Vehicle-bridge coupling vibration of curved beam bridge under earthquake excitation

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Abstract: Based on the non-inertial reference system, the vehicle model of four freedom degrees is used to establish the vehicle-bridge coupling dynamic system model of the curved beam in space. Considering the vehicle-bridge coupling vibration characteristics under the non-uniform seismic excitation at the two supports of the bridge, the three-dimension space curvilinear-vehicle coupling dynamic control equation is established based on the Hamiltonian principle. Then the coupling dynamic control equation of the vehicle and bridge is numerically analyzed by the method of separating variables.

1. Introduction
Curvilinear Bridges have been widely used in engineering due to their linear elegance, large span, high space utilization rate and small terrain restriction, etc. However, due to the coupling of bending moment and torque in the beam under the influence of curvature, the dynamic research of curvilinear bridge vehicle-bridge coupling system is getting more complex[1].

Traditional vehicle-bridge coupling dynamics analysis focuses on the coupling dynamics of vehicle-bridge relative motion, but does not pay attention to the coupling dynamics of vehicle-bridge system with respect to the foundation reference system from the perspective of relative motion[2]. However, when the vehicle-bridge coupling system is under the action of occasional strong excitation earthquake, and seismic source transfers through foundation, capture, bridge and car body, the amplitude and the phase of seismic wave change in a larger area, which results in the spatial variation of seismic ground motion that will consequently lead to the bridge structure of the linear distribution at different supports could in the same earthquake experience significant differences. For example, two bridge abutment displacement differences lead to the motion of the bridge relative to the original foundation reference system[3]. At this time, the beam body movement includes both the deformation movement and the space movement of the bridge system. Therefore, it is more appropriate to study the vehicle-bridge coupling dynamic system from the non-inertial reference perspective[4][5].

2. Vehicle-bridge coupling model and control equation of curved bridge
The space curved beam vehicle-bridge coupling model under the perspective of non-inertial reference is set up as shown in Fig.1. The $\vec{O}x'y'z'$ is an inertial coordinate system and the $\vec{O}xyz$ is a relative coordinate system. $\vec{r}_0$ is the radius vector from the relative coordinate system to the inertial coordinate system. $\vec{r}_a$ is the radius vector of the arbitrary point $a$ in the beam after deformation movement to the relative coordinate system. $\vec{r}_a$ is the radius vector of the arbitrary point $a$ after deformation movement to the inertial coordinate system. $R$ is the curvature radius of curved beam.
Fig. 1 the space curved beam vehicle-bridge coupling model under the perspective of non-inertial reference

The adopted vehicle model is a 4-dof vehicle model. \( v \) is the vehicle velocity and \( 2d \) is the distance between the front and rear wheels. The vehicle dynamic balance equation is

\[
m_i \ddot{x}_i + \sum_{i=1}^{2} c_{i1} (\dot{x}_i + \eta_i d \theta_i - \dot{z}_i) + \sum_{i=1}^{2} k_{i1} (z_i + \eta_i d \theta_i - z_i) = 0
\]

\[
l_i \ddot{\theta}_i + \sum_{i=1}^{2} c_{i1} (\eta_i d \theta_i + d^2 \theta_i - \eta_i d \dot{z}_i) + \sum_{i=1}^{2} k_{i1} (\eta_i d \dot{z}_i + d^2 \theta_i - \eta_i d z_i) = 0
\]

\[
m_{ti} \ddot{z}_i + c_{ti} (\dot{z}_i - \eta_i d \theta_i - \dot{z}_i) + c_{wi} (\dot{z}_i - \dot{z}_i) + k_{ti} (z_i - \eta_i d \theta_i - z_i) + k_{wi} (z_i - z_{wi}) = 0
\]

\( m \)-- mass, \( I \)-- moment of inertia, \( k \)-- stiffness, \( c \)-- damping, \( z \)-- vertical displacement, \( \theta \)-- Angle of rotation, \( \eta_1 = 1, \eta_2 = -1 \)

\[
\int_0^1 (\delta T - \delta V + \delta W) \, dt = 0
\]

\( \delta T, \delta V, \delta W \) are the variations of the kinetic energy, deformation potential energy and external work of the system respectively.

The displacement component of the arbitrary point \( \alpha \) on the beam structure in relative coordinates is set as

\[
u(x, y, z, t) = u_0(x, t) - y \left( v_{0,x}(x, t) - \frac{u_0(x, t)}{R} \right) - z w_{0,x}(x, t)
\]

\[
v(x, y, z, t) = v_0(x, t) + 2 \theta_b(x, t)
\]

\[
w(x, y, z, t) = w_0(x, t) - y \theta_b(x, t)
\]

\( u_0, v_0, w_0 \)--the surface displacement, \( \theta_b \)--beam section angle

According to Hamiltonian variation principle, the variation of the curved beam kinetic energy is

\[
\delta T = \frac{1}{2} \int \rho (c_0)^2 dV = \rho A \int \left( \ddot{u}_i + \frac{d}{dt} \left( \frac{\ddot{u}_i}{\alpha} \right) \right) \delta u_i dx (i = 1, 2, 3)
\]

