Detuning Enhanced Cavity Spin Squeezing

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The unconditionally squeezing of the collective spin of an atomic ensemble in a laser driven optical cavity [I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Phys. Rev. Lett 104, 073602 (2010)] is studied and analyzed theoretically. Surprisingly, we find that the largely detuned driving laser can improve the scaling of cavity squeezing from $S^{-2/5}$ to $S^{-2/3}$, where $S$ is the total atomic spin. Moreover, we also demonstrate that the experimental imperfection of photon scattering into free space can be efficiently suppressed by detuning.

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Introduction.- Large ensembles of atoms are good platforms for quantum information processing [1–3], as well as test beds of fundamental physics [4], atomic clocks [5–8], magnetometers [9, 10] and gravitational wave detectors [11]. An important benchmark for the protocols in high-precision measurements is spin squeezing arising from the entanglement of atoms [12]. The squeezed spin state (SSS) [11] is a quantum correlated state with reduced fluctuations in one of the collective spin components, which attracts considerable interest for both fundamental and practical reasons.

Pumping the atomic ensemble by squeezed light [12–15] has been proposed by quantum-state transfer from light to the atomic spin. In this method, the degree of spin squeezing is determined by the quality of the squeezed light, which is the source of spin squeezing. The quantum nondemolition measurement [16–19] is another method of generating spin squeezing and has already been performed by several groups [20]. The atomic ensembles can be squeezed conditioned on the measurement results [21], which are related to the performance of the detector. The last and very promising system is the cavity squeezing [22, 23], without requiring measurement of the light field. Cavity squeezing relies on the off-resonant interaction between an ensemble of atoms and a light field circulating in an optical resonator cavity, and the ensemble spin imprints its quantum fluctuations on the light, which acts back on the spin state to reduce those fluctuations. When considering cavity squeezing [22, 23], the strong atom-cavity coupling is usually required and the effect of detuning between the light and the cavity is not discussed.

In this paper, we theoretically study the detuning dependence of cavity spin squeezing for the experimental scheme demonstrated in Ref. [28] (Fig. 1a). Comparing with the near resonance case [22], it is surprising to find that the scaling of cavity squeezing on atom number can be significantly improved from $S^{-2/5}$ to $S^{-2/3}$ for large detuning. In addition, we find that the spin squeezing will be enhanced if the atoms are weakly coupled to the cavity or the laser detuning is very large. From our numerical solutions and analytical analysis, the large detuning is very important as the squeezing originates from the laser induced spin state dependent geometry phase [29, 30]. Finally, we study the influence of scattering of photon into free space due to imperfect Raman scattering, and demonstrate that the optimal spin squeezing can be obtained with appropriate detuning. This improvement of spin squeezing by detuning is very feasible for experiments, without the requirement of preparation or post-selection of photon state. The detuning enhanced cavity spin squeezing can also be applied to other systems, such as nitrogen-vacancy centers in diamond, to prepare SSS for quantum metrology.

Model.- The system (Fig. 1b) is an ensemble of $N$ identical three-level atoms trapped inside an optical Fabry-Pérot cavity. There are two stable ground states $|\downarrow\rangle$ and $|\downarrow\rangle$, which are coupled to the excited state $|e\rangle$ via optical fluctuations.
transitions of frequencies $\omega_c \pm \omega_i / 2$. The cavity resonance frequency $\omega_i$ is chosen so that the detunings to transitions $|\uparrow\rangle \leftrightarrow |e\rangle$ and $|\downarrow\rangle \leftrightarrow |e\rangle$ are opposite in sign but having the same magnitude $\Delta = \omega_i / 2$. For simplicity, we only consider the case where the two transitions have equal single-photon Rabi frequency $2g$ and all atoms are uniformly coupled to the cavity. The Hamiltonian of the system reads ($\hbar = 1$)

$$H_{\text{cav}} = \omega_c c^\dagger c + \sum_{i=1}^N \left( \frac{\omega_i}{2} |\uparrow\rangle_i \langle\uparrow|_i + |\downarrow\rangle_i \langle\downarrow|_i + \omega_c |e\rangle_i \langle e|_i \right) + g [c |e\rangle_i \langle\uparrow|_i + |\downarrow\rangle_i \langle e|_i + H.c.] .$$  

