Right-handed sneutrinos as self-interacting dark matter in supersymmetric economical 3-3-1 model

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Abstract:
In this work we show that the supersymmetric economical $SU(3)_C \otimes SU(3)_L \otimes U(1)_X(3-3-1)$ gauge model has a realistic candidate for self-interacting dark matter. In the model under consideration, the right-handed sneutrino is in bottom of the triplet, which is a singlet of the Standard Model $SU(2)_L$ group. In addition, the right-handed sneutrino is the lightest slepton. By these properties, the right-handed sneutrino is weakly interacting with the Standard Model and stable without introduction of extra symmetry. From the Spergel-Steinhardt condition, the typical mass limit $\leq 10$ MeV is derived. With self-interacting coupling constant fixed by supersymmetry, this limit is deduced without any approximation. The condition for thermal generated self-interacting dark matter in the Universe is also obtained.

Keywords: Particle-theory and field-theory models of the early Universe, Dark matter, Supersymmetry, Supersymmetric partners of known particles

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1. Introduction

One of the themes of the history of physics has been the discovery that the world familiar to us is only a tiny part of an enormous and multi-faceted Universe. Over the past ten years, astronomers have recognized that the stuff that we are made of accounts for only 4% of the total content of the Universe.

Until a few years ago, the more satisfactory cosmological scenarios were ones composed of ordinary matter, cold dark matter and a contribution associated with the cosmological constant. To be consistent with inflationary cosmology, the spectrum of density fluctuations would be nearly scale-invariant and adiabatic. However, in recent years it has been pointed out that the conventional models of collisionless cold dark matter (CCDM) lead to problems with regard to galactic structures. N-body simulations with CCDM indicate that galaxies should have singular halos with large numbers of subhalos. The CCDM predictions for the Tully-Fisher relation and the stability of galactic bars in high surface brightness spiral galaxies are not in agreement with what is observed, indicating lower density galaxy cores than predicted by CCDM. A number of other inconsistencies, which we will not describe here, are discussed in

In order to overcome the possible difficulties of CCDM, one suggestion has been that the cold dark matter particles have a non-dissipative self-interaction, and it has been shown that such cold, non-dissipative self-interacting dark matter (SIDM) can be effective in alleviating the various problems of CCDM. One should notice that self-interacting models lead to spherical halo centers in clusters, which is not in agreement with
ellipsoidal centers indicated by strong gravitational lensing observations and by Chandra ones. However, SIDM models are self-motivated as alternative models. The key property of this kind of matter is that, although its annihilation cross-section is suppressed, its scattering cross section is enhanced.

Several authors have proposed models in which a specific scalar singlet that satisfies the SIDM properties is introduced in the Standard Model (SM) in an \textit{ad hoc} way \cite{3,8}. To be stable, this scalar cannot interact strongly with the SM particles and it is guaranteed by introduction of an extra symmetry (usually an \textit{U}(1)).

The first gauge model for SIDM were found by Fregolente and Tonasse \cite{8} in the minimal 3-3-1 model. The next version of SIDM is the 3-3-1 model with right-handed neutrinos \cite{9} (For alternative direction in which the singlet Higgs fields are WIMP, see Ref. \cite{10}).

One of the main motivations to study the 3-3-1 models is an explanation in part of the generation number puzzle. In the 3-3-1 models, each generation is not anomaly free; and the model becomes anomaly free if one of quark families behaves differently from other two \cite{11,12}. Consequently, the number of generations is multiple of the color number. Combining with the QCD asymptotic freedom, the generation number has to be three.

In one of the 3-3-1 models, the right-handed neutrinos are in bottom of the lepton triplets \cite{13} and three Higgs triplets are required. It is worth noting that in the version with right-handed neutrinos, there are two Higgs triplets with \textit{neutral components in the top and bottom}. In the earlier version, these triplets can have vacuum expectation value (VEV) either on the top or in the bottom, but not in both. Assuming that all neutral components in the triplet can have VEVs, we are able to reduce number of triplets in the model to be two \cite{14,15}. Such a scalar sector is minimal, therefore it has been called the economical 3-3-1 model \cite{16}. In a series of papers, we have developed and proved that this non-supersymmetric version is consistent, realistic and very rich in physics \cite{13,16,17,18}.

It is known that the economical (non-supersymmetric) 3-3-1 model does not furnish any candidate \cite{16} for SIDM with the condition given by Spergel and Steinhardt \cite{3}. In the other hands, supersymmetry \cite{19} contains interesting Higgs physics \cite{20}, where Higgs masses are constrained by supersymmetry. While earlier one might have viewed the Higgs fields as just one of many features of low energy supersymmetric models, the constraints on the Higgs mass are now problematic. With a larger content of the scalar sector, the supersymmetric version is expected to have a candidate for the self-interaction dark matter. The scalar Higgs sector in the supersymmetric economical 3-3-1 model does not provide the candidate for SIDM \cite{21}. In this paper, we show that the right-handed sneutrinos are good candidates for the SIDM.

This paper is organized as follows. In Sec. \ref{sec:2} we recapitulate the necessary elements of the model under consideration. The couplings of SIDM are presented in Sec. \ref{sec:3}, while in Sec. \ref{sec:4} we derive the lower mass limit for the SIDM. In Sec. \ref{sec:5} we get the condition for thermal generation of SIDM. Finally, the last section - Sec. \ref{sec:6} is devoted to our conclusions.
2. Basic elements

In this section we first recapitulate the basic elements of the model \[21\], which are related to our analysis below.

