Superluminal Neutrinos from Gauge Field

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Abstract

We consider a possibility that the recent OPERA results on neutrinos’ superluminality could be caused by a local effect of a new gauge field sourced by the earth. If neutrinos couple to this gauge field via a gauge-invariant but non-renormalizable interaction, the coupling effectively changes a background metric, thereby leading to superluminality of neutrinos. This possibility naturally might explain why neutrinos from CERN CNGS beam to Gran Sasso Laboratory become superluminal while those from SN1987A to Earth become subluminal.

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The OPERA experiment has recently announced a sensational result, that is, it was observed that neutrinos travel faster than the speed of light [1]. Although this remarkable discovery needs further experimental scrutiny in future, in the theoretical side it might call for consideration beyond standard physics, such as violation of the Lorentz invariance or extra-dimensional scenarios.

However, it is known that the breakdown of relativity could give us many theoretical pathologies. For instance, one could construct perpetuum mobile devices employing black holes which would have different horizons for different particle species. Moreover, one should recall that the Lorentz invariance provides a strong constraint on the selection of actions in quantum field theories. Indeed, under the situation of the absence of the Lorentz invariance, one has no idea how to determine actions. It is the manifest Lorentz invariance that supports the success of Standard Model at the fundamental level.

Thus, it is worth pursuing some possibilities for explaining the OPERA’s results as well as SN1987A ones [2] at the same time from a Lorentz-invariant theory. Actually, there have appeared such theories based on the Lorentz invariance where the cause of superluminal propagation of neutrinos is sought in our earth itself and the gravitational field. Put differently, the main idea is that superluminal neutrinos are local effects triggered by a new massive spin-2 tensor field [3, 4] and a new spin-0 scalar field of the Galileon type [5] which exist in the neighborhood of our earth. Then it is a natural question to ask ourselves whether or not the similar theories could be constructed in terms of a new spin-1 gauge field since the gauge field is on an equal footing with the tensor and scalar fields as elementary particles. In this article, we will see that this is the case, and we shall make an explicit theory using the magnetic field sourced by the earth.

We shall begin with the Lagrangian density of our theory :

\[ \mathcal{L} = \bar{\psi} (i \gamma^{\mu} \slashed{D}_\mu - m) \psi - i \frac{1}{M^*} F_{\mu}^{\alpha} F_{\alpha \nu} \bar{\psi} \gamma^{\mu} \slashed{D}^{\nu} \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}, \]  

(1)

where \( M^* \) is a mass scale which controls the strength of the coupling and the gauge field strength \( F_{\mu \nu} \) is defined by \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) as usual. Moreover, we have defined \( \bar{\psi} \gamma^{\mu} \slashed{D}_\mu \psi \equiv \frac{1}{2} (\bar{\psi} \gamma^{\mu} \partial_\mu \psi - \partial_\mu \bar{\psi} \gamma^{\mu} \psi) \). Because of this definition, the Lagrangian density (1) is manifestly unitary. The fields \( A_\mu \) and \( \psi \) denote a new spin-1 gauge field and the spin-1/2 neutrino field, respectively. For simplicity of presentation, we shall set mass of neutrinos to be vanishing, i.e., \( m = 0 \) in what follows. Note that compared to the conventional system of spinor plus gauge fields, as a feature of the Lagrangian density (1) there exists a non-renormalizable but gauge-invariant coupling (therefore, unitary without a ghost), which provides the following effective metric that neutrinos see

\[ g^{(\nu)}_{\mu \nu} = \eta_{\mu \nu} - \frac{1}{M^*} F_{\mu}^{\alpha} F_{\alpha \nu}. \]

(2)

Incidentally, one could pick up a linear coupling of the gauge field strength to neutrinos,

\[ \bar{\psi} \gamma^{\mu} A_\mu \psi \]
that is, \(-i\frac{1}{M_4^2}F_{\mu\nu}\bar{\psi}\gamma^\mu\partial^\nu\psi\), but it is easy to show that this simpler coupling does not yield an effective metric since \(F_{\mu\nu}\) is an anti-symmetric tensor rather than a symmetric one.

