Investigation of suspended nanoliquid flow of Eyring–Powell fluid with gyrotactic microorganisms and density number

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Abstract
The aim of the current investigation is to discuss the behavior of mixed convection magnetohydrodynamic flow of Eyring–Powell nanoliquid subjected to gyrotactic microorganisms over a stretchable cylinder. Energy communication is developed through the first law of thermodynamics and deliberated in the manifestation of viscous dissipation. Furthermore, Brownian motion and thermophoresis effects are also considered. Nonlinear system of partial differential equations is altered into ordinary one due to employing transformations. The given systems are then solved through ND-solve technique. Impact of influential variables on velocity, motile microorganism’s temperature, and concentration is deliberated graphically. Skin friction coefficient, mass transfer rate, density number, and Nusselt number are numerically computed versus different influential variables. Velocity and temperature have opposite impact for curvature parameter. For higher estimation of fluid parameter, temperature and velocity fields boost up.

Keywords
Eyring–Powell nanofluid, gyrotactic microorganisms, stretching cylinder, MHD, mixed convection, Brownian motion, thermophoresis effects, density number

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Introduction
Study of rheological characteristics of non-Newtonian liquids is very monotonous than compared to viscous liquids. A single constitutive equation is not appropriate to scrutinize the non-Newtonian liquids because of their complex and diverse behaviors. Recently, numerous investigators and scientists have focused their consideration on non-Newtonian liquids. It is because of their vast applications in engineering, biology, and industries like fiber sheets, wire drawing, glass formation, paper production, and crystal growth. Common examples of non-Newtonian liquids are shampoo, ketchup, yogurt, mud, greases, pasta, certain oils, paints, and so on. Initially, in 1944, Eyring and Powell proposed a Eyring–Powell model which is based on kinetic theory of fluids. Influence of radiation on magnetohydrodynamic (MHD) flow of Eyring–Powell liquid flow due to a stretchable surface is highlighted by Hayat et al.¹ Hosseinzadeh et al.² worked on hybrid nanoparticles in a hexagonal triplex latent heat with fine effects. Impact of heat flux on an Eyring–Powell liquid flow due to shrinking surface is examined by Ara et al.³ Entropy optimization in reactive flow of Eyring–Powell liquid with variable thermal conductivity by a stretchable surface is discussed by Salawu et al.⁴

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Some fruitful researches about nanomaterials in flow of non-Newtonian and Newtonian fluids are highlighted in some of the studies.\textsuperscript{5-16} Colloidal suspension of nano-sized particles (oxides, metals, carbides, or carbon nanotubes) and conventionally working materials (water, oil, and ethylene glycol) are known as nanomaterial. Nanomaterials have innovative behaviors that make them more significant in various applications in thermal transmission like medicinal procedures, domestic refrigerators, hybrid-powered engines, fuel cells, heat exchangers, and automobile thermal management. Choi and Eastman\textsuperscript{17} are the first who theoretically proved that heat conduction phenomenon of conventionally working materials can be increased by inserting nano-sized particles. Khan et al.\textsuperscript{18} discussed the behavior of thermophoresis and Brownian diffusion in Prandtl–Eyring nanoliquids with entropy optimization and cubic autocatalysis chemical reaction. Some investigations made by numerous researchers are presented in some of the studies.\textsuperscript{2,19-26}

Bioconvection is a phenomenon in which microorganisms are inserted in nanoliquids for nanoparticles movement. It is presumed that nanoparticles have no impact on the spinning direction and motion of microorganisms. Initially, Kuznetsov\textsuperscript{27} studied fluid layers of finite depth with suspended gyrotactic microorganisms. Impact of magnetic field, stratification phenomenon, and gyrotactic microorganisms on Maxwell nanoliquid flow is highlighted by Khan et al.\textsuperscript{28} Khan et al.\textsuperscript{29} explored the behavior of Darcy-Forchheimer mixed convective over a curved sheet with activation energy and entropy generation. Heat and mass transport over a convective stretched sheet with gyrotactic microorganisms and stratification phenomena is examined by Alsaeedi et al.\textsuperscript{30} Khan et al.\textsuperscript{31} investigated the effect of gyrotactic microorganisms and activation energy on natural bioconvectional flow of Sisko nanofluids.

