The static potential in $\mathcal{N}=4$ supersymmetric Yang-Mills at weak coupling

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We compute the static potential associated to the locally 1/2 BPS Wilson loop in $\mathcal{N}=4$ supersymmetric Yang-Mills theory with $\mathcal{O}(\lambda^2/r)$ accuracy. We also resum the leading logarithms, of $\mathcal{O}(\lambda^{n+1}\ln^n \lambda/r)$, and show the structure of the renormalization group equation at next-to-leading order in the multipole expansion. In order to obtain these results it is crucial the use of an effective theory for the ultrasoft degrees of freedom. We develop this theory up to next-to-leading order in the multipole expansion. Using the same formalism we also compute the leading logarithms, of $\mathcal{O}(\lambda^{n+3}\ln^n \lambda/r)$, of the static potential associated to an ordinary Wilson loop in the same theory.

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There is a huge interest in the study of the Yang-Mills theory with $\mathcal{N}=4$ supersymmetry in four dimensions. One of the reasons is the conjectured existence of a correspondence between the $\mathcal{N}=4$ supersymmetric Yang-Mills and the type IIA superstring theory on an $\text{AdS}_5 \times S^5$ background [1]. This duality is known as the AdS/CFT correspondence and has importance consequences, since it allows to compute the strong 't Hooft coupling limit ($\lambda \equiv \frac{N_c g^2}{4\pi}$) of certain correlators in the $\mathcal{N}=4$ supersymmetric Yang-Mills theory for large $N_c$. This is so because this limit becomes equivalent to the classical limit on the string theory side for which computational techniques exist.

Checks of this conjecture are difficult to obtain, since the only quantitative approach to gauge theories is based on computations at weak coupling. In some cases it is possible to check the conjecture with the result at weak coupling. This usually happens when non-renormalization theorems exists that permit to perform the computation on the perturbative side exactly.

In other cases no such checks exist and, usually, one can only study both the weak and strong coupling limit with increasing degree of accuracy hoping to gain further input on how the extrapolation from weak to strong coupling takes place. In this paper we concentrate in one of such correlators, the locally 1/2 BPS static Wilson loop, or, more specifically, the large time limit of its logarithm. Its strong 't Hooft coupling limit for large $N_c$ has been computed using the AdS/CFT correspondence in [2]. On the other hand the question whether the understanding of the infrared structure of the static potential in the weak coupling regime may shed some light on the AdS/CFT correspondence has been addressed in Refs. [4] [5]. In these references some infrared divergences were found, which allowed the authors to obtain the leading logarithmic correction to the tree level result. These infrared divergences have a similar origin to those found in the QCD static potential at weak coupling [6], which, however, in this case first appear at three loops. They are due to the existence of degrees of freedom with energy of $\mathcal{O}(\lambda/r)$, which we will call ultrasoft in what follows. This scale is much smaller than the soft scale $\sim 1/r$ at weak coupling. In any case, it is somewhat surprising that, after so much work, the static potential has not even been computed with next-to-leading-order (NLO), ie. $\mathcal{O}(\lambda^2/r)$, accuracy yet. The reason is that, at this order, ultrasoft effects enter into the game and an infinite resummation of diagrams is needed in order to obtain the desired accuracy. However, this problem can be revisited from an effective field theory perspective, as it has already been done for the QCD case [7]. As we will see, by doing so, the problem trivializes and we will able to compute the static potential with $\mathcal{O}(\lambda^2/r)$ accuracy. The use of effective theories will also allow us to write renormalization group equations for the static potential. By solving them we will also obtain the static potential with leading-log (LL), ie. $\mathcal{O}(\lambda^{n+1}\ln^n \lambda/r)$, accuracy.

We will also use this example to give full details of how the factorization between the soft and ultrasoft scale takes place in the static potential for a specific computation including finite pieces. In the case of QCD this factorization takes places at three loops and the full computation does not exist yet.

Finally, using the same formalism we will also compute the static potential associated to an ordinary Wilson loop with NNLL accuracy up to the two-loop matching condition.

