Effects of unconventional breakup modes on incomplete fusion of weakly bound nuclei

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The incomplete fusion dynamics of $^6\text{Li} + ^{209}\text{Bi}$ collisions at energies above the Coulomb barrier is investigated. The classical dynamical model implemented in the PLATYPUS code is used to understand and quantify the impact of both $^6\text{Li}$ resonance states and transfer-triggered breakup modes (involving short-lived projectile-like nuclei such as $^5\text{Be}$ and $^5\text{Li}$) on the formation of incomplete fusion products. Model calculations explain the experimental incomplete-fusion excitation function fairly well, indicating that (i) delayed direct breakup of $^6\text{Li}$ reduces the incomplete fusion cross-sections, and (ii) the neutron-stripping channel practically determines those cross-sections.

Different types of models have been used to investigate low-energy fusion dynamics of weakly bound nuclei, ranging from classical to quantum mechanical methods. Ref. [3] provides a critical survey of different theoretical approaches. New studies on the inclusive non-elastic breakup cross section may provide a quantum mechanical route to the calculation of the ICF cross section of weakly bound nuclei [13]. Another interesting quantum mechanical framework is the time-dependent wave-packet (TDWP) method [14, 15]. This method calculates the incomplete and complete fusion cross-sections unambiguously [13], which is a challenge using the continuum discretized coupled-channels (CDCC) method [16, 18]. The TDWP approach is currently undergoing further development to be implemented using a three-dimensional reaction model.

Some of the challenges of the quantum mechanical models can be overcome via the use of the three-dimensional classical dynamical model [19, 21, 22]. This model is implemented using the PLATYPUS code [22], which uses classical trajectories in conjunction with stochastic breakup [15, 21]. This is done through the input, which includes a breakup function determined from sub-barrier breakup measurement [8, 10], that undergoes Monte-Carlo sampling [15, 21]. This breakup function encodes the effect of the Coulomb and nuclear interactions that cause the breakup, making this approach a quantitative dynamical model for relating the sub-barrier NCBU to the above-barrier ICF and CF of weakly bound nuclei, rather than a breakup model [19, 21, 22]. It is important to note however that this fusion model only works at energies above the Coulomb barrier generated between the projectile and target. This is due to the absence of quantum tunneling that is the primary way of fusion at sub- and near-barrier energies.

Different types of models have been used to investigate low-energy fusion dynamics of weakly bound nuclei, ranging from classical to quantum mechanical methods. A recent attempt to amend this classical model by adding a correction at sub- and near-barrier energies, to take into account quantum tunneling. This was done by incorporating a tunneling factor based on the WKB approximation [23]. This improved the results outputted from the model, relative to experimental sub-barrier fusion measurements [23]. Additional modifications have been suggested for interpreting sub-barrier breakup measurements [12].

The dynamics surrounding prompt and delayed breakup is important to understand [11], and this is explored in the present work using PLATYPUS. For instance, prompt breakup happens in the instant the excitation of the $^6\text{Li}$ projectile is chosen to take place. At this point $^6\text{Li}$ is converted into its cluster fragments (alpha-deuteron) and then the fragments and target propagate according to the defined interactions between them [21]. Delayed breakup is induced by reaching the $1^+, 2^+$ or $3^+$ resonant states in $^6\text{Li}$, which then triggers the dissociation of $^6\text{Li}$ with the delay coming from the half-life of the resonant $^6\text{Li}$ state. The $3^+$ resonant state has a much longer half-life than the $2^+$ and $1^+$ resonant states, so the $^6\text{Li}$ breakup takes place at the outgoing branch of its trajectory, far away from the target nucleus, not affecting fusion. So the effect of the $3^+$ resonance on fusion will be neglected in the calculations below. The $1^+, 2^+$ or $3^+$ resonant states in $^6\text{Li}$ can be described by resonant alpha-deuteron $d$-states with energy and width of $(E_{res}, \Gamma_{res}) \equiv (4.18, 1.5), (2.84, 1.7)$ and $(0.716, 0.024)$. 