2.1 Particle content

The superfield content in this paper is defined in a standard way as follows

\[
\hat{F} = (\tilde{F}, F), \quad \hat{S} = (S, \tilde{S}), \quad \hat{V} = (\lambda, V),
\]

(2.1)

where the components \( F, S \) and \( V \) stand for the fermion, scalar and vector fields while their superpartners are denoted as \( \tilde{F}, \tilde{S} \) and \( \lambda \), respectively \[19, 22\].

The superfields for the leptons under the 3-3-1 gauge group transform as

\[
\hat{L}_aL = \left( \tilde{\nu}_a, \tilde{\nu}_a^c, \tilde{\nu}_a^c \right)_L \sim (1, 3, -1/3), \quad \hat{\nu}_a^c \sim (1, 1, 1),
\]

(2.2)

where \( \tilde{\nu}_a^c = (\tilde{\nu}_R)^c \) and \( a = 1, 2, 3 \) is a generation index. Here and in the following, the values in the parentheses denote quantum numbers based on the \( (SU(3)_C, SU(3)_L, U(1)_X) \) symmetry.

The superfields for the left-handed quarks of the first generation are in triplets

\[
\hat{Q}_{1L} = \left( \tilde{u}_1, \tilde{d}_1, \tilde{u}'_1 \right)_L \sim (3, 3, 1/3),
\]

(2.3)

where the right-handed singlet counterparts are given by

\[
\hat{\tilde{u}}_{1L}^c, \quad \hat{\tilde{d}}_{1L}^c \sim (3^*, 1, -2/3), \quad \hat{\tilde{d}}_{1L} \sim (3^*, 1, 1/3).
\]

(2.4)

Conversely, the superfields for the last two generations transform as antitriplets

\[
\hat{Q}_{\alpha L} = \left( \tilde{u}_\alpha, -\tilde{u}_\alpha, \tilde{d}'_\alpha \right)_L \sim (3, 3^*, 0), \quad \alpha = 2, 3,
\]

where the right-handed counterparts are in singlets

\[
\hat{\tilde{u}}_{\alpha L}^c \sim (3^*, 1, -2/3), \quad \hat{\tilde{d}}_{\alpha L}^c \sim (3^*, 1, 1/3).
\]

(2.5)

The primes superscript on usual quark types \( u' \) with the electric charge \( q_{u'} = 2/3 \) and \( d' \) with \( q_{d'} = -1/3 \) indicate that those quarks are exotic ones. The mentioned fermion content, which belongs to that of the 3-3-1 model with right-handed neutrinos \[13, 15\] is, of course, free from anomaly.

The two superfields \( \hat{\chi} \) and \( \hat{\rho} \) are at least introduced to span the scalar sector of the economical 3-3-1 model \[16\]:

\[
\hat{\chi} = (\hat{\chi}_1^0, \hat{\chi}_2^0, \hat{\chi}_2^0 T \sim (1, 3, -1/3),
\]

\[
\hat{\rho} = (\hat{\rho}_1^+, \hat{\rho}_2^0, \hat{\rho}_2^+ T \sim (1, 3, 2/3).
\]

(2.6)
To cancel the chiral anomalies of Higgsino sector, the two extra superfields $\tilde{\chi}'$ and $\tilde{\rho}'$ must be added as follows

$$
\tilde{\chi}' = (\tilde{\chi}'_0, \tilde{\chi}'_1, \tilde{\chi}'_2)^T \sim (1, 3^*, 1/3), \\
\tilde{\rho}' = (\tilde{\rho}'_1^- , \tilde{\rho}'_0 , \tilde{\rho}'_2^-)^T \sim (1, 3^*, -2/3).
$$

(2.7)

In this model, the SU(3)$_L \otimes$ U(1)$_X$ gauge group is broken via two steps:

$$
SU(3)_L \otimes U(1)_X \xrightarrow{w, w'} SU(2)_L \otimes U(1)_Y \xrightarrow{v, v', u, u'} U(1)_Q,
$$

(2.8)

where the VEVs are defined by

$$
\sqrt{2}\langle \chi \rangle^T = (u, 0, w), \\
\sqrt{2}\langle \chi' \rangle^T = (u', 0, w'), \\
\sqrt{2}\langle \rho \rangle^T = (0, v, 0), \\
\sqrt{2}\langle \rho' \rangle^T = (0, v', 0).
$$

(2.9)

The VEVs $w$ and $w'$ are responsible for the first step of the symmetry breaking while $u$, $u'$ and $v$, $v'$ are for the second one. Therefore, they have to satisfy the constraints:

$$
u, \ u', \ v, \ v' \ll w, \ w'.
$$

(2.10)

The vector superfields $\hat{V}_c, \hat{V}$ and $\hat{V}'$ containing the usual gauge bosons are, respectively, associated with the SU(3)$_C$, SU(3)$_L$ and U(1)$_X$ group factors. The colour and flavour vector superfields have expansions in the Gell-Mann matrix bases $T^a = \lambda^a/2$ ($a = 1, 2, ..., 8$) as follows

$$
\hat{V}_c = \frac{1}{2} \lambda^a \hat{V}_{ca}, \quad \hat{V}_c = - \frac{1}{2} \lambda^{a*} \hat{V}_{ca}; \\
\hat{V} = \frac{1}{2} \lambda^a \hat{V}_a, \\
\hat{V}' = \frac{1}{2} \lambda^{a*} \hat{V}'_a,
$$

where an overbar $-$ indicates complex conjugation. For the vector superfield associated with U(1)$_X$, we normalize as follows

$$
X \hat{V}' = (XT^g) \hat{B}, \quad T^g = \frac{1}{\sqrt{6}} \text{diag}(1, 1, 1).
$$

(2.11)

In the following, we are denoting the gluons by $g^a$ and their respective gluino partners by $\lambda^a_c$, with $a = 1, \ldots, 8$. In the electroweak sector, $V^a$ and $B$ stand for the SU(3)$_L$ and U(1)$_X$ gauge bosons with their gaugino partners $\lambda^a_B$ and $\lambda_B$, respectively.

