Reconstructing Quantum Mechanics Without Foundational Problems

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Abstract

I present a reconstruction of general Hamiltonian action mechanics that eliminates all foundational problems of quantum mechanics. The key advance is the completion of Hamiltonian mechanics to the universal mechanics of particles based on action-waves, consistent with the inclusive validity of the principle of stationary action. It is found that irreducible indeterminism is intrinsic and universal at all scales of dynamics. The new action-wave equation is the complete description of single dynamical histories, dissolving the classical-quantum divide. The statistical theory of quantum mechanics emerges as the ensemble average of modified action dynamics. The ensemble average of the new action mechanics leads to a hybrid function consisting of the action-waves and the probability density of the ensemble. This hybrid wavefunction obeys the Schrödinger equation, which is not a single particle dynamical equation. The reconstructed mechanics without matter waves is free of the cardinal problem known as the collapse of the wavefunction and with that, the vexing issue of quantum measurement is resolved. Another significant advance is the correct decoding of quantum entanglement and purging of nonseparability and nonlocality in quantum correlations. The action-waves do not carry the burden of divergent zero-point energy. The reconstructed mechanics is in complete agreement with all empirical requirements and in harmony with credible physical ontology.

Introduction

Quantum mechanics is believed to be a universal theory of physical phenomena, with a simple mathematical structure that is well established. Its empirical success and reach are unprecedented. Yet, there is no consensus
on a satisfactory interpretation and understanding of quantum mechanics [1, 2, 3]. The theory is beset with serious foundational problems that have been discussed and debated for nearly a century.

In this paper, I reconsider the theoretical structure of quantum mechanics (QM) and find that it is factually an emergent theory, obtained as the ensemble average of a more fundamental and universal theory of single particle dynamics. I present a reconstruction of quantum mechanics (RQM) and the Schrödinger equation, based on the new physical input of the completion of the action mechanics of Hamilton [4]. This is not a re-interpretation of the Schrödinger quantum mechanics; it is a reconstruction requiring a fundamental conceptual change at the very foundation of particle mechanics based on ‘carriers of action’. Thus, what we conventionally call ‘classical mechanics’ requires a fundamental modification motivated by the universal validity of Hamilton’s principle of stationary action [5]. The key finding is that instead of the Hamiltonian action equation $\frac{\partial S}{\partial t} = -H$, the complete and correct dynamical equation is $\frac{\partial \zeta(S)}{\partial t} = -i\varepsilon H \zeta$, where the action-wave is $\zeta(x, t) = \exp(iS(x, t)/\varepsilon)$ with $\varepsilon$ as a fundamental scale of action. It will be shown that the action-wave is necessarily complex valued. The action-wave mechanics already predicts that a fundamental irreducible indeterminism is present universally in mechanics at all scales. The new mechanics removes the distinction between classical and quantum mechanics, and eliminates all foundational issues associated with quantum mechanics in one sweep. The vexing issues of the collapse of the wavefunction and the quantum measurement problem are eliminated, while single particle interference effects are correctly reproduced. Entanglement of particle states turns out to be an ensemble notion. Correct quantum correlations without nonlocality and nonseparability of particle states result from the local interference of action-waves.

Statistical average of the particle dynamics obeying the new action-wave equation over the ensemble of dynamical histories results in a probability density. This positive real probability density $\rho(x, t) = \chi^* \chi$ combined with the action-wave $\langle \zeta(x, t) \rangle$ gives a hybrid ensemble function $\psi(x, t) = |\chi| \langle \zeta(x, t) \rangle$, where $\chi = Ae^{i\phi}$. It is this ensemble function $\psi(x, t)$ that obeys the Schrödinger equation and exact Born’s rule. The Hilbert space structure with the square-integrability of this hybrid complex function is emergent and refers to the ensemble of dynamical histories and not to single histories. Thus, the Schrödinger equation is not an equation for single particle or single dynamical history. It is an ensemble equation. All previous attempts to understand the foundational structure of quantum mechanics searched in vain for solutions in the analytical interpretations of the Schrödinger equation and its wavefunction. Whereas, in fact, the complete resolution of the foundational problems lay outside formal quantum mechanics, in the modified Hamiltonian
action dynamics, and not in the interpretation of the Schrödinger equation or its wavefunction. It turns out that this is naturally accomplished without changing the mathematical formalism and statistical structure of quantum mechanics. Therefore, I am able to eliminate all foundational problems of quantum mechanics without affecting its well tested statistical predictions.

The reconstruction of mechanics is based on identifying two distinct physical entities in dynamics at all scales – the material particle, which is the carrier of dynamical quantities like energy and momentum, and the associated ‘action-waves’ (or ‘phase-waves’) that are the carriers of ‘action’, with no energy or momentum. The completion of Hamiltonian action mechanics to a universal mechanics of particles based on action-waves does not differentiate classical and quantum, relativistic and nonrelativistic, microscopic and macroscopic etc. This is logically demanded by the physical reason behind the universal applicability of the principle of stationary action, namely that a wave-like entity capable of interference is present in all dynamics. Therefore, the new Hamiltonian action-wave mechanics, rather than quantum mechanics and the Schrödinger equation, is the universal basis of all mechanics. The particle’s dynamics is linked to the phase of the action-waves in a wave-particle connection. The existence of a fundamental scale $\varepsilon$ of the action in the action-wave defines the fundamental uncertainty in action, which reflects in all dynamics. This scale is identified empirically as the Planck’s constant $\hbar$.

I will show that both classical and quantum dynamics of a particle are described by the same equation of evolution of the action-wave, $\frac{\partial \zeta(S)}{\partial t} = -\frac{i}{\varepsilon} H \zeta$. Hamilton’s action equation $\frac{\partial S}{\partial t} = -H$ (called the Hamilton-Jacobi equation [6]) descends from this master wave-equation for dynamics. Material particles do not behave as waves or superpose; there are no matter waves. Quantum mechanics is factually the ensemble average of the new Hamiltonian action mechanics. The wavefunction of the Schrödinger dynamics is really a hybrid entity involving the ensemble averaged probability density for the material particle, without superposition, and action-waves of quantum possibilities that can superpose and interfere event by event. The Schrödinger wavefunction does not pertain to single quantum history, as believed hitherto, but to the ensemble average of such histories. Thus, both the probabilistic particle dynamics and the vital quantum interference are intact. There is a radical change in the physical paradigm and interpretation, while the mathematical super-structure and operations remain intact because the quantum mechanical calculations pertain to the averages over the (virtual) ensemble. This explains all features of quantum mechanics.
1 Foundational Problems of QM

The standard theory of quantum mechanics is based on the Schrödinger equation for the time evolution of a wavefunction, which refers to single dynamical history from preparation to observation, in all interpretations. The wavefunction is a single functional dependent on the coordinates of the particle, in the case of a single particle, and on the 3N coordinates for N particles. Though the concept evolved from de Broglie’s matter-waves, the multiparticle wavefunction evades consistent physical interpretations. However, all standard approaches treat the wavefunction as having a space-time support because only then one can understand how local interactions in space and time can alter the wavefunction. Also, the entire notion that quantum mechanics has nonlocal features is dependent on changes that happen to the wavefunction across spatially separated regions.

The characteristic and essential trait of quantum mechanics is the inherent indeterminism; the theory has only probabilistic predictions, despite being described by the deterministic Schrödinger equation of evolution. This indeterminism is linked to the wave-particle duality and the uncertainty principle, encoded mathematically in the commutation relations between observables. Indeterminism results in different and random outcomes in measurements or observations on the quantum states described by the same wavefunction. In practise, this is manifest in the statistically distributed results on identically prepared ensemble of quantum systems, represented by the same wavefunction or the quantum state. As an example, in the physical state of an electron represented by $|z+\rangle$, its spin component is known with certainty to be in the positive z direction. When measured along the direction $z'$, such that $z \cdot z' = \cos \theta$, the probability for observing the electron in the state $|z'+\rangle$ is $\cos^2(\theta/2)$. Each measurement on the identical states $|z+\rangle$ then returns random results of $|z'+\rangle$ and $|z'-\rangle$, with a fraction $\cos^2(\theta/2)$ in the + direction.

The conventional formalism of QM leads to several vexing problems, debated as its ‘birth defects’ [2, 7, 8]. Foremost is the core issue known as the collapse of the wavefunction during an observation. The quantum physical state is represented by a state vector in Hilbert space, $|\psi\rangle$. This can be linear superposition of component states $|\psi\rangle = \sum c_i |\psi_i\rangle$, with $\sum |c_i|^2 = 1$. The coefficients $c_j$ determine the probability $p_j$ to be observed in the particular state $|\psi_j\rangle$ through Born’s rule $p_j = |c_j|^2$, with $\sum |c_j|^2 = 1$. An unknown general state could be represented similarly with values of $c_i$ unspecified. When a measurement is made, only one (or a subset) of the possible results $i = n$ materializes, stochastically. The quantum state is said to collapse or reduce uniquely to $|\psi_n\rangle$, with other components disappearing instantaneously. Since Born’s rule dictates that $|c_n|^2 = 1$ after observing the system in the state
ψ_n⟩, consistency demands the instantaneous vanishing of all other components. Therefore, any space-time representation of the wavefunction implies a nonlocal reduction of some entity in space and time. On the other hand, the interpretation of the wavefunction as a purely mathematical entity to represent the state of the particle in the abstract Hilbert space, without a space-time counterpart, fails in the physical explanation of even simple interference experiments involving local interactions (as illustrated in the next section).

The problem of collapse gets more difficult for multi-particle systems. For a two-particle two-state system, with possible states |+⟩ and |−⟩, the joint state may be a linear superposition \( \psi_e = c_a|+1⟩|−2⟩ + c_b|−1⟩|+2⟩ \), called an entangled state. This has no quantum mechanical description as the joint state (product state) of two independent quantum systems, |s_1⟩|s_2⟩. The most general states of the particles (1 and 2) with two base-states are |s_1⟩ = a_1|+1⟩ + b_1|−1⟩ and |s_2⟩ = a_2|+2⟩ + b_2|−2⟩. Then the most general product state is

\[
\psi_{1,2} = (a_1|+1⟩ + b_1|−1⟩) \otimes (a_2|+2⟩ + b_2|−2⟩)
\]  

But the state \( \psi_e \) cannot be written as \( \psi_{1,2} \). Then, there is no definite quantum mechanical state for either particle! Each particle is not in the state |+⟩, not in the state |−⟩, or not in a general superposition \( a|+⟩ + b|−⟩ \). The particles have their quantum mechanical existence only as a single non-separable system. However, each particle is available to the experimenter as separate systems, possibly in widely separated laboratories. Measurement on one particle returns a definite value + or − and a corresponding quantum mechanical state, say |−1⟩. This means that \( \psi_e \) has collapsed to the state |−1⟩|+2⟩. Then, instantaneously and nonlocally, the other particle acquires a definite new state |+2⟩ without any interaction or observation. An observation on one particle has created individual states for both, from a situation in which neither had quantum state. The conceptual difficulty arising due to this induced nonlocal collapse is the basis of the Einstein-Podolsky-Rosen (EPR) discussion [9] and the results about the conflict between quantum mechanics and Einstein locality. (EPR assumed Einstein locality and that the physical state of matter could not be changed nonlocally from a spatially separated region. Then they concluded that quantum mechanics was not complete – wavefunction description is not in one-to-one correspondence with the physical state, because the wavefunction of a particle can be changed by a measurement on another particle. See appendix 4 for details).

The ensuing quantum measurement problem [8, 10] is closely linked to the collapse of the wavefunction. If two physical states of a system are represented as |1⟩_s and |2⟩_s the physical system can also be in the state \( a|1⟩_s + b|2⟩_s \) with \( |a|^2 + |b|^2 = 1 \). During a measurement, another physical
system acting as the ideal measurement ‘apparatus’ with pointer states $|1\rangle_A$ and $|2\rangle_A$ interacts with the system and forms the correlated and entangled state

$$|S_{sA}\rangle = a|1\rangle_s|1\rangle_A + b|2\rangle_s|2\rangle_A$$

Then, *neither the microscopic system nor the macroscopic apparatus has any individual physical state.* The bizarre situation is unavoidable in standard QM irrespective of the size and mass of the ‘apparatus’. It should be understood correctly that neither of the physical systems is in any superposition of the two allowed states; each system has no physical state of its own within QM [11] (this crucial point is often not adequately understood, diluting the severity of the problem). This is the same as the much discussed problem of the Schrödinger’s cat, where the ‘apparatus system’ is macroscopic and possibly living [12].

