Towards Understanding the Temperature and Current Sensitivities of Transition-Edge Sensors

Yu Zhou$^{1,2}$

$^1$ Tsinghua University, 100084, Beijing, China
E-mail: zhouyu@ac.jaxa.jp

Abstract. The transition-edge sensor (TES) technology is widely applied to X-ray spectroscopy or imaging applications at wavelengths ranging from infrared to sub-mm, with the aim of potentially achieving unprecedented spectral resolution and detection sensitivity. As a critical component of the X-ray microcalorimeter, the TES affects the energy resolution via two main parameters: temperature sensitivity and current sensitivity. Tremendous efforts have been made to fabricate TESs with high temperature sensitivity and low current sensitivity, in order to enhance the energy resolution of the microcalorimeters. However, since the resistance of TESs is a complex function of temperature, current, and magnetic field, it is difficult to optimize the operational point of the detector from the first principle. We conducted an experiment to map the parameter space of a sample of MoAu TESs in the transition phase. The results show that the current sensitivity depends only on the resistance of the TESs, which is in line with the two-fluid model. The figure of merit of energy resolution dependence on the quasiparticle diffusion length has been compared with the prediction of the two-fluid model, which indicates that the time-averaging critical current of phase-slip centers is not a constant throughout the superconducting transition. The magnetic field could potentially enhance the energy resolution by reducing the charge imbalance relaxation time.

1. Introduction

Superconductor in phase transition (Transition-Edge Sensor, TES) behaves like an exquisitely sensitive thermometer, which enables them a wide application in X-ray microcalorimeters and sub-mm bolometers due to their excellent expected sensitivity and energy resolution. TESs used in X-ray microcalorimeters have a typical length scale of $\sim 100 \mu$m and are often operated at the low-temperature environment of $\sim 100$ mK in order to suppress the Johnson noise. Normal metal and superconducting material are deposited into thin-film bilayers to design the transition temperature as a whole, taking advantage of the proximity effect. For TESs that is fabricated by normal-superconducting bilayers, describing the transitions need more than one single $T_c$, since they are usually in a broadened range of temperature. Physics models have been brought up trying to explain their resistive mechanism and predict the detector performance. Two main competitive models are phase-slip line model and weak-link model. Phase-slip line model is based on the two-fluid model, which considers the current components as superconducting and normal parts separately, and the total resistance consists of many individual voltage units [1, 2, 3]. Weak-link model assumes that the TES in the transition phase and the superconducting wires

$^2$ Present address: Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, 3-1-1 Yoshino-dai, Sagamihara, Kanagawa 252-5210, Japan.
on both sides together have composed a Josephson junction [4, 5]. Each one of the models has unveiled partial properties of the TES. Nevertheless, since the TES resistance in the transition is a complex function of temperature, current, and magnetic field, how to optimize the operational conditions to achieve the best energy resolution remains a question.

The thermoelectric circuit analysis of X-ray microcalorimeter has suggested the TES energy resolution is proportional to \( \sqrt{\frac{4k_B T^2 C}{\alpha}} \times \text{Noise} \), assuming the resistance in superconducting transition is also a function of the current [6]. Given that temperature \( T \) and heat capacity \( C \) are usually limited by ADR cooling power and the design of detection efficiency, TES temperature sensitivity (\( \alpha = \frac{\partial \ln R}{\partial \ln T} \)) and current sensitivity (\( \beta = \frac{\partial \ln R}{\partial \ln I} \)) are the two main parameters for optimizing the detector performance. Various experiments have proved the fact that increasing bias current of the TES will drive the \( R - T \) curve to a broader extension toward lower temperature [7], consistent with the fact that current sensitivity degrades the energy resolution. In order to achieve optimized operation of TESs, a proper combination of temperature, current and field for bias condition is needed to have high temperature sensitivity and relatively low current sensitivity, or minimum \( \sqrt{1 + 2\beta/\alpha} \). The difficulty is that the correlation between \( \alpha \) and \( \beta \) has not been clearly understood. Many works measuring \( I - V \) curves find both \( \alpha \) and \( \beta \) increase as resistance decreases monotonically[8, 9, 3, 10], whereas there are also exceptions, for example: Jethava et al (2009) shows the \( \beta \) having no correlation with \( \alpha \) when \( \beta > 0.2 \), but correlating with \( \alpha \) when \( \beta < 0.2 \)[11]. Another experiment from Lindeman et al (2008) suggests that \( \beta \) has been decreasing with TES resistance, while \( \alpha \) stays rather constant for 20%~80% normal resistance[7]. In Smith et al (2013), both \( \alpha \) and \( \beta \) were found to oscillate with an applied magnetic field and stay correlated with each other[5]. All of these results were only presenting one projection of the entire parameter space and could not answer the question of whether a sweet spot exists to minimize \( \sqrt{1 + 2\beta/\alpha} \), as the figure of merit of energy resolution. Therefore, we experimented to map the temperature sensitivity and current sensitivity of TESs throughout the whole transition and calculate for \( \sqrt{1 + 2\beta/\alpha} \), to understand the physics implication of the temperature sensitivity and current sensitivity and their dependency of the operational conditions.

