Holographic model for heavy-vector-meson masses

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Abstract – The experimentally observed spectra of heavy-vector-meson radial excitations show a dependence on two different energy parameters. One is associated with the quark mass and the other with the binding energy levels of the quark-antiquark pair. The first is present in the large mass of the first state while the other corresponds to the small mass splittings between radial excitations. In this article we show how to reproduce such a behavior with reasonable precision using a holographic model. In the dual picture, the large energy scale shows up from a bulk mass and the small scale comes from the position of anti-de Sitter (AdS) space where field correlators are calculated. The model determines the masses of four observed \textit{S}-wave states of charmonium and six \textit{S}-wave states of bottomonium with 6.1\% rms error. In consistency with the physical picture, the large energy parameter is flavor dependent, while the small parameter, associated with quark-antiquark interaction, is the same for charmonium and bottomonium states.

Introduction. – Vector mesons made of light quarks present an approximately linear relation between the mass squared and the radial excitation number \(n\), the so-called radial Regge trajectory: \(m^2_n \sim \alpha n\). So, the mass spectrum can be approximately described by just one dimensional parameter (energy scale), related to the interaction between the quark and the antiquark.

For heavy vector mesons the situation is different, as one can see from the experimental values of the masses of the \(n = 1, 2, 3, 4\), \textit{S}-wave states of charmonium \([1]\)
\begin{align*}
m_1 &= 3097 \text{ MeV}, & m_2 &= 3686 \text{ MeV}, & m_3 &= 4039 \text{ MeV}, \nonumber\n\text{ and } m_4 &= 4441 \text{ MeV}. \end{align*}

The mass gaps \((m_{n+1} - m_n)\) are much smaller than the mass of the first state. This suggests the existence of two different energy scales. A large one, associated with the heavy constituent quark masses and a smaller one, that appears in the gaps between radial excitations, related to the energy levels of quark-antiquark interaction (binding energy). A similar behavior is observed for bottomonium states, where the difference in the scales is even more evident. The Regge trajectories for heavy quarkonium are discussed, for example, in \([2]\).

AdS/QCD models, like the hard wall \([3–5]\), are motivated by gauge string duality \([6–8]\) and provide nice descriptions of mass spectra of glueballs and vector mesons made of light quarks. A recent review of hard-wall and other holographic models developed afterward can be found in \([9]\).

Heavy vector mesons have been discussed in the context of AdS/QCD models in refs. \([10–18]\). However, concerning the mass spectra of \textit{S}-wave states of heavy quarkonium, there is no accurate predictive AdS/QCD model available in the literature. The previous studies either provide masses with large errors with respect to experimentally observed data, or depend on many parameters, thus lacking of predictivity. In particular, the simplest picture that one could draw for the heavy-quarkonium states is that the mass spectrum should depend on the quark mass, that is flavor dependent, and on the quark-antiquark interaction, that is flavor independent. This simple physical picture is absent in these previous works and will emerge in the present article.

We present here a holographic AdS/QCD model that describes the masses of the \textit{S}-wave states of charmonium and bottomonium with just 3 parameters that have a very clear physical interpretation: one is associated with the mass of the \(c\)-quark, the other with the mass of the \(b\)-quark...
and the third with the flavor-independent quark interaction. The model reproduces the masses of ten states of charmonium and bottomonium with good precision, characterized by 6.1% rms error.

**Holographic picture of heavy vector mesons.**

The current \( J^\mu = \bar{q} \gamma^\mu q \) associated with a heavy vector meson is assumed to be dual to the \( V_\mu \) components of a massive vector field \( V_m = (V_\mu, V_z) \) \((\mu = 0, 1, 2, 3)\) living in anti-de Sitter space:

\[
ds^2 = \frac{R^2}{z^2} (-dt^2 + d\vec{x} \cdot d\vec{x} + dz^2),
\]

where \((t, \vec{x}) \in \mathbb{R}^{1,3}\) and \(z \in (0, \infty)\) is called radial coordinate. For the action, we choose

\[
I = \frac{1}{2g^2} \int d^4x dz \sqrt{-g} \left\{ -\frac{1}{2} F_{mn} F^{mn} - \mu^2 (z) V_m V^m \right\},
\]

where \(F_{mn} = \partial_m V_n - \partial_n V_m\) and the bulk mass has the form \(\mu^2 (z) = M^2 z^2 / R\) where \(M\) is a mass parameter that plays the role of introducing the (heavy) quark mass in the model. In the conformal (AdS/CFT) case, where the mass \(M\) is zero, the action (2) is gauge invariant and one can choose the gauge \(V_z = 0\). Then the remaining components \(V_\mu\) of the vector field play the role of generators of correlators of the boundary currents \(J^\mu\). In the present case, there is no gauge invariance, but we assume that solutions of the vector field satisfying the condition \(V_z = 0\) work, as in the conformal case, as the sources of the current correlators.

