Flavor-specific behavior of conserved charge fluctuations at the QCD confinement transition

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Abstract. We propose measurements of fluctuation observables, which are sensitive to the chemical freeze-out of the matter produced in heavy-ion collisions, in order to discriminate a potential flavor-specific behavior of light and strange quarks at the QCD confinement transition. This complements our former considerations which were restricted to the strange quark sector only.

1. Introduction

The QCD confinement transition from a state with deconfined, colored and (almost) massless partonic degrees of freedom to a state containing color-neutral and massive hadrons and resonances, in which the partons are confined, is known from first-principles to be an analytic crossover at vanishing net baryon-number density [1]. Accordingly, this phase transformation of QCD matter takes place over a broad temperature $T$-interval, where an exact value for the pseudo-critical temperature $T_c$ cannot be pinned down unambiguously. In fact, depending on the considered order-parameter-like observable, values of $T_c = (154 \pm 9)$ MeV [2], or $T_c = (147 \pm 5)$ MeV and $T_c = (155 \pm 6)$ MeV [3] are found.

Within large-scale world-wide efforts, the phase diagram of QCD matter is sought to be explored experimentally. Envisaged goals are the investigation of the confinement/deconfinement transition in the laboratory and a determination of the properties of deconfined matter and of the nature of its degrees of freedom. In the relativistic heavy-ion collisions (HIC) being performed at the LHC (Large Hadron Collider) and at RHIC (Relativistic Heavy Ion Collider), indeed, the deconfined phase of QCD matter is transiently produced. Observables suitable for indicating this are, for example, fluctuations in conserved charges, which are sensitive to the microscopic structure of the matter [4]. The conserved charges relevant in HIC are the net baryon-number, net electric-charge and net strangeness. A rapidly changing, non-monotonous behavior in the fluctuations of these charges can provide clear signatures for the confinement/deconfinement transition. For example, in [5] the ratio of fourth- to second-order cumulants of net electric-charge fluctuations was proposed as a possible observable.

2. Indications for a flavor-hierarchy at confinement

In the gauge field theory of the strong interaction, QCD, the conserved charges are instead the net quark-numbers of individual quark flavors. Fluctuations in these charges can be quantified in the

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A function of observations formerly made in [9]. Thus, the first-principle results reported in [8] provided a dependent result obtained for the liberation temperature of strange quarks coincided with the difference in the behavior between light and strange quarks was observed [8], where the model—this HRG model prediction may be interpreted as the onset of a liberation of quarks. Again, a combinations exactly vanish. Thus, any deviation of the corresponding lattice QCD data from model, which properly describes QCD thermodynamics in the hadronic phase, some of these of uncorrelated hadrons and resonances as described by the Hadron Resonance Gas (HRG) of freedom carrying the quantum numbers of light or of strange quarks. For a system composed of non-monotonous behavior with $T$ in independent ratio of fourth- to second-order quark-number susceptibilities $\chi$ order susceptibilities were studied in lattice QCD simulations in [8]. In particular, the volume—up and strange quarks [6]. This observation was interpreted in [7] as signal for a flavor-hierarchy $T$ showed a clear difference in the behavior of the net quark-number fluctuations with showed $T$ at a smaller temperature than for strange quarks, cf. [8]. Relating to the light quark chemical potential $\mu$ to the equilibrium pressure $P$ by means of lattice QCD calculations. Quark-number susceptibilities are related to the grand canonical ensemble description of the system by susceptibilities and are therefore accessible from first-principles by means of lattice QCD calculations. Quark-number susceptibilities are related to the equilibrium pressure $P$ by appropriate derivatives reading

$$\chi_{l}^{uds} = \frac{\partial^{j+m+n}(P/T^4)}{\partial(\mu_u/T)^l \partial(\mu_d/T)^m \partial(\mu_s/T)^n} \bigg|_{\mu_u=\mu_d=\mu_s=0},$$

where $\mu_u$, $\mu_d$ and $\mu_s$ represent the up, down and strange quark chemical potentials associated with the net quark-numbers of up, down and strange quarks, respectively.

