An extension of the generalized nonlocal theory for the mode analysis of plasmonic waveguides at telecommunication frequency

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Abstract
We present an extension of the generalized nonlocal (GNL) optical response theory for the mode analysis of several plasmonic waveguides. We show that, compared with the local description, the imaginary part of the effective mode index is enlarged using the GNL response model. We ascribe this enlargement to the ‘effective’ surface modification and the induced charge diffusion. This result is quite different from that of the hydrodynamic model, where the imaginary part becomes smaller compared with that of the local model. Further, we investigate the influence of geometry parameters on propagation properties and find that the nonlocal effects are much more remarkable for smaller gap and sharper tip. Although the introduction of diffusion has a negative impact on the propagation length, it reveals the true physical insight and should be taken care when dealing with nanoplasmonic waveguide for photonic integration applications.

Keywords: nonlocal effects, hydrodynamic model, plasmonic waveguide, nanostructures, nanophotonics

(Some figures may appear in colour only in the online journal)

1. Introduction
Plasmonic devices, based on surface plasmons [1, 2] (i.e. charge density oscillations propagate along metal-dielectric interface [3]), are capable of squeezing light into regions much smaller than the diffraction limit [4–6], thus becoming a promising candidate for integration of photonic and electronic circuits on the nanometer scale [7, 8]. So far, various plasmonic waveguiding structures have been proposed and investigated, including metal slot waveguide [9], dielectric-loaded plasmon waveguide [10], long-range plasmon waveguide [11], channel plasmon waveguide [12], metal wedge plasmon waveguide [13], metallic nanowire waveguide [14], to mention a few. However, these waveguides suffer from high propagation loss, hindering the applications where both subwavelength confinement and long-range propagation are required. Recently, the hybrid plasmonic waveguide is proposed [15, 16] and attracts a growing amount of interest [17–30], as it can achieve long propagation length and subwavelength confinement simultaneously. Based on the remarkable performances, many optical components have been proposed and demonstrated, such as laser [31–34], coupler [35], splitter [36], nanodisk [37], and modulator [38]. In addition, as the hybrid plasmonic waveguide offers giant field enhancement in the gap, it has been used to enhance optical forces [39, 40] as well as molecular fluorescence [41].

It is noteworthy that the modeling of these plasmonic structures is generally based on classical electromagnetic theory, where the local dielectric response [42] is used to describe the electromagnetic properties of metal. This local picture has proved to be accurate enough for large structures.
However, as the sizes of plasmonic devices shrink down to the size of about 10 nm or smaller, the quantum nature of the electrons and the nonlocal effects associated with them significantly change the plasmonic response, thus the classical theory can no longer describe the electromagnetic properties accurately. The quantum tunneling across subnanometer gap of dimers has been investigated by full quantum simulation [43, 44]. Also a semi-classical quantum-corrected model (QCM) [45–49] has been proposed to incorporate the tunneling effects into classical theory, and it is in excellent agreement with full quantum mechanical calculations for small plasmonic structures. For gap distance larger than the ‘threshold tunnel-distance’ $d_{th}$ (about 0.4 nm for noble metal particles) [45, 47, 50] while smaller than several nanometers, the nonlocal effects are the dominant quantum effects on the optical response [50]. In this nonlocal regime, the hydrodynamic (HD) model [51–61], which introduces a quantum pressure term in the equation of motion for the electron fluid, has been widely used to model nonlocal effects in the field of plasmonics. Also effort has been made to model tunneling effects and nonlocal effects simultaneously [46, 62]. Moreover, transformation optics provides a different angle to understand the nonlocal effects [63].

