Elliptic flow of charm hadrons is investigated based on the quark coalescence model. Due to the large difference between the charm quark and light quark masses, hadrons containing both light and charm quarks show a qualitatively different $v_2(p_\perp)$ from hadrons containing only light quarks. Simple relations are proposed to infer quark elliptic flow from those of hadrons. The effects of the finite momentum spread of hadron wavefunctions are also studied, and are found to be small for charm hadrons.

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I. INTRODUCTION AND CONCLUSIONS

During the early stage of ultra-relativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the future Large Hadron Collider (LHC), a deconfined phase, often called the quark-gluon plasma, is expected to be formed. As the dense matter expands, the parton degrees of freedom in the deconfined phase convert to hadrons. Quark coalescence is a simple model of hadronization, where it is assumed that gluons, which are abundant at high temperatures, either convert to quark-antiquark pairs or serve to “dress” quarks into constituent quarks near hadronization. The effective degrees of freedom are constituent quarks and anti-quarks, and mesons form from a quark-antiquark pair, while baryons from three quarks according to their valence quark compositions. Quark coalescence has been applied to heavy ion collisions in the algebraic coalescence rehadronization (ALCOR) and microscopic coalescence rehadronization (MICOR) models to describe hadron abundances and in a multiphase transport model (AMPT) with string melting to describe the elliptic flow at RHIC. Recently it has been used to address the elliptic flow of hadrons of different flavors, and the large $p/\pi$ ratio observed at RHIC.

Elliptic flow, $v_2 \equiv \langle \cos(2\phi) \rangle$, the second Fourier moment of the azimuthal momentum distribution, is one of the important experimental probes of collective dynamics in $A + A$ reactions. It results from the spatial asymmetry in the transverse plane in non-central collisions, which is largest at early times. Therefore, elliptic flow is sensitive to the properties of dense matter, such as its equation of state or the effective scattering cross section of partons produced in the collisions, Measurements of elliptic flow at high transverse momentum provide important constraints about the density and effective energy loss of partons.

Charm quark elliptic flow holds the key to resolve a recent controversy regarding charm dynamics. It was pointed out in Ref. that both i) initial perturbative QCD charm production without final state interactions (as modeled in PYTHIA) and ii) complete thermal equilibrium for charm hadrons are consistent with the single electron spectra from PHENIX. Information on the magnitude of charm quark elliptic flow is essential to distinguish between these two extreme scenarios.

In this study we determine the elliptic flow of charm mesons and baryons based on quark coalescence and investigate how to extract the charm quark elliptic flow from that of charm hadrons. We extend the simple quark coalescence formalism to the case of different constituent quark masses, which is relevant for heavy flavors, and also consider the effect of the finite momentum width of hadron wavefunctions.

We find that relations between hadron and parton elliptic flow are modified for hadrons with constituent quarks of unequal masses. As a result, charm hadron elliptic flow is predicted to increase much slower with transverse momentum and saturate at a much higher momentum scale than the flow of hadrons with only lighter flavors. A very slow increase of charm hadron elliptic flow with $p_\perp$ is a strong indication for a small charm quark elliptic flow. The effect of the finite momentum spread in hadron wavefunctions is relatively small for charm hadrons. In this case, simple relations exist to unfold both light and charm quark $v_2(p_\perp)$ from hadron elliptic flow. Comparing these predictions with upcoming experimental data will tell to what degree charm quarks rescatter in the medium and provide valuable insights into the dynamics and hadronization of the dense partonic matter in heavy ion collisions.

II. MASS AND WAVEFUNCTION EFFECTS IN QUARK COALESCENCE

A convenient starting point to describe meson production via coalescence $\alpha \beta \rightarrow M$ is the relation between the phase space distributions of constituent quarks $\alpha$ and $\beta$, the meson wavefunction $\psi_p$, and the invariant momentum distribution of produced mesons. Here $\vec{p} \equiv \vec{p}_\alpha + \vec{p}_\beta$, $\vec{q} \equiv \vec{p}_\alpha - \vec{p}_\beta$, $q_M$ is the statistical factor for the meson formation, and the integration runs over a 3D space-time hypersurface parameterized by
\( \sigma^\mu(x) \). The expression is valid if coalescence is a rare process. When the coalescence probabilities are high for the constituents, instead of the quadratic (cubic) scaling of the meson (baryon) number with the quark number implied by the above equation, the scaling should be linear \( B C \). Eq. \( (1) \) also neglects the hadronic binding energy. Because of the large charm quark mass, this assumption is better met for charm hadrons than for lighter ones.

