High-resolution simulations of the head-on collision of white dwarfs

D. García-Senz,1,2 ★ R. M. Cabezón,3 A. Arcones,4,5 A. Relaño1 and F. K. Thielemann3

1 Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, c/Compte d’Urgell 187, E-08036 Barcelona, Spain
2 Institut d’Estudis Espacials de Catalunya, c/Gran Capità 2-4, E-08034 Barcelona, Spain
3 Departement Physik, Universität Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland
4 Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany
5 GSI Helmholtzzentrum für Schwerionenforschung, Planckstraße 1, D-64291 Darmstadt, Germany

Accepted 2013 September 25. Received 2013 September 21; in original form 2013 March 26

ABSTRACT

The direct impact of white dwarfs has been suggested as a plausible channel for Type Ia supernovae. In spite of their (a priori) rarity, in highly populated globular clusters and in galactic centres, where the amount of white dwarfs is considerable, the rate of violent collisions between two of them might be non-negligible. Even more, there are indications that binary white dwarf systems orbited by a third stellar-mass body have an important chance to induce a clean head-on collision. Therefore, this scenario represents a source of contamination for the supernova light-curve sample that it is used as standard candles in cosmology, and it deserves further investigation. Some groups have conducted numerical simulations of this scenario, but their results show several differences. In this paper, we address some of the possible sources of these differences, presenting the results of high-resolution hydrodynamical simulations jointly with a detailed nuclear post-processing of the nuclear abundances, to check the viability of white dwarf collisions to produce significant amounts of 56Ni. To that purpose, we use a 2D axisymmetric smoothed particle hydrodynamic code to obtain a resolution considerably higher than in previous studies. In this work, we also study how the initial mass and nuclear composition affect the results. The gravitational wave emission is also calculated, as this is a unique signature of this kind of events. All calculated models produce a significant amount of 56Ni, ranging from 0.1 to 1.1 M⊙, compatible not only with normal-branch Type Ia supernova but also with the subluminous and super-Chandrasekhar subset. Nevertheless, the distribution mass function of white dwarfs favours collisions among 0.6−0.7 M⊙ objects, leading to subluminous events.

Key words: supernovae: general – white dwarfs.

1 INTRODUCTION

There is growing evidence supporting that the astrophysical scenario leading to a Type Ia supernova (SNIa) explosion does not necessarily have to be unique. This assessment is probably true in light of the diversity shown by the light curves of SNIa as a function of the galaxy type, with the brightest events associated with galaxies with high star formation rate. The inferred amount of 56Ni from observations points to the fact that two or even more different progenitors may exist (Howell et al. 2006; Hicken et al. 2007; Bianco et al. 2011). Given the importance of SNIa in modern astrophysics, it is of great relevance to discern between the various potential channels that produce them.

★ E-mail: domingo.garcia@upc.edu

© 2013 The Authors
Published by Oxford University Press on behalf of the Royal Astronomical Society
In this work, we use a 2D axisymmetric SPH code (AXISSPH) to simulate the head-on collision of two WDs along the symmetry axis (i.e. with impact parameter $b = 0$). The motivation is to rule out resolution problems as the cause for the production (or absence) of high $^{56}$Ni yields, which might partially explain the different results obtained among previous works. We also post-processed the nuclear abundances obtained from the numerical simulation, using the extended BasNet reaction network (Thielemann et al. 2011). In this way, we want to validate the results obtained with the simple alpha network, which is typically used in hydrodynamical calculations. A comparison between models 2D7755 and 3D7755 in Table 1 indicates that the maximum resolution achieved by AXISSPH using $N = 90,000$ particles ($\approx 40\text{ km}$ for model 2D7755 at $t = 0\text{ s}$) is much better than that of current 3D calculations using several hundred thousand particles ($\approx 150\text{ km}$ for model 3D7755 at $t = 0\text{ s}$). This allows us to invest more computational effort in the combustion process itself by taking tiny time steps, often smaller than 1$\mu$s. In the direct collision scenario, the nuclear combustion takes place, for the most part, in the quasi-statistical equilibrium (QSE) regime, and the complete statistical equilibrium of the species cannot be assumed (as in the Chandrasekhar-mass models of SNIa). Instead, individual reactions have to be tracked in detail using very small time steps. Thus, in practice, it is necessary to compute a large number of models to reasonably describe the nuclear combustion. The main aim of this work is to determine the main features of these explosions and the nucleosynthetic yields performing numerical simulations of the head-on collisions of WDs with high spatial and temporal resolution. The axisymmetric hypothesis helps to make the calculation feasible.

This paper also brings information concerning other parameters which were not addressed in previous works. For example, we explored the sensitivity of the ensuing nucleosynthetic yields with respect to the initial carbon content of the WDs. The gravitational radiation resulting from this kind of stellar encounters is also given, as it is an important distinct signature to identify these progenitors in the future.

The paper is organized as follows. In Section 2.1, we describe the numerical setting that we used to perform the simulations and the nuclear network used for the post-processing (Section 2.2). In Section 3, we present the results of the calculations, which include the description of the overall evolution of the nominal case of 0.7 $M_\odot + 0.7 M_\odot$ (Sections 3.1 and 3.2), the impact of the WD masses (Section 3.3), the dependence of the results on the initial carbon abundance (Section 3.4), the comparison of the results with the post-processed $^{56}$Ni yields (Section 3.5) and the calculation of the resulting gravitational wave emission for the 0.7 $M_\odot + 0.7 M_\odot$ and 1.06 $M_\odot + 0.81 M_\odot$ scenarios (Section 3.6). Finally, Section 4 is devoted to the conclusions and final remarks.

### Table 1. Settings of the runs performed in this work. The name of the run states the type of simulation, the mass of the WDs and the initial nuclear composition in mass fractions of $^{12}$C and $^{16}$O. During the collision, there is a factor of $\approx 2.5$ of improvement in the maximum resolution achieved in the 2D simulations.

| Run       | $M_1 (M_\odot)$ | $M_2 (M_\odot)$ | $d_{12}^a$ (km) | $v_1^a$ (km s$^{-1}$) | $v_2^a$ (km s$^{-1}$) | $X_{12C}/X_{16O}$ | Type   | $N$  | Res. (km)$^a$ |
|-----------|-----------------|-----------------|-----------------|---------------------|---------------------|-----------------|--------|-----|---------------|
| 3D7755    | 0.7             | 0.7             | 32 000          | $-1700$             | 1700                | 0.5 / 0.5       | 3D     | 200 000 | 150           |
| 2D7755    | 0.7             | 0.7             | 32 000          | $-1700$             | 1700                | 0.5 / 0.5       | 2D     | 88 560 | 40            |
| 2D7755Res | 0.7             | 0.7             | 32 000          | $-1700$             | 1700                | 0.5 / 0.5       | 2D     | 177 120 | 29            |
| 2D6655    | 0.6             | 0.6             | 36 000          | $-1487$             | 1487                | 0.5 / 0.5       | 2D     | 88 560 | 46            |
| 2D6673    | 0.6             | 0.6             | 36 000          | $-1487$             | 1487                | 0.7 / 0.3       | 2D     | 88 560 | 46            |
| 2D6637    | 0.6             | 0.6             | 36 000          | $-1487$             | 1487                | 0.3 / 0.7       | 2D     | 88 560 | 46            |
| 2D10855   | 1.06            | 0.81            | 69 600          | $-1156$             | 1510                | 0.5 / 0.5       | 2D     | 88 560 | 24            |

$^a$This is the maximum resolution before the impact.

