We present here measurements of the thermopower, thermal conductivity, and electrical resistivity of the newly reported compound CaFe$_4$As$_3$. Evidence is presented from specific heat and electrical resistivity measurements that a substantial fraction of the Fermi surface survives the onset of spin density wave (SDW) order at the Néel temperature $T_N=88$ K, and its subsequent commensurate lockin transition at $T_2=26.4$ K. The specific heat below $T_2$ consists of a normal metallic component from the ungapped parts of the Fermi surface, and a Bardeen-Cooper-Schrieffer (BCS) component that represents the SDW gapping of the Fermi surface. A large Kadowaki-Woods ratio is found at low temperatures, showing that the ground state of CaFe$_4$As$_3$ is a strongly interacting Fermi liquid. The thermal conductivity $\kappa$ of CaFe$_4$As$_3$ is an order of magnitude smaller than those of conventional metals at all temperatures, due to a strong phonon scattering. The thermoelectric power $S$ displays a sign change from positive to negative indicating that a partial gap forms at the Fermi level with the onset of commensurate spin density wave order at $T_2=26.4$ K. The small value of the thermopower $S$ and the enhancements of the resistivity due to gap formation and strong quasiparticle interactions offset the low value of the thermal conductivity $\kappa$, yielding only a modest value for the thermoelectric figure of merit $Z \leq 5 \times 10^{-6}K^{-1}$ in CaFe$_4$As$_3$. The results of ab initio electronic structure calculations are reported, confirming that the sign change in the thermopower at $T_2$ is reflected by a sign change in the slope of the density of states at the Fermi level. Values for the quasiparticle renormalization $Z$ are derived from measurements of the specific heat and thermopower, indicating that as $T \rightarrow 0$, CaFe$_4$As$_3$ is among the most strongly correlated of the known Fe-based pnictide and chalcogenide systems.

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INTRODUCTION

Although thermoelectric materials have received enduring attention over the past half century for their ability to convert thermal energy to electrical energy and vice versa, the growing need to develop new materials with enhanced properties for applications has led to a renewed interest in recent years. [1] The potential performance of a given thermoelectric material is expressed in terms of the thermoelectric figure of merit $Z = S^2/\rho\kappa$, where $S$ is the thermoelectric power, $\rho$ is the electrical resistivity, and $\kappa$ is the thermal conductivity. High thermoelectric power must be combined with low thermal conductivity and low electrical resistivity to optimize the figure of merit. Large values of $Z$ have been observed in the filled skutterudites AT$_4$Sb$_{12}$ ($A$ = rare earths or alkaline earth metals, $T$ = transition metals), which combine large values of the thermopower $S$, which can be as large as 10-100 $\mu$V/K, with low thermal conductivity $\kappa$, as small as a few $\text{W/K-m}$. [2, 3] The low $\kappa$ of these systems reflects an unusually small phonon contribution, originating with enhanced scattering of heat-carrying phonons from the low energy rattling and tunneling modes of filler “$A$” atoms in the $T_4$Sb$_{12}$ cage-like structures characteristic for these compounds. [4, 5] Consequently, it is potentially of great interest to identify and explore other classes of correlated materials that similarly form in open and cage-like structures.

Recently, a new Fe-As system, CaFe$_4$As$_3$, has been reported, [6, 7] that crystallizes in orthorhombic structure (space group $Pbnm$). In this structure, the Ca atoms are confined inside nearly rectangular tunnels, oriented along the $b$ axis, whose walls are constructed from a network of Fe-As tetrahedra which are similar to the FeAs planes in the layered iron-pnictide superconductors. The combination of confined Ca atoms and the open structure of the FeAs framework in CaFe$_4$As$_3$ is reminiscent of the cages in the filled skutterudites RFe$_4$As$_{12}$ ($R$ = Ce and Pr). [8, 9] and it is possible that here too a low thermal conductivity might be realized. We note as well that large thermopowers are generally found in metals where strong correlations lead to enhanced densities of states near the Fermi level. CaFe$_4$As$_3$ undergoes an antiferromagnetic transition at $T_N = 88$ K, which has been classified as an incommensurate spin density wave (SDW). [10] The SDW becomes commensurate with the underlying lattice below a second transition that occurs at $T_2 = 26.4$ K. However, the SDW does not appear to gap the entire Fermi surface, since metallic Fermi liquid behavior is found at the lowest temperatures, where the electrical resistivity $\rho(T) = \rho_0 + AT^2$ and the electronic specific heat $C_{el} = \gamma T$. Intriguingly, the Kadowaki-Woods ra-
EXPERIMENTAL DETAILS

