Effects of viscoelastic bonding layer on performance of piezoelectric actuator attached to elastic structure

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Abstract

In the context of the finite elements analysis, the mechanical performance of viscoelastically bonded smart structures is investigated and analyzed. Three different models are considered and compared. In the 1st model, the actuator is glued to the host structure. On the other hand, in the two other models the actuator is glued to the bonding layer which is glued to the host structures. To explore the effect of the bonding layer characteristics on the mechanical behavior of the host structure, both elastic and viscoelastic layers are considered. The Prony’s series are utilized to simulate the viscoelastic constitutive response. The mathematical formulation of the coupled problem is presented and the dynamic finite elements equations of motion of the coupled electromechanical systems are introduced. The proposed methodology is verified by comparing the obtained results with the available results in the literature and good consentience is observed. Both static and dynamic vibration behaviors are studied incorporating the interfacial shear stresses between the bonding layer and the host structure as well as the displacements as a comparison criterion to determine the performance controlling function of the host structure. Parametric study of piezoelectric properties showed that permittivity is required in solving such systems but does not affect the performance. On the other hand, the piezoelectric characteristics have significant effects on the mechanical performance of smart structures and can be used in the optimum selection of combination just like mechanical properties and geometry. Additionally, the obtained results show that the model with viscoelastic bonding layer has an overall static performance nearly half of elastic bonding layer model while it has a slight effect on the dynamic behavior compared with the corresponding elastic bonding layer. The proposed methodology with the obtained results is supportive in the applications of structure health monitoring and dynamics of smart structural systems. The proposed procedure could be extended in a future work to include the coupled electromagnetic effects on the dynamic behavior of smart structures in hygrothermal environment.

1. Introduction

Piezoelectric materials are considered as the smart materials that have abilities to change their physical properties when they are under specific external provocateur. Piezoelectric materials have two abilities, one is known as direct effect in which the material produces electric charges when it is under stress, the other is the inverse effect where the material produces stress or change in dimensions when it is under electric field. Since their discovery, they have been widely used in many applications either as sensor based on direct effect or as actuator based on inverse effect. One of these applications is a structure health monitoring, in which piezoelectric is used to control vibrations on structures due to their rapid reaction and affordability (Kuna 2006, Nguyen-Xuan et al 2009, Nanthakumar et al 2013, Chen et al 2019, Gao and Zhang 2020).
Studying piezoelectric actuators in structures controlling have been under consideration for a long time, including analytical, numerical or hybrid studies (Eltaher et al 2019, 2020a, 2020b). Reasons of the extensive studies includes the fact of the discontinuity found between the host structure and the actuator resulting in complicated stresses especially at the tip of the actuator which may lead to debonding effects. Therefore, actuators should be well studied to predict their behavior accurately. The aim of this study is to investigate the actual performance of actuators attached to structure, as possible, in the setting of structures health monitoring (SHM). As will be mentioned, most studied either examine the performance of piezoelectric actuators without considering bonding layer or considering the bonding layer but as elastic materials which in reality they are viscoelastic materials. In addition, the effects of piezoelectric properties mainly, permittivity and piezoelectric constants on the designing and material selection are studied to show their importance as many studies focused on geometry and mechanical properties. Some studies focused in developing analytical models to evaluate the performance of piezoelectric actuator based on either shear-lag theory, pin-forced model, or elasticity equation-based model. (Bailey and Hubbard 1985, Crawley and De Luis 1987, Im and Atluri 1989, Lee and Moon 1989, Crawley and Anderson 1990, Wang and Rogers 1991, Wang and Meguid 2000, Wang and Huang 2001).

