The formation and evolution of protostellar discs; three-dimensional adaptive mesh refinement hydrosimulations of collapsing, rotating Bonnor–Ebert spheres

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ABSTRACT
We present a detailed study of the collapse of molecular cloud cores using high-resolution three-dimensional adaptive mesh refinement (AMR) numerical simulations. In this first in a series of investigations our initial conditions consist of a spherical molecular core obeying the hydrostatic Bonnor–Ebert profile with varying degrees of initial rotation. Our simulations cover both the formation of massive discs, in which massive stars form, and low-mass discs. We use a customized version of the FLASH code the AMR technique, which allows us to follow the formation of a protostellar disc and protostellar core(s) through more than 10 orders in density increase, while continuously resolving the local Jeans length (i.e. obeying the Truelove criterion). Our numerical simulations also incorporate the energy loss due to molecular line emission in order to obtain a more realistic picture of the protostellar core and disc formation.

Our initial states model systems of mass 168 and $2.1\,M_\odot$ that will form high- and low-mass stars, respectively. We follow many features such as the development complex shock structures, and the fragmentation of the disc. We find that slowly rotating cores ($\Omega_1 t_{\text{ff}} = 0.1$) produce discs in which a strong bar develops but does not fragment. Faster initial rotation rates ($\Omega_1 t_{\text{ff}} = 0.2$) result in the formation of a ring, which may fragment into two protostellar cores. The size of the rings found in our simulated discs agree with the observations of similar systems.

Key words: accretion, accretion discs – hydrodynamics – methods: numerical – ISM: clouds – ISM: evolution.

1 INTRODUCTION
The formation of protostars and their surrounding discs from collapsing, dense, molecular cores within molecular clouds is still an actively debated topic. Early pioneering works by Bodenheimer & Sweigart (1968), Larson (1969) and Penston (1969) addressed this problem using numerical simulations for various initial conditions. Bodenheimer & Sweigart studied collapses of isothermal spherical symmetric gas clouds that are initially out of equilibrium. They pointed out that the solutions depend only very weakly on the choice of the initial density profile (homogeneous or Chandrasekhar-type). Larson (1969) and Penston (1969) included the effect of energy dissipation by radiation in their one-dimensional (1D) simulations and found that cooling during the initial collapse phase is efficient enough to keep the gas at its initial temperature. Although both studies start with different initial conditions (Larson used a homogeneous density distribution, whereas Penston used a Bonnor–Ebert, BE, profile) they obtained similar results during the isothermal collapse. Starting with a highly unstable, initial singular isothermal sphere, Shu (1977) obtained a different evolution scenario: a self-similar inside-out collapse and claimed that the former solutions are physically artificial. Hunter (1977) in his study of unstable isothermal spheres pointed out that both investigations address the same problem but refer to two different stages of the isothermal collapse. The Larson–Penston studies investigate the collapse of molecular clouds prior to the formation of the protostellar core, whereas Shu’s study applies to the accretion phase of a non-rotating cloud on to a central protostar.

In reality, molecular cores have some initial angular momentum so that most of the collapsing material ends up in a rotating accretion disc (due to their non-vanishing initial angular momentum). This significantly alters the dynamics of the evolution of the protostellar core. More recently, this issue was addressed with the help of two- (2D) and three-dimensional (3D) numerical simulations by several groups (for a review see Bodenheimer et al. 2000). For instance Burkert & Bodenheimer (1993) studied the fragmentation of
a rotating sphere of uniform density with an $m = 2$ perturbation on a fixed nested grid. Their simulations showed that a bar formed, which then fragmented into two low-mass binaries and several lighter secondary fragments. Truelove et al. (1998) used a 3D adaptive mesh refinement (AMR) technique that resolves the local Jeans length throughout the simulation to reinvestigate the fragmentation problem. These authors found that a bar-like structure collapses instead to a single filament without further fragmentation. These differences can be shown to depend upon the ability to resolve the local Jeans length during the collapse (see, e.g., the review by Bodenheimer et al. 2000). An equivalent requirement for smoothed particle hydrodynamic (SPH) simulations was found by Bate & Burkert (1997) who showed that the minimal resolvable mass must be smaller than the Jeans mass to avoid numerical fragmentation. However, these authors also noted that the inclusion of artificial viscosity and heating, which increases the Jeans length and slows the collapse, lead to physical fragmentation of bars, which is predicted in the work of Inutsuka & Miyama (1992). Clouds with an initial Gaussian density profile were studied by Boss (1993) who concluded that the most sensitive parameter for clouds to fragment is $\alpha$, the ratio of the thermal to gravitational energy.

Matsumoto & Hanawa (2003) did an extensive parameter study of rotating Bonnor–Ebert spheres with different rotation profiles and rotation speeds. Their investigations are based on a three-dimensional nested grid technique, which resolves the local Jeans length and the use of two different equation of states (EOSs) depending on the central density (an isothermal EOS in the low-density regime and a barotropic EOS in the high-density regime). They showed that the final structure of the collapsed cloud (bar, ring or binary system) depends sensitively on the parameter $t_{\text{ff}} \Omega$, where $t_{\text{ff}}$ and $\Omega$ are the free-fall time and the angular velocity, respectively.

Another important physical process that alters the dynamics of the core formation and disc evolution is cooling during the collapsing phase. An often used simplification to account for the decreasing cooling efficiency is the previously mentioned sudden switch of the EOS, depending on the local density. We do not follow this approach but rather include the loss of energy due to molecular line emission by collisional excitations. In this work we show that the effective equation of state is a complex function of time, density and space, which influences the structure of the protoplanetary disc and its possible fragmentation.

The study of Bonnor–Ebert spheres as initial states for star formation is of particular interest because they resemble marginal stable molecular cores in pressure equilibrium and have flat-topped rather than singular density profiles. Bonnor–Ebert-type cores are observed in a variety of molecular clouds (e.g. Alves, Lada & Lada 2001; Harvey et al. 2001; Racca, Gómez & Kenyon 2002) and in numerical simulations of star cluster formation in molecular clouds (Tilley & Pudritz 2004). The observation of massive discs undergoing high-mass star formation (Chini et al. 2004) suggests that a similar scenario pertains to massive star formation (e.g. Yorke & Sonnhalter 2002).

In this paper we investigate low- and high-mass star formation from rotating molecular cores through AMR simulations. This paper is organized as follows: in Section 2 we explain our numerical scheme including the cooling procedure, and summarize the properties of the static Bonnor–Ebert sphere. Our results of the collapse in the isothermal and non-isothermal regime are given in Section 3. This section includes also a review of the pure isothermal collapse of an overcritical Bonnor–Ebert sphere. The formation of discs in low- and high-mass rotating cores are discussed in Section 4, and in Section 5 we present our results on the formation of rings and bars in the disc and their possible fragmentation. Finally, we summarize and discuss our findings in Section 6.

