Cognitive Honeypots against Lateral Movement for Mitigation of Long-Term Vulnerability

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Abstract

Lateral movement of advanced persistent threats (APTs) has posed a severe security challenge. Static segregation at separate times and spatial locations is not sufficient to protect valuable assets from stealthy and persistent attackers. Defenders need to consider time and stages holistically to discover the latent attack path across a large time-scale and achieve long-term security for the target assets. In this work, we propose a random time-expanded network to model the stochastic service requests in the enterprise network and the persistent lateral movement over stages. We design cognitive honeypots at idle production nodes to detect and deter the adversarial lateral movement and protect the target node proactively and persistently. To increase the honeypots’ stealthiness, the location of the honeypot changes randomly at different times and stages. Based on the probability of service links and the likelihood of successful compromises, the defender can design the optimal honeypot policy that minimizes the long-term cyber risks of the target assets and the probability of interference and roaming cost. We propose an iterative algorithm and approximate the vulnerability with the union bound for computationally efficient deployment of honeypots. The vulnerability analysis results under the optimal and heuristic honeypot policies demonstrate that the proposed cognitive honeypot can effectively protect the target node from the lateral movement attack.

Index Terms

Advanced persistent threats Lateral movement Time-expanded network Cognitive honeypot Dynamic security Risk analysis

I. INTRODUCTION

Advanced Persistent Threats (APTs) have recently emerged as a critical security challenge to enterprise networks due to their stealthy, persistent, and sophisticated nature. The life cycle of APT attacks consists of multiple phases [1], [2]. After the initial intrusion by phishing emails, social engineering, or an infected USB, the attacker enters the enterprise network from an external network domain. Then, the attacker establishes a foothold, escalates privileges, and moves laterally in the enterprise network to search for valuable assets as his final target. The targeted assets can be either a database with confidential information or a controller for an industrial plant as shown in the instance of APT27 [3] and Stuxnet, respectively. Since valuable assets are usually segregated and cannot be compromised by an attacker from the external domain directly in the initial intrusion phase, it is indispensable for the attacker to exploit the internal network flows to move laterally from the location of the initial intrusion to the final target of the valuable assets.

Early detection of the adversarial lateral movement can deter APTs in the cradle yet is challenging due to the persistence and stealthiness of an APT attack and alert fatigue. First, an APT attacker is persistent. The long time duration between the initial intrusion and the final target compromise makes it difficult for the defender to relate alarms over a time-scale of years and piece together shreds of evidence to identify the attack path. Second, an APT attack is stealthy. Each time the attacker has compromised a new network entity such as a host and obtained its root privilege, he does not launch any subversive actions to the compromised entity and remains “under the radar”. These entities are only used as the attacker’s stepping stones for the final target compromise. Third, the high volume of network traffic during regular operation generates a considerable number of false alarms, and thus significantly delay and reduce the accuracy of adversary detection. Without accurate and timely detection of adversarial lateral movement, defensive methods such as patching and resetting all suspicious entities with high frequency become cost-prohibitive and largely reduce operational efficiency as all those entities become unavailable for the incoming service requests.

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Honeypot is a promising active defense method by deception. A honeypot is a monitored and regulated trap that is disguised to be a valuable asset for the attacker to compromise. Since legitimate users do not have the motivation to access a honeypot, any inbound network traffic directly reveals the attack with negligible false alarms. The off-the-shelf honeypots are applied at fixed locations and on isolated machines that are never involved in the regular operation. Configuring honeypots at fixed locations are simple and inexpensive while isolating the honeypot completely from the production system can achieve a zero risk of an attacker using the honeypot as a pivot node [4]. Despite the above two benefits, honeypots at fixed and isolated locations can be easily identified by sophisticated attackers, such as APTs, [5] and become ineffective.

Motivated by the concept of cognitive radio [6] and the roaming honeypot [7], we develop the concept of cognitive honeypots to mitigate the long-term cyber risks of a target asset during the adversarial lateral movement. Contrary to the off-the-shelf honeypots, the cognitive honeypots are applied to idle machines of the production system with random changing locations to make the honeypot indecipherable and unpredictable for the attacker. Since the defender reconfigures a part of the production systems as honeypots, she needs to guarantee that the no honeypot configuration interferes with service requests generated during the regular operation. Also, the defender needs to balance security with the cost of changing the honeypot location and reconfiguring the involving entities. We manage to consider the above three factors, i.e., the level of stealthiness/indecipherability, the probability of interference, and the cost of roaming, in determining the optimal honeypot policy that minimizes the target node’s long-term vulnerability (LTV).

In this work, we model the adversarial lateral movement in the enterprise network as a time-expanded network [8], where the additional temporal links reveal the attack path explicitly. We consider the scenario where service links happen randomly at each time stage and the attacker can exploit these service links for lateral movement with a given success probability. To compute the optimal policy for the cognitive honeypot efficiently, we propose an iterative algorithm and approximate the LTV by its upper and lower bounds, which results in pessimistic and optimistic honeypot policies, respectively. Besides these optimal policies, we also analyze the LTV under other heuristic honeypot policies. The analysis provides critical thresholds and quantitative bounds which can help the defender to analyze the LTV under the given system parameters such as the probability of service links and the attacker’s success probability. Moreover, if the defender can control the arrival probability of service links and affect the success probability, the analysis further indicates the direction to modify these parameters to achieve long-term security.

A. Related Works

1) Lateral Movement and Long-term Security: The attacker’s stealthy and persistent lateral movement poses a severe security challenge. Since the attacker can remain undetected in the compromised nodes for a long time, a network that is secure at any separate stages may become insecure if the times and the spatial locations are considered holistically. Thus, defenders need to pursue long-term security in lieu of static security and aim to reduce the long-term vulnerability of their valuable assets. Previous works, such as [9], [10], and [11], which focus on the detection of, the risk analysis under, and the response to lateral movement, respectively, have considered the dynamic aspects yet not explicitly. Our work manages to capture the explicit relationship between the time stages and the spatial locations of the legitimate network flows and the adversarial lateral movement, which enables the defender to compute the target node’s LTV under an arbitrary length of the time window.

2) Cognitive Honeypots: Honeypots as a defensive deception method have been widely studied from various aspects. The authors in [4], [12] have investigated the optimal timing and actions to attract and engage attackers in the honeypot. The authors in [13] have investigated the optimal honeypot configuration and the signaling mechanism to incentivize attackers and disincentivize legitimate users to access a honeypot simultaneously. All these honeypots are assumed to have fixed and segregated locations. In this work, we consider cognitive honeypots that use the idle machines of the production system to increase the stealthiness of honeypots. The terminology of “cognitive honeypots” has appeared in [14] but refers to a cognition of the suspicion level. The authors in [15] have investigated the optimal honeypot locations during the adversarial lateral movement to prevent the attacker from compromising the target node. Their honeypot policy requires a partial observation of the state, which may not be available as a result of the attacker’s stealthiness. Our work assumes that the defender does not know whether, when, or where the initial intrusion and the lateral movement occur in the network. Without real-time feedback information such as
TABLE I: Summary of notations.

| Variable | Meaning |
|----------|---------|
| $\mathcal{V} \equiv \{ \mathcal{V}_U, \mathcal{V}_H \}$ | Node set of users and hosts |
| $N = |\mathcal{V}|$ | Number of user and host nodes |
| $\mathcal{V}_I \subseteq \mathcal{V}$ | The node-set of potential initial intrusion |
| $\mathcal{V}_D \subseteq \mathcal{V}$ | The node-set that can be reconfigured as honeypots |
| $\mathcal{V}_S \subseteq \mathcal{V}$ | The set of all the subsets of $\mathcal{V}$ |
| $n_{j0} \in \mathcal{V} \setminus \mathcal{V}_I$ | The target node that contain valuable assets |
| $\Delta k \in \mathbb{Z}_+^+$ | The length of the time window |
| $\rho_i$ | The probability that the initial intrusion occurs at node $n_i \in \mathcal{V}_I$ |
| $\beta$ | The probability / frequency of service links |
| $\lambda$ | The probability of a successful compromise |
| $\gamma$ | The probability of honey links |

alerts of node compromise, the cognitive honeypot is a proactive defense method to protect the target node against adversarial lateral movement.

