Intrinsic detection efficiency of superconducting nanowire single photon detector in the modified hot spot model

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Abstract
We theoretically study the dependence of the intrinsic detection efficiency (IDE) of a superconducting nanowire single photon detector on the applied current, $I$, and magnetic field, $H$. We find that the current, at which the resistive state appears in the superconducting film, depends on the position of the hot spot (a region with suppressed superconductivity around the place where the photon has been absorbed) with respect to the edges of the film. This circumstance leads to inevitable smooth dependence IDE($I$) when IDE $\sim 0.05 - 1$, even for a homogeneous straight superconducting film and in the absence of fluctuations. For IDE $\lesssim 0.05$, a much sharper current dependence comes from the fluctuation-assisted vortex entry to the hot spot, which is located near the edge of the film. We find that a weak magnetic field strongly affects IDE when the photon detection is connected with fluctuation-assisted vortex entry to the hot spot (IDE $\ll 1$), and it weakly affects IDE when the photon detection is connected with the current-induced vortex nucleation in the film with the hot spot (IDE $\sim 0.05 - 1$).

Keywords: superconducting nanowire single photon detector, modified hot spot model, detection efficiency

(Some figures may appear in colour only in the online journal)

1. Introduction

There are currently several proposed phenomenological models [1–7] describing the detection mechanism of superconducting nanowire single photon detectors (SNSPD) (To compare part of these models, see [8].) In this paper, we study how intrinsic detection efficiency (IDE), which is one of the main characteristics of SNSPD, depends on the current, $I$, and magnetic field, $H$, in the modified hot spot model. In comparison with the system detection efficiency, which defines the probability of detecting the photon by the whole device, IDE defines the probability of detecting the photon, which produces a voltage pulse, when it is absorbed by the main element of the SNSPD: a superconducting current-carrying film in the form of a meander [9].
to half of the depairing current (see figure 19(b) in [10]). This result coincides with our result for the finite-width film with the normal spot (see equation (12) in [4] in the limit \( \gamma \to 0 \) when \( w \gg R \) (i.e., when the film becomes formally infinite). The finite width of the film changes this simple result. The critical current now depends on the position of the normal spot, or notch, with respect to the edges of the film.

Our modified hot spot model has two main differences from previous hot spot models [1–3, 6, 7]: We solve the current continuity equation, \( \text{div} j = 0 \), in the film with the hot spot, which automatically gives us the maximal value of the current density near the hot spot. We also take into account the back effect of the current redistribution on the superconducting order parameter in the film with the hot spot. For example, it gives us the nucleation of the vortex-antivortex pair inside the hot spot, located far from the edges of the film, in the place where the supervelocity reaches its maximal value. The proposed model cannot relate the energy of the photon with the size of the hot spot and with the suppression of superconductivity inside it; these are the two control phenomenological parameters of our model. But we demonstrate that the radius of the hot spot and the level of suppression of the superconductivity only affect the dependencies IDE(I) and IDE(H) quantitatively.

In our previous work [4], we consider two locations of the hot spot: in the center and at the edge of the film. We find that in these two limiting cases, the resistive state starts (in the absence of fluctuations) at different critical currents, called detection currents, \( I_{\text{det}} \), via the appearance of the current-induced vortices and their motion across the film. We stress here that the vortex nucleation is the direct consequence of the spatially nonuniform distribution of the superconducting order parameter and the supercurrent in the superconducting film with the hot spot.

In this work, we find \( I_{\text{det}} \) at different locations of the hot spot in the film. We show that \( I_{\text{det}} \) reaches the minimal value, \( I_{\text{det}}^{\text{min}} \), when the hot spot is located near the edge of the film, and reaches its maximal value when the hot spot is located in the center of the film. We also calculate the energy barrier for the vortex entry to the superconducting film with the hot spot when \( I < I_{\text{det}}^{\text{min}} \), and we find the rate of fluctuation-assisted vortex entry as a function of the current. These results allow us to calculate the dependence IDE(I) in a wide range of currents, and we argue that vortices play important role both when IDE \( \approx 1 \) and when IDE \( \ll 1 \). Application of the magnetic field locally decreases the current density in one half of the film, and increases it in the other half, because of field-induced screening currents. We show that it leads to a strong increase of IDE when the photon detection is governed by the fluctuation-assisted vortex entry to the film (IDE \( \ll 1 \)), and the effect becomes much weaker when the photon detection is determined by the current-induced vortex entry (IDE \( \approx 1 \)).

