Structure formation in a nonlocally modified gravity model

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Outline

1. A nonlocally modified gravity model which describes the late time acceleration

2. Is it also consistent with the growth of structure in the universe?
Describing the expansion history by $a(t)$

Universe in large scales ($> 100\text{Mpc}$):
- Homogeneous, isotropic and spatially flat
- Described by FRW (Friedmann-Robertson–Walker) metric

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}$$

- Expansion history can be described by the scale factor
  $$\dot{a} > 0 \text{ expanding} \quad \ddot{a} > 0 \text{ accelerating}$$

- Current phase of acceleration: the Hubble rate is approaching to a constant, meaning accelerating
  $$\dot{H} \sim 0 \quad \frac{\ddot{a}}{a} = \dot{H} + H^2 > 0$$
Late time acceleration: a surprise, not expected from General Relativity

- **Einstein equation**
  
  \[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]
  
  spacetime curvature energy and momentum

  Specialize it to the FRW (homogeneous, isotropic, spatially flat) geometry:

- **Friedmann equation**
  
  \[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho \]
  
  expansion rate energy density

If we consider only matter (including both ordinary and dark matter), what observation tells us:

- **LHS** \( H(t) = \frac{\dot{a}}{a} \) approaching to a const. but **RHS** \( \rho_{\text{matter}}(t) = \frac{\rho_0}{a^3(t)} \) falling off
  
  \( \dot{H} \sim 0 \Rightarrow \frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \)

- Two approaches to this problem:
  
  **Modify LHS:** Modified Gravity or **Add more energy to RHS:** Dark Energy
A MG model: nonlocally modified gravity

- **Model:** Deser and Woodard, PRL 99 (2007) 111301, arXiv:0706.2151 proposed to explain late time acceleration without DE using a nonlocal Lagrangian

\[
\mathcal{L} = \frac{1}{16\pi G} \sqrt{-gR} \left[ 1 + f\left(\frac{1}{\Box} R\right) \right] = \mathcal{L}_{E-H} + \Delta \mathcal{L}
\]

- What does \(\frac{1}{\Box} R\) mean?

For example, for the FRW geometry

\[ds^2 = -dt^2 + a^2(t)d\bar{x} \cdot d\bar{x}\]

\[\Box F(x) = \left( -\partial_t^2 - 3H\partial_t + \frac{\nabla^2}{a^2} \right) F(x)\]

If F is a function of time only,

\[
\frac{1}{\Box} F(t) = -\int_{t_i}^{t} \frac{dt'}{a^3(t')} \int_{t_i}^{t'} dt'' a^3(t'') F(t'') \quad \Rightarrow \quad \frac{1}{\Box} R = -\int_{t_i}^{t} \frac{dt'}{a^3(t')} \int_{t_i}^{t'} dt'' a^3(t'')(6\dot{H} + 12H^2)
\]

In general, find a Green's function for \(\Box\)

\[
\frac{1}{\Box} F(t, \bar{x}) = \int d^4 x' G(x, x') F(t', \bar{x}')
\]
Retarded Green’s function for □ for the FRW background

\[ \square G_{ret}(x; x') = \left( -\dot{a}^2 - 3H\dot{a} + \frac{\nabla^2}{a^2} \right) G_{ret}(x; x') = \delta^4(x - x') \]

can be constructed using the massless, minimally coupled scalar mode functions \( u(t, k) \) for arbitrary \( a(t) \)

\[ \ddot{u} + 3H(t)\dot{u} + \frac{k^2}{a^2(t)} u = 0 \]

No general solution for the mode function \( u(t, k) \) but the sub-horizon limit, use the WKB approximation to find

\[ u(t, k) = \frac{1}{\sqrt{2k}} \exp \left[ -ik \int^t_{t'} \frac{dt'}{a(t')} \right] \]

This will be used for the perturbation eqns involving acting \( \frac{1}{\square} \) on \( \Phi(t, \vec{x}) \) & \( \Psi(t, \vec{x}) \).
Main features of this nonlocally modified gravity

- $\frac{1}{R}$ is dimensionless: doesn’t introduce a new mass parameter
- $\frac{1}{R}$ is extremely small for the Solar System: $\frac{1}{R} = \frac{GM}{c^2 r} \to 10^{-9}$ at the surface of the earth

- Two built-in delays of the onset of acceleration
  - $R \approx 0$ for radiation-domination $\rightarrow$ no modification until $t \sim 10^5 \text{yrs}$
  - Grows logarithmically after that: $\frac{1}{R} \approx \frac{4}{3} \ln \left( \frac{t}{t_{eq}} \right) \to \frac{1}{R}\Big|_{t_{now}=10^{10} \text{yrs}} \sim -15$

  don’t need huge fine-tuning for the parameter function $f$
  to match with the LCDM expansion history

\[
\begin{align*}
\text{Deffayet and Woodard,} \\
\text{JCAP 08 (2009) 023, arXiv:0904.0961}
\end{align*}
\]
Modified field eqn gives the expansion history

