Diquark electromagnetic form factors in a Nambu–Jona-Lasinio model†

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Abstract

Electromagnetic properties of diquarks are investigated in the framework of a color–octet Nambu–Jona-Lasinio model, which describes baryons as bound states of diquarks and quarks. We calculate the electromagnetic form factors of scalar and axial vector diquark bound states using the gauge–invariant proper–time regularization. The nucleon charge radii and magnetic moments are estimated in a simple additive diquark–quark model. This picture reproduces the qualitative features such as the negative charge radius of the neutron. Within this model axial vector diquarks are seen to be important.

† Supported by COSY under contract 41170833 and by the DFG under contract Re 856/2–1

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1. Introduction

A prime objective of hadronic physics is the understanding of baryons as bound states of QCD. Since the direct application of QCD in the nonperturbative regime is extremely difficult, models for baryons have been constructed which preserve certain features of QCD believed to be essential. Such models include the non-relativistic quark model, in which baryons are described as constituent quarks moving in a phenomenological potential, or soliton models based on the limit \( N_C \to \infty \). In both these approaches the description of baryons is essentially of a static nature, which entails serious difficulties due to the lack of proper quantization of the translational motion. A fully relativistic description of baryons can be accomplished in an approach in which baryons are considered as bound states of diquarks and quarks. Evidence for correlated diquark states in baryons is found in deep–inelastic lepton scattering [1] and in hyperon weak decays [2]. Attempts have been made to describe diquarks and baryons in non-local approximations to QCD [3]. A more tractable approach is based on a local effective quark model, a Nambu–Jona-Lasinio (NJL–) model with a color–octet current–current interaction. This model arises as the leading term in a current expansion of the quark–quark interaction and can also be motivated semiclassically [4]. It describes the spontaneous breaking of chiral symmetry, i.e., the dynamical generation of quark masses. Furthermore, after Fierz transformation of the interaction the color-singlet part of the model can be bosonized exactly and provides a realistic description of meson properties [5, 6, 7]. Diquark bound states have been considered in [8, 9]. In order to study baryons, the NJL model with color-octet interaction has been transformed into an effective theory of meson and baryon fields by use of functional integral techniques [10]. In this theory bound states of diquarks and quarks occur due to quark exchange. In [11] the Faddeev–type equation for the binding of a scalar diquark and a quark has been solved using a static approximation for the quark exchange interaction. The resulting masses of the \( \frac{1}{2}^+ \)–baryon octet are in satisfactory agreement with experiment.

Our aim is to extend this approach to the study of the low–energy electromagnetic structure of baryons. As a first step towards describing baryon form factors in the effective hadron theory derived from the NJL model, we calculate here the on-shell electromagnetic form factors of the constituent diquarks. The relevant diquark bound states occur in the scalar (0\(^+\)) and axial vector (1\(^+\)) channel. The diquark form factors are a crucial ingredient in the baryon form factor and contain information about the sizes of the correlated diquark states. To ensure gauge invariance, the NJL quark loop is regularized using Schwinger’s proper–time method [13]. We also compare with the results obtained with the usual sharp–cutoff regularization. With the diquark form factors we then estimate the nucleon charge radii and magnetic moments in an additive model of extended diquarks and pointlike valence quarks, using the spin–flavor wave functions of the non-relativistic quark model. Such a picture may be regarded as a crude approximation to the baryon wave function.

2. The model

The basis of our description is an effective quark Lagrangian of the NJL type,

\[
\mathcal{L}_q = \bar{q}(i\partial - m^0)q - \frac{1}{2}g j^A \gamma^\mu j^\mu A,
\]

where \( j^\mu A = \bar{q} \gamma^\mu \gamma^A q \) is the color-octet current of the quark field, \( g \) an effective coupling.