2. Model

In our hot spot model (as in other hot spot models [1–3, 6, 7]), it is assumed that wherever the photon is absorbed, there is a nonequilibrium (heated) distribution of the quasiparticles over the energy that locally suppresses the superconductivity and leads to redistribution of the current density in the film. Calculation of the actual nonequilibrium distribution function of quasiparticles, \( f_{\text{eq}} \), needs the solution of a kinetic equation; it is a very complicated problem [16] that we do not study in this paper. Our aim is to study how the presence of the region with locally suppressed superconductivity (the superconducting order parameter) affects the value of the critical current, at which the superconducting state of the film with the hot spot (HS) becomes unstable. For this purpose, we numerically solve the Ginzburg–Landau equation for the complex superconducting order parameter, \( \Delta = |\Delta| e^{i\phi} \)

\[
\xi_s^2 \left( \mathbf{V} - \frac{2e\mathbf{A}}{\hbar c} \right) \Delta + \left( 1 - \frac{T_{\text{bath}}}{T_c} + \Phi_1 - \frac{|\Delta|^2}{\Delta_{\text{GL}}} \right) \Delta = 0 \tag{1}
\]

with the additional term

\[
\Phi_1 = \int_{|\Delta|}^\infty \frac{2(f_0 - f_{\text{eq}})}{\sqrt{\epsilon^2 - |\Delta|^2}} d\epsilon, \tag{2}
\]

which takes into account the impact of the nonequilibrium quasiparticle distribution function, \( f(\epsilon) \neq f_0(\epsilon) = 1/\exp(e/k_B T_{\text{bath}}) + 1 \). In equation (1), \( \Delta \) is a vector potential, \( \xi_s^2 = \pi\kappa D/8k_B T_c \), and \( \Delta_{\text{GL}} = 8\pi^2(k_B T_c)^2/\xi \) are the zero-temperature Ginzburg–Landau coherence length and the corresponding order parameter; \( D \) is a diffusion coefficient. Note that the imaginary part of equation (1) leads to the current continuity equation \( \text{div} j = 0 \) (\( j_s \) is a superconducting current density), which allows us to find the proper distribution of the current density in the film with the HS.

In numerical calculations, it is convenient to use dimensionless units. Therefore, we scale the length in units \( \xi = (k_B T_{\text{bath}})^{1/2} \), \( \Delta \) in units \( \Delta_{\text{GL}} = (1 - T_{\text{bath}}/T_c)^{1/2} \), and \( A \) in units \( \Phi_0/2\pi \xi^2 \) (\( \Phi_0 \) is a magnetic flux quantum). In these units, equation (1) has the following form

\[
(\mathbf{V} - i\hbar \mathbf{A}^\perp) \Delta + \left( \alpha - |\Delta|^2 \right) \Delta = 0, \tag{3}
\]

where \( \alpha = (1 - T_{\text{bath}}/T_c + \Phi_1)/(1 - T_{\text{bath}}/T_c) \).

In equation (3), the effect of the absorbed photon on the superconducting properties of the film is described by the parameter \( \alpha \) (in equilibrium \( \alpha = 1 \)), which is determined by \( f_{\text{eq}}(\epsilon) \). In our model, we put \( \alpha = \text{const} < 1 \) inside the HS region, which leads to local suppression of \( |\Delta| \) both inside and outside the HS due to the proximity effect, arising from the term with a spatial derivative in equations (1) and (3). Surely this assumption oversimplifies the real situation where \( \alpha \) depends on the coordinate, and we cannot expect that our results are quantitatively valid. But below we demonstrate that qualitatively, the obtained results do not depend on the actual value of \( \alpha \), which governs the suppression of \( |\Delta| \) inside...
the HS, and we expect that they are valid in the real situation with coordinate-dependent \( \alpha(r) \).

