Can the Red Shift be a consequence of the Dilaton Field?

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Abstract

The possibility that the expansion rate of the Universe, as reflected by the Red Shift, could be produced by the existence of the dilaton field is explored. The analysis starts from previously studied solutions of the Einstein equations for gravity interacting with a massive scalar field. It is firstly underlined that such solutions can produce the observed values of the Hubble constant. Since the Einstein-Klein-Gordon lagrangian could be expected to appear as an effective one for the dilaton in some approximation, the mentioned solutions are applied to study this field. Therefore, the vacuum expectation value for the dilaton is selected to be of the order of the Planck mass, as it is frequently fixed in string phenomenology. Then, it follows that the value of its effective mass should be as low as $m = 3.9 \times 10^{-29} \text{ cm}^{-1}$ in order to produce the observed expansion rate. The discussion can also predict a radius of the Universe of the order of $10^{29} \text{ cm}$. Finally, after adopting the view advanced in a previous work, in which these mentioned solutions are associated to interior configurations of collapsed scalar fields, a picture of our Universe as a black hole interior is suggested.
I. INTRODUCTION

The relevance for the physical world of the string theory is a central issue to be decided for the future research activity in this basic area of modern theoretical physics [1,2,3]. At present the experimental limitations make unclear the perspectives for the clarification of this question [4,5]. Therefore, it becomes of central interest the search for any signal in Nature from the string structure of matter.

As a preamble it could be remarked the construction of realistic models in four dimensions from superstring theory, the breakdown of various symmetries in the low energy limit play a very crucial role. At these low energies, it is commonly expected that a field theoretical effect should be dominant in order to generate the hierarchy of scales currently observed. In one of the preferred views: the two steps scenario, string effects dominate to lift vacuum degeneracy and field theory effects are responsible for to break supersymmetry. In this approach, the vacuum expectation values (vev) of the dilaton and other vacuum fields are fixed at high energies with values near to Planck mass. A problem here resides in the implementation of this scenario due to no yet complete understanding of nonperturbative string effects. The search of a deeper knowledge about these effects is currently very active due to the findings of equivalence of the various string theories under duality [6].

In this note, our purpose is to explore the possibility for the existence of a macroscopic signal form the string structure of matter. it would arise from the relevant scalar field arising in string theory: the dilaton, which reflects the condensation of zero mass strings [1]. The idea is the following: As it was recalled above, for the construction of string phenomenology, the mean vev of the dilaton is fixed at values of the order of the Planck mass [4,5]. Then we may consider the properties of the dilaton as input parameters of a particular type of solution of the Einstein equations for massive scalar field interacting with gravity [7,8]. Such solutions are regular at a centre of spherical symmetry and show a decreasing gravitational potential away from the center. The decaying potential could be interpreted as reflecting the expansion rate of the Universe. Consider also the possibility that a small value of the
mass for the dilaton can be generated due to combined non-perturbative string corrections and the breaking of supersymmetry. Then, it seems a sensible issue to determine the value of such a mass which can predict the observed Hubble constant when the dilaton vev is taken of the order of the Planck mass. The resulting mass values calculated here turn to be extremely small. This outcome seems at least compatible with the masses of the dilaton field in the perturbative string theory[1]. Such mass values, up to our knowledge, could not be rejected under the basis of the current understanding of string phenomenology.

It should be recognized that the picture here examined would introduce radical changes in the most accepted views at present about the dynamical evolution of the Universe [9]. Therefore, a further inspection of its implication is in need.

It should be also said that the above mentioned solutions corresponds to the internal part of the extended solutions investigated in [7]. In that work the central aim was to obtain indications about the possibility of interpreting such field configurations as internal space times of black holes. The external metric in the proposed solutions is the Schwartzschild one with a null scalar field. An important circumstance in such an interpretation is that the proper mass of the interior scalar field is exactly coinciding with the proper mass of a Schwartzschild solution having the horizon at the precise radius in which the internal metric become singular [7]. Work is being directed to rigorously argument that these global field configurations solve the Einstein-Klein-Gordon equation in some generalized mathematical sense. The complete validation of these considerations then, would indicate the interpretation of our Universe as an internal space time of a black hole of collapsed dilaton matter.