The idea of using a bulk mass that varies with the radial coordinate as a type of infrared cut off in the gauge theory has the following interpretation: the radial coordinate of AdS space is associated with the energy of the gauge theory. A bulk mass increasing quadratically with \(z\) implies that low energies are represented by bulk fields with large mass. The limit of zero energy corresponds to an infinitely massive field. So, the bulk mass term suppresses the low energies of the gauge theory.

It is important to remark that the sign of the bulk mass term \(-\mu^2 V_m V^m\) in eq. (2), with our metric convention, is the opposite of the usual mass term of a vector field. That means, it is like an imaginary mass term. If one choose an action like (2) but with an opposite mass term \(+\mu^2 V_m V^m\) it is known [19,20] that such a term can be eliminated by a field redefinition. This way, an action like (2) but with the a mass term \(+\mu^2 V_m V^m\) can be mapped into a soft-wall action [21] without mass. In the present case, this is not possible due to our choice of sign. Therefore, our action is not equivalent to a soft-wall action. Thus, the new model that we are proposing for heavy vector mesons is not equivalent to a soft-wall model.

The second energy parameter that is needed to represent the heavy vector mesons, corresponding to the quark-antiquark interaction, is introduced in the model as the inverse of the position \(z = z_0\) of the radial AdS coordinate where the correlation functions of the gauge theory currents are calculated. We take the prescription:

\[
\langle 0 | J_\mu (x) J_\nu (y) | 0 \rangle = \frac{\delta}{\delta V_\mu (x)} \frac{\delta}{\delta V_\nu (y)} \exp (-I_{\text{on shell}}),
\]

where the source of the current operator is the value of the bulk field at the finite location \(z_0\): \(V_\mu (x) = \lim_{z \to z_0} V_\mu (x, z)\) and the on-shell action is obtained by constraining the action of eq. (2) to the AdS slice \(z < \infty\). This means the on-shell action is

\[
I_{\text{on shell}} = - \frac{1}{2g^2} \int d^4x \left[ \frac{1}{z} V_\mu \partial_\nu V^\mu \right]_{z = z_0} \to \infty,
\]

where we introduced \(g^2 = g^2 / R\), the relevant dimensionless coupling of the bulk vector field. A similar calculation of two-point functions at a non-vanishing position of the radial AdS coordinate was discussed in refs. [22,23], in the context of an AdS/QCD model with an exponential dilaton background.

One can represent the vector field \(V_\mu (x, z)\) in momentum space and use the decomposition

\[
V_\mu (p, z) = v(p, z) V_\mu^0 (p),
\]

where \(v(p, z)\) is the bulk-to-boundary propagator that satisfies the equation of motion:

\[
\partial_z \left( \frac{1}{z} \partial_z v(p, z) \right) + \left( \frac{p^2}{z} - M^4 z \right) v(p, z) = 0.
\]

The boundary condition

\[
\lim_{z \to z_0} v(p, z) = 1
\]

imposes that \(V_\mu^0 (p)\) acts as the source of current-current correlators. The upper limit of the on-shell action in eq. (4) is cancelled imposing Neumann boundary condition at infinity:

\[
\lim_{z \to \infty} \left( \frac{\partial v}{\partial z} \right) = 0.
\]

It is important to remark that when one uses a massless vector field to generate the correlators of the gauge current by means of a coupling term of the form: \(\int d^4x V^\mu J_\mu\), gauge invariance of the field implies conservation of the current. In the present case, where we use a massive vector field, the coupling to the currents does not guarantee by itself the conservation of the current. We will assume that the current is conserved, since it comes from a gauge-invariant theory. The vector field is just as an external source that generates the expectation values of the currents. So, in momentum space the currents are transversal and the vacuum expectation value of product of currents has the structure:

\[
\int d^4x e^{-ipz} \langle 0 | J_\mu (x) J_\nu (0) | 0 \rangle = \left( \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) G(p^2).
\]
The AdS/CFT prescription, proposed in refs. [7,8], is to take the on-shell action as the generator of the gauge theory correlators. Correspondingly, the bulk fields at \( z = 0 \) play the role of the generators of the boundary theory correlators. Here an analogous prescription is used. The difference is that we take the bulk fields at the finite position \( z = z_0 \) as the sources for the gauge theory correlators and assume that the same relations between bulk fields and boundary operators of the AdS/CFT correspondence are valid. Correspondingly, the generator of correlation function is

\[
I_{\text{on shell}} = -\frac{1}{2g_5^2} \int d^4x \left[ \frac{1}{z} V_\mu \partial_\nu V^{\nu} \right]_{z = z_0}.
\] (10)

This holographic prescription provides an expression for the two-point function in terms of the bulk-to-boundary propagator:

\[
G(p^2) = -\lim_{z \to z_0} \left( \frac{1}{g_5^2 z} \frac{\partial v(p, z)}{\partial z} \right).
\] (11)

The equation of motion (6) with Neumann boundary condition at infinity, has the solution

\[
v(p, z) = C(p) \exp(-M^2z^2/2)U(p^2/4M^2, 0, M^2z^2),
\] (12)

where \( U(a, b, x) \) is the confluent hypergeometric Kummer function as defined in [24] and \( C(p) \) is an arbitrary factor that does not depend on \( z \). Following a similar procedure as in [23], one can build the bulk-to-boundary propagator, satisfying \( v(p, z_0) = 1 \), by simply choosing \( C(p) \) appropriately:

\[
v(p, z) = \frac{e^{-M^2z^2/2}e^{-M^2z^2/2}}{M^2z^2/2U(p^2/4M^2, 0, M^2z^2)}.
\] (13)

Inserting the bulk-to-boundary propagator (13) in our holographic two-point function of eq. (11) one finds

\[
G(p^2) = \frac{1}{g_5^2} \times \frac{M^2U(p^2/4M^2, 0, M^2z_0^2) - \frac{p^2}{2}U(1 + p^2/4M^2, 0, M^2z_0^2)}{U(p^2/4M^2, 0, M^2z_0^2)}.
\] (14)

We associate the poles of \( G(p^2) \) with the masses of the states of the theory: \( p_n^2 = -m_n^2 \).

At this point it is interesting to compare the approach developed here with the hard-wall model [4,5]. In the hard-wall case, the space ranges from \( z = 0 \) to a maximum value \( z = z_{\text{HW}} \). The position \( z = 0 \), where one calculates the correlation functions, plays the role of an ultraviolet boundary corresponding to infinite energy. The position \( z_{\text{HW}} \), represents a hard infrared cutoff (hard wall), where the field solutions corresponding to the states satisfy Dirichlet or Neuman boundary conditions. There is just one energy parameter in the hard-wall model: \( 1/z_{\text{HW}} \).

\[
\begin{array}{|c|c|}
\hline
\text{State} & \text{Mass (MeV)} \\
\hline
1S & 3096.916 \pm 0.011 \\
2S & 3686.109 \pm 0.012 \\
3S & 4039 \pm 1 \\
4S & 4421 \pm 4 \\
\hline
\end{array}
\]

Table 1: Experimental masses for the charmonium S-wave resonances from [1].

\[
\begin{array}{|c|c|}
\hline
\text{State} & \text{Mass (MeV)} \\
\hline
1S & 9460.3 \pm 0.26 \\
2S & 10023.26 \pm 0.32 \\
3S & 10355.2 \pm 0.5 \\
4S & 10579.4 \pm 1.2 \\
5S & 10860 \pm 11 \\
6S & 11019 \pm 8 \\
\hline
\end{array}
\]

Table 2: Experimental masses for the bottomonium S-wave resonances from [1].

In contrast, the model developed here is defined in the region \( z_0 \leq z < \infty \). The position \( z = z_0 \) is a boundary where we calculate the correlation functions. The masses are defined as the poles of eq. (14) that come from the zeroes of the denominator of this equation. So, the solutions of the equation of motion, that have the form \( \exp(-M^2z^2/2)U(p^2/4M^2, 0, M^2z^2) \) vanish at \( z = z_0 \), for \( p^2 = -m_n^2 \). Note that the normalization condition of eq. (7) is achieved by writing

\[
v(p, z) = C(p) \exp(-M^2z^2/2)U(p^2/4M^2, 0, M^2z^2)
\]

and choosing \( C(p) \) to be the inverse of the value of the function \( \exp(-M^2z^2/2)U(p^2/4M^2, 0, M^2z^2) \) at \( z = z_0 \). The fact that the solutions of the equation of motion corresponding to the physical states vanish at \( z = z_0 \) means that this position represents a hard wall. On the other hand, the present model has also a smooth infrared cutoff represented by the mass term \( \mu(z) = M^2z^2/R \). So, there are two energy parameters: \( 1/z_0 \) and \( M \).

