Probing up-down quark matter (udQM) via Gravitational Wave

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Recently, it was shown that quark matter with only $u$, $d$ quarks (udQM) can be the ground state of matter for baryon number $A > A_{\text{min}}$ with $A_{\text{min}} \gtrsim 300$. In this paper, we explore $ud$ quark stars (udQSs) that are made of udQM in the context of the gravitational-wave probe of the tidal deformability in binary star mergers, in contrast to previous studies of hadronic stars (HSs) and strange quark stars (SQSs). We give complete analyses of the two-families scenario, which includes cases of udQS-udQS and udQS-HS mergers. The obtained values of the tidal deformability at 1.4 solar mass and the average tidal deformability are in good compatibility with the GW170817 and AT2017gfo constraints.

I. INTRODUCTION

In the conventional picture of nuclear physics, quarks are confined in the state of hadrons. However, it is also possible that quark matter, a state consists of deconfined quarks, exists. Bodmer [1], Witten [2] and Terazawa [3] proposed the hypothesis that quark matter with comparable numbers of $u$, $d$, $s$ quarks, also called strange quark matter (SQM), might be the ground state of baryonic matter. However, this is based on the MIT bag model that can not adequately model the flavour-dependent feedback of the quark gas on the QCD vacuum. Improved models have shown that two flavour quark matter is more stable than three flavour case [4–7]. However, the possibility of absolutely stable quark matter with only $u$, $d$ quarks (udQM) was commonly dismissed because of the observed stability of ordinary nuclei. However, in [6], we found that the udQM can be more stable than ordinary nuclear matter and strange quark matter when the baryon number $A$ is sufficiently large above $A_{\text{min}} \gtrsim 300$. The large $A_{\text{min}}$ ensures the stability of ordinary nuclei. This possibility was demonstrated with a phenomenological quark-meson model that can satisfy all the masses and decay widths constraints of the light mesons.

The udQM laying beyond the periodic table has a large positive charge. Recently, a search for such high-electric-charge objects is attempted using 34.4 $fb^{-1}$ of 13 TeV $pp$ collision data collected by the ATLAS detector at the LHC during 2015 and 2016 [8]. One can also look for the evidence of it from gravitational-wave (GW) detection experiment. The binary merger

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of compact stars produces strong gravitational wave field, which encodes the information of the tidal deformation that is sensitive to the matter equation of state (EOS). For example, stars with stiff EOSs can be tidally deformed easily due to their large radii. Stars that are made of quark matter like \textit{udQM} have distinct GW signature of tidal deformability.

The GW170817 event detected by LIGO [9] is the first merger event of compact stars, together with the subsequent detection of the electromagnetic counterpart, AT2017gfo/GRB 170817A [10], lead us to the era of “multi-messenger astronomy”. The chirp mass of the binary is determined to be $M_c = 1.188 \, M_\odot$. For low spin case, the binary mass ratio $q = M_2/M_1$ is constrained to the range of $0.7 \sim 1.0$, and an upper bound is placed on the tidal deformability at 1.4 solar mass $\Lambda(1.4M_\odot) \lesssim 800$, and on the average tidal deformability $\bar{\Lambda} \leq 800$ at 90\% confidence level, which exclude very stiff EOSs [9]. Lower bounds of tidal deformability can be placed from the UV/optical/infrared counterpart of GW170817(AT2017gfo) with kilonova models [11–13]. To the author’s knowledge, the current lower bounds are $\Lambda(1.4M_\odot) \gtrsim 200$ [12] and $\bar{\Lambda} \gtrsim 242$ [13].

Conventionally, the binary mergers are studied in the one-family scenario where it is assumed that all compact stars are with one family of hadronic matter EOS [9, 14, 15]. However, the discovery of the large two solar mass $M_\odot$ pulsar ruled out a large amount of soft EOSs that were expected with the presence of hyperons and $\Delta$ resonances in the interiors. Moreover, the lower bound of $\bar{\Lambda}$ in one-family scenario excludes compact stars with small radii, which causes tension with what the X-ray analyses suggest [15, 16]. Therefore, it is likely that the stars with mass above $2 \, M_\odot$ are actually the quark stars (QSs), and the ones with small maximum mass are the hadronic stars (HSs). This possibility is the so-called “two-families” scenario, which is based on the Bodmer-Witten-Terazawa hypothesis or the recent finding of stable \textit{udQM}, and that the hadronic stars can coexist with quark stars [17]. Especially, \textit{ud} quark stars (\textit{udQSs}) that are made of \textit{udQM} can satisfy the two solar mass constraint more easily than the strange quark stars (SQSs) since \textit{udQM} gives smaller effective bag constant.

