Configuration-picture description of quantum-dot carrier scattering for the interaction with LO phonons

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Abstract.
A theory of scattering processes between the electronic configurations of a single quantum dot (QD) in interaction with LO phonons is presented. Polaronic renormalizations of the QD carriers are taken into account using a Green’s function approach and play an essential role in the calculation of the scattering rates. Moreover, the rates are configuration-dependent, so that additional carriers in the QD, that do not take part in a transition, influence its efficiency. The QD carrier correlations are fully included by using a von Neumann-Lindblad formalism in the configuration basis. Numerical results for scattering rates and QD carrier dynamics are shown to illustrate the theory.

1. Introduction
Semiconductor quantum dots (QDs) provide a three-dimensional carrier confinement with atom-like discrete density of states. In the weak excitation regime, optical properties are determined by multi-exciton states. Single QD spectroscopy allows to follow the temporal dynamics of excitation and cascaded emission from various multi-exciton configurations [1]. This leads to the question to what extent the QD carrier kinetics can be described in terms of one-particle occupation probabilities, that are averaged over the QD ensemble and/or over repeated measurements, or whether the system needs to be analyzed in terms of many-body configurations. Related to this is the importance of carrier correlations and their inclusion in the theoretical models.

Due to the finite number of confined carriers in a QD, we are able to address the many-body dynamics of the QD carriers in terms of the von Neumann equation. This provides a description of carrier scattering processes in which the QD carriers enter via multi-exciton configurations. For Coulomb scattering processes, the QD carriers can be coupled to carriers in delocalized states via Lindblad terms in the Born-Markov
Figure 1: Schematic drawing of confined energy levels in the quantum-dot (QD) on wetting-layer (WL) system. The quasi-continuum of WL states (grey area) has larger interband transition energies than the discrete QD states for holes ($|1\rangle$, $|2\rangle$) and electrons ($|3\rangle$, $|4\rangle$).

approximation [2]. In this paper, we consider carrier scattering due to interaction with LO phonons, which cannot be treated in the Born approximation. We overcome this problem by including polaronic quasiparticle properties of the QD carrier configurations, which are obtained from solutions for the retarded Green’s functions.

Numerical results for the thus obtained rates of LO-phonon-assisted processes and for the dynamics of the QD system due to these processes, as obtained from the von Neumann-Lindblad (vNL) equation, are presented.

2. Model system
We consider a QD-WL system in the limit of a small QD density with the QD containing a confined ground state (GS) and first excited state (ES) in each band. To simplify the theoretical description, we assume that any excitation of carriers in the WL or the QD is spin-polarized and that spin-flip processes can be neglected on the time scales discussed in this paper. Hence only one spin-subsystem will be considered. Moreover, we assume that further degeneracies of the excited state are sufficiently lifted due to an asymmetric QD shape, which for instance results from strain effects. Thus, we end up with the simplest model system that contains all features we would like to illustrate, including two confined states for holes ($|1\rangle$, $|2\rangle$) and electrons ($|3\rangle$, $|4\rangle$), as shown in Fig. 1. The theory presented in the next section, however, is more general and allows for an arbitrary number of single-particle states in the QD.
3. Theory
3.1. Configuration picture
Carrier kinetics is concerned with the time evolution of the populations \( f_i \) of QD single-particle states, given by the expectation values of the number operators \( \hat{n}_i \). However, the populations carry only a partial information about the state of the QD, whose full description is provided by its density operator \( \rho(t) \), acting in the many-particle Hilbert space spanned by all configurations \( |I\rangle = |n_1n_2n_3n_4\rangle, \ n_i \in \{0,1\} \). The configurations contain information about the simultaneous occupancies of each single-particle state in the QD. In the process of scattering of a QD carrier, the presence and distribution of “spectator” carriers, which do not take part in the transition, influence the scattering event through the Pauli principle, which limits the availability of unoccupied states. This way the exclusion principle induces “Pauli correlations” which modify the carrier dynamics. Additional correlations are generated by the Coulomb interaction between the QD carriers. All these effects are taken into account exactly in the configuration basis of the QD.

On the other hand, a description only in terms of single-particle expectation values, e.g. by a Boltzmann equation for \( f_i \), represents an approximate treatment, as every carrier in the QD reacts only to an independently averaged “mean-field” distribution of collision partners. As we have discussed in Ref. [2], this corresponds to a Hartree-Fock-like factorization of many-particle expectation values in the equations of motion for the QD single-particle state occupancies, which amounts to neglecting certain correlations between the QD carriers. We also showed that, as expected, there are situations where the QD as a system with a small number of single-particle state populations can not be described properly in the single-particle picture. For these reasons, in what follows we address the QD-carrier dynamics using the configuration picture.

