Membership-function-dependent approach to design filter for non-linear systems with time-varying delay via interval type-2 fuzzy model

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Abstract
This paper addresses the problem of $H_{\infty}$ filtering for non-linear systems with time-varying delay via the interval type-2 (IT2) fuzzy model and the membership-function-dependent approach. By taking the time-varying delay and uncertainties into consideration, the non-linear plant is modelled as an IT2 fuzzy delayed system, which fully captures the parameter uncertainties of the systems by using upper and lower membership functions. An IT2 fuzzy filter is designed to estimate the unknown information of the non-linear systems, which does not require the same premise variables as the plant. By combining the IT2 fuzzy delayed system and the IT2 fuzzy filter, a filtering error system is established. Based on the Lyapunov theory, a fuzzy-dependent Lyapunov function is constructed. Then, the membership-function-dependent approach, delay analysis approach, and some slack matrix variables are employing to analyse the derivative terms of the Lyapunov function, and a less conservative condition is derived to guarantee the filtering error system is stable. Moreover, the standard conditions of the filter are given in the form of linear matrix inequalities via the matrix decoupling technique. Finally, a numerical example is given to demonstrate the effectiveness and merits of the proposed method.

1 | INTRODUCTION

It is well known that the fuzzy systems in the form of Takagi–Sugeno (T-S) model have received considerable attention from researchers and engineers [1, 2], which is classified as type-1 T-S model. By converting non-linear systems to some sub-local linear dynamic systems with weighting summation, the scheme of T-S models has certified to be an effective method for non-linear systems. Recently, a number of issues are investigated for the type-1 T-S model. Stability analysis has always been a hot topic in the T-S fuzzy literature, including piecewise Lyapunov functions [3, 4], fuzzy Lyapunov functions [5, 6], polynomial Lyapunov functions [7, 8], and so on. Relying on the development of stability analysis, other research topics have also received extensive attention. For example, the output feedback controller of fuzzy systems with switch scheme and non-linear perturbations is studied in [9]. Based on the event-triggered mechanism, sliding mode observer design for uncertain fuzzy systems is used in [10]. The output tracking control problem of robotic systems in the form of fuzzy is addressed in [11]. A new sample-control problem is investigated for T-S fuzzy systems with time-delay in [12].

It should be noticed that the parameter uncertainties are inevitable in practical control systems [13–17]. However, the type-1 fuzzy sets can not deal with the parameter uncertainties problem directly [18]. Fortunately, in recent years, the type-1 fuzzy model is extended to interval type-2 (IT2) fuzzy one, which has a powerful ability to tackle uncertainties in membership function [19, 20]. By employing a lower and upper membership functions approach to solve uncertain grade of membership, IT2 fuzzy logic has more comparative advantages to cope with the parameter uncertainties of non-linear systems. Since the IT2 fuzzy model was proposed, it has received full attention in the research field. The problem of sliding mode controller design with the guaranteed-cost performance for IT2 fuzzy uncertain systems is concerned in [21]. In a network
environment with uncertainties and delays, the state feedback controller based on state estimation is designed in [22]. Taking actuator failure and stochastic quantisation into consideration, the fuzzy control problem with dissipativity is studied in [23]. Based on the sum-of-squares lemma, the design of state-feedback control for IT2 polynomial fuzzy systems is investigated in [24–26]. The tracking control problem for IT2 fuzzy systems is dealt with in [27–29]. An IT2 fuzzy controller is designed to cope with the problems of systems subject to pole constraint and control saturation in [30]. Based on Bessel–Legendre integral inequality, a new method of stability analysis and stabilisation is obtained for IT2 fuzzy delay plant in [31]. In [32] and [33], the problem of dynamic output feedback control for IT2 fuzzy systems is investigated.

During the last decades, considerable results have appeared on filter designing and state estimation for in signal processing, computer and control systems [34–41]. In [38], a new H∞ filter is designed for the fuzzy system, but the parameter uncertainties are not considered. By employing the IT2 fuzzy model, the authors in [39] investigates the problem of filter design for discrete-time non-linear systems with parameter uncertainties and intermittent measurements. For continuous-time non-linear systems, the authors in [40] proposed a filter design approach based on the IT2 fuzzy model. Notice that time-delay is inescapable in various complex non-linear systems. However, the problem of filtering for IT2 fuzzy system with time-delay is not considered in [39] and [40]. Up to now, few studies intensively handle such problems associated with IT2 fuzzy systems with time-delay. As an exception, the authors in [41] investigates the filtering problem of the IT2 fuzzy system with fixed time-delay, wherein the inconsistent premise variables between the plant and filter also are considered. Nevertheless, it is worth pointing out that comparatively little progress has been made toward addressing the problem of IT2 fuzzy systems with time-varying delay and mismatched premise variables.

Motivated by the discussion above, the problem of $H_{\infty}$ filtering is solved for IT2 fuzzy system with time-varying delay and mismatched premise variables by using the membership-function-dependent (MFD) approach [19, 42–44]. The main contributions of this paper are summarised as follows.

(i) Different from the existing works [34–38], the parameter uncertainties are considered in the entire analysis process. Based on the IT2 fuzzy model, the parameter uncertainties of the systems are represented in the membership function by using upper and lower membership functions. Moreover, the problem of time-varying delay for IT2 fuzzy systems is investigated in this paper.

(ii) An IT2 fuzzy filter is designed to estimate the unavailable state information of the non-linear systems. Compared with the existing works [36, 37], the designed filter is not required the same premise variables as the plant.

(iii) By adopting some slack matrices and combining the MFD approach, a less conservative condition is derived to guarantee the stability of the system and satisfy the desired performance.

