PP-Wave Holography for Dp-Brane Backgrounds

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Abstract

As an extension of the so-called BMN conjecture, we investigate the plane-wave limit for possible holographic connection between bulk string theories in non-conformal backgrounds of Dp-branes and the corresponding supersymmetric gauge theories for $p < 5$. Our work is based on the tunneling picture for dominant null trajectories of strings in the limit of large angular momentum. The tunneling null trajectories start from the near-horizon boundary and return to the boundary, and the resulting backgrounds are time-dependent for general Dp-branes except for $p = 3$. We develop a general method for extracting diagonalized two-point functions for boundary theories as Euclidean (bulk) S-matrix in the time-dependent backgrounds. For the case of D0-brane, two-point functions of supergravity modes are shown to agree with the results derived previously by the perturbative analysis of supergravity. We then discuss the implications of the holography for general cases of Dp-branes including the stringy excitations. All the cases ($p \neq 3, p < 5$) exhibit interesting infra-red behaviors, which are different from free-field theories, suggesting the existence of quite nontrivial fixed-points in dual gauge theories.

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1. Introduction

One of remarkable developments in string theory in recent years has been the formulation of ‘holographic’ connection between string theories in bulk space-times and supersymmetric gauge theories defined on their boundaries. The most typical and best known case is the AdS/CFT correspondence between type IIB string theory around the large number \( (N) \) of D3-branes and maximally supersymmetric \( \mathcal{N} = 4 \) Yang-Mills theory in four dimensions in the large \( N \) strong coupling limit \( R^4 = g_{YM}^2 N \to \infty \). This case is very special in the sense that the system is governed by exact superconformal symmetry on both sides of bulk and boundary space-times, which is quite restrictive in constraining the dynamics of the systems. There are many reasons to expect that the (super)conformal symmetry is not prerequisite for such holographic connections. In the case of general \( D_p \)-branes, there exists no conformal symmetry and hence the comparison of both sides necessarily requires much more deeper understanding about dynamics especially on the side of gauge-theories. On the side of bulk string theories, what we need is the study of propagating closed strings among source-and-probe \( D_p \)-branes. In supergravity approximation, this is in principle straightforward, though full stringy treatment is in general very hard, owing to the complicated structure of background space-times. However, once we have definite results on the bulk side, we would have predictions for the dynamics of large \( N \) strong \( (g_{YM}^2 N \gg 1) \) coupling gauge theories from closed string theories.

From this point of view, an important step is the proposal made in ref. [1] concerning the gauge invariant operators (BMN operators) of the \( \mathcal{N} = 4 \) super Yang-Mills theory which correspond to the higher stringy modes in the bulk for a special class of states with infinitely large orbital angular momentum \( J \sim R^2 \). The effect of stringy excitations is reflected to anomalous conformal dimensions of the (non BPS) BMN operators, which are predicted to behave as \( \sqrt{1 + \frac{R^4 n^2}{J^2}} \), since the world-sheet string theory in the same limit is described as a free massive field theory with mass of order \( J^2 / R^4 \), as derived by taking the so-called Penrose limit [2] around the null geodesics describing the trajectories of strings in the limit \( R^2 \sim J \to \infty \).§ This prediction is based on the usual assumption that the energy with respect to global time coordinate of AdS geometry corresponds to conformal dimension of CFT. If this is correct, the conformal dimension \( \sqrt{1 + \frac{R^4 n^2}{J^2}} = 1 + \frac{R^4 n^2}{2 J^2} + \cdots \)

§In the present paper, we always use the unit \( \sqrt{\alpha'} = \ell_s = 1 \).
for stringy operators \( n \neq 0 \) has a smooth analytic behavior with respect to the 't Hooft coupling \( g_{YM}^2 N \sim R^4 \), and hence is accessible by perturbative computations on the gauge theory side. A number of explicit computations using perturbation theory have been reported on the gauge-theory side, and consistency with the above predictions has been largely confirmed, although some crucial issues related to the interpretation of holography and to the derivation of correlation functions are being in controversy. For a (partial) list of works on this subject, we refer the reader to reviews appeared recently [3].

It is tempting to extend this proposal to a more general case of Dp-branes [4] and to see whether or not similar behaviors can be expected for nonconformal cases. In particular, for D0-branes, that would give some important predictions on the behavior of the corresponding gauge theory, namely, M(atrix) theory in a large \( N \) strong coupling limit, with respect to the stringy degrees of freedom in terms of the matrix variables. In M(atrix) theory, a simple perturbative analysis on the gauge theory side cannot give reliable results on the large \( N \) strong coupling behavior, because of severe infrared problems. Even two-point functions can have very nontrivial behavior in the absence of conformal symmetry. On the other hand, it is well known that the Penrose limit [5] of general Dp-brane backgrounds leads to world-sheet theories with intrinsically time-dependent masses except for \( p = 3 \) [6]. Thus, the absence of conformal symmetry corresponds to the time dependence, and one of the main problems to be overcome in extending the BMN conjecture boils down to extracting definite predictions from string theories with time dependent mass terms. This is by itself an interesting question as an example in extending the usual treatments of string theory in simple time-independent backgrounds to time-dependent cases. To our knowledge, from the viewpoint of holography, no concrete results have been reported in the literature along this line: We do not know how the correspondence between conformal dimensions of boundary theory and light-cone energy of bulk theory should be generalized to nonconformal and time-dependent world sheet theories, respectively, for general Dp-branes. Actually, even in the typical case of D3-branes, the usual treatments do not give any definite prescription on how the two-point functions of boundary theory is computed directly from the bulk theory, in the absence of concrete holographic dictionaries between bulk and boundary theories.

In fact, for general Dp-branes there is a pseudo-symmetry called the ‘generalized conformal symmetry’, as proposed in [7]. In the case of D0-branes for definiteness, the general-
ized conformal symmetry combined with some natural assumptions related to holography predicts [3] that two-point functions for at least supergravity states and the corresponding gauge-theory operators have a general form

$$\langle O_{I}(t_{1})O_{I}(t_{2}) \rangle \sim \frac{1}{g_{s}^{2}\ell_{s}^{8}}(g_{s}N\ell_{s}^{7})^{(\Delta_{I}+6)/5}|t_{1} - t_{2}|^{-(7\Delta_{I}+12)/5}$$

under the following assumptions:

1. They are diagonalized, and the operators $O_{I}$ have definite scaling dimensions in the sense that

$$O_{I}(t) \to O'_{I}(t') = \lambda^{\Delta_{I}}O_{I}(t), \quad t \to t' = \lambda^{-1}t, \quad g_{s} \to g'_{s} = \lambda^{3}g_{s}. \quad (1.2)$$

The operators are normalized such that their engineering dimensions with respect to length are equal to $-1$.

2. The two-point correlation functions should be inversely proportional to 10-dimensional Newton constant $G_{10} \sim g_{s}^{2}\ell_{s}^{8}$ and, apart from this prefactor, the only possible parameter entering in the two-point functions is the length scale $(g_{s}N\ell_{s}^{7})^{1/7}$.

The second assumption comes from the basic holographic relation that two-point functions are obtained by diagonalizing linearized fluctuations in the bulk supergravity fields around the D0-brane background, as reviewed briefly in Appendix A. Recall that the gravitational potential around $N$ D0’s is $(g_{s}N\ell_{s}^{7})/r_{7}$, which implies that the near-horizon dynamics should be governed by the length scale $(g_{s}N)^{1/7}\ell_{s}$. Note that this scale is different from the elementary D0-scale $\ell_{M} \sim g_{s}^{1/3}\ell_{s}$ which is nothing but the M-theory scale. This assumption demands that the two-point functions take the form $G_{10}^{-1}f((g_{s}N\ell_{s}^{7})^{1/7},|t - t'|)$. The transformation rule in (1.2) has its origin in the invariance of the M(atrix) theory action and also of the D0-background under the transformation $X(t) \to X'(t') = \lambda X(t), \quad t' = \lambda^{-1}t, \quad g_{s} \to g'_{s} = \lambda^{3}g_{s}$, which is motivated by the space-time uncertainty relation (see ref. [3]), $\Delta t\Delta X \geq \ell_{s}^{2}$, and is equivalent to the kinematical Lorentz boost along the M-theory direction. The power-law behavior satisfied by this prediction reflects the absence of mass gap in this system, as it should be in any theory of gravity. The spectrum for the dimensions $\{\Delta_{I}\}$ has been derived in previous works [10], by performing detailed supergravity analyses which have confirmed the above prediction. The results, $\Delta_{I} = 4\ell_{I}/7 + 2n - 3$ (or $\Delta_{I} = 4\ell_{I}/7 + 2n - 3/2$ for fermionic operators) with $\ell_{I}$ being the
angular momentum and \( n \) being non-negative integers, are consistent with the behavior of M(atrix) theory operators known approximately from one-loop analysis in \([11]\), with some slight but puzzling corrections as discussed in \([8, 10]\). In view of this, it seems reasonable to expect that certain appropriate extensions of the BMN correspondence exist for non-conformal case of Dp-branes.

The purpose of the present paper is to provide a first step along this line. We develop a general method of extracting diagonalized two-point functions at the boundary as the Euclidean S-matrix in the bulk. Based on this general method, we derive the spectrum of \( \{\Delta_I\} \), confirming and generalizing the results of the previous supergravity analyses at least in the case of bosonic excitations. As we will see, holography predicts that the behaviors of stringy BMN operators in nonconformal cases are in general quite different from the conformal case of D3-branes.

For performing the required analysis in a clear space-time picture, it is very crucial to base our discussion of the Penrose limit on the tunneling picture, as proposed in refs. \([12, 13]\) for the case of D3-branes, which allows us to avoid singularities at the horizon for nonconformal Dp-branes and to establish a direct connection between bulk and boundary. In fact, to our knowledge, there has been no other proposals which have given concrete prescriptions on the direct relationship between bulk amplitudes and boundary correlators in such a general way as allowing extensions to non-conformal cases. Therefore, in the next section, we begin by recapitulating main points of this proposal adapted to the general Dp-branes. Namely, we argue that, in the limit of large angular momentum \( J \), two-point functions in dual gauge theories should be described by transition amplitudes (which we call collectively the Euclidean S-matrix) defined along tunneling null trajectories traversing from boundary to boundary in the bulk. Then we are led to world-sheet theories with time dependent backgrounds, since the effective masses of strings are in general not constant along the trajectories. A general quantum theory with time-dependent mass is given in section 3. The discussion is sufficiently general and might be interesting in its own right apart from our specific application treated in the present work. In section 4, we derive diagonalized two-point functions for Dp-branes on the basis of the general formalism of section 3. We compare the results with previous supergravity analysis in the case of D0-branes and further consider the cases of general Dp-branes including stringy excitations. In particular, we discuss the predictions from holography on the infra-red
limit of the dual gauge theories of Dp-branes. In section 5, we conclude by summarizing the present work and by mentioning possible future directions. We also point out very interesting possible implications from our results on the infra-red behaviors for the cases $p = 1$ and $p = 4$, suggesting the shifts $d = p + 1 \to d_{\text{eff}} = d + 1$ of effective dimensionalities. A brief summary of field-theory analyses in supergravity [10] with partial extensions to general case of Dp-brane backgrounds are given in Appendix A for the purpose of making the present exposition reasonably self-contained. Appendix B is devoted to a side remark on an alternative effective PP-wave description of the supergravity fluctuations in the case of D0 backgrounds. This description is useful to confirm our results in the case of non-stringy supergravity modes from a perspective which is slightly different from the main text.

2. PP-wave holography and tunneling null geodesics

2.1 Null geodesics in the Dp-brane backgrounds

As is well known, the Penrose limit can be regarded as a semiclassical limit [15] for the propagation of particles or strings along null geodesics with large (angular) momentum. Let us recall the classical metric around $N$ Dp-branes in the near horizon limit ($-\pi/2 \leq \psi \leq \pi/2$)

$$ds^2 = q_p^{1/2} \left[ H^{-1/2}(-dt^2 + d\tilde{x}_a^2) + H^{1/2}(dr^2 + r^2 d\psi^2 + r^2 \cos^2 \psi d\Omega_{7-p}^2) \right],$$  

(2.1)

where $H = 1/r^{7-p}$ and $q_p = 2^{7-2p} \pi^{(9-3p)/2} \Gamma((7-p)/2) g_{YM}^2 N$. We have rescaled the coordinates along the D-branes $(t, \tilde{x}_a) \to q_p^{1/2}(t, \tilde{x}_a)$ $(a = 1, \ldots, p)$ from the usual convention, such that the characteristic dimensional parameter $q_p^{1/2}$ appears as the overall prefactor in the metric. The metric can also be written in the form

$$ds^2 = q_p^{1/2} e^{2\tilde{\phi}/(7-p)} \left\{ \left( \frac{2}{5-p} \right)^2 \frac{-dt^2 + d\tilde{x}_a^2 + dz^2}{z^2} + d\psi^2 + \cos^2 \psi d\Omega_{7-p}^2 \right\},$$  

(2.2)

$$e^{\tilde{\phi}} \equiv H^{(3-p)/4} = \left( \frac{5-p}{2} z \right)^{(7-p)(3-p)/(2(5-p))}.$$  

(2.3)

The radial coordinate $z$ is defined by

$$z = \frac{2}{5-p} \sqrt{\frac{1}{r^{5-p}}}.$$  

(2.4)
For $p = 3$, (2.2) is the AdS$_5 \times$ S$^5$ metric for the Poincaré patch. The dilaton background is given by $e^\phi = g_s q_p^{(3-p)/4} e^{\tilde{\phi}}$. We can think of the Weyl factors $q_p^{1/2} e^{2\tilde{\phi}/(7-p)}$ as defining position-dependent effective length scales for these backgrounds.