In our model, we use the static approach for the HS, (with \( \alpha(t) = \text{const} \) inside the hot spot. We can do this because of the different time scales existing in this problem. Indeed, during a very short initial time period, the hot quasiparticles appear after photon absorption, as seen in [16], undergo the downconversion cascade to the energy level just above \( \Delta_{\text{eq}} \), and a further relaxation process develops for much longer. The low-energy nonequilibrium quasiparticles diffuse in space, which is a relatively slow process due to the small group velocity of quasiparticles sitting at the energies close to the energy gap, but they simultaneously suppress \( \Delta \) locally below its equilibrium value. This results in the appearance of quasiparticles with energy less than \( \Delta_{\text{eq}} \), which already cannot diffuse, so the quasiparticles trapped by the HS [16]. These quasiparticles, can relax to the equilibrium via electron-phonon scattering, with characteristic inelastic electron-phonon relaxation time, \( \tau_{\text{e-ph}} \). Alternatively, they can scatter with quasiparticles, which have energy \( \epsilon > \Delta_{\text{eq}} \) and which can already diffuse out of the HS region; this process provides the relaxation to the equilibrium with the characteristic inelastic electron-electron relaxation time, \( \tau_{\text{e-e}} \). The minimum of these two times governs the final stage of evolution of the hot spot. But the time change of \( \Delta \) is a much faster process, and at low temperatures it is proportional to \( \hbar/\Delta_{\text{eq}} \), which is much shorter than \( \tau_{\text{e-ph}} \) or \( \tau_{\text{e-e}} \). Therefore, on the time scale of change of \( \Delta \), one may consider the quasiparticle distribution function as a static object at the final stage of its time evolution.

To provide insight on the possible values of \( \alpha \), one can use the local temperature approach, which implies that \( f_{\text{loc}}(\epsilon) \) can be described by the Fermi–Dirac function with the local temperature \( T_{\text{loc}} \neq T_{\text{bath}} \). With the help of equation (2), it is easy to show that in this limit

\[
\alpha(\vec{r}, t) = \left(1 - T_{\text{loc}}(\vec{r}, t)/T_{\text{c}}\right)/(1 - T_{\text{bath}}/T_{\text{c}}).
\]  

The area where \( T_{\text{loc}} > T_{\text{bath}} \) increases in time due to the diffusion of hot quasiparticles from the place where the photon was absorbed. From equation (3), it follows that the order parameter is suppressed in the place where \( T_{\text{loc}} \geq T_{\text{c}} \) and \( \alpha \leq 0 \). At some moment in time, the region where \( T_{\text{loc}} > T_{\text{c}} \) reaches its maximal size, and it is natural to model the HS by the circle with radius, \( R \), and put \( \alpha = 0 \) inside the circle. In this case, the radius of the spot, \( R \), and the energy of the absorbed photon, \( \hbar \omega/\lambda \), are roughly related as

\[
\eta \approx \frac{\hbar \omega}{\lambda} \approx \frac{dnR^2}{8\pi}H_{\text{em}}^2
\]

where \( H_{\text{em}} = \Phi_0/2\sqrt{2}\pi\xi\lambda_L^2 \) is the thermodynamic magnetic field, \( \lambda_L \) is the London penetration depth, \( d \) is the thickness of the film, and \( H_{\text{em}}^2/8\pi \) is the superconducting condensation energy per unit of volume. The coefficient \( 0 < \eta < 1 \) takes into account that only part of the energy of the photon is delivered for the suppression of \( \Delta \), and the rest of the photon’s energy heats the quasiparticles and phonons.

When the photon is absorbed at the edge of the film, the nonequilibrium quasiparticles cannot leave the sample, and we model the HS by the semicircle with a larger radius, \( R' = \sqrt{2}R \), to keep the volume of the HS unchanged.

From equation (3), it follows that for the spots with \( R \gg \xi \), the order parameter inside the HS is \( \approx \sqrt{\alpha}\Delta_{\text{eq}} \) when \( \alpha \gg 0 \), and it is equal to zero when \( \alpha < 0 \). Note that a different \( \alpha \) corresponds to a different level of the nonequilibrium inside the HS; in the local temperature approximation, this relation is given by equation (4). Due to the proximity effect, the order parameter is also suppressed partially at \( r > R \), and it becomes larger than \( \sqrt{\alpha}\Delta_{\text{eq}} \) inside the HS when the radius is about the coherence length \( \xi \).

In numerical calculations, we consider the film of finite width, \( w \), and length, \( L = 4w \), with different locations of the HS (the region where \( \alpha < 1 \)) across the film, as see in figure 1. We also add to the right-hand side of equation (3) the term with time derivative \( d\Delta/\partial t \), which allows us to find not only the value of the critical current, but also the place in the film where the vortices nucleate.