3) Time-Expanded Network: Time-expanded networks have been applied in transportation [16], satellite communications [17], social networks [18], and network security [19]. Since the transportation planning and satellite communications follow a timetable, the time-expanded networks in these applications usually have time-varying links that are deterministic and known at all time stages. We consider a large enterprise network and assume that the defender does not know which service requests will occur at each future stage. Thus, we consider a time-expanded network with random topology. Comparing to high-level dynamic and stochastic models such as [1], [15], the time-expanded network provides an explicit temporal dimension to understand the timing of service requests and lateral movement.

B. Notation and Organization of the Paper

Throughout this paper, we use the pronoun ‘he’ for the attacker and ‘she’ for the defender. The superscript represents the time index. The calligraphic letter $\mathcal{V}$ represents a set and $\mathcal{V} \setminus \mathcal{V}_I$ means the set of elements in $\mathcal{V}$ but not in $\mathcal{V}_I$. We summarize important notations in Table I for readers’ convenience. The rest of the paper is organized as follows. Section II introduces the time-expanded network to model lateral movement, the random arrival of the service links, and the cognitive honeypot. In Section III, we compute the optimal honeypot policy regularized by the level of stealthiness, the probability of interference, and the cost of roaming. The LTV of the target node is then analyzed. Section 6 concludes the paper.

II. CHRONOLOGICAL ENTERPRISE NETWORK MODEL

As shown in Fig. 1, we model the normal operation of an enterprise network as a series of user-host networks in chronological order. Nodes U1 and U2 represent the two users’ client computers. Nodes H1, H2, and H3 represent the three hosts in the network. In particular, host H3 stores confidential information or serves as a controller for a critical actuator. Define $\mathcal{V} := \{ \mathcal{V}_U, \mathcal{V}_H \}$ as the node set where $\mathcal{V}_U, \mathcal{V}_H$ is the set of the user nodes and hosts, respectively. The solid arrows represent two types of service links, i.e., the user-host connections and the host-host communications through an application such as HTTP [20]. Users such as U1 and U2 can access non-confidential hosts, such as H1 and H2, through their client computers for upload and/or download requests. However, to prevent data theft and physical damages, host H3 is inaccessible to users; e.g., there are no service links from U1 or U2 to H3 at any time stage $k$. Since the normal operation requires data exchanges among hosts, directed network flows exist among hosts at different times; e.g., H3 has an outbound connection to H2 at time $k = k_0$ and an inbound connection from H2 at time $k = k_0 + 3$. We assume that both types of service links occur randomly and last for a random but finite duration. Whenever there is a change of network topology, i.e., adding or deleting the user-host and host-host links, we define it as a new stage. Thus, we can characterize the chronological network as a series of user-host networks at discrete times $k = k_0, k_0 + 1, \cdots, k_0 + \Delta k$, where the initial stage $k_0 \in \mathbb{Z}^+$ and $\Delta k \in \mathbb{Z}_0^+$.

A. Time-Expanded Network and Random Service Links

We abstract the series of networks in Fig. 1 from $k \in \{k_0, \cdots, k_0 + \Delta k\}$ as a time-expanded network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \Delta k)$ in Fig. 2.
In the time-expanded network, we distinguish the same user or host node by the stage $k$ and define $n_k^i \in \mathcal{V}$ as the $i$-th node in set $\mathcal{V}$ at stage $k \in \{k_0, \ldots, k_0 + \Delta k\}$. We drop the superscript $k$ if we refer to the node rather than the node at stage $k$ or the time does not matter. We can assume that the number of nodes $N := |\mathcal{V}|$ does not change with time without loss of generality as we can let $\mathcal{V}$ contain all the potential users and hosts in the enterprise network over $\Delta k$ stages. The link set $\mathcal{E} := \{\mathcal{E}_{k_0}, \ldots, \mathcal{E}_{k_0 + \Delta k}\} \cup \{\mathcal{E}_{C_{k_0}}, \ldots, \mathcal{E}_{C_{k_0 + \Delta k - 1}}\}$ consists of two parts. On the one hand, the user-host and host-host connections at each stage $k \in \{k_0, \ldots, k_0 + \Delta k\}$ are represented by the set $\mathcal{E}^k := \{e(n_k^i, n_k^j) \in \{0, 1\} | n_k^i, n_k^j \in \mathcal{V}, i \neq j, \forall i, j \in \{1, \ldots, N\}\}$. On the other hand, set $\mathcal{E}^C_{k} := \{e(n_k^i, n_{k+1}^i) = 1 | n_k^i, n_{k+1}^i \in \mathcal{V}, \forall i \in \{1, \ldots, N\}\}$ contains the virtual temporal links from stage $k$ to $k + 1$. A link exists if $e(\cdot, \cdot) = 1$ and does not if $e(\cdot, \cdot) = 0$. The time-expanded network $\mathcal{G}$ is always a directed graph due to the temporal causality represented by the set $\mathcal{E}_{C_{k}}, k \in \{k_0, \ldots, k_0 + \Delta k - 1\}$.

Since the user-host and the host-host connections happen randomly at each stage, we assume that a service link from node $n_k^i \in \mathcal{V}$ to node $n_k^j \in \mathcal{V} \setminus \{n_k^i\}$ exists with probability $\beta_{i,j} \in [0, 1]$ for any stage $k \in \{k_0, \ldots, k_0 + \Delta k\}$. If a connection from node $n_k^i$ to $n_k^j$ is prohibitive; e.g., U1 cannot access H3 in Fig. 1, then $\beta_{i,j} = 0$. We can define $\boldsymbol{\beta} := \{\beta_{i,j}, i, j \in \{1, \ldots, N\}$, as the service-link generating matrix without loss of generality by letting

\[
\begin{align*}
\beta_{i,j} &= \begin{cases} 
1 & \text{if } e(n_k^i, n_k^j) = 1 \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]

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\[ k = k_0 \quad k = k_0 + 1 \quad k = k_0 + 2 \quad k = k_0 + 3 \quad k = k_0 + \Delta k \]

Fig. 2: Time-expanded network \( G = \{ \mathcal{V}, \mathcal{E}, \Delta k \} \) for the adversarial lateral movement and the cognitive honeypot configuration. The solid, dashed, double-lined arrows represent the service links, the temporal connections, and the honey links to honeypots, respectively. The shadowed nodes reveal the attack path from U1 to H3 explicitly over \( \Delta k = 3 \) stages.

\[ \beta_{i,i} = 0, \forall i \in \{1, \cdots, N\} \]. The service links at each stage may only involve a small number of nodes and leave other nodes idle defined as follows.

