Effect of operating frequency and phase angle on performance of Alpha Stirling cryocooler driven by a novel compact mechanism

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Abstract. Amongst the mechanical cryocoolers in use, Stirling cycle cryocoolers exhibit the desirable features such as high efficiency, low specific power consumption, small size and mass and large mean time before failure. Stirling cycle cryocooler of Alpha configuration exhibits better theoretical performance as compared to Gamma. However, the theory could not be put into practice due to unavailability of compatible drive mechanism for Alpha cryocooler providing large stroke to diameter ratio. The concept of novel compact drive mechanism can be made functional to operate miniature Alpha Stirling cryocoolers. It allows the use of multi-cylinder system while converting rotary motion to reciprocating. This permits the drive mechanism to be employed for driving different configurations of Stirling cryocooler simultaneously. This drive is capable of providing large stroke to diameter ratio compared to other drive mechanisms generally in use for the purpose. A stroke to diameter ratio of three is chosen in the present work and the drive dimensions are calculated for four piston-cylinder arrangements with 90° phase difference between adjacent arrangements providing two Alpha Stirling cryocoolers working simultaneously. It has to be noted that the coolers operate at half the frequency of the motor used. As the two coolers operate at phase difference of 180°, during compression stroke of one unit, the suction stroke occurs for the other unit. Due to power output of second unit, the combined peak torque requirement falls by 26.81% below the peak torque needed when one unit is operated separately. This allows for use of a comparatively lower torque motor. The practicability of the drive ensuring smooth operation of the system is decided based on comparison between torque availability from the motor and torque requirement of the complete unit.

The second order method of cyclic (or thermodynamic) analysis provides a simple computational procedure useful for the design of Stirling cryocooler and is adopted for the present theoretical investigations. An appropriate choice of the equations to compute different losses, from available co-relations, is made in accordance with the conditions existing in the present system. The effects of operating frequency and phase angle between compressor and expander pistons are presented in this paper. The cryocooler performance enhances with increase in operating frequency. However, cryocooler operation at 24 Hz (motor operation at 48 Hz) is considered for theoretical performance prediction. The maximum net refrigeration effect as well as COP is available at phase angle of 81°. However, it is essential to fix the phase angle at 90° for both the cryocoolers for the positive functioning of drive mechanism.

1. Introduction

The cryocooler performance depends on the mass flow rate of working fluid and the pressure ratio, which in turn depend on the strokes of reciprocating members for a given cylinder diameter. The
larger the stroke, higher is the mass flow rate of working fluid and the pressure ratio. So, the drive mechanism that can offer larger stroke can enhance the cryocooler performance. In addition, the drive should be compact, efficient and free from side thrust. These requirements are satisfied by the novel compact drive mechanism [1]. Its uniqueness lies in its functioning and its ability to drive various Stirling cryocooler configurations simultaneously, without changing the mechanism hardware. This is an entirely original concept and can be developed and activated to drive the new generation cryocoolers.

2. Novel compact drive mechanism

This drive mechanism is a kinematic friction drive [1] that converts rotary motion of driving disc into reciprocating motion of pistons in their respective cylinders. It comprises of driving and stationary discs placed parallel to each other along the same central axis and having a similar groove at the periphery. A driven disc (in form of a slice of a hemi-sphere) rotates in the peripheral grooves of these two discs. The convex surface of the driven disc is engaged in the matching concave surfaces of the rotary and stationary discs. The schematic of two positions of driven disc 180° apart are shown in figure 1. A vertical piston limb and a horizontal load-bearing limb connected to each other at 90° make one L-shaped reciprocating member. Four such members are located at an angle of 90° from each other on a pitch circle diameter. This positioning of the reciprocating members lessens the possible difficulties of dynamic balancing of driven disc. All the piston limbs reciprocate with same stroke in their respective cylinders rigidly fitted to the stationary disc. A pair of adjacent piston-cylinder arrangements with a regenerator in between can form one Stirling cycle cryocooler of Alpha configuration. Two such cryocoolers can operate in a single ensemble, driven by the solitary novel compact drive mechanism as shown in figure 1. This compact drive mechanism is competent enough to drive any configuration of Stirling cryocooler (based on mechanical arrangement) viz. Alpha, Beta or Gamma.