3. Detection current

Let us now discuss the mechanism of destruction of the superconducting state in the superconducting film with the HS. When the HS is located at the edge of the film, the vortex enters the HS via the edge of the film and then it passes through the film, as seen in the sketch in figure 2(a), if the current exceeds the critical value, \( I_{\text{pass}} \). In the local temperature approach in [4], we find that the vortex motion may strongly heat the superconductor, leading to the appearance of the normal domain. In the following, we assume that passage of even a single vortex through the film is sufficient to destroy the superconducting state in the film biased at the current, which is not much smaller than the depairing current.
We also find that for a relatively large radius of the HS \((R \geq 3\xi\) when \(\alpha = 0\)) at currents \(I_{en} < I < I_{pass}\), the vortex enters the HS, but cannot leave it. A similar effect was found in [17], which explored the effect of a special kind of edge defect on the vortex penetration to the superconducting film. The HS, as a region with suppressed superconductivity, could be considered a photon-induced pinning center, and the vortex becomes unpinned only at the current \(I \geq I_{pass}\). But, in contrast with the usual pinning center, the HS exists only for a short period of time, \(\min(\tau_{r-ph}, \tau_{r-ce})\), as seen in the discussion in section II.

Therefore, in the range of the currents \(I_{en} < I < I_{pass}\), the vortex is temporarily pinned, and after dissociation of the HS, it becomes unpinned and can pass through the film or exit via the nearest edge due to interaction with its image outside the film. Our numerical simulations, with the help of the time-dependent Ginzburg–Landau equation and the time-dependent \(\alpha(t)\), confirm both scenarios. Starting from the HS state with \(\alpha = 0\) and a pinned vortex, we gradually increase \(\alpha\) in time up to its equilibrium value of \(\alpha = 1\) (no HS state). We find that when the current is just above \(I_{en}\) the vortex exits via the nearest edge, while at relatively larger currents (less than \(I_{pass}\)) the vortex passes through the film.

When the spot is located near the edge of the film, as seen in figure 2(b), the same characteristic currents \(I_{en}\) and \(I_{pass}\) are present; the former corresponds to the vortex entrance to the HS, and the latter to its unpinning. In contrast with the case drawn in figure 2(a), when the HS dissociates, the vortex passes the film in the whole current interval, \(I_{en} < I < I_{pass}\).

We explain this by the larger distance from the nearest edge and the smaller attraction force coming from the image of the vortex.

When the distance, \(\delta w\) in figures 2(b) and (c), between the edge of HS and the edge of the film exceeds \(\sim 2\xi\), the vortex/antivortex pair is nucleated inside the HS at the current \(I_{pair}\). Again, if the \(R \geq 3\xi\) vortex and antivortex become unpinned at a larger current, \(I_{pass} > I_{pair}\). In this case, the dissociation of the HS leads to the annihilation of the vortex-antivortex pair, and they pass through the film only at \(I \geq I_{pass}\).

By increasing the radius of the HS, the gap between currents \(I_{en}\) (or \(I_{pair}\)) and \(I_{pass}\) increases. For spots with \(R \leq 3\xi\) and \(\alpha = 0\), these two currents coincide, which is connected to the relatively large value of \(|\Delta|\) inside the spot and a lesser ability to pin the vortices. We also check that with the variation of \(\alpha\) and the width of the film values of \(\Delta_{en}, I_{pair}\) and \(I_{pass}\) change quantitatively but the mechanism of destruction of the superconducting state stays the same.

The physical origin for the results is the following. Due to the current-crowding effect, the current density reaches its maximal value near the HS, as seen in figure 3(b). But simultaneously, there is an enlargement of the supervelocity, \(v_\alpha \sim j_{\alpha}/|\Delta|^2\), inside the HS, as seen in figure 3(c). The last effect is crucial because the superconducting state becomes unstable when the velocity of superconducting electrons exceeds some...
Indeed, when the HS approaches the edge of the film, the current crowding increases in the narrowest sidewalk, the supervelocit... when the photons with the same energy) and the current crowding decreases. This leads to an increase in the applied current at which the vortex can enter the HS.

How are the results shown in figure 4 related to the intrinsic detection efficiency of SNSPD? Consider, for example, the photon which creates the HS with a radius of $R = 4\xi$. Let the transport current $I$ equal 0.5$I_{\text{dep}}$, shown by the dashed line in figure 4. Then the part of the film in which HS causes $I_{\text{det}} < I$ will detect the absorbed photon, while the rest of the film, including the regions near the edges and the center of the film, won’t. This will provide IDE $< 1$. Only when the current exceeds the threshold value, $I_{\text{det}} = I_{\text{det}}^\text{max} \approx 0.58I_{\text{dep}}$, does the whole film participate in the detection of photons, and IDE $= 1$.

