Resonance width oscillation in the bi-ripple ballistic electron waveguide

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Interference of quasi bound states is studied in a ballistic electron ripple waveguide with two ripple cavities whose distance apart can be varied. This system is the waveguide analog of Dicke’s model for two interacting atoms in a radiation field. Dicke’s model has resonances whose widths change in an oscillatory manner as the distance between the atoms is varied. Resonances that form in a bi-ripple waveguide behave in a manner that has some similarity to Dicke’s system, but also important differences. We numerically investigate the behavior of resonance widths in the waveguide as the distance between the two ripple cavities changes and we find that the resonance widths oscillate with variation of distance, but the coupling does not decrease as it does in Dicke’s system. We discuss differences between our waveguide system and other systems showing the analogous of Dicke effect. We also study S-matrix pole trajectories and find that they rotate in counterclockwise direction on a circle in the complex energy plane.

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Interference is a distinctive property of waves in both quantum and classical mechanics. It plays an especially important role in quantum mechanics because it demonstrates the wave nature of particles. Matter-wave interference has been observed in atomic, nuclear, and solid state systems. However, these systems are often too difficult to control to allow investigation of a variety of interference phenomena. Recent developments of nano technology now enable us to study interference phenomena using ballistic electron waveguides, carbon nanotubes, quantum dots. Indeed, quantum dots are often called “artificial atoms” because they have discrete energy levels due to the confinement of the electron wavefunction. One can change the electron state inside a quantum dot with external controls such as gate voltages. The Coulomb blockade conductance profile shows the discreteness of energy levels inside a quantum dot.

Ballistic electron waveguides have been used to study the interaction of quantum systems with their environment. They have provided an important tool for studying the effects of quantum chaos on scattering phenomena.\textsuperscript{1,2} In waveguides, Coulomb blockade does not occur because discrete energy levels don’t exist. However, even though there is no discrete energy spectrum, quasi bound states can build up in waveguides and significantly affect the electron transmission. Since there exist both non-resonant and resonant channels for electron transmission, resonances show the Fano profile, indicating the interaction between resonant and non-resonant channels.

Dicke proposed that spontaneous radiation from two interacting atoms would have a much longer wavelength than the distance between the two atoms.\textsuperscript{3} This radiation is associated with a transition from a symmetric collective state of the atoms to the ground state. It is called superradiance, because the symmetric state is strongly coupled to the external photon fields and its decay occurs more rapidly than the decay of excited states of independent atoms. There is also an antisymmetric collective state which is weakly coupled to the photon fields. Its decay is called subradiance because it decays more slowly than independent atoms. The different coupling to the photon fields results in broadening and narrowing of the resonance widths, which is called Dicke effect. The analogous effect can be found in mesoscopic systems because quantum dots or quasi bound states in a waveguide behave like atoms. The Dicke effect in mesoscopic systems was first found by Shahbazyan and Raikh for two channel resonant tunneling in a system with two impurities. In their system, the bound state of each impurity is indirectly coupled through the electron wavefunction in external leads. The analogy to Dicke effect has since been found in several other mesoscopic systems, such as quantum dots coupled via a common phonon field and a quantum wire with side coupled quantum dots.\textsuperscript{9} The Dicke effect has also been seen in the spontaneous radiation from collectively interacting quantum dots in an experiment by M. Scheibner et al. in 2007.\textsuperscript{11}

In a previous study, we found the analog of the Dicke effect in a multi-ripple ballistic electron waveguide. Quasi bound states in each cavity of the waveguide interact through the internal leads to form the collective states which result in different coupling to the external leads. We found the broadening and narrowing of resonance widths in electron transmission (conductance) as more ripple cavities are added to the waveguide. However, we could not determine how the resonance widths change as the distance between ripple cavities varies, which is one of the key features of the Dicke effect. In this brief report, we study the dependence of resonance widths on the distance between the two cavities of a bi-ripple waveguide the Generalized Scattering Matrix Method (GSM)\textsuperscript{13,14}.

