CNOT gate on reverse photon modes in a ring cavity

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Abstract
Photon modes of the reverse rotation in a ring QED cavity coupled with a single atom are considered. By applying the Schrieffer–Wolf transformation for the off-resonant light–atom interaction, an effective Hamiltonian of the photon modes evolution is obtained. Heisenberg equations for the input–output photon mode operators are written, and the expression for the wave function of the system is found. The analytical solution shows the condition of the control NOT quantum gate implementation on chiral photon modes. A possible on-chip experimental implementation and recommendations for the construction of an optical quantum computer using this gate are considered.

Keywords Two-photon gate · Ring cavities · Photon–photon interaction · Off-resonant light–atom interaction · Femtosecond-laser-written waveguides

1 Introduction
Photonic two-qubit gates are important instruments for quantum processing, optical quantum communication, etc. However, their effective experimental implementation remains a quite difficult problem. Since the photons do not interact with each other in free space, the photonic two-qubit gates are implemented in the probabilistic way using linear optical elements [1]. It is possible to enhance the photon–photon interaction in nonlinear media. In particular, the authors of paper [2] have proposed using the additional nonlinear two-photon frequency up- and down-conversion elements. The nonlinear interaction is rather weak but can be considerably enhanced in micro- and nanocavities due to the Purcell effect. Moreover, this indirect interaction can be replaced by the resonant interaction of photons with atoms. Therefore, the construc-
tion of photon–photon two-qubit gates using cavities containing atoms is a subject of the intensive study.

The CPhase photon–photon gate using the interaction of polarization-encoded photon qubits with a cavity containing a three-level atom was presented in [3]. The CPhase gate on the nitrogen-vacancy center in this configuration was considered in [4,5]. The three-qubit control SWAP (Toffoli) gate was proposed in [6]. The deterministic photon–photon \( \sqrt{\text{SWAP}} \) gate using a \( \Lambda \)-type atom–cavity system with polarization encoding of qubits was proposed theoretically in [7]. The CPhase photon–photon gate in this approach was also proposed in [8]. CNOT gates were considered for a photon qubit interacting with a cavity containing atom [9], quantum dot [10,11], diamond nitrogen-vacancy centers [12] and collective magnon system [13]. Recently, the implementation of the original scheme [3] has successfully been demonstrated in experiment [14]. The experimental implementation of the SWAP gate using a nanofiber-coupled microsphere resonator coupled to a single Rb atom was presented in [15]. The CNOT gate on a quantum dot in a photonic crystal cavity strongly coupled to polarization-encoded photon qubits was demonstrated in [16].

Thus, a large number of theoretical and experimental works concerning photon–photon cavity-based quantum gates have been performed, although the quantum efficiency and fidelity of the two-qubit gates remain insufficiently high [14], which requires further searching for new ways to improve the practical implementation of two-photon gates. In addition, the aforementioned approaches used polarization encoding of photon qubits and it is interesting to study the possibility to use other encodings. Here, we propose to use the dispersion interaction of light modes with the atom in order to suppress the effect of the atomic decoherence processes. Moreover, we propose the photon–photon CNOT gate using encoding of qubits on counterpropagating photon modes in a ring cavity with a controlling two-level atom and consider the on-chip scheme for its experimental implementation.

2 Schrieffer–Wolf transformation and effective Hamiltonian

The analyzed scheme is shown in Fig. 1, where a resonant atom is placed in a ring cavity connected via a waveguide with a multi-qubit quantum memory cell. The ring

![Fig. 1 On-chip scheme for the photon–photon CNOT gate scheme; the reverse whispering-gallery modes of the optical cavity interact with a two-level atom with the effective coupling constant \( \mu \) and with each other with coupling constant \( \eta \) by scattering in the Bragg grating situated closely to the cavity surface](image-url)
cavity supplies two counterpropagating (chiral) light modes coupled with each other due to scattering on the tuned Bragg grating. We assume that the atom interacts with two photons oppositely rotating in the ring cavity.

