Superheavy particles either for UHECR or for muon anomaly

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Abstract

We show that, according to the scheme of spontaneous breakings starting from a GUT with symmetry $E_6$, it is possible that either a superheavy particle without ordinary interactions is source of ultra-high energy cosmic rays or a not so heavy lepton mixes with muon explaining the recently observed discrepancy of the anomalous magnetic moment of the latter.

1 Introduction

To the already much discussed problem of the roughly isotropic cosmic rays with energy above $10^{20}$ eV [1], one may add the recently observed discrepancy [2] in the muon anomalous magnetic moment (MAM) as requiring theories beyond the Standard Model (SM) of elementary particles.

There are many explanations of ultra-high energy cosmic rays (UHECR) based on decaying superheavy objects with lifetime larger than the universe age [3], as well as possible contributions of new particles to add to the theoretical evaluation of MAM and fill the $2.6−\sigma$ gap up to the experimental value [4]. But since the simplest solution of the latter problem is to include a heavy lepton which mixes with muon, one may inquire whether the same scheme offers a superheavy particle which might be origin of UHECR. In the frame of Grand Unification Theories (GUT) the most convenient symmetry is $E_6$, which may come down from the more fundamental superstring theory, because it contains for each generation a heavy charged lepton and one particle without ordinary interactions and therefore with possible great stability.

The feasibility of the model depends on details of the Higgs fields which produce the breaking of symmetry from $E_6$ down to QCD and electromagnetism giving mass subsequently to superheavy particles and to ordinary ones.
We will show that there are essentially two interesting alternative chains. In the first the particle without ordinary interactions is heavier than the exotic lepton and can decay in it through non-standard gauge bosons, having therefore a short lifetime with no possibility of explaining the UHECR. But since the exotic doublet of leptons mixes strongly with the ordinary one, the muon acquires a relevant flavour-changing coupling with Higgs which may give a contribution to the MAM of the order of the present discrepancy with the experimental value. The second possible chain gives an exotic lepton with a mass larger than that of the particle without ordinary couplings and with a weak mixing with light particles, so that it contributes negligibly to MAM. But now the particle without ordinary couplings decays through virtual exotic fermions whose extremely low mixing with light ones may produce a lifetime as large as the universe age allowing it as source of UHECR.

2 $E_6$ and its breaking

The GUT model based on the symmetry of the exceptional group $E_6$ has 78 gauge bosons of which 45 are those predicted by $SO(10)$ and the rest will be called $X$. The left-handed fermions are normally placed in the fundamental 27-dimensional representation, being the ordinary ones including $\nu^c$ in the representation 16 of $SO(10)$, an exotic lepton doublet $\begin{pmatrix} N \\ E \end{pmatrix}$ together with the singlets $N^c$ and $E^c$ and those of charge $-\frac{1}{3}$ quarks $D$ and $D^c$ in a representation 10, and finally a fermion $L$ without interactions with the 45 $SO(10)$ bosons in the trivial representation 1. As it is usual, we work with the charge conjugated left components instead of the right-handed ones.

$E_6$ symmetry must break at a high scale to that of the SM, and the latter to the present one of QCD and electromagnetism at low electroweak (EW) scale. We will assume a detailed high scale chain passing through the maximum subgroups involving the intermediate GUT symmetries $SO(10)$ and $SU(5)$ that is a relevant scheme for producing cosmic strings, which are also of cosmological interest, because of the breaking of the accompanying abelian groups $[5]$. Obviously to unify couplings of SM according to $SU(5)$ also contributions of supersymmetry (SUSY) are needed $[6]$. Therefore the succession of symmetries that we consider is

$$E_6 \rightarrow SO(10) \times U(1) \rightarrow SO(10) \rightarrow SU(5) \times \tilde{U}(1) \rightarrow$$

$$SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1) \rightarrow SU(3)_C \times U(1)_{em}$$

(1)

The fermions in the fundamental 27-plet are distributed $[7]$ according to the representations of $SO(10) \times U(1)$

$$27 = 16^{1/4} + 10^{-1/2} + 1^1$$

(2)

and then to those of $SU(5) \times \tilde{U}(1)$ through

$$16 = S^{3/2} + 10^{-1/2} + 1^{-5/2}$$
The gauge bosons are in the self-adjoint representation 78 which decomposes in \( SO(10) \times U(1) \) as

\[
78 = 45^0 + 1^0 + 16^{-3/4} + \overline{16}^{3/4}
\]

and the subsequent \( SU(5) \times \overline{U}(1) \) components

\[
45 = 24^0 + 10^2 + 10^{-1/2} + 1^0
\]

\[
\overline{16} = 1^{5/2} + 10^{1/2} + 5^{-3/2}
\]

