Towards a mathematical model of plant growth

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Abstract. Mankind has long been interested in the growth of plants, even in the Middle Ages, Leonardo da Vinci observed the seasonality of growth and some features of plant forms. In recent years, many agricultural producers have begun to implement a coordinate (precise) farming system. In this regard, there is a need to improve methods for assessing and managing the growth of agricultural crops. To assess the functioning of the system “plant – soil – air” (P–S–A), the concept of plant growth potential is introduced, which is the ratio of the power spent on the formation of a unit of vegetation mass. The paper considers the theoretical prerequisites for determining the growth potential. Since the operational management of the formation of the crop yield is an important task in crop production, the functioning of the P–S–A system during the growing season is proposed to be evaluated by dimensionless coefficients, which represent the ratio of the substance mastered by plants to the incoming one. The product of these coefficients in terms of light-heat-food-gas-and moisture supply represents the reliability of the P–S–A system, and allows you to evaluate and effectively manage the technological process of the growth of cultivated plants during the growing season.

1. Introduction
The variety of methods and approaches to the mathematical modelling of the plant – soil – air (P–S–A) system functioning can be divided into the following groups: the theory of stability and bifurcations [1, 2], catastrophes [3, 4], thermodynamic approach [5-7], automata [1, 2, 8], and the application of the principles of optimality in biology [9]. In particular, there has been presented a system of differential equations of interaction between a plant and a nutrient solution [10]. Moreover, the cation absorption model was derived from Ohm's law in combination with Goldman's equation. And the anion absorption model is similar to the chemical reaction rate model. The application of the network topology of the interaction of plants and nutrient solutions seems appropriate and in fact reflects the third law of thermodynamics of Nernst. There are also a number of works on mathematical modelling of plant interaction with soil containing heavy metals [11], physiological and biological indicators [12], studying plant growth by shooting phenotyping. Of particular interest are works on modelling competitive interactions and feedback between plants and soil during the growing season [13]. Numerical modelling reveals the occurrence of spatio-temporal patterns in the field of the influence of Hopf bifurcation and the rate of biomass propagation. Without dwelling on questions about the advantages of a particular method or approach, we note that some of the elements listed above can be mutually successfully applied. Nevertheless, it should be noted that the thermodynamic approach is more promising, since to describe the extremely complex nature of energy and mass transfer processes in the P–S–A system (the total number of environmental factors and related indicators that determine the value and quality of the...
crop can reach three hundred) [10], non-equilibrium thermodynamics of irreversible processes proceeds from more general ideas about the object and, most importantly, this method allows you to measure a number of quantities included in differential equations [6]. As noted by the founder of nonequilibrium thermodynamics Prof. I. R. Prigozhin [6] "irreversible processes are as real as reversible ones, and do not correspond to the additional restrictions that we have to impose on the laws that are reversible in time", "irreversible processes play an essential constructive role in the physical world. They are the basis of important coherent processes that are particularly clearly manifested at the biological level", "irreversibility is deeply connected with dynamics. We can say that irreversibility occurs where the basic concepts of classical or quantum mechanics cease to correspond to the observed ones."

Thus, taking into account the requirements of the coordinate (exact) system of agriculture on the one hand, the reliability of obtaining the planned crop yield on the other, it is possible to formulate the purpose of the research: improving the mathematical model of the functioning of the plant–soil–air system for making operational management decisions.

2. Mathematical model

To assess the functioning of the “plant – soil – air” system, starting from sowing to harvesting, it is proposed to use the plant growth potential, which is the ratio of the power spent on the formation of a unit of vegetation mass:

\[ \xi = \frac{\Delta N}{\Delta m} \] (1)

where \( \Delta N \) is the power spent on forming a unit of plant mass \( \Delta m \).

In such a statement, the question of plant growth will be reduced to determining or measuring the values of \( \Delta N \) and \( \Delta m \). In this paper, we propose an analytical definition.

The power spent on forming a unit of plant mass can be roughly determined from the fundamentals of non-equilibrium thermodynamics of irreversible processes, since kinetic equations of the form are performed near the equilibrium state for irreversible processes [6, 7, 14]:

\[ \sum_{i=1}^{n} L_{ij} X_{j} = 1, 2, 3, 4 \] (2)

where \( I_i \) is thermodynamic flows, \( X_j \) – thermodynamic forces, and \( L_{ij} = const \) – phenomenological coefficients.

