involved here are much larger. This allows the right-handed neutrino mass scale to be at the relatively low flavor scale we are discussing.}

\footnote{Using numbers similar to those in (17), and accounting for the anomalous scaling enhancement built into those numbers, can yield neutrino masses in the eV range. We also note that in comparing these operators to those containing quark fields, the latter operators may be dynamically favored due to QCD effects.}
The whole set of 4-fermion operators which can contribute in this way to neutrino masses are:

\[
\begin{bmatrix}
E_{L1}E_{R1}N_{L2}N_{R2} & B_{\nu} \\
E_{L1}E_{R1}N_{L2}N_{R2} & C_{\nu} \\
E_{L1}E_{R1}N_{L1}N_{R2} & D_{\nu} \\
E_{L1}E_{R1}N_{L1}N_{R1} & E_{\nu} \\
E_{L1}E_{R1}N_{L2}N_{R1} & F_{\nu} \\
E_{L1}E_{R1}N_{L2}N_{R1} & G_{\nu}
\end{bmatrix}
\]  

(20)

After labelling the heavy right-handed neutrino masses as

\[
\begin{bmatrix}
N_{R2}^2 & m_1 \\
N_{R2}^2 & m_2 \\
N_{R2}N_{R2} & m_3 \\
N_{R1}N_{R1} & m_4
\end{bmatrix}
\]  

(21)

we find for the light neutrino mass matrix the following.

\[
\begin{bmatrix}
B_{\nu}^2 \\
B_{\nu}C_{\nu} \\
B_{\nu}D_{\nu} \\
C_{\nu}^2 \\
C_{\nu}D_{\nu} \\
D_{\nu}^2
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2 \\
m_3 \\
m_2 \\
m_2 \\
m_2
\end{bmatrix}
\]

(22)

We see that this matrix bears no resemblance to quark or charged lepton mass matrices. Large mixings are expected, with masses unrelated to the family hierarchy. CP violation is also expected, and it is to that topic which we now turn.

5 CP violation

We first note that the quantity most sensitive to possible CP violation in new 4-fermion effects is the $\varepsilon$ parameter in the $K-K$ system. In other words we have a natural setting for a superweak model of CP violation. Superweak models are especially attractive in the context of the strong CP problem, since they allow for the quark mass matrix to be real, or very close to it, which would account for why the strong CP violating parameter $\theta$ is close to vanishing. When we consider how this can arise in the present model, we find that CP violation in the quark sector may arise in a way similar to neutrino masses; that is, via 6-fermion operators only. This provides
a natural suppression mechanism which can go a long way towards suppressing strong CP violation to acceptable levels.

In our picture we assume that above the flavor scale we have a CP invariant gauge theory of massless fermions. Our dynamical assumption is that lepton-number violation, $SU(2)_V$ breaking and CP violation all originate in the right-handed neutrino condensates (both bilinear and multilinear). We may then consider the operators which feed CP violation feed into the quark sector. It can be shown that they must violate lepton-number or $SU(2)_V$ or both. This in turn requires 6-fermion operators, of which the following are two examples.

\[
\overline{D}_{L2} D_{R2} \overline{D}_{L2} D_{R2} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{D}_{L2} D_{R2} \overline{N}_{L1} \overline{N}_{L1}
\]

From the mass matrices we have given it can be seen that in the presence of the heavy $\nu^\tau$ mass, these generate the $\Delta S = 2$ operators $(\bar{d}_{LSR})^2$ and $(\bar{s}_{LSR})^2$. If the 6-fermion operators have coefficients of order $1/(100 \text{ TeV})^5$ and $\langle \overline{N}_{L1}^2 \rangle \approx (1 \text{ TeV})^3$, then the coefficients of the $\Delta S = 2$ operators are of the right size to give $\varepsilon$ in $K-\overline{K}$ mixing. $\varepsilon'$ on the other hand requires $d-s$ mass mixing, and thus is negligible.

The following operators

\[
\overline{D}_{L2} D_{R1} \overline{D}_{L2} D_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} D_{R2} \overline{D}_{L1} D_{R2} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} D_{Rj} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

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The following operators

\[
\overline{D}_{L2} D_{R1} \overline{D}_{L2} D_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} D_{R2} \overline{D}_{L1} D_{R2} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L2} D_{R2} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

\[
\overline{D}_{L1} \epsilon_{ij} \overline{E}_{L1} E_{R1} \overline{N}_{L1} \overline{N}_{L1}
\]

10
Thus the CP violating parts of quark masses can be of similar magnitude to the neutrino masses. This by itself does not sufficiently suppress $\theta$, but the detailed structure of the quark mass matrices can lead to further suppression.

6 Conclusion

There can be many other effects of the new flavor physics, through nonrenormalizable effects and in particular through the effects of the $X$ boson. For example we can expect anomalous couplings of standard model gauge bosons to the third family. Flavor changing effects may surface in $B-\overline{B}$ and $D-\overline{D}$ mixing, with the result that the $B$ factories may uncover flavor changing effects rather than CP violation.

In conclusion, we have bypassed the usual approaches to electroweak symmetry breaking and proceeded straight to the flavor problem. We have suggested that there is a dynamically broken flavor gauge symmetry around 100 to 1000 TeV which generates a wide variety of multi-fermion operators. Close to a TeV the remnant flavor symmetry breaks, fourth family masses arise, and electroweak symmetry breaking occurs. We have explored the interplay between quark and lepton sectors in the generation of mass matrices. We have also seen how the suppression of CP violation in the quark sector is similar to the suppression of neutrino masses. One of the first signals of this picture could be the absence of CP violation at B factories.

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References

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What do we do with the Higgs sector?

Technicolor

Supersymmetry

What breaks supersymmetry?
- strong interactions?

Fermion masses?
- extended TC
- new sectors

Top mass?
- new sector

Flavor changing neutral currents?

Where is the theory of flavor?

Figure 1

Figure 2

Figure 3