(1)

Here, $c$ and $c^\dagger$ are the photon annihilation and creation operators for the cavity mode, and the index $i$ labels the individual atoms. As we are interested in the linear and dispersive regime of atom-field interactions, we assume the excited state population is negligible. The assumption requires a large detuning $|\Delta| \gg \kappa, \Gamma$, and sufficiently low intracavity photon number $\langle n \rangle \ll (\Delta / g)^2$, where $\kappa$ is the cavity linewidth, $\Gamma$ is the excited state decay rate. After adiabatically eliminating the excited state of atom and considering external continuum fields [31, 32], we obtain the effective Hamiltonian for the system

$$H_{\text{eff}} = (\delta + \Omega S_z) c^\dagger c + \int_{-\infty}^{\infty} \omega b_b^\dagger b_\omega d\omega$$

$$+ \sqrt{\kappa} \left[ \beta_{\text{in}}^*(t) c + c^\dagger \beta_{\text{in}}(t) \right]$$

$$+ \sqrt{\frac{\kappa}{2\pi}} \int_{-\infty}^{\infty} \left( b_b^\dagger c + c^\dagger b_\omega \right) d\omega ,$$

(2)

where $\delta = \omega_c - \omega_i$ is the detuning between the resonator mode and the driving light, $S_z = \frac{1}{2} \sum_{i=1}^N (|\uparrow\rangle_i \langle\uparrow|_i - |\downarrow\rangle_i \langle\downarrow|_i)$ is the $z$-component of the total spin, $\Omega = 2g^2 / |\Delta|$ is the dispersive frequency shift due to spin-photon interaction, $\beta_{\text{in}}(t)$ is the driving, and $b_b$ ($b_b^\dagger$) is the annihilation (creation) operator of the continuum.

Under coherent laser driving, the intracavity field is the coherent state with spin-dependent phase shift. Assume the system is in state

$$|\psi\rangle = \sum_m C_m |\varphi_m(t), m\rangle \prod_\omega |\varphi_{\omega,m}(t)\rangle ,$$

(3)

$$\text{where } m \text{ is the quantum number associated with } S_z, \text{ which is conserved during the evolution. } |\varphi_m(t)\rangle \text{ is the cavity photon state and the } |\varphi_{\omega,m}(t)\rangle \text{ is the state of continuum. By solving the Schrodinger equation } i\hbar \frac{\partial}{\partial t} |\psi\rangle = H_{\text{eff}} |\psi\rangle , \text{ the time dependent intracavity field }$$

$$\varphi_m(t) = -i \sqrt{\pi} \int_0^t \beta_{\text{in}}(t') e^{-i(\Delta + \Omega m)(t-t')} e^{-\kappa(t-t')/2} dt',$$

(4)

$$\text{and the continuum modes are }$$

$$|\varphi_{\omega,m}(t)\rangle = e^{-i \int_0^t \text{Re}(\sqrt{\pi}\beta_{\text{in}}(t') |\varphi_m(t')\rangle dt')} |\varphi_{\omega,m}(t)\rangle ,$$

$$|\varphi_{\omega,m}(t)\rangle = -i \sqrt{\frac{\kappa}{2\pi}} \int_0^t \varphi_m(t') e^{-i\omega(t-t')} dt',$$

(5)

In general, the cavity photon, continuum and atomic spin states are entangled [Eq. (3)]. If the output field is not measured, the density matrix of cavity photon and the atomic spin can be written as

$$\rho_{\text{m,atom}} = \sum_{m,n} C_m C_n e^{\varphi_m(t)} |\varphi_m(t), m\rangle \langle \varphi_n(t), n|$$

