Compaction of a granular material under cyclic shear

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Abstract. In this paper we present experimental results concerning the compaction of a granular assembly of spheres under periodic shear deformation. The dynamic of the system is slow and continuous when the amplitude of the shear is constant, but exhibits rapid evolution of the volume fraction when a sudden change in shear amplitude is imposed. This rapid response is shown to be to be uncorrelated with the slow compaction process.

PACS. 45.70.Cc Granular compaction – 61.50.-f Crystalline state

1 Introduction

Powder compaction has recently attracted the attention of physicists as the prototype of a disordered system without thermal fluctuations. For millimetric grains, thermal energy is negligible compared to gravitational energy. Once poured in a box, the grain packing is trapped in a metastable configuration and stay in this state unless an external excitation such as vibrations is imposed. During such a process, the particles assembly exhibits a slow and complex evolution towards a more compact configuration. This evolution has been studied experimentally by Knight et al [1] and Nowak et al [2] in the case of a vertically vibrated packing of spheres.

Their experiment consisted in a vertical cylinder full of monodispersed beads submitted to successive distinct vertical taps of controlled acceleration. The mean volume fraction was recorded after each tap. The experiments were first performed with taps of constant amplitude [1]. They have shown that the increase in volume fraction is a very slow process well fitted by the inverse of a logarithm of the number of taps. The more energetic taps are the more efficient for compaction. Nowak et al [2] have studied the compaction under taps of variable amplitude, and showed that irreversible processes occur during the compaction. Starting from a loose packing, the evolution of the volume fraction is not the same when increasing the amplitude of vibration as when decreasing. The first branch is irreversible, whereas the second is reversible. The highest volume fraction is obtained by first increasing the tap acceleration then decreasing it back to zero. Recently, Josserand et al [3] have investigated the evolution of the packing to sudden change of tap amplitude. They showed that the response of the system depends on the history of compaction.

These experimental results have motivated numbers of theoretical and numerical studies trying to find a minimal model which could exhibit qualitatively similar features. Most of the models suggest that the rearrangement of particles in a more compact state needs more and more cooperative reorganization as the volume fraction increases. Absorption-desorption model [2,4,5], geometrically frustrated lattice model [6,7,8], random walk model [9] or excluded volume model have been proposed [10,11]. Most of these approaches aspire to describe the vertically vibrated system. To our knowledge the question of how the compaction dynamics depends on the method of excitation has not yet been raised. In absence of thermal fluctuations, one can legitimately wonder how the compaction dynamic depends on the way the system is excited.

An alternative way of excitation is horizontal vibration. In a previous experiment we have shown that very compact and crystalline packings of monodispersed beads can be obtained this way [12] whereas vertical vibrations only yield random close packings. However, under horizontal vibration, only the top layers of the packing are affected and indeed submitted to a periodic horizontal shear. The goal of this paper is the study of the response to shear of the whole packing. A parallelepiped box full of beads is submitted to a horizontal shear through the periodic motion of two parallel walls. The resulting compaction is analyzed in detail in this paper.

Deformation of granular material under cyclic solicitation has been extensively studied in soil mechanics [13]. Cyclic loading in a triaxial test shows a compaction but no systematic study of the dynamic of compaction for long time behavior has been investigated. Cyclic shear is also evoked by Scott et al [14] in a short note where they show that this deformation applied to monodispersed beads packing could lead to a partially ordered structure.
A shear apparatus has been also developed by Bridgwater and Scott [15,16] in order to study the segregation dynamic during shear. Cyclic shear has been recently applied to thermally activated systems such as assembly of spherical colloidal particles. A periodic shear deformation applied to a colloidal hard spheres glass yields the formation of ordered regions [17,18]. However, both the thermal excitation and the imposed deformation are in this case present. The experiments presented in this paper can be seen as the analog of the colloidal experiments at zero temperature. The paper is presented as follows. The experimental setup is described in section 2. In section 3 the compaction under constant shear amplitude is investigated and the crystallisation process and the slow relaxation are discussed. The section 4 reports the study of the response of the system to sudden change in the shear amplitude. Finally we discuss in section 5 the analogies and differences between compaction under cyclic shear and vertically vibrated system before giving concluding remarks in section 6.