Binary merger in two-families scenario includes three cases: HS-HS [18], HS-QS [15] and QS-QS [19]. The possibility of HS-HS is disfavoured for GW170817 based on the consideration of prompt collapse [18]. The possibility of QS-QS system sometimes is disfavoured because of the kilonova observation from nuclear radioactive decay [20]. However, \textit{udQM} has a large $A_{\min}$ so that it is possible that the \textit{udQM} ejected is quickly destabilized by the finite-size effects and convert into ordinary or heavy nuclei. The conversion is far more rapid for \textit{udQM} than SQM, due to a large $A_{\min}$ and the non-strange composition so that there is no need to involve extra weak interaction to convert away strangeness.

With these motivations, we explore the properties of \textit{udQS} and its gravitational-wave probe in the two-families scenario, including the binary merger cases \textit{udQS-udQS} and \textit{udQS-HS}.
II. PROPERTIES OF udQS

The EOS of udQM can be approximated as the following linear form

\[ p = \frac{1}{3}(\rho - \rho_s) \]  

(1)

where \( \rho_s \) is the finite density at the surface. The coefficient 1/3 is modified by the strange quark mass effect in the SQM scenario. In the region of interest for udQM, we can take the relativistic limit where energy per baryon number takes the form [6]:

\[ \varepsilon = \frac{\rho}{n_A} = \frac{(\chi N_C p_F^4)}{4\pi^2 + B_{\text{eff}}} = \frac{3}{4} N_C p_F \chi + 3\pi^2 \chi B_{\text{eff}} / p_F^3 \] 

(2)

where \( N_C = 3 \) is the color factor and \( \chi = \sum_i f_i^{4/3} \) is the flavour factor, with \( f_u = 1/3 = 1/2 f_d \) for udQM. The effective Fermi momentum \( p_F = (3\pi^2 n_A)^{1/3} \). \( B_{\text{eff}} \) is the effective bag constant that accounts for the QCD vacuum contribution. Note that in realistic models [4–6], \( B_{\text{eff}} \) has dependence on density and flavour composition, but the dependence only cause a substantial effect when strangeness turns on at very large density. Therefore, in this udQM study, we can approximate it as an effective constant. Minimizing the energy per baryon number with respects to \( p_F \) for fixed flavour composition gives

\[ \frac{E}{A} = 3\sqrt{2\pi} \left( \chi^3 B_{\text{eff}} \right)^{1/4}, \] 

(3)

at which \( p = 0, \rho = \rho_s = 4B_{\text{eff}} \). It was shown in [4–6] that \( B_{\text{eff}} \) has smaller value in the two-flavour case than the three-flavour case, so that udQM is more stable than SQM at the bulk. From Eq. (3), The absolute stability of udQM implies \( E/A < 930 \text{ MeV} \), which corresponds to \( B_{\text{eff}} \lesssim 60 \text{ MeV/fm}^3 \) for the two flavour case. In general, larger \( E/A \) or \( B_{\text{eff}} \) gives a larger \( A_{\text{min}} \). The stability of current periodic table elements against udQM requires \( A_{\text{min}} \gtrsim 300 \), which translates to \( E/A \gtrsim 900 \text{ MeV} \) or \( B_{\text{eff}} \gtrsim 50 \text{ MeV/fm}^3 \) for quark-meson model with quark-vacuum surface tension \( \sigma \sim (91 \text{ MeV})^3 \) [6].

