de Broglie-Bohm interpretation for wave function of Reissner-Nordström-de Sitter black hole

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Abstract

We study the canonical quantum theory of the Reissner-Nordström-de Sitter black hole (RNdS). We obtain an exact general solution of the Wheeler-DeWitt equation for the spherically symmetric geometry with electromagnetic field. We investigate the wave function from a viewpoint of the de Broglie-Bohm interpretation. The de Broglie-Bohm interpretation introduces a rigid trajectory on the minisuperspace without assuming an outside observer or causing collapse of the wave function. In our analysis, we obtain the boundary condition for the wave function which corresponds to the classical RNdS black hole and describe the quantum fluctuations near the horizons quantitatively.

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I. INTRODUCTION

The canonical form of general relativity is presented by Dirac [1], and by Arnowitt, Deser and Misner (ADM) [2]. In the formulation, the dynamics of the gravitational field is given as the totally constraint system. For quantization of the gravity, the constraints are used as operator restrictions on the state, and the Hamiltonian constraint gives the Wheeler-DeWitt (WD) equation [3,4]. The canonical quantum gravity based on the Wheeler-DeWitt approach has mathematical and conceptual difficulties. The WD equation is a functional differential equation with respect to the three dimensional metric components $g_{\mu\nu}$ and includes the product of the functional derivatives. When the space is inhomogeneous, one can hardly solve this equation of infinite degrees of freedom. It has also an operator ordering ambiguity. If one reduce the degrees of freedom to finite by symmetry, the WD equation becomes a greatly simple and solvable equation. Kuchař studied the geometrodynamics of Schwarzschild black holes [5]. The black hole mass is shown to be a dynamical variable on the phase space. Nakamura, Konno, Oshiro and Tomimatsu applied the canonical formulation to the inside of the horizon of the Schwarzschild black hole, and obtained an exact solution of the WD equation under a condition of a mass eigenstate equation. In the WKB region, their solution correspond to a classical black hole solution and they derived a semiclassical picture from the exact solution [6]. The WKB approximation breaks down if a variation of a potential is considerably large over a wavelength. In this situation the frequency of the wave function is very high and the amplitude violently fluctuates. Therefore they were inhibited from interpreting the wave function near the horizon. In the region where the scalar curvature becomes very large such as a horizon, a sensible quantum effect of the spacetime is expected to appear prominently. We cannot neglect the influence of the spacetime fluctuation on the Hawking radiation [7,8]. In our preceding paper [9], we solved the WD equation for a minisuperspace and obtained the wave function for the Schwarzschild black hole. Then we applied the de Broglie-Bohm (dBB) interpretation to the wave function in order to estimate the quantum fluctuations instead of the WKB approach. Further we investigated the operator ordering ambiguity on the quantum effect. In the dBB approach [10–12], by introducing a rigid trajectory picture thorough the phase factor of the wave function, we can make quantitative estimations of quantum effects near horizons and make a natural derivation of the (semi)classical picture from the quantum theory without having to assume an outside observer and having to cause the collapse of the wave function. The emergence of the classical picture from quantum systems is a problem under debate by several authors [13]. In regard to the conceptual problem of time, that is, there is no time evolution in the WD equation, we can also get the parametric time in a natural way, although the general covariance spontaneously breaks down at a quantum level.

The purpose of our work is to investigate static states of quantum geometry near the horizons. We solve the WD equation for the Reissner-Nordström-de Sitter (RNdS) black hole. The RNdS geometry has some specific features such that the Cauchy horizon and the timelike singularity exist. In our minisuperspace, the dBB trajectory picture shows that quantum effects are dominant near all horizons while the trajectory asymptotically approaches to the classical one in an infinite region or the singularity.
II. CANONICAL FORMALISM

The general form of the Einstein-Maxwell action with the cosmological constant is written as

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ (\mathcal{R} - 2\lambda) - F_{\mu\nu}F^{\mu\nu} \right].$$  \hspace{1cm} (1)$$

In this expression, \(\mathcal{R}\) is the four-dimensional Riemann curvature scalar and \(\lambda\) denotes the cosmological constant. We take the natural unit \(G = c = 1\). \(F_{\mu\nu}\) is the electromagnetic field strength