II. 1. THE SPECIAL SOLUTIONS

The coupled set of Klein-Gordon and Einstein equations in spherical coordinates which was considered in Ref. [7] has the explicit form

\[
\frac{u'(\rho)}{\rho} - \frac{1 - u(\rho)}{\rho^2} = - (8\pi k/c^4) \left( \frac{u(\rho) \varphi'(\rho)^2}{2} + \frac{(mc/\hbar)^2 \varphi(\rho)^2}{2} \right)
\] (1)
\[
\frac{u(\rho)}{v(\rho)} \frac{v'(\rho)}{\rho} - \frac{1 - u(\rho)}{\rho^2} = + (8\pi k/c^4) \left( \frac{u(\rho)\varphi'(\rho)^2}{2} - \frac{(mc/\hbar)^2 \varphi(\rho)^2}{2} \right)
\]  
(2)

\[
m^2 \varphi(\rho) - u(\rho) \varphi''(\rho) = \varphi'(\rho) \left( \frac{u(\rho) + 1}{\rho} - (8\pi k/c^4) \frac{\rho(mc/\hbar)^2 \varphi(\rho)^2}{2} \right)
\]  
(3)

where the invariant interval has been taken as defined by the spherical coordinates form

\[
ds^2 = v(\rho)dx_o^2 - u(\rho)^{-1}d\rho^2 - \rho^2 (\sin^2 \theta d\phi^2 + d\theta^2).
\]

\[dx_o = cdt\]

The scalar field \(\varphi\) is real and its mass \(m\) can be absorbed in the definition of a new radial variable \(r = mcp/\hbar\). Moreover, the scalar field also can been scaled as \(\varphi = \phi/\sqrt{8\pi k/c^4}\) in order to absorb the factor \(8\pi k/c^4\) multiplying the energy momentum tensor in it. After these changes, the equations take the form

\[
\frac{u'(r)}{r} - \frac{1 - u(r)}{r^2} = - \frac{u(r) \varphi'(r)^2}{2} - \frac{\phi(r)^2}{2}
\]  
(4)

\[
\frac{u(r)}{v(r)} \frac{v'(r)}{r} - \frac{1 - u(r)}{r^2} = + \frac{u(r) \varphi'(r)^2}{2} - \frac{\phi(r)^2}{2}
\]  
(5)

\[
\phi(r) - u(r) \varphi''(r) = + \phi'(r) \left( \frac{u(r) + 1}{r} - \frac{r\phi(r)^2}{2} \right)
\]  
(6)

It can be observed that equations (4) and (6) are closed because they do not depend on \(v(r)\). In place of \(v(r)\), below it will be sometimes used the variable

\[
\nu(r) = \log (v(r)).
\]

The set of equations (4)-(6) have solutions showing a leading asymptotic behavior near the origin \(r = 0\) of the following form

\[
u(r) = 1 - \frac{\phi_0 r^2}{6} + ...
\]  
(7)

\[
\phi(r) = \phi_0 \left( 1 + \frac{r^2}{6} \right) + ...
\]  
(9)

The behavior of this solutions motivated the present note. As it can be noticed from them, the gravitation potential decreases away from the origin of coordinates with
a quadratic dependence with the radial distance. Near the origin, the metric is approximately lorentzian. Then, some questions are directly suggested: Could these solutions be applied to construct a model of the expansion rate of the Universe?, assuming that so is the case: What is the nature of the scalar field being considered?. Similar questions have been addressed in the literature in connection with the possible role of a cosmological constant in Robertson-Walker (WR) cosmologies [9]. The existence of physical sources of such cosmological constants has been a limitation for the introduction of a cosmological term in such models (It may be useful to say that the massive term for the scalar field plays a similar role in the dynamical equations, that a cosmological constant if the field is space-time independent in some region). However, at present there are new theoretical ideas related with string theory phenomenology which assume the existence of a vacuum expectations of scalar fields. These values could be suspected to be connected the parameter $\phi_0$ in equations (7)-(9).

Concretely, as it was mentioned before, in one of the adopted views about the construction of string phenomenological theories, a high value of the vev for the dilaton field of the order of the Planck mass is fixed [4,5]. On another hand, the perturbative value of the dilaton mass is null, a fact that can cast doubts on the present proposal. However, it can be taken into account the possibility that a mass can be generated by string non perturbative corrections and also by the various spontaneous symmetry breaking effects that are expected to occur in order to reproduce the observable experience. Therefore, it seems reasonable to assume non vanishing values for the dilaton effective mass. As it will be seen, the magnitude of the product of the dilaton mass and its vev is determined from the condition of fixing the observable value of the Hubble constant.

Let us consider the equation (8) which defines the gravitation potential in the vecinity of the origin when written in the form

$$v(\rho) = 1 + 2\Phi(\rho)/c^2,$$

(10)

where $\Phi(\rho)$ is the gravitational potential.

Then, for $\Phi$ it follows
\[ \Phi(\rho) = -(4\pi/6)G_N\phi_0^2(m/h)^2\rho^2. \]  

(11)

The Hubble law is now expressed by the relation

\[ E_{Pot}(0) = 0 = E_{Pot}(\rho) + E_{Kin}(\rho) = M \Phi(\rho) + M \frac{V^2}{2} \]  

(12)

where the l.h.s. is the potential energy of a galaxy of mass \( M \) at the origin of coordinates and \( V(\rho) = d\rho/dt \) is its velocity at the radial distance \( \rho \).

