Adaptive finite-time consensus tracking control for coopetition flexible joint multi-manipulators with full-state constraints

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ABSTRACT In this paper, the adaptive full-state constraints bipartite consensus tracking problem for coopetition flexible joint multi-manipulator systems with finite-time convergence is investigated. Based on the finite-time convergent method, the command filtered backstepping technique is introduced to ensure the excellent convergence performance without the influence of the repeated differential problem which is called explosion of complexity, and the error compensation mechanism is proposed to reduce the error caused by the filtering progress. Furthermore, by using the barrier Lyapunov functions, the state of the system and error compensation signals are proved to be constrained in their respective expected ranges. It is worth mentioning that the flexible joint multi manipulator system is in the mode of coopetition. Finally, the practicability of the proposed algorithm is exhibited in the simulation.

INDEX TERMS Command filtered backstepping, Flexible joint multi-manipulator, Full-state constraints, Finite time convergence, Coopetition

I. INTRODUCTION

In the application of practical engineer, the trajectory tracking control method of robotic manipulator has always been an enduring theme, since it can involve many fields that cannot be solved by manpower. The researches of rigid links and rigid joints satisfy a part of the control requirements [1], however without considering the harmonic gear transmission and joint torque sensor, this kind of assumption is insufficient. In order to simulate the actual system more accurately, the flexible joint manipulators (FJMs) has become a key topic for researchers. The control schemes in [2], [3] design the backstepping method to FJMs for high order system creatively, further more complex but also closer to practical application conditions, such like deadzone [4] and output constraints [5], are taken into account due to the features of the nonlinear, friction, flexibility and strong coupling. However, only the single manipulator system are considered in the above studies, and the requirements of flexible joint multi-manipulators system (FJMMS) in practice are ignored.

To achieve the function of sort, assembly, allocation and so on, the FJMMS based on multi-agent systems (MASs) are apply to industrial assembly line widely. The studies [6], [7] show the solution to the problem of multi-manipulators in the early stage, but they only apply to the condition of rigid joints and rigid links. In fact, the FJMMS need to apply the processing method of high-order system, therefore we should shift the focus of our research on the MASs. The theoretical development of MASs has become more and more mature, especially for leader-following system [8], [9], since it provides a clear convergence goal, so that the stability and energy saving of the system are significantly improved. Backstepping is a appropriate solution to the high-order system, which becomes a key to study unknown dynamics after combining with the adaptive method [10]–[12]. However, the explosion of complexity (EOC) is a following problem due to the demand of using the states as the virtual input signals, and each step in backstepping requiring the derivation of virtual signal. Therefore, the dynamic surface control (DSC) is proposed as a response to eliminate the defective influence of EOC [13]–[15], but the error caused by the introduction of filtering process is not properly handled. The command filtered backstepping (CFB) method is adopted for its more effective characteristics, since the advantages of DSC are
retained and the filtering error is compensated at the same time [16]–[18]. But in our opinions, to match the FJMMS better, the control characteristics considered in the above article are still insufficient.

In the practical physical systems, the engineers always pursue quick responding, precise tracking and strong anti-disturbance. To satisfy these requirements, the finite-time control method is applied widely [19]–[21]. On the other hand, in the actual robot control system, due to the limitation of work environment and some technical indicators, such as position, velocity, voltage, researchers always need to give an upper bound to ensure the safety performance, so that the topic of state constraints is proposed [22]–[24]. Barrier Lyapunov functions (BLFs) is an effective technique to study this kind of problem, since it can constrain the control signal through the mathematical properties of the signal function, for example, the logarithmic and tangent function. Combined with adaptive neural network, the authors in [37] investigate the integral BLF to implement joint space constraints, but only apply to single robotic manipulator. The consensus tracking problem of time-varying joint with tangent-type BLF and input saturation is studied in [38]. However, the above control schemes are only apply to single robotic manipulator without finite-time convergence. What’s more, in some requirements of confrontation and negation, the cooption system can play its excellent role [25]–[27]. The CFB finite-time method combined with full-state constraints for MASs is investigated in [28], [29], but they do not consider the cooption system and manipulator. On the contrary, the authors in [30]–[32] solve the bipartite consensus tracking problem for cooperation MASs, but the condition of finite-time convergent and states constraints are not handled. The problem for FJMs with output constraints is solved in [33], while the research for FJM based on backstepping control method is designed in [34], [35], but only adopt to single manipulator asymptotically convergent system. Up to now, to our best knowledge, for cooperation FJMMS consisted of full-state constraints and finite-time characteristic, the research of control method still needs to be improved urgently.

Synthesize all the viewpoints in the above discussions and directions, we will propose a proper CFB control law for cooperation FJMMS with finite-time convergent and full-states constraints. In this article, the control purpose is to make the cooperative manipulators in the FJMMS effectively track the desired trajectory, and the competitive manipulators track the desired trajectory in the reverse direction. The explosion of complexity problem is eliminated by the second order sliding mode differentiator, and the error compensation signals is designed to compensate the error in filtering process. The barrier Lyapunov function is adopt to limit the range of state changes for cooperation system, and the performance of finite-time convergent is guaranteed by virtual control signal based on exponential algorithm. The summaries of the proposed research is shown as follow:

1) Compared with the studies for single-maneupulator in [1]–[5] and the backstepping methods for MASs in [10]–[18], we establish a CFB control law for FJMMS, so that the mature technology for MASs is applied to multi-manipulators. It is worth mentioning that the proposed cooperation system has the character of bipartition, which means the manipulators can track the same object with opposite direction.

2) Compared with the performance of asymptotically convergent based on the traditional backstepping methods in [10]–[12], the DSC methods in [13]–[15] and the CFB methods in [16]–[18], the finite-time convergent CFB algorithm is adopt in this paper to achieve the advantages of responding more accurate and faster, and the problem of EOC and filtering error is solved proper. Besides, the barrier Lyapunov function is applied to give the regions for the system states, so that the full-state constraint problem, which is better than the output constraint [5], [22] performance, has been further improved.

II. SYSTEM BASIC PRINCIPLE

A. GRAPH THEORY

This paper investigates a FJMMS which composed of one expected tracking signal as leader and N manipulator as followers. For convenience of description, we introduce the knowledge of graph theory to show the relationship between the manipulators.