We consider a waveguide system in which two ripple cavities are connected to each other by a flat waveguide whose length is $WL$. The length of each ripple...
cavity is \( W \), and the upper wall of each cavity is described by an analytic function, \( y = d - \cos(2\pi x/W) \) (See Fig. 1). The outer ends of each cavity is connected to a semi-infinite lead whose height is \( L = (d - a) \). We use R-matrix theory to calculate the S-matrix for a single ripple cavity [12, 11]. R-matrix theory was originally developed by Wigner and Eisenbud in 1950s for the study of nuclear scattering. Recently, R-matrix theory has been used to study electron transmission in ballistic electron waveguides [11, 5, 6, 12]. We consider a waveguide built in a 2DEG (two dimensional electron gas) made of a GaAs/AlGaAs heterostructure at a very low temperature. We then use the GaAs effective mass of the electron \( m^* = 0.061m_e \), where \( m_e \) is the free electron mass. We use parameters \( a = 13.846 \) Å, \( d = 47.269 \) Å, and \( W = 300 \) Å for the waveguide in Fig. 1. \( E_1 = \frac{h^2}{2m^*L^2} = 0.5034074eV \) is the threshold energy to open the first propagating mode in the semi-infinite waveguide. We use \( E_1 \) as the unit of energy through this report. We only consider the incident electron energies \( E \) which allow one propagating mode so that \( E_1 \leq E \leq 4E_1 \).

Since electrons freely propagate in the flat waveguide between the ripple cavities, the electron wavefunction acquires the phase factor \( e^{ik_1WL} \), where \( k_1 = \sqrt{2m^*/\hbar^2(E-E_1)} \). The S-matrix for phase acquisition is a diagonal matrix,

\[
S_{WL} = \begin{pmatrix} s_{WL} & 0 \\ 0 & s_{WL} \end{pmatrix}.
\]

For a single ripple waveguide, the S-matrix is written in terms of transmission and reflection coefficients such as

\[
S_1 = \begin{pmatrix} r_1 & t'_1 \\ t_1 & r'_1 \end{pmatrix}.
\]

The S-matrix for the second ripple waveguide \( S_2 \) can be obtained in the same way. If two ripple waveguides are identical, \( S_1 \) and \( S_2 \) are same. We consider only identical ripples in this report. The overall scattering matrix \( S_T \) is obtained by the GSE [13, 14],

\[
S_T = \begin{pmatrix} r_1 + t'_1s_{WL}U_2r_2s_{WL}t_1 & t'_1s_{WL}U_2r'_2 \\ t_2s_{WL}U_1t_1 & r'_2 + t_2s_{WL}U_1r'_1s_{WL}t'_2 \end{pmatrix}.
\]

where \( U_1 = (1 - r'_1s_{WL}r_2s_{WL})^{-1} \) and \( U_2 = (1 - r'_1s_{WL}r_2s_{WL})^{-1} \).

Fig. 2 shows the transmission (conductance) of an electron through the bi-ripple waveguide for different flat waveguide lengths \( WL \). The dashed line is the electron transmission for a waveguide with a single ripple cavity. We can see the resonance width narrowing and broadening (Dicke effect) depending on \( WL \).

Fig. 3 shows how the S-matrix poles change position in a complex energy plane. The two poles induce the two
resonances in the transmission plot (Fig. 2). The two poles rotate on a circle (which is centered at the pole position for a waveguide with a single cavity (red * in Fig. 3) as the distance WL is varied. The behavior of the poles indicates that not only the widths but also the resonance energy (real part of the pole) changes with WL. The pair of poles is located on opposite sides of the circle, which means they have a \( \pi \) phase difference. As WL increases, one of the poles gets closer to the real axis while the other moves away from the real axis (See Fig. 3). As one of the poles gets closer to the real axis, its corresponding resonance becomes sharper (long-lived state, \textit{subradiance}) while the other resonance becomes broader (short-lived state, \textit{superradiance}). The narrowing and broadening of the resonance width is shown in Fig. 2 (b) \( \sim \) (f).

As the poles rotate, it becomes possible for one of the poles to reach the real axis and then the width of the resonance collapses, indicating that the lifetime of the resonance has become infinite. When this happens, the resonance state is completely decoupled from external leads. This phenomena has been called “bound state in continuum” (BIC) in other studies \[15, 16\].

Let us express the S-matrix pole position for a waveguide with a single ripple cavity as \( E = E_0 - i \Gamma_0 \) (red * in the Fig. ref fig3). Since S-matrix poles for the bi-ripple waveguide rotate on a circle in the complex energy plane, the position of the poles can be written as a sinusoidal function of WL such that,

\[
E_1 = E_+ - i \Gamma_+ \quad \text{and} \quad E_2 = E_- - i \Gamma_-
\]

