Coexisting Coverage and Throughput of Multi-RAT Wireless Networks with Unlicensed Band Access

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Abstract—In this letter, the coexisting coverage and throughput of a wireless network consisting of multiple subnetworks of different radio access technologies (RATs) are investigated. The coexisting coverage probability that is defined as the sum of the density-weighted coverage probabilities of all subnetworks is found in closed-form and it will be shown to have the concavity over the number of channels in the unlicensed band. The optimal deployment densities of all access points (APs) of different RATs that maximize the coexisting coverage probability will be shown to uniquely exist and found under the derived constraint on network parameters. The coexisting throughput is defined as the sum of the throughputs of all subnetworks and numerical results show that it is significantly higher than the throughput of the unlicensed band only accessed by WiFi APs.

Index Terms—Coexistence, coverage probability, unlicensed band, Poisson point process, Martén hard-core point process

I. INTRODUCTION

With the proliferation of wireless smart handsets, cellular data traffic is expected to considerably grow to meet huge and different throughput demands entailed by versatile networking services. To make cellular networks jump over the high throughput hurdle due to limited licensed spectrum (band), deploying small cells with unlicensed band access is a promising mean of mitigating the spectrum crunch crisis. However, when small cells extend their services to the unlicensed band, how to mitigate or even eliminate interfering between small cells and the existing unlicensed access points (APs) (such as WiFi APs) is extremely pivotal since it is the root of reaching the well-coexisting goal of a wireless network with multiple radio access technologies (RATs) [1].

The most important and fundamental issue that intrinsically exists in a wireless network consisting of multiple subnetworks of different RATs is the coexistence performance between these subnetworks. Specifically, the coverage and throughput of all the coexisting subnetworks are the two most important performance metrics that need to be first investigated since they can indicate whether or not coexistence essentially benefits the entire network. To simply and thoroughly delve the coverage and throughput performances in the multi-RAT network, the concept of coexisting coverage is first proposed in this letter for the small cell and WiFi subnetworks with unlicensed band access. The coverage probabilities studied in all prior works are only for single-RAT cellular networks (typically see [2]) so that their results cannot completely characterize the coverage probability of a multi-RAT network.

Under the assumption that WiFi APs use carrier-sense-multiple access (CSMA) and small cells use random access protocols to access all channels in the unlicensed band, our first contribution is to find the close form expression for the coverage probabilities of the two subnetworks and use them to characterize the coexisting coverage probability that is essentially defined as the sum of all the density-weighted coverage probabilities. To the best of our knowledge, the coexisting coverage probability was first proposed in this work. Our second contribution is to theoretically show that there exist a unique optimal number of channels and a unique optimal ratio of the small cell density to the WiFi density that jointly maximize the coexisting coverage probability if the derived constraint on network parameters holds. Finally, the coexisting throughput that is the sum of the (per-channel) mean spectrum efficiencies of all subnetworks was proposed and numerical results show that it is significantly higher than the throughput of the unlicensed band only accessed by WiFi APs.

II. SYSTEM MODEL

Consider a large-scale wireless network in which there coexist two subnetworks of different RATs: one is a small cell (cellular) network and the other is a WiFi network. In the subnetwork of RAT “τ”, all access points (APs) form a marked homogeneous Poisson point process (PPP) of density \( \lambda_{\tau} \) denoted by

\[
\Phi_{\tau} \equiv \{(A_{r,i}, H_{r,i}, P_{r}) : A_{r,i} \in \mathbb{R}^2, P_{r}, H_{r,i} \in \mathbb{R}_+ \},
\]

where \( \forall i \in N_{+}, r \in \mathcal{R} \equiv \{s, w\} \) in which “s” denotes the small cell subnetwork and “w” stands for the WiFi subnetwork, \( P_{r} \) is the transmit power of all APs of RAT \( r \). All \( H_{r,i} \)'s characterize the fading and/or shadowing channel power gains in the downlink between access points and their serving users and they are independent identical distributes. random variables with unit mean for all \( i \) and \( r \in \mathcal{R} \). In particular, the users of the small cell subnetwork can access both the licensed and unlicensed bands, yet the users of the WiFi subnetwork only can access the unlicensed band.