Single crystals of CaFe₄₃As were grown from a Sn flux, forming in a rodlike morphology with the crystallographic b axis along the rod axis. Details of the sample preparation are described in Ref. [6]. Using the thermal transport option (TTO) of a Quantum Design Physical Property Measurement System (PPMS) over the range of temperatures with 2 K < T < 300 K, the thermal conductivity \( \kappa(T) \), thermoelectric power \( S(T) \), and resistivity \( \rho(T) \) were simultaneously measured on a 0.5 \( \times \) 0.5 \( \times \) 3 mm³ sample using a two-probe method, where the temperature gradient across the sample was always less than 3% of the background temperature during the measurement and an AC current of 1 mA with frequency of 17 Hz was applied for the resistivity measurement. Specific heat measurements were also carried out using the thermal relaxation technique implemented on the PPMS for 5 K < T < 150 K.

EXPERIMENTAL RESULTS AND DISCUSSION

The filled circles in Fig. 1(a) show the temperature dependence of the thermal conductivity \( \kappa \), measured with the heat flow along the b axis of a single crystal of CaFe₄₃As. The temperature dependence of the electrical resistivity \( \rho(T) \) is presented in Fig. 1(b), showing an overall metallic character, with a sharp drop at \( T_2 \) and a slope discontinuity at \( T_N \). The electronic contribution to the thermal conductivity \( \kappa_{el} \) was determined using the Wiedemann-Franz law \( \kappa_{el} = L_0 T/\rho \), where \( L_0 \) is the Sommerfeld value 2.45 \( \times \) 10⁻⁸ WΩ/K² for the Lorenz ratio \( L = \rho S/k_B T \). The phonon contribution \( \kappa_{ph} \) was subsequently determined as the difference between \( \kappa \) and \( \kappa_{el} \). \( \kappa \), \( \kappa_{el} \), and \( \kappa_{ph} \) are compared in Fig. 1(a). We note that values for \( \kappa \) above \( \sim 200 \)K are likely to be \( \sim 10-30\% \) higher than the actual values, due to systematic discrepancies in the PPMS/TTO radiation corrections. [11, 12].

The overall thermal conductivity \( \kappa \) of CaFe₄₃As is an order of magnitude smaller at all temperatures than the values found in conventional metals, which are typically \( \approx 10-30 \) W/K-m. However \( \kappa \) is comparable to the values measured in the filled skutterudites \( AT_{4}X_{12} \). [3, 8, 12] For example, \( \kappa \) of \( RFe_{4}As_{12} \) (\( R = Ce \) and \( Pr \)) is no more than \( 3 \sim 4 \) W/K-m at all temperatures, [3, 8] comparable to the values found in CaFe₄₃As above \( T_N \) (Fig. 1(a)), and as much as a full order of magnitude larger than at the lowest temperatures. By analogy to the filled skutterudites, the characteristically small value of \( \kappa \) in CaFe₄₃As can be attributed to its cage-like structure, described above. [3, 7] The equivalent isotropic atomic displacement parameter (ADP) of the Ca atoms is more than 40% larger than the averaged equivalent isotropic ADP of the Fe and As atoms in CaFe₄₃As, [6] and is comparable to the amplitudes of motion realized in the skutterudites, where the ADPs of the filler atoms are \( \approx 30\% \) larger than those of the other atoms. [14] If the low thermal conductivity observed in CaFe₄₃As arises from
the rattling of Ca atoms in the open tunnel-like structure, then the phonon mean free path should be comparable to either the 6 Å cage dimension, or to the 3.7 Å nearest Ca-Ca separation distance. We have estimated the phonon mean free path \(d\) using the expression \(\kappa_{\text{ph}} = \frac{1}{3}C_v v_s d\), where \(C_v\) at 300 K is 189 J/mol K, a Debye temperature \(\theta_D\) is taken from the ADP of CaFe₄As₃ as 328 K, and the averaged sound velocity \(v_s\) is estimated from the Debye model to be 2784 m/s. The total mean free path \(d \sim 12\) Å, which is substantially larger than either the dimension of the tunnel-like structure in CaFe₄As₃ or the spacing of the Ca ions contained in the cage. We conclude that the small thermal conductivity likely doesn’t originate with a rattling mode, but instead with strong phonon scattering due to the complicated crystal structure and 32 atom unit cell, with multiple Fe and As site symmetries.

