DARK ENERGY IN MODIFIED SUPERGRAVITY

Sergei V. Ketov $^{a,b}$ and Natsuki Watanabe $^a$

$^a$ Department of Physics, Graduate School of Science, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-0397, Japan

$^b$ Kavli Institute for Physics and Mathematics of Universe, The University of Tokyo, Kashiwa-shi, Chiba 277-8568, Japan

ketov@phys.se.tmu.ac.jp, watanabe-natsuki1@ed.tmu.ac.jp

Abstract

We propose a supersymmetric extension of the dynamical dark energy function and the scalar (super)potential in $F(R)$ supergravity. Our model is viable in the Einstein approximation, and also has an analytic (regular) scalar potential. The hidden sector responsible for spontaneous supersymmetry breaking is given too.
1 Introduction

The Standard (ΛCDM) Model of cosmology provides the simplest description of the current cosmic acceleration in agreement with all known observational data [1]. However, it does not explain its origin and its value. The next simple models are given by dynamical Dark Energy (DE), such as quintessence and \( f(R) \) gravity. The \( f(R) \) gravity is known as the non-trivial, viable and consistent alternative to a cosmological constant, while it is classically equivalent to the quintessence [2]. An embedding (or derivation) of a viable \( f(R) \) gravity from a more fundamental theory of gravity is unknown, whereas its consistency with particle physics can only be established in the context of a unified theory beyond the Standard Model of elementary particles. The leading proposals for such unified theory are supersymmetry, supergravity and superstrings.

The supersymmetric extension of \( f(R) \) gravity in curved superspace was proposed in ref. [3], where it was dubbed \( F(R) \) supergravity. Its structure was studied in refs. [4, 5, 6, 7, 8, 9, 10], whereas its consistency and viability for inflation and reheating in the early universe was established in refs. [11, 12]. The first application of \( F(R) \) supergravity to the current DE was given in ref. [13]. It is worth mentioning that the cosmological ΛCDM Model cannot be naively extended to supergravity, since pure (Einstein) supergravity can only have a negative or vanishing cosmological constant.

A viable description of the current dark energy in \( f(R) \) gravity imposes certain constraints on the function \( f(R) \) (see ref. [2] for our notation):

\[
\left| f(R) - \left( -\frac{1}{2} R \right) \right| \ll |R|, \quad \left| f'(R) - \left( -\frac{1}{2} \right) \right| \ll 1 \quad \text{and} \quad |R| f''(R) \ll 1, \quad (1)
\]

where the primes denote the derivatives with respect to the argument \( R \) (the scalar curvature of spacetime), as well as the stability conditions

\[
f'(R) < 0 \quad \text{and} \quad f''(R) > 0 \quad (2)
\]

In particular, it means that all those models must be close to the Standard ΛCDM Model, while some fine-tuning is required to get the observed value of the present cosmic acceleration. Still, there is considerable (functional) freedom in the choice of the function \( f(R) \) satisfying all the criteria, see e.g., refs. [14, 15, 16] for some explicit viable examples. In this paper we impose more theoretical constraints on the functions \( F(R) \) and \( f(R) \) via the corresponding scalar potential in the low space-time curvature regime relevant to the Einstein approximation.

Our paper is organised as follows. In Sec. 2 we briefly review the classical correspondence between \( f(R) \) gravity and quintessence [17, 18, 19, 20]. In Sec. 3 we replace the Appleby-Battye (AB) scalar potential [14] by an Uplifted-Double-Well (UDW) scalar potential. In Sec. 4 we relate it to \( F(R) \) supergravity. In Sec. 5 and propose a model of spontaneous supersymmetry (SUSY) breaking that gives rise to the UDW scalar potential. Sec. 6 is our conclusion.
2 $f(R)$ Gravity and Quintessence

The action of $f(R)$ gravity is given by

$$S = \int d^4x \sqrt{-g} \, f(R)$$  \hspace{1cm} (3)

We use the natural units $\hbar = c = M_{Pl} = 1$, where $M_{Pl}$ is the (reduced) Planck mass, and the spacetime signature is $(+,−,−,−)$. In our notation, a de Sitter space has a negative constant scalar curvature $R_0 < 0$.