Assume there are \( m \) channels available for each AP with unlicensed band access. In order to prevent a small cell (AP) from retaining a channel too long and giving rise to a significant impact on the channel access opportunity of WiFi APs, each small cell is assumed to randomly and equally likely switch among the \( m \) channels as a new transmission time slot starts, which means the small cells using the same channel is a thinning PPP of intensity \( \lambda_{s}/m \). The channel access scheme of each WiFi AP is the carrier sense multiple access (CSMA) protocol in which each WiFi AP can retain a transmission.
opportunity if there is at least one channel available in its circular sensing area of radius $R$. Namely, every WiFi AP is assumed to be able to detect all channel access activities within its sensing area. Due to the CSMA protocol, the resulting transmitting WiFi APs actually become a Matérn hard-core point process (HCPP), and its probability of retaining a transmission opportunity (i.e., transmitting probability) for $m$ channels and a sensing area of $\pi R^2$ can be found as

$$\eta_m = 1 - \left(1 - \left[1 - \exp\left(-\pi R^2 \lambda_m\right)\right]\right)^m. \quad (2)$$

Since the resulting transmitting WiFi APs are a Matérn HCPP, they are spatially correlated and no longer a PPP any more so that theoretically the probability generating functional of a PPP cannot be applied to them. This exaggerates the analysis of some performance metrics, such as coverage probability, average link rate, etc. Nonetheless, for the tractability of analysis we still assume all transmitting WiFi APs as a homogeneous thinning PPP of density $\lambda_m \eta_m$. Such an assumption is actually very accurate and validated since in general $\lambda_m$ is fairly small. Note that $\eta_m$ is a monotonic increasing and concave function of $m$ and $\lim_{m \to \infty} \eta_m = 1$, and the transmitting probability of a small cell is one throughout this letter (i.e., $\eta_m = 1$) since a small cell is assumed to always have data to transmit and not perform the CSMA protocol.

III. ANALYSIS OF COEXISTING COVERAGE

Provided that every user in the subnetwork of RAT $r$ associates with an AP, its coverage probability based on its location at the origin is defined by

$$\rho_r = \mathbb{P}\left[H_r P_r \left| A_{r,0}\right| \leq \theta_t, t \in \mathbb{R}\right], \quad (3)$$

where $\|X - Y\|$ denotes the Euclidean distance between two nodes $X$ and $Y$, $\alpha > 2$ is the path loss exponent, $A_{r,0}$ is the associated AP of the user, $\theta_t > 0$ is the signal-to-interference ratio (SIR) threshold for successful decoding, and $I_t$ is the interference from the subnetwork of RAT $t$ given by

$$I_t = \left\{ \begin{array}{ll}
\left\{ A_{r,i} : A_{r,i} \in \Phi \land A_{r,0}, P_i G_{r,i} \|A_{r,i}\|^{-\alpha}, & \text{if } t = r \\
A_{r,i} : A_{r,i} \in \Phi \land A_{r,0}, P_i G_{r,i} \|A_{r,i}\|^{-\alpha}, & \text{otherwise}
\end{array} \right. \right\},$$

where $G_{r,i}$’s represent the (random) interference channel gains due to fading and/or shadowing. Since all APs are able to access the unlicensed band, they all receive the sum interference $\sum_{t \in \mathbb{R}} I_t$ from the subnetworks of different RATs.

The coverage probability in (3) depends on how a user associates with their AP. The following theorem gives an explicit result of $\rho_r$ for nearest AP association and Rayleigh-fading in communication channels.

**Theorem 1:** If all users of the subnetwork of RAT $r$ associate with their nearest transmitting AP and their communication channels undergo Rayleigh fading, their coverage probability is given by

$$\rho_r = \left\{1 + \frac{\theta_t^2}{m} \frac{\tau_\alpha}{\lambda_m} \frac{\eta_m \lambda_m}{\eta_r \lambda_r} \left(\frac{P_r}{P_t}\right)^2 - \ell(\theta_t)\right\}^{-1}, \quad (4)$$

where $\eta_r$ is the transmitting probability of an AP of RAT $t$, $\ell(x) = \int_0^x (1 - \mathcal{L}_{G}(t^{-\alpha})) dt$, $\mathcal{L}_{G}(s) = \mathbb{E}[e^{-sG}]$ is the Laplace transform of random variable $Z$ and $\tau_\alpha = \Gamma\left(1 - \frac{2}{\alpha}\right) \mathbb{E}[G^{\frac{2}{\alpha}}]$. Furthermore, if all interference channels also undergo Rayleigh, $\ell(x) = \int_0^x \frac{dt}{t^{1 + \frac{2}{\alpha}}}$ and $\tau_\alpha = \Gamma\left(1 - \frac{2}{\alpha}\right) \mathbb{E}[G^{\frac{2}{\alpha}}].$