We shall show that in this paper we neglect the quantum tunneling effects and focus on the nonlocal effects. By now, there have been lots of reports on nonlocal effects, such as waveguiding [51–53], scattering [54–58], nanofocusing [59, 60], etc. Contrary to local classical model, the nonlocal effects result in large blueshift of the localized surface plasmon resonance frequency, additional resonances above plasma frequency, and field enhancement reduction [60, 61]. Recent reports [64, 65] show that the HD model can interpret the size-dependent blueshift (or gap size-dependent redshift) of the resonances by using the pressure related nonlocal effects while cannot explain the broadening [66, 67] of the resonances. Therefore, researchers developed a semi-classical generalized nonlocal (GNL) response model, which contains both the quantum pressure and the induced charge diffusion. The results show that the HD response causes the blueshift, whereas the diffusion causes the line broadening.

In this paper, we follow the approach outlined in [64, 65], and present an extension of the GNL optical response theory for the mode analysis of several plasmonic waveguides. We investigate the generalized nonlocal response on waveguiding properties of a nanowire, nanowire dimers, and a hybrid plasmonic waveguide. The metal nanowire waveguide has analytical solutions and others have more degree of freedom. Also the tunneling effects between metal and dielectric in a hybrid waveguide are weaker compared with that of metal-gap-metal structures [47, 52]. The minimum gap size is chosen to be 0.5 nm, which is larger than $d_{th}$, thus neglecting the quantum tunneling effects. Results show that the GNL model enforces the imaginary part of the effective mode index of the fundamental mode compared with the local Drude model, while the real part becomes smaller. This is quite different from the HD model, where both the imaginary part and real part become smaller compared with that of the local Drude model [52]. However, the real parts and field profiles

Figure 1. The real parts of the effective indices (a) and the propagation length (b) with respect to wavelength of the fundamental mode of an Ag nanowire with $R = 2$ nm based on different models. (c) and (d) for Re($n_{eff}$) and $L$ with respect to $R$ at $\lambda = 1550$ nm. Red and black lines stand for the local analytical and simulation results, respectively. Blue line represents the result based on the HD model and green line for the GNL model. The legend in the inset applies throughout this paper.
The electrical field in spatially dispersive media is given by [64]
\[
\nabla \times \nabla \times \mathbf{E} = \left( \frac{\omega}{c} \right)^2 \int \mathbf{e}(\mathbf{r}, \mathbf{r}') \mathbf{E}(\mathbf{r}'),
\]
where \( \mathbf{e}(\mathbf{r}, \mathbf{r}') \) is the nonlocal response function. In the HD model, taking the quantum pressure term into account, the coupled equations are [56, 58]:
\[
\nabla \times \nabla \times \mathbf{E} = \left( \frac{\omega}{c} \right)^2 \mathbf{E} + i\omega \mu_0 \mathbf{J},
\]
where \( \mathbf{J} \) is the current density, \( \beta = (3/5)^{1/2} \nu_F \) is the nonlocal parameter with \( \nu_F \) being the Fermi velocity. \( \omega \) is the angular frequency, and \( c \) is the speed of light in vacuum. Equations (3) and (4) are coupled equations in the HD model, and one can use them to solve arbitrary geometries by different numerical methods. Furthermore, one can obtain another equation of motion for the electron density by only taking the charge diffusion into account [64], and it is
\[
\frac{D}{i\omega} \nabla (\nabla \cdot \mathbf{J}) + \mathbf{J} = \sigma_0 \mathbf{E},
\]
where \( D \approx 4A\nu_F^2/\omega_p \) is the diffusion constant. Apparently, equations (4) and (5) have the same mathematical form, and the only difference is the prefactor of \( \nabla (\nabla \cdot \mathbf{J}) \) originating from different physical insights. Then by considering both pressure term and diffusion, rewriting the nonlocal \( \nabla (\nabla \cdot \mathbf{J}) \) modification to Ohm’s law as a Laplacian modification to \( \varepsilon_D \), one can obtain that the wave equation in the metal is [51, 64, 65]
\[
\nabla \times \nabla \times \mathbf{E} = \left( \frac{\omega}{c} \right)^2 \left[ \varepsilon_D + \eta^2 \nabla^2 \right] \mathbf{E},
\]
where \( \eta \) is the GNL parameter, and given as
\[
\eta^2 = \frac{\beta^2}{\omega (\omega + i\gamma)} + \frac{D}{i\omega}.
\]
One can solve equation (6) for different structures to investigate the GNL effects. Here an additional boundary condition \( \mathbf{J} \cdot \mathbf{n} = 0 \) is imposed at the metal-dielectric boundary. Notice that when one sets \( D = 0 \), the GNL model degenerates to the HD model. Further, it degenerates to the local Drude model by setting \( D = \beta = 0 \).

