Migration of Protoplanets in Radiative Disks

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ABSTRACT

Context. In isothermal disks the migration of protoplanets is directed inward. For small planetary masses the standard type I migration rates are so fast that this may result in an unrealistic loss of planets into the stars.

Aims. We investigate the planet-disk interaction in non-isothermal disks and analyze the magnitude and direction of migration for an extended range of planet masses.

Methods. We have performed detailed two-dimensional numerical simulations of embedded planets including heating/cooling effects as well as radiative diffusion for realistic opacities.

Results. In radiative disks, small planets with \( M_{\text{planet}} < 50 M_{\text{Earth}} \) do migrate outward with a rate comparable to absolute magnitude of standard type I migration. For larger masses the migration is inward and approaches the isothermal, type II migration rate.

Conclusions. Our findings are particularly important for the first growth phase of planets and ease the problem of too rapid inward type-I migration.

Key words. accretion disks – planet formation – hydrodynamics

1. Introduction

Planets form in disks surrounding young stars. The growing protoplanets undergo an embedded phase where the gravitational interaction with the ambient gaseous disk results in a change of its orbital elements. For protoplanets with masses below about 30 Earth masses the disk is not disturbed too strongly and the interaction can be treated in the linear approximation. Calculations of the total disk torques acting on the planet lead generally for these small masses to a reduction of the semi-major axis, i.e. to an inward migration (Goldreich & Tremaine 1979; Ward 1997; Tanaka et al. 2002; Tanaka & Ward 2004). It soon turned out that the inward drift of this type-I migration is very fast and the planets might be lost before they can grow to larger objects (Korycansky & Pollack 1993). This problem has become more visible after comparing population synthesis models with the characteristics of observed planetary systems (Alibert et al. 2004; Ida & Lin 2008). To avoid this rapid phase of inward migration, alternative scenarios have been sought. In a turbulent disk, migration occurs stochastically with inward and outward phases which slows down the migration (Nelson 2005). Departures from the linear regime at around 10-20 \( M_{\text{Earth}} \) can also lead to reduced inward migration (Masset et al. 2006). However, both processes are not sufficient to solve the problem. The planet trap scenario to halt planetary migration (Masset et al. 2006) requires a positive density gradient which may not be given in general.

To simplify the calculations, nearly all of the analytical and numerical studies devoted to study the planet-disk interaction process have focussed on isothermal disks, where the temperature is a given function of the position in the disk. Early work on non-isothermal disks focussed on high mass embedded planets and did not notice a strong effect on migration (D'Angelo et al. 2003; Klahr & Kley 2006). Using a fully three-dimensional radiative calculations of an embedded small mass planet, Paardekooper & Mellema (2006) have shown in a very important work that migration can be significantly slowed down or even reversed when thermal effects are included. Subsequent analysis indicate that this behaviour is related to a radial entropy gradient in the flow (Baruteau & Masset 2008; Paardekooper & Mellema 2008). Recently, Paardekooper & Papaloizou (2008) have shown that for small planet masses, the combination of radiative and viscous diffusion may allow for long-term unsaturated positive torques and possible outward migration.

In this letter we investigate this possibility in more detail for a whole range of planetary masses, which will allow us to estimate its effect on the long term evolution of the planet. For that purpose we perform two-dimensional numerical hydrodynamical simulations of embedded planets in radiative disks. A method to treat the three-dimensional radiative transfer approximately in these 2D simulations will be outlined in the next section. Our results on the migration rate for various masses (in Section 3) indicate that for masses smaller than about 50 \( M_{\text{Earth}} \) the torques remain unsaturated in the long run and migration is indeed directed outward, while larger planets drift inward. The consequence for the migration process and the overall evolution of planets in disks is discussed.