If the transport current $I < I_{\text{det}}^\text{min} \approx 0.35I_{\text{dep}}$, then IDE goes to zero in the absence of fluctuations. Fluctuations favor the creation of the vortices, and they may provide the finite IDE even at $I < I_{\text{det}}^\text{min}$. To distinguish this process from the current-induced vortex penetration at the current $I > I_{\text{en}}$, we use the term ‘fluctuation-assisted vortex penetration’ at $I < I_{\text{en}}$. Because the barrier for the vortex entry increases rapidly with decreasing current [5, 19, 20] the main contribution to the fluctuation-assisted IDE $\neq 0$ comes from the photons, which create the HS near the edge of the film, where $I_{\text{det}}$ is minimal. For this location of the HS, the vortex enters via the edge of the film, and one needs to calculate the energy barrier for the vortex entry to the HS when $I < I_{\text{en}} = I_{\text{det}}$. In figure 5(a), we show the calculated barrier, $\Delta F$ (the energy is scaled in units of $E_0 = \Phi_0^2/d/\pi^2\lambda^2$), for the different locations of the HS with $R = 4\xi$. $\Delta F$ is found using the numerical procedure from [20]. The energy barrier increases rapidly below some current $I^*(\delta w) < I_{\text{det}}(\delta w)$, as seen in figure 5(a), because at currents $I \leq I^*$, the vortex cannot be pinned by the HS and $\Delta F$ is roughly determined by the vortex entry to the film outside the HS region.

One can see in figure 5(a) that for a given value of the current, there is a minimal barrier for the vortex entry, $\Delta F_{\text{min}}$, when the HS is located at a specific distance from the edge of the film. Figure 5(b) shows the dependence, $\Delta F_{\text{min}}(I)$, found for different radii of the HS and $\alpha$. Note that at $I \sim I_{\text{det}}^\text{min}$, the energy barrier increases much slower with the current decrease when compared to the result from the London model [5, 19] ($\Delta F_{\text{min}} E_0 \sim (1 - 1/I_{\text{dep}})$, see the dashed line in figure 5(b)) or the Ginzburg–Landau model [20] ($\Delta F_{\text{min}} E_0 \sim A(w)(1 - 1/I_{\text{dep}})$, with $A(w) \sim 1.5 - 1.8$ for films with $w = 7 - 30\xi$) for vortex entry to the film without HS. Physically, it is connected with a suppressed-order parameter in the sidewalk between the HS and the edge of the film, as seen in figure 3(a). As a result, it costs less energy for

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**Figure 4.** Dependence of the detection current on the location of the HS with different radii in the film with $w = 20\xi$. 

Critical value [17]: a similar instability appears in flowing superfluid helium when its velocity exceeds critical value. In the superconductor with the uniform distribution of the order parameter, it coincides with the condition that the current density reaches the depairing current density, $J_{\text{dep}}$. But inside and around the HS there is a gradient of $\Delta I$, as seen in figure 3(a), and the vortices nucleate in the place where the supervelocitiy (not the superconducting current) reaches the maximal value. Note that when the vortex enters the HS, or when the vortex/antivortex pair is nucleated inside the HS, the maximal current density near the spot is approximately the same as $J_{\text{dep}}$, as seen in figure 3(b).

With these findings, we define the photon detection current, $I_{\text{det}}$, as the current at which at least one vortex can pass through the film after the appearance of the photon-induced HS. This current is equal to $I_{\text{pass}}$ when the vortex/antivortex pair is nucleated inside the HS; it is equal to $I_{\text{en}}$ when the single vortex enters the HS via the edge of the film. For the HS located at the edge of the film, shown in figure 2(a) we also take into account that the vortex passes through the film at the current a bit larger than $I_{\text{en}}$.