The total Hamiltonian of the studied system is written as follows $H = H_0 + H_1$, where $H_0 = H_a + H_r$ is the main Hamiltonian and $H_1 = H_{r-a}^{cw} + H_{r-a}^{ccw}$ is the perturbation Hamiltonian. In this scheme, we focus on considering only a two-level atom, but the obtained results can be also applied to the case of a three-level atom, where the resulting specific features and advantages will be briefly discussed below. The main Hamiltonian (in units $\hbar$) consists of the atomic Hamiltonian $H_a = \omega_0 S^z$, where $\omega_0$ is the frequency of the atomic transition in the two-level model, $S^z$ is the operator of the effective spin $z$-component and the radiation Hamiltonian $H_r = \omega_{cw} a_{cw}^{\dagger} a_{cw} + \omega_{ccw} a_{ccw}^{\dagger} a_{ccw} + \eta(a_{cw}^{\dagger} a_{ccw} + a_{ccw}^{\dagger} a_{cw})$, where $\omega_{cw}$ and $\omega_{ccw}$ are frequencies of the clockwise and counterclockwise photons in the ring cavity, $a_{cw}^{\dagger}(ccw)$ and $a_{cw}(ccw)$ are the creation and annihilation operators for the clockwise (counterclockwise) photon, $\eta$ is the clockwise–counterclockwise photon interaction constant. The coupling of the counterpropagating modes can be implemented by forming the Bragg grating with parameters (period and amplitude) determining the appropriate $\lambda$ value.

The perturbation Hamiltonian includes the atomic interaction with clockwise and counterclockwise photons $H_{r-a}^{cw} = g S^+ a_{cw} + g^* S^- a_{cw}^{\dagger}$ and similarly $H_{r-a}^{ccw} = g S^+ a_{ccw} + g^* S^- a_{ccw}^{\dagger}$, where $S^+$ and $S^-$ are raising and lowering operators in the two-level model and $g$ is the interaction constant. Below we consider the off-resonant interaction between light modes and the two-level atom by assuming some detuning $\Delta$ of the atomic frequency $\omega_0$ from the cavity mode frequency $\omega_{ccw}$. In this case, we get the dispersive light–atom interaction, where the atomic decoherent processes are suppressed by the factor $\frac{\Delta}{\omega} \ll 1$ [16]. Performing the Schrieffer–Wolf transformation [17] $e^s$ for this case, we get for the effective Hamiltonian $H_s = H_0 + \frac{1}{2} [H_1, s]$ providing $H_1 + [H_0, s] = 0$ that gives the following formula for the transformation operator:

$$s = \alpha^{cw} g S^+ a_{cw} + \beta^{cw} g^* S^- a_{cw}^{\dagger} + \alpha^{ccw} g S^+ a_{ccw} + \beta^{ccw} g^* S^- a_{ccw}^{\dagger},$$

where $\Delta = \omega_0 - \omega_{cw}$, $\alpha^{cw} = -\beta^{cw} = -\frac{1}{\Delta - \eta}$, $\alpha^{ccw} = -\beta^{ccw} = -\frac{1}{\Delta - \eta}$ and it is assumed that $\beta^{ccw} g \ll 1$, $\beta^{cw} g \ll 1$ and $\Delta > g, \eta, \gamma$ leads to the dispersive interaction of light with the atom, $\gamma$ is a decay constant of the atomic coherence (i.e., we neglect the decoherent processes in the atomic dynamics), $\omega_{cw} = \omega_{ccw} = \omega_r$, $\beta^{cw} = \beta^{ccw} = \beta$.