### 2.2 Higgs content

One of the most important things in studying Higgs sector is recognition the SM Higgs boson. Since it is electrically neutral, we are interested in only neutral Higgs bosons. Expansion of Higgs fields in the model under consideration, is

$$
\chi^T = \left( \frac{w + S_1 + i A_1}{\sqrt{2}}, \chi^-, \frac{w + S_2 + i A_2}{\sqrt{2}} \right), \\
\rho^T = \left( \rho_1^+, \frac{v + S_5 + i A_5}{\sqrt{2}}, \rho_2^+ \right), \\
\chi'^T = \left( \frac{w' + S_1 + i A_1}{\sqrt{2}}, \chi'^+, \frac{w' + S_2 + i A_2}{\sqrt{2}} \right), \\
\rho'^T = \left( \rho'_1^-, \frac{v' + S_5 + i A_5}{\sqrt{2}}, \rho'_2^- \right).
$$

(2.12)
The weak eigenstates and physical eigenstates are related through the following matrix

\[
\begin{pmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{pmatrix} =
\begin{pmatrix}
c_\beta s_\theta & -s_\beta c_\theta & -c_\beta c_\theta & -s_\alpha s_\beta s_\theta & -c_\alpha s_\beta s_\theta & 0 \\
c_\beta c_\theta & s_\beta c_\theta & c_\beta c_\theta & -s_\alpha s_\beta c_\theta & -c_\alpha s_\beta c_\theta & 0 \\
s_\theta & -c_\theta & c_\theta & -s_\alpha s_\theta & -c_\alpha s_\theta & 0 \\
s_\beta c_\theta & c_\beta c_\theta & s_\beta c_\theta & -s_\alpha c_\beta c_\theta & c_\alpha c_\beta c_\theta & 0 \\
0 & 0 & 0 & -c_\alpha c_\gamma & s_\alpha c_\gamma & s_\gamma \\
0 & 0 & 0 & s_\alpha s_\gamma & -c_\alpha s_\gamma & c_\gamma
\end{pmatrix}
\begin{pmatrix}
S'_{1a} \\
\phi_{S_{2a}} \\
H \\
\phi_{S_{3a}} \\
S'_5 \\
S'_5
\end{pmatrix}
\tag{2.13}
\]

\[t_\theta \equiv \frac{u}{w} = \frac{u'}{w'}, \ t_{2\alpha} \equiv \frac{-2m_{36a}^2}{m_{33a}^2 - m_{35a}^2} \propto \frac{v}{w}, \ t_\beta \equiv \frac{w}{w'}, \ cot_\gamma \equiv \frac{v}{v'}.
\tag{2.14}\]

Pursuing interactions of the scalar Higgs bosons with the SM gauge ones, it was recognized that the following \(H\) is the SM Higgs boson [23]:

\[H = s_\alpha S_3 + c_\alpha S_6',\]

\[m_H^2 = \frac{1}{2} \left[m_{33a}^2 + m_{66a}^2 - \sqrt{(m_{33a}^2 - m_{66a}^2)^2 + 4m_{36a}^2} \right],\]

where

\[m_{33a}^2 = \frac{18g^2 + g'^2}{54c_\theta^2}(w^2 + w'^2), \quad m_{66a}^2 = \frac{9g^2 + 2g'^2}{27}(v^2 + v'^2),\]

\[m_{36a}^2 = \frac{(9g^2 + 2g'^2)\sqrt{(v^2 + v'^2)(w^2 + w'^2)}}{54c_\theta}.
\tag{2.16}\]

From (2.16) and (2.17), we have

\[t_\alpha \propto \frac{v}{w} \Rightarrow t_\alpha \gg t_\theta.
\tag{2.18}\]

Taking into account \(\alpha = \frac{e^2}{4\pi} = \frac{1}{125}, \ s_W^2 = 0.2312, \) we have

\[m_H \simeq 91.4 \text{ GeV}.
\]

This value is very close to the lower limit of 89.8 GeV (95% CL) given in Ref. [24] p. 32.

It is interesting to note that this mass is also closed to the Z boson mass.

### 2.3 Right-handed sneutrinos - SIDM candidates

In Ref. [21], we have introduced all of the possible soft terms to break supersymmetry. As a result, our effective Lagrangian of supersymmetric breaking is the most general. The different sources of supersymmetric breaking such as Fayet-Iliopoulos (D-term), O’Raifeartaigh (F-term), gauge-mediated,... lead to the Lagrangian given in Eq. (18) of Ref. [21].

