Mass splitting of vector meson and spontaneous spin polarization under rotation

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In the present paper, we study the effect of the rotation on the masses of scalar meson as well as vector meson in the framework of 2-flavor Nambu–Jona-Lasinio model. The existence of rotation causes a tedious quark propagator and corresponding polarization function. Applying the random phase approximation, the meson mass is calculated numerically. It is found that the behavior of scalar and pseudoscalar meson masses under the angular velocity $\omega$ is similar to that at finite chemical potential, both rely on the behavior of constituent quark mass and reflect the property related to the chiral symmetry. However, masses of vector meson $\rho$ have more profound relation with rotation. After tedious calculation, it turns out that at low temperature and small chemical potential, the mass for spin component $s_z = 0, \pm 1$ of vector meson under rotation shows very simple mass splitting relation $m_{s_z}(\omega) = m_{\rho}(\omega = 0) - \omega s_z$, similar to the Zeeman splitting of charged meson under magnetic fields. Especially it is noticed that the mass of spin component $s_z = 1$ vector meson $\rho$ decreases linearly with $\omega$ and reaches zero at $\omega_c = m_{\rho}(\omega = 0)$, this indicates the system will develop $s_z = 1$ vector meson condensation and the system will be spontaneously spin polarized under rotation.

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I. INTRODUCTION

In non-central heavy-ion collision (HIC), large vorticity and strong magnetic field are expected to be generated in extremely hot quark gluon plasma (QGP). Straightforward electromagnetic (EM) computation shows the magnetic field would reach about $O(10^{14}) T$\textsuperscript{1} in the early stage of HIC, while kinetic and hydrodynamic simulations\textsuperscript{2,3} indicate the local vorticity would exceed $0.5 f m^{-1}$ with the total angular momentum of QGP at a range of $O(10^5) - O(10^6)h$. Known as the Barnett and magnetization effects, spin particles are polarized by these pseudo vector field and thus distribute differently from the normal thermal distributions. Besides chiral effects induced by such pseudo vector fields\textsuperscript{4–6}, studies on these distribution modifications would be helpful to understand the hadronization mechanism of the strong interaction as well. Inspired by the large amplitude and retention by the angular momentum conservation, vorticity has attracted more and more interests recently.

Comparing with magnetic field effects, the rotation-related effects are electric charge blind, and only involve kinetic properties of the QGP and strong interaction which we are mostly interested in. Experimentally, in order to screen out the EM effects neutral particles with finite spin numbers are chosen as carriers of the vorticity polarization effects. As it is difficult to detect the chargeless particle directly the distribution of its charged daughter particle serves as an alternative observable for the global polarization effect. With the help of the $\Lambda$ hyperon the average magnitude of the vorticity of QGP has been extracted by the STAR collaboration\textsuperscript{7}. In these measurements the expectation of $\Lambda$ polarization as well as the vorticity behavior of collision energy have been confirmed as well. All the results seem to be understandable by considering the energy shift induced by the vorticity polarization to spins. However the theory became a little vague when the $K^{*0}$ and $\phi$ mesons' measurements were presented in \textsuperscript{8}. The mismatch between these measurement indicates the fine structure of hadrons may play an non-negligible role in polarization processes.

The mass is one of the most fundamental attributes of a hadron. For a composite particle it will be modified by the single-particle dispersion relation of the fundamental degree of freedom as well as the interaction among them. Studies of hadron masses would help us to discover many clues of the environment where hadrons are born. As a well-known example, $\sigma$ meson and pion masses would change with the growing temperature and chemical potential because of chiral restoration\textsuperscript{9}. And recently people have studied the vector meson $\rho$ mass in external magnetic field as well by taking the polarization effect on quarks into account. A lattice calculation demonstrates that charged $\rho$ meson mass decreases firstly and increases finally, leaving a minimum around $eB \simeq 1 GeV^2$\textsuperscript{10}. And by using effective models,

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such as the Nambu–Jona-Lasinio(NJL) model with vector channel, the $\rho$ meson with different spin components have been studied \cite{11,12}. Therefore a natural question is what about the mass behavior under a background vorticity field which is a little like the magnetic case at the first sight. For the rotating effect, the co-rotating frame \cite{13} is usually adopted and a nontrivial connection term will be introduced \cite{13}, which serves as a polarization term for angular momentums. With this extended NJL model it is suggested that chiral phase transition would take place as angular velocity increasing \cite{15}. Furthermore, people have established more complicated phase diagrams which combine rotation and other physical conditions such as chemical potential, isospin and magnetic field \cite{16,18}. In those NJL models, at the quark level, the rotation always behaves as an effective chemical potential. This analogy has been understood with a Hamiltonian shifting $\hat{H}$ → $\hat{H} - \vec{\omega} \cdot \vec{J}$ and the latter term may be corresponding to an effective chemical potential \cite{15,18}. At the same quark level, holographic models also contribute to elaborate the property of rotating quark matter by setting up a four-dimensional AdS-Kerr-Newman black hole to construct a rotation-magnetism analogy \cite{20}. While for the composite hadrons, such as vector mesons, there have been few works on the mass behaviors.

In this paper, we focus on the scalar and the vector meson and investigate their masses under the rotation at finite chemical potential. In Sec. II in order to deal with both the finite temperature and density cases, we introduce the two-flavor NJL model with vector channel in the co-rotating frame. In this framework, we generate the dynamical quark mass with chiral symmetry spontaneous breaking and construct scalar and vector masons with the dressed quark propagator and extract the corresponding masses with the well-known random phase approximation(RPA) in Sec. III and show their numerical results in Sec. IV. Because of the rich phase structure at large chemical potential we only study the range of $\mu_q < 200$MeV in this work and leave the discussion of the rotating color superconductivity in our following works. We have found that masses of scalar mesons are controlled by the chiral phase transition which could be driven by temperature, density and rotation. While the vector meson, which carries net angular momentum, is governed by the polarization effect on the total angular momentum before chiral symmetry restoration. At large angular velocity the mass of spin component $s_z = 1$ for the vector meson vanishes. This indicates the macroscopic condensate of spin component $s_z = 1$ of vector meson ($\rho^{1+1}$) thus spontaneous spin polarization would be induced in the ultra-fast rotating system. In Sec. V we summarize our main results and give an outlook.