In this article, we investigated the behavior of mixed convective MHD flow of Eyring–Powell nanoliquid subjected to gyrotactic microorganisms over a stretchable cylinder. Energy attribution is developed through the first law of thermodynamics. Brownian diffusion and thermophoretic effects are also accounted. The gyrotactic microorganisms concept is used to control the random motion of fluid nanoparticles. Heat, motile microorganisms, and mass transfer rates are examined subjected to stratification effects. Partial differential system is altered to ordinary system by suitable transformations and then tackle through numerical built in ND-solve method.\textsuperscript{32-39} Features of influential variables on velocity, motile microorganisms, temperature, and concentration are examined through graphs. Surface drag force, gradient of temperature, Sherwood, and density numbers are numerically computed and discussed.

Mathematical modeling

Consider incompressible, two-dimensional, and steady MHD mixed convective flow of Eyring–Powell nanomaterials by a stretchable surface of cylinder. Microorganisms are exploited to control the motion of fluid nanoparticles. Furthermore, dissipation is taken into consideration in modeling of heat equation. Heat, mass, and motile microorganisms transfer rates are discussed in the presence of stratification effects. Let $u_w (= w_0z/l)$ be the stretching velocity along $z$-direction. A constant magnetic field of strength $(B_0)$ is exerted at an inclination of $\alpha$ to the cylinder. The flow diagram is highlighted in Figure 1.

The governing layer expressions in view of aforementioned assumptions are

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0$$ \hspace{1cm} (1)

$$\rho \left\{ \frac{u \partial v}{\partial r} + \frac{w}{r} + \frac{\partial u}{\partial z} \right\} = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \frac{\mu v}{r} - \frac{\mu v}{r^2} + \frac{\mu v}{r^2}$$

$$+ \left( \frac{\partial^2 v}{\partial r^2} - \frac{1}{r} \frac{\partial v}{\partial r} \right) - \left( \frac{1}{\beta e^2 \partial r^2} + \frac{1}{\partial r^2} \right)$$

$$\times \left( \frac{\partial^2 v}{\partial r^2} - \frac{1}{r} \frac{\partial v}{\partial r} + \frac{v}{r^2} \right) - \left( \frac{1}{\beta e^2 \partial r^2} + \frac{1}{\partial r^2} \right)$$

$$\times \left( \frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial w}{\partial r} + \frac{w}{r^2} \right) - \left( \frac{1}{\beta e^2 \partial r^2} + \frac{1}{\partial r^2} \right)$$

$$\times \left( \frac{\partial w}{\partial r} - \frac{w}{r} \right) - \frac{2}{\beta e^2 r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)$$

$$+ 2 \frac{\beta e^2}{\beta e^2 r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right) - \frac{2}{\beta e^2 r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^3$$

$$- \frac{2}{\beta e^2 r} \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^2$$ \hspace{1cm} (2)

![Figure 1. Flow diagram.](image)
\[
\rho_f \left[ \frac{\partial w}{\partial r} + \frac{w}{r} \frac{\partial v}{\partial z} \right] = \rho_{f*} g \beta_T (T - T_\infty) - g \beta_r (\rho_p - \rho_{f*}) (C - C_{\infty}) - \frac{g}{\beta_m} (\rho_m - \rho_{f*}) (n - n_{\infty}) - \sigma w B_0^2 \sin^2 \theta
\]

\[
+ \frac{1}{\beta c} \frac{\partial^2 w}{\partial r^2} + \left( \frac{\mu}{r} + \frac{1}{\beta c} \right) \frac{\partial v}{\partial r} - \frac{1}{\beta c^3} \left( \frac{\partial^2 w}{\partial r^2} \right)^2 + \frac{\partial^2 w}{\partial r^2} \frac{\partial^2 v}{\partial r^2} - \frac{2}{r} \frac{\partial v}{\partial r} \frac{\partial^2 w}{\partial r^2}
\]