3.2. System-reservoir interaction
The QD system is described by the system Hamiltonian

\[
H_S = H_0 + H_{\text{Coul}},
\]

consisting of a “free” part

\[
H_0 = \sum_i \hat{n}_i \varepsilon_i^0,
\]

with the confinement energies \( \varepsilon_i^0 \), and an interacting part

\[
H_{\text{Coul}}^{\text{QD}} = \frac{1}{2} \sum_{ijkl} V_{ijkl} a_i^\dagger a_j^\dagger a_k a_l,
\]

which accounts for Coulomb interaction between the QD carriers and leads to shifts of the configuration energies as well as configuration mixing. The picture is simplified by noticing [3] that the most important terms in \( H_{\text{Coul}}^{\text{QD}} \) are those which are diagonal in the configuration basis. In other words, the dominant Coulomb effects correspond to (direct
and exchange) energy renormalizations of the configurations, while configuration mixing is negligible.

\[ H_{\text{Coul}}^{\text{QD}} \approx H_{\text{Coul}}^{\text{diag}}, \quad \left( H_0 + H_{\text{Coul}}^{\text{diag}} \right) |I\rangle = \varepsilon_I |I\rangle . \] (4)

As an open system, the QD interacts with carriers in the WL and with phonons. Carriers can be captured from the WL into the QD or released back with certain rates. Thus, the WL is part of the QD environment as a fermionic bath with a given temperature and chemical potential. Similarly, the phonon system constitutes a bosonic energy reservoir, taken at the same temperature. While in Ref. [2] we considered the system dynamics only due to Coulomb interaction with the WL reservoir, we will focus here on carrier-phonon interaction.

To obtain the evolution of the reduced density matrix \( \rho(t) \) of the QD system (S) in contact with the reservoirs (R), we consider the evolution generated by the total Hamiltonian

\[ H = H_S + H_R + H_{SR} \] (5)

in the Born-Markov limit [4]. \( H_R \) is the reservoir Hamiltonian, describing the quasi-free WL carriers and phonons with energies \( \varepsilon_k \) and \( \hbar \omega_q \), respectively:

\[ H_R = \sum_k \varepsilon_k a_k^\dagger a_k + \sum_q \hbar \omega_q b_q^\dagger b_q . \] (6)

In polar semiconductors the dominant electron-phonon interaction is provided by the LO phonons. Therefore we limit our discussion to carrier-LO-phonon scattering, and accordingly we assume, as it is customary, a dispersionless spectrum \( \omega_q = \omega_{LO} \). The interaction between system and reservoir is described by the Hamiltonian

\[ H_{SR} = \sum_{i,j,q} g_{ij}^q a_i^\dagger a_j (b_q + b_q^\dagger) , \] (7)

with the matrix elements \( g_{ij}^q \) as given by the Fröhlich coupling [5].

Here we have to distinguish between two cases. First, when both electronic states \( i, j \) correspond to QD states, the term describes the \( |j\rangle \rightarrow |i\rangle \) scattering inside the QD assisted by the emission or absorption of a phonon. This is a redistribution of carrier population, like e.g. the relaxation of an electron from the excited state into the ground state of the QD. The second case involves a QD state and a WL one, and corresponds to the capture from or reemission into the WL of a carrier, again assisted by phonons.

Expressing the interaction, Eq. (7), into the configuration basis is straightforward but requires some care. We begin by discussing the first case. The operator \( a_i^\dagger a_j \) produces transitions between QD-configuration differing only by the occupancies of states \( i \) and \( j \). Such transitions are of the form \( |J\rangle \rightarrow |I\rangle \), in which \( |J\rangle = a_j^\dagger |\Phi\rangle \) and \( |I\rangle = a_i^\dagger |\Phi\rangle \), with \( |\Phi\rangle \) a reference state which specifies the “spectator” carriers and in which, of course, the states \( i, j \) are left empty. Such a process is present in \( H_{SR} \) by a term \( s_{IJ} \Gamma_{IJ} \).
where \( s_{IJ} = |I\rangle \langle J| \) represents the system and \( \Gamma_{IJ} = \sum_q g_{i,j}^q (b_q + b_{-q}^\dagger) \) represents the reservoir. A similar treatment holds for the terms in which the number of QD carriers is not conserved. For instance, in the case of a carrier capture \( |I\rangle = a_i^\dagger |\Phi\rangle \), as before, but now \( |J\rangle = |\Phi\rangle \) and the annihilation operator belongs to the reservoir factor \( \Gamma_{IJ} = \sum_{q,k} g_{i,j}^{q,k} a_k (b_q + b_{-q}^\dagger) \).