3. GNL effects in a metallic nanowire and dimers

As one example, we first perform a FEM GNL solution of Maxwell’s equations for a cylindrical Ag nanowire surrounded by air. As the translational invariance of the infinite Ag nanowire in \( z \) direction simplifies the calculation, the simulation domain becomes a two-dimensional cross-section. In all calculations, the local Drude constants \( h\omega_p = 8.2953 \) eV and \( h\gamma = 0.055 \) eV are taken from the fitting to Palik’s experimental data [68] for silver in the wavelength range from 1 to 2 \( \mu \)m. The nonlocal parameter \( \beta = 0.0036c \) and \( D = 3.0661 \times 10^{-4} \) m\(^2\)/s\(^{-1}\) for \( A = 0.5 \) and

- **Figure 2.** The comparison of the normalized (a) \( E_x \) and (b) \( E_y \) using local Drude model (black line), HD model (blue line), and GNL model (green line). (c)-(e) Show the normalized electric field \( |E| \) distributions based on GNL model, HD model, and local Drude model, respectively. The wavelength is 1550 nm, and \( R = 2 \) nm.

2. The GNL response model

First, we recall the local Drude metal permittivity [42], which is expressed as
\[
\varepsilon_D(\omega) = \varepsilon_\infty + \frac{i\sigma_D}{\varepsilon_0 \omega},
\]
where \( \sigma_D = i\omega_p^2 \omega_0 / (\omega + i\gamma) \) is the Drude conductivity. \( \varepsilon_\infty \) is the vacuum permittivity, \( \gamma \) and \( \omega_p \) are the damping coefficient and the plasma frequency, respectively. We neglect the interband effects, and set \( \varepsilon_\infty = 1 \).

Based on the HD and GNL model have no significant difference.

The paper is organized as follows. Firstly, we introduce the GNL model, and define the related parameters. Secondly, we perform a FEM GNL solution of Maxwell’s equations for a metallic nanowire, and compare the simulation results with the analytical results, followed by studying nanowire dimers. Then we investigate the GNL effects in a hybrid plasmonic waveguide based on an elliptical metallic nanowire. At last, we will make some discussions and conclude the paper.
The definition of the effective mode index of the fundamental mode can be seen in [69, 70]. First, we compare the effective mode indices $n_{\text{eff}}$ based on different models (local Drude model, $\beta = 0$ and $D = 0$, HD model, $\beta \neq 0$ and $D = 0$, GNL model, $\beta \neq 0$ and $D \neq 0$). Then we compare the field distributions. Here, $n_{\text{eff}} = k_{\text{app}}/k_0$, $k_{\text{app}}$ is the propagation constant and $k_0$ is the wavenumber in the vacuum. The propagation length is $L = 1/(2k_0 \text{Im}(n_{\text{eff}}))$ with $\text{Im}(n_{\text{eff}})$ being the imaginary part of $n_{\text{eff}}$.