## 2 Numerical Scheme and Initial Conditions

For our studies of collapsing gas clouds we used the FLASH code (Fryxell et al. 2000), which solves the coupled gravito-hydrodynamic equations on an adaptive mesh. FLASH is based on a block structured AMR technique, which is implemented in the PARAMESH library (Olson et al. 1999). This AMR technique allows us to follow the core formation over more than the 10 orders of magnitude in density increase without violating the Truelove criterion (Truelove et al. 1997). For this purpose we implemented a new refinement criterion to the FLASH code, which assures that the local Jeans length

$$\lambda_J = \left(\frac{\pi c^2}{G\rho}\right)^{1/2}$$

is at least resolved by $N > 4$ grid points, where $c$, $G$ and $\rho$ are the isothermal speed of sound, the gravitational constant and the local mass density, respectively. The runs we present in this work are performed with an even smaller Jeans number $J = \Delta x / \lambda_J$ of $1/8$ or $1/12$ ($\Delta x$ is the grid spacing in one dimension at the point $x$).

### 2.1 Properties of Bonnor–Ebert spheres

We choose as an initial setup a non-magnetized spherical gas cloud in a marginal stable hydrostatic equilibrium, i.e. a Bonnor–Ebert sphere (Ebert 1955; Bonnor 1956). With the definitions

$$\rho(\xi) \equiv \rho_0 e^{-\Phi(\xi)} ,$$

$$\xi \equiv \frac{r}{r_0} , \quad r_0 = \frac{c}{\sqrt{4\pi G \rho_0}} ,$$

the radial density profile is given by the solution of the Lane–Emden equation (cf. Chandrasekhar 1967):

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\Phi}{d\xi}\right) = e^{-\Phi} ,$$

$$\Phi(0) = \Phi'(0) = 0 ,$$

where $\rho_0$ is the initial central density, $c$ is the thermal speed of sound, $r_0$ is the characteristic radius of the gas cloud and a prime denotes the differentiation with respect to $\xi$. The solution is characterized by a density profile with a flat core of density $\rho_0$ and an envelope, which falls off with radius roughly as $r^{-2}$ until truncated by the pressure of the surrounding medium. Note that the function $\Phi(\xi)$ is up to an integration constant – equal to the gravitational potential $\psi$ (cf. equation 11).

We use a fourth-order Runge–Kutta numerical scheme to solve equation (4) for the potential $\Phi(\xi)$ and the acceleration $\Phi'(\xi)$, which give the density profile $\rho(\xi)$ and the mass parameter $q(\xi)$. Fig. 1 summarizes the radial dependences of the dimensionless parameters that determine the Bonnor–Ebert sphere with the cut-off radius of $\xi_c = 6.5$.

The total mass within a Bonnor–Ebert sphere at the radius $\xi$ is given by
where \( \phi \equiv \phi(\xi) \), \( \phi' \equiv \phi'(\xi) \) and \( q \equiv q(\xi) \). Therefore, \( \alpha = 3/2c^2/|\psi| \) is given by

\[
\alpha = \frac{3}{2} \frac{1}{\phi + q - \phi}.
\]

With \( \xi = 6.5 \) (\( \phi = 2.66, q = 15.85 \)) \( \alpha \) is in the range of:

\[
0.081 \leq \alpha \leq 0.094.
\]

The rotational to gravitational energy \( \beta \) for a rigidly rotating Bonnor–Ebert sphere is given by

\[
\beta = \frac{2}{5} \frac{\psi}{c^2} \frac{1}{\phi + q - \phi} = \frac{16}{15\pi^2} \xi^2 \left( \frac{t_\mathrm{ff}}{t_{\Omega} \Omega} \right)^2,
\]

where \( v_c \) and \( \Omega \) are the toroidal velocity and angular velocity, respectively. For instance, a sphere rotating with \( t_{\Omega} \Omega = 0.1 \) has values of \( \beta \) that span the range

\[
0 \leq \beta \leq 0.0029
\]

and scales roughly proportional to \( \xi^2 \), while \( \beta \) takes its maximum at the edge of the sphere and is given by \( \beta_{\max} = 0.11 (\phi')^{-1} (t_{\Omega} \Omega)^2 \).

### 2.2 Initial conditions: low- and high-mass cores

In this paper we present the results of four simulations that aim to study both low- and high-mass pre-stellar cores. We start each simulation with an overcritical Bonnor–Ebert sphere, which rotates initially with a constant angular velocity \( \Omega \). Three of these initial models have the same initial core density \( \rho_0 \), but different initial angular velocities \( \Omega \) (run A1–A3) and a mass of 168 \( M_\odot \), characteristic of massive star formation (Chini et al. 2004), and one low-mass model starting with a core density according to the observed Bok globule Barnard 68 by Alves et al. (2001) with a mass of 2.1 \( M_\odot \) and a medium angular velocity (run B68). The initial parameters of these runs are summarized in Table 1.

One particularly interesting object for low-mass star formation is the Bok globule Barnard 68, which was extensively studied by Alves et al. (2001). The density profile of this cloud exhibits a close to perfect Bonnor–Ebert profile with \( \xi = 6.9 \pm 0.2 \) corresponding to a physical radius \( R = 1.25 \times 10^4 \) au, a core density of \( \rho_0 = 1.0 \times 10^{-18} \text{ g cm}^{-3} \), a total mass of \( M_{\text{B68}} = 2.1 M_\odot \) and a temperature of \( T = 16 \text{ K} \) (we set up this run with an external pressure of \( P_{\text{ext}}/k_T = 2.4 \times 10^8 \text{Kcm}^{-3} \)). This corresponds to the model marked by a diamond shown in Fig. 2. One can see that this observed system is nearly immediately scalable from the initial state of our simulations A1–A3, defined by the triangle. To simulate the collapse of a realistic molecular cloud we initialize run B68 with the physical parameters given above. According to Lada et al. (2003), Barnard 68 might rotate slightly with a ratio of the rotational to gravitational energy of a few per cent. We initialize the sphere with \( t_{\Omega} \Omega = 0.2 \) corresponding to \( \beta_{\max} \approx 0.01 \).

#### Table 1

| Run | \( t_{\Omega} \Omega \) (s) | \( \Omega \text{ (rad s}^{-1}) \) | \( t_{\Omega} \Omega \) | \( \beta_{\max} \) |
|-----|-----------------|-----------------|-----------------|-----------------|
| A1  | 3.63 \times 10^{13} | 2.755 \times 10^{-15} | 0.1 | 2.88 \times 10^{-3} |
| A2  | 3.63 \times 10^{13} | 5.509 \times 10^{-15} | 0.2 | 1.15 \times 10^{-2} |
| A3  | 3.63 \times 10^{13} | 8.265 \times 10^{-15} | 0.3 | 2.59 \times 10^{-2} |
| B68 | 2.12 \times 10^{12} | 9.430 \times 10^{-14} | 0.2 | 1.21 \times 10^{-2} |

\( \equiv 2029 \) (15)

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1 Note, as \( \xi = 2\pi r/\lambda_s \) this instability criterion is almost equal to the Jeans instability criterion, i.e. \( r > \lambda_s \), and the amount of Jeans masses within the sphere is \( 3/2q \sim 1.53 \).
Figure 2. Scaling of the physical parameters of critical Bonnor–Ebert spheres. The solid line shows the parameters for a sphere with $T = 20$ K, and the upper and lower dashed lines show the relations for a sphere with $T = 10$ and 30 K, respectively (note that the axis labelling for $P_{\text{ext}}$ is only valid for $T = 20$ K and has to be rescaled for different temperatures). The triangle (△) refers to our model parameters for A1–A3, the square (□) shows the Coalsack globule 2 observed by Racca et al. (2002) and the diamond (○) marks the properties of Barnard 68 (our model B68) as given in Alves et al. (2001).

