Supersymmetry projection rules on exotic branes

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We study the supersymmetry projection rules on exotic branes in type II string theories and M-theory. They justify the validity of the exotic duality between standard branes and exotic branes of codimension two. By virtue of the supersymmetry projection rules on various branes, we can consider a system that involves multiple nonparallel exotic branes in the nongeometric background.

Subject Index B10, B20, B23

1. Introduction

Exotic branes [1–5] should play a central role in the investigation of the nonperturbative dynamics in string theory and gauge theory. This is because the exotic branes originate from standard branes such as fundamental strings (or F-strings, for short), Neveu–Schwarz fivebranes (NS5-branes) and Dirichlet branes (D-branes) via the string dualities in lower dimensions. The standard branes have contributed to understanding the nonperturbative dynamics in string theory and gauge theory [6]. However, compared with the standard branes, the dynamical feature of the exotic branes is still unclear. The main reason is that the transverse space of an exotic brane has a nontrivial monodromy due to the string dualities [5,7,8].

Consider, for instance, an exotic $5_2^2$-brane. This object comes from an NS5-brane via the T-duality along two transverse directions of it. The transverse space has the $SO(2, 2; \mathbb{Z}) = SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z})$ monodromy structure, which originates from the T-duality [5,9–13]. This nontrivial monodromy structure often prevents us from analyzing excitations of the $5_2^2$-brane. This is completely different from the case of the standard branes¹.

The exotic branes are also characterized by their masses. They are different from those of the standard branes. For instance, the masses of an exotic $b_n^c$-brane and an exotic $b_n^{(d,c)}$-brane are described as

\begin{align}
b_n^c : M &= \frac{R_1 \cdots R_b (R_{b+1} \cdots R_{b+c})^2}{g_s^{n-2} f_s^{b+2c+1}}, \quad (1.1a) \\
b_n^{(d,c)} : M &= \frac{R_1 \cdots R_b (R_{b+1} \cdots R_{b+c})^2 (R_{b+c+1} \cdots R_{b+c+d})^3}{g_s^{n-2} f_s^{b+2c+3d+1}}. \quad (1.1b)
\end{align}

¹ Strictly speaking, a D7-brane also has an $SL(2, \mathbb{Z})$ monodromy originating from the string S-duality [7].
Here the labels $b$, $c$, and $n$ indicate the spatial dimensions, the number of isometry directions in the transverse directions, and the power of the string coupling constant in the tension of the exotic brane under consideration, respectively. The symbol $(d, c)$ also indicates the $c$ and $d$ isometry directions. $g_s$ and $\ell_s$ are the string coupling constant and the string length, respectively. Each $R_i$ indicates the radius or size in the $i$th direction. $R_1 \cdots R_b$ represents the volume of the brane expanded along the spatial $12\cdots b$ directions. Unlike those of the standard branes, the mass formulae (1.1) possess multiple powers of radii such as $(R_{b+1})^2$. Expressions (1.1) are derived from those of the standard branes via the string dualities:

\[
\begin{align*}
\text{T}_y\text{-duality} & : \quad R_y \rightarrow \frac{\ell_s^2}{R_y}, \quad g_s \rightarrow \frac{\ell_s}{R_y} g_s, \\
\text{S-duality} & : \quad g_s \rightarrow \frac{1}{g_s}, \quad \ell_s \rightarrow \frac{1}{g_s^{1/2}} \ell_s.
\end{align*}
\] (1.2a)

We note that $T_y$ implies the T-duality transformation along the $y$-direction. For instance, consider the $5_2^7$-brane again. As mentioned above, this comes from an NS5-brane whose mass is

\[
M = \frac{R_1 R_2 \cdots R_5}{g_s^2 \ell_s^6},
\] (1.3)

when the NS5-brane is expanded along the $12345$-directions. Following the nomenclature in Ref. [10], we refer to this as NS5(12345). Performing the T-duality along the $89$-directions, we obtain the $5_2^7(12345,89)$-brane whose mass is

\[
M = \frac{R_1 R_2 \cdots R_5 (R_8 R_9)^2}{g_s^2 \ell_s^{10}}.
\] (1.4)

Another feature of the exotic branes is that the codimension, i.e., the difference between the bulk spacetime dimensions and the worldvolume dimensions, is less than three. This implies that the single exotic brane does not have a well-defined background geometry in the supergravity framework. For instance, in the case of a standard brane of codimension $k > 3$, its background geometry is governed by a harmonic function of $r^{-k+2}$, where $r$ indicates the distance from the core of the brane. If the codimension is two or one, the harmonic function becomes logarithmic or linear, respectively.

Due to the above features, it is often difficult to analyze the global structure of the exotic branes. However, since the exotic branes are cousins of the standard branes, their Bogomol’nyi–Prasad–Sommerfield (BPS) conditions should be characterized in the same way as those of the standard branes. For instance, a $D_p$-brane stretched along the $12\cdots p$ directions preserves supercharges of the form $\epsilon_L Q_L + \epsilon_R Q_R$ with

\[
\epsilon_L = \Gamma^{012\cdots p} \epsilon_R,
\] (1.5)

where $\epsilon_L$ and $\epsilon_R$ are the supersymmetry parameters given by the Majorana–Weyl fermions with different chiralities $\Gamma \epsilon_L = +\epsilon_L$, $\Gamma \epsilon_R = -\epsilon_R$ in type IIA theory, or the same chirality $\Gamma \epsilon_{L,R} = +\epsilon_{L,R}$ in type IIB theory. $\Gamma$ is the chirality operator in ten dimensions. $Q_L$ and $Q_R$ are the corresponding left and right supercharges, and $\Gamma^a$ is the $a$th Dirac gamma matrix. The chirality operator $\Gamma$ is described by the Dirac gamma matrices $\Gamma = \Gamma^{012\cdots 9}$. We refer to (1.5) as the supersymmetry projection rule. In this paper, we will explore the supersymmetry projection rules on various exotic branes in type II string theories and M-theory [4,10].

Before moving to the main part of this paper, we also mention an interesting relation among defect branes of codimension two [14]. There are various defect branes in $D$-dimensional spacetime. We show them in Table 1.
Table 1. Defect branes in D-dimensional spacetime. Here the integer $n$ in the first row indicates the power of the string coupling constant in each brane's mass. $D_p$ means the $D_p$-brane, while $b_c^n$ and $b_n^{(d,c)}$ represent the exotic branes. F1 and P denote the F-string and the pp-wave, which also behave as defect branes in four and three dimensions, respectively. The labels in brackets represent the tensor fields coupling to the corresponding defect branes. We refer to the branes of $n = 0, 1, 2$ as fundamental, Dirichlet, and solitonic branes, respectively [14].

| $D$ | $n = 0$ | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ |
|-----|--------|--------|--------|--------|--------|
| IIB | D7 $[C_8]$ | $7_3 [E_8]$ | $6_1^4 [E_8]$ | $5_2^3 [E_8]$ | $4_3^4 [E_8]$ |
| 9   | D6 $[C_7]$ | $6_1^4 [E_8]$ | $5_2^3 [E_8]$ | $4_3^4 [E_8]$ | $3_4^5 [E_8]$ |
| 8   | D5 $[C_6]$ | $5_2^3 [E_8]$ | $4_3^4 [E_8]$ | $3_4^5 [E_8]$ | $2_5^6 [E_8]$ |
| 7   | D4 $[C_5]$ | $4_3^4 [E_8]$ | $3_4^5 [E_8]$ | $2_5^6 [E_8]$ | $1_6^7 [E_8]$ |
| 6   | D3 $[C_4]$ | $3_4^5 [E_8]$ | $2_5^6 [E_8]$ | $1_6^7 [E_8]$ | $0_7^8 [E_8]$ |
| 5   | D2 $[C_3]$ | $2_3^4 [E_8]$ | $1_6^7 [E_8]$ | $0_7^8 [E_8]$ | $4_9^{10} [F_8]$ |
| 4   | F1 $[B_2]$ | $1_6^7 [E_8]$ | $0_7^8 [E_8]$ | $4_9^{10} [F_8]$ | $3_10^{11} [F_8]$ |
| 3   | P      | $0_7^8 [E_8]$ | $3_10^{11} [F_8]$ | $4_9^{10} [F_8]$ | $5_11^{12} [F_8]$ |

Fig. 1. Exotic duality, S-duality, and T-duality along $k$ directions (labeled as E, S, and T$_{(k)}$, respectively) among the standard branes and the exotic branes [14,15].

There exists an $SL(2, Z)$ duality group under which the standard branes of $n = 0, 1$ are mapped to the exotic branes of $n = 4, 3$ and vice versa, and the solitonic branes of $n = 2$ are mapped to other solitonic branes. This duality group is a subgroup of the U-duality group in each dimension. This is referred to as the exotic duality [14,15]. Even though the U-duality group in a certain space-time dimension is different from that of a different dimension, any exotic duality is described by $SL(2, Z)$. This duality is illustrated in Figs. 1 and 2.

The exotic duality of a single brane was suggested in Ref. [14] from the viewpoint of the string duality groups and their representations. This was also analyzed in Ref. [16] by virtue of the $E_{11}$
supergravity technique. Furthermore, in the framework of other extended supergravity such as \( \beta \)-supergravity \([17,18]\) and its extended version \([15,19]\)\(^2\), the exotic duality was further investigated. In this work, we would like to confirm the validity of the exotic duality from the viewpoint of the supersymmetry projection rules such as \((1.5)\), discussed in Refs. \([4,10]\), and apply it to new brane configurations that involve \textit{multiple} nonparallel (exotic) branes.

