 Decay of Z-string due to the fermion emission.

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Abstract

The question of classical topological stability of the gauge vortex defects is reanalyzed upon taking into account the quantum perturbative effects. The purely Abelian Higgs string remains stable while the coupling of the effectively Abelian Z-string (with the fixed zero upper component of the scalar doublet) to the fermions of the minimal standard model is shown to result in its decay, via the fermionic pair emission. The decay rate is scaled as $10^{-1}$ of the decay rate of $Z^0$ boson. The influence of the surrounding charge-asymmetric fermionic matter is considered, demonstrating the dependence of the decay rate on the contour shape and its suppression at sufficiently large charge asymmetry.

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I. INTRODUCTION

The interest to the electroweak (EW) defects in the form of Z-strings [1] that could be produced at the phase transition in early universe is attributed, in particular, to their possible role in the processes of baryogenesis at the EW scale [2]. First, they are treated as possible carriers of nonzero baryon number [3,4], so that the change of this number \( \Delta B \) is related to the change of the Chern-Simons (helicity) number \( \Delta n_{\text{CS}} \) of the Z-string configuration via the integrated anomaly equation for the baryon number current [5]:

\[
\Delta B = \Delta n_{\text{CS}} = N_f \frac{\bar{g}^2}{(4\pi)^2} \cos(2\theta_W) \Delta \int d^3_x Z \cdot (\nabla \times Z),
\]

provided the background electromagnetic field is absent. Here \( g \) and \( g' \) are, respectively, the SU(2) and U(1) coupling constants, \( \bar{g} = \sqrt{g^2 + g'^2} \), \( \theta_W \) is the Weinberg angle, and \( N_f \) is the number of the fermionic families. The last integral in Eq. (1.1) is recognized to be the helicity number \( h_Z \) of the EW Z-string configuration. Second, EW strings were suggested to be the source [6] of nonequilibrium [7] required in any model of baryogenesis, to produce the baryon density via the evolution of the EW string network.

Although the simplest variant of Z-string solution is known to be unstable [1] towards to the development of either the upper component of the Higgs field doublet [8,9], or to W condensation [10], these instabilities can be possibly neutralized by particles bound to Z-string [9,11] or by imposing external magnetic field [12], respectively. The two Higgs doublet generalization of the Minimal Standard Model (MSM) admits classically stable Z-string [13].

In the papers cited above the possible quantum properties of Z-strings were neglected despite the fact that these defects possess the microscopic transverse size of the order of \( \sim m_Z^{-1} \) and could behave similar to \( Z^0 \) bosons and, in particular, become unstable via the emission of fermions. The latter point should be clarified from the very start. Indeed, the Z-string with the fixed zero upper component of the Higgs field can be treated as an effectively Abelian Abrikosov-Nielsen-Olesen (ANO) string [14] in the Higgs model with the action

\[
S = \int d^4x \left[ -\frac{1}{4} F^2_{\mu\nu} + |(\partial_\mu + igA_\mu)|^2 - \frac{\lambda^2}{2} (|\phi|^2 - \frac{\eta^2}{2})^2 \right].
\]

This string is known to be topologically stable. The formal proof is reduced to the statement that the homotopy \( f(\rho)[(1 - u)\exp(i\theta) + u] \) interpolating between the two static configurations of the scalar field \( \phi \) with different winding numbers \( n = 1 \) at \( u = 0 \) and \( n = 0 \) at \( u = 1 \), is not admissible since the energy barrier \( \Delta E \propto u^2(1 - u)^2 \int_0^\infty d\rho \rho^4 f(\rho) \), separating these two configurations with the finite energy per unit length is infinite. Here \( g \) is the U(1) coupling constant, the profile of the scalar field \( f(\rho) \) has the asymptotic \( f \to \eta/\sqrt{2} \) at \( \rho \to \infty \), \( \theta \) is the polar angle in the \( xy \) plane, \( \rho = \sqrt{x^2 + y^2} \). In quantum case one examines the path integral

\[
\langle \Psi_f | \Psi_i \rangle = \int d[\phi_f] \Psi_i^* \phi_f \int_{\phi(t_i) = \phi_f}^{\phi(t_f) = \phi_i} d[\phi] \exp(iS[\phi]) \Psi_i[\phi_i] d[\phi_i],
\]

(\( \phi \) is the shorthand notation for all relevant fields) which determines the overlap of two wave functionals. The latter is claimed to be vanishing, if the fields \( \phi_i \) and \( \phi_f \) belong to distinct
homotopical classes characterized in the present case by different winding numbers of the scalar field, since the infinite energy barrier makes the exponent to be rapidly oscillating. How is evaded this topological veto in the course of particle emission?

To this end one should emphasize the following. First, the above homotopy determines the field configuration that does not obey the equations of motion, except for the border values $u = 0$ and $u = 1$, provided $u$ is taken as dynamical variable. Admitting it as the quantum trajectory which does not oblige to obey the classical equations of motion, one encounters the infinite energy barrier making the amplitude vanishing. So, strictly speaking, only this specific type of the field deformation is not allowed quantum mechanically. On the other hand, the time dependent winding number $n(t)$ could interpolate between different topologies without going beyond the class of fields with finite energy per unit length, $\phi = \eta \exp[in(t)\theta]/\sqrt{2}$. This is still the solution of the equations of motion, provided the time component of the vector potential is induced via the condition of the vanishing covariant time derivative of the scalar field. (See Sec. II below). Classically, however, the winding number being the degree of mapping should be an integer which would seem to leave only the known possibility of the conserved $n$. One of the purposes of the present paper is to point out that this conclusion can be evaded by treating this number as quantum dynamical variable $n \equiv n(t)$. To this end we go beyond the approximation of the classical background field and quantize the vortex field configuration. The latter, in the approximation of the Higgs boson mass being much greater than the gauge boson mass called sometimes as the London limit, is known to be characterized solely by the spacetime dependent phase of the scalar field. The demand of the scalar field to be single valued should be imposed then only on the classical field configuration. Precisely, the expectation value of $n$ in the quantum state $\Psi$ should be an integer. The wave function of this state $\Psi$ will be found explicitly. The next step is to study the evolution of this wave function under the influence of the interaction of the vortex background with the propagating excitations. It will be shown that the wave function in its dependence on time and the winding number acquires the components peaked at successively diminishing winding numbers which is interpreted as violation of the topological stability of the classical string configuration. The lifetime of the string with the unit winding number turns out to be proportional to the lifetime of the gauge boson of the model, with the factor depending on the string contour. The purely Abelian string in the model survives this mechanism due to kinematical reason, since the net mass of the final particles entering the vertex of their interaction with the string background is greater than the energy splitting of the quantum levels between which the transition occurs followed by the particle emission. The emergence of the levels results from treating the winding number as quantum variable. It should be emphasized that this type of instability is by no means include the transition via the infinite energy barrier mentioned above.

