Calculation Model of Sound and Vibration Propagation in a Building Fragment Based on the Method of Statistical Energy Analysis

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Abstract. The theory sound insulation based on the matching of the wave fields by M. S. Sedov and Statistical energy analysis (SEA) are used to model sound transmission through single and double leaf partitions. A fragment of a building is considered as a set of acoustically connected rooms and structures. On the basis of the method of statistical energy analysis, the equations of energy balance are written, the solution of which allows to determine the energy of sound in the rooms and the energy of sound vibration in the structures. A complete model of the SEA, including bending, longitudinal and shear waves in structures and sound waves in the air volumes of the rooms, is proposed. The coupling loss factors from room to room (non-resonant sound transmission through partitions) are calculated on the theory by M. S. Sedov. It is given the calculation of the coupling loss factors, internal losses and modal densities. The results of vibro-acoustic calculations and measurements on models of the building fragments are presents.

1. Introduction
Reducing noise and providing acoustic comfort in a multi-story apartment buildings is an actual problem especially when there are built-in acoustic and vibrational sources of noise. To ensure sound insulation in a building, it is necessary to take into account not only the design parameters of the actual partitions structures separating the rooms, but also the conditions for the propagation of sound vibrations through adjacent structures.

2. Formatting the title, authors and affiliations
To determine the sound insulation of adjacent rooms, taking into account the indirect sound transmission, the flow-energy method is widespread [1, 2, 3]. It allows a simplified assessment of the sound insulation of adjacent rooms, taking into account the indirect sound transmission.
In contrast to the flow-energy method, the SEA method allows one to simulate the propagation of sound in a fragment of a building, taking into account the wave parameters of elements of a common
In the classical formulation, the SEA method, when determining the energy coupling coefficients, takes into account only resonant forms of sound transmission, neglecting non-resonant ones [4]. The coupling loss factors from room to room as non-resonant sound transmission through partitions were presented by R.J.M. Craik [5, 6]. The main theory sound insulation of single and double partitions based on the matching of the wave fields with regard non-resonant sound transmission through partition given by M. S. Sedov [7, 8].

In addition to classical formulation of the SEA method in [9, 10] all structural elements were they were presented as a set of subsystems with the energy of bending, longitudinal and shear waves. Energy is exchanged between structural elements between three fields of waves contained in each of them, and the exchange between acoustic subsystems (rooms) and structural subsystems occurs between fields of flexural waves in structures and fields of longitudinal waves in indoor air. Calculations of coefficients of passing and transformation of waves are given for calculation of coupling loss factors.

3. Formatting the text
This paper proposes a model that describes sound and vibration propagation in a building fragment based on a statistical energy analysis method with the representation of non-resonant sound transmission in the form of energy coupling coefficients between rooms in the energy balance equations, based on theory by M. S. Sedov. Calculations and measurements of noise and vibration in the building fragment were carried out to verify the proposed method of calculation.

4. The theoretical foundations of the SEA taking into account non-resonant sound transmission
To represent the energy interconnection of subsystems, including between acoustic ones, in the complete model of the SEA, we consider a system that includes two panels (subsystems with indices $i$ and $j$) and two air volumes of a room (acoustic subsystems with indices $k$ and $m$). As suggested in [9], let us assume that each panel consists of 3 independent subsystems carrying energy of bending, longitudinal, and shear waves. Let us represent the energy interaction of the enclosing structures and rooms with each other in the form of a diagram in the figure 1. In the scheme $W_i$, $W_j$, $W_k$ and $W_m$ are the energy in the subsystems, $P_i$, $P_j$, $P_k$ and $P_m$ are the powers of external sources coming into the subsystems, $P_{i \text{diss}}$, $P_{j \text{diss}}$, $P_{k \text{diss}}$ and $P_{m \text{diss}}$ are the power losses in the subsystems dissipation, $P_{ij}$, $P_{ji}$, $P_{ik}$ and $P_{jk}$, $P_{km}$ and $P_{mk}$ are the powers of the energy exchange between the subsystems. The superscripts of the parameters in the diagram indicate the types of waves and the direction of flow of energy exchange: $A$ – these are acoustic fields in rooms, $b$ – fields of bending waves in panels; $l$ – fields of longitudinal waves; $s$ – fields of shear waves.
Let us now present the computational model of a part building (or its fragment) as a set of $M$ rooms of rectangular shape, fenced with flat single-layer panels, the total number of which is $N$. To denote the rooms we will use the subscripts $i$ and $j$, and to designate panels exchanging the energy of bending fields, longitudinal and shear waves, use the subscripts $k$ and $m$, and to denote panels exchanging the energy of bending fields, longitudinal and shear waves, use the subscripts $i$ and $j$. The sequence of these indices will show the direction of energy flow.

