Structure behind Mechanics: Overview

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Abstract

This letter proposes a new scenario to solve the structural or conceptual problems remained in quantum mechanics, and gives an overview of the theory proposed in quant-ph/9906130 (including quant-ph/9909025 and quant-ph/0001015).

PACS numbers: 03.65.Ca, 03.65.Bz, 11.10.Ef, 02.40.-k

Keywords: Quantum Mechanics, Classical Mechanics.

Submitted to Physics Letters A.

Although quantum mechanics in twentieth century has succeeded to predict the correct results of large numbers of experiments in the process to find new fundamental particles in the nature, it seems to have left some fundamental open problems, structural or conceptual ones:

1. the structural problems of
   - the operator ordering [1, 2],
   - the analyticity at the exact classical-limit of $\hbar = 0$ [3, 4] and
   - the semantics of the regularization in quantum field theories [5, 6];

2. the conceptual problems of
   - the universal validity [7],
   - the wave-reduction in measurement processes [8] and
   - the compatibility with causality [9, 10, 11].

These difficulties come from the problem how and why quantum mechanics relates itself with classical mechanics: the relationship between the quantization that constructs quantum mechanics based on classical mechanics and the classical-limit that induces classical mechanics from quantum mechanics as an approximation with Planck’s constant $\hbar$ taken to be zero; the incompatibility between the ontological feature of classical mechanics and the epistemological feature of quantum mechanics.

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mechanics in commonly accepted interpretations. The present theory\textsuperscript{1} originated by the previous letter \textsuperscript{2} aims to solve all of the above listed problems in quantum mechanics. Let me here present just an overview of the theory to inform the vital conclusions obtained from the theory without the mathematical and/or semantic complexities, while detail descriptions will be made in a series of full papers \textsuperscript{3}.

The proposed theory on physical reality, named as \textit{Structure behind Mechanics} (SbM), supposes that a field or a particle $X$ on the four-dimensional spacetime has its internal-time $\tilde{o}_{P(t)}(X) \in S^1$ relative to a domain $P(t)$ of the four-dimensional spacetime, whose boundary and interior represent the present and the past at ordinary time $t$, respectively. For $\hbar = \hbar/2$, the classical action $\hbar S_{P(t)}(X) \in \mathbb{R}$ realizes internal-time $\tilde{o}_{P(t)}(X)$ in the following relation:

$$\tilde{o}_{P(t)}(X) = e^{iS_{P(t)}(X)}.$$ \hfill (1)

Object $X$ also has the external-time $\tilde{o}^*_{P(t)}(X) \in S^1$ relative to $P(t)$ which is the internal-time of all the rest but $X$ in the universe. It gains the actual existence on $P(t)$ if and only if the internal-time coincides with the external-time in a quasi-periodic way:

$$\tilde{o}_{P(t)}(X) = \tilde{o}^*_{P(t)}(X).$$ \hfill (2)

This condition discretizes or quantizes the ordinary time passing from the past to the future, and mathematically supports Whitehead’s epochal theory of time \textsuperscript{4}. It also shows that object $X$ has its actual existence only when it is exposed to or has the possibility to interact with the rest of the world, and illustrates that such existence can become the empirical one through the actual interaction with the open system as in a measurement process. The indeterministic influence of the outside on object $X$ would cause the irreversibility of time on the fundamental level. In this way, the present theory can presuppose that there exist three levels of existence:

1. the ideal existence: the immutable being or potentiality,
2. the actual existence: the becoming or emergence,
3. the empirical existence: the appearing or detected.

The ordinary dogma of quantum mechanics has admitted only the third level of the indeterministic existence, while the deterministic realism in classical mechanics has accepted the first category. The present theory considers that both mechanics’ really refer the second kind of existence, \textit{emergence}, which can be defined here as deterministic process from the inside in terms of the ideal existence and that must be described later as indeterministic process from the outside including an observer in terms of the empirical existence. As such, it reconciles two different ideas of reality in general relativity and quantum theory; it also provides the regularization method in quantum field theories with the semantics that a regularization parameter is corresponding to the time-interval of the emergences of a particle. The both sides of relation (2) further obey the variational principle as

$$\delta \tilde{o}_{P(t)}(X) = 0, \quad \delta \tilde{o}^*_{P(t)}(X) = 0.$$ \hfill (3)

\textsuperscript{1} Almost all the information of the introduced theory has appeared in the newest version of quant-ph/9906130 on the LANL preprint server.

