The amplitude of mass fluctuations and mass density of the Universe constrained by strong gravitational lensing

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Received 2003 month day; accepted 2003 month day

Abstract We investigate the linear amplitude of mass fluctuations in the universe, \( \sigma_8 \), and the present mass density parameter of the Universe, \( \Omega_m \), from the statistical strong gravitational lensing. We use the two populations of lens halos model with fixed cooling mass scale \( M_c = 3 \times 10^{13} h^{-1} M_\odot \) to match the observed lensing probabilities, and leave \( \sigma_8 \) or \( \Omega_m \) as a free parameter to be constrained by data. Another varying parameter is the equation of state of dark energy \( \omega \), and its typical values of \(-1, -2/3, -1/2 \) and \(-1/3 \) are investigated. We find that \( \sigma_8 \) is degenerate with \( \Omega_m \) in a way similar to that suggested by present day cluster abundance as well as cosmic shear lensing measurements: \( \sigma_8 \Omega_m^{0.6} \approx 0.33 \) (Bahcall & Bode 2003 and references therein). However, both \( \sigma_8 \leq 0.7 \) and \( \Omega_m \leq 0.2 \) can be safely ruled out, the best value is when \( \sigma_8 = 1.0, \Omega_m = 0.3 \) and \( \omega = -1 \). This result is different from that obtained by Bahcall & Bode 2003, who gives \( \sigma_8 = 0.98 \pm 0.1 \) and \( \Omega_m = 0.17 \pm 0.05 \). For \( \sigma_8 = 1.0 \), higher value of \( \Omega_m = 0.35 \) requires \( \omega = -2/3 \) and \( \Omega_m = 0.40 \) requires \( \omega = -1/2 \).

Key words: cosmology: theory — cosmological parameters — gravitational lensing

1 INTRODUCTION

The amplitude of mass fluctuations, denoted as \( \sigma_8 \) when referring to the rms linear density fluctuation in spheres of radius \( 8h^{-1}\text{Mpc} \) at \( z = 0 \), is a fundamental cosmological parameter that describes the normalization of the linear spectrum of mass fluctuations in the early universe. Assuming Gaussian initial fluctuations, the evolution of structure in the universe depends exponentially on this parameter (for an excellent review see Bahcall & Bode 2003).

Recent observations suggest an amplitude that ranges in value from \( \sigma_8 \sim 0.7 \) to a high value of \( \sigma_8 \sim 1.1 \). The low amplitude values of \( \sigma_8 \sim 0.7 \) are suggested by current observations of the cosmic microwave background (CMB) spectrum of fluctuations (Netterfield et al. 2002; Sievers et al. 2003; Bond et al. 2002; Ruhl et al. 2003) and by recent observations of the present cluster abundance as well as cosmic shear lensing measurements (Jarvis et al. 2003; Hamana et al. 2003; Seljak 2002). However, these determinations of \( \sigma_8 \) are degenerate with...
other parameters like mass density parameter $\Omega_m$. The evolution of cluster abundance with time, especially for the most massive clusters, breaks the degeneracy between $\sigma_8$ and $\Omega_m$ (e.g., Peebles, Daly & Juszkiewicz 1989; Eke, Cole & Frenk 1996; Oukbir & Blanchard 1997; Bahcall, Fan & Cen 1997; Carlberg et al. 1997; Bahcall & Fan 1998; Donahue & Voit 1999; Henry 2000). This evolution depends strongly on $\sigma_8$, and only weakly on $\Omega_m$ or other parameters. Bahcall & Bode (2003) used the abundance of the most massive clusters observed at $z \sim 0.5-0.8$ to place a strong limit on $\sigma_8$ and found that $\sigma_8 = 0.98\pm0.1$, $\Omega_m = 0.17\pm0.05$, and low $\sigma_8$ values ($\lesssim 0.7$) are unlikely. In the model of one population of halos (Navarro-Frenk-White, NFW) combined with each galactic halo a central point mass, the lensing probabilities are shown to be sensitive to $\sigma_8$ (Chen 2003a, hereafter, paper I).