The null trajectories which have nonzero angular momentum $J \equiv EH^{1/2} r^2 \dot{\psi}$ for the direction of the angle $\psi$ and traverses along the radial direction of $r$ satisfy

$$\dot{r}^2 = H^{-1} (-\ell^2 r^{-2} + H),$$

where $\ell \equiv J/E$ and $\dot{r}$ is the velocity with respect to the affine time $\tau$ satisfying the relation

$$EH^{-1/2} \dot{\tau} = E \rightarrow \dot{\tau} = H^{1/2}. \tag{2.6}$$

The parameter $E$ introduced here can be interpreted as energy with respect to the target time $t$. These conventions for the target energy and angular momentum are appropriate if the particle theory is understood as the limit from the string theory as formulated in later subsections. Providing that we restrict ourselves to the case $p < 5$, the allowed region is

$$r \leq r_0, \quad r_0 = \ell^{-2/(5-p)}. \tag{2.7}$$

Namely, the null trajectories never reach the near-horizon boundary $r \sim q_p^{1/(7-p)}$ for large $g_s N$ and fall to the horizon at $r = 0$ which is a singular point except for $p = 3$. In particular, the trajectories always go to the horizon in a finite affine time. This is problematical for at least two reasons. First, hitting singularity invalidates the classical approximation itself, and secondly, the separation of the trajectories from the boundary makes holographic correspondence very obscure, since the identification of the bulk modes and the operators on the boundary is based on the behavior of the former near the boundary. For $p = 3$, we can extend the trajectories without encountering singularities to globally defined AdS space-time. However, this does not solve the second problem since it is difficult to associate global coordinate to physical events on the boundary. For $p = 6$ the situation is opposite: The allowed region is $r \geq \ell^{2/(p-5)}$. The case $p = 5$ is marginal: There is no trajectory, either real or tunneling, which traverses from boundary to boundary. In the present paper, we restrict our considerations to $p < 5$. 

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2.2 Tunneling null geodesics and holography for D3-branes

To explain the problem in a simplest possible setting, let us for the moment specialize ourselves to the conformal case $p = 3$. The null geodesic is given by

$$z = \frac{\ell}{\cos \ell \tau}, \quad t = \ell \tan \ell \tau, \quad \psi = \ell \tau.$$  \hspace{1cm} (2.8)

Since it is easy to check that $\ell \tau$ is nothing but the time coordinate of the global AdS metric, periodicity with respect to $\tau$ reflects the fact that the global AdS space-time is a universal cover of hyperboloid with topology $S^1 \times \mathbb{R}^4$. Because this periodicity would affect all physical amplitudes defined around this background, it seems very difficult to adopt the usual interpretation of the affine time as proposed in [1] that $\tau$ should be identified with radial time of the boundary theory as $\vec{x} \sim e^{-\tau} \vec{x} (\vec{x} \equiv (\bar{x}_a, t))$. It is hard to imagine any periodicity in the dynamics of super Yang-Mills theory in radial quantization on the boundary. In fact, the only clue for the holography in terms of physical amplitudes in the AdS/CFT correspondence is the famous GKP-Witten relation [17] between from-boundary-to-boundary amplitudes in the bulk space-times and the correlation functions of super Yang-Mills theory defined at the boundary. The correspondence between states $\{\phi_i^0(\vec{x})\}$ at the boundary and the fluctuating fields $\{\phi_i(z, \vec{x})\}$ in the bulk is based on the boundary condition

$$\lim_{z \to 0} \phi_i^j(z, \vec{x}) = z^{4 - \Delta_i} \phi_i^j(\vec{x}).$$ \hspace{1cm} (2.9)

The states $\phi_i^j(\vec{x})$ couple to a set of local gauge-invariant operators $\{\mathcal{O}_i(\vec{x})\}$ with definite conformal dimensions $\Delta_i$ at the boundary. Clearly, it is impossible to directly use this correspondence for the propagation of strings along the above trajectory. From the viewpoint at the boundary, the relevant region of affine time seems to be only a finite segment $-\pi/2 < \ell \tau < \pi/2$ corresponding to $-\infty < t < \infty$.

The proposal made in ref. [12] in order to resolve this difficulty is simply to consider a complex trajectory which represents a tunneling in the semi-classical picture from boundary to boundary, instead of the real trajectory going from horizon to horizon. Formally, this corresponds to using purely imaginary affine time, $\tau \to -i \tau$, and also to Wick-rotating the target time and angles, $t \to -it$ and $\psi \to -i \psi$. Thus the tunneling trajectory is now given by

$$z = \frac{\ell}{\cosh \ell \tau}, \quad t = \ell \tanh \ell \tau, \quad \psi = \ell \tau.$$ \hspace{1cm} (2.10)
This drastically changes the global structure of the trajectory such that it never reaches the horizon. It now traverses from boundary \((\tau \to -\infty)\) to boundary \((\tau \to \infty)\) with an infinite time interval. In this case, we cannot identify the affine time \(\tau\) with the radial time of boundary theory, since near the boundary the tunneling trajectory is orthogonal to D3-brane target space-time. Instead, it must be identified with the coordinate \(z\) by \(z \sim e^{-\ell |\tau|}\) near the boundary. This is consistent with the identification of energy with respect to \(\tau\) as the conformal dimensions at the boundary, since it is well known that the variable \(z\) plays the role of effective cutoff parameter for the short distance structure of the dual gauge theory at the boundary. It is also important that the parameter \(2\ell\) is now directly interpreted, from the second in the equations (2.8), as the distance of two end-points, corresponding to the insertion of operators for the boundary theory.

We emphasize that this change of trajectory solves another obvious puzzle which has been ignored in the recent literature. The conjecture in [11] assumes that the transverse directions to the null propagation consist of 8 directions, of which 4 are directions of \(S^5\) orthogonal to the \(\psi\)-direction and of which the remaining 4 are nothing but the directions of the world-volume of D3-branes. The latter fact is manifested in the derivatives \(D_iZ\) \((i = 1, \ldots, 4)\), which are identified as the latter 4 transverse degrees of freedom, expressed in terms of the complex field \(Z = \phi_5 + i\phi_6\) corresponding to the \(U(1)\) R-charge directions. However, since the affine-time direction along the world-sheet (or world line) must be orthogonal to the transverse excitations of strings, we cannot think of any directions of the base space of the boundary theory as affine-time \(\tau\), including a rather familiar identification of global time \(\ell \tau\) with the time of radial quantization of Yang-Mills theories. Clearly, the appearance of the derivatives along the base-space directions representing transverse excitations becomes a contradiction if one insists that the affine-time flows along any one direction of the D3-brane world volume. The fact that the Hamiltonian with respect to the affine time is essentially the dilatation operator at the boundary should be interpreted as the (gravitational) Hamiltonian constraint describing the dynamics of this theory, and should be discriminated from the unjustified kinematical identification of the affine time with the radial time.

Another related remark at this juncture is that, because of the drastic change of the global structure of trajectories, connecting two pictures, real propagation or complex tunneling, is not straightforward, though the analytic continuation with respect to the
target time $t$ is of course allowed after computing correlators. For example, we would encounter integrations over the affine time in computing various physical amplitudes. However, we would not be allowed to deform the integration contour to real affine time, since we would then have divergent results in general because of the periodicity as warned above, obstructing the deformation of the integration contours. For further discussions related to this problem, see [14].

Though the main subject of the present paper is the semi-classical quantization of the fluctuations around tunneling null geodesics and the derivation of transition amplitudes for them, it is useful here to discuss the two-point amplitude in the classical limit taking the example of a massless point particle on the D3-brane background. The part of the action containing $(t, z, \psi)$ is

$$ S = \frac{1}{2} \int d\tau \frac{1}{\eta} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu $$

$$ = \frac{1}{2} \int d\tau \frac{1}{\eta} \left\{ \frac{R^2}{z^2} (\dot{t}^2 + \dot{z}^2) - R^2 \dot{\psi}^2 \right\} \tag{2.11} $$

where $R = q_3^{1/4}$ is the common scales of both AdS$_5$ and S$^5$, and we have kept the einbein $\eta$ unfixed. For representing the amplitude for a fixed angular momentum

$$ J = \frac{\delta S}{\delta \psi} = -R^2 \frac{\dot{\psi}}{\eta}, \tag{2.12} $$

we perform a Legendre transformation to Routh function

$$ S = S - \int d\tau P_\psi \dot{\psi} $$

$$ = \frac{1}{2} \int d\tau \left\{ \frac{1}{\eta} \frac{R^2}{z^2} (\dot{t}^2 + \dot{z}^2) + \frac{\eta}{R^2} J^2 \right\}. \tag{2.13} $$

Solving the equation of motion, taking the null-constraint into account, and substituting the result to the Routh function, we obtain

$$ \bar{S} = \frac{J}{R} \int d\tau \sqrt{\frac{R^2}{z^2} (\dot{t}^2 + \dot{z}^2)}. \tag{2.14} $$

Note that use of the Routh function corresponds to a Fourier transformation of the wave function to the $J$-basis.\(^*\) Thus the amplitude between states with definite $(t, z, J)$ is given by

$$ \langle t_f, 1/\Lambda, J; T | t_i, 1/\Lambda, J; -T \rangle = \int \mathcal{D}t \mathcal{D}z e^{-\bar{S}[t, z, J]}. \tag{2.15} $$

\(^*\)For related treatments of D-particles using Routh functions, see [18].
In the classical approximation, the result is therefore
\[
\langle t_f, 1/\Lambda, J; T|t_i, 1/\Lambda, J; -T \rangle_{\text{class}} = e^{-S} = e^{-2J\ell T} = \left(\frac{1}{\Lambda|t_f - t_i|}\right)^{2J}, \tag{2.16}
\]
where the last equality is derived as follows: From (2.10), we see that the distance of two boundary points where the trajectory begins and ends is \(|t_f - t_i| = 2\ell\). It also follows that the cutoff \(\Lambda\) for the radial coordinate \((1/\Lambda \leq z \leq z_0 = \ell)\) and the time interval \(T\) with respect to \(\tau (-T \leq \tau \leq T)\) are related as
\[
\frac{1}{\Lambda} = 2\ell e^{-\ell T}. \tag{2.17}
\]
Thus apart from the cutoff-dependent factor which should be renormalized as has been already exhibited in the boundary condition (2.9), the two-point amplitude reduces to the expected form of the 2-pt correlator for an operator of conformal dimension \(\Delta \sim J\) for large \(J \sim R^2\). As discussed in \cite{12} in the WKB approximation, inclusion of linearized fluctuations leads to the shift \(J \rightarrow J + 4\), which is appropriate for a scalar operator, whose origin in the present context is the zero-point energies, \(4 = 8 \times 1/2\), of 8 transverse directions in the 10D space-time.

### 2.3 Tunneling null geodesics in the Dp-brane backgrounds

We now extend the above discussion to general Dp-branes \((p < 5)\). Because of the singularities at the horizon, it is crucial to base our discussions on the tunneling picture. After the formal Wick rotations \((\tau \rightarrow -i\tau, t \rightarrow -it, \psi \rightarrow -i\psi)\), the equation determining the trajectories becomes
\[
\dot{r} = \pm H^{-1/2} \sqrt{\ell^2 r^{-2} - H} = \pm r_0^{-\left((5-p)/2\right)} \sqrt{r_0^{5-p} - r^{5-p}}. \tag{2.18}
\]
The allowed region is \(r \geq r_0\) for \(p < 5\). Comparing with the conformal case of D3-branes, there is a notable difference when \(p < 3\). To see this, let us examine the equations for the trajectory using the \(z\) coordinate
\[
\dot{z} = \mp \left(\frac{5-p}{2}\right)^2 e^{-\frac{2\pi}{5-p} z} \sqrt{z_0^2 - z^2}, \quad \dot{t} = \left(\frac{5-p}{2}\right)^2 z^2 e^{-\frac{2\pi}{5-p} z}, \quad z_0 = \frac{2\ell}{5-p}, \tag{2.19}
\]
where \(e^{\phi}\) is given in (2.3). This shows that, when \(p < 3\), the trajectory starting from the asymptotic boundary returns to the boundary in a finite affine time \(2T_b\)
\[
T_b = \left(\frac{2}{5-p}\right)^{\frac{7-p}{5-p}} \int_0^{z_0} \frac{dz}{\sqrt{z_0^2 - z^2}}. \tag{2.20}
\]
The corresponding (coordinate) distance between two end-points at the boundary is
\[ |t_f - t_i| = \int_{-T_b}^{T_b} d\tau \dot{t} = 2 \int_0^{z_0} dz \frac{z}{\sqrt{z_0^2 - z^2}} = 2z_0 = \frac{4}{(5 - p)} \ell. \tag{2.21} \]

We have to set a cut-off in approaching the boundary, such that \(1/\Lambda \leq z \leq z_0\). The reason is that, in order for the near-horizon approximation to be valid, \(1/\Lambda\) is assumed to be of order \(q_p^{(5-p)/2(7-p)} \rightarrow 0 \text{ as } g_s N \rightarrow \infty\) corresponding to \(r < q_p^{1/(7-p)}\),
as discussed in detail in [10] for the case of D0-branes. The affine time interval is
\(-T \leq \tau \leq T\) (with \(z(\pm T) = 1/\Lambda\) where
\[ T_b - T = \left( \frac{2}{5 - p} \right)^{\frac{7-p}{5-p}} \int_0^{1/\Lambda} dz \frac{z^{-\frac{3-p}{5-p}}}{\sqrt{z_0^2 - z^2}} \sim \frac{5 - p}{3 - p} \left( \frac{2}{5 - p} \right)^{\frac{7-p}{5-p}} \frac{1}{z_0} \Lambda^{-\frac{3-p}{5-p}}. \tag{2.22} \]

We see below that the two-point amplitudes in general exhibit similar nonanalyticity appeared here with respect to the cutoff \(\Lambda\).