By using (1), we get the effective Hamiltonian:

$$H_s = \omega_{cw} a_{cw}^{\dagger} a_{cw} + \omega_{ccw} a_{ccw}^{\dagger} a_{ccw} + \eta(a_{cw}^{\dagger} a_{ccw} + a_{ccw}^{\dagger} a_{cw}) + (\omega_0 + 2\beta |g|^2) S^z + \mu S^z (a_{cw}^{\dagger} a_{cw} + a_{ccw}^{\dagger} a_{ccw} + a_{cw} a_{cw}^{\dagger} + a_{ccw} a_{ccw}^{\dagger}),$$

where the coupling between the cavity modes is controlled by the atomic state via $\mu S^z$ with $S^z = \pm 1/2 (\mu = 2\beta |g|^2 = 2|g|^2/(\Delta - \eta))$. Below we analyze the dynamics of the field modes.
3 System dynamics and CNOT gate

Taking into account the coupling of the cavity with an external waveguide and using the well-known approach of quantum optics [15], we obtain two coupled equations:

$$\frac{d}{dt} a_{cw} = -\left( i \omega_{cw} + \frac{\kappa}{2} \right) a_{cw} - i \left\{ \eta + \mu S^z \right\} a_{ccw} + \sqrt{\kappa} a_{in,1}(t),$$  \hspace{1cm} (2)

$$\frac{d}{dt} a_{ccw} = -i \left( \omega_{ccw} + \frac{\kappa}{2} \right) a_{ccw} - i \left\{ \eta + \mu S^z \right\} a_{cw} + \sqrt{\kappa} a_{in,2}(t),$$  \hspace{1cm} (3)

where $\kappa$ is the coupling constant of the cavity mode with free modes, $a_{in,(1,2)}$ is the input field of the m-cavity mode launched from the quantum memory cell (see Fig. 1).

The linear equations for the field operators $a_{cw}(t)$ and $a_{ccw}(t)$ are easily solved for its Fourier components and yield for the two atomic states $(m = \pm 1/2)$:

$$a_{cw}(\omega) = \frac{-i \left\{ \eta + \mu m \right\} \sqrt{\kappa} a_{in,2}(\omega) + \left( \frac{\kappa}{2} - i (\omega - \omega_{ccw}) \right) \sqrt{\kappa} a_{in,1}(\omega)}{(\eta + \mu m)^2 + \left( \frac{\kappa}{2} - i (\omega - \omega_{cw}) \right)^2},$$  \hspace{1cm} (4)

$$a_{ccw}(\omega) = \frac{-i \left\{ \eta + \mu m \right\} \sqrt{\kappa} a_{in,1}(\omega) + \left( \frac{\kappa}{2} - i (\omega - \omega_{cw}) \right) \sqrt{\kappa} a_{in,2}(\omega)}{(\eta + \mu m)^2 + \left( \frac{\kappa}{2} - i (\omega - \omega_{ccw}) \right)^2},$$  \hspace{1cm} (5)

Using these solutions and taking into account the initial state for the input light fields $a_{in,1}(t)$ and $a_{in,2}(t)$, we find the output fields $a_{out,1}(t)$ and $a_{out,2}(t)$ from relations [18] $a_{out,1}(t) = \sqrt{\kappa} a_{cw}(t) - a_{in,1}(t), a_{out,2} = \sqrt{\kappa} a_{ccw} - a_{in,2}$. By assuming $\omega_{cw} = \omega_{ccw} = \omega_r$, we get:

$$a_{out,1}(\omega) = B_{1,1}(m, \omega) a_{in,1}(\omega) + B_{1,2}(m, \omega) a_{in,2}(\omega),$$  \hspace{1cm} (6)

$$a_{out,2}(\omega) = B_{2,1}(m, \omega) a_{in,1}(\omega) + B_{2,2}(m, \omega) a_{in,2}(\omega),$$  \hspace{1cm} (7)

where

$$B_{1,2}(m, \omega) = B_{2,1}(m, \omega) = \frac{-i \left\{ \eta + \mu m \right\} \kappa}{(\eta + \mu m)^2 + \left( \frac{\kappa}{2} - i (\omega - \omega_r) \right)^2},$$

and

$$B_{1,1}(m, \omega) = B_{2,2}(m, \omega) = -\left\{ \frac{(\eta + \mu m)^2 - \frac{\kappa^2}{4} - (\omega - \omega_r)^2}{(\eta + \mu m)^2 + \left( \frac{\kappa}{2} - i (\omega - \omega_r) \right)^2} \right\}. $$

