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Determination of the rigid rotation plastic hinge point in SENB specimens in different strain hardening steels

W Khor1,2,4, P L Moore3, H G Pisarski3 and C J Brown1
1 College of Engineering, Design and Physical Sciences, Brunel University London, UB8 3PH, UK
2 National Structural Integrity Research Centre, CB21 6AL, UK
3 TWI, CB21 6AL, UK

Abstract. According to some standards, fracture toughness tests loaded under bending are assumed to deform around a fixed plastic hinge point within the ligament ahead of the notch tip. The rotation factor, \( r_p \), defines the proportion of the ligament ahead of the crack tip to where this hinge point is located. In this paper, the concept of an SENB specimen bending about a fixed rotational point under loading was investigated. Experimental SENB tests were carried out on three different strain hardening steels, and the geometrical point of rotation was determined experimentally throughout the tests using a double clip gauge and the similar triangles principle. The experimental results were then used to develop and validate a series of different strain hardening property numerical models. By extracting the rotational factor from the different strain hardening property models, a relationship between strain hardening and a strain hardening corrected rotational factor, \( r_{p,sh} \) was established. This corrected rotational factor function was used to propose an improved equation for the calculation of CTOD and CTOD R-curves, which gave good estimations of CTOD when compared to values measured experimentally from sections through silicone replicas of the specimen crack-tip. The improved R-curve equation will be proposed for future amendments to the ISO 12135 standard.

1. Introduction

The Crack Tip Opening Displacement (CTOD) is one of the best known elastic-plastic fracture toughness parameters in fracture mechanics. The concept was originally developed at TWI, Cambridge in the 1960’s, and is commonly used in the oil and gas industry. CTOD is advantageous in cases where the concept of linear elastic fracture mechanics is insufficient to account for ductile deformation, such as in pressure vessels, offshore platforms and pipelines.

There are several different definition for CTOD, but all describe CTOD as a material parameter relative to the physical opening at the original crack tip region. In general, CTOD is described as the fracture toughness in a specimen at the point of maximum load, initiation for stable crack extension or unstable crack extension, as appropriate. Notched specimens, sharpened by fatigue pre-cracking are loaded, and the load-displacement data are used in the calculation of CTOD.

Fracture toughness testing (often described simply as ‘CTOD testing’) became standardised in the 1970s, and is currently represented in a number of standards BS 7448, ISO 12135, ASTM E1820, and WES1108. However, different assumptions about the determination of CTOD are used, which can lead to different values of CTOD. Whilst BS 7448, WES1108 and single point determination of

4 Corresponding author, weeliam.khor@gmail.com
CTOD in ISO 12134 assume a rotational point, ASTM 1820 and R-curve determination in ISO 12135 do not; instead they derive CTOD from $J$.

It is important to define the value of CTOD with accuracy, particularly when CTOD is being used to determine tolerable flaw sizes for the assessment of structural integrity. Under-estimates of CTOD could lead to either rejection of the material or unnecessary repairs being required, resulting in significant burdens on the cost of fabrication. On the other hand, over-estimates of CTOD might lead to potentially unsafe structures being assessed as fit-for-service. This study examines the effect of strain hardening on the determination of CTOD in Single Edge Notch Bend (SENB) specimens manufactured from three different steels.

2. The concept of rigid rotational factor for the determination of CTOD
When a SENB specimen is loaded under three-point bending, the crack tip experiences tensile stress, whereas there is a region in the un-cracked ligament which experiences compression. In the calculation of the plastic component of CTOD in BS 7448-1, WES1108, earlier versions of ISO 12135, ASTM E1290-93 and E1820-01, it is assumed that the specimen rotates about a stationary point within the un-cracked ligament ahead of the crack tip. The rotation point is a distance ahead of the crack tip equal to $r_p \times B_0$, where $r_p$ is the rotational factor and $B_0$ is the remaining ligament ahead of the crack tip. This concept was introduced by Dawes, based on a 2-D derived plastic hinge model assuming plane strain condition in the equation.

