Meta-stable brane configuration of product gauge groups

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Abstract
Starting from the \( \mathcal{N} = 1 SU(N_c) \times SU(N'_c) \) gauge theory with fundamental and bifundamental flavors, we apply the Seiberg dual to the first gauge group and obtain the \( \mathcal{N} = 1 \) dual gauge theory with dual matters including the gauge singlets. By analyzing the \( F \)-term equations of the superpotential, we describe the intersecting type IIA brane configuration for the meta-stable nonsupersymmetric vacua of this gauge theory. By introducing an orientifold 6-plane, we generalize to the case for \( \mathcal{N} = 1 SU(N_c) \times SO(N'_c) \) gauge theory with fundamental and bifundamental flavors. Finally, the \( \mathcal{N} = 1 SU(N_c) \times Sp(N'_c) \) gauge theory with matters is also described very briefly.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is well known that the \( \mathcal{N} = 1 SU(N_c) \) QCD with fundamental flavors has a vanishing superpotential before we deform this theory by mass term for quarks. The vanishing superpotential in the electric theory makes it easier to analyze its nonvanishing dual magnetic superpotential. Sometimes by tuning the various rotation angles between NS5-branes and D6-branes appropriately, even if the electric theory has nonvanishing superpotential, one can make the nonzero superpotential to vanish in the electric theory. Two procedures, deforming the electric gauge theory by adding the mass for the quarks and taking the Seiberg dual magnetic theory from the electric theory, are crucial to find out meta-stable supersymmetry breaking vacua in the context of dynamical supersymmetry breaking [1, 2]. Some models of dynamical supersymmetry breaking can be studied by gauging the subgroup of the flavor symmetry group by either field theory analysis or using the brane configuration.

1 For the type IIA brane configuration description of \( \mathcal{N} = 1 \) supersymmetric gauge theory, see the review paper [3].
In this paper, starting from the known $\mathcal{N}=1$ supersymmetric electric gauge theories, we construct the $\mathcal{N}=1$ supersymmetric magnetic gauge theories by brane motion and linking number counting. The dual gauge group appears only on the first gauge group. Based on their particular limits of corresponding magnetic brane configurations in the sense that their electric theories do not have any superpotentials except the mass deformations for the quarks, we describe the intersecting brane configurations of type IIA string theory for the meta-stable nonsupersymmetric vacua of these gauge theories.

We focus on the cases where the whole gauge group is given by a product of two gauge groups. One example can be realized by three NS5-branes with D4- and D6-branes, and the other by four NS5-branes with D4- and D6-branes. For the latter, the appropriate orientifold 6-plane should be located at the center of this brane configuration in order to have two gauge groups. Of course, it is also possible, without changing the number of gauge groups, to have the brane configuration consisting of five NS5-branes and orientifold 6-plane, at which the extra NS5-brane is located, with D4- and D6-branes, but we will not do this particular case in this paper.

In section 2, we review the type IIA brane configuration that contains three NS5-branes, corresponding to the electric theory based on the $\mathcal{N}=1SU(N_c) \times SU(N'_c)$ gauge theory [4–6] with matter contents and deform this theory by adding the mass term for the quarks. Then we construct the Seiberg dual magnetic theory which is $\mathcal{N}=1SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets by brane motion and linking number counting. We do not touch the part of second gauge group $SU(N'_c)$ in this dual process.

In section 3, we consider the nonsupersymmetric meta-stable minimum by looking at the magnetic brane configuration we obtained in section 2 and present the corresponding intersecting brane configuration of type IIA string theory, along the line of [7–11] (see also [12–14]) and we describe M-theory lift of this supersymmetry breaking type IIA brane configuration.

In section 4, we describe the type IIA brane configuration that contains four NS5-branes, corresponding to the electric theory based on the $\mathcal{N}=1SU(N_c) \times SO(N'_c)$ gauge theory [15] with matter contents and deform this theory by adding the mass term for the quarks. Then we take the Seiberg dual magnetic theory which is $\mathcal{N}=1SU(\tilde{N}_c) \times SO(N'_c)$ gauge theory with corresponding dual matters as well as various gauge singlets, by brane motion and linking number counting. The part of second gauge group $SO(N'_c)$ in this dual process is not changed under this process.

In section 5, the nonsupersymmetric meta-stable minimum is constructed by looking at the magnetic brane configuration we obtained in section 4 and we present the corresponding intersecting brane configuration of type IIA string theory and describe M-theory lift of this supersymmetry breaking type IIA brane configuration, as we did in section 3.

In section 6, we describe the similar application to the $\mathcal{N}=1SU(N_c) \times Sp(N'_c)$ gauge theory [15] briefly and make some comments on future directions.

2. The $\mathcal{N}=1$ supersymmetric brane configuration of $SU(N_c) \times SU(N'_c)$ gauge theory

After reviewing the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N}=1SU(N_c) \times SU(N'_c)$ gauge theory [4–6], we construct the Seiberg dual magnetic theory which is $\mathcal{N}=1SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory.
2.1. Electric theory with $SU(N_c) \times SU(N'_c)$ gauge group

The gauge group is given by $SU(N_c) \times SU(N'_c)$ and the matter contents [4–6] are given by

- $N_f$ chiral multiplets $Q$ are in the fundamental representation under the $SU(N_c)$, $N'_f$ chiral multiplets $\tilde{Q}$ are in the antifundamental representation under the $SU(N'_c)$ and then $Q$ are in the representation $(N_c, 1)$ while $\tilde{Q}$ are in the representation $(\overline{N_c}, 1)$ under the gauge group.
- $N'_f$ chiral multiplets $Q'$ are in the fundamental representation under the $SU(N'_c)$, $N_f$ chiral multiplets $\tilde{Q}'$ are in the antifundamental representation under the $SU(N_c)$ and then $Q'$ are in the representation $(1, N'_c)$ while $\tilde{Q}'$ are in the representation $(1, \overline{N'_c})$ under the gauge group.
- The flavor singlet field $X$ is in the bifundamental representation $(N_c, \overline{N_c})$ under the gauge group and its complex conjugate field $\tilde{X}$ is in the bifundamental representation $(\overline{N_c}, N_c)$ under the gauge group.

In the electric theory, since there exist $N_f$ quarks $Q$, $N'_f$ quarks $\tilde{Q}$, one bifundamental field $X$ which will give rise to the contribution of $N'_c$, and its complex conjugate $\tilde{X}$ which will give rise to the contribution of $N'_c$, the coefficient of the beta function of the first gauge group factor is

$$b_{SU(N_c)} = 3N_c - N_f - N'_c$$

and similarly since there exist $N'_f$ quarks $Q'$, $N_f$ quarks $\tilde{Q}'$, one bifundamental field $X$ which will give rise to the contribution of $N_c$ and its complex conjugate $\tilde{X}$ which will give rise to the contribution of $N_c$, the coefficient of the beta function of the second gauge group factor is

$$b_{SU(N'_c)} = 3N'_c - N'_f - N_c.$$  

The anomaly free global symmetry is given by $[SU(N_f) \times SU(N'_f)]^2 \times U(1)^3 \times U(1)_R$ [4–6] and let us denote the strong coupling scales for $SU(N_c)$ as $\Lambda_1$ and for $SU(N'_c)$ as $\Lambda_2$. The theory is asymptotically free when $b_{SU(N_c)} = 3N_c - N_f - N'_c > 0$ for the $SU(N_c)$ gauge theory and when $b_{SU(N'_c)} = 3N'_c - N'_f - N_c > 0$ for the $SU(N'_c)$ gauge theory.

The type IIA brane configuration for this theory can be described by $N_c$ color D4-branes (01236) suspended between a middle NS5-brane (012345) and the right $NSS^7_4$-brane (012389) (denoted by $NSS^7_{41}$-brane) along $x^6$ direction, together with $N_f$ D6-branes (0123789) which are parallel to $NSS^7_4$-brane and have nonzero (45) directions. Moreover, an extra left $NSS^7_4$-brane (denoted by $NSS^7_{32}$-brane) is located at the left-hand side of a middle NS5-brane along the $x^6$ direction and there exist $N'_c$ color D4-branes suspended between them, with $N'_f$ D6-branes which have zero (45) directions. These are shown in figure 1 explicitly. See also [3] for the brane configuration.