The final (conscious) experience about the measurement is, however, either the state $|1\rangle_s|1\rangle_A$ or the state $|2\rangle_s|2\rangle_A$. This is the collapse of a nonseparable entangled state into a joint product state; it is the collapse of the superposition of multi-particle states to one specific separable product state in which each system now has a definite individual state. The process by which this ‘state reduction’ and the appearance of a unique pointer state happen is a mystery.

The quantum measurement problem is more severe than it looks in the case of the ‘system and apparatus’. We saw that there is no QM state for the apparatus until observed. However, to observe the apparatus, an observer $O$ is required, which might be a more complex apparatus or a human being, eventually. But, if QM is universally applicable, all that can happen is a more complex entanglement

$$|S_{sAO}\rangle = a|1\rangle_s|1\rangle_A|1\rangle_O + b|2\rangle_s|2\rangle_A|2\rangle_O$$

and not a collapse to any definite state for the observer, or the apparatus, or the system. It is an infinite regress, until an agent that does not obey the law of linear superposition in quantum mechanics is artificially introduced. This is what is done by the standard interpretation, when it assumes a ‘classical apparatus’ for measurements. Desperate desire for the resolution of the quantum measurement problem has prompted some physicists to take the extreme speculative view that an undefined entity called ‘consciousness’ is involved in breaking the system-apparatus entanglement. Such proposals are inconsistent because they invoke structures built out of matter that do not obey the laws of quantum mechanics.

Finally, despite many proposals, there is no satisfactory understanding of the emergence of the classical macroscopic world from what is believed to

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4In this general review paper written in 2005, I had already indicated the empirical factors that hold the key to eliminating the foundational issues in QM. That the theoretical advance needs modifying Hamilton’s action mechanics was not expected then.
be the more fundamental quantum world. Are there two separate physical worlds with a transition zone, with a fundamental quantity like mass as the parameter, or is dynamics based on a universal principle and single evolution equation? The answer is in the solution to the measurement problem.

The Reconstructed Quantum Mechanics (RQM), derived from Reconstructed Hamiltonian Action Mechanics (RHAM), is free from all these foundational problems.

2 Reconstructing Hamiltonian Mechanics

There are two essential empirical facts in microscopic dynamics, represented in figure 1 that are important for the correct physical theory. Quantum particle dynamics is often about multiple possibilities that can interfere. For such a situation, the spatial paths could be well separated. What is clear is that the particle cannot be in both paths without violating all fundamental conservation laws because all experimental results are as if the particle remains as a single whole at all times, with its intrinsic properties (mass, electric charge, spin etc.) not divided or smeared out. Therefore, a virtual copy would violate the conservation laws and division of any sort would not be consistent with the general empirical results. However, the interference phenomena, characteristic of quantum dynamics, demand that there is some entity that divides and be physically present in the multiple paths.

In a Mach-Zehnder interferometer, the relative phase in the two paths determines the probability for the particle to exit through either of the output ports. When the phase difference is $2n\pi$, every particle will exit via one port and for $(2n + 1)\pi$, via the other port, deterministically, particle by particle.
However, the total phase difference depends on independent phase shifts in the two paths, due to the positions of the mirrors, local phase shifters etc. Therefore, it is obvious and unavoidable that for each particle there is some non-matter physical entity in both paths, which can account for the different and independent phase shifts that finally combine to give the phase difference that determines the port of exit.

The important new finding is that taking care of these two empirically essential features provides a unique reconstruction of quantum mechanics that is free of all the foundational problems. However, this reconstruction has to start from a reconstruction of classical Hamiltonian mechanics itself, because it turns out that classical mechanics in the present form is not complete. When W. R. Hamilton introduced the principle of stationary action in 1832, it was generalised from Fermat’s least action principle for optics. He developed a universal and “general method for expressing the paths of light, and of the planets”. Hamilton also discussed the rationale involved in the principle, through Huygens’ theory of wave optics. In the subsequent papers that described the method for mechanics based on the ‘characteristic function’, or the action function $S(x,t)$, he wrote the general equation for dynamics as

$$\frac{\partial S}{\partial t} = -H, \quad H = T + V$$  \hspace{1cm} (4)$$

The momentum is derived from the action as $\partial S/\partial x = p$. The equation was called the Hamiltonian equation by C. G. J. Jacobi, who developed it further mathematically. However, this equation does not explicitly acknowledge the action as a central property residing in a wave-like entity and does not represent the core reason for the universal validity of the principle of stationary action! The principle is operative because action is physically manifest in a wave property where the phase of the wave is the scaled action. Hence, I complete Hamilton’s action mechanics by modifying the equation to a new universal form

$$\frac{\partial \zeta(S)}{\partial t} = -i \frac{\varepsilon}{\varepsilon} H \zeta$$  \hspace{1cm} (5)$$

where the function $\zeta(x,t) = \exp(iS(x,t)/\varepsilon)$. I call this function the ‘Action-wave’, since it has a direct physical presence. This equation is the key step in the correct description of single particle, single dynamical history in the reconstructed universal dynamics. The parameter $\varepsilon$ is the fundamental scale of action that is to be determined empirically. This equation contains Hamilton’s equation $\partial S/\partial t = -H$.

The first feature I note is that the action-wave is necessarily complex valued. For, the relation between the first order time evolution and the Hamiltonian involves second order spatial derivatives. Therefore, the dynamical equation cannot be built on a real periodic function like $\cos(S/\varepsilon)$ or on a
real linear combination of sinusoidal functions; the action wave needs both quadratures as a complex valued function, \( \cos(S/\varepsilon) + i\sin(S/\varepsilon) \). Therefore, the fundamental equation for dynamics involves complex numbers because complex valued action waves are the basis of dynamics at all scales. This feature is then inherited by quantum mechanics. The action-wave has unit amplitude, dictated by the dynamical equations \( \partial S/\partial t = -H \) and \( \partial S/\partial x = p \).

We note the immediate main result that the new Hamiltonian action-wave equation predicts intrinsic indeterminism in dynamics at all scales. With \( H = (p^2/2m) + V \), and \( p = \partial S/\partial x \) we get,
\[
i\varepsilon \frac{\partial \zeta(S)}{\partial t} = -\zeta \frac{\partial S}{\partial t} = \left(-\frac{\varepsilon^2}{2m} \nabla^2 + V\right) \zeta = \zeta \left(\frac{p^2}{2m} + V\right) - \zeta \frac{i\varepsilon}{2m} \nabla^2 S \tag{6}
\]

In addition to the familiar Hamiltonian mechanics in the first term, there is an additional term proportional to \( \varepsilon \) and to the concentration of the action in space, \( \nabla^2 S \). The equation is the single particle, single dynamical history equation. Hence, this term points to the variations in an ensemble of action-waves, for the dynamics of a single particle. When \( \varepsilon \) is tiny, this term is negligible. But it will become significant for microscopic dynamics. To see this clearly, consider the situation when \( p \approx 0 \). Then, we get
\[
\frac{\partial S}{\partial t} = \frac{i\varepsilon}{2m} \nabla^2 S \tag{7}
\]

This resembles the diffusion equation, but the diffusion constant is pure imaginary. Therefore, this is not dissipative diffusion, but dephasing of the action waves (fig. 2). The action can be specified only to the precision of \( \varepsilon \); thus the fundamental uncertainty in action is the quantity \( \varepsilon \). Therefore, dynamics at all scales have the same unavoidable intrinsic uncertainty in action. This is empirically determined as the Planck’s constant, \( \varepsilon \equiv \hbar \approx 1.055 \times 10^{-34} \) J-s, which is a miniscule amount of action. We have found that the fundamental uncertainty is not limited to microscopic world and quantum dynamics. It is universal, but negligible for macroscopic dynamics. This resolves the primary fundamental puzzle discussed in the context of quantum mechanics, namely the origin of indeterminism. Its source is action-wave dynamics, and it is universal. Classical mechanics is not deterministic. Classical mechanics is dynamics in which the irreducible indeterminism is negligible. Hence, we have also answered another foundational question, about a quantum-classical divide and a transition domain. There is no such divide. The new dynamics predicts that the experiments that search for evidence of a transition, either at Planck mass scale or any other scale in mesoscopic systems, will have a definite null result.

Figure 2 shows how the dephasing of the action waves translates to the diffusion of the amplitude of the waves. The effect is non-dissipative and
progresses to reduce the concentration of action, $\nabla^2 S$. This manifests in the linear diffusion of the amplitude of the action-waves, which is of crucial importance when we discuss the reconstruction of quantum mechanics later.

Surprisingly, with the Reconstructed Hamiltonian Action-wave Mechanics (RHAM), all foundational problems of quantum mechanics are eliminated, even before we discuss the reconstruction of quantum mechanics by ensemble averaging the action-wave dynamics. This is because the Schrödinger equation is not the single particle equation, whereas the foundational problems mainly concern quantum effects related to the dynamical history and observation of single physical systems. The new Hamiltonian action-wave equation governs all dynamics, classical and quantum, and the solutions for the foundational problems are to be sought there. Later, I will show that the Schrödinger equation is obtained as the ensemble average of Hamiltonian action-wave dynamics and that the wavefunction in quantum mechanics is a hybrid entity, with its real positive amplitude obtained as an ensemble averaged probability density, combined with the complex action-wave. Therefore, the wavefunction of quantum mechanics and the Schrödinger equation do not describe single particle dynamics (single dynamical history), contrary to the present interpretations of quantum mechanics. However, all interference and correlations in quantum mechanics are correctly reproduced from the local interference of the action-waves.

### 3 Elimination of All Foundational Problems

I have already discussed the universal nature of indeterminism and the absence of a classical-quantum divide revealed by the new Hamiltonian action-wave mechanics. The physical picture provided by the modified action-wave mechanics is transparent. The particle, which is the carrier of dynamical quantities like energy and momentum, has the dynamics linked to the temporal and spatial derivatives of the action carried by associated action-waves.
Action-waves carry action \( S(p_i, x^i) \propto \int p_i \, dx^i \), and not the dynamical quantities \( p_i \). In particular, action-waves do not possess energy or momentum. The fundamental scale of action \( \varepsilon \) limits the precision to which the dynamical quantities can be specified in combination with the coordinates. Particle or matter itself does not have wave property. There are no matter waves. What exists is a wave-particle connection, as given by the Hamiltonian action-wave equation, and not wave-particle duality. The long-held notion that matter behaves as waves, or as matter-waves, was far from the actual physical fact, and it was a red herring. Now I discuss each of the foundational problems and its definite solution naturally arising from the single step of modifying the Hamiltonian action mechanics to the equation \( i\varepsilon \frac{\partial \zeta(S)}{\partial t} = H\zeta \). It is significant that we do not need the Schrödinger equation and the wavefunction resulting from the ensemble average to discuss how the foundational problems are eliminated, because they are not relevant for analysing the dynamical history of single systems. The wavefunction of standard QM turns out to be an ensemble averaged mathematical entity with no physical counterpart for single dynamical history in space and time. Hence, I refer to the ‘collapse of the wavefunction’ as ‘collapse of the state’ in further discussion.

### 3.1 Collapse of the State and Interference

In RHAM, the dynamics of the particle contains significant scatter related to the dephasing term in the dynamical equation when the experimental situation involves multiple possibilities of dynamics comparable to the quantum of action \( \hbar \). We label these possibilities as \( |\psi_i\rangle \), anticipating convenience of notation later. (But, there is no distinction between classical and quantum, for these states). As the material particle encounters a fork of possibilities \( |\psi_i\rangle \), it takes exactly one \( i = k \) with the probability determined by the experimental set up (beam splitters, slits etc.) and the initial action-phase of the associated action-wave at that instant. This initial phase is random and unknowable within the quantum of action, and it has the intrinsic uncertainty \( \hbar \). This uncertainty translates to the choice of a random path. Since the material particle has no wave property, it is never in superposition of physical states. The particle is in one unique path (state) at any instant, in a given event. The action-wave splits as \( a_i \exp(iS_i/\hbar) \) and propagates in all paths or possibilities. The amplitude coefficients \( a_i \) are determined by the experimental set up (like a beam splitter or an aperture). The split action-waves obey the amplitude relation \( \sum a_i^2 = 1 \), as any other wave-like entity. Thus, for a symmetric beam splitter \( a_i = 1/\sqrt{2} \), but this applies to only the action-waves and not for the particle. The interference of action-waves determines the subsequent probability of partition into other states (see calculation below). When a measurement is done for any physical quantity, the
result is the factual state of the particle. *When a measurement of position is done, the particle is found where it actually is. There is no collapse of the state*, because the action-waves do not collapse and the particle is not in a superposition. The action-waves, which do not carry energy or momentum, continue their propagation, but it is irrelevant for further detail of the dynamics of the particle. The relevant fact about the physical quantity ‘action’ is that it is an accumulated integral entity, linked to the space-time coordinate durations in the dynamical history. Only the local relative action at the point of overlap of multiple possible histories matters for interference and probabilities, at the scale of the fundamental action of $\hbar$. The interactions of the action-waves involve exchange of only action. This is the essence of wave-particle connection in RHAM. No conservation law is affected.

