1. INTRODUCTION

The low-energy spectral lines detected between \( \approx 20 \) and 50 keV in the spectra of some gamma-ray bursts (GRBs) provide the most powerful means by which an analyst can probe the burst source environment. This is because the study of lines, unlike the study of continuum spectra, can yield precise information about the temperature and column depth of the line formation region, the velocity and orientation of that region, and the strength and orientation of any magnetic field in that region.

Candidate lines in GRB spectra were first reported by Mazets et al. (1980, 1981), who detected single absorption-like dips and troughs in the spectra of \( \approx 15\% \) of bursts observed by the Konus detectors of \textit{Venera 11} and 12.

Hueter (1987) reported similar absorption-like features in the spectra of two bursts observed by the \textit{HEAO 1} A-4 detector, and the \textit{Ginga} Gamma-Ray Burst Detector (GBD) observed three bursts—GRB 870303, GRB 880205, and GRB 890929—the spectra of which exhibited harmonically spaced absorption-like line candidates with centroid energies between \( \approx 20 \) and 50 keV (Murakami et al. 1988; Fenimore et al. 1988; Yoshida et al. 1991; Graziani et al. 1992, 1993; Freeman et al. 1999a, hereafter Paper I). A spectrum from an earlier epoch of GRB 870303, denoted S1, was subsequently found to exhibit a single absorption-like line candidate at \( \approx 20 \) keV (Graziani et al. 1992, who used S2 to denote the other line candidate spectrum of GRB 870303; Graziani et al. 1993; Paper I). Fits with phenomenological continuum-plus-line(s) models demonstrate that the \textit{Ginga} line candidates have statistical significance from \( \sim 10^{-3} \) to \( \sim 10^{-8} \). The enhanced low-energy coverage of the \textit{Ginga} GBD \( (E_{\text{low}} \approx 1.5 \text{ keV}, \text{compared with } E_{\text{low}} \gtrsim 20 \text{ keV for Konus and HEAO 1 A-4}), \) allowed analysts to rule out the possibility that these line candidates could be explained by a sudden change in continuum shape.

Mazets et al. posited that the Konus line candidates were formed by either cyclotron absorption or cyclotron scattering in the strong magnetic fields of Galactic neutron stars \( (\sim 10^{12} \text{ G}) \). The observation of harmonically spaced line candidates gave further credence to this hypothesis because the quantization of an electron's energy perpendicular to a strong field can lead to the formation of evenly spaced lines with energies \( E_n \approx 11.6nB_{12} \) keV. Rigorous proof of the
viability of this hypothesis came when a number of analysts used theoretical models of cyclotron scattering (and not cyclotron absorption) to produce emergent spectra from line-forming regions, which were found to compare favorably with the Ginga data (Wang et al. 1989a; Alexander & Mészáros 1989; Nishimura & Ebisuzaki 1992; Wang, Wasserman, & Lamb 1993). For instance, Wang et al. (1989a) used a Monte Carlo radiative transfer code (Wang, Wasserman, & Salpeter 1988, 1989b; Lamb et al. 1989) to demonstrate that the harmonic line candidates in the GRB 880205 spectrum could be produced in a line-forming layer with a magnetic field \(1.7 \times 10^{-12}\) G and column density \(1.2 \times 10^{21}\) cm\(^{-2}\). Freeman et al. (1992) later demonstrated that deepening the scattering region behind the continuum source led to a substantially better fit to these same data.

The cyclotron scattering interpretation for line candidates went largely unchallenged when it was consistent with the prevailing view that GRB sources were neutron stars residing in a thick disk in the Milky Way (with scale height \(\approx 2\) kpc; see, e.g., Higdon & Lingenfelter 1990; Harding 1991). However, recent developments, while not challenging the cyclotron scattering picture per se, have led many to question whether the reported line candidates actually exist at all. First, the discovery of optical transients (OTs) associated with GRBs (e.g., van Paradijs et al. 1997 and references therein), and the apparent determination of redshifts for five of them—GRB 970508 (Metzger et al. 1997), GRB 971214 (Kulkarni et al. 1998), GRB 980613 (Djorgovski et al. 1999), GRB 980703 (Djorgovski et al. 1998), and GRB 990123 (Kelson et al. 1999)—have indicated that some (if not all) GRBs occur at cosmological distances. Second, there have been no reports of definitive detections of line candidates in spectra of the Burst and Transient Source Experiment Spectroscopy Detectors (BATSE SDs) on the Compton Gamma-Ray Observatory (Palmer et al. 1994; Band et al. 1996; Briggs et al. 1996, 1998). These developments, combined with the apparent difficulty of forming lines in the circumstellar environments of cosmological burst sources (cf. Stanek, Paczynski, & Goodman 1993 and Ulmer & Goodman 1995, who attempt to account for lines by invoking gravitational femtolensing), have led many to adopt the viewpoint that all bursts are cosmological and that the reported line candidates, for whatever reason, are not real.

This viewpoint may be intuitively reasonable on its surface. However, while we do not seek to disprove the posited cosmological origin of the GRBs listed above, we feel that there are several reasons that we should continue to test the cyclotron scattering model. While none are completely compelling by themselves, they cumulatively indicate that the complete solution of the GRB mystery may have not yet been provided by the discovery of GRB redshifts.

The first reasons are the suggestive pieces of direct observational evidence that there are two or more classes of classical gamma-ray bursts. These pieces of evidence range from the well-established to the speculative. Well-established is the identification of two classes with differing light-curve morphologies (Kouveliotou et al. 1993, using burst duration; Lamb, Graziani, & Smith 1993, using light-curve variability). Further studies lend credence to the reality of separate classes (Kouveliotou et al. 1996; Katz & Canel 1996), but it is not yet proved that the correlation of burst properties cannot be explained using a single underlying mechanism.

Less well-established, but still highly suggestive, evidence is provided by burst repetition (e.g., Quashnock & Lamb 1993; Wang & Lingenfelter 1995). Unlike classes based on light-curve morphology, repetition indicates directly that there must be a Galactic component to the overall GRB source population, since it is generally considered impossible for cosmological bursters caused by one-time events such as neutron star–neutron star or neutron star–black hole mergers to repeat. Evidence for repetition is absent from the Third BATSE (3B) catalog (Meegan et al. 1996; Tegmark et al. 1996), but Graziani & Lamb (1996) have questioned how the systematic errors of this catalog are computed. (For details on the BATSE burst location algorithm, see Pendleton et al. 1999). Also, in 1996 October, after the publication of the 3B catalog, BATSE observed four bursts in less than 2 days that all came from directions consistent with a single source. Graziani, Lamb, & Quashnock (1998) perform simulations that indicate that the probability of such a spatial and temporal coincidence of four distinct bursts from four different sources is \(3.1 \times 10^{-5}\). The probability is increased to \(1.6 \times 10^{-3}\) if only three bursts occurred, but one of the three would have to be the longest ever observed by BATSE.

The most speculative piece of evidence is the apparent lack of consistency that recently localized bursts show when examined in other wavebands. For instance, since the beginning of 1997, there have been 11 GRBs observed with the BeppoSAX WFC and/or the RXTE ASM detectors whose positions have been refined through the use of the Interplanetary Network (IPN).\(^9\) Of these 11 bursts, X-ray afterglows, OTs, and radio afterglows were detected for eight, six, and three of them, respectively. (Two have measured redshifts and are thus conclusively cosmological—GRB 971214 and GRB 990123.) The lack of counterparts may be, of course, more simply explained by invoking, e.g., selection effects, rather than by invoking a separate class of bursters the broadband behavior of which is such that they do not show optically transient emission. Also, if there is a separate class of bursters that are defined by counterpart behavior, it would not necessarily have to be associated with the Milky Way.

Another reason for us to continue to test the cyclotron scattering model is the result of the exacting statistical analysis of the 3B catalog performed by Loredo & Wasserman (1995, 1998a, 1998b). They determine that the 3B catalog is consistent with the hypothesis that there is a component of the GRB source population residing within the Galaxy, with that component comprised of either dim local halo sources at distances \(\lesssim 1\) kpc (accounting for up to \(\approx 60\%\) of all bursts), or luminous halo sources at distances \(\gtrsim 50\) kpc (accounting for up to \(\approx 10\%\) of all bursts). Studies show that neutron stars that receive high initial kick velocities when they are formed can populate the halo (e.g., Li &Dermer 1992; Podziadlowski, Rees, & Ruderman 1995; Bulik, Lamb, & Coppi 1998). Such “hybrid” models with both Galactic and cosmological bursters fit to the 3B data better than models with only cosmological bursters, in part because the existence of a highly isotropic, extragalactic burst component weakens greatly the isotropy constraints on any anisotropic Galactic component. However, the fit is

\(^9\) This information was compiled using http://www.aip.de/People/JGreiner/grbgen.html, which contains complete references, including texts of circulars, for each burst.
not so much better as to rule out models containing only cosmological bursters.

A final reason to continue to test the cyclotron scattering model is the fact that the BATSE SDs may lack the low-energy spectral sensitivity that is necessary for low-energy lines to be easily detected. The gain settings on the individual SDs differ; those with the highest gain settings can, in principle, observe GRBs at energies $\gtrsim 10$ keV. An electronic artifact discovered after launch affects energy calibration such that spectra are distorted in the first $\approx 10$ channels above the low-energy cutoff (the so-called SLED effect; see Band et al. 1992). Despite this, studies using simulated Ginga line candidate spectra indicated that the SDs were still capable of detecting low-energy line candidates (Band et al. 1995), and no line candidates were definitively detected during initial visual searches of those spectra with the largest signal-to-noise ratios (Palmer et al. 1994; Band et al. 1996). An automated line candidate search algorithm designed by the BATSE SD team (Briggs et al. 1996) was then applied to spectra in 117 bright bursts for which there is at least one spectrum with signal-to-noise ratio greater than 5 at $\approx 40$ keV (Briggs et al. 1998). This automated search, which is considerably more sensitive than a visual search, yielded 12 candidate spectral line candidates for which the change in $\chi^2$ between the continuum and continuum-plus-line fits is greater than 20 (significance $< 5 \times 10^{-3}$). Perhaps problematically, all the line candidates except one are emission-like lines observed at $\approx 40$ keV, with the one exception being an absorption-like line candidate observed at $\approx 60$ keV. Briggs et al. (1998) cannot rule out the possibility that sharp breaks in the continua, instead of lines, cause the observed features, but such breaks would be inconsistent with low-energy Ginga GRB data. Briggs et al. estimate the ensemble chance probability of the most significant feature as $\lesssim 10^{-3}$ and state that few, if any of these features result from statistical fluctuations. However, these lines should not be considered definitively detected, as the contemporaneous data from other SDs are still being examined (Briggs et al. 1999).

We feel that the aforementioned reasons give us ample justification to test the cyclotron scattering model further. In this paper, we use it to examine the lines$^{10}$ exhibited by GRB 870303 S1 and S2. Successful fits of the cyclotron scattering model to these data, when paired with the successful fits of cyclotron scattering models to the data of GRB 880205 by Wang et al. (1989a) and Freeman et al. (1992), would further strengthen the evidence supporting our contention that some (though not all) GRBs are Galactic in origin.

In § 2.1, we describe the spatial geometry of the line formation region. We assume it to be a static plane-parallel slab of electrons threaded by a strong magnetic field, which can be oriented at an arbitrary angle relative to the slab normal (unlike in Wang et al. 1989a, where the field was parallel to the slab normal). The assumption of a static slab is strictly valid only for bursters located within $\sim 100$ pc, if line formation occurs at the magnetic pole, since otherwise the inferred burst luminosity would exceed the Eddington limit and radiation forces would blow the line-forming layer off in a wind (see Isenberg, Lamb, & Wang 1998a, who modify the code to examine the theoretical ramifications of a wind). As noted above, such distances are consistent with current observational limits (Loredo & Wasserman 1998b).

