The model of fiber-reinforced concrete element deformation under the conditions of plane stress state and subjected to radiation and its identification

R B Garibov and I G Ovchinnikov
Balakovo Institute of Engineering and Technology of the National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 140 Chapaeva Street, Saratov Region, Balakovo, 413853, Russia
garibovr@mail.ru

Abstract. A model of fiber-reinforced concrete element deformation was presented under the conditions of plane stress and subjected to radiation, leading to the change of fibre-concrete deformation diagram and radiation swelling. They proposed a model identification technique, which use determined the coefficients for the cases of nonlinear and linear approximation of deformation diagrams, and the approximation error for these cases was estimated. Using the given physical relationships and a number of additional hypotheses, a non-linear integral-differential resolution equation of a fiber-reinforced concrete plate on an elastic foundation is obtained under radiation conditions.

The work constructions under the conditions of radiation exposure are often made of fiber-reinforced concrete and are a combination of plate or shell elements [1, 2]. As a design scheme of a plate-like element, a rectangular plate model is used usually with one or another type of contour support, because of a plane stress state material. Let's consider the physical relationships for a plane stress state for fiber-reinforced concrete, taking into account the effect of radiation exposure.

The physical relationships for this case take the following form, taking into account the effect of radiation:

\[
\begin{align*}
\sigma_{ij}^{\delta\delta} &= \frac{\gamma_j^{\delta\delta}(e_a, \Phi)}{1 - v_j^{\delta\delta}(e_a, \Phi)} \left[ e_i + v_j^{\delta\delta}(e_a, \Phi) \cdot e_j \right] - \left[1 + v_j^{\delta\delta}(e_a, \Phi) \right] \gamma_j^{\delta\delta}(e_a, \Phi) \cdot e_{\delta\delta} \\
\sigma_i^{\delta\delta} &= \frac{\gamma_j^{\delta\delta}(e_a, \Phi)}{1 - v_j^{\delta\delta}(e_a, \Phi)} \left[ e_i + v_j^{\delta\delta}(e_a, \Phi) \cdot e_j \right] - \left[1 + v_j^{\delta\delta}(e_a, \Phi) \right] \gamma_j^{\delta\delta}(e_a, \Phi) \cdot e_{\delta\delta} \\
\varepsilon_{\delta\delta}^{\delta\delta} &= \frac{\gamma_j^{\delta\delta}(e_a, \Phi)}{2(1 + v_j^{\delta\delta}(e_a, \Phi))} \varepsilon_{\delta\delta}
\end{align*}
\]

Here \(\sigma_{ij}^{\delta\delta}, \sigma_i^{\delta\delta}, \varepsilon_{\delta\delta}^{\delta\delta}\) are the components of the stress tensor, \(e_i, e_j, e_{\delta\delta}\) – are the components of the strain tensor, \(e_a\) – deformation intensity, \(e_{\delta\delta}^{\delta\delta}\) – radiation swelling, determined by the following expression:
where $e_{\text{max}}$ is the maximum value of radiation deformations for fiber concrete of a given composition, $\delta, \nu$ – the empirical coefficients, depending on the radiation deformation of a filler and the energy spectrum of the neutron flux. The kinetics of its change is shown schematically in Figure 1.

The remaining functions have the following form: 

$$
\psi_j^{\phi\delta} = \frac{\sigma_{u}^{\phi\delta}(e_u, \Phi)}{e_u},
$$

$$
\nu_j^{\phi\delta}(e_u, \Phi) = \frac{1}{2} - \frac{2\nu_0}{2E_0} \psi_j^{\phi\delta}(e_u, \Phi),
$$

![Figure 1. The kinetics of radiation deformation (swelling) change (scheme).](image)

For $\psi_j^{\phi\delta}$ they use the following dependence (Figure 2, 3):

$$
\psi_j^{\phi\delta} = \begin{cases} 
(A_j - B_j e_u^j), & \Phi < \Phi_{1\text{upp}} \\
(A_j - B_j e_u^j) f(\Phi), & \Phi \geq \Phi_{1\text{upp}} 
\end{cases}
$$