In this paper, we use the model of the two populations of lens halos to calculate the lensing probabilities in flat quintessence cold dark matter (QCDM) cosmology with different cosmic equations of state $\omega$ (Chen 2003b, hereafter, paper II), leaving $\sigma_8$ and $\Omega_m$ as free parameters to be constrained from the Jodrell-Bank VLA Astrometric Survey (JVAS) and the Cosmic Lens All-Sky Survey (CLASS; Browne et al. 2000; Helbig 2000; Browne et al. 2003; Myers et al. 2002).

## 2 LENSONG PROBABILITIES

When the quasars at the mean redshift $<z_q>=1.27$ are lensed by foreground CDM halos of galaxies and clusters of galaxies, the lensing probability with image separations larger than $\Delta \theta$ and flux density ratio less than $q_\ell$ is (Schneider et al. 1992)

$$P(\Delta \theta > \Delta \theta_0, q_\ell) = \frac{dD_L(z)}{dz} \int_0^{z} \frac{dD_L(z)}{dz} \int_0^{\infty} n(M, z)\sigma(M, z)B(M, z)dM,$$

where $D_L(z)$ is the proper distance from the observer to the lens located at redshift $z$. And $\dot{n}(M, z)$ is the physical number density of virialized dark halos of masses between $M$ and $M + dM$ at redshift $z$ given by Jenkins et al. (2001). The cross section $\sigma(M, z)$ is mass and redshift dependent, and is sensitive to flux density ratio of multiple images $q_\ell$ for SIS halos,

$$\sigma(M, z) = \pi \xi_0^2 \dot{\theta}(M - M_{\min}) \times \begin{cases} \frac{y_{\ell}^2}{\Delta \theta_0^2}, & \text{for } \Delta \theta \leq \Delta \theta_0; \\ y_{\ell}^2 - \Delta \theta^2, & \text{for } \Delta \theta_0 \leq \Delta \theta < \Delta \theta_{y_{\ell}}; \\ 0, & \text{for } \Delta \theta \geq \Delta \theta_{y_{\ell}}, \end{cases}$$

where $\dot{\theta}(x)$ is a step function, and $M_{\min}$ is the minimum mass of halos above which lenses can produce images with separations greater than $\Delta \theta$. It is shown (in paper II) that the contributions from galactic central supermassive black holes can be ignored when $q_\ell \leq 10$, so the lensing equation for SIS halos is simply $y = x - |x|/x$, where $x = |x|$ and $y = |y|$, which are related to the position vector in the lens plane and source plane as $\xi = x\xi_0$ and $\eta = y\eta_0$, respectively. The length scales in the lens plane and the source plane are defined to be $\xi_0 = 4\pi(\sigma_v/c)^2(D_L^A/D_L^B)/D_S^A$ and $\eta_0 = \xi_0 D_S^A/D_L^A$. Since the surface mass density is circularly symmetric, we can extend both $x$ and $y$ to their opposite values in our actual calculations for convenience. From the lensing equation, an image separation for any $y$ can be expressed as $\Delta \theta(y) = \xi_0 \Delta x(y)/D_L^A$, where $\Delta x(y)$ is the image separation in lens plane for a given $y$. So in Eq. (2), the source position $y_{\Delta \theta}$, at which a lens produces the image separation $\Delta \theta$, is the reverse of this expression. And $\Delta \theta_0 = \Delta \theta(0)$ is the separation of the two images which are just on the Einstein ring; $\Delta \theta_{y_{\ell}} = \Delta \theta(\ell y_{\ell})$ is the upper-limit of the separation above which the flux ratio of the two images will be greater than $q_\ell$. Note that since $M_{DM}$ is related to $\Delta \theta$ through $\xi_0$ and $\sigma_\ell^2 = GM_{DM}/2r_{vir}$, we can formally write $M_{DM} = M_{DM}(\Delta \theta(y))$ and determine $M_{\min}$ for galaxy-size lenses by $M_{\min} = M_{DM}(\Delta \theta(y_{\ell}))$.

