Torsion driven Inflationary Magnetogenesis

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Abstract

We show that breaking of the conformal invariance of electromagnetic Lagrangian which is required for inflationary magnetogenesis arises naturally in the Poincaré Gauge Theory. We find that in the minimal version of this model the generated magnetic field is too small to explain the observations. We propose some generalizations of the minimal model which may lead to sufficiently large magnetic fields.
I. INTRODUCTION

Magnetic fields are found in almost all bound structures, such as, galaxies, stars, star clusters, and even some planets like Earth [1–3]. Recent observational evidences [4–7] suggest that even cosmic voids contain magnetic fields of order $10^{-16}$ Gauss with correlation lengths of 1 Mpc or more. The upper limit on the magnetic field on such scales is of order $10^{-9}$ Gauss [8]. Such large scale correlation is hard to explain by any astrophysical process. This is because one would expect astrophysical processes to generate these fields during radiation domination phase and small Hubble radius during this epoch cannot account for such large scale correlations. Even large scale magnetic fields in galaxies [9] and clusters of galaxies in the distant past are not easy to explain. One possibility is that these fields are actually a relic from the inflationary era [10–12]. That explains why they are supposedly present in the voids. The same mechanism also explains the origin of strong magnetic field in bound structures by serving as the seed field that was amplified via dynamo mechanism [13] in these structures.

There exist many models of inflationary magnetogenesis in which the magnetic field is amplified many folds during inflation [10, 14–22]. All these model break conformal invariance of electromagnetic Lagrangian which is required to amplify the vacuum fluctuations of electromagnetic fields. In the model developed by Ratra [14], the electromagnetic Lagrangian is coupled with the inflaton field to break the conformal invariance. It turns out that for some parameter values this model suffers from strong coupling or the back reaction problem [23, 24]. There exist several proposals [25–27] to generate cosmic magnetic fields of the required strength without facing either of these problems, if conformal invariance is broken during inflation. However there also exist additional constraints from CMB which impose further limits on the magnetic field that can be generated [28–30].

Although there exist several proposals [10, 15–22] which break conformal invariance, these do not explain the coupling between electromagnetic and scalar field assumed in the Ratra model [14]. So far this coupling has been put in by hand. The only exception is the string inspired model [16] which has some similarity to the Ratra model. We propose that such coupling naturally arises in Poincaré Gauge Theory, when torsion, a dynamical variable of the theory is taken into account. It turns out that by a specific choice of torsion, sourced by a scalar field, referred to as “tlaplon” in the literature [31], one can create a scenario where both inflation and magnetogenesis is driven by the tlaplon itself.

The article is structured in the following manner. In Section (II), we discuss in brief the possibility of primordial origin of magnetic fields, the Ratra’s model [14] and its drawbacks. This is followed by a brief introduction to the Poincaré Gauge Theory of Gravity in Section (III). It is found that generically coupled torsion (a dynamical variable in Poincaré Gauge Theory) to electromagnetic field breaks gauge invariance. But this can be avoided by choosing a restricted form of torsion, as has been suggested in [31]. This choice of torsion leads to some problems as has been pointed out in [32]. In later sections we see that how these and similar problems can be evaded. Finally, in Section (VI) we see how magnetogenesis can be brought about by assuming the restricted form.

II. PRIMORDIAL ORIGIN OF MAGNETIC FIELDS

The basic idea is that the magnetic fields are a relic of inflation. This means that the vacuum fluctuations of the electromagnetic field get amplified during inflation and subsequently became classical fluctuations in the later phases of the evolution of the universe. This inflationary paradigm
also accounts for the origin of classical density perturbations [33]. However, it is known that the electromagnetic Lagrangian is conformally invariant which is tantamount to saying that the equations of motion for the fields are not modified by curvature induced due to a metric that is conformally related to the Minkowski metric. Since FRW metric is conformally related to the Minkowski metric, hence no amplification of vacuum fluctuations can take place in this case. In fact, due to spatial expansion, the energy density of the electromagnetic field varies as $a^{-4}$. Hence, one needs to break the conformal invariance of electromagnetism if one wants to bring about any amplification.

In [14] Ratra proposed a coupling of the form

$$\sqrt{-g}e^{2\alpha \phi}F^{\mu\nu}F_{\mu\nu}$$

(II.1)

between the inflaton field ($\phi$) and electromagnetic field $F_{\mu\nu}$ with $\alpha$ as a free parameter. He further showed that under slow roll inflation conditions, this causes amplification of fields. However, it appears to pose some problems. For slow roll inflation, let us assume that $e^{\alpha \phi}$ evolves as $a^\beta$. It turns out that sufficient amount of scale invariant amplification is achieved when $\beta$ is either 2 or $-3$ [25]. The physical electronic charge scales as $1/a^\beta$. Assuming that it is of order unity by the end of inflation, for $\beta = 2$ it will necessarily be very large at the beginning of inflation. Hence the results cannot be trusted since they lie in the strong coupling regime which cannot be handled reliably.

For $\beta = -3$ case, the electric field grows too fast and back reacts, due to which it’s energy density becomes comparable to the inflaton field before the end of inflation. In fact electric field shows this tendency for any $\beta < -2$. Hence, we either end up facing the strong coupling or back reaction or not enough amplification for some of the values in between. A nice description of both these problems can be found in [25]. In an alternate treatment of this problem, however, it is possible to get sufficiently strong magnetic fields with no back reaction [26, 27]. In our analysis we shall follow the formalism developed in these papers.