---

for early studies and Röpke et al. 2012 for a more recent approach). In the second scenario, two WDs collide (Pakmor et al. 2010). This collision can occur either by the loss of angular momentum by gravitational wave emission and unstable mass accretion in the coalescence scenario (Dan et al. 2011) or by the more violent head-on encounter (Katz & Dong 2012). This last case has been studied by Benz, Thielemann & Hills (1989), Rosswog et al. (2009), Raskin et al. (2009, 2010) and Hawley, Athanassiadou & Timmes (2012) being also the matter of study in this work.\footnote{See also the recent paper by Kushnir et al. (2013) simulating the head-on scenario with an Eulerian axisymmetric code.} 

A rough estimation of the direct collision rate for WDs in a typical globular cluster and in the Galactic centre was done by Benz et al. (1989). For a typical globular cluster, they inferred a total of $\approx 120$ WD collisions in the galaxy within the Hubble time. The number of collisions in the Galactic centre was estimated to be an order of magnitude higher, $\approx 1870$. Taking both contributions, and assuming that all the collisions end in an explosion, gives an upper limit for the rate of this channel: $\approx 2 \times 10^{-7}$ events yr$^{-1}$, considerably less than the observed SNIa rate $\approx 2 \times 10^{-3}$ yr$^{-1}$ (Cappellaro, Evans & Turatto 1999). Nevertheless, these rates might have been underestimated under the light of the work of Katz & Dong (2012). They showed that binary WD systems that are orbited by a distant perturber have a few per cent chance of a clean head-on collision within 5 Gyr, by calculating approximately 10 000 three-body integrations for a wide range of initial conditions and different kinds of perturbers.

The collisional channel merits to be investigated at least by two reasons. First, it will be useful to clean the template sample of standard SNIa from outliers in order to reduce the dispersion. The second reason is that this channel provides a firm mechanism to get high amounts ($\gtrsim 0.7 M_\odot$) of $^{56}$Ni, difficult to obtain in other scenarios. The much lower rate observed for these super-Chandrasekhar-mass explosions could be more easily conciliated to the expected rate of the collisional channel. The numerical studies of the head-on scenario were pioneered by Benz et al. (1989), and have been revisited by Rosswog et al. (2009), Raskin et al. (2009, 2010) and Hawley et al. (2012). In those works, the supernova is triggered by the direct encounter of a pair of WDs with masses in the range $0.4 \leq M_{\text{WD}} \leq 1.1 M_\odot$. The results of these studies vary from low $^{56}$Ni production ($\approx 0.01 M_\odot$) to large yields ($\gtrsim 1 M_\odot$). The most complete study to date was that carried out by Raskin et al. (2010), using a 3D smoothed particle hydrodynamics (SPH) code, where they explored different WD masses, mass ratios and impact parameters. The main conclusion of the study was that, for a reasonable choice of parameters, it was not difficult to obtain $0.3 \sim 0.8 M_\odot$ of $^{56}$Ni.
2 METHOD

2.1 Numerical setting

In terms of computational power, it is too costly to improve the resolution in 3D simulations beyond previous works and up to a level that really makes a difference. Thus, we decided to use an axisymmetric SPH code and study the \( b = 0 \) collision with very high resolution. All calculations were carried out with the code AXISPH, developed and detailed in García-Senz et al. (2009) and Relañó (2012). Given the importance of the hydrodynamic method used to carry out the simulations, we give a summary of the mathematical formulation of the numerical scheme as well as of their most relevant features in Appendix A.

We also simulated a 3D low-resolution collision of two twin 0.7 \( M_\odot \) WDs to compare the results with the 2D high-resolution calculation and corroborate previous convergence studies. To that purpose, we used our 3D-SPH code (García-Senz & Bravo 2005; Bravo & García-Senz 2008) conveniently adapted to handle the present scenario.

Both codes include an equation of state (EOS) consisting of radiation plus a mixture of ions, treated as an ideal gas with Coulomb corrections including plasma polarization effects, and relativistic electrons with any degree of degeneracy. The energy release due to nuclear combustion is taken into account using a 14-nuclei alpha network (from \( ^4\text{He} \) to \( ^{40}\text{Zn} \)) that also includes the binary reactions \( ^{12}\text{C} + ^{12}\text{C}, ^{12}\text{C} + ^{16}\text{O} \) and \( ^{16}\text{O} + ^{16}\text{O} \) (Cabezón, García-Senz & Bravo 2004). One key ingredient of this network is that the evolution of nuclear species at each time step is implicitly coupled with the temperature evolution. The values of both, the molar fractions of the species and temperature, are simultaneously found at step \( n + 1 \) as a function not only of their known values at step \( n \) but also on their final, unknown values, at step \( n + 1 \). Starting from the values \( y^n \) and \( T^n \) of the molar fractions and temperature, this is done by solving the coupled set of equations \( dy/dt = F(y^{n+1}, T^{n+1}) \); \( dT/dt = G(y^{n+1}, T^{n+1}) \) with the Newton–Raphson technique, where \( F \) describes the evolution of the 14 nuclei belonging to the \( \alpha \)-chain and \( G \) is the energy equation (or, equivalently, the temperature equation). This feature allows us to handle in a very robust way the combustion stages of the mixture, from the nuclear statistical equilibrium (NSE) regime to normal combustion, including the QSE and final freeze-out of the species. A network of similar size was used by Rosswog et al. (2009) but without the implicit coupling to the temperature evolution. Raskin et al. (2010) used a hybrid hydrostatic/self-heating network to compute the nuclear combustion.

Actually, the correct handling of nuclear reactions is the most challenging piece of this kind of calculations. To understand why, Fig. 1 depicts the characteristic time \( \tau_T \) of temperature increase due to the \( ^{12}\text{C} + ^{12}\text{C} \) reaction at a constant density \( \rho = 5 \times 10^7 \) g cm\(^{-3} \). This time is here defined as \( \tau_T = (\frac{dT}{dnuc})^{-1} \eta \) (i.e. the time step leading to an increase of temperature by the relative fraction \( \eta \)) being \( S_{nuc} \) the instantaneous nuclear energy generation rate, \( \eta = 10^{-1} \) and \( c_v \), the specific heat at each temperature, computed using the EOS. As we can see, when the temperature exceeds two billion degrees, the time step falls down to 0.01\( \mu s \) which makes it difficult to handle the hydrodynamics jointly with the nuclear burning in multidimensional calculations. This means that even using an \( \alpha \)-network, it is necessary to adopt a clever approach to handle the combustion. A good starting point is to consider the implicit coupling between nuclear reactions and temperature via the energy equation (Müller 1986; Cabezón et al. 2004), because it stabilizes the combustion, especially when photodisintegrations take over. Nevertheless, the implicit thermal coupling is not a sufficient condition to make the calculation feasible in practice, owing to the very small time steps. Thus, we have made use of two additional hypotheses. First, when the current hydrodynamical time step becomes larger than the instantaneous thermal time \( \tau_T \), we make use of an operator splitting technique. In this situation, the combustion algorithm decouples the nuclear network from the dynamics until the current hydrodynamical time step is recovered. This procedure allows us to compute the energy release using effective time steps close to \( 10^{-10} \) s, small enough to describe the nuclear combustion of most of the models shown in Table 1. Our second approach concerns massive WDs (i.e. only in our model 2D10855), where the central density is high and \( \tau_T(\rho^{-2}) \) becomes too small even for the splitting technique. Only for this run, we allow the material to jump to complete NSE if \( \rho \geq 5 \times 10^7 \) g cm\(^{-3} \) and \( T \geq 3.5 \times 10^9 \) K. Our NSE routine has the same 14 nuclei as the \( \alpha \)-network, with exactly the same nuclear parameters (such as partition functions and binding energies) so that they are totally compatible. Once the transition to NSE has been completed, the further evolution of nuclear abundances is again calculated with the implicit \( \alpha \)-network, so that the freeze-out of the species is computed in a realistic way. Even with these simplifications, the number of iterations needed to simulate an explosion is rather large, ranging from \( \approx 2.5 \times 10^4 \) for case 2D6637 to \( \approx 4 \times 10^5 \) for case 2D10855 (see Table 1 for further details on the names of the runs).