By realizing that the two outer $NSS^7_{41}$-branes are perpendicular to a middle NS5-brane and the fact that $N_f$ D6-branes are parallel to $NSS^7_4$-brane and $N'_f$ D6-branes are parallel to $NSS^7_{32}$-brane, the classical superpotential vanishes. However, one can deform this theory. Then the classical superpotential by deforming this theory by adding the mass term for the quarks $Q$ and $\tilde{Q}$, along the lines of [1, 7–11], is given by

$$W = mQ\tilde{Q}$$  

and this type IIA brane configuration can be summarized as follows:

- One middle NS5-brane with worldvolume (012345).
- Two $NSS^7_3$-branes with worldvolume (012389).

2 We introduce two complex coordinates $v = x^4 + ix^9$ and $w = x^8 + ix^9$ for simplicity.
Figure 1. The $N = 1$ supersymmetric electric brane configuration of $SU(N_c) \times SU(N'_c)$ with $N_f$ chiral multiplets $Q$, $N'_f$ chiral multiplets $\tilde{Q}$, $N_f$ chiral multiplets $Q'$, $N'_f$ chiral multiplets $\tilde{Q}'$. The flavor singlet bifundamental field $X$ and its complex conjugate bifundamental field $\tilde{X}$. The $N_f$ D6-branes have nonzero $v$ coordinates where $v = m$ for equal massive case of quarks $Q, \tilde{Q}$ while $Q'$ and $\tilde{Q}'$ are massless.

- $N_f$ D6-branes with worldvolume (0123789) located at the positive region in $v$ direction.
- $N_c$ color D4-branes with worldvolume (01236). These are suspended between a middle NS5-brane and $NS5'_R$-brane.
- $N'_c$ color D4-branes with worldvolume (01236). These are suspended between $NS5'_L$-brane and a middle NS5-brane.

Now we draw this electric brane configuration in figure 1 and we put the coincident $N_f$ D6-branes in the nonzero $v$ direction. If we ignore the left $NS5'_L$-brane, $N'_c$ D4-branes and $N'_f$ D6-branes, then this brane configuration corresponds to the standard $N = 1$ SQCD with $SU(N_c)$ gauge group.

2.2. Magnetic theory with $SU(\tilde{N}_c) \times SU(N'_c)$ gauge group

One can consider dualizing one of the gauge groups regarding as the other gauge group as a spectator. One takes the Seiberg dual for the first gauge group factor $SU(N_c)$ while remaining the second gauge group factor $SU(N'_c)$ unchanged. Also we consider the case where $\Lambda_1 \gg \Lambda_2$, in other words, the dualized group’s dynamical scale is far above that of the other spectator group.

Let us move a middle NS5-brane to the right all the way past the right $NS5'_R$-brane. For example, see [7–14]. After this brane motion, one arrives at figure 2. Note that there exists a creation of $N_f$ D4-branes connecting $N'_f$ D6-branes and $NS5'_R$-brane. Recall that the $N_f$ D6-branes are perpendicular to a middle NS5-brane in figure 1. The linking number [16] of NS5-brane from figure 2 is $L_5 = N'_f - \tilde{N}_c$. On the other hand, the linking number of NS5-brane from figure 1 is $L_5 = -\frac{N_f}{2} + N_c - N'_f$. Due to the connection of $N'_c$ D4-branes with $NS5'_R$-brane, the presence of $N'_c$ in the linking number arises. From these two relations,
one obtains the number of colors of dual magnetic theory
\[
\tilde{N}_c = N_f + N'_c - N_c.
\]
(2.2)

The linking number counting looks similar to that in [7] where there exists a contribution from the O4-plane.

Let us draw this magnetic brane configuration in figure 2 and recall that we put the coincident \( N_f \) D6-branes in the nonzero \( v \) directions in the electric theory, along the lines of [7–14]. The \( N_f \)-created D4-branes connecting between D6-branes and \( NS5'_R \)-brane can move freely in the \( w \) direction. Moreover since \( N'_c \) D4-branes are suspended between two equal \( NS5'_L,R \)-branes located at different \( x^6 \) coordinates, these D4-branes can slide along the \( w \) direction also. If we ignore the left \( NS5'_L \)-brane, \( N'_c \) D4-branes and \( N'_f \) D6-branes (detaching these from figure 2), then this brane configuration corresponds to the standard \( \mathcal{N} = 1 \) SQCD with the magnetic gauge group \( SU(\tilde{N}_c = N_f - N_c) \) with \( N_f \) massive flavors [12–14].

The dual magnetic gauge group is given by \( SU(\tilde{N}_c) \times SU(N'_c) \) and the matter contents are given by

- \( N_f \) chiral multiplets \( q \) are in the fundamental representation under the \( SU(\tilde{N}_c) \). \( N_f \) chiral multiplets \( \bar{q} \) are in the antifundamental representation under the \( SU(\tilde{N}_c) \) and then \( q \) are in the representation \((\tilde{N}_c, 1)\) while \( \bar{q} \) are in the representation \((\tilde{N}_c, 1)\) under the gauge group.
- \( N'_f \) chiral multiplets \( Q' \) are in the fundamental representation under the \( SU(N'_c) \). \( N'_f \) chiral multiplets \( \bar{Q}' \) are in the antifundamental representation under the \( SU(N'_c) \) and then \( Q' \) are in the representation \((1, N'_c)\) while \( \bar{Q}' \) are in the representation \((1, N'_c)\) under the gauge group.
- The flavor singlet field \( Y \) is in the bifundamental representation \((\tilde{N}_c, N'_c)\) under the gauge group and its complex conjugate field \( \bar{Y} \) is in the bifundamental representation \((\tilde{N}_c, N'_c)\) under the gauge group.

There are \((N_f + N'_c)^2\) gauge singlets in the first dual gauge group factor as follows:

- \( N_f \)-fields \( F' \) are in the fundamental representation under the \( SU(N'_c) \), its complex conjugate \( N_f \)-fields \( \bar{F}' \) are in the antifundamental representation under the \( SU(N'_c) \) and
then $F'$ are in the representation $(1, N'_f)$ under the gauge group while $\tilde{F}'$ are in the representation $(1, \tilde{N}_f)$ under the gauge group.

These additional $N_f SU(N'_f)$ fundamentals and $N'_f SU(N'_f)$ antifundamentals are originating from the $SU(N_c)$ chiral mesons $\tilde{X}Q$ and $X\tilde{Q}$ respectively. It is clear to see that from figure 2, since the $N_f$ D6-branes are parallel to the $NSL,D$-brane, the newly created $N'_f$ D4-branes can slide along the plane consisting of these $D6$-branes and $NSL,D$-brane arbitrarily. Then strings stretching between the $N_f$ D6-branes and $N'_f$ D4-branes will give rise to these additional $N_f SU(N'_f)$ fundamentals and $N'_f SU(N'_f)$ antifundamentals.

- $N^2_f$-fields $M$ are in the representation $(1, 1)$ under the gauge group.

  This corresponds to the $SU(N_c)$ chiral meson $Q\tilde{Q}$ and the fluctuations of the singlet $M$ correspond to the motion of $N_f$ flavor D4-branes along (789) directions in figure 2.

- The $N^2_c$-fields $\Phi$ is in the representation $(1, N^2_c - 1) \oplus (1, 1)$ under the gauge group.

  This corresponds to the $SU(N_c)$ chiral meson $X\tilde{X}$ and note that $X$ has a representation $\tilde{N}_f$ of $SU(N'_f)$ while $\tilde{X}$ has a representation $N'_c$ of $SU(N'_c)$. The fluctuations of the singlet $\Phi$ correspond to the motion of $N'_c$ D4-branes suspended two $NSL, R$-branes along the (789) directions in figure 2.