$f(\Phi) = 1 - \alpha_0 (\text{lg} \beta_0 \Phi)$, $\Phi$ is the dose of neutron irradiation. Here $\nu_0, E_0$ is the coefficient of transverse deformation and the modulus of elasticity at small linear deformations. The dependence (3) can be used in the cases when the effect of irradiation leads to a similar change in a generalized strain curve $\sigma_0^{\phi\delta}(e_u, \Phi), \sigma_0^{\phi\delta}(e_u, \Phi)$. If the irradiation effect leads to an inconvenient change in a generalized strain curve, the relationship of the following form can be used:

$$
\psi_j^{\phi\delta} = \begin{cases} 
(A_j - B_j e_u^j), & \Phi < \Phi_{1\text{upp}} \\
(A_j (\Phi) - B_j (\Phi) e_u^j), & \Phi \geq \Phi_{1\text{upp}} 
\end{cases}
$$

where $A_j(\Phi) = A_{j0}(1 - \alpha_{j1}(\text{lg} \beta_{j1} \cdot \Phi), \ B_j(\Phi) = B_{j0}(1 - \alpha_{j1}(\text{lg} \beta_{j1} \cdot \Phi)$.

In these relations, the nonlinearity of fiber concrete deformation and its unequal work on tension and compression are taken into account, it is assumed that any point is in the stretched state ($j=1$) if the average stress is $\sigma_0 \geq 0$, and in the compressed state ($j=2$), if $\sigma_0 < 0$.

$$
\sigma_0^{\phi\delta} = (\sigma_0^{x\phi\delta} + \sigma_0^{y\phi\delta}) / 3
$$
To use the physical relationships (3), it is necessary to have the experimental values of the coefficients in the function $\psi_{j}(\varepsilon, \Phi)$, as well as the experimental data on transverse deformation coefficient $\nu_{j}(\varepsilon, \Phi)$. The coefficients for the dependence of the form (4) can be determined by the approximation of the experimental fibre-concrete deformation diagram by the following function:

$$
\sigma = \begin{cases}
A_{1}^{\phi\delta}(\Phi) \cdot e - B_{1}^{\phi\delta}(\Phi) \cdot e^{\phi}, & \sigma \geq 0 \\
A_{2}^{\phi\delta}(\Phi) \cdot e - B_{2}^{\phi\delta}(\Phi) \cdot e^{\phi}, & \sigma < 0
\end{cases}
$$

(5)

The coefficients $A_{i}^{\phi\delta}(\Phi)$, $B_{i}^{\phi\delta}(\Phi)$ in the assumption, that $n=3$ are determined from the conditions of the functional minimum:

$$
I = \sum_{i=1}^{N} (A_{i}^{\phi\delta} \varepsilon_{i} - B_{i}^{\phi\delta} \varepsilon_{i}^{3} - \sigma_{i})^{2} \rightarrow \min,
$$

(6)

Which lead to the following formulae:

$$
A_{i}^{\phi\delta} = \frac{\sum_{i=1}^{N} \sigma_{i} \varepsilon_{i}}{\sum_{i=1}^{N} \varepsilon_{i}^{3} - \sum_{i=1}^{N} \varepsilon_{i}^{4}},
$$

(7)

$$
B_{i}^{\phi\delta} = \frac{\sum_{i=1}^{N} \sigma_{i} \varepsilon_{i}^{3} - \sum_{i=1}^{N} \sigma_{i} \varepsilon_{i}^{4}}{\left(\sum_{i=1}^{N} \varepsilon_{i}^{2}\right)^{2}},
$$

(8)
Where $N$ is the number of experimental points on the deformation curves. In these formulas, the index $j$ corresponding to the stretching ($j = 1$) or compression ($j = 2$) is omitted for simplicity. Having determined the values of the coefficients $A_j^{\Phi\epsilon}(\Phi), B_j^{\Phi\epsilon}(\Phi)$ for several influence values $\Phi_k$ and approximating the obtained values $A_j^{\Phi\epsilon}(\Phi_k), B_j^{\Phi\epsilon}(\Phi_k)$ by the most suitable functions, we find the required dependences $A_j^{\Phi\epsilon}(\Phi), B_j^{\Phi\epsilon}(\Phi)$.