Recent continuum-extrapolated lattice QCD results for $\chi^2$ and $\chi^4$ at physical quark masses showed a clear difference in the behavior of the net quark-number fluctuations with $T$ between up and strange quarks [6]. This observation was interpreted in [7] as signal for a flavor-hierarchy at the QCD confinement transition. In order to shed more light onto this possibility, higher-order susceptibilities were studied in lattice QCD simulations in [8]. In particular, the volume-independent ratio of fourth- to second-order quark-number susceptibilities $\chi^4_L/\chi^2_L$ of flavor $f$ was found to depict a non-monotonous behavior with $T$, which occurs for light (up and down) quarks at a smaller temperature than for strange quarks, cf. [8]. Relating $\mu_u$ and $\mu_d$ via $\mu_u = \mu_d = \mu_L/2$ to the light quark chemical potential $\mu_L$, the light quark-number susceptibilities $\chi^2_L$ and $\chi^4_L$ can be obtained from up and down quark-number susceptibilities as $\chi^2_L = (\chi^2_u + 2\chi^2_d + \chi^2_s)/4$ and $\chi^4_L = (\chi^4_u + 4\chi^4_{ud} + 6\chi^4_{udd} + 4\chi^4_{ud} + \chi^4_s)/16$. The corresponding lattice QCD results [8] for $\chi^4_L/\chi^2_L$ as a function of $T$ are shown in Fig. 1.

Moreover, particular combinations of higher-order cumulants of fluctuations in the conserved charges and of correlations among them were considered in [8] which probe directly the degrees of freedom carrying the quantum numbers of light or of strange quarks. For a system composed of uncorrelated hadrons and resonances as described by the Hadron Resonance Gas (HRG) model, which properly describes QCD thermodynamics in the hadronic phase, some of these combinations exactly vanish. Thus, any deviation of the corresponding lattice QCD data from this HRG model prediction may be interpreted as the onset of a liberation of quarks. Again, a difference in the behavior between light and strange quarks was observed [8], where the model-dependent result obtained for the liberation temperature of strange quarks coincided with the observations formerly made in [9]. Thus, the first-principle results reported in [8] provided further indications for a flavor-specific behavior in the confinement transition of QCD.

In the HRG model, the equilibrium pressure reads as

$$P(T, \mu_B, \mu_Q, \mu_S) = \sum_j (-1)^{B_j+1} \frac{d_j T}{(2\pi)^3} \int d^3 k \ln \left[ 1 + (-1)^{B_j+1} e^{-\left( \sqrt{k^2+m_j^2}-\mu_j \right)/T} \right],$$

where $L = 0.5, 1.5, 2.5, 3.5, 4.5, 5.5$. The corresponding lattice QCD results [8] for $\chi^4_L/\chi^2_L$ are shown in Fig. 1.

**Figure 1.** Comparison of the HRG model result (long-dashed curve) for $\chi^4_L/\chi^2_L$ with the continuum-extrapolated lattice QCD data at physical quark masses (circles) from [8]. The change in the behavior of the $\chi^4_L/\chi^2_L$-data with $T$ as indicator for the onset of the deconfinement of light quarks happens at about $T \simeq (146 \pm 3)$ MeV (dotted band). Given the large errors in the data from [8] within the transition region, this is also where the HRG model starts to deviate significantly from lattice QCD.
where the sum runs over all the hadronic and resonance states included in the model. In Eq. (2), \( d_j, m_j \) and \( \mu_j \) denote the degeneracy factor, the mass and the chemical potential of the excitation \( j \), where the latter is given by \( \mu_j = B_j \mu_B + Q_j \mu_Q + S_j \mu_S \) with \( B_j, Q_j \) and \( S_j \) as quantum numbers of \( j \) in baryonic charge, electric charge and strangeness, respectively, while \( \mu_B, \mu_Q \) and \( \mu_S \) denote the chemical potentials associated with the corresponding charge-numbers being conserved in HIC. Fluctuations in these charges can be quantified by the susceptibilities \( \chi_{\min} \) readily obtained from Eq. (1) by replacing \((u, d, s)\) by \((B, Q, S)\). Relating the chemical potentials via \( \mu_B = \mu_u + 2\mu_d, \mu_Q = \mu_u - \mu_d \) and \( \mu_S = \mu_d - \mu_s \), cf. [10], the ratio \( \chi_4^L/\chi_2^L \) can be determined from the susceptibilities calculable in the HRG model as

