Time Iteration for Service Life Prediction of Concrete Structure in Marine Environment

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Abstract. On the basis of chloride diffusion model, the time-varying and randomness of chloride diffusion in concrete are considered. Based on the reliability method, the service life of cross sea bridge structure can be predicted by the double cycle iteration of reliability and service life, and the service life of concrete of cross sea bridge structure can be directly calculated by the double iteration. In order to reduce the iterative calculation, through the improvement of the time iteration strategy, the tangent slope is replaced by the cut line slope, which not only makes the iterative convergence fast and accurate, but also solves the problem of service life and It is difficult to derive the implicit function of reliability.

1. Introduction

For the concrete structure exposed to chloride environment, when the chloride ion concentration in the concrete reaches a critical value around the reinforcement, it will lead to the reinforcement corrosion, resulting in the degradation of the strength and durability of reinforced concrete components. The time required for the concentration of chloride ions around the steel bars in concrete to reach the critical value is called the corrosion induction period, which can account for more than 80% of the corrosion damage process of concrete structures [1]. Therefore, in the structural analysis and design, the corrosion induction period is usually regarded as the service life of reinforced concrete structures under chloride environment. Therefore, the study of chloride diffusion process and concentration distribution is the key to the analysis of service life of structural concrete in chloride environment. Among them, the diffusion coefficient of chloride affects an important material parameter of chloride diffusion process [2], and also directly affects the accuracy of service life prediction of concrete structures.

Since the 1950s, scholars in Europe and America began to pay attention to the durability of concrete structure, the service life of concrete structure has always been a problem of great concern. Yu Hongfa [3], Tao Qi [4] and other scholars use analytical model to predict the service life of concrete in chloride environment. Sang [5], Shi Yanghang [6], Yang Lufeng [7] and other scholars use finite element method, finite difference method and Boundary element method to solve the numerical model to predict the service life of concrete structures in chloride environment. The analytical model is simple and convenient, which is suitable for considering one-dimensional chloride diffusion from the perspective of materials; the numerical model can better adapt to complex boundary conditions and concrete material properties, and is suitable for considering two-dimensional and three-dimensional
chloride diffusion [8]. In the actual process, the climate environment, materials and construction process will cause uncertainty, while the analytical model and numerical model do not consider the randomness of each parameter in the process of predicting the service life of concrete structure, so the predicted service life may have some deviation.

At the same time, many scholars study the service life of concrete structures under the uncertain chloride environment, and put forward many service life prediction models based on reliability index [9-11]. At present, Monte Carlo method [12] and reliability service life prediction model [7-8] Based on the analytical model of chloride diffusion of Fick's second law are commonly used. The service life prediction model based on reliability is to determine the time when the structure durability is lower than the specified reliability index by establishing the relationship between durability reliability index and erosion time. As the end time of durability life of chloride ion erosion, the corresponding reliability is calculated by changing the service time continuously, until the service time consistent with the target reliability is determined as the service life, and the reliability value corresponding to each service time is obtained through cyclic calculation, which is actually a trial and error method, and at the same time it increases the amount of calculation.

In this paper, considering the randomness of the attenuation coefficient of chloride diffusion coefficient of concrete, combined with the first-order second-order moment method, a reliability analysis model of service life of concrete structure based on chloride diffusion is established. When calculating the service life, it is contrary to the traditional method. Through the known reliability of service life, based on the reliability algorithm, the relationship between service life and reliability is established, and the initial stage is given. Through double iteration, the final iteration self converges, and the output result is also the service life under the reliability index. The iteration result of this method has high accuracy, and self converges after 5 steps of iteration, and the selection of the initial service life has little influence on the final iteration result and iteration steps.

2. The analytical model of chloride diffusion in concrete structure

The chloride ion in concrete structure diffuses mainly, which satisfies Fick's second law. The chloride diffusion equation considering the age decay of concrete diffusion coefficient can be expressed as [13]:

\[
\frac{\partial C}{\partial t} = D_0 \left( \frac{t_0}{t_0 + t} \right)^n \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)
\]

(1)

Where, \( C \) represents the concentration of chloride ion in concrete (%), the percentage of chloride ion mass in concrete mass; \( D_0 \) represents the diffusion coefficient of chloride ion in concrete at the time of initial age \( t_0 \) (\( t_0 = 28 \text{d} \) in this paper), \( n \) represents the decay coefficient of chloride ion diffusion coefficient in age; \( t \) represents the time of exposure of concrete to chloride ion environment (a); If the thickness of the concrete cover is \( d \), and the thickness of the reinforcement in the corner of the concrete is equal in both directions, the chloride concentration on the surface of the reinforcement can be expressed as follows:

\[
C(d, t) = C_s - (C_s - C_0) \left[ \text{erf} \left( \frac{d}{2 \sqrt{D_0 T}} \right) \right]^m
\]

