Out Of This World Supersymmetry Breaking

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Abstract

We show that in a general hidden sector model, supersymmetry breaking necessarily generates at one-loop a scalar and gaugino mass as a consequence of the super-Weyl anomaly. We study a scenario in which this contribution dominates. We consider the Standard Model particles to be localized on a (3+1)-dimensional subspace or “3-brane” of a higher dimensional spacetime, while supersymmetry breaking occurs off the 3-brane, either in the bulk or on another 3-brane. At least one extra dimension is assumed to be compactified roughly one to two orders of magnitude below the four-dimensional Planck scale. This framework is phenomenologically very attractive; it introduces new possibilities for solving the supersymmetric flavor problem, the gaugino mass problem, the supersymmetric CP problem, and the $\mu$-problem. Furthermore, the compactification scale can be consistent with a unification of gauge and gravitational couplings. We demonstrate these claims in a four-dimensional effective theory below the compactification scale that incorporates the relevant features of the underlying higher dimensional theory and the contribution of the super-Weyl anomaly. Naturalness constraints follow not only from symmetries but also from the higher dimensional origins of the theory. We also introduce additional bulk contributions to the MSSM soft masses. This scenario is very predictive: the gaugino masses, squark masses, and $A$ terms are given in terms of MSSM renormalization group functions.
1 Introduction

If string theory is correct, fundamental physics is higher dimensional and has a large amount of supersymmetry. Because we live in four dimensions with no supersymmetry, something nontrivial must necessarily happen with the other dimensions, which are compactified or truncated with boundaries. In general, reducing the dimensionality can reduce the amount of supersymmetry. In fact, for extended supersymmetry, the breaking is necessarily of a geometric nature [1]. For $N = 1$ supersymmetry however, the breaking can be effectively four-dimensional and field theoretical, independent of the geometry of the small compact dimensions.

In this paper, we investigate the not so radical assumption (in light of the above) that the mechanism for breaking the final $N = 1$ supersymmetry of the MSSM is also intimately tied to the geometry of the extra dimensions. We assume (for reasons we will later explain) that the MSSM itself is confined to a (3+1)-dimensional subspace, or “3-brane”, of the higher dimensional spacetime. Supersymmetry is then broken somewhere else. Known possible mechanisms include Scherk-Schwarz compactification [2] [3], rotated branes [4] [5] [6], and gaugino condensation or dynamical supersymmetry breaking on a separate 3-brane along the lines of [7, 8]. They all share an important common feature: supersymmetry is preserved locally on the visible sector 3-brane, but is broken globally. In the decompactification limit, supersymmetry breaking would disappear from the visible world. We refer to such supersymmetry breaking as having originated in a “sequestered sector”.

What this means from the vantage point of four dimensions is that the ultraviolet convergence is improved. Indeed the supersymmetry-breaking effects we compute are finite, cut off by the compactification scale. This is remarkable given the well known non-renormalizability and ultraviolet sensitivity of (super)gravity, especially in higher dimensions. But because supersymmetry breaking in the visible world requires communication across the extra dimensions, the diagrams contributing to supersymmetry-violating effects are finite. This was previously pointed out in the context of Scherk-Schwarz supersymmetry breaking in Refs. [9] [10], and in the toy model of Ref. [11]. This gives a starting point, in the context of field theory, for solving the supersymmetric flavor problem. Generally, the problem involves the fact that there are arbitrary flavor dependent higher dimension operators in the Kahler potential which upon supersymmetry breaking give flavor dependent masses. In fact, upon running the squark masses using known flavor violation, one finds a logarithmic dependence on the renormalization scale, indicating the necessity for such counterterms. It could be that there is a definite scale at which the masses are degenerate related to the string scale, and string “miracles” solve the flavor problem. However, it is much more compelling to see the solution in the low-energy field theory. We argue that sequestered supersymmetry breaking eliminates the necessity for such counterterms. Essentially the higher dimensional theory generalizes the notion of naturalness; the absence of counterterms might not be obvious from the standpoint of symmetries of the low-energy theory, but can nonetheless be guaranteed by assumptions in the higher dimensional theory.

We will show that supersymmetry breaking in higher dimensions allows for a successful
phenomenology of supersymmetry breaking masses. One can generate a realistic spectrum, potentially solve the flavor problem, and address other problems that plague alternative mechanisms of supersymmetry breaking. We identify a mechanism for communicating supersymmetry breaking through the super-Weyl anomaly [12] [13] [14] which is always present in hidden sector models in which supersymmetry breaking is communicated via supergravity. This is our chief result from which many phenomenological conclusions follow. We also show how separating the supersymmetry breaking sector introduces a physical cutoff on the theory (below the Planck scale) and therefore permits perturbative control of supergravity loops relevant to supersymmetry breaking.

It is important to keep in mind that the framework we develop in this paper can be applied to any specific theory of higher dimensional supersymmetry breaking. We will identify the important assumptions for such a theory to agree with our framework.

Our work may have applications to the M-theory scenario of Horava and Witten [7] [8], which is partly motivated by the possible unification of gauge and gravitational couplings in higher dimensions. More work on this scenario appears in Refs. [9, 15-24]. It should also apply to string theory models employing D-brane constructions [25] [26]. In addition, there have been several papers devoted to extra dimensions with compactification scales of order the weak scale or smaller [27-41]. The present paper is unrelated to these proposals; as we will see our compactification scale is roughly of order the GUT scale.

The outline of the paper is as follows. In Section 2, we review the requirements of a successful theory of supersymmetry breaking. We discuss existing proposals, and the advantages of the scenario we present. In Section 3, we outline in more detail the higher dimensional framework we will consider and contrast it to existing suggestions for breaking supersymmetry in higher dimensions. In Section 4, we review the particulars of the supergravity formalism which we will need. We identify the distinguishing features of a theory derived from higher dimensions according to our proposal. In Section 5, we discuss the flavor problem in more detail. We demonstrate why it is endemic to standard “hidden” sector models, but not to “sequestered” sector theories. In Section 6, we present the heart of our paper, in which we explain why supersymmetry breaking is necessarily communicated to the superpartners (including gauginos) at the one-loop level via the super-Weyl anomaly. In Section 7, we discuss additional contributions to soft masses from bulk modes. In Sections 8 and 9, we show that the mu problem and the supersymmetric CP problem can also be solved. In Section 10, we discuss the predictions of our scenario. One of the exciting aspects of our proposal is that it is very constrained, and therefore very predictive. We conclude in Section 11.

After completing this work we became aware of Ref. [42] which overlaps with some of our results in Section 6.

2 Supersymmetry Breaking: Constraints and Goals

Given that supersymmetric partners are yet to be discovered, it is remarkable how constrained a theory of supersymmetry breaking is if it is to naturally explain observed features
of the low-energy world. In this section, we enumerate the desired features of a theory of supersymmetry breaking. We briefly review how the two standard scenarios, hidden sector supersymmetry breaking and gauge-mediated supersymmetry breaking fare under these requirements. We then discuss in a qualitative manner why the sequestered supersymmetry breaking scenario we have in mind has the potential to meet the necessary standards. In the remainder of the paper, we will discuss these issues in greater detail in the context of the higher dimensional theories.

1. The first requirement of any successful supersymmetry breaking theory is to give the correct masses to the superpartners. Given known constraints (and ignoring experimental loopholes [43], we require masses to be above about 100 GeV. On the other hand, given naturalness constraints, one does not want the Higgs, stop, or gaugino masses to be far in excess of a TeV if the theory is to be reasonably natural. Roughly speaking, this means that at least some scalar masses and gaugino masses must be of the same order of magnitude; for example, one should not be loop-suppressed with respect to the other. Another nontrivial requirement of a model is that the scalar mass squared for the superpartners turns out positive.

2. The \( \mu \) parameter, multiplying \( H_u H_d \) in the superpotential, should also be between about 100 GeV and a TeV. The lower bound comes from the chargino mass, whereas the upper bound is a naturalness constraint. Meeting this requirement is a challenge since the \( \mu \) parameter appears in a supersymmetric term; one would like a way to connect its magnitude to that of supersymmetry breaking. The parameter \( \mu B \), multiplying the supersymmetry breaking scalar mass term \( H_u H_d \) should not be substantially larger than \( \mu^2 \), although it can be smaller.

3. Flavor changing neutral current constraints provide some of the most severe restrictions on the supersymmetric spectrum. The most stringent constraint (if there is no additional CP violation from the supersymmetry breaking sector) is for the squarks of the first two generations. In general the constraint depends on several parameters. For the case where the gluinos are roughly as heavy as the squarks the bound is given by [44],

\[
\delta_{sq} \left( \frac{\text{TeV}}{m_{sq}} \right) < 6 \times 10^{-3},
\]

where \( \delta_{sq} \) denotes the fractional flavor-violation in the squark mass-squared matrix, and \( m_{sq} \) denotes the average squark mass. The constraints from lepton flavor changing events are also quite severe, in particular coming from the experimental limit on \( \mu \rightarrow e\gamma \). Again the exact bound is parameter dependent. For \( \tan\beta \sim O(1) \), and a photino with roughly half the slepton mass, the bound is [45],

\[
\delta_{sl} \left( \frac{300\text{GeV}}{m_{sl}} \right)^2 < 4 \times 10^{-2},
\]

where \( \delta_{sl} \) is the fractional flavor violation in the slepton mass-squared matrix, and \( m_{sl} \) is the average slepton mass. For large \( \tan\beta \) there is another important contribution to
\( \mu \rightarrow e\gamma \). For \( \tan\beta \sim 50 \), a mu-term of 500 GeV, and a photino with roughly half the slepton mass, the bound is given by,

\[
\delta_{sl}(\frac{300\text{GeV}}{m_{sl}})^3 < 4 \times 10^{-5}.
\] (3)

Of course the infrared squark and slepton mass matrices are to be determined by starting with the mass-matrix obtained at the scale at which flavor and flavor violation originates and then running it down to low energies using the MSSM renormalization group. Why the ultra-violet squark mass matrix and running effects should be such as to produce the extreme level of flavor degeneracy demanded phenomenologically by eqs. (1, 2, 3) is the Supersymmetric Flavor Problem. Its resolution is critical for any candidate realistic theory of supersymmetry breaking and electroweak symmetry breaking.

4. The phases of the \( A \) and \( B \) parameters are constrained to be small for consistency with the electric dipole moment of the neutron. That is, CP should be approximately conserved.

5. A less technical requirement for a credible theory (barring experimental verification) is simplicity.

6. A desirable (though not essential) feature is testability.

Hidden Sector models are defined by the fact that the only couplings between the supersymmetry breaking sector and the standard sector are in the Kahler potential and are suppressed by \( 1/M_{Pl} \).

1. The chief success of hidden sector models is that it is straightforward to generate positive scalar mass squared. It is less straightforward to generate the gaugino mass; as emphasized in Ref. [47], a singlet VEV which breaks \( R \) symmetry and supersymmetry is required. This can be a nontrivial requirement in a dynamical supersymmetry breaking model. It is also a nontrivial requirement for the weakly coupled heterotic string, where the Green-Schwarz anomaly induced coupling of the moduli field to the gauginos is too small compared to the scalar mass [48] [15]. (Even when the scalar mass is through the anomaly, it is the mass-squared rather than the mass which is suppressed, thereby enhancing the scalar relative to the gaugino mass).

2. The \( \mu \) problem can be solved through the Giudice-Masiero mechanism [49]. That is there can be a term in the Kahler potential, \( f d^4\theta \Sigma H_1 H_2/M_{Pl} \), where \( \Sigma \) is a singlet which breaks supersymmetry.

3. CP is generally a problem, without additional symmetries. (See Ref. [50] and references therein.)
4. When the above problems are not solved, the theory appears simple; however attempts to address these inadequacies necessarily lead to more complicated models.

5. Until we know how the problems are solved, it is difficult to definitely determine the spectrum. However, to the extent that the gauge couplings are unified, one can predict the gaugino mass parameters corresponding to SU(3), SU(2), and U(1) to be in the ratio of gauge coupling squared.

More recently gauge-mediated models have received a good deal of attention. (See Ref. [51] for a review and references.) In these models, supersymmetry breaking is ultimately communicated through gauge couplings. The chief objective of these models was to address the flavor problem.

1. The success of the spectrum is model-dependent, the greatest danger being negative mass squared. However, many successful models were constructed with a desirable spectrum. One of the very nice features of gauge-mediation is that it is automatic that gaugino masses arise at one loop, while scalar mass squared arise at two, so that scalar and gaugino masses are competitive.

2. There is no compelling solution to the $\mu$ problem, though there are nice suggestions [65] [66] [67] [68]. What makes it so difficult is that even if one accepts a small parameter to suppress $\mu$ in the superpotential, the same small parameter naturally multiples the mass squared parameter $\mu B$, in which case the $\mu$ term is too small compared to $\mu B$.

3. The chief success of gauge-mediated models is that they have no flavor problem.

4. They generally have a CP problem.

5. They are often very complicated. Particularly when one incorporates the requirement of a $\mu$ parameter, there is no model which seems a likely candidate for describing the real world.

6. The models are predictive. They give the same prediction for the ratio of gaugino mass as the hidden sector models. However, the spectrum of scalar masses is determined by gauge couplings and charges and is therefore distinctive.

We now introduce the idea of a “sequestered” sector, which is basically a hidden sector that is truly hidden. Supersymmetry breaking can occur anywhere except on the 3-brane where the standard model particles are confined. Gravity in the bulk will permit the communication of supersymmetry breaking. If the “compactification” scale, determined by the separation of the supersymmetry breaking from the visible sector 3-brane is sufficiently small (that is the separation exceeds the Planck length), the theory is well under control. There are no direct couplings between the sequestered sector and the standard sector above the compactification scale. All gravity communication can be addressed perturbatively, since
gravity mediated loops are suppressed by the ratio of the compactification to Planck scale. There can however be direct couplings between the sectors, depending on what matter resides in the bulk. We will discuss later the possibilities both with and without bulk matter which couples to both sectors (aside from gravity).