Most of our simulations are designed to study the formation of massive accretion discs in which massive (O, B) stars may form. The collapse of massive (up to 120 $M_\odot$), initially singular cores has been simulated by Yorke & Sonnhalter (2002), who included a careful treatment of radiative transfer effects. Recent observations by Chini et al. (2004) have discovered a massive (20-$M_\odot$) star that is being formed within a very massive, large accretion disc (at least 100 $M_\odot$) of gas in the young star-forming region of M 17. We therefore chose the initial setup for the study of massive star and disc formation corresponding to runs A1–A3 as follows: the cut-off radius is $\xi_0 = 6.5$ corresponding to a physical radius of $R = 1.62$ pc ($R = 3.34 \times 10^6$ au), an unperturbed core density $\rho_0 = 3.35 \times 10^{-21}$ g cm$^{-3}$, and speed of sound $c = 0.408$ km s$^{-1}$ ($T = 20$ K). These parameters, with a 10 per cent overdensity, result in a total mass of the gas cloud of $M = 168$ $M_\odot$ and an external pressure $P_{\text{ext}}/k_B = 3.2 \times 10^9$ K cm$^{-3}$. The initial free-fall time $t_{\text{ff}} = \sqrt{3\pi/32 G \rho_0}$ of our Bonnor–Ebert sphere is $1.1 \times 10^9$ yr.$^2$. The edge of the sphere is defined by a density decrease of the ambient medium by a factor of 100, whereas the pressure is continuous at the edge of the sphere. Therefore, the ambient low-density gas is a hundred times warmer than the pressure is continuous at the edge of the sphere.

Radiative cooling by molecular excitation lines is very efficient in the high-density regime of temperature, i.e. contracts isothermally (cf. discussion in Section 3). Therefore, we can apply our results to observed molecular clouds with different initial physical parameters.

All runs are set up with a 10 per cent, $m = 2$ density perturbation on top of a slightly enhanced Bonnor–Ebert profile, i.e.

$$\rho = \rho_{\text{BE}} [1.1 + 0.1 \cos(2\varphi)],$$

where $\rho_{\text{BE}}$ obeys equation (4) and $\varphi$ is the azimuthal angle (the 10 per cent overall density enhancement guarantees the collapse of a critical BE-sphere as it overwhelms the relatively small amount of rotation). All spheres rotate initially with a rigid body rotation profile with the amount of rotation given in Table 1.

### 2.3 Radiative cooling

Earlier (for a review see Bodenheimer et al. 2000) and recent (e.g. Matsumoto & Hanawa 2003) simulations of protostellar disc formation and fragmentation typically use two types of equations of state: (i) an isothermal equation of state throughout, which assumes that radiative cooling is always efficient enough to keep the gas at the same initial temperature, or (ii) an isothermal EOS that is switched to an adiabatic one at sufficiently high density. We did some simulations of the latter approach and found problems not only caused by numerical artefacts but also by physical difficulties: the sudden change of the equation of state violates energy conservation (an isothermal EOS corresponds to an infinite heat bath, whereas an adiabatic EOS assumes a finite thermal energy).

To achieve a more realistic, and more physically correct picture of the formation of protostellar discs, in our simulations we account for the effect of cooling by collisional excitations of gas molecules. We use cooling functions provided by Neufeld & Kaufman (1993) and Neufeld et al. (1995). These authors computed the radiative cooling rates as a function of the gas temperature, density and optical depth for the temperature and density range of $10^3 \leq T \leq 2500$ K and $10^3 \leq n(H_2) \leq 10^{10}$ cm$^{-3}$. Using steady-state molecular abundances for the most important coolants in molecular clouds (H$_2$, H$_2$O, CO, O$_2$, HCl, C and O) they provide a total cooling rate, which can be used to calculate the energy loss due to radiative cooling. We use their optical depth parameter for a singular isothermal sphere (which is appropriate for a sphere with a 1/$r^2$ density profile),

$$\tilde{\Lambda}_{\text{HIS}} = 5.1 \times 10^{19} \left( \frac{n}{\text{cm}^{-3}} \right)^{1/2} \text{cm}^2 \text{per km s}^{-1},$$

to read off the total cooling rate from the provided cooling data base. The cooling rates provided by Neufeld & Kaufman (1993) and Neufeld et al. (1995) are available for densities $n(H_2) \leq 10^{10}$ cm$^{-3}$. Because the cooling power per H$_2$ molecule in the high-temperature ($T > 100$ K) and high-density ($n > 10^{10}$ cm$^{-3}$) regime is nearly independent of the density, we extrapolate their data in the high-density range assuming that the cooling power is only a function of temperature. As the maximum temperatures in our simulations are significantly below the dissociation temperature of H$_2$ ($T_{\text{dis}} \sim 2000$ K) we do not account for this cooling process in the present simulations. We also did not consider cooling processes due...
to gas–grain interactions, which we will implement in future studies. However, our results are similar in character to those of Yorke, Bodenheimer & Laughlin (1995) who did include dust cooling.

The treatment of the thermal cooling of the gas allows one to calculate the internal energy density, $\epsilon$, at any position and time. The pressure, $P$, at each time-step is then calculated by:

$$P = \epsilon(1 - \gamma),$$

where $\gamma$ is the adiabatic index. Throughout this study we use a constant $\gamma$ of 5/3. Including cooling in our simulations the thermal energy is not only altered by compressional heating during the collapsing phase but also by the loss of energy due to radiative emission. We found it is useful to compare our approach, which uses full molecular cooling, with the earlier adiabatic model studies by calculating an effective EOS, i.e. $\gamma_{\text{eff}} \equiv dP/d\rho$, at each time-step.

### 3 CORE FORMATION AND EVOLUTION

#### 3.1 Pure isothermal collapse of Bonnor–Ebert spheres

Early theoretical work on star formation (e.g. Hayashi & Nakano 1965) has already pointed out that cooling during the initial stage of collapsing molecular cloud cores is efficient enough to ensure constant temperatures of the gas over 4–5 orders of magnitude in density increase. Hence, the isothermal collapse proceeds as a free-falling contraction of the gas. Our simulations – including molecular cooling – resemble this early isothermal phase until $n \sim 10^7$ cm$^{-3}$.

