Determination of the periodicity and synchronization of anticipative agent based supply-demand model

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Abstract. The paper presents the transformation of cobweb model by including the anticipation about the supply and demand. Developed transformation leads to oscillatory behaviour. The periodic conditions of the model have been analytically determined by the application of z-transform. Periodic solutions of the system are presented in the form of an inverse Farey tree, where the Golden Ratio path could be observed. The table of periodic conditions is given up to period 8. The agent-based system was developed in order to show the possibility of controlling the system by varying the key parameter, which determines the frequency response of agents and their interaction. A note on application in the stock market has been provided.

1. Introduction
Controlling supply and demand in proper balance becomes more challenging in the new, unpredictable economy [4], [18], [15]. In the classical cobweb model the functions of demand $Q^d_k$ and supply $Q^s_k$ are specified in the following form:

$$Q^d_k = a + bP^k_k$$

$$Q^s_k = c + dP^k_{k-1}$$

where $a, b, c$ and $d$ are parameters of supply and demand specific to individual markets [21]. The price $P^k_k$ and supply $Q^s_k$ should be restricted to the positive values. In the cobweb model it is assumed that in any one time period producers supply a given amount (determined by the previous time period’s price) and then the price adjusts so that all the products supplied are bought by customers. Initial cobweb equation could be restated by introducing additional time step and factors $A, B, C$ and $D$:

$$P^k_{k+2} = \frac{d}{b}(A^k_k - (\frac{bB^k_k - c + a}{d}))$$

$$Q^s_{k+2} = \frac{d}{b}(C^k_k - a - \frac{b}{d}(D^k_k - c))$$
with initial conditions:

\[
P_{k+1} = \frac{p-a}{b} \tag{5}
\]

\[
P_k = \frac{bP_{k+1} + a-c}{d} \tag{6}
\]

\[
Q_{k+1}^i = p \tag{7}
\]

\[
Q_k^i = a + \frac{b}{d} (Q_{k+1}^i - c) \tag{8}
\]

Terms \(A_k\) and \(B_k\) in Eq. (3) could be replaced by the terms \(P_{k+1}\) or \(P_k\), similarly \(C_k\) and \(D_k\) in Eq. (4) by \(Q_{k+1}^i\) or \(Q_k^i\). This yields 16 different system combinations defined by Eq. (3) and Eq. (4) that should be studied. The system combination further examined will have the following terms: \(A_k = P_{k+1}, B_k = P_k, C_k = Q_{k+1}^i\) and \(D_k = Q_k^i\). Proposed replacement changes the difference \(\Delta P\) and \(\Delta Q\) to state equations for \(P\) and \(Q\). This yields the following set of equations:

\[
P_{k+2} = \frac{d}{b} \left( P_{k+1} - \frac{bP_k - c + a}{d} \right) \tag{9}
\]

\[
Q_{k+2}^i = \frac{d}{b} \left( Q_{k+1}^i - (a + \frac{b}{d} (Q_k^i - c)) \right) \tag{10}
\]

In the performed modification we are considering the dynamics which is dependant on two different time state values in Eq. (9) rather than one. If we reformulate Eq. (9) and Eq. (10), the dependency of the future-present-past events could be observed:

\[
P_k = \frac{bP_{k+1} + a-c}{d} + \frac{b}{d} P_{k+1} \tag{11}
\]

\[
Q_k^i = \frac{b}{d} Q_{k+1}^i + \frac{b}{d} Q_{k+1}^i + a - \frac{bc}{d} \tag{12}
\]

Eq. (11) and Eq. (12) state that the value of the present is dependent on the past as well as on the future. In this case we anticipate future supply and demand.

