Magnetized rotational neutron star and the MR relations

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Abstract. Radii and masses of neutron stars are calculated for various EoSs using a perturbative approach. Also increased masses are calculated due to magnetic fields. Moreover, the radius of a neutron star is calculated as a function of its increased total mass by rotation. As for the EoSs, we adopt 5 hadronic EoSs in relativistic mean field (RMF) theory to calculate the mass-radius relations (MR relations). It is found that the total masses are over twice the solar mass for all 5 hadronic EoSs in the presence of magnetic fields together with rotation. Three obtained EoSs (GM1, TM2-ωρ-a and TM2-ωρ-b) are found to be in the range of the observation. Hybrid stars are calculated to have masses more than twice the solar mass in a rapid rotation for 5 EoSs.

1. Introduction
A rotating neutron star, called pulsar, was observed in 1967. Since then, many pulsars and magnetars have been observed as neutron stars. A neutron star is an object of radius about 10 km, and most have masses of approximately 1.4 times the solar mass (M⊙). However, the neutron star PSR J1614-2230 forming a binary system with a white dwarf was discovered in 2010, whose observed mass was 1.97 ± 0.04 M⊙[1]. In 2013, the neutron star PSR J0348+0432 with mass of 2.01 ± 0.04 M⊙ was observed, and the existence of neutron stars which have masses twice the solar mass is established beyond doubt [2]. Recently an analysis of the gravitational wave indicates a massive neutron star with mass between 2.01 ± 0.04 M⊙ and 2.16 ± 0.17 M⊙[3]. Such very heavy neutron stars give a strong limit on equations of state, for which typical mass of 1.4 M⊙ is standard.

The presence of such massive neutron stars can be easily described if the neutron star matter consists of only nucleons interacting through two or three body repulsive nuclear forces. However, hyperons should appear naturally in the inner core of the neutron star where its density is a few times higher than the nuclear saturation density. The appearance of hyperons softens the equation of state (EoS) of neutron star matter, and makes it difficult to explain the presence of the massive (2M⊙) neutron stars. This problem is referred to as the hyperon puzzle.

In our previous study[5], the mass-radius (MR) relation of deformed neutron stars in the axially symmetric poloidal magnetic field was calculated. The MR relations were obtained by solving the Hartle equations, whereas the relation for spherical stars was obtained by solving the Tolman-Oppenheimer-Volkoff equations. The anisotropic effects of the poloidal magnetic fields were found to be non-negligible for a strong magnetic field with more than 3 × 10¹⁸G at the...
center of a neutron star. In this study, we consider magnetic fields or/and rotation. From the observation, there exist strong magnetized neutron stars, called magnetars, having a magnetic field of about $2 \times 10^{15}$ G on the surface [4]. Also a rotational neutron star with 716 Hz frequency has been observed [6]. Moreover, observed radii of neutron stars also give a good indication for giving NS a strong limit. We denoted the radius of a neutron star as $R_{NS}$. $R_{NS}$ is less than 13.6 km, which is derived from the observation of the GW170817 gravitational wave event [7]. We calculate radii and masses for 5 hadronic EoSs. We compare the mass-radius relation of various EoSs for magnetized neutron stars and rotational neutron stars. We also calculate radii and masses of hybrid stars. We use the MIT bag model for quark matter (inner core). The MIT bag model is one of the models describing the properties of hadrons, which was proposed in 1974 by a group of researchers in the MIT (Massachusetts Institute of Technology).

2. Formulations

2.1. Equation of State

In this paper the neutron star matter is assumed to be static and uniform in the high density region, which is described in the relativistic mean field (RMF) theory based on the nonlinear Walecka model. The Lagrangian is given as [8, 9, 10, 11, 12, 14, 15, 16]

$$\mathcal{L} = \sum_b \mathcal{L}_b + \mathcal{L}_m + \sum_l \mathcal{L}_l + \mathcal{L}_{em},$$  (1)

where

$$\mathcal{L}_b = \bar{\psi}_b \left( i\gamma_\mu \partial^\mu - m_b + g_{\sigma b} \sigma + g_{\tau b} \tau \cdot \mathbf{P}^\mu + g_{\omega b} \gamma_\mu \omega^\mu - g_{\phi b} \gamma_\mu \phi^\mu - g_{\rho b} \gamma_\mu \rho^\mu \right) \psi_b,$$

$$\mathcal{L}_m = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) + \frac{1}{2} \left( \partial_\mu \sigma^\ast \partial^\mu \sigma^\ast - m_{\sigma^\ast}^2 \sigma^{\ast 2} \right) + \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \phi_\mu \phi^\mu - \frac{1}{4} \Phi_{\mu \nu} \Phi^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} \mathbf{P}^{\mu \nu} \cdot \mathbf{P}_{\mu \nu} - \frac{1}{3} b m_n \left( g_{\sigma \sigma} \right)^3 - \frac{1}{4} c g_{\sigma \sigma},$$

$$\mathcal{L}_l = \bar{\psi}_l \left( i\gamma_\mu \partial^\mu - m_l \gamma_\mu A^\mu - m_l \right) \psi_l,$$

$$\mathcal{L}_{em} = -\frac{1}{4} \mathbf{F}^{\mu \nu} \mathbf{F}_{\mu \nu}.$$  (5)