In the dual theory, since there exist $N'_c$ quarks $\tilde{q}$, $N_f$ quarks $\tilde{Q}$, one bifundamental field $Y$ which will give rise to the contribution of $N'_c$, its complex conjugate $\tilde{Y}$ which will give rise to the contribution of $N'_c$, the coefficient of the beta function for the first gauge group factor [6] is

$$b^\text{mag}_{SU(N_f)} = 3\tilde{N}_c - N_f - N'_c = 2N_f + 2N'_c - 3N_c,$$

where we inserted the number of colors given in (2.2) in the second equality and since there exist $N'_c$ quarks $\tilde{Q}', N'_f$ quarks $\tilde{Q}'$, one bifundamental field $Y$ which will give rise to the contribution of $\tilde{N}_c$, its complex conjugate $\tilde{Y}$ which will give rise to the contribution of $\tilde{N}_c$, $N_f$ fields $\tilde{F}'$, its complex conjugate $\tilde{N}_c$, $N'_f$ fields $\tilde{F}'$ and the singlet $\Phi$ which will give rise to $N'_c$, the coefficient of the beta function of second gauge group factor [6] is

$$b^\text{mag}_{SU(N'_c)} = 3N'_c - N'_f - \tilde{N}_c - N_f - N'_c = N'_c + N_c - 2N_f - N'_f.$$

Therefore, both $SU(\tilde{N}_c)$ and $SU(N'_c)$ gauge couplings are IR free by requiring the negativity of the coefficients of beta function. One can rely on the perturbative calculations at low energy for this magnetic IR free region $b^\text{mag}_{SU(\tilde{N}_c)} < 0$ and $b^\text{mag}_{SU(N'_c)} < 0$. Note that the $SU(N'_c)$ fields in the magnetic theory are different from those of the electric theory. Since $b_{SU(N'_c)} > 0$, $SU(N'_c)$ is more asymptotically free than $SU(N'_c)_{\text{mag}}$ [6]. Neglecting the $SU(N'_c)$ dynamics, the magnetic $SU(\tilde{N}_c)$ is IR free when $N_f + N'_c < \frac{3}{2}N_c$ [6].

The dual magnetic superpotential, by adding the mass term (2.1) for $Q$ and $\tilde{Q}$ in the electric theory which is equal to put a linear term in $M$ in the dual magnetic theory, is given by

$$W_{\text{dual}} = (Mq\tilde{q} + YF\tilde{q} + \tilde{Y}q\tilde{F} + \Phi Y\tilde{Y}) + mM,$$

where the mesons in terms of the fields defined in the electric theory are

$$M \equiv \tilde{Q}Q, \quad \Phi \equiv X\tilde{X}, \quad F' \equiv \tilde{X}Q, \quad \tilde{F}' \equiv X\tilde{Q}.$$
The classical moduli space of vacua can be obtained from $F$-term equations

$$
q \tilde{q} + m = 0, \quad \tilde{q} M + \tilde{F} \tilde{Y} = 0, \\
M q + Y F' = 0, \quad F' \tilde{q} + \tilde{Y} \Phi = 0, \\
\tilde{q} Y = 0, \quad q \tilde{F}' + \Phi Y = 0, \\
\tilde{Y} q = 0, \quad Y \tilde{Y} = 0.
$$

Then, it is easy to see that there exist three reduced equations

$$
\tilde{q} M = 0 = M q, \quad q \tilde{q} + m = 0
$$

and other $F$-term equations are satisfied if one takes the zero vacuum expectation values for the fields $Y, \tilde{Y}, F'$ and $\tilde{F}'$. Then the solutions can be written as follows:

$$(q) = \left( \sqrt{m} e^{\varphi} \mathbf{1}_{N_c} \right), \quad (\tilde{q}) = \left( \sqrt{\tilde{m}} e^{-\varphi} \mathbf{1}_{\tilde{N}_c} \right), \quad (M) = \left( \begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{array} \right),$$

$$(Y) = (\tilde{Y}) = (F') = (\tilde{F}') = 0.
$$

Let us expand around a point on (2.4), as done in [1]. Then the remaining relevant terms of the superpotential are given by

$$W_{\text{dual}}^{\text{rel}} = \Phi_0 (\delta \varphi \delta \tilde{q} + m) + \delta Z \delta \varphi \tilde{q}_0 + \delta \tilde{Z} \tilde{q}_0 \delta \tilde{\varphi}
$$

by following the same fluctuations for the various fields as in [9]:

$$q = \left( q_0 \mathbf{1}_{N_c} + \frac{1}{\sqrt{2}} (\delta \chi_+ + \delta \chi_) \mathbf{1}_{\tilde{N}_c} \right), \quad \tilde{q} = \left( \tilde{q}_0 \mathbf{1}_{\tilde{N}_c} + \frac{1}{\sqrt{2}} (\delta \chi_+ - \delta \chi_-) \mathbf{1}_{\tilde{N}_c} \delta \tilde{\varphi} \right),
$$

$$M = \left( \begin{array}{cc} \delta Y & \delta Z \\ \delta \tilde{Z} & \Phi_0 \mathbf{1}_{N_f - \tilde{N}_c} \end{array} \right)
$$

as well as the fluctuations of $Y, \tilde{Y}, F'$ and $\tilde{F}'$. Note that there also exist three kinds of terms, the vacuum $\langle q \rangle$ multiplied by $\delta \tilde{Y} \delta \tilde{F}'$, the vacuum $\langle \tilde{q} \rangle$ multiplied by $\delta F' \delta Y$ and the vacuum $\langle \Phi \rangle$ multiplied by $\delta Y \delta \tilde{Y}$. However, by redefining these, they do not enter the contributions for the one-loop result, up to quadratic order. As done in [17], one gets that $m^2 q_0$ will contain $(\log 4 - 1) > 0$ implying that these are stable.

### 3. Non-supersymmetric meta-stable brane configuration of $SU(N_c) \times SU(N'_f)$ gauge theory

Now we recombine $\tilde{N}_c$ D4-branes among $N_f$ flavor D4-branes connecting between D6-branes and $NS5'_f$-brane with those connecting between $NS5'_c$-brane and NS5-brane and push them in the $+v$ direction from figure 2. After this procedure, there are no color D4-branes between the $NS5'_f$-brane and NS5-brane. For the flavor D4-branes, we are left with only $(N_f - \tilde{N}_c)$ flavor D4-branes.

Then the minimal energy supersymmetry breaking brane configuration is shown in figure 3, along the lines of [7–14]. If we ignore the left $NS5'_c$-brane, $N'_f$ D4-branes and $N'_f$ D6-branes (detaching these from figure 3), as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the
Figure 3. The nonsupersymmetric minimal energy brane configuration of $SU(\tilde{N}_c = N_f + N'_f - N_c) \times SU(N'_f)$ with $N_f$ chiral multiplets $q$, $N'_f$ chiral multiplets $\tilde{q}$, $N'_f$ chiral multiplets $Q'$, $N'_f$ chiral multiplets $\tilde{Q}'$, the flavor singlet bifundamental field $Y$ and its complex conjugate $\tilde{Y}$ and various gauge singlets.

$N = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c = N_f - N_c)$ with $N_f$ massive flavors [12–14].

The type IIA/M-theory brane construction for the $N = 2$ gauge theory was described by [18] and after lifting the type IIA description to M-theory, the corresponding magnetic M5-brane configuration with equal mass for the quarks where the gauge group is given by $SU(\tilde{N}_c) \times SU(N'_f)$, in a background space of $xt = v^{N_f} \prod_{k=1}^{N'_f} (v - e_k)$ where this four-dimensional space replaces (45610) directions, is described by

$$t^3 + (v^{\tilde{N}_c} + \cdots) t^2 + (v^{N'_f} + \cdots) t + v^{N_f} \prod_{k=1}^{N'_f} (v - e_k) = 0, \quad (3.1)$$

where $e_k$ is the position of the D6-branes in the $v$ direction (for an equal massive case, we can write $e_k = m$) and we have ignored the lower power terms in $v$ in $t^2$ and $t$ denoted by $\cdots$ and the scales for the gauge groups in front of the first term and the last term, for simplicity. For fixed $x$, the coordinate $t$ corresponds to $y$.

