Nonlocal spin Hall effect and spin-orbit interaction in nonmagnetic metals

S. Takahashi*, S. Maekawa

Institute for Materials Research, Tohoku University, Sendai, 980-8577, Japan
CREST, Japan Science and Technology Agency, Kawaguchi, 332-0012, Japan

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Abstract

Spin Hall effect in a nonlocal spin-injection device is theoretically studied. Using a nonlocal spin-injection technique, a pure spin current is created in a nonmagnetic metal (N). The spin current flowing in N is deflected by spin-orbit scattering to induce the Hall current in the transverse direction and accumulate charge at the edges of N, yielding the spin-current induced Hall effect. We propose a method for extracting the spin-orbit coupling parameter in nonmagnetic metals via the nonlocal spin-injection technique. © 2021 Elsevier B.V. All rights reserved.

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There has been growing interest in spin transport in magnetic nanostructures, because of potential applications to spin electronic devices [1]. Recent experimental studies have demonstrated that the spin polarized carriers injected from a ferromagnet (F) into a nonmagnetic material (N) such as a normal metal [2,3,4,5] and superconductor [6,7] create a spin accumulation in N. In this paper, we consider a nonlocal spin-injection Hall device, and discuss the anomalous Hall effect (AHE) in the presence of spin current (or charge current) flowing in N, taking into account side jump and skew scattering.

The basic mechanism for AHE is the spin-orbit interaction in N, which causes a spin-asymmetry in the scattering of conduction electrons by impurities; up-spin electrons are preferentially scattered in one direction and down-spin electrons in the opposite direction. Spin injection techniques makes it possible to induce AHE in nonmagnetic conductors. When spin-polarized electrons are injected from F to N, these electrons moving in N are deflected by the spin-orbit scattering to induce the Hall current in the transverse direction and accumulate charge at the edges of N, yielding the spin-current induced spin Hall effect (SHE) [8,9,10].

Using the Boltzmann transport equations which incorporates the spin-asymmetric scattering of conduction electrons by nonmagnetic impurities in N within the Born approximation, we can derive the “total” spin and charge currents flowing in N [10,11]

\[ \mathbf{J}_s = \mathbf{j}_s + \mathbf{j}_s^H, \quad \mathbf{J}_q = \mathbf{j}_q + \mathbf{j}_q^H. \]  

where \( \mathbf{j}_s = -\left(\sigma_N/e\right)\nabla \delta \mu_N \) and \( \mathbf{j}_q = \sigma_N \mathbf{E} \) are the longitudinal spin and Ohmic currents, \( \sigma_N = 2e^2N(0)D \) is the electrical conductivity, \( \delta \mu_N = 1/2(\mu_N^+ - \mu_N^-) \) is the chemical potential shift, \( \mu_N^+ \) is the chemical potential of electrons with spin \( \sigma \), and \( D \) is the diffusion constant. The second terms in Eq. (1) are the transverse spin and charge Hall currents caused by spin-orbit scattering:

\[ \mathbf{j}_s^H = \alpha_H [\mathbf{z} \times \mathbf{j}_q], \quad \mathbf{j}_q^H = \alpha_H [\mathbf{z} \times \mathbf{j}_s] = -\frac{\alpha_H \sigma_N}{e} (\mathbf{z} \times \nabla \delta \mu_N), \]  

with \( \alpha_H = \alpha_{H,SS} + \alpha_{H,SS} \), where \( \alpha_{H,SS} = \hbar \eta_{so}(3mD) \) is the side jump (SJ) contribution, and \( \alpha_{H,SS} = (2\pi/3) \eta_{so} N(0)V_{imp} \) is the skew scattering (SS) contribution, \( \eta_{so} = \frac{k_F^2}{m^*} \eta_{so} \) is the dimensionless spin-orbit coupling parameter, \( k_F \) is the Fermi momentum, and \( V_{imp} \) is the impurity potential.

Equations (2) and (3) indicate that the spin current \( \mathbf{j}_s \) induces the transverse charge current (charge Hall current) \( \mathbf{j}_q^H \), whereas the charge current \( \mathbf{j}_q \) induces the transverse spin current (spin Hall current) \( \mathbf{j}_s^H \). Equation (1) is expressed in the matrix forms

\[ \begin{bmatrix} J_{s,x} \\ J_{s,y} \\ J_{s,z} \\ J_{q,x} \\ J_{q,y} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{bmatrix} \begin{bmatrix} E_x \\ -\nabla_y \delta \mu_N/e \end{bmatrix}, \]

\[ \begin{bmatrix} J_{s,x} \\ J_{s,y} \\ J_{s,z} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{bmatrix} \begin{bmatrix} E_x \\ -\nabla_y \delta \mu_N/e \end{bmatrix}, \]
where $\sigma_{xx} = \sigma_N$ is the longitudinal conductivity and $\sigma_{xy}$ is the Hall conductivity contributed from SJ and SS: $\sigma_{xy} = (\alpha_{H}^{SJ} + \alpha_{H}^{SS}) \sigma_N = \alpha_{xy}^{SJ} + \alpha_{xy}^{SS}$ with

$$\sigma_{xy}^{SJ} = \frac{e^2}{h} \eta_{0} n_{e} \epsilon_T, \quad \sigma_{xy}^{SS} = \alpha_{H}^{SS} \frac{n_{imp}}{N(0)V_{imp}} [N(0)V_{imp}]^{-1},$$

where $n_{e}$ is the (electron) density and $n_{imp}$ is the impurity concentration. Note that $\sigma_{xy}^{SJ}$ is independent of impurity concentration $n_{imp}$.

