Relationships between Particle Size and Fracture Energy for Single Particle Crushing

Yoshiteru Kanda
Department of Chemical Engineering,
Yamagata University*

Shigeru Sano
Ichinoseki Technical College**

Fumio Saito
Department of Chemical Engineering,
Yokohama National University***

Saburo Yashima
Research Institute of Mineral Dressing and Metallurgy,
-Tohoku University****

Abstract

An experimental study of single-particle crushing at slow compression rate was carried out for two kinds of glassy and five kinds of natural materials. The specimens were almost spherical particles of 0.5 to 3.0 cm in diameter.

The relationships between particle size and fracture energy (strain energy) were calculated by using the results of the size effect ranging from about 10 cm to 0.0030 cm of single particle crushing, as shown in the previous papers. The experimental results are summarized as follows:

1) The fracture energies obtained were larger than the values calculated from the theoretical equations for limestone, marble and gypsum.

2) For natural materials, the specific fracture energies rapidly increased with decreasing particle size within the range of particle size smaller than about 500 μm.

Introduction

The importance of fracture characteristics in determining optimum design or operating condition in comminution process has been extensively recognized in various industries. The energy required for grinding materials or the grinding resistance is generally expressed as a function of particle size through size reduction. As well accepted, however, prolonged comminution has the tendency to show the finite limit of size reduction peculiar to the equipment used. This is not only due to the absorption of the energy into powder bed and the reduction of energy distribution for each fractured fragment, but also due to the increase in fracture strength resulted from size reduction, that is, the effect of size on strength.

In the present paper, the relationship between compressive strength of spherical specimens of 0.5 to 3 cm in nominal diameter and fracture energy was experimentally investigated. Furthermore, the energy required for single particle crushing and the specific fracture energy was calculated by use of the experimentally obtained relationship between compressive strength of spheres with an approximate diameter range of 30 μm to 10 cm presented in the literatures. The result produced was that specific fracture energy in-

† This report was originally printed in Kogaku Kogaku Ronbun-shu, 10, 108-112 (1984) in Japanese, before being translated into English with the permission of the editorial committee of the Soc. Chemical Engineers, Japan.
increased with increasing particle size, showing a remarkable rise especially within the finer range of the particle size.

The experimental or calculated results obtained in this work were based on the condition where spherical or near-spherical specimens were fractured under static compressive load\(^9\). However, the actual fracture is accompanied by the phenomenon that irregularly shaped particles are fractured under dynamic or impact load. Therefore, the results of this paper will be available in practice as far as the difference between the ideal and actual phenomenon is taken into consideration to reasonable extent.

1. Sample and experimental method

The specimens used and their mechanical properties are listed in Table 1. The relationship between compressive strength of spheres and fracture energy is studied by experiment and calculation by use of the nominal diameter and number of specimens\(^2\) prepared as indicated in Table 2. Fracture experiments were carried out with Simadzu universal testing machine, Model REH-30, to obtain load-displacement variation which was presented on an X-Y recorder through differential transducer. The manufacturing procedure and its accuracy of the specimens, and the experimental method were fully described in the literatures\(^3,9\).

2. Calculation of fracture energy

When the sphere of \(x \) in diameter is compressed between parallel plates as shown in Fig. 1, the elastic theory provides the displacement \(\Delta \) as a function of the load \(P\) as follows:

\[
\Delta = 2 \left( \frac{9}{16} \cdot \frac{1}{x} \cdot \left( \frac{1 - \nu^2}{Y} \right) \cdot P^2 \right)^{\frac{1}{3}}
\]

where \(\nu\) is Poisson's ratio and \(Y\) is Young's modulus. For an overall or net energy for fracture, various definitions have been proposed\(^4,5\), though the final concept has been never obtained yet. In this paper, the term energy required for fracture is defined as elastic strain energy accumulated in a particle until the fracture will take place. That is:

\[
E = \int Pd(\Delta) = 0.832 \left( \frac{1 - \nu^2}{Y} \right) x \cdot \frac{1}{3} P^{\frac{5}{3}}
\]