II. NJL MODEL IN CO-ROTATING FRAME

NJL model is an effective model with 4-fermion interaction which is widely used to study quark-quark and quark-antiquark pairing which corresponding to chiral phase transition, superfluidity and superconductivity and so on. Besides the usual scalar channels we take account the vector channels in order to construct the vector $\rho$ mesons. The Lagrangian of the two-flavor NJL model in the co-rotating frame is given by \cite{16,21}:

$$\mathcal{L} = \bar{\psi} [i \gamma^\mu (\partial_\mu + \Gamma_\mu) - m] \psi + G_S (\bar{\psi} \psi)^2 + (\bar{\psi} \gamma_5 \tau^3 \psi)^2 - G_V (\bar{\psi} \gamma_\mu \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 \psi)^2,$$

where $m$ is the current quark mass. $G_S$ and $G_V$ are the coupling constants in the scalar and vector channels, respectively. In the curved co-rotating frame the gamma matrices $\hat{\gamma}_\mu$ should be defined according to the corresponding Clifford algebra. The curved gamma matrices are connected with the flat ones with the vierbein as $\hat{\gamma}_\mu = e_\mu^\alpha \gamma^\alpha$ and where $e_\mu^\alpha$ should be chosen to satisfy $g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b$, where $\eta_{ab}$ is the metric of flat space-time and $\gamma^\alpha$ is flat gamma matrices. In our case a simple enough choice is $e_\mu^a = \delta_\mu^a + \delta_\mu^b \delta_\nu^a v_i$ and $e_\mu^a = \delta_\mu^a - \delta_\mu^b \delta_\nu^a v_i$, where $v_i$ is the linear velocity $\vec{v} = \vec{\omega} \times \vec{x}$ under the presence of a constant angular velocity $\vec{\omega}$. The so-called spinor connection is given by $\Gamma_\mu = \frac{i}{2} \epsilon^{\alpha \beta} \gamma_\mu [\gamma^\alpha, \gamma^\beta]$ where $\Gamma_{\alpha \beta} = \eta_{ac} e^c_\sigma G^\alpha_{\mu \nu} e^{b^c \sigma} - e^{a^c \sigma} \partial_\nu e^{c \sigma}$ and $G^\alpha_{\mu \nu}$ is the usual Christoffel connection determined by $g_{\mu\nu}$ \cite{15,18}. In the slow velocity limit $|\vec{\omega} \times \vec{x}| \ll c$ we could only keep the $O(\vec{v})$ terms which can be reduced to the ordinary polarization form as $\vec{\omega} \cdot \vec{J}$, where $\vec{J} = \vec{x} \times \vec{p} + \vec{S}$ is the total angular momentum \cite{14,15} and $\vec{S} = \frac{1}{4} \left( \vec{\sigma} 0 0 \vec{\sigma} \right)$ is the spin operator.

Applying the mean field approximation and choosing the direction of rotation as the z-axis, the bilinear part of the Lagrangian at finite chemical potential is given by \cite{16}

$$\mathcal{L} = \bar{\psi} [i \gamma^\mu \partial_\mu + \gamma^0 (\omega \hat{J}_z + \mu) - M] \psi - \frac{(M - m)^2}{4G_S},$$

where $J_z$ is the third component of total angular momentum $\hat{J}$, and $\mu$ is the quark chemical potential, it is seen that the angular velocity plays similar role as the chemical potential, and $M$ is the constituent quark mass which is given
by the chiral condensate as \( M = m - 2G_S \langle \bar{\psi}\psi \rangle \). The general grand potential is given by \([15, 16]\):

\[
\Omega(T, \mu, M, \omega) = \int d^3r \left\{ \frac{(M-m)^2}{4G_S} - \frac{N_cN_f}{16\pi^2} T \sum_n \int dk_i^2 \right. \\
\left. \left[ J_n(k_ir)^2 + J_{n+1}(k_ir)^2 \right] \left[ \ln(1 + e^{(E_k-(n+\frac{1}{2})\omega-\mu)/T}) + \ln(1 + e^{-(E_k-(n+\frac{1}{2})\omega-\mu)/T}) + \ln(1 + e^{(E_k+(n+\frac{1}{2})\omega+\mu)/T}) \right] \right\}.
\]

(3)

where \( E_k = \sqrt{k_t^2 + k_z^2 + M^2} \) and \( k_{t,z} \) are the transverse and longitudinal momentum respectively. Obviously the local potential approximation \( \partial_t M(r) \approx 0 \) has been adopted during solving the eigen modes. In the following computation we choose \( N_c = 3 \) and \( N_f = 2 \). In this work we will neglect the four-fermion contributions to the ground state, which means the chiral condensate is completely computed by the gap equation as \( \frac{\partial \Omega}{\partial m} = 0 \) with the constraint \( \frac{\partial^2 \Omega}{\partial M^2} > 0 \).

In Sec. [IV] we will show the numerical result of the constituent quark mass \( M \). It serves as the environment where mesons are given birth, and thus modifies their masses. In the mean field approximation the gap equation is just the one-loop diagram of the quark propagator which reads as

\[
S(\tilde{r}; \tilde{r}') = \frac{1}{(2\pi)^2} \sum_n \int \frac{dk_0}{2\pi} \int k_tdkt \int dk_z \frac{e^{in(\phi-\phi')e^{-ik_0(t-t') + ik_z(z-z')}}}{[k_0 + (n + \frac{1}{2})\omega]^2 - k_t^2 - k_z^2 - M^2 + i\epsilon}
\times \left\{ [\overline{\psi} \gamma^0 \psi \gamma^3 + M [J_n(k_ir)J_n(k_ir')]\mathcal{P}_+ + e^{i(\phi-\phi')} J_{n+1}(k_ir)J_{n+1}(k_ir')\mathcal{P}_-] - i\gamma_1 k_t e^{i\phi} J_{n+1}(k_ir)J_{n+1}(k_ir')\mathcal{P}_+ - \gamma_2 k_t e^{-i\phi} J_{n+1}(k_ir)J_{n+1}(k_ir')\mathcal{P}_- \right\},
\]

(4)

where \( \mathcal{P}_\pm = \frac{1}{2}(1 \pm i\gamma_1\gamma_2) \) are projection operators and \( \tilde{r} = (t, r, \theta, \phi) \) are the coordinates in the cylindrical frame.

### III. SCALAR AND VECTOR MESON MASS UNDER ROTATION

#### A. The scalar meson

In the NJL model, meson is regarded as \( q\bar{q} \) bound states or resonances, which can be obtained from the quark-antiquark scattering amplitude \([22, 24]\). In the random phase approximation (RPA), the full propagator of \( \sigma \) meson \( D_\sigma(q^2) \) can be expressed to leading order in \( 1/N_c \) as an infinite sum of quark-loop chains:

\[
D_\sigma(q^2) = \frac{2G_S}{1 - 2G_S\Pi_\sigma(q^2)},
\]

(5)

where \( \Pi_\sigma(q^2) \) is the quark one-loop polarization function and takes the form of

\[
\Pi_\sigma(q) = -i \int d^4\tilde{r} Tr_{sfc} [\bar{\psi}(0; \tilde{r})\gamma^0 \gamma^3 \psi(\tilde{r}; 0)] e^{iq\tilde{r}},
\]

