Abstract

The sigma model action described in this paper differs in four important features from the usual sigma model action for the four-dimensional Green-Schwarz heterotic superstring in a massless background. Firstly, the action is constructed on an N=(2,0) super-worldsheet using a Kahler potential and an Ogievetsky-Sokatchev constraint; secondly, the target-space background fields are unconstrained; thirdly, the target-space dilaton couples to the two-dimensional curvature; and fourthly, the action reduces in a flat background to a free-field action.

A conjecture is made for generalizing this N=(2,0) sigma model action to the ten-dimensional Green-Schwarz heterotic superstring in a manner that preserves these four new features.
I. Introduction

The construction of two-dimensional sigma model actions for the bosonic and Neveu-Schwarz-Ramond strings in massless backgrounds has been a useful tool for determining string-corrected equations of motion for the massless fields. These equations of motion are obtained by calculating the $\beta$-functions of the non-linear sigma model perturbatively in $\alpha'$ and demanding that they vanish. Although in principle, the string-corrected equations of motion can also be determined from the S-matrix scattering amplitudes, the $\beta$-function technique has proven to be much more effective.

However in order for the perturbative expansion in $\alpha'$ of the $\beta$-function to make sense, the sigma model action must become a two-dimensional free-field action when $\alpha' = 0$. By using a normal coordinate expansion for the background fields, this condition implies that the sigma model action in a flat background must also be a free-field action. Note that such a free-field condition is also needed for calculating S-matrix scattering amplitudes.

For the case of the Green-Schwarz superstring, the usual sigma model action does not satisfy this free-field condition. One attempt at avoiding this problem is to classically gauge-fix all of the fermionic Siegel symmetries to semi-light-cone gauge where $\gamma^+ \theta = 0$, and then rescale $\gamma^- \theta$ to $\sqrt{\partial_+ x^+ (\gamma^- \theta)}$. Although the resulting sigma model action in a flat background is a free-field action, it contains several strange features.

Firstly, because the semi-light-cone gauge choice is not possible at points where $\partial_+ x^+ = 0$, the target-space background fields must satisfy the light-cone gauge condition that their $+$ components vanish and that they are independent of $x^-$. This condition allows $x^+$ to have classical solutions with $\partial_+ x^+$ nowhere vanishing.† Secondly, the requirement that the original sigma model action is classically invariant under Siegel transformations imposes certain torsion constraints on the background fields. (In ten dimensions, these torsion constraints force the background fields on-shell, so vanishing of the one-loop $\beta$-function does not further restrict them.) In four dimensions, these torsion constraints do not force the background fields on-shell, however vanishing of the one-loop $\beta$-function is not enough to determine their equations of motion. Thirdly, the target-space dilaton field does not couple to the two-dimensional curvature, so one does not get the usual relationship between the string coupling constant and the dilaton zero mode. And fourthly, the conformal anomaly of this free-field action is $c = d + \frac{d}{2} (d - 2) - 26$, which is non-zero even when $d = 10$.

An alternative approach to the sigma model action for the Green-Schwarz heterotic superstring was developed more recently using two-dimensional super-worldsheets. After imposing an STVZ-like constraint on the target-space coordinates, sigma model actions for the Green-Schwarz heterotic superstring have been constructed with $N=(1,0)$, $(2,0)$, $(4,0)$, and $(8,0)$ worldsheet supersymmetry. Although all of these actions coincide classically with the usual Green-Schwarz sigma model action, only the $N=(2,0)$ action has been shown to reduce in a flat background to a free-field action which can be consistently

† The authors of reference 5 disagree that this light-cone gauge condition is necessary, and instead impose only the restriction that $R_{abc+} = R_{abc-} = 0$. 
quantized in ten dimensions.\textsuperscript{19,20} The disadvantage of these actions is that the STVZ-like constraint requires the background fields to satisfy the same torsion constraints as in the usual Green-Schwarz sigma model, and the dilaton field still does not couple to the two-dimensional curvature.

In this paper, a new sigma model action with $N=(2,0)$ worldsheet supersymmetry is constructed for the Green-Schwarz heterotic superstring. After replacing the STVZ-like constraint with the Ogievetsky-Sokatchev constraint, this sigma model action is constructed out of a Kahler potential with no extra constraints on the background fields and with the standard Fradkin-Tseytlin coupling of the target-space dilaton field to the two-dimensional curvature. Although this $N=(2,0)$ action can be constructed for four, six, and ten-dimensional target spaces, only the four-dimensional case will be investigated in detail and shown to describe a background of minimal $d=4$ $N=1$ supergravity and super-Yang-Mills coupled to an antisymmetric tensor field. It is conjectured in the conclusion of this paper that the analogous $N=(2,0)$ sigma model action for a ten-dimensional target space describes a background of $d=10$ $N=1$ supergravity and super-Yang-Mills.