\(^a\)Recently, direct numerical solutions to the Faddeev equation for the nucleon have been obtained [12], which support the static approximation [11].
constant and $m^0$ the current quark mass matrix. By way of Fierz transformations in color, flavor and Dirac space the interaction in eq.(1) can be rewritten equivalently as an interaction in the color-singlet ($1_C$) quark–antiquark and color-antitriplet ($\bar{3}_C$) quark–quark channel [10],

$$-\frac{3}{2} g J^A \mu = g_1 (q \Lambda^a q) (q \Lambda^a q) + g_2 (q \Gamma^a C q^T) (q^T C \Gamma^a q).$$

Here, $C = i \gamma^2 \gamma^0$ is the charge conjugation matrix and the vertices $\Lambda^a, \Gamma^a$ are matrices in color, flavor and Dirac space,

$$\Lambda^a = 1_C \left( \frac{\lambda^a}{2} \right) F^a, \quad \Gamma^a = \left( \frac{i \epsilon^a}{\sqrt{2}} \right) C \left( \frac{\lambda^a}{2} \right) F^a, \quad \alpha = (A, a, a),$$

where

$$O^a \in \left\{ 1, i \gamma_5, \frac{i \gamma^\mu}{\sqrt{2}}, \frac{i \gamma^\mu \gamma_5}{\sqrt{2}} \right\}$$

and $(\epsilon^A)_{BC} = \epsilon_{ABC}$ are the antisymmetric generators of the $\bar{3}$–representation of $SU(3)_C$. From the Fierz transformation one obtains $g_1 = g_2 = g$. However, the two terms in eq.(2) are independently chirally invariant, so that chiral symmetry does not preclude different effective coupling constants in the meson and diquark channel.

By introducing collective meson and baryon fields, the generating functional defined by the quark lagrangian, eq.(1), has been rewritten as an effective hadron theory [10]. In the course of this reformulation one also introduces collective fields for diquarks, which form building blocks for baryons. One obtains an effective action of meson and diquark fields, $\phi^a$ and $\Delta^a$, coupling to a baryon source, $\chi$,

$$S = -\frac{3}{4g} \int d^4x \left( \phi^a \phi^a + \Delta^a \Delta^a \right) - \frac{1}{2} \text{Tr} \log G^{-1} - \int d^4 x \left( \bar{\chi} \chi^T \right) G_B[\Delta] \left( \begin{array}{c} \chi \\ \chi^T \end{array} \right).$$

Here, $G$ is the quark Green’s function in the Nambu–Gorkov formalism of superconductivity [4],

$$G^{-1} = \begin{pmatrix} G^{-1} & \Delta C^\dagger \\ -C \Delta & -G^{-1} T \end{pmatrix}, \quad G^{-1} = i\phi - m_0 - \phi - \frac{3}{g} \int d^4 x \bar{\chi} \chi,$$

and the collective meson and diquark fields couple to the quarks through the vertices $\phi = \phi^a \Lambda^a$ and $\Delta = \Delta^a \Gamma^a$. In particular, the vacuum value of the scalar meson field generates the constituent quark mass, $M$. The form of the baryon propagator, $G_B[\Delta]$ is given in [10]. After integrating out the diquark fields one obtains from eq.(5) an effective hadron theory, in which diquarks and quarks interact through quark exchange [10] and form bound baryon states [11].

The quark loop in eq.(5) is defined with a cutoff procedure, which is an important physical ingredient of this effective model. A simple regularization scheme is a sharp Euclidean cutoff, applied to the momentum integrals resulting from the expansion of $\text{Tr} \log G^{-1}$ in meson and diquark fields. This method in general breaks gauge invariance,

\footnote{Following ref. [10] we denote $2 \times 2$–matrices in the Nambu–Gorkov formalism by bold sans-serif letters.}
when an electromagnetic field is introduced. A gauge invariant regularization is provided by the proper–time method of Schwinger [13], which defines the real part of the quark loop, after continuation of \( G \) to Euclidean space, as

\[
\text{Re} \text{Tr}_A \log G_E = \frac{1}{2} \int_{\Lambda^2}^\infty \frac{ds}{s} \text{Tr} \exp(-s G^*_E G_E).
\]

The imaginary part of the Euclidean quark loop, which generates anomalous (intrinsic parity–violating) meson and diquark processes, is finite and left unregularized [3].

