A critical discussion is given of the results for baryon electromagnetic and axial form factors obtained from relativistic constituent quark models in the framework of Poincaré-invariant quantum mechanics. The primary emphasis lies on the point-form approach. First we summarize the predictions of the Goldstone-boson-exchange constituent quark model for the electroweak nucleon structure when using a spectator-model current in point form. Then the influences of different dynamics inherent in various kinds of constituent quark models (Goldstone-boson-exchange, one-gluon-exchange, instanton-induced interactions) are discussed. Finally the point-form results are compared to analogous predictions calculated in instant form. Relativistic effects are always of sizeable magnitude. A nonrelativistic approach is ruled out. The instant-form results are afflicted with severe shortcomings. In the spectator-model approximation for the current, only the point-form results appear to be reasonable a-priori. In fact, the corresponding quark model predictions provide a surprisingly good description of all elastic electroweak observables in close agreement with existing experimental data, specifically for the Goldstone-boson-exchange constituent quark model.

1. Introduction

Constituent quark models (CQMs) have become a reliable concept for the description of hadron spectroscopy. Specifically the low-lying spectra of the light and strange baryons have experienced a reasonable explanation by respecting the spontaneous breaking of chiral symmetry of quantum chromodynamics (QCD) in the dynamics employed for the effective interaction between constituent quarks. In this respect, the so-called Goldstone-boson-exchange (GBE) CQM\(^1\) has proven especially adequate\(^2\). Consequently it appears essential to include the relevant symmetries of low-energy QCD in the construction of any CQM. It is of similar importance to observe the symmetry requirements of special relativity. In order to obtain relativistic predictions for observables, any CQM must be based on a dynamical
concept (e.g., a relativistic mass operator or an equivalent Hamiltonian) invariant under the transformations of the Poincaré group.

In following a relativistic quantum-mechanical treatment of few-quark systems one must make a choice of the formalism to be applied. The different approaches are distinguished by the specific stability subgroups of the Poincaré group in case of an interacting system\textsuperscript{3,4}. The point form is characterized by four generators dependent on interactions, namely, the components of the four-momentum. The stability subgroup of the instant form has the same dimension (with the Hamiltonian and the three generators of the Lorentz boosts dependent on interactions). In case of the front form, only three generators are interaction-dependent.

Until a few years ago the point form had been the approach least frequently followed, even though it has specific advantages. For instance, one can easily and accurately apply Lorentz boosts, since their generators remain purely kinematical. Following the works by Klink et al.\textsuperscript{5}, the Graz-Pavia collaboration has applied the point form to the calculation of electromagnetic and axial form factors of the nucleon\textsuperscript{6,7,8}. One has obtained very remarkable results. The direct predictions of the GBE CQM, calculated with the nucleon wave functions just as obtained from the quark model, have been found in close agreement with experimental data in all instances. The behaviour of these results is therefore rather distinct from corresponding results obtained before in other approaches such as the front form (see, for example, refs.\textsuperscript{9}). There, one needed quark form factors in order to bring the theoretical predictions into the vicinity of the experimental data.

Below I first summarize the characteristics of the point-form results for the electroweak structure of the nucleons. Then I compare the covariant results with the nonrelativistic ones, consider different CQMs (wave functions), and contrast the point form to the instant form. In the discussion a few observations are made also with regard to relativistic invariance (frame independence) and current conservation.

2. Point-Form Results

Let us first have a look at the predictions of the GBE CQM\textsuperscript{1} for the nucleon electromagnetic form factors in figs. 1 and 2 and for the axial as well as induced pseudoscalar form factors in fig. 3. There the covariant results obtained in point-form spectator approximation (PFSA) are displayed. The direct predictions of the GBE CQM are immediately found in reasonable
agreement with the available experimental data up to momentum transfers of $Q^2 \sim 4 \text{ GeV}^2$. On the other hand, the results calculated in nonrelativistic impulse approximation (NRIA) fall short in every respect. In order to demonstrate the boost effects we also show the results that come out if one uses a nonrelativistic current but includes the boosts according to the point form (PFSA-NRC). For the axial form factor, instead, we give the results for
the case when a relativistic current is employed but no boosts are included (RC/no boosts). With regard to the induced pseudoscalar form factor a comparison is given to the case when the pion pole is neglected; evidently, one then misses contributions of more than an order of magnitude. From all of these results one learns that relativity is of utmost importance and the pion degrees of freedom play an essential role.
Figure 3. Nucleon axial and induced pseudoscalar form factors as predicted by the GBE CQM\(^1\).

