Sterile Neutrinos in Neutrinoless Double Beta Decay: An Update

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We revisit the mechanism of neutrinoless double beta (0νββ) decay mediated by the exchange with the heavy Majorana neutrino N of arbitrary mass mN, slightly mixed ~ UN with the electron neutrino νe. By assuming the dominance of this mechanism we update the well known 0νββ-decay exclusion plot in the mN − UN plane taking into account recent progress in calculation of nuclear matrix elements within quasiparticle random phase approximation and improved experimental bounds on the 0νββ-decay half-life of 76Ge and 136Xe. We also consider the known formula approximating the mN dependence of the 0νββ-decay nuclear matrix element in a simple explicit form. We analyze its accuracy and specify the corresponding parameters allowing one to easily calculate the 0νββ-decay half-life for arbitrary mN for all the experimentally interesting isotopes without resorting to real nuclear structure calculations.

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I. INTRODUCTION

After the triumph of the neutrino oscillation and the LHC experiments in discovering two longly awaited key elements of the Nature: neutrino mass and mixing as well as Higgs boson, the next breakthrough of comparable magnitude may happen in neutrinoless double beta (0νββ) decay searches. This hope is fed from both theoretical and the experimental sides. Lepton Number Violation (LNV) is forbidden in the Standard Model (SM) and, therefore, observation of any LNV process would unambiguously whiteness of Majorana nature of neutrinos [1, 2], indicate the existence of a new high-energy LNV scale and related new physics [3], provide a basis for solution of the problem of matter-antimatter asymmetry of the Universe via the leptogenesis [4]. Among the LNV processes 0νββ-decay is widely recognized as the most promising candidate for experimental searches. Another possible probe of LNV, which, as it has been recently realized, could be competitive or complementary to 0νββ-decay is the like-sign dilepton searches at the LHC [7,11]. However, this option still requires detailed studies to clarify its status. On the experimental side of the 0νββ-decay one expects a significant progress in the sensitivities of near future experiments, stimulating the hopes for observation of this LNV process (for recent review, c.f. [12]).

The theory of 0νββ-decay deals with three energy scales associated with rather different physics, namely: (1) the LNV scale and underlying quark-level mechanisms of 0νββ-decay, (2) hadronic scale ~ 1 GeV and QCD effects including nucleon form factors, (3) nuclear scale pF ~ (100-200) MeV and nuclear structure arrangement (pF is the nucleon Fermi momentum in a nucleus). In the literature all these three structure levels have been addressed from different perspectives (c.f. [13-14]).

In the present paper we revisit the mechanisms of 0νββ-decay mediated by Majorana neutrino N exchange with an arbitrary mass mN [15]. Our goal is to update and extend the analysis [16] of the case with several mass eigenstates N dominated by “sterile” neutrinos νs and with an admixture UN of the active flavor νe. Massive neutrinos N have been considered in the literature in diverse contexts (c.f. Ref. [17]) with the masses mN ranging from the eV to the Planck scale. Their phenomenology have been actively studied from various perspectives including their contribution to particle decays and production in collider experiments (for a recent review, c.f. [18,19]). The corresponding searches for N have been carried out in various experiments [20]. An update of the previous analysis of Ref. [16] is needed, because of the recent progress in calculation of the double beta decay nuclear matrix (NME) elements, which includes contrains on the nuclear Hamiltonian from the two-neutrino double beta decay half-life [21,22], self-consistent description of the two-nucleon short-range correlations [23] and the restoration of isospin symmetry [24]. There is also a significant progress in 0νββ-decay experiments [12] especially for 76Ge [24] and 136Xe [26] isotopes, which allows improvements of the previous limits in the neutrino sector.

The paper is organized as follows. In the next section [11] we set up the formalism underlying our analysis of the Majorana exchange mechanism of 0νββ-decay. Then we calculate the corresponding NME. Section [11] deals with an approximate formula for the NME explicitly representing their dependence on mN for arbitrary values of...
this parameter. In section [5] we extract the 0νββ-decay limits in the parameter plane \( m_N - |U_{eN}|^2 \) and compare them with other existing limits [20].