Following the procedure explained for the case of D3-branes, we evaluate the Routh function for the tunneling null trajectory in the point-particle case as
\[ \bar{S} = S - \int d\tau J \dot{\psi} = \frac{q_p^{1/2}}{2} \int d\tau \left\{ \frac{1}{\eta} e^{-2\phi/(5-p)} \frac{1}{z^2} (\dot{t}^2 + \dot{z}^2) + \left( \frac{2}{5 - p} \right)^2 q_p^{-1} \eta e^{-2\phi/(5-p)} J^2 \right\}. \tag{2.23} \]

Substituting the classical solution, we find
\[ \bar{S} = \left( \frac{2}{5 - p} \right)^2 \frac{J^2}{E} \int_{-T}^{T} d\tau e^{-\frac{2}{5-p} \phi} = \frac{4}{5 - p} Jz_0 \int_{1/\Lambda}^{z_0} dz \frac{1}{z \sqrt{z_0^2 - z^2}} = -\frac{4}{5 - p} J \left\{ \log \frac{1}{z_0 \Lambda} - \log(1 + \sqrt{1 - \frac{1}{z_0 \Lambda}}) \right\}. \tag{2.24} \]

In the limit \(1/\Lambda \ll z_0\), the first term dominates, and we obtain
\[ \bar{S} \sim \frac{4}{5 - p} J \log(z_0 \Lambda). \tag{2.25} \]

Using (2.21), the classical contribution to the amplitude becomes
\[ \langle t_f, 1/\Lambda, J; T_b | t_i, 1/\Lambda, J; -T_b \rangle_{\text{class}} = e^{-\bar{S}} = \left( \frac{2}{\Lambda |t_f - t_i|} \right)^{\frac{1}{5-p} J}. \tag{2.26} \]

The power law behavior with respect to the target (coordinate) distance \(|t_f - t_i|\) conforms to the predictions of generalized conformal symmetry, indicating that the leading behavior
of $\Delta$'s for large $J (\sim q_p^{1/2})$ are $4J/7$, for the case of the D0-branes. This is consistent with the supergravity result mentioned in the Introduction, and already suggests that the theories at the boundary corresponds to nontrivial infra-red fixed points. Note also that in spite of the difference from the D3-case with respect to the affine time interval, the behavior of two-point functions with respect to the cutoff parameter shows similar power-law structure. In view of this power-law behavior, we can think of $\ln z \rightarrow \ln (1/\Lambda)$, rather than the original affine time, as the effective time parameter along the tunneling trajectory near the boundary.

2.4 Fluctuations around the tunneling null geodesics

We now derive the action for fluctuations around the tunneling null geodesic. In the present paper, we study only the bosonic part, postponing the treatment of fermionic excitations and supersymmetry to a forthcoming work. It is convenient to use the following form of the space-time metric

$$ds^2 = q_p^{1/2} \left[ -dv \left( 2du - H^{-1/2} dv + 2\ell H^{-1/2} dx \right) + (\ell^2 H^{-1/2} - H^{1/2} r^2) dx^2 + \frac{H^{1/2}}{r^2} \cosh^2\psi d\Omega^2 \right].$$

(2.27)

The metric (2.21) (doubly Wick rotated $(t,\psi) \rightarrow -i(t,\psi)$) can be brought to the above form [5] by the coordinate transformations,

$$u = u(r), \quad v = t + \ell \psi + a(r), \quad x = \psi + b(r),$$

with

$$du \over dr = \pm \frac{H^{1/2}}{\sqrt{\ell^2 r^2 - H}}, \quad da \over dr = \mp \sqrt{\frac{\ell^2}{r^2} - H}, \quad db \over dr = \mp \frac{\ell}{r^2} \frac{1}{\sqrt{\ell^2 r^2 - H}}.$$

Substituting this metric into the standard string (bosonic) action

$$S = \frac{1}{4\pi} \int d\tau \int_0^{2\pi} d\sigma (\partial_\tau x^\mu \partial_\tau x^\nu g_{\mu\nu} + \partial_\sigma x^\mu \partial_\sigma x^\nu g_{\mu\nu} + i \epsilon^{\alpha\beta} \partial_\tau x^\mu \partial_\sigma x^\nu B_{\mu\nu}),$$

(2.28)

it is easy to check that the trajectory defined by

$$u = u^{(0)} \equiv \tau, \quad v = v^{(0)} \equiv 0, \quad x = x^{(0)} \equiv 0, \quad \tilde{x}_a = \tilde{x}_a^{(0)} \equiv 0,$$

(2.29)

satisfies the equations of motion and the Virasoro constraint

$$- \partial_\tau x^\mu \partial_\tau x^\nu g_{\mu\nu} + \partial_\sigma x^\mu \partial_\sigma x^\nu g_{\mu\nu} = 0,$$

(2.30)
\[ \partial_\tau x^\mu \partial_\sigma x^\nu g_{\mu\nu} = 0. \tag{2.31} \]

We choose the string length parameter \( \alpha \sim P^+ \) to be proportional to the target-space energy, \( \alpha = E/q_p^{1/2} = J/(\ell q_p^{1/2}) \), such that the definitions of angular momentum \( J \) and the target energy \( E \) coincide with the convention as introduced in subsection 2.1.

The fluctuations around the trajectory are treated by performing expansion

\[ x_\mu = x_\mu^{(0)} + Lx_\mu^{(1)} + L^2x_\mu^{(2)} + \cdots. \]

Then, the \( \mathcal{O}(L^0) \) part (classical part) of the action vanishes due to the constraints\(^\|\) and \( \mathcal{O}(L^1) \) part vanishes since \( x^{(0)} \) satisfies the equation of motion. To the next order, the Virasoro constraints require

\[ v^{(1)'} = \dot{v}^{(1)} = 0, \]

and the action reduces to

\[
S^{(2)} = \frac{1}{4\pi} \int d\tau \int_0^{2\pi} d\sigma \left \{ H^{-1/2}(\ddot{x}_a^2 + \ddot{x}_a'^2) + (\ell^2 H^{-1/2} - H^{1/2}r^2)(\dot{x}^2 + x'^2) + H^{1/2}r^2 \cosh^2 b(\dot{y}_l^2 + y_l'^2) \right \},
\]

where we have suppressed the superscript (1) on the fields, and \( y_l \) \( (l = 1, \ldots, 7 - p) \) are the coordinates along the sphere \( (S^{7-p}) \) directions. We set the expansion parameter as \( L = q_p^{-1/4} \). The higher order terms are neglected in the limit of large \( q_p \). Here, \( H \) and \( r \) are evaluated on the classical trajectory and hence depend on \( \tau \). Performing the field redefinition

\[ \ddot{x}_a \rightarrow H^{1/4}\dddot{x}_a, \quad y_l \rightarrow \frac{1}{H^{1/4}r \cosh b} y_l, \quad x \rightarrow \frac{1}{\sqrt{\ell^2 H^{-1/2} - H^{1/2}r^2}} x, \]

and partial integration, the action becomes

\[ S^{(2)} = \frac{1}{4\pi} \int d\tau \int_0^{2\pi} d\sigma \left \{ \dddot{x}_a^2 + \dddot{x}_a'^2 + m_x^2(\tau)\dddot{x}_a^2 + \dddot{x}_a'^2 + m_x^2(\tau)\dddot{x}_a^2 + \dddot{y}_l^2 + y_l'^2 + m_y^2(\tau)y_l'^2 \right \}, \tag{2.32} \]

where

\[ m_x^2 = m_x^2 = -\frac{(7 - p)}{16r^2} \left \{ (3 - p) + (3p - 13)\ell^2r^{5-p} \right \}, \tag{2.33} \]

\[ m_y^2 = -\frac{(7 - p)}{16r^2} \left \{ (3 - p) - (p + 1)\ell^2r^{5-p} \right \}. \tag{2.34} \]

\(^\|\)For the present purpose, we consider the Routh function, which gives non-zero contribution at \( \mathcal{O}(L^0) \), as we have seen. Since the term \( -J \int d\theta \partial_\tau (\psi_0 + L\psi^{(1)} + \cdots) \) added to the original action in order to convert to the Routh function is a total derivative, it does not affect the bulk action for the fluctuations.
These formula for the mass parameters have been previously obtained \[6\] in the real affine-time approach. Here \( r = r(\tau) \) is given by the classical solutions determined by (2.18). Hence the masses are time dependent except for \( p = 3 \). In particular, for \( p < 3 \), the masses asymptotically increase indefinitely, while for \( 5 > p > 3 \) they vanish asymptotically, suggesting physical consequences drastically different from the conformal case for both cases. Intuitively, the former corresponds to the fact that the external tidal force defeats the string tension and the latter to the converse that the effect of the tidal force becomes negligible, as the strings approach the boundary.

There remain no fields with negative metric in the Euclidean sense. On the other hand, the squared masses can become negative in the region deep inside the bulk. Near the boundary they are always positive for \( 0 \leq p \leq 4 \) and asymptotically become infinite. By rewriting the above expressions using turning point \( r_0 \),

\[
m_x^2 = m_x^2 = \frac{(7 - p)\ell^2}{16r^2}[2(5 - p)r_0^{5-p} + (13 - 3p)(r_5^{5-p} - r_0^{5-p})],
\]

\[
m_y^2 = \frac{(7 - p)\ell^2}{16r^2}[2(p - 1)r_0^{5-p} + (p + 1)(r_5^{5-p} - r_0^{5-p})],
\]

we see that \( m_y^2 \) is positive for \( 1 \leq p \leq 4 \) in the allowed region. It should be emphasized here that if we remain in the usual picture of real null geodesics we would have been confronted by the difficulty that (mass)\(^2\) are negatively infinite as we approach the horizon.

Note that the action (2.32) has a global \( SO(p + 1) \times SO(7 - p) \) symmetry and also more importantly that the action is invariant under the scaling transformation

\[
\begin{align*}
    r &\rightarrow \lambda r, & \ell &\rightarrow \lambda^{-(5-p)/2}\ell, & (\tau, \sigma) &\rightarrow (\lambda\tau, \lambda\sigma), & \alpha &\rightarrow \lambda\alpha
\end{align*}
\]

(2.35)

with all the fluctuating fields being of zero dimension. This property of course reflects the pseudo-symmetry under the generalized scaling transformation of the original metric, which for general \( p \) takes the form \( X_i \rightarrow \lambda X_i, \quad \vec{x} \rightarrow \lambda^{-1}\vec{x}, \quad g_s \rightarrow \lambda^{3-p}g_s \), where \( \vec{x} \propto (t, \vec{x}_a) \) and \( X_i \) are base space and transverse coordinates, respectively. The scaling of \( \ell \left( \sim |\vec{x}_1 - \vec{x}_2| \equiv |t_f - t_i| \right) \) is the consequence of our rescaling of the base space coordinate \( \vec{x} \rightarrow q_p^{1/2}\vec{x} \). As in the ordinary conformal case, the cutoff \( \Lambda \) in general breaks this scaling property. However, after the dependence on the cutoff is suitably eliminated by wave function renormalization, the correlation functions should be symmetric under these scaling transformations. This is indeed satisfied by the results of supergravity analyses \[10\]
for \( p = 0 \). The symmetry under the scaling transformation \( 2 \cdot 3.3 \) will play an important role in the present work too.

Finally, we note that, for the purpose of the present paper discussing the two-point transition amplitudes within the string-tree approximation, the dilaton background \( \phi \) can be ignored. The reason is that its coupling to strings occurs only through the world-sheet curvature term \( \int d\tau d\sigma \sqrt{h(2)} \phi R(2) \) which in the limit \( L \to 0 \) reduces to \( \int d\tau d\sigma \sqrt{h(2)} \phi_{\text{classical}} \times R(2) \) and hence does not couple with the fluctuating fields. But it would play an important role for ensuring the world-sheet conformal symmetry by cancelling the anomaly, especially when we discuss the string-loop effects. Note also that, provided the world-sheet conformal symmetry is valid, the world-sheet metric can be chosen such that \( R(2) = 0 \) for the world sheets of cylinder topology.

3. General theory of harmonic oscillators with time dependent masses

In the conformal case of \( p = 3 \), the quantization of our system \([12]\) is completely straightforward since the mass is constant, and the 2-pt amplitude takes the trivial form

\[
\langle t_f, 1/\Lambda, J, \{N_n\}; T|t_i, 1/\Lambda, J, \{N_n\}; -T \rangle = \left( \frac{1}{\Lambda |t_f - t_i|} \right)^{2(J+\Delta)},
\]

where, as in the usual formulation \([1]\) using a real null geodesic, \( \Delta = \sum_n N_n \sqrt{1 + \frac{n^2 R^4}{\ell^2}} \) is the quantum contribution to conformal dimension, corresponding to the frequency \( \omega_n = \sqrt{\ell^2 + \frac{R^4 n^2}{\ell^2}} \) of the modes of strings, each of which contributes to the amplitude as \( e^{-2 \omega_n T} \). Here we have subtracted the zero-point energies, since they cancel after taking into account the fermionic excitations. In the nonconformal cases \( p \neq 3 \), the situation is much more nontrivial, owing to the complicated time dependence of mass functions. Thus, to study the fluctuations around the classical trajectory, we now have to develop a general quantum theory of harmonic oscillators with time-dependent potential of the form \( m(\tau)^2 x(\tau)^2 / 2 \). We hope that our discussion will be useful for other cases, such as cosmological applications \([19]\), with time dependent backgrounds than the present specific example. Hasty readers may wish to skip general formalism below and go directly to the final formula \( 3.34 \), which is astonishingly simple.