For repetitions of the particle dynamics with the state of the particle prepared identically, the action-waves remain the same except for the uncertainty $\hbar$ in action. This uncertainty results in the microscopic stochasticity in the particle dynamics. Different histories in the ensemble have different values for the physical observables. Now the ensemble average defines the probabilities $\rho_i$ for the different $|\psi_i\rangle$. The superposition like $a_1|\psi_1\rangle + a_2|\psi_2\rangle$ is only applicable to the action-waves. For the particle, $\rho_i = A_i^2$ with $\sum \rho_i = 1$, where the real quantities $A_i$ refers to an ensemble of dynamical histories of the particle in the state $|\psi_i\rangle$. It is clear that the amplitudes $a_i$ of splitting of the action-waves at the experimental set up are related to the ensemble averaged particle probabilities as $\sqrt{\rho_i} = A_i = a_i$.

If the particle is allowed to propagate without interruption by detection to the point where the action-waves are recombined, the phase difference after interference determines the subsequent dynamics, stochastically, but only at the tiny scale of $\varepsilon$.

Consider a generic symmetric interferometer for neutrons, figure 3 with the field $B$ and its gradient $B'$ in the z-direction. At the point of entry in the apparatus, the random initial phase of x-polarized neutrons results in...
the random choice of $|z+\rangle$ or $|z-\rangle$ state and the path a particular neutron actually takes. The particle propagates exactly in one of the paths and unique state ($|z+\rangle$ or $|z-\rangle$) in each event, with no superposition. Measurement before path closure results in the factual state of the particle; there is no collapse. The action-waves corresponding to both states are present in every event, accumulating the dynamical phase $\int pdx$, and the phase from the interaction $V(x)/\hbar = \mu \cdot B/\hbar$. If there are spin flippers in the paths, the actual spin flip with the energy exchange $\Delta E = 2\mu_nB$ happens only for the particle, where it is actually propagating. The action-wave changes its phase by $\pi/2$ as it passes the spin flipper, but there is no exchange of energy or ‘flip of spin’.

The initial action-waves corresponding to the possibilities of the particle states $|x+\rangle$ or $x-\rangle$ would split at the entrance port of a Stern-Gerlach interferometer as

$$|x+\rangle = \frac{1}{\sqrt{2}} |z+\rangle + \frac{1}{\sqrt{2}} |z-\rangle$$

$$|x-\rangle = \frac{1}{\sqrt{2}} |z+\rangle - \frac{1}{\sqrt{2}} |z-\rangle$$

This is very similar to a Mach-Zehnder interferometer with a polarizing beamsplitter and two orthogonal input states. However, we stress that the particle enters in only one of the path-states, $|z+\rangle$ or $|z-\rangle$.

The time evolved action-waves superpose at the exit as

$$|x'+\rangle = \frac{1}{\sqrt{2}} |z+\rangle e^{i\omega_+ t} + \frac{1}{\sqrt{2}} |z-\rangle e^{i\omega_- t}$$

The quantity $\omega_{\pm} \equiv \pm \mu_n B/\hbar$. Note that the probability for the particles to be in the $|z+\rangle$ or the $|z-\rangle$ does not change due to the phase evolution, before interference at exit. Now the probability for getting the particle in the state $|x-\rangle$ can be calculated,

$$P(x+ \rightarrow x-) = |\langle x | x'-\rangle|^2 = \frac{1}{2} \left( e^{i\omega_+ t} - e^{i\omega_- t} \right)^2 = \frac{1}{2} \left( 1 - \cos \frac{2\mu_n B t}{\hbar} \right)$$

We have reproduced the most important feature of single particle quantum interference, while avoiding the collapse of the quantum state. This is the feature that Feynman described as the ‘only mystery’ in quantum mechanics. When the particle emerges out, the resultant phase of the action-waves from both the paths determines the subsequent probability to be found in states $|x\pm\rangle$, $|y\pm\rangle$ etc. The probability for the other possibility is $P(x+ \rightarrow x+) = 1 - P(x+ \rightarrow x-)$. 

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Figure 4: The separable particle states in the situation that is conventionally treated as entangled and non-separable. The particles have definite physical states in each dynamical history. Two pairs of correlated action-waves are operative in each history and their local interferences correctly give the quantum correlation without nonlocality (section 3.4).

As other examples, a massive neutrino of one flavour is in one and only one of the three mass states and never in superposition of the three. The action-waves corresponding to the mass states superpose and interfere, resulting in oscillations in the probabilities for different flavour states. An electron neutrino can be in only one of the three mass states at origin, and remains so throughout its propagation. The interference of the action-waves changes only the probabilities for flavour states. (The details are worked out in the appendix 1, for the case of two states). This example is important in the context of the theory of gravity and quantum gravity, because RHAM makes clear that there is no superposition of different masses, energies, and long-range gravitational fields. Similarly, an atom is never in a superposition of two positions or two (or more) energy states; only the action-waves are, which carry no energy.

The reconstruction generalizes to the relativistic situation and the Dirac equation \( i\hbar \dot{\psi} = \hat{H}_D \psi \), with the same Hamiltonian action-wave equation as the basis for relativistic particle dynamics as well. In fact, the fundamental requirement that the time evolution is of first order arises from the parent action-wave equation. Later I will show how this naturally incorporates the time evolution of probability and the Born’s rule, independent of the action-wave equation of dynamics. This resolves the foundational issue of ‘zitterbewegung’, identifying it as the ensemble average, and not as a feature of single particle dynamics (see section 4.2 and appendix 2).

A multi-particle ‘entangled’ system in RHAM is equally simple. The superposition represented as \( c_1|+\rangle_1|\rangle_2 + c_2|\rangle_1|+\rangle_2 \), and conventionally interpreted as the non-separable entangled state, is applicable only to the associated action-waves (fig. 4). The joint physical state of the two particles is always a unique unentangled separable state, like \( |+\rangle_1|\rangle_2 \) or \( |\rangle_1|+\rangle_2 \). The single dynamical history of a pair of particles states is not entangled. The probabilities for measurement results in a general basis are determined by the local relative phase of the interference of the correlated pairs of action-waves.
(see section 3.4). Hence there is no collapse of the state. Nor is any nonlocality. Ensemble average gives \( |c_1| \) and \( |c_2| \) for the average particle states and determines the hybrid wavefunction; ‘entanglement’ of particle states is apparent and an ensemble statement. RHAM and the ensemble averaged RQM deconstruct the enigma of quantum entanglement, restoring the physicality and separability of the particle states. I will show explicitly (section 3.4) that the action-waves interfere locally for each particle of the pair to give all correlations correctly, after addressing the vital quantum measurement problem.

### 3.2 Quantum Measurement Problem

A quantum system that can be in the superposition \( \sum c_i |\psi_i\rangle \) is conventionally thought to be in the correlated entangled state after interaction with an apparatus with pointer states \( |P_j\rangle \),

\[
|\psi\rangle_{sA} = \sum c_k |\psi_k\rangle |P_k\rangle
\]

However, we have already seen in the example of single particles or a pair of particles that only the action-waves corresponding to the different possible physical states are in superposition, and that the matter states are unique and distinct at every event. Entanglement between the system and apparatus, or between any two systems, is an ensemble apparition. The superposition in equation (11) is applicable only to the action waves; for particle states it is in fact an ensemble averaged expression. In each trial, a particular state \( |\psi_n\rangle \) of the system, determined by the local interference of the action-waves, results in a unique correlated pointer state \( |P_n\rangle \) of the apparatus after the interaction, giving the joint product state \( |\psi_n\rangle |P_n\rangle \). The joint state after interaction at any measurement event is exactly one of the possible \( |\psi_k\rangle |P_k\rangle \). Since there is no collapse of the state of either the system or the apparatus, the quantum measurement problem is solved completely. The joint state \( |\psi_k\rangle |P_k\rangle \) changes from trial to trial. The ensemble average of such multiple measurements has the relative probability \( |c_k|^2 \). (It is this set of probabilities that decoherence models reproduce, and not what happens in single measurement events). The macroscopic apparatus is characterised by the stable action-phase of its state, because the quantum uncertainty in the phase amounting to a few \( \hbar \) is insignificant compared to the gross action of the large physical system. Otherwise, there is no difference between macroscopic and microscopic matter, in RHAM and RQM.
3.3 The Quantum-Classical Divide

The starting point of the reconstruction was the generalized Hamiltonian action-wave mechanics, with universal scope of all dynamics. Quantum mechanics is just the ensemble average of universal mechanics. Therefore, there is no classical-quantum divide in particle mechanics. For a single particle or dynamical history, universal mechanics of Hamiltonian action-wave equation is applicable. Averaged over the ensemble of histories, we get the Schrödinger equation and quantum mechanics (see section 4). The action-wave equation is more fundamental. There is no change or transition of the dynamical law when we pass from the macroscopic world to the microscopic world. Neither is any involvement of the observer and ‘consciousness’, beyond what is obvious and familiar in the macroscopic world. We already discussed how the perceived entanglement in measurement pertains to the ensemble and not to the individual act of observation. There is no chain of observation entanglements, to be eventually resolved by a conscious observer. All speculations over the role of the observer are reduced to the microscopic disturbances to the action-waves, involving a few quanta of action. The quantum phase uncontrollably changes during the exchange of a quantum of action. This explains all cases of inherent indeterminism during measurement and the effects on quantum interference, without invoking direct perturbations on the dynamical content like the momentum of the particle [14]. The action-waves of the macroscopic systems interfere over tiny spatio-temporal intervals, much smaller than atomic scales, and thus quantum interference is not effective in determining their dynamics. Hence, for macroscopic objects the Hamiltonian action wave equation \( i\hbar \frac{\partial \zeta}{\partial t} = H\zeta \) can be reduced to the approximate Hamilton-Jacobi dynamical equation \( \frac{\partial S}{\partial t} = -H \).

3.4 Entanglement and Correlations

We conclude the discussion of the elimination of foundational problems with the solution of the hard problem of the correlations of the two-particle entangled system in which the enigma of quantum mechanics appears in the most pronounced way. It is this problem that gave rise to most debates on quantum mechanics, initiated by the Einstein-Podolsky-Rosen incompleteness argument that established the incompatibility between Einstein locality and the wavefunction description of physical states in QM (see appendix 4). As well known, two-particle correlations were discussed also in the context of certain classical statistical theories with ‘hidden’ statistical variables, which were advanced to replace the theory of quantum mechanics, with an upper limit on such correlations represented by the Bell’s inequalities [15]. I have shown elsewhere that such local hidden variable theories are incompatible with the fundamental conservation laws, and hence unphysical [16][17].
any case, all debates on multi-particle correlations can rest now, in the light of the factual physical situation revealed by RHAM.

Consider a two-particle state that is correlated in momentum, but restricted to two possible values ± (two paths) in an interference experiment, as shown in Fig. 5A. The particles are always in some definite correlated physical state, either |+⟩₁|−⟩₂ or |−⟩₁|+⟩₂, in a given pair-event. There is no superposition of two-particle states. Two pairs of action-waves, ( +₁, −₁ ) and ( +₂, −₂ ), associated with the two particles are present in each event. The action-waves are correlated at the source (or interaction point) through the conservation laws [16, 17, 18]. When the dynamical quantity \( p_s \) is conserved, it reflects in the phase of the action-waves as \( \exp(i p_s x_s) \), with \( p_s = p_1 + p_2 \). The sum of the initial phases are fixed by the conservation constraint, but the individual phases are random. We will see that this directly translates to the feature that individual local measurements on each particle give random results. But their correlation visibility can be 100%, depending on the degree to which the conservation constraint \( p_s = p_1 + p_2 \) is maintained during the generation and propagation of the particles from the source.