The vacuum solutions to the theory (3) with $R = R_0$ satisfy

$$R_0 f'(R_0) = 2f(R_0)$$  \hspace{1cm} (4)

The action (3) can be transformed to the Einstein frame, by first rewriting it to the form

$$S = \int d^4x \sqrt{-g} \left[ f'(\phi)(R - \phi) + f(\phi) \right] ,$$  \hspace{1cm} (5)

where the scalar field $\phi$ has been introduced. Its equation of motion reads

$$f''(\phi)(R - \phi) = 0$$  \hspace{1cm} (6)

so that we get $\phi = R$ and, hence, the original action (3) back.

After the conformal transformation, $\tilde{g}_{\mu\nu} = g_{\mu\nu}f'(\phi)$, the action (5) reads

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{R} + \frac{3}{4(f')^2} \tilde{g}^\mu\nu \partial_\mu f'(\phi) \partial_\nu f'(\phi) - V(f'(\phi)) \right]$$  \hspace{1cm} (7)

with the scalar potential

$$V(\phi) = \frac{\phi f'(\phi) - f(\phi)}{f'(\phi)^2}$$  \hspace{1cm} (8)

The canonically normalised scalar field is given by

$$\sigma = -\sqrt{\frac{3}{2}} \ln f'(\phi) , \quad \text{or} \quad f'(\phi) = -\exp \left[ -\sqrt{\frac{2}{3}} \sigma \right] \equiv e^{-y}$$  \hspace{1cm} (9)

Equation (8) is a quadratic equation w.r.t. $f'(\phi)$, so that it can be rewritten to the form

$$f'(R) = \frac{R \pm \sqrt{R^2 - 4Vf}}{2V}$$  \hspace{1cm} (10)

where $V = V(R)$ and $f = f(R)$. Eq. (11) is the inverse problem for fixing the $f(R)$ function by a given scalar potential. For instance, in the case of slow-roll inflation, we have $V \approx \text{const.}$, so that $f'(R) \propto R$ approximately, which gives rise to the Starobinsky model of $(R + R^2)$ inflation \cite{21}. The present DE corresponds to the case $f'(R) \approx -\frac{1}{2}$ or $f(R) \approx -\frac{1}{2}R - \Lambda$ with $\Lambda > 0$. Here we are interested in the case of the upper sign choice and $4Vf << R^2$ in eq. (10). Then it reduces to $f' \approx f/R$ so that the Einstein term $(-\frac{1}{2}R)$ dominates in $f(R)$. It is enough for viability of our model in the Einstein approximation $|R_0| << |R| << 1$, where $R_0$ is the present cosmic value of the scalar curvature.
3 AB Function and its UDW Alternative

The trial dark energy function proposed by Appleby and Battye in ref. [14],
\[ f_{AB}(R) = -\frac{1}{2} R + \frac{1}{2a} \ln \left[ \cosh(aR) + \tanh(b) \sinh(aR) \right], \tag{11} \]
has \( f(0) = 0 \) and is viable in the Einstein approximation. It has two positive parameters, \( a \) and \( b \), which must be fine tuned to meet observations. In the Newtonian limit one finds [14]
\[ f_{AB}(R) \approx -\frac{1}{2} R - \frac{1}{4a} \ln (1 + e^{2b}) + O(e^{-2a|R|+2b}) \tag{12} \]
so that one gets the effective cosmological constant \( \Lambda \approx b/(2a) \approx |R_0| = 12H_0^2 \) in the limit \( b \gg 1 \). To meet observations, one should take \( \Lambda^{1/4} \approx 0.0024 \text{eV} \), and \( b \geq 30 \) from imposing the local gravity constraints [22].

The inverse function to \( f'_{AB}(R) = -\frac{1}{2} \left[ 1 - \tanh(aR + b) \right] \tag{13} \)
is available in an analytic form, so that it is not difficult to calculate the effective scalar potential from eq. (8). We find
\[ V_{AB}(y) = \frac{1}{2a} e^{2y} \left[ \ln (1 - e^{-y}) - e^{-y} \ln (e^{y} - 1) + 2be^{-y} + C \right], \tag{14} \]
where the constant \( C \) is given by \( C = \ln \left( e^{b} + e^{-b} \right) - b \).

The parameter \( a \) appears only as the overall factor in the scalar potential [14], so we introduce \( v_{AB} = V_{AB}/V_0 \) with \( V_0 = (2a)^{-1} \). The function \( v_{AB}(y) \) has only one parameter \( b \), and its profile (at \( b = 1.5 \)) is shown in Fig. 1.

