Multi-Label Learning with Label Enhancement

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Abstract

Multi-label learning deals with training instances associated with multiple labels. Many common multi-label algorithms are to treat each label in a crisp manner, being either relevant or irrelevant to an instance, and such label can be called logical label. In contrast, we assume that there is a vector of numerical label behind each multi-label instance, and the numerical label can be treated as the indicator to judge whether the corresponding label is relevant or irrelevant to the instance. The approach we are proposing transforms multi-label problem into regression problem about numerical labels which can reflect the hidden label importance. In order to explore the numerical label, one way is to extend the label space to a Euclidean space by mining the hidden label importance from the training instances. Such process of transforming logical labels into numerical labels is called Label Enhancement. Besides, we give three assumptions for numerical label of multi-label instance in this paper. Based on this, we propose an effective multi-label learning framework called MLL-LE, i.e., Multi-Label Learning with Label Enhancement, which incorporates the regression loss and the three assumptions into a unified framework. Extensive experiments validate the effectiveness of MLL-LE framework.

Introduction

In traditional supervised learning, one instance only has a single label. However, in many real-world tasks, an instance may simultaneously be associated with multiple labels. Thus, to deal with the such problem, multi-label learning has become the hot research topic in recent years. During the past years, multi-label learning has been widely applied in various fields such as text (Rubin et al. 2012), video (Wang et al. 2011), image (Cabral et al. 2011), audio (Lo et al. 2011), etc.

Formally speaking, let \( \mathcal{X} = \mathbb{R}^d \) be the \( d \)-dimensional feature space and \( \mathcal{Y} = \{ y_1, y_2, \ldots, y_l \} \) be the label space with \( l \) possible class labels. Given a multi-label training set \( \mathcal{D} = \{ (x_i, y_i) \} \) \( 1 \leq i \leq n \), where \( x_i \in \mathcal{X} \) is \( d \)-dimensional feature vector and \( y_i \subseteq \mathcal{Y} \) is the set of labels associated with \( x_i \), the task of multi-label learning is to learn a multi-label predictor mapping from the space of feature vectors to the space of label vectors (Gibaja and Ventura 2015, Tsoumakas, Katakis, and Vlahavas 2010, Zhang and Zhou 2014). Each element of the label vector \( y_i \) is a logical indicator of whether the corresponding label is relevant or irrelevant to the instance \( x_i \). On the contrary, we assume that there is a vector of numerical label behind each multi-label instance, and numerical label which can reflect the hidden label importance can be treated as indicator to judge whether the corresponding label is relevant or irrelevant to the instance.

In this paper, we give a multi-label learning approach by exploring the numerical labels behind the multi-label instance. In order to learn the numerical label, the label space should be extended to a Euclidean space. Because of the space extension, the numerical label carries more semantic information to describe the instance than logical label and the hidden label importance of the training instance can be reflected from the numerical label. The process of transforming logical label into numerical label is called Label Enhancement.

To find proper numerical label of each instance, we have three assumptions: 1) the numerical label should be close enough to the original label in the Euclidean space, but we don’t expect the numerical label to be the same as the original; 2) the numerical label should obey the smoothness assumption (Zhu, Lafferty, and Rosenfeld 2005), which says that the points close to each other are more likely to share a label. With the extension from the logical label space to the Euclidean label space, we can easily induce from the smoothness assumption that the local topological structure can be transferred from the feature space to the numerical label space. So the feature space and the label space should share similar local topological structure; 3) the numerical label should be smooth enough. That is, we don’t expect the numerical label too large or too small.

Based on the above assumptions of numerical label, we propose an efficient approach named Multi-Label Learning with Label Enhancement (MLL-LE), which tries to explore numerical label vector behind multi-label instance. The target of MLL-LE is to find a mapping from feature space to numerical label space. In our approach, we formulate the problem by incorporating the mapping loss and the above three assumptions into a unified framework, and develop an alternating solution for the optimization.

Because of the three assumptions, the MLL-LE brings the following advantages: 1) it helps to explore the correlation among the labels because of the transference of the topological structure from the feature space; 2) it extends the tra-
tional logical label to the numerical label, which can describe the instance in greater details and thus may bring better performance; 3) it helps to make more complicated decisions based on the numerical labels after prediction. And the advantages of MLL-LE are validated in the following experiments.