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and 5 Li. Consequently, for those transfer channels, 5 Li and 8 Be are considered initial projectiles with modified incident energies to match the distance of closest approach \( R_{\text{min}} \), i.e., \( E_0 + Q_{\text{tr}} \), being \( E_0 \) and \( Q_{\text{tr}} \) the 6 Li incident energies and the average transfer Q-values, respectively. The \( Q_{\text{tr}} \) values are -1.059 MeV (n-stripping) and 13.338 MeV (d-pickup). Effects on \( R_{\text{min}} \), due to uncertainties of \( Q_{\text{tr}} \) caused by excitations of both projectile-like and target-like nuclei are not included.

The measured (prompt) breakup functions deduced from the Q-value spectra of the studied channels \( \sigma_{10} \) are characterized by two constants, \( (\alpha, \beta) \), of the exponential function \( P_{\text{BU}}(r_{\text{min}}) = \exp(\alpha R_{\text{min}} + \beta) \). These values are \((-0.353738, 1.26228), (-0.799239, 8.03581)\) and \((-1.57787, 14.3403)\) for 6 Li + 209 Bi, 6 Li + 210 Bi and 8 Be + 207 Pb systems, respectively \([22]\). Refs. \([10, 11, 12]\) explain how these functions are experimentally determined.

The parameters of the Woods-Saxon nuclear interaction between the projectile fragments and the target are determined by approximately matching the corresponding Sao-Paulo potential barriers \([27]\). Table II presents those parameters, while for the Coulomb interaction the potential of a uniformly charged sphere has been used. All the radius parameters provide a distance determined by \( r_0 A^{1/3} \), where \( A \) is the heaviest mass in the corresponding binary system.

TABLE I: Parameters of the Woods-Saxon nuclear potential for different systems in the present calculations as well as the radius parameter of their Coulomb interactions (last column).

| Systems        | \( V_0 \) (MeV) | \( r_0 \) (fm) | \( a_0 \) (fm) | \( r_{\text{cm}} \) (fm) |
|----------------|-----------------|---------------|---------------|-------------------------|
| 209 Bi + 6 Li  | -51.761         | 1.545         | 0.659         | 1.2                    |
| 209 Bi + 4 He  | -32.931         | 1.461         | 0.605         | 1.2                    |
| 209 Bi + 2 H   | -26.000         | 1.465         | 0.668         | 1.2                    |
| 4 He + 2 H     | -78.460         | 1.150         | 0.700         | 1.465                  |
| 210 Bi + 6 Li  | -51.761         | 1.513         | 0.663         | 1.2                    |
| 210 Bi + 4 He  | -32.931         | 1.459         | 0.608         | 1.2                    |
| 210 Bi + 1 H   | -9.900          | 1.320         | 0.679         | 1.2                    |
| 4 He + 1 H     | -52.350         | 1.100         | 0.378         | 1.2                    |
| 207 Pb + 8 Be  | -120.903        | 1.430         | 0.762         | 1.2                    |
| 207 Pb + 4 He  | -62.000         | 1.385         | 0.620         | 1.2                    |
| 4 He + 4 He    | -16.696         | 1.200         | 0.620         | 1.2                    |

Table II presents the centroid \( (d_{012}) \) and variance \( (\sigma_{012}) \) of the Gaussian distributions for the radial g.s. probability density of the different two-body projectiles \([22]\). These parameters describe the radial probability density inside the radius of the Coulomb barrier between the fragments. This radial probability density is derived from the projectile g.s. wave-function. In sampling breakup, the maximal internal energy and angular momentum of the two-body projectiles are \( \epsilon_{\text{max}} = 6 \text{ MeV} \) and \( l_{\text{max}} = 4 \hbar \), as in Ref. \([10]\). Resonant breakup events are also sampled in this energy and angular momentum window, as explained in point (i). Prompt breakup for partial waves other than a resonant wave is included in the direct breakup of 6 Li.
the energy region of a resonance. Only even $l$ are included when the breakup fragments are identical. These $\epsilon_{\text{max}}$ and $l_{\text{max}}$ values as well as the values of the orbital angular momenta of the projectiles ($L_0 \leq 100\hbar$) guarantee the convergence of the above-barrier ICF and NCBU cross-sections.

