Prospective teachers’ understanding on students’ learning hypotheses in solving proportion problem

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Abstract. Prospective mathematics teachers need to comprehend mathematics competencies along with how they would teach it. This article describes prospective teacher skills in predicting students’ learning hypothesis regarding proportion problem. An explorative qualitative research was carried out to investigate how the students who were mathematics teacher candidates understand mathematical learning hypotheses. These research participants were 12 students during the third year of their undergraduate study. Data was collected from students’ written work and field notes during a focus group discussion about understanding hypothetical learning trajectory (HLT). The discussion was a part of Indonesian realistic mathematics education course. It raised a topic specifically how junior high school students’ thinking toward proportion problem. The result of this research highlighted that most of the prospective teachers comprehend the content aspect of proportion problem as supported by their answers. However, they found it difficult to predict various ways of students’ thinking. They could only propose at most three possible ways to solve the problem. They might be lack of pedagogical aspect in teaching proportion such that they could not think on how the junior high school thinking to approach the problem.

1. Introduction
Teachers’ training is one important issue in mathematics education. Research on teacher training has been conducted through decades to support teacher quality improvement. It not only included prospective teacher (university students from education department) [1, 2] but also involved pre-service [3–5] and in-service teacher [6, 7]. The research topic varied from motivation, teaching skills [8], conceptual understanding and mathematical communication [9], even research on mathematics anxiety its relation with teachers [10]. It all shaped and promoted how the university prepared the prospective teachers.

In Indonesia, there were four competencies required to become a teacher [11]. These were pedagogical competencies, personality, professionalism, and social competencies. Teachers’ understanding of content or subject matter categorized as professional competency while knowledge on how a teacher would teach the content included in pedagogical competencies. Both of them promote various research topics on mathematics education [11, 12].

Regarding a popular proverb, the way teachers have been taught shaped their tendencies in how they would teach. Nicol and Crespo [13] stated that mathematical understanding might influence pre-service teacher while using textbooks or teacher’s guide. In fact, acquiring mathematical knowledge for teaching was complicated. Ball [14] proposed to reform teacher education based on research findings which highlight assumptions by teacher candidates. It was revealed that the teachers learn mathematics only to prepare them with subject matter knowledge. Instead of considering mathematical
knowledge was independent of context, researchers presented teachers should perceive mathematics situated in teaching practice [15].

Hence, it raises a discourse on how prospective teachers were prepared to comprehend such knowledge. Teacher education should be exposed with mathematics content and context within the plan on how they supposed to teach it. Widjajanti [16] tried a collaborative problem based learning to develop mathematics proficiency. It revealed that the strategy potentially promoted the prospective mathematics teacher competencies including conceptual understanding, procedural fluency, and adaptive reasoning.

In addition, Simon [17] introduced a Hypothetical Learning Trajectory (HLT) as an alternative for discussing the content as teaching preparation. The HLT could help mathematics educators plan up to evaluate the learning process. Starting from the learning goal setting, HLT required a teacher to design a specific mathematical task. The task was completed with hypotheses on how the students would respond it. This prediction would enable a teacher to plan questions, hints, or other feedback so that suit on the students’ need. By doing so, the teacher would learn not only the mathematics content but also the pedagogical aspect.

Using Gudmundsdottir and Shulman [18] terminology, HLT considered as a form of pedagogical content knowledge. From Vygotsky’s perspective, an HLT promote students’ learning at their zone of proximal development (ZPD). Several components contained in an HLT were the goal for the students’ learning, the mathematical task used to support their learning, and the hypotheses about the students’ learning process. This HLT usually found within the discussion on design research in mathematics education. Teacher or researcher used HLT as part of instructional design using Realistic Mathematics Education (RME) Approach [19–21].

However, educational research in Indonesia rarely discussed the potential effect of using HLT or pedagogical content knowledge to improve prospective teacher competencies preparing mathematics teaching. In fact, the research merely focused on analyzing the pedagogical content knowledge ability of prospective teachers [22–25]. It did not specifically expose how a mathematical task being answered by students from the perspective of the prospective teacher. It also did not address the hypotheses on how students would respond to the mathematical task.

This present study aims to investigate the prospective teachers’ understanding on students’ learning hypotheses. Therefore the research questions addressed in this study is how prospective teachers’ understanding on students’ learning hypotheses. The topic proportion was chosen since understanding different types of proportion and the solution was important as prospective mathematics teachers. It would help their students to acquire mathematical thinking especially in thinking proportionally.

2. Methods
This research is an explorative research which aims to investigate mathematics teacher candidates’ understanding on mathematical learning hypotheses. The participants were undergraduate students from mathematics education department at the second semester of their third year in university. There were 12 males students participated in this study. The participants were grouped in pairs to solve a problem-solving task. The focus group discussion was held later on to clarify their solution. Since the data collection phase was part of PMRI (Indonesian RME) course, they would get feedback on their hypotheses. Analysis of prospective teachers’ understanding on students’ learning hypotheses was based on their written work and field notes.

