Low dimensional bound entanglement with one-way distillable cryptographic key

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We provide a class of bound entangled states that have positive distillable secure key rate. The smallest state of this kind is \(4 \otimes 4\), which shows that peculiar security contained in bound entangled states does not need high dimensional systems. We show, that for these states a positive key rate can be obtained by \textit{one-way} Devetak-Winter protocol. Subsequently the volume of bound entangled key-distillable states in arbitrary dimension is shown to be nonzero. We provide a scheme of verification of cryptographic quality of experimentally prepared state in terms of local observables. Proposed set of 7 collective settings is proven to be optimal in number of settings.

Quantum cryptography is one of the very interesting practical phenomena within quantum information theory \([1, 2, 3, 4]\). There were in general two ideas to produce cryptographic key. The first was based on sending nonorthogonal states \([5]\), the second - on specially chosen measurements of maximally entangled pairs \([6]\). They have been shown to be equivalent in general \([7]\) including most general eavesdropper attack. An important ingredient of the protocol was so called Quantum Privacy Amplification \([8]\) based on distillation of EPR pairs \([9]\). Despite of natural expectations, that distillability is much wider than class \(\gamma_{AA'BB'}\), where \(AA'\) is hold by Alice, \(BB'\) by Bob, and measurement of \(AB\) in standard basis provides one bit of perfect key (ie. possible outcomes are either \(\{00\} \) or \(\{11\}\) with equal probability \(1/2\) and they are completely independent on measurements on physical system different than \(AA'BB'\)). Following \([10]\) we will write \(\gamma\) in matrix representation in \(ABA'B'\) order of subsystems, so that its \(4 \times 4\) structure corresponds to two qubit subsystem \(AB\).

We shall start the construction of key distillable BE states by providing a class of states with \(K_D > 0\):

**Proposition 1.** Consider two private bits \(\gamma_1, \gamma_2\) and take any biased mixture of the form:

\[
\rho = p_1 \gamma_1 + p_2 \sigma_x^A \gamma_2 \sigma_x^A
\]

with, say, \(p_1 > p_2\) and \(\sigma_x^A = (\sigma_x)_A \otimes I_{A'BB'}\). The distillable key \(K_D(\rho)\) fulfills \(K_D(\rho) \geq 1 - h(p_1)\) where \(h(p_1)\) is the binary entropy of distribution \(\{p_1, p_2\}\).

Before we prove the proposition, we have to recall a technique called "privacy squeezing" \([11]\) that allows to investigate privacy of states of type \(\rho_{AA'BB'}\). The state is purified to \(\psi_{ABA'B'E}\) so that Eve holds the subsystem \(E\) of \(\psi\). To draw key, Alice and Bob will measure systems \(AB\) in standard basis, and will process the outcomes by public discussion. The systems \(A'B'\) will not be actively used, so the relevant state is

\[
\rho_{ABE}^{(eq)} = \sum_{ij} p_{ij} |ij\rangle \langle ij|_{AB} \otimes \rho_{E}^{(ij)}
\]

where \(\rho_{E}^{ij}\) are Eve’s states given the outcome was \(ij\). The state is called eq (two registers are classical). From such state by Devetak-Winter \([12]\) protocol one can get

\[
K_{DW} = I(A : B) - I(A : E)
\]
bits of key where \( I(A : B)_{\rho} = S(A) + S(B) - S(AB) \), \( S(X) \) being von Neumann entropy of \( X \) subsystem of state \( \rho \). We will now provide a different ccq state \( \sigma^{(ccq)}_{ABE} \), which is no better, in the sense that Eve can obtain it from \( \rho_{ABE} \) by some operation on her system. Clearly, such \( \sigma^{(ccq)} \) can give no more key than \( \rho^{(ccq)} \). The state \( \sigma^{(ccq)} \) we produce as follows. First, from \( \rho_{ABE} \) we obtain its privacy squeezed (p-squeezed) version \( \sigma_{AB} \). To this end (i) we apply so-called twisting [10] i.e. a unitary transformation controlled by standard basis on \( AB \) \( U_T = \sum_{ij} |ij\rangle\langle ij| AB \otimes U_{ij}^{AB} \), and subsequently (ii) we trace over systems \( A'B' \). One finds that p-squeezed state \( \sigma_{AB} \) has a property that the ccq state \( \sigma^{(ccq)}_{ABE} \), emerging after measuring it in standard basis \( |ij\rangle \) is no better than \( \rho^{(ccq)}_{ABE} \). However, if twisting is properly chosen, then it may still produce much key. The privacy is now squeezed to solely two systems \( AB \).