$v_F = 1.39 \times 10^6 \text{ m s}^{-1}$. In figures 1(a) and (b), we show the effective mode index and the propagation length for an Ag nanowire with radius $R = 2$ nm in the wavelength range from 1 to 2 $\mu$m. Red and black lines stand for the local analytical and numerical simulation results, respectively. These two curves overlap almost completely. As shown in figure 1(a), the real parts of $n_{\text{eff}}$ based on the HD (blue line) and GNL (green line) model have little difference and both smaller compared with the local situation, which is consistent with earlier report [51, 52].
However, there exists a big difference in the imaginary parts, as shown in figure 1(b). The imaginary parts based on the GNL model are much larger (shorter propagation length) than that of the local Drude model. This is quite different from that of the HD model, where the imaginary part becomes smaller compared with that of the local Drude model [32]. We attribute this enlargement of $\text{Im}(n_{\text{eff}})$ to taking the induced charge diffusion into account, which magnifies the Joule heating loss. Further, figures 1(c) and (d) give the dependence of $n_{\text{eff}}$ on $R$. As the radius of nanowire increases, the real parts of $n_{\text{eff}}$ based on GNL model approach that of the local situation. The relative differences of the imaginary parts between the GNL model and local Drude model are 2.8% and 1.2% for $R = 1$ nm and 3 nm, respectively. These results indicate that nonlocal effects weaken as $R$ increases.

Figure 5. The real parts of the effective indices (a) and the propagation length (b) for the hybrid mode based on different models with respect to wavelength. Black lines stands for local simulation result. Blue line represents the result based on the HD model and green line for the GNL model. The parameters are the same as in figure 4.

Figure 6. The real parts of the effective indices (a) and the propagation length (b) for the hybrid mode based on different models with respect to $h$ at $\lambda = 1550$ nm. $a = 1$ nm, $b = 5$ nm, $W_1 = 20$ nm, $W = 600$ nm, $h_{\text{Si}} = 50$ nm, $h_{\text{SiO}_2} = 200$ nm, and $h_{\text{sub}} = 100$ nm.

Figure 7. The real parts of the effective indices (a) and the propagation length (b) for the hybrid mode based on different models with respect to $b/a$ at $\lambda = 1550$ nm. $b = 5$ nm, $h = 0.5$ nm, $W_1 = 20$ nm, $W = 600$ nm, $h_{\text{Si}} = 50$ nm, $h_{\text{SiO}_2} = 200$ nm, and $h_{\text{sub}} = 100$ nm.
$R = 2 \text{ nm}$. The green line overlap with the blue one suggests that the field profiles have no difference between the GNL and HD model. However, there is a discrepancy between the local and nonlocal model. The radial electric field profiles in figure 2(a) shows that the introduction of nonlocal effects (spatial dispersion) removes the electric field discontinuity at the boundaries of the metallic nanowire. This is only valid for metal without interband effects and surrounded by air [51]. The continuity of the electric field can be clearly seen in figures 2(c) and (d), and field penetrates into the metal by $\delta \approx 0.1 \text{ nm}$, the order of the Fermi wavelength of silver. The longitudinal wave penetration reveals the non-vanishing extension of the induced charges into the metal by the incident fields, which shows that the usual assumption of a delta function distribution of the induced charges in the metal is inaccurate.

As stated above, adding the induced charge diffusion into HD model enhances the loss. From another point view, in the nonlocal model (both HD and GNL) the induced charge is moved inside the geometrical surface of metal by $\delta \approx 0.1 \text{ nm}$. This means that the nonlocal effects result in an ‘effective’ modification [50, 67] of the metal interface boundaries, reducing the ‘effective’ radius of the metal wire to $R' = R - \delta$, which is smaller than $R$. As we can obtain from figure 1(d) that under the same condition, the plasmonic mode of Ag nanowire with smaller radius has larger loss. The surface modification effect thus indicates that the blue (HD) and green lines (GNL) should lie below the black lines in figures 1(b) and (d), however, the blue lines fail.

Nanowire dimers are an interesting system, which also have attracted lots of attention and been used to investigate the impact of nonlocal effects on scattering properties based on HD model [56, 58]. As the gap dimensions of the dimers are reduced, the field is more concentrated in the gap. When the gap is reduced to a few nanometers or smaller, the non-local effects begin to take effect. Figure 3 gives the effective mode index and the propagation length with respect to

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**Figure 8.** The configuration of a coupling system (a), $E_y$ for the symmetric and antisymmetric modes based on the HD (b) and GNL model (c) with $S = 35 \text{ nm}$. (d) Dependence of the coupling length on the separation. Other parameters are the same as in figure 4.

