NEUTRINO OSCILLATIONS: HIERARCHY QUESTION

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The only experimentally observed phenomenon that lies outside the standard model of the electroweak interaction is neutrino oscillations. A way to try to unify the extensive neutrino oscillation data is to add a phenomenological mass term to the Lagrangian that is not diagonal in the flavor basis. The goal is then to understand the world’s data in terms of the parameters of the mixing matrix and the differences between the squares of the masses of the neutrinos. An outstanding question is what is the correct ordering of the masses, the hierarchy question. We point out a broken symmetry relevant to this question, the symmetry of the simultaneous interchange of hierarchy and the sign of θ_{13}. We first present the results of an analysis of data that well determine the phenomenological parameters but are not sensitive to the hierarchy. We find \( θ_{13} = 0.152 \pm 0.014, θ_{23} = 0.25 \pm 0.03 \) π and \( Δ_{32} = 2.45 \pm 0.14 \times 10^{-3} \text{ eV}^2 \), results consistent with others. We then include data that are sensitive to the hierarchy and the sign of θ_{13}. We find, unlike others, four isolated minimum in the \( χ^2 \)-space as predicted by the symmetry. Now that Daya Bay and RENO have determined θ_{13} to be surprisingly large, the Super-K atmospheric data produce meaningful symmetry breaking such that the inverse hierarchy is preferred at the 97.2 \% level.

Keywords: neutrino, neutrino oscillations, mixing parameters
1. Introduction

Neutrinos undergo the phenomenon of flavor oscillations; a neutrino with a particular flavor (electron, mu, or tau) will change its flavor, a phenomenon not included in the standard model of the electroweak interaction. A possible explanation of this behavior is neutrino oscillations. The neutrinos are assigned masses. The created neutrino has definite flavor, but these flavor eigenstates are not the physical free particles, the mass states. For three neutrinos, this leads to a phenomenology that is parameterized by three mixing angles, a CP violating phase, and two mass-squared differences, $\Delta_{ij} =: m_i^2 - m_j^2$. A goal of the field is to unite the experimental data through the determination of these parameters.

Neutrino oscillations approximately break into two uncoupled oscillations, solar and atmospheric oscillations. The solar oscillations are predominantly governed by the solar data that determine the mixing angle $\theta_{12}$ and the KamLAND reactor data that determine the mass-squared difference $\Delta_{21}$. We fix these two parameters to values determined by others. With them fixed, we perform a full three-neutrino analysis.

An outstanding question in neutrino phenomenology is that of hierarchy,
whether the small mass-squared difference lies below (normal hierarchy) or above (inverse hierarchy) the large mass-squared difference. A related question, as we will show, is whether the value of $\theta_{13}$ is positive or negative. We utilize the convention on the bounds of $\theta_{13}$ of $-\frac{\pi}{2} \leq \theta_{13} < +\frac{\pi}{2}$ and $0 \leq \delta < \pi$. These are the more convenient bounds for discussing the symmetry of interest here. Recent long-baseline reactor experiments have measured the value of $\theta_{13}$ and found it to be large, $\sin^2(2\theta_{13}) \approx 0.1$. Before these measurements, the world’s data demonstrated very little preference for either the hierarchy or the sign of $\theta_{13}$. A goal of this work is to examine the effect of the knowledge of $\theta_{13}$ on the hierarchy question and the sign of $\theta_{13}$. This report is preliminary in that we make use of the atmospheric analysis of Ref. 6 by including its constraints on $\theta_{23}$ and $\theta_{13}$ separately, ignoring correlations. No significant evidence for CP violation has been found so for now we set the CP phase $\delta$ to zero.