Next, let us consider what configuration the new gauge field takes in the local neighborhood of our earth. To do that, it is interesting to proceed by analogy to the real electro-magnetic field existing on the earth although this consideration is not always necessary. From the physical point of view, there is not so much electric charge on the earth since the excessive positive or negative electric charge would be almost neutralized on the earth (Of course, there is a feeble electric current on the earth, which is called "earth current" or "telluric current" [6]). Indeed, by following the similar line of argument mentioned later, it is of interest that one can prove that the static electric potential \(A_t = \frac{\alpha}{r}\) with \(\alpha\) being a constant does not lead to superluminal propagation of neutrinos.

On the other hand, we are empirically familiar with the fact that there is the earth’s magnetic field, which is dubbed "geomagnetism" and is generated in the fluid outer core by a self-exciting dynamo process. The earth’s magnetic field is known to be approximately described by a small magnetic dipole sitting at the center of the earth. Let us assume for simplicity that the magnetic field takes a value in the direction of the \(z\) axis \(^3\) and ask if neutrinos propagate superluminally because of an effective metric which is induced by the magnetic field. In this case, the magnetic field is given by

\[ F_{12} \equiv B_z = \frac{\alpha}{r^2}, \]  

where \(\alpha\) is a constant and its sign is not important since it appears in a quadratic form in the formulae below, and \(r\) is the distance from the center of the earth. Then, the equation of motion for the spinor field reads

\[ \gamma^0 \partial_t \psi + \gamma^1 (1 + \frac{1}{M_4^2} \frac{\alpha^2}{r^4}) \partial_x \psi + \gamma^2 (1 + \frac{1}{M_4^2} \frac{\alpha^2}{r^4}) \partial_y \psi + \gamma^3 \partial_z \psi - \frac{2\alpha^2}{M_4^2 r^6} (x\gamma^1 + y\gamma^2) \psi = 0, \]  

where the last term denotes a field-dependent mass term which is neglected in what follows since the size is very tiny. \(^4\)

Comparing this with the Dirac equation in a curved space-time

\[ e_A^\mu \gamma^A \partial_\mu \psi = 0, \]  

where \(e_A^\mu\) is the vierbein, and \(\mu\) and \(A\) are curved and flat indices, respectively, we can easily read off non-vanishing components of the vierbein

\[ e^t_0 = e^z_3 = 1, \quad e^x_1 = e^y_2 = 1 + \frac{1}{M_4^2} \frac{\alpha^2}{r^4}. \]  

\(^3\)Geomagnetism is not parallel to the surface of the earth except region near the equator.

\(^4\)Using (11) and \(R_E = 6.4 \times 10^6\)m, this field-dependent mass is approximately \(\hbar c \frac{\alpha^2}{M_4^2 R_E^5} = 10^{-5} \hbar c \frac{\alpha}{R_E} = 10^{-16} eV\).
Next, using the relation between the metric tensor and the vierbein
\[ g^{\mu\nu} = e^\mu_A e^\nu_B \eta^{AB}, \]  
the effective metric takes the form
\[ g^{tt} = 1, \quad g^{xx} = g^{yy} = -(1 + \frac{1}{M_4^4 r^4})^2, \quad g^{zz} = -1. \]  

Accordingly, the effective space-time on which neutrinos propagate has the line element
\[ ds^2 = dt^2 - \frac{1}{(1 + \frac{1}{M_4^4 r^4})^2} (dx^2 + dy^2) - dz^2. \]  

Then, along the null trajectory (since neutrinos are assumed to be massless) with the fixed \( y \) and \( z \) directions, the velocity of neutrinos is given by
\[ v \equiv \frac{dx}{dt} = 1 + \frac{1}{M_4^4 r^4}. \]  

The result of OPERA gives us the condition for the dimensionless quantity \( \beta \)
\[ \beta \equiv \frac{v - c}{c} = \frac{1}{M_4^4 r^4} = 10^{-5}. \]  

Now we have to specify the coupling of the new gauge field \( A_\mu \) to other particles in Standard Model in order to gain information on the mass scale \( M_* \). Let us assume that magnetic charge of the earth sources the gauge field and creates a local magnetic field. This field modifies a local gravitational background which neutrinos see and as a result the velocity of neutrinos becomes larger than that of light. Thus, we assume that the coupling to the rest of particles is given by \( A_\mu J^\mu \) where \( J^\mu \) is the electro-magnetic current which does not include neutrinos.