I will now derive the two-particle correlation from entirely local interference of action-waves, retaining the independent physical states of the particles, eliminating nonseparability and nonlocality, and answering the EPR query.

Referring to Fig. 5A, the source emits the particles with opposite momenta \((p_{+1}, p_{-2})\) or \((p_{-1}, p_{+2})\). An important requirement on the source of a correlated pair of particles is that the physical nature of the source should not limit the correlation. In the case of correlation in momenta, the size of the source should be larger than \( h/\delta p \) for getting correlated particles, where \( \delta p \) signifies the degree of the lack of correlation (this requirement is present both in RHAM and conventional quantum mechanics). Thus, the point of origin \( y_s \) of the particles varies by more than \( h/\Delta p \) event to event, stochastically. Now, two pairs of correlated action-waves from \( y_s \) are present in each event, one pair for each particle; \( p_{+1} \) and \( p_{-1} \) associated with particle #1 and
$p_{+2}$ and $p_{+2}$ with particle #2. They interfere locally at the detector D1 at $y_1$ and at the detector D2 at $y_2$.

$$
|1, y_1\rangle = \frac{1}{\sqrt{2}} e^{ip_{+1}(y_1-y_2)/\hbar} + \frac{1}{\sqrt{2}} e^{ip_{-1}(y_1-y_2)/\hbar}
$$

$$
|2, y_2\rangle = \frac{1}{\sqrt{2}} e^{ip_{-2}(y_2-y_1)/\hbar} + \frac{1}{\sqrt{2}} e^{ip_{+2}(y_2-y_1)/\hbar}
$$

(12)

The action-wave $p_{+1}$ is always coupled with the $p_{-2}$ wave and the $p_{-1}$ wave with $p_{+2}$ wave, since they are correlated; there are no $(p_{+1}, p_{+2})$ or $(p_{-1}, p_{-2})$ combinations. Because of the conservation constraint, the $p_{+1}, p_{-2}$ action-waves are like a single coherent phase-wave; so are $p_{-1}, p_{+2}$. Also, the correlation requires that the transverse momenta obey $p_{+} = -p_{-}$. The detected intensity at the detector D1 of area $dS$ is $||1, y_1||^2 dS$.

$$
||1, y_1||^2 = 1 + \frac{1}{2} e^{ip_{+1}(y_1-y_2)/\hbar} e^{-ip_{-1}(y_1-y_2)/\hbar} + \frac{1}{2} e^{ip_{-1}(y_1-y_2)/\hbar} e^{-ip_{+1}(y_1-y_2)/\hbar}
$$

$$
= 1 + \frac{1}{2} e^{i(p_{+}-p_{-})(y_1-y_2)/\hbar} + \frac{1}{2} e^{-i(p_{+}-p_{-})(y_1-y_2)/\hbar} = 1 + \cos (\Delta k (y_1 - y_2))
$$

(13)

We have written $p/\hbar = k$. For particle #2, we get a similar expression $||2, y_2||^2 = 1 + \cos (\Delta k (y_2 + y_s))$. Though these are the familiar cosine forms of interference, there is no single particle interference because the source point $y_s$ is a stochastic quantity in the extended source, and $y_s \Delta k \geq 2\pi$. Thus, the average intensity at any detector position is $\langle ||1, y_1||^2 \rangle = \langle ||2, y_2||^2 \rangle = 1$, a uniform intensity.

However, for the two-particle coincidence detection in the two detectors involves simultaneous, but independent, local interference of the action-waves at location 1 and 2, as a simple product of the two amplitudes,

$$
|1, y_1\rangle|2, y_2\rangle = \left(\frac{1}{\sqrt{2}} e^{ik_{+1}(y_1-y_2)} + \frac{1}{\sqrt{2}} e^{ik_{-1}(y_1-y_2)}\right)_1 \left(\frac{1}{\sqrt{2}} e^{ik_{-2}(y_2-y_3)} + \frac{1}{\sqrt{2}} e^{ik_{+2}(y_2-y_3)}\right)_2
$$

(14)

Keeping in mind that there is no $(k_{+1}, k_{+2})$ or $(k_{-1}, k_{-2})$ combinations, we get

$$
|1, y_1\rangle|2, y_2\rangle = \frac{1}{\sqrt{2}} e^{ik_{+1}(y_1-y_2)} \frac{1}{\sqrt{2}} e^{ik_{-2}(y_2-y_3)} + \frac{1}{\sqrt{2}} e^{ik_{-1}(y_1-y_2)} \frac{1}{\sqrt{2}} e^{ik_{+2}(y_2-y_3)}
$$

(15)

This equation holds the important physical point. There is no ‘entanglement’ in the physical states of each pair of particles. Entanglement is just the correlation of the action-waves, explicit in equation (15). Particle states are separable.
The detected intensity-intensity correlation is

\[ \langle |1, y_1 \rangle |2, y_2 \rangle \rangle \langle |2, y_2 \rangle |1, y_1 \rangle \rangle = \frac{1}{2} + \frac{1}{4} e^{i \Delta k_y (y_1 - y_2)} + \frac{1}{4} e^{-i \Delta k_y (y_1 - y_2)} = \frac{1}{2} (1 + \cos (\Delta k_y (y_1 - y_2))) \]

(16)

This is an interference pattern with 100% visibility, visible by fixing one detector and scanning the other. There is no collapse of the state and there is no nonlocality. A measurement at D1 has no influence whatsoever over what is detected at D2. The two are independent local interferences and respect strict Einstein locality. However, when correlated with time stamps, perfect interferometric correlation results. Any mixture or lack of correlation in the action-waves results in the reduction of visibility, eventually dropping to 50% for the thermal state, as in the Hanbury Brown-Twiss correlation.

The correlation of the particles of a spin singlet that epitomises all the quantum mysteries is treated similarly. The only difference is that the correlation is in angular momentum (spin). When the pair of particles emerge from the source with total spin zero, they are randomly but entirely in only one of the two possible joint states \( |S_\pm \rangle = |+\rangle_1 |-\rangle_2 \) or \( |S_\mp \rangle = |-\rangle_1 |+\rangle_2 \), with no superposition. The action-waves corresponding to both states are present in every event (similar to two anti-correlated helicity states). The entangled state in conventional QM is the ensemble averaged hybrid wavefunction \( \langle |S = 0 \rangle \rangle \rangle = (|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2) / \sqrt{2} \) in any basis, as explained in the next section. This entanglement is not a statement on each pair, but over the ensemble.

For the case of spins, the initial random angular orientation \( \varphi \) in fig. 5B is like the random origin \( y_s \) in fig. 5A. The spin analyzer orientation \( \theta_i \) corresponds to a stable phase, exactly as the detector position in the example of the interference. Action-waves associated with each particle superpose and interfere, locally at analysers 1 and 2. Instead of the factor \( k_{+1} (y_1 - y_s) \) etc. in the two-path interference case earlier, here we have \( s_+ (\theta_i - \varphi) \) and \( s_- (\theta_i - \varphi) \) in the local interferences at analysers #1 and #2.

\[ |1, \theta_1 \rangle = \frac{1}{\sqrt{2}} |+\rangle_1 e^{i s_+ \theta_1} e^{-i s_+ \varphi} + \frac{1}{\sqrt{2}} |-\rangle_1 e^{i s_- \theta_1} e^{-i s_- \varphi} \]

\[ |2, \theta_2 \rangle = \frac{1}{\sqrt{2}} |+\rangle_2 e^{i s_+ \theta_2} e^{-i s_+ \varphi} + \frac{1}{\sqrt{2}} |-\rangle_2 e^{i s_- \theta_2} e^{-i s_- \varphi} \]

(17)

For independent detection, say particle #1 in the state \( |+\rangle \), with spin analyzer oriented at \( \theta_1 \) the situation is analogous to interferometer in fig. 3, but now with a random phase \( \varphi \). We get the probability
\[ P(1+, \theta_1) = |\langle 1 + |1, \theta_1 \rangle|^2 = \left( \frac{1}{2} e^{i s_+ \theta_1} e^{-i s_+ \varphi} + \frac{1}{2} e^{i s_- \theta_1} e^{-i s_- \varphi} \right)^2 \]
\[ = \frac{1}{2} + \frac{1}{2} \cos(\Delta s (\theta_1 - \varphi)) \] (18)

The probability \( P(1-, \theta_1) = 1 - P(1+, \theta_1) \). Since \( \varphi \) is stochastic in the interval \( (0, 2\pi) \), the average of \( \cos \Delta s (\varphi - \theta_1) \) is zero and there is uniform probability \( 1/2 \) for detecting \( +1 \) and \( -1 \) for the spin projections, for any setting of the apparatus.

The joint detection is with the constraint that the \( |+\rangle_1 \) action-wave is correlated with the \( |-\rangle_2 \) wave and the \( |\rangle_1 \) action-wave is with the \( |+\rangle_2 \) wave. Analogous to a double-interferometer, as in fig. 5A, there are no \( |+\rangle_1 |+\rangle_2 \) wave or \( |-\rangle_1 |-\rangle_2 \) combinations.

\[ |1, 2\rangle = \frac{1}{2} |+\rangle_1 e^{i s_+ \theta_1} e^{-i s_+ \varphi} |-\rangle_2 e^{i s_- \theta_2} e^{-i s_- \varphi} + \frac{1}{2} |-\rangle_1 e^{i s_- \theta_1} e^{-i s_- \varphi} |+\rangle_2 e^{i s_+ \theta_2} e^{-i s_+ \varphi} \] (19)

The probability for joint measurement of \( (+1, -1) \) or \( (-1, +1) \) for the spin projections with their product \(-1\) is

\[ P_{+-,-+} = |\langle 1_2+ 2_1- \rangle|^2 = \frac{1}{2} + \frac{1}{4} e^{-i(s_+ - s_-)(\theta_1 - \theta_2)} + \frac{1}{4} e^{i(s_+ - s_-)(\theta_1 - \theta_2)} \]
\[ = \frac{1}{2} (1 + \cos(\Delta s (\theta_1 - \theta_2))) = \cos^2(\theta_1 - \theta_2) / 2 \] (20)

Then the probability to get \( (+1, +1) \) or \( (-1, -1) \) for the spin projections with their product \(+1\) is

\[ P_{++,-=} = |\langle 1_2+ 2_1- \rangle|^2 = 1 - \cos^2(\theta_1 - \theta_2) / 2 = \sin^2(\theta_1 - \theta_2) / 2 \] (21)

The final spin correlation function for particles from a spin singlet \( |S = 0\rangle \) is

\[ C(\theta_1 - \theta_2) = -1 \times P_{+-,-+} + 1 \times P_{++,-=} = -\cos(\theta_1 - \theta_2) \] (22)

I have derived the singlet correlation from the local interference of the action waves of RHAM, without nonlocality and collapse of the states. The derivation did not even refer to the conventional entangled wavefunction. When \( (\theta_1 - \theta_2) = 0 \), we get perfect anti-correlation event by event for every pair of particles, irrespective of the individual setting of the analyzers or the distance between the particles. Resolution of this problem, considered the greatest quantum mystery, shows the correctness and scope of the action-wave mechanics and the reconstructed quantum mechanics. All familiar results involving bipartite and multi-particle correlations, like teleportation, happen through the action-wave correlation, while particle states remain separable and unentangled. There are no exceptions.
4 Reconstruction of Quantum Mechanics

4.1 General Description

Deepening dynamics by modifying the Hamiltonian action mechanics with the new wave equation $i\hbar \partial \zeta(x, t)/\partial t = H \zeta$ enabled the complete elimination of all the foundational problems that plagued quantum mechanics. It remains to show that the Schrödinger equation and quantum mechanics emerge naturally and entirely from the ensemble average of the dynamical histories of Hamiltonian action-wave mechanics. There is no difference in the dynamical equation between macroscopic and microscopic dynamics. There is a fundamental uncertainty in all dynamics, because the action is specified only up to the fundamental scale, with the uncertainty $\Delta \int p_i dx^i = \hbar$. Therefore, the equation $\nabla S(x, t) = p$ also is an approximation, ignoring the fundamental uncertainty due to the quantum of action $\hbar$. Classical mechanics, or mechanics in its core form, is not deterministic, contrary to the generally held view over centuries. The momentum of the particle is related to the action-wave, $p \zeta = -i\varepsilon \nabla \zeta$. Therefore, the dynamics of one particle differs from another identical one by such a difference in the action. This irreducible stochasticity manifests significantly only at microscopic scale, in the dynamics of fundamental particles, atoms and light.