If the nonlinearity is not taken into account during the modeling of fiber-reinforced concrete behavior, and only the heterodimodity is taken into account, then the dependence (4) takes the following form:

$\psi_j = \begin{cases} E_j^0 (\Phi < \Phi_{\text{inap}}), & j=1 \text{ at } \sigma_0^j \geq 0, \ j=2 \text{ at } \sigma_0^j \leq 0, \\ E_j^0 (\Phi \geq \Phi_{\text{inap}}), & \end{cases}$  \hspace{1cm} (9)

where $E_j$ is the modulus of fiber concrete elasticity during tension $E_1^{\Phi\epsilon}(\Phi)$ and compression $E_2^{\Phi\epsilon}(\Phi)$. In this case, in order to find $E_j$ the diagram of fibro-concrete deformation is approximated by the following function:

$\sigma = \begin{cases} E_1^{\Phi\epsilon}(\Phi) \cdot e, & \sigma \geq 0, \\ E_2^{\Phi\epsilon}(\Phi) \cdot e, & \sigma < 0, \end{cases}$ \hspace{1cm} (10)

and the coefficients $E_j$ are determined from the condition of the functional minimum:

$I = \sum_{i=1}^{N} (\sigma_i - E e_i)^2 \rightarrow \min,$ \hspace{1cm} (11)

which provides the following:

$E = \left( \sum_{i=1}^{N} \sigma_i e_i \right) / \left( \sum_{i=1}^{N} e_i^2 \right),$ \hspace{1cm} (12)

Due to the experimental data lack on the effect of irradiation on the fiber concrete behavior, we confine ourselves to the determination of the coefficients for fiber-reinforced concrete with the TSMID addition in normal non-radiative conditions [3]. Table 1 and 2 below are the results of deformation curve approximation of fiber-reinforced concrete with the addition of TSMID under tension and compression.

It can be seen that the error in experimental data approximation during the use of the linear model (10) reaches 20% on average. (Large values of errors at small values of stresses can be ignored). The use of the nonlinear model (5) leads to the approximation error not exceeding 5%.

**Table 1.** Coefficient values: $A_f=30.71 \times 10^3$ MPa, $B_f=11.79 \times 10^3$ MPa, $E_f=22.21 \times 10^3$ MPa, (stretch).

| Experimental stresses, MPa | Experimental deformation, $\epsilon$, % | Theoretical stress by non-linear model, MPa | Theoretical stress by linear model, MPa | Error by non-linear model, MPa | Error by linear model, MPa | Error by non-linear model, % | Error by linear model, % |
|---------------------------|----------------------------------------|------------------------------------------|----------------------------------------|-----------------------------|-----------------------------|--------------------------------|-----------------------------|
| 1.00                      | 0.00003                                | 0.98                                     | 0.71                                   | -0.02                       | -0.29                       | -1.71                          | -28.65                       |
| 2.00                      | 0.00005                                | 1.52                                     | 1.11                                   | -0.48                       | -0.89                       | -23.99                         | -44.50                       |
| 3.00                      | 0.00010                                | 3.05                                     | 2.30                                   | 0.05                        | -0.70                       | 1.62                           | -23.36                       |
| 4.00                      | 0.00014                                | 3.96                                     | 3.09                                   | -0.04                       | -0.91                       | -1.06                          | -22.70                       |
| 5.00                      | 0.00020                                | 5.20                                     | 4.44                                   | 0.20                        | -0.56                       | 3.95                           | -11.21                       |
| 6.00                      | 0.00032                                | 5.96                                     | 7.14                                   | -0.04                       | 1.14                        | -0.73                          | 18.92                        |
When the deformation equation is derived for a fiber-reinforced concrete plate, taking into account the effect of irradiation, we will use physical relations in the form (1), and it will be considered the Kirchhoff-Love hypotheses as fair ones:

$$e_x = e_x + \chi_x z; \quad e_y = e_y + \chi_y z; \quad e_{xy} = e_{xy} + 2\chi_{xy} z;$$

(13)

where $e_x, e_y, e_{xy}$ – the deformation of the middle surface points, $\chi_x, \chi_y, \chi_{xy}$ – the curvatures at these points, $z$ – the coordinates of the points considered from the middle surface.

