The energy dependence of the hard exclusive diffractive processes in pQCD as the function of momentum transfer.

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Abstract

We predict the dependence on energy of photo(electro) production processes: $\gamma (\gamma^*) + p \rightarrow V + X$ with large rapidity gap at small $x$ and large momentum $-t$ transferred to $V$ in pQCD. Here $V$ is a heavy quarkonium ($J/\psi$, $\Upsilon$) or longitudinally polarized light vector meson (in the electroproduction processes), etc. In the kinematics of HERA we calculate the dependence on energy of cross sections of these processes as the function of momentum transfer $t$, photon virtuality $Q^2$ and/or quarkonium mass. In the kinematical region $Q_0^2 \leq -t \ll Q^2 + M_V^2$ the nontrivial energy dependence of the cross section for the vector meson production due to the photon scattering off a parton follows within QCD from the summing of the double logarithmic terms. In the second regime $-t \geq Q^2 + M_V^2$ within DGLAP approximation in all orders of perturbation theory the $q\bar{q}$ parton elastic cross section is energy independent. We show that the correct account of the double logarithmic terms and of the gluon radiation including kinematical constraints removes the disagreement between pQCD calculations and recent HERA experimental data. The explicit formula for the dependence of the differential cross section $d^2\sigma/dtdx_J$ of these processes on $s_{\gamma^*N}$ is obtained. We show that perturbative Pomeron type behavior may reveal itself only at energies significantly larger than those available at HERA. In addition we evaluate the energy dependence of DVCS processes.
I. INTRODUCTION.

The hard inelastic exclusive photo(electro) production processes: $\gamma^* p \rightarrow V + \text{rapidity gap} + X$ with a large rapidity gap and proton dissociation into hadrons draw a lot of attention recently (V is vector $\rho$– meson, charmonium $J/\psi$, or bottomium). (Such processes can be dubbed hard inelastic diffractive (HID) processes, in order to distinguish them from the diffractive processes, when the final particle remain a proton: $X = p$). These processes with $V = J/\psi$ were extensively studied experimentally at HERA [1–3] due to a reasonably large cross section and a clean final state - production of an isolated charmonium state with large transverse momentum which is separated from the proton remnants by a large rapidity gap. Theoretically this process is advantageous, since the large charmonium mass $m_V^2$ makes it possible to evaluate cross section of the process in perturbative QCD, for different values of transverse momentum transfer $-t$ and virtuality of the photon $Q^2$ [4–6]. Moreover, the QCD factorization theorem has been proved [7, 8] for diffractive photo (electro) production of vector mesons at least for the case of the longitudinal photon putting the calculations of diffractive cross sections on the solid theoretical basis. Processes where proton remains intact are strongly suppressed at large $t$ by the nucleon form factor and will be considered briefly only.

Moreover, the charmonium photoproduction is to be probed soon at ultraperipheral collisions at LHC [9] for significantly larger energies. The CMS and ATLAS detectors are well suited for such an investigation since they cover large rapidity intervals. The ALICE detector may be capable of studying this process in a certain rapidity range as well.

Phenomenological analysis of the H1 and ZEUS data on diffractive charmonium production [1, 2], carried out in ref. [10] within framework of the QCD factorization theorem has demonstrated that the experimental data can be described if the energy dependence of two body pQCD scattering amplitude would be significantly slower at large $-t$, than at $t = 0$. Recently, a new experimental data on the photoproduction process $\gamma p \rightarrow J/\psi + \text{rapidity gap} + X$ was reported [3]. This data also indicate a rather dramatic slowing down of the dependence of the cross section of this process on the energy with increase of $-t$. This conclusion is consistent within the experimental uncertainties with the phenomenological analysis of ref. [10]. The observed behavior differs [3] from the predictions which assumed that the energy dependence of the amplitudes describing two body processes $f(s, M_X^2, t, Q^2)$ can be evaluated in terms of the BFKL approximation with the coupling constant independent of $t$ [5, 6], or with $\alpha_{\text{eff}} = \alpha_s(t)$ [11].

The novel QCD effect discovered in this paper is that the pattern of energy dependence of cross section of the HID on $t$, observed at HERA, follows from the DGLAP approximation of pQCD. The amplitude of the hard two body collisions has a rather simple form in a wide interval of high energies where the double logarithmic (DL) approximation is legitimate:

$$f = f_{DL}(x_1, x_2, M_X^2, t, Q^2, M_V^2)F(x, M_X^2, t). \quad (1.1)$$

Within the DGLAP approximation [12–14] the function $f_{DL}$ is obtained by summing large $\alpha_s(N_c/2\pi) \ln(x_1/x) \ln(Q^2 + M_X^2)/(Q_0^2 - t)$ terms, that arise in the integration over parton transverse momenta in the domain where $\alpha_s \ll 1$. Here $x_1 = -t/M_X^2$ is the fraction of the proton momentum carried by the parton involved in the large $t$ elastic scattering. Variables $x_i$ are the fractions of the proton momentum carried by gluons exchanged in $t$ channel which are attached to a parton of the proton (For the explicit definition of $x_1, x_2$ see section 2.). Since at large $t$ transverse momenta of exchanged gluons are large, $x_i$ are not vastly different.
Note that it has been demonstrated in ref. [15], see also refs. [13, 16], that increasing with energy DL terms provide a good description of the structure functions of proton measured at small $x$ at HERA. This observation suggests that at HERA energies $F = 1$. (We normalise $f_{DL}$ and $F$ in such a way that $F = constant = 1$ at low energies.)

The function $F = 1$ within the LO DGLAP approximation to pQCD, but it is a function of energy within the BFKL and resummation models [17–19]. This is because these approaches take into account the contributions that increase with the energy but do not contain double logarithmic (DL) terms. We estimate energy dependence of $F$ in the paper. The effect is small at HERA energies but may become noticeable at significantly higher energies.

