$\beta$ delayed emission of a proton by a one-neutron halo nucleus

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Abstract

Some one-neutron halo nuclei can emit a proton in a $\beta$ decay of the halo neutron. The branching ratio towards this rare decay mode is calculated within a two-body potential model of the initial core+neutron bound state and final core+proton scattering states. The decay probability per second is evaluated for the $^{11}$Be, $^{19}$C and $^{31}$Ne one-neutron halo nuclei. It is very sensitive to the neutron separation energy.

1. Introduction

Some neutron-rich halo nuclei can emit a proton. This process is possible if the neutron separation energy is very small. Indeed a weakly bound halo neutron may $\beta$ decay, producing a proton which can be emitted, possibly together with neutrons. Processes where this proton is bound with one or two neutrons have been observed in the $\beta$ delayed deuteron and triton decays of $^6$He and $^{11}$Li [1-8]. Recently we have calculated the branching ratio of an even rarer process where the proton remains unbound but is accompanied by a free neutron [9]. This decay is uniquely possible for $^{11}$Li, among nuclei with known separation energies. The study has been performed in a three-body model with a simplified description of the continuum. An even simpler process is however possible.

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A one-neutron halo nucleus can be viewed as a normal nucleus, the core, to which a neutron is bound in an orbital with a large radius. The $\beta$ decay of the bound halo neutron may occur, releasing the proton, under the condition of energy conservation

$$S_n < (m_n - m_p - m_e)c^2 \approx 0.782 \text{ MeV},$$

where $S_n$ is the neutron separation energy of the decaying nucleus and $m_n$, $m_p$ and $m_e$ are the neutron, proton and electron masses, respectively. Among one-neutron halo nuclei for which $S_n$ is known with sufficient precision, this decay is allowed at least for $^{11}\text{Be}$ and $^{19}\text{C}$, and probably for $^{31}\text{Na}$. It should be observable if the branching ratio is large enough. This decay mode of $^{11}\text{Be}$ has been considered by Horoi and Zelevinsky but the results do not seem to have been published [10]. Here we study this rare decay mode within a two-body potential model. The initial halo nucleus is treated as a core+neutron bound state. The final states lie in the core+proton continuum. How rare is this decay is the main question raised in the present exploratory study.

2. Decay probability for $\beta$ delayed proton emission

The $\beta$ decay of the halo neutron releases the resulting proton from the core. The distribution of decay probability per time unit as a function of the energy $E < Q$ of the relative motion of the two particles is given by

$$\frac{dW}{dE} = \frac{1}{2\pi^3} \frac{m_ec^2}{\hbar} G_\beta^2 f(Q - E) \left( \frac{dB(F)}{dE} + \lambda^2 \frac{dB(GT)}{dE} \right),$$

where $G_\beta \approx 2.996 \times 10^{-12}$ is the dimensionless $\beta$-decay constant and $\lambda \approx -1.268$ is the ratio of the axial-vector to vector coupling constants. The Fermi integral $f(Q - E)$ depends on the kinetic energy $Q - E$ available for the electron and antineutrino with

$$Q = (m_n - m_p - m_e)c^2 - S_n.$$  \hspace{1cm} (3)

The total decay probability per time unit $W$ is obtained by integrating (2) from zero to $Q$. The branching ratio can than be derived as

$$\mathcal{R} = W t_{1/2} / \ln 2,$$

where $t_{1/2}$ is the half life of the halo nucleus.
In the present model, the halo nucleus is described as a two-body core+neutron system in its ground state with total angular momentum $J_i$ resulting from the coupling of the orbital momentum $l_i$ of the relative motion and the neutron spin $s = 1/2$. The spin of the core is assumed to be zero. The parity of the initial state is $(-1)^{l_i}$. The radial wave function is denoted as $u_{d_i,l_i}^i$ with the normalization $\int_0^\infty |u_{d_i,l_i}^i(r)|^2 dr = 1$. It is obtained from a potential $V_i$ adjusted to reproduce the experimental neutron separation energy $S_n$.

The final scattering state of the core and the proton is a distorted wave with wave vector $k$. Because of selection rules, only some partial waves with total angular momentum $J_f$ resulting from the coupling of the orbital momentum $l_f$ and the proton spin $s$ are allowed. The radial wave functions $u_{k,l_f}^{k_f}$ for a wave number $k = \sqrt{2\mu E/h^2}$ where $\mu$ is the core-proton reduced mass are obtained with a potential $V_f$ describing the core+proton system. They are normalized according to $\int_0^\infty u_{k,l_f}^{k_f}(r)u_{k',l_f}^{k_f}(r)dr = \delta(k - k')$. The potential $V_f$ is usually poorly known when the core is unstable.