I. THE STATIC SINGLET ENERGY

The $\mathcal{N}=4$ SUSY Lagrangian reads

$$\mathcal{L}_{\mathcal{N}=4} = -\frac{1}{4} F_{\mu \nu}^a F^{\mu \nu \, a} + \frac{1}{2} \sum_{i=1}^{6} (D_\mu \Phi_i)^a (D^\mu \Phi_i)^a$$
\[-\frac{i}{2} \bar{\psi} \gamma_{\mu} (D^\mu \Psi)^a + \cdots. \quad (1)\]

\(\Phi\) and \(\Psi\) represent a scalar and majorana particle respectively. The adjoint covariant derivative reads: \(D_\mu^a)^a = \partial_\mu^a)^a - g f^{abc} A_\mu^b (\cdot)^c\), \(i\) range from 1 to 6, and the dots represent interactions between the scalars and the majorana particles or self-interactions between the scalars.

We now take the locally \(1/2\) BPS Wilson loop (for a motivation of this definition see \([8]\))

\[W_C = \frac{1}{N_c} \text{Tr} P e^{-ig \int \mathcal{D}(\nu, \chi \Phi) \left(\chi_{\mu}^\dagger \partial_{\mu} \chi + \Phi_{\mu} \chi_{\mu}^\dagger \right)} , \quad (2)\]

where \(\Phi_n \equiv \Phi \cdot \hat{n}\), \(n\) is a six-dimensional vector with \(n^2 = 1\), and \(C\) represents the path followed by the Wilson loop. In our case we consider a static Wilson loop and its associated singlet static energy

\[E_s(r) = \lim_{T \to \infty} \frac{i}{T} \ln(W_{\square}), \quad (3)\]

where \(W_{\square}\) is the rectangular Wilson loop with edges \(x_1 = (T/2, r/2), x_2 = (T/2, -r/2), y_1 = (-T/2, r/2)\) and \(y_2 = (-T/2, -r/2)\). The symbol \(\langle \rangle\) means the average over the massless fields.

In order to describe the static Wilson loop at the dynamical level we consider the Lagrangian \([11]\) plus static sources in the appropriate representation

\[\mathcal{L} = \mathcal{L}_{N=4} + \mathcal{L}_{\text{stat}} , \quad (4)\]

where

\[\mathcal{L}_{\text{stat.}} = \bar{\psi}^\dagger (i \partial_\mu - g A_\mu - g \Phi_n) \psi + \bar{\chi}_{\mu}^\dagger (i \partial_\mu + g A_\mu - g \Phi_n) \chi^\mu . \quad (5)\]

\(\psi\) and \(\chi\) (the conjugated field of \(\chi\)) correspond to static sources in the fundamental and anti-fundamental representation respectively. The case with only one static source has been considered in Ref. \([2]\).

With this Lagrangian the static singlet energy can be obtained from the following Green function

\[I \equiv \langle 0 | \chi(x_2) W(x_2, x_1) \psi(x_1) \psi(x_2) \chi(y_2) | 0 \rangle \]

\[\times \bar{\psi}^\dagger (y_1) W(y_1, y_2) \chi(y_2) | 0 \rangle . \quad (6)\]

where \(W(x_2, x_1) = W_C\) for a straight path \(C\) with initial and final points \(x_1\) and \(x_2\) respectively.

**II. EFFECTIVE THEORY**

The use of effective field theories allows us for an efficient description of the infrared structure of the static potential at weak coupling. This has been shown to be so for the QCD static potential \([7, 10]\). In that case the effective theory was \(p\)NRQCD \([11]\). For a review see \([12]\). Here we would like to construct the \(N = 4\) supersymmetric version of \(p\)NRQCD in the static limit. We will do so at next-to-leading order in the multipole expansion. As we will see the main difference is the existence of massless scalars. On the other hand the heavy quark and antiquark rearrange in a singlet or octet configuration under (ultrasoft) gauge transformations as in QCD. The simplification arises from the existence of two disparate scales: the soft \(\sim 1/r\) and the ultrasoft \(\sim \lambda/r\) scales, and that we only aim for a description of the dynamics at energies of order \(\sim \lambda/r\). Therefore, degrees of freedom with energy \(\sim 1/r\) can be integrated out and one can factorize the physics associated to each scale. Note that we live in the opposite limit to strong coupling. For us \(E_s(r) \ll 1/r\), whereas in the strong coupling limit \(E_s(r) \gg 1/r\).