**Proof:** Since the interferences from different RATs are independent and $H_i$ is exponentially distributed with unit mean, the coverage probability in (3) can be rewritten as

$$\rho_r = \mathbb{E}[R_t] \prod_{t \in \mathbb{R}} \mathbb{E}[e^{-\theta_t R_t^2 / P_r}], \quad (5)$$

where $R_t$ denotes $|A_{r,0}|$. If $t \neq r$, $R_t$ and $I_t$ are independent and following the results of the outage probability in [5], [6] gives

$$\mathbb{E}\left[e^{-\theta_t R_t^2 / P_r} \mid R_t\right] = \exp\left(-\pi \eta_t R_t^2 / \theta_t \tau_\alpha \frac{P_r}{P_t}\right).$$

However, if $t = r$, $R_t$ and $I_t$ are no longer independent and using the result in (4) for a given $R_t = x$ leads to

$$\mathbb{E}\left[e^{-\theta_t x^2 / P_t} \mid R_t = x\right] = \mathbb{E}\left[\exp\left(-\theta_t \sum_{A_{r,i} : A_{r,i} \in \Phi \land A_{r,0}, P_i G_{r,i} \|A_{r,i}\|^{-\alpha}} \frac{x^2 G_{r,i}^2}{\|A_{r,i}\|^2}\right)\right]$$

$$= \mathbb{E}\left[\exp\left(-\theta_t \sum_{A_{r,i} : A_{r,i} \in \Phi \land A_{r,0}, P_i G_{r,i} \|A_{r,i}\|^{-\alpha}} \frac{x^2}{2 + \|A_{r,i}\|^2}\right)\right] = \exp\left(-\pi \eta_t x^2 / \theta_t \tau_\alpha \left|\tau_\alpha - \ell(\theta_t)\right| / m\right).$$

Then substituting the two results in above into (5) yields

$$\rho_r = \mathbb{E}[R_t] \left\{e^{-\theta_t R_t^2 / P_t \frac{2}{\alpha} \left|\tau_\alpha - \ell(\theta_t)\right| / m}\right\}. \quad (6)$$

Since $A_{r,0}$ is the nearest AP of the user using RAT $r$, the pdf of $R_t^2$ is $f_{R_t^2}(x) = \pi \eta_t x e^{-\pi \eta_t x}$ and using it to calculate $\rho_r$ in above results in (4). Finally, if all interference also undergo Rayleigh fading, $G$ is an exponential random variable with unit mean and variance and thus $\ell(x)$ and $\tau_\alpha$ can be further explicitly found.

According to Theorem 1 specifically the coverage probabilities for small cells and WiFi APs can be inferred and explicitly given as follows

$$\rho_{sc} = \left\{1 + \frac{\theta_s^2}{m} \frac{\tau_\alpha}{\lambda_s} \frac{\eta_s \lambda_s}{\eta_r \lambda_r} \left(\frac{P_s}{P_r}\right)^2 - \ell(\theta_s)\right\}^{-1}, \quad (6)$$

where $\eta_s = 1$ and $\eta_r > 0$. To evaluate the coexisting coverage of the wireless network with multiple RATs, the following **coexisting coverage probability** for the multi-RAT network is proposed:

$$\rho_{cc} = \frac{\sum_{r \in \mathbb{R}} \eta_r \lambda_r \rho_r}{\sum_{r \in \mathbb{R}} \eta_r \lambda_r} = \omega_s \rho_s + \omega_r \rho_r, \quad (7)$$

where $\omega_s$ and $\omega_r$ are the weights of the small cells and WiFi APs, respectively.
where \( \omega_r \triangleq \eta_r \lambda_r / \sum_{r \in \mathcal{R}} \eta_r \lambda_r \). The coexisting coverage probability is defined as the average of all coverage probabilities of all subnetworks of different RATs and weighting probability \( \omega_r \) can be interpreted as the probability that the nearest transmitting AP associated by a user is from the subnetwork of RAT \( r \). The definition of the coexisting probability is motivated by the concept of spacial throughput which is usually defined as the product of the transmitting node density and the coverage probability assuming each communication link has unit spectrum efficiency \([8]\). In other words, the spacial throughput of a multi-RAT wireless network can be essentially found by the sum of the spacial throughputs of all subnetworks, which is divided by the total density to obtain the coexisting coverage probability of the entire network.

Since the retaining (transmitting) probability \( \eta_w \) of a WiFi AP is monotonically increasing with the number of channels, \( \rho_s \) and \( \rho_w \) are monotonically decreasing and increasing, respectively. This indicates that there exists a unique optimal \( \rho \) that maximizes \( \rho_{ce} \) as shown in the following theorem.

**Theorem 2:** For a given set of the deployment densities of all APs, the coexisting coverage probability \( \rho_{ce} \) in \([8]\) is a monotonic and concave function of the channel number \( m \) so that its maximum achieves at \( \rho_{ce}(m) \).

