Pionic coupling constants of heavy mesons in the quark model

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We analyse pionic couplings of heavy mesons combining PCAC with the dispersion quark model to calculate the relevant transition form factors. Ground states and radial excitations are considered. For the ground state coupling constants the values \( \hat{g} = 0.5 \pm 0.02 \) in the heavy quark limit, and \( g_{B^+D^0} = 40 \pm 3, g_{D^0D^+} = 16 \pm 2 \) are obtained. A sizeable suppression of the coupling constants describing the pionic decays of the radial excitations is observed.

Pionic coupling constants of heavy mesons are basic quantities to understand the heavy meson lifetimes: they govern strong decays of heavy mesons by the emission of a pion which in practice turn out to be most important strong decay modes. The knowledge of the pionic coupling constants of the ground-state as well as the excited heavy mesons is timely in particular in view of the new experimental data on the excited heavy mesons \( 1 \).

The heavy quark (HQ) symmetry provides important relations between the coupling constants of pionic transitions of various mesons containing a heavy quark. In the leading \( 1/m_Q \) order (LO) the heavy quark spin decouples from other degrees of freedom \( 2 \). Thus in the limit of an infinitely heavy spectator quark, the respective pionic decay rates for mesons with different spins but same quantum numbers of the light degrees of freedom (total momentum of the light degrees of freedom \( j \), angular momentum \( L \), etc) \( 3 \) are equal.

Note, however that the heavy quark expansion which provides many rigorous results for semileptonic transitions between heavy mesons, turns out to be much less efficient in the case of the strong transitions. In fact, these two types of processes have quite different dynamics. Semileptonic transitions of heavy mesons are induced by the heavy quark weak transition and in this case HQET predicts not only the structure of the \( 1/m_Q \) expansion of the long-distance (LD) contributions to the form factors, but also provides important absolute normalization of the LO form factors. On the other hand, in the case of strong pionic decay the heavy quark remains spectator and the dynamics of the process is determined by the light degrees of freedom. As a consequence HQ symmetry does not provide any absolute normalization, thus making the pionic decays of heavy mesons a more complicated problem to analyse compared with semileptonic decays.

The amplitudes of the strong pionic vector-to-vector and pseudoscalar-to-vector transitions have the structure

\[
\begin{align*}
(V(p_2)\pi(q)|P(p_1)) &= g_\nu \epsilon_\nu(p_2)g_{V\pi} \\
(V(p_2)\pi(q)|V(p_1)) &= \epsilon_\nu(p_1)\epsilon_\nu(p_2)p_{1\alpha}p_{2\beta} \epsilon^{\alpha\beta} g_{VV\pi},
\end{align*}
\]

where \( \epsilon \) denotes the polarization vector of the vector meson. The coupling constants can be expanded in a \( 1/m_Q \) series as follows

\[
\begin{align*}
\frac{f_\pi}{2\sqrt{\mp M_P M_V}} g_{V\pi} &= \hat{g}_{V\pi} = \hat{g} + \hat{g}_{V\pi}^{(1)} m_Q + \ldots, \\
\frac{f_\pi}{2} g_{VV\pi} &= \hat{g}_{VV\pi} = \hat{g} + \hat{g}_{VV\pi}^{(1)} m_Q + \ldots,
\end{align*}
\]

and due to the HQ symmetry in the leading \( 1/m_Q \) order both constants are governed by the same quantity \( \hat{g} \), whereas the higher order terms are different.

The pionic coupling constants for heavy mesons \( g_{DD^\ast\pi} \) and \( g_{BB^\ast\pi} \) have been analysed within various versions of the constituent quark model \( 4 \), sum rules \( 5 \) and using the lattice simulations \( 6 \).

The results of the quark model strongly depend on the particular version of the QM used: the values of \( \hat{g} \) range from 1 (nonrelativistic quark model) to 1/3 (the Salpeter equation with massless quarks) \( 1 \). LCSR \( 5 \) obtained a small value \( 0.32 \pm 0.02 \). Recent lattice simulations reported 0.42 with however large statistical errors \( \pm 0.04 \pm 0.08 \). Thus at the moment no reliable predictions are available even for the coupling constants of the ground state mesons. An analysis of the pionic decays of radially excited states is almost lacking. This calls for further investigation and a better understanding of the problem.