Away from \( y = 0 \) on its right-hand-side (when \( b > 1 \)) the function \( v_{AB}(y) \) has two minima and one maximum, so it can be well approximated by an Uplifted-Double-Well (UDW) scalar potential of the Higgs-type (see Fig. 1),
\[ v_{UDW}(y) = \frac{1}{4} [(y - y_0)^2 - v^2]^2 + \frac{\mu^2}{2} [(y - y_0) - v]^2, \tag{15} \]
where we have introduced three real parameters \((y_0, v, \mu)\). The extrema of a quartic scalar potential are given by roots of a cubic equation. In the case (15) one root \((y_3)\) is given by \( y_3 = y_0 + v \), whereas the remaining two roots \( y_\pm \) are the roots of a quadratic equation. We find
\[ y_\pm - y_0 = \frac{1}{2} \left[ -v \pm \sqrt{v^2 - 4\mu^2} \right] \tag{16} \]
By demanding the local minima of \( v_{UDW} \) to coincide with those of \( v_{AB} \) we find
\[ \mu^2 = b^{-1} - \frac{1}{2} b^{-4}, \quad y_0 = b - \frac{1}{2} b^{-2}, \quad v = b + \frac{1}{2} b^{-2} \tag{17} \]
For large \( b \gg 1 \) the potential barrier between the two vacua (de Sitter and Minkowski) exponentially grows as \( e^{2(b-1)} \), while the constant \( C \) goes to zero.

The UDW scalar potential (15) is analytic, is bounded from below, and is non-negative. It has the absolute minimum corresponding to the flat (Minkowski) vacuum, and another minimum corresponding to the de Sitter vacuum that can be identified with an accelerating universe. Those vacua are separated by the high potential barrier, so that the lifetime of the universe in the meta-stable de Sitter vacuum can be larger than its age.

The AB scalar potential (14) has a non-analytic behaviour at \( y = 0 \) which corresponds to the infinite scalar curvature \( R \). The UDW scalar potential (15) is regular at \( y = 0 \) (and also for \( y < 0 \)), while \( y = 0 \) corresponds to the low space-time curvature of the order \( R_0 \). Hence, the AB and UDW functions are drastically different in the regime of the high space-time curvature where both models cannot be trusted. However, they are almost the same in the Einstein regime \( |R_0| << |R| << 1 \), which is enough for our purpose.

4 \( F(R) \) Supergravity and Quintessence

The action of \( F(R) \) supergravity in the chiral (curved) \( \mathcal{N} = 1 \) superspace of \((1 + 3)\)-dimensional spacetime, which was proposed in ref. [3], reads

\[
S = \int d^4x \, d^2\theta \, \mathcal{E} \, F(R) + \text{H.c.} ,
\]
where \( F(\mathcal{R}) \) is a holomorphic function of the covariantly chiral scalar curvature superfield \( \mathcal{R} \), and \( \mathcal{E} \) is the chiral superspace density. The scalar curvature \( R \) appears as the field coefficient at the \( \theta^2 \) term in the superfield \( \mathcal{R} \).\(^1\) The action (18) is equivalent to

\[
S = \int d^4x \, d^2\theta \, \mathcal{E} \left[ -\mathcal{Y} \mathcal{R} + Z(\mathcal{Y}) \right] + \text{H.c.},
\]

where we have introduced the new covariantly chiral scalar superfield \( \mathcal{Y} \) and the new holomorphic function \( Z(\mathcal{Y}) \) related to the function \( F \) as

\[
F(\mathcal{R}) = -\mathcal{R} \mathcal{Y}(\mathcal{R}) + Z(\mathcal{Y}(\mathcal{R})).
\]

The equation of motion of the superfield \( \mathcal{Y} \), which follows from the variation of the action (19) with respect to \( \mathcal{Y} \), has the algebraic form

\[
\mathcal{R} = Z'(\mathcal{Y})
\]

so that the function \( \mathcal{Y}(\mathcal{R}) \) is obtained by inverting the function \( Z' \). Substituting the solution \( \mathcal{Y}(\mathcal{R}) \) back into the action (19) yields the original action (18) because of Eq. (20). We also have

\[
\mathcal{Y} = -F'(\mathcal{R})
\]

The inverse function \( \mathcal{R}(\mathcal{Y}) \) always exist under the physical condition \( F'(\mathcal{R}) \neq 0 \).