In order to fix the notations, let us consider the neutral current piece of the lagrangian density of MSM assuming for a while a single fermionic family:

$$\mathcal{L}_{NC} = \mathcal{L}_{\text{boson}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}},$$

where the bosonic part is

$$\mathcal{L}_{\text{boson}} = -\frac{1}{4} Z_{\mu\nu}^2 + |(\partial_\mu + \frac{i}{2} g Z_\mu) \Phi|^2 - \frac{1}{2} \lambda^2 (|\Phi|^2 - \frac{1}{2} \eta^2)^2,$$

(1.4)
and the upper component of the Higgs doublet is taken to be zero; the fermionic part is

\[ \mathcal{L}_{\text{fermion}} = i(\bar{\psi}_+L, \bar{\psi}_-L) \left[ \partial \bar{\psi}_+L - i\bar{\psi}_-L(T_3 - Q_L)\hat{Z} \right] \left( \psi_+L \right) + \\
i\bar{\psi}_+R \left[ \partial \bar{\psi}_+R - i\bar{\psi}_-R(-Q_R)\hat{Z} \right] \psi_+R + i\bar{\psi}_-R \left[ \partial \bar{\psi}_-R - i\bar{\psi}_+R(-Q_R)\hat{Z} \right] \psi_+R; \]  

(1.5)

while

\[ \mathcal{L}_{\text{Yukawa}} = -h_-(\bar{\psi}_-L\psi_+L\Phi + \text{c.c.}) - h_+(\bar{\psi}_+L\psi_+R\Phi^* + \text{c.c.}). \]  

(1.6)

gives the mass to the fermions and describes their interaction with the Higgs boson. Note that \( h_+ = 0 \) in the case of leptons. Hereafter \( \partial \equiv \gamma^\mu \partial_\mu \) etc., \( Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \xi = \sin^2 \theta_W, \) \( T_3 \) is the third component of the weak SU(2) isospin, and \( Q_L = \text{diag}(Q_+L, Q_-L) \) is the charge matrix of the left fermions, \( Q_{\pm L(R)} \) being corresponding electric charge.

The subsequent material is organized as follows. Section II contains the derivation of the effective reduced action for the gauge vortex state in terms of the winding number, including the discussion of the contour motion omitted in Ref. [16]. It is argued there why the dynamics of the radial part (modulus) of the scalar field is inessential for evaluation of the lifetime of the string in the London limit. The quantum state of the string configuration and the criterion of its stability against the particle emission is discussed in Sec. III. The purely Abelian Higgs model admits the string stable against this mechanism. This is not the case for effectively Abelian Z-string which possesses the coupling with almost massless fermions. The calculation of the corresponding decay rate of the state of Z-string having arbitrary geometric shape, including the parity-odd contours, is presented in Sec. IV, with the taking into account of the effects of the nonzero fermionic density. The damping of the energy, Z-flux and helicity (Chern-Simons) number of Z-string is discussed in Sec. V. Sec. VI is devoted to the discussion of the validity of assumptions adopted in the paper, and to the conclusion drawn from the present study.

**II. THE ACTION OF Z-STRING IN TERMS OF THE WINDING NUMBER**

The fact that the Z-string solution [1] is an embedding into SU(2)\( \times \)U(1) gauge group of the known U(1) ANO vortex solution [14] [compare Eqs. (1.2) and (1.4)] permits one to refer to our earlier derivation [16] of the reduced effective action. When so doing, it is very comfortable to use the space Fourier transforms of the fields, which are elementary functions rather than the special ones used in writing down the original solution [1].

The nonstationary field configuration of Z-string is expressed through the spacetime dependent phase \( \chi \equiv \chi(x, t) \) of the Higgs field \( \Phi(x, t) = \eta \exp(i\chi) / \sqrt{2} \) in the London limit. This is the limit of \( m_H \gg m_Z, \ln m_H/m_Z \) is also large, where \( m_Z = \bar{g}\eta/2 \) and \( m_H = \lambda\eta \) are the masses of the \( Z^0 \) and Higgs bosons; \( \lambda \) and \( \eta/\sqrt{2} \) are the Higgs field self-coupling and magnitude. Let us remind briefly why the dynamics of the phase \( \chi \) of the scalar field, not the dynamics of its radial part (modulus) \( f \), is the only one essential in this limit. To this end one should rewrite the action Eq. (1.2) in terms of these variables,

\[ S = \int d^4x \left\{ \frac{1}{2}(-\partial_t A - \nabla A_t)^2 - \frac{1}{2} (\nabla \times A)^2 + f^2 (\dot{\chi} + gA_t)^2 - (\nabla \chi - gA)^2 \right\} \]
\begin{equation}
+ f^2 - (\nabla f)^2 - \frac{\lambda^2}{2} \left( |\phi|^2 - \frac{\eta^2}{2} \right)^2 \right),
\end{equation}

and to convince that the contribution of the radial part to the path integral Eq. (2.1) is factored out in the London limit. Indeed, an exact factorization could be broken, in principle, in the following two cases. First, there is the mixed third term in the first line of Eq. (2.1). But for large distances \( \rho \gtrsim m^{-1}_H \) one may set \( f \simeq \eta/\sqrt{2} \), which leaves only the phase, while at the short distances \( \rho < m^{-1}_H \) the amplitude of the Higgs condensate behaves as \( f \sim \rho^n \) \cite{14}, so this term gives negligible contribution. The conclusion remain valid in the case of oscillating \( n \) (see below), since one should average the contribution over the period of oscillations after which it is not the winding number itself but its amplitude that enters into expression leaving the suppression at \( \rho \to 0 \) intact. Second, the term \( \dot{f}^2 \) is proportional to \( \dot{n}^2 \) at the short distances, however, its contribution to the action is suppressed as \( (\eta / m_H)^2 \) as compared to the logarithmically enhanced \( (\ln m_H / m_Z \gg 1) \) contribution of large distances \( \rho \gtrsim m^{-1}_H \) coming from the terms containing the phase. So, one can ignore the details of the Higgs field profile \( f \), taking it to be uniform \( \eta/\sqrt{2} \) in all coordinate space except the vortex line where it approaches zero at characteristic distances \( \sim m^{-1}_H \). The contribution of the radial part is then factored out as redefinition of the measure in the path integration over \( n(t) \).

The equation for the Z-magnetic field,

\begin{equation}
\nabla \times H_Z = \frac{2m^2_Z}{g} \nabla \chi - m^2_Z \mathbf{Z},
\end{equation}

is solved to give the Z-magnetic field strength,

\begin{equation}
H_Z(k, t) = \frac{4\pi n_a}{g} \cdot \frac{m^2_Z}{k^2 + m^2_Z} \oint d\sigma X'_a \exp(-ik \cdot X_a),
\end{equation}

and the vector potential:

\begin{equation}
Z(k, t) = \frac{4\pi n_a}{g} \cdot \left( \frac{1}{k^2} - \frac{1}{k^2 + m^2_Z} \right) \oint d\sigma i[k \times X'_a] \exp(-ik \cdot X_a).
\end{equation}