### III. A BRIEF REVIEW OF POINCARÉ GAUGE THEORY

The three fundamental interactions – weak, electromagnetic and strong are described within the framework of relativistic quantum field theories on flat Minkowski spacetime. These quantum fields reside in the spacetime but are aloof from it [34]. Moreover all of these theories are gauge theories. Gravity seems to be different from these in that (a) it is a classical theory & (b) it is based on the deformation the spacetime itself. Upon quantization, one faces the problem of non-renormalizability [35, 36]. It is suggested that gauging, in addition to providing many other features, might provide a renormalizable version of gravity [37]. Besides these reasons, it is natural to inquire whether gravitation can also be based on local gauge invariance [36, 38]. In addition to this it has been shown in the singularity theorems of Hawking and Penrose that a cosmology based on Riemannian geometry would either force universe to fall into or come out of singularity. The simplest way of avoiding such consequences is to assume non-Riemannian geometries [37, 39]. Poincaré gauge theory based on the local gauge invariance of the Poincaré group provides one such paradigm.

Mathematically speaking, Poincaré group is a semi direct product[40] of the spacetime translations and the Lorentz group, i.e., $P(1,3) \equiv SO(1,3) \rtimes T(1,3)$ [41]. When we gauge this group, we find that instead of one, two gauge fields are obtained. Historically, for the first time Utiyama applied gauge principle to generate gravitational interactions [42] by gauging the Lorentz group.
Later Kibble gauged the whole Poincaré group [38]. Here we must emphasize that different space-time geometries are obtained upon gauging different groups. For example if one considers only the translational group then one obtains Weitzenböck geometry, a geometry with torsion but no curvature [43]. Poincaré gauge theory also assumes the metricity condition:

\[ \nabla_\mu g_{\rho\sigma} = \partial_\mu g_{\rho\sigma} - \Gamma^\tau_{\mu\rho} g_{\tau\sigma} - \Gamma^\tau_{\mu\sigma} g_{\rho\tau} = 0, \]  

(III.1)

here the covariant derivative is taken w.r.t. the affine connection \( \Gamma^\alpha_{\beta\gamma} \). We point out that there are more general classes of theories known as metric affine theories where even metricity condition (i.e. Eq. III.1) isn’t assumed. The reader is referred to Refs. [37, 44] for more details.

According to general relativity, Minkowskian spacetime in the presence of matter becomes Riemannian. The geometry of a manifold is encoded in the connection \( \Gamma^\alpha_{\beta\gamma} \) which in general can be written as [45]

\[ \Gamma^\alpha_{\beta\gamma} = \dot{\Gamma}^\alpha_{\beta\gamma} + K^\alpha_{\beta\gamma}, \]  

(III.2)

here the quantity \( K^\alpha_{\beta\gamma} \) is called the contortion and \( \dot{\Gamma}^\alpha_{\beta\gamma} \) is the usual Christoffel symbol. Furthermore, the quantity

\[ T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta}, \]  

(III.3)

is called the torsion, which is clearly antisymmetric w.r.t. to its last two indices. In addition to the condition III.1, we also assume \( \dot{\nabla}_\mu g_{\rho\sigma} = 0 \). This, together with \( \dot{\Gamma}^\alpha_{\beta\gamma} = \dot{\Gamma}^\alpha_{\gamma\beta} \) allows us to solve for \( \dot{\Gamma}^\alpha_{\beta\gamma} \) in terms of the derivative of the metric tensor. Further more using \( \dot{\nabla}_\mu g_{\rho\sigma} = 0 \) in Eq. III.1 gives the symmetry property of the contortion tensor

\[ K_{\sigma\mu\rho} = -K_{\rho\mu\sigma}, \]  

(III.4)

thus the contortion tensor is antisymmetric with respect to its first and third indices. Next, using Eq. (III.2) in (III.3) we get

\[ T^\alpha_{\beta\gamma} = K^\alpha_{\beta\gamma} - K^\alpha_{\gamma\beta}. \]  

(III.5)

Equation III.5 using condition III.4 can be inverted and contortion can be expressed in terms of torsion as follows

\[ K^\alpha_{\beta\gamma} = \frac{1}{2} \left( T^\alpha_{\beta\gamma} + T^\alpha_{\gamma\beta} + T^\alpha_{\gamma\beta} \right). \]  

(III.6)

In general relativity the connection is torsion free but on account of gauging the Poincaré group it becomes endowed with torsion and spacetime manifold becomes Riemann Cartan [41, 43]. So if we consider Lagrangian proportional to Ricci scalar, which can be written as [46]

\[ R = \dot{R} + \left[ 2 \nabla_\rho K^{\rho\sigma}_{\alpha\beta} + K^{\rho\alpha\nu} K_{\alpha\rho\nu} - K^{\rho}_{\rho\alpha} K^{\alpha\sigma}_{\gamma} \right], \]  

(III.7)

then \( R \) gets contribution from torsion as well. The resulting theory is called Einstein-Cartan-Sciama-Kibble (ECSK) Theory [39]. This is a theory which approximately resembles Einstein’s general relativity but also predicts additional effects which arise due to torsion.

**IV. MINIMAL COUPLING AND TORSION IN POINCARE GAUGE THEORY**

The effects of torsion are expected to be very small in the weak field limit. However these might play a significant role in the early Universe [47, 48]. Introducing electromagnetism and other
gauge interactions in this framework turns out to be difficult because it breaks gauge invariance. As a remedy, a minimal coupling procedure has been prescribed in [31]. This model, however, is ruled out by solar observations [32]. We will see how by coupling torsion with non-abelian gauge fields this problem can be evaded. We also find it fascinating that the model leads to precisely the same form of interaction (i.e., Eq. II.1) that was proposed by Ratra.