(6)

where \( Tr_{sfc} \) means trace in spin, flavor and color space. After a tedious calculation in Appendix [A] the polarization function could be simplified as this form

\[
\Pi_\sigma(q^2) = -2iN_f N_c \int \frac{d^4p}{(2\pi)^4} \times \left\{ \left( p_0 + q_0 + \frac{1}{2}\omega \right) \left( p_0 + \frac{1}{2}\omega \right) + M^2 - (\vec{p} + \vec{q}) \cdot \vec{p} \quad \frac{\left( p_0 + q_0 + \frac{1}{2}\omega \right)^2 - (\vec{p} + \vec{q})^2 - M^2}{\left( p_0 + \frac{1}{2}\omega \right)^2 - \vec{p}^2 - M^2} \right\},
\]

(7)
If we use finite temperature theory with chemical potential \[ \mu \], the polarization function will be:

\[
\Pi_s(\tilde{q}, iv_\nu) = 2N_f N_c T \sum_{s=\pm} \sum_N \int \frac{d^3\bar{p}}{(2\pi)^3} \left[ (i\tilde{\omega}_N + iv_\nu) + \frac{1}{2} s\tilde{\omega} + \mu \right] \left[ i\tilde{\omega}_N + \frac{1}{2} s\tilde{\omega} + \mu \right] - (\bar{p} + \tilde{q}) \cdot \bar{p} - (\bar{p} + \tilde{q}) \cdot \bar{p} - M^2 - (\bar{p} + \tilde{q}) \cdot \bar{p} - M^2 - (\tilde{\omega}_N + \frac{1}{2} s\tilde{\omega} + \mu)^2 - \bar{p}^2 - M^2 \right],
\]

where \( \tilde{\omega}_N = (2N + 1)\pi T \) is Matsubara frequency. Considered analytic continuation \( \Pi_s(\tilde{q}, \nu) = \Pi_s(\tilde{q}, iv_\nu)|_{\nu + i0} \) and set \( \tilde{q} = 0 \), an explicit form of \( \Pi_s(0, \nu) \) is shown in Appendix A.

From the pole of above propagator in Eq. (5), the \( \sigma \) mass can be obtained by solving:

\[
1 - 2G_S \Pi_s(0, \nu) = 0,
\]

We have similar operation for pseudoscalar meson \( \pi \). The operators in polarization functions are defined as \( \tau^\pm = \frac{1}{\sqrt{2}}(\tau_1 \pm i\tau_2) \) where \( \tau_i \) are Pauli Matrice. In polarization functions, we choose \( \tau^a = \tau^a, \tau^b = \tau^b \) for neutral pion and \( \tau^a = \tau^+, \tau^b = \tau^- \) for charged pion. However, polarization functions have the same form for different charged mesones.

\[
\Pi_{ps}(\tilde{q}, iv_\nu) = -i \int d^4\bar{r} Tr_{sfc} [i\gamma^\nu S(0; \bar{r}) \gamma^\tau S(\bar{r}; 0)] e^{i\tilde{q} \cdot \bar{r}} = 4iN_f N_c \int d^4p \left\{ \frac{(p_0 + q_0 + \frac{1}{2}\omega)}{(p_0 + \frac{1}{2}\omega)^2 - (\bar{p} + \tilde{q})^2 - M^2} \left[ (p_0 + \frac{1}{2}\omega)^2 - \bar{p}^2 - M^2 \right] \right\},
\]

For finite temperature formalism with chemical potential, the polarization function will be:

\[
\Pi_{ps}(\tilde{q}, iv_\nu) = -4N_f N_c T \sum_{s=\pm} \sum_N \int \frac{d^3\bar{p}}{(2\pi)^3} \left[ (i\tilde{\omega}_N + iv_\nu) + \frac{1}{2} s\tilde{\omega} + \mu \right] \left[ i\tilde{\omega}_N + \frac{1}{2} s\tilde{\omega} + \mu \right] - (\bar{p} + \tilde{q}) \cdot \bar{p} - (\bar{p} + \tilde{q}) \cdot \bar{p} - M^2 - (\tilde{\omega}_N + \frac{1}{2} s\tilde{\omega} + \mu)^2 - \bar{p}^2 - M^2 \right],
\]

Considered analytic continuation \( \Pi_{ps}(\tilde{q}, \nu) = \Pi_{ps}(\tilde{q}, iv_\nu)|_{\nu + i0} \) and set \( \tilde{q} = 0 \), an explicit form of \( \Pi_{ps}(0, \nu) \) is shown in Appendix A.

From the pole of above propagator, the pion mass can be obtained by solving:

\[
1 - 2G_S \Pi_{ps}(0, \nu) = 0.
\]

B. The \( \rho \) meson

Following the Ref. [11], we construct the vector meson in a similar way with the rotation-modified quark propagators. For the 2-flavor model we take the vector \( \rho \) meson for example, its 1-loop polarization function reads as

\[
\Pi^{\mu, ab}(q) = -i \int d^4\bar{r} Tr_{sfc} [i\gamma^\mu \tau^a S(0; \bar{r}) i\gamma^\nu \tau^b S(\bar{r}; 0)] e^{i\tilde{q} \cdot \bar{r}}.
\]

As there is no isospin breaking in the quark propagators \( S(0; \bar{r}) \), the polarization functions of charged and neutral \( \rho \) mesons are supposed to be the same under rotation. Nonzero elements of the matrix reads as

\[
\Pi_{\rho}^{\mu} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \Pi^{11} & \Pi^{12} & 0 \\
0 & \Pi^{21} & \Pi^{22} & 0 \\
0 & 0 & 0 & \Pi^{33}
\end{pmatrix}.
\]

The explicit expressions of matrix elements are shown in Appendix [B]. The analysis of the Lorentz structure suggests the tensor can be decomposed according to its polarization directions as follows

\[
\Pi_{\rho}^{\mu} = A_1^{\mu} P^{\mu} + A_2^{\mu} L^{\mu} + A_3^{\mu} u^\mu u^\nu + A_4^{\mu} u^\mu u^\nu,
\]
where \( u^\mu \) is the four momentum in the rest frame. \( u^\mu = (1, 0, 0, 0) \) is a unit vector. And the projection operators are given as:

\[
P_{1\mu \nu} = -\epsilon_{1\mu}^\mu \epsilon_{1\nu}^\nu, \quad (s_z = -1 \text{ for } \rho \text{ meson }),
\]
\[
P_{2\mu \nu} = -\epsilon_{2\mu}^\mu \epsilon_{2\nu}^\nu, \quad (s_z = +1 \text{ for } \rho \text{ meson }),
\]
\[
L_{\mu \nu} = -b_{\mu} b_{\nu}^*, \quad (s_z = 0 \text{ for } \rho \text{ meson }).
\]

(16)

where in flat frame \( \epsilon_{1\mu}^\mu = \frac{1}{\sqrt{2}} (0, 1, i, 0) \) and \( \epsilon_{2\mu}^\mu = \frac{1}{\sqrt{2}} (0, 1, -i, 0) \) are the right and left-hand polarization vectors respectively. And \( b_{\mu} = (0, 0, 0, 1) \) is the direction of rotation. As a result the \( \rho \) meson propagator can be decomposed in the similar way as:

\[
D_{\rho \mu \nu}(q^2) = D_1(q^2)P_{1\mu \nu} + D_2(q^2)P_{2\mu \nu} + D_3(q^2)L_{\mu \nu} + D_4(q^2)u_\mu u_\nu,
\]