\( \alpha \)--tensor expression for the displacement component

Considering the linear relationship between strain and displacement, the strain component at arbitrary point in the beam is expressed as

\[
\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{v}{R} \frac{R}{R+y}
\]

\[
\varepsilon_{xy} = \frac{\partial v}{\partial x} - \frac{u}{R} \frac{R}{R+y} + \frac{\partial u}{\partial y}
\]

\[
\varepsilon_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial y}
\]

\[
M_x = \int (y \sigma_{xx} - 2 \sigma_{xy}) dA = -GJ \left( \theta_{0,x} + \frac{w_{0,x}}{R} \right)
\]

\[
M_y = -\int \sigma_{yx} \cdot x dA = \int \left( u_{0,x} + \frac{y^2 + v_0}{R} - y \left( v_{0,x} - \frac{u_{0,x}}{R} \right) - z \left( w_{0,xx} - \frac{\theta_{0,x}}{R} \right) \right) \cdot x \cdot \left( 1 + \frac{y}{R} \right) dA
\]
\[ M_x = \int \sigma_{xx} \cdot y \, dA = \int \left( u_{xx} + \frac{v_0^2 + v_0}{R} - y \left( \frac{w_{0,xx}}{R} \right) - \frac{z \left( w_{0,xx} \right)^2}{R} \right) \cdot y \cdot \left( 1 + \frac{y}{R} \right) \, dA \]

\[ f = \int (y + x^2) \, dA \]  

(7)

The variation of the curved beam deformation energy is

\[ \delta V = \int \sigma \delta e \, dV = \int \left( \sigma_{xx} \delta e_{xx} + \sigma_{xy} \delta e_{xy} + \sigma_{zz} \delta e_{zz} \right) \, dV \]

\[ = \int \left( N_x \delta u_{xx} + \frac{r_0^2 + v_0}{R} \right) - M_x \delta \left( \frac{r_0^2 + v_0}{R} \right) + M_x \delta \left( \frac{w_{0,xx}}{R} \right) + \frac{M_x (\theta_b + \frac{w_{0,xx}}{R})}{R} \, dx \]

\[ = \int \left( -N_x \delta u_0 + \left( \frac{N_x}{R} - \frac{M_{xx}}{R} \right) \delta v_0 + \left( M_{xx} + \frac{M_{xx}}{R} \right) \delta \theta_b + \frac{M_{xx} - \frac{M_{xx}}{R}}{R} \delta \theta_b \right) \, dx \]

(8)

The variation of the external beam virtual work is

\[ \delta W = Q_i \cdot \delta u_i \quad (i = 1, 2, 3) \]

By combining the above equation and applying the independent variable component property, the motion control equations of the curved beam in space can be obtained as

\[ \rho A (r_0^2 + \dot{v}_0) - E A \left( u_{0,xx} + \frac{v_0^2}{R} \right) - Q_1 = 0 \]

\[ \rho A (r_0^2 + \dot{v}_0) + \frac{E A}{R} \left( u_{0,xx} + \frac{r_0^2}{R} + v_0 \right) + E I_x \left( v_{0,xxxx} + \frac{2}{R^2} v_{0,xx} + \frac{1}{R^4} v_0 \right) - Q_2 = 0 \]

\[ \rho A (r_0^2 + \dot{v}_0) + E I_x \left( w_{0,xxxxx} - \frac{\theta_b}{R} \right) - \frac{G J}{R} \left( \theta_{b,xx} + \frac{w_{0,xx}}{R} \right) - Q_3 = 0 \]

(9)

3. \textbf{Curved beam vehicle-bridge coupling system interaction}

Close contact model is adopted for vehicle wheel set and bridge deck, then the vehicle-bridge coupling control equation in the non-inertial system can be

\[
\begin{bmatrix}
M_{bb} & 0 \\
0 & M_{vv}
\end{bmatrix}
\begin{bmatrix}
\ddot{X}_b \\
\ddot{X}_v
\end{bmatrix}
+
\begin{bmatrix}
C_{bb} & C_{bv} \\
C_{vb} & C_{vv}
\end{bmatrix}
\begin{bmatrix}
\dot{X}_b \\
\dot{X}_v
\end{bmatrix}
+
\begin{bmatrix}
K_{bb} & K_{bv} \\
K_{vb} & K_{vv}
\end{bmatrix}
\begin{bmatrix}
X_b \\
X_v
\end{bmatrix}
=
\begin{bmatrix}
F_b \\
F_v
\end{bmatrix}
\]

(10)

4. \textbf{Equation solving}

Assume that the bridge deck only bears the load of vehicles vertically, the vehicles remain in contact with the bridge deck during the operation without disengaging, the vehicle wheelset displacement is consistent with the bridge deck displacement, the wheelset displacement is replaced by the bridge deck displacement, and the wheelset mass is ignored, then the wheelset load borne by the bridge can be expressed as

\[ f_w = \sum \left( c_{wi} ( \xi - (\gamma_b^2 + \omega_0^2) ) + k_{wi} ( \xi - (\gamma_b^2 + \omega_0^2) ) \right) + \left( 0.5 m_w + m_z \right) g \]

(11)

\[ \gamma_b, \dot{\gamma}_b, \ddot{\gamma}_b \] -- vertical displacement of beam body in the inertial reference frame caused by earthquake excitation, velocity and acceleration of beam body in inertial reference frame caused by earthquake excitation.

The static displacement at both ends of the simply supported beam is assumed as \( a(t) \) and \( b(t) \) respectively and the change of seismic wave along the radial direction of the beam is ignored. The static displacement at any part of the beam can be expressed as formula 12.

\[ \gamma_b = a(t) + \left( b(t) - a(t) \right)^\frac{n}{2} \]

(12)
5. Conclusion

Based on the non-inertial reference, a general space curved beam vehicle-bridge coupling dynamic model is established, which takes into account the non-uniform seismic excitation at the two supports of the bridge, and takes the first three modes of vibration model for solution and analysis.

The torsion angle of the beam is only coupled with the vertical displacement of the beam, so the torsion angle has the same vibration form as the mid-span displacement of the beam.

The torsion angle of the beam is significantly affected by the radius of curvature, and the larger the curvature of the beam is, the more likely the shear failure of structure is to happen.

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