In the kinematic region $Q_0^2 < -t < Q^2 + M_V^2$, the energy dependence of hard amplitudes is determined by $f_{DL}$. The equation (1.1) differs in this kinematic range from the Pomeron exchange expression $(s/s_0)^{\alpha_P(t) - 1}$ often used to describe the data. In particular, DL approximation predicts strong dependence of phenomenological $\alpha_P(t)$ on $Q^2$ and on incident energy (i.e. non universality, dependence on the external conditions). This non universality of $\alpha_P$ specific for DGLAP approximation has been observed at HERA at $t = 0$ [20]:

$$F_2(x, Q^2) \sim \exp(r(Q^2) \log(1/x)), r(Q^2) \sim 0.05 \log(Q^2/\Lambda^2).$$

(1.2)

In the kinematical region $-t \geq Q^2 + M_V^2$, $f_{DL}$ is energy independent. Hence the energy dependence of the cross section is determined by the function $F$ which does not contain large logarithms from the integration over parton transverse momenta and which is equal to one at HERA energies. Thus pQCD predicts a sharp decrease with $-t$ of the rate of the rise with energy of the photo(electro) production cross sections as compared to forward scattering at HERA energies.

The estimates of the kinematic range dominated by double logarithms [18, 21] show that the universal Pomeron behavior may be valid only for the energies well beyond the kinematical region: $\ln(x_f/x) \sim (2 \div 3)\log(Q^2/(-t + Q_0^2))$ occupied by double logarithms, i.e. when the DL terms disappear. At $Q^2 + M_P^2 \geq 10 GeV^2$ this condition corresponds to the kinematics far beyond the kinematical range of HERA. The explicit analysis of the phase-space constraints on the multi Regge kinematics due to the energy-momentum conservation shows that even at ultraperipheral processes at the LHC (where $x$ down to $10^{-6}$ can be reached) these constraints limit the possibility of the onset of the Pomeron behavior to the kinematic range $-t \geq Q^2 + M_P^2$. It will be very interesting to look for onset of a Pomeron behavior in pQCD in ultraperipheral collisions in this particular kinematic region.

Let us note that the dominance of the DL terms at small $x$ in a wide kinematical range is a common feature of DGLAP, one of saddle points in the improved BFKL, and the resummation models (see the discussion in refs. [18, 19, 22]). In the improved BFKL approach [23] double logarithmic terms are accounted properly in the same way as in the resummation models. Thus the energy dependence given by $f_{DL}$ should disappear with increase of $-t$ in all approaches as long as the contribution of the BFKL saddle point can be neglected. In other words, our results will remain quantitatively correct also in these approaches if one treats correctly the collinear terms and includes the energy momentum conservation constraints. The difference may reveal itself only at the energies far beyond HERA. On the contrary the contribution of the BFKL saddle point [17] in the inverse Mellin transformation has no DL terms and therefore it does not lead to a peculiar dependence on $t$ of the rate of increase of the cross section with energy discussed in the paper.

We solve the DGLAP evolution equations in the form of the DL approximation in a large interval of $t$. It is known that the DL approximation gives quantitative description of the
HERA data on the structure functions of a proton \[15\]. Technically, the main effect related to the increase of \(-t\) is the following. In the case of \(t = 0\) the strong energy dependence of QCD cross section arises because the energy dependent terms \(\log(x_0/x)\) are enhanced by the factor \(\log(Q^2/\Lambda^2)/\log(Q_0^2/\Lambda^2)\) where \(Q_0^2\) is an initial virtuality. This factor originates from the integration over transverse momenta in the range \(Q_0^2 \ll k_t^2 \ll Q^2\). For \(-t \gg Q_0^2\) the effective range of integration changes to \(-t \ll k_t^2 \ll Q^2 + M_V^2\), leading to the decrease of the rate of the energy increase with the increase of \(-t\). At \(-t \sim Q^2 + M_V^2\) the DL terms disappear all together. Then the energy dependence may arise due to either nonperturbative initial condition for QCD evolution or the terms that do not contain integration over large transverse momenta.

Since the intermediate state invariant mass of the \(c\bar{c}\) system at large \(-t\) is significantly larger then the masses of charmonium states we can approximate \(f\) at \(t \neq 0\) with unequal \(x_i\) by the nonforward parton distribution with \(x = (x_1 + x_2)/2\). It was shown that at small \(x\) and large scale controlling hardness of the process this approximation does not introduce significant uncertainties \[7, 24, 25\].

The nonforward parton distributions for diffractive Z boson production were studied also in ref. \[26\] in the kinematical region: \(\log(Q^2/Q_0^2) > \log(x_0/x)\) which differs from the kinematical region of HID \((\log(Q^2/Q_0^2) \ll \log(x_0/x))\) considered in this paper. Cross section of this process is strongly suppressed at large \(t\) as compared to that for HID by the square of nucleon form factor. Moreover impulse approximation=QCD factorization should be modified at large \(t\) where two gluon scattering off two different partons becomes preferable. Besides, the dependence on the running coupling constant derived in our paper within DGLAP approximation, differs from the one suggested in ref. \[24\].

The onset of the black disc limit for the processes with large rapidity gap may occur in the kinematics where \(x_J = -t/M_X^2\) would be sufficiently small, i.e. in the kinematics where the gap between \(V\) and the system \(X\) is relatively small. The distinctive signature of this regime is a significantly slower decrease of cross section with \(-t\) as compared to pQCD regime. Note also that in the black regime inelastic diffraction is suppressed. In the discussed process this will lead to a further slowing down of the increase of the cross section with energy at fixed \(x_J, t\) or perhaps even to a decrease of the cross section with energy. This effect is amplified with increase of \(x_J\) since the transverse distribution of gluons becomes more localized in the transverse plane.

The paper is organized in the following way. In Sec. II we discuss the kinematics of the HID process. In Sec. III we consider the DGLAP evolution equations for the nonforward GPD and solve them in the double logarithmic approximation. In Sec. IV we evaluate the energy dependence of the amplitudes of hard diffractive processes for the kinematics achieved at HERA. In Sec. V we discuss briefly dependence on energy of the cross section of DVCS process as a function of \(t\). In the conclusion we discuss the directions for the future progress.