The task consisted of 2 proportion problems. It was designed to investigate their understanding regarding students’ learning hypotheses. The task’s demand described in Table 1. Responses for the first problem was evaluated to find out the variation occurred in the solution. Possible variation was expected since the problem could be solved using at least two different concepts of proportion. The first number was more explicit than the second one in addressing the comparison between two ratios. On the other hand, the focus for the second problem was a proportional relation within the context of dividing and reselling cheese into the various smaller amounts. Nevertheless, both problems can be categorized as a numerical comparison [26]
Table 1. The description of problem given to the prospective teachers

| Unit      | Description                                                                 | Source                      |
|-----------|-----------------------------------------------------------------------------|-----------------------------|
| Strawberry jam | Given two jars of strawberry jam and each jar contains different net weight from different brand. Determine which jam is cheaper. Explain the possible answer given by junior high school students. | Modified from [27]         |
| Cheese    | A brand of cheese sells the product in a kilogram. The seller usually divides the cheese into smaller amount. Determine the possible amount to be sold and predict the answer of junior high school. | Developed by author         |

3. Results and Discussion

Cramer and Post [26] have categorized three distinct types of problem to explore proportionality. Those were (1) missing value, (2) numerical comparison, and (3) qualitative prediction and comparison. The missing-value problem three units from two equal ratios were given while another one is missing. This type of problem was usually solved by using cross-product algorithms. Numerical comparison gave away two completes rates. It required reasoning instead of the numerical answer. On the other hand, qualitative prediction and comparison depended on comparison rather than numerical values. Each proportion task might require a different strategy for solving it. Students might apply unit-rate strategy, calculate the multiplier (factor-of-change), use fraction, or even utilize cross-product algorithm [26, 28].

Table 2. The variety of prospective teacher prediction on students’ thinking

| Possible strategies | The number of prospective teachers |
|---------------------|-----------------------------------|
| Unit-rate strategy  | 12                                |
| Fraction Strategy   | 6                                 |
| Ratio Table         | 2                                 |

The prospective students showed at most three predictions of students’ thinking in solving the jam problem. Table 2 shows the variety of predictions for the first problem. As shown in Table 2, the most common prediction occurred was the unit-rate strategy. By dividing the price over the net weight for each jar of jam, the students’ was predicted to get the price of one gram for each jam. Then, it was easy to decide which product/brand was cheaper than another.

Half of the participants proposed a fraction strategy. They perceived the two ratios as fractions. They predict that the students might compare those ratios by converting two fractions into the same denominator. By doing so, it is required only to compare one unit. In fact, the unit chosen to be denominator was the net weight. Those ratios were converting into the same amount of weight such that the price could be compared. Moreover, there was no participant chose the price as the common unit. It shows their awareness of context. Since the question was which jam has a cheaper price, they used the weight as the unit rate.

The factor of change strategy did not come up because this problem type still required a numerical answer before used it to reason. It was different from the typical numerical comparison problem, for instance comparing ratio between a mixture of chocolate and milk [29]. High school students used functional strategy to compare the relationship between two variables. They represented the data as division then determine the invariant for each ratio even though did not perceive it as proportion constant [29].
Moreover, in discussing the prediction the prospective teachers used various representations as shown in Figure 1. Each representation showed the respective strategies. The comparison between two fractions was not shown as a final calculation. The interesting part of this answer sheet was the existence of ratio table [27]. Even though the answer was still missing one more ratio table, this prospective teacher noticed that the table might be useful as a tool to solve the problem. Furthermore, the students might also come up with the table by themselves.

However, there were no participants predicted a possible incorrect solution of the problem. In fact, for nine grade students, solving numerical comparison was more difficult than solving missing value task [30]. The component of HLT should include all possible way of students’ thinking even if it is incorrect. It can be inferred that the participants comprehend the mathematical aspect but failed to see it as context to build students’ understanding on proportion.

![Figure 1: Various Representations](image)

Different from the first problem, the selling cheese problem “only” required the prospective teacher to determine the possible amount and price of the cheese. It asked them to approach the problem contextually. Table 3 shows the number of various amounts that was predicted occurs in the students’ solution.

| Table 3. The various amount of cheese |
|--------------------------------------|
| **Possible amount** | **The number of prospective teacher** |
| 2                     | 4                     |
| 11                    | 2                     |
| 7                     | 4                     |
| 8                     | 2                     |

There were two prospective teachers in a group approach the problem as they have to divide the 1 kg amount of cheese evenly. Their solution was 4 pack of 100 gram and 4 pack of 150 gram. Another pair who wrote two amount of cheese understood the problem correctly by dividing it into 35 and 50 gram. There were two groups proposed 7 possible amount of cheese although they have different
variations. The same amount occurred was 50 and 250 gram. The others were, 10, 20, 25, 50, 100, 125, 150, 175, 200, 500 gram. Eleven possible amount of cheese was proposed by one group starting from 25, 50, and then increased by 50 until reached 500 gram as shown in Figure 2. The unique solution comes from a group that proposed 8 different amount since they included 12.5 gram, 37.5 gram, 62.5 gram, and 75 gram.

Based on those numbers, it can be inferred that the participants predicted that students might play with numbers without making it related to the context. In Indonesia, the common amount of cheese could be found in the supermarket were 500 gram, 250 gram, 100 gram, and surprisingly 35 gram.

![Figure 2: Eleven solutions](image)

This type of problem was expected to sharpen the idea of using context in proportion problem as a trigger to learn mathematics. Students with the guidance from the teacher might discuss the context as they calculate the price for each amount. Discussion about the rationality of selling each amount of cheese and the factor of change from one amount to another might lead to a more formal tool such as ratio table. The constant ration and product relationship could be explored if the ratio table was used. It helped the prospective teacher to reason about the proportional relationship when cross multiplication cannot be applied to the problem [31].

4. Conclusion
The research revealed that prospective teachers comprehend the content aspect of the proportion problem. Most of them could solve the first problem correctly. In addition, understanding the selling cheese context led them to propose various numbers that could be thought by students. This could be used to build students’ thinking toward proportionality. Most of the hypotheses revealed the correct solution instead of exploring any other possible way. They were less aware that students might solve the problems incorrectly. It showed a real challenge for them to predict how the students would think. From pedagogical content knowledge theory, they might acquire the competency of mathematics but lack understanding the way to teach it.

Acknowledgements
We would like to thank Ministry of Higher Education and Research for funding the research under PDP (Penelitian Dosen Pemula) Grant in 2018.
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