(17)

where coefficients \( D_i \) have the RPA summation forms as:

\[
D_i(q^2) = \frac{2G_V}{1 + 2G_V A_i^2}.
\]

(18)

Again the momentum poles here are corresponding to masses of vector \( \rho \) mesons which are solutions to equations:

\[
1 + 2G_V A_i^2 = 0,
\]

(19)

where

\[
A_1^2 = -(\Pi_{11} - i\Pi_{12}), \quad (s_z = -1 \text{ for } \rho \text{ meson }),
\]
\[
A_2^2 = -\Pi_{11} - i\Pi_{12}, \quad (s_z = +1 \text{ for } \rho \text{ meson }),
\]
\[
A_3^2 = -\Pi_{33}, \quad (s_z = 0 \text{ for } \rho \text{ meson }).
\]

(20)

IV. NUMERICAL RESULTS AND DISCUSSION

In order to evaluate the mass of \( \rho \) meson at finite chemical potential and relatively large vorticity, we choose the soft cut-off scheme to avoid the leakage of the energy scale. The cut-off function is:

\[
f_\Lambda(p) = \frac{\Lambda^{10}}{p^{10} + \Lambda^{10}},
\]

(21)

where \( \Lambda = 582\text{MeV} \). In numerical calculation, Momentum integrals are understood as follows [26]

\[
\int \frac{dp}{2\pi} \rightarrow \int \frac{dp}{2\pi} f_\Lambda(p).
\]

(22)

The other parameters are chosen as those in Ref [11], i.e. \( G_S A^2 = 2.388 \) and \( G_V A^2 = 1.73 \) and the current quark mass \( m_0 = 5\text{MeV} \).

By neglecting mesons’ fluctuations it is easy to solve the gap equation of chiral condensate at finite temperature as well as chemical potential under rotation. As the phase diagram shown in Ref. [15, 16, 18] the vorticity serves as another kind of chemical potential which would weaken the chiral condensate at finite temperature case and complement the chemical potential at finite density case. As shown in Fig. (2), there is a crossover at medium temperature along the angular velocity. While at low temperature the increase of chemical potential will change the 1st order chiral restoration to a crossover in Fig. (2a), (2b) and (2d). As the phase structure determines the macroscopic properties of the system it is reasonable to expect that the dependence of meson masses on the angular velocity would be smooth at medium temperature and density systems, while kinked at the 1st order point for the low density systems.

A. The scalar meson

Because of carrying no net angular momentum, the profile of scalar meson mass is completely determined by the chiral symmetry in our model. For the zero chemical potential case shown in Fig. (2a) and (2b), as angular
velocity increases the chiral condensate behaves the same as that in the [15]. At extremely low temperature the chiral restoration is 1st order and thus the masses keep invariant and then jump together at the critical angular velocity. While in hot matter the condensate keeps melting slowly until the crossover range $\omega \sim 0.6\text{GeV}$. As the consequence, $\sigma$ meson mass stays almost static and pions serve as Goldstone particles in the chiral breaking phase. When the $\omega$ close to the crossover range they approach each other and eventually become almost degenerate because of the chiral symmetry restoration. The behavior at finite density could be understood with chiral symmetry as well by noticing the order of phase transition. As Fig.(2a), (2c) and (2d) shown, at low density, i.e. $\mu < 100\text{MeV}$ and zero temperature, there is a 1st order gap at $\omega \simeq 0.8\text{GeV}$ for the dependence of the chiral condensate on angular velocity. After that the pion would break the constraint of Goldstone theorem, that is the mass increases to meet that of $\sigma$ meson which driven by the chiral symmetry. As the chemical potential increase further, the phase transition would be weaken into the crossover, and the mass dependence on the angular velocity would become more and more smooth as shown in Fig. (2c) and (2d).

![FIG. 1: The constituent quark mass as a function of angular velocity for different chemical potentials.](image)

The constituent quark mass calculated from $M = m - 2G_S \langle \bar{\psi}\psi \rangle$ as a function of angular velocity is shown in Fig. 1 for different chemical potentials. It is seen that the chiral condensate shows 1st order phase transition at large angular velocity for small chemical potentials and at small angular velocity for large chemical potentials, this is in agreement with the results in [10], where it has been observed that the 1st order phase transition shows up in two corners of the 3D $T - \mu - \omega$ phase diagram.

From the numerical result it is clear that the angular velocity and chemical potential are complementary to each other when driven the chiral restoration. At low chemical potential the critical/crossover angular velocity is larger and become smaller when the chemical potential is larger. However it is obvious that the chemical potential and angular velocity are not exactly equivalent to each other. Because physically the chemical potential is the energy shift from the difference between particle and anti-particle, while the shift induced by the rotation polarization is from the spin up and down difference. From this aspect the $\pm \frac{1}{2}\omega$ could be treated as the spin chemical potential. Analytically the difference could explicitly observed in the gap equation and the polarization functions as follows.
FIG. 2: scalar meson mass as a function of angular velocity at different chemical potential and temperature.

\[
\Pi_s(0,i\nu_n) = N_f N_c \sum_{s=\pm} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left[ \text{Res}_1(\vec{p},\nu_n) \theta \left( -\mu + \frac{s\omega}{2} - E_p \right) n_f \left( E_p + \mu - \frac{s\omega}{2}, T \right) 
+ \text{Res}_3(\vec{p},\nu_n) \theta \left( -\mu + \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) 
- \text{Res}_1(\vec{p},\nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) n_f \left( -E_p + \mu + \frac{s\omega}{2}, T \right) 
- \text{Res}_2(\vec{p},\nu_n) n_f \left( E_p + \mu + \frac{s\omega}{2}, T \right) 
+ \text{Res}_1(\vec{p},\nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) + \text{Res}_3(\vec{p},\nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) 
- \text{Res}_1(\vec{p},\nu_n) - \text{Res}_3(\vec{p},\nu_n) \right],
\]

(23)

where \( \text{Res}_1(\vec{p},\nu_n), \text{Res}_2(\vec{p},\nu_n), \text{Res}_3(\vec{p},\nu_n) \) and \( \text{Res}_4(\vec{p},\nu_n) \) are residues in Eq. (A14).
And the polarization function $\Pi_{ps}$ is given as:

$$\Pi_{ps}(0, i\nu_n) = N_f N_c \sum_{s=\pm} \int \frac{d^3 p}{(2\pi)^3} \left[ \text{Res} 1'(\vec{p}, \nu_n) \theta \left( -\mu - \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) \right. \\
+ \text{Res} 3'(\vec{p}, \nu_n) \theta \left( -\mu - \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) \\
- \text{Res} 1'(\vec{p}, \nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) n_f \left( -E_p + \mu + \frac{s\omega}{2}, T \right) - \text{Res} 2'(\vec{p}, \nu_n) n_f \left( E_p + \mu + \frac{s\omega}{2}, T \right) \\
- \text{Res} 3'(\vec{p}, \nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) n_f \left( -E_p + \mu + \frac{s\omega}{2}, T \right) - \text{Res} 4'(\vec{p}, \nu_n) n_f \left( E_p + \mu + \frac{s\omega}{2}, T \right) \\
+ \text{Res} 1'(\vec{p}, \nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) + \text{Res} 3'(\vec{p}, \nu_n) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) \\
- \text{Res} 1'(\vec{p}, \nu_n) - \text{Res} 3'(\vec{p}, \nu_n) \right].$$

where $\text{Res} 1'(\vec{p}, \nu_n), \text{Res} 2'(\vec{p}, \nu_n), \text{Res} 3'(\vec{p}, \nu_n)$ and $\text{Res} 4'(\vec{p}, \nu_n)$ are residues in Eq.\ref{eq:A17}.