If line formation occurs away from the magnetic pole, then the assumption of a static slab may be valid even for luminous halo bursters at distances $\sim 100$ kpc because of the confinement provided by closed magnetic field lines (e.g., Zheleznyakov & Serber 1994, 1995). We examine two slab geometries, which we denote "1-0" and "1-1," where the numbers represent relative electron column densities above and below the continuum photon source plane. A slab illuminated from below represents a line-forming region in the magnetosphere of a neutron star, while a source plane embedded within a slab corresponds to line formation within a semi-infinite neutron star atmosphere.

In § 2.2, we summarize the physics of radiation transfer in strong magnetic fields that is incorporated into the Monte Carlo code we use to generate spectra (Wang et al. 1988, 1989b; Lamb et al. 1989; Lamb, Wang, & Wasserman 1990; and references therein), and in § 2.3, we provide examples of these spectra. As noted above, resonant cyclotron scattering, and not cyclotron absorption, describes the peculiar properties of the Ginga lines—the comparable strengths of first and second harmonics, the absence of third and higher harmonics, and the narrowness of the lines. The appearance of the lines is greatly affected by Raman scattering, i.e., resonant cyclotron scattering in which the electron is excited from the ground state directly to the second or higher Landau level, but then de-excites back to the ground state indirectly via intermediate energy levels. In Raman scattering, the original photon is destroyed, and two or more photons are created (or spawned) as the electron de-excites. Spawned photons with energies near that of the first harmonic line alter its profile and reduce its equivalent width to a value similar to that of the second harmonic line. Because the majority of photons that undergo scattering at the second and higher harmonics are destroyed, the lines have an approximately absorption-like profile. The third harmonic is not seen because its optical depth is small compared to that of the second harmonic ($\tau_3 \approx 0.05 \tau_2$). Narrow lines result from the fact that the line-forming region is optically thin to all continuum photons except for those with energies equal to the first and second harmonic energies. Thus scattered photons at $\approx 20$ keV, and not continuum photons $\gtrsim 1$ MeV, dictate the temperature of the line-forming region, and the line width.

In § 3 we summarize the statistical concepts that we use in this paper. These concepts are described in more detail in Paper I and Freeman et al. (1999b, hereafter Paper III). In those works, we present general, rigorous methodologies that address the problem of establishing the existence of a line in a spectrum, that are based upon both the so-called frequentist, and Bayesian, paradigms of statistical inference. In this work, instead of establishing the existence of lines, our statistical goal is to compare azimuthally symmetric models of line formation, i.e., models in which the magnetic field is parallel to the slab normal, with models having arbitrary magnetic field orientations. If the GRB source is a neutron star with a simple dipole field, these two classes may be interpreted as representing line formation at the magnetic pole, or elsewhere. We use a rigorous method of model comparison (described in § 3.2) to select between

$^{10}$ The rigorous statistical analysis carried out in Paper I on the data of GRB 870303 S1 and S2 demonstrates that when the data are considered jointly, the significance of the continuum-plus-lines model is $\sim 10^{-3}$. Thus we feel that we may drop the word "candidates" when referring to these lines throughout the remainder of this paper.
polar cap and more general models, demanding, e.g., that the increase in the quality of the fit of general models be sufficiently great to justify considering of the ramifications of line formation away from the pole. Once best-fit models are selected, we use a rigorous method of parameter estimation (described in § 3.3) to place limits on each of the model parameters.

We apply the cyclotron scattering model to the data of GRB 870303 S1 and S2 in § 4. S1, showing 4 s of data, exhibits a saturated line at $\approx 20$ keV, while S2, showing 9 s of data, exhibits two harmonically spaced lines at $\approx 20$ and 40 keV. The midpoints of the S1 and S2 time intervals are 22.5 s apart. In Paper I, we establish the evidence for the GRB 870303 lines using simple phenomenological models. In that work, we establish the frequentist significance of, and Bayesian odds favoring, the S1 line to be $3.6 \times 10^{-5}$ and 114:1, respectively; for S2, the respective figures are $1.7 \times 10^{-4}$ and 7:1, while for the combined (S1 + S2) data, they are $4.2 \times 10^{-8}$ and 40,300:1. In joint fits to the combined (S1 + S2) data in this work, we find that the best-fit values of $\mu$ and/or $\phi$ change during the 22.5 s between S1 and S2 for both the 1-0 and 1-1 geometries, meaning that the orientation of the observer changes with time. We interpret the change in orientation by invoking neutron star rotation, and in § 4.5 (and the Appendix) we describe how we place limits on the rotation period.

In § 5, we discuss our results.

2. CYCLOTRON SCATTERING IN STRONG MAGNETIC FIELDS

In this paper, we assess the hypothesis that the lines exhibited at $\approx 20$ keV in GRB 870303 S1 and $\approx 20$ and 40 keV in GRB 870303 S2 were formed within a strong magnetic field ($B \approx 10^{12} \text{ G}$). We make the further assumption that the lines were formed on the surface of, or near, a neutron star, the only astronomical object where such strong field strengths have been observed. We assess the hypothesis by generating spectra with a Monte Carlo code that numerically treats radiation transfer in strong fields (Wang et al. 1988) and fitting these spectra to the observed data. In this section, we describe the spatial geometry of the line-forming region and then summarize the physics of radiation transfer included in the Monte Carlo code. We then provide examples of spectra generated with the code, so as to build the reader’s intuition before we discuss the results of model fitting in § 4.

2.1. Spatial Geometry of the Line-forming Region

We assume that line formation occurs in a plane-parallel slab of electrons, which has infinite horizontal extent and height similar to a neutron star atmospheric scale height ($h \sim 10^{-3} R_{\text{NS}}$). This slab contains an electron-proton plasma, threaded with a magnetic field $B$ that is oriented at angle $\Psi$ with respect to the slab normal $\hat{n}$ (Fig. 1).\textsuperscript{11} We neglect variations in the magnitude and direction of $B$ within the slab.

Continuum photons are injected into the slab at a source plane, travel through it, and emerge from one of its faces. We refer to photons that emerge from the top of the slab as “transmitted” and those that emerge from the bottom as

\textsuperscript{11} Wang et al. (1989a) consider only $\Psi = 0$ in their fits to the Ginga data of GRB 880205.

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**Fig. 1.**—Coordinate system used in this work. The infinite plane-parallel slab is threaded by a magnetic field $B$ oriented at an angle $\Psi$ relative to the slab normal $\hat{n}$. The angles $\theta$ (or angle cosine $\mu = \cos \theta$) and $\phi$, the azimuthal angle from the projection of $B$ onto the slab (Fig. 1).

The location of the source plane with respect to the slab determines the geometry of the system. We apply the nomenclature “1-x” to denote geometries; $1$ and $x$ (a number) represent the relative values, above and below the source plane, of the electron column density $N_e$ between the photon source plane and the observer. We examine two geometries in this work (Fig. 2). The 1-0 geometry is a slab illuminated from below; this geometry is used by Wang et al. (1989a). This geometry represents a line formation region physically separated from an isotropically emitting source of continuum photons, e.g., an illuminated flux tube. We assume that the reflected photons return to the neutron star surface, where they are thermalized, i.e., absorbed by non-resonant inverse magnetic bremsstrahlung. We use a 1-1 geometry to model line formation in an isothermal semi-infinite neutron star atmosphere. Slater, Salpeter, & Wasserman (1982) and Wang et al. (1988) determine that the mean number of scattering events between the source and the top edge that a resonant photon experiences prior to its escape approaches a limiting value as the atmosphere becomes semi-infinite (i.e., a 1-$\infty$ geometry). The number of scattering events experienced in the 1-1 geometry is within $\approx 10\%$ of its limiting value, while the number experienced in the 1-4 geometry (used, e.g., in Freeman et al. 1992) matches the limiting value (Isenberg, Lamb, & Wang 1998b). We do not use the 1-4 geometry, despite its greater

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**Fig. 2.**—Two plane-parallel slab geometries that we examine in this work, denoted “1-0” and “1-1.” The numbers represent the relative electron column densities above and below the continuum photon source plane. Injected photons travel through the slab and emerge from one of the plane-parallel faces of the slab. The observer is situated above the source plane.
accuracy, because the overall number of scattering events (both resonant and nonresonant) is approximately three times that using the 1-1 geometry; the computer time required for the 1-4 simulation increases proportionally. Also, in the 1-1 geometry, the reflection symmetry of the line-forming region allows us to limit injection of photons to a hemisphere facing the observer. The sum of the resulting transmitted and reflected spectra is equivalent to the spectrum which would result from spherically isotropic photon injection. By using hemispherical input, we reduce the amount of computer time per simulation by a factor of 2.

2.2. The Physics of Resonant Cyclotron Scattering

Here, we summarize the physics of strong field radiation transfer that is incorporated into the Monte Carlo code. Unless we specify otherwise, the reader may find more details on any of the elements of the code described below in Wang et al. (1988).

We assume that the line-forming plasma is cold: \( kT_{e,\parallel} \ll E_B = (\hbar/e) m_e c B \approx (\hbar/m_e c) B \), where \( \gamma \) is the Lorentz factor \((\approx 1 \text{ for the electron energies assumed in this work})\), and the symbol \( \parallel \) indicates that it is only the energies parallel to \( B \) that follow the (continuous) Maxwell-Boltzmann distribution. The allowed electron energies perpendicular to the field are the Landau levels. The spacing of these levels is assumed to be much larger than typical electron energies, so that they are not collisionally populated, and the photon densities are assumed to be small, so that they are also not radiatively populated. Thus, in each scattering event, the initial and final electron energy state is the Landau ground state \((n = 0)\).

Electrons that are excited to states \( n \geq 2 \) may either de-excite directly to the ground state (in which case the photon has undergone resonant scattering), or it may reach the ground state via intermediate excitation levels (Raman scattering). Raman scattering is dominant in weak fields \((B/B_1 \ll 1, \text{ where } B_1 \text{ is the critical field strength, } 4.4 \times 10^{13} \text{ G})\), at which the first-harmonic energy matches the electron mass energy \( 511 \text{ keV} \)). For instance, for the magnetic field strengths considered in this paper, electrons excited to \( n = 2 \) have probability \( \approx 1 - (E_B/m_e c^2) = 1 - (B/B_1) \approx 0.95 \) of de-exciting to the first excited state (Daugherty & Ventura 1977). When this occurs, the incident photon with energy \( E \approx 2E_B \) is destroyed, and two photons with energies \( E \approx E_B \) are spawned. The second and higher harmonics thus have an absorption-like line profile in weak fields, while the spawned photons act to reduce the equivalent width of the first harmonic line from the value it would have had if Raman scattering did not occur. Because the equivalent widths of the second and higher harmonic lines are \( \propto (B/B_1)\gamma^{-2} \), the code does not treat excitation to states \( n \geq 4 \).

The code uses scattering cross sections provided by Herold (1979) and Daugherty & Ventura (1977). Herold states how to formulate exact relativistic scattering cross sections and provides leading-order expressions for the case in which the initial and final electron states are the Landau ground state, valid in the limit

\[
\left( \frac{E}{m_e c^2} \right)^2 \frac{B_1}{B} \ll 1 .
\]

The code uses these expressions, which are valid in the energy and magnetic field strengths regimes that are of interest in this work. However, these expressions make no provision for the natural line width of the resonances, and as a consequence the scattering amplitudes diverge at the resonances. To correct the divergence, the natural line width, \( \Gamma_n = n \pi \xi E_{\parallel}^2 / 3 m_e c^2 \), is added (Wasserman & Salpeter 1980; Harding & Daugherty 1991; Graziani 1993; \( \xi \) is the fine-structure constant, and the line width is given in units of energy). The code uses second and third harmonic resonant scattering cross sections derived in the nonrelativistic limit by Daugherty & Ventura, who use Fermi’s Golden Rule. The first-harmonic cross section given by Daugherty & Ventura does not as accurately portray the line wing profile; hence the Herold cross section, with finite line width added, is used instead.