1. One can obtain a successful spectrum. The key ingredient is the as yet neglected effects of the super-Weyl anomaly, which always introduces a coupling of the auxiliary field of the gravitational multiplet to both the gaugino and scalar masses. The auxiliary field obtains a vacuum expectation value through its coupling to the supersymmetry breaking sector. It couples in turn to the standard model fields because of the breaking of scale invariance. The couplings are determined through the supersymmetric renormalization group functions. In a usual hidden sector scenario, these couplings are present but are suppressed relative to the leading term. We assume the bulk matter is such that these are the dominant contributions to the gaugino and squark mass squared (the sleptons are lighter and have competitive additional contributions). One satisfying aspect of this coupling is that there is no one-loop contribution to the scalar mass squared; with this “anomaly-mediation” the gaugino and scalar mass (for fields with the appropriate gauge charge) are comparable. We find however that for fields which transform only under nonasymptotically free gauge theories, the mass squared due to the anomaly contribution is negative. This implies the necessity of additional mass contributions to the scalar mass squared. These can arise from many sources that will be discussed in Section 7.

2. The $\mu$ parameter can be generated without generating an excessively large $\mu B$ parameter due to the constraints from the coupling of the gravitational multiplet.

3. CP suppression is straightforward. Even without any further analysis, it is straightforward to have CP violated on our 3-brane, but nowhere else. We then find the absence of new phases in the $A$ and $B$ terms. This implies a natural solution of the SUSY strong CP problem.

4. Flavor violation is automatically suppressed by the dominant anomaly-mediated contribution to the squark mass squared. Additional scalar mass contributions may or may not be flavor preserving. It is straightforward to make models in which flavor preservation is automatic.

5. The “model” is straightforward. In fact there is little model building required. Most of the results we present in this paper are straightforward consequences of our fundamental assumption that we derive supersymmetry-breaking from a higher dimensional theory.

6. Sequestered supersymmetry breaking is very predictive. The ratio of gaugino masses depends on the beta functions, rather than simply the gauge coupling as for the other two scenarios. There is a nearly degenerate wino/zino LSP, of which the zino is the lighter. We predict $A$-terms proportional to the corresponding Yukawa couplings.
3 The Framework

The scenario we consider involves supersymmetry breaking in a higher dimensional theory. The first question this brings up is where are the standard model fields? We have implicitly assumed until this point that the standard model fields are confined to a four dimensional boundary. This is the only consistent assumption if we are to avoid rapid blow-up of their couplings in the higher dimensional theory and a very low string scale. The basic problem is that if one assumes a coupling of order $M^{d-4}$ in a compactified $d$-dimensional theory, where $M$ is some fundamental scale such as the $d$-dimensional Planck scale, it would induce a dimensionless coupling of order $(r_c M)^{d-4}$ in the effective four-dimensional theory, where $r_c$ is the compactification radius. When the compactification scale is well below $M$, this would imply a small four-dimensional coupling. However, we know that several of the standard model couplings are of order unity. Therefore, there cannot be a large separation between the compactification scale and the scale $M$ when there are standard model fields in the bulk. In this case, one runs the risk of uncontrolled flavor violation, since there is no large scale separation. So in this paper we choose to examine a scenario in which the standard model fields are confined to a four-dimensional subspace, or “3-brane” in the higher dimensions. This is the “visible sector”.

The next question is what we assume about the ratio of the higher dimensional compactification radii. In fact, our results can be examined without explicitly assuming any particular scheme. There might be six large dimensions (recall here large means within two orders of magnitude of the Planck scale), there might be one, or various other possibilities in between.

A further question is what we assume to be in the bulk higher dimensional spacetime. We will first consider the effects of the gravitational multiplet, and then consider the effects of additional bulk matter fields. It is very important that we assume there are no bulk fields that have large flavor-dependent couplings to matter fields. What large means will be made clear later on.. This is nonetheless a strong assumption. For example, in the Horava-Witten setup, it would mean assuming the dilaton is heavy. It is hard to assess the likelihood of this assumption being correct, since we simply do not yet know how moduli states obtain a potential. Presumably, the states do have a potential and are massive with a well-defined minimum. Our assumption is that this mass is above the compactification scale. It could be that these states obtain mass only after supersymmetry breaking (though cosmology argues against that [47]). In that case our assumption would simply not be true.

There are then two scenarios for supersymmetry breaking in the bulk which we have in mind when setting up our framework. Supersymmetry breaking can occur on another 3-brane via a truly hidden sector. Alternatively, supersymmetry breaking can occur through a nonlocal effect in the bulk. That is the conditions for supersymmetry can be different in different slices of the bulk. There are several examples where this is the case, including Scherk-Schwarz [1], and rotated branes [1] [2] [3]. From the point of view of the low-energy theory below the compactification scale, these theories will look very similar. There will be an effective supersymmetry breaking sector whose fields do not have direct couplings to
those of the standard model fields. The theories will differ in the precise field responsible for supersymmetry breaking, the precise form of the low-energy potential for this field, and in the relation between fundamental scales of the high dimensional theory and the supersymmetry breaking scale observed in the visible sector. These details are not relevant to the results we find in the four-dimensional effective field theory analysis, which would apply to any of these models. It is nonetheless of interest to explicitly analyze the high-energy theories to see how well they accommodate the assumptions we outline later. A more detailed discussion of the non-local supersymmetry breaking scenarios from the four-dimensional effective field theory will be presented in a subsequent paper [52].

A final example of a theory involving supersymmetry breaking in higher dimensions is that of Horava and Witten [7, 8]. It is interesting in that in the presence of the full dilaton multiplet, supersymmetry breaking is nonlocal; one can find a supersymmetric solution on each wall. However, if the dilaton multiplet is heavy, supersymmetry breaking could be broken dynamically on the invisible boundary, making it a truly hidden sector.

There have already been analyses of the low-energy effective field theory for some models derived from string/M-theory. Ref. [9] evaluated the loop-induced contribution to the scalar and gaugino masses. Whereas the former were of order $m^{3/2}_{3/2}/M_{Pl}$, the latter were much smaller of order $m^{3/2}_{3/2}/M_{Pl}^2$. This is the standard small gaugino mass problem [53]. Ref. [15] also included the “dilaton”. This is important because the direct coupling of the large compactification modulus (with the supersymmetry breaking auxiliary component) generates at tree level neither scalar nor gaugino mass. However, through mixing between the dilaton and the compactification modulus, one generates a coupling of order $\alpha$ for both the gaugino mass and the scalar mass squared. Here $\alpha$ is a parameter which determines the effective vacuum energy on the boundaries and bulk, and arises from the internal components (assuming Calabi Yau compactification of six of the dimensions) of the three-form tensor necessary to stabilize the configuration. In the weakly coupled heterotic string, where $\alpha$ arises from the Green-Schwarz mechanism, the gaugino mass is too small. In the M-theory context, Ref. [15] requires that the gauge and gravity couplings are unified, which gives $\alpha$ of order unity, permitting a phenomenologically acceptable ratio of the gaugino and scalar masses. However, if indeed this is the dominant contribution to the scalar mass squared, it is difficult to imagine a resolution of the flavor problem. In general, arbitrary renormalization effects at the Planck scale would generate flavor-dependent couplings for the dilaton (or other shape moduli which would also mix), implying nondegenerate scalar mass squared.

In this paper, we will show there is generally an additional anomaly-induced contribution to both the scalar and gaugino mass squared. We assume that the content and scales of the theory are such that this is the dominant squark mass contribution. For most of the paper we will assume that this is because flavor dependent contributions mediated by light bulk states are simply absent. We will further discuss the significance of the flavor constraints in Section 10.

It is important to recognize that the framework we establish here can be applied to any specific model. For any given example, one would need to establish if the assumptions we find necessary apply. In a subsequent paper, [12] we will apply these considerations to “nonlocal”
theories of supersymmetry breaking.

4 The Four-Dimensional Effective Theory

In order to derive the low-energy consequences of the existence of higher dimensions, one could take several approaches. One could start from a fundamental theory and derive in the higher dimensions the exact mechanism for communicating supersymmetry breaking via auxiliary components in the higher dimensional theory. This approach was taken in a toy model in Ref. [11]. Alternatively, one can assume that supersymmetry breaking is indeed communicated and derive the effective low-energy theory consistent with this and other facts we know about a specific higher dimensional theory, while accounting for general covariance of the low-energy theory. In this section we follow the latter approach and derive the special form of the effective four-dimensional supergravity Lagrangian below the compactification scale, $\mu_c$, corresponding to the higher-dimensional scenario in which the visible and hidden sectors live on separate 3-branes. We begin by reviewing the general form of four-dimensional supergravity coupled to matter, as detailed for example in Ref. [54]. This will allow us to point out the important features and to set notation.

4.1 General Four-Dimensional $N = 1$ Supergravity

There are several formulations of supergravity; one can write the Lagrangian explicitly in terms of $E$ and $\mathcal{E}$, the vielbein superfields along the lines of [54] or one can use a compensator formalism, along the lines of Refs. [55], [14]. We choose to use the Wess and Bagger formalism but we explicitly isolate the complex spin-0 auxiliary field which can acquire a supersymmetry-violating but Lorentz invariant expectation value. It will be convenient for us to formally define a flat-space chiral superfield, $\Phi$, to house this auxiliary field,

$$\Phi \equiv 1 + F_{\Phi} \theta^2. \quad (4)$$

We stress that $\Phi$ is not a separate chiral superfield in curved superspace, but rather just a formal device for separating out the scalar auxiliary field of the off-shell supergravity multiplet. This device will be very convenient later when we consider supersymmetry breaking in the the flat space limit. In terms of $\Phi$ we have,

$$\mathcal{E} = e\Phi^3 + ... \quad (5)$$

where $e$ is the determinant of the vielbein and the ellipsis contains supergravity fields of non-zero spin. This same field $\Phi$ appears in the kinetic terms since we can rewrite [56], [55] (again dropping fields with non-zero spin),

$$E \propto \Phi \Phi^i. \quad (6)$$

For this reason, the coupling of the field $\Phi$ is determined by that of the metric determinant, and it will enter in a way precisely determined by the conformal scaling of any operator in the Lagrangian.
The general Lagrangian (up to two-derivative order in the low-energy expansion) for supergravity coupled to matter can then be written,

\[ \mathcal{L} = \sqrt{-g}\{ \int d^4\theta f(Q, e^{-V} Q) + \int d^2\theta (\Phi^3 W^2 + \tau(\Phi)) + \text{h.c.} \} - \frac{1}{6} f(q, \tilde{q}) (R + \text{vector auxiliary terms + gravitino terms}), \]

where we have employed flat-superspace notation in the first line, with the understanding that derivatives of the matter fields are to be made covariant with respect to gravity and to the auxiliary gravitational vector field. We denote the matter chiral superfields by \( Q \), their lowest bosonic components by \( \tilde{q} \), vector superfields by \( V \), and their supersymmetric field strengths by \( W_\alpha \). We have dropped color and flavor indices for notational simplicity. \( R \) is the spacetime curvature scalar. The function \( f \) can be written in the form,

\[ f \equiv -3M^2_{\text{Pl}} e^{-K/3M^2_{\text{Pl}}}, \]

where \( M_{\text{Pl}} \) denotes the four-dimensional reduced Planck mass, and where \( K \) is defined to be the supergravity Kahler potential. With this definition of \( K \), a canonical \( K = qq^\dagger \) leads to normalized fields with no higher dimensional two derivative terms, as shown below in Eq. (10).

The supergravity lagrangian eq. (7) is not in the canonical (and perhaps more familiar) form where the Einstein action has a field-independent coefficient of \( M^2_{\text{Pl}}/2 \). To obtain the canonical form we have to eliminate the field-dependence by redefining the metric by a Weyl transformation,

\[ g_{\mu\nu} \to e^{K/3M^2_{\text{Pl}}} g_{\mu\nu}. \]

This transformation then changes the \( K \)-dependence (or \( f \)-dependence) of the remaining terms in the lagrangian. After Weyl transforming and integrating out auxiliary fields, the resulting Lagrangian is

\[ \mathcal{L} = \sqrt{-g}\{ \frac{M_{\text{Pl}}^2}{2} R + K_{ij}(\tilde{q}^i, \tilde{q}) D_\mu \tilde{q}^i D^\mu \tilde{q}^j - V(\tilde{q}^i, \tilde{q}) \}
- \tau(\tilde{q})(F_{\mu\nu}F^{\mu\nu} + iF_{\mu\nu}F^{\mu\nu}) + \text{h.c. + fermion terms}, \]

where the Kahler metric is given by,

\[ K_{ij}(\tilde{q}^i, \tilde{q}) \equiv \frac{\partial}{\partial \tilde{q}^i} \frac{\partial}{\partial \tilde{q}^j} K, \]

and the scalar potential is given by

\[ V = e^{K/M^2_{\text{Pl}}}(\frac{\partial W}{\partial \tilde{q}^i} + \frac{W}{M^2_{\text{Pl}}} \frac{\partial K}{\partial \tilde{q}^i}) K^{-1} \{ \frac{\partial W}{\partial \tilde{q}^j} + \frac{W}{M^2_{\text{Pl}}} \frac{\partial K}{\partial \tilde{q}^j} \} - 3 \frac{|W|^2}{M^2_{\text{Pl}}} + \frac{g^2}{2} (\frac{\partial K}{\partial \tilde{q}^a} \tau \tilde{q})^2, \]

where the last term is the gauge D-term scalar potential. This is the more familiar form of supergravity lagrangian. However, this form obscures many of the features we are interested in, and we will therefore mostly employ eq. (7).
In this formalism the field-dependent gravitino mass is given by,

\[ m_{3/2} = e^{K/2M_P^2} \frac{|W|}{M_P^2}. \]  

(13)

This becomes the physical gravitino mass when we replace all fields by their vacuum expectations and after tuning the cosmological constant to zero.

It is also edifying to observe the origin of the scalar mass squared in Eq. (10). When supersymmetry is broken, there are two things which carry this information; the VEV of the field in the goldstino multiplet \( \Sigma \) and the auxiliary component of the \( \Phi \) field (which translates into dependence on \( W \) in Eq. (10)). From Eq. (10), one can verify explicitly or by rescaling that there is no tree-level contribution to the \( \tilde{q} \) scalar mass from \( F_\Phi \). The source of mass can then only be terms which couple \( \Sigma \) and \( Q \) directly in \( f \) or from the terms proportional to \( R \). Clearly in a flat-space background in which the cosmological constant has been cancelled, the second term does not contribute. So the only source of tree-level scalar mass is the “curvature” terms in \( f \) which introduce direct couplings between the so-called hidden sector and the visible sector. This fact is obscured somewhat in Eq. (10), but is nonetheless true. It is manifested by the fact that when \( W \) is chosen to cancel the cosmological constant, the source of the scalar mass squared is \( \partial W/\partial \Sigma \), and not \( W \).