- Varying the action w.r.t the metric we get the modified field eqn:

\[
G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}
\]

\[
\Delta G_{\mu\nu} = \left[ G_{\mu\nu} + g_{\mu\nu} - D_{\mu}D_{\nu} \right] \left\{ f\left(\frac{1}{\bar{R}}\right) + \frac{1}{\bar{R}} \left[ R f'(\frac{1}{\bar{R}}) \right] \right\} + \left[ \delta_{\mu}^{(\rho)} \delta_{\nu}^{(\sigma)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \partial_{\rho} \left(\frac{1}{\bar{R}}\right) \partial_{\sigma} \left(\frac{1}{\bar{R}} \right) R f'(\frac{1}{\bar{R}}) \right]\]

- The 0th order eqns for the FRW background

\[
3H^2 + \Delta G_{00} = 8\pi G \rho
\]

\[
-2\dot{H} - 3H^2 + \frac{1}{3a^2} \delta^{ij} \Delta G_{ij} = 8\pi G \rho
\]

- Construction of the nonlocal distortion function \( f \):

  The free parameter \( f \) can be chosen to fit any expansion history:

  In particular, the function \( f \) is fitted to mimic the expansion history of LCDM Deffayet and Woodard, JCAP 08 (2009) 023, arXiv:0904.0961.

\[
f(X) \sim \frac{1}{4} \left[ \tanh \left( \frac{1}{3}X + \frac{5}{2} \right) - 1 \right]
\]

What about the growth history?
Perturbations in modified gravity

To see the growth of structure, perturb the metric around the FRW background; perturbations encoded in the two potentials, which depend on space and time

\[ ds^2 = -(1 + 2\psi)dt^2 + a^2(t)(1 - 2\Phi)d\vec{x} \cdot d\vec{x} \]

Generally, differences between GR and MG

| GENERAL RELATIVITY | MODIFIED GRAVITY |
|--------------------|------------------|
| \( \Phi + \Psi = 0 \) | \( \Phi + \Psi \neq 0 \) |
| \( \nabla^2 \Phi = -4\pi G\rho_m a^2 \delta \) | \( \nabla^2 \Phi = -4\pi G_{\text{eff}}\rho_m a^2 \delta \) |
Perturbation Equations

- The modified field equations (generically)

\[
\begin{align*}
G_{00} + \Delta G_{00} &= 8\pi G T_{00} \\
\bar{G}_{00} + \Delta \bar{G}_{00} + \delta \left( G_{00} + \Delta G_{00} \right) &= 8\pi G \left( \bar{T}_{00} + \delta T_{00} \right) \\
G_{ij} + \Delta G_{ij} &= 8\pi G T_{ij} \\
\bar{G}_{ij} + \Delta \bar{G}_{ij} + \delta \left( G_{ij} + \Delta G_{ij} \right) &= 8\pi G \left( \bar{T}_{ij} + \delta T_{ij} \right)
\end{align*}
\]

- The FRW background equations govern the expansion history

\[
\bar{G}_{00} + \Delta \bar{G}_{00} = 8\pi G \bar{T}_{00}, \quad \bar{G}_{ij} + \Delta \bar{G}_{ij} = 8\pi G \bar{T}_{ij}
\]

- Perturbations equations govern the growth history

\[
\delta \left( G_{00} + \Delta G_{00} \right) = 8\pi G \delta T_{00}, \quad \delta \left( G_{ij} + \Delta G_{ij} \right) = 8\pi G \delta T_{ij}
\]

Arranging these in a more conventional form (for the far sub-horizon modes, \( k >> H_a \)):

- Poisson equation

\[
k^2 \Phi + k^2 \Phi \left\{ f(\bar{X}) + \frac{1}{\bar{X}} \left[ \bar{R} f'(\bar{X}) \right] \right\} + \frac{k^2}{2} \left\{ f'(\bar{X}) \frac{1}{\bar{X}} \delta R + \frac{1}{\bar{X}} \left[ f'(\bar{X}) \delta R \right] \right\} = 4\pi G a^2 \bar{\rho} \delta
\]

\[
k^2 \Phi + k^2 E[\Phi] = 4\pi G a^2 \bar{\rho} \delta
\]

- Gravitation slip equation

\[
(\Phi + \psi) = 8\pi G \delta T_B - (\Phi + \psi) \left\{ f(\bar{X}) + \frac{1}{\bar{X}} \left[ \bar{R} f'(\bar{X}) \right] \right\} - f'(\bar{X}) \frac{1}{\bar{X}} \delta R - \frac{1}{\bar{X}} f'(\bar{X}) \delta R
\]
Solution for the slip equation