It is clear that the functions depend on both the $\mu \pm \omega/2$ combinations. However it is also reasonable that the critical behavior would take place at one of the angular velocities which satisfy $E_p - \mu \pm \frac{\omega}{2} = 0$. If we choose both the chemical potential and angular velocity positive, the $\omega = 2(\mu - \frac{\omega}{2})$ part would dominate the critical behavior. Hence the chemical potential and angular velocity appear to be complementary to each other on the determination of the critical point.

### B. The $\rho$ meson

Taking the direction of rotation as the $z$-axes, and the three components of a massive vector meson can be represented as $s_z = \pm 1$ and $s_z = 0$. And the nonzero spin ones would be polarized by the so-call Barnett effect which introduces the shift as $-\vec{\omega} \cdot \vec{S}$ to the energy levels under rotation. In our 2-flavor model we take the $\rho$ meson for example to explore the rotation-induced energy shift with the self-consistent numerical calculations at the quark level. Fig\ref{fig:3} shows the numerical results for $\rho$ masses with $s_z \pm 1$ and $s_z = 0$ as functions of angular velocity at temperature $T = 10$ MeV. It is obvious that the splitting mass curves have shown the different influence of rotation. For the $s_z = 0$ case there is no net angular momentum for the particle polarization by the rotation. This makes the mass dependence on the rotation is almost the same as the scalar case which stay invariant as the chiral condensate below the critical angular velocity. While for the $s_z = \pm 1$ cases the rotation polarization would generate the energy shift $\mp \omega$ to the corresponding masses. This is confirmed by the numerical results in Fig\ref{fig:3}. The mass dependence on the angular velocity is two straight lines for the $s_z = \pm 1$ components. The behavior could be analytically proven with explicit form of the polarization functions. In the pole approximation the masses are determined by the pole of the meson propagators as Eq.\ref{eq:19}. With straightforward computation in the Appendix the polarization functions of vector meson satisfy

$$\frac{1}{2} A_1^2 (m_\rho + \omega) + \frac{1}{2} A_2^2 (m_\rho - \omega) = A_3^2 (m_\rho).$$

This means the $m_\rho(\omega = 0) - s_z \omega$ are exactly the masses of $s_z = 0, \pm 1$ components. The mass of $s_z = 1$ spin component decreases linearly with the angular velocity, and reaches zero at the critical angular velocity $\omega_c = m_\rho(\omega = 0)$. Beyond the critical angular velocity $\omega_c$, the $s_z = 1$ spin component of vector meson will develop condensation in the vacuum and this indicates that the system will be spontaneously spin polarized under strong rotation.

### V. CONCLUSION

Using the NJL model with vector channel interaction we have calculated the scalar, pseudoscalar and vector mesons’ masses at finite temperature, chemical potential and angular velocity. In the RPA and pole approximation the mesons are treated as the effective degree of freedoms which transmit the interaction between quarks. And the masses are determined by the polarization functions. This approximation could preserve the Goldstone theorem explicitly although the back reaction of meson to the phase transition is neglected. Because of the four-fermion point interaction and pole approximation the microscopic details of mesons have been lost. And all of them behave as fundamental particles which are polarized by rotation according to their net spin angular momentum. For the scalar and pseudoscalar cases the mass spectra are controlled by the chiral condensate which is the main mechanism
generating the hadron mass in NJL model. At low temperature and chemical potential the chiral restoration is 1st order which make the meson masses a sudden jump at the critical angular velocity. While as the temperature or chemical potential increasing the phase transition would degenerate to crossovers which also smoothen the mass curves of mesons along the angular velocity. It is easy to expect that at large enough angular velocity the vector condensate vacuum would be preferred and the corresponding effective mass should be zero. That is why we have only studied the vector meson’s mass behavior below the $\omega = m_{\rho}$. It is found that although the polarization function computation is complicated masses of the three components $s_z = 0, \pm 1$ are the same as the result by treating them as the fundamental particles, that is $m_{\rho}$ and $m_{\rho} \pm \omega$. Once the chiral restored the vector condensate would emerge simultaneously which will be studied in our next work.

In non-central heavy-ion collisions, the created system carries large angular momentum. The properties of particles will be changed under rotating medium. In this paper, we investigated the behavior of scalar and vector meson mass under the rotation. It is found that the behavior of scalar and pseudoscalar meson masses under the angular velocity $\omega$ is similar to that at finite chemical potential, both rely on the behavior of constituent quark mass and reflect the property related to the chiral symmetry. However, masses of vector meson have more profound relation with rotation. After tedious calculation, it turns out that at low temperature and small chemical potential, the mass for spin component $s_z = 0, \pm 1$ of vector meson under rotation shows very simple mass splitting relation $m_{\rho}^s(\omega) = m_{\rho}(\omega = 0) - \omega s_z$, similar to the Zeeman splitting of charged meson under magnetic fields. Especially it is noticed that the mass of spin component $s_z = 1$ vector meson $\rho$ decreases linearly with $\omega$ and reaches zero at $\omega_c = m_{\rho}(\omega = 0)$, this indicates the system will develop $s_z = 1$ vector meson condensation and the system will be spontaneously spin polarized under rotation. It deserves further study to compare the spin polarization with $s_z = 1$ vector meson condensation and the spin polarization defined by the condensation of $< \bar{\psi} i \sigma^{\mu \nu} \psi >$ proposed in [27].