From this curve (3.1) of the cubic equation for $t$ above, the asymptotic regions for three NS5-branes can be classified by looking at the first two terms providing the NS5-asymptotic region, the next two terms providing the $NS'_L$-brane asymptotic region and the final two terms giving the $NS'_R$-brane asymptotic region as follows:

1. $v \rightarrow \infty$ limit implies $w \rightarrow 0$, $y \sim v^{\tilde{N}_c} + \cdots$ NS asymptotic region.

2. $w \rightarrow \infty$ limit implies $v \rightarrow m$, $y \sim w^{N_f + N'_f - N_c} + \cdots$ $NS'_L$ asymptotic region, $v \rightarrow m$, $y \sim w^{N'_f - \tilde{N}_c} + \cdots$ $NS'_R$ asymptotic region.

Here the two $NS'_L,R$-branes are moving in the $+v$ direction holding everything else fixed instead of moving D6-branes in the $+v$ direction, in the spirit of [14]. The harmonic function

3 The M5-brane lives in (0123) directions and is wrapped on a Riemann surface inside (4568910) directions. The Taub-NUT space in (45610) directions is parametrized by two complex variables $v$ and $y$ and the flat two dimensions in (89) directions by a complex variable $w$. See [14] for the relevant discussions.
sourced by the D6-branes can be written explicitly by summing over two contributions from the $N_f$ and $N'_f$ D6-branes and a similar analysis that will solve the differential equation and find the nonholomorphic curve can be done [7–10, 14]. An instability from a new M5-brane mode arises.

4. The $\mathcal{N} = 1$ supersymmetric brane configuration of $SU(N_c) \times SO(N'_c)$ gauge theory

After reviewing the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1SU(N_c) \times SO(N'_c)$ gauge theory [15], we describe the Seiberg dual magnetic theory which is a $\mathcal{N} = 1SU(N'_c) \times SO(N_c)$ gauge theory.

4.1. Electric theory with $SU(N_c) \times SO(N'_c)$ gauge group

The gauge group is given by $SU(N_c) \times SO(N'_c)$ and the matter contents [15] (similar matter contents are found in [4]) are given by

- $N_f$ chiral multiplets $Q$ are in the fundamental representation under the $SU(N_c)$, $N_f$ chiral multiplets $\tilde{Q}$ are in the antifundamental representation under the $SU(N_c)$ and then $Q$ are in the representation $(N_c, 1)$ while $\tilde{Q}$ are in the representation $(\overline{N_c}, 1)$ under the gauge group.
- $2N'_f$ chiral multiplets $Q'$ are in the fundamental representation under the $SO(N'_c)$ and then $Q'$ are in the representation $(1, N'_c)$ under the gauge group.
- The flavor singlet field $X$ is in the bifundamental representation $(N_c, N'_c)$ under the gauge group and the flavor singlet $\tilde{X}$ is in the bifundamental representation $(\overline{N_c}, N'_c)$ under the gauge group.

In the electric theory, since there exist $N_f$ quarks $Q$, $N_f$ quarks $\tilde{Q}$, one bifundamental field $X$ which will give rise to the contribution of $N'_c$, and its complex conjugate $\tilde{X}$ which will give rise to the contribution of $N'_c$, the coefficient of the beta function of the first gauge group factor is

$$b_{SU(N_c)} = 3N_c - N_f - N'_c$$

and similarly, since there exist $2N'_f$ quarks $Q'$, one bifundamental field X which will give rise to the contribution of $N'_c$ and its complex conjugate $\tilde{X}$ which will give rise to the contribution of $N_c$, the coefficient of the beta function of the second gauge group factor is

$$b_{SO(N'_c)} = 3(N'_c - 2) - 2N'_f - 2N_c.$$ 

The anomaly free global symmetry is given by $SU(N_f)^2 \times SU(2N'_f) \times U(1)^2 \times U(1)_R$ and let us denote the strong coupling scales for $SU(N_c)$ as $\Lambda_1$ and for $SO(N'_c)$ as $\Lambda_2$, as in the previous section. The theory is asymptotically free when $b_{SU(N_c)} > 0$ for the $SU(N_c)$ gauge theory and when $b_{SO(N'_c)} > 0$ for the $SO(N'_c)$ gauge theory.

The type IIA brane configuration of $\mathcal{N} = 2$ gauge theory [19] consists of four NS5-branes (012345) which have different $x^6$ values, $N_c$ and $N'_c$ D4-branes (01236) suspended between them, $2N_f$ and $2N'_f$ D6-branes (0123789) and an orientifold 6-plane (0123789) of positive Ramond charge\(^4\). According to $\mathbf{Z}_2$ symmetry of orientifold 6-plane (O6-plane) sitting at $v = 0$ and $x^6 = 0$, the coordinates $(x^4, x^5, x^6)$ transform as $-(x^4, x^5, x^6)$, as usual. See also [3] for the discussion of O6-plane.

\(^4\) There are many different brane configurations with O6-plane in the literature and some of them are present in [20–24].
By rotating the third and fourth NS5-branes which are located at the right-hand side of O6-plane, from \(v\) direction toward \(-w\) and \(+w\) directions respectively, one obtains the \(\mathcal{N} = 1\) theory. Their mirrors, the first and second NS5-branes which are located at the left-hand side of O6-plane, can be rotated in a \(\mathbb{Z}_2\) symmetric manner due to the presence of O6-plane simultaneously. That is, if the first NS5-brane rotates by an angle \(-\omega\) in \((v, w)\) plane, denoted by \(NS5_{-\omega}\)-brane [3], then the mirror image of this NS5-brane, the fourth NS5-brane, is rotated by an angle \(\omega\) in the same plane, denoted by \(NS5_\omega\)-brane. If the second NS5-brane rotates by an angle \(\theta\) in \((v, w)\) plane, denoted by \(NS5_\theta\)-brane [3], then the mirror image of this NS5-brane, the third NS5-brane, is rotated by an angle \(-\theta\) in the same plane, denoted by \(NS5_{-\theta}\)-brane. For more details, see figure 4.5

We also rotate the \(N_f\) D6-branes which are located between the second NS5-brane and an O6-plane and make them be parallel to \(NS5_{-\omega}\)-brane and denote them as \(D6_{-\omega}\)-brane with zero \(v\) coordinate (the angle between the unrotated D6-branes and \(D6_{-\omega}\)-branes is equal to \(\frac{\pi}{2} - \theta\)) and its mirrors \(N_f\) D6-branes appear as \(D6_{-\omega}\)-branes between the O6-plane and third NS5-brane. There is no coupling between the adjoint field and the quarks since the rotated \(D6_{-\omega}\)-branes are parallel to the rotated \(NS5_{\omega}\)-brane [3, 5]. Similarly, the \(N_f\) D6-branes which are located between the third NS5-brane and the fourth NS5-brane can be rotated and we can make them be parallel to \(NS5_{-\omega}\)-brane and denote them as \(D6_{-\omega}\)-branes with nonzero \(v\) coordinate (the angle between the unrotated D6-branes and \(D6_{-\omega}\)-branes is equal to \(\frac{\pi}{2} - \omega\)) and its mirrors \(N_f\) D6-branes appear as \(D6_{-\omega}\)-branes between the first NS5-brane and the second NS5-brane.

Moreover the \(N_c\) D4-branes are suspended between the first NS5-brane and the second NS5-brane (and its mirrors) and the \(N_c\) D4-branes are suspended between the second NS5-brane and the third NS5-brane.

For this brane setup\(^5\), the classical superpotential is given by [15]

\[
W = -\frac{1}{4} \left[ \frac{1}{4 \tan(\omega - \theta)} + \frac{1}{\tan 2\theta} \right] \text{tr}(X \tilde{X})^2 + \frac{\text{tr} X \tilde{X} X \tilde{X}}{4 \sin 2\theta} + \frac{(\text{tr} X \tilde{X})^2}{4 N_c \tan(\omega - \theta)},
\]

\(^6\) The angles of \(\theta_1\) and \(\theta_2\) in [15] are related to the angles \(\theta\) and \(\omega\) as follows: \(\theta = \theta_1\) and \(\omega = \theta_2\).