The kinetic terms of \( \mathcal{Y} \) are obtained by using the (Siegel) identity

\[
\int d^4x \, d^2\theta \, \mathcal{E} \, \mathcal{Y} \mathcal{R} + \text{H.c.} = \int d^4x \, d^4\theta \, E^{-1}(\mathcal{Y} + \mathcal{\bar{Y}}),
\]

where \( E^{-1} \) is the full curved superspace density. Therefore, the Kähler potential reads

\[
K = -3 \ln (\mathcal{Y} + \mathcal{\bar{Y}})
\]

and gives rise to the kinetic terms

\[
\mathcal{L}_{\text{kin}} = \frac{\partial^2 K}{\partial \mathcal{Y} \partial \mathcal{Y}} \bigg|_{\mathcal{Y} = \mathcal{Y}} \partial_\mu \mathcal{Y} \partial^\mu \mathcal{\bar{Y}} = 3 \frac{\partial_\mu \mathcal{Y} \partial^\mu \mathcal{\bar{Y}}}{(\mathcal{Y} + \mathcal{\bar{Y}})^2}
\]

The kinetic terms (25) represent the non-linear sigma model \(^{[23]}\) with the hyperbolic target space of (real) dimension two, whose metric is known as the standard (Poincaré) metric with \( SL(2, \mathbb{R}) \) isometry.

In the decoupling limit of supergravity, the effective scalar potential \( V(\mathcal{Y}, \mathcal{\bar{Y}}) \) of a complex scalaron \( \mathcal{Y} \) is derived from eq. (19) when keeping only scalars (i.e. ignoring their spacetime derivatives together with all fermionic contributions).

\(^1\)See Ref. \(^{[9]}\) for details about our notation and \( F(\mathcal{R}) \) supergravity.
and eliminating the auxiliary fields near the minimum of the scalar potential. One finds \[12\]

\[ V = \frac{21}{2} |Z'(Y)|^2 = \frac{21}{2} |R(Y)|^2 \]  

(26)

that gives rise to the chiral superpotential

\[ W(Y) = \sqrt{\frac{21}{2}} Z(Y). \]  

(27)

The superfield equations (24) and (27) are model-independent, i.e. they apply to any function \( F(R) \) in the large \( M_{Pl} \) limit, near the minimum of the scalar potential with the vanishing cosmological constant. After the holomorphic superfield redefinition

\[ Y = \exp \left( \sqrt{\frac{2}{3}} \Phi \right) \]  

(28)

the kinetic terms (25) begin with the canonical term \( \bar{\Phi} \Phi \) or \( \partial_{\mu} \bar{\Phi} \partial^\mu \Phi \) where \( \Phi| = \phi \).

Accordingly, the chiral superpotential of \( \Phi \) is given by \( W(\Phi) = W(Y(\Phi)) \).

5 Spontaneous SUSY breaking model

The DE in the present universe, like any other (positive) non-vanishing vacuum energy, requires a spontaneous SUSY breaking. The latter can occur in the hidden matter sector (beyond the MSSM), which must include the scalaron superfield \( \Phi \) in our scenario. Since scalaron has the universal interaction with the gravitational strength to all matter fields, it is also the natural messenger of the gravitational mediation of the SUSY breaking from the hidden sector to the visible (matter) sector.\[\text{\footnotesize $^2$}\]

The simplest (Wess-Zumino-type) model of the hidden sector, leading to spontaneous SUSY breaking, consists of three chiral superfields, \( \Phi_1, \Phi_2 \) and \( \Phi_3 \). Let us choose the chiral superpotential of \( \Phi_1 \) in the form

\[ W_1(\Phi_1) = l^{1/2} \left( \frac{1}{6} \Phi_1^3 - \frac{1}{2} v^2 \Phi_1 \right), \]  

(29)

with two real (positive) parameters \( l \) and \( v \). It gives rise to the scalar potential

\[ V_1(\phi_1) = \left| \frac{\partial W_1}{\partial \Phi_1} \right|^2 = \frac{l}{4} |\phi_1^2 - v^2|^2 \]  

(30)

where \( \Phi_1| = \phi_1 \). Similarly, the chiral superpotential of \( \Phi_2 \) in the form

\[ W_2(\Phi_2) = \frac{\mu}{\sqrt{2}} \left( \frac{1}{2} \Phi_2^2 - u \Phi_2 \right), \]  

(31)

\[\text{\footnotesize $^2$The idea of the gravitational mediation of SUSY breaking was proposed in ref. [24].}\]
with two real (positive) parameters $\mu$ and $u$, gives rise to the scalar potential

$$V_2(\phi_2) = \left| \frac{\partial W_2}{\partial \Phi_2} \right|^2 = \frac{\mu^2}{2} |\phi_1 - u|^2$$  \hfill (32)