The remainder of this paper can summarise as follows: The problem statement and preliminaries are given in Section 2. The stability analysis and filter design approach are presented in Section 3. A numerical example is given to verify the proposed method in Section 4. Finally, Section 5 concludes this paper.

Notations: Throughout this paper, let superscript “T” and “−1” denote the transpose and inverse of a matrix, respectively. Let $\mathbb{R}^{n}$ and $\| \cdot \|$ denote, respectively, the n-dimensional Euclidean space and the Euclidean norm of a vector. Let $\mathcal{B}_{2}(0, \infty)$ denote the space of square-integrable vector function over $[0, \infty)$. The notations $\text{diag} \{ \cdot \}$ and “∗” are denoted as the diagonal matrix and a symmetric term in a matrix.

2 | PRELIMINARIES AND PROBLEM FORMULATION

In this section, consider the non-linear systems with time-varying delay and parameter uncertainties, by using IT2 fuzzy model, which can be represented as follows:

**Plant Rule:** $i$: IF $\ell_{1}(x(t))$ is $\bar{\Theta}_{1}$, $\ell_{2}(x(t))$ is $\bar{\Theta}_{2}, \ldots$, AND $\ell_{p}(x(t))$ is $\bar{\Theta}_{p}$, THEN

$$
\begin{align*}
\dot{x}(t) &= A_{i}x(t) + A_{z}x(t - \tau(t)) + B\omega(t) \\
y(t) &= C_{i}x(t) + C_{z}x(t - \tau(t)) + D_{i}\omega(t) \\
z(t) &= L_{i}x(t)
\end{align*}
$$

(1)

where $\bar{\Theta}_{d}$ is IT2 fuzzy set, $d = 1, 2, \ldots, p$, $i = 1, 2, \ldots, r_{s}, x(t) \in \mathbb{R}^{n}$ denotes the state vector of the systems; $y(t) \in \mathbb{R}^{n}$ represents output vector measured by sampler; $z(t) \in \mathbb{R}^{n}$ represents the signal to be estimated; $\omega(t) \in \mathbb{R}^{n_{o}}$ denotes the disturbance signal satisfying $\omega(t) \in \mathcal{B}_{2}(0, \infty)$; $A_{i}, A_{z}, B_{i}, C_{i}, C_{z}, D_{i}$, and $L_{i}$ are system matrices with appropriate dimensions. Assume that all the systems delays are lump into $\tau(t)$, which satisfies $0 \leq \tau(t) \leq \tau_{0}$ and $0 \leq \dot{\tau}(t) \leq \alpha$. The firing strength of rule i-th in the fuzzy set can be expressed as the following interval set:

$$
M_{i}(x(t)) = \left[ M_{l_{i}}(x(t)), M_{r_{i}}(x(t)) \right],
$$

(2)

where

$$
\begin{align*}
M_{l_{i}}(x(t)) &= \prod_{d=1}^{p} \mu_{d}(\ell_{d}(x(t))) \geq 0, \\
M_{r_{i}}(x(t)) &= \prod_{d=1}^{p} M_{d}(\ell_{d}(x(t))) \geq 0, \quad M_{l_{i}}(x(t)) \geq M_{r_{i}}(x(t)) \geq 0,
\end{align*}
$$

$M_{l_{i}}(x(t))$ and $M_{r_{i}}(x(t))$ stand for the lower bound and the upper bound of the membership, respectively. $\mu_{\bar{\Theta}_{d}}(\ell_{d}(x(t)))$ and $M_{d}(\ell_{d}(x(t)))$ represent the lower bound and the upper bound of the membership functions, respectively. Therefore, the IT2 fuzzy model of the non-linear systems (1) can be
expressed as:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} \mathcal{W}_i(x(t)) [A_ix(t) + A_{ij}x(t) - \tau(t)] + B_i\omega(t) \\
\gamma(t) &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))[C_ix(t) + C_{ij}x(t) + D_i\omega(t)] \\
\zeta(t) &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))L_i\omega(t),
\end{align*}
\]  

\[ (3) \]

where \( \mathcal{W}_1(x(t)) = \tilde{\kappa}_i(x(t)) + \kappa_i(x(t)) + x_i(x(t)) \geq 0, \]

in which \( \sum_{i=1}^{r} \mathcal{W}_i(x(t)) = 1 \) and \( \sum_{i=1}^{r} \mathcal{W}_i(x(t)) = 0; \kappa_i(x(t)), \]

\( \tilde{\kappa}_i(x(t)) \) are non-linear function to describe the parameters of the systems [19], which satisfies \( 0 \leq \kappa_i(x(t)), \tilde{\kappa}_i(x(t)) \leq 1 \),

\( \kappa_j(x(t))) + \tilde{\kappa}_j(x(t)) = 1. \]

Because the system is affected by external disturbance, the state of the system could not be accurately obtained. Therefore, the following filter is employed to estimate the unknown system state:

**Filter rule** \( j \): IF \( \varphi_1(x(t)) \) is \( \tilde{\kappa}_1, \) AND \( \varphi_2(x(t)) \) is \( \tilde{\kappa}_2, \ldots, \) AND \( \varphi_q(x(t)) \) is \( \tilde{\kappa}_q, \) THEN

\[
\begin{align*}
\dot{\tilde{x}}_j(t) &= A_{ij}\tilde{x}_j(t) + B_{ij}\gamma(t) \\
\tilde{\zeta}_j(t) &= C_{ij}\tilde{x}_j(t),
\end{align*}
\]

\[ (4) \]

where \( \tilde{x}_j(t) \in \mathbb{R}^{n_x} \) represents the filter state vector, \( \gamma(t) \in \mathbb{R}^{n_y} \)

denotes filter input vector, \( \tilde{\zeta}_j(t) \in \mathbb{R}^{n_x} \) is the estimated value of \( \zeta(t); A_{ij}, B_{ij}, \) and \( C_{ij} \) are the filter parameter matrices to be designed. \( \tilde{\kappa}_j, \) is IT2 fuzzy set, \( j = 1, 2, \ldots, r, \lambda = 1, 2, \ldots, q. \)