Integrating out the soft scale, \(1/r\), from \([4]\) we are left with an effective theory where only ultrasoft degrees of freedom remain dynamical. The surviving fields are the \(\psi\)-\(\bar{\chi}\) states (with ultrasoft energy) and the massless ultrasoft gluons, scalars and fermions. The \(\psi\)-\(\bar{\chi}\) states can be decomposed into a singlet (S) and an octet (O) under colour transformation. The relative coordinate \(\mathbf{r} = x_1 - x_2\), whose typical size is the inverse of the soft scale, is explicit and can be considered as small with respect to the remaining (ultrasoft) dynamical lengths in the system. Hence the massless fields can be systematically expanded in \(\mathbf{r}\) (multipole expansion), and the effective Lagrangian is constructed order by order in \(\mathbf{r}\). As a typical feature of an effective theory, all the non-analytic behaviour in \(\mathbf{r}\) is encoded in the matching coefficients, which can be interpreted as potential-like terms.

In order to have the proper free-field normalization in the colour space we define

\[S \equiv \frac{1}{\sqrt{N_c}} S \quad O \equiv \frac{T^a}{\sqrt{T_F}} O^a , \quad (7)\]

where \(T_F = 1/2\).

The effective Lagrangian density that can be constructed with these fields and that is compatible with the symmetries of Eq. \([4]\) is given at the next-to-leading order in the multipole expansion by:

\[\mathcal{L}_{US} = \text{Tr} \left\{ \sum \psi^\dagger (i \partial_\mu - V_s(r) + \ldots) S + \sum \bar{\psi}^\dagger (i \partial_\mu - V_s(r) + \ldots) O \right\} \]

\[-2g V_{\Phi}(r) \text{Tr} \left\{ \sum (\Phi_n + S \Phi_n) \right\} \]

\[-g V_{\Phi}(r) \text{Tr} \left\{ \sum (\Phi_n, O \} \right\}\]
+ gV_A(r) \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O \right\} \nonumber \\
+ g \frac{V_B(r)}{2} \text{Tr} \left\{ O^\dagger \left\{ \mathbf{r} \cdot \mathbf{E}, O \right\} \right\} \nonumber \\
- g \frac{V_C(r)}{2} \text{Tr} \left\{ O^\dagger \left\{ \mathbf{r} \cdot (\mathbf{D}\Phi_n), O \right\} \right\}, \quad (8)

where \( \mathbf{R} \equiv (x_1 + x_2)/2, \ S = S(r, \mathbf{R}, t) \) and \( \mathbf{O} = \mathbf{O}(r, \mathbf{R}, t) \) are the singlet and octet wave functions respectively. All the gluon and scalar fields, as well as the derivative of them, in Eq. (8) are evaluated in \( \mathbf{R} \) and \( t \). In particular \( F^{00} \equiv \mathbf{E} = \mathbf{E}(\mathbf{R}, t) \) and \( iD_0 \mathbf{O} = i\partial_0 \mathbf{O} - g[A_0(\mathbf{R}, t), \mathbf{O}] \). \( V_X(r) \) are the matching coefficients of the effective Lagrangian. They are determined by matching the effective and the underlying theory at a scale \( \nu \) smaller than \( 1/r \) and larger than the ultrasoft scales. Since we are at weak coupling, the matching can be done perturbatively. At the lowest order in the coupling constant we get \( \alpha_{\nu_c} = \alpha_{\nu_c} = \alpha_s = \frac{1}{2} \). \( V_A = V_B = V_\phi = V_{\Phi_o} = 1 \), where we have defined \( (C_A = N_c, C_F = (N_c^2 - 1)/(2N_c)) \)

\[ V_s(r) \equiv -2C_F \frac{\alpha_{\nu_c}}{r}, \quad (9) \]
\[ V_o(r) \equiv 2 \left( \frac{C_A}{2} - C_F \right) \frac{\alpha_{\nu_c}}{r}, \]

which correspond to the singlet and octet heavy \( \psi - \bar{\chi} \) static potential respectively.

