Operative Time Definition and Principal Uncertainty

Martin Schönh

Department of Theoretical Physics and Astrophysics
Faculty of Science, Masaryk University
Kotl!sk! 2, BRNO, Czech Republic

PACS numbers:
01.55 + b, 04.20 Cv, 04.60 + n, 03.30 + p,

ABSTRACT

Arguments are given that time must be defined in an operative manner, i.e., by constructing devices which can serve as clocks. The investigation of such devices leads to the conclusion that there is a principal uncertainty of time if one considers periods which are not large compared with the Planck time. Thus, according to the old (classical) concept, time cannot be well-defined at this scale. The uncertainty of time leads to a breakdown of Special and General relativity in the Planck regime; the same happens with causality. We present arguments that the classical concept of time, which treats $t$ simply as a real parameter, must be replaced by a new one.

†) e-mail address: schoen%elanor.sci.muni.cs@csbrmu11.bitnet
1 Introduction

There have been many attempts to quantize gravity but so far there is no well-defined, satisfying theory of Quantum Gravity. Such a theory, however, is desirable because the classical general relativistic description breaks down at the very beginning of our Universe or, e.g., at the final stage of the formation of a black hole. It is also required if we agree that the aim of physics is to provide an uniform picture of nature.

Gravitation is closely connected with the structure of space and time, and it seems that the role of time is crucial for each approach to Quantum Gravity and should hence be clarified before trying to construct the quantized theory. There are indications that the present treatment of time is not adequate. Arguments for that, as far as the canonical quantization approach is concerned, can be found in Ref. 1. Also, the constraints of general relativity generate dynamical trajectories which in general do not admit a global time function (see e.g. Ref. 2). These observations raise the question about the true nature of time. The old, so far used time concept, which treats time simply as a parameter (let us call it the classical one), must be subjected to a critical investigation.

In this paper we inquire the existence of a well-defined time quite generally, i.e., also in the presence of non-gravitational fields or in the absence of any field. For that purpose we investigate the construction of clocks which serve to define time. We call that procedure an *operative definition of time*. "Operative" means that we are able to construct at least in principle some device which allows us to measure the considered quantity (here time) with sufficient exactness (i.e., the uncertainty of \( t \) must be small compared with \( \Delta t \)).

But why must time be defined operatively? Obviously, there are theories which contain elements which cannot be defined in such a way. If we take e.g. the state vector ("wavefunction") \( \psi \) of Quantum Theory, there is no device which defines directly that state vector. However, there is a fundamental difference between the concept "state vector" and the classical concept of time: the underlying theory.

Each element of any physical theory has to be interpreted, i.e., there must exist a prescription how to "map it into reality" by showing that object (or group of objects) of the real world which corresponds to the considered element. Let us consider e.g. the state vector \( \psi \). Quantum Theory provides us with the knowledge of how to construct out of \( \psi \) measurable results.
provides us further with methods how to generate some particular $\psi$ by constructing special devices. But it does not claim that $\psi$ is directly realized in nature. All we know is the correspondence between $\psi$ and derived measurable results. Each of those (like e.g. the position of a flash) can be defined operatively. Thus we know how to map $\psi$ into reality, and, hence, there is no problem of definition present. Let us now consider time. According to the classical concept (= underlying theory), we can construct out of the time $t$ essentially just one measurable result, namely $t$ itself. Moreover, in the case of time the underlying theory (tacitly) claims that $t$ is directly measurable. Hence, either it must be possible to define $t$ operatively or we have to change the underlying theory, i.e., replace the classical time concept by a new one.

In this paper we are going to show that the time cannot be well-defined operatively if the considered time period is not large compared with the Planck time. At that scale a principal uncertainty is showing up which is of the same order as the time period. This result holds independently of the physical situation of the clock’s neighbourhood, i.e., independent of the presence of fields. We will arrive at that conclusion by studying various types of possible clocks and investigating their behaviour taking into account that they have to be material objects which obey physical laws. We will see that Heisenberg’s Uncertainty Principle and the existence of a Schwarzschild horizon cause limitations concerning the size of the clock. These limitations lead directly to our result.