Equations (6) and (7) demonstrate the dependence of the output fields on the state of the atom. For the input single-photon state

$$|\Psi_{in}\rangle = \sum_m a_m \int_{-\infty}^{\infty} d\omega \{ C_{1}^{(m)}(\omega) a_{in,1}^\dagger(\omega) + C_{2}^{(m)}(\omega) a_{in,2}^\dagger(\omega) \} |0\rangle_1 |0\rangle_2 |m\rangle_c,$$

(8)
we get the output quantum state of light after the photon–atom interaction:

$$|\psi_{at+ph}\rangle = \sum_{m} \alpha_{m} \int_{-\infty}^{\infty} d\omega \{ C_{1, out}^{(m)} (\omega)a_{out, 1}^{(m)\dagger} (\omega) + C_{2, out}^{(m)} (\omega)a_{out, 2}^{(m)\dagger} (\omega) |0\rangle_{1} |0\rangle_{2} |m\rangle_{c} \}.$$ 

where

$$C_{1, out}^{(m)} (\omega)a_{out, 1}^{(m)\dagger} (\omega) = C_{1}^{(m)} (\omega) B_{1, 1}^{*} (m)a_{in, 1}^{\dagger} (\omega) + C_{2}^{(m)} (\omega) B_{1, 2}^{*} (m)a_{in, 2}^{\dagger} (\omega),$$

$$C_{2, out}^{(m)} (\omega)a_{out, 2}^{(m)\dagger} (\omega) = C_{1}^{(m)} (\omega) B_{2, 1}^{*} (m)a_{in, 1}^{\dagger} (\omega) + C_{2}^{(m)} (\omega) B_{2, 2}^{*} (m)a_{in, 2}^{\dagger} (\omega).$$

(9)

We assume the input photon state $C_{1, 2}^{m} (\omega)$ is characterized by sufficiently narrow spectral width $\frac{\delta \omega_{in}}{\kappa} \ll 1$, where $B_{1, 2}^{*} (m) = B_{1, 2}^{*} (m) = \frac{i |\eta + \mu \omega|}{(\eta + \mu)^{2} + \frac{\kappa^{2}}{\omega^{2}}}$, $B_{1, 1}^{*} = B_{2, 2}^{*} = -\frac{(\eta + \mu)^{2} - \frac{\kappa^{2}}{\omega^{2}}}{(\eta + \mu)^{2} + \frac{\kappa^{2}}{\omega^{2}}}$. By taking into account the fulfillment of the impedance matching conditions:

$$\eta = \frac{\mu}{2}, \mu = \frac{\kappa}{2},$$

(11)

we find for the field amplitudes: $B_{1, 2}^{*} (-1/2) = B_{2, 1}^{*} (-1/2) = 0$, $B_{1, 1}^{*} = B_{2, 2}^{*} = 1$ if $m = -1/2$, and $B_{1, 2}^{*} (1/2) = B_{2, 1}^{*} (1/2) = i$, $B_{1, 1}^{*} = B_{2, 2}^{*} = 0$ if $m = 1/2$. Finally, the wave function of the output fields is

$$|\psi_{at+ph}\rangle = \alpha_{1/2} \int_{-\infty}^{\infty} d\omega \{ C_{1}^{(-1/2)} (\omega)a_{in, 1}^{\dagger} (\omega) + C_{2}^{(-1/2)} (\omega)a_{in, 2}^{\dagger} (\omega) \} |0\rangle_{1} |0\rangle_{2} |1/2\rangle_{c}$$

$$+i\alpha_{1/2} \int_{-\infty}^{\infty} d\omega \{ C_{1}^{(1/2)} (\omega)a_{in, 2}^{\dagger} (\omega) + C_{2}^{(1/2)} (\omega)a_{in, 1}^{\dagger} (\omega) \} |0\rangle_{1} |0\rangle_{2} |1/2\rangle_{c}.$$ 