The set-up discussed here consists of two Alpha configuration Stirling cryocoolers in the same ensemble, driven by the solitary novel compact drive mechanism. The drive provides movement to four piston limbs (or rods) with two compressor pistons and two expander pistons. A mechanical phase difference of 90° is maintained by the drive mechanism between the compressor piston and the corresponding expander piston adjacent to it. The two compressor pistons move with phase difference of 180° between them. Similarly, the two expander pistons also have phase difference of 180°. The compressor pistons of larger diameter are mounted on the piston rods and move in the respective compressor cylinders. The desired mass flow rate is obtained by maintaining the compressor piston diameter more than its rod diameter. In the expander cylinders, the piston rods themselves act as the expander pistons. It means that the compression space is more than the expansion space in each cryocooler. The stroke maintained in each cylinder, either compressor or expander is same due to positioning of all the piston limbs on same pitch circle diameter. The regenerators of same size are placed between each pair of compressor and expander. The space below each compressor piston is connected to each other with an inter-connected volume in between. The schematic of drive mechanism with the cryocoolers is as shown in figure 1.
Figure 1. Schematic diagram of the drive mechanism at two different positions of driven disc with two simultaneously driven Alpha Stirling cryocoolers

The maximum load on any piston in the drive is when that piston is completely inside its respective cylinder and when the maximum pressure acts on it. At this instance, the load-bearing limb is also completely inside the guide sleeve thus providing the maximum area for load bearing. The line contact between the spherical shaped power transmitting surfaces results in rolling motion and ensures low frictional loss. This can result in a high efficiency friction drive [2]. Figure 2 shows the three dimensional picture of drive mechanism driving two Alpha Stirling cryocoolers for understanding purpose. Here the piston rods are shown to act as pistons.
Second order method [3] is used for cyclic analysis of miniature Alpha Stirling cryocooler driven by a novel compact drive mechanism during the present work. The analysis consists of calculating the ideal and net values of refrigeration effect and input power. The ideal values are modified by considering different losses to obtain the net values. The losses due to fluid friction and mechanical friction affect the input power. These losses are computed and added to the ideal input power to find the net power input to the cryocooler. The losses due to imperfect heat transfer, fluid friction and heat conduction etc. affect the refrigeration effect. These losses are estimated and subtracted from the ideal refrigeration effect to get the net refrigeration effect available.

Bapat [4] has applied the second order method for Alpha configuration with same compression and expansion space volumes resulting in improved refrigeration effect. The present analysis also considers same compression and expansion space volumes. The co-relations for losses, available in literature, are modified suitably to match the conditions in the system under consideration during present study [5]. The system is driven by a novel compact mechanism that has an inherent feature of operating more than one cryocoolers simultaneously in a single ensemble. In the present arrangement considered, two Alpha configuration cryocoolers can operate simultaneously using the novel compact drive. Hence, the total values of refrigeration effect and input power with two Alpha units are finally calculated.

The cycle is assumed to start from the reference condition when the expander piston is farthest from the cold end or with maximum expansion space volume available. As the cycle initiates, the expander piston travels towards its inner dead centre. The phase shift between the expander and compressor pistons is considered as 90° with expander piston leading the compressor piston as it is actually achievable with the novel compact mechanism to be used to drive the cryocoolers. Due to this phase difference, the compressor piston is exactly midway of its stroke length when the expander piston is at its outer dead centre at the beginning of the cycle. The compressor piston moves towards
its outer dead centre with the commencement of the cycle. The magnitude and direction of
displacement of both the pistons over the cycle are essential to calculate the instantaneous property
values at different intermediate positions (or states) during the cycle. The cycle is considered to be
split into 144 intervals, each interval corresponding to 2.5° angular movement of the crank.