In the previous work [25], we have shown that the right-handed sneutrinos are the lightest sfermions. Let us remind some definitions. In the base \((\tilde{\nu}_{1L}, \tilde{\nu}_{0R}) = (\tilde{\nu}_{1L}, \tilde{\nu}_{2L}, \tilde{\nu}_{3L}, \tilde{\nu}_{1R}, \tilde{\nu}_{2R}, \tilde{\nu}_{3R})\), the mass matrix is given by

\[
\begin{pmatrix}
A_{ab} & E_{ab} \\
E_{ab} & G_{ab}
\end{pmatrix}
\]

\[\tag{2.19}\]
where

\[ A_{ab} = \frac{g^2}{2} \delta_{ab} \left( N_3 + \frac{1}{\sqrt{3}} N_8 - \frac{2t^2}{3} N_1 \right) + M_{ab}^2 + \frac{1}{4} \mu_0 \mu_0 \]  

\[ + \frac{1}{18} v^2 (\lambda_a \lambda_b + 4 \lambda'_c \lambda'_e) + \frac{1}{18} \lambda_a \lambda_b w^2, \]

\[ G_{ab} = -g^2 \delta_{ab} \left( \frac{1}{\sqrt{3}} N_8 + \frac{t^2}{3} N_1 \right) + M_{ab}^2 + \frac{1}{4} \mu_0 \mu_0 \]

\[ + \frac{1}{18} v^2 (\lambda_a \lambda_b + 4 \lambda'_c \lambda'_e) + \frac{1}{18} \lambda_a \lambda_b u^2, \]

\[ E_{ab} = -\frac{\sqrt{2}}{2} \left( \varepsilon_{ab} v + \frac{1}{6} \mu_0 \lambda'_a \lambda'_e \right), \]  

(2.20)

and [25]

\[ N_3 = -\frac{1}{4} \left( \frac{u^2 \cos 2\beta}{s^2_\beta} + \frac{v^2 \cos 2\gamma}{c^2_\gamma} \right), \]

\[ N_8 = \frac{1}{4\sqrt{3}} \left[ \frac{v^2 \cos 2\gamma}{c^2_\gamma} - \left( u^2 - 2w^2 \right) \frac{\cos 2\beta}{s^2_\beta} \right] \]

\[ N_1 = \frac{1}{6} \left[ \left( u^2 + w^2 \right) \frac{\cos 2\beta}{s^2_\beta} + 2v^2 \frac{\cos 2\gamma}{c^2_\gamma} \right] . \]  

(2.21)

As usual, we assume that there is substantial mixing among \((\tilde{\tau}_L, \tilde{\tau}_R)\) only [26]. Then eigenstates and eigenmasses in this case are given in Table [1].

**Table 1: Masses and eigenstates of sneutrinos**

| Eigenstate | $\tilde{\nu}_{1L}$ | $\tilde{\nu}_{2L}$ | $\tilde{\nu}_{3L}$ | $\tilde{\nu}_{1R}$ | $\tilde{\nu}_{2R}$ | $\tilde{\nu}_{3R}$ |
|------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| (Mass)$^2$ | $A_{11}$ | $A_{22}$ | $A_{33}$ | $G_{11}$ | $G_{22}$ | $G_{33}$ |

The mass splittings for the sleptons are governed by sum-rules [23]

\[ m_{\tilde{\nu}_{1L}}^2 - m_{\tilde{\nu}_{1L}}^2 = m_{\tilde{\nu}_{2L}}^2 - m_{\tilde{\nu}_{2L}}^2 = -g^2 T_3 = \frac{g^2}{4} \left( \frac{v^2 \cos 2\gamma}{c^2_\gamma} + u^2 \frac{\cos 2\beta}{s^2_\beta} \right), \]  

\[ = m_{\tilde{\nu}_W}^2 \cos 2\gamma + \frac{g^2 u^2 \cos 2\beta}{s^2_\beta}, \]  

\[ (2.23) \]

\[ m_{\tilde{\nu}_{1L}}^2 - m_{\tilde{\nu}_{1R}}^2 = m_{\tilde{\nu}_{2L}}^2 - m_{\tilde{\nu}_{2R}}^2 = \frac{g^2}{2} \left( T_3 + \sqrt{3} T_8 \right) = \frac{g^2}{4} \left( w^2 - u^2 \right) \frac{\cos 2\beta}{s^2_\beta}. \]  

(2.24)

In the limit $u \approx 0$, Eq.(2.23) is consistent with those in the Minimal Supersymmetric Standard Model. Assuming further $\cos 2\beta > 0$, we obtain: $m_{\tilde{\nu}_{1L}}^2 > m_{\tilde{\nu}_{1R}}^2$. Since no experimental data on supersymmetric partners, we have a right to assume that.

To finish this section, we note that the right-handed sneutrinos are the lightest sfermions (in company with suggestion $\cos 2\beta > 0$). So they are stable. In addition, since they are...
singlet of the SM $SU(2)_L$ gauge group, they do not interact with the ordinary particles of the SM. For some range of the parameters, they pose the right abundance for CDM (see below). Hence they are realistic candidate for DM. Concerning $\tilde{\nu}_{aL}$ stability, notice that they carry lepton number $L = -1$, so final state of their decay must be slepton and scalar Higgs boson. However, this is forbidden due to the smallness of their masses. For the short, let us call the right-handed sneutrinos as dark matter and denote $\tilde{\nu}_{aL}$ by S.