4.2 Effective Supergravity in the 3-Brane Scenario

Let us now specialize to the effective theory describing physics of an initially higher dimensional theory below the compactification scale, \( \mu_c < M_{Pl} \) in the 3-brane scenario. For simplicity, in this section, we neglect bulk fields other than four-dimensional supergravity.

In this case, the effective four-dimensional theory consists of supergravity coupled to the visible and sequestered sector fields. The essential point in deriving the constraints imposed from the above assumptions is that there is no direct coupling in the higher dimensional theory between the sequestered sector fields and the visible sector fields. We are quite literally hiding the hidden sector. This decoupling means that there are no allowed operators (at the level of \( 1/M_{Pl}^2 \)) at tree level with direct couplings between sequestered and visible sector fields. As we will see, this is the key to the resolution of the flavor problem. Nevertheless, there will be couplings generated at the radiative level between the two sectors. That will be the subject of the sections that follow.

The above observation gives a powerful constraint on the form of the four-dimensional supergravity Lagrangian effective below \( \mu_c \), namely that if the four-dimensional supergravity fields are formally switched off,

\[ g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi = 1, \]  

(14)

1Exactly how the higher dimensional theory reduces to four-dimensional supergravity is an interesting and subtle issue. For a detailed explicit discussion of the dimensional reduction in the (simpler) case where the bulk fields are gauge fields rather than supergravity, see ref. [11].
then the visible and hidden sectors should decouple. It follows that Eq. (7) must take the special form

\[ f = -3M_{Pl}^2 + f_{vis} + f_{hid} \]
\[ W = W_{vis} + W_{hid} \]
\[ \tau W^2 = \tau_{vis} W_{vis}^2 + \tau_{hid} W_{hid}^2 \]

(15)

where the functions \( f_{vis}, W_{vis}, \tau_{vis} \) depend only visible sector fields, and \( f_{hid}, W_{hid}, \tau_{hid} \) are functions of only hidden sector fields.

As can be seen, the fact that the visible and hidden sectors are physically separated in the underlying higher dimensional theory is reflected very simply in the special form of Eq. (7) given above. On the other hand, the Kahler potential defined by eq. (8) is given by

\[ K = -3M_{Pl}^2 \ln(1 - \frac{f_{vis}}{3M_{Pl}^2} - \frac{f_{hid}}{3M_{Pl}^2}) \]

(16)

and is not simply additive for the two sectors. The induced couplings between the two sectors are consequences of the fact that both sectors couple to a common gravitational multiplet and because of the Weyl rescaling done to obtain a canonical \( M_{Pl} \). It is important to recognize that the separation is manifest in \( f \), and not \( K \).

One can use this Kahler potential to derive masses in the low-energy theory; however, it clearly obscures the fact that the theory originated in two decoupled sectors. Of course, in principle a four-dimensional theory could have a Kahler potential of this very special form; however, there is no symmetry which would maintain it in the presence of radiative corrections. Here this separation is nonetheless “natural” in that it is enforced by the geometry. It is clearly simpler to work directly with \( f \) since the Weyl-rescaling given by Eq. (9) to obtain the canonical form of Eq. (10) completely obscures the simplicity of Eq. (15) by introducing many apparent interactions between the two sectors. Many of the special properties of our scenario then appear as the result of “miraculous” cancellations. We will therefore work with the non-canonical form of the lagrangian, Eq. (7), where the special properties are manifest.

All the subsequent conclusions follow from the above special form of the gauge kinetic function, superpotential, and Kahler potential and therefore can be understood solely in four-dimensional language. In fact, one can ask whether we really exploit the higher dimensions. The answer is we do not see any way to guarantee the above form without such an assumption. It is clearly not protected by symmetries of the four-dimensional world. Without the assumption of a deep underlying reason for separating the elements of the Lagrangian in this way, it would be an ad hoc assumption not stable to radiative corrections (at least for the Kahler potential).

For the purposes of the rest of the paper, we will take \( f_{hid} \) and \( W_{hid} \) to contain only a chiral multiplet \( \Sigma \) in which the Goldstino resides as the fermionic component. It will become clear that our general conclusions rely on the assumption of the factorization of \( K \) and not on the specific form of \( f_{hid} \). It is therefore simplest to take the minimal sequestered sector
and to ignore any other potential light fields which might reside there. The superfield $\Sigma$ may be composite or fundamental.

We assume that both the lowest component and auxiliary components of $\Sigma$ can acquire nonvanishing expectation values.

$$\langle \Sigma \rangle = \langle \sigma \rangle + \Lambda_H^2 \theta^2.$$  

(17)

Here $\Lambda_H$ is the scale of supersymmetry breaking and $\langle \sigma \rangle$ is assumed to be well below the Planck scale (otherwise we would work in terms of redefined fields). The VEV of the lowest component $\langle \sigma \rangle$ will not play an important role in any case.

5 The Supersymmetric Flavor Problem

In the previous section, we remarked that the squark mass squared must arise (classically) from direct couplings between the so-called hidden and visible sectors. This clearly permits the possibility of dangerous flavor changing couplings due to dimension four operators in the Kahler potential which couple hidden sector and visible sector fields. In fact, from a field theory perspective, we know these flavor violating couplings are present. This is because radiative corrections in the MSSM require Yukawa dependent counterterms, due to the logarithmically divergent running.

This also makes it clear why higher dimensional supersymmetry breaking has the potential to eliminate dangerous flavor violation. First of all, the sectors are decoupled at tree-level. Second, the addition of higher dimensions and the consequent separation between the sequestered and visible sectors actually improves convergence in the effective theory. This is clear because there is no counterterm involving hidden and visible sector fields. Any coupling between the two sectors arises from a finite supergravity calculation. In fact, the situation is even better because the introduction of a cutoff scale (the compactification scale) beneath the Planck scale means that higher loops are further suppressed by powers of $\mu_c/M_{Pl}$.

Let us see how the flavor problem manifests itself in the context of purely four-dimensional hidden sector models of supersymmetry breaking. The general supergravity Lagrangian in four dimensions will have Planck-suppressed interactions between the hidden and visible sectors given by

$$f = (1 - \frac{h}{M_{Pl}^2} \Sigma^\dagger \Sigma) Q^\dagger e^{-V} Q + ...$$

$$\tau = (\frac{1}{g^2} + \frac{k}{M_{Pl}} \Sigma) + ...,$$  

(18)

where $h, k$ are order one dimensionless constants. Even if one were to posit that these interactions were absent at tree level, they would certainly be induced at loop level at the Planck scale. There need be no suppression by a small dimensionless parameter because gravity is strong at the Planck scale. On the other hand it is consistent to assume that there
is no interaction between the visible and hidden sectors in the superpotential,

\[ W = W_{\text{vis}} (Q) + W_{\text{hid}} (\Sigma), \quad (19) \]

since this relation is radiatively stable due to the non-renormalization theorem. After putting in the hidden sector VEV, we find soft visible sector masses,

\[ m^2_{\tilde{q}} = \frac{h \Lambda_H^4}{M_{Pl}^2}, \]

\[ m_{\text{gaugino}} = k \frac{\Lambda_H^2}{M_{Pl}}. \quad (20) \]

If \( \Lambda_H \sim \sqrt{v M_{Pl}} \), where \( v \) denotes the weak scale, we obtain weak-scale soft masses as desired.

The flavor problem resides in the fact that there is no reason for the \( h \) couplings of the squarks to the hidden sector, and consequently the ultraviolet squark masses, to respect flavor. It is very difficult to understand why the string or Planck scale physics (implicitly integrated out to yield the effective supergravity lagrangian) should very precisely respect flavor when the lower-energy physics certainly does not. In early papers the flavor problem was obscured by working in terms of the Kahler potential, \( K \), instead of \( f \), and assuming,

\[ K = |\Sigma|^2 + Q^I e^{-V} Q. \quad (21) \]

The proposed justification was that this form of Kahler potential leads to the minimal renormalizable kinetic terms in Eq. (10). Formally, it appears that there are no direct interactions between the hidden and visible sectors in both the Kahler potential and superpotential, and therefore any visible soft terms that result must be purely gravity-mediated. Since gravitational couplings are flavor-blind it was expected that the soft terms should be as well. This argument is quite false (as is now generally appreciated). Using Eq. (8) we find that the proposed Kahler potential corresponds to

\[ f = -3 M_{Pl}^2 + |\Sigma|^2 + (1 + \frac{|\Sigma|^2}{M_{Pl}^2}) Q^I e^{-V} Q + ... \quad (22) \]

which is flavor-blind. However, we see that the soft masses arise out of a direct, albeit Planck-suppressed, coupling between the visible and hidden sector fields, not as the result of a supergravitational exchange. The real source of this coupling is that we have implicitly integrated out Planck-scale (string) states which may couple to both the visible and hidden sectors. There is absolutely no reason to believe that their couplings are flavor-blind; on the contrary one would expect at least some of them to distinguish the flavors [57]. The second point is that it is absurd to expect renormalizable matter kinetic terms after Weyl rescaling in a non-renormalizable supergravity theory. Therefore the special form of eq. (21) is highly unnatural and the flavor problem prevails in the context of four-dimensional hidden sector models.
Let us now turn to our 3-brane scenario. From Eq. (15) it is clear that there is no direct
coupling between the hidden and visible sectors, and that gravity really is the only interme-
diary. Of course just as in purely four-dimensional scenarios, we have implicitly integrated
out any Planck-mass bulk states in getting to the higher dimensional effective field theory.
One expects some of these states to couple to visible sector matter in a flavor-dependent
way. However, in this scenario, the couplings induced by heavy states are exponentially sup-
pressed. This can be more precisely understood by thinking of any such potential effects as
due to the exchange of one or more bulk propagators. Since the 3-branes break translation
invariance in the extra dimensions, it is useful to consider a bulk propagator in position
space. It must extend from the visible sector 3-brane out to a point roughly a distance
$r_c = 1/\mu_c$ away in the extra dimensions. This could be a point on a separate 3-brane if
we are considering supersymmetry breaking localized there, or a typical point in the bulk
in “non-local” supersymmetry breaking scenarios. Therefore, a coordinate space propagator
for a bulk state of mass $m$ will be suppressed by $e^{-mr_c} \sim e^{-m/\mu_c}$. This is one of the central
distinguishing features of the seqestered sector scenario: Bulk states with masses $m \gg \mu_c$
will have their contributions to visible sector supersymmetry breaking sharply cut off. If all
bulk states with flavor-dependent couplings to the visible sector satisfy $m > \mu_c$, then their
flavor-dependent supersymmetry breaking effects are suppressed. In purely four-dimensional
theories there is no such “switch” for turning off flavor-dependent supersymmetry breaking
effects.

Until Sections 7 and 8, we will assume that the only bulk states lighter than $\mu_c$ belong
to four-dimensional supergravity. Since masses (and off-shell momenta) larger than $\mu_c$ are
irrelevant for communication of supersymmetry-breaking to the visible sector, we can study
that process in the four-dimensional supergravity theory effective below $\mu_c$. The only super-
gravitational field that enters into nonderivative terms and can therefore give rise to soft
visible masses is the chiral field $\Phi$. Indeed it generally has a supersymmetry-breaking $F$-
term expectation value, as illustrated in Appendix 1. In general,

$$F_\Phi \sim \mathcal{O}(F_\Sigma/M_{Pl}) = \Lambda_H^2/M_{Pl}. \quad (23)$$

It is clear that only through this term can supersymmetry breaking be communicated to the
visible sector. However it is easy to see that in fact soft visible masses are not generated
classically. First since $\Phi$ does not couple to the gauge field strength the gaugino masses
vanish.

It is clear that the superpotential is irrelevant to the soft scalar mass squared (in the
absence of fundamental mass terms or visible sector VEVs). The only term which is then
relevant is

$$\mathcal{L}_{\text{visible kinetic}} = \int d^4\theta f(Q, Q^\dagger)\Phi \Phi^\dagger. \quad (24)$$

It is clear that this term does not generate a soft scalar mass since $\Phi$ can simply be super-
symmetically redefined away by $Q\Phi \rightarrow Q$. This can also be readily found explicitly by
evaluating the $d^4\theta$ integral which gives

$$\mathcal{L}_{\text{aux}} = f_{\text{vis}}(\tilde{q}^\dagger, \tilde{q})|F_\Phi|^2 + \frac{\partial^2 f_{\text{vis}}}{\partial \tilde{q} \partial \tilde{q}^\dagger}|F_Q|^2 + \frac{\partial f_{\text{vis}}}{\partial \tilde{q}} F_Q F_\Phi^\dagger + 3W_{\text{vis}}(\tilde{q})F_\Phi + \frac{\partial W_{\text{vis}}}{\partial \tilde{q}} F_Q + \text{h.c.} + \ldots \quad (25)$$
Integrating out $F_Q$ gives a potential for $\tilde{q}$ of the form,

$$V_{\text{vis}}(\tilde{q}) = \frac{1}{\partial_{\tilde{q}}^2 f_{\text{vis}}}{\partial^2 f_{\text{vis}}}{\partial \tilde{q} \partial \tilde{q}^\dagger} - 6 \text{Re} \ W_{\text{vis}}(\tilde{q}) F_\Phi - f_{\text{vis}} |F_\Phi|^2.$$  \hspace{1cm} (26)

It is clear that the mass term cancels. This may appear to be an unfortunate accident. We will see in the next section that this is not an accident and that it is far from unfortunate. Masses will indeed be generated at the radiative level, and the leading contribution to the squark mass matrix will be flavor-symmetric. Clearly, the absence of tree level terms in the fundamental Lagrangian which give mass means that the mass generation in the effective low-energy theory must give a finite calculable (or potentially vanishing) squark mass.

Before proceeding, let us briefly investigate other potential flavor-violating operators. There could be higher dimension operators involving purely visible sector fields. Because this are suppressed by the Planck scale, they are negligible. Other possible operators suppressed only by the compactification scale could be generated by integrating out Kaluza Klein modes of bulk moduli. Again, these are clearly sufficiently suppressed if the compactification scale is sufficiently high which it will be. Finally, flavor violating masses can and will be generated in loops in the low-energy theory due to flavor-violating Yukawa couplings. However, these will be finite and cut-off by the compactification scale. Flavor violation in the low-energy theory is calculable. For the known Yukawa couplings, we are safe. The problem which plagues ordinary hidden sectors is the unknown counterterms, which are absent in the higher dimensional scenario.