II. KINEMATICS OF THE DIFFRACTIVE PRODUCTION AT HIGH ENERGIES.

The theoretical framework for the calculation of inelastic diffractive processes in the high energy processes has been developed in the seventies and it is the so called triple-Pomeron limit \[27\]. The original approach used independence of Pomeron trajectory on external conditions - the property of the soft QCD. It has been understood later \[28\] how these ideas
can be adjusted to evaluate the cross sections of the hard processes with large rapidity gap directly in pQCD where the QCD factorization substitutes the Pomeron pole factorization. The analogue of the triple "Pomeron" vertex is calculable within the pQCD (see Fig. 1 and discussion below). The natural variables in these processes are the invariant mass of hadronic states produced in the proton dissociation $M_X$, the square of the transverse momentum transfer $-t = -(p_\gamma - p_V)^2$ and the square of the invariant energy for $q\bar{q}$ parton $j$ elastic scattering:

$$s' \equiv s_{q\bar{q}+\text{parton } j'} = x_J s - Q^2,$$

(2.1)

$s = W^2_{\gamma p}$ is an invariant energy squared of a full proton-photon system. These quantities are connected by the kinematic relation

$$x_J = -t / (M_X^2 - m^2_p - t),$$

(2.2)

The rapidity gap for sufficiently large momentum transfer $-t$ and invariant energy $s$ is given by

$$\delta y = \ln \frac{s}{\sqrt{(M^2_V - t)(M_X^2 - t)}}.$$  

(2.3)

Finally, there exists a kinematical boundary on $-t$. For the forward scattering in the kinematics where $M_X^2 \gg Q^2$:

$$-t_{\text{min}} = (M_X^2 - m^2_p)(M_X^2 + Q^2)/2s.$$  

(2.4)

This boundary is however irrelevant from the practical point of view, since in the kinematics characteristic for the processes with large gap in rapidity $M_X^2/s \ll 1$. Therefore in the essential kinematical domain investigated in this paper: $|t| \gg |t|_{\text{min}}$ and corrections due to $t_{\text{min}}$ can be neglected.

Note that though in principle $M_X$ can be measured using energy-momentum conservation in terms of the momentum carried by vector meson, in practice it can be determined from the measurement of the rapidity interval occupied by system $X$.

Cross section of the HID processes $\frac{d\sigma}{dt dx_J}$ is calculable at large $-t$ within pQCD as the consequence of the QCD factorization theorem cf. [4, 5, 10]:

$$\frac{d\sigma}{dt dx_J} = \frac{d\sigma_{\gamma+p \rightarrow V+j}}{dt} \left( (81/16) G(x_J, t) + \sum_i (q_i(x_J, t) + \bar{q}_i(x_J, t)) \right).$$

(2.5)

The cross section is the product of two factors, both of which dependent on $W_{\gamma p}$: $G(x_J, t)$ and the cross section of photon scattering on a parton $j$ given by a nonforward amplitude $f$ (see Fig. 1) (Here $\frac{d\sigma_{\gamma+p \rightarrow V+j}}{dt}$ is the cross section of scattering off a parton $j$ ). In order to evaluate the energy dependence of the cross section one needs to take into account the energy dependence of both factors in eq. 2.5.

The first factor in eq. 2.5 is

$$\frac{d\sigma}{dt} = \frac{|f|^2}{16\pi}.$$  

(2.6)

Here the amplitude $s' f$ is a hard amplitude of a photo/electro production of a system $V$ when a virtual photon scatters off a parton with 4-momentum $x_J p$. The amplitude $f$ is a convolution of impact factor describing transition $\gamma^* \rightarrow V$ with the generalized parton
distribution $D$ where the initial condition for the QCD evolution is the amplitude of the scattering off a single parton. This amplitude depends on four parameters $-t$, $Q^2, M^2$ and the effective energy of parton-photon system (eq. 2.1). In addition $D$ depends on two arguments $x_i$ rather than on $x_J$:

$$x_1 = (M^2 + Q^2)/(s' + Q^2); x_2 = (M^2 - M^2_V)/(s' + Q^2). \quad (2.7)$$

Here $M^2$ is the invariant mass of intermediate state in the impact factor. In the charmonium production $M^2$ is approximately the invariant mass of $c\bar{c}$ pair:

$$M^2 = \frac{k_t^2 + m_c^2}{z(1-z)}, \quad (2.8)$$

and $z$ is the fraction of the photon momentum carried by one of the charmed quarks, $m_c$ is the running charmed quark mass. $k_t$ is typical momentum of the charmed quark. To account for nonzero $t = -\Delta^2$ one should substitute in the above formulae $k_t$ by $k_t - z\Delta$. As the consequence of large mass of $c$ quark the contribution of the end point ($z \sim 0, z \sim 1$) is negligible. So essential $z \approx 1/2$.

Since $M^2$ is significantly larger than $M^2_V$ for large $\Delta^2$, $x_i$ are not very different. So it can be demonstrated following ref. [7, 25] that to a good approximation $D$ is equal to a diagonal parton distribution, but with $x = (x_1 + x_2)/2$:

$$x_1 D(x_1, x_2, t, Q^2, Q_0^2) \approx [(x_1 + x_2)/2]D((x_1 + x_2)/2, t, Q^2, Q_0^2). \quad (2.9)$$

$$x \sim (2M^2 - M^2_V + Q^2)/2x_J. \quad (2.10)$$

Technically this result is due to the fact that the initial condition for the generalized parton distribution for small $x$ practically coincides with a diagonal one with $x = (x_1 + x_2)/2$. The main contribution to skewness comes from the cell in the Feynman diagram of Fig. 1 closest to the vector meson. It is easy to check that this is a property of the DGLAP dynamics in the kinematics $-t \ll Q^2 + M^2_V$. For $-t \geq Q^2 + M^2_V$ there are no logarithms from the integration over transverse momenta and nonforward distributions discussed below do not depend on energy in the DGLAP approximation. Thus the energy dependence of $f$ and nonforward diagonal distribution are the same.

The study of the first factor in eq. 2.5 will be the central subject of our paper. The total cross section is given by the integral

$$\frac{d\sigma}{dt} = \int_{\text{kinematical cuts}} dx_J \frac{d\sigma}{dt dx_J}. \quad (2.11)$$

We carry on our calculations for photoproduction in the kinematic region $-t_{\text{min}} \ll -t < M^2_V$ and for electroproduction in the kinematic range $-t_{\text{min}} \ll -t \ll M^2_V + Q^2$. We shall see below that for complimentary kinematic range of $-t > M^2_V + Q^2$ the energy dependence is quite different.