In Section 2 we discuss the symmetry that is relevant for the investigation of the hierarchy question. In Section 3 we present results of our analysis of data that do not have sensitivity to the hierarchy or the sign of $\theta_{13}$. We then include data that are sensitive to the hierarchy and the sign of $\theta_{13}$ in Section 4. We finish with a discussion of the significance of this work and point out future work.
2. Symmetry

Extracting neutrino mixing parameters from data is complicated by the existence of symmetries that lead to degeneracies,$^{10}$ i.e. different mixing parameters yield the same oscillation probabilities. The MINOS,$^{11}$ T2K,$^{12}$ and the future NO$
u$A$^{13}$ experiments are at an $L/E$ where such a broken symmetry exists. In the limit of $\theta_{13}=0$ and no matter effects, these experiments are insensitive to the hierarchy. The symmetry pertinent to these experiments is the simultaneous interchange of hierarchy and the sign of $\theta_{13}$, giving a four-fold degeneracy. In Fig. 1 we depict the vacuum oscillation probability $P_{\mu e}$ for the T2K experiment for full three-neutrino mixing and non-zero $\theta_{13}$. The four-fold degeneracy is partially broken by non-zero $\theta_{13}$. The solid curve represents both the normal hierarchy, positive $\theta_{13}$ as well as the inverse hierarchy, negative $\theta_{13}$ results while the dashed curve represents the inverse hierarchy, positive $\theta_{13}$ and the normal hierarchy, negative $\theta_{13}$ results. Non-zero $\theta_{13}$ breaks the four fold degeneracy down to two two-fold degeneracies.

We present, in Fig. 2, $P_{\mu e}$ versus neutrino energy including the MSW matter effects for the T2K experiment.$^{12}$ The matter interaction of the neutrino breaks the two fold symmetries by altering the magnitude of the
oscillations while leaving the position of the peaks nearly unchanged. Spectral information that measures the position of the peaks maintains approximately the two-fold degeneracy of the vacuum oscillations. The magnitude of the signal then breaks this symmetry to distinguish the hierarchy. The combination of T2K and NOνA will be particularly powerful as they have vastly different baselines and thus different matter effects. This will be helpful in disentangling the hierarchy, the sign of $\theta_{13}$, and CP violation.

3. Parameters

In this section we present the results of a global analysis that includes the following experiments: 1) The long-baseline muon disappearance experiments, MINOS neutrino, MINOS anti-neutrino and T2K. 2) The one kilometer baseline reactor experiments, Daya Bay and RENO. 3) The constraints on $\theta_{23}$ from Super-K as taken from the analysis of Ref. 6.

In Fig. 3 we present the results of our analysis for $\Delta_{32}$. Throughout this work we calculate $\chi^2(\theta_{13}, \theta_{23}, \Delta_{32})$ and marginalize over the non-displayed parameters. The solid blue curve is the full analysis. The long-baseline MINOS neutrino disappearance experiment dominates this results. The dashed red curve is the result excluding the MINOS anti-neutrino and T2K neutrino experiments. We see they move the minimum upwards. Our results are $\Delta_{32} = 2.45 \pm 0.14 \times 10^{-3}$ eV$^2$, errors are the 90% errors.

**Fig. 4.** $\chi^2$ versus $\theta_{13}$. The solid blue curve is the result using all the experiments mentioned in the text. The red dashed curve excludes RENO.
In Fig. 4 we present the results of our global analysis for the mixing angle $\theta_{13}$. The solid blue curve is the full analysis. Daya Bay dominates this result. The red dashed curve is the full analysis minus RENO. It is seen that RENO reduces the low error bar. Our results are $\theta_{13} = 0.152 \pm 0.014$.

In Fig. 5 we present the result of our global analysis for the mixing angle $\theta_{23}$. The solid blue curve is the full analysis. The Super-K solar results dominate this result. The red dashed curve is the full analysis minus the three long-baseline muon disappearance experiments. We see that they have little effect. Our result remains the same as Ref. 6, $\theta_{23} = 0.251^{+0.03}_{-0.05} \pi$.

