I. INTRODUCTION

Degenerate dense plasmas have become a subject of important research over the last few years as those can be achieved in the laboratories\textsuperscript{12,13} and can be useful for understanding the salient features of collective plasma oscillations in superdense astrophysical bodies like white dwarfs, neutron stars, magnetars etc.\textsuperscript{2,3} In these plasma environments, the typical number density of charged particles (mainly electrons/positrons/holes) becomes extremely high, i.e., \( n \gtrsim 10^{30} \text{ cm}^{-3} \), and their thermodynamic temperature becomes low (\( T \sim 10^{5} - 10^{7} \text{ K} \)), so that the particles follow the Fermi-Dirac statistics. The degeneracy condition in these plasmas (where the Fermi energy is higher than the thermal energy) is satisfied for \( T \lesssim 10^{7} \text{ K} \). However, in metals \( (n \sim 10^{23} \text{ cm}^{-3}) \) electrons are degenerate at \( T \lesssim 10^{5} \text{ K} \). Furthermore, in regimes, e.g., in an outer mantle of white dwarfs, where electrons may be nonrelativistic degenerate, the number density and the temperature satisfy \( n \gtrsim 10^{26} \text{ cm}^{-3} \) and \( T \lesssim 10^{7} \text{ K} \)\textsuperscript{4} and the magnetic field is \( B_{0} \lesssim 10^{8} \text{ T} \). However, for ultra-relativistically degenerate electrons (e.g., in the core of white dwarfs) we can have the density \( n \gtrsim 10^{45} \text{ cm}^{-3} \) and the temperature \( T \lesssim 10^{9} \text{ K} \). In degenerate dense plasmas there appear a time scale in the units of plasma period and a typical length scale \( \lambda_{F} = v_{F}/\omega_{p} \), where \( v_{F} \) is the Fermi velocity and \( \omega_{p} \) is the plasma oscillation frequency. Just like the Debye length in classical plasmas, \( \lambda_{F} \) represents the scale length of electrostatic screening in quantum plasmas. In the latter, the typical quantum mechanical (Such as tunneling associated with the quantum Bohm potential) as well as the statistical (Fermi-Dirac pressure for degenerate species) effects play crucial roles for the collective plasma dynamics. Furthermore, in these high-density plasmas, the equilibrium density of charged particles and the static ambient magnetic field can be nonuniform with finite scale lengths. Thus, there appear the \( \mathbf{E} \times \mathbf{B} \) drift (where \( \mathbf{E} \) is the electric field and \( \mathbf{B} \) is the magnetic field), diamagnetic drift and polarization drift velocities due to the density and magnetic field inhomogeneities, as well as the quantum drift velocity caused by the strong density correlations due to quantum fluctuations. It naturally becomes a challenge to understand the properties of drift waves as well as the formation of localized structures in degenerate dense plasmas\textsuperscript{5–11} as the quantum effects begin to modify the physical process at high densities.

Drift waves are typically low-frequency (compared to the ion gyrofrequency) electrostatic or electromagnetic waves that exist in a spatially nonuniform magnetoplasma. Such waves are caused by the guiding center drifts of the charged particles in presence of a density gradient across the external magnetic field. Various types of drift waves, e.g., the electrostatic drift waves and coupled drift–Alfvén waves have been reported in classical [See, e.g., Refs.\textsuperscript{12,13} as well as quantum plasmas [See, e.g., Refs.\textsuperscript{5–11}] which play a crucial role in cross-field plasma particle transport\textsuperscript{14–16} and the formation of coherent structures like solitons in space\textsuperscript{10,17} laboratory\textsuperscript{14,15} as well as degenerate dense plasmas that are magnetized. Note that both the drift–Alfvén waves can be excited by free energy sources that are stored in the equilibrium pressure gradient and in magnetic field inhomogeneity. In astrophysical environments, since the parametric cascading of electromagnetic signals occurs, the drift waves may generate sufficient electromagnetic disturbance that should be considered in telescopic observations. Furthermore, since the cores or outer mantles of compact astrophysical stars (e.g., white dwarfs) are dense and may consist of a plasma of unbounded nuclei and electrons, i.e., a plasma composed of positively charged ions providing almost all the mass (inertia) and the pressure, as well as electrons providing the pressure (restoring force) but none of the mass (inertialess), and the magnetic field in these environments can be strong, the drift–wave excitation in degenerate dense plasmas could be useful for identifying the modulated drift–wave packets that may spontaneously emerge in magnetized white dwarfs, neutron stars etc. Tasso\textsuperscript{15} and later Oremskii \textit{et al.}\textsuperscript{20} had in-
vestigated the nonlinear interactions of one-dimensional drift waves in presence of a uniform magnetic field as well as the density and temperature gradients in electron-ion plasmas. They reported the formation of drift-wave turbulence in magnetized plasmas. However, two- or three-dimensional effects are essential for the drift-wave turbulence in classical, magnetically confined plasmas (tokamak) for the anomalous transport of charged particles.