Here the integral over \( \sigma \) comes from the equation for the phase \( \chi \) read off from Ref. \cite{17}, with the proper continuation to the Minkowski spacetime:

\begin{equation}
\nabla \times \nabla \chi(x, t) = 2\pi n_a \oint d\sigma X'_a \delta^3(x - X_a(\sigma, t)),
\end{equation}

where \( X_a \equiv X_a(\sigma, t) \) is the evolving closed string contour \( a \) parametrized by the arclength \( \sigma \). Hereafter the prime over \( X \) will denote the derivative with respect to corresponding parameter along the contour, while the overdot will do the time derivative. The case of many contours is embraced by taking the sum over individual contributions in the right hand side of Eq. (2.3). It is argued in Ref. \cite{18} that possible fermionic zero modes \cite{19} do not perturb the string profile. Recall that the winding number \( n_a \equiv n_a(t) \) of the scalar field is directly related with the number of quanta of the magnetic-like flux, in the present case...
the Z-flux, via the condition of the vanishing covariant derivative of the Higgs field deep inside in the Higgs condensate,

$$\oint Z \cdot dl = \frac{2}{g} \oint \nabla \chi \cdot dl = \phi_0 n_a,$$

(2.6)

where $\phi_0 = \frac{4\pi}{g}$ is the quantum of Z-flux. It is essential that the first term in the parentheses of Eq. (2.4) is in fact a pure gauge one. Indeed, the Fourier component of

$$v(x, t) \equiv \frac{2}{g} \nabla \chi$$

found from Eq. (2.5), is

$$v(k, t) = \frac{4\pi n_a}{g k^2} \oint d\sigma [k \times X'_a] \exp(-i k \cdot X_a).$$

(2.7)

Locally, in a plane transverse to the tangent vector $X'_a$, Eq. (2.7), after going back to the coordinate space, leads to the transverse components of $v$ to be

$$v_i(x, t) \propto \varepsilon_{ij} \partial_j \ln |\rho| = \partial_i \ln |\rho| / |\rho|; X'_a \cdot \rho = 0.$$

To specify the dynamical part of the problem, one should write down the Z-electric field $E_Z = -\nabla Z_t - \partial_t Z$, where

$$Z_t = -\frac{2}{g} \partial_t \chi$$

(2.8)

replaces the condition $Z_t = 0$ appropriate in the static case. One has

$$E_Z = \frac{2}{g} \nabla \partial_t \chi - \partial_t Z = \frac{2}{g} (\nabla \partial_t - \partial_t \nabla) \chi + \partial_t \left( \frac{2}{g} \nabla \chi - Z \right).$$

(2.9)

The commutator of the derivatives is nonzero in view of the singular character of the phase $\chi$, so the Fourier component of $E_Z$ becomes

$$E_Z(k, t) = -\frac{4\pi n_a}{g} \oint d\sigma (\dot{X}_a \times X'_a) \exp[-i k \cdot X_a(\sigma, t)]$$

$$+ \frac{k^2}{k^2 + m_Z^2} \partial_t \nabla (k \cdot v(k, t)).$$

(2.10)

Note that the vector potential $Z$ and the magnetic $H_Z$ type field strength can also be expressed through the gradient of the singular phase $\chi$ as

$$Z(k, t) = \left( 1 - \frac{k^2}{k^2 + m_Z^2} \right) v(k, t),$$

$$H_Z(k, t) = \frac{m_Z^2}{k^2 + m_Z^2} [k \times v(k, t)].$$

(2.11)

In what follows we will omit the encounters of the nearby string segments. Important are in the processes of the string rearrangements, they cannot be described in the framework of
the London limit and demands the numerical integrations of the full set of the equations of motion. Substituting Eqs. (2.3), (2.4) and (2.10) into the lagrangian \( L_{\text{boson}} = \int d^3x L_{\text{boson}} \) one obtains, with the help of the relation
\[
\int d^3x H_z^2(x) = \int d^3k|H_z(k)|^2/(2\pi)^3
\]
etc, the expression for the action of the single gauge vortex:
\[
S_{\text{vortex}} = \frac{\phi_0^2}{2(2\pi)^3} \int \frac{d^3k}{(k^2 + m_Z^2)^2} \int dt \int d\sigma_1 d\sigma_2 \exp \left\{ ik \cdot [X(\sigma_1) - X(\sigma_2)] \right\}
\times \left\{ n_a^2 k^2 - m_Z^2 [X'(\sigma_1) \cdot X'(\sigma_2)] + n_a^2 (k^2 + 2m_Z^2) [k \cdot (\dot{X}(\sigma_1) \times X'(\sigma_1))] \right.
\left. \cdot \dot{X}(\sigma_2) \times X'(\sigma_2) \right\}.
\]
Let us make a step apart and show with the method similar to those of P. Orland, Ref. [17] and [20] how the known Nambu-Goto (NG) action results from Eq. (2.12). To this end one should take the limit of the fixed winding number \( n_a \) and set the Z-boson mass \( m_Z \to \infty \) before the momentum integration. Then the term \( \propto n_a \) drops out, and the action becomes, in the gauge \( t = \tau \),
\[
S_{\text{NG}} = \frac{\phi_0^2}{2} \int d^2s_1 d^2s_2 \delta^{(4)} [X(s_1) - X(s_2)] \left\{ -X'(s_1) \cdot X'(s_2)
\right.
\left. + [\dot{X}(s_1) \times X'(s_1)] \cdot [\dot{X}(s_2) \times X'(s_2)] \right\},
\]
where \( s_{1,2} = (\tau_{1,2}, \sigma_{1,2}) \) is the two- dimensional vector. Using the Gaussian regularization of the \( \delta \) function and the expansion
\[
X(s_2) \simeq X(s_1) + (s_2 - s_1)^A \partial_A X
\]
valid under the condition \( |X''(\sigma)| \ll m_Z \), one obtains
\[
S_{\text{NG}} = \frac{1}{2} \left( \frac{\phi_0}{2\pi \Lambda^2} \right)^2 \int d^2s_1 d^2z \exp \left( -\frac{1}{2\Lambda^2} z^A z^B \partial_A X^\mu \partial_B X_\mu \right) (-\dot{X}^2 + [\dot{X} \times \dot{X}']^2)
\]
\[
= \frac{\phi_0^2}{4\pi \Lambda^2} \int d^2s \sqrt{\det \partial_A X^\mu \partial_B X_\mu},
\]
where \( \Lambda^{-1} \to \infty \) is an ultraviolet cutoff, \( \partial_A = \partial/\partial z^A \) and \( \det \partial_A X^\mu \partial_B X_\mu = -\dot{X}^2 + [\dot{X} \times \dot{X}']^2 \) in the chosen gauge. Up to an overall factor, the last equality in Eq. (2.13) is recognized to be the NG-action.

Coming back to the case of large but finite \( m_Z \), one obtains the action as the sum \( \sum_a S^{(a)}_{\text{vortex}} \), where
\[
S^{(a)}_{\text{vortex}} = \frac{4\pi}{g^2} \ln \frac{m_H}{m_Z} \int dt \int d\sigma \left\{ (\dot{\eta}_a^2 - m_{Z_a}^2) X_a^2 + m_{Z_a}^2 \eta_a^2 [\dot{X}_a \times \dot{X}_a']^2 / c_0^3 \right\}.
\]
Here the first and the third terms come from the electric-type field, while the second term comes from the kinetic energy of the scalar field, thus demonstrating an intimate interplay
of the vector and scalar fields in the string solution. The energy of $Z$-magnetic field is not enhanced logarithmically in the London limit and by this reason is dropped. Further, 
\[ c_0^2 = \frac{4m_Z^2}{O(1)m_H^2} \ln \frac{m_H}{m_Z} \ll 1 \]
is the velocity squared which characterizes the classical string motion in the case of finite masses, the factor $O(1)$ reflects the ignorance of the true Higgs field profile, and $m_H$ appears as the natural upper limit of the integration over momentum. We retain only the terms in the action that refer to the string background. Omitted are the terms which correspond to the propagating $Z$ and Higgs bosons. They result in the renormalization of the parameters of the action Eq. (2.16) and do not contribute, due to the energy conservation, to its imaginary part (see below), which will be further of our main concern. Eq. (2.16) is valid in the case of large but finite $m_Z$, so the terms corresponding to the interaction are exponentially small, and only nearby segments of the string contour give an appreciable contribution to the integral over arclength.