According to the model of two populations of halos, cluster-size halos are modeled as NFW profile: $\rho_{NFW} = \rho_\ell r_\ell^3/[r(r + r_\ell)^2]$, where $\rho_\ell$ and $r_\ell$ are constants. We can define the mass of
a halo to be the mass within the virial radius of the halo \( r_{\text{vir}} = M_{\text{DM}} / 4\pi \rho_c r_{\text{vir}}^3 \), where \( f(c_1) = \ln(1 + c_1) - c_1 / (1 + c_1) \), and \( c_1 = r_{\text{vir}} / r_s = 9(1 + z)^{-1}(M/1.5 \times 10^{13} h^{-1} M_\odot)_{-0.13} \) is the concentration parameter, for which we have used the fitting formula given by Bullock et al. (2001).

The lensing equation for NFW lenses is as usual \( y = x - \mu_s g(x)/x \) (Li & Ostriker 2002), where \( y = |y|, \eta = y D^2_A / D^2_L \) is the position vector in the source plane, in which \( D^2_A \) and \( D^2_L \) are angular-diameter distances from the observer to the source and to the lens respectively, \( x = |x| \) and \( x = \xi / r_s \), \( \xi \) is the position vector in the lens plane. The parameter \( \mu_s = 4\rho_s r_s / \Sigma_{\text{cr}} \) is independent, in which \( \Sigma_{\text{cr}} = (c^2 / 4\pi G)(D^2_A / D^2_L D^2_{\text{LS}}) \) is critical surface mass density, with \( c \) the speed of light, \( G \) the gravitational constant and \( D^2_{\text{LS}} \) the angular-diameter distance from the lens to the source. The function \( g(x) \) has a analytical expression originally given by Bartelmann (1996). The cross section for the cluster-size NFW lenses is well studied (Li & Ostriker 2002).

The lensing equation is \( y = x - \mu_s g(x)/x \) and the multiple images can be produced only if \(|y| \leq y_{\text{cr}}\), where \( y_{\text{cr}} \) is the maximum value of \( y \) when \( x < 0 \), which is determined by \( dy/dx = 0 \), and the cross section in the lens plane is simply \( \sigma(M, z) = \pi y_{\text{cr}}^2 r_s^2 \).

As for the magnification bias \( B(M, z) \), we use the result given by Li & Ostriker (2002) for NFW lenses. For singular isothermal sphere (SIS) model, the magnification bias is \( B_{\text{SIS}} \approx 4.76 \).

We consider In this paper the spatially flat QCDM cosmology models. The density parameter \( \Omega_m \) ranges from 0.2 to 0.4 as suggested by all kinds of measurements (e.g., Peebles & Ratra 2003 and the references therein). We investigate the varying parameter \( \sigma_8 \) within its entire observational range from 0.7 to 1.1 (e.g., Bahcall & Bode 2003). The Hubble parameter is \( h = 0.75 \). Three negative values of \( \omega \) in equation of state \( \rho_q = \omega \rho_Q \), with \( \omega = -1 \) (cosmological constant), \( \omega = -2/3 \), \( \omega = -1/2 \) and \( \omega = -1/3 \) are investigated. We use the conventional form to express the redshift \( z \) dependent linear power spectrum for the matter density perturbation and the linear growth suppression factor of the density field in QCDM cosmology established by Ma et al. (1999), which are needed in Eq. (1).

3 DISCUSSION AND CONCLUSIONS

Since the lensing rate is sensitive to the source redshift \( z_s \), results can be affected considerably by including the redshift distribution into calculations (e.g., Sarbu, Rusin & Ma 2001). However, since its distribution in the JVAS/CLASS survey is still poorly understood, we use the estimated mean value of \( < z_s > = 1.27 \) (Marlow et al. 2000 Chae et al. 2002 Oguri 2003 Huterer & Ma 2004 Paper I).