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4. Dependence of $I_{\text{det}}$ on the location of the HS at $H = 0$

In figure 4, we present the dependence of $I_{\text{det}}$ on the coordinate of the center of the HS with different radii. $R = 2\xi$, $4\xi$, and $5\xi$, which roughly correspond to the photons with $\lambda/\eta = 25\mu m$, $6.3\mu m$, and $4.0\mu m$, respectively. Note that $\eta \approx 0.1 - 0.4$, according to previous estimations [2, 18], and for the calculation of $\lambda$ with the help of equation (5), we use the parameters of a TaN film [18] with the thickness $d = 3.9\text{ nm}$. One can see that the minimum of the dependence $I_{\text{det}}(y)$ is reached when the HS touches the edge of the film, and $I_{\text{det}}$ is maximal when the HS sits in the center of the film. We argue that this result is the consequence of different currents crowding at different locations of the HS in the film.
the creation of the vortex or vortex nucleus [20] when compared to the film without HS.

IDE in the fluctuation region could be found with help from the Arrhenius law, \( \text{IDE} = \beta \exp (-\Delta F_{\text{min}} / k_B T) \), where the coefficient \( \beta \) in front of the exponent is equal to IDE at \( I = I_{\text{det}}^{\text{min}} \). We choose \( \beta = 0.05 \) because of the rapid decrease of \( I_{\text{det}} \) near the \( I_{\text{det}}^{\text{min}} \), shown in figure 4). Using the parameters of the TaN film [18] (\( \lambda_L = 560 \text{ nm}, d = 3.9 \text{ nm} \) and \( T = 4 K \), we find \( E_0/k_B T \approx 62 \). In figure 6, we plot IDE as a function of the current for photons with different wavelengths, which create the HS with different radii. In the same figure, we plot the sketch of dependence IDE(I), which follows from the hot belt model [5] (dotted curve). In the hot belt model, IDE < 1 is explained exclusively by the effects of fluctuations, which provide the fast drop of IDE with a current decrease. A much smoother change of IDE from 1 up to \( \approx 0.05 \) in the modified HS model exists even at \( T = 0 \), and it changes with much faster decay when IDE \( \leq 0.05 \), where it is finite only due to fluctuations.

We also calculate dependence IDE (\( I \)) at a fixed current. For this purpose, one needs to find the part of the film where \( I_{\text{det}}(y) \) is smaller than the transport current for the chosen \( R \) (i.e., wavelength). Because we know the barrier for the vortex entry to the HS, we can calculate the fluctuation-induced IDE, too. In figure 7, we present the results of our calculations. As in dependence IDE(I), one may distinguish two regions: a relatively smooth variation of IDE with \( \lambda \) when it varies in the range \( \sim 0.05 \rightarrow 1 \), and a much faster decay of IDE at larger wavelengths, where IDE is finite only due to fluctuation-assisted vortex entry to the HS. The qualitatively found results

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**Figure 5.** (a) Energy barrier for the vortex entry to the HS with radius \( R = 4 \xi (\alpha = 0) \) located at different distances, \( \delta \omega \), from the edge of the film. Solid line shows the minimal energy barrier at given value of the current. (b) Current dependence of the minimal barrier for the vortex entry to the HS with different radii and \( \alpha \). Dashed line corresponds to the dependence, \( \Delta F/E_0 = 1 - I / I_L \), following from the London model for the vortex entry to the film without the HS (at \( I \sim I_c \)).

**Figure 6.** Dependence of intrinsic detection efficiency on applied current, which follows from the nonmonotonic dependence \( I_{\text{det}}(y) \) (at \( I > I_{\text{det}}^{\text{min}} \)) and the finite probability for vortex entry due to fluctuations at \( I < I_{\text{det}}^{\text{min}} \) (area below the dashed line). Dotted curve corresponds to dependence IDE(I) following from the hot belt model [5] for one energy of the photon (qualitative presentation).

**Figure 7.** Dependence of intrinsic detection efficiency on the wavelength for the superconducting film with \( w = 20 \xi \) and different transport currents. In calculations, we use parameters of TaN film from [18].
resemble experimentally observed dependence IDE (\( \lambda \)). (See, for example, [8, 9, 18].)