In order to simulate the collision, we built 2D and 3D initial models of the WDs, being properly relaxed to ensure good mechanical equilibrium before the simulation starts. We place both stars with an initial separation \( d_{1/2}^i \) (see Table 1) and give them an initial velocity corresponding to point-mass free-fall from infinity, up to that distance. In many of the 2D simulations, we used \( N_{2D} = 88,560 \) particles. It is worth noting that to achieve the same resolution in 3D, roughly \( N_{3D} \approx 2.6 \times 10^7 \) particles would be needed. In order to have a similar resolution as in previous works, we used 200,000 particles for the 3D simulation, same as Raskin et al. (2010). Table 1 shows a summary of the main features of the initial models used in this work.

2.2 Extended nuclear network and post-processing

In general, the alpha network used in the simulations provides only a rough estimation of the energy generation and of the amount of...
the most abundant isotopes (Timmes, Hoffman & Woosley 2000). However, a more accurate calculation of the nucleosynthesis requires a more extended reaction network. In a post-processing step, we have compared the results of the alpha network in the simulations with those obtained from the BasNet network (see e.g. Thielemann et al. 2011), which includes in the present application more than 1500 nuclei and all relevant reactions. We use the REACLIB compilation, containing theoretical statistical-model rates by Rauscher & Thielemann (2000) (NON-SMOKER) and experimental rates by Angulo et al. (1999) plus further updates (https://groups.nscl.msu.edu/jina/reaclib/db/). Where available, experimental beta-decay rates are used from NuDat2 supplemented by theoretical ones (Möller, Pfeiffer & Kratz 2003). The theoretical weak interaction rates are taken from Langanke & Martínez-Pinedo (2001).

The Lagrangian nature of SPH allows all particles used in the calculation to behave as tracers from the nucleosynthetic point of view. Nevertheless, instead of running the extended network over the whole sample of particles, we select a smaller but representative set of particles for the post-processing nucleosynthesis calculations, utilizing the density and temperature evolution from the hydrodynamic (SPH) simulation. Their initial composition is thus $X(^{12}\text{C}) = X(^{16}\text{O}) = 0.5$. For selecting these representative tracer particles, the full set of particles is ordered with respect to the maximum temperatures encountered during their evolution, which functions as a criterion for the expected nucleosynthesis features. We used 21 bins, from $T = 2$ up to six in intervals of $\Delta T = 0.2$, with the exception of the last bin which ranges within $T = 6–7$. From each temperature bin, we randomly select 50 particles, obtaining a total of 1050 trajectories. We tested whether this procedure assures convergence in the abundances obtained by performing the nucleosynthesis post-processing also for all tracers in two temperature bins, one responsible for the major production of intermediate-mass elements (IME) and one for the dominant production of Fe-group elements. The result is that we find at most a 5 per cent deviation in abundances of relevance in this zone. This error can go up to 30 per cent for elements with a negligible abundance in the relevant mass zones. Thus, when integrating over all ejecta, a maximum error of 5 per cent for the composition is assured, in agreement with Seitenzahl et al. (2010), which is suitable for the scope of this paper.

3 RESULTS

3.1 The baseline case: collision of two 0.7 M⊙ WDs (models 2D7755 and 3D7755)

The baseline case simulates the head-on collision of two 0.7 M⊙ WDs with an initial composition of $^{12}\text{C}$ and $^{16}\text{O}$ at equal parts. The numerical setting is made as described at the end of Section 2.1.

The sequence of events is summarized in the series of snapshots depicted in Fig. 2, and its continuation in Fig. 3, where we plot all particles from the 2D7755 run. The color represents, from left to right, density ($10^3$ g cm$^{-3}$), temperature ($10^9$ K), mass fraction of $^4\text{He}$ + $^{12}\text{C}$ + $^{16}\text{O}$, mass fraction of IME (from $^{20}\text{Ne}$ to $^{40}\text{Ca}$) and mass fraction of Fe-group elements (from $^{44}\text{Ti}$ to $^{60}\text{Zn}$). It has to be noted that the paucity of particles close to the $z$-axis in Figs 2 and 3 is a natural consequence of the axisymmetric hypothesis. A point in the 2D Cartesian plane is a ring in the 3D space. At the same distance to the centre of each star, a volume element at the equator is more massive than the same element near the pole. This produces a dilution of the mass points near the symmetry axis when particles with the same mass are used to describe the hydrodynamics.

The simulation starts before what is represented in Figs 2 and 3. The main parameters of the configuration at $t = 0$ s are given in Table 1. When the stars get closer, their shape is deformed by tidal forces. Nevertheless, this deformation is not strong because of the short time available before contact. At $t = 4.66$ s, both stars reach contact and the collision starts. At that time, the relative velocity of the stars is $v \simeq 4800$ km s$^{-1}$, higher than the speed of sound, even at their centre ($c_s \simeq 4000$ km s$^{-1}$). A shock wave is formed at the contact region with increasing density and temperature. In the first row of Fig. 2 ($t = 5.63$ s), the shock moves slowly outwards from the contact interface, until it stalls due to the ram kinetic pressure exerted by the fresh infalling material. The stagnation of the accretion shock has been also described not only in other papers dealing with the collision scenario (Rosswog et al. 2009; Raskin et al., 2010), but also in the abundant literature devoted to Type II supernova explosions (for see example Woosley & Weaver 1986, and references therein). The matter that has gone through the shock has increased its density and temperature, leading to combustion. As soon temperature reaches $10^9$ K and density $\simeq 10^9$ g cm$^{-3}$, the $^{12}\text{C}$ + $^{16}\text{O}$ reaction settles around the collision plane, producing alpha particles that react with $^{12}\text{C}$ and $^{16}\text{O}$, climbing up through the alpha network, producing IME (second row of Fig. 2, $t = 6.19$ s). As can be seen, the combustion is limited to a small region along the contact interface, because those particles, with high enough temperature, still have a relatively low density. At the typical peak densities reached during the collision, the conductive and radiative heat transport processes are inefficient at evacuating the thermal energy accumulated in the detonated zone before disruption (Raskin et al. 2010). In consequence, the energy liberated by nuclear reactions cannot leave the pocket of ashes that is left behind the shock, because it is surrounded by higher density regions, which induce a steep positive gradient of pressure in all directions. The combustion region remains then confined during a time $t \simeq 1$ s, large enough to allow the synthesis of approximately $0.06 M_\odot$ of IME.

At approximately $t = 6.30$ s (third row of Fig. 2), the temperature and density at the edge of the shock front are high enough to initiate carbon burning. In that region, energy can be freely released, giving rise to a detonation front that propagates all along the stalled shock front. This detonation is so fast that usually emerges almost simultaneously from single particles and induces a rapid spontaneous burning, especially for those particles close to/at the stalled shock. On one hand, the detonation wave that propagates into the low-density unburnt material (region out of the stalled shock) produces a big amount of IME ($\simeq 0.8 M_\odot$). On the other hand, the wave propagating through the already hot burnt material raises the density and temperature of that material even more, producing Fe-group elements. Because the system is not absolutely symmetric (the stars are not initially mirrored, but displaced), the detonation may start at slight different times in both hemispheres. To quantify the impact of the initial degree of symmetry in the final outcome, a model was run starting from a totally symmetric initial configuration. The evolution of this model was similar to that of model 2D7755 but

$^3$ Is still unclear whether the resolution is enough or not to resolve the initiation of the detonation (even more when dealing with the axisymmetric hypothesis), but the convergence tests reported in this work suggest that the main observables deduced from the simulations are robust.
some differences are worth commenting. The symmetric case gives $0.33 \, M_\odot$ of $^{56}$Ni (2D7755 produced $0.36 \, M_\odot$), and a bit more $^{28}$Si, $0.42 \, M_\odot$ ($0.408 \, M_\odot$ from model 2D7755), so that the energetics of the explosion of the mirrored and displaced cases remained similar. The absolute temperature peak achieved during the collision was a little higher for the symmetric model ($\approx 8$ per cent) but the density at the temperature peak was lower ($\approx 6$ per cent). A slightly higher combustion temperature and reduced density may be the cause of the nucleosynthetic discrepancies between both models, as photodisintegrations are most favoured by high temperatures and lower densities. We should keep in mind, however, that starting from a totally ordered set of points is not always advisable when working with a particle code. An excessive symmetry may produce numerical artefacts when a particle meets with itself. A little amount of asymmetry is often beneficial in SPH, being this the reason behind the initial setting of models shown in Table 1.