The modulational instability (MI) is a well-known mechanism for the energy localization of wave packets in a nonlinear dispersive medium. It signifies the exponential growth of a small plane wave perturbation while propagating in the medium. This gain leads to the amplification of the sidebands, and thereby breaking up the uniform wave into a train of pulses. Thus, the MI acts as a precursor for the formation of bright envelope solitons or highly energetic rogue waves in plasmas. The amplitude modulation of a finite amplitude drift wave by zonal flows has been considered by Jovanovic et al. in a nonuniform magnetoplasma. They showed that the full nonlinear system of equations governing drift-wave zonal flow interactions can be reduced to a cubic nonlinear Schrödinger equation (NLSE), which possesses localized envelope soliton solutions. It is to be noted that the nonlinear interaction between zonal flows and drift waves as in Ref. involves one-dimensional propagation that belong to the convective cell mode (with a non-Boltzmann electron distribution) rather than to the drift-mode spectrum. Though, such nonlinear interaction is energetically more favorable than one with zonal flows that belong to the drift-wave spectrum, there may be the situation when the convective-cell part of the spectrum does not appear, e.g., in the dynamics of Rossby-wave turbulence in rotating fluid. However, it has been shown that the presence of immobile charged dust grains may lead to the appearance of an additional term (proportional to the dust-density gradient) in the ion vorticity equation [See, e.g., Eq. (13) in Ref. which is associated with the Shukla-Varma mode (convective cell) in degenerate dense electron-ion plasmas. Furthermore, the convective cell mode is not affected by the degeneracy pressure of electrons and the quantum force associated with the Bohm potential and the degeneracy pressure of electrons. It is shown that the dynamics of the modulated drift-wave packet is governed by a modified NLSE, which depicts the formation of dark and bright envelope solitons, as well as drift-wave rogons.

II. DERIVATION OF QUANTUM DRIFT-WAVE EQUATION

We consider a nonuniform quantum electron-ion magnetoplasma in presence of the inhomogeneities of the equilibrium density and the external magnetic field \(zB_0(x)\). Thus, at equilibrium, the gradient of the total energy (Fermi energy density plus the magnetic energy density) vanishes, i.e.,

\[
\frac{\partial}{\partial x} \left[ n_0(x) T_{Fe}(x) + \frac{B_0^2(x)}{8\pi} \right] = 0, \tag{1}
\]

where \(n_0(x)\) is the unperturbed electron or ion number density and \(T_{Fe}(x) \equiv (\hbar^2/2k_B m_e) (3\pi^2 n_0(x))^{2/3}\) is the electron Fermi temperature in which \(h = h/2\pi\) is the reduced Planck’s constant, \(m_e\) is the electron mass and \(k_B\) is the Boltzmann constant.

In the propagation of the low-frequency (in comparison with the ion gyrofrequency \(\omega_{ci} = eB_0/m_ec, \) where \(e\) is the magnitude of the electron charge, \(m_i\) is the ion mass, and \(c\) is the speed of light in vacuum), long-wavelength (in comparison with the ion-thermal gyroradius \(\rho_i = c/\omega_{ci}\), where \(c_\perp = \sqrt{2k_B T_{Fe}/m_i}\) is the quantum ion-acoustic speed) electrostatic (with the field \(E = -\nabla \phi\), where \(\phi\) is the electrostatic potential) drift waves in nonuniform quantum magnetoplasmas, the perpendicular (to \(\mathbf{z}\)) components of the electron and ion fluid velocities are

\[
\mathbf{v}_{e\perp} \approx \frac{c}{B_0(x)} \hat{z} \times \nabla \phi - \frac{c}{eB_0(x) n_e} \hat{z} \times \nabla P_e - \frac{c}{eB_0(x)} \hat{z} \times \nabla \psi_q \equiv \mathbf{V}_E + \mathbf{V}_D + \mathbf{V}_Q, \tag{2}
\]

\[
\mathbf{v}_{i\perp} \approx \frac{c}{B_0(x)} \hat{z} \times \nabla \phi - \frac{c}{B_0(x) \omega_{ci}} \frac{d}{dt} \left( \nabla \phi \right) \equiv \mathbf{V}_E + \mathbf{V}_P, \tag{3}
\]

where we have assumed that \(|d/dt| \ll \nu_{ci} \ll \omega_{ce}\), with \(d/dt \equiv \partial/\partial t + \mathbf{V}_E \cdot \nabla\) and \(\nabla \perp \equiv \hat{z}(\partial/\partial x) + \hat{y}(\partial/\partial y)\).

Here \(\nu_{ci}\) is the electron-ion collision frequency and \(\omega_{ce} = eB_0/m_ec\) is the electron gyrofrequency. Also, \(\psi_q = \psi q\) .
$-(\hbar^2/2m_e) (\nabla^2\sqrt{n_e}/\sqrt{n_e})$ is the quantum Bohm potential, and $V_E$, $V_D$, $V_P$ and $V_Q$ are, respectively, the $E \times B$ drift, diamagnetic drift, polarization drift and the quantum drift (due to tunneling) velocities. The pressure $P_e$ for weakly relativistic degenerate electrons is given by the following equation of state: \[ P_e = \frac{m_e V_{Fe}^2}{5n_0^2}(x)n_e^{5/3} = \frac{1}{5} \left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{m_e} n_e^{5/3}, \] where $V_{Fe} = \sqrt{2k_B T_{Fe}/m_e}$ is the electron Fermi-thermal speed.