Many recent researchers exploit finite element method (FEM) either for pure analyzing or validating results of experiments and analytical models of composite, coupled materials and complex structures and materials (Safaei 2020, Wang et al 2021, Yang et al 2021, Liu et al 2022, Zhang et al 2022). And this method have been used in many researches. Ong et al (2003) created a one-dimension analytical model based on shear lag between piezoelectric patch and host structure. Results of the model were validated by numerical analysis. Sun and Tong (2002) focused on effects of debonding on the dynamic behavior of beams with piezoelectric actuators. They developed a semi-analytical model based on the interfacial displacement and shear to evaluate the frequency response of the beams. Luo and Tong (2002) studied peel stresses of piezoelectric plate by formulating FEM. Taking a lower shear and elastic moduli for the adhesive layer to model peel stresses. Nienwenhui et al (2005) developed FEM consists of a piezoelectric patch attached to an isotropic plate to study the performance as wafer transducer to produce waves considering pulser reactions. Liu and Giurgiutiu (2007) developed a FE model to simulate piezoelectric wafer active sensors PWAS in various structures for structural health monitoring (SHM).

Many research works were done based on hybrid techniques, where piezoelectric part is solved by FEM to evaluate excitation and used as input for analytical model, then, evaluate the host structure response. (Moulin et al 2008, Guo and Jin 2009, Xiang and Shi 2009). Among those various studies with different methods, some studies considered effects of bonding layer, as the fact that piezoelectric patches are attached with host structures by bonding methods which can be detached due aging or excess applied forces. More accurate prediction of the performance can be obtained if bonding layer is considered in models. Jin and Wang (2011) developed a static model to include adhesive layer effects. They examined the effect of different adhesive layer properties and different geometries on the performance of the actuator, especially the shear stress developed at the interfacial between the actuator and host structure. Benjeddou et al (2013) studied dynamically a piezoelectric patch bonded to a plate of carbon composite to find optimum piezoelectric and dielectric behaviors. Rao et al (2014) investigated debonding effects on piezoelectric sandwich beam bending behavior. Yu and Wang (2016) studied the effects of material properties, bonding layer, and loading frequency by developing a pure finite element model in ANSYS based on the outcomes of Jin and Wang (2011). Their results agreed to some extend to Wang and Meguid (2000), Wang and Huang (2001), and Jin and Wang (2011) with the fact of singularity is present at the tip of the actuator. They also found the increase of elasticity modulus of host structure will increase shear stresses along actuator, but the displacement is reduced. Singh et al (2017) investigated analytically effects of imperfect bonded piezoelectric layer on the propagation of love-type wave. Daşdemir (2017) constructed a mathematical model of imperfect bonding of a sandwich plate with piezoelectric middle layer to evaluate the dynamic response. The model was developed by FEM.

Singh et al (2018) developed an analytical model to study the effect of loose bonding between functionally graded piezoelectric layer and a functionally graded fiber-reinforced structure, based on love-type wave propagation. Peijian Chen et al (2018) studied the performance of piezoelectric actuator bonded perfectly and imperfectly by adhesive layer with graded half plane, which its shear modulus varies along its depth, considering interfacial stresses. Yousefsani and Tahani (2018) focused on the interfacial stresses between adhesively bonded piezoelectric actuator and composite joints by developing analytical models based on layer wise theory. Kumar et al (2019) developed an analytical model based on micropolar theory to investigate the effect of imperfect bonded piezoelectric layer over a micropolar elastic half space. Liu et al (2019) studied debonding of piezoelectric actuators and sensor in SHM systems using signal-based method, they found that with larger debonding the phase offset will raise proportionally. Meng et al (2019) developed the mathematical models of PZT sensor to include damping resulted from adhesive layer by correcting these models with experimental studies results. Rautela and Bijudas (2019) detected and recognized the debonding of piezoelectric transducers of a structure by constructing a 1 DOF model of electromechanical admittance. Under non-slipping JKR model,
Luo and Zhou (2021) studied analytically adhesive contact between piezoelectric biomaterials, and derived solutions of stress and electric displacement.

