Distortion Characteristics and Experimental Research on Transformer Windings under Harmonic Condition

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Abstract. Firstly, the limitations of the existing transformer harmonic models both at home and abroad are analyzed. Based on this, the influence of the skin effect and the proximity effect on the windings under harmonics conditions is considered. Based on the electromagnetic field principle, the paper analyses the distortion characteristics of winding resistance parameters, introduces the AC power group coefficient and establishes of winding harmonic loss model. Compared with the traditional engineering calculation method, we can see that the theoretical model proposed in this paper can improve the accuracy and make the transformer winding loss calculation more accurate, Therefore, the model has certain guiding value for engineering calculation.

1. Introduction

The operation of power transformers is not only affected by the harmonics of the power grid, but also is one of the most important harmonic sources in the power system due to the nonlinearity of the iron core. Harmonic current flowing through the transformer will make the transformer temperature increases and the total loss increases, which not only will cause damage to the transformer insulation, but also lead to transformer load capacity decline, thus it affects its safety and stable operation. Due to the effective use of transformers for a long time, even very small losses, the annual energy consumption is also huge. The total transformer losses account for about 8% of the total generating capacity, and most of them are transformer harmonic losses. According to a Japanese company reports that when the transformer harmonic current reaches 10% of rated current, the transformer loss than the rated loss of 10%.

Research of China's harmonic loss model of the transformer is less, the main use of conventional IEEE transformer model[1-3]. These models do not deal with the resistance parameters of the transformer windings in the harmonic environment or do not handle well enough and are often calculated by multiplying the DC resistance instead of the AC resistance. This simplification has little effect on the circuit calculation with fundamental only, but under harmonic conditions the equivalent resistance of the transformer windings at different frequencies is much larger than the DC resistance due to skin and proximity effects, making the calculation model inaccurate. In addition, useing improved parameter method about the transformer harmonic loss calculation is easy to implement and more widely, the calculation is relatively simple, but can not well reflect the phase angle between the voltage and current Change, in practice, it can be used as a quick estimate method[4-8].

Based on the Maxwell's equations and the Poynting's theory, this paper considers winding.

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parameter distortion under harmonic current about skin effect and proximity effect, introduces the AC winding coefficient, and establishes the winding loss calculation model to make the harmonic loss more accurate calculate. In addition the paper sets up the experiment platform, carry out the transformer winding resistance measurement experiment and observe the variation of harmonic resistance with frequency, at last, compared with the calculation model and the DC resistance used to study the equivalent AC resistance model , We can see that the calculation model of harmonic loss in this paper is more accurate.

2. Analysis of Transformer Winding Harmonic Loss
Transformer windings through the exchange of current, if you need to accurately calculate the transformer losses according to the winding layer of wire as a whole analysis considering skin effect and proximity effects. Set the transformer winding radius of r, the axial direction of z, the current direction of flow $\phi$, the establishment of coordinate system. as shown in figure 1. Such as $i_k$, $i_\phi$, $i_r$, $k$, $\phi$, r direction of the current, $H_-$ and $H_+$ are the magnetic field strength which parallel to the transformer winding inner and outer surface. Assuming that the current flowing through the circular conductor inside the winding satisfies the relationship of the electric field intensity E ($\phi$ axis direction) and the magnetic field strength H of the winding layer in the cylindrical coordinates[9-11]

$$\frac{\partial H_+}{\partial r} = \sigma E_\phi$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) = -j \omega \mu H_\phi$$

Where: $\sigma$ is the conductivity of copper conductor; $\mu$ is the permeability of copper conductor; $\omega$ is the angular frequency.

Since the magnetic field strength and electric field strength are all functions of the windings axis distance $r$ at any point on the windings, the solution Bessel function equation is obtained:

$$H(r) = c_1 J_0 (mr) + c_2 K_0 (mr)$$

$$E(r) = -m / \gamma \left[ c_1 J_1 (mr) - c_2 K_1 (mr) \right]$$

Where $J_0$ and $J_1$ are Bessel functions of the first kind, $K_0$ and $K_1$ are Bessel functions of the second kind, and the coefficients $c_1$ and $c_2$ are determined by the boundary conditions.

Figure 1. Hollow cylindrical conductor

Figure 1 is the coordinate system of hollow cylindrical conductor. For $a$ layer of winding. Since the electric field strength E and the magnetic field strength H are orthogonal, the energy flux per unit area passing through a unit time is represented by the Poynting vector as $E (r) \times H (r)$. From the energy loss point of view, Layer winding unit length of energy:

The total dissipative power of the $n_{th}$ winding is
\[ P_c = P_a + P_{ac} = \frac{N^2 T m}{\sigma w} \left[ \frac{(2n^2 - 2n + 1) \coth(md_n)}{n^2 - n} \left( \sqrt{1 + d_n/r_n^+} + \sqrt{1 - d_n/r_n^-} \right) \right] \]  

(5)

Where \( W \) is the width of the winding; \( N \) is the number of turns of the coil contained in each winding; \( d_n \) is the thickness of the \( n \)th winding; and \( r_n \) is the distance between the \( n \)th winding and the winding axis. The use of Taylor expansion of equation (5) to simplify, available \( n \)-layer winding loss:

Assuming that the thickness of each layer of the winding is \( d \), the loss of the \( n \)-th layer winding can be obtained by using the Taylor expansion of equation (5).

\[ P_n = \text{Re} \left\{ \frac{N^2 T m}{\sigma w} \left[ \frac{\coth(md)}{2(n^2 - n) \tanh(md/2)} \right] \right\} \]

(6)

Assuming that the direct current in the windings is \( I \), the DC power loss per unit length of the \( n \)-th winding is:

\[ F_{hr} = \frac{R_{ac}}{R_{dc}} \]

(7)

\[ F_{hr} = \text{Re} \left\{ K_h (1+j) \left[ \frac{\coth(K_h(1+j))}{2(n^2 - 1) \tanh(K_h/2)} + \frac{9}{45} K_h^4 \right] \right\} \]

(8)

Where: \( K_h \) is the ratio of the thickness \( d \) of the winding layer to the skin depth \( \delta \) of the \( h \)th harmonic

\[ K_h = \frac{d}{\delta_h} \]

and \( N \) is the number of winding layers.

The formula (8) series expansion to simplify, get the first \( h \) harmonic AC resistivity:

\[ F_{hr} = 1 + \frac{5n^2 - 1}{45} K_h^4 \]

(9)

The total harmonic loss caused by the transformer winding is:

\[ P_{cu} = I_{ac}^2 R_{ac} + R_{ac} \sum_{h=1}^{\infty} F_{hr} I_h^2 \]

(10)

3. Experimental Research on Transformer Wingding Harmonic Resistance

In order to effectively carry out the transformer harmonic loss test research. This paper is based on the laboratory of power quality comprehensive experiment research platform. The platform grid simulator can simulate the output voltage harmonics. The tested transformer used in this experiment is SG-150/0.38 dry isolation transformer. Experiment platform is shown in Figure 2.

![Experiment platform](image-url)
Carrying on short circuit test, the transformer side of the grid simulator, issued by the network simulator each harmonic voltage, the secondary short, respectively, measuring the number of different harmonics, different harmonic voltage transformer copper loss. In order to observe the change of the harmonic resistance of the transformer with the change of the frequency of the harmonic current, the harmonic voltages of 1, 3, 5, 7... 49 times are respectively applied to the transformer to measure the RMS current flowing through the transformer and the active current flowing through the primary side of the transformer Power, calculate the transformer active power loss and then calculate the harmonic resistance value. It is easy to carry out experiments and as far as possible to avoid errors. In this experiment, take the transformer through the ends of the active power in addition to the square of the current form harmonic transformer winding resistance calculation. In this paper, the measurements are measured at room temperature 20 ℃, the experimental results are shown in Table 1.

| Frequency | Voltage RMS/V | Current RMS/A | power/W | Measuring resistance/Ω |
|-----------|--------------|---------------|---------|------------------------|
| 1         | 7.380        | 4.892         | 32.984  | 1.379                  |
| 3         | 10.35        | 4.521         | 30.56   | 1.489                  |
| 5         | 12.266       | 4.410         | 28.756  | 1.479                  |
| 7         | 17.3         | 4.741         | 40.404  | 1.794                  |
| 9         | 15.67        | 3.86          | 33.44   | 2.3115                 |
| 11        | 14.014       | 3.944         | 42.61   | 2.7393                 |
| 13        | 9.150        | 4.305         | 62.021  | 3.3465                 |
| 15        | 13.46        | 4.25          | 72.7    | 4.025                  |
| 17        | 11.632       | 4.974         | 124.05  | 5.014                  |
| 19        | 13.270       | 5.406         | 169.01  | 5.796                  |
| 21        | 13.262       | 4.352         | 131.65  | 6.958                  |
| 23        | 14.50        | 4.560         | 169.78  | 8.165                  |
| 25        | 15.052       | 4.962         | 232.18  | 9.43                   |
| 27        | 14.79        | 4.05          | 173.54  | 10.58                  |
| 29        | 14.712       | 4.044         | 189.52  | 11.845                 |
| 31        | 17.652       | 4.713         | 314.2   | 14.145                 |
| 33        | 20.345       | 4.832         | 351.741 | 15.065                 |
| 35        | 23.044       | 4.935         | 408.9074| 16.79                  |
| 37        | 18.780       | 4.166         | 323.334 | 18.63                  |
| 39        | 18.5         | 4.0           | 314.824 | 19.6765                |
| 41        | 16.938       | 3.375         | 233.8211| 20.5275                |
| 43        | 22.878       | 4.183         | 398.4178| 22.77                  |
| 45        | 23.502       | 3.994         | 391.112 | 24.518                 |
| 47        | 24.051       | 3.904         | 410.3164| 26.9215                |
| 49        | 24.444       | 3.886         | 443.184 | 29.348                 |

**Figure 3.** The relationship between harmonic equivalent resistance and frequency

Figure 3 shows the relationship between the equivalent harmonic resistance of the winding and the number of harmonics. It can be seen that the equivalent resistance of the winding increases obviously
with the increase of the number of harmonics, showing a positive correlation, indicating that the resistance is affected by the skin Effects and proximity effects and gradually increase.