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \hspace{1cm} (2)$$

The action principle gives the Einstein-Maxwell’s equation in vacuum. A static spherically symmetric solution of it is known as the Reissner-Nordström-de Sitter (RNdS) metric

$$ds^2 = -(1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{\lambda}{3} R^2) dT^2 + \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{\lambda}{3} R^2\right)^{-1} dR^2 + R^2 d\Omega^2, \hspace{1cm} (3)$$

which describes a black hole with the mass \(M\) and the electric charge \(Q\) in the background of the static de Sitter space. \(d\Omega^2\) denotes the line element on the unit sphere.

We reduce the gravitational degrees of freedom by spherical symmetry anzats. Let us restrict our attention to the inside of the horizon, where the radial coordinate plays the role of the time coordinate. We denote the coordinate as \(t\). We introduce two metric variables \(U\) and \(V\) which depend only on \(t\) as

$$ds^2 = -\frac{\alpha(t)^2}{U(t)} dt^2 + U(t) dR^2 + V(t) d\Omega^2, \hspace{1cm} (4)$$

where \(\alpha\) is the lapse function. Because of spherical symmetry, the shift vector \(N^a\) except the radial component \(N^t\) must be zero. We have chosen the radial component \(N^t\) to be zero taking account of the static spacetime. Using the metric Eq.(4) the action Eq.(1) is decomposed into the ADM hypersurface action

$$S_\Sigma = \int dt \int dr L, \hspace{1cm} (5)$$

where the Lagrangian is

$$L = \frac{1}{4} \left( -\dot{V}\dot{U} - \frac{UV^2}{2\alpha V} + 2\alpha \right) + \frac{V}{2\alpha} F_{01}^2 - \frac{\lambda}{2} \alpha V, \hspace{1cm} (6)$$

with

$$F_{01} = \dot{A} - A', \hspace{1cm} (7)$$

which is the \((0,1)\) component of the field strength \(F_{\mu\nu}\). Here the notations are \(\cdot = \partial/\partial t\) and \(\cdot' = \partial/\partial r\). Although we have assumed that the electromagnetic field \(A_1(\equiv A)\) depends only on time variable \(t\), the field \(A_0\) is a redundant degree of freedom and we need not to impose the symmetry on \(A_0\).
The Euler-Lagrange equation for the system Eq.(3) gives the classical solution:
\[ \alpha = 1, \]  
\[ U = - \left( 1 - \frac{2m}{\sqrt{V}} + \frac{Q^2}{V} - \frac{\lambda}{3} V \right), \]  
\[ \sqrt{V} = t, \]  
\[ VF_{01} = Q\alpha (\text{const}), \]  
which correspond to the extension of the RNdS spacetime Eq.(3). The integration constant \( m \) represents the asymptotically observed mass of a spherically symmetric matter. The point \( V^{1/2} = t = 0 \) is a real singularity. There are three horizons at the values \( t_1, t_2, \) and \( t_3 \) for which \( U = 0. \)