The present work is considered as a natural extension to the mentioned works by using FE modeling to discover the mechanical performance of smart structures considering different piezoelectric properties and considering viscoelastic bonding layer rather than elastic one. The Prony’s series is adopted to model the viscoelastic constitutive relations of the bonding layer. A comprehensive model is provided including the mathematical equations and boundary conditions simplifications that control the physical phenomena. Based on the mathematical formulation, the coupled dynamic FE model is presented and implemented using ANSYS commercial software. The efficiency and accuracy of the proposed model is examined by comparing the obtained results with the corresponding available different models found in literature, including models with direct bonding between actuator and hosts structure, models with high host structure elasticity moduli, and model including bonding or adhesive layer. Numerical studies are performed to demonstrate the capability of the proposed computational procedure in studying and analyzing the static as well as the dynamic behavior of smart structures. Throughout these numerical experiments, the interfacial shear stresses and displacements are considered as the criteria of the mechanical performance of piezoelectric smart structure system with viscoelastic adhesive layer. Based on the obtained results, concluding remarks and recommendations for a future work are summarized.

2. Mathematical formulation

The utility of piezoelectric actuators in structure health monitoring (SHM) can take different shapes depending on the structure size and geometry. Therefore, to simulate and model piezoelectric actuator that is very thin, 2D plane strain condition could be considered. The bonding layer is considered thinner than the actuator but with the same length. The poling direction of the piezoelectric actuator is taken at $y$-axis with applied voltage on the same direction. Figure 1 shows the structure in two cases without and with bonding layer, respectively. The mathematical formulation of the problem is presented as follows (Han et al 2009, Jin et al 2010, Gao et al 2021):

It is assumed that, the poling direction of the actuator is along the $y$-axis. The applied voltage between the upper and lower electrodes of the actuator results in an electric frequency $\omega$ along the poling direction of the actuator. The electric field intensity can be given by (Han et al 2009, Jin et al 2010, Gao et al 2021)

$$E_y = \frac{(V^- + V^+)}{h}$$

with $V^-$ and $V^+$ refer to the electric potential at the lower and upper surface of the actuator; respectively, $h$ is the actuator thickness.

The dynamic equations of motion of the host structure can be expressed as

$$\sigma_{ij} + f_i = \rho \ddot{U}$$

where $\sigma_{ij}$, $f_i$, $\rho$ are respectively the Cauchy stress tensor, the body force per unit volume, and the mass density. $\dot{U}$ refers to the host structure acceleration. The plane strain dynamic displacement field in the homogeneous isotropic elastic host structure is given by, Nowacki (2018)

$$u_x = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y}, \quad u_y = \frac{\partial \Phi}{\partial y} - \frac{\partial \Psi}{\partial x}$$

in which $\Phi$ and $\Psi$ are two displacement potentials, which satisfy Nowacki (2018)

$$(\nabla^2 + K^2)\Phi = 0$$

where $\nabla^2$ is the Laplacian operator; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. While $K$ and $k$ are two wave numbers which are defined as Nowacki (2018)

$$K = \frac{\omega}{C_L}, \quad k = \frac{\omega}{C_T}$$

with $C_L$ and $C_T$ are the longitudinal and transverse shear wave speed of the elastic medium, respectively. The corresponding boundary conditions at the interface are expressed as Nowacki (2018)

$$\begin{align*}
\tau_{xy}(x, 0) &= -\tau \frac{|x|}{a}, & \sigma_y(x, 0) &= 0, \quad |x| > a; \\
\tau_{xy}(x, 0) &= 0, & \sigma_y(x, 0) &= 0, \quad |x| < a.
\end{align*}$$

with $a$ refers to the half length of contact between the actuator, bonding layer and the host structure. With bonding layer having a span of $2a$ and thickness $l$ both the geometry and the material characteristics of the bonding layer significantly affect the interfacial shear stress between the layer and the host structure.
Assuming linear viscoelastic constitutive response, the material constitutive relations can be expressed as (Abdelrahman et al 2020, Abdelrahman and Elshafei 2021)

\[
\sigma_{ij}(x, t) = \int_{s=0}^{t} R_{ijkl}(t - s) \frac{\partial E_{kl}(x, s)}{\partial s} ds = R_{ijkl}(0) \varepsilon_{kl}(x, t) \\
= -\int_{0}^{t} \frac{dR_{ijkl}(t - s)}{ds} \varepsilon_{kl}(x, s) ds
\]  