**Definition 1.** A node \( n_i^k \in \mathcal{V} \) is said to be **idle** if it does not participate in any service links at stage \( k \), i.e., \( e(n_i^k, n_j^k) = 0 \) and \( e(n_j^k, n_i^k) = 0 \), \( \forall n_j^k \in \mathcal{V} \).

**B. Attack Model of Lateral Movement over Large Time Scale**

We assume that the initial intrusion can only happen at a subset of \( N \) nodes \( \mathcal{V}_I \subseteq \mathcal{V} \) due to the network segregation. Take Fig. 1 as an example, if all hosts in the enterprise network are segregated from the Internet, the initial intrusion can only happen to the client computer of U1 or U2 through phishing emails or social engineering. Although network segregation narrows down the potential location of initial intrusion from \( \mathcal{V} \) to the subset \( \mathcal{V}_I \) that can be a single node, it is still challenging for the defender to prevent the nodes in \( \mathcal{V}_I \) from an initial intrusion as the defender cannot determine when the initial intrusion happens; i.e., the value of \( k_0 \) is unknown. In this work, we assume that the initial intrusion only happens to one node in set \( \mathcal{V}_I \) at a time; i.e., no concurrent intrusions happen. Once the attacker has entered the enterprise network via the initial intrusion from an external network domain, he does not launch new intrusions from the external domain to compromise more nodes in \( \mathcal{V}_I \). Instead, the attacker can exploit the internal service links to move laterally over time, which is much stealthier than intrusions from external network domains. For example, after the attacker has controlled U1’s computer by phishing emails, he should not send phishing emails to other users, which increase his probability of being detected. We define \( \rho_i \in [0, 1] \) as the probability that the initial intrusion happens at node \( n_i^{k_0} \in \mathcal{V}_I \) for all \( k_0 \in \mathbb{Z}^+ \) and \( \sum_{i \in \mathcal{V}_I} \rho_i = 1 \) holds. This probability can be estimated based on the node’s vulnerability assessed by the Common Vulnerability Scoring System (CVSS) [21].
After the initial intrusion, the attacker can exploit service links at different stages to move laterally. Once the attacker compromises a node by obtaining its root privilege, the privilege is retained for the attacker in all the following stages. Thus, the attacker can launch simultaneous attacks from all the compromised nodes to move laterally whenever there are outbound service links from them. If there are multiple service links from one compromised node, the attacker can also compromise all the sink nodes of these service links within the stage. Note that the only objective of the attacker is to search for valuable nodes such as H3 by lateral movement, compromise it, and then launch subversive attacks for data theft and physical damages. Thus, we assume that the attack does not launch any subversive attacks in all the compromised nodes except at the target node to remain stealthy. Even though the attacker retains the root privileges of all these compromised nodes, he only uses them as stepping stones to reach the target node.

The persistent lateral movement over a long time period enables the attacker to reach and compromise segregated nodes that are not in the initial intrusion set $\mathcal{I}_I$. In both Fig. 1 and Fig. 2 although the network has no direct service links, represented by solid arrows, from U1 to H3 at each stage, the cascade of static security in all stages does not result in long-term security over $\Delta k = 3$ stages. After we add the temporal links represented by the dashed arrows, we can see the attack path from the initial intrusion node U1 to the target node H3 over $\Delta k = 3$ stages as highlighted by the shadows in Fig. 2 by considering the times and the spatial locations holistically. The temporal order of the service links affects the likelihood that the attacker can compromise the target node. For example, if we exchange the services links that happen at stage $k_0 + 1$ and stage $k_0 + 2$, then the attacker from node U1 cannot reach H3 in $\Delta k = 3$ stages. However, the attacker can always compromise the target node after a sufficiently long time as long as the attacker can move laterally along service links, compromise nodes stealthily without triggering alerts, and control the compromised nodes persistently.

The adversarial exploitation of service links is not always successful. Moreover, the attacker may choose not to compromise a service link due to the existence of the honey link as introduced in Section II-C. Thus, if the attacker has compromised nodes $n_i^k \in \mathcal{V}, k' < k$, before stage $k$ and a service link from $n_i^k$ to $n_j^k \in \mathcal{V} \setminus \{n_i^k\}$ exists at stage $k$, i.e., $e(n_i^k, n_j^k) = 1$, we can define $\lambda_{i,j} \in [0,1]$ as the probability that the attacker at node $n_i^k$ chooses to compromise node $n_j^k$ at any stage $k \in \{k_0, \ldots, k_0 + \Delta k\}$ and successfully obtain the root privilege of node $n_j^k$ within stage $k$.

C. Cognitive Honeypot

The lateral movement of persistent and stealthy attacks makes the enterprise network insecure in the long run. The high rates of false alarms and the miss detection of the initial external intrusion and the following internal compromise make it challenging for the defender to identify the set of nodes that have been compromised. Thus, the defender needs to patch and reset all suspicious nodes at all stages to deter the attacks, which can be cost-prohibitive.

Honeypots are a promising active defense method to detect and deter these persistent and stealthy attacks by deception. Since regular honeypots are implemented at fixed locations and on machines that are never involved in the regular operation, advanced attacks such as APTs can identify the honeypots and avoid accessing them. Motivated by the roaming honeypot [7] and the fact that the service links at each stage only involve a small number of nodes, we develop the following cognitive honeypot configuration that utilizes and reconfigures different idle nodes at different stages as honeypots.

Let $\mathcal{D}_D \subseteq \mathcal{V}$ be the subset of nodes that can be reconfigured as honeypots when idle. At each stage $k$, the defender randomly selects a node $n_i^w \in \mathcal{D}_D$ to be the potential honeypot and creates a random honey link from other nodes to $n_i^w$. Since disguising a honeypot as a normal node requires emulating massive services and the continuous monitoring of all inbound network flows are costly, we assume that the defender sets up at most one honeypot and monitors one honey link at each stage.

As shown in Fig. 2, U1, H2, and H3 are idle at stage $k_0 + 1$ and U1 is reconfigured as the honeypot. The link from H3 to U1 is the honey link which is monitored by the defender. At stage $k_0$, U2 is the only idle node and it is reconfigured as the honeypot with a honey link from U1 to U2. As stated in Section II-B, the attacker who has obtained U1’s root privilege at stage $k_0$ does not interfere with any normal operations to remain stealthy. Thus, the defender can still reconfigure U1 as a honeypot at stage $k_0 + 1$. However, the honeypot of U1 at stage $k_0 + 1$ cannot identify the attacker by monitoring all the inbound traffic as he has already compromised U1. On the contrary, the honeypots at stage $k_0$ and $k_0 + 2$ can trap the attacker as he has compromised U1 and may mistake the honey links as service links for lateral movement. The defender should avoid applying honey links from the target node...
to the honeypot. If the attacker has not compromised the target node H3 as shown in stage \(k_0 + 1\), the honeypot cannot capture the attacker. If the attacker has compromised the target node as shown in stage \(k_0 + 3\), then the late detection cannot reduce the loss that has already been made. Theoretically, the honeypot can achieve zero false alarms as all legitimate network flows should occur only at the service links. For example, although the existence of the honey link at stage \(k_0\) enables legitimate users at U1 to access another user’s computer U2, a legitimate user aiming to finish the service request from U1 to H1 should not access any irrelevant nodes other than host H1. On the other hand, an attacker at U1 cannot tell whether the links from U1 to H1 and H2 are service links or honey links. Thus, only an attacker at U1 has the probability to access the honeypot H2 at stage \(k_0\).

