Expansion of the domain of existence of the cylindrical involute gears

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Abstract. The method for optimal geometric synthesis of gearing of cylindrical involute gears is proposed in this paper. A new geometry settlements system has been proposed. The technique developed on the basis of this settlements system can significantly expand the domain of existence of the solution to the synthesis problem, as well as the domain of existence of gear.

1. Introduction
The geometric synthesis is always an urgent task for the design of gear transmission. When applying to the geometric synthesis, it is necessary to determine the basic geometric parameters of the gears, ensuring transmission performance and the fulfillment of a number of conditions for quality characteristics. For the geometric synthesis of cylindrical gears, a locking loop method was proposed in the paper [1]. This is the most common method in the design of gears, allowing purposefully choose the addendum modification coefficients of the wheels. The method of blocking contours is quite simple, clear and included in the current standards. The disadvantage of this method when designing gears is the impossibility of building block circuits for all possible combinations of numbers of teeth and tool parameters, as well as the limitations of the synthesis of several gears, for example of internal gears with a small difference of numbers of teeth of gear wheels.

The domain of existence of a gear transmission on blocking contours is the range of values of addendum modification coefficients of wheels in which the gear wheel and a gear pair can exist. The shape and size of the field of the blocking contour are significantly influenced by the type of engagement, the number of teeth, the choice of the calculation system, the method of cutting, etc. In geometric synthesis, the idea of expanding the field of the blocking circuit was indicated in the paper [1] by various methods, including a change in the settlements system, refusing to maintain standard radial clearances.

In order to expand the area of existence of the solution to the problem of geometric synthesis as well as the area of existence of gear transmission, a new geometry calculation system was developed in which the radial clearances are assigned to the output parameters of the synthesis [2]. That is the composition of the output parameters of geometric synthesis consists of displacement coefficients and radial clearances.

It is known that for geometric synthesis of internal gearing with a small difference in the number of teeth, such a settlements system allows several times to expand the area of existence of the solution to the synthesis problem, and in some cases, the domain is missing while maintaining standard spaces [3]

The solution to the synthesis problem is greatly complicated by the geometric synthesis of a four-bar gear transmission, in which one of the gears simultaneously participates in two gearings since the
number of synthesis conditions increases several times, and the number of output synthesis parameters increases from three to six. Furthermore, with a separate synthesis of two gears, the geometrical parameters of the spurious gear can be different, which requires solving two interrelated problems.

2. Theoretical part

When calculating the geometric parameters of gears the following parameters are considered given: module \( m \); numbers of gear teeth \( z_1, z_2, z_3 \); the addendum modification coefficients \( x_1, x_2, x_3 \); the clearance coefficients \( c_{12}^*, c_{21}^*, c_{23}^*, c_{32}^* \). Designations of geometric parameters and calculation formulas correspond to GOST 16532-60 and GOST 19274-73.

With non-standard radial clearances, the diameters of the vertex circles should satisfy the dependencies

\[
\begin{align*}
d_{a1} &= 2a_{w12} - d_{f2} - 2c_{12}^* m; \\
d_{a2} &= 2a_{w12} - d_{f1} - 2c_{21}^* m; \\
d_{a2} &= d_{f3} - 2a_{w23} - 2c_{23}^* m; \\
d_{a3} &= d_{f2} + 2a_{w23} + 2c_{32}^* m.
\end{align*}
\]

where \( a_{w12}, a_{w23} \) – the working interaxle distance of the external and internal gearing; \( d_{f1}, d_{f2}, d_{f3} \) – the diameters of the circles of the depressions of the wheels.

Since the satellite \( z_2 \) simultaneously in two gears: external and internal, the condition shall be met

\[
2a_{w12} - d_{f1} - 2c_{21}^* m = d_{f3} - 2a_{w23} - 2c_{23}^* m.
\]  

(2)

This condition leads to a decrease in the number of output synthesis parameters to six, and the coefficient of radial clearance \( c_{23}^* \) is determined by the equation (2).

The problem of geometric synthesis for a coaxial 2K-H type planetary gear is more complicated, since it requires the fulfillment of a number of conditions and limitations of the geometric synthesis of planetary gears. One of the important conditions is the condition of the alignment of the transmission, which interconnects the interaxial distances of the external and internal engagement [4].

