Lessons from the $B_{s}^{0,+} \to K^{*0,+} \mu^{+}\mu^{-}$ angular analyses

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We perform an analysis within the Standard Model of $B_{s}^{0,+} \to K^{*0,+} \mu^{+}\mu^{-}$ decays in light of the recent measurements from the LHCb experiment, showing that new data strengthen the need for sizable hadronic contributions and correlations among them. We then extend our analysis to new physics via the Standard Model effective theory, and carry out a state-of-the-art fit of available $b \to s\ell^{+}\ell^{-}$ data, including possible hadronic contributions. We find the case of a fully left-handed operator standing out as the simplest scenario with a significance of almost $6\sigma$.

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After the observation of the Higgs boson [1,2], indirect searches for physics beyond the Standard Model (SM) are playing an increasingly important role in the program of the Large Hadron Collider (LHC), as the recorded luminosity increases. In addition to precision electroweak and Higgs physics, LHC is also providing a huge amount of high-precision data in the flavor sector, in particular on rare and CP-violating decays of heavy mesons. In this context, $b \to s\ell^{+}\ell^{-}$ transitions have recently been under the spotlight, not only because of their potential sensitivity to new physics (NP) [3–6], but also because of the current experimental hints of deviations from the SM, see, e.g., [7–18]. As any other indirect search for NP, the quest for NP in $b \to s\ell^{+}\ell^{-}$ decays requires not only high experimental precision, but also a robust estimate of theoretical uncertainties in the SM prediction. From this point of view, the set of experimental results which hint at NP in $b \to s\ell^{+}\ell^{-}$ transitions can be divided in two broad classes. The first contains ratios of decay branching ratios (BRs) for different leptons in the final state; the second contains absolute BRs and angular distributions. The former is particularly clean from the theoretical point of view [19–21], but experimentally challenging,1 while the latter is also subject to sizable theoretical uncertainties [26,27]. Indeed, while the calculation of decay amplitudes for exclusive $b \to s\ell^{+}\ell^{-}$ transitions is well defined in the infinite $b$ and $c$ mass limit [28–30], and while in the same limit the uncertainty from decay form factors can be eliminated by taking suitable ratios of observables [31,32], in the real world amplitude calculations must cope with power corrections, which can be sizable or even dominant in several kinematic regions [33–37]. For example, the operator product expansion is known to fail altogether for resonant $B \to K^{(*)}\ell^{+}\ell^{-}$ transitions [38], and its accuracy is questionable close to the $c\bar{c}$ threshold. For this reason, estimating corrections to QCD factorization in the low dilepton invariant mass (low-$q^{2}$) region of $B \to K^{(*)}\ell^{+}\ell^{-}$ decay amplitudes is a crucial step toward a reliable assessment of possible deviations from SM predictions in these decay channels. Unfortunately, first-principle calculations of these power corrections are not currently available, and a theoretical breakthrough would be needed to perform such calculations, see, e.g., the discussion in [27,39,40]. Waiting for this breakthrough, the only reliable option is to use data-driven methods to account for the theoretical uncertainties and to quantify possible deviations from the SM. Obviously, data-driven methods are (much) less NP sensitive than (bold) theoretical assumptions, but as more and more data become

1Ratios of angular observables as the ones proposed in [22–24] and measured by Belle in [25] may also be considered in this category.
available the road to a robust test of the SM becomes viable. In this context, the very recent angular analysis of the $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ decay [41], together with the recent update on the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ one [42], represents a milestone in the effort to discern possible NP contributions from long-distance QCD effects. In this paper, we exploit these recent data to perform a detailed study of QCD pollution in angular observables, and to assess the compatibility of $B^{0,+} \rightarrow K^{*0,+} \mu^+ \mu^-$ with the SM. We then combine angular observables with lepton flavor universality (LFU) violating ones to provide the best estimate of possible NP contributions to $b \rightarrow s \ell^+ \ell^-$ transitions. The lesson we learn from the present analysis is twofold: (i) Within the SM, experimental data on angular analyses can be reproduced with sizable hadronic contributions, including a possible contribution that mimics NP effects; (ii) In the Standard Model effective theory (SMEFT) [43,44], the significance of NP from the global $b \rightarrow s \ell^+ \ell^-$ analysis increases with the inclusion of new data, reaching a maximum of almost 6σ for the simple scenario of a nonvanishing $C_{2223}^{LQ}$, always taking into account hadronic effects (see Eq. (8) below for the definition). All details of our treatment of hadronic uncertainties and of our Bayesian analysis technique can be found in Refs. [13,33,36]; here we limit ourselves to a concise review of the necessary ingredients. The main contributions to the $B \rightarrow K^{(*)} \ell^+ \ell^-$ decay amplitudes come from the following operators:

\[ Q_{7g} = \sqrt{\frac{\alpha_e}{64\pi}} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R, \]

\[ Q_{9V,\ell} = \frac{\alpha_e}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell), \]

\[ Q_{10A,\ell} = \frac{\alpha_e}{4\pi} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma^5 \ell), \]

\[ Q_8^s = (\bar{s}_L \gamma_\mu \gamma_5 \ell_L)(\bar{\ell} \gamma^\mu b_L). \]

Following [26,45], SM decay amplitudes can be conveniently decomposed in the helicity basis:

\[ H_{7g} \propto \left\{ C_{9}^{SM} \tilde{V}_{L\tilde{L}} + \frac{m_b^2}{q^2} \left[ \frac{2m_b}{m_B} C_{9}^{SM} \tilde{V}_{L\tilde{L}} - 16\pi^2 h_1 \right] \right\}, \]

\[ H_{7g} \propto C_{10}^{SM} \tilde{V}_{L\tilde{L}}, \quad H_{7g} \propto \frac{m_b m_\ell}{q^2} C_{10}^{SM} \left( \tilde{S}_L - \frac{m_\ell}{m_b} \tilde{S}_R \right) \]

(5)

with $\lambda = 0, \pm$ and $C_{9,10}^{SM}$ the SM Wilson coefficients of the operators in Eqs. (1)–(3), normalized as in Ref. [13]. The factorizable part of the amplitudes corresponds to seven independent form factors, $\tilde{V}_{0,\pm}$, $\tilde{T}_{0,\pm}$, and $\tilde{S}$, smooth functions of $q^2$ [46,47]. At first order in $\alpha_e$, nonlocal effects arise from the insertion of the operator in Eq. (4) yielding nonfactorizable power corrections in $H_{7g}$ via the hadronic correlator $h_1(q^2)$ [27,33,48], receiving the main contribution from the time-ordered product of:

\[ \frac{e_{\rho}^{\nu}(\lambda)}{m_B} \int d^4x e^{iqx} \langle K^+ | T \{ \bar{c}(x) \gamma^\nu c(x) Q_8^s(0) \} | B \rangle. \]

(6)

Within different setups and assumptions, most recent attempts to estimate the charm-loop contribution of Eq. (6) [39,49,50] find agreement with the outcome of the light-cone sum-rule computation in [51]. However, a reliable estimate of nonfactorizable effects encoded in $h_{0,\pm}(q^2)$ remains theoretically challenging in the full kinematic region of interest. In this work, we adopt a data-driven method based on the following general parametrization of the hadronic contributions:

\[ H_\nu \propto \frac{m_b^2}{q^2} \left[ \frac{2m_b}{m_B} (C_{7g}^{SM} + h(0)^{(0)}) \tilde{T}_{L\bar{L}} - 16\pi^2 h^{(2)}(q^2) \right] \]

\[ + (C_9^{SM} + h(1)^{(0)}) \tilde{V}_{L\bar{L}}, \]

\[ H_\nu \propto \frac{m_b^2}{q^2} \left[ \frac{2m_b}{m_B} (C_{7g}^{SM} + h(0)^{(0)}) \tilde{T}_{L\bar{L}} - 16\pi^2 h^{(0)} \right] \]

\[ + h(1)^{(2)} h^{(2)}(q^2) + (C_9^{SM} + h(1)^{(0)}) \tilde{V}_{L\bar{L}} \]

\[ H_\nu \propto \frac{m_b^2}{q^2} \left[ \frac{2m_b}{m_B} (C_{7g}^{SM} + h(0)^{(0)}) \tilde{T}_{L\bar{L}} - 16\pi^2 \sqrt{q^2} h^{(0)} \right] \]

\[ + h(1)^{(2)} \sqrt{q^2} + (C_9^{SM} + h(1)^{(0)}) \tilde{V}_{L\bar{L}}. \]

(7)

It is evident from Eq. (7) that the coefficients $h(0)^{(0)}$ and $h(1)^{(0)}$ can mimic LFU effects of NP, contributing to $C_7$ and $C_9$ respectively. Consequently, the extraction of NP contributions to $C_{7g}$ from angular observables crucially depends on the theoretical assumption on the size of $h(0,1)$. However, precise experimental data can in principle lead to the determination of all $h$‘s, improving our knowledge of hadronic contributions and strengthening or weakening our confidence on the estimates of Refs. [39,49–51]. In this context, it is very interesting to quantify the impact of the new data on the determination of the $h$‘s. Using the HEPFI code [52,53], we compare the results of a SM fit to the data in Refs. [25,41,42,54–68] with the one omitting the most recent data in Refs. [41,42].