For quark coalescence into baryons \( \alpha \beta \gamma \rightarrow B \), Eq. \( (1) \) can be generalized in a straightforward manner.

Note, Eq. \( (1) \) is valid for collectively expanding sources, provided the parton distribution functions, \( f_i(\vec{p}, x) \), change little over the spatial size of the hadron wavefunctions \((\sim 1 \text{ fm})\). However, it has to be modified if flow velocity gradients are very large as shown by Eqs. \( (3.19-20) \) in Ref. \( [24] \). We leave the discussion of this and other space-momentum correlation effects to further studies.

For a hadron at rest, \( \psi_0(\vec{r}) \) has a small momentum space extension of \( \Lambda_{QCD} \sim 1/\text{fm} \) (based on the uncertainty principle) and therefore Eq. \( (1) \) reduces to the simple formula considered in Ref. \( [3] \). For a fast moving hadron, on the other hand, the wavefunction can change significantly due to Lorentz boost. We estimate this effect assuming that \( \psi_0 \) is dominated by the contributions of dressed valence quarks. In this case, e.g., for mesons, \( |\psi_0(\vec{q}')|^2 \) is the probability density for finding quark/antiquark \( \alpha \) and \( \beta \) with momenta \((q'_\alpha, q'_\beta) = (+q'/2, -q'/2)\) in the hadron rest frame (primed quantities refer to the hadron rest frame throughout this study). For a hadron at midrapidity with momentum \( \vec{p} = (p_\perp \vec{n}_\perp, 0) \) in the LAB frame, the transverse momenta of valence quarks along the transverse boost direction \( \vec{n}_\perp \) are given by

\[
p_{\perp i} = \frac{E'_i}{m_i} p_{\perp} + \vec{q}'_{\perp i} \cdot \vec{n}_\perp \sqrt{\frac{p_{\perp}^2 + m_i^2}{m_i}}.
\]

Here \( E'_i \equiv (m^2_i + |q'_i|^2)^{1/2}, m_i \) is the effective mass of valence quark \( i \) and \( m_i \) is the meson mass.

Suppose now that the rest-frame wavefunction was sufficiently narrow so that one can take \( \vec{q}' \rightarrow 0 \). For the weakly bound system assumed, \( m_i \approx m_\alpha + m_\beta \) and Eq. \( (2) \) then gives \( p_{\perp i} = p_{\perp} m_i / (m_\alpha + m_\beta) \). This relation also holds in general for the average constituent momenta, provided \( |\vec{q}'_{\perp i}|^2 \ll m_i^2 \). Introducing the constituent momentum fractions \( z_i \equiv p_{\perp i}/p_{\perp} \), for mesons and baryons we then have, respectively,

\[
z_i = \frac{m_i}{m_\alpha + m_\beta}, \quad \bar{z}_i = \frac{m_i}{m_\alpha + m_\beta + m_\gamma}.
\]

If the effective masses of constituent quarks are similar, \( \bar{p}_{\perp \alpha} = \bar{p}_{\perp \beta} = \bar{p}_{\perp} / 2 \) for a meson, which is the case, e.g., for pions or the \( J/\psi \). On the other hand, for \( D \) mesons \( m_\alpha < m_\beta \) and therefore most of the hadron momentum is carried by the heavy quark. The asymmetric momentum configuration arises because coalescence requires the constituents to have similar velocities, not momenta.

The Lorentz boost also affects the width of the hadron wavefunction. From Eq. \( (2) \) the spreads of valence quark momentum fractions in the LAB frame are

\[
delta z_i = \frac{q'_{\perp i} \cdot \vec{n}_\perp}{m_i} \frac{\sqrt{p_{\perp}^2 + m_i^2}}{p_{\perp}} \approx \frac{q'_{\perp i} \cdot \vec{n}_\perp}{m_i},
\]

if the hadron is moving relativistically. For massive hadrons, such as charm hadrons, \( \delta z_i \approx q'_{\perp i} \cdot \vec{n}_\perp / m_i \ll 1 \) because \( q'_{\perp} \) is typically on the order of \( \Lambda_{QCD} \).

On the other hand, for hadrons with masses comparable to \( \Lambda_{QCD} \), e.g., pions, \( \delta z_i \) is always on the order of unity regardless of the transverse momentum. However, note that for such light mesons the binding energy cannot be neglected and hence the conventional coalescence formalism may not be reliable.