The Higgs fields responsible for the breakings shown in Eq.(1) may be in the representations 27 and 78, but it is necessary at least one more to give masses to all the fermions through Yukawa terms according to

\[
27 \times 27 = \overline{27} + 351 + 351'
\]

which, for the purposes to be discussed in the next Sections, is the 351 with \( SO(10) \times U(1) \) components

\[
351 = 144^{1/4} + 126^{-1/2} + 54^1 + 16^{-5/4} + 10^{-1/2} + 1^{-2}
\]

that in terms of \( SU(5) \times \overline{U}(1) \) are

\[
144 = 45^{-3/2} + 40^{1/2} + 24^{5/2} + 15^{1/2} + 10^{1/2} + 5^{-3/2} + 5^{-7/2}
\]

\[
126 = 50^{-1} + 45^1 + 15^{3/2} + 10^{-3} + 5^{-1} + 1^{-5}
\]

\[
54 = 24^0 + 15^2 + 15^{-2}
\]

3 Alternative useful for muon anomalous magnetic moment

For the six breakings of Eq.(1) we use eight expectation values of Higgs fields, which for economy will be taken in the representations 78 and 351, to give masses to all fermions and mixing of ordinary with exotic ones.

78 has no influence on fermions as is seen in Eq.(6), but is needed to break \( E_6 \) because it contains \( 1^0 \) which is invariant under \( SO(10) \times \overline{U}(1) \) and to break \( SO(10) \) through 450 which contains \( 1^0 \) of \( SU(5) \times \overline{U}(1) \).

We use all the \( SO(10) \times \overline{U}(1) \) representations of 351. \( 1^{-2} \) is necessary to break \( SO(10) \times \overline{U}(1) \) giving mass to \( L \) which will be therefore lighter than \( X \) gauge bosons, with the possible exception of the \( \overline{Z} \) associated to \( \overline{U}(1) \), but heavier than the other 10 exotic fermions. \( 126^{-1/2} \) is used to break \( SU(5) \times \overline{U}(1) \) giving mass to \( \nu^c \), noting that one has to take for the Higgs the complex conjugate of its \( SU(5) \times \overline{U}(1) \) representations according to Eq.(6).
To complete the breakings at GUT scale of $SU(5)$ we use $54^1$ and $144^{1/4}$ which give mass to exotic fermions and mix them with ordinary ones, respectively. This is because they both contain, according to Eq.(8), a 24 of $SU(5)$ which has a component invariant under $SU(3)_C \times SU(2)_L \times U(1)$ that gives the Yukawa couplings

$$\phi(54, 24)(D^c D - \frac{3}{2} E^c E - \frac{3}{2} N^c N)$$
$$\phi(144, 24)(d^c D - \frac{3}{2} E^c e - \frac{3}{2} N^c \nu)$$

Due to the fact that mass and mixing terms are analogous, which would not happen in case of including $351'$, at GUT scale the massless states apart from $u$ will be

$$d_0 = d, \quad d^c_0 = d e^c \cos \theta \pm D^c \sin \theta$$
$$e_0 = e, \quad e^c_0 = e \cos \theta \pm E \sin \theta$$
$$\nu_0 = \nu \cos \theta \pm N \sin \theta$$

with orthogonal heavy mass states $\hat{E}$ etc.

The peculiar feature of equal mixing, even with large $\theta$, of $e$ and $\nu$ and no mixing of $d$ and $u$ leads to unchanged charged weak interactions for ordinary fermions, and additionally Eq.(10) produces no change $\phi(54, 24)$ in the neutral current ones.

At this stage we may already anticipate that this scheme will not allow $L$ to be source of UHECR. In fact, if the mass of $Z$ is of the same order but lower than that of $L$ the latter will decay according to Fig.1 provided there is mixing of $L$ and $\nu^c$ because of Eqs.(2, 4). With the approximation $M_{\nu^c} << M_L$, the lifetime $\tau_L$ would be

$$\tau_L^{-1} \simeq \frac{k}{4} M_L \frac{1}{\lambda}(1 - 3\lambda^2 + 2\lambda^3), \quad \lambda = \frac{M_Z^2}{M_L^2},$$

where $k$ will depend on the mixing. In case that this is large $k/4 \sim 10^{-2}$ because it contains a coupling $\alpha_{GUT}$ and with $M_L \sim 10^{16}$ GeV the result would be $\tau_L \sim 10^{-38}\text{sec}$!