We start from considering the quantization of coordinate operator satisfying the \( (Eu-

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The Euclidean equation of motion in the Heisenberg representation,
\[ \frac{d^2}{d\tau^2} X(\tau) = m(\tau)^2 X(\tau). \] (3.1)

In our cases, by choosing the origin of time appropriately, we can assume \( m(\tau) = m(-\tau) \).

The general solution can be expressed in the form
\[ X(\tau) = f_+(\tau)a + f_-(\tau)a^\dagger \] (3.2)

with the normalization condition for the Wronskian
\[ f_+ \frac{df_-}{d\tau} - f_- \frac{df_+}{d\tau} = 1, \] (3.3)

which is possible since
\[ \frac{d}{d\tau} (f_+ \frac{df_-}{d\tau} - f_- \frac{df_+}{d\tau}) = 0. \] (3.4)

The solutions \( f_{\pm}(\tau) \) are chosen such that they satisfy the boundary condition
\[ f_{\pm}(\tau) \to 0, \quad \tau \to \pm T_b, \] (3.5)

near the boundary \( r \to \infty \) in the large \( q_p \) limit. They correspond to positive and negative frequency solutions, respectively, when the Euclidean affine time \( \tau \) is formally Wick-rotated to the real affine time \( (\tau \to it) \) in the case of constant mass. We stress that naive approximate methods for obtaining \( f_{\pm}(\tau) \), such as adiabatic or WKB-like treatments, are not allowed, since we cannot assume that \( dm(\tau)/d\tau \ll m(\tau)^2 \): For large \( r \), \( dm(\tau)/d\tau \) and \( m(\tau)^2 \) are in general of the same order \( O(r^{3-p}) \) \( (p \neq 3) \).

Note that \( a, a^\dagger \) are assumed to be independent of \( \tau \). By using the time reflection symmetry, we can set
\[ f_-(\tau) = f_+(-\tau), \] (3.6)

which means that \( X \) satisfies the reflection condition
\[ X(\tau)^\dagger = X(-\tau), \] (3.7)

instead of the ordinary condition of hermiticity in the real-time formulation. Here we have assumed that the solutions \( f_{\pm} \) are real. When \( m(\tau)^2 < 0 \) for some regions of time as in our cases, it could be that we were forced to use complex solutions. In such a situation, the time-reflection condition should be replaced by
\[ f_-(\tau) = \overline{f_+(-\tau)}, \] (3.8)

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keeping the reflection condition (3.7). Actually, it will turn out that we can assume real solutions for our later applications.

The quantization condition is expressed as

\[ [X(\tau), P(\tau)] = i, \]  

(3.9)

by defining the momentum operator

\[ P(\tau) = i \frac{d}{d\tau} X(\tau) = i(\frac{df_+}{d\tau} a + \frac{df_-}{d\tau} a^\dagger), \]  

(3.10)

which satisfies the same reflection condition as the coordinate operator, \( P(\tau)^\dagger = P(-\tau) \). The quantization condition is equivalent to

\[ [a, a^\dagger] = 1, \]  

(3.11)

because of the Wronskian condition (3.3).

Let us now try to transform this formalism into the Hamiltonian picture. First, we can rewrite the equation of motion in the first order form as

\[ i \frac{dX}{d\tau} = P, \quad i \frac{dP}{d\tau} = -m(\tau)^2 X \]  

(3.12)

which is expressed using commutator as

\[ \frac{dX}{d\tau} = [H(\tau), X], \quad \frac{dP}{d\tau} = [H(\tau), P] \]  

(3.13)

with the time-dependent Hamiltonian,

\[ H(\tau) = \frac{1}{2}(P^2 + m(\tau)^2 X^2). \]  

(3.14)

We can then introduce the transition operators by, depending respectively on positive and negative \( \tau \),

\[ U_+(\tau, 0) = \mathcal{T}_+ \exp \left[ \int_0^\tau d\tau' H(\tau') \right] \]  

(3.15)

or

\[ U_-(\tau, 0) = \mathcal{T}_- \exp \left[ -\int_0^\tau d\tau' H(-\tau') \right] \]  

(3.16)

for \( \tau > 0 \), with \( \mathcal{T}_\pm \) being the time or anti-time ordering operation, respectively. They satisfy

\[ X(\pm \tau) = U_\pm(\pm \tau, 0) X(0) U_\pm(\pm \tau, 0)^{-1}, \quad P(\pm \tau) = U_\pm(\pm \tau, 0) P(0) U_\pm(\pm \tau, 0)^{-1}, \]  

(3.17)
for $\tau \geq 0$. The reflection condition for the transition operator takes the form

$$U_\pm(\pm\tau, 0)^\dagger = U_\pm(0, \mp\tau). \quad (3.18)$$

By definition, we have also

$$U_-(\tau, 0)^{-1} = U_+(0, -\tau). \quad (3.19)$$

In the case of ordinary time-independent harmonic oscillator, we have $U_\pm(\pm\tau, 0) = \exp(\pm\tau H) = U_\pm(0, \mp\tau)$; namely $U_\pm$ are diagonalized and hermitian. In the general time-dependent cases, however, the transition operators are neither diagonalized, nor hermitian in the usual sense.

The relation between Heisenberg and Schrödinger pictures is formulated as

$$\langle \psi_1 | O(\tau) | \psi_2 \rangle = \langle \psi_1(\tau) | O | \psi_2(\tau) \rangle. \quad (3.20)$$

Then the Euclidean out and in (ket) states in the Schrödinger picture are defined using the above transition operators as

$$| \psi_f(\tau) \rangle_{\text{out}} = U_+(\tau, 0)^{-1} | \psi_f \rangle, \quad (3.21)$$

$$| \psi_i(\tau) \rangle_{\text{in}} = U_-(\tau, 0)^{-1} | \psi_i \rangle, \quad (3.22)$$

for $\tau = T \to T_b$. Note that the bra-states $\langle \psi(\pm\tau) |$ are in general not the conjugate of the ket-states $| \psi(\pm\tau) \rangle$ in the above relation, since the former are defined as

$$\langle \psi(\tau) | = \langle \psi | U_+(\tau, 0)^{-1}, \quad \langle \psi(\tau) | = \langle \psi | U_-(\tau, 0)^{-1}. \quad (3.23)$$

Note however that these bra-and-ket states are defined at the common time $\tau = 0$. The orthonormality condition can then be expressed in terms of this internal product, which is by definition independent of $\tau$, as

$$\langle \psi_1 | \psi_2 \rangle = \langle \psi_1(\pm\tau) | \psi_2(\pm\tau) \rangle = \delta_{12}. \quad (3.24)$$

The initial and final states, $| \psi_i \rangle$ and $| \psi_f \rangle$, are defined at times, $\tau = -T$ and $\tau = T$, respectively. In this circumstance, the natural definition of the S-matrix is

$$S_{fi} \equiv \langle \psi_i(\tau) | \psi_f(\tau) \rangle_{\text{out}} = \langle \psi_i | U_-(\tau, 0)U_+(\tau, 0)^{-1} | \psi_f \rangle$$

$$= \langle \psi_i | (U_+(\tau, 0)U_-(\tau, 0)^{-1})^{-1} | \psi_f \rangle = \langle \psi_i | U_+(\tau, -\tau)^{-1} | \psi_f \rangle \quad (3.23)$$
with
\[ U_+(T, -T) = \mathcal{T}_+ \exp[\int_{-T}^{T} d\tau H(\tau)] \equiv S^{-1}(T). \] (3.24)

Thus,
\[ S(T) = \mathcal{T}_- \exp[-\int_{-T}^{T} d\tau H(\tau)]. \] (3.25)

Due to the time-reflection condition, the $S$-operator is hermitian, in contrast to general transition operators $U$'s. For time independent case, this leads to the naive Euclideanized $S$-operator $S = \exp(-2TH)$.

The main task for obtaining the two-point functions is then to ‘diagonalize’ the $S$-matrix operator, $S(T)$. Our strategy toward this goal is as follows: Since the time dependent Hamiltonian is quadratic in the time-independent $(a, a^\dagger)$ basis, we can always express the $S$-operator in the following normal-ordered form:
\[ S(T) = N(T) : \exp[\frac{1}{2}A(T)(a^\dagger)^2 + B(T)a^\dagger a + \frac{1}{2}C(T)a^2] : \] (3.26)

Because of hermiticity, $A = C$ and $B$ is real. This can further be converted to the exponential form
\[ S(T) = \tilde{N}(T) \exp[\frac{1}{2}\tilde{A}(T)(a^\dagger)^2 + \tilde{B}(T)a^\dagger a + \frac{1}{2}\tilde{C}(T)a^2], \] (3.27)

which can then be transformed to a ‘diagonalized’ form
\[ S(T) \rightarrow N(T) \exp[-\Omega(T)b^\dagger(T)b(T)] \] (3.28)

with
\[ [b(T), b^\dagger(T)] = 1 \] (3.29)

by a suitable $T$-dependent Bogoliubov transformation ($DG - EF = 1$),
\[ (a, a^\dagger) \rightarrow (b(T) = D(T)a + E(T)a^\dagger, b^\dagger(T) = F(T)a + G(T)a^\dagger). \] (3.30)

This immediately leads to an expression for $\Omega(T)$ and the normalization,
\[ \Omega = \sqrt{\tilde{B}^2 - \tilde{A}\tilde{C}}, \] (3.31)
\[ \mathcal{N} = \tilde{N} \exp[-\frac{\tilde{B}(T) + \Omega(T)}{2}]. \] (3.32)

Thus, the $S$-operator can always be diagonalized for arbitrary $T$ when the initial ket-states $|\psi_i\rangle$ and the final bra-states $\langle\psi_f|$ are represented in the Fock bases $\{b^\dagger(T)^n|0\}_b\}$. 

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and \( \{ b(0|b(T)^n \} \), respectively, where the ‘vacuum’ states are defined by \( b(T)|0\rangle_b = 0 = b(0|b^+(T) \). In terms of the original creation and annihilation operators, these vacua are ‘squeezed’ states such as \( |0\rangle_b \sim \exp(-\frac{E(T)}{\beta(T)}(a^1)^2)|0\rangle \). Because of time-dependent potential, quanta corresponding to \( a, a^\dagger \) are being continuously created and annihilated during propagation along the trajectory. But, the transition amplitudes can be diagonalized in suitable bases prepared at the both initial and final states constructed as above. To avoid possible confusion, we emphasize that what we are doing here is entirely different from the diagonalization of the Hamiltonian itself at each given time, which does not provide any help for our purpose. Our procedure takes the dynamical transition of states from \( \tau = -T \) to \( \tau = T \) into full account. This kind of ‘integrated’ diagonalization has not been familiar in the preceding literature. However, this is the analogue in the first-quantized approach to the diagonalization of linearized fluctuations in arbitrary backgrounds in (second-quantized) field theory formalism.

Here we collect some of key formulae for performing these manipulations. In terms of the solutions \( f_{\pm} \), the coefficient functions of the normal ordered form are given by solving the following differential equations with the boundary conditions \( N(0) = 1, A(0) = B(0) = 0 \):

\[
\frac{d}{dT} \ln(1 + B) = \frac{d}{dT} \ln N^2 = -2\beta - 2\alpha A, \tag{3.33}
\]

\[
\frac{d}{dT} A = - (\gamma + 2\beta A + \alpha A^2) - \alpha (1 + B)^2, \tag{3.34}
\]

with

\[
\alpha = m^2 f_-^2 - \dot{f}_-^2, \quad \beta = m^2 f_+ f_- - \dot{f}_+ \dot{f}_-, \quad \gamma = m^2 f_+^2 - \dot{f}_+^2, \tag{3.35}
\]

which follow from the equation

\[
\frac{dS(T)}{dT} = - H^+(T) S(T) - S(T) H(T). \tag{3.36}
\]

We have assumed that the solutions \( f_{\pm} \) are all real.

In fact, apart from the normalization function \( N(T) \), the transition operator can be algebraically determined by the following simple trick. First note that by its definition the \( S \) operator satisfies the commutation relations with the coordinate and momentum operators,

\[
X(T) = S(T)^{-1} X(-T) S(T), \quad P(T) = S(T)^{-1} P(-T) S(T), \tag{3.37}
\]
which, using the normal order form of the operator $S(T)$ with $A = C$, reduce to

$$1 + B - A = \frac{f_+(T)}{f_-(T)}, \quad 1 + B + A = -\frac{\dot{f}_+(T)}{f_-(T)},$$

(3.38)

respectively. It is easy to check that these forms indeed satisfy the above differential equations with the initial condition $A(0) = B(0) = 0$. Then, combining these formulae with the result for the normalization function obtained from the differential equation, we have the completely explicit form for the normal-ordered $S$-operator as

$$N^2 = 1 + B = \frac{1}{2f_-(T)f_-(T)},$$

(3.39)

$$A = -\frac{1}{2}\left(\frac{f_+(T)}{f_-(T)} + \frac{\dot{f}_+(T)}{f_-(T)}\right).$$

(3.40)

The coefficient functions of the exponential form are then determined by the following general formulae relating normal and exponential forms, (3.26) and (3.27);

$$\exp\left(\begin{array}{cc} -\tilde{B} & -\tilde{A} \\ \tilde{C} & \tilde{B} \end{array}\right) = \begin{pmatrix} \frac{1}{1+B} & \frac{-A}{1+B} \\ \frac{1}{1+B} & 1 + B - \frac{CA}{1+B} \end{pmatrix},$$

(3.41)

$$\tilde{N}(T) = N(T)\sqrt{1+B} \exp(-\tilde{B}/2) = (1+B) \exp(-\tilde{B}/2).$$

(3.42)

For example, the former equation is derived by computing

$$\left( \begin{array}{c} SaS^{-1} \\ Sa^\dagger S^{-1} \end{array} \right) = \mathcal{L} \left( \begin{array}{c} a \\ a^\dagger \end{array} \right)$$

using the two (normal and exponentiated) expressions of $S$ and equating the results, where $\mathcal{L}$ is the $2 \times 2$ matrix defining the Bogoliubov transformation corresponding to the S-operator. In particular, the eigenvalue function $\Omega$ for the diagonalized $S$-matrix is given by

$$\cosh \Omega = \frac{1}{2} \left( 1 + B + \frac{1 - AC}{1+B} \right).$$

(3.43)

All these formulae are exact for arbitrary $T$, and, if we wish, can trivially be converted to real affine-time formulation. In the special case of constant mass, we have the familiar formulas as

$$\alpha = \gamma = 0, \quad \beta = m \quad \rightarrow \quad A = C = 0 = \tilde{C} = \tilde{A}, \quad \Omega = -\tilde{B}, \quad e^{\tilde{B}} = 1 + B = e^{-2mT},$$

and hence $S = e^{-m(2a^\dagger a + 1)^T}$. 22
Finally, we can see the crucial role played by the boundary conditions for \( f_\pm \). Since \( f_+(T) = f_-(-T) \rightarrow 0 \) in the limit of large \( r(T) \) \((T \rightarrow T_b)\), \[3.38\] shows that \( A(T) \sim 1 + B(T) \rightarrow 0 \) asymptotically near the boundary. This leads to the simple exponential form of the boundary-to-boundary \( S \)-operator,

\[
S(T) \rightarrow (1 + B)^{a^\dagger a + 1/2} = (2f_-(T)f_-(T))^{-(a^\dagger a + 1/2)}.
\]

Namely, the time-independent creation and annihilation operators \((a^\dagger, a)\) themselves diagonalize the 2-pt functions as in the case of constant mass; \((b^\dagger(T), b(T)) \rightarrow (a^\dagger, a)\), as we approach the boundary \( T \rightarrow T_b \). This is remarkable in view of the complicated time dependence, but is owing to our choice of representations for the coordinate and the momentum with our boundary condition for \( f_+ \) together with the time-reflection symmetry. For this remarkably simple result, the Euclidean nature of our system is very important.

