Construction of Rate \((n - 1)/n\) Non-Binary LDPC Convolutional Codes via Difference Triangle Sets

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Abstract—This paper provides a construction of non-binary LDPC convolutional codes, which generalizes the work of Robinson and Bernstein. The sets of integers forming an \((n - 1, w)\)-difference triangle set are used as supports of the columns of rate \((n - 1)/n\) convolutional codes. If the field size is large enough, the Tanner graph associated to the sliding parity-check matrix of the code is free from 4 and 6-cycles not satisfying the full rank condition. This is important for improving the performance of a code and avoiding the presence of low-weight codewords and absorbing sets. The parameters of the convolutional code are shown to be determined by the parameters of the underlying difference triangle set. In particular, the free distance of the code is related to \(w\) and the degree of the code is linked to the “scope” of the difference triangle set. Hence, the problem of finding families of difference triangle set with minimum scope is equivalent to find convolutional codes with small degree.

I. INTRODUCTION

The aim of this paper is to construct a family of non-binary low-density parity-check (NB-LDPC) convolutional codes suitable for iterative decoding. The class of LDPC block codes was introduced by Gallager [8]. Their name is due to the fact that they have a parity-check matrices that is sparse. Similarly to LDPC block codes, one can construct LDPC convolutional codes as codes whose sliding parity-check matrices are sparse, which allows them to be decoded using iterative message-passing algorithms.

In the last few years, some attempts to construct binary LDPC convolutional codes were done. However, most of the constructions are for time-varying convolutional codes, see for instance [2], [14], [18].

In 1967, Robinson and Bernstein [15] used difference triangle sets for the first time to construct binary recurrent codes, which are defined as the kernel of a binary sliding parity-check matrix. At that time, the theory of convolutional codes was not yet developed and the polynomial notation was not diffused, but now, we may regard recurrent codes as a first version of convolutional codes. This was the first time that a combinatorial object was used to construct convolutional codes. Three years later, Tong in [16], used diffuse difference triangle sets to construct self-orthogonal diffuse convolutional codes, defined by Massey [12]. The aim of these authors was to construct codes suitable for iterative decoding and their result was a rudimental version of binary LDPC convolutional codes.

In this paper, we exploit the structure of difference triangle sets to construct non-binary LDPC convolutional codes, whose parity check matrices are free from 4-cycles and 6-cycles not satisfying the so called full rank condition. Our construction may be regarded as a generalization over \(\mathbb{F}_q\) of the construction of Robinson and Bernstein. We describe a close link between the properties of the difference triangle set and the parameters of the code. Moreover, we derive information on the column distances and on the free distance of the constructed codes, by exploiting the structure of the underlying difference triangle set.

The paper is structured as follows. In Section II, we first give some useful basics of the theory of convolutional codes and then we define difference triangle sets and their scope. In Section III, we define non-binary LDPC block codes and non-binary LDPC convolutional codes. In Section IV, we give a new construction of rate \((n - 1)/n\) non-binary LDPC convolutional codes, starting from an \((n - 1, w)\) difference triangle set. We show how the parameters of the code are related to the properties of the triangle set and we point out that several research works in combinatorics can be exploited to improve our construction. We derive some distance properties of the codes and the exact formula for computing their density. We conclude with further comments and future research directions in Section V.

II. PRELIMINARIES

A. Convolutional Codes

Let \(q\) be a prime power, \(\mathbb{F}_q\) be the finite field of order \(q\) and \(k, n\) be positive integers, with \(k \leq n\). A rate-\(k/n\) convolutional code over \(\mathbb{F}_q\) is a submodule \(C\) of \(\mathbb{F}_q[z]^n\) of rank \(k\), such that there exists a \(k \times n\) polynomial generator matrix \(G(z) \in \mathbb{F}_q[z]^{k \times n}\) which is basic and reduced, i.e., it has a right polynomial inverse and the sum of the row degrees of \(G(z)\) attains the minimal possible value such that