With this assumption, the magnetic field on the earth is about \( 0.5 \times 10^{-4} \) Tesla, so we have the relation
\[ \frac{\alpha}{R_E^2} = 0.5 \times 10^{-4}, \]  

where \( R_E \) is the radius of the earth and takes the value \( R_E = 6.4 \times 10^6 m \). Recovering dimensional factors, it turns out that the mass scale is described as
\[ M_*^4 = \frac{\hbar^3}{\mu_0 c^5} \left( \frac{\alpha}{R_E^2} \right)^2 \times 10^5, \]  

where \( \mu_0 \) denotes magnetic permeability of the vacuum. From this expression, we arrive at the value of the mass scale in the theory at hand
\[ M_* \approx 10^{-6} MeV. \]
This scale is so low that it is tempting to identify the new gauge field with the conventional electro-magnetic field, but to do so would need more investigation in future. Here we shall assume that this conjecture holds and move on to the issue of subluminal neutrinos from SN1987A to Earth. If the new gauge field were the familiar electro-magnetic field, the speed of light would remain the velocity of light even if the velocity of neutrinos becomes superluminal by the mechanism mentioned thus far. In this sense, our theory might give a nice resolution to the problem why neutrinos from CERN CNGS beam to Gran Sasso Laboratory become superluminal while those from SN1987A to Earth become subluminal.

Finally, let us comment on the relation between the present theory and Cohen-Glashow one [7]. The superluminal interpretation of the OPERA results has been recently refuted theoretically by Cherenkov-like radiation [7]. However, more recently three different resolutions to the Cohen-Glashow result have been proposed by three groups [8, 9, 10]. In particular, it has been shown in the last article [10] that if the velocity of an electron is equal to that of a neutrino, the threshold neutrino energy for the Cherenkov-like radiation becomes infinite and the process will not occur kinematically since the threshold neutrino energy is given by

$$E_\nu = \frac{2m_e}{\sqrt{v_\nu^2 - v_e^2}}.$$  

In our work, when we generalize the spinor field \( \psi \) to the SU(2) doublet \( L = (\nu_e, e)^T \) where \( \nu_e \) and \( e \) are respectively the neutrino spinor field and electron one, the velocity of the neutrino and the electron takes the same value owing to the mechanism of our theory (Recall that neutrino oscillation requires the three kinds of neutrinos to have the almost same velocity). Thus, our theory amounts to realizing the resolution proposed by Evslin [10]. Furthermore, one might fear that the interaction term

$$-i \frac{1}{M_4} F_\mu \Gamma^\nu \psi \gamma^\mu \partial^\nu \psi$$

in (1) would induce the bremsstrahlung of photons, for instance. Since this term is non-renormalizable and makes sense of only for energies less than the mass scale \( M_* \), one can show that such the bremsstrahlung of photons does not make a large contribution as follows: For simplicity, let us calculate the tree amplitudes of \( \psi A^2 \rightarrow \psi A^2 \) at the lowest order whose result takes the form

$$m^2 \left( \frac{1}{M_4} \right)^2 \frac{(p^2)^2}{p^4} |_{p < E} \sim m^2 E^5 \frac{m^2}{M_4^2}.$$  

Since we assume that \( E < M_* \) and the mass of a neutrino is small, this contribution is rather small. Moreover, one finds that the higher-order tree amplitudes become smaller than this lowest one, so we can safely ignore such the bremsstrahlung as long as we stay in the weak coupling region and we assume the mass of a neutrino to be small.

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