\[
\frac{\chi_4^L}{\chi_2^L} = \frac{\langle \mu \rangle^2}{\langle \mu \rangle^2 + 3 \langle \mu \rangle} = \frac{81 \chi_4^B + 108 \chi_3^B + 54 \chi_2^B + 12 \chi_1^B + 3 \chi_0^B}{4 (9 \chi_4^B + 6 \chi_3^B + \chi_2^B)}.
\]

The corresponding result for a HRG model containing excitations up to a mass of 2 GeV is shown in Fig. 1 to agree fairly well with the lattice QCD data from [8] for \( T \lesssim 146 \text{ MeV} \).

### 3. Verifying a possible flavor-hierarchy experimentally

The ratios of those susceptibilities, which can be associated with fluctuations in the conserved charges, form in principle measurable, volume-independent observables. They can be related to higher-order moments of multiplicity distributions [11], which are fixed at the chemical freeze-out of the matter produced in HIC. Assuming that the chemical freeze-out takes place subsequent to the confinement transition, a flavor-specific behavior should leave a fingerprint in the higher-order moments of flavor-sensitive particle-identified multiplicity distributions. Hence, a flavor-hierarchy at confinement should be testable experimentally.

In [12], we proposed a measurement suitable for determining the freeze-out characteristics of strange quarks. Here, we want to extend this idea to the light quark sector. Given the spectrum of hadrons identifiable with the ALICE detector at the LHC, we propose to determine for an ensemble composed of \( K^+, K^0, p \) and \( \bar{p} \) the statistical moments of the combined multiplicity distribution \( N_L = \frac{3}{2} N_p + \frac{1}{2} N_{K^0} - \frac{3}{2} N_{K^+} - \frac{3}{2} N_{K^-} \). In this ensemble, pions are disregarded because they do neither contribute to the net light quark-number nor to its fluctuations. By comparison of the product between kurtosis \( \kappa \) and variance \( \sigma^2 \) of this distribution with the susceptibility ratio \( \chi_4^L/\chi_2^L \) determined via Eq. (3) from the pressure in Eq. (2), where the sum is restricted to the above proposed ensemble, a freeze-out temperature specific to light quarks might be deducible from the fluctuation observables \( \kappa \) and \( \sigma^2 \) in a model-dependent way, see Fig. 2 (left).

For the strange quark sector, instead, we suggested in [12] to consider an ensemble composed of \( K^+, \Lambda^0, \Xi^- \) and \( \Omega^- \) as well as their anti-particles. By determining \( k \sigma^2 \) for the combined multiplicity distribution \( N_s = N_{K^-} + N_{\Lambda^0} + 2 N_{\Xi^-} + 3 N_{\Omega^-} - N_{K^+} - N_{\Lambda^0} - 2 N_{\Xi^+} - 3 N_{\Omega^+} \) and comparing with \( \chi_4^S/\chi_2^S \equiv \chi_4^B/\chi_2^B \) obtained from Eq. (2) restricted to this ensemble, the corresponding freeze-out temperature for strange quarks may be deduced, see Fig. 2 (right). A possible difference in the freeze-out behavior between light and strange quarks found from the fluctuation observables would provide an experimental verification for a flavor-hierarchy in the QCD confinement transition.

