Electroweak Phase Transition in the $U(1)'$-MSSM

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Abstract

In this work, we have investigated the nature of the electroweak phase transition in the $U(1)$ extended minimal supersymmetric standard model without introducing any exotic fields. The effective potential has been estimated exactly at finite temperature taking into account the whole particle spectrum. For reasonable values of the lightest Higgs and neutralino, we found that the electroweak phase transition could be strongly first-order due to: (1) the interactions of the singlet with the doublets in the effective potential, and (2) the evolution of the wrong vacuum, that delays the transition.

Keywords: baryogenesis, electroweak phase transition, extra singlet, extra gauge boson.

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1 Introduction

The matter-antimatter asymmetry in the Universe is observed to be $n_b/n_\gamma \sim 10^{-10}$ [1]. If this asymmetry is to be explained by microphysics rather than initial conditions, then there must be processes occurring in the early Universe that violate baryon number and CP and which occur out of thermal equilibrium. It appears that the standard model (SM) satisfies all three conditions [2, 3]; the baryon number is not conserved at quantum level due to the $B + L$ anomaly [4], a CP violation source does exist in the quark sector ($CKM$ matrix), and a departure from thermal equilibrium could in principle be achieved through a strong first-order phase transition [3]. However, detail calculations show that the SM fails to generate the
observed baryon asymmetry due to the smallness of the CP violation effect and the weakness of the electroweak phase transition (EWPT) [5].

In gauge theories, a first-order phase transition takes place if the vacuum of the theory does not correspond to the global minimum of the potential. Since it is energetically unfavored, the field changes its value to the true vacuum (i.e., the absolute minimum of the potential). Because of the existence of a barrier between the two minima, this mechanism can happen by tunneling or thermal fluctuations via bubble nucleation. The electroweak baryogenesis scenario is realized when the $B$ and CP violating interactions pass through the bubble wall. These interactions are very fast outside the bubbles but suppressed inside. Then a net baryon asymmetry results inside the bubbles which are expanding and filling the Universe at the end.

In the SM, the EWPT is too weak [6] unless the Higgs mass is less than 45 GeV [7], which is in conflict with present data [8]. But a departure from thermal equilibrium without being in conflict with this severe bound on the Higgs mass, is possible when extending the SM with additional gauge singlets [9, 10], new heavy fermions [11], an extra Higgs doublet [12], or in some supersymmetric extensions of the SM.

In spite of its success and popularity, the minimal supersymmetric standard model (MSSM) with R-parity still has two major problems: the $\mu$-problem [13] and the fast proton decay due to dimension 5 operators [14]. A natural solution to these problems would probably require the extension of MSSM by a new mechanism or a new symmetry. The $U(1)'$-extended MSSM ($USSM$, $UNMSSM$ or $UMSSM$) [15] is a straightforward extension of the MSSM with a nonanomalous TeV scale Abelian gauge symmetry. This simple enlargement of the gauge sector is well motivated in string construction [16], in grand unified theories [17] such as $SO(10)$ and $E_6$, in models of dynamical symmetry breaking [18] and little Higgs models [19]. The $\mu$-problem and the dangerous dimension 5 operator can be solved naturally with an appropriate $U(1)'$ charge assignment. Furthermore, these $U(1)'$ models can provide a new candidates for dark matter that are not excluded by direct dark matter searches and with interesting signatures at colliders [20, 21].

In the MSSM, the EWPT could be strongly first-order if the light stop is lighter than the top quark [22]. In the singlet extended MSSM [23], such as the NMSSM [24], the EWPT get stronger easily for a large range of parameters [25]. In gauge extensions of the MSSM, such as UMSSM, the EWPT is also strongly first-order but with the price of introducing 3 new extra singlet scalars [26], or by adding new extra heavy singlet fermions [27].

The main reason that makes it less easier to have a strong EWPT in the UMSSM compared to NMSSM, is that the former contains a new gauge interaction which results in strong constraint on the mixing between the SM gauge boson $Z$ and the new one $Z'$ [28]

\[ 2M_{ZZ'}^2/(M_{Z'Z'}/M_{ZZ}) < 10^{-3}, \]

and the bound on the heavy $Z'$ mass [29]

\[ M_{Z'} > (500 - 800) \text{ GeV}, \]

which implies serious constrains on the vacuum expectation value (vev) of the singlet and the new $U(1)'$ gauge coupling $g'$.

However, both models have similar form for the scalar potential, where the singlet can play the same role during the EWPT dynamics. In this type of model, the singlet vev within the wrong vacuum could be nonzero, i.e., $\langle S \rangle = x \neq 0$, and therefore, is temperature dependant during the EWPT dynamics. This feature could delay the EWPT, i.e., lowers the critical temperature, and enhances the parameters that define the strong first-order phase transition criterion [30]:

\[ v(T_c)/T_c > 1, \]

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where $T_c$ is the critical temperature and $v(T)$ is the temperature dependent scalar vev.

In this work, we will investigate the possibility of getting a strong first-order phase transition within the minimal gauge extension of the MSSM, UMSSM without adding any new field beside the usual singlet.

This paper is organized as follows: in the second section, we give a brief review of the UMSSM model, define the effective potential and discuss different constraints on the parameters. After that, we discuss the EWPT dynamics and show how to get a first-order phase transition. In the fourth section, we discuss our numerical results. Finally, we summarize our results. The different field-dependant masses used in the estimation of the effective potential are given in Appendix A.

## 2 The UMSSM model

The $U(1)'$-MSSM (or UMSSM) is based on the gauge group $G = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ with the couplings $g_3$, $g_2$, $g_1$ and $g'$, respectively [15] and the superpotential is given by

$$W = \lambda S \epsilon_{ij} H_1^i H_2^j + Y_U \epsilon_{ij} Q^i U^j H_2^j + Y_D \epsilon_{ij} Q^i D^j H_1^j + Y_L \epsilon_{ij} L^i E^j H_1^j,$$

where $\epsilon_{ij}$ is the antisymmetry $2 \times 2$ tensor, $Y_U$, $Y_D$ and $Y_L$ are Yukawa couplings, and $\lambda$ is a coupling constant in which $\lambda \langle S \rangle$ generates the $\mu$-term in the MSSM. The particle content of this model is given by the left-handed chiral superfields $L \sim (1, 2, -1/2, Q_L)$, $E^c \sim (1, 1, 1, Q_E)$, $Q_i \sim (3, 2, 1/6, Q_Q)$, $U^c \sim (\bar{3}, 1, -2/3, Q_U)$, $D^c \sim (\bar{3}, 1, 1/3, Q_D)$, $H_1 \sim (1, 2, -1/2, Q_1)$, $H_2 \sim (2, 1/2, Q_2)$ and $S \sim (1, 1, 0, Q_S)$, where the $U(1)'$ charges, $Q'$s, are model dependent. For instance, in a class of $E_6$ gauge models, the group can be broken in two steps to its $SO(10)$ and $SU(5)$ subgroups:

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\psi \times U(1)_\chi,$$

and in this case, the $U(1)'$ generator is given in terms of the $U(1)_\psi$ and $U(1)_\chi$ generators and mixing angle $\theta_{E_6}$ [15]

$$Q' = Q_\psi \cos \theta_{E_6} + Q_\chi \sin \theta_{E_6}.$$  

Although the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ model is motivated in the $E_6$ framework, we will not single out a particular charge pattern and we only require that the model be anomaly free. However, with the above particle content, one can easily show that the invariance of the Yukawa terms in the superpotential under $U(1)'$ and the absence of the $SU(3)_C - SU(3)_C - U(1)'$ anomaly implies that $Q_s = 0$, and the $\mu$ problem arises again. Thus, anomaly cancelation requires exotic representations beyond those of the MSSM. The simplest extension for the anomaly to vanish is to assume three generations of heavy (few TeV) vectorlike pairs of chiral fields $K_i$ and $K_i^c$ which transform as $(3, 1)$ and $(\bar{3}, 1)$ under $SU(3)_C \times SU(2)_L$ and opposite hypercharges [31].

### 2.1 The effective potential

In case where both squarks or/and sneutrinos do not develop vevs, the scalar potential is a combination of the so-called $D$, $F$ and soft terms, which are given by

$$V_D = \frac{g_3^2 + g_1^2}{8} (H_2^+ H_2 - H_1^+ H_1)^2 + \frac{g_2^2}{2} |H_1^+ H_2|^2 + \frac{g'^2}{2} |Q_1 H_1^+ H_1 + Q_2 H_2^+ H_2 + Q_S |S|^2|^2,$$

$$V_F = |\lambda|^2 \{ |\epsilon_{ij} H_1^i H_1^j|^2 + |S|^2 \left[ H_1^+ H_1 + H_2^+ H_2 \right] \},$$

$$V_{soft} = m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2 + m_S^2 |S|^2 + \{ A_S \epsilon_{ij} H_1^i H_2^j + h.c. \}.$$  

(6)
Here \( m_1^2, m_2^2, m_3^2 \) and \( A_\Lambda \) are usually called the SUSY soft parameters. The charges \( Q \)'s should be chosen in such a way that the anomaly cancelations are ensured [15] [20].

The structure of the tree-level potential (8) seems to allow two explicit relative \( CP \) violating phases between the doublets and the singlet. However, gauge invariance dictates that the only allowed phase in the potential, is that of the combination \( S e_{ij} H_1^i H_2^j \), since gauge rotations can be used to set the charged Higgs vev \( \langle H_1^+ \rangle = 0 \), and the condition \( \langle H_2^- \rangle = 0 \) implies that the physical charged Higgs is nontachyonic (\( M_{H^\pm}^2 > 0 \)) [15]. Therefore, the general form of the ground state could be written as

\[
\langle H_1 \rangle = \frac{v_1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{v_2}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \langle S \rangle = \frac{v_x e^{i\theta}}{\sqrt{2}}.
\]

Here \( v = (v_1^2 + v_2^2)^{1/2} = 246 \text{ GeV} \), and \( \theta \) is the relative \( CP \) violating phase between the singlet and the doublets; and \( \theta_0 \) its value at the ground state. The tree-level scalar potential in terms of the neutral components is given by

\[
V(h_1, h_2, S) = \left[ \left( g_1^2 + g_2^2 \right)/4 + g^2 Q_1^2 \right] h_1^2/8 + \left[ \left( g_1^2 + g_2^2 \right)/4 + g^2 Q_2^2 \right] h_2^2/8 + g^2 Q_S^2 S^4/8
+ \left[ \lambda^2 - \left( g_1^2 + g_2^2 \right)/4 + g^2 Q_1 Q_2 \right] h_1^2 h_2^2/4 + \left[ \lambda^2 + g^2 Q_1 Q_S \right] h_1^2 S^2/4
+ \left[ \lambda^2 + g^2 Q_2 Q_S \right] h_2^2 S^2/4 + m_1^2 h_1^2/2 + m_2^2 h_2^2/2 + m_S^2 S^2/2 + \{ A_\lambda S h_1 h_2 e^{i\delta}/2\sqrt{2} + h.c. \}.
\]

The neutral scalar sector in this model contains three \( CP \)-even scalars and one \( CP \)-odd scalar. The tadpole minimization conditions at the ground state along the \( CP \)-odd scalar forces the relative \( CP \) phase \( \theta \), to be canceled by the phase of the parameter \( A_\Lambda \) in (8), which must be taken as \( A_\Lambda = \pm |A_\Lambda| e^{-i\theta_0} \). This makes the ground state at tree-level independent of this relative phase, and the phase \( \delta \) in (8) is just \( \delta = \theta - \theta_0 \). It is clear that the dependence of (8) on the phases disappears at the ground state. However, when considering the one-loop corrections in the tadpole minimization condition along the \( CP \)-odd scalar, the tree-level phase cancelation is no longer valid at one-loop, and the argument of \( A_\Lambda \) differs slightly than \(-\theta_0 \) (or \(-\theta_0 + \pi \)).

The structure of the tree-level potential (8) implies the presence of a mixing between the submatrices (\( CP \)-odd scalars, Goldstone bosons, and \( CP \)-even scalars) at tree-level for any values of the scalar fields except at the ground state, however this mixing is proportional to \( \sin \delta \). This will ensure the \( CP \) conservation at tree-level in the scalar sector. Indeed, in addition to the phase appearance in the scalar sector due to the deviation of \( \text{Arg}(A_\Lambda) \) from \(-\theta_0 \), the \( CP \) violation effect could be seen in this model at one-loop [32], and also through the dependence on \( \theta \) and not \( \delta \) can be seen in the superpartner masses (23), (26) and (27).

The parameters \( m_1^2, m_2^2, \) and \( m_3^2 \) can be eliminated by taking \((v_1, v_2, v_x e^{i\theta_0})\) as a minimum of the effective potential. The vacuum stability of (8) requires the condition:

\[
\lambda^4 - \lambda^2 \left[ (g_1^2 + g_2^2)/2 + 2g^2 (Q_S^2 - Q_1 Q_2) \right] + \frac{3}{4} g^2 \left( g_1^2 + g_2^2 \right) Q_S^2 > 0,
\]

where the \( U(1)' \) gauge invariance condition \( Q_1 + Q_2 + Q_S = 0 \) is taken into account.

