Hole-burning in an Autler-Townes doublet and in superluminal (subluminal) Electromagnetically induced transparency of a light pulse via a joint nonlinear coherent Kerr effect and Doppler broadening

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We investigate the behavior of light pulse propagation in a 4-level double Lambda atomic system under condition of electromagnetically induced transparency. The Fano type interference effect and spectral hole burning appears in the the dynamics of the absorption-dispersion spectra caused by the joint nonlinear coherence Kerr effect and Doppler broadening. The coherent Kerr effect exhibits an enhancement (reduction) in superluminal (subluminal) in negative (in positive) group index while the Doppler broadening generates multiple hole burning in the Autler-Townes like spectra of this system. The hole burning in addition with coherent Kerr effect on the spectral profile influences the dynamics of subluminal and superluminal of the probe pulse through the medium. The characteristics of superluminality and subluminality modified by considering cold-Kerr-free medium and hot-Kerr-dependent mediums. The light pulse delays and advances in different regions of dispersion medium with the Doppler broadening and coherent Kerr effect. Consequently, the pulse delays by 49\(\mu\)s, while advance by \(-91\mu\)s, for a same set of parameters [note: a revised version is under preparation]

I. INTRODUCTION

The amplitude and the phase control of the group velocity of light in optical media have attracted a lot of attention in the recent years. High degree control over this speed is possible and it can be made much smaller than \(c\), greater than \(c\), or even negative [1]. The manipulation of light pulse propagation in an optical medium can be realized by changing the the dispersive properties of the medium. The exact control over the optical properties of the medium gives rise to the observation of some interesting phenomena based on quantum coherence and quantum interference[2, 3]. Examples of some fascinating phenomena, are Coherent population Trapping (CPT), [2, 4] Lasing without Inversion (LWI)[5, 6] Electromagnetically Induced Transparency (EIT)[8, 11] Multi-wave mixing [12, 14] Enhancing Kerr nonlinearity[15] Optical Soliton [16, 17]. The development of theoretical and experimental techniques for control the light pulse propagation through optical media are the results of fast few decades. Electromagnetic fields are used to create large atomic coherence and it is possible to tailor the amplification, absorption, dispersion properties of multilevel atoms[18, 19]. The intrusting application of these techniques is to adjust the group velocity of light pulses to propagate it very slowly or very fast. The region of normal dispersion is the one in which group velocity is lesser than the vacuum velocity of light \((v_g < c)\), while the region of anomalous dispersion is the one in which \(v_g > c\). In the normal dispersion region subluminal and in the anomalous dispersion region superluminal propigation of light occurs [20]. The well known approach to slow down the light pulse propagation in atomic vapor is the technique of electromagnetically induced transparency. Electromagnetically induced transparency(EIT) and Spontaneously generated coherence(SGC) change the steady state response of the medium[21]. Experimentally Hau, et al slow down the group velocity of light pulse to 17\(ms^{-1}\) in bose Einstein condensate. In Rb vapor, the group velocity of light was reduced to (90\(ms^{-1}\), 8\(ms^{-1}\))[22, 26]. Agarwal et al received a group index of 10\(^3\) in two level atomic system in Doppler broadened configuration of saturated absorption spectroscopy[27]. Shang-qi kuang [28] theoretically present slow light propagation on coherent hole burning in a Doppler broadened three level A-type atomic configuration. They use the coherent hole burning dip and achieved the slow light propagation at resonance condition. A large numbers of experiments on slow light propagation in Doppler broadened medium are reported
by M.M. Kash and A. Kasapi. It is pointed out that the Kerr nonlinearity could be increased by several orders of magnitude by taking advantage of the EIT. Kang and Zhu proposed a large enhancement in the Kerr nonlinearity with vanishing linear susceptibility in coherent prepared four-level Rb atoms. Quantum coherence and interference manifested by EIT, suppresses the linear susceptibility and greatly enhances the nonlinear susceptibility at low light intensity. Large Kerr nonlinearity has also been reported using EIT, in Wang et al. proposed the use of double EIT schemes for the optimal production of the cross phase modulation. G.S. Agarwall reduced the group velocity of light pulse by Kerr effect under the condition of EIT and found that the group velocity considerably reduces in a dispersive medium. Kocharovsky stopped the light pulse in hot gases. In another experiment light has been stopped and stored in atomic coherence. In a classic paper, slow light has many potential applications in optical delay lines, quantum entanglement of slow photons, non-classical squeezing state entangled atomic ensembles, optical black hole. L.J. Wang et al demonstrated superluminal light propagation for light using the region of lossless anomalous dispersion between two closely spaced gain lines in a double-peaked Raman gain medium. Saharai et al. proposed a scheme based on four-level EIT atomic medium and controlled the normal/anomalous dispersion of light via phase of the driving field. Theoretically four level gain atomic system is extend to polychromatic pump fields and multiple anomalous region of superluminality were observed in the gain doublet region as well as between the two pairs of the doublet regions. Moti Fridman et al used the superluminal group velocity for temporal cloaking follow the spatial cloaking idea in the temporal domain. A hole is created space and time windows to hide the object and information by the manipulation of positive negative group index. For best cloaking, it is require to increase the time gap to microsecond and to millisecond. The manipulation of negative group velocity is also used for the quality of imaging. However in their experiment there is also a lack of best quality of the images arising due to low values of the negative group index as discussed in thier experimental paper. According to these facts it is necessary to manipulate slow and fast light for better communication and better technology of imaging in cloaking. In this article we explore the mechanism for slow and fast manipulation by the jointly effect of kerr nonlinearity and Doppler broadening in a double lambda atomic configuration. We show that how the slow and fast light can be significantly influence by the jointly effect of kerr nonlinearity and Doppler broadening.