The way of communication between followers can be described as $G = (V, E, A)$, where $V = [1, 2, \ldots, N]$ represents the set of node, $E \subseteq V \times V$ represents the set of edges, the adjacency matrix of $G(i, j) \in \mathcal{E}$ is named $A$, which represents that $i$ can receive information about $j$, in other words, there is an edge from $j$ to $i$, and $N_j = \{j \mid (j, i) \in E\}$ represents the neighbor set of $i$. $A = [a_{i,j}] \in R^{N \times N}$ with $a_{i,j} \neq 0$ for $(j, i) \in E$, $a_{i,j} = 0$ for $(j, i) \notin E$, and $a_{i,i} = 0$ for $(i, i) \in V$. The case of $a_{i,j}a_{j,i} < 0$ is not included in this article. We denoted the direct path from node $i$ to $j$ that an ordered set of edges transform information as $\{(i, k), (k, l), \ldots, (m, j)\}$, then a directed graph with a spanning tree is defined as the case where $i$ is the root node, $j$ is any node, and there is a direct path from $i$ to $j$. The Laplacian matrix $L$ is denoted as $L = D - A$, where $D = \text{diag}\{d_1, d_2, \ldots, d_N\}$, $d_i = \sum_{j=1}^{N} a_{i,j}$, $i = 1, 2, \ldots, N$. The system includes one leader and N followers, which this leader is defined as 0, that is to say the graph is $G = (\tilde{V}, \tilde{E})$. $\tilde{V}$ is defined as $\tilde{V} = \mu \cup \{0\}$, and $\tilde{E}$ is defined as $\tilde{V} \times \tilde{V}$. $B$ is defined as $B = \text{diag}\{b_1, b_2, \ldots, b_N\}$, if there is a link from leader 0 to the $j$, then $b_i > 0$, otherwise $b_i = 0$. We define the matrix $H$ as $H = L + B$. If $G$ is bisected $\{V_1, V_2\}$ that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = \mathcal{V}$ and $a_{i,j} \leq 0$ for $\forall j \in V_c$, $j \in V_d$, $c \neq d$, $c \in \{1, 2\}$, $a_{i,j} \geq 0$ for $\forall i, j \in V_c(i, c \in \{1, 2\})$, $G$ is called structural balance.

This article assumes that $G$ is structurally balanced. We define a matrix $\Xi = \text{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$, when $i \in V_1$, $\xi_i = 1$, and when $i \in V_2$, $\xi_i = -1$. This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2022.3158972, IEEE Access

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B. MANIPULATOR DYNAMIC DESCRIPTION

The model for FJMMS is given as follows:

\[
\begin{align*}
N_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) + F_i\dot{q}_i + \hat{K}_i(q_i - \bar{q}_i) &= 0 \\
y_i &= \bar{q}_i
\end{align*}
\]

(1)

where \( x_{i,s} = [x_{i,s,1}, x_{i,s,2}, \cdots, x_{i,s,n}]^T \), all \( x_{i,s,p} \) must satisfy \( |x_{i,s,1}| \leq k_{i,s,q} \) and \( k_{i,s,q} > 0, i = 1, \cdots, N, s = 1, \cdots, 4, p = 1, \cdots, n \). Further define \( \bar{x}_i = [x_{i,1,1}, \cdots, x_{i,n}]^T \in \mathbb{R}^n \) as the desired trajectory to be tracked, where \( \bar{x}_i \) and \( \dot{x}_i \) are both smooth and bounded known signals, and must satisfy \( |\bar{x}_i| \leq r \leq \bar{l}_{i,1} \).

The control task of this article is to make the end-effectors of the cooperative manipulators track the expected trajectory, and end-effectors the competitive manipulators track the trajectory opposite to the expected trajectory.

Assumption 1: The proposed FJMMS can be regarded as strongly connected, and information from the leader manipulator can be communicated to any follower manipulator through the spanning tree.

Assumption 2: During the coordinated movement, the manipulator will not enter any single configuration.

Remark 2: These conditions are relatively common in robot dynamics control. But in actual control, Assumptions 1 and 2 may indeed vary from device to device due to communication problems. If these assumptions cannot be satisfied in some special cases, the event-triggered strategy in [39] can be adopted. But these assumptions are generally considered satisfied in the academic study of robot dynamics [6].

C. SOME LEMMAS

Lemma 1: [20] For any positive scalar \( a, b \) and \( a > 0, b > 0 \), then

\[
|q|^a |p|^b \leq \frac{a\varphi(q,p)}{a + b} + \frac{b\varphi(q,p) - \bar{\varphi}}{a + b}|q|^a + |p|^b
\]

(5)

where \( \varphi(q,p) \) is a real valued function and \( \varphi(q,p) > 0 \).

Lemma 2: [23] When the establishment of Assumption 1 achieves, then all eigenvalues of matrix \( H \) have positive real parts, which is defined as

\[
H = D - A + B
\]

(6)

Lemma 3: [20] For \( u_i \in \mathbb{R}^N, i = 1, 2, \cdots, N, \) if \( 0 < n \leq 1 \), then

\[
\left( \sum_{s=1}^{N} |u_i|^n \right)^n \leq \sum_{s=1}^{N} |u_i|^n \leq N^{1-n} \left( \sum_{s=1}^{N} |u_i|^n \right)^n
\]

(7)

Lemma 4: [22] For any \( \delta > 0 \), if \( \omega \in \mathbb{R} \) exists and satisfies \( |\omega| < \delta \), then the following inequality holds

\[
\log \frac{\delta^2}{\delta^2 - \omega^2} \leq \frac{\omega^2}{\delta^2 - \omega^2}
\]

(8)

Lemma 5: [19] \( \forall k_1, k_2, k_3 \in R, \) if \( k_1 > 0, k_2 > 0, 0 < k_3 < 1 \) and satisfies \( V + k_1 V + k_2 V^2 \leq 0 \) where \( V(t) \) represents a continuous function, then in a period of finite time

\[
T \leq t_0 + \frac{1}{k_1 (1 - k_3)} \log [1 + k_1 k_2^{1-k_3} (t_0)]
\]

(9)

\( V(t) \) will converge into the equilibrium point, where \( t_0 \) is the initial time.
Lemma 6: [10] For a continuous function $\Lambda(V) \in R^n$ and it is restricted to a compact set $\Gamma_V \subset R^n$, then the fuzzy logic system $\Omega^T Y(V)$ satisfies

$$|\Lambda(V) - \Omega^T Y(V)| \leq \nu_0$$

where $\nu_0 > 0$ is the approximation error, $\Omega = [s_1, s_2, \cdots, s_r]^T \in R^r$, $\epsilon > 0$ is the constant weight vector in ideal condition,

$$Y(V) = \left[\begin{array}{c} v_1(V), v_2(V), \cdots, v_r(V) \end{array}\right]^T$$

represents the basis function vector of the

$$v_m(V) = \exp \left[\frac{(V - \theta_m)^T (V - \theta_m)}{\hat{\theta}^2} \right]$$

where $\theta_m = [\theta_{1,m}, \theta_{2,m}, \cdots, \theta_{r,m}]^T$ represents the center and $\hat{\theta}$ represents the width, $i = 1, 2, \cdots, r$.

III. PROOF PROCESS

A. BACKSTEPPING PROCESS DESIGN

The finite-time bipartite output tracking are as follows:

$$\lim_{t \to T} x_i = r, i \in \gamma_1$$

$$\lim_{t \to T} x_i = -r, i \in \gamma_2, i = 1, 2, \cdots, N$$

Denote the output consensus tracking errors

$$f_{i,1} = \sum_{j=1}^{N} \left[ x_{i,1} - \text{sign} \left( a_{i,j} \right) x_{j,1} \right] + b_i (x_{i,1} - \varepsilon_i r)$$

$$f_{i,s} = x_{i,s} - z_{i,s}, s = 2, 3, 4$$

where $z_{i,p} = [\zeta_{i,s,1}, \cdots, \zeta_{i,s,n}]^T$, $p = 2, 3, 4$, $j$ represents other robotic arms related to $i$. The output of the following second-order finite-time sliding mode differentiator is given as [36]:

$$\dot{\zeta}_{i,s,1,p} = \dot{\theta}_{i,s,p}$$

$$\dot{\theta}_{i,s,1,p} = -\gamma_{i,s,1,p} [\zeta_{i,s,1,p} - \delta_{i,s-1,p}] + \frac{1}{\hat{\theta}} \text{sign} (\zeta_{i,s,1,p} - \delta_{i,s-1,p}) + \zeta_{i,s,2,p}$$

$$\dot{\zeta}_{i,s,2,p} = -\gamma_{i,s,2,p} \text{sign} (\zeta_{i,s,2,p} - \theta_{i,s,p})$$

$$i = 1, \cdots, N, s = 2, 3, 4, p = 1, \cdots, n$$

where $\delta_{i,s-1} = [\delta_{i,s-1,1}, \cdots, \delta_{i,s-1,n}]^T$ is the virtual control signal as the input of the system.