A. Torsion and Electromagnetism

In the presence of torsion, the electromagnetic gauge covariant derivative can be expressed as

\[
\nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\alpha_{\mu \nu} A_\alpha = \partial_\mu A_\nu - \tilde{\Gamma}^\alpha_{\mu \nu} A_\alpha - K^\alpha_{\mu \nu} A_\alpha = \tilde{\nabla}_\mu A_\nu - K^\alpha_{\mu \nu} A_\alpha.
\]

Furthermore, using this, electromagnetic field tensor can be written as

\[
F_{\mu \nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu - T^\alpha_{\mu \nu} A_\alpha.
\]

Due to an extra term proportional to torsion, electromagnetic gauge invariance is not preserved. It has been suggested [31] that if we choose a specific form for torsion, and slightly modify the gauge transformation conditions, we can restore gauge invariance and still have a restricted version of torsion. We impose the following form of torsion

\[
T^\alpha_{\beta \gamma} = \delta^\alpha_{\gamma} \partial_\beta \phi - \delta^\alpha_{\beta} \partial_\gamma \phi,
\]

where \(\phi\) is scalar field. Using Eq. (IV.3) in (III.6), the contortion will have the following form

\[
K^\alpha_{\beta \gamma} = g_{\beta \gamma} \partial_\alpha \phi - \delta^\alpha_{\beta} \partial_\gamma \phi.
\]

Using this expression in Eq. (III.7), the Ricci scalar in terms of \(\phi\) is found to be

\[
R = \hat{R} - 6 \partial_\mu \phi \partial^\mu \phi + \frac{2}{\sqrt{-g}} \partial_\rho \left( \sqrt{-g} K^\rho_{\sigma} \right),
\]

where \(\hat{R}\) is the standard Ricci scalar computed using the Christoffel connection \(\tilde{\Gamma}^\alpha_{\beta \gamma}\). The gauge transformation gets modified to

\[
A_\mu \rightarrow A_\mu + e^\phi \partial_\mu \epsilon,
\]

where \(\epsilon\) is the transformation parameter. We see that we obtain an extra contribution equal to \(e^\phi\) which vanishes in the absence of torsion. In this case, it can be easily checked that the field tensor, Eq. (IV.2), and hence the Lagrangian remains invariant.

The modified form of gauge transformation can now be extended to charged matter fields. Let \(\psi\) denote a complex scalar field which transforms under \(U(1)\) gauge transformation as \(\psi \rightarrow \psi' = \exp(ie\epsilon) \psi\). The minimal coupling prescription [31] leads to the covariant derivative \(D_\mu \psi = \partial_\mu \psi - ie \exp(-\phi) A_\mu \psi\). Hence we find that the effective electromagnetic coupling in this case is \(e/\exp(\phi)\). Using the modified form of torsion in the gravitational Lagrangian, we obtain

\[
S = \int d^4x \sqrt{-g} \left[ -\frac{M^2_{\text{pl}}}{16\pi} \left( \hat{R} - 6 \partial^\rho \phi \partial_\rho \phi \right) - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} + (D_\mu \psi)^* D^\mu \psi \right],
\]

(IV.6)
where the effects of torsion have been explicitly displayed in terms of the field $\phi$ and $M_{\text{pl}}$ is the Planck mass. Here we have performed an integration by parts and dropped a total divergence. In order to relate this to the form given in Eq. (II.1), we perform the transformation,

$$A^\mu = V^\mu e^\phi$$  \hspace{1cm} (IV.7)

In terms of the field $V_\mu$, the gauge transformation of Eq. (IV.5) becomes $V_\mu \rightarrow V_\mu + \partial_\mu \varepsilon$ and the field tensor becomes,

$$F_{\mu \nu}(A) = e^\phi F_{\mu \nu}(V).$$  \hspace{1cm} (IV.8)

The covariant derivative $D_\mu \psi = \partial_\mu \psi - ieA_\mu \psi$ now takes the standard form and the action can be written as

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_{\text{pl}}^2}{16\pi} \left( \dot{R} - 6\partial_\rho \phi \partial^\rho \phi \right) - \frac{1}{4}e^{2\phi}F^{\mu \nu}F_{\mu \nu} + (D_\mu \psi)^* D_\mu \psi \right]$$  \hspace{1cm} (IV.9)

Now, we can see the similarity with the Lagrangian in Ratra model [14] during inflation. It is clear that the coupling term $f^2$ which had to be put by hand in the Ratra model has automatically come out. We point out that this term could be inserted in the coupling to matter fields or directly as a coupling with the gauge kinetic term in the Ratra model also exactly in analogy with Eqs. IV.6 and IV.9.

However the Hojman et al minimal prescription model [31] (Eq. IV.9) is in conflict with solar data [32]. As we shall see in the next two subsections this problem is evaded since the minimal coupling procedure extended to non-abelian gauge fields naturally generates effective potential terms for the field $\phi$. Alternatively we may go beyond the minimal coupling prescription [31] by adding kinetic and potential terms in the field $\phi$ while preserving Poincaré gauge invariance.