Since 1919, a number of equations have been created for the mathematical description of the growth of an individual plant organ, an entire plant, or the entire vegetation cover [4, 15-17]. In our opinion, the most effective one should be considered the equation of Dr. Yu. K. Ross [16]:

\[ \frac{dm}{d\tau} = k_F F_j - k_R R_j - V_j + \sum_{i=1}^{4} (A_{ij} - A_{ji}) + \sum_{i=1}^{4} (B_{ij} - B_{ji}) \] (3)

where \( k_F \) and \( k_R \) are the proportionality coefficients for photosynthesis and respiration, respectively; \( F_j \) and \( R_j \) are intensities of photosynthesis and respiration, respectively, of the \( j \)-th organ (1 means leaves, 2 – stems, 3 – roots, and 4 – reproductive organs); \( V_j \) – the rate of falling off of dead parts of the \( j \)-th organ; \( A_{ij} \) and \( B_{ij} \) – the rate of outflow of "fresh" and "old" assimilates, respectively, from the \( i \)-th organ to the \( j \)-th organ and \( A_{ji} - A_{ij} \) and \( B_{ji} - B_{ij} \) – the rate of exchange of "fresh" and "old" assimilates between the \( i \)-th and \( j \)-th organs, respectively. "Fresh" assimilates mean the biomass created during photosynthesis per unit of time, and "old" – the biomass created before that.

By integrating equations (2) and (3) with respect to \( dt \) and substituting in (1), we obtain the numerical value \( \xi \).

Considering the representativeness of dimensionless quantities for monitoring and operational management, for example, in protected ground conditions, for the formation of crop yields, we introduce [10] dimensionless coefficients for light–heat–food–gas and moisture supply.
2.1. The coefficient of light availability of me in the P–S–A system on the slopes

The radiation balance \( R \), which is formed on the underlying surface, consists of the part of \( R^{(o)} \) that reaches the soil surface, and the remainder of \( R^{(h)} \), which is formed at the upper boundary of H plants.

Since the slopes are located in different exposures (southern, northern, eastern, western, north-eastern, south-eastern, north-western, south-western), the value of \( m_e \) for different exposures of the slope will be recorded:

\[
m_e = \frac{R^{(H)}}{R^{(H_e)} + R^{(n_e)}}
\]

For horizontally located agricultural fields, each element \( R^{(o)} \) and \( R^{(h)} \) can be written in expanded form: the equation of the thermal balance at the level of the soil surface \( X=0 \):

\[
R^{(o)} = \lambda \frac{\partial T}{\partial x} \bigg|_{x=0} + C_p \rho k(x) \frac{\partial T}{\partial x} \bigg|_{x=0} + L \rho k(x) \frac{\partial q}{\partial x} \bigg|_{x=0}
\]

equation of the heat balance at the height of the plant crowns \( X=H \):

\[
R = R^{(H)} + R^{(o)} = -C_p \rho k(x) \frac{\partial T}{\partial x} \bigg|_{x=H-0} + C_p \rho k(x) \frac{\partial T}{\partial x} \bigg|_{x=H+0} - L \rho k(x) \frac{\partial q}{\partial x} \bigg|_{x=H-0} - L \rho k(x) \frac{\partial q}{\partial x} \bigg|_{x=H+0} + R^{(n)}
\]

where \( \lambda \) is the coefficient of thermal conductivity of the soil, W/(m·K); \( k(x) \) – the coefficient of turbulence, m²/s; \( C_p \) – the thermal conductivity of the air, J/(kg·K); \( \rho \) – air density, kg/m³; \( L \) – latent heat of water vaporization, J/kg; \( q \) – specific air humidity; \( \partial T/\partial x \) – vertical gradient of air temperature; \( \partial q/\partial x \) – vertical gradient of air humidity; \( H \) – plant height, m; \( X \) – a single designation for the temperature of the soil – air system \( T \) along the \( x \) coordinate, the positive value of which is counted towards the air \( x>0 \), and the negative value – towards the soil.