Figs 5 and 7 from our simulation A1 (see below) clearly show the evolution in the isothermal phase at low densities. Because the solutions of collapsing isothermal Bonnor–Ebert spheres are rather general we devote the following paragraph to review the collapse of a pure isothermal cloud.

A detailed 1D numerical investigation of a collapsing isothermal Bonnor–Ebert sphere was performed by Foster & Chevalier (1993). The results of our 3D simulations in the isothermal regime are very similar to the results found by Foster & Chevalier (1993). Fig. 3 shows the time evolution of the density profile and the radial velocity profile, which have the same trend as the 1D results: after a sound wave propagates through the sphere at a sound crossing time of $t_{\text{sc}} = \xi/\sqrt{4\pi G \rho_0}$ ($\sim 3.8 \times 10^5$ yr in the case of runs A0 and $\sim 2.4 \times 10^5$ yr in the case of B68) the cloud starts to collapse from outside-in. Initially, the gravitational acceleration of the Bonnor–Ebert sphere,

$$g = \frac{G M}{r^2} = c_s^2 \frac{\rho_0}{\sqrt{4\pi G \rho_0}},$$

is largest at $\xi = 3.0$, $(\phi(3.0) = 0.517)$, and decreases only slightly with radius $(\phi(6.5) = 0.375)$. Soon after the low-density material from the outer part of the cloud is accelerated towards the centre, the density profile of the sphere starts to change: a high-density flat core with a $1/r^2$ envelope builds up. The $1/r^2$ law in the envelope reflects the fact that the sphere maintains a state that is close to hydrostatic equilibrium in the envelope. Now the gravitational acceleration is highest at the edge of the flat core and decreases with $1/r$. Therefore, material at the core boundary is constantly accelerated, whereas material in the envelope nearly stays at a constant infalling velocity because the acceleration becomes very low in this region. As the core density increases the core shrinks continuously and the $1/r^2$ envelope profile becomes more dominant. The maximum of the radial velocity appears around the edge of the flat core region and moves inward with time. Although, the infalling material becomes supersonic, no shock occurs in this isothermal collapsing phase because the central gravitational force accelerates the gas in front of the supersonic material fast enough to prevent a velocity discontinuity. The radial velocity stays close to zero at the outer edge of the sphere and at the centre of the cloud.

We emphasize that the collapse of an overcritical Bonnor–Ebert sphere differs from the expansion wave solution of Shu (1977) as the latter is an inside-out collapse of an initially singular isothermal sphere (SIS). The collapse of an SIS proceeds in a self-similar fashion leading to a velocity profile that increases with decreasing radial distance to the centre (cf. fig. 2 of Shu 1977) whereas the velocity of a collapsing Bonnor–Ebert sphere always possesses a finite maximum and goes to zero at the centre of the cloud.

To obtain a quantitative estimate of the isothermal collapse described above we approximate the density profile shortly after the initial collapse by:

$$\rho(r) \approx \begin{cases} \rho_{\text{core}} & r \leq r_{\text{core}} \\ \rho_{\text{core}} \left(\frac{r}{r_{\text{core}}}\right)^{-2} & r \gg r_{\text{core}} \end{cases},$$

where $\rho_{\text{core}}$ is the core density and $r_{\text{core}}$ is the radial distance to the core edge. This approximation is valid as long as the collapse is isothermal. The core becomes non-isothermal after the density increased above the critical density of $n \sim 10^7$ cm$^{-3}$ (see Section 3.2).
Using conservation of the mass of the sphere, $M$, we can estimate the core radius for a given core density (with the critical density of $\rho_{\text{core}} = 10^{-16} \, \text{g cm}^{-3}$ and a sound velocity of $c = 0.41 \, \text{km s}^{-1}$):

$$r_{\text{core}} \approx \sqrt[3]{\frac{M}{4\pi \rho_{\text{core}} R}} = \frac{c}{\sqrt[3]{4\pi G \rho_{\text{core}}}} \left(\xi, \phi'\right)^{1/2},$$

$$\sim 4.5 \times 10^2 \, \text{au},$$

(21)

where $R$ is the radius of the sphere and we assumed $r_{\text{core}} \ll R$. Using this approximations the total core mass is given by

$$M_{\text{core}} \approx \frac{4\pi}{3} r_{\text{core}}^3 \rho_{\text{core}} = \frac{1}{3} M \frac{r_{\text{core}}}{R}$$

$$= \frac{1}{3} \frac{c^3}{\sqrt[3]{4\pi G \rho_{\text{core}}}} \left(\xi, \phi'\right)^{3/2}$$

$$\sim 0.1 \, \text{M}_\odot.$$  

(22)

3.2 Collapse with molecular cooling: initial isothermal phase

Theoretical models of collapsing protostars predict that the formation of a protostar occurs in several stages (e.g. Larson 1969; Larson 2003). One can infer already from Fig. 4, which compares the cooling time-scale $t_{\text{cool}}$ and the local free-fall time $t_{\text{ff}}$, that a gravitationally unstable low-density cloud will undergo an isothermal collapse until the cooling by molecular line emission becomes inefficient when $t_{\text{cool}} > t_{\text{ff}}$. For instance, a 20-K cloud cannot be efficiently cooled if its core density exceeds more than $10^{11} \, \text{cm}^{-3}$. After the core density reaches this critical value, the high-density region of the gas cloud contracts almost adiabatically on a much longer time-scale. Another prediction one can infer from Fig. 4 is the evolution trajectory of the core in the temperature–density plane, which follows the track where $t_{\text{ff}} \sim t_{\text{cool}}$ in the inefficient cooling regime. Our simulations are in good agreements with this prediction.

Initially, due to effective cooling, the unstable sphere maintains its initial temperature, while collapsing on its local free-fall time. Fig. 5 shows the evolution trajectory of the core of the cloud in the temperature–density plane from our simulation A1. The initial contraction stage clearly indicates the regime where cooling is fast enough to stay at the initial temperature of the cloud, while core density increases. A quantification of this stage is shown in Figs 6 and 7 where we compute the effective equation of state, $\gamma_{\text{eff}} = \frac{d \log p}{d \log \rho}$, in the core-forming region as a function of the core density (from run A1).
From Fig. 4 and from our simulations (Figs 5 and 7) we obtain a core density (i.e. of material interior to radius \( r_{\text{core}} \)) at the end of the isothermal phase of \( n_{\text{core}} \sim 10^{13} \text{ cm}^{-3} \) and \( \rho_{\text{core}} \sim 10^{-16} \text{ g cm}^{-3} \). Using equations (21) and (22) gives a core radius of \( r_{\text{core}} \sim 450 \text{ au} \) and a core mass of \( M_{\text{core}} \sim 0.1 \text{ M}_\odot \). Note that these values are independent of the initial mass \( M \) and radius \( R \) as long as the initial sphere obeys the Bonnor–Ebert profile given by equation (4). Other investigations of spherical collapses of molecular clouds (for a review see Larson 2003) also predict a first ‘hydrostatic core’ when the density reaches \( \sim 10^{-10} \text{ g cm}^{-3} \), where the core mass is almost independent of the initial mass or radius of the cloud. The predicted ‘hydrostatic core’ mass is approximately 0.01 \text{ M}_\odot \). Our findings give a larger core mass at the end of the isothermal collapsing phase as cooling becomes inefficient at lower densities.