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Appendix A: The polarization function under rotation

For the σ meson, substituting the Eq. [4] into the definition of polarization function, the scalar proper polarization function under rotation is given as

\[
\Pi_s(q) = -i \int d^4\tilde{r} Tr_{sfc} [iS(0; \tilde{r})iS(\tilde{r}; 0)] e^{iq\cdot\tilde{r}}
\]

\[
= -iN_f \sum_{n=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \int d^4\tilde{r} \int \frac{dk_0dk_z}{(2\pi)^2} \int_0^{+\infty} k_idk_i \int \frac{dp_0dp_z}{(2\pi)^2} \int_0^{+\infty} p_idp_i \frac{2\pi}{2\pi} \times Tr(A_n B_l) \times e^{-ik_0t+ik_zz} e^{i\phi} e^{-t\phi} [k_0 + (n + \frac{1}{2})\omega]^2 - k_l^2 - k_z^2 - M^2 + \frac{i\epsilon}{2} \times [p_0 + (l + \frac{1}{2})\omega]^2 - p_l^2 - p_z^2 - M^2 + \frac{i\epsilon}{2} \times e^{ip_0t-ip_zz} e^{-t\phi}, \tag{A1}
\]

where

\[
A_n = \begin{pmatrix}
    (k_0 + M + \left(n + \frac{1}{2}\right)\omega) J_n(k_0) J_{n+1}(0) & 0 & -k_z J_n(k_0) J_{n+1}(0) & -k_z J_n(k_0) J_{n+1}(0) & i\epsilon J_n(k_0) J_{n+1}(0) \\
    k_z J_n(k_0) J_{n+1}(0) & (k_0 + M + \left(n + \frac{1}{2}\right)\omega) J_n(k_0) J_{n+1}(0) & -k_z J_n(k_0) J_{n+1}(0) & -k_z J_n(k_0) J_{n+1}(0) & i\epsilon J_n(k_0) J_{n+1}(0) \\
    k_z J_n(k_0) J_{n+1}(0) & k_z J_n(k_0) J_{n+1}(0) & (k_0 - M + \left(n + \frac{1}{2}\right)\omega) J_n(k_0) J_{n+1}(0) & -k_z J_n(k_0) J_{n+1}(0) & i\epsilon J_n(k_0) J_{n+1}(0) \\
    k_z J_n(k_0) J_{n+1}(0) & k_z J_n(k_0) J_{n+1}(0) & k_z J_n(k_0) J_{n+1}(0) & (k_0 - M + \left(n + \frac{1}{2}\right)\omega) J_n(k_0) J_{n+1}(0) & -k_z J_n(k_0) J_{n+1}(0) \end{pmatrix},
\]

\[
B_l = \begin{pmatrix}
    (M + p_0 + \left(l + \frac{1}{2}\right)\omega) J_l(p_0) J_{l+1}(0) & 0 & -p_z J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) \\
    p_z J_l(p_0) J_{l+1}(0) & (M + p_0 + \left(l + \frac{1}{2}\right)\omega) J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) \\
    p_z J_l(p_0) J_{l+1}(0) & p_z J_l(p_0) J_{l+1}(0) & (M + p_0 + \left(l + \frac{1}{2}\right)\omega) J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) \\
    p_z J_l(p_0) J_{l+1}(0) & p_z J_l(p_0) J_{l+1}(0) & p_z J_l(p_0) J_{l+1}(0) & (M + p_0 + \left(l + \frac{1}{2}\right)\omega) J_l(p_0) J_{l+1}(0) & -p_z J_l(p_0) J_{l+1}(0) \end{pmatrix} \tag{A2}
\]

Here we give a matrix form instead of the summation of projection operators in Eq. [4]. The symbol "Tr_{sfc}" stands for evaluate the trace on spinor, flavor and color space. After a tedious calculation, we get

\[
Tr(A_n B_l) = \frac{1}{2}(2k_0 + 2n\omega + \omega)(2\omega + 2p_0 + \omega) + 2M^2 - 2k_zp_z J_l(k_0) J_{l+1}(0) J_n(k_0) J_{n+1}(0) J_{l+1}(p_0) J_1(p_0) \tag{A3}
\]

\[
+ \frac{1}{2}(2k_0 + 2n\omega + \omega)(2\omega + 2p_0 + \omega) + 2M^2 - 2k_zp_z J_l(k_0) J_{l+1}(0) J_n(k_0) J_{n+1}(0) J_{l+1}(p_0) J_1(p_0) \]

\[
- 2k_z p_z J_1(k_0) J_{l+1}(0) J_{n+1}(0) J_n(k_0) J_{l+1}(0) J_1(p_0) J_1(p_0) \]

When \( n \neq 0 \), it’s obvious that \( J_n(0) = 0 \) and \( J_0(0) = 1 \). As a consequence, the result of the summation will have finite terms

\[
\Pi_s(q) = -i \int d^4\tilde{r} Tr_{sfc} [S(0; \tilde{r})S(\tilde{r}; 0)] e^{iq\cdot\tilde{r}}
\]

\[
= -iN_f \sum_{n=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \int d^4\tilde{r} \int \frac{dk_0dk_z}{(2\pi)^2} \int_0^{+\infty} k_idk_i \int \frac{dp_0dp_z}{(2\pi)^2} \int_0^{+\infty} p_idp_i \frac{2\pi}{2\pi} \times \left\{ \frac{1}{2}(2k_0 + \omega)(2p_0 + \omega) + 2M^2 - 2k_zp_z \times \left\{ J_0(k_0) J_0(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 + \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 + \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
+ \frac{1}{2}(2k_0 - \omega)(2p_0 - \omega) + 2M^2 - 2k_zp_z \times \left. \left\{ J_0(k_0) J_0(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 - \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 - \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
- 2k_z p_z \times \left\{ J_1(k_0) J_1(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 + \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 + \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
- \frac{1}{2}(2k_0 - \omega)(2p_0 - \omega) + 2M^2 - 2k_zp_z \times \left. \left\{ J_1(k_0) J_1(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 + \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 + \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
- \frac{1}{2}(2k_0 + \omega)(2p_0 + \omega) + 2M^2 - 2k_zp_z \times \left. \left\{ J_1(k_0) J_1(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 - \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 - \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
- \frac{1}{2}(2k_0 - \omega)(2p_0 - \omega) + 2M^2 - 2k_zp_z \times \left. \left\{ J_1(k_0) J_1(p_0) e^{-ik_0t+ik_zz} e^{ip_0t-ip_zz} \times \left[ (k_0 - \frac{1}{2}\omega)^2 - k_l^2 - k_z^2 - M^2 + i\epsilon \right] \left[ (p_0 - \frac{1}{2}\omega)^2 - p_l^2 - p_z^2 - M^2 + i\epsilon \right] \right. \\
\right) \times e^{ip_0t}$$. \tag{A4}

Applying the integral representation of Bessel functions, the polarization function can be simplified. In integral
representation, Bessel functions are expressed as:

\[ J_n(r) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{(r \sin \theta - n \theta)} d\theta, \]

\[ J_0(r) = \frac{1}{2\pi} \int_{0}^{2\pi} e^{\pm ir \cos \theta} d\theta, \]

\[ J_1(r) = \frac{1}{2\pi i} \int_{0}^{2\pi} e^{ir \cos \theta + i\theta} d\theta, \]

\[ J_1'(r) = -\frac{1}{2\pi i} \int_{0}^{2\pi} e^{-ir \cos \theta + i\theta} d\theta. \]

(A5)