In models that include singlets, like the present one, one needs to be careful about whether the minimum \((v_1, v_2, v_x e^{i\theta_0})\) is the absolute one. This can be checked by comparing the effective potential value at this minimum with its value at the wrong vacuum, which could be, in singlet models, \((0, 0, x)\) rather than the origin \((0, 0, 0)\). The wrong vacuum can be defined as the minimum of the effective potential in the direction where all the \( SU(2) \) doublets vanish. At tree-level, it is given by

\[
|x|^2 = v_x^2 + \left[ Q_1/Q_S + \lambda^2/g^2 Q_S^2 \right] v_1^2 + \left[ Q_2/Q_S + \lambda^2/g^2 Q_S^2 \right] v_2^2 + \frac{\sqrt{2}}{g^2 Q_S^2} A_\Lambda v_x v_1 v_2.
\]
The value of $x$ in (10) should be relaxed by including the one-loop corrections given below in Eq. (11). In the case where the right hand side of (10) is positive, the wrong vacuum $(0, 0, x)$ does exist, otherwise it should be $(0, 0, 0)$. In both cases, $(v_1, v_2, v_x e^{i\theta_0})$ must be the absolute minimum for the effective potential. This condition imposes additional constraints on the parameters of the model.

The one-loop effective potential at zero temperature is given in the $\overline{DR}$ scheme by [33]

$$V^{1-l}(h_1, h_2, S) = V(h_1, h_2, S) + \sum_i \frac{n_i m_i^4}{64\pi^2} \left( \log \frac{m_i^2}{\Lambda^2} - \frac{3}{2} \right),$$

(11)

where $m_i(h_1, h_2, S)$ are the field-dependant masses, which are given in Appendix A. $\Lambda$ is the renormalization scale, which is taken to be $\Lambda = \tilde{\nu} = 246$ GeV, and $n_i$ are the fields multiplicities: $n_W = 6$, $n_Z = n_{Z'} = 3$, $n_{\tilde{h}_0} = n_{\tilde{h}_2} = n_S = n_{A^0} = n_G = 1$, $n_t = -12$, $n_{i_L} = n_{i_R} = 6$, $n_{\chi} = -2$, $n_{\tilde{C}} = -4$, where $A^0$, $G$, $i_{L,R}$, $\tilde{\chi}$ and $\tilde{C}$ denote the $CP$-odd Higgs, Goldstone boson, left- and right-handed squarks, neutralinos and charginos, respectively.

The thermal corrections to the effective potential can be computed using the known techniques [34]. The one-loop effective potential at finite temperature is given by

$$V_{\text{eff}}(h_1, h_2, S, T) = V^{1-l}(h_1, h_2, S) + T^4 \sum_i n_i J_{B,F} \left( m_i^2(h_1, h_2, S) / T^2 \right),$$

(12)

with

$$J_{B,F}(\alpha) = \frac{1}{2\pi^2} \int_0^\infty x^2 \log(1 + \exp(-\sqrt{x^2 + \alpha})),$$

(13)

$n_i$ are given above and $m_i^2(h_1, h_2, S)$ are given in Appendix A. In this work, to take into account all the heavy and light fields, we will evaluate this integral numerically.

### 2.2 The parameters

In this model, we have many parameters, some of which are free like: $g'$, $\lambda$, $v_x$, $\tan\beta = v_1/v_2$, and the soft terms: $m_Q$, $m_U$, $A_t$, $A_\lambda$, $M_2$, $M_1$, and $M'_1$; and others that are fixed by a measured physical quantities such as: $g_1$, $g_2$, $v$ and $y_t$; or can be conditions like the elimination of $m_1^2$, $m_2^2$, and $m_3^2$ in (8).

In scanning the parameter space of this model, we take into account the constraint $Q_1 + Q_2 + Q_3 = 0$, conditions from the minimization of the potential, the perturbativity of the quartic couplings in (8), and the vacuum stability [3]. Another constraint could be derived from the upper bound on the mixing between the gauge boson $Z$ and the new one $Z'$ [11], and the lower bound on the $Z'$ mass [2]. The condition (2), could be achieved by considering relatively large $v_x$ or large $(g'Q')$. The condition (11) could be fulfilled if the mixing term $M^2_{ZZ'}$ is vanishing, i.e.,

$$Q_1 = Q_2 \tan^2 \beta,$$

(14)

which leads to a fine tuning in the values of $Q_{1,2}$ and $\tan\beta$. The second possibility is making $M^2_{ZZ'} >> M^2_{ZZ}, M^2_{Z'Z'}$, which roughly means

$$g' |Q_S| v_x \gtrsim (500 - 800) \text{ GeV}.$$  

(15)

In our search for the parameter’s space that fulfills the strong first-order phase transition criterion, $v (T_c) / T_c > 1$, we will focus on the two following regions :

1. Moderate values for the parameters $Q_{1,2}$ and $\tan\beta$, where (11) is nearly satisfied. In this case, the singlet vev $v_x$ can be of order $v$ or relatively smaller.
The two terms $M_{ZZ}^2$ and $M_{ZZ}^2$, in (22), are suppressed with respect to the mass term $M_{ZZ}^2$. In this case, the values of $U'(1)$ charge and the vev of the singlet, $Q_x$ and $\nu_x$, must be large enough.

We should also distinguish between the two cases where the minimum $(0,0,x \neq 0)$ of the potential does exist or not. The second case could be ensured by choosing the parameter $A_\lambda$ in (10) to be extremely negative, then $x$ does not exist. This is easier to satisfy in region (1). In both cases, the condition (2) is, almost, automatically fulfilled within (1).

Also, one needs to know the effect of the phase $\theta$ on the EWPT strength. At tree-level (3), this phase is not relevant due to the choice of $A_\lambda$, but the thermal corrections depend on this phase through all the fields masses expect gauge bosons and top quark. Therefore, we consider both cases, the condition (2) is, almost, automatically fulfilled within (1).

Below this temperature, the new minimum becomes the absolute one, and the system has to move from the old (false) vacuum to the new (true) one. In the case where a barrier does exist.