A question can be raised that, is there a way to enhance the group velocity of a fast laser light in a dispersive medium. If the answer is yes then, what phenomenon is responsible for the increase in the group velocity. The answer of this interesting question is explored in the present article. In order to answer this question, we propose a five-level atomic scheme, and the effect of Kerr nonlinearity on a fast light is studied. The scheme is based on EIT in which the group index can be enhanced significantly in the negative domain due to the Kerr effect. This enhancement boosts superluminality in a dispersive medium.

II. MODEL AND EQUATION

We consider experimental 4-level double lambda type atomic-configuration driven by two appropriate coherent control fields and a probe field [see Fig. 1]. The lower ground levels and are coupled with the upper excited level by two control fields of Rabbi frequencies , whereas the lower level is coupled with level by a probe field of Rabbi frequency . To observe the condition of atomic motion relative to the frequencies of the driving fields, we modified the system interaction due to flexible environment. Therefore we consider the atomic velocity linear in the response function of the medium of the system respectively. To explain the equations of motion and optical properties of the system, we proceed with the following interaction picture Hamiltonian in the dipole and rotating wave approximations:

\[ H(t) = -\frac{\hbar}{2} \Omega_1 \exp[-i\Delta_1 t] |1\rangle \langle 4| + \frac{\hbar}{2} \Omega_2 \exp[-i\Delta_2 t] |3\rangle \langle 4| - \frac{\hbar}{2} \Omega_p [-i\Delta_p] |2\rangle \langle 4| + H.c. \] (1)

The angular frequencies three fields are related to the atomic states frequencies as: , , and . The general form of density matrix equation is given by the following relation:

\[ \frac{d\rho_{i}}{dt} = -\frac{i}{\hbar} [H_{t}, \rho_{i}] - \frac{1}{2} \Gamma_{ij} \sum (\sigma^{\dagger} \sigma_{j} + \rho_{i} \sigma^{\dagger} \sigma - 2 \sigma_{j} \sigma_{k}) \] (2)

where is raising operator , is levering operator for the three decays. After straight forward calculation we obtain the three coupled rate equations in the first order as:

\[ \dot{\rho}_{24} = [i\Delta_p + \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3)]\rho_{24} - \frac{i}{2} \Omega_1 \rho_{21} - \frac{i}{2} \Omega_2 \rho_{23} + \frac{i}{2} \Omega_p (\rho_{44} - \rho_{22}), \] (3)

\[ \dot{\rho}_{21} = [i(\Delta_p - \Delta_1)]\rho_{21} - \frac{i}{2} \Omega_1 \rho_{24} + \frac{i}{2} \Omega_p \rho_{41}, \] (4)
\[
\tilde{\rho}_{23} = \frac{i(\Delta_p - \Delta_d)}{2} \bar{\rho}_{23} - \frac{i}{2} \Omega_p \tilde{\rho}_{24} + \frac{i}{2} \Omega_p \tilde{\rho}_{43},
\]

(5)