Recalling the definition of the Hubble constant \( H_0 \) through \( (V/\rho)^2 = H_0^2 \) the following relation between the mass and the vev of the scalar field follows

\[ m^2 \phi_0^2 = \frac{h^2H_0^2}{4\pi k} \]  

(13)

After assuming the values for \( H_0 = 75 \times 10^5 \text{ cm/(sMpc)} \) and the gravitation constant \( k = 6.67 \times 10^{-8} \text{ cm/gs}^2 \) the product of the mass and the vev takes the value (in rationalized units: \([m] = \text{cm}^{-1}, [\phi] = \text{cm}^{-1}\))

\[ m\phi_0 = 24584 \text{ cm}^{-2}. \]  

(14)

The relationship (14) has been gotten starting from a solution for a massive scalar field in interaction with the gravity, within the context of general classical relativity. However, as it was discussed in Ref.[8] the Einstein Klein Gordon equations can be expected to become a reasonable effective action for the dilaton field in a consistent context of gravity based in superstring theory, if the dilaton field acquires mass through any mechanism.

Finally after setting the vev of the dilaton to 1 in the Planck scale, i.e.

\[ <\phi_0> = 0.63 \times 10^{33} \text{ cm}^{-1} \text{ using (14) it is obtained} \]

\[ m = 3.9 \times 10^{-29} \text{ cm}^{-1}. \]  

(15)

Therefore, if the nonperturbative corrections or symmetry breaking effects are able to produce a tiny non vanishing dilaton mass, it would be sufficient to generate the observed expansion rate of the galaxies.
Let us consider now an additional implication of the possibility being discussed.

As it was mentioned in the introduction, the approximate solutions near \( r = 0 \) can be numerically extended away from the origin by taking as the initial conditions the values at small radial distance of \( u(r), v(r), \phi(r) \) and \( \phi'(r) \). The value of the vev is taken as \( \phi_0 = 4.5 \) in (7)-(9) which is determined for a fixed magnitude of the dilaton vev of the order of the Planck mass. The initial values and the first derivative of the scalar field necessary for the numerical algorithm were calculated also from (7)-(9) at the radial position \( r = 0.003 \).

The results of [7] show a decreasing behavior of the function \( u(r) \), which approaches zero linearly at some radius \( r_0 = 0.502416 \). The picture for the variation of \( v(r) \), while similarly decreasing, tends to approach a constant value differing from zero when \( r \) approaches \( r_0 \). The scalar field, on another hand, increases away from the origin and approaches a constant value at \( r_0 \), but with a fast growing slope diverging at \( r_0 \).

The asymptotic behavior near \( r_0 \) of the numerical solution was also determined in [7] to be given by

\[
\begin{align*}
    u(r) &= (r_0\phi_0^2 - 2/r_0)(r_0 - r) + \ldots \\
    \phi(r) &= \phi_0 - 2\sqrt{r_0 - r}/\sqrt{r_0} + \ldots
\end{align*}
\]

It was shown in [7] that the singular behavior near \( r_0 \) of the numerical solution is well reproduced by this asymptotic expressions. An important property of the dependence with the radial distance of \( u(r) \) is the fact that this quantity tends to vanish at \( r = r_0 \). This fact allows to show that the proper mass of the scalar field in the interior region exactly coincides with the proper mass of a Schwartzschild solution having its horizon at the same radius \( r_0 \). This circumstance strongly suggest the interest of a global field configuration defined by the here considered one at the internal region after extending it to coincide with the Schwartzschild solution at exterior by also taking null values of the scalar field (as suggestd by the non-hair theorems). Such fields evidently solve the dynamical equations everywhere except at of the shell \( r = r_0 \). Then, if some localized
sources axist at the boundary, they should not contribute to the total value of the proper mass. This remarks strongly indicate that such fields should correspond to solutions in all the space in some generalized mathematical sense. Work on the verification of this possibility is under development.

III. CONCLUSIONS

An alternative for the physical interpretation of the observed expansion rate of the Universe is explored. It rests in the assumption that the dilaton field in string theory gets a small mass under non-perturbative corrections or the symmetry breakings needed for string theory to become a realistic theory. The validity of the picture would implicate the interpretation of the Universe as the interior region of a black hole formed in the collapse of dilaton field matter. It is clear that the formulation of the proposed model should undergo a closer scrutiny in connection with its ability to explain the many observed cosmological data. The investigation on these questions would be considered elsewhere.

The discussion has some resemblance with the analysis of Robertson-Walker models under assuming a non-vanishing cosmological constant [9]. It could be imagined that what is done here is to explore the possibility that the vev of the dilaton field play a similar role that a cosmological constant in determining the expansion rate [9].
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