Lemma 7: [36] When the values of $\gamma_{i,s,1,p}$ and $\gamma_{i,s,2,p}$ are selected properly, the following equation holds.

$$\zeta_{i,s,1,p} = \delta_{i,s-1,p} \theta_{i,s,p} = \hat{\delta}_{i,s-1,p}$$

The filtering error $z_{i,s+1} - \delta_{i,s}$ will be defined in the introduction of the command filter. In order to eliminate this error from the command filter, the following error compensation mechanism is used:

$$\dot{h}_{i,1} = -\rho_{i,1} h_{i,1} + (d_i + b_i)(z_{i,2} - \delta_{i,1} + h_{i,2})$$

$$\dot{h}_{i,2} = -\rho_{i,2} h_{i,2} + w_{i,2}(z_{i,3} - \delta_{i,2} + h_{i,3})$$

$$\dot{h}_{i,3} = -\rho_{i,3} h_{i,3} + z_{i,4} - \delta_{i,3} + h_{i,4} - \xi_{i,3} F_{i,3}$$

$$\dot{h}_{i,4} = -\rho_{i,4} h_{i,4} - \xi_{i,4} F_{i,4}$$

where $\rho_{i,s}$, $w_{i,2}$, $w_{i,3}$ are all positive constants, $\xi_{i,s}, \mu_{i,s} = 0$ are defined as proportional gain, $F_{i,s} = [h_{i,s,1}, \cdots, h_{i,s,n}]^T$, $s = 1, \cdots, 4$.

The weight vector $\tau_{i,s,p}, s = 2, 4, p = 1, \cdots, n$ is unknown. In order to approximate the weight vector, $\varphi = \max \{\|\tau_{i,s,p}\|^2\}$ is selected, the adaptive update law is determined by (18).

$$\dot{\varphi}_i = -2 \lambda_i \lambda_i \varphi_i + \sum_{p=1}^{n} \left( \frac{1}{2c_{i,p}^2} \lambda_i K_{i,2,s,s} \rho_{i,2,s} \rho_{i,2,s} + \frac{1}{2c_{i,s}^2} \lambda_i K_{i,4,s,s} \rho_{i,4,s} \rho_{i,4,s} \right)$$

where $\lambda_i, \lambda_i, c_{i,s}^2$ and $c_{i,s}^2$ are positive constant, $\rho_{i,2,s}, \rho_{i,4,s}$ are basis function vectors.

For convenience of description, denote

$$K_i, s, p = \frac{\omega_i, s, T I_{p}}{k_i, s, p k_i, s, - \omega_i, s, I_{p} \omega_i, s}$$

$$k_i, s = [k_{i,1,s}, k_{i,2,s}, \cdots, k_{i,n,s}]^T$$

$$\omega_i, s = [\omega_{i,1,s}, \omega_{i,2,s}, \cdots, \omega_{i,n,s}]^T$$

$$I_1 = \text{diag}(1, 0, \cdots, 0)$$

$$I_2 = \text{diag}(0, 1, \cdots, 0)$$

$$\vdots$$

$$I_n = \text{diag}(0, 0, \cdots, 1)$$

$s = 1, 2, 3, 4, i = 1, \cdots, N$

where $k_i, s$ are the upper boundaries for $\omega_i, s$. From (19) we can get:

$$K_{i,1,s} = [k_{i,1,s}, 0, \cdots, 0]$$

$$K_{i,2,s} = [0, k_{i,2,s}, 0, \cdots, 0]$$

$$\vdots$$

$$K_{i,n,s} = [0, 0, \cdots, K_{i,n,s}]$$
The virtual control signals are defined as:

\[
\delta_{i,1} = \frac{1}{d_i + b_i} \left( - \pi_{i,1} f_{i,1} + b_i \varepsilon_i \dot{r} - \phi_{i,1} S_{i,1} \right)
- \frac{\xi_{i,1} M_{i,1}}{\alpha + 1} + \sum_{j=1}^{N} a_{i,j} x_{j,2}
\]

\[
\delta_{i,2} = \frac{1}{w_{i,2}} \left[ - \pi_{i,2} f_{i,2} + \dot{z}_{i,2} - \frac{1}{2c_i^2} Q_{i,2} - \frac{1}{2} R_{i,2}
- \phi_{i,2} S_{i,2} - \frac{\xi_{i,2} M_{i,2}}{\alpha + 1} - (d_i + b_i) P_{i,2} \right]
\]

\[
\delta_{i,3} = - \pi_{i,3} f_{i,3} + \dot{z}_{i,3} - g_{i,3} P_{i,3} - \phi_{i,3} S_{i,3} - \frac{\xi_{i,3} M_{i,3}}{\alpha + 1}
\]

\[
\delta_{i,4} = \frac{1}{w_{i,4}} \left( - \pi_{i,4} f_{i,4} + \dot{z}_{i,4} - P_{i,4} - \frac{1}{2c_i^2} Q_{i,4}
- \frac{1}{2} R_{i,4} - \phi_{i,4} S_{i,4} - \frac{\xi_{i,4} M_{i,4}}{\alpha + 1} \right)
\]

(21)

where

\[
S_{i,s} = \left[ K_{i,s,1,1}^{\frac{\alpha+1}{2}} \omega_{i,s,1,1}, \cdots, K_{i,s,n,n}^{\frac{\alpha+1}{2}} \omega_{i,s,n,n} \right]^T, i = 1, 2, 3, 4
\]

\[
M_{i,s} = \left[ K_{i,s,1,1}, \cdots, K_{i,s,n,n} \right]^T, i = 1, 2, 3, 4
\]

\[
P_{i,s} = \left[ K_{i,s-1,1}, \omega_{i,s,1,1}, \cdots, \frac{K_{i,s-1,n,n}}{K_{i,s,n,n}} \omega_{i,s,n,n} \right]^T, s = 2, 3, 4
\]

\[
Q_{i,s} = \left[ -K_{i,s,1,1} \beta_{i,s,1,1} \rho_{i,s,1,1}, \cdots, -K_{i,s,n,n} \beta_{i,s,n,n} \rho_{i,s,n,n} \right]^T, s = 2, 4
\]

\[
R_{i,s} = \left[ K_{i,s,1,1}, \cdots, K_{i,s,n,n} \right]^T, s = 2, 4
\]

and the compensated error tracking signal \( \omega_{i,s} \) are as follows:

\[
\omega_{i,s} = f_{i,s} - h_{i,s}, s = 1, 2, 3, 4
\]

(23)

Define the compact set \( \Omega_{\omega_{i,s}} \) as \( \{ |\omega_{i,s} | < k_{i,s} \} \), and the designed backstepping control process is given as follow.