From these arguments we can infer that we need a new concept of time which provides us with some framework how to construct out of it results also measurable at the Planck scale ($\sim 10^{-44}\text{sec}$). The old time concept has too poor structure to fit to reality at that scale.

Additionally, we are going to investigate implications of this result upon Special and General Relativity (as well as upon causality). We will see that all break down at Planck scale. That result is not new (at least as far as General Relativity is concerned); it can be obtained also by considering metric fluctuations (see e.g. Ref. 3) or, alternatively, by an analysis of measurability of field quantities similar to that performed by Bohr and Rosenfeld (cf. Ref. 4). The derivation presented in this paper is from the mathematical point of view especially simple; all we need are some primitive estimations.

However, the main goal of that paper is not to give a new argument that something fundamental must happen at Planck scale but to investigate the conceptual nature of time and to motivate trying approaches to Quantum Gravity with some fundamentally changed time concept. The plan of this paper is as follows. In section 2 we study the possibility of
defining time in an operative manner by considering elementary clocks. In section 3 we discuss some immediate consequences concerning Special Relativity and causality. In section 4 we investigate the principle of equivalence in the light of the results obtained in the preceding sections. Section 5 concludes this paper.

2 An Operative Definition of Time

Let us start by considering an arbitrary spacetime which is equipped with a system of ideal light clocks. Each of these clocks consists of two spherical "mirror" masses situated in a distance $l$ between which a photon is oscillating. Every time when the photon is returning to the sphere from which it first started a counter is turned one unit further. We denote such a time unit by $t$. The time which is recorded by the counter is called time at the position of the (first) sphere.

The clock works appropriate only if the following conditions are satisfied. First of all, the photon must be able to leave the first sphere and to reach the second one. Thus, $l$ must be greater than the Schwarzschild radius of the sphere. We obtain:

$$l \geq \frac{2Gm}{c^2} \quad (1)$$

where $G$ is the gravitational constant, $c$ is the velocity of light, and $m$ is the mass of one sphere (let both have equal masses). One can even strengthen that condition by replacing "$\geq"$ by "$\gg"$; then we obtain a weak field condition which must be satisfied if we demand that the gravitational field between the spheres shall be weak. But for our purpose (1) is sufficient. Note that (1) is only a necessary condition; to be a sufficient one would require additionally, that the radius of the sphere is greater than the Schwarzschild radius. Again, the above form is good enough for the following.

Uncertainty of position and momentum of the mirror spheres cause an uncertainty $\Delta t$ in the time $t$ defined by such a clock. In order to have a "reasonable" definition of time we should demand:

$$\Delta t \ll t = \frac{2l}{c} \quad (2)$$

(2) is equivalent to

$$\Delta l \ll l \quad (3)$$
We are now going to calculate the minimal uncertainty $\Delta l_{\text{min}}$ in $l$ which is compatible with Heisenberg’s Uncertainty Principle. For the uncertainty $\Delta p$ in the momentum $p$ of the mirror sphere holds: $\Delta p \approx m\Delta v$, where $v$ is the velocity of that mass. Since $\Delta v$ causes an uncertainty $\Delta l$ during the time interval $t$ which is equal to $t\Delta v$, we get (according to $(\Delta l\Delta p)_{\text{min}} \approx \hbar$)

$$\frac{m(\Delta l_{\text{min}})^2}{t} \approx \hbar \tag{4}$$

where $\hbar$ is the Planck constant. Since $t = 2l/c$ we obtain finally

$$\Delta l_{\text{min}} \approx \sqrt{\frac{2\hbar l}{mc}} \tag{5}$$

This could be made arbitrarily small by $m \to \infty$ but we have to bear in mind condition (1).

(3) together with (5) implies

$$m \gg \frac{\hbar}{lc} \tag{6}$$

(3) together with condition (1) yields:

$$l^2 \gg \frac{\hbar G}{c^3} = l_{\text{Planck}}^2 \tag{7}$$

where $l_{\text{Planck}}$ is the so-called Planck length (cf.e.g. Ref. 3). From (7) we can infer that the size of our clock must be large compared with the Planck length. Since $t = 2l/c$ we obtain finally (neglecting factors like 2):

$$t \gg \sqrt{\frac{\hbar G}{c^5}} = t_{\text{Planck}} \tag{8}$$

Thus, the time unit $t$ defined by an ideal clock must be large compared with the Planck time $t_{\text{Planck}}$, if $t$ shall be defined accurately.