III. QUANTUM STATE OF THE STRING CONFIGURATION AND THE CRITERION OF ITS STABILITY AGAINST THE PARTICLE EMISSION

It is seen that under the condition $|\dot{X}_a| \ll c_0$ assumed hereafter, the dynamics of the winding number becomes decoupled from the contour dynamics and is governed by the oscillator-like action,
\[ S_{vortex}^{(a)} = \frac{1}{2} \int dt M_v (\dot{n}_a^2 - \omega^2 n_a^2), \]where the frequency and effective mass, in the gauge $X'^2 = 1$, are, respectively, $\omega = m_Z$ and
\[ M_v = \frac{8\pi L}{\bar{g}^2} \ln \frac{m_H}{m_Z}. \]Hereafter $L = \int d\sigma$ is the length of $Z$-string in the chosen gauge. The quantization subjected to the constraint $\langle \psi|n|\psi \rangle = n_0$, where $n_0$ is an integer, is performed with the help of the indefinite Lagrange multipliers and gives the energy levels of the system
\[ E_N = \varepsilon_v Ln_0^2 + m_Z(N + \frac{1}{2}), \]where $\varepsilon_v = 4\pi m_Z^2 \bar{g}^{-2} \ln m_H/m_Z$ is the energy per unit length of the vortex with the unit winding number, $N = 0, 1 \cdots$ (do not confuse with the winding number $n$) \[21\]. The wave functions are the oscillatory ones, $\psi_N(n) = \psi_N^{(osc)}(n - n_0),$
\[ \psi_N^{(osc)}(n - n_0) = \left( \frac{2\varepsilon_v L}{\pi m_Z} \right)^{1/4} \frac{1}{\sqrt{2^N N!}} \exp \left[ -\varepsilon_v L / m_Z (n - n_0)^2 \right] H_N \left( \sqrt{2\varepsilon_v L / m_Z (n - n_0)} \right), \]($H_N$ is the Hermit polynomial) displaced to $n_0$. They are sharply peaked at the integer numbers. Specifically, the wave function of the vacuum state without string is $\psi_0^{(osc)}(n)$, irrespective of the string contour $X$ and the radial part of the scalar field.
Now one can evaluate the overlap integral (1.3) for the string with different winding numbers, \( n_i \) at the moment \( t = 0 \) and \( n_f \) at some later moment \( t > 0 \). The string will be assumed to be in the ground states, \( N = 0 \), so that its wave functions at those moments are \( \psi_i = \psi^{(osc)}(n_i) \) and \( \psi_f = \psi^{(osc)}(n_f) \), respectively. The result depends crucially on the spectrum of the propagating excitations, since the string interaction with the latter may result in an imaginary correction to the energy levels Eq. (3.3). As it will be shown below, such a correction emerges as the imaginary part of the frequency of the \( n \)-oscillator, so that the wave functional of the initial string configuration acquires a correction that gives rise to a logarithmic damping of the string.

The expression for the probability averaged over the period \( T = 2\pi m^{-1}_Z \) looks as

\[
\langle f|n_i \rangle = \frac{-\varepsilon_{V} L}{2m_Z} \left[ n_i^2 + n_f^2 - 2n_i n_f \exp(-\Gamma t/2 - i m_Z t) \right].
\]

The expression for the probability averaged over the period \( T = 2\pi m^{-1}_Z \) looks as

\[
\langle w_{f_1} \rangle = \mathcal{N} \exp \left[ -\frac{\varepsilon_{V} L}{m_Z} (n_i^2 + n_f^2) \right] I_0 \left[ \frac{2\varepsilon_{V} L}{m_Z} n_i n_f \exp(-\Gamma t/2) \right] 
\approx \mathcal{N} \exp \left\{ -\frac{\varepsilon_{V} L}{m_Z} [n_f - n_i \exp(-\Gamma t/2)]^2 \right\}.
\]

[\( I_0(z) \) is the modified Bessel function of order zero], where the contribution of the path integral over the radial part of the scalar field, as is argued in Sec. [2], is taken into account in the form of the constant normalization factor \( \mathcal{N} \).

If the damping were absent, \( \Gamma = 0 \), the expression for the relative probability would have a sharp maximum at \( n_i = n_f \), thus reproducing at the quantum level the topological conservation of the string winding number. On the contrary, at \( \Gamma \neq 0 \) the maximum is achieved at the configurations with the decaying winding number. The character of the decay is established by reconciling the integers \( n_i \) and \( n_f \) with the continuous time dependence in Eq. (3.4). Indeed, somewhere within the time interval \( \Delta t \) from the start, such that \(-2\Gamma^{-1} \ln (1 - 1/2n_i) < \Delta t < -2\Gamma^{-1} \ln (1 - 3/2n_i)\), the nearest integer \( n_f \) for which the probability is nonzero, is \( n_f = n_i - 1 \), and so forth down to \( n_i = 1 \). In particular, the lifetime of the U(1) gauge string with \( n = 1 \) is \( 2\Gamma^{-1} \). The classical topological veto is evaded, because due to the damping the wave functional of the initial string configuration acquires the components of the vacuum state. Note that the topology change is faster for initially multiple winding numbers, since the time of awaiting of \( n = n_i - 1 \) is \( \Delta t \sim 1/\Gamma n_i \ll \Gamma^{-1} \), so that the total decay time is \( T \sim 2\Gamma^{-1} \sum_{k=1}^{n_i} k^{-1} \sim 2\Gamma^{-1} \ln n_i \). The last approximate expression has the same form as if the winding number would change continuously in the course of the decay.

Let us confront the result of Eqs. (3.3) and (3.6) with the usual approach to the string stability. What is implied in the present approach is the preparation of the string in the quantum state in which the expectation value of the winding number is kept fixed. It is perfectly the case that takes place for the Abrikosov string in type II superconductors immersed into external magnetic field, while no specific mechanism has not yet been devised for the Z-string. Now remove the constrain sustaining the string. Then the state with the fixed winding number becomes the superposition of the stationary states of an unconstrained oscillator, so that the amplitude of finding it in the state with different expectation value \( n_f \)
of the winding number at a later moment is given by Eq. (3.5). The feature of the present approach is thus the quantum mechanical treatment of the background field configuration. It should be recalled in this respect that the notion of the classical background field is itself an approximation aimed to represent the condensate of indefinite number of quanta of corresponding quantum field. One of the possible representatives of such a condensate is the coherent state, in which the expectation value of the field operator, in the present case the phase of the Higgs field, is nonzero. The wave function \( \psi_0^{(osc)}(n - n_0) \) used in the above calculation is just the wave function of the coherent state in which the winding number of the scalar field is the integer \( n_0 \). In this sense the usual topological stability of the classical configuration would correspond to the permanently constrained string resulting in the amplitude \( \langle f | i \rangle \propto \delta_{n_i n_f} \) irrespective of the damping rate \( \Gamma \). The rôle of the damping in this case is reduced to relaxing the field configuration to that determined by external conditions.

