Oblique Corrections in Deconstructed Higgsless Models

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ABSTRACT

In this talk, using deconstruction, we analyze the form of the corrections to the electroweak interactions in a large class of “Higgsless” models of electroweak symmetry breaking, allowing for arbitrary 5-D geometry, position-dependent gauge coupling, and brane kinetic energy terms. Many models considered in the literature, including those most likely to be phenomenologically viable, are in this class. By analyzing the asymptotic behavior of the correlation function of gauge currents at high momentum, we extract the exact form of the relevant correlation functions at tree-level and compute the corrections to precision electroweak observables in terms of the spectrum of heavy vector bosons. We determine when nonoblique corrections due to the interactions of fermions with the heavy vector bosons become important, and specify the form such interactions can take. In particular we find that in this class of models, so long as the theory remains unitary, $S - 4 \cos^2 \theta_W T > O(1)$, where $S$ and $T$ are the usual oblique parameters.

1. The Model and Its Relatives

Recently, “Higgsless” models of electroweak symmetry breaking have been proposed [2]. Based on five-dimensional gauge theories compactified on an interval, these models achieve unitarity of electroweak boson self-interactions through the exchange of a tower of massive vector bosons [3]. Precision electroweak constraints [4] arising from corrections to the $W$ and $Z$ propagators in these models have previously been investigated [5] in the continuum and, in the case of weak coupling, [6,7] using deconstruction [8].

The model we analyze, shown diagrammatically (using “moose notation” [9,8]) in Fig. 1, incorporates an $SU(2)^{N+1} \times U(1)^{M+1}$ gauge group, and $N + 1$ nonlinear ($SU(2) \times U(1)$) nonlinear terms.

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$a$Talk presented by M.K. at SUSY 2004: The 12th International Conference on Supersymmetry and Unification of Fundamental Interactions, held at Epochal Tsukuba, Tsukuba, Japan, June 17-23, 2004. This talk is based on the work done in [1].
Figure 1: Moose diagram for the class of models analyzed in this talk. $SU(2)$ gauge groups are shown as open circles; $U(1)$ gauge groups as shaded circles. The fermions couple to gauge groups 0 and $N + 1$. The values of the gauge couplings $g_i$ and $f$-constants $f_i$ are arbitrary.

$SU(2))/SU(2)$ sigma models adjacent to $M$ ($U(1) \times U(1))/U(1)$ sigma models in which the global symmetry groups in adjacent sigma models are identified with the corresponding factors of the gauge group. The Lagrangian for this model at $O(p^2)$ is given by

$$L_2 = \frac{1}{4} \sum_{j=1}^{N+M+1} f_j^2 \text{tr} \left( (D_\mu U_j)^\dagger (D^\mu U_j) \right) - \frac{1}{2g_j^2} \sum_{j=0}^{N+M+1} \text{tr} \left( F^j_{\mu\nu} F^{j\mu\nu} \right),$$

with

$$D_\mu U_j = \partial_\mu U_j - iA_j^{\mu-1}U_j + iU_jA_j^\mu,$$

where all gauge fields $A^j_\mu$ ($j = 0, 1, 2, \ldots, N + M + 1$) are dynamical. The first $N + 1$ gauge fields ($j = 0, 1, \ldots, N$) correspond to $SU(2)$ gauge groups; the other $M + 1$ gauge fields ($j = N + 1, N + 2, \ldots, N + M + 1$) correspond to $U(1)$ gauge groups.

The fermions in this model take their weak interactions from the $SU(2)$ group at $j = 0$ and their hypercharge interactions from the $U(1)$ group with $j = N + 1$, at the interface between the $SU(2)$ and $U(1)$ groups. The neutral and charged current couplings to the fermions are thus written as

$$J_3^\mu A_0^\mu + J_\mu^\mu A_{N+1}^\mu, \quad \frac{1}{\sqrt{2}} J_{\pm}^\mu A_0^{\mu\mp}. \tag{3}$$

This model includes the model analyzed in [6,7] and a deconstructed version of models in [5]. In our analysis, generalizing [7], we leave the gauge couplings and $f$-constants of the deconstructed model arbitrary and parameterize the electroweak corrections in terms of the masses of the heavy vector bosons. We therefore obtain results which describe arbitrary 5-D geometry, position-dependent couplings, or brane kinetic energy terms.