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Although the present QM results are strongly scattered, one feature of this approach still seems to be very attractive: namely, various processes can be connected to each other via the wave functions of the mesons involved, once a proper formulation of the QM is used. We believe that such a proper formulation should (i) be based on a relativistic consideration and (ii) reproduce rigorous QCD results in the known limits, e.g. in the limit when the meson decay is induced by the HQ transition. The dispersion formulation of ref [3] was shown to satisfy these requirements. Recently, we have applied this dispersion approach to analyse the $B \rightarrow \pi$ transition form factors and determined the relevant quark model parameters and the wave function of the $B$ meson [3]. Once the whole framework and the numerical parameters are fixed we expect to perform a reliable analysis of the pionic coupling constants of the heavy mesons.

An additional attractive feature of this approach is that it is straightforward to incorporate the excited states, in particular radial excitations.

The dispersion approach is based on the spectral representations of the form factors through the wave functions of the initial and final mesons. The double spectral densities are obtained from the relevant Feynman graphs, whereas the subtraction terms remain apriori ambiguous and should be determined from some other arguments. In ref [7] the subtraction terms have been determined such that the correct structure of the $1/m_Q$ expansion in the leading and subleading orders is reproduced. Such a procedure of determining the subtraction terms provides a reliable description of the form factors if the decaying quark is heavy for both heavy and light quarks produced in the weak decay. However such a strategy might be not efficient for the case when the interacting quark is light but the spectator is heavy, that is exactly the case of the pionic decay of a heavy meson, and requires proper modification.

In this letter we analyse the pionic transitions of the ground state and radially excited heavy mesons, studying in parallel the $V \rightarrow V\pi$ and $V \rightarrow P\pi$ cases. We make use the PCAC which allows one to reduce the calculation of the complicated amplitude involving three hadrons to a considerably more simple amplitudes of the $P \rightarrow V$ and $V \rightarrow V$ transitions induced by the light-quark axial-vector current. The corresponding amplitudes have the following structure

$$
\langle V(p_2)|A_\mu|V(p_1)\rangle = i\epsilon_\alpha(p_1)\epsilon_\beta(p_2)\epsilon_{\mu\alpha\beta}P_\nu h(q^2)/2 + \ldots,
\langle V(p_2)|A_\mu|P(p_1)\rangle = i\epsilon_\nu(p_2) [g_{\mu\nu}f(q^2) + a_+ (q^2)] P_\mu q_\nu + a_-(q^2)q_\mu q_\nu],
$$

where $q = p_1 - p_2$, $P = p_1 + p_2$, and the dots denote terms transverse with respect to $q_\mu$ the particular form of which is not important for us. The form factors $a_-$ and $h$ contain a pole at $q^2 = m^2_\pi$ due to the contribution of the intermediate pion state in the $q^2$-channel, and the residues in these poles are expressed through the pionic coupling constants $g_{VP\pi}$ and $g_{VV\pi}$ as follows

$$
h(q^2) = \frac{f_{\pi}q^2g_{VV\pi}}{m^2_\pi - q^2} + \tilde{h}(q^2)
$$

$$
a_-(q^2) = \frac{f_{\pi}g_{VV\pi}}{m^2_\pi - q^2} + \tilde{a}_-(q^2),
$$

where $\tilde{h}$ and $\tilde{a}_-$ are regular at $q^2 = m^2_\pi$.