\(^5\) For arbitrary angles \(\theta\) and \(\omega\), the superpotential for the \(SU(N_c)\) sector is given by \(W = X \phi \tilde{X} + \tan(\omega - \theta) \text{tr} \phi^2\) where \(\phi\) is an adjoint field for \(SU(N_c)\). There is no coupling between \(\phi\) and \(N_f\) quarks because \(D6_{-\omega}\)-branes are parallel to \(NS5_{-\omega}\)-branes. The superpotential for the \(SO(N_c')\) sector is given by \(W = X \phi_1 \tilde{X} + \phi_2 \tilde{X} + \tan(\omega - \theta) \text{tr} \phi_1^2 - \frac{1}{\text{tr} \phi_2^2}\) where \(\phi_1\) and \(\phi_2\) are an adjoint field and a symmetric tensor for \(SO(N_c')\) [25]. After integrating out \(\phi, \phi_1\) and \(\phi_2\), the whole superpotential can be written as in (4.1).
It is easy to see that when $\theta$ approaches 0 and $\omega$ approaches $\frac{\pi}{2}$, then this superpotential vanishes.

Now one summarizes the supersymmetric electric brane configuration with their worldvolumes in type IIA string theory as follows.

- $NS_{5-\omega}$-brane with worldvolume by both (0123) and two spatial dimensions in the $(v, w)$ plane and with negative $x^6$.
- $NS_{5\theta}$-brane with worldvolume by both (0123) and two spatial dimensions in the $(v, w)$ plane and with negative $x^6$.
- $NS_{5-\phi}$-brane with worldvolume by both (0123) and two spatial dimensions in the $(v, w)$ plane and with positive $x^6$.
- $NS_{5\phi}$-brane with worldvolume by both (0123) and two spatial dimensions in the $(v, w)$ plane and with positive $x^6$.
- $N_f D_6\theta$-branes with worldvolume by both (01237) and two spatial dimensions in the $(v, w)$ plane and with negative $x^6$ and $v = 0$.
- $N_f D_6-\omega$-branes with worldvolume by both (01237) and two spatial dimensions in the $(v, w)$ plane and with positive $x^6$ and $v = 0$.
- $N_f D_6\omega$-branes with worldvolume by both (01237) and two spatial dimensions in the $(v, w)$ plane and with positive $x^6$. Before the rotation, the distance from $N_c$ color D4-branes in the $+v$ direction is nonzero.
- $N_f D_6-\omega$-branes with worldvolume by both (01237) and two spatial dimensions in the $(v, w)$ plane and with positive $x^6$. Before the rotation, the distance from $N_c$ color D4-branes in the $-v$ direction is nonzero.
- $O_6$-plane with worldvolume (0123789) with $v = 0 = x^6$.
- $N_c$ D4-branes connecting $NS_{5-\omega}$-brane and $NS_{5\phi}$-brane, with worldvolume (01236) with $v = 0 = w$ (and its mirrors).
- $N_f'$ D4-branes connecting $NS_{5\theta}$-brane and $NS_{5-\phi}$-brane, with worldvolume (01236) with $v = 0 = w$.

We draw the type IIA electric brane configuration in figure 4 which was basically given in [15] already but the only difference is to put $N_f$ D6-branes in the nonzero $v$ direction in order to obtain nonzero masses for the quarks which are necessary to obtain the meta-stable vacua.

4.2. Magnetic theory with $SU(\tilde{N}_c) \times SO(N'_c)$ gauge group

One takes the Seiberg dual for the first gauge group factor $SU(N_c)$ while keeping the second gauge group factor $SO(N'_c)$, as in previous case. Also we consider the case where $\Lambda_1 \gg \Lambda_2$, in other words, the dualized group’s dynamical scale is far above that of the other spectator group.

Let us move the $NS_{5-\phi}$-brane to the right all the way past the right $NS_{5\omega}$-brane (and its mirrors to the left). After this brane motion, one arrives at figure 5. Note that there exists a creation of $N_f$ D4-branes connecting $N_f D_6\omega$-branes and $NS_{5\omega}$-brane (and its mirrors). Recall that the $N_f D_6\omega$-branes are not parallel to the $NS_{5-\phi}$-brane in figure 4 (and its mirrors). The linking number of $NS_{5-\phi}$-brane from figure 5 is $L_5 = \frac{N_f}{2} - \tilde{N}_c$. On the other hand, the linking number of $NS_{5-\phi}$-brane from figure 4 is $L_5 = -\frac{N_f}{2} + N_c - N'_c$. From these, one gets the number of colors in dual magnetic theory

$$\tilde{N}_c = N_f + N'_c - N_c.$$

(4.2)
Let us draw this magnetic brane configuration in figure 5 and remember that we put the coincident $N_fD_6$-branes in the nonzero $\nu$ directions (and its mirrors). The $N_f$-created D4-branes connecting between $D_6$-branes and NS$5_\omega$-brane can move freely in the $\omega$ direction, as in previous case. Moreover, since $N'_f$ D4-branes are suspended between two unequal NS$5_{\pm\omega}$-branes located at different $\chi^6$ coordinates, these D4-branes cannot slide along the $\omega$ direction, for arbitrary rotation angles. If we are detaching all the branes except the NS$5_{\omega}$-brane, NS$5_{\omega}$-brane, $D_6$-branes, $N_f$ D4-branes and $N'_f$ D4-branes from figure 5, then this brane configuration corresponds to $\mathcal{N} = 1$ SQCD with the magnetic gauge group $SU(\tilde{N}_c) = N_f - N_c$ with $N'_f$ massive flavors with tilted NS5-branes.

The dual magnetic gauge group is given by $SU(\tilde{N}_c) \times SO(N'_c)$ and the matter contents are given by

- $N_f$ chiral multiplets $\tilde{q}$ are in the fundamental representation under the $SU(\tilde{N}_c)$, $N_f$ chiral multiplets $q$ are in the antifundamental representation under the $SU(\tilde{N}_c)$ and then $q$ are in the representation $(\tilde{N}_c, 1)$ while $\tilde{q}$ are in the representation $(\tilde{N}_c, 1)$ under the gauge group.
- $2N'_f$ chiral multiplets $Q'$ are in the fundamental representation under the $SO(N'_c)$ and then $Q'$ are in the representation $(1, N'_c)$ under the gauge group.
- The flavor singlet field $Y$ is in the bifundamental representation $(\tilde{N}_c, N'_c)$ under the gauge group and its complex conjugate field $\tilde{Y}$ is in the bifundamental representation $(\tilde{N}_c, N'_c)$ under the gauge group.

There are $(N_f + N'_f)^2$ gauge singlets in the first dual gauge group factor as follows:

- $N_f$-fields $F'$ are in the fundamental representation under the $SO(N'_c)$, $N_f$-fields $\tilde{F}'$ are in the fundamental representation under the $SO(N'_c)$ and then $F'$ are in the representation $(1, N'_c)$ under the gauge group while $\tilde{F}'$ are in the representation $(1, N'_c)$ under the gauge group.

These additional $2N_fSO(N'_c)$ vectors originate from the $SU(N_c)$ chiral mesons $X\bar{Q}$ and $\bar{X}Q$ respectively. It is easy to see that from figure 5, since the $D_6$-branes are parallel to the $NS5_{\omega}$-brane, the newly created $N_f$ D4-branes can slide along the plane consisting of $D_6$-branes and $NS5_{\omega}$-brane arbitrarily (and its mirrors). Then strings connecting the $N_fD_6$-branes and $N'_f$ D4-branes will give rise to these additional $2N_fSO(N'_c)$ vectors.
- $N'_f$-fields $M$ are in the representation $(1, 1)$ under the gauge group.
This corresponds to the $SU(N_f)$ chiral meson $Q\bar{Q}$ and the fluctuations of the singlet $M$

correspond to the motion of $N_f$ flavor D4-branes along (789) directions in figure 5.

- The $N_f^2$ singlet $\Phi$ is in the representation (1, adj) $\oplus$ (1, symm) under the gauge group.

This corresponds to the $SU(N_c)$ chiral meson $X\bar{X}$ and note that both $X$ and $\bar{X}$ have
the representation $N_c^*$ of $SO(N_c^*)$. In general, the fluctuations of the singlet $\Phi$ correspond to
the motion of $N_f^*$ D4-branes suspended two $NS$-branes along the (789) directions in figure 5.

