Hyperbolic and Fibonacci numbers in genetic biomechanics

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Abstract. One-dimensional Fibonacci numbers and their applications are actively used in various fields of mathematics, computer technology and computer science, biology and economics. The article presents the results of the study of additive series of two-dimensional hyperbolic numbers with Fibonacci coordinates. Particular attention is paid to the matrix representation of such hyperbolic Fibonacci numbers. Some applications of new additive sequences in the field of genetic biomechanics are shown, including the laws of morphogenesis of phyllotaxis and the basic psychophysical law of Weber-Fechner. The important role of hyperbolic numbers in the modeling of biological phenomena is noted. The described results lead to new applications of Fibonacci numbers in various fields of science and technology.

1. Introduction

Biomechanical systems are genetically inherited. It is natural to think that all inherited physiological systems must be structurally consistent with the genetic coding system in order to be inherited by the next generation and survive in the course of biological evolution. However, in modern monographs and textbooks on biomechanics, this relationship of biomechanical (muscle, cardiovascular, sensorimotor, etc.) systems with the genetic system is not not considered. To fill this gap, the Laboratory of Biomechanical Systems Research, IMASH RAS, is developing "genetic biomechanics" that takes into account and studies the structural relationships of the genetic system and many interrelated physiological systems that form a single organism based on inherited control algorithms and coordination of its various parts.

In the course of this study, the deep connections of the structure of living organisms - as information entities - with systems, which are mathematically modeled by formalisms of the theory of resonances of oscillatory systems with many degrees of freedom and quantum informatics, are revealed. From a genetic point of view, organisms are informational entities. All organisms have the same fundamentals of molecular genetic coding, using structured DNA and RNA molecules that carry sequences of nitrogenous bases as informational texts. As has long been known, a number of inherited biological phenomena, including the morphogenetic laws of phyllotaxis, are associated with one-dimensional Fibonacci numbers, which also have diverse applications in other fields of knowledge: the creation of Fibonacci computers, network topology for parallel computing, economics, the generation of pseudorandom sequences, etc. [1-11].

In our article, two-dimensional hyperbolic numbers are determined and considered, the coefficients of the basis units of which are Fibonacci numbers and which can be called "hyperbolic Fibonacci numbers". They can significantly enrich the set of applications of Fibonacci numbers, including in
genetic biomechanics. One of the promising areas in genetic biomechanics is the mathematical modeling of inherited genetic and other biological structures from the perspective of multidimensional numerical systems used in mathematics, physics, computer science, etc. Various types of multidimensional numbers - complex numbers, hyperbolic numbers, double numbers, dual numbers, quaternions and other hypercomplex numbers - are used in various fields of modern science. They play the role of a magic tool for the development of theories and calculations in the physics of heat, light, sounds, vibrations, elasticity, gravity, magnetism, electricity, fluid flows, quantum-mechanical phenomena, special relativity, nuclear physics, etc. For example, in physics of the XX century, thousands of works were devoted to the Quaternions of Hamilton (their bibliography is in [12]).

A number of data that we obtained in addition to the previously known ones allows us to consider it particularly interesting to use hyperbolic numbers (also called double numbers, Lorentz numbers, split-complex numbers, perplexes) in conjunction with their algebraic extensions for modeling in the field of genetic biomechanics. This type of multidimensional numbers is used, for example, in the special theory of relativity, where it is associated with Lorentz transformations. Hyperbolic rotations are a special case of this kind of numbers. On the way of using hyperbolic numbers in genetic biomechanics is possible a docking with ideas by V.I. Vernadsky on the non-Euclidean geometry of living matter [13] is possible. In biology, the connection of biostructures with hyperbolic numbers and hyperbolic rotations was previously noted in studies of the morphogenetic laws of phyllotaxis, that is, the helical arrangement of leaf organs of plants and animal body parts in accordance with Fibonacci numbers [14, 15]. The author of these works drew attention to the fact that ontogenetic transformations of phyllotaxis lattices in plants are adequately modeled by hyperbolic rotations. On this basis, he expressed the opinion that the geometry of living bodies has a structural connection with the Minkowski geometry. Our article provides additional data on the relationship of genetic and biological structures with hyperbolic numbers. Moreover, considerable attention is paid to the biological laws of phyllotaxis associated with Fibonacci numbers.

