Rates of K–shell Electron Capture Decays of $^{180}$Re and $^{142}$Pm Atoms

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We propose a theoretical analysis of the experimental data on the time behaviour K–shell electron capture (EC) decays of atoms $^{180}$Re and $^{142}$Pm in solid targets, obtained recently by Fästermann et al., Phys. Lett. B 672, 227 (2009) and Vetter et al., Phys. Lett. B 670, 149 (2008). We show that the absence of the time modulation in these data rules out the explanation of the “GSI Oscillations” (Yu. A. Litvinov et al., Phys. Lett. B 664, 162 (2008)) by means of two closely spaced ground mass–eigenstates of mother nuclei. PACS: 12.15.Ff, 13.15.+g, 23.40.Bw, 26.65.+t

Introduction

Measurements of the K–shell electron capture (EC) and positron ($β^+$) decay rates of the H–like heavy ions $^{142}$Pm$^{60+}$, $^{140}$P$_{t}^{58+}$, and $^{122}$I$^{52+}$ with one electron on their K–shells have been recently carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [1]–[4]. The measurements of the rates of the number of daughter ions $^{142}$Nd$^{90+}$, $^{140}$Ce$^{58+}$ and $^{122}$Te$^{52+}$ showed a time modulation of exponential decays with periods $T_{EC} = 7.10(22) s$, $7.06(8) s$ and $6.11(3) s$ and modulation amplitudes $a_{d}^{EC} = 0.23(4)$, $0.18(3)$ and $0.22(2)$ for $^{142}$Pm$^{60+}$, $^{140}$P$_{t}^{58+}$ and $^{122}$I$^{52+}$, respectively.

Since the rates of the number $N_d^{EC}(t)$ of daughter ions are defined by

$$\frac{dN_d^{EC}(t)}{dt} = \lambda_d^{EC} (t) N_m(t),$$

(1)

where $\lambda_d^{EC}(t)$ is related to the EC–decay rate and $N_m(t)$ is the number of mother ions at time $t$ after the mother ions are produced, the time–modulation of $dN_d^{EC}(t)/dt$ can be described in terms of a periodic time–dependence of the EC–decay rate $\lambda_d^{EC}(t)$ [1]–[4]

$$\lambda_d^{EC}(t) = \lambda_{EC} (1 + a_{d}^{EC} \cos(\omega_{EC}t + \phi_{EC})),$$

(2)

where $\lambda_{EC}$ is the EC–decay constant, $a_d^{EC}$, $T_{EC} = 2\pi/\omega_{EC}$ and $\phi_{EC}$ are the amplitude, period and phase of the time–dependent term [1]. Furthermore it was shown that the $β^+$–decay rate of $^{142}$Pm$^{60+}$, measured simultaneously with its modulated EC–decay rate, is not modulated with an amplitude upper limit $a_{β^+} < 0.03$ [2]–[4].

The important property of the periods of the time modulation of the EC–decay rates is their proportionality to the mass number $A$ of the nucleus of the mother H–like heavy ion. Indeed, the periods $T_{EC}$ can be described well by the phenomenological formula $T_{EC} = A/20 s$. The proportionality of the periods of the EC–decay rates to the mass number $A$ of the mother nuclei and the absence of the time modulation of the $β^+$–decay branch [2]–[4] can be explained, assuming the interference of neutrino mass–eigenstates caused by a coherent superposition of monochromatic neutrino mass–eigenstates with electron lepton charge [5,6]. Indeed, nowadays the existence of massive neutrinos, neutrino–flavour mixing and neutrino oscillations is well established experimentally and elaborated theoretically [7]. The observation of the interference of massive neutrino mass–eigenstates in the EC–
decay rates of the H–like heavy ions sheds new light on the important properties of these states.

According to atomic quantum beat experiments [8], the explanation of the “GSI oscillations”, proposed in [5,6], bears similarity with quantum beats of atomic transitions, when an excited atomic eigenstate decays into a coherent state of two (or several) lower lying atomic eigenstates. In the case of the $EC^+$–decay one deals with a transition from the initial state $|m\rangle$ to the two–body final state $|d\nu_e\rangle$ with a daughter ion $d$ in its stable ground state and an electron neutrino $\nu_e$, which is a coherent superposition of two neutrino mass–eigenstates with the energy difference equal to $\omega_{21} = \Delta m^2_{21}/2M_m$ related to $\omega_{EC}$ in the moving ion system with a Lorentz factor $\gamma = 1.43$ as $\omega_{EC} = \omega_{21}/\gamma$, where $\Delta m^2_{21} = m_2^2 - m_1^2$ is the difference of squared neutrino masses $m_2$ and $m_1$ of mass–eigenstates $\nu_2$ and $\nu_1$, respectively.