If we consider the case of real affine time, the coefficient function \( A \) provides an oscillating contribution whose frequency is of the same order as that of \( 1 + B \), and hence it is not clear whether the corrections coming from \( A \) can safely be neglected.

4. Two-point PP-wave \( S \)-matrix for \( Dp \)-brane backgrounds

We now proceed to the derivation of diagonalized two-point \( S \)-matrices for \( Dp \)-brane backgrounds on the basis of general formalism given in the previous section. We first consider the case of \( D0 \)-brane within supergravity approximation ignoring stringy excitations and confirm that the results are consistent with the previous field-theory analysis. The case of general \( p (\leq 5) \) including the stringy modes will then be treated.

4.1 Case of \( D0 \) in supergravity approximation without stringy excitations

The equation of motion we have to solve is

\[
\frac{d^2}{d\tau^2} X(\tau) = m(\tau)^2 X(\tau),
\]

with mass function

\[
m(\tau)^2 = \frac{7}{16} \frac{\ell^2 r^5 - 3}{r^2} (= m_y^2),
\]

or

\[
m(\tau)^2 = \frac{7}{16} \frac{13\ell^2 r^5 - 3}{r^2} (= m_x^2),
\]

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and radial function \( r = r(\tau) \) satisfying

\[
\frac{dr}{d\tau} = \begin{cases} \sqrt{\ell^2 r^5 - 1} & (\tau > 0), \\ -\sqrt{\ell^2 r^5 - 1} & (\tau < 0). \end{cases}
\] (4.4)

The turning point is \( r_0 = \ell^{-2/5} \). Since \( \tau \) is only given as an implicit function, it is more convenient to convert the equation of motion in terms of the radial coordinate \( r \). We obtain

\[
(\ell^2 r^5 - 1) \frac{d^2 X}{dr^2} + \frac{5}{2} \ell^2 r^4 \frac{dX}{dr} - m(r)^2 X = 0.
\] (4.5)

For the mass functions above, this can be solved as

\[
X_y(r) = C_1 r^{-3/4} \exp \left( \ell \int \frac{r^{3/2}}{\sqrt{\ell^2 r^5 - 1}} \, dr \right) + C_2 r^{-3/4} \exp \left( -\ell \int \frac{r^{3/2}}{\sqrt{\ell^2 r^5 - 1}} \, dr \right)
\] (4.6)

and

\[
X_x(r) = C_1 r^{7/4} + C_2 \frac{\sqrt{\ell^2 r^5 - 1}}{r^{3/4}},
\] (4.7)

respectively.

We have to choose the basis of the solutions satisfying our boundary conditions required in the previous section. Let us first take the second case with the mass function \( m_x \). As a function of \( \tau \), the solution must be continuous at \( \tau = 0 \). To obtain the solution conforming to this requirement, we can set \( X_x(\tau) = \tilde{C}_1 r^{7/4} + \tilde{C}_2 \frac{dr}{d\tau} r^{-3/4} \), and determine the coefficients such that the boundary condition is satisfied. We find

\[
f_x^+(\tau) = \sqrt{\frac{\ell}{5}} \left( r^{7/4} - \frac{1}{\ell} \frac{dr}{d\tau} r^{-3/4} \right),
\] (4.8)

\[
f_x^-(\tau) = \sqrt{\frac{\ell}{5}} \left( r^{7/4} + \frac{1}{\ell} \frac{dr}{d\tau} r^{-3/4} \right).
\] (4.9)

Here and in what follows, we adopt the same normalization condition as in the previous section. Similarly, for the case of mass function \( m_y \), we find

\[
f_y^+(\tau) = \frac{1}{\sqrt{2\ell}} r^{-3/4} \exp \left( -\ell \int_0^\tau r(\tau)^{3/2} \, d\tau \right),
\] (4.10)

\[
f_y^-(\tau) = \frac{1}{\sqrt{2\ell}} r^{-3/4} \exp \left( \ell \int_0^\tau r(\tau)^{3/2} \, d\tau \right).
\] (4.11)

Here the integral on the exponential can be performed, giving (for \( \tau > 0 \))

\[
\ell \int_0^\tau r(\tau)^{3/2} d\tau = \int_{r_0}^r \frac{\ell r^{3/2}}{\sqrt{\ell^2 r^5 - 1}} \, dr = \frac{2}{5} \ln \left[ \sqrt{\ell^2 r^5} + \sqrt{\ell^2 r^5 - 1} \right] \rightarrow \ln(\ell^{2/5} r) \quad (r \rightarrow \infty).
\] (4.12)
We remark that, in spite of the fact that the masses squared become negative near the turning point, the exact solutions remain as real functions in the whole allowed range of the radial coordinate $r$.

Given these solutions, we can immediately apply the general formulae of the previous section. The near-boundary behaviors of the coefficient functions $A, B$ are

\[ A \sim \frac{3}{28\ell^2} r^{-5} + O(\ell^{-4} r^{-10}), \quad (4.13) \]
\[ B + 1 (= N^2) \sim \frac{5}{14\ell^2} r^{-5} + O(\ell^{-4} r^{-10}), \quad (4.14) \]

and

\[ A \sim 3 (4\ell^2 r^5)^{-2/5} + O((4\ell^2 r^5)^{-7/5}), \quad (4.15) \]
\[ B + 1 (= N^2) \sim 4 (4\ell^2 r^5)^{-2/5} + O((4\ell^2 r^5)^{-7/5}), \quad (4.16) \]

for the cases of $m_x$ and $m_y$, respectively. Thus the contribution of each supergravity mode to two-point functions is, using the relation between $r(T)$ and the UV cutoff $\Lambda$,

\[ \frac{1}{\Lambda} = \frac{2}{5-p} r(T)^{-(5-p)/2}, \]

\[ S_x(T) \sim (\ell^2 r(T)^5)^{\frac{1}{2}[a^1a + \frac{1}{2}]} \sim (|t_i - t_f| \Lambda)^{-\frac{2}{5}[a^1a + \frac{1}{2}]}, \quad (4.17) \]
\[ S_y(T) \sim (\ell^2 r(T)^5)^{\frac{3}{2}[a^1a + \frac{1}{2}]} \sim (|t_i - t_f| \Lambda)^{-\frac{4}{5}[a^1a + \frac{1}{2}]}, \quad (4.18) \]

depending on the $SO(1)$ or $SO(7)$ direction, respectively.

In order to make comparison with the supergravity analysis given in [10], we have to multiply these results over all modes of transverse directions. Therefore, fixing precisely the over-all constant term of the exponents requires an accurate evaluation of zero-point contributions including the fermionic coordinates. We leave such a full treatment of fermions and supersymmetry to a forthcoming work, and in the present work we will be satisfied by checking consistency with [10], ignoring the constant part. Note that, once the dependence with respect to the target distance is fixed, the scaling symmetry guarantees that the two-point functions, with a prescribed normalization condition as discussed in the Introduction after removing the cutoff dependent factors, have the correct behavior with respect to the coupling constant $q_0 \sim g_s N$. 

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Comparing the dependence on the target distance \(|t_i - t_f|\) in the expressions (4.17) and (4.18) for the contributions to the S-operator with the general form of the two-point functions
\[
\langle \mathcal{O}_I(t_1) \mathcal{O}_I(t_2) \rangle \sim \frac{1}{g_s^2 \ell_8^8} (g_s N \ell_8^8)^{(\Delta_I + 6)/5} |t_1 - t_2|^{-(7\Delta_I + 12)/5},
\]
we see that a single excitation along the SO(1) or SO(7) direction contribute to the generalized conformal dimension as
\[
\Delta_x = 10/7 \quad \text{or} \quad \Delta_y = 4/7,
\]
respectively. On the other hand, as we have already mentioned in the Introduction, the field-theory analysis predicts the spectrum for the generalized conformal dimension of the form
\[
\Delta_I = 4\ell_I/7 + 2n_I - 1
\]
where \(\ell_I\) is the number of orbital excitations \(\tilde{X}_i = X_i/\sqrt{q_0} (i = 1, 2, \ldots, 9)\) of transverse modes in the sense of M-theoretical 11 dimensions and \(n_I = 1 - n_+ + n_-\) measures the numbers of 11-dimensional (upper) light-cone indices. For example, \(n_I\) and \(\ell_I\) for the moments of the 11-dimensional energy-momentum tensor
\[
T_{i_1i_2\ldots i_k}^{++} = \text{STr}(\tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_k} + \cdots)/g_s,
T_{i_1i_2\ldots i_k}^{++} = \text{STr}((D_0 X_i) \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_k} + \cdots)/g_s,
T_{i_1i_2\ldots i_k}^{++} = \text{STr}(\frac{1}{2}(D_0 X_i)(D_0 X_i) \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_k} + \cdots)/g_s,
\]
are
\[
(n_I, \ell_I) = (-1, k), (0, k), (1, k), \ldots etc
\]
For complete details on this correspondence, we refer the reader to [10]. For studying the BMN-type operators, we choose the direction of the angular momentum \(J\) to be \(i = 8, 9\) such that the BMN ‘Z’ coordinate is
\[
Z = (X_8 + iX_9)/\sqrt{2}
\]
and, correspondingly, the excitation modes along \(SO(7)\) and \(SO(1)\) are
\[
\tilde{X}_i \text{ with } i = 1, 2, \ldots, 7 \quad \text{or} \quad D_0Z,
\]
respectively: The BMN-ground state with a definite angular momentum \( J \) corresponds to \( T_{ZZ\ldots Z}^{++} \) (\( J = \ell_I \)). Then, starting from this ground state, single excitations along the \( SO(7) \) and \( SO(1) \) contribute to the shifts \((n_I, \ell_I) \to (n_I, \ell_I + 1)\) and \((n_I, \ell_I) \to (n_I + 1, \ell_I - 1)\), respectively. Here the shift of \( n_I \) in the second case originates from either \( n_+ \to n_+ - 1 \) or \( n_- \to n_- + 1 \), depending on operators in consideration. Hence, the shift of the generalized conformal dimensions are \( \frac{4}{7} \) and \( \frac{2 - 4}{7} = \frac{10}{7} \), respectively, in agreement with the results \(^{[4,20]}\) of the particle-string picture in the PP-wave limit.

### 4.2 General case of \( Dp \) (\( p < 5 \)) with stringy excitations

Passing through the important nontrivial test that the PP-wave limit correctly reproduces the supergravity results for \( p = 0 \) case, we are now in the position to study the general case of \( Dp \)-brane backgrounds with stringy excitations. We treat each Fourier mode separately. Thus for the \( n \)-th stringy modes, we set

\[
X(\tau, \sigma) \sim \frac{1}{\sqrt{\pi \alpha'}} \cos(\frac{n}{\alpha} \sigma)X(\tau) \quad \text{or} \quad \frac{1}{\sqrt{\pi \alpha'}} \sin(\frac{n}{\alpha} \sigma)X(\tau),
\]

and the equation of motion takes the form,

\[
\left( \frac{d^2}{d\tau^2} - \frac{n^2}{\alpha^2} \right) X(\tau) = m(\tau)^2 X(\tau)
\]  (4.24)

with mass functions

\[
m(\tau)^2 = -\frac{(7 - p)}{16r^2} [(3 - p) + (3p - 13)\ell^2 r^{5-p}] \quad (= m^2_{p,x}),
\]  (4.25)

or

\[
m(\tau)^2 = -\frac{(7 - p)}{16r^2} [(3 - p) - (p + 1)\ell^2 r^{5-p}] \quad (= m^2_{p,y}),
\]  (4.26)

where \( r = r(\tau) \) is given by

\[
\frac{dr}{d\tau} = \begin{cases} \sqrt{\ell^2 r^{(5-p)}} - 1 & (\tau > 0), \\ -\sqrt{\ell^2 r^{(5-p)}} - 1 & (\tau < 0). \end{cases}
\]  (4.27)

Note that in our convention the component field \( X(\tau) \) has always scaling dimension \( 1/2 \) under the scaling transformation \(^{[2.35]}\). This is necessary for the consistency with the normalization condition for \( X(\tau) \) assumed in the previous section.

