Transient Convective Heating Transport of the Micropolar Fluid Flow Between Asymmetric Channel with Activation Energy

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Abstract. Here modelling and computations are presented to introduce the concept of activation energy involved in the micropolar fluid between the porous vertical channel with the boundary conditions of third kind. The fluid is considered to be gray, absorbing-emitting but non scattering medium. The resulting system of equations are solved numerically by Crank-Nicolson implicit finite difference method. The effect of various physical parameter such as activation energy, micropolar parameter, reaction rate, Reynolds number, Schmidt number, heat and mass transfer Biot numbers on the velocity, temperature and concentration field are discussed by means of pictorial representation.

1. Introduction
The hydrodynamics of the fluids are based on the fact that the fluid fragments does not have any intrinsic structure, this behaviour results in the known Navier-Stokes equations which describes a lot of hydrodynamic phenomena. Micropolar fluids may embody the fluids comprises of rigid randomly sized particles. Moreover, the particles are suspended in viscous medium where the deformation of fluid particle is ignored. These fluids are helpful in describing the detailed behaviour of colloidal solutions, polymeric suspension, liquid crystals, animal blood, etc. Eringen [1] developed the concept of Micropolar fluid theory which is designed to meet the needs of the system that do not comfort the Navier-Stokes equation. The Reynolds number for the three dimensional case is generalized by Singh and Sinha [2]. Sharma and Gupta [3] analyzed the impact of medium permeability in thermal convection of micropolar fluids. They have analyzed the thermal instability of micropolar fluids in porous medium. Łukaszewicz [4] presented mathematical aspects of the micropolar fluid flow theory in his book. From this concept, he described the non-Newtonian behaviour of certain fluids. By using the Optimal Homotopy Analysis method, Rashid et al. [5] discussed the heat dissipation and transverse magnetic fields effects on micropolar rotating fluid flow between two lateral plates.

Thermal radiation affects the heat dissipation effects and the temperature distribution of the micropolar fluid in a channel. Radiation effects on a micropolar fluid flowing through a porous medium with and without magnetic field has been considered by many authors [6-9]. On the other hand, Muhammed et al. [10] investigated the influence of activation energy in non-linear
radiative stagnation point flow of nanofluid. They investigated about the species concentration which enhances for higher estimation of activation energy. The activation energy is characterized as a meager amount of energy required to initiate a specific chemical reaction. The chemical reactions requiring lesser number of activation energy are referred to as spontaneous reactions. The role of the activation energy is also highly recommended in the areas of chemical engineering, food processing and mechanics of oil water synthesis [11-13].

The intent of the commentary is to investigate the consequence of activation energy on the fully refined free convection flow, mass and heat dissipation of micropolar fluid amidst the two lateral porous vertical plates. The numerical results for the flow, heat and mass transfer attributes are furnished by means of graphs and tables.

2. Mathematical Formulation

In this paper, we consider a laminar flow of a micropolar fluid which flows inbetween two vertical plates. These two erect plates are situated at \(y = 0\) and \(y = L\), where the breadth of the channel is given by \(L\). The solute concentration of the internal surface of the left and right plate is marked by \(C_1\) and \(C_0\). \(T_1\) and \(T_0\) are the uniform reference temperature of the left and right plate accordingly. The plates are feigned to have a trifling compactness and dissipates the heat by means of convection and mass transfer co-efficients \(h_1, h_2, h_3\) and \(h_4\). The micropolar fluid is pretended to be a gray but non-scattering medium, which is analogously implanted or disparate with the celerity \(v_0\) to or going back to the walls.

![Physical geometry of the problem](image)

The velocity of Microrotation is indicated as \((0, 0, n)\), where the portion of the microrotation is represented by \(n\) and it is perpendicular to the \(x - y\) plane in which the microrotation is attained by grabbing the curl of velocity vector at microscopic level. The physical properties describing the fluid are reckoned to be fixed except the variation in density which gooses the buoyancy force. The governing equations for this problem is given by:

\[
\frac{\partial v}{\partial y} = 0
\]
where at time \( t \), the temperature of the walls are enacted as \( T \). The last term in the conservation of mass presents due to the essence of Activation of the microinertial effect is given by \( j \), \( T_0 \) is the temperature whereas \( C \) and \( C_0 \) are the concentration. The molecular diffusivity is given by \( D \). The suction or injection of the fluid is denoted by \( v_0 \). \( v_0 \leq 0 \) represents the injection at \( y = L \) and suction at \( y = 0 \), while the contradiction occurs for \( v_0 > 0 \). The last term in the conservation of mass presents due to the essence of Activation of energy, in which \( T_\infty \) is the ambient temperature. \( k_r, n, E_a \), are the reaction rate, fitted rate constant and activation energy respectively.