However, we have considered just one special clock. Could it not be possible that some appropriately constructed clock enables us to define a time unit which is smaller than that of (8)? In order to answer this question we consider now more general clocks.

Let us first investigate an arbitrary clock consisting of some device of size $l$ and mass $m$. Let time again be defined by some light ray which is moving periodically within that device. Again, we have the condition (1); otherwise the clock cannot work. Moreover, the estimations based on Heisenberg’s Uncertainty Principle leading to condition (3) are still valid. Thus, we obtain
again our result $t \gg t_{Planck}$.

One might also consider some device which is operating with particles of velocity $v$ instead of photons (“mechanical clock”). Then condition (7) is still valid (it can even be replaced by a stronger condition, but we do not need that strong condition). In condition (7) $c$ is replaced by $v$; thus we get instead of (7) $t^2 \gg \hbar G/c^2 v$. But since $1/v \geq 1/c$, we get again the result (7), and hence, also (8).

So far, we have considered clocks which work according to the same principle as our ideal light clock. Let us now study clocks which might be really different from those considered above. We are going to consider a ”radioactive” clock. That is a device consisting of a sample of some material which decays with half-life time $t_H$. That sample is situated in the center of a hollow sphere which serves to collect all emitted photons. The recorded rate can be used for a definition of time. If we equip this hollow sphere with sufficient mass we can decrease its position uncertainty as much as desired. Hence, it seems that in such a manner it is possible to construct an unlimited definition of time. But let us consider that clock in greater detail.

We assume that the material consists of two-energy-levels systems. The energy levels are denoted by $E_1$ and $E_0$, respectively. For such a situation the well-known relation

$$t_H \Delta E \geq \hbar$$

means, that if the system has a half-lifetime $t_H$, then the difference $E_1 - E_0$ can be at best determined with an uncertainty $\Delta E$. An upper boundary for $\Delta E$ is the total energy of the sample. Hence, we obtain:

$$\Delta E \leq mc^2$$

where $m$ is the sample mass. Let the sample have the size $l$. Then, again condition (1) holds, for otherwise the emitted photon would be unable to reach the hollow sphere. On the other hand, if we denote the minimal time period which can be measured by our clock by $t$, we get the condition $l \ll ct$, for otherwise the point where the decay took place is such uncertain that we cannot infer from the recorded rate the time within the exactness given by $t$. Namely, if the size of the sample is of the same order as the distance travelled by the photon within that time $t$, the non-centralization of the decay events cause perturbations of the recorded rate of order $t$. (9) and (10) together with that condition yield:

$$tt_H \gg \frac{G\hbar}{c^5}$$

But since the half-lifetime $t_H$ cannot be much bigger than the minimal measurable time unit $t$ (otherwise the clock would not be exact enough to detect $t$), we obtain again result (8). Thus, also the ”radioactive” clock fails to define time units which are not large compared with the Planck time.
In this section we have considered different types of clocks. All of them showed a principal uncertainty of the time defined thereby if the time unit is not large compared with the Planck time. Of course, we have not considered all possible clocks. But since the clocks investigated in this section are very elementary, it is quite reasonable to assume that the principal uncertainty is a fundamental one and a basic feature of nature, and is hence present for all clocks.

3 Consequences concerning Special Relativity and Causality

Special Relativity is based on the behaviour of systems of synchronized clocks, which form a reference frame. Let us therefore investigate under which conditions such a synchronization can be performed successfully. Consider two clocks which are separated from each other by a distance $d$. Let us try to synchronize them by sending some light signal from one clock to the other one. Heisenberg’s Uncertainty Principle leads again to some minimal uncertainty $(\Delta d)_{\text{min}}$ in the distance which is approximately equal to $\sqrt{\frac{\hbar}{mc}}$ (see (1)). In order to be able to perform such a synchronization procedure the distance $d$ must exceed the Scharzschild radius of the clock, i.e., $Gm/c^2 \leq d$. Putting both results together we obtain $(\Delta d)_{\text{min}} \geq \sqrt{\frac{\hbar G}{c^3}}$ (neglecting factors like 2). Thus, the minimal uncertainty in $d$ is at least equal to the Planck length. This causes an uncertainty in the synchronization $(\Delta t)_{\text{syn}}$ which is equal to $(\Delta d)_{\text{min}}/c$. Thus, we get the result

$$(\Delta t)_{\text{syn}} \geq t_{\text{Planck}}$$

(12)

Again, the Planck time turns out to be the fundamental limit: it is not possible to synchronize clocks more perfectly than $t_{\text{Planck}}$.