Another mechanism of the “GSI oscillations” has been proposed by Giunti [9,10] and Kienert et al. [11]. The authors [9–11] assume the existence of two closely spaced ground mass–eigenstates of the nucleus of the H–like heavy ion of unknown origin as the initial state of the $EC^+$–decay and describe it by a coherent superposition of the wave functions of two closely spaced ground mass–eigenstates

$$|m\rangle = \cos \theta |m'\rangle + \sin \theta |m''\rangle,$$

where $|m'\rangle$ and $|m''\rangle$ are two closely spaced ground mass–eigenstates of the nucleus of the mother ion with masses $M_m'$ and $M_m''$, respectively, $\theta$ is a mixing angle. By definition of eigenstates the mass–eigenstates of the nucleus of the H–like heavy ion $|m'\rangle$ and $|m''\rangle$ should be orthogonal $\langle m'|m''\rangle = 0$.

Unlike our analysis [5,6], the authors [9–11] draw an analogy of the “GSI oscillations” with quantum beats of atomic transitions [8], when an atom, excited into a state of a coherent superposition of two closely spaced energy eigenstates, decays into a lower lying energy eigenstate. According to [8], the intensity of radiation, caused by a transition from such a coherent state into a lower energy eigenstate, has a periodic time–dependent term with a period inversely proportional to the energy–difference $\Delta E_{m'm''}$ between two closely spaced energy eigenstates. In the approach, proposed in [9–11], the period of the time modulation is equal to $T_{EC} = 2\pi\gamma/\Delta E_{m'm''}$, where $\Delta E_{m'm''} = M_{m'} - M_{m''}$ is the energy difference fixed for $T_{EC} = 7.06(8)$ s and $\gamma = 1.43$, the period of the time modulation of the $EC^+$–decay rate and the Lorentz factor of the H–like $^{140}$Pr$^{58+}$ ion [11].

Such an explanation of the “GSI oscillations” is ruled out by the experimental data on the time modulation of (i) the $EC^+$–decay rates of the H–like $^{122}$F$^{52+}$ ions, showing the time modulation with the period $T_{EC} = 6.11(3)$ s and obeying the so–called $A$–scaling $T_{EC} = A/20$ s of periods of the time modulation of the $EC^+$–decay rates, where $A$ is the mass number of mother nuclei [2–4], and (ii) the positron ($\beta^+$) decay rates of the H–like $^{142}$Pm$^{60+}$ ions, showing no time modulation [2–4]. Indeed, as has been shown in [12], in case of the existence of two closely spaced ground mass–eigenstates of the nucleus of the H–like mother ion the $\beta^+$–decay rates of the H–like heavy ions should be modulated with a period $T_{\beta^+}$ equal to the period of the time modulation of the $EC^+$–decay rate, i.e. $T_{\beta^+} = T_{EC} = 7$ s.

In this letter we give one more reason for the exclusion of the explanation of the “GSI oscillations” by means of two closely spaced ground mass–eigenstates of the nuclei of mother ions, proposed in [9–11].

**Experimental analysis of $EC^+$–decay rate of $^{180}$Re atoms**

Recently [13] the experimental data on the time modulation of the $EC^+$–decay rate of $^{180}$Re atoms with quantum numbers $I^+ = 1^-$, where $I$ is the nuclear spin, have been declared as a new test for theoretical approaches for the explanation of the “GSI oscillations”. In [13] $^{180}$Re, produced in the reaction $^{181}$Ta($^3$He, 4$n$) in the 50 mg/cm$^2$ tantalum foil with a 33 MeV $^3$He beam from the Munich MLL tandem accelerator [13], are unstable under the $EC^+$–decay $^{180}$Re($^{180}$Re → $^{180}$W$^{180}$W$^{180}$W$^{180}$W + $\nu_e$ with nuclei $^{180}$W$^{180}$W$^{180}$W$^{180}$W in the excited state, which then decay $^{180}$W$^{180}$W$^{180}$W$^{180}$W + $\gamma$ with emission of photons [14]. According to the experimental $A$–scaling of the periods of time modulation [23],
the rate of the number of daughter atoms or the radiated intensity should show a time modulation with a period $T_{EC} = A/20\gamma = 6.3\text{s}$. However, the experimental time spectrum of the EC–decays of $^{180}$Re shows no time modulation (see Fig. 1) [13].