where \(\sigma_{ij}(x, t)\) and \(\varepsilon_{kl}(x, t)\) are respectively the Cauchy stress and the infinitesimal strain tensors; \(R_{ijkl}(t - s)\) is the relaxation modulus, and \(s\) is a time variable which defines the time at which the excitation input is applied. According to the generalized Maxwell model, the relaxation modulus can be repressed by Abdelrahman and Elshafei (2021)

\[
R_{ijkl}(t - s) = E_{ijkl} \left(1 - \sum_{n=1}^{N} E_{ijkl} \left(1 - \exp \left(-\frac{(t - s)}{\tau_{n}}\right)\right)\right)
\]  

where \(E_{ijkl}\) and \(E_{ijkl}^{\infty}\) are the instantaneous and transient moduli of the viscoelastic material, \(\tau_{n}\) is the material retardation time constant and \(N\) is the number of Maxwell chains in the generalized Maxwell model. These constants are determined experimentally through either creep or relaxation tests. On the other hand the interfacial shear stress \(\bar{\tau}\) distributed in the layer is determined by the following constitutive relation Han et al (2009)

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**Figure 1.** Schematic to actuator attached to host structure without bonding layer while (a) with bonding layer (b).
\[ -\tau = \mu_b \varepsilon_x + C_b \dot{\varepsilon}_x \]  

with the subscript ‘b’ refers to the bonding layer, \( \mu_b \) and \( C_b \) are the shear modulus and the viscosity coefficient of the bonding layer. \( \varepsilon_x \) and \( \dot{\varepsilon}_x \) are the longitudinal strain and the longitudinal strain rate in the bonded layer, respectively.

Assuming thin actuator, the actuator equation of motion can be written as, (Han et al 2009)

\[
\frac{d\sigma_x^a}{dx} + \frac{\tau(x)}{h} = -\rho_a \omega^2 u^a
\]

where the superscript ‘a’ refers to the actuator; \( \sigma_x^a \), \( \rho_a \), and \( u^a \) are respectively the axial stress, the mass density and the axial displacement of the actuator, respectively.

Taking direct piezoelectric effect, additional electric term will be added to the equation to fully represent piezoelectric materials, incorporating the piezoelectric effect. The strain stress relation can be expressed as, Vijaya (2012)

\[
\varepsilon_i = S_{ij}^E \sigma_j + d_{mi} E_m
\]

Considering the same techniques with electric displacement formula we get, the inverse piezoelectric effect equation Vijaya (2012)

\[
D_m = d_{mi} \sigma_i + \xi_{ik} E_k
\]

where, \( \sigma \) is stress vector (N/m²), \( \varepsilon \) is strain vector (m/m), \( E \) is vector of applied electric field (V/m), \( \xi \) is permittivity (C/Vm), \( d \) is matrix of piezoelectric strain constants (m/V), \( S \) is compliance matrix (m²/N) which is equal to the inverse of stiffness matrix \( c \), and \( D \) is vector of electric displacement (C/m²). The \( E \) over compliance coefficient refers to constant electric field, while \( \sigma \) over permittivity refers to constant stress, and \( i \), and \( j \) refers to the direction of electric potential. Equations (8) and (9) are the constitutive equations that governing piezoelectric structures.

3. Finite elements formulation

The finite elements technique, as an efficient numerical method, is adopted to obtain the numerical solution of the coupled electromechanical system. Utilizing the advantage of the problem symmetry, the problem can be simplified by taking only a quarter of it to be modelled, through dividing it through \( y \)-axis and at middle of host structure. The omitted parts are represented by symmetry boundary conditions as shown in figure 2.