Comparing the value of the core mass at the end of the isothermal phase with the local Jeans mass, \( M_J = \pi^{3/2}/6c^3/\sqrt{G \rho_{\text{core}}} \approx 0.56 \text{ M}_\odot \), (which is independent of the sphere parameters) gives an estimate of gravitational strength at this point. The fact that \( M_{\text{core}}/M_J < 1 \) in the fast contracting isothermal phase is the reason that the collapsing cloud has not yet broken up into several fragments.

With the approximation of the radial density profile equation (20) the total mass within a sphere of radius \( r \gg r_{\text{core}} \) is given by

\[
M(r) \approx \frac{c^2}{G} \left( \xi, \phi' \right) r \approx 1 \text{ M}_\odot \left( \frac{T}{20 \text{ K}} \right) \left( \frac{r}{3.3 \times 10^{16} \text{ cm}} \right),
\]

where we used \( \xi = 6.5 \) for the numerical example. Comparing the core mass \( M_{\text{core}} \) to total mass of the sphere, \( M \), we obtain

\[
\frac{M_{\text{core}}}{M} \approx \frac{1}{3} \left( \frac{\phi'}{\phi} \right)^{1/2} \left( \frac{\rho_{\text{core}}}{\rho_0} \right)^{-1/2} \approx 0.08 \left( \frac{\rho_{\text{core}}}{\rho_0} \right)^{-1/2}.
\]

Using equation (23) we can compute the mass accretion at different radii:

\[
\dot{M}(r) \approx -\frac{c^2}{G} \left( \xi, \phi' \right) v_r,
\]

where \( v_r \) is the radial infall velocity. The gravitational acceleration outside the core region is \( g \approx c^2 (\xi, \phi') r \) and the radial velocity \( v_r \approx c^2 (\xi, \phi') (4\pi G \rho_{\text{core}})^{-1/2} / r \) assuming that the velocity changes on the dynamical time of the core region. Together with the size of the core region equation (21) we obtain a good estimate of the mass accretion on to the core:

\[
M_{\text{core}} \approx \frac{c^3}{G} \left( \xi, \phi' \right)^{3/2} \sim 6.1 \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1} \left( \frac{T}{20 \text{ K}} \right)^{3/2}.
\]

Equation (26) shows that the mass accretion of a collapsing Bonnor–Ebert sphere is independent of its initial mass and radius. This result

\[
\text{is applicable to non-merging, i.e. low-mass, cloud cores, and similar to the case of a collapsing SIS (Shu 1977).}
\]

Although equation (26) suggests that the mass accretion is constant in time we observe a slowly increasing \( M \), which is due to the steady increase of the radial velocity. At a given radius \( r \gg r_{\text{core}} \) in the envelope the mass accretion approaches a constant value. The mass accretion has its maximum at the core radius and drops quickly towards the core centre as the radial velocity sharply decreases at radii \( r < r_{\text{core}} \). Fig. 8 shows the evolution of the mass accretion in the isothermal regime.

3.3 Collapse with cooling: post-isothermal phase

When the central density exceeds the critical density \( n_{\text{crit}} \approx 10^{13} \text{ cm}^{-3} \) thermal energy produced by gravitational contraction is no longer radiated efficiently and the core starts to heat up. The resulting increase of the thermal pressure slows down the contraction of the central gas region and the effective equation of state becomes stiffer. This is reflected in increasing the effective equation of state, \( \gamma_{\text{eff}} \). Our simulations (cf. Fig. 7) show a steep rise of \( \gamma_{\text{eff}} \) from 1.0 to \( \approx 1.3 \) after the core density rises above the critical cooling density. As a result of the complex cooling process \( \gamma_{\text{eff}} \) does not stay at a fixed value but varies with time with a trend to decrease in the regime \( 10^8 \lesssim n_{\text{core}} \lesssim 10^{12} \text{ cm}^{-3} \). The strong variability of \( \gamma \) in Fig. 7 also reflects the multiple shock occurrence during the collapse (cf. Fig. 19 below). Also note that the data points in this plot do not correspond to the time-steps in the simulation, but to the larger time-steps at which we obtained output data files. Sampling the data points on a higher time resolution might result in a smoother graph.

In the post-isothermal phase, the density increase in the centre of the gas cloud is dictated by the cooling time, \( t_{\text{cool}} \), as opposed to the dynamical time, \( t_{\text{d}} \). Fig. 9 shows the evolution time-scale of the core region, i.e. \( n_{\text{core}}/M_{\text{core}} \) as a function of the density. After the core density reaches \( \sim 10^9 \text{ cm}^{-3} \) the collapse proceeds on a time-scale that is determined by the cooling process. At densities of \( \gtrsim 10^{10} \text{ cm}^{-3} \) the core density increases on a nearly constant time-scale much larger than the local free-fall time marking a phase of ‘hydrostatic equilibrium’ (cf. also Larson 1969). At later times the temperature of the core (at \( n \gtrsim 10^{13} \text{ cm}^{-3} \) and \( T \gtrsim 250 \text{ K} \)) rises to a point when cooling by molecular excitations becomes more efficient (due to the
abundant production of H$_2$O and its multitude of excitation levels) and the collapse proceeds again almost on the local free-fall time.

The slower contraction of the warmer core region, as compared with the isothermal case, causes the supersonic infalling material to shock at the edge of the warm core. These first shocks occur preferentially above and below the disc plane, where the gas is not rotationally supported at a typical distance of $\sim$500 au for the runs A1 from the centre. Temperatures at this point reach 70–80 K and the temperature profile is discontinuous rising from 30 to 70 K between the outer pre-shock region and the shocked region within a few tens of au (cf. Fig. 10). The velocities at this point reach values of 1.5 km s$^{-1}$ ($\mathcal{M} \approx 2.5$). As infalling gas piles up at the shock boundary, a density discontinuity also develops in the post-shock region, which separates a low-density envelope and a high-density pre-protostellar disc. This post-shock region slowly moves towards the centre of the gas cloud and its temperature increases further, while supersonic gas from the outside envelope hits this shock boundary.

Gas inside this first shock region is subsonic and therefore not shock heated leading to a lower temperature region behind the shock. As the core density and the core temperature continue to increase, gas inside the pre-protostar region becomes supersonic and a secondary shock develops at $\sim$100 au above and below the disc plane (cf. lower panel of Fig. 10). Typical temperatures at the secondary inner shock boundaries are $\sim$150 K, whereas the core stays cooler. This stage (the core density is now $\sim$10$^{13}$ cm$^{-3}$) marks the first development of the protostellar disc.