For \( n = 0 \), the equation of motion \(^{[1,24]}\) for each \( m_{p,x} \) or \( m_{p,y} \) is exactly solvable with the results for \( \tau \geq 0 \) as

\[
f^x_\pm(\tau) = (5-p)^{-\frac{1}{2}} \ell^{-\frac{1}{2-p}} u^{\frac{p-3}{2-p}} (\sqrt{u} + \sqrt{u-1})^{\mp 1},
\]  (4.28)

\[
f^y_\pm(\tau) = 2^{-\frac{1}{2}} \ell^{-\frac{1}{5-p}} u^{\frac{p-3}{5-p}} (\sqrt{u} + \sqrt{u-1})^{\mp 2},
\]  (4.29)
where \( u = \ell^2 r^{5-p} \). These general expressions are valid for all \( p < 5 \) including \( p = 0 \) and \( p = 3 \). The asymptotic behaviors of the above solutions for \( r \to \infty \) (\( \tau \to T_b \)) are given respectively as

\[
f_+^x \sim \frac{1}{2} (5-p)^{-\frac{3}{2}} \ell^{\frac{3}{2}} r^{-\frac{13-3p}{4}}, \quad f_+^y \sim 2 (5-p)^{-\frac{3}{2}} \ell^{\frac{3}{2}} r^{\frac{(5-p)}{4}},
\]

\[
f_+^y \sim (2\ell)^{-\frac{9-3p}{2(5-p)}} r^{-\frac{7-p}{4}}, \quad f_-^y \sim (2\ell)^{\frac{p+1}{2(5-p)}} r^{\frac{p+1}{4}}.
\]

Thus, from (3.44),

\[
S_x(T) \sim (\lvert t_i - t_f \rvert \Lambda)^{-2[a^t a + \frac{1}{2}]},
\]

\[
S_y(T) \sim (\lvert t_i - t_f \rvert \Lambda)^{-\frac{4}{5}[a^t a + \frac{1}{2}]},
\]

The behavior of the \( SO(7-p) \) directions is consistent with the field-theory analysis for general \( p \) in supergravity discussed in Appendix A.2. On the other hand, the behavior of the \( SO(1+p) \) excitations conforms to the generalized BMN conjecture that they correspond to the derivatives \( D_i Z \) of the field \( Z \) along the base-space directions of \( D_p \)-branes.

For \( p \neq 3 \) and \( n \neq 0 \), it is difficult to solve this equation exactly. By recalling that the two-point S-matrix is governed by the asymptotic behaviors of the solutions \( f_\pm \) for large \( r \) or \( T \to T_b \), we can try to extract some information for the solutions after making the large-\( r \) approximation for the differential equations themselves as

\[
\ell^2 r^{5-p} \frac{d^2 X}{dr^2} + \frac{5-p}{2} \ell^2 r^{4-p} \frac{dX}{dr} - \left( m_{\text{app}}^2 + \frac{n^2}{\alpha^2} \right) X = 0,
\]

where

\[
m_{\text{app}}^2 = \frac{(7-p)(13-3p)}{16} \ell^2 r^{3-p} \quad (= m_{\text{app},p,x}^2),
\]

or

\[
m_{\text{app}}^2 = \frac{(7-p)(p+1)}{16} \ell^2 r^{3-p} \quad (= m_{\text{app},p,y}^2),
\]

for \( SO(1+p) \) and \( SO(7-p) \) directions, respectively. This is valid when

\[
\ell^2 r^{(5-p)} \gg 1, \quad \frac{n^2}{\alpha^2} \gg \frac{1}{r^2}.
\]

The equation (4.34) for \( p \neq 3 \) can be converted into the modified Bessel equation

\[
w^2 \frac{d^2 Y}{dw^2} + w \frac{dY}{dw} - (\nu^2 + w^2) Y = 0,
\]
with
\[ \nu = \left| \frac{p - 5}{p - 3} \right| \quad \text{or} \quad \nu = \left| \frac{2}{p - 3} \right|, \]
for the \( SO(1 + p) \) and \( SO(7 - p) \) directions, respectively, by making the redefinition
\[ X(\tau) = w^{1/2}Y(w), \quad w = \frac{2n}{|3 - p|\ell\alpha}r^{(p - 3)/2}. \]
Thus the asymptotic form of the solutions \( f_\pm \) is described by the following general form
\[ X = C_1 r^{-\frac{3+p}{4}} I_\nu \left( \frac{2n}{|3 - p|\ell\alpha} r^{-\frac{3+p}{4}} \right) + C_2 r^{-\frac{3+p}{4}} K_\nu \left( \frac{2n}{|3 - p|\ell\alpha} r^{-\frac{3+p}{4}} \right). \]
The coefficients \( C_1, C_2 \) in general depend on \( n/\alpha \) and \( p \). Since we have the exact solutions for \( n = 0 \), these coefficients are determined by matching the above form in the limit \( n \to 0 \). When \( p = 0 \), for example, it gives
\[ f_{app,p=0}^{+} = \sqrt{5} 3^{-\frac{1}{4}} \Gamma \left( \frac{2}{3} \right) \left( \frac{n}{\alpha} \right)^{-\frac{3}{8}} \ell^{1/6} r^{-\frac{3}{4}} I_{\frac{3}{8}} \left( \frac{2n}{3\ell\alpha} r^{-\frac{3}{4}} \right) \sim \frac{\sqrt{5}}{10} \ell^{-\frac{3}{4}} r^{-\frac{11}{4}}, \]
\[ f_{app,p=0}^{-} = \frac{2\sqrt{5} 3^{\frac{3}{4}}}{15\Gamma \left( \frac{7}{3} \right)} \left( \frac{n}{\alpha} \right)^{\frac{3}{8}} \ell^{-\frac{7}{4} \nu} r^{-3/4} K_{\frac{3}{8}} \left( \frac{2n}{3\ell\alpha} r^{-\frac{3}{4}} \right) + C r^{-\frac{3}{4}} I_{\frac{3}{8}} \left( \frac{2n}{3\ell\alpha} r^{-\frac{3}{4}} \right) \sim \frac{2\sqrt{5}}{5} \ell^{\frac{1}{4}} r^{\frac{3}{4}} \]
for the \( SO(1) \) direction. In the last line, a constant \( C \) remains unfixed. This can be fixed by using the next-to-leading contribution of \( f_- \) for \( n = 0 \). However, we can set \( C = 0 \) since only the leading behavior is important for our purpose.

In the general case with \( p < 3 \), we can set the asymptotic forms for \( \tau \to T_b \ (r \to \infty) \) as
\[ f_+(\tau) = c_+^{\nu} w^{1/2} I_\nu(w), \quad f_-(\tau) = c_-^{\nu} w^{1/2} K_\nu(w). \]
The Wronskian normalization condition requires
\[ c_+^{\nu} c_-^{\nu} = \alpha/|n|. \]
If we interpolate these asymptotic solutions deep inside the bulk and directly impose the continuity condition at \( \tau = 0 \), these underdetermined coefficients \( c_+^{\nu} \) are fixed by requiring the condition of time-reflection symmetry. In the limit \( n \to 0 \), this can indeed be done as exemplified above for \( p = 0 \), and the resulting general asymptotic form of the coefficients is
\[ \lim_{n \to 0} c_+^{\nu} \to \sqrt{\frac{\alpha \nu}{2|n|}} 2^{-(3-p)\nu/(5-p)} \Gamma(\nu)(p - 3)^{\nu} |\frac{n}{\alpha}|^{-\nu} \ell^{2\nu/(5-p)}. \]
For general $n$, the symmetry under the scaling $\ell \to \sqrt{\alpha |n|} \ell$ demands that the coefficients take the form

$$c^+ = \sqrt{\frac{\alpha}{|n|}} c_\nu(s), \quad c^- = \sqrt{\frac{\alpha}{|n|}} (c_\nu(s))^{-1}.$$  \hspace{1cm} (4.45)

Here the argument in the undertermined function $c_\nu(s)$ is given as

$$s \equiv \frac{|n|}{\alpha} \ell^{-2/(5-p)} = |n| q_p^{1/2} \ell^{(3-p)/(5-p)} / J \hspace{1cm} (4.46)$$

in terms of the angular momentum $J$ and the distance $\ell = (5-p)|t_f - t_i|/4$. Similarly, the argument of the modified Bessel function at the cutoff scale is given by

$$w = \frac{2|n|}{(3-p) \ell \alpha} r(T)^{(p-3)/2} = \frac{2}{3-p} \left( \frac{2}{5-p} \right)^{(p-3)/(5-p)} \frac{|n| q_p^{1/2}}{J} \Lambda^{(p-3)/(5-p)} = \frac{|n|}{\alpha} (T_b - T), \hspace{1cm} (4.47)$$

which contains the cutoff parameter $\Lambda$, but is actually independent of the target-space distance $\ell$. We note that the appearance of these new variables is basically due to the existence of new scale $q_p^{1/2}$, playing the role of effective string tension. Remember that $q_p^{1/2}$ appears as the over-all prefactor of the metric tensor.

We can understand why the cutoff-dependent variable $w$ is associated with the stringy excitations as follows: First, recall that the effective scale of the background metric is of order $R_c \equiv \sqrt{q_p^{1/2} r^{-(3-p)/2}}$. Near the boundary this is of order $\sim q_p^{1/(7-p)}$, since $r \sim q_p^{1/(7-p)}$. The contribution of Kaluza-Klein modes to the squared mass $M^2$ is then of order $M_{KK}^2 \sim (J/R_c)^2$, while the contributions of the stringy excitations is of order $M_{st}^2 \sim n$. In terms of $M^2$, the generalized conformal dimensions of order $O(J)$ take the form

$$\Delta_I \sim \frac{M_{KK}^2}{(J/R_c^2)} \sim J + O(1).$$

The corrections by the stringy excitations are therefore expected to enter through the form $M_{st}^2 \sim n R_c^2 \ell / J \sim w$ at least in the regime where the background curvature is regarded as small.

From these facts, we can see that, firstly, the cutoff dependence always enters in the S-operator in a factorized form. Thus, the cutoff dependence can be removed (or renormalized) by assuming a suitable normalization condition for 2-pt correlation functions, just as we have already seen for supergravity modes ($n = 0$) without stringy excitations. Secondly, the expression (4.46) implies that the long-distance behavior of the asymptotic solutions for $p < 3$ and hence of the S-operator with respect to the distance $\ell \to \infty$ (with
fixed $n$) at the boundary is governed by the large $|n|q_p^{1/2}/J$ behavior of the solutions. Namely, we can study the infra-red properties of the dual gauge theories of $Dp$-brane ($p<3$) by examining the limit of sufficiently large $n$ for fixed $\ell$ such that

$$n^2 q_p/J^2 \gg \ell^{-2(3-p)/(5-p)}.$$  

Therefore let us consider the behavior of the solutions in the large $n$ limit. Since the cutoff of $r$ is of order $q_p^{1/(7-p)}$, we can neglect the mass term arising from the background comparing with the mass corresponding to the stringy excitation, if we consider the limit of extremely large $n^2 q_p/J^2$ such that

$$n^2 \alpha^2 = n^2\ell^2 q_p/J^2 \gg \ell^2 q_p^{(3-p)/(7-p)} \rightarrow n^2 q_p/J^2 \gg q_p^{(3-p)/(7-p)}.$$  

Though this (cutoff dependent) condition is different from the above condition (4.48) which is sufficient for the infra-red behavior, the scaling symmetry allows us to assume that no new scale enters in the computation of the coefficient function $c_\nu(s)$. Then, the original differential equation is reduced to the trivial one $[\frac{d^2}{dt^2} - (\frac{\alpha}{\alpha})^2]X = 0$, and thus the solutions are now approximated by

$$f_+ = \sqrt{\frac{\alpha}{2|n|}} e^{-\frac{|n|}{\alpha} t},$$  

$$f_- = \sqrt{\frac{\alpha}{2|n|}} e^{\frac{|n|}{\alpha} t},$$  

which should be matched with the large $n$ limit of (4.32). Using the last of various equivalent expressions for $w$ in (4.47) and the asymptotic form of the modified Bessel functions ($I_\nu(w), K_\nu(w)) \rightarrow (e^w/\sqrt{2\pi w}, e^{-w}/\sqrt{2w/\pi})$, we obtain the asymptotic form of the function $c_\nu(s)$ as

$$c_\nu(s) \sim \sqrt{\pi} e^{-|n|T_b/\alpha}$$  

for large $n$. Since $T_b \sim \ell^{-2/(5-p)}$ for $p<3$, the argument in the exponential is proportional to $s$ as it should be. This shows that the contribution to the $S$-operator from the stringy excitations with nonzero $n$ in the large distance limit of the boundary theory is of the form

$$S(T)_{p<3} \sim \exp\{-\hat{c}|n|q_p^{1/2}/\ell t_j - t_i(t^{(3-p)/(5-p)}(a^i a + 1/2))$$  

where $\hat{c}$ is a numerical constant. This two-point function is exponentially damped compared with the non-stringy modes. In other words, the infra-red limits of the dual gauge
theories for $p < 3$ are described by non-stringy supergravity modes, and the stringy modes are decoupled.

If we repeat the above computation in the case $p = 3$, we get the correct 2-pt function for the large $n$ limit, $S \sim \left( \frac{1}{|n|^{\alpha}} \right)^{2p/4} a^1 (a^{1/2} n) / \Lambda$, which is of course power-behaved with anomalous conformal dimensions $(\sqrt{1 + (n^2 R^4 / J^2)} \sim q_p^{1/2} |n| / J)$ and hence the stringy modes cannot be neglected in the infra-red limit for $p = 3$. In contrast to this, the stringy excitations for $p < 3$ cannot be described simply by the shift of generalized conformal dimension to anomalous generalized conformal dimensions.