In the dual theory, since there exist $N_f$ quarks $q$, $N_f$ quarks $\bar{q}$, one bifundamental field $Y$
which will give rise to the contribution of $N_c^*$ and its complex conjugate $\bar{Y}$ which will give rise
to the contribution of $N_c^*$, the coefficient of the beta function of the first gauge group factor
with (4.2) is

$$b_{SU(N_c^*)}^\text{mag} = 3N_c^* - N_f^* - N_c^* = 2N_f^* + 2N_c^* - 3N_c^*$$

and since there exist $2N_f^*$ quarks $Q^*$, one bifundamental field $Y$ which will give rise to the
contribution of $N_c^*$, its complex conjugate $\bar{Y}$ which will give rise to the contribution of $N_c^*$,$N_f^*$
fields $F'$, its complex conjugate $\bar{F}'$ and the singlet $\Phi$ which will give rise to $N_c^*$, the
coefficient of the beta function is

$$b_{SO(N_c^*)}^\text{mag} = 3(N_c^* - 2) - 2N_f^* - 2N_c^* - 2N_f^* - 2N_c^* = -N_c^* + 2N_c^* - 4N_f^* - 2N_f^* - 6.$$ 

Therefore, both $SU(N_c^*)$ and $SO(N_c^*)$ gauge couplings are IR free by requiring the negativeness
of the coefficients of beta function. One can rely on the perturbative calculations at low
energy for this magnetic IR free region $b_{SU(N_c^*)}^\text{mag} < 0$ and $b_{SO(N_c^*)}^\text{mag} < 0$. Note that the
$SO(N_c^*)$ fields in the magnetic theory are different from those of the electric theory. Since
$b_{SO(N_c^*)} < b_{SO(N_c^*)}^\text{mag} > 0$, $SO(N_c^*)$ is more asymptotically free than $SO(N_c^*)^\text{mag}$. Neglecting the
$SO(N_c^*)$ dynamics, the magnetic $SU(N_c^*)$ is IR free when $N_f + N_c^* < 3/2 N_c^*$, as in previous case.

The dual magnetic superpotential, by adding the mass term for $Q$ and $\bar{Q}$ in the electric
theory which is equal to put a linear term in $M$ in the dual magnetic theory, is given by

$$W_\text{dual} = [(\Phi^2 + \cdots) + Q^* \Phi Q^* + M q \bar{q} + Y \bar{F}' \bar{q} + \bar{Y} q F' + \Phi Y \bar{Y}] + m M,$$

where the mesons in terms of the fields defined in the electric theory are

$$M \equiv Q \bar{Q}, \quad \Phi \equiv X \bar{X}, \quad F' \equiv \bar{X} Q, \quad \bar{F}' \equiv X \bar{Q}.$$ 

We abbreviated all the relevant terms and coefficients appearing in the quartic superpotential
for the bifundamentals in electric theory (4.1) and denote them here by $\Phi^2 + \cdots$. Here
$q$ and $\bar{q}$ are fundamental and antifundamental for the gauge group index, respectively, and
antifundamentals for the flavor index. Then, $q \bar{q}$ has a rank $N_c^*$ and $m$ has a rank $N_f^*$. Therefore, the
$F$-term condition, the derivative of the superpotential $W_\text{dual}$ with respect to $M$, cannot be satisfied if the rank $N_f^*$ exceeds $N_c^*$, and the supersymmetry is broken. Other $F$-term equations
are satisfied by taking the vacuum expectation values of $Y, \bar{Y}, F', \bar{F}'$ and $Q'$ to vanish.

The classical moduli space of vacua can be obtained from $F$-term equations and one gets

$$q \bar{q} + m = 0, \quad Q^* \Phi Q^* + M q \bar{q} + Y \bar{F}' \bar{q} + \bar{Y} q F' + \Phi Y \bar{Y} + m M = 0.$$ 

A mismatch appears between the number of colors from field theory analysis and those from brane motion when
we take the full dual process on the two gauge group factors simultaneously [15]. By adding $4N_f^*$ D4-branes to
the dual brane configuration without affecting the linking number counting, this mismatch can be removed. Similar
phenomena occurred in [5, 26]. It turned out that there exists a deformation $\Delta W$ generated by the meson $Q'X\bar{X}Q'$.
This is exactly the second term, $Q'\Phi Q'$, in (4.3). In the previous example, there is no such deformation term in (2.3).
Then, it is easy to see that there exists a solution
\[ \tilde{q} M = 0 = M q, \quad q \tilde{q} + m = 0. \]

Other \( F \)-term equations are satisfied if one takes the zero vacuum expectation values for the fields \( Y, \tilde{Y}, F', \tilde{F}' \). Then the solutions can be written as
\[ \langle q \rangle = \left( \sqrt{m} e^{\phi} 1_N \right), \quad \langle \tilde{q} \rangle = \left( \sqrt{m} e^{-\phi} 1_N \right), \quad \langle M \rangle = \left( \begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 1_{N_f - \tilde{N}_5} \end{array} \right), \quad (4.4) \]
\[ \langle Y \rangle = \langle \tilde{Y} \rangle = \langle F' \rangle = \langle \tilde{F}' \rangle = \langle Q' \rangle = 0. \]

Let us expand around a point on (4.4), as done in [1]. Then the remaining relevant terms of superpotential are given by
\[ W_{\text{dual}}^{\text{rel}} = \Phi_0 (\delta \phi \delta \tilde{\phi} + m) + \delta Z \delta \phi \tilde{q} q_0 + \delta \tilde{Z} q_0 \delta \tilde{\phi} \]

by following the similar fluctuations for the various fields as in [9]. Note that there also exist four kinds of terms, the vacuum \( \langle q \rangle \) multiplied by \( \delta \tilde{Y} \delta F' \), the vacuum \( \langle \tilde{q} \rangle \) multiplied by \( \delta F' \delta \tilde{Y} \), the vacuum \( \langle \Phi \rangle \) multiplied by \( \delta \tilde{Y} \delta \tilde{Y} \) and the vacuum \( \langle \tilde{\Phi} \rangle \) multiplied by \( \delta Q' \delta Q' \).

However, by redefining these, they do not enter the contributions for the one-loop result, up to quadratic order. As done in [17], one gets that \( m_{\tilde{B}_4}^2 \) will contain \((\log 4 - 1) > 0\) implying that these are stable.

5. Nonsupersymmetric meta-stable brane configuration of \( SU(N_c) \times SO(N_f') \) gauge theory

Since the electric superpotential (4.1) vanishes for \( \theta = 0 \) and \( \omega = \frac{\pi}{2} \), the corresponding magnetic superpotential in (4.3) does not contain the terms \( \Phi^2 + \cdots \) and it becomes
\[ W_{\text{dual}} = \langle Q' \Phi Q' + M q \tilde{q} + Y \tilde{F}' \tilde{q} + \tilde{Y} q F' + \Phi \tilde{Y} Y \rangle + m M. \]

Now we recombine \( \tilde{N}_5 \) D4-branes among \( N_f \) flavor D4-branes connecting between \( D6_{\omega = -} = D6 \)-branes and \( N_55_{\omega = +} = NS5' \)-brane with those connecting between \( NS5' \)-brane and \( NS5_{-\phi = 0} = NS5 \)-brane (and its mirrors) and push them in the \(+v\) direction from figure 5.

Of course their mirrors will move to the \(-v\) direction in a \( Z_2 \) symmetric manner due to the \( O6^+\)-plane. After this procedure, there are no color D4-branes between the \( NS5' \)-brane and the \( NS5 \)-brane. For the flavor D4-branes, we are left with only \( (N_f - \tilde{N}_5) \) D4-branes (and its mirrors).

Then the minimal energy supersymmetry breaking brane configuration is shown in figure 6. If we ignore all the branes except \( NS5' \)-brane, \( NS5 \)-brane, \( D6 \)-branes, \( (N_f - \tilde{N}_5) \) D4-branes and \( \tilde{N}_5 \) D4-branes, as observed already, then this brane configuration corresponds to the minimal energy supersymmetry breaking brane configuration for the \( N' = 1 \) SQCD with the magnetic gauge group \( SU(\tilde{N}_5) \) with \( N_f' \) massive flavors [12–14]. Note that \( N_f' \) D4-branes can slide along the \( w \) direction for this brane configuration.