2 Experimental setup

The experimental setup is sketched in Fig. 1. The shear cell was parallelepiped, and the volume occupied by the beads was typically 10.5 cm high, 7.9 cm wide and 10.2 cm deep. The bottom of the cell was a flat PVC plate attached to a horizontal linear displacement device driven by a stepper motor. The two mobile lateral walls were aluminum plates linked with two hinges to the bottom plate. These plates were covered with a thin plastic sheet to insulate the packing from metallic dust due to particle/wall friction. The front and back sides were glass plates fixed on the bottom plate.

Fig. 1. Schematic of the shear cell and position sensor. $D$ is the horizontal displacement of the bottom plate and $h$ is the height of the packing. Angle of shear is $\theta = \arctan(D/h)$.

The granular packing was confined on the top by a rectangular plate mounted on a vertical displacement rail. This plate was independent from the rest of the shear cell and was free to move vertically only, no horizontal displacement being allowed. The periodic shear deformation was obtained by imposing a periodic horizontal displacement to the bottom plate, the two mobile lateral walls being constrained by two cables to remain in contact with the top plate.

The granular material we used were spherical glass beads. Experiments were performed with either monodisperse or bidisperse set of beads. For experiments with monodisperse particles, beads were $2.97 \pm 0.06$ mm in diameter with a density of $2.52 \pm 0.02$ g cm$^{-3}$. The cell was filled with 1450 g of particles. The bidisperse set of particles was constituted of 725 g of the 3 mm beads and 725 g of 2 mm beads of the same density. In order to prevent the surface deterioration of the beads during long-time experiments, beads were coated with silicon oil Rhodorsil 47V100 (viscosity hundred times viscosity of water). Note that the same quantitative results were observed without lubrication, but lifetime of the beads was in this case quite shorter.

We measured the volume fraction during the compaction process by recording the vertical position of the top plate, which went down (resp. up) when compaction (resp. dilatation) occurred. Its position was accurately measured by a linear position sensor Novotechnik T50 (potentiometric transducer). The position resolution was $3 \times 10^{-4}$ cm corresponding to a $2 \times 10^{-4}$ resolution in volume fraction. Both the data acquisition and the horizontal displacement of the cell were controlled by the same PC computer. Data could be recorded continuously during the shear, or once every cycle for the study of the long time evolution.

The experimental procedure was the following. The top plate was removed, the mobile side walls were put the vertical position and the particles were poured into the cell through a hopper. The mean initial volume fraction of the packing obtained by this procedure $\phi_0 = 0.592 \pm 0.008$. The top plate was then slowly put into contact with the packing. Once the initial random packing was ready, the periodic shear deformation was imposed. The lateral plates were inclined to an angle $+\theta$, followed by an inclination to an angle $-\theta$. The plates were then put back in the vertical position and the volume fraction was recorded. A new cycle of shear was then applied and so on. The volume fraction could also be continuously recorded.

During the experiments, we have observed that the inclination angle $\theta$ was the only pertinent control parameter. Change in the velocity of the bottom plate (from 1.9 to 3.8 cm/s) or change in the weight exerted by the top plate on the packing (from 1.8 to 3.8 kg) did not affect any of the measurements. We also performed experiments using asymmetric motion of the lateral plates: the plates were oscillating between $\theta_{medium} + \theta$ and $\theta_{medium} - \theta$. In the range accessible by our set-up ($\theta < 12^\circ$) no influence of the medium position has been observed. The compaction process was then only controlled by the amplitude of the cyclic shear.
Fig. 2. Continuous evolution of volume fraction as a function of number of cycles; \( \theta = 5.4^\circ \).

3 Cyclic shear of constant amplitude

We have first studied the evolution of a packing submitted to a periodic shear of constant amplitude \( \theta \). A typical evolution of the packing volume fraction is presented in Fig. 3. In this figure the volume fraction \( \phi \) was recorded with a resolution of 325 samples/cycle. This plot clearly shows a slow and continuous evolution from the initial random packing towards higher volume fraction. Supposed to this slow evolution, a quasi-periodic oscillation is observed, corresponding to the dilatation occurring during one cycle: the packing is denser in the vertical position and alternatively dilates when the walls are inclined at \( \theta \) and \(-\theta\). This behavior is also observed when the medium position is not the vertical. In this case a local maximum of the volume fraction is measured at the medium position \( \theta_{\text{medium}} \), whereas a local minimum is observed when the walls are inclined at \( \theta_{\text{medium}} + \theta \) and \( \theta_{\text{medium}} - \theta \).