(3)

\[
\frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = \frac{k_f}{(\rho c)_f} \left[ \frac{1}{r} \frac{\partial T}{\partial r} \right] + \frac{\partial^2 T}{\partial r^2} + \frac{1}{T_{\infty}} \left( \frac{\partial^2 T}{\partial r^2} \right)^2
\]

\[
+ \tau \left[ D_B \left( \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + D_T \frac{\partial^2 T}{\partial r^2} \right)^2 \right]
\]

\[
+ \mu \left( \frac{\partial^2 w}{\partial r^2} \right) + \frac{1}{\beta c} \frac{\partial^2 w}{\partial r^2} - \frac{1}{\beta c^3} \frac{\partial^2 v}{\partial r^2}
\]

(4)

\[
\frac{\partial C}{\partial r} + \frac{w}{r} \frac{\partial C}{\partial z} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) + D_T \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
\]

(5)

\[
\frac{\partial n}{\partial r} + \frac{1}{r} \frac{\partial n}{\partial z} + \frac{b W c}{C_w - C_0} \left( \frac{\partial}{\partial r} \left( \frac{n C}{r} \right) + \frac{\partial C}{\partial r} \right) = D_n \left( \frac{1}{r} \frac{\partial n}{\partial r} + \frac{\partial^2 n}{\partial r^2} \right)
\]

(6)

with

\[
u = \nu_0 \frac{w_0}{\nu}, \quad u = 0, \quad T = T_w = T_0 + \frac{a n z}{T}, \quad C = C_w = C_0 + \frac{n n}{n_0 + \frac{c z}{T}} \quad \text{at } r = R,
\]

\[
w \to 0, \quad T = T_\infty = T_0 + \frac{a n z}{T}, \quad C = C_\infty = C_0 + \frac{b z}{T}, \quad n = n_\infty = n_0 + \frac{c z}{T} \quad \text{as } r \to \infty
\]

(7)

where \(u, w\) show the velocity components in \(r\)- and \(z\)-direction, respectively; \(\mu\) is the dynamic viscosity; \(\rho_f\) is the density of nanoparticles; \(\rho_{f*}\) is the ambient density of nanofluid; \(\rho_m\) is the density of microorganisms; \(\beta\) and \(c\) are the Eyring–Powell fluid parameters; \(g\) is the gravitational acceleration; \(\beta_r\) is the concentration expansion coefficient; \(\beta_T\) is the thermal expansion coefficient; \(\rho_p\) is the density of nanoparticles; \(T\) is the temperature; \(T_\infty\) is the ambient temperature; \(T_0\) is the reference temperature; \(T_w\) is the surface temperature; \(C\) is the concentration; \(C_{\infty}\) is the ambient concentration; \(C_0\) is the reference concentration; \(C_w\) is the surface concentration; \(n\) is the concentration of microorganisms; \(n_{\infty}\) is the ambient concentration of microorganisms; \(n_0\) is the reference concentration of microorganisms; \(n_\infty\) is the surface concentration of microorganisms; \(b\) is the chemotaxis constant; \(\gamma\) is the average volume of microorganisms; \(D_B\) is the coefficient of Brownian diffusion; \(D_T\) is the thermophoresis; \(W_c\) is the maximum speed of microorganisms cells; \(\sigma\) is the electrical conductivity; \(R\) is the radius of the cylinder; \(a_1, b_1, c_1, a_2, b_2, \) and \(c_2\) are the dimensionless constants; and \(P\) is the pressure vector.

Considering

\[
\begin{align*}
\eta &= \sqrt{\frac{w_0}{\nu_T}} \left( \frac{R}{r} \right) f(\eta), \quad \eta = \sqrt{\frac{w_0}{\nu_T}} \left( \frac{r^2 - R^2}{2R} \right),
\end{align*}
\]

\[
T = T_0 + a_n z + a_n z 0(\eta), \quad w = w_0 f(\eta),
\]

\[
C = C_0 + b_n z + b_n z 0(\eta) \phi(\eta), \quad n = n_0 + c_n z + c_n z 0(\eta) \chi(\eta)
\]

(8)

