An Accelerated Expansion Model in the Absence of the Cosmological Constant

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Summary

Based on some observations, the apparent energy, associated with gravity, of vacuums is defined, with that of normal vacuums to be zero and that of the vacuums losing some energy to be negative. An important application of the energy is its contribution to Einstein’s equation. A cosmological model, accounting for recent observations of the accelerated expansion of the universe, in the absence of the cosmological constant, can be well constructed. In a certain case, the expansion of the universe would be decelerated at its early epoch and accelerated at its late epoch. The curvature of the universe would depend on the ratio of matter energy to total energy. The missing mass problem does no longer exist in this model. Most negative apparent energy vacuums might be contained in voids, then the spacetime of galaxy clusters or that of the solar system would not be significantly affected by this kind of energy.

PACS number: 98.80.Bp, 98.80.Dr, 98.80.Es, 98.80.Ft

Recent observations showed that the expansion of the universe is accelerated rather than decelerated [1–3]. An economic approach to this phenomenon is to adopt the cosmological constant
which is often referred to the vacuum energy produced by the phase transitions the universe undergoes as it cools. However, there are some reasons against this scenario [4]. For example, the amount of vacuum energy produced by all the phase transitions can be about $10^{420}$ times greater than the density of all the matter in the universe [5]; in particular, the constant corresponds to a universal energy density but its influence on the nearby spacetime has never been observed. These facts suggest that vacuum energy acts as something like potential energy and the apparent energy associated with gravity, of the vacuums in the nearby spacetime might be zero.

Recently, the Casimir force was detected in laboratory [6]. It is reasonable that a work done by the force would extract more or less energy from vacuums. We call a vacuum not losing any energy a normal vacuum and that losing some a deficit vacuum. According to the above comprehension, we define the apparent energy associated with gravity, of normal vacuums to be zero and that of deficit vacuums to be negative. The energy is assumed to contribute to Einstein’s equation the way matter energy does.

Now we consider a cosmological model of the Robertson-Walker metric following Einstein’s equation and the conservation equation of the energy-momentum tensor. The difference is that, we take

$$\rho = \rho_m + \rho_v$$

(1)

and

$$p = p_m + p_v$$

(2)

with that $\rho_v$ can be negative, where $m$ denotes matter and $v$ represents vacuums.

For the Robertson-Walker metric, Einstein’s equation gives [7]

$$3 \dddot{R} = -4\pi G(\rho + 3p)R,$$

(3)

$$R \dddot{R} + 2 \dddot{R} + 2k = 4\pi G(\rho - p)R^2,$$

(4)

and the conservation equation of the energy-momentum tensor yields

$$\dot{p}R^3 = \frac{d}{dt}[R^3(\rho + p)],$$

(5)

where $\dot{R} = dR/dt$. From (3) and (4) one can obtain

$$\dddot{R} + k = \frac{8\pi G}{3} \rho R^2$$

(6)
\[ 2R \dddot{R} + \ddot{R}^2 + k = -8\pi G \rho R^2. \]  

Let us define

\[ H \equiv \frac{\dot{R}}{R}, \]  

\[ q \equiv -\frac{R \dddot{R}}{\ddot{R}}, \]  

\[ \rho_c \equiv \frac{3H^2}{8\pi G}, \]  

and

\[ \Omega \equiv \frac{\rho}{\rho_c}. \]

Then equations (6) and (7) can be written as

\[ \frac{k}{R^2} = H^2(\Omega - 1) \]  

and

\[ \frac{k}{R^2} = H^2(2q - 1 - \frac{3p}{\rho} \Omega), \]

respectively.

The observation of \( q < 0 \) suggests that, at least at the present time, the following condition must be satisfied (see equation (3)):

\[ \rho + 3p < 0. \]  

One means to meet condition (14) is to consider a universe containing both deficit and normal vacuums (then on the average, \( \rho_v < 0 \)), and to assume that deficit vacuums act as negative energy photons (then \( p_v = \rho_v/3 < 0 \)). (According to quantum electrodynamics, normal vacuums are full of all kinds of electromagnetic modes.)