Accordingly, the fracture energy per unit mass \(E/M\) is given by

\[
\frac{E}{M} = 4.99 \frac{1}{\pi \rho} \left( \frac{1 - \nu^2}{Y} \right) (\frac{P}{x^2})^{\frac{5}{3}}
\]

Table 1 Properties of samples

| Samples          | \(\rho\) [kg/m\(^3\)] | \(Y\) [Pa] | \(\nu\) [-] | \(W_t\) [kWh/t] |
|------------------|------------------------|------------|------------|-----------------|
| Quartz glass     | 2.20 \times 10^3       | 7.35 \times 10^{10} | 0.16       | 14.8            |
| Borosilicate     |                        |            |            |                 |
| glass            | 2.33 \times 10^3       | 6.12 \times 10^{10} | 0.21       | 15.2            |
| Quartz           | 2.62 \times 10^3       | 8.71 \times 10^{10} | 0.16       | 13.3            |
| Feldspar         | 2.55 \times 10^3       | 5.87 \times 10^{10} | 0.26       | 12.4            |
| Limestone        | 2.70 \times 10^3       | 6.80 \times 10^{10} | 0.32       | 9.4             |
| Marble           | 2.70 \times 10^3       | 5.34 \times 10^{10} | 0.30       | 6.7             |
| Gypsum           | 2.30 \times 10^3       | 3.78 \times 10^{10} | 0.32       | 6.3             |

Table 2 Nominal diameter and number of specimens prepared

| Samples          | Nominal diameter [cm] |
|------------------|-----------------------|
|                  | 0.5       | 1.0       | 1.5       | 2.0       | 2.5       | 3.0       |
| Quartz glass     | 20        | 20        | 20        | 20        | 20        |           |
| Borosilicate glass| 20        | 20        | 20        | 20        | 20        |           |
| Quartz           | 20        | 20        | 20        | 20        | 20        |           |
| Feldspar         | 20        | 20        | 20        | 20        | 20        |           |
| Limestone        | 20        | 20        | 20        | 20        | 20        |           |
| Marble           | 20        | 20        | 20        | 20        | 20        |           |
| Gypsum           | 20        | 20        | 20        | 20        | 20        |           |

* Yamaguchi\(^8\) reported that 20 specimens must be required for one point to obtain the confidence limit up to 95% by use of granite. Our study is for the time being based on his results which might depend on the material kind.

Fig. 1 Crushing of sphere
where \( \rho \) is the particle density.

The compressive strength of spheres \(^2\) was presented by Hiramatsu et al.\(^2\) as

\[
S = \frac{2.8P}{\pi x^2}
\]  

(4)

Substituting Eq. (4) into Eq. (3), \( E/M \) is determined from the compressive strength as follows:

\[
\frac{E}{M} = 0.897 \rho^{-1} \pi^{\frac{2}{3}} \left( \frac{1 - \nu^2}{Y} \right)^{\frac{2}{3}} \nu^{\frac{5}{3}} \]

(5)

The strength of material, which is well known to be sensitive to its structure\(^4\) and thus dependent upon its size, is generally expressed as\(^1,7\)

\[
S = (S_0 V_0^{\frac{1}{m}}) V^{-\frac{1}{m}}
\]

(6)

where \( S_0 \) is the strength of a specimen having a unit volume \( V_0 \) and \( m \) is Weibull's coefficient of uniformity with the value greater than 1. As shown in Eq. (6), the effect of specimen volume on strength decreases with increasing \( m \), and the material becomes perfectly uniform in the limiting case as \( m = \infty \).