One can get

\[
\begin{align*}
&\frac{1}{(1 + 2\gamma^2)(1 + 2\beta^2) f'' + 2\gamma^2(1 + 2\beta^2) f'''} - H_a^2 \sin^2\theta f'' - 2\beta \gamma^2(1 + 2\gamma^2)^2 f'' - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2 - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2
\end{align*}
\]

\[
\begin{align*}
&+ G_0 \theta - G_0 \phi - R_0 \zeta = 0
\end{align*}
\]

(9)

\[
\begin{align*}
&\frac{1}{(1 + 2\gamma^2)(1 + 2\beta^2) f'' + 2\gamma^2(1 + 2\beta^2) f'''} - H_a^2 \sin^2\theta f'' - 2\beta \gamma^2(1 + 2\gamma^2)^2 f'' - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2 - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2
\end{align*}
\]

\[
\begin{align*}
&+ G_0 \theta - G_0 \phi - R_0 \zeta = 0
\end{align*}
\]

(10)

\[
\begin{align*}
&\frac{1}{(1 + 2\gamma^2)(1 + 2\beta^2) f'' + 2\gamma^2(1 + 2\beta^2) f'''} - H_a^2 \sin^2\theta f'' - 2\beta \gamma^2(1 + 2\gamma^2)^2 f'' - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2 - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2
\end{align*}
\]

\[
\begin{align*}
&+ G_0 \theta - G_0 \phi - R_0 \zeta = 0
\end{align*}
\]

(11)

\[
\begin{align*}
&\frac{1}{(1 + 2\gamma^2)(1 + 2\beta^2) f'' + 2\gamma^2(1 + 2\beta^2) f'''} - H_a^2 \sin^2\theta f'' - 2\beta \gamma^2(1 + 2\gamma^2)^2 f'' - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2 - 2\beta \gamma^2(1 + 2\gamma^2) (f'')^2
\end{align*}
\]

\[
\begin{align*}
&+ G_0 \theta - G_0 \phi - R_0 \zeta = 0
\end{align*}
\]

(12)

with

\[
\begin{align*}
\phi(\eta) &= 1, \quad \phi(\eta) = 0, \quad \theta(\eta) = 1 - S_1, \quad \phi(\eta) = 1 - S_2, \\
\chi(\eta) &= 1 - S_1 \quad \text{at } \eta = 0, \quad \phi(\eta) = 0, \\
\delta(\eta) &= 0, \quad \phi(\eta) = 0, \quad \chi(\eta) \to 0 \quad \text{as } \eta \to \infty
\end{align*}
\]

(13)
where $\gamma^* = 1/R(\sqrt{\nu f/\nu_0})$ is the curvature parameter; $\lambda = w_0^2 \zeta^2 / 2c^2 \Omega^2 \beta^3$ and $\beta^* = 1/\mu \beta c$ are the Eyring–Powell fluid material parameters; $Ha = (\sigma \beta_0^2 l / \nu_0 \nu_f)$ is the magnetic parameter; $Pr = (\nu_f (\rho c_p) / \kappa_f)$ is the Prandtl number; $\alpha = g \beta T(\rho_f w_0^2 (T_w - T_0))$ is the thermal Grashof number; $G_c = g \beta_c (\rho_f w_0^2 (\rho_p - \rho_\infty)) (C_w - C_0)$ is the concentration Grashof number; $Rb = (\beta_\lambda (\rho_f w_0^2) (n_w - n_0) (\rho_m - \rho_f))$ is the bioconvection Rayleigh number; $Nt = \tau \Delta T (T_w - T_0) / \nu_0 T_\infty$ is the thermophoresis parameter; $Nb = \tau \Delta B (C_w - C_0) / \nu_0$ is the Brownian motion parameter; $Ec = w_0^2 / \nu_p (T_w - T_0)$ is the Eckert number; $Pe = (b W / D_n)$ is the bioconvection Peclet number; $S_1 = \alpha_2 / \alpha_1$ is the thermal stratification parameter; $S_2 = b_2 / b_1$ is the mass stratification parameter; $S_3 = \alpha_2 / \alpha_1$ is the motile density stratification parameter; $Lb = \nu_f / D_n$ is the bioconvection Lewis number; $Sc = \nu_f / D_n$ is the Schmidt number; and $\Omega = n_\infty / (n_w - n_0)$ is the concentration difference of microorganisms.