III. DGLAP EVOLUTION EQUATIONS FOR NONZERO $-t$.

A. General structure.

Let us now consider the nonforward parton distribution that enters in eq. 2.5. In this section we calculate it in the LO DGLAP approximation and explain how to generalize these
results to the NLO approximation of pQCD. It was explained above that the generalized parton distribution can be approximated at small $x$ by the parton distribution with the argument $x = (x_1 + x_2)/2$ and with the transverse momentum transfer $t = -\Delta^2$. We now proceed to calculate this distribution. First, it is easy to show that in the class of gauges $(CA) = 0$, where $C^\mu = c_1 p^\mu + c_2 q^\mu$ ($c_i$ are numbers of the order 1), used in ref. [13, 16] only ladder diagrams depicted in Fig. 2 contribute to the parton distribution. (This is not true in other gauges, in particular in the Feynman gauge). To achieve similarity with the parton model description of this ladder we will take in the paper $c_2 = 0$. Throughout this section we will consider the partons in the ladder to be massless and $Q^2$ is parameter characterizing hardness of the process: photon virtuality and/or large mass of quarkonium.

The calculation of the nonforward DGLAP ladder goes in the same way as for forward DGLAP ladder. In order to find the contribution of a given ladder cell $i$ we use the Sudakov variables:

$$k_i = \alpha_i q + \beta_i p + k^i_t$$

The gluon propagator in the gauge $CA = 0$ is given by

$$D_{\mu\nu}^{ab}(k) = \frac{1}{k^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{C^\mu k^\nu + C^\nu k^\mu}{C k} + k^\mu k^\nu \frac{C^2}{(kC)^2} \right].$$

This gauge is known to be free of ghosts. Moreover, it was shown in ref. [16] that in the leading logarithmic approximation (LLA) the contribution of the term $k^\mu k^\nu$ is zero. As a result, since we are working in a LLA we can use the propagator (3.3)

$$D_{\mu\nu}^{ab}(k) = \frac{1}{k^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{C^\mu k^\nu + C^\nu k^\mu}{C k} \right].$$

The proof [16] is valid for $-t_{\text{min}} \ll -t \ll Q^2 + M_V^2$. In the kinematical region $s \gg -t \gg Q^2 + M_V^2$ this prove should be valid as well since the contribution of the new structures $\propto \Delta_\mu$ is suppressed by the power of energy.

We now perform the standard algebraic calculations using the propagator (3.3) for the cell $i$. In the DGLAP approximation $k^2_i \gg k^2_{i(i-1)}$, and the integrals over longitudinal and transverse momenta decouple. The use of the gauge invariance allows to demonstrate that only one tensor structure, $g^\prime_{\mu\nu} \equiv g_{\mu\nu} - (p^\mu q^\nu + p^\nu q^\mu)/pq$ leads to the contribution containing terms $\propto \ln(Q^2/Q_0^2)$. There are additional tensor structures that appear for the nonforward ladder but they do not lead to the logarithmic contributions in the integral over the transverse momenta. Moreover, integrals over transverse momenta of gluons exchanged between different cells do not produce $\ln(Q^2/Q_0^2)$. Thus the same structure leads to large logarithms in both forward ($t = 0$) and nonforward cases.

Moreover, the direct calculation shows that the $t$ dependence is present in the integrand only in transverse momenta of propagators in the cell. Hence the DGLAP kernels $\Phi(z)$ do not depend on $t$. The calculation of integral over $\alpha_i$ is done by taking residues. As a result the contribution of a single cell $i$ in the DGLAP ladder can be written in the form

$$dP_i = \alpha_i(Q^2)dz\Phi(z)I(\Delta)g^i_{\mu\nu}.$$

Here $z_i = \beta_i/\beta_{i-1}$. In deriving eq (3.4) we took into account the gauge invariance, and summed the $s$ and $u$ channel contributions (that gives a factor of 2). The integral over transverse
momenta $I(\Delta)$ has the form:

$$I(\Delta) = \frac{1}{2\pi} \int d^2 k_t \left[ \frac{1}{(k_t^2 - \Delta^2)^2} + \frac{1}{k_t^2 - k_t^2 (\Delta - k_t^2)^2} \right]$$

$$= \log(Q^2/(Q_0^2 - t)) + \text{non logarithmic terms},$$

(3.5)

where $t = -\Delta^2$. The integral over $k_t^2$ is over the range $Q_0^2$ to $Q^2$. However the presence of large $-t$ leads to a cancellation of the contribution of small $k_t^2 \leq -t$. Hence the contribution of a single cell $i$ to the ladder has the form:

$$dP_i = \alpha_s dz \Phi(z) \log(Q^2/(Q_0^2 - t + Q_0^2)) g_{\mu\nu}^i,$$

(3.6)

instead of

$$dP_i = \alpha_s dz \Phi(z) \log(Q^2/Q_0^2) g_{\mu\nu}^i,$$

(3.7)

for the DGLAP ladder with $t = 0$. Here $\Phi(z)$ is the conventional DGLAP kernel. The full answer for the ladder is the convolution:

$$\sum_n (\frac{\alpha_s \log(Q^2/(Q_0^2 - t))}{4\pi})^n \prod \int \frac{d\beta_i}{\beta_i} \Phi(z_i)$$

(3.8)

The main difference (besides the definition of $x$ -cf. Eq. 2.10) between nonforward and forward distributions is the change of the argument of the logarithm arising from the integration over transverse momenta.

Note that the latter expressions were derived in the kinematics $Q^2 \gg -t \gg Q_0^2$, while in the kinematical region $Q^2 \sim -t$ one cannot single out the logarithmic contributions since they cannot be considered any longer as a large parameter. This means that the upper limits of integration in the integral of eq. 3.5 must be taken in such a way that for $-t \sim Q^2$ the argument of the logarithm is equal to one. Such choice is allowed since we work in the LLA and all integrals are calculated with logarithmic accuracy. In addition, the expression for the nonforward distribution should smoothly match with the forward distribution as $-t \to 0$. Thus the arguments of the logarithms that appear in the nonforward DGLAP distribution are indeed $\log(Q^2/Q_0^2 - t)$.

We calculated only the imaginary part of the nonforward ladder. The real part is also nonzero and can be reconstructed using the dispersion relation over the energy or more rapidly using the Gribov-Migdal formula [29]. However, the contribution of the real part of $f$ into cross section is numerically small and thus can be neglected.