But even without mixing of $L$ and $\nu^c$ or if $M_X > M_L$, the decay of $L$ would be fast enough through Fig.2 which would not require mixing of $L$ and $E$ with ordinary fermions $O$ if gauge boson $X$ belongs to 16 according to Eqs.(2, 4). But mixing $O$ and $E$ is necessary for the subsequent decay of the latter. With $\lambda = M_X^2/M_L^2$, $G = \Gamma_X/M_L^2$ where $\Gamma_X$ is the $X$ width and in the approximation $M_L >> M_E, m_O$ the lifetime is

$$\tau_L^{-1} \simeq \frac{\alpha_{GUT}^2 M_L^2}{24\pi} \frac{1}{M_X^2} \lambda^2 \int_0^\infty dx x^2 \left(\frac{3 - 4x}{(1 - 2x - \lambda)^2 + \lambda G}\right),$$

which, taking $M_X \sim 10^{17}$ GeV, gives $\tau_L \sim 10^{-32}\text{sec}$. 


Going to the breaking of SM, this may be done by a Higgs in $10^{-1/2}$ giving mass to ordinary fermions and mixing $L$ with $N$ according to

$$H(10, \bar{5}) (d^c d + e^c e + N^c L) + H(10, 5)(u^c u + \nu^c \nu + LN) \quad (13)$$

The mixing $L - N$ allows another possible channel of decay of the former but less important than those of Fig.1, in case of large $k$, and Fig.2 because it necessarily occurs at EW scale.

Moreover the $16^{-5/4}$ through its $SU(5) \times \tilde{U}(1)$ component $1^{-5/2}$ gives a mixing of $L$ with $\nu^c$ which is small if it occurs at this EW scale, but it might be large if it was produced at $SU(5) \times \tilde{U}(1)$ breaking where such expectation value was possible. However $16^{-5/4}$ is the only component of $351$ which is not necessary for this alternative.

The eventual contribution of new particles [10] to the MAM has renewed its interest due to the $2.6 - \sigma$ discrepancy between experimental and SM calculation

$$\Delta a_\mu = \Delta \left( \frac{g-2}{2} \right)_\mu \approx 4 \times 10^{-9}. \quad (14)$$

Recent explanations come from SUSY [11] and perhaps extra dimensions [12] but also from the exotic fermion [13] of $E_6$.

In fact with the present scheme of breakings based on $351$, the Yukawa couplings for $e$ and $E$ of Eqs.(9, 13) through the mixing of Eq.(10) will give a "flavour" changing coupling of the physical four-component $e_0$ and $\hat{E}$ with the Higgs field $h$ added to the vacuum expectation value

$$\mathcal{L}_{FC} = \kappa e_0 (\alpha - \beta \gamma_5) \hat{E} h + h.c. \quad , \quad (15)$$

where $\kappa$ depends on the mixing angle and on the parameters necessary to give the value of the lepton masses. An analogous term for the second generation of fermions will produce a contribution [14] to MAM according to Fig.3

$$\Delta a_\mu \simeq \frac{1}{8\pi^2 M_M} \frac{m_\mu}{\kappa^2 F(M_h/M_M)} \quad , \quad (16)$$

where, for $M_h << M_M$, $F(0) = \frac{1}{2}$. Therefore, being $\kappa \approx 1$, to fill the present discrepancy Eq.(14) it is necessary that the mass of the exotic lepton $M_M \lesssim 10^5$GeV. Since in the present scheme the mixing is large because it occurs at GUT scale, it is not unreasonable to have the above maximum values of $\kappa$ and $M_M$.

It is interesting that in a different approach in which light leptons are coupled to heavy leptons and pions [15] a contribution to $\Delta a_\mu$ analogous to Eq.(16) may be consistent to UHECR due to the increase of $\nu$-nucleon cross-section [16].

We may remark that the law of Eq.(16) evades the treatment of Ref. [4] because our coupling Eq.(15) requires mixing of two Higgs. On the contrary, the effective couplings coming from SUSY or extra dimensions would give $\Delta a_\mu \sim \frac{m_\mu}{\Lambda^2 (\sqrt{\Lambda})^2}$ which, to satisfy the experimental Eq.(14), would require a scale for new physics $\Lambda < 1$ TeV. Therefore, corrections of MAM like Eq.(16) are not
pressed by a too close new physics scale, though for the alternative explanations different masses of SUSY partners or sum of Kaluza-Klein modes in extra dimension make their situation easier.

4 Alternative useful for ultra-high energy cosmic rays

We choose now to break the symmetries of Eq.(1) using eight expectation values of Higgs in all the $SO(10) \times U(1)$ components of $27$ plus the minimum needed ones of $78$ and $351$.

Regarding $78$, we use as before its components $1^0$ to break $E_8$ and $45^0$, with $1^0$ of $SU(5) \times \tilde{U}(1)$, to break $SO(10)$. In addition we use again $45^0$, but with its $24^0$ of $SU(5) \times \tilde{U}(1)$, to break $SU(5)$ keeping the SM symmetry and without influence on fermions.

Also two components of $351$ are necessary: $1^{-2}$ to give mass to $L$ and $126^{-1/2}$ for $\nu^c$ as before.

