Partially Independent Robust Control for Thrust-Limited Rendezvous in Near-Circular Orbits

Neng Wan and Ming Liu

Abstract

Space environment is so complex and changeable that an accurate model of spacecraft rendezvous is difficult to obtain, which consequently complicates the rendezvous control task. In order to realize the robust stability as well as to optimize the time and fuel cost of the rendezvous process, a linearized rendezvous model with parametric uncertainty and a partially independent robust controller are proposed in this paper. In-plane and out-of-plane motion models are established separately in the presence of external disturbance and non-circularity of the target’s orbit. The partially independent controller is integrated with an in-plane robust guaranteed cost controller and an out-of-plane robust $H_\infty$ controller. A numerical rendezvous example is presented to illustrate the usefulness and advantages of the partially independent robust control scheme.

1 Introduction

Widely applied to crew exchange, large-scale assembly, spacecraft maintenance, docking, interception, formation flying and other astronautic missions involving more than one spacecraft, autonomous rendezvous of spacecraft has been regarded as a crucial operational technology in astronautic engineering. As the autonomous control scheme is the cardinal and decisive issue that determines the success of the rendezvous, it has been and continues to be an engaging area of study.

The dynamic models mostly used in studying rendezvous problems are usually derived from the two-body problem. Among these models, the Clohessy-Wiltshire equations [1] were favored by many researchers because of their concise and linearized form; nonetheless, the application of these equations is very limited, for they are only suitable and precise enough for the rendezvous in circular orbits. In order to settle this problem, De Vries [2] and Tschauner [3] put forward the models for rendezvous in elliptical orbits, but nonlinear terms were introduced in these models, which restricted their further implementations. Considering the fact that most of the general rendezvous missions were accomplished in near-circular orbits with small eccentricities, researchers began to search for some eclectic models, which are linearized and precise enough for most rendezvous missions. Although linearizing the gravitational force function is the most direct linearization manner, most literatures focused on the linearization performed with respect to orbital elements and figuring out the closed-form state transition matrices, which is easier and more practical; in [4], Carter conducted a comprehensive survey on these linearized rendezvous models. But most of the results given in survey [4] are in terms of either the true or eccentric anomaly of one spacecraft, which require the solution of the Kepler problem for each particular application. To overcome this problem, a time-explicit representation of the equations of relative motion was first given by Anthony and Sasaki [5], and more recent development was the efforts of Melton [6].

Robust guaranteed cost control was first raised by Chang and Peng [7] to optimize preassigned performance criteria of system with parameter uncertainties. Since the 1990’s, on the basis of Chang and Peng’s results, many works have been carried out. Petersen and McFarlane [8] synthesized a state feedback guaranteed cost controller via a Riccati equation approach. Yu and Chu [9] designed a guaranteed cost controller for linear uncertain time-delay systems via a linear matrix inequality (LMI) method. Esfahani and Petersen [10] solved the guaranteed cost output feedback control problem in a matrix substitution manner. More recently, Guan and Chen [11], and Wu et al. [12] investigated the guaranteed cost control methods for time-delay systems. Zhang et al. [13] studied a guaranteed cost control scheme for a class of uncertain stochastic nonlinear systems with multiple time delays. Tanaka et al. [14] presented a guaranteed cost control for polynomial fuzzy systems via a sum of squares approach.

Robust $H_\infty$ control technique is usually used in synthesizing guaranteed cost controllers for systems with external disturbances. This technique was first proposed by Zames [15], but due to the lack of an efficient solution, the focus of
the $H_\infty$ control problem quickly shifted from its application to the formulations of the $H_\infty$ control problems and figuring out their solutions. Time domain approach [16], frequency domain approach [17] and Riccati equation approach [18] were the three main methods before the widely usage of LMI approach [19, 20] in solving the $H_\infty$ control problems. More recent works on robust $H_\infty$ control can be found in [21, 22].

Optimal control problem of spacecraft rendezvous has attracted numerous researchers. Some previous works on this area have been introduced in our recent works [23] and the references therein. During the last decade, a spate of scholars tried to improve the automation of spacecraft rendezvous by adopting advanced control theories. Based on the sliding mode control theory, Ebrahimi et al. [24] developed an optimal guidance laws for spacecraft rendezvous. Gao et al. [25] investigated a multi-object robust $H_\infty$ control scheme for rendezvous in circular orbits. Li et al. [26] proposed a sample-data control technique for rendezvous via a discontinuous Lyapunov approach. Yang and Gao [27] synthesized a robust reliable controller for thrust-limited rendezvous in circular orbits by using the fault-tolerant control theory. Gao et al. [28] studied a robust $H_\infty$ control approach for rendezvous in elliptical orbits. In [29], Yang et al. considered the spacecraft rendezvous with thrust nonlinearity and sampled-data control. In [23], we put forward a robust tracking control method for relative position holding and rendezvous in near-circular orbits with upper bounds on control forces; and in [30], we provided an observer-based control scheme for thrust-limited rendezvous in near-circular orbits. Other recent works on optimal spacecraft rendezvous problem can be found in [31–33]. However, to the best of the authors knowledge, all the existing works either synthesized a coupled rendezvous controller that realized the in-plane and the out-of-plane motion controls jointly, or neglected the control task of the out-of-plane motion. Therefore, up to now, an efficient control scheme which is customized according to the different mechanics and engineering properties of the in-plane and the out-of-plane motion has not been proposed yet.

According to the dynamic models and the control theories introduced above. In this paper, a linearization manner is provided to establish a linearized time-explicit rendezvous model that is accurate for the near-circular rendezvous and facilitative for the controller synthesis. As the features of the in-plane and the out-of-plane motions are quite different, in order to guarantee the stability and performance of the rendezvous system, the in-plane and the out-of-plane motion controllers are synthesized respectively. For in-plane motion, which is usually propelled by high-thrust engines and consumes more fuel than out-of-plane motion, a robust guaranteed cost controller is utilized to optimize an integrated performance index, which is a measurement of the time cost, the fuel cost and the smoothness of the trajectory; and a less conservative saturation control law is used to limit the in-plane control forces of the chase vehicle. For out-of-plane motion, which is usually driven by low-thrust engines and therefore sensitive to external perturbations, a robust $H_\infty$ control controller is employed to suppress the disturbance. A partially independent controller is then synthesized by solving two convex optimization problems subject to LMI constraints. To verify the functions and advantages of our proposed controller, a numerical rendezvous demonstration is presented with a comparison between a coupled robust controller and a partially independent controller.