**B. Account of the running coupling constant.**

In order to obtain the full answer for the DGLAP approximation we also need to account for the radiative corrections to the propagators and vertices. An effective method to account for these corrections is to use the dispersion relations over the mass of the produced parton in the same way as in ref. [13, 16, 30]. Our interest is in the kinematical domain: $-t = \Delta^2 \geq Q_0^2$ otherwise dependence on $t$ can be legitimately neglected. The integrand in eq. 3.5 has two potentially important kinematical regions: $k_t^2 \leq \Delta^2$ and $k_t^2 \gg \Delta^2$. In the first kinematic
region the integrand is strongly suppressed and it is of order $1/\Delta^2$. The reason is that the IR singularities in the integrand are cancelled out when $\vec{k}_t \rightarrow 0, \vec{k}_t \rightarrow \vec{\Delta}$. As a result, the entire logarithmically enhanced contribution in the integrand comes from the second kinematic region. However, in this region

$$\frac{2(k_t, k_t - \Delta)}{(\vec{k}_t - \vec{\Delta}_t)^2(k_t^2)} = \frac{1}{(\vec{k}_t - \vec{\Delta}_t)^2} + \frac{1}{k_t^2} - \frac{\Delta_t^2}{k_t^2(\Delta_t - \vec{k}_t)^2} \sim \frac{2}{k_t^2} + O\left(\frac{-t}{k_t^4}\right). \quad (3.9)$$

The factor $(k_t, k_t - \Delta)$ follows from gauge invariance and accounts for the gluon polarizations $[27]$. This means that within the logarithmic accuracy the integral over transverse momenta is the same as in the forward case, the only difference is that the integration starts at $k_t^2 = -t$. Hence we can directly use the results of ref. [16]. The renormalization of the ladder is given by diagrams of Fig. 3. Consider first the contribution of the exchanged gluon and two effective vertices. Taking the sum of discontinuities over two vertices and an exchanged gluon we obtain for the integral over transverse momenta

$$(1/\pi) \int (d^2k_t/k_t^2)dk_t^2 \frac{\Gamma(k_2, k)\Gamma(k_2, k - \Delta)dG(k_2)}{k_t^2 + i\epsilon}. \quad (3.10)$$

Here as usual $d_G, d_F$ account for the multiplicative renormalization of the gluon propagator

$$1/k^2 \rightarrow d_G(k^2, \sigma)/k^2, \quad (3.11)$$

and renormalization of the propagator of the fermion

$$1/k^2 \rightarrow d_F(k^2, \sigma)/k^2. \quad (3.12)$$

(Above we omitted the additional terms proportional to $\delta(k_2^2)$ since they are not important for large $t$.) Therefore we can use the same approach as in ref. [16]. Namely, we substitute $k_t^2 \rightarrow -k_t^2(1 - z)$ and use kinematic identity:

$$k^2 = -k_t^2/(1 - z) + zk_2^2/(1 - z). \quad (3.13)$$

Then the integral over $k_2^2$ in the dispersion relation (3.10) is determined by the pole at $k_2^2 = -k_t^2(1 - z)/z$. Thus in the Feynman integral for a renormalized cell we must put $k_2^2 = -k_t^2(1 - z)/z$.

Since all renormalized vertices are explicitly written in ref. [16], the only new element to take into account is that the left and the right vertices and the propagators $d_F$ depend now on different arguments. However since $k_t^2 \gg \Delta^2$ integration does not introduce any changes relative to the case $-t = 0$. The factors $\alpha_s(Q^2), d_G, d_F$ all combine together to the running coupling constant $\alpha_s(k_t^2)$. Then the integral over transverse momenta in the ladder is given by

$$\int_{-t}^{Q^2} d^2k_t \alpha_s(k_t^2)/k_t^2 \equiv \chi' = \frac{1}{b} \log\left(\frac{\log(Q^2/\Lambda^2)}{\log(-t + Q_0^2)/\Lambda^2}\right), \quad (3.14)$$

where

$$\alpha_s(k_t^2) = \frac{4\pi}{b \log(k_t^2/\Lambda^2)}, \quad b = 11 - 2N_f/3. \quad (3.15)$$

We included $Q_0^2$ in eq. (3.14) to match parton distributions in the limit $t \rightarrow 0$. The same result can be obtained by calculating the integral in transverse momentum plane in the
whole region of $k_i$ explicitly, combining all three terms in the integrand together using the Feynman parametrization:

$$I = \int_0^1 dk_i^2 dx \frac{(\vec{k} - (1 - x)\vec{\Delta})^2 + t(1 - x)^2}{(\vec{k} - (1 - x)\vec{\Delta})^2 - tx(1 - x)}.$$  (3.16)

Finally let us note that we cannot distinguish in the LLA whether the argument in the coupling constant is $k_i^2 t$ or $k_i^2$. In the paper we follow suggestion of ref. [30], that choice of $k_i^2$ minimizes higher order corrections.

The above results show that effectively the nonforward DGLAP distribution differs from the forward one only by the substitution of the variable $\chi = \frac{1}{\beta} \log(\log{Q^2/\Lambda^2})$ by $\chi'$ given by eq. 3.14. Then the nonforward PDF satisfy the evolution equation:

$$\frac{dG_B^A(x, Q_0^2, Q^2, t)}{d\log(Q^2 - t)} = \frac{\alpha_s(Q^2 - t)}{4\pi} \sum_C \int_0^1 \frac{dz}{z} \Phi_B^C(z) G_C^A(z, x, Q_0^2, Q^2, t).$$  (3.17)

Here $\Phi_B^C(z)$ are the standard DGLAP kernels, $G'$s are the nonforward ladders, $A, B, C$ denote the parton species. Eq. 3.17 is valid for all massless partons $A, B, C$. The solution of this equation can be obtained from the solution for the forward distributions by the substitution $\chi \rightarrow \chi'$.

The expression obtained in the previous subsection for the single cell in the approximation when the running of the coupling constant is neglected coincides with the expression in ref. [26]. This is because such DL terms are the same within DGLAP and BFKL approaches. [13]. In difference from ref. [26] we include the running coupling constant within the DGLAP framework and derive an evolution equation.

C. Nonforward parton distributions for the hard inelastic exclusive diffractive processes within the LO DGLAP approximation.

In order to evaluate the amplitude $f$ we need to solve eq. 3.17 for the case of scattering off a parton. All we need to do is to solve the corresponding DGLAP evolution for zero $t$ and then substitute $\chi \rightarrow \chi'$. The corresponding parton distributions for forward case were derived in refs. [13, 16, 31]: gluon distributions in the gluon, $D_G^G(x, Q^2)$, and in quark $D_Q^G(x, Q^2)$. Since $D_Q^G \ll D_G^G$ at small $x$ [16] we shall need only the $D_G^G$ function.