**Proof:** The coverage probabilities \( \rho_s \) and \( \rho_w \) can be equivalently expressed as follows

\[
\rho_s = \frac{m}{m + a_s \eta_w + b_s}, \quad \rho_w = \frac{m}{m + a_w \eta_w + b_w},
\]

where \( a_r \)'s and \( b_r \)'s are pertaining to \( \lambda_r \), \( \eta_r \), \( P_r \) and \( \alpha \) for \( r \in \mathcal{R} \). Thus, \( \rho_s \) and \( \rho_w \) are concave for all \( m \in \mathbb{N}_+ \). The coexisting coverage probability \( \rho_{ce} \) be neatly expressed as

\[
\rho_{ce} = \max_{\rho_s + \rho_w} \left( \frac{m}{m + a_s \eta_w + b_s} \right),
\]

which is a monotonic and concave function of \( m \) since it is a linear combination of \( \rho_w \) and \( \rho_s \). Therefore, the maximum of \( \rho_{ce} \) occurs at \( m \).

The simulation and theoretical results of the coexisting coverage probabilities for different values of \( m \) are shown in Fig. 1. Since the density of WiFi APs is larger than that of small cells in the simulation and certain amount of interference is reduced due to multiple channels, both the coverage probabilities of the two subnetworks increase along the number of channels, as expected. As a result, the coexisting coverage probability eventually increases and attains its maximum at the channel number \( m \). In addition, simulated and theoretical results perfectly coincide each other so that the validness of \([6, 7] \) and \([8]\) is verified. The coexisting coverage probability also can be optimized via deploying the two subnetworks with appropriate densities, as shown in the following section.

**IV. OPTIMALITY OF COEXISTING COVERAGE PROBABILITY AND THROUGHPUT**

According to the definition of the coexisting coverage probability, in a multi-RAT wireless network with \( m \) channel bands the following (per-channel) **coexisting throughput** is proposed and defined as

\[
C_{ce} = \frac{1}{m} \sum_{r \in \mathcal{R}} \int \rho_r (2^{\eta_r} - 1) d\eta_r,
\]

where \( |\mathcal{R}| \) is the cardinality of set \( \mathcal{R} \) and \( \Theta \triangleq \{(\eta_r), r \in \mathcal{R}\} \). Essentially, \( C_{ce} \) is the sum of the throughputs of all subnetworks. Hereupon for a given set of SIR threshold \( \Theta \) maximizing the coexisting coverage probability over all network parameters definitely helps enhance the mean channel capacity of each user. The following theorem shows the fundamental constraints on the network parameters for achieving the optimality of the coexisting coverage probability and deployment densities.

**Theorem 3:** For a given SIR threshold set \( \Theta \) and a given channel number \( m \), the coexisting coverage probability in \([8]\) can be maximized if the following inequality holds

\[
\min \left\{ c_w, c_s \right\} > \sqrt{d_w \sqrt{d_w}}, \tag{10}
\]

where \( c_r \triangleq (1 + \eta_r \theta_r^2) (|\tau_r - \ell(\eta_r)|/m) / \omega_r \), \( d_r \triangleq \eta_r \theta_r^2 \tau_r / \omega_r m \) and \( \omega_r \) is defined in \([8]\), \( \forall r \in \mathcal{R} \), and the optimal density ratio of \( \lambda_s \) to \( \lambda_w \) is given by

\[
\left( \frac{\lambda_s}{\lambda_w} \right)^* = \eta_n \frac{P_w}{P_s} \frac{d_w}{d_s} \left( c_w - \sqrt{d_w \sqrt{d_w}} \right). \tag{11}
\]

For the special case of \( \theta_r = \theta \) for all \( r \in \mathcal{R} \), the constraint \([10]\) further reduces to

\[
\min \left\{ \left( \frac{\lambda_s}{\eta_n \lambda_w} \right) (m + \eta_w \kappa), \left( \frac{\lambda_w}{\lambda_s} \right) (m + \kappa) \right\} > \theta \tau_s \tag{12}
\]

where \( \kappa \triangleq \theta \tau_s \) and \( \Theta \triangleq \{\tau_s, \ell(\theta)\} \), and \([11]\) becomes

\[
\left( \frac{\lambda_s}{\lambda_w} \right)^* = \frac{P_w}{P_s} \frac{d_w}{d_s} \left( (m + \eta_w \kappa) / \sqrt{\eta_w} - \theta \tau_s / \sqrt{\eta_w} \right). \tag{13}
\]