2. The Low-Energy $\rho$ Parameter

In discussing low-energy interactions it is conventional to write the neutral-current Lagrangian in terms of weak and electromagnetic currents as

$$L_{nc} = -\frac{1}{2} A(Q^2)J_3^\mu J_{3\mu} - B(Q^2)J_0^\mu J_{Q\mu} - \frac{1}{2} C(Q^2) J_Q^\mu J_{Q\mu}. \tag{4}$$

On the other hand, the charged-current Lagrangian is written as

$$L_{cc} = -\frac{1}{2} [G_{CC}(Q^2)]_{\mu\nu} J_\mu^\mu J_{\nu}. \tag{5}$$
The coefficients $A(Q^2), B(Q^2), C(Q^2)$ and $[G_{CC}(Q^2)]_{WW}$ are obtained by calculating the neutral and the charged gauge field propagator matrices. From the explicit form of these coefficients, we find \[1\] that the low-energy $\rho$ parameter is identically 1 in the present model:

$$\rho \equiv \frac{A(Q^2 = 0)}{[G_{CC}(Q^2 = 0)]_{WW}} = 1.$$  \hspace{1cm} (6)

This is an extension of the result in \[10\] and is due to our specifying that the fermion hypercharges arise from the $U(1)$ group with $j = N + 1$. \footnote{In \[11\], the linear moose model with fermions coupling to the $p$th ($0 < p < N + 1$) $SU(2)$ gauge group and the $N + 1$st $U(1)$ group was analyzed. We can see that $\rho = 1$ is satisfied for arbitrary $p$ due to our specifying that the fermion hypercharges arise from the $U(1)$ group with $j = N + 1$.}

### 3. Electroweak Parameters

From the calculation of the gauge boson propagator and corresponding residues, we see that the size of the coupling of the heavy resonances to fermions depends crucially on the isospin asymmetry of these states. (See the explicit form of residues in \[1\].)

In the case of large isospin violation, we expect that the exchange of heavy neutral resonances may be important and result in an extra four-fermion contribution to low-energy exchange proportional to $J^\mu J^\nu$. In the case of small isospin violation, we also expect that the exchange of heavy resonances may be important and result in an extra four-fermion contribution to both low-energy neutral and charged current exchange proportional to the square of the weak $SU(2)$ currents $\vec{J} \cdot \vec{J}$. Given the constraint that the low-energy $\rho$ parameter must equal to one, the matrix element for four-fermion neutral and charged current processes are characterized by

$$M_{NC} = e^2 \frac{Q' Q}{Q^2} + \frac{(I_3 - s^2 Q)(I'_3 - s^2 Q')}{\left(\frac{s^2 c^2}{e^2} - \frac{S}{16 \pi}\right) Q^2} + \frac{1}{4 \sqrt{2} G_F} \left(1 - \alpha T e^2 + \frac{\alpha \delta}{4 s^2 c^2}\right)$$

$$+ \sqrt{2} G_F \frac{\alpha \delta}{s^2 c^2} I_3 I'_3 - 4 \sqrt{2} G_F \alpha T (I_3 - s^2 Q)(I'_3 - s^2 Q'), \hspace{1cm} (7)$$

$$M_{CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{\left(\frac{s^2}{e^2} - \frac{S + U}{16 \pi}\right) Q^2} + \frac{1}{4 \sqrt{2} G_F} \left(1 + \frac{\alpha \delta}{4 s^2 c^2}\right)$$

$$+ \sqrt{2} G_F \frac{\alpha \delta}{s^2 c^2} \frac{(I_+ I'_- + I_- I'_+)/2}{2}. \hspace{1cm} (8)$$

Here $I_a^{(l)}$ and $Q^{(l)}$ are weak isospin and charge of the corresponding fermion, $\alpha = e^2/4\pi$, $G_F$ is the usual Fermi constant, and the weak mixing angle (as defined by the on-shell $Z$ coupling) is denoted by $s^2$ ($c^2 \equiv 1 - s^2$). From these analysis, we can see when nonoblique corrections become important, and we can estimate the size of them.

We can obtain expressions for $S, T,$ and $U$ by comparing the residues as calculated from \[7\], \[8\] with their values as calculated in terms of the gauge boson spectrum. Independent of the amount of isospin violation, we have found the following constraint on the
oblique electroweak parameters $S$ and $T$

$$\alpha S - 4 \cos^2 \theta_W \alpha T = 4 \sin^2 \theta_W \cos^2 \theta_W M^2 W \sum_{n=1}^{N} \frac{1}{M^2 W n},$$

(9)

where $M_{W n}$ is the mass of the $n$th heavy charged resonances. In any unitary theory, we expect the mass of the lightest vector $M_{W 1}$ to be less than $\sqrt{8 \pi v}$ ($v \approx 246$ GeV). Evaluating eqn. (9) we see that we expect $S - 4 \cos^2 \theta_W T$ to be of order one-half or larger, generalizing the result of [7].

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