Reply to Comment on ”Dispersion Velocity of Galactic Dark Matter Particles” by Evans.

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In his comment on our Letter [1], Evans [2] has expressed concern that the value of \( \langle v^2 \rangle_{DM}^{1/2} = 600 \text{ km s}^{-1} \) derived by us may not be valid in general because of our assumption of the Maxwellian form for the distribution function (DF). He also concludes that there are undetected numerical errors in our work. Neither of his points is valid. Below, we argue that our results are quite general and are only weakly sensitive to the precise form of the DF assumed in the analysis. In addition, we show that Evans’ worries about numerical errors are unfounded.

We have based our analysis on two of the simplest and most widely used DFs, namely, (a) the Maxwellian and (b) the ‘lowered isothermal’ or ‘King model’ [3]. The latter has the property that both the spatial density and the velocity dispersion vanish at the “tidal radius” \( r_t \), and \( \langle v^2 \rangle_{DM}^{1/2} \) depends on the galactocentric distance \( R \); it decreases from \( \langle v^2 \rangle_{DM}^{1/2} \sim \sqrt{3\sigma^2} \) at \( R = 0 \) to \( \langle v^2 \rangle_{DM}^{1/2} = 0 \) at \( R = r_t \), where \( \sigma \) is the velocity parameter of the model [3].
Fig. 1 shows the rotation curves for the two DFs. With the King model the best fit to the rotation curve is obtained for $\sigma = 330\,\text{km\,s}^{-1}$ which corresponds to the solar neighborhood value of dark matter velocity dispersion, $\langle v^2 \rangle_{\text{DM,\,\odot}}^{1/2} \sim 570\,\text{km\,s}^{-1}$, not significantly different from $600\,\text{km\,s}^{-1}$ for the Maxwellian DF (hence our comment in Ref. 18 of [1]). The robust nature of this result can be understood as follows: In the absence of streaming ($\langle v \rangle = 0$), the leading moment, $\langle v^2 \rangle$, appears as the pressure term in the Jeans equations (see Eq. 4-27 of [3]). Thus for all pressure supported halos the value of $\langle v^2 \rangle_{\text{DM,\,\odot}}^{1/2}$ will not be significantly different from the value of $\sim 600\,\text{km\,s}^{-1}$ derived by us [1].

Concerning the question of correctness of our numerical code, we note from Fig. 1 that for the case of the Maxwellian DF, even though the asymptotic relation for the circular velocity $V_{c,\infty} = \sqrt{\left( \frac{\pi}{2} \right) \langle v^2 \rangle_{\text{DM,\,\odot}}^{1/2}}$ is strictly valid only for single-component isothermal spheres, $V_c$ does indeed reach an asymptotic value implied by the above relation at $R \sim 1000\,\text{kpc}$. Moreover, even though it is difficult to anticipate fully the behavior of the solutions to the exponentially non-linear differential equations involved in the problem, the expectation of spherical symmetry at large distances is confirmed by our numerical calculations. At small $R$, the known visible matter dominates the gravitational potential, allowing us to have an approximate analytic expression for the dark matter density $\rho_{\text{DM}}$ (see Eq. (3) of Ref. [1]) which indicates that $\rho_{\text{DM}}$ decreases with increasing $R$ more rapidly than for a single component isothermal sphere. As already noted in [1], we have cross-checked our numerical results against these expectations, and in all cases we find excellent conformity to them.

Finally we would like to emphasise the advantage of using data on circular velocities, in deriving the relevant halo phase space model parameters; $V_c^2$ directly yields $R_{\text{eff}}$, where $\phi$ is the total gravitational potential. In this context, note from Fig. 1 that while both Maxwellian and King forms of the DF predict essentially identical $V_c(R)$ up to $\sim 20\,\text{kpc}$, the curves are very different at large $R$. This underscores the need to measure with greater precision $V_c$ as a function of $R$, especially at large galactocentric distances, to fix the parameters describing the dark matter halo more exactly.
In summary, we have shown that Evans’ [2] criticisms of our Letter [1], are not valid and our result of \( \langle v^2 \rangle_{\text{DM}}^{1/2} = 600 \text{ km s}^{-1} \) is robust and stands unaffected.
REFERENCES

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[2] N. W. Evans, astro-ph/9606153.
[3] J. Binney and S. Tremaine, Galactic Dynamics (Princeton Univ. Press, Princeton, 1987), p. 232.