A problem with this “hybrid” procedure that mixes exact relativistic resonant cross sections and nonrelativistic higher harmonic cross sections is that in the treatment of electron-photon scattering, there is the implicit approximation

\[
\sum_i |a_i|^2 \approx |a_{0-0} + a_{0-1-0}|^2 + \sum_{i \neq 0-0-0-1-0} |a_i|^2 ,
\]

where \( a_i \) is the matrix element for the ith scattering channel. This approximation is exact for scattering at the first harmonic both in the line core (within a few Doppler widths of line center) and wings (far from the line center). For higher harmonic scattering, this approximation is valid only near the line center. If there is significant scattering at energies between the first and second harmonics, then the matrix cross-term that is missing from this expression may be important. Consequently, we limit use of the code to modeling slabs that are not optically thick at the first harmonic line wings (see below; also Wasserman & Salpeter 1980; Lamb et al. 1989):

\[
a_1 \tau_1 = \frac{\Gamma_1}{2E_p(v_{th}/c)} \tau_1 \approx 0.34 \left( \frac{kT_{e,\parallel}}{\text{keV}} \right)^{-1} \left( \frac{N_e}{10^{21} \text{ cm}^{-2}} \right) \lesssim 1 ,
\]

where \( a_1 \) is the ratio of the natural line width to the Doppler energy width for the first harmonic, \( v_{th} \) is the thermal electron velocity along the magnetic field lines \([=(2kT_{e,\parallel}/m_e)^{1/2}]\), \( N_e \) is the column density of electrons in the slab, and \( \tau_1 \) is the polarization-, angle-, and frequency-averaged optical depth in the first harmonic. For relevant temperatures \( kT_{e,\parallel} \approx 5 \text{ keV} \); see below), \( N_e \approx 10^{22} \text{ cm}^{-2} \).

(If \( N_e \lesssim 10^{12} \text{ cm}^{-2} \), the line-forming slab may be moderately optically thick to photons at the harmonic energies but will be optically thin to continuum photons. Thus scattered photons at \( \approx 20 \text{ keV} \), and not continuum photons \( \gtrsim 1 \text{ MeV} \), dictate the temperature of the line-forming region. Lamb et al. 1990 derive what they dub the Compton temperature, \( T_C \) [which equals \( T_{e,\parallel} \) in this work, but not generally], for this density regime, and find that \([1]\) slabs reach this temperature on timescales \( t \sim 10^{-8} \text{ s} \), and \([2]\) \( kT_C \) is a fraction \([\approx 25\%]\) of the first-harmonic energy. Thus the lines are narrow. This result demonstrates the appropriateness of the originally stated assumption that lines are formed in a cold plasma.)

The resonant cross sections are averaged over the initial polarization states and summed over the final states. Polarization-averaged cross sections are appropriate for first-harmonic scattering in optically thick media when the vacuum contribution to the dielectric tensor dominates the
plasma contribution, a condition we may state as \[\frac{[n_e/10^{22}\text{ cm}^{-3}][B/10^{12}\text{ G}]}{4} \lesssim 1\], where \(n_e\) is the electron number density. This condition holds for the physical conditions that we explore here and is, in fact, less limiting than the requirement that line-forming regions be optically thin in the line wings.

The line profile at each harmonic is created by making a Lorentz transformation to the lab frame and averaging over \(f(p)\), the one-dimensional electron momentum distribution (along \(B\)). The total scattering profile is assumed to be the sum of the profiles for the individual Landau levels (including the continuum contribution). Profiles are averaged over azimuthal angle, \(\phi\), despite the fact that for \(\Psi \neq 0\), azimuthal symmetry is broken. The \(\phi\)-dependent part of the scattering cross section is significant only in the line wings and continuum, and in the present work we consider only line-forming regions that are not optically thick in the line wings.

While we have used the terminology “line core” and “line wings,” whose meanings are intuitively well known, we can define them mathematically as fulfilling the conditions \(|x_0/\mu_b| \ll 1\) (core) and \(|x_0| \gg 1\) (wings), where \(\mu_b\) is the cosine of the angle between \(B\) and the direction of propagation of the photon, and \(x_0 \equiv [(E-E_c)/E_h(n_e/c)]\) is a dimensionless energy shift (with the denominator being the Doppler width associated with the nth harmonic energy \(E_n = nE_h\); Wasserman & Salpeter). In the line core, the thermal electron distribution dominates the profile so that it is \(\propto \exp[-(x_0/\mu_b)^2]\), while in the wings, the tail of the Lorentzian distribution dominates so that the profile is \(\propto a_n x_0^{-2}\), where \(a_n\) is again the ratio of the natural line width to the Doppler energy width. If \(\mu_b = 0\) the line wings extend to \(x(\mu_b) = 0\), i.e., the line profile is purely Lorentzian for all \(x\). This is because the Doppler effect vanishes, to first order in \(v/c\), for photons propagating perpendicularly to \(B\). Wasserman & Salpeter showed that for the first harmonic, the core-wing boundary appears at \(x/\mu_b \approx 2.62 - 0.19 \log(100a/|\mu_b|)\). We refer to the wings at energies below and above the line center as the red and blue wings respectively.

Lamb et al. (1989) showed that relativistic kinematics has a significant effect on the shape of the absorption profile, even in the limits \(E, kT_{e,i} \ll m_e c^2\). For zero natural line width, relativistic kinematics prohibits scattering at the nth harmonic above a cutoff energy \(E_c = [(\sqrt{1 + 2b - 1)m_e c^2/\sqrt{1 - \mu_b^2}]\), where \(b \equiv B/B_s\) (Daugherty & Ventura 1978; Harding & Daugherty 1991; see the Appendix of Wang et al. 1993 for a physical derivation). Wasserman & Salpeter show that for the physical conditions in which electron recoil is important, photons will escape more readily in the red wing than in the blue. This result motivated Lamb et al. (1989), Wang et al. (1989b), and Wang et al. (1993) to make the simplistic assumption that the resonant scattering profile was zero for \(E > E_c\), i.e., to ignore the effects of the finite natural line width. For \(E \lesssim E_c\), the effects of finite line width were included. The simplification led to the appearance of spikes in some cyclotron scattering spectra at energies just above that of the first harmonic (e.g., as \(\mu \to 0\) for \(\Psi = 0\)). This spike contained both photons that were scattered at the first harmonic and spawned photons resulting from Raman scattering at the second and higher harmonics, which then immediately escaped from the slab. The code used in this work does include the effect of finite natural line width for \(E \gg E_c\) so that the resonant scattering profile is now small but finite above \(E_c\). With this enhancement, the spikes are smeared by scattering and no longer appear. We stress that even though the scattering profile is finite above the cutoff energy when the effects of natural line width are properly treated, the profile still falls off sharply above \(E_c\), leading to a strongly asymmetric line shape in some spectra.

2.3. Examples of Monte Carlo Spectra

To help build the reader’s intuition before describing the results of spectral fits to the data of GRB 870303 S1 and S2, we present examples of spectra produced with the Monte Carlo code. In Figures 3 and 4, we show output counts spectra produced with \(\Psi = 0\), for the 1-0 and 1-1 geometries.
emission in a particular direction is (Daugherty & an electric dipole transition, and the probability of photon
maximized. Radiative decay to the Landau ground state is
\[ N \] for all \( k \).

If then the photon travels perpendicularly to

\[ k \] and are not shown). Each spectrum is produced assuming

\( 1.7 \) and \( N_{e,21} = 0.6 \). The relative strengths of the harmonics in each figure may be understood as arising from the interplay of line-of-sight column density (i.e., whether we observe line formation from above or the side) and the angular dependence of the resonant scattering cross section of the \( N \)th harmonic:

\[ \sigma_N \propto (1 + \mu_b^2)(1 - \mu_b^2)^{N-1} \].

If \( \mu_b = 0 \), then the photon travels perpendicularly to \( B \), and \( \sigma_N \) is nonzero for all \( N \), and a maximum for all \( N \geq 1 \). If \( \mu_b = 1 \), then the photon travels along \( B \), and \( \sigma_N \) equals zero for all \( N \), except \( N = 1 \), for which case the cross section is maximized. Radiative decay to the Landau ground state is an electric dipole transition, and the probability of photon emission in a particular direction is \( \propto 1 + \mu_B^2 \) (Daugherty & Ventura 1977); the emitted photon is thus most likely to be emitted in the direction of \( B \).

If we compare the line profiles in Figures 3–5, we see that they are broadened if the observer is oriented along \( B \). This is because scattering will occur only if the energy of the photon, as calculated in the electron rest frame, matches the resonant energy, i.e., if \( E_r(1 - \beta \mu_b) = E_v \), where \( \beta = v_e/c \), \( E_v \) is the photon lab frame energy, and \( E_v \) is the line centroid energy for the first harmonic. Hence if \( \mu_b = 0 \), only those photons with exactly the resonant energy can scatter; if \( \mu_b = 1 \), then there is a range of \( E_v \) such that scattering is possible, so that the scattering profile is broadened.

If we compare Figures 3a and 4a, we see distinctive “shoulders” arising in the red and blue wings of the first harmonic in the 1-1 geometry. They appear most prominently when the observer is oriented along \( \hat{n} \) (\( \mu \rightarrow 1 \)), regardless of the angle \( \Psi \) or the geometry (Isenberg et al. 1998b). Line shoulders are a predicted element of radiation transfer in strong fields (Wasserman & Salpeter), and many authors discuss their appearance (Alexander & Mészáros 1989; Nishimura & Ebisuzaki 1992; Araya & Harding 1996; Isenberg et al. 1998b; cf. Nishimura 1994, whose results contradict those of Alexander & Mészáros and Isenberg et al.). If \( \alpha_1 \), the ratio of the natural line width to the Doppler energy width for the first harmonic, \( \rightarrow 0 \), then the energy of a scattered photon must lie within the regime of the optically thick line core. That photon will thus only escape after \( \sim \tau_1^2 \) scatters. If \( \alpha_1 \neq 0 \), then with each scatter there is a probability \( \sim \alpha_1 \) that a photon will be redistributed into the optically thin line wings, so that after \( \sim \alpha_1 \) scatters, that photon will escape. Photons in the first-harmonic core will, on average, scatter at least this many times for the 1-1 geometry, but not for the 1-0 geometry, for the values of \( N_e \) considered in this paper. This is due to the fact that in the 1-0 geometry, photons that scatter in the line core may cross the source plane, whereupon they are lost from the calculation. The size of the shoulders will change depending upon the importance of Raman scattering in the line-forming region, since this dictates the number of photons spawned within the first-harmonic line core. Their relative size in the red and blue wings is determined by the relationship between \( T_{\text{keV}} \) and \( T_c \); because we assume these temperatures to be equal, the equivalent widths of the two shoulders will be equal.

3. STATISTICAL METHODOLOGY

In this work, we examine two general classes of models: the azimuthally symmetric model class, which has free parameters \( B, N_e, \) and \( \mu \) (\( \Psi = 0 \) and \( \phi \) is undefined); and a general model class, which includes \( \Psi \) and \( \phi \) as additional free parameters. If the GRB source is a neutron star with a simple dipole field, these two classes may be interpreted as representing line formation at the magnetic pole, or elsewhere. Hence we will refer to the azimuthally symmetric models as “polar cap” models throughout the remainder of this work. Because many physical processes are known to occur at the magnetic pole (e.g., accretion of gas from a companion star onto a neutron star), the association of simpler models with the magnetic pole has intuitive appeal.