Next, we neglect the ion motion parallel to $\hat{z}$ as well as the compressional magnetic field perturbation (i.e., we discard the coupling between the drift waves and the quantum ion-acoustic waves)\cite{5, 22}. Such an approximation is valid in a low-$\beta$ plasma with wave frequency satisfying $|\partial/\partial t| \gg v_F |\partial/\partial z|$. Thus, inserting the parallel component of the electron density perturbation, we obtain the modified neutrality condition $n_e \| \rho_e \|$. In order to obtain the modified equation for the electron plasma oscillation frequency, the following modified (by the quantum effects) Boltzmann law for the electron number density perturbation\cite{31, 32}

\[
\frac{n_{e1}}{n_0(x)} = \left[ 1 + \varphi + \frac{\hbar^2}{2} \left( 1 + \varphi \right)^{3/4} \nabla^2 \left( 1 + \varphi \right)^{3/4} \right]^{3/2}
\]

\[\approx 1 + \frac{3}{2} \varphi^2 + \frac{3}{8} \varphi^2 + 9 \hbar^2 \varphi^2 \left[ \nabla^2 \varphi - \frac{1}{4} \nabla^2 \varphi - \frac{1}{2} \varphi \nabla^2 \varphi \right],\] (5)

where $n_{e1} \ll n_0$, $\varphi = e\phi/k_B T_{Fe} \ll 1$ and $\mathcal{H} = H \omega_{ci}/\omega_{pi}$ with $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$ denoting the ion plasma oscillation frequency and $H = \hbar \omega_{pi}/k_B T_F$ the ratio of the plasmon energy density to the Fermi energy density. In deriving Eq. (5), we have considered the semiclassical limit $\hbar^2 \ll 1$.

Substituting Eq. (5) into the ion continuity equation, we obtain\cite{22}

\[
\frac{\partial n_{i1}}{\partial t} + \nabla \cdot \left[ (n_0(x) + n_{i1}) V_E \right] \approx \frac{c n_{0i}(x)}{B_0(x)} \frac{d\nabla^2 \varphi}{dt}. \] (6)

We can now combine Eqs. (5) and (6) under the quasineutrality condition $n_{i1} = n_{e1}$, which holds for a dense magnetized plasma with $\omega_{pi} \gg \omega_{ci}$, to obtain the modified quantum drift-wave equation in one-space dimension

\[
\left(1 - \frac{\partial^2}{\partial y^2}\right) \frac{\partial \varphi}{\partial t} + \left(\alpha - \beta \mathcal{H}^2 \frac{\partial^2 \varphi}{\partial y^2}\right) \frac{\partial \varphi}{\partial y} + \frac{1}{2} \frac{\partial \varphi}{\partial y} - \gamma \varphi \frac{\partial \varphi}{\partial y} \right) \right] = 0. \] (7)

where

\[
\alpha = -\frac{2}{3} \frac{\partial}{\partial x} \ln \left[ \frac{n_0(x)}{B_0(x)} \right] > 0, \] (8)

\[
\beta = \frac{1}{4} \left(\frac{3}{2} \right)^{5/2} \frac{\partial}{\partial x} \ln \left[ \frac{n_0^{-1/3}(x)}{B_0(x)} \right], \] (9)

\[
\gamma = \sqrt{\frac{3}{2}} \frac{\partial}{\partial x} \ln \left[ \frac{n_0^{1/3}(x)}{B_0(x)} \right], \] (10)

and $\delta = 9/16$. In Eq. (7), the time $t$ is normalized by the ion gyroperiod $\omega_{ci}^{-1}$, the space coordinates $x$ and $y$ are normalized by $\rho_s$ and $\sqrt{(2/3)} \rho_s$, respectively. Furthermore, the drift-wave equation (7) has been modified due to both the quantum mechanical and statistical effects. The latter also modify the expressions for $\alpha$, $\beta$ and $\gamma$, which appear due to the density and magnetic field gradients, and can change their signs with the choice of the scale lengths of inhomogeneity. In the formal limit of $\mathcal{H} = 0$, i.e., simply disregarding the quantum Bohm potential term (tunneling effect), one can recover the similar expression as in Ref.\cite{22} except some factors which appear due to the different pressure law for degenerate electrons.