The remainder of this paper is organized as follows. Section 2 sets up linearized models for both in-plane and out-of-plane motion and formulates the rendezvous control problems; Section 3 synthesizes the partially independent controllers; Section 4 shows a numerical demonstration; and Section 5 draws a conclusion.

**Notation.** The notations used throughout this paper are given below. $\| \cdot \|_2$ refers to either the Euclidean vector norm or the induced matrix 2-norm. $\text{diag}(\cdots)$ stands for a block-diagonal matrix. In symmetric block matrices or complex matrix expressions, an asterisk (*) is used to represent a term that is induced by symmetry. For a matrix $A$, $A^T$ stands for the transpose of $A$; and $\text{sym}(A)$ stands for $A + A^T$, when $A$ is a square matrix. For a real symmetric matrix $B$, the notation $B > 0$ ($B < 0$) is used to denote its positive- (negative-) definiteness. $I$ and $0$ respectively denote the identity matrix and zero matrix with compatible dimension. If the dimensions of matrices are not explicitly stated, they are assumed to be compatible for algebraic operation.

## 2 Motion Analysis and Problem Formulation

In this section, the control problems of each plane are formulated. In-plane and out-of-plane motion models are established and linearized on the basis of Melton’s and our previous works [6, 23, 30]. According to the requirements and features of each motion, the control tasks for each plane are stated respectively.
2.1 Relative Motion Model

Suppose that a target vehicle is moving on a near-circular orbit with an adjacent chase vehicle approaching it. It is assumed that the spacecraft is only influenced by a central gravitational source; the target vehicle do not maneuver during the rendezvous; and propulsive forces of the chase vehicle are continues and independent along each axis.

To describe the relative motion between the spacecraft systematically, a relative Cartesian coordinate system is defined in Figure 1. The system’s origin is at the centroid of the target vehicle. The $x$-axis is parallel to the vector $r$ from the Earth’s centroid to the target’s centroid. The $z$-axis is aligned with the target orbit’s angular momentum vector, and the $y$-axis completes a right-handed coordinate system.

![Figure 1: Relative Cartesian coordinate system for spacecraft rendezvous.](image)

Define the state vector as $x(t) = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$, which contains the relative positions and velocities of the chase vehicle; and define the input vector as $u(t) = [f_x, f_y, f_z]^T$, where $f_i$ for $i = x, y, z$ are the control forces of the chase vehicle along each axis. Relative motion model for spacecraft rendezvous in all kinds of orbits can be expressed in a matrix form as

$$\dot{x}(t) = A_n x(t) + B u(t), \quad (1)$$

where

$$A_n = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2\mu/r^3 + \omega^2 & \dot{\omega} & 0 & 0 & 2\omega & 0 \\ -\dot{\omega} & -\mu/r^3 + \omega^2 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\mu/r^3 & 0 & 0 & 0 \end{bmatrix}, \quad B = \frac{1}{m} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $\mu$ is the gravitational parameter; $r$ is the radius of the target orbit; $\omega$ is the angular rate of the target vehicle; $\dot{\omega}$ is the angular acceleration of the target vehicle; and $m$ is the mass of the chase vehicle. As can be seen in (1), system matrix $A_n$ includes some nonlinear terms, which complicate the task of control. To linearize equation (1), generalized Lagranges expansion theorem is utilized and introduced in the following lemma.

**Lemma 1** (see [34]). Let $y$ be a function of $x$ in terms of a parameter $\alpha$ by

$$y = x + \alpha \phi(x).$$

Then for sufficiently small $\alpha$, any function $F(y)$ can be expanded as a power series in $\alpha$,

$$F(y) = F(x) + \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} \frac{d^{n-1}}{dx^{n-1}} \left[ \phi(x)^n \frac{dF(x)}{dx} \right].$$

Kepler’s time equation is

$$E = M + e \sin E,$$  \quad (4)

where $e$ denotes the eccentricity of the target orbit; $E$ denotes the eccentric anomaly of the target vehicle; $M = n(t - t_p)$ is the mean anomaly of the target vehicle; $n = \sqrt{\mu/a^3}$ is the mean motion of the target vehicle; $a$ is the semimajor axis.
of the target orbit; and \(t_p\) is the time of periapsis passage. When the eccentricity of target orbit \(e\) is sufficiently small, according to Lemma 1 and Kepler’s time equation (4), any function \(F(E)\) can be expanded as a power series in constant \(e\), which is very helpful in linearizing the nonlinear terms in (1).

By equation

\[
r = a(1 - e \cos E),
\]

the nonlinear terms in system matrix \(A_n\) can be written as the functions of \(E\) as

\[
\frac{\mu}{r^3} = n^2 \left( \frac{a}{r} \right)^3 = n^2 \left( \frac{1}{1 - e \cos E} \right)^3,
\]

\[
\omega = \frac{h}{r^2} = n \left( \frac{1}{1 - e \cos E} \right)^2,
\]

\[
\omega^2 = \frac{h^2}{r^4} = n^2 \left( \frac{1}{1 - e \cos E} \right)^4,
\]

\[
\dot{\omega} = -2 \frac{h}{r^3} = -2n^2 \frac{e \sin E}{(1 - e \cos E)^4},
\]

where \(h\) is the angular momentum of the target orbit. By Lemma 1, when eccentricity \(e\) is sufficiently small, the expressions above can be expanded as

\[
\frac{\mu}{r^3} = n^2 \left\{ \frac{1}{(1 - e \cos M)^3} - \frac{3e^2 \sin^2 M}{(1 - e \cos M)^4} + \frac{e^2}{2} \left[ \frac{12e^2 \sin^4 M}{(1 - e \cos M)^4} - \frac{9e \cos M \sin^2 M}{(1 - e \cos M)^4} \right] + \cdots \right\}, \tag{7a}
\]

\[
\omega = n \left\{ \frac{1}{(1 - e \cos M)^2} - \frac{2e^2 \sin^2 M}{(1 - e \cos M)^3} + \frac{e^2}{2} \left[ \frac{6e^2 \sin^4 M}{(1 - e \cos M)^4} - \frac{6e \cos M \sin^2 M}{(1 - e \cos M)^4} \right] + \cdots \right\}, \tag{7b}
\]

\[
\omega^2 = n^2 \left\{ \frac{1}{(1 - e \cos M)} - \frac{4e^2 \sin^2 M}{(1 - e \cos M)^2} + \frac{e^2}{2} \left[ \frac{20e^2 \sin^4 M}{(1 - e \cos M)^3} - \frac{12e \cos M \sin^2 M}{(1 - e \cos M)^3} \right] + \cdots \right\}, \tag{7c}
\]

\[
\dot{\omega} = -2n^2 \left\{ \frac{e \sin M}{(1 - e \cos M)} + \frac{e^2 \sin M}{(1 - e \cos M)^2} - \frac{4e \sin^2 M}{(1 - e \cos M)^3} \right\} + \cdots \right\}. \tag{7d}
\]