Adding up all the terms leads to the Hamiltonian

\[
H_{SR} = \sum_{IJ} s_{IJ} \Gamma_{IJ},
\]

(8)

whose Born-Markov treatment can proceed in the standard way [4]. The result is a vNL equation of the form

\[
\frac{\partial}{\partial t} \rho = -\frac{i}{\hbar} [H_S, \rho] + \sum_{I,J} \gamma_{IJ} \frac{2}{2} \left[ 2s_{IJ}^\dagger \rho s_{IJ} - s_{IJ}^\dagger s_{IJ} \rho - \rho s_{IJ}^\dagger s_{IJ} \right].
\]

(9)

The commutator part represents the quantum-mechanical evolution of the system, whereas the second part describes, by means of the so-called Lindblad terms, the irreversible, dissipative kinetics due to the contact with the reservoir. Our task is to calculate the \( |J\rangle \rightarrow |I\rangle \) transition rates

\[
\gamma_{IJ} = \frac{2}{\hbar^2} \text{Re} \int_{-\infty}^t dt' e^{i\omega_{IJ}(t-t')} \langle \Gamma_{IJ}(t) \Gamma_{IJ}(t') \rangle_R,
\]

(10)

where \( \hbar \omega_{IJ} = \hbar (\omega_J - \omega_I) \) is the difference of the corresponding configuration energies, and the time dependence of the operators corresponds to the interaction picture with respect to \( H_S + H_R \).

As an example consider again the case in which both \( i \) and \( j \) are QD states. Then Eq. (10) reads

\[
\gamma_{IJ} = \frac{2}{\hbar^2} \text{Re} \int_{-\infty}^t dt' e^{i(\tilde{\omega}_j - \tilde{\omega}_i)(t-t')} \sum_q |g_{i,j}^q|^2 \left\{ (1 + N_{LO}) e^{-i\omega_{LO}(t-t')} + N_{LO} e^{i\omega_{LO}(t-t')} \right\},
\]

(11)

in which \( N_{LO} \) is the phonon number at the given temperature and \( \hbar \tilde{\omega}_i = \varepsilon_i^\phi \), \( \hbar \tilde{\omega}_j = \varepsilon_j^\phi \) are the carrier energies of the two states involved, calculated in the presence of the “spectator” carriers as specified in the reference configuration \( |\Phi\rangle \). It becomes clear that, in the present model, the expressions for the transition rates are very similar to the single-particle case, the only difference being that the energies involved are Coulomb renormalized by the presence of the other carriers. Similarly, the rate of the capture from the WL into the QD state \( i \) is given by

\[
\gamma_{IJ} = \frac{2}{\hbar^2} \text{Re} \int_{-\infty}^t dt' e^{-i\omega_{i}(t-t')} \sum_{q,k} f_{k} e^{i\omega_{k}(t-t')} |g_{i,k}^q|^2 \left\{ (1 + N_{LO}) e^{-i\omega_{LO}(t-t')} + N_{LO} e^{i\omega_{LO}(t-t')} \right\},
\]

(12)
with $f_k$ the thermal equilibrium occupancy of the WL state with energy $h\omega_k$, from which the capture took place.

The time integrations in the Eqs. (11) and (12) lead to strict energy-conserving $\delta$-functions, requiring a perfect compensation by the reservoir of the energy $h\omega_{J\ell}$ exchanged in the system transition. This is a consequence of the low-order perturbative treatment of the system-reservoir interaction. While this is not a limitation for transitions enabled by Coulomb scattering processes, the constant LO-phonon frequency makes LO-phonon-assisted transitions between the energetically discrete system configurations in general impossible. This is a well-known problem, referred to in the literature as the “phonon bottleneck” [6–9]. It was shown [10–12] that by taking into account higher order effects, non-vanishing transition rates can be obtained. A systematic way of doing this is provided by Green’s function (GF) techniques.

In what follows we will use transition rates obtained by GF methods in the Lindblad terms of Eq. (9). In this way not only the bottleneck is circumvented but also the correlation effects appearing in the kinetics are correctly accounted for.