**Proof of Proposition.** Using the above method, we will apply p-squeezing to the state \( \rho \), and show that from the \( \sigma^{(ccq)} \) the DW protocol gives \( 1 - h(p_1) \) rate of key. A basic fact about private bits [11] is that there exists twisting which brings them to the form \( \psi^{AB}_0 \otimes \rho_{A'B'} \) where \( \rho_{A'B'} \) is some state, \( \psi_0 \) is one of four Bell states

\[
|\psi_{0,1}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad (4)
\]

\[
|\psi_{2,3}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \quad (5)
\]

and in the twisting \( U_{01} = U_{10} = I \). Using this, we immediately obtain a twisting which, followed by partial trace over \( A'B' \), turns the state \( \rho \) of eq. [11] into a mixture of \( \psi_0 \) and \( \psi_2 \). I.e. the p-squeezed state of [11] is \( \sigma_{AB} = p_1|\psi_0\rangle\langle \psi_0| + p_2|\psi_2\rangle\langle \psi_2| \). Its purification is of the form

\[
\psi^{\dagger}_{ABE} = \sqrt{p_1}|\psi_0\rangle_{AB}|e_1\rangle_E + \sqrt{p_2}|\psi_2\rangle_{AB}|e_2\rangle_E \quad (6)
\]

so that by measuring it in standard basis Alice and Bob obtain the ccq state

\[
\sigma^{(ccq)}_{ABE} = \frac{p_1}{2}[P_{00} + P_{11}] \otimes P_{e_1} + \frac{p_2}{2}[P_{01} + P_{10}] \otimes P_{e_2} \quad (7)
\]

with \( P_{ij} = |ij\rangle\langle ij| \) and \( P_{e_i} = |e_i\rangle\langle e_i| \). For this state, we have \( I(A : B) - I(A : E) = 1 - h(p_1) \) Thus by DW protocol [13] we get this amount of key. Because this state is no better than our state of interest \( \rho^{(ccq)} \) (the ccq state obtained by measuring AB systems of initial state \( \rho \)) we obtain that the distillable key of \( \rho \) satisfies

\[
K_D(\rho) \geq 1 - h(p_1) \quad \text{which ends the proof of proposition.}
\]

Let us note that the key is here drawn by one-way protocol. We have found that applying two-way recurrence protocol will not increase the key rate.

**Construction of small dimensional bound entangled states with \( K_D \) \( > \) 0 First let us recall, that any private bit can be represented in its \( X \)-form [11]:

\[
\gamma_{AB'A'B'} = \frac{1}{2} \begin{bmatrix}
\sqrt{XX'} & 0 & 0 & X \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
X^\dagger & 0 & 0 & \sqrt{XX'}
\end{bmatrix}
\quad (8)
\]

where \( X \) is arbitrary operator with trace norm one, and which completely represents the pbit. Then, we obtain that state \( \rho \) is of the form

\[
\rho = \frac{1}{2} \begin{bmatrix}
p_1\sqrt{X_1X_1^\dagger} & 0 & 0 & p_1X_1 \\
0 & p_2\sqrt{X_2X_2^\dagger} & p_2X_2 & 0 \\
0 & p_2X_2^\dagger & p_2\sqrt{X_2^\dagger X_2} & 0 \\
p_1X_1^\dagger & 0 & 0 & p_1\sqrt{X_1^\dagger X_1}
\end{bmatrix}
\quad \text{(9)}
\]

We assume now that \( A' \) and \( B' \) are systems both described by \( C^d \). Now the essential part of the construction is the following substitution: \( X_1 = \frac{W_U}{||W_U||} \) where

\[
W_U = \sum_{ij} u_{ij} |ij\rangle\langle ji|
\quad (10)
\]

and \( u_{ij} \) are unitary matrix elements of some matrix \( U \) on \( C^d \). Note that \( ||W_U|| = \sum_{ij} |u_{ij}| \) (here we use the trace norm of the matrix). The second operator we choose \( X_2 = \frac{W_U^\dagger}{||W_U^\dagger||} \) with \( \Gamma \) being partial transposition on subsystem \( B' \). In this case one has just \( ||W_U^\dagger|| = d \)

The corresponding mixing probabilities are

\[
p_1 = \frac{||W_U||}{||W_U|| + ||W_U^\dagger||} \quad p_2 = \frac{||W_U^\dagger||}{||W_U|| + ||W_U^\dagger||} \quad (11)
\]

respectively.