By use of Eqs. (5) and (6), the fracture energy and the specific fracture energy at size \( x \) are rewritten respectively as follows:

\[
E = 0.15 \cdot 63^{\frac{5m}{3}} \cdot \pi^{\frac{2m-5}{3m}} \cdot \left( \frac{1 - \nu^2}{Y} \right)^{\frac{2}{3}} \cdot (S_0 V_0^{\frac{1}{m}})^{\frac{1}{3}} \cdot x^{\frac{5}{m}}
\]

(7)

\[
\frac{E}{M} = 0.897 \cdot 63^{\frac{5m}{3}} \cdot \rho^{-1} \pi^{\frac{2m-5}{3m}} \cdot \left( \frac{1 - \nu^2}{Y} \right)^{\frac{2}{3}} \cdot (S_0 V_0^{\frac{1}{m}})^{\frac{1}{3}} \cdot \nu^{\frac{5}{m}}
\]

(8)

The ratio \( E/M \) increases over the finer region of \( x \) with a decrease in the coefficient \( m \) or the lack of uniformity of the material in question.

3. Experimental results and discussion

3.1 Experimental results of compressive strength of spheres and fracture energy

The typical values of \( E/M \) of borosilicate glass, quartz, feldspar, and marble plotted against the strength \( S \) are shown in Figs. 2 and 3 by using all the specimens of Table 2. The straight lines in these diagrams represent calculated solutions derived from the values \( \rho, Y, \) and \( \nu \) in Table 1\(^9\) which were experimentally obtained.

It is clear from Figs. 2 and 3 that the agreement between the experiment and the calculation seems to be satisfactory and thus these materials can be considered as semi-elastic solids. The similar tendency was also given for quartz glass. For marble, on the other hand, its experimental values are greater than the calculated line as indicated in Fig. 3. This reason was stated in the previous paper\(^12\) as follows; the authors numerically integrated the load-displacement curve to obtain the whole fracture energy and determined the ratio of the energy required for plastic deformation to the whole energy by use of extrapolation. The calculated ratio in the case of marble was about 0.63, and this implies that the fracture energy measured would consist of the large amount of plastic deformation. From this reason, the experimental values might be greater than the calculated solutions.

For limestone and gypsum, the ratios were 0.55 and 0.78 respectively and both of their experimental results gave greater values than

---

\(*\) The value \( S \) can be calculated under compressive load which is applied on points even if the shape of particles are irregular.

\(*\ast\) This does not imply ideal material without flaw alone, but uniform one which has the various kinds of flaw.
Summarizing the above discussion is that when the material has plastic characteristics, the fracture energy obtained experimentally is greater than the calculated solution. Therefore Eq. (5) will be available for estimating the fracture energy, only when the portion of plastic deformation energy and elastic strain energy can be determined in advance as stated in the previous paper12).

3.2 Relationship between particle size and fracture energy

By use of Eqs. (7) and (8), Weibull's coefficient of uniformity \( m \) yields the fracture energy of a single particle \( E \) and the specific fracture energy \( E/M \). A typical relationship between the specimen volume and the strength presented in the previous paper13) is indicated in Fig. 4, which shows a set of line segments obtained by the least squares method. Such a relation was found to be expressed as a single straight line with a constant slope for glass materials and as a set of line segments with plural slopes for natural materials12. In any case, however, the results imply that the strength will increase with decreasing particle volume or particle size.

The calculated solutions of \( E \) and \( E/M \) by use of Eqs. (7) and (8) and the values of \( m \) in the previous paper13) are plotted against the particle size \( x \) in Figs. 5, 6 and 7. The values of \( E \) and \( E/M \) are expressed as a straight

Fig. 3 Relationships between strength \( S \) and specific fracture energy \( E/M \)

It seems that the relationship between volume and strength for glass material shows the similar tendency when the same manufacturing procedure as that of natural materials are employed. The detail of this is under examination.
Fig. 6 Relationships between $x$ and $E/M$ or $E$

line for glass material as shown in Fig. 5. On the other hand, they are revealed as a set of line fragments with different slopes for the natural materials as indicated in Figs. 6 and 7. Such a discontinuous distribution is considered to result from the following procedure; first, the relationship between specimen volume and strength was determined in view of a representative straight line which indicates the most acceptable points in the fluctuated values obtained in this work. Then the line segments which were to result in the minimum probable error over each size range in question were determined. From this reason, the end points of each line segment were separated although the particle size distribution was continuous. In practice, however, the correlated line may pass continuously near these end points.