In the $V \rightarrow V$ case, we find that the function $\Phi_V(q^2) = h(q^2)(m^2_\pi - q^2)$ is smooth between the point $q^2 = 0$ and $q^2 = m^2_\pi$ and due to the smallness of the pion mass we assume that $\Phi_V(0) \simeq \Phi_V(m^2_\pi)$. This yields the relation

$$
g_{VV\pi} = h(0)/f_{\pi}.
$$

For the $P \rightarrow V$ case an additional step is necessary: taking first the divergence of the axial-vector current we find

$$
\langle V(p_2)|q_\mu A_\mu|P(p_1)\rangle = q_\nu \epsilon_\nu(p_2) \Phi_P(q^2) \frac{m^2_\pi}{m^2_\pi - q^2}
$$

with $\Phi_P(q^2) = [f(q^2) + (p^2_1 - p^2_2)a_+(q^2) + q^2\tilde{a}_-(q^2)] (m^2_\pi - q^2) + q^2f_{\pi}g_{VP\pi}$. A standard PCAC assumption $\Phi_P(m^2_\pi) \simeq \Phi_P(0)$ yields

$$
g_{PV\pi} = \frac{1}{f_{\pi}} [f(0) + (p^2_1 - p^2_2)a_+(0)].
$$

Notice that the pion should not be necessarily soft and the relations [3] and [4] can be readily applied to the case when the masses $M_1$ and $M_2$ are substantially different as e.g. for the transition between the ground-state and radially-excited heavy mesons.

The relations [3] and [4] represent the coupling constants of interest through the transition form factors, for which we apply the dispersion approach.
I. SPECTRAL REPRESENTATIONS OF THE COUPLING CONSTANTS

We start with considering the spectral representation of the form factors along the lines of ref [7]. We apply this method to the case of the heavy spectator quark and propose proper modifications to calculate the pion coupling constants.

The dispersion approach gives the transition form factors of the meson $M_1$ to the meson $M_2$ as double relativistic spectral representations in terms of the soft wave functions $\psi_1(s_1) = G_1(s_1)/(s_1 - M_1^2)$ and $\psi_2(s_2) = G_2(s_2)/(s_2 - M_2^2)$, of the initial and final mesons, respectively. To be specific, in the case of the weak decay $P_Q \to V'_q$ induced by the weak quark transition $Q \to q'$ where $q'$ might be both heavy and light the form factor can be represented as follows

\[
f(q^2) = \int \frac{ds_1}{s_1 - M_1^2} \frac{ds_2}{s_2 - M_2^2} \left[ \hat{f}_D(s_1, s_2, q^2) + ((s_1 - M_1^2) + (s_2 - M_2^2)) \hat{g}_D(s_1, s_2, q^2) + (s_1 - M_1^2)\xi_1(s_1, s_2) + (s_2 - M_2^2)\xi_2(s_1, s_2) \right],
\]

where $\hat{f}_D = \text{disc}_s \text{disc}_x f_D(s_1, s_2, q^2)$ and $\hat{g}_D = \text{disc}_s \text{disc}_x g_D(s_1, s_2, q^2)$ are directly calculated from the triangle Feynman graph with pointlike vertices as double spectral densities of the relevant form factors. The functions $\xi_1$ and $\xi_2$ are not known precisely but are known to behave as $1/m_Q$ in the limit $m_Q \to \infty$, where $m_Q$ is the mass of the initial heavy quark. It is important to point out at the hierarchy of different terms in eq (7) in case the initial active quark is heavy: namely, the term proportional to $\text{disc}_s \text{disc}_x f_D$ contributes in the LO (and all other orders as well), the subtraction term proportional to $\text{disc}_s \text{disc}_x g_D$ contributes starting from the subleading order, and the subtraction terms proportional to $\xi$ contribute only in the higher orders. So this expansion works well if the initial active quark is heavy but the spectator quark is light, both for the cases of the light and heavy active quark in the final state.

However in case the spectator is heavy and the active quark is light, the subtraction term proportional to $\hat{g}_D$ as well as the functions $\xi$ contain powers of the spectator mass, and all terms have the same order of magnitude and thus should be considered on an equal footing. Hence, the expansion (7) becomes ineffective in practice and the procedure requires a modification.