### B. Torsion and Non-abelian Fields

In this section we generalize the principle of minimal coupling [31] to make torsion compatible with non-abelian fields as well. Consider a matter field $\psi$ which under a non abelian group $G$ transforms as

$$\psi'(x) = U(x)\psi(x)$$  \hspace{1cm} (IV.10)

where $U \in G$ is a non-abelian transformation given by

$$U(x) = \exp[i\alpha'(x)T^i],$$  \hspace{1cm} (IV.11)

$\alpha'^i$s are transformation parameters and $T^i$s are generators of the group $G$. In analogy with Eq. IV.5 we propose the following transformation for the gauge fields $W^i$,

$$W'^i_\mu T^i = U \left( W^i_\mu T^i + e^\phi \frac{i}{g} \partial_\mu \right) U^\dagger,$$  \hspace{1cm} (IV.12)

with the gauge derivative given by

$$D_\mu \psi = \left( \partial_\mu - ig e^{-\phi} W^i_\mu T^i \right) \psi.$$  \hspace{1cm} (IV.13)

The covariant derivative transforms as

$$(D_\mu \psi)' = U(x)D_\mu \psi.$$  \hspace{1cm} (IV.14)
Furthermore, we obtain the field strength tensor by computing $[D_\mu, D_\nu]$. This leads to

$$[D_\mu, D_\nu] \psi = -ige^{-\phi} F^i_{\mu\nu} T^i \psi \quad \text{(IV.15)}$$

where

$$F^i_{\mu\nu} T^i = \partial_\mu W^i_\nu T^i - \partial_\nu W^i_\mu T^i - ige^{-\phi} [W^i_\mu T^i, W^i_\nu T^j] - W^i_\nu T^i \partial_\mu \phi + W^i_\mu T^i \partial_\nu \phi \quad \text{(IV.16)}$$

One can easily check that this is just the expression obtained by replacing the derivatives by gravitational gauge covariant derivatives (see Eq. IV.1), i.e.,

$$F^i_{\mu\nu} T^i = \nabla_\mu W^i_\nu T^i - \nabla_\nu W^i_\mu T^i - ige^{-\phi} [W^i_\mu T^i, W^i_\nu T^j] \quad \text{(IV.17)}$$

We can now make the transformation of the non-abelian vector potential $W^i_\mu$ analogous to Eq. IV.7. This leads to the standard form of the field strength tensor up to an overall factor of $e\phi$. Hence the kinetic energy term of the non-abelian fields becomes $-(1/4)e^2\phi F^i_{\mu\nu} F^{i\mu\nu}$ as in the case of abelian gauge theory.

The coupling of $\phi$ with non-abelian gauge fields is interesting since it generates an effective potential for $\phi$. Consider a $SU(3)$ gauge field analogous to QCD. After dynamical symmetry breaking, the operator $F^i_{\mu\nu} F^{i\mu\nu}$ acquires an expectation value, i.e.

$$\langle F^i_{\mu\nu} F^{i\mu\nu} \rangle = \Lambda^4$$

where $\Lambda$ is a parameter. At leading order we may therefore replace the term in the Lagrangian as

$$e^{2\phi} F^i_{\mu\nu} F^{i\mu\nu} \rightarrow e^{2\phi} \langle F^i_{\mu\nu} F^{i\mu\nu} \rangle$$

which leads to an effective potential term for $\phi$. We will also get additional contribution from the topological term $e^{\mu\nu\alpha\beta} F^{i}_{\mu\nu} F_{i\alpha\beta}$ which will also pick up an overall factor $\exp(2\phi)$. We point out that other potential terms also get generated, the details of which depend on the model being considered. We shall discuss an explicit model below.

The potential terms are expected to lead to a background value $\phi_0$ of the field $\phi$. As the Universe evolves, we assume that $\phi$ also undergoes a slow cosmological evolution and takes a value $\phi_0$ at the current time. The evolution should be sufficiently slow in order that it is not in conflict with constraints on time dependence of fundamental parameters. We next expand $\phi$ about its background value such that

$$\phi = \phi_0 + \hat{\phi} \quad \text{(IV.20)}$$

Furthermore we scale $\phi$ in order to convert its kinetic energy term into the canonical form. The background factor $e^{\phi_0}$ gets absorbed into redefinition of parameters. With this expansion we obtain a mass term for $\hat{\phi}$ as well as higher order terms. This field acquires an effective mass given by

$$m_\phi \sim \frac{\Lambda^2}{\beta M_{pl}}$$

Hence we generate a mass term as well as higher order potential terms for the field $\phi$. We discuss this in more detail below.
C. Evading the Solar Constraints

As has been discussed in literature [32], the minimal coupling model discussed above is in conflict with constraints from the solar system. It turns out that the torsion or the scalar field $\phi$ generated by the Sun is sufficiently large that it leads to observable differences between the gravitational accelerations of particles with different electromagnetic energy content. Non-observation of this difference rules out the minimal coupling model [31] discussed above. However the constraints can be evaded if we add a suitable potential $V(\phi)$ to the action in Eq. (V.2) such that the scalar field acquires mass. As we have shown in the previous section, we may not need to explicitly add potential terms since an effective potential also gets generated by non-abelian gauge fields. Once the field acquires mass, its equation of motion can be expressed as

$$\nabla^2 \phi - m^2_\phi \phi = \frac{1}{3} (B^2 - E^2).$$

(IV.22)

This is a generalization of Eq. 29 of [32]. The electromagnetic energy content of the Sun acts as a source of the field $\phi$. We see that if the mass $m_\phi$ is sufficiently large then the field $\phi$ will decay exponentially and will be negligible at Earth. Assuming $\beta$ in Eq. (IV.21) of order unity and $\Lambda$ of order 1 GeV corresponding to the QCD scale, we obtain $m_\phi$ of order $10^{-10}$ eV. This is large enough to completely suppress the signal arising due to Sun.