Further evidence for the strong phonon scattering comes from the temperature dependence of \(\kappa\), which displays a broad shoulder centered around 30 K in CaFe₄As₃. Ordinarily, a broad maximum in \(\kappa_{\text{ph}}\) is expected at \(T \approx \theta_D/10\), resulting from the crossover between phonon-boundary and/or phonon-point defect scattering at low temperatures \((T \ll \theta_D/10)\) and phonon-phonon Umklapp scattering at high temperatures \((T \gg \theta_D/10)\). In principle, the absence of a maximum in \(\kappa(T)\) in CaFe₄As₃ might be ascribed to particularly strong phonon scattering from point-like defects. However, the broad maximum is found even in polycrystalline filled skutterudites, where defects are expected to be plentiful. A shoulder in \(\kappa(T)\) is found only in skutterudites with small filler atoms, whose large amplitude rattling leads to particularly strong phonon scattering. The shoulder observed in \(\kappa(T)\) in CaFe₄As₃ implies that the phonon scattering is very strong here as well.

The temperature dependencies of the thermal conductivities \(\kappa\) and the electrical resistivity \(\rho\) reveal the presence of a SDW transition into an antiferromagnetically ordered state below the Néel temperature \(T_N=88\) K. Recent neutron diffraction measurements confirm that the 88 K transition in CaFe₄As₃ is indeed to an incommensurate SDW state. Figure 1(a) shows that the thermopower \(S\) with the heat flow along the \(b\) axis.

FIG. 2: Temperature dependence of the thermoelectric power \(S\) with the heat flow along the \(b\) axis.

The temperature dependence of the thermoelectric power \(S(T)\) depicted in Fig. 2 also confirms the strong phonon scattering in CaFe₄As₃. In general, the thermoelectric power \(S\) is the sum of the phonon drag and diffusion terms, \(S_g\) and \(S_d\), respectively. Phonon-drag in metals leads to a prominent peak in \(S_g\) at \(\sim \theta_D/5\), which is caused by a crossover between different phonon scattering mechanisms at higher and lower temperatures. The small arrow in Fig. 2 shows at most a vanishingly small peak in \(S(T)\) near 70 K \(\approx \theta_D/5\), indicating that the phonon drag peak, and indeed \(S_g(T)\) itself, are suppressed in CaFe₄As₃, presumably by the strong phonon scattering. Consequently, we take \(S_g \ll S_d\), so that \(S \approx S_g + S_d \approx S_d\) for CaFe₄As₃.

The thermopower \(S\) is surprisingly small in CaFe₄As₃, considering the rather large magnitude of the electrical resistivity \(\rho\). Nonetheless, the magnitudes of \(S\) and \(\rho\) are comparable to those found in CaFe₂As₂, where \(\rho \approx 0.4\) mΩcm and \(S \approx 1-2 \mu V/K\) above the structure/magnetic transition temperature at 170 K. This correspondence suggests that the small magnitude of \(S\) in both
compounds results from a near balance of the electron and hole concentrations, and this result is confirmed for CaFe$_2$As$_2$ in first-principle electronic structure calculations. [22] We will show below that our own electronic structure calculations, carried out using DFT, support a similar conclusion in CaFe$_4$As$_3$, and that this particle-hole symmetry is expected in compounds with SDW transitions.