**Figure 9.** The permittivity of silver with respect to wavelength. Red crosses denote the data from Palik and solid lines for our fitting. The Drude fitting parameters are $\varepsilon_\infty = 1$, $h\omega_p = 8.2953 \text{ eV}$, and $h\gamma = 0.055 \text{ eV}$. Other parameters are the same as in figure 4.
wavelength and gap. The radii of the two Ag nanowires are \( R_1 = R_2 = 2 \text{ nm} \). The gap is 1 nm for figures (a) and (b). The relative difference of propagation length between the GNL and local Drude model reduces as wavelength increases. The relative difference is 35.7% at \( \lambda = 1 \mu \text{m} \) and reduces to 5.8% at 2 \( \mu \text{m} \). Compared with that (about 7.3% at \( \lambda = 1 \mu \text{m} \) for \( R = 2 \text{ nm} \)) of a metal nanowire, the huge difference indicates that the nonlocal effects are much more remarkable for dimers with gap = 1 nm. Figures 3(c) and (d) show the real parts of effective mode indices and propagation length with respect to gap at \( \lambda = 1550 \text{ nm} \). Clearly, one can see that nonlocal effects weaken as the gap increases. The imaginary parts based on the GNL model are much larger (shorter propagation length) than that of the local Drude model. These results are similar to that of the nanowire waveguide. In section 4, we will show that nonlocal effects between metal and dielectric in a hybrid waveguide are weaker compared with that of metal-gap-metal structures (dimers here) [47, 52].

4. GNL effects in a hybrid plasmonic waveguide

To further investigate the GNL effects, we consider a hybrid plasmonic waveguide based on an elliptical metallic nanowire, which has more than one degree of freedom. One can change the long axis or short axis to obtain a sharp tip, which is naturally rounded. The cross-section of the proposed structure is shown in figure 4(a), which consists of a high-permittivity dielectric (Si) on insulator (SiO\(_2\)), a substrate Si, and an elliptical metallic nanowire above Si. Remove the metal wire, the structure becomes a conventional dielectric waveguide. With the metal wire located at a distance \( h \) above the high-index dielectric Si, the two photonic and SP modes couple to form a hybrid mode, which has the subwavelength character of a plasmonic mode and lossless property of a photonic mode, achieving subwavelength confinement and relative long propagation length simultaneously.

Figure 4(b) shows the electric field distribution of the hybrid mode based on the GNL model at \( \lambda = 1550 \text{ nm} \). The parameters are \( a = 1 \text{ nm}, b = 5 \text{ nm}, h = 0.5 \text{ nm}, W_1 = 20 \text{ nm}, W = 600 \text{ nm}, h_{\text{Si}} = 50 \text{ nm}, h_{\text{SiO}_2} = 200 \text{ nm}, \) and \( h_{\text{sub}} = 100 \text{ nm} \). We take the same parameters in section 3, that is, \( h_{\text{sub}} = 8.2953 \text{ eV} \) and \( \hbar \gamma = 0.055 \text{ eV} \) for silver. The permittivities of Si and SiO\(_2\) are chosen to be 12.25 and 2.25, respectively. The nonlocal parameter \( \beta = 0.0036c \), diffusion constant \( D = 3.0661 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \) for \( A = 0.5 \) and \( v_F = 1.39 \times 10^8 \text{ m s}^{-1} \). Unlike the modal field distribution based on the local Drude model, the modal field distribution based on the GNL model relates to the boundary condition and the induced charge distribution. Apparently, spatial dispersion smooths the charge density singularity and results in field penetration into the metal.