\textsuperscript{2} The author of letter \textsuperscript{2}, "Tosch Ono," is the same person as that of the present paper, "Toshihiko Ono."
These equations produce the equations of motion in the deduced mechanics.

Theory of SbM, the description of a system from inside, provides a foundation for quantum mechanics and classical mechanics as the description of a system from outside, named as protomechanics. The space $M$ of all the objects over present hypersurface $\partial P(t)$ have an mapping $o_t : TM \rightarrow S^1$ for the position $(x_t, \dot{x}_t)$ in the cotangent space $TM$ corresponding to an object $X$:

$$ o_t (x_t, \dot{x}_t) = \hat{o}_{P(t)} (X). \quad (4) $$

For the velocity field $v_t$ over $M$ such that $v_t (x_t) = \frac{dx_t}{dt}$, we will introduce a section $\eta_t$ and call it synchronicity over $M$:

$$ \eta_t (x) = o_t (x, v_t (x)). \quad (5) $$

thereby, synchronicity $\eta_t$ has an information-theoretical sense, as defined for a collective set of objects that have different initial conditions from one another. On the other hand, the emergence-frequency $f_t (\eta_t)$ represents the frequency that object $X$ satisfies condition (2) on $M$; and the true probability measure $\nu_t$ on $TM$, representing the ignorance of the initial position, defines the emergence-measure $\mu_t (\eta_t)$ as follows:

$$ d\mu_t (\eta_t) (x) = d\nu_t (x, v_t (x)) \cdot f_t (\eta_t) (x). \quad (6) $$

Through a measurement process, the above defined emergence-measure becomes the probability measure for the detection of a particle if positive everywhere. The emergence-measure for the observables measured in indirect ways can partially have negative values since there are two cases that internal-time $\hat{o}_t (X)$ exceed $\hat{o}_t^* (X)$ for condition (9) and viceversa. Such negativity partially causes the strange behaviors of non-commuting observables [15] and the breaking of Bell’s inequality [10]. The induced Hamiltonian $H_t^{TM}$ on $T^*M$, further, redefines the velocity field $v_t$ and the Lagrangian $L_t^{TM}$ as follows:

$$ v_t (x) = \frac{\partial H_t^{TM}}{\partial p} (x, p (\eta_t) (x)) \quad (7) $$

$$ L_t^{TM} (x, v(x)) = v(x) \cdot p (\eta_t) (x) - H_t^{TM} (x, p (\eta_t) (x)), \quad (8) $$

where mapping $p$ satisfies the modified Einstein-de Broglie relation:

$$ p (\eta_t) = -i\hbar \eta_t^{-1} d\eta_t. \quad (9) $$

The equation of motion is the set of the following equations:

$$ \left( \frac{\partial}{\partial t} + L_{v_t} \right) \eta_t (x) = -i\hbar^{-1} L_t^{TM} (x, v_t (x)) \eta_t (x), \quad (10) $$

$$ \left( \frac{\partial}{\partial t} + L_{v_t} \right) d\mu_t (\eta_t) = 0. \quad (11) $$

Protomechanics has the statistical description of the set $\Gamma$ of all the synchronicities on space $M$. To investigate such a description, we will introduce the related group. The group $D(M)$ of