2. Periodicity of the anticipative cobweb model

Different modes of cyclic behaviour response could be observed as the system’s dynamic response, when parameter \(d\) is varied. Synchronization patterns are named by the shape of the Poincaré first-return map representing the values of \(P_k, P_{k+1}\). The vertices of the solution converge to the edge point of the pentagram. Points on the vertices form the line at the periodic condition values for parameter \(d\). The system is in transition to the next full polygon synchronization, which is estimated as the quad synchronization, where parameter \(d\) is near 0 (or hexagon, depending on the direction of parameter \(d\) variation).

The analytical determination of periodicity conditions is provided by the application of \(z\)-transform, which is the basis of an effective method for the solution of linear constant-coefficient difference equations. It essentially automates the process of determining the coefficients of the various geometric sequences that comprise a solution [16]. The application of \(z\)-transform on Eq. (9) and Eq. (10) with initial conditions stated by Eqs. (5)–(8) gives:
\[ Y(z) = -y_0 z + y_0 dz - y_0 z^2 \]

Inverse \( z \)-transform yields the following solution:

\[
Y^{-1}(z) = 2^{-1-n} y_0 \left( d - \sqrt{-4 + d^2} \right)^n - y_1 \left( d - \sqrt{-4 + d^2} \right)^n + 2^{-1-n} y_0 \left( d + \sqrt{-4 + d^2} \right)^n + \\
y_1 \left( d + \sqrt{-4 + d^2} \right)^n - 2^{-1-n} y_0 d \left( d + \sqrt{-4 + d^2} \right)^n
\]

In order to gain conditions for the periodic response of the system the following equation should be solved:

\[ Y^{-1}(z) = y_0 \]  

(15)

Let us compute a numerical example of periodic solution applying the \( z \)-transform. The period examined will be the period of \( 9 \), i.e. \( n = 9 \). In Eq. (15) one should put the condition \( n = 9 \). One of the possible solutions for the initial condition worth examining is the following:

\[ d = \frac{1}{2(-1+i\sqrt{3})^\frac{1}{3}} + (\frac{1}{2}(-1+i\sqrt{3}))^\frac{1}{3} \]

(16)

The term \((-1+i\sqrt{3})^\frac{1}{3}\) (let us denote the term as \( z^* \)) could be expressed in the following way by three different imaginary values in polar form:

\[ z_1^* = \sqrt{2}(\cos \frac{2\pi}{9} + i \sin \frac{2\pi}{9}) \]

(17)

\[ z_2^* = \sqrt{2}(\cos \frac{8\pi}{9} + i \sin \frac{8\pi}{9}) \]

(18)

\[ z_3^* = \sqrt{2}(\cos \frac{14\pi}{9} + i \sin \frac{14\pi}{9}) \]

(19)

By putting Eqs. (17), (18) and (19) into Eq. (16) and performing trigonometric reduction, one gets the following solutions:

\[ d_1 = 2 \cos \frac{2\pi}{9}, \quad d_2 = 2 \cos \frac{4\pi}{9}, \quad d_3 = 2 \cos \frac{8\pi}{9} \]

(20)

By inspecting Eq. (16) and considering the equation for the roots of complex numbers [13]:
the general form of the solution for parameter \( d \) could therefore be defined as:

\[
d = 2 \cos \frac{2\pi m}{n}
\]

(22)

where \( n \) is the period and \( m = 1, 2, 3, \ldots, n - 1 \). A similar procedure could be performed for the arbitrary period \( n \). A more general solution, which applies parameter \( b \), which was fixed for the purpose of determining solutions, is:

\[
d = 2b \cos \frac{2\pi m}{n}
\]

(23)