For convenience we change the variables from \( U \) and \( V \) to \( z_+ \) and \( z_- \) as
\[ z_+ \equiv U\sqrt{V}, \quad z_- \equiv \sqrt{V}. \]  
By using these new variables, the Lagrangian Eq.(6) becomes a simpler and symmetric form
\[ L = \frac{1}{2} \left( -\frac{1}{\alpha} \dot{z}_+ \dot{z}_- + \alpha \right) + \frac{z_+^2}{2}\alpha F_{01}^2 - \frac{\lambda}{2} \alpha z_-^2. \]  
The canonical momenta conjugate to \( z_+, z_- \) and \( A \) are obtained from the Lagrangian:
\[ \Pi_+ \equiv \frac{\partial L}{\partial \dot{z}_+} = -\frac{1}{2\alpha} \dot{z}_-, \]  
\[ \Pi_- \equiv \frac{\partial L}{\partial \dot{z}_-} = -\frac{1}{2\alpha} \dot{z}_+, \]  
\[ \Pi_A \equiv \frac{\partial L}{\partial A} = \frac{z_+^2}{\alpha} (\dot{A} - A_0'). \]  
Since the Lagrangian Eq.(13) do not contain the terms \( \dot{A} \) and \( \dot{A}_0 \), the corresponding momenta vanish trivially and they yield the primary constraints. The Legendre transformation gives the Hamiltonian and the action for this system is
\[ S_\Sigma = \int dt \int dr (\Pi_A \dot{A} + \Pi_+ \dot{z}_+ + \Pi_- \dot{z}_- - \alpha H - A_0 H_A), \]  
where \( H \) and \( H_A \) are
\[ H = -2 \Pi_+ \Pi_- - \frac{1}{2} + \frac{\Pi_+^2}{2z_+^2} + \frac{\lambda}{2} z_-^2, \]  
\[ H_A = - (\Pi_A)' \]  
Here we integral out over the unit sphere and treat the integral \( \int_0^\infty dr \) to be finite. \( \Pi_\alpha \) and \( \Pi_{A_0} \) have vanishing Poisson brackets with the Hamiltonian and also generate the secondary constraints which give the Hamiltonian constraints: \( H \approx 0 \) and \( H_A \approx 0 \). The secondary constraints do not generate further constraints, because the Poisson brackets with the Hamiltonian weakly vanish. Then the Hamiltonian constraints are the first class. We also introduce
the black hole mass as a dynamical variable on canonical data. Following the calculation of the Schwarzschild mass by Kuchař, the RNdS black hole mass is expressed as

$$M = 2 \, \Pi_+ \, z_+ \, \Pi_+ + \frac{z_+}{2} + \frac{\Pi_+^2}{2 z_-} - \frac{\lambda}{6} z_3^3. \quad (20)$$

Next we quantize the black hole system in the Schrödinger representation. The canonical momenta are quantized as usual differential operators:

$$\hat{\Pi}_+ = -i \hbar \frac{\partial}{\partial z_+}, \quad \hat{\Pi}_- = -i \hbar \frac{\partial}{\partial z_-}, \quad \hat{\Pi}_A = -i \hbar \frac{\partial}{\partial A}. \quad (21)$$

We take the Weyl ordering in the following calculations. Following Dirac’s canonical quantization procedure, we impose operator restrictions on the state vector $\Psi$ as constraints. For the Hamiltonian constraint Eq. (18),

$$\hat{H} \Psi = \left( -2 \hat{\Pi}_+ \hat{\Pi}_- - \frac{1}{2} + \frac{\hat{\Pi}_+^2}{2 z_-} + \frac{\lambda}{2} z_2^2 \right) \Psi(z_+, z_-, A) = 0, \quad (22)$$

which is called the Wheeler-DeWitt (WD) equation. The mass operator $\hat{M}$ is weakly commutable with the Hamiltonian in the Weyl ordering:

$$[\hat{H}, \hat{M}] \Psi = 2i \hbar \, \hat{\Pi}_+ \hat{H} \Psi = 0. \quad (23)$$

In addition to the WD equation, we also impose another two restrictions on the state vector $\Psi$ as constraint equations, the mass operator $\hat{M}$ with the mass eigenvalue $m$ and the Hamiltonian $H_A$ with the eigenvalue of the charge $Q$:

$$\hat{M} \Psi = \left( 2 \hat{\Pi}_+ \, z_+ \, \hat{\Pi}_+ + \frac{z_+}{2} + \frac{\hat{\Pi}_+^2}{2 z_-} - \frac{\lambda}{6} z_3^3 \right) \Psi(z_+, z_-, A) = m \, \Psi(z_+, z_-, A), \quad (24)$$

$$\hat{H}_A \, \Psi(z_+, z_-, A) = Q \, \Psi(z_+, z_-, A). \quad (25)$$

Eq. (25) is equivalent to the charge conservation law on the state vector $\Psi$. First we solve the equation (23)

$$\left( -i \hbar \frac{\partial}{\partial A} + Q \right) \Psi = 0. \quad (26)$$

The solution is

$$\Psi = \psi(z_+, z_-) e^{-iQA/\hbar}, \quad (27)$$

where $\psi$ is a general function of $z_+$ and $z_-$. Next we shall solve the two equations Eqs. (22) and (24) to determine $\psi$. Instead of solving the mass eigenvalue equation directly, we consider an eigenvalue equation derived from the linear combination of the Hamiltonian and the mass operators:

$$\hat{L} \Psi \equiv [\Pi_+ \, z_+ \, \hat{H} + \Pi_- \, (\hat{M} - m)] \Psi = 0, \quad (28)$$
and then, after making a variable transformation from \( z \) to \( \tilde{z} \):
\[
\tilde{z}_- = \frac{1}{z_-} \left( z_-^2 - 2mz_- - Q^2 - \frac{\lambda}{3} z_-^4 \right),
\]
we can obtain the first order differential equation
\[
\left( \tilde{z}_- \frac{\partial}{\partial \tilde{z}_-} - z_+ \frac{\partial}{\partial z_+} \right) \psi(z_+, z_-) = 0.
\]
This equation is the symmetric with respect to the variables \( z_+ \) and \( \tilde{z}_- \). Then we further transform the variables \( z_+ \) and \( \tilde{z}_- \) to \( y \) and \( z \):
\[
y \equiv \frac{1}{\hbar} \sqrt{-z_+/\tilde{z}_-}, \quad z \equiv \frac{1}{\hbar} \sqrt{-z_+ \tilde{z}_-},
\]
Using the variables Eq. (31), Eqs. (28) and (22) become the form:
\[
y \frac{\partial}{\partial y} \psi(y, z) = 0,
\]
\[
\left( \frac{1}{z} \frac{\partial}{\partial z} \frac{\partial}{\partial z} + 1 \right) \psi(y, z) = 0.
\]
The equation (33) for \( \psi \) is the Bessel’s differential equation with zeroth order. Then we finally obtain an exact solution of the RNdS quantum black hole:
\[
\Psi(z, A) = \left( c_1 H_0^{(1)}(z) + c_2 H_0^{(2)}(z) \right) e^{-iQA/\hbar},
\]
\[
z = \sqrt{-z_+ \tilde{z}_-} = \left[ -u \left( v - 2m\sqrt{v} + Q^2 - \frac{\lambda}{3} v^2 \right) \right]^{1/2},
\]
where \( c_1 \) and \( c_2 \) are integration constants. The Hankel functions \( H_0^{(1)} \) and \( H_0^{(2)} \) are linearly independent and complex conjugate to each other. If the charge \( Q = 0 \) and the cosmological constant \( \lambda = 0 \), the quantum RNdS solution agrees with the quantum Schwarzschild one [9].

III. DE BROGLIE-BOHM INTERPRETATION

In the de Broglie-Bohm(dBB) interpretation the quantum mechanics is explained as follows. First, the wave function \( \Psi \) is given as the solution of the Schrödinger equation. In our case the corresponding equation is the Wheeler-DeWitt(WD) equation, which has no time evolution and whose eigen state vector is considered as a stationary state with zero energy. Next, the wave function is decomposed into two real functions, the amplitude \( R \) and the phase \( S \) according to the expression \( \Psi(z_+, z_-, A) = R(z_+, z_-, A) \exp[iS(z_+, z_-, A)/\hbar] \). Substituting this expression into the WD equation (22), we can rewrite it in the real part and the imaginary part equations:
\[
2 \frac{\partial S}{\partial z_+} \frac{\partial S}{\partial \tilde{z}_-} + \frac{1}{2} + V_Q = 0, \quad (36)
\]
\[
\frac{\partial}{\partial z_+} (z_+ R^2) \frac{\partial S}{\partial z_+} = 0, \quad (37)
\]
where the function $V_Q$ is

$$V_Q = -\frac{2\hbar^2}{R} \frac{\partial^2}{\partial z_+ \partial z_-} R. \quad (38)$$

Here we use the constraint equation for the charge Eq.(25) in advance. As a result the dependence of the function $V_Q$ and the equations (36) and (37) on the variable $A$ is removed.