The firing strength of rule \( j \)-th in the fuzzy set can be expressed as the following interval set:

\[
\mathcal{W}_j(x(t)) = \left[ \mathcal{W}_j(x(t)), \mathcal{W}_j(x(t)) \right],
\]

\[ (5) \]

where

\[
\mathcal{W}_j(x(t)) = \prod_{\lambda=1}^{q} \mu_{\kappa\lambda}(\varphi_\lambda(x(t))) \geq 0,
\]

\[
\mathcal{W}_j(x(t)) = \prod_{\lambda=1}^{q} \mu_{\tilde{\kappa}\lambda}(\varphi_\lambda(x(t))) \geq 0,
\]

\[
0 \leq \mu_{\kappa\lambda}(\varphi_\lambda(x(t))) \leq \mu_{\tilde{\kappa}\lambda}(\varphi_\lambda(x(t))) \leq 1,
\]

\[
0 \leq \mathcal{W}_j(x(t)) \leq \mathcal{W}_j(x(t)) \leq 1.
\]

Inspired by [19], we can derive the following IT2 fuzzy filter model:

\[
\begin{align*}
\dot{\tilde{x}}_j(t) &= \sum_{i=1}^{r} \mathcal{W}_i(x(t)) [A_{ij}\tilde{x}_j(t) + B_{ij}\gamma(t)] \\
\tilde{\zeta}_j(t) &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))C_{ij}\tilde{x}_j(t),
\end{align*}
\]

\[ (6) \]

where

\[
\mathcal{W}_j(x(t)) = \mathcal{W}_j(x(t)) + \mathcal{W}_j(x(t)) = \sum_{j=1}^{r} \left( \mathcal{W}_j(x(t)) + \mathcal{W}_j(x(t)) \right),
\]

\( \mathcal{W}_j(x(t)) \geq 0, \)

\[
\sum_{j=1}^{r} \mathcal{W}_j(x(t)) = 1,
\]

\[
0 \leq \mathcal{W}_j(x(t)), \mathcal{W}_j(x(t)) \leq 1, \mathcal{W}_j(x(t)) = 1.
\]

For the sake of convenience, we define \( \mathcal{W}_1 = \mathcal{W}_1(x(t)), \)

\( \mathcal{W}_j = \mathcal{W}_j(x(t)), \) and \( \epsilon(t) = \tilde{\zeta}_j(t) - \zeta_j(t), \) the filtering error system can be expressed as follows:

\[
\begin{align*}
\dot{\xi}_j(t) &= \tilde{A}_{ij}\xi_j(t) + \tilde{A}_{ij}\xi_j(t) - \tau(t) + B_i\omega(t) \\
\epsilon(t) &= \mathcal{E}_{ij}\xi_j(t),
\end{align*}
\]

\[ (7) \]

where

\[
\begin{align*}
\tilde{A}_{ij} &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))A_{ij} \\
B_{ij} &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))B_{ij} \\
\mathcal{E}_{ij} &= \sum_{i=1}^{r} \mathcal{W}_i(x(t))E_{ij}.
\end{align*}
\]

In this paper, we aim at designing an IT2 fuzzy filter (6) to make the filtering error system (7) meet the following conditions:

(1) When \( \omega(t) = 0, \) the filtering error system (7) is asymptotically stable.

(2) For \( \omega(t) \in \mathcal{L}_2[0, \infty], \) the filtering error systems (7) is asymptotically stable and meets the given performance index \( \gamma, \)

\[
\int_{0}^{\infty} \|\epsilon(t)\|^2 dt \leq \gamma \int_{0}^{\infty} \|\omega(t)\|^2 dt.
\]

Before the end of the section, the following lemma is given to facilitate stability analysis.

**Lemma 1** ([Jensen inequality] [45]). There exists scalar \( \alpha > 0, \)

for any symmetric positive definite matrix \( G > 0, \) and the function \( g \) satisfies
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**3 | MAIN RESULTS**

In this section, the stability analysis and filter design will be given based on the MFD.

### 3.1 | Stability analysis

**Theorem 1.** Given scalars \( u > 0 \) and \( \tau_0 \geq 0 \), assume that

\[
|\mathfrak{M}_i| \leq \mathfrak{K}_i,
\]

where \( \mathfrak{K}_i \geq 0, i = 1, 2, \ldots, r \), and membership function satisfying \( \mathfrak{M}_j - \partial \mathfrak{M}_j \geq 0 \) (0 < \( \mathfrak{K}_j \leq 1 \)), the IT2 fuzzy filtering error system (7) is asymptotically stable with prescribed performance \( \gamma > 0 \), if there exist appropriate matrices \( \mathbf{P}_i > 0 \), \( \tilde{Q}_1 > 0 \), \( \tilde{Q}_2 > 0 \), \( \tilde{S}_1 > 0 \), \( \tilde{S}_2 > 0 \), \( \tilde{M}_1 > 0 \), \( \mathbf{Y}_1 \), \( \mathbf{Y}_2 \), and \( \mathbf{G}_1 \), \( i = 1, 2, \ldots, r \), such that the following inequalities hold:

\[
\begin{bmatrix}
\mathbf{Y}_1 & \mathbf{M}_1 \\
\mathbf{M}_2 & \mathbf{Y}_2
\end{bmatrix} \geq 0,
\]

\[
\mathbf{Y}_{l,j} < 0,
\]

\[
\mathbf{Y}_{l,j} < 0,
\]

\[
\mathbf{Y}_{l,j} + \mathbf{Y}_{l,j} < 0, \quad i < j
\]

where

\[
\begin{align*}
\mathbf{Y}_{l,j} &= \mathbf{Y}_{l,j} - \mathbf{G}_1, \\
\mathbf{Y}_{l,j} &= \partial_j \mathbf{Y}_{l,j} - \partial_j \mathbf{G}_1 + \mathbf{G}_1,
\end{align*}
\]

\[
\begin{align*}
\mathbf{Y}_{l,j} &= \partial_j \mathbf{Y}_{l,j} - \partial_j \mathbf{G}_1 + \mathbf{G}_1, \\
\mathbf{Y}_{l,j} &= \partial_j \mathbf{Y}_{l,j} - \partial_j \mathbf{G}_1 + \mathbf{G}_1,
\end{align*}
\]

\[
\mathbf{Y}_{l,j} = \mathbf{H}_j \mathbf{G}_j \mathbf{Y}_{l,j} + \mathbf{H}_j \mathbf{G}_j \mathbf{Y}_{l,j}
\]

\[
\begin{bmatrix}
\varphi_{11} & \varphi_{12} & 0 & \varphi_{14} \\
\varphi_{21} & \varphi_{22} & \tilde{R} & \varphi_{24} \\
0 & \tilde{Q}_2 - \tilde{R} & 0 & \varphi_{44}
\end{bmatrix}.
\]

**Remark 1.** It should be noticed that the filtering error system is modelled based on the IT2 model. Compared with the existing works [34–38], it can capture the uncertainties caused by membership function.

**Remark 2.** Notice that the mismatched premise constraint is considered in the design of the IT2 fuzzy filter, namely, the synchronous premise assumption in [36, 37] is not required in this paper.

**Proof.** We construct the following Lyapunov function:

\[
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),
\]

where

\[
\begin{align*}
V_1(t) &= \xi_t^T(t) P_i \xi(t), \\
V_2(t) &= \int_{t-\tau(t)}^t \xi_t^T(t) \tilde{Q}_1 \xi(t) d\theta + \int_{t-\tau(t)}^t \xi_t^T(t) \tilde{Q}_2 \xi(t) d\theta, \\
V_3(t) &= \tau_0 \int_{t-\tau_0}^t \xi_t^T(t) \tilde{R} \xi(t) d\theta, \\
V_4(t) &= 2 \tau_0 \int_{t-\tau_0}^t \xi_t^T(t) \tilde{S}_2 \xi(t) d\theta.
\end{align*}
\]

Calculating the derivative of (13), it can be obtained that

\[
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t),
\]

where

\[
\begin{align*}
\dot{V}_1(t) &= 2 \xi_t^T(t) P_i \xi(t) + \sum_{j=1}^r \mathbf{H}_j \xi_t^T(t) P_i \xi(t), \\
\dot{V}_2(t) &= \xi_t^T(t) \tilde{Q}_1 \xi(t) - (1 - \xi(t)) \xi_t^T(t) \tilde{Q}_1 \xi(t) + (1 - \xi(t)) \xi_t^T(t) \tilde{Q}_1 \xi(t) + \tau_0 \int_{t-\tau_0}^t \xi_t^T(t) \tilde{R} \xi(t) d\theta, \\
\dot{V}_3(t) &= \tau_0 \int_{t-\tau_0}^t \xi_t^T(t) \tilde{R} \xi(t) d\theta, \\
\dot{V}_4(t) &= \tau_0 \int_{t-\tau_0}^t \xi_t^T(t) \tilde{S}_2 \xi(t) d\theta.
\end{align*}
\]
\[ + \tau_0 \xi^T(t) \tilde{\zeta}_2 \xi(t) - 2\tau_0^2 \int_{-\tau_0}^{t} \int_{t+\theta}^{t+\tau_0} \xi^T(s) \tilde{\zeta}_2 \xi(s) ds d\theta. \]

For given matrix \( \bar{M}_1 > 0 \) with suitable dimensions, one has
\[ \bar{G}_1 - 2 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \xi^T(s) \bar{M}_1 \xi(s) ds d\theta = 0, \] (15)
where \( \bar{G}_1 = \tau_0 \xi^T(t) \bar{M}_1 \xi(t) - \int_{t+\theta}^{t+\tau_0} \xi^T(s) \bar{M}_1 \xi(s) ds. \) According to (14), (15), and 0 \( \leq t(t) \leq \tilde{\alpha} \), we can obtain
\[ \dot{V}(\xi(t)) \leq 2\xi^T(t) \tilde{P}_1 \xi(t) + \sum_{i=1}^{r} \psi_i \xi^T(t) \tilde{P}_1 \xi(t) \]
\[ - (1 - \alpha) \xi^T(t - \tau(t)) \tilde{Q}_2 \xi(t - \tau(t)) \]
\[ - \xi^T(t - \tau_0) \tilde{Q}_2 \xi(t - \tau_0) + \xi^T(t)(\tau_0 R + \tau_0^2 \tilde{\zeta}_3) \xi(t) \]
\[ - 2\tau_0^2 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \xi^T(s) \tilde{\zeta}_3 \xi(s) ds d\theta \]
\[ - 2\tau_0^2 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \xi^T(s) \bar{M}_1 \xi(s) ds d\theta + 2\tau_0^2 \xi^T(t) \bar{M}_1 \xi(t) \]
\[ - 2\tau_0^2 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \xi^T(s) \bar{M}_1 \xi(s) ds d\theta + 2\tau_0^2 \xi^T(t) \bar{M}_1 \xi(t) \]
\[ - 4\tau_0^2 \int_{-\tau_0}^{0} \int_{t+\theta}^{t} \xi^T(s) \bar{M}_1 \xi(s) ds d\theta \]
\[ - \tau_0 \int_{t+\tau_0}^{t} \xi^T(s) \tilde{R} \xi(s) ds + \xi^T(t - \tau(t)) \tilde{Q}_2 \xi(t - \tau(t)). \]
By applying Lemma 1 to deal with the item \( -\tau_0 \int_{t+\tau_0}^{t} \xi^T(s) \tilde{R} \xi(s) ds \) in (16), it can be derived that
\[ -\tau_0 \int_{t+\tau_0}^{t} \xi^T(s) \tilde{R} \xi(s) ds \leq \begin{bmatrix} \xi(t) \\ \xi(t - \tau(t)) \\ \xi(t - \tau_0) \end{bmatrix}^T \bar{G}_2 \begin{bmatrix} \xi(t) \\ \xi(t - \tau(t)) \\ \xi(t - \tau_0) \end{bmatrix}, \] (17)
where
\[ \bar{G}_2 = \begin{bmatrix} -\bar{R} & \bar{R} & 0 \\ * & -2\bar{R} & \bar{R} \\ * & * & -\bar{R} \end{bmatrix}. \]