\[
C := \{u(z)G(z) \mid u(z) \in \mathbb{F}_q[z]^k\} \subseteq \mathbb{F}_q[z]^n.
\]
If $G(z)$ is a reduced, basic generator matrix for $C$, there exists a parity-check matrix $H(z) \in \mathbb{F}_q[z]^{(n-k)\times n}$ with $H_0$ full rank such that

$$C := \{v(z) \in \mathbb{F}_q[z]^n \mid H(z)v(z)^\top = 0\}.$$  

We define the degree $\delta$ of $C$ as the highest degree of the $k \times k$ full minor of $G(z)$. We denote a convolutional code of rank $k/n$ degree $\delta$ by $(k/n, \delta)$. For a polynomial vector $v(z) = \sum_{i=0}^{\mu} v_i z^i \in \mathbb{F}_q[z]$, we define the weight of $v(z)$ as $w_t(v(z)) := \sum_{i=0}^{\mu} w_t(v_i) \in \mathbb{N}_0$, where $w_t(v_i)$ denotes the Hamming weight of $v_i \in \mathbb{F}_q^n$. The free distance of a convolutional code $C$, $\text{d}_{\text{free}}(C)$, is defined as the minimum of the nonzero weights of the codewords in $C$. The parameters $\delta$ and $\text{d}_{\text{free}}$ are needed to determine respectively the decoding complexity and the error correction capability of a convolutional code with respect to some decoding algorithm. For this reason, for any given rate $k/n$ and field size $q$, the aim is to construct convolutional codes with “small” degree $\delta$ and “large” free distance $\text{d}_{\text{free}}$.

Remark 1. There is a natural isomorphism between $\mathbb{F}_q[z]^n$ and $\mathbb{F}_q^d$ that allows to consider a generator and a parity-check matrix of a convolutional code as polynomials whose coefficients are matrices. In particular, we will consider $H(z) \in \mathbb{F}_q^{(n-k)\times n}[z]$, such that $H(z) = H_0 + H_1 z + \ldots + H_{\mu} z^\mu$, with $\mu > 0$. With this notation, we can expand the kernel representation $H(z)v(z)^\top$ in the following way:

$$H v^\top = \begin{bmatrix} H_0 & \ldots & \ldots & \ldots & \ldots & H_\mu \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \vdots \\ \vdots & & & & & \ddots \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_\mu \end{bmatrix} = 0 \quad (1)$$

We will refer to the representation of the parity-check matrix of $C$ in equation (1) as sliding parity-check matrix.

For any $j \in \mathbb{N}_0$ we define the $j$-th column distance of $C$ as

$$d_j^C(C) := \min_{v_0 \neq 0} \left\{ w_t(v_0 + v_1 z + \ldots + v_j z^j) \mid v(z) \in C \right\}$$

$$= \min_{v_0 \neq 0} \left\{ w_t(v_0 + \ldots + v_j z^j) \mid H_j^c[v_0 \ldots v_j]^\top = 0 \right\}$$

with $H_j^c := \begin{bmatrix} H_0 & \ldots & \ldots & \ldots & \ldots & H_\mu \\ H_1 & H_0 & \ldots & \ldots & \ldots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \vdots \\ \vdots & & & & & \ddots \end{bmatrix}$.

We recall the following result.\n
Theorem 2. [9 Proposition 2.2] Let $d \in \mathbb{N}$. Then the following properties are equivalent.

1) $d_j^C = d$.

2) None of the first $n$ columns of $H_j^c$ is contained in the span of any other $d - 2$ columns and one of the first $n$ columns of $H_j^c$ is in the span of some other $d - 1$ columns of that matrix.

B. Difference Triangle Sets

A difference triangle set is a collection of sets of integers such that any integer can be written in at most one way as difference of two elements in the same set. Difference triangle sets find application in combinatorics, radio systems, optical orthogonal codes and other areas of mathematics [3, 4, 10]. We refer to [5] for a more detailed treatment. More formally, we define difference triangle sets in the following way.

