A simple inert model solves the little hierarchy problem and provides a dark matter candidate

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(Dated: May 2, 2014)

We discuss a minimal extension to the standard model in which two singlet scalar states that only interact with the Higgs boson is added. Their masses and interaction strengths are fixed by the two requirements of canceling the one-loop quadratic corrections to the Higgs boson mass and providing a viable dark matter candidate. Direct detection of the lightest of these new states in nuclear scattering experiments is possible with a cross section within reach of future experiments.

PACS numbers: 11.30.Qc, 12.60Fr, 14.80.Bn, 95.35.+d

Physics at LEP taught us how electroweak (EW) precision measurements prefer a Higgs boson mass between 100 and 200 GeV [1] and—not having seen new particles beyond those of the standard model (SM)—a cutoff $\Lambda$ for higher order operators encoding new physics larger than 5 TeV [2]. Both these results seem to be confirmed by the early runs of the LHC. In particular, the possible determination of the Higgs boson mass around 125 GeV [3] comes in support to the first, while the absence of new states propagating below the TeV scale [4] would support the second one.

When taken together, the two statements above rise the problem of the little hierarchy: for the Higgs boson mass (and the electroweak vacuum expectation value) to be in the 100 GeV range—that is, roughly between one and two orders of magnitude smaller than the cutoff—quadratic renormalization effects must be canceled to an unnaturally high accuracy. This cancellation may either come from a symmetry—as it is the case in supersymmetric models—or be an accident in which the various terms conspire to cancel against each other. In the latter case, the cancellation is best thought as the effect of a dynamical mechanism, at work beyond the cutoff, which arises from new physics that we do not know and simulate by fixing by hand some of the terms in the effective lagrangian.

In this letter, we come back to the little hierarchy problem by following such an empirical approach and discuss a possible solution based on the presence of inert scalar states [5], that is, scalar particles only interacting with the Higgs boson (and gravity) which acquire no vacuum expectation value.

The simplest realization is based on the addition to the SM of just two states: two real scalars $S_a$ and $S_b$ transforming as the singlet representation of the EW gauge group $SU(2) \times U(1)$ (and similarly not charged under the color group).

In addition we impose a $Z_2$ symmetry under which $S_{a,b}$ are odd and all the SM fields are even. In this way, the new states couple to the SM Higgs doublet only through quartic interactions in the scalar potential.

By construction, our model is inert and therefore we only look for solutions with $\langle S_{a,b} \rangle = 0$, thus $Z_2$ is unbroken and after EW symmetry breaking the lightest singlet state can potentially be a viable cold dark matter (DM) candidate.

We solve the little hierarchy problem by assuming that the Veltman condition [6] is satisfied, namely that the new scalar sector couples to the SM Higgs boson just so as to make the one-loop quadratic divergences to the SM Higgs boson bare mass vanish (see [7] for an earlier application of the same idea in a non-inert model and [8] for a multi-scalar implementation studying solutions different from ours). Once one-loop quadratic divergent terms are canceled, the Higgs boson mass is only renormalized by one-loop finite terms and higher-loop corrections and can therefore be naturally smaller than the cutoff.

In this approach, we may also ask whether there are natural solutions for which the bare masses of $S_{a,b}$ are of the same order of their one-loop quadratic correction, or, in other words, whether there are solutions for which $S_{a,b}$ has a mass of the order $m \gtrsim \Lambda/4\pi$—that is, $m \gtrsim 700$ GeV for $\Lambda \sim 10$ TeV. Such mass values would then be natural thus removing the need of adding additional particles (fermions, in this case) to cancel à la Veltman the one-loop quadratic divergences to the masses of $S_{a,b}$. As we shall see, this is indeed the case.

The lagrangian of the model is given by the kinetic and Yukawa terms of the SM with a scalar potential given by

$$V(H, S_a, S_b) = \mu^2_a (H^\dagger H) + \mu^2_b (H^\dagger H) + \lambda_1 (H^\dagger H)^2 + \lambda_2 a^4 S_a^4 + \lambda_3 b^4 S_b^4 + \lambda_4 a^2 b^2 S_a^2 S_b^2 + \lambda_5 a^4 S_a^4 + \lambda_6 b^4 S_b^4 + \lambda_7 a^2 b^2 S_a^2 S_b^2 .$$

Notice that in eq. (1) the quadratic term $S_a S_b$ is not present because eliminated by a field redefinition. A subset of the independent parameters of the potential in eq. (1) is constrained by requiring:

- the Veltman condition for the SM Higgs doublet renormalization;
- tree-level unitarity for $S_a S_b$ scattering;
- correct relic density for the lightest $S_i$ as a cold DM candidate.
We impose no non-triviality condition.