Here $b, m, l,$ and $em$ indicate baryons, mesons, leptons, and photons, respectively. The field strengths are explicitly given as

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$  (6)

$$\Omega_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,$$  (7)

$$\Phi_{\mu \nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu,$$  (8)

$$\mathbf{P}_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - g_\rho \rho_\mu \times \rho_\nu.$$  (9)

Here, $\tau/2$ represents the isospin operator and $\sigma_{\mu \nu} = i/2 [\gamma_\mu, \gamma_\nu]$. The baryon octet $\{p, n, \Lambda, \Sigma^0, \Sigma^\pm, \Xi^0, \Xi^\pm\}$, the electron, and the muon are taken into account for fermions.
Table 1. Properties of each EoS [12, 29].

|       | GM1 | TM1-a | TM1-b | TM2-ωρ-a | TM2-ωρ-b |
|-------|-----|-------|-------|----------|----------|
| ρ (fm$^3$) | 0.154 | 0.146 | 0.146 | 0.146    | 0.146    |
| $B/A$ (MeV) | -16.3 | -16.3 | -16.3 | -16.4    | -16.4    |
| $K$ (MeV)   | 300.7 | 281.2 | 281.2 | 281.7    | 281.7    |
| $J$ (MeV)   | 32.5  | 36.9  | 36.9  | 32.1     | 32.1     |
| $L$ (MeV)   | 94.4  | 111.2 | 111.2 | 54.8     | 54.8     |
| $K_{sym}$ (MeV) | 18.1 | 33.8  | 33.8  | -70.5    | -70.5    |

In this model, the scalar-meson $\sigma$, the vector-meson $\omega$, and the vector-isovector-meson $\rho$ with masses of $m_\sigma = 511.198$ MeV, $m_\omega = 783.0$ MeV, and $m_\rho = 770.0$ MeV are introduced. The coupling constants of nucleons with these mesons, $g_{\sigma N}$, $g_{\omega N}$, and $g_{\rho N}$, and some self-interactions among mesons are determined by fitting the physical quantities at the saturation density [8, 9, 10]. The coupling constants of hyperons with these mesons and hidden-strangeness mesons, $\sigma^*$ and $\phi$, are determined by fitting the properties of hypernuclei in the quark model [12]. The RMF EoS is used to describe the denser region, where its density is over the neutron drip density $\rho_{ND} = 2.51 \times 10^{-4}$ fm$^{-3}$. The neutron drip density is predicted by the HFB-25 Brussels-Montreal nuclear mass model [17]. The neutron drip density might be changed in the presence of a strong magnetic field, but the MR relations of neutron stars are not so sensitive to the neutron drip density. In order to describe the lower density region, the Baym-Pethick-Sutherland (BPS) EoS is used [18] with the atomic masses given in Ame2012 [19, 20] and HFB-24 [22].

2.2. Magnetic fields

In this paper we adopt a density-dependent magnetic field strength given by [21, 16]

$$B(\rho) = B_s + B_0 \left[ 1 - \exp \left\{ -\alpha \left( \frac{\rho}{\rho_0} \right)^\gamma \right\} \right],$$

(10)

where $B_s$ indicates the strength on the surface and $B_0$ indicates the one in a much denser region than that of the saturation number density $\rho_0$ (0.153 fm$^{-3}$). Here the parameters $\alpha = 0.05$ and $\gamma = 2$ are adopted [16]. In the following the value of $B_s$ is fixed constant to be $10^{15}$ G. We use spherically symmetric magnetic pressure (SSMP) for magnetic fields [5].

2.3. Hartle Equations

A theoretical method to calculate masses and eccentricities of axially deformed objects due to slow rotations was first introduced by J. B. Hartle and others in Ref. [24, 25, 26, 27] in the framework of General Relativity. The metric for such an object can be written as

$$ds^2 = -e^\nu \left[ 1 + 2 \{ h_0 + h_2 P_2(\cos \theta) \} \right] dt^2 + e^\lambda \left[ 1 + 2 \frac{e^\lambda}{r} \{ m_0 + m_2 P_2(\cos \theta) \} \right] dr^2 + \sigma^2 \left[ 1 + 2k_2 P_2(\cos \theta) \right] \left[ d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2 \right],$$

(11)
where $\omega(r, \theta)$ represents the local angular velocity of a rotating star, and $h_0(r)$, $h_2(r)$, $m_0(r)$, $m_2(r)$, and $k_2(r)$ are the second order perturbative terms with respect to the angular velocity $\Omega$. The second order Legendre polynomial is given as $P_2(\cos \theta) = \frac{1}{2}(3\cos^2 \theta - 1)$.

3. Results and Discussion

3.1. Hyperon star with and without rotation and magnetic fields

In this study, we employ various kinds of EoSs. We choose 5 hadronic EoSs, namely, GM1, TM1-a, TM1-b, TM2-$\omega$-$\rho$-a, and TM2-$\omega$-$\rho$-b [10, 11, 12, 13], where the basic properties of various EoSs are given. Figure 1 shows the mass versus radius relation of a neutron star for each EoS in various situations. These figures show the total mass of a neutron star as a function of its radius. These are so-called the MR relations.