Figure 5 gives the effective mode index and the propagation length with respect to wavelength. One can obtain the same change trend of \( n_{\text{eff}} \) as for the metal nanowire in section 3. The real parts based on the GNL and HD model overlap with each other. The relative difference of propagation length between the GNL and local Drude model reduces as wavelength increases. For the hybrid plasmonic waveguide, the relative difference is 21% at \( \lambda = 1 \mu \text{m} \) and reduces to 3% at 1.8 \( \mu \text{m} \). As we have shown in section 3, the relative difference is 35.7% at \( \lambda = 1 \mu \text{m} \) for nanowire dimers. This means one can achieve enhanced or more evident nonlocal effects by using metal-gap-metal structures. Meanwhile, one can see that the influence of nonlocal effects on \( \text{Im}(\varepsilon_{\text{eff}}) \) is much more remarkable for higher frequency. While the difference of propagation length between the local Drude model and HD model changes a little. Unlike the HD model, the GNL model gives a larger \( \text{Im}(\varepsilon_{\text{eff}}) \) compared with the local Drude model.

Further, we study the influence of the gap \( h \) and \( b/a \) on the effective mode index of the hybrid mode. First, we set \( a = 1 \text{ nm}, b = 5 \text{ nm} \) and sweep \( h \) from 0.5 to 5 \( \text{nm} \) at \( \lambda = 1550 \text{ nm} \). Then we set \( h = 0.5 \text{ nm}, b = 5 \text{ nm} \) and sweep \( b/a \) from 1 to 5.

One can obtain from figure 6 that the difference of both real parts and imaginary parts between the GNL model and local Drude model become smaller as \( h \) increases. For \( h = 0.5 \text{ nm} \) and 5 \( \text{nm} \), the relative differences of the imaginary parts between the GNL model and local Drude model are 5.8% and 2.4%, respectively. The relative differences of the real parts between the GNL model and local Drude model are 3% and 1.1%, respectively. Figure 7 shows the propagation properties of the hybrid mode with respect to \( b/a \) at \( \lambda = 1550 \text{ nm} \). Here we set \( b = 5 \text{ nm} \) and vary \( a \) from 5 to 1 \( \text{nm} \) (correspondingly \( b/a \) from 1 to 5). As \( b/a \) increases (i.e. decrease of metal size and much more sharper elliptical tip), the relative difference of both real parts and imaginary parts between the GNL model and local Drude model increase monotonously. For \( b/a = 1 \) and 5, the relative differences of the real parts between the GNL model and local Drude model are 1.9% and 3%, respectively. The relative differences of the imaginary parts between the GNL model and local Drude model are 3.1% and 5.8%, respectively. From figures 6 and 7, one can deduce that the spatial dispersion has a greater influence on effective mode index for smaller \( h \) and larger \( b/a \) (or sharper tip).

Here, the surface modification effect [67] results in a larger \( h \), however, smaller \( b \) and \( a \). In the local situation, the increase of \( h \) reduces the loss when one keeps \( b \) and \( a \) invariable; The reduction of \( b \) and \( a \) increases the loss when one keeps \( h \) invariable. Numerical calculations show that the loss reduced by \( h \) increase cannot compensate the loss increased by \( b \) and \( a \) reduction. For the local model, \( \text{Im}(\varepsilon_{\text{eff}}) \) is 0.60689 with \( h = 0.5 \text{ nm}, a = 1 \text{ nm}, \) and \( b = 5 \text{ nm} \). \( \text{Im}(\varepsilon_{\text{eff}}) \) is 0.63057 with \( h' = 0.6 \text{ nm}, d' = 0.9 \text{ nm}, \) and \( b' = 4.9 \text{ nm} \) at \( \lambda = 1550 \text{ nm} \). For the GNL model, \( \text{Im}(\varepsilon_{\text{eff}}) \) is 0.64212 with \( h = 0.5 \text{ nm}, a = 1 \text{ nm}, \) and \( b = 5 \text{ nm} \). Notice that 0.64212 > 0.63057 means there is another factor (induced charge diffusion) causes the increase of loss in GNL model. Then the enlargement of loss based on the GNL model could be qualitatively explained by the superposition of surface modification (\( h \) increasing while \( a, b \) decreasing) effects as well as the induced charge diffusion, which is the key to the GNL model.

