Bose-Einstein Correlations in one and two dimensions have been studied in charged current muon-neutrino interaction events collected with NOMAD. In one dimension the Bose-Einstein effect has been analyzed with the Goldhaber and the Kopylov parametrizations. The two-dimensional shape of the source has been investigated in the longitudinal co-moving frame. A significant difference between the transverse and the longitudinal sizes is observed.

1 Introduction

This short communication is based on my Master’s Degree thesis work and on the consequent paper done in collaboration with the NOMAD group in Pisa. Further details can be found in these papers. Here only the main results and conclusions will be presented. Bose-Einstein correlations (BEC) are an “apparent attraction” in momentum space between identical bosons. This effect is due to the symmetrization of the quantum mechanical wave function of two identical bosons with respect to particle exchange. BEC effect is relevant if and only if these particles are close to each other in momentum space. BEC in momentum space are related to the spatial dimensions of the production source. In fact the shape of BEC depends on the spatial and temporal distributions of the boson source and on its degree of coherence. So studies of BEC may lead to a better understanding of the dynamics of the particle interactions yielding like-sign bosons in the final state.

2 The phenomenology of BEC

The BEC effect can be parametrized in terms of the two particle correlation function $R$ defined as:

$$ R(p_1, p_2) = D(p_1, p_2)/D_0(p_1, p_2) $$

(1)
where \( p_{1,2} \) are the particle four-momenta, \( D(p_1, p_2) \) is the measured two-particle density and \( D_0(p_1, p_2) \) the particle “reference density”. BEC are usually parametrized using the Goldhaber parametrization, which assumes a spherical Gaussian density function for the emitting sources:

\[
R(Q) = 1 + \lambda \exp \left( -R_G^2 Q^2 \right)
\]

where \( Q^2 = -(p_1 - p_2)^2 = M_{\pi\pi}^2 - 4m_\pi^2 \), with \( M_{\pi\pi} \) the invariant mass of the pion pair and \( R_G \) the source size. The chaoticity parameter \( \lambda \) measures the degree of coherence in pion production, i.e. the fraction of pairs of identical particles that undergo interference (\( 0 \leq \lambda \leq 1 \)).

The Kopylov-Pogdoretskii (KP) parametrization corresponds to a radiating spherical surface of radius \( R_{KP} \) with pointlike oscillators of lifetime \( \tau \):

\[
R(Q_t, Q_0) = 1 + \lambda \left[ 4J_1^2(Q_t R_{KP})/(Q_t R_{KP})^2 \right] / [1 + (Q_0 \tau)^2]
\]

where \( J_1 \) is the first-order Bessel function, \( \vec{p} = \vec{p}_1 + \vec{p}_2 \), \( \vec{q} = \vec{p}_1 - \vec{p}_2 \), \( Q_t = |\vec{q} \times \vec{p}| / |\vec{p}| \). This parametrization is not Lorentz invariant and the variables are calculated in the c.m. of the hadronic final state.

The shape of the hadronic source can be measured by studying BEC as a function of the components of the vector \( \vec{q} \). This study is performed in the longitudinal centre of mass system (LCMS) in order to avoid possible effects caused by Lorentz boost. This reference system is defined for every particle pair as that where \( \vec{p} = \vec{p}_1 + \vec{p}_2 \) is perpendicular to the axis defined by the hadronic jet direction. In the analysis we use the longitudinal component \( Q_\parallel \) and the perpendicular one \( Q_\perp \) to the hadronic jet axis. The parametrization of the correlation is then performed separately for the two \( Q_\parallel \) and \( Q_\perp \) components:

\[
R(Q_\parallel, Q_\perp) = 1 + \lambda \exp \left( -Q_\parallel^2 R_\parallel^2 - Q_\perp^2 R_\perp^2 \right)
\]

3 Analysis and Results

A critical point of the analysis is the definition of the “reference density” \( D_0(p_1, p_2) \): it should be identical to \( D(p_1, p_2) \) except for the lack of BEC effect. The common methods used to build the reference sample from data events are: “Unlike-sign pairs”, “Mixed event” and “Reshuffled \( P_i \)” techniques. We have carefully tested these three methods with a full Monte Carlo simulation. We have required that, in absence of BEC, the distributions in \( R(Q) \) should be flat or have no structure at small \( Q \) (\( \leq 0.2 \text{ GeV} \)) which would distort the study of \( R \). In consequence of this requirement, the “mixed event” and “reshuffled \( P_i \)” methods have been rejected because of the presence of an excess and a defect at small \( Q \) respectively. The unlike-sign sample has been found to be the most adequate in the BEC region. BEC effects are then investigated by looking in the data at the following ratio:

\[
R(Q) = \frac{\text{“like-sign” pion-pairs}}{\text{“unlike-sign” pion-pairs}} = \frac{N_{++}(Q) + N_{--}(Q)}{N_{+-}(Q)}
\]