To find such proper modification it is convenient to start with considering the $V_1 \to V_2$ transition induced by the axial-vector current. In the quark model the process is represented by the triangle graph with relevant vertices describing the quark structure of the vector meson states. The procedure described in detail in ref [7] yields for the $g_{V_1V_2\pi} = h(q^2 = 0)/f_\pi$ the following spectral representation

\[
g_{V_1V_2\pi} = \frac{m_1}{4\pi^2 f_\pi} \int_{(m_1 + m_3)^2}^\infty ds \Psi_{V_1}(s)\Psi_{V_2}(s) \times \left[ m_1 \log(r) + (m_3 - m_1)^{1/2} s + \frac{1}{\sqrt{s + m_1 + m_3}} \left( s + m_3^2 - m_3^2 \lambda^{1/2} - \frac{m_1^2}{2} \log(r) \right) \right],
\]

with $r = (s + m_1^2 - m_3^2 + \lambda^{1/2})/(s + m_1^2 - m_3^2 - \lambda^{1/2})$, $\lambda = (s - m_1^2 - m_3^2)^2 - 4m_1^2m_3^2$, and $m_1$ and $m_3$ the masses of the active and the spectator constituent quarks, respectively (we follow the notations of ref [8]).

This spectral representation can be rewritten in a more conventional form as an integral over the light-cone variables as follows

\[
g_{V_1V_2\pi} = \frac{m_1}{2\pi^2 f_\pi} \int \frac{dxdk^2}{x(1-x)^2} \Psi_{V_1}(s)\Psi_{V_2}(s) \left[ m_1 x + m_3 (1-x) + \frac{2k^2}{\sqrt{s + m_1 + m_3}} \right].
\]

Here $s = \frac{m_1^2}{2} + \frac{m_3^2}{2} + \frac{k^2}{x(1-x)}$, and $x$ and $1-x$ are the fraction of the light-cone momentum carried by the spectator and the active quark, respectively.

The expression (8) is obtained as a double dispersion representation in the invariant masses of the initial and final $qq$ pairs, the spectral density of which is calculated from the triangle Feynman graph. At $q^2 = 0$ it is reduced to a simple form (8). In principle subtraction terms can be added to this double spectral representation. However, in the case of the $V \to V$ transitions there are no reasons dictating a necessity of subtractions, and we assume that the subtraction terms are absent.

The representation (9) describes the $\pi_0 \to 2\gamma$ decay if we set $m_1 = m_3$ and take the quark-photon vertex in the form $\bar{q} \gamma_\mu q$. In this case only the first term in (9) is present, and the photon wave function corresponding to the
pointlike interaction reads \( \Psi_\gamma(s) = 1/s \). So, \( g_{V\gamma\pi} \) becomes independent of the quark mass and we simply reproduce the value of the axial anomaly \(^{10}\) from the imaginary part of the triangle graph (cf \(^{11}\) and refs therein).

The axial-vector current satisfies the equation of motion \( \partial_\mu A_\mu = 2m_J j_5 \), so the coupling constant \( g_{V,V\pi} \) can be equivalently calculated directly from the amplitude \( \langle V_3(p_2)|2m_J j_5(0)|V_1(p_1) \rangle = \varepsilon_\alpha(p_1)\varepsilon_\beta(p_2)\delta_{\alpha\beta} g_{P\pi} h(q^2)/2 \).

We proceed the same way to analyse the \( P \to V \) transition: instead of calculating the full representations for \( f \) and \( a_+ \) and then taking their linear combination \(^{12}\), we focus on the amplitude \( \langle V(p_2)|2m_J j_5|P(p_1) \rangle \). Similar to the \( V \to V \) case, this amplitude provides the correct double spectral density of the spectral representation for \( g_{P\pi} \) which takes the form

\[
g_{P\pi} = \frac{m_1}{4\pi^2 f_\pi} \int_{(m_1 + m_3)^2}^\infty ds \frac{\Psi_P(s)\Psi_V(s)}{(s-(m_1-m_3)^2)^{1/2}} \times \left[ (s-(m_1-m_3)^2)\log(r) - \left( 1 + \frac{2m_1}{\sqrt{s} + m_1 + m_3} \right) \left( (s+m_1^2-m_3^2)\log(r) - 2\lambda^{1/2} \right) \right]
\]