The structure of this paper is as follows. In Sect. 2, we first list the supersymmetry projection rules on the standard branes. Extracting the string dualities acting on the supersymmetry parameters, we explicitly write down the supersymmetry projection rules on the exotic branes. We find that their expressions justify the exotic duality from the supersymmetry viewpoint. In order to check the consistency, we apply the supersymmetry projection rules on the exotic branes to certain brane configurations that contain exotic branes. In Sect. 3, we consider the exotic duality applied to multiple nonparallel branes. By virtue of the supersymmetry projection rules discussed in Sect. 2, we find various interesting configurations. Section 4 is devoted to the conclusion and discussions. In Appendix A the detailed computations to derive the supersymmetry projection rules on various exotic branes are explicitly described.

2. Supersymmetry projection rules

In this section, we first exhibit the supersymmetry projection rules such as \((1.5)\) on the standard branes in type II string theories and M-theory. Following the rules, we introduce the string dualities acting on the supersymmetry parameters. Using the string dualities, we write down the rules on various exotic branes. To avoid complications, we do not write down the concrete derivation of each exotic brane in this section. It is summarized in Appendix A. Next, we apply the supersymmetry projection rules to certain brane configurations derived from an F-string ending on a D3-brane. Analogous to the string dualities on the mass formulae of branes \((1.2)\), we do not seriously consider their global structures.

2.1. Rules on standard branes

First of all, we gather the supersymmetry projection rules on standard branes. These are very common and can be seen in the literature (see, for instance, Refs. \([6,20]\)):

- Standard branes in IIA theory:
  \[
  \Gamma \epsilon = \sigma_3 \epsilon, \quad \text{or equivalently} \quad \Gamma \epsilon_L = +\epsilon_L, \quad \Gamma \epsilon_R = -\epsilon_R, \tag{2.1a}
  \]
  \[
  P(1) : \pm \epsilon = \Gamma^{01} \epsilon, \tag{2.1b}
  \]
  \[
  F(1) : \pm \epsilon = \Gamma^{01} \epsilon, \quad \text{NS5}(12345) : \pm \epsilon = \Gamma^{012345} \epsilon, \tag{2.1c}
  \]
  \[
  Dp(12 \cdots p) : \pm \epsilon = \Gamma^{012 \cdots p} A_p(\sigma_3) \epsilon, \quad A_p \equiv \begin{cases} 1 : & p = 2, 6, \\ \Gamma : & p = 0, 4, 8. \end{cases} \tag{2.1d}
  \]

- Standard branes in IIB theory:
  \[
  \Gamma \epsilon = \epsilon, \quad \text{or equivalently} \quad \Gamma \epsilon_L = +\epsilon_L, \quad \Gamma \epsilon_R = +\epsilon_R, \tag{2.2a}
  \]
  \[
  P(1) : \pm \epsilon = \Gamma^{01} \epsilon, \tag{2.2b}
  \]

\(^2\)The extended version of \( \beta \)-supergravity \([15,19]\) involves the Ramond–Ramond potentials \( C_p \) and their string dualized objects \( \gamma^p \). This formulation may be referred to as “\( \gamma \)-supergravity”.


\[ F1(1) : \pm \epsilon = \Gamma^{01}(\sigma_3)\epsilon, \quad \text{NS5}(12345) : \pm \epsilon = \Gamma^{012345}(\sigma_3)\epsilon, \]
\[ D_p(12 \cdots p) : \pm \epsilon = \Gamma^{012 \cdots p} B_p \epsilon, \quad B_p \equiv \begin{cases} \sigma_1 : & p = 1, 5, 9, \\ i\sigma_2 : & p = 3, 7. \end{cases} \]  
\[ (2.2c) \]

\[ o \text{ Standard branes in M-theory:} \]
\[ M2(12) : \pm \eta = \Gamma^{012} \eta, \]
\[ M5(12345) : \pm \eta = \Gamma^{012345} \eta. \]  
\[ (2.3a) \]

In order to make the discussion clear, we introduce a "doublet" of two supersymmetry parameters \( \epsilon_L \) and \( \epsilon_R \) in such a way that \( \epsilon \equiv (\epsilon_L, \epsilon_R)^T \). This is a Majorana spinor in type IIA theory, while it can be interpreted as an \( SL(2, \mathbb{Z}) \) doublet in type IIB theory. Due to this description, we also introduce the Pauli matrices \( \sigma_i \) acting on the doublet \( \epsilon \). We also note that \( \Gamma \) is the chirality operator defined as \( \Gamma = \Gamma^{0123456789} \) in terms of the Dirac gamma matrices \( \Gamma^a \). They are subject to the Clifford algebra \( \{ \Gamma^a, \Gamma^b \} = 2\eta^{ab} \). In this work we use the mostly plus signature \( \eta_{ab} = \text{diag}(-++ \cdots +) \).

In the case of M-theory, the supersymmetry parameter \( \eta \) is a Majorana spinor. Dirac gamma matrices in eleven dimensions are the same as those in ten dimensions, while the chirality operator \( \Gamma \) is uplifted to the eleventh gamma matrix \( \Gamma^y \) in 11D theory.

### 2.2. Dualities on supersymmetry parameters

Since various standard branes are related to each other via the string dualities, there should exist duality transformation rules on the supersymmetry parameters \( \epsilon_L \) and \( \epsilon_R \). Here, without the derivation, we exhibit the T-duality and S-duality [21] (for instance, see Ref. [22]):

\[ \text{T}_y \text{-duality :} \quad \epsilon_L \rightarrow \epsilon_L, \quad \epsilon_R \rightarrow \Gamma^y \Gamma \epsilon_R, \]  
\[ \text{S-duality :} \quad \epsilon \rightarrow S \epsilon, \quad S = \frac{1}{\sqrt{2}} (\mathbb{1}_2 - i\sigma_2). \]  
\[ (2.4a) \]
\[ (2.4b) \]

We have a couple of comments on the above rules. In the case of the \( \text{T}_y \)-duality, the operator \( \Gamma^y \Gamma \) generates the parity transformation along the \( y \)-direction such as \( (\Gamma^y \Gamma)^{-1} \Gamma^y (\Gamma^y \Gamma) = -\Gamma^y \), while this behaves as an identity operator acting on the other \( \Gamma^i (i \neq y) \) as follows: \( (\Gamma^y \Gamma)^{-1} \Gamma^i (\Gamma^y \Gamma) = \Gamma^i \). In the S-duality case, this rule does not change the supersymmetry projection rule on the D3-brane. This is expressed by \( S^{-1}(i\sigma_2)S = i\sigma_2 \). On the other hand, the operator \( S \) transforms \( \sigma_1 \) and \( \sigma_3 \) in such a way that \( S^{-1}\sigma_1 S = \sigma_3 \) and \( S^{-1}\sigma_3 S = -\sigma_1 \). Under this transformation, the F-string and the NS5-brane are mapped to the D-string and the D5-brane, and vice versa. Geometrically, the S-duality transformation means a rotation along the second axis of the 3D \( SL(2, \mathbb{Z}) \) space.

### 2.3. Rules on exotic branes

Applying the string dualities to the supersymmetry projection rules on the standard branes, we can derive those of various exotic branes in a straightforward way. As mentioned before, the explicit computations are listed in Appendix A. Here we summarize the supersymmetry projection rules on the solitonic branes and defect branes discussed in Refs. [4,10], and the rules on the domain walls, which are new. First we exhibit the rules on the solitonic branes in type IIA and IIB theories, respectively.
Solitonic five-branes in IIA theory:

NS5(12345) : \( \pm \epsilon = \Gamma^{012345} \epsilon, \)  

KK5(12345, 9) : \( \pm \epsilon = \Gamma^{012345} \Gamma \epsilon, \)  

\( 5^{2}_{2}(12345, 89) \) : \( \pm \epsilon = \Gamma^{012345} \epsilon, \)  

\( 5^{3}_{2}(12345, 789) \) : \( \pm \epsilon = \Gamma^{012345} \Gamma \epsilon, \)  

\( 5^{4}_{2}(12345, 6789) \) : \( \pm \epsilon = \Gamma^{012345} \epsilon. \)  

Solitonic five-branes in IIB theory:

NS5(12345) : \( \pm \epsilon = \Gamma^{012345}(\sigma_{3}) \epsilon, \)  

KK5(12345, 9) : \( \pm \epsilon = \Gamma^{012345} \epsilon, \)  

\( 5^{2}_{2}(12345, 89) \) : \( \pm \epsilon = \Gamma^{012345}(\sigma_{3}) \epsilon, \)  

\( 5^{3}_{2}(12345, 789) \) : \( \pm \epsilon = \Gamma^{012345} \epsilon, \)  

\( 5^{4}_{2}(12345, 6789) \) : \( \pm \epsilon = \Gamma^{012345}(\sigma_{3}) \epsilon. \)  

We find that the transverse directions with isometry (i.e., the 6789-directions) do not contribute to the supersymmetry projection rules. On the other hand, analogous to the NS5-brane, the hyperplanes in which the five-branes are stretched provide the projections. The above expressions guarantee that the defect \((p, q)\) five-brane, a bound state of a defect NS5(12345) and an exotic \(5^{2}_{2}(12345, 89)\), is a 1/2-BPS object in string theory [11–13], while a bound state of a KK5(12345,9) and a \(5^{2}_{2}(12345, 89)\) breaks supersymmetry.

Next, we gather the supersymmetry projection rules on the defect branes [5,10,14] in Table 1.