Thermodynamic analysis is done for cold tip temperature of 70 K. The design parameters obtained
after optimization and the other geometrical parameters kept at fixed values due to mechanical
strength and geometrical considerations are summarised below. The same are employed to know the
effects of different parameters like frequency and phase angle on cryocooler performance.

| Table 1. Design parameters [5] |
|-----------------------------|----------------|
| Parameter                   | Value  |
| Compressor piston diameter (mm) | 7.5    |
| Compressor piston stroke (mm) | 22.5   |
| Expander piston diameter (mm) | 7.5    |
| Expander piston stroke (mm)  | 22.5   |
| Compressor side connecting tube diameter (mm) | 1.5   |
| Radial clearance (µm)        | 20.0   |
| Regenerator length (mm)      | 30.0   |
| Regenerator diameter(mm)     | 8.0    |
| Regenerator matrix mesh size (Mesh No. – wire gauge) | 250 – 47 |
| Regenerator matrix material  | Stainless Steel 316 |

| Table 2. Other geometrical parameters [5] |
|-----------------------------|----------------|
| Parameter                   | Value  |
| Compressor piston length (mm) | 105.0   |
| Expander piston length (mm)  | 105.0   |
| Cylinder length (mm)         | 75.0    |
| Cylinder thickness (mm)      | 0.15    |
| Compressor side connecting tube length (mm) | 120.0 |

The drive mechanism designed for mechanical safety is practical only when it can fulfill the
torque requirement criterion. So, the function of a mechanically safe novel compact drive is to provide
the torque required by the system. The total torque requirement includes the maximum required torque
in the cycle by the system and the frictional torque consumed by the drive mechanism itself. The
maximum required torque by the system is calculated from the value of maximum required input
power to the system at a particular operating frequency. The torque availability is calculated from
the motor power considering operating frequency of the motor and mechanical efficiency of the novel
compact drive. The required torque is compared with the available torque and the designed drive is
practicable only if the torque requirement is less than the available torque.

The parameters like frequency and phase angle between compressor and expander certainly have
an effect on the cooler performance. So, the cryocooler cooling capacity required at a particular
temperature and the input power required for it govern the choice of their values. The effects of variations carried out in these parameters are identified and presented in next section.

4. Effect of operating frequency

In the present system, the speed of motor decides the cryocooler operating frequency. The motor can operate at maximum of 50 Hz that is the power supply frequency. As a result of an inherent feature of the novel compact mechanism (that is intended to drive the cryocooler), the pistons reciprocate at half the frequency of driving motor. So, the cryocooler can operate at the maximum frequency of 25 Hz. The effect of change in frequency on the cryocooler performance in terms of net refrigeration effect and net input power is observed in table 3. It shows that the ideal refrigeration effect increases proportional to the frequency. The input power requirement also increases with frequency. The different losses also increase with frequency because of increase in mass flow rates leading to rise in fluid friction and imperfect heat transfer. The torque requirement criterion allows the cryocooler to be operated at the frequencies above 22 Hz for the considered mean cycle pressure.

| Operating frequency (Hz) | 21.0 | 22.0 | 23.0 | 24.0 | 25.0 |
|--------------------------|------|------|------|------|------|
| Ideal RE (W)             | 7.700| 8.067| 8.434| **8.861** | 9.167 |
| Total loss in RE (W)     | 5.206| 5.367| 5.534| **5.732** | 5.885 |
| Net RE (W)               | 2.494| 2.700| 2.900| **3.129** | 3.282 |
| Ideal IP (W)             | 36.734| 38.483| 40.232| **42.273** | 43.731 |
| Total loss in IP (W)     | 7.188| 7.602| 8.024| **8.521** | 8.887 |
| Net IP (W)               | 43.922| 46.085| 48.256| **50.794** | 52.618 |
| COP                      | 0.0568| 0.0586| 0.0601| **0.0616** | 0.0624 |
| Refrigeration η (%)      | 18.659| 19.247| 19.748| **20.245** | 20.496 |
| Available torque (N-m)   | 2.665| 2.913| 3.173| **3.490** | 3.727 |
| Required torque (N-m)    | 2.982| 3.091| 3.170| **3.342** | 3.445 |

5. Effect of phase angle

The phase angle between the expander and compressor pistons is a sensitive parameter influencing the cryocooler performance. In the present system, it depends on the system geometry and can be altered only if the system hardware is modified. A small change in dead volume of the system alters the phase angle for a given set of design parameters. As it is not changeable from outside the system during operation, it cannot be treated as an operating parameter of the present system. The effect of change in phase angle for the optimized design condition is seen in table 4. The phase angle increment results in the drop of maximum cycle pressure and rise in its minimum value. This reduces the pressure ratio with increase in phase angle. The maximum net refrigeration effect as well as COP is available at phase angle of 81° as observed from table 4 for present thermodynamic design. The general optimum value for an Alpha configuration is 90° when it follows an ideal Stirling cycle.
6. Conclusion

The magnitude of vibrations in the system may not practically allow the cryocooler to operate at 25 Hz. So, an optimum operating frequency can be experimentally determined. The value of 24 Hz is considered for theoretical performance prediction.