II. The Sigma Model Action in a Flat Target Space

In four dimensions, the new sigma model action is a curved target-space generalization of the $N=(2,0)$ twistor-string action first described by Ivanov and Kapustnikov.\textsuperscript{16} This action in a flat background was constructed using a four-dimensional complex target space, $[X^m, \bar{X}^m, \Theta^\mu, \bar{\Theta}^{\dot{\mu}}]$ where $m = 0$ to 3 and $\mu, \dot{\mu} = 1$ to 2, together with the reality condition,

$$X^m - \bar{X}^m = i \Theta^{\mu} \gamma_{\mu \dot{\mu}} \bar{\Theta}^{\dot{\mu}}. \quad (II.1)$$

As two-dimensional superfields on the $N=(2,0)$ super-worldsheet, these target-space coordinates satisfy the chirality conditions,

$$\bar{D}X^m = D\Theta^\mu = D\bar{X}^m = D\bar{\Theta}^{\dot{\mu}} = 0, \quad (II.2)$$

where $D = \partial_\kappa + \frac{i}{2} \bar{\kappa} \partial_\bar{z}$, $\bar{D} = \partial_{\bar{\kappa}} + \frac{i}{2} \kappa \partial_z$, and the two-dimensional Minkowski-space super-worldsheet is parameterized by the coordinates $[z, \kappa, \bar{\kappa}; \bar{z}]$ with $\kappa$ the complex conjugate of $\bar{\kappa}$, but $z$ and $\bar{z}$ independent real variables.

In the presence of these reality and chirality constraints, the flat target-space action of Ivanov and Kapustnikov is:

$$S = i \int d^2z d^2\kappa [X_m \partial_\bar{z} \bar{X}^m + i (X_m \bar{\Theta}^\mu \gamma_{\mu \dot{\mu}}) \partial_\bar{z} \Theta^\mu - i (\bar{X}_m \Theta^\mu \gamma_{\mu \dot{\mu}}) \partial_\bar{z} \bar{\Theta}^{\dot{\mu}}], \quad (II.3)$$

which takes the following form after using the reality constraints to express all superfields in terms of $X^{1+2}$, $\bar{X}^{1-2}$, $X^{3-0}$, $\bar{X}^{3-0}$, $\Theta^1$, and $\bar{\Theta}^1$ (e.g., $X^{1-2} = \bar{X}^{1-2} + i \bar{\Theta}^1 \Theta^2$, $\Theta^2 = i D\bar{X}^{1-2} / D\bar{\Theta}^1$, etc.):\textsuperscript{19}

$$S = i \int d^2z d^2\kappa [X^{1+2} \partial_\bar{z} \bar{X}^{1-2} + i W \partial_\bar{z} \Theta^1 - i W \partial_\bar{z} \bar{\Theta}^1], \quad (II.4)$$

where $W = \bar{X}^{3-0} \Theta^1$ and $\bar{W} = X^{3-0} \bar{\Theta}^1$. 

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The only remaining constraints implied by equation (II.1) are:

\[ D\Theta^1 \bar{D}\bar{\Theta}^1 - \frac{i}{2}(\Theta^1 \partial \bar{\Theta}^1 + \bar{\Theta}^1 \partial \Theta^1) = \frac{1}{2} \partial_x (X^{3+0} + \bar{X}^{3+0}) \quad \text{and} \quad -i\Theta^2 \bar{\Theta}^2 = X^{3-0} - \bar{X}^{3-0}. \quad (II.5) \]