Let's write the energy balance equations for each $i$-th constructive subsystem:

$$\begin{align*}
P^b_i &= \omega \cdot \eta^b_i \cdot W^b_i - \omega \left[ \sum_{j=1}^{N} (\eta^{b,j}_i \cdot W^b_j) + \sum_{j=1}^{N} (\eta^{b,j}_i \cdot W^b_j) + \sum_{k=1}^{M} \eta^{b,k}_i \cdot W^b_k \right] \\
\end{align*}$$

(1)

where $\eta^b_i = \eta^b_j + \sum_{j=1}^{N} \eta^{b,j}_i + \sum_{j=1}^{N} \eta^{b,j}_i + \sum_{k=1}^{M} \eta^{b,k}_i$; $j \neq i$

$$\begin{align*}
P^l_i &= \omega \cdot \eta^l_i \cdot W^l_i - \omega \left[ \sum_{j=1}^{N} (\eta^{l,j}_i \cdot W^l_j) + \sum_{j=1}^{N} (\eta^{l,j}_i \cdot W^l_j) + \sum_{j=1}^{N} (\eta^{l,j}_i \cdot W^l_j) \right] \\
\end{align*}$$

(2)

where $\eta^l_i = \eta^l_j + \sum_{j=1}^{N} \eta^{l,j}_i + \sum_{j=1}^{N} \eta^{l,j}_i + \sum_{j=1}^{N} \eta^{l,j}_i$; $j \neq i$
\[
P_i' = \omega \cdot \eta_i^{bb} \cdot W_i' - \omega \left[ \sum_{j=1}^{N} (\eta_{ji}^{bb} \cdot W_j') + \sum_{j=1}^{N} (\eta_{ji}^{bl} \cdot W_j') + \sum_{j=1}^{N} (\eta_{ji}^{ls} \cdot W_j') \right]
\]  
(3)

where \( \eta_i^{bb} = \eta_i^{bl} + \sum_{j=1}^{N} \eta_{ji}^{bb} + \sum_{j=1}^{N} \eta_{ji}^{bl} + \sum_{j=1}^{N} \eta_{ji}^{ls} ; j \neq i \)

Now we write the energy balance equations for each acoustic subsystem (rooms), taking into account the energy coupling coefficient and the non-resonant sound transmission from adjacent acoustic subsystems:

\[
P_k^A = \omega \cdot \eta_k^{bb} \cdot W_k^A - \omega \cdot \sum_{i=1}^{N} \eta_{ki}^{bb} \cdot W_i^A - \omega \cdot \sum_{m=1}^{N} \eta_{km}^{bb} \cdot W_m^A,
\]

(4)

where \( \eta_k^{bb} = \eta_k^{bb} + \sum_{i=1}^{N} \eta_{ki}^{bb} + \eta_{km}^{bb} \)

\[
P_m^A = \omega \cdot \eta_m^{bb} \cdot W_m^A - \omega \cdot \sum_{j=1}^{N} \eta_{jm}^{bb} \cdot W_j^A - \omega \cdot \sum_{k=1}^{N} \eta_{km}^{bb} \cdot W_k^A
\]

(5)

where \( \eta_m^{bb} = \eta_m^{bb} + \sum_{j=1}^{N} \eta_{jm}^{bb} + \eta_{km}^{bb} \)

\( \eta_{bb} \) – energy coupling coefficients, taking into account non-resonant sound transmission from adjacent acoustic subsystems.