### III. DERIVATION OF NLSE AND MODULATIONAL INSTABILITY

We derive the governing nonlinear equation for the amplitude-modulated low-frequency ($\omega < \omega_{ci} \ll \omega_{pi}$) drift-wave packets in quantum magnetoplasmas. Note that the wave equation (7), which describes the evolution of the electrostatic perturbation $\varphi$ has a harmonic wave solution $\varphi = \varphi_0 \exp(i k y - i \omega t)$ in the small-amplitude limit $\varphi_0 \ll 1$. However, whenever the wave amplitude becomes non-negligible, a nonlinear mechanism for the generation of harmonic waves comes into play. In order to study the amplitude modulation and associated stability/instability profiles of these electrostatic drift waves we assume that $\varphi$ takes the form of a modulated wave packet, i.e., the composition of a fast carrier wave with a slow variation in amplitude. Initial drift-wave packets are modulated by the nonlinear effects. If we see the packet on the coordinate frame moving at the group velocity $v_g$ (to be determined by the linear dispersion relation), the time variation of the packet looks slow and hence the space and the time variables are stretched as $\xi = (y - v_g t)$, $\tau = t^2$, where $\epsilon$ is a small free (real) parameter $0 < \epsilon < 1$) representing the weakness of perturbation. Following the standard reductive perturbation technique\cite{33}, we expand $\varphi$ about its equilibrium value as

\[
\varphi = 0 + \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \cdots \] (11)
where the perturbation \( \varphi^{(n)} \), \( n = 1, 2, 3, \ldots \) can be considered as a sum of infinite number of Fourier modes:

\[
\varphi^{(n)} = \sum_{l=-\infty}^{\infty} \varphi_l^{(n)}(\xi, \tau) \exp[i(lky - \omega t)].
\]

Thus, we express the wave potential \( \varphi \) as

\[
\varphi = \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} \varphi_l^{(n)}(\xi, \tau) \exp[i(lky - \omega t)],
\]

where \( \varphi_{-l}^{(n)} = \varphi_l^{(n)*} \) holds for real physical variables and the asterisk denotes the complex conjugate. In practice, only the terms with \( l \leq n \) contribute in the summation, i.e., (up to) first harmonics are expected for \( n = 1 \), up to second harmonics for \( n = 2 \) etc.

Next, to derive an evolution equation of NLSE-type, we follow the similar method as, e.g., in Refs. [35,36]. In Ref. [36], the modulational instability and the evolution of localized wave envelopes in space and dusty plasmas has been studied. Here, starting from a set of fluid equations, and using a standard reductive perturbation technique, a NLSE has been derived which was shown to admit bright or dark envelope solitons. Furthermore, the occurrence of freak waves or rogue waves associated with the propagation of electrostatic wave packets in quantum electron-positron-ion plasmas has been investigated by Mckerr et al. [35]. Using a multiscale technique, the authors have shown that the evolution of the wave envelopes can be described by a NLSE, which admits envelope solitons as well as localized breathers. Thus, following Refs. [35,36], we substitute the expansion Eq. (13) into Eq. (7), and equate different powers of \( \epsilon \).

For \( n = l = 1 \), equating the coefficient of \( \epsilon \), we obtain the following linear dispersion law for the quantum drift waves (Since we are interested in the modulation of a plane wave with frequency \( \omega \) and wave number \( k \), we put \( \varphi_1^{(1)} = 0 \) for all \( l \) except \( l = \pm 1 \))

\[
\omega = \frac{\alpha k}{1 + k^2(1 - \delta \mathcal{H}^2)}. \tag{14}
\]

Since \( \delta \mathcal{H}^2 < 1 \), it follows that the frequency of the carrier drift-wave mode decreases as the quantum parameter \( \mathcal{H} \) (\( < 1 \)) decreases from a certain value, say \( \mathcal{H} = 0.3 \). This is the consequence of relatively high-density regimes for plasmas in which a drift-wave carrier modes propagate with lower frequencies. The effect of \( \mathcal{H} \) on the wave modes is more pronounced when the wave numbers \( k \) approaches 1. However, in the limit \( k \to 0 \), the wave becomes dispersionless.

From the second-order expressions for the first harmonics, i.e., for \( n = 2, l = 1 \), we obtain an equation in which the coefficient of \( \varphi_1^{(1)} \) vanishes by the dispersion equation (14), and the coefficient of \( \partial \varphi_1^{(1)} / \partial \xi \), after equating to zero, gives the following compatibility condition:

\[
v_g \equiv \frac{\partial \omega}{\partial k} = \frac{\alpha (1 - k^2 + \delta \mathcal{H}^2 k^2)}{(1 + k^2 - \delta \mathcal{H}^2 k^2)^2}. \tag{15}
\]

Thus, the group velocity of the drift-wave packet is also modified by the term \( \sim \mathcal{H} \). Its value increases (decreases) with increasing (decreasing) values of \( \mathcal{H} \). Evidently, \( v_g > 0 \) for \( \alpha > 0, \delta \mathcal{H}^2 < 1 \) and for wave numbers satisfying \( k < 1/\sqrt{1 - \delta \mathcal{H}^2} \).