For comparison, we plot in Fig. 1 the lensing probability versus image separation angle for each set of parameters of cosmology and lens halo models with the same values as those taken in the right panel of Fig. 1 in paper I, except the mean value of the redshift of quasars \( < z_s > \), the amplitude of mass fluctuations \( \sigma_8 \) and the mass density parameter \( \Omega_m \). This is a model of one population of halos (NFW) combined with each galactic halo a central point mass (\( M_{\text{eff}} \)). Other than a higher value of \( < z_s > = 1.5 \) used in paper I, we use an estimated value of \( < z_s > = 1.27 \) in this paper. We use a slightly higher value of \( \sigma_8 = 1.0 \) (while in paper I, this value is \( \sigma_8 = 0.95 \)). The histogram represents the results of JVAS/CLASS; the solid, dash-dotted, dashed and dotted lines (from top downwards) stand for, respectively, the matched values of the pair \( (q_t, M_{\text{eff}}/M_\bullet) \) (\( M_\bullet \) is a galactic central black hole mass) of (10, 200), (100, 100), (1000, 50) and (10000, 30). Five values of \( \Omega_m \) ranging from 0.2 to 0.4 (as explicitly indicated in each panel) are chosen to see its effect on lensing probabilities. We find that lensing probability is sensitive to \( \Omega_m \), however, the best fit parameters are only when \( \omega = -1 \) and \( \Omega_m = 0.4 \), which is different from the result given in paper I. The reason is that the lensing probabilities are quite sensitive to the mean redshift of quasars \( < z_s > \), the higher redshift will produce a larger value of lensing probability. This means that the NFW+point-mass model for galaxy size lens halos indeed reduces the probabilities considerably when small image flux density ratio is taken.
into account, which can be confirmed when compared with the SIS model and the discussion below.

As just mentioned, when we use NFW+point-mass to model galaxy size lens halos, the predicted lensing probabilities can match observations only when a higher value of \(< z_s >\) is used. We pointed out in paper II that a two populations of lens halos model with mass distributions NFW \((M_{DM} > M_c)\) and SIS \((M_{DM} < M_c)\), can match observations better, even when a reasonable lower value of \(< z_s >\) and \(M_c\) are used. We chose the cooling mass scale to be \(M_c = 3.0 \times 10^{13} h^{-1} M_{\odot}\) in this paper rather than \(M_c = 5.0 \times 10^{13} h^{-1} M_{\odot}\) used in paper I.

So it would be interesting to investigate both the \(\Omega_m\) and \(\sigma_8\) dependent lensing probability with the combined SIS and NFW model. In each panel of fig. 2 the parameters are: \(q_t = 10\), \(\sigma_8 = 1.0\). And \(\Omega_m\) takes five values as in fig. 1 (as explicitly indicated). We find that, lensing probability is also sensitive to \(\Omega_m\), and clearly, \(\Omega_m = 0.2\) can be safely ruled out. For \(\omega = -1\) (cosmological constant), the best fit value of mass density parameter is \(\Omega_m = 0.3\). This result is different from those obtained with other methods (see Bahcall & Bode 2003). Higher value of \(\Omega_m = 0.35\) requires \(\omega = -2/3\) and \(\Omega_m = 0.40\) requires \(\omega = -1/2\).