In general, the alignment condition is reduced to [4]

\[
\frac{z_1 + z_2}{\cos \alpha_{w12}(x_1, x_2)} = \frac{z_3 - z_2}{\cos \alpha_{w23}(x_2, x_3)}
\]  

(3)

In the particular case when \( z_1 + z_2 = z_3 - z_2 \) through the involute of link angles, the alignment condition will have the form

\[
x_1 + x_2 = x_3 - x_2 \text{ or } x_3 = x_1 + 2x_2
\]  

(4)

In this case, the number of output synthesis parameters is reduced to five, and the addendum modification coefficient \( x_2 \) determined by equation (3) or in a particular case – (4).

For the synthesis of efficient gears, it is necessary to fulfill the following additional conditions: the absence of teeth undercutting, the absence of interference of the teeth, the absence of cutting of the teeth, the absence of sharpening of the teeth, represented as synthesis restrictions. Transmission performance is characterized by overlap factors as \( \varepsilon_{12}, \varepsilon_{23} \) and engagement angles \( \alpha_{w12}, \alpha_{w23} \).
3. Algorithm and software development

To solve the optimization problem in geometric synthesis, the paper [5] highlights a technique was proposed for the optimal geometric synthesis of a four-link mechanism with associated cylindrical gears participating in one external and one internal gearing. The work [6] proposed a technique for coaxial 2K-H type planetary gear.

Since the selected qualitative characteristics of the transmission are contradictory, a compromise solution of the optimal geometric synthesis problem is required. The objective function is adopted as:

\[ F(x_1, x_2, c_{12}, c_{21}, c_{32}) = k_1 \frac{1}{e_{\alpha_{12}}} + k_2 \frac{1}{e_{\alpha_{23}}} + k_3 k'_3 \alpha_{w_{12}} + k_4 k'_4 \alpha_{w_{23}} \]

where \( k_1, k_2, k_3, k_4 \) – weighting coefficients, the values of which are assigned depending on the importance of the criteria in solving a specific problem, wherein \( k_1 + k_2 + k_3 + k_4 = 1; \ k'_3, k'_4 = 2.3 \) are normalization factors.

To solve the problem of nonlinear mathematical programming, the library of Optimization Toolbox functions included in MatLab was used. The program allows to find the minimum of the objective function while ensuring the conditions and constraints of synthesis in the form of equalities and inequalities. The main window of the program is shown in figure 1.

![Program main window](image)

**Figure 1.** Program main window.

The program allows you to enter the initial data of the gears and the tool, the area of permissible values of the optimized parameters, the values of the weight coefficients in the dialog mode or from a prepared Microsoft Excel table. In the field of "Optimization" displays the optimal values of the output parameters of the synthesis, and in the field of "Restrictions" – the values of quality characteristics and reserves on the constraints of the synthesis.

4. Algorithm and software development

The table 1 shows a comparison of the results of calculating the transmission parameters with one external and one internal gearing for the optimal option and options in which the values of the components of the vector of the output synthesis parameters were changed by (compared to optimal) 5-20%.
Initial data for calculations: module \( m = 2.5 \); gear tooth numbers \( z_1 = 15 \), \( z_2 = 30 \), \( z_3 = 80 \). For cutting gears with external teeth a shaper cutter 2536-0109 is used, wheels with internal teeth a shaper cutter 2530-0168 in accordance with GOST 9323-79.