As we already stressed above the pQCD evolution effects at small $x$ are reduced to the double logarithmic(DL) approximation. Indeed, it was shown in ref. [15] that the PDFs evaluated using the DL approximation for gluon and quark distributions gives very good description of data at HERA energies. Moreover the double logarithmic approximation gives a good description of the parton distributions even at relatively large $x_B \leq 10^{-2}$ [16]. As a result for the theoretical description of the small $x$ inelastic diffraction it should be sufficient to use the double logarithmic expressions for parton distributions. In order to stress that we work with a gluon distribution with a single parton boundary conditions we shall denote this PDF as $D(x, Q^2, t)$ below.

The simplest description of the parton distribution in the DLA is achieved after the Mellin transform:

$$D(j, t, Q^2, Q_0^2) = \int_0^1 z^{j-1} D(x, t, Q^2, Q_0^2).$$  (3.18)
The solution of the relevant DGLAP equation has the form \[13, 16\]:

\[ D(j, \chi) = \exp(\gamma(j)\chi), \quad (3.19) \]

where

\[ \gamma(j) = \frac{4N_c}{j - 1} - a, \quad a = (11/3)N_c + (2/3)N_f. \quad (3.20) \]

For our case \( N_c = 3, N_f = 3 \). The solution in the \( x, \chi \) space is obtained after the inverse Mellin transform:

\[ xD(x, Q^2, t) = 8N_c\chi' I_1(u)/u, \quad (3.22) \]

where

\[ u = \sqrt{16N_c \log(x/x_J)\chi'}. \quad (3.23) \]

The function \( I_1 \) is the modified Bessel function of the first kind. For \( t = 0 \) this is just the formula for \( D_G^C \) in the DLA, first obtained in ref. [31]. For very small \( x \) we have the asymptotic expansion:

\[ xD(x, \chi') = \frac{\chi'^{1/4}N_c^{1/4}}{\sqrt{2\pi \log(x_J/x)^{3/4}}} \exp(u), \quad (3.24) \]

where \( x_J \) is given by eq. 2.2.

The formulae 3.22 and 3.24 give the solution for nonforward parton distributions that will be building blocks for the diffraction amplitude \( f \).

D. The nonforward parton distributions beyond the DGLAP approximation.

In the previous subsections we derived the formulae for the amplitude \( f \) within the DGLAP approximation (eq 3.22) which predicts the zero rate of the increase with energy for the nonforward PDF in the kinematical range \(-t \geq Q^2 + M_V^2\). Formally we derived this result within LO DGLAP approximation. But account of NLO, NNLO, ...approximations will not change this prediction because all DL terms disappear in this kinematical domain.

The DGLAP approximation ignores the possible contribution of \( \ln(x_0/x) \) terms that are not enhanced by large \( \log(Q^2/Q_0^2 - t) \) terms. These terms are connected with the gluon radiation in the multi Regge kinematics. The BFKL and resummation models [17, 18, 32] take into account both such terms and the double logarithmic terms [13, 17]. However, at present the direct numerical comparison between the BFKL and DGLAP results is not possible since the BFKL based models do not include the phase space constraints on the multi Regge gluon radiation, that follow from the energy-momentum conservation, and in addition neglect the running of the coupling constant. Recently the attempt was made to include such effects using the so called resummation models approach [18, 22, 32]. However, these were models developed for \( t = 0 \) only. The interrelation between the Pomeron behavior
and the double logarithmic approximation in the multi Regge kinematics needs an additional investigation.

In order to take into account the terms that do not contain large logarithms originating from the integration over parton transverse momenta it is convenient to represent a parton distribution as the product of DL contribution and the contribution that is not included in DLA. We may parametrize the nonforward parton distribution in the form as:

\[ f(x, Q^2, t) = f_{DL}(x, Q^2, t)F(x, t). \] (3.25)

As we already mentioned, the Pomeron behavior due to emission of gluon in the multi Regge kinematics is delayed till very high energies as a consequence of the strong constraints due to the local energy momentum conservation.

Consider first the HERA energies. The interval in rapidity necessary for the emission of the gluons adjacent in rapidity in the multi Regge kinematics is at least \( \sim 2 \div 2.5 \) \cite{33,35}. Since \( \ln(x_0/x) \geq \ln(M_V^2/Q_0^2) \), the photon fragmentation region occupies within the DLA at least \( \geq \log(M_V^2/Q_0^2) \gg 2 \div 3 \) units in rapidity. The proton dissociation in the triple reggeon limit occupies at least three units in rapidity (due to acceptance of the HERA detectors). Rapidity span for the kinematics of HERA is \( \sim 8 \div 9 \) units. Thus in the HERA kinematics it is hardly possible to emit even one additional gluon in the multi Regge kinematics. This means that at HERA energies if \( -t \leq Q^2 + M_V^2 \) there is no enough phase space for multi Regge corrections, and the resulted energy dependence is given the DGLAP terms, i.e. \( F = 1 \). For the kinematic range \( -t \geq Q^2 + M_V^2 \) the same kinematic analysis shows that because of the diminishing of the photon fragmentation region due to disappearance of DL terms where maybe a room for the radiation of one gluon within the multi Regge kinematics. We obtain:

\[ F(x, t) = 1 + \beta(Y - \Delta Y)\theta(Y - \Delta Y). \] (3.26)

Here \( \Delta Y \sim 4 \div 5 \) is the minimal rapidity needed for the start of a gluon radiation in a a multi Regge kinematics. Numerically we expect that the coefficient of the logarithmic term \( \beta(t) \ll \alpha s(t) \) - 1 since the existence of large logarithmic corrections of this type at HERA energies would contradict to a good agreement between DL asymptotics and experiment in the entire kinematic range of HERA \cite{15}. The intercept of Pomeron trajectory \( \alpha s(t) \) is two - three times smaller in the resummation models, than in the LO BFKL approximation because of the accurate account of the DL terms \cite{18,32}. Thus with a good accuracy we can put \( F = 1 \) at HERA also for \( -t \geq Q^2 + M_V^2 \).