In order to study the influence of the nonlocal effects on the crosstalk between adjacent structures, we calculate the coupling lengths of two identical structures [71]. In figure 8(a),
we show schematically the configuration of a coupling system that consists of two separated hybrid waveguides with edge-to-edge distance of $S$. The $E_z$ components for the symmetric and antisymmetric modes are shown in figures 8(b) and (c) for the HD and GNL model, which illustrating relatively weak modal overlap for $S = 35$ nm. Figure 8(d) shows the coupling lengths ($L_c = \lambda / (2\Delta n)$) for various waveguide separations, where $\Delta n = |\text{Re}(n_{\text{eff}}^s) - \text{Re}(n_{\text{eff}}^a)|$ is the difference of the effective refractive indices of the symmetric and antisymmetric modes. As one can be clearly see in figure 8(d), the coupling lengths based on the local Drude model are the same as that of the HD and GNL model. Although the gap size is 0.5 nm here, the nonlocal effects seem to have little influence on the coupling length. When the separation is decreased to $S = 20$ nm, the coupling length is only about 1 $\mu$m. One can obtain reduced crosstalk by enlarging separation distance. When $S > 80$ nm, $\Delta n$ approaches zero, illustrating that the two modes are decoupled.

5. Discussions

As the HD model and GNL model are based on the assumption of free electron gas model, we need to guarantee that the local Drude response is valid in the operating frequencies [72]. We choose wavelength in the range from 1 to 2 $\mu$m. Here, we also need to declare that the dielectric parameters are taken from the fitting to Palik’s experimental data [68] for silver in the range from 1 to 2 $\mu$m, as shown in figure 9. These results have bigger imaginary parts compared with the data from Johnson and Christy [73]. For instance, the permittivity of silver at $\lambda = 1550$ nm is $-129 + 3.3i$ for data from Johnson and Christ and $-106 + 7.35i$ for our fitting. This means that the propagation length can be improved by choosing different permittivity data.

Up to now, the mode area has not yet been mentioned. In the literature [74], different measurements of mode area are used and these measurements for the same waveguide mode can even result in different performances. Among which, the peak energy density related measurement is widely used. However, as one takes the nonlocal effects into account, the expression for peak energy density becomes different [51, 52]. Since we find that the mode field distributions for the GNL model and the HD model are indistinguishable from each other, we deduce they have the same mode areas, or at least the same order of magnitude.

In the simulation of nonlocal effects, we add an external current density to the metal domain, without adopting the weak form of equations (3) and (4). This method is easier and more transparent than using weak form. As we have shown above, the electric field penetrates into to the metal about 0.1 nm, which is smaller than the Fermi wavelength ($\lambda_F = 0.5$ nm for silver). Then one should take care of the computational grid when numerically investigating nonlocal effects. In our simulation, the metal structure size is far smaller than the wavelength, and the maximum grid of the metal is chosen to be 0.1 nm so as to ensure the accurateness of the results. Also a convergence analysis was implemented to ensure both the real and imaginary parts of the complex effective mode indices varied by less than 1%.

6. Conclusion

In summary, we have presented an extension of the GNL optical response theory for the mode analysis of three plasmonic waveguides, a pure metal nanowire, nanowire dimers, and a hybrid plasmon waveguide. Results show that the imaginary part of the effective mode index increases compared with the local Drude model. This is because diffusion causes the enhancement of loss. The results are quite different from that of the HD model. Whereas the real parts and field profiles based on the HD and GNL model have no significant difference. Contrary to the local description, results also show that the electric field penetrates into the metal about 0.1 nm in the nonlocal model, which smooths the singularity of the induced charge distribution at the metal surface. Further investigations show that the nonlocal effects are much more remarkable for smaller gap and sharper tip. And nonlocal effects in a metal-gap-metal structure are more evident than that in pure nanowire and metal-gap-dielectric structures. Therefore one should take the nonlocal effects into account when dealing with nanoplasmonic waveguide for practical applications. The obtained results may play a role when one utilizes subwavelength plasmonic waveguides in high density photonic integration.

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