However, equations (7a-d) are still nonlinear, and further linearization should be implemented. Compute the Taylor series expansions of (7a-d) around point \(e = 0\), we have

\[
\frac{\mu}{r^3} = n^2 \left[ 1 + 3e \cos M + \frac{e^2}{2} (9 \cos 2M + 3) + \frac{e^3}{8} (53 \cos 3M + 27 \cos M) + O (e^4) \right], \tag{8a}
\]

\[
\omega = n \left[ 1 + 2e \cos M + \frac{e^2}{2} (5 \cos 2M + 1) + \frac{e^3}{4} (13 \cos 3M + 3 \cos M) + O (e^4) \right], \tag{8b}
\]

\[
\omega^2 = n^2 \left[ 1 + 4e \cos M + e^2 (7 \cos 2M + 3) + \frac{e^3}{2} (23 \cos 3M + 17 \cos M) + O (e^4) \right], \tag{8c}
\]

\[
\dot{\omega} = -2n^2 \left[ e \sin M + \frac{5e^2}{2} \sin 2M + e^3 \left( \frac{23}{8} \sin 3M + 4 \cos 2M \sin M + \frac{19}{8} \sin M \right) + O (e^4) \right]. \tag{8d}
\]

By now, we have linearized all the nonlinear terms in (1). Truncate the expansions (8a-d) at order \(e\), the linearized relative motion model can be expressed as

\[
\dot{x}(t) = (A + \Delta A)x(t) + Bu(t), \tag{9}
\]

where

\[
A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}, \quad B = \frac{1}{m} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\]
Then according to (9), dynamic model of in-plane motion can be expressed as

\[ \dot{x} = A_x x + B_x u, \]

where the norm-bounded uncertain matrix \( \Delta A_x \), which is defined as the non-circular uncertainty of the rendezvous model, contains the shape information of the target orbit.

Compared with the rendezvous models for circular target orbit [1], model (9) contains the non-circularity of the target orbits \( \Delta A_x \), which makes the model more precise and more generalized to engineering applications; while compared with the nonlinear models for rendezvous in elliptical orbits, model (9) is linearized, which makes it friendlier to the controller designers. However, any mathematical model is erroneous due to the perturbations that are not considered, so is model (9).

For in-plane motion, high-thrust engines can be implemented to compensate the errors caused by these perturbations; but for out-of-plane motion, which is usually driven by low-thrust engines, these perturbations cannot be neglected. Therefore, according to the different features of each motion and demands on the control system, we synthesized in-plane and out-of-plane motion controllers independently for optimal performance. To synthesize this partially independent controller, model (9) is decomposed into a model for in-plane motion and a model for out-of-plane motion, and the rendezvous control problems are formulated respectively in the rest of this section.

### 2.1.1 In-Plane Motion Model

For in-plane motion model, define the state vector as \( p(t) = [x, y, x, y]^T \) and the input vector as \( u_p(t) = [f_x, f_y]^T \).

Then according to (9), dynamic model of in-plane motion can be expressed as

\[ p(t) = (A_p + \Delta A_p)p(t) + B_p u_p(t), \]

where

\[
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
3n^2 & 0 & 0 & 2n \\
0 & 0 & -2n & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
10en^2 \cos M & -2en^2 \sin M & 0 & 4en \cos M
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
2en^2 \sin M & en^2 M & -4en \cos M & 0
\end{bmatrix},
\]

where the norm-bounded matrix \( \Delta A_p \) is the uncertain matrix of the in-plane motion model and can be factorized as

\[ \Delta A_p = E_{p1} A_p E_{p2}, \]

where \( E_{p1} \) and \( E_{p2} \) are two constant matrices with proper dimensions; and \( A_p \) is a time-variant matrix bounded by \( A_p^T A_p < I \).

### 2.1.2 Out-of-Plane Motion Model

For out-of-plane motion model, the state vector is defined as \( q(t) = [z, \dot{z}]^T \); the input vector is \( u_q(t) = f_z \). In order to design an out-of-plane motion controller robust to the external disturbance, the perturbation force \( w_q(t) \) are considered and shares the same mechanics mechanism with the control force \( u_q(t) \). Then the out-of-plane motion model can be described as

\[ q(t) = (A_q + \Delta A_q)q(t) + B_{qw} u_q(t) + B_{qw} w_q(t), \]

where

\[
\begin{bmatrix}
0 & 1 \\
-n^2 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 \\
-3en^2 \cos M & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix},
\]

where

\[ A_q = \begin{bmatrix} 0 & 1 \\ -n^2 & 0 \end{bmatrix}, \quad \Delta A_q = \begin{bmatrix} 0 & 0 \\ -3en^2 \cos M & 0 \end{bmatrix}, \quad B_{qw} = B_{qw} = B_q = \frac{1}{m} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \]

\[ B_{qw} = B_{qw} = B_{qw} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \]

\[ B_{qw} = B_{qw} = B_{qw} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \]

5
where the norm-bounded matrix $\Delta A_q$ is the uncertain matrix of the out-of-plane motion model and can be factorized as

$$\Delta A_q = E_{q1} A_q E_{q2},$$

where $E_{q1}$ and $E_{q2}$ are two constant matrices with proper dimensions; and $A_q$ is a time-variant matrix bounded by $A_q^T A_q < I$.

### 2.2 Problem Formulation

Robust stability of the rendezvous systems, bounded control forces, optimal fuel and time cost are three essential requirements on the partially independent controller. Based on these requirements, in-plane and out-of-plane rendezvous control problems are formulated successively as follows.

#### 2.2.1 Control Problem of In-Plane Motion

To assess the performance of in-plane motion controller, define the in-plane quadratic cost function as

$$J_p = \int_0^\infty \left[ p^T(t) Q_p p(t) + u^T_p R_p u_p(t) \right] \, dt,$$

where state weighting matrix $Q_p$ is a positive symmetric matrix related to the convergence rate of the in-plane states and the smoothness of the in-plane trajectory; and control weighting matrix $R_p$ is a positive symmetric matrix related to the fuel cost of the in-plane maneuver. By introducing two auxiliary matrices, $U_{px} = [1, 0]^T$ and $U_{py} = [0, 1]^T$, the thrust constraints for in-plane maneuver can be formulated as

$$|f_i| = |U_{pi} u(t)| \leq u_{pi,max}, \quad (i = x, y),$$

where $u_{pi,max}$ is the maximum control forces that propellers can generate along the $i$-axis. Then according to the in-plane motion model (10) and the requirements presented above, the in-plane motion control problem to be investigated is stated below.