**Related Work**

Multi-label approaches can be roughly grouped into three types based on the thought of order of label correlations (Zhang and Zhou 2014). The simplest ones are the first-order approaches which decompose the problem into a series of binary classification problems, each for one label (Boutell et al. 2004; Zhang and Zhou 2007). However the first-order approaches neglects the fact that information of one label may be helpful for the learning of another related label. On the contrary, second-order approaches consider the correlations between pairs of class labels (Eliseeff and Weston 2002; Fürnkranz et al. 2008), and the high-order approaches consider the correlations among label sub-sets or all the class labels (Di et al. 2010; Read et al. 2011; Tsoumakas, Katakis, and Vlahavas 2011). For all of them, the common approaches are to treat each label in a crisp way. In contrast, MLL-LE treats the label as numerical.

There have been some related works transforming the logical label into the numerical label. For instance, according to (Tai and Lin 2012), the computational complexity is reduced by seeking the principle correlations between labels and the bases of the Euclidean space are the combinations of the logical label vectors. Another related work (Sun, Ji, and Ye 2011) projects the feature space and the label space to a new space where the algorithm maximizes the correlation between the projections of the two spaces. Label Distribution Learning (LDL) (Geng 2016), another related work, gives each label a numerical degree to describe the instance. However, LDL requires the availability of the label distributions in the training set, so LDL doesn’t always satisfy the real-world applications. Note that MLL-LE doesn’t generate numerical label first and then train the model. In MLL-LE, the model training and the generating of numerical label are simultaneous.

The rest of this paper is organized as follows. We propose the MLL-LE approach in the next section, and then present the results of comparative studies, followed by the conclusion.

**The MLL-LE Approach**

**The MLL-LE Framework**

Let $X = \mathcal{R}^d$ be the input space and the label space with $l$ labels can be expressed as $Y = \{-1, +1\}^l$. Note that -1 is used to represent irrelevant to the example instead of 0 in label vector. The training set of multi-label learning can be described as $D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$. According to the above sections, we assume that there is a numerical label vector behind every multi-label instance. We use $u_i \in \mathcal{U} = \mathcal{R}^l$ to denote the numerical label vector of $x_i$. The aim of MLL-LE Approach is to learn a mapping model from the input space to the numerical label space, i.e., $f: \mathcal{X} \rightarrow \mathcal{U}$, which can predict a numerical label vector for a test multi-label instance.

According to the above sections, the numerical label should satisfy the three assumptions: 1) the numerical label should be close enough to the original label in the Euclidean space; 2) the feature space and the label space should share similar local topological structure; 3) the numerical label should be not too large or too small. Aiming at finding a proper mapping from input space to numerical label space and keeping the three assumptions of numerical label, we optimize both $f$ and $U$ by minimizing the following function:

$$
\min_{f, U} L_1(X, f, U) + \alpha L_2(f) + \beta L_3(U, Y) + \gamma L_4(W, U) + \delta L_5(U),
$$

(1)

where $X = \{x_1, \ldots, x_n\}^T$ is the feature matrix, $Y = \{y_1, \ldots, y_n\}^T$ is the original label matrix, $U = \{\mu_1, \ldots, \mu_n\}^T$ is the numerical label matrix, $W$ is the matrix containing the topological structure information of feature space, $L_1$ loss is a mapping loss function, $L_2$ loss is a regularizer to control the complexity of the model $f$, $L_3$ loss makes sure numerical label close enough to original label in Euclidean space, $L_4$ loss controls numerical label to share similar local topological structure of feature space, and $L_5$ loss is a regularizer to control the numerical label not too large or too small.

**The Mapping Model Design** In this subsection, we will give a sample design of mapping model. Finding a mapping model from input space to numerical label space is a multi-output regression problem. There have been some efficient multi-output regression algorithms proposed such as k-nearest neighbor regression (KNNR) (Burba, Ferraty, and Vieu 2009), multi-output support vector regression (MSVR) (Pérez-Cruz et al. 2002; Tuia et al. 2011) and structured output-associative regression (SOAR) (Bo and Smichischescu 2009). In this paper, we assume that $f$ is MSVR. So $L_1$ loss is set as:

$$
L_1(X, f, U) = \sum_{i=1}^{n} L_r(r_i)
$$

(2)

where $r_i = \|\xi_i\| = \sqrt{\xi_i^T \xi_i}$, $\xi_i = \mu_i - \varphi(x_i)\Theta - b$, $\Theta = [\theta^1, \ldots, \theta^l]$, $b = [b^1, \ldots, b^l]^T$, and $\varphi(x)$ is a nonlinear transformation of $x$ to a higher dimensional feature space $\mathcal{R}^H$.