Since the Cogley-Vincent-Gilles mechanism is adopted to formulate the radiation portion of heat dissipation. The heat flux term in Eq. (4) is interpreted as

\[
\frac{\partial q_r}{\partial y} = 4(T - T_0) \int_0^\infty K_{\lambda h} \left( \frac{\partial e_{\lambda p}}{\partial T} \right)_L d\lambda_3
\]

where \( K_{\lambda h} \) is the absorption coefficient, \( \lambda_3 \) is the wave length, \( e_{\lambda p} \) is the Planck’s function, whereas at time \( t \leq 0 \), the temperature of the walls are enacted as \( T \).

After using Eq. (7) in Eq. (4), the energy equation (4) can be modified as,

\[
\rho c_p \left( \frac{\partial T}{\partial t} \right) + \rho c_p v_0 \left( \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} \right) - 4(T - T_0)I
\]

where

\[
I = \int_0^\infty K_{\lambda h} \left( \frac{\partial e_{\lambda p}}{\partial T} \right)_L d\lambda_3
\]

The dimensionless variables are introduced to non-dimensionalize the governing equations which is given by,

\[
Y = \frac{y}{L}, \quad U = \frac{u \rho g \beta L^2}{k}, \quad \tau = \frac{tv}{L^2}, \quad \theta = \frac{(T - T_0)\rho^2 g^2 \beta^2 L^4}{k^4}, \quad \phi = \frac{(C - C_0)\rho^2 g^2 \beta^2 L^4}{k^4},
\]

\[
\frac{\partial u}{\partial t} + \left( \frac{\partial u}{\partial y} \right) v_0 = \frac{\partial^2 u}{\partial y^2} \left( \nu + \frac{K}{\rho} \right) + (T - T_0) \beta g + (C - C_0) \beta g + \frac{\partial n}{\partial y} \left( \frac{K}{\rho} \right)
\]

\[
\left( \frac{\partial n}{\partial t} \right) \rho j + \left( \frac{\partial n}{\partial y} \right) v_0 j = \left( \frac{\partial^2 n}{\partial y^2} \right) \gamma - (2n + \frac{\partial u}{\partial y}) K
\]

\[
\left( \frac{\partial T}{\partial t} \right) \rho c_p + \left( \frac{\partial T}{\partial y} \right) \rho c_p v_0 = \left( \frac{\partial^2 T}{\partial y^2} \right) k - \left( \frac{\partial q_r}{\partial y} \right)
\]

\[
\left( \frac{\partial C}{\partial t} \right) + \left( \frac{\partial C}{\partial y} \right) v_0 = D \left( \frac{\partial^2 C}{\partial y^2} \right) - k_r (c - c_\infty) \left( \frac{T}{T_\infty} \right)^n \exp \left( -\frac{E_a}{k_0 T} \right)
\]

and the suitable initial and boundary conditions are given by

For \( t > 0 \)

\[
u = 0, \quad n = 0, \quad T = T_0, \quad C = C_0
\]

For \( t > 0 \)

\[
u = 0, \quad v = v_0, \quad n = 0, \quad k_w \frac{\partial T}{\partial y} = h_1 [T_1 - T], \quad -D_w \frac{\partial C}{\partial y} = h_3 [C_1 - C]
\]

\[
u = 0, \quad v = v_0, \quad n = 0, \quad -k_w \frac{\partial T}{\partial y} = h_2 [T - T_0], \quad -D_w \frac{\partial C}{\partial y} = h_4 [C - C_0]
\]
\[ B = \frac{j}{L^2}, \quad N = \frac{npg\beta L^3\nu n}{k}, \quad Pr = \frac{\nu}{\alpha}, \quad Re = \frac{\nu_0\rho L}{\mu}, \quad \lambda_1 = \frac{K}{\mu}, \]
\[ \lambda_2 = \frac{\gamma}{\mu L^2}, \quad Sc = \frac{\nu}{D}, \quad \sigma = \frac{k_r^2}{c}, \quad \epsilon_1 = \frac{k\mu}{T_0\rho^2\gamma^2\beta^2L^4} \]

where \( \lambda_1 \) can be the micropolar parameter, \( \lambda_2, B \) be the micropolar constants and the Reynolds number is represented as \( Re \).