Consider the symmetry of a deformed SENB specimen (Figure 1), the distance of the rotational point from the crack mouth opening displacement (CMOD), $H$ is defined in terms of rotational factor, $r_p B_0 + a_0$. From Figure 1, $V_{g1}$ and $\delta$ can be related using the similar triangles assumption, given below:

\[ V_{g1} = 2[r_p(W - a_0) + a_0 + z_1] \sin \theta \quad (1) \]
\[ \delta = 2r_p(W - a_0) \sin \theta_p \quad (2) \]

Rearranging Equation (1) into Equation (2) gives

\[ \delta = \frac{r_p(W - a_0) V_{g1}}{r_p(W - a_0) + a_0 + z_1} \]

The similar triangles assumption is used for the determination of the plastic CTOD in BS 7448 Parts 1, 4 and ISO 12135 used a constant value of $r_p = 0.4$ in the calculation of CTOD.

3. Methodology

3.1. Finite element modelling methods
A Geometrically and Materially Non-linear Analysis (GMNA) FE model was used to predict CTOD in SENB specimens with the same geometry and material properties as the experimental tests. Fully three-dimensional quarter SENB models were simulated using commercially available software (ABAQUS v6.14) with a blunted crack tip of 0.03mm radius. Figure 2 shows the outline geometry of the SENB specimen in the model. 8-noded linear brick elements (C3D8R) were used to model the SENB specimens as no difference in results were observed using C3D20R.
A standard convergence test was performed based on varying the element size distributed across the crack tip. CTOD was determined from the numerical model based on both the opening of the original crack tip, which is the original concept of CTOD and equivalent to the experimentally-cast silicone replica measurements, described below (Figure 2).

Additional FE analyses were undertaken based on the techniques described above with idealized tensile properties. The idealized tensile properties were generated employing the modified Ramberg-Osgood power law for $0.44 \leq \sigma_y / \sigma_{uts} \leq 0.98$. The equation used is given in Equation (3)

$$\varepsilon = \frac{\sigma}{E} + \alpha \left( \frac{\sigma}{\sigma_{ys}} \right)^{n-1}$$

(3)

Where $\alpha = 0.002$, $E = 207$GPa and $\sigma_{ys} = 400$MPa. Relatively, decreasing $n$ would lead to the increase of strain hardening (decreasing tensile ratio). The true stress-strain curve obtained based on Equation (3) is shown in Figure 3.

3.2. Experimental methods

For the experimental tests, three different steels were chosen to cover a range of strain hardening behaviour:
- M01 Low strain hardening: High strength steel SA-543-GrB-Cl1, \( \sigma_{ys} = 850 \text{MPa} \) and \( \sigma_{ys}/\sigma_{uts} = 0.93 \)
- M02 Medium strain hardening: Structural steel S355J2, \( \sigma_{ys} = 421 \text{MPa} \) and \( \sigma_{ys}/\sigma_{uts} = 0.72 \)
- M03 High strain hardening: Austenitic stainless steel SS316, \( \sigma_{ys} = 286 \text{MPa} \) and \( \sigma_{ys}/\sigma_{uts} = 0.45 \)

SENB specimens of cross-section 20mm x 40mm were machined from the steel plates, notched and fatigue pre-cracked to give a nominal \( a_0/W \) ratio of 0.5. A total of eight specimens were tested: two each for M01 and M03, four from M02. A crack casting method using silicone replica compound was used to cast the crack for physical measurement of CTOD. The specimens were loaded to the selected displacement points based on the clip gauge opening, then held in displacement control. The silicone compound was slowly injected into the specimen notch from one side of the specimen and allowed to cure for 5 minutes. After curing time, the specimen was loaded to the next point of interest and held in constant displacement again. The replica was then extracted and the casting procedure was repeated. Typically, around ten silicone replica casts (SRCs) were obtained at different displacement levels for each test.

