Spectrum Results with Kogut-Susskind Quarks

Doug Toussaint

aDepartment of Physics, University of Arizona, Tucson, AZ 85721, USA

I summarize recent developments in spectrum calculations using Kogut-Susskind quarks. Theoretical developments include one-loop computations with improved actions. I present some recent simulation results, mostly from a MILC collaboration project using three flavors. Effects of dynamical quarks are clearly seen in the isovector $0^{++}$ meson propagator and in the mass ratio $\frac{J}{\rho}$.

The Kogut-Susskind formulation of lattice fermions, often called “staggered fermions”, has long been a popular formulation for QCD simulations including dynamical quarks. The basic reason for this is the single remaining exact chiral symmetry, which is sufficient to protect the quark mass from additive renormalization. From a practical viewpoint, an important consequence is a lower limit on the eigenvalues of $M^\dagger M$, which insures that the simulation will be well behaved and not encounter “exceptional configurations.” Also, but less importantly, there are simply fewer fermionic degrees of freedom to handle. The combined effect of these advantages is that we can use our limited computer power to push to smaller dynamical quark masses, or larger physical sizes, or perhaps to larger statistics, by using Kogut-Susskind quarks. The price that we pay is that it can be a painful exercise to figure out the lattice implementation of your desired operator and, more importantly, flavor symmetry is broken by effects of order $a^2$.

Although they are of order $a^2$, in practice the effects of flavor symmetry breaking are unpleasantly large at accessible lattice spacings. The situation can be greatly improved by the use of an improved action which suppresses the coupling of the quarks to high momentum gluons, as well as fixing up the quarks' dispersion relation. See Ref. [1] for some relevant references. The most important point is that exchange of a gluon with momentum near $\pi/a$ can scatter a low momentum quark from one corner of the Brillouin zone into another, resulting in a mixing of different flavors. “Fattening” the links, by averaging paths in the parallel transport, effectively puts a form factor into the quark-gluon vertex which suppresses such exchanges.

At this conference we have seen the first results of one-loop calculations done with improved Kogut-Susskind actions. In Ref. [2] the quark mass renormalization is calculated, and in Ref. [3] renormalization constants for several operators...
Figure 2. The static quark potential with $a \approx 0.13$ fm. The octagons are the quenched potential and diamonds the three flavor potential at $m_s$. The lines are fits to “Coulomb plus linear plus constant”, and the rulers show the lattice spacings and units of 0.1 fm.

are shown. The results are encouraging, with none of the coefficients coming out surprisingly large. In contrast, the conventional Kogut-Susskind action often has large one-loop corrections. This can again be understood in terms of a form factor suppressing coupling to high momentum gluons[2,3] so that a diagram like Fig. 2b of Ref. [3] will not get large contributions when the quark is in another corner of the Brillouin zone. In other words, we suppress the unwanted doublers in the loop diagrams.

Figure 1 shows the result of one scaling test of the improved Kogut-Susskind action. In this figure we plot the $\rho$ mass in units of the static quark potential as a function of lattice spacing. For the length scale we use $r_1$, the distance where $r_1^2 F(r_1) = 1.0$. All these points are from quenched simulations, and they are all interpolated to the quark mass where $m_\pi r_1 = 0.807$, or

$m_\pi \approx 460$ MeV.

Next one is motivated to ask whether it is practical to further improve the Kogut-Susskind action. At this conference DiPierro and Mackenzie reported on experiments with empirically tuning the coefficients of the paths in the action[9]. The gains were limited, which they interpret as evidence of the need for four quark operators. In Ref. [8] a one-loop improved action was presented, and methods for handling these four quark operators by introducing auxiliary bosonic fields were sketched. Taking a different approach, A. Hasenfratz and Knechtli combined fattening of the links with a projection back onto unitary SU(3) matri-
Figure 4. Squared pion mass divided by quark mass. The octagons are quenched $a \approx 0.13$ fm results, diamonds two flavor, and squares three flavor. The bursts are preliminary quenched and three-flavor results at $a \approx 0.09$ fm.

Figure 5. Quenched Goldstone pion masses from Kim and Ohta[17] and MILC[12]. The axes have been arbitrarily rescaled to match the values at the strange quark mass, which is approximately the low point on the curves.

ces in the “HYP”, for “Hypercubic blocking” action[10]. This action produces smaller mass splittings among the pions than the $a^2$ tadpole action. These authors also describe an algorithm for dynamical simulation of this action, which cannot be expressed in a simple way as a sum over paths.

In another theoretical development presented at this conference, Levkova and Manke have worked out an action for unimproved dynamical Kogut-Susskind quarks on an anisotropic lattice[11]. While their primary motivation is high temperature QCD, this approach could be useful for spectroscopy of glueballs, hybrids and excited states.