Therefore, the use of the chiral superpotential

$$W(\Phi_1, \Phi_2, \Phi_3) = W_1(\Phi_1) + W_2(\Phi_2) + \Phi_3(\Phi_2 - \Phi_1)$$  \hfill (33)

gives rise to the scalar potential

$$V(\phi) = V_1(\phi) + V_2(\phi) = \frac{1}{4} \left| \phi^2 - v^2 \right|^2 + \frac{\mu^2}{2} |\phi - u|^2$$  \hfill (34)

where $\phi = \Phi$ and $\Phi = \Phi_1 = \Phi_2$. The scalar potential (34) is the complex extension of the UDW scalar potential (15), where $\text{Re}(\phi) = y - y_0$ and $u = v$.

The superpotential (33) is the particular example of the O’Raifeartaigh-type models of spontaneous SUSY breaking [25]. Indeed, the system of equations

$$\frac{\partial W_1}{\partial \Phi_1} = \frac{\partial W_2}{\partial \Phi_2} = \frac{\partial W_3}{\partial \Phi_3} = 0$$  \hfill (35)

does not have a solution when $u \neq v$, which gives rise to a positive vacuum energy. When $u = v$, we have a stable (Minkowski) vacuum with unbroken SUSY and the vanishing vacuum energy, but also a metastable (de Sitter) vacuum with DE.

6 Conclusion

Our main results are given by eqs. (10), (14), (15), (17) and (33). We replace the effective scalar potential (14) associated with the ad hoc AB function (11) by the Higgs-type scalar potential (15) that gives rise to a meta-stable accelerating universe. We propose the specific (O’Raifertaigh-type) model of the hidden sector leading to spontaneous SUSY breaking and the UDW scalar potential, in terms of three chiral scalar superfields with the chiral superpotential (33). In our approach the chiral scalaron superfield is the universal messenger of the gravitational mediation of SUSY breaking to the visible sector (Standard Model) of elementary particles.

Acknowledgements

SVK is supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. SVK is grateful to A. D. Dolgov and T. T. Yanagida for discussions.
References

[1] E. Komatsu et al., Astrophys. J. Suppl. 192 (2011) 18
[2] A. De Felice and S. Tsujikawa, Living Rev. Rel. 13 (2010) 3
[3] S. J. Gates, Jr. and S. V. Ketov, Phys. Lett. B 674, 59 (2009)
[4] S. J. Gates, Jr., S. V. Ketov and N. Yunes, Phys. Rev. D80 (2009) 065003
[5] S. V. Ketov, Class. and Quantum Grav. 26 (2009) 135006
[6] S. V. Ketov, Phys. Lett. B 692, 272 (2010)
[7] S. V. Ketov, AIP Conf. Proc. 1241, 613 (2010); arXiv:0910.1165 [hep-th]
[8] S. V. Ketov and N. Watanabe, JCAP 1103, 011 (2011)
[9] S. V. Ketov, e-Print: arXiv:1201.2239 [hep-th]
[10] S. V. Ketov and A. A. Starobinsky, JCAP 08 (2012) 022
[11] S. V. Ketov and A. A. Starobinsky, Phys. Rev. D 83, 063512 (2011)
[12] S. V. Ketov and S. Tsujikawa, Phys. Rev. D86 (2012) 023529
[13] S. V. Ketov and N. Watanabe, Phys. Lett. B705 (2011) 410
[14] A. Appleby and R. Battye, Phys. Lett. B654 (2007) 7
[15] W. Hu and I. Sawicki, Phys. Rev. D76 (2007) 064004
[16] A.A. Starobinsky, JETP Lett. 86 (2007) 157
[17] B. Whitt, Phys. Lett. B145 (1984) 176
[18] J. D. Barrow and S. Cotsakis, Phys. Lett. B214 (1988) 515
[19] K.-I. Maeda, Phys. Rev. D39 (1989) 3159
[20] S. Kaneda, S. V. Ketov and N. Watanabe, Mod. Phys. Lett. A25 (2010) 2753
[21] A. A. Starobinsky, Phys. Lett. B91 (1980) 99
[22] L. Amendola and S. Tsujikawa, Phys. Lett. B660 (2008) 125
[23] S. V. Ketov, Quantum Non-linear Sigma-models, Springer-Verlag, 2000
[24] A. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970
[25] L. O’Raifeartaigh, Nucl. Phys. B96 (1975) 331.