Table 2 is a partial treatment of the experimental measurements to calculate the AC group coefficient that is the ratio of the value of each harmonic resistance and DC resistance, and use the formula (9) to calculate the harmonic coefficient of each AC group, and compared.

Table 2. Comparison of Experiment and Theoretical Calculation of AC Power Factor

| Frequency | Measuring resistance/Ω | Experiment AC resistance coefficient | Skin depth/mm | Theory AC resistance coefficient | $\sqrt{n}$ |
|-----------|-------------------------|--------------------------------------|---------------|----------------------------------|------------|
| 1         | 1.379                   | 1.02                                 | 14.2          | 1.011                            | 1          |
| 3         | 1.489                   | 1.22                                 | 8.2           | 1.2                              | 1.73205    |
| 5         | 1.479                   | 1.286                                | 6.35          | 1.276                            | 2.2361     |
| 7         | 1.794                   | 1.56                                 | 5.37          | 1.54                             | 2.6457     |
| 9         | 2.3115                  | 2.01                                 | 4.73          | 1.9                              | 3          |
| 11        | 2.7393                  | 2.382                                | 4.28          | 2.34                             | 3.3166     |
| 13        | 3.3465                  | 2.91                                 | 3.94          | 2.865                            | 3.6056     |
| 15        | 4.025                   | 3.5                                  | 3.67          | 3.4774                           | 3.873      |
| 17        | 5.014                   | 4.36                                 | 3.444         | 4.2                              | 4.123      |
| 19        | 5.796                   | 5.04                                 | 3.26          | 4.98                             | 4.359      |
| 21        | 6.958                   | 6.05                                 | 3.1           | 5.8664                           | 4.582576   |
| 23        | 8.165                   | 7.1                                  | 2.96          | 6.8545                           | 4.795832   |
| 25        | 9.43                    | 8.2                                  | 2.84          | 7.91                             | 5          |
| 27        | 10.58                   | 9.2                                  | 2.733         | 9.056                            | 5.196152   |
| 29        | 11.845                  | 10.3                                 | 2.64          | 10.252                           | 5.385165   |
| 31        | 14.145                  | 12.3                                 | 2.55          | 11.63                            | 5.567764   |
| 33        | 15.065                  | 13.1                                 | 2.472         | 13.035                           | 5.744563   |
| 35        | 16.79                   | 14.6                                 | 2.4           | 14.546                           | 5.91608    |
| 37        | 18.63                   | 16.2                                 | 2.334         | 16.144                           | 6.082763   |
| 39        | 19.6765                 | 17.11                                | 2.3           | 17.06                            | 6.244998   |
| 41        | 20.5275                 | 17.85                                | 2.274         | 17.807                           | 6.403124   |
| 43        | 22.77                   | 19.80                                | 2.22          | 19.503                           | 6.557439   |
| 45        | 24.518                  | 21.32                                | 2.17          | 21.268                           | 6.708204   |
| 47        | 26.9215                 | 23.41                                | 2.12          | 23.25                            | 6.855655   |
| 49        | 29.348                  | 25.52                                | 2.07          | 25.478                           | 7          |

Figure 4. Comparison between the calculated value of the theoretical formula and the experimental measurement value.

Figure 5. The calculated value of the theoretical formula is compared with the $\sqrt{n}$ times DC resistance value.
Figure 4 compares the AC resistivity measured in the experiment with the AC resistivity in the theoretical formula proposed in this paper. As the number of harmonic currents flowing through the transformer windings increases, the harmonic resistance increases more significantly. The figure shows that although the theoretical formulas in this paper are underperforming at low frequencies, the overall trend is more in line with the facts. And at high frequencies, i.e., the skin effects and proximity effects become more pronounced, the theoretical calculations proposed in this paper are more accurate.

Figure 5 is calculated by the theoretical formula of the AC power coefficient and the previous study of the DC resistance to the equivalent AC resistance comparison can be seen from the figure, the harmonic less than 20 times, the transformer winding resistance by The effect of skin effect and adjacent effect is not obvious, and the change of resistance value is insignificant, and the difference between the two methods is insignificant. However, when the harmonic frequency is higher than 20 times, as the harmonic frequency increases, the skin is affected by skin effect and The effect of the proximity effect is more pronounced, and the harmonic resistance increases significantly. It can be seen from the figure that the equivalent resistivity of the theoretical calculation model is more accurate than the equivalent DC resistance is more reasonable.

4. Conclusion
Based on the Maxwell's equations and the Poynting's theorem, this paper considers the winding parameter distortion characteristics under harmonic currents and the skin effect and the proximity effect, introduces the AC winding coefficient and the consumption and builds the winding loss calculation model, Calculate. And set up the experimental platform, the harmonic current in the background of the transformer winding resistance measurement experiment, observation of harmonic resistance with frequency variation, compared with the calculation model and the model has been studied, we can see the harmonic loss calculation model more accurate This model has certain guiding value for engineering calculation.

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