### Results and discussions

In this article, we have employed Newton built-in shooting method to progress numerical results for the obtained nonlinear differential system. Furthermore, the salient effect of pertinent parameters on velocity, temperature, and concentration of microorganisms is examined through graphs. In this section, skin friction coefficient ($Cf_z$), gradient of temperature ($Nu_z$), Sherwood number ($Sh_z$), and density number ($Nn_z$) are numerically computed through various interesting parameters.

### Physical quantities

Coefficient of skin friction ($Cf_z$), Nusselt number ($Nu_z$), Sherwood number ($Sh_z$), and density number ($Nn_z$) are expressed as

$$
Cf_z = \left( \frac{\tau_x}{\rho c^2 \nu} \right)_{r=R}, \quad Nu_z = \frac{z q_w}{k (T_w - T_0)}, \quad Sh_z = \frac{2 q_m}{D_B (C_w - C_0)}, \quad Nn_z = \frac{2 q_s}{D_n (n_w - n_0)}
$$

(14)

where $\tau_x$ is the shear stress, $q_w$ is the heat flux, $q_m$ is the mass flux, and $q_s$ is the density flux expressed as

$$
\tau_x = \mu \frac{\partial v}{\partial r} + \frac{1}{\beta c} \left( \frac{\partial w}{\partial r} \right) - \frac{1}{6 \beta c^3} \left( \frac{\partial w}{\partial r} \right)^3, \quad q_w = -\left( k \frac{\partial T}{\partial r} \right)_{r=R}, \quad q_m = -\left( D_B \frac{\partial C}{\partial r} \right)_{r=R}, \quad q_s = -\left( D_n \frac{\partial n}{\partial r} \right)_{r=R}
$$

(15)

Finally, we can write

$$
Cf_z = \left( Re_z \right)^{-0.5} \left( 1 + \beta^* \right) \frac{f'(0)}{f''(0) \Omega(0)^3}, \quad Nu_z Re_t^{-0.5} = \theta'(0), \quad Sh_z Re_t^{-0.5} = -\phi'(0), \quad Nn_z Re_t^{-0.5} = -\xi'(0)
$$

(16)

where $Re_z = (z^2 \nu_0 / \nu_f)$ shows the local Reynolds number.

### Velocity

Characteristics of pertinent parameters like curvature parameter ($\gamma^*$), Eyring–Powell fluid material parameters ($\beta^*$), magnetic or Hartmann number ($Ha$), thermal Grashof number ($G_c$), solutal Grashof number ($G_s$), and bioconvection Rayleigh number ($Rb$) on velocity ($f'(\eta)$) are delineated in Figures 2–7, respectively. Characteristic of $\gamma^*$ on $f'(\eta)$ is portrayed in Figure 2. For larger $\gamma^*$, the radius of the cylinder decreases and consequently fluid contact area with cylinder decreases and as a result velocity decreases. Figure 3 shows the characteristic of fluid parameter ($\beta^*$) on $f'(\eta)$. For rising values of $\beta^*$, viscosity of fluids decreased and as a result $f'(\eta)$ boosts up. Behavior of $Ha$ on velocity is sketched in Figure 4. For higher values of $Ha$, Lorentz force increases which creates more disturbance to the fluid motion and consequently $f'(\eta)$ decreases. Behavior of $G_c$ on $f'(\eta)$ is displayed in Figure 5. Clearly, $f'(\eta)$ is a decreasing function of $G_c$. Figure 6 discusses the effect of $G_s$ on velocity ($f'(\eta)$). As expected, velocity increases when an enhancement occurs in the concentration of Grashof number. Impact of bioconvection Rayleigh number ($Rb$) on $f'(\eta)$ is portrayed in Figure 7. One can find that $f'(\eta)$ decreased with rising values of bioconvection Rayleigh number ($Rb$).