2. Hyperbolic numbers and their matrix representations

In abstract algebra, a hyperbolic number is written in the form $z = x + yj$, where $x$ and $y$ are real coefficients, and $j$ is the imaginary unit that is not equal to ± 1, but whose square is equal to +1 that is $j^2 = +1$ [16]. The set of all hyperbolic numbers forms an algebra over the field of real numbers and is located on the hyperbolic plane. This algebra contains zero divisors. Addition and multiplication of hyperbolic numbers are determined by the expressions (1), (2):

\[(x + yj) + (u + vj) = (x + u) + (y + v)j\]  
\[(x + yj)(u + vj) = (xu + yv) + (xv + yu)j\]  

(1)  
(2)

The hyperbolic number $z = x + yj$ has a matrix representation in the form of a bisymmetric matrix $Z$ in (3), where $E$ is the identity matrix representing the real basis unit, and the matrix $J$ represents the imaginary basis unit. If $x^2 - y^2 = 1$, then the matrix $Z$ defines hyperbolic rotations, known in the special theory of relativity as Lorentz transformations. Hyperbolic rotations are usually expressed by the symmetric matrix (4) through the hyperbolic cosine “cosh” and the hyperbolic sine “sinh”, since $\cosh \alpha^2 - \sinh \alpha^2 = 1$ [17]:

\[Z = \begin{bmatrix} x & y \\ y & x \end{bmatrix} = xE + yJ, E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\]  
\[\begin{bmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{bmatrix}\]  

(3)  
(4)

The hyperbolic number Bisymmetric matrices that represent hyperbolic numbers have real eigenvalues and orthogonal eigenvectors (which distinguishes them from asymmetric matrix representations of complex numbers). It can be recalled that symmetric matrices form the basis of the theory of resonances of oscillatory systems with many degrees of freedom, and are also metric tensors.
from the point of view of Riemannian geometry. Fibonacci numbers $F_n$ form an additive sequence in which each number is the sum of the previous two: $F_n = F_{n-1} + F_{n-2}$ (see Table 1).

### Table 1. Fibonacci series.

| n=1 | n=2 | n=3 | n=4 | n=5 | n=6 | n=7 | n=8 | n=9 | n=10 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| $F_n$ | 1   | 1   | 2   | 3   | 5   | 8   | 13  | 21  | 34   | 55   | ...  |

Fibonacci numbers are closely related with the golden ratio $\phi = (1 + \sqrt{5})/2 = 1.618 \ldots$ The Binet formula (5) expresses the $n$th Fibonacci number in terms of $n$ and the golden ratio and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio with increasing $n$:

$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\phi - (-\phi)^{-1}}$$  \hspace{1cm} (5)

In biology, it has long been known that, for example, in many plant objects the spiral arrangement of their parts forms ordered patterns (shoots of plants and trees, seeds in the heads of sunflowers, flakes of coniferous cones and pineapples, etc.). These patterns are determined by overlapping left and right oriented spiral lines - parastichi. Two parameters are usually indicated to characterize phyllotaxis: the number of left spirals and the number of right spirals that are observed on the surface of phyllotaxis objects. Phyllotaxis of structures with such patterns is described by the relations of neighboring Fibonacci numbers [8, 14, 15, 18]:

$$F_{n+1}/F_n: 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, \ldots$$  \hspace{1cm} (6)

$$(F_{n+1}/F_n) \to (F_{n+2}/F_{n+1}): 2/1 \to 3/2 \to 5/3 \to 8/5 \to 13/8 \to 21/13 \ldots$$  \hspace{1cm} (7)

The sequence (6) is termed the “parastichic sequence” [8, 18]. It seems natural to use 2-dimensional hyperbolic numbers for modeling these 2-parametric patterns in phyllotaxis objects and their ontogenetic transformations. In this approach, proposed by the author, the sequence (6) of phyllotaxis ratios is transformed into additive sequences (8, 9) reflecting linear notation of appropriate numbers and their matrix presentations (we term sequences (8, 9) as parastichic sequences of hyperbolic numbers):

$$F_{n+1} + F_n j: 2 + j, 3 + 2j, 5 + 3j, 8 + 5j, 13 + 8j, 21 + 13j, 34 + 21j, \ldots$$  \hspace{1cm} (8)