Defect branes in IIA theory:

\( 6^{1}_{3}(123456, 7) \) : \( \pm \epsilon = \Gamma^{0123456}(\sigma_{1}) \epsilon, \)  

\( 4^{3}_{3}(1234, 567) \) : \( \pm \epsilon = \Gamma^{01234}(\sigma_{1}) \epsilon, \)  

\( 2^{5}_{3}(12, 34567) \) : \( \pm \epsilon = \Gamma^{012}(\sigma_{1}) \epsilon, \)  

\( 0^{7}_{3}(1, 1234567) \) : \( \pm \epsilon = \Gamma^{0}(\sigma_{1}) \epsilon, \)  

\( 1^{6}_{4}(1, 234567) \) : \( \pm \epsilon = \Gamma^{01} \epsilon, \)  

\( 0^{(1,6)}_{4}(, 234567, 1) \) : \( \pm \epsilon = \Gamma^{01} \epsilon. \)  

Defect branes in IIB theory:

\( 7^{3}_{3}(1234567) \) : \( \pm \epsilon = \Gamma^{01234567}(i \sigma_{2}) \epsilon, \)  

\( 5^{2}_{3}(12345, 67) \) : \( \pm \epsilon = \Gamma^{012345}(\sigma_{1}) \epsilon, \)  

\( 3^{4}_{3}(123, 4567) \) : \( \pm \epsilon = \Gamma^{0123}(i \sigma_{2}) \epsilon, \)  

\( 1^{6}_{3}(1, 234567) \) : \( \pm \epsilon = \Gamma^{01}(\sigma_{1}) \epsilon, \)  

\( 1^{6}_{4}(1, 234567) \) : \( \pm \epsilon = \Gamma^{01}(\sigma_{1}) \epsilon, \)  

\( 0^{(1,6)}_{4}(, 234567, 1) \) : \( \pm \epsilon = \Gamma^{01} \epsilon. \)
We have summarized the supersymmetry projection rules on all the defect branes in Table 1. We have comments on the defect branes. The supersymmetry projection rule on each defect $p_3^{-p}$-brane coincides with that of the D$p$-brane. These two branes are exotic dual. The rules on the $1_5^0$-brane and the $0_4^{(1,0)}$-brane in type IIA/IIB theories are also exactly the same as those of the F-string and the pp-wave, respectively. The former exotic branes are exotic dual of the latter standard branes [14,15].

Applying the additional string dualities to the defect branes in Table 1, we obtain the domain walls represented as $b_3^{(1,c)}$-branes and $b_4^{(d,3)}$-branes [23].

- **Domain walls in IIA theory:**
  \[
  \begin{align*}
  7^{(1,0)}_3(1234567, 9) : \quad \pm \epsilon &= \Gamma^{012345679}_{1} \Gamma(\sigma_1) \epsilon, \\
  5^{(1,2)}_2(12345, 67, 9) : \quad \pm \epsilon &= \Gamma^{0123459}_{1} \sigma_1 \epsilon, \\
  3^{(1,4)}_2(123, 4567, 9) : \quad \pm \epsilon &= \Gamma^{01239}_{1} \epsilon, \\
  1^{(1,6)}_1(1, 234567, 9) : \quad \pm \epsilon &= \Gamma^{019}_{1} \epsilon, \\
  5^{5}_{4}(12345, 789) : \quad \pm \epsilon &= \Gamma^{012345}_{1} \epsilon,
  \end{align*}
  \]

- **Domain walls in IIB theory:**
  \[
  \begin{align*}
  6^{(1,1)}_3(123456, 7, 9) : \quad \pm \epsilon &= \Gamma^{01234569}_{1} (i \sigma_2) \epsilon, \\
  4^{(1,3)}_2(1234, 567, 9) : \quad \pm \epsilon &= \Gamma^{012349}_{1} (i \sigma_1) \epsilon, \\
  2^{(1,5)}_2(12, 34567, 9) : \quad \pm \epsilon &= \Gamma^{0129}_{1} (i \sigma_2) \epsilon, \\
  0^{(1,7)}_1(1234567, 9) : \quad \pm \epsilon &= \Gamma^{09}_{1} \epsilon, \\
  5^{5}_{4}(12345, 789) : \quad \pm \epsilon &= \Gamma^{012345}_{1} \epsilon,
  \end{align*}
  \]

We have comments on the rules on the domain walls. Analogous to D-branes, there exist BPS $(2b+1)^{(1,c)}_3$-branes in type IIA theory, and BPS $(2b)^{(1,c)}_3$-branes in type IIB theory. On the other hand, there exist BPS $b_4^{(d,3)}$-branes with $b + d = 5$ in both IIA and IIB theories. It is noticed that the above is not a complete list of domain walls. There exist other kinds of domain walls in type II string theories. For instance, we find a type IIB $5_5^4(12345, 6789)$-brane originating from
a type IIB NS5(12345)-brane via the string ST6789-dualities. More complicatedly, a type IIB 3\textsuperscript{1,2,3}(123, 678, 45, 9)-brane can be derived from the type IIB NS5(12345)-brane under the string T\textsubscript{459}ST\textsubscript{678}-dualities. In principle, we can obtain the supersymmetry projection rules on the 5\textsuperscript{2}-brane and the 3\textsuperscript{1,2,3}-brane under the duality rules (2.4) in the same way as the derivation of their masses by using (1.2).

As listed above, the supersymmetry projection rules on the exotic branes are quite similar to those of the standard branes. In particular, we emphasize that the rules on the defect branes (2.7) and (2.8) coincide with those of the standard branes (2.1) and (2.2), respectively. Hence they justify the validity of the exotic duality illustrated in Figs. 1 and 2 from the supersymmetry viewpoint.

2.4. Exotic branes in M-theory

Once we have understood the supersymmetry projection rules in type IIA theory, we can uplift them to those in M-theory. This procedure is simple because the type IIA supersymmetry parameter \( \epsilon \) becomes a Majorana spinor \( \eta \) in M-theory. Uplifting type IIA theory to M-theory, we can also interpret the chirality operator \( \Gamma \) as the eleventh Dirac gamma matrix \( \Gamma^\natural \). Furthermore, we have also identified the relation between the string coupling \( g_s \), the string length \( \ell_s \), the radius of the M-theory circle \( R_\natural \), and the 11D Planck length \( \ell_P \) in the literature:

\[
\begin{align*}
\epsilon &= \eta, \quad \Gamma = \Gamma^\natural, \\
\frac{g_s}{12} \ell_s &= R_\natural, \quad g_s^{1/3} \ell_s = \ell_P.
\end{align*}
\]  

(2.11a)  

(2.11b)

Applying the uplift to various exotic branes in type IIA theory, we obtain the exotic branes and their supersymmetry projection rules:

\[
\begin{align*}
\text{KK6}(12345\natural, 9) : \quad &\pm \eta = \Gamma^{012345\natural} \eta, \\
5^3(12345, 89\natural) : \quad &\pm \eta = \Gamma^{012345} \eta, \\
5^{(1,3)}(12345, 789, \natural) : \quad &\pm \eta = \Gamma^{012345\natural} \eta, \\
2^6(1\natural, 234567) : \quad &\pm \eta = \Gamma^{015} \eta, \\
0^{(1,7)}(12345\natural, 1) : \quad &\pm \eta = \Gamma^{01} \eta.
\end{align*}
\]

(2.12a)  

(2.12b)  

(2.12c)  

(2.12d)  

(2.12e)

We again summarize the detailed computations in Appendix A4. Here, for simplicity, we skip consideration of the uplift of the domain walls in (2.9).

2.5. Brane configurations as a consistency check

In order to check the consistency of the supersymmetry projection rules on the exotic branes, we apply them to a certain brane configuration. In this paper we focus on the system in which a brane is ending on another brane. A typical example is the system of an F-string ending on a D3-brane, shown in Fig. 3.

It is easy to confirm that the supersymmetry projection rules on the D3-brane and the F-string (2.2) preserve a quarter of the supersymmetry of the system in Fig. 3. Next, we apply the string dualities to the system in Fig. 3 and obtain various configurations. Performing the S-duality and the T-dualities along the 1234-directions, followed by the S-duality again, we obtain the configuration in which the 7\textsubscript{3}-brane is involved (see Fig. 4).

We note that the 7\textsubscript{3}-brane is also called the NS7-brane or the (0,1) sevenbrane. Since the configuration in Fig. 4 is derived from that in Fig. 3 via the string ST\textsubscript{1234}S-dualities, this should also preserve...
Fig. 3. F-string ending on D3-brane. The F-string is stretched along the 8th direction, while the D3-brane is expanded in the 567-plane.

|   | IIB | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|-----|---|---|---|---|---|---|---|---|---|---|
| D3 |    | - | - | - | - | - | - | - | - | - | - |
| F1 |    | - | - | - | - | - | - | - | - | - | - |

Fig. 4. NS5-brane ending on 73-brane. This is the ST\textsubscript{1234}S-dual of the configuration in Fig. 3.

|   | IIB | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|-----|---|---|---|---|---|---|---|---|---|---|
| 73 |    | - | - | - | - | - | - | - | - | - | - |
| NS5 |   | - | - | - | - | - | - | - | - | - | - |

Fig. 5. 5\textsubscript{34}-brane ending on 4\textsubscript{(1,3)}-brane. This is the ST\textsubscript{4679}-dual of the configuration in Fig. 4. The symbols \textbullet\textsuperscript{2} and \textbullet\textsuperscript{3} in the table indicate that the mass of the brane depends on the corresponding direction with the power described (see the general mass formulae (1.1)).

|   | IIB | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|-----|---|---|---|---|---|---|---|---|---|---|
| 4\textsubscript{(1,3)} \textsuperscript{534}(1235,467,9) | - | - | - | - | \textbullet\textsuperscript{2} | - | \textbullet\textsuperscript{2} | \textbullet\textsuperscript{2} | - | - |
| 5\textsubscript{34}(12348,679) | - | - | - | - | \textbullet\textsuperscript{2} | \textbullet\textsuperscript{2} | - | - | - | - |

the 1/4-BPS condition. This can be confirmed by using the supersymmetry projection rules on the IIB NS5-brane in (2.2) and the 7\textsubscript{3}-brane in (2.8). The former is given as $\epsilon = \Gamma^{01234567} \epsilon$ and the latter is $\epsilon = \Gamma^{01234567} \epsilon$. These two conditions provide the equation $\epsilon = \Gamma^{5678} \epsilon$. Since $\Gamma^{5678}$ is traceless and its square becomes the identity, we can choose the +1 eigenvalue of the supersymmetry parameter $\epsilon$. Thus it turns out that the configuration preserves a quarter of the supersymmetry.