At $t = 6.39$ s (first row of Fig. 3), both detonation fronts have overcome each other, shocking again the burnt material of the contact region, and producing even more Fe-group elements, up to an amount of $0.38 \, M_\odot$. From those, $0.36 \, M_\odot$ is $^{56}$Ni. In the second row of Fig. 3 ($t = 6.80$ s), the release of nuclear energy is large enough to overpower the gravitational potential and the kinetic pressure of the infalling material, leading to a rapid expansion of the star.
At $t = 12.95$ s (last row of Fig. 3), the system is disrupted, and the decline of temperature and density freezes the nuclear abundances in a sort of distorted onion-layer structure, with lighter elements in the outer shells and heavier elements in the internal ones. Table 2 shows the total energy of the gas: $1.62 \times 10^{51}$ erg after freezing, similar to the $1.66 \times 10^{51}$ erg obtained by Kushnir et al. (2013) for the same scenario, and a little higher than $1.3 \times 10^{51}$ erg of model W7 by Nomoto et al. (1984).

One advantage of using a Lagrangian method to compute the hydrodynamics is that the evolution of individual mass elements can be easily tracked. In Fig. 4, we represent the evolution in the plane $\rho-T$ of four selected particles of the upper WD depicted in Figs 2 and 3. Roughly speaking, the particle sample first undergoes a phase of adiabatic compression, until the temperature exceeds a billion degrees. Afterwards, nuclear release takes over and temperature rapidly climbs to 3–5.5 GK, the precise value depending on the initial location of the particle. Nevertheless, the nuclear ashes do not expand, and remain stagnated and confined by the kinetic pressure of the infalling material. Once the released nuclear energy is large enough to overcome the kinetic pressure of the material, it gradually settles in the long adiabatic expansion line crossing Fig. 4, until their total dilution. Particles 1 and 2 have a richer behaviour than that of particles 3 and 4 in the upper part of their trajectories, because they describe additional small loops during the...
Table 2. Kinetic energy at infinity and group abundances of the calculated models after the freezing of nuclear reactions.

| Run        | KE (∞) (10^{51} erg) | α (M⊙)  | 12C + 16O (M⊙) | IME (M⊙) | Fe group (M⊙) |
|------------|-----------------------|---------|----------------|-----------|---------------|
| 3D7755     | 1.71                  | 0.0013  | 0.177          | 0.81      | 0.42          |
| 2D7755     | 1.62                  | 0.0032  | 0.205          | 0.81      | 0.39          |
| 2D7755Res  | 1.63                  | 0.0026  | 0.197          | 0.82      | 0.39          |
| 2D6655     | 1.33                  | 0.0062  | 0.228          | 0.71      | 0.27          |
| 2D6673     | 1.34                  | 0.0033  | 0.245          | 0.80      | 0.15          |
| 2D6637     | 1.10                  | 0.0020  | 0.263          | 0.71      | 0.23          |
| 2D10855    | 2.06                  | 0.0081  | 0.195          | 0.63      | 1.05          |

We also conducted a 3D calculation of the same scenario using the same input physics and initial setting, in order to compare with the results of the 2D code and perform a convergence test. We used 200,000 particles for run 3D7755, so resolution is roughly a factor of 5 lower than that in run 2D7755.

The evolution in 3D is very similar to that calculated in 2D using AXISSPH. We show the evolution of internal, kinetic and gravitational energies and the nuclear species in Fig. 5, in comparison with those of run 2D7755. Despite the different code conception and resolution, the outcome of the simulations in 2D and 3D is strikingly similar. Minor differences can be seen in the evolution of energies and abundances. The production of IME is a bit delayed in the 2D calculation, which can be due to the lower value of the smoothing length in this run (corresponding to the higher resolution). This delays the contact between both stars in about \( t \approx 4h/v_{rel} \), being \( v_{rel} \) the relative velocity at the impact moment. Run 3D7755 synthesizes a bit more elements of the Fe group, while the total number of IME is practically equal in both calculations. The energetics of both simulations are consistent with the nucleosynthesis trend: the 3D calculation has a slightly higher final kinetic energy than the 2D run. The total amount of synthesized \(^{56}\text{Ni}\) in 3D and 0.36 M⊙ in 2D (see Table 3).

Therefore, the comparison between the 2D and 3D calculations supports the conclusion of Raskin et al. (2010) that \( \approx 2 \times 10^5 \) particles in 3D would be enough to capture the main features of the collision.

It is important to take into account that these limited nuclear networks that are used coupled with hydrocodes provide a reasonable energy generation rate, but the final abundances should not be taken as being precise values. In order to obtain more realistic yields, we would need either a more extended network, which is prohibitively expensive when coupled with an hydrodynamical calculation, or to post-process the final result using the thermodynamical trajectories of the particles. Hence, in order to gain insight in the final outcome of the head-on collision scenario, we post-processed the evolution with a larger nuclear network and compared the results in Section 3.5.

3.2 Convergence of the results: model 2D7755Res

Model 2D7755Res simulates the same scenario than model 2D7755 but using twice the number of particles (177,120). In this way, we increase the spatial resolution and, more importantly, the mass resolution by a factor of 2. Then, we can compare the results between both calculations and establish a convergence of the outcome with resolution within the 2D approximation. The evolution of the collision is depicted in Figs 6 and 7, following the same colour profiles as in Figs 2 and 3, and showing the snapshots at similar times. Comparing the results, it is clear that the general evolution is very similar, with the exception that the slight lack of symmetry in the

![Figure 4](https://example.com/figure4.png)

Figure 4. Path followed by four selected particles in the \( \rho-T \) plane for the 0.7 M⊙ − 0.7 M⊙ collision (model 2D7755). Particles 1, 2, 3 and 4 belong to the upper WD shown in Figs 2 and 3. At \( t = 0 \), they were located at (comoving) coordinates (0, 0), (0, −0.5RWD), (0.5RWD, 0) and (0.5RWD, −0.5RWD), respectively, where RWD is the radius of the WD. Point 1 is just at the centre of the WD. The lower panel shows a zoomed section of the high-temperature and high-density region, where particles are shocked several times by the detonation fronts.

combustion phase (see Fig. 4, lower panel). These loops point to the existence of trains of shock waves moving and reflecting within the detonated volume, modifying the thermodynamic trajectory of the particles. The analysis of the trajectories of similar mass elements in model 2D7755Res, calculated with higher resolution, supports this conclusion.

It is important to take into account that these limited nuclear networks that are used coupled with hydrocodes provide a reasonable energy generation rate, but the final abundances should not be taken as being precise values. In order to obtain more realistic yields, we would need either a more extended network, which is prohibitively expensive when coupled with an hydrodynamical calculation, or to post-process the final result using the thermodynamical trajectories of the particles. Hence, in order to gain insight in the final outcome of the head-on collision scenario, we post-processed the evolution with a larger nuclear network and compared the results in Section 3.5.
As a consequence, the energetics are also very similar and a very similar amount of IME (0.815 M⊙) with mass 0.6 M⊙, according to Table 2, changing the mass of the WDs has a sizeable impact. This last case would give rise to a considerably bright event, in agreement with other studies, for model 2D6655.

The amount of radioactive nickel ejected in the explosion also follows a similar trend going from low, 0.25 M⊙, for model 2D6655 in Table 3, to high, 1.02 M⊙, for model 2D10855. Therefore, near-head-on collisions of canonical (≈0.6 M⊙) WDs would give rise to a subluminous SNIa explosion, in agreement with other studies (Rosswog et al. 2009; Raskin et al. 2010).