The Monte Carlo samples are also used to estimate possible spurious BEC effects from non-pion contaminations present in the sample. In fact, after the event and track quality cuts, we have looked for the purity of the track identification. The tracks investigated are those obtained from the full Monte Carlo simulation. We have found that the positive and negative samples of particles used for BEC studies contain respectively \( \approx 61\% \) of \( \pi^+ \) and \( \approx 77\% \) of \( \pi^- \). In the analysis, all secondary charged particles have been assumed to be pions, unless identified
as muons, electrons or protons. We have studied how the BEC could be changed by these misidentified tracks and by the use of the unlike-sign sample as a reference. We have found that contaminations in the pions sample due to kaons and protons produce only a variation of the overall normalization of the distribution in $R(Q)$ and no distortion of its shape for $Q \leq 0.2$ GeV. Therefore they do not affect the measurement of the radius of the emitting source. We have observed that the unlike-sign pair distribution as reference sample has the essential property of reproducing faithfully the non-BEC distribution of like-sign pairs (the ratio is reasonably flat). However, it is dangerously affected by meson resonances and by electron-positron pairs from photon conversions. For this reason the $Q$ intervals $0 \leq Q \leq 0.04$ GeV ($e^+e^-$ pairs from photon conversions), $0.3 \leq Q \leq 0.45$ GeV ($K^0$ decay) and $0.6 \leq Q \leq 0.825$ GeV ($\rho$ decay) have been excluded from the analysis.

In order to take into account the contribution due to the long-range correlations outside the BEC region, we have introduced in Eq. 2, 3 and 4 a second degree polynomial factor $(1 + aQ + bQ^2)$. This choice inevitably affects the results of the BEC analysis and contributes to the systematic errors on $\lambda$ and the radius parameters. In the literature linear and quadratic forms have been also used. We have found that the linear parametrization is inadequate to be used in the present analysis of the experimental data, while the quadratic parametrization reproduces almost exactly the results of the polynomial form.

The Goldhaber parametrization gives for the radius of the pion emission region the value $R_G = 1.01 \pm 0.05(stat) \pm 0.09(sys)$ fm and for the chaoticity parameter the value $\lambda = 0.40 \pm 0.03(stat) \pm 0.06(sys)$. Moreover the BEC parameters are substantially independent of the particle pair charge.

Using the Kopylov-Podgoretskii parametrization yields $R_{KP} = 2.07 \pm 0.04(stat) \pm 0.14(sys)$ fm and $\lambda_{KP} = 0.29 \pm 0.06(stat) \pm 0.04(sys)$. Also in this case the results are independent of the particle pair charge.

Performing a two-dimensional analysis in the LCMS frame yields $R_\parallel = 1.32 \pm 0.14(stat)$ fm and $R_\perp = 0.98 \pm 0.10(stat)$ fm. We have found an elongation factor $(R_\parallel - R_\perp)/R_\parallel \approx 35\%$. Our measurements confirm the LEP results that in the LCMS reference frame the longitudinal size of the pion source is 30-40% larger than the transverse one.

Figure 1: $R$ as a function of the Goldhaber variable $Q$ (a) and of the KP variables $Q_t$ and $Q_0$ (b).
two contributions should be fairly well separated in the c.m. frame of the hadronic jet. But the source radius $R_G$ shows no differences for particles emitted at different rapidities, demonstrating that the typical hadronization scale is much longer than the interaction radius, resulting in a unique hadron source, independent of the detail of the quark interactions.

![Diagram](image)

Figure 2: Compilation of results obtained by various experiments for the chaoticity parameter $\lambda$ (a) and the Goldhaber radius $R_G$ (b).

In the present analysis, it has not been possible to verify the Goldhaber radius increase with the event charged multiplicity $N_{ch}$ observed at LEP experiments. This effect is not visible in our data probably because of NOMAD low multiplicity ($N_{ch} \leq 10$) compared to that at LEP ($N_{ch} \gg 10$).

It may be interesting to compare our results with those in the $\pi\pi$ channel for lepton-induced reactions. The results on $\lambda$ show that there are two groups of experiments ($\lambda \approx 0.5$ and $\lambda \approx 1$) which are consistent within each group, but not between them. The results on $R_G$ show that the value of $R_G$ computed with a linear model is systematically lower ($R_G \approx 0.6$ fm) than the one computed with a quadratic or polynomial form ($R_G \approx 0.9$ fm).

4 Conclusions

The size and the chaoticity of the pion source are about 1 fm and about 0.4 respectively, quite independent of the final state rapidity sign of the emitted pions. A difference of about 35% is found between the longitudinal and transverse dimensions of the source. The final state hadronization processes have universal features with little dependence on the type or energy of the interacting particles.

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