We note that the coupling constant $\eta$ can be varied by changing the contrast of the Bragg grating and its distance to the surface of the ring cavity as it has been used recently for the nanofiber cavity containing one atom [19] (see Fig. 1). By performing an additional rotation of the atom state by the angle $\pi/2$, we get:

$$|\psi_{out}\rangle = \exp \left\{-i \frac{S_{z} \pi}{2} \right\} |\psi_{at+ph}\rangle = \exp \left\{i \pi/4 \right\}.$$ 

$$[\alpha_{1/2} \int_{-\infty}^{\infty} d\omega \{ C_{1}^{(-1/2)} (\omega)a_{in, 1}^{\dagger} (\omega) + C_{2}^{(-1/2)} (\omega)a_{in, 2}^{\dagger} (\omega) \} |0\rangle_{1} |0\rangle_{2} |1/2\rangle_{c}$$

$$+\alpha_{1/2} \int_{-\infty}^{\infty} d\omega \{ C_{1}^{(1/2)} (\omega)a_{in, 2}^{\dagger} (\omega) + C_{2}^{(1/2)} (\omega)a_{in, 1}^{\dagger} (\omega) \} |0\rangle_{1} |0\rangle_{2} |1/2\rangle_{c}.$$ 

(12)
Equation (12) shows that the atom in the state \( m = -1/2 \) preserves the initial quantum state of the two-mode photon state, while we have the NOT gate for the photon state if \( m = 1/2 \). This indicates the performance of CNOT gate in the case of the single-mode cavity. Bearing in mind that the atomic state (evaluation of the atomic amplitudes \( \alpha_{-1/2} \) and \( \alpha_{1/2} \)) can be regulated by the controlling photon, the photon–photon CNOT gate can be constructed using this approach.

4 Possibilities of experimental implementation

In order to construct the considered CNOT gate, we should satisfy the two matching conditions (11). The first condition \( \eta = \mu/2 = |g|^2/(\Delta - \eta) \) can be fulfilled by selecting the photon–atom frequency detuning \( \Delta = \omega_0 - \omega_{ccw} \) for the given coupling constant of the circulating cavity light modes \( \eta \). In turn, the coupling constant \( \eta \) is determined by the parameters of the fabricated Bragg grating (see Fig. 1). For the fulfillment of (11), we can adjust the coupling coefficient \( \kappa \) of the cavity with the external waveguide by tuning the refractive index in the coupling region. If the system parameters are chosen properly, we can efficiently implement the proposed CNOT gate by following this scenario. First, we use a controlling photon qubit for the excitation of the controlling atom situated in the ring cavity. The effective photon transfer to the atomic state can be implemented by the appropriated choice of the temporal photon mode. In particular, this process can be performed for the two-level or three-level lambda-type atom interacting with additional laser field on the adjoined atomic transition [20]. Moreover, the implementation of the considered CNOT gate on the three-level atom could provide the longer coherence time on the two ground levels that make it easier to perform. Then, we load the target photon qubit in the cavity that is not in resonance with the control atom at this stage. Finally, we introduce the CNOT gate operation cavity in resonance with its atom and perform the CNOT gate as it was described above.

The whispering-gallery mode optical cavity with the high quality factor of up to \( Q = 10^{11} \) [21] can be used to construct the CNOT gate with the enhanced coupling constant \( \mu \). In addition to the controlling two-level atom (e.g., Rb), we also suggest a quantum dot, a nanoparticle with a single active center attached to the surface of the cavity or a single atom or ion implanted in the cavity surface. Optical tapered fiber is widely used for coupling the light field with the cavity mode [22–24]. For the critical coupling, a very thin fiber is necessary with the diameter of \( D < \lambda \), while such fibers are usually too unstable and can be damaged easily. Recently, integrated photonic waveguides have been used for the efficient coupling of the atom with light [25].