4. Hierarchy

Two long-baseline experiments, the MINOS and T2K $\nu_{\mu} \rightarrow \nu_e$ experiments, were specifically designed to be at an $L/E$ where $P_{\mu e}$ is sensitive to the hierarchy, the sign of $\theta_{13}$, and CP violation. MINOS has taken its final data. We have not completed an analysis of this data and here present results from earlier data. This data was not taken with an off-axis beam causing the background to be much larger than the signal. Also, the value of $L/E$ for MINOS does not place it at the peaks seen in Figs. 1, 2. It is sensitive to the hierarchy only. The data from T2K contain only six counts. We add these two data sets to the data utilized in the previous
section and present the results in Fig. 6. The blue curves depict the normal hierarchy, red the inverse hierarchy. Solid curves represent positive $\theta_{13}$, dashed curves negative $\theta_{13}$. There is only sensitivity to the hierarchy, with the inverse hierarchy preferred at about one sigma. Since T2K is presently running and NO$\nu$A is just starting, the future looks quite promising.

Atmospheric data covers an extremely large range of $L/E$ values including those that discriminate between the hierarchies, the value and sign of $\theta_{13}$, and the amount of CP violation. In Fig. 7 we present $\chi^2$ versus $\theta_{13}$ for Super-K as taken from Ref. 6. The blue curve is the normal hierarchy while the red curve is the inverse hierarchy. The inverse hierarchy is preferred over the normal hierarchy. The source of this is the lack of excess electron neutrinos in the energy region from 3 to 7 GeV. This is the region where there are MSW matter resonances for the normal hierarchy but not for the inverse hierarchy. The lack of an excess of electron neutrinos implies no resonances and hence favor the inverse hierarchy. The effect is quadratic in $\theta_{13}$. The statistics at these high energies are presently not so good. With the now known large value of $\theta_{13}$, should the lack of excess electron neutrinos persist in future data, the atmospheric data alone could prove that the inverse hierarchy is the correct choice.

The linear terms in $\theta_{13}$ occur when there is interference between the two

![Fig. 6. $\chi^2$ versus $\theta_{13}$ for all of the experiments as explained in the text plus the two long-baseline muon appearance experiments, MINOS and T2K. The blue curves are the normal hierarchy, the red the inverse hierarchy. The solid curves are positive $\theta_{13}$, the dashed negative $\theta_{13}$.](image-url)
mass-squared differences. This extends from the $L/E$ of T2K and NOνA of 590 km/GeV to where the effect is maximum\textsuperscript{20} at about $1.5 \times 10^4$ km/GeV. For atmospheric data this occurs for data near the largest $L$ and the smallest $E$. Note that the slope of the linear terms (the inverse hierarchy is dominated by the linear terms) also favors inverse hierarchy, plus favor negative $\theta_{13}$.

We, unlike others, find four distinct and isolated minima, one for each value of the hierarchy and the sign of $\theta_{13}$ as implied by the symmetry. In Fig. 8 we present the values of $\chi^2$ versus $\theta_{13}$ for the four cases. We find that the negative hierarchy is favored over the positive hierarchy and that negative $\theta_{13}$ is favored over positive $\theta_{13}$. The probability that each of the four probabilities is correct is given in Table I. We use the Bayesian method proposed in Ref. 21 to obtain these probabilities.

5. Conclusions

We have investigated the implications of the recent knowledge of the value of $\theta_{13}$ on the determination of the physical hierarchy and the related question of the sign of $\theta_{13}$. We find that the inverse hierarchy is preferred at the 97.2\% level, while negative $\theta_{13}$ is preferred at the 92.5\% level. The sensitivity that leads to these conclusions arises from the Super-K atmospheric data.

We are in disagreement with the results from Refs. 8 and 9, which differ
Fig. 8. $\chi^2$ versus $\theta_{13}$ for the four solutions corresponding to hierarchy and the sign of $\theta_{13}$. The blue curves depict the normal hierarchy, the red curves the inverse hierarchy. The solid curves depict positive $\theta_{13}$, the dashed curves negative $\theta_{13}$.