Since the coefficients and values included in equation (5) and (6) for slopes of different steepness and exposure are different, their influence on the coefficient of light availability of the \( m_e \) seems appropriate to take into account the generalized coefficient of steepness and exposure of the slopes of the \( k_{ke} \):

\[
m_e = k_{ke} \frac{R^{(H)}}{R^{(H)} + R^{(n)}}
\]

2.2. Coefficient of moisture availability of n_e in the P-S-A system on slopes

The coefficient of moisture availability of \( n_e \), which is a fraction, for slopes of different steepness and exposure can be determined by the expression (8):

\[
n_e = \frac{U_e + C_e + \Phi_e \downarrow}{W_e + O_e + \Phi_e \uparrow}
\]

where \( U_e \) – total evaporation by the system (water evaporation by the soil and evaporation from the surface of plant leaves or transpiration); \( C_e \) – slope runoff of precipitation; \( \Phi_e \downarrow \) – vertical filtration (from top to bottom); \( W_e \downarrow \) – initial moisture reserves in the soil; \( O_e \) – precipitation; \( \Phi_e \uparrow \) – vertical filtration (from bottom to top).

Obviously, if \( n_e=0 \), the system is not provided with moisture, and if \( n_e=1 \), all available moisture is used by the system P–S–A.
2.3. Coefficient of heat supply of pe in the P–S–A system on slopes

The coefficient of heat supply $p_e$ in the plant–soil–air system on the slopes can be determined by the formula (9):

$$p_e = \frac{(C_{b\Delta T_e})_{be} + (C_{p\Delta T_e})_{pe} + (C_{n\Delta T_e})_{ne} + \lambda \frac{T_e}{\Delta x}|_{x=a}}{Q_{ce}^H + Q_{qe}^H}$$

where $(C_{b\Delta T_e})_{be}$ – heat accumulation in the plant growth layer, J/(m$^2$ꞏs); $C_b$ – volumetric heat capacity of the air layer between plants, J/(m$^3$ꞏsꞏK); $\Delta T_e$ – temperature change in the plant growth layer during the observation period, K/m; $(C_{p\Delta T_e})_{pe}$ – heat storage of plants, J/(m$^2$ꞏs); $C_p$ – volumetric heat capacity of plant mass ($\Delta T_e$ in this combination $(C_{p\Delta T_e})_{pe}$ represents the change in plant temperature, i.e. the average temperature of leaves of different tiers, stems, etc., K/m); $(C_{n\Delta T_e})_{ne}$ – heat content of the upper root layer (0 - a), containing soil and plant roots, J/(m$^2$ꞏs); $C_n$ – volumetric heat capacity of the soil of the root layer (0 - a), J/(m$^3$ꞏsꞏK); $\Delta T_e$ in combination $(C_{n\Delta T_e})_{ne}$ represents a change in the temperature of the upper root layer (0 - a), K/m; $\lambda \frac{T_e}{\Delta x}|_{x=a}$ – heat flow into the soil starting from the depth $x=a$, J/(m$^2$ꞏs); $\lambda$ – coefficient of thermal conductivity of the soil in the layer (a=∞), W/(mꞏK); $\Delta T_e$ in combination $\lambda \frac{T_e}{\Delta x}|_{x=a}$ represents the change in temperature in the layer (a=∞), K/m; $Q_{ce}^H + Q_{qe}^H$ – the total short-wave and long-wave radiation coming from the atmosphere to the upper level of plants, J/(m$^2$ꞏs).

In the given expression (9), the numerator is when subtracting from the total arrival of solar heat $(Q_{ce}^H + Q_{qe}^H)$ the value of the energy accumulated by the system (plant – soil – air), and the denominator – heat flows to it. Therefore, the change of the $p_e$ is possible in the range from 0 to 1.

2.4. The coefficient of food security of re in the P–S–A system on the slopes

The coefficient of food availability of $r_e$ on sloping lands of different steepness and exposure from the coefficient of $r_f$ will differ primarily from the additional component of the $M_e$, which takes into account the change in all the elements of nutrition as a result of erosion processes:

$$r_e = \frac{\sum M_{(i,j,v,k,q.l)e}}{\sum M_{(i,j,v,k,q.l)e} + \sum M_{(i,j,v,k,q.l)e} - \sum M_{(i,j,v,k,q.l)e} + \sum M_{(i,j,v,k,q.l)e} - \sum M_{(i,j,v,k,q.l)e}}$$

(10)