In this section, we summarize the methods of statistical inference that we use to fit models from each class to the data, to compare the best-fitting models from each class, and to place limits on values of the free parameters of the models which fit best overall. The reader will find fuller
We create Monte Carlo spectra at discrete points on a grid of values for the model parameters $B_{12}$ and $N_{e,21}$, and, if applicable, $\Psi$. We show these values in Table 1.\footnote{We can analytically shift the line profiles in $E$ (or equivalently $B_{12}$) by up to $\pm10\%$, with no loss in accuracy, during fits.} For each given set of parameter values, we determine the Compton temperature $T_c = T_{\gamma}/B_{12}^2$ by carrying out a series of Monte Carlo calculations for several different temperatures and determining at which temperature heating and cooling balance within the slab (see, e.g., Freeman et al. 1992; Isenberg et al. 1998b). Hence, $T_c$ is not a free parameter of the fit.

Photons emerging from the line-forming region are binned. We generally use eight bins spanning $\mu = [0, 1]$ (where $\mu = \cos \theta$; an exception made in fits to GRB 870303 S1). As for $\phi$, if $\Psi \neq 0$ and $\Psi \neq \pi/2$, we may take advantage of symmetry across the $(B, n)$-plane to use eight bins spanning $\phi = [0, \pi]$. If $\Psi = \pi/2$, further symmetry across the plane perpendicular to the $(B, n)$-plane allows us to reduce the number of bins to four, spanning $\phi = [0, \pi/2]$.

We find that a spectrum is sufficiently accurate, i.e., the counts standard deviation in each output bin is sufficiently low, if $\sim 10^6$ photons are injected into the line-forming region. We also find that the number of photons needed to portray the low-energy line profiles accurately is roughly an order of magnitude fewer than the number needed to portray the high-energy continuum accurately ($E \gtrsim 100$ keV). We cannot ignore the high-energy continuum, because the Ginga GBD recorded the energy lost by an impinging photon (e.g., a 500 keV photon may have lost only 20 keV while passing through the GBD, leading to the recording of a count within the energy regime of interest). Thus we run the code with continuum photons sampled at energies $\lesssim 100$ keV and attach to the resulting spectrum a separately created high-energy continuum ($E \approx 100-1479$ keV). Each spectrum is properly weighted so that the overall spectrum is smoothly continuous at the matching point.

For a given set of input parameters, many spectra will be generated, one for each binned value of $(\mu, \phi)$. In principle, we would assess the goodness of fit of each generated spectrum using the Poisson likelihood function $\mathcal{L}$, and select that spectrum for which $\mathcal{L}$ is maximized. However, for reasons given below, we fall back upon the understanding that in the limit of a large number of counts $n$ in a bin, we can use Pearson’s $\chi^2$ statistic, an approximation of $L = \log \mathcal{L}$:

$$s^2 = \sum_{i=1}^{N} \frac{(m_i - n_i)^2}{\sigma_i^2}. \quad (5)$$

The fit extends over $N$ data bins, and $m_i$ and $n_i$ are the model amplitude and data in bin $i$, respectively. The best-fit parameters are those for which $s^2$ is minimized.\footnote{We follow the notation of Lampton, Margon, & Bowyer (1976), reserving the symbol $\chi^2$ for a statistic which is explicitly sampled from the $\chi^2$ distribution.} We set $\sigma_i^2 = m_i$ (“model variance”), and hereafter denote the fitting statistic as $s_m^2$.

### 3.2. Model Comparison

In Paper I, we describe both frequentist and Bayesian methods of model comparison and apply both methods to determine the frequentist significance of, and the Bayesian odds favoring, a spectral model with exponential Gaussian absorption lines. The calculation of the Bayesian odds generally requires the analyst to integrate numerically the Poisson likelihood function $\mathcal{L}$ over parameter space. This can be computationally intensive when the number of parameters is large. However, we can determine the odds analytically if the shape of the likelihood surface in parameter space is similar to that of a multidimensional Gaussian (the Laplace approximation). In Paper I, we were able to reparameterize the exponentialized Gaussian line model so that we could use the Laplace approximation. Unfortunately, the function $\mathcal{L}(\mu, N_e, \Psi, \phi)$ does not have such the required Gaussian shape, nor can we reparameterize the model so that $\mathcal{L}$ will have this shape. Thus in this work we use only the frequentist method of model comparison.

The frequentist comparison of two models, the null hypothesis $H_0$, and the alternative hypothesis $H_1$, is carried out by constructing a test statistic $T$, which is usually a function of the goodness-of-fit statistics for both models. There are two probability distribution functions (PDFs) that indicate the a priori probability that we would observe the value $T$, which are computed assuming the truth of $H_0$, and $H_1$, respectively. The test significance $s$ is calculated by computing the tail integral of the $H_0$ PDF from $T$ to infinity. The resulting number represents the probability of selecting $H_1$ when in fact $H_0$ is correct; if $s$ is sufficiently small, we reject $H_0$ in favor of $H_1$. A common threshold for rejecting the null hypothesis is $s \leq 0.05$; in this work, we use the more conservative threshold $s \leq 0.01$. The selection of the threshold value is subjective; we use a more conservative value than is used normally because, e.g., we would demand that any increase in the quality of fit provided by general models be sufficiently great to justify considering the ramifications of line formation taking place outside the polar cap.

The preferred model comparison statistic $T$ would be the ratio of the maximum Poisson likelihoods of $H_1$ and $H_0$, but then simulations are required to estimate $s$. We can make an analytic estimate of $s$ that is approximately correct using the $\chi^2$ maximum likelihood ratio ($\chi^2$ MLR) test (Eadie et al. 1971, pp. 230–232). The test statistic is $T = \Delta s^2_m = s_m^2(H_0) - s_m^2(H_1)$. The PDF $p(\Delta s^2_m | H_0)$ is then assumed to be the $\chi^2$ distribution for $\Delta P = P_1 - P_0$ degrees of freedom, where $P_0$ and $P_1$ are the number of free param-
For a single data set, we directly compare fits of the three-parameter polar cap model, and five-parameter general model, to the data. When jointly fitting two (or more) data sets, the process of model selection becomes more complicated, as there is more than one model per class. For instance, for fits to S1 and S2, there are eight polar cap models, with the simplest specifying that \( B_{S1}, N_{e,S1}, \mu_{S1} = (B_{S2}, N_{e,S2}, \mu_{S2}) \), and the next three simplest specifying that one of the parameters changes between the epochs of S1 and S2, etc. (Analogously, there are 32 models in the general class.) We illustrate how we select models within a model class in Figure 6. We begin by comparing the simplest model \((M_i)\) with all models that have a greater number of free parameters \((M_{i-1} \ldots M_N)\) in Fig. 6, computing the significance of the additional parameters of each alternative model \((\xi_{1,2} \ldots \xi_{1,n})\). We then adopt the simplest alternative model for which \(\alpha_{2\text{MLR}} \leq 0.01\) as our new null hypothesis \(M_i\). (If two or more models with the same number of free parameters fulfills this criterion, we would select the one with the smallest \(s_m^2\), or equivalently, the smallest \(\alpha_{2\text{MLR}}\)) We repeat the comparison process, until either no alternative model is adopted, or the most complex model is selected.

### 3.3. Parameter Estimation

In Paper I, we describe both frequentist and Bayesian methods of parameter estimation and apply both methods to determine the confidence (frequentist) and credible (Bayesian) intervals on the free parameters of the best-fit exponentiated Gaussian absorption line models. If the surface defined by \(s_m^2\) in parameter space has paraboloidal shape, then the \(\alpha\) confidence interval for an individual parameter is given by those values of that parameter for which \(s_m^2 = \hat{s}_{m,\text{min}} + \chi^2\), where the values of all other free parameters are allowed to vary to new best-fit values. Otherwise, simulations are needed to determine the appropriate confidence intervals (Eadie et al., pp. 190–201). Since the \(s_m^2\) surfaces for fits in this work do not have the required paraboloidal shape, we limit ourselves to computing Bayesian credible intervals.

We may determine a credible interval for a particular parameter \(x\) of model \(M_i\), without reference to the other, “uninteresting,” parameters, collectively denoted \(x',\) by marginalizing the Bayesian posterior function \(p(x, x' | D, I)\) over the space of parameters \(x':\)

\[
p(x | D, I) = \int dx' p(x, x' | D, I)
\]

\[
\propto \int dx' p(x, x' | I)p(D | x, x', I) . \tag{6}
\]

\(D\) and \(I\) represent the data and background information about the experiment (such as the detector bandpass) respectively, while \(p(x, x' | I)\) is the prior probability (a quantitative statement of our state of knowledge about the relative probability of each possible value of \(x\) and \(x'\) before the data \(D\) are examined). \(p(D | x, x', I)\) is simply the likelihood, \(\mathcal{L}(x, x')\), which is assumed to be \(\propto \exp (-s_m^2/2)\). If the prior is assumed to be uniform, then \(p(x | D, I)\) is simply proportional to \(\mathcal{L}(x, x')\). The credible interval is then the range \([x_1, x_2]\) such that

\[
z = \left[ \frac{\int_{x_1}^{x_2} dx p(x | D, I)}{\int_{x_1}^{x_2} dx p(x | D, I)} \right], \tag{7}
\]

where \(z\) is the desired probability content (e.g., 0.683 for 1 \(\sigma\) bounds), and \(p(x_1 | D, I) = p(x_2 | D, I)\). (Note that if the likelihood surface is not well behaved and exhibits a number of modes, then the credible “interval” may actually consist of a number of intervals.)

We note two items that affect the interpretation of these regions. First, our use of the Monte Carlo code is limited to \(N_{e,21} \leq 10\). The upper limit can potentially cut off highly probable regions of parameter space, affecting the computation of credible intervals. Second, it is not computationally feasible to create models using all \(\Psi\) values when creating credible intervals for the parameters of the general model. Thus the quoted marginal distribution for each model parameter other than \(\Psi\) is of the form \(p(x, \Psi = \Psi_{\text{best-fit}} | D, I)\), and we do not compute the marginal distribution for \(\Psi\) itself.

### 4. APPLICATION OF THE CYCLOTRON SCATTERING MODEL TO GRB 870303

#### 4.1. The Data

The Los Alamos/ISAS Gamma-Ray Burst Detector (GBD; Murakami et al. 1989) on Ginga detected GRB 870303 at 16:23 UT on 1987 March 3. Figure 7 shows burst-mode time history data for the GBD Proportional Counter (PC), which covered 1.4–23.0 keV, and the GBD Scintillation Counter (SC), which covered 16.1–335 keV. The GBD continuously recorded burst-mode data at 0.5 s intervals. These data were not stored in memory until a burst was detected, at which time the data from 16 s prior to the burst trigger until 48 s after the burst trigger were...
The burst detector on Pioneer Venus Orbiter (PV O) also observed GRB 870303, allowing the use of photon time-of-arrival information to limit the possible source location to an annulus upon the sky. A. Yoshida (1995, private communication, correcting Yoshida et al. 1989) reports this annulus to lie in the range $11.2 < \theta_{\text{inc}} < 57.6$. Since the shape and amplitude of a model counts spectrum that is derived from a given photon spectrum depends sensitively on $\theta_{\text{inc}}$, we treat this angle as a freely varying model parameter in Paper I. However, testing the cyclotron scattering hypothesis for more than one angle of incidence is not (currently) computationally feasible. Thus, we assume $\theta_{\text{inc}} = 37.7$ in this work; this is the angle assumed by others who analyze GBD data (e.g., Murakami et al. 1988; Fenimore et al. 1988; Wang et al. 1989a; Graziani et al. 1992, 1993).