By using Eq.(10), the modified form of governing equations (1)-(5) are termed by,

\[ Re \left( \frac{\partial U}{\partial Y} \right) + \left( \frac{\partial U}{\partial \tau} \right) = \frac{\partial^2 U}{\partial Y^2} + \lambda_1 \left( \frac{\partial^2 U}{\partial Y^2} \right) + \theta + \phi + \lambda_1 \left( \frac{\partial N}{\partial Y} \right) \]  
(11)

\[ Re B \left( \frac{\partial N}{\partial Y} \right) + B \left( \frac{\partial N}{\partial \tau} \right) = \lambda_2 \left( \frac{\partial^2 N}{\partial Y^2} \right) - \lambda_1 \left( \frac{\partial U}{\partial Y} + 2N \right) \]  
(12)

\[ Re Pr \left( \frac{\partial \theta}{\partial Y} \right) + Pr \left( \frac{\partial \theta}{\partial \tau} \right) = \left( \frac{\partial^2 \theta}{\partial Y^2} \right) - N_r \theta \]  
(13)

\[ Re Sc \left( \frac{\partial \phi}{\partial Y} \right) + Sc \left( \frac{\partial \phi}{\partial \tau} \right) = \left( \frac{\partial^2 \phi}{\partial Y^2} \right) - \sigma Sc(1 + \epsilon_1\theta)^2 exp \left( \frac{-E}{1 + \epsilon_1\theta} \right) \phi \]  
(14)

the dimensionless form of the boundary conditions (6) is given by,

\[ \tau \leq 0 \; ; \; U = 0, \; N = 0, \; \theta = 0, \; \phi = 0 \]  
(15)

\[ \tau > 0 \; ; \; U = 0, \; N = 0, \]  
\[ -\frac{\partial \theta}{\partial Y} = BiH_1[\xi - \theta], \; -\frac{\partial \phi}{\partial Y} = BiM_1[\eta - \phi] \]  
\[ \text{at} \; \; Y = 0 \]  
(16)

\[ \tau > 0 \; ; \; U = 0, \; N = 0, \]  
\[ -\frac{\partial \theta}{\partial Y} = BiH_2[\theta - \epsilon_1\xi], \; -\frac{\partial \phi}{\partial Y} = BiM_2[\phi - \eta\xi] \]  
\[ \text{at} \; \; Y = 1 \]  
(17)

where \( \xi = \frac{PrGr\beta gL}{\epsilon \nu}, \; \eta = \frac{PrGm\beta gL}{\epsilon \nu}, \) are the dimensionless groups, \( \epsilon = \frac{T_2-T_0}{T_1-T_0}, \; \zeta = \frac{C_2-C_0}{C_1-C_0}, \) are the non-dimensional parameters, \( k_w = \) temperature conductivity, \( D_w = \) molecular diffusivity, \( N_R = \frac{4L^2}{k} \) is the parameter for thermal radiation, the Grashof number is given by \( Gr = \frac{\beta gL^3(T-T_0)}{\nu} \), the modified Grashof number is expressed as \( Gm = \frac{\beta gL^3(C-C_0)}{\nu} \), \( BiH_1 = \frac{h_1L}{k_w}, \; BiH_2 = \frac{h_2L}{k_w}, \)

\( BiM_1 = \frac{h_1L}{D_w}, \; BiM_2 = \frac{h_2L}{D_w}, \) are the heat and mass dissipation Biot numbers.