II. FORMALISM

We assume that in addition to the three conventional light neutrinos there exist other Majorana neutrino mass eigenstates \( N \) of an arbitrary mass \( m_N \), dominated by the sterile neutrino species \( \nu_s \) and with some admixture of the active neutrino weak eigenstates, \( \nu_{e,\mu,\tau} \) as

\[
N = \sum_{\alpha = e,\mu,\tau} U_{N\alpha} \nu_\alpha. \tag{1}
\]

The phenomenology of the intermediate mass sterile neutrinos \( N \) in various LNV processes have been actively studied in the literature (for a recent review, c.f. [18,19]) and limits in the \( |U_{eN}|^2 - m_N \)-plane have been derived. It has been shown that 0νββ-decay limits for \( |U_{eN}|^2 - m_N \) are the most stringent in comparison with the limits from the other LNV processes except for a narrow region of this parametric plane [16,19,27].

We study the possible contribution of these \( N \) neutrino states to 0νββ-decay via a nonzero admixture of \( \nu_e \) weak eigenstate. From non-observation of this LNV process we update the stringent limits on the \( \nu_e - \nu_e \) mixing matrix element \( U_{eN} \) in a wide region of the values of \( m_N \). We compare these limits with the corresponding limits derived from the searches for some other LNV processes. We also discuss typical uncertainties of our calculations originating from the models of nucleon and nuclear structure.

The contribution of Majorana neutrino state, \( N \), to the 0νββ-decay amplitude is described by the standard neutrino exchange diagram between the two β-decaying neutrons. Assuming the dominance of this LNV mechanism the 0νββ decay half-life for a transition to the ground state of final nucleus takes the form

\[
\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} g_A^4 \sum_N \left(U_{eN} m_N \right) m_p \left|M^{0\nu}(m_N, g_A^{\text{eff}})\right|^2,
\tag{2}
\]

The proton mass is denoted by \( m_p \). The phase-space factor \( G^{0\nu} \) is tabulated for various 0νββ-decaying nuclei in Ref. [28]. In the above formula \( g_A \) and \( g_A^{\text{eff}} \) stand for the standard and “quenched” values of the nucleon axial-vector coupling constant, respectively. Their meaning will be discussed in what follows. The nuclear matrix element in question \( M^{0\nu} \) is given by

\[
M^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p \times e^{ip \cdot (x-y)} \\
\left| \langle 0_f^+ | J^{\mu}(x) | n \rangle \langle n | J^{\mu}_l(y) | 0_f^+ \rangle \right| \sqrt{p^2 + m_N^2 + E_n - \frac{E_l - E_f}{2}}.
\tag{3}
\]

Here, \( R \) and \( m_e \) are the nuclear radius and the mass of electron, respectively. We use as usual \( R = r_0 A^{1/3} \) with \( r_0 = 1.2 \text{ fm} \). Initial and final nuclear ground states with energies \( E_i \) and \( E_f \) are denoted by \( |0_f^+\rangle \) and \( |0_i^+\rangle \), respectively. The summation runs over intermediate nuclear states \( |n\rangle \) with energies \( E_n \). The dependence on \( g_A^{\text{eff}} \) enters to \( M^{0\nu} \) through the weak one-body nuclear charged current \( J^{\mu}_l \) given by

\[
J^{\mu}_l(r) = \sum_{i=1}^A \tau^+_i J^{\mu}_l \delta(r - r_i), \tag{4}
\]

\[
J^{\mu}(r) = \sum_{i=1}^A \mathbf{J}^+ \delta(r - r_i), \tag{5}
\]

where the sum is taken over the total number \( A \) of nucleons in a nucleus. The operators with subscript \( i \) act only on the \( i^{\text{th}} \) nucleon. The isospin rising operator \( \tau^+_i \) converts neutron to proton. The coordinates of beta decaying nucleons are denoted by \( r_i \). In the leading order of non-relativistic approximation one has

\[
J^{\mu}_l = g_V(p^2),
\]

\[
J^{\mu} = -g_A(p^2) \sigma + g_V(p^2) \frac{\mathbf{P} \cdot \mathbf{P}}{2m} - i (g_V(p^2) + g_M(p^2)) \frac{\sigma \times \mathbf{P}}{2m}. \tag{6}
\]

Here, \( \mathbf{p} = p_n - p_p \) with \( p_n \) and \( p_p \) being the initial neutron and the final proton 3-momenta, respectively. For the nucleon electroweak form factors we use the standard parameterization:

\[
g_V(p^2) = \left(1 + \frac{p^2}{M_V^2}\right)^{-2}, \quad g_A(p^2) = g_A^{\text{eff}} \left(1 + \frac{p^2}{M_A^2}\right)^{-2},
\]

\[
g_M(p^2) = (\mu_p - \mu_n) g_V(p^2), \quad g_V(p^2) = 2m_p g_A(p^2) (p^2 + m_n^2)^{-1},
\]

where \( (\mu_p - \mu_n) = 3.70, M_V = 850 \text{ MeV}, M_A = 1086 \text{ MeV}, \) and \( m_\pi \) is the pion mass. For the induced pseudoscalar form factor \( g_P(p^2) \) the standard Goldberger-Treiman PCAC relation is assumed.

