Study on Scattered Data Points Interpolation Method Based on Multi-line Structured Light

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Abstract. Aiming at the range image obtained through multi-line structured light, a regional interpolation method is put forward in this paper. This method divides interpolation into two parts according to the memory format of the scattered data, one is interpolation of the data on the stripes, and the other is interpolation of data between the stripes. Trend interpolation method is applied to the data on the stripes, and Gauss wavelet interpolation method is applied to the data between the stripes. Experiments prove regional interpolation method feasible and practical, and it also promotes the speed and precision.

1. Introduction
Data visualization technology has been applied widely in many fields such as nature science, engineering technology and communication. Spatial interpolation is a very important step for data visualization [1]. Spatial interpolation can compensate the loss data and enhance the data density, which is necessary for data visualization and 3D reconstruction of original object.

Now, there are a lot of spatial interpolation methods. Many foreign scholars have compared the interpolation methods according to different situation [2-3]. In our country, Li Xin [4] has classified spatial interpolation methods and introduced the applicability, arithmetic, excellence and shortcoming of each method. He has pointed out that there wasn’t absolutely optimal spatial inner interpolation method; optimal method should be chose according to the data spatial distributing features. This paper put forward a regional interpolation method according to the memory features of scattered range data obtained by multi-line structured light.

2. Ways of obtaining scattered data and its distributing features
The principle of obtaining scattered range data by multi-line structured light is as figure 1 shows.

\[
Z = B \left\{ \begin{array}{c}
\cot \alpha_i + \cot \beta_0 + \left( \frac{\tan \beta_1}{N} \right)_n \times \left( 1 - \left( \frac{\cot \beta_0 \times \tan \beta_1}{N} \right)_n \right)^{-1} \\
\end{array} \right\}^{-1} \tag{1}
\]
The spatial coordinate values of each pixel can be got from formula (1), (2) and (3). Therefore, the 3D coordinate values of all sectional points of the object can be got after computing each point on the stripes.

Scattered data is distributed in the form of stripe in the space, and is deposited in the form of matrix in range image. In rang image, the matrix is (m, n), m and n present the row value and the column value of the point respectively. Each element in the matrix corresponds to a vector (X, Y, Z) of each 3D spatial point. The vector is the coordinate values of the practical scanning point corresponding to the pixel in the world coordinate.

3. Stripe scattered points interpolation study
Because of the limit of the system such as shelter characteristic, the affect of object surface characteristic and thinning operation, some stripes are not integrate, like disconnection points and disconnection stripes, they loss some spatial information, which will cause it hard to reconstruct the 3D object precisely. Spatial interpolation is needed to get the lost information. According to the distributing form of the data, this paper use trend interpolation method, which has the excellence of reflecting the change of the object surface rapidly such as cycle and trend.

3.1. Interpolation principle
The basic thought of trend fitting technology [5] is to use surface presented by functions to fit the trend change of the phenomena feature. Polynomials regressive analysis is the simplest way to describe long distance gradual change feature. When 2D space fit, suppose the spatial coordinates of the data point, namely X and Y, are independent variables, coordinate Z that presents the feature value is a dependent variable, so the binary regressive function is:

\[ Z = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \] (4)

In the formula mentioned above, \( a_0, a_1, a_2, a_3, a_4, a_5 \) are polynomials coefficients. Given n sample points, the measurement value is \( Z_i \), and the estimated value is \( Z_i' \), when the difference square summation of \( Z_i \) and \( Z_i' \) is minimum, namely

\[
X = Z \cot \alpha_i = B \cot \alpha_i \left\{ \cot \beta_0 + \left( \frac{\tan \beta_1}{N} \right)_n \times \left[ 1 - \left( \frac{\cot \beta_0 \times \tan \beta_1}{N} \right)_n \right]^{-1} \right\}^{-1} 
\]

\[
Y = \frac{m}{M} \tan \beta_2 \left( B \cos \beta_0 - x \cos \beta_0 + Z \sin \beta_0 \right) 
\]
then regressive equation matches the fitting line or surface best. The coefficients can be got from it.

3.2. Interpolation thought
The degree of regressive function is not the higher the better, generally, 2 or 3 would be alright. Polynomials with high degree can approach the measurement point well, but it makes the computation more complex and reduces the effect of separating trend to the inner interpolation and outer deduction effect, therefore it separate the holistic trend and less reflect the trend principle.

Interpolation steps are as follows:
1. Check the number of lost point on the stripes, set the part where the number of lost point is less than 9 or equal to 9 as point loss, and set the part where the number of lost point is more than 9 as segment loss.
2. For the part where some point is lost, take the data points around the lost point to make regressive analysis, determine the coefficients of the regressive function according to the minimum difference square summation principle, and compute the lost data.
3. For the part where segment is lost, adopt the data of the two ends of the lost segment to make regressive analysis respectively, and determine the best coefficients. Then the two functions do interpolation from the two ends until all the lost data is compensated.
4. Interpolation isn’t finished until there isn’t data lost on the whole stripe.

3.3. Interpolation effect
Intercept the parts that need trend interpolation from the image. The following two figures are before interpolation figure and after interpolation figure. Through compare, we can find that trend can reflect the change trend of the stripe well.