We can conclude that Special Relativity is limited by the condition that the considered times are large compared with the Planck time. The reason is the principal synchronization uncertainty (12): the possibility to synchronize a clock frame breaks down when we approach the Planck time. The concept time of a reference frame becomes senseless.

Now let us study what happens with causality at Planck scale. Two states are said to be causally connected if the first one, say $A$, taking place at time $t_A$ determines the second one called $B$, taking place at time $t_B$. The concept of causality is quite unproblematic in classical physics, but can also be maintained in Relativity and Quantum Theory, if the concept of state is defined...
appropriately. However, as far as the latter is concerned, this is not completely true. There is one process which violates causality: the reduction of the wavefunction due to some measurement. But even in that case it is possible to save the causal relation if we give up determinism and replace it by some weak causal principle which states that A causes B with some probability.

In any case, if A and B are related by a causal relation, then $t_A$ cannot be bigger than $t_B$. Now, let us approach the Planck scale. If $t_B - t_A \sim t_{\text{Planck}}$, it is impossible to say which event happens first. It becomes senseless to say that one causes the other. On the other hand, it is also not reasonable to say that they cause each other mutually, for there is still some asymmetry between A and B: unless $t_B - t_A = 0$, $t_B$ is more often observed to occur "later" (according to necessarily unsharp clocks) as $t_A$. However, with some new time concept that asymmetry could appear in a new light; and perhaps it is then possible to reformulate the causal principle in such a way that it holds also for the Planck scale. But according to the present concept we must conclude that causality breaks down or loses its sense in the Planck regime.

4 Consequences concerning Gravity

In this section we will investigate whether the equivalence principle which is essential for General Relativity can be applied if the gravitational field is such strong that it changes significantly across a distance which is not large compared with the Planck length. Such a situation was realized near Big Bang when the age of the Universe was of order of magnitude of the Planck time. The equivalence principle tells us that it is possible to find space-time coordinates such that locally Special Relativity holds. One can formulate this principle in greater detail and distinguish between strong and weak principle but for our purpose the above form is sufficient. Let us now try to apply this principle to some region of a strong gravitational field. The region which can be described locally in terms of Special Relativity by choosing appropriate coordinates must be smaller than the size of significant change of the field strength. Since we are considering the case that this size is of order of magnitude of the Planck length our local neighbourhood cannot be equipped with clocks of size much larger than that length. In other words, Einstein's comoving box is too small for accurate clocks which are necessarily much larger than $l_{\text{Planck}}$, as we have seen in section 2 (see eq. (7)).

Let us now imagine some hypothetical observer within that comoving box. He can use only very small clocks which are necessarily very uncertain. Moreover, according to our investigations performed in the previous section, for such
small time units Special Relativity breaks down. It is thus meaningless to say that for our observer the laws of Special Relativity hold. Hence, the equivalence principle cannot be applied near the Planck regime.

But without being able to apply the equivalence principle General Relativity cannot be an appropriate description of strong gravitational fields. That is a well-known result which is also confirmed by arguments different from those considered in this paper (see Ref. 3, 4). Since, as we have seen above, this breakdown is connected with the rather serious impossibility of maintaining the ”classical” concept time at one point we should first replace that concept by a new one before trying to quantize gravity. That new, so far unknown concept must be one which reflects the principal uncertainty of time in the Planck regime.