$^3$Please allow me to name the proposed theory in this way for convenience tentatively. It can not be regarded, of course, as a new mechanics in a physical sense until it suffers the several experimental and theoretical tests.
all the $C^\infty$-diffeomorphisms of $M$ and the abelian group $C^\infty(M)$ of all the $C^\infty$-functions on $M$ construct the semidirect product $S(M) = \mathcal{D}(M) \times_{semi} C^\infty(M)$, and define the multiplication · between $\Phi_1 = (\varphi_1, s_1)$ and $\Phi_2 = (\varphi_2, s_2) \in S(M)$ as
\[
\Phi_1 \cdot \Phi_2 = (\varphi_1 \circ \varphi_2, (\varphi_2^* s_1) \cdot s_2),
\]
for the pullback $\varphi^*$ by $\varphi \in \mathcal{D}(M)$ (consult [14]). We shall further introduce the group $Q(M) = Map(\Gamma, S(M))$ of all the mapping from $\Gamma$ into $S(M)$, that has the Lie algebra $q(M) = Map(\Gamma, s(M))$ and its dual space $q(M)^* = Map(\Gamma, s(M)^*)$. Synchronicity $\eta_t^\tau(\eta)$, such that the labeling time $\tau$ satisfies $\eta_t^\tau(\eta) = \eta$, has the momentum $\eta_t^\tau(\eta) = -i\hbar\eta_t^\tau(\eta)^{-1} d\eta_t^\tau(\eta)$ and the emergence-measure $\mu_t^\tau(\eta)$ such that
\[
\bar{\mu}_t(p^* F_t) = \int_{\Gamma} d\mathcal{M}(\eta) \bar{\mu}_t(\eta) (p^* F_t(\eta))
\]
\[
= \int_{\Gamma} d\mathcal{M}(\eta) \bar{\mu}_t^\tau(\eta) (p^* F_t(\eta)),
\]
where $\mathcal{M}$ is a probability measure on $\Gamma$. The introduced labeling time $\tau$ can always be chosen such that $\eta_t^\tau(\eta)$ does not have any singularity within a short time for every $\eta$. The emergence-momentum $\mathcal{J}_t^\tau \in q(M)^*$ such that
\[
\mathcal{J}_t^\tau(\eta) = d\mathcal{M}(\eta) (\bar{\mu}_t^\tau(\eta) \otimes p_t^\tau(\eta), \bar{\mu}_t^\tau(\eta))
\]
satisfies the following relation for the functional $F_t : q(M)^* \to \mathbb{R}$:
\[
F_t(\mathcal{J}_t^\tau) = \bar{\mu}_t (p^* F_t),
\]
whose value is independent of labeling time $\tau$. For Hamiltonian operator $\hat{H}_t^\tau = \frac{\partial}{\partial \mathcal{J}_t^\tau} (\mathcal{J}_t^\tau) \in q(M)$ corresponding to Hamiltonian $p^* H_t(\eta)(x) = H_t^{TM}(x, p(\eta))$, equations (10) and (11) of motion becomes Lie-Poisson equation (consult [13]):
\[
\frac{\partial \mathcal{J}_t^\tau}{\partial t} = ad_{\hat{H}_t^\tau} \mathcal{J}_t^\tau.
\]

Classical mechanics requires the local dependence on the momentum for functionals, while quantum mechanics needs the wider class of the functions that depend on their derivatives. For the derivative operator $D = \hbar dx^j \partial / \partial x^j$, the space of the classical functionals and that of the quantum functionals are defined as
\[
C_{cl}(\Gamma) = \{ p^* F \ | \ p^* F (\eta)(x) = F(x, p(\eta)(x)) \}
\]
\[
C_q(\Gamma) = \{ p^* F \ | \ p^* F (\eta)(x) = F(x, p(\eta)(x), ..., D^n p(\eta)(x), ...) \},
\]
and related with each other as
\[
C_{cl}(\Gamma) \subset C_q(\Gamma).
\]
In other words, the classical-limit indicates the limit of $\hbar \to 0$ with fixing $|p(\eta)(x)|$ finite at every $x \in M$, or what the characteristic length $|x|$ and momentum $|p|$ such that $x/|x| \approx 1$ and $p/|p| \approx 1$ satisfies
\[
[p]^{-n-1} D^n p(\eta)(x) \ll 1.
\]
In this way, the protomechanics realizes the \textit{analyticity} of the exact classical-limit. The dual spaces make an decreasing series of subsets:

\[ C_c \circ (\Gamma)^* \supset C_q \circ (\Gamma)^*. \]  

(22)

Thus, quantum mechanics allows more restricted class of the emergence measures such as the density matrices for discrete eigen wave-functions than classical mechanics, while it has considerably wider class of observables.