3 The electroweak phase transition

Due the condition (1) and (2), there could exist a hierarchy between the vev of the singlet and those of the doublets, i.e., $\nu_x \gg \nu_{1,2}$. In this case, the gauge symmetry could be broken in two step. However, in the case where the mixing is extremely suppressed as in region (1) (11), the singlet vev could be low as $\sim 500$ GeV. In this case, the gauge symmetry could be broken just in one step. In the two-step symmetry breaking case, one notices that above a certain high temperature, the singlet vev was zero $\nu_x (T_{\nu_x}) = 0$. This can be seen by putting $\nu_{1,2} = 0$ in (11), and taking only the thermal correction of $Z'$. Then at lower temperatures, it is not sure that the system moves directly from $(0,0,0)$ to $(\nu_1, \nu_2, \nu_x e^{i\theta_0})$, or via an intermediate step: $$(0,0,0) \rightarrow (0,0,x) \rightarrow (\nu_1, \nu_2, \nu_x e^{i\theta_0})$$

This will depend, in general, on the theory parameters, especially the value of the singlet vev. If it is comparable to the EW vev $\nu$, the phase transition could occur once, however if it is much larger than $\nu$, the phase transition will occur in two steps.

Since the singlet dynamics does not affect the $SU(2)$ sphalerons processes, we will not be interested in distinguishing between the one- and two-steps symmetry breaking. We will treat our field dynamics using the effective potential where the singlet is replaced by a temperature-dependant vev, i.e.,

$$V(h_1, h_2, T) = V_{\text{eff}}(h_1, h_2, x(T), T).$$

At higher temperatures, the effective potential admits only one minimum where $\nu_{1,2}(T) = 0$. As the Universe cools down, the effective potential acquires a new minimum $\nu_{1,2}(T) \neq 0$; but it is not the absolute one. At such temperature, the critical temperature $T_c$, the two minima get degenerate

$$V_{\text{eff}}(\nu_1^c, \nu_2^c, \nu_x^c, T_c) = V_{\text{eff}}(0, 0, x^c, T_c).$$

Below this temperature, the new minimum becomes the absolute one, and the system has to move from the old (false) vacuum to the new (true) one. In the case where a barrier does exist.
between the two minima, this transition has to occur via tunneling through bubbles nucleation at certain points, which expand and the whole space by the new vacuum \( v_{1,2}(T) \neq 0 \), i.e., the symmetry is broken.

If the effective potential (12) is expanded as powers of \( m/T \) (in the limit \( m \ll T \)) in a similar way to the SM in the doublet's direction, the two leading terms are of order \( h^2T^2 \) and \( h^3T \). The first term determines the temperature when the barrier between the potential two minima disappears, while the second term is relevant to the strength of the first-order phase transition. It turns out that a resummation of the so-called Daisy diagrams [35], leads to a contribution of order \( h^3T \) which exactly cancels the contribution from certain particles (e.g., Higgs and longitudinal gauge bosons). Therefore this screening effect could weaken the strength of the EWPT. For that, we will check the importance of this effect by using a resummed effective potential where (12) is modified by replacing the bosonic field-dependant masses by their thermally corrected values \[36\]. The thermal corrections to the bosonic masses are given in Appendix B.

The \( B + L \) anomalous interactions [4] that violate the baryon number do not have the same rate in the symmetric and broken phases (i.e., at both sides of the bubble wall). In the symmetric phase, this rate behaves like \( \sim T^3 \) \[37\], and is suppressed as \( \exp(-E_{Sp}/T) \) \[35\], in the broken phase, where \( E_{Sp} \) is the system static energy within such field configuration called the sphaleron \[39\]. Therefore any generated baryon number at the symmetric phase will get erased, unless these interactions are switched off in the broken phase, which translates to the condition (3). In reality, the \( B + L \) anomalous interactions should be switched off at a temperature \( T_0 < T_n < T_c \), where \( T_n \) is the temperature at which bubbles start to nucleate, and \( T_0 \) is the temperature at which the barrier between the two minima completely disappears. Then the condition in (3) should be fulfilled at \( T_n \), but, in general, the two values are significantly close, and if \( v(T_c)/T_c > 1 \), it is necessarily satisfied at \( T_n \). Indeed, in some earlier works (e.g. [40, 41]), the condition (3) at \( T_0 \) was used in defining the phase transition. This means that the anomalous \( B + L \) violating processes should be switched off at the end of the phase transition, i.e., the Universe is filled by the true vacuum by the expanded bubbles. The value \( T_0 \), usually called the lower metastability temperature or lower spinodial decomposition point [41], can be determined using the Jacobian \( \text{det} \{\partial^2V(h, T_0)/\partial h_i \partial h_j \}|_{h=0} = 0 \). However, in our work, it is safer to consider the condition (3) at \( T_c \), that is defined in (20).

Since the singlet field does not play an important role in the sphaleron processes [10], the criterion of a strong first-order phase transition in our case is given by

\[
v(T_c)/T_c \equiv \sqrt{v_1^2(T_c) + v_2^2(T_c)/T_c} > 1.
\]

In the general case where the relative phase \( \theta \neq 0 \), the field ground state at nonzero temperature should be written as \( \{v_i\}_{i=1,4} = (v_1, v_2, v_x \cos \theta, v_x \sin \theta) \) instead of \( (v_1, v_2, v_x) \) and the relative phase. These 4 variables should be treated independently when looking for \( v_{1,2,x} \) and \( \theta \) at any temperature \( T \). Then the phase transition could be defined through the equations

\[
\frac{\partial}{\partial v_i} V_{\text{eff}} (v_i, T_c) = 0, V_{\text{eff}} (v_1, v_2, v_3, v_4, T_c) = V_{\text{eff}} (0, 0, x_1, x_2, T_c),
\]

where \( x_{1,2} \) are the real and imaginary parts of the \( T \)-dependent \( x \) given in (10).

4 Numerical results

In the following figures, we show the dependence of the quantity \( v(T_c)/T_c \) in (19) on the lightest Higgs mass (in Fig. (1) left), and on the lightest neutralino mass (in Fig. (1) right), for a random choice of about \( 10^6 \) cases in both regions (1) and (2). We find that only about 8% of the
benchmarks fulfill the required conditions, and only 2.5% of the survived benchmarks give a strong first-order phase transition.

Figure 1: The dependence of the quantity $\nu_c/T_c$ on the lightest Higgs mass, on the lightest neutralino mass, and the corresponding critical temperature, taking into account different values of the parameters in (16) is shown. The red points refer to benchmarks from region (1), and the green ones to benchmarks from region (2).