- **GR:** $\Phi + \Psi = 0$

- In this nonlocally modified gravity $\Phi + \Psi \neq 0$

Gravitational slip as a function of redshift in the nonlocal model. The two curves, \textit{barely distinguishable}, are for $k=0.03$ (red) and $k=0.3$ $h$/Mpc (blue)

Park and Dodelson, PRD \textbf{87}, 024003 (2013)
arXiv: 1209.0836
Solution for the Poisson equation

- **GR:** \( G_{\text{eff}} = G \)
- In this nonlocally modified gravity, the Poisson eqn becomes
  \[
  k^2 \Phi + k^2 E[\Phi] = 4\pi G \alpha^2 \tilde{\rho} \delta
  \]
- Define the effective Newton’s constant as
  \[
  k^2 \Phi \equiv 4\pi G_{\text{eff}} \alpha^2 \tilde{\rho} \delta
  \]
  which is
  \[
  G_{\text{eff}} \frac{k^2 \Phi + k^2 E[\Phi]}{G} = \frac{1}{1 + \frac{E[\Phi]}{\Phi}}
  \]

The fractional change to Newton’s constant as a function of redshift \( z \). The two curves, which depict the evolution for \( k=0.03 \) (red) & \( k=0.3 \) h/Mpc (blue) is virtually scale-independent.

Why a dip?

Park and Dodelson, PRD 87, 024003 (2013) arXiv: 1209.0836
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  ➡️ **Modified Gravity** should make \( G \) grow to compensate the dropping of energy density!
Why a dip in the curve of \( \frac{G_{\text{eff}}}{G} \)?

0\textsuperscript{th} order eqn has 1 effect: rescaling of G (the same as Blue curve)

1\textsuperscript{st} order eqn has 2 effects: Blue curve: rescaling of G

\[
\frac{G_{\text{eff}}}{G} = \frac{1}{1 + \frac{E[\Phi]}{\Phi}} \cdot \left( f(\bar{X}) + \frac{1}{\Phi} \left[ Rf'(\bar{X}) \right] \right) + \frac{1}{2\Phi} \left( f'(\bar{X}) \frac{1}{\Phi} \delta R(\Phi) + \frac{1}{\Phi} \left[ f'(\bar{X}) \delta R(\Phi) \right] \right)
\]

Red curve: against the rescaling
"A discriminating probe of gravity at cosmological scales”
Zhang, Liguori, Bean and Dodelson PRL 2007

\[ E_G \sim \frac{\text{Galaxy position-lensing correlation}}{\text{Galaxy position-redshift space correlation}} \]

\[ \sim \frac{\text{Laplacian of Newtonian potential}}{\text{Peculiar velocity divergence}} \]

\[ E_G \sim \frac{\left< \delta_g \kappa \right>}{\left< \delta_g \theta \right>}, \quad \frac{\Phi - \Psi}{\ddot{\delta}}, \quad \frac{\Omega_{0m} \tilde{G}_{\text{eff}}}{\beta} \]

\[ \tilde{G}_{\text{eff}} = \frac{G_{\text{eff}}}{G} \quad \beta = \frac{d \ln D}{d \ln a} \]
Distinguishing between modified gravity and LCDM

Zhang, Liguori, Bean and Dodelson PRL 2007

$E_{G_{\text{nonlocal}}} (z = 0.5) = 0.37$
$E_{G_{\text{nonlocal}}} (z = 1.0) = 0.30$
$E_{G_{\text{nonlocal}}} (z = 1.5) = 0.28$
$E_{G_{\text{nonlocal}}} (z = 2.0) = 0.26$
$E_G(k)$  As a function of $k$, $z = 1.5$ fixed

Nonlocal

ADEPT+LSST  $1.3 < z < 1.7$

SKA
As a function of $z$, $k=0.03 \, h/\text{Mpc}$ fixed

**Diagram**

- **GR**
- **Nonlocal**

**Axes**
- $z$-axis
- $EG$-axis

**Data**
- The graph shows the behavior of $EG$ as a function of $z$ with $k=0.03 \, h/\text{Mpc}$.
- The data points or curves for GR and Nonlocal are indicated on the graph.

**Analysis**
- The plot illustrates how $EG$ changes with $z$ for the given $k$ value.
- Comparing GR and Nonlocal models, differences in behavior can be observed.

**Conclusion**
- The graph provides insights into the nonlocal effects compared to GR for a specific value of $k$.
Summary and Conclusion

- A nonlocally modified gravity model proposed by Deser and Woodard gives an explanation for current cosmic acceleration.
- This model predicts a pattern of growth that differs from standard general relativity (+ dark energy) at the 10-30% level.
- These differences will be easily probed by the next generation of galaxy surveys, so the model should be tested shortly.