The solution could in some cases be expressed in an alternative algebraic or trigonometric form. Table 1 shows the solutions for parameter \( d \) up to the period \( n = 8 \). Alternative solutions could be expressed as the roots of the polynomial. Table 1 incorporates the Shape symbols, which are important in the study of the response of dynamical systems. This is especially the case in the examination of complex nonlinear dynamical systems [27]. One of the important conditions gained by the proposed inspection is the value of the period \( n = 10 \), which correlates closely with the period \( n = 5 \). The value of parameter \( d \) is \( d = \frac{1}{2} (1 + \sqrt{5}) \) with numerical value 1.61803... This solution represents the Golden Ratio (usually denoted with \( \Phi \)) [9]. Some of the different representations of the solution for the parameter \( d \) value at period \( n = 10 \) are:

\[
d_{10} = \Phi = 2 \cos \frac{\pi}{5} = \frac{1}{2} (1 + \sqrt{5}) = 1.61803398874989484820...
\]

(24)

The first solution of parameter \( d \) at period \( n = 10 \) connects the considered discrete system with the Fibonacci numbers given by the infinite series:

\[
d_{10} = \Phi = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{F_n F_{n+1}}
\]

(25)

The fact, that the periodicity conditions of the examined discrete system incorporates the Golden Ratio number \( \Phi \) could be observed in other studies of complex nonlinear expansions of the basic cobweb systems, e.g. [3] Almost Homoclinic Tangency Lemma. One should expect that the symmetric response in \( n \)-mapping to follow the pattern with a match at a certain point to the solution of synchronization values, for example, \( \Phi \). The value that inevitably emerges with Golden Ratio \( \Phi \) is in our case present at period \( n = 5 \), where the value for parameter \( d = \frac{\sqrt{5} - 1}{2} = 0.61803... \), often called the Golden Ratio Conjugate (usually denoted with \( \phi \)). Table 1 is similar to the gained parameter values for the domain of 2-D dynamic attraction by [24]. The mentioned set of parameters is augmented with the value for the periods 8, which is not stated in [24].

**Table 1.** Synchronization parameter \( d \) values of periodicity conditions up to period 8.
Table 1 (*) indicates that since period 2 is on the boundary of the solutions interval, the periodic response of the system depends on the initial conditions. An example of numerical values of the period-2 response is: $a = 1, b = 1, c = -1, d = -2, p = 1$. The value for the tetragon marked with (***) in Table 1 is taken in the limit since the system of equations returns the undefined value when $d = 0$. Therefore one should consider the tetragon period condition as the value approaching zero, i.e. $d \to 0$. In this case, the system response is undetermined ($\frac{0}{0}$) at its critical point.

Numerical values of the solutions for parameter $d$ confirm the findings of [25, 24] characterized by the following Prop. 1 about the domain of attraction for 2-D dynamics by $n$-dimensional linear bifurcation analysis. Determination of the periodicity condition for the anticipative system of cobweb linear difference equations leads to the following proposition, which provides the interconnection between the periodicity cycles in the considered anticipative cobweb model and general bifurcation conditions for the nonlinear discrete dynamical systems:

**Proposition 1** Periodicity conditions of the anticipative cobweb model $d = 2b\cos\frac{2\pi m}{n}$ equals bifurcation condition on the flutter boundary determining q-periodic fixed points of the nonlinear discrete dynamical system $trJ = 2\cos 2\pi\Omega$.

Here, $\Omega$ represents a rational fraction $\Omega = \frac{p}{q}; b = 1$. Prop. 1 provides dynamical interpretation of bifurcation periodicity conditions, which are important in the analysis of non-linear systems where manifestations of chaos and turbulence might occur [25]. There is a significant effort being made to further determine the conceptual framework of bifurcation analysis for which the central problem is bifurcation control. Prop. 1 provides a possible simplification of bifurcation condition representations. Geometrical visualization of such conditions is an important factor in the analysis of nonlinear discrete dynamical systems [27].

The emergence of system periodic stability in the shape of an $n$-sided polygon could be observed not only in economic systems [20]; the $n$-sided polygon and the Farey tree organization of the equilibriums could be observed in technical systems [26, 5] as, for example, in laser control as the paradigm of the chaotic system [11]. Data analysis in the field of economics by the Poincaré first-return map, which exercises the periodic character, is presented in [1].