The linear feature of quark matter EOS Eq. (1) makes it possible to perform a dimensionless rescaling on parameters [21, 22]

\[ \bar{\rho} = \frac{\rho}{4 B_{\text{eff}}}, \bar{p} = \frac{p}{4 B_{\text{eff}}}, \bar{r} = r \sqrt{4 B_{\text{eff}}}, \bar{m} = m \sqrt{4 B_{\text{eff}}}, \] 

(4)

which enter the TOV equation

\[ \frac{dp(r)}{dr} = - \frac{[m(r) + 4\pi r^3 p(r)] [\rho(r) + p(r)]}{r(r - 2m(r))}, \quad \frac{dm(r)}{dr} = 4\pi \rho(r)r^2. \] 

(5)
so that the rescaled solution is also dimensionless, and thus is independent of any specific value of \( B_{\text{eff}} \). The results for \( \bar{M} = M \sqrt{4B_{\text{eff}}} \), \( \bar{R} = R \sqrt{4B_{\text{eff}}} \) is shown in Fig. 1a.

![Diagram showing rescaled \( \bar{M} \) vs radius \( \bar{R} \) of udQS. The black dot at \((\bar{M}, \bar{R}) = (0.0517, 0.191)\) denotes the maximum mass configuration.](image)

(a) The rescaled \( \bar{M} \) vs radius \( \bar{R} \) of udQS. The black dot at \((\bar{M}, \bar{R}) = (0.0517, 0.191)\) denotes the maximum mass configuration.

![Diagram showing M-R of udQS. Lines with darker color are with larger effective bag constant \( B_{\text{eff}} \), which samples \((45, 50, 60, 63)\) MeV/fm\(^3\) respectively.](image)

(b) M-R of udQS. Lines with darker color are with larger effective bag constant \( B_{\text{eff}} \), which samples \((45, 50, 60, 63)\) MeV/fm\(^3\) respectively.

Figure 1

The TOV solution of any other EOS with different \( B_{\text{eff}} \) value can be obtained directly from rescaling the dimensionless solution back with Eq. (4). In this way, the maximum mass and corresponding radii are \( M_{\text{max}} \approx 15.17/\sqrt{B_{\text{eff}}} \) \( M_{\odot} \), at which \( R \approx 82.8/\sqrt{B_{\text{eff}}} \) km, where \( B_{\text{eff}} \) is in units of MeV/fm\(^3\). Therefore, the possibility of udQS being larger than 2 solar mass corresponds to \( B_{\text{eff}} \lesssim 57.5 \) MeV/fm\(^3\), as shown in Figure 1b. We adopt this as the upper bound of \( B_{\text{eff}} \) since it is more strict than the \( B_{\text{eff}} \lesssim 60 \) MeV/fm\(^3\) from the udQM’s stability. We take ten percent departure for the lower and upper bounds obtained to account for other possible uncertainties like finite mass and other effects outside the model. Then the window we probe is

For udQM :  \( B_{\text{eff}} \in [45, 63] \) MeV/fm\(^3\) \( \approx [136^4, 148.3^4] \) MeV\(^4\)  \( (6) \)

Some SQM studies [19] exploited similar small bag constant value to have star mass above 2 solar mass, but pQCD effect has to be included to guarantee the stability of normal nuclei against non-strange quark matter, which is opposite to our picture of absolute stable udQM. The finite strange quark mass effect also affects their results to a level of noticeably different from our udQM study here.

One can do an interpolation of Fig. 1a to get following analytical expression as function of \( C = M/R = \bar{M}/\bar{R} \):

\[
C = -0.1938 + \frac{1}{5.159 - 42.91M^{2/3} + 277.657M^{4/3} - 889.395M^2}.
\]  \( (7) \)
The matching of Eq. (7) with exact numerical result is shown in Fig. 2a.

(a) $C = \frac{M}{R}$ as function of rescaled mass $\bar{M}$. Numerical result (solid) vs analytical formula Eq. (7) fitting (dashed)

(b) Love number $k_2$ as function of $C = \frac{M}{R}$. Numerical result (solid) vs analytical fitting Eq. (12).

Figure 2

The response of compact star to external disturbance is characterized by the Love number $k_2$ [23–26]:

$$k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y_R - 1) - y_R] \times \left\{ 2C[6 - 3y_R + 3C(5y_R - 8)] + 4C^3[13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] \right\}^{-1},$$  

where $C = \frac{M}{R}$, and $y_R$ is $y(r)$ evaluated at the surface. The function $y(r)$ can be obtained by solving the following equation [26]:

$$ry'(r) + y(r)^2 + y(r)e^{\lambda(r)} \left[ 1 + 4\pi r^2(p(r) - \rho(r)) \right] + r^2Q(r) = 0,$$  

with boundary condition $y(0) = 2$. Here

$$Q(r) = 4\pi e^{\lambda(r)} \left( 5\rho(r) + 9p(r) + \frac{p(r) + \rho(r)}{c_s^2(r)} \right) - 6\frac{e^{\lambda(r)}}{r^2} - (\nu'(r))^2,$$  

and

$$c_s^2(r) = \left[ 1 - \frac{2m(r)}{r} \right]^{-1}, \quad \nu'(r) = 2e^{\lambda(r)}m(r) + 4\pi p(r)r^3.$$