Accordingly, the shear and couple strains, the heat flow rate intensity and the mass flux on the walls are assigned as follows,

\[ \tau_w = (\mu + K) \left. \frac{\partial u}{\partial y} \right|_{y=0,1}, \; \tau_m = \gamma \left. \frac{\partial n}{\partial y} \right|_{y=0,1}, \; q_w = -k \left. \frac{\partial T}{\partial y} \right|_{y=0,1}, \; q_m = -D \left. \frac{\partial C}{\partial y} \right|_{y=0,1} \]  
(18)

3. Numerical Procedure

The non-linearity equations (11)-(14) along with the conditions (15)-(17) are numerically solved by Crank-Nicolson methodology. The domain \((0 < \tau < \infty) - (0 < Y < 1)\) is divided into a web of lines which is collateral to \( \tau \) and \( Y \) coordinates. After applying the Crank-Nicolson methodology for equations (11)-(14) the governed equations followed by the boundary conditions are altered into the algebraic equations which are as follows:
\[
\left( \frac{U_{i+1}^j - U_i^j}{\Delta \tau} \right) + Re \left( \frac{U_{i+1}^j - U_{i+1}^{j+1} + U_{i+1}^j - U_{i+1}^j}{4\Delta Y} \right) = (1 + \lambda_1) \left( \frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{2(\Delta Y)^2} \right) \\
+ \frac{1}{2} \left( \theta_{i+1}^j + \theta_i^j + \phi_{i+1}^j + \phi_i^j \right) + \lambda_1 \left( \frac{N_{i+1}^j - N_{i+1}^{j+1} + N_{i+1}^j - N_{i+1}^j}{4\Delta Y} \right)
\]

\[
B \left( \frac{N_{i+1}^j - N_i^j}{\Delta \tau} \right) + ReB \left( \frac{N_{i+1}^j - N_{i+1}^{j+1} + N_{i+1}^j - N_{i+1}^j}{4\Delta Y} \right) = \lambda_2 \left( \frac{N_{i+1}^j - 2N_i^j + N_{i-1}^j + N_i^j - 2N_i^j + N_{i-1}^j}{2(\Delta Y)^2} \right) \\
- \lambda_1 \left( \frac{U_{i+1}^j - U_{i+1}^{j+1} + U_{i+1}^j - U_{i+1}^j}{4\Delta Y} \right) - \lambda_2 (N_{i+1}^j + N_i^j)
\]

\[
Pr \left( \frac{\theta_{i+1}^j - \theta_i^j}{\Delta \tau} \right) + RePr \left( \frac{\theta_{i+1}^j - \theta_{i+1}^{j+1} + \theta_{i+1}^j - \theta_{i+1}^j}{4\Delta Y} \right) = \left( \frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j + \theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{2(\Delta Y)^2} \right) \\
- N_R \left( \theta_{i+1}^j + \theta_i^j \right)
\]

\[
Sc \left( \frac{\phi_{i+1}^j - \phi_i^j}{\Delta \tau} \right) + ReSc \left( \frac{\phi_{i+1}^j - \phi_{i+1}^{j+1} + \phi_{i+1}^j - \phi_{i+1}^j}{4\Delta Y} \right) = \left( \frac{\phi_{i+1}^j - 2\phi_i^j + \phi_{i+1}^j + \phi_{i+1}^j - 2\phi_i^j + \phi_{i-1}^j}{2(\Delta Y)^2} \right) \\
- \sigma S \left( \phi_{i+1}^j + \phi_i^j \right) \left( 1 + \epsilon_1 (\theta_{i+1}^j + \theta_i^j) \right)^{\eta_1} \exp \left( -\frac{E}{1 + \epsilon (\theta_{i+1}^j + \theta_i^j)} \right)
\]

the initial and boundary conditions are given by,

\[
U_i^1 = 0, \quad N_i^1 = 0, \quad \theta_i^1 = 0, \quad \phi_i^1 = 0
\]

\[
U_i^j = 0, \quad N_i^j = 0, \quad \frac{\theta_i^j - \theta_i^j}{\Delta Y} = BiH_1[\xi - \theta_i^j], \quad \frac{\phi_i^j - \phi_i^j}{\Delta Y} = BiM1[\eta - \phi_i^j]
\]

\[
U_N^j = 0, \quad N_N^j = 0, \quad \frac{\theta_N^j - \theta_N^j}{\Delta Y} = BiH_2[\theta_N^j - \epsilon \xi], \quad \frac{\phi_N^j - \phi_N^j}{\Delta Y} = BiM2[\phi_N^j - \eta \xi]
\]

For Y and time directions, the mesh sizes are represented by \( \Delta Y \) and \( \Delta \tau \) respectively. To validate the present numerical results, we compared these results with previously published results [14] in Table 1 and we found that it was in good agreement.