At the late epoch of the universe, the pressure of matter particles is negligible. Then \( p = p_v \), \( 3p = 3p_v = \rho_v \). Condition (14) leads to \( \rho < -\rho_v \), which allows \( \rho > 0 \) (note \( \rho_v < 0 \)). Hence, it is possible that a positive energy density (where \( \rho_m > -\rho_v \)) may lead to an accelerated expansion of the universe at its late epoch so long as \( \rho < -\rho_v \) or \( \rho_m < -2\rho_v \). For \( \rho > 0 \), we find \( -\rho_v < \rho_m < -2\rho_v \). Equation (13) leads to \( k/R^2 = H^2(2q - 1 - \alpha\Omega) \), where

\[ \alpha \equiv \frac{\rho_v}{\rho}. \]
This together with (12) yield $\Omega = \frac{2q}{(1 + \alpha)}$. As $\rho < -\rho_v$, for $\rho > 0$, we find $\alpha < -1$. For $q < 0$, this indicates that $\Omega > 0$. The curvature of the universe depends on $\alpha$: when $\alpha > -(1-2q)/\Omega$, then $k = -1$ and $0 < \Omega < 1$; when $\alpha = -(1-2q)/\Omega$, then $k = 0$ and $\Omega = 1$; when $\alpha < -(1-2q)/\Omega$, then $k = +1$ and $\Omega > 1$. Equation (15) shows, if $\alpha$ is a constant, the sign of $\rho$ will remain unchanged.

For a flat universe, $\alpha = -(1-2q)$ at the late epoch. When $\alpha$ is known (e.g., determined by the late epoch acceleration), $\rho$ and $\rho_v$ may be known since $\rho_m$ is measurable.

At the early epoch of the universe, $p_m = \rho_m/3$. Then $p = \rho/3$ (as $p_v = \rho_v/3$). For $\rho > 0$, we find from (3) that $\ddot{R} < 0$ (then $q > 0$), indicating that the expansion of the universe is decelerated.

Equation (13) leads to $k/R^2 = H^2(2q-1-\Omega)$. This together with (12) yield $\Omega = q$. Since $q > 0$, then $\Omega > 0$. When $k = -1$, then $2q - 1 < \Omega < 1$, $0 < q < 1$; when $k = 0$, then $\Omega = 1$, $q = 1$; when $k = +1$, then $1 < \Omega < 2q - 1$, $q > 1$.

We find in this model that the universe can possess a positive energy density ($\rho > 0$, where $\rho_m > -\rho_v$) and a positive energy parameter ($\Omega > 0$). The curvature of the universe depends on the ratio of vacuum energy (or matter energy) to total energy. As the pressure of matter particles becomes less important as time goes on, the expansion of the universe will change. When $p_m = ((\alpha + 1)/3(\alpha - 1))\rho_m$, we find from (3) that $\ddot{R} = 0$. It is at this moment the expansion changing from deceleration to acceleration.

It is obvious that deficit vacuums, as they possess negative energy, will be expelled by gravitation. Then there will seldom be deficit vacuums remained in galaxy clusters, and therefore the spacetime of galaxy clusters and that of the solar system will not be significantly affected. We suspect that the place in the universe containing most deficit vacuums might be voids. In the above model, deficit vacuums are assumed to act as negative energy photons. These photons will be deflected towards the center of voids and then the voids might be crowded with them. Within the voids, if the amount of vacuum apparent energy (negative) is over that of matter energy, the spacetime might be somewhat like that of the Schwarzschild solution with (more or less) negative mass, and then matter objects would experience (strong or weak) anti-gravitation. Many matter objects must have been driven to the sheets around the voids, and the number of those remained will be small. In addition, any matter objects would undergo the negative pressure $(p_{\text{void}}) \simeq p_v(\text{void}) = \rho_v(\text{void})/3 < 0$ in the voids. Those survived would be that with their components being firmly connected. The cloud structure of matter must finally be disintegrated.
and some other matter structures must be reduced. As a result, the amount of matter in the sheets would grow and that in the voids would reduce (the disintegrated matter would be easier to be expelled to the sheets by anti-gravitation). In this model, we can assign the observed matter density to be $\rho_m$ (e.g., at the present time, taking $\rho_m = 0.3\rho_c$ [8]), then the missing mass problem will no longer exist. (We suggest that many conventional problems should be reexamined in this new model.)

It might be possible that deficit vacuums act as negative mass particles, or some act as negative mass particles while others act as negative energy photons. Situations in these cases will be different.

**ACKNOWLEDGEMENTS**

The author is grateful to Professors G. Z. Xie, Xue-Tang Zheng and Shi-Min Wu for their guide and help. This work was supported by the United Laboratory of Optical Astronomy, CAS, the Natural Science Foundation of China, and the Natural Science Foundation of Yunnan.
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