To establish if \( \Gamma \) is zero or not and hence the condition of the string stability, one should examine the spectrum of the propagating excitations and their couplings with the string background. Expanding the action of purely Abelian Higgs model Eq. (1.2) into the contributions of the string background and the excitations, which are the massive neutral Higgs \( \varphi_H \) and vector \( A \) bosons, one can show that the only relevant coupling is the string \( \rightarrow \varphi_H + A \) one. Since the emission of the excitations occurs in the course of the transitions between the levels spaced, with the evident replacement, by \( \omega \approx m_A \), its contribution to \( \Gamma \) is forbidden by the energy conservation. Thus, the stability of the Abelian Higgs string in the model Eq. (1.2) with the neutral scalar field is merely the kinematical consequence of rather limited spectrum of excitations [23]. It well may be not the case in other models. In particular, the effectively Abelian Z-string with the fixed upper component of the Higgs doublet illustrates this since possesses the interaction with almost massless fermions.

IV. EVALUATION OF THE DECAY WIDTH OF Z-STRING.

The fermionic loop correction to the action Eq. (3.1) can, in principle, be evaluated by the integrating out the propagating fermions from the total action of the standard model. However, we need only the imaginary part of the resulting effective action. Since this imaginary part arises due to real intermediate states on the mass shell, one can use the unitarity relation for its evaluation and, in turn, \( \text{Im} \omega \), allowing for the quantum transitions between the energy levels of the oscillator Eq. (3.1).

In order to obtain the lagrangian of interaction of Z-string with the physical fermions, one should rotate away the phase \( \chi \) from the mass term of the Lagrangian of the standard model Eq. (1.6), with zero upper component of the Higgs field. It can be accomplished by the phase rotation of the chiral fermions,

\[
\begin{align*}
\psi_L & \rightarrow \psi_L \exp[-2i\chi(T_3 - Q_L\xi)], \\
\psi_{\pm R} & \rightarrow \psi_{\pm R} \exp(2i\chi Q_{\pm R}\xi),
\end{align*}
\]

with the phases proportional to their respective Z charges,

\[
Q_Z = T_3 - Q \sin^2 \theta_W;
\]
\(Q\) and \(T_3\) standing, respectively, for the electric charge and the third component of weak isospin of the chiral fermion. The pure gauge term of the background field \(Z\) presented as the first term in the parentheses of the expression for \(Z\) in Eq. (2.11) absorbs the gradient of the rotation phase thus leading to no shift of the lagrangian. The term describing the interaction of the single chiral fermion, say the left one, with the string becomes

\[ \mathcal{L}_{\text{int}} = \bar{\psi}_L \hat{Z}^{(1)} Q_Z \psi_L, \]

where the Fourier transform of the space components of the short range piece of the background \(Z\)-field \(Z^{(1)}\) is given by the second term in parentheses of expression for \(Z\) in Eq. (2.11), while the time component is zero in the London limit. Note that taking the limit \(m_Z \to \infty\) results in the decoupling of \(Z\)-flux from external fermions.

The matrix element of the emission of a pair of chiral fermions by \(Z\)-string in the course of the quantum transition between the levels \(N\) and \(N - 1\) shifted by \(m_Z\) is

\[ \mathcal{M}_N = -2\pi \bar{\psi}_L \gamma_i \mathbf{V} \gamma_i \psi_L \langle N - 1 | n_a | N \rangle, \]

as is evident from the oscillator-like character of the action Eq. (3.1). The following calculation of the width \(\lambda_N\) of the \(N\)th level includes the effects of the particle-antiparticle asymmetry in the surrounding fermionic matter by means of the Fermi blocking factors in the expression for the density of final states. Inserting the identity \(1 = f \int d^3k \delta(k - p_1 - p_2)\), one obtains

\[ \lambda_N = \frac{N \tilde{g}^2 Q_Z^2}{8 M_v m_Z (2\pi)^5} \int d^3k \frac{d^3p_1 d^3p_2}{\varepsilon_1 \varepsilon_2} \delta(m_Z - \varepsilon_1 - \varepsilon_2) \delta(k - p_1 - p_2) \]

\[ \times \sum_{\text{spins}} |\bar{u}_L(p_1)\gamma_i \mathbf{V} \psi_L(p_2)|^2 [1 - f(\varepsilon_1 - \mu)][1 - f(\varepsilon_2 + \mu)] \]

\[ = \frac{N \tilde{g}^2 Q_Z^2}{16\pi M_v m_Z} \int \frac{d^3k}{(3\pi)^2 |k|} \theta(m_Z - |k|) \int_{(m_Z - |k|)/2}^{(m_Z + |k|)/2} d\varepsilon \left\{ |\mathbf{V}|^2 (k^2 - m_Z^2 + 4m_Z\varepsilon) \right\} \]

\[ + (k^2 - m_Z^2)2\varepsilon - m_Z) k \cdot [\mathbf{V} \times \mathbf{V}^*] / k^2 \}

\[ \times [1 - f(\varepsilon - \mu)][1 - f(m_Z - \varepsilon + \mu)]. \]

Here \(T\), \(\mu\) are, respectively, the temperature and chemical potential, \(\mathbf{V} \equiv \mathbf{V}(k)\), \(f(\omega) = (\exp \frac{\omega}{T} + 1)^{-1}\) is the Fermi distribution, the fermions are taken massless, and the temperature dependence of \(m_Z\) is assumed to be included. The linear dependence of \(\lambda_N = N\Gamma\) on \(N\) means a nonzero imaginary part of the frequency \(\text{Im} \omega = -\Gamma / 2\), hence the damping of the quantum mechanical probability, with the rate \(\Gamma\).

Internal integral over \(\varepsilon\) in Eq. (4.4) can be written as

\[ G(\mu, |k|) \equiv \int_{(m_Z - |k|)/2}^{(m_Z + |k|)/2} d\varepsilon \cdots = |\mathbf{V}|^2 (k^2 + m_Z^2) I_1(|k|) \]

\[ + 2(2m_Z |\mathbf{V}|^2 + (k^2 - m_Z^2) k \cdot [\mathbf{V} \times \mathbf{V}^*] / k^2) I_2(|k|), \]

(4.5)
where

\begin{align*}
I_1(|k|) &= \frac{T}{1 - \exp(-m_Z/T)} \ln \frac{\cosh T + \cosh \frac{m_Z + |k|}{2T}}{\cosh T + \cosh \frac{m_Z - |k|}{2T}}, \\
I_2(|k|) &= \frac{1}{2} \frac{\mu}{T} \exp \frac{m_Z}{2T} \int \frac{d\varepsilon}{|k|/2} \left( \frac{\cosh m_Z \varepsilon}{2T} + \cosh (\varepsilon - \mu) \right) \left( \cosh m_Z \varepsilon + \cosh (\varepsilon + \mu) \right).
\end{align*}

(4.6)

Because of \( V \) is rather complicated function of the momentum, further evaluation of the decay width cannot be performed explicitly. Instead, we will obtain useful approximate results valid in the situations of the physical interest.