The first of these constraints can be solved by bosonizing the components of \( \Theta^1 = \theta^1 + \kappa \lambda^1, \bar{\Theta}^1 = \bar{\theta}^1 + \bar{\kappa} \bar{\lambda}^1, \)

\[ DW = w + \bar{\kappa} \bar{\varepsilon}, \quad \text{and} \quad \bar{D}W = \bar{w} + \kappa \varepsilon \]

in the following way:20

\[ \lambda^1 = (\partial_x x^{3+0} + \frac{1}{2} \theta^1 \partial \bar{x}^1 + \frac{1}{2} \bar{\theta}^1 \partial \theta^1) e^h + e^{-\bar{h}}, \quad \bar{\lambda}^1 = e^{-h}, \quad (II.6) \]

where \( x^{3\pm 0} \) is the lowest component of \( \frac{1}{2}(X^{3\pm 0} + \bar{X}^{3\pm 0}) \) and \( h, \bar{h} \) are chiral bosons with screening charge \(-1\) that satisfy \( h(y)\bar{h}(z) \to \log(y - z) \) as \( y \to z \). Because the operator-products of \( \lambda^1, \bar{\lambda}^1, w, \) and \( \bar{w} \), are just free-field operator products, the action of equation (II.4) remains a free-field action when expressed in terms of \( X^{1+2}, \bar{X}^{1-2}, x^{3+0}, x^{3-0}, h, \bar{h}, \theta^1, \bar{\theta}^1, \varepsilon, \) and \( \bar{\varepsilon} \) (\( \bar{\varepsilon} \equiv \varepsilon - \frac{1}{2} \partial_x x^{3-0} \theta^1 - x^{3-0} \partial_x \theta^1 \) has no singularities near \( x^{3+0} \)).

The second constraint of equation (II.5) can be rewritten,

\[ DX^{1+2} \bar{D}X^{1-2} + iDW \bar{D}\Theta^1 - i\bar{D}W D\Theta^1 = 0, \quad (II.7) \]

which is just the N=(2,0) stress-energy tensor for the free-field action of equation (II.4). The conformal anomaly of this stress-energy tensor in \( d \) dimensions is \( c = (d - 2) + \frac{1}{2}(d - 2) + 2(-2 - 1), \) which cancels the conformal anomaly contribution of the N=(2,0) ghosts \( (c = -6) \) when \( d = 10.19 \)

Although manifest SO(3,1) super-Poincaré invariance has been broken, this free-field action and stress-energy tensor can easily be checked to be Lorentz invariant by explicitly constructing the SO(3,1) super-Poincaré generators out of the free fields. For a discussion of this procedure, see section III of reference 21 where these generators were explicitly constructed for the ten-dimensional case.

### III. The Massless Background Fields

The first step in generalizing to a curved target space is to replace the reality constraint of equation (II.1) with the constraint,

\[ X^m - \bar{X}^m = iH^m (X + \bar{X}, \Theta, \bar{\Theta}), \]

where \( H^m \) is a real superfield. This reality constraint first appeared in the work of Ogievetsky and Sokatchev in their description of conformal supergravity,22 and more recently was used by Delduc and Sokatchev to write an N=2 worldline supersymmetric action for the four-dimensional superparticle in a supergravity and super-Yang-Mills background.23

It is convenient to define fermionic target-space derivatives,

\[ \nabla_\mu = \partial_\theta^\mu + i\tilde{f}_\mu^m \partial_{X^m} \quad \text{and} \quad \bar{\nabla}_\mu = \partial_{\bar{\theta}}^\mu + i\tilde{f}_\mu^m \partial_{\bar{X}^m}, \quad (III.1) \]
where \( f^m_\mu = A^m_\nu \partial_\nu H^\mu \), \( \tilde{f}^m_\mu = -\tilde{A}^m_\nu \partial_\nu H^\mu \), and \( A^m_\nu \) is the inverse matrix of \((\delta^m_\nu - i\partial_X H^\nu)\). \( \tilde{A}^m_\nu \) is the inverse matrix of \((\delta^m_\nu + i\partial_X H^\nu)\). Note that \( f^m_\mu \) and \( \tilde{f}^m_\mu \) are uniquely determined by requiring that

\[
[\nabla_\mu, X^m - \tilde{X}^m - iH^m] = [\nabla_\mu, X^m - \tilde{X}^m - iH^m] = 0. \quad (III.2)
\]

Under the holomorphic coordinate transformations, \( X^m \rightarrow X'^m(X, \Theta) \) and \( \Theta^\mu \rightarrow \Theta'^\mu(X, \Theta) \), it is easy to check that \( \nabla_\mu \rightarrow \nabla'_\mu = A^\mu_\nu \nabla_\nu \), where \( H^m \rightarrow H'^m = H^m - i(X'^m - \tilde{X}^m) + i(X^m - \tilde{X}^m) \) and \( A^\mu_\nu \) is the inverse matrix of \( \nabla_\nu \Theta'^\mu \).