When we come to microscopic dynamics, the dephasing term in the time evolution of action, $\partial S/\partial t = i\hbar \nabla^2 S/2m$, becomes significant and the dynamical histories vary considerably from particle to particle. Then, we have no choice, but to describe dynamics in a statistical framework, ensemble averaged. Averaging over the particle dynamics leads to probability densities $\rho_i$ for different physical states $|s_i\rangle$. Since the action-wave amplitudes in different possibilities obey $a_i^2 = \rho_i$, the ensemble average of the action-wave equation has the same basic form, $i\hbar \partial f(x, t)/\partial t = H f(x, t)$, but now with a new ensemble averaged function $f(x, t) = \sqrt{\rho(x, t)} \langle \zeta(x, t) \rangle$. The Schrödinger equation describes the time evolution of the hybrid ensemble function $f(x, t)$.

The discussion of foundational problems in quantum mechanics in the past were all focussed on the analysis of the Schrödinger equation, because it was assumed that it was the microscopic fundamental equation from which macroscopic mechanics would emerge as an effective theory. This expectation was not realised, and the reason is clear now. The Hamiltonian action-wave equation is the fundamental equation for all dynamics and the Schrödinger equation is the result of its ensemble averaging.

4.2 Details of Reconstruction

Once we are in the domain of statistical description of ensemble averaged dynamics, which is the only way to describe dynamics in the microscopic world,
we need to consider the time evolution of the probability density explicitly, along with the time evolution of dynamical quantities. I will show now that the ensemble equation in which both are integrated is the Schrödinger equation.

The classical conservation (continuity) equation for the probability density is

$$\frac{\partial \rho(x, t)}{\partial t} = -\nabla \cdot (\rho v) = -\rho \nabla \cdot v - v \cdot \nabla \rho \quad (23)$$

Thus, dynamics enters the continuity equation through the particle velocities. With the modified action-wave equation, this continuity equation is not complete because of the dephasing term \((i\hbar \nabla^2 S/2m)\) in the action-wave equation contributes an additional microscopic term related to \(\nabla^2 S = \nabla \cdot v\).

We have already seen how dephasing causes a dissipationless ‘diffusion’ of the amplitude of the ensemble of action-waves (refer to figure 2). Therefore, we get the corresponding diffusion equation for the amplitude of the action-waves, with the same imaginary diffusion constant,

$$\frac{\partial a(x, t)}{\partial t} = \frac{i\hbar}{2m} \nabla^2 a \quad (24)$$

Now we note the important connection between the amplitude \(a_i\) of the action-waves and the ensemble averaged probabilities of particle dynamics, \(\sqrt{\rho_i} = A_i = a_i\), discussed in section 3.1.

Since the positive real \(\rho(x, t) = A^2(x, t)\), the continuity equation (23) for \(A(x, t)\) is completed by adding the term in equation (24)

$$\frac{\partial A(x, t)}{\partial t} = -\frac{A}{2} \nabla \cdot v - v \cdot \nabla A \rightarrow -\frac{A}{2m} \nabla^2 S - \frac{1}{m} \nabla S \cdot \nabla A + \frac{i\hbar \chi}{2mA} \nabla^2 A \quad (25)$$

We can recast this equation of constraint as a ‘wave equation’ by defining the pseudo-wave \(\chi(x, t) = \langle \rho^{1/2} \rangle \exp(i\phi) \equiv A \exp(i\phi)\). The mathematical independence of \(\rho\) from the dummy phase \(\phi\) is ensured with \(\rho = A^2 = \chi \chi^*\). This is the exact Born’s rule. We stress that it is universally applicable, for dynamics at all scales.

The appropriate equation for the function \(\chi(x, t)\) that contains all the terms in the equation for \(\partial A/\partial t\) is

$$\partial \chi/\partial t = i\chi \frac{\partial \phi}{\partial t} + \frac{\chi}{A} \frac{\partial A}{\partial t} = i\chi \frac{\partial \phi}{\partial t} - \frac{\chi}{2m} \nabla^2 S - \frac{\chi}{Am} \nabla S \cdot \nabla A + \frac{i\hbar \chi}{2mA} \nabla^2 A \quad (26)$$

The time evolution \(\partial A/\partial t\) is represented by the last three terms in the evolution equation for \(\partial \chi/\partial t\). The first term concerns the evolution of the dummy phase of the complex function representing \(\sqrt{\rho}\). Up to this point, the discussion was general and no aspect of dynamics was specifically involved, except
expressing the velocity in term of the gradient of the action-wave in the en-
semble continuity equation. In particular, \( \phi \) has no connection to dynamics
or probability. Now we take the step that takes us directly to the theory of
quantum mechanics.

The term \( \partial \phi/\partial t \) in \( \partial \chi/\partial t \) is redundant as it stands because \( \phi \) is a dummy
phase with no relation to dynamics. It is striking that the ensemble of particle
dynamics and the continuity of its probability density can be combined if
we elevate the dummy quantity \( \phi \) to be the action \( S(x,t)/\hbar \) in a hybrid
function \( \psi(x,t) = A(x,t)\zeta(x,t) \). Both \( A(x,t) \) and \( \zeta(x,t) \) are averaged over
the ensemble of dynamical histories. For dynamics starting from the same
physical state, the action-waves remain the same, but the particle dynamics
is stochastic, as I discussed in section 3.

The new hybrid ‘wavefunction’ is \( \psi(x,t) = A(x,t) \exp(iS(x,t)/\hbar) \). Then

\[
\frac{i\hbar \partial \psi}{\partial t} = -\frac{\partial S}{\partial t} + \frac{i\hbar \partial A}{A \partial t} = -\frac{\partial S}{\partial t} - \frac{i\hbar \psi}{2m} \nabla^2 S - \frac{i\hbar \psi}{Am} \nabla S \cdot \nabla A - \frac{\hbar^2 \psi}{2m A} \nabla^2 A
\]  

Here, \( \bar{A} \equiv A = \sqrt{\rho} \) is an ensemble averaged quantity and \( S(x,t)/\hbar \) is the
phase of the action-wave of single particle dynamics. We see clearly how
the ensemble average of the time evolution equation of the action-wave over
dynamical histories naturally leads to the time evolution equation for the
amplitude of the action-waves, and through that the time evolution of prob-
abilities.

Thus, we fuse the action-wave and the ensemble averaged probability
density (its square root) into a new abstract wave-function, representing the
ensemble average of many dynamical histories with the same action-waves.
This has no existence in space and time due to the fact that the real amplitude
\( \bar{A}(x,t) \) is an ensemble averaged quantity. Thus, the wavefunction is not the
action-wave. Nor is it a probability wave. Since there are only action-waves in
reality and not matter-waves, wavefunction is not a matter-wave either. The
wavefunction \( \psi(x,t) \) an abstract ensemble entity with no direct ontological
significance. Its mathematical properties are determined by the way it is
synthesised; it is complex valued (action-wave) and square integrable (finite
probabilities).

It is remarkable that the fundamental Hamiltonian action-wave equation
(eq. 5) with the non-relativistic Hamiltonian, if extended to the hybrid wave-
function \( \psi(x,t) \), contains both the dynamical content for the time evolution
of the action \( S(x,t) \) and all the terms in the conservation and diffusion equa-
tion for the probability amplitude \( A(x,t) \).

\[
\frac{i\hbar \psi}{\partial t} = H\psi(x,t) = H[A(x,t)\zeta(x,t)]
\]  

The connection between \( i\hbar \partial \psi/\partial t \) on the left and the dynamical quanti-
ties is given by equation (27). The first term represents the single system dynamics (action-wave equation) and the matching Hamiltonian term on the right should be the kinetic term $p^2/2m$, which does not contain the quantum signature $\hbar$. The dephasing term in the action-wave equation is not observable, because the action-waves are not directly observable. Dephasing manifests through the diffusion of the probability amplitude, as depicted in the fig. 2. What is remarkable is that all the three ensemble terms signifying the macroscopic and microscopic stochasticity are also contained within this Hamiltonian.

$$\nabla^2 \psi = \nabla \cdot \left( A \frac{i}{\hbar} e^{iS/\hbar} \nabla S + e^{iS/\hbar} \nabla A \right) = \nabla \cdot \left( \frac{i}{\hbar} \psi S + \frac{\psi}{A} \nabla A \right)$$

(29)

$$= -\frac{1}{\hbar^2} \psi (\nabla S)^2 + \frac{i}{\hbar} \psi \nabla^2 S + \frac{i}{\hbar A} \nabla A \cdot \nabla S + \frac{\psi}{A} \nabla^2 A + \frac{i}{\hbar A} \nabla A \cdot \nabla S$$

(30)

Therefore, we get the matching terms in

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = \frac{\psi}{2m} (\nabla S)^2 + \frac{-i\hbar}{2m} \psi \nabla^2 S + \frac{-i\hbar}{m} \psi \nabla S \cdot \nabla A + \frac{-\hbar^2}{2m} \frac{\psi}{A} \nabla^2 A$$

(31)

The first term is the particle momentum. The last three terms concern the ensemble averaged time evolution, in which the third contains the intrinsic uncertainty related to the quantum of action. For a given single event, the particle state is well defined within the small uncertainty with the scale $\hbar$. The characteristic feature of quantum dynamics is the Heisenberg uncertainty, encoded as the uncertainty in the phase $S/\hbar$ as $\delta S \simeq \hbar$. This manifests in the scatter in $p$ and $x$ in the statistical ensemble of particle events as $\Delta p \Delta x \simeq \hbar$. When many histories of particle dynamics are averaged, the description is in terms of the probability density, characterized by $A(x, t)$ and its derivatives.

Combining all the terms we arrive at the equation for the time evolution of the ensemble averaged hybrid quantity $\psi$,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi$$

(32)

This is the Schrödinger equation, which generalizes to the universal equation for particle quantum mechanics $i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$, but with a radically new inner structure and interpretation. In this bottom up construction, we have discovered that the Schrödinger equation is an ensemble equation. It is not an equation for matter waves or probability waves. When an ensemble of Hamiltonian action-wave dynamics is averaged, we get the Schrödinger equation.
Hence, quantum mechanics is derived from the universal mechanics described by Hamiltonian action-wave mechanics.

Though Schrödinger was inspired by Hamilton’s action mechanics [19], since he developed the wave equation for de Broglie’s matter-waves, it was assumed that the equation dealt with single particles or single dynamical histories. The severe problems of interpretation surfaced after the equation became successful in its statistical predictions. The reconstruction achieved by combining the ensemble averaged probability density of particle dynamics and the action-waves shows the factual situation.

"It is now clear that the particle is always in a unique physical state, as a single whole, and not in superposition of states. What is superposed in single particle dynamics is the action-waves associated with the particle. The action-waves associated with the particle states $|s_i\rangle$ are present with every realization of the evolution in the ensemble, causing superposition and interference. $|\psi\rangle = \langle \rho \rangle^{1/2} |s\rangle \exp(iS/\hbar)$ is the entire ensemble representation of the physical state. This wavefunction is a mathematical entity synthesised from the ensemble of dynamical histories and it is not a physical entity.

It may be easily verified that the ensemble averaged hybrid wavefunction with the action-wave content is consistent with the basic axioms of quantum mechanics. It belongs to the Hilbert space and it obeys exact Born’s rule by construction. However, being ensemble averaged, there is no ‘collapse of the wavefunction’ during individual measurements. The connections between the Heisenberg formulation of quantum mechanics, Schrödinger mechanics, the Hamilton’s equations for dynamics, and the general time evolution in terms of Poisson brackets etc. are more transparent in terms of the new action-wave mechanics. These relations are ensemble relations. In fact, the very situation that these two equivalent formulations of QM were possible was due to the true ensemble nature of the Schrödinger equation. Some of these formal aspects will be explored in another paper.