Above $T_2$, $S$ is positive with a broad maximum, reaching a value of 3.3 $\mu$V/K at $\approx$ 170 K. Only a vanishingly weak anomaly is found at $T_N$, while a sharp drop is found at $T_2$ where $S$ changes sign from positive to negative with decreasing temperature. $S$ is proportional to the energy derivative of the density of states (DOS). The sign change of $S$ at $T_2$ indicates that a drastic change of the DOS accompanies the commensurate SDW transition at $T_2$, and we will show below that this too is consistent with the electronic structure calculations.

Figure 3(a) shows the temperature dependence of the specific heat $C_p/T$ measured for CaFe$_4$As$_3$ for $5 < T < 150$ K. $C_p/T$ is not linear in $T^2$ for $T \leq 35$ K, suggesting that there may be an electronic and magnetic component of the specific heat for $T \leq T_2$, in addition to the phonon contribution. To investigate this possibility, we have modeled $C_p$ as the sum of a Sommerfeld term $\gamma_g T$, a phonon contribution $C_{ph} = \beta T^3$, and a BCS term $C_{BCS} = A_g \exp(-\Delta/T)/T$ that reflects the temperature dependence in the specific heat resulting from the formation of the SDW gap:

$$C_p/T = \gamma_g/T + C_{ph}/T + C_{BCS}/T = \gamma_g + \beta T^2 + A_g \exp(-\Delta/T)/T$$

The best fit to the data for $T \leq T_2$ is shown in Fig. 3, where the three different components of the fit are compared. The parameters of this fit are $\gamma_g = 0.03$ J/Fe-mol K$^2$, $\beta = 1.27 \times 10^{-4}$ J/Fe-mol K$^3$, $A_g = 9.37$ J/Fe-mol K, and $\Delta = 52.80$ K. The ratio $\Delta/k_B T_2 = 2.0$ is very similar to the value of 2.3 found for the incommensurate SDW in Cr, [19] and is also close to the minimum value of 1.764 found in the two-band model of itinerant antiferromagnetism. [23] We note that the Debye expression using the same value of $\theta_D = 312$ K determined from this fit also describes the specific heat above $T_N$ well (Fig. 1(a)), where the electronic and magnetic contributions are expected to be very small. This value of $\theta_D = 312$ K is also in good agreement with the value of 328 K that was determined from the ADP. The internal consistency of these different estimates of $\theta_D$ gives added weight to our conclusion that a broad peak at $\sim 40$ K in $C_p - C_{ph}$ is electronic and magnetic in origin, and does not reflect anomalous features in the phonon density of states that are not described by the Debye model. The electronic and magnetic part of the specific heat $C_p - C_{ph}$ displays a sharp peak at $T_N$ (Fig. 1(a)), although only a weak peak marks the onset of commensurate SDW order at $T_2$ (Fig. 3). Both SDW ordering anomalies are superposed on the broad peak that is centered near $\sim 40$ K. Although we do not show the data here, this broad peak is unchanged when magnetic fields as large as 9 T are applied, indicating that it is not likely to be a Schottky anomaly. Its insensitivity to fields suggests instead an intrinsic electronic excitation of the incommensurate SDW with an energy scale $\Delta/3 \approx 0.45 k_B T_N$.

