Fast-time and slow-time processing of the pseudorandom amplitude-phase-shift keyed signals

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Abstract The two-dimensional raw data structure is used for modern pulse-Doppler radars. Fast-time and slow-time processing of radar return signals is performed. The matched filter compresses each received pulse in fast time. The FFT-based spectral processing of the compressed pulses is then performed in slow time. The two-dimensional structure of raw data has specific features in radars with the transmission and reception of pseudorandom amplitude-phase-shift keyed (APSK) signals to a common aerial. It is formed when the coherent processing interval of the APSK signal is divided into subintervals. The article describes the fast-time and slow-time processing of the APSK signal subintervals. The structure of the signal in the subintervals is also analyzed. The choice of the subinterval duration is discussed. The possible energy losses during the processing of the reflected signals are estimated. The results of the processing modeling of the additive sum of APSK signals with different Doppler frequencies are presented.

1 Introduction

Radar systems use various signal processing techniques to extract useful information from raw radar returns. According to one of them, a target is detected when the return signal is received, processed at the coherent processing interval (CPI), and the processing result exceeds the detection threshold. The delay of this signal determines the target’s range, and its Doppler frequency determines the target’s speed. They are unknown. Therefore, signals are processed by a multi-channel delay and Doppler frequency shift correlation device.

Radar signals are often processed using digital technology. The complex envelope of the received raw signal is allocated. It is sampled at a rate determined by the signal bandwidth. The sampling interval $t_b$ is usually associated with the range resolution sample. Samples of the complex envelope obtained at the CPI can be represented by the matrices with fast and slow-time samples. [1, 2]

Pulse-Doppler radars usually transmit multiple periodic pulses. The medium pulse repetition frequency (PRF) is used for simultaneously measure the target's range and speed. The pulse repetition interval is greater than the maximum processed delay of the reflected signal. The reflected signal is processed with a Doppler frequency lower than the pulse repetition frequency. The radar receives the return signals in the pauses between transmission pulses. Radar return samples are inserted in the two-dimensional matrix. The data recorded at each reception interval between the pulse transmissions forms the rows of the matrix. The number of rows in the data matrix is equal to the number of pulse repetition intervals per CPI. Sample rate of the received signal determines a fast time. Pulse repetition interval determines a slow time. Signal processing is performed in two dimensions. First, each pulse is compressed on the matched filter (or correlator) in the fast time. Then the compressed pulses are processed in the slow time as Doppler filtering for each range bin.
When a radar transmits and receives signals with pseudorandom amplitude and phase-shift keying (APSK) on common aerial [3, 4], the question arises how to form a data matrix. The probing signal of such radar is a pseudorandom train of pulses with random duration. The intervals between pulse transmissions are random and significantly shorter than the instrumental radar range. The probing signal pulses have different phase-shift keying. The pulses of reflected signals with different delays partially overlap in time. The receiver is blanked during the pulses transmission of the APSK signal. Some of the return pulses are lost. Reflected signals with different delays lose different pulses. Therefore, the data matrix rows are not associated with the reception intervals between APSK signal pulses. Filters that are matched with individual pulses of the APSK signal are not used for its processing.

The simplest way is to implement processing of reflected APSK signals on correlators. Each correlator accumulates the product of the blanked return signal and the reference APSK signal with delays $\tau_n = m \cdot t_b$, $m=1,2,3,...$ and the Doppler frequency $F_v = v T$, $v=0,\pm1,\pm2,...$ . In this case, the matrix data of the received signal consists of a single string that is fed to all correlators. The length of the data matrix string is equal to the number of APSK signal samples per CPI. Correlators process the signal in fast time at a rate equal to the width of the APSK signal spectrum.

Another method of coherent processing of the received signal is based on a fast Fourier transform (FFT) over the product of samples of the received signal and the reference APSK signal with the $m$-th time shift. In this case, the FFT dimension will be equal to the signal length of several tens or hundreds of thousands. The FFT results determine the spectral processing of the reflected signal in a range equal to the width of its spectrum. This is significantly more than the instrumental range of Doppler frequencies.