| Parameter | \( \chi_1 \) | \( \chi_2 \) | \( \chi_3 \) | \( c_{12} \) | \( c_{21} \) | \( c_{23} \) | \( c_{32} \) | \( \varepsilon_{\alpha 12} \) | \( \varepsilon_{\alpha 23} \) | \( \alpha_{w12}, \text{deg} \) | \( \alpha_{w23}, \text{deg} \) | Constraint satisfaction | Object function (\( F \)) |
|-----------|--------|--------|--------|---------|---------|---------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| Options   | Optimal | 1      | 2      | 3       | 4       | 1      | 2      | 3       | 4       | 1      | 2      | 3       | 4       | 1      | 2      | 3       | 4       |
| \( \chi_1 \) | 0.0374 | 0.0380 | 0.0410 | 0.0320  | 0.0430  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \chi_2 \) | -0.1417 | -0.1000 | -0.1100 | -0.1600 | -0.1300 |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \chi_3 \) | 0.0535 | 0.0500 | 0.0460 | 0.0560  | 0.0520  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( c_{12} \) | 0.2000 | 0.2100 | 0.2200 | 0.2300  | 0.2240  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( c_{21} \) | 0.2000 | 0.2300 | 0.2100 | 0.2400  | 0.2370  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( c_{23} \) | 0.2000 | 0.2264 | 0.2065 | 0.2426  | 0.2354  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( c_{32} \) | 0.4093 | 0.4200 | 0.4400 | 0.4340  | 0.4600  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \varepsilon_{\alpha 12} \) | 1.6610 | 1.6211 | 1.6314 | 1.6212  | 1.6142  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \varepsilon_{\alpha 23} \) | 1.8013 | 1.7587 | 1.7528 | 1.7453  | 1.7125  |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \alpha_{w12}, \text{deg} \) | 19.2399 | 19.5558 | 19.5043 | 19.0574 | 19.3703 |        |        |        |        |        |        |        |        |        |        |        |        |
| \( \alpha_{w23}, \text{deg} \) | 21.1546 | 20.8999 | 20.9342 | 21.2702 | 21.0812 |        |        |        |        |        |        |        |        |        |        |        |        |
| Constraint satisfaction | done | done | done | done | done |        |        |        |        |        |        |        |        |        |        |        |        |        |
| Object function (\( F \)) | 0.6947 | 0.7024 | 0.7017 | 0.7022  | 0.7068  |        |        |        |        |        |        |        |        |        |        |        |        |

Table 1. Program testing

Table 2. Comparison with existing settlement system

| Name of quality indicators | Option 1 | Option 2 | Option 3 |
|----------------------------|----------|----------|----------|
| Engagement factor          | \( \varepsilon_{\alpha 12} \) | 1.7153   | 1.3594   | 1.2980   |
|                           | \( \varepsilon_{\alpha 23} \) | 1.7362   | 1.7021   | 1.4500   |
| Working pressure angle (degree) | \( \alpha_{w12} \) | 19.3366  | 25.2461  | 25.3711  |
|                           | \( \alpha_{w23} \) | 19.3366  | 25.2461  | 25.3711  |
| Constraint satisfaction    | done     | done     | done     | done     |
| Profile shift coefficient   | \( \chi_1 \) | -0.2237  | 1.0000   | 0.5000   |
|                           | \( \chi_2 \) | 0.1220   | 0.1000   | 0.6340   |
|                           | \( \chi_3 \) | 0.0204   | 1.2000   | 1.7680   |
|                           | \( c_{12} \) | 0.2000   | 0.2500   | 0.2500   |
| Clearance coefficient      | \( c_{21} \) | 0.2051   | 0.2500   | 0.2500   |
|                           | \( c_{23} \) | 0.2000   | 0.2500   | 0.2500   |
|                           | \( c_{32} \) | 0.4132   | 0.2500   | 0.2500   |
| Object function            | \( F \) | 0.6779   | 0.8375   | 0.8635   |
Table 2 compares the results of calculating the planetary transmission 2K-H parameters for the new calculation system (option 1) and for existing calculation systems (option 2 [1], option 3 [7]).

Initial data for calculations: module $m = 2.5$; gear tooth numbers $z_1 = 20; z_2 = 30; z_3 = 80$. For cutting gears with external teeth a shaper cutter 2536-0109 is used, wheels with internal teeth a shaper cutter 2530-0168 in accordance with GOST 9323-79.

Tables 1 and 2 show that at the optimal point with the new settlements system all the restrictions are met, and the minimum value of the objective function is achieved in comparison with other points and the existing settlements system.

4. Summary and Conclusions

The proposed method for geometrical synthesis of gear transmission, based on the new calculation system, allows to increase the number of output synthesis parameters from three to five and to expand the domain of the existence of geometric synthesis problem solution, to quickly select the most optimal gear option, ensuring the fulfillment of all synthesis constraints and obtaining specified values of quality indicators corresponding to the specific problem being solved.

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