The zeroth harmonic mode appears due to the nonlinear self-interaction of the drift-wave modes. Thus, equating the coefficient of \( \epsilon^2 \) for \( n = 2, l = 0 \), we obtain

\[
\varphi_0^{(2)} = \left( \frac{\gamma + v_g/2 + \beta \mathcal{H}^2 k^2}{\alpha - v_g} \right) \left| \varphi_1^{(1)} \right|^2 = \lambda \left| \varphi_1^{(1)} \right|^2. \tag{16}
\]

Next, we consider the second-order harmonic mode for \( n = l = 2 \), and equate the coefficient of \( \epsilon^2 \) to obtain

\[
\varphi_2^{(2)} = \frac{\omega + 2\gamma k + 2H^2 k^2(3\omega /2 - \beta k)}{4(\alpha k - \omega (1 + 4k^2) + 2\omega \delta \mathcal{H}^2 k^2)} \left| \varphi_1^{(1)} \right|^2
\]

and for wave numbers \( \delta \mathcal{H}^2 < 1 \), \( \delta \mathcal{H}^2 < 1 \), we obtain an equation for the third-order first-harmonic mode in which the coefficients of \( \varphi_1^{(3)} \) and \( \partial \varphi_1^{(2)} / \partial \xi \) vanish by the dispersion relation and the group velocity expression, respectively. In the reduced equation we substitute the expressions for \( \varphi_0^{(2)} \) and \( \varphi_2^{(2)} \) from Eqs. (16) and (17) to obtain the following NLSE

\[
i \frac{\partial \Phi}{\partial \tau} + P \frac{\partial^2 \Phi}{\partial \xi^2} + Q |\Phi|^2 \Phi = 0, \tag{18}
\]

where \( \Phi = \varphi_1^{(1)} \) is the potential perturbation, or in the original frame of reference

\[
i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial y} \right) \Phi + P \frac{\partial^2 \Phi}{\partial y^2} + Q |\Phi|^2 \Phi = 0, \tag{19}
\]

with \( \Phi \sim \epsilon \varphi_1^{(1)} \). The coefficients of the drift-wave group dispersion and the nonlinearity are

\[
P \equiv \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} = \frac{(\delta \mathcal{H}^2 - 1)(\omega + 2kv_g)}{(1 + k^2 - \delta \mathcal{H}^2 k^2)^2}, \tag{20}
\]

and

\[
Q = \frac{(\lambda + \mu)(\omega /2 + \gamma k) + \mathcal{H}^2 k^2 Q_0}{1 + k^2 - \delta \mathcal{H}^2 k^2}, \tag{21}
\]

where \( Q_0 = 2\beta \mu k + \omega \delta (\lambda + 7\mu)/2. \)

In a generic manner, the modulated drift-wave packet whose amplitude is governed through the NLSE (18) can be stable (unstable) to a plane wave perturbation.
if \( PQ < 0 \) (\( > 0 \)) (See, e.g., Ref\(^{22}\)). From Eq. (20) we find that \( P \) is always negative for \( \delta \mathcal{H}^2 < 1 \). Thus, the sign of \( PQ \) changes with only the sign change of the nonlinear coefficient \( Q \). The latter depends not only on the range of values of the wave number \( k < 1 \), but also on the quantum parameter \( \mathcal{H} \) as well as the scale lengths of inhomogeneities. It turns out that the stable/ unstable domains of \( k \) get modified with the inclusion of the quantum effects. Following, e.g., Ref\(^{22}\) we find that to a plane wave perturbation with frequency \( \Omega \) and wave number \( K \), the product \( PQ \) is zero according to when \( K \leq K_* \), where \( K_* = \sqrt{2(Q/P)|\Phi_0|} \) is the critical value of \( K \) with \( \Phi_0 \) denoting the potential of the drift-wave pump. The instability growth rate is given by

\[
\Gamma = |P|K^2 \sqrt{K_*^2 - 1}, \tag{22}
\]

with a maximum \( \Gamma_{\text{max}} = |Q||\Phi_0|^2 \).

Next, we numerically investigate the range of values of \( k \) in which the drift waves become stable or unstable to the modulation for different values of the parameter \( \mathcal{H} \) as well as the scale length of inhomogeneity. For convenience, we assume the equilibrium density and the magnetic field with the scale lengths \( L_n \) and \( L_b \) to be of the forms \( n_0(x) = n_0(0) \exp(-x/L_n) \) and \( B_0(x) = B_0(0) \exp(-x/L_b) \) respectively. We also assume that \( L_b > L_n \) so that \( \alpha > 0 \). The sign of \( \gamma \) will depend on the choice of values of \( L_b \) larger than \( L_n \). The stable \( (PQ < 0) \) and unstable \( (PQ > 0) \) regions are plotted against \( k \) as shown in Fig. 1. It is seen that at higher values of \( \mathcal{H} \), which correspond to relatively low-density regimes, the instability occurs in a larger domain of \( k \). However, the stable regions remain almost unchanged with \( \mathcal{H} \). We find that for \( \mathcal{H} = 0.1 \), \( PQ > 0 \) occurs in \( 0.3149 \lesssim k \lesssim 0.83 \), while \( 0.3148 \lesssim k \lesssim 0.88 \) represents the domain in which \( PQ > 0 \) for \( \mathcal{H} = 0.3 \). Typically, for a ratio \( \omega_{ci}/\omega_{ce} = 0.1 \), the value \( \mathcal{H} = 0.1 \) corresponds to the plasma number density \( n_0 \sim 10^{24} \text{ cm}^{-3} \), while \( n_0 \sim 10^{21} \text{ cm}^{-3} \) is for \( \mathcal{H} = 0.3 \). On the other hand, the larger the scale length of inhomogeneity of the magnetic field, the greater is the instability domain. From Fig. 1 it is also found that there exists a critical value of the scale length \( L_b \) below which the drift wave is modulationally stable.