2. Physical modelling

The protoplanetary disk is treated as a two-dimensional, non-self-gravitating gas that can be described by the Navier-Stokes equations. The embedded planet is modelled as a point mass that orbits the central star on fixed, circular orbit. For the planetary potential we use a smoothing length of \( e = 0.6H \) where \( H \) is the vertical scale height of the disk. To calculate the gravitational torques acting on the planet we apply a tapering function to exclude the inner parts of the Hill sphere of the planet, where the transition lies at 0.8 Hill radii (see Crida et al. 2008, Fig. 2).
2.1. Energy Equation

In the present setup we utilise fully radiative models with an improved thermodynamic treatment using the thermal energy equation in the following form

$$\frac{\partial \Sigma_c T}{\partial t} + \nabla \cdot (\Sigma_c T \mathbf{u}) = -p \nabla \cdot \mathbf{u} + D - Q - 2H\nabla \cdot \mathbf{F}.$$  \hspace{1cm} (1)

Here $\mathbf{u} = (u_r, u_\phi)$ is the two-dimensional velocity, $\Sigma$ the density, $p$ the pressure, $T$ the (mid-plane) temperature of the disk, and $c_s$ is the specific heat at constant volume. On the right hand side, the first term describes compressional heating, $D$ the (vertically averaged) dissipation function, $Q$ the local radiative cooling from the (vertical) surfaces of the disk, and $\mathbf{F}$ denotes the two-dimensional radiative flux in the $(r, \phi)$-plane. For all models, $H$ is calculated from the sound-speed as $H(r) = c_s / \Omega_K(r)$, where $\Omega_K$ is the Keplerian angular velocity, and $c_s = \sqrt{\gamma p/\Sigma}$, $\gamma = 1.43$ being the adiabatic index. The 2D-pressure is given by $p = \rho \gamma^n \Sigma / \mu$ with the mean molecular weight $\mu = 2.35$.

To calculate the radiative losses $Q$ (from the two sides of the disk) we follow D’Angelo et al. (2003) and Kley et al. (2005).

$$Q = 2\sigma_R T^4 \tau_{eff},$$

where $\sigma_R$ is the Stefan-Boltzmann constant and $\tau_{eff}$ is an estimate for the effective temperature (Hubeny 1990), given by:

$$\tau_{eff} = T^4 / \tau = 3 / 8 (\tau) + \sqrt{\tau} / 4 + 1 / 4\tau.$$  \hspace{1cm} (2)

For our two-dimensional disk we approximate the mean vertical optical depth by $\tau = (1/2)\Sigma$, where for the Rosseland mean opacity $\kappa$ the analytical formulæ by Lin & Papaloizou (1985) are adopted. The radiative transport in the plane of the disk is treated in the flux-limited diffusion approximation where the flux is given by:

$$F = -\frac{\lambda c}{\rho k} 4aT^3 \nabla T.$$  \hspace{1cm} (3)

Here $c$ is the speed of light, $a$ the radiation constant, $\rho = \Sigma/(2H)$ the mid-plane density, and $\lambda$ the flux-limiter (see Kley 1989).

In the following we present results of numerical simulations using various sub-parts of the energy equation, Eq. (1). If only the first term on the right hand side is used, the model is called adiabatic; when only the last term is omitted, it is a model with local heating and cooling; and the usage of the full energy equation is termed fully radiative. To make contact with previous results we will also use isothermal models where a pre-described radial temperature distribution is held fixed and no energy equation is solved at all. Please note that in the full version, the radiative treatment (in Eq. (1)) incorporates full 3D effects of the radiative transport in that the vertical part ($z$-direction) is taken care of by the local cooling term, $Q$, and the horizontal part through $F$.

2.2. Reference Model

The two-dimensional $(r - \phi)$ computational domain consists of a complete ring of the protoplanetary disk centred on the star, extending from $r_{\text{min}} = 0.4$ to $r_{\text{max}} = 2.5$ in units of $r_0 = d_{\text{Jup}} = 5.2$AU. The mass of the central solar mass, and the total disk mass inside $[r_{\text{min}}, r_{\text{max}}]$ is $M_{\text{disk}} = 0.01 M_\oplus$. For the present study we have kept the kinematic viscosity constant with $v = 10^{15}$cm$^2$/s, a value which relates to an equivalent $\alpha$ at $r_0$ of 0.004 for a disk aspect ratio of $H/r = 0.05$, where $v = \alpha H^2 \Omega_K$.

To construct a joint reference model for the different types of approximations to the energy equation (isothermal, adiabatic, etc.) we construct a fully radiative model with no embedded planet, using the above layout.