2. Experiment Setup and Data Analysis

Figure 1 shows an overview of the experimental setup. The experiments were performed in an adiabatic demagnetization refrigerator (ADR) which cools down the TES detectors to their critical temperature at \( \sim 100 \) mK. Four TES devices made of 140 \( \mu \)m square Mo/Au bilayer (50 nm for Mo and 260 nm for Au in thickness) were tested, named as MXW0203, MXW1213, MXW4647, MXW4950, respectively [12]. Devices MXW1213 and MXW4647 have three extra gold stripes deposited transversely on top of the normal metal layer as a special geometry design, while the devices MXW0203 and MXW4950 don’t have any normal finger structure with them and are designed as bare bilayers. TES devices MXW4647, MXW4950 are grown onto silicon nitride membrane so that they have relatively low thermal conductances(\( G \sim 0.3 \) nW/K) to the heat sink. In comparison, devices MXW0203 and MXW1213 are directly deposited onto the silicon wafer so their thermal conductances to the heat sink are relatively high(\( G \sim 3 \) nW/K). The thermal conductances between the TES devices and heat bath were estimated by fitting the power dissipation in TES at differently controlled bath temperature, while the temperature of the TES was kept at the same as the corresponding bias point at around 90% normal resistance were picked out, where the current sensitivity is negligible. The fitting error of thermal conductances for four TESs are within 2%. \( I - V \) curves were densely measured at different and steady controlled bath temperature, with a step of 0.2 mK. The raw data contains thousands of data points for each \( I - V \) curve, binning of data was applied to average out the noise with 50 data points for each group. For binned data, temperature in TES for every point
on \( I - V \) curves were calculated from corresponding bath temperature and thermal conductance. Based on the definition, \( \alpha \) was calculated as the derivative of temperature at the same current value, and \( \beta \) was calculated as the derivative of current at the same temperature (\( \alpha = \frac{R_0}{T_0} \frac{\Delta R}{\Delta T} |_{T_0}, \beta = \frac{I_0}{R_0} \frac{\Delta R}{\Delta I} |_{T_0} \)). The resistance difference for derivative calculation is as small as 0.1 m\( \Omega \). For further investigation into \( \alpha \) and \( \beta \)'s dependence on TES temperature and current, the binned data were reassigned into 11 different resistance groups ranging from 0.05\( R_N \) to 0.6\( R_N \) at an interval of 0.05\( R_N \). In every resistance group, the measured quantities are averaged and their scattering levels are reflected in errors that are calculated as the standard deviation. A uniform B-field can be generated perpendicularly to the TES plane via a 60-turn NbTi field coil inside the shielded test box.

\[ J - J_s(X_1) = J_n(X_1) = -\frac{1}{e\rho} \frac{d\mu}{dx} \approx \frac{\Delta\mu}{2e\rho\Lambda_*} \] (1)

where \( J_s(X_1) \) denotes for time-average superconducting current density, \( J_n(X_1) \) for normal current density, \( J \) for total current density, \( \rho \) for resistivity, \( \Delta\mu \) for normal electrochemical

![Figure 1. Experimental setup for the TES measurements.](image)

3. Two-fluid Model
In the two-fluid model, the quantum phase-slip centers are considered as basic voltage units[13]. A gradient of the chemical potential of normal electrons \( \mu \) must exist to drive the normal current and lead to a voltage drop across each phase-slip center[14], while the chemical potential of cooper pairs, \( \mu_p \), drives the phase of superconducting order parameter to grow linearly with time, according to the Josephson relation. As superconducting density grows, once the local critical value is reached, phase slip occurs and turns cooper pairs into normal current, while the superconducting phase itself vanishes to zero and starts acceleration again. The typical length scale of this cyclic steady-state to generate the voltage drop is defined as the quasiparticle diffusion length \( \Lambda_* \)[15, 16].