We point out that besides the potential term generated through QCD (Eq. IV.19), quantum corrections due to electroweak, gravity and other beyond the Standard Model fields would also generate other terms in the effective potential for the $\phi$ field. Hence it is natural to include potential terms for this field.

V. GENERALIZED GRAVITATIONAL ACTION

The minimal coupling model implied by the Poincaré symmetry has the necessary ingredients to generate magnetogenesis. It naturally produces the coupling of a scalar field $\phi$ with the electromagnetic field similar to that assumed in [14]. As we have discussed in Section (IV.B) that a potential for the field $\phi$ also gets generated naturally. Here we shall not necessarily assume that $\phi$ is also the inflaton field. It is primarily responsible for magnetogenesis. In the next section, we shall study the generation of magnetic fields within the minimal model. As we shall see, the minimal model Eq. (IV.9) can lead to an enhancement of the magnetic fields. In this section, we study generalized models which may lead to a modification of the kinetic energy term of the field $\phi$. This may be useful in avoiding the back reaction problems.

Before discussing the generalized models, we point out that it is possible to add kinetic and potential terms in the field $\phi$ while preserving Poincaré and the electromagnetic gauge invariance which are the guiding principles in constructing this action. In particular we can add terms in gravitational action including the field $\phi$, such as,

$$L_T = \tilde{\beta} g^{\gamma \sigma} T^\alpha_{\beta \gamma} T^\beta_{\alpha \sigma}.$$  

(V.1)

This directly leads to a kinetic term for $\phi$. In contrast we cannot generate a potential term for $\phi$ by adding terms involving the torsion tensor. Such terms have to be added directly in terms of $\phi$. However as we have seen, an effective potential is generated by non-abelian QCD like fields.
After adding such terms, the final action can be expressed as:

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{M_{Pl}^2}{16\pi} \left( \hat{R} - 6\beta^2 \partial_\rho \phi \partial_\rho \phi \right) - \frac{1}{4} e^{2\phi} F^{\mu \nu} F_{\mu \nu} - V(\phi) \right] \]  

(V.2)

where the added kinetic term has been accommodated by introducing the parameter \( \beta \).

Following Campanelli \cite{27}, we find that this action can generate magnetic field of the required strength. Depending on the model, i.e. the choice of potential, it may be necessary to choose \( \beta \) to be very small. This will require fine tuning since the additional kinetic energy term has to be chosen very precisely in order to obtain a small value of \( \beta \). This fine tuning may be evaded if we further generalize the gravitational action such that it becomes,

\[ S_g = \int d^4x \sqrt{-g} \left[ -e^{2\phi} \frac{M_{Pl}^2}{16\pi} R - \frac{1}{6} e^{2\phi} M_T^2 \bar{g}^{\mu \nu} T^\alpha_{\mu \beta} T^\beta_{\nu \alpha} - V(\phi) \right] \]  

(V.3)

Here the full form of \( R \) is given in Eq. (IV.4). We next make a conformal transformation such that \( \bar{g}_{\mu \nu} = e^{2\phi} g_{\mu \nu} \)

In terms of the new metric \( \bar{g}_{\mu \nu} \), the gravitational action becomes

\[ S_g = \int d^4x \sqrt{-\bar{g}} \left[ -e^{2\phi} \frac{M_{Pl}^2}{16\pi} \hat{R} + \frac{1}{2} M_T^2 \bar{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \]  

(V.5)

Here we do not need to fine tune the parameter \( M_T \). Now we may choose the potential such that \( \phi \) may act as the inflaton field. The matter action is chosen by the principle of minimal coupling as prescribed in \cite{31}. The full action has precisely the form which leads to both inflation and magnetogenesis following the analysis presented in \cite{14,27,27}.

Due to the conformal transformation Eq. (V.4) the matter action also undergoes some change. The gauge kinetic energy terms remain unaffected. However terms involving scalar and spinor fields may change. Lets us consider the effect on the Higgs field \( H \). With the transformation Eq. (V.4), the Higgs action becomes

\[ S_H = \int d^4x \sqrt{-\bar{g}} \left[ e^{-2\phi} \bar{g}^{\mu \nu} D_\mu H H^\dagger D_\nu H - m_H^2 e^{-4\phi} H^\dagger H - \lambda e^{-4\phi} (H^\dagger H)^2 \right] \]  

(V.6)

where \( D_\mu \) is the gauge covariant derivative. Similar terms are generated for all scalar fields which have non-zero vacuum value. Furthermore the fermion field condensates are expected to generate additional terms in the effective potential for \( \phi \). In order to reduce Higgs kinetic energy term to its canonical form, we may transform the Higgs field such that

\[ \tilde{H} = e^{-\phi} H \]  

(V.7)

This transformation, however, leads to complicated derivative terms for the \( \phi \) field. In any case, we find the appearance of additional potential terms for \( \phi \) from the scalar field action besides the terms discussed in Section (IV B).

