Closing the Gap on $R_{D^*}$ by including longitudinal effects

J. E. Chavez-Saab and Genaro Toledo

Instituto de Física, Universidad Nacional Autónoma de México, AP20-364, Ciudad de México 01000, Mexico.

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Measurements of the $R_{D^*} \equiv \text{Br}(B \to \tau \nu D^*)/\text{Br}(B \to \ell \nu D^*)$ parameter remain in tension with the standard model prediction, despite recent results helping to close the gap. The standard model prediction it is compared with considers the $D^*$ as an external particle, even though what is detected in experiments is a $D\pi$ pair it decays into, from which it is reconstructed. We argue that the experimental result must be compared with the theoretical prediction considering the full 4-body decay ($B \to \ell \nu D^* \to \ell \nu D\pi$). We show that the longitudinal degree of freedom of the off-shell $D^*$ helps to further close the disagreement gap with experimental data. We find values for the ratio $R_{D^*} \equiv \text{Br}(B \to \tau \nu, D\pi)/\text{Br}(B \to \ell \nu, D\pi)$ of $R_{D^*} = 0.271 \pm 0.003$ and $R_{D^*} = 0.273 \pm 0.003$, where the uncertainty comes from the uncertainty of the form factors parameters. Comparing against $R_{D^*}$ reduces the gap with the latest LHCb result from 0.94σ to 0.37σ, while the gap with the latest Belle result is reduced from 0.40σ to just 0.04σ and with the world average results from 3.4σ to 2.2σ.

I. INTRODUCTION

The formulation of the Standard Model (SM) incorporates the three families of leptons with universal couplings to gauge bosons, such that the differences in similar processes for different leptons are only of kinematical origin. This lepton flavor universality has been tested in the weak sector, for example, by studying the semileptonic decays of heavy mesons. The parameter that is expected to reflect the universality property is defined as the ratio of similar processes into two different leptons, namely

$$R_X \equiv \frac{\text{Br}(P \to \tau \nu, X)}{\text{Br}(P \to \ell \nu, X)},$$

(1)

where $P$ is the decaying particle, typically a pseudoscalar meson, $l = e, \mu$ and $X$ is the hadronic product.

Early measurements of $R_D$ [12] from $B$ meson decays prompted the question of possible lepton flavour violation after finding significant discrepancies with the SM predictions, although a more recent measurement has obtained $R_D = 0.375 \pm 0.064 \pm 0.026$ [3] which is in better agreement with the SM prediction of $R_{D}^{SM} = 0.300 \pm 0.008$ [4].

For $R_{D^*}$, several B-factories conducted experiments which have also consistently measured values of $R_{D^*}$ that are higher than the SM prediction [1,3,5,6]. The most recent results from the LHCb [1] and Belle [8] collaborations, that study the process through hadronic channels of the $\tau$ decay, have reduced this disagreement gap to a statistical significance of just 0.94σ and 0.42σ, respectively. However, a discrepancy with combined statistical significance of 3.4σ [2] is still found when considering the previous experiments which use other τ reconstruction methods [1,2,3,5,6]. Since the deviation is still large in the $R_{D^*}$ case, we can inquire at which extent the nature of the hadronic particle in the final state affects the result. The $B \to \ell \nu D^*$ decay is represented by a single tree-level diagram, shown in Fig. 1. Since the $D^*$ decay width is relatively small, its decay process is usually considered not to play any role in the estimation of $R_{D^*}$ (since it is common to both leptonic decay modes, it should cancel in the ratio). Thus, it is common to consider simply this 3-body decay, from which a value of $R^{SM}_{D^*} = 0.252 \pm 0.003$ is obtained [10]. The error bar accounts for the uncertainties on the form factors of the vertex connecting the $B$, $W$ and $D^*$ particles as measured by Belle [11].

We argue that the tension of the theoretical prediction with experimental measurements is due in part to the fact that $R_{D^*}$ is not the proper quantity that the results should be compared to, since in all the experiments the $D^*$ is never measured directly but through its decay into a $D\pi$ pair. Therefore, it is adequate to consider the ratio

$$R_{D^*} \equiv \frac{\text{Br}(B \to \tau \nu, D\pi)}{\text{Br}(B \to \ell \nu, D\pi)},$$

(2)

obtained from the full 4-body diagram shown in Fig. 2 as the proper value to compare to. Upcoming experiments will help to settle down the experimental value, and a solid SM prediction to compare to is mandatory.