Let \( \vec{k'} = (k_t, \phi + \theta) = (k_x, k_y), \vec{r} = (r, \phi) = (x, y) \), and then we have the transformation formulae:

\[ \int_{0}^{\infty} \frac{k_t dk_t}{2\pi} \int_{0}^{2\pi} \frac{d\theta}{2\pi i} i k_t e^{i\phi} e^{ik_t r \cos \theta + i\theta} = \int \frac{dk_x dk_y}{(2\pi)^2} (k_x + i k_y)e^{i\vec{k'} \cdot \vec{r}}, \]

(A6)

\[ \int_{0}^{\infty} \frac{k_t dk_t}{2\pi} \int_{0}^{2\pi} \frac{d\theta}{2\pi i} i k_t e^{-i\phi} e^{-ik_t r \cos \theta - i\theta} = \int \frac{dk_x dk_y}{(2\pi)^2} (k_x - i k_y)e^{-i\vec{k'} \cdot \vec{r}}. \]

(A7)

Applied the transformation formulae, the polarization function for scalar meson can be expressed without Bessel function. It will be more efficient for numerical calculation.

\[ \Pi_s(q) = -i N_f N_c \int d^4\vec{r} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \times \]

\[ \left\{ \begin{array}{c}
\left[ \frac{1}{2} (2k_0 + \omega)(2p_0 + \omega) + 2M^2 - 2k_z p_z \right] \\
+ \left[ \frac{1}{2} (2k_0 - \omega)(2p_0 - \omega) + 2M^2 - 2k_z p_z \right] \end{array} \right\} \times \frac{e^{-ik_z r \cdot e^{ip \cdot \vec{r}}}}{e^{-ik_z \cdot e^{ip \cdot \vec{r}}}} \\
- 2(k_x + i k_y)(p_x - i p_y) \times \frac{\left[ (k_0 + \frac{1}{2} \omega)^2 - k^2 - M^2 + i\epsilon \right]}{\left[ (p_0 + \frac{1}{2} \omega)^2 - p^2 - M^2 + i\epsilon \right]} \times \frac{e^{-ik_z r \cdot e^{ip \cdot \vec{r}}}}{e^{-ik_z \cdot e^{ip \cdot \vec{r}}}} \\
- 2(k_x + i k_y)(p_x - i p_y) \times \frac{\left[ (k_0 - \frac{1}{2} \omega)^2 - k^2 - M^2 + i\epsilon \right]}{\left[ (p_0 - \frac{1}{2} \omega)^2 - p^2 - M^2 + i\epsilon \right]} \times \frac{e^{-ik_z r \cdot e^{ip \cdot \vec{r}}}}{e^{-ik_z \cdot e^{ip \cdot \vec{r}}}} \times e^{iq_z r} \right. \]

(A8)

Furthermore, integrating the \( r \) and \( k \) analytically, we can get the polarization function as following:

\[ \Pi_s(q) = -i N_f N_c \int \frac{d^4p}{(2\pi)^4} \times \]

\[ \left\{ \begin{array}{c}
\left[ 2 \left( p_0 + q_0 + \frac{1}{2} \omega \right)(p_0 + \frac{1}{2} \omega) + 2M^2 - 2(p_z + q_z) p_z \right] - 2 \left( [p_x + q_x] + i(p_y + q_y) \right)(p_x - i p_y) \\
\left[ (p_0 + q_0 + \frac{1}{2} \omega)^2 - (\vec{p} + \vec{q})^2 - M^2 + i\epsilon \right] \left[ (p_0 + \frac{1}{2} \omega)^2 - \vec{p}^2 - M^2 + i\epsilon \right] \\
\left[ (p_0 + q_0 - \frac{1}{2} \omega)(p_0 - \frac{1}{2} \omega) + 2M^2 - 2(p_z + q_z) p_z \right] - 2 \left( [p_x + q_x] + i(p_y + q_y) \right)(p_x - i p_y) \\
\left[ (p_0 + q_0 - \frac{1}{2} \omega)^2 - (\vec{p} + \vec{q})^2 - M^2 + i\epsilon \right] \left[ (p_0 - \frac{1}{2} \omega)^2 - \vec{p}^2 - M^2 + i\epsilon \right] \end{array} \right\} \]

(A9)
Due to symmetric analysis for integration, the expression can be simplified as following:

\[
\Pi_s(q^2) = -2iN_f N_c \int \frac{d^3 \vec{p}}{(2\pi)^3} \left\{ \frac{(p_0 + q_0 + \frac{1}{2} \omega)}{(p_0 + q_0 + \frac{1}{2} \omega)^2 - (\vec{p} + \vec{q})^2 - M^2} \right\} \cdot \left\{ \frac{(p_0 + q_0 - \frac{1}{2} \omega)}{(p_0 + q_0 - \frac{1}{2} \omega)^2 - (\vec{p} + \vec{q})^2 - M^2} \right\}.
\]  

(A10)

For finite temperature formalism:

\[
p_0 \to i\tilde{\omega}_N, \quad q_0 \to \nu_n, \quad \int \frac{p_0}{2\pi} \to iT \sum_N, \quad \tilde{\omega}_N = (2N + 1)\pi T.
\]  

(A11)

The polarization function at finite temperature and chemical potential under rotation can be rewritten as:

\[
\Pi_s(\vec{q}, \nu_n) = 2N_f N_c T \sum_{s = \pm} \sum_N \int \frac{d^3 \vec{p}}{(2\pi)^3} \left\{ (i\tilde{\omega}_N + \nu_n) \left( \mu - \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) \right\}.
\]

(A12)

Setting \( \vec{q} = 0 \), Matsubara Summation will give us a result in term of residue theorem:

\[
\Pi_s(0, \nu_n) = N_f N_c \sum_{s = \pm} \int d^3 \vec{p} \left\{ \text{Res1}(\vec{p}, \nu_n)\theta \left( -\mu - \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) + \text{Res3}(\vec{p}, \nu_n)\theta \left( -\mu - \frac{s\omega}{2} + E_p \right) n_f \left( E_p - \mu - \frac{s\omega}{2}, T \right) - \text{Res1}(\vec{p}, \nu_n)\theta \left( \mu + \frac{s\omega}{2} - E_p \right) n_f \left( -E_p + \mu + \frac{s\omega}{2}, T \right) - \text{Res3}(\vec{p}, \nu_n)\theta \left( \mu + \frac{s\omega}{2} - E_p \right) n_f \left( -E_p + \mu + \frac{s\omega}{2}, T \right) + \text{Res1}(\vec{p}, \nu_n)\theta \left( \mu + \frac{s\omega}{2} - E_p \right) + \text{Res3}(\vec{p}, \nu_n)\theta \left( \mu + \frac{s\omega}{2} - E_p \right) \right\}.
\]