In addition to this approach, we propose a femtosecond-laser-written waveguide in a fused silica plate [26,27]. The robust construction of this device is shown in Fig. 1. The laser written waveguide has a small region where waveguide draw nearer (several \( \mu \text{m} \)) to the surface of fused silica plate. This region can be used for coupling with whispering-gallery light modes through the evanescent part of the light field. The coupling constant can be adjusted by varying the refractive index between the cavity and the plate or between the waveguide and the surface of the fused silica plate using short laser pulses. Moreover, the printed waveguide can be implemented in an
inorganic crystal doped with rare earth ions [28–30] for on-chip quantum storage of photonic qubits as is depicted in Fig. 1. It is worth noting that the device can be efficiently connected with resonator quantum memory cells [30–36] for the operation with a large number of photonic qubits.

During the fabrication of a waveguide, the laser forms the shell of the waveguide (two or more damage tracks) with the decreased refractive index. In addition, these damage tracks produce the stress field that leads to the increase in the core refractive index [27]. It was shown that the spectroscopic and coherency properties of rare earth ions in material perspective for the implementation of a quantum computer did not change drastically [26, 37]. Our measurements in the waveguides fabricated in the $^{7}\text{LiYF}_4 : Er^{3+}$ show that erbium ions in these waveguides have the coherent time sufficiently long for quantum informatics and the waveguides with the large diameter ($\sim 100 \mu m$) have the relatively low waveguide propagation losses (0.6 dB/cm).

However, with the decrease in the waveguide diameter, these losses are enhanced and the coherence time is somewhat shortened. Therefore, we studied another type of waveguides that are fabricated in $Y_2SiO_5$ (the most popular material for the quantum informatics applications [17]). The laser beam produced a weak damage at the focal volume with the relevant increase in the refractive index and it is a position of waveguide core. After fabrication of the waveguides, the difference of refractive indices of the core with the diameter $d_{core}$ and the coating is approximately $\Delta n=0.001$. Such waveguides have demonstrated a sufficiently broad radiation profile in the transverse dimension. We found that the transverse radiation distribution is about 40 $\mu m$ at the core diameter of $d_{core}=5 \mu m$. At the abrupt inclination of the waveguide, the radiation gets out of the waveguide as it is also shown by the numerical simulations (Fig. 2).

Therefore, the waveguide should have a rather large bending radius $R$. The waveguide transmission can reach unity at the optimal parameters of bending, $d_{core}$, etc. The light transmission through the waveguide with the diameter of $d_{core}=5 \mu m$ and 40 $\mu m$ bending size is shown in Fig. 3. For example, the transmission can reach $Tr=97.5\%$ in the operation zone at the radius $R$ of about 0.08 m that corresponds to the inclination angle of 1.28 degrees. In this case, the bending length is about 3.6 mm. Figure 1 shows that a series of waveguides with the core size of $5 \mu m$ was fabricated.

![Fig. 2 Behavior of radiation with the $E_y$ polarization in the bent waveguide with the core diameter of 5 $\mu m$ and waveguide length of $L = 0.591 \mu m$ and $L = 2.41 \mu m$](image)
in quartz glass with the length of 11.8 mm by using the femtosecond-laser writing method. The repetition rate of laser pulses producing the waveguides was 1 MHz, the pulse length was 400 fs and the average radiation power was 82 mW. The laser output was frequency-doubled to meet optimal focusing conditions with a Mitutoyo Plan APO 100X microscope objective [38]. The two bending zones have the total length of 7.2 mm, which determines the maximum length of 4.6 mm of the interaction zone between the cavity and the waveguide. The production of a curved waveguide was carried out at a depth of 55 μm with the subsequent rise of the waveguide core to the surface of the optical chip to a depth of 15 μm. Such shape of the waveguides ensured the steady propagation of light and its localization near the waveguide at a distance of 40 μm (as it was found by the numerical calculations). After fabricating the waveguides, the ends of the optical chip were sanded to a depth of 150 μm and polished to provide the best input/output ratio of the optical signal. The curvature radius was 80 mm, which provided a transfer efficiency in a double-bending waveguide structure of more than 90% in accordance with the numerical calculation for 2 bend \((Tr)^2 = 95\%\); see Fig. 3.