1) Random Honeypot Configuration and Identification: Since the defender cannot predict future service links nor determine the set of compromised nodes at the current stage, she needs to develop a time-independent policy \(\gamma := \{\gamma_w\}, \forall n^k_l, n^k_w \in \mathcal{Y}\), to determine the honeypot location and the honey link at each stage \(k\) to minimize the risk that an attacker from the node of the initial intrusion can compromise the target node after \(\Delta k\) stages. Each policy element \(\gamma_w\) is the probability that the honeypot is node \(n^k_w\) and the honey link is from node \(n^k_l\) to \(n^k_w\) at stage \(k \in \{k_0, \cdots, k_0 + \Delta k\}\). Note that \(\gamma_i = 0, \forall i \in \mathcal{Y}\), and we can let \(n_l, n_w\) belongs to the entire node set \(\mathcal{Y}\) without loss of generality because if a node \(n_w \notin \mathcal{Y}_D\) is not reconfigurable, then we can let the probability \(\gamma_w\) be zero. Define \(n_j = 0 \in \mathcal{Y}\) as the target node to protect for all stages and the target node is segregated from the set of potential initial intrusion. Then, defender should avoid honey links from node \(n_j\) for all stages, i.e., \(\gamma_{j,w} = 0, \forall n_w \in \mathcal{Y}\). If a honey link from \(n_l\) to \(n_w\) is not available for all stages due to the physical segregation or other security methods, e.g., a link from U1 to H3, then \(\gamma_{w} = 0\). Since at most one link is allowed, we have the constraint \(\sum_{n_l, n_w \in \mathcal{Y}} \gamma_{w} = 1\). In this work, we assume that the honeypot policy \(\gamma\) is not affected by the realization of the service links at each stage \(l\) and thus can interfere with the service links as defined in Definition 2. If the honeypot \(n^k_w\) selected by the policy \(\gamma\) is interfering, then the defender does not monitor or filter the inbound network flows to avoid interference with the normal operation.

**Definition 2.** A honeypot \(n^k_w\) ∈ \(\mathcal{Y}\) at stage \(k\) is said to be interfering if \(n^k_w\) is the source or the sink node of any service link at stage \(k\).

Although we increase the difficulty for the attacker to identify the honeypot by applying it to idle nodes in the network and change its location at every stage, we cannot eliminate the possibility of advanced attackers identifying the honeypot \([5]\). If the attacker has compromised node \(n_l\) before stage \(k\) and there is a honey link from node \(n^k_l\) to \(n^k_w\) at stage \(k\), then we assume that the attacker has probability \(q_{l,w} \in [0, 1]\) to identify the honey link and choose not to access the honeypot. If the honeypot is not identified, then the attacker accesses the honeypot and he is detected by the defender. We assume the defender can deter the lateral movement completely after a detection from any single honeypot by patching or resetting all nodes at that stage.

As stated in Section II-B, the attacker can move simultaneously from all the compromised nodes to multiple nodes through service links that connect them. For example, the attacker at stage \(k_0 + 2\) can compromise H2 and H1 through the two service links and may also reach the honeypot if the attacker attempts to compromise H3 from U1. However, we assume that the attacker at a compromised node does not move consecutively through multiple service links, or honey links defined in Section II-C as the attacker cannot distinguish honey links from service links, in a single stage to remain stealthy. Contrary to the persistent lateral movement over a long time period, consecutive attack moves within one stage make it easier for the defender to connect all the indicators of compromise (IoC) and attribute the attacker. Take Fig. 2 as an example. Suppose that there are two links, e.g., H1 to U2 and U2 to H2 at a stage \(k\), where each link can be either a service link or a honey link. If the attacker has only compromised H1 among these three nodes, then he only attempts to compromise node U2 rather than both U2 and H2 during stage \(k\).

2) Interference, Stealthiness, and Cost of Honeypot Roaming: In this section, we define three critical security metrics for a cognitive honeypot to achieve low interference, low cost, and high stealthiness. Define \(\mathcal{S}_3\) as the set of all the subsets of \(\mathcal{Y}\). Define a series of binary random variables \(x^k_{v,v'} \in \{0, 1\}, v, v' \in \mathcal{S}_3, n^k_v \in \mathcal{Y}\), where \(x^k_{v,v'}\) means that there are no direct service links from any node \(n^k_v\) to node \(n^k_{v'}\) and from \(n^k_v\) to \(n^k_{v'}\) at stage \(k\). Thus, \(\text{Pr}(x^k_{v,v'} = 1) = \prod_{n^k_v \in \mathcal{V}}(1 - \beta_v) \prod_{n^k_{v'} \in \mathcal{V}}(1 - \beta_{v'})\) represents the probability that the honeypot at \(n^k_w\) does not

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1 The future work would consider a feedback honeypot policy based on the information of the service links at the current stage. The information set is finite yet huge; i.e., there are \(2^N\) possible combinations for the realization of service links at a stage.
interfere with any service link whose source node is in set $v$ and sink node is in $v'$. Then, we can define $H_{Pol}(\gamma)$ as the probability of interference in Definition 3. A cognitive honeypot requires the defender to utilize only idle nodes as honeypots with a low probability of interfering. To reduce $H_{Pol}(\gamma)$, the defender can design $\gamma$ based on the value of $\beta$, i.e., the frequency/probability of all potential service links.

**Definition 3.** The probability of interference (Pol) for any honeypot policy $\gamma$ is

$$H_{Pol}(\gamma) := \sum_{n_h \in \mathcal{F}} \sum_{n_v \in \mathcal{P} \setminus \{n_h\}} \gamma_{h,v} (1 - \Pr(x^k_{y \setminus \{n_v\}, v \setminus \{n_v\} = 1}))$$

$$= \sum_{n_w \in \mathcal{V}} (1 - \Pr(x^k_{y \setminus \{n_w\}, v \setminus \{n_w\} = 1})) \sum_{n_h \in \mathcal{F} \setminus \{n_w\}} \gamma_{h,w}. \tag{1}$$

Since the attacker can learn the honeypot policy $\gamma$, the defender prefers the policy to be as random as possible to increase the stealthiness of the honeypot. A fully random policy that assigns equal probability to all possible honeypot links provides forward and backward security; i.e., even if an attacker identifies the honeypot at stage $k$, he cannot use that information to deduce the location of the honeypots in the following and previous stages. We use $H_{SL}(\gamma)$, the entropy of $\gamma$ in Definition 4, as a measure for the stealthiness level of the honeypot policy where we define $0 \cdot \log 0 = 0$.

**Definition 4.** The stealthiness level (SL) for any $\gamma$ is $H_{SL}(\gamma) := \sum_{n_h,n_w \in \mathcal{V}} \gamma_{h,w} \log(\gamma_{h,w})$.