Electrical resistivity measurements can be used to estimate how much of the Fermi surface in CaFe$_4$As$_3$ is gapped by the incommensurate and commensurate SDW transitions. Analyses carried out in other SDW systems have used high magnetic fields to collapse the SDW gap, revealing the underlying background resistivity $\rho_{BG}$ of the gapless state. [24, 25] This approach is not possible in CaFe$_4$As$_3$, where the small measured magnetoresistance implies that very large fields would be required to close the SDW gap. Instead, we determine $\rho_{BG}$ in CaFe$_4$As$_3$ by a linear extrapolation of the measured resistivity for $T \geq T_N$, as shown in Fig. 1(b). We subtracted this estimate of $\rho_{BG}$ from $\rho$, and normalized by $\rho$ to obtain $(\rho - \rho_{BG})/\rho = \Delta \rho/\rho$, which represents the percentage change in the resistivity that can be associated with SDW formation. This is in turn proportional to the ratio of the Fermi surface volumes in the gapped and ungapped states. [24] The temperature dependence of $\Delta \rho/\rho$ is plotted in Fig. 1(b). $\Delta \rho/\rho$ increases in an order parameter-like fashion below $T_N$, extrapolating to a value of $\sim 10%$ as $T \to 0$ in the absence of the commensurate transition at $T_2$. This result implies that the incommensurate SDW in CaFe$_4$As$_3$ gaps about 10% of the Fermi surface.

Our analysis of the specific heat is also consistent with a Fermi surface in CaFe$_4$As$_3$ that is only partially gapped...
by SDW formation. We note that $\gamma_0$ is slightly enhanced, indicating that the quasiparticles that survive the Fermi surface gapping at both $T_N$ and $T_2$ have substantial interactions that increase their effective masses and lead to the heavy fermion behavior already noted for CaFe$_4$As$_3$.[7] We point out that the $T = 0$ value of the quasiparticle gap associated with SDW formation, $\Delta$ is very similar to the 40 K electronic specific heat scale deduced from the broad peak in the specific heat $C_p - C_{ph}$ above $T_2$, suggesting a common origin.

Our measurements confirm the general scenario of Fermi surface gapping that is accomplished by incommensurate SDW formation at $T_N$, followed by a second transition at $T_2$, where the SDW becomes commensurate with respect to the underlying lattice. The temperature dependence of the specific heat below $T_2$ is well described within the BCS theory, and we present evidence for a possible collective mode of the SDW gapped state, having a characteristic energy of $\approx 0.25 \Delta$. The overall success of this analysis, which assumes an itinerant nature for the magnetism in CaFe$_4$As$_3$, is somewhat surprising, considering that the neutron scattering measurements found a rather large Fe moment in the paramagnetic state. Inelastic scattering experiments are needed to assess the possibility that $\Delta$ instead represents an anisotropy gap in the transverse spin excitations, suggestive of a more localized picture of the magnetism.

We will combine these experimental results with electronic structure calculations to elucidate the strength of the correlations in CaFe$_4$As$_3$, and to provide a phenomenological understanding of the thermal and electrical properties described above. The electronic structure of CaFe$_4$As$_3$ was first considered in its paramagnetic state.[8] It was subsequently shown[10] that the large unit cell leads to a much larger Fermi surface with a different topology than those found in the 1111 and 122 families of iron pnictides. Our analysis shows that the strength of the correlations in CaFe$_4$As$_3$, at least in the SDW phase, is substantially higher than in other iron pnictides such as the 1111 and the 122 families, while being comparable to the 111, and 2322 families, as well as the 11 family of iron chalcogenides.[26][27] Our calculations correctly reproduce the sign change in the density of states at the Fermi level, predicted by the temperature dependence of the thermopower. Finally, we ascribe the observed low thermoelectric coefficient to a substantial particle hole symmetry present near the Fermi level, and suggest compositional modifications that may improve the thermoelectric performance of CaFe$_4$As$_3$.

An alternative interpretation of the thermoelectric power data can be provided within a picture where all the electrons are itinerant. In this view, all five Fe d bands are important in the electronic structure of CaFe$_4$As$_3$, and we assume that the correlation strength is intermediate. By this we mean that the correlations are not large enough to make the material insulating, but also not so small that the wave function renormalization $Z$ is less than 0.5. These initial assumptions are consistent with previous calculations on LaO$_{1-x}$Fe$_2$As$_2$[28], which found a mass renormalization between 3 and 5 ($Z \sim 0.2-0.3$), in good agreement with subsequent optical experiments.[29] Rather than calculating $Z$ from first principles for CaFe$_4$As$_3$, we extract it from an analysis that combines the results of the thermopower and specific heat measurements described above with ab initio electronic structure calculations carried out in both the paramagnetic and ordered states. The unit cell needed to describe the incommensurate order for $T_2 \leq T \leq T_N$ is prohibitively large, so we will restrict our calculations to the paramagnetic state with $T \geq T_N$ and the commensurate SDW state with $T \leq T_2$.