In order to efficiently transfer radiation into the cavity, we need to find the appropriate parameters of the working area near the surface with the optimal depth of the waveguide and the length (see Fig. 4). To ensure the interaction of the optical field of the waveguide with the microcavity, the chip surface could be polished, which determines the actual depth of the waveguide region interacting with the microcavity. We simulated numerically the radiation properties in such waveguide systems. The light is reflected from the crystal–air interface at the approach of radiation to the operative region (where the ring cavity is located). As a result, the maximum of the radiation distribution in the transverse direction of the asymmetric waveguide is shifted deeper into the crystal. This effect is due to the small difference of refractive indices. This waveguide structure has an optimal distance for the radiation transfer from the wave-
Fig. 4  Numerical simulation of the ring cavity excitation by the waveguide field. (The waveguide mode is excited from the right side mode.) Diameter of the cavity is \( D_{\text{cavity}} = 110 \, \mu\text{m} \), and the wavelength is \( \lambda = 808 \, \text{nm} \). a Diameter of the waveguide is \( d_{\text{core}} = 4 \, \mu\text{m} \). b Diameter of the waveguide is \( d_{\text{core}} = 2 \, \mu\text{m} \). The enhancement of the ring cavity mode and out waveguide mode with the decrease in waveguide diameter \( d_{\text{core}} \) up to 2 \( \mu\text{m} \) is observed and the field is scattered partially in air.

uider to the crystal–air interface \( d_{\text{surf}} \) and then to the resonator. In this case, the initial diameter of the waveguide \( d_{\text{core}} \) should be also reduced to some optimal value. For instance, the excitation of the resonator (with the diameter \( D_{\text{cavity}} = 110 \, \mu\text{m} \) and the refractive index of the crystal \( n_{\text{cavity}} = 1.47 \)) is shown in Fig. 4 for the radiation wavelength \( \lambda = 808 \, \text{nm} \) and the polarization along the vertical axis \( E_y \) at the distance from the waveguide to the interface \( d_{\text{surf}} = 1 \, \mu\text{m} \). Thus, these calculations showed that the effective excitation of the resonator with the diameter of 110 \( \mu\text{m} \) is possible at the waveguide core diameter of 2 \( \mu\text{m} \) and the waveguide bending radius of 0.1 m.

5 Conclusion

We have proposed the photon–photon CNOT gate on counterpropagating photon modes in a ring cavity containing a two-level atom. It was found theoretically that the CNOT gate can be implemented at the optimal coupling constants of the photon–atom interaction in the ring cavity and the coupling constant of the photons propagating clockwise and counterclockwise. It is shown that the CNOT gate can be implemented deterministically with a small number of operation steps in a simple optical scheme. In our scheme, in addition to the proposed photon–photon CNOT gate, the single-qubit gates can also be implemented by the additional control of the atomic state which is necessary for the universal quantum computing.

We have considered the possible implementation of the proposed CNOT operation in an on-chip waveguide scheme. We have conducted a numerical simulation for such a scheme and carried out the experimental studies of the light propagation in the femtosecond-laser-written bending waveguide fabricated in quartz glass. The optimal parameters of the waveguide systems were found for the 90% efficiency of waveguide light transmission to the cavity area. Thus, these studies demonstrated possible ways for the on-chip implementation of the proposed photon–photon CNOT gate.
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