TABLE II: Centroid and variance of the Gaussian distributions for the radial g.s. probability density of the different two-body projectiles.

| Projectiles | $d_{012}$ (fm) | $\sigma_{012}$ (fm) |
|-------------|----------------|----------------------|
| $^6\text{Li}(^4\text{He} + ^2\text{H})$ | 3.12 | 1.05 |
| $^7\text{Li}(^4\text{He} + ^1\text{H})$ | 2.95 | 1.78 |
| $^8\text{Be}(^4\text{He} + ^4\text{He})$ | 1.80 | 1.00 |

Results. Figure 1(a) shows a clear difference between the ICF cross-sections for delayed (dashed line) and prompt (solid line) direct breakup of $^6\text{Li}$. The lower cross-sections for delayed breakup can be explained by the fact that when the projectile is excited to a resonant state the projectile flies past the target, due to the half-life associated with the resonant state, before it breaks up. Most breakup events happen close to the distance of minimal approach [19, 21]. The half-lives for the $1^+$ and $2^+$ states are of $4.2 \times 10^{-22}\text{s}$ and $3.7 \times 10^{-22}\text{s}$, respectively. The typical collision time is $\sim 10^{-21}\text{s}$, therefore these short-lived states can have a significant effect on the ICF process. This causes the probability of an ICF reaction taking place to decrease, as it is less likely for a fragment to be absorbed when resonance states ($1^+$, $2^+$) are excited near the target. Consequently, the ICF and NCBU cross-sections are anticorrelated as observed in Fig. 1(b). When the projectile is excited to a resonant state, the projectile breaks up in the outgoing branch of its trajectory, making it less likely for a fragment to be absorbed by the target.

Figure 2 shows the alpha (dashed line) and deuteron (thin solid line) contributions to the direct ICF excitation function (thick solid line). The deuteron contribution is higher than the alpha contribution due to the respective difference in Coulomb barriers between the alpha and the target ($\sim 21.25\text{ MeV}$) and the deuteron and the target ($\sim 10.1\text{ MeV}$). For instance, Fig. 3 shows, for the ICF process, the relative kinetic energy distributions for $\alpha-^{209}\text{Bi}$ and $d-^{209}\text{Bi}$ immediately following the prompt direct breakup of $^6\text{Li}$ at the incident energy of $E_{\text{c.m.}} = 50.5\text{ MeV}$. Although the position of the maxima of these distributions qualitatively agrees with a simple partition of the $^6\text{Li}$ incident energy between the clusters according to their masses (arrows), as depicted in Fig. 3(a), the role of the individual Coulomb barriers in the respective ICF process is crucial, as observed in Fig. 3(b). In Fig. 3(b), we can see that the yield of deuterons with a positive kinetic energy (dashed line) is substantially larger than that for alpha particles (solid line), explaining the alpha and deuteron contributions in Fig. 3. The trends in Fig. 3 are also observed in Fig. 1.

**FIG. 1:** (a) Incomplete-fusion excitation function when direct breakup of $^6\text{Li}$ into alpha and deuteron happens in collisions with the $^{209}\text{Bi}$ target: prompt breakup (solid line), delayed breakup (dashed line). (b) The same but for the no-capture breakup excitation curve.

**FIG. 2:** (Color online) Alpha and deuteron contributions to the incomplete-fusion excitation function for prompt direct breakup of $^6\text{Li}$ in collisions with $^{209}\text{Bi}$.
FIG. 3: (a) Relative kinetic-energy distributions for $\alpha^{-209}$Bi and \textit{d}--$^{209}$Bi immediately following the prompt direct breakup of $^6$Li in collisions with $^{209}$Bi at $E_{c.m.} = 50.5$ MeV. (b) The same but the energy is measured relative to the height of the Coulomb barriers between each cluster and the $^{209}$Bi target. The arrows denote the partition of the $^6$Li incident energy between these clusters according to their masses.

FIG. 4: (Color online) Experimental incomplete-fusion cross-sections for $^6$Li + $^{209}$Bi [24] are compared with \textsc{platypus} calculations at above-barrier (arrow) energies. Direct breakup as well as transfer-triggered breakup channels are included. (a) For \textit{delayed} direct breakup of $^6$Li, \textit{delayed} breakup of $^8$Be after d-pickup, and \textit{delayed} breakup of $^5$Li after n-stripping. (b) The same but for \textit{prompt} breakup processes.