Definition 3. An $(N, M)$-difference triangle set (DTS) is a set $\mathcal{T} := \{T_1, T_2, \ldots, T_N\}$, where for any $1 \leq i \leq N$, $T_i := \{a_{i,j} \mid 1 \leq j \leq M\}$ is a set of nonnegative integers such that $a_{i,1} < a_{i,2} < \ldots < a_{i,M}$ and all the differences $a_{i,j} - a_{i,k}$, with $1 \leq i \leq N$ and $1 \leq k < j \leq M$ are distinct. When $N = 1$, we will refer to a $(1, M)$-DTS simply as DTS.

An important parameter characterizing an $(N, M)$-DTS $\mathcal{T}$ is the scope $m(\mathcal{T})$, that is defined as

$$m(\mathcal{T}) := \max\{a_{i,M} \mid 1 \leq i \leq N\}.$$  

Observe that, a very well-studied problem in combinatorics is finding families of $(N, M)$-DTSs with minimum scope. In this work, we will use the sets in a DTS as supports of the columns in the sliding parity-check matrix of a convolutional code. We will relate the scope of the DTS with the degree of the code. Since we want to minimize the degree of the code, it is evident that the mentioned combinatorial problem plays a crucial role also here.

III. Low-Density Parity-Check Codes

A. Non-Binary LDPC Codes

In this section we briefly introduce LDPC block codes and we focus in particular on their non-binary version. We extend then the notion to LDPC convolutional codes.

LDPC codes are known for their performances near the Shannon-limit over the additive white Gaussian noise channel [11]. Their non-binary (NB-LDPC) version was first investigated by Davey and Mackay in 1998 in [6]. In [7], it was observed that NB-LDPC codes defined over a finite field with $q$ elements can have better performances than the binary ones. A NB-LDPC code is defined as the kernel of an $N \times M$ sparse (at least 1/2 of the entries are zeros) matrix $H$ with entries in $\mathbb{F}_q$. We can associate to $H$ a bipartite graph $G = (V, E)$, called Tanner graph, where $V = V_v \cup V_c$ is the set of vertices. In particular, $V_v = \{v_1, \ldots, v_N\}$ is the set of variable nodes and $V_c = \{c_1, \ldots, c_M\}$ is the set of check nodes. $E \subseteq V_v \times V_c$ is the set of edges, with $e_{n,m} = (v_n, c_m) \in E$ if and only if $h_{n,m} \neq 0$. The edge $e_{n,m}$ connecting a check node and a variable node is labelled by $h_{n,m}$, that is the corresponding permutation node. For an even integer $\ell$, we call a simple closed path consisting of $\ell/2$ check nodes and $\ell/2$ variable nodes in $G$ an $\ell$-cycle. The length of the shortest cycle is called the girth of $G$ or girth of $H$. It is proved that having higher girth decreases the decoding failure of the bit flipping
Hence, it is a common problem to construct NB-LDPC codes. A matrix whose error-correcting properties of the code are determined by the decoding of the first block (see also \[17\]). In particular, it can be represented by an \(\ell \times \ell\) block matrix:

\[
A = \begin{bmatrix}
a_1 & a_2 & 0 & \cdots & 0 \\
0 & a_3 & a_4 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & a_{\ell-3} & a_{\ell-2} \\
a_\ell & 0 & \cdots & 0 & a_{\ell-1}
\end{bmatrix},
\]

where \(a_i \in \mathbb{F}_q^\times\). The cycle does not satisfy the FRC if \(\det(A) = 0\). In this case, the cycle gives an absorbing set. Hence, it is a common problem to construct NB-LDPC codes in which the shortest cycles satisfy the FRC.

The convolutional counterpart of NB-LDPC block codes is given by convolutional codes defined over a finite field \(\mathbb{F}_q\) whose sliding parity-check matrix is sparse.

### IV. Construction of Rate \((n-1)/n\) NB-LDPC Convolutional Codes

In this section we will provide a construction of NB-LDPC convolutional codes over \(\mathbb{F}_q\), with the aid of difference triangle sets. In a certain sense, this could be regarded as an extension over \(\mathbb{F}_q\) of the construction given by Robinson and Bernstein.

Let \(\mathbb{F}_q\) be the finite field of order \(q = p^N\), where \(p\) is a prime number.