In this paper we do not study the dilatancy properties in detail but rather focus on the long time evolution of the packing volume fraction. For this purpose, the volume fraction is recorded only once a cycle when the walls are brought back to the vertical position (stars in Fig. 2). A monotonous curve is then obtained, representing the evolution of the packing volume fraction. For this purpose, the volume fraction is measured at the medium position \( \theta_{\text{medium}} \) whereas a local minimum is observed when the walls are inclined at \( \theta_{\text{medium}} + \theta \) and \( \theta_{\text{medium}} - \theta \).

Fig. 3. Compaction under different shear angles : \( \theta = 2.7^\circ \) (star), \( \theta = 5.4^\circ \) (square), \( \theta = 10.7^\circ \) (circle). Insert: volume fraction of the packing versus shear angle for increasing number of cycles; \( n = 10^4 \) cycles (asterisks), \( n = 3 \times 10^4 \) cycles (crosses), \( n = 6 \times 10^4 \) cycles (triangles).

The first remark is that the compaction process is a very slow process. After \( 6 \times 10^4 \) cycles (one week experiment), the volume fraction still slowly increases. This slow compaction appears to be more efficient for large shear amplitude than for small shear amplitude as shown by insert in Fig. 3 representing the volume fraction as a function of the shear amplitude \( \theta \) after \( 10^4 \) (asterisks), \( 3 \times 10^4 \) (crosses), and \( 6 \times 10^4 \) cycles (triangles).

Although curves of Fig. 3 present very small fluctuations for an individual run, the final volume fraction can vary significantly (approximately 10%) from one experiment to another carried out at the same shear angle. For example, Fig. 3 shows two compaction curves obtained for \( \theta = 5.4^\circ \) starting from almost the same initial volume fraction (0.594 \( \pm \) 0.0008) prepared by the same procedure. Evolution of the packing thus seems to be rather sensitive to the initial random structure of the grains assembly. Insert of Fig. 3 presents the same curves in semi-logarithmic scale. We have tried to fit our experimental results by the inverse logarithmic function proposed by Knight et al for the compaction under vertical taps (Eq. 3 of [1]). However, no reasonable agreement with both the short and long time behavior could be found (Fig. 3). Other proposed fits like the two exponential function [20] or the stretched exponential ([1] and references therein) do not provide more convincing agreements.

A major difference between the periodic shear compaction and the vertically vibrated system is the large volume fraction up to 0.693 that can be attained by the former. Such a value obtained for monodispersed particles is much larger than the volume fraction of a random close packing, which is 0.63, according to Scott and Kilgour [21]. This means that crystalline arrangements are created during the compaction, which perhaps explains why the log \( t \) behavior is not observed in our system. The crystal structure of the packing is indeed observed at the wall of the cell as shown in Fig. 3. Moreover we checked that crystallisation is also present in the bulk by carefully removing the particles layer after layer at the end of an experiment. The picture in Fig. 3 shows a structure observed in the bulk. The orientation of the crystal observed in the bulk is not always parallel to the walls, suggesting that the order is not only wall-induced but nucleates and grows in the bulk.

In contrast, no crystalline alloy structure has been observed with the bidispersed material made of mixture of 2 and 3 mm beads. However the evolution of the volume...
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Fig. 4. Compaction curves for $\theta = 5.4^\circ$ for two different runs. Insert: semi-logarithmic scale. Dotted line is the inverse logarithmic fit $\phi = \phi_\infty - (\phi_\infty - \phi_0)/[1 + B \ln(1 + n/\tau)]$ with $\phi_\infty = 0.74$, $\phi_0 = 0.594$, $B = 0.1$ and $\tau = 0.8$.

Fig. 5. Photographs of the 3 mm beads packing. (a) Front view of the packing. (b) Ordered structure in the bulk of the packing (upper layers of the packing were removed).