3. Diquark electromagnetic form factors

An important ingredient in the electromagnetic structure of baryons are the electromagnetic form factors of the constituent diquarks. To describe the electromagnetic interactions of diquarks, we couple an electromagnetic field to the quark fields of eq. (4), by way of minimal substitution, \( i \bar{\psi} \rightarrow i \bar{\psi} - QA_\mu \gamma^\mu \), where \( Q = \frac{1}{2} e (\lambda^3 + \frac{1}{2} \lambda^8) \) is the quark charge matrix. We then expand the effective action, eq. (5), in the background of the electromagnetic field to second order in the diquark field. (Here, the baryon source is set to zero, \( \chi = 0 \), and the meson field is left at its vacuum value.) In Minkowski space and in momentum representation, this expansion is of the form

\[
S = S_0 + \int \frac{d^4p}{(2\pi)^4} \Delta^*_\alpha(-p) \mathcal{D}^{-1\alpha\beta}(p) \Delta_\beta(p) + \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \Delta^*_\alpha(-p - \frac{i}{2} q) \Delta_\beta(p - \frac{i}{2} q) A_\mu(q) \mathcal{F}^{\alpha\beta\mu}(p, q),
\]

where \( \mathcal{D} \) is the diquark propagator and \( \mathcal{F} \) the electromagnetic vertex function. The masses of the diquark bound states are determined as the zeros of the inverse propagator. On-shell diquark form factors are then obtained by evaluating the vertex function for appropriate incoming and outgoing four-momenta \( p \pm \frac{i}{2} q \) on the diquark mass shell and normalizing the fields in eq. (8) to unit residue of the propagator. Note that the electromagnetic couplings of the diquark fields come entirely from the quark loop of eq. (3). The diquark propagator and electromagnetic vertex function are diagonal in color and of the form

\[
\mathcal{D}^{-1\alpha\beta}(p) = \delta^{AB} \sum_{ij} \left( \frac{\lambda^a}{2} \right)_{ij} \left( \frac{\lambda^b}{2} \right)_{ji} \left( -\frac{3}{2g_2} g^{ab} + I_{ij}^{ab}(p) \right),
\]

\[
\mathcal{F}^{\alpha\beta\mu}(p, q) = \delta^{AB} \sum_{ij} \left( \frac{\lambda^a}{2} \right)_{ij} \left( \frac{\lambda^b}{2} \right)_{ji} \left( Q_i J_{ij}^{ab\mu}(p, q) - Q_j J_{ji}^{ba\mu}(p, q) \right).
\]

In cutoff regularization, the functions \( I_{ij}^{ab} \) and \( J_{ij}^{ab\mu} \) are given by the loop integrals

\[
I_{ij}^{ab}(p) = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr}_{\text{Dirac}} \left[ G_i(k - \frac{i}{2} p) O^a G_j(k + \frac{i}{2} p) O^b \right],
\]

\[
J_{ij}^{ab\mu}(p, q) = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr}_{\text{Dirac}} \left[ G_i(k - \frac{i}{2} p) O^a G_j(k + \frac{i}{2} p - \frac{i}{2} q) \gamma^\mu G_j(k + \frac{i}{2} p + \frac{i}{2} q) O^b \right],
\]

where \( G_i(p) = (\not{p} - M_i)^{-1} \) is the quark propagator. In proper–time regularization corresponding expressions are derived using standard techniques for the expansion of the time–ordered exponential, eq. (7) [13]. Only the final expressions will be given below.

\[\text{The symbol } \delta^{ab} \text{ means the metric tensor if } a, b \text{ are Lorentz indices, otherwise } \delta^{ab}.\]
Table 1: The masses of the scalar ud– and the axial vector uu–diquark, \( m_0^+ \) and \( m_1^+ \), and their electromagnetic charge radii, for various values of the effective diquark coupling constant, \( g_2 \). Also shown is the magnetic moment of the axial uu–diquark, \( \mu_1^+ \), in units of \( 2Q_ue / 2M \). Proper–time regularization is used; the parameters are \( M = 400 \text{ MeV} \), \( \Lambda = 630 \text{ MeV} \), \( m_0 = 17 \text{ MeV} \).