We can consider a more complicated brane configuration. Applying the ST\textsubscript{4679}-dualities to the system in Fig. 4, we obtain the brane configuration in which the domain wall 5\textsubscript{34}-brane is ending on the domain wall 4\textsubscript{(1,3)}-brane (see Fig. 5).

We consider the supersymmetry projection rules on the type IIB 4\textsubscript{(1,3)}-brane and the 5\textsubscript{34}-brane in (2.10). The former is $\epsilon = \Gamma^{012359}(\sigma_3) \epsilon$, while the latter is $\epsilon = \Gamma^{01234567} \epsilon$. Splitting them into equations for $\epsilon_L$ and $\epsilon_R$, we obtain $\epsilon_L = -\Gamma^{4589} \epsilon_L$ and $\epsilon_R = +\Gamma^{4589} \epsilon_R$. Since $\Gamma^{4589}$ is traceless and its square is the identity, we can choose the eigenvalues $-1$ and $+1$ on the parameters $\epsilon_L$ and $\epsilon_R$, respectively. Hence we can again find that the configuration preserves a quarter of the supersymmetry in type IIB theory.

### 3. Exotic mappings of multiple branes

In the previous section, we established the supersymmetry projection rules on the exotic branes as well as those of the standard branes. As mentioned before, the exotic duality was first discussed in Ref. [14] and further developed in Ref. [15]. This is the duality between a single standard brane and a single exotic brane, as illustrated in Figs. 1 and 2. In this section, we explore a mapping of multiple nonparallel (exotic) branes. This is a procedure in which a certain brane configuration in the conventional framework is mapped to a new brane configuration that is realized in the nongeometric
framework. The new configuration cannot be described in ordinary supergravity theories. It should be given in terms of the $\beta$-supergravity or its extended version, which govern the string theory of nongeometric backgrounds [15,17–19]. From now on, we refer to this mapping as exotic mapping.

First we apply the exotic mapping of a D$p$-brane ending on a D$(p + 2)$-brane in Fig. 6.

Under the exotic duality, the D$p$-brane and the D$(p + 2)$-brane are mapped to the $p^7_3 - p^3_3$-brane and the $(p + 2)_5^5 - p^3_3$-brane, respectively. Here we should notice that the exotic duality from D$p$ to $p^7_3 - p^3_3$ is given as the $T(7 - p)$ST$(7 - p)_3$-dualities, while the exotic duality from D$(p + 2)$ to $(p + 2)_5^5 - p^3_3$ is the $T(5 - p)_3$ST$(5 - p)_3$-dualities, as illustrated in Fig. 1. We should notice that we cannot map the left configuration in Fig. 6 to the right one via the string dualities. The configuration in the right figure is realized only in the nongeometric background. However, since we have already understood that the supersymmetry projection rules on the D$p$-brane and the D$(p + 2)$-brane are exactly same as those of the $p^7_3 - p^3_3$-brane and the $(p + 2)_5^5 - p^3_3$-brane, we would be able to consider the existence of the right configuration in Fig. 6 in the nongeometric framework.

Similarly, we can also consider the exotic mapping of the configuration in which a solitonic brane is ending on a D$p$-brane, as in Fig. 7.

Here the integer $c$ is restricted to $c = 0, 1, 2$ in order to avoid the emergence of a domain wall. This map would also be applicable because the supersymmetry projection rule on the $S^5_2$-brane coincides with that of the $S^5_2^{2 - c}$-brane. Furthermore, we can consider the exotic mapping of the D3–F1 system in Fig. 8.

---

3 As introduced in Sect. 1, the terminology $T(k)$ indicates the T-duality along $k$ directions.
Fig. 9. Exotic mapping from the Dp–F1 system to the $p_3^{7-p} - 1_4^5$ system.

Again, the supersymmetry projection rule on the $1_4^5$-brane is equal to that of the F-string. Once we recognize the validity of the exotic mapping in Fig. 8, we can immediately apply the T-dualities to this system and obtain the configurations in Fig. 9.

Since both the F-string and the $1_4^5$-brane exist in type IIA theory as well as in type IIB theory, the exotic mapping of any integer $p$ in Fig. 9 is applicable.

We should able to consider a certain property from the two configurations in Fig. 9. In the left picture, we can read off the excitations of the Dp-brane in terms of the mode excitations of the (open) F-string ending on the Dp-brane in the small string coupling regime $g_s \to 0$. By the same analogy, we should be able to read off the excitations of the defect $p_3^{7-p}$-brane in terms of the “mode excitations” of the (open) defect $1_4^5$-brane in the strong coupling limit $g_s \to \infty$. Unfortunately, however, we have not understood any mode excitations of the $1_4^5$-brane. Thus, it seems quite difficult to evaluate the excitations of the defect $p_3^{7-p}$-branes with our current understanding.

We have studied the validity of the exotic mapping applied to the configurations of multiple nonparallel (exotic) branes. In order to prove this validity completely, we have to take care of the non-trivial monodromy structure of each brane. Since this task is beyond the scope of this work, we would like to study this issue in the framework of supergravity theories for nongeometric backgrounds [15,17–19] in future work.

4. Conclusion and discussions

In this paper, we have studied the supersymmetry projection rules on various exotic branes in type II string theories and M-theory. Following the string dualities acting on the mass formulae and the supersymmetry parameters, we obtained explicit expressions for the projection rules. By virtue of these rules, we discussed the exotic duality among the defect branes in type II theories. Furthermore, we considered the mapping from a configuration of multiple nonparallel standard branes to that of multiple nonparallel exotic branes. Applying the exotic mapping to the configurations in which a brane is ending on another brane, we could read off the situations in which (exotic) branes can be ending on (exotic) branes. Although this is analogous to the case of the standard branes, we should stress that the latter configuration cannot be obtained from the former via the string dualities. This is because the latter configuration is only realized in nongeometric backgrounds in string theory. This should be analyzed in terms of the extended versions of supergravity theories, called $\beta$-supergravity or $\gamma$-supergravity.

In this work we have not seriously considered the global structure of the spacetime modified by the nontrivial monodromy caused by the string dualities, and the back reactions originating from the strong tensions of the exotic branes. However, we can trust the supersymmetry projection rules on the exotic branes as far as they concern the supersymmetric configurations.

In the configuration in which the exotic $1_4^5$-brane is ending on the exotic $p_3^{7-p}$-brane, we would expect that the excitations on the $p_3^{7-p}$-branes could be evaluated in terms of the mode expansions...
of the exotic $T^5_4$-brane in the string coupling region $g_s \to \infty$. This is a naive analogy from the configuration that the F-string is ending on the D-brane. In order to analyze this issue, extended supergravity theories such as $\beta$-supergravity and its further extension, which contains the dualized Ramond–Ramond potentials, might be a strong framework.

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Appendix A. Computations

In this appendix we exhibit the supersymmetry projection rules on various exotic branes derived from the string dualities (1.2), (2.4), and (2.11).

A.1. SUSY projection rules on solitonic branes

Descendants from the IIA NS5-brane

○ IIA NS5(12345) $\xrightarrow{T_9} \text{IIB KK5}(12345,9)$:

\[
\begin{align*}
\text{IIA NS5(12345)} : & \quad \pm \epsilon = \Gamma^{012345} \epsilon, \\
i.e., \quad \pm \epsilon_L &= \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\text{T}_9\text{-duality} : & \quad \epsilon_R \rightarrow \Gamma^9 \epsilon_R, \\
 & \quad \pm \epsilon_R = \Gamma^9 \Gamma^{012345} \Gamma^9 \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\therefore \quad \text{IIB KK5}(12345,9) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
namely, \quad \pm \epsilon &= \Gamma^{012345} \epsilon. \quad (A1)
\end{align*}
\]

○ IIB KK5(12345,9) $\xrightarrow{T_8} \text{IIA } S^5_2(12345,89)$:

\[
\begin{align*}
\text{IIB KK5(12345,9)} : & \quad \pm \epsilon = \Gamma^{012345} \epsilon, \\
i.e., \quad \pm \epsilon_L &= \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\text{T}_8\text{-duality} : & \quad \epsilon_R \rightarrow \Gamma^8 \epsilon_R, \\
 & \quad \pm \epsilon_R = \Gamma^8 \Gamma^{012345} \Gamma^8 \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\therefore \quad \text{IIA } S^5_2(12345,89) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
namely, \quad \pm \epsilon &= \Gamma^{012345} \epsilon. \quad (A2)
\end{align*}
\]
Descendants from the IIB NS5-brane

○ IIA $5^2_2(12345, 89) \xrightarrow{T_7} \text{IIB } 5^3_2(12345, 789)$:

\[
\begin{align*}
\text{IIA } 5^2_2(12345, 89) : & \quad \pm \epsilon = \Gamma^{012345} \epsilon, \\
\text{i.e., } \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
T_7\text{-duality : } & \quad \epsilon_R \rightarrow \Gamma^7 \epsilon_R, \\
\therefore \quad \text{IIB } 5^3_2(12345, 789) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\text{namely, } & \quad \pm \epsilon = \Gamma^{012345} \epsilon. \quad (A3)
\end{align*}
\]