3. Interaction of the DM candidate

It is well-known that to be candidate for DM, particles do not interact with the SM fields except, with the Higgs boson. In the model under consideration, the couplings arise in both $F$- and $D$-term contributions. The scalar potential of the model is a result of summation over $F$ and $D$ terms [24]:

$$V = F^{\phi*}F_{\phi} + \frac{1}{2} \sum_{a} D^{a}D_{a}. \quad (3.1)$$

1. Coupling from $F$-terms

Here we display only the $F$-terms giving necessary interactions [25]:

$$L_{F} = \frac{1}{9} \lambda_{a}\lambda_{b}[\tilde{\nu}_{aL}\tilde{\nu}_{bL}](\rho^{*}\rho) - (\tilde{\nu}_{aL}\rho)(\rho^{*}\tilde{\nu}_{bL})]
+ \frac{1}{9} \lambda_{a}\lambda_{b}[\tilde{\nu}_{aL}\tilde{\nu}_{bL}](\chi^{*}\chi) - (\tilde{\nu}_{aL}\chi)(\chi^{*}\tilde{\nu}_{bL})]
+ \frac{4}{9} \lambda'_{ca}\lambda'_{cb}[\tilde{\nu}_{aL}\tilde{\nu}_{bL}](\rho^{*}\rho) - (\tilde{\nu}_{aL}\rho)(\rho^{*}\tilde{\nu}_{bL})]
+ \frac{1}{9} \gamma_{ac}\gamma_{be}[(\tilde{\nu}_{aL}\rho')(\tilde{\nu}_{bL}\rho')]^{*}. \quad (3.2)$$

Notations in this section is given in Ref. [25].

From (3.2), we get couplings of the right-handed sneutrinos with neutral scalar Higgs bosons:

$$L_{SSH}^{F} = \frac{1}{9} \lambda_{a}\lambda_{b}[\tilde{\nu}_{aL}\tilde{\nu}_{bL}](\chi^{0*}\chi^{0} + \chi^{0*}\chi^{3} + \rho^{0*}\rho^{0})] + \frac{4}{9} \lambda'_{ca}\lambda'_{cb}[\tilde{\nu}_{aL}\tilde{\nu}_{bL}](\rho^{0*}\rho^{0}). \quad (3.3)$$

It is worth noting that $\lambda_{a}$ is a coefficient of $R$-parity violating interactions (see [25]), hence they have to be very small. Therefore, the main contribution in (3.3) is the last term. It was known that the mentioned term provides mass for neutrinos, so it has to be much smaller as compared to $\gamma_{ac}$ [18]: $\lambda'_{ca} \ll \gamma_{ac}$.

2. Coupling from $D$-terms

As before, we display the terms giving necessary contribution only. It also exists in $D$-term forms:

$$D^{a} = -g \left( \sum_{sfermions} \tilde{f}^{\dagger}T^{a}\tilde{f} + \sum_{Higgs} H^{\dagger}T^{a}H \right). \quad (3.4)$$
Since $T_a = T_a^\dagger$, we have

$$(D^a)^* D_a = \left( \sum_{\text{sfermions}} \tilde{f}^\dagger T_a \tilde{f} \right)^2 + 2g^2 \left( \sum_{\text{sfermions}} \tilde{f}^\dagger T_a \tilde{f} \right) \left( \sum_{\text{Higgs}} H^\dagger T_a H \right) + \cdots, \quad (3.5)$$

where $\cdots$ are the terms which do not contribute to sfermion masses. The first term gives sfermion self-interactions. The factor 2 in the second term in (3.5) is the Newton’s binomial coefficient. Since sneutrino masses and interactions are our interest, therefore, in the second factor at the last line of (3.5), only the diagonal $T_8$ satisfies this purpose. This factor is given by:

$$H_8 \equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > T_8 < H > = \frac{1}{2\sqrt{3}} \left( \chi_1^0 \chi_1^0 - 2\chi_3^0 \chi_3^0 - \chi_1^0 \chi_1^0 + 2\chi_3^0 \chi_3^0 + \rho^0 \rho^0 - \rho^0 \rho^0 \right). \quad (3.6)$$

Here we have taken into account that for antitriplets, $T_8$ changes a sign. Let us consider the first factor of the about mentioned term in (3.5). Since the singlet fields do not give contribution, hence for sleptons we have:

$$SL_8 \equiv \tilde{L}_a^\dagger T_8 \tilde{L}_a = \frac{1}{\sqrt{3}} \left( \frac{1}{2} \tilde{\nu}_{aL} \tilde{\nu}_{aL} + \frac{1}{2} \tilde{\nu}_{aL} \tilde{\nu}_{aL} - \tilde{\nu}_{aL} \tilde{\nu}_{aL} \right). \quad (3.7)$$

Thus, the contribution from $SU(3)_L$ subgroup is:

$$g^2 SL_8 \times H_8. \quad (3.8)$$

So, sneutrino self-interaction arisen from $SL_8$ is given by:

$$\frac{g^2}{6} (\tilde{\nu}_{aL} \tilde{\nu}_{aL}^c)^2. \quad (3.9)$$

Now we are looking at $U(1)_X$ subgroup:

First, for the Higgs part, we have

$$H_1 \equiv \sum_{H=\chi,\chi',\rho,\rho'} < H^\dagger > X < H > = -\frac{1}{3} \left[ (\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0 - (\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - 2(\rho^0 \rho^0 - \rho^0 \rho^0) \right]. \quad (3.10)$$