A tradeoff of roaming honeypots is the cost to reconfigure the idle nodes when the defender changes the location of the honeypot and the honey link. Define the term $C(\gamma_{h_1,w_1}, \gamma_{h_2,w_2})$, $\forall n_{h_1}, n_{h_2}, n_{w_1}, n_{w_2} \in \mathcal{V}$, as the cost of changing a $(n_{h_1} - n_{w_1})$ honey link to a $(n_{h_2} - n_{w_2})$ honey link. If only the location change of honeypots incurs a cost, we can let $C(\gamma_{h_1,w_1}, \gamma_{h_2,w_2}) = 0$, $\forall h_1 \neq h_2, \forall w_1 \in \mathcal{V}$, without loss of generality. Then, we can define the cost of roaming in Definition 5.

**Definition 5.** The cost of roaming (CoR) for any honeypot policy $\gamma$ is

$$H_{CoR}(\gamma) := \sum_{n_{h_1} \in \mathcal{V} \setminus \{n_{h_1}\}} \gamma_{h_1,w_1} (1 - \Pr(x^k_{y \setminus \{n_{w_1}\}, v \setminus \{n_{w_1}\} = 1}))$$

$$\cdot \sum_{n_{h_2} \in \mathcal{V} \setminus \{n_{h_2}\}} \gamma_{h_2,w_2} (1 - \Pr(x^k_{y \setminus \{n_{w_2}\}, v \setminus \{n_{w_2}\} = 1})) \cdot C(\gamma_{h_1,w_1}, \gamma_{h_2,w_2}) \tag{2}$$

**III. COGNITIVE HONEYPOT FOR LTV MINIMIZATION**

Throughout the entire operation of the enterprise network, the defender does not know whether, when, and where the initial intrusion has happened. The defender also cannot know the attack path of the lateral movement until a honeypot detects the attack. Therefore, the honeypot policy $\gamma$ aims to reduce the vulnerability of the target node proactively and persistently, once the adversarial lateral movement happens after the initial intrusion at an unknown initial stage $k_0$.

Given the target node $n_{j_0} \in \mathcal{V} \setminus \mathcal{Y}_I$, a subset $v \in \mathcal{V}_I$, and the defender’s honeypot policy $\gamma$, we define $g_{j_0}(v, \gamma, \Delta k)$ as the probability that an attacker who has compromised the set of nodes $v$ can compromise the target node $n_{j_0}$ within $\Delta k$ stages. Since the initial compromise happens only to a single node $n_i \in \mathcal{Y}_I$ independently with probability $\rho_i$ as argued in Section II-B, the $\Delta k$-stage vulnerability of the target node $n_{j_0}$ defined in Definition 5 equals $\bar{g}^{\Delta k}_{j_0, \gamma}(\gamma) := \sum_{n_i \in \mathcal{Y}_I} \rho_i g_{j_0}(\{n_i\}, \gamma, \Delta k)$.

**Definition 6.** Define the $\Delta k$-stage vulnerability of the target node $n_{j_0}$ as the probability that an attacker with the initial intrusion set $\mathcal{Y}_I$ can compromise the target node $n_{j_0}$ within a time window of $\Delta k$ stages.

The length of the time window represents the time-sensitivity of the defender’s demand for long-term security. For example, suppose that the defender can detect and deter the attacker after the initial intrusion yet with a delay due to the high rate of false alarms. If the delay can be contained within $\Delta k_0$ stages, then the defender should choose the honeypot policy to minimize the $\Delta k_0$-stage vulnerability. Consider a given threshold $T_0 \in [0, 1]$, we define the concept of level-$T_0$ stage-$\Delta k$ security for node $n_{j_0}$ and honeypot policy $\gamma$ in Definition 7.

**Definition 7.** Policy $\gamma$ achieves level-$T_0$ stage-$\Delta k$ security for node $n_{j_0}$ if $\bar{g}^{\Delta k}_{j_0, \gamma}(\gamma) \leq T_0$. 
Finally, we define the defender’s decision problem of a cognitive honeypot that can minimize the LTV for the target node with a low PoI, a high SL, and a low CoR in (3). The coefficients $\alpha_{PoI}, \alpha_{SL}, \alpha_{CoR}$ represent the tradeoffs of $\Delta k$-stage vulnerabilities with PoI, SL, and CoR, respectively.

$$\min_{\gamma} \sum_{n_h \in \mathcal{Y}} \gamma_{h,w} = 1,$$

$$\gamma_{h,w} = 0, \forall n_h \in \mathcal{Y}, \forall n_w \in \mathcal{V} \setminus \mathcal{Y},$$

(3)

### A. Immediate Vulnerability

We first compute the probability that an initial intrusion at node $n_i \in \mathcal{Y}_I$ can compromise the target node $n_j \in \mathcal{V}_I$ within $\Delta k = 0$ stages. The term $\gamma_{w,1}(1 - q_{i,w})$ is the probability that the attacker with initial intrusion at node $i$ is trapped by the honeypot at node $n_w$. Since the attacker does not take consecutive movements in one stage to remain stealthy as stated in Section 1-I-B, $g_{j_0}(\{n_i\}, \gamma, 0)$ equals the product of the probability that attacker exploits the service link from $n_i$ to $n_j$ successfully and the probability that the attacker is not trapped by the honeypot, i.e.,

$$g_{j_0}(\{n_i\}, \gamma, 0) = \beta_{i,j_0} \gamma_{i,j_0} (1 - \sum_{w \neq i,j_0} \text{Pr}(x_{\gamma \setminus \{n_w\}, \gamma, 0} = 1) \gamma_{w,1} (1 - q_{i,w})), \forall n_i \in \mathcal{Y}_I.$$

(4)

### B. $\Delta k$-stage Vulnerability

Define $\mathcal{Y}_{i,j_0} \subseteq \mathcal{Y}_S$ as the set of all the subsets of $\mathcal{V} \setminus \{n_i, n_{j_0}\}$. For each $v \in \mathcal{Y}_{i,j_0}$, define $\mathcal{Y}_{v,i,j_0}$ as the set of all the subsets of $\mathcal{V} \setminus \{n_i, n_{j_0}, v\}$. Define the shorthand notation $f_{v,a}(\beta, \lambda) := \prod_{n_h \in v, i} \beta_{i,h_1} \lambda_{h_1} \lambda_{h_2} (1 - \lambda_{h_2})$ as the probability that the attacker with initial intrusion at node $n_j$ has compromised the service links from $n_i$ to all nodes in set $v \in \mathcal{Y}_{i,j_0}$, yet fails to compromise the remaining service links from $n_i$ to all nodes in set $u \in \mathcal{Y}_{i,j_0}$. We can compute $g_{j_0}(\{n_i\}, \gamma, \Delta k)$ based on the following induction, i.e.,

$$g_{j_0}(\{n_i\}, \gamma, \Delta k) = g_{j_0}(\{n_i\}, \gamma, 0) + (1 - \beta_{i,j_0} \lambda_{i,j_0}) \sum_{v \in \mathcal{Y}_{i,j_0}} \sum_{u \in \mathcal{Y}_{v,i,j_0}} f_{v,a}(\beta, \lambda)(1 - \sum_{n_u \in \mathcal{V} \setminus \{n_{i,u}\}} \text{Pr}(x_{\gamma \setminus \{n_{i,u}\}, \gamma, 0} = 1) \gamma_{w,1} (1 - q_{i,w})), \forall n_i \in \mathcal{Y}_I.$$

(5)