Let us briefly touch on the opposite short-distance behavior of 2-pt functions for $p < 3$. Clearly, we expect that the short-distance structure is encoded in the small $s$ behavior of the function $c_\nu(s)$. Because the cutoff $\Lambda$ associated with $r$ is expected to play the role for the short distance cutoff also for the boundary theory as $\ell > 1 / \Lambda \sim q_p^{- (5-p) / 2 (7-p)}$, $s$ near the UV cutoff behaves as $s \sim |n| q_p^{1/2} q_p^{-(3-p) / 2 (7-p)} / J \sim |n| q_p^{2 / (7-p)} / J$. Therefore, for nonzero $n$, the asymptotic form for small $n$ can be used for the short distance behavior when

$$|n| q_p^{2 / (7-p)} / J \ll J.$$  \hspace{1cm} (4.53)

It would be an interesting problem to study the function $c_\nu(s)$ for small $s$, beyond its asymptotic form given above, by solving the differential equation perturbatively in $n^2 / \alpha^2$.

In the intermediate regime, the behavior of two-point functions are thus very nontrivial.

Finally, let us consider the case $p = 4$. One of the marked differences of this case from the cases $p < 3$ is that the affine time interval from the boundary to boundary is infinite, as in the conformal case $p = 3$. So near the boundary, we have

$$w = \frac{4 |n| q_p^{1/2}}{J} \Lambda = \frac{|n|}{\alpha} \tau \to \infty,$$  \hspace{1cm} (4.54)

in contrast to $w \to 0$ for $p < 3$. Also, the mass functions arising from the background vanishes asymptotically near the boundary. Using these properties, we derive the asymptotic solutions for the stringy modes $n \neq 0$ as

$$f_+ \to \sqrt{\frac{\alpha}{\pi |n|}} e^{-w}, \quad f_- \to \sqrt{\frac{\pi \alpha}{|n|}} e^w.$$  \hspace{1cm} (4.55)

It is easy to check that these asymptotic behaviors can be matched with the asymptotic forms obtained from the modified Bessel equation. The difference from the cases $p < 3$ is that we have to replace the roles of $I_\nu(w)$ and $K_\nu(w)$.
The above form of the asymptotic solutions implies that the contributions \((2f_\ell f_-)^{-(a^\dagger a+\frac{1}{2})}\) of the stringy modes to the S-operator does not contain any \(\ell\) dependence, and hence does not modify the behaviors of 2-pt correlation functions determined by the lowest supergravity modes with \(n = 0\). Thus, the 2-point functions of the stringy BMN operators of the boundary theory, \((4+1)\)-dimensional Yang-Mills theory with maximal supersymmetry, are completely degenerate with those of supergravity operators without stringy excitations. This is almost the free-field behavior, but is not quite so, since the contributions of supergravity modes are

\[
S_x(T) \sim \ell^{-(2a^\dagger a+1)}, \quad S_y(T) \sim \ell^{-4(a^\dagger a+\frac{1}{2})}.
\]  

(4.56)

The free-field behavior would correspond to \(S_y(T) \sim \ell^{-3(a^\dagger a+\frac{1}{2})}\). This strongly suggests that the infra-red behavior of this system is governed by a nontrivial fixed point. The degeneracy will be lifted when we take into account the interactions of various stringy modes including higher-loop effects, since the string vertices have complicated dependence on the external string states.

5. Conclusion

To summarize, we have discussed how the tunneling picture proposed in ref. [12] is utilized to extend the holographic interpretation of the PP-wave limit to general nonconformal backgrounds of Dp-branes. We have then developed a general quantum theory with time dependent masses, in order to extract predictions for two-point correlation functions of the holographically dual gauge theories. The behaviors of the resulting two-point functions for non-stringy supergravity modes are consistent with available field-theoretical analyses using supergravity. The behavior of the stringy BMN states turned out to be very different from that of D3-branes, though roughly the behaviors are not in contradiction with the conventional expectations for superrenormalizable \((p < 3)\) and nonrenormalizable \((p = 4)\) gauge theories. The implications we found for the dual gauge theories seem to be important, since for all \(p < 5\) \((p \neq 3)\) the infra-red structure is very nontrivial. In particular, we pointed out that our results may be interpreted as a strong indication for the existence of nontrivial fixed points in non-conformal super Yang-Mills theories of Dp-branes.

We hope that our work laid a foundation for further investigations of holography.
for general Dp branes using the PP-wave limit. Apart from an immediate extension of the present formalism to include fermionic excitations, there are many directions of further researches along the line of our work, such as treatments of higher-point correlation functions and string-loop effects, considerations of other possible classical backgrounds including spinning strings [15] [16] and extension to 11-dimensional theories in connection with supermembranes.

Another important task is of course to develop nonperturbative methods for studying correlation functions directly within the framework of holographically dual gauge theories themselves. Some sort of generalized mean-field approaches, extending those pursued in [20] [21], seems to be a promising direction, in view of the fact that the infra-red behaviors of the systems are predicted to be almost free-field like for $p = 4$, while for $p < 3$ the corrections to free-field behaviors with respect to stringy modes are sufficiently dramatic to decouple the stringy excitations in the infra-red from the supergravity modes. Note also that the powers exhibited in 2-pt S-operators with respect to the distance $\ell$ are different from the free field behavior in all the cases except for the conformal case $p = 3$.

The shifts of exponents from those expected from free-field theories should correspond, at least in some qualitative sense, to the presence of nontrivial mean fields for $p \neq 3$. It is of some interest to define effective dimensionality from the shifted exponents. Since free massless theories with $d_{\text{eff}}$ base-space dimensions would behave as $\ell^{-(d_{\text{eff}}-2)}$, the $SO(7-p)$ S-operator implies

$$d_{\text{eff}} = 2 + \frac{4}{5 - p}.$$  

For $p = 1$ and $p = 4$, remarkably, we have integer effective dimensions $d_{\text{eff}} = 3$ and $d_{\text{eff}} = 6$, respectively, which are related to the true base-space dimensions $d$ by $d_{\text{eff}} = d + 1$. It would be an interesting dynamical question whether this phenomenon can be interpreted from the viewpoint of M-theory as being related to M2 and M5 branes, respectively. For instance, it has been shown in a previous work by two of the present authors [23] that the supermembrane and IIA matrix-string theory [24] in the large $N$ limit can be directly related by a new particular matrix regularization. From the viewpoint of the matrix-string, the Yang-Mills coupling constant is reversed. Therefore, the weak coupling limit $g_s \sim g_{YM}^2 \to 0$ in the context of the present work must correspond to the M-theory limit $R_{11}(\sim 1/\sqrt{g_{YM}}) \to \infty$ by which the 11th dimension is de-compactified. In this sense, the above result $d_{\text{eff}} = 3$ for $p = 1$ may have a natural interpretation. Also, in the case $p = 4$,
it would be very interesting if \( d_{\text{eff}} = 6 \) is related to the argument in [25] on the relevance of the 6-dimensional fixed-point theory with (2,0) supersymmetry for the infra-red limit of 5-dimensional maximally supersymmetric Yang-Mills theory.

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A Supergravity analysis of two-point functions

In this Appendix, we summarize briefly the calculation of the two-point functions based on the more standard supergravity analysis. We apply the general prescription proposed by Gubser, Klebanov and Polyakov and by Witten [17] to the non-conformal theories. In A.1, we review the calculation for the D0-brane background performed in the previous works [10]. In A.2, we discuss the form of the two-point function of the operators corresponding to supergravity modes with large angular momentum for the case of general Dp-branes.

A.1 Two-point functions for the D0-branes

In [10], complete spectrum of the supergravity fluctuations around the near-horizon D0-brane background has been worked out. By expanding the fields into the spherical harmonics on \( S^8 \), and diagonalizing the linearized equations of motion, it was found that each physical mode of the bosonic fluctuations is described by the following scalar action in the \((t, z)\) space**:

\[
S = \frac{1}{8\kappa^2} Cq^{3/2} \int dt dz \left\{ \left( \partial_0 \psi_{I,\ell I} \right)^2 + \left( \partial_z \psi_{I,\ell I} \right)^2 + \frac{\nu^2_{\ell I} \psi_{I,\ell I}^2}{z^2} \right\}.
\]

\[(A.1)\]

**Note that the normalization of the coordinates \( t \) and \( z \) used in this paper differs from the conventional one used in [10] by dimensionful factor. The relation is \((t, z)_{\text{here}} = q^{-1/2}(t, z)_{\text{conventional}}\).
where we are considering the Euclidean space-time and \( q_0 \) is replaced by \( q \) for notational brevity. \( C \) is a numerical constant and \( \kappa^2 = g_s \ell_s^8 \) is the Newton constant in 10 dimensions. We have denoted the field collectively as \( \psi_{I,\ell_I} \), where \( I \) labels the modes and, \( \ell_I \) is the total angular momentum on \( S^8 \). The constant \( \nu_{I,\ell_I} \) is given by \( \nu_{I,\ell_I} = 2\ell_I/5 + c_I \), where \( c_I \) is determined by the explicit diagonalization. For example, for the traceless symmetric tensor modes on \( S^8 \), we have \( c_I = 7/5 \). See [10] for the spectrum of \( \nu_{I,\ell_I} \). (In the following, we will suppress the subscript \((I,\ell_I)\) for brevity.)

The equation of motion for the Fourier mode \( \psi_\omega (t,z) = \int d\omega e^{i\omega t} \psi_\omega (z) / (2\pi) \) is

\[
\left[ \partial_z^2 + \frac{1}{z} \partial_z - (\omega^2 + \frac{\nu^2}{z^2}) \right] \psi_\omega (z) = 0.
\]

This is solved by the modified Bessel function. We assume that the solution is regular at the origin \((z \to \infty : r \to 0)\), and we impose a boundary condition at the end of the near-horizon region \((z = q^{-5/14} \to 0: r \sim q^{1/7} \to \infty)\):

\[
\psi_\omega (q^{-5/14}) = \lambda_\omega. \quad (A.2)
\]

The solution which satisfy these conditions is written as

\[
\psi_\omega (z) = \lambda_\omega \frac{K_\nu (z|\omega|)}{K_\nu (q^{-5/14}|\omega|)}. \quad (A.3)
\]

We assume the relation between the classical supergravity action and the generating functional of the gauge-theory correlators, in the form with cut off as proposed by Gubser, Klebanov and Polyakov:

\[
e^{-S_{cl}[g]}|_{\psi(q^{-5/14},t)=\lambda(t)} = \langle e^{\int dt \lambda(t) O(t)} \rangle. \quad (A.4)
\]

Here, \( S_{cl} \) is the classical value of the supergravity action which is a functional of the boundary value of the field. By evaluating the action \( (A.1) \) with the classical solution \( (A.1) \), we obtain the two-point function

\[
\langle O(t_1) O(t_2) \rangle = \int \frac{d\omega_1}{2\pi} \int \frac{d\omega_2}{2\pi} e^{i\omega_1 t_1} e^{i\omega_2 t_2} \langle O(\omega_1) O(\omega_2) \rangle
\]

\[
= - \int \frac{d\omega_1}{2\pi} \int \frac{d\omega_2}{2\pi} e^{i\omega_1 t_1} e^{i\omega_2 t_2} (2\pi) \frac{\delta}{\delta \lambda_{\omega_1}} (2\pi) \frac{\delta}{\delta \lambda_{\omega_2}} S_{cl}[\lambda]|_{\lambda=0}
\]

\[
= \frac{C}{8\kappa^2 g^{3/2}} \int d\omega e^{i\omega (t_1-t_2)} q^{-5/14} \partial_z K_\nu (z|\omega|)|_{z=q^{-5/14}} \frac{K_\nu (q^{-5/14}|\omega|)}{K_\nu (q^{-5/14}|\omega|)}.
\]
In order to study the behavior in the region of large \(|t_1 - t_2|\), we expand the integrand in powers of \(|\omega|\). In the case of \(\nu \neq \) integer, we have

\[
\langle \mathcal{O}(t_1)\mathcal{O}(t_2) \rangle = \frac{C}{8\kappa^2} q^{8/7} \int d\omega e^{i\omega(t_1-t_2)} \left[-\nu + \cdots - \frac{2\nu}{q^{-5/14}} \frac{\Gamma(-\nu + 1)}{\Gamma(\nu + 1)} \left(\frac{|\omega| q^{-5/14}}{2}\right)^{2\nu} (1 + \cdots)\right] \tag{A.5}
\]

where the dots represent the terms which are of positive integer powers in \((|\omega| q^{-5/14})^2\).

The Fourier integral in the above expression is performed using the formula

\[
\int_{-\infty}^{\infty} d\omega e^{-i\omega(t_1-t_2)} |\omega|^{2\nu} = \frac{\Gamma(2\nu + 1)}{\Gamma(-\nu)} \frac{2^{\nu+1} \sqrt{\pi}}{|t_1 - t_2|^{2\nu+1}},
\]

which is valid for \(\nu \neq 0, 1, 2, \ldots, -\frac{1}{2}, -\frac{3}{2}, \ldots\) Integral of the terms analytic in \(\omega\) are divergent, and correspond to the derivatives of the delta function \(\delta(t_1 - t_2)\). Ignoring these divergences, the leading term of the correlator when \(|t_1 - t_2| \gg q^{-5/14}\) is\(^{11}\)

\[
\langle \mathcal{O}(t_1)\mathcal{O}(t_2) \rangle = \frac{C}{8\kappa^2} 2^{-\nu+2} \sqrt{\pi} \frac{\Gamma(2\nu + \frac{1}{2})}{\Gamma(-\nu)} \frac{q^{3/2-5\nu/7}}{|t_1 - t_2|^{2\nu+1}}. \tag{A.6}
\]

The right hand side has scaling dimension \(\Delta = -1 + 10\nu/7\) with respect to the generalized conformal symmetry. For each supergravity modes, we can consistently determine the corresponding operators \(\mathcal{O}\) which have the correct \(\Delta\), as shown in the tables in \[10\]. The scaling dimension of an operator is determined solely from its 11-dimensional tensor structure, and is given by (4.23) in the text.