If the CPI of duration $T$ is divided into subintervals with a duration $T_S = 1/F_{\text{max}}$, where $F_{\text{max}}$ - maximum Doppler frequency, samples of the received APSK signal can be stored in a two-dimensional matrix, like data matrix in a pulse-Doppler radar. The signal samples on the subintervals form rows of the data matrix. During coherent processing, they are compressed in fast time. Then Doppler filtering of compressed subintervals of the same range bin is performed in slow time based on the FFT of dimension $N/N_8$.

We should take into account the features of the APSK signal data matrix. The subintervals contain a different number of received pulses of the reflected APSK signal due to their pseudorandom duration and pseudorandom interval between transmissions. Therefore, subinterval compression is performed on correlators.

The ambiguity function (AF) of the APSK signal is thumbtack shaped and has uniform sidelobe level in time delay and Doppler frequency shift plane: $\tau \in [-T/2,-t_b] \cup [t_b,T/2]$, $F_v \in [-1/2t_b, 1/2t_b]$, $t_b$ is the duration of the coded bit. The signals do not have ambiguity of target detection by delay and Doppler frequency. The instrument delay and Doppler frequency ranges can be any of the range $\tau_{\text{max}} < T/2$ and $|F_{\text{max}}| < 1/2t_b$. Therefore, the value of $\tau_{\text{max}}$ can be less than or greater than $T_S$.

This article describes fast-time and slow-time processing of the pseudorandom APSK signals.

2. Description of the APSK signal and its processing

The complex envelope $u(t)$ of the APSK signal of duration $T$ is given by

$$u(t) = \frac{1}{\sqrt{t_b}} \sum_{n=0}^{N-1} w_n \cdot \text{rect} \left[ \frac{t - nt_b}{t_b} \right] \quad 0 \leq t < T,$$

where $w_n \in \{0, \pm1\}$, $n=0..N-1$, is the amplitude-phase code, associated with $u(t)$; $t_b$ is the duration of the coded bit.

Usually $w_{i+kx} = x_i z_{i+kx}$, $x_i \in \{1,0\}$, $z_i \in \{\pm1\}$, $i=0..Nx-1, k=0..kx-1, N=Nx \cdot kx$. The duration of the signal can be defined as $T=N \cdot t_b$, or $T=Nx \cdot tx$, $tx=kx \cdot t_b$. The code $z_i$ is such that the APSK signal phase changes from pulse to pulse.

Target signal detection is based on cross-correlation function calculation.
where \( s(t) \) is the received signal;

\[
u_{\text{blank}}(t) = \sum_{n=0}^{N-1} (1 - |w_n|) \cdot \text{rect} \left( \frac{t - n \cdot T_b}{T_b} \right)
\]

is the receiver blanking signal during the transmission of the APSK probing signal pulses;

the delay \( \tau_m = m \cdot T_b \), \( m=1,2,3,.. \), \( m_{\text{max}} \) and the Doppler frequency \( F_i = v/T \), \( v=\pm 1, \pm 2,.., \pm v_{\text{max}} ; \)

* - the complex conjugation sign.

\[
R_{m,n} = \int_{0}^{T} s(t) u_{\text{blank}}(t) u^*(t - \tau_m) \exp(-j2\pi F_i t) \, dt
\]

(2)

The values of the signal \( s(t) \) sampled with an interval \( T_b \), form a data stream, \( s_n=s(n \cdot T_b) \), \( n=0..N-1 \), for their discrete processing. This data stream is collected at a rate equal to the signal bandwidth by the Nyquist criterion. This dimension is referred to fast time. Multichannel correlation receiver compresses the data stream, \( s_n \), at the CPI.

\[
R_{m,n} = \sum_{n=0}^{N-1} s_{n} (1 - |w_n|) w_{n-m} \exp \left( -j \frac{2\pi}{N} v n \right)
\]

(3)

Processing according to expressions (2) or (3) will be called classical.