Working with the field \( H \), i.e. without transforming to \( \tilde{H} \), we may express the potential terms as

\[ V(\phi) = ae^{-4\phi} + be^{2\phi} + \ldots \]  

(V.8)
The factors $a$ and $b$ represent the contributions due to the vacuum expectation value of the Higgs field and the QCD condensates. These are not really constant since these will also change as $\phi$ evolves with time. In order to determine the vacuum expectation value of $\phi$ we can treat $a$ and $b$ as constants. Keeping only these two terms the minimum of the potential is found to be

$$\phi_0 = \frac{1}{6} \ln(2a/b)$$ (V.9)

By expanding around the minimum and rescaling $\phi$ such that $\tilde{\phi} = M_T \phi$ we obtain a mass term for $\tilde{\phi}$. The overall terms, i.e. $\exp(-4\phi_0)$ and $\exp(2\phi_0)$, should be absorbed into the Higgs vacuum expectation value and the QCD condensates respectively. The $\phi$ mass is found to be order

$$\frac{1}{\sqrt{M_T}} \max \left\{ \sqrt{a}, \sqrt{b} \right\}.$$

We expect $a$ to be at least as large as the electroweak scale and $M_T < M_{pl}$. This generates a mass larger than $10^{-6}$ eV which is sufficient to evade the solar constraints.

For generation of primordial magnetic fields, however, the situation is more complicated. This is because now we need to study the time evolution of the Higgs as well as the QCD field. In case of QCD this will require a rather complicated quantum analysis of non-abelian fields. We do not pursue this in the present paper and simply assume a potential for $\phi$ which can lead to primordial magnetogenesis. As discussed earlier, quantum corrections would generate additional terms in the effective potential for $\phi$ which need to be included for a complete analysis.

The introduction of the factor $e^{2\phi}$ in Eq. V.3 is partially justified by considering an $f(R)$ gravity model such as the Starobinsky model [49]. In that case the gravitational action becomes very complicated due to the presence of the $R^2$ term which involves four derivative terms of the field $\phi$. However if we generalize it such that the action reads,

$$S_{\text{Staro}} = \int d^4x \sqrt{-g} \left[ -\frac{M_{pl}^2}{16\pi} \left( e^{2\phi} R - \frac{R^2}{6M_T^2} \right) + e^{2\phi} M_T^2 g^\mu_\nu T^\alpha_T \partial_\mu T^\beta_T \right]$$ (V.10)

then after the conformal transformation the action reduces to the standard Starobinsky action along with an additional kinetic energy term for the field $\phi$ given in Eq. V.2. Only with the introduction of this extra factor $e^{2\phi}$ do we get a simple action in the Einstein frame.

An alternative approach is to demand global conformal invariance. In this case the mass scale $M_{pl}$ gets replaced by $a\Phi$ where $\Phi$ is a scalar field and $a$ is a parameter, see for example [50]. We do not introduce the extra kinetic energy term proportional to $M_T$. The resulting gravitational action may be expressed as,

$$S_g = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{(a\Phi)^2}{16\pi} \hat{R} + \frac{1}{2} g^\mu_\nu \partial_\mu \Phi \partial_\nu \Phi - V(\Phi, \phi) \right]$$ (V.11)

In this case we do not need to introduce the extra factor $e^{2\phi}$ in the gravitational action. This action is invariant under the global conformal transformation $g_{\mu\nu} \rightarrow g_{\mu\nu} / \Lambda^2$, $\Phi \rightarrow \Lambda \Phi$. We impose this symmetry also on the matter action. We next make the transformation $\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}$, $\tilde{\Phi} = e^{-\phi} \Phi$. The action now becomes

$$S_g = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{(a\Phi)^2}{16\pi} \hat{R} + \mathcal{L}_\mathcal{K} - V(\tilde{\Phi}, \phi) \right]$$ (V.12)
where
\[ \mathcal{L}_K = \frac{1}{2} \tilde{g}^{\mu \nu} \left( \partial_\mu \tilde{\Phi} \partial_\nu \tilde{\Phi} + \tilde{\Phi}^2 \partial_\mu \phi \partial_\nu \phi - 2 \tilde{\Phi} \partial_\mu \tilde{\Phi} \partial_\nu \phi \right) \] (V.13)

The conformal symmetry may be broken softly by the dynamical mechanism analogous to the one described in [51–55]. Here one assumes the existence of a dark strongly coupled sector. The condensate formation in this sector leads to dynamical breakdown of conformal invariance which also triggers the electroweak spontaneous symmetry breaking. Here we assume some strongly coupled sector with very high mass scale such that these particles decay at some early time during the evolution of the Universe. The condensate formation in this sector leads also to a vacuum expectation value of the field \( \tilde{\Phi} \). We assume that this field undergoes negligible evolution since the beginning of inflation and hence we can simply set it equal to its vacuum expectation value. Alternatively we need to go to the Einstein frame. In the present case the additional terms generated by going from Jordan to Einstein frame are assumed to be negligible at leading order due to our assumption that \( \tilde{\Phi} \) evolves negligibly during inflation. We therefore set \( a \tilde{\Phi} \) equal to its vacuum expectation value which is assumed to be equal to \( M_{pl} \). Ignoring the derivatives of \( \tilde{\Phi} \), we generate a kinetic energy term for \( \phi \) whose overall normalization is equal to \( \tilde{\Phi}^2 \). We assume that this value is sufficiently small in comparison to \( M_{pl} \) so that it does not lead to back reaction during inflation. This requires us to set the parameter \( a \) of the order of \( 10^3 \). Hence with an appropriate choice of the parameter \( a \) this leads to exactly the action given in Eq. V.5 with the scale \( M_T \) being generated by \( \tilde{\Phi} \).

