Investigation of hydraulic fracture growth near a mine opening

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Abstract. The behavior of a hydraulic fracture near the mine opening in a plane-parallel formulation has been investigated. The case was considered, when the rock was under hydrostatic pressure, and the initiating crack was oriented in the direction of mine opening. It is found that the fracture path is greatly influenced by the ratio of hydrostatic pressure and critical tensile stress of rock. It is shown that when the hydrostatic pressure increases with respect to critical tensile stress, the fracture begins to deviate from the mine opening earlier and does not reach its surface. It is also shown that under various conditions, fractures cannot bypass the mine opening completely due to the occurrence of severe strains in the area between the opening and fracture.

1. Introduction
Hydraulic fracturing technology is an effective approach widely used as part of mineral resource development strategies. Formation of a crack with optimal geometry and orientation is critical in hydraulic fracturing (HF). The stress-strain state and properties of rocks, along with characteristics of the loading exerted by downhole equipment, fluid injection rate and its properties, etc. have the most significant impact on the crack evolution. Understanding the hydraulic fracturing behavior under in-mine conditions is critical, inasmuch as mine opening can be located in close proximity to the HF-induced cracks and affect them. One of the approaches to study HF crack behavior in such conditions is numerical modeling.

This work aims to study the behavior of a hydraulic fracturing-induced crack near the mine opening on the basis of numerical experiments. The problem was solved in a plane-parallel formulation, when the rock was under hydrostatic pressure and the initiating crack was oriented in the direction towards the mine opening. The numerical model equations were solved using the finite element method.

2. Basic equations of the model
Let’s consider the problem of crack growth in a homogeneous elastic medium whose deformations are determined by the equations as follows:

\[
\sigma_y = 2\mu\varepsilon_y + \lambda\varepsilon_0\delta_y, \tag{1}
\]
where $\sigma_{ij}$ is stress tensor of a rigid body; $\varepsilon_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2$ is strain tensor of a rigid body; $u_i$ is the body deformation; $\lambda$, $\mu$ are Lamé parameters; $\varepsilon_0$ is volumetric strain. The fluid flow within the fracture will be described by the flow continuity equation:

$$\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \bar{q} = 0,$$

(2)

where $d$ is the fracture opening; $\bar{q} = (q_1, q_2, q_3)$; $q_i = d \bar{k} (\partial p / \partial x_i)$; $\bar{k}$ is effective permeability of fracture in the $i$-th direction. Within frames of this problem, we assume that the fluid flow within fractures is a Poiseuille flow of Newtonian fluid. In this case, the effective permeability of crack is defined as

$$\bar{k} = -\frac{d^2}{12\eta},$$

(3)

where $\eta$ is dynamic fluid viscosity.

Modeling of crack growth was carried out on the basis of the cohesive zone model (CZM) [1]. The application of CZM suggests that a damage zone (material softening) develops ahead of the crack tip, which is associated with tensile stresses $\sigma$ integration and deformations (displacements) $u$ according to a certain law (Figure 1a). As such, this approach does not require determining the critical stress intensity factor $K_c$. The two independent parameters $\sigma_c$ (critical strain stress) and $G_c$ (critical energy released during the damage) are set, instead, which are calculated as:

$$G_c = \int \sigma du,$$

where $u_c$ is the displacement jump associated with complete failure of the material. When the cohesive damage zone is small compared to the crack length, the stress intensity factor and released cohesive fracture energy are bound in formula [2]

$$K_c = \sqrt{\frac{G_c E}{1 - \nu^2}},$$

(4)

where $E$ is elasticity modulus of the material (Young's modulus (MPa)); $\nu$ is Poisson's ratio.

The material damage is triggered when critical tensile stress $\sigma_c$ is attained. Then new values of critical tensile stress $\bar{\sigma}_c$ and elasticity modulus are calculated according to the specified damage mechanism $\bar{E}$. The material damage mechanism is determined by the damage variable $D$, which is largely controlled by the strain beyond the limit of the linear elastic stiffness zone. Figure 1b shows an example of the function $D(u)$, where $D = 0$ corresponds to the intact state, and $D = 1$ corresponds to complete damage.

![Figure 1](image-url)  

**Figure 1.** Dependence of tensile stress $\sigma$ in the material on deformations $u$ (a) and the adhesive damage function $D(u)$ (b): 1—exponential, 2—linear damage evolution laws.
The calculation of the new state takes into account the function $D(u)$ using the formulas:

$$\sigma = (1 - D)\sigma_c, \quad E = (1 - D)E.$$  

This process is performed until the state is described as $\sigma_c = 0, \quad E = 0$, which means complete damage of the material and crack initiation. For initiation of the destruction process, the maximal principal stress criterion is applied.

The equations of system (1) will be solved numerically using the eXtended Finite Element Method (XFEM) [3–5]. The XFEM method enables us to obtain solutions containing a displacement jump using discontinuous functions (representing the gap between the crack surfaces), while solutions near the crack tip are based on specific asymptotic functions, i.e. enrichment functions described as asymptotic (to capture the singularity at the crack tip). The method allows simulating the process of the crack initiation and growth in the direction, which is determined from the stress state analysis near its tip and therefore can be arbitrary.