As it is clear from Fig. 1, the EWPT could be strongly first-order in the two regions (1) and (2), for a large range of the of the values of the lightest Higgs mass, and lightest neutralino mass. These masses are estimated at tree-level, which are expected to be boosted to larger values when considering one-loop corrections. One remarks that the phase transition strength does depend on the lightest Higgs and neutralino masses. One also remarks that the majority of the benchmarks that give a weak phase transition have the value of $\nu(T_c)/T_c \sim 0.14 - 0.18$, and their corresponding critical temperature is between $T_c \sim 300 - 600$ GeV. While all the corresponding values of $T_c$ to the strong phase transition are $T_c \leq 180$ GeV.

The mixing with the singlet will modify the doublets couplings to the fermions, and since the lightest Higgs contains a singlet amount, its mass could be (much) smaller than SM bound $\sim 114$ GeV. One remarks from Fig. 1 (right), that the EWPT could be strongly first-order for small values of the lightest neutralino masses; $m_{\chi_1} < 15$ GeV. Such a light mass of neutralino with a spin-independent cross section of order $10^{-5}$ pb could be a possible interpretation of the recent observations of CoGeNT and DAMA/LIBRA [42]. In particular, the lightest neutralino can have a non negligible component of the superpartner of the $U(1)'$ gauge boson [43].
The critical temperature is, in general, higher when comparing with minimal SM (∼100 GeV), the generic value is larger than 300 − 600 GeV. This is a consequence of the interaction of the doublets with the singlet that has, in general, a very large vev. However for the benchmarks, giving a strong first-order EWPT, $T_c$ is relatively smaller than the generic value. In fact, for points (1) as $T_c$ is smaller than about 180 GeV, the stronger the EWPT becomes.

In order to understand this point, we take a benchmark from Fig. 1 and study the dependence of the scalar vevs on the temperature $T$ below and just above the critical temperature. We will also check how could this behavior be changed with respect to the charges $Q'$s, and other parameters like $A_t$, and $M'_1$, that appear in the effective potential at one-loop level. Therefore we consider the benchmarks in Table-1.

|   | (a)          | (b)          | (c)          | (d)          |
|---|--------------|--------------|--------------|--------------|
| $\lambda$ | 0.0235       | 0.0235       | 0.0235       | 0.0235       |
| $\tan\beta$ | 2.0566       | 2.0566       | 2.0566       | 2.0566       |
| $A_\lambda$ | -208.9569    | -208.9569    | -208.9569    | -208.9569    |
| $\nu_x$ | 1173.3560    | 1173.3560    | 1173.3560    | 1173.3560    |
| $Q_1$ | 0.3575       | 1.5          | 1.6147       | 1.6147       |
| $Q_2$ | 1.6147       | 1.6147       | 1.6147       | 1.6147       |
| $\theta_0$ | 0.5888       | 0.5888       | 0.5888       | 0.5888       |
| $M_1$ | 857.9520     | 857.9520     | 857.9520     | 1500         |
| $M_2$ | 398.7435     | 398.7435     | 398.7435     | 398.7435     |
| $M'_1$ | 881.3455     | 881.3455     | 881.3455     | 881.3455     |
| $A_t$ | 274.4727     | 274.4727     | 274.4727     | 274.4727     |
| $T_c$ | 103.6387     | 326.2436     | 302.9225     | 100.8061     |
| $\nu_c/T_c$ | 2.5836       | 0.1940       | 0.1976       | 2.6356       |
| $m_{h_1}$ | 172.1886     | 163.3732     | 173.0714     | 172.1589     |
| $m_{\chi_1}$ | 5.6907       | 5.6569       | 5.6907       | 6.2573       |

Table 1: The values of the parameters used to study the scalar vevs dependence with respect to the temperature are shown. The mass-dimension parameters are given in GeV.

From this table, it is clear that the EWPT strength is extremely sensitive to the charges $Q'$s which represent in a way the strength of the $U(1)'$ interactions since the $Q'$ charges always appear multiplied by $g'$. However, the soft parameter $A_t$ and $M_1$ that appear in the effective potential at one loop could have an important effect on the strength of the EWPT for some particular choices of the parameter space. The dependence of the EWPT strength on the relative phase $\theta_0$, could be seen by taking changing its value of the benchmark (a) in Table-1. We find that its effect is extremely negligible [difference in the value of $\nu(T_c)/T_c$ is < 0.1%].

In Fig. 2 we show the dependence of the ground state on the temperature below the critical temperature for the benchmark (a) in Table-1 and its modifications (b), (c) and (d). One remarks that the common feature between all these cases is that the dependence of the singlet vev on the temperature is very weak around and below the critical temperature. This could be understood due to the fact that the singlet vev is much larger than this temperature $[x(T \ll x) \sim x(0)]$, and also, due to a possible two-stages phase transition realization, where the first stage $(0,0,0) \rightarrow (0,0,x(T))$, takes place around the temperature $T_c \sim x$; therefore at lower temperatures $T \ll T_c$, the singlet vev becomes almost temperature independent.

Another important remark, is that the Higgs vevs for this benchmark [Fig. 2-a] are increasing when the Universe gets cooled unlike the SM [9], or MSSM [11], and similar remark for (d). In cases (b) and (c), the Higgs vevs decrease, but with slower pace than in the SM or MSSM, which
Figure 2: The dependence of the scalar vevs on the temperature $T$ is shown. These quantities are scaled by their zero temperature values. The solid curves refer to the broken phase, where $(\nu_1, \nu_2, \nu_x)$ represents the global minimum. While the dashed ones refer to the symmetric phase, where the global minimum is $(0,0,x(T))$.

makes the critical temperature larger than in the SM. The modified cases (b) and (c) belong to the majority of the benchmarks mentioned before.

The effect of the increasing doublets' vev values at high temperatures with respect to their zero temperature values has been mentioned in a similar work [27]. This behavior and the slow decrease of the doublets' vevs with respect to the temperature, is a consequence of the interaction of the singlet with the doublet Higgs. These interactions have the effect of relaxing the shape of the potential in the direction of the doublets, and therefore enhancing the ratio in (19). This is a common feature for models with singlets [10].

From (20), the phase transition is defined as the degeneracy of the two vacua. Unlike the SM and MSSM, the wrong vacuum $(0,0,x(T))$, is evolving with respect to the temperature, and could be reached through a phase transition at very high temperature. Therefore, its evolution could be a very important factor which can strengthen the EWPT. It is defined from the effective potential as the local minimum

$$V'(T) = V_{\text{eff}}(0,0,x(T),T), \quad \frac{\partial}{\partial S}V_{\text{eff}}(0,0,x(T),T) = 0,$$

which was a global one before the phase transition, i.e., above the critical temperature. We need to check that the evolution of (21) is responsible of lowering the critical temperature in (a) and
which does not play the same role in (b) and (c). In Fig. 3, we show how does the evolution of the wrong vacuum with respect to the temperature decrease the critical temperature for the benchmarks (a) versus its modification (b).