Applying the virtual displacement principle, the weak form can be expressed as (Abdelrahman and Mahmoud 2016, Abdelrahman et al 2016)

\[
\int_N (\varepsilon : \sigma - \rho \delta u.b) dv - \int_T \delta u.tds = \int_T \rho \delta u dv
\]

where \( \varepsilon \) and \( \sigma \) respectively refer to the strain and stress tensors. \( b \) is the body force per unit volume, \( t \) is the surface traction vector, while \( \rho \) is the mass density. Neglecting the effect of body force, based on the discretized finite element form, the displacement and strain displacement fields can be expressed as Abdelrahman et al 2016

\[
U = Nq^e \text{ and } \varepsilon = Bq^e
\]

where \( N \) and \( B \) are the shape function and the strain displacement gradient matrices, respectively, and \( q^e \) is the element nodal displacement vector.

Evaluating integrals, the coupled dynamic FE equations can be expressed as Vijaya (2012)

\[
\begin{align}
[m_e'] \ddot{u}^e + [d_{uu}'] \dot{u}^e + [k_{uu}'] \dot{u}^e + [l_{uu}'] \ddot{\varphi}^e & = (f_b)^e + (f_e)^e + (f_p)^e \quad \text{(12a)} \\
[k_{u\varphi}'] \dot{u}^e + [k_{\varphi\varphi}'] \dot{\varphi}^e & = (q_e)^e + (q_p)^e \quad \text{(12b)}
\end{align}
\]

The overall finite elements equations of motion for the coupled electromechanical system can be expressed as Vijaya (2012)

\[
\begin{align}
\sum_{e=1}^{NE} ([M_e'] \ddot{u}^e) & + [D_{uu}'] \dot{u}^e + [K_{uu}'] u^e + [K_{u\varphi}'] \dot{\varphi}^e = \sum_{e=1}^{NE} (f_b)^e + (f_e)^e + (f_p)^e \quad \text{(13)} \\
\sum_{e=1}^{NE} ([K_{u\varphi}'] u^e + [K_{\varphi\varphi}'] \dot{\varphi}^e) & = \sum_{e=1}^{NE} (Q_e)^e + (Q_p)^e \quad \text{(14)}
\end{align}
\]
The overall coupled FE system of equations can be written in the following matrix form as Vijaya (2012)

\[
\begin{bmatrix}
M & 0 \\
D_{uu} & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\dot{\varphi}
\end{bmatrix}
+\begin{bmatrix}
D_{uu} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{\varphi}
\end{bmatrix}
+\begin{bmatrix}
K_{uu} & K_{u\varphi} \\
K_{u\varphi} & K_{\varphi\varphi}
\end{bmatrix}
\begin{bmatrix}
u \\
\varphi
\end{bmatrix}
=\begin{bmatrix}
F_{b} + F_{S} + F_{P} \\
Q_{S} + Q_{P}
\end{bmatrix}
\]  

(15)

where \( NE \) refers to the total number of finite elements, \( d_{uu} \) is the mechanical damping matrix, \( k_{uu} \) is the mechanical stiffness matrix, \( k_{u\varphi} \) is the piezoelectric stiffness matrix, \( k_{\varphi\varphi} \) is the dielectric stiffness matrix, \( f_{b}, f_{S}, \) and \( f_{P} \) are the body, surface and point external forces affecting the element, correspondingly. While \( q_{S} \) and \( q_{P} \) are the electrical charges at the surface and point on the element, sequentially. \( \ddot{u}, \dot{u}, u, \varphi, F_{b}, F_{S}, F_{P}, Q_{S}, \) and \( Q_{P} \) are the assembled global field quantities which are vectors. \( M, D_{uu}, K_{uu}, K_{u\varphi}, \) and \( K_{\varphi\varphi} \) are the assembled matrices.

4. Verification of the numerical simulation

The structure is modelled and simulated in ANSYS software. Dimensions of the structure are taken from Yu and Wang (2016), host structure has 100 mm in length and 50 mm in width. The actuator length is 10 mm and thickness of 1 mm. For a bonding layer model, the thickness of the layer is 0.25 mm. The structure is meshed using two elements. Plane 13 element for piezoelectric actuator with activating plane strain behavior and 3 degrees of freedoms \( Ux, Uy \) and \( Volt \). Plane 182 is used for host structure and the bonding layer. The meshing of actuator and bonding layer has fixed element size while the host structure is more accurate near actuator and bonding layer and less accurate far from them.