(4)

where

\[
E_+ = E_0 + \Gamma_0 \sin(kL + \delta_0) \quad \text{(5)}
\]

\[
E_- = E_0 - \Gamma_0 \sin(kL + \delta_0)
\]

and

\[
\Gamma_+ = \Gamma_0 + \Gamma_0 \cos(kL + \delta_0) = \Gamma_0 (1 + \alpha) \quad \text{(6)}
\]

\[
\Gamma_- = \Gamma_0 - \Gamma_0 \cos(kL + \delta_0) = \Gamma_0 (1 - \alpha).
\]

The quantity \( \alpha = \cos(kL + \delta_0) \) is a measure of the coupling between the cavities, \( k \) is a wave number at a resonance energy, and a phase factor, \( \delta_0 \) is used because a pole is not on the real axis when \( WL = 0 \). Fig. 4 shows the pole trajectories in the complex energy plane as WL increases up to 250 A. It confirms that the pair of poles always stays on a circle and it verifies Eq. 5 and 6.

As was shown in Refs. \[11 \] and \[12 \], a sequence of resonances occurs as energy in the interval \( E_1 \leq E \leq 4E_1 \) is increased. Fig. 5(a) shows the width of the narrow resonance as a function of the distance WL for the resonance at energy \( E = 1.3278E_1 \) and 5(b) shows the width of the resonance at energy \( E = 1.5830E_1 \). The resonance widths are clearly sinusoidal functions of WL, which indicates that the coupling between the two cavities varies sinusoidally with increasing WL. Fig. 5(b) oscillates slightly faster than 5(a) because of its higher resonance energy. In Eq. 5 the coupling parameter \( \alpha \) is a sinusoidal function which gives the oscillatory behavior of the width in Fig. 2. This behavior of the coupling parameter in our 1D system is different from that seen in higher dimension. In Ref. \[8 \], the coupling between a collective state of two impurities and external electron wavefunction (a 2D system) is a Bessel function \((J_0(s_{12}/\lambda_f)) \) (See Eq. (19) in Ref. \[8 \]). In the double quantum dots coupled through the common phonon fields \[17 \] (a 3D system), the coupling parameter is a zeroth order Bessel function \((\sin(Qd)/Qd)\). In these 2D and 3D systems, the coupling parameter decays with the distance between two atoms, as is the case for the Dicke system (a 3D system). This dependence of the coupling on zeroth order Bessel functions (which have their largest values at long wavelength), means that in 2D and 3D systems the superradiance occurs predominantly with the wavelengths much longer than the atomic distance. However, in our 1D waveguide system (the transverse direction is constrained), the coupling parameter \( (\alpha) \) does not decay. Therefore, the superradiant resonance exists regardless of the wavelength of the electron. It is possible that a superradiant resonance appears with a very large distance between two cavities, but it could not exceed the coherence length of electrons in the waveguide. In 2DEGs made of GaAs/AlGaAs, for example, at a very low temperature the coherence length is about 10\( \mu \)m.

As we have seen, the two S-matrix poles induce two resonances in the electron transmission. The behavior of these poles is associated with the symmetry of the scattering wavefunction at resonance energies. In Dicke’s model, \textit{subradiant resonance} is related to the antisymmetric collective state of the atomic system and the \textit{superradiant resonance} is related to the symmetric collective state. In the bi-ripple waveguide, as the pair of poles rotate in a counterclockwise direction (with increasing distance between the two ripple cavities), they each
FIG. 5: (Color online) The resonance width $\Gamma_-$ for a bi-ripple waveguide. $\Gamma_-$ oscillates as $WL$ is varied. The blue line (a) is for the resonance near $E = 1.3278E_1$. The red line (b) is for the resonance near $E = 1.5830E_1$.

In conclusion, we have studied the behavior of quasi bound states in a bi-ripple waveguide as the distance between the two ripple cavities changes. We found that the widths and positions of the resonances associated with the quasi bound state poles change in a sinusoidal way with variation of the distance between the ripple cavities. In 2D and 3D models of the Dicke effect, the coupling parameter decays when the distance between two atoms (impurities or quantum dots) is longer than the wavelength of photon or electron. In the ripple waveguide system, the coupling parameter does not decay because the waveguide is a quasi 1-D system. We also studied the trajectories of S-matrix poles in the complex energy plane. We found that the pair of poles that give rise to the super- and subradiant resonances in the electron transmission, rotate on a circle centered on the S-matrix pole for the waveguide with a single cavity and are phase shifted by $\pi$. Therefore, superradiant and subradiant resonances appear in oscillatory manner as the distance between cavities is changed. Furthermore, as the S-matrix poles rotate, the symmetry of the electron state associated with the superradiant (subradiant) resonance alternates between symmetric and anti-symmetric with increasing distance between the two ripple cavities.

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