Dynamical suppression of large instantons∗

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We investigate the distribution of instanton sizes in the framework of a simplified model for ensembles of instantons. This model takes into account the non-diluteness of instantons. The infrared problem for the integration over instanton sizes is dealt with in a self-consistent manner by approximating instanton interactions by a repulsive hard core potential. This leads to a dynamical suppression of large instantons. The characteristic features of the instanton size distribution are studied by means of analytic and Monte Carlo methods. We find a power law behaviour for small sizes, consistent with the semi-classical results. At large instanton sizes the distribution decays exponentially. The results are compared with those from lattice simulations.

1. INTRODUCTION

Instantons are field configurations of non-abelian SU(N) gauge theories, which lead to non-perturbative effects. In recent years they have been studied in lattice gauge theories by means of Monte Carlo calculations by different groups [1–4].

In the dilute gas approximation the logarithm of the partition function contains an integral over instanton sizes, \( \int d\rho \rho^{-5} (\rho \Lambda)^{b} \), where \( b = 11N/3 \) and \( \Lambda \) is the scale parameter. The integrand increases with \( \rho \), leading to an infrared divergence. This is an artifact of using the semiclassical approximation after it has become invalid, i.e. for \( \rho \geq 1/\Lambda \). If the semiclassical approximation is meaningful at all, a solution of this problem in the context of the full instanton ensemble is required.

The simplest way is to cut the integrations off at some ad-hoc value \( \rho_c \). But the dominant contribution comes from large \( \rho_j \) near the cut-off where the assumption of diluteness fails. Moreover, the introduction of an ad-hoc cut-off leads to inconsistencies with the renormalization group [5].

In order to solve the problem it has been proposed that instanton sizes are cut off in a dynamical way [6]. The cut-off should originate from configurations where instantons start to overlap. The interaction is expected to suppress overlapping instantons and to result in a self-consistent cut-off.

In connection with the dynamical cut-off the distribution of instanton sizes is of central importance. For small sizes the distribution is predicted to be

\[
\begin{align*}
n(\rho) & \sim \rho^{b-5} \\
& \text{by the dilute gas approximation. For large sizes } \rho, \text{ where the dynamical cut-off is in effect, not much is known about the distribution. There are arguments } \begin{cases} & \text{in favour of a suppression like} \\
& n(\rho) \sim \exp(-c\rho^p) \quad \text{with } p = 2. \end{cases}
\end{align*}
\]

We have investigated the distribution of instanton sizes in a model where the instanton interactions are approximated by a repulsive hard core potential. The radius of an instanton core varies proportional to the size \( \rho_j \) of the instanton. Although this approximation appears to be crude, the general features of the instanton ensemble with a dynamical cut-off are present.

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2. SIZE DISTRIBUTION

In $d = 1$ dimensions the distribution of instanton sizes is exactly given by

$$n(\rho) = \frac{C}{b} \rho^{b-2} e^{-c\rho}.$$

(3)

where we recognize an exponential suppression of large instanton sizes.

In higher dimensions, $d > 1$, we obtained approximate expressions by means of a van der Waals type approximation $[6]$,

$$n(\rho) = \frac{C_d}{b} \rho^{b-d-1} \exp(-c\rho^d).$$

(4)

For small $\rho$ it grows powerlike with the semiclassical exponent $\alpha = b - d - 1$. The value of the exponent $p = d$ in the exponential decay at large $\rho$ should be considered with reservations, because the saddle point approximations are of uncertain quality there.

We have studied the size distribution also by means of grand canonical Monte Carlo simulations of the simplified instanton gas model. In the case of $d = 1$ dimensions the Monte Carlo data agree very well with the available exact result $[4]$.

In the more interesting case of four space-time dimensions ($d = 4$) we consider $\alpha = 7/3$, which is the value for SU(2) gauge theory. The resulting size distribution shows the expected behaviour. For small instanton radii a power law with exponent $\alpha$ can be confirmed. In order to study the behaviour of $n(\rho)$ for large $\rho$ we considered the ratio $F(\rho) = n(\rho)/\rho^\alpha$ and tried fits of the form $F_{\text{fit}}(\rho) = a \exp(-c\rho^p)$. In agreement with the theoretical results they showed that $c$ depends on $\alpha$, while $p$ is nearly independent of it. In Fig. 1 the result of a fit in the interval $[0, 2.25]$ is shown.

The main interest is in the exponent $p$. We find $a \approx 0.89$, and the fit leads to $c = 3.3 \pm 0.2$ and $p = 1.9 \pm 0.2$. For $\alpha = 6$, the SU(3) case, the results for $p$ are the same within the present errors.

In recent years much effort has been devoted to lattice Monte Carlo calculations of properties of the instanton ensemble, and some quantitative statements have been given. For small $\rho$, lattice calculations appear to support the power law $[5]$.

For the large-$\rho$ distribution, de Forcrand et al. predict an exponential decrease with $p = 3 \pm 1$ from their SU(2) lattice data $[2]$. In contrast to this, Smith and Teper conclude from their SU(3) simulations a decay according to $\rho^{-\xi}$ with $\xi \approx 10 \ldots 12$ $[3]$.

To conclude, fits to our numerical Monte Carlo results suggest a behaviour like

$$n(\rho) \overset{\rho \to \infty}{\sim} \rho^\alpha \exp(-c\rho^p).$$

(5)

The results indicate that our simplified model reproduces the main features of instanton ensembles with a dynamical infrared cut-off.

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