For a meaningful comparison of experimental results with the HRG model, however, strong resonance decays have to be included into the considerations. This will be discussed elsewhere. Moreover, detector acceptance cuts have to be taken into account. They can be included into the HRG model approach by replacing in Eq. (2) the measure \( d^3k \rightarrow k_T \sqrt{k_T^2 + m_j^2} \cosh(y) \, dk_T \, dy \, d\phi \) and \( \sqrt{k^2 + m_j^2} \rightarrow \sqrt{k_T^2 + m_j^2} \cosh(y) \), where \( k_T \) is the transverse momentum and \( y \) the rapidity of the identified particles. The influence of acceptance cuts on the HRG model results is shown in Fig. 2 (solid curves). The applied cuts read \( |y| < 0.5 \) as well as \( 0.2 < k_T(\text{GeV}/c) < 2 \) and \( 0.5 < k_T(\text{GeV}/c) < 3 \) for the light and strange quark specific ensembles, respectively.
Figure 2. Determination of the freeze-out temperature for light (left panel) and strange quarks (right panel) from comparing fluctuation observables. Possible results depend on the potentially measured $\kappa \sigma^2$-values for the specific multiplicity distributions $N_L$ and $N_s$ of primary particles (dotted bands). Long-dashed and short-dashed curves show the HRG model results for $\chi_f^4/\chi_f^2$ ($f = L, s$), where excitations of mass up to 2 GeV or only the particles contained in the specific ensembles are considered in Eq. (2), respectively, while the solid curves depict the impact of detector acceptance cuts on the specific ensemble results.

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References
[1] Aoki Y, Endrodi G, Fodor Z, Katz S D and Szabo K K 2006 The order of the quantum chromodynamics transition predicted by the standard model of particle physics *Nature* 443 675-8
[2] Bazavov A et al 2012 The chiral and deconfinement aspects of the QCD transition *Phys. Rev. D* 85 054503
[3] Borsanyi S, Fodor Z, Hoelbling C, Katz S D, Krieg S, Ratti C and Szabo K K 2010 Is there still any $T_c$ mystery in lattice QCD? Results with physical masses in the continuum limit III *J. High Energy Phys.* JHEP09(2010)073
[4] Asakawa M, Heinz U W and Müller B 2000 Fluctuation probes of quark confinement *Phys. Rev. Lett.* 85 2072-5
[5] Ejiri S, Karsch F and Redlich K 2006 Hadronic fluctuations at the QCD phase transition *Phys. Lett.* B 633 275-82
[6] Borsanyi S, Fodor Z, Katz S D, Krieg S, Ratti C and Szabo K K 2012 Fluctuations of conserved charges at finite temperature from lattice QCD *J. High Energy Phys.* JHEP01(2012)138
[7] Ratti C, Bellwied R, Christoforetti M and Barbaro M 2012 Are there hadronic bound states above the QCD transition temperature? *Phys. Rev. D* 85 014004
[8] Bellwied R, Borsanyi S, Fodor Z, Katz S D and Ratti C 2013 Is there a flavor hierarchy in the deconfinement transition of QCD? *Preprint* arXiv:1305.6297 [hep-lat]
[9] Bazavov A et al 2013 Strangeness at high temperatures: from hadrons to quarks *Phys. Rev. Lett.* 111 082301
[10] Cheng M et al 2009 Baryon number, strangeness and electric charge fluctuations in QCD at high temperature *Phys. Rev. D* 79 074505
[11] Karsch F and Redlich K 2011 Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations *Phys. Lett.* B 695 136-42
[12] Alba P, Alberico W M, Bluhm M, Ratti C and Bellwied R 2013 Flavor hierarchy in the confinement transition of QCD *Proc. of Science* PoS(CPOD2013)060