## 6 Anomaly-Mediated Supersymmetry Breaking

Although we have argued that there are no tree-level supersymmetry breaking visible sector masses, this is not true at the quantum level. We will show that certain anomalous rescaling violations provide a previously unrecognized mechanism for communicating supersymmetry breaking. It will turn out that “anomaly-mediated” supersymmetry breaking generates a one-loop gaugino mass and degenerate two-loop scalar mass squareds. We stress that this communication of supersymmetry breaking is not optional; it operates within any hidden sector model. In most hidden sector models, there are larger tree-level contributions, at least to the scalar mass squared. As we have discussed, this is not true for sequestered supersymmetry breaking, so the gaugino and squark masses we derive here will be the dominant contribution to the masses. However, as we will see, in order to obtain a realistic spectrum, comparable bulk loop effects of the type discussed in Section 7 must also be present.

We remind the reader of the super-Weyl anomaly and discuss its implications for the gaugino mass. We will then extend the analysis to the effects of rescaling anomalies on the scalar sector, where we will show that scalar masses are also generated.

Let us begin with a visible sector consisting of pure super-Yang-Mills theory. From Eq. (21) it appears that the visible sector Lagrangian does not couple to $\Phi$. This corresponds to a classical invariance of the visible sector Lagrangian under super-Weyl transformations, where
Φ (or more precisely the determinant of the super-vielbein) is multiplied by an arbitrary chiral superfield. In particular this allows us to completely Weyl-transform away ⟨Φ⟩ from the supergravity multiplet which couples to the Yang-Mills theory, thereby obtaining a canonical supersymmetry preserving gravitational background. If the classical super-Weyl invariance were exact this would imply that the visible gauginos are massless. However it is known that in the presence of the super-Yang-Mills sector, the super-Weyl transformation is anomalous, and results in the shift

\[ \tau \rightarrow \tau - 2b_0 \ln(\Phi) \]  

(27)

where \( b_0 \) is the gauge theory one-loop beta function coefficient reflecting the anomaly in Weyl (scale) invariance.

Now it is known that the dependence on the leading component of \( \Phi \) goes away; since we have already set the leading component to 1, this is manifest. However, there is nontrivial dependence on the auxiliary component of \( \Phi \). Explicitly, after evaluating the \( \theta \) integral there is a gaugino mass term

\[ m_{gaugino} = -b_0 g^2 F_\Phi \]  

(28)

where \( g \) is the gauge coupling.

This is a critical result. It means that even when there is no direct coupling of fields of the supersymmetry breaking sector to matter, there is nonetheless a gaugino mass which appears at one loop. This is true quite generally. It is most important of course when there is no larger term. In the absence of tree-level couplings, this would be the dominant gaugino mass contribution. The prediction in such models is that gaugino masses appear in the ratio of their beta functions.

There are many ways to derive the above anomalous coupling but it can be thought of as a result of a violation of super-Weyl invariance. This is associated with the cutoff dependence of a non-finite theory and should have effects wherever cutoff effects appear. Although the renormalizable physics is independent of the detailed nature of the cutoff, we point out that in the present scenario there is a physical cutoff provided by the string scale (or whatever scale marks the onset of new gravitational physics). In fact results of the form of Eq. (28) were derived in certain string-based models by accounting for string-threshold corrections [58]. Here we will extend these results and demonstrate their generality within effective field theory. Soft masses proportional to beta functions have also been derived in the string-based models discussed in Refs. [59].

We now consider the further consequences of the scale anomaly and derive the scalar mass squared. For now, we treat the the bulk classically. That is, we neglect supergravity loops as well as those of other fields which might be present in the bulk. It should be clear (up to \( M_{Pl} \) suppressed interactions induced by Weyl-rescaling) that the hidden sector dynamics decouples from the visible sector, and can be integrated out. The only effect of the hidden sector dynamics will be through the \( \Phi \) field (that is gravity) which couples to both sectors.

\[ \mathcal{L}_{eff}(M) = \int d^4\theta Q^\dagger e^{-V} Q \Phi^\dagger \Phi + \int d^2\theta [\Phi^3(m_0Q^2 + y_0Q^3) + \frac{1}{g_0^4} \mathcal{W}_{al}^2] + h.c. + \mathcal{O}(1/M) \]  

(29)
where $M$ is the higher dimensional Planck scale (which will be somewhat lower than the four-dimensional Planck scale $M_{Pl}$). The field $\langle \Phi \rangle = 1 + \langle F_\phi \rangle \theta^2 \sim \Lambda_H^2 / M_{Pl}$, appears here as a supersymmetry-breaking background for the visible sector.

We can attempt to eliminate $\Phi$ dependence by rescaling $Q$ according to,

$$Q \Phi \to Q.$$  \hspace{1cm} (30)

Since $Q$ is a physical chiral superfield and $\Phi$ is a background chiral superfield this transformation respects supersymmetry. The naive result is then,

$$L_{\text{eff}}(M) = \int d^4 \theta Q^\dagger e^{-V}Q + \int d^2 \theta (m_0 \Phi Q^2 + y_0 Q^3 + \frac{1}{g_0^2} W^2) + \text{h.c.}$$  \hspace{1cm} (31)

We see that $\Phi$ only appears beside visible sector mass parameters, as would be expected from the conformal coupling of $\Phi$. The MSSM gauge symmetries allow only one such mass parameter, namely the $\mu$-term. We will treat the $\mu$ term separately in Section 8 and assume for now there are no explicit mass parameters in the Lagrangian.

Classically the $\Phi$-dependence and supersymmetry-breaking are again absent in the visible sector after the rescaling. However the quantum functional integral measure is not invariant under the rescaling, and the resulting $\Phi$ dependence is described by the Konishi anomaly \cite{60}. The most familiar manifestation of this is the well-known axial anomaly, following from the rescaling if we imagined the chiral superfield, $\Phi$, to be a pure phase.

The origin of the super-Weyl and Konishi anomalies can be understood as follows. We saw above that after rescaling, $\Phi$ appears beside any mass terms. Although there are no explicit mass terms, the ultraviolet cutoff $\Lambda_{UV}$ and through it the renormalization scale $\mu$ (not to be confused with the $\mu$ term which we are neglecting in this section) always provide an implicit mass scale for the theory. In Appendix 2 we review how this anomaly arises in supersymmetric QED using an explicit supersymmetric Pauli-Villars regularization where the regulator provides an explicit mass term in the Lagrangian. More general theories are discussed in Ref. \cite{14} and Ref. \cite{61}. Here, we will simply proceed by using the fact that anomalous $\Phi$ dependence will multiply cutoff dependence.

If we renormalize our cut-off visible sector theory at an infrared scale $\mu$ (that is, we integrate out all modes above some experimentally accessible scale $\mu$), the Wilsonian effective Lagrangian must take the general form,

$$L_{\text{eff}} = \int d^4 \theta Z(\frac{\mu}{\Lambda_{UV} \Phi}, \frac{\mu}{\Lambda_{UV} \Phi^t})Q^\dagger e^{-V}Q + \int d^2 \theta y_0 Q^3 + \tau(\frac{\mu}{\Lambda_{UV} \Phi})W^2 + \text{h.c.}.$$  \hspace{1cm} (32)

The Lagrangian’s dependence on $\mu/\Lambda_{UV}$ follows from dimensional analysis and the fact that renormalizability of the theory in which higher dimension terms have been dropped means that cutoff dependence can be absorbed in the couplings. As mentioned above, $\Phi$ accompanies any cutoff dependence. Non-renormalization of the superpotential coupling $y_0$ implies that cutoff dependence is limited to the Kahler potential and gauge coupling (which only renormalizes at one-loop). Since $\Phi$ is formally a chiral superfield, supersymmetry
ensures that $\tau$ is a holomorphic function of $\Phi$. On the other hand, $Z$ must depend on both
the chiral and anti-chiral superfields, $\Phi$ and $\Phi^\dagger$.

We can get considerably more information about the anomalous $\Phi$ dependence by relating
it to the well-understood chiral anomaly in the classical $R$-symmetry. Before the rescaling,
Eq. (29) has an exact formal $R$-symmetry under which $R[\Phi] = 2/3, R[Q] = 0$. (The
symmetry is formal because the lowest component of $\Phi$ is fixed to be one.) This symmetry
is valid even in the presence of mass terms, so it is exact even in the presence of ultraviolet
regulator fields. After the rescaling, we have $R[Q] = 2/3$. This $R$ symmetry gives us
additional information about how $\Phi$ couples in $Z$ and $\tau$ (and in the process we rederive Eq.
(27)).

The fact that the $R$-symmetry must be formally exact in the presence of $\Phi$ forces
$Z$ to depend only on the $R$-invariant combination,

$$|\Phi| \equiv (\Phi^\dagger \Phi)^{1/2}.$$  \hspace{1cm} (33)

Therefore we will consider $Z$ to be a function of $\mu/\Lambda_{UV}|\Phi|$ from now on.

Now in the absence of $\Phi$ (that is $\Phi = 1$), the $R$-symmetry is anomalous. An $R$-symmetry
transformation results in a shift in the $\theta$-angle, $\text{Im} \tau$. When $\Phi$ is fixed to one, $R$-symmetry
transformations will cause $\tau$ to shift anomalously. However, when $\Phi$ is present, it is rotated
by the $R$-transformation and its logarithm will cancel the anomalous shift to give back an
exact symmetry. In order for the $R$-symmetry to be exact when the $\Phi$ field is included, the
gauge kinetic function $\tau$ evaluated at the scale $\mu$ must have the form,

$$\tau(\frac{\mu}{\Lambda_{UV} \Phi}) = \frac{1}{g_0^2} + 2b\ln(\frac{\mu}{\Lambda_{UV} \Phi}),$$ \hspace{1cm} (34)

where $b$ is an as-yet determined constant. This constant is easily identified since from the
$\mu$-dependence it must be the one-loop supersymmetric $\beta$-function coefficient, $b_0$, coefficient,

$$\beta(g) \equiv \frac{dg}{d\ln \mu} = -b_0 g^3 + ...$$ \hspace{1cm} (35)

Notice this result has the following interpretation. The $\Phi$-dependent rescaling of the field
has created a cutoff $\Lambda \Phi$. To obtain the couplings at a scale $\mu$, we would run according to the
renormalization group from $\Lambda \Phi$ to $\mu$. With this formula, one can derive the gaugino mass
of Eq. (23).

We now proceed to derive the scalar masses. Recall that $\Phi = 1 + F_\phi \theta^2$, and Taylor-expand
the $\ln \Phi = F_\phi \theta^2$ dependence in Eq. (32). First,

$$\ln Z(\frac{\mu}{\Lambda_{UV} |\Phi|}) \equiv \ln Z(\frac{\mu}{\Lambda_{UV}}) - \frac{1}{2} F_\phi \theta^2 \frac{d\ln Z}{d\ln \mu}(\frac{\mu}{\Lambda_{UV}}) + \text{h.c.}$$

$$+ \frac{1}{4} |F_\phi|^2 \theta^2 \frac{d^2\ln Z}{d(\ln \mu)^2}(\frac{\mu}{\Lambda_{UV}}),$$

$$\equiv \ln Z(\frac{\mu}{\Lambda_{UV}}) - \frac{1}{2} \gamma(g, y)(F_\phi \theta^2 + \text{h.c.}) + \frac{1}{4} |F_\phi|^2 \theta^2 (\frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y),$$ \hspace{1cm} (36)

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where we have used the renormalization group functions defined by

\[
\begin{align*}
\gamma(g, y) &\equiv \frac{\partial \ln Z}{\partial \ln \mu} \\
\beta_g(g, y) &\equiv \frac{\partial g}{\partial \ln \mu} \\
\beta_y(g, y) &\equiv \frac{\partial y}{\partial \ln \mu},
\end{align*}
\]

having explicitly isolated the \(\Phi\)-dependence. We can take the couplings \(g\) and \(y\) to be the renormalized couplings in the exactly supersymmetric theory after \(Z\) has been rescaled to unity. (Recall that because of the superpotential non-renormalization theorem, the renormalization of the Yukawa couplings is entirely due to the associated wavefunction renormalizations.)

Let us now compute the soft masses to one-loop order in the visible sector couplings. We begin by observing that as far as mass terms are concerned (see below for \(A\)-term trilinear scalar couplings) all the terms in \(\ln Z\) can be rescaled away except for the term proportional to \(|F\Phi|^2\), by the superfield redefinition,

\[
\exp\left\{\frac{1}{2}\ln Z\left(\frac{\mu}{\Lambda_{UV}}\right) - \frac{1}{4} \gamma(g, y) F\Phi \theta^2 \right\} \Phi \rightarrow \Phi.
\]

Of course this rescaling also suffers from a formally one-loop anomaly, but since the transformation parameter is already of one-loop order, the real anomaly is of higher order and is neglected here. Therefore having made this transformation, we effectively have

\[
Z\left(\frac{\mu}{\Lambda_{UV}}\Phi\right) = 1 + \frac{1}{4} |F\Phi|^2 \theta^2 \theta^2 \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right).
\]

From this the scalar mass-squared renormalized at \(\mu\) follows;

\[
m^2_{\tilde{q}}(\mu) = -\frac{1}{4} |F\Phi|^2 \theta^2 \theta^2 \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right).
\]

An important feature of this result is that the scalar mass squared only arises at two-loops. As discussed in Section 2, this is a very desirable property of the spectrum. Now to compute scalar masses to one-loop order we must clearly retain terms up to two-loop order in the scalar mass-squared. Therefore each of \(\gamma\), \(\beta_g\) and \(\beta_y\) must be kept to one-loop order. They have the general forms,

\[
\begin{align*}
\gamma &= c_0 g^2 + d_0 y^2 \\
\beta_g &= -b_0 g^3 \\
\beta_y &= y (c_0 y^2 + f_0 g^2).
\end{align*}
\]

From this it follows that the scalars have mass-squareds

\[
m^2_{\tilde{q}} = \frac{1}{2} \left\{ c_0 b_0 g^4 - d_0 y^2 (c_0 y^2 + f_0 g^2) \right\} |F\Phi|^2.
\]
From eq. (42) we can see a general feature of anomaly-mediated supersymmetry breaking. Since for all gauge representations, \( c_0 > 0 \), the contribution to scalar mass-squareds from purely gauge field loops is positive for asymptotically free gauge theories and negative for infrared free gauge theories. This poses a phenomenological danger for scalars whose visible sector couplings are dominated by infrared free gauge interactions. Other sources of supersymmetry-breaking must be sufficiently large to counter the negative contributions found here. We will discuss these contributions in the following section.