Model 2D10855 is the only super-Chandrasekhar-mass model considered in this paper. Even supposing a high rate of head-on impacts they would, for the most part, be the product of canonical 0.6−0.7 M⊙ WDs. The direct encounter of two WDs with masses as high as 0.81 and 1.06 M⊙ would be rare, with a low probability to be realized in nature. Still it is an interesting limiting case worth exploring. In Fig. 8, there are represented different time slices, similar to those of model 2D7755 presented in Section 3.1.
In this scenario, first contact happens at higher speeds than in case 2D7755. Nevertheless, due to its higher densities, the collision occurs subsonically for the most massive star, while it is supersonic for the less massive one. This immediately triggers an asymmetric detonation shock that expands through the upper star (Fig. 8, first and second rows). IME and Fe-peak elements are created in the less massive star while its companion remains without much alteration apart from a pressure wave that slowly compresses and heats the material, but not enough to switch on the nuclear reactions yet. In the third row of Fig. 8, two new detonation fronts can be seen in the most massive WD. The first one is formed when the pressure wave crosses the centre of the massive WD, while the second one happens close to the oblique contact layer between both stars (indicated by a red arrow). Both detonations shock the unburnt material twice, rising the temperature and the density enough to trigger a very fast carbon burning, and leading to a creation of an enormous amount of Fe-peak elements (first and second rows of Fig. 9). These late detonations cross over the whole system, catching up with the initial fronts and releasing energy enough to disrupt the stars (third row of Fig. 9). Interestingly, layers with different compositions start to mix in the last stages, due to the shape of the contact layer between both stars and the subsequent interaction between layers moving at different velocities.
3.4 Dependence on the initial carbon abundance (models 2D6637 and 2D6673)

The nuclear composition of the C+O WDs participating in the explosion is often taken uniform with mass fractions $X_C = X_O = 0.5$. Nevertheless, the precise composition profile along the star could also be a function of the mass of the WD and of the not yet well-known $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate. In order to explore the sensitivity of the results with the initial carbon abundance, we have simulated two additional explosions varying this parameter: cases labelled 2D6673 and 2D6637 in Tables 1–3. The results of the simulations are summarized in Table 2 and Fig. 10. The most remarkable conclusion is that there is not a uniform trend in the results as the initial abundance of carbon changes, a clear signature that we are facing a highly non-linear phenomenon. For example, the lowest final kinetic energy corresponds to model 2D6637 ($X_C = 0.3$) but the kinetic energies of cases $X_C = 0.7$ and $X_C = 0.5$ are virtually the same. Fig. 10 represents the evolution of four nuclear groups: $\alpha$, C+O, IME and Fe, for the three choices of the initial carbon abundance. As we can see, the larger the initial carbon abundance, the earlier the synthesis of the different groups begins. There is a considerable delay in the time at which the Fe nuclei begin to be synthesized, of almost 1 s, between models 2D6673 and 2D6637. Thus, for the $X_C = 0.3$ model combustion takes place, on average, at higher densities than for model with $X_C = 0.7$, leading to a stronger detonation but smaller abundances of IME. Regarding the $^{56}\text{Ni}$ yields, these two effects (strength of the detonation and number
Head-on collisions of WD

Figure 8. Snapshots of the evolution of model 2D10855. From left to right: colour scale indicates density (10^7 g cm^{-3}), temperature (10^9 K), mass fraction of 4He + 12C + 16O, mass fraction of IME (from 20Ne to 40Ca) and mass fraction of Fe-group elements (from 44Ti to 60Zn). From top to bottom: t = 16.87, 17.12, 17.53 s. All boxes have the same size, 10^9 cm in the X-direction and 2 × 10^9 cm in the Y-direction. The red arrow in the last row points to the second detonation (see the text for further details).

of IME produced) work in opposite directions, partially compensating mutually. The stronger detonation in the case of the system with lower amount of 12C should lead to a high amount of 56Ni, but the yields are partially suppressed by the lower number of seeds (12C) for creating IME. This might explain why an intermediate case (like 2D6655) is more efficient at producing 56Ni than the other two more extreme cases, where one of the two mechanisms dominates.

The uncertainties in the initial carbon abundance lead to a new scatter in explosion energies and nickel yields, to be added to those due to the initial mass of the WDs and unknown impact parameter.

Note that, for a given WD mass, the scatter in the yields of 56Ni caused by the initial 12C content is, in many cases, of similar size as the dispersion shown in Table 4 summarizing the calculations reported by other groups.

3.5 Comparison with an extended network

We compare the results of the alpha network and the post-processed nucleosynthesis for model 2D7755 (baseline case, Section 3.1). In the post-processing calculation, the amount of nickel ejected is 0.40 M⊙, compared to 0.36 M⊙ from the alpha network in the
simulation. This difference mainly comes from the opening of new channels which pump matter from IME to iron group, due to the combined action of \((\alpha, p)(p, \gamma)\) reactions which are typically faster than the \((\alpha, \gamma)\) reactions at very high temperature. A similar conclusion was reached by Timmes et al. (2000) in their comparative study of nuclear networks. We have also checked that the large and the reduced networks provide similar amounts of released nuclear energy. For the total, time-integrated, nuclear energy released up to the freezing of the abundances, the relative difference between both calculations is \(\simeq 3\) per cent. Although electron captures are also included in the post-processing network, they are negligible at the typical densities reached during the collision.

Fig. 11 shows a comparison of the abundance evolution for one particle calculated with the full post-processing network (dashed lines) and the reduced alpha network employed in the simulations (solid lines). This specific particle experiences temperature conditions which are at the borderline of explosive O-burning and incomplete Si-burning in nucleosynthesis terms, i.e. O (and C and lighter nuclei) are burnt, Si and intermediate-mass nuclei are the dominant products, and Fe-group elements can already be produced to some extent. It can be seen that there is a non-negligible number of protons produced. These originate mostly from \((\alpha, p)\) reactions. Such reactions are not included in the reduced alpha network, which only contains \((\alpha, \gamma)\) reactions for IME and beyond. Besides producing
protons, these reactions also proceed faster than \((\alpha, \gamma)\) reactions, permitting a faster build-up of Fe-group nuclei. Additional reaction channels also permit a more complete burning of oxygen. Therefore, the amounts of (mostly burnt) oxygen are overpredicted (upper panel), while \(^{52}\text{Fe}\) and \(^{56}\text{Ni}\) are significantly underproduced (bottom panel). This particle was selected because it gives an extreme difference in \(^{56}\text{Ni}\); particles which experience higher temperatures and complete Si-burning will produce significant and comparable amounts of Ni in the reduced alpha as well as the post-processing network. Thus, the amount of \(^{56}\text{Ni}\) obtained in the simulations can be used as a reliable lower estimate.

