Optimization of Interval Type-2 Fuzzy Logic Controller for Uncertain Inverted Pendulum System

Qing Wu\textsuperscript{a}, Jiahao Liu\textsuperscript{b} and Tao Zhao\textsuperscript{b}

1. College of Electrical Engineering and Information Technology, Sichuan University, Chengdu 610065, China
2. College of Electrical Engineering and Information Technology, Sichuan University, Chengdu 610065, China
3. College of Electrical Engineering and Information Technology, Sichuan University, Chengdu 610065, China

\textsuperscript{a}15882181576@163.com; \textsuperscript{b}2017223030076@stu.scu.edu.cn; \textsuperscript{c}zhaotaozhao@126.com

Abstract. In this paper, an uncertain inverted pendulum system is controlled by an IT2 fuzzy logic controller (IT2 FLC) which was optimized by the particle swarm optimization (PSO) algorithm. The IT2 FLC, as an extension of the type-1 fuzzy logic controller (T1 FLC), has better control performance in dealing with uncertainties than T1 FLC. However, IT2 FLC has more complexity because the parameters of the IT2 FLC are more than the parameters of T1 FLC, so it is difficult to select appropriate parameters to get better performance. The PSO algorithm is used to optimize several parameters of IT2 FLC via two cost functions. According to simulations, the performance of the IT2 FLC is optimized by the PSO algorithm is better than the IT2 FLC without optimization, and the IT2 FLC has better performance in dealing with uncertainties than T1 FLC.

1. Introduction
The type-1 fuzzy logic controller (T1 FLC), presented by Prof. Lotfi Zadeh in 1965, has been studied for decades by many experts, and has many applications in the fields of industry. T1 FLC has the ability to deal with the uncertainties and nonlinearity. With the increase of uncertainty of the object, the interval type-2 fuzzy logic controller (IT2 FLC) is presented on the foundation of the T1 FLC. Compared with the T1 FLC, the T2 FLC has better ability to handle uncertainties, but the IT2 FLC is much more complex than the T1 FLC. The membership function is a mathematical tool for characterizing fuzzy sets and also is one of the most important parts of the IT2 FLC, whose parameters will have a great effect on the performance of the system. To get the better performance, we need to tune the parameters of membership function. However, the number of membership function is too many, so it is difficult to tune it with general methods.

The particle swarm optimization (PSO) algorithm is a kind of evolutionary algorithm developed by J. Kennedy and R.C. Eberhart in 1995, which starts from the random solution to optimal solution by iterative computation. The PSO algorithm seeks the global optimum by following the current searched optimal value, so we can use PSO algorithm to seek the optimal parameters of membership function in a certain range of space. Furthermore, the PSO algorithm has attracted much attention from the academic circles because of its advantage of easy to achieve, high precision and fast convergence.
This paper studies the optimal design of IT2 FLC for the uncertain inverted pendulum system by PSO algorithm. Simulation results show the effectiveness of the proposed method. The paper is organized as follows. Section 2 introduces preliminaries. In Section 3, the optimal design of IT2 FLC is presented. Section 4 provides numerical simulation to demonstrate the effectiveness of the proposed approach. The conclusion is drawn in the last section.

2. Preliminaries

2.1. Interval type-2 fuzzy sets

An interval type-2 fuzzy set (IT2 FS) \( \tilde{A} \) can be expressed as

\[
\tilde{X} = \int_{x \in D_x} \int_{u \in J_u \subseteq [0,1]} u_x(x, \mu) / (x, u),
\]  

(1)

Where \( x \) and \( u \) are the primary variable and secondary variable respectively; and \( J_x \) is called the support of the secondary membership function; and the amplitude of \( u_x(x, u) \), called a secondary grade of \( \tilde{X} \), equals 1 for \( \forall x \in D_x \) and \( \forall u \in J_u \subseteq [0,1] \) [1].