We now evaluate eqs. (9, 10) for the scalar and the axial vector diquark channel, which are relevant to the description of baryons. We use throughout the proper–time regularization and only compare the numerical results with those obtained with cutoff regularization. Since we shall consider only the nucleon later we assume the isospin limit, \( M_u = M_d = M \).

Let us first consider the scalar ud–diquark (\( O_a^1 = i\gamma^5 \)). In proper–time regularization one finds, after continuation to Minkowski space \( d \),

\[ I_0^a(p) = p^2 A_0(p^2) + 2M^2B, \]

with

\[ A_0(p^2) = \frac{1}{16\pi^2} \int_0^1 d\alpha \Gamma(0, \frac{M^2 - \alpha(1-\alpha)p^2}{\Lambda^2}), \]

\[ B = \frac{1}{16\pi^2} \Gamma(-1, \frac{M^2}{\Lambda^2}). \]

The second term in eq. (12) is related to the quark condensate, \( \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = 4N_C M^3 B \). The scalar diquark mass, \( m_0^+ \), is shown in table 1 for different values of the effective diquark coupling constant, \( g_2 \). Here, the coupling in the meson channel, \( g_1 \), is determined by the constituent quark mass through the gap equation, the cutoff is fixed by fitting the pion decay constant, and \( m_0 \) is determined from the pion mass. Diquark couplings of \( g_2/g_1 \sim 2 \) are required to reproduce the masses of the spin–\( \frac{1}{2} \) baryons if quark exchange is included \[d\]. A ratio \( g_2/g_1 \sim 2.5 \) is needed in an additive diquark–quark model, see below. Similar values are also required to obtain sufficiently bound axial diquarks.

To obtain the electromagnetic form factor for an on–shell scalar diquark of mass \( m_0^+ \), we evaluate the vertex function, eq. (10), for incoming and outgoing momenta \( p \pm \frac{1}{2} q \) with \( (p \pm \frac{1}{2} q)^2 = m_0^+ \), which entails \( p \cdot q = 0 \). On the mass shell, the vertex function is transverse to the photon momentum,

\[ J_0^\mu(p, q) = 2F_0^+(q^2) p^\mu, \]

and the normalized diquark form factor is defined as \( f_0^+(q^2) = F_0^+(q^2)/Z_0^+ \), with \( Z_0^+ = \partial / \partial p^2 I_0^+|_{p^2 = m_0^2} \). In proper–time regularization one finds

\[ F_0^+(q^2) = A_0(m_0^2) + (m_0^2 - \frac{1}{2} q^2) C_1(q^2) + \frac{1}{2} q^2 C_2(q^2) \]

\[d\text{We suppress the flavor indices on } J^{ab}_0 \text{ and } J^{ab} \text{ in the following.} \]
The scalar diquark form factor as a function of the photon momentum, $q^2$, is shown in fig. 1, for a diquark mass of $m_0^+ = 318$ MeV. In particular, $f_0^+(0) = 1$, i.e., the total charge is conserved as a consequence of the gauge–invariant regularization. The scalar diquark r.m.s. charge radius, $\langle r^2 \rangle_0^+ = -6 \partial^2 \partial q^2 f_0^+ |_{q^2=0}$ is given in table 1.

In sharp cutoff regularization the scalar diquark mass and form factor are evaluated by continuing the integrals (11) to Euclidean space, introducing Feynman parameters and applying the cutoff after a shift of the integration variable. Due to the breaking of gauge invariance by the cutoff, $f_0^+(0) \neq 1$ if $m_0^+ > 0$, so that we have divided $f_0^+(q^2)$ by $f_0^+(0)$ to obtain meaningful results. Determining the parameters as above in the meson sector, somewhat larger ratios $g_2/g_1$ are required to obtain the same diquark masses as in proper–time regularization. Nevertheless, comparing the charge radii for scalar diquarks of the same mass, the results are quite close in both regularization schemes, cf. table 2. The virtue of the gauge–invariant proper–time regularization, eq.(7), is that it is applied at the level of the action, rather than at the level of individual loop integrals, which is necessary for the electromagnetic Ward identities to be satisfied. In particular, with this method the electromagnetic vertices are also defined off the diquark mass shell.