(7)

by tracing the continuum modes out, where

$$\varphi_{m,n}(t) = -i \int_0^t \sqrt{\pi} \text{Re}(\beta_{\text{in}}(t') |\varphi_m(t')\rangle - \beta_{\text{in}}(t') |\varphi_m(t')\rangle dt' - \kappa \int_0^t |\varphi_m(t')|^2 dt' / 2 - \kappa \int_0^t |\varphi_m(t')|^2 dt' / 2 + \kappa \int_0^t \varphi_m(t')^* |\varphi_m(t')| dt'.$$

(8)

The spin squeezing is evaluated by squeezing parameter [11]

$$\xi_s^2 = \min \left( \frac{\Delta S_{n_s}^2}{S / 2} \right) .$$

(9)

Where $\Delta S_{n_s}^2$ is the variance of spin operators along direction perpendicular to the mean-spin direction $n_0 = \frac{\vec{s}}{|\langle \vec{s} \rangle |}$, which is determined by the expectation values $\langle S_n \rangle$, with $n \in \{x, y, z\}$. For an atomic system initialized in a coherent spin state (CSS) [33] along the $x$ axis, satisfying $S_x |\psi(0)\rangle_{\text{atom}} = S |\psi(0)\rangle_{\text{atom}}$, we have $C_m = 2^{-S} \sqrt{(2S)! / (S-m)! (S+m)!} \text{ and } \Delta S_{n_s}^2 \leq S / 2$. Thus, for squeezed spin states we have $\xi_s^2 < 1$.

Detuning enhanced squeezing.- Now, we study the cavity spin squeezing with continuous drive $\beta_{\text{in}}(t) = i \sqrt{\pi} \beta_0$ with a small detuning $\delta = -\kappa / 2$. For easier illustration, it is useful to introduce the dimensionless shearing strength [22]

$$Q = \frac{4S|\beta_0|^2 \Omega^2 t}{\kappa},$$

(10)

which is proportional to the transformation degree from the optical field to the atomic spin. In Fig. 2(a), we plot the spin squeezing parameter $\xi_s^2$ as a function of shearing strength $Q$ for various coupling $\Omega$. It clearly shows that the spin squeezing parameter has a minimal value for certain optimal $Q$, and it takes longer time for smaller coupling $\Omega$. The minimal value of spin squeezing parameter increases with the coupling $\Omega$, because there are higher order effects associated with $\Omega$ that will limit the squeezing.

To study the effect of the detuning $\delta$ on spin squeezing, we set $\delta = -x \kappa / 2$, and the dimensionless shearing strength can be generalized as

$$Q_x = 4Q x / (1 + x^2)^2 ,$$

(11)

In Fig. 2(b), we plot the spin squeezing parameter $\xi_s^2$ as a function of shearing strength $Q_x$ for various detuning
detuning $\delta = -x\kappa/2$ can be written as

$$\varphi_m = \frac{\kappa\beta_0}{\kappa/2 + i(\delta + \Omega m)}. \tag{12}$$

From Eq. (8), we solve the phase factor as

$$\phi_{m,n}(t) = i\frac{\varphi_m^2|\varphi_n|^2\Omega t}{\kappa\beta_0^2} \times \left\{ \frac{\kappa^2 + \delta^2}{\Omega\kappa} (n - m) + \frac{\delta}{\kappa} (n^2 - m^2) + \frac{\Omega}{\kappa} nm(n - m) + i\frac{(n - m)^2}{2} \right\}. \tag{13}$$

The first term accounts for the coefficient that approximately proportional to $Q_x$, and the terms within the brace are the linear, quadratic and higher order couplings of spin $z$-component. The quadratic term corresponding to spin squeezing interaction $S_z^2$, while the last two terms give rise to disorder and decoherence of spin states. It’s obvious that the detuning is essential in the cavity induced spin squeezing, as there is no squeezing at all for zero detuning $\delta = 0$. The parameters $\frac{\delta}{\kappa} \gg 1$ and $\frac{\delta}{\kappa} \gg \frac{\Omega}{\kappa}$ should be satisfied. This can explain the results the dependence of optimal spin squeezing on $\delta$ and $\Omega$ shown in Figs. 2(a) and 2(b): (1) For very large $Q$ or $Q_x$, the disorder and decoherence dominate over the coherent process. (2) Larger $\delta$ helps to suppress both disorder and dissipation. (3) Smaller $\Omega$ can suppress the high order terms, thus can enhance the squeezing.