To consider all dimensions into a unique restriction and yield a single support vector for all dimensions, we use the Vapnik ε-insensitive loss based on the L2 norm for $L_r$ loss (Tuia et al. 2011), i.e.,

$$
L_r(r) = \begin{cases} 
0 & r^2 - 2r\varepsilon + \varepsilon^2 \\
(r - \varepsilon)^+ & r \geq \varepsilon 
\end{cases}
$$

(3)

which will create an insensitive zone determined by $\varepsilon$ around the estimate, i.e., the loss of $r$ less than $\varepsilon$ will be ignored. For $\varepsilon = 0$, the MSVR reduces to a kernel ridge regression.
(KRR) for each component, but for nonzero \( \varepsilon \) value, the solution takes into account all outputs to construct each individual regressor. This way, the cross-output relations are exploited, thus leading to possibly more accurate predictions. Furthermore, the MSVR can return a multidimensional and sparse solution.

And \( L_2 \) loss is a regularizer of \( \Theta \) to control the complexity of MSVR:

\[
L_2(\Theta) = \|\Theta\|_F^2. \tag{4}
\]

**Label Enhancement** In this subsection, we will discuss the design of the loss \( L_3, L_4 \) and \( L_5 \). The three loss functions can transform the logical label into the numerical label. And the transformation from logical label to numerical label is named for Label Enhancement by us. For \( L_3 \) loss, to make sure numerical label matrix \( U \) close enough to the original logical label matrix \( Y \), we choose the square of the Frobenius norm of the two matrices' difference. So we define \( L_3 \) loss as:

\[
L_3(U, Y) = \|U - Y\|_F^2. \tag{5}
\]

For the loss \( L_4 \), the topological structure of feature space can be expressed by a full-connected graph \( G = (V, E, W) \), where \( V \) is the vertex set of training examples, i.e., \( V = \{x_i|1 \leq i \leq n\} \), \( E \) is the edge set in which \( e_{ij} \) represents the relationship between \( x_i \) and \( x_j \), and \( W \) is the weight matrix in which each element \( W_{ij} \) represents the weight of the edge \( e_{ij} \). We treat \( W \) as the matrix containing the topological structure information of feature space. Due to the smoothness assumption, the topological structure of feature space can be transferred to the numerical label space by utilizing the local neighborhood information of every instance. To estimate the topological structure of feature space, local neighborhood information of every instance should be used to construct the full-connected graph \( G \). According to Local linear embedding (LLE) \cite{Roweis2000}, each point can be reconstructed by a linear combination of its neighbors. So we can get the approximation of the topological structure of the feature space by minimizing the following target:

\[
\min_W \sum_{i=1}^{n} \left\| x_i - \sum_{j \neq i} W_{ij} x_j \right\|^2 \tag{6}
\]

\[
s.t. \sum_{i=1}^{n} W_{ij} = 1, \quad W_{ij} \geq 0, \forall 1 \leq i \leq n, 1 \leq j \leq n,
\]

where \( W_{ij} = 0 \) if \( x_j \) is not in \( x_i \)'s \( K \)-nearest neighbor points. \( \sum_{j=1}^{n} W_{ij} = 1 \) is constrained because of the translation invariance. Furthermore, for the later optimization convenience, we constrain that each \( W_{ij} \) should be bigger than 0. The approximation of Eq. (6) can be transformed into the following \( n \) least square programming problem:

\[
\min_{W_i} \quad W_i^T G_i W_i \tag{7}
\]

\[
s.t. \quad 1^T W_i = 1, \quad -I W_i \leq 0,
\]

where \( G_i \) is the local Gram matrix at point \( x_i \), with \( G_{ij} = (x_i - x_j)^T(x_i - x_j) \), \( I \) is an identity matrix, \( 1 \) is a vector of all ones and \( 0 \) is a vector of all zeros. Because the feature space and the label space should share similar local topological structure, we can use the same way to reconstruct the numerical label space, then we define the \( L_4 \) loss as:

\[
L_4(U, W, Y) = \|U - W U\|_F^2. \tag{8}
\]

As for \( L_5 \) loss, we choose the square of the Frobenius norm of \( U \) to control the each element in \( U \):

\[
L_5(U) = \|U\|_F^2, \tag{9}
\]

so the loss \( L_3, L_4 \) and \( L_5 \) can complete the task of Label Enhancement, the transformation from original logical label to numerical label.