We performed density functional theory (DFT) calculations in both the low temperature SDW and the high temperature paramagnetic phases of CaFe$_4$As$_3$. The calculation is done using the projector augmented wave (PAW) method[30][31] as implemented in VASP[32] and the PBE exchange correlation functional.[33] The lattice constants $a = 11.852$ Å, $b = 3.7352$ Å, and $c = 11.5490$ Å as well as the atomic coordinates are taken from the 15 K SDW state.[3] For the paramagnetic state, we use a $8 \times 24 \times 8$ dense mesh and a energy cutoff of 300 eV. To simulate the low temperature SDW state with a $Q = (0, 3/8 \pi/b, 0)$ wave vector,[34] we use a $1 \times 8 \times 1$ supercell of the PM unit cell and perform a non-collinear magnetic
calculation using a $6 \times 2 \times 6$ mesh and an energy cutoff of 250 eV. The magnitude and arrangement of the magnetic moments in our calculations agree well with those deduced from experimental measurements.\cite{34}

The density of states that results from these calculations in both the high temperature paramagnetic state and in the low temperature SDW state is presented in Fig. 5. We note with interest that the energy derivative of the density of states $N(\varepsilon)$ changes sign from negative in the paramagnetic phase to positive in the SDW phase. The density of states at the Fermi level is approximately twice as large in ungapped and paramagnetic CaFe$_3$As$_3$ than it is in SDW CaFe$_2$As$_3$, indicating that the SDW gap has removed a large fraction of the high temperature Fermi surface.

![Fig. 5](image_url)  
**FIG. 5:** (Color online) Upper panel: The density of states (DOS) for both the high temperature paramagnetic (PM) phase and low temperature SDW phase of CaFe$_2$As$_3$. An expanded view near the Fermi level is presented in the lower panel.

It is our intention to combine experimental measurements of the thermopower $S(T)$ with these DFT calculations of $N(\varepsilon)$ and its energy derivative $N'(\varepsilon)$ to estimate the quasiparticle renormalization factor $Z_S$. An independent determination of $Z = Z_F$ can be extracted from the Sommerfeld coefficient $\gamma_g$, although this is only possible in the SDW phase since the large phonon contribution to the specific heat $C_p$ prohibited an accurate determination of $\gamma_g$ in the paramagnetic state. We obtain expressions for the thermopower $S$ and Sommerfeld coefficient $\gamma_g$ with the aid of the corresponding local Fermi equations.\cite{35} Ignoring the asymmetry in the self energy,\cite{36} and estimating the slope of the value of a transport function $\Phi_{\varepsilon_F}$ from the derivative of the density of states calculated using DFT. $Z$ is understood to be an average quasiparticle renormalization weight.

$$S = \frac{k_B k_BT \Phi'_{\varepsilon_F} E_F^2}{|e| Z_S \Phi_{\varepsilon_F} E_0^2}$$

(1)

$$\gamma_g = \frac{\pi^2 k_B^2}{3} Z_S N(\varepsilon_F)$$

(2)

where $k_B/|e| = 8.6 \times 10^{-5}$ V/K, $N(\varepsilon_F)$ is the density of states at the Fermi energy $\varepsilon_F$, and $E_F^2 = 1.75$ and $E_0 = 0.82$.\cite{34} The experimental input is listed in Table II and consists of the experimental value of the SDW value of $\gamma_g(T \to 0)$ as well as two values for the temperature derivative of $S$, one for the paramagnetic state, which we approximate as $S(T_N)$ and one which represents the gapped SDW state, which we set equal to $S(T_2)$. Both are subsequently extrapolated to $T = 0$, incurring substantial error bars. We summarize the results of our analysis in Table II.