**Step 1: Construct the Lyapunov function as**

\[
W_{i,1} = \frac{1}{2} \sum_{p=1}^{n} \log \frac{K_{i,1,p}^{T} p_{i,1,p}}{K_{i,2,p}^{2} - \omega_{i,1,p}^{2}}
\]

(24)

We can get

\[
W_{i,1} = \frac{1}{2} \sum_{s=1}^{n} \log \frac{k_{i,s,1}^{T} f_{i,s} k_{i,s,1}}{-\omega_{i,1}^{T} f_{i,s} \omega_{i,1}}
\]

(25)

Derivation of (25) can be obtained

\[
\dot{W}_{i,1} = \sum_{p=1}^{n} K_{i,1,p} \left( \dot{p}_{i,1,p} \right)
= \sum_{p=1}^{n} \left\{ \sum_{j=1}^{N} \left| a_{i,j} \right| \left[ \dot{x}_{i,j} - \text{sign}(a_{i,j}) \dot{x}_{j,1} \right] \right\}
+ b_{i} \left( \dot{x}_{i,1} - \varepsilon \dot{r} \right) - h_{i,1}
\]

(26)

substitute \( \delta_{i,1} \) and \( h_{i,1} \) into formula (26) to get

\[
\dot{W}_{i,1} = \sum_{p=1}^{n} K_{i,1,p} \left[ (d_i + b_i) \dot{x}_{i,1} - \sum_{j=1}^{N} a_{i,j} \dot{x}_{j,1} \right]
- b_{i} \varepsilon \dot{r} + \pi_{i,1} h_{i,1} - (d_i + b_i) (\dot{z}_{i,2} - \delta_{i,1})
+ h_{i,2} + \xi_{i,1} F_{i,1}
\]

\[
= \sum_{p=1}^{n} K_{i,1,p} \left[ (d_i + b_i) \dot{x}_{i,2} - \sum_{j=1}^{N} a_{i,j} \dot{x}_{j,2} \right]
- b_{i} \varepsilon \dot{r} + \pi_{i,1} h_{i,1} - (d_i + b_i) (\dot{z}_{i,2} + h_{i,2})
- \pi_{i,1} F_{i,1} + b_{i} \varepsilon \dot{r} - \phi_{i,1} S_{i,1}
- \frac{\xi_{i,1}}{\alpha + 1} M_{i,1} + \sum_{j=1}^{N} a_{i,j} \dot{x}_{j,2} + \xi_{i,1} F_{i,1}
\]

\[
= \sum_{p=1}^{n} K_{i,1,p} \left[ (d_i + b_i) \omega_{i,2} - \pi_{i,1} \omega_{i,1} - \phi_{i,1} S_{i,1}
- \frac{\xi_{i,1}}{\alpha + 1} M_{i,1} + \xi_{i,1} F_{i,1} \right]
\]

(27)

Using Lemma 2 is not difficult to get

\[
\xi_{i,1} K_{i,1,p} h_{i,1}^{\alpha} \leq \frac{\xi_{i,1}}{\alpha + 1} M_{i,1} + \frac{\xi_{i,1} \alpha}{\alpha + 1} h_{i,1}^{\alpha + 1}
\]

(28)

substitute (28) into (27) to get

\[
\dot{W}_{i,1} \leq \sum_{p=1}^{n} \left[ - \pi_{i,1} \omega_{i,1} K_{i,1,p} + (d_i + b_i) K_{i,1,p} \omega_{i,2}
+ \xi_{i,1} \alpha \frac{\xi_{i,1} h_{i,1}^{\alpha + 1}}{\alpha + 1} \right]
\]

(29)

**Step 2:** Construct the Lyapunov function as

\[
W_{i,2} = W_{i,1} + \frac{1}{2} \sum_{p=1}^{n} \log \frac{K_{i,2,p}^{2}}{K_{i,2,p}^{2} - \omega_{i,2,p}^{2}}
\]

(30)

Derivation of (30) can be obtained

\[
\dot{W}_{i,2} = \dot{W}_{i,1} + \sum_{p=1}^{n} K_{i,2,p} \omega_{i,2} \dot{\omega}_{i,2}
\]

\[
= \dot{W}_{i,1} + \sum_{p=1}^{n} K_{i,2,p} \left[ c_{i,2} + w_{i,2} x_{i,3} - \dot{z}_{i,2}
+ \pi_{i,2} h_{i,2} - w_{i,2} (\dot{z}_{i,3} - \delta_{i,2} + h_{i,3})
+ \xi_{i,2} F_{i,2} \right]
\]
where $\delta_{i,2}$ contains uncertainty, we use the fuzzy logic system to approximate it, then $e_{i,2,p}, p = 1, \ldots, n$ can be approximately expressed as:

$$e_{i,2,p} = \gamma_{i,2,p}^{\top} \hat{\theta}_{i,2,p} + \tilde{e}_{i,2,p}$$  \hspace{1cm} (32)

Substitute $\delta_{i,2}$ and $\hat{h}_{i,2}$ into formula (31) to get

$$\dot{W}_{i,2} = W_{i,1} + \sum_{p=1}^{n} K_{i,2,p} \left[ \dot{x}_{i,2} - \dot{\tilde{z}}_{i,2} + \pi_{i,2} \dot{h}_{i,2} \right.
\left. - w_{i,2}(\zeta_{i,3} + h_{i,3}) - \pi_{i,2} f_{i,2} + \dot{\tilde{z}}_{i,2} - \frac{Q_{i,2}}{2c_{i,2}^{2}} \right.
\left. - \frac{R_{i,2}}{2} - \phi_{i,2}(\dot{S}_{i,2} - \frac{\xi_{i,2}}{\alpha + 1} M_{i,2} + \xi_{i,2} F_{i,2} \right.
\left. - (d_{i} + b_{i}) \rho_{i,2} \right] \hspace{1cm} (34)

Substituting (34) to get

$$\dot{W}_{i,2} \leq \dot{W}_{i,1} + \sum_{p=1}^{n} \left[ w_{i,2} K_{i,2,p} \dot{\omega}_{i,3,p} - \pi_{i,2} K_{i,2,p} \dot{\omega}_{i,2,p} \right.
\left. - K_{i,1,p} \dot{\omega}_{i,2,p} - \phi_{i,2} K_{i,2,p} \dot{\omega}_{i,2,p} \right]
\left. + \frac{Q_{i,2}}{2c_{i,2}^{2}} \dot{\varphi}_{i,2}^{\top} \dot{\rho}_{i,2,p} \rho_{i,2,p} - \frac{1}{2} R_{i,2} \right.
\left. - \phi_{i,2} K_{i,2,p} \dot{\omega}_{i,2,p} + \frac{\xi_{i,2}}{\alpha + 1} \dot{h}_{i,2,p} + \frac{\xi_{i,2}}{\alpha + 1} \dot{h}_{i,2,p} \right] \hspace{1cm} (35)

**Step 3:** Construct the Lyapunov function as

$$W_{i,3} = W_{i,2} + \frac{1}{2} \sum_{p=1}^{n} \log \frac{K_{i,3,p}^{2}}{K_{i,3,p}^{2}} \omega_{i,3,p}^{2}$$  \hspace{1cm} (36)