3.3. Physical measurements of CTOD
The SRCs extracted from the test specimens were sliced in the middle and CTOD was measured using an optical microscope. The location of the initial fatigue crack tip was identified from the initial crack length, and the width of the silicone replica was measured at this point to give the value of CTOD from the replica, \( \delta_{SRC} \) (Figure 4). This method takes CTOD at the mid specimen thickness, which corresponds to the plane strain condition assumed in the equations\(^{10,11}\). The CTOD values obtained from the SRCs were considered to represent the actual physical CTOD, and thus were treated as the baseline for comparison to other methods.

4. The determination of the rotational factor based on the similar triangles method
The concept of a rigid rotational point has been successfully implemented for the determination of CTOD, where the fixed value of \( \rho_p = 0.4 \) is found to be most accurate\(^{10,12}\) for material within the range of \( 0.65 < \sigma_{ys}/\sigma_{uts} < 0.7 \). To investigate the rotational factor, \( \rho_p \) for SENB specimens, an opened crack was investigated based on the similar triangles concept (Figure 1). This method extrapolates the crack face angles into the unbroken ligament ahead of the crack tip, where the intersection of the angles is defined as the rotational point.

To simplify the derivation, the following terms are defined: \( \rho_pB_0 = Y, z_1 + a_0 = C, \) and \( a_0 + z_2 = D \). To relate the lower and upper clip gauge opening, \( V_{g1} \) and \( V_{g2} \) respectively to the point of rotation,

\[
\sin \theta = \frac{V_{g1}}{C+Y} = \frac{V_{g2}}{D+Y}
\]

Leading to

\[
\frac{V_{g2}}{V_{g1}} = \frac{D+Y}{C+Y}
\]

Expanding \( D \) and factoring \( C+Y \) gives
\[ \frac{V_{g2}}{V_{g1}} = \frac{C + Y + (z_2 - z_1)}{C + Y} = 1 + \frac{(z_2 - z_1)}{C + Y} \]

Rearranging the equation leads to

\[ Y = r_p B_0 = \left( \frac{z_2 - z_1}{V_{g2}/V_{g1} - 1} \right) - (z_1 + a_0) \]

Where the rotation factor, \( r_p \) based on \( V_{g1} \) and \( V_{g2} \) is given as

\[ r_p = \left( \frac{z_2 - z_1}{V_{g2}/V_{g1} - 1} \right) - (z_1 + a_0) \times \frac{1}{B_0} \quad (4) \]

Equation (4) allow the rotational factor to be calculated based on two clip gauges positioned at different heights above the crack mouth in standard laboratory tests. Similarly, the rotational factor based on the plastic displacement can be obtained by simply replacing the lower and upper clip gauge displacement, \( V_{g1} \) and \( V_{g2} \) with the plastic lower and upper clip gauge displacement, \( V_{p1} \) and \( V_{p2} \).

To include the effects of strain hardening, \( r_{p,sh} \) is extracted based on the intersection the line extrapolated from the lower clip gauge opening or CMOD and CTOD to the symmetry line (Figure 5). Equation (4) was therefore modified for the calculation of \( r_{p,sh} \), described as

\[ r_{p,sh} = \left( \frac{z_1 + a_0}{(CMOD/\delta) - 1} \right) \times \frac{1}{B_0} \quad (5) \]

5. Results and discussion

Conceptually, the strain hardening based rotational factor, \( r_{p,sh} \) accounts for the effects of strain hardening based on the measured CTOD, which allows the rigid rotational plastic hinge concept to estimate CTOD for any given material strain hardening property. Based on Equation (5), \( r_{p,sh} \) was derived using the CTOD from the FE models with idealised tensile properties (Figure 6). Data were extracted for 0.2mm< FE CTOD< 1.0mm to minimise the influence of the elastic CTOD and large deformation on the model results, while ensuring sufficient loads such that the point of rotation has stabilised.