Similarly, for sleptons

$$SL_1 \equiv -\frac{1}{3} (\tilde{\nu}_{aL} \tilde{\nu}_{aL} + \tilde{\nu}_{aL} \tilde{\nu}_{aL} + \tilde{\nu}_{aL} \tilde{\nu}_{aL}) + \tilde{\nu}_{aL} \tilde{\nu}_{aL}. \quad (3.11)$$

The contribution from subgroup $U(1)_X$ is

$$g^2 \times SL_1 \times H_1 = g^2 t^2 \times SL_1 \times H_1. \quad (3.12)$$
Again, sneutrino self-interaction is given by
\[
\frac{g^2t^2}{18}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)^2,
\]
with \[25\]
\[
t = \frac{g'}{g} = \frac{3\sqrt{2}SW}{\sqrt{4c^2_W - 1}}.
\]

The total contribution is a result of summation over two above mentioned subgroup parts. Thus, the dark matter - Higgs boson interactions are given by
\[
\mathcal{L}_{SSHH}^D \in (SL_8.H_8 + SL_1.H_1)
\]
\[
= -\frac{g^2}{6}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)(\chi_1^0 \chi_0^0 + 2\chi_3^0 \chi_3^0 - \chi_1^0 \chi_1^0 + 2\chi_3^0 \chi_3^0 + \rho^0 \rho^0 - \rho^0 \rho^0)
\]
\[
+ \frac{g^2t^2}{9}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)[(\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - (\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - 2(\rho^0 \rho^0 - \rho^0 \rho^0)].
\]

Hence the total DM-Higgs interaction Lagrangian is the following
\[
\mathcal{L}_{int} = \mathcal{L}_{SSHH}^F + \mathcal{L}_{SSHH}^D
\]
\[
= \frac{1}{9}\lambda_a \lambda_b (\tilde{\nu}_{aL}^c \tilde{\nu}_{bL}^c) (\chi_1^0 \chi_0^0 + \chi_3^0 \chi_3^0 + \rho^0 \rho^0)
\]
\[
+ \frac{1}{18}\lambda_{ca} \lambda_{cb} (\tilde{\nu}_{aL}^c \tilde{\nu}_{bL}^c)(\rho^0 \rho^0)
\]
\[
+ \frac{g^2}{6}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)[(\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - (\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - 2(\rho^0 \rho^0 - \rho^0 \rho^0)].
\]

Substitution of (2.12) into (3.16) yields quartic couplings
\[
\mathcal{L}_{SSHH} = \frac{1}{18}\lambda_a \lambda_b (\tilde{\nu}_{aL}^c \tilde{\nu}_{bL}^c) (S_1^2 + S_2^2 + S_3^2 + A_1^2 + A_2^2 + A_3^2)
\]
\[
+ \frac{1}{18}\lambda_{ca} \lambda_{cb} (\tilde{\nu}_{aL}^c \tilde{\nu}_{bL}^c)(S_5^2 + A_5^2)
\]
\[
+ \frac{g^2}{12}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)(S_1^2 - 2S_2^2 - S_3^2 + 2S_4^2 + S_5^2 - S_6^2)
\]
\[
+ A_1^2 - 2A_2^2 - A_3^2 + 2A_4^2 + A_5^2 - A_6^2)
\]
\[
+ \frac{g^2t^2}{18}(\tilde{\nu}_{aL}^c \tilde{\nu}_{aL}^c)[(S_1^2 + S_2^2) - (S_3^2 + S_4^2) - 2(S_5^2 - S_6^2)
\]
\[
+ (A_1^2 + A_3^2) - (A_2^2 + A_4^2) - 2(A_5^2 - A_6^2)].
\]

We remind that $A_5, A_6$ are Goldstone bosons (massless) \[21\] and three massless states are mixing of
\[
A'_1 = s_\beta A_1 - c_\beta A_3,
\]
\[
A'_2 = s_\beta A_2 - c_\beta A_4,
\]
\[
\varphi_A = s_\theta A'_3 + c_\theta A'_4,
\]
\[
(3.18)
\]
where
\[ A'_3 = c_\beta A_1 + s_\beta A_3, \quad A'_4 = c_\beta A_2 + s_\beta A_4. \] (3.19)

One massive eigenstate
\[ \phi_A = e_\theta A'_3 - s_\theta A'_4, \] (3.20)

with mass equal to those of the X bilepton \[21\]
\[ m_{\phi_A}^2 = \frac{g^2}{4}(1 + t_\theta^2)(w^2 + w'^2) = m_X^2. \] (3.21)

Expressing \( S_i, A_i, i = 1, 2, \ldots, 5 \) through physical fields by \((2.13)\), we will get quartic DM-DM-Higgs-Higgs interactions. However, we are just interested in the coupling of the SM Higgs boson \( H \). It reads
\[
\mathcal{L}_{SSH} = \frac{1}{18} \lambda_a \lambda_b (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c) H^2 (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) + \frac{4}{18} \lambda'_{ca} \lambda'_{cb} (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c) H^2 (c_\alpha^2 c_\gamma^2)
+ \frac{g^2}{12} (\bar{\nu}_{aL}^c \bar{\nu}_{aL}^c) H^2 [s_\alpha^2 (1 - 3 c_\theta^2)(1 - 2 c_\beta^2) + c_\alpha^2 (1 - 2 s_\gamma^2)]
- \frac{2t^2}{3} [s_\alpha^2 c_\beta^2 + 2 c_\alpha^2 c_\gamma^2]). \] (3.22)