The MI growth rate \( \Gamma \) is exhibited in Fig. 2. We find that the growth rate of instability is enhanced with higher values of \( \mathcal{H} \), i.e., as one enters from higher density regimes (say, \( n_0 \sim 10^{24} \text{ cm}^{-3} \)) to relatively lower ones (say, \( n_0 \sim 10^{21} \text{ cm}^{-3} \)), \( \Gamma \) tends to attain its maximum value. In other words, the growth rate of instability is suppressed when the quantum effects are more pronounced in high-density regimes. The increasing trend of \( \Gamma \) is also seen for a fixed \( \mathcal{H} \), but at lower values of the scale length of inhomogeneity \( L_b \). In each of these cases, the cut-offs of \( \Gamma \) occur at higher values of the wave number of modulation \( K \).

In the ranges of values of \( k \) for which the drift-wave packet becomes unstable \( (PQ > 0) \), Eq. (18) can be rewritten as

\[
i \frac{\partial \Phi}{\partial \hat{\tau}} + \frac{1}{2} \frac{\partial^2 \Phi}{\partial \hat{\xi}^2} + |\Phi|^2 \Phi = 0, \tag{23}
\]

where \( \hat{\tau} = Q\tau \) and \( \hat{\xi} = \sqrt{Q/2P} \xi \). Thus, Eq. (23) have the following rogue wave solution that is located on a non-
been recently obtained by Akhmediev et al.\textsuperscript{39} higher amplitudes. The analytic form of these waves have
generation of another highly energetic rogue waves with
first-order rogue waves is also possible, and can cause the
amplification of the amplitudes of the wave packets are
modulated by the nonlinearity during the evolution of the wave pack-
and time, are generated due to the MI of the coher-
ent quantum drift-wave packets in the limit of infinite
energy is concentrated in a relatively small area in space
and time, is generated at a speed $V$ and oscillating at a frequency $\Omega$
which represents a localized region of hole (void) traveling at a speed $U$.
The pulse width $W_b$ of the pulse is given by $W_b = \sqrt{2P/Q}\Psi_{a0}$, where $\Psi_{a0}$ is the constant amplitude.

On the other hand, for $PQ < 0$, the quantum drift-
wave packets are modulational stable which may propagate in the form of dark-envelope solitons characterized by a depression of the drift-wave potential around $\xi = 0$.

Typical form of this solution is given by

$$
\Psi = \Psi_{a0} \tanh^2 \left( \frac{\xi - V\tau}{W_d} \right), \quad \theta = \frac{1}{2P} \left[ U\xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right],
$$

where $G_n$, $H_n$ and $D_n$ ($\neq 0$) are some polynomial functions of $\xi$ and $\tau$, and $n = 1, 2, 3, \ldots$, is the order of the solution. The first-order ($n = 1$) solution corresponds to the Peregrine soliton\textsuperscript{39} in which $G_1 = 4$, $H_1 = 8\tau$ and $D_1 = 1 + 4\xi^2 + 4\tau^2$. However, superposition of two
first-order rogue waves is also possible, and can cause the generation of another highly energetic rogue waves with higher amplitudes. The analytic form of these waves have been recently obtained by Akhmediev et al.\textsuperscript{39} using the deformed Darboux transformation approach. This second order ($n = 2$) rogue solution has the same form as
Eq. (24) where the polynomials $G_2$, $H_2$ and $D_2$ are given by\textsuperscript{[39,40]}

$$
G_2 = -\left( \xi^2 + \tau^2 + \frac{3}{4} \right) \left( \xi^2 + 5\tau^2 + \frac{3}{4} \right) + \frac{3}{4},
$$

$$
H_2 = \tau \left[ 3\xi^2 - \tau^2 - 2 \left( \xi^2 + \tau^2 \right)^2 - \frac{15}{8} \right],
$$

$$
D_2 = \frac{1}{3} \left( \xi^2 + \tau^2 \right)^3 + \frac{1}{4} \left( \xi^2 - 3\tau^2 \right)^2 + \frac{3}{64} \left( 12\xi^2 + 44\tau^2 + 1 \right).
$$

FIG. 3. The evolution of (a) first (upper panel) and (b) second (lower panel) order rogue solutions of Eq. (23) at $k = 0.5$ for $L_n = 0.01$, $L_b = 6L_n$ and $H = 0.1$. Here, $\xi = \tilde{\xi}$ and $\tau_1 = \tilde{\tau}$. The corresponding values of $P$, $Q$, $\alpha$, $\beta$ and $\gamma$ are $P = -0.2391$, $Q = -0.0028$, $\alpha = 68$, $\beta = 34$ and $\gamma = -20$.