According to Table 1 gained by the z-transform, the classification of the periodic solution could be drawn, as shown in Fig. 1.
Figure 1. Periodicity of the incursive cobweb model in the form of an inverse Farey tree, where paths of Golden Ratio could be observed (greyed fractions)

The structure in Fig. 1 shows the inverse Farey tree which corresponds to the values of the periodic conditions gained by the z-transform. One of the questions that arose in the analysis of similar 2-D systems is the question concerning the rule and emergent organizational properties that determines periodicity [23]. In our case, the change of parameter \( d \) causes the system to switch between different periodic equilibriums. The ordering of the equilibriums is determined by the general Eq. (23). The rational fraction \( \frac{m}{n} \), which is in our case transformed by Eq. (23) to the value of parameter \( d \), corresponds to the Farey sequence, which could be represented by the inverse Farey tree in Fig. 1. The greyed fractions in the Farey tree point to the sequences of Fibonacci numbers \( \{F_i/F_{i+1}, i \in \mathbb{N}\} \) and \( \{F_i/F_{i+2}, i \in \mathbb{N}\} \). A periodic parameter space region of system response is determined by the condition \( det > 0 \) and periodicity by \( det < 0 \). The classification at \( d < 0 \) specifies the angles, which are determined by the three points in the 2-D map in our case, \( \alpha_n < \frac{\pi}{2} \); in the case where \( d > 0 \), the angles of the map are \( \alpha_n > \frac{\pi}{2} \). The strongest periodicity points are determined by the polygon structures in 2-D mapping starting with digon, triangle, etc. Other periodicity is the subset of the main sections, which is determined by the \( \sum \alpha_n \) and the Farey tree. The region of non-periodicity determines the response of the model that is equal to the conventional cobweb model.

One of the properties of the hyperincursive, nonlinear and chaotic systems, which are of special interest in the analysis of complex economic systems [2, 6, 17, 12], is that they have cycles of every length. It is important to be aware of the organizational structure of the periodic solutions provided by the emergent Farey tree incorporating the anticipative cobweb model and according to Prop. 1 also for the bifurcation conditions of nonlinear systems.

3. Equilibrium analysis
Equilibrium conditions of the studied systems corresponds to the periodic solutions and are therefore of primary importance for the description of system response. The equilibrium condition for the \( P \)
segment of the system provides an interesting response of the \(Q^s\) segment, which exhibits an hexagonal shape in the two dimensional Poincaré first-return map. Since hexagonal topology has a special meaning in the analysis of discrete dynamical systems [20], the equilibrium at \(n=3\) and \(n=6\) is examined. Other conditions are stated by the previously stated periodicity conditions, which are summarized in Table 1.

The equilibrium condition for the \(P\) segment of the anticipative cobweb system could be stated as:

\[
\frac{d}{b} \left( \frac{p-a}{b} - \frac{b(p-c)}{d} + a \right) = \frac{p-a}{b} = \frac{p-c}{d}
\]  

(26)

The equilibrium values of the parameters for the \(P\) segment of the system are: \(a = c = p\) and \(b = d \neq 0\) and, in this case \(P = 0\).

The equilibrium condition for the \(Q^s\) segment of the anticipative cobweb model could be stated as:

\[
\frac{d}{b} (p-a) - \frac{b(p-c)}{d} (a + \frac{b}{d} (p-c) - c)) = p = a + \frac{b}{d} (p-c)
\]  

(27)

The \(Q^s\) segment of the system has no solution. When the equilibrium conditions for the \(P\) segment of the system are considered in fact in all the cases, the \(Q^s\) segment of the system could not be in a stable state. A graphic presentation of the equilibrium conditions \(a = c = p\) and \(b = d\) is shown in Fig. 2. The hexagonal shape is known as the possible optimum shape in the context of spatial economics [20]. The existence of hexagonal shapes is known in space economy and explained by the structural stability [20].