Similar to (26)-(29), we get:

$$W_{i,3} \leq \sum_{p=1}^{n} \left[ \sum_{m=1}^{n} \left( - \pi_{i,m} K_{i,m,p} \dot{\omega}_{i,m} + \frac{\xi_{i,m}}{\alpha + 1} h_{i,m,p} \right) \right.
\left. - \phi_{i,m} K_{i,m,p} \dot{\omega}_{i,m} + \frac{\xi_{i,m}}{\alpha + 1} h_{i,m,p} \right]
\left. + K_{i,3,p} \dot{\omega}_{i,3,p} \right]
\left. + \frac{\eta_{i,2}^{2} + \xi_{i,2}^{2}}{2} + \frac{K_{i,2,p}^{2}}{2c_{i,2}^{2}} \left( \|\tau_{i,2,p}\|^{2} - \dot{\varphi}_{i}^{\top} \rho_{i,2,p} \rho_{i,2,p} \right) \right] \hspace{1cm} (37)

**Step 4:** Construct the Lyapunov function as

$$W_{i,4} = W_{i,3} + \frac{1}{2} \sum_{p=1}^{n} \log \frac{K_{i,4,p}^{2}}{K_{i,4,p}^{2}} \omega_{i,4,p}^{2}$$  \hspace{1cm} (38)

Similar to (31)-(35) and $u_{i} = \delta_{i,4}$, we get:

$$W_{i,4} \leq \sum_{p=1}^{n} \left[ \sum_{s=1}^{n} \left( - \pi_{i,s} K_{i,s,p} \dot{\omega}_{i,s} + \frac{\xi_{i,s}}{\alpha + 1} h_{i,s,p} \right)
\left. - \phi_{i,s} K_{i,s,p} \dot{\omega}_{i,s} + \frac{\xi_{i,s}}{\alpha + 1} h_{i,s,p} \right]
\left. + K_{i,4,p} \dot{\omega}_{i,4,p} + \frac{\eta_{i,2}^{2} + \xi_{i,2}^{2}}{2} + \frac{K_{i,2,p}^{2}}{2c_{i,2}^{2}} \dot{\varphi}_{i,s}^{\top} \dot{\rho}_{i,2,p} \rho_{i,2,p} \right] \hspace{1cm} (39)

where $\dot{\omega}_{i,m} = \|\tau_{i,m,p}\|^{2} - \dot{\varphi}_{i,m}, m = 2, 4$.

**Remark 3:** In order to ensure the Lyapunov function $W_{i,4}$ is positive definite, we assume a domain of definition $\Omega_{\omega_{i,s,p}}$, for compensated tracking error signal $\omega_{i,s,p}$, which is a general premise for barrier Lyapunov function in [22].

**B. THE PROOF OF STABILITY**

**Theorem 1:** For the FJMMS (2) that satisfies the assumptions 1-2, this paper is based on the command filter (15), and selects the designed virtual control signal (21), adaptive law (18) and error compensation mechanism (17), which can make the tracking error (14) convergence to a small neighborhood containing the origin in finite time, satisfying the full state constraints of the system.

**Proof:**

Define the Lyapunov function for (18) as

$$\dot{W} = \frac{1}{2} \sum_{s=1}^{n} \sum_{i=1}^{N} h_{i,s}^{\top} h_{i,s}$$  \hspace{1cm} (40)
Taking the derivative of (40), we get

\[
\dot{W} = \sum_{s=1}^{4} \sum_{i=1}^{N} h_{i,s}^{T} \dot{h}_{i,s} \\
= \sum_{i=1}^{N} \left[ \left( h_{i,1}^{T} \dot{h}_{i,1} + h_{i,2}^{T} \dot{h}_{i,2} + h_{i,3}^{T} \dot{h}_{i,3} + h_{i,4}^{T} \dot{h}_{i,4} \right) \\
+ (d_i + b_i) h_{i,1}^{T} \dot{h}_{i,1} + w_i \dot{h}_{i,2} + \sum_{p=1}^{n} \xi_{i,s} \dot{\phi}_{i,s,p} \right] \\
+ \left( \sum_{s=1}^{4} \sum_{i=1}^{N} \left( -\pi_{i,s} h_{i,s}^{T} h_{i,s} - \sum_{p=1}^{n} \xi_{i,s} \dot{\phi}_{i,s,p} \right) \right) \\
+ \sum_{i=1}^{N} \left( \sum_{s=1}^{4} \left( \frac{d_i + b_i}{2} \chi_{i,1} (h_{i,1}^{T} \dot{h}_{i,1} + h_{i,2}^{T} \dot{h}_{i,2}) \\
+ \frac{w_i}{2} \chi_{i,2} (h_{i,2}^{T} \dot{h}_{i,2} + h_{i,3}^{T} \dot{h}_{i,3}) \\
+ \chi_{i,3} (h_{i,3}^{T} \dot{h}_{i,3} + h_{i,4}^{T} \dot{h}_{i,4}) \right) \right) \\
= \frac{1}{2} \sum_{s=1}^{4} \sum_{i=1}^{N} \left[ (d_i + b_i) \chi_{i,1} (2 - 2\pi_{i,1}) h_{i,1}^{T} \dot{h}_{i,1} \\
+ \left( w_i \chi_{i,2} + (\chi_{i,2} - 2\pi_{i,2}) \right) h_{i,2}^{T} \dot{h}_{i,2} \\
+ \left( w_i \chi_{i,3} + (\chi_{i,3} - 2\pi_{i,3}) \right) h_{i,3}^{T} \dot{h}_{i,3} \\
+ \left( \chi_{i,3} - 2\pi_{i,4} \right) h_{i,4}^{T} \dot{h}_{i,4} - \sum_{p=1}^{2} \xi_{i,s} \dot{\phi}_{i,s,p} \right) \\
\leq \frac{1}{2} \sum_{i=1}^{N} \sum_{s=1}^{4} \left( \dot{\chi}_{i,s} h_{i,s}^{T} h_{i,s} - \sum_{p=1}^{n} \xi_{i,s} \dot{\phi}_{i,s,p} \right) \\
\leq -\dot{\chi}_{\text{min}} W - \sum_{i=1}^{N} \sum_{s=1}^{4} \sum_{p=1}^{n} \xi_{i,s} \dot{\phi}_{i,s,p} \\
\text{where} \\
\dot{\chi}_{i,s} = \min \left[ 2\pi_{i,1} - (d_i + b_i) \chi_{i,1} \pi_{i,1}, \\
2\pi_{i,2} - (d_i + b_i) \chi_{i,1} + w_i \chi_{i,2}, \\
2\pi_{i,3} - w_i \chi_{i,2} + \chi_{i,3} - 2\pi_{i,4} - \chi_{i,3} \right] \\
\dot{\chi}_{\text{min}} = \min \left[ \dot{\chi}_{1,1}, \dot{\chi}_{2,1}, \dot{\chi}_{3,1}, \dot{\chi}_{4,1} \right] \\
\text{It is worth noting that} 2\pi_{i,1} - (d_i + b_i) \chi_{i,1} \pi_{i,1}, 2\pi_{i,2} - (d_i + b_i) \chi_{i,1} + w_i \chi_{i,2}, \\
2\pi_{i,3} - w_i \chi_{i,2} + \chi_{i,3} - 2\pi_{i,4} - \chi_{i,3} \text{ and} 2\pi_{i,4} - \chi_{i,3} \text{ in} (44) \text{ are all positive constants.}
\]