As stated above, except for the presence of the “spectator” carriers, the formalism describing each transition is similar to single-particle scattering. What is not taken into account is the fact that the phonons are not only providing scattering but are also changing the bare electrons into polarons. In order to include polaronic effects, the quantum kinetic approach [11] replaces systematically the simple exponentials, oscillating with the one-particle frequency $\tilde{\omega}_i = \varepsilon_i^\Phi / \hbar$, by the polaronic retarded GFs, since the latter represent the carrier dressed with the LO-phonon interaction (polarons) in the same way as the exponentials correspond to the bare-particle GF. As a consequence, in the transition rates Eqs. (11) and (12) we change the exponentials according to

$$\frac{1}{i\hbar} e^{-i\tilde{\omega}_i (t-t')} \Rightarrow G_r^i(t-t').$$

(13)

Of course, the corresponding retarded GF should be calculated in the presence of the same reference state $|\Phi\rangle$. This problem is addressed in more detail below.

### 3.3. Spectral problem

As indicated in the previous section, renormalizations, that are due to the interaction of the carriers with the phonon reservoir, are included by calculating the time evolution of the retarded GFs under the influence of a polaron self-energy. Moreover, Hartree-Fock shifts, that do not emerge from the interaction of QD carriers, but also from the interaction with WL carriers, shall be taken into account. The retarded GFs are defined as expectation values with respect to a certain reference configuration $|\Phi\rangle$:

$$G_r^i(t-t') = \theta(t-t') \left[ G_r^>^i(t-t') - G_r^<^i(t-t') \right],$$

$$G_r^>^i(t-t') = -\frac{i}{\hbar} \langle \Phi | a_i(t) a_i^\dagger(t') | \Phi \rangle,$$

$$G_r^<^i(t-t') = \frac{i}{\hbar} \langle \Phi | a_i^\dagger(t') a_i(t) | \Phi \rangle.$$
Thus the spectrum of a single-particle state $i$ is probed by creating an electron (a hole) in the reference configuration at one time, annihilating it at another time and projecting the result back on the reference configuration. To calculate all possible transition rates in the above discussed scheme, we have to solve the Dyson equation for retarded GFs of the QD-WL system with a given self-energy for each reference configuration, which means all configurations but that of a full QD.

The retarded GFs obey the spectral Kadanoff-Baym equation [13, 14]

$$\left( i\hbar \frac{\partial}{\partial t} - \tilde{\varepsilon}_i \right) G^r_i(t) = \delta(t) + \int_0^t dt' \Sigma^r_i(t - t') G^r_i(t') \tag{15}$$

with the self-energy

$$\Sigma^r_i(t - t') = \delta(t - t') \Sigma^{HF}_i + \theta(t - t') \left[ \Sigma^>^r_i(t - t') - \Sigma^<_i(t - t') \right]. \tag{16}$$

Here, $\Sigma^{HF}_i$ contains instantaneous screened Hartree-Fock contributions due to Coulomb interaction between all carriers in the QD-WL system and the propagators $\Sigma^>_i$ and $\Sigma^<_i$ are taken in RPA [13]:

$$\Sigma^>_i(t - t') = \sum_j (1 - f_j) G^r_j(t - t') D^>_i,j(t - t'),$$
$$\Sigma^<_i(t - t') = -\sum_j f_j G^r_j(t - t') D^<_i,j(t - t'), \tag{17}$$

with the phonon propagators in thermal equilibrium

$$D^>_i,j(t - t') = \sum_q |g^{i,j}_q|^2 d^{>}(t - t'),$$
$$d^{>}(t - t') = \left\{ (1 + N_{LO}) e^{\mp i\omega_{LO}(t-t')} + N_{LO} e^{\pm i\omega_{LO}(t-t')} \right\}. \tag{18}$$

The occupancies of WL states $j$ are assumed to be the equilibrium values. If $i$ is a QD single-particle state, the QD state occupancies in the self-energy depend on the reference state, in which the spectral problem is solved. On the other hand, if $i$ is a WL single-particle state, the QD state occupancies are in principle given by the average over the occupancies of all QD configurations, which in general depends on time. Assuming that the influence of the QD ensemble on the reservoir is negligible, we completely neglect renormalizations of WL states due to QD state population. Thus the spectral problem can be solved for each reference configuration beforehand, independently of the carrier kinetics, and then used for the calculation of modified Lindblad rates. As the RPA self-energy (17) itself contains the full retarded GFs again, the Kadanoff-Baym equation (15) is solved self-consistently.
3.4. Scattering rates