$c_s^2(r) \equiv dp/d\rho$ denotes the sound speed squared. For star with a finite surface density like QS, a matching condition is used at the boundary $y_R^{ext} = y_R^{int} - 4\pi R^3\rho_s/M$ [27]. The $\rho(r)$ and $p(r)$ in Eq. (9) are obtained from the coupled TOV equations Eq. (5). We can scale
Eq. (9) with respect to Eq. (4). A polynomial fit to the solution gives:

\[ k_2 = -13.679 C^4 + 0.2033 C^3 + 8.0979 C^2 - 4.5133 C + 0.7482 \]  

(12)

We notice that similar fit for \( k_2(C) \) was obtained in reference [28]. The dimensionless tidal deformability \( \Lambda \) as an analytical function of mass \( \bar{M} \) are thus obtained from the definition

\[ \Lambda = \frac{2k_2}{3C^5} \]  

(13)

with substitution of Eq. (12) and Eq. (7). The result is shown in Fig. 3.

![Figure 3: \( \Lambda vs \bar{M} \) for \( udQM \). Numerical exact result (solid) vs analytical fitting (dashed) are shown. The grey band represents the allowed region of \( udQM \) with \( B_{\text{eff}} \in [45, 63] \text{ MeV/fm}^3 \) with \( M = 1.4M_\odot \). The blue band is the GW170817/AT2017gfo constraint \( 200 \lesssim \Lambda(1.4M_\odot) \lesssim 800 \).](image)

For \( M = 1.4M_\odot \) and \( B_{\text{eff}} \in [45, 63] \text{ MeV/fm}^3 \), one has \( \bar{M} = M\sqrt{4B_{\text{eff}}} \in [0.320, 0.385] \). Mapping this range to Fig. 3 gives \( \Lambda(1.4M_\odot) \in [340, 856] \), which has a large overlapping region with the GW170817/AT2017gfo constraint \( 200 \lesssim \Lambda(1.4M_\odot) \lesssim 800 \). Especially, the point where \( \Lambda(1.4M_\odot) \) reaches the upper bound \( \Lambda(1.4M_\odot) \sim 800 \) maps to \( B_{\text{eff}} \) value that almost coincides with the lower bound of \( udQM \ B_{\text{eff}} \), so that a more strict constraints will push \( udQM \) to a larger \( E/A \) than 900 MeV and a larger \( A_{\text{min}} \) than 300.

III. BINARY MERGER IN THE TWO-FAMILIES SCENARIO

The average tidal deformability of a binary system is defined as:

\[ \tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4}{(M_1 + M_2)^5} \Lambda(M_1) + (1 \leftrightarrow 2) \]
\[
\frac{16}{13} \frac{(1 + 12q)}{(1 + q)^5} \Lambda(M_1) + (1 \leftrightarrow 2, q \leftrightarrow 1/q),
\]

where \(M_1\) and \(M_2\) are the masses of the binary components. \(q = M_2/M_1\), with \(M_2\) being the smaller mass so that \(0 < q \leq 1\). Then for given chirp mass \(M_c = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}\), we have \(M_2 = (q^2(q + 1))^{1/5} M_c\) and \(M_1 = M_2(q \rightarrow 1/q)\).

\[\text{A. \textit{udQS-udQS Merger}}\]

In this case, the average tidal deformability can be expressed as function of the rescaled mass parameter \(\bar{M} = M \sqrt{4B_{\text{eff}}}\):

\[
\tilde{\Lambda}(q, \bar{M}_c) = \frac{16}{13} \frac{(1 + 12q)}{(1 + q)^5} \tilde{\Lambda}(\bar{M}_1) + (1 \leftrightarrow 2, q \leftrightarrow 1/q).
\]