Based on plant model (10), design a robust guaranteed cost controller with input saturation such that the following requirements are met during the rendezvous.

(i) In-plane motion system (10) is asymptotically stable at $p(t) = 0$, which means the chase vehicle is able to rendezvous with the target vehicle in the target’s orbital plane.

(ii) Quadratic cost function (14) should be the minimal. In other words, the in-plane rendezvous should be an optimal compromise between the fuel cost, the time cost and the smoothness of the in-plane trajectory.

(iii) Control forces along the $x$- and $y$-axis should not exceed the upper bounds $u_{pi,max}$ ($i = x, y$) given in (15).

#### 2.2.2 Control Problem of Out-of-Plane Motion

To evaluate the performance of out-of-plane motion controller, define the out-of-plane quadratic cost function as

$$J_q = \int_0^\infty \left[ q^T(t) Q_q q(t) + u^T_q R_q u_q(t) \right] \, dt,$$

where $Q_q$ is a positive symmetric matrix related to the convergence rate and the smoothness of the out-of-plane trajectory; and $R_q$ is a positive scalar related to the fuel cost of the out-of-plane maneuver.

After phasing, a phase before terminal rendezvous [23], the out-of-plane distance between two spacecraft is usually very small, so it is reasonable for us to assume that the initial out-of-plane distance and velocity between the spacecraft are zero at the outset of the terminal rendezvous.

It is well known that the maximum control thrust along the $z$-axis is mainly circumscribed by the sum of perturbation forces out of the orbital plane. Therefore, once the out-of-plane thrusters are chosen properly, the limited-thrust requirement on the out-of-plane controller can be omitted.

Summing up the out-of-plane motion model (12) and some demands given above, in order to be robust to the perturbation forces, robust $H_\infty$ control technique is utilized, and the out-of-plane motion control problem is stated below.

Based on plant model (12), design a robust $H_\infty$ controller such that the following requirements during the rendezvous.

(iv) Out-of-plane motion system (12) is robustly stable at $q(t) = 0$, which means the chase vehicle can be stabilized in the targets orbital plane in spite of the perturbation force $w_q(t)$ and the parameter uncertainty $\Delta A_q$.

(v) Quadratic cost function (16) is the minimal subject to $w_q(t)$ and $\Delta A_q$, which means the out-of-plane rendezvous is also an optimal compromise between the fuel cost, the time cost and the smoothness of the out-of-plane trajectory.
3 Partially Independent Controller

In this section, a robust guaranteed cost controller with input saturation for in-plane motion control and a robust $H_\infty$ controller for out-of-plane motion control will be synthesized.

A lemma needed by the subsequent derivation is given, whose proof and application can be found in [18].

**Lemma 2** (see [18]). Given matrices $Y = Y^T$, $D$ and $E$ of appropriate dimensions,
\[
Y + DFE + E^T F^T D^T < 0,
\]
for all $E$ satisfying $E^T E \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that
\[
Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.
\]

### 3.1 In-Plane Motion Controller

Consider the following state feedback control law
\[
u_p(t) = -K_p p(t),
\]
where $K_p$ is the state feedback gain matrix of in-plane motion controller. Substituting equation (19) into the plant model (10), we get the closed-loop in-plane motion model
\[
p(t) = (A_p + \Delta A_p - B_p K_p) p(t).
\]

Sufficient condition for the existence of a robust guaranteed cost controller with input saturation is given below.

**Theorem 1.** Consider the closed-loop system (20) with the state feedback control law in (19). For a given in-plane initial state vector $p(0)$, if there exist a positive symmetric matrix $X_p$, a matrix $Y_p$ with proper dimensions, positive scalars $\varepsilon_p$ and $\rho$ satisfying
\[
\begin{bmatrix}
sym(A_p X_p - B_p Y_p) + \varepsilon_p E_{p1} E_{p1}^T X_p E_{p2}^T & Y_p^T & X_p \\
\ast & -\varepsilon_p I & 0 \\
\ast & \ast & -R_p^{-1} \\
\ast & \ast & \ast & -Q_p^{-1}
\end{bmatrix} < 0,
\]
then there exists a proper controller such that the closed-loop system (20) is asymptotically stable at $p(t) = 0$; the positive scalar $\rho$ is an upper bound of the quadratic cost function (14), and the control forces are limited within $u_{pi,max}$ for $i = x, y$.

**Proof.** Consider the Lyapunov function $V_p(t) = p^T(t) P_p p(t)$, where $P_p \in \mathbb{R}^{4 \times 4}$ is a positive symmetric matrix. Substituting (20) into the derivative of $V_p(t)$, we obtain
\[
\dot{V}_p(t) = \text{sym} \left[ p^T(t) P_p (A_p + \Delta A_p - B_p K_p) p(t) \right].
\]

In order to optimize the cost function and guarantee the stability of the in-plane motion, let the following inequalities hold
\[
V_p(t) < -\left[ p^T(t) Q_p p(t) + u_p^T(t) R_p u_p(t) \right] < 0.
\]

Integrating (25) from 0 to $\infty$ and noticing that $p(t) \to 0$ as $t \to \infty$, we get
\[
0 < J_p = \int_0^\infty \left[ p^T(t) Q_p p(t) + u_p^T(t) R_p u_p(t) \right] dt \leq V_p(0).
\]

From (26), we can see that $V_p(0) = p^T(0) P_p p(0)$ is an upper bound of the quadratic cost function $J_p$, when inequalities (25) holds. Substituting (11), (19) and (24) into (25), inequalities (25) can be ensured by
\[
\Psi_p + (P_p E_{p1}) A_p E_{p2} + E_{p2}^T A_p^T (P_p E_{p1})^T < 0,
\]

7
where
\[ \Psi_p = \text{sym} \left[ P_p (A_p - B_p K_p) \right] + Q_p + K_p^T R_p K_p \, . \]

As \( \Psi_p \) is a symmetric matrix, and \( A_p \) satisfies \( A_p^T \leq I \), by Lemma 2, there exists a positive scalar \( \varepsilon_p \) ensuring (27) by
\[ \Psi_p + \varepsilon_p (P_p E_{p1}) (P_p E_{p1})^T + \varepsilon_p^{-1} E_{p2}^T E_{p2} < 0 \, . \]

According to Schur complement, (28) is equivalent to
\[
\begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
* & \Pi_{22}
\end{bmatrix} < 0, \tag{29}
\]
where
\[
\Pi_{11} = \text{sym} \left[ P_p (A_p - B_p K_p) \right] + \varepsilon_p P_p E_{p1} E_{p1}^T P_p^T,
\]
\[ \Pi_{12} = \begin{bmatrix} E_{p2}^T & K_p^T & I \end{bmatrix}, \]
\[ \Pi_{22} = \text{diag} \left( -\varepsilon I, -R_p^{-1}, -Q_p^{-1} \right) . \]

Defining \( X_p = P_p^{-1}, Y_p = K_p P_p^{-1} \), pre- and post-multiplying (29) with \( \text{diag} (P_p^{-1}, I) \), we obtain LMI (21).