This paper deals with the implementation of XFEM method based on the phantom nodes [5] in concert with the cohesive damage law [6]. In this case, the general solution for displacements $u(x)$ including fracturing will be written as

$$u(x) = \sum_{i=1}^{N} [N_i(x)u_i + H(x)a_i], \quad (5)$$

where $N_i(x)$ is interpolation functions; $N$ is the number of nodes in the element; $u_i$ is nodes shifts; $a_i$ additional (phantom) nodes; $H(x)$ is the enriched Heaviside function. The first term in its totality describes the continuous constituent of the solution, while the second term accounts for the part of the solution associated with the fracturing. This hydraulic fracturing modeling technique is effective owing to its computational complexity which yields results that are consistent with the experimental data [7].

3. Numerical experiments

Let’s consider the problem of a relationship between a crack caused by hydraulic fracturing and mine opening in plane-parallel formulation. Let there be a two-dimensional domain containing a mine opening in the form of a circle (Figure 2). Let the initiating crack be oriented in the direction of the mine opening and located from it at a distance which equals the length of one diameter (Figure 2).

At the outer boundary of the region, we set the conditions that provide hydrostatic pressure near the mine opening, whose boundary (which is a circle, in this case) is free. For numerical calculations, we select the following parameters: rock elasticity modulus $E = 3.6$ GPa, Poisson’s ratio $\nu = 0.3$, dynamic viscosity of the fluid $\eta = 0.001$ Pa·s, the calculation domain size: 110×65 m, the mine opening radius: 2.5 m; critical damage energy $G_c = 280$ N/m.

Discretization of computational domain is shown in Figure 2. The smallest element size in the calculations was 0.17 m.

**Figure 2.** Discretization of two-dimensional domain with the mine opening area shown as a circle. The left part of the image shows the initiating crack, oriented towards the mine opening.
It was found that the relationship between hydrostatic pressure \( P_h \) in the medium and critical tensile stress \( \sigma_c \) affects significantly the crack growth near the mine opening. Figure 3 shows the crack trajectories at different \( P_h \) and \( \sigma_c \) ratios, which were obtained using the hydrostatic pressure \( P_h = 15 \times 10^5 \text{Pa} \). A series of numerical experiments was performed under different levels of \( \sigma_c \), i.e. with variations in \( P_h / \sigma_c \) ratio.

![Figure 3](image_url)

**Figure 3.** The cracks propagation trajectories in proximity to the mine opening, which is round (a) or square (b) in shape. The numbers indicate the curves obtained with different \( P_h / \sigma_c \) ratios: 1—\( \sigma_c = 0.125P_h \); 2—\( \sigma_c = 0.25P_h \); 3—\( \sigma_c = 0.5P_h \); 4—\( \sigma_c = 1.0P_h \); 5—\( \sigma_c = 2.0P_h \).

Figure 3a shows that the lower the value of \( \sigma_c \) relative to \( P_h \), the sooner hydraulic fracture begins to deviate from the mine opening. An increase in the tensile stress causes the crack to propagate into the mine. We performed similar experiments for a mine opening shaped as a square with a side of 5 m (Figure 3b), and the results were very much alike. In the case of a square configuration of mine opening, the cracks started to deviate earlier and exhibited a greater bypass distance.

![Figure 4](image_url)

**Figure 4.** Rock mass displacements near the round-shaped mine opening, m: (a) in the horizontal direction; (b) in the vertical direction.
Figure 5. Rock mass displacements near the square-shaped mine opening, m: (a) in the horizontal direction; (b) in the vertical direction.

It was found that hydraulic fracturing cracks cannot bypass the mine opening completely. Numerical experiments have shown that after a certain volume of injected working fluid, the crack stops growing. What a further fluid injection brings about is an increase in the crack opening and amount of deformations in the span between the mine opening and the HF-induced crack. Dark colors in Figures 4 and 5 indicate areas of severe deformations for round and square-shaped mine workings at \( P_h = 15 \times 10^5 \text{ Pa} \) and \( \sigma_c = 7.5 \times 10^5 \text{ Pa} \) (\( \sigma_c = 0.5 P_h \)). In both these cases, the same volume of fluid was injected. Note that Figure 3a shows the resulting cracks, which showed no growth under further fluid injection. In the case of a square configuration of the mine opening, the length of the cracks was longer, however, their growth was also arrested at a certain moment, with subsequent increase in their opening and rock deformations.

4. Conclusions
The hydraulic fracturing crack behavior near the mine opening has been investigated. We considered a plane-parallel formulation of the problem and the case when the initiating crack is oriented in the direction of the mine opening. It is established that when the rocks are under hydrostatic pressure, the crack caused by hydraulic fracturing deviates from the mine opening. The degree of deviation is determined primarily by the relationship between the hydrostatic pressure and critical tensile stress of rocks. The higher the hydrostatic pressure relative to the critical tensile stress, the sooner the crack begins to deviate from the mine opening. Numerical experiments have shown that due to the strain-softening in the area between the mine and the propagating crack, the latter cannot completely bypass the mine opening. The presence of the strain softening zone leads to the fact that after a certain volume of injected working fluid, the cracks will stop growing. Further fluid injection leads only to an increase in the crack opening and amount of deformations between the crack and mine opening.

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