The non-gauge degrees of freedom in the Ogievetsky-Sokatchev superfield, \( H^m \), can be determined by using some of these holomorphic coordinate transformations to gauge-fix to the form:

\[
H^m = \Theta^\mu \tilde{\Theta}_\mu \epsilon^m_{\mu\bar{\nu}} + \Theta^\mu (\tilde{\Theta})^2 \epsilon^m_{\mu\bar{\nu}} + \tilde{\Theta}_\mu (\Theta)^2 \bar{\epsilon}^m_{\bar{\mu}\bar{\nu}} + (\Theta)^2 (\tilde{\Theta})^2 a^m. \quad (III.3)
\]

The remaining holomorphic coordinate transformations can be used to further gauge away 11 components of \( e^m_{\mu\bar{\nu}} \) (4 coordinate + 6 Lorentz + 1 scale), 8 components of \( \xi^m_{\mu} \) and \( \bar{\xi}^m_{\bar{\mu}} \) (4 Q-supersymmetries and 4 S-supersymmetries), and one component of \( a^m \) (1 chiral), leaving the usual conformal supergravity multiplet of 8 bosons and 8 fermions.\(^2\)

The next step in constructing an \( N=(2,0) \) sigma model action is to introduce a Kahler potential on the complex four-dimensional target space, \( K_M(X, \bar{X}, \Theta, \bar{\Theta}) \), and its complex conjugate, \( \bar{K}_M(X, \bar{X}, \Theta, \bar{\Theta}) \), where \( M \) takes four complex bosonic values \((m = 0 \text{ to } 3)\) and two complex fermionic values \((\mu = 1 \text{ to } 2)\). As usual in Kahler geometry,\(^2\) the Kahler metric and torsion potential will be defined in terms of \( K_M \) by:

\[
G_{MN} = \partial_M \bar{K}_N + (-1)^{s(M)s(N)} \partial_N K_M \quad \text{and} \quad B_{MN} = \partial_M \bar{K}_N - (-1)^{s(M)s(N)} \partial_N K_M, \quad (III.4)
\]

where \( s(M) = 0 \text{ (or 1) if } M \text{ takes bosonic (or fermionic) values} \).

Because \( K_M \) and \( H^m \) both describe the target-space geometry, they are not independent superfields. The constraint that relates them is:

\[
h_{\mu\bar{\nu}} \equiv G_{\mu\bar{\nu}} + if^m_{\mu} G_{m\bar{\nu}} - if^m_{\bar{\nu}} G_{m\mu} - \tilde{f}^m_{\mu} \tilde{f}^m_{\bar{\nu}} G_{m\bar{\nu}} = 0, \quad (III.5)
\]

where \( G_{MN} \) and \( f^m_\mu, \tilde{f}^m_\mu \) are defined in equations (III.4) and (III.1). Under holomorphic coordinate transformations, \( h_{\mu\bar{\nu}} \rightarrow h'_{\mu\bar{\nu}} = A^\nu_\mu \tilde{A}^\bar{\nu}_{\bar{\mu}} h_{\nu\bar{\nu}} \) where \( A^\nu_\mu \) is the inverse matrix of \( \nabla_\nu \Theta'^\mu \), so the constraint is coordinate-independent.

Under the following transformations of \( K_M \), \( \delta G_{MN} = 0 \) and \( \delta B_{MN} = \partial_M \partial_N F \) (which leaves the field strength \( \partial_P B_{MN} - (-1)^{s(M)s(P)} \partial_M \partial_P B_{PM} \) unchanged):

\[
\delta K_M = \Lambda_M(X, \Theta) + i\partial_M F(X, \bar{X}, \Theta, \bar{\Theta}), \quad \delta \bar{K}_M = \bar{\Lambda}_{\bar{M}}(X, \bar{\Theta}) - i\partial_{\bar{M}} F(X, \bar{X}, \Theta, \bar{\Theta}), \quad (III.6)
\]

where \( \Lambda_M \) is holomorphic and \( F \) is real.
Using the $\Lambda_\mu$ and $\Lambda_m$ transformations, it is possible to gauge-fix $\hat{K}_\mu \equiv K_\mu + i f^m_{\mu} K_m$ to the following form:

$$\hat{K}_\mu = (\Theta)^2 \phi_\mu + \Theta^\nu (\Theta)^2 t_{\mu\nu} + i (\Theta)^2 (\bar{\Theta})^2 \psi_\mu. \quad (III.7)$$