The blanking signal \( u_{\text{blank}}(t) \) makes differences in the reference signals of the range cells of the multi-channel receiver. Therefore, a correlator is preferable to a matched filter for APSK signal processing.

The correlation receiver contains \( 2m_{\text{max}} v_{\text{max}} \) channels.

Typically, the delay operating range is much less than the CPI, and the Doppler frequency operating range is much less than the APSK signal spectrum width. The APSK signal provides an unambiguous measurement of the range and speed of the target in the range up to \( D_{\text{max}}=Tc/2 \) and \( V_{\text{max}}=\lambda/(4T_b) \), \( c \) – the speed of light, \( \lambda \) – the wavelength. For example, the APSK signal with a duration of 10msec and a spectrum width of 10 MHz provides \( D_{\text{max}}=1500 \) km and \( V_{\text{max}}=75000 \) m/s in radar with \( \lambda=3 \) cm. The signal length is \( N=100000 \). This signal provides a range resolution cell of 15 m and a Doppler resolution cell of 100 Hz.

Let’s assume that it is necessary to detect a moving pinpoint target. The target range can be up to 15 km. The target speed can be up to \( V_S=9.6 \) m/s. To do this, a multi-channel receiver of the radar with a 3 cm wavelength must contain \( m_{\text{max}}=1000 \) and \( v_{\text{max}}=64 \) channels. But \( m_{\text{max}}<<N \) and \( v_{\text{max}}<<N \). The maximum speed \( V_S \) of the target determines the maximum value of the Doppler frequency \( F_S=2V_S/\lambda \) of the signal reflected from it.

The phase change of the return signal with the maximum Doppler frequency shift will not exceed \( \pi \) for a time interval equal to \( T_S=1/2F_S \). Then the CPI with duration \( T \) consists of \( K_S=T/T_S \) subintervals of the duration \( T_S=N \tau_{b} \), \( N \) – the number of signal samples per subinterval.

Let \( k=0..K_S-1 \) be the number of the subinterval on the CPI, and \( i \) be the number of the signal sample within the subinterval. After replacing \( n=k \cdot N_S+i \), the expression (3) can be replaced with the expression

\[
R_{m,v} = \sum_{k=0}^{K_S-1} \exp \left[ -j \frac{2\pi}{K_S} v k \right] \sum_{i=0}^{N_S-1} s_{kN_S+i} (1 - |w_{kN_S+i}|) w_{kN_S+i-m}^* \exp \left[ -j \frac{2\pi}{N_S} v i \right]
\]

(4)

The internal sum of the expression (4) describes the fast-time processing of signal subintervals.

\[
r_{m,k} = \sum_{i=0}^{N_S-1} s_{kN_S+i} (1 - |w_{kN_S+i}|) w_{kN_S+i-m}^* \exp \left[ -j \frac{2\pi}{N_S} \frac{v}{K_S} i \right]
\]

(5)
Then expression (6) describes slow-time processing of the APSK signal. It consists of spectral processing of compressed signal subintervals.

\[ R_{m,v} = \sum_{k=0}^{K_S-1} r_{m,k}^{v} \exp \left[ -j \frac{2\pi}{K_S} v k \right] \]  

(6)

If we ignore the linear phase progression of the reflected signal on the subinterval duration, then the expression (5) becomes simpler

\[ r_{m,k} = \sum_{j=0}^{N_S-1} s_{kN_S+i} \left( 1 - |w_{kN_S+i}| \right) w_{kN_S+i-m}^{*} \]  

(7)

The \( r_{m,k} = r_{m,k}^{0} \) value is used instead of \( r_{m,k}^{v} \) for any \( m \) in expression (6).

Expressions (5) and (6) describe a two-stage implementation of classical processing of received APSK signals. Processing at each stage is also performed by \( 2 \text{m}_{\text{max}} \text{v}_{\text{max}} \) correlators. The amount of calculations for two-stage classical processing is the same as for one-stage classical processing.