Collecting two simple observations, namely that: (i) \( |W_UW_U^\dagger| = |W_U^\dagger W_U| \) which after normalisation gives separable, PPT-invariant state), (ii) \( |W_UW_U^\dagger| = |W_U^\dagger W_U| \) (again after normalisation giving PPT-invariant separable state) we get immediately that \( \rho \) with parameters defined like above is PPT invariant. At the same time we have desired security condition \( p_1 > p_2 \) if only

\[
\frac{p_1}{p_2} = \frac{||W_U||}{||W_U^\dagger||} = \frac{\sum_{ij} |u_{ij}|}{d} > 1.
\quad (12)
\]

The latter is satisfied for any unitary \( U \) which written in \{\|ij\|\} basis has more than \( d \) nonzero entries.

Thus we have a large class of states that contain secure key and are at the same time PPT invariant states. Of course they are entangled since entanglement is a precondition of secure key distillation [13].

**Observation 1** The ratio of \( p_1 \) and \( p_2 \) in [11] which is related to \( K_{DW} \), key rate achieves the highest value
for unimodular unitaries $U$ (ie such that $|u_{ij}| = \frac{1}{\sqrt{d}}$ irrespectively of indices $\{i,j\}$). Then it amounts to $[p_1]_{optimal} = \sqrt{d}$.

Indeed, by use of Lagrange multipliers with slightly more general constraints $\sum_{ij} |u_{ij}|^2 = d$ we get that optimal $U$ is unimodular.

Example of small bound entangled states with $K_D > 0$ on $4 \otimes 4$ system. Putting $d = 2$ we get the smallest secure BE states in our construction. An easy example is a state with $U$ equal to 1-qubit Hadamard gate (H).

Note, that in this case, the state $\gamma_2$ coincides with the so called “flower state” with $U = H$, which exhibited locking of entanglement cost $[10]$. The total state can be written as a mixture of Bell states on $AB$ subsystem of state, that are classically correlated with some other states on $A'B'$. Namely we have

$$\rho_H = \sum_i q_i |\psi_i\rangle_{AB} \otimes \rho_A^{(i)}_{AB'},$$

(13)

where the correlated states are the following:

$$\rho^{(0)} = \frac{1}{2} [P_{\psi_0} + P_{\psi_2}]$$

$$\rho^{(1)} = \frac{1}{2} [P_{\psi_1} + P_{\psi_3}]$$

$$\rho^{(2,3)} = P_{\chi_{\pm}},$$

(14)

with $P_{\psi_i}$ being projectors onto corresponding Bell states and $P_{\chi_{\pm}}$ projectors onto pure states

$$\chi_{\pm} = \frac{1}{2} (\sqrt{2 \pm \sqrt{2}} |00\rangle \pm \sqrt{2 \mp \sqrt{2}} |11\rangle)$$

(15)

respectively. The mixing distribution $\{q_i\}_{i=0}^3$ is $(\frac{1}{2}, \frac{1}{8}, \frac{1}{8}, \frac{1}{4})$. Since $d = 2$, one has $p_1 = \sqrt{\frac{7}{4+\sqrt{2}}}$, so by proposition 1 a positive key rate can be gained from this 4-qubit PPT state. It reads:

$$K_D^{DM}(\rho_H) = 1 - h(p_1) = 0.0213399$$

(16)

per copy of $\rho_H$. Note again, that it automatically means, that $\rho_H$ is bound entangled. Indeed we have shown, that state of that construction is PPT invariant, and it can not be separable, as it has non-zero distillable key. We show now the next property of this state.