Minimizing the upper bound \( V_p (0) \), a positive scalar \( \rho \) is introduced and meets
\[ V_p (0) = p^T (0) P_p p (0) \leq \rho \, . \tag{30} \]

Applying Schur complement, inequality (30) is equivalent to
\[
\begin{bmatrix}
\rho & p^T (0) \\
* & -P_p^{-1}
\end{bmatrix} < 0. \tag{31}
\]

By the definition, \( X_p = P_p^{-1}, Y_p = K_p P_p^{-1} \), inequality (31) can be transformed into LMI (22). The previous derivation gives the sufficient condition for the existence of the controller that meets requirements (i) and (ii). To fill requirement (iii), squaring both sides of (15), substituting (19) into the result and dividing both sides of the result by \( u_{pi,\max}^{2} \), there is
\[ u_{pi,\max}^{-2} [U_p K_p p (t)]^T U_p K_p p (t) \leq 1 \, . \tag{32} \]

The control forces along the x- and y-axis can be limited within \( u_{pi,\max} \, (i = x, y) \), when (32) holds. Dividing both sides of (30) by \( \rho \) and considering \( V (t) < 0 \), we have
\[ \rho^{-1} V_p (t) < \rho^{-1} V_p (0) \leq 1 \, . \tag{33} \]

Utilizing inequalities (33), we can ensure inequality (32) by
\[ u_{pi,\max}^{-2} [U_p K_p p (t)]^T U_p K_p p (t) < \rho^{-1} p^T (t) P_p p (t) \, . \tag{34} \]

By Schur complement, inequality (34) is equivalent to
\[
\begin{bmatrix}
-\rho^{-1} I & U_p K_p \\
* & -u_{pi,\max}^{-2} P_p
\end{bmatrix} < 0. \tag{35}
\]

Pre- and post-multiplying (35) with \( \text{diag} (I, X_p) \), we obtain LMI (23). This completes the proof. \( \square \)

From (30), we can infer that by minimizing positive scalar \( \rho \), the upper bound of quadratic cost is also minimized. In order to solve the optimal controller from the result of Theorem 1, a positive scalar \( w \) is introduced to convert Theorem 1 into a minimization problem which can be readily solved by commercial software. The positive scalar \( w \) is supposed to meet \( \rho < w \), which is equivalent to
\[ \begin{bmatrix} -w & 1 \\
1 & -\rho^{-1} \end{bmatrix} < 0 \, . \tag{36} \]

Then the robust guaranteed cost controller with input saturation for in-plane motion can be solved by the following convex optimization problem,
\[
\min_{\varepsilon_p, \rho^{-1}, X_p, Y_p} w \, , \tag{37}
\]
An optimal solution that consists of $\varepsilon_q$, $\rho$, $w$, $X_p$ and $Y_p$ can be obtained, and the in-plane state feedback gain matrix $K_p$ is then determined by $K_p = Y_p X_p^{-1}$. As no preassigned constant is required and because of its concise format, the improved saturation control law embedded in Theorem 1 is less conservative and more practical than those used in [25, 27, 31].

3.2 Out-of-Plane Motion Controller

For out-of-plane motion controller, consider the following state feedback control law

$$u_q(t) = -K_q q(t) ,$$

where $K_q$ is the state feedback gain matrix. Substituting (38) into the plant model (12), we get the closed-loop out-of-plane motion model

$$q(t) = (A_q + \Delta A_q - B_q K_q) q(t) + B_q w_q(t) .$$

In order to guarantee the robustness of $J_q$ to the external disturbance, define the controlled output as

$$z_q(t) = Q^{-1}_q q(t) + R^{-1}_q u_q(t) .$$

Then requirement (v) can be satisfied, if (40) or $\|z_q(t)\|_2$ is the minimal. Assuming that the cost function (16) is limited by

$$\|z_q(t)\|_2 \leq \gamma \|w(t)\|_2 ,$$

where $\gamma$ is the $H_\infty$ performance. Sufficient condition for the existence of a robust guaranteed cost controller is given below.

**Theorem 2.** Consider the closed-loop system (39) with the state feedback control law in (38). If there exist a positive symmetric matrix $X_q$, a matrix $Y_q$ with proper dimensions and a positive scalar $\varepsilon_q$ satisfying

$$
\begin{bmatrix}
\text{sym} (A_q X_q - B_q Y_q) + \varepsilon_q E_q T E_q^T & B_q & X_q X_q^T & 0 & Y_q^T & X_q^T
\end{bmatrix}
\begin{bmatrix}
\gamma^2 I & 0 & 0 & 0
* & -\varepsilon_q I & 0 & 0
* & * & -\varepsilon_q I & 0
* & * & * & -\varepsilon_q I
* & * & * & * & -Q^{-1}_q
\end{bmatrix} < 0 ,
$$

then there exists a proper controller such that system (39) is robust stable at $q(t) = 0$, and the quadratic cost (16) is guaranteed by (41) subject to the parameter uncertainty and external disturbance.

**Proof.** Consider the Lyapunov function $V_q(t) = q^T(t) P_q q(t)$, where $P_q \in \mathbb{R}^{2 \times 2}$ is a positive symmetric matrix. Substituting (20) into the derivative of $V_q(t)$, there is

$$V_q(t) = \left[ q(t) \begin{array}{c} \varepsilon_q \end{array} \right]^T \begin{bmatrix}
\text{sym} [P_p (A_q + \Delta A_q - B_q K_q)] & P_p B_q
0 & 0
\end{bmatrix} \begin{bmatrix}
q(t) \varepsilon_q
\end{bmatrix} .$$

Despite external disturbance $w_q(t)$, the derivative of $V_q(t)$ is

$$V_{q0}(t) = \text{sym} \left[ q^T(t) P_q (A_q + \Delta A_q - B_q K_q) q(t) \right] .$$