**Table 1.** Comparison of temperature gradient at both walls for certain limiting condition \((E = 0, \sigma = 0)\)

| Re  | \( \frac{\partial T}{\partial Y} \) | \( \frac{\partial T}{\partial Y} \) | \( \frac{\partial T}{\partial Y} \) |
|-----|----------------|----------------|----------------|
| 0.1 | -0.0900        | -0.0974        | -0.090035      | -0.097420      |
| 1   | 0.4971         | -0.3283        | 0.497104       | -0.328367      |
| 2   | 0.8996         | -0.4741        | 0.899663       | -0.474103      |
| 10.0| -0.0001        | -0.1695        | -0.000190      | -0.169547      |
| 1   | 0.1341         | -0.2701        | 0.134108       | -0.270184      |
| 2   | 0.2601         | -0.3566        | 0.260124       | -0.356601      |
4. Results and Discussion
The resultant governing equations (11)-(14) along with boundary conditions which is numerically solved by finite difference method. The results that we obtained by using the numerical methods are plotted and discussed for the flow and heat dissipation attributes against the pertinent parameters in this section. The default parameters ($\sigma = 4$, $n = 0.5$, $B = 0.001$, $Pr = 0.72$, $\lambda_1 = 5$, $\lambda_2 = 1$, $\epsilon_1 = 1$, $Bi_1 = 10$, $Bi_2 = 1$, $Re = 0.1$, $Sc = 1$, $N_R = 1$, $E = 1$) are fixed unless otherwise specified.

**Figure 2.** Velocity and Angular velocity profile for high and low Reynolds number with external heat in (a) left wall and (b) right wall.

**Figure 3.** Temperature and Concentration profile for high and low Reynolds number with external heat in (a) left wall and (b) right wall.

Fig.2 shows the effect of Reynolds number on the velocity and angular velocity for the external heating on both plates of the channel. It is shown that by incrementing the Reynolds number values there is an increase in the velocity contour for both the cases of heating, whereas the
angular velocity profile decreases adjacent to the left wall whereas increases nearer to the right wall. The effect of Reynolds number on the temperature and concentration profile have been plotted in Fig.3. It is observed that the temperature and concentration profile increased with adding resistance to the fluid as expected. Also, in the case of highly viscous fluid ($Re << 1$), the temperature and concentration change between the two plates are more than in the case of inviscid fluid ($Re > 1$).

![Figure 4](image)

**Figure 4.** Temperature and Concentration profile for radiation with external heat in (a) left wall and (b) right wall.

The influence of radiation parameter imposing on temperature and concentration profile is displayed in fig.4. It is seen that an increment in the radiation source is to decrease the temperature and increase the concentration for both cases of external heating at the plates. By setting the values of higher radiation parameter, the temperature and the concentration change is observed more in the absence of radiation source.

Fig.5 is sketched to visualize the deviations of concentration with respect to activation parameter with different cases of highly viscous/inviscid fluid. It is identified that an enhancement in the activation energy results in the growth of concentration profile. Generally in the case of highly viscous fluid, the solutal concentration may be lesser than in the case inviscid fluid. The expected same trend has been observed in fig.5.

In order to visualize the flow characteristics with time and space variables, we have plotted the graphs against some pertinent parameters in the case of highly viscous fluid which is shown in fig.6. The drastic change has been noted in highly viscous fluid than in inviscid fluid. The contour plot for temperature and concentration have been shown in fig.7 and fig.8. From fig.7, it is pointed out that with the inclusion of radiation source, the amount of heat exchange from the left hot wall to the micropolar fluid is more than in the absence of radiation parameter. From fig.8, it is observed that there is a gradual improvement in the transient state which is more in the case of absence of activation energy but in the presence of activation energy there is a marginal depression in the steady state of the fluid.

5. Conclusion
The modelling and computations have been implemented to predict the combined effect of activation energy and radiation on the flow and heat transfer trait in the low and high Reynolds number regime with the assumption of external heating on both plates of the channel. The
Figure 5. Concentration profile for radiation with variation in activation energy parameter.

Figure 6. Velocity, Angular velocity, Temperature and Concentration profile for highly viscous fluid.
Figure 7. Temperature distribution for different values of radiation parameter ($N_R$)

Figure 8. Concentration layout for different values of Activation energy

non-dimensional governing equations were solved numerically by Crank-Nicolson implicit finite difference method and the comparison has been done with the previously published results. It is concluded that the impression of pertinent parameters on the flow are more significant.

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