For more intuitive understanding, we obtain the spin squeezing parameter $\xi_s^2$ from the Heisenberg equation \[22\] under certain approximation

$$\frac{\Omega}{\kappa}|S_z| \frac{1 + |x|}{1 + x^2} \leq \frac{\Omega}{\kappa} \sqrt{S^2(1 + |x|)} \ll 1, \tag{14}$$

$$1 \ll |Q_x| \ll S, \tag{15}$$

$$\xi_s^2 = \frac{1}{Q_x} + \frac{2}{Q_x^2} + \frac{Q_x^4}{24S^2}, \ x \neq 0. \tag{16}$$

When $(5/2)^{5/4}12^{-1/4}S^{-1/2} \leq x \ll 12^{1/6}S^{1/3}$, we obtain the optimal cavity squeezing $\xi_{s,\text{min}}^2 = (5/2)12^{-1/3}S^{-2/5}x^{-4/5}$ at the point $Q_x = 12^{1/3}S^{2/5}x^{-1/5}$. When the detuning is very large $x \gg 12^{1/6}S^{1/3}$, the squeezing limit is $\xi_{s,\text{min}}^2 = (3/2)12^{-1/3}S^{-2/3}$ with $Q_x = 12^{1/6}S^{1/3}$. The detuning is the source of the effect nonlinear interactions between the atomic spin and the optical mode, and the large detuning means that the $1/Q_x^2$ is the main factor of spin squeezing rather than the part $2/(Q_xx)$. We can improve the scaling of cavity squeezing to $(3/2)12^{-1/3}S^{-2/3}$ with sufficient detuning.

**Imperfections.** In previous studies, we have neglected the scattering of photon into free space, which is an unavoidable process that deteriorates squeezing performance \[21\]. Any atoms scattering photon into free space...
which depends on the single-atom cooperativity $\eta=2, 20, \infty$. (b) The optimal squeezing parameter $\xi_s^2$ as a function of detuning $x$ for the various cooperativity $\eta = 1, 2, 20$. (c) The solid lines are optimal squeezing parameter $\xi_s^2$ as a function of the cooperativity $\eta$ for the fixed detuning $x = 1$ (green) and optimized detuning (black), and the dashed lines are results for ideal condition $\eta = \infty$ for $x = 1$ (red) and $x = 200$ (blue). The atomic spin is $S = 10^4$.

will acquire a random phase, so that it no longer contributes to the mean spin length. The Raman transitions $|\uparrow\rangle \to |e\rangle \to |\downarrow\rangle$ or $|\downarrow\rangle \to |e\rangle \to |\uparrow\rangle$ reduce the correlation between the time average $\bar{S}_z$ during the cavity squeezing process. The average photon number emitted into free space per atom is given by

$$R_x = Q_x(1 + x^2)/(8xS\eta),$$

which depends on the single-atom cooperativity $\eta = 4g^2/(\kappa\Gamma)$. This expression indicates that very large collective cooperativity $S\eta \gg 1$ is required to suppress the scattering of cavity photon into free space. We extend the solution previously obtained in \cite{21} to the large detuning, and obtain the spin squeezing parameter:

$$\xi_s^2 = \frac{\langle S_y^2 \rangle + \langle S_z^2 \rangle - \sqrt{\langle S_z^2 \rangle^2 - \langle S_z^2 \rangle} + W^2}{S},$$