In Section 3.2, we provided expressions for transition rates due to LO-phonon-assisted carrier capture and relaxation, as they result from the Born-Markov treatment of system-reservoir interaction. If we perform the replacement (13) in Eq. (12), we obtain the following expression for the rate of carrier capture into a QD state \( i \) from the WL:

\[
\gamma_{IJ} = 2\text{Re} \int_{-\infty}^{t} dt' \sum_k G^r_i(t-t') (G^r_k(t-t'))^* f_k D^>_k(t-t').
\]  

(19)

For the reverse process, \( G^r(t-t') \) has to be replaced by \( (G^r(t-t'))^* \) and \( f_k \) by \( (1-f_k) \). Replacing the exponential factors in Eq. (11) yields the following expression for the rate of relaxation from a QD state \( j \) into another QD state \( i \):

\[
\gamma_{IJ} = 2\text{Re} \int_{-\infty}^{t} dt' G^r_i(t-t') (G^r_j(t-t'))^* D^>_i,j(t-t').
\]  

(20)

4. Numerical results

In this section, we present numerical results that are obtained from the solution of the spectral problem formulated in Eqs. (15)-(17) and the subsequent evaluation of the vNL equation (9) with scattering rates that are modified according to Eqs. (19) and (20).

4.1. Model parameters

For the numerical results in this paper, we consider an InGaAs system consisting of a flat lens-shaped QD on a WL. We assume effective electron and hole masses \( m_e = 0.067 m_0 \) and \( m_h = 0.15 m_0 \) [15], respectively, and the static dielectric constant \( \varepsilon = 12.5 \) as well as the high-frequency dielectric constant \( \varepsilon_\infty = 10.9 \) and the LO-phonon energy \( \omega_{LO} = 36 \text{meV} \). A WL thickness of 2.2 nm and additional 2.1 nm QD height are used, leading to large subband energy spacing in z-direction, so that only the energetically lowest confined state in this direction is considered. The finite height of the confinement potential for electrons and holes is taken to be 350 and 170 meV, respectively, so that equal z-confinement wave functions for electrons and holes can be adopted. Furthermore, we assume equal QD in-plane wave functions for electrons and holes, where the in-plane confinement potential is modeled as a harmonic oscillator potential with oscillator length \( l = 5.4 \text{nm} \). Using these assumptions as well as an envelope approximation, the wave functions of the combined QD-WL system and with them the Coulomb matrix elements can be constructed following the procedure described in [16], where a similar system geometry is adopted. The ground state energies for electrons (holes) are taken to be 80 meV (30 meV) and the excited state energies are taken to be 40 meV (15 meV) below the WL continuum.

4.2. Spectral functions

The first step of the numerical procedure is to solve the polaron problem in each reference configuration \( |\Phi\rangle \) to obtain the time evolution of the polaronic retarded GFs of all QD and WL single-particle states. Although the GFs are used in their time-dependent form to calculate scattering rates, the spectral properties of the single-particle states can
Figure 2: Spectral functions of the electronic ground state (GS) and excited state (ES) with (a) the empty QD ($|0X\rangle$) and (b) the ES exciton ($|1X_{ES}\rangle$) as reference state $|\Phi\rangle$ for a WL-carrier density of $n_{WL} = 5 \cdot 10^{11} \text{ cm}^{-2}$ and a temperature $T = 300 \text{ K}$.

be illustrated by means of their spectral functions, which are obtained as the Fourier transform of the retarded GFs:

$$\hat{G}_i(\omega) = -\frac{1}{\pi} \text{Im} G^R_i(\omega). \quad (21)$$

As we consider only retarded GFs in thermal equilibrium, the spectral functions are not time-dependent. On the other hand, their shape clearly depends on the reference state in which the spectral problem is solved, as different QD state occupancies enter in the self-energy (17). In Fig. 2 we show spectral functions of the QD electron states for two different reference configurations. In both cases, the spectral functions exhibit the characteristic features of phonon replica and hybridisation that were discussed in literature [11]. Nevertheless, the actual form of these features is modified by the presence of additional carriers in the QD. The latter also cause Hartree-Fock shifts, which are reflected in the spectral positions of the main peaks but are small due to efficient screening by the WL carriers. These differences in the spectral properties of the QD states are reflected by the fact that the rate of a certain scattering process depends on the reference configuration it takes place in.
4.3. Scattering rates

In this section, we present numerical results for the temperature dependence of selected phonon-assisted scattering rates, that are important in our model system, for a WL-carrier density of $n_{WL} = 5 \cdot 10^{11} \text{cm}^{-2}$. For the results shown in Fig. 3, we used retarded GFs that were calculated with the empty QD as reference state. As discussed in the previous section, a different reference state will in general modify the rates, but they are not likely to drastically change their overall behaviour.