(A13)

where \( n_f(x, T) = \frac{1}{e^{\pi T} + 1} \) is distribution function, and four residues are given as following:

\[
\text{Res1}(\vec{p}, \nu_n) = -\frac{i\nu_n E_p + E_p^2 + M^2 - \vec{p}^2}{E_p (-\nu_n^2 - 2i\nu_n E_p)},
\]

\[
\text{Res2}(\vec{p}, \nu_n) = -\frac{i\nu_n E_p - E_p^2 - M^2 + \vec{p}^2}{E_p (-\nu_n^2 + 2i\nu_n E_p)},
\]

\[
\text{Res3}(\vec{p}, \nu_n) = \frac{+i\nu_n E_p + E_p^2 + M^2 - \vec{p}^2}{E_p (-\nu_n^2 + 2i\nu_n E_p)},
\]

\[
\text{Res4}(\vec{p}, \nu_n) = \frac{-i\nu_n E_p - E_p^2 - M^2 + \vec{p}^2}{E_p (-\nu_n^2 - 2i\nu_n E_p)}.
\]

(A14)

We should notice that \( E_p = \sqrt{\vec{p}^2 + M^2} \) and quark mass \( M \) is a function of angular velocity \( \omega \). For pseudoscalar meson, the finite temperature version polarization function is:

\[
\Pi_{ps}(\vec{q}, \nu_n) = -4N_f N_c T \sum_{s = \pm} \sum_N \int d^3 \vec{p} \left\{ (i\tilde{\omega}_N + \nu_n) \left( \mu + \frac{s\omega}{2} + \mu \right) \left( i\tilde{\omega}_N + \frac{1}{2} s\omega + \mu \right) - M^2 - (\vec{p} + \vec{q}) \cdot \vec{p} \right\}.
\]

(A15)
Setting \( \bar{q} = 0 \), Matsubara Summation gives:

\[
\Pi_{\rho s}(0, i\nu_n) = N_f N_c \sum_{s=\pm} \int \frac{d^4p}{(2\pi)^4} \left\{ \text{Res} \left( \frac{p}{2} + E_p \right) \theta \left( -\mu - \frac{s\omega}{2} + E_p \right) \right. \\
+ \text{Res}' \left( \frac{p}{2} + E_p \right) \theta \left( -\mu - \frac{s\omega}{2} + E_p \right) \right. \\
- \text{Res} \left( \frac{p}{2} + E_p \right) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) \right. \\
- \text{Res}' \left( \frac{p}{2} + E_p \right) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) \\
+ \text{Res} \left( \frac{p}{2} + E_p \right) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) \\
- \text{Res}' \left( \frac{p}{2} + E_p \right) \theta \left( \mu + \frac{s\omega}{2} - E_p \right) \left\}.
\]

where

\[
\text{Res} \equiv \frac{iv_n E_p + E_p^2 + M^2 + \vec{p}^2}{E_p (-\nu_n^2 - 2i\nu_n E_p)},
\]

\[
\text{Res}' \equiv \frac{-iv_n E_p - E_p^2 + M^2 + \vec{p}^2}{E_p (-\nu_n^2 + 2i\nu_n E_p)}.
\]

Appendix B: The polarization function for vector meson under rotation

For \( \rho \) meson, the polarization function with one loop contribution can be expressed as

\[
\Pi^{\mu\nu,ab} = -i \int d^4\bar{r} \text{Tr}_{x_f c} \left[ i\gamma^\mu \tau^a S(0; \bar{r})i\gamma^\nu \tau^b S(\bar{r}; 0) \right] e^{i\bar{r}\bar{r}}.
\]

Using the approach introduced in Appendix A. It is obvious that the charge of \( \rho \) meson will make on difference with polarization function under rotation. We can get the nonzero elements of the matrix

\[
\Pi^{\mu\nu}_\rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi^{11}_{11} & \Pi^{12}_{12} & 0 \\ 0 & \Pi^{21}_{21} & \Pi^{22}_{22} & 0 \\ 0 & 0 & 0 & \Pi^{33}_{33} \end{pmatrix}
\]

Using the same method in Appendix A and setting \( \bar{q} = 0 \), we will get the nonzero elements which is given by:

\[
\Pi^{11}(q_0) = N_f N_c \int \frac{d^4p}{(2\pi)^4} \\
\times \left\{ \frac{2M^2 - 2 \left( p_0 + \frac{\omega}{2} \right) \left( p_0 + q_0 - \frac{\omega}{2} \right) - 2p_x^2 + 2p_y^2 + 2p_z^2}{\left( p_0 + \frac{\omega}{2} \right)^2 - p^2 - M^2 \left( p_0 + q_0 - \frac{\omega}{2} \right)^2 - p^2 - M^2} \right. \\
- \left. \frac{2M^2 - 2 \left( p_0 - \frac{\omega}{2} \right) \left( p_0 + q_0 + \frac{\omega}{2} \right) - 2p_x^2 + 2p_y^2 + 2p_z^2}{\left( p_0 - \frac{\omega}{2} \right)^2 - p^2 - M^2 \left( p_0 + q_0 + \frac{\omega}{2} \right)^2 - p^2 - M^2} \right\},
\]

\[
\Pi^{12}(q_0) = -iN_f N_c \int \frac{d^4p}{(2\pi)^4} \\
\times \left\{ \frac{2M^2 - 2 \left( p_0 + \frac{\omega}{2} \right) \left( p_0 + q_0 - \frac{\omega}{2} \right) - 2p_x^2 + 2p_y^2 + 2p_z^2}{\left( p_0 + \frac{\omega}{2} \right)^2 - p^2 - M^2 \left( p_0 + q_0 - \frac{\omega}{2} \right)^2 - p^2 - M^2} \right. \\
- \left. \frac{2M^2 - 2 \left( p_0 - \frac{\omega}{2} \right) \left( p_0 + q_0 + \frac{\omega}{2} \right) - 2p_x^2 + 2p_y^2 + 2p_z^2}{\left( p_0 - \frac{\omega}{2} \right)^2 - p^2 - M^2 \left( p_0 + q_0 + \frac{\omega}{2} \right)^2 - p^2 - M^2} \right\},
\]
\[ \Pi^{21}(q_0) = i N_f N_c \int \frac{d^4p}{(2\pi)^4} \times \left\{ \frac{2M^2 - 2 \left( p_0 + \frac{\omega}{2} \right) \left( p_0 + q_0 - \frac{\omega}{2} \right) + 2p_x^2 - 2p_y^2 + 2p_z^2}{\left[ \left( p_0 + \frac{\omega}{2} \right)^2 - \vec{p}^2 - M^2 \right] \left[ \left( p_0 + q_0 - \frac{\omega}{2} \right)^2 - \vec{p}^2 - M^2 \right]} - \frac{2M^2 - 2 \left( p_0 - \frac{\omega}{2} \right) \left( p_0 + q_0 + \frac{\omega}{2} \right) + 2p_x^2 - 2p_y^2 + 2p_z^2}{\left[ \left( p_0 - \frac{\omega}{2} \right)^2 - \vec{p}^2 - M^2 \right] \left[ \left( p_0 + q_0 + \frac{\omega}{2} \right)^2 - \vec{p}^2 - M^2 \right]} \right\}, \]
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