Materials properties and boundary conditions are imposed according to Yu and Wang (2016), with PZT-4 as the piezoelectric actuator material, Epoxy E51-618 as bonding layer, and host structure material properties are provided in table 1.
Table 1. Shows materials properties of host structure, bonding layer, and actuator.

| Host structure material properties | Actuator material (PZT-4) | Bonding layer material (Epoxy) |
|------------------------------------|---------------------------|------------------------------|
| Young’s Modulus GPa                | Elastic Stiffness Parameters (GPa) | $\epsilon_1 = 139,$ |
|                                    |                           | $\epsilon_2 = 67.8$ |
|                                    |                           | $\epsilon_3 = 74.3$ |
|                                    |                           | $\epsilon_4 = 115$ |
|                                    |                           | $\epsilon_{44} = 25.6$ |
| Poisson’s Ratio 0.3                | Permittivity ($\frac{C}{m}$) | $\epsilon_{11} = 6.45 \times 10^{-9}$ |
| Density kg m$^{-3}$ 7800           | Density kg m$^{-3}$      | $\epsilon_{33} = 5.62 \times 10^{-9}$ |
| Piezoelectric Constant (C m$^{-2}$) |                           | Poisson’s Ratio 0.38 |
|                                    | $\epsilon_{33} = -5.2$, $\epsilon_{13} = 15.1$ | |
|                                    | $\epsilon_{13} = 12.7$ | |

To make the model more generalized and can be used for different problems, host structure is modeled as half-plane. To illustrate the most common condition found in smart structure in which the actuator thickness is very thin compared to the host structure.

Quarter of the complete structure is simulated, by slicing at $y$-axis and at middle of the host structure, the remaining parts are replaced with symmetry boundary conditions along $y$-axis and at the middle of the host structure as shown in figure 2. Considering also there is no mechanical loads applied to host structure or the actuator. Electric voltage is applied at poling direction of the actuator by setting 100 V at the top surface of actuator and at bottom surface of the actuator is grounded (0 V). The attachments between the three parts is perfect. Gluing command is applied to attach them to each other. Actuator glued to host structure for model without bonding layer. In model with bonding layer, actuator is glued to bonding layer, then the bonding layer is glued to the host structure.

The mechanical deformation of the actuator, generated from the applied voltage, causes concentrated surface forces on the host structure leading it to deform. And this is what known by actuation effect in integrated smart structures. These concentrated forces are the transferred loads from the actuator to the host structure at interface due to the presence of material discontinuity between the actuator and the host structure.

The criteria of analyses are the interfacial shear stress, as it denotes the load transferred between the actuator and the host structure. Also, the vertical displacement represents the deformations happened to the host structure. Both shear stress and displacement are taking versus the ratio $X/\alpha$, where $X$ is the interfacial position at which the displacement or shear stress measured and $\alpha$ is the half length of the actuator. The ratio is taking to make the model more general and able to represent different problems.

The results of the developed models are compared to the corresponding results obtained by Jin and Wang (2011), and Wang and Huang (2001). In static analysis of perfectly bonded model without bonding layer the actuator showed a clear singularity of interfacial shear stress at actuator tip which agrees with the results of previous studies Jin and Wang (2011), and Wang and Huang (2001) as demonstrated in figure 3 verifying the proposed numerical procedure.

To investigate the effect of the host structure stiffness on the distribution of the interfacial shear stress as well as the vertical displacement distributions throughout the contact interface, the static analysis on the model without bonding layer is performed to evaluate the shear stress when the host structure material has elastic modulus that is 10 times of the original model. The results of the developed model is matched with the results of other models in literature qualitatively, as the interfacial shear stress is increased while the vertical displacement in $y$-direction is reduced, figures 4 and 5 clarify the outcomes.