We are going to construct a sliding parity-check matrix as in equation (1). Observe that the decoding of a convolutional code \(C\) is done sequentially by blocks of length \(n\), hence, the error-correcting properties of the code are determined by the decoding of the first block (see also \[17\]). In particular, it is sufficient to analyze the portion of the sliding parity-check matrix \(H\) which affects the decoding of the first block, namely

\[
\mathcal{H} := H_{\mu}^T = \begin{bmatrix}
H_0 & H_0 & \cdots & H_0 \\
H_1 & H_{\mu} & H_{\mu-1} & H_0 \\
\vdots & \vdots & \ddots & \vdots \\
H_{\mu} & H_1 & H_0 & \cdots & H_0
\end{bmatrix}.
\]

First of all, observe that since \(H_0\) is full rank, one can perform Gaussian elimination on the block

\[
\begin{bmatrix}
H_0 & H_1 & \cdots & H_{\mu}
\end{bmatrix},
\]

which results in the following block matrix:

\[
\bar{H} = \begin{bmatrix}
A_0 & I_{n-k} \\
A_1 & 0 \\
\vdots & \vdots \\
A_\mu & 0
\end{bmatrix},
\]

where \(A_i \in \mathbb{F}_q^{(n-k) \times k}\) for \(i = 1, \ldots, \mu\). With an abuse of notation, we will still write \(H_0\) for indicating \([A_0 | I_{n-k}]\), and \(H_i\) for the matrices \([A_i | 0]\).

Note that it is important to construct the sliding parity-check matrix \(H\) of a NB-LDPC convolutional code such that the Tanner graph \(\mathcal{G}\) associated to \(H\) does not contain short cycles not satisfying the FRC. It is easy to see that \(H\) satisfies this property if and only if \(\mathcal{H}\) does. By the discussion of the previous section, this is equivalent to construct \(\mathcal{H}\), such that all the \(2 \times 2\) and \(3 \times 3\) minors that are non-trivially zero, are non-zero.

In the following we focus on the construction of rate \((n-1)/n\) NB-LDPC convolutional codes. In particular, we will construct the matrices \(A_i \in \mathbb{F}_q^{I \times (n-1)}\), such that the resulting matrix \(\mathcal{H}\) does not contain 4-cycles and 6-cycles, not satisfying the FRC.

#### A. Construction

Let \(n, w\) be positive integers. Consider an \((n-1, w)\)-DTS \(\mathcal{T} := \{T_1, \ldots, T_{n-1}\}\). Each \(T_i\) will give the positions of the non-zero elements of the first \(n - 1\) columns of the matrix \(H\) of equation (1); the last column will be simply given by the vector \([1, 0, \ldots, 0]^T\).

**Definition 4.** With the notation above, define the matrix \(\bar{H}^T \in \mathbb{F}_q^{mT \times n}\), in which the \(k\)-th column has weight \(w\) and support \(T_k := \{a_{1,1}, \ldots, a_{k,w}\}\). Formally, let \(\alpha\) be a primitive element for \(\mathbb{F}_q\), so that any non-zero element of \(\mathbb{F}_q\) can be written as power of \(\alpha\). For any \(1 \leq i \leq m(T)\), \(1 \leq k \leq n - 1\),

\[
\bar{H}_{i,k}^T = \begin{cases}
\alpha^k & \text{if } i \in T_k \\
0 & \text{otherwise}
\end{cases}.
\]

The last column of \(\bar{H}^T\) is given by \([1, 0, \cdots, 0]^T\). Derive the matrix \(\bar{H}^T\) by “shifting” the columns of \(H^T\) and then a sliding matrix \(H^T\) of the form of equation (1). Finally, define \(C^T := \ker(\bar{H}^T)\) over \(\mathbb{F}_q\). Note that here \(\mu = m(T) - 1\).