**Experimental analysis of EC–decay rate of $^{142}$Pm atoms**

In Ref.[15] the authors have investigated a time modulation of the EC–decay rate of atoms $^{142}$Pm with quantum numbers $I^\pi = 1^+$ in the transitions $^{142}$Pm$_{I^\pi = 1^+}$ → $^{140}$Nd$^+_{I^\pi = 0^+} + \nu_e + \gamma$ and $^{142}$Pm$_{I^\pi = 0^+} + \gamma + \nu_e$, where the K–holes in the atoms $^{140}$Nd$^+$ are occupied by the electrons from the $L\!L\!L$–shell with the emission of photons. The atoms $^{142}$Pm were produced in the $^{124}$Sn($^{23}$Na, $5n$)$^{142}$Pm reaction at the Berkeley 88–inch Cyclotron with a bombarding time short compared to the periods of the time modulation measured in [1], i.e. of order of $T_{EC}/1.43 \approx 5\text{s}$. A time spectrum of $X$–rays from the transition $^{142}$Nd$^+_{I^\pi = 0^+} → ^{142}$Nd$_{I^\pi = 0^+} + \gamma$ has been measured. As has been found the number of counts can be fitted well by exponential decay curve and shows no time modulation (see Fig. 2).

There has been also found no time modulation for the $\beta^+$–decay rates of atoms $^{142}$Pm. This agrees well with the prediction given in [6] for the H–like heavy ions $^{142}$Pm$^{60+}$.

Below we give a theoretical analysis of the experimental data on the EC–decays of $^{180}$Re and $^{142}$Pm atoms.

**Transition amplitudes and rates**

For the analysis of the time modulation of the decay rates of the $^{180}$Re$_{I^\pi = 1^+} → ^{180}$W$_{I^\pi = 2^+} + \nu_e$ and $^{142}$Pm$_{I^\pi = 1^+} → ^{142}$Nd$^+_{I^\pi = 0^+} + \gamma + \nu_e$ decays we follow time–dependent perturbation theory [16] and the procedure, proposed in [17], for the theoretical analysis of the chains of the transitions.

The transitions are caused by the Hamilton operators $H_W(t'')$ and $H_{elm}(t')$ of weak and electromagnetic interactions

$$H_W(t'') = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3 x'' \left[ \bar{\psi}_n (x'') \gamma^\mu (1 - g_A \gamma^5) \psi_p (x'') \right] \times \left[ \bar{\psi}_\nu_e (x'') \gamma_\mu (1 - \gamma^5) \psi_e (x'') \right],$$

$$H_{elm}(t') = e \int d^3 x' \vec{J}(x') \cdot \vec{A}(x')$$

with standard notation [18,19]. $\vec{J}(x')$ and $\vec{A}(x')$ are the nuclear electromagnetic current and the electromagnetic vector potential, respectively [19].
The amplitudes of the decays under consideration we define as follows:  

\[ A(m \rightarrow d_+ \gamma \nu_e)(t) = - \int_{-\infty}^{t} dt' \langle \gamma d_+ | H_{\text{el}}(t') | d_+ \rangle \times \int_{-\infty}^{t} dt'' \langle \nu_e d_- | H_W(t'') | m \rangle, \]  

(5)

where \( m \) is the mother atom \( ^{180}\text{Re} \) or \( ^{142}\text{Pm} \), \( d_- \) is the daughter atom \( ^{180}\text{W}_{I^* = 2^+} \) or \( ^{142}\text{Nd}_{I^* = 0^+} \), \( \nu_e \) is the electron neutrino, \( d_+ \) is the daughter atom \( ^{180}\text{W}_{I^* = 2^+} \) or \( ^{142}\text{Nd}_{I^* = 0^+} \) and \( \gamma \) is a photon. For the calculation of the integrals over time we use the \( \varepsilon \)-regularisation \[6\].