This model is obtained from an approximate initial state (with $H/r = 0.05$) that is then relaxed to its equilibrium by performing a time evolution of the disk solving the full Navier-Stokes equations including the energy. The initial disk stratification at the start of this process is given by $\Sigma(r) \propto r^{-1/2}$, $T(r) \propto r^{-3/2}$, and pure Keplerian rotation ($u_r = 0, u_\phi = (GM_\star/r)^{1/2}$). The evolution towards the equilibrium takes about 150 orbits and the resulting density and temperature distribution is displayed in Fig. 1. The density has a $\Sigma \propto r^{-1/2}$ profile which follows for constant $v$ directly from the angular momentum equation. The temperature has a $T \propto r^{-1.6}$ profile which follows from $Q = D$ for the used opacity. These relations are superimposed in Fig. 1 for small radii (higher temperatures) the opacity law has different temperature dependence and the slope $T(r)$ changes. The relative scale height for this radiative model is not constant but falls off with radius. At $r = 1$ we find $H/r \approx 0.045$ and at the outer radius $H/r \approx 0.03$.

![Fig. 1. Density and temperature vs. radius for the relaxed reference model. For illustrative purpose, simple comparison power laws have been added.](image)

2.3. Numerics

We work in a rotating reference system, rotating with the orbital frequency of the planet. For our standard cases we use an equidistant grid in $r$ and $\phi$ with a resolution of $128 \times 384$. In an effort to ensure a uniform environment for all models and minimize disturbances (wave reflections) from the radial boundaries we impose at $r_{\text{min}}$ and $r_{\text{max}}$ damping boundary conditions where both velocity components are relaxed towards their initial state on a timescale of the order of the local orbital period. As the initial radial velocity is vanishing, this damping routine ensures that no mass flows through the boundaries such that the total disk mass remains constant. The angular velocity is relaxed towards the Keplerian values. For the density and temperature we apply closed radial boundary conditions. In the azimuthal direction, periodic boundary conditions are imposed for all variables.

The numerical details of the used finite volume code (RH2D) relevant for these disk simulations have been described in Kley (1999), where we have additionally implemented the FARGO method (Masset 2000) that speeds up the numerical computation of differentially rotating flows. The energy equation Eq. (1) is solved explicit-implicitly applying operator-splitting. The heating and cooling term $D - Q$ is treated as one sub-step in this procedure (see Günther et al. 2004), and the additional radiative
diffusion part in the energy equation is solved through an implicit method to avoid possible time step limitations.

3. Results

3.1. A model with 20 \( M_{\text{Earth}} \)

To illustrate the influence of the 4 different formulations of the energy equation on the torque, we present a model with an embedded 20 \( M_{\text{Earth}} \) planet. For this particular mass, a recent 3D-study to analyze the effects of an isothermal disk on all orbital elements, indicated negative rates for migration as well as for eccentricity and inclination changes (Cresswell et al. 2007). The starting configuration (at \( t = 0 \)) for the 4 cases has been the equilibrated reference model corresponding to the fully radiative case as described above (see Fig. 1). In Fig. 2 we display the time evolution of the torque acting on the planet, for the different formulations of the energy equation. As expected, the isothermal condition leads after about 50 orbital periods to a constant negative torque implying inward migration. The adiabatic model has an initial phase of positive torques, before settling to negative values of about 40% of the isothermal case. This behaviour has been suggested by Baruteau & Masset (2008) who argue that in the adiabatic case the torques will be unsaturated and positive during the onset phase, while long-term calculations should converge to negative values.

In contrast, both radiative models settle to constant positive values. Here, the fully radiative case yields a 25% smaller value due to the included heat diffusion in the disk plane. In all cases, we have continued the models with double and quadruple resolution. In general, we find (for this planet mass) similar results and a convergence of the torques at about double resolution. Only the model with pure heating/cooling (i.e. no radiative diffusion) apparently requires much higher resolution for convergence, a fact we attribute to the highly local character of this type of energy equation. The addition of radiative diffusion in the disk plane eases the numerical requirements and makes at the same time the simulations physically more realistic.

3.2. Variation of the planet mass

To estimate the influence of the planet mass we performed a sequence of models with masses varying from \( q = 10^{-5} \) (3\( M_{\text{Earth}} \)) up to \( q = 10^{-3} \) (\( M_{\text{Jupiter}} \)). The resulting equilibrium specific torques are displayed in Fig. 4 for the fully radiative and isothermal case. Clearly, for small planetary masses up to about 50\( M_{\text{Earth}} \), the torques exerted on the planet is positive for the radiative model and turns negative above this mass value (see bottom panel). On the contrary, for the isothermal models the torques are negative throughout and follow the expected well known results. From the superimposed dotted line, it is clear that the specific torque is proportional to \( q \), as predicted by linear theory (eg. Tanaka et al. 2002).