Figure 3: The evolution of the effective potential values at the minima \((0,0,x)\) and \((v_1,v_2,v_x)\) with respect to the temperature \(T\) in units of \(v^4\) is shown. In this temperature range, the value of the effective potential at \((0,0,0)\) is about \(5.879 \sim 6.620\) for benchmark (a), and \(7.322 \sim 8.131\) for benchmark (b) in units of \(v^4\).

From Fig. 3, one remarks that the minimum \((v_1, v_2, v_x)\) becomes the true one at low temperature for case (a), and at large temperature for case (b) due to the evolution of the wrong minimum \((0,0,x)\). This can be identified from the intersection of the thermal effective potential at \((v_1,v_2,v_x)\) and its value at \((0,0,x)\). If one completely ignores the minimum \((0,0,x)\), the EWPT, which is defined in this case by the intersection of this value with effective potential at \((v_1,v_2,v_x)\) and its value at \((0,0,0)\), will take place at very high temperatures, normally of the order \(\sim O(v_x)\). It is clear that the wrong vacuum evolution has the main role in lowering the critical temperature in case (a) compared with case (b). It might seem that the EWPT is less sensitive to some parameters such as \(m_Q, m_U, A_t, M_2, M_1,\) and \(M'_1\), since they appear in the effective potential at one-loop. However, we have shown in Table-1 and in Fig. 3 that the EWPT dynamics is sensitive to these inputs more than expected. In fact, if one adds a small perturbation to these parameters, then the wrong vacuum, \((0,0,x)\), becomes the absolute one at zero temperature, through the one-loop corrections in \((11)\), which will rule out our benchmark.

In order to estimate the screening effect on the EWPT strength, we consider the modified effective potential \([30]\) by replacing the longitudinal gauge bosons and squarks masses in \((12)\) by their thermally corrected expression \((30)\) and \((31)\) in Appendix B. We ignore the scalar contributions because they are less relevant to the EWPT dynamics. One can distinguish two different types of the benchmarks in Fig. 1; the first variant: the benchmarks with increasing doublet vevs with respect to the temperature \([like the benchmark (a) in Fig. 2]\), and the second variant are the benchmarks with decaying doublet vevs with respect to the temperature like most of the benchmarks in Fig. 1.

For the second variant, it is expected to have a similar behavior as in the (MS)SM-like models where the screening effect weakens significantly the strength of the EWPT. The benchmarks whose generic values of \(v_c/T_c\) in the interval \(0.14 - 0.18\) get reduced to \(0.10 - 0.12\), that is about 30% effect. In fact, even the benchmarks that give a strong first-order EWPT but with (slowly) decaying doublet vevs with respect to the temperature become weak EWPT benchmarks after including the daisy-diagrams contribution.
Without including the daisy-diagrams, the first variant correspond to benchmarks giving a strong first-order phase transition whose critical temperatures are less than $T_c < 180 \sim 200$ GeV. Once we include the daisy diagrams contribution, we find that the doublets vevs become sharply increasing with respect to the temperature, and this results in a very low critical temperature ($T_c \sim 32 - 36$ GeV), and very strong ($v_c/T_c \sim 7 - 9$) EWPT. This unusual effect is due to the breakdown of the approximation in which one neglects the $mT$ term in the expression of the thermal mass. In other words, one should expect that at low temperatures the thermal mass is approximately given by couplings $\times$ zero temperature mass instead of the $T^2$ term \[35\]. For perturbative couplings, this is smaller than the zero temperature mass of the particle and can be neglected. Thus, unlike the second variant, we expect that the screening effects for the first variant to be negligible and do not significantly reduce the strength of the phase transition found earlier (which is first-order). To estimate how small the screening effect is, requires taking into account the exact thermal corrections in the effective potential which is beyond the scope of this paper.

It is worth noticing that large values of $v(T_c)/T_c$ [i.e., larger than $v(T_c)/T_c > 3.5$] can be easily obtained. This corresponds to a severe suppression of the sphaleron ($B + L$ violating) processes inside the bubbles, and not necessary to the freezing of the Universe in the wrong vacuum. The decay of the wrong vacuum is related to the bubbles’ dynamics, not to the (non-)efficiency of the $B + L$ violating processes. This point requires a special careful investigation to put constraints on the theory parameters from the fact that the wrong vacuum must decay into the true one.

5 Conclusion

In this work, the nature of the electroweak phase transition within the minimal $U(1)$ extension of the MSSM (UMSSM) without including exotic particles, has been investigated. We found that the EWPT could be strongly first-order for a large range of the lightest Higgs and neutralino masses, without adding extra singlet scalars or fermions. We evaluated the effective potential at one-loop taking into account the whole particle spectrum, and its temperature-dependant corrections were estimated exactly using the known techniques.

We found that the strength of the EWPT could be enhanced due to two factors: first, the interactions of the singlet scalars with the doublets which relax the shape of the effective potential in the doublets directions, and therefore, lead to a large value for the ratio $v(T)/T$ at the critical temperature. The second factor is that the temperature-dependant local minimum, $(0, 0, x(T))$, could play an important role during the EWPT dynamics. It can delay the phase transition until relatively low temperatures (even below 100 GeV), which favor the ratio $v(T_c)/T_c$ to be large enough, without conflicting the usual severe experimental constraints of the SM and MSSM.

During this dynamics, the doublets vevs could be decaying with respect to the temperature but slower than in the case of the SM or MSSM. Another unusual behavior is that the doublets vevs could be increasing with respect to the temperature, which leads to a very strong first-order EWPT. We found that the inclusion of the ring contribution does weaken the EWPT if the doublets vevs are decaying with respect to the temperature (the second variant), and do not change significantly the strength of phase transition in the opposite case.

We also mention that the strength of the EWPT, as well as the reliability of the theory benchmarks, are very sensitive to the input parameters that appear in the effective potential at one-loop.
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A  Field-dependent masses

Here, we will present field-dependent masses with the existence of $CP$ phases; and by vanishing these phases, we get the $CP$ concerning case.