This represents a localized pulse traveling at a speed $U$ and oscillating at a frequency $\Omega_0$ at rest. The width $W_b$ of the pulse is given by $W_b = \sqrt{2P/Q}\Psi_{a0}$, where $\Psi_{a0}$ is the constant amplitude.

As is known, MI of wave envelopes also gives rise to the formation of bright envelope solitons of Eq. (18), whose exact analytic form can be obtained by considering $\Psi = \sqrt{V} \exp(it\theta)$, where $\Psi$ and $\theta$ are real functions to be determined (see for details, e.g., Ref.\textsuperscript{12}). Thus, for $PQ > 0$ Eq. (18) has the following bright-envelope soliton solution

$$
\Phi_n(\xi, \tau) = \left[ (-1)^n + \frac{G_n(\xi, \tau) + iH_n(\xi, \tau)}{D_n(\xi, \tau)} \right] \exp(it),
$$

The typical forms of these rogue solutions given by Eq. (24) for $n = 1, 2$ are shown in Fig. 3. These highly energetic rogue waves in which a significant amount of energy is concentrated in a relatively small area in space and time, are generated due to the MI of the coherent quantum drift-wave packets in the limit of infinite wave modulation period. In particular, they significantly amplify the carrier wave amplitudes, and hence increase the nonlinearity during the evolution of the wave packets. The amplification factor of the amplitude of the $n$-th order rational solution [Eq. (24)] at $\xi = \tilde{\xi} = 0$ is, in general, of $2n + 1$. Hence, localized quantum drift waves that are modelled by the higher-order breather solutions can also cause the formation of super rogue waves. To mention, the first-order Peregrine soliton has been recently observed experimentally in plasmas\textsuperscript{26}. However, the second-order rogue soliton, which has been observed in water waves\textsuperscript{31} is yet to be observed in plasmas.

As is known, MI of wave envelopes also gives rise to the formation of bright envelope solitons of Eq. (18), whose exact analytic form can be obtained by considering $\Psi = \sqrt{V} \exp(it\theta)$, where $\Psi$ and $\theta$ are real functions to be determined (see for details, e.g., Ref.\textsuperscript{12}). Thus, for $PQ > 0$ Eq. (18) has the following bright-envelope soliton solution

$$
\Psi = \Psi_{a0} \tanh^2 \left( \frac{\xi - V\tau}{W_d} \right), \quad \theta = \frac{1}{2P} \left[ U\xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right],
$$

where $G_n$, $H_n$ and $D_n$ ($\neq 0$) are some polynomial functions of $\xi$ and $\tau$, and $n = 1, 2, 3, \ldots$, is the order of the solution. The first-order ($n = 1$) solution corresponds to the Peregrine soliton\textsuperscript{39} in which $G_1 = 4$, $H_1 = 8\tau$ and $D_1 = 1 + 4\xi^2 + 4\tau^2$. However, superposition of two
first-order rogue waves is also possible, and can cause the generation of another highly energetic rogue waves with higher amplitudes. The analytic form of these waves have been recently obtained by Akhmediev et al.\textsuperscript{39} using the deformed Darboux transformation approach. This second order ($n = 2$) rogue solution has the same form as
Eq. (24) where the polynomials $G_2$, $H_2$ and $D_2$ are given by\textsuperscript{[39,40]}

$$
G_2 = -\left( \xi^2 + \tau^2 + \frac{3}{4} \right) \left( \xi^2 + 5\tau^2 + \frac{3}{4} \right) + \frac{3}{4},
$$

$$
H_2 = \tau \left[ 3\xi^2 - \tau^2 - 2 \left( \xi^2 + \tau^2 \right)^2 - \frac{15}{8} \right],
$$

$$
D_2 = \frac{1}{3} \left( \xi^2 + \tau^2 \right)^3 + \frac{1}{4} \left( \xi^2 - 3\tau^2 \right)^2 + \frac{3}{64} \left( 12\xi^2 + 44\tau^2 + 1 \right).
$$
the number density of charged particles varies in the rage $10^{21} - 10^{26}$ cm$^{-3}$ with the temperature $T \lesssim 10^7$ K and the magnetic field $B_0 \sim 10^8$ T or more.

**ACKNOWLEDGEMENT**

This research was partially supported by the SAP-DRS (Phase-II), UGC, New Delhi, through sanction letter No. F.510/4/DRS/2009 (SAP-I) dated 13 Oct., 2009, and by the Visva-Bharati University, Santiniketan-731 235, through Memo No. Aca-R-6.12/921/2011-2012 dated 14 Feb., 2012.