The value of the nucleon axial-vector coupling constant in vacuum is \( g_A = 1.269 \). In the nuclear medium this constant is expected to be renormalized to some smaller, the so called, “quenched” value \( g_A^{\text{eff}} \) [29]. This is motivated, in particular, by the fact that the calculated values of the strength of the Gamow-Teller β-decay transitions to individual final states are significantly larger than the experimentally measured ones. Theoretically the Gamow-Teller strength is a monotonically increasing function of \( g_A \). Therefore, this discrepancy with experiment can be rectified by a proper adjustment of \( g_A \) to some smaller “quenched” value \( g_A^{\text{eff}} \). It was shown in Ref. [21], that this value is compatible with the quark axial-vector coupling \( g^{\text{eff}} = g_A^{\text{quark}} = 1 \). In some recent works \( g_A^{\text{eff}} < 1 \) has been advocated [30,31]. To our opinion this sort of strong quenching still requires a more firm justification.
Therefore in our analysis we consider the following two options:

\[
g_{A}^{\text{eff}} = g_{A} = 1.269 \quad \text{(7)}
\]

\[
g_{A}^{\text{eff}} = g_{\text{quark}} = 1 \quad \text{(8)}
\]

We calculated the NME defined in Eq. (3) within the QRPA with partial restoration of isospin symmetry \cite{24}. Two different types of NN-potentials (CD-Bonn and Argonne) as well as quenched and unquenched values of the nucleon axial-vector constant \((g_{A}^{\text{eff}} = 1.0 \text{ and } 1.269)\). The dashed lines and the area between them correspond to results obtained with the approximate formula in Eq. (13).

**III. “INTERPOLATING” FORMULA**

We have also carried out the calculations of the NME in Eq. (3) for the two conventional limiting cases: the light \(m_{N} \ll p_{F}\) and the heavy \(m_{N} \gg p_{F}\) Majorana neutrino exchange mechanisms, where \(p_{F} \sim 200 \text{ MeV}\) is the characteristic momentum transferred via the virtual neutrino, which is of the order of the mean nucleon momentum of Fermi motion in a nucleus. For these limiting cases the half-life formula \(T_{1/2}^{\nu}\) is reduced to:

\[
[T_{1/2}^{\nu}]^{-1} = G_{A}^{\nu} \left( \sum_{N} U_{eN}^{2} \frac{m_{N}}{(p^{2} + m_{N}^{2})} \right)^{2}, \quad \text{(13)}
\]

where

\[
A = G_{A}^{\nu} \left| M_{\nu}^{0\nu}(g_{A}^{\text{eff}}) \right|^{2}, \quad \text{(14)}
\]

\[
\langle p^{2} \rangle = m_{p}m_{e} \left( \frac{M_{\nu}^{0\nu}(g_{A}^{\text{eff}})}{M_{\nu}^{0\nu}(g_{A}^{\text{eff}})} \right)^{2}, \quad \text{(15)}
\]

with the values of the matrix elements \(M_{\nu}^{0\nu}(g_{A}^{\text{eff}})\) and the parameters \(g_{A}^{\text{eff}}\) and \(A\) given for various isotopes in Table I. In order to estimate the accuracy of the approximate formula \(13\), we compare it with the “exact” QRPA results in Fig. 1 for \(^{76}\text{Ge}\) and \(^{136}\text{Xe}\) where the dotted curves correspond to the interpolating formula \(13\). As seen it is a rather good approximation of the “exact” QRPA result except for the transition region where the accuracy is about 20% - 25%.