Fig. 6. Example of angle variation effect during the compaction process. Insert shows a close-up of the first jump (point A).

fraction as a function of number of cycles and shear amplitude presents the same trend using mono or bidispersed material. In this case, the maximum volume fraction obtained for $\theta = 10.7^\circ$ after $6 \times 10^4$ cycles is 0.65 to be compared with 0.693 for the monodispersed packing.

4 Response to a change in shear amplitude

After the study of compaction under constant shear angle, we investigated the effects of a sudden change of angle. A periodic shear with inclination angle $\theta_1$ is first imposed to a random packing, and at a given time, the shear amplitude is suddenly changed to another value $\theta_2 = \theta_1 + \Delta \theta$. A typical experiment is presented in Fig. 6. Starting with a shear amplitude $\theta_1 = 2.7^\circ$, the angle is switched to $\theta_2 = 10.7^\circ$ at $n = 5000$, and then switched back to $\theta_1$ at $n = 10^4$. As can be seen, increasing the shear angle at $n = 5000$ produces a rapid fall of volume fraction $\phi$ followed by a slow and continuous increase. When shear angle is decreased back at $n = 10^4$, a rapid increase of $\phi$ occurs, followed by a slower one.

Such a response is non-trivial. It first shows that memory effects exist in the compaction process. Points A and B in Fig. 6 correspond to packings having the same volume fraction. However, their responses to the same shear amplitude $\theta = 10.7^\circ$ are different: packing A becomes looser whereas packing B pursues its compaction. This means that knowing the volume fraction of the packing is not sufficient to predict the evolution of the system. Such memory effects have been recently put in evidence in vertically shaken granular packings.

Another important observation is the rapid character of the packing response. Whereas the compaction from initial random state is a slow process (Sec. 3), a sudden variation of shear amplitude induces a rapid change of volume fraction, which can be regarded as a discontinuity or a jump. The amplitude $\Delta \phi$ of this jump can be accurately...
Jump amplitude versus age of packing. $\Delta \theta = 5.4^\circ$ (circles), $\Delta \theta = -5.4^\circ$ (stars).

Fig. 7. Jump amplitude versus age of packing. $\Delta \theta = 5.4^\circ$ (circles), $\Delta \theta = -5.4^\circ$ (stars).

defined and measured with the help of the tangent lines as shown on the insert of Fig. 6. This definition is also valid for positive jumps ($n = 10^4$ on Fig. 6). We have systematically studied how the volume fraction jumps depend on the different parameters.

First, we have investigated how the discontinuity $\Delta \phi$ is influenced by the age of the packing, namely the time elapsed before the angle change is applied. Fig. 8 shows the response $\Delta \phi$ to the same angle change $\Delta \theta = 5.4^\circ$ as a function of the elapsed time. The jump $|\Delta \phi|$ increases with age and eventually saturates for $n > 2000$. The same trend is observed for positive jumps ($\Delta \theta = -5.4^\circ$). After 2000 cycles, the response does not depend on the age anymore.

Secondly, we have observed that the initial angle does not play any role. Changing the shear amplitude from $\theta_1$ to $\theta_1 + \Delta \theta$ yield a response $\Delta \phi$ in volume fraction independent of $\theta_1$. In fine, the response of the system for sufficiently aged packings only depends on the shear change $\Delta \theta$.

Fig. 8 summarizes jump measurements carried out for the number of cycles $n > 3 \times 10^4$. Data points align along a straight line crossing the origin:

$$\Delta \phi = -\alpha \Delta \theta$$  \hspace{1cm} (1)

with $\alpha = 1.45 \times 10^{-3}$ and $\Delta \theta$ in degrees. In conclusion, the response of the packing to sudden change amplitude appears to be simpler than first thought. The rapid variation of the volume fraction induced by the change is simply proportional and opposite to the angle change.

Fig. 8. Relation between volume fraction jumps and angle change for aged packings ($n > 3 \times 10^4$).