Second, we observe that the superfield transformation, Eq. (38), induces trilinear \( A \)-terms which, using the superpotential non-renormalization theorem, are proportional to the \( \beta \) functions of the corresponding Yukawa couplings. In order to avoid confusion we will restore indices \( i, j, k, ... \) labelling the various chiral superfields. Then we find,

\[
A_{ijk} = \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k)y_{ijk}F_\Phi, \tag{43}
\]

where each \( \gamma_i \) has the form given in Eq. (41).

We wish to discuss a few general features of the derivation of the gaugino and scalar masses. First of all, although the results we derived here all occur at loop level, they should be thought of as the initial conditions for a renormalization group running in the MSSM. We have not yet done renormalization group running below any SUSY threshold. The final low-energy spectrum would include these effects as well. However, this running is not from the Planck or compactification scale, but only from the scale of a SUSY breaking mass. Other running has already been included.

Second, our results appear similar in spirit to those of Refs. [62] [63], who computed higher loop masses in terms of one-loop parameters of the beta function and anomalous dimension in the context of gauge-mediation. This similarity can be understood as follows. In gauge-mediation, the messengers of supersymmetry breaking are vector-like multiplets which appear in loops. In our calculations, the “messengers” of supersymmetry breaking can be thought of as the regulator fields. This interpretation in terms of loops of regulator fields is made explicit in Appendix B. This leads to quite distinct results however. For example, the beta function dependence in the gaugino and scalar mass squared arises because not only charged matter, but gauge bosons themselves have regulator fields which directly couple to supersymmetry breaking. This leads to a very different spectrum from gauge-mediated models. However, from the vantage points both of diagrammatic perturbation theory and the more formal analysis presented above, it can be useful to use the similarity of the results. For example, we can understand the sign of the soft scalar mass squared. Because the regulator fields yield loops of the opposite sign, the sign should be the opposite of that given by the physical messenger fields, which is to say the opposite sign of the beta function. This analogy is also useful in understanding how loops are cut-off at a high scale, where the supersymmetry violating regulator mass goes away.

Finally, let us consider the effect of having supersymmetric mass thresholds, such as a GUT scale, in the visible sector. As discussed earlier, the dominant tree-level effect is that

\[2\]This is corrected from our original version and confirms the result of Ref. [42].
the massive fields will acquire supersymmetric splittings because their mass parameters in the lagrangian, $m_0$, will be multiplied by $\Phi$ after the rescaling. However, assuming that $m_0$ is much larger than any experimentally accessible energy, we are really interested in how the supersymmetry-breaking splittings among the massive states feeds down radiatively to the light visible sector fields. The central point is that once we integrate out the massive fields, the effective Lagrangian for the light fields must be of the general form of eq. (32), with the substitutions,

$$
\begin{align*}
Z\left(\frac{\mu}{\Lambda_{UV}}\phi\right) & \rightarrow Z\left(\frac{\mu}{\Lambda_{UV}}\phi, \frac{m_0}{\Lambda_{UV}}\right) \\
\tau\left(\frac{\mu}{\Lambda_{UV}}\phi\right) & \rightarrow \tau\left(\frac{\mu}{\Lambda_{UV}}\phi, \frac{m_0}{\Lambda_{UV}}\right).
\end{align*}
$$

(44)

We see that the new $m_0$ dependence does not enter at all into the considerations that determined the soft masses in terms of the renormalized couplings at $\mu$, since these only involved derivatives with respect to $\mu$. The only effect of the $m_0$ threshold is that it changes the renormalized couplings themselves as functions of the ultraviolet couplings above $m_0$. Since we directly measure and work in terms of the infrared couplings and are ignorant of the ultraviolet ones, the presence of the $m_0$ threshold is irrelevant for our soft mass calculation. This provides a sort of ultraviolet “immunity” for our results. There are corrections to this picture coming from possible non-renormalizable terms in the effective lagrangian, suppressed by $m_0$. If $m_0$ is very large then these effects can be neglected, but clearly this would not be the case if there were a supersymmetric threshold close to the weak scale. Diagrammatically, the irrelevance of heavy states can be understood in terms of a cancellation between loops of massive states and their regulators, which both have the same coupling to the supersymmetry breaking $\Phi$ field.

7 Additional Contributions to the Soft Scalar Masses

In the previous section, we evaluated the anomaly-mediated contribution to the gaugino and scalar masses. We found the remarkable fact that although gaugino masses arise at one-loop, the scalar mass squared only arises at two. However, we also showed that the spectrum had a serious problem, namely the negative mass squared for the sleptons. However, there are other ways scalars can obtain masses; once supersymmetry is broken, one expects this to be communicated at the radiative level to the scalar mass squared since no symmetry prevents such a mass. In this section, we will see that in the presence of additional bulk fields, there are indeed additional contributions to the slepton (and squark) masses. In Section 10, we will describe the requirements for these to be of the type and order of magnitude to generate a reasonably natural spectrum consistent with flavor-violating constraints.
7.1 Bulk Radiative Corrections

Let us reconsider our higher dimensional scenario with some number and types of bulk fields with non-renormalizable Planck-suppressed couplings to the visible and hidden sector fields. Examples of these are bulk scalars coupling (in higher dimension operators) to both visible and sequestered sector fields, 3-brane fields appearing in the gauge kinetic functions of bulk gauge fields, or supergravity fields themselves. We assume that there are no couplings of the bulk to the visible sector which give rise to renormalizable strength interactions (after compactification). So for example we exclude consideration of 3-brane fields which are charged under the bulk gauge fields, or Yukawa couplings of bulk and 3-brane fields. If the fundamental theory contained any such couplings it is assumed that the associated bulk fields have Planck-scale masses and have decoupled.

Let us begin by considering the form of radiative corrections to the soft masses from bulk loops, within the framework of the four-dimensional effective theory below $\mu_c$. Here it is very important that we consider only the regime $\mu_c < M_{Pl}$. Unlike conventional supergravity, where everything is confined to a single four-dimensional spacetime, there will be a well-defined perturbative loop expansion when computing supersymmetry-violating effects with a well-defined cut-off for divergent integrals. This follows from the assumption of couplings suppressed by the Planck scale and dimensional analysis. Here we show explicitly how this arises.

It might appear curious that using higher dimensions has actually improved the convergence of supergravity. This is of course true only for supersymmetry-violating operators that require communication across the compact dimension. To actually compute these corrections, one can use the results of a conventional supergravity calculation using our special Kahler potential. However, it is edifying to understand the loop calculation directly in position space in the higher dimensions and before Weyl rescaling. We consider here the case with one extra dimension for concreteness. One can then see directly that the separation $r_c \sim 1/\mu_c$ effectively acts as a (physical) point-splitting regulator for the one-loop corrections and therefore one obtains a finite result. This is most easily seen by considering the form of a one-loop corrections in position-space. We assume that the bulk fields contain only Planck mass suppressed couplings to both the visible and sequestered sectors. In the case of the gravity multiplet, a loop diagram would require two Planck mass suppressed vertices with two derivatives. A bulk scalar would couple to matter in the form

$$ S \supset \int d^5x \int d^4\theta \left( c_1 \frac{\Sigma \Sigma B B^\dagger}{M_{Pl}^2} \delta(x_5 - r_c) + c_2 \frac{QQ^\dagger B B^\dagger}{M_{Pl}^2} \delta(x_5) \right) $$

where $B$ is a bulk field. $M$ (as opposed to $M_{Pl}$) denotes the five-dimensional Planck scale. Note that we have taken quite a general form for the coupling between the bulk and sequestered and visible sectors, assuming only that it was suppressed by $M_{Pl}$. One could in principle incorporate also single $B$ couplings if $B$ is a singlet; however these will only contribute to irrelevant derivative couplings.

The above interaction gives rise to a two-derivative coupling of $B$ to $Q$. We therefore
expect a loop-integral of the form
\[ m_q^2 \sim \frac{1}{M^3} \int d^4 x \partial_\mu G(x, r_c) \text{Str} \Delta(x, r_c) \partial_\mu G(x, r_c), \] (46)

where, \( G \) is the five-dimensional massless scalar Euclidean Green function which connects the bulk interaction with \( \tilde{q} \) at the spacetime origin to any point on the hidden sector 3-brane at which there is a supersymmetry-breaking interaction, \( \Delta \), which can split the masses of the bulk modes in the four-dimensional effective theory. The derivatives arise from the higher dimension operator which couples the bulk fields to the visible sector and \( \Delta \) could be a supersymmetry breaking mass-squared insertion for the bulk field for example. Since the bulk couplings to the hidden sector are Planck-suppressed (by powers of \( M \) in five-dimensions) we have,
\[ \langle \Delta \rangle \sim \mathcal{O}(\frac{F^2}{M^3}). \] (47)
Since \( m_{\text{bulk}}^2 \sim F_S^2/M_{\text{Pl}}^2 \) and \( M_{\text{Pl}}^2 \sim r_c M^3 \) we find,
\[ \text{Str} \langle \Delta \rangle \sim r_c \text{Str} m_{\text{bulk}}^2. \] (48)

Now an ultraviolet divergence would normally occur when the two interaction points coincide. However in the present case they cannot get closer than \( r_c \) and therefore the \( x \)-integral is finite, and by dimensional analysis is of order \( \mu_c^4 \). Our final result is then,
\[ m_q^2 \sim \frac{\text{Str} m_{\text{bulk}}^2 \mu_c^2}{16 \pi^2 M_{\text{Pl}}^2} \sim \frac{|F_S|^2}{16 \pi^2 M_{\text{Pl}}^4} \mu_c^2. \] (49)
This is the key point which we emphasized in the introduction; loops are finite and cutoff by \( \mu_c \).

One can generate small \( A \)-terms from the bulk as well.

If we now assume \( F_\phi \sim F_S/M_{\text{Pl}} \), we see that a positive slepton mass squared requires \( \mu_c/M_{\text{Pl}} \sim 1/10 \).

One can obtain the exact result of this calculation by substituting our special form of the Kahler potential including the additional contributions from Eq. (45) using a physical cutoff \([31]\) of the compactification scale.

In a Scherk-Schwarz scenario, the authors of Ref. [9] computed precisely the above loop effects and found
\[ m_q^2 \sim 0.88 K^{-1} QQ^\dagger \left( R_{Q^1Q} - K_{Q^1Q} \right) \frac{\mu_c^4}{M_{\text{Pl}}^4}, \] (50)
where we have taken \( m_{3/2} = \pi/2 \mu_c \). Notice in this scenario, the bulk mass is not Planck-suppressed. So the analogy to the above formula requires taking \( m_{\text{bulk}} \sim F_\phi \sim \mu_c \). We then find the compactification scale should be given as \( \mu_c/M_{\text{Pl}} \sim 1/100 \).

There are a few further observations about this result. The second term, representing the pure gravity contribution, is negative. This means there must be additional bulk contributions.
A particularly interesting such case is if the bulk scalars arise from the dimensional reduc-
tion of higher dimensional supergravity. This is interesting because such bulk contributions,
like those of gravity, can be flavor-blind. For example, in the Horava-Witten setup, the mod-
ulus which determines the scale of the eleventh dimension has flavor-blind couplings. One
can compute the contribution from a single such modulus; the result is that any individual
such scalar gives a contribution $1/3$ the size of gravity. Therefore, with at least four such
scalars, one can obtain a positive flavor-independent bulk contribution to the soft scalar
mass squared.

Another interesting thing to observe from the above formula is the preferred choice of
compactification scale, which is approximately $0.01 M_{Pl}$ in order to compete with the loop-
induced anomaly-mediated negative slepton mass squared. Depending on the number of
dimensions of size $r_c$, one finds the higher dimensional Planck scale ranges from about
$3 \cdot 10^{16}$ GeV to $2 \cdot 10^{17}$ GeV, where we have varied the number of large dimensions from one
to six. So we see this allows for the possibility of unifying gauge and gravitational couplings,
while giving a consistent positive flavor-degenerate contribution to the soft scalar masses! Of
course in this case there is only a small separation between the compactification and string
scales. Nonetheless these simple dimensional analysis estimates indicate a very intriguing
possibility.

We observe that this is different from the strongly coupled heterotic string setup for a
couple of reasons. One is that there were two distinct scales in that case: the Calabi-Yau
radius and the 11-dimensional radius. That can perhaps be accommodated; we assumed com-
mon radii for simplicity. Furthermore, one can only analyze the heterotic string in the weakly
or strongly coupled limits; in the regime of intermediate string coupling, the scales could be
different. Furthermore, there can be additional bulk states which can contribute to the soft
scalar masses. The more problematic aspect is that flavor is established geometrically; this
means that some scalars arising from the Kaluza-Klein reduction of the gravitational multi-
plet would not be expected to have flavor-blind couplings. The additional difference in the
specific phenomenology which has been done is the assumption that the dilaton is present
in the low-energy theory; this gave very different contributions to the scalar and gaugino
and cannot address the flavor problem. With the dilaton present, there are direct tree-level
couplings in the four-dimensional effective theory between the supersymmetry breaking and
visible sectors.

We close this section with an aside. One might have thought one can readily generate
a flavor-blind contribution to the soft scalar mass squared by the one-loop correction to
the vacuum energy, which would require a different constant superpotential term than that
determined at tree-level. In fact, such loop corrections have been considered in Ref. [64,
[18]. However, we see no such corrections before Weyl rescaling. The only bulk radiative
corrections to the soft scalar masses involve the direct couplings of the bulk fields to the
visible sector fields. This follows from the general analysis of Section 4. There we saw
the apparent contributions to the mass squared from $W$ must and did cancel (which was
more apparent before integrating out the auxiliary field and Weyl rescaling). This is still true
when one-loop corrections are included. The contribution from Weyl rescaling which appears
in the kinetic term cancels against the potential contribution. Now one has the option of rescaling the Planck mass to eliminate the higher dimension kinetic terms; this then puts the additional contribution in the Kahler potential as higher dimension terms. Therefore, additional contributions associated with vacuum energy which are enhanced by the number of chiral multiplets really arise from an assumed flavor-independent higher dimension operator in the Kahler potential, of the form $|Q|^2/M_{Pl}^2$. It is worth noting that the large one-loop mass correction of [64, 48] relies on this assumed form of the Kahler potential. In our case, loops of matter fields give too small a contribution to the vacuum energy in our case since the mass arose only at one-loop. This means that only bulk fields that couple directly to matter in higher dimension terms give a contribution to the scalar mass squared.