In the dBB interpretation, we introduce the trajectories $Z_+(T), Z_-(T)$ and $A(T)$ on which the particles are assumed to move with the momenta:

$$\Pi_+ = -\frac{1}{2} \dot{Z}_+ = \frac{\partial S}{\partial z_+} \bigg|_{z_+ = z_+, z_- = z_-; A = A}, \quad (39)$$

$$\Pi_- = -\frac{1}{2} \dot{Z}_- = \frac{\partial S}{\partial z_-} \bigg|_{z_+ = z_+, z_- = z_-; A = A}, \quad (40)$$

$$\Pi_A = \dot{A} = \frac{\partial S}{\partial A} \bigg|_{z_+ = z_+, z_- = z_-; A = A}. \quad (41)$$

Here $\cdot = \partial/\partial T$ is a derivative with respect to a parameter introduced through the phase factor $S$. These trajectories are assumed to be a statistical ensemble of the probability distribution given by $R^2$. In this interpretation, Eqs.(36) and (37) indicate the Hamilton-Jacobi equation and the continuity equation of the probability, respectively. In the Eq.(36), there is a term $V_Q$ which is not present in the classical Hamilton-Jacobi equation. The trajectories are modified by this term quantitatively. If $V_Q$ is negligible compared with the classical potential, the quantum trajectory approaches to the classical one and indeed this situation is what we call classical. In this sense we call the function $V_Q$ quantum potential. We recall that the quantum theory itself is applied to the system of an ensemble, that is, one must perform many measurements to one particle to obtain the wave function. In the ordinary Copenhagen interpretation, the system is divided into the external observer described by the classical mechanics and the quantum system, and the notion of the probability enters the theory. In the de Broglie-Bohm interpretation, on the other hand, there is no division between the classical observer and the quantum system and no collapse of the wave function. The predictability of the quantum mechanics stems from the notion of the distribution of a statistical ensemble of well defined quantum trajectories, which are modified by a quantum effect produced by the quantum potential.

For our minisuperspace model, we calculate the quantum potential and the quantum trajectories. We select the second term of the solution Eq.(34)

$$\Psi(z, A) = NH_0^{(2)}(z)e^{-iQA} \equiv R(z, A)e^{iS(z, A)/\hbar}, \quad (42)$$

since $H_0^{(2)}$ is a outgoing wave from the singularity at the origin to the outside. From the dBB point of view, it will be shown to correspond to the classical black hole (Fig.2). On the contrary, the superposition of $H^{(1)}$ and $H^{(2)}$ in Eq.(35) does not approach any classical solution. The equations on the velocities Eqs.(36), (37) and (39) are obtained as

$$\dot{Z}_+ = \frac{2\hbar}{\pi} \frac{Z_+^2 - \lambda^2 Z_+ - Q^2}{Z_+^2 Z_+ |H_0^{(2)}(Z)|^2}, \quad (43)$$
\[ \dot{Z}_- = \frac{2\hbar}{\pi} \frac{1}{Z_+ |H_0^{(2)}(Z)|^2}, \]
\[ \dot{A} = Q. \]  
(44)  
(45)

The ratio of Eq.(43) to Eq.(44) gives a functional relation \((Z_+, Z_-)\).

\[ Z_+ = c_0 \left( Z_- - 2m + \frac{Q^2}{Z_-} - \frac{\lambda}{3} Z_-^3 \right), \]
(46)

where \(c_0\) is an integration constant. With the choice of \(c_0 = -1\) this relation is translated back to that of the original metric variables \(U\) and \(V\) in Eq.(4):

\[ U = -\left( 1 - \frac{2m}{V^{1/2}} + \frac{Q^2}{V} - \frac{\lambda}{3} V \right), \]
(47)

which corresponds to the classical relation in Eq.(3). There are three horizons corresponding to the null surfaces \(U = 0\).

The quantum potential Eq.(38) is expressed as

\[ V_Q = -\frac{1}{2} \left( 1 - \frac{4}{\pi^2 Z^2 |H_0^{(2)}(Z)|^4} \right), \]  
with \( Z = \sqrt{-Z_+ Z_-} = \sqrt{|V| - 2m + \frac{Q^2}{\sqrt{V}} - \frac{\lambda}{3} V^{3/2}|}. \)
(48)

The behavior in the flat region \((Z \gg 1)\) and near the horizons \((Z \approx 0)\) is approximately

\[ V_Q \approx \begin{cases} 
0 & \text{for } Z \gg 1 \\
\frac{1}{8Z^2(lnZ)^4} & \text{for } Z \approx 0.
\end{cases} \]
(49)

\(V_Q\) approaches to zero at infinity, where the amplitude \(R\) becomes a constant. Near the horizon, on the other hand, \(V_Q\) takes an infinite value.