Taking \(\Omega_p\), in the first order, while \(\Omega_1\) and \(\Omega_2\), in all order of perturbation and consider the atoms initially in the ground state \(|2\rangle\), while the population initially in the other states are zero then the following conditions are applicable to the density matrix. \(\tilde{\rho}_{22}^{(0)} = 1, \tilde{\rho}_{44}^{(0)} = 0, \tilde{\rho}_{14}^{(0)} = 0, \tilde{\rho}_{43}^{(0)} = 0\). The above set of equations can be solved for \(\tilde{\rho}_{24}^{(1)}\) using the relation.

\[
Z(t) = \int_{-\infty}^{t} e^{-M(t-v)} Pdv = -M^{-1}Q,
\]

(6)

where \(Z(t)\) and \(Q\) are column matrices while \(Q\) is a 3x3 matrix. The solution is written by:

\[
\tilde{\rho}_{24}^{(1)} = \frac{4i(\Delta_p - \Delta_d)(\Delta_p - \Delta_d)\Omega_p}{(\Delta_2 - \Delta_p)[4A_1(\Delta_p - \Delta_1) - i\Omega_p^2] + i(\Delta_p - \Delta_1)\Omega_p^2},
\]

(7)

To add the Doppler broadened effect in the atomic configuration, we replace the detuning parameters by: \(\Delta_1 = \Delta + \alpha_1 k_1v, \Delta_2 = \Delta + \alpha_2 k_2v, \Delta_p = \Delta + kv,\) where, \(\alpha_{i=1:2} = 1\), indicate co-propagation direction of coherent fields to the probe field, and counter propagating directions, while \(\alpha_{i=1:2} = -1\), show counter-propagation direction of coherent fields to the probe field. Here \(k_1, k_2, k_p\) are the wave vectors of the two coherent fields and a probe field and for simplicity we put \(k_1 = k_2 = k_p = k\).

\[
\tilde{\rho}_{24}^{(1)(kv)} = \left[\frac{-4Ti(\Delta_p - \Delta_d + kv - \alpha_2 k v)}{i\Omega_p^2 T + A(\Delta_2 - \Delta_p - kv + \alpha_2 k v)}\right] \Omega_p,
\]

(8)

\[
A = -i\Omega_1^2 + 4B(\Delta_p - \Delta_1 + kv - \alpha_1 k v)
\]

(9)

\[
A_1 = \frac{2i(\Delta_p + \gamma_1 + \gamma_2 + \gamma_3)}{2}
\]

(10)

\[
B = \frac{2i(\Delta_p + kv) + \gamma_1 + \gamma_2 + \gamma_3}{2}
\]

(11)

\[
T = [kv - \Delta_1 + \Delta_p - \alpha_1 k v]
\]

(12)

### III. SUSCEPTIBILITY AND GROUP INDEX

The susceptibility is a response function of medium due to an applied electric field. The susceptibility of our driven hot atomic system is calculated to the first order in the probe field fields and to the all order in the control field. The calculated susceptibility for our hot atomic system is written as:

\[
\chi(kv) = \beta \left[\frac{-4Ti(\Delta_p - \Delta_d + kv - \alpha_2 k v)}{i\Omega_p^2 T + A(\Delta_2 - \Delta_p - kv + \alpha_2 k v)}\right],
\]

(13)

Where \(\beta = \frac{2N|\mu_2|^2}{\hbar^2}\) and \(N\), is the atomic number density of the medium and \(|\mu_2|\), is dipole moment between the level \(|2\rangle\), and \(|4\rangle\). To introduce the effect of kerr field in the system we expand \(\chi_v\), with the intensity of the control field \(\Omega_1\) in the following passion in the perturbation limit[?].

\[
\chi^k(kv) = \chi^{(0)}(kv) + \frac{1}{2} \frac{\partial}{\partial t} \chi(kv)|_{t\to0} : (14)
\]

Where \(\chi^{(0)}(kv)\), is the probe equation of motion without the kerr field field \(I = |\Omega_1|^2\), intensity and \(\frac{1}{2} \frac{\partial^2}{\partial t^2}\), is the term contributed due to the kerr field. The Kerr field effect susceptibility in the presence of the Doppler broadening is the following:

\[
\chi^k(kv) = \beta \left[\frac{-4TiZ}{i\Omega_p^2 T - 4BTZ} + \frac{-4\Omega_p^2 T Z^2}{(i\Omega_p^2 T - 4T Z B)}\right]
\]