For large planets (\( q \approx 2.5 \times 10^{-4} \)), there is little difference between radiative and isothermal disks, and the torque is well approximated by the model given by Crida & Morbidelli (2007): the dot-dashed curve in the top panel comes from their Eq. (15), with \( v_r \) replaced by \( \frac{2}{3} \). This model refines the type II migration rate for planets whose gap is not completely empty; therefore, it applies here for \( q \gtrsim 3 \times 10^{-4} \) (see the gap profiles in Fig. 5). We find that this model, and type II migration in general, is also valid in radiative disks.

The two-dimensional density distribution for the two cases (isothermal and fully radiative) is displayed in Fig. 4 for a planetary mass of \( q = 10^{-4} \) (33\( M_{\text{Earth}} \)) which has the maximum positive torque. In the radiative model the higher temperature results in slightly larger opening angles of the spiral arms with a re-
Two-dimensional grey-scale plot of the density (\(\propto \Sigma^{1/4}\)) scaling between 150 and 350 g/cm\(^2\)) in the vicinity of the planet for the isothermal model (left) and the fully radiative model (right). The planet mass in this case refers to \(10^{-4} M_{\oplus}\) or \(33 M_{\oplus}\). For these plots we have used double resolution models (256 \(\times\) 768). 

A comparison of the radial density stratification of the isothermal and radiative cases (Fig. 5) indicates shallower gaps for the latter, in particular the region inside of the planet (\(r < 1\)) is less cleared. Larger planet masses lead to gap opening with similar gap depths for the isothermal and radiative case. For the largest planet mass the gap seems to be wider in the isothermal case, possibly due to lower temperatures, but boundary effects (at \(r_{\text{min}}\) and \(r_{\text{max}}\)) begin to become visible. For larger planet masses a radially increased domain is clearly required.

4. Summary

We have performed an investigation of the migration of planets in disks using two-dimensional numerical simulation including heating/cooling effects as well as radiative diffusion.

Using different formulations of the energy equation, we first show that for a planet mass of \(20 M_{\oplus}\), migration is directed inwards in the isothermal and adiabatic situation, while inclusion of radiative effects leads to an outward migration of the planet. This finding supports the torque reversal mechanism in radiative disks due to corotation effects as suggested by [Baruteau & Masset (2008)]. A detailed parameter study for planetary masses in the range between \(10^{-5}\) and \(10^{-3} M_{\oplus}\) shows that the effect is limited to planets in the low mass regime, \(M_p \leq 50 M_{\oplus}\) where corotation effects are indeed important. Larger mass planets open up gaps in the disk and the migration rate becomes similar to the isothermal case.

Our findings are particularly important for the first growth phase of planets and may ease the problem to too rapid inward type I migration. Depending on the mass accretion rate onto the planet the growing planetary embryos can spend an extended time span in an outward migration phase and avoid loss into the star. However, close-in planets exist for a range of planetary masses, according to the observations. Thus, a significant and long outward migration phase may create new difficulties. Whether this problem really exists can only be answered by following the actual long-term migration of planets through the disk including its mass growth.

Constructing the necessary migration histories of planetary cores, to be used in population synthesis models, requires suitable scaling laws for the migration process as a function of disk parameter (\(\Sigma(r), T(r)\)) for realistic accretion disks with net mass flow. The present study can be used as a starting point for these larger parameter studies. The inclusion of three-dimensional effects and additional physics (MHD, self-gravity, mass accretion) will make the models even more realistic in the future.

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Fig. 4. Two-dimensional grey-scale plot of the density (\(\propto \Sigma^{1/4}\)) scaling between 150 and 350 g/cm\(^2\)) in the vicinity of the planet for the isothermal model (left) and the fully radiative model (right). The planet mass in this case refers to \(10^{-4} M_{\oplus}\) or \(33 M_{\oplus}\). For these plots we have used double resolution models (256 \(\times\) 768).

Fig. 5. Azimuthally averaged density for isothermal and radiative cases for different planet masses.
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