Taking into account this expression, it will have the following form for the moments and the forces in a rectangular coordinate system:

$$M_{x}^{\phi \phi} = \int_{-h/2}^{h/2} \begin{vmatrix} \sigma_{x} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{y} \end{vmatrix} \frac{\partial \tilde{z}}{\partial x} dz = \int_{-h/2}^{h/2} \sigma_{x} \tilde{z} \frac{\partial \tilde{z}}{\partial x} dz$$

$$N_{y}^{\phi \phi} = \int_{-h/2}^{h/2} \begin{vmatrix} \sigma_{y} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{x} \end{vmatrix} \frac{\partial \tilde{z}}{\partial y} dz = \int_{-h/2}^{h/2} \sigma_{y} \tilde{z} \frac{\partial \tilde{z}}{\partial y} dz$$

$$M^{\phi \phi} = \int_{-h/2}^{h/2} \begin{vmatrix} \sigma_{x} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{y} \end{vmatrix} \frac{\partial \tilde{z}}{\partial x} \frac{\partial \tilde{z}}{\partial y} \begin{vmatrix} \sigma_{x} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{y} \end{vmatrix} \frac{\partial \tilde{z}}{\partial x} dz = \int_{-h/2}^{h/2} \sigma_{x} \tilde{z} \frac{\partial \tilde{z}}{\partial x} dz$$

$$N^{\phi \phi} = \int_{-h/2}^{h/2} \begin{vmatrix} \sigma_{y} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{x} \end{vmatrix} \frac{\partial \tilde{z}}{\partial y} \frac{\partial \tilde{z}}{\partial x} \begin{vmatrix} \sigma_{x} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{y} \end{vmatrix} \frac{\partial \tilde{z}}{\partial x} dz = \int_{-h/2}^{h/2} \sigma_{y} \tilde{z} \frac{\partial \tilde{z}}{\partial y} dz$$

(14)

here $z_0$ is the equation of the neutral surface, determined from the condition $\sigma_0=0$ and separating the stretched zone of the plate from the compressed one; $i, j$ – the indices characterizing the compressed and the stretched zone of a plate. With the stretched bottom zone of the bending plate $j=1$, $i=2$, with the stretched upper zone and the compressed lower one $j=2$, $i=1$, $h$ – the plate thickness. The expression for $z_0$ is obtained from the condition $\sigma_0=0$, which is reduced to the following form:

$$e_x + e_y = 2(1 - v_i^2) \cdot e_0^{\phi \phi}.$$  

(15)

or taking into account (13)

$$z_0 = e_x + e_y + \frac{2(1 - v_j^2) \cdot e_0^{\phi \phi}}{\chi_x + \chi_y}.$$  

(16)
There is some uncertainty here associated with the choice of the value \( v_j \). Since the coefficient of transverse deformation cannot change abruptly in the zone of stress sign change due to nonphysicality, then in the formula (16) we will use the following value instead of \( v_j \):

\[
v_{wp} = (v_j + v_i) / 2.
\]  

Finally,

\[
z_0 = \frac{\varepsilon_x + \varepsilon_y + 2(1 - v^2) \cdot \varepsilon_{\phi}}{\chi_x + \chi_y}.
\]  

Substituting (1) into (14) after some transformations, we will obtain the following expressions:

\[
M_{x}^{\phi} = \varepsilon_x J_1^{\phi} + \varepsilon_y I_1^{\phi} + \chi_x J_2^{\phi} + \chi_y I_2^{\phi} + \Delta M_{\phi}^{\phi};
\]

\[
M_{y}^{\phi} = \varepsilon_y J_1^{\phi} + \varepsilon_x I_1^{\phi} + \chi_y J_2^{\phi} + \chi_x I_2^{\phi} + \Delta M_{\phi}^{\phi};
\]

\[
H^{\phi} = \varepsilon_{xy} T_1^{\phi} + 2 \chi_{xy} T_2^{\phi};
\]

\[
S^{\phi} = \varepsilon_{xy} T_0^{\phi} + 2 \chi_{xy} T_1^{\phi};
\]

\[
N_x^{\phi} = \varepsilon_x J_0^{\phi} + \varepsilon_y I_0^{\phi} + \chi_x J_1^{\phi} + \chi_y I_1^{\phi} + \Delta N_x^{\phi};
\]

\[
N_y^{\phi} = \varepsilon_y J_0^{\phi} + \varepsilon_x I_0^{\phi} + \chi_y J_1^{\phi} + \chi_x I_1^{\phi} + \Delta N_y^{\phi};
\]  