Using the \(U - \sqrt{V}\) relation (47), the remaining relation \(T - \sqrt{V}\) is obtained in the integral form:

\[ T = \frac{\pi}{2} \int Z|H_0^{(2)}(Z)|^2 d\sqrt{V}. \]
(50)

We can show that the \(T - \sqrt{V}\) relation approaches to the classical relation \(T = \sqrt{V}\) in the semiclassical region. We estimate the \(T - \sqrt{V}\) relation (50) near the horizons as

\[ T - T_0 \approx \begin{cases} 
\sqrt{V} & \text{for } Z \gg 1 \\
\frac{1}{2} (ZlnZ)^2 \sqrt{V} & \text{for } Z \approx 0,
\end{cases} \]
(51)

where \(T_0\) is an integration constant. The numerical estimation of Eq.(50) is shown in Fig.2. We fixed \(T = 0\) at \(\sqrt{V} = 0\) and carried out the integral in Eq.(50). The rate of change of \(dT/d\sqrt{V}\) of the dBB trajectory is diminishing as approaching to the horizons \(\sqrt{V} = \)
\( \sqrt{V_1}, \sqrt{V_2}\) and \( \sqrt{V_3} \) \((Z = 0)\). Near the horizons, \( T \) of the quantum trajectory shows flat behavior and takes a finite value. We connect the opposite sides of the horizon smoothly on the dBB trajectory. Fig.2 shows that the radial coordinate \( T \) of (at) the horizons of the quantum black hole is smaller than that of the classical one, while the quantum surface area is identical with the classical one \((4\pi V_1, 4\pi V_2 \text{ and } 4\pi V_3)\).

Now we consider apparent horizons. The horizon is characterized by the expansions for the ingoing and outgoing null rays \( \theta^- \) and \( \theta^+ \). Classically the equation \( \theta^+ = 0 \) expresses the apparent horizon. Therefore it has a local meaning, while the event horizons which exist at the null surface \( U = 0 \) has a global meaning. In our black hole model based on the canonical approach, we have introduced the black hole mass as a dynamical variable on canonical data. The black hole mass have a classical relation to the expansion \( \theta^- \theta^+ \):

\[
\theta^- \theta^+ = U V^{-1}(\sqrt{V})^2 = -\frac{1}{4V} \left( 1 + \frac{H_A^2}{V} - \frac{\lambda}{3}V - \frac{2M}{\sqrt{V}} \right). \tag{52}
\]

In classical level the apparent horizon \( \theta^- \theta^+ = 0 \) agrees with the event horizon through the relation Eq.(9). In quantum level, the apparent and the event horizon may be allowed not to coincide by quantum fluctuations. Following ref. [6] we define the apparent horizons in quantum level by \( \Psi^* \hat{\theta}^+ \hat{\theta}^- \Psi = 0 \). The mass operator \( \hat{M} \) and the electromagnetic momenta \( \hat{H}_A \) in this definition are reduced to the eigen values \( m \) and \( Q \) respectively, and therefore \( U = 0 \) must be satisfied on the quantum apparent horizons. The quantum apparent horizon agrees with the quantum event horizon obtained by the dBB calculus. Thereby the separation of the event and the apparent horizon is not found from the dBB point of view.