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**Figure 2.** $f'(\eta)$ through $\gamma^*$. 
The impact of $S_1$ on $\theta(\eta)$. For higher values of $S_1$, the $\theta(\eta)$ decreases. Figures 10 and 11, respectively, examine the effect of $Nb$ and $Nt$ on temperature ($\theta(\eta)$). Here, we observed temperature enhancing through Brownian diffusion variable ($Nb$) and thermophoresis parameter ($Nt$).
Figure 12 shows the behavior of curvature parameter on $\theta(\eta)$. Clearly note that $\theta(\eta)$ decreases with increasing $\gamma^*$. Behavior of $\beta^*$ on $\theta(\eta)$ is portrayed in Figure 13. Here, it is noted that $\theta(\eta)$ increases when an increment occurs in $\beta^*$. Characteristic of $Ec$ on $\theta(\eta)$ is highlighted in Figure 14. Here, increasing values of $Ec$ leads to an increment in $\theta(\eta)$.

**Concentration**

Figure 15 is plotted to study the behaviors of $Sc$ on concentration ($\phi(\eta)$). For higher $Sc$, mass diffusivity decreases and thus $\phi(\eta)$ is diminished. Figure 16 is sketched to examine the impact of $S_2$ on concentration ($\phi(\eta)$). One can observe that concentration declines with $S_2$. Figure 17 depicts the effect of Brownian...
movement variable \( (Nb) \) on concentration. This figure manifests that with the increasing value of \( Nb \), the concentration decreases. Figure 18 shows the effect of \( Nt \) on concentration \( (\varphi(\eta)) \). Clearly, \( \varphi(\eta) \) increases with higher estimation of \( Nt \).

**Motile density**

Salient behaviors of \( Pe, Lb, S_3 \), and \( \Omega \) on \( \chi(\eta) \) are discussed in Figures 19–22, respectively. Impact of \( Pe \) on \( \chi(\eta) \) is depicted in Figure 19. As expected, motile density decreased when an increment occurs in the bioconvection Peclet number. Behavior of \( Lb \) on \( \chi(\eta) \) is displayed in Figure 20. It is observed that motile density decreases against bioconvection Lewis number. Figure 21 shows the behavior of \( S_3 \) on \( \chi(\eta) \). Here, one can find that motile density decreases for larger \( S_3 \). Characteristic of \( \Omega \) on \( \chi(\eta) \) is sketched in Figure 22. It is scrutinized that an increase in \( \Omega \) increases the
concentration of microorganisms in the ambient liquid and as a result \( \chi(\eta) \) decreases.

**Engineering quantities**

In this article, the influence of influential parameters on \( Cf, Sh_z, Nu_z, \) and \( Nn_z \) is examined. The behavior of \( Cf_z \) versus involved parameters is displayed in Table 1. Here, skin friction coefficient increases with curvature variable \( \beta^* \), fluid parameter \( \gamma^* \), Hartmann number \( Ha \), thermal Grashof number \( G_t \), and bioconvection Rayleigh number \( Rb \), while decreases for higher concentration of solutal Grashof number \( G_c \).

From Table 2, one can find that \( Nu_z \) diminishes with

![Figure 21. \( \chi(\eta) \) through \( S_3 \).](image1)

![Figure 22. \( \chi(\eta) \) through \( \Omega \).](image2)

**Table 1.** Numerical results of skin friction coefficient.

| \( \gamma^* \) | \( \beta^* \) | \( Ha \) | \( G_t \) | \( G_c \) | \( Rb \) | \( -Cf_z \) |
|----------------|--------------|---------|--------|--------|--------|----------|
| 0.1            | 0.2          | 0.8     | 0.5    | 0.5    | 1.0    | 0.957054 |
| 0.5            |              |         |        |        |        | 1.28163  |
| 1.0            |              |         |        |        |        | 1.66342  |
| 0.2            |              |         |        |        |        | 0.957054 |
| 0.6            |              |         |        |        |        | 1.19265  |
| 1.0            |              |         |        |        |        | 1.42633  |
| 0.2            |              |         |        |        |        | 0.957054 |
| 0.5            |              |         |        |        |        | 1.03085  |
| 0.8            |              |         |        |        |        | 1.10197  |
| 0.4            |              |         |        |        |        | 0.957054 |
| 0.8            |              |         |        |        |        | 0.796156 |
| 1.2            |              |         |        |        |        | 0.63926  |
| 0.4            |              |         |        |        |        | 0.957054 |
| 0.8            |              |         |        |        |        | 1.04636  |
| 1.2            |              |         |        |        |        | 1.13686  |
| 0.1            |              |         |        |        |        | 0.957054 |
| 0.5            |              |         |        |        |        | 1.10842  |
| 1.0            |              |         |        |        |        | 1.30079  |