5. Effect of the magnetic field

How do dependencies \( I_{\text{det}}(y) \) and IDE(I) change in the presence of the applied magnetic field? In figure 8, we plot the current density distribution in the superconducting film with and without the perpendicular magnetic field when there is no HS. One can see that in the presence of a low magnetic field, the current density increases in the left half of the film and it decreases in the right half of the film; for the opposite direction of H, the situation is the opposite. Here, under a low magnetic field, we mean fields \( H < H_s \), where \( H_s = \Phi_0 / 4\pi w \) is a magnetic field at which the surface barrier for vortex entry to the straight superconducting film is suppressed [21]. For the film with \( w = 20\xi \), \( H_s / 2 \approx 0.025H_c2 \), where \( H_c2 \) is a second critical magnetic field. Using the results of section 4, one may expect that the detection current becomes smaller, in comparison to the case \( H = 0 \), for an HS appearing in the part of the film with locally enhanced current density, and vice versa in the opposite case. Our numerical calculations confirm this idea as seen in figure 9. Note that \( I_{\text{det}} \) slightly changes when the HS is located in the central part of the film because of the relatively small change of the current density there (see figure 8).

In addition, we calculate the minimal energy barrier for the vortex entry to the HS located near the left/right edges of the film at different magnetic fields. From figure 10, it follows that the shape of dependence, \( \Delta F_{\text{min}}(I) \), weakly changes at low magnetic fields, \( H \ll H_s \), while the detection current in the left and right minima (\( I_{\text{det}}^{L,R} \)) varies linearly with the magnetic field (see the inset in figure 10).

With the help of our results, we calculate the dependence IDE(I) at different magnetic fields as seen in figure 11. Because \( I_{\text{th}} \) stays practically unchanged (see figure 9) while \( I_{\text{det}}^\text{min} \) decreases with the increase in \( H \) (see the inset in figure 10), the strongest change in IDE occurs at \( I < I_{\text{det}}^\text{min} \), when IDE \( \lesssim 0.05 \). Note that the hot belt model [5] predicts a relatively large change in IDE in the whole range of 0 < IDE < 1, and a linear decrease of the threshold current when \( H \) increases (see the dash-dotted curves in figure 11). Indeed, in the hot belt model, \( I_{\text{th}} \) is equal to the critical current of the film with the hot belt, which decreases linearly at the weak magnetic field, like \( I_{\text{det}} \) does, as seen in inset in figure 10.

6. Discussion

The vortex-assisted mechanism of the photon detection has been discussed in several works [4, 5, 9, 22]. In contrast with [9, 22], we argue that the vortices play an important role in all range of 0 < IDE < 1, and not only when IDE \( \lesssim 1 \) is...
determined by the fluctuation-assisted vortex entry or the unbinding of the vortex-antivortex pair. From the hot belt model [5], it follows that threshold current $I_{th}^{\alpha}$, at which IDE is about of unity, decreases linearly with an increase of the magnetic field, while we predict very weak dependence of $I_{th}^{\alpha}$ at low magnetic fields, $H \lesssim \Phi_0/4\pi\xi_w$.

Our model predicts that at current $I \gtrsim I_{th}^{\alpha}$, where $I_{th}^{\alpha}$ depends on the radius of the HS and, hence, on the energy of the photon (see figure 4), the photon count rate varies much less with the magnetic field than it does at smaller currents; some signs of this effect were observed in [26], and figure 3 therein. To observe this effect experimentally it is preferable to use the materials with threshold current $I_{th}^{\alpha}$ that is much smaller than the critical current of the superconducting film, as it is in the materials studied in [18, 23–25], because in this case one can vary the magnetic field in a wide range without overcoming the critical current, $I_s^c(H)$. Experimentally, $I_{th}^{\alpha}$ for each photon’s wavelength could be determined from the dependence of detection efficiency (DE) on the current if it saturates and DE(I) has a plateau at large currents. According to our calculations, at $I = I_{det}^{\min}$ the DE $\approx 0.05$ DE$\text{plateau}$ (which corresponds to IDE $\approx 0.05$). But in our model, we consider only straight homogenous film, while real SNSPD are based on the superconducting meanders, which have bends and different inhomogeneities, such as structural defects, and variations in thickness and/or width. Therefore, the minimal detection current may correspond to the HS located in the weakest part of the superconducting meander, which determines its critical current. To demonstrate this effect, in figure 12 we show calculated dependence, $I_{det}(y)$, for the film with a 90° degree bend (see the inset in figure 12). In figure 12, one can also see that the photon absorbed near the inner corner of the bend (the current density is maximal there due to current crowding) could be detected at the smallest transport current. By comparing figure 12 with figure 9, one can also see that the bend acts like a weak magnetic field. This similarity is not an accident, because in both cases there are places in the film where the local current density is maximal and the minimal detection current corresponds to the absorption of the photon near that location.