Gauge bosons. In this model, the gauge bosons masses do not depend on $CP$ phases. They have similar masses as in the MSSM, but the $Z$ boson is mixed with the new $Z'$

$$
M_W^2 = \frac{1}{4} g'^2 \left( h_1^2 + h_2^2 \right),
$$

$$
M^{2}_{Z-Z'} = \left( \begin{array}{ccc}
\frac{1}{2} (g_1^2 + g_2^2) (h_1^2 + h_2^2) & \frac{1}{2} g'\sqrt{g_1^2 + g_2^2} (Q_1 h_1^2 - Q_2 h_2^2) \\
\frac{1}{2} g'\sqrt{g_1^2 + g_2^2} (Q_1 h_1^2 - Q_2 h_2^2) & g'^2 \left( Q_1^2 h_1^4 + Q_2^2 h_2^4 + Q_2^2 S^2 \right)
\end{array} \right). 
$$

(22)

Tops and Stops. The stop masses are the same as in the MSSM with replacing $\mu$ but its effective value $\mu = \frac{\lambda S}{\sqrt{2}}$:

$$
m^2_t = \frac{1}{2} y_t^2 h_2^2 \\
M^2_{\tilde{t}} = \left( \begin{array}{ccc}
m^2_Q + m^2_t + \frac{1}{3} g_1^2 (h_1^2 - h_2^2) / 8 \\
A_t h_2 / \sqrt{2} + \lambda h_1 S e^{i \theta} / 2 \\
A_t h_2 / \sqrt{2} + \lambda h_1 S e^{i \theta} / 2
\end{array} \right),
$$

(23)

Scalars. The spectrum of physical Higgses after symmetry breaking consists of three neutral $CP$ even scalars, a mixture between one $CP$ odd pseudoscalar ($A^0$), and two Goldstone bosons that are absorbed by $Z$ and $Z'$ respectively, and a mixture between a charged Higgs and the Goldstone bosons that are absorbed by $W^\pm$. In the ground state $h_i = \langle h_i \rangle$, there is no mixture between the real and imaginary parts of the complex fields of the scalar sector, and the squared-mass matrix should be represented in two independent $3 \times 3$ matrices in the two basis $[\sqrt{2} Re(H^0_1), \sqrt{2} Re(H^0_2), \sqrt{2} Re(S)]$ and $[\sqrt{2} Im(H^0_1), \sqrt{2} Im(H^0_2), \sqrt{2} Im(S)]$. However, in general case where $h_i \neq \langle h_i \rangle$, such a mixing does exist and it is proportional to $\sin \delta$, as shown in Sec. 2.1, and this mixing vanishes at the ground state. Then the elements of the scalar field-dependent mass-squared matrix in the basis $[\sqrt{2} Re(H^0_1), \sqrt{2} Re(H^0_2), \sqrt{2} Re(S), \sqrt{2} Im(H^0_1), \sqrt{2} Im(H^0_2),$
\[ \sqrt{2}Im(S) \], are given by

\[
M_{11} = (g_1^2 + g_2^2) (3h_1^2 - h_2^2) / 8 + g^2Q_1 (3Q_1 h_1^2 + Q_2 h_2^2 + Q_s S^2) / 2 + \lambda^2 (h_1^2 + S^2) / 2 + m_1^2,
\]

\[
M_{12} = (2\lambda^2 - 4g_1^2 + g_2^2) / 4 + 4g^2Q_1 Q_2 h_1 h_2 + Re(Q) S,
\]

\[
M_{13} = (\lambda^2 + g^2Q_1 Q_2) h_1 S + Re(Q) h_2, M_{14} = 0, M_{15} = Im(Q) S, M_{16} = Im(Q) h_2,
\]

\[
M_{22} = (g_1^2 + g_2^2) (3h_1^2 - h_2^2) / 8 + g^2Q_2 (3Q_2 h_2^2 + Q_1 h_1^2 + Q_s S^2) / 2 + \lambda^2 (h_1^2 + S^2) / 2 + m_2^2,
\]

\[
M_{23} = (\lambda^2 + g^2Q_2 Q_1) h_2 S + Re(Q) h_1, M_{24} = Im(Q) S, M_{25} = 0, M_{26} = Im(Q) h_1,
\]

\[
M_{33} = g^2 Q_s (3Q_s S^2 + Q_1 h_1^2 + Q_2 h_2^2) / 2 + \lambda^2 (h_1^2 + h_2^2) / 2 + m_3^2, M_{34} = Im(Q) h_2,
\]

\[
M_{35} = Im(Q) h_1, M_{36} = 0,
\]

\[
M_{44} = (g_1^2 + g_2^2) (h_1^2 - h_2^2) / 8 + g^2Q_1 (Q_1 h_1^2 + Q_2 h_2^2 + Q_s S^2) / 2 + \lambda^2 (h_1^2 + S^2) / 2 + m_1^2,
\]

\[
M_{45} = -Re(Q) S, M_{46} = -Re(Q) h_2,
\]

\[
M_{55} = (g_1^2 + g_2^2) (h_2^2 - h_1^2) / 8 + g^2Q_2 (Q_2 h_2^2 + Q_1 h_1^2 + Q_s S^2) / 2 + \lambda^2 (h_1^2 + S^2) / 2 + m_2^2,
\]

\[
M_{56} = -Re(Q) h_1, M_{66} = g^2 Q_s (Q_s S^2 + Q_1 h_1^2 + Q_2 h_2^2) / 2 + \lambda^2 (h_1^2 + h_2^2) / 2 + m_3^2,
\]

\[
(24)
\]

with \( Q = A \lambda e^{i(\theta - \theta_0)}/\sqrt{2} \). The masses of the charged scalars are given in the basis \((H_1^+, H_2^+)\)

\[
M_{11} = ((g_1^2 + g_2^2) h_1^2 - (g_1^2 - g_2^2) h_2^2) / 8 + g^2Q_1 [Q_1 h_1^2 + Q_2 h_2^2 + Q_s S^2] / 2 + \lambda^2 S^2 / 2 + m_1^2,
\]

\[
M_{12} = (g_1^2 - 2\lambda^2) h_1 h_2 / 4 - Q^*, M_{21} = (M_{12}^*)^*,
\]

\[
M_{22} = ((g_1^2 + g_2^2) h_2^2 - (g_1^2 - g_2^2) h_1^2) / 8 + g^2Q_2 [Q_1 h_1^2 + Q_2 h_2^2 + Q_s S^2] / 2 + \lambda^2 S^2 / 2 + m_2^2.
\]

\[
(25)
\]

**Charginos and Neutralinos.** The chargino masses are the same as in the MSSM with replacing \( \mu \) by its effective value \( \mu_{eff} = \lambda S \). The two chargino \( \tilde{\chi}_{1,2}^+ \) masses are given by the MSSM formula

\[
m_{\tilde{\chi}_{1,2}^+}^2 = \frac{1}{2} \left[ \lambda^2 S^2 / 2 + M_2^2 + g_2^2 (h_1^2 + h_2^2) \mp \left\{ \left( \lambda^2 S^2 / 2 - M_2^2 + g_2^2 (h_1^2 - h_2^2) \right)^2 \right\} \right] + 2g_2^2 \left( \lambda^2 h_2^2 S^2 / 2 + M_2^2 h_1^2 \right) + 2\sqrt{2} g_2^2 \lambda M_2 h_1 h_2 S \cos \theta \right]^{1/2},
\]

\[
(26)
\]

where \( M_2 \) is the \( SU(2) \) gaugino mass.