Where they noted:

\[
J_k^{\phi} = \int_{z_0}^{z_0 + h/2} \alpha_j z^k \partial z + \int_{z_0}^{z_0 + h/2} \alpha_i z^k \partial z, \text{ at } k = 0, 1, 2;
\]

\[
I_k^{\phi} = \int_{z_0}^{z_0 + h/2} \alpha_j v_i z^k \partial z + \int_{z_0}^{z_0 + h/2} \alpha_i v_i z^k \partial z, \text{ at } k = 0, 1, 2;
\]

\[
T_k^{\phi} = \int_{z_0}^{z_0 + h/2} \beta_j z^k \partial z + \int_{z_0}^{z_0 + h/2} \beta_i z^k \partial z, \text{ at } k = 0, 1, 2;
\]

where:

\[
\alpha_j = \frac{\Psi_j}{1 - v_j^2}, \quad \alpha_i = \frac{\Psi_i}{1 - v_i^2}, \quad \beta_j = \frac{\Psi_j}{2(1 + v_j)}, \quad \beta_i = \frac{\Psi_i}{2(1 + v_i)}.
\]

\[
\Delta M_{\phi}^{\phi} = - \int_{z_0 - h/2}^{z_0} (1 + v_j) \psi_j \varepsilon_{\phi} \partial z - \int_{z_0}^{h/2} (1 + v_j) \psi_j \varepsilon_{\phi} \partial z,
\]

\[
\Delta N_x^{\phi} = - \int_{z_0 - h/2}^{z_0} (1 + v_j) \psi_j \varepsilon_{\phi} \partial z - \int_{z_0}^{h/2} (1 + v_j) \psi_j \varepsilon_{\phi} \partial z.
\]  

We assume that normal and shear stresses act on the plate contour:

\[
N_x = N_x^\Gamma, \quad N_y = N_y^\Gamma, \quad S = S^\Gamma.
\]  

Then from the conditions (23) we obtain the following expressions to determine the deformations \( \varepsilon_x, \varepsilon_y, \varepsilon_{xy} \):

\[
\varepsilon_x = f_1 \cdot \chi_x + f_2 \cdot \chi_y + \Delta \varepsilon_x^*,
\]

\[
\varepsilon_y = f_2 \cdot \chi_x + f_1 \cdot \chi_y + \Delta \varepsilon_y^*.
\]
\[
E_{xy} = -\left(\frac{(2T_1^{\phi\phi})^2}{T_0^{\phi\phi}}\right)\chi_{xy} + \frac{S^\Gamma}{T_0^{\phi\phi}}.
\]  
(24)

In these expressions:
\[
f_1 = \frac{I_0^{\phi\phi} I_1^{\phi\phi} - J_0^{\phi\phi} J_1^{\phi\phi}}{(J_0^{\phi\phi})^2 - (I_0^{\phi\phi})^2}, \quad f_2 = \frac{I_0^{\phi\phi} J_1^{\phi\phi} - J_0^{\phi\phi} I_1^{\phi\phi}}{(J_0^{\phi\phi})^2 - (I_0^{\phi\phi})^2},
\]
\[
\Delta e^x = f_3 \cdot \Delta N_1^{\phi\phi} + f_4 \cdot N_x^\Gamma + f_5 \cdot N_y^\Gamma, \quad \Delta e^y = f_3 \cdot \Delta N_2^{\phi\phi} - f_5 \cdot N_x^\Gamma + f_4 \cdot N_y^\Gamma,
\]
(25)

where
\[
f_3 = \frac{I_0^{\phi\phi} - J_0^{\phi\phi}}{(J_0^{\phi\phi})^2 - (I_0^{\phi\phi})^2}, \quad f_4 = \frac{J_0^{\phi\phi}}{(J_0^{\phi\phi})^2 - (I_0^{\phi\phi})^2}, \quad f_5 = \frac{I_0^{\phi\phi}}{(J_0^{\phi\phi})^2 - (I_0^{\phi\phi})^2}.
\]
(26)

Substituting the expression (24) into the formula for the moments \(M_x, M_y, H\) from (19) we will obtain the following expression for the bending and twisting moments after some transformations:
\[
M_x^{\phi\phi} = D_1 \cdot \chi_x + D_2 \chi_y + \Delta M^*_x;
\]
\[
M_y^{\phi\phi} = D_2 \cdot \chi_x + D_1 \chi_y + \Delta M^*_y;
\]
\[
H^{\phi\phi} = D_0 \chi_{xy} + \Delta H^*;
\]
(28)