![Figure 5. The effect of crack tip blunting due to strain hardening.](image)

**Figure 5.** The effect of crack tip blunting due to strain hardening.

![Figure 6. Strain hardening rotational factor vs. FE CTOD.](image)

**Figure 6.** Strain hardening rotational factor vs. FE CTOD.
Figure 6 shows the dependency of $r_{p\,sh}$ on tensile ratio, as the overall $r_{p\,sh}$ decreases with the increase of strain hardening, where strain hardening is simply expressed as the tensile ratio ($\sigma_{ys}/\sigma_{uts}$). Based on the mean $r_{p\,sh}$ for the respective tensile ratio, a linear relationship was obtained to describe the effect of $r_{p\,sh}$ on strain hardening for the range of materials studied (Figure 7), given as:

$$r_{p\,sh} = 0.4668 \frac{\sigma_{ys}}{\sigma_{uts}} + 0.0996$$ (6)

The implication of this equation is that for steels with low $\sigma_{ys}/\sigma_{uts}$ ratio (high strain hardening steels), SENB specimens plastically rotate about a point closer to the crack tip than specimens with high $\sigma_{ys}/\sigma_{uts}$ ratio (low strain hardening steels).

Based on the $r_{p\,sh}$ findings, a CTOD equation considering strain hardening was obtained for SENB specimens with $a_{0}/W$ of 0.5, using the elastic CTOD from WES 1108 and plastic CTOD based on $r_{p\,sh}$ and the similar triangles concept, thereafter described as $\delta_{sh}$, given as

$$\delta_{sh} = K^2 \frac{(1-v^2)}{E_m J_{WES} \sigma_{ys} E} + \frac{r_{p\,sh} R_p V_p}{r_{p\,sh} R_p V_p + a_0}$$ (7)

where $m_{J_{WES}} = 4.9 - 3.5(\sigma_{ys}/\sigma_{uts})$

CTOD values measured from the silicone replicas were used to validate the standard CTOD equations (BS 7448-1, ASTM E1820 and WES 1108), including CTOD from Equation (7) (Figure 8). All three standards underestimated CTOD for the low and medium strain hardening steels, M01 and M02, with $\delta_{sh}$ being most accurate and ASTM E1820 giving the lowest estimation. In the higher strain hardening steel, M03, ASTM and WES gave similar values and lower estimation of CTOD, whereas $\delta_{sh}$ gave the most accurate estimation despite overestimating CTOD in several instances. Some scatter was observed from the higher strain hardening M03. BS 7448 overestimated CTOD for M03 as it does not consider the effects of strain hardening.

The calculated CTOD values were normalised to the measured CTOD to indicate the accuracy of the equations (Figure 9). $\delta_{sh}$ and BS 7448 gave similar average estimation, followed by WES 1108 and ASTM E1820 (-6.4%, -6.8%, -15.6% and -30.9% respectively). $\delta_{sh}$ gave the best accuracy and precision compared to all three standards. The WES and ASTM give lower estimations of CTOD as they were validated to determine CTOD from $J$ based on the 45 degree intercept concept of CTOD, which gives lower values to CTOD from the original crack tip.

Figure 7. Relationship between the strain hardening rotational factor, $r_{p\,sh}$ and tensile ratio, $\sigma_{ys}/\sigma_{uts}$.
Figure 8. CTOD from silicone replica vs. CTOD from equations, M01, M02 and M03, full plot and expanded view (a, b, c and d respectively)
6. Conclusion

Based on the experimental measurements of CTOD, it was found that:

- The rigid rotational concept is practical method for the estimation of CTOD
- The strain hardening corrected rotational factor, $r_{sh}$ increases linearly from 0.30 at $\sigma_y/\sigma_{uts}=0.44$ to 0.56 at $\sigma_y/\sigma_{uts}=0.98$, given as
  $$r_{sh} = 0.4668 \frac{\sigma_y}{\sigma_{uts}} + 0.0996$$
- In steel SENB specimens with a nominal crack length of $a_0/W=0.5$, the estimated CTOD with the highest accuracy and precision compared to BS 7448-1, ASTM E1820 and WES 1108 is given by the following equation:
  $$\delta_{sh} = K^2 \left( \frac{1-v^2}{m_{JWES} \sigma_{ys} E} \right) + \frac{r_{psht}\beta_o\nu_p}{r_{psht}\beta_o+a_0}$$
  Where $m_{JWES}=4.9-3.5(\sigma_y/\sigma_{uts})$

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