Expressions (7) and (6) describe coherent processing of the APSK signal with pre-compression of subintervals. We will call it fast-time and slow-time processing.

The subinterval compression procedure is performed by a set of \( m_{\text{max}} \) correlators, each of which is set to a fixed delay and zero Doppler frequency. As a result, the number of correlators at this stage is reduced by a factor of \( K_S \) compared to the classical APSK signal processing.

Spectral processing can be implemented by calculating the fast Fourier transform (FFT) in each range channel. The functional diagram of the subinterval processing of the APSK signal is shown in Figure 1.
Simplifying the subinterval compression procedure leads to losses during coherent processing of the APSK signal. The Doppler frequency of the detected signal determines the amount of these losses. Let's estimate their value.

Compare the change in the value of \(|R_{m,v}|\) when the compression of subintervals according to expression (5) is replaced by compression according to expression (7). Let the samples of the complex envelope of the processed signal be described by the expression \(s_{n}=w_{n,m}-\exp[2\pi v s_{n}/N]\). The signal associated with \(s_{n}\) has a single amplitude, delay \(\tau_{n}=m s_{n} t_{b}\), Doppler frequency \(F=\nu s / T\), and zero initial phase. This data stream describes the signal of a point target.

The output response in the \((m,s,v)\) processing channel with pre-compression of the received signal subintervals is described by the expression

\[
R_{m,v}|_{m=ms,v=vs} = \sum_{k=0}^{K_{s}-1} \sum_{i=0}^{N_{s}-1} |w_{kN_{s}+i-m}| \left(1 - |w_{kN_{s}+i}| \right) \exp \left[ j \frac{2\pi v}{N} i \right]
\]

The output response in the \((m,s,v)\) channel of classical processing is described by

\[
R_{m,v}|_{m=ms,v=vs} = \sum_{k=0}^{K_{s}-1} \sum_{i=0}^{N_{s}-1} |w_{kN_{s}+i-m}| \left(1 - |w_{kN_{s}+i}| \right)
\]

The delay-averaged response \(|R_{m,v}|\) at \(m=ms\), \(v=vs\) takes the following values:

– the processing with pre-compression of the received signal subintervals

\[
\overline{R}(v)|_{v=vs} = (1-C)CN \left| \sin \left( \frac{\pi v}{K_{s}} \right) \right| \left| \frac{\pi v}{K_{s}} \right|,
\]

– the classical processing

\[
\overline{R}(v)|_{v=vs} = (1-C)CN ,
\]

where the parameter \(C=\sum_{n=0}^{N_{s}-1} |w_{n}| / N\) is the probability of the phase coded pulse transmission of the APSK signal.

Neglecting the Doppler frequency of the processed signal on the subinterval of the \(T_s\) duration has two consequences.

A comparison of expressions (10) and (11) shows that the response value decreases according to the function \(|\sin(\pi v / K_{s})|/|\pi v / K_{s}|\). The loss from neglecting the Doppler frequency of the processed signal on the subinterval of \(T_s\) duration does not exceed 4 dB in the operating range of Doppler frequencies \(|F|<K_{s}/2T\). Signals with a Doppler frequency \(F>2K_{s}/T\) will respond in frequency channels with \(v=0..K_{s}-1\).

3. Simulation results

Let's look at an example of fast-time and slow-time processing of the APSK signal.

Figure 2 shows complex envelope fragments of \(4T_s\) duration of the following signals:

1) the probing signal \(u(t)\) with \(N=32000\), \(C=1/5\), \(t_x=4 t_b\);

2) the real component of the complex envelope of two reflected APSK signals \(s_{1}(t)\) and \(s_{2}(t)\) with unit amplitudes, delays \(r_{1}=100 t_{b}\) and \(r_{2}=600 t_{b}\), Doppler frequencies \(F_{1}=0\) and \(F_{2}=31/T\), and initial phases \((-\pi / 4)\) and \(2\pi / 3\), respectively;

3) the receiver blanking signal \(u_{blank}(t)\) is the inverse of the envelope of the probing signal.
4) the real component of the complex envelopes of signals \( s_1(t) \) and \( s_2(t) \) after blanking in the receiving path by the \( u_{blank}(t) \) signal.