The clear advantage of the formula \(13\) is that it shows explicitly the \(m_{N}\) dependence of the \(0\nu\beta\beta\) amplitude or the half-life. Therefore, it can be conveniently used for an analysis of any contents of the neutrino sector without engaging the sophisticated machinery of the nuclear structure calculations. Also any upgrade of nuclear structure approaches typically bringing out asymptotical NMEs for \(m_{N} \ll p_{F}\) and \(m_{N} \gg p_{F}\) allows one to immediately reconstruct with a good accuracy updated NMEs for arbitrary \(m_{N}\).
TABLE I. The values of the nuclear matrix elements for the light and heavy neutrino mass mechanisms defined in Eqs. (11), (12) and the parameters \((p^2)\), \(A\) of “interpolating formula” specified in Eqs. (13)-(15). The calculations have been carried out within the QRPA with partial restoration of isospin symmetry [24]. Two different types of NN-potential (CD-Bonn and Argonne) as well as quenched \((g_A = 1.00)\) and unquenched \((g_A = 1.269)\) values of the nuclear axial-vector constant have been considered. The cases presented are: a - Argonne potential, \(g_A = 1.00\); b - Argonne, \(g_A = 1.269\); c - CD-Bonn, \(g_A = 1.00\); d - CD-Bonn, \(g_A = 1.269\).

| nucleus | \(M^{0\nu}_p\) | \(M^{0\nu}_h\) | \(\sqrt{(p^2)}\) [MeV] | \(A \times 10^{-10} \text{yrs}^{-1}\) |
|---------|----------------|----------------|--------------------------|---------------------------|
|         | a   | b   | c   | d   | a   | b   | c   | d   | a   | b   | c   | d   | a   | b   | c   | d   |
| \(^{48}\text{Ca}\) | 0.463 | 0.541 | 0.503 | 0.594 | 29.0 | 40.3 | 49.0 | 66.3 | 173. | 189. | 216. | 231. | 0.541 | 1.05 | 1.55 | 2.83 |
| \(^{76}\text{Ge}\) | 3.886 | 5.157 | 4.211 | 5.571 | 204. | 287. | 316. | 433. | 159. | 163. | 190. | 193. | 2.55  | 5.05 | 6.12 | 11.5 |
| \(^{82}\text{Se}\) | 3.460 | 4.642 | 3.746 | 5.018 | 186. | 262. | 287. | 394. | 161. | 165. | 192. | 194. | 9.12  | 18.1 | 21.7 | 40.9 |
| \(^{96}\text{Zr}\) | 2.154 | 2.717 | 2.341 | 2.957 | 132. | 184. | 202. | 276. | 171. | 180. | 203. | 212. | 9.30  | 18.1 | 21.8 | 40.7 |
| \(^{100}\text{Mo}\) | 4.185 | 5.402 | 4.525 | 5.850 | 244. | 342. | 371. | 508. | 167. | 174. | 198. | 204. | 24.6  | 48.3 | 56.8 | 107. |
| \(^{110}\text{Pd}\) | 4.845 | 5.762 | 4.856 | 6.255 | 238. | 333. | 360. | 492. | 160. | 166. | 189. | 194. | 7.07  | 13.8 | 16.2 | 30.2 |
| \(^{116}\text{Cd}\) | 3.086 | 4.040 | 3.308 | 4.343 | 150. | 209. | 222. | 302. | 153. | 157. | 179. | 183. | 9.74  | 18.9 | 21.3 | 39.5 |
| \(^{124}\text{Sn}\) | 2.797 | 2.558 | 3.079 | 2.913 | 146. | 184. | 224. | 279. | 158. | 186. | 217. | 214. | 5.00  | 7.94 | 11.8 | 18.2 |
| \(^{128}\text{Te}\) | 3.445 | 4.563 | 3.828 | 5.084 | 215. | 302. | 331. | 454. | 173. | 178. | 204. | 207. | 0.705 | 1.39 | 1.67 | 3.14 |
| \(^{130}\text{Te}\) | 2.945 | 3.888 | 3.297 | 4.373 | 189. | 264. | 292. | 400. | 175. | 180. | 206. | 209. | 13.2  | 25.7 | 31.4 | 59.0 |
| \(^{136}\text{Xe}\) | 1.643 | 2.177 | 1.847 | 2.460 | 108. | 152. | 166. | 228. | 178. | 183. | 208. | 211. | 4.41  | 8.74 | 10.4 | 19.7 |