### A.2 Large \(J\) behavior of the two-point functions for the \(Dp\)-branes

To our knowledge, supergravity spectrum on \(Dp\)-brane backgrounds for \(p \neq 0, 3\) has not been analyzed previously, except for the partial results for \(p = 1\) reported in \[22\]. Instead of deriving the precise correspondence between gauge theory operators and supergravity modes, which of course deserves for a separate work, we discuss here the form of the correlators in the large angular momentum limit by an illustrative calculation.

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\(^{11}\)Two-point function in the conventional normalization of \(t\) is obtained by the rescaling \(t \rightarrow q^{-1/2}t\) and \(\mathcal{O} \rightarrow q^{1/2}\mathcal{O}\) (since we are considering the operators with length dimension \(-1\)):

\[
\langle \mathcal{O}(t_1)\mathcal{O}(t_2) \rangle_{\text{conventional}} = \frac{C}{8\kappa^2} 2^{-\nu+2} \sqrt{\pi} \frac{\Gamma(2\nu + \frac{1}{2})}{\Gamma(-\nu)} \frac{q^{1+2\nu/7}}{|t_1 - t_2|^{2\nu+1}}.
\]
Let us consider a massless scalar field \( \varphi \) which couple to the metric and the dilaton of Dp-brane background

\[
S = \frac{1}{8\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi,
\]

(A.7)

and calculate the gauge theory two-point function, by applying the GKP-Witten prescription. Strictly speaking, this action simply for a usual scalar field \( \phi \) may not really describe the fluctuations around the Dp-brane (except for \( p = 3 \), where the dilaton background is constant), but the dependence on \( J \) will be inferred from this analysis.

Substituting the Dp-brane metric (2.2) and the dilaton \( e^{\phi} = q_p^{(3-p)/4} e^{\tilde{\phi}} \) with \( e^{\tilde{\phi}} \) given in (2.3), the action becomes

\[
\tilde{C}_1 q_p^{(p+1)/2} \int d^{10}x z \sqrt{\tilde{g}} \left\{ \delta_{mn} \partial_m \varphi \partial_n \varphi + \partial_z \varphi \partial_z \varphi + \left( \frac{2}{5 - p} \right)^2 \frac{1}{z^2} \tilde{g}^{ij} \partial_i \varphi \partial_j \varphi \right\},
\]

where the base space of the gauge theory \( x^m (m, n = 0, 1, \ldots, p) \) is Euclidean, and \( \tilde{g}_{ij} \) is the metric of unit \( S^{8-p} \). \( \tilde{C}_1, \tilde{C}_2, \tilde{C}_3 \) below are numerical constants. Note that when \( \varphi \) is a spherical harmonics on \( S^8 \) with angular momentum \( J \), the last term becomes

\[
\int d^{8-p} \tilde{x} \sqrt{\tilde{g}} \tilde{g}^{ij} \partial_i \varphi \partial_j \varphi = \int d^{8-p} \tilde{x} \sqrt{\tilde{g}} J (J + 7 - p) \varphi^2
\]

where \( \tilde{x}_i \) are the coordinate on \( S^{8-p} \). By performing a field redefinition \( \varphi = q_p^{1/2} z^{(7-p)/(5-p)} \tilde{\varphi} \), and ignoring total derivative, the action becomes

\[
S = \frac{\tilde{C}_1}{8\kappa^2} q_p^{(p+3)/2} \int d^{p+1}x z dzz \left\{ \delta_{mn} \partial_m \tilde{\varphi} \partial_n \tilde{\varphi} + \partial_2 \tilde{\varphi} \partial_2 \tilde{\varphi} + \nu^2 \tilde{\phi}^2 \right\},
\]

(A.8)

where

\[
\nu^2 = \left( \frac{2}{5 - p} \right)^2 J (J + 7 - p) + \left( \frac{7 - p}{5 - p} \right).
\]

In the large \( J \) limit, we have

\[
\nu \sim \frac{2}{5 - p} J.
\]

(A.9)

It seems reasonable to assume that this leading behavior aside perhaps from the constant part is true for all the physical modes on the Dp-background, as we have confirmed in the case of the D0-brane background [10].

We calculate the two-point function by evaluating the supergravity action classically, in the same way as for the D0-branes. We expand the fields into Fourier mode \( \tilde{\varphi}_k \)

\[
\tilde{\varphi}(z, x^m) = \int \frac{dp^{p+1}}{(2\pi)^{p+1}} e^{ikm \cdot x} \tilde{\varphi}_k(z),
\]

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and impose the boundary condition at the end of the near-horizon region \((z = q_p^{-((5-p)/(14-2p))})\). We take the solution of the equation of motion

\[ \hat{\phi}_k(z) = \frac{\lambda_k}{K_\nu(p^{-((5-p)/(14-2p))} k)} K_\nu(z_k |k|), \]  

(A.10)

and evaluate the action \((A.8)\) classically. Following the GKP-Witten prescription

\[ e^{-S_{cl}[\gamma]} |_{\psi(q_p^{-((5-p)/(14-2p))},x) = \lambda(x)} = \langle e^{\int dp^{+1} x \lambda(x) O(x)} \rangle, \]  

(A.11)

we obtain the two-point function

\[ \langle O(x) O(x') \rangle = \hat{C}_2 \frac{q_p(p+3)/2}{(2\pi)^{p+1}} \int \frac{d^{p+1} k}{(2\pi)^{p+1}} e^{ikm(x_m - x'_m)} q_p^{-((5-p)/(14-2p))} \frac{\partial_z K_\nu(z_k |k|)}{K_\nu(q_p^{-((5-p)/(14-2p))} k)} \]

\[ = \hat{C}_2 \frac{q_p(p+3)/2}{(2\pi)^{p+1}} \int \frac{d^{p+1} k e^{ikm(x_m - x'_m)} q_p^{-((5-p)/(14-2p))} \left( \frac{|k| q_p^{-((5-p)/(14-2p))} (1 + \cdots) \right)^{2\nu}}{(2\pi)^{p+1}} \]

where the dots represents terms with positive integer powers in \((k^2 q_p^{-((5-p)/(14-2p))})\). (We have assumed \(\nu \neq \text{integer} \) when expanding the Bessel functions.) The first line of the right hand side gives the delta function divergences, which we ignore. Performing the \(k\) integral, the leading part of the two-point function becomes

\[ \langle O(x) O(x') \rangle = \hat{C}_3 \frac{q_p^{1+2\nu/p}}{8\kappa^2 |x - x'|^{p+1+2\nu/p}}. \]

From \((A.9)\), we see that the correlator of the operator \(O = \text{Tr}(X^J)\) (with large \(J\)), which would correspond to the ground state of the string in the PP-wave limit take the form

\[ \langle O(x) O(x') \rangle \sim q_p^{J/(5-p)} \frac{1}{|x - x'|^{p+1+2\nu/p}}. \]  

(A.12)

Also, when we excite one 'y-oscillator' in the particle picture, the exponent will increase by \(4/(5 - p)\). These results are consistent with those we find using the PP-wave analysis in the text.

B Particle amplitude on the 'effective metric' of the D0-branes

In this Appendix, we argue that the diagonalized supergravity fluctuations around the D0-branes can be regarded alternatively as the fields propagating on an effective metric,
which is the direct product of AdS and sphere. We study the particle amplitude on that effective background. Unfortunately, the description cannot be applied for stringy modes.

The effective action of the supergravity fluctuations (A.1) can be regarded as the s-wave part of a massive scalar action in an effective metric \( \hat{g}_{\mu \nu} \), (without a dilaton coupling):

\[
S = \frac{1}{8\kappa^2} \int d^{10}x \sqrt{-\hat{g}} \left\{ \hat{g}^{\mu \nu} \partial_\mu \hat{\psi}_{I,\ell_1} \partial_\nu \hat{\psi}_{I,\ell_1} + \hat{m}^2_{I,\ell_1} \hat{\psi}^2_{I,\ell_1} \right\}. \tag{B.13}
\]

Here, \( \hat{g}_{\mu \nu} \) is related to the original D0-brane metric \( g_{\mu \nu} \) by a Weyl transformation \( \hat{g}_{\mu \nu} = \left( q^{3/4} e^{\tilde{\phi}} \right)^{-2/7} g_{\mu \nu} \) and is just the (Euclidean) AdS\(_2 \times S^8\) metric:

\[
\hat{ds}^2 = \hat{g}_{\mu \nu} dx^\mu dx^\nu = q^{2} \left\{ \left( \frac{2}{5} \right)^2 \frac{dt^2 + dz^2}{z^2} + d\psi^2 + \sin^2 \psi d\Omega_7^2 \right\}. \tag{B.14}
\]

The scalar field \( \hat{\psi}_{I,\ell_1} \) and mass \( \hat{m}_{I,\ell_1} \) are defined by

\[
\hat{\psi}_{I,\ell_1} = q^{5/28} z^{1/2} \psi_{I,\ell_1}, \quad \hat{m}^2_{I,\ell_1} = \left( \frac{25}{4} \nu_{I,\ell_1}^2 - \frac{25}{16} \right) q^{-2/7}, \tag{B.15}
\]

where \( e^{\tilde{\phi}} \) is given in (2.3). This effective action is equivalent with the one given in the first reference in [10].

We have ignored the total derivative terms when rewriting (A.1) in the form (B.13). When calculating the gauge theory correlators, such terms contribute only to the ‘delta function’ divergences, mentioned in A.1. Also, the field redefinition (B.15) does not affect the two-point function. Namely, if we impose the boundary condition to \( \hat{\psi} \), we obtain the same result for the leading part of the two-point function (A.6).

Note that when the angular momentum \( \ell_1 = J \) is large, the mass is given by \( \hat{m}^2_{I,J} \sim J^2 q^{-2/7} \). This allows us to regard \( \hat{\psi}_{I,J} \) modes (with large \( J \)) alternatively as the higher partial waves of a scalar field \( \hat{\psi}_{I,\ell_1=0} \) on the space-time (B.14). Indeed, if we assume \( \hat{\psi}_{I,J} \) is a spherical harmonics with angular momentum \( J \), the derivative along \( S^8 \) gives

\[
\int d^{10}x \sqrt{-\hat{g}} \hat{g}^{ij} \partial_i \hat{\psi}_{I,J} \partial_j \hat{\psi}_{I,J} = \int d^{10}x \sqrt{-\hat{g}} q^{-2/7} J(J+7) \hat{\psi}^2_{I,J},
\]

which agrees with the mass term in the \( J \to \infty \) limit. This fact suggests that the effective metric (B.14) has a meaning in the 10-dimensional sense, at least in this limit. We expect that there is also a semi-classical picture based on the particle on this effective metric.

Let us therefore study the particle amplitude on our effective D0-brane metric. Since the space-time is of the AdS\( \times S \) form, the amplitude is obtained in the similar way as for
the D3-brane case discussed in the text. To obtain the classical contribution, consider the 
\((t, z, \psi)\) part of the massless particle action on (B.14):
\[
S = \frac{1}{2} \int d\tau \frac{1}{\eta} \left\{ \frac{R^2}{\xi^2} (\dot{t}^2 + \dot{z}^2) - \left( \frac{5}{2} \right)^2 R^2 \dot{\psi}^2 \right\}, \tag{B.16}
\]
where \(R \equiv 2q^{1/7}/5\), and the double Wick-rotation has been performed. Conjugate 
momentum for \(\psi\) is \(J = -(5R/2)^2 \dot{\psi}/\eta\), and the Routh function becomes
\[
\bar{S} = \frac{1}{2} \int d\tau \left\{ \frac{1}{\eta} \frac{R^2}{\xi^2} (\dot{t}^2 + \dot{z}^2) + \left( \frac{2}{5} \right)^2 \frac{\eta}{R^2} J^2 \right\},
\]
which is identical to the one for the D3-brane case if we replace \(J \rightarrow 2J/5\) in the latter. 
Proceeding similarly to that case, we then obtain
\[
\langle t_f, 1/\Lambda, J; T | t_i, 1/\Lambda, J; -T \rangle_{\text{class}} = e^{-\bar{S}} = \left( \frac{1}{\Lambda | t_f - t_i |} \right)^{\frac{4}{5}J}. \tag{B.17}
\]
The action for the fluctuations of particle on the effective metric (B.14) is found to be
\[
S = \frac{1}{2} \int d\tau \left\{ \dot{x}^2 + \dot{y}_l^2 + \ell^2 (x^2 + \frac{4}{25} y_l^2) \right\} \tag{B.18}
\]
where \(l = 1, \ldots, 7\). The amplitude including the contributions from the fluctuations be-
comes
\[
\left( \frac{1}{\Lambda | t_f - t_i |} \right)^{\frac{4}{5}J + \frac{4}{5} n_y + 2n_x}, \tag{B.19}
\]
where \(n_x, n_y\) are the occupation numbers for the \(x\)- and \(y\)- oscillators, respectively. This 
is consistent with the result from the supergravity analysis, and also agrees with the one 
obtained from the particle on the true (string-frame) D0-brane background. However,
our analysis in the text clearly shows that this kind of effective theory is not meaningful
for stringy excitations modes as it stands, though there may be some different ways of
extension to stringy excitations.

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