4.2. Results: GRB 870303 S1

In Paper I, we describe fits to these data using exponentiated Gaussian absorption lines. We fit the data in PC bins 8–15 and SC bins 2–31 with a power-law model ($dN/dE \propto E^{-1.55}$), onto which is multiplied one line with equivalent width $W_E \approx 10$ keV. For this model, $s_m^2 = 22.3$ for 34 degrees of freedom (dof). The frequentist significance of this line is $1.2 \times 10^{-5}$, and the Bayesian odds favoring the model with a line is 120:1. (These numbers differ from those given above, and in Paper I, in that $\theta_{\text{inc}}$ is assumed to be 37.7)

When we use the 1-0 geometry, the mode of the polar cap model places the observer along $B$, where the first-harmonic equivalent width is maximized, while higher harmonic

...
equivalent widths are minimized. For this model, $s_m^2 = 33.0$ (33 dof), with $B_{12} = 1.78$, $N_{e,21} = 5$, and $0.875 \leq \mu \leq 1$. This fit compares unfavorably with the best phenomenological model fit ($s_m^2 = 22.3$). We decrease $\Delta \mu$ and examine $\mu \approx 1$ to maximize the first-harmonic strength and to minimize the effect of the weak second harmonic. If we use a bin width $\Delta \mu = 1/256$, $s_m^2 = 26.7$, with $N_{e,21} = 10$ (the upper limit) (see Fig. 9 and Tables 2 and 3). Smaller bin widths do not lead to further reduction in $s_m^2$. We note that tests using values of $N_{e,21}$ above the upper limit indicate that $s_m^2$ increases, owing to increasingly apparent shoulders in the red and blue wings of the line. This fit indicates that we observe line formation from a privileged position above the polar cap if the line formation region is illuminated isotropically.

The difference $\Delta s_m^2$ between this best-fit and that of Paper I is due to the fact that at the polar cap, we cannot create a first harmonic line with a sufficiently large equivalent width ($W_{\text{B}} \approx 10$ keV). To determine the equivalent width of the best-fit line model, we model the line profile using three exponentiated Gaussians, two representing the shoulders and one representing the line core:

$$\frac{dN}{dE}(E) = C(E) \times \prod_{i=1}^{3} \exp \left\{ -\beta_i \exp \left[ -\frac{(E - E_{\text{c},i})^2}{2\sigma_i^2} \right] \right\}.$$  \hspace{1cm} (8)

$C(E)$ is the continuum photon flux at energy $E$, $\beta_2 = \beta_{\text{core}} > 0$, and $\beta_1$ and $\beta_3 < 0$. The equivalent width of the line is $W_{\text{B}} \approx 5$ keV, a value that depends only weakly upon $N_e$ in the vicinity of the mode. (This value is roughly 2 $\sigma$, or $\Delta s_m^2 \approx 4.4$, away from the best-fit phenomenological value $W_{\text{B}} \approx 10$ keV.) The equivalent width of the line core is $\approx 7$ keV, while each of the shoulders has equivalent width $\approx -1$ keV.

One can increase the equivalent width by limiting photon input into the line-forming region to a cone with axis parallel to $\hat{n}$. This decreases the number of photons spawned into the first-harmonic line core by greatly reducing the angle-averaged cross section of the second and third harmonics. Such a model would be consistent with the hypothesis that photons are beamed during continuum formation (Share et al. 1986; Ho & Epstein 1989; Ho, Epstein, & Fenimore 1990; Dermer 1990). A typical beam cone opening angle for 20–40 keV continuum photons is $\theta_{\text{cone}} \approx 45^\circ$. Such a model is also consistent with the hypothesis that the line formation region is suspended far above the polar cap (Dermer & Sturmer 1991; Sturmer & Dermer 1994). In this case, only photons that travel along the magnetic polar axis will interact with the localized region of line formation ($\theta_{\text{cone}} \to 0^\circ$).

We compute spectra for these two test cases, assuming $B_{12} = 1.76$ and $N_{e,21} = 0.6$ (6% of the best-fit value for isotropic photon input). We find that both hypotheses are consistent with the data; in both cases, $s_m^2 = 23.0$, and the line equivalent width is increased to $\approx 10$ keV.

We find that the general cyclotron scattering model, with $\Psi$ and $\phi$ as additional free parameters, fits the data better than the polar cap model, with $s_m^2 = 23.6$ (31 dof). The best fit for this model occurs for $\Psi = \pi/2$; we may interpret this as meaning that line formation occurs at the neutron star magnetic equator. The observer is oriented along $B$ (i.e., is looking along the surface of the slab); the line-of-sight opacity in the first harmonic is thus maximized (see Figs. 3a)

\[14\] Another, similar, hypothesis that cannot be directly tested with static models is that the line-forming region is driven off the star in a wind by radiation pressure (Miller et al. 1991, 1992; Isenberg et al. 1998a).

### TABLE 2

| SPECTRUM | $s_m^2$ Polar Cap | $s_m^2$ General | $\Delta s_{\text{MLR}}$ |
|----------|--------------------|-----------------|------------------------|
| 1-0      |                    |                 |                        |
| S1       | 26.7 (33)$^*$      | 23.6 (31)       | 0.21                   |
| S2       | 33.6 (33)$^*$      | 31.7 (31)       | 0.39                   |
| S1 + S2  | 70.8 (69)$^*$      | 59.6 (66)$^*$   | 0.01                   |
| 1-1      |                    |                 |                        |
| S1       | 37.7 (33)          | 24.4 (31)$^*$   | 1.3 $\times 10^{-3}$   |
| S2       | 33.9 (33)$^*$      | 32.9 (31)       | 0.61                   |
| S1 + S2  | 73.7 (69)$^*$      | 59.9 (65)$^*$   | 8.0 $\times 10^{-3}$   |

*Note.—The number of degrees of freedom is given in parentheses.

* Selected model. For both the 1-0 and 1-1 geometries, the data do not conclusively select either the polar cap model or general model.
TABLE 3

S1: BAYESIAN CREDIBLE INTERVALS

| PARAMETER | POLAR CAP | GENERAL (Ψ = π/2) |
|-----------|-----------|-------------------|
|           | 1σ        | 2σ                | 3σ            | 1σ          | 2σ          | 3σ          |
|  \( B_{12} \) (G) | 1.76 \( ^{+0.16}_{-0.14} \) | 1.76 \( ^{+0.25}_{-0.17} \) | 1.76 \( ^{+0.51}_{-0.39} \) | 1.83 \( ^{+0.12}_{-0.11} \) | 1.83 \( ^{+0.28}_{-0.25} \) | 1.83 \( ^{+0.45}_{-0.36} \) |
| \( \log N_{e,21} \) (cm\(^{-2}\)) | 1.00 \( ^{+0.09}_{-0.14} \) | 1.00 \( ^{+0.19}_{-0.14} \) | 1.00 \( ^{+0.39}_{-0.27} \) | -1.52 \( ^{+0.06}_{-0.05} \) | -1.52 \( ^{+1.39}_{-1.38} \) | -1.52 \( ^{+2.08}_{-2.08} \) |
| \( \mu \) | 0.990 \( ^{+0.088}_{-0.990} \) | 0.990 \( ^{+0.088}_{-0.990} \) | 0.990 \( ^{+0.088}_{-0.990} \) | 0.06 \( ^{+0.06}_{-0.06} \) | 0.06 \( ^{+0.06}_{-0.06} \) | 0.06 \( ^{+0.06}_{-0.06} \) |
| \( \phi \) (rad) | 0.20 \( ^{+0.20}_{-0.20} \) | 0.20 \( ^{+0.20}_{-0.20} \) | 0.20 \( ^{+0.20}_{-0.20} \) | 0.20 \( ^{+0.20}_{-0.20} \) | 0.20 \( ^{+0.20}_{-0.20} \) | 0.20 \( ^{+0.20}_{-0.20} \) |

* The credible region does not include marginalization over Ψ.

b \( \log N_{e,21} = 1 \) is a parameter boundary.

c The 1σ credible region does not include \( \log N_{e,21} = (0.32, 0.47) \).

d \( \mu_{\text{best}} \in [0.985, 0.992] \).

e The 1σ credible region does not include \( \mu = 0.09, 0.61 \).

f The 2σ credible region does not include \( \mu = (0.18, 0.31) \).

g The 3σ credible region does not include \( \mu = (0.187, 0.194) \).

h \( \mu_{\text{best}} \in [0.000, 0.125] \).

i \( \phi_{\text{best}} \in [0, 0.39(\pi/8)] \).

Comparing the polar cap and general models, we find that the significance of the extra two parameters in the general model is \( \chi^2_{2\text{MLR}} = 0.21 \); we select the polar cap model. It is, however, of great theoretical interest that models in which lines form far from a polar cap can fit the data so well, a point to which we return in § 5.1.

For the 1-1 geometry, the polar cap and general models fit to the data with \( s_m^2 = 37.0 \) (33 dof) and 24.4 (31 dof), respectively. Polar cap model spectra do not fit well to the S1 data because of the presence of either line shoulders (which have maximum size at \( \mu = 1 \)) or a strong second-harmonic line (which has maximum strength at \( \mu = 0 \)). Comparing the polar cap and general models, we find that the significance of the extra two parameters of the general model is \( \chi^2_{2\text{MLR}} = 1.6 \times 10^{-3} \); we select the general model (see Fig. 10 and Tables 2 and 3). The S1 data thus indicate strongly that line formation does not occur at the magnetic polar cap, within the context of a semi-infinite atmosphere.

FIG. 10.—Left: Data and predicted counts (upper panel), photon spectrum (middle panel), and residuals of the fit in units of \( \sigma \) (lower panel), for the best cyclotron scattering line model fit to the data of GRB 870303 S1 for the 1-1 geometry. Center and right: Two-dimensional 1σ, 2σ, and 3σ Bayesian credible regions for the fit.
4.3. Results: GRB 870303 S2

In Paper I, we describe fits to these data using exponentiated Gaussian absorption lines. We fit the data in PC bins 7–15 and SC bins 2–31 with a power-law model that is cut off exponentially \((dN/dE \propto E^{-1.22} \times \exp [E(\text{keV})/137.3])\). To model the lines, we multiply onto this continuum two lines with equivalent width \(W_E \approx 2.3\) and 4.6 keV. For this model, \(s_m^2 = 33.6\) (35 dof). The frequentist significance of these lines is \(4 \times 10^{-5}\), and the Bayesian odds favoring the model with lines is 17:1. (Again, these numbers differ from those given above, and in Paper I, in that they are derived assuming \(\theta_{\text{inc}} = 37.7\)).

For the 1-0 geometry, the polar cap and general models fit to the data with \(s_m^2 = 33.6\) (33 dof) and 31.7 (31 dof), respectively. We find that the relatively weak lines of S2 can form at any location on a magnetized neutron star, i.e., for each \(\Psi\), there exist some values of \(\mu\) and \(\phi\) such that the data are fitted well with the cyclotron scattering model. The significance of the two extra parameters of the general model is \(\alpha_{2,\text{MLR}} = 0.39\); we select the polar cap model (see Fig. 11 and Tables 2 and 4).

For the 1-1 geometry, the polar cap and general models fit to the data with \(s_m^2 = 33.9\) (33 dof) and 32.9 (31 dof), respectively. As is the case for the 1-0 geometry, we find that the S2 lines can form anywhere on a magnetized neutron star. The significance of the two extra parameters of the general model is \(\alpha_{2,\text{MLR}} = 0.61\); we select the polar cap model (see Fig. 12 and Tables 2 and 4). We note that \(\mu \to 0\) for this geometry, unlike for the 1-0 geometry; this is directly related to the presence of shoulders in spectra as \(\mu \to 1\).

4.4. Results: The Combined (S1 + S2) Data

In Paper I, we describe fits to combined (S1 + S2) data using exponentiated Gaussian absorption lines. We select a line model parameterized by the first-harmonic energy \((E_c \approx 21.6\text{ keV})\), two first-harmonic equivalent widths \((W_{E,1,S1} \approx 10.3\text{ keV} \text{ and } W_{E,1,S2} \approx 2.5\text{ keV})\), and one second-harmonic equivalent width \((W_{E,2,S1} = W_{E,2,S2} \approx 3.1\text{ keV})\). For this model, \(s_m^2 = 58.0\) (68 dof). The frequentist significance of the lines, evaluated jointly, is \(3.1 \times 10^{-8}\), and the Bayesian odds favoring the model with lines is 8080:1. (Again, these numbers differ from those given above, and in Paper I, in that they are derived assuming \(\theta_{\text{inc}} = 37.7\)).