Our main results in the SM are presented in Figs. 1–2, where the impact of the new data on the determination of the hadronic contributions (including $h(1)^{(0)} \equiv \Delta C_9$) can be clearly seen. In particular, in Fig 1 we show how the latest experimental information on $P_5$, see Ref. [69], can be accommodated in the SM once sizable hadronic effects as the ones obtained for $B^{0,+} \rightarrow K^{*0,+} \ell^+ \ell^-$ in Fig. 2 are taken into account. In the left panel of Fig. 2 we present an update of our analysis of Ref. [13], studying all available $b \rightarrow s \ell^+ \ell^-$ data at low $q^2$ previous to the LHCb measurements.
new data globally strengthen the evidence of nonvanishing $h$’s, introducing a slight preference for purely hadronic contributions.

Generalizing our analysis to the SMEFT, we consider the following additional operators:

\[ O_{LQ}^{(1)}(2223) = \bar{L}_2 \gamma^\mu L_2 (\bar{Q}_2 \gamma^\mu Q_3), \]
\[ O_{LQ}^{(3)}(2223) = \bar{L}_2 \gamma^\mu \tau L_2 (\bar{Q}_2 \gamma^\mu \tau Q_3), \]
\[ O_{Qe}^{(2322)}(2223) = (\bar{Q}_2 \gamma^\mu Q_3) (\bar{e}_2 \gamma^\mu e_2), \]
\[ O_{Ld}^{(2223)}(2223) = (\bar{L}_2 \gamma^\mu L_2 (\bar{d}_2 \gamma^\mu d_3)), \]
\[ O_{ed}^{(2223)}(2223) = (\bar{e}_2 \gamma^\mu e_2)(\bar{d}_2 \gamma^\mu d_3), \]

where $\tau^A=1,2,3$ are Pauli matrices (a sum over $A$ in the equations above is understood), weak doublets are in upper case and $SU(2)_L$ singlets are in lower case, and flavor indices are defined in the basis of diagonal down-type quark Yukawa couplings. Since in our analysis operators $O_{LQ}^{(1,3)}$ always enter as a sum, we collectively denote their Wilson coefficient as $C_{LQ}^{2223}$. We normalize SMEFT Wilson coefficients to a NP scale $\Lambda = 30$ TeV. With this normalization, after electroweak symmetry breaking $C_9$ receives contributions from $\mathcal{N}_\Lambda (C_{LQ}^{2223} + C_{Qe}^{2223})$, $C_{10}$ from $\mathcal{N}_\Lambda (-C_{LQ}^{2223} + C_{Qe}^{2223})$ and the chirality-flipped operators $C_9$ from $\mathcal{N}_\Lambda (C_{ed}^{2223} + C_{Ld}^{2223})$, $C_{10}$ from $\mathcal{N}_\Lambda (C_{ed}^{2223} - C_{Ld}^{2223})$, with $|N_\Lambda| \approx 0.7$. To quantitatively compare different NP scenarios, where different sets of SMEFT Wilson coefficients are allowed to float, to the SM, we compute the information criterion (IC) [70]:

\[ \text{FIG. 1. Result from a fit in the SM to the up-to-date experimental } b \rightarrow s \ell^+\ell^- \text{ data at low } q^2 \text{ for the binned angular observable } \mathcal{P}_0 \text{ [69]. We show the obtained 95\% highest probability density interval (HPDI) adopting the parametrization in Eq. (6), together with the most recent measurements from the LHCb angular analyses in [41,42]. Quark-spectator effects distinguishing the outcome for } B^{0,+} \text{ decays are at the percent level.} \]