Similar to [45], the following slack matrices are introduced
\[ \sum_{i=1}^{r} \sum_{j=1}^{r} \psi_i \psi_j \xi^T(t)(\bar{Y}_1 + \bar{Y}_2) \xi(t) = 0, \] (18)
where \( \bar{Y}_1 = \bar{Y}_1^T \) and \( \bar{Y}_2 = \bar{Y}_2^T \). Then, based on (8), and (15) – (18), we have
\[ \dot{V}(t) + c^T(t) e(t) - \gamma^2 \omega^T(t) \omega(t) < 0, \]
(22)
Integrating from 0 to \( \infty \) under zero condition, one can get
\[ \int_{0}^{\infty} \|e(t)\|^2 dt \leq \gamma^2 \int_{0}^{\infty} \|\omega(t)\|^2 dt. \]
(23)
This completes the proof. It is necessary to point out the obtained stability condition can guarantee the filtering error system (7) is said to be asymptotically stable, which also satisfies the given performance index \( \gamma \).
Proof. A conservative condition is derived because more system items can be constructed Lyapunov function (13) is fuzzy-dependent. By using the MFD approach and introducing the slack matrix, a less conservative condition is derived because more system items can be obtained. Moreover, Corollary 1 is given in the following via the traditional Lyapunov function.

**Corollary 1.** Given scalars $u > 0$ and $\tau_0 \geq 0$, the IT2 fuzzy filtering error system (7) is asymptotically stable with prescribed performance $\gamma > 0$, if there exist appropriate matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$ and $R > 0$, such that the following inequalities hold:

$$\Psi_{ii} < 0, \quad i = 1, 2, \ldots, r,$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, r,$$  \hspace{1cm} (24)

(25)

where

$$\Psi_{ij} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & \Psi_{14} \\ * & \Psi_{22} & R & \Psi_{24} \\ * & * & \Psi_{33} & 0 \\ * & * & * & \Psi_{44} \end{bmatrix}.$$  \hspace{1cm} (26)

From (24) and (25), we can derive that

$$\dot{V}_r(t) + e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) < 0.$$  \hspace{1cm} (27)

Under zero condition, we can compute that $\int_0^\infty \|e(t)\|^2 dt \leq \gamma^2 \int_0^\infty \|\omega(t)\|^2 dt$. This completes the proof. \hspace{1cm} \(\square\)

### 3.2 Filter design

Theorem 1 gives a stability condition for the filtering error system (7), but there are some coupling matrices, which can not be solved by standard numerical software. Thus, in this section, we will provide a filter design approach to derive the parameter existence condition of the filter in the form of LMIs.

**Theorem 2.** Given scalars $u > 0$, $\tau_0 \geq 0$, and $\delta > 0$, under the assumption (8), the filtering error system (7) is asymptotically stable with prescribed performance $\gamma > 0$, if there exist appropriate matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, $\delta_1 > 0$, $\delta_2 > 0$, $M_1 > 0$, $M_2 > 0$, such that the following inequalities hold:

$$\Psi_{ii} \leq 0, \quad i = 1, 2, \ldots, r,$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i < j, \quad i, j = 1, 2, \ldots, r.$$  \hspace{1cm} (28)

\[\begin{bmatrix} S_1 & M_1 \\ * & S_2 \end{bmatrix} \geq 0.\]  \hspace{1cm} (29)

$$\Psi_{ii} \leq 0, \quad i = 1, 2, \ldots, r.$$  \hspace{1cm} (30)

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i < j.$$  \hspace{1cm} (31)