○ IIB $5^3_2(12345, 789) \xrightarrow{T_6} \text{IIA } 5^4_2(12345, 6789)$:

\[
\begin{align*}
\text{IIB } 5^3_2(12345, 789) : & \quad \pm \epsilon = \Gamma^{012345} \epsilon, \\
\text{i.e., } \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
T_6\text{-duality : } & \quad \epsilon_R \rightarrow \Gamma^6 \epsilon_R, \\
\therefore \quad \text{IIA } 5^4_2(12345, 6789) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \\
\text{namely, } & \quad \pm \epsilon = \Gamma^{012345} \epsilon. \quad (A4)
\end{align*}
\]

Descendants from the IIB NS5-brane

○ IIB NS5(12345) $\xrightarrow{T_9} \text{IIA KK5}(12345, 9)$:

\[
\begin{align*}
\text{IIB NS5}(12345) : & \quad \pm \epsilon = \Gamma^{012345} (\sigma_3) \epsilon, \\
\text{i.e., } \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \\
T_9\text{-duality : } & \quad \epsilon_R \rightarrow \Gamma^9 \epsilon_R, \\
\therefore \quad \text{IIA KK5}(12345, 9) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \\
\text{namely, } & \quad \pm \epsilon = \Gamma^{012345} \epsilon. \quad (A5)
\end{align*}
\]

○ IIA KK5(12345, 9) $\xrightarrow{T_8} \text{IIB } 5^2_2(12345, 89)$:

\[
\begin{align*}
\text{IIA KK5}(12345, 9) : & \quad \pm \epsilon = \Gamma^{012345} \Gamma \epsilon, \\
\text{i.e., } \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \\
T_8\text{-duality : } & \quad \epsilon_R \rightarrow \Gamma^8 \epsilon_R, \\
\therefore \quad \text{IIB } 5^2_2(12345, 89) : & \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \\
\text{namely, } & \quad \pm \epsilon = \Gamma^{012345} (\sigma_3) \epsilon. \quad (A6)
\end{align*}
\]


\(\circ\) IIB \(\mathcal{S}_2(12345, 89) \xrightarrow{T_7} \text{IIA } \mathcal{S}_2(12345, 789)\):

\[
\text{IIB } \mathcal{S}_2(12345, 89) : \quad \pm \epsilon = \Gamma^{012345}(\sigma_3)\epsilon, \\
n\text{i.e., } \pm \epsilon_L = \Gamma^{012345}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

\(T_7\)-duality:

\[
\epsilon_R \rightarrow \Gamma^7\Gamma\epsilon_R, \\
\pm \epsilon_R = -\Gamma^7\Gamma^{012345}\Gamma^7\Gamma\epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

\(\therefore\) IIA \(\mathcal{S}_2(12345, 789) : \quad \pm \epsilon_L = \Gamma^{012345}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

namely, \(\pm \epsilon = \Gamma^{012345}(\sigma_3)\epsilon\). \(\text{(A7)}\)

\(\circ\) IIA \(\mathcal{S}_2(12345, 789) \xrightarrow{T_6} \text{IIB } \mathcal{S}_2(12345, 6789)\):

\[
\text{IIA } \mathcal{S}_2(12345, 789) : \quad \pm \epsilon = \Gamma^{012345}\Gamma\epsilon, \\
n\text{i.e., } \pm \epsilon_L = \Gamma^{012345}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

\(T_6\)-duality:

\[
\epsilon_R \rightarrow \Gamma^6\Gamma\epsilon_R, \\
\pm \epsilon_R = -\Gamma^6\Gamma^{012345}\Gamma^8\Gamma\epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

\(\therefore\) IIB \(\mathcal{S}_2(12345, 6789) : \quad \pm \epsilon_L = \Gamma^{012345}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

namely, \(\pm \epsilon = \Gamma^{012345}(\sigma_3)\epsilon\). \(\text{(A8)}\)

\subsection*{A.2 SUSY projection rules on defect branes}

\textit{Descendants from the IIB D7-brane}

\(\circ\) IIB \(\mathcal{S}_7(1234567) \xrightarrow{S} \text{IIB } \mathcal{S}_3(1234567)\):

\[
\text{D7}(1234567) : \quad \pm \epsilon = \Gamma^{01234567}(i\sigma_2)\epsilon, \\
n\text{S-duality: } \epsilon \rightarrow S\epsilon,
\]

\(\therefore\) IIB \(\mathcal{S}_3(1234567) : \quad \pm \epsilon = \Gamma^{01234567}S^{-1}(i\sigma_2)S\epsilon \\
= \Gamma^{01234567}(i\sigma_2)\epsilon.
\] \(\text{(A9)}\)

\(\circ\) IIB \(\mathcal{S}_3(1234567) \xrightarrow{T_7} \text{IIA } \mathcal{S}_5^1(123456, 7)\):

\[
\text{IIB } \mathcal{S}_3(1234567) : \quad \pm \epsilon = \Gamma^{01234567}(i\sigma_2)\epsilon, \\
n\text{i.e., } \pm \epsilon_L = \Gamma^{01234567}\epsilon_R,
\]

\(T_7\)-duality:

\[
\epsilon_R \rightarrow \Gamma^7\Gamma\epsilon_R, \\
\pm \epsilon_L = \Gamma^{01234567}\Gamma^7\Gamma\epsilon_R = -\Gamma^{0123456}\epsilon_R,
\]

\(\therefore\) IIA \(\mathcal{S}_5^1(123456, 7) : \quad \pm \epsilon_L = -\Gamma^{0123456}\epsilon_R, \\
n\text{namely, } \pm \epsilon = -\Gamma^{0123456}(\sigma_1)\epsilon\).
\] \(\text{(A10)}\)

On the right-hand side, there is a negative sign that should not distract readers because this merely comes from the anticommutation relations among the Dirac gamma matrices. In later computations we often encounter the same situation.
\( \circ \) IIA \( 6^1(123456, 7) \rightarrow \text{IIA} 5^2_3(12345, 67) \):

\[
\text{IIA} 6^1(123456, 7) : \quad \pm \epsilon = -\Gamma^{0123456}(\sigma_1)\epsilon, \\
\text{i.e.,} \quad \pm \epsilon_L = -\Gamma^{0123456}\epsilon_R,
\]

\( T_6 \)-duality : \( \epsilon_R \rightarrow \Gamma^6\epsilon_R \),

\[
\pm \epsilon_L = -\Gamma^{0123456}\Gamma^6\epsilon_R = -\Gamma^{012345}\epsilon_R,
\]

\( \therefore \) IIB \( 5^2_3(12345, 67) : \quad \pm \epsilon_L = -\Gamma^{012345}(\sigma_1)\epsilon. \quad (A11)
\]

\( \circ \) IIB \( 5^2_3(12345, 67) \rightarrow \text{IIA} 4^3_3(1234, 567) \):

\[
\text{IIB} 5^2_3(12345, 67) : \quad \pm \epsilon = -\Gamma^{012345}(\sigma_1)\epsilon, \\
\text{i.e.,} \quad \pm \epsilon_L = -\Gamma^{012345}\epsilon_R,
\]

\( T_5 \)-duality : \( \epsilon_R \rightarrow \Gamma^5\epsilon_R \),

\[
\pm \epsilon_L = -\Gamma^{012345}\Gamma^5\epsilon_R = +\Gamma^{01234}\epsilon_R,
\]

\( \therefore \) IIA \( 4^3_3(1234, 567) : \quad \pm \epsilon_L = \Gamma^{01234}\epsilon_R, \quad (A12)
\]

\( \circ \) IIA \( 4^3_3(1234, 567) \rightarrow \text{IIA} 3^4_3(123, 4567) \):

\[
\text{IIA} 4^3_3(1234, 567) : \quad \pm \epsilon = \Gamma^{01234}\Gamma(\sigma_1)\epsilon, \\
\text{i.e.,} \quad \pm \epsilon_L = \Gamma^{01234}\epsilon_R,
\]

\( T_4 \)-duality : \( \epsilon_R \rightarrow \Gamma^4\epsilon_R \),

\[
\pm \epsilon_L = \Gamma^{01234}\Gamma^4\epsilon_R = +\Gamma^{0123}\epsilon_R,
\]

\( \therefore \) IIB \( 3^4_3(123, 4567) : \quad \pm \epsilon_L = \Gamma^{0123}\epsilon_R, \quad (A13)
\]

\( \circ \) IIB \( 3^4_3(123, 4567) \rightarrow \text{IIA} 2^5_3(12, 34567) \):

\[
\text{IIB} 3^4_3(123, 4567) : \quad \pm \epsilon = \Gamma^{0123}(i\sigma_2)\epsilon, \\
\text{i.e.,} \quad \pm \epsilon_L = \Gamma^{0123}\epsilon_R,
\]

\( T_3 \)-duality : \( \epsilon_R \rightarrow \Gamma^3\epsilon_R \),

\[
\pm \epsilon_L = \Gamma^{0123}\Gamma^3\epsilon_R = -\Gamma^{012}\epsilon_R,
\]

\( \therefore \) IIA \( 2^5_3(12, 34567) : \quad \pm \epsilon_L = -\Gamma^{012}\epsilon_R, \quad (A14)
\]
\[ \text{\( T_{2}\)-duality} : \quad \epsilon_{R} \to \Gamma^{2} \epsilon_{R}, \]
\[ \pm \epsilon_{L} = - \Gamma^{012} \epsilon_{R}^{2} \Gamma \epsilon_{R} = - \Gamma^{01} \epsilon_{R}, \]
\[ \therefore \quad \text{IIA } 1^{3}_{3}(1, 234567) : \quad \pm \epsilon = - \Gamma^{01} \epsilon_{R}. \]
Descendants from the IIB $1^6_4$-brane via T-duality