Furthermore, choosing $F = \Theta^\nu \bar{\Theta}^\mu c_{\mu\nu} + \Theta^\mu (\Theta)^2 c_{\mu\nu} + \bar{\Theta}^\mu (\Theta)^2 c_{\mu\nu} + (\Theta)^2 (\bar{\Theta})^2 c + i (X^m - \bar{X}^m - i H^m) C_m (\Theta, \bar{\Theta})$, one can gauge away $t_{\mu\nu} e^{\mu\nu} - \bar{\ell}_{\mu\nu} e^{\mu\nu}$, three components of the antisymmetric tensor field, $b_{mn} = \gamma^\mu_{\mu\nu} \gamma^\nu_{\mu\nu} ((t_{\mu\nu} + t_{\nu\mu}) c_{\mu\nu} + (\bar{t}_{\mu\nu} + \bar{t}_{\nu\mu}) c_{\mu\nu})$, all of $\phi_\mu$ and $\bar{\phi}_\mu$, and the complete superfield $K_m + \bar{K}_m$ (note that $\hat{K}_\mu$ and $\hat{K}_\mu$ are unaffected by the gauge transformation involving the real superfield $C_m$). Since $0 = h_{\mu\bar{\mu}} = -i \bar{K}_m \nabla_\mu f^m_{\mu} + i K_m \nabla_\mu f^m_{\mu} + \nabla_\mu \hat{K}_\mu - \nabla_\bar{\mu} \hat{K}_{\bar{\mu}}$, the remaining components of $K_m$ and $\bar{K}_m$ are determined from $\hat{K}_\mu$, $\hat{K}_{\bar{\mu}}$, and $H^m$, and therefore the only non-gauge degrees of freedom in $K_M$ are a real scalar, $\sigma \equiv t_{\mu\nu} e^{\mu\nu} + \bar{t}_{\mu\nu} e^{\mu\nu}$, a gauge-fixed antisymmetric tensor, $b_{mn}$, and complex spinors, $\psi_\mu$ and $\bar{\psi}_\mu$, totalling four bosons and four fermions.

Although the usual interpretation of the $\sigma$, $\psi_\mu$, and $\bar{\psi}_\mu$ fields is as the target-space dilaton and dilatinos, it is more natural to identify them as the determinant of the vierbein, $det(e^m_{\mu\bar{\mu}})$, and the gamma-matrix traces of the gravitinos, $\gamma^m_{\mu\bar{\mu}} e^m_{\mu\bar{\mu}}$ and $\gamma^m_{\mu\bar{\mu}} \xi^m_{\mu\bar{\mu}}$. This is because using the holomorphic coordinate transformations that give rise to scale and S-supersymmetries, $\Theta^\mu \rightarrow a \Theta^\mu + i \Theta^\mu \alpha^\mu$, one could have gauge-fixed these $\sigma$ and $\psi_\mu$ components of $\hat{K}_\mu$ instead of gauge-fixing the vierbein and gravitino components of the $H^m$ superfield.\(^{10}\)

With this identification of $\sigma$ and $\psi_\mu$, one now needs a superfield containing the target-space dilaton and dilatinos. This can be accomplished with a complex holomorphic scalar bosonic superfield, $\Phi(X, \Theta)$, and its complex conjugate, $\Phi(X, \bar{\Theta})$. Expanding in components, $\Phi = \phi + i \Theta^\mu \chi_\mu + i (\Theta)^2 \rho$, where $\phi + \bar{\phi}$ is the dilaton, $\chi_\mu$ and $\bar{\chi}_\mu$ are the dilatinos, and $i(\phi - \bar{\phi})$, $\rho$, $\bar{\rho}$ are bosonic auxiliary fields. So the total non-gauge degrees of freedom in $H^m$, $K_M$, and $\Phi$ are 16 bosons and 16 fermions, as in the usual minimal formulation of N=1 Poincaré supergravity coupled to an antisymmetric tensor field.