**Example 5.** Let \(\mathbb{F}_q := \{0, 1, \alpha, \ldots, \alpha^{q-2}\}\) and \(\mathcal{T}\) be a \((2, 3)\)-DTS, such that \(T_1 := \{1, 2, 6\}\) and \(T_2 := \{1, 2, 4\}\). Then, with the notation above,

\[
\bar{H}^T = \begin{bmatrix}
\alpha & \alpha^2 & 1 \\
0 & 0 & 0 \\
\alpha^6 & 0 & 0 \\
0 & 0 & \alpha^6 \\
\alpha^6 & 0 & 0
\end{bmatrix},
\]

which leads to the sliding matrix in Figure 1.

**Example 6.** Let \(\mathbb{F}_q := \{0, 1, \alpha, \ldots, \alpha^{q-2}\}\) and \(\mathcal{T}\) be a \((2, 3)\)-DTS, such that \(T_1 := \{1, 2, 6\}\) and \(T_2 := \{2, 3, 5\}\). Then, with the notation above,

\[
\bar{H}^T = \begin{bmatrix}
\alpha & 0 & 1 \\
\alpha^2 & \alpha^4 & 0 \\
0 & \alpha^6 & 0 \\
0 & 0 & \alpha^{10} \\
\alpha^6 & 0 & 0
\end{bmatrix}.
\]
which leads to the sliding matrix in Figure 2.

**Proposition 7.** Let $T$ be an $(n - 1, w)$-DTS with scope $m(T)$. Then, the code $C^T$ given as in Definition 2 is an $(n, n - 1, m(T) - 1)_q$ convolutional code.

**Remark 8.** As already mentioned, an interesting problem in combinatorics is to find families of difference triangle sets having minimum scope [3], [5], [10]. This is a difficult task in general. For our application, it is desirable to have a difference triangle set $T$ whose scope is as small as possible so that the degree of $C^T$ is small as well. This is desirable for convolutional codes because the complexity of the decoding algorithm increases with $\delta$.

**Theorem 9.** Let $T$ be an $(n - 1, w)$-DTS and consider the matrix $[A_0^T \cdots A_{j-1}^T]^\top$ defined as in the previous construction. Denote by $w_j$ the minimal column weight of $[A_0^T \cdots A_{j-1}^T]^\top$. For $I \subseteq \{1, \ldots, \mu + 1\}$ and $J \subseteq \{1, \ldots, n(\mu + 1)\}$ we define $[H^T]_{I,J}$ as the submatrix of $[H^T]$ with rows index $I$ and column indices $J$. Assume that for all $I, J$ with $|I| = |J| \leq w$ and $j_1 := \min(J) \leq n - 1$ and $I$ containing the indices where column $j_1$ is nonzero, we have that the first column of $[H^T]_{I,J}$ is not contained in the span of the other columns of $[H^T]_{I,J}$. Then

(i) $d_{\text{free}}(C^T) = w + 1,$
(ii) $d_j^c = w_j + 1.$

**Proof.** (i) Without loss of generality, we can assume that the first entry of $H_0$ is nonzero. Let $M \subseteq \{1, \ldots, \mu + 1\}$ with $|M| = w$ be the set of positions where the first column of $H$ (and hence also the first column of the sliding parity-check matrix) has nonzero entries. Denote the values of these nonzero entries by $d_1, \ldots, d_w$. Then, $v(z) = \sum_{i=0}^w v_i z^i$ with $v_0 = [1 \ 0 \ \cdots \ 0 - d_1]$ and $v_i = \begin{cases} [0 \cdots 0] & \text{for } i \neq I \notin M \\ [0 \cdots 0 - d_{i+1}] & \text{for } i \in M \end{cases}$ for $i \geq 1$ is a codeword with $\text{wt}(v(z)) = w + 1$. Hence $d_{\text{free}} \leq w + 1.$

Assume by contradiction that there exists a codeword $v \neq 0$ with weight $d \leq w$. We can assume that $v_0 \neq 0$, i.e. there exists $i \in \{1, \ldots, n\}$ with $v_0,i \neq 0$. One knows $\text{wt}(H^T v^\top) = 0$. Of this homogeneous system of equations, where we consider the nonzero components of $v_0, v_1, \ldots, v_{\text{deg}(v)}$ as variables, we take only the rows where column $i$ of $H^T$ has nonzero entries.

