New constraints on neutrino physics from Boomerang data

Steen Hannestad

NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
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We have performed a likelihood analysis of the recent data on the Cosmic Microwave Background Radiation (CMBR) anisotropy taken by the Boomerang experiment. We find that this data places a strong upper bound on the radiation density present at recombination. Expressed in terms of the equivalent number of neutrino species the 2\sigma bound is \(N_\nu \leq 13\), and the standard model prediction, \(N_\nu = 3.04\), is completely consistent the the data. This bound is complementary to the one found from Big Bang nucleosynthesis considerations in that it applies to any type of radiation, i.e. it is not flavour sensitive. It also applies to the universe at a much later epoch, and as such places severe limits on scenarios with decaying neutrinos. The bound also yields a firm upper limit on the lepton asymmetry in the universe.

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Introduction — The standard Big Bang model has been remarkably successful in describing the observed features in our universe. The latest and most impressive confirmation of the model comes from observations of anisotropies in the cosmic microwave background radiation (CMBR). These anisotropies are predicted by the Big Bang model, and were first discovered by the COBE satellite in 1992. Subsequently it was realized that precision measurements of these anisotropies can yield very detailed information about the fundamental cosmological parameters, and accordingly a vast number of papers have investigated the potential of upcoming experiments to measure these parameters (see for instance and references therein).

Now we have the first results which may rightly be called precision CMBR measurements. They stem from the balloon-borne experiment Boomerang which was flown over the Antarctica in 1999.

The results indicate that the universe is flat, and are essentially a confirmation of the standard Big Bang model (1). The data is so good that it can also be used to constrain physics beyond the particle physics standard model. A strong indication of such exotic physics would be additional radiation energy in the universe at the time of recombination. This could for instance be caused by additional light neutrinos, or some other exotic particle which decoupled at very high temperature. In the present letter we use the data from Boomerang to place a strict upper limit on the radiation density present at the time of recombination \((T \simeq 1\,\text{eV})\). The standard way of expressing the energy density in light, non-interacting species, is in terms of the equivalent number of neutrinos:

\[
N_{\text{eff,i}} = \frac{\rho_i}{\rho_{\nu,0}}, \tag{1}
\]

where \(\rho_{\nu,0}\) is the energy density in a standard neutrino species.

From Big Bang nucleosynthesis one can also infer a limit to the effective number of neutrino species. By observing the primordial abundances of \(^3\text{He}, \, ^4\text{He}\) and \(^7\text{Li}\), and comparing them to the theoretically predicted values, one can infer an upper limit to \(N_\nu\) of \(\lesssim 4\) (2).

The CMBR limit can be viewed as complementary to the BBN limit because the limit from BBN applies to the radiation energy density present when the temperature of the universe was about 1 MeV, whereas the CMBR limit applies at a temperature of 1 eV. Furthermore, the BBN limit is also flavour sensitive. If the extra energy density is in the form of electron neutrinos, it changes the weak reaction rates for the beta processes that interconvert neutrons and protons. With some fine tuning, even a very substantial amount of energy can be hidden in the neutrino sector while still yielding the same outcome for BBN.

That is not the case for CMBR. In this case, extra energy density is detectable because the CMBR spectrum changes with the addition of radiation. After recombination, the CMBR fluctuations can still change. If the universe is completely matter dominated and flat, the photons see a constant gravitational potential (in the linear approximation), and thus travel with constant energy, except for the overall redshifting. However, immediately after recombination the universe was not completely matter dominated so that the gravitational potential was not constant. This leads to an enhancement of the first acoustic peak in the power spectrum and is known as the early Integrated Sachs-Wolfe (ISW) effect. This effect is only sensitive to energy density and not to the specific nature of the radiation.