from each other. We can only speculate on the source of the differences. That we find four distinct minima in the $\chi^2$-space as implied by the symmetry while others do not is encouraging for us. This could be caused by the use of the mass-squared dominance approximation that, as we have pointed out,\cite{7,19,20} does not converge to the correct answer in precisely the region where there is interference between the two mass-squared difference oscillations. Alternatively, the second minimum for each hierarchy could have accidentally been overlooked. The preference for the inverse hierarchy comes from the lack of excess electron neutrinos in the region of the MSW matter resonances. A lack of excess electron neutrinos implies a lack

| hierarchy | sign $\theta_{13}$ | $\theta_{13}$ | % probable |
|-----------|-------------------|--------------|------------|
| normal    | $+$               | $0.148 \pm 0.015$ | 0.3 %      |
| normal    | $-$               | $0.151 \pm 0.013$ | 2.5 %      |
| inverse   | $+$               | $0.152 \pm 0.013$ | 7.2 %      |
| inverse   | $-$               | $0.153 \pm 0.014$ | 90.0 %     |
of such resonances which favors the inverse hierarchy, something that has
been known for some time.

We previously presented\textsuperscript{22} an even more preliminary result that is consistent with the present results. The upgraded work here incorporates the latest Daya Bay data, and more importantly marginalizes over parameters rather than fixing them. We still need to update the two MINOS experiments to include data that came out after this talk. Most importantly, we are reviewing our atmospheric analysis. It is this data that produce the different results for each of the four solutions. We will also add CP violation.

References
1. D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. D} \textbf{71}, 017301 (2005).
2. Y. Abe \textit{et al.} [Double-Chooz Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 131801 (2012).
3. F. P. An [Daya Bay Collaboration], arXiv:1210.6327 [hep-ex] (2012).
4. J. Ahn \textit{et al.} [RENO Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 191802 (2012).
5. J. Escamilla-Roa, D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. C} \textbf{82}, 028501 (2010).
6. J. E. Roa, D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. Lett.} \textbf{103}, 061804 (2009); J. Escamilla-Roa, D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. C} \textbf{81}, 015501 (2010).
7. D. C. Lattimer and D. J. Ernst, \textit{Phys. Rev. C} \textbf{72}, 045502 (2005).
8. D. V. Forereou, M. Tortóla and J. W. F. Valle, \textit{Phys. Rev. D} \textbf{86}, 073012 (2012).
9. G. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunnon, \textit{Phys. Rev. D} \textbf{86}, 103012 (2012).
10. V. Barger, D. Marfatia and K. Wisnant, \textit{Phys. Rev. D} \textbf{65}, 073023 (2002); D. C. Latimer and D. J. Ernst, \textit{Mod. Phys. Lett. A} \textbf{20}, 1663 (2005).
11. P. Adamson \textit{et al.} [MINOS Collaboration], \textit{Phys. Rev. Lett.} \textbf{107}, 181802 (2011).
12. K. Abe \textit{et al.} [T2K Collaboration], \textit{Phys. Rev. Lett.} \textbf{107}, 041801 (2011).
13. J. Nowak [NO\nu A Collaboration], \textit{AIP Conf. Proc.} \textbf{1441}, 423 (2012).
14. P. Adamson \textit{et al.} [MINOS Collaboration], \textit{Phys. Rev. Lett.} \textbf{106}, 181801 (2011).
15. P. Adamson \textit{et al.} [MINOS Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 191801 (2012).
16. Y. Abe \textit{et al.} [T2K Collaboration], \textit{Phys. Rev. D} \textbf{85}, 031103 (2012).
17. J. Hosaka \textit{et al.} [Super-K Collaboration], \textit{Phys. Rev. D} \textbf{74}, 032002 (2006).
18. P. Adamson \textit{et al.} [MINOS Collaboration], arXiv:1301.4581 [hep-ex] (2013).
19. D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. C} \textbf{71}, 062501 (2005).
20. D. C. Latimer, J. Escamilla and D. J. Ernst, \textit{Phys. Rev. C} \textbf{76}, 055502 (2007).
21. H. R. Burroughs, B. K. Cogswell, J. Escamilla-Roa, D. C. Latimer and D. J. Ernst, \textit{Phys. Rev. C} \textbf{85}, 068501 (2012).
22. D. J. Ernst, B. K. Cogswell, H. R. Burroughs, J. Escamilla-Roa and D. C. Latimer, \textit{J. Phys. Conf. Ser.} \textbf{403}, 012040 (2012).