Expression in \((3.22)\) can be rewritten in the form
\[
\mathcal{L}_{SSH} = \frac{\lambda_{Sab}}{18} (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c) H^2, \] (3.23)

where
\[
\lambda_{Sab} = \lambda_a \lambda_b (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) + 4 \lambda'_{ca} \lambda'_{cb} c_\alpha^2 c_\gamma^2
+ \frac{3 \delta_{ab} g^2}{2} [s_\alpha^2 (1 - 3 c_\theta^2)(1 - 2 c_\beta^2) + c_\alpha^2 (1 - 2 s_\gamma^2)]
- \frac{2t^2}{3} [s_\alpha^2 c_\beta^2 + 2 c_\alpha^2 c_\gamma^2]). \] (3.24)

We turn now to the triple DM-DM Higgs boson interaction. Substitution \((2.12)\) into \((3.16)\) yields
\[
\mathcal{L}_{SSH} = \frac{1}{9} \lambda_a \lambda_b (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c) (uS_1 + wS_2 + vS_5)
+ \frac{4}{9} \lambda'_{ca} \lambda'_{cb} (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c) (vS_5)
- \frac{g^2}{6} (\bar{\nu}_{aL}^c \bar{\nu}_{aL}^c) (uS_1 - 2wS_2 - u'S_3 + 2w'S_4 + vS_5 - v'S_6)
+ \frac{g^2 t^2}{6} (\bar{\nu}_{aL}^c \bar{\nu}_{aL}^c) (uS_1 + wS_2 - u'S_3 - w'S_4 - 2vS_5 + 2v'S_6). \] (3.25)

Expressing \( S_i, i = 1, 2, 3, \ldots, 6 \) through physical Higgs fields by \((2.12)\) yields the necessary couplings. Then, we can write triple DM-DM-Higgs couplings in the form:
\[
\mathcal{L}_{SSH} = \lambda_H H (\bar{\nu}_{aL}^c \bar{\nu}_{bL}^c), \] (3.26)
where
\[
\lambda_H = \frac{1}{9} \left[ \lambda_\alpha \lambda_\beta (u s_\alpha s_\beta s_\theta + w s_\alpha s_\beta c_\theta + v c_\alpha c_\gamma) + 4 \lambda'_\alpha \lambda'_\beta v c_\alpha c_\gamma \right] + \frac{\delta_{ab} g^2}{6} \left[ 2 w s_\alpha c_\theta \frac{c_\alpha}{c_\gamma} - \frac{t^2}{3} \left( w s_\alpha c_\theta - 2 v c_\alpha c_\gamma \right) \right].
\] (3.27)

Here we have taken into account \( u \simeq u' \) [23]. As mentioned above, both kinds of couplings constants in the \( F \)-terms are small. Thus, the main contribution in (3.27) is one from the \( D \)-terms.

The \( D \)-terms give also dark matter self-interaction. This kind of interaction exists only in \( D \)-terms. Summation over (3.9) and (3.13) yields quartic DM self-interaction
\[
\mathcal{L}_{SSSS} = \frac{g^2}{6} (\bar{\nu}_L^a \nu_L^b) (\bar{\nu}_L^d \nu_L^c) \left( 1 + \frac{t^2}{3} \right).
\] (3.28)

Next, we turn on application of the above mentioned interactions to physical processes relevant to the SIDM.

4. Limit on sneutrino mass

With self-interaction in (3.28) we can get a limit for DM mass. The Spergel-Steinhardt condition on self-interaction cross-section of \( SS^+ \to SS^+ \) has a form [3, 5]
\[
r_S = \frac{\sigma}{M} = (2.05 \times 10^3 \div 2.57 \times 10^4) \text{ GeV}^{-3}.
\] (4.1)

From (3.28), it follows that
\[
\sigma(SS^+ \to SS^+) + \sigma(SS \to SS) = \frac{3}{128 \pi m_S^2} \left[ \frac{2 g^2}{3} \left( 1 + \frac{t^2}{3} \right) \right]^2.
\] (4.2)

Combination of (4.1) and (4.2) implies that
\[
m_S = 35.8 \alpha_\eta^{1/3} \left( \frac{2.05 \times 10^3 \text{ GeV}^{-3}}{r_S} \right)^{1/3} \text{ MeV}
\] (4.3)

where
\[
\alpha_\eta = \frac{1}{9 \pi} g^4 \left( 1 + \frac{t^2}{3} \right)^2 = \frac{16 m_W^4}{9 \pi v^4} \left( 1 + \frac{t^2}{3} \right)^2 = 0.027.
\] (4.4)

Here we have used \( m_W = 80.388 \text{ GeV}, \ v = 246 \text{ GeV} \) [24]. Note that \( \alpha_\eta \) in the model under consideration is quite fair for perturbative theory and this is in good agreement with estimation in Ref. [3]. Thus
\[
m_S = \alpha_\eta^{1/3} (15.4 - 35.8) \text{ MeV} \simeq (9 \div 22) \text{ MeV}.
\] (4.5)

So sneutrino mass limit is in the Spergel-Steinhardt mass range \( \sim 30 \text{ MeV} \) [3].
5. Thermal generation of self-interacting dark matter

The cosmic density of light gauge singlet scalars has been calculated in Ref. [5] and is given by

\[ \Omega H = 2g(T_\gamma)T_\gamma^3 \frac{\sum n_i \Theta_i}{\rho_c g(T)} \]  