As noted in §3.2, there are eight polar cap models for joint fits to two data sets, which have a minimum of three, and maximum of six, free parameters. After selecting those models with the lowest values of \(s_m^2\) for each possible number of free parameters, we find \(s_m^2 = 70.8\) (69 dof), 66.5 (68 dof), 64.4 (67 dof), and 60.3 (66 dof), respectively for the 1-0 geometry (Table 5). In no case is \(\Delta s_m^2\) sufficient to reject the null hypothesis; the significance of the three additional parameters of the most complex model is \(\alpha_{2,\text{MLR}} = 0.015\). The additional reduction in \(s_m^2\) would be necessary for \(\alpha_{2,\text{MLR}}\) to reach our selection criterion of 0.01 is \(\approx 0.9\). Because the variation of \(s_m^2\) between Monte Carlo manifestations of input models is \(\approx 1\), there is only a small possibility that the six-parameter model would be selected more often than the three-parameter model if we were to repeat the fit with an infinite number of Monte Carlo spectra. Thus we select the simplest three-parameter polar cap model (see Tables 2 and 6 and Fig. 13; the credible

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**TABLE 4**

| Parameter         | 1-0 MODEL: POLAR CAP | 1-1 MODEL: GENERAL (\(\Psi = 0\)) |
|-------------------|----------------------|-----------------------------------|
| \(B_{1.3} (\text{G})\) | 1.96 ± 0.07          | 1.96 ± 0.15                       | 1.96 ± 0.23                       |
| \(\log N_{e,21} (\text{cm}^{-3})\) | -0.22 ± 0.22         | -0.22 ± 0.25                      | -0.22 ± 0.91                      |
| \(\mu\) | 0.31 ± 0.09          | 0.31 ± 0.49                       | 0.31 ± 0.67                       |

\(^*\) \(\mu_{\text{inc-de}} \in [0.250, 0.375]\)  
\(^{**}\) The 2\(\sigma\) credible region does not include \(\mu = (0.56, 0.58)\)  
\(^{***}\) \(\mu_{\text{inc-de}} \in [0.000, 0.125]\)
regions, which we do not present, are similar to those in Fig. 11).

Fits of the 32 general models (with minimum five, and maximum 10, free parameters) to the combined (S1 + S2) data for the 1-0 geometry yield $\Delta s_m^2 = 70.8$ (67 dof) for the simplest five-parameter model and 59.6 (66 dof) for the six-parameter model for which $\mu_{S1} \neq \mu_{S2}$ (Table 5). The significance of the additional parameter is $\chi^2_{\text{MLR}} = 1.7 \times 10^{-3}$; we select the six-parameter model. No more complicated model is favored over this model (see Tables 2 and 6 and Fig. 14); in this figure, we show only the credible regions for the parameter subset [$\mu_{S1}, \mu_{S2}$]).

Comparing best-fit polar cap and general models, we find that $\Delta s_m^2 = 11.2$ for $\Delta P = 3$. The significance of the additional parameters is $\chi^2_{\text{MLR}} = 0.01$. We conclude that on the basis of the current evidence, we cannot choose between these models.

The application of the polar cap models to the combined (S1 + S2) data for the 1-1 geometry yields best-fits of $s_m^2 = 73.7$ (69 dof) for the simplest model, through $s_m^2 = 71.3$ (66 dof) for the most complex model; $\chi^2_{\text{MLR}} = 0.49$ (Table 5). We select the simplest model three-parameter model (see Tables 2 and 7, and Fig. 15; the credible regions, which we do not present, are similar to those in Fig. 11).

The application of the general class of models yields $s_m^2 = 71.6$ (67 dof), 65.0 (66 dof); and 59.9 (65 dof; $\mu_{S1} \neq \mu_{S2}$), respectively; fitting models with eight or more free parameters offers little further reduction.

| TABLE 5 |
| S1 + S2: Frequentist Model Fits |

| $P$ | $s_m^2$ Polar Cap | $s_m^2$ General | $s_m^2$ Polar Cap | $s_m^2$ General | $s_m^2$ PAPER | $\Delta P$ |
|-----|------------------|-----------------|------------------|-----------------|--------------|-----------|
| 3... | 70.8$^a$         | ...             | 73.7$^c$         | ...             | ...          | 58.0      |
| 4... | 66.5             | ...             | 72.2             | ...             | ...          | 58.0      |
| 5... | 64.4             | 69.5            | 71.9             | 71.6            | ...          | 58.0      |
| 6... | 60.3             | 59.6$^c$        | 71.3             | 65.0            | ...          | 59.9$^c$  |
| 7... | ...              | ...             | ...              | ...             | ...          | ...       |

| $^a$ The number of free parameters. |
| $^b$ Using exponentiated Gaussian absorption line profiles (Paper I). |
| $^c$ Selected model. |
The model to which we would compare this model is the best-fit three-parameter polar cap model.

Thus, again, we would compare this model to the seven-parameter model and find that in the general model on the basis of the current evidence. (Best-fit values, credible intervals, etc., for the seven-parameter general model may be found in Tables 2 and 7 and Fig. 16; note that in this figure, we show only the credible regions for the parameter subset \([\mu_{S1}, \mu_{S2}, \phi_{S1}, \phi_{S2}]\).)

### 4.5. Limits on Neutron Star Rotation Period

The combined \((S1 + S2)\) data indicate that if \(\Psi = \pi/2\), the best-fit values of \(\mu/\Psi\) and/or \(\phi/\Psi\) change during the 22.5 s between \(S1\) and \(S2\) for both the 1-0 and 1-1 geometries. In other words, the orientation of the observer relative to the line-forming region changes with time. The simplest way to interpret this result is to invoke neutron star rotation (see, e.g., Lamb, Wang, & Wasserman 1992).

For the 1-0 geometry, the best-fit model has \(\mu\) changing as a function of time, but not \(\phi\) (Table 6). Because of the symmetries that exist for the particular case of \(\Psi = \pi/2\), \(\phi \in \{-(\pi/8), (\pi/8]\) or \(\phi \in \{7\pi/8), (9\pi/8]\). However, if \(\phi \neq 0 \text{ or } \pi\), then \(\phi\) must change as \(\mu\) changes, violating the original model assumption that \(\phi\) does not change. This condition leads us to posit a model of the rotating neutron star in which (1) the rotation axis is perpendicular to the observer’s line of sight; (2) the star may rotate in either the 1-0 or 1-1 geometry.

### Table 6

**S1 + S2: Bayesian Credible Intervals for 1-0 Geometry**

| PARAMETER | 1 \(\sigma\) | 2 \(\sigma\) | 3 \(\sigma\) | 1 \(\sigma\) | 2 \(\sigma\) | 3 \(\sigma\) |
|-----------|--------------|--------------|--------------|--------------|--------------|--------------|
| \(B_{12} (G)\) | 1.95 ± 0.05 | 1.95 ± 0.11 | 1.95 ± 0.15 | 1.87 ± 0.08 | 1.87 ± 0.14 | 1.87 ± 0.20 |
| \(\log N_{e21} (cm^{-2})\) | -0.22 ± 0.25 | -0.22 ± 0.55 | -0.22 ± 0.97 | -0.22 ± 0.44 | -0.22 ± 0.58 | -0.22 ± 0.67 |
| \(\mu\) | 0.31 ± 0.50c,0.12 | 0.31 ± 0.594,0.31 | 0.31 ± 0.69,0.31 | 0.19 ± 0.14,0.07 | 0.19 ± 0.27,0.12 | 0.19 ± 0.35,0.15 |
| \(\mu_{S1}\) | 0.02c | 0.05 | 0.09 | 0.06 | 0.09 | 0.12 |
| \(\mu_{S2}\) | 0.02c | 0.05 | 0.09 | 0.06 | 0.09 | 0.12 |
| \(\phi_{S1}\) (rad) | 0.20 ± 0.19,0.10 | 0.20 ± 0.19,0.10 | 0.20 ± 0.19,0.10 | 0.20 ± 0.20,0.20 | 0.20 ± 0.20,0.20 | 0.20 ± 0.20,0.20 |
| \(\phi_{S2}\) (rad) | 0.19 ± 0.11 | 0.19 ± 0.11 | 0.19 ± 0.11 | 0.19 ± 0.11 | 0.19 ± 0.11 | 0.19 ± 0.11 |

* The credible region does not include marginalization over \(\Psi\).

### Table 7

**S1 + S2: Bayesian Credible Intervals for 1-1 Geometry**

| PARAMETER | 1 \(\sigma\) | 2 \(\sigma\) | 3 \(\sigma\) | 1 \(\sigma\) | 2 \(\sigma\) | 3 \(\sigma\) |
|-----------|--------------|--------------|--------------|--------------|--------------|--------------|
| \(B_{12} (G)\) | 1.96 ± 0.06 | 1.96 ± 0.12 | 1.96 ± 0.19 | 1.94 ± 0.05 | 1.94 ± 0.12 | 1.94 ± 0.20 |
| \(\log N_{e21} (cm^{-2})\) | -0.52 ± 0.30 | -0.52 ± 0.33 | -0.52 ± 0.39 | -0.22 ± 0.19 | -0.22 ± 0.14 | -0.22 ± 0.10 |
| \(\mu\) | 0.06 ± 0.06 | 0.06 ± 0.06 | 0.06 ± 0.06 | 0.19 ± 0.02c,0.15 | 0.19 ± 0.08,0.19 | 0.19 ± 0.16,0.19 |
| \(\mu_{S1}\) | 0.06 | 0.06 | 0.06 | 0.31 ± 0.13 | 0.31 ± 0.10 | 0.31 ± 0.12 |
| \(\mu_{S2}\) | 0.06 | 0.06 | 0.06 | 0.20 ± 0.14,0.10 | 0.20 ± 0.10,0.20 | 0.20 ± 0.08,0.20 |
| \(\phi_{S1}\) (rad) | 0.19 | 0.19 | 0.19 | 0.59 ± 0.11 | 0.59 ± 0.10 | 0.59 ± 0.09 |
| \(\phi_{S2}\) (rad) | 0.59 | 0.59 | 0.59 | 0.59 | 0.59 | 0.59 |

* The credible region does not include marginalization over \(\Psi\).
(3) the magnetic axis lies within the neutron star–observer plane; and (4) line formation is localized and occurs only where the magnetic equator intersects the neutron star–observer plane. This model is shown pictorially in Figure 17. We assume that the neutron star rotates less than once between $S_1$ and $S_2$, since otherwise we could expect to see evidence for line(s) during the time period separating the two spectra.

The rotation period is given by

$$t_{\text{rot}} = \frac{2\pi}{|\theta_{S_1} - \theta_{S_2}|} \times 22.5 \text{ s} ,$$

where $\theta_{S_1} = \cos^{-1}(\mu_{S_1})$ and $\theta_{S_2} = \cos^{-1}(\mu_{S_2})$. Generally, there are two possible rotation periods that we may derive, depending upon the direction of the star’s rotation. However, for $\Psi = \pi/2$, there are four possible times (which we hereafter denote as $t_{\text{rot, n}}$), because of the symmetry between $\phi = 0$ and $\phi = \pi$ (see Fig. 17).

To define credible intervals for the rotation periods, we repeatedly sample values of the cosines $\mu_{S_1}$ and $\mu_{S_2}$ from the posterior distribution $p(\mu_{S_1}, \mu_{S_2} | \Psi = \pi/2, \phi = 0$ or $\pi, D)$ and determine for each sampled cosine pair the possible rotation periods $t_{\text{rot, n}}$. (This distribution is slightly different from that shown in Fig. 14, which includes marginalization over $\phi$.) We determine probability distributions from $10^6$ values of $t_{\text{rot, n}}$, and use these distributions to estimate the 1, 2, and 3 $\sigma$ credible intervals given in Table 8.