(2)

Where, \( m \) represents the parameters related to the diffusion dimension of chloride ions. For one-dimensional diffusion, \( m = 1 \); for two-dimensional diffusion, \( m = 2 \). \( C_s \) represents the surface concentration (%); \( C_0 \) represents the initial concentration (%). \( \text{erf} \) is the error function, \( T \) is time-varying equivalent service time (a), and the relationship between \( T \) and service time \( t \) is as follows:

\[
T = \frac{(t_0)^n}{1 - n} \left[ (t_0 + t)^{1 - n} - (t_0)^{1 - n} \right]
\]

(3)
3. Reliability of concrete structure
In the process of concrete structure analysis and design, the corrosion induction period is usually taken as the service life of reinforced concrete structure under chloride environment [14], and the chloride content on the surface of reinforcement in concrete reaches the critical concentration as the limit state of the durability of concrete structure, so as to establish the normal service limit state function of concrete structure under chloride corrosion:

$$g = C_r - C(d,t)$$

(4)

The limit state function $g = g(X)$ is also known as a function of the basic random vector $\{X\}$. Only independent normal random variables $\{X\} = [X_1, X_2, \cdots, X_M]^T$ are considered here. Then, by transforming $\xi = (X_i - \bar{X}_i)/\sigma_i$, $\bar{X}_i$ and $\sigma_i$ respectively represent the mean value and standard deviation of $X_i$. The limit state function in the standard normal space can be obtained $G(\{\xi\}) = g(\{X\})$. Furthermore, the first order second moment method of reliability analysis is used to obtain the reliability index $\beta$ of concrete structure at equivalent diffusion time $T$, which is equal to the shortest distance from the origin of standard normal coordinate system to the surface of limit state:

$$\beta = \sqrt{\{\xi^*\}^T \{\xi^*\}}$$

(5)

Where, $\{\xi^*\}$ is the design point vector in the uncorrelated standard normal space, which can be determined by[8]:

$$\{\xi\}_{i+1} = [\{\xi\}_i]^T \{\alpha\} + b \{\alpha\}$$

(6)

Where, $\{\xi\}_{i+1}$ and $\{\xi\}_i$ are denote the design points for two adjacent iteration steps. $b$ and $\{\alpha\}$ denote respectively the step size and the search direction vector defined by:

$$b = \frac{G(\{\xi\}_i)}{\| \nabla G(\{\xi\}_i) \|}$$

(7)

$$\{\alpha\} = -\frac{\nabla G(\{\xi\}_i)}{\| \nabla G(\{\xi\}_i) \|}$$

(8)

Where, $\nabla G(\{\xi\}_i)$ is the gradient vector of the limit state surface in the standard normal space. The iteration will be stopped when $|\beta_{i+1} - \beta_i|/\beta_i \leq 10^{-3}$ is satisfied, and $\beta_{i+1}$ will be considered as the convergence value of the reliability index.

4. Service life of structural concrete based on Reliability

4.1 Relationship between service life and reliability
After obtaining the reliability index corresponding to a specific service time through the reliability iterative calculation, it is still unable to predict the service life of concrete, because the service time needs to be given before calculating the reliability index, but the assumed service time is not exactly the service life.
With the extension of exposure time of concrete structure to chloride corrosion environment, the reliability index of concrete structure will continue to decline. It is assumed that the reliability indexes of concrete structures at $t_k$ and $t_{k+1}$ ($t_{k+1} > t_k$) are $\beta_k$ and $\beta_{k+1}$, respectively. When the target reliability index $\beta_t$ of concrete structure satisfies $\beta_t > \beta_k > \beta_{k+1}$, then the time corresponding to the target reliability index can be obtained by interpolation:

$$t_s = t_k + \frac{\beta_k - \beta_t}{\beta_k - \beta_{k+1}} (t_{k+1} - t_k) \tag{9}$$

### 4.2 Iterative model of service life based on Newton method

Based on the equation of reliability index service time curve, the equation of reliability index service time curve can be expressed as the following expression:

$$\beta = f(X,t) \tag{10}$$

Where, $\beta$ represents reliability index, $X$ represents parameter vector in diffusion model, $t$ represents service time.