Similar to the outer first shock, the inner shock fronts move slowly towards the disc plane, while the local Mach number rises to $\mathcal{M} \sim 4$, corresponding to a shock velocity of 3 km s$^{-1}$. Again, the collapsing core region is fed only by gas inside the inner shock fronts, whereas gas falling from outside onto the shocks is stalled at the shock boundaries and heats up the gas in these shock regions.

Fig. 11 shows the time evolution of the density and temperature profiles, respectively, in the non-isothermal regime from simulation A1. The second line from the bottom in the density plot marks the final stage of the isothermal collapse phase. In the non-isothermal phase the density profile deviates from the $r^{-2}$ profile because of the slower contraction of the core during this regime. The density falls off more steeply than $r^{-2}$ with increasing radius in the heated up region and approaches an $r^{-2}$ profile in the isothermal regime at $r > 10^{15}$ cm. The transition region appears at a radius of $\sim 7 \times 10^{15}$ cm, which is in good agreement with the theoretical prediction of equation (21). The temperate profile rises steeply with decreasing radius, roughly as $r^{-2}$, whereas the core region has a flat temperature profile. The profiles $\rho(r)$ are radially binned spherical averages, i.e. $\rho(r) = \int d\Omega \rho(x)$, where $d\Omega = d\cos \theta \, d\phi$ is the differential solid angle.

Fig. 12 shows the evolution of radial infall velocity as a function of radius from run A1. The peak velocity increases with time and ‘moves’ towards smaller radii (the first line from the top refers to the second line from the bottom of Fig. 11, which marks the end of the isothermal regime). As a result of the shock that develops at the edge of the core the velocity profile becomes steeper with time, whereas the peak velocity approaches a constant value of $\sim 3c$ in the spherical collapse phase. The velocity at the centre always drops to zero.

In Fig. 13 we show the evolution of the 1D density profiles for run B68. Because the initial core density, $\rho_0$, in this case is higher than for the runs A1–A3 ($\rho_{\text{init}}/\rho_{A1} \sim 300$) the isothermal region with $\rho \propto r^{-2}$ is smaller, but the kink in the profile, which separates the non-isothermal core region from the isothermal envelope, appears at the same physical radius of $\sim 7 \times 10^{15}$ cm. This result is again in agreement with the theoretical prediction. Finally, we note that a similar double shock structure has been seen in 2D collapse simulations by Yorke et al. (1995), which feature dust as the coolant.

**Figure 9.** Evolution time-scale, $\text{revol} \equiv \dot{n}_{\text{core}}/\rho_{\text{core}}$ as a function of the core density $\rho_{\text{core}}$. The solid line shows the local free-fall time, i.e. $t \propto \dot{n}_{\text{core}}^{1/2}$ (from run A1).

**Figure 10.** Temperature line profiles along the z-axis through the centre of the gas cloud for run A1. The upper panel shows the line profile after the first shock develops when the central density reached $2 \times 10^9$ cm$^{-3}$. The lower panel shows the temperature profile at a later stage when $n_{\text{core}} \sim 10^{11}$ cm$^{-3}$. At this time a second shock in the inner core region develops when supersonic material falls on to the protoplanetary disc (from run A1).
Figure 11. Radial profiles of the density (upper panel) and temperature (lower panel) at different times. As the simulation resolution increases with time due to the Jeans refinement criterion the line profiles also increase in resolution. The dashed short line shows an $r^{-2}$ density profile. The data are compiled from run A1.

Figure 12. Radial profiles of the radial velocity at different times (these profiles correspond those shown in Fig. 11). During the first stages of the spherical collapse the radial velocity increases steadily with time until it reaches the maximum value of $\sim 3c$ and the peak velocity is moving towards the inner part of the cloud. After the temperature in core region begins to increase the maximum of the radial Mach number stays at $r \sim 10^{16}$ cm (from run A1).

Figure 13. Similar to Figs 11 and 12 except that the profiles are for run B68. The dashed line in the density panel shows an $r^{-2}$ profile.

4 DISC FORMATION

As described in the previous section, the appearance of the secondary shock establishes the initial state for the formation of the protostellar disc. The protostellar disc builds up and achieves a column density of $\Sigma \approx 10^{15}$ g cm$^{-2}$ and a disc height $h \sim 200$ au. As the shock fronts move towards the equatorial plane, the density and temperature increase on a time-scale slower than the local free-fall time. Fig. 14 (run A1) and Fig. 17 (run B68, see below) show the time evolution of the column density, which we compute as follows:

$$\Sigma(R) = \int dz \rho(x, y, z),$$

(27)

where $R = \sqrt{x^2 + y^2}$ is cylindrical radius and we evaluate the integral (27) throughout the entire simulation box. Similar to the radial density profile the column density profile develops an envelope and a flat core region. The non-isothermal core is separated from the envelope by a steep increase in density, which leads to a kink in the radial density profile. In general, we find that the column density falls off roughly as $R^{-2}$. The disc height decreases, while the disc becomes denser and reaches a value of $\sim 10$ au when the column density becomes $\sim 10^3$ g cm$^{-2}$.

In the case of run B68 the ring structure is clearly visible in latest density profile.

Once a disc-like object forms, one can define the disc height, $h$, by

$$h(R) = \frac{\Sigma(R)}{\rho(R)},$$

(28)

Fig. 14 shows the evolution of $h$ for run A1 with time. The initial thick protostellar disc flattens to a solar nebular disc with a disc height of a few tens of au. At the time when the protostellar disc first forms, it rotates close to a solid body with an angular velocity
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Figure 14. Radial profiles of the column density, $\Sigma$, and disc height, $h$, at different times. For comparison, the dashed line shows an $R^{-2}$ profile. The profiles correspond to the density shown in Fig. 11 (run A1).

of $\Omega \sim 10^{-11}\text{ rad s}^{-1}$ whereas the outer envelope follows a rotation law according to its density distribution, i.e. $\Omega \propto r^{-1}$. Fig. 15 shows the evolution of the angular velocity for run A1: while the disc is slowly accreting gas to a core density of $\rho \sim 10^{-11}\text{ g cm}^{-3}$ its core spins up to $\Omega \sim 10^{-9}\text{ rad s}^{-1}$. These numbers show that the core region is rotationally supported against gravitational collapse, i.e. $\Omega \sim \sqrt{4\pi G \rho/3}$.