8Be excited states can impact on the ICF cross-sections as its g.s. half-life is very long ($\sim 10^{-16}$s) compared to the collision time. The $^8$Be g.s. breakup has been neglected, but both its prompt breakup and delayed breakup via its first $2^+$ resonant state with $(E_{res}, \Gamma_{res}) \equiv (3.0, 1.5)$ MeV [29] are included. In contrast to $^8$Be, $^5$Li has a short-lived g.s. ($\sim 3.3 \times 10^{-22}$s) whose decay can substantially affect the ICF cross-sections as shown in Fig. 4 (a) (dashed line). The n-stripping channel provides the dominant contribution to the ICF cross-sections. Figs. 4(a) and 4(b) depict extreme cases of breakup (delayed and prompt), so the real scenario is somewhere in between, which reasonably agrees with the observations. The present classical dynamical model does not treat quantum tunneling, so the description of the experimental data at energies very close to the Coulomb barrier ($\sim 29.8$ MeV) is not reliable.

Conclusions. The incomplete fusion process for $^6$Li + $^{209}$Bi collisions at energies above the Coulomb barrier has been investigated with the classical dynamical model im-

the simplified TDWP calculations discussed in Ref. [13], and disagree with those in Ref. [28] where the ICF treatment neglects the competition and correlation between the alpha- and deuteron-capture processes. In Ref. [28] it is assumed that all the observed ICF products [24] (i.e., actinium and polonium isotopes) were originated from the $^6$Li direct breakup process, which is neither the main observed breakup channel [10–12] nor the dominant ICF route as demonstrated below.

Figure 4 presents the different contributions of various reaction processes to the total ICF cross-sections (thick solid line) which are compared with experimental data [24]. Together with the direct (delayed and prompt) breakup of $^6$Li, transfer-triggered breakup of projectile-like nuclei such as $^8$Be (after d-pickup) and $^5$Li (after n-stripping) occurs during collisions of $^6$Li and $^{209}$Bi at energies near the Coulomb barrier. These projectile-like nuclei are unstable and their short-lived states may affect the formation of ICF products. Only prompt breakup of $^8$Be excited states can impact on the ICF cross-sections as its g.s. half-life is very long ($\sim 10^{-16}$s) compared to the collision time. The $^8$Be g.s. breakup has been neglected, but both its prompt breakup and delayed breakup via its first $2^+$ resonant state with $(E_{res}, \Gamma_{res}) \equiv (3.0, 1.5)$ MeV [29] are included. In contrast to $^8$Be, $^5$Li has a short-lived g.s. ($\sim 3.3 \times 10^{-22}$s) whose decay can substantially affect the ICF cross-sections as shown in Fig. 4 (a) (dashed line). The n-stripping channel provides the dominant contribution to the ICF cross-sections. Figs. 4(a) and 4(b) depict extreme cases of breakup (delayed and prompt), so the real scenario is somewhere in between, which reasonably agrees with the observations. The present classical dynamical model does not treat quantum tunneling, so the description of the experimental data at energies very close to the Coulomb barrier ($\sim 29.8$ MeV) is not reliable.

Conclusions. The incomplete fusion process for $^6$Li + $^{209}$Bi collisions at energies above the Coulomb barrier has been investigated with the classical dynamical model im-
implemented in the PLATYPUS code. The main conclusions of the present work are as follows:

1. The resonant states ($1^+$, $2^+$) of $^6$Li play an important role in the direct ICF and NCBU cross-sections, due to the respective half-life of these resonant states ($\sim 10^{-22}$s) relative to the collision time ($\sim 10^{-21}$s). Delayed breakup via excitation of those resonance states reduce the theoretical ICF cross-sections.

2. The deuteron contribution to the direct component of ICF cross-sections (this component is much smaller than the n-stripping component) is significantly higher than the alpha contribution because of the much smaller Coulomb barrier between the deuteron and the $^{209}$Bi target (by $\sim 11$ MeV).

3. The n-stripping channel involving the projectile-like nucleus $^5$Li clearly dominates the formation of ICF products. This is the central result of the present work. In contrast most quantum mechanical fusion calculations assume that the direct breakup of $^6$Li is the dominant ICF channel.

4. Using information from sub-barrier breakup measurements, PLATYPUS provides a comprehensive and insightful explanation of the ICF excitation function at above-barrier energies.

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