Consider now the energies significantly larger than the ones reached at HERA. In the ultraperipheral processes at the LHC one may reach \( x \sim 10^{-6} \), i.e. up to 10 units in rapidity available for the HID process, after the exclusion of the proton fragmentation region, that corresponds to 3 units in rapidity at least (see above). The good agreement between the results of DGLAP and resummation models up to \( x \sim 10^{-4} \div 10^{-5} \) for \( Q^2 \sim 30 \text{ GeV}^2 \) (\( \alpha s \sim 0.2 \)) indicate that DL terms occupy the region of at least \( (2 \div 3)\log(Q^2/Q_0^2) \) in rapidity, i.e. \( 4 \div 7 \) units for \( Q^2 \sim M_V^2 \sim 10 \text{ GeV}^2 \) for charmonium production. The DL terms define region in rapidity occupied by the photon fragmentation region and result in a reduction of the rapidity interval corresponding to the multi Regge kinematics. This means that only \( 3 \div 5 \) units in rapidity at most may be available for the emission of gluons in the multi Regge kinematics – less than the minimal region \( \Delta Y \sim 5 \). Thus in the ultraperipheral processes which could be studied at the LHC for small \( -t < M_V^2 \), our equations should be applicable, at least qualitatively.
For the kinematic range \(-t \geq Q^2 + M_V^2\) the double logarithms are absent. In this case the rapidity range available for the gluon emission could reach \(8 \div 9\) units for multi Regge kinematics (after subtracting the photon fragmentation region (impact factor occupies \(\sim 2\) units in rapidity). Hence in this case \(2 \div 3\) gluons could be emitted in the multi Regge kinematics.

Thus the DL behavior will continue to reveal itself in the entire interval of energies available at ultraperipheral collisions at the LHC. Only at the maximum energies available at the LHC and \(-t > M^2\) there is a sufficient phase space for the multi Regge gluon emission. In the kinematics of HERA the energy dependence is given by DGLAP approximation. For LHC energies, at the maximum energies to be probed in ultraperipheral processes, and at \(-t \sim M^2\) it will be interesting to look for the Pomeron behavior which will compete with the onset of the black disk regime.

IV. THE ENERGY DEPENDENCE OF CROSS SECTIONS OF HARD INELASTIC PROCESSES WITH PROTON DISSOCIATION.

Let us apply our results to the exclusive inelastic production of charmonium. The QCD factorization theorem \([7, 8]\) allows us to evaluate the diffractive cross section in terms of the convolution of nonforward PDF discussed in the previous section with the parton distribution describing proton dissociation. The corresponding Feynman graph are shown at Fig. 1. We shall argue that the obtained above energy dependence of nonforward amplitude can be used as the interpolation formula correct also for \(-t \sim Q^2 + M^2\).

Let us consider first the kinematic range \(Q^2_0 \ll -t \ll Q^2 + M_V^2\). Since \(-t \ll Q^2 + M^2\) we may use the dipole approximation \([7]\). We expand the impact factor into Taylor series over exchanged gluons transverse momenta, and then follow the steps used in the proof of QCD factorisation theorem. Thus the amplitude \(f\) of the process is proportional to

\[
f(-t, M_V, Q) = K \int_0^1 du \int d^2 r (\phi_V(u, r, \Delta) \Delta t \phi_\gamma(r, u, \Delta)) x D(x, 4r^2, t).
\]  

(4.1)

The proportionality constant \(K\) is energy independent. It is matrix in the space of photon and vector meson polarizations: \(L \to L, T \to T, L \to T, T \to L\), where \(L, T\) are longitudinally and transversely polarized quarkonium and incident virtual photon.

\(D\) is a nonforward PDF, discussed in the previous sections. The argument of the nonforward parton distribution \(x\) is given by eq. \([2.10]\)

\[
x \sim (x_1 + x_2)/2 \sim 3(M_V^2 + Q^2)/(2x_js). \tag{4.2}
\]

Let us stress that the only assumption used in the derivation of eq. \([4.1]\) is the possibility to approximate impact factor by a first term in the Taylor expansion in exchanged gluon transverse momentum \(q_1\). Numerically such approximation is rather effective. The characteristic momenta of quarks in the impact factor \(r^2 \sim (Q^2 + M^2)/4\) (it is possible to neglect here the \(t\)-dependent term since \(-t \ll Q^2 + M^2\)). In addition \(u \approx 1/2\) \([36]\). This means that the expression \([4.2]\) can be actually approximated as

\[
f = x D((M_V^2 + Q^2)/(2x Js), (M_V^2 + Q^2), t) \Phi(t, Q^2, M_V^2), \tag{4.3}
\]

where

\[
\Phi(t, Q^2, M^2) \sim \int d^2 r \int_0^1 du (\phi_V(u, r, \Delta) \Delta t \phi_\gamma(r, u, \Delta)). \tag{4.4}
\]
\( \Phi \) depends on the quarkonium wave functions and influences the t-dependence of the cross section but it is independent of the incident energy. The energy dependence of the diffraction amplitude is entirely given by the convolution of nondiagonal forward distribution \( D((M_p^2 + Q^2)/(2x s), M^2 + Q^2, t) \) described in detail in the previous section with a parton distribution measured in DIS. Thus in the kinematical range \( Q_0^2 < -t < Q^2 + M_p^2 \) the energy dependence of the cross section at fixed \( x_j = -t/M_X^2 \) is determined solely by the function \( D \) (eq. 3.22) and does not depend on details of the photon and quarkonium wave functions.

If momentum transfer \(-t \sim Q^2 + M_p^2\) is large, the factorization theorem in the form described above should be modified. This is because the diffractive process is no more dominated by the DGLAP kinematics and the dipole approximation is not valid. The amplitude is still given by the convolution of the impact factor and the gluon distribution. However in this kinematic region there is only one transverse scale \(-t\), and the large logarithms originated due to QCD evolution are absent. Hence in the DGLAP approximation all amplitudes are reduced to energy independent impact factor. One would obtain the same result if we would simply extrapolate the energy dependence obtained above to this region. Thus we can use the energy dependence given by eqs. 3.22, 3.25 in the entire kinematic range \( Q_0^2 \leq -t \leq Q^2 + M_p^2 \). Beyond the DGLAP approximation the gluon distribution may contain the energy dependent terms which are not enhanced by large logarithms related to the \( Q^2 \) evolution. However it was argued in the previous section that such terms can give only small correction due to the energy-momentum constraints.