The set of energy balance equations for all subsystems gives a system of linear algebraic equations for unknown wave energies in the subsystems \( W_i^b, W_i^l, W_i^s, W_i^A \). Avoiding cumbersome writing, let us present the system of equations in a matrix form, breaking the matrix of unknowns, coefficients with them and free members into sub matrices:

\[
\begin{bmatrix}
W_i^b \\
W_i^l \\
W_i^s \\
W_i^A
\end{bmatrix}
\begin{bmatrix}
A_{bb}^{bb} & A_{bb}^{hl} & A_{bb}^{ls} & A_{bb}^b \\
A_{bl}^{bb} & A_{bl}^{hl} & A_{bl}^{ls} & A_{bl}^l \\
A_{ls}^{bb} & A_{ls}^{hl} & A_{ls}^{ls} & A_{ls}^s \\
A_{A}^{bb} & A_{A}^{hl} & A_{A}^{ls} & A_{A}^A
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(6)

The power of external sources is set in the matrix of free members, and the matrix of coefficients is formed from the coefficients of energy communication and the coefficients of total losses, reflecting, ultimately, the planning and design features of the building. The system of real algebraic equations in the matrix form (6) is a complete mathematical model of sound propagation through a building or its fragment. Neglecting the longitudinal and shear waves, one can move from the complete SEA model to the truncated, well-known form described above, which takes into account only the energy of bending waves in the structures. In the matrix view, it will look like this:

\[
\begin{bmatrix}
W_i^b \\
W_i^A
\end{bmatrix}
\begin{bmatrix}
A_{bb}^{bb} & A_{bb}^A \\
A_{A}^{bb} & A_{A}^A
\end{bmatrix}
= \begin{bmatrix}
P_i^b \\
P_i^A
\end{bmatrix}
\]

(7)

The choice of a complete or truncated SEA model depends on the size of the design fragment of the building, on the material and size of the structures, which predetermines the sufficiency of the number of vibration modes in the calculated frequency bands. The use of the complete (including all types of
waves) model of the SEA for vibro-acoustic calculation of buildings can significantly increase the accuracy of the solution, which is the higher, the greater the number of structures and rooms included in the design fragment of the building.

In the work [10] it was shown that the SEA model, which takes into account only bending waves in structures, ensures acceptable accuracy of sound insulation calculation for adjacent rooms only, and for calculating a building fragment that includes at least 3 rooms and enclosing structures, it should be used the model of the SEA, more completely taking into account the physical pattern of propagation and transformation of waves of all basic types.

The solution is to form the power matrix of external sources, the matrix of energy coupling coefficients and loss coefficients and the calculation of the unknown energies \( W^b_i, W^l_i, W^s_i, W^A_k \) in each subsystem. The levels of sound intensity are determined by the formula:

\[
L_k = 10 \log \frac{W^A_k \cdot c_o}{P_o \cdot V_k}
\]

where \( c_o = 344 \text{ m/s} \) – sound velocity in the air; \( P_o = 10^{-12} \) – power threshold; \( V_k \) – volume of the \( k \)-th room, \( \text{m}^3 \).

The root-mean-square vibration acceleration on the \( i \)-th panel in the frequency band under study can be obtained using the formula:

\[
a_i^2 = \frac{\omega^2 \cdot W^b_i}{\rho_i \cdot h_i \cdot S_i}
\]

where \( \rho_i, h_i, S_i \) – accordingly, the density of the material, thickness and area \( i \)-th of the panel.