For completeness let us give the \(0\nu\beta\beta\)-decay half-life formula for a generic neutrino spectrum, which incorporates a popular scenario \(\nu\text{MSM} [33, 34]\) offering a solution of the DM and baryon asymmetry (BAU) problems via massive Majorana neutrinos. Let the neutrino spectrum contain: (i) three light neutrinos \(\nu_k=1,2,3\) with the masses \(m_{\nu(k)} \ll p_F \sim 200 \text{ MeV} \) dominated by \(\nu_{e,\mu,\tau}\), (ii) a number of the dark matter (DM) candidate neutrinos \(\nu^{DM}_j\) with the masses \(m^{DM}_j\) at the keV scale, (iii) a number of heavy neutrinos \(\nu_h\) with the masses \(m_N \gg p_F\), (iv) plus several intermediate mass \(m_h\) neutrinos \(\nu_h\) among which there could be a pair highly degenerate in mass needed for the generation of the BAU via leptogenesis [34]. In this case the “interpolating” formula [13] allows us to write down for the half-life of any \(0\nu\beta\beta\)-decaying isotope

\[
[T^{0\nu}_{1/2}]^{-1} = A \left[ \frac{mp}{(p^2)^3} \sum_{k=1}^{3} U_{ek}^2 m_k + \frac{mp}{(p^2)^3} \sum_i (U_{e_i}^{DM})^2 m_i^{DM} + m_p \sum_N U_{eN}^2 \right] \left[ \frac{mp}{(p^2)^2} \sum_h U_{eh}^2 m_h \right]^2 \text{MeV}^{-1}. \tag{16}
\]

Here due to typically very small mixing between the light and massive neutrino mass eigenstates \(|U_{eN}^2|, |U_{eN}|, |U_{eh}| \ll |U_{ek}|\) the mixing matrix of the light neutrinos \(\nu_k\) to a good accuracy can be identified with the element of the PMNS mixing matrix \(U_{ek} \approx U_{e_k}^{PMNS}\).

Finally, the following observation might be of interest. Note that the parameter \((p^2)\) with the typical value \(\sim (200 \text{ MeV})^2\) can be interpreted as the mean Fermi momentum of nucleons \(p_F\) in a nucleus. This is suggested by the structure of the NME in Eq. (3). In fact, we can schematically write for the \(m_N\) dependence

\[
M^{0\nu}(m_N) \simeq \text{const} \cdot \int_0^\infty \frac{h(p^2)}{\sqrt{p^2 + m_N^2}(\sqrt{p^2 + m_N^2} + \overline{E}_n)} \frac{p^2 \, dp}{p^2 + m_N^2} \equiv \text{const} \cdot \frac{1}{p^2 + m_N^2}. \tag{17}
\]

Here \(\overline{E}_n = E_n - (E_p - E_F)/2 \sim 10 \text{ MeV}\) is a small value in comparison with the so defined “mean” neutrino momentum \(\overline{p^2}\) taking into account the smearing effect of the nucleon form factors and the nuclear wave function codified in \(h(p^2)\) factor (for definitions see Ref. [22]). In the
last step in Eq. (17) we identified $p^2$ with the parameter $\langle p^2 \rangle$ in Eq. (13) as suggested by the comparison of Eq. (13) with Eq. (17). Kinematically the mean momentum transfer such as $\sqrt{\langle p^2 \rangle}$ is expected to be of the order of the mean nucleon Fermi momentum $p_F$ in a nucleus.

Although $\langle p^2 \rangle$ is just a parameter of the parametrization (13) tabulated in Table I its rather small variation over the isotopes supports the above physical interpretation. On top of that we show in Fig. 2 the normalized momentum transfer distribution $C(p)$ defined in Ref. [24]. It characterizes the contribution of the momentum $p$ to the NME for several values of $m_N$ and two options for the NN potential. As seen from Fig. 2 for the intermediate mass $m_N = 200$ MeV corresponding to the transition region of the “interpolating” formula in Eq. (13) the NME is dominated by the mean value of the virtual neutrino momentum $p \approx 200$ MeV. This fact again indicates that the parameter $\sqrt{\langle p^2 \rangle}$ is correlated with the mean momentum transfer and, consequently, with $p_F$. The above given interpretation could be useful for gross estimates analyzing systems whose NMEs are unavailable.