The transition amplitudes \( A(m \rightarrow d_+ \gamma \nu_e)(t) \) we calculate below for (i) the initial state of the mother ion \( m \), defined by a coherent superposition of two closely spaced ground mass–eigenstates \( |m'\rangle \) and \( |m''\rangle \), given by Eq. (3), and massless electron neutrino \( |\nu_e\rangle \), and (ii) the mother ion \( m \) in the one initial state and the electron neutrino, given by a coherent superposition of neutrino mass–eigenstates \( |\nu_j\rangle \) as \( |\nu_e\rangle = \sum_j U_{e j} |\nu_j\rangle \), where \( U_{e j} \) are matrix elements of the mixing matrix \( U \) \[7\]. In the last case the Hamilton operator of weak interactions is changed as \( H_W(t'') = \sum_j U_{e j} H_W^{(j)}(t'') \), where \( H_W^{(j)}(t'') \) is \[10\]

\[ H_W^{(j)}(t'') = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3 x'' \left[ \bar{\psi}_n(x'') \gamma^\mu (1 - g_A \gamma^5) \psi_p(x'') \right] \times \left[ \bar{\psi}_{\nu_j}(x'') \gamma_\mu (1 - \gamma^5) \psi_e(x'') \right]. \]  

(6)

The rate \( \lambda_{EC}^{(\gamma)}(t) \) of the \( m \rightarrow d_+ + \gamma + \nu_e \) transition is defined by

\[ \lambda_{EC}^{(\gamma)}(t) \propto \int \sum \frac{d^2}{dt^2} |A(m \rightarrow d_+ \gamma \nu_e)(t)|^2 d\rho_f, \]  

(7)

where we sum over all polarisations of interacting particles and \( d\rho_f \) is the element of the phase volume of the final state \( f = d_+ \gamma \nu_e \).

**EC-decay from two closely spaced mass–eigenstates of mother nucleus \( ^{180}\text{Re} \)**

In the approach, based on the wave function of the mother atom given by Eq. (3), for the integrand of the EC-decay rate of the \( m \rightarrow d_- + \nu_e \rightarrow d_+ + \gamma + \nu_e \) decay we get the following expression

\[ \lim_{\varepsilon \rightarrow 0} \sum \frac{d^2}{dt^2} |A(m \rightarrow d_+ \gamma \nu_e)(t)|^2 \propto \delta(\omega + E_{d+}(\vec{k}_+)) \times \delta(E_{\nu_e}(\vec{k}_{\nu_e}) + E_{d_-}(\vec{k} + \vec{k}_+) - M_m) \times \delta(\vec{k} + \vec{k}_+ + \vec{k}_{\nu_e}) \times (1 + \sin 2\theta \cos(\Delta E_{m' m''} t)). \]  

(8)

Substituting Eq. (8) into Eq. (7) and integrating over the phase volume we get

\[ \lambda_{EC}^{(\gamma)}(t) = \lambda_{EC}^{(\gamma)}(1 + \sin 2\theta \cos(\Delta E_{m' m''} t)). \]  

(9)

For the calculation of the transition rate \( \lambda_{EC}^{(\gamma)} \) of atom \( ^{180}\text{Re} \) we have to take into account not only K–shell electron captures but also L–, M–, N– and so on shell electron captures. According to \[20\], the transition rate \( \lambda_{EC}^{(\gamma)} \) is proportional to the wave function \( |\psi_n(0)|^2 = Z^3 \alpha^3 m_e^5 / n^2 \), summed over the principal quantum number \( n \). For atom \( ^{180}\text{Re} \) the contribution of the L–, M–, N– and so on shell electron captures relative to the K–shell electron capture makes up of about 19%, since the upper shell corresponds to \( n = 6 \) \[7\]. However, the contribution of the electron captures from the higher shells does not influence the frequency of the time modulation.