For the 1-1 geometry, both $\mu$ and $\phi$ change as a function of time, which greatly complicates the derivation of rotation periods. In particular, the orientations of the magnetic and
rotation axes, \(\hat{H}\) and \(\hat{R}\), can be arbitrary. In the Appendix, we describe a Bayesian method that we use to derive rotation periods once we sample values of \(\mu\) and \(\phi\) for S1 and S2. We determine the probability distributions from \(10^4\) values of \(t_{\text{rot,n}}\), and use these distributions to estimate the 1, 2, and 3 \(\sigma\) credible intervals given in Table 8.

5. DISCUSSION

In this paper, we demonstrate that Monte Carlo models of cyclotron scattering in the strong magnetic field \((B \sim 10^{12} \text{ G})\) of a Galactic neutron star can successfully account for the positions and strengths of the lines exhibited at \(\approx 20\) keV in GRB 870303 S1 and \(\approx 20\) and 40 keV in GRB 870303 S2. Our results are robust to changes in slab geometry and magnetic field orientation. Given that physically rigorous models of line formation within the cosmological burst environment do not yet exist (Stanek et al. 1993 and Ulmer & Goodman 1995, e.g., invoke femtolensing), our results, when paired with the successful fits of cyclotron scattering models to the data of GRB 880205 by Wang et al. (1989a) and Freeman et al. (1992), provide strong support for the hypothesis that some (though not all) GRBs are Galactic in origin.

5.1. Galactic Source Population

5.1.1. Static Line Formation at the Polar Cap

We model static line formation at the magnetic polar cap of a neutron star with a simple dipole field by setting \(B\) parallel to the slab normal \(\hat{n}\), i.e., by setting \(\Psi = 0\). We find that while we can easily fit this model to the data of GRB 870303 S2, acceptable fits to the data of GRB 870303 S1 can be made only with difficulty (1-0 geometry) or cannot be made at all (1-1 geometry). For the 1-0 geometry, we conclude that either the observer is oriented directly along \(B\) (\(\mu = 1\)) or that the line formation region is levitating far above the polar cap, where it presents a small solid angle to continuum photons, with the result that the equivalent widths of the second and third harmonics are reduced rela-
tive to the width of the first harmonic. The latter conclusion is consistent with the suggestion by Dermer & Sturmer (1991) and Sturmer & Dermer (1994) that a scattering atmosphere with a geometry similar to the 1–0 may be present in both accretion-powered pulsars and GRBs, with present that are optically thick to line scattering but thin to continuum scattering forming within a few stellar radii of the neutron star center. For the 1–1 geometry, we find that prominent emission-like shoulders on either side of the first harmonic prevent a good fit to the S1 data if the observer is oriented along \( B (\mu = 1) \); by decreasing \( \mu \), we can reduce the magnitude of the shoulders, but second and third harmonics form, preventing an acceptable fit to these data.\(^{15}\)

The results of fits to the S1 data argue against static line formation at the magnetic polar cap. If line formation does indeed occur above the polar cap in an outflowing plasma, the neutron star probably resides within the Galactic halo at a distance of \( \gtrsim 50 \) kpc. Two lines of reasoning point to this conclusion. First, as pointed out by Lamb et al. (1990), a static polar cap line formation region will be disrupted on timescales \( \sim 10^{-6} \) s unless the burst flux is below the magnetic Eddington limit. At the nonmagnetic Eddington limit, the radiative pressure exerted by both line and continuum photons upon electrons in the line-forming layer balances the gravitational force upon the protons which are electrostatically coupled to the electrons; the magnetic Eddington limit is determined by replacing the Thomson cross section of the electron with the resonant cyclotron scattering cross section of the first harmonic. This lowers the luminosity limit from \( \sim 10^{38} \text{ergs s}^{-1} \) to \( \sim 10^{36} \text{ergs s}^{-1} \). The upper limit on the distance to the burst source with an electron-proton line-forming layer is thus \( \sim 100 \) pc.

Second, Loredo & Wasserman (1998b) determine that the data of the 3B catalog (Meegan et al. 1996) is consistent with the hypothesis that there is a component of the source population residing within the Galaxy, with that component either being comprised of dim local halo sources at distances \( \lesssim 1 \) kpc, or luminous halo sources at distances \( \gtrsim 50 \) kpc. They make this conclusion by using Bayesian methods (developed in Loredo & Wasserman 1995) to compare a model in which sources residing in a Bahcall–Soneira halo with core size 2 kpc (Bahcall & Soneira 1980) are mixed with cosmological GRB sources, with the best-fit cosmological GRB source model from Loredo & Wasserman (1998a). Each class is assumed to contain standard candle bursters, with the further assumptions for the cosmological bursters that the comoving burst rate is homogeneous and independent of redshift. (Loredo & Wasserman 1998a demonstrate that this simple cosmological model fits the data as well as more complicated models with either an inhomogeneous burst rate or a power-law burst luminosity function.) The best fit of the halo model is better than that of the purely cosmological model, for both dim local and luminous halo bursters. The best-fit fractions of dim local halo sources are 0.59 (64 ms 3B catalog data) and 0.36 (1024 ms data), and the bursts are limited to distances \( \lesssim 1 \) kpc; the respective fractions for luminous halo sources are 0.11 and 0.073, with inferred distances \( \gtrsim 50 \) kpc. However, the 3B data lack the power to constrain the model parameters tightly, and thus the credible intervals for each fraction include zero at a significance level \( \lesssim 2 \sigma \), or 95.5%. (Despite the lack of constraining power, the data do not support the bursts residing at distances intermediate between 1 and 50 kpc.) Because of this lack of constraining power, the Bayesian odds favoring the Galactic component models are not large: 2.5 and 6.7 (64 and 1024 ms) for dim local halo sources, and 0.45 and 0.25 for luminous halo sources. Odds of greater than 10–20 would be considered strong evidence in favor of an alternative model (see the review by Kass & Raftery 1995 and references therein).

We note that if lines exhibited by accretion-powered pulsar (APP) spectra are also formed via cyclotron scattering, the lack of shoulders in the 12 known APPs with lines (Mihara 1995) places a lower limit on column densities that is much larger than the column densities determined in our analyses, within the context of the 1-1 geometry. This result is shown by Isenberg et al. (1998b), who consider cylindrical line formation regions with photon injection along the axis, which represent the canonical model of the emission region of accretion-powered pulsars. They show that if \( N_{e,21} \gtrsim 1000 \), the shoulders disappear. This implies that even if the 1-1 geometry is an acceptable approximation for both APPs and GRBs, the mechanisms underlying line formation must be significantly different.

### 5.1.2. Static Line Formation at the Magnetic Equator

The most theoretically intriguing result of our analyses is the possibility that line formation may occur away from the magnetic polar cap (i.e., for \( \Psi \neq 0 \)). The S1 data provide strong evidence that, for the 1-1 geometry, line formation does not occur at the polar cap, with the best-fit location being the magnetic equator (\( \Psi = \pi/2 \)). The combined (S1 + S2) data provide marginal evidence, for both geometries, favoring equatorial line formation. Line formation at a magnetic equator may be static even if the burst luminosity greatly exceeds the critical Eddington luminosity in a strong magnetic field, since the closed magnetic field lines may effectively trap the plasma (see, e.g., Zheleznyakov &

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\(^{15}\) However, we note that shoulders are not always inconsistent with observed GRB line profiles: Freeman et al. (1992) show that the 1-1 polar cap model, with shoulders, fits to the data of GRB 880205 better than a 1-0 polar cap model without shoulders; \( \lesssim 5 \) falls from 43.9 to 36.6.
Serber 1994, 1995). Thus the assumption of line formation in static layers may be acceptable even if the progenitor neutron star resides in the Galactic halo at distances \( \gtrsim 50 \) kpc.

5.1.3. Line Formation in an Outflow

If line formation occurs at the magnetic polar cap of a luminous halo burst, the line formation region will initially flow outward along \( B \). In an outflow, the variation of the magnetic field and plasma velocity with altitude will tend to broaden the lines. Thus the relevant question is whether we can still observe narrow lines in the spectra of distant halo GRBs. The answer appears to be yes. Miller et al. (1991, 1992) calculated the properties of the second and third harmonics, approximating Raman scattering with cyclotron absorption (see § 5.2). They showed that narrow lines can be formed at these harmonics, and they successfully fitted outflow absorption models to the second harmonic of GRB 880205. Chernenko & Mitrofanov (1995) calculate the properties of the first-harmonic line in an outflow, but they assume the line to be formed via absorption; they find that narrow first-harmonic lines are possible. Isenberg et al. (1998a) show that the cyclotron scattering model can create narrow lines at all harmonics. They treat this problem using a variant of the Monte Carlo code applied in this work. They assume a hot spot at the magnetic pole of a neutron star and apply the radiation force calculation of Mitrofanov & Tsygan (1982) to determine the plasma outflow velocity. They conclude that cyclotron scattering lines can form within a relativistic outflow with properties similar to those of GRB 870303 and GRB 880205, provided the hot spot is a small fraction of the stellar surface (\( r_{\text{hot}} \lesssim 0.1 R_{\text{NS}} \)).

A natural consequence of the fact that the line-forming layer may move along field lines is that the height of the line-forming layer may change with time, as, for example, the burst intensity or hot spot radius changes. A change in height would be accompanied by a change in inferred magnetic field strength (i.e., by an inferred shift in line-centroid energy). Yoshida et al. (1992) report that the magnetic field strength shows a declining trend within the 9 s interval in which the line candidates of GRB 880205 have maximum significance, which indicates that the line-forming region is flowing outward from the neutron star. We find that the combined (\( S_1 + S_2 \)) data are nearly consistent with the hypothesis that the line-forming region moves inward during the \( \approx 20 \) s between line epochs. Within the context of the most complex six-parameter polar cap model, which the data almost favor over the simplest three-parameter model, \( B_{12} \) changes from 1.76 to 1.96 from S1 to S2, indicating that S2 line formation might occur closer to the polar cap than S1 line formation.

5.2. Cyclotron Scattering versus Cyclotron Absorption

Many authors (e.g., Fenimore et al. 1988; Graziani et al. 1992) fit data with the cyclotron absorption model, motivated by its computational simplicity and by the fact that approximating cyclotron Raman scattering by cyclotron absorption is approximately valid for the second and higher harmonics. The cyclotron absorption model spectrum is

\[
CA(E) = C(E) \exp \left[ - \sum_{n=1}^{m} A_n G_n(E) \right],
\]

where \( C \) is the continuum spectrum, \( m \) is the number of harmonics, and

\[
G_n(E) = \frac{1}{\sqrt{\pi \Delta E_n}} \exp \left[ - \frac{(E - E_n)^2}{\Delta E_n^2} \right]
\]

is the absorption line profile for the \( n \)th harmonic. The line widths \( \Delta E_n \) are given by \( E_n/(2kT_{\text{los}} \mu^2/m_c^2)^{1/2} \), where \( \mu \) is the cosine of the angle between the line of sight and \( B \). The line amplitudes \( A_n \) are given by \( N_{\text{los}} \alpha \), where \( N_{\text{los}} \) is the column density along the line of sight, and

\[
\alpha_n \approx 5.5 \times 10^{-20} \left( \frac{n^2 B}{2 B_\text{los}} \right)^{n-1} (1 + \mu^2)^{-1} (n - 1)!
\]

\[
\times (1 - \mu^2)^{n-1} \text{keV cm}^{-2}
\]

is the absorption coefficient of the \( n \)th harmonic. Because modeling the first-harmonic line with cyclotron absorption is not valid, we must allow different temperatures and column densities when fitting to the first, and higher, harmonics. If, for example, we fit two harmonic lines to the data, the relationships between the cyclotron absorption model parameters and the physical parameters of the line-forming region are

\[
E_1 \propto B, A_1 \propto N_{\text{los}} (1 - \mu^2), A_2 \propto BN_{\text{los}} (1 - \mu^4), \text{ and } \Delta E_1 = E_1(kT_{\text{los}} \mu^2/m_c^2) \text{ and } \Delta E_2 = 2E_1(kT_{\text{los}} \mu^2/m_c^2). \]

N_{\text{los}} and \( kT_{\text{los}} \mu^2/m_c^2 \), however, have no direct physical meaning.