Now considering $27$, $1^1$ is required to give mass to the 10 exotic fermions according to

$$\phi(1,1)(D^c D + E^c E + N^c N)$$

Since presumably the expectation value of $1^1$ will appear at the same scale of $1^{-2}$ corresponding to the breaking of $SO(10) \times U(1)$, it will depend on the constants of the Yukawa couplings if $L$ is heavier or lighter than the exotic fermions. It would be also possible that one of these expectation values develops at a lower scale.

To break SM we need $10^{-1/2}$ which will give mass to ordinary fermions and mixing of $L$ and $N$ as in Eq.(13).

Finally $16^{1/4}$ mixes ordinary and exotic fermions, which is necessary because otherwise the latter would be stable, according to

$$H(16, 1)(d^c D + E^c e + N^c \nu) + H(16, 5)(D^c d + e^c E + \nu^c N)$$

The expectation value of $SU(5)$ $\bar{5}$ may appear breaking SM at the EW scale.

On its hand the $1^{-5/2}$ component of the first term of Eq.(18) might appear earlier, at the breaking of $SU(5) \times \tilde{U}(1)$. If this occurs together with Eq.(17) it would produce a mixing of the same type of Eq.(10) but with smaller angle because of the higher scale of the mass term. Therefore a discussion analogous to that of Sect.3 will lead to a contribution to MAM smaller than the experimental discrepancy Eq.(14) due to smaller mixing and larger $M_M$ in Eq.(16).

Alternatively, if $H(16, 1)$ appears at the same EW scale of $H(16, \bar{5})$ the mixing of exotic and ordinary fermions will be even smaller giving way to negligible corrections of charged and neutral weak interactions and of MAM.

But now the situation regarding UHECR will be different because $L$ cannot decay to $\nu^c$ since there is no mixing between them, and if $M_L < M_E$, the decay of $L$ will be given by $L \rightarrow OO \overline{O} V$ with $V$ vector boson of $SO(10)$ 45 which
includes the twelve of SM as is seen in Fig.4. The coupling $\mathcal{E}OV$ is possible, because of $U(1)$ charge conservation of Eqs.(2, 4), only if there is mixing of $\mathcal{E}$ and $O$.

If e.g. the scale of $27(1^1)$ is of the order of breaking of $SO(10) \times U(1)$ but that of $351(1^2)$ is smaller, it is possible to have $M_L \sim 10^{12}$ GeV corresponding to UHECR whereas $M_E \sim 10^{16}$ GeV and $M_X \sim 10^{17}$ GeV due to breaking of $E_6$. Therefore the decay of Fig.4 could be replaced by an effective coupling $\sim \frac{1}{M_X^2 M_E} \overline{\mathcal{O}} \gamma^\mu \mathcal{L} \mathcal{O}_{\gamma^\mu\nu} O_{\nu \gamma}$. Since $M_V \ll M_L$, an estimation of $L$ lifetime is

$$\tau_L^{-1} \sim \alpha_{\text{GUT}}^2 \alpha_M 10^{-7} \frac{M_L^8}{M_X^4 M_E^2 M_V}. \quad (19)$$

Introducing the above masses and $M_V \sim 10^2$ GeV Eq.(19) would give

$$\tau_L^{-1} \sim \alpha_M \frac{10^7}{\text{sec}}, \quad (20)$$

of the order of universe age $t_0 \sim 10^{18}$ sec for $\alpha_M \sim 10^{-25}$, not unreasonable for the square of the extremely small mixing between EW and GUT scales. Then $L$ might be origin of UHECR.

5 Conclusions

We have seen that one possible alternative for Higgs responsible for the breaking of $E_6$ symmetry passing through $SO(10)$ and $SU(5)$ gives a strong mixing of ordinary and exotic leptons which might allow a not too high mass of the latter and consequently a correction of the $\mu$ magnetic moment of the order of the discrepancy between experimental measurement and Standard Model calculation. The qualitative difference with other explanations based on supersymmetry or extra dimensions is that in our case the scale of new physics is well above the TeV region.

Another alternative of Higgs for the same chain of symmetry breakings starting from $E_6$ produces very heavy fermions in the $SO(10)$ decuplet with extremely small mixing with ordinary ones, so that the $L$ particle in $SO(10)$ singlet with mass $\sim 10^{12}$ GeV may have a lifetime of the order of universe age and explain the ultra high energy cosmic rays. It is clear that to avoid present overclosure of universe $L$ particles must have been produced non-thermally to be now a small fraction of dark matter.

Our proposal does not require non-renormalizable interactions for the decay of $L$ at variance from other hypothetical superheavy particles like cryptons [17], protected by half-integer electric charge and a hidden gauge invariance, or those coming from string models in a sector with broken GUT like unitons and singletons, protected by a discrete symmetry [18].

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