- IIB $1^6_4(1, 234567) \xrightarrow{T_2} \text{IIA } 1^6_4(1, 234567)$:
  \[
  \begin{align*}
  \text{IIB } 1^6_4(1, 234567) : & \quad \pm \epsilon = -\Gamma^{01}(\sigma_3)\epsilon, \\
  \text{i.e., } & \quad \pm \epsilon_L = -\Gamma^{01}(\sigma_3)\epsilon_L, \quad \pm \epsilon_R = +\Gamma^{01}(\sigma_3)\epsilon_R, \\
  \text{T}_2\text{-duality : } & \quad \epsilon_R \to \Gamma^2\epsilon_R, \\
  \pm \epsilon_R = +\Gamma^2\Gamma^{01}\Gamma^2\epsilon_R = +\Gamma^{01}\epsilon_R, \\
  \therefore \quad & \text{IIA } 1^6_4(1, 234567) : \quad \pm \epsilon_L = -\Gamma^{01}\epsilon_L, \quad \pm \epsilon_R = +\Gamma^{01}\epsilon_R, \\
  \text{namely, } & \quad \pm \epsilon = -\Gamma^{01}\epsilon. \quad (A20)
  \end{align*}
  \]

- IIB $1^6_4(1, 234567) \xrightarrow{T_1} \text{IIA } 0^{(1,6)}_4(1, 234567, 1)$:
  \[
  \begin{align*}
  \text{IIB } 1^6_4(1, 234567) : & \quad \pm \epsilon = -\Gamma^{01}(\sigma_3)\epsilon, \\
  \text{i.e., } & \quad \pm \epsilon_L = -\Gamma^{01}\epsilon_L, \quad \pm \epsilon_R = +\Gamma^{01}\epsilon_R, \\
  \text{T}_1\text{-duality : } & \quad \epsilon_R \to \Gamma^1\epsilon_R, \\
  \pm \epsilon_R = +\Gamma^1\Gamma^{01}\Gamma^1\epsilon_R = -\Gamma^{01}\epsilon_R, \\
  \therefore \quad & \text{IIA } 0^{(1,6)}_4(1, 234567, 1) : \quad \pm \epsilon_L = -\Gamma^{01}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{01}\epsilon_R, \\
  \text{namely, } & \quad \pm \epsilon = -\Gamma^{01}\epsilon. \quad (A21)
  \end{align*}
  \]

- IIA $1^6_4(1, 234567) \xrightarrow{T_1} \text{IIB } 0^{(1,6)}_4(1, 234567, 1)$:
  \[
  \begin{align*}
  \text{IIA } 1^6_4(1, 234567) : & \quad \pm \epsilon = -\Gamma^{01}\epsilon, \\
  \text{i.e., } & \quad \pm \epsilon_L = -\Gamma^{01}\epsilon_L, \quad \pm \epsilon_R = +\Gamma^{01}\epsilon_R, \\
  \text{T}_1\text{-duality : } & \quad \epsilon_R \to \Gamma^1\epsilon_R, \\
  \pm \epsilon_R = +\Gamma^1\Gamma^{01}\Gamma^1\epsilon_R = -\Gamma^{01}\epsilon_R, \\
  \therefore \quad & \text{IIB } 0^{(1,6)}_4(1, 234567, 1) : \quad \pm \epsilon_L = -\Gamma^{01}\epsilon_L, \quad \pm \epsilon_R = -\Gamma^{01}\epsilon_R, \\
  \text{namely, } & \quad \pm \epsilon = -\Gamma^{01}\epsilon. \quad (A22)
  \end{align*}
  \]

A.3. SUSY projection rules on domain walls

Domain walls from defect branes via T-duality

- IIB $7_3(1234567) \xrightarrow{T_9} \text{IIA } 7^{(1,0)}_3(1234567), 9$:
  \[
  \begin{align*}
  \text{IIB } 7_3(1234567) : & \quad \pm \epsilon = \Gamma^{01234567}(i\sigma_2)\epsilon, \\
  \text{i.e., } & \quad \pm \epsilon_L = \Gamma^{01234567}\epsilon_L, \\
  \text{T}_9\text{-duality : } & \quad \epsilon_R \to \Gamma^9\epsilon_R, \\
  \pm \epsilon_R = \Gamma^{01234567}\Gamma^9\epsilon_R = -\Gamma^{012345679}\epsilon_R, \\
  \therefore \quad & \text{IIA } 7^{(1,0)}_3(1234567), 9) : \quad \pm \epsilon_L = -\Gamma^{012345679}\epsilon_L, \\
  \text{namely, } & \quad \pm \epsilon = -\Gamma^{012345679}\Gamma(\sigma_1)\epsilon_R. \quad (A23)
  \end{align*}
  \]
\begin{itemize}
  \item IIA $6_3^1(123456, 7) \xrightarrow{T_9} \text{IIB } 6_3^{(1,1)}(123456, 7, 9)$:
    \begin{align*}
      \text{IIA } 6_3^1(123456, 7): & \quad \pm \epsilon = -\Gamma^{0123456}(\sigma_1)\epsilon, \\
      \text{i.e., } & \quad \pm \epsilon_L = -\Gamma^{0123456}\epsilon_R, \\
      \text{T}_9\text{-duality: } & \quad \epsilon_R \rightarrow \Gamma^9\epsilon_R, \\
      & \quad \pm \epsilon_L = -\Gamma^{0123456}\Gamma^9\epsilon_R = -\Gamma^{01234569}\epsilon_R, \\
    \end{align*}
    \therefore \quad \text{IIB } 6_3^{(1,1)}(123456, 7, 9): \quad \pm \epsilon_L = -\Gamma^{01234569}\epsilon_R, \\
    \text{namely, } \quad \pm \epsilon = -\Gamma^{01234569}(i\sigma_2)\epsilon. \tag{A24}
  \\
  \item IIB $5_3^2(12345, 67) \xrightarrow{T_9} \text{IIA } 5_3^{(1,2)}(12345, 67, 9)$:
    \begin{align*}
      \text{IIB } 5_3^2(12345, 67): & \quad \pm \epsilon = -\Gamma^{012345}(\sigma_1)\epsilon, \\
      \text{i.e., } & \quad \pm \epsilon_L = -\Gamma^{012345}\epsilon_R, \\
      \text{T}_9\text{-duality: } & \quad \epsilon_R \rightarrow \Gamma^9\epsilon_R, \\
      & \quad \pm \epsilon_L = -\Gamma^{012345}\Gamma^9\epsilon_R = +\Gamma^{0123459}\epsilon_R, \\
    \end{align*}
    \therefore \quad \text{IIA } 5_3^{(1,2)}(12345, 67, 9): \quad \pm \epsilon_L = \Gamma^{0123459}\epsilon_R, \\
    \text{namely, } \quad \pm \epsilon = \Gamma^{0123459}(\sigma_1)\epsilon. \tag{A25}
  \\
  \item IIA $4_3^3(1234, 567) \xrightarrow{T_9} \text{IIB } 4_3^{(1,3)}(1234, 567, 9)$:
    \begin{align*}
      \text{IIA } 4_3^3(1234, 567): & \quad \pm \epsilon = \Gamma^{01234}(i\sigma_2)\epsilon, \\
      \text{i.e., } & \quad \pm \epsilon_L = \Gamma^{01234}\epsilon_R, \\
      \text{T}_9\text{-duality: } & \quad \epsilon_R \rightarrow \Gamma^9\epsilon_R, \\
      & \quad \pm \epsilon_L = \Gamma^{01234}\Gamma^9\epsilon_R = \pm\Gamma^{012349}\epsilon_R, \\
    \end{align*}
    \therefore \quad \text{IIB } 4_3^{(1,3)}(1234, 567, 9): \quad \pm \epsilon_L = \Gamma^{012349}\epsilon_R, \\
    \text{namely, } \quad \pm \epsilon = \Gamma^{012349}(\sigma_1)\epsilon. \tag{A26}
  \\
  \item IIB $3_3^4(123, 4567) \xrightarrow{T_9} \text{IIA } 3_3^{(1,4)}(123, 4567, 9)$:
    \begin{align*}
      \text{IIB } 3_3^4(123, 4567): & \quad \pm \epsilon = \Gamma^{0123}(i\sigma_2)\epsilon, \\
      \text{i.e., } & \quad \pm \epsilon_L = \Gamma^{0123}\epsilon_R, \\
      \text{T}_9\text{-duality: } & \quad \epsilon_R \rightarrow \Gamma^9\epsilon_R, \\
      & \quad \pm \epsilon_L = \Gamma^{0123}\Gamma^9\epsilon_R = -\Gamma^{01239}\epsilon_R, \\
    \end{align*}
    \therefore \quad \text{IIA } 3_3^{(1,4)}(123, 4567, 9): \quad \pm \epsilon_L = -\Gamma^{01239}\epsilon_R, \\
    \text{namely, } \quad \pm \epsilon = -\Gamma^{01239}(\sigma_1)\epsilon. \tag{A27}
\end{itemize}
\[ \text{IIA}_2^{5}(12, 34567) \xrightarrow{T_9} \text{IIB}_3^{(1.5)}(12, 34567, 9) : \]

\[ \pm \epsilon = -\Gamma^{012}(\sigma_1)\epsilon, \]

i.e., \[ \pm \epsilon_L = -\Gamma^{012}\epsilon_R, \]

\[ \text{T}_9\text{-duality : } \epsilon_R \rightarrow \Gamma^9\epsilon_R, \]