Since the constituent quark momentum components perpendicular to the hadron momentum \((i.e., \vec{n}_\perp \) at mid-rapidity) are small \((\sim \Lambda_{QCD})\), we further simplify Eq. \( (1) \) by considering only the quark momentum component along \( \vec{n}_\perp \). Then the integrals over the wavefunction can be recast, for mesons for example, as \( \int d^3q \psi_0(\vec{q})^2 = \int d\zeta_\alpha |\Phi_M(\zeta_\alpha)|^2 \). As seen from Eq. \( (4) \), \( \delta z_i \) is independent of \( p_{\perp} \) for large \( p_{\perp} \) and thus \( \Phi_M(\zeta_\alpha) \) only depends on \( \zeta_\alpha \).

In this case Eq. \( (1) \) becomes, with \( z_\beta = 1 - z_\alpha \),

\[
E(\vec{N}(\vec{p})) = g_M \int \frac{d\sigma_{\perp}}{(2\pi)^3} \int d\zeta_\alpha |\Phi_M(\zeta_\alpha)|^2 f_\alpha(z_\alpha, \bar{p}, x) f_\beta(z_\beta, \bar{p}, x).
\]

III. ELLIPTIC FLOW FOR CHARM HADRONS

A. Narrow wavefunction case (the limit of zero momentum spread)

Because for charm hadrons the spread \( \delta z_i \) is small, it is a good approximation to consider only the mean quark momentum fraction \( \bar{z}_i \) in Eq. \( (3) \). For simplicity, we shall neglect the spatial variation of \( f_i \) on the hypersurface and assume that in non-central heavy ion collisions the \( \cos(2\phi) \) component is the only non-trivial term in the quark azimuthal distribution, i.e.,

\[
f_i(p_{\perp}) \equiv (2\pi)^3 \frac{dN_i}{dp_{\perp}} = h_i(p_{\perp}) [1 + 2v_{2i}(p_{\perp}) \cos(2\phi)].
\]

Eq. \( (4) \) then relates \( v_2(p_{\perp}) \) of mesons or baryons to those of the constituent quarks as \( [5] \)

\[
v_2^M(p_{\perp}) = \frac{v_{2,\alpha} + v_{2,\beta}}{1 + 2v_{2,\alpha}} \approx v_{2,\alpha} + v_{2,\beta},
\]

\[
v_2^B(p_{\perp}) = \frac{v_{2,\alpha} + v_{2,\beta} + v_{2,\gamma} + 3v_{2,\alpha}v_{2,\beta}v_{2,\gamma}}{1 + 2v_{2,\alpha} + v_{2,\beta} + v_{2,\gamma}} \approx v_{2,\alpha} + v_{2,\beta} + v_{2,\gamma}.
\]

where \( v_{2,i} \equiv v_{2i}(\bar{z}_i p_{\perp}) \) is the elliptic flow of valence quark \( i \) at its average momentum. As a result, for hadrons consisting of both light and charm quarks, elliptic flow at a given \( p_{\perp} \) contains the light quark elliptic flow at a much smaller \( p_{\perp} \).
If the light quark density is so high that the coalescence probability for a charm quark is unity, Eq. (7) breaks down, and charm hadrons just inherit the flow contribution from charm quarks [16]. Preliminary experimental data indicate that $p_{\perp}$ scale for light quarks below which this occurs might be as low as 1 GeV [17]. Also note that our study does not include the independent fragmentation of partons, which eventually dominates over coalescence at very large $p_{\perp}$ [18, 19, 20]. Due to the power-law shape of the parton spectra at high $p_{\perp}$, since the fragmentation function of heavy quarks in vacuum is much harder than that of light quarks, the $p_{\perp}$ scale above which independent fragmentation starts to dominate can be different for charm quarks than those for light quarks.

When Eq. (11) is valid and the effect of the momentum spread is small, it is possible to unfold the quark elliptic flow from hadron elliptic flow using Eq. (7). For example, if $v_2(p_{\perp})$ is known for $D$ and $\Lambda_c$, we have

$$v_2^q(p_{\perp}) = v_2^D ((2 + r)p_{\perp}) - v_2^D ((1 + r)p_{\perp}),$$
$$v_2^c(p_{\perp}) = 2v_2^D \left( \frac{1 + r}{r} p_{\perp} \right) - v_2^c \left( \frac{2 + r}{r} p_{\perp} \right),$$

where $r \equiv m_c/m_q$ is the ratio of the charm quark and light ($u$ and $d$) quark effective mass.