\[
= \frac{m_1}{4\pi^2 f_\pi} \int \frac{dxdk^2}{x(1-x)^2} \frac{\Psi_P(s)\Psi_V(s)}{x(1-x)^2} \left[ (s-(m_1-m_3)^2) - \frac{k_1^2}{1-x} \left( 1 + \frac{2m_1}{\sqrt{s} + m_1 + m_3} \right) \right]. \quad (10)
\]

The form of the relations \(^{13}\) and \(^{14}\) however prompts that in distinction to the \( V \to V \) transition, in the \( P \to V \) transition the subtraction term is nonzero. The latter cannot be determined uniquely within the dispersion approach. It is very important however that the HQ symmetry ensures the subtraction term to contribute only in the subleading \( 1/m_Q \) order. Namely, the HQ symmetry predicts the LO relation between the coupling constants \( g_{P\pi} = m_Q g_{V\pi} \). The double spectral densities of the representations for \( g_{V\pi} \) \(^{12}\) and \( g_{P\pi} \) \(^{15}\) satisfy this relation. Hence the subtraction terms in \( g_{V\pi} \) and \( g_{P\pi} \) should also have the same LO \( 1/m_Q \) behavior. Since the subtraction term in \( g_{V\pi} \) is absent, the subtraction term in \( g_{P\pi} \) does not contribute in the LO. Although we cannot determine the subtraction term uniquely, several reasonable ways of fixing this subtraction term yield a numerical uncertainty in \( g_{P\pi} \) to be not more than 10%.

The normalization condition for the radial wave functions \( \Psi \) of the ground state and the radial excitation of the vector and the pseudoscalar mesons has the form

\[
\frac{1}{8\pi^2} \int ds \Psi_i(s) \Psi_j(s) \frac{\lambda^{1/2}}{\delta} \left[ 1 - \frac{(m_1-m_3)^2}{(s-(m_1-m_3)^2)^{1/2}} \right] = \frac{1}{8\pi^2} \int \frac{dxdk^2}{x(1-x)^2} \Psi_i(s)\Psi_j(s)[s-(m_1-m_3)^2] = \delta_{ij}. \quad (11)
\]

It is easy to see that in the nonrelativistic (NR) limit \( |\vec{k}| = \lambda^{1/2}/2\sqrt{s} \ll m_1, m_3 \) the coupling constants take the values \( g_{P\pi}^{NR} = 2M/f_\pi \) and \( g_{V\pi}^{NR} = 2/f_\pi \). Moreover, in the NR limit the coupling constants of the transition between the ground state and the radial excitation vanish, i.e. \( g_{P\pi}^{NR} = g_{V\pi}^{NR} = 0 \).

For the analysis of the HQ expansion it is convenient to introduce a new variable \( z \) such that \( s = (z + m_1 + m_3)^2 \) and use the fact that the soft wave functions \( \phi_0 \) are localized in the region \( z \simeq \Lambda_{QCD} \). Performing the HQ expansion of all quantities including \( \Psi \) in the inverse powers of \( m_3 = m_Q \to \infty \) and keeping \( m_1 = O(\Lambda_{QCD}) \) (see \(^{3}\)) we find

\[
\hat{g}_{ij} = \frac{1}{2} \int dz \phi^i_0(z) \phi^j_0(z) m_1 \left[ m_1 \log \left( \frac{z + m_1 + \sqrt{z(z + 2m_1)}}{z + m_1 - \sqrt{z(z + 2m_1)}} \right) + 2 \sqrt{z(z + 2m_1)} \right]. \quad (12)
\]

In this expression \( \phi^i_0(z) \) are the LO soft wave function of the ground state and the radially excited mesons which satisfy the normalization condition

\[
\int dz \phi^i_0(z) \phi^j_0(z) \sqrt{z(z + 2m_1)^{3/2}} = \delta_{ij} \quad (13)
\]