The difference in levels of transverse vibration accelerations can be obtained:

\[
\Delta L_{ij}^a = 10 \log \frac{a_i}{a_j} = 10 \log \frac{W^b_i \cdot \rho_i \cdot h_i \cdot S_i}{W^b_j \cdot \rho_j \cdot h_j \cdot S_j}
\]

5. Practical relevance, implementation results, experimental research results

Modeling the propagation of sound and vibration by statistical energy analysis involves the development of design schemes that establish relationships between building elements and the determination of energy coupling factors and subsystem total loss coefficients. Figure 2 shows the design diagram of a fragment of a building consisting of 3 rooms. The scheme includes 3 air spaces of the rooms (acoustic subsystems), 15 flat single-layer panels and one double leaf panel (structural subsystems). The number of unknown values of wave energy in structures and rooms according to the truncated model of the SEA is 19, and the complete model of the SEA – 51. The description of the computed fragment includes real geometric and physical mechanical properties of a residential large-panel house, which corresponded to the program of vibro-acoustic experimental studies conducted in-situ. The object of the field experiment was chosen fragment in a newly built panel building in the city of Tomsk.
The building has a distance between the longitudinal axes of 5.4 m. The thickness of the internal interior panels is 80 mm, double interroom panels 170 mm, single-layer continuous floors 160 mm, exterior walls 400 mm, loggia plates 80 mm. The material of supporting structures is heavy concrete with a density of 2500 kg / m$^3$. The external walls are made of expanded clay concrete with a volume weight of 1800 kg / m$^3$. Calculation and tests were carried out in a fragment of a building with living rooms, the first room of 5.54 m x 6.54 m x 2.9 m, the second and third room of 5.54 m x 3.23 m x 2.9 m.

Sound pressure level measurements were made by sound level meters, and vibration acceleration amplitudes were measured using accelerometers BC 111 of the ICP standard and ZET-LAB analyzers. The noise source was an omnidirectional noise source, which was installed in the second room. Vibration measuring instruments determined the vibration levels in the one-third octave bands in the range from 100 to 3150 Hz on the walls and on the floor in the rooms. Measurements were made on 5 points in rooms and 10 points on the panel, followed by averaging the obtained values. Figure 3 shows the calculated and measured sound pressure levels in rooms I and II.

![Figure 2. The design scheme of a fragment of the building.](image)

![Figure 3. Calculated and measured frequency dependences of the sound pressure level in rooms I and II.](image)
Figure 4 presents the theoretical and experimental results of a study of airborne sound insulation of a double partition between adjacent rooms.

![Figure 4](image)

**Figure 4.** Calculated and measured frequency dependences of the sound insulation of the internal enclosing structure installed between room I and room II.

Figure 5 shows the frequency dependences of the difference in vibration acceleration levels between the excited floor panel and wall panels obtained from the experiment and by calculation.

![Figure 5](image)

**Figure 5.** The frequency characteristics of the difference in vibration acceleration levels between the slab overlap and the walls (the lower indices denote the numbers of the panels on which the vibration accelerations were measured).

6. **Conclusion**

In the calculation of sound insulation of partitions on the basis of the theory of SEA the non-resonance sound transmission can be taken into account on the basis of the theory of matching the fields of waves of M. S. Sedov by adding to the equation of the energy balance of the energy coupling coefficient between the rooms.

Various models of statistical energy analysis (SEA) can be used in the calculations of sound insulation of partitions in dependence on the size of the calculated fragment of building. For a
fragment of a building consisting of one or two rooms, a «truncated» model can be applied, taking into account only the energy of bending waves in structures. For a fragment of a building consisting of three or more rooms, one should use the «complete» model of the SEA, taking into account the energy of bending, longitudinal and shear waves in the structures.

To describe the process of sound and vibration propagation in a fragment of a building, a model is proposed, represented by a system of linear algebraic energy balance equations written for each of the subsystems. The model includes the conditions that the energy flowing into the structures and rooms outside the fragment in question is taken into account when determining the total loss of structures that limit the fragment. The non-resonant form of sound transmission through a structure separating two adjacent rooms is also taken into account.

Calculations of the building fragment from 3 rooms allowed to calculate the sound pressure levels in the premises and the mean square values of vibration accelerations on the enclosing panels as an example. To check the method of vibroacoustic calculation of a building fragment, measurements of noise levels and vibrations were carried out in a newly built panel building. The experiment confirmed the high enough accuracy of the proposed vibroacoustic calculation based on the method of statistical energy analysis and the theory of matching the fields of waves.

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