Squaring both sides of (41), there is

$$z_{qT}(t) z_q(t) - \gamma^2 w_{qT}(t) w_q(t) \leq 0 .$$

Integrating (45) from 0 to $\infty$, we have

$$\int_0^\infty [z_{qT}(t) z_q(t) - \gamma^2 w_{qT}(t) w_q(t) + V_q(t)] dt + V_q(0) - V_q(\infty) \leq 0 .$$

According the assumption of zero-initial condition and $V_q(\infty) > 0$, inequality (46) can be ensured by

$$z_{qT}(t) z_q(t) - \gamma^2 w_{qT}(t) w_q(t) + V_q(t) \leq 0 .$$
For more general situation where zero-initial condition is not met, the robust $H_\infty$ control method can refer to [35–38]. Substituting (40) and (43) into (47), inequality (47) can be rewritten as

$$\begin{bmatrix} \text{sym} [PP_q (A_q + \Delta A_q - B_q K_q)] + Q_q + K_q^T R_q K_q & P_q B_q \\ * & -\gamma^2 I \end{bmatrix} < 0. \tag{48}$$

Using Schur complement, inequality (48) is equivalent to

$$\Theta_1 < \Theta_2, \tag{49}$$

where

$$\Theta_1 = \text{sym} [PP_q (A_q + \Delta A_q - B_q K_q)],$$

$$\Theta_2 = -Q_q - K_q^T R_q K_q - \gamma^{-2} P_q B_q (P_q B_q)^T.$$

From (49), it is easy to see that $\Theta_1 < \Theta_2 < 0$. According to (44), $\Theta_1 < 0$ is equivalent to $V_q < 0$, which fills requirement (iv).

Substituting (13) into (48), we have

$$\Psi_q + \Delta_q \Phi_q E_q + E_q^T \Phi_q^T \Delta_q < 0, \tag{50}$$

where

$$\Psi_q = \begin{bmatrix} \text{sym} [PP_q (A_q - B_q K_q)] + Q_q + K_q^T R_q K_q & P_q B_q \\ * & -\gamma^2 I \end{bmatrix},$$

$$\Delta_q = \begin{bmatrix} P_q E_q & 0 \\ 0 & 0 \end{bmatrix}, \quad \Phi_q = \begin{bmatrix} \Lambda_q & 0 \\ 0 & 0 \end{bmatrix}, \quad E_q = \begin{bmatrix} E_{q1} & 0 \\ 0 & 0 \end{bmatrix}.$$

As $\Psi_q$ is a symmetric matrix and $\Phi_q^T \Phi_q < I$, by Lemma 2, there exists a positive scalar $\varepsilon_q$ ensuring (50) by

$$\Psi_q + \varepsilon_q \Delta_q \Lambda_q^T + \varepsilon_q^{-1} E_q^T E_q < 0. \tag{51}$$

By Schur complement, inequality (51) is equivalent to

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} < 0, \tag{52}$$

where

$$\Omega_{11} = \begin{bmatrix} \text{sym} [PP_q (A_q - B_q K_q)] + \varepsilon_q P_q E_q E_q^T P_q^T & P_q B_q \\ * & -\gamma^2 I \end{bmatrix}, \quad \Omega_{12} = \begin{bmatrix} E_{q1}^T & 0 & K_q^T & I \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{22} = \text{diag} (-\varepsilon I, \varepsilon I, -R_q^{-1}, -Q_q^{-1}).$$

Defining $X_q = P_q^{-1}$ and $Y_q = K_q P_q^{-1}$, pre- and post-multiplying (52) with $\text{diag}(P_q^{-1}, I)$, we obtain LMI (42). This completes the proof.

In order to get the optimal controller, which meets the requirement (iv) and (v), the following convex optimization problem needs to solve

$$\min_{\varepsilon_q, X_q, Y_q, \gamma} \gamma,$$

s.t. (42).

An optimal solution that consists of $\gamma$, $\varepsilon_q$, $X_q$ and $Y_q$ can be obtained, and the out-of-plane state feedback gain matrix $K_q$ is then determined by $K_q = Y_q X_q^{-1}$.
4 Illustrative Example

In this section, a numerical example that compares a coupled controller with our proposed partially independent controller is given to illustrate the usefulness and advantages of the latter. All the results are obtained from a simulation program built on the two-body problem.

Consider a scenario of spacecraft rendezvous as follows. A target vehicle is in a low earth orbit (LEO) with eccentricity $e = 0.05$ and semimajor axis $a = 7082.253$ km; then we can figure out the mean motion of this target vehicle is $n = 1.06 \times 10^{-3}$ rad/s. The initial positions of the chase vehicle are -5000 m, 5000 m and 0 m from the target vehicle along the $x$-, $y$- and $z$-axis; thus the initial state vector of the rendezvous system is $x(0) = [-5000, 5000, 0, 5, -5, 0]$. The mass of the chase vehicle is 200 kg; the maximum control forces of the chaser along the $x$-, $y$- and $z$-axis are 30 N, 30 N and 10 N; and the perturbation force is set to

$$w_q(t) = 5 \sin (0.1t).$$

All the weighting matrices ($Q_p, Q_q, R_p$ and $R_q$) are assigned to be indentity matrices. With the parameters assigned above, an optimal partially independent controller and an optimal coupled controller are solved in the following.

4.1 Optimal Partially Independent Controller

The optimal partially independent controller is integrated with the solutions of (37) and (53). From the simulation parameters presented at the beginning of this section, the initial state vector of the in-plane motion is $p(0) = [-5000, 5000, 5, -5]^T$. According to (11), we can assign the matrices $E_{p1}, E_{p2}$ and $A_p$ as

$$E_{p1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2e & 4e & 0 \\ 2e & 0 & 0 & 4e \end{bmatrix}, \quad E_{p2} = \begin{bmatrix} n^2 & 0 & 0 & 0 \\ 0 & n^2 & 0 & 0 \\ 2.5n^2 & 0 & 0 & n \\ 0 & 0.25n^2 & -n & 0 \end{bmatrix}.$$

$$A_p = \text{diag} (\sin M, -\sin M, \cos M, \cos M),$$

where the mean anomaly $M = nt$. Then we are now able to obtain the in-plane part of the optimal controller by solving (37). The state feedback gain matrix of in-plane motion controller is

$$K_p = \begin{bmatrix} K_{p,11} & K_{p,12} \end{bmatrix} = \begin{bmatrix} 0.0040 & -0.0024 & 0.5331 & -0.1432 \\ -0.0004 & 0.0033 & -0.0881 & 0.9718 \end{bmatrix}. \tag{56}$$

where $K_{p,11}$ and $K_{p,12} \in \mathbb{R}^{2 \times 2}$.