By substituting Eqs. (2), (3), (4), (5), (8) and (9) into Eq. (1), we can get the final objective function:

\[
\min_{\Theta, b, U} \sum_{i=1}^{n} L_r(r_i) + \alpha \|\Theta\|_F^2 + \beta \|U - Y\|_F^2 + \gamma \|QU\|_F^2 + \delta \|U\|_F^2 \tag{10}
\]

\[
s.t. \quad Q = I - W
\]

\[
r_i = \|\xi_i\| = \sqrt{\xi_i^T \xi_i}
\]

\[
\xi_i = \mu_i - \varphi(x_i) \Theta - b
\]

\[
L_r(r) = \begin{cases} 0 & r < \varepsilon \\ r^2 - 2r\varepsilon + \varepsilon^2 & r \geq \varepsilon. \end{cases}
\]

**The Alternative Solution of the Optimization**

Solve the optimization problem in Eq. (10) by optimizing two of the three variables with the other one fixed. Fix \( U \) to optimize \( \Theta \) and \( b \), and the target function of Eq. (10) can be rewritten as:

\[
L(\Theta, b) = \sum_{i=1}^{n} L_r(r_i) + \alpha \|\Theta\|_F^2
\]

\[
r_i = \|\xi_i\| = \sqrt{\xi_i^T \xi_i}
\]

\[
\xi_i = \mu_i - \varphi(x_i) \Theta - b
\]

\[
L_r(r) = \begin{cases} 0 & r < \varepsilon \\ r^2 - 2r\varepsilon + \varepsilon^2 & r \geq \varepsilon. \end{cases}
\]

Notice that Eq. (11) is a MSVR with the \( L2 \) norm-based Vapnik \( \varepsilon \)-insensitive loss. So \( \Theta \) and \( b \) can be optimized by training a MSVR \cite{Tuia2011}.

When we fix \( \Theta \) and \( b \) to solve \( U \), the optimization target becomes:

\[
L(U) = \sum_{i=1}^{n} L_r(r_i) + \beta \|U - Y\|_F^2 + \gamma \|QU\|_F^2 + \delta \|U\|_F^2
\]

\[
Q = I - W
\]

\[
r_i = \|\xi_i\| = \sqrt{\xi_i^T \xi_i}
\]

\[
\xi_i = \mu_i - \varphi_i
\]

\[
L_r(r) = \begin{cases} 0 & r < \varepsilon \\ r^2 - 2r\varepsilon + \varepsilon^2 & r \geq \varepsilon. \end{cases}
\]
The quadratic approximation is further constructed. Firstly, a MSVR model is trained using the training instance $x$ and $\xi$. Iterative Re-Weighted Least Square (IRWLS) is an algorithm (Nocedal and Wright 2006) to minimize $L(U)$, where $L(U)$ is the numerical label matrix.

**Algorithm 1: The IRWLS Procedure of Calculating $U$**

**Input:** the re-prediction result matrix $P$, the label matrix $Y$ and the edge weight matrix $W$  
**Output:** the numerical label matrix $U$  
1. Initialize $U^{(0)}$ with the full zero matrix and initialize the iterator $k = 0$.  
2. while not convergence do  
3. Calculate $D_{a}^{(k)}$ with $P$ and $U^{(k)}$ via Eq. (16).  
4. Calculate $U^*$ by Eq. (20) with $P$, $Y$, $W$ and $D_{a}^{(k)}$. Define a descending direction for Eq. (12) as $F^{(k)} = U^* - U^{(k)}$.  
5. Obtain the next step solution, i.e., $U^{(k+1)} = U^{(k)} + \eta(k)F^{(k)}$, computing the step size $\eta(k)$ using a backtracking algorithm (Nocedal and Wright 2006).  
6. $k = k + 1$.  
7. Return $U^{(k+1)}$.  

where  
$$p_i = \varphi(x_i)\Theta + b,$$  
and $p_i$ can be regarded as the result of re-prediction which means using the training instance $x_i$ to predict via the trained MSVR model.