We show the mass accretion rate of the disc, $\dot{M}$, in Fig. 16 (run A1) and Fig. 17 (run B68). This quantity is defined by the standard vertically averaged continuity equation:

$$\frac{dM}{dt} = -2\pi v_r R \Sigma,$$

where $v_r$ is the radial velocity in the disc. In the case of run A1 after the protostellar disc forms the mass accretion at the outer edge of the disc is $\sim 5 \times 10^{-4}\text{ M}_\odot\text{ yr}^{-1}$, and increases slightly with time to $\sim 10^{-3}\text{ M}_\odot\text{ yr}^{-1}$ while the disc radius decreases. For run B68 the mass accretion is highest at the outer radius of the ring and reaches a maximal value of $3 \times 10^{-4}\text{ M}_\odot\text{ yr}^{-1}$ there. Most of the infalling material is deposited at this outer rim of the disc and the mass accretion decreases towards the disc centre as $\dot{M} \propto r$. The infalling gas outside of the disc reaches a constant terminal velocity of the order of the speed of sound (cf. the radial velocity plot of Fig. 12), which results in a constant mass accretion in time in this outer region. In the lower panel of Fig. 16 we compile the evolution of the total mass in the disc, $M_{\text{disc}}(R) = 2\pi \int dR R \Sigma$, for run A1, which reaches a few solar masses.

5 DISC EVOLUTION: BARS, RINGS AND FRAGMENTATION

Although we started all runs with a 10 per cent, $m = 2$ density perturbation, we did not observe any signs of fragmentation or non-central collapse in the isothermal regime. This is due to low mass (compared with the Jeans mass) in the collapsing core region (cf. Section 3.2). The first features that appear are a bar- or ring-type structure. They are first evident after the stage of disc formation during which the non-isothermal core slowly accretes the surrounding gas.

It is known that thin discs are unstable to fragmentation if its Toomre $Q$-parameter (Toomre 1964),

$$Q = \frac{c K}{\pi G \Sigma},$$

where $c$ is the gas scale height, $K$ the Bondi-Hoyle infall velocity and $\Sigma$ the surface density of the disc.
becomes smaller than unity, where \( \kappa \) is the epicyclic frequency:

\[
\kappa = \left( 4 \Omega^2 + R \frac{d\Omega^2}{dR} \right)^{1/2}.
\]

The first unstable modes that are expected to grow and possibly persist for some dynamical times are the \( m = 0 \) ring-mode and \( m = 2 \) bar-mode. We find that a ring-type structure develops in the case with \( t_{ff}/\Omega_1 = 0.2 \) (runs A1 and B68) with a dimensionless radius of \( \xi_{\text{ring}} \sim 6 \times 10^{-3} \). For run A2 this corresponds to a physical radius of \( \sim 300 \) au and for run B68 to \( \sim 18 \) au. In both cases the ring-structure persists only a few dynamical times and either fragments into a binary system (run A2) or collapses to a bar (run B68). In the case with \( t_{ff}/\Omega_1 = 0.1 \) (run A1) and \( t_{ff}/\Omega_1 = 0.3 \) (run A3) no such ring structures develop in the simulations. Instead a bar-like structure with spiral arms forms without an intermediate formation of a ring structure. A ring structure in the Coalsack globule 2 was recently discovered by Lada et al. (2004). Our simulations may pertain to this system; in particular, as this cloud can be described well with a subcritical Bonnor–Ebert profile.

We show 2D images of the density, the temperature and the \( Q \)-parameter from runs An and B68 at different times in Figs 18–29. The bar that forms in the slow rotating run A1 shows no signs of fragmentation although the \( Q \)-parameter drops below unity when the bar density is very high, \( >10^{-8} \) \( \text{g cm}^{-3} \). Slight instabilities in this high-density and high-temperature (600–800 K) regime cannot grow quickly enough to lead to fragmentation as the inefficient cooling prevents a fast collapse of overdensities. In the case of faster rotation (run A2), instabilities form earlier resulting in a smaller-\( Q \) value than in run A1. These instabilities are large enough to lead to the fragmentation of the ring-type structure. In this situation, we observe the breakup into two fragments where each of them has a disc with spiral structure. These discs are surrounded with a circumstellar disc, and are connected by a tidal stream. The separation between the members of this binary system is \( \sim 260 \) au, which is roughly twice the size of the spiral structures itself. Each of these protostellar systems has a total mass of \( \sim 1.3 \) \( \text{M}_\odot \). Run A3 with \( t_{ff}/\Omega_1 = 0.3 \) (Figs 24–26) does not form a ring structure but collapses to very twisted spiral arms and shows no sign of fragmentation. The simulation of Barnard 68 (run B68, Figs 27–29) with \( t_{ff}/\Omega_1 = 0.2 \) results in a short lived ring structure, which collapses to a bar. This bar has a size of \( \sim 130 \) au and a total mass of \( \sim 0.1 \) \( \text{M}_\odot \).

The double-shock structure of these discs is easily seen in the density plots, the temperature structure and velocity fields in the \( x-z \) plane cuts.

The complex temperature structure due to cooling and the resulting pressure response in the non-isothermal regime prevents a simple analysis from deciding what kind of structure one can expect from a rotating collapsing cloud. From our simulations we conclude that during the formation of the pre-stellar disc a ring structure appears at a radius of

\[
\xi = \xi_{\text{ring}}(t_{ff}/\Omega_1),
\]

whenever the cloud rotates fast enough. In order to decide whether such a possibly formed ring will fragment into several pieces, one
Figure 18. 2D slices of the mass density (logarithmic scale in g cm$^{-3}$) and velocity field at different times for run A1. After the protostellar disc forms, a spiral feature builds up and collapses to a thin filament. The left- and right-hand panels show the evolution in the disc plane and perpendicular to the disc plane, respectively. Note that the highest resolution areas correspond (from top to bottom) to 256, 512 and 1024 pixels in each dimension.
Figure 19. Evolution of the temperature field (grey-scale in K) and density (contour lines). The panels correspond to those of Fig. 13 (run A1).
Figure 20. The evolution of the Toomre Q-parameter (left-hand panel, logarithmic scale) and the angular velocity $\Omega$ (right-hand panel, logarithmic scale in rad s$^{-1}$). The panels correspond to those of Fig. 13 (run A1).
Figure 21. 2D slices of the mass density (logarithmic scale in g cm$^{-3}$) and velocity field at different times for run A2.
Figure 22. Evolution of the temperature field (grey-scale in K) and density (contour lines). The panels correspond to those of Fig. 21 (run A2).
Figure 23. The evolution of the Toomre $Q$-parameter (left-hand panel, logarithmic scale) and the angular velocity $\Omega$ (right-hand panel, logarithmic scale in rad s$^{-1}$). The panels correspond to those of Fig. 21 (run A2).
Figure 24. 2D slices of the mass density (logarithmic scale in g cm$^{-3}$) and velocity field at different times for run A3.
Figure 25. Evolution of the temperature field (grey-scale in K) and density (contour lines). The panels correspond to those of Fig. 24 (run A3).
Figure 26. The evolution of the Toomre $Q$-parameter (left-hand panel, logarithmic scale) and the angular velocity $\Omega$ (right-hand panel, logarithmic scale in rad s$^{-1}$). The panels correspond to those of Fig. 24 (run A3).
Figure 27. 2D slices of the mass density (logarithmic scale in g cm$^{-3}$) and velocity field from the simulation B68 at different times. From top to bottom: $t = 5.35, 5.40, 5.41 t_{ff}$ corresponding 3.597, 3.628, 3.633 $\times 10^5$ yr.
Figure 28. Evolution of the temperature field (grey-scale in K) and density (contour lines). The panels correspond to those of Fig. 27 (run B68).
Figure 29. The evolution of the Toomre $Q$-parameter (left-hand panel, logarithmic scale) and the angular velocity $\Omega$ (right-hand panel, logarithmic scale in rad s$^{-1}$). The panels correspond to those of Fig. 27 (run B68).
has to compare this radius with the size of the non-isothermal core where the enhanced thermal pressure stabilizes the core region against fragmentation. In Section 3.1 we showed that the core radius of a collapsing Bonnor–Ebert sphere is (almost) independent of the initial cloud parameters and only determined by the density where $t_\text{ff} \sim t_\text{cool}$ (cf. equation 21). For a ring to fragment its size must not be much smaller than this warm core size, which gives the condition