(15)

\[
Z = (kv - \Delta_2 + \Delta_p - \alpha_2 k v)
\]

(16)

When \(v = 0\), there is no Doppler broadening effect in the system. The system is called cold atomic system, if there is Doppler broadening effect in the system, then it is called hot atomic system. For cold atomic system, we represent susceptibility without kerr effect is \(\chi\), and in the presence of kerr effect is \(\chi^k\). These susceptibility are from the eq13 and eq15, when one put \(v = 0\). The Doppler susceptibilities are the average of \(\chi_{kv}\), over the Maxwellian, distribution and is describe bellow:

\[
\chi^{(d)} = \frac{1}{V_D \sqrt{\pi}} \int_{-\infty}^{\infty} \chi(kv) e^{-\frac{(kv)^2}{V_D^2}} d(kv)
\]

(17)

\[
\chi^{(dk)} = \frac{1}{V_D \sqrt{\pi}} \int_{-\infty}^{\infty} \chi^k(kv) e^{-\frac{(kv)^2}{V_D^2}} d(kv)
\]

(18)

Where \(V_D = \sqrt{K_B T \omega^2 M c^2}\), is the Doppler width. Where \(\chi^{(d)}\) and \(\chi^{(dk)}\) are the Doppler broadened susceptibilities without the kerr nonlinearity as well as in the presence of kerr nonlinearity.

\[
N_g = 1 + 2\pi Re[\chi] + 2\pi \omega_{24} Re \frac{\partial \chi}{\partial \Delta},
\]

(19)

\[
\tau_d = \frac{L(N_g - 1)}{c}
\]

(20)

These are the mean results which will be analyzed and discussed in details. \(\tau_d\) is group delay/advance time. When its value is positive it is called delay and if its value is negative it is called advance time. To observe the nature of the pulse shape at the output we used the transfer function. The output pulse \(S_{out}(\omega)\), after propagating through the medium can be related to the input pulse \(S_{in}(\omega)\) by the relation: \(S_{out}(\omega) = H(\omega)S_{in}(\omega)\). We choose a Gaussian input pulse of the form:

\[
S_{in}(t) = \exp[-t^2/\tau_0^2] \exp[i(\omega_{24} + \xi)t],
\]

(21)
IV. RESULTS AND DISCUSSION

We explain our main results in the Eqs.(13,15,17,18,19,20,21,23) for absorption, dispersion, group index, time delay and pulse shape distortion, when there is no kerr effect but no Doppler broadening effect in the system. We also present our results for Absorption, Dispersion, Group index and for the system when there is its maximum kerr and broadening effects. When we put $v = 0$, in the eq13, we obtain the optical results of double lambda configuration for cold atomic system. Further if we turn off one of the control field in our system [$\Omega_1 = 0$, or $\Omega_2 = 0$], the optical behavior of our system concise with the famous A-type atomic configuration. Next, we focus on the main results of our atomic system and are committed (1) to discuss the subluminal and superluminal behavior of the light pulse propagating through their associated dispersive regions, (2) to discuss the incoherent Doppler broadening effect in the system, (3) to discuss the kerr effect in the system. The kerr effect is coherent effect and significant contribution to optical properties. Experiment can be easily adjusted on kerr effect. The Doppler broadening effect is temperature dependent incoherent and experiment is not so easy as compare to kerr effect. In Fig2, the plots are traced for the system, when there is no kerr effect neither Doppler broadened. The absorption spectrum
FIG. 5: Absorption, Dispersion, Group index and group delay/advance time versus $\Delta_p$, such that $\gamma = 1 MHz$, $\gamma_1 = \gamma_3 = \gamma_5 = \gamma_7 = 1\gamma$, $\omega_2 = 10^3\gamma$, $\Delta_1 = 0\gamma$, $\Delta_2 = 0\gamma$, $\Omega_1 = 0.5\gamma$, $\Omega_2 = 0.3\gamma$, $V_D = 1.5\gamma$. $\alpha_{\Omega_i=1.2} = -1$. Neither Doppler nor Kerr effect black solid line. Doppler effect but no Kerr effect black dashed line. Kerr effect but no Doppler effect red dashed line. Both Doppler and Kerr effect dashed blue line.