### C. Optimal Pessimistic and Optimistic Honeypot Policy

For a given $\gamma$, we can write out the explicit form of $g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1)$ as in (4) and (5). However, the complexity increases dramatically with the cardinality of set $v$ because the event that the attacker can compromise target node $n_{j_0}$ within $\Delta k$ stages from node $n_i$ is not independent of the event that the attacker can achieve the same compromise from node $n_h \neq n_i$. Thus, we use the union bound

$$g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k) \geq \max_{n_j \in \{n_i\} \cup v} g_{j_0}(\{n_j\}, \gamma, \Delta k),$$

$$g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k) \leq \min(1, \sum_{n_j \in \{n_i\} \cup v} g_{j_0}(\{n_j\}, \gamma, \Delta k)).$$

to simplify the computation and provide a upper bound and a lower bound for $g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k), v \neq \emptyset$, in (6) and (7), respectively. With the initial condition $g_{j_0}^{lower}(\{n_i\}, \gamma, 0) = g_{j_0}^{upper}(\{n_i\}, \gamma, 0) = g_{j_0}(\{n_i\}, \gamma, 0), \forall n_j \in \{n_i\} \cup v$, we obtain the following induction for all $\Delta k \in \mathbb{Z}^+$, i.e.,

$$g_{j_0}^{lower}(\{n_i\}, \gamma, \Delta k) = g_{j_0}(\{n_i\}, \gamma, 0) + (1 - \beta_{i,j_0} \lambda_{i,j_0}) \sum_{v \in \mathcal{Y}_{i,j_0}} \sum_{u \in \mathcal{Y}_{v,i,j_0}} f_{v,a}(\beta, \lambda)(1 - \sum_{n_u \in \mathcal{V} \setminus \{n_{i,u}\}} \text{Pr}(x_{\gamma \setminus \{n_{i,u}\}, \gamma, 0} = 1) \gamma_{w,1} (1 - q_{i,w}))) \max_{n_j \in \{n_i\} \cup v} g_{j_0}^{lower}(\{n_j\}, \gamma, \Delta k - 1),$$

(6)

$$g_{j_0}^{upper}(\{n_i\}, \gamma, \Delta k) = g_{j_0}(\{n_i\}, \gamma, 0) + (1 - \beta_{i,j_0} \lambda_{i,j_0}) \sum_{v \in \mathcal{Y}_{i,j_0}} \sum_{u \in \mathcal{Y}_{v,i,j_0}} f_{v,a}(\beta, \lambda)(1 - \sum_{n_u \in \mathcal{V} \setminus \{n_{i,u}\}} \text{Pr}(x_{\gamma \setminus \{n_{i,u}\}, \gamma, 0} = 1) \gamma_{w,1} (1 - q_{i,w}))) \min(1, \sum_{n_j \in \{n_i\} \cup v} g_{j_0}^{upper}(\{n_j\}, \gamma, \Delta k)).$$

(7)
Let us define the term $\bar{g}_{\Delta k}^{\text{lower}}(\gamma) := \sum_{n_i \in \mathcal{Y}_i} \rho_{j_0,n_i} g_{j_0,n_i}^{\text{lower}}(\{n_i\}, \gamma, \Delta k)$ and $\bar{g}_{\Delta k}^{\text{upper}}(\gamma) := \sum_{n_i \in \mathcal{Y}_i} \rho_{j_0,n_i} g_{j_0,n_i}^{\text{upper}}(\{n_i\}, \gamma, \Delta k)$ as the pessimistic and optimistic estimates of the $\Delta k$-stage vulnerability of the target node $n_{j_0}$, respectively. Thus, replacing $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ in (5) with $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ and $\bar{g}_{\Delta k}^{\text{upper}}(\gamma)$, we obtain the optimal pessimistic and optimistic honeypot policy $\gamma^{\text{lower}}$ and $\gamma^{\text{upper}}$, respectively.

We propose the following iterative algorithm to compute these two honeypot policies. We use $\gamma^{\text{lower}}$ as an example and $\gamma^{\text{upper}}$ can be computed in the same fashion. First, we consider any feasible honeypot policy $\gamma$ and compute $g_{j_0}^{\text{lower}}(\{n_i\}, \gamma, \Delta k), \forall n_i \in \mathcal{Y}_i, \forall \Delta k \in \{1, \cdots, \Delta k\}$, via (6). Then, we solve (3) by replacing $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ with $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ and plugging in $g_{j_0}^{\text{lower}}(\{n_i\}, \gamma, \Delta k), \forall n_i \in \mathcal{Y}_i$, as constants. Since $\bar{g}_{\Delta k}^{\text{lower}}(\gamma), H_{\text{PoI}}(\gamma), H_{\text{CoR}}(\gamma)$ are all linear with respect to $\gamma$, the objective function of the constrained optimization in (3) is a linear function of $\gamma$ plus the entropy regularization $H_{\text{SL}}(\gamma)$. Then, we can solve the constrained optimization in closed form and update the honeypot policy from $\gamma$ to $\gamma^{t+1}$. Given a small error threshold $\epsilon > 0$, the above iteration process can be repeated until there exists a $T_1 \in \mathbb{Z}^+$ such that a proper matrix norm is less than the error threshold, i.e., $\|\gamma^{T_1+1} - \gamma^T\| \leq \epsilon$. Then, we can output $\gamma^{T_1+1}$ as the optimal pessimistic policy $\gamma^{\text{lower}}$.

Algorithm 1: Optimal Pessimistic (and Optimistic) Honeypot Policy

1. Initialization $\mathcal{Y}_i, n_{j_0} \in \mathcal{Y} \setminus \mathcal{Y}_i, \Delta k \in \mathbb{Z}^+, \epsilon > 0, \gamma^0$;
2. while $\|\gamma^{t+1} - \gamma^t\| > \epsilon$ do
3. $t := t + 1$;
4. for $\Delta k = 1, \cdots, \Delta k$ do
5. for $i \in \mathcal{Y}_i$ do
6. Compute $g_{j_0}^{\text{lower}}(\{n_i\}, \gamma^t, \Delta k)$ via (6);
7. end
8. end
9. Replace $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ with $\bar{g}_{\Delta k}^{\text{lower}}(\gamma)$ and plug in $g_{j_0}^{\text{lower}}(\{n_i\}, \gamma^t, \Delta k), \forall n_i \in \mathcal{Y}_i$;
10. Obtain $\gamma^{t+1}$ as the solution of (3);
11. if $\|\gamma^{t+1} - \gamma^t\| \leq \epsilon$ then
12. $T_1 = t$;
13. Terminate
14. end
15. Output $\gamma^{\text{lower}} = \gamma^{T_1+1}$.

D. $\Delta k$-Stage Vulnerability Analysis under Heuristic Policies

Besides the optimal pessimistic and optimistic policies, the defender can also apply honeypots according to other heuristic policies. In Section [III-D2 and III-D3], we analyze the $\Delta k$-stage vulnerability for a target node $n_{j_0} \in \mathcal{Y} \setminus \mathcal{Y}_i$ under two types of honeypot policies with or without direct honey links from node $n_i \in \mathcal{Y}_i$, respectively.