Thus, the explanation of the “GSI oscillations”, based on the hypothesis of the existence of two closely spaced ground mass–eigenstates of the mother nuclei, predicts the time modulation of the EC–decay rates of the \( ^{180}\text{Re}_{I^* = 1^+} \rightarrow ^{180}\text{W}_{I^* = 2^+} + \nu_e \rightarrow ^{180}\text{W}_{I^* = 0^+} + \gamma + \nu_e \) and \( ^{142}\text{Nd}_{I^* = 1^+} \rightarrow ^{142}\text{Nd}_{I^* = 0^+} + \nu_e \rightarrow ^{142}\text{Nd}_{I^* = 0^+} + \gamma + \nu_e \) decays with a period of about \( T_{EC} \simeq 5 \text{ s} \). The time spectra for \( ^{180}\text{Re} \) and \( ^{142}\text{Pm} \), obtained after a single irradiation by setting a narrow gate on the 903 keV gamma line \[13\] and for \( ^{142}\text{Nd}_{K\alpha} \) X–rays, are shown in Fig. 1 and Fig. 2, respectively. The time axes in Fig. 1 and Fig. 2 are binned with 1 s/bin and 0.5 s/bin, respectively. During 1 s/bin and as well as 0.5 s/bin the time modulation of the EC–decay rates of atoms \( ^{180}\text{Re} \) and \( ^{142}\text{Pm} \) with periods \( T_{EC} \simeq 5 \text{ s} \) should be measured well. However, such a time modulation is not shown on the experimental time spectra and the prediction of such a time
modulation disagrees with the experimental data.

EC−decay of mother atoms $^{180}$Re and $^{142}$Pm in a theory of weak interactions with massive neutrinos

In a theory of weak interactions with massive neutrinos a time modulation of the EC−decay rates of atoms $^{180}$Re and $^{142}$Pm can appear only due to a coherence of the processes of the emission of massive neutrinos $m \rightarrow d_+ + \nu_1 \rightarrow d_+ + \gamma + \nu_1$ and $m \rightarrow d_+ + \nu_2 \rightarrow d_+ + \gamma + \nu_2$. This is unlike the approach, based on the existence of two closely spaced mass−eigenstates of the mother nuclei $m$, where a coherence in the initial state Eq.1 cannot be destroyed by the subsequent transitions $m \rightarrow d_+ + \nu_e \rightarrow d_+ + \gamma + \nu_e$.

A coherence of the processes of the emission of massive neutrinos $m \rightarrow d_+ + \nu_1 \rightarrow d_+ + \gamma + \nu_1$ and $m \rightarrow d_+ + \nu_2 \rightarrow d_+ + \gamma + \nu_2$ can be destroyed by the broadening of neutrino mass−eigenstates energies caused by (i) a very short lifetime of the K−hole, created in the electron capture process, (ii) short lifetimes of the excited nuclear states, (iii) the excited phonon spectra in the solid target and so on. So there exist enough physical reasons to argue that in a theory of weak interactions of massive neutrinos the EC−decay rates of atoms $^{180}$Re and $^{142}$Pm, measured in [1315], should not show any time modulation.

Nevertheless, we would like to show that even if a coherence of the processes of the emission of massive neutrinos is not destroyed, the EC−decay rates of atoms $^{180}$Re and $^{142}$Pm, calculated in such an approach, should not show an observable time modulation.

In the approach, assuming the interference of massive neutrinos for the explanation of the “GSI oscillations”, the integrand of the EC−decay rate Eq.7 of the $m \rightarrow d_+ + \nu$ → $d_+ + \gamma + \nu_e$ decay can be obtained in analogy with that given by Eq.8. The interference term takes the form $2U_{e2}U_{e1} \cos(\omega_{EC}t) = \sin 2\theta_{12} \cos(\omega_{EC}t)$, where $\theta_{12}$ is the mixing angle [7] and $\omega_{EC}$ is the energy difference of neutrino mass−eigenstates $\nu_2$ and $\nu_1$ equal to $\omega_{EC} = \epsilon_2(\bar{k} + \bar{k}_+) - \epsilon_1(\bar{k} + \bar{k}_+)$.