### 8 The $\mu$ Problem

No theory of supersymmetry breaking would be complete without a solution to the $\mu$ problem. Although the $\mu$ term in the superpotential preserves supersymmetry, we know the scale is essentially the same as that of supersymmetry breaking so is presumably connected to it. Therefore one expects the mechanism which breaks supersymmetry to also be responsible for inducing the $\mu$ parameter.

The lack of solution to the $\mu$ problem is one of the least satisfying aspects of gauge-mediated supersymmetry breaking. Although solutions exist [65] [66] [67] [68], they are not compelling. The basic problem is that in gauge-mediated models, supersymmetry breaking is induced at loop-level. So a term in the effective superpotential which induces $\mu$ is generally too large without some additional suppression factor; in this case $\mu B$ is too large. For example, if there is a singlet $S$ which has nonvanishing auxiliary component, one can write $\epsilon \int d^2 \theta S H_u H_d$. If $\epsilon \sim 10^{-2}$, one can successfully obtain a $\mu$ parameter of the correct size. But then $\mu B$, the scalar mass squared term, is too large. On the other hand, there is a simple solution in hidden sector models; one can construct a term in the Kahler potential [49] $\Sigma H_1 H_2/M_{Pl}$.

In our models, we can clearly not simply fine-tune the problem away. If we permit a Peccei-Quinn (PQ) symmetry-breaking term $\int d^2 \theta H_1 H_2 \Phi^3$ (where we have included the essential dependence on $\Phi$), one finds that $\mu B$ is far too large. However, this problem might be readily solved by a miraculous cancellation if it is assumed the $\mu$ term arises from the following higher dimensional term in the Kahler potential:

$$L_{\mu-term} = \alpha \int d^4 \theta \frac{1}{M_{Pl}} \left( \Sigma + \Sigma^\dagger \right) H_1 H_2 \Phi^\dagger \Phi + \text{h.c.}$$

(51)

Here $\Sigma$ is the supersymmetry breaking hidden sector field. After the rescaling, Eq. (30), (applied only to the visible sector fields) one obtains,

$$L_{\mu-term} = \alpha \int d^4 \theta \frac{1}{M_{Pl}} \left( \Sigma + \Sigma^\dagger \right) H_1 H_2 \frac{\Phi^\dagger}{\Phi} + \text{h.c.}$$

(52)
Notice that with this operator, there is no classical contribution to $\mu B$ when $F_\phi \propto F_\Sigma$. It is straightforward to see that there is an anomaly-mediated contribution however resulting from Eq. (53) given by

$$B = \frac{1}{2} (\gamma H_1 + \gamma H_2) F_\phi.$$  (53)

One can ask where such an operator as Eq. (51) arises, and if there are couplings of $\Sigma$ which produce other operators that do generate a $B$ term. We provide a simple example, with a five-dimensional fundamental spacetime for concreteness.

Assume there is a massive bulk vector field $V$ with the following four dimensional Lagrangian

$$L_V = \int d^4 \theta m^2 V^2 + aV(\Sigma + \Sigma^\dagger)M^{1/2} + \frac{b}{M^{1/2}} VH_1 H_2 + \text{h.c.}$$  (54)

Here, $M$ is the Planck scale of the five-dimensional theory (this can however be generalized to higher dimensions), $a$ and $b$ are numbers, and $m$ is the vector mass in the five-dimensional theory. Notice that we have written the most general term allowed at this order in a $1/M$ expansion.

One can then work out the contribution to the operator above Eq. (51). It scales like $ab/r mc$. For $m$ of order the compactification scale and $a$ and $b$ somewhat less than one, one finds an acceptable $\mu$ parameter.

This model serves as an existence proof. It is probably not as complicated as the generation of a $\mu$ term in gauge-mediated models, but is not as straightforward as the mechanism in hidden sector models with no additional constraints. One could construct other models; the ones we have so far constructed are less natural in that they would permit additional couplings and one would need to assume relations among parameters. Since we have not made very unreasonable assumptions, we expect that a fundamental high energy theory might allow for a vector field of the type we have described. However, there might be a yet more compelling solution to the $\mu$ problem which we have overlooked.

### 9 CP Violation

We claim this scenario is also very advantageous from the vantage point of CP violation. There are two types of potential CP problems [50]. The first is the SUSY strong CP problem which requires the phase of the $A$ and $B$ parameters to be small, of order $10^{-2}$ [50]. Clearly, this problem is solved if $A$ and $B$ are only radiatively generated. The existing phases can be rotated away. In our models, the problem with the phase of $A$ is automatically solved. The problem with the phase of $B$ depends on the precise solution to the $\mu$ problem. In our example, the problem is solved.

The other CP problem is that $\epsilon_K$ can be too large [50]. This problem is distinct and tied up with the solution to the flavor problem. In the case that squark masses are degenerate, there is clearly no problem. Furthermore, in a higher dimensional theory, it is possible that there are no operators which communicate CP violation between our 3-brane and the bulk. Then it could be natural for the SUSY breaking sector to preserve CP.
In this section we will assemble and examine our results for the soft breaking terms in the MSSM. These will be determined by the anomaly-mediated contribution and an unknown bulk contribution. We will work in the approximation that all Yukawa couplings vanish except for $y_t$ and $y_b$.

We first derive the anomaly-mediated contribution to the masses, which will dominate for all but the sleptons and Higgses. We remind the reader that although the formulae below are loop-level results, they should be considered as matching conditions for a full renormalization group analysis in the MSSM below the superpartner mass scale.

Recall, the formula for the anomaly-mediation contribution to the soft scalar mass squared

$$m_{\tilde{q}}^2 = -\frac{1}{4}|F_\phi|^2 \left( \frac{d\gamma}{dy_t} \beta_g + \frac{d\gamma}{dy_b} \beta_g \right)$$

(55)

where $\gamma \equiv d\log Z(\mu)/d\log \mu$. When we take $\gamma = cg^2 + dy_t^2$, $\beta_g = -bg^2$, we can work out the masses of the squarks and gauginos in terms of the above parameters.

We then have for the anomaly-mediated contribution for $\tilde{q} = \tilde{t}_R$ and the $H_u$ doublet of scalars,

$$m_{\tilde{q}}^2 = \frac{1}{2} \{ cbg^4 + \frac{d\gamma}{dy_t} \beta_t \} |F_\phi|^2.$$ 

(56)

Similarly, for $\tilde{q} = \tilde{b}_R$ and the $H_d$ doublet of scalars,

$$m_{\tilde{q}}^2 = \frac{1}{2} \{ cbg^4 + \frac{d\gamma}{dy_b} \beta_b \} |F_\phi|^2,$$

(57)

and for $\tilde{q} = \tilde{t}_L$, $\tilde{b}_L$,

$$m_{\tilde{q}}^2 = \frac{1}{2} \{ cbg^4 + \frac{d\gamma}{dy_t} \beta_t + \frac{d\gamma}{dy_b} \beta_b \} |F_\phi|^2.$$ 

(58)

For the remaining scalars,

$$m_{\tilde{q}}^2 = \frac{1}{2} cbg^4 |F_\phi|^2.$$ 

(59)

It should be understood in the equation above that the gauge terms indicate the sum over the appropriate terms according to the gauge charge of the scalar; the Higgs and top squark mass are of course not the same.

The numerical value of $y_t$ and $y_b$ is uncertain because of the unknown value of $\tan \beta$. For simplicity, we give the values of the masses for vanishing $\beta_t$ and $\beta_b$. We take the values of the parameters at about 1 TeV from Ref. [70]: $\alpha_Y = 0.01$, $\alpha_2 = 0.032$, $\alpha_3 = 0.1$. With the well-known renormalization group coefficients [70], we find the scalar masses

$$m_{\text{sleptons}}^2 = -1.3 \times 10^{-5}|F_\phi|^2 + m_{\text{bulk}}^2$$

$$m_{\text{squarks}}^2 = 5.5 \times 10^{-4}|F_\phi|^2 + m_{\text{bulk}}^2$$

$$m_H^2 = -1.3 \times 10^{-5}|F_\phi|^2 + m_{\text{bulk}}^2$$

(60)

(61)
while the gauginos acquire soft mass-squareds are

\[
\begin{align*}
    m^2_{\text{gluino}} &= 6.1 \times 10^{-4} F_\phi^2 \\
    m^2_{\text{wino}} &= 6.4 \times 10^{-6} F_\phi^2 \\
    m^2_{\text{bino}} &= 7.0 \times 10^{-5} F_\phi^2.
\end{align*}
\]

(62)

Even without knowing the bulk contributions of \( F_\phi \), there are several interesting features of the spectrum which we can identify.

- The ratio of gaugino masses is determined. We find \( m_3 : m_2 : m_1 = 3.0 : 0.3 : 1 \).
- The wino/zino are the lightest of the gauginos. Furthermore, assuming a bulk contribution to the slepton mass at least of order of magnitude of the bulk contribution, the wino/zino are the lightest supersymmetric particles!
- The squarks and gauginos are the heaviest particles, and are quite heavy; they are an order of magnitude heavier than the wino.

In fact, the wino and zino are nearly, but not exactly, degenerate. Refs. [74] have studied some of the phenomenology of such a situation, based on the string-derived models of Ref. [58]. In the approximation \( M_2 < M_1 < \mu \), one can approximately determine the mass difference [71] [72] [73] to find

\[
m_{\tilde{\omega}} - m_\tilde{z} \sim \frac{m_2}{2} \left( \frac{m_\mu}{\mu} \right)^4
\]

(63)

This is fascinating. The wino and zino are so nearly degenerate that the wino lifetime should be quite large. Of course this depends on parameters, but for reasonable parameters, one finds the lifetime of the wino is comparable to that of a muon, since the lifetime scales as \((\Delta m)^5\). This means supersymmetric events should have a very striking signature. Sometimes the LSP will manifest itself as a zino, and look like a “typical” LSP. However, sometimes it will appear as a wino. The wino event will not deposit energy in the calorimeter, appearing as a missing energy event, but the wino will be detected in the muon chamber. With good time of flight, one should even be able to learn about the mass. In any case, this striking signature of these events should be sufficient to identify the sequester-sector scenario.

There are a couple of comments on the spectrum so far. First, away from the fixed point, \( H_u \) can have a largish contribution which is a function of \( y_t \), and \( H_d \) could also have a largish contribution if \( y_b \) is large. However, these contributions are proportional to \( \beta_{y_t} \) and \( \beta_{y_b} \). There is a large \( \tan \beta \) fixed point [74]. If the world sits at the fixed point of \( y_t \), the extra contribution to the \( H_u \) mass is reduced to zero. Similarly, the \( H_d \) mass contribution is very sensitive to \( y_b \) near the fixed point. So although the Higgs mass seems to need some tuning, this is not the case if one is at a fixed point. However, since we have less parameter freedom in our model than would be allowed in a general hidden sector, one would have to check the consistency of the whole picture. This is an interesting problem for future study.
The second comment is the fact that the stop squark mass is large. Naturalness bounds constrain the stop mass since it feeds directly into the Higgs mass; large stop implies large Higgs mass. However, these masses are being given at a low scale, the supersymmetry breaking scale of order 1 TeV. Below this scale, one would apply the standard renormalization to determine the spectrum. Since we are starting the scaling at such a low energy scale, there is no large logarithm. This means the naturalness constraints on the stop mass (and the gluino mass) are less severe. Masses of 2 TeV are perfectly consistent. In fact, one might even be willing to accommodate heavier masses; if true, this could be discouraging from the point of view of finding these superpartners. The upshot is that the naturalness bounds and reach of the collider needs to be reconsidered in light of our different spectrum.

Finally, we comment on the gaugino mass spectrum. It is commonly understood that a necessary consequence of gauge unification is that the gaugino mass parameters associated with SU(3), SU(2), and U(1) must arise in the ratio $g_3^2 : g_2^2 : g_1^2$. However, our scenario is perfectly consistent with unification. One way to think of why this happens is that there are large threshold corrections. The heavy GUT mass particles receive tree-level supersymmetry breaking, whereas the light states only receive the anomaly-mediated one-loop contribution. So the corrections from integrating out the heavy GUT states are so large as to give the prediction we have made here.

Notice that the gaugino masses depend on the beta functions. So the light wino is understood in terms of the fact that SU(2) scales relatively slowly.

We now consider the size of bulk effects. These are more model-dependent, so we simply constrain them by the necessary phenomenological criteria. The first essential requirement is that bulk contributions are sufficiently large to make the slepton mass squared positive. This means that they should be at least of order $10^{-5}|F_\phi|^2$.

The second requirement is that flavor violation is sufficiently suppressed. There are two possibilities; one is that sleptons are sufficiently heavy to suppress flavor violation, as proposed in Ref. [76]. This would require slepton masses of order $1 - 10$ TeV. The other possibility is that the sleptons are relatively light but sufficiently degenerate to suppress flavor violation, due to flavor-conserving bulk couplings.

Now one might expect a comparable bulk contribution to the squark mass squared. However, with no flavor symmetry requirement on the bulk contribution, an additional bulk contribution of order 1 TeV would be far too large, and introduce unacceptable flavor violation into the squark mass matrix. So it is clear that the bulk states that couple to quarks must give a significantly smaller contribution than their leptonic counterparts. This is naturally obtained with the “switch” described in Section 5. If the states coupling to squarks have mass in excess of the compactification scale, their contribution to the squark mass will exponentially decouple. One other potentially unsatisfactory feature of the heavy slepton scenario is that one might also expect the Higgs to then have a relatively heavy mass, decreasing the naturalness of the scenario.

The second possibility is that the sleptons are light. Then they must necessarily be fairly degenerate. Recall that this could happen if the light bulk moduli arise from the Kaluza-Klein reduction of the gravitational multiplet as discussed in Section 7. One would then
expect a small correction to the anomaly-mediated contribution to the squark mass matrix (recall the large QCD-induced contribution to the squark masses).

Notice that in both scenarios, under the assumptions outlined above, the squark mass is dominated by the anomaly-mediated contribution. So we have one more prediction assuming small slepton mass which is

- The ratio of gaugino to squark mass is 1.05.

To give a general idea of what these spectra look like, we present two “typical” examples. In the first, the light chargino is near the experimental bound, and in the second, the squarks and gluinos are towards the upper limit of the experimentally detectable range at the LHC.