3.6 Gravitational radiation from the 0.7 M\(_{\odot}\) + 0.7 M\(_{\odot}\) and 1.06 M\(_{\odot}\) + 0.81 M\(_{\odot}\) WD collision

Under the light of a recent work (Katz & Dong 2012), the head-on collision of WDs might not be as rare as is generally believed. If that is the case, the rate of WD collisions may have an important impact in the sample of SNIa light curves used in cosmology. Therefore, there is a necessity of clearly identifying outliers from the sample. In order to do that, a gravitational wave detection will be a clear distinct signature that will differentiate among head-on collisions and other channels of supernova production (Lorén-Aguilar, Isern & García-Berro 2010). With that in mind, we calculated the gravitational emission of cases 2D7755 and 2D10855, using the reduced quadrupole approximation. Axisymmetric scenarios have only one polarization in the gravitational wave emission (Finn & Evans 1990). In order to evaluate gravitational radiation in 2D cylindrical SPH, we need to calculate the reduced quadrupole in cylindrical coordinates and integrate over the azimuthal angle (Cabezón 2010). For example, for the \(xx\) component we have

\[
-I_{xx} = \frac{1}{6} \int \int \int \rho \left[ (x^2 - \frac{1}{3} R^2) \right] dV = \frac{1}{6} \int \int 2\pi \rho \left( r^2 - 2z^2 \right) dr dz d\phi.
\]

As we can see in García-Senz et al. (2009), \(2\pi \rho r\) is the 2D density \(\eta\), and multiplied by \(dr dz\) gives the mass element, which within the SPH technique is the mass of each particle. Hence, we can write a discretized version of equation (1) as

\[
I_{xx} = \frac{1}{6} \sum_{i=1}^{N} m_i \left( r_i^2 - 2z_i^2 \right),
\]
Following the same procedure, we can find the following relations, where the summation is extended over all the particles of the system.

\[ I_{xx} = I_{yy} = -\frac{1}{2} I_{zz}, \quad (3) \]

\[ I_{xy} = I_{xz} = I_{yz} = 0. \quad (4) \]

Therefore, for an observer located along the \( Y \) axis \((\theta = \phi = \frac{\pi}{2})\) at a distance \( D = 10 \text{ kpc} \), we obtain the gravitational wave amplitudes for both polarizations,

\[ h_+ = -\frac{3}{D} \frac{G}{c^4} I_{xx}, \quad (5) \]

\[ h_{\times} = 0. \quad (6) \]

There are different direct methods for evaluating the components of \( \mathbf{I} \). Nevertheless, time derivatives cause numerical difficulties due to two main reasons: the numerical noise introduced by discretization and the magnification of the high-frequency components of the noise. To avoid these problems, Finn & Evans (1990) proposed three methods which avoid direct time derivatives of \( \dot{I} \) to evaluate \( \ddot{I} \) that weights the mass contribution by the moment arm \( r \) (rather than \( r^2 \)); and the ‘stress formula’ that uses the stress tensor to derive an expression of \( h_{\times} \) that involves accelerations and no explicit time derivatives. In this work, we use the method proposed by Centrella & McMillan (1993) which is similar to the ‘stress formula’ of Finn and Evans, and at the same time takes advantage of the Lagrangian nature of SPH,

\[ I_{xx} = \frac{1}{3} \sum_{i=1}^{N} \rho_i \left[ \dot{r}_i^2 + r_i \dot{r}_i - 2 (\dot{z}_i^2 + z_i \ddot{z}_i) \right]. \quad (7) \]

The results show a single strong burst of gravitational emission, followed by a slow fade-out of the signal (see Fig. 12). The peak value is rather high for events happening in our galaxy, even comparable to those obtained by neutron star mergers at the Mpc scale, but with a much longer time-scale. Nevertheless, being a single event lacking in periodicity, its detection will be difficult. The long-term relaxation of the emission can span over several hundreds (even a thousand) of seconds, but without any trace of a ring-down phase, as expected from a system where there is no remnant. Fig. 12 also shows the gravitational emission of the 2D10855 case. The emission in this scenario is considerably stronger, in spite of showing very similar properties compared to that of scenario 2D7755. The lack of symmetry in the emission of this case is due, as expected, to the different masses of the two WDs involved in this collision.

**4 CONCLUSIONS**

In this paper, we have studied the head-on collision scenario for WDs. These events might represent a possible channel to SNIa explosions because they trigger energetic explosions and yield reasonable amounts of \( ^{56}\text{Ni} \), compatible in most cases with faint supernova events. We presented seven simulations that explore mainly two parameters: initial mass of the WDs and initial nuclear composition, the last one never being studied before. We conducted a 2D versus 3D comparison and a 2D versus 2D with twice the number of particles, as convergence tests for both spatial and mass resolutions. In both cases, we obtained very good agreement, especially in the 2D comparisons, where the mass resolution was tested and showed a convergence for both nucleosynthetic and energetic output. We confirm the results of previous works, where the yields of \( ^{56}\text{Ni} \) increase with the initial mass of the system, obtaining 0.25 M\( _\odot \) of \( ^{56}\text{Ni} \) for the less massive scenario and 1.02 M\( _\odot \) for the super-Chandrasekhar simulation. The final kinetic energy of the fluid, in all cases, is well beyond 10\(^4\) erg, making these collisions an interesting alternative to the standard scenarios to reproduce SNIa explosions.

**Table 4.** Comparison of the outcomes for the \( ^{56}\text{Ni} \) yields of head-on collisions of WD from previous works with those of this work.

| Calculation          | Masses (M\( _\odot \)) | Code      | Particles (Res., km\(^a\)) | Nuclear network                           | \( ^{56}\text{Ni} \) (M\( _\odot \)) |
|----------------------|-------------------------|-----------|----------------------------|--------------------------------------------|----------------------------------------|
| Benz et al. (1989)   | 0.60+0.60               | 3D-SPH    | 5000 (–)                   | 14-isotope\(^b\)                           | 0.008                                  |
| Rosswog et al. (2009)| 0.60+0.60               | FLASH     | – (49)                     | 19-isotope\(^c\)                           | 0.16                                   |
| Raskin et al. (2009) | 0.60+0.60               | 3D-SPH    | 8 × 10\(^4\) (–)          | 7-isotope\(^d\) + post-processing\(^e\)   | 0.32                                   |
| Raskin et al. (2010) | 0.64+0.64               | 3D-SPH    | 2 × 10\(^5\) (–)          | 13-isotope\(^c\)                           | 0.34                                   |
| Hawley et al. (2012) | 0.64+0.64               | FLASH     | – (130)                    | 13-isotope\(^c\)                           | 0.32                                   |
| Kushnir et al. (2013)| 0.60+0.60               | FLASH     | – (8.5)                    | Not specified                             | 0.29                                   |
|                     | 0.70+0.70               | FLASH     | – (7.6)                    |                                            | 0.50                                   |
|                     | 1.00+0.80               | FLASH     | – (6.8)                    |                                            | 0.89                                   |
|                     | 0.60+0.60               | 2D-SPH    | 88 560 (46)                |                                            | 0.25                                   |
|                     | 0.70+0.70               | 3D-SPH    | 2 × 10\(^5\) (150)        | 14-isotope\(^f\) + temperature coupling\(^f\) | 0.39                                   |
| This work (2013)    | 0.60+0.60               | 2D-SPH    | 88 560 (40)                |                                            | 0.36                                   |
|                     | 0.70+0.70               | 2D-SPH    | 177 120 (29)               |                                            | 0.36                                   |
|                     | 1.06+0.81               | 2D-SPH    | 88 560 (24)                |                                            | 1.02                                   |
|                     | 0.70+0.70               | 2D-SPH    | 88 560 (–)                 | Post-processing\(^f\)                      | 0.40                                   |

\(^{a}\) For grid-based codes, the maximum resolution in km is shown (in parentheses) as well as the maximum resolution at \( t = 0 \text{ s} \) in SPH calculations when available. \(^{b}\) Benz et al. (1989). \(^{c}\) Timmes (1999). \(^{d}\) Hix et al. (1998). \(^{e}\) Raskin et al. (2010). \(^{f}\) Cabezón et al. (2004). \(^{g}\) Thielemann et al. (2011).
The dependence of the outcome on the initial nuclear composition is more complicated. In this case, there are two mechanisms acting in opposite directions. When the initial carbon abundance is low, the detonation happens at higher densities and temperatures, triggering high yields of $^{56}$Ni. However, having less $^{12}$C produces a smaller number of IME and leads to a partial suppression of the yields. This combination between the strength of the detonation and initial fuel leads to a non-linear dependence of the final yields on the initial nuclear composition.

In Table 4, there is a comparison of the yields of $^{56}$Ni obtained in this work with those of other groups. We summarize there the main available information about the head-on collision of WDs with masses as close as possible to those considered in these simulations.

