Nuclei cross sections in Extensive Air Showers

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Abstract Cross sections for proton inelastic collision with different nuclei are described within the Glauber approximation. The significant difference between approximate “Glauber” formula and exact calculations with geometrical scaling assumption is shown for very high energy cross section calculations. According to obtained results values of proton–proton cross section reported by the Akeno and Fly’s Eye experimental groups are about 10% overestimated.

The rise of the proton-proton cross section (total, inelastic) as the interaction energy increases is an important feature of the strong interaction picture. The growth itself is established quite well both from theoretical and experimental point of view. However the question how fast do cross sections rise is discussed permanently. A definite answer is still lacking. Theoretical predictions agree well with one another and with accelerator data in the region where data exists ($\sqrt{s} \approx 20 \div 2000$ GeV) but they differ above. Before the LHC shifts the direct measurements limit to $10 \sim 14$ TeV the only existing information can be derived from the cosmic ray extensive air shower (EAS) data.

The important difference between the collider and EAS proton–proton cross section measurements is that in fact the proton–air interactions are involved in the EAS development thus the value which is real measured is the cross section for the interactions with air nuclei. The value of proton–proton cross section is obtained from it using a theory for nuclei interactions. In most of recent papers concerning this subject it can be found just a few sentences like “calculations have been made in the standard Glauber formalism” or something very similar [1, 2].

The original Glauber paper [3] has been published about forty years ago. Rather complicated equations for scattering cross sections can be simplified significantly applying some additional assumptions which validity is limited. We would like to compare results of calculations with and without these simplifications. The exact Glauber formalism will be used to evaluate the proton–proton cross section values from the Akeno and Fly’s Eye data.

For the scattering of particle on the close many particle system (nucleus), if each interaction can be treated as a two particle one, the overall phase shift for incoming wave is a sum of all two-particle phase shifts.

$$\chi_A(b, \{d\}) = \sum_{j=1}^{A} \chi_j(b - d_j)$$

where $\{d\}$ is a set of nucleon positions in the nucleus ($d_j$ is a position of the $j$th nucleon in the plane perpendicular to the interaction axis). The equation (1) is the essence of Ref.[3] and in fact it defines the Glauber approximation. The scattering amplitude is thus given by

$$S(t) = \frac{i}{2\pi} \int e^{itb} d^2b \int |\psi(\{d\})|^2 \left\{ 1 - e^{i\chi_A(b, \{d\})} \right\} \prod_{j=1}^{A} d^2d_j,$$  (2)
where \( \psi \) describes the wave function of the nucleus with nucleons distributed according to \( \{d\} \). If one neglect position correlations of the nucleons and denotes by \( \varrho_j \) each single nucleon density and if all interactions can be described by the same phase shift function \( \chi \) then

\[
S(t) = \frac{i}{2\pi} \int e^{itb} d^2b \left\{ 1 - \prod_{j=1}^{A} \varrho_j(b_j) e^{i\chi(b-d_j)_j} d^2d_j \right\}.
\]

(3)

On the other hand, the scattering process can be treated as a one collision process with its own nuclear phase shift \( \chi_{\text{opt}}(b) \)

\[
S(t) = \frac{i}{2\pi} \int e^{itb} \left\{ 1 - e^{i\chi_{\text{opt}}(b)} \right\} d^2b.
\]

(4)

The comparison with Eq. (2) gives

\[
e^{i\chi_{\text{opt}}(b)} = \left\langle \psi(\{d\}) \right| e^{i\sum_{j=1}^{A} \chi_j(b-d_j)_j} \prod_{j=1}^{A} d^2d_j \right\rangle \left( e^{i\chi(b, \{d\})} \right),
\]

(5)

where the \( \langle \rangle \) means the averaging over the all possible configuration of nucleons \( \{d\} \). To go further with the calculations of \( \chi_{\text{opt}} \) the commonly used assumption has to be made. If we assume that the number of scattering centers \( (A) \) is large (with the transparency of the nucleus as a whole constant) then

\[
\chi_{\text{opt}}(b) = i \int d^2d \rho_A(d) \left[ 1 - e^{i\chi(b-d)} \right].
\]

(6)

where \( \rho_A \) is the distribution of scattering centers (nucleons) positions in the nucleus (\( \sum \varrho_j \)). When the individual nucleon opacity \( |1 - e^{i\chi(b)}| \) is a very sharply peaked compared with \( \rho_A \) then with the help of the optical theorem the simple formula for the scattering amplitude can be found. The proton nucleus inelastic cross section is thus

\[
\sigma_{\text{inel}}^{\text{pp}} = \int d^2b \left[ 1 - e^{-\sigma_{\text{inel}}^{\text{pp}} \rho_A(b)} \right] = \int d^2b \left\{ 1 - \left[ 1 - \sigma_{\text{tot}}^{\text{pp}} \rho_A/A \right]^A \right\}.
\]

(7)

This result is often but not quite correctly called “the Glauber approximation”.