**Spectrum I:** $m_{\text{gluino}} = 980 \text{ GeV}$, $m_{\text{wino}} = 89.974 \text{ GeV}$, $m_{\text{zino}} = 89.348$, $m_{\text{bino}} = 330 \text{ GeV}$, $m_{\text{squark}} = 930 \text{ GeV}$, $m_{\text{slepton}} = 200 \text{ GeV}$. Here we took $\mu = 250 \text{ GeV}$. For this value, $\Delta m \equiv m_{\text{wino}} - m_{\text{zino}} = 600 \text{ MeV}$. If, on the other hand, one takes $\mu = 800 \text{ GeV}$, one finds $\Delta m = 9 \text{ MeV}$.

**Spectrum II:** $m_{\text{gluino}} = 1950 \text{ GeV}$, $m_{\text{wino}} = 194.23 \text{ GeV}$, $m_{\text{zino}} = 194.112 \text{ GeV}$, $m_{\text{bino}} = 660 \text{ GeV}$, $m_{\text{squark}} = 1850 \text{ GeV}$, $m_{\text{slepton}} = 300 \text{ GeV}$. Here we took $\mu = 500 \text{ GeV}$, which gives $\Delta m = 120 \text{ MeV}$. For $\mu = 1600 \text{ GeV}$, one finds $\Delta m < 10 \text{ MeV}$.

The values for $\mu$ were motivated by a positive higgs mass squared with and without the Yukawa-dependent contribution. We have not included these to distinguish the top and bottom squark masses, but away from the fixed point, one should.

As emphasized above, a remarkable feature of the spectrum is the light wino and the small mass splitting from the zino.

In fact, these splittings have not included further radiative corrections. Particularly for very small splitting, custodial SU(2) violations should modify these numbers. We expect that this corrects our result at the level of 0.1% of the wino mass, but a calculation is necessary to confirm this.

The lifetime of the wino, for small mass splitting from the zino, is

$$\tau = \left( \frac{m_\mu}{\Delta m} \right)^5 \times 10^{-8} \text{ sec} \quad (64)$$

If we take $\beta \gamma$ of order unity, and assume a lifetime of 10 nsec is required to get to the muon chambers, this translates into a mass splitting of about 160 MeV in order to obtain our “smoking gun” signature. It is clear that the mass splitting is in a very interesting range. Much of the time $\Delta m$ is small compared to this number, or within a factor of 10, in which case one might hope to see a track sufficiently distinctive from a tau to distinguish it. Only for small $m_\mu$ and $m_{\text{wino}}$ was the lifetime sufficiently short to decay before reaching the muon chambers. Clearly, a more detailed analysis is in order to assess the reliability of this signature over the allowed parameter range.

It might seem remarkable that our spectrum is so predictive. The reason is our assumption that there are no direct couplings between the supersymmetry breaking and visible sectors. It is true that this assumption is not necessarily associated with the existence of higher dimensions. In fact, at one-loop, the anomaly-mediated contributions to both scalar
and gaugino masses will arise in any hidden sector model. However, without the assumption of higher dimensions, the nonexistence of tree-level masses, at least for the scalars, would be entirely unnatural. What we have is a mechanism for naturalness which is not implicit in the low-energy theory. We therefore expect this spectrum is associated with the existence of higher dimensions. Without this assumption, one would find an unacceptable hierarchy between the gaugino and scalar mass. This gives the amazing possibility of testing directly for the existence of higher dimensions in the mass spectrum of supersymmetric partners. If we are not unlucky, the wino will be “long-lived” and there will be the “smoking gun” signature described above. This would be later confirmed by a measurement of the spectrum. If the wino decays before the muon chamber, it will look tau-like and might therefore be difficult to identify. However, measurements of the spectrum will always serve to confirm or reject this scenario.

11 Conclusions

Clearly physics permits new possibilities if we live in a higher dimensional universe. One important feature from the low-energy point of view is that the notion of naturalness gets extended. Not only symmetries, but also the geometry of space-time can forbid operators from appearing in the low-energy Lagrangian. Many of the standard problems with hidden sector models have new potential resolutions in this framework.

We point out that our model is perfectly consistent with grand unification of gauge couplings. However, whether or not we can unify the gauge and gravitational coupling is more model-dependent. Since we have a preferred value of the compactification scale to generate scalar masses, one is not necessarily free to choose the mass scales to provide this further unification. However, we have seen in Section 7 that it could be possible for the scales to permit gauge-gravity unification. Given the uncertainty in the estimates, we find this an intriguing possibility.

Amazingly, the assumption of a sequestered sector is extremely predictive. The spectrum is different from any other model which accommodates gauge unification. This is due to the large threshold corrections when crossing a heavy particle mass scale. We have found predictions for the ratio of gaugino masses, and the ratio to the squark mass. We also have pointed out that the small mass splitting of the two lightest superpartners permits the potential for a dramatic signature. One might question the direct connection between these signatures and the assumption of higher dimensions; most follow from the gaugino mass contribution present in any hidden sector. However, it is difficult to envision any other hidden sector which naturally gives scalar masses of the same order of magnitude without arbitrary and unnatural assumptions. Furthermore the ratio of gaugino to squark mass should be unique to our scenario; any other scenario should give additional contributions to the squark mass.

Our assumptions are quite general, but there are some requirements for the high energy model. There must be at least one higher dimension which is stabilized at a mass scale near but below the Planck scale. Bulk fields which are lighter than the compactification scale
are required to provide the positive contribution to the slepton mass squared. Furthermore, the light states should not introduce large flavor dependence in the squark or slepton mass squared. Clearly it is difficult to assess the likelihood of these assumptions without a better understanding of the mechanism of moduli stabilization.

There is much which remains to be understood. In a future paper, we will present the effective theory of particular sequestered sectors, in particular those representing nonlocal supersymmetry breaking. It is also of interest to extend the analysis of [11] to allow one to match the higher dimensional theory directly to the four-dimensional theory. Furthermore, a detailed understanding of the possibilities of models derived from string theory, and whether they can permit our assumptions would be important. Further analysis of chiral theories derived from D-branes in the presence of gravity is essential.

Finally, there is much to be better understood in the phenomenology. One would want to include higher order loop corrections, and also to perform the full renormalization group running. There are interesting questions with respect to the viability of a large tan beta fixed point in our scenario and a better understanding of the implications of naturalness.

We find it encouraging that ultimately our scenario is testable. There is little model dependence to our results; general features of the spectrum reflect our underlying assumption that the source of supersymmetry breaking resides in the extra dimensions, sequestered from the visible world.

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Appendix A: A Minimal Sequestered Sector

In this Section, we give an example of a sequestered sector. This illustrates how one treats the theory from the low-energy four-dimensional vantage point. It will furthermore serve to illustrate the existence of a nonzero VEV of \( F_\phi \).

In this Appendix, we assume that supersymmetry breaking is caused by some strong dynamics in a hidden sector at an “intermediate” scale \( \Lambda_H \), much larger than the weak scale. The details of the hidden dynamics are irrelevant to the weak-scale supersymmetry breaking effects inherited by the visible sector. Here we make the simplest assumption that the only massless state produced by the hidden sector is the Goldstino, and that all other hidden sector states have typical masses of order \( \Lambda_H \). (It would however not significantly alter our analysis to include some extra massless hidden sector states.) It therefore makes sense to imagine integrating out all the massive (non-perturbative) \( \Lambda_H \)-scale physics. So we replace the unknown (and presumably strongly coupled and complicated) hidden sector by a simple
model with a single chiral superfield whose auxiliary component expec ation value breaks supersymmetry spontaneously at $\Lambda_H$.

We begin by switching off supergravity and considering a flat-superspace hidden sector given by,

$$\mathcal{L} = \int d^4 \theta \Lambda_H^2 \ln(1 + |\Sigma|^2 / \Lambda_H^2) + \int d^2 \theta \Lambda_H^2 \Sigma + \text{h.c.} \quad (65)$$

We straightforwardly derive the scalar potential,

$$V = \Lambda_H^4 (1 + |\sigma|^2 / \Lambda_H^2)^2. \quad (66)$$

We see immediately that,

$$\langle \sigma \rangle = 0$$
$$m_\sigma^2 = 2\Lambda_H^2$$
$$\langle F_\Sigma \rangle = \Lambda_H^2$$
$$m_{\psi \Sigma} = 0. \quad (67)$$

That is, supersymmetry is broken at the required scale and the only massless state is the Goldstino. The scalar $\sigma$ is the analog of the massive $\sigma$ mode of the linear $\sigma$ model. The non-renormalizability of the model is unimportant since it is to be used as a low-energy effective theory. All high-energy degrees of freedom have already been integrated out.

Let us now couple just this sector to supergravity by taking eq. (7) to be given by,  

$$f = -3M_{Pl}^2 + \Lambda_H^2 \ln(1 + |\Sigma|^2 / \Lambda_H^2)$$

$$W = \Lambda_H^2 (\Sigma + c), \quad (68)$$

where the constant $c$ has been included to cancel the cosmological constant. Our model now resembles the Polonyi model, but with a non-canonical Kahler potential which gives the scalars $\sigma$ a $\Lambda_H$-scale mass. Because the scalars are massive, the weak gravitational couplings will only negligibly alter the VEVs found above for $\Sigma$. We can use these VEVs in eq. (7) to solve for $\langle F_\Phi \rangle$. The $F_\Phi$ equation of motion simply yields,

$$\langle F_\Phi \rangle = \frac{\Lambda_H^2 c}{M_{Pl}^2}. \quad (69)$$

The vacuum energy following from the auxiliary part of eq. (7) is then given by,

$$\langle V \rangle = \Lambda_H^4 (1 - 3\frac{c^2}{M_{Pl}^2}). \quad (70)$$

Now the Weyl rescaling discussed in Section 4 would normally necessitate multiplying the naive vacuum energy from eq. (7) by $\exp(2\langle K \rangle / 3M_{Pl}^2)$, but at $\sigma = 0$ one can see by Eq.
that this factor is unity. The vacuum energy we have computed is nothing but the cosmological constant and must be tuned to vanish. Thus we must have,

\[
F_\phi = \sqrt{\frac{\Lambda_H^2}{M_{Pl}}} \\
c = \sqrt{3 M_{Pl}}.
\] (71)

The super-Higgs mechanism will cause the Goldstino to be eaten by the gravitino, which will therefore acquires a mass Eq. (13),

\[
m_{3/2} = \sqrt{\frac{\Lambda H}{M_{Pl}}}.
\] (72)

**Appendix B: Regulating SQED**

Here we present a regularization for the simplest gauge theory, SQED, coupled to the supersymmetry-breaking background gravitational superfield, \( \Phi \). It will concretely illustrate how anomalous \( \Phi \)-dependence appears after the rescaling of Eq. (30). A similar discussion appears in Ref. [63]. More general supersymmetric gauge theories are more difficult to explicitly regulate and are treated in Ref. [14].

The naive lagrangian for SQED coupled to the \( \Phi \) background is straightforwardly given by,

\[
\mathcal{L}_{SQED} = \int d^4 \theta [Q_+^\dagger e^V Q_+ |\phi|^2 + Q_-^\dagger e^V Q_- |\Phi|^2] + (\int d^2 \theta \frac{1}{g_0^2} \mathcal{W}^2 + \text{h.c.}) + \text{gauge fixing.} \tag{73}
\]

The fact that the gauge multiplet terms are \( \Phi \)-independent is a reflection of the classical Weyl-invariance of their action. This property is shared by the standard gauge-fixing terms, which we do not write explicitly here. Of course the Feynman diagrams that follow from this naive lagrangian are ultraviolet-divergent. We can gauge-invariantly regulate closed loops of charged fields using the time-honored Pauli-Villars procedure of adding massive charged regulator fields with the “wrong” statistics,

\[
\mathcal{L}_{PV} = \int d^4 \theta [Q_+^{reg} \dagger e^V Q_+^{reg} |\Phi|^2 + Q_-^{reg} \dagger e^V Q_-^{reg} |\phi|^2] + (\int d^2 \theta \Phi^3 \Lambda_{UV} Q_+^{reg} Q_-^{reg} + \text{h.c.).} \tag{74}
\]

For sufficiently many such fields with oscillating statistics and masses of order \( \Lambda_{UV} \) any divergence from charged loops can be regulated. For simplicity, we will retain just one pair of charged regulator fields in this discussion.

There will still be divergences in loops containing internal gauge-multiplet lines. For these we need to soften the behavior of the gauge propagators above the cutoff scale. For example, we can regulate the (Feynman gauge) photon propagator as follows,

\[
\frac{\eta_{\mu \nu}}{p^2} \rightarrow \frac{\eta_{\mu \nu}}{p^2 (1 - p^2/\Lambda_{UV}^2)}.
\] (75)
By multiplying by sufficiently many such cutoff “form-factors” we can regulate any loops containing internal photon lines. Again for simplicity we will just consider one such form-factor. The regulated propagator can be thought of as coming from a regulated lagrangian containing higher derivatives,

\[ L_{\text{photon}}^{\text{reg}} = -\frac{1}{4} F_{\mu\nu}(1 + \frac{\partial^2}{\Lambda_{\text{UV}}^2}) F^{\mu\nu} + \text{gauge} - \text{fixing}. \] (76)

While this is a satisfactory gauge-invariant regulated lagrangian, its supersymmetrized form also involves higher derivatives and therefore poses a needless complication. To avoid this we will separate the regulated photon propagator into the difference of a naive propagator and a massive propagator,

\[ \frac{\eta_{\mu\nu}}{p^2(1 - p^2/\Lambda_{\text{UV}}^2)} = \frac{\eta_{\mu\nu}}{p^2} - \frac{\eta_{\mu\nu}}{p^2 - \Lambda_{\text{UV}}^2}. \] (77)

The supersymmetrized gauge mass term is simply given by [54],

\[ \delta L_{\text{gauge mass}} = \int d^4\theta \frac{1}{g_0^2} \Lambda_{\text{UV}}^2 V^2. \] (78)

For an abelian gauge theory this mass term effectively does not break gauge invariance since we can imagine it arising from the Higgs mechanism, where the Higgs charge is infinitesimally small and the Higgs VEV is infinitely large, in such a way that their product yields a \( \Lambda_{\text{UV}} \) mass. In this limit, the physical Higgs degrees of freedom decouple and the Goldstone fields are eaten away, leaving only the gauge mass term. This trick cannot be used for non-abelian gauge fields because charge is quantized and therefore cannot be taken infinitesimal. This mechanism for giving mass to the gauge multiplet also makes it clear how \( \Phi \)-dependence must appear when coupled to supergravity. Since the mass term is effectively a conventional Higgs kinetic term in unitary gauge it must appear multiplied by \( |\Phi|^2 \) in the supergravity formalism. That is,

\[ \mathcal{L} = \int d^4\theta \frac{1}{g_0^2} \Lambda_{\text{UV}}^2 V^2|\phi|^2 + \left( \int d^2\theta \frac{1}{g_0^2} W^2 + \text{h.c.} \right) + \text{gauge} - \text{fixing}. \] (79)

Let us then summarize the algorithm for regulating a general Feynman supergraph: (i) Write the integral following from Eq. (73) plus Eq. (74). The latter’s contributions will regulate all charged loop (sub-)divergences. (ii) Subtract from the naive super-propagator for each internal gauge line, the super-propagator arising from the massive gauge lagrangian, Eq. (79).