![Diagram](image)

**Figure 2.** Example of forming a data matrix for two APSK signals with different delays.

Figure 2 also shows the four subintervals of the signals \([s_1(t) \cdot u_{blank}(t)], [s_2(t) \cdot u_{blank}(t)]\) and the reference signal of the \( m_1=100 \) and \( m_2=600 \) range channels. These subintervals form rows of data matrices for processing the APSK signal. The values of the elements in the matrix row determine the
signal at a fast time, \(i=0..N_S-1\). The matrix columns define the signal at a slow time, \(k=0..K_S-1\). Sample streams of the reference APSK signal with different time shifts form reference data matrices.

The APSK signal of length \(N=32000\) consists of \(K_S=64\) subintervals. The length of the subinterval is \(N_S=500\) in the example. Pseudorandom amplitude and phase-shift keying of the probing signal at the CPI allows detecting reflected signals with a delay exceeding the subinterval duration.

Let's perform a simulation of processing the additive sum of signals \(s_1(t), s_2(t)\) and noise. The signal \(\eta(t)\) is the noise with the normal distribution law and power \(P_0=-40 \text{ dB}\) in the signal band. The selected noise level after processing will be comparable to the level of interference from the side lobes of the signal ambiguity function. So it can be ignored during processing.

Figure 3 shows quadrature components of the subinterval compression results of the APSK signal. Rows \(m=100\) and \(600\) of the matrix \(r_{mk}\) contain correlation peaks. Their amplitudes change according to the value of the Doppler frequency of signals \(s_1\) and \(s_2\), respectively. Other lines contain data proportional to the side lobes of the correlation function of the APSK signal subintervals.

Figure 4 shows two-dimensional graphs of the modulus of the \(R_{m,v}\) function normalized to its maximum value \(R_{\text{max}}\). The amplitudes of the signals are the same. Therefore, the responses after classical processing are the same. However, the Doppler frequency of the second signal is \((K_S/2-1)/T, \nu_2=31\). Therefore, its response is reduced by 4\,\text{dB} after fast and slow time processing.
If the Doppler frequency shift of the return signal is greater than \((K_S/2T)\), then the response will be in the frequency channel with a multiple of \((K_S/T)\). Its value will be proportional to the value of the \(\sin(x)/x\) function. For example, if the Doppler shift of the frequency of the reflected signal increases to \((N_S + K_S/2 - 1)/T\), \(v_2=95\), the response appears in the 31st frequency channel after fast and slow time processing (figure 5.a). Its value is 13 dB less than the response value after classical processing (figure 5.b). Thus, a target with a speed outside the analyzed range can create false peaks. As a result, it will be taken for a weak low-speed target. To avoid this, you should increase the operating range of Doppler frequencies, that is, increase \(K_S\) and decrease \(N_S\).

![Figure 5](image.png)

**Figure 5.** Results of processing the sum of signals \(s_1(t)\) and \(s_2(t)\) with Doppler shifts \(v_1=0\) and \(v_2=31\), respectively a) fast and slow time processing, b) classical processing.

When the law of amplitude manipulation does not change from one subinterval of duration \(T_s\) to another, the APSK signal has a periodic envelope. There is an ambiguity in the measurement of the Doppler frequency.

4. Conclusion
The article describes the fast-time and slow-time processing of the APSK signal subintervals. The subintervals of the APSK signal have a different structure, as does the entire signal. The duration of the subinterval is inversely proportional to the operating range of the Doppler frequencies. The energy losses during the processing of the reflected signals depend on its Doppler frequency. For signals with a Doppler frequency from the operating range, the energy loss does not exceed 4 dB. Signals with a Doppler frequency exceeding the operating range are displayed in the operating range with an attenuation of at least 13 dB. The simulation results confirm the theoretical results.

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