How important are the specific dynamics prevailing in a certain CQM? In figs. 4 and 5 we give a comparison of the PFSA predictions of the GBE CQM\(^1\), of the one-gluon-exchange (OGE) CQM after Bhaduri-Cohler-Nogami\(^10\) in the relativistic parametrization by Theufl et al.\(^11\), and of the instanton-induced (II) CQM by the Bonn group\(^12\), which is treated in a Bethe-Salpeter approach; in addition the case with the confinement inter-
Figure 4. Comparison of proton and neutron electric form factors as predicted by the GBE$^1$, OGE$^{11}$, and II$^{12}$ CQMs and the case with the confinement potential only.

action only is shown. One sees that the dynamical influences are rather weak once a realistic nucleon wave function is produced. In particular, the kind of hyperfine interaction (GBE or OGE or II) is not so decisive, at least not for the nucleon ground state. If only the confinement interaction is present, however, one faces severe shortcomings especially with respect
Figure 5. Comparison of proton and neutron magnetic form factors as predicted by the GBE\textsuperscript{1}, OGE\textsuperscript{11}, and II\textsuperscript{12} CQMs and the case with the confinement potential only.

to the neutron form factors. Above all the neutron electric form factor (fig. 4) is dependent on a small mixed-symmetry spatial component in the wave function. If it is absent, like in the case with the confinement interaction only, one practically gets a zero result. In this context we have not shown a comparison for the induced pseudoscalar form factor. As explained
above it requires the pion-pole contributions, which cannot consistently be implemented neither for the OGE nor the II CQMs.

We have not addressed the electric radii and the magnetic moments here. They follow from the electric and magnetic form factors in the limit $Q^2 \to 0$. The corresponding results have already been calculated not only for the nucleons but also for all other octet and decuplet baryon ground states\cite{13}. Again the direct predictions (of the GBE CQM) in PFSA are immediately found to be reasonable and in good agreement with experiment in all cases whenever data exist. Relativistic effects are of considerable importance also for the electric radii and magnetic moments. This may appear strange at first sight, since we deal here with observables in the limit of zero momentum transfer. Nevertheless boost effects bring about sizeable contributions, and a nonrelativistic theory is bound to fail even for these quantities\cite{13}.

3. Comparison of Point-Form and Instant-Form Results

In view of the solid performance of the relativistic approach along the point form one has to ask why these surprising results come out (whenever a realistic wave function is employed). One has to bear in mind that the theory is by no means complete, since only a model current is used, namely, the so-called PFSA current. Of course, this model current is certainly not a
one-body current but still the corresponding calculation may lack sizeable contributions from further types of few-body currents. In order to elucidate the properties of the point form in the spectator model, we have performed a completely analogous study in instant form. In figs. 7 and 8 we present a comparison of the results obtained with the GBE CQM in PFSA and in instant-form spectator approximation (IFSA); in addition the NRIA (from figs. 1 and 2) is repeated. It is seen that the IFSA results remain far away from a reasonable description of the nucleon electromagnetic form factors.

Figure 7. Comparison of proton and neutron electric form factors of the GBE CQM calculated in PFSA and IFSA as well as in NRIA.
In fact, the IFSA results fall closer to the NRIA than to the PFSA (and thus to the experimental data), especially for the electric form factors. While this comparison is given for the Breit-frame calculations, one has to note that the instant-form results in the spectator-model approximation are frame-dependent. This makes them particularly questionable. A serious requirement of a relativistic theory is thus violated. In contrast, the point-form results are frame-independent. They are manifestly covariant even in the spectator-model approximation for the current.
Another criterion for a reliable theoretical approach to electromagnetic form factors is current conservation. We have checked the fulfillment of the continuity equation in case of the PFSA. In the range of momentum transfers considered here, the violation of current conservation remains below 1%. This is a satisfying observation though it does not definitely tell that two- and three-body currents would ultimately be small.

4. Conclusions

From the present studies one can learn several important lessons. First of all it is evident that a nonrelativistic CQM is by no means adequate to describe the properties of hadrons, not even in the domain of low energies or momentum transfers. Second, an approach following relativistic (Poincaré-invariant) quantum mechanics turns out to be justified and convenient. It allows to implement the symmetry requirements of special relativity and is not confronted with the problems of a field-theoretic approach (such as truncations of infinite series, discretizations of integrations, etc.). Specifically the point-form approach seems to bring about a number of advantages. It guarantees a-priori for covariance, allows to solve the dynamical equations rigorously, and keeps the violation of current conservation very small; in practice, it is negligible in the domain of momentum transfers considered here. The IFSA, on the other hand, is affected by severe theoretical shortcomings. Most embarrassing is the frame dependence. It makes the instant-form approach in the spectator approximation very questionable if not completely inadequate.

Certainly, at this instance, we are also left with a number of open problems. Even though the PFSA results provide a consistent description of all aspects of the electroweak structure of the light and strange baryon ground states, one must not forget that the approach relies on simplifying assumptions and is by no means complete. It is also distinct from a field-theoretic treatment. Obviously, one may ask for the contributions of two- and three-body currents. The approximate fulfillment of current conservation in the PFSA may be taken as a hint that these contributions might indeed be small. However, this must still be proven by performing calculations with a more elaborate or even the complete current operator. Until this problem is settled one can also not definitely conclude on a possible structure of constituent quarks and/or a finite extension of the interaction vertices. It is also clear that due to their unitary equivalence all forms of relativistic quantum mechanics must lead to the same results once a full calculation is
performed. The contributions missing beyond the present spectator-model calculations should then turn out of different magnitudes in the point and instant forms (and, of course, also in the front form).

Beyond the elastic form factors a number of further observables remain to be studied. The framework of Poincaré-invariant quantum mechanics is also applicable to inelastic processes such as transition form factors etc. Further important insights in the performance of relativistic CQMs and the adequacy of the relativistic quantum-mechanical approach may thus be obtained.

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