The initial state vector of the out-of-plane motion is $q(0) = [0, 0]^T$. According to equation (13), we can assign the matrices $E_{q1}, E_{q2}$ and $A_q$ as

$$E_{q1} = \begin{bmatrix} 0 & 0 \\ 6e & 0 \end{bmatrix}, \quad E_{q2} = \begin{bmatrix} n^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_q = \begin{bmatrix} -0.5 \cos M & 0 \\ 0 & 0 \end{bmatrix}. \tag{57}$$

Then we are now able to get the out-of-plane part of the controller by solving (53). The optimal $H_{\infty}$ performance $\gamma = 1.0014106$, and the state feedback gain matrix of out-of-plane motion controller is

$$K_q = \begin{bmatrix} K_{q,11} & K_{q,12} \end{bmatrix} = \begin{bmatrix} 77.8897 & 8.5170 \times 10^3 \end{bmatrix}. \tag{58}$$

Integrate (56) and (58) together, for the optimal partially independent controller, the three-axis control signal can be generated by $u(t) = K_{pic}x(t)$, where the integrated state feedback gain matrix $K_{pic} \in \mathbb{R}^{3 \times 6}$ is

$$K_{pic} = \begin{bmatrix} K_{p,11} & 0 & K_{p,12} & 0 \\ 0 & K_{q,11} & 0 & K_{q,12} \end{bmatrix}. \tag{59}$$
4.2 Optimal Coupled Controller

In section 3.1, we designed a in-plane motion controller by considering the motion along the $x$- and $y$-axis together. Using the same method, when consider the motions along all three axes together, we can synthesize a coupled robust guaranteed cost controller to make a comparison with the partially independent controller. For brevity, the detailed derivations will not be presented in this paper, but similar procedures for synthesizing the coupled controller can be found in [23, 30]. For the coupled controller, the three-axis control input vector can be generated by $u(t) = K_{cc}x(t)$, where the state feedback gain matrix $K_{cc}$ is

$$
K_{cc} = \begin{bmatrix}
0.0041 & -0.0023 & 0.0004 & 0.5867 & -0.0962 & 0.1239 \\
-0.0004 & 0.0032 & 0.0002 & -0.0718 & 0.9779 & 0.0509 \\
0.0007 & 0.0002 & 0.0015 & 0.1928 & 0.1060 & 0.7455
\end{bmatrix},
$$

which is determined by solving a convex optimization problem similar to (37).

4.3 Simulation Results

All the simulation data presented in this subsection are collected from a nonlinear two-body model, which is more accurate and precise than the plant model (9). Simulation results of the in-plane motion and those of the out-of-plane motion will be shown successively.

4.3.1 In-Plane Motion

Figure 2 depicts the in-plane trajectories of the chase vehicles controlled by controller (59) and (60). As the difference between these two trajectories is not obvious, for clarity, only position states and propulsive thrusts generated by the optimal partially independent controller are shown in Figure 3 and Figure 4.

From Figure 2 and Figure 3, it can be seen that the state vector converges to $0$ asymptotically with both controllers, thus the performances of these two controllers are similar in the orbital plane. Moreover, from the curves and the maximum magnitudes of the control forces shown in Figure 4, we can conclude that with the partially independent control law, the control forces of the chase vehicle are restricted under the upper bounds given in equation (15) with little conservation.

4.3.2 Out-of-Plane Motion

Figure 5 illustrates the out-of-plane distance between two spacecraft when controlled by different controllers; the out-of-plane control forces generated by different control framework are given in Figure 6; and Figure 7 depicts the out-of-plane cost functions of different control schemes during the rendezvous.

Comparing the amplitudes of the relative distances in Figure (5a) and (5b), it can be concluded that the partially independent controller performs much better than the coupled controller in suppressing the external disturbance. In Figure 6, we can find that both of the controllers generate bounded control signals, which justifies the prediction made in subsection 2.2.2; nevertheless, Figure 6 also explicates that the input signal can track the disturbance well only with the partially independent control scheme. From Figure 7, we can see that although the partially independent control method costs more fuel in compensation for disturbance, the overall cost function $J_q$ of the partially independent controller is much lower than that of the coupled controller.

5 Conclusions

In sum, this paper has proposed a partially independent control proposed for thrust-limited rendezvous in near circular orbits. Based on the two-body problem, a linearization manner to establish linear model of rendezvous in near-circular orbits has been given. The derivation of the partially independent controller has been explicated. An illustrative example has been presented, which showed the advantages of the controller introduced in this paper. Due to the robust stability, optimal performance and input constraints of this partially independent controller, it has a wide range of application in spacecraft rendezvous.

Acknowledgments

The authors specially acknowledge the staff of arXiv.org and the readers of this paper.
Figure 2: In-plane rendezvous trajectories during the first 3000s.

Figure 3: Relative distance between two spacecraft in orbital plane.

Figure 4: Propulsive thrust of chase vehicle in orbital plane.
Figure 5: Relative distance between two spacecraft out of orbital plane.

Figure 6: Propulsive thrust of chase vehicle out of orbital plane.
Partially independent controller
Coupled controller

Figure 7: Difference in out-of-plane quadratic cost function.

References

[1] W. H. Clohessy and R. S. Wiltshire, “Terminal guidance system for satellite rendezvous,” Journal of the Aerospace Sciences, vol. 31, no. 11, pp. 1516-1521, 1960.
[2] J. P. De Vries, “Elliptic elements in terms of small increments of position and velocity components,” AIAA Journal, vol. 1, no. 11, pp. 2626-2629, 1963.
[3] J. Tschauer, “Elliptic orbit rendezvous,” AIAA Journal, vol. 5, no. 6, pp. 1110-1113, 1967.
[4] T. E. Carter, “State transition matrices for terminal rendezvous studies: brief survey and new example,” Journal of Guidance, Control, and Dynamics, vol. 21, no. 1, pp. 148-155, 1998.
[5] M. L. Anthony and F. T. Sasaki, “Rendezvous problem for nearly circular orbits,” AIAA Journal, vol. 3, no. 9, pp. 1666-1673, 1965.
[6] R. G. Melton, “Time-explicit representation of relative motion between elliptical orbits,” Journal of Guidance, Control, and Dynamics, vol. 23, no. 4, pp. 604-610, 2000.
[7] S. S. L. Chang and T. K. C. Peng, “Adaptive guaranteed cost control of systems with uncertain parameters,” IEEE Transactions on Automatic Control, vol. 17, no. 4, pp. 474-483, 1972.
[8] I. R. Petersen and D. C. McFarlane, “Optimal guaranteed cost control and filtering for uncertain linear systems,” IEEE Transactions on Automatic Control, vol. 39, no. 9, pp. 1971-1977, 1994.
[9] L. Yu and J. Chu, “An LMI approach to guaranteed cost control of linear uncertain time-delay systems,” Automatica, vol. 35, no. 6, pp. 1155-1159, 1999.
[10] S. H. Esfahani and I. R. Petersen, “An LMI approach to the output-feedback guaranteed cost control for uncertain time-delay systems,” International Journal of Robust and Nonlinear Control, vol. 10, no. 3, pp. 157-174, 2000.
[11] X. P. Guan and C. L. Chen, “Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays,” IEEE Transactions on Fuzzy Systems, vol. 12, no. 2, pp. 236-249, 2004.
[12] L. Wu, J. Lam, X. Yao, and J. Xiong, “Robust guaranteed cost control of discrete-time networked control systems,” Optimal Control Applications and Methods, vol. 32, no. 1, pp. 95-112, 2011.
[13] H. Zhang, Y. Wang, and D. Liu, “Delay-Dependent Guaranteed Cost Control for Uncertain Stochastic Fuzzy Systems with Multiple Time Delays,” IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 38, no. 1, pp. 126-140, 2008.
[14] K. Tanaka, H. Ohtake, and H. O. Wang, “Guaranteed cost control of polynomial fuzzy systems via a sum of squares approach,” *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 38, no. 2, pp. 561-567, 2009.