i) Empty space, \( \mu = 0, T = 0 \). The charge conjugation \((C)\)-even, parity \((P)\)-odd, \( CP \)-odd structure \( i \mathbf{k} \cdot [\mathbf{V} \times \mathbf{V}^*]/k^2 \) drops after the integration over final states as it should, since no \( CP \) nonconserving effects remain in the charge symmetric situation, in the absence of an explicit \( CP \) nonconservation in the lagrangian. The necessary expression for \(|V(k)|^2\),

\[ |V(k)|^2 = \left( \frac{4\pi}{\bar{g}} \right)^2 \frac{k^2}{(k^2 + m_Z^2)^2} \int d\sigma_1 d\sigma_2 (X'_1 \cdot X'_2) \exp[-i \mathbf{k} \cdot (\mathbf{X}_1 - \mathbf{X}_2)], \]

where \( X_1 \equiv X(\sigma) \) etc, can be obtained for sufficiently smooth contours whose curvature satisfy the condition \(|X''|/m_Z \ll 1\). Then one can use the expansion

\[ X(\sigma_2) = X(\sigma_1) + \frac{z}{1!} X'(\sigma_1) + \frac{z^2}{2!} X''(\sigma_1) + \frac{z^3}{3!} X'''(\sigma_1) \cdots, \]

(4.7)

where \( z = \sigma_2 - \sigma_1 \), to show that upon neglecting the terms with the second and higher derivatives of the contour the following approximate expression holds:

\[ |V(k)|^2 \simeq 2\pi \left( \frac{4\pi}{\bar{g}} \right)^2 \frac{k^2}{(k^2 + m_Z^2)^2} \int d\sigma \delta(\mathbf{k} \cdot \mathbf{X}'_a). \]

(4.8)

When obtaining Eq. (4.8), the integration over \( z \) can be extended to \( \pm \infty \). Choosing the local (at given \( \sigma \)) coordinate system \( \mathbf{k} = (\mathbf{k}_\perp, \mathbf{k} \cdot \mathbf{X}') \), the \( \mathbf{k} \) integration is easily performed. The summation over all fermionic species, with the expression for the total width of \( Z^0 \)-boson

\[ \Gamma_Z = \bar{g}^2 m_Z \sum_{\text{fermions}} Q^2_Z/24\pi, \]

gives the damping rate

\[ \Gamma = \kappa (1 - \ln 2) \Gamma_Z. \]

(4.9)

Hereafter the notation

\[ \kappa = \frac{3}{4 \ln m_H/m_Z} \]

is used. The length of the string \( L \) drops from the final expression, since the factor \( L^{-1} \) coming from the probability of the quantum jump \(|\langle N - 1 | n_a | N \rangle|^2\) [see Eqs. (3.2) and (4.3)], is cancelled by another factor \( L \) [see Eq. (4.8)] arising due to incoherent emission of a fermionic pair by arbitrary point along the string. Since \( \ln m_H/m_Z \) is conjectured to be
large, say 3, the damping becomes about $10^{-1}$ of the decay rate of $Z^0$ boson. Note that the emission of the Higgs boson and of a pair of $Z$-bosons is forbidden by the energy conservation, since in the lowest order adopted here the energy released in the decay is $m_Z$. The photon emission occurs only in higher orders, hence corresponding partial rate is suppressed.

ii) Cold, strongly charge-asymmetric fermionic matter, $|\mu|/T \gg 1$, $|\mu| > m_Z$. This is the case of the strong degeneracy of the fermions, so one can write

$$[1 - f(\varepsilon - \mu)][1 - f(m_Z - \varepsilon + \mu)] \simeq \begin{cases} \exp(\varepsilon - \mu)/T, & \mu > 0 \\ \exp(m_Z - \varepsilon - |\mu|)/T, & \mu = -|\mu| < 0. \end{cases} \tag{4.10}$$

Consider for the definiteness the case $\mu > 0$. Then the expression for $G(\mu, |k|)$ reads

$$G(\mu, |k|) \simeq T \exp \left( \frac{m_Z/2 - \mu}{T} \right) \left\{ |V|^2 \left[ (|k| + m_Z)^2 \exp \left( \frac{|k|}{2T} \right) - (|k| - m_Z)^2 \exp \left( -\frac{|k|}{2T} \right) \right] + 2|k| |k| \cdot (k^2 - m_Z^2) i \vec{k} \cdot \left[ \vec{V} \times \vec{V}' \right]/k^2 \right\}. \tag{4.11}$$

Since the dominant contribution comes from $|k| \sim m_Z$, and $m_Z \gg T$, one may keep only the rising exponents in the calculation. Furthermore, all the polynomial $|k|$-dependent expressions in the denominators of the $V$-dependent expressions can be approximated by their values at $|k| = m_Z$. The evaluation of the $CP$-odd contribution uses the expression

$$\frac{i \vec{k} \cdot [\vec{V} \times \vec{V}']/k^2}{k^2} = \left( \frac{4\pi}{\bar{g}} \right)^2 \int d\sigma_1 \int d\sigma_2 \frac{i \vec{k} \cdot \left[ \vec{X}'_1 \times \vec{X}'_2 \right]}{(k^2 + m_Z^2)^2} \exp(-i \vec{k} \cdot \vec{X}_{12}),$$

where $\vec{X}_{12} \equiv \vec{X}(\sigma_1) - \vec{X}(\sigma_2)$, together with the dropping of the terms, which are suppressed by the powers of $m_Z/|\chi_{12}| \gg 1$. The integration over $\vec{k}$ and the inclusion of the case $\mu < 0$ results in the damping rate:

$$\Gamma \simeq \kappa \left( \frac{T}{m_Z} \right)^2 \langle \exp\left( \frac{m_Z - |\mu|}{T} \right) \rangle \left( 1 + \frac{TF_1}{m_Z} \right) \Gamma_Z, \tag{4.12}$$

where

$$\langle (\cdots) \rangle = \sum_f (\cdots) Q^2_Z \sum_f Q^2_Z,$$

is the average over $Z$-charges, and

$$F_1 = \frac{8}{\pi L} \int d\sigma_1 d\sigma_2 \frac{\vec{X}_{12} \cdot \left[ \vec{X}'_1 \times \vec{X}'_2 \right]}{X_{12}^2} \sin m_Z |\chi_{12}| \tag{4.13}$$

is the parity-odd factor which depends on the geometry of the string contour.