The present theory also explains how protomechanics deduces classical mechanics and quantum mechanics, respectively. They will consider the space of the synchronicities such that

\[ \Gamma_A = \left\{ \eta \mid p_j (\eta) (\vec{x}) = \h^A k_j \in \mathbb{R} \text{ at a fixed point } \vec{x} \right\}, \]  

(23)

which requires \( A = 0 \) and \( A = 1 \) for classical case and quantum case, respectively. The choice of reference point \( \vec{x} \) does not affect the deduced mechanics. A Lagrange foliation \( \vec{p} \) in \( TM \) further has a synchronicity \( \vec{\eta}[k] \in \Gamma_A \):

\[ \vec{p}[k] = p (\vec{\eta}[k]); \]  

(24)

and it separates every synchronicity \( \eta[k] \in \Gamma_A \) into two parts:

\[ \eta[k] = \vec{\eta}[k] \cdot \xi. \]  

(25)

where \( \xi \in \Gamma_0^A \). Compress all the infinite information of back ground \( \xi \) finally produces classical mechanics and quantum mechanics. In the classical-limit, Lie-Poisson equation (17) deduces the classical Liouville equation for the induced probability density function \( \rho_t^{T^* M} \) on cotangent space \( T^* M \):

\[ \frac{\partial}{\partial t} \rho_t^{T^* M} = \{ \rho_t^{T^* M}, H_{T^* M} \}. \]  

(26)

For canonical Hamiltonians, Lie-Poisson equation (17) deduces the following quantum Liouville equation for the density matrix \( \hat{\rho}_t \) and the corresponding Hamiltonian operator \( \hat{H} \):

\[ \frac{\partial}{\partial t} \hat{\rho}_t = [\hat{\rho}_t, \hat{H}] / (-i \hbar). \]  

(27)

If the Hamiltonian is not canonical and has the operator-ordering problem, it will not be expressed in the summation of finite numbers of polynomials of position observable \( \hat{x} \) and momentum observable \( \hat{p} \) in general [17]. Even so, the protomechanics has no such trouble for the concrete calculations on the level of expression [17] before deducing operator expression [27]. In addition, the present theory proves valid also for the half-spin of a particle as a rigid spherical rotor in a well-known way [18], while it may also enable the conventional geometric interpretation of a spinor since such an observable is not directly measured as discussed later.

The present theory shares a modal interpretation with the de Broglie-Bohm theory [18]. A prototypical experiment would always substitute the measurement of the \textit{position} of an object \( Y \) not only for that of the position itself but also for that of an observable \( \hat{F} \) as the spin, the momentum, or the energy representing the state of another object \( X \). Object \( Y \) can be a prepared particle to be scattered, a radiated particle like a photon or a classical object such as a detector’s pointer, while object \( X \) is another particle but \( Y \) or an internal freedom of a particle \( Y \) for the observation of the spin of \( Y \). The following three processes constitute such an experiment:
1. the preparing process to select an appropriate initial state for $Y$,
2. the translating process to decompose a spectrum of $Y$, and
3. the detecting process to detect a particle $Y$ but not $X$.

On the first stage of preparation, let us suppose that observable $\hat{F}$ of a particle or a field $X$ has discrete igen vectors $|X; j\rangle$ such that $\hat{F}|X; j\rangle = j|X; j\rangle$ for every discrete igen values $j$. The initial wave function would be prepared as $|\psi_{in}\rangle = \sum_j c_j |X; j\rangle \otimes |Y; \phi\rangle$ for a wave vector $|Y; \phi\rangle$ of object $Y$ whose emergence frequency is positive everywhere:

$$f_Y(\eta)(x) \geq 0.$$

(28)

This relation requires the positivity of the Wigner function corresponding to vector $|Y; \phi\rangle$:

$$\int_{\mathbb{R}^3} d^3k \left\langle Y; \phi \left| k - \frac{k'}{2} \right\rangle e^{ik' \cdot x} \left\langle k + \frac{k'}{2} \right| Y; \phi \right\rangle \geq 0.$$ 

(29)

Initial wave function $|\psi_{in}\rangle$, on the second stage, will be changed through the spectral decomposition into $|\psi_{out}\rangle = \sum_j c_j |X; j\rangle \otimes |Y; \phi_j\rangle$, where $|Y; \phi_j\rangle$ represents the spatial wave function of $Y$ moving toward the $j$-th detector. On the third stage, the wave-reduction occurs as the decoherence that the density matrix loses its nonorthogonal parts after the interaction with the measuring apparatus and/or its environment:

$$\hat{\rho}_{out} = \sum_{j,k} c_j^* c_k |X; j\rangle \langle X; k| \otimes |Y; \phi_j\rangle \langle Y; \phi_k|$$