where $W = \langle S_y S_z + S_z S_y \rangle$ and the mean value of spin operators $\bar{S}$ are solved approximately in the rotating frame as

$$\langle \bar{S}_y \rangle = \langle S_z \rangle = 0, \langle \bar{S}_y^2 \rangle = \frac{S}{2},$$

$$\langle \bar{S}_y^2 \rangle = \frac{S}{2} [1 + S e^{-4Rx} (1 - e^{-U})],$$

$$\langle \bar{S}_y S_z + S_z \bar{S}_y \rangle = S (1 - R_x) Q_x e^{-V},$$

with parameters $U = \frac{Q_x}{x^2 S} + \frac{Q_x^2 (1 - 2Rx/3)}{S}$, $V = \frac{Q_x}{x^2 S} + \frac{Q_x^2 (1 - 2Rx/3)}{4S}$.

Although $\xi_s^2$ is a complicated function of $\eta, x$ and $Q_x$ due to imperfection, the spin squeezing can be optimized for a given $\eta$ by adjusting the laser detuning and pump power and interacting time. Fig. 3(a) shows the squeezing parameter $\xi_s^2$ as a function of $Q_x$ for various values of the cooperativity $\eta$ and fixed large detuning $x = 200$. And in Fig. 3(b), the optimized spin squeezing parameters for certain $Q_x$ is calculated against detuning $x$ for given cooperativity $\eta$. These results indicate that the squeezing parameter is very sensitive to the value of the cooperativity $\eta$, and better spin squeezing can be achieved for larger $\eta$ and appropriate detuning $x$. Shown in the Fig. 3(c) is the optimal squeezing parameter $\xi_s^2$ as a function of the $\eta$. Green and black solid lines are the results for fixed detuning ($x = 1$) and optimized detuning. With increasing $\eta$, $\xi_s^2$ is reduced and trend to be saturated at certain value. Compared with the fixed detuning, the optimal detuning is always better, indicating that the detuning regulation can efficiently enhance the spin squeezing. When the cooperativity is not too small $\eta > 0.1$, the squeezing by optimized detuning can be even better than the result of fixed detuning with $\eta = \infty$.

To lowest order expansion of $R_x \ll 1$ and ignoring curvature effects for the moment, the asymptotic solution of the squeezing parameter [Eq. (18)] can be written as

$$\xi_s^2 = \frac{Q_x^2}{Q_x x} + \frac{2}{Q_x x} + \frac{Q_x (x^2 + 1)}{6x S\eta}.$$

When the $\delta$ is very small, the squeezing variance suppressed by the square of the shearing strength is neglected. Consequently, there exist an optimum shearing strength $Q_{scatt} = \sqrt{125\eta/(x^2 + 1)}$, to achieve the optimum squeezing $\xi_s^2 = \sqrt{\frac{3(x^2 + 1)}{4S\eta}}$. For very large detuning that satisfies $x \gg 12^{1/6} S^{1/3}$, we have optimum squeezing $\xi_s^2 = 3 \left(\frac{12x^2 S\eta}{4S^{2/3}}\right)^{2/3}$ for the shearing strength $Q_{scatt} = \left(\frac{12x^2 S\eta}{4S^{2/3}}\right)^{1/3}$. The squeezing is thus possible even for very weakly coupled resonator and atoms with single photon-atom coupling cooperativity $\eta \ll 1$, as long as the collective cooperativity $S\eta \gg 1$.

Conclusion.- We have theoretically analyzed the experimental method to squeeze the collective spin of an atomic ensemble in a driven optical cavity unconditionally. We find that strong atom-cavity coupling weakens the spin squeezing and the large detuned laser driving can improve the scaling of spin squeezing to $S^{-2/3}$, which is the ultimate limit of the ideal one-axis twisting spin squeezing. The imperfection of light scattering into free space can be efficiently suppressed by optimal detuning, which can be tested experimentally and may further improve the sensitivity of quantum metrology based on the SSS.

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