$$\left| \begin{array}{c}
F_{n+1} \\
F_n \\
F_{n+1} \\
F_n \\
\end{array} \right| = \left| \begin{array}{c}
\sqrt{5}/2 \ 
1 \ 
3 \ 
5 \ 
8 \ 
13 \ 
21 \ 
34 \ 
\end{array} \right| = \left| \begin{array}{c}
0 \ -1 \\
1 \ 0 \\
5 \ -8 \\
8 \ -13 \\
\end{array} \right| = \left| \begin{array}{c}
\sqrt{5}/2 \ 1/2 \\
3 \ 2 \ 3/2 \\
5 \ 3 \ 2 \ 3/2 \\
8 \ 5 \ 3 \ 2 \ 3/2 \\
13 \ 8 \ 5 \ 3 \ 2 \ 3/2 \\
21 \ 13 \ 8 \ 5 \ 3 \ 2 \ 3/2 \\
34 \ 21 \ 13 \ 8 \ 5 \ 3 \ 2 \ 3/2 \\
\end{array} \right|, \ldots$$  \hspace{1cm} (9)

In this approach, to define a hyperbolic number $u + vj$, which transforms a hyperbolic number $F_{n+1} + F_n j$ into its neighboring hyperbolic number $F_{n+2} + F_{n+1} j$ from the sequence (8), the following equation (10) should be solved:

$$(F_{n+1} + F_n j)(u + vj) = F_{n+2} + F_{n+1} j$$  \hspace{1cm} (10)

The solution to this equation (10) gives the following expressions (11) for components of the desired hyperbolic number $u + vj$:

$$u = \frac{F_{n+1}}{F_n} + \frac{(-1)^{n+1}F_{n-1}}{F_n(F_n^2 - F_{n-1}^2)} \quad v = \frac{(-1)^n}{F_n^2 - F_{n-1}^2}$$  \hspace{1cm} (11)

In the case of such components (11), $u^2 - v^2 \neq 1$ and the appropriate matrix $\left| \begin{array}{cc}
u & -j \\
v & j \\
\end{array} \right|$ does not present a hyperbolic rotation in the sense of expression (4). But this matrix can be rewriting into the form (12) where the matrix of a hyperbolic rotation (in the sense of expression (4)) is multiplied by a coefficient $\sqrt{u^2 - v^2}$:
Now let us describe results of our study of eigenvalues of the symmetric matrices in the parastichic sequence (9). Each of these matrices \( \| F_{n+1} \; F_{n} \; F_{n-1} \| \) has two eigenvalues, which are equal to two Fibonacci numbers again: \( F_{n+2} \) and \( F_{n-1} \). One can noted that these eigenvalues are the sum and the difference of the Fibonacci components of the original hyperbolic number \( F_{n+1} + F_{n} \) since \( F_{n+2} = F_{n+1} + F_{n} \) and \( F_{n-1} = F_{n+1} - F_{n} \). The ratio \( F_{n+2}/F_{n-1} \) of such eigenvalues defines a new sequence (13) of Fibonacci ratios, which tend to \( \phi \) as \( n \) increases:

\[
F_{n+2}/F_{n-1} : 3/1, 5/1, 8/2, 13/3, 21/5, 34/8, 55/13, \ldots
\]

By analogy with expressions (6, 8, 9) such pair of eigenvalues \( F_{n+2} \) and \( F_{n-1} \) can be considered as components of a new hyperbolic number \( F_{n+2} + F_{n-1} \). In this case the sequence of ratios (13) is transformed into additive sequences (14, 15) reflecting linear notation of appropriate hyperbolic numbers and their matrix presentations:

\[
\| F_{n+2} \; F_{n-1} \; F_{n+1} \| : 3/1, 5/1, 8/2, 13/3, 21/5, 34/8, 55/13, \ldots
\]

\[
\| F_{n+2} \; F_{n-1} \; F_{n+1} \| : 3/1, 5/1, 8/2, 13/3, 21/5, 34/8, 55/13, \ldots
\]

Each of symmetric matrices \( \| F_{n+1} \; F_{n} \; F_{n+1} \| \) of the sequence (15) has two eigenvalues, which are again equal to two Fibonacci numbers multiplied by a factor 2 (twice the Fibonacci numbers): \( 2F_{n+1} \) and \( 2F_{n} \). Ratios \( 2F_{n+1}/2F_{n} \) of such eigenvalues form a sequence, which is identical to the initial parastichic sequence (6). Using the Binet’s formula (5), all members of these sequences can be additionally expressed through the golden ratio \( \phi \) in integer powers.