4.3 Summary Comments on RQM

Quantum mechanics is really the fusion of two fundamental features: stochastic particle dynamics and event by event quantum interference of action-waves. Thus, Schrödinger equation is a hybrid evolution equation, obtained as the ensemble average of the Hamiltonian action-wave equation. This is the crux of reconstructed quantum mechanics (RQM). All mechanics is included in the Hamiltonian action-wave equation, and when averaged over many dynamics histories we get the Schrödinger equation, which has an ensemble averaged ‘wavefunction’ instead of the action-wave. The ensemble average of stochastic particle dynamics solely determines the real positive amplitude and the action waves are not involved. This gives exact Born’s rule for the
Figure 6: Matter dynamics and action-wave interference in RQM: statistical results of measurements as well as quantum interference are correctly reproduced, without collapse and other foundational issues. A) Conventional QM. B) RQM: Particle dynamics has two distinct possibilities inside the interferometer that do not superpose, and two at the exit (black arrows). The probabilities at the exit port are determined by the relative phase of the two interfering action-waves (broken arrows), which are present in every event.

probability density. The relative phase of all interfering action-waves determines the probability for particle dynamics at the location of interference. The particle is in a definite physical state at every instant and does not participate in superposition. Both the particle and its action-waves exist in real space and time and the hybrid wavefunction is in the Hilbert space of QM. I stress that the action-waves do not carry or exchange energy or momentum. The foundational problems were already eliminated with the Hamiltonian action-wave dynamics, and RQM is free of such birth defects. An example of a matter interferometer in RQM and its comparison with standard quantum mechanics are depicted in figure 6, as illustration.

We note in passing that the failure of attempts like the de Broglie-Bohm (dBB) formulation to deal with the foundational problems can be traced to treating the Schrödinger equation as the fundamental equation for single particle dynamics and the basis for interpretation, whereas factually it is the equation for ensemble averaged dynamics. The unphysical features, apart from nonlocality, of the dBB theory can be traced to misinterpreting the ensemble term \((h^2/2mA) \nabla^2 A\) as a quantum potential acting on single particle dynamics. In fact, such attempts took us farther from the right path because the dBB approach was trying to restore determinism to quantum mechanics, with position as hidden variable and the momentum through the action equation \(\partial S/\partial x = p\), both specified, whereas the complete solution was in the action-wave dynamics with universal indeterminism. More details may be found in appendix 3.
4.4 Indeterminism as a Fundamental Feature

We saw in the road to the reconstruction that there is inherent indeterminism in mechanics because particle dynamics is governed by the action-wave and the fundamental (minimum) scale of action in the action-wave, $\varepsilon = \hbar$. This results in the dephasing of the action-waves associated with particle mechanics. Such indeterminism is not specific to quantum mechanics; it is part of universal mechanics, including classical (macroscopic) mechanics, independent of the physical scales and the form of Hamiltonian. Dephasing of action-waves manifests as the (dissipation-less) diffusion of the probability density. All dynamics is indeterministic, at the scale of action $\hbar$. That is the fundamental lesson from the reconstruction. When $\varepsilon \nabla^2 S \ll (\nabla S)^2$, dynamics according to the approximate Hamilton-Jacobi equation cannot be distinguished from the dynamics of the complete Hamiltonian action-wave equation. Then the small statistical spread in the different dynamical histories cannot be discerned. Similar condition is used for the WKB approximation in quantum mechanics,

$$\left( \frac{\hbar}{p^2} \frac{\partial p}{\partial x} \right) \ll 1$$

Here, we see clearly that the celebrated Hamilton-Jacobi equation and the dynamics it represents (including Newton’s equation of motion) are approximations to a more complete classical mechanics described by the action-wave equation. The determinism in classical mechanics is only apparent, and not fundamental.

Since the wavefunction is the result of ensemble average of the histories particle dynamics and the action-waves, I have cleared the mystery why the Schrödinger equation is a differential equation for the probabilities of observables, while accurate on the interference and correlation effects. The long held hope and search for a deterministic dynamics turn out to be a mirage-chase. *Indeterminism is not a foundational problem; it is a foundational feature.*

4.5 Quantum Information

The reconstructed mechanics makes it clear that a single quantum system, like a two-level atom, in factually in only one of the states at any instant, and not in a superposition of the two states. What are superposable are the action-waves corresponding to the states. Thus, the physical aspect of a quantum-bit is very different in RQM, compared to conventional QM. All interference effects that enable quantum information processing and quantum computing are traceable to the action-waves. As stated before, entanglement of particle states is an ensemble statement. The superposition represented
as $c_1|+\rangle_1|-\rangle_2 + c_2|-\rangle_1|+\rangle_2$ is applicable only to the associated action-waves. Quantum information processing then deals with the physical states of matter and their evolution, as described by the action-wave dynamics; it does not have any special role or physical priority in the understanding of matter and its dynamics.

4.6 Fundamental Quantum Scales

The reconstruction of mechanics that I outlined has one unique scale or quantum of action, which is empirically determined as $\hbar$. There is no additional quantization scales associated with the dynamical quantities like energy and momentum, or with the coordinate variables like position. Even the fundamental quantization of spin is in fact just the quantization of action arising in the ‘looped’ nature of the action-waves. This is significant when we consider speculations based on synthesized scales like the Planck quantities. In RQM, such scales are not expected to be fundamental. RQM predicts that there is no quantum to classical transition associated with the scale of Planck mass or any macroscopic mass scales. Since there is no quantum measurement problem and the notion of the collapse of the wavefunction in RQM, speculations of new physics in quantum mechanics at some such scale associated with mass or gravity are rendered redundant. In particular, the speculations on spontaneous localization of the wavefunction are no more relevant, because the wavefunction is a hybrid mathematical entity and not a physical entity in space and time.

The other Planck scales that are frequently mentioned are the Planck length and Planck time, both being in the extreme microscopic compared to what is practically accessible. RQM based on the single scale of action does not expect new quantum effects at these scales.

The de Broglie relations for the hypothetical matter waves were based on applying the optical wave relations to the relativistic energy of a particle. Then, $E = h\omega$ and $p = h\kappa = h/\lambda$. The phase velocity of de Broglie’s matter waves is $v_p = E/p = c^2/v$, where $v$ is the velocity of the particle. The group velocity $d\omega/dk$ coincides with the particle velocity. The action-wave relations in the modified Hamiltonian action dynamics are different, and physically more reasonable. The action relations are

$$\frac{\partial S}{\partial t} = -H, \quad \frac{\partial S}{\partial x} = p$$

Both have the additional terms arising due to the fundamental scale of action $\hbar$. This defines a time and length scales for the action-waves $\delta t_0 = h/H$ and $\delta x_0 = h/p$. The action has multiple contributions including those unrelated to motion. These terms must be subtracted to get the terms related to true
motion, to find the ratio $E/p$. The action contribution from the rest mass-energy is one such. Thus, both the phase velocity and group velocity are below $c$ in RHAM.

4.7 Divergence of Zero-Point Energy

A serious problem with quantum mechanics arises when we consider multi-particle wavefunctions and its limit in a quantum field theory, like quantum electrodynamics or even quantum optics. Since the conventional ‘waves’ in quantum mechanics (associated with real photons) have energy and momentum, even the wave modes of the electromagnetic vacuum have the zero-point energy of $\hbar\omega/2$ per wave mode of each polarisation. This results in divergent energy density in space, in clear contradiction with the observed slow dynamics of the universe, accurately determined in the Hubble parameter. This problem of the very large ‘cosmological constant’ and such divergences in quantum fields are solved in the modified Hamiltonian action dynamics because the waves in reality are the action-waves that do not have energy or momentum. The particle states represent a finite number of particles with a finite zero-point energy, which is in fact negligible. This is a significant conceptual advance with further implications to quantum field theories and the interpretation of second quantization. In particular, there are no vacuum modes with zero-point energy in quantum optics based on RQM. This is well-supported by recent homodyne experiments done in our laboratory in a configuration that avoided the conventional symmetric beam splitter in the balanced homodyne set up [20].

4.8 General Relativity and Action-Wave Mechanics

The perceived incompatibility between the general theory of relativity (GTR) and quantum mechanics is well known [21]. This is a serious impediment to completing the quantum theory of gravitational phenomena. The new action-wave mechanics changes this grim scenario positively. First of all, Schrödinger equation is not the equation for each dynamical history; it is an ensemble equation. Therefore, the incompatibility of the two theories is not determined by the Schrödinger equation. Since I have shown that the Hamiltonian action mechanics has the universal irreducible uncertainty, general relativity that obeys the principle of stationary action will not be an exception. GTR will necessarily inherit this microscopic uncertainty in action, related to the fundamental scale $\hbar$, in a suitable modification. The variational principle is applicable to gravitational physics through the geometric action that contains the Ricci scalar curvature $R$ and the determinant.
of metric $g_{\alpha\beta}$,

$$S_{GR} = \frac{c^4}{16\pi G} \int R\sqrt{-g}d^4x$$  \hspace{1cm} (35)

With the modification to Hamiltonian mechanics, the time evolution of this action is modified as well, with the additional microscopic term involving $\hbar$. The major conceptual change is that matter-energy and its gravitational fields corresponding to different physical states do not superpose. This is applicable irrespective of whether the particle is treated as an extended entity or not. The action-waves that superpose do not carry potentially divergent energy-momentum or its gravity. With the seemingly impenetrable wall between the ‘classical’ and the quantum world dissolved, and the quantum features of superposition confined to the action-waves, a quantum theory of gravity looks more feasible than ever [22].

But this needs conceptual revisions at two different fronts. One is to reconstruct general relativity to incorporate the action-wave basis, as for any classical dynamical theory that respects the principle of stationary action. This will integrate the irreducible uncertainty of the quantum of action with general relativity, already providing the universal equation for single dynamical histories. Due to the tiny magnitude of the quantum of action, none of the empirical results of GTR would be affected. We have seen that the matter states and the states of gravitational fields do not form superpositions; only related action-waves can be in superposition. Hence, there is no more the speculated superposition of ‘geometries’. Also, the Schrödinger equation deals with the ensemble dynamics, and not single quantum histories. This new feature needs to be formally incorporated.

The second front involves moving closer to true physical dynamical variables rather than working in an effective geometric description. In electromagnetism, the effect of the sources (charges and their currents) is described in terms of their potential fields. For gravity, it is the effect of such potential fields on spatial and temporal intervals that are taken as the primary dynamical variable. (Electromagnetism does not affect neutral matter, whereas gravity affects all matter with mass-energy, with ‘equivalence’). As a consequence, the gravitational potentials and the space-time intervals between matter states are identified. However, the metric-references are invariably constructed out of matter. There is no clock or physical time without matter. Also, there is only one matter-energy filled universe where all physical phenomena are actualized. It is easy to prove, with both logical arguments and several experimental results, that such a universe has the role of a master determining frame [23]. Then GTR itself and Einstein’s equations need a crucial modification to make the theory consistent with its singular relevance.
in this universe \[24\]. This ‘Centenary Einstein’s Equations’ (CEE) is

\[ R_{ik} - \frac{1}{2} g_{ik} R - \frac{8 \pi G}{c^4} U_{ik} = \frac{8 \pi G}{c^4} T_{ik} \] (36)

The energy-momentum tensor of the matter-energy in the universe, \( U_{ik} \), is the integral part of this equation. Therefore, the possibility of the vacuum Einstein’s equations has disappeared. Apart from being in complete agreement with all experimental tests and observations, this equation sacrifices general coordinate invariance to gain many new predictions \[23, 25\], consistent with the fact that all observable gravitational phenomena unfolds in this one, and only one, universe.

I believe that the road to a theory of quantum gravity is straighter and clearer with Hamiltonian action-wave mechanics, RQM, and CEE as the basis.

**Concluding remarks**

I have reconstructed and generalized Hamiltonian action mechanics incorporating the action-waves that are responsible for the universal validity of the principle of action in a new action-wave equation. This is the universal equation for single dynamical histories and it contains the microscopic irreducible uncertainty arising from a fundamental quantum of action. This key step solves all vexing foundational and conceptual problems of quantum mechanics. The theory of quantum mechanics based on the Schrödinger equation turns out to be an emergent theory that results from ensemble averaging the action-wave dynamics. The wavefunction of the Schrödinger equation is a hybrid entity encoding the ensemble averaged stochastic dynamics of particles with distinct physical states and the action-waves that accumulate the dynamical phases in all possibilities, event by event. There are no matter-waves. The new wave-particle connection, rather than duality, in the action-wave mechanics eliminates the collapse of the wavefunction, the measurement problem, and the quantum-classical divide. It restores separable independent particle states in quantum entanglement and eliminates nonlocality. The reconstruction extends to relativistic QM, quantum optics, and the Dirac equation, enabling the resolution of some additional foundational issues. With the action-waves as phase carriers, the divergent energy that is usually associated with the quantum vacuum modes is no more troublesome. It is hoped that RHAM and the emergent RQM will result in the definite closure of the philosophical debates anchored on the conceptual issues of QM \[26\]. RHAM makes it clear that matter-energy states and their long-range fields do not superpose. The entity in superposition, the action-waves, do not have energy-momentum, nor associated fields. In particular,
the ‘classical’ theory of gravity, which respects the action principle, will need
the same Hamiltonian action-wave modification and the ensemble average
of such a theory will be important for the theory of quantum gravity. We
note that there is no physical sense in the ensemble average over dynamical
histories when there is only a single realisation of a material system, like
the universe. Therefore, RHAM will rechart the conceptual trajectory in the
fields of quantum gravity and quantum cosmology.