\[ \text{FIG. 2. Inference of hadronic contributions from a fit in the SM to the available experimental } b \rightarrow s \ell^+\ell^- \text{ dataset at low } q^2, \text{ adopting the parametrization in Eq. (6), omitting (left panel in green) or using (right panel in red) new data from Refs. [41,42]. Contours correspond to smallest regions of 68\%, 95\%, 99.7\% probability. For marginalized one-dimensional posterior distributions the 68\% highest probability density interval (HPDI) is explicitly reported, highlighted by vertical bands.} \]
TABLE I. Mean and standard deviation (std) of the posterior distribution of the SMEFT Wilson coefficients from a fit to the full set of most recent \( b \to s \ell^+\ell^- \) data at low \( q^2 \) in the NP scenarios A, B, C along with \( \Delta IC \equiv IC_{\text{SM}} - IC_{\text{NP}} \). Results in white lines are obtained allowing for hadronic contributions as in the parametrization in Eq. (6), while results in gray lines are obtained using the \( q^2 \) extrapolation of the QCD sum-rule estimates of [51].

| NP scenario | Mean(\text{std}) | \( \Delta IC \) |
|-------------|------------------|-----------------|
| A: \( C_{2223}^{LQ} \) | 0.77(13) 0.92(12) | 29 58 |
| B: \( \{ C_{2223}^{LQ}, C_{3222}^{LQ} \} \) | 0.80(18), 0.05(30) 1.03(12), 0.71(13) | 26 81 |
| C: \( \{ C_{2223}^{LQ}, C_{3222}^{LQ}, C_{2322}^{LQ}, C_{6222}^{LQ} \} \) | 1.11(23), 0.49(36), −0.42(23), −0.28(43) 1.10(12), 0.83(15), −0.33(19), 0.04(37) | 26 89 |

\[
\Delta IC \equiv -2\log L + 4\sigma^2_{\log L},
\]

where the first and second terms respectively represent mean and variance of the log likelihood posterior distribution. Model selection between two scenarios proceeds according to the smallest IC value reported and the extent to which a model should be preferred over another one follows the canonical scale of evidence of Ref. [71], related in this context to (positive) IC differences. For convenience we always report \( \Delta IC \equiv IC_{\text{SM}} - IC_{\text{NP}} \).

In the simplest NP scenario considered (scenario A), we just allow for NP contributions to appear in \( C_{2223}^{LQ} \), corresponding to \( \Delta C_{9,\mu} = -\Delta C_{10,\mu} \). We then generalize to the case of nonvanishing \( C_{2223}^{LQ} \) and \( C_{3222}^{LQ} \) (scenario B), which allows for independent NP contributions to \( C_{9,\mu} \) and \( C_{10,\mu} \). Finally, we also switch on \( C_{2322}^{LQ} \) and \( C_{6222}^{LQ} \), thus allowing for NP to modify independently also the chirality-flipped operators \( C_{9,\mu}^{c} \) and \( C_{10,\mu}^{c} \) (scenario C). The results of our fit in the three scenarios described above are summarized in Table I and Fig. 3. Our main conclusion is that the preferred scenario is the simplest one, namely a NP contribution to \( C_{2223}^{LQ} \), or equivalently \( \Delta C_{9,\mu} = -\Delta C_{10,\mu} \) leading to \( \Delta IC = 29 \). The fitted value of \( C_{2223}^{LQ} = 0.77 \pm 0.13 \) corresponds to \( \Delta C_{9,\mu} = -\Delta C_{10,\mu} = -0.54 \pm 0.09 \) for a NP scale \( \Lambda \) of 30 TeV, deviating from the SM with a significance of \( \sim 6\sigma \). Scenarios B and C, in spite of the increase in model complexity, do not produce a sizable improvement in the fit.

The conclusion would be very different if a less conservative approach to hadronic uncertainties was taken, using QCD sum-rule estimates of the hadronic contributions and extrapolating them to the whole kinematic range up to the largest \( q^2 \) bin in Fig. 2. Then, the simplest scenario would not lead to an optimal description of experimental data, and additional operators would be needed. From the grey lines in Table I, the four-operator scenario including chirality-flipped operators achieves the best result, reproducing a NP pattern similar to the one...
with simultaneously nonvanishing $(C_{0_{\mu}}, C'_{0_{\mu}})$ highlighted, e.g., in [13,16]. We stress again that a conservative treatment of hadronic uncertainties is therefore crucial to obtain an unbiased picture of the kind of NP that may lie behind these intriguing experimental results.

Future updates of the present fit with forthcoming experimental data from LHC experiments [72], particularly with the LHCb phase II upgrade [73], and from Belle II [74], will further clarify the current picture. This will hopefully lead both to a clearer evidence for NP, possibly supported by other complementary set of measurements [75–78], and to an improved understanding of the QCD dynamics of charm contributions.

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