Finally, coupling the superstring to a super-Yang-Mills background can be accomplished in the same way as for the superparticle,\(^{23}\) by introducing a real scalar superfield, $V^I (X, \bar{X}, \Theta, \bar{\Theta})$, with the gauge invariance $\delta V^I = \Lambda^I (X, \Theta) + \bar{\Lambda}^I (\bar{X}, \bar{\Theta})$ where $\Lambda^I$ is a holomorphic superfield and $I$ labels the group generators. Since $V^I$ can be gauge-fixed to the form, $V^I = \Theta^\mu \bar{\Theta}^\nu v^I_{\mu\nu} + \Theta^\mu (\Theta)^2 w^I_{\mu\nu} + \bar{\Theta}^\mu (\Theta)^2 \bar{w}^I_{\mu\nu} + (\Theta)^2 (\bar{\Theta})^2 y^I$, where $v^I_{\mu\nu}$ contains the further gauge invariance, $\delta v^I_{\mu\nu} = \gamma^m_{\mu\bar{\nu}} \partial_m \lambda^I$, $V^I$ contains the usual non-gauge degrees of freedom of $4g$ bosons and $4g$ fermions of N=1 super-Yang-Mills ($g$ is the dimension of the group).

IV. The Sigma Model Action in a Curved Target Space

After putting the two-dimensional super-vierbein into superconformal gauge such that

$$D_\kappa = e^{\ell(z - \frac{i}{2} \kappa \bar{\kappa}; \bar{\kappa}; z)} (\partial_\kappa + \frac{i}{2} \bar{\kappa} \partial_z) \quad \text{and} \quad \bar{D}_{\bar{\kappa}} = e^{\ell(z + \frac{i}{2} \kappa \bar{\kappa}; \bar{\kappa}; z)} (\partial_{\bar{\kappa}} + \frac{i}{2} \kappa \partial_z) \quad (IV.1)$$
the sigma model action for the four-dimensional Green-Schwarz heterotic superstring in a massless background is:

\[ S = i \int d^2z d^2\kappa [K_m \partial_2 X^m + K_\mu \partial_2 \Theta^\mu - \bar{K}_m \partial_2 \bar{X}^m - \bar{K}_\mu \partial_2 \bar{\Theta}^\mu + iV^I j_I + \kappa \bar{\kappa} k + \alpha'(\Phi \partial_2 \bar{L} - \bar{\Phi} \partial_2 L)], \quad (IV.2) \]

where \( X^m - \bar{X}^m = iH^m , H^m \) is determined by \( K_M \) and \( \bar{K}_M \) from the constraint \( h_{\mu \bar{\nu}} = 0 \) of equation (III.5), \( j_I \) is any right-moving current satisfying the commutation relations \([j_I , j_J ] = f^K_{IJ} j_K \), \( k \) is the kinetic energy term for the two-dimensional right-moving fields in \( j_I \), and \( L , \bar{L} \) are chiral and anti-chiral two-dimensional superfields containing the \( N=(2,0) \) superconformal degrees of freedom. Note that the left-moving stress-energy tensor is \( \bar{\Omega} \) superfields containing the \( N=(2,0) \) superconformal degrees of freedom. Note that the left-moving stress-energy tensor is \( \bar{D}_N K_m D_N X^m + \bar{D}_N \bar{K}_m D_N \Theta^\mu - D_N \bar{K}_m \bar{D}_N \bar{X}^m - D_N K_\mu \bar{D}_N \bar{\Theta}^\mu = - h_{\mu \bar{\nu}} D_N \Theta^\mu \bar{D}_N \bar{\Theta}^\mu = 0 \). In a flat background, \( K_\mu = - \frac{1}{2} \bar{X}_m \) and \( K_\mu = iX^m \gamma^m_{\mu \bar{\mu}} \bar{\Theta}^\mu \), so the lowest component of \( -i \bar{D}_N K_\mu \) and \( -i D_N \bar{K}_\mu \) can be interpreted as a curved-space generalization of the twistor fields, \( \bar{w}_\mu = x_m \gamma^m_{\mu \bar{\mu}} \bar{\lambda}^\mu \) and \( w_\mu = \bar{x}_m \gamma^m_{\mu \bar{\mu}} \lambda^\mu \).