**Proof:** Since directly finding the optimal densities is too complex to be tractable, instead we resort to first finding the optimal solution of transmit powers maximizing the coexisting coverage probability. Then the duality between power and density can be explored to acquire the optimal densities if it exists. Using the definitions of \( c_r \) and \( d_r \), the coexisting coverage probability \( \rho_{ce} \) in \([8]\) can be concisely written as

\[
\rho_{ce}(x) = \frac{x}{c_s x + d_s} + \frac{1}{c_w + d_w x} = \frac{1}{c_s} - \frac{d_s / c_s^2}{x + d_s / c_s} + \frac{1}{d_w} / x + c_w / d_w.
\]
where $x \triangleq \left(\frac{P}{\theta}\right)^{1/\alpha}$. Thus $p_{ce}(x)$ is a linear combination of a monotonic convex and concave functions and its $n$th derivative w.r.t. $x$ is

$$p_{ce}^{(n)}(x) = \frac{(-1)^{n+1}d_{w}n!}{c_{w}^{2}(x + d_{w}/c_{w})^{n+1}} + \frac{(-1)^{n}n!}{d_{w}(x + c_{w}/d_{w})^{n+1}}.$$  

Hence, since $c_{r} > d_{r}$ for all $r \in \mathcal{R}$, the set $\{x \in \mathbb{R}_{++} : p_{ce}^{(1)}(x) = 0, p_{ce}^{(1)}(x) < 0\}$ is not empty provided that $c_{w}^{2} > d_{w}d_{w} + \sqrt{c_{w}^{2}} > d_{w}d_{w}$ hold (i.e. the constraint in (10) holds). The optimal value of $x$ can be solved from $p_{ce}^{(1)}(x) = 0$ under condition (10), which generates the duality result of the density ratio given in (11). Also, in the case of $\theta_{r} = \theta$ for all $r \in \mathcal{R}$, the result in (13) is obtained by substituting $c_{r}$’s and $d_{r}$’s with $\theta_{r} = \theta$ into (11).

The inequality constraint in (10) provides us some insights into the design of the network parameters, including the number of channels, deployment densities and SIR thresholds such that there exists a set of deployment densities to maximize the coexisting coverage probability. The simulation and theoretical results of the coexisting coverage probability for different density ratios of the two subnetworks are illustrated in Fig. 2. The simulation network parameters listed in the figure satisfy the constraint (10) for all $\frac{\lambda_{w}}{\lambda_{s}} > 0$, whereas the optimal ratio of $\lambda_{w}$ to $\lambda_{s}$ given in (13) for the simulation network parameters is about 14.5 and this coincides with the simulation result in Fig. 2. Note that $\left(\frac{\lambda_{w}}{\lambda_{s}}\right)^{*}$ in (10) converges to $\left(\frac{P_{w}}{P_{s}}\right)^{2/\alpha}$ as $m$ approaches to infinity, which means that a large number of channels makes channel contention between WiFi APs almost disappear and this further indicates that the optimal density ratio only depends on the power ratio if all APs in a network can arbitrarily access a channel.

According to Theorems 1 and 3 the coexisting throughput $C_{ce}$ defined in (2) can be jointly optimized over channel number $m$ and density ratio $\frac{\lambda_{w}}{\lambda_{s}}$ as well. However, its optimal results are analytically intractable. Nevertheless, Fig. 3 shows the simulation results of the mean throughputs for a fixed $m$ and it verifies the optimality of $C_{ce}$ over density ratio $\frac{\lambda_{w}}{\lambda_{s}}$. As shown in the figure, the throughput of the unlicensed band only accessed by WiFi APs is a constant around 1.25 (bps/Hz), whereas coexisting throughput $C_{ce}$ is between 1.47 (bps/Hz) and 1.62 (bps/Hz). Thus, the throughput gain of two coexisting subnetworks is at least 20%. Most importantly, there indeed exists a unique optimal density ratio $\left(\frac{\lambda_{w}}{\lambda_{s}}\right)^{*} \approx 1.5$ that maximizes $C_{ce}$ and achieves almost 30% throughput gain, which is a fairly significant improvement.

V. CONCLUSION

The coexisting coverage probability is first investigated for a wireless network specifically consisting of two WiFi and small cell subnetworks. For given deployment densities, it is a monotonic concave function of the number of channels, whereas the optimal ratio of AP densities maximizing it is shown to uniquely exist and found for a given number of channels if the AP densities satisfy the derived constraint on the network parameters. The coexisting throughput characterized by the sum of the throughputs of all subnetworks is proposed and numerical results show that it is significantly larger than the throughput of the unlicensed band accessed only by WiFi APs and can be jointly maximized over the AP densities as well as the number of channels.

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