Observation 2 The state $\rho_H$ is extremal in the set of PPT states.

$\rho_H$ is a mixture of form (10) with $X_1 = W_H$ and $X_2 = X_1^T$. It is straightforward to check, that any mixture with the weight of $\gamma_1$ different then value $p_1 = \sqrt{\frac{7}{4+\sqrt{2}}}$ leads to NPT state. In fact the same argument proves extremity of the state from our construction with $X = W_U$, if only $X$ is hermitian operator, and either $X$ or $X^T$ has some positive eigenvalue.

Although the states $\rho_U$ lay on the edge of the set of PPT states, basing on this family of states we are able to show, that the set of PPT key distillable states has non-zero volume in the set of all PPT $4 \otimes 4$ states.

Observation 3 The set of PPT distillable key of the form $\rho_U$ has nonzero volume in $4 \otimes 4$.

The proof of this observation bases on the fact, that Devetak-Winter lower bound (33) is continuous in $\rho$, as mutual information is continuous function of $\rho$. If we consider now $p_{mix} = (1 - p_{mix}^E)$, for suitably small $p_{mix}$ there exists suitably small $\epsilon > 0$ for which the ball of all states with the center at $\rho_U$ and radius $\epsilon$ lays within the set of PPT states. By continuity argument, one gets the thesis. In fact a similar argument gives this result in any dimension higher than $4 \otimes 4$.

Upper bound for $K_D$.- Similarly as private bit is analogue of singlet from entanglement distillation theory, the state $\rho$ of (11) is analogue of mixture of two singlets (which is actually just the $p$-squeezed state $\sigma_p$). The distillable entanglement of the latter state is just $1 - h(p_1)$, and moreover it is achieved, by coherent application of the DW protocol. Therefore there is a question, if $K_D(\rho)$ is just equal to $1 - h(p_1)$. In 10 11 we show that relative entropy of entanglement $E_r$ is an upper bound for $K_D$. We have analysed the $4 \otimes 4$ PPT state $\rho_H$, and have found a separable state which gives $E_r(\rho_H) \leq 0.116$.

Preparation of $\rho_U$ $4 \otimes 4$ states.- As it is shown in example 13 $\rho_U$ $4 \otimes 4$ states are only classically correlated along $AB$ versus $A'B'$ cut. Thus they can be created by preparing randomly according to $\{q_i\}$ distribution, separately two states: a Bell state $\psi_i$ (for $AB$ subsystem) and $\rho^{(i)}$ (for $A'B'$ subsystem). According to observation 2 $\rho_U$ lays on the boundary of PPT set, any small perturbation can destroy this property. However in spirit of observation 3, one can construct PPT states that are to some extent robust against perturbations.

Key distillability verification for experimentally prepared state.- We now address the question of verification whether a state prepared experimentally in many copies has nonzero distillable key. In the spirit of proof of the proposition 11 instead of estimating whole $\rho_{AB'A'B'}$ we suggest to estimate only few parameters of its privacy squeezed state $\sigma_{AB}$ and subsequently compute some lower bound on the value of DW rate (3) for the ccq state $\sigma_{ccq}^{(ccq)}$ of the latter.

Once we do not estimate whole state $\rho_{AB'A'B'}$, the formula 34 can not be used directly to decide its quality. Instead we first consider a lower bound for distillable key from ccq state. Namely

$$K_D(\rho^{(ccq)}) \geq I(A:B)_{\rho^{(ccq)}} - S(E)_{\rho^{(ccq)}},$$

(17)

which is a consequence of formula 34. Indeed, $I(A:E)$ for ccq state is equal to $S(\rho_E) - \sum_i p_i S(\rho_{Ei})$, which is Holevo function of ansamble $\{p_i, \rho_{Ei}\}$. This however can not be greater then just entropy of Eve’s subsystem $S(\rho_E)$ and the assertion follows.

Using this lower bound we provide now the one which is a function of only diagonal and antidiagonal matrix elements of (2-qubit) privacy squeezed state $\sigma_{AB}$. Note,
that \( S(\sigma_E) = S(\sigma_{AB}) \), as the total \( ABE \) state is pure. To estimate \( S(\sigma_E) \) we will consider state \( \sigma_{AB} \) subjected to twirling \( U \) which projects onto Bell basis. Twirling can not decrease the entropy, and commutes with measurement in computational basis, so one has that

\[
K_D^{(\sigma_{AB})} (\sigma_{AB}^{twir}) \geq I(A : B) - S(\sigma_{AB}^{twir})
\]

This is desired lower bound as it is a function of only those parameters, which we suggest to estimate experimentally.