The MILC collaboration has used the $a^2$ tadpole improved action for a set of hadron spectrum calculations[12]. These calculations used three flavors of dynamical quarks, as well as a quenched run and one two-flavor run, on lattices tuned to match the lattice spacing at about 0.13 fermi. For masses larger than the strange quark mass $m_s$ three degenerate flavors were used, while for smaller masses one quark mass was held fixed at $m_s$. Quark masses down to $m_q = 0.14 m_s$, or $m_x/m_\rho = 0.35$, were used. The lattice size was $20^3 \times 64$, or $L \approx 2.6$ fm. In addition, some preliminary results at a finer lattice spacing of $a \approx 0.09$ fm are available.

The big advantage of using lattices matched in lattice spacing and physical size for different numbers and masses of quarks is that the effects of dynamical quarks can be convincingly exposed. The simplest quantity that shows this is the static quark potential. Figure 2 shows the potential in quenched QCD and in three-flavor QCD with all the quarks at the strange quark mass. The slopes of these two potentials agree at the point chosen to define the length scale, but their overall shape is different. Notice that we do not see, and do not expect to see, string breaking in the distance range shown here. To quantify the change in shape we look at dimensionless quantities such as $r_0\sqrt{\sigma}$ or $r_0/r_1$. Figure 3 shows $r_0\sqrt{\sigma}$ for the three flavor runs. It also contains two flavor results with...
Wilson quarks from CPPACS and SESAM, and
a two-flavor improved Kogut-Susskind point from
MILC.

One aspect of lattice spectroscopy where sea
quarks are expected to have important effects is
in the chiral behavior of hadron masses. In the
case of the Goldstone pion, at lowest order the
squared pion mass is proportional to the quark
mass, so corrections to this behavior can be
displaying by plotting the squared pion mass divided
by the quark mass. Figure 4 is such a plot for
the MILC runs in both full and quenched QCD.
The increase at the right side of the graph is understood as the transition from chiral behavior to heavy quark
behavior, while the much sharper upturn at low
quark mass is the expected chiral logarithm. Al-
though the quenched and three flavor curves look
similar, the theoretical expectation is that the three flavor line has a finite limit as $m_q \to 0$, while the quenched line diverges logarithmically. The
quenched pion masses in Fig. 4 are similar to ear-
lier quenched results of Kim and Ohta using the
conventional Kogut-Susskind action at a lattice
spacing of about 0.46 fm. A crude comparison is in Fig. 4, where I have arbitrarily rescaled both the vertical and horizontal axes to match the results at the strange quark mass.

While it is tempting to simply fit Fig. 4 to the
chiral behavior predicted in Ref. [17], it is prob-
ably necessary to take account of the remaining
flavor symmetry violations. Claude Bernard has been
fitting this data both directly to the con-
tinuum form, and to a form that takes into ac-
count flavor symmetry violations using the
Lagrangian of Lee and Sharpe with empirically
determined non-Goldstone pion masses in each
loop, to calculate corrections to the pion mass. In
the quenched case, where the coefficient of the
chiral logarithm can be a parameter, reasonable
fits can be obtained both with and without the
flavor symmetry breaking corrections. However,
the coefficient of the logarithm comes out about
0.06 with the simple fitting, and about 0.14 when
the flavor symmetry breaking is included. When
the three flavor data is fit, with the coefficients of
the logarithms determined by the chiral theory,
good fits can only be obtained when the flavor
symmetry breaking is included.

An application of these chiral corrections to
the pion mass was presented at this conference
by the Ohio State group [21]. Here a particu-
lar combination of chiral lagrangian parameters
was computed and found to be inconsistent with
$m_{up} = 0$. The Boulder “HYP” action was
used for the valence quarks in this project. In
view of Bernard’s observations on the effects of
flavor symmetry breaking, it is appropriate that
they used a valence quark action which makes
these effects as small as possible.

The MILC spectrum calculations on matched
quenched and three flavor lattices found some
interesting differences between the quenched and
full QCD spectra (and at least one interesting
lack of differences). The most striking difference
is in the coupling of hadrons to two particle in-
termediate states, which is presumably present
in full QCD calculations but is represented only
by “hairpin” diagrams in quenched calculations.
Figure 5 shows masses obtained for the isovector

Figure 6. Masses for $0^{++}$ propagators
in quenched QCD (octagons) and full QCD
(squares). The straight line is a linear fit to the
large mass points, intended to represent the mass
of a $q\bar{q}$ state, and the curved line is the mass of a
two particle $\pi + \eta$ state.
Figure 7. “J” with improved Kogut-Susskind quarks. The octagons are quenched values, squares three flavor and the diamond a two flavor point. The burst is the real world value, and the cross the UKQCD quenched value[22]. The horizontal scale parameterizes the quark mass, with lighter quarks to the right.