We conclude that the energy dependence of the total diffractive cross section in the DGLAP approximation can be explicitly calculable in the kinematic range \(-t \leq Q^2 + M_p^2\) as

\[
\frac{d\sigma}{dt dx_j} = \Phi(t, Q^2, M_p^2) \left( \frac{4N_c^2 I_1(u)}{\pi u^2} \right) ((81/16)G(x_j, t) + \sum_i (q_i(x_j, t) + \bar{q}_i(x_j, t))).
\]

Here

\[
u = \sqrt{16 N_c \log(x/x_j)} \chi', \quad \chi' = \frac{1}{b} \log(\frac{\log(Q^2/\Lambda^2)}{\log(-t + Q_0^2)/\Lambda^2}),
\]

\[x_j = -t/(M_X^2 - m^2_p - t), \quad x \sim 3(Q^2 + M_p^2)/(2x s), \quad b = 11 - 2/3 N_f, N_c = 3,
\]

and \( \Phi(t, Q^2, M_p^2) \) is the energy independent function, which depends on the details of the wave functions of the produced quarkonium and a photon.

For the kinematic region \(-t \sim Q^2 + M_p^2\) the cross section becomes energy independent in the kinematics covered by HERA, up to logarithmic corrections of the order \((1 + 2\beta(t) \log(x/x_j))\) for \(\log(x_j/x) > 4 \div 5\), and \(\beta(t) \ll \alpha_F(t) - 1\).

The important case of charmonium photoproduction can be obtained from the previous formulae just by putting \(Q^2 = 0\).

To illustrate the pattern of the variation with \( t \) of the dependence of the cross section on the rapidity gap interval we present in Fig. 4 the logarithmic derivative \( \frac{d \log(d^2\sigma/dt dx_j)}{d \log(x/x_j)} \) for the \( J/\psi \) photoproduction at \(-t = 0, 4, 9 \text{ GeV}^2\). It corresponds to the effective "local" value of \(2\alpha_F(t) - 2\). One can see that this quantity rapidly decreases with increase of \(-t\). Note that the discussed approximation somewhat overestimates effective \( Q^2 \) for photoproduction.
leading to a somewhat stronger energy dependence than a more realistic analyses of the $t = 0$ photoproduction of $J/\psi$ \cite{36}.

Hence we conclude that for a fixed $x_J$ and $-t \geq 4 \text{ GeV}^2$ the discussed process should depend very weakly on the rapidity gap interval at the HERA energies. This is consistent with the recent HERA findings \cite{3} and phenomenological analysis of the earlier data \cite{10}.

It is worth mentioning that at sufficiently high energies where one may expect an increase of the amplitude due to the BFKL type dynamics the absorptive effects may strongly modify the energy dependence of the cross section. Really the process we consider is an example of the inelastic diffractive process. Such processes disappear in the black disk regime. In particular if $x_J$ is kept fixed and sufficiently large (say $\geq 0.05$) the contribution of the scattering at large impact parameters would be suppressed, while for the small impact parameters the probability of interaction with a gap in rapidity goes to zero. Since such effects strongly depend on the size of the $Q\bar{Q}$ dipole they would also result in a weaker dependence of the cross section on $t$. These effects will be considered elsewhere.

V. DVCS SCATTERING.

The knowledge of the nonforward parton distribution evaluated above allows to calculate dependence on energy of the process of diffractive photon production: $\gamma^*(Q^2) + p \rightarrow \gamma + p$ -DVCS. Although this process is hardly possible to observe at large $t$ we present our results for completeness.

Amplitude of this process is

$$xG(x, Q^2, t) = F(t, x, Q^2) xG(x, \chi'),$$  \hspace{0.5cm} (5.1)

where $xG(x, \chi)$ is the gluon distribution at HERA, whose energy dependence is well approximated by \cite{13}

$$xG(x, \chi) \sim \exp(\sqrt{16N_c \log(x_0/x)}\chi), x_0 \sim 0.1, Q_0^2 \sim 1 \text{GeV}^2,$$  \hspace{0.5cm} (5.2)

and $\chi'$ is given by eq. 3.14.

$$F_{2g}(t, x) = 1/(1 + t/m_g^2(x, Q^2))^2,$$  \hspace{0.5cm} (5.3)

is dipole approximation for the two gluon form factor of the nucleon \cite{37}. (We suppressed here the weak dependence of $F_{2g}$ on $Q^2$). $F_{2g}(t, x)$ may depend on energy in a restricted range of energies because of pion cloud around nucleon carrying small fraction of proton momentum. So its contribution at $x \geq 0.1$ should be negligible, But hard interaction rapidly increases with interaction energy and at sufficiently small $x$ pion tail should reveal itself \cite{38}.

The expression 5.1 had been derived in the impulse approximation, which is questionable at large $-t$. However this effect is important for the calculation of absolute value of amplitude but it will not change the energy dependence.

VI. CONCLUSIONS

We found different QCD dynamics in the cross section of HID at HERA energies for fixed $x_J = -t/M_X^2$ in the two kinematical regions. In the first region $Q_0^2 \leq -t \leq Q^2 + M_X^2$ we
calculated the energy dependence of cross section within the DL approximation - eq. 4.5. In the second region $-t \geq Q^2 + M_V^2$ we showed that the cross section of the HID processes is energy independent. This result is exact within the DGLAP approximation - valid within any order of LO, NLO,...NNLO approximations. Our calculations explains observed in the recent HERA data on the HID processes significant decrease of the intercept of pQCD "Pomeron" with increase of $-t$ as compared to the intercept at $t=0$. The corrections to DGLAP approximation due to gluon radiation in the multi-Regge kinematics are small at the kinematics of HERA, as it follows both from the analysis of energy-momentum constraints on the multi Regge gluon radiation, and the analysis of ref. [15] that DLA gives a good description of the HERA data.

We argued that in ultraperipheral collisions at LHC the DL will give at least qualitatively good description of energy dependence of HID processes in most of the kinematic space available. However there may be a kinematic region $x \sim 10^{-6}, -t \geq M_V^2$, that lies in the borderline of available energies where it will be very interesting to look for the onset of the pomeron behavior.

In this paper we focused only on the energy dependence of the diffractive cross sections. The detailed numerical studies of the cross sections, including their absolute values and $t$-dependence will be presented elsewhere [39].

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FIG. 1: The Feynman diagram describing the double diffractive process in the triple reggeon limit in pQCD (there is also a cross diagram, not depicted explicitly.)
FIG. 2: The nondiagonal ladder
FIG. 3: The renormalization of the ladder.
FIG. 4: The logarithmic derivative of the differential cross section as a function of energy for $-t = 0, 4, 9 \text{ GeV}^2$. 