Here, the values included in (10) are indicated by the index "e", which means different exposures and steepness of the slopes; $\sum M_e$ – the sum of the contents of each food element on slopes of different steepness and exposure; the "-" sign is placed when the nutrients are washed out together with the soil as a result of erosion processes; the "+" sign is placed when the accumulation of erosion products on different parts of the slope results in an influx of nutrients in all forms; $\sum M_{(i,j,v,k,q.l)e}$ – the part of the food that the system has mastered for all the elements in the agricultural field; $\sum M_{(i,j,v,k,q.l)e}$ – the sum of the initial contents of each battery ($i=1,2,3\ldots$) in an agricultural field; $\sum M_{(i,j,v,k,q.l)e}$ – the sum of all the amounts of nutrition introduced into the soil, differing by type ($j=1,2,3\ldots$) and the form ($v=1,2,3\ldots$) in an agricultural field; $\sum M_{(i,j,v,k,q.l)e}$ – the sum of all the amounts of nutrition resulting from leaching and volatilization in the agricultural field. 

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2.5. The coefficient of gas supply of $s_e$ in the $P$–$S$–$A$ system on the slopes

The coefficient of gas supply of $s_e$ on slopes of different steepness and exposure is obviously determined by the expression (11):

$$
s_e = \frac{\rho_c k_e(x) \left[ \frac{\partial C_e}{\partial x} \bigg|_{x=H+0} + \frac{\partial C_e}{\partial x} \bigg|_{x=H-0} \right]}{C_e}
$$

(11)

Here, the values included in the above equation (11) are indicated by the index "e", which means different exposures and steepness of the slopes; $\rho_c$ – carbon dioxide density CO$_2$; $k_e(x)$ – coefficient of turbulent diffusion in an agricultural field; $\frac{\partial C_e}{\partial x} \bigg|_{x=H+0}$ – vertical gradient of the average volume concentration of carbon dioxide from the atmosphere at the level of plant growth in an agricultural field; $\frac{\partial C_e}{\partial x} \bigg|_{x=H-0}$ – vertical gradient of the average volume concentration of carbon dioxide from the soil in an agricultural field; $C_e$ – the amount of carbon dioxide produced at the level of plant growth in the agricultural field.

Note that the determination of the coefficients of $m_e$, $n_e$, $p_e$, $r_e$ and $s_e$ can be significantly facilitated if they are expressed in terms of corresponding coefficients for horizontally located areas and on the basis of standard meteorological and hydrological data. The experiments made it possible to introduce corrections to each of these coefficients of $m_e$, $n_e$, $p_e$, $r_e$ and $s_e$, taking into account the features associated with different exposure and slope steepness.

3. Results and discussion

The dynamics of changes in the yield of grain crops (figure 1) shows that the relatively low yields of the European part of Russia are primarily due to soil degradation, erosion processes and emergency situations of natural and climatic conditions (frost, drought, floods, etc.). Jumps in yields in Russia and Germany are explained by wars. As noted above, the number of factors affecting the yield can reach three hundred. It is quite difficult to take into account and generalize the above. Therefore, we consider the proposed assessment of the functioning of the “plant–soil–air” system, starting from sowing to harvesting, to be more appropriate, with the plant growth potential, which is the ratio of the power spent on the formation of a unit of vegetation mass. From the point of view of nonequilibrium thermodynamics of irreversible processes, this approach is the most objective, since: if the sown seed has not risen, then it will die; the ascended plant successfully develops or dies as a result of natural and climatic conditions, that is, it can be represented in the first approximation by the bifurcation equations (figure 2). It should be noted that the presented graph of the plant growth function from time to time is consistent with a number of studies [2-5, 10, 14].

For control and operational management, for example, in conditions of protected soil, for the formation of crop yields, dimensionless coefficients for light-heat-food-gas and moisture supply have been introduced, representing the ratio of the substance mastered by plants to the incoming one.
Figure 1. Changes in the yield of grain crops.

Figure 2. Plant growth function over time: 1 – freezing, 2 – drought, 3 – floods, n – other catastrophic conditions.

‘Figure 3’ shows the results of studies [18, 19] of plant growth depending on soil moisture (withering moisture), which practically coincide with the bifurcation diagram given in [10].