Concerning calculations carried out with the SPH technique, for the collision of two $0.6M_\odot$ WDs, Rosswog et al. (2009) obtained around 0.32 $M_\odot$ of $^{56}$Ni, after post-processing the hydrodynamic output. A similar amount of $^{56}$Ni was reported in Raskin et al. (2009) using an $\alpha$-network of 13 nuclei. A larger amount of radioactive nickel, 0.51 $M_\odot$, was, however, found in Raskin et al. (2010), this time with a pair of 0.64 $M_\odot$ WDs (Table 4).

A calculation carried out with a different hydrodynamic code has been reported by Rosswog et al. (2009). Using the FLASH AMR code, the amount of radioactive nickel ejected after the collision of two 0.6 $M_\odot$ WDs was considerably lower (0.16 $M_\odot$) than the yields obtained with the SPH method. Nevertheless, Hawley et al. (2012) have also recently studied the head-on scenario with FLASH, and they reported higher yields of $^{56}$Ni (0.32 $M_\odot$, for 0.64 + 0.64 WDs), more compatible with our results, stating that the discrepancy with the previous work might be due to time-step handling during the combustion stage.

Three points may explain these differences: (1) resolution issues, (2) the treatment of the nuclear reactions and (3) a possible dependence on the numerical technique used for calculating the hydrodynamical evolution.

Resolution issues can be related either to spatial, temporal and/or mass resolution. A lack of spatial resolution can lead to a poor description of the fluid properties in sensitive regions, like the shock front. A poor temporal resolution makes more difficult the integration of the equations for the nuclear reactions, losing precision or obtaining even non-physical results. And low mass resolution produces very massive fluid elements that, when undergoes nuclear burning, always assume homogeneous burning and generate an enormous amount of energy that can artificially trigger a too early detonation. In this work, we used a 2D axisymmetric SPH code (AXISSPH) that provides a spatial resolution five times better than the standard 3D calculations of this scenario done so far. The price to pay is that we only can simulate the $b=0$ collision (i.e. pure head-on), which, based on previous studies, has to be taken as a limiting case, where the energetics and $^{56}$Ni production are maximal. This imposed symmetry allowed us to decrease considerably the number of particles involved in the calculation without losing spatial resolution. In this way, we could put a stronger computational effort in the coupled nuclear network that evaluates the abundances and energetic evolution of the fluid due to nuclear reactions.

The nuclear network and its implementation are the second suspect. In our case, we used an implicit 14-nuclei alpha network that solves the equations of the nuclear species evolution and the

![Figure 11. Evolution of abundances for an example particle based on a reduced alpha network (solid lines) and on an extended network (dashed line). Different panels correspond to various groups of elements as indicated in the legends.](image1)

![Figure 12. $h_\perp$ polarization of the gravitational emission from models 2D7755 and 2D10855, as seen by an observer in the collision plane and at a distance $D = 10$ kpc.](image2)
temperature change, consistently. We used the operator splitting technique to allow the network to reach the small time steps it needs, which are computationally prohibitive for the hydrodynamic step. In this way, we can calculate the whole evolution with high temporal resolution, from the normal burning to the NSE, and the posterior freeze-out. In order to check the outcome of our reduced nuclear network, we have post-processed over 1000 particles for model 2D7755 and obtained 0.40 M\(_\odot\) of nickel compared to the 0.36 M\(_\odot\) resulting from the alpha network in the hydrodynamical simulations. This difference is due to the many other channels to burn the material up to nickel in the extended network, which are absent in the reduced one. Among these, for example, the (\(\alpha, p\)) reactions rates are significantly faster than the (\(\alpha, \gamma\)) reactions for elements heavier than Ca. Therefore, the amount of nickel provided by the simulations may be underestimated by 10 per cent and can be used as a lower limit.

The last possibility is that there is a dependence on the hydrodynamic code employed to do the calculation. There is an inherent difficulty for hydrocodes to trigger a detonation, because, nowadays, it is not possible to resolve the microinstabilities which might be the real onset of the detonation. Therefore, the promptness with which a hydrodynamic code can detonate relays on how efficiently it can reach sustained detonation conditions within the employed resolution. For example, as stated in Raskin et al. (2010), the conditions to have a sustained detonation in SPH are more difficult to fulfil than in mesh codes. In order to trigger the detonation, a particle that reaches the proper conditions has to be capable of delivering enough energy to its neighbours. This leads to a delay in the detonation, which then occurs at more extreme conditions, enhancing the nuclear output. Nevertheless, the results of Hawley et al. (2012), who used FLASH to perform their simulations, are very similar to those obtained in this work, even to the 3D7755 low-resolution calculation, pointing to the fact that the code dependence might be lower than previously thought, and that the real sources for the spread of the results are either resolution or the nuclear network.

In spite of the scatter in the specific value, all the calculations point to the fact that significant amounts of \(^{56}\text{Ni}\) are produced naturally by WD collisions. Overall, this scenario has interesting properties to explain both faint supernova explosions and very bright events. The rate at which these collisions may occur in nature is the last possibility that deserves further research.

ACKNOWLEDGEMENTS

The authors acknowledge useful comments of Cody Raskin and Frank Timmes concerning nucleosynthetic yields in the collisional scenario. DG and AR were supported by the Spanish MEC grants AYA2010-15685 and AYA2008-04211-C02-C01, and DURSI of the Generalitat de Catalunya. RMC was supported by the Swiss Platform for High-Performance and High-Productivity Computing within the supernova project. FKT was supported by the Swiss National Science Foundation (SNSF) and the EuroGENESIS and CompStar programmes. AA was supported by a Feodor Lynen Fellowship (Humboldt Foundation) combined with support from SNSF, and with the Helmholtz-University Young Investigator grant no. VH-NG-825.

REFERENCES

Angulo C. et al., 1999, Nucl. Phys. A, 656, 3
Benz W., Thielemann F.-K., Hills J. G., 1989, ApJ, 342, 986
Bianco F. B. et al., 2011, ApJ, 741, 20
Bravo E., García-Senz D., 2008, A&A, 478, 843
Cabezón R. M., 2010, PhD thesis, Polytechnical University of Catalonia
Cabezón R. M., García-Senz D., Bravo E., 2004, ApJS, 151, 345
Cappellaro E., Evans R., Turatto M., 1999, A&A, 351, 459
Centrella J. M., McMillan S. L. W., 1993, ApJ, 416, 719
Dan M., Rosswog S., Guillochon J., Ramirez-Ruiz E., 2011, ApJ, 737, 89
Finn L. S., Evans C. R., 1990, ApJ, 351, 588
García-Senz D., Bravo E., 2005, A&A, 430, 585
García-Senz D., Relaño A., Cabezón R. M., Bravo E., 2009, MNARS, 392, 346
Hawley W. P., Athanassiadou T., Timmes F. X., 2012, ApJ, 759, 39
Hicken M., Garnavich P. M., Prieto J. L., Blondin S., DePoy D. L., Kirshner R. P., Parrent J., 2007, ApJ, 669, L17
Hillebrandt W., Niemeyer J. C., 2000, ARA&A, 38, 191
Hix W. R., Khokhlov A. M., Wheeler J. C., Thielemann F.-K., 1998, ApJ, 503, 332
Howell D. A. et al., 2006, Nat., 443, 308
Iben I., Jr, Tutukov A. V., 1984, ApJS, 54, 335
Katz B., Dong S., 2012, preprint (arXiv:1211.4584)
Kushnir D., Katz B., Dong S., Livne E., Fernández R., 2013, preprint (arXiv:1303.1180)
Langanke K., Martínez-Pinedo G., 2001, At. Data Nucl. Data Tables, 79, 1
Loren-Aguilar P., Isern J., García-Berro E., 2010, MNRAS, 406, 2749
Möller P., Pfeiffer B., Kratz K.-L., 2003, Phys. Rev. C, 67, 055802
Monaghan J. J., 1992, ARA&A, 30, 543
Müller E., 1986, A&A, 162, 103
Nomoto K., 1982, ApJ, 253, 798
Nomoto K., Thielemann F.-K., Yokoi K., 1984, ApJ, 286, 644
Pakmor R., Kromer M., Röpke F. K., Sim S. A., Ruiter A. J., Hillebrandt W., 2010, Nature, 463, 61
Raskin C., Timmes F. X., Scannapieco E., Diehl S., Fryer C., 2009, MNRAS, 399, L156
Raskin C., Scannapieco E., Rockefeller G., Fryer C., Diehl S., Timmes F. X., 2010, ApJ, 724, 111
Rauscher T., Thielemann F.-K., 2000, At. Data Nucl. Data Tables, 75, 1
Relaño A., 2012, PhD thesis, Polytechnical University of Catalonia, available at: http://hdl.handle.net/10803/83525
Röpke F. K. et al., 2012, ApJ, 750, L19
Rosswog S., Kasen D., Guillochon J., Ramirez-Ruiz E., 2009, ApJ, 705, L128
Seitenzahl I. R., Röpke F. K., Fink M., Pakmor R., 2010, MNRAS, 407, 2297
Thielemann F.-K. et al., 2011, Prog. Part. Nucl. Phys., 66, 346
Timmes F. X., 1999, ApJS, 124, 241
Timmes F. X., Hoffman R. D., Woosley S. E., 2000, ApJS, 129, 307
Whelan J., Iben I., Jr, 1973, ApJ, 186, 1007
Woosley S. E., Weaver T. A., 1986, ARA&A, 24, 205