1) Indirect Honeypot Links: For an initial intrusion at node $n_i \in \mathcal{Y}_i$, honey links with source nodes other than $n_i$ are able to capture the attacker in the following stages due to the adversarial lateral movement. Thus, in this subsection, we analyze the $\Delta k$-stage vulnerability when there are no direct honey links from the initial intrusion node $n_i$, i.e., $\gamma_{w} = 0, \forall n_w \in \mathcal{Y}$, or the attacker can always identify the honey links, i.e., $\gamma_{w} = 1, \forall n_w \in \mathcal{Y}$. This indirect honeypot policy represents the scenario where the defender has excluded node $n_i$ mistakenly from the set $\mathcal{Y}_i$ that the initial intrusion can happen. Or it represents a powerful attacker who can identify all the honey links from node $n_i$. Due to the capability mismatch between the honeypot and the attacker under the indirect honeypot policy, the vulnerability of any target node increases strictly with the length of the time window and converges to the maximum value of 1 as shown in Proposition [1]. This means that the attacker can always compromise the target node given sufficiently long stages of lateral movement.

Proposition 1. If $\gamma_{w}(1 - q_{w}) = 0, \forall n_w \in \mathcal{Y}$, then $g_{j_0}(\{n_i\}, \gamma, \Delta k) \in [0, 1]$ is an non-decreasing function regarding variable $\Delta k$ for all $n_{j_0} \in \mathcal{Y} \setminus \mathcal{Y}_i, n_i \in \mathcal{Y}_i$. The value of function does not increase to 1 as $\Delta k$ increases to infinity if and only if $\beta_{i,j_0} = 0$ and $g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1) = g_{j_0}(\{n_i\}, \gamma, \Delta k - 1), \forall v \in \mathcal{Y}_i, \forall \Delta k \in \mathbb{Z}^+$. 
Proof. If \( \gamma_{w}(1 - q_{i,w}) = 0, \forall n_{w} \in \mathcal{Y} \), we can use the facts that \( g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1) \geq g_{j_0}(\{n_i\}, \gamma, \Delta k - 1) \), \( \forall \gamma, n_{j_0} \in \mathcal{Y}, n_i \in \mathcal{Y}_1, \Delta k \geq 0, \forall v \in \mathcal{Y}_S \), and
\[
\sum_{v \in \mathcal{Y}_{i,j_0}} \sum_{u \in \mathcal{Y}^*_{i,j_0}} f_{v,u}(\beta, \lambda) \equiv 1, \forall \beta, \lambda,
\]
to obtain
\[
g_{j_0}(\{n_i\}, \gamma, \Delta k) = \beta_{i,j_0} \lambda_{i,j_0} + (1 - \beta_{i,j_0} \lambda_{i,j_0}) \sum_{v \in \mathcal{Y}_{i,j_0}} \sum_{u \in \mathcal{Y}^*_{i,j_0}} f_{v,u}(\beta, \lambda) g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1)
\geq \beta_{i,j_0} \lambda_{i,j_0} + (1 - \beta_{i,j_0} \lambda_{i,j_0}) g_{j_0}(\{n_i\}, \gamma, \Delta k - 1) \geq g_{j_0}(\{n_i\}, \gamma, \Delta k - 1),
\]
for all \( \Delta k \in \mathbb{Z}^+ \). The inequality is an equality if and only if \( \beta_{i,j_0} \lambda_{i,j_0} = 0 \) and \( g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1) = g_{j_0}(\{n_i\}, \gamma, \Delta k - 1) \), \( \forall v \in \mathcal{Y}_S, \forall \Delta k \in \mathbb{Z}^+ \). \( \square \)

Besides honeypot policy \( \gamma \), the \( \Delta k \)-stage vulnerability also depends on the probability/frequency of service links, i.e., the value of \( \lambda \). In the constrained optimization problem \( 3 \), we assume that the defender cannot design \( \beta \) and \( \lambda \) and they are given as known parameters. In the rest of this subsection, we briefly investigate the possibility of design \( \beta \) and \( \lambda \) to reduce the \( \Delta k \)-stage vulnerability. To reduce \( \beta_{j_1,j_2}, \forall n_{j_1}, n_{j_2} \in \mathcal{Y} \), the defender needs to sacrifice operation efficiency and reduce the arrival probability/frequency of the service link from \( n_{j_1} \) to \( n_{j_2} \). To reduce the probability of a successful compromise from node \( n_{j_1} \) to \( n_{j_2} \), i.e., \( \lambda_{j_1,j_2}, \forall n_{j_1}, n_{j_2} \in \mathcal{Y} \), the defender can enhance the service link from \( n_{j_1} \) to \( n_{j_2} \) or demotivate the attacker to initiate the compromise by disguising the service link as a honey link.

The term \( r := (1 - \beta_{i,j_0} \lambda_{i,j_0}) \sum_{u \in \mathcal{Y}^*_{i,j_0}} f_{0,u}(\beta, \lambda) \in (0, 1) \) represents the probability that there is no direct link from \( n_i \) to target \( n_{j_0} \) and the attacker at node \( n_{j_0} \) does not attempt to or fails to compromise all the service links from node \( n_i \). In \( 8 \) where \( \gamma_{w}(1 - q_{i,w}) = 0, \forall n_{w} \in \mathcal{Y} \), we can upper bound the term \( g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1) \) by 1 for all \( v \neq 0 \), which leads to
\[
g_{j_0}(\{n_i\}, \gamma, \Delta k) \leq (1 - r) \cdot g_{j_0}(\{n_i\} \cup v, \gamma, \Delta k - 1) + r \cdot g_{j_0}(\{n_i\}, \gamma, \Delta k - 1)
\leq (1 - r) + r \cdot g_{j_0}(\{n_i\}, \gamma, \Delta k - 1)
= 1 - r^{\Delta k} + r^{\Delta k} g_{j_0}(\{n_i\}, \gamma, 0) = 1 - r^{\Delta k} (1 - \beta_{i,j_0} \lambda_{i,j_0}),
\]
where the final line results from solving the first-order linear difference equation iteratively by \( \Delta k - 1 \) times.

If \( r = 1 \), i.e., \( \beta_{i,j_0} \lambda_{i,j_0} = 0, \lambda_{i,j} = 0, \forall n_j \in \mathcal{Y} \), then the \( \Delta k \)-stage vulnerability remains the same as the 0-stage vulnerability, i.e., \( g_{j_0}(\{n_i\}, \gamma, \Delta k) = g_{j_0}(\{n_i\}, \gamma, 0) = 0 \) for any finite \( \Delta k \in \mathbb{Z}^+ \). Thus, no lateral movement can happen from the initial compromise node \( n_i \in \mathcal{Y} \) for any finite \( \Delta k \in \mathbb{Z}^+ \) and we achieve 0 vulnerability for the target node \( j_0 \) for any length of the time window \( \Delta k \in \mathbb{Z}^+ \). However, \( r = 1 \) is a restrictive condition which requires the attacker to fail the compromise from node \( n_i \) to any node \( n_j \) with probability 1, i.e., \( \lambda_{i,j} = 0, \forall n_j \in \mathcal{Y} \). In Proposition 2, we relax the \( r = 1 \) to \( r \geq 1 - m/\Delta k \) for any finite \( m \ll \Delta k \) to guarantee a level-1 (\( \beta_{i,j_0} \lambda_{i,j_0} \) stage-\( \infty \) security defined in Definition 7). The proof follows directly from a limit analysis based on (9).