[15] G. Zames, “Feedback and optimal sensitivity: model reference transformation multiplicative seminorms, and approximate inverse,” *IEEE Transactions on Automatic Control*, vol. 26, no. 2, pp. 301-320, 1981.

[16] B. R. Zames, “Stabilization of uncertain systems via linear control,” *IEEE Transactions on Automatic Control*, vol. 28, no. 8, pp. 848-850, 1983.

[17] B. A. Francis, *A Course in $H_\infty$ Control Theory*, Springer, Berlin, 1987.

[18] P. P. Khargonekar, I. R. Petersen, and K. Zhou, “Robust stabilization of uncertain linear system: quadratic stability and $H_\infty$ control theory,” *IEEE Transactions on Automatic Control*, vol. 35, no. 3, pp. 356-361, 1990.

[19] S. P. Boyd, *Linear Matrix Inequalities in System and Control Theory* (Vol. 15), SIAM, Philadelphia, 1994.

[20] M. Chilali and P. Gahinet, “$H_\infty$ design with pole placement constraints: an LMI approach,” *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 358-367, 1996.

[21] M. Liu, J. You, and X. Ma, “$H_\infty$ filtering for sampled-data stochastic systems with limited capacity channel,” *Signal Processing*, vol. 91, no. 8, pp. 1826-1837, 2011.

[22] L. Wu, X. Su, and S. Peng, “Mixed $H_2/H_\infty$ approach to fault detection of discrete linear repetitive processes,” *Journal of the Franklin Institute*, vol. 348, no. 2, pp. 393-414, 2011.

[23] N. Wan, M. Liu, and H. R. Karimi, “Robust tracking control for rendezvous in near-circular orbits,” *Mathematical Problems in Engineering*, vol. 2013, Article ID 726945, 11 pages, 2013.

[24] B. Ebrahimi, M. Bahrami, and J. Roshanian, “Optimal sliding-mode guidance with terminal velocity constraint for fixed-interval propulsive maneuvers,” *Acta Astronautica*, vol. 62, no. 10, pp. 556-562, 2008.

[25] H. Gao, X. Yang, and P. Shi, “Multi-Objective Robust $H_\infty$ Control of Spacecraft Rendezvous,” *IEEE Transactions on Control Systems Technology*, vol. 17, no. 4, pp. 794-802, 2009.

[26] Z. Li, M. Liu, H. R. Karimi, and X. Cao, “Sampled-data control of spacecraft rendezvous with discontinuous Lyapunov approach,” *Mathematical Problems in Engineering*, vol. 2013, Article ID 814271, 10 pages, 2013.

[27] X. Yang, and H. Gao, “Robust reliable control for autonomous spacecraft rendezvous with limited-thrust,” *Aerospace Science and Technology*, vol. 24, no. 1, pp. 161-168, 2013.

[28] X. Gao, K. L. Teo, and G. R. Duan, “Robust $H_\infty$ Control of Spacecraft Rendezvous on Elliptical Orbit,” *Journal of the Franklin Institute*, vol. 349, no. 8, pp. 2515-2529, 2012.

[29] X. Yang, X. Cao, and H. Gao, “Sampled-data control for relative position holding of spacecraft rendezvous with thrust nonlinearity,” *IEEE Transactions on Industrial Electronics*, vol. 59, no. 2, pp. 1146-1153, 2012.

[30] N. Wan, M. Liu, and H. R. Karimi, “Observer-based robust control for spacecraft rendezvous with thrust saturation,” *Abstract and Applied Analysis*, vol. 2014, Article ID 710850, 10 pages, 2014.

[31] D. Sheng, X. Yang, and H. R. Karimi, “Robust control for autonomous spacecraft evacuation with model uncertainty and upper bound of performance with constraints,” *Mathematical Problems in Engineering*, vol. 2013, Article ID 589381, 18 pages, 2013.

[32] Z. Li, M. Liu, and H. R. Karimi, “Observer-based stabilization of spacecraft rendezvous with variable sampling and sensor nonlinearity,” *Mathematical Problems in Engineering*, vol. 2013, Article ID 902452, 11 pages, 2013.

[33] Z. Li, X. Yang, and H. Gao, “Autonomous impulsive rendezvous for spacecraft under orbital uncertainty and thruster faults,” *Journal of the Franklin Institute*, vol. 350, no. 9, pp. 2455-2473, 2013.

[34] R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics* AIAA, Ohio, 1999.

[35] P. P. Khargonekar, K. M. Nagpal, and K. R. Poolla, “$H_\infty$ control with transients,” *SIAM Journal of Control and Optimization*, vol. 29, no. 6, pp. 1373-1393, 1991.

[36] T. Namikawa, M. Fujita and R. S. Smith, “A generalized $H_\infty$ control system design attenuating initial state uncertainties,” in *Proceedings of the American Control Conference*, pp. 2204-2209, 2002.
[37] A. V. Savkin, P. N. Pathirana, and F. A. Faruqi. “Problem of precision missile guidance: LQR and $H_\infty$ control frameworks,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 3, pp. 901-910, 2003.

[38] Y. K. Foo. “$H_\infty$ control with initial conditions,” *IEEE Transactions on Circuits and System*, vol. 53, no. 9, pp. 867-871, 2006.