The ratio of the SJ and SS Hall contributions is

$$\frac{\sigma_{xy}^{SJ}}{\sigma_{xy}^{SS}} = \frac{2 n_{imp}}{n_{e}} N(0)V_{imp} = \frac{3}{4\pi} \frac{\hbar}{\epsilon_T \tau_{imp}} N(0)V_{imp},$$

where $\tau_{imp}$ is the momentum scattering time and $\epsilon_T$ is the Fermi energy. In ordinary non-magnetic metals, the ratio is very small because $n_{imp} \ll n_{e}$ and $N(0)V_{imp} \sim 1$, so that SS gives the dominant contribution to SHE. However, in very dirty metals or in low-carrier materials such as doped semiconductors with $n_{imp} \sim n_{e}$, the SJ conductivity is comparable to or even larger than the SS conductivity in SHE.

In the following, we consider a spin-injection Hall device shown in Fig. 1, and concentrate on the spin-current induced SHE. The magnetization of F electrode points to the $z$ direction. When the current $I$ is sent from F to the left side of N, the spin-polarized electrons are injected to create a pure spin current $j_s$ in N on the right side, where the total charge current is expressed as

$$J_q = -(\alpha_{H} \rho_{N}/e)(\hat{z} \times \nabla \delta_{MN}) + \sigma_N E.$$  \hspace{1cm} (8)

where the first term is the Hall current induced by $J_q$, the second term is the Ohmic current induced by surface charge, and $\alpha_{H} \sim \eta_{0} N(0)V_{imp}$ (skew scattering). In the open circuit condition in the transverse direction, where $J_q$ vanishes, the nonlocal Hall resistance $R_H = \rho_{N}/I$ becomes

$$R_H = \frac{1}{2} \alpha_{H} P_T (\rho_N/d_N) e^{-L/\eta_0},$$  \hspace{1cm} (9)

in the case of tunnel junction, where $P_T$ is the tunneling spin polarization, $\rho_{N}$ is the resistivity, $l_N$ is the spin-diffusion length, and $d_N$ is the thickness of N. In the case of metallic-contact junction

![Fig. 1. Spin injection Hall device (top view). The magnetic moment of F is aligned perpendicular to the plane. The spin-current induced Hall voltage $V_H = V_H^+ - V_H^-$ is induced in the transverse direction by injection of pure spin current $j_s$.](image)

Table 1

| Spin-orbit coupling parameter $\eta_0$ of Cu, Al, and Ag. |
|-------------------|-----------------|-----------------|
| $l_N$ (nm)        | $\rho_N$ (\(\mu\Omega\)cm) | $\tau_{imp}/\tau_{sf}$ | $\eta_0$ |
| Cu$^a$            | 1000            | 1.43            | 0.70 $\times 10^{-3}$ | 0.040        |
| Cu$^b$            | 1500            | 1.00            | 0.64 $\times 10^{-3}$ | 0.037        |
| Cu$^c$            | 546             | 3.44            | 0.41 $\times 10^{-3}$ | 0.030        |
| Al$^d$            | 650             | 5.90            | 0.36 $\times 10^{-4}$ | 0.009        |
| Ag$^e$            | 195             | 3.50            | 0.50 $\times 10^{-2}$ | 0.110        |

$^a$Ref. [2], $^b$Ref. [3], $^c$Ref. [4], $^d$Ref. [2], $^e$Ref. [5].

where $R_H = \frac{1}{2} \alpha_{H} P_T (\rho_N/d_N) R_F \rho_N \sinh^{-1}(L/l_N)$,

$$R_H = \frac{1}{2} \alpha_{H} P_T (\rho_N/d_N) R_F \rho_N \sinh^{-1}(L/l_N),$$

(10)

where $p_F$ is the spin polarization of F, $R_N$ and $R_F$ are the spin resistances of the N and F electrodes: $R_N = (\rho_N l_N)/A_N$ and $R_F = (\rho_F l_F)/A_f$ with $A_N$ the cross-sectional area of N and $A_f$ the contact area between N and F. Usually, $R_N$ is one or two orders of magnitude larger than $R_F$ [12]. Recently, the spin-current induced AHE have been measured using spin injection techniques [13,14].

It is worthwhile to make the product $\rho_N l_N$, which is related to the spin-orbit coupling parameter $\eta_0$ as

$$\rho_N l_N = \frac{3 \sqrt{3} \pi \hat{R}_K}{2 \pi} \sqrt{\tau_{sf}/\tau_{imp}} = \frac{3 \sqrt{3} \pi \hat{R}_K}{4} \frac{1}{\pi}$$

(11)

where $\hat{R}_K = h/e^2 \sim 25.8 \, \text{k}\Omega$ and $\tau_{sf}$ is the spin-flip scattering time. The formula (11) provides a method for extracting the physical parameters of spin-orbit scattering in nonmagnetic metals. Using experimental data of $\rho_N$ and $l_N$ in Eq. (11), we obtain the value of the spin-orbit coupling parameter $\eta_0 = 0.01–0.04$ in Cu, Al, and Ag as listed in Table 1. Therefore, Eqs. (9) and (10) yields $R_H$ of the order of 0.1–1 m\Omega, indicating that the spin-current induced SHE is observable by using nonlocal spin-injection Hall devices.

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