The damping is completely prohibited in this case. In fact, the energy of emitted fermion spreads from 0 to $m_Z$, so at $|\mu| > m_Z$ it necessarily hits the filled states inaccessible by the Pauli exclusion principle. The conditions of the complete suppression are not likely to be achieved in the universe at large scale. To be sure, the chemical potentials of all fermions should obey the condition $|\mu| > m_Z$. But only for neutrinos there is a rather high cosmological upper bound $[\sum_\nu (\mu_\nu/T)^1/4] < 45$ for the net contribution and considerably...
more stringent bound from nucleosynthesis, $|\mu_e|/T < 0.2$, for the electronic neutrino. The baryonic and leptonic asymmetries are small, $\sim 10^{-8} - 10^{-10}$. So the averaged fermionic asymmetry of the universe is insufficient to prohibit the decay strongly. Yet the smaller scale inhomogeneities in chemical potential of all types of fermions at the EW epoch could not be excluded by this argument.

iii) Cold, weakly charge-asymmetric fermionic matter, $|\mu| \ll T \ll m_Z$. One has

$$G(\mu, |\mathbf{k}|) \simeq |\mathbf{k}| \left\{ |\mathbf{V}|^2 \left( \mathbf{k}^2 + m_Z^2 + 8\mu m_Z \exp \frac{-m_Z}{2T} \right) + 4\mu \exp \frac{-m_Z}{2T} \cdot \mathbf{ch} \left( \frac{|\mathbf{k}|}{2T} \right) \cdot (\mathbf{k}^2 - m_Z^2) \mathbf{i} \cdot [\mathbf{V} \times \mathbf{V}^*]/\mathbf{k}^2 \right\}, \quad (4.14)$$

which results in the damping rate:

$$\Gamma \simeq \kappa \left( 1 - \ln 2 + \frac{2 \langle \mu \rangle T}{m_Z^2} + \frac{\langle \mu \rangle T^2 F_1}{m_Z^2} \right) \Gamma_Z, \quad (4.15)$$

with the same geometric factor $F_1$ as in previous case.

iv) Hot, weakly charge-asymmetric fermionic matter, $\mu/T \ll 1, m_Z/T \ll 1$. Here the above $G$ is

$$G(\mu, |\mathbf{k}|) \simeq \frac{1}{4} |\mathbf{k}| \left\{ |\mathbf{V}|^2 \left[ \mathbf{k}^2 \left( 1 + \frac{m_Z \mu}{6T^2} \right) + m_Z^2 \right] + \frac{\mu k^2}{12T^2} \cdot (\mathbf{k}^2 - m_Z^2) \mathbf{i} \cdot [\mathbf{V} \times \mathbf{V}^*]/\mathbf{k}^2 \right\}, \quad (4.16)$$

The evaluation of the $CP$-even contribution looks similar to the previous cases, while the $CP$-odd term requires some care. One has

$$\int G_{CP-odd}^{\mu, |\mathbf{k}|} \frac{d^3k}{(2\pi)^3 |\mathbf{k}|} = \frac{\mu}{12T^2} \left( \frac{4\pi}{\bar{g}} \right)^2 \oint d\sigma_1 \oint d\sigma_2 ([\mathbf{X}_1' \times \mathbf{X}_2'] \cdot \nabla_{12})$$

$$\times (-1) \int \frac{d^3k}{(2\pi)^3} \theta(m_Z - |\mathbf{k}|) \left( 1 - \frac{3m_Z^2}{\mathbf{k}^2 + m_Z^2} + \frac{2m_Z^4}{(\mathbf{k}^2 + m_Z^2)^2} \right)$$

$$\times \exp(-\mathbf{i} \cdot \mathbf{k} \cdot \mathbf{X}_{12}).$$

When evaluating the integral over $m_Z$-dependent terms, one should have in mind that the upper integration limit can be set to infinity. Indeed, the oscillating factor $\sin |\mathbf{k}|\mathbf{X}_{12}$ resulting from the integration over the polar angle, shows that the most essential contribution comes from $|\mathbf{k}| \sim |\mathbf{X}_{12}|^{-1}$, which, in view of inequality $m_Z|\mathbf{X}_{12}| \gg 1$, means that the suggested approximation is true. All this yields

$$G_{CP-odd}^{\mu, |\mathbf{k}|} \simeq \frac{\mu m_Z^2}{24\pi T^2} \left( \frac{4\pi}{\bar{g}} \right)^2 \oint d\sigma_1 \oint d\sigma_2 \frac{\mathbf{X}_{12} \cdot [\mathbf{X}_1' \times \mathbf{X}_2']}{|\mathbf{X}_{12}|}$$

$$\times \left( -\frac{\sin m_Z|\mathbf{X}_{12}|}{\pi |\mathbf{X}_{12}|^2} + \frac{m_Z^2}{2} \exp(-m_Z|\mathbf{X}_{12}|) \right).$$

The integral over the first term in the parentheses cannot be further simplified, but the second one can. Making use of the expansion Eq. (4.7) and extending the limits of integration
over \( z \) to \( \pm \infty \), in view of the fast convergence, one arrives at the expression for the damping rate:

\[
\Gamma \simeq \frac{\kappa}{4} \left[ 1 - \ln 2 + \left( \frac{\langle \mu \rangle m_Z}{T^2} \right) \left( 1 - \frac{4}{3} \ln 2 \right) + \frac{\langle \mu \rangle m_Z F_2}{3T^2} \right] \Gamma_Z, \tag{4.17}
\]

with another geometric \( P \)-odd factor

\[
F_2 = \int d\sigma \frac{X' \cdot [X'' \times X''']}{m_Z^3 L} - \int d\sigma_1 d\sigma_2 \frac{X_{12} \cdot [X'_1 \times X'_2]}{4\pi L m_Z X_{12}^3} \sin m_Z |X_{12}|. \tag{4.18}
\]

Note that for contours with a smooth dependence on \( \sigma \) of the radius of curvature \( R \), the first term in \( F_2 \) is reduced to

\[
\int d\sigma X' \cdot [n \times n']/m_Z^3 L R^2,
\]

where \( n \) is the normal to the contour, and the integral over \( \sigma \) is \( 2\pi \) times the so called \textit{twist} \cite{26} of the contour, while the second term, up to the factor \( 1/L m_Z \), looks similar to the \textit{writhing number} \cite{26} (see them in Sec. \text{[IV]}, if it were not the oscillating factor in the integrand coming from the nontrivial distribution of Z-magnetic field.

V. DAMPING OF THE BASIC CHARACTERISTICS OF Z-STRING

Assume that the Z-string with the winding number \( n_0 \) be formatted. Then its energy is \( E_0 \simeq \varepsilon v L n_0^2 \). After that it goes to the state with the winding number \( n = 1 \) via either the fermion pair emission, after the time duration \( T_0 \simeq 2\Gamma^{-1} \ln n_0 \), or by some another mechanism mentioned in \cite{22}, whose rate is unknown. In turn, the state with the single winding number decays only via the emission of the fermions. Since this state has the energy \( \varepsilon v L/m_Z \) and is the superposition of the stationary states whose excitation quantum numbers \( N \) are distributed according to the Poisson formula with the mean \( N_0 = \varepsilon v L/m_Z \), the number of emitted fermions is by a factor of \( \ln N_0 \gg 1 \) greater than that in the case of the single \( Z^0 \) boson. Since Z-flux is directly related to the winding number, it decays with the rate \( \Gamma/2 \). The energy depends quadratically on Z-flux, so its damping rate is \( \Gamma \).