The Veltman condition consists in the vanishing of the one-loop quadratic divergence contribution to the Higgs mass. The most unambiguous way to compute it is by using dimensional regularization (DR) and extract the pole in $D = 2$. Other renormalization schemes, and in particular those with a sharp cutoff, give rise to scheme-dependent divergent logarithmic terms which obscure the result. Accordingly we find

$$\delta \mu_H^2 \propto \frac{1}{16\pi^2} \left[ \frac{9}{4} g'^2 + \frac{3}{4} g^2 - 6 \lambda_1 + \lambda_3a + \lambda_3b - 12 g'^2 \right],$$

(2)

where $g$ and $g'$ are the EW gauge couplings and $y_i$ is the top Yukawa couplings—having neglected all the other much smaller Yukawa couplings. As usual the minimization of the scalar potential is obtained by imposing that at the minimum $\langle H \rangle = v_W/\sqrt{2}$ and, as already mentioned, by requiring that $S_{a,b}$ be inert with vanishing vacuum expectation values. In this way, we obtain three simple relations between the masses of the physical scalars, namely $h, S_a$ and $S_b$, and the parameters of the scalar potential $V(H, S_a, S_b)$:

$$\lambda_1 = \frac{m_h^2}{2v_W^2}, \quad \lambda_3a + \lambda_3b = \frac{m_{S_a}^2 + m_{S_b}^2 - 2 \mu_a^2 - 2 \mu_b^2}{v_W^2}. \quad (3)$$

Accordingly, the Higgs boson mass quadratic divergent contribution becomes proportional to

$$\delta \mu_H^2 \propto \frac{1}{16\pi^2 v_W^2} \left[ 3m_h^2 + 3m_{S_a}^2 + 6m_{S_b}^2 + m_{S_a}^2 + m_{S_b}^2 - 2 \mu_a^2 - 2 \mu_b^2 - 12 m_h^2 \right]. \quad (4)$$

Consistently we compute the one-loop finite contributions to the Higgs boson mass using dimensional regularization with renormalization scale $\mu$. The SM particle contributions are negligible while the new scalars, the mass of which is not fixed at this level, contribute with

$$\delta m_{h}^2 (\mu^2) \simeq \frac{1}{16\pi^2} \left[ \lambda_3a m_{S_a}^2 \log \frac{m_{S_a}^2}{\mu^2} + \lambda_3b m_{S_b}^2 \log \frac{m_{S_b}^2}{\mu^2} \right],$$

(5)

where we are neglecting terms proportional to $\lambda_3v_W/(m_a^2 - m_b^2)$ because they are small for the solutions we are interested in.

By imposing the Veltman condition $\delta \mu_H^2 = 0$ we obtain the sum $m_{S_a}^2 + m_{S_b}^2$ in terms of the unknown parameter $\mu_{a,b}^2$ and of the input physical quantities $m_h$, $m_t$, $v_W$ and $m_Z$ which, by substituting their experimental values (taking for $m_h$ the value of 125 GeV), yields

$$m_{S_a}^2 + m_{S_b}^2 = 500^2 + 2 \mu_a^2 + 2 \mu_b^2,$$

(6)

with an overall uncertainty—given by the neglected lighter quark masses contribution—of about 5%.

Since $S_{a,b}$ are inert, $\mu_{a,b}^2 > 0$ and we automatically have a lower bound for $m_{S_a}^2 + m_{S_b}^2$ given by 500 GeV$^2$. Moreover, the Higgs boson discovery put a constraint on the Higgs decay into light singlet scalars $h \to S_a S_b$, which gives an individual lower bound for $m_{S_a, S_b}$ of roughly 60 GeV—well below the mass range we are interested in.

In addition, the sum $\lambda_3a + \lambda_3b$ is fixed by eq. (3) to be

$$\lambda_3a + \lambda_3b = \frac{500^2}{v_W^2} \approx 4.1,$$

(7)

a rather large value which is however still within the perturbative regime since $(\lambda_3a, \lambda_3b)^2/4 \pi^2 < 1$ and the triviality bounds on the Higgs potential.

Tree-level unitarity for the scattering of the additional scalars can be verified by examining the partial wave unitarity for the two-particle amplitudes for energies $s \geq M_{W}^2, M_{Z}^2$. We write the $J = 0$ partial wave amplitude $a_0$ in terms of the tree-level amplitude $T$ as

$$a_0(s) = \frac{1}{32\pi} \int_{-1}^{1} d\cos \theta T(s) = \frac{1}{16\pi} F(\lambda_i).$$

(8)

We can compute the amplitudes by using only the scalar potential because of the equivalence theorem [9]. Accordingly, in the high-energies regime, the only relevant contributions come from the quartic couplings in the potential and $F$ in eq. (9) is a function of the $\lambda_i$ in the potential in eq. (1). The combination of the $\lambda_i$ entering in $F$ is constrained by unitarity which requires $a_0(s) < 1$. If we take $\lambda_1, \lambda_3a, \text{and } \lambda_3b$ according to eq. (3) and eq. (7), the unitarity constraint is satisfied by taking all the other $\lambda_i$ of order one.

The EW precision measurements are automatically satisfied by the singlets inert model because of the value of the Higgs boson mass and the non-interaction of the new states with the rest of the SM particles.