In this work, we show that the corrections that arise from the full process, corresponding to adding the longitudinal degree of freedom of the off-shell $D^*$, are not negligible and help to get the experimental and theoretical results in better agreement. In section [11] we elaborate on the origin of these corrections and contrast with the shortened 3-body decay. Then, we present our result for

![Diagram](image-url)
Here, the transversal correction is proportional to the polarizations the squared amplitude becomes:

\[ |M|^2 = M_{3\nu}^* M_{2\nu}^* \left( \frac{m_{D^*}}{m_{D^*}^2} g_\nu + \frac{\rho_\nu}{m_{D^*}} \right)^2 \]

where \( \rho_\nu \) denotes the polarization tensor of the \( D^* \) and \( M_{3\nu} \) and \( M_{2\nu} \) are the 3-body \( D^* \) production and 2-body \( D^* \) decay amplitudes, respectively, with the polarization tensor factored out. By summing over the vector meson polarizations the squared amplitude becomes:

\[ |M|^2 = M_{3\mu} M_{3\alpha}^* \left( -g_{\mu\nu}^{\mu\nu} + \frac{\rho_\mu^{\mu\nu}}{m_{D^*}^2} \right) \times \frac{m_D^2}{m_{D^*}^2} |M_{2\nu}^* M_{2\beta}^*|^2, \]

where \( p_D \) and \( m_{D^*} \) are the momentum and mass of the \( D^* \), respectively.

The complete 4-body amplitude, on the other hand, is given by

\[ M = M_{3\mu} D_{\mu\nu} M_{2\nu}, \]

where \( D_{\mu\nu} \) is the \( D^* \) propagator which, upon considering the absorptive correction (dominated by the \( D\pi \) mode [12]), can be set in terms of the transverse and longitudinal part as follows [13 14]:

\[ D_{\mu\nu} = \frac{-i T_{\mu\nu}}{p_D^2 - m_{D^*}^2 + i m_{D^*} \Gamma_D^*} + \frac{i L_{\mu\nu}}{m_{D^*}^2 - i m_{D^*} \Gamma_L}, \]

with the corresponding projectors:

\[ T_{\mu\nu} \equiv g_{\mu\nu} - \frac{\rho_\mu^{\mu\nu}}{p_D^2} \text{ and } L_{\mu\nu} \equiv \frac{\rho_\mu^{\mu\nu}}{p_D^2}. \]

Here, the transversal correction is proportional to the full decay width, \( i m_{D^*} \Gamma_D^* = \sqrt{p_D^2 \Gamma_D^* (m_{D^*}^2 - m_D^2)} \), while the longitudinal function \( i m_{D^*} \Gamma_L \) is proportional to the \( D - \pi \) mass difference, as will be discussed below.

The transversal part has a pole at \( p_D^2 = m_{D^*}^2 \). Since \( \Gamma_D^* \equiv \Gamma_D^* (m_{D^*}^2) \) is relatively small, the only relevant contribution is just around the pole and a narrow width approximation can be used. This allows us to rewrite the transversal part of the squared amplitude as

\[ |M_T|^2 = M_{3\mu} M_{3\alpha}^* T_{\mu\nu} T_{\alpha\beta} M_{2\nu} M_{2\beta}^* \frac{\pi \delta(p_D^2 - m_{D^*}^2)}{m_{D^*} \Gamma_D^*}. \]

We note that the delta function forces the transversal part of the \( D^* \) to remain on-shell, thus recovering the same structure from \( (3) \), with global factors in \( (5) \) compensated by the splitting of the phase space in \( (3) \). Therefore, taking only the transversal part of the \( D^* \) propagator is equivalent to working with the on-shell scheme that is often used.

On the other hand, the longitudinal part of the propagator remains off-shell because it lacks a pole, and gives place to two new terms in the squared amplitude (one purely longitudinal and one of interference) that cannot be accounted for in the \( B \to \ell \nu D^* \) process.