It is shown that the capture rate for holes into the excited state is much larger than that for electron capture, as the hole states are energetically much closer to the WL continuum wherefrom the carriers are captured. Moreover, the electron capture rate depends stronger on temperature. The reason is, that in our model system, the electronic excited state is more than a phonon energy away from the WL continuum, so that electron capture relies on the spectral broadening of single-particle states, which is reduced at low
temperatures. Furthermore, the probability of phonon emission as an assisting process decreases with decreasing temperature. Both rates exhibit non-monotonous behaviour, which can be assigned to the fact that not only polaronic properties play a role here, but also other effects, that do not necessarily act in the same direction. These are the temperature dependence of the WL carrier population at the band edge, from where carriers are captured, as well as that of Hartree-Fock shifts. The relaxation rates of both electrons and holes decrease significantly with the temperature, as they do not depend on WL-carrier, but only on phonon populations as well as on spectral broadening. However, the electron relaxation is much faster than the hole relaxation, which is due to the fact that the energetic separation of electronic states in our model is much closer to the phonon energy than that of hole states.

Figure 4: Time evolution of single-particle state occupancies in the QD at a temperature \( T = 300 \text{K} \) in the presence of a WL-carrier density of \( n_{\text{WL}} = 5 \cdot 10^{11} \text{cm}^{-2} \) for different initial conditions: (a) Excited state exciton. (b) Empty QD. Note the different time scales.

4.4. Carrier dynamics

Finally, we use the renormalization-modified scattering rates to calculate the time evolution of the QD system given by the vNL equation. To demonstrate the efficiencies of different types of scattering processes, we consider two different initial situations. As a
first example, the QD is prepared to be in a state of unity probability for the ES exciton, which is described as \( \rho_0 = |1X_{ES}\rangle \langle 1X_{ES}| = |1001\rangle \langle 1001| \) in the configuration picture. The results are shown in Fig. 4(a). As expected, the occupancies of the excited electron and hole states decrease on a very short time scale due to fast carrier relaxation, whereas the GS is filled. While the hole ES reaches its thermal equilibrium occupancy after a ps, the electron ES is refilled on a much longer timescale due to slow carrier capture from the WL. As a second example, we use the empty QD as initial state, which is given by \( \rho_0 = |0X\rangle \langle 0X| = |0000\rangle \langle 0000| \) in the configuration picture. The corresponding results are shown in Fig. 4(b). We see that while holes are captured into the QD and relax into the GS within a ps, the electron relaxation is strongly inhibited by the slow electron capture from the WL. As a consequence, the thermalization of the electrons is ten times slower than that of the holes. Note that the electron occupancies shown in Fig. 4(a) and (b) have not reached their thermal values after 2 ps and 20 ps, respectively. Nevertheless, in both examples, under von Neumann-Lindblad evolution the system relaxes to a thermal state, which is uniquely defined by the Lindblad rates and thus independent of the specific initial density matrix of the system. For low temperatures, one can expect a slower thermalization of carriers especially in the first situation, where relaxation is the dominant process. On the other hand, a low WL-carrier density will not change much in the first situation, while thermalization of an initially empty QD will become extremely slow.

5. Conclusions
We have presented a method to combine the description of a single QD in the configuration picture in terms of the vNL equation with many-particle renormalizations, that are calculated in a GF approach. Thus we fully take into account correlations between QD carriers and at the same time include polaronic properties, that arise due to their interaction with a phononic bath. This enables us to obtain finite rates for LO-phonon-assisted scattering processes between QD configurations, which is not possible in the usual Born-Markov treatment of system-reservoir interaction due to the discreteness of the QD spectrum. Moreover, the rates are configuration-dependent, so that additional carriers in the QD, that do not take part in a transition, influence its efficiency. We have shown numerical results for the temperature dependence of selected scattering rates as well as for the time evolution of a QD system due to phonon-assisted scattering processes for different initial conditions. We found a strong temperature dependence of carrier relaxation rates, while carrier capture rates are more or less constant. Moreover, it shows that the capture from the WL continuum represents a decelerating factor for the thermalization of electrons, which is much faster after an initial excitation of the QD.

The numerical results illustrate how the “phonon bottleneck” is efficiently circumvented by using a GF approach to include higher-order effects in the treatment of system-reservoir coupling.

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