Then, combining (27) and (29), we have

$$\dot{V}_r(t) + e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) \leq \eta^T(t) \left( \sum_{i=1}^r \sum_{j=1}^r \Psi_{ij} \right) \eta(t),$$  \hspace{1cm} (32)

where

$$\sum_{i=1}^r \sum_{j=1}^r \bar{\Psi}_{ii} \bar{\Psi}_{jj} \Psi_{ij} = \sum_{i=1}^r \bar{\Psi}_{ii} \bar{\Psi}_{jj} \Psi_{ij} + \sum_{i=1}^r \sum_{j=i+1}^r \bar{\Psi}_{ii} \bar{\Psi}_{jj} (\Psi_{ij} + \Psi_{ji}).$$  \hspace{1cm} (33)
\[ \alpha_{11} = \alpha_{11} + \bar{\alpha}_{11} + \mathcal{Q}_1 - R + 2\tau_v M_1 + \hat{h}_{11}, \]

\[ \hat{h}_{11} = \tau_0 F_1 + \sum_{i=1}^r \mathcal{S}_1 (T_1 + T_2 + \mathcal{P}_i), \]

\[ \alpha_{22} = -(1 - u) \mathcal{Q}_1 + \mathcal{Q}_2 - 2R, \]

\[ \alpha_{11} = \begin{bmatrix} P_i A_i + B_i C_i & A_j \\ F_i A_i + B_i C_i & A_j \end{bmatrix}, \]

\[ \alpha_{12} = \begin{bmatrix} P_i A_i + B_i C_i & 0 \\ F_i A_i + B_i C_i & 0 \end{bmatrix}, \]

\[ \alpha_{13} = \begin{bmatrix} P_i B_i + B_i D_i \\ F_i B_i + B_i D_i \end{bmatrix}, \]

\[ \alpha_{15} = \begin{bmatrix} L_1^T \\ -C_j^T \end{bmatrix}, \]

\[ \alpha_{55} = \begin{bmatrix} P_i & F_j \\ F_i & F_j \end{bmatrix}. \]

and the designed IT2 fuzzy filter parameters are given by

\[ A_{ij} = F_i^{-1} A_{ij}, B_{ij} = F_i^{-1} B_{ij}, C_{ij} = \bar{C}_{jj}, \quad (34) \]

Proof. Decomposing matrix \( \bar{P}_i \) into \( \bar{P}_i = \begin{bmatrix} P_i & \Lambda_i \\ * & W_i \end{bmatrix} \), where \( P_i \geq 0, W_i > 0 \), and \( \Lambda_i \) is an invertible matrix. By using the Schur complement lemma, \( Y_{ij} \) can be represented by

\[ Y_{ij} = \begin{bmatrix} \varphi_1 & -\mathcal{Q}_1 + \mathcal{Q}_2 - 2R & 0 \\ * & * & -\mathcal{Q}_2 - R \\ * & * & * \\ * & * & * \end{bmatrix}, \]

\[ \begin{bmatrix} \bar{P}_i B_j & \tau_0 A_{ij} P_i & \tau_0^2 A_{ij} P_i & \\ 0 & \tau_0 A_{ij} P_i & \tau_0^2 A_{ij} P_i & \\ 0 & 0 & 0 & \\ -\gamma^2 I & \tau_0 B_j P_i & \tau_0^2 B_j P_i & \\ \gamma & \hat{h}_{13} & 0 & \\ * & * & * -I \end{bmatrix}. \]

where

\[ \varphi_1 = \bar{P}_i A_{ij} + \sum_{i=1}^r \mathcal{S}_1 (T_1 + T_2 + \mathcal{P}_i) + \hat{h}_{12}, \]

\[ \hat{h}_{12} = \mathcal{Q}_1 - R + 2\tau_v \bar{M}_1 + \tau_0^2 \hat{h}_1. \]

Notice that there exist non-convex terms \( -\bar{P}_i R^{-1} \bar{P}_i \) and \( -\bar{P}_i \Lambda_i^{-1} \bar{P}_i \) in (35). Inspired by [46], there exists any given scalar \( \delta > 0 \) such that

\[ -\bar{P}_i R^{-1} \bar{P}_i \leq -2\delta \bar{P}_i + \delta^2 \bar{R}. \quad (36) \]

In this way, the item \( -\bar{P}_i R^{-1} \bar{P}_i \) in \( Y_{ij} \) can be replaced by \( -2\delta \bar{P}_i + \delta^2 \bar{R} \). Similarly, \( -\bar{P}_i \Lambda_i^{-1} \bar{P}_i \) can be replaced by \( -2\delta \bar{P}_i + \delta^2 \bar{S}_2 \). Then, we can derive

\[ 
\begin{bmatrix}
\varphi_1 & \bar{P}_i A_{ij} + \bar{R} & 0 \\
* & -(1 - u) \mathcal{Q}_1 + \mathcal{Q}_2 - 2R & \bar{R}
\end{bmatrix},
\begin{bmatrix}
\gamma & \hat{h}_{13} & 0 \\
* & * & * \\
* & * & -I
\end{bmatrix}. \]

where

\[ \hat{h}_{13} = -2\delta \bar{P}_i + \delta^2 \bar{R} \quad \text{and} \quad \hat{h}_{14} = -2\delta \bar{P}_i + \delta^2 \bar{S}_2. \quad \text{Let} \quad \bar{H} = \begin{bmatrix} I & 0 \\ 0 & \Lambda_i W_i^{-1} \end{bmatrix}, \quad \text{and define} \]

\[ F_i = \Lambda_i W_i^{-1} \Lambda_i^T, \quad \mathcal{Q}_1 = H \bar{Q}_1 H^T, \quad \mathcal{Q}_2 = H \bar{Q}_2 H^T, \]

\[ M_1 = H \bar{M}_1 H^T, \quad R = H \bar{R} H^T, \quad S_1 = H \bar{S}_1 H^T, \quad S_2 = H \bar{S}_2 H^T, \]

\[ \mathcal{A}_{ij} = \Lambda_i A_{ij} W_i^{-1} \Lambda_i^T, \quad B_{ij} = \Lambda_i B_{ij}, \quad C_{ij} = C_{ij} W_i^{-1} \Lambda_i^T, \]

\[ J_1 = \text{diag}(H, H, H, I, H, H, I) J_2 = \text{diag}(H, H), \]

\[ T_1 = H \bar{T}_1 H^T, \quad T_2 = H \bar{T}_2 H^T, \]

\[ \mathcal{G}_i = J_1 (\mathcal{E}_i J_1^T)^{-1}. \]

Then, pre- and post-multiply (9) by \( J_2 \) and its transpose \( J_2^T \), and pre- and post-multiply (37) by \( J_1 \) and its transpose \( J_1^T \), we can obtain the inequalities (30) and (31). Similarly, we can obtain (32) and (33).