We end up with a homogeneous system with $w$ equations and $d$ variables, whose coefficient matrix has full column rank according to the assumptions of the theorem. This implies $v = 0$, what is a contradiction.

(ii) The result follows from Theorem 2 with an analogue reasoning as in part (i).

**Remark 10.** With the assumptions of Theorem 9 one has $d_j^c = d_{\text{free}}(C^T)$ for $j \geq \mu$. Moreover, one achieves higher column distances (especially for small $j$) if the elements of $T$ are small.

**Proposition 11.** If $N$ is the maximal message length, i.e. for any message $v$, $\text{deg}(v) + 1 \leq N/n$, then the sliding parity-check matrix of a convolutional code derived in Definition 2 has density

$$\frac{w(n - 1) + 1}{\mu n + N}.$$

**Proof.** To compute the density of a matrix, one has to divide the number of nonzero entries by the total number of entries. The result follows immediately.

**Theorem 12.** Let $T$ be an $(n - 1, w)$-DTS with scope $m(T)$ and $\mathbb{F}_q$ be the finite field with $q$ elements with $q > (n - 1)\delta + 1 = (n - 1)(m(T) - 1) + 1$. Let $C^T$ be the rate $(n - 1)/n$ convolutional code defined over $\mathbb{F}_q$ from $T$, with $H^T$ as defined in (3). Then, all the $2 \times 2$ minors of $H^T$ that are non-trivially zero are non-zero.

**Proof.** The only $2 \times 2$ minors to check are the ones of the form $\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}$. By definition of DTS, the support of any column of $H^T$ intersects the support of its shift at most once. This
The determinant can not be zero because $M$ from $r, s > 0$, with $r \neq s$ and $2 \leq i + r, l + r, t + r \leq \delta - 1$ and $4 \leq i + r + s, l + r + s, t + r + s \leq \delta$, the minor is given by

$$\begin{vmatrix}
\alpha_i & \alpha_j & \alpha_k \\
\alpha_{i+r} & \alpha_{l+r} & \alpha_{m+r} \\
\alpha_{i+r+s} & \alpha_{l+r+s} & \alpha_{m+r+s}
\end{vmatrix}.$$

This determinant is 0 if and only if $rk = rj = 0 \pmod{(q-1)}$. Hence we have that

$$a_1 a_2 a_3 = a_4 a_5 a_6, a_7 a_8 a_9,$$

with $a_i \neq 0$ for any $i$. As we observed in Theorem 12 in this case all the columns are shifts of three different columns from $H_T$. Hence we have that, given $1 \leq i, l, t \leq \delta - 3, r, s > 0$, with $r \neq s$ and $2 \leq i + r, l + r, t + r \leq \delta - 1$ and $4 \leq i + r + s, l + r + s, t + r + s \leq \delta$, the minors are given by

$$\begin{vmatrix}
a_1 a_2 a_3 \\
a_4 a_5 a_6 \\
a_7 a_8 a_9
\end{vmatrix} = \begin{vmatrix}
a_i & a_j & a_k \\
a_{i+r} & a_{l+r} & a_{m+r} \\
a_{i+r+s} & a_{l+r+s} & a_{m+r+s}
\end{vmatrix}.$$

This determinant is 0 if and only if

$$\alpha_{i+r+m+r+s} + \alpha_{r+m+r+s+j+r} + \alpha_{r+m+r+s+sk} = 0.$$

Without loss of generality we can assume that $j < k < m$ and it turns out that the maximum exponent in equation (5) is $rk + rm + sm$ while the minimum is $rk + rj + sj$. Let $M := rk + rm + sm - (rk + rj + sj)$. We immediately see that the maximum value for $M$ is $(\delta - 1)(n - 2)$ hence this determinant can not be zero because $\alpha$ is a primitive element for $\mathbb{F}_q$ and, by assumption, $q = p^N$, where $N > M$.

**Case II.** The $3 \times 3$ minors are of the form

$$\begin{vmatrix}
a_1 & a_2 & 0 \\
0 & a_3 & a_4 \\
a_6 & 0 & a_5
\end{vmatrix}.$$
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