Although the formula is useful for one-way key distillable states, knowing the diagonal and antidiagonal elements of \( \sigma_{AB} \) is enough to decide if a two-way recurrence protocol can make key rate non-zero.

We give now the observables, which measured on \( \rho_{ABA'B'} \) reveals desired elements of its p-squeezed state. These are:

\[
\begin{align*}
O_1 &= U_1^\dagger \{ \sigma_x \otimes \sigma_z \}_{AB} \otimes I_{A'B'} U_\tau \\
R_{1,2} &= U_1^\dagger \{ P_{\psi_0,2} - P_{\psi_1,1} \}_{AB \otimes I_{A'B'}} U_\tau \\
I_{1,2} &= U_1^\dagger \{ P_{\psi_0,2} - P_{\psi_1,1} \}_{AB \otimes I_{A'B'}} U_\tau
\end{align*}
\]

where \( P_{\psi} \) are Bell states with relative phase \( \pm i \). The observable \( O_1 \) reveals the diagonal elements of state \( \sigma_{AB} \). In fact, it is just equal to \( \{ \sigma_x \otimes \sigma_z \}_{AB} \otimes I_{A'B'} \) as any twisting (here \( U_1 \)) commutes with the measurement in basis which it controls. \( O_1 \) needs therefore just one setting. The \( R_{k,1} (I_k) \) observables reveals real (imaginary) parts of (possibly complex) coherences on antidiagonal of \( \sigma_{AB} \). Indeed, one has for example

\[
\begin{align*}
\text{Tr} R_1 \rho_{ABA'B'} &= \\
\text{Tr} U_1^\dagger \{ P_{\psi_0} - P_{\psi_1} \}_{AB \otimes I_{A'B'}} U_\tau \rho_{ABA'B'} = \\
\text{Tr} U_1^\dagger \{ P_{\psi_0} - P_{\psi_1} \}_{AB} \otimes I_{A'B'} U_\tau \rho_{ABA'B'} U_\tau^\dagger = \\
\text{Tr} [P_{\psi_0} - P_{\psi_1} |U_{A'B'} \rho_{ABA'B'} U_{A'B'}^\dagger = \\
\text{Tr} [P_{\psi_0} - P_{\psi_1}] |U_{A'B'} \rho_{ABA'B'} U_{A'B'}^\dagger = \\
\text{Tr} [P_{\psi_0} - P_{\psi_1}] \sigma_{AB}
\end{align*}
\]

where second equality is by property of trace, and third by definition of subsystem of a quantum state. Last equality uses definition of privacy squeezed state \( \sigma_{AB} \) which is obtained by acting on \( \rho_{ABA'B'} \) with some twisting \( U_\tau \) and tracing out \( A'B' \) subsystem.

Local decomposition of verification observables for \( \rho_H \). In case of \( \rho_H \) twisting which realises privacy squeezing is equal to:

\[
U_\tau = |00\rangle \langle 00| \otimes W_H + |01\rangle \langle 01| \otimes P_H + |10\rangle \langle 10| + |11\rangle \langle 11| \otimes I
\]

where by \( P_H = \sum_{ij} h_{ij} \langle i|i \rangle + \sum_{ij} h_{ij} \langle ij |ij \rangle \) for \( h_{ij} \) elements of Hadamard and \( W_H = P_H^\dagger \). For this \( U_\tau \), the observables \( R_1 \) and \( I_1 \) can be decomposed into Pauli operators \( \{ I, \sigma_x, \sigma_y, \sigma_z \} \) in the following way:

\[
R_1 = \frac{1}{4} [\sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y]_{AB} \otimes \\
\otimes [(I \otimes \sigma_z + \sigma_z \otimes I) + (\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y)]_{A'B'}
\]