![Figure 2: Response of the \(Q^s\) segment of the system while the \(P\) segment is in equilibrium](image)

**Proposition 2** The equilibrium condition for the \(P\) segment of the anticipative cobweb system defined by the equations from Eq. (5) to Eq. (10) is: \(a = c = p\) and \(b = d\). Under these conditions, the \(Q^s\) segment of the system has the response of a hexagonal shape with vertices

\[\{(a,0),(a,a),(0,a),(-a,0),(-a,-a),(0,-a)\}\]

in \(Q^s_{k}, Q^s_{k+1}\) mapping.

While the response of the system for the \(P\) segment is in equilibrium, the \(Q^s\) segment of the system has a hexagonal-like shape with a significant dimension of parameter \(a\) value and edge dimensions of \(a\) and \(a\sqrt{2}\).
Proposition 3  **Triangular (△), i.e. three-period response in 2-D mapping is determined by the condition** \( b = -d \).

In order to gain the term for the period in the \( P \) values, one should apply periodic condition. The values for times \( 1,\ldots,4 \) should be symbolically expressed. By inserting Eq. (5) and Eq. (6) into Eq. (9) the following term is gained:

\[
P_{k+2} = \frac{d}{b} \left( \frac{p - a}{b} - \frac{a - c + \frac{b(p - c)}{d}}{d} \right)
\]  

(28)

By repetition of a similar procedure for the equation for \( P_{k+3} \), considering the period 4 condition, i.e. \( P_{k+3} = P_k \), one should get the following equation:

\[
d \left( \frac{d}{b} \left( \frac{p - a}{b} - \frac{a - c + \frac{b(p - c)}{d}}{d} \right) - \frac{p - c}{d} \right) = 0
\]  

(29)

with solution \( b = -d \in \mathbb{R}/\{0\} \).

Proposition 4  **Hexagon (○), i.e. six-period response in 2-D mapping is determined by the condition** \( b = d \).

The values for times \( 1,\ldots,6 \) should be symbolically expressed. For example, determination of \( P_{k+2} \) is based on expressions for \( P_{k+1} \) and \( P_k \), determination of \( P_{k+3} \) is based on expressions for \( P_{k+2} \) and \( P_{k+1} \), etc. Periodic condition is expressed as \( P_{k+6} = P_k \) with solution \( b = d \in \mathbb{R}/\{0\} \).

4. **Agent-based anticipative cobweb model**

This section describes the interactions between several economic entities modelled as agents. Agent interaction represents the alternative control mechanism, which should provide standing oscillations and global equilibrium-seeking behaviour found in real world cases. The initial *feedback-anticipative* cobweb model oscillates only for a certain parameter set, which represents a thin borderline in parameter space. If the system parameters are not on this border, the model either decays or produces explosive behaviour [19]. There are several approaches to fixing the mentioned problem. One of the methods for solving this problem is application of the *floor-roof* principle, which should limit capital stock values as well as raw materials, etc. [19]. In a developed *agent-based* system we move from simple linear dynamics to incorporation of the nonlinear rules as the delayed and anticipated information concept. The anticipated response of the system is therefore complex in behaviour due to the coevolving nature of the system [7]. In the proposed agent-based formulation one should assume that each agent has an individual, local view of the problem posed, which is addressed by the *feedback-anticipative* principle.