It is instructive to compare the form factor of the scalar $ud$–diquark to that of the charged pion [14]. The only difference between the diquark and the meson vertex function is the sign of the quark charges in eq.(10) and a factor $N_C$ multiplying the quark loop.

where

$$C_{1,2}(q^2) = \frac{1}{16\pi^2} \int_0^1 d\beta \int_0^{1-\beta} d\alpha X_{1,2} \frac{\exp(-Y^2/\Lambda^2)}{Y^2},$$

$$X_1 = \alpha, \quad X_2 = 1, \quad Y^2 = M^2 - \alpha(1-\alpha)m_0^2 - \beta(1-\alpha-\beta)q^2$$

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Table 2: Comparison of the diquark charge radii for proper–time and cutoff regularization. Here, the ratio \( g_2/g_1 \) is chosen independently in both schemes in order to obtain the given scalar diquark masses.

| \( m_0^+ /\text{MeV} \) | \( m_1^+ /\text{MeV} \) | \( \langle r^2 \rangle_{0^+} /\text{fm} \) | \( \langle r^2 \rangle_{1^+} /\text{fm} \) | \( m_0^+ /\text{MeV} \) | \( \langle r^2 \rangle_{0^+} /\text{fm} \) | \( \langle r^2 \rangle_{1^+} /\text{fm} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 400            | 778            | .48            | .90            | 785            | .43            | 1.02           |
| 300            | 761            | .48            | .74            | 770            | .42            | 0.79           |

However, since the same factor \( N_C \) occurs also in the meson propagator, this factor cancels if we consider the normalized on–shell pion form factor. Thus, in the isospin limit the pion form factor is given by the expression for the diquark form factor, eq.(15), evaluated with \( m_{0^+}^2 \rightarrow m_{\pi}^2 \). The curve for the pion form factor as a function of \( q^2 \) is almost identical to that of the scalar diquark shown in fig. 1.

One may consider the scalar diquark (or pion) r.m.s. charge radius in table 1 as a function of the bound state mass. For strongly bound diquarks the charge radius is essentially independent of the diquark mass and thus practically identical to that of the pion. It is therefore, to good approximation, given by the result of a gradient expansion around \( p^2 = 0 \) [13], which in proper–time regularization reads

\[
\langle r^2 \rangle_{0^+} = \frac{N_C}{4\pi^2f_{\pi}^2}\exp(-M^2/\Lambda^2).
\]

In the other extreme, near \( m = 2M \), where the diquark would become unbound, the charge radius grows like \( \langle r^2 \rangle_{0^+} \sim (m - 2M)^{-1} \). This is the behaviour expected for a weakly bound state below a continuum threshold [16]. Such divergent behaviour would be absent in a model which incorporates quark confinement.

Let us now consider the axial vector diquark \( (O^{a,b} = -iF_{\rho}\sigma/\sqrt{2}) \). With the gauge–invariant proper–time regularization and in the absence of flavor symmetry breaking, the axial diquark polarization operator deriving from the quark loop is transverse,

\[
I_{1^+}^{\rho\sigma}(p) = p^2A_1(p^2)(g^{\rho\sigma} - \frac{p^\rho p^\sigma}{p^2}),
\]

with

\[
A_1(p^2) = \frac{1}{16\pi^2} \int_0^1 d\alpha 2\alpha(1 - \alpha) \Gamma \left( 0, \frac{M^2 - \alpha(1 - \alpha)p^2}{\Lambda^2} \right).
\]