We may use the cyclotron absorption model to describe the line formation region if at least both the second and third harmonics are strong, i.e., the data request that each of these harmonics be fitted with a line shape parameterized by centroid energy, equivalent width, and full width (see Paper I for a description of this particular parameterization). Only then is the number of required fit parameters, five (we assume \( E_3 = (3/2)E_2 \), larger than the number of physical parameters needed to describe the region \( B, T_{\text{c},1,2}, N_{\text{c},2}, \mu \)). Otherwise, such as when only the first and second harmonics are observed, radiative transfer calculations are required. No simple description can be used to explain the first-harmonic line, the appearance of which depends critically on the outcome of the multiple resonant scatters required in order for individual photons to escape, as well as on the introduction of spawned photons at energies near that of the first harmonic.

The cyclotron scattering model can describe the line-forming region using fewer free parameters than the cyclotron absorption model (because of the relationship between \( T_{\text{c},1,2} \) and \( B \) in the scattering model) and can use the first-harmonic data to help forge the description. However, we have found that moderate-resolution GRB data may require an even smaller number of free parameters than the minimum three of the cyclotron scattering model. In Table 9, we list the number of model parameters required in fits to the GRB 870303 data with the exponentiated Gaussian absorption-line model (Paper I) and the cyclotron absorption and scattering models. Each harmonic in these data can be adequately modeled with a saturated line shape parameterized by centroid energy and equivalent width; the full width is set to be proportional to the equivalent width. The number of free parameters is then \( N_{\text{harm}} + 1 \), where \( N_{\text{harm}} \) is the number of observed harmonics (\( N_{\text{harm}} \) line width parameters and one harmonic line energy). This is an upper limit. For instance, the S2 data are adequately fitted.
using two, rather than three, parameters (by setting $W_{E,2} = 2W_{E,1}$), because the second harmonic data lack constraining power. Lamb (1992) argues that physically based models that have parameters not required by the data will (1) adequately fit the data but (2) have reduced diagnostic power because of a lack of constraint on the individual model parameters. This is certainly true for the cyclotron absorption model. For both GRB 870303 (Graziani et al. 1992) and GRB 880205 (Fenimore et al. 1988), the cyclotron absorption model determines the magnetic field well (since it is a function of only $E_1$), but $\mu$ is undetermined and $kT_{e,1}$ and $N_e^{\text{los}}$ are therefore poorly constrained. However, we determine in this work that photon spawning and the presence of line shoulders give the cyclotron scattering model greater diagnostic power than we would have predicted. (Note that such the absorption model fits adequately because large shoulders are not observed in GRB 870303 and GRB 880205; if large shoulders were present, no model using absorption-like profiles would adequately fit the data.) Spawned photons fill the first harmonic in the polar cap model, decreasing its ability to adequately fit the S1 data in the 1-0 geometry; the addition of shoulders in the 1-1 geometry makes no adequate fit possible. While polar cap models adequately fit the S2 data, the line shoulders that appear in the 1-1 geometry greatly reduce the range of $\mu$ consistent with the data (cf. the 1-0 result, for which there is no constraint on $\mu$ at the 3 $\sigma$ limit).

Fits to the combined (S1+S2) data in Paper I clearly indicate that the data prefer a four-parameter model in which $W_{E,1}$ changes between S1 and S2. A cyclotron scattering model having different $\mu$ and/or $N_e$ values for S1 and S2, and at least four parameters overall, would thus be expected to provide the best fit to the data. Fits of the general model to the joint data fulfill that expectation, with six- and seven-parameter models chosen. Surprisingly, however, the simplest three-parameter polar cap models adequately fit the data for both the 1-0 and 1-1 geometries; spawned photons and line shoulders prevent the data from selecting more complex models over the simplest model, even if up to six parameters are allowed to float freely (Table 5).

| Spectrum | Exponentiated Gaussian Absorption* | Cyclotron Absorption | Cyclotron Scattering |
|----------|-----------------------------------|----------------------|----------------------|
| S1 ........... | 2 | 3(1)$^b$ | 3 (1-0) |
| S2 ........... | 2 | 5(3)$^b$ | 3 (1-0) |
| S1+S2...... | 4 | 7(3)$^b$ | 3 or 6$^c$ (1-0) |

* See Paper I for description.
$^b$ The number in parentheses represents the number of physically relevant parameters.
$^c$ The data do not conclusively select either the polar cap model or general model.

the observation of one harmonic during S1 and two harmonics during S2 to suggest that line formation is observed along $B$ for S1 and perpendicular to $B$ for S2. Invoking both localized line formation and line formation along an equatorial arc, they derive a rotation period $45$ s $\leq t_{\text{rot}} \leq 180$ s. Lamb et al. (1992) invoke similar arguments to explain the changes in line-centroid energy, strength, and width of the line candidate exhibited by the HEAO 1 A-4 data of GRB 780325 (Hueter 1987) and to predict $40$ s $\leq t_{\text{rot}} \leq 80$ s.

Examining the real-time data of GRB 870303, Yoshida et al. (1989) find an additional peak in the PC time history beyond the two seen in burst mode (Fig. 7). The peaks have periodicity $\approx 30$ s. They conclude that the data in the third peak can be adequately fitted by an $\approx 10$ keV thermal bremsstrahlung spectrum, an $\approx 1.6$ keV blackbody spectrum, or an $\approx 0.3$ keV thermal cyclotron spectrum. No spectral features are apparent, but the spectrum is perhaps too soft for lines to be unambiguously detected. Such a periodicity is perhaps barely consistent with $t_{\text{rot,3}}$ in the 1-0 geometry but is consistent with both $t_{\text{rot,3}}$ and $t_{\text{rot,4}}$ in the 1-1 geometry. If we hypothesize that the neutron star completes more than half a rotation between S1 and S2, it follows that a localized region of line formation disappears behind the stellar disk between the two epochs. Graziani et al. (1993) consider a model in which lines appear at time $t_1$, disappear at time $t_2$, and are constant in between. They find that the uncertainties $\sigma_{t_1}$ and $\sigma_{t_2}$ in the times at which the lines appear and disappear are large, and they cannot exclude the possibility that lines are present throughout GRB 870303.

Rotation periods $\gtrsim 2.5$ s are longer than those that would be expected for a neutron star born with rotation period 1 s and that has undergone magnetic braking for a Hubble time ($P_{\text{max}} \approx 5$ s). Mass transfer from companions slows some accretion-powered X-ray pulsars, which have rotation periods up to $\approx 800$ s (see, e.g., Nagase 1989). Similarly, accretion of matter onto high-velocity neutron stars on the way to or within a Galactic corona may act to slow a neutron star and provide fuel for GRBs. Repeated GRBs (Graziani et al. 1998) may in turn cause losses in angular momentum, slowing the neutron star, if mass is ejected from the source.

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APPENDIX

GENERAL DERIVATION OF THE NEUTRON STAR ROTATION PERIOD

The orientation of the observer relative to the slab where line formation takes place is described by a polar cosine, $\mu$, and azimuth angle, $\phi$ (see Fig. 1). If both $\mu$ and $\phi$ change with time, we cannot use the simple model described in §4.5, in which $\phi$ is constant in time, to place limits on the rotation period(s) of the underlying neutron star. Here, we outline a general Bayesian method for deriving credible intervals for the rotation period(s).

We first write out a marginalized posterior probability distribution for $\mu$ and $\phi$ for the best-fit model, adding as free parameters the magnetic field axis orientation at the epoch of the first spectrum, $\vec{H}_1$, and the rotation axis, $\vec{R}$:

$$p(\mu_1, \mu_2, \phi_1, \phi_2, \vec{H}_1, \vec{R} \mid D, I, \Psi = \pi/2) = p(\mu_1, \mu_2, \phi_1, \phi_2, \vec{H}_1, \vec{R} \mid I) \int dB dN e p(B \mid I) p(N_e \mid I) \times \frac{\mathcal{L}(\mu_1, \mu_2, \phi_1, \phi_2, \vec{H}_1, \vec{R}, B, N_e, \Psi = \pi/2)}{p(D \mid I)}. \quad (A1)$$

(Here, we assume that $\Psi = \pi/2$; we do not marginalize over this parameter.) We determine the rotation periods $t_{\text{rot}, n}$ by sampling from this posterior distribution. (There are four possible rotation periods that can be derived, for reasons given in §4.5 and illustrated in Fig. 17.) However, we must deal with the Bayesian prior, the first factor on the right-hand side of equation (A2). We expand it:

$$p(\mu_1, \mu_2, \phi_1, \phi_2, \vec{H}_1, \vec{R} \mid I) = p(\vec{R} \mid I)p(\vec{H}_1 \mid \vec{R}, I) \cdots (\phi_2 \mid \phi_1, \mu_2, \mu_1, \vec{H}_1, \vec{R}) = p(\vec{R} \mid I)p(\vec{H}_1 \mid \phi_1)p(\phi_1 \mid \mu_1, \vec{H}_1, \vec{R})p(\phi_2 \mid \mu_2, \vec{H}_1, \vec{R}, I). \quad (A2)$$

We randomly sample $\mu_1, \mu_2, \vec{H}_1$, and $\vec{R}$ from uniform distributions. Because of polar symmetry, the values $\mu_1$ and $\mu_2$ define circles centered on the visible disk of the neutron star where line formation may take place. Given $\vec{H}_1$, we can define the magnetic equator, which either intersects the line formation circle twice, or not at all. Since we assume $\Psi = \pi/2$, the vector pointing out of the star at the intersection points is simply $\hat{n}_1$ (Fig. 1). At the intersection points, $\hat{O} \cdot \vec{n}_1 = \mu_1$, where $\hat{O}$ is the vector pointing to the observer. If we define $\hat{O}$ as $(0, -1, 0)$, $\mu_1 = -\mu_2$. The only unknown is then $n_1$, which we can solve for numerically. Given the location of line formation, we project $\hat{O}$ onto the local surface tangent (i.e., onto a vector which is perpendicular to $\hat{n}_1$). The dot product of this projected vector with the vector $-\vec{H}_1$ (the direction of the field at the magnetic equator) is the cosine of $\phi_1$.

We may determine $\hat{n}_2$ by using the information that $\vec{R} \cdot \hat{n}$ is constant and $\hat{O} \cdot \vec{n}_2 = \mu_2$. Either two solutions to these equations exist, or none at all. Given a solution, we next determine the possible local field direction $-\vec{H}_2$. The vectors $\vec{R}, \vec{n}_1,$ and $\vec{n}_2$ allow us to use the law of cosines to determine $\beta$, the angle through which the neutron star rotates between the two epochs. We transform the Cartesian coordinate system such that $\vec{R} = (0, 0, 1)$, rotate the system through angle $\beta$, and undo the transformation. We then solve for $\phi_2$, as above.

The likelihood $\mathcal{L}$ does not depend upon $\vec{H}_1$ or $\vec{R}$, so the integrated expression in equation (A2) is proportional to the four-dimensional posterior probability distribution $p(\mu_1, \mu_2, \phi_1, \phi_2, \vec{H}_1, \vec{R} \mid I, \Psi = \pi/2)$. For the particular case of interest in this paper, this distribution is known: two dimensional “slices” of it are shown in Figure 16. We use this distribution as a rejection function: we assess the relative probability of our solution with respect to the best-fit parameters of the model, sample a random number $r$, and accept the solution if $r < P_{\text{solution}}/P_{\text{best-fit}}$. If we accept the solution, we then solve for the rotation period $t_{\text{rot}, n}$.

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