Likelihood analysis — The CMBR fluctuations are usually described in terms of the power spectrum, which is again expressed in terms of \(C_l\) coefficients as \(l(l+1)C_l\), where
The $a_{lm}$ coefficients are given in terms of the actual temperature fluctuations as
\[ T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi). \tag{4} \]

Given a set of experimental measurements, the likelihood function is
\[ \mathcal{L}(\Theta) \propto \exp \left( -\frac{1}{2} x^T [C(\Theta)^{-1}] x \right), \tag{5} \]
where $\Theta = (\Omega, \Omega_b, H_0, n, \tau, \ldots)$ is a vector describing the given point in parameter space. $x$ is a vector containing all the data points and $C(\Theta)$ is the data covariance matrix. This applies when the errors are Gaussian. If we also assume that the errors are uncorrelated, this can be reduced to the simple expression, $\mathcal{L} \propto e^{-\chi^2/2}$, where
\[ \chi^2 = \sum_{i=1}^{N_{\text{max}}} \frac{(C_{\text{obs}} - C_{\text{theory}})^2}{\sigma^2} \tag{6} \]
is a $\chi^2$-statistics and $N_{\text{max}}$ is the number of power spectrum data points. In the present letter we use Eq. (3) for calculating $\chi^2$.

The procedure is then to calculate the likelihood function over the space of cosmological parameters. The likelihood function for $N_\nu$ is then obtained by keeping $N_\nu$ fixed and maximizing $\mathcal{L}$ over the remaining parameter space. The fundamental free parameters which we allow to vary are: $\Omega_m$, the matter density; $\Omega_b$, the baryon density; $h$, the Hubble parameter; $n$, the spectral index; $\tau$, the optical depth to reionization; and $Q$, the overall normalization of the spectrum given in terms of the quadrupole moment $Q_5-40$. The range in which they are allowed to vary are given in Table I. We assume a flat universe with $\Omega_0 = \Omega_b + \Omega_m + \Omega_\Lambda = 1$. This is the value strongly suggested by the Boomerang experiment [6]. Also, the final result of the analysis does not vary much even if $\Omega_0$ is allowed to vary. For a given value of $N_\nu$, we maximize the likelihood over the remaining parameter space by using the non-linear optimization method called simulated annealing [12]. The data set which we use is the one given in de Bernardis et al. [1], and we use the publicly available CMBFAST package for calculating theoretical power spectra [13].

We show the result of the likelihood analysis in Fig. 1 in terms of $\chi^2$. The absolute minimum $\chi^2$ is 5.27 at

\[ N_\nu = 1.6. \] The number of Boomerang data points is 12 [9], and we allow 6 parameters to vary. Thus, the number of degrees of freedom in our fit is 6, so that the obtained best fit $\chi^2$ falls within 1$\sigma$ of the expected for an acceptable fit. The CMBR constraint on $N_\nu$ is given by
\[ N_\nu \leq \begin{cases} 6.2 & 1\sigma, \vspace{0.5em} \\ 13 & 2\sigma. \end{cases} \tag{7} \]

While this constraint is clearly much weaker than the $N_\nu \leq 4$ obtained from BBN considerations [2], it applies to any type of relativistic energy density.

\begin{table}[h]
\centering
\caption{The free parameters used in the present analysis, as well as the range in which they are allowed to vary.}
\begin{tabular}{|l|c|}
\hline
Parameter & range \\
\hline
$Q$ & 5-40 $\mu$K \\
$\Omega_m$ & 0.2-1 \\
$\Omega_b h^2$ & 0.002-0.050 \\
h & 0.30-0.9 \\
n & 0.7-1.3 \\
$\tau$ & 0-1 \\
\hline
\end{tabular}
\end{table}

\[ \text{FIG. 1.} \quad \chi^2 \text{ for the Boomerang data as a function of } N_\nu. \quad \text{The curve has been obtained by minimizing } \chi^2 \text{ over all other free parameters.} \]

Discussion—We have calculated a strong upper bound on radiation density present at recombination. Next, we may ask what bounds can be placed on neutrino properties from this.

The standard model prediction is that $N_\nu = 3.04$ [8]. Although the absolute minimum for $\chi^2$ is at $N_\nu = 1.6$, the standard model is completely consistent with the

*Note that there is an estimated 10% calibration uncertainty in the overall normalization of the Boomerang data. However, this effect is completely degenerate with varying $Q$ because we use only one data set. Therefore we do not need to worry about it.
Boomerang data. There is no indication in the data of neutrino physics beyond the standard model.