(5.1)

with

\[ \Theta_i \equiv \frac{n_i}{T^3} = \frac{\eta \Gamma_i^2}{4\pi^3 K m_H^3} \]  

(5.2)

where \( T_\gamma = 2.4 \times 10^{-4} \) eV is the present photon temperature, \( g(T) = 2 \) is the photon degree of freedom, \( g(T) = g_B + \frac{7}{8} g_F \) \((g_B \) and \( g_F \) are the relativistic boson and fermion degree of freedom, respectively), \( \rho_c = 7.5 \times 10^{-47} h^2 \) GeV\(^4 \) is the critical density of the Universe (\( h \approx 0.71 \) is the Hubble constant in units of 100 km s\(^{-1} \) Mpc\(^{-1} \)), \( \eta = 1.87 \), \( K^2 = 4\pi^3 g(T)/45m_{pl}^2 \) and \( m_{pl} = 1.2 \times 10^{19} \) GeV is the Planck mass. For non-supersymmetric 3-3-1 model \( g(T) \approx 130 \) [8], and for the supersymmetric one, following Ref. [28], we take \( g(T) \approx 260 \). We will take \( T \geq m_H \geq T_{ew} \) [5], where \( T_{ew} \geq 1.65 m_H \) [29].

Decay rate for \( H \rightarrow SS^+ \) is

\[ \Gamma_H = \frac{\lambda_H^2}{16\pi m_H}, \text{ for } m_H \gg m_S, \]  

(5.3)

where \( m_H, m_S \) are masses of Higgs boson and DM, respectively.

Numerical estimation yields

\[ \lambda_H \approx 1.89 \times 10^{-6} \left( \frac{g(T)}{260} \right)^{\frac{2}{3}} \left( \frac{m_H}{90 \text{ GeV}} \right)^{\frac{1}{2}} \left( \frac{m_S}{30 \text{ MeV}} \right)^{-\frac{1}{4}} . \]  

(5.4)

Note that at the tree level, the SM Higgs boson has mass

\[ m_H^2 \approx (0.206 - 0.067/e^2)(v^2 + v'^2) = (0.206 - 0.067/e^2) v_{SM}^2, \]  

(5.5)

where \( v_{SM} = 246 \) GeV. Taking into account the upper limit [13]: \( \sin^2 \theta \leq 0.0064 \), we get a mass of the SM Higgs boson at the tree level: \( m_H \approx 91.573 \) GeV. It is expected that the radiative correction will give positive contribution to the Higgs boson mass.

For the right-handed sneutrinos, we have

\[ m_S = (A_{11})^{\frac{1}{2}}, \]  

(5.6)

where \( A_{aa} \) is given by (2.20).

Combining (3.27) and (5.4) yields

\[ -\frac{1}{9} \left[ \lambda_a \lambda_b (u s_\alpha s_\beta s_\theta + w s_\alpha s_\beta c_\theta + v c_\alpha c_\gamma) + 4\lambda'_\alpha \lambda'_\beta v c_\alpha c_\gamma \right] \]

\[ + \frac{2\delta_{ab} m_W^2}{3v_{SM}^2} \left[ 2w s_\alpha c_\theta s_\beta - v c_\alpha c_\gamma - \frac{\tilde{t}^2}{3} (w s_\alpha c_\theta - 2v c_\alpha c_\gamma) \right] \]

\[ = 1.89 \times 10^{-6} \left( \frac{g(T)}{260} \right)^{\frac{2}{3}} \left( \frac{m_H}{90 \text{ GeV}} \right)^{\frac{1}{2}} \left( \frac{m_S}{30 \text{ MeV}} \right)^{-\frac{1}{4}} . \]  

(5.7)
By suitable choice, the condition (5.7) for the SIDM in the model under consideration can be easily satisfied. Thus, a system of three equations (5.5), (5.6) and (5.7) are the constraint conditions to guarantee that the SIDM does not overpopulate the Universe.

6. Conclusion

In this paper we have shown that the supersymmetric economical 3-3-1 model has natural candidates for the SIDM. It is the light right-handed sneutrinos. The reason behind this choice relies on the fact that the right-handed sneutrinos are singlets of the SM $SU(2)_L$ group and the lightest slepton. The first reason prevents interactions of the DM candidates with particles in the SM, except for the Higgs boson $H$. The second one stabilizes the DM without imposition extra symmetry.

In difference with the previous SIDM candidates which are scalar Higgs bosons, the right-handed sneutrinos in this case are superpartners of leptons with $L = -1$. It is interesting to note that in Ref. [30], the right-handed neutrinos are a possible candidate of warm dark matter.

In order to be able to account for the observed properties of dark matter halos (the Spergel-Steinhardt condition), the right-handed sneutrinos have to be light with mass of ten MeV. It is emphasized that the DM self-interaction is fixed from $D$-terms, hence the above mentioned limit was obtained without any assumption. Meanwhile they do not overpopulate the Universe with $\Omega_H = 0.3$. This dark matter arises naturally in the model without imposition of extra symmetry.

Finally, we would like to mention that the economical 3-3-1 model contains the minimal Higgs sector (economical) with very rich phenomenology, specially in neutrino sector. Its supersymmetric generalization has almost the same properties such as Higgs sector and is very constrained. In addition, in this supersymmetric version, the candidates for self-interacting dark matter exist naturally.

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