The massive neutrinos have equal 3−momenta due to momentum conservation, described by the $\delta$−function $\delta^{(3)}(\bar{k} + \bar{k}_+ + \bar{k}_j)$, where $\bar{k}_j$ is a 3−momentum of neutrino mass−eigenstate $\nu_j$. Since 3−momenta of massive neutrinos are equal, the energy difference $\omega_{EC} = \epsilon_2(\bar{k} + \bar{k}_+) - \epsilon_1(\bar{k} + \bar{k}_+)$ is inversely proportional to the $Q$−value of the EC−decay

$$\omega_{EC} = \frac{m_2^2 - m_1^2}{E_2(\bar{k} + \bar{k}_+) + E_1(\bar{k} + \bar{k}_+)} \approx \frac{\Delta m_{21}^2}{2Q_{EC}} \tag{10}$$

where $Q_{EC} = 3800$ keV and $Q_{EC} = 4870$ keV are the $Q$−values of the EC−decays of atoms $^{180}$Re and $^{142}$Pm [14], respectively, and $E_2(\bar{k} + \bar{k}_+) \approx E_1(\bar{k} + \bar{k}_+) \approx Q_{EC} [15,18]$.

Thus, the rates of the $^{180}$Re, $^{142}$Pm decay rates read

$$\lambda^{(\gamma)}_{EC}(t) = \lambda^{(\gamma)}_{EC}(1 + \sin 2\theta_{12} \cos(\omega_{EC}t)) \tag{11}$$

where the frequencies are equal to

$$\omega_{EC} = \left\{ \begin{array}{ll}
4.38 \times 10^4 \text{s}^{-1}, & ^{180}\text{Re} \\
3.42 \times 10^4 \text{s}^{-1}, & ^{142}\text{Pm},
\end{array} \right. \tag{12}$$

calculated for $\Delta m_{21}^2 = 2.19 \times 10^{-4}$ eV$^2$ [21]. The periods of time modulation are

$$T_{EC} = \frac{2\pi}{\omega_{EC}} = \left\{ \begin{array}{ll}
1.43 \times 10^{-4} \text{s}, & ^{180}\text{Re} \\
1.84 \times 10^{-4} \text{s}, & ^{142}\text{Pm},
\end{array} \right. \tag{13}$$

During 1 s/bin and 0.5 s/bin the periodic terms of the EC−decay rates of atoms $^{180}$Re and $^{142}$Pm, calculated in a theory of weak interactions with neutrino mass−eigenstates, make of about 7000 and 2700 oscillations, respectively, and, of course, cannot be observed by the reported experiments. This agrees well with the experimental time spectra in Fig. 1 and Fig. 2, measured in [1315].

Conclusive discussion

We have shown that following the hypothesis of the existence of two closely spaced ground mass−eigenstates of the mother nuclei of atoms $^{180}$Re and $^{142}$Pm one gets the EC−decay rates of atoms $^{180}$Re and $^{142}$Pm, modulated with periods $T_{EC} \approx 5$ s. This disagrees with the experimental data, obtained in [1315].
The absence of time modulation of the EC–decay rates of atoms $^{180}\text{Re}$ and $^{142}\text{Pm}$, calculated in a theory of weak interactions with neutrino mass–eigenstates, can be related to a violation of a coherence of the processes of the emission of massive neutrinos. Such a decoherence can have some physical reasons caused by short lifetimes of the K–holes, produced by the captured electrons, excited nuclei, excited phonon spectra of solid targets and so on [22]. Nevertheless, we have shown that even if a coherence of the processes of the emission of massive neutrinos is not destroyed, a time modulation of the EC–decay rates of atoms $^{180}\text{Re}$ and $^{142}\text{Pm}$ appears with periods of order of $T_{EC} \sim 10^{-4}$ s, which cannot be observed at the present experiments. This agrees well with the experimental data on the time spectra of the EC–decays of $^{180}\text{Re}$ and $^{142}\text{Pm}$ atoms [13,15].

Since the experimental data on the time modulation of the number of the EC–decay rates of $^{180}\text{Re}$ have declared as the test for the theoretical approaches for the explanation of the “GSI oscillations”, measured in [1]–[4], the results, obtained above, allow us to argue that the time modulation of the EC–decay rates of the H–like heavy ions $^{142}\text{Pm}^{60+}$, $^{140}\text{Pr}^{58+}$ and $^{122}\text{I}^{52+}$ and the $\beta^+$–decay rates of the H–like $^{142}\text{Pm}^{60+}$ can be explained only due to the interference of neutrino mass–eigenstates [6] (see also [5]).

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