| $N(\varepsilon_F)$ | $N'(\varepsilon_F)$ | $dS/dT$ | $Z_S$ | $\gamma_g$ | $Z_r$ |
|------------------|---------------------|---------|------|----------|------|
| SDW ($T \leq T_2$) | 1.5                 | -0.8(3) | 0.16(8) | 0.03     | 0.12 |
| PM ($T \geq T_N$)  | 3.2                 | -6(2)   | 0.08(3) | 0.37(20) |     |

The values of $Z_S$ that are extracted from this analysis indicate that CaFe$_2$As$_3$ is very strongly correlated in the paramagnetic state, and that the quasiparticle mass is more than doubled with the onset of commensurate SDW order, an observation that is echoed in the large Kadowaki-Woods ratio found in the low temperature Fermi liquid phase. The values of $Z_S$ and $Z_r$ are in reasonable agreement in the SDW phase, and support the conclusion that the SDW removes less correlated parts of the Fermi surface, leaving quasiparticles that are much more strongly correlated in the SDW phase of CaFe$_2$As$_3$ than in other members of the iron pnictide families. This is quite different from what is found in other iron pnictide and chalcogenide superconductors, in which the magnetic phases are more coherent and less.
correlated than the paramagnetic phases. The presence of a large Sommerfeld coefficient implies that the SDW gap $\Delta$ does not extend over the entire Fermi surface, although the quasiparticles associated with the ungapped Fermi surface appear to be strongly interacting at the lowest temperatures.

The large value of the Sommerfeld coefficient $\gamma_g$ in the SDW phase suggests that CaFe$_4$As$_3$ may have a large value of the Seebeck coefficient via the Behnia-Flouquet ratio $\frac{S}{\gamma_g} = \frac{3}{\pi^2 k_B} \frac{1}{N(\varepsilon_F)} \frac{\Phi'(\varepsilon_F)}{\Phi(\varepsilon_F)} \frac{E_1^2}{E_0^2}$. However, we find instead that the thermoelectric power $S$ is small at all temperatures, due to the weak energy dependence of the density of states near the Fermi level. Although we find that CaFe$_4$As$_3$ has a rather small thermal conductivity $\kappa$ and as well an electrical resistivity $\rho$ that is somewhat larger than expected in a good metal, perhaps due to the strong quasiparticle interactions implied by the small values of $Z$, these factors are not sufficient to overcome the small values of $S$, giving a disappointingly small value for the thermoelectric figure of merit $Z = S^2 / \kappa \rho$ in CaFe$_4$As$_3$. We find that the largest values of $ZT$ occur at 200 K, where $ZT = 1.7 \times 10^{-4}$, indicating that similar thermoelectric performances are found on high temperature range in both CaFe$_4$As$_3$ and some skutterudites, such as PrFe$_4$As$_{12}$. Clearly, the figure of merit $Z$ in CaFe$_4$As$_3$ is limited by the relatively small thermopower, and Fig. 5 shows that the reason is a surprising particle-hole symmetry that limits $\Phi(0)$. Provided that the strength of the electronic correlations is not reduced, we believe that electron doping should lead to a considerable enhancement of the thermopower. For instance, the DFT density of states suggest that the thermopower in the paramagnetic state changes sign and increases by about an order of magnitude at the following doping levels: 9-14 % on the Fe site, 12-19 % on the As site, and 35-50 % on the Ca site. However, if many-body renormalizations are included, the required doping levels can be expected to be substantially reduced from the levels predicted by DFT. Similarly, we speculate that superconductivity might be induced in CaFe$_4$As$_3$ by improving its metallic character, perhaps by a dopant that reduces the lattice constant. Further experimental and theoretical work is needed to establish whether a localized or itinerant picture is more appropriate for CaFe$_4$As$_3$.

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