Before ending this section we propose a generalization of the Starobinsky model by demanding conformal symmetry. The resulting action can be written as
\[ S_C = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{16\pi} \left( a^2 \Phi^2 R - \xi^2 R^2 \right) + \frac{1}{2} \tilde{g}^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi, \phi) \right] \] (V.14)

We again make the transformation \( \tilde{g}_{\mu \nu} = e^{2\phi} g_{\mu \nu} \), \( \Phi = e^{-\phi} \Phi \). The resulting action reads
\[ S_C = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{16\pi} \left( a^2 \Phi^2 \tilde{R} - \xi^2 \tilde{R}^2 \right) + \mathcal{L}_K - V(\Phi, \phi) \right] \] (V.15)

As discussed earlier the field \( \tilde{\Phi} \) acquires vacuum expectation value such that \( a \tilde{\Phi} = M_p \). Assuming that \( \tilde{\Phi} \) does not evolve significantly with time during inflation we can replace it with its vacuum expectation value. Then the first two terms on the right hand side yield the standard Starobinsky model. The remaining terms involve the kinetic and potential terms for the field \( \phi \) and \( \tilde{\Phi} \). Along with the coupling with the electromagnetic field these can lead to magnetogenesis.

To summarize this section, we have shown that the model for magnetogenesis which was proposed by Ratra [14] naturally appears in the Poincaré invariant theory with the minimal prescription principle proposed in [31]. The minimal model, however, requires fine tuning of the kinetic energy term and does not contain a potential term for scalar field \( \phi \) which generates torsion. We have argued that potential terms for this field appear naturally and it is easy to generalize the model such that it does not suffer from fine tuning. This is best accomplished by imposing conformal invariance. Hence the model has all the ingredients required for primordial magnetogenesis. We have also described a conformal generalization of the Starobinsky model which will lead to both inflation and magnetogenesis.

### VI. MAGNETOGENESIS

The basic formalism for magnetogenesis in the Ratra model has been developed in several papers [25, 27]. Our basic point is that Poincaré gauge theory naturally leads to models similar to
the Ratra model. In particular the minimal coupling model leads to a specific form of the coupling of a scalar field \( \phi \) to the electromagnetic fields as well as the kinetic term for \( \phi \). As we have discussed in the previous section, both the kinetic and potential terms in \( \phi \) may be substantially modified in comparison to what we obtain by the minimal coupling procedure. Hence depending on the model, different scenarios described in [27] could be realized. Hence it is clear that we can always generate magnetic field of the required strength if we go beyond the minimal coupling model. In this section we discuss how far the minimal coupling model is successful in generating magnetic fields.

The minimal coupling model, proposed in [31], is given in Eq. IV.9. Here we have only displayed the coupling of \( \phi \) with the electromagnetic field. Similar terms also should be included for all gauge fields, abelian and non-abelian. As described above, an effective potential term for the scalar field \( \phi \) of the form

\[
V = M_1^4 e^{2\phi}
\]  

(VI.1)

naturally appears due to the vacuum condensates formed by QCD like non-abelian fields. Here we assume existence of some QCD like fields with very high mass scale \( M_1 \). The equation of motion for \( \phi \), neglecting electromagnetic effects, can be expressed as

\[
12M_{pl}^2 \beta^2 (\dot{\phi} + 3H \phi) + \frac{\partial V}{\partial \phi} = 0
\]  

(VI.2)

with \( \beta = 1 \). Here we assume that inflation is caused by some field other than \( \phi \). Let us assume that during inflation the time dependent part of this field is small. In that case we can approximate \( \exp(2\phi) \approx 1 + 2\phi \). This leads to a linear potential \( V(\phi) \approx b\phi \) for \( \phi \) and admits a solution of the form

\[
e^{\phi} = C \left( \frac{a}{a_i} \right)^{\alpha}
\]  

(VI.3)

with constant \( \alpha \ll 1 \). Ignoring the constant part of \( \phi \), we obtain \( \phi \approx \alpha H t \). The solution starts to break down as \( \phi \) approaches unity. This corresponds to the small \( \phi \) (\( \phi \) is same as \( \alpha \) in our notation) limit of the models discussed in [27]. In this limit the model leads to magnetic fields of order \( 10^{-32} \) G which is rather small.

We next discuss the more general case in which the field \( \phi \) is not necessarily small while working within the framework of the minimal coupling model. In this case we use the full form of the potential given in Eq. VI.1. Assuming that \( \ddot{\phi} \) is negligible, the solution for \( \phi \) becomes

\[
e^{2\phi} = \frac{1}{2m(t + t_0)}
\]  

(VI.4)

where \( m = M_1^4 / (18M_{pl}^2 H) \) and \( t_0 \) is an integration constant. We set \( t_0 = N_0 / H \) and find that \( \ddot{\phi} \) is negligible if

\[
Ht + N_0 \gg 1
\]  

(VI.5)

which holds for a wide range of choices of \( N_0 \). We point out that this requires \( N_0 \) to be sufficiently larger than unity. Furthermore this condition holds independent of the value of \( \beta \). With this condition we also find that the kinetic energy term for \( \phi \) does not cause back reaction for the inflationary potential. The effective value of \( \alpha \) in this case is found to be

\[
\alpha = -\frac{1}{2(Ht + N_0)}
\]  

(VI.6)
which varies slowly with time but is necessarily small throughout inflation. Hence we find that
with this choice of potential we are forced to have small values of $\alpha$ which effectively also imply
a small value of the time dependent part of $\phi$. This implies that the minimal coupling model does
not lead to sufficiently large magnetic fields which can provide seeds for the galactic dynamo. This
can be modified only if we allow a generalized potential which is permissible in our framework
but goes beyond the minimal coupling model.