The voltage drop across one unit of phase-slip center can be written as:

\[ J - J_s(X_1) = J_n(X_1) = -\frac{1}{e\rho} \frac{d\mu}{dx} \approx \frac{\Delta\mu}{2e\rho\Lambda_*} \] (1)
potential difference across the phase-slip center, and $\Lambda^*$ stands for the quasiparticle diffusion length. The voltage drop across the phase-slip center equals to:

$$V = \Delta \mu/e \approx 2 \rho \Lambda^*[J - J_s(X_1)]$$

(2)

where $\Lambda^*$ denotes for the cross-section area that current flows through. Two-fluid model assumes a number of superconducting phase-slip centers contributing to the total voltage drop. The current is split into superconducting and normal components:

$$I(T) = c_I I_c(T) + \frac{V}{c_R R_n}$$

(3)

where $I_c$ denotes for the critical current of TES and $c_I = I_c/I_c$ defines the ratio of the time-averaging critical current divided by the local critical current. $R_n$ denotes for normal resistance of the TES. To further derive the temperature and current sensitivity, the critical current dependence on temperature $I_c(T) = I_{c0}(1 - T/T_c)^{3/2}$ needs to be introduced from Ginzburg-Landau theory, although the relation is valid only when temperature approaches $T_c$. Then the temperature sensitivity can be written in an analytical form:

$$\alpha_I = \frac{R_0}{R_0} \frac{\delta R}{\delta T} |_{T_0} = \frac{T_0}{R_0 I_{c0}} \frac{\delta V}{\delta I} = \frac{3}{2} c_I c_R \frac{R_n}{R_0} \frac{I_{c0}}{T_0} \frac{T_0}{T_c} (1 - \frac{T_0}{T_c})^{1/2}$$

(4)

When the temperature stays constant $\delta V/\delta I = R_0 + I_{c0}(\delta R/\delta I)$, the current sensitivity ends up with a simple expression:

$$\beta_I = \frac{1}{\delta I} \frac{\delta V}{\delta I} - 1 = c_R \frac{R_n}{R_0} - 1$$

(5)

Combining equation 2 and 3, one can obtain $c_R = 2N \rho_n \Lambda^*/(A R_n) = 2 \Lambda^* N/L$, where $\rho_n$ is for normal resistivity and $L$ for detector length, and $N$ stands for the total number of the phase-slip centers (or phase-slip lines). In this way, $N \Lambda^*$ can be estimated from the measurement of current sensitivity.

4. Results and Discussion

4.1. Temperature and Current Sensitivity Distribution

The measured temperature and current sensitivity distribution is shown in Fig 2. For all TES devices, $\alpha$ substantially increases with TES temperature and decreases with TES current, indicating that it requires less current and higher temperature at the same bias point to achieve better energy resolution. For MXW0203, data are sufficient to show that $\alpha$-T correlation becomes weaker as TES turns normal, which is plausible since $\beta$ drops towards zero at high resistance bias point so that TES resistance is less modulated by the current.

On the other hand, $\beta$ does not show significant difference when either current or temperature changes, but seems to depend only on the TES resistance, which is consistent with the prediction of current sensitivity from the two-fluid model that $\beta$ is determined by $N \Lambda^*/L$ and $R_{TES}$. $L$ is detector length which is a constant for one device. If $N \Lambda^*$ does not or only weakly depend on the TES bias point, the current sensitivity is expected to be determined only by the resistance.

4.2. Quasiparticle Diffusion Length and the Energy Resolution

The product of quasiparticle diffusion length and number of phase slip centers $N \Lambda^*$ can be estimated from the current sensitivity according to equation 5. Since Fig 2 has demonstrated that $\beta$ does not depend on either temperature or current except for TES resistance, all data points within one resistance group are adopted for $N \Lambda^*$ evaluation at that bias point. Fig 3 plots
the measured $N\Lambda_\ast$ as a function of TES resistance, in which the error bars are estimated by the standard deviation of the data points in each group. For all devices, $N\Lambda_\ast$ ranges from about 20 $\mu$m to 40 $\mu$m and is in general correlated with TES resistance above 0.3 $R_N$. Nevertheless, this trend seems to turn over at low resistance end for TESs with bare bilayer design (MXW0203 and MXW4950) that $N\Lambda_\ast$ tends to increase as TESs turn superconducting below 0.3 $R_N$. For MXW1213, the feature of this turn-over is not as significant but the slope of correlation has leveled off in low resistance range.