The type IIA/M-theory brane construction for the \( N' = 2 \) gauge theory was described by [19] and after lifting the type IIA description we explained so far for M-theory, the corresponding magnetic M5-brane configuration with equal mass for the quarks where the gauge group is given by \( SU(\tilde{N}_5) \times SO(N_f') \), in a background space of \( x_k = (-1)^{N_f + N_f'} v^{2N_f + 2} \prod_{k=1}^{N_f} (v^2 - \epsilon_k^2) \) where this four-dimensional space replaces (45610) directions, is characterized by
\[ t^4 + (v^{\tilde{N}_5} + \cdots) t^3 + (v^{N_f} + \cdots) t^2 + (v^{\tilde{N}_5} + \cdots) t + v^{2N_f + 4} \prod_{k=1}^{N_f} (v^2 - \epsilon_k^2) = 0. \]
Figure 6. The nonsupersymmetric minimal energy brane configuration of $SU(\tilde{N}_f = N_f + N'_f - N_c) \times SO(N'_c)$ with $N_f$ chiral multiplets $q$, $N_f$ chiral multiplets $\tilde{q}$, $2N'_f$ chiral multiplets $Q'$, the flavor singlet bifundamental field $Y$ and its complex conjugate bifundamental field $\tilde{Y}$ and gauge singlets. The $N'_c$ D4-branes and $2(N_f - \tilde{N}_c)$ D4-branes can slide in the $w$ direction freely in a $Z_2$ symmetric way.

From this curve of the quartic equation for $\tau$ above, the asymptotic regions can be classified by looking at the first two terms providing the $NS5_R$-brane asymptotic region, the next two terms providing the $NS5'_L$-brane asymptotic region, the next two terms providing $NS5'_R$-brane asymptotic region and the final two terms giving the $NS5_L$-brane asymptotic region as follows:

1. $v \to \infty$ limit implies
   $\begin{align*}
   w &\to 0, \quad y \sim v^{\tilde{N}_c} + \cdots \quad NS5_R \text{ asymptotic region}, \\
   w &\to 0, \quad y \sim v^{2N_f + 2N'_f - \tilde{N}_c + 4} + \cdots \quad NS5_L \text{ asymptotic region}.
   \end{align*}$

2. $w \to \infty$ limit implies
   $\begin{align*}
   v &\to -m, \quad y \sim w^{\tilde{N}_c - N'_c} + \cdots \quad NS5'_L \text{ asymptotic region}, \\
   v &\to +m, \quad y \sim w^{N_c - \tilde{N}_c} + \cdots \quad NS5'_R \text{ asymptotic region}.
   \end{align*}$

Now the two $NS5'_L$-branes are moving in the $\pm v$ direction holding everything else fixed instead of moving D6-branes in the $\pm v$ direction. Then the mirrors of the D4-branes are moved appropriately. The harmonic function sourced by the D6-branes can be written explicitly by the summing of three contributions from the $N_f$ and $N'_f$ D6-branes (and its mirrors) plus an O6-plane, and a similar analysis to solve the differential equation and find out the nonholomorphic curve can be done [7–10, 14]. In this case also, we expect an instability from a new M5-brane mode.

6. Discussions

So far, we have dualized only the first gauge group factor in the gauge group $SU(N_c) \times SO(N'_c)$. What happens if we dualize the second gauge group factor $SO(N'_c)$? (For the case $SU(N_c) \times SU(N'_c)$, the behavior of the dual for the second gauge group will be the same as when we take the dual for the first gauge group factor.) This can be done by moving the $NS5_0$-brane and $N'_f$ D6$_0$-branes, that can be located at the nonzero $v$ coordinate for massive quarks $Q'$, to the right passing through O6-plane (and their mirrors to the left). According to the linking number counting, one obtains the dual gauge group $SU(N_c) \times SO(\tilde{N}'_f = 2N_c + 2N'_f - N'_c + 4)$. One can easily see that there is a creation of $N'_f$ D4-branes connecting $NS5_0$-brane and
D6_φ-branes (and its mirrors). Then from the brane configuration, there exist the additional $2 N_f' S O(N'_c)$ quarks originating from the $S O(N'_c)$ chiral mesons $Q X \equiv F'$ and $Q' X \equiv F'$. The deformed superpotential $\Delta W = Q X X Q'$ can be interpreted as the mass term of $F' \bar{F}'$. Then one can write the dual magnetic superpotential in this case. However, it is not clear how the recombination of color and flavor D4-branes and splitting procedure between them in the construction of meta-stable vacua arises since there is no extra NS5-brane between two $N S 5_{3, \delta, \theta}$-branes. If there exists an extra NS5-brane at the origin of our brane configuration (then the gauge group and matter contents will change), it would be possible to construct the corresponding meta-stable brane configuration. It would be interesting to study these more in the future.

As already mentioned in [8] and section 4, the matter contents in [4] are different from those in section 4 with the same gauge group. In other words, the theory of $S U(N_c) \times S O(N'_c)$ with $X$, which transform as fundamental in $S U(N_c)$ and vector in $S O(N'_c)$, an antisymmetric tensor $A$ in $S U(N_c)$, as well as fundamentals for $S U(N_c)$ and vectors for $S O(N'_c)$ can confine either the $S U(N_c)$ factor or $S O(N'_c)$ factor. This theory can be described by the web of branes in the presence of $O 4^-$-plane and orbifold fixed points. With two NS5-branes and an $O 4^-$-plane, by modifying out $Z_3$ symmetry acting on $(v, w) \rightarrow (v \exp (\frac{2 \pi i}{3}), w \exp (\frac{2 \pi i}{3}))$, the resulting gauge group will be $S U(N_c) \times S O(N'_c) + 4$ with the above matter contents [27]. Similar analysis for $S U(N_c) \times S p(N'_c - 2)$ gauge group with opposite $O 4^-$-plane can be done. Then in this case, the matter in $S U(N_c)$ will be a symmetric tensor $S$ and other matter contents are present also. It would be interesting to see whether this gauge theory and the corresponding brane configuration will provide a meta-stable vacuum.

Let us comment on another possibility where the gauge group is given by $S U(N_c) \times S p(N'_c)$ and the matter contents are given by

- $N_f$ chiral multiplets $Q$ are in the fundamental representation under the $S U(N_c)$, $N_f$ chiral multiplets $\bar{Q}$ are in the antifundamental representation under the $S U(N_c)$ and then $Q$ are in the representation $(N_c, 1)$ while $\bar{Q}$ are in the representation $(N_c, 1)$ under the gauge group.
- $2 N'_f$ chiral multiplets $Q'$ are in the fundamental representation under the $S p(N'_c)$ and then $Q'$ are in the representation $(1, 2 N'_c)$ under the gauge group.
- The flavor singlet field $X$ is in the bifundamental representation $(N_c, 2 N'_c)$ under the gauge group and the flavor singlet $\tilde{X}$ is in the bifundamental representation $(N_c, 2 N'_c)$ under the gauge group.

One can compute the coefficients of beta functions of the each gauge group factor, as we did for previous examples.

The type IIA brane configuration of an electric theory is exactly the same as figure 4 except the RR charge O6-plane with negative sign. The classical superpotential\(^8\) is given by [15]

$$W = -\frac{1}{4} \left[\frac{1}{4 \tan(\omega - \theta)} + \frac{1}{\tan 2 \theta}\right] \text{tr}(X \tilde{X})^2 - \frac{\text{tr} X \tilde{X} \bar{X} X}{4 \sin 2 \theta} + \frac{(\text{tr} X \tilde{X})^2}{4 N_c \tan(\omega - \theta)}. \quad (6.1)$$

In this case, when $\theta$ approaches $\frac{\pi}{2}$ and $\omega$ approaches 0, then this superpotential vanishes.