The present system consists of four pistons placed on same pitch circle diameter forming two simultaneously operating Alpha cryocoolers. The requirement of the drive mechanism is that these pistons should be equally spaced to synchronize their reciprocation and balance the forces exerted on them during operation. This is possible only if the consecutive pistons (together working as one cryocooler) are placed at an angle of 90°. These two pistons working together will make one cryocooler with a 90° phase angle between their motions. Hence, allowing for the positive functioning of drive mechanism, it is essential to fix the phase angle at 90° for both the cryocoolers. So, the present analysis considers constant phase angle of 90° though the optimum phase angle is 81°.

| Phase angle (degree) | 78.0 | 79.0 | 80.0 | 81.0 | 82.0 | 83.0 | 84.0 | 90.0 |
|----------------------|------|------|------|------|------|------|------|------|
| Maximum cycle pressure (bar) | 43.254 | 43.137 | 43.020 | **42.903** | 42.786 | 42.668 | 42.550 | **41.841** |
| Minimum cycle pressure (bar) | 9.514 | 9.543 | 9.572 | **9.601** | 9.631 | 9.660 | 9.691 | **9.874** |
| Pressure ratio | 4.546 | 4.521 | 4.495 | **4.469** | 4.443 | 4.417 | 4.391 | **4.237** |
| Ideal RE (W) | 8.616 | 8.651 | 8.683 | **8.713** | 8.740 | 8.765 | 8.786 | **8.861** |
| Total loss in RE (W) | 5.371 | 5.411 | 5.450 | **5.461** | 5.500 | 5.537 | 5.545 | **5.732** |
| Net RE (W) | 3.245 | 3.240 | 3.233 | **3.252** | 3.240 | 3.228 | 3.242 | **3.129** |
| Ideal IP (W) | 41.961 | 42.060 | 42.145 | **42.217** | 42.276 | 42.321 | 42.353 | **42.273** |
| Total loss in IP (W) | 8.286 | 8.317 | 8.346 | **8.376** | 8.400 | 8.420 | 8.441 | **8.521** |
| Net IP (W) | 50.247 | 50.377 | 50.491 | **50.593** | 50.676 | 50.741 | 50.794 | **50.794** |
| COP | 0.0646 | 0.0643 | 0.0640 | **0.0643** | 0.0639 | 0.0636 | 0.0638 | **0.0616** |
| Refrigeration η (%) | 21.218 | 21.134 | 21.039 | **21.123** | 21.004 | 20.899 | 20.970 | **20.245** |
| Available torq. (N-m) | 3.490 | 3.490 | 3.490 | **3.490** | 3.490 | 3.490 | 3.490 | **3.490** |
| Required torq. (N-m) | 3.394 | 3.393 | 3.391 | **3.389** | 3.386 | 3.383 | 3.379 | **3.342** |
References

[1] Bapat S L, 2006, A compact drive mechanism of a reciprocating machine, Indian Patent No. 200858.

[2] Sant K D and Bapat S L, 2010, Investigations on Alpha Stirling cryocooler driven by a novel compact mechanism, 23rd National Symposium on Cryogenics (NSC – 23), N.I.T. Rourkela.

[3] Martini W, 1978, Stirling engine design manual, NASA report no. CR 135-382.

[4] Bapat S L, 2000, Feasibility of alpha configuration for miniature Stirling cycle coolers, Proceedings of 18th International Cryogenic Engineering Conference (ICEC-18), Mumbai, 611 – 614.

[5] Sant K D, 2012, Investigations on twin Alpha Stirling cryocoolers driven by a novel compact drive mechanism with preliminary experimental results, Ph.D. Thesis, Department of Mechanical Engineering, I.I.T. Bombay.