Due to the reason of \( f_{i,s} = \omega_{i,s} + h_{i,s} \), if \( f_{i,s} \) must meet the condition of converging to the desired area in a finite time, then \( \omega_{i,s} \) and \( h_{i,s} \) must be required to converge to the desired area in a finite time. Therefore, we add \( \dot{\varphi} \) to the Lyapunov function of the system, and the global Lyapunov function is shown in (45):

\[
W = \sum_{i=1}^{N} W_i + \dot{W} + \sum_{i=1}^{N} 2\beta_i \dot{\varphi}_i^2 \tag{45}
\]

Derivation of (45) and substituting (18) can be obtained:

\[
\dot{W} = \sum_{i=1}^{N} \dot{W}_{i,4} + \dot{W} - \sum_{i=1}^{N} \frac{1}{\beta_i} \dot{\varphi}_i \dot{\varphi}_i \\
\leq \sum_{i=1}^{N} \sum_{s=1}^{4} \left( -\pi_{i,s} \dot{K}_{i,s,p} \omega_{i,s} + \frac{\alpha \xi_{i,s}}{\alpha + 1} h_{i,s}^{T} \dot{h}_{i,s} \\
- \dot{\phi}_{i,s} \dot{K}_{i,s,p} \omega_{i,s} + \frac{\alpha \xi_{i,s}}{\alpha + 1} h_{i,s}^{T} \dot{h}_{i,s} \right) \right) - \sum_{i=1}^{N} 2\beta_i \dot{\varphi}_i \dot{\varphi}_i \\
+ \sum_{i=1}^{N} \left( c_{i,2}^2 + \eta_{i,2}^2 + c_{i,4}^2 + \eta_{i,4}^2 \right) - \dot{\chi}_{\text{min}} \dot{W} \\ 
= \sum_{i=1}^{N} \sum_{s=1}^{4} \sum_{p=1}^{n} \left( -\pi_{i,s} \dot{K}_{i,s,p} \omega_{i,s} + \frac{\alpha \xi_{i,s}}{\alpha + 1} h_{i,s}^{T} \dot{h}_{i,s} \\
- \dot{\phi}_{i,s} \dot{K}_{i,s,p} \omega_{i,s} + \frac{\alpha \xi_{i,s}}{\alpha + 1} h_{i,s}^{T} \dot{h}_{i,s} \right) \right) - \sum_{i=1}^{N} 2\beta_i \dot{\varphi}_i \dot{\varphi}_i \\
\leq \frac{1}{2} \sum_{i=1}^{N} \sum_{s=1}^{4} \left( c_{i,2}^2 + \eta_{i,2}^2 + c_{i,4}^2 + \eta_{i,4}^2 \right) - \dot{\chi}_{\text{min}} W - \sum_{i=1}^{N} 2\beta_i \dot{\varphi}_i \dot{\varphi}_i \\
\text{Based on Lemma 3:}
\]

\[
\beta_i \dot{\varphi}_i \dot{\varphi}_i \leq \frac{1}{2} \beta_i \dot{\varphi}_i^2 - m_i \varphi_i^2 \\
m_i = \frac{\mu_i \beta_i (2\beta_i - 1)}{2} \beta_i > \frac{1}{2} 
\]

Based on Lemma 4:

\[
\log \left( k_{i,s}^{T} I_k h_{i,s} - \omega_{i,s}^{T} I_k \omega_{i,s} \right) \leq \log \left( k_{i,s}^{T} I_k h_{i,s} - \omega_{i,s}^{T} I_k \omega_{i,s} \right) 
\]
Substitute (48) and (49) into (47) to get:

\[
\dot{W} \leq \sum_{i=1}^{N} \sum_{s=1}^{4} \sum_{p=1}^{n} \left[ -\pi_{i,s} \log \frac{k_{i,s}^2}{k_{i,s}^2 - \omega_{i,s}^2} - 2^{\frac{\alpha+1}{2}} \phi_{i,s} \left( \frac{1}{2} \log \frac{k_{i,s}^2}{k_{i,s}^2 - \omega_{i,s}^2} \right)^{\frac{\alpha+1}{2}} - \frac{\xi_{i,s}}{\alpha + 1} \left( \frac{h_{i,s}^2}{\omega_{i,s}^2} \right)^{\frac{\alpha+1}{2}} + \sum_{i=1}^{N} \left( \beta_{i} \frac{\dot{\varphi}_{i}^2}{\mu_{i}} + m_{i} \frac{\alpha+1}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right) \left( c_{i,2}^2 + n_{i,2}^2 + c_{i,4}^2 + n_{i,4}^2 \right) - \hat{\chi}_{\min} \dot{W} \right] \\
\]

Using this conclusion, (50) can be rewritten as:

\[
\dot{W} \leq \sum_{i=1}^{N} \sum_{s=1}^{4} \sum_{p=1}^{n} \left[ -\pi_{i,s} \log \frac{k_{i,s}^2}{k_{i,s}^2 - \omega_{i,s}^2} - 2^{\frac{\alpha+1}{2}} \phi_{i,s} \left( \frac{1}{2} \log \frac{k_{i,s}^2}{k_{i,s}^2 - \omega_{i,s}^2} \right)^{\frac{\alpha+1}{2}} - \frac{\xi_{i,s}}{\alpha + 1} \left( \frac{h_{i,s}^2}{\omega_{i,s}^2} \right)^{\frac{\alpha+1}{2}} + \sum_{i=1}^{N} \left( \beta_{i} \frac{\dot{\varphi}_{i}^2}{\mu_{i}} + m_{i} \frac{\alpha+1}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right) \left( c_{i,2}^2 + n_{i,2}^2 + c_{i,4}^2 + n_{i,4}^2 \right) - \hat{\chi}_{\min} \dot{W} \right] \\
\]

If \( \frac{m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \geq 1 \), we can get

\[
\left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} - \left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} \leq 0 \quad (51)
\]

else if \( 0 < \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} < 1 \), we can get

\[
\left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} - \left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} \leq 1 - \left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} < 1 \quad (52)
\]

Then we can get a general conclusion

\[
\sum_{i=1}^{N} \left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} - \sum_{i=1}^{N} \left( \frac{2m_{i}}{\mu_{i}} \frac{\varphi_{i}^2}{\varphi_{i}^2} \right)^{\frac{\alpha+1}{2}} \leq N_{c} \quad (53)
\]
in a period of finite time

\[ T_1 \leq t_0 + \frac{2}{\alpha q_1(1-\alpha)} \log \left[ 1 + \frac{q_1}{q_2} W^{\frac{1-\alpha}{\alpha}} (t_0) \right] \]  

Similarly, for (57), if \( W \frac{q_3}{q_1} > \frac{q_2}{(1-\alpha)} \), we get \( W \leq -q_1 W - \alpha q_2 W^{\frac{1-\alpha}{\alpha}} \), then the signals \( \hat{\omega}_{i,s,p}, \hat{h}_{i,s,p}, \hat{\varphi}_i \) converge to the neighborhood

\[
\left \{ W(T) \mid W(T) \leq \left[ \frac{q_3}{q_2(1-\alpha)} \right]^{\frac{1}{2\alpha}} = V_2 \right \}
\]

in a period of finite time

\[ T_2 \leq t_0 + \frac{2}{\alpha q_1(1-\alpha)} \log \left[ 1 + \frac{q_1}{q_2} W^{\frac{1-\alpha}{\alpha}} (t_0) \right] \]