For rough numerical estimates, we consider the conditions in semi-peripheral heavy ion collisions at the top RHIC energy, $\sqrt{s} = 200$ A GeV. With the assumption of Eq. (6), one obtains

$$E \frac{dN(p)}{dp} \propto \int dz_\alpha \Phi_M(z_\alpha)^2 h_\alpha(z_\alpha p_{\perp}) h_\beta(z_\beta p_{\perp}) \times [1 + 2v_{2,\alpha}(z_\alpha p_{\perp}) \cos(2\phi)] [1 + 2v_{2,\beta}(z_\beta p_{\perp}) \cos(2\phi)].$$

We recover Eq. (7) when the momentum spread is zero, i.e., $|\Phi_M(z_\alpha)|^2 = \delta(z_\alpha - \bar{z}_\alpha)$. Including the momentum spread but neglecting small corrections of higher orders in $v_{2,i}$, the mean elliptic flow is

$$v_2^M(p_{\perp}) = \frac{\int dz_\alpha w_M(z_\alpha, p_{\perp}) [v_{2,\alpha}(z_\alpha p_{\perp}) + v_{2,\beta}(z_\beta p_{\perp})]}{\int dz_\alpha w_M(z_\alpha, p_{\perp})},$$

where the weight function for mesons is given by

$$w_M(z_\alpha, p_{\perp}) = |\Phi_M(z_\alpha)|^2 h_\alpha(z_\alpha p_{\perp}) h_\beta(z_\beta p_{\perp}).$$

For baryons, the similar expression for $v_2^M(p_{\perp})$ involves two integrals, over $z_\alpha$ and $z_\beta$, where $w_B(z_\alpha, z_\beta, p_{\perp})$ contains the product of three quark distributions.

For the hadron wavefunctions, for simplicity we take the form used in the valon model [23]:

$$|\Phi_M(z_\alpha)|^2 \propto z_\alpha^{a_\alpha} z_\beta^{b_\alpha}, |\Phi_B(z_\alpha, z_\beta)|^2 \propto z_\alpha^{a_\alpha} z_\beta^{b_\beta} z_\gamma^{b_\gamma},$$

where $z_\beta = 1 - z_\alpha$ for mesons and $z_\gamma = 1 - z_\alpha - z_\beta$ for baryons. The normalization constants play no role in Eq. (10) for the elliptic flow and have been omitted in the above. We take $\Lambda_QCD = 0.2$ GeV, and constituent quark masses $m_u = m_d = 0.3$ GeV, $m_s = 0.5$ GeV, $m_c = 1.5$ GeV. Observing Eq. (4) in the large $p_{\perp}$ limit, we take $\sum \delta z_i^2 = n_v (\Lambda_QCD/m_q)^2/3$. With $z_i$ given by Eq. (4), the exponents $a$ and $b$ are then determined. With

FIG. 1: Comparison of $v_2(p_{\perp})$ for quarks and charm hadrons.

| $z_\alpha^{a_\alpha} z_\beta^{b_\alpha}$ | $(1 - z_\alpha)^{a_\alpha} (1 - z_\beta)^{b_\alpha}$ |
|-----------------|-----------------|
| $(1 - z_\gamma)^{a_\alpha} (1 - z_\beta)^{b_\alpha}$ | $(1 - z_\gamma)^{a_\alpha} (1 - z_\beta)^{b_\alpha}$ |

For light and heavy hadrons, respectively.

For harder than that of light quarks, the hadron elliptic flow increases with $n_{v_{\perp}}^2$, as found in the Molnar's parton transport model (MPC) [18].
the convention that $a$ is for the lightest and $b$ (or $d$ for a baryon) is for the heaviest quark in a hadron, the values of $(a, b)$ are $(0.25, 1.1)$ for kaons, $(8.2, 8.2)$ for $\phi$, $(4.9, 28)$ for $D$, and $(88, 88)$ for $J/\psi$. In general, $a$ and $b$ increase for heavier mesons, reflecting their narrower spread in $z$. For pions the above method gives a singular solution, i.e., $a = b < -1$ leading to divergence in $\int dz_a |\Phi_M(z_a)|^2$. Therefore, for pions we take $a = b = 0$ [6], i.e., a flat distribution in $z$, for simplicity. For baryons, the $(a, b, d)$ values are $(3.6, 3.6, 3.6)$ for protons, $(4.2, 4.2, 7.7)$ for $\Lambda$, $(5.3, 9.5, 9.5)$ for $\Xi$, $(14, 14, 14)$ for $\Omega$, and $(7.2, 7.2, 40)$ for $\Lambda_c$.

FIG. 2: Weight function $w(z_a, p_\perp)$ of $\pi$ and $\phi$ at different $p_\perp$.