5 Discussion

Two kinds of evolution are observed in the system of cyclic sheared packing: a slow and continuous compaction when the shear amplitude is constant and a rapid response when a change in shear amplitude is imposed. The striking feature is that the rapid dynamic is uncorrelated with the slow compaction: the variation of the volume fraction $\Delta \phi$ induced by a sudden change $\Delta \theta$ in shear amplitude is independent of the state $\phi$ of the system, and depends linearly on $\Delta \theta$. This observation suggests to split the evolution of the volume fraction in two terms:

$$\phi(n) = \phi_{\text{slow}}(n) - \alpha \theta.$$  \hspace{1cm} (2)

The first term $\phi_{\text{slow}}(n)$ corresponds to a slow and continuous evolution and the second term is proportional to angle $\theta$. This description is compatible with Fig. 8 when a sudden change in $\theta$ is imposed, one observes a corresponding jump $\alpha \Delta \theta$ in volume fraction, $\phi_{\text{slow}}$ being a continuous function.

One way of interpreting these results is to say that the periodic shear deformation plays two roles. On one hand, it allows for the particles to slowly and continuously rearrange to form a more compact packing. It thus profoundly affects the structure of the packing in an irreversible way, by introducing more and more order. This contribution is $\phi_{\text{slow}}$. On the other hand, it introduces an additional disorder superimposed to the ordered structure, represented in Eq. 2 by the negative contribution $-\alpha \theta$. These two contributions are uncorrelated. In this framework, the slow and rapid dynamics would then correspond to changes in the packing structure that are of complete different nature.

However, such interpretation remains speculative as long as we are not able to relate the macroscopic behavior of the volume fraction to local rearrangement of the particles. An important question is the discrimination between the rearrangements involved in the slow compaction process from those involved during the rapid compaction or dilatation subsequent to a sudden change in shear amplitude. Some numerical simulations suggest the existence of collective motion of cluster and of individual motion of particles which could perhaps be related to the slow and rapid dynamics.

At this stage, it is interesting to compare the results presented here about the compaction of a granular pack-
ing under cyclic shear with the results obtained previously in the case of a vertically vibrated packing. In order to do so, we have carried out the annealing experiment analog to the one described in \cite{2} for the tap excitation: the amplitude of shear is continuously increased then decreased and increased again. The evolution of the volume fraction is plotted in Fig. 9. As observed in the tap experiment, one get an irreversible branch and a reversible one.

However, in our case, the reversible branch is simply a straight line parallel to \(-\alpha \theta\). In the light of the above interpretation about the response of the system to sudden shear changes, this behavior can be explained as follows: slowly increasing the shear amplitude yields a quasi-saturated state, where \(\phi_{\text{slow}}(n)\) does no longer vary with \(n\). By then changing the angle \(\theta\), one explores the linear variation of \(\phi\) with \(\theta\). The slight shift between the decreasing and increasing branches in the reversible regime comes from the slight evolution of \(\phi_{\text{slow}}(n)\). Hence this shift increases (resp. decreases) when decreasing (resp. increasing) the rate of change in angle.

There thus exist several similarities between the two modes of compaction. However, some differences have to be noted. First, cyclic shear induces the crystallisation of an assembly of monodispersed beads, whereas such order is not reported in the experiments of vertical taps. The small aspect ratio (10 particles in a cross-section) of the tap experiment could perhaps inhibit the crystalisation. A second difference is the amplitude of the fluctuations. According to \cite{2}, vertical taps induce fluctuations from one cycle to another of the order of 0.8% of the mean volume fraction, whereas in our shear experiment the variation of volume fraction is less than 0.1%. Periodic shear is perhaps a less violent way of exploring the different packing configurations.

\section{Conclusion}

In this paper we have presented experiments about the compaction of a granular media under cyclic shear. First we have put in evidence that this method of compaction leads to crystals when the material is made of monodispersed particles. By studying the response of the system to sudden change in shear amplitude we have shown that discontinuities in volume fraction are observed, with amplitudes proportional to the angle change. The evolution of the packing is thus composed of two dynamics: a slow, continuous and monotonous compaction and a rapid evolution occurring during the shear jump. This observation suggests that the rearrangement of the particles in the packing associated to the two dynamics are of different nature. However, the next important step would be to experimentally relate the compaction process with the evolution of the internal structure of the packing.

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