This procedure clarifies the claims made in Section 6. In the naive Lagrangian Eq. (73), we can easily rescale away \( \Phi \)-dependence altogether. This same rescaling can also be applied to the Pauli-Villars regulator fields, but now we see that \( \Phi \)-dependence is retained in the cutoff mass-term,

\[ \mathcal{L}_{\text{PV}} = \int d^4\theta [Q_+^{\text{reg}} \dagger e^V Q_+^{\text{reg}} + Q_-^{\text{reg}} \dagger e^V Q_-^{\text{reg}}] + \left( \int d^2\theta \Phi \Lambda_{\text{UV}} Q_+^{\text{reg}} Q_-^{\text{reg}} + \text{h.c.} \right). \] (80)
Similarly, the gauge multiplet regulator mass-squared is also multiplied by $|\Phi|^2$. That is, the rescaled regulated theory depends on $\Lambda_{UV}$ and $\Phi, \Phi^\dagger$, precisely through the combinations $\Lambda_{UV}\Phi, \Lambda_{UV}\Phi^\dagger$. Now let us consider the R-symmetry before the rescaling under which $R[\Phi] = 2/3$ and $R[Q] = 0$. This is clearly a valid symmetry even in the presence of the regulator. After the rescaling of course $R[Q] = 2/3$. We still have a valid symmetry as long as we remember the $\Phi$-dependence appearing with the cutoff masses. If we simply go to the flat space theory now by imposing $\Phi = 1$, then we must reproduce the fact that the R-symmetry is anomalous. We conclude that in the renormalized theory, anomalous $\Phi$ dependence must be present so as to cancel the anomaly that would otherwise be there. This only involves the phase of the lowest component of $\Phi$, but because $\Phi$ couples as a background superfield in regulated SQED, the couplings of its various components after renormalization will be related by supersymmetry. More explicitly at one-loop and after rescaling, we can see that the usual logarithmic divergence of the gauge vacuum polarization must come in the form $\int d^2\theta \ln(\Lambda_{UV}\Phi)\mathcal{W}^2 + \text{h.c.}$, since the regularization is purely due to Eq. (80), while the usual logarithmic divergence in the wavefunction of the charged fields must come in the form $\int d^4\theta \ln(\Lambda^2_{UV}|\Phi|^2)|Q|^2$, since the regularization is purely due to Eq. (79).

References

[1] J. Scherk and J. H. Schwarz, Phys. Lett. B82 (1979) 60.

[2] J. Scherk and J. H. Schwarz, Nucl. Phys. B153 (1979) 61; E. Cremmer, J. Scherk and J. H. Schwarz, Phys. Lett. B84 (1979) 83; P. Fayet, Phys. Lett. B159 (1985) 121, Nucl. Phys. B263 (1986) 649.

[3] R. Rohm, Nucl. Phys. B237 (1984) 553; H. Itoyama and T. Taylor, Phys. Lett. B186 (1987) 129; C. Kounas and M. Porrati, Nucl. Phys. B310 (1988) 355; S. Ferrara, C. Kounas, M. Porrati and F. Zwirner, Nucl. Phys. B318 (1989) 75; C. Kounas and B. Rostand, Nucl. Phys. B341 (1990) 641.

[4] A. Brandhuber, J. Sonnenschein, S. Theisen and S. Yankielowicz, *Brane Configurations and 4-D Field Theory Dualities*, Nucl.Phys.B502 (1997) 125, hep-th/9704044.

[5] E. Witten, *Branes and the Dynamics of QCD* Nucl.Phys. B507 (1997) 658, hep-th/9706109.

[6] N. Evans and M. Schwetz *The Field Theory of Nonsupersymmetric Brane Configurations*, Nucl.Phys. B522 (1998) 69, hep-th/9708122.

[7] P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506, hep-th/9510203; E. Witten, Nucl. Phys. B471 (1996) 135, hep-th/9602070; P. Horava and E. Witten, Nucl. Phys. B475 (1996) 94.
[8] P. Horava, *Gluino Condensation in Strongly Coupled Heterotic String Theory*, Phys. Rev. D54 (1996) 7561, hep-th/9608019.

[9] I. Antoniadis and M. Quiros, *Supersymmetry Breaking in M-theory and Gaugino Condensation*, Nucl.Phys. B505 (1997) 109, hep-th/9705037.

[10] I. Antoniadis, S. Dimopoulos and G. Dvali, *Millimetre-Range Forces in Superstring Theories with Weak-Scale Compactification*, Nucl.Phys. B516 (1998) 70, hep-9710204.

[11] E. A. Mirabelli and M. E. Peskin, *Transmission of Supersymmetry Breaking From a Four-dimensional Boundary*, Phys. Rev. D58 (1998) 065002, hep-th/9712214.

[12] J. P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145 and Phys. Lett. B271 (1991) 307.

[13] G. L. Cardoso and B. Ovrut, Nucl. Phys. B369 (1992) 351 and Nucl. Phys. B392 (1993) 315.

[14] V. Kaplunovsky and J. Louis, Nucl.Phys. B422 (1994) 57, hep-th/9402005.

[15] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry Breaking and Soft Terms in M Theory*, Phys. Lett. B415 (1997) 24, hep-th/9707143.

[16] Z. Lalak and S. Thomas, *Gaugino Condensation, Moduli Potential and Supersymmetry Breaking in M Theory Models*, Nucl. Phys. B515 (1998) 55, hep-th/9707223.

[17] E. Dudas, Phys.Lett. B416 (1998) 309, hep-th/9709043.

[18] A. Lukas, B. A. Ovrut and D. Waldram, *On the Four-Dimensional Effective Action of Strongly Coupled Heterotic String Theory*, hep-th/9710208.

[19] K. Choi, H. B. Kim and C. Munoz, *Four-Dimensional Effective Supergravity and Soft Terms in M Theory*, hep-th/9711158.

[20] A. Lukas, B. A. Ovrut and D. Waldram, *Gaugino Condensation in M-theory on S^2/Z_2*, Phys.Rev. D57 (1998) 7529, hep-th/9711197.

[21] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry Breakdown at a Hidden Wall*, hep-th/9801030.

[22] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *The Universe as a Domain Wall*, hep-th/9803235.

[23] J. Ellis, Z. Lalak, S. Pokorsky and W. Pokorski, *Five Dimensional Aspects of M-Theory Dynamics and Supersymmetry Breaking*, hep-ph/9805377.

[24] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *Heterotic M Theory in Five-Dimensions*, hep-th/9806051.
[25] Z. Kakushadze, *A Three Family SU(4) × SU(2) × SU(2) Type I Vacuum*, hep-th/9806044.

[26] J. Lykken, E. Poppitz and S. P. Trivedi, *Branes with GUTS and Supersymmetry Breaking*, hep-th/9806080.

[27] I. Antoniadis, Phys. Lett. B246 (1990) 377; I. Antoniadis and C. Kounas, Phys. Lett. B261 (1991) 369; I. Antoniadis, C. Munoz and M. Quiros, Nucl. Phys. B397 (1993) 515.

[28] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The Hierarchy Problem and New Dimensions at a Millimeter*, hep-ph/9803315.

[29] K. R. Dienes, E. Dudas and T. Ghergetta, *Extra Spacetime Dimensions and Unification*, hep-ph/9803466.

[30] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV*, hep-ph/9804398.

[31] G. Shiu and S.-H. H. Tye, *TeV Scale Superstring and Extra Dimensions*, hep-th/9805157.

[32] R. Sundrum, *Effective Field Theory for a Three-Brane Universe*, hep-ph/9805471.

[33] A. Pomarol and M. Quiros, *The Standard Model from Extra Dimensions*, hep-ph/9806263.

[34] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D59 (1999) 086004, hep-ph/9807344.

[35] R. Sundrum, *Compactification for a Three-Brane Universe*, hep-ph/9807348.

[36] P. C. Argyres, S. Dimopoulos, J. March-Russell, Phys. Lett. B441 (1998) 96, hep-th/9808138.

[37] Z. Kakushadze and S.-H. H. Tye, *Brane World*, hep-th/9809147.

[38] K. Benakli, *Phenomenology of Low Quantum Gravity Scale Models*, hep-ph/9809582.

[39] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, *Stabilization of Submillimeter Dimensions: The New Guise of the Hierarchy Problem*, hep-th/9809124.

[40] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Soft Masses in Theories with Supersymmetry Breaking by TeV Compactification*, hep-ph/9810410.

[41] N. Arkani-Hamed and S. Dimopoulos, *New Origins for Approximate Symmetries from Distant Breaking in Extra Dimensions*, hep-ph/981153.
[42] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, *Gaugino Mass without Singlets*, hep-ph/9810442.

[43] G. R. Farrar, *Status of Light Gaugino Scenarios*, Nucl. Phys. Proc. Suppl. 62 (1998) 485, hep-ph/9710277.

[44] S. Raby and K. Tobe, *The Phenomenology of SUSY Models with a Gluino LSP*, hep-ph/9807281.

[45] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A Complete Analysis of FCNC and CP Constraints in General SUSY Extensions of the Standard Model*, Nucl. Phys. B477 (1996) 321, hep-ph/9604387.

[46] R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B445 (1995) 219, hep-ph/9501334.

[47] T. Banks, D. B. Kaplan and A. E. Nelson, *Cosmological Implications of Dynamical Supersymmetry Breaking*, Phys. Rev. D49 (1994) 779, hep-ph/9308292.

[48] A.. Brignole, L.E. Ibanez and C. Munoz, *Towards a Theory of Soft Terms for the Supersymmetric Standard Model*, Nucl. Phys. B422 (1994) 125, Erratum-ibid. B436 (1995) 747, hep-ph/9308271.

[49] G. F. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480.

[50] See the review by, Y. Grossman, Y. Nir and R. Rattazzi, *CP Violation Beyond the Standard Model*, hep-ph/9701231.

[51] G.F. Giudice, R. Rattazzi, *Theories with Gauge Mediated Supersymmetry Breaking*, hep-ph/9801271.

[52] L. Randall and R. Sundrum, in preparation.

[53] Y.J. Ahn and J. D. Breit, Nucl. Phys. B273 (1986) 75; P. Binetruy, S. Dawson and I. Hinchliffe, Phys. Lett. B179 (1986) 262; J. Ellis, D. V. Nanopoulos, M. Quiros and F. Zwirner, Phs. Lett. B180 (1986) 83; J. Ellis, A. B. Lahanas, D. V. Nanopoulos, M. Quiros and F. Zwirner, Phys. Lett. B188 (1987) 408.

[54] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd ed., Princeton Univ. Press (1992).

[55] W. Siegel and S.J. Gates, Jr., Nucl. Phys. B147 (1979) 77; S.J. Gates, M.T. Grisaru, M. Rocek and W. Siegel, *One Thousand and One Lessons in Supersymmetry*, Benjamin/Cummings (1983) Reading, USA.

[56] J. Bagger, E. Poppitz and L. Randall, *Destabilizing Divergences in Supergravity Theories at Two Loops*, Nucl. Phys. B455 (1995) 59, hep-ph/9505244.
[57] J. Louis and Y. Nir, *Some Phenomenological Implications of String Loop Effects*, Nucl. Phys. B447 (1995) 18, hep-ph/9411429.

[58] L. Ibanez and D. Lust, Nucl. Phys. B382 (1992) 305.

[59] P. Binetruy, M. K. Gaillard and Yi-Yen Wu, Phys. Lett. B412 (1997) 288; Nucl. Phys. B493 (1997) 27; Nucl. Phys. B481 (1996) 109.

[60] T. E. Clark, O. Piguet and K. Sibold, Nucl. Phys. B159 (1979) 1; K. Konishi, Phys. Lett. B135 (1984) 439.

[61] M. K. Gaillard, *One Loop Pauli-Villars Regularization of Supergravity 1. Canonical Gauge Kinetic Energy*, hep-th/9806227.

[62] G. F. Giudice and R. Rattazzi, *Extracting Supersymmetry Breaking Effects from Wave Function Renormalization*, Nucl. Phys. B511 (1998) 25, hep-ph/9706540.

[63] N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, *Supersymmetry Breaking Loops from Analytic Continuation into Superspace*, hep-ph/9803290.

[64] K. Choi, J. E. Kim and H. P. Nilles, Phys. Rev. Lett. 73 (1994) 1758, hep-ph/9404311.

[65] M. Dine and A. E. Nelson, Phys. Rev. D48 (1993) 1277.

[66] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1996) 2658.

[67] G. Dvali, G. F. Giudice and A. Pomarol, Nucl. Phys. B478 (1996) 31.

[68] S. Dimopoulos, G. Dvali and R. Rattazzi, Phys. Lett. B413 (1997) 336, hep-ph/9707537.

[69] R. Rattazzi and U. Sarid, *Large Tan Beta in Gauge Mediated SUSY Breaking Models*, Nucl. Phys. B501 (1997) 297, hep-ph/9612464.

[70] P.M. Ferreira, I. Jack and D.R.T. Jones, Phys. Lett. B387 (1996) 80, hep-ph/9605440.

[71] S. P. Martin and P. Ramond, *Sparticle Spectrum Constraints*, Phys. Rev. D48 (1993) 5365, hep-ph/9306314.

[72] B. Grinstein, J. Polchinski and M. B. Wise, Phys. Lett. B130 (1983) 285.

[73] H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75.

[74] C. H. Chen, M. Drees and J. F. Gunion, Phys. Rev. D55 (1997) 330; Phys. Rev. Lett. 76 (1996) 2002.

[75] J.A. Casas, J.R. Espinosa and H.E. Haber, *The Higgs Mass in the MSSM Infrared Fixed Point Scenario*, Nucl. Phys. B526 (1998) 3, hep-ph/9801363.

[76] A. G. Cohen, D.B. Kaplan and A.E. Nelson, *The More Minimal Supersymmetric Standard Model*, Phys. Lett. B388 (1996) 588, hep-ph/9607394.