In the neutralino sector, there is an extra \( U(1)' \) zino and the higgsino \( \tilde{S} \) as well as the four MSSM neutralinos. The 6 x 6 mass matrix reads, in the basis \((\tilde{B}', B, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S})\)

\[
M_{\tilde{c}c} = \begin{pmatrix}
M_1^2 & 0 & 0 & g'Q_1 h_1 & g'Q_2 h_2 & g'Q_s S e^{i\theta} \\
0 & M_1 & 0 & -\frac{1}{2} g_1 h_1 & \frac{1}{2} g_1 h_2 & 0 \\
0 & 0 & M_2 & \frac{1}{2} g_2 h_1 & -\frac{1}{2} g_2 h_2 & 0 \\
g'Q_1 h_1 & -\frac{1}{2} g_1 h_1 & \frac{1}{2} g_2 h_1 & 0 & -\frac{1}{\sqrt{2}} S e^{i\theta} & -\frac{1}{\sqrt{2}} h_2 \\
g'Q_2 h_2 & \frac{1}{2} g_1 h_2 & -\frac{1}{2} g_2 h_2 & -\frac{1}{\sqrt{2}} S e^{-i\theta} & 0 & -\frac{1}{\sqrt{2}} h_1 \\
g'Q_s S e^{-i\theta} & 0 & 0 & -\frac{1}{\sqrt{2}} h_2 & -\frac{1}{\sqrt{2}} h_1 & 0
\end{pmatrix},
\]

\[
(27)
\]

where \( M_1 \) and \( M_1' \) are the gaugino masses associated with \( U(1) \) and \( U(1)' \), respectively.

**B Thermal corrections to the bosonic masses**

All the bosonic fields acquire thermal mass corrections from the three typical diagrams in (2a)-(2c). In Fig. (3c), we show the fermionic contributions to the scalar mass-squared matrix elements,
while scalar contributions could be evaluated from diagrams in Fig. (4-a) and (4-b). The gauge contributions could be deduced from Fig. (4-a) and (4-b) just by replacing the internal scalar legs by gauge ones. The thermal corrections to gauge field masses could be obtained similarly to the scalar ones by replacing the scalar external legs by the gauge ones.

![Diagram](a) ![Diagram](b) ![Diagram](c)

Figure 4: The one-loop diagrams that contribute to the scalar thermal corrections.

When comparing the leading terms from each diagram, $T^2/12$, $-mT/8\pi$ and $-T^2/24$ respectively, one easily finds that the integral contribution of (4-b) is less important with respect to the others, and therefore one could ignore it when taking the high temperature expansion. The high temperature expansion is assumed to be a good approximation for temperatures $m/T < 2.2$ for bosonic and $m/T < 1.6$ for fermionic masses with an error less than 5%.

In our model, we are interested in temperatures around the EW scale where the singlet has already developed its vev, then all the masses and vertices should be evaluated under the condition

$$ (h_1, h_2, S) = (0, 0, \nu_x e^{i\theta_0}). \quad (28) $$

Under this condition, many fields will decouple and therefore do not contribute to the thermal corrections like $\sqrt{2}Re(S)$, $\sqrt{2}Im(S)$, $Z'$, the stops, the neutralinos and the charginos, and our case meets exactly the MSSM case [40]. The internal lines inside the loops should be mass eigenstates, then the vertices required to evaluate (4) need to be modified by the induced mixing between the interaction states and the mass eigenstates under the condition (28). In many models, scalar contributions to the effective potential are generally neglected due their less relevance with respect to the gauge contributions, therefore, we will not consider their thermal corrections here. In what follows, we give the thermal corrections to the scalar, gauge bosons, and squarks.

**Gauge bosons.** The thermal correction to the charged gauge bosons $W^\pm$, is given by

$$ \frac{11}{4} g^2 T^2, \quad (29) $$

while the corrections to the mass-squared matrix elements in the basis $\{\gamma_\mu, Z_\mu, B'_\mu\}$ are given by

$$ M^2_{11} = \frac{T^2}{6} g_2^2 (17g_1^2 + 15g_2^2)/(g_1^2 + g_2^2)T^2, $$

$$ M^2_{12} = \frac{T^2}{6} g_1 g_2 \sqrt{g_1^2 + g_2^2} + \frac{T^2}{6} g_1 g_2 (g_2^2 - g_1^2)/(g_1^2 + g_2^2), $$

$$ M^2_{13} = \frac{T^2}{12} g_1 g_2 g' (2Q_1 - 2Q_2 - 3Q_3)/\sqrt{g_1^2 + g_2^2}, $$

$$ M^2_{22} = \frac{T^2}{6} g_1 g_2^2 /\sqrt{g_1^2 + g_2^2} + \frac{T^2}{36} (11g_1^4 + 8g_2^2 g_1^2 + 9g_2^4)/(g_1^2 + g_2^2), $$

$$ M^2_{23} = \frac{T^2}{12} g_1 g_2 g' + \frac{T^2}{12} g' (Q_1 - Q_2) (3g_1^2 + g_2^2) + 3Q_1 g_2^2)/\sqrt{g_1^2 + g_2^2}, $$

$$ M^2_{33} = \frac{T^2}{2} g^2 (Q_1^2 + Q_2^2 + Q_3 Q_T), \quad (30) $$

where $Q_{l,T}$ is the right-, left-handed top charge under $U(1)'$. One can obtain the thermal corrections for the MSMM by putting $g' = 0$ in (30), however the result is slightly different than in [40], because here we consider the scalar contributions.
Squarks. The left- and right-handed squarks are not mixed under the condition (28), and their thermal corrections to the diagonal elements are

\[
m^2_{\tilde{t}_L} = \frac{4}{9} g_s^2 T^2 + \frac{1}{4} g_2^2 T^2 + \frac{1}{108} g_s^2 T^2 + \frac{1}{6} y_t^2 T^2,
\]
\[
m^2_{\tilde{t}_R} = \frac{4}{9} g_s^2 T^2 + \frac{4}{27} g_2^2 T^2 + \frac{1}{3} y_t^2 T^2,
\]

where \(g_s\) and \(y_t\) are the strong and Yukawa couplings.

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