Then denote \( V = \text{min}\{Q_1, Q_2\} \), we have

\[
\frac{1}{2} \log \frac{k_{i,s,p}^2}{\omega_{i,s,p}^2 - \omega_{i,s,p}^2} \leq V
\]

\[
\frac{1}{2} h_{i,s,p}^2 \leq V
\]

in finite time \( T = \text{max}T_1, T_2 \). Thus, we have

\[
|\hat{\omega}_{i,s,p}| \leq \min \{k_{i,s,p} \sqrt{1 - e^{-2V}} \} \leq k_{i,s,p} |\hat{h}_{i,s,p} | \leq V
\]

Since the compensation signal \( h_{i,s,p} \) is guaranteed to be bounded, if we want to ensure that \( |\hat{h}_{i,s,p}| \leq h_0 \) is true, then there must be an upper limit for \( h_{i,s,p} \), where \( h_0 > 0 \). According to equation (16), we can get \( |\hat{f}_{i,s,p}| \leq k_{i,s,p} + h_0 \).

Based on the above formulas, for \( f_{i,s,p} \in \nu \), denote \( \Omega = \{f_{1,1}, f_{2,1}, \ldots, f_{N,1}\}^T \), \( \Theta = [y_1 - r, y_2 - r, \ldots, y_N - r]^T \), considering (23) and Lemma 1, we have that \( \Omega = R \Theta \) and have \( \Omega^T \Theta = \Theta^T (R^T R) \Theta \), we then get

\[
\sigma_0^2 \sum_{i=1}^{N} (y_i - r)^2 \leq \sum_{i=1}^{N} f_{i,1}^2
\]

where \( \sigma_0 \) refers to the minimum singular value of \( H \). So we have

\[
|y_i - r| \leq \sqrt{\sum_{i=1}^{N} (y_i - r)^2}
\]

\[
\leq \frac{1}{\sigma_0} \sqrt{\sum_{i=1}^{N} f_{i,1}^2} \leq \frac{\sqrt{N}}{\sigma_0} |f_{i,1}|_{\text{max}}
\]

In order to ensure \( |y_i| \leq \hat{k}_{i,1} \), it only needs \( |y_i| \leq |r| + \frac{\sqrt{N}}{\sigma_0} (\hat{k}_{1,1} + h_0)_{\text{max}} \leq (\hat{k}_{1,1})_{\text{min}} \), and consider \( r < \hat{k}_0 \), we have \( (\hat{k}_{1,1})_{\text{max}} \leq -h_0 + \frac{\sqrt{N}}{\sigma_0} ((\hat{k}_{1,1})_{\text{min}} - k_0) \).

Due to \( \delta_{i,1} \) is a function that constituted of \( f_{i,1} \omega_{i,1}, \hat{\varphi}_{i,1}, \hat{r} \), \( \delta_{i,1} \) is also bounded. Therefore there is \( \hat{\varphi}_{i,2} > 0 \) such that \( |z_{i,2}| > \hat{\varphi}_{i,2} \), from \( |z_{i,2}| \leq |\omega_{i,1}| + |h_{i,2}| + |z_{i,2}| < k_{i,2} + h_0 + \hat{\varphi}_{i,2} \), in order to ensure \( |z_{i,2}| < \pi_{i,2} \), it only needs \( k_{i,2} \leq \pi_{i,2} - h_0 - \hat{\varphi}_{i,2} \). Similarly, for \( |z_{i,s}| \leq \hat{\varphi}_{i,s} \leq 3, \ldots, 4, \) the result \( |x_{i,s}| < \hat{\pi}_{i,s} \) can be guarantee if \( k_{i,s} \leq \hat{\pi}_{i,s} - h_0 - \hat{\varphi}_{i,s} \). This completes the proof.

**Remark 4:** From the Lyapunov condition of (54), it is worth noting that the closed-loop system can be stable in finite time, and the convergence radius can theoretically be adjusted to infinitesimal. From the definition of (63) and (57), it can be seen that if better convergence is required larger \( \pi_{i,s,p}, \phi_{i,s,p}, \hat{\xi}_{i,s,p}, \hat{\xi}_{i,s,p}, \) \( \hat{\lambda}_i, \gamma_1, \gamma_{1,1}, \gamma_{1,2} \) in equation (18) can be appropriately selected. On the other hand, from the definition of \( \hat{\lambda}_{i,s,p} \), we can also conclude that the directed graph information \( (d_i + b_i) \) needs to be considered when selecting the \( \pi_{i,s,p} \) value. Finally, referring to [29], we can get that \( h_0 < \frac{\sigma_0}{\sqrt{N}} ((\hat{k}_{1,1})_{\text{min}} - k_0) \), then \( h_0 + \hat{\varphi}_{i,s,p} \leq \hat{k}_{i,s,p} \) and \( \xi_{i,s,p} \leq \pi_{i,s,p} \) need to be satisfied in the process of selecting \( \pi_{i,s,p}, \phi_{i,s,p}, \hat{\xi}_{i,s,p}, \hat{\xi}_{i,s,p}, \hat{\lambda}_i, \gamma_1, \gamma_{1,1}, \gamma_{1,2} \).

**Remark 5:** In practical multi-agent control systems, some excellent methods have been proposed due to practical issues such as communication and system mode switching. As mentioned in this work [39], the authors effectively implement multi-agent consensus tracking control using event-triggered strategy and switched stochastic nonlinear systems. Under this control method, the problems of multi-agent communication and multi-mode control are effectively solved. By way of contrast, our research can complete the convergence faster by using finite-time, and the full-state constraints are used to realize the position and velocity constraints of FJMMs in the actual control. However, [39] provides a new research direction for us to solve multi-agent communication and mode switching problems in practical control.

**Remark 6:** The algorithm proposed in this paper can realize the finite-time consistent tracking of FJMMs under the premise of full-state constraints and co-opetition. FJMMs need to satisfy both Assumptions 1 and 2, and its convergence time is affected by the initial value, and does not consider input feedback and Medium event-triggered strategies and switched stochastic systems [39].

**Remark 7:** It is worth noting that in this research, we establish the error compensating signals and virtual control signals based on the fractional exponential power, so that the expect object of finite-time convergence is achieved. However, in [10]–[18], the application of non-negative weights graph in [10]–[18], the control topic in our research is more challenging, since we expect that the follower manipulators can track the leader signal with the same trajectory and opposite symbols, which conforms to the actual engineering requirement closer.

**IV. SIMULATION**

In this section, the proposed algorithm is applied to FJMMs, and simulation analysis to verify the effectiveness of the
TABLE 1. Parameters of the FJ robot systems.

| Parameter | Description              | Values | Unit       |
|-----------|--------------------------|--------|------------|
| $M_i$     | the mass of link         | 0.4 kg | kg         |
| $L$       | the length of link       | 1 m    |            |
| $g$       | gravity acceleration     | 10 m/s |            |
| $K$       | joint stiffness          | 1.5 N/m |            |
| $B$       | damping coefficient      | 0.015  | N·m/rad    |
| $B_1$     | friction coefficient     | 0.02   | none       |
| $J$       | joint flexibility        | 3.4 m/s²|            |

proposed control algorithm.