The nucleon to rho mass ratio is a much studied quantity where lattice simulations typically disagree with the real world number. Many groups have looked at this, and it has become clear that this quantity is especially sensitive to effects of the lattice spacing. Thus it is interesting to use the matched lattices to look for effects of sea quarks. Figure 8 shows this ratio in the MILC calculations, with no discernable difference between the quenched and full QCD curves. This is in contrast to UKQCD calculations on matched lattices with two flavors of Wilson quarks, where the two flavor numbers are larger than the quenched (See Fig. 9 of Ref. [16]). The two plusses in Fig. 8 are quenched points at \( a \approx 0.09 \) fm. Preliminary three-flavor results at \( a \approx 0.20 \) and 0.09 fm show a similar trend. Although this lattice spacing dependence is much less than seen with the conventional Kogut-Susskind action, it is clear that a careful continuum extrapolation will still be required for this quantity.

\[ J = m_K \cdot \frac{\partial m_V}{\partial m_{PS}} \approx \frac{m_K \cdot (m_\phi - m_\rho)}{2 \cdot (m_K^2 - m_\pi^2)} \]
Acknowledgements

I am grateful to the organizers of Lattice-01 for the opportunity to present this talk. I thank Claude Bernard, Joachim Hein, Takashi Kaneko, Francesco Knechtli, Kostas Orginos, and Howard Trottier for providing results used in this talk. This work was supported by the U.S. DOE.

REFERENCES

1. S. Naik, Nucl. Phys. B316, 238 (1989); C. Bernard et al. (MILC), Phys. Rev. D 58, (1998) 014503; C. Bernard et al. (MILC), Phys. Rev. D 55, 1133 (1997); G.P. Lepage, Nucl. Phys. (Proc. Suppl.) 60A 267 (1998); J.F. Lagae and D.K. Sinclair, Phys. Rev. D 59, (1999) 014511; Nucl. Phys. (Proc. Suppl.) 63, 892; K. Orginos and D. Toussaint, Phys. Rev. D 59:014501 (1999); Nucl. Phys. B (Proc. Suppl.) 73, 909 (1999); K. Orginos, R.L. Sugar and D. Toussaint, Phys. Rev. D 60 (1999) 054503, Nucl. Phys. B (Proc. Suppl.) 83, 878 (2000); Y.-B. Luo, Phys. Rev. D 57, 265 (1998); G.P. Lepage, Phys. Rev. D 59, (1999) 074502; C. Bernard et al. (MILC), Phys. Rev. D 61, 111502 (2000); U.M. Heller, F. Karsch and B. Sturm, Phys. Rev. D 60, (1999) 114502; F. Karsch, Nucl. Phys. B (Proc. Suppl.) 60A, 169 (1998); B. Beinlich, A. Bicker, F. Karsch, E. Laermann, A. Peikert, Nucl. Phys. (Proc. Suppl.) 63 (1998) 895; O. Kaczmarek, F. Karsch, E. Laermann, Nucl. Phys. (Proc. Suppl.) 73 (1999) 441; F. Karsch, E. Laermann, A. Peikert, Ch. Schmidt, S. Stickan, Nucl. Phys. (Proc. Suppl.) 94 (2001) 411.
2. M. Golterman, Nucl. Phys. (Proc. Suppl.) 73 (1999) 906.
3. J. Hein, Q. Mason, G.P. Lepage and H. Trottier, these proceedings.
4. P. Mackenzie, P. Lepage and H. Trottier, these proceedings; M. Nobes, these proceedings.
5. C. Bernard et al., Phys. Rev. D 61 (2000) 111502; Phys. Rev. D 64 (2001) 054506.
6. C. Bernard et al. Phys. Rev. Lett. 81, 3087 (1998).
7. Sara Collins, Robert G. Edwards, Urs M. Heller and John Sloan, Nucl. Phys. (Proc. Suppl.) 53, 877 (1997); private communication; K.C. Bowler et al., Phys. Rev. D 62 (2000) 054506.
8. T. Blum et al., hep-lat/0007038. Kostas Orginos, private communication.
9. M. DiPierro and P. Mackenzie, these proceedings.
10. A. Hasenfratz and F. Knechtli, Phys. Rev. D 63 (2001) 114502; Phys. Rev. D 64 (2001) 034504, hep-lat/0106014. F. Knechtli, these proceedings.
11. L. Levkova and T. Manke, these proceedings.
12. C. Bernard et al., Phys. Rev. D 64 (2001) 054506.
13. C. Bernard et al., Phys. Rev. D 62, 034503, 2000.
14. A. Ali Khan et al., hep-lat/0105015. T. Kaneko, private communication.
15. G.S. Bali et al., Phys. Rev. D 62 (2000) 054503.
16. C.R. Allton et al., hep-lat/0107021.
17. S. Kim and S. Ohta, Nucl. Phys. (Proc. Suppl.) 47 (1996) 350; Nucl. Phys. (Proc. Suppl.) 63 (1998) 185; Nucl. Phys. (Proc. Suppl.) 73 (1999) 195; Phys. Rev. D 61 (2000) 074506.
18. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.
19. W. Lee and S. Sharpe, Phys. Rev. D 60, 114503 (1999).
20. C. Bernard, in preparation.
21. G. Kilcup, G. Fleming and D. Nelson, these proceedings.
22. P. Lacock and C. Michael, Phys. Rev. D 52, 5213 (1995).
23. JLQCD collaboration: S. Aoki et al., Nucl. Phys. (Proc. Suppl.) 94 (2001) 233; also these proceedings.