Substitute the results of Eq. (13) into the formula above, we obtain Fig. 4. As the figure shows, for chirp mass \(M_c = 1.188 M_\odot\) and \textit{udQM} effective bag constant \(B_{\text{eff}} \in [45, 63]\) MeV/fm³, the constraint \(242 \lesssim \tilde{\Lambda} \lesssim 800\) translates to \(0.4 \leq q \leq 1\), which agrees with \(q = 0.7 \sim 1\) [9]. Here I give a few remarks that can be observed from this figure:

- Only symmetric system has a large \(\bar{M}_c = M_c \sqrt{4B_{\text{eff}}}\) window.
- The more symmetric the two masses are, the larger \(\tilde{\Lambda}\) the system has.
• Larger $\tilde{M}_c = M_c \sqrt{4B_{\text{eff}}}$ gives smaller $\tilde{\Lambda}$.

• A lower upper bound of $\tilde{\Lambda}$ will lift the lower bound of $ud$QM effective bag constant value, which points to $E/A$ larger than 900 MeV and $A_{\text{min}}$ larger than 300.

B. udQS-HS Merger

In this case, we need the information of the hadronic matter EOS, which has large uncertainties in the intermediate-density region. Based on nuclear physics alone, the EOS should match the low-density many-body calculation and high-density pQCD result [29]. Here we use three benchmarks of hadron matter EOSs, SLy [30, 31] Bsk19, Bsk21 [32], that have unified representations from low density to high density. Bsk19 is an example of soft EOS. Stars with Bsk19 have maximum mass $M_{\text{max}} = 1.86M_\odot < 2M_\odot$ and $R_{1.4M_\odot} = 10.74 \text{km} < 11 \text{km}$. The feature of small mass and small radii is preferred for typical QS-HS studies of two-families scenario. For illustration, we also give benchmarks of hard EOS Bsk21 ($M_{\text{max}} = 2.27M_\odot$ and $R_{1.4M_\odot} = 12.57 \text{km}$), and EOS SLy which is moderate ($M_{\text{max}} = 2.05M_\odot$ and $R_{1.4M_\odot} = 11.3 \text{km}$). With Eq. (14) and the $\Lambda(M)$ of $ud$QM and the hadron EOS benchmarks, we obtain the average tidal deformability $\tilde{\Lambda}$ of the udQS-HS system, as shown in Fig. 5.

![Figure 5: Average tidal deformability $\tilde{\Lambda}$ vs $q = M_2/M_1$ in QS-HS merger, with $M_2$ being the mass of hadronic star. $M_c = 1.188 M_\odot$ and $q = 0.7 \sim 1$ are considered for the GW170817 event. For HS EOS, SLy (blue), Bsk19 (red), Bsk21 (black) are used. For $ud$QS EOS, lines with darker color are with larger bag constant $B_{\text{eff}}$, which samples $B_{\text{eff}} = (45, 50, 60, 63) \text{MeV/fm}^3$ respectively.](image)

The order of $\tilde{\Lambda}$ for different hadron EOSs matches the expectation from the general rule that HSs with stiffer EOSs and QSs with smaller effective bag constant have large radii, and thus have large deformability. We also see a good compatibility with current GW170817 constraint $242 \lesssim \tilde{\Lambda} \lesssim 800$. 
IV. CONCLUSIONS

Motivated by the distinct properties that make ud quark star a good candidate for the two-families scenario, we have studied the related tidal deformability of binary star merger including the udQS-udQS and HS-udQS cases. With the dimensionless rescaling method used, the analyses can be straightforwardly generalized to arbitrary binary chirp mass and effective bag constant for current and future gravitational-wave events study. In particular, we have shown the compatibility between the cases of udQS-udQS and udQS-HS with the GW170817/AT2017gfo constraint on $\Lambda(1.4M_\odot)$ and $\tilde{\Lambda}$. Possible implications on constraining the effective bag constant of udQM and the associated $E/A$ and $A_{\text{min}}$ have been discussed.

Note Added: As we were finalizing this paper, we became aware of the new paper [33]. This paper has some discussions on the 1.4 solar mass tidal deformability $\Lambda(1.4M_\odot)$ of non-strange quark star in the context of NJL model with proper-time regularisation, and it is also found that this quantity is in good compatibility with the experiment constraint.

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