To minimize $L(U)$, we use an iterative quasi-Newton method called Iterative Re-Weighted Least Square (IRWLS) (Pérez-Cruz et al. 2002; Tuia et al. 2011). Firstly, $L_r(r_i)$ is approximated by its first order Taylor expansion at the solution of the current $k$-th iteration, denoted by $U^{(k)}$.  

$$L'_r(r_i) = L_r(r_i^{(k)}) + \frac{dL_r(r)}{dr}\bigg|_{r_i^{(k)}}(\xi_i^{(k)})^T r_i^{(k)} - (\xi_i - \xi_i^{(k)}),$$  
(14)

where $\xi_i^{(k)}$ and $r_i^{(k)}$ are calculated from $U^{(k)}$. Then a quadratic approximation is further constructed  

$$L''_r(r_i) = L_r(r_i^{(k)}) + \frac{dL_r(r)}{dr}\bigg|_{r_i^{(k)}}r_i^2 - (r_i^{(k)})^2 2r_i^{(k)} = a_i r_i^2 + \tau,$$  
(15)

and $\tau$ is a constant term that does not depend on $U^{(k)}$. Combining Eqs. (12), (15) and (16) can get:  

$$L''(U) = \sum_{i=1}^{n}a_i r_i^2 + \beta \|U - Y \|^2_F + \gamma \|QU\|^2_F + \delta \|U\|^2_F,$$  
(16)

where $\Xi = [\xi_1, \ldots, \xi_n]^T = U - P$, $P = \{p_1, \ldots, p_n\}^T$ and $[D_{a}^{(k)}]^i = a_i \Delta_i^T$ ($\Delta_i^T$ is the Kronecker's delta function). Eq. (17) can be transformed into a simpler form:  

$$L''(U) = \text{trace}(U^T A U - 2B U),$$  
(17)

$$A = D_{a} + (\beta + \delta)I + \gamma(I - W)^T(I - W),$$  
(18)

$A$ is a positive definite matrix, so minimization of Eq. (18) is a convex optimization problem and we can solve this unconstrained quadratic programming problem by setting the

**Algorithm 2: The MLL-LE Algorithm**

**Input:** the multi-label training set $\{X, Y\}$ and the maximum number of iteration $m$  
**Output:** the numerical label matrix $U$ and the parameters $\Theta$ and $b$ of the MSVR model  
1. Initialize $U^{(0)}$ with the full zero matrix and initialize the iterator $t = 1$.  
2. Construct the weight matrix $W$ by optimizing Eq. (7).  
3. while $t \leq m$ do  
4. **Training the MSVR:** Calculate $\Theta^{(t)}$ and $b^{(t)}$ by training a MSVR with $X$ and $U^{(t-1)}$.  
5. **Re-prediction:** Calculate each element of $P^{(t)}$ by Eq. (13) with $X$, $\Theta^{(t)}$ and $b^{(t)}$.  
6. **Label Enhancement:** Update $U^{(t)}$ via Algorithm 1 with $Y$, $W$ and $P^{(t)}$.  
7. $t = t + 1$.  
8. Return $U^{(t)}$, $\Theta^{(t)}$ and $b^{(t)}$.  

Table 1: Characteristics of the seven multi-label data sets.

| Data set   | $|S|$ | dim($S$) | $L(S)$ | $F(S)$ | $LCard(S)$ | $LDen(S)$ | $DL(S)$ | $PDL(S)$ | Domain     |
|------------|------|----------|--------|--------|------------|-----------|---------|----------|------------|
| CAL500     | 502  | 68       | 174    | numeric | 26,044     | 0.150     | 502     | 1.000    | audio      |
| medical    | 978  | 1,449    | 45     | nominal | 1,245      | 0.028     | 94      | 0.096    | text       |
| llog       | 1,460| 1,004    | 75     | nominal | 1,180      | 0.016     | 286     | 0.196    | text       |
| enron      | 1,702| 1,001    | 53     | nominal | 3,378      | 0.064     | 753     | 0.442    | text       |
| image      | 2,000| 294      | 5      | numeric | 1,236      | 0.247     | 20      | 0.010    | images     |
| yeast      | 2,417| 103      | 14     | numeric | 4,237      | 0.303     | 198     | 0.082    | biology    |
| slashdot   | 3,782| 1,079    | 22     | nominal | 1,181      | 0.054     | 156     | 0.041    | text       |

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then we can have
\[
\frac{\partial L''(U)}{\partial U} = 2AU - 2B^T = 0,
\]
then we can have
\[
U = A^{-1}B^T = (D_n + (\beta + \delta)I + \gamma(I - W)^T(I - W))^{-1}(D_nP + \beta Y).
\]

The direction of Eq. (20) is used as the descending direction for the optimization of Eq. (12), and the solution for the next iteration \(U^{(k+1)}\) is obtained via a line search algorithm along this direction.

The IRWLS procedure of calculating \(U\) is presented in Algorithm 1. The value of \(n(k)\) is computed using a backtracking algorithm [Nocedal and Wright 2006] that initially sets \(n(k) = 1\) and checks \(L(U^{(k+1)}) < L(U^{(k)})\). If the condition is not met, \(n(k)\) is multiplied by a positive constant \(\rho\) lower than 1 (in this paper, \(\rho\) is set to 0.1), and the procedure is repeated until a decrease is achieved in the minimizing functional.