The 2-body decay amplitude is \( M_{2\nu} = -i g(p_D - p_\pi)_\nu \), where \( p_D \) and \( p_\pi \) are the momenta of the \( D \) and the \( \pi \), respectively, and \( g \) is the \( D^* - D - \pi \) coupling. Thus, the longitudinal part of the amplitude can be written as

\[ M_L = ig M_{3\mu} \frac{p_D^\mu}{p_D^2} \frac{m_D^2 - m_{D^*}^2}{m_{D^*}^2 - i m_{D^*} \Gamma_L}. \]

Hence, the longitudinal corrections are modulated by a dimensionless mass-difference parameter \( \Delta^2 \equiv (m_{D^*}^2 - m_{D^*}^2)/m_{D^*}^2 \). This is a known result for any vector meson that decays into two pseudo-scalars, and \( \Delta^2 \) is usually invoked as a suppression factor. However, due to the large mass difference between the \( D \) and \( \pi \) mesons, here it happens to be a relatively large value of \( \Delta^2 = 0.86 \). Thus, the longitudinal corrections that are missing in the \( B \to \ell \nu D^* \) case may carry an important weight.

The 3-body decay amplitude can be written as

\[ M_3^\mu = \frac{G_F}{\sqrt{2}} V_{cb} \langle D^*(p_D^\mu, \epsilon_\mu)|J^{\mu\nu}|B(p_B) \rangle l_\lambda, \]

where \( l_\lambda \equiv \bar{u}_\gamma \lambda (1 - \gamma^5) u_\nu \) is the leptonic current, \( V_{cb} \) is the CKM matrix element and the hadronic matrix element can be parameterised in terms of four form factors [15 16] (see appendix). The parameters of such form factors have been obtained from a heavy quark analysis of \( B^0 \) decays with electron and muon products measured by the Belle collaboration [11].

\[ R_{D\pi} \] including the uncertainties estimate. The discussion and concluding remarks are presented in section [III].
In calculating the new decay widths, both the transverse and interference parts of the squared amplitudes have been integrated as being on-shell through the narrow width approximation, while the longitudinal part was integrated off-shell, in the full 4-body phase space as given in [17] and implemented with the Vegas subroutine. We have also found that the interference term makes a slight distinction between the $l = e$ and $l = \mu$ cases. Thus, we quote our final result separately as

$$R_{\bar{D}\pi}^l = 0.271 \pm 0.003$$

and

$$R_{D\pi}^l = 0.273 \pm 0.003,$$

where the first uncertainty comes from the uncertainties on the measurement of the form factors ($V$, $A_0$, $A_1$ and $A_2$) that characterize the hadronic vertex between the $B$, $D^*$ and $W$ [11] in agreement with [10]. Notice that $A_0$ is not independent, but derived from $A_2$ as discussed in the appendix. Thus, an anti-correlation on the form factors is present, which turns out to bring the uncertainties lower.

In Table I we show the contribution to the branching ratio from the transversal, interference and longitudinal parts of the amplitude for all three lepton flavor products, which are consistent with Belle measurements for the electron and muon [11]. Within parenthesis we quote the errors coming from the uncertainties in the form factor parameters and $V_{cb}$ [11]. In the last two rows we show $R_{\bar{D}\pi}$ for the electron and the muon as each part is added, namely, transversal, interference and longitudinal parts. We note that the interference part is suppressed, mainly due to the transversality property, but this is not exact as antisymmetric contributions from the $B - D - W$ vertex and the leptonic current tensor break this condition. Also, due to the cancellation of global factors in the ratio, $R_{D\pi}$ has a much higher precision than the individual branching ratios.

An early estimation of the contribution of the $D^*$ longitudinal polarization to the $B \to l \nu D^*$ process [10] quotes a value for the pure longitudinal rate of 0.115(2), which is in agreement with our result of 0.110(3) for the electron and 0.111(3) for the muon.

|          | Transversal | Interference | Longitudinal |
|----------|-------------|--------------|--------------|
| Electron | 4.6(3)      | 7.6(6) × 10^{-8} | 9.0(7) × 10^{-2} |
| Muon     | 4.6(3)      | 1.6(1) × 10^{-3} | 9.0(7) × 10^{-2} |
| Tau      | 1.16(8)     | 1.02(7) × 10^{-1} | 1.00(8) × 10^{-3} |
| $R_{\bar{D}\pi}^e$ | 0.252       | 0.274         | 0.271         |
| $R_{D\pi}^\mu$ | 0.252       | 0.275         | 0.273         |