The energy and Z-flux are nonzero for string configurations of arbitrary shape, including the straight strings. On the contrary, the helicity,

\[
h_Z = \int d^3 x Z \cdot (\nabla \times Z) = \int \frac{d^3 k}{(2\pi)^3} \frac{i k \cdot [v(k, t) \times v^*(k, t)]}{(k^2 m_Z^2 + 1)^2} =
\]

\[
\left( \frac{4\pi}{\bar{g}} \right)^2 \int \frac{d^3 k}{(2\pi)^3} \left( \frac{m_Z^2}{k^2 + m_Z^2} \right)^2 \sum_{a, b} \int d\sigma_a d\sigma_b \exp[-ik \cdot (X_a - X_b)] n_a n_b
\]

\[
\times i k \cdot [X'_a \times X'_b]/k^2
\]

\cite{27}, is nonzero only for the configurations of Z-strings which are not invariant under the space inversion. Indeed, the terms with \( a \neq b \), after the momentum integration, give the linking number,
of two contours, with the exponentially small corrections. The contribution of the typical term with \( a = b \), after the momentum integration, reads

\[
h_Z(a = b) \propto W[a] - \frac{1}{4\pi} \oint d\sigma_1 \oint d\sigma_2 \frac{X_{12} \cdot [X'_1 \times X'_2]}{|X'_{12}|^3} \left( 1 + m_Z |X_{12}| + \frac{1}{2} m_Z^2 |X_{12}|^2 \right) \exp(-m_Z |X_{12}|),
\]

(5.2)

where \( X_{12} = X_a(\sigma_1) - X_a(\sigma_2) \) refers to the same contour \( a \), and

\[
W[a] = \frac{1}{4\pi} \oint d\sigma_1 \oint d\sigma_2 \frac{X_{12} \cdot [X'_1 \times X'_2]}{|X'_{12}|^3}
\]

(26) is the writhing number of the contour \( a \). The \( m_Z \)-dependent term in Eq. (5.2) is evaluated with the help of the expansion (13) to give

\[-\frac{1}{2\pi m_Z^2} \oint d\sigma X'_a \cdot [X''_a \times X'''_a].\]

In the case of sufficiently smooth contours the latter can be represented as \(-T[a]/(m_Z R)^2\), where

\[
T[a] = \frac{1}{2\pi} \oint d\sigma X' \cdot [n \times n']
\]

(26) is the twist number of the contour \( a \) whose normal vector is \( n \) and the radius of curvature is \( R \). So the twist contribution to the helicity is suppressed as \((R m_Z)^{-2}\), and the resulting expression for the helicity can be written as (3 21 27)

\[
h_Z = \left( \frac{4\pi}{g} \right)^2 \left\{ \sum_a n_a^2 W[a] + 2 \sum_{a < b} n_a n_b L[a, b] \right\}.
\]

(5.3)

Since \( h_Z \) is just Chern-Simons number of the \( Z \)-string field configuration, it characterizes possible processes with anomalous nonconservation of the baryon number (2 4).

The rate of the change in the helicity is evaluated semiclassically. To this end one should first take the expectation value of Eq. (5.1) in the quantum state discussed in Sec. III. Further differentiation with respect to time is then performed similar to the case of the damped classical oscillator, where the averaging over period is implied, giving the contribution due to the variable \( Z \)-flux to be \( \dot{h}_Z^{\text{Z-flux}} = -\Gamma h_Z \). The contribution coming from the slow classical contour motion is calculated with the help of the relation

\[
\frac{\partial}{\partial t} \oint d\sigma [k \times X'] \exp(-ik \cdot X) = i \oint d\sigma k \times ([k \times X'] \exp(-ik \cdot X),
\]

which can be verified by a straightforward calculation. One finds

\[
\dot{h}_Z^{\text{contour}} = \left( \frac{4\pi}{g} \right)^2 \int \frac{d^3k}{(2\pi)^3} \left( \frac{m_Z^2}{k^2 + m_Z^2} \right)^2 \sum_{ab} \langle n_a n_b \rangle \oint d\sigma_a \oint d\sigma_b (\dot{X}_a - \dot{X}_b) [X'_a \times X'_b] \times \exp(-ik \cdot X_{ab})
\]

\[
= \frac{2\pi m_Z^3}{g^2} \sum_{ab} \langle n_a n_b \rangle \oint d\sigma_a \oint d\sigma_b (\dot{X}_a - \dot{X}_b) [X'_a \times X'_b] \exp(-m_Z |X_{ab}|).
\]

(5.4)
It is clear that the terms with \( a \neq b \) give exponentially small contribution. This is natural, since the analogous terms in the expression for \( h_Z \) give the contribution to the linking number \( L[a, b] \) known to be the topological invariant. The contribution of the terms with \( a = b \) is calculated with the help of Eq. (4.7). In contrast to the case of helicity itself, their contribution to the time derivative of the latter is independent of \( m_Z \). In total, one obtains

\[
\dot{h}_Z \simeq -\Gamma h_Z + \frac{8\pi}{g^2} \sum_a \langle n_a^2 \rangle \oint \sigma \mathbf{\dot{X}}_a \cdot [\mathbf{X}_a' \times \mathbf{X}_a'''].
\] (5.5)

The second term in the right hand side is of purely classical origin. Its inclusion is justified in the case when the decay by the fermionic pair emission is prohibited.

**VI. CONCLUSION**

The results obtained in this paper shed a new light on the issue of stability of the gauge vortex defects in that the latter can be considered as purely classical field configurations only under the definite circumstances. The winding number of the scalar field is intimately interrelated with the gauge field so that if the latter decays the former will do the same, provided the spectrum of excitations allowed to couple to the string background satisfies the specific threshold condition. The radial part \( f \) (modulus) of the scalar field is irrelevant in the London limit. The above threshold condition is satisfied for \( Z \)-strings coupled to fermions. The interaction with fermions of these extended objects is innate and will result in their decay even in the models that admits metastability at the classical level [12,13]. The suppression of this decay could be possible, in principle, at the nonzero fermionic density in surrounding matter. The main reason of instability of a sufficiently long \( Z \)-string in empty space is not the collapse (in the case of closed contour), but a much more fast process of the fermion emission. The numerical estimate for the critical length \( L_0 \) is obtained from the condition

\[
\frac{L_0}{v} \sim \frac{1}{\Gamma} \ln \left( \frac{4\pi m_Z v}{g^2 \Gamma \ln \frac{m_H}{m_Z}} \right).
\]

Taking the velocity of the shrinkage \( v \sim 0.5 \), one gets \( L_0 m_Z \sim 2 \times 10^3 \). The loops with the length greater than 2000 Compton lengths of \( Z \)-boson will evaporate before their shrinkage.
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[21] Note that emergence of $m_Z$ in Eq. (3.3) as the frequency of an effective oscillator is the feature of the London limit. In general, the frequency squared in the case of the straight string is expressed through the exact profiles of the scalar and vector fields $f(\rho)$ and $v(\rho)$ possessing the $\rho \to \infty$ asymptotics to be, respectively, $f \to \eta/\sqrt{2}$ and $v \to -g^{-1}$ as

$$\omega^2 = \frac{\int_0^\infty dp \rho^{-1}[v'^2 + 2g^2(v + g^{-1})^2f^2]}{\int_0^\infty dp \rho^{-1}(v + g^{-1})^2}.$$ 

Under the replacement $f \to \eta/\sqrt{2}$ the above expression gives $m_Z^2$ with the logarithmic accuracy.

[22] There could be another decay mode of the state with multiple winding number $n > 1$ referred usually as the ‘classical instability against the decay into the strings with the unit winding number’. However, its mechanism has never been specified.

[23] The transitions with the changing topology of the purely Abelian Higgs strings are possible upon the inclusion of the instanton-like solutions [23] however, their probability is exponentially small. See S. W. Hawking, S. F. Ross, Phys. Rev. Lett. 75, 3382 (1995); M. Emparan, ibid., p.3386;

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