It is clear from Figs. 6 and 7 that the ratio $E/M$ of natural materials remarkably increase with a particle size less than 500 $\mu$m, and this implies the large amount of energy required for producing fine particles.

The results obtained in this work are based on the ideal fracture experiment under static, compressive load. It should be noticed that a practical fine comminution is accompanied by a fracture of irregular shaped particles under dynamic or impact load and the values $\nu$, $Y$ and $S$ in Eqs. (6), (7) and (8) are dependent upon the loading speed. It should be also remarked that these results will be available for practical use only if the condition used is reasonably restricted.

Conclusion

The relationship between compressive strength of spheres and fracture energy was experimentally investigated under static compressive load by using 7 kinds of specimens of 0.5 to 3 cm in diameter. Furthermore, the values of fracture energy per a single particle and unit mass of the particles with a diameter range of 0.0030 to 10 cm were calculated by
use of the relationship between particle volume (particle diameter) and strength which had been previously reported. Summarizing the results of this study:

1) The fracture energies were larger than the values calculated from the theoretical equations for plastic-like materials such as limestone, marble and gypsum, while these values were in reasonable agreement for quartz glass, borosilicate glass, quartz, and feldspar.

2) For natural materials, the specific fracture energies rapidly increased with decreasing particle size within the range of particle size smaller than about $500 \mu m$, and this implies the large amount of energy required for producing fine particles.

Although the results obtained in this work are based on the ideal fracture behavior and seem to differ from the actual one, the resultant tendency may be reasonably available for practical use.

**Nomenclature**

- $E$: fracture energy [J]
- $E/M$: specific fracture energy [J/kg]
- $m$: Weibull's coefficient of uniformity [-]
- $P$: load [N]
- $S$: compressive strength of sphere [Pa]
- $S_0$: compressive strength of sphere per unit volume $V_0$ [Pa]
- $V$: volume of specimen [$cm^3$] or [$m^3$]
- $W_f$: Bond’s work index [kWh/t]
- $x$: particle size [cm] or [m]
- $Y$: Young's modulus [Pa]
- $\Delta$: deformation of specimen [m]
- $\nu$: Poisson’s ratio [-]
- $\rho$: density of specimen [kg/m$^3$]

**References**

1) Epstein, B.: *J. Appl. Phys.*, 19, 140 (1948).
2) Hiramatsu, Y., T. Oka and H. Kiyama: *J. Min. Inst., Japan*, 81, 1024 (1965).
3) Kanda, Y., S. Yashima and J. Shimoizaka: *J. Min. Inst., Japan*, 85, 1024 (1969).
4) Nakagawa, Y., K. Matsui and S. Okuda: *Kagaku Kōgaku*, 18, 146 (1954).
5) Nishimatsu, Y., S. Ōkubo, T. Yamaguchi and S. Koizumi: *J. Min. Inst., Japan*, 97, 1163 (1981).
6) Tanaka, T.: *Kagaku Kōgaku*, 18, 180 (1954).
7) Weibull, W.: *Ing. Vetenskaps Akad. Handl.*, No.151 (1939).
8) Yamaguchi, U.: *J. Soc. Materials Sci., Japan*, 14, 198 (1965).
9) Yashima, S., S. Morohashi, O. Awano and Y. Kanda: *Kagaku Kōgaku*, 34, 210 (1970).
10) Yashima, S., Y. Kanda, T. Izumi and T. Shinozaki: *ibid.*, 36, 1017 (1972).
11) Yashima, S., Y. Kanda, T. Sasaki, M. Iijima and F. Saito: *ibid.*, 27, 1218 (1973).
12) Yashima, S., F. Saito, T. Sagawa, T. Numata, S. Sano and Y. Kuwahara: *J. Min. Inst., Japan*, 91, 535 (1975).
13) Yashima, S. and F. Saito: *J. Res. Assoc. Powder Tec., Japan*, 16, 714 (1979).
14) Yokobori, T.: “*Zairyo Kyōdōgaku*”, Gihōdō, Tokyo, 86 (1955).