So far we have assumed that $\phi$ is not the inflaton field. However since it varies slowly, the effective
potential remains approximately constant during most of the evolution. Hence it is possible to
consider $\phi$ also as the inflaton field, as long as we choose $N_0$ to be sufficiently large. However it is
not clear how to exit inflation and enter the reheating phase in this framework. It may be possible
to enhance the model in order to accomplish this. For example, we may also add the term $F_{\mu \nu} \tilde{F}^{\mu \nu}$
for the non-abelian fields which also acquires a vacuum expectation value. This term has an effective
coupling to an axion like field $\chi$ and would also pick up a factor $e^{2\phi}$. Let us assume that
this condensate dominates the evolution during inflation. Furthermore the axion field $\chi$ remains
constant during inflation but undergoes evolution towards the end which effectively ends inflation
and leads to reheating.

A. Beyond the Minimal Model

We have already seen that within the framework of the minimal model we are unable to generate
the magnetic field of the required strength. As discussed earlier, the required kinetic energy term
in $\phi$ can be generated naturally within the minimal model, as demonstrated, for example, in Eqs.
V.11 and V.12. However we are unable to generate the required potential term. Here we go beyond
the minimal model by adding the following potential term:

$$V(\phi) = 6\beta^2 M_p^2 m^2 \phi^2$$

(VI.7)

where $m$ is a parameter with dimensions of mass. The kinetic energy term in $\phi$ is given in Eq. V.2
or equivalently in Eq. V.13 with $\langle \Phi \rangle = \sqrt{3/4\pi} \beta M_{pl}$. In this case $\phi$ also acts as the inflaton.

Imposing the slow roll condition $\ddot{\phi} \ll 3H\dot{\phi}$ in the equation of motion Eq. VI.2 we obtain the solution

$$\phi = \phi_0 e^{-t/\tau}$$

(VI.8)

where $\phi_0$ and $\tau$ are constants, such that

$$\tau = \frac{3H}{m^2}$$

(VI.9)

The slow roll condition implies,

$$\tau \gg \frac{1}{H}$$

(VI.10)

which also leads to

$$\frac{m^2}{H^2} \ll 1$$

(VI.11)

From Eq. VI.3, we obtain

$$\alpha = -\frac{\phi_0}{\tau H} e^{-t/\tau}$$

(VI.12)

Now, let us estimate the value of $\phi_0$. The Einstein's equations of motion give us

$$3H^2 = \frac{3}{2}\beta^2 \phi^2 + 3\beta^2 m^2 \phi^2$$

(VI.13)
Here, for kinetic term to be negligible, we need

$$\tau >> \frac{1}{m}$$  \hspace{1cm} (VI.14)

Ignoring the kinetic energy term, we obtain, $\phi_0 \approx H/\beta m$.

Let us now assume that $\alpha \approx -2$ and approximately constant over much of inflation. The value $\alpha = -2$ is required in order to generate scale invariant magnetic field within the framework developed in [27]. This can be accomplished by requiring that $t << \tau$ during inflation. This is equivalent to requiring that $H\tau >> N$, where $N$ is the number of e-folds during inflation. For example if we take $H\tau \approx 500$, assuming that $N \approx 60$, it will assure that the necessary conditions are met over much of inflation and we would produce a nearly scale invariant spectrum for a wide range of values of $k$ of the required strength [27]. The parameter values for this case would be $m \approx H\sqrt{3/500}$ and $\beta \approx 1/(-\alpha m) \approx 0.5/\sqrt{1500} << 1$. Hence with this choice of parameters we are able to generate the required magnetic field.

\section*{VII. CONCLUSIONS}

The problem of generating magnetic fields during inflation can be resolved by breaking the conformal invariance of the electromagnetic Lagrangian [14]. The required coupling between Torsion and Electromagnetism however had to be put in by hand. We have shown that torsion naturally leads to this coupling within the framework of the model developed by Hojman et al [31] which satisfies electromagnetic gauge invariance. The model is based on a minimal version of torsion such that the torsion field can be expressed in terms of a scalar field $\phi$ called tlaplon in the literature [31]. The main problem with this model is that it is ruled out by constraints due to solar data [32]. We have shown that due to coupling with non-abelian fields, the minimal model acquires an effective potential term for the scalar field and evades the solar constraints. We have studied the generation of magnetic fields within the framework of the minimal model. We find that these are equivalent to the small $p$ (or $\alpha$) limit of the models discussed in [27]. Hence the magnetic field generated in this case is relatively small. The minimal model also leads to a rather large contribution from the kinetic energy of $\phi$ unless $p$ is close to 0.

We have also discussed several generalizations of the minimal model. We have argued that quantum corrections will generate additional potential terms and hence there is no reason to restrict oneself to the minimal model. We can add potential terms in the scalar tlaplon while maintaining Poincare symmetry. With these terms it is possible to generate larger magnetic fields in comparison to the minimal model. In particular we have considered a model which displays invariance under global conformal transformation $g_{\mu\nu} \rightarrow g_{\mu\nu}/\Lambda^2$, $\Phi \rightarrow \Lambda \Phi$, where $\Phi$ is a scalar field. By imposing this symmetry on a generalized Starobinsky model we find that we can naturally suppress the kinetic energy terms of $\phi$. With this suppression it possible to have significant variation of $\phi$ during inflation which is required for magnetogenesis. By assuming a simple form of the potential for $\phi$ we have explicitly demonstrated the generation of nearly scale invariant magnetic field of required strength. However we still need a mechanism to generate the required potential which has so far simply been assumed. For this purpose it may be useful to study the conformal model in more detail using the effective potential approach which includes corrections due to loop contributions.
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