Note that we distinguish between the singlet static energy \( E_s(r) \) and the singlet static potential \( V_s(r) \). The reason for that has been discussed in detail in Ref. [9]. \( V_s(r) \) corresponds to the matching coefficient that appear in the effective theory and it is the proper object to be introduced in a Schroedinger equation in case we were working with particles with large but finite mass. On the other hand \( E_s(r) \) directly corresponds to Eq. (3) and represents the energy of two static particles (one could do a similar distinction for the octet static potential and energy, which in the QCD case corresponds to the hybrid energy).

Charge conjugation proves to be a very useful symmetry to eliminate operators in the effective Lagrangian. The effective Lagrangian is invariant under charge conjugation plus particle \( \leftrightarrow \) antiparticle exchange. In particular singlet, octet, gluon and scalar fields transform as:

\[
S(t, r, \mathbf{R}) \rightarrow \sigma^2 S(t, -r, \mathbf{R})^T \sigma^2, \\
O(t, r, \mathbf{R}) \rightarrow \sigma^2 O(t, -r, \mathbf{R})^T \sigma^2, \\
A_\mu(t, \mathbf{R}) \rightarrow -A_\mu(t, \mathbf{R})^T, \quad \Phi_n(t, \mathbf{R}) \rightarrow \Phi_n(t, \mathbf{R})^T.
\]

Asking for invariance under the above transformations the following operators are eliminated:

\[
\delta \mathcal{L} = -gV_D \text{Tr} \left\{ O^\dagger [\Phi_n, O] \right\}, \quad (11)
\]

One can also easily check perturbatively up to order \( \alpha_s^3 \) that the matching coefficient of these operators are zero. Therefore, they do not affect the results obtained in this paper for the singlet static potential and energy.

We have to mention that we have not explored all the constraints supersymmetry (or any other remaining underlying symmetry) may impose on the matching coefficients of the effective Lagrangian and, accordingly, on the structure of the renormalization group equation. This would need a dedicated study that goes beyond the aim of this work. In the case of QCD some analysis have been performed for the underlying Poincare symmetry of pNRQCD in Ref. [13].

Finally, let us note that the octet singlet potential is \( 1/N_c^2 \) suppressed compared with the singlet potential. Therefore, it can be neglected in the large \( N_c \) limit.

### III. RG

The renormalization group equation of the singlet static potential has the following structure

\[
\nu \frac{d}{d\nu} V_s = \gamma_1(V_s - V_o) + \gamma_3 r^2 (V_s - V_o)^3 + \cdots, \quad (14)
\]

where \( \nu \) is the factorization scale, and the dots refer to higher orders in the multipole expansion. This structure follows from the multipole expansion and the mass gap between the octet and singlet static potential. The anomalous dimensions \( \gamma_i \) have structure themselves. \( \gamma_1 \) can be computed as an expansion in \( \alpha_s \) (note that some powers of \( \sqrt{\alpha_s} \) get associated a power of \( V_\phi \) or \( V_{\Phi_o} \)). For \( \gamma_3 \) we also have dependence on \( V_A^2 \) (but in a very specific way):

\[
\gamma_3 = V_A^2(2C_F\alpha_s)F(\alpha_s, V_\phi, V_{\Phi_o}) \quad (15)
\]

For both anomalous dimensions we can easily obtain the lowest non-trivial contribution. They read

\[
\gamma_1 = \frac{2C_F\alpha_s}{\pi} V_\phi^2 + \mathcal{O}(\alpha_s^2), \quad (16)
\]
\[
\gamma_3 = \frac{2C_F\alpha_s}{3\pi} V_A^2 + \mathcal{O}(\alpha_s^2). \quad (17)
\]

They come from the computation of the ultraviolet behavior of the diagram in Fig. 1 with the \( V_\phi \) or \( V_A \).
We obtain
\[ V_s = -\frac{\lambda^{1+2\frac{d}{c}}}{r} \frac{1}{\sqrt{1 + \frac{\lambda^{3/2}}{r}(1 - \lambda^{1+\frac{d}{c}})}}. \]  

For this result is not possible to take the strong coupling limit, since it becomes imaginary.