This procedure of analysis of the eigenvalues of new and new sequences of symmetric matrices, representing hyperbolic numbers by analogy with sequences (8, 9, 14, 15), can be repeated as long as desired, obtaining a hierarchy of eigenvalues of the matrices based on Fibonacci numbers multiplied by a factor 2 at corresponding steps of the iterative processes.

The following important point should be emphasized. In contrast to the traditional additive series of one-dimensional Fibonacci numbers, the author introduces an additive series of two-dimensional hyperbolic numbers and an additive series of \((2 \times 2)\)-matrices representing these numbers and defining an additional additive series of eigenvalues of these matrices (8, 9, 14, 15). As far as we know, such Fibonacci series of two-dimensional numbers have not been described in the literature by anyone, and therefore they can be considered new in the extensive subject matter of Fibonacci numbers and their applications (some of author's results of the study of additive series of 4-dimensional hyperbolic Fibonacci numbers will be presented below).

Similar results are obtained by considering the additive series of two-dimensional hyperbolic Lucas numbers and the additive series of their matrix representations, which determine the additive series of eigenvalues of these symmetric matrices (these results are being published in a separate article). Here one can remind that one-dimensional Lucas numbers form the series \( L_{n+2} = L_{n} + L_{n+1} \): 2, 1, 3, 4, 7, 11, 18, ... , which is also known in phyllotaxis laws [8, 9]. A study of additive series of complex numbers, whose components are Fibonacci numbers, and of their ordinary representations by non-symmetric matrices gives also interesting additive series of their eigenvalues but in form of complex numbers.

It should be noted that the study of the eigenvalues of symmetric matrices has special meaning due to the fact that in the theory of oscillations symmetric matrices are matrix representations of oscillatory systems with many degrees of freedom. Moreover, the eigenvalues of such a matrix determine the resonant frequencies of the corresponding oscillatory system. The described results on the properties of inherited phyllotaxis phenomena with their Fibonacci ratios, represented by symmetric matrices and their matrix eigenvalues, are important, in particular, for the concept of multi-resonance genetics, which connects structural features of molecular-genetic systems with resonances of oscillatory systems [19].
3. Hyperbolic numbers and the Weber-Fechner law

It is profitable for an organism, which is a single whole, to have the same typical algorithms at different levels of its functioning for a mutual optimal coordination of its parts. By this reason we study possibilities to simulate different innate phenomena on the general basis of hyperbolic numbers and its algebraic extensions. This Section is devoted to the main psychophysical law by Weber-Fechner and its structural connection with phyllotaxis laws through hyperbolic numbers. The innate Weber-Fechner law states that the intensity of the perception is proportional to the logarithm of stimulus intensity; it is expressed by the equation:

\[ p = k \ln \frac{x}{x_0} = k(\ln x - \ln x_0) \]  \hspace{1cm} (16)

where \( p \) - the intensity of perception, \( x \) – stimulus intensity, \( x_0 \) - threshold stimulus, \( \ln \) – natural logarithm, \( k \) – a weight factor. It is known that different types of inherited sensory perception are subordinated to this law: sight, hearing, smell, touch, taste, etc. Because of this law, the power of sound in physics and engineering technologies is measured on a logarithmic scale in decibels.

One can suppose that the innate Weber-Fechner law is the law especially for nervous system. But it is not so since its meaning is much wider because it holds true in many kinds of lower organisms without a nervous system in them: “this law is applicable to chemo-tropical, helio-tropical and geo-tropical movements of bacteria, fungi and antherozoids of ferns, mosses and phanerogams ... . The Weber-Fechner law, therefore, is not the law of the nervous system and its centers, but the law of protoplasm in general and its ability to respond to stimuli” [20].