Notice that the Isgur-Wise function is directly expressed through \( \phi_0 \) (see \(^{3}\)). These formulas provide a possibility to evaluate the coupling constants of interest if the numerical parameters of the model are known.

\[\text{II. NUMERICAL ANALYSIS}\]

We now provide the numerical estimates of the LO quantity \( \hat{g} \) for the ground-state and radially excited mesons. The spectral representation for this quantity is completely fixed and we need to choose the proper numerical parameters of the model. As found in many applications of the dispersion approach (see \(^{3,5}\) and refs therein), an approximation
of the soft wave function by an exponent provides reasonable estimates. Assuming the wave function to have the form
\[ \Psi(s) \approx \exp(-\tilde{K}^2/2\beta^2), \]
the LO radial wave function of the ground state reads
\[ \phi(z) \approx \sqrt{\frac{z + m_1}{z + 2m_1}} \exp\left(-\frac{z(z + 2m_1)}{2\beta^2}\right). \] (14)

Similarly, we assume for the wave function of the radial excitation the form
\[ \phi_r(z) \approx \left(1 - C_r \frac{z(z + 2m_1)}{2\beta^2}\right) \exp\left(-\frac{z(z + 2m_1)}{2\beta^2}\right), \] (15)
where the normalization factor and the coefficient \( C_r \) are fixed by the normalization conditions and the orthogonality between the wave functions of the ground state and the radial excitation.

The light-quark mass was determined from the numerical matching of the transition form factors in the \( B \to \pi, \rho \) decay to the available lattice data, and was found to be tightly restricted to the range \( m_1 = 0.23 \pm 0.01 \text{ GeV} \). Therefore the only unknown parameter is \( \beta_\infty \). The analysis of the leptonic decay constants of heavy mesons shows that simple approximate relations
\[ \beta_P(m_Q) = \beta_\infty (1 - C_P/m_Q), \quad \beta_V(m_Q) = \beta_\infty (1 - C_V/m_Q) \] (16)
with \( C_P \approx 0.1, C_V \approx 0.2 \) and \( \beta_\infty = 0.5 \) yield the values of \( \beta \) which provide reasonable leptonic constants of the charm and beauty mesons (see Table 1) calculated within the quark model through formulas given in [7,12]. The slope parameter of the IW function for such \( \beta_\infty \) is \( \rho^2 = 1.2 \pm 0.03 \) [9]. The eq (12) then yields
\[ \hat{g} = 0.5, \quad \hat{g}_r = 0.11. \] (17)

Thus for the LO quantity the orthogonality of the radial wave functions provides a strong suppression of \( \hat{g}_r \).

The results of calculating the pionic coupling constants of the \( B \) and \( D \) mesons are given in Table 2. Whereas the \( g_{lV,\pi} \) is determined quite reliably within the dispersion approach, the \( g_{lV,\rho} \) suffers from the intrinsic uncertainty of the approach based on the impossibility to fix the subtraction terms. Assuming several reasonable choices of subtraction terms in the form factors \( f, a_+ \) and \( g \) which provide the correct behaviour in the limit of the heavy active quark, but differ for the case of the heavy spectator we have found that in all cases, the difference in the coupling constants of the ground state is not more than 10%. This minor difference is easily understood taking into account that the subtraction terms contribute only in the subleading order.