![Figure 2. Stripes before trend interpolation.](image1)
![Figure 3. Stripes after trend interpolation.](image2)

4. Interpolation study between stripes
Although multi-line structured light method can obtain the object surface’s information rapidly, due to the effect of projective distance, the distance between stripes is long, which makes a great deal of object information lost, and that is adverse for reconstruction and data visualization. Gauss wavelet interpolation method [6] can interpolate data between stripes rapidly, turn sparse points into dense points, which lay a firm foundation for visualization and reconstruction.

4.1. Interpolation principle
2D Gauss wavelet function is adopted as interpolation function, so we get

$$z = f(x, y) = k \exp \left[ -\frac{1}{2} \left( \frac{x - a}{u} \right)^2 \right] \exp \left[ -\frac{1}{2} \left( \frac{y - b}{v} \right)^2 \right]$$

(6)

In the formula, $K$ is a wavelet coefficient, $x$ and $y$ are the plane coordinate values of the interpolation points, $z$ is the corresponding depth value, $a$ is a translation factor in the direction $x$, $b$ is a translation factor in the direction $y$, $u$ is a flex factor in the direction $x$, $v$ is a flex factor direction in the direction $y$. $K, a$ and $b$ can be got from the following formulas:
\[
\begin{align*}
a &= \frac{(y_2 - y_3)\left[u^2(x_2^2 - x_3^2) + u^2(y_2^2 - y_3^2) + 2u^2v^2(\ln z_2 - \ln z_3)\right] - 2v^2[(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)]}{2v^2[(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)]} \\
b &= \frac{(x_2 - x_3)[v^2(x_2^2 - x_3^2) + u^2(y_2^2 - y_3^2) + 2u^2v^2(\ln z_2 - \ln z_3)] - 2u^2[(y_1 - y_3)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)]}{2u^2[(y_1 - y_2)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)]} \\
K &= z_1 \exp\left[-\frac{1}{2}\left(\frac{x_1 - a}{u}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{y_1 - b}{v}\right)^2\right]
\end{align*}
\]

In the formulas, \((x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)\) are coordinate value of the points on the two neighboring stripes.

4.2. Interpolation thought

The interpolation point is between two stripes. Three points are drew from the two stripes each time, one point from one stripe, the other two points from the other one stripe. Set the values of the three points as known quantity and put them into formulas from (7) to (9), so the translation factors and the flex factors of the wavelet functions can be got. Then put the translation factors, the flex factors and the plane coordinate of the interpolation point into wavelet function (6) and get the depth data of the interpolation point. The plane coordinate values of the point can be got from the following formulas:

\[
\begin{align*}
x &= \frac{1}{3}(x_1 + x_2 + x_3) \\
y &= \frac{1}{3}(y_1 + y_2 + y_3)
\end{align*}
\]

The principles of choosing points are that firstly, set the first points of the two neighboring stripes to be datum mark, then compute the distances between the second points of each stripe and the two datum marks respectively and set the point whose distance between itself and the two datum marks is minimum to be the third point. Then, set the second point of the stripe with two points and the first point of the other stripe to be new datum marks, find the third point according to the same way. The course of choosing points isn’t finished until all the points of one of the stripe is used up. The coordinate of these interpolation points computed in this way can embody the information of neighboring points on the surface of the object fully.

The flex factors \(u\) and \(v\) are got from experiments. Generally, when \(u\) is about 11 and \(v\) is about 16, the interpolation effect is good. In this paper, \(u\) is 10 and \(v\) is 15.

The interpolation steps are as follows:

1. Find the first point and determine the third point.
2. Compute the coordinate value \((x, y)\) of the interpolation point according to the plane coordinate values \((x_i, y_i)(i=1,2,3)\) of the three points to determine the location of the interpolation point.
3. Put \((x_i, y_i)(i=1,2,3)\) into formulas from (7) to (9), get the translation factors and wavelet coefficients.
4. Put the coefficients into formula (6), get the depth value of the interpolation point.
5. Find the location of another interpolation point according to the principle mentioned above, and then skip to step 2.
6. Interpolation isn’t finished until all the points of one of the two stripes is used up.
4.3. Interpolation effect
The following figures are image before Gauss wavelet interpolation and image after Gauss wavelet interpolation. Through compare, we can find that dense points can be got after interpolation.

![Figure 4. Image before Gauss wavelet interpolation.](image1)

![Figure 5. Image after Gauss wavelet interpolation.](image2)

5. Experiment
From Figure 8 and Figure 7, we can find that image after interpolation can reflect the surface shape of the object well, and get more surface information than that before interpolation. Therefore it provides more spatial information for reconstruction and data visualization.

![Figure 6. Range image.](image3)

![Figure 7. Image after thinning.](image4)

![Figure 8. Image after interpolation.](image5)

6. Conclusion
In this paper, when we do interpolation for the range image obtained by multi-line structured light, trend-fitting method is adopted to do interpolation for the points on the stripes and Gauss wavelet interpolation method is adopted for the points between stripes. This interpolation method adopts different interpolations aiming at different region not only can do interpolation rapidly, but also has a better effect.

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