To explore the effect of the bonding layer characteristics on the interfacial shear stress and the vertical displacement distributions, analysis of a model with bonding layer has been performed. Due to the presence of the layer, which causes additional material discontinuity between the actuator and the host structure, resulting in reduction of shear stresses near the tip of the actuator and increasing the shear stress along the actuator. That is observed especially, when the modulus of the bonding layer is lower than the modulus of host structure and the actuator. The developed model detected these effects successfully. Figure 6 illustrates the outcomes.

Figure 7 demonstrates that incorporating the bonding layer effect, causes a reduction in the interfacial vertical displacement throughout the contact interface. In another words, considering bonding layer will reduce the actuation effect.

The developed model is compatible with the results found in literature, as it predicts the singularity at tip of the actuator, anticipates the raise in shear stress when using material having a higher modulus of elasticity for the host structure. Finally, it is illustrated the performance of actuator when considering bonding layer which results in tremendous reduction of shear stress near tip.
5. Numerical results and discussions

5.1. Effects of piezoelectric properties

Most of the previous studies focused on evaluating the performance of the actuator based only on different combinations of geometry and mechanical properties. A Parametric study is carried out to predict the behavior of the actuator attached to elastic host structure considering various values of permittivity and piezoelectric

![Figure 3](image1.png)

**Figure 3.** Shear stress distribution throughout the half contact interface of the actuator and the host structure.

![Figure 4](image2.png)

**Figure 4.** Shear stress distribution throughout the contact interface for two different values of the elastic modulus of the host structure.

![Figure 5](image3.png)

**Figure 5.** The vertical displacement distribution throughout the contact interface for two different values of the elastic modulus of the host structure.

5. Numerical results and discussions

5.1. Effects of piezoelectric properties

Most of the previous studies focused on evaluating the performance of the actuator based only on different combinations of geometry and mechanical properties. A Parametric study is carried out to predict the behavior of the actuator attached to elastic host structure considering various values of permittivity and piezoelectric
constant to evaluate their effects on the displacement and interfacial shear stress. Their values will be taken as half and doubled in the model without bonding layer to get clear effects.

Figures 8 and 9 show that, the permittivity has no effects on both the interfacial shear stress and the displacement, as it does not affect the actuator performance significantly. Nevertheless, the model is not solvable with absence of the permittivity of actuator or its dielectric constant.

Figures 10 and 11 show that piezoelectric constant matrix has great effects on the interfacial shear stress and the vertical displacement throughout the contact interface. As it can be seen, a raise or reduction in the constant will result in equivalent changes in the interfacial shear stress and the vertical displacement along actuator contact interface. Therefore, piezoelectric material with high piezoelectric constant can compensate for some geometrical and mechanical properties of the actuator and host structure. Piezoelectric constant should be one of the criteria used to select optimum combination of actuator and host structure.

5.2. Viscoelastic bonding layer
Most bonding materials are viscoelastic materials such epoxy. Viscoelastic material is material that possesses both elastic and viscous properties. The bonding layer was represented, in previous models in literature, as elastic materials with a spring element that caused retraction. On the other hand, viscoelastic material is modeled by a combination of both spring and dashpot elements those have the capability to store and dissipate energy. To get realistic predictions of the actuator performance, viscoelastic properties should be considered.

Viscoelastic materials behave with strain rate dependent manner. A model with viscoelastic bonding layer is developed to perform static and dynamic analyses. The viscoelastic constitutive response is modeled with the Prony series. The elastic and viscoelastic properties of epoxy are presented in table 2. Additional two models are
developed, the first includes elastic bonding layer with the properties provided in table 2, the second is the same model with same materials properties but including the Prony series to simulate viscoelastic constitutive behavior. Additionally, seeking for fair and clear comparison, plane 182 is one of the elements recommended to
model and simulate both elastic and viscoelastic materials with the same meshing size. Then we will get the same meshing, and element applied in elastic model but with viscoelastic behavior leading to very apparent judgment.