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Appendix 1: Massive Neutrinos

The discovery that neutrino flavours show quantum interference effects, by
which the probability to detect them in any particular flavour ‘oscillates’, is
explained in conventional quantum mechanics by representing each flavour
state ($|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$) as a linear superposition of three mass eigenstates
$(m_1, m_2, m_3)$. Thus, the physical picture of a neutrino that is emitted in a
nuclear reaction is as a particle propagating as a superposition of three mass
states. This implies that each neutrino propagates with a superposition of
3 distinct gravitational fields. Modified action-wave mechanics and RQM
clarifies that the quantum mechanics of individual neutrinos is factually very
different from this picture. Every neutrino, irrespective of its flavour, is in
one of the mass eigenstates and not in a superposition. This (random) state
is retained from emission to detection. This also implies that the gravita-
tional field of a particular neutrino corresponds to only one of the possible
masses, and not a superposition of three possibilities of fields or geometries.
The action-waves corresponding to the three mass-energy states copropagate
with the neutrino and generate all interference effects. When observed, the
probability to be in each flavour is determined by the local relative phase of
the action-waves, exactly similar to the case of an interferometer. I illustrate
the ‘neutrino oscillation’ phenomena in RQM, with the simpler case of a two
neutrino flavours ($|\nu_e\rangle, |\nu_\mu\rangle$) and two mass states ($m_1, m_2$).

Consider a neutrino generated as an electron neutrino $\nu_e$, which has two
possible mass states $m_1$ and $m_2$. The neutrino particle is randomly in one
and only one of these mass states, say $m_2$ and not in a superposition of
states. The action-waves associated with $m_1$ and $m_2$ are $|1\rangle \exp(iEt/h)$ and
\[ |\nu(t)\rangle = |1\rangle c_{m1}e^{i\omega_1t} + |2\rangle c_{m2}e^{i\omega_2t} \]  

\(|c_{m1}|^2 + |c_{m2}|^2 = 1 \text{ and } \omega_i = E_i/\hbar. \]  

The probability to remain an electron neutrino when observed at time \( t \) is

\[ P_{ee} = |\langle \nu_e | \nu(t) \rangle|^2 = |c_{m1}|^2 e^{i\omega_1t} + |c_{m2}|^2 e^{i\omega_2t}|^2 \]

\[ = |c_{m1}|^4 + |c_{m2}|^4 + |c_{m1}|^2 |c_{m2}|^2 (e^{i(\omega_2 - \omega_1)t} + e^{-i(\omega_2 - \omega_1)t}) \]

\[ = 1 - 2 |c_{m1}|^2 |c_{m2}|^2 + 2 |c_{m1}|^2 |c_{m2}|^2 \cos(\omega_2 - \omega_1) t \]

\[ = 1 - 4 |c_{m1}|^2 |c_{m2}|^2 \sin^2(\omega_2 - \omega_1) t/2 \]  

The probability of the electron neutrino to be detected as a muon neutrino is

\[ P_{e\mu} = |\langle \nu_\mu | \nu(t) \rangle|^2 = 1 - |\langle \nu_e | \nu(t) \rangle|^2 = 4 |c_{m1}|^2 |c_{m2}|^2 \sin^2(\omega_2 - \omega_1) t/2 \]

Since

\[ E = (p^2 c^2 + m^2 c^4)^{1/2} = pc \left(1 + \frac{m^2 c^4}{2p^2 c^2}\right) \approx E + \frac{m^2 c^4}{2E} \]

We have \((\omega_2 - \omega_1) t = \frac{c^3}{2\hbar E} (m_2^2 - m_1^2) \frac{L}{c} = \frac{\beta^2}{2\hbar E} \Delta m^2 L\). If \(c_{m1} = \cos \theta\) and \(c_{m2} = \sin \theta\), as conventionally coded, then

\[ |\langle \nu_\mu | \nu(t) \rangle|^2 = 4 \sin^2 \theta \cos^2 \theta \sin^2 \left(\frac{c^3}{4\hbar E} \Delta m^2 L\right) = \sin^2 2\theta \theta^2 \sin^2 \left(\frac{c^3}{4\hbar E} \Delta m^2 L\right) \]

We see that the particular neutrino particle that started out as an electron neutrino remained in a single mass state (with mass \(m_2\) here) throughout its propagation. The interference of action-waves corresponding to the two mass states determined the probability of detecting the neutrino in either of the flavour states at time \(t\) after emission. Another electron neutrino might be in mass state \(m_1\) or \(m_2\), randomly and with probabilities \(p_1\) and \(p_2\). In RQM, \(|c_{m1}| = \sqrt{p_1}\) and \(|c_{m2}| = \sqrt{p_2}\).

**Appendix 2: The Dirac Equation**

Quantum mechanics reconstructed from the universal action-wave mechanics naturally extends to more general Hamiltonians. The dynamical equation of
the particle is always given by the time evolution of the action wave $\zeta(x,t) = \exp(iS/\hbar)$,

$$ih\frac{\partial \zeta}{\partial t} = H\zeta$$

(44)

Therefore, the extension to the ensemble averaged wavefunction is a first order time evolution equation, for any Hamiltonian.

Consider a non-relativistic example of a spin-1/2 particle with a magnetic moment. The magnetic energy in an applied field $B$ is $E_m = \mu_0 s \cdot B$, where the spin $s$ is two-valued. We have the two-valued energy

$$E_m = \pm \mu |B| = \pm \frac{\mu_0}{2} \left( B_x^2 + B_y^2 + B_z^2 \right)^{1/2}$$

(45)

As well known in the context of the Pauli equation (generalized from the Schrödinger equation), $ih\frac{\partial \psi}{\partial t} = H_{ij}\psi_j$, the linear Hamiltonian corresponding to this is

$$H = \mu_0 \sigma \cdot B = \mu_0 \left( \sigma_x B_x + \sigma_y B_y + \sigma_z B_z \right)$$

(46)

where $\sigma_i$ are the Pauli $2 \times 2$ matrices. Explicitly written,

$$H_{ij}\psi_j = \mu_0 \left[ \begin{array}{cccc} B_z & B_x - iB_y & 0 & 0 \\ B_x + iB_y & -B_z & 0 & 0 \\ \mu_0 B_z & \mu_0(B_x - iB) & -\hbar\Omega & 0 \\ \mu_0(B_x + iB) & -\mu_0B_z & 0 & -\hbar\Omega \end{array} \right] \left[ \begin{array}{c} \psi_u \\ \psi_d \end{array} \right]$$

(47)

We are also familiar with the common physical situation in which a particle in a symmetric double well potential has the tunneling probability characterized by a frequency $\Omega$ and the energy scale $\hbar\Omega$. The bare energy levels split into a doublet $\pm \hbar\Omega$. Then we have

$$H_{mn}\psi_n = \hbar \left[ \begin{array}{cc} \Omega & 0 \\ 0 & -\Omega \end{array} \right] \left[ \begin{array}{c} \psi_+ \\ \psi_- \end{array} \right]$$

(48)

Now I consider the spin-1/2 particle in a magnetic field, in the double well potential with tunneling probability characterized by $\hbar\Omega$. The energy levels split such that the total energy is

$$E_\pm = \pm \left( \hbar^2\Omega^2 + \mu^2 B_x^2 + \mu^2 B_y^2 + \mu^2 B_z^2 \right)^{1/2}$$

(49)

Because of the independent two-valuedness of the two physical situations, and the linearity of action-wave mechanics, the Hamiltonian now have to be 4-dimensional, with a 4-component wavefunction. We can write it by inspection, to get the correct eigenvalues of the energy $E_\pm$. We get

$$H_{ij}\psi_j = \left[ \begin{array}{cccc} \hbar\Omega & 0 & \mu_0 B_z & \mu_0(B_x - iB) \\ 0 & \hbar\Omega & \mu_0(B_x + iB) & -\mu_0B_z \\ \mu_0 B_z & \mu_0(B_x - iB) & -\hbar\Omega & 0 \\ \mu_0(B_x + iB) & -\mu_0B_z & 0 & -\hbar\Omega \end{array} \right] \left[ \begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{array} \right]$$

(50)
We cannot get the right energy expression with a $2 \times 2$ Hamiltonian because we need to maintain the two-valuedness of energy for any linear combination of the spin projections. We can arrive at this Hamiltonian more formally by demanding that the total energy should match the expression for the linear Hamiltonian. Extending the Pauli Hamiltonian to include the tunneling, we get

$$H = \mu \mathbf{\alpha} \cdot \mathbf{B} + \beta \hbar \Omega$$

(51)

where the matrices $\alpha$ are similar to the Pauli matrices and the matrix $\beta$ incorporates the tunneling term. The total energy should match the expression for the Hamiltonian,

$$\hbar^2 \Omega^2 + \mu^2 \left( B_x^2 + B_y^2 + B_z^2 \right) = (\mu \mathbf{\alpha} \cdot \mathbf{B} + \beta \hbar \Omega) \cdot (\mu \mathbf{\alpha} \cdot \mathbf{B} + \beta \hbar \Omega)$$

(52)

This determines the the matrices $\alpha$ and $\beta$. We get

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 2 \delta_{ij}$$

(53)

$$\beta^2 = 1; \quad \alpha_i \beta + \beta \alpha_i = 0$$

(54)

The anti-commuting nature originates in the requirement that the energy has two independent contributions, without cross terms. *This non-relativistic Hamiltonian has the same structure as the Dirac Hamiltonian and the matrices $\alpha_i$ and $\beta$ are the Dirac matrices.* We see at once that it is Hamiltonian action-wave dynamics and not relativity that determines the structure and form of the Dirac equation, even though, historically, relativity was the motivating force. The equation for dynamics of the particle remains identical in all cases,

$$i \hbar \frac{\partial \zeta}{\partial t} = H \zeta \rightarrow i \hbar \frac{\partial \psi}{\partial t} = H \psi$$

(55)

Only the form of the Hamiltonian changes.

The particle in RQM tunnels back and forth, stochastically, without being in a superposition. The probability is determined by the relative phase of the action-waves, as usual. When $B = 0$, the action-waves corresponding to the two energy states are $\exp(i\Omega t)$ and $\exp(-i\Omega t)$. The ensemble averaged symmetric wavefunction with its amplitude as $A = \sqrt{\rho}$ is

$$\psi(x,t) = \frac{1}{\sqrt{2}} (\chi_+ \exp(i\Omega t) + \chi_- \exp(-i\Omega t))$$

(56)

As I demonstrated for other examples, the particle is not in a superposition of two states; only the action-waves are.

The expression for the total energy in equation (49) reduces to

$$E_\pm = \pm \hbar \Omega \left( 1 + \frac{\mu^2 B^2}{2\hbar \Omega} \right)$$

(57)
in the limit of small magnetic field, $|\mu B| \ll \hbar \Omega$. Similarity to the expression for non-relativistic kinetic energy $p^2/2m$ is evident. One may check that the solutions split into a large component $\psi_L$ and a small component $\psi_S \sim \frac{\mu^2 B^2}{2m} \psi_L$, as it happens in the case of the Dirac equation.

The Dirac equation for the relativistic electron is

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

where the Hamiltonian corresponds to the relativistic energy

$$E = \pm \left( m^2 c^4 + p^2 c^2 \right)^{1/2}$$

The relevant Hamiltonian is similar to the non-relativistic Hamiltonian we considered,

$$H = \alpha \cdot pc + \beta mc^2$$

Since the particle is never in a superposition of different states, the factual situation in RQM in relativistic mechanics does not involve difficulties of conventional Dirac quantum mechanics, like ‘zitterbewegung’. The interferences are between the action-waves and not between the (non-existent) matter-waves. The relative phases are relevant for the probabilities during observation and not for the actual dynamics of the particle. (It is significant that the zitterbewegung motion was highlighted by Schrödinger, who was a strong advocate for real matter-waves). In summary, the ‘zitter motion’ is an ensemble result and a single particle phenomenon. Analogue experiments confirm this as an ensemble averaged observable quantity [27].