\[ \pm \epsilon_L = -\Gamma^{012}\Gamma^9\epsilon_R = -\Gamma^{0129}\epsilon_R, \]

\[ \therefore \text{IIB}_2^{3(1.5)}(12, 34567, 9) : \]

\[ \pm \epsilon_L = -\Gamma^{0129}\epsilon_R, \]

namely, \[ \pm \epsilon = -\Gamma^{0129}(i\sigma_2)\epsilon. \]

\[ (A28) \]

\[ \text{IIA}_2^{16}(1, 234567) \xrightarrow{T_9} \text{IIB}_3^{(1.6)} (1, 234567, 9) : \]

\[ \pm \epsilon = -\Gamma^{01}(\sigma_1)\epsilon, \]

i.e., \[ \pm \epsilon_L = -\Gamma^{01}\epsilon_R, \]

\[ \text{T}_9\text{-duality: } \epsilon_R \rightarrow \Gamma^9\epsilon_R, \]

\[ \pm \epsilon_L = -\Gamma^{01}\Gamma^9\epsilon_R = +\Gamma^{019}\epsilon_R, \]

\[ \therefore \text{IIA}_1^{(1.6)} (1, 234567, 9) : \]

\[ \pm \epsilon_L = +\Gamma^{019}\epsilon_R, \]

namely, \[ \pm \epsilon = +\Gamma^{019}(\sigma_1)\epsilon. \]

\[ (A29) \]

\[ \text{IIA}_0^{7}(1, 1234567) \xrightarrow{T_9} \text{IIB}_3^{(1.7)} (1, 1234567, 9) : \]

\[ \pm \epsilon = \Gamma^0\Gamma(\sigma_1)\epsilon, \]

i.e., \[ \pm \epsilon_L = \Gamma^0\epsilon_R, \]

\[ \text{T}_9\text{-duality: } \epsilon_R \rightarrow \Gamma^9\epsilon_R, \]

\[ \pm \epsilon_L = \Gamma^0\Gamma^9\epsilon_R = +\Gamma^{09}\epsilon_R, \]

\[ \therefore \text{IIB}_0^{3(1.7)} (1, 1234567, 9) : \]

\[ \pm \epsilon_L = +\Gamma^{09}\epsilon_R, \]

namely, \[ \pm \epsilon = +\Gamma^{09}(\sigma_1)\epsilon. \]

\[ (A30) \]

**Descendants from the IIB \(5_2^3\)-brane**

\[ \text{IIB}_2^{5}(12345, 789) \xrightarrow{S} \text{IIB}_3^{5}(12345, 789) : \]

\[ \pm \epsilon = \Gamma^{012345}\epsilon, \]

\[ \text{S-duality: } \epsilon \rightarrow S\epsilon, \]

\[ \therefore \text{IIB}_2^{5}(12345, 789) : \]

\[ \pm \epsilon = \Gamma^{012345}S^{-1}S\epsilon = \Gamma^{012345}\epsilon. \]

\[ (A31) \]
\[ \text{IIB } 5^3_{4}(12345, 789) \xrightarrow{T_5} \text{IIA } 4^{(1,3)}_{4}(1234, 789, 5): \]

\[ \pm \epsilon = \Gamma^{012345} \epsilon, \]

i.e., \[ \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \]

\[ T_5\text{-duality:} \quad \epsilon_L, \quad \epsilon_R \rightarrow \Gamma^5 \Gamma \epsilon, \]

\[ \pm \epsilon_R = \Gamma \Gamma^5 \Gamma^{012345} \Gamma^5 \Gamma \epsilon_R = -\Gamma^{012345} \epsilon_R. \]

\[ \therefore \quad \text{IIA } 4^{(1,3)}_{4}(1234, 789, 5): \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \]

namely, \[ \pm \epsilon = \Gamma^{012345} \Gamma \epsilon. \quad (A32) \]

\[ \text{IIA } 4^{(1,3)}_{4}(1234, 789, 5) \xrightarrow{T_4} \text{IIB } 3^{(2,3)}_{4}(123, 789, 45): \]

\[ \pm \epsilon = \Gamma^{012345} \Gamma \epsilon, \]

i.e., \[ \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \]

\[ T_4\text{-duality:} \quad \epsilon_L, \quad \epsilon_R \rightarrow \Gamma^4 \Gamma \epsilon, \]

\[ \pm \epsilon_R = -\Gamma \Gamma^4 \Gamma^{012345} \Gamma^4 \Gamma \epsilon_R = \Gamma^{012345} \epsilon_R, \]

\[ \therefore \quad \text{IIB } 3^{(2,3)}_{4}(123, 789, 45): \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \]

namely, \[ \pm \epsilon = \Gamma^{012345} \epsilon. \quad (A33) \]

\[ \text{IIB } 3^{(2,3)}_{4}(123, 789, 45) \xrightarrow{T_3} \text{IIA } 2^{(3,3)}_{4}(12, 789, 345): \]

\[ \pm \epsilon = \Gamma^{012345} \epsilon, \]

i.e., \[ \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \]

\[ T_3\text{-duality:} \quad \epsilon_L, \quad \epsilon_R \rightarrow \Gamma^3 \Gamma \epsilon, \]

\[ \pm \epsilon_R = \Gamma \Gamma^3 \Gamma^{012345} \Gamma^3 \Gamma \epsilon_R = -\Gamma^{012345} \epsilon_R, \]

\[ \therefore \quad \text{IIA } 2^{(3,3)}_{4}(12, 789, 345): \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = -\Gamma^{012345} \epsilon_R, \]

namely, \[ \pm \epsilon = \Gamma^{012345} \Gamma \epsilon. \quad (A34) \]

\[ \text{IIA } 2^{(3,3)}_{4}(12, 789, 345) \xrightarrow{T_2} \text{IIB } 1^{(4,3)}_{4}(1, 789, 2345): \]

\[ \pm \epsilon = \Gamma^{012345} \Gamma \epsilon, \]

i.e., \[ \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \]

\[ T_2\text{-duality:} \quad \epsilon_L, \quad \epsilon_R \rightarrow \Gamma^2 \Gamma \epsilon, \]

\[ \pm \epsilon_R = -\Gamma \Gamma^2 \Gamma^{012345} \Gamma^2 \Gamma \epsilon_R = \Gamma^{012345} \epsilon_R, \]

\[ \therefore \quad \text{IIB } 1^{(4,3)}_{4}(1, 789, 2345): \quad \pm \epsilon_L = \Gamma^{012345} \epsilon_L, \quad \pm \epsilon_R = \Gamma^{012345} \epsilon_R, \]

namely, \[ \pm \epsilon = \Gamma^{012345} \epsilon. \quad (A35) \]
○ IIB $1_{4}^{(4,3)}(1, 789, 2345) \xrightarrow{T_{1}}$ IIA $0_{4}^{(5,3)}(789, 12345)$:

\[
\text{IIB } 1_{4}^{(4,3)}(1, 789, 2345) : \quad \pm \epsilon = \Gamma^{012345} \epsilon, \\
\text{i.e., } \pm \epsilon_{L} = \Gamma^{012345} \epsilon_{L}, \quad \pm \epsilon_{R} = \Gamma^{012345} \epsilon_{R}, \\
T_{1}\text{-duality} : \quad \epsilon_{R} \rightarrow \Gamma^{1} \Gamma \epsilon_{R}, \\
\pm \epsilon_{R} = \Gamma^{1} \Gamma^{012345} \Gamma^{1} \Gamma \epsilon_{R} = -\Gamma^{012345} \epsilon_{R}, \\
\therefore \quad \text{IIA } 0_{4}^{(5,3)}(789, 12345) : \quad \pm \epsilon_{L} = \Gamma^{012345} \epsilon_{L}, \quad \pm \epsilon_{R} = -\Gamma^{012345} \epsilon_{R}, \\
\text{namely, } \pm \epsilon = \Gamma^{012345} \Gamma \epsilon. \quad (A36)
\]

Descendants from the IIB $4_{3}^{(1,3)}$-brane

○ IIB $4_{3}^{(1,3)}(1234, 567, 9) \xrightarrow{S} \text{IIB } 4_{4}^{(1,3)}(1234, 567, 9)$:

\[
\text{IIB } 4_{3}^{(1,3)}(1234, 567, 9) : \quad \pm \epsilon = \Gamma^{012349} (\sigma_{1}) \epsilon, \\
S\text{-duality} : \quad \epsilon \rightarrow S \epsilon, \\
\therefore \quad \text{IIB } 4_{4}^{(1,3)}(1234, 567, 9) : \quad \pm \epsilon = \Gamma^{012349} S^{-1} (\sigma_{1}) S \epsilon \\
\quad = \Gamma^{012349} (\sigma_{3}) \epsilon. \quad (A37)
\]

○ IIB $4_{4}^{(1,3)}(1234, 567, 9) \xrightarrow{T_{9}}$ IIA $5_{4}^{3}(12349, 567)$:

\[
\text{IIB } 4_{4}^{(1,3)}(1234, 567, 9) : \quad \pm \epsilon = \Gamma^{012349} (\sigma_{3}) \epsilon, \\
\text{i.e., } \pm \epsilon_{L} = \Gamma^{012349} \epsilon_{L}, \quad \pm \epsilon_{R} = -\Gamma^{012349} \epsilon_{R}, \\
T_{9}\text{-duality} : \quad \epsilon_{R} \rightarrow \Gamma^{9} \Gamma \epsilon_{R}, \\
\pm \epsilon_{R} = -\Gamma^{9} \Gamma^{012349} \Gamma^{9} \Gamma \epsilon_{R} = \Gamma^{012349} \epsilon_{R}, \\
\therefore \quad \text{IIA } 5_{4}^{3}(12349, 567) : \quad \pm \epsilon_{L} = \Gamma^{012349} \epsilon_{L}, \quad \pm \epsilon_{R} = \Gamma^{012349} \epsilon_{R}, \\
\text{namely, } \pm \epsilon = \Gamma^{012349} \epsilon. \quad (A38)
\]

