Addenda 2008 to variations QCD2007 (2nd)

On concise hypotheses for the interpretation of a wide scalar resonance as gauge boson binary in QCD $\rightarrow$ some new analyses

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after QCD2008, Montpellier, 6.-13. July 2008

This work aims to give some answers to questions raised at QCD2008 [1]. Fig. 1 plots the s-wave phase shifts versus $K = \frac{1}{2} (M_{\pi\pi}^2 - 4 m_{\pi}^2)^{1/2}$.

- from ref. [2] Colangelo, Gasser and Leutwyler,
- interpolates $\bullet$, $+$: from ref. [3] Na48/2 coll. corrected for isospin breaking , $\blacksquare$: from ref. [4] Protopopescu et al., $\blacksquare$: from ref. [5] CERN-Munich coll.; W. Ochs, thesis 1973.

$\blacksquare$ - $\blacksquare$: minimal meromorphic parametrization of the influence of $f_0(980)$ → $\rightarrow$ - $\rightarrow$ linear approximations $\delta_{00} = 0.5 a_{00}K$ , $\leftrightarrow$ $a_{00} m_{\pi} = 0.22$ , $a_{00} m_{\pi} = 0.16$.

ideal in the caption to figure 1 refers to the limit $e = 0$, $m_d = m_u$.

The rapid phase variation induced by $f_0(980)$ defines two fringes, denoted low and high, the two regions

$$\begin{align*}
\text{low} : & \quad 0 \leq K \leq \sim 0.9 \text{ GeV} \\
& \quad 2m_{\pi} \leq \sqrt{s} \leq \sim 0.94 \text{ GeV} \\
\text{high} : & \quad \sim 1.0 \text{ GeV} \leq K \leq \sim 1.6 \text{ GeV} \\
& \quad \sim 1.04 \text{ GeV} \leq \sqrt{s} \leq \sim 1.625 \text{ GeV}
\end{align*}$$

(1)

The minimal meromorphic parametrization is defined from the complex pole position on the second s-sheet, the K-plane with $\Im K < 0$

$$\begin{align*}
C_R^2 &= (K_R - \frac{1}{2} i \gamma_R)^2 = M_R^2 - s_0 = (M_R - \frac{1}{2} i \Gamma_R)^2 - s_0 \\
S_{\text{mmp}}(K_R, \gamma_R; K) &= \frac{|C_R|^2 - K^2 + i \gamma_R K}{|C_R|^2 - K^2 - i \gamma_R K}
\end{align*}$$

(2)

The analytically correct derivations from solving the Roy equations in the range limited by Lehmann ellipses are reviewed in ref. [6]. The combination of scattering data, used through absorptive parts between $0.8 \text{ GeV} \leq M_{\pi\pi} \leq 2 \text{ GeV}$ with ideal $\pi\pi$ scattering lengths, accurately determined through chiral expansions, lead to an apparently most definite prediction and evaluation of pole parameters in the $I=0$, s-wave channel in refs. [7] Caprini, Leutwyler and compared with results obtained in ref. [8] Kaminski, Pelaez and Yndurain in eqs. [4] and [4] below.
While the absolute systematic errors differ by a factor 3 - 4, this is by far not a proof of the correctness of these results, as discussed subsequently, and in any case does not change the apparent excellent agreement of deduced phase shifts as displayed in figure 1.

The evaluations following Caprini yield 4 sets compared below with results from ref. [8]

\[
\begin{align*}
M_\sigma &= 446 \pm 6 \text{ (stat)} \pm^{40}_4 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 534 \pm 12 \text{ (stat)} \pm^{88}_6 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 455 \pm 6 \text{ (stat)} \pm^{31}_5 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 556 \pm 12 \text{ (stat)} \pm^{68}_8 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 458 \pm 6 \text{ (stat)} \pm^{31}_3 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 506 \pm 12 \text{ (stat)} \pm^{66}_3 \text{ (syst)} \text{ MeV} \\
M_\sigma &= 496 \pm 6 \text{ (stat)} \pm^{31}_3 \text{ (syst)} \text{ MeV} \\
\Gamma_\sigma &= 516 \pm 16 \text{ (stat)} \pm^{78}_2 \text{ (syst)} \text{ MeV}
\end{align*}
\]

(3)

(4)

To figure 2:

\[ M_{\pi\pi} \equiv \sqrt{s} \text{ throughout} \]

\[ \delta_{00} = 0.5 \text{ a00 K} \]

\[ \mathcal{I} : \text{ from ref. [3] as in figure 1, with enlarged errors for systematics.} \]

\[ \mathcal{I} : \text{ from ref. [9] with statistical errors, lowest } M_{\pi\pi} \text{ bin only.} \]

\[ \mathcal{I} : \text{ minimal meromorphic phase from the superposition of f0(980) and gb with masses and widths as indicated in the figure.} \]

\[ \mathcal{I} : \text{ background relative to the minimal meromorphic phase, chosen to follow the lower boundary along the low fringe permitted by } \mathcal{I} \text{ and to maintain optimal agreement in the threshold- and high fringe regions.} \]

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\[
S_{mmp}(K) = \prod_{\alpha=1}^{N} S_{mmp}(K_{R_\alpha}, \gamma_{R_\alpha}; K)
\]

(5)
The background introduced above for $J^{PC} = 0^{++}, I = 0; \pi\pi \rightarrow \pi\pi$ is defined relative to $S_{mmp}^N$ given in eq. (5)

$$S = S_{bg}^N S_{mmp}^N ; S_{bg}^N (K) = \eta_{bg}^N (K) \exp \left( 2i \delta_{bg}^N (K) \right) \quad (6)$$