Let us end this section with a very undetailed proposal about the construction of a future theory which is able to describe gravity in the Planck regime and, hence, the very early universe. The proposal is quite a conservative one for it tries to preserve the essence of the equivalence principle. It consists in performing the following steps:

1. Find some generalization of Special Relativity which is valid also for short time periods. For that purpose a new time concept is required which reveals a principal and unescapable uncertainty near Planck time. Note that it is not sufficient to replace $t$ by some operator $\hat{t}$ and to construct in such a way some ”quantum theory of time” because then we can choose an eigenstate of that operator and get an exact time value. But we saw that time alone and for itself becomes uncertain.

2. Apply a slightly modified equivalence principle which states that it is possible to find coordinates such that locally the theory to be constructed in the first step holds.

3. Try to quantize that new general relativistic theory.

Perhaps, there are close relations between step 1 and step 3 such that they must be performed together before applying the equivalence principle. But these are speculations which we do not want to continue now.
5 Conclusions and Outlook

We have seen that an operative definition of time is not possible when we are considering time periods which are not large compared with the Planck time (\( \sim 10^{-44} \text{sec} \)). Any possible clock will become uncertain and unsharp when we decrease its size down to the Planck length such that the thereby defined time unit becomes comparable with the Planck time. Thus, we must give up the concept of a continuous, exactly measurable time and have to replace it by a new time concept which reflects this principal uncertainty for small periods. Note that this new concept cannot simply be some replacement of the classical parameter \( t \) by an operator \( \hat{t} \) for in this case the measured value could be made arbitrarily exact by choosing some eigenstate of \( \hat{t} \). The principal time uncertainty is more serious and requires a mathematical framework which is different from the ordinary operator language of the present quantum theory.

Note also that this principal uncertainty is not merely a problem of time measurement. We have adopted the point of view that if a physical quantity is present in the underlying theory then it must be possible in principle either to measure this quantity directly or to construct out of it measurable results. The latter is e.g. true in Quantum Theory, where the state vector cannot be "verified" directly in nature by some suitable device. However, Quantum Theory tells us how we can derive from the state vector operatively defined quantities. On the other hand, the classical "time theory" tells us (at least tacitly) that time, contrary to the state vector, can be measured directly. According to this concept, time is an "elementary" quantity. Hence, it must be possible to define it operatively. Thus, the observed principal uncertainty at Planck scale shows that the classical time is not well-defined.

This principal time uncertainty shows up regardless whether some field is present or not. Thus, the Planck regime requires fundamental changes not only for strong gravitational fields but also for all other fields as well as for flat spacetimes without any field. Moreover, we have seen that Special Relativity breaks down near the Planck regime also due to a second reason: it is then impossible to synchronize a system of clocks forming a frame of reference and hence to define the common time of that system. We have further considered the consequences of that principal uncertainty concerning causality. We found, that there is no well-defined causal relation at Planck scale. The same happens with General Relativity. We saw that for strong gravitational fields which change significantly across some distance which is not large compared with the Planck length the equivalence principle cannot be applied. Hence, General Relativity must break down at Planck regime; a result which is also confirmed by other observations (cf. Refs. 3, 4).

However, there is some hope that a generalisation of these theories can be
found which is also valid in the Planck regime if we construct an appropriate new time concept. Perhaps, such a concept could be the clue to find some satisfying theory of Quantum Gravity. We believe that the time problem is the key problem for constructing such a theory.

Of course, all our results are based on the assumption that Heisenberg’s Uncertainty Principle as well as condition (1) arising from the existence of a Schwarzschild horizon are at least approximately valid (i.e., sufficient for our estimations performed in section 2). If this is not true, then perhaps the classical concept of time could be maintained but at least one of these basic laws must be altered fundamentally in the Planck regime. However, it seems to be quite reasonable to assume that the estimations performed in the previous sections are justified. In this case, the classical concept of time cannot be maintained.

Acknowledgement

The author wishes to express his thanks to the hospitality of the Department of Theoretical Physics and Astrophysics, Masaryk University Brno, Czech Republic, where the main part of this work has been carried out.

References

1. K.V. Kuchar in *Quantum Gravity 2*, edited by C. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1981)
2. P. Hajicek, Phys. Rev. D 34, 1040 (1986)
3. J. A. Wheeler, Ann. Phys. (N.Y.) 2, 604 (1957)
4. B. S. de Witt in *Gravitation: An Introduction to Current Research*, ed. by L. Witten (John Wiley & Sons, Inc., New York, 1962)