A brief description of the experiments is as follows. Soil drought was simulated in the tillering phase, for lettuce – at the age of 25 days; the experiment lasted 7-24 days. Soil moisture at the beginning of the experiment was equal to 24-25%, i.e. close to the field (total) moisture capacity. In the future, it decreased, especially rapidly under oats and lettuce, which had the largest leaf surface, Wheat and especially millet, whose leaf area is small, absorbed moisture 2-3 times slower, respectively. Soil moisture at the end of the experiment, when the plants experienced steady wilting, varied from 15% (for lettuce) to 7% (for oats, wheat and millet) (figure 3a).
Moisture pressure, as well as soil moisture, also decreased throughout the experiment, but less evenly than humidity (figure 3b). At the beginning of the experiment, it decreased very slowly, and then faster and faster. This is due to the fact that the differential moisture capacity (dW/dP) of the soil also decreases more sharply as humidity decreases. The initial level of moisture pressure in the soil for all crops was close to zero (–6...–3 atm), the final level reached –16 atm for lettuce, –35 atm for wheat and –40 atm for oats and millet.

The moisture pressure in plant leaves also decreased rapidly during the experiment: from –8 to –20 atm for lettuce, from –15 to –45 atm for oats, from –15 to –60 atm for wheat and up to –65 atm for millet (figure 3c). Thus, the lower level of the moisture pressure in the leaves achieved during drought correlates well with the degree of xerophytic plant: when withering, the moisture pressure in the leaves of xerophytes is 3 times lower than that of hygrophytes. This undoubtedly shows the hereditary nature of xerophyte plants (their genotype), which has developed in the process of their age-old adaptation to the conditions of annual soil moisture deficiency during the growing season. In fact, the more the moisture pressure in the leaves (and other plant organs) can decrease, the greater the moisture pressure drop between the soil and plants, the flow of moisture from the soil to the roots and further to other plant organs and, consequently, transpiration.

Relative transpiration (figure 3d) is maintained at level 1 for some time, and then falls, reaching very low values when plants wither (0.4 for lettuce and 0.2 for other plants). These characteristic features of the dynamics of relative transpiration become more understandable if additional parameters are used in the analysis: soil moisture conductivity coefficient and moisture pressure drop between plants and soil.

Since there are quite a lot of emergency situations during the growing season of plants, we consider it appropriate to apply the concept of reliability.

Depending on the biological time t of plant development (seed germination, seedling formation, tillering, stemming, ear formation, earing, flowering, development of milk ripeness, development of wax ripeness, development of full ripeness for cereals; it is possible to consider for other types of agricultural crops), the values of the coefficients are variable, then the entry will be more fair:

\[ H_{\text{pub}}(t) = m_c(t) \cdot n_c(t) \cdot p_c(t) \cdot r_c(t) \cdot s_c(t). \]  

\[ (12) \]
If any coefficient of light-heat-food-gas-and moisture supply in (12) is zero, the plant will die. Thus, the successful development and formation of the crop will be when $H_{suhr}>0$.

4. Conclusion
Catastrophic droughts in the 1970s and 1980s, which affected almost all continents of the world, negative temperature fluctuations associated with global warming make us reconsider the trends in the development of modern crop production. The tendency of increasing dependence of the variability of the size and quality of the crop yield on weather fluctuations forms the task of increasing the stability of crop production in terms of light, heat, food, gas and moisture supply. In this article, we tried to apply a thermodynamic approach to describe the functioning of the “plant–soil–air” system.

Based on the nonequilibrium thermodynamics of irreversible processes, to assess the functioning of the system of coefficients for light–heat–food–gas and moisture availability during the growing season, the plant growth potential is proposed, which is the ratio of the power spent on the formation of a unit of vegetation mass. From the point of view of nonequilibrium thermodynamics of irreversible processes, this approach is the most objective, since: if the sown seed has not risen, then it will die; a plant that has risen successfully develops or dies as a result of, for example, drought or other natural and climatic phenomena, that is, it can be represented in the first approximation by bifurcation equations. For the control and operational management of the formation of the crop yield, dimensionless coefficients for light-heat- food- gas and moisture supply have been introduced, representing the ratio of the mastered substance by plants to the incoming one. The product of these coefficients in terms of light-heat- food-gas and moisture supply represents the reliability of the functioning of the plant–soil–air system. Finally, an additional goal of the research is to determine the limits of variation of the coefficients of light–heat–food–gas and moisture availability for the sustainable functioning of the “plant–soil–air system”.

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