1. M. Tabak, J. Hammer, M. Glinsky, W. Krueer, S. Wilks, J. Woodward, E. Campbell, M. Perry and R. Mason, Phys. Plasmas 1, 1626 (1994).
2. Z. Son, and N. J. Fisch, Phys. Rev. Lett. 95, 225002 (2005).
3. P. K. Shukla and B. Eliasson, Phys. Usp. 53, 51 (2010).
4. A. P. Misra and P. K. Shukla, Phys. Rev. E 85, 026409 (2012).
5. A. P. Misra and C. Bhowmik, Phys. Plasmas 16, 012103 (2009).
6. P. K. Shukla and L. Stenflo, Phys. Lett. A 357, 229 (2006).
7. P. K. Shukla and S. Ali, Phys. Plasmas 13, 082101 (2006).
8. Z. Wu, H. Ren, J. Cao and P. K. Chu, Phys. Plasmas 15, 082103 (2008).
9. M. Salimullah, M. Jamil, I. Zeba, Ch. Uzma, and H. A. Shah, Phys. Plasmas 16, 034503 (2009).
10. Z. Wu, H. Ren, J. Cao and P. K. Chu, Phys. Plasmas 15, 082103 (2008).
11. Ali, N. Shukla and P. K. Shukla, Europhys. Lett. 78, 45011 (2007).
12. B. B. Kadomtsev, Plasma Turbulence (Academic, New York, 1965).
13. J. Weiland, Collective Modes in Inhomogeneous Plasma: Kinetic and Advanced Fluid Theory (Institute of Physics, Bristol, 2000).
14. W. Horton, Rev. Mod. Phys. 71, 735 (1999); W. Horton and A. Hasegawa, Chaos 4, 227 (1994).
15. V. I. Petviashvili and O. A. Pokhotelov, Sov. J. Plasma Phys., 12, 657 (1986).
16. D. Sundqvist, V. Krasnoselskikh, P. K. Shukla, A. Vaivads, Mats André, S. Buchert and H. Rème, Nature (London) 436, 825 (2005); D. Sundqvist and S. D. Bale, Phys. Rev. Lett. 101, 065001 (2008).
17. N. Vianello, M. Spolacore, E. Martines, R. Cavazzana, G. Serianni, M. Zuin, E. Spada and V. Antoni, Nucl. Fusion 50, 042002 (2010).
18. C. Theiler, L. Furno, J. Loizu and A. Fasoli, Phys. Rev. Lett. 108, 065005 (2012).
19. H. Tasso, Phys. Lett. A 24, 618 (1967).
20. V. N. Oremskii, H. Tasso, and H. Wobig, in Proc. 3rd International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, USSR, 1968, International Atomic Energy, Vienna, Vol. 1, p. 671 (1969).
21. V. I. Petviashvili, Sov. J. Plasma Phys. 3, 150 (1977).
22. P. K. Shukla and A. P. Misra, Phys. Lett. A 376, 2591 (2012).
23. W. M. Moslem, R. Sabry, S.K. El-Labany, P. K. Shukla, Phys. Rev. E 84, 066402 (2011).
24. P. K. Shukla and A. P. Misra, Phys. Lett. A 376, 2591 (2012).
25. P. K. Shukla and W. M. Moslem, Phys. Lett. A 376, 1125 (2012).
26. H. Bajung, S. K. Sharma and Y. Nakamura, Phys. Rev. Lett. 107, 255005 (2011).
27. D. Jovanovic, P. K. Shukla and B. Eliasson, J. Plasma Phys. 76, 665 (2010).
28. D. Jovanovic and P. K. Shukla, Phys. Lett. A 374, 2048 (2010).
29. P. K. Shukla and R. K. Varma, Phys. Fluids B 5, 236 (1993).
30. H-F Liu, S-Q Wang, K-H Li, Z-H Wang, W-B Zhang, Z-L Wang, Q-Xiang, K-Huang, Y-Liu, S-Li, F-Z Yang and L-Chang, Phys. Plasmas 20, 044502 (2013).
31 S. Chandrasekhar, Mon. Not. R. Astron. Soc. 95, 207 (1935).
32 A. P. Misra and P. K. Shukla, Phys. Rev. E 85, 026409 (2012).
33 A. P. Misra and A. R. Chowdhury, Phys. Plasmas 13, 072305 (2006).
34 T. Taniuti, N. Yajima, J. Math. Phys., 10, 1369 (1969).
35 I. Kourakis and P. K. Shukla, Nonlinear Process. Geophys. 12, 407 (2005).
36 M. Mckerr, I. Kourakis and F. Haas, Plasma Phys. Control. Fusion, 56, 035007 (2014).
37 K. Nishikawa and C. S. Liu, Advances in Plasma Physics, Vol. 6, p. 59, Wiley, New York (1976).
38 W-P Zhong, M. R. Belic and T. Huang, Phys. Rev. E 87, 065201 (2013).
39 D. H. Peregrine, J. Aust. Math. Soc., Ser. B, Appl. Math. 25, 16 (1983).
40 N. Akhmediev, A. Ankiewicz, M. Taki, Phys. Lett. A 373, 675 (2009).
41 A. Chabchoub, N. Hoffmann, M. Onorato, N. Akhmediev, Phys. Rev. X 2, 011015 (2012).
42 I. Kourakis and P.K. Shukla, Nonlinear Processes in Geophysics, 12, 1 (2005); R. Fedele, H. Schamel and P. K. Shukla, Phys. Scr. T98, 18 (2002); idem, Eur. Phys. J. B 27, 313 (2002).