It follows from the meromorphic structure of $S_{mmp}^N$, that the presence in $T = \frac{1}{2} (S - 1)$ of an Adler 0, for $-K^2 = \kappa^2 = 4m_\pi^2 - s > 0; \kappa > 0$ requires a nontrivial background $\rightarrow S_{bg}^N \neq 1$. For we use the parametrization

$$\delta_{bg}^2 (K) = (K / K_1)^3 e^{-B K^2} ; K_1 = 0.59 \text{ GeV} , B = 4.2 \text{ GeV}^{-2}$$

$$\eta_{bg}^2 (K) = 1; \text{ with modifications particularly for } N = 2 \rightarrow N = 3 \quad (7)$$

centering on the low fringe region, and coming back to inelasticities in the high fringe below in conjunction with the third resonance $f_0(1500)$ and figure 3. I follow the hypotheses and derivations presented in refs. [11] and concerning the role of $f_0(1500)$ in the decays $B \rightarrow K \pi\pi$, $K K\pi\pi$ [12] in collaboration with Wolfgang Ochs.

**To figure 3:**

This is an extension of figure 2 to include the influence of three resonances $f_0(980)$, gb and $f_0(1500)$.

- : as in figure 1.
- : as in figure 2 except for the color.
- : minimal meromorphic phase from the superposition of gb and $f_0(980)$ but with different $f_0$ mass $m f_0 = 0.99 \text{ GeV}$, same ratio $\Gamma f_0 / m f_0 = 0.055$.
- : background phase added with same mass and width parameters as for and $K_1 = 0.62 \text{ GeV} (\text{eq. 7})$.
- : as in figure 2, with $m f_0 = 0.98 \text{ GeV}, \Gamma f_0 / m f_0 = 0.055$.
- : minimal meromorphic phase from the superposition of gb, $f_0(980)$ and $f_0(1500)$ with mass and width parameters $m f_0(1500) = 1.51 \text{ GeV}, \Gamma f_0(1500) / m f_0(1500) = 0.07$, and background parameters $\eta_{bg}^2 = 1$ to keep qualitative features of $f_0(1500)$ only and $K_1 = 0.62 \text{ GeV} , B = 4.2 \text{ GeV}^{-2} (\text{eq. 7})$.

The rise of the s-wave phase towards the end of the high fringe region was remarked in ref. [8].

It formed the entry point of the discussion in ref. [11].
To figure 4:
Here we present aspects of the absorptive part \( \Im t_{00} \)
with \( t_{00} = (M_{\pi\pi}/K) \frac{1}{2\pi} \left( S_{bg}^N S_{mmp}^N - 1 \right) ; N = 2, 3 \)
and compare with the analyses of [13] Au, Morgan and Pennington,
[2] Colangelo, Gasser and Leutwyler and the solution to the
Roy equations [14] Ananthanarayan, Colangelo, Gasser and Leutwyler.
The resonance parameters used for \( N = 2 \) and \( N = 3 \) are
\[
\begin{align*}
m_{f_0} &= 0.99 \text{ GeV} & \Gamma_{f_0}/m_{f_0} &= 0.055 \\
m_{g_{bg}} &= 1.0 \text{ GeV} & \Gamma_{g_{bg}}/m_{g_{bg}} &= 0.9 \\
m_{f_0(1500)} &= 1.51 \text{ GeV} & \Gamma_{f_0(1500)}/m_{f_0(1500)} &= 0.07
\end{align*}
\]
The inelasticity is extended to include ( for \( \pi\pi \) elastic ) two \( I = 0 \) \( \pi\pi \) and \( KK \) two-body channels
\[
\begin{align*}
\eta_{bg}^{2,3}(K) &= \partial(K_{th} - K) + \partial(K - K_{th}) a e^{-b K^{'}/K^{'15}} \\
K^{'15} &= (m_{f_0(1500)}^2 - 4 m_{K}^2)^{1/2}; m_{K} = 0.49565 \text{ GeV} \\
K_{th} &= (4 m_{K}^2 - 4 m_{\pi}^2)^{1/2}
\end{align*}
\]
with parameters fixed at \( a = 1 \); \( b = \log 0.6 = 0.5108 \); \( m_{\pi} = 0.13957 \text{ GeV} \).
No data is used to determine the elasticity parameter \( \eta_{bg}^{2,3}(M_{\pi\pi} = m_{f_0(1500)}) \sim 0.6. \)

To figure 4 (continued)

\( \Im t_{00} \) from ref. [13].
\( \Im t_{00} \) from ref. [2].

Concluding remarks

1) the main analyses of reactions \( \pi N \rightarrow \pi\pi N (\Delta) \) in refs. [4], [5] cannot
be taken at face value for the derived elastic \( \pi\pi \) s-waves within the quoted
errors, in both low and high fringe regions
( defined in eq. (4) )

2) derivations and hypotheses discussed in refs. [11], [12] are basically correct.

3) claims of a scalar resonance pole in the region within a radius of at least
150 MeV around the position \( \sqrt{s} = 500 - \frac{i}{2} 500 \text{ MeV} \) on the second
s-sheet of elastic \( \pi\pi \) scattering are incorrect.

I wish to dedicate this work to the memories of Jan Stern, Francisco
Yndurain and Peter Schlein.
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