IV. EXPERIMENTAL LIMITS

Having the nuclear matrix element $M^{0\nu}(m_N)$ calculated, we can derive the $0\nu\beta\beta$-decay limits on the mass $m_N$ of the N neutrino and its mixing $U_{eN}$ with the $\nu_e$ neutrino weak eigenstate. Here we assume no significant cancellation between different terms in Eq. (2) or (13). In other words we consider only one term in Eqs. (2), (13). Applying the presently best lower bounds on the $0\nu\beta\beta$-decay half-life of $^{76}$Ge (combined GERDA + Heidelberg-Moscow) [25] and $^{136}$Xe (combined EXO+KamlandZEN) [20],

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq T_{1/2}^{0\nu-exp}(^{76}\text{Ge}) = 3.0 \times 10^{25} \text{ yrs, (18)}$$

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) \geq T_{1/2}^{0\nu-exp}(^{136}\text{Xe}) = 3.4 \times 10^{25} \text{ yrs,}$$

we derived from Eq. (2) the $|U_{eN}|^2 - m_N$ exclusion plot shown in Fig. 3. Alternatively, as we demonstrated in section III the same could be done on the basis of the interpolating formula in Eq. (13) without visible changes in Fig. 3.

In Fig. 3 we also show typical domains excluded by some other experiments summarized in Refs. [18] [20]. These domains are just indicative, because most of the previous bounds were obtained for some fixed values of $m_N$. For convenience, we interpolated this set of experimental points by continuous curves in different intervals of $m_N$. As seen from Fig. 3 the $0\nu\beta\beta$-decay limits exclude the parts of the $|U_{eN}|^2 - m_N$ parameter space previously unconstrained by the laboratory experiments except for a very small interval $m_N = 300-400$ MeV.

However, the following comment here is in order. The constraints listed in Refs. [20] are based on the searches for peaks in differential rates of various processes and the direct production of N states followed by their decays in a detector. In Ref. [19] [35] it was pointed out that in this case the results of data analysis depend on the total decay width of N, including the neutral current decay channels. The latter have not been properly taken into account in the derivation of the mentioned experimental constraints. However, the neutral current N-decay channels introduce the dependence of the final results on all the mixing matrix elements $U_{eN}, U_{\mu N}$ and $U_{\tau N}$. In this situation one cannot extract individual limits for these matrix elements without some additional assumptions, introducing a significant uncertainty. In contrast, our $0\nu\beta\beta$-decay limits involve only $U_{eN}$ mixing matrix element and, therefore, are free of the mentioned uncertainty. This is because in $0\nu\beta\beta$-decay intermediate Majorana neutrinos are always off-mass-shell states and their decay widths are irrelevant. On the other hand the above derived $0\nu\beta\beta$-decay constraints may be significantly weakened in the presence of the CP Majorana phases $\alpha^{CP} \neq 2\pi n$, for an integer $n$. This is because in that case in Eqs. (2), (13), (16) there may happen a cancellation between different terms.

V. SUMMARY

We updated the $0\nu\beta\beta$-decay limits in the plane $|U_{eN}|^2 - m_N$ for the updated nuclear matrix elements [24] and experimental data [18]. Our limits are shown in Fig. 3. We studied some uncertainties endemic to the nuclear structure calculations in general and for the QRPA in particular. These are the choice of the NN-potential
and the value of the nucleon axial-vector coupling $g_A^{\text{eff}}$ in nuclear matter. In Fig. 3 we compared the $0\nu\beta\beta$-decay limits with the corresponding limits from other searches and shown that the former confidently override the latter for all $m_N$ values except for a narrow interval around $\sim 300$ MeV where certain improvement of the $0\nu\beta\beta$-decay limits is needed. We also commented on the reliability of both the experimental results shown in Fig. 3 as “Other searches” and the $0\nu\beta\beta$-decay limits themselves disclosing some assumptions incorporated in their derivation.

We analyzed the “interpolating” formula Eq. (13) from the viewpoint of its accuracy and usefulness in phenomenological analysis of neutrino models in the part of their predictions for $0\nu\beta\beta$-decay. This formula allows one to easily update $0\nu\beta\beta$-decay limits for $|U_{eN}|^2 - m_N$ once either new experimental data for the $0\nu\beta\beta$-decay half-life or updated NMEs for the light and heavy Majorana exchange mechanisms are released. As an application of this formula we gave an approximate representation of the $0\nu\beta\beta$-decay half-life in Eq. (16) for the neutrino spectrum of the presently popular MSM scenario.

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