\[
I_1 = - \frac{1}{4} [\sigma_x \otimes \sigma_y + \sigma_y \otimes \sigma_x]_{AB} \otimes \\
\otimes [(I \otimes \sigma_z + \sigma_z \otimes I) + (\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y)]_{A'B'}
\]

and the same for \( R_2 \) and \( I_2 \), which read

\[
R_2 = \frac{1}{4} [\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y]_{AB} \otimes [(I \otimes I - \sigma_z \otimes \sigma_z) + \\
\frac{1}{\sqrt{2}} \{|I \otimes I - \sigma_z \otimes \sigma_z + (\sigma_y \otimes \sigma_y - \sigma_x \otimes \sigma_x)\}|_{A'B'}
\]

\[
I_2 = - \frac{1}{4} [\sigma_y \otimes \sigma_x - \sigma_x \otimes \sigma_y]_{AB} \otimes [(I \otimes I - \sigma_z \otimes \sigma_z) + \\
\frac{1}{\sqrt{2}} \{|I \otimes I - \sigma_z \otimes \sigma_z + (\sigma_y \otimes \sigma_y - \sigma_x \otimes \sigma_x)\}|_{A'B'}
\]

Generalizing approach for two-qubit case of to four qubit case, one can easily show, that decomposition for \( R_2 \) and \( I_2 \) is optimal in the sense, that it needs 6 settings. The set of these 6 collective settings is enough for both \( R_2, I_2 \) and \( R_1, I_1 \) (for the latter it needs only different classical post-processing). Since this set of settings is optimal for determining \( R_2, I_2 \) and suffices for determining all \( R_1, I_1 \), we conclude, that it is optimal for our task. Together with one setting for \( O_1 \) one needs 7 different collective settings to verify with lower bound if the state \( \rho_H \) has non zero distillable key.

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[1] D. Bouwmeester, A. K. Ekert, and A. Zellinger, The physics of quantum information : quantum cryptography, quantum teleportation, quantum computation (Springer, New York, 2000).
[2] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
[3] G. Alber, T. Beth, M. Horodecki, P. Horodecki, R. Horodecki, M. Rotteler, H. Weinfurter, R. Werner, and A. Zellinger, Quantum Information: An Introduction to Basic Theoretical Concepts and Experiments (Springer, 2001).
[4] J. Gruska, Quantum Computing (McGraw-Hill, London, 1999).
[5] C. H. Bennett and G. Brassard, in Proceedings of the IEEE International Conference on Computers, Systems
and Signal Processing (IEEE Computer Society Press, New York, Bangalore, India, December 1984, 1984), pp. 175–179.

[6] A. Ekert, Phys. Rev. Lett 67, 661 (1991).

[7] P. W. Shor and J. Preskill, Phys. Rev. Lett 85, 441 (2000), quant-ph/0003004.

[8] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett 77, 2818 (1996), quant-ph/9604039.

[9] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. Smolin, and W. K. Wootters, Phys. Rev. Lett 78, 2031 (1997), quant-ph/9511027.

[10] K. Horodecki, M. Horodecki, P. Horodecki, and J. Oppenheim, Phys. Rev. Lett 94, 160502 (2005), quant-ph/0309110.

[11] K. Horodecki, M. Horodecki, P. Horodecki, and J. Oppenheim, quant-ph/0506189.

[12] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett 80, 5239 (1998), quant-ph/9801069.

[13] I. Devetak and A. Winter (2003), quant-ph/0306078.

[14] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).

[15] M. Curty, M. Lewenstein, and N. Lütkenhaus, Phys. Rev. Lett 92, 217903 (2004), quant-ph/0307151.

[16] K. Horodecki, M. Horodecki, P. Horodecki, and J. Oppenheim, Phys. Rev. Lett 94, 200501 (2005).

[17] C. H. Bennett, D. P. DiVincenzo, J. Smolin, and W. K. Wootters, Phys. Rev. A 54, 3824 (1997), quant-ph/9604024.

[18] O. Guhne, P. Hyllus, D. Bruss, A. Ekert, M. Lewenstein, C. Macchiavello, and A. Sanpera, JMO 50 (6-7), 1079 (2003), quant-ph/0210134.

[19] O. Guhne and P. Hyllus, Int. J.Theor.Phys. 42, 1001 (2003), quant-ph/0301162.