show a typical transparency for the propagating probe field at resonance $\Delta_p = 0$, as shown in Fig. 2a. The slope of dispersion in transparency window is anomalous as shown in Fig. 2b. The group index in the anomalous region is negative. The value of group index and group advance time at the parameters $\gamma_1 = \gamma_2 = \gamma_3 = 2\gamma$ and $\Omega_1 = \Omega_2 = 2\gamma$ are $-3000$ and $-200\mu s$ in the medium as shown in Fig. 2(c, d). When the intensity of the control field $\Omega_2$ is increased from $2\gamma$ to $4\gamma, 6\gamma$, the transparency width is increased and the negative group index and time advancement are degraded due to the less anomalous dispersion seen in Fig. 2(c, d). Solid black line. Doppler free Kerr free system by solid black line. The Doppler free Kerr free system by dashed black line. The Doppler effect but Kerr free system by dashed red line. The Doppler as well as Kerr effect system by dashed blue line. The joint effect of Kerr nonlinearity and Doppler broadened introduced in the system, show significant special profiles of absorption and dispersion. The absorption in the resonance point $\Delta_p = 0$, in all the cases are approach to zero. At this point the light are totally transmitted. Around the resonance point the absorption is a function of probe detuning $\Delta_p$, and different spectral profiles with the Kerr and Doppler broadened effect. Closed to the resonance point at $\Delta_p = \pm 2.5$, there are large symmetric negative absorption called Electromagnetically induced transparency Amplification with Kerr nonlinearity, if both control and Kerr field co-propagates ($\alpha_i = 1$), with the probe field as shown in Fig. 3(a). The slope of dispersion at resonance point $\Delta_p = 0$, is anomalous in the Kerr free system (cold and hot atomic system with no Kerr effect). When the Kerr effect is switch in the system the slope of dispersion is reversed (normal) at resonance $\Delta_p = 0$, both in Doppler free and Doppler effect system. Near the resonance point at $\Delta_p = \pm 2.5$, the slope of dispersion are anomalous in all the cases but steep anomalous with Kerr effect Fig. 3(b). The normal dispersion show slow light propagation, while anomalous dispersion describe fast light propagation. The control and Kerr field counter propagate ($\alpha_1 = -1$), to the probe field in the cell, the negative absorption is reduced and vanished at a certain intensity of the Kerr field. The absorption are increase with the Doppler effect and small symmetric lamb dip are appears on both sides of the central absorption peak. The central absorption peak and Lamb dips are enhances with the strength of Kerr effect, see in Fig. 3(c). The dispersion slopes are normal at the resonance in the presence of Kerr effect, while anomalous without Kerr effect for both cold and hot medium. To studies the important and detail physic of normal and anomalous dispersion and their variation with Doppler as well as Kerr effect we plots the group index and group delay times near the resonance point against the control field $\Omega_2$, as shown in Fig. 4(a,b,c,d), when the control and Kerr field co-propagating ($\alpha_i=1,2 = 1$), to the probe field. The group index as well as time delay/advance are the function of...
control field keeping the probe detuning $\Delta_p = 0.5\gamma$. At
low intensity, bellow $\Omega_2 = 2\gamma$, the group index and time
delay are positive. The values of group index for all the
four cases at $\Omega_2 = 1.5\gamma$ are written here $N_g = 986.90$
$N^k_g = 2945.95$, $N^d_g = 1244.91$, $N^{d,k}_g = 2472.8$ as shown
in Fig4[a]. The corresponding group delay time are
t$_{ad} = 19\mu s$, $t_{kd}^k = 58\mu s$, $t_d^d = 24\mu s$, $t_{d,k}^{d,k} = 49\mu s$, see in
the Fig4[b]. At the positive group index, group velocity
is $v_g = c/N_g$, is positive and varies with the group
index. The pulses of all the four cases are different
group velocities of $[c/986.90$, $c/2945.95$, $c/1244.91,$
$c/2472.8]$. These results have large potential application
in telecommunication systems. It is important to the
preservation of two light pulses. If both pulses are arrive
at the same time to the detectors, they will accumulate
only one of them, and the information of the other will
be lost, which slow down the over all flow of information.
Activating the slow light medium, one of the pulse is
delay from the other, the importation of both the pulses
are preserved, while the flow of information is speed up.
To discus the advantage of negative group index and
advance time observe the same two Fig4[a,b]. Above the
intensity of the control field $\Omega_2 = 2\gamma$, the group index
and time delay are negative(advance time) values. The