**Proposition 2.** Consider the scenario where \( \gamma_{w}(1 - q_{i,w}) = 0 \) for all \( n_{w} \in \mathcal{Y} \) and \( r \) is a function of \( \Delta k \) with parameters \( m \in \mathbb{R} \setminus \{0\} \) and \( n \in \mathbb{R}^+ \), i.e., \( r = 1 - m \Delta k^{-n} \).

1. If \( (1 - r)/m \) is of the same order with \( 1/\Delta k \), i.e., \( n = 1 \), then the limit of the upper bound \( \lim_{\Delta k \to \infty} 1 - r^{\Delta k}(1 - \beta_{i,j_0} \lambda_{i,j_0}) \) is a constant \( 1 - e^{-m}(1 - \beta_{i,j_0} \lambda_{i,j_0}) \in (0, 1) \).
2. If \( (1 - r)/m \) is of higher order, i.e., \( n > 1 \), then the limit of the upper bound is \( g_{j_0}(\{n_i\}, \gamma, 0) = \beta_{i,j_0} \lambda_{i,j_0} \in (0, 1) \).
3. If \( (1 - r)/m \) is of lower order, i.e., \( n < 1 \), then the limit of the upper bound is 1.

The equality holds if and only if \( \beta_{i,j_0} \lambda_{i,j_0} = 1 \). Based on the fact that \( 1 - e^{-m}(1 - \beta_{i,j_0} \lambda_{i,j_0}) \geq \beta_{i,j_0} \lambda_{i,j_0} \), we can conclude that if \( r \geq 1 - m/\Delta k \) for any finite \( m \ll \Delta k \), then the \( \infty \)-stage vulnerability of target node \( n_{j_0} \) is upper bounded by \( \beta_{i,j_0} \lambda_{i,j_0} \) and thus achieves the level-1 (\( \beta_{i,j_0} \lambda_{i,j_0} \) stage-\( \infty \) security as defined in Definition 7). Note that if the target node is segregated from the initial intrusion set, i.e., there is no direct service link from node \( n_i \) to the target node \( n_{j_0} \), then \( \beta_{i,j_0} \lambda_{i,j_0} = 0 \) and the target node \( n_{j_0} \) can achieve a zero vulnerability for infinite length of time window.
The term $T_r$.

Based on the first equality in (10), we obtain that if $g_{j_0}(\{n_i, n_{j_0}\})$, which applies a direct honeypot from the initial intrusion node $n_i$ to node $n_{j_0}$. Then, we obtain the corresponding $\Delta k$-stage vulnerability and an explicit lower bound in (10) based on (5) by using the inequality $g_{j_0}(\{n_i, n_{j_0}\}) \leq g_{j_0}(\{n_i, n_{j_0}\})$. Define shorthand notations $k_1 := \prod_{l \neq w_0} (1 - \beta_{l,w_0})(1 - \beta_{l,w_0})$, and $k_2 := \sum_{v \in \{n_{j_0} \setminus n_{w_0}\} \setminus \{n_{j_0}\}} f_{v,u}(\beta, \lambda) - \sum_{v \in \{n_{j_0} \setminus n_{w_0}\} \setminus \{n_{j_0}\}} f_{v,u}(\beta, \lambda) = 1$. Note that $k_1 = 0$ is a very restrictive condition as it requires that both the honeypot $n_{j_0}$ is not interfering as defined in Definition 2 and the attacker can never identify the honeypot link from $n_i$ to $n_{j_0}$, i.e., $q_{i,j} = 0$.

Define a shorthand notation $r_2 := (1 - \beta_{l,w_0})k_1$ for all $\Delta k \in \mathbb{Z}^+$. According to the first equality in (10), we also obtain an explicit upper bound for $g_{j_0}(\{n_i, n_{j_0}\}), \forall \Delta k \in \mathbb{Z}^+$, in Lemma 1 by using the inequality $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k) \leq 1$. Note that we can obtain a tighter upper bound with a more complicated explicit form by bounding the term $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k - 1)$ by 1 only if $\gamma$ is not empty and then solve the resulting linear difference equation correspondingly.

Lemma 1. If $\gamma_{j_0} = 1, w_0 \neq j_0$ and $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k)$ is upper bounded by $T_{2, upper} := 1 - \beta_{l,w_0}k_1 = 1 - \beta_{l,w_0}k_2(1 - \beta_{l,w_0})$ for all $\Delta k \in \mathbb{Z}^+$. The bound $T_{2, upper}$ is non-trivial if $\beta_{l,w_0}k_1 \neq 1$, and $k_2(1 - \beta_{l,w_0}) = 0$.

Proposition 3. If $\beta_{l,w_0}k_1 \neq 0$ and $\gamma_{j_0} = 1, w_0 \neq j_0$, then

1. The term $T_{2, lower} := (1 - \beta_{l,w_0}k_1) \in [0, 1]$ is strictly less than 1 and the value 0 is achieved only if $k_1 = 1$.
2. If $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k - 1) < T_{2, lower}$, then $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k) > g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k - 1)$.
3. The term $\lim_{\Delta k \to \infty} g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k)$ is lower bounded by $\max(T_{2, lower}, T_{2, lower}^2)$. 

Proof. Based on the first equality in (10), we obtain that if $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k - 1) < T_{2, lower}^2$, then $g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k) > g_{j_0}(\{n_i, n_{j_0}\}, \gamma, \Delta k - 1)$. Since the above is true for all $\Delta k \in \mathbb{Z}^+$, we know that the $\Delta k$-stage vulnerability increases with $\Delta k$ strictly until it has reached $T_{2, lower}^2$. When $\beta_{l,w_0}k_1 \neq 0$ and $k_1 \neq 1$, $T_{2, lower}^2$ strictly greater than 0 is a non-trivial lower bound. The other lower bound $T_{2, lower}^1$ comes from Lemma 1.

In this subsection, we consider an advantageous scenario for the defender as the initial intrusion can only happen to a single node $n_i$. Lemma 1 shows that if the defender applies a direct honeypot from $n_i$ in a deterministic fashion, then the $\Delta k$-stage vulnerability is always upper bounded. However, this direct policy cannot reduce the $\infty$-stage vulnerability to zero and the $\Delta k$-stage vulnerability still increases with $\Delta k$ until the value is greater than the threshold $T_{2, lower}^2$. After combining the results with the ones in Proposition 1, we can conclude that comparing with the indirect honeypot policy, the direct policy can mitigate the LTV of the target node yet within a certain degree.

IV. Conclusion

Stealthy and persistent lateral movement of APTs poses a severe security challenge to enterprise networks. Honeypots, as a promising deceptive defense method, can detect lateral movement attacks at their early stages with
a negligible false positive rate. Since advanced attackers, such as APTs, can identify the honeypots located at fixed machines not in the production system, we propose a cognitive honeypot mechanism which reconfigures different idle production nodes as honeypot at different stages based on the probability of service links and the probability of successful compromise. The time-expanded network is used to model the time of the random service occurrence and the adversarial compromise explicitly. Besides the main objective of reducing the long-term vulnerability of the target node against lateral movement attacks, we also consider the level of stealthiness, the probability of interference, and the cost of roaming as three tradeoffs of the long-term security. To reduce the computation complexity, we propose an iterative algorithm and approximate the vulnerability with the union bound. The results of the vulnerability analysis under the optimal and heuristic honeypot policies demonstrate that the proposed cognitive honeypot can effectively protect the target node from the lateral movement attack.

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