Within this model, the Fermi reduced decay probability is given by

$$\frac{dB(F)}{dE} = \frac{1}{\hbar v} |I_{l_i,l_i}^i|^2$$

and the Gamow-Teller reduced decay probability by

$$\frac{dB(GT)}{dE} = \frac{6}{\hbar v} \sum_{J_f}(2J_f + 1) \left\{ \frac{J_f}{s} \frac{s}{J_i} \frac{l_i}{1} \right\}^2 |I_{l_i,l_i}^f|^2$$

with the relative velocity $v = \hbar k/\mu$ and the radial integrals

$$I_{l_i,l_f} = \int_0^\infty u_{k,l_f}^{k_f}(r)u_{d_i,l_i}^i(r)dr.$$  \hspace{1cm} (7)

If the final wave function does not depend on $J_f$, the Gamow-Teller term simplifies as

$$\frac{dB(GT)}{dE} = 3 \frac{dB(F)}{dE}. \hspace{1cm} (8)$$

The reduced decay probability can then also be written as

$$\frac{dW}{dE} = W_n \frac{f(Q - E)}{f_n} \frac{dB(F)}{dE}, \hspace{1cm} (9)$$
where $W_n$ is the free-neutron $\beta$ decay probability per second and $f_n$ is the corresponding Fermi integral.

With respect to a free neutron, the decay probability is affected in two ways. First, the ratio $f(Q - E)/f_n$ is small due to the reduction of phase space, since $f_n \equiv f(Q + S_n)$. It becomes extremely small when $E$ tends to $Q$. The $\beta$ delayed proton emission is favoured by very small separation energies $S_n$. Second, the reduced decay probability appearing in (9) is proportional to the square of a radial integral (7). Because of the Coulomb repulsion and the smallness of the $Q$ value, the scattering waves are small and, when $E$ tends to zero, tend to zero as $k^{1/2} \exp(-\pi \eta)$ [11], where $\eta = Z c^2 / \hbar v$ is the Sommerfeld parameter. They become thus smaller with increasing charge $Z_c$ of the core. They also become smaller with increasing orbital momentum. Hence, at given $Q$ value, we expect the decay probability to be largest for the lightest halo nuclei and for the halo neutron in the s wave.

3. Results and discussion

Before making explicit calculations, we have to specify the choice of potentials. The Fermi strength is proportional to the square of an overlap integral (7) between the initial and final radial wave functions. In order to have a realistic overlap, it is useful to have a correct node structure for these wave functions. Indeed, the presence of nodes leads to an integrand that changes sign one or several times and thus to a reduction of the overlap. Spectroscopic factors can also affect the size of the Fermi strength but given the limited knowledge on these quantities, we choose to ignore them in the present exploratory study. Finally, absorption in the core+proton optical potential might also play a role. However, the energies of the states after decay are lower than, or comparable to, the energy of the Coulomb barrier. Absorption should be weak and can safely be neglected.

Hence, we shall use real potentials $V_i$ and $V_f$ which should be deep enough to provide a realistic node structure of the initial and final radial wave functions. To keep the model simple we only use central Woods-Saxon potentials with range $r_0 A_c^{1/3}$ where $A_c$ is the mass number of the core. The depth is adapted to the separation energy for the core+n system. The same form factor with an additional point-sphere Coulomb potential is employed for the final core+p elastic scattering. Because of the small energies, the phase shifts are small and the sensitivity to $V_f$ is weak. Now let us consider explicit cases.
The best documented case is $^{11}$Be. Its $1/2^+$ ground state has a separation energy of about 501 keV \cite{11} and its half life is 13.8 s \cite{13}. The halo neutron is described by an $s$ wave. The parameters of the Woods-Saxon potential are taken as $r_0 = 1.2$ fm, $a = 0.6$ fm and $V_{i0} = 62.52$ MeV \cite{14}. In the $s$ wave, this potential possesses one unphysical forbidden state. The same parameters are used for the final potential except $V_{f0}$. The $^{11}$B nucleus has a proton separation energy $S_p \approx 11.228$ MeV \cite{15}. Its lowest $1/2^+$ state is located at the excitation energy $E_x \approx 6.79$ MeV. In the $s$ wave, $V_{f0} = 84.1$ MeV is adjusted so that the potential possesses one forbidden state and one bound state fitted to the energy $E_x - S_p \approx -4.52$ MeV with respect to the $^{10}$Be+p threshold. Bound and scattering states should thus have a reasonable node structure.