values of group index for all the four cases at $\Omega_2 = 3\gamma$
are $N_g = -862.84$, $N^k_g = -1367.93$, $N^d_g = -703.013$,
$N^{d,k}_g = -1104.11$, see in Fig4[a]. The corresponding
group advance times are $t_{ad} = -17.27\mu s$, $t_{kd}^k = -27.37\mu s$,
$t_d^d = -14.80\mu s$, $t_{d,k}^{d,k} = -22.10\mu s$, see in Fig4[b]. The
negative time delay (advance time) are very important
for spacial modes images. The degree of spacial modes
images are quantified and increase in qualities, when
the advance time is increase. Advance time are also
potential application in spacial and temporal cloaking
devices, therefore our results will be easily adjusted to
improve the current technology of telecommunication
systems and images as well as cloaking. At high
sufficient value of control field the group index and
group delay are then saturated. Fig5 show variation of
absorption, dispersion, group index and group de-
lay/advancement with probe detuning. When the rabbi
($\Omega_i=1,2$), frequencies of control and kerr field are smaller
then decay rates ($\gamma_{i=1,2,3}$), while the kerr and control
field counter-propagate ($\alpha_i = -1$) to the probe field.
The Dark lines are appears on the spectrum due to
fano type quantum interference. These dark lines are
more dominant with doppler effect. These different dark
lines arise due to quantum interference effect among
the excitation probability amplitudes of the probe field.
The probe field follows different paths in the presence of
Doppler broadened, by the dressed states of the excited
real energy level created by the atom-field interaction
of the system. The Lorentzian line shape of the spectra
split into different components, which depend on the
numbers of dark lines and its sum of the FWHM, obey
the Weiskopf-Wigner theory, see in Fig5[a,b]. The group
index close to the resonance region at $\delta_p = 0.01$, are
negative for all the cases. The values of group index
for all the four cases at this nearest resonance point of
the probe detuning, are $N_g = -67580$, $N^k_g = -459616$
$N^d_g = -25046$, $N^{d,k}_g = -172503$, see in Fig5[c]. $\gamma$
The group velocity($c/N_g$) at this region are $v_g = -c/67580$,
v$^k_g = -c/459616$, $v^d_g = -c/25046$, $v^{d,k}_g = -c/172503$. It
mean that the kerr effected medium without Doppler
broadened (cold kerr atomic medium) is more superl-
unal. The significant advance times of the pulse for
the four cases are $t_{ad} = -13\mu s$, $t_{kd}^k = -91\mu s$, $t_d^d = -5\mu s$,
$t_{d,k}^{d,k} = -34\mu s$, see in Fig5[d]. These results are applicable
to increase the quality of spacial modes images. Further
the results are potential application for the temporal
cloaking devices to increase the time gaps. In Fig6,
the graphs are traced for absorption and dispersion
versus probe detuning. The absorption and dispersion
show intrustion behavior with the Doppler and kerr
effect. The topical Lamb dip at the resonance point are
more dominant with small value of the Doppler width.
When the Doppler width is increase the lamb dip is
reduced. The superluminal behavior are degraded with
the decrease of Lamb dip. The Dark line are appear
on the spectrum when the coherent fields are counter
propagate to the probe field. These dark lines are en-
hanced value with the kerr nonlinearity. The subluminal
and superluminal behavior are enhance, when the light
of dark lines are increase. Fig7 show Gaussian pulse
shape at the input and out put of the medium. All
the pulses are the same shape, when the coherent fields
counter propagate to the probe (a) and co-propagate
to the probe but slightly broaden. Where $\tau_0$ is input pulse
width. In signal processing, optical memories,
data synchronization, optical buffers, cloaking devices,
spacial modes images required slow and fast light pulse
delay/advance times. For significant process the
delay/advance $t_d$, times need to be very large as compare
to the pulse with $\tau_0$. Other requirement is that the
pulse not be distorted. These two condition oppose each
other. Large delay occur with great distortion. Hoverer
distortion can be minimized. Therefore the system
is sufficient distortion, which does not effect the sub-
luminal and superluminal propagation. In conclusion,
we proposed a double lambda 4-level atomic system
driven by two coherent control fields, and a probe field.
The proposed scheme displays interesting results of
subluminal and superluminal light with the jointly effect
of kerr nonlinearity and Doppler broadening. These
results are various advantages in the telecommunication
process as well as spacial modes image and temporal
cloaking devices.

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