Where they noted:
\[
\Delta M^*_x = D_3 \Delta N_3^{\phi\phi} + D_3 N_x^\Gamma + D_4 N_y^\Gamma + \Delta M_5^{\phi\phi};
\]
\[
\Delta M^*_y = D_3 \Delta N_4^{\phi\phi} + D_3 N_x^\Gamma + D_4 N_y^\Gamma + \Delta M_5^{\phi\phi};
\]
\[
\Delta H^* = \frac{S^\Gamma \cdot T_1^{\phi\phi}}{T_0^{\phi\phi}};
\]
(29)

in which
\[
D_1 = f_1 J_1^{\phi\phi} + f_2 I_1^{\phi\phi} + J_2^{\phi\phi}, \quad D_2 = f_2 J_1^{\phi\phi} + f_1 I_1^{\phi\phi} + I_2^{\phi\phi}, \quad D_3 = f_1 J_1^{\phi\phi} + f_2 I_1^{\phi\phi},
\]
\[
D_4 = f_2 J_1^{\phi\phi} - f_1 I_1^{\phi\phi}, \quad D_5 = f_4 I_1^{\phi\phi} - f_5 J_1^{\phi\phi}, \quad D_6 = 2T_2^{\phi\phi} - 2 \left(\frac{T_1^{\phi\phi}}{T_0^{\phi\phi}}\right)^2.
\]
(30)

Taking into account the fact that the curvatures can be expressed through the deflection of W plate in the following way:
\[
\chi_x = -\frac{\partial^2 W}{\partial x^2}; \quad \chi_y = -\frac{\partial^2 W}{\partial y^2}; \quad \chi_{xy} = -\frac{\partial^2 W}{\partial x \partial y}.
\]
(31)

The expressions for the moments will take the following form:
\[
M_x^{\phi\phi} = -D_1 \frac{\partial^2 W}{\partial x^2} - D_2 \frac{\partial^2 W}{\partial y^2} + \Delta M^*_x,
\]
\[
M_y^{\phi\phi} = -D_2 \frac{\partial^2 W}{\partial x^2} - D_1 \frac{\partial^2 W}{\partial y^2} + \Delta M^*_y,
\]
\[ H^{\phi_b} = -D_b \frac{\partial^3 W}{\partial x \partial y^2} + \Delta H^*. \] (32)

Considering a plate element of the size \( \partial x \cdot \partial y \) and the height \( h \) under the action of the intensity \( p \), normal to the load surface, one can obtain the following differential equation connecting the bending moments and external loads:

\[ \frac{\partial^2 M^{\phi_b}}{\partial x^2} + 2 \frac{\partial^2 H^{\phi_b}}{\partial x \partial y} + \frac{\partial^2 M^{\phi_b}}{\partial y^2} = -p. \] (33)

Substituting the expressions (32) in (33) we obtain the following resolving differential equation:

\[ \frac{\partial^2}{\partial x^2} \left( -D_1 \frac{\partial^2 W}{\partial x^2} - D_2 \frac{\partial^2 W}{\partial y^2} + \Delta M^* \right) + 2 \frac{\partial^2}{\partial x \partial y} \left( -D_b \frac{\partial^2 W}{\partial x \partial y} + \Delta H^* \right) + \frac{\partial^2}{\partial y^2} \left( -D_2 \frac{\partial^2 W}{\partial x^2} - D_1 \frac{\partial^2 W}{\partial y^2} + \Delta M^* \right) = -p. \] (34)

If you enter the following notation: \( q = \frac{\partial^2 \Delta M^*}{\partial x^2} + 2 \frac{\partial^2 \Delta H^*}{\partial x \partial y} + \frac{\partial^2 \Delta M^*}{\partial y^2} \).

where \( q \) is the fictitious radiation load (“additive”), the final nonlinear integral-differential resolution equation of the fiber-reinforced concrete plate under the conditions of radiation exposure relative to the deflection \( W \) will be the following one:

\[ \frac{\partial^2}{\partial x^2} \left( D_1 \frac{\partial^2 W}{\partial x^2} \right) + \frac{\partial^2}{\partial x^2} \left( D_2 \frac{\partial^2 W}{\partial y^2} \right) + 2 \frac{\partial^2}{\partial x \partial y} \left( D_b \frac{\partial^2 W}{\partial x \partial y} \right) + \frac{\partial^2}{\partial y^2} \left( D_2 \frac{\partial^2 W}{\partial x^2} + D_1 \frac{\partial^2 W}{\partial y^2} \right) = p + q. \] (35)

Here \( p \) is the intensity of the external load; \( q \) is the dummy “radiation load”, \( D_1, D_2, D_b \) are the integral stiffnesses that take into account the nonlinearity, different resistance of fiber concrete and the effect of radiation.

References

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