The dual magnetic gauge group is given by $S U(N_c) = N_f + 2 N'_f - N_c \times S p(N'_c)$ with the same number of colors of dual theory as those in previous cases and the matter contents are given by

\(^8\) The superpotential for the $S p(N'_c)$ sector is given by $W = X \phi_3 X + X \phi_4 X + \tan \theta \text{tr} \phi_2^3 - \frac{1}{\tan \theta} \text{tr} \phi_2^4$, where $\phi_3$ and $\phi_4$ are an adjoint field (symmetric tensor) and an antisymmetric tensor for $S p(N'_c)$ [25]. Note that there is a sign change in the second trace term of the superpotential in (6.1), compared to (4.1).
Figure 7. The nonsupersymmetric minimal energy brane configuration of $SU(\tilde{N}_c = N_f + 2N'_f - N_c) \times Sp(N'_c)$ with $N_f$ chiral multiplets $q$, $N_f$ chiral multiplets $\tilde{q}$, $2N'_f$ chiral multiplets $Q'$, the flavor singlet bifundamental field $Y$ and its complex conjugate bifundamental field $\tilde{Y}$ and gauge singlets. Note the RR charge of O6-plane is negative and its charge is equivalent to $-4$ D6-branes. The $2N'_c$ D4-branes and $2(N_f - \tilde{N}_c)$ D4-branes can slide in the $\omega$ direction freely in a $Z_2$ symmetric way.

- $N_f$ chiral multiplets $q$ are in the fundamental representation under the $SU(\tilde{N}_c)$, $N_f$ chiral multiplets $\tilde{q}$ are in the antifundamental representation under the $SU(\tilde{N}_c)$ and then $q$ are in the representation $(\tilde{N}_c, 1)$ while $\tilde{q}$ are in the representation $(\tilde{N}_c, 1)$ under the gauge group.
- $2N'_f$ chiral multiplets $Q'$ are in the fundamental representation under the $Sp(N'_c)$ and then $Q'$ are in the representation $(1, 2N'_c)$ under the gauge group.
- The flavor singlet field $Y$ is in the bifundamental representation $(\tilde{N}_c, 2N'_c)$ under the gauge group and its complex conjugate field $\tilde{Y}$ is in the bifundamental representation $(\tilde{N}_c, 2N'_c)$ under the gauge group.

There are $(N_f + 2N'_f)^2$ gauge singlets in the first dual gauge group factor
- $N_f$-fields $F'$ are in the fundamental representation under the $Sp(N'_c)$, $N_f$-fields $\tilde{F}'$ are in the fundamental representation under the $Sp(N'_c)$ and then $F'$ are in the representation $(1, 2N'_c)$ under the gauge group while $\tilde{F}'$ are in the representation $(1, 2N'_c)$ under the gauge group.
- $N'_f$-fields $M$ are in the representation $(1, 1)$ under the gauge group.
- The $4N'_2$ singlet $\Phi$ is in the representation $(1, \text{adj}) \oplus (1, \text{antisym})$ under the gauge group.

The dual magnetic superpotential for arbitrary angles is given by $(4.3)$ with appropriate $Sp(N'_c)$ invariant metric $J$. The stability analysis can be done similarly.

After following the procedure from figures 4–5 with the opposite RR charge for the O6-plane and by taking the limit where $\theta \to \frac{\pi}{2}$ and $\omega \to 0$, the minimal energy supersymmetry breaking brane configuration is shown in figure 7.

Compared to the previous nonsupersymmetric brane configuration in figure 6, the role of NS5-brane and $NS5'$-brane is interchanged with each other: undoing the Seiberg dual in the context of [13]. This kind of feature of recombination and splitting between color D4-branes and flavor D4-branes occurs in [8]. At the electric brane configuration, $N_f$ D6-branes are perpendicular to the NS5-brane and this leads to the coupling between the quarks and adjoint in the superpotential. However, the overall coefficient function including these extra terms vanishes and eventually the whole electric superpotential will vanish according to the above limit we take.
From the quartic equation with the presence of the opposite RR charge for the O6-plane, in a background space of $xt = (-1)^{N_f + N_f'} v^{2N_f - 4} \prod_{k=1}^{N_f} (v^2 - e_k^2)$,

$$t^4 + (v \bar{R} i + \cdots) t^3 + (v N_i + \cdots) t^2 + (v \bar{N} i + \cdots) t + v^{2N_f - 4} \prod_{k=1}^{N_f} (v^2 - e_k^2) = 0,$$

the asymptotic regions can be classified as follows:

1. $v \to \infty$ limit implies
   
   $w \to 0, \quad y \sim v^{N_f - \bar{R} i} + \cdots \quad NS5_R$ asymptotic region,
   
   $w \to 0, \quad y \sim v^{\bar{N}_i - N_i} + \cdots \quad NS5_L$ asymptotic region.

2. $w \to \infty$ limit implies
   
   $v \to -m, \quad y \sim w^{2N_i + 2N_f - \bar{N}_i - 4} + \cdots \quad NS5_R$ asymptotic region,
   
   $v \to +m, \quad y \sim w^{\bar{N} i} + \cdots \quad NS5_L$ asymptotic region.

In [28], the $SU(7) \times \tilde{Sp}(1)$ model and the $SU(9) \times \tilde{Sp}(2)$ model can be obtained by dualizing the $SU(7) \times SU(2)$ model with a bifundamental and two antifundamentals for $SU(7)$ and a fundamental for $SU(2)$ and the $SU(9) \times SU(2)$ with a bifundamental and two antifundamentals for $SU(9)$ and a fundamental for $Sp(1)$ respectively (note that $Sp(1) \sim SU(2)$). The matter contents in an electric theory are different from those in the previous paragraph. The matter contents in the magnetic description are given by an antisymmetric tensor and a fundamental in the first gauge group as well as a bifundamental, a fundamental in the second gauge group and two antifundamentals in the first gauge group. There exists a nonzero dual magnetic superpotential. Also the dual description the $SU(7) \times \tilde{Sp}(1)$ model and $SU(9) \times \tilde{Sp}(2)$ model can be constructed from the antisymmetric models of Affleck–Dine–Seiberg by gauging a maximal flavor symmetry and adding the extra matter to cancel all anomalies and extra flavor.

On the other hand, the models $SU(2N_c + 1) \times SU(2)$ has its brane box model description in [29] where the above examples correspond to $N_c = 3$ and $N_f = 4$ respectively. In particular, the case where $N_c = 1$ (the gauge group is $SU(3) \times SU(2)$, i.e., $(3, 2)$ model [30]) was described by the brane box model with superpotential or without superpotential. Then it would be interesting to obtain the Seiberg dual for these models using the brane box model and look for the possibility of having meta-stable vacua for these models. Moreover, this gauge theory was generalized to the $SU(2N_c + 1) \times Sp(N'_c)$ model with a bifundamental and $2N'_c$ antifundamentals for $SU(2N_c + 1)$ and a fundamental for $Sp(N'_c)$ and its dual description $SU(2N_c + 1) \times Sp(N'_c = N_c - N'_c - 1)$ with a bifundamental and $2N'_c$ antifundamentals for $SU(2N_c + 1)$ and a fundamental for $Sp(N'_c)$ as well as two gauge singlets [28]. For the particular range of $N_c$, the dual theory is IR free, not asymptotically free.

According to [31], $SU(2N_c)$ with antisymmetric tensor and antifundamentals can be described by two gauge groups $Sp(2N_c - 4) \times SU(2N_c)$ with bifundamental and antifundamentals for $SU(2N_c)$. Some of the brane realization with zero superpotential was given in the brane box model in [29]. Similarly from the result of [32] by following the method of [31], the dual description for $SU(2N_c + 1)$ with antisymmetric tensor and fundamentals can be represented by two gauge group factors. This dual theory breaks the supersymmetry at the tree level. Similar discussions are present in [33]. Then it would be interesting to construct the corresponding Seiberg dual and see how the electric theory and its magnetic theory can be mapped into each other in the brane box model.
There are also different directions concerning the meta-stable vacua in different contexts and some of the relevant works are present in [34–43], where some of them use anti D-branes and some of them describe the type IIB theory and it would be interesting to find out how similarities, if any, appear and what are the differences in what sense between the present work and those works.

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