Let us show that hyperbolic numbers are related to the Weber-Fechner law, which is based on the natural logarithm (16). Historically the natural logarithm was formerly termed the hyperbolic logarithm, as it corresponds to the area under a hyperbola [21, 22]. History of hyperbolic logarithms is described for example in the book [21]. As known, the natural logarithm can be defined for any positive real number “\( a \)” as the area under the hyperbola \( y = 1/x \) from 1 to \( a \) (figure 1a). It means that two points of the hyperbola with their coordinates \((x, 1/x)\) and \((x_0, 1/x_0)\), where \( x > 1 \) and \( x_0 > 1 \), define values of natural logarithms \( \ln(x) \) and \( \ln(x_0) \). Subtraction \( \ln(x) - \ln(x_0) = \ln(x/x_0) \) expresses the intensity of perception \( p \) in the expression (16) of the Weber–Fechner law (figure 1b).

![Figure 1](image_url)  

**Figure 1.** Natural logarithm as the area under the hyperbola \( y = 1/x \). a) \( \ln(a) \) is equal to the area under the hyperbola from 1 to \( a \). b) \( \ln(x/x_0) \) is equal to the area under the hyperbola from \( x_0 \) to \( x \).

A plane of the hyperbola \( y=1/x \) can be naturally considered as the hyperbolic plane where points \((x, 1/x)\) and \((x_0, 1/x_0)\) on this hyperbola are defined as hyperbolic numbers \( x + \frac{1}{x}i \) and \( x_0 + \frac{1}{x_0}i \). From the standpoint of the expression (16) of the Weber-Fechner law, any transformation of a stimulus intensity \( x \) \((x > x_0)\) into a new stimulus intensity \( x_2 \) \((x_2 > x_0)\) corresponds to the case that the hyperbolic number \( x + \frac{1}{x_2}i \) is transformed into a new hyperbolic number \( x_2 + \frac{1}{x_2}i \) on the same hyperbola \( y=1/x \) by means of multiplication of the first hyperbolic number with another hyperbolic number \( u + vj \) that is \( (x + \frac{1}{x_2}i)(u + vj) = x_2 + \frac{1}{x_2}j \) where \( u = \frac{x_2^2 - x}{x_2(x^4 - 1)} \) and \( v = \frac{x(x^2 - 1)}{x_2(x^4 - 1)} \).
This analysis gives evidences that our sensory perception obeys the same structural principles as morphogenesis with its phyllotaxis laws and that these principles can be effectively modelling on the basis of hyperbolic numbers.

4. Some concluding remarks

The development of modern mathematical natural sciences is based on the use of certain mathematical tools. Mathematical tools of theoretical research can be compared with glasses for a visually impaired person: adequate glasses provide a person with a clear and beautiful picture of reality, which he had previously seen as blurred and hidden by fog. This article attracts attention of researches to interesting features of additive sequences of 2^n-dimensional hyperbolic numbers, having Fibonacci coordinates. These multidimensional numbers can be used for modeling some biological structures including phyllotaxis phenomena.

In contrast to the traditional study of sequences of Fibonacci numbers in the framework of one-dimensional real numbers, the study of hyperbolic numbers with Fibonacci coordinates proposed by authors additionally introduces important mathematical objects: eigenvalues and eigenvectors of matrix representations of the named multidimensional hyperbolic numbers.

The matrix form of presentation of hyperbolic numbers deserves special attention by the following reasons:

1) This presentation form is based on symmetric matrices, which are closely related with the theory of resonances of oscillatory systems, having many degrees of freedom, and also with Punnett squares from Mendelian genetics of inheritance of traits in living organisms [19, 23];
2) These symmetric matrices can be interpreted as metric tensors, which are main invariants in Riemannian geometry and which can be used in the theory of morpho-resonance morphogenesis [9, 10, 19];
3) These symmetric matrices are related with hyperbolic rotations \(|cosh \alpha \ sinh \alpha; \ cosh \alpha \ sinh \alpha|\), which are particular cases of hyperbolic numbers and are connected with the theory of biological phyllotaxis laws, with problems of locomotion control [24] and also with Lorenz transformations in the special theory of relativity;
4) These symmetric matrices are related with the theory of solitons of sine-Gordon equation. Such known solitons are the only relativistic type of solitons; they were put forward for the role of the fundamental type of solitons of living matter in the book [25].

The proposed approach connecting Fibonacci numbers with multi-dimensional hyperbolic numbers. It expands modern possibilities of constructing new applications and theories based on Fibonacci numbers and matrix representations of multidimensional hyperbolic numbers. One of the interesting directions is the application of hyperbolic numbers with Fibonacci coordinates in generalized crystallography, genetic biomechanics and biochemical aesthetics [9, 26-33]

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