In this paper, the Newton-Raphson iterative method is used to solve the equation $f(X,t) = \beta$. The function $f(X,t)$ is expanded by Taylor series at the initial service time $t_0$:

$$f(X,t) = f(X,t_0) + f'(X,t_0)(t - t_0) + O(t^2) \tag{11}$$

When $f(X,t) = \beta$, ignoring the terms higher than second-order, there are:

$$f(X,t_0) + f'(X,t_0)(t - t_0) = \beta \tag{12}$$

Calculating the service time of the above equation:

$$t = t_0 + \frac{\beta - f(X,t_0)}{f'(X,t_0)} \tag{13}$$

Because the expansion of function $f$ only keeps the first-order term, the service life obtained is not an accurate value of service life. The real service life can be approached only by iteration. The iteration formula is as follows:

$$t_{k+1} = t_k + \frac{\beta_k - f(X,t_k)}{f'(X,t_k)} \tag{14}$$

However, function $f(X,t)$ has no real expression, so it is difficult to solve derivative $f'(X,t_{k+1})$. In this paper, secant slope is used instead of tangent slope $f'(X,t_{k+1})$, seen in Figure 1, which can avoid the difficulty of solving derivative:

$$f'(X,t_{k+1}) \approx \frac{f(X,t_k) - f(X,t_{k-1})}{t_k - t_{k-1}} \tag{15}$$

The iterative formula becomes:

$$t_{k+1} = t_k + \frac{t_k - t_{k-1}}{f(X,t_k) - f(X,t_{k-1})} \left[ \beta_k - f(X,t_k) \right] \tag{16}$$

Until the expression $\left| \frac{(t_{k+1} - t_k)}{t_k} \right| \leq 10^{-3}$ is satisfied, the service life of concrete structure is $t_{k+1}$.
5. Case study
Considering a bridge across the sea in Beibu Gulf of China. The thickness of concrete cover $d$, the action level of marine chloride corrosion environment (usually expressed by surface chloride concentration $C_s$), critical chloride concentration $C_r$, chloride diffusion coefficient $D_0$ and decay coefficient $n$ at the initial exposure time are random. Refer to China’s Guide for durability design and construction of concrete structures (CCES01-2004), Code for durability design of concrete structures (GB/T50476-2008) and JCSS probabilistic model code [15]. See Table 1 for specific values of random parameters.

| Variable | Mean Value | Variation Coefficient | Distribution |
|----------|------------|------------------------|--------------|
| $D_0$ (mm$^2$/a) | 29.6 | 0.2 | Normal |
| $d$ (mm) | 45 | 0.15 | Normal |
| $C_s$ (%) | 0.18 | 0.2 | Normal |
| $C_r$ (%) | 0.05 | 0.2 | Normal |
| $n$ | 0.4 | 0.1 | Normal |

In order to verify the accuracy of this method, three different groups of service time ($t_1 = 1$ a, $t_2 = 10$ a), ($t_1 = 1$ a, $t_2 = 80$ a) and ($t_1 = 100$ a, $t_2 = 80$ a) are selected as the initial secant slope. The service life iteration model established in this paper is used. In this case, the service life of the bridge pier is calculated by using the target reliability index $\beta_t = 1.5$, $m = 2$. At the same time, the Monte Carlo Method is also used to calculate the service life of the bridge pier, which is compared with the calculation results of this method. The calculation results are shown in Figure 2. It can be seen from the figure that the service life value obtained by the iterative calculation in this paper is consistent with the result of Monte Carlo Method, which shows that the method established in this paper has high calculation accuracy. For different initial values, the calculation results converge to the same result, which shows that the sensitivity of this method to the initial value is not high, and the calculation results are stable convergence, which can be converged by 4-6 iteration steps, with better calculation efficiency. At the same time, it can be seen from the convergence process that the initial value will affect the convergence speed to a certain extent. How to select the appropriate initial value of iteration to ensure the fastest convergence speed remains to be further analyzed.
6. Conclusion
When using the traditional reliability calculation method to calculate the service life of concrete structures in chloride environment, it is necessary to constantly try to calculate the corresponding reliability index values of different service times and determine the final service life by comparing with the target reliability index. This trial and error process often brings a lot of calculation costs for the solution of practical problems. The iterative process of service life established in this paper can converge to the service life value, and can effectively avoid the calculation cost caused by trial and error.

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