In the next section we show the mass spectra for charmonium and bottomonium S-wave states, obtained from the poles of eq. (14).

\section*{Model vs. experimental data.}

The experimental values for the masses of S-wave states of charmonium and bottomonium from the Particle Data Group Collaboration [1] are shown in tables 1 and 2, with the corresponding uncertainties. The best fit of the model for the masses of the heavy vector mesons is obtained for the choice of parameters:

\[
M_c = 0.74 \text{ GeV;} \quad M_b = 1.35 \text{ GeV;} \quad 1/z_0 = 0.25 \text{ GeV},
\]

where \( M_c \) and \( M_b \) are the values of the parameter \( M \) of the model used for the cases of charmonium and bottomonium.
remarkable agreement is found. Comparison of the masses from this reference and the results 
BaBar published in ref. [26]. We present in table 5 a com-

Table 4: Masses of bottomonium

| State | Mass (MeV) |
|-------|------------|
| 1S    | 3075.5 (0.68%) |
| 2S    | 3664.5 (0.58%) |
| 3S    | 4118.2 (1.20%) |
| 4S    | 4502.5 (1.84%) |

The percentages are the deviations with respect to average experimental values.

bottomonium, respectively. The energy parameter $1/z_0$ represents the energy levels of the interaction between the quark and the antiquark, that is expected to be dominated by color interaction, so for consistency we use the same value for the two flavors of vector mesons.

We show in tables 3 and 4 the results of the holographic model, with the percentage deviations with respect to (average) experimental data. As a measure of the predictability of the model, one can define the rms error for estimating $N$ quantities using a model with $N_p$ parameters as

$$\delta_{rms} = \left( \frac{1}{N-N_p} \sum_{i} \left( \frac{\delta O_i}{O_i} \right)^2 \right)^{1/2}, \quad (15)$$

where $O_i$ is the average experimental value and $\delta O_i$ is the deviation of the value given by the model. We find for our estimate of 10 states with 3 parameters: $\delta_{rms} = 6.1\%$. In particular, considering separately the charmonium states, we have an rms error of $\delta_{rms} = 1.7\%$.

It is interesting to mention that although the experimental data available for charmonium at present time are conclusive only for the masses of the first four states: 1S–4S, there is also some indication from experimental data on $e^+e^-$ annihilation analyzed by BaBar collaboration about higher S-wave states. In ref. [25] masses for 5S up to 8S states are estimated, based on data from BaBar published in ref. [26]. We present in table 5 a comparison of the masses from this reference and the results of the holographic model for these states, where again a remarkable agreement is found.

### Table 5: Possible higher S-wave charmonium states compared with the results of the holographic model using $M_c = 0.74$ GeV and $1/z_0 = 0.25$ GeV. The percentages in parenthesis are the deviations of the model with respect to estimates from [25].

| State | Possible mass (MeV) | Holographic result |
|-------|---------------------|-------------------|
| 5S    | 4780                | 4842 (1.36%)      |
| 6S    | 5090                | 5150 (1.18%)      |
| 7S    | 5440                | 5434.4 (0.18%)    |
| 8S    | 5910                | 5699.3 (3.56%)    |

Final comments. – The results of the tables presented in the previous section and the rms errors of 6.1% show that the model proposed here is indeed capturing the behavior of the mass spectra of heavy-vector-meson radial excitations. The large energy scale, related to the quark mass, was introduced as a varying bulk mass while the small scale associated with quark-antiquark interaction showed up from the position of anti-de Sitter space where operator expectation values are calculated.

For completeness, we mention that the calculation of correlation functions at a finite position in AdS space appeared, in the context of exponential (soft-wall) dilaton background, in [23] and was used recently in [27] to describe the observed behavior of decay constants of vector mesons.

As we have shown here, it is the combination of the varying bulk mass with the definition of the gauge theory correlators at a finite position of AdS space that provides the appropriate description of heavy-vector-mesons masses.

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