Our model prefers a higher value of \(\Omega_m \geq 0.3\). In Fig. 2 we have already used higher values of \(\sigma_8 (= 1.0)\) and \(M_c (= 3.0 \times 10^{13} h^{-1} M_{\odot})\). Lower values of \(\Omega_m\) require more higher values of these two parameters, which would be out of the range suggested by other measurements. In order to see the effect of \(\sigma_8\) on lensing probability, we thus fix the value of \(\Omega_m\) to be 0.3 and 0.35, respectively, and vary \(\sigma_8\) from 0.7 to 1.1 in each case. The results are shown in Fig. 4 and Fig. 5. We find that, when \(\Omega_m = 0.3\), lensing probabilities are only slightly sensitive to \(\sigma_8\) at small image separations \((0.3'' < \Delta \theta < 3'')\), where JVAS/CLASS survey has a well-defined sample suitable for analysis of the lens statistics. The lensing probabilities are sensitive to \(\sigma_8\) at larger image separations, but no sample suitable for analysis exists in this range. While \(\sigma_8 \leq 0.7\) seems unlikely, all the values in the range \(0.8 \leq \sigma_8 \leq 1.1\) are possible. For a larger value of \(\Omega_m = 0.35\), as shown in Fig. 4 the lensing probabilities are more sensitive to \(\sigma_8\) than in Fig. 3. In this case, even \(\sigma_8 = 0.7\) is acceptable (it predicts 12.5 lenses with image separation \(\geq 0.3''\), while the observed value is 13), and \(\sigma_8 = 1.1\) matches a value of equation of state of dark energy to be \(\omega = -2/3\).

Note that Chae et al. (2002) reported the main results on cosmological parameters (matter density \(\Omega_m\) and equation of state for dark energy \(\omega\)) from a likelihood analysis of lens statistics, they gave \(\Omega_m = 0.31 \pm 0.07\) and \(\omega = 0.55 \pm 0.11\), both at 68\% confidence level. Our results, although not precise, are in agreement with theirs. However, Since Chae et al. (2002) used the Schechter luminosity function rather than Press-Schechter mass function to account for the mass distribution, they didn’t refer to \(\sigma_8\). Precise results using the same model of this paper from a likelihood analysis will be presented in another paper.

Acknowledgements The author thanks the anonymous referee for useful comments and constructive suggestions. This work was supported by the National Natural Science Foundation of China under grant No.10233040.

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Fig. 1 Predicted lensing probability with image separations $\Delta \theta$ and flux density ratios $q_r$ in $\Lambda$CDM cosmology. The cluster-size lens halos are modelled by the NFW profile, and galaxy-size lens halos by NFW+BULGE. Instead of SIS, we treat the bulge as a point mass, its value $M_{\text{eff}}$ is so selected for each $q_r$ that the predicted lensing probability can match the results of JVAS/CLASS represented by histogram. In each panel, the solid, dash-dotted, dashed and dotted lines (from top downwards) stand for, respectively, the matched values of the pair $(q_r, M_{\text{eff}}/M_*)$ ($M_*$ is a galactic central black hole mass) of $(10, 200)$, $(100, 100)$, $(1000, 50)$ and $(10000, 30)$. $< z_s > = 1.27$ and $\sigma_8 = 1.0$ for all panels here, and from left to right, $\Omega_m$ is $0.2$, $0.25$, $0.3$, $0.35$ and $0.4$, respectively.
Fig. 2 The integral lensing probabilities with image separations larger than $\Delta \theta$ and flux density ratio less than $q_r$, for quasars at mean redshift $< z_s > = 1.27$ lensed by NFW ($M_{DM} > M_c$) and SIS ($M_{DM} < M_c$) halos. In each panel, $q_r = 10.0$ and $\sigma_8 = 1.0$, and from left to right, $\Omega_m$ is 0.2, 0.25, 0.3, 0.35 and 0.4, respectively. The solid, dashed, dash-dotted and dotted lines stand for $\omega = -1$, -2/3, -1/2 and -1/3, respectively.
Fig. 3  Same as Fig. 2, except the value of $\sigma_8$, the value of which, from left to right, is 0.7, 0.8, 0.9, 1.0 and 1.1, respectively. In each panel, $\Omega_m = 0.3$. 
Fig. 4  Same as Fig. 3, except that $\Omega_m = 0.35$. 