By performing the integration over \( \kappa \) and \( \bar{\kappa} \), one obtains the following action:

\[ S = \int d^2z [\Omega \eta_{ab} E^a_\mu E^b_\nu \partial_2 y^\mu \partial_2 y^\nu + b_{MN} \partial_2 y^M \partial_2 y^N + A^I_M j_I \partial_2 y^M + k \]

\[ + \alpha' ((\Phi + \bar{\Phi}) R + i(\Phi - \bar{\Phi}) F + i(\nabla_\mu \Phi) \lambda^\mu \zeta + i(\nabla_\mu \bar{\Phi}) \bar{\lambda}^\mu \bar{\zeta})], \quad (IV.3) \]

where \( M \) and \( N \) range over four real bosonic values (\( m , n = 0 \) to 3) and four real fermionic values (\( \mu , \nu = 1 \) to 2, \( \mu , \bar{\nu} = 1 \) to 2), \( y^m = x^m \), \( y^\mu = \theta^\mu \), \( y^\bar{\mu} = \bar{\theta}^\mu \), \( E^A_\mu \) is the inverse super-vierbein obtained from the covariant fermionic derivatives \( \nabla_\mu \) and \( \nabla_\bar{\mu} \) of equation (III.1) (e.g., \( E^a_\mu = \delta^a_\mu \) and \( E^\bar{a}_\mu = f^\bar{a}_\mu \)), \( b_{MN} \) and its field strength \( \Omega \) are obtained from the chiral spinor prepotential \( \Sigma_\mu = (\nabla)^2 \bar{K}_\mu \) (e.g., \( b_{\mu \bar{\nu}} = \gamma_{\mu \bar{\nu}} \bar{\Sigma} \), \( \Omega = \nabla_\mu \Sigma_\nu \epsilon^{\mu \nu} + \bar{\nabla}_\bar{\mu} \Sigma_\bar{\nu} \epsilon^{\bar{\mu} \bar{\nu}} \)), \( A_M \) is obtained from the gauge-covariant derivatives \( e^{-V} \nabla_\mu e^V \) and \( \bar{\nabla}_\bar{\mu} \), \( 26 \) \( R \) is the two-dimensional curvature, \( F \) is the field strength of the two-dimensional gauge field, \( \zeta \) and \( \bar{\zeta} \) are the field strengths of the two-dimensional gravitinos, and \( \lambda^\mu \) is the lowest component of \( D_\mu \Theta^\mu \).

The first part of this action is equivalent to the usual four-dimensional Green-Schwarz sigma model action, except for the new torsion constraints on \( E^A_\mu \) that \( T^{\bar{\alpha}}_{\bar{a} \alpha} = T^{\bar{\alpha}_{\bar{a}}}_{\alpha} = 0 \) (as was shown in reference 10, this "superconformal" Green-Schwarz action is invariant under Siegel transformations).

The second part of the action is fundamentally different from the usual Green-Schwarz action in that it contains couplings to the \( N=(2,0) \) geometry. Note that not only the genus coupling constant, but also the instanton theta parameter, can now be absorbed into the zero modes of background fields.

**V. Conclusion**

As discussed in the introduction, the usual Green-Schwarz sigma model action has many unpleasant features that are not present in the action of equation (IV.2). Because this new action reduces to a free-field action in a flat background, it should be possible to calculate the \( \beta \)-function perturbatively and find string-corrected equations of motion for the \( d=4 \) \( N=1 \) supergravity fields.
One possible generalization would be to construct a similar sigma model action for the four-dimensional Green-Schwarz closed superstring in an N=2 supergravity background. It seems likely that one should use an N=(2,2) super-worldsheet and introduce a Kahler potential which depends on complex target-space coordinates which are either chiral or twisted-chiral two-dimensional superfields, i.e., are chiral or anti-chiral independently in the left and right-handed directions.

A more interesting generalization would be to construct an N=(2,0) sigma model action for the ten-dimensional Green-Schwarz heterotic superstring. An obvious guess for this action is to simply replace the four-component complex vectors and two-component complex spinors with ten-component complex vectors and sixteen-component complex spinors. The constraint \( h_{\mu\nu} = 0 \) of equation (III.5) now contains 256 components which restrict not only the imaginary part of \( X^m \), but also components of \( \Theta^\mu \) and \( \bar{\Theta}^\mu \). One reason for believing this naive guess is that in a flat background, where \( h_{\mu\nu} = \gamma^m_{\mu\nu} (X_m - \bar{X}_m) + i (\gamma^n_{\mu\rho} \Theta^\rho (\gamma^n_{\nu\sigma} \bar{\Theta}^\sigma) \), the action reduces to a free-field action that has been used successfully to calculate S-matrix scattering amplitudes for the ten-dimensional superstring. Furthermore, the physical vertex operators that were used to calculate these scattering amplitudes couple in the appropriate way to their background fields in this sigma model action.

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