The inert scalars $S_i$, being gauge singlets, only interacts with the SM particles through the Higgs boson $h$. The point-like interaction $\lambda_3a/2 S_i S_j h h$ which contribute to the cross section $S_i S_j \to h h$. The Higgs boson $h$ also mediates the scattering processes $S_i S_j \to f f$, $S_i S_j \to W^+ W^-$, $S_i S_j \to ZZ$.

It has been shown [10] that a single inert singlet that couples with the Higgs boson with a small coupling is a realistic cold DM candidate with a mass $\lesssim v_W$. In our case, the lightest singlet may account for the correct relic density in the opposite regime where its mass is $\gg v_W$ and its coupling with the Higgs boson relatively large. In this case, fixing $S_a$ to be the lightest and assuming that $S_b$ is sufficiently heavier not to take into account coannihilation processes, the scattering amplitude is dominated by the pointlike $S_a S_a \to h h$ vertex which gives a contribution to the total cross section equal to

$$\langle \sigma v \rangle \approx \frac{1}{16\pi} \frac{\lambda_3a^2}{m_{S_a}^2}.$$ 

(9)
FIG. 1: The allowed region for the singlet masses $m_{S_a}$ and $m_{S_b}$ satisfying all the constraints discussed in the text. The upper line comes by requiring finite one-loop corrections to the Higgs boson mass of order 100 GeV when the lightest singlet state provides the correct amount of relic density in the universe. The diagonal line corresponds to the condition that $S_b$ is sufficiently heavier than $S_a$ so as to allow neglecting coannihilation processes when computing the relic density. Finally the vertical line on the left is the lowest mass value for $S_a$ which is still natural for an effective theory below 10 TeV.

To estimate the viability of $S_a$ as DM candidate, we make use of the approximated analytical solution \[11\]. The relic abundance $\frac{n_{DM}}{s}$ is written as

$$\frac{n_{DM}}{s} = \sqrt{\frac{180}{\pi g_* M_{pl} T_f (\sigma v)}},$$

(10)

where $M_{pl}$ is the Planck mass, $T_f$ is the freeze-out temperature, which for our and similar candidates is given by $m_{S_a}/T_f \sim 26$. The constant $g_*$ = 106.75 + 2 counts the number of SM degrees of freedom in thermal equilibrium plus the additional degrees of freedom related to the singlets, $s$ is their total entropy density. Current data fit within the standard cosmological model give a relic abundance with $\Omega_{DM} h^2 = 0.112 \pm 0.006 \ [12]$ which corresponds to

$$\frac{n_{DM}}{s} = (0.40 \pm 0.02) \times 10^3 m_{S_a}/\text{GeV},$$

(11)

Under the hypothesis that the two singlets have a mass $\geq 800$ GeV not to be protected at the one-loop level, that the $S_b$ decays mainly through $S_b \rightarrow S_a hh$, and that the finite contribution in eq. (3) is of order the Higgs boson mass, eqs. (10) – (11) determine an allowed region in the singlet masses space as shown in Fig. 1 and roughly delimited by

$0.7 \leq m_{S_a} (\text{TeV}) \leq 0.9 \quad \text{and} \quad 0.8 \leq m_{S_b} (\text{TeV}) \leq 1.1$.

The uncertainty here is dominated by the approximations built in the analytical expression given by eq. (10) which can be estimated to be of the order of 10%.

Notice that had we introduced only one singlet state, the solution of the little hierarchy problem would have given it a mass and a coupling to the Higgs boson leading to a relic abundance too large to agree with current determinations. This is the motivation behind the introduction of two rather than just one additional scalar.

We verified—by using the software of [13]—that photon and charged particle fluxes are two or more orders of magnitude smaller than those corresponding to a typical EW scale DM candidate. For this reason, there seems to be little hope of testing the model in such indirect DM searches.

On the other hand, there exists a possibility of detecting the inert scalar $S_a$ in nuclear scattering experiments. The $\lambda_3$ quartic term in eq. (1) gives rise also to the three fields interaction $SShh$ which yields the effective singlet-nucleon vertex

$$f_N \frac{\lambda m_N}{m_h^2} S_a S_a \bar{\psi}_N \psi_N.$$
The (non-relativistic) cross section for the process is given by

\[ \sigma_N = f_N^2 m_r^2 \frac{\lambda_s^2}{4\pi} \left( \frac{m_r}{m_s, m_h^2} \right)^2, \]

where \( m_r \) is the reduced mass for the system which is, to a very good approximation in our case, equal to the nucleon mass \( m_N \); the factor \( f_N \) contains many uncertainties due to the computation of the nuclear matrix elements and it can vary from 0.3 to 0.6 [15]. Substituting the values to the computation of the nuclear matrix elements and correcting relic density for the lightest neutral state, finite contributions to the Higgs boson mass are too large. Moreover, the mass splittings among the components of \( R \) have to be controlled so as not to affect EW precision measurements, implying that not all the coannihilation effects can be neglected.

Acknowledgments

We thank P. Ullio and N. Fornengo for explaining to us some aspects of DM physics. MF thanks SISSA for hospitality.