○ IIB $4_{4}^{(1,3)}(1234, 567, 9) \xrightarrow{T_{4}}$ IIA $3_{4}^{2,3}(123, 567, 49)$:

\[
\text{IIB } 4_{4}^{(1,3)}(1234, 567, 9) : \quad \pm \epsilon = \Gamma^{012349} (\sigma_{3}) \epsilon, \\
\text{i.e., } \pm \epsilon_{L} = \Gamma^{012349} \epsilon_{L}, \quad \pm \epsilon_{R} = -\Gamma^{012349} \epsilon_{R}, \\
T_{4}\text{-duality} : \quad \epsilon_{R} \rightarrow \Gamma^{4} \Gamma \epsilon_{R}, \\
\pm \epsilon_{R} = -\Gamma \Gamma^{4} \Gamma^{012349} \Gamma^{4} \Gamma \epsilon_{R} = \Gamma^{012349} \epsilon_{R}, \\
\therefore \quad \text{IIA } 3_{4}^{2,3}(123, 567, 49) : \quad \pm \epsilon_{L} = \Gamma^{012349} \epsilon_{L}, \quad \pm \epsilon_{R} = \Gamma^{012349} \epsilon_{R}, \\
\text{namely, } \pm \epsilon = \Gamma^{012349} \epsilon. \quad (A39)
A.4. Uplifting to M-theory

Here we uplift type IIA branes to those of M-theory. The mass formulae of type IIA branes are rewritten as those of M-theory branes via the relation \((2.11)\).

Uplifting IIA solitonic five-branes

- IIA NS5(12345) \(\xrightarrow{\text{uplift}}\) M5(12345):

\[
M_{\text{NS5}} = \frac{R_1 R_2 R_3 R_4 R_5}{g_s^2 \ell_6^6}, \quad \pm \epsilon = \Gamma^{012345} \epsilon,
\]

\[
\rightarrow M_{\text{MS5}} = \frac{R_1 R_2 R_3 R_4 R_5}{\ell_6^6}, \quad \pm \eta = \Gamma^{012345} \eta. \quad (A43)
\]
We also uplift the defect branes in type IIA theory (2.7).

- IIA KK5(12345,9) $\xrightarrow{\text{uplift}}$ KK6(123456,9):
  \[
  M_{KK5} = \frac{R_1 R_2 R_3 R_4 R_5 (R_9)^2}{g_s^2 \ell_s^5}, \quad \pm \epsilon = \Gamma^{012345} \epsilon,
  \]
  \[
  M_{KK6} = \frac{R_1 R_2 R_3 R_4 R_5 R_7 (R_9)^2}{\ell_p^9}, \quad \pm \eta = \Gamma^{0123456} \eta. \tag{A44}
  \]

- IIA $5_2^2(12345,89)$ $\xrightarrow{\text{uplift}}$ $5^3(12345,89\bar{5})$:
  \[
  M_{5_2^2} = \frac{R_1 R_2 R_3 R_4 R_5 (R_8 R_9)^2}{g_s^2 \ell_s^5 10}, \quad \pm \epsilon = \Gamma^{012345} \epsilon,
  \]
  \[
  M_{5^3} = \frac{R_1 R_2 R_3 R_4 R_5 (R_8 R_9 R_7)^2}{\ell_p^{12}}, \quad \pm \eta = \Gamma^{0123456} \eta. \tag{A45}
  \]

- IIA $5_2^3(12345,789)$ $\xrightarrow{\text{uplift}}$ $5^{(1,3)}(12345,789,\bar{5})$:
  \[
  M_{5_2^3} = \frac{R_1 R_2 R_3 R_4 R_5 (R_7 R_8 R_9)^2}{g_s^2 \ell_s^5}, \quad \pm \epsilon = \Gamma^{012345} \Gamma \epsilon,
  \]
  \[
  M_{5^{(1,3)}} = \frac{R_1 R_2 R_3 R_4 R_5 (R_7 R_8 R_9)^2 (R_5)^3}{\ell_p^{15}}, \quad \pm \eta = \Gamma^{0123456} \eta. \tag{A46}
  \]

**Uplifting IIA defect branes**

We also uplift the defect branes in type IIA theory (2.7).

- $1_4^6(1,234567)$ $\xrightarrow{\text{uplift}}$ $2^6(1\bar{5},234567)$:
  \[
  M_{1_4^6} = \frac{R_1 R_2 R_3 R_4 R_5 R_6 R_7^2}{g_s^4 \ell_s^{14}}, \quad \pm \epsilon = \Gamma^{01} \Gamma \epsilon,
  \]
  \[
  M_{2^6} = \frac{R_1 R_2 R_3 R_4 R_5 R_6 R_7^2}{\ell_p^{15}}, \quad \pm \eta = \Gamma^{01} \eta. \tag{A47}
  \]

- $0_4^{(1,6)}(234567,1)$ $\xrightarrow{\text{uplift}}$ $0^{(1,7)}(234567\bar{5},1)$:
  \[
  M_{0_4^{(1,6)}} = \frac{(R_2 R_3 R_4 R_5 R_6 R_7)^2 (R_1)^3}{g_s^2 \ell_s^{16}}, \quad \pm \epsilon = \Gamma^{01} \epsilon,
  \]
  \[
  M_{0^{(1,7)}} = \frac{(R_2 R_3 R_4 R_5 R_6 R_7)^2 (R_1)^3}{\ell_p^{18}}, \quad \pm \eta = \Gamma^{01} \eta. \tag{A48}
  \]

Other defect branes in (2.7) are uplifted to some of the (exotic) branes in M-theory that have already appeared.

**References**

[1] M. Blau and M. O'Loughlin, Nucl. Phys. B 525, 182 (1998) [arXiv:hep-th/9712047] [Search INSPIRE].
[2] N. A. Obers and B. Pioline, Phys. Rept. 318, 113 (1999) [arXiv:hep-th/9809039] [Search INSPIRE].
[3] E. Eyras and Y. Lozano, Nucl. Phys. B 573, 735 (2000) [arXiv:hep-th/9908094] [Search INSPIRE].
[4] E. Lozano-Tellechea and T. Ortín, Nucl. Phys. B 607, 213 (2001) [arXiv:hep-th/0012051] [Search INSPIRE].
[5] J. de Boer and M. Shigemori, Phys. Rev. Lett. 104, 251603 (2010) [arXiv:1004.2521 [hep-th]] [Search inSPIRE].

[6] A. Giveon and D. Kutasov, Rev. Mod. Phys. 71, 983 (1999) [arXiv:hep-th/9802067] [Search inSPIRE].

[7] B. R. Greene, A. D. Shapere, C. Vafa, and S. T. Yau, Nucl. Phys. B 337, 1 (1990).

[8] E. A. Bergshoeff, J. Hartong, T. Ortín, and D. Roest, J. High Energy Phys. 0702, 003 (2007) [arXiv:hep-th/0612072] [Search inSPIRE].

[9] T. Kikuchi, T. Okada, and Y. Sakatani, Phys. Rev. D 86, 046001 (2012) [arXiv:1205.5549 [hep-th]] [Search inSPIRE].

[10] J. de Boer and M. Shigemori, Phys. Rept. 532, 65 (2013) [arXiv:1209.6056 [hep-th]] [Search inSPIRE].

[11] T. Kimura, Nucl. Phys. B 893, 1 (2015) [arXiv:1410.8403 [hep-th]] [Search inSPIRE].

[12] T. Kimura, S. Sasaki, and M. Yata, J. High Energy Phys. 1503, 076 (2015) [arXiv:1411.3457 [hep-th]] [Search inSPIRE].

[13] T. Okada and Y. Sakatani, J. High Energy Phys. 1503, 131 (2015) [arXiv:1411.1043 [hep-th]] [Search inSPIRE].

[14] E. A. Bergshoeff, T. Ortín, and F. Riccioni, Nucl. Phys. B 856, 210 (2012) [arXiv:1109.4484 [hep-th]] [Search inSPIRE].

[15] Y. Sakatani, J. High Energy Phys. 1503, 135 (2015) [arXiv:1412.8769 [hep-th]] [Search inSPIRE].

[16] A. Kleinschmidt, J. High Energy Phys. 1110, 144 (2011) [arXiv:1109.2025 [hep-th]] [Search inSPIRE].

[17] D. Andriot and A. Betz, J. High Energy Phys. 1312, 083 (2013) [arXiv:1306.4381 [hep-th]] [Search inSPIRE].

[18] D. Andriot and A. Betz, J. High Energy Phys. 1407, 059 (2014) [arXiv:1402.5972 [hep-th]] [Search inSPIRE].

[19] C. D. A. Blair and E. Malek, J. High Energy Phys. 1503, 144 (2015) [arXiv:1412.0635 [hep-th]] [Search inSPIRE].

[20] Y. M. Cho and S. Nam, Dualities in Gauge and String Theories: Proceedings (World Scientific, Singapore, 1998), p. 146, [arXiv:hep-th/9705011] [Search inSPIRE].

[21] J. H. Schwarz, Nucl. Phys. B 226, 269 (1983).

[22] Y. Imamura, Soryushiron Kenkyu 96, 187 (1998) [in Japanese].

[23] E. A. Bergshoeff, A. Kleinschmidt, and F. Riccioni, Phys. Rev. D 86, 085043 (2012) [arXiv:1206.5697 [hep-th]] [Search inSPIRE].