The important point of this paper is to show how the point-nucleon approximation changes the results. Formulas given in Eq. (6) are in fact in agreement with the factorization hypothesis for individual nucleon–nucleon \( \chi \) function \( \chi(s, b) = i\omega(b)f(s) \). According to it nucleons are getting blacker as the interaction energy increases. However, for some time (see, e.g., Ref. [4]) it is known that this is not the case. Experimental data strongly favour the geometrical scaling hypothesis \( \chi(s, b) = i\omega(b/b_0(s)) \) (see, e.g., Ref. [5]) which treats nucleons as getting bigger. Nucleus profiles obtained using exact Glauber formula (Eq. (6)) differ from the \( A\sigma_{\text{inel}}^{\text{pp}} \rho_A/A \) suggested by Eq. (7). The difference can be seen clearly in Fig. 1.

The significant change of the nucleus size have to influence the value of inelastic cross section. Fig. 2 shows the change of inelastic cross section of proton–nucleus with the interaction energy calculated using geometrically scaled and factorized nucleus profiles. As it can be seen the difference at very high energies is remarkable.

The results presented above indicate the importance of reexamination of the proton–proton cross section estimation based on proton–air data measured in EAS experiment.
The conversion from proton–air to proton–proton cross section presented in Fig. 3 is obtained using the exact Glauber formalism. The original Fly’s Eye estimation of proton–proton total cross section given in Ref. [2] is 120 mb while according to results given in Fig. 3 it is equal to 110 mb. The same procedure should be applied to the Akeno data.

In Fig. 4 the calculated proton–air cross section energy dependence is given. The solid line represents result obtained using exact Glauber formula [Eq. (5)] and proton–proton phase shift $\chi$ function described in Ref. [6]. The outcome of the “simplified Glauber” approach [Eq. (7)] is given for a comparison by the long dashed line.

Concluding, we have shown that the geometrical scaling hypothesis with the exact Glauber formalism gives the value of proton–proton total cross section at about 30 TeV about 10% smaller than that reported in original Fly’s Eye and Akeno papers. Results presented in this work have been obtained using the following cross section energy dependence

$$\sigma_{\text{inel}}(s) = 32.4 - 1.2 \ln(s) + 0.21 \ln^2(s),$$

which fits quite well accelerator measurements as well as the EAS points diminish by $\sim 10%$.

The rise of the $\sigma_{A\text{-air}}$ predicted by the geometrical scaling hypothesis with the exact Glauber formalism is significantly faster than the one which can be obtained using simplified formulas (Fig. 2). This can change the physical conclusions based on Monte Carlo simulations of the EAS development at very high energies.

References

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Fig.1. Nitrogen nucleus profile (phase shift: $\chi_{p-N}$) obtained using the exact Glauber formula with geometrical scaling (solid lines) and factorization hypothesis (simplified Glauber) (dashed lines) for different interaction energy (per nucleon–nucleon collision).

Fig.2. Cross section of collisions of different nuclei with the “air nucleus” calculated using exact Glauber formula with geometrical scaling (solid lines) and simplified formulas (dashed lines) as a function of interaction energy.

Fig.3. Relationship between inelastic proton–air cross section and the value of proton–proton cross section (inelastic - solid curve and total - dashed one) calculated using exact Glauber formula with geometrical scaling. Solid horizontal line represents the value measured in Fly's Eye [2] experiment (dashed area shows 1σ bounds).

Fig.4. Inelastic proton–air cross sections calculated using exact Glauber formula with geometrical scaling as a function of interaction energy (per nucleus). Experimental points are from Akeno and Fly’s Eye experiments (squares and the circle, respectively). Results of calculations with the “simplified Glauber” formula with the same proton–proton cross section energy dependence are given by the long dashed line. The default CORSIKA [7] proton–air cross section is shown for a comparison by the short dashed line.