5.2.1. Static analysis

The viscoelastic bonding layer affected the resulted interfacial shear stress and interfacial vertical displacement dramatically, as the shear stress reduced due to creep effect of the viscoelastic material and no singularity appeared near the tip of the actuator compared to the model without layer. While model with viscoelastic layer achieved nearly the half of interfacial shear stress of elastic layer model, see figure 12. For the vertical displacement viscoelastic layer model is the lowest, as indicated in figure 13.

5.2.2. Dynamic analysis

Seeking for deep investigation of the viscoelastic bonding layer on the mechanical performance of smart structures, the dynamic analysis under harmonic excitation has been performed. In addition to the same

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**Table 2.** Epoxy viscoelastic properties, [Kulkarni et al (2018)].

| Elastic Properties | Prony series |
|--------------------|--------------|
| Elasticity Modulus $E_0$ | 4060.1 MPa | $E_i$ | 0.0738 | 436.4 |
| Shear modulus $G_0$ | 1481.8 MPa | $\tau_i$ | 0.06407 |
| Poisson’ ratio, $\nu$ | 0.37 | 0.1470 | 0.06407 |
| Density, $\rho$ | 1180 kg m$^{-3}$ | 0.3154 | 0.0001163 |
|                 | 0.3786       | 7.321e-7 |

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Figure 11. The vertical displacement of actuator throughout the contact interface for different values of the piezoelectric constant.

Figure 12. The interfacial shear stress distribution throughout the contact interface for different models.
boundary conditions as used in static analyses, a frequency analysis including applied voltage 0 V at bottom
surface and 100 V at top surface, frequency with range of 0 to 100 KHz has been developed. This wide range is
considered to cover the frequencies that are used in SHM applications. As it can be seen in figures 14 and 15,
viscoelastic layer model has the lowest interfacial shear stress and vertical displacement, but it is very close to
elastic layer model except at some natural frequencies as elastic layer model is slightly higher. This means that the
presence of bonding layer, elastic or viscoelastic, will reduce the shear stress transferred from the actuator to the
host structure under different frequencies. While the two models; the elastic and the viscoelastic at natural
frequencies have shear stress and displacement that are lower than the model with no layer by very large range,
meaning the later model’s prediction is far away from realistic for such systems.

6. Conclusions

In the framework of FEM, the mechanical performance of viscoelastically bonded layered smart structure is
analyzed with including the effects of piezoelectric parameters on the performance of smart structures. As most
previous studies investigated elastically bonded smart structure and considered effects of mechanical properties
on smart structure performance. Based on Prony series, the viscoelastic constitutive response is simulated by the
hereditary integral. The mathematical formulation of the coupled electromechanical system is developed. Based
on plane strain analysis, the finite elements model for the electromechanical system is developed. The developed
finite element simulation is verified and compared with the available results in the literature. The model
developed has achieved results that are highly aligned and agreed with the results found in literature including the presence of singularity near actuator’s tip, and the raise in shear stress when host structure modulus of elasticity is increased. Adding to that, the singularity is disappeared when considering bonding layer between the actuator and the elastic structure. The conducted parametric studies revealed the following remarks:

- The dielectric constant, or the permittivity of piezoelectric materials has no effect on actuator performance, but it is mandatory for solving piezoelectric problems. While piezoelectric constants have significant effects on the performance of actuator as they have positive relationship with interfacial shear stress and displacement. Increase of the value of piezoelectric constants will raise them in the same amount.

- Material with high piezoelectric constant has the ability to compensate bad mechanical and geometrical aspects.

- Considering viscoelastic properties of the bonding layer under static analysis, the shear stresses and the vertical interfacial displacement are reduced heavily due to the creep property of viscoelastic material.

- In dynamic analysis the viscoelastic properties also resulted in the lowest shear stresses and the interfacial vertical displacement, but not far from model with only elastic properties of the bonding layer. It is better to consider bonding layer than neglected it, to get accurate results.

- Seeking for extension of the presented method in a future work, the coupled electromagnetic effects on the dynamic behavior of smart structures in hygrothermal environment could be discussed and analyzed.

**Data availability statement**

The data that support the findings of this study are available upon reasonable request from the authors.

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