Real economic systems are determined by their periodic response. Let us assume that there are many different economic systems in our environment which interact. Here the economic system will be represented as agent \( A \) interacting with other agents with the goal of reaching general systemic equilibrium by differentiating parameter \( d \), which determines the frequency response of the system. Here, the question arises: “Is it possible to control the economic system by changing the control parameter \( d \), which actually alters the frequency response of the system?” The deviation in prices could be understood as the change of the system frequency response. In other words, if we change the price, the frequency of the system alters. Therefore, one should try to control the system by changing its frequency response.
Consider the following agent-based anticipative cobweb model of price $P$ dynamics derived from equation Eq. (11):

$$P_k = \frac{b P_{k-1} + a - c}{d_k} + \frac{b}{d_k} P_{k+1}$$

(30)

Initial conditions for Eq. (30) should be stated in matrix form by Eqs. (5, 6). In the above equation matrix annotation represents column vectors, which have the same arbitrary dimension $n$ determined by the number of agents. For the computation of the new values of price $P$, shift operator $\rho$ on sequence $P$ is applied, which shift sequence $p \in P$ one step to the left:

$$\rho(\langle P \rangle) = \langle P_{n+1} \rangle$$

(31)

providing the forward shifted values for $P_{k-1}$ and $P_k$ in Eq. (30). The decision of change in parameter $d$ will be dependant on the sum of two price values at time $k + 1$ and time $k - 1$. Here, the relative value of the price by taking the range of system response in the denominator will be considered:

$$e = \frac{\xi_{k+1} + \xi_{k-1}}{|[\xi_k] - [\xi_k]|}$$

(32)

In Eq. (32) $\xi$ represents the estimation chain for $r$ time steps computed in a similar manner to $P$ in Eq. (30) except for the initial conditions, which are stated in matrix form as in Eqs. (5, 6) for time 0, while for $\xi(0)$ shift operator $\rho$ is applied forcing the anticipation principle as $\xi_k = f(P_{k+1})$. Besides the notation for absolute value in the denominator, the roof and floor operators are applied. In order to perform the control by variation of parameter $d$, where $n$ agents are present, the following state equation with the adaptive rule for $\Delta d_k$ is introduced:

$$d_{k+1} = d_k + \Delta d_k$$

(33)

where $\Delta d$ determines the change in control parameter $d$:

$$\Delta d_k = \begin{cases} 
\beta & \text{if } e = \lfloor e \rfloor \\
-\beta & \text{if } e = \lceil e \rceil 
\end{cases}$$

(34)

In the above definition of the agent’s rule, the floor and ceiling functions over a vector of relative prices $e$ considers only a finite number of lags. One should notice that the mentioned floor-roof operators are applied on vector $d$ rather than on vector $P$, which would mean the strict, conventional implementation of the floor-roof principle [19]. Parameter $\beta$ is the intensity of agents reaction to the market disequilibrium; $\beta \in (0,1)$. Initialization of vector $d$ is determined by random value $r_i \in [-2,2]$, which falls within the interval of periodic solutions for the anticipative cobweb system. Certainly, one could also assign an arbitrary value for $d$ as this will also be considered.

The idea captured in the above definition considers a situation where an economic system, of which the market price in the past and estimated future is the highest, should be controlled by increasing the value of control parameter $d$, thus changing the frequency response of the system. The case at the lower end of the market price is inverse.
5. A note on application
Possible application of the developed agent-based model is in the field of stock exchange analysis and control where several economic systems interact. Here it is our intention to show the manifestation of periodicity and control rule as described in previous sections on the real case. Ten top ranked stocks of the Ljubljana Stock Exchange [14] will be considered, namely: AELG, GRCG, HDOG, IEKG, ITBG, KRKG, LKPG, MELR, PETG and SAVA (Fig. 3).

![Figure 3: Normalized stock values (upper) and average probability of floor-ceiling rule (lower)](image)