The left upper panel of Fig.1 shows those cases without rotation and magnetic fields, the right upper panel shows that with rotation, the left bottom panel shows that with magnetic fields, and the right bottom panel is that with both rotation and magnetic fields. Here, in the rotational case, $\Omega$ is assumes to be $0.03$ km$^{-1}$. In the magnetized case, $B_0$ is assumed to be $2 \times 10^{18}$ G. The yellow solid line and orange solid line indicate the pulsars, PSR J1614-2230 and PSR J0348+0432, respectively. The colors of lines indicate EoSs as follows, dark blue; GM1 EoS, green; TM1-a EoS, light blue; TM1-b EoS, purple; TM2-$\omega$-$\rho$-a EoS, red; TM2-$\omega$-$\rho$-b EoS.

We obtain neutron star masses more than twice the solar mass ($>2M_\odot$) either in the strong magnetic field of $2 \times 10^{18}$ G in the center or in the rapid rotation of $\Omega = 0.03$ km$^{-1}$ in the gravitational unit in the TM2-$\omega$-$\rho$-b EoS. Furthermore, we obtain neutron stars with masses more than $2M_\odot$ both with a strong magnetic field and in a rapid rotation for 5 hadronic EoSs. However, this rotation ($\Omega = 0.03$ km$^{-1}$) corresponds to a neutron star revolving at about 13,700 Hz at maximum mass, so it is not realistic. We also assume $\Omega$ to be $0.01$ km$^{-1}$ and the mass in rotation slightly increases. This case does not give masses over twice the solar mass. It is apparent, however, that if one makes the frequency larger, the mass increases more.

Now, let us summarize the neutron star radius at $1.4$ M$_\odot$ in Table 2. Here, $R$ indicates radius without rotation and magnetic fields, $R_{\text{rot}}$, radius with rotation, $R_{\text{mag}}$, radius with magnetic fields, and $R_{\text{rot&mag}}$, radius with both rotation and magnetic fields for each EoS. From the observation [7], the maximum of the upper limit radius is 13.6 km. Therefore from Table 2, TM1-a and TM1-b EoSs are out of range of observation.

| EoS            | $R$ (km) | $R_{\text{rot}}$ (km) | $R_{\text{mag}}$ (km) | $R_{\text{rot&mag}}$ (km) |
|----------------|----------|------------------------|------------------------|-----------------------------|
| GM1            | 13.46    | 13.34                  | 13.77                  | 13.49                       |
| TM1-a          | 14.07    | 14.12                  | 14.33                  | 14.26                       |
| TM1-b          | 14.10    | 14.12                  | 14.33                  | 14.26                       |
| TM2-$\omega$-$\rho$-a | 13.23    | 13.02                  | 13.47                  | 13.21                       |
| TM2-$\omega$-$\rho$-b | 13.24    | 13.02                  | 13.47                  | 13.21                       |

3.2. Hybrid star with and without rotation

Here, we discuss about hybrid stars. Hybrid star is the neutron star which has quark matter in the inner core and hadronic matter in the outer core. We use the MIT bag model for the quark matter. The left panel of Fig.2 shows the MR relation of hybrid star and the right panel shows the MR relation of hybrid star with rotation. Here, for the rotation, $\Omega$ is assumed to be 0.02 km$^{-1}$. From the right panel of Fig.2, we can see that all 5 EoSs give a mass over twice the solar mass.
Figure 1. Various MR relations of hadronic stars. The left upper panel shows those cases without rotation and magnetic fields, the right upper panel is that with rotation ($\Omega = 0.03\text{ km}^{-1}$), the left bottom panel is that with magnetic fields ($B_0 = 2 \times 10^{18}\text{ G}$), and the right bottom panel is that with both rotation ($\Omega = 0.03\text{ km}^{-1}$) and magnetic fields ($B_0 = 2 \times 10^{18}\text{ G}$).

Figure 2. Various MR relations of hybrid stars. The left panel shows the case without rotation, and the right panel shows the one with rotation. The colors of the line is the same as Fig.1.

4. Summary
We calculated the mass-radius relations for magnetized and rotating neutron stars using various kinds of EoSs. We obtained neutron star masses more than twice the solar mass ($2M_\odot$) either in
the strong magnetic field of $2 \times 10^{18} \text{ G}$ in the center or in the rapid rotation of $\Omega = 0.03 \text{ km}^{-1}$ in the TM2-$\omega_{\rho}$-b EoS. Furthermore, we obtained neutron stars with masses more than $2M_\odot$ both with a strong magnetic field and in a rapid rotation for 5 hadronic EoSs. From radius of $1.4M_\odot$, GM1, TM2-$\omega_{\rho}$-a and TM2-$\omega_{\rho}$-b EoSs are in the range of observation. Moreover, we obtained hybrid stars with masses more than $2M_\odot$ in a rapid rotation for all 5 EoSs.

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