Notice that there is a possibility of an alternative determination of the \( g_{lV,B\pi} \) from the form factors of the semileptonic \( B \to \pi \) transition near zero recoil. Namely,
\[ f_0(M_B^2) \approx f_0(M_{B^*}^2) = \frac{g_{lV,B\pi} f_{B^*}}{2M_{B^*}}(1 + r'(1)), \] (18)
where \( r(q^2) \) is defined as follows
\[ r(q^2) = \frac{f_-(q^2)}{f_+(q^2)} \frac{M_B^2}{M_{B^*}^2 - M_B^2}. \] (19)
The function \( r(q^2) \) satisfies the condition \( r(1) = 1 \) and is known numerically from the results of the lattice simulations in the region \( q^2 \leq 0.7 \) [11,12], in particular \( r(0.7) \approx 0.9 \). The function \( r(q^2) \) is regular near \( q^2 = 1 \) and thus there are no reasons for fast variations of this function near \( q^2 = 1 \). Then, taking into account the lattice data we can estimate \( r'(1) \leq 0.8 \). Using the current algebra relation \( f_0(M_B^2) = f_B/f_{\pi} \), neglecting the \( 1/m_b \) corrections which are small, and taking the ratio of the leptonic constants \( f_{B^*}/f_B \approx 1.2 \) we find
\[ \hat{g} = \frac{1}{f_{B^*}/f_B(1 + r'(1))} \geq 0.45 \] (20)
\[ 11 \text{Notice that in contrast to the PCAC based approaches, the NR quark-model calculations based on taking into account the quark structure of the emitted pion through the pion wave function yield bigger values of } \hat{g} \text{ and do not yield a suppression of } \hat{g}_r \text{ since the product of the orthogonal wave function is smeared by the integration with the pion wave function and as the result the orthogonality is not efficient}[13].
in agreement with [14] and with the recent result of the lattice simulation [15]. We would like to point out that the value $\hat{\gamma} \simeq 0.27$ proposed in [14] as the preferable solution found from the description of the $D^*$ decays falls far out of this estimate.

The calculation of the coupling constants of the pionic transition between the ground state and the radial excitation is more involved. In this case the LO term is itself strongly suppressed because of the orthogonality of the wave functions. So the dependence on the parameters of the wave function of the radial excitation in the case of the $g_{V\pi\pi}$ and, in addition to this, also the dependence on the particular choice of the subtraction procedure in $g_{P\pi\pi}$ is more sizeable. Table 2 presents numerical results. The slope parameter of the Gaussian wave function of the radial excitation at finite masses is not known, so for estimating $g_{V\pi\pi}$ we assume that this parameter lies within a 10% interval around the corresponding ground-state slope. In the case of $g_{P\pi\pi}$ the relation (8) prompts that the $1/m_Q$ corrections numerically might be sizeable if the $M^2_{V\pi} - M^2_{P\pi}$ is big as it is for the radial excitations of heavy mesons. In this case we cannot obtain a reliable estimate and Table 2 provides the lower bounds found from the spectral representations without subtractions.

III. CONCLUSION

We have analysed the pionic coupling constants of heavy mesons within the framework based on the combination of PCAC with the dispersion approach and obtained the following results:

1. Spectral representations of the coupling constants in terms of the wave functions of the initial and final heavy mesons have been obtained. The double spectral densities of these spectral representations for $g_{P\pi\pi}$ and $g_{V\pi\pi}$ are equal in the HQ limit in agreement with the HQ symmetry.

In the case of the $V \rightarrow V$ transition the calculation of the $g_{V\pi\pi}$ is equivalent to the calculation of the amplitudes $\langle V|2m_j|V \rangle$. There is no reason dictating a necessity of any subtraction term in the spectral representation for $g_{V\pi\pi}$ and thus the latter is determined unambiguously within our approach.

In the $P \rightarrow V$ case the double spectral density of $g_{P\pi\pi}$ can be also found from the amplitude $\langle V|2m_j|P \rangle$. The spectral representation for $g_{P\pi\pi}$, however, contains a subtraction term which cannot be fixed within our approach. Important is that due to the HQ symmetry this subtraction term does not contribute in the leading $1/m_Q$ order and thus numerically is not essential at least for the $g_{P\pi\pi}^*$. 

2. A spectral representation for the LO $1/m_Q$ coupling constant $\hat{\gamma}$ which describes the pionic transition in the HQ limit has been obtained. The $\hat{\gamma}$ is represented through the LO wave functions of the heavy mesons which determine also the Isgur-Wise function. The $\hat{\gamma}$ is determined quite reliably within the dispersion approach since it is not affected by the uncertainties in the subtraction procedure.