III. DISCUSSION

We have argued that the experimental information for $R_{D\pi}$ must be compared with the theoretical $R_{D\pi}$, since the experimental information relies on the reconstruction of the full 4-body decay process. We have shown that the longitudinal correction from the $D^*$ propagator introduces a correction to the branching ratios, which produces a value of $R_{\bar{D}\pi}^e = 0.271 \pm 0.003$ and $R_{D\pi}^l = 0.273 \pm 0.003$, where the uncertainty comes from the experimental measurement of the form factor parameters. Within the error bars, $R_{D\pi}$ can be considered as flavor-independent. This contrasts with the value of $R_{D\pi} = 0.252 \pm 0.003$ [10] often used to compare with. The two quantities are distinguishable from each other at the current precision, with a statistical significance of 1.3$\sigma$ and therefore, the correction introduced is meaningful when comparing with the experimental result. Namely, by comparing with $R_{D\pi}$ instead, we find that the difference with the latest results from LHCb [7] goes down from 0.94$\sigma$ to 0.37$\sigma$, while the difference with the latest Belle results [8] goes down from 0.40$\sigma$ to just 0.04$\sigma$, and the difference with the world average results [9] goes down from 3.4$\sigma$ to 2.2$\sigma$. In all cases the agreement with the experiments is improved, but there still remains some tension with the world average results that cannot be explained by the longitudinal corrections alone.

In order to exhibit the role of the form factors, in Fig. 3 we show the contribution of each of them to the three-body differential decay width for the case of the tau (upper panel) and electron (lower panel). The vector-like contribution $A_1$ is the dominant in each case, while the other contributions compete among themselves. Lattice calculations have provided information on this form factor at zero recoil which is consistent with the current experimental information [18]. The $A_0$ form factor accounts for the longitudinal projector for the transferred momentum $q$, corresponding to states with helicity zero for the lepton-neutrino system. Because such a state is forbidden in the zero-mass limit for both the lepton and the neutrino, the term is heavily suppressed for both the electron and the muon, but not the tau. Measurements of the vertex have only been done for electric and muonic flavors [11], where it is undetectable. Instead, $A_0$ is obtained from $A_2$ by invoking an approximate relation derived from heavy quark effective theory [10] [19] [20] and used as such in the tau system. The drastically different behavior that $A_0$ has between flavors might be important for understanding the experimental results. Thus, it is important to measure the form factors and in particular $A_0$ using tau decay channels, to shed light on the nature of possible deviations from the SM.

It has also been suggested that other observables defined in terms of the longitudinal contribution may be useful to search for new physics signals [21]. Early calcu-
and neutral vector mesons [23][25] systems have shown that they are expected to be important at the few percent level in the branching ratios. Thus, upcoming improvements on the experimental side will require accounting for them in the theoretical prediction.

Appendix: Hadronic vertex

For the 3-body decay, the hadronic vertex connecting the $B$, $D^*$ and $W$ is characterized by four form factors as follows [10]:

$$
\langle D^*(p_{D^*}, \epsilon_\mu) | J^{\lambda \mu} | B(p_B) \rangle =
\frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon^{\lambda \mu \alpha \beta} (p_B) (p_{D^*})\beta - 2m_{D^*} A_0(q^2) \frac{q^\lambda q^\mu}{q^2}
- (m_B + m_{D^*}) A_1(q^2) \left( q^\lambda - \frac{q^\mu q^\alpha}{q^2} \right)
+ \frac{A_2(q^2) q^\mu}{m_B + m_{D^*}} \left( (p_B + p_{D^*})\lambda - m_B^2 - m_{D^*}^2 q^\lambda \right),
$$

where $J^{\lambda \mu}$ is the weak current, $\epsilon_\mu$ the polarization vector of the $D^*$, and $q = p_B - p_{D^*}$ the transferred momentum. These terms constitute the most general doubled-indexed Lorentz structure that can be constructed with the available variables, for an on-shell $D^*$. In the 4-body decay, terms proportional to $p_{D^*}^\mu$ may be present, coupled to the longitudinal part of the $D^*$ propagator. Since the form factors have been derived from Belle data [11] without including these terms, a new analysis should be necessary.

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