Notice that the matrix \( \bar{P}_i = \begin{bmatrix} P_i & \Lambda_i \\ * & W_i \end{bmatrix} > 0 \), according to Schur complement lemma, \( \bar{P}_i \) is equivalent to \( \bar{P}_i - \Lambda_i W_i^{-1} \Lambda_i^T > 0 \). By using the matrix equivalent transformation function \( \Lambda_i^T W_i \mathcal{E}_i (t) \), the parameter matrices of the filter (6) can be obtained as follows:
We consider the IT2 fuzzy system made up of two subsystems in the following:

\[
\begin{align*}
A_{ff} &= \Lambda_i^{-T} W_i (A_i^{-1} A_{ij}^{-1} A_i^{-T} W_i) W_i^{-1} \Lambda_i^{-T} = F_i^{-1} A_{ij}, \quad B_{ff} = \\
&= \Lambda_i^{-T} W_i (A_i^{-1} B_{ij}) = F_i^{-1} B_{ij}, \quad C_{ff} = (C_i A_i^{-T} W_i) W_i^{-1} \Lambda_i^{-T} = C_i. \quad \text{This completes the proof.}
\end{align*}
\]

4 | SIMULATION

In this section, we consider a numerical example to verify the effectiveness of the proposed MFD IT2 filter design methods.

**Example.** We consider the IT2 fuzzy system made up of two subsystems in the following:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{2} \mathbb{M}_i(x(t))[A_i x(t) + A_{ii} x(t - \tau(t)) + B_i \omega(t)] \\
y(t) &= \sum_{i=1}^{2} \mathbb{M}_i(x(t))[C_i x(t) + C_{ii} x(t - \tau(t)) + D_i \omega(t)] \\
\tilde{z}(t) &= \sum_{i=1}^{2} \mathbb{M}_i(x(t))L_i x(t)
\end{align*}
\]

with

\[
\begin{align*}
[A_1 & B_1] = \begin{bmatrix} -2.1 & 0.1 & 1 \\ 1 & -2 & -0.2 \\ 1 & 0 & 0.3 \end{bmatrix}, \\
[C_1 & D_1] = \begin{bmatrix} -1.9 & 0 & 0.3 \\ -0.2 & -1.1 & 0.1 \\ 0.5 & 0.6 & 0.6 \end{bmatrix}, \\
[A_2 & B_2] = \begin{bmatrix} -1.1 & 0.1 & 1 \\ -0.8 & -0.9 & -0.5 \\ -0.8 & 0.6 & 0 \end{bmatrix}, \\
[C_2 & D_2] = \begin{bmatrix} -0.9 & 0 & -0.2 \\ -1.1 & -1.2 & 0.3 \\ -0.2 & 1 & 0 \end{bmatrix}.
\end{align*}
\]

The corresponding membership function of the IT2 fuzzy system and the filter are chosen as follows:

\[
\begin{align*}
\mathbb{M}_1(x(t)) &= 1 - \frac{1}{1 + e^{-(x(t) + 0.1)T}}, \\
\mathbb{M}_2(x(t)) &= 1 - \mathbb{M}_1(x(t)), \\
\mathbb{M}_3(x(t)) &= 1 - \frac{1}{1 + e^{-x(t)/1.2}}, \quad \mathbb{M}_4(x(t)) = 1 - \frac{1}{1 + e^{-x(t)/0.25}}, \\
\tilde{z}(t) &= \begin{bmatrix} -0.25 & 0.1 \end{bmatrix}, \quad \tilde{z}_f(t) = \begin{bmatrix} 0.1 \sin(t), \quad 0 \end{bmatrix},
\end{align*}
\]

where \( \mathbb{M}_i(x(t)) \) is the uncertain parameter. Assume that the disturbance of the original system is

\[
\omega(t) = \begin{cases} 
0.1 \sin(t), & t \leq 4; \\
0, & 4 \leq t \leq 10.
\end{cases}
\]

For given \( u = 0.2, \tau_0 = 0.5 \) and \( \delta = 10, \) by solving the LMIs of Theorem 2, we can obtain the parameters of the filter as follows:

\[
A_{f1} = \begin{bmatrix} -2.0582 & 0.0309 \\ 0.0246 & -1.6082 \end{bmatrix}.
\]
FIGURE 3  The curve of $e(t)$

FIGURE 4  The curve of $z(t)$ and $z_f(t)$

TABLE 1  Obtained minimal performance $\gamma$ for $u = 0.2$ and $\tau_0 = 0.5$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ | $\delta = 10$ | $\delta = 20$ |
|-----------------|----------------|--------------|--------------|--------------|--------------|--------------|
| In [48]         | 0.2575         | 0.2471       | 0.2379       | 0.2358       | 0.2365       | 0.2405       |
| Theorem 2       | 3.4842         | 5.1438       | 3.3818       | 1.4063       | 0.0022       | 0.1581       |
| in our work     | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ |

TABLE 2  Obtained minimal performance $\gamma$ for $u = 0.2$ and $\tau_0 = 0.6$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ | $\delta = 10$ | $\delta = 20$ |
|-----------------|----------------|--------------|--------------|--------------|--------------|--------------|
| In [48]         | 0.2685         | 0.2531       | 0.2421       | 0.2388       | 0.2411       | 0.2501       |
| Theorem 2       | 1.6091         | 5.2870       | 4.2746       | 1.4546       | 0.0005       | 0.0096       |
| in our work     | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-5}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ |

TABLE 3  Obtained minimal performance $\gamma$ for $u = 0.2$ and $\tau_0 = 0.8$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ | $\delta = 10$ | $\delta = 20$ |
|-----------------|----------------|--------------|--------------|--------------|--------------|--------------|
| In [48]         | 0.3613         | 0.2695       | 0.2481       | 0.2442       | 0.2485       | 0.3449       |
| In [49]         | 0.2194         | --           | --           | --           | --           | --           |
| Theorem 2       | 6.2144         | 3.1215       | 8.9178       | 2.1042       | 0.0015       | 0.0036       |
| in our work     | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-5}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ |

TABLE 4  Obtained minimal performance $\gamma$ for $u = 0.2$ and $\tau_0 = 1$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ | $\delta = 10$ | $\delta = 20$ |
|-----------------|----------------|--------------|--------------|--------------|--------------|--------------|
| In [48]         | 0.6675         | 0.3105       | 0.2585       | 0.2531       | 0.2585       | 0.4485       |
| In [49]         | 0.2297         | --           | --           | --           | --           | --           |
| Theorem 2       | 4.8172         | 6.6728       | 4.9506       | 5.5557       | 0.0017       | 0.0070       |
| in our work     | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ | $\times 10^{-4}$ |

$A_{f_2} = \begin{bmatrix} -1.9829 & 0.0029 \\ 0.0013 & -1.7353 \end{bmatrix},$

$B_{f_1} = \begin{bmatrix} 0.2637 \\ -1.1500 \end{bmatrix},\quad B_{f_2} = \begin{bmatrix} 0.4724 \\ 0.1847 \end{bmatrix},$

$C_{f_1} = \begin{bmatrix} -0.1868 & -0.0498 \end{bmatrix},$

$C_{f_2} = \begin{bmatrix} 0.0282 & -0.0444 \end{bmatrix}.$

In addition, we derive the minimal performance index $\gamma = 0.0022$, which is smaller than in [47]. Under the obtained parameters and initial state $x(0) = x_f(0) = [1 \ -2]^T$, some results can be described as follows. Figures 1 to 4 show the response of the states of the system and filter. Figure 3 depicts the filtering error $e(t)$. Figure 4 describes the response of $z(t)$ and $z_f(t)$. From Figures 1 to 4, it can be seen that in the case of external interference in the system, the designed filter can well estimate the signal of the original system, which demonstrates the designed filter is effective and feasible.

TABLE 5  Obtained maximal $\tau_0$ with $u = 0.2 \gamma = 0.3$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ |
|-----------------|----------------|--------------|--------------|--------------|
| In [48]         | 0.72           | 0.77         | 0.78         | 0.72         |
| Theorem 2 in our work | 2.96 | 2.68 | 2.05 | 2.07 |

TABLE 6  Obtained maximal $\tau_0$ with $u = 0.2 \gamma = 0.4$

| Method          | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ |
|-----------------|----------------|--------------|--------------|--------------|
| In [48]         | 1.00           | 1.09         | 1.09         | 1.01         |
| Theorem 2 in our work | 3.12 | 3.00 | 2.31 | 2.26 |
To further demonstrate the effectiveness and merits of our work, we make some comparisons with some existing works on the problem of $H_\infty$ filtering for non-linear fuzzy systems with time delay. Tables 1–4 depict the obtained performance index $\gamma$ for different $\delta$. Moreover, the maximum value of $\tau_0$ for different cases are depicted in Tables 5–7. We can see that the proposed algorithm in our work can derive smaller $\gamma$ than those in [48, 49], and the bigger $\tau_0$ are obtained than those in [48].

**Remark 4.** Compared with the type-1 fuzzy filtering approaches [34–38], the proposed approach can solve the parameter uncertainties problem of non-linear systems. Moreover, different from the result of the fixed time-delay in [41], the problem of non-linear systems with time-varying delay is addressed in this paper. In addition, our proposed approach can derive more relax results than the existing ones.

## 5 CONCLUSION

In this paper, the fuzzy filter design for IT2 fuzzy non-linear systems with time-varying delay has been presented. The non-linear plant with time-varying delay has been described by the IT2 fuzzy model, while the uncertainties are captured by the IT2 fuzzy set. A filtering error system has been constructed based on the IT2 fuzzy model. A fuzzy filter has been designed, which needs not to share the same premise variables as the plant. By using the MFD approach and introducing some slack matrices, the stability condition has been derived, which can not only guarantee the filtering error system is asymptotically stable but also possess a less conservativeness. A numerical example has been provided to verify the effectiveness of the proposed method. Although the problem of filtering for IT2 fuzzy with time-varying delay has been investigated in this paper, packet loss and cyber-attack often occur in the practical system. Thus, the problem of filtering for IT2 fuzzy systems with packet loss and cyber-attack will be considered in our future work.

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## TABLE 7 Obtained maximal $\tau_0$ with $a = 0.2 \gamma = 0.5$

| Method                  | $\delta = 0.7$ | $\delta = 1$ | $\delta = 2$ | $\delta = 5$ |
|------------------------|----------------|--------------|--------------|--------------|
| In [48]                | 1.14           | 1.16         | 1.17         | 1.06         |
| Theorem 2 in our work  | 3.32           | 3.18         | 3.11         | 2.56         |
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