2.2. IT2 FLC

As the Figure 1 showed, an IT2 FLC contains five parts, called fuzzifier, interference engine, rulebase, type-reducer and defuzzifier respectively [2]. The fuzzifier can covert a crisp input to an IT2 FS by the membership function. In general, the shape of membership function is Triangle, Gauss and trapezoid. The defuzzifier can covert an IT2 FS to a crisp output. The rulebase is the heart of the IT2 FLC, which has fuzzy if-then rules of the form:

\[
\text{IF } x_i \text{ is } \tilde{X}_{i}^n \text{ and } L \text{ and } x_j \text{ is } \tilde{X}_{j}^n, \text{THEN } y \text{ is } Y^n \quad n=1,2,\cdots, N
\]

where \( \tilde{X}_{i}^n (i=1,\ldots,I) \) are IT2 FSs, and \( Y^n \) is an interval which can be the centroid of an IT2 membership function or the simplest TSK model [3]. The choice of rulebase is depend on the experience of experts or the input-output data with intelligence algorithms. The inference engine uses minimum or product t-norm generally. To simplify the calculation, we use product t-norm in this paper [3]. The type-reducer is unique for type-2 FLC, converting IT2 FSs into T1 FSs[4].

2.3. PSO algorithm

In this paper, we use PSO algorithm to optimize the IT2 FLC for the inverted pendulum system. In the PSO algorithm, each particle can be viewed as a point and all the particles make up a ‘swarm’. The particle has two information, called weight and velocities. Each individual seek the most optimal rely on the information of its own and other individuals. At each iteration, the variables of ith particle update by
\[ v_i = \omega v_i + c_i r_i (pbest_i - w_i) + c_2 r_2 (gbest - w_i) \]
\[ w_i = w_i + v_i \]

(2)

where the \( w_i \) and \( v_i \) represents the position and velocity respectively; \( pbest_i \) is the optimal weight in all iteration for a particle and \( gbest \) is the optimal weight in all iteration for all particles; and the \( c_i \) and \( c_2 \) are positive constant learning rates; and the \( \omega \) is called inertia weight; the \( r_1 \) and \( r_2 \) are random numbers between 0 and 1[5].

2.4. Dynamic model of the inverted pendulum system

The inverted pendulum system is a typical multi-variable, nonlinear and fast-moving system that can effectively reflect many keys issues in control, whose state space model can be represented as:

\[ \begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = \frac{g \sin x_1 - \frac{m l^2 x_2^2 \cos x_1 \sin x_1}{m_c + m} + \frac{\cos x_1}{m_c + m} l \left( \frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \right)}{l} + \frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m} \end{cases} \]

(3)

where \( g \) is the gravity acceleration; and \( m_c \) is the mass of the car; and \( m \) is the mass of the inverted pendulum; and \( l \) is the half of length of the inverted pendulum; and \( u \) is the control force; and \( x_1 \) is the angle of the inverted pendulum, and \( x_2 \) is the derivative of \( x_1 \)[3].

3. Optimization of the FLC for the inverted pendulum system

![Figure 2. Overall control diagram for the inverted pendulum system](image)

Table 1. The rulebase of the inverted pendulum system

| \( x_2 / x_1 \) | NB1     | NS1     | ZE1     | PS1     | PB1     |
|----------------|---------|---------|---------|---------|---------|
| NB2            | [-10.1 -9.9] | [-7.1 -6.9] | [-5.1 -4.9] | [-5.1 -4.9] | [-0.1 0.1] |
| NS2            | [-7.1 -6.9] | [-5.1 -4.9] | [-3.1 -2.9] | [-0.1 0.1] | [2.9 3.1] |
| ZE2            | [-5.1 -4.9] | [-3.1 -2.9] | [-0.1 0.1] | [2.9 3.1] | [4.0 5.1] |
| PS2            | [-3.1 2.9] | [-0.1 0.1] | [2.9 3.1] | [4.0 5.1] | [6.9 7.1] |
| PB2            | [-0.1 0.1] | [2.9 3.1] | [4.0 5.1] | [6.9 7.1] | [9.9 10.1] |
The overall control diagram for the inverted pendulum system shown in Figure.2 can be divided into two parts. The first part is FLC system and the second part is about optimization of PSO algorithm. The IT2 FLC has two inputs and one output, which represents two states of the inverted pendulum system and control force of the inverted pendulum system. The two inputs of FLC are represented by five linguistic terms NB, NS, ZE, PS, and PB, as shown in Figure.3. The fuzzy rulebase is showed in Table.1, which is determined by the experience of experts. The second part is about the optimization of IT2 FLC by PSO algorithm. The flow diagram of PSO algorithm is shown in Figure.4. In each iteration, the fitness value was computed by cost function, which was used to evaluate the weight of particles. The cost functions of PSO used as mean of root of squared error (MSE) and mean of absolute magnitude of the error (MAE) are represented as follow:

\[ \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (x_i - x_d)^2 \]  
\[ \text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |x_i - x_d| \]

where \(x_d\) is the desired output of the inverted pendulum system. The selecting of \(pbest\) and \(gbest\) is relay on the fitness value [6, 7, 8].

**Figure 3.** The membership function of IT2 FLC

**Figure 4.** The flow diagram of PSO algorithm

4. Simulation results
Let the initial value of $x_1$ and $x_2$ be equal to -0.6 and 0.25 respectively. The desired balance position is $x_d = 0$.

![Figure 5. Simulation result of the angle of the inverted pendulum](image)

The simulation result is showed as Figure 5. Because the initial membership functions is not suitable for the controlled system, the angle of the inverted pendulum shake around the balanced position. After optimization, the angle of the inverted pendulum is stable at the balanced position. The different cost functions for the IT2 FLC can be viewed in Table 2.

| Cost functions | IT2 FLC without PSO | IT2 FLC with PSO |
|----------------|---------------------|------------------|
| MSE            | 0.0059              | 0.0058           |
| MAE            | 0.0197              | 0.0139           |

In the following analysis, a disturbance with a sine signal whose amplitude is 0.8, is increased to the inverted pendulum system. The T1 FLC and IT2 FLC are designed for the uncertain inverted pendulum system, respectively. The corresponding simulation result is shown in Figure 6. It can be seen from Figure 6 that the IT2 FLC has better performance in dealing with uncertainties than T1 FLC.

![Figure 6. The control for the angle of the inverted pendulum](image)

5. Conclusions
In this paper, an IT2 FLC for the uncertain inverted pendulum system is designed. The proposed IT2 FLC is optimized by PSO algorithm. The simulation results illustrate that the performance of system after being optimized is better than the system without optimization. When disturbance exists in system, the IT2 FLC has more robustness than T1 FLC.

Acknowledgments
This work is supported by the National Natural Science Foundation of China (61703291) and the Applied Basic Research Program of Science and Technology Department of Sichuan Province, China (2016JY0085).

References
[1] Atanassov K, Gargov G. Interval valued intuitionistic fuzzy sets[M]. Elsevier North-Holland, Inc. 1989.
[2] Mendel J M. Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions, 2nd Edition[M]. Springer Publishing Company, Incorporated, 2017.
[3] Wang L X. A course in fuzzy systems and control[M]. Prentice-Hall, Inc. 1996.
[4] Wu D. Approaches for Reducing the Computational Cost of IT2 Fuzzy Logic Systems: Overview and Comparisons[J]. IEEE Transactions on Fuzzy Systems, 2013, 21(1):80-99.
[5] Eberhart R, Kennedy J. A new optimizer using particle swarm theory[C]// International Symposium on MICRO Machine and Human Science. IEEE, 2002:39-43.
[6] Castillo O, Melin P, Kacprzyk J, et al. Type-2 Fuzzy Logic: Theory and Applications[M]. Springer Publishing Company, Incorporated, 2008.
[7] Melin, Patricia, et al. "Optimal design of type-2 and type-1 fuzzy tracking controllers for autonomous mobile robots under perturbed torques using a new chemical optimization paradigm." Expert Systems with Applications40.8(2013):3185-3195.
[8] Bingül Z, Karahan O. A Fuzzy Logic Controller tuned with PSO for 2 DOF robot trajectory control[J]. Expert Systems with Applications, 2011, 38(1):1017-1031.