The complete procedure of the proposed MLL-LE approach is presented in Algorithm 2. \(U\) is initialized by the full zero matrix, and then \(U\), \(\Theta\) and \(B\) are updated alternately. After optimization, we can get the MSVR model from the input space to the numerical label space. Given a test instance, the MSVR can predict a vector of numerical label, then the numerical label bigger than zero is considered as relative label and the one smaller than zero is irrelative to the example.

Noted that we are proposing a framework of multi-label
learning, the mapping model can be replaced with other regressors, like KNNR, SOAR, Artificial Neural Network (Haykin 1994), Regression Tree, etc. $L_4$ loss can be replaced with other manifold assumptions like Laplacian Eigenmaps (Belkin and Niyogi 2003), Locality Preserving Projection (He and Niyogi 2004), etc. The other loss functions in Eq. (1) can be replaced with other forms.

**Experiment**

**Experiment Configuration**

**Data Sets** For comprehensive performance evaluation, the seven multi-label data sets in Mulan (Tsoumakas et al. 2011) and Weka are used for our experiment. We use $|S|$, $\dim(S)$, $L(S)$, $F(S)$, $LCard(S)$, $LDen(S)$, $DL(S)$ and $PDL(S)$ to represent its number of examples, number of features, number of class labels, feature type, label cardinality, label density, distinct label set and proportion of distinct label sets respectively. Table 1 summarizes detailed characteristics of the real data sets, which are roughly organized in ascending order of the number of examples $|S|$.

**Comparing Algorithms** In this paper, we choose to compare the performance of MLL-LE against five state-of-the-art algorithms, including two first-order approaches binary relevance (BR) (Boutell et al. 2004) and ML-$k$NN (Zhang and Zhou 2007), one second-order approach calibrated label ranking (Fürnkranz et al. 2008), and two high-order approaches Ensemble of Classifier Chains (ECC) (Read et al. 2011) and Random k-labelsets (RAkEL) (Tsoumakas, Katakis, and Vlahavas 2011). Furthermore, the parameters suggested in the literatures are used for ECC and RAkEL (ECC: ensemble size 30; RAkEL: ensemble size 2i with k=3). The $k$ value of ML-$k$NN is fixed to 10. For MLL-LE, the number of neighbors $K$ is set to 10, which is the same $k$ value as ML-$k$NN. The parameters $\varepsilon$ in MLL-LE is chosen among $\{0.01, 0.1, 1\}$ and the parameters $\alpha, \beta, \gamma$ and $\delta$ in MLL-LE are all chosen among $\{0.001, 0.01, 0.1, 1, 10, 100, 1000\}$. The maximum number of MLL-LE iterations is set to 100. RBF kernel function is used in MLL-LE.

**Evaluation Metrics** Five widely-used evaluation metrics are used in this paper, Hamming loss, One-error, Coverage, Ranking loss and Average precision (Zhang and Zhou 2014). Furthermore, for Average precision, the larger the values the better the performance; While for the other four metrics, the smaller the values the better the performance. These metrics serve as good indicators for comprehensive comparative studies as they evaluate the performance of the learned models from various aspects.

**Experiment Results**

Table 2 shows the detailed experimental results of each comparing algorithm on the seven data sets. On each data set, 50% examples are randomly sampled without replacement to form the training set, and the rest 50% examples are used to form the test set. The sampling process is repeated for ten times and the average predictive performance across ten training/testing trials are recorded. For each evaluation metric, ↓ indicates the smaller the better while ↑ indicates the larger the better. Furthermore, the best performance among the five comparing algorithms is shown in boldface.

From Table 2 we can see, on these seven data sets, MLL-LE ranks 1st in all five metrics except Hamming loss. In Hamming loss metrics, MLL-LE ranks 1st in 71.4% cases. Thus MLL-LE achieves competitive performance against the well-established multi-label learning algorithms across the seven data sets and diverse evaluation metrics, which validate the effectiveness of our framework for multi-label learning.

**Conclusion**

This paper propose the framework of multi-label learning by exploring the numerical labels behind the multi-label instances. Extensive comparative studies clearly validate the advantage of the MLL-LE framework against the state-of-the-art multi-label learning approaches. In the future, we will explore if there exists better ways to improve the performance of our framework for multi-label learning.

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