**Appendix 3: A Note on Bohmian Mechanics**

The differences between the robust basis of action-wave mechanics and all previous paths taken to understand quantum mechanics, especially interpretational ones, are transparent. All attempts to resolve the foundational issues and to get a consistent and rational interpretation mistook the Schrödinger equation and the wavefunction as describing the single dynamical history. Einstein’s departure from this universal view did not go far either. RQM actualises what was crucially missing in Einstein’s speculation on the statistical interpretation of the wavefunction. A purely statistical interpretation of the wavefunction cannot reproduce single particle interference. In RQM, the real amplitude part of the wavefunction is statistical and ensemble averaged, while the action-wave can remains invariant (coherent over the ensemble). Thus, the empirically and conceptually crucial single particle interference is reproduced in RQM. Since matter itself has no wave property in RQM, it is
obviously different from all approaches that identify the Schrödinger wavefunction with any single history with ontological status in space and time or in configuration space.

Since the basis of action-wave dynamics refers to the universal method invented by Hamilton, expressed through the action-wave \( \zeta(x,t) = \exp(iS(x,t)/\hbar) \), there is the possibility that RQM may be confused with a totally different approach named after L. de Broglie and D. Bohm that uses the Schrödinger wave function in the polar form \( \psi_{dBB} = R \exp(iS/\hbar) \). It is obvious that the functions \( \zeta(x,t) \) and \( \psi_{dBB} \) are very different. The de Broglie-Bohm (dBB) theory is based on analysing the Schrödinger equation as the fundamental equation for single particle dynamics. I have already shown that this is factually incorrect and that the Hamiltonian action-wave equation is the fundamental equation. Schrödinger equation and its wavefunction pertain to the ensemble. Since the dBB theory uses this ensemble wavefunction to calculate what it calls the ‘quantum potential’ that controls the single particle dynamics in the theory, there is an inadvertent conceptual mix-up that renders the theory physically deficient.

The dBB theory splits the Schrödinger equation into its constituents of action dynamics and continuity of probability density. The dynamical equation is the classical Hamilton-Jacobi equation \( \partial S/\partial t = -H \), now with a nonlocal quantum potential \( V_q = -(\hbar^2/2mR)\nabla^2 R \). This is the vestige of wrongly taking the Schrödinger equation as the single particle equation; thus the ensemble term is misinterpreted as a potential (without sources) for single particle dynamics! Central to the dBB approach is the real existence of the wavefunction in the multi-particle configuration space for each individual history of the quantum evolution. Then, of course, the issues of nonlocality, inseparability etc. are inherent the dBB theory. This alone shows the drastic difference between RQM and dBB approach.

The dBB theory uses position of the particle as hidden variables and a classical guiding equation that specifies the velocity of each particle. Thus, with both position and velocity specified, there are real trajectories in dBB scheme, as in classical mechanics. Positions are given as initial conditions and the velocity in terms of \( \nabla S \). However, these trajectories are controlled by the quantum potential \( V_q = -(\hbar^2/2mR)\nabla^2 R \) defined by the whole nonlocal wavefunction \( R \exp(iS/\hbar) \). One can either work with the quantum potential or directly in terms of the equation \( v_j = -i\hbar/2m_j \rho (\psi^* \nabla_j \psi - \psi \nabla_j \psi^*) \), but in either case the probability density given by the whole wavefunction nonlocally determines each single particle velocity. The identifying feature of Bohmian approach is nonlocality, as Bohm stressed. RQM has no such irrational features because the wavefunction, being an ensemble averaged abstract hybrid construct, has no role in the single particle behaviour. RQM has no guiding equation and no classical trajectories. In fact, RQM emerged
from the basic assertion that even classical mechanics has inherent indeterminism specified by the Hamiltonian action-wave equation. RQM respects the uncertainty relation at the single particle level, arising in the quantum of action; thus the position and momentum of the particle cannot be specified simultaneously.

The failure of the dBB approach, traced to misinterpreting an ensemble quantity as a single system potential $V_q$, can be illustrated vividly with an example that was originally pointed out by Einstein in 1953 [30]. A particle in a definite energy state in a 1-d box ($-l, +l$) has the Schrödinger wavefunction as a superposition of two harmonic waves moving in opposite directions,

$$\psi(x, t) = \frac{1}{2} A e^{-i\omega t} (e^{i\alpha x} + e^{-i\alpha x}) = A e^{-i\omega t} \cos \alpha x$$  \hspace{1cm} (61)

The box is assumed to be much larger than the de Broglie wavelength. All the statistical results of standard quantum mechanics are consistent, in the statistical sense. However, the dBB approach has a glaring inconsistency even in this simple instance, because of its mixing the Schrödinger wavefunction and the single particle dynamics. The central guiding equation gives $p/m = \partial S/\partial x = a - a = 0$. At all positions, the particle does not move - neither to the left nor to the right! This is independent of whether the particle is microscopic or macroscopic. Then, when observed, the wavefunction and the quantum potential (which holds the particle stationary) change instantaneously, allowing the particle to move at $p/m = \pm a$. The acceleration involved can be arbitrarily large. Thus, though the problem of collapse and conflict with Born’s rule is avoided in dBB, the particle behaviour is still discontinuous and more bizarre than in standard QM (standard QM bypasses the issue by dealing with only statistical averages in observations).

The situation with a pair of “entangled particles” shows the difference even more clearly. The dBB wavefunction, while dealing with a particular pair, is the same entangled wavefunction of standard QM with its nonlocal nature persistent over arbitrary spatial separation. Now the wavefunction depends on the coordinates of both the particles. A measurement on one particle affects the dynamics of the other through the nonlocal quantum potential. There is no such irrational feature in RQM, as we have already shown.

**Appendix 4: The EPR Argument**

The Einstein-Podolsky-Rosen argument on the incompleteness of the wavefunction description in QM has an ‘EPR version’, as published in the article in Physical Review [9], and a more clear and concise ‘Einstein version’, available in print through his many articles [31] and also published letters (to
E. Schrödinger, M. Born, K. Popper etc.). I present the essential argument about the lack of one-to-one correspondence between the quantum mechanical state represented by the wavefunction and the physical state of material particles. Consider two spin-1/2 particles that have moved apart into spacelike separated regions after splitting from the spin-zero composite state.

Let us assume that the particles are in some unknown physical state of spin. We do not assume that spin projection values in more than one direction can be specified. This is very important for avoiding the common confusion about the correct EPR argument (see below). In other words, we do not assume that the particle has definite spin projections in multiple directions simultaneously, as in classical theories and classical hidden variable theories. So, we do not make any assumption that violates non-commutativity of spin projections along different directions. To stress again, we just assume that particles have some physical state in which some observables possibly may have definite values, without violating the quantum mechanical restrictions on non-commuting observables. Hence, there is no assumption or mention of ‘physical reality’ or ‘elements of reality’. (These notions are superfluous in the core argument as Einstein himself clarified [31]. According to him the “main argument was buried in erudition”).

The joint quantum mechanical state of the particles is specified by a wavefunction. In the basis \( |x\rangle \), it is given by

\[
\psi_{1,2} = \frac{1}{\sqrt{2}} |+x,-x\rangle - |-x,+x\rangle \tag{62}
\]

The same state can also be represented in the non-commuting basis \( |y\rangle \) of the orthogonal y-direction or in any basis \( n \) as

\[
\psi_{1,2} = \frac{1}{\sqrt{2}} |+n,-n\rangle - |-n,+n\rangle \tag{63}
\]

A measurement of the spin projection on either particle will give random results \( \pm \), but the results on the two particles will be in perfect anti-correlation, pair by pair, if measured along the same direction.

Now I state the only crucial assumption in the analysis, of Einstein locality. The physical state of one particle cannot be influenced or changed by a measurement on the other spatially separated particle. I stress that the assumption is about the physical state and not about its representation in quantum mechanics by a wavefunction. The QM state can of course be changed by a distant measurement. If the spin is measured along the any arbitrarily chosen direction on particle 1, the result will be either + or -. The wavefunction of the particle after the measurement is definitely \( |+n_1\rangle \) (or \(|-n_1\rangle \)). The corresponding wavefunction of the other particle on which no measurement has been done becomes \(|-n_2\rangle \) (or \(|+n_2\rangle \)). Since the direction
is arbitrary, two different choices for \( n \) correspond to two different wavefunctions, and distinct QM physical states. This implies that the quantum mechanical state of a particle can be changed by a measurement on another particle in a spatially separated region. However, we have assumed that the physical state cannot be affected in a spatially separated region. Now there is a conflict; the physical state cannot change, but the quantum mechanical state can be forced to be one of the allowed states in a basis of free choice. Therefore, there is no one-to-one correspondence between the physical state and the QM state. Hence, QM is incomplete.

That is the precise content of the Einstein version of the EPR argument. Nowhere in this proof, Einstein assumed that the physical state of the particle is specified by definite values for non-commuting observables. On the contrary, I explicitly stated that the physical state respects the restrictions of non-commutativity.

If there is any lingering doubt about the strength of this proof, I can state a new version that is stronger. Going back to the form of the entangled state

\[
\psi_{1,2} = \frac{1}{\sqrt{2}} |+x, -x\rangle - |-x, +x\rangle
\]

we note that neither particle has its own physical state whatsoever, within QM description. For, the general state is \( a|x+\rangle + b|x-\rangle \) for the first particle and \( c|x+\rangle + d|x-\rangle \) for the second. Since the entangled state is not the product of these general states, and since there no single particle states other than these, neither particle has a quantum state. Measurement on one particle endows it with a state, and then the spatially separated unobserved particle gets a definite QM state from a situation it has no state whatsoever. Therefore, QM states can be changed factually by measurements in a spatially separated region. So, we have proved that wavefunction description of the physical state is not compatible with Einstein locality. If, on the other hand, we insist that the QM state is the faithful representation of the physical state, Einstein locality is violated. This proof does not need any additional empirical support.

The action-wave mechanics and the reconstructed quantum mechanics clear all aspects of the EPR problem. RQM shows that the particle states are always separable and the notion of entanglement of particle states is an ensemble concept. The correlation of the action-waves and their local interference reproduces the exact quantum correlations. Einstein locality is strictly respected. Measurement reveals only the result of local interference of action-waves. Individually taken, these results are random because the action-waves corresponding to each particle have an initial random phase. However, the sum of their phases are fixed by a conservation constraint and this random phase is irrelevant for the correlations. The entangled wavefunction of standard quantum mechanics does not refer to a single pair of
particles. Naturally it is not a faithful representation of the factual physical state of the pair of particles. This ‘incompleteness’ is due to the ensemble averaged nature of the wavefunction and not because of any feature of quantum dynamics per se. The summary is that with a complete picture of universal dynamics contained in the Hamiltonian action-wave equation and the ensemble averaged Schrödinger equation, there is nothing to add or change in quantum mechanics – as the statistical description of microscopic dynamics, and only as that, the theory is complete.

I will conclude this appendix with a comment. J. S. Bell derived the maximum correlation possible in any classical statistical theory that respects Einstein locality, with hidden stochastic variables and definite prior values for the observables of a particle [15]. Bell found an upper bound for the spin correlation of two spin-1/2 particles with total spin zero. The quantum theoretical result for the same correlation in the spin singlet state exceeds this upper bound. *This implies that quantum theory is not a classical statistical theory* [16]. (We knew this already; otherwise Heisenberg would not have written quantum mechanics in terms of non-commuting relations). There is no information about *Einstein locality* in the experimental tests; they just measure the correlation and show that it exceeds the upper bound set for the classical statistical theories with hidden variables. There is an unfounded impression that the EPR argued for completing quantum mechanics by replacing it with a classical statistical theory with hidden variables, of the kind considered by Bell. The EPR paper does not contain phrases like hidden variables, statistical theory etc., as can be easily checked. Any such impression is gross distortion of the content of EPR paper and Einstein’s views. Besides, it has been shown that the local hidden variable theories of the kind Bell considered are incompatible with the fundamental conservation laws; hence they are unphysical theories [16, 17]. This result was unfortunately not known when Bell’s theorem was widely discussed. Thus, the experimental tests were about the viability of theories that were incompatible with the central conservation laws of physics! In any case, with the new Hamiltonian action-wave mechanics and RQM, all discussions on completing quantum mechanics can rest.

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