The period of the observation will be from 6.1.1997 to 5.11.2009 (3213 trading days) incorporating the critical fall in values from 2008 onward; several stocks came in to market after the initial date i.e. the last stock observed entered the market on 6.1.2000 all of which present 30,059 data points. The normalized values of mentioned stocks are shown in the upper part of the Fig. 3; here equity rates are considered. The proposed floor-ceiling rule stated by Eqs. (32-34) provides a meaningful interaction between several economic systems which occurrence in Slovenian top ten stocks is shown by Average probability in the bottom of Fig. 3; here the moving average of one year was considered. It is surprising, that the rule stated by Eqs. (32-34) for agent based model occurs with $p = 0.46$; $\sigma = 0.06$. The values are shown from 2001 onward since several stocks were not yet listed on the stock exchange market. Regarding the periodicity and the model stated by Eq. (11) one could ask, why is the determination of price at time $t$ dependant on price at $t-1$ and $t+1$? In the stock exchange market this would be the case, when the price on the current day ($t$) could not be determined. The current price is not known before the closing on the end of the same day. Therefore for time $t$, only estimated price could exist, which is made on previous price ($t-1$) and expected price ($t+1$). There are methods to enhance the estimation of current price to extend the number of time-lags considered [8] which is the topic of rational expectations and intertemporal equilibrium. As the example regarding the periodicity of agents, the manifestation of period six in Slovenian stock...
The exchange market is shown in Fig. 4; here the moving average of one year is taken. On y-axis the number of period six occurrences are shown on average in one year for ten considered stocks, with $\bar{x} = 34; \sigma = 11$. That would mean, that the period six occurs on average every two weeks in one of the ten stock values. Period six is shown here as one possible manifestation of equilibrium as shown by Prop. 2. From Fig. 4 one could observe, that the Number of period six occurrences is falling from the year 2004 onward which might be one of the indicators of ongoing instability in the stock exchange market.

The importance of the question, how to control several interacting agents is apparent in the case of the global crisis which is indicated in 2008. It is a matter of further research to investigate the control strategies on the real cases which would provide global equilibrium or at least compensate for the global disturbances.

6. Discussion
Understanding the emergence of periodicity is a prerequisite for economic system control. The basic theory developed in this field considers the analysis of eigen vectors and eigen values [24]. The equality of the gained solutions determined by Prop. 1 for the anticipative cobweb model and general periodic solution for discrete nonlinear systems [24] provides a view of the Farey tree structure, which is [24] universal in nonlinear expansions of the considered systems.

Several anticipative cobweb systems were interconnected in order to form an agent-based model where interaction was determined by the discrete rule. By developing an agent-based model, the methodological platform is provided for a further examination of the interaction of several feedback-anticipative systems. The developed system provides promising results obeying the rules of interaction and self-synchronization found in real world economics systems. Another important fact is that agents with more interaction provide better conditions for general system stability. A detailed analysis of the agent-based model provides proof of system stability, which is one of the key conditions that should be meet by agent-based models simulating complex economic systems. The experimental examination of the two agent model indicates, that the entire model could be set in the global equilibrium mode. All the stated characteristics of the agent-based model as well as the response of the system for eight agents provides a promising methodological platform for studying the interaction between several economic entities such as agents. The proposed model provides the means for analyzing interaction, feedback, anticipation, frequency response, synchronization, standing oscillations and system equilibrium. An introduction of feedback-anticipative systems interconnection and control by varying the parameter, which influences system frequency response, represents a new perspective for the
analysis of complex systems. The key values in the Farey tree hold for the agent-based model and have provided a possible starting-point for the search of analytical solutions in the parameter space.

The present study provides interconnection of Kaldor’s cobweb model paradigm with an anticipative approach and agent-based paradigm. The common denominator as a thread of the paper is the evolutionary emergence principle hidden in the form of the Farey tree incorporating the Golden Ratio path.

The developed system confirms the applicability of the control rule where system response is controlled by the variation of parameter $d$, which influences the frequency response of a particular agent.

By examining the developed agent-based system, it could be concluded that interaction of discrete oscillatory agents by itself results in stable working conditions. In the present case the rule of interaction provided system stability and a synchronization property which is found in real world systems [22] and significantly determines interacting economic systems.

Acknowledgment
This research was supported by the Slovenian Research Agency (Programme No. P5-0018 & Proj. No. RU/16-18-040).

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