![Distribution of decay probability per second for the $\beta$ delayed np decay of $^{11}$Be, $^{19}$C and $^{31}$Ne.](image)

The $Q$ value \cite{3} is small, 0.281 MeV. The distribution of decay probability is displayed in Fig. 1. The most probable energies of the relative motion are in the interval 0.1-0.2 MeV. The total decay probability $1.5 \times 10^{-9}$ s$^{-1}$ leads to a branching ratio $3.0 \times 10^{-8}$.
The $^{19}\text{C}$ $1/2^+$ ground state has a separation energy of $580 \pm 90$ keV [15] and a half life $t_{1/2} = 46.2$ ms [13]. As a simple picture, we consider a neutron in the $s$ wave with one forbidden state and no spectroscopic factor. The parameters of the Woods-Saxon potential are $r_0 = 1.25$ fm, $a = 0.62$ fm and $V_{i0} = 41.42$ MeV giving a $Q$ value of 202 keV. For the final $^{18}\text{C}+p$ system, the $s$ wave possesses one forbidden state. We assume a possible $1/2^+$ bound state near $E_x = 2.1$ MeV [16]. With $S_p \approx 16.35$ MeV [15], we take $V_{f0} = 77.2$ MeV which gives a bound state at $-14.2$ MeV. The distribution of decay probability is displayed in Fig. 1. It is much smaller than for $^{11}\text{Be}$ because of the larger charge of the core and the smaller $Q$ value. The total decay probability $2.7 \times 10^{-12}$ s$^{-1}$ leads to a branching ratio $1.8 \times 10^{-13}$.

A candidate for delayed proton emission is $^{31}\text{Ne}$. Its neutron separation energy is poorly known: $0.33 \pm 1.07$ MeV [15]. Its half life is $t_{1/2} = 3.4$ ms [13]. This nucleus belongs to an island of inversion where its ground state should be an intruder state. Its one-neutron removal cross section [17] is too large for agreeing with the quantum numbers $0f7/2$ of the naive shell model. The ground state could be described with a $1p3/2$ orbital [18] although a $2s1/2$ orbital has also been considered [20]. Here we assume a $p$ wave ground state at $-0.33$ MeV giving $Q = 0.45$ MeV. It can be reproduced with the parameters $r_0 = 1.25$ fm, $a = 0.75$ fm and $V_{i0} = 48.86$ MeV [18]. This potential has one forbidden state in the $p$ wave. Little is known about the $^{30}\text{Ne}+p$ scattering. One can also expect an intruder $3/2^-$ state in the vicinity of the ground state. Hence we choose $V_{f0} = 90.0$ MeV which provides a forbidden state and a bound state at $-16.1$ MeV, not far above $-S_p \approx -17.7$ MeV.

The distribution of decay probability is displayed in Fig. 1. It is smaller than for $^{11}\text{Be}$ because of the larger charge of the core and the higher orbital momentum, but these effects are partly compensated by the larger $Q$ value. The most probable energies $E$ lie between 0.25 and 0.35 MeV. The total decay probability $3.3 \times 10^{-10}$ s$^{-1}$ leads to a branching ratio $1.6 \times 10^{-12}$. For an $s$ ground state with two forbidden states ($V_{i0} = 69.27$ MeV), the decay probability $W \approx 1.6 \times 10^{-9}$ would be five times larger.

The separation energy of $^{31}\text{Ne}$ is quite uncertain. The one-neutron removal cross section can be interpreted as arising from $S_n \approx 0.6$ MeV but this assumption is weakened by the lack of knowledge of spectroscopic factors [18]. Hence we display in Fig. 2 the dependence of the decay probability on the separation energy $S_n$. One observes that it varies very strongly. If $S_n$ is around 0.6 MeV, the decay probability is reduced by about six orders
of magnitude. On the contrary, the decay probability can be larger by five orders of magnitude if the separation energy is very small.

Finally, let us note that an estimate of the order of magnitude (in general within a factor of two) can be obtained with the simple approximation

$$I_{l, l', j'} = C \frac{2}{\pi} \int_a^{\infty} F_l(\eta, kr) e^{-\kappa r} dr$$  \hspace{1cm} (10)

where \(a = 5 \text{ fm}\), \(\kappa = \sqrt{2\mu S_n/\hbar^2}\) and \(F_l\) is a regular Coulomb function. Under the same conditions as in Fig. 1, the asymptotic normalization constant \(C\) is 0.83, 0.96, 0.69 \text{ fm}^{-1/2}\) for \(^{11}\text{Be}, ^{19}\text{C}, ^{31}\text{Ne},\) respectively.

4. Conclusion

As a summary, we have evaluated the order of magnitude of the decay probability per second for the \(\beta\) delayed proton emission by one-neutron halo nuclei. The best candidate for observing such a decay is \(^{11}\text{Be}\) in spite of the fact that its separation energy is not very small. The probability of this delayed decay is smaller than for the neutron-and-proton delayed decay of
$^{11}\text{Li}$ by an order of magnitude. Because of a longer lifetime, the branching ratio is larger by two orders of magnitude. The observation of this $\beta$ delayed decay mode of $^{11}\text{Be}$ would thus require high radioactive beam intensities and long measurement times.

The neutron separation energies of the other candidates, $^{19}\text{C}$ and $^{31}\text{Ne}$, are less well known and the decay probabilities are thus more uncertain. We have shown that the decay probability varies strongly with the neutron separation energy. A very small $S_n$ would be advantageous for the study of this decay mode. This advantage however decreases when the charge of the core increases. The best candidate for observing such a decay would be a not too heavy one-neutron halo nucleus with a very small separation energy.

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