$$\xi_\text{ring} \sim \left( \frac{\rho_0}{\rho_{\text{core}}} \right)^{1/2} \left( \xi_0 \phi' \right)^{1/2}. \quad (33)$$

We find that ring structures that do not obey the condition (33) collapse to a single bar rather than fragmenting into two or more pieces. Figs 27 and 28 show the time evolution of the density and temperature of the simulation B68 where a first formed ring condenses to a bar. Also note that the core region rotates with $\Omega_1$ whereas the disc rotates differentially outside the core region (cf. Fig. 15), where shearing effects enhance instabilities. Another mechanism that facilitates the fragmentation of a ring structure is the appearance of a shock at the core edge. A ring structure of similar size as the core can accrete more material than smaller rings increasing the possibility of fragmentation.

In the case where $t_\text{ff} \Omega_1 = 0.1$ (run A1), we find that a two-armed spiral structure develops when $\rho_{\text{core}} \sim 10^{-11} \text{ g cm}^{-3}$ and $T \sim 200 \text{ K}$. If the initial sphere rotates with $t_\text{ff} \Omega_1 = 0.2$ (run A2) a ring structure forms earlier when the density and temperature are, respectively, $\sim 10^{-14} \text{ g cm}^{-3}$ and $\sim 85 \text{ K}$. This confirms the general picture that slowly rotating clouds collapse to bars and thin filaments, whereas faster rotating objects form a ring-type structure. A recent numerical investigation of rotating Bonnor–Ebert spheres by Matsumoto & Hanawa (2003) showed that spheres with $t_\text{ff} \Omega_1 \sim 0.2$ collapse to a ring, which fragments into several pieces.

The theory of bars in disc-like structures (see Binney & Tremaine 1987) predicts that the bar velocity $\Omega_b$ must be larger than the so-called pattern velocity,

$$\Omega_p \equiv \Omega - \frac{1}{2} \frac{\kappa}{\Omega}, \quad (34)$$

due to fragmentation. In Section 3.4 we showed that $\Omega_b$ must be larger than $\Omega_p$ for fragmentation to occur. A ring structure that forms earlier than the core may thus collapse to a ring rather than to a bar. This might be due to the fact that the bar formation time is much longer than the rotation period of the system. A ring that forms early may thus be able to accrete more material than a bar formed late, allowing it to fragment into a ring rather than a bar.

Finally, we note that the column density profiles of the discs formed in our simulations are rather steep, obeying

$$\Sigma \propto R^{-1.95 \pm 0.05}. \quad (36)$$

This is steeper than Hayashi models (Hayashi 1981) but in good agreement with disc models inferred from the measurement by Kuchner (2004) of planets in extrasolar systems, who found $\Sigma \propto R^{-2.0 \pm 0.5}$.

6 SUMMARY AND DISCUSSION

In this work we studied the collapse of rotating marginal stable Bonnor–Ebert spheres using 3D hydrodynamical simulations based on the AMR technique. Compared with former numerical studies of collapsing molecular clouds, we included the effect of radiative cooling by molecular line emissions. This more realistic approach shows that the effective equation of state is a complex function of density and time during the collapsing phase and cannot be approximated by a time-independent EOS. Initially, the gas cloud collapses isothermally on a free-fall time until the core density reaches the critical point where cooling becomes less efficient (i.e. $t_\text{cool} > t_\text{ff}$) and $t_\text{cool} < t_\text{ff}$ at $n \sim 10^{5.5} \text{ cm}^{-3}$. During this isothermal phase the cloud collapses from outside in, where the density profile approaches a flat core region with an $r^{-2}$ envelope. The infall velocity peaks at the edge of the core where it reaches $M \sim 3$ and drops to zero at the centre of the gas cloud. Subsequently, the temperature in core region starts to rise and an increasing pressure support stabilizes the gas clump preventing immediate fragmentation. As cold gas from the envelope falls on to the warm core region shock fronts build up, separating the cold envelope from the heated up core. All our simulations show a similar double shock structure: an early outer shock and a later inner shock. Both shock fronts move slowly toward the core centre where the inner shock region sets the conditions for the first appearance of the protostellar disc. The appearance of these shocks also determines the accretion rate on to the core (respectively, bar) as material outside the shock region is stalled at the shock boundaries. We find peak accretion rates for the low- and high-mass systems of $3 \times 10^{-4}$ and $\sim 10^{-3} M_\odot \text{ yr}^{-1}$, respectively.
respectively. In this work we did not keep track of the composition of the gas during the collapse of the molecular cloud, but it is known that the chemistry might be altered within the shock region (e.g. Jørgensen et al. 2004).

Depending on the initial rotation and density a ring mode or a bar develops in the disc plane. Only one of our four simulations result in the fragmentation of the protostellar disc. In this particular case, a ring structure developed first, which then fragments into a binary system in which each system contains a bar structure. The preferred appearance of bars in our simulations supports theoretical predictions that the most likely growing instability mode in discs is the $m = 2$ bar mode.

As a result of the limitations of our numerical scheme we are not able to follow the bar evolution for several rotation periods as the time-scales of the bar rotation and core evolution diverge (i.e. $t_{\text{rot}} \gg t_{\text{ff}}$) shortly after the bar forms. Further fragmentation of the bars might happen during the phase of hydrogen dissociation ($T > 2000$ K) where the effective equation of state drops below 4/3 or when tidal interactions of the spiral arms become stronger.

A comparison of our A2 simulations – which produces a distinct ring that fragments into a binary – with the recent observation of a massive disc in M 17 (Chini et al. 2004) is interesting. The observations reveal an inner torus (within a $100$-$\text{M}_\odot$ disc) the radius of which is $\sim 3.8 \times 10^{16}$ cm. The radius of the dense initial ring that forms in our simulation A2 is of the same order ($\sim 10^{16}$ cm, see Fig. 21), which is in good agreement with these observations.

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