Three identical manipulators were selected as experimental objects. The dynamics equation of the FJMMS is shown in (1), where $F(\dot{q}) = B_1 \cos(\dot{q})$, assuming that $G(q) = 0$ and the parameters are shown in Table 1 below. The expected tracking signal is chosen as $r = 0.5 \cos(2t)$, and the initial states of follower manipulators are chosen as $x_1(0) = 0.6, x_2(0) = -0.6, x_3(0) = 0.7$. Figure 1 showed that the interactions between three followers and one leader, the Laplacian matrix is $L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$, the matrix of the signed weights is $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Define the constraints of $x_{i,1}, x_{i,2}, x_{i,3}$ and $x_{i,4}$ as $l_{i,1,1} = 0.8, l_{i,2,1} = 2, l_{i,3,1} = 0.7$ and $l_{i,4,1} = 5, i = 1, 2, 3$; The restriction conditions of $\omega_{i,1}, \omega_{i,2}, \omega_{i,3}$ and $\omega_{i,4}$ are respectively $l_{i,1,2} = 1.1, l_{i,2,2} = 0.3, l_{i,3,2} = 0.3$ and $l_{i,4,2} = 5, i = 1, 2, 3$; Select the proportional gain as $\pi_{i,1,1} = 0.4, \pi_{i,1,2} = 15, \pi_{i,1,3} = 15, \pi_{i,1,4} = 15, \pi_{i,2,1} = 0.4, \pi_{i,2,2} = 20, \pi_{i,2,3} = 20, \pi_{i,2,4} = 20, \pi_{i,3,1} = 0.4, \pi_{i,3,2} = 20, \pi_{i,3,3} = 20, \pi_{i,3,4} = 20$; And the parameters for selecting the filter are $\gamma_{i,p,1,d} = 35, \gamma_{i,p,2,d} = 35, i = 1, 2, 3, 4; g_{1,2} = 1, g_{1,4} = 1, g_{2,2} = 1, g_{2,4} = 1, g_{3,2} = 1, g_{3,4} = 1, \phi_{1,1} = 1, \phi_{1,2} = 1, \phi_{1,3} = 7, \phi_{1,4} = 2, \phi_{2,1} = 1, \phi_{2,2} = 1, \phi_{2,3} = 7, \phi_{2,4} = 2, \phi_{3,1} = 1, \phi_{3,2} = 1, \phi_{3,3} = 7, \phi_{3,4} = 2, \xi_{1,1} = 0.1, \xi_{1,2} = 3, \xi_{1,3} = 2, \xi_{1,4} = 1, \xi_{2,1} = 0.1, \xi_{2,2} = 3.8, \xi_{2,3} = 2, \xi_{2,4} = 1, \xi_{3,1} = 0.1, \xi_{3,2} = 3, \xi_{3,3} = 2, \xi_{3,4} = 1; r = \frac{3}{5}$. 

FIGURE 1. The interactions among agent

FIGURE 2. The trajectories of $r$ and $x_{i,1}, i = 1, 2, 3$ in proposed algorithm

FIGURE 3. The trajectories of $r$ and $x_{i,1}, i = 1, 2, 3$ without finite-time convergence
Figure 4. The trajectories of $r$ and $x_{1,i}$, $i = 1, 2, 3$ in Reference 28

Figure 5. The trajectories of $x_{1,2}$, $x_{1,3}$, and $x_{1,4}$ in proposed algorithm

Figure 6. The trajectories of $x_{1,2}$, $x_{1,3}$, and $x_{1,4}$ without finite-time convergence

Figure 7. The trajectories of $x_{1,2}$, $x_{1,3}$, and $x_{1,4}$ in Reference 28

Figure 8. The trajectories of $x_{1,2}$, $x_{1,3}$, and $x_{1,4}$ in proposed algorithm

Figure 9. The trajectories of $x_{1,2}$, $x_{1,3}$, and $x_{1,4}$ without finite-time convergence
FIGURE 10. The trajectories of $x_{1,2}, x_{1,3}$ and $x_{1,4}$ in Reference 28

FIGURE 11. The trajectories of $x_{2,2}, x_{2,3}$ and $x_{2,4}$ in proposed algorithm

FIGURE 12. The trajectories of $x_{2,2}, x_{2,3}$ and $x_{2,4}$ without finite-time convergence

FIGURE 13. The trajectories of $x_{2,2}, x_{2,3}$ and $x_{2,4}$ in Reference 28

FIGURE 14. The trajectories of $x_{3,2}, x_{3,3}$ and $x_{3,4}$ in proposed algorithm

FIGURE 15. The trajectories of $x_{3,2}, x_{3,3}$ and $x_{3,4}$ without finite-time convergence
In order to intuitively reflect the advantages of the proposed algorithm, in the case of the same parameters, choose two algorithms for comparison. One is an algorithm without finite time, when other parameters in the algorithm remain unchanged, let $\gamma = 0$, and the obtained system is asymptotically convergent. The other is finite-time command filter backstepping without full-state constraints in [28]. Figures 2-4 are the trajectories of $r$ and $x_{1,1}, i = 1, 2, 3$. Figures 5-13 are the response curves of $x_{1,s}, i = 1, 2, 3, s = 2, 3, 4$. Figure 14 is the torques response curves of $\psi_{1,1}, i = 1, 2, 3$. Figures 15-17 are the time responses of $\delta_{1,s}$ and $\zeta_{1,s+1}, i = 1, 2, 3, s = 1, 2, 3$ under proposed algorithm. Figure 18 shows the comparison of the overall tracking error (OTE). By comparing Figures 2-4, it can be seen that the algorithm proposed in this paper has better convergence effect than the algorithm in [28], and the algorithm without finite-time is the worst. Figures 5-13 show the position states of each robotic arm system $x_{1,s}, i = 1, 2, 3, s = 2, 3, 4$. Figure 5, 6, 8, 9, 11, 12 do not exceed the set limits, but in Figures 7, 10 and 13, we can know that $x_{1,3}, x_{1,4}, x_{2,3}, x_{2,4}$ and...
The trajectories of the OTE under different convergent speed $x_{\hat{i},s}$ have exceeded the set constraints, which is the effect of full state constraints in the proposed algorithm. $\delta_i$, $\zeta_{i,s+1}$, $i = 1, 2, 3$, $s = 1, 2, 3$ in Figures 15-17 indicate the filter. The filtering speed is very fast and the effect is very obvious. It can be seen from Figure 18 that when the system in the proposed algorithm, the system converges faster, the convergence effect is better after stabilization, and the error is the smallest. On the contrary, although the convergence speed of the algorithm in [28] is close to the proposed algorithm, the OTE is larger than the proposed algorithm, and the algorithm without finite-time has a slower convergence speed. At the same time, it is also revealed that the algorithm in this paper can perfectly solve the co-opetition problem of multi-agent robots arms.

V. CONCLUSION

A novel FJMMs control strategy based on CFB and barrier Lyapunov function is proposed in this article, whose purpose is to solve the finite-time consensus tracking problem under the characteristics of cooperation and full state constraints. The defect of computational explosion in the traditional backstepping process is overcome by the command filter, and combined with the error compensating signals, the problem of filtering error is also solved. It is proved that the performance of state constraints is guaranteed excellently, while the consensus tracking problem is also solved, especially the system features the cooperation. Further study will make an attempt on how to apply the proposed algorithm on switching topology.

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