We would like to finish this section with some remarks. The order at which infrared logarithms appear in the static potential is easily visualized in the effective theory. In the case of QCD infrared logarithms first appear at \( O(\alpha_s^2/r) \). This follows from the multipole expansion suppression of the interaction of the octet and singlet field through the chromoelectric field. There is no such suppression in the supersymmetric case, as we can see from the third line of Eq. \( \text{(20)} \), which explains the appearance of such effects already at \( O(\alpha_s^2/r) \). On the other hand these effects can not come from gluons alone, since it would correspond to the pure QCD case. Therefore, the logarithms have to be associated with the scalars. This is seen quite clearly in the effective theory.

IV. THE STATIC POTENTIAL AT ONE LOOP

In this section we compute the singlet static potential and energy at one loop (as well as some other matching coefficients). This computation is necessary for an eventual complete NLL evaluation. Moreover, this example will allow us to visualize how the factorization between the soft and ultrasoft scale takes place in dimensional regularization including finite pieces.

We first compute the soft piece, i.e. the static potential. Diagrams with self energy cancel with diagrams with internal vertices \[ \text{[6].} \] Therefore, only the ladder diagram (and the crossed one if we go beyond the large \( N_c \)) has to be considered. Note that this is only true in the Feynman gauge. In any case, the final result is gauge independent. Therefore, the expression of the bare static singlet potential in \( D = 4 + 2\epsilon \) dimensions in momentum space reads
\[ \tilde{V}_{s,B} = -2C_F g^2 \frac{1}{k^2} \left\{ 1 - C_A g^2 k^2 \frac{\Gamma[1 - \epsilon]\Gamma[2\epsilon]}{2^{1+2\epsilon}\pi^{2+\epsilon}\Gamma[2\epsilon]} \right\} + \mathcal{O}(g^4) \]  

After subtraction of the divergent piece in the \( \overline{\text{MS}} \) scheme one can take the \( D \to 4 \) limit in the potential. It reads
\[ \tilde{V}_{s,\overline{\text{MS}}} = -2C_F g^2 \frac{1}{k^2} \left\{ 1 - 2\frac{\lambda}{\pi} \ln(k/r) + \mathcal{O}(\lambda^2) \right\}. \]
We see that there is no finite piece associated to the logarithm in the \( \overline{\text{MS}} \) scheme. Eq. (29) is the initial condition of the singlet static potential for the eventual renormalization group equation at NLL. At NLL we would also need the initial conditions for \( V_\Phi \) and \( V_A \). Using arguments analogous to those in Ref. [E] we can conclude that there are no one loop corrections to those matching coefficients and

\[
V_\Phi = 1 + \mathcal{O}(\lambda^2),
\]

\[
V_{\Phi o} = 1 + \mathcal{O}(\lambda^2),
\]

\[
V_A = 1 + \mathcal{O}(\lambda^2).
\]

The bare potential in position space reads

\[
V_{s,B} = - \frac{2 C_F g^2 r^{-2 \epsilon}}{4 \pi} \frac{1}{r^{3/2 + \epsilon}} \left\{ 1 - \frac{C_A g^2 r^{-2 \epsilon}}{2 \pi^{2 + \epsilon}} \frac{\Gamma[1/2 + 2 \epsilon] \Gamma^2[2 \epsilon] \Gamma[1/2 + \epsilon]}{\Gamma[1/2 + 2 \epsilon] \Gamma[2 \epsilon] \Gamma[1/2 + \epsilon]} + \mathcal{O}(g^4) \right\}.
\]

\[
V_{s,\overline{\text{MS}}} = - \frac{2 C_F g^2 r^{-2 \epsilon}}{4 \pi} \frac{1}{r^{3/2 + \epsilon}} \left\{ 1 + \frac{C_A g^2 r^{-2 \epsilon}}{2 \pi^{2 + \epsilon}} \frac{\Gamma[1/2 + 2 \epsilon] \Gamma^2[2 \epsilon] \Gamma[1/2 + \epsilon]}{\Gamma[1/2 + 2 \epsilon] \Gamma[2 \epsilon] \Gamma[1/2 + \epsilon]} + \mathcal{O}(g^4) \right\},
\]

which in four dimensions reduces to

\[
V_{s,\overline{\text{MS}}} = - \frac{2 C_F \overline{\alpha}_s}{r} \left\{ 1 + \frac{2 \alpha_s}{\pi} \ln(n \nu) + \frac{\gamma_E}{2} + \lambda^2 \right\} + \mathcal{O}(g^4).
\]

\[
\delta V_{s, \overline{\text{MS}}} = 2 C_F g^2 (V_o - V_s) \left( 1 + \frac{\lambda^2}{\pi} \right) \ln(n \nu) + \frac{\gamma_E}{2} + \lambda^2 + \mathcal{O}(g^4).
\]