For the transition between the ground-state heavy mesons we have found $\hat{\gamma} = 0.5 \pm 0.02$. We argue that the behavior of the $B \rightarrow \pi$ transition form factors near zero recoil provide independent arguments in favour of such considerably large value of $\hat{\gamma}$.

For the pionic transition between the ground state and the radial excitation in the HQ limit a strong suppression $\hat{g}_r = 0.11 \pm 0.02$ is observed as a consequence of the orthogonality of the corresponding radial wave functions.

3. Using the QM parameters obtained from the analysis of the $B \rightarrow \pi$ decay we have calculated the pionic coupling constants of the ground state charm and bottom mesons. Our final numerical estimates are listed in Table 2.

For the $V \rightarrow V$ transition higher-order $1/m_Q$ effects in $g_{V\pi\pi}$ are found to be small and not to exceed 5-6%.

In the $P \rightarrow V$ case, the coupling constant $g_{P\pi\pi}$ depends on the choice of the subtraction procedure which cannot be fixed unambiguously. However, numerically the uncertainty estimated by using several reasonable subtraction procedures is not more than 10%.

4. We estimated the pionic couplings of the radially excited states $B^*\pi$ and $D^*\pi$. The errors turn out to be much bigger compared with the corresponding ground states: the LO contribution is strongly suppressed and thus the $1/m_Q$ effects are found to be numerically more important. As a result the coupling constants are sensitive to the specific form of the radial-excitation wave function and $g_{P\pi\pi}$, in addition to this, strongly depends on the the details of the subtraction procedure. Therefore only lower bounds on $g_{P\pi\pi}$ are given.

\footnote{Strictly speaking, we cannot completely exclude a possibility of subtraction terms both in $\hat{g}_{V\pi\pi}$ and $\hat{g}_{P\pi\pi}$ which give the same contributions to both quantities in the LO. However, unless no reasons for such subtraction terms are found we do not include them into consideration. In a recent analysis of the $B \rightarrow \pi$ form factors [8] we have found that the behavior of $f_B$ at zero recoil is compatible with $\hat{g}_{B^*\pi} \simeq 0.5 - 0.7$. The values of $\hat{g}_{B^*\pi}$ as big as $0.6 - 0.7$ can be obtained only by assuming the subtraction terms both in $g_{B^*\pi}$ and $g_{B^*\pi}$ which give a nonvanishing contribution already in the leading $1/m_Q$ order.}
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TABLE I. Quark masses and the slope parameters of the soft meson wave functions and the calculated leptonic decay constants (in GeV)

| $m_b$ | $m_c$ | $m_u$ | $\beta_{\infty}$ | $\beta_B$ | $\beta_{B^*}$ | $\beta_D$ | $\beta_{D^*}$ | $f_B$ | $f_{B^*}$ | $f_D$ | $f_{D^*}$ |
|-------|-------|-------|------------------|----------|-------------|----------|-------------|-------|-----------|-------|----------|
| 4.85  | 1.4   | 0.23  | 0.5              | 0.49     | 0.49        | 0.46     | 0.43        | 0.16  | 0.175     | 0.2   | 0.22     |

TABLE II. Pionic coupling constants of heavy mesons. The error bars correspond to the variations of the slope parameters $\beta$ around the average values of Table I yielding a 10% variation of the leptonic decay constants.

|         | $\hat{g}_{VV^\pi}$ | $\hat{g}_{VP^\pi}$ | $\hat{g}_{V'V^\pi}$ | $\hat{g}_{V'P^\pi}$ |
|---------|---------------------|---------------------|---------------------|---------------------|
| $D, D^*$ | 0.53±0.03           | 0.53±0.05           | 0.15±0.03           | > 0.14              |
| $B, B^*$ | 0.5±0.02            | 0.5±0.04            | 0.12±0.03           | > 0.12              |
| HQ-limit | 0.5±0.02            | -                   | 0.11±0.02           | -                   |