To study the dependence of $\frac{\alpha I}{\sqrt{1+2\beta I}}$ on $N\Lambda_\ast$, data at certain temperature range are selected to calculate for $\frac{\alpha I}{\sqrt{1+2\beta I}}$(MXW0203-[107.6,107.8]mK, MXW1213-[103,103.2]mK, MXW4647-[101.2,101.4]mK, MXW4950-[97.4,97.6]mK) and $N\Lambda_\ast$ uses all data points inside each resistance group to obtain an averaged value, as shown in Fig 4. According to the analytical form of $\alpha$ and $\beta$ in the frame of the two-fluid model, $\frac{\alpha I}{\sqrt{1+2\beta I}}$ can be expressed as:

$$\frac{\alpha I}{\sqrt{1+2\beta I}} = \frac{3}{2} \frac{I_0 T_0}{V_0 T_c} (1 - \frac{T_0}{T_c})^{1/2} \left\{ \frac{1}{NR_0 R_n/L} \cdot \frac{1}{\Lambda_\ast} - \left( \frac{1}{2NR_0 R_n/L} \right)^2 \frac{1}{\Lambda_\ast^2} \right\}^{1/2} \tag{6}$$

Equation 6 shows that $\frac{\alpha I}{\sqrt{1+2\beta I}}$ is anti-correlated to a quadratic function of $1/\Lambda_\ast$, which reaches minimum at $\Lambda_\ast = (\frac{R_0}{R_n}) \frac{L}{2N}$, where the energy resolution is maximized. Nevertheless, by definition
$R_0$ equals to $\frac{2N\Lambda_s}{L}(1 - \frac{T_s}{T_c})R_n$ so that $\Lambda_s$ is always larger than $\frac{(R_0/R_n)^2L}{2N}$. Therefore, $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ is expected to be monotonically increasing with $N\Lambda_s$ as long as the term other than the quadratic function remains constant, which is not necessarily true throughout the TES transition. The largest uncertainty emerges from $c_f$, which is poorly explored and usually simplified as a constant in literatures. Fig 4 overplots the model in dash lines assuming a series value of $c_f$ and $R_0/R_n$, with $I_{c0} = 60\ mA$, $I_0 = 30\ \mu A$, $\frac{T_s}{T_c} = 0.98$, $L = 120\ \mu m$ and $R_n = 10\ m\Omega$ fixed. The model predicts higher $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ as TES is biased at smaller resistance or if $c_f$ is larger. Using the measured $R_0/R_n$ and $I_0$ of MXW0203 and with $c_f = 0.5$ assumed, the model-predicted values of $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ are recalculated for MXW0203 in the transition, which is highlighted as the red line in Fig 4. Above $0.3R_N$ the model-predicted $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ is much larger than the measured ones and below which $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ is systematically underestimated. Adjusting $c_f$ can compensate for this discrepancy so that the model-predicted values match with the measurements, which leads to a smaller $c_f$ at higher resistance(from 0.1@0.6$R_N$ to 0.9@0.1$R_N$).

Figure 3. $N\Lambda_s$ versus the TES resistance. All data points in resistance range are averaged for $N\Lambda_s$ calculation. Errors are evaluated via deviation as shown in the shaded regions.

Figure 4. $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ versus $N\Lambda_s$ plotted against the model prediction(dashed line). $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ are estimated within certain temperature range and errors are obtained via deviation. The uncertainty of $N\Lambda_s$ is indicated by the shaded regions.

4.3. The Influence from Magnetic Field

By comparing the quasiparticle diffusion length and the figure of merit of the energy resolution of TES MXW0203 with different B-field setup, it is found that applying the magnetic field to TES can suppress $N\Lambda_s$ which shows a potential in enhancing the energy resolution, as shown in Fig 5. To quantify the effect of the field on TES parameters, the gain of $N\Lambda_s$ and $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ at various TES temperature-controlled bias points were calculated throughout the transition. Deducing the gain of the field application requires $N\Lambda_s$ and $\frac{\alpha_f}{\sqrt{1+2\beta_f}}$ measured with net 0.7mG B-field to be divided by the values obtained with the net-zero field, the errors of which are evaluated.
Figure 5. The gain of $N\Lambda_*$ (a) and $\frac{\alpha I}{\sqrt{1+2\beta I}}$ (b) with B-field application for TES MXW0203 at various bias points is calculated and averaged according to the TES temperature range. Errors are evaluated via standard deviation and plotted as the shadowed regions above.

according to error propagation relation. Differed from the stereotype that the existence of B-field always leads to a broaden transition region or suppresses the temperature sensitivity in situ, the results suggest that if the same operational bias resistance is achieved at a same controlled TES temperature, the value of $N\Lambda_*$ would become smaller when a magnetic field is present. Furthermore, the energy resolution can even become better when B-field is applied, based on the fact that the gain of $\frac{\alpha I}{\sqrt{1+2\beta I}}$ has revealed a general trend beyond one, which is more significant at low bias points, as shown in Fig 5(b). The suppression of $N\Lambda_*$ in the presence of B-field is consistent with the scenario where the pair breaking perturbation of the magnetic field reduces charge imbalance relaxation time for the superconducting film, as measured in SIN junctions via magnetic field dependence of the excess current[17]. This effect might also dominate the enhancement of $\frac{\alpha I}{\sqrt{1+2\beta I}}$ as a result of the accordingly inhibited current sensitivity, which could provide a new way to improve detector performance in the future.