(30)

$$\rightarrow \quad \hat{\rho}^f = \sum_j c_j^* c_j |X; j\rangle \langle X; j| \otimes |Y; \phi_j\rangle \langle Y; \phi_j|.$$ 

(31)

To realize decoherence (30), Machida and Namiki [20] considered that a macroscopic device is an open system that interacts with the external environment, and describes the state of the measuring apparatus by introducing the continuous super-selection rules for Hilbert spaces. The state of the $j$-th detector is described for continuous measure $P$ on the region $L \subset M$ occupied with considerable number of atoms constituting the detector:

$$\hat{\rho}^{(j)} = \int_L dP(l) \hat{\rho}(l)^{(j)} \in \Omega^M.$$ 

(32)

They further utilized the Riemann-Lebesgue Lemma to induce the decoherence of the density matrix $\hat{\rho}^f$ or makes all the off-diagonal part zero through the interaction between the particle and the detector. The present theory does not only allows the continuous super-selection rules but also justifies the utilized approximation or limiting process that takes the particle number consisting the detector as infinite, without serious problems of the objectification [21] since objects always have their own reality. The relative frequency that particle $Y$ appears in the $j$-th detector should be proportional to the coefficient $|c_j|^2$ since the emergence measure of $X$ is conserved through the experiment since object $X$ itself is not measured.

As discussed so far, the present theory is a good candidate to solve all the remained problems in quantum mechanics. It meets our ordinary feeling that a celebrated Schödinger cat must know
himself that he is alive if so. It also revises the nonconstractive idea that the fundamental theory must be valid independently of the describing scale, and that classical mechanics can share an ontology with quantum mechanics. It is expected to provide the mathematical basis for the mechanics in the intermediate region between classical scale and quantum scale, or applied to the quantum phenomena of a gravitational field. In addition, the proposed mechanism discretizing the ordinary time may be utilized to calculate the nonperturbative effects of the quantum field theories.

References

[1] H.J. Groenwald, Physica 12 (1946) 405.
[2] van Hove, Mem. Acad. Roy. Belg. 26 (1951) 61.
[3] V.P. Maslov and M.V. Fedoriuk, Semi-Classical Approximation in Quantum Mechanics (Rei- del, Dordrecht, 1981).
[4] A. Truman, J. Math. Phys. 17 (1976) 1852.
[5] W. Pauli and F. Villars, Rev. Mod. Phys. 21 (1949) 434.
[6] G. ’t Hooft and M. Vertman, Nucl. Phys. B 44 (1972) 189.
[7] P. Mittelstaedt, The Interpretation of Quantum Mechanics and the Measurement Process (Cambridge University Press, 1998).
[8] Paul Busch, Pekka J. Lahti adn Peter Mittelstaedt, The Quantum Theory of Measurement (Springer-Verlag, Berlin Heidelberg, 1994), second edition.
[9] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47 (1935) 777.
[10] J.S. Bell, Physics 1 (1964) 195; Speakable and unspeakable in quantum mechanics (Cambridge University Press, 1987).
[11] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49 (1982) 91.
[12] T. Ono, Phys. Lett. A 230 (1997) 253; Phys. Lett. A 233 (1997) 493; the doctoral dissertation in University of Tokyo (1997).
[13] T. Ono, submitted to Found. Phys. (1999); quant-ph/9909023.
[14] A.N. Whitehead, Process and Reality, edited by D.R. Griffin and D.W. Sherburne (The Free Press, New York and London, 1979).
[15] J.E. Moyal, Proc. Camb. Phil. Soc. 45 (1949) 99.
[16] J. Marsden, T. Ratiu, and A. Weinstein, Trans. Am. Math. Soc. 281 (1984) 147.
[17] R. Abraham and J. Marsden, Foundation of Mechanics (Addison-Wesley, Reading, MA, 1978), second edition.
[18] P.R. Holland, *The Quantum Theory of Motion* (Cambridge University Press, 1993).

[19] E. Wigner, Phys. Rev. 40 (1932) 749.

[20] S. Machida and M. Namiki, Prog. Theor. Phys. 63 (1980) 1457; Prog. Theor. Phys. 63 (1980) 1833.

[21] B. d’Espagnat, *Reality and the Physicist* (Cambridge University Press, 1989), Addendum.