Finally, the energy of two static sources in the fundamental and anti-fundamental representation in \( \mathcal{N} = 4 \) gluodynamics reads

\[
E_s(r) = V_{s,B} + \delta V_{s,B} = V_{s,\overline{\text{MS}}} + \delta V_{s,\overline{\text{MS}}},
\]

\[
E_s(r) = \frac{2 C_F \overline{\alpha}_s}{r} \left\{ 1 + \frac{2 \lambda}{\pi} \ln(2 \lambda) + \gamma_E - 1 \right\} + \mathcal{O}(\lambda^2),
\]

\[
V. \text{ ORDINARY WILSON LOOP AND ITS ASSOCIATED STATIC ENERGY}
\]

Besides the locally 1/2 BPS Wilson loop considered in the previous sections, one could also consider the ordinary Wilson loop, which we define by eliminating the interaction of the static sources with the scalars in Eq. (2), i.e.

\[
W_C = \frac{1}{N_c} \text{Tr} \left\{ e^{-ig} f_c \mu A_\mu \right\}.
\]

One can then redo the analysis of the previous sections for this case. Here we would like to compute the leading logarithmic corrections to the static energy and potential. In order to do so we can use the effective Lagrangian in Eq. (5) eliminating the terms proportional to the scalars fields\(^2\), rescaling by a factor 1/2 the static potentials in Eq. (29), and redefining \( \lambda \equiv N_c \overline{\alpha}_s / 2 \). This implies that, to the order of interest, \( \gamma_1 = 0 \) and \( \gamma_3 \) is 1/2 the value quoted in Eq. (17). From this exercise, we learn the important lesson that the leading in \( \alpha_s \) correction to the ordinary Wilson loop static energy in \( \mathcal{N} = 4 \) supersymmetric Yang-Mills theory is the same to the one obtained in QCD [F]. The difference with QCD is due to the fact that \( \alpha_s \) itself does not run. This implies that there are no \( \mathcal{O}(\alpha_s \ln(\alpha_s)) \) terms in the static singlet potential except for \( n = 1 \) (unlike in QCD [H]). These findings can be summarized in the following equation, which is correct with NNLL accuracy,

\[
E_s(r) = - \frac{C_F \overline{\alpha}_s}{r} \left( 1 + \alpha_1 \alpha_s + \alpha_2 \overline{\alpha}_s^2 + \frac{C_A^3 \overline{\alpha}_s^3}{12 \pi} \ln(C_A \alpha_s) \right).
\]

The coefficient \( \alpha_1 = N_c / \pi \) has been obtained in Ref. [E]. We confirm this result. The coefficient \( \alpha_2 \) is at present unknown.

For the static singlet potential one should replace \( C_A \alpha_s \) by \( \nu / r \) in the logarithmic term, where \( \nu \) would correspond to the factorization scale.

VI. CONCLUSIONS

Eqs. (19), (32) and (33) are the main results of our paper.

We have obtained the singlet static energy (and potential) with LL and NLO accuracy for the 1/2

\(^2\) At present, we can not claim that those coefficients vanish at any order in \( \alpha_s \) but, at most, they are \( \mathcal{O}(\alpha_s^2) \). This implies that the contribution from those terms is suppressed compared with those due to the Tr\{OES\} term.
BPS static Wilson loop. We have provided with expressions at arbitrary dimensions, as well as with the formalism (based on effective field theories), that could be relevant for future computations. We have illustrated how the merge of the soft and ultrasoft contribution takes place in the case of the singlet static energy including finite pieces. This may be of help in order to visualize how things will work in the case of QCD, where this mixing takes place at three loops.

A naive interpolation to strong coupling of the renormalization group improved LL result does not agree with the supergravity conjecture.

For the ordinary Wilson we have computed the singlet static energy and potential with NLO accuracy and we have also performed the resummation of logarithms with NNLL accuracy up to the initial matching condition $a_2$.

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