5. Conclusion
Understanding the physics mechanism of TES thin-film bilayers and how to optimize the operational point is essential in the application of the X-ray microcalorimeter. This work has studied the temperature sensitivity and current sensitivity distribution in the TES transition systematically. The experimental evidence that $\beta$ depends only on TES resistance supports the prediction of the two-fluid model. A correlation between $N\Lambda_*$ and $\frac{\alpha I}{\sqrt{1+2\beta I}}$ is expected for TES devices if an identical bias condition is provided. During the phase transition, higher $c_I$ is required at lower bias point to compensate for the discrepancy between model and measurement, which needs further study or theoretical modification for a comprehensive explanation. The magnetic field could enhance $\frac{\alpha I}{\sqrt{1+2\beta I}}$ by suppressing $N\Lambda_*$, which is potentially a new clue for TES optimization in the future.
Acknowledgements
I would like to thank Professor Dan McCammon, Felix Jaeckel from UW-Madison, Professor Wei Cui and Hua Feng from Tsinghua University for supporting this research, and all students who have contributed to the experimental facilities. I also thank ISAS external funding acquisition incentive system for supporting the publication and travel expenses.

References
[1] Bennett D A, Swetz D S, Schmidt D R and Ullom J N 2013 Phys. Rev. B 87 020508
[2] Bennett D A, Schmidt D R, Swetz D S and Ullom J N 2014 Appl. Phys. Lett. 104 042602
[3] Ullom J N and Bennett D A 2015 Supercond. Sci. Tech. 28 084003
[4] Smith S J, Bandler S R, Brown A D, Chervenak J A, Figueroa-Feliciano E, Finkbeiner F, Iyomoto N, Kelley R L, Kilbourne C A and Porter F S et al 2008 J. Low Temp. Phys. 151 195–200
[5] Smith S J, Adams J S, Bailey C N, Bandler S R, Busch S E, Chervenak J A, Eckart M E, Finkbeiner F M, Kilbourne C A and Kelley R L et al 2013 J. Appl. Phys. 114 074513-074513-24
[6] Irwin K D and Hilton G C 2005 Transition-Edge Sensors (Enss, C.) p 63
[7] Lindeman M A, Barger K A, Brandl D E, Crowder S G, Rocks L, McCammon D and Hoevers H F C 2008 J. Low Temp. Phys. 151 190–194
[8] Bennett D A, Swetz D S, Horansky R D, Schmidt D R and Ullom J N 2012 J. Low Temp. Phys. 167 102–107
[9] Smith S J, Adams J S, Bandler S R, Busch S E, Chervenak J A, Eckart M E, Finkbeiner F M, Kelley R L, Kilbourne C A and Lee S J et al 2014 J. Low Temp. Phys. 176 356–362
[10] Wakeham N A, Adams J S, Bandler S R, Chervenak J A, Datesman A M, Eckart M E, Finkbeiner F M, Kelley R L, Kilbourne C A and Minussi A R et al 2018 J. Low Temp. Phys. 193 231–240
[11] Jethava N, Ullom J N, Irwin K D, Doriese W B, Beall J A, Hilton G C, Vale L R and Zink B 2009 American Inst. Phys. (Conf. Series vol 1185) ed Young B and Cabrera B et al pp 31–33
[12] Zhou Y, Anbarish C V, Gruenke R, Jaeckel F T, Kripps K L, McCammon D, Morgan K M, Wulf D and Zhang S et al 2018 J. Low Temp. Phys. 193 321–327
[13] Skocpol W J, Beasley M R and Tinkham M 1974 J. Low Temp. Phys. 16 145–167
[14] Pippard A B, Shepherd J G and Tindall D A 1971 Proc. Roy. Soc. London Series A 324 17–35
[15] Irwin K D, Hilton G C, Wollman D A and Martinis J M 1998 J. Appl. Phys. 83 3978–3985
[16] Tinkham M 1979 J. Low Temp. Phys. 35 147–151
[17] Tsuboi K and Yagi R 2010 Physica C 470 S877–S878