If sterile neutrino degrees of freedom exist, then they can be exited by oscillations in the early universe. However, a sterile neutrino can at most contribute \( N_{\nu,\text{sterile}} = 1 \) so that our CMBR bound does not even exclude this possibility at the 1σ level.

Next, we have no direct measurement of the lepton asymmetry in the universe, and quite a large lepton asymmetry could in fact be hidden in the neutrino sector. The lepton asymmetry in neutrinos is usually expressed in units of \( \xi_{\nu} \equiv \mu_{\nu}/T_{\nu} \). The neutrino distribution function is then given by

\[ f = \frac{\exp(E/T_{\nu} \pm \xi_{\nu}) + 1}{(0.49 \pi)^{1/4}} \]

where + applies to neutrinos and − to antineutrinos. From BBN, one obtains the bound

\[ \xi_{\nu} \in [-0.06, 1.1] \quad (8) \]

\[ |\xi_{\nu,\tau}| \leq 6.9. \quad (9) \]

This is in the absence of any oscillations. For massless neutrinos, a chemical potential is equivalent to a change in the effective number of species

\[ N_{\nu,\text{eff}} = 3 + \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4. \quad (10) \]

Using this, we can translate our 2σ upper bound on \( N_{\nu} \) to a bound on \( \xi_{\nu} \)

\[ |\xi_{\nu,\tau}| \leq 3.7 \quad (2\sigma). \quad (11) \]

This bound applies if only one species has a chemical potential. If the asymmetry is equally shared then \( |\xi_{\nu}| \leq 2.4 \). Lesgourgues and Peloso have also discussed a cosmological lepton asymmetry as a possible explanation of the relatively low amplitude of the second acoustic peak in the Boomerang data, but they did not perform a likelihood analysis of the data.

Interestingly this bound is at the brink of excluding a scenario for the production of ultra high energy cosmic rays (UHECRs), proposed by Gelmini and Kusenko. In this scenario, very high energy neutrinos interact with a degenerate sea of neutrinos within about 50 Mpc of earth. These interactions produce charged particles such as protons which would then be the observed UHECR primaries. However, because of the small \( \nu\nu \) interaction cross section this scenario only works if the cosmic neutrino background is degenerate with \( \xi \approx 4 \). At present, the CMBR bound disfavours this model, but cannot exclude it completely. It has been estimated that with the upcoming MAP and Planck experiments, it will be possible to constrain \( \xi \) with a precision of about 0.1. This will definitively confirm or rule out scenarios like this.

Neutrino decays to massless secondaries prior to recombination are also seen in the CMBR fluctuations as an increased \( N_{\nu} \). The effective number of neutrinos is given roughly by

\[ N_{\nu} \approx 3 + 0.516 \alpha^{2/3}, \quad (12) \]

where \( \alpha \) is the decay “relativity” parameter

\[ \alpha \equiv 3.50 \left( \frac{m_{\nu}}{1\text{keV}} \right)^2 \left( \frac{\tau}{1\text{y}} \right). \quad (13) \]

Again we can translate our bound on \( N_{\nu} \) into a bound on \( \alpha \) and thereby on neutrino decays

\[ \alpha \leq 85.3 \quad (2\sigma). \quad (14) \]

In Fig. 2 we show what this translates into in terms of neutrino lifetime and mass.

Note that CMBR measurements can also be used to constrain late neutrino decays, which take place after recombination. However, such decays are detectable because of the late ISW effect they produce at small \( l \). Since Boomerang does not detect fluctuations below about \( l = 50 \), we have not calculated any bound on late decays.

In conclusion, Boomerang has provided us with the first precision CMBR data. This data can be used to constrain many different types of exotic physics, not just in the neutrino sector. The Boomerang data is only the first indication of what we can expect with the new generation of CMBR experiments. In the next few years we will have data from the MAP and Planck satellites, which is expected to be an order of magnitude better than that from Boomerang.
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