This result shows that in the bent film without intrinsic inhomogeneities, the area near the bend determines both $I = I_{det}^{\min}$ and minimal IDE, which is not connected with the fluctuation-assisted vortex entry. For example, at $I = 0.28 I_{dep}$, which is a little above the minimal detection current for the HS with $R = 4\xi$ located near the bend, as seen in figure 12, the minimal energy barrier for the vortex entry to the straight part of the film with the HS is about 0.29$E_0$, as seen in figure 5(a). Taking into account that typically $E_0/k_B T \gtrsim 50$, one easily finds that the vortex penetration to the straight part of the film is suppressed by the factor $\exp (-\Delta F/k_BT) \lesssim 10^{-6}$ which is much smaller than the ratio between the area near the bends and the rest of the superconducting meander (which is about $10^{-2} - 10^{-3}$ depending on how one estimates the active area near the bend). Therefore, we expect that in real SSPD, the fluctuation-assisted vortex entry contributes to the finite IDE when it becomes smaller than $\lesssim 10^{-3} = 10^{-2}$.

The actual boundary between the fluctuation-assisted and the current-induced vortex penetrations (leading to the detection of the photon) could be deduced from the experiment with the magnetic field. Indeed, because the first mechanism is more sensitive to the magnetic field (see figure 11), the IDE at currents $I < I_{det}^{\min}$ should increase much faster than at larger currents.

The effect discussed above could explain the absence of dependence of the photon count rate (PCR) on the magnetic field, which was experimentally found in [27]. Indeed, in that work, the field dependence was studied in the current interval where the PCR decreased from its maximal value by two orders of magnitude, which is equivalent to a similar change in IDE. Therefore, it might be that the minimal current was
still larger than $I < I_{\text{det}}^{\text{min}}$. As a result, at the magnetic fields $H < 100 \, \text{Oe}$ and $H_t \sim 4000 \, \text{Oe}$, which is called $H^*$ in [27], the PCR could change by no more than several percent following the change of $r_{\text{det}}^{\text{min}}$ seen in figure 9.

On the contrary, in [26], the strong dependence of PCR on the magnetic field was observed at low currents. Comparison of the results with the predictions of the hot belt model [5] demonstrated good qualitative but bad quantitative agreement. To fit the experimental results, the authors of [26] used the special dependence of the characteristic vortex energy (see equation (3) in [26]) on the wavelength and current (see the insets in figures 2(b) and 3 in [26]), which does not follow from the theory of [5]. Our model also predicts the strong dependence of PCR on the magnetic field, but only at the current $I < I_{\text{det}}^{\text{min}}(\lambda)$. Because the current dependence of the energy barrier for the vortex entry is not linear (see figures 5(b) and 10) we expect a quasi-exponential increase of PCR at low magnetic fields, which differs from the exponential law (see, for example, equation (2) in [26] or equation (5) in [27]) following from the linear dependence, $\Delta F(I)$, in the London model. To make the quantitative comparison with [26], one needs to calculate the IDE(I) for the film with the bends using the same procedure presented in this paper for the straight film, and find the energy barrier for the vortex entry to the HS, located close to the bend. Our present model gives only the qualitative prediction that some current $r_{\text{det}}^{\text{min}}$ exists (at this current IDE $\sim 10^{-3} - 10^{-2}$), above which the PCR weakly depends on $H$, and at smaller currents it depends much more strongly on weak magnetic fields (quasi-exponentially).

7. Conclusion

In the framework of the modified HS model, we find that the intrinsic detection efficiency of an SNSPD gradually changes with the current. The change of IDE from 1 to $\sim 0.05$ occurs due to the dependence of the current, at which the resistive response appears, on the location of the HS in the film. The resistive state starts from the vortex entry when the hot spot is located near or at the edge of the film, or from the nucleation of the vortex-antivortex pair when the hot spot is located far from the edge, and its/motion across the film. The change of IDE from $\sim 0.05$ up to 1 when the hot spot is connected with the fluctuation-assisted vortex entry to the HS located near the edge of the film, when the transport current itself cannot cause the vortex entry.

A weak applied magnetic field ($H \ll \Phi_0/4\pi\xi^2$) strongly affects (increases) IDE $\leq 1$ when it is nonzero only due to fluctuation-assisted vortex entry to the hot spot. At the currents close to the threshold current, at which IDE $\approx 1$, the applied magnetic field weakly affects the detection ability.

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