Comments on “Composite Fermion (CF) model of quantum Hall effect - Two-dimensional electron system in high magnetic fields, S. S. Mandal, M. R. Peterson and J. K. Jain, Phys. Rev. Lett. 90, 106403 (2003).

Keshav N. Shrivastava

School of Physics, University of Hyderabad, Hyderabad 500046, India

The flux quanta attachment to the electrons creates composite fermions (CFs). The mass, the size and the density of the CF are inconsistent with real material. The sequence of fractional charges which suggest formation of CF agrees with the data but there are no monopoles in GaAs. Hence the CF model is internally inconsistent. There are two options at this stage. (1) The flux quanta are attached to the electrons. This is a theoretical possibility with nothing to do with any experiment ever performed within the last seventy five years. It will violate Maxwell equations and create unreal objects. (2) The CF give a sequence which is deduced from the experimental data and hence agrees with the data. In this case, the masses, the sizes and the densities are internally inconsistent.

Corresponding author: keshav@mailaps.org
Fax: +91-402-301 0145. Phone: 301 0811.

1. Introduction

The paper by Mandal et al shows that even number of magnetic flux quanta are attached to the electron. We find that this flux attachment is internally inconsistent. If there is a contact between the sequence and the data then the model does not have Lorentz invariance and mass and size are inconsistent. If the flux quanta are attached to the electron then there is no contact of this flux-attached model with the experimental data on quantum Hall effect because there are no monopoles in GaAs. By using the experimental numbers, Jain[1-3] constructed a sequence which gives the correct plateaus in the quantum Hall effect. From these sequences, effort is made to construct a formula for the magnetic field. This field formula is interpreted in terms of flux quanta attachment to the electron but it is internally inconsistent.

We find that whenever effort is made to compare this composite fermion (CF) model with the data, some thing or the other is always wrong. There are many different number of flux quanta required to reach agreement with the experimental data. These are denoted by $^6$CF, $^8$CF and $^{10}$CF.

Several questions arise. This CF model is correct and should be accepted as new physics. It will be immediately obvious that CF model is not internally consistent and hence should be discarded. Let us examine a few points.

2. Comments.
(i) Abundance of flux quanta and Monopoles

Usually the flux is generated by the flow of current in a coil. In the CF model, the current which generates the flux quanta is missing. On page 106403-2, left column, para 2, it is mentioned that

“N electrons are confined to the two dimensional surface of a sphere, moving under the influence of a radial magnetic field produced by a magnetic monopole of strength, Q, at the center. According to Dirac’s quantization condition, Q can be either an integer or a half integer, and produces a total flux of 2Q\phi_o ...” [Surely, Dirac’s monopoles do not exist in GaAs/AlGaAs and hence the CF model does not apply]. Dirac symmetrized the Maxwell equations and predicted the monopoles in 1931 but these monopoles have not been seen in any of the experiments. Although ’tHooft[2] removed the string of the Dirac’s monopole, it is still not found in nature. In the ’tHooft’s theory, the Maxwell equations remain unchanged except at one point. Therefore, whereas there are no monopoles in the Maxwell theory, there is a monopole in ’tHooft’s. There is a report of finding a monopole, the mass of which may be equal to that of an atom of atomic number 200. In that case, the difference between a heavy atom and a monopole was not resolved. In that case the mass of the monopole will be about 10^5 times larger than that of an electron. Naturally impossible in GaAs. The number of such monopoles will be very small but ^8CF or ^10CF require far too many monopoles to be present. As far as the number of monopoles is concerned for ^10CF, 10 times more monopoles are required than the number of electrons and what will be the mass of such a large number of monopoles? It is obvious that the CF model is demanding far too many flux quanta than can be available. Therefore, on the grounds of abundance of free flux quanta, the CF model can not be accepted as a new theory.

(ii) Mass.

What is the mass of a vortex? The vortices are found in superconductors with unit flux \phi_o=hc/2e. In the normal state we need not take the charge as 2e but e. Hence the normal unit flux is \phi_o=hc/e. The mass was calculated by Suhl and found to be 10^4 per unit length in the units of the mass of the electron. If we take an order of magnitude of a flux mass as 10^4, then no such free mass is available in GaAs. If the CF model is correct, then the mass of GaAs after CF formation will become 10^4 times the mass before CF formation. Therefore, CF model is surely inapplicable. Let us examine this mass from another view point. The magnetic length is \ell_o=(\phi_o/B)^{1/2}. At 8 Tesla, the magnetic length is about 10^{-6} cm so that the wave vector is 10^6 cm^{-1}. The mass due to kinetic energy is the mass per unit length \text{mk}_F \sim m \times 10^6. The wave vector of the electron is about 10^8 cm^{-1} so that for the same energy, which is of the order of cyclotron energy, the mass of the vortex should be 100 times larger than the electron mass. Therefore, when ^10CF are formed the mass of the CF will be 1000 times the mass of an electron but there is no evidence of such a large massive quasiparticle.

(iii) Free flux.