In order to investigate the property of the horizon, we also calculate the radial motion of the light ray. Here we treat the light ray as a classical object in the quantum background geometry. The coordinate of the light ray on the dBB trajectory is

\[
R = \pm \frac{\pi}{2} \int \sqrt{V} |H_0^{(2)}(Z)|^2 d\sqrt{V} \quad \text{with} \quad Z = \left| \sqrt{V} - 2m + \frac{Q^2}{\sqrt{V}} - \frac{\lambda}{3}V^{3/2} \right|. \tag{53}
\]

For comparison, the light ray in the classical Reissner-Nordström-de Sitter black hole is

\[
R_{\text{cl}} = \pm \int \left( 1 - \frac{2m}{\sqrt{V}} + \frac{Q^2}{V} - \frac{\lambda}{3}V \right)^{-1} d\sqrt{V} \quad \text{and} \quad \sqrt{V} = T. \tag{54}
\]

Using Eq.(50), the approximate behavior near the horizon \( Z \approx 0 \) of Eq.(53) is estimated as

\[
R - R_0 \approx \begin{cases} 
R_{\text{cl}} & \text{for} \quad Z \gg 1 \\
\mp \frac{\pi}{2} \sqrt{V} (\ln Z)^2 & \text{for} \quad Z \approx 0,
\end{cases} \tag{55}
\]

where \( R_0 \) denotes an integration constant. In Fig.3, the numerical estimation of Eq.(53) is shown. The integration constant \( R_0 \) is fixed at the origin \((T = 0)\) and the infinity \((T = \infty)\) in order that the light ray on the quantum trajectory coincides with that on the classical trajectory. The light ray forms a cups and reaches the horizons at finite \( R \).
IV. SUMMARY AND DISCUSSION

We studied the quantum geometrodynamics of the vacuum spacetime with spherical symmetry through a device of solving the Wheeler-DeWitt (WD) equation in the mini-superspace. We estimate the wave function from a viewpoint of the de Broglie-Bohm (dBB) interpretation. The distinctive feature of the dBB approach based on well defined quantum trajectories is that we can quantitatively estimate all quantum effects including the gravitation, even if the norm of the wave function cannot be defined. In a region where all quantum effects vanish, these trajectories are approach to classical ones. In this manner, we can always resolve the quantum theory into the (semi)classical one without having to assume an external observer and having to invoke the collapse of the wave function. In particular these properties are agreeable to quantum cosmology.

In our calculation, we do not address a boundary contribution for the Hamiltonian which gives the ADM energy. For the system of coordinates on the hypersurface $\Sigma$ which is asymptotically Cartesian, the ADM energy of a black hole observed at the right and the left infinity is its black hole mass. J. Mäkelä and P. Repo studied the dynamical aspects of the quantum Reissner-Nordström black hole. In their model, the ADM mass and the electric charge spectra of the black hole are discrete. Moreover their semiclassical analysis showed that the horizon area spectrum of black holes is closely related to the Bekenstein's proposal. Our calculation shows that the quantum potential becomes zero at infinity. If there are the boundary terms, the quantum potential at the boundary will be significant.

York introduced the idea of the quantum ergosphere that the apparent and the event horizon are separated quantum mechanically and then Nakamura et al proposed that the gravitational fluctuation spontaneously induces the phenomena. Our analysis shows that the mass eigen state does not distinguish the two horizons. Rather, the quantum effect of the gravitational field makes the photon propagating along the radial coordinate reach the horizon within a finite time $R$. This may indicate that the causal connection between the opposite sides of the horizon is tied more strongly than in the case of the classical black hole. We can also propose that the quantum effect enhances the Hawking radiation. Further analysis is needed to judge these suggestions.
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FIG. 1. The quantum potential for Reissner-Nordström-de Sitter space. There are three singular points $\sqrt{V} = \sqrt{V_1}$, $\sqrt{V_2}$ and $\sqrt{V_3}$ for which $Z = 0$. On each horizon, the quantum potential diverges and takes a positive infinite value.

FIG. 2. The $T - \sqrt{V}$ relation is shown. The classical relation is denoted by the dashed line and the quantum one by the solid line. The rate of change $dT/d\sqrt{V}$ of the dBB trajectory is diminishing as approaching to the horizons $\sqrt{V} = \sqrt{V_1}$, $\sqrt{V_2}$ and $\sqrt{V_3}$ ($Z = 0$). $T$ shows flat behavior and takes a finite value on the horizons.

FIG. 3. The light ray on the dBB trajectory is shown by the solid line. The dashed line is the light ray in the classical geometry. The light ray on the dBB trajectory forms a cups and reaches the horizons at finite $R$. 

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