APPENDIX A: MAIN FEATURES OF AXISSPH

The hydrocode AXISSPH is a two-dimensional SPH code written in axisymmetric geometry which incorporates several improvements over existing versions of these hydrodynamic codes (García-Senz et al. 2009). Its main advantage relies in the more accurate treatment of particles moving close to the symmetry z-axis which is a singular line.

All axisymmetric SPH codes assume that fluid mass elements are tori characterized by its mass \(m\) and distance \(r\) to the symmetry axis. The basic hydrodynamic SPH equations can be obtained from the 3D-SPH equations after a simple substitution of the interpolating kernel \(W_{3D}\) by its two-dimensional counterpart \(W_{2D}\) and using the following relationship between the volumetric \(\rho\) and surface \(\eta\) densities,

\[
\rho = \frac{\eta}{2\pi r}.
\]
To avoid the underestimation of $\eta$ in the neighbourhood of the symmetry axis, it is usual to add reflective particles to the numerical scheme. Still, this is not sufficient to ensure the correct behaviour of the dynamics near the $z$-axis because any error in the SPH interpolations may lead to a large error in $\eta$ when $\xi = \frac{r}{R} \leq 2$, enforcing $\eta \rightarrow 0$ when $r \rightarrow 0$. To elude that problem, $\text{AXISSPH}$ makes use of a correction factor $f_i$ which reduces the value of $\eta$ when $\xi = \frac{r}{R} \leq 2$, enforcing $\eta \rightarrow 0$ when $r \rightarrow 0$. A similar function $f_2$ works to ensure the adequate behaviour of the $r$-component of velocity near the symmetry axis. The ensuing basic Euler equations are (see Relaño 2012 for the detailed derivation of these equations)

mass equation:

$$\dot{\eta}_i = \sum_{j=1}^{N} m_j W_{ij} f_1^j = \eta_i f_1^i \quad (A2)$$

momentum equations:

$$\dot{r}_i = 2\pi \frac{P_i}{\eta_i} - 2\pi \sum_{j=1}^{N} \left[ m_j \left( \frac{P_{rj}}{\eta_j} \cdot f_1^i(\xi_j) + \frac{P_{rj}}{\eta_j} + \Pi_1^{2D} \right) \frac{\partial W_{ij}}{\partial r_i} \right] - \left( \frac{2\pi P_i}{\eta_i} \right) \frac{df_1(\xi_i)}{dr_i}, \quad (A3)$$

energy equation:

$$\frac{du_i}{dr} = -2\pi \frac{P_i}{\eta_i} v_i + 2\pi \frac{P_i r_i}{\eta_i} \frac{d\eta_i}{dr} + \frac{1}{2} \sum_{j=1}^{N} m_j \Pi_1^{2D}(v_i - v_j) \cdot \nabla_i^{2D} W_{ij}, \quad (A5)$$

being

$$\frac{d\eta_i}{dr} = \sum_{j=1}^{N} m_j (f_1^i v_i - f_2^j v_j) \frac{\partial W_{ij}}{\partial r_i},$$

$$+ \sum_{j=1}^{N} m_j \left( \frac{\partial f_1^i}{\partial r_i} v_i - \frac{\partial f_2^j}{\partial r_j} v_j \right) W_{ij}$$

$$+ f_2^i \sum_{j=1}^{N} m_j (v_i - v_j) \frac{\partial W_{ij}}{\partial z_i}.$$ \quad (A6)

where $\dot{\eta}$ is the corrected value of the surface density, $\Pi_1^{2D}$ is the artificial viscosity and the remaining symbols have their usual meaning in SPH (Monaghan 1992). The first terms on the right-hand side of equations (A3) and (A5), called the hoop-stress terms, are specific to the axisymmetric geometry, and they can be a source of troubles if the surface density $\eta$ is not well evaluated. The correction factors, only valid for the cubic-spline interpolating kernel, affecting the density $\eta$ and $v_i$ are

$$f_1^i(\xi) = \begin{cases} \left[ \frac{7}{15} \xi_i^{-1} + \frac{2}{5} \xi_i - \frac{1}{5} \xi_i^3 + \frac{1}{50} \xi_i^4 \right]^{-1} & 0 < \xi_i \leq 1, \\ \frac{\xi_i^2}{5} - \frac{4}{5} \xi_i + \frac{1}{2} \xi_i^3 & 1 < \xi_i \leq 2, \\ 1 & \xi_i > 2, \end{cases} \quad (A7)$$

$$f_2^i(\xi) = \begin{cases} \left[ \frac{14}{45} \xi_i^{-2} + \frac{7}{15} \xi_i - \frac{1}{5} \xi_i^3 + \frac{1}{10} \xi_i^4 \right]^{-1} & 0 < \xi_i \leq 1, \\ \frac{\xi_i^2}{5} - \frac{2}{5} \xi_i + \frac{6}{5} \xi_i^3 - \frac{1}{2} + \frac{3}{5} \xi_i & 1 < \xi_i \leq 2, \\ 1 & \xi_i > 2, \end{cases} \quad (A8)$$

with $\xi_i = \frac{r_i}{R}.$

The basic scheme described above was conveniently modified to describe the hydrodynamics of the encounter of two WDs. Instead of the energy equation (A5), we used a similar expression based on the temperature equation, which is more appropriate to handle degenerate structures such as WDs. The rate of released nuclear energy obtained with the alpha network was added to the right of the temperature equation, and they can be a source of troubles if the surface density $\eta$ is not well evaluated. The correction factors, only valid for the cubic-spline interpolating kernel, affecting the density $\eta$ and $v_i$ are

$$\frac{du_i}{dr} = -2\pi \frac{P_i}{\eta_i} v_i + 2\pi \frac{P_i r_i}{\eta_i} \frac{d\eta_i}{dr} + \frac{1}{2} \sum_{j=1}^{N} m_j \Pi_1^{2D}(v_i - v_j) \cdot \nabla_i^{2D} W_{ij}, \quad (A5)$$

being

$$\frac{d\eta_i}{dr} = \sum_{j=1}^{N} m_j (f_1^i v_i - f_2^j v_j) \frac{\partial W_{ij}}{\partial r_i},$$

$$+ \sum_{j=1}^{N} m_j \left( \frac{\partial f_1^i}{\partial r_i} v_i - \frac{\partial f_2^j}{\partial r_j} v_j \right) W_{ij}$$

$$+ f_2^i \sum_{j=1}^{N} m_j (v_i - v_j) \frac{\partial W_{ij}}{\partial z_i}.$$ \quad (A6)

This paper has been typeset from a TeX/TeX file prepared by the author.