**Table 2.** Numerical values of Nusselt number.

| \( \gamma^* \) | \( Pr \) | \( Ec \) | \( Nb \) | \( Nt \) | \( S_1 \) | \( -Nu_z \) |
|----------------|--------|--------|--------|--------|--------|----------|
| 0.1            | 0.816768 |
| 0.5            | 0.940968 |
| 1.0            | 1.10267 |
| 1.0            | 0.816768 |
| 1.0            | 0.8668  |
| 1.5            | 0.993042 |
| 0.8            | 0.816768 |
| 0.8            | 0.721198 |
| 1.2            | 0.628704 |
| 0.4            | 0.816768 |
| 0.5            | 0.787432 |
| 0.7            | 0.758133 |
| 0.2            | 0.816768 |
| 0.5            | 0.762758 |
| 0.7            | 0.729464 |
| 0.1            | 0.816768 |
| 0.3            | 0.703044 |
| 0.5            | 0.6499  |

**Table 3.** Numerical values of Sherwood number.

| \( \gamma^* \) | \( Nb \) | \( Nt \) | \( S_2 \) | \( Sc \) | \( -Sh_z \) |
|----------------|--------|--------|--------|--------|----------|
| 0.1            | 0.503581 |
| 0.5            | 0.58214 |
| 0.9            | 0.647421 |
| 0.3            | 0.503581 |
| 0.5            | 0.59675 |
| 0.7            | 0.648399 |
| 0.2            | 0.503581 |
| 0.5            | 0.274481 |
| 0.7            | 0.128945 |
| 0.2            | 0.503581 |
| 0.5            | 0.503581 |
| 0.7            | 0.679454 |
| 0.7            | 0.503581 |
| 1.0            | 0.672272 |
| 1.3            | 0.829945 |
Table 4. Numerical values of density number.

| $\gamma^*$ | $Pe$ | $Lb$ | $S_3$ | $\Omega$ | $-\text{Nn}_2$ |
|------------|------|------|-------|----------|----------------|
| 0.1        | 0    | 0.686044 |     |        |                |
| 0.5        | 0.926125 | 1.52178 |     |        |                |
| 1.0        | 0    |       | 0.686044 | 1.52178 |                |
| 0.5        | 0    |       | 0.686044 | 1.52178 |                |
| 1.0        | 0    |       | 0.686044 | 1.52178 |                |

Conclusion

From this study, the following conclusions can be drawn:

- The temperature field, mass concentration, and motile density decrease with increasing curvature variable, while reverse effect is observed in the case of velocity;
- Velocity has opposite behavior for $\gamma^*$ and $Ha$;
- For larger $\beta^*$, velocity ($f'(\eta)$) decreases;
- $f'(\eta)$ decreases with increasing values of $G_1$, $G_c$, and $Rb$;
- $Nt$ and $Nb$ have similar effects on temperature;
- The temperature field has opposite effect for curvature parameter and fluid material parameter;
- Concentration decreases through $Sc$ and $S_2$;
- For larger $Nt$, the $\phi(\eta)$ increases, whereas opposite effect is observed for $Nb$;
- Motile density decreases through $Lb$, $Pe$, and $\Omega$;
- $Cf_z$ increases with higher estimation of $\gamma^*$ and $Ha$;
- $Nn_z$ boosts up with $\gamma^*$, $Nt$, and $Nb$;
- $Sh_z$ has opposite effect for $Nt$ and $Nb$;
- For higher estimation of $Pe$ and $Lb$, the $Nn_z$ increases.

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