For heavy ion collisions at RHIC energies, the parton momentum distributions at mid-rapidity, $h_i(p_\perp)$, may be parameterized by the sum of an exponential soft part and a power-law hard component. Assuming these have identical shapes for light quarks ($u, d, \bar{u}, \bar{d}, s$ and $\bar{s}$), we take $h_i(p_\perp) \propto e^{-p_\perp/T} + c_H/(1 + p_\perp/\Lambda_H)^w$ with $T = 0.18$ GeV, $c_H = 0.36$, $\Lambda_H = 1.3$ GeV, and $w = 8.3$. For charm (and anti-charm) quarks, we use $h_c(p_\perp) \propto (p_\perp + 0.5$ GeV$)^2/(1 + p_\perp/6.8$ GeV$)^3$, which reasonably parameterizes the transverse momentum spectrum of primary charm quarks at mid-rapidity in $pp$ collisions at $\sqrt{s} = 200$ GeV from PYTHIA [27]. We note that medium effects for charm quarks in $A + A$ collisions such as energy loss [28, 29] are not considered in this study.

Fig. 2 shows the weight functions [11] for pions and $\phi$ mesons at $p_\perp = 1$, 4 and 8 GeV, with all maxima normalized to one. For the pion, they are not centered around the mean value $z_\pi = 0.5$ at higher $p_\perp$ values. Instead the dominant momentum configuration for the two constituent quarks is around $z_q \sim 0$ and 1. This asymmetric momentum configuration at high $p_\perp$ for wavefunctions that are flat in $z$ is a result of the power-law enhancement of the parton spectra at high $p_\perp$, as shown in Ref. [3]. For massive hadrons, such as the $\phi$, the wavefunction becomes narrower. Actually, for large $a$ and $b$ the meson wavefunction in Eq. (12) becomes a Gaussian, $\exp[-(z_a - \bar{z}_a)^2/(2\delta z_a^2)]$, where $\bar{z}_a \simeq a/(a + b)$ and $\delta z_a^2 \simeq ab/(a + b)^3$. The narrowness of the wavefunction then dominates over the momentum distributions in Eq. (11) so that symmetric momentum configurations are usually favored. Indeed, as shown in Fig. 2 the weight functions for $\phi$ center at $z_\phi = 0.5$ for all three $p_\perp$ values.

![FIG. 3: Elliptic flows of hadrons including momentum spread in wavefunctions: a) for mesons and b) for baryons.](image)

We now study the effect of momentum spread on hadron elliptic flow via Eq. (11) for mesons and the analogous equation for baryons. For light quarks including $s$ and $\bar{s}$, the elliptic flow is parameterized as $v_2, a(p_\perp) = v_{2,a}^{\max}\tanh(p_\perp/p_0)$ with $v_{2,a}^{\max} = 0.05$ and $p_0 = 0.75$ GeV, based on parton transport simulations using MPC. We consider $v_2, b(p_\perp) = v_2, a(p_\perp)$ unless specified otherwise. As shown in Fig. 3 all qualitative features for charm hadrons discussed earlier remain valid. However, in general hadron elliptic flow is reduced relative to Eq. (17) because the concave shape of the quark $v_2, b(p_\perp)$ ansatz tends to penalize any momentum spread (the average of $v_2, a(p_\perp)$ at two different $p_\perp$ values is lower than the value at the average $p_\perp$). For example, the value of elliptic flow at $p_\perp = 6$ GeV is lower than Eq. (17) by 36% for pions, 32% for kaons, 11%(24%) for $D$ mesons with(without) charm flow, 9% for protons, 10% for $\Lambda$, 5% for $\Xi$, 10%(17%) for $\Lambda_c$ with(without) charm flow. These corrections are especially large for pions and ka!ons at high $p_\perp$ and cause the decrease in their $v_2, a(p_\perp)$ above $\sim 2$ GeV seen in Fig. 3. The reason for this suppression is that the elliptic flow of the slower constituent quark of a pion or a kaon is still far below the saturation value because of the very asymmetric momentum configuration $\bar{s}$ (see Fig. 2). However, note that the flows of pions and kaons may be significantly modified by resonance contributions and the binding energy, which have been neglected in this study. We also found that the momentum spread has a negligible effect on the elliptic flow of massive hadrons, especially those with quarks of similar masses. The flow reductions are 0.8% for $\phi$, 0.01%
for $J/\psi$ and 1% for $\Omega$, thus these hadrons reflect more
directly the partonic elliptic flow.

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