Let us take a positive approach to the problem. The fluxes that get attached to the electrons in any of the various composite fermion models are not physical magnetic fields and therefore need no persistent currents to create them. They are either short hand...
for rearrangement of purely electronic states or a mathematical transformation that does
nothing but change the description of the system but no such transformation is known.
The general motivation for the “attached flux” language is that one imagines putting a
real magnetic solenoid through a point (or electron) in the 2DEG and gradually increases
the field in the solenoid until it contains one unit of flux, (it may never happen so). The
electric field due to the changing magnetic field acts on the other electrons outside the
solenoid and changes their state. The final unit flux, however, is invisible to all electrons
since only fractional fluxes produce a Bohm-Aharonov effect. It is therefore discarded
and the word “attached flux” is retained as the appropriate language to describe the
rearranged many-electron state created by this mechanism. [Jain does not do this any
way and this mechanism will not produce the experimental sequence of plateaus]. In a
Chern-Simon field, a unit flux does nothing physical at all. It allows us to describe the
Laughlin state as a kind of Bose condensate of the purely mathematical composite boson.

A good analogy for these purely mathematical transformations is provided by the
two-dimensional Ising model. A Jordan-Wigner transformation which is similar in char-
acter to a “flux attachment”, turns the spins into Majorana fermions. (Majorana means
that which exchanges the space coordinates of two interacting particles). These are free
fermions so the model is easily solved in this language. All these “attached fluxes” are
loosely related to the operation but all have in common that no real magnetic flux is being
attached to anything. Similarly, Holstein-Primakoff transformation transforms the spin
operators into bosons so that the boson Hamiltonian can be solved by usual methods.
The fermion transformation does not introduce internal inconsistencies nor does the bo-
son transformation. In Jain’s CF there are too many inconsistencies and the whole thing
does not work like a transformation. Therefore, even if the “flux attachment” model is
correct, the series which agree with the quantum Hall effect data are unphysical. Hence
when “flux attachment” models are solved, they will have nothing to do with the experi-
mental data. In the case of Chern-Simons fields, the scalar potential term has been set
to zero and then only the vector potential is changed so there is no Lorentz condition.
Therefore, Jain’s CF model has nothing to do with the experimental work on GaAs. The
electron scattering or attachment to magnetic monopoles may be interesting by itself as
a good problem but it will have nothing to do with quantum Hall effect. Dyakonov[5]
and Farid[6] have also shown that there is a lack of theoretical basis in CF model.

(iv) Exactness.

Some of the parts of the calculation are exact but Laughlin has used $\frac{1}{a^2}$ in place of
$\frac{1}{a^2}$ where $Ba^2=\phi_o$. Therefore, $e$ becomes fractional or $B$, cannot be resolved. Laughlin
performed the calculation of energy for $m=1/3$ and $1/5$ but suggested that the ground
state may cross the charge-density wave at $1/7$. Now, Mandal et al[3] have extended the
calculation to $1/7$, $1/9$ and $1/11$ and exactness is preserved. This means that $6^{\text{CF}}$, $8^{\text{CF}}$
and $10^{\text{CFs}}$ are correct but attaching 10 flux quanta to one electron has no chances of
agreeing with size or mass requirements and the density of $10^{\text{CFs}}$, on physical grounds
can not be equal to that of electrons and hence CF model becomes internally inconsistent.
Is there enough room in GaAs to keep all these flux quanta? The answer is, no. There
is no room to keep so many flux quanta. So what is an exact answer is also a wrong
answer. So if there is a model of flux attachment to electrons, then it will have nothing to do with the experimental data. If the series agrees with the data, then it has nothing to do with flux attachment.

3. Conclusions

The attachment of flux quanta can work like a Holstein-Primakopf transformation but Jain has not found the transformation. As and when such a transformation is found, it will not be a theory of GaAs like real material. The sequences which are made to agree with the experimental data create internal inconsistencies. The exactness is limited to the product of charge and the field and hence can not resolve whether charge or field has been fractionalized. The experimental data is not consistent with 1/odd fractions. It is important to learn the numerator also. The mass of the vortices is not consistent with GaAs. The abundance of the vortices is also not consistent with what is possible in GaAs. Pan et al[7] have also noted that the experimental data are not consistent with the sequences suggested by Jain[1-3]. There are too many inconsistencies in the CF model and hence they can not be resolved. If the flux attachment transformation is worked out it will not be relevant to experimental work. Hence, CF model should be discarded in as much as its comparison with the experimental data is concerned [8].

About the author: Keshav Shrivastava has obtained Ph.D. degree from the Indian Institute of Technology and D. Sc. from Calcutta University. He is a member of the American Physical Society, Fellow of the Institute of Physics (U.K.) and Fellow of the National Academy of Sciences, India. He has worked in the Harvard University, University of California at Santa Barbara, the University of Houston and the Royal Institute of Technology Stockholm. He has published 170 papers in the last 40 years. He is the author of two books. Shrivastava’s paper with Roy Anderson, published in J. Chem. Phys. 48, 4599(1967) was found to be useful by M. C. R. Symons, F. R. S. The letter with K. W. H. Stevens, J. Phys. C 3, L 64 (1970) was useful to D. I. Bolef, the paper with Vincent Jaccarino, Phys. Rev. B13, 299(1976) was useful to J. B. Goodenough, F. R. S. His paper, J. Phys. C 20, L789 (1987) on the microwave absorption, was useful to Nobel Laureate K. Alex Müller. His paper published in the Proc. Roy. Soc. A419, 287-303(1988), communicated by B. Bleaney, F. R. S. represents original work of interest to Nobel Laureate J. H. van Vleck. He discovered the flux quantized energy levels in superconductors and the correct theory of 1/3 charge in quantum Hall effect. The correct theory of quantum Hall effect is given in ref.9.

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