The longitudinal part of eq.(16) is thus given entirely by \((3/2g_2)g^{\rho\sigma}\). Axial diquark masses are given in table 1 for various values of \( g_2 \). In contrast to the scalar diquark the axial diquark is only weakly bound for reasonable values of \( g_2 \). The electromagnetic vertex function, eq.(10), for on–shell axial vector diquarks of incoming and outgoing four-momenta \( p \pm \frac{1}{2}q \), with \( (p \pm \frac{1}{2}q)^2 = m_{1^+}^2 \), is of the form [17]

\[
F_{1^+}^{\rho\mu}(p,q) = 2g^{\rho\sigma}p^\mu F_{1^+}^{c}(q^2) - \left[ g^{\rho\mu}(p + \frac{1}{2}q)^\sigma + g^{\sigma\mu}(p - \frac{1}{2}q)^\rho \right] F_{1^+}^{c}(q^2).
\]

In proper–time regularization one finds\(^\dagger\)

\[
F_{1^+}^{c}(q^2) = \frac{1}{2}F_{0^+}(q^2) - H_1(q^2),
\]

\(^\dagger\)In eq.(20) we have omitted a term \( \propto (p - \frac{1}{2}q)^\sigma(p + \frac{1}{2}q)^\rho \), since we are interested only in the diquark charge and magnetic moment form factors.
\[ F_{1+}^{m}(q^2) = \frac{1}{2} A_0(m_{1+}^2) - \frac{1}{2} A_0(q^2) - \frac{1}{2} q^2 C_1(q^2) + \frac{1}{2} m_{1+}^2 C_2(q^2) - H_2(q^2), \]  

where
\[ H_{1,2} = \frac{1}{16\pi^2} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta W_{1,2} \Gamma \left( 0, \frac{Y^2}{\Lambda^2} \right), \quad W_1 = \alpha, \quad W_2 = 2\beta, \]  

and \( Y^2 \) and \( F_0^m(q^2) \) are as defined eqs.\(^{(16, 15)}\) and evaluated with \( m_0^{+} \rightarrow m_{1+}^{+} \). The normalized electric and magnetic form factors of the axial vector diquark are obtained as \( f_{e,m}^{1+}(q^2) = F_{e,m}(q^2)/Z_{1+} \), where \( Z_{1+} = \partial/\partial p^2 (p^2 A_1)|_{p^2=m_{1+}^2} \). In particular, \( f_{e}^{1+}(0) = 1 \) and \( f_{m}^{1+}(0) \) is the magnetic moment of the \( ij \)-axial vector diquark in units of \((Q_i + Q_j)e/2m_{1+}\) \((i,j = u,d)\). The electric form factor describes the charge distribution in the axial diquark and is shown in fig. 1. The charge radius of the axial diquark is larger than that of the scalar diquark, see table 1. This fact is mainly due to the weaker binding of the axial diquark. However, even strongly bound axial diquarks would be larger than scalar diquarks of the same mass. In gradient expansion at \( m_0^{+} = m_{1+}^{+} = 0 \) one would find \( \langle r^2 \rangle_{1+} = \frac{22}{20} \langle r^2 \rangle_{0+} \). Larger charge radii for the axial diquarks are also found in cutoff regularization, cf. table 2. The axial diquark magnetic moment is shown in table 1, in units of the sum of the constituent quark magnetic moments, \((Q_i + Q_j)e/2M\). The diquark magnetic moment is seen to be lowered compared to the non-relativistic value as a consequence of the diquark binding.

4. Estimating the baryon form factor

Given the diquark electromagnetic form factors we now wish to describe the electromagnetic interactions of baryons as bound states of constituent diquarks and quarks. To calculate the baryon electromagnetic form factor in the effective hadron theory \([10]\) based on the NJL model, one would extract from eq.\((5)\) the baryon electromagnetic vertex, which describes the coupling of the photon to the diquarks, the quarks and to the quark exchange. One would then take the matrix element of this vertex between on–shell baryon states, i.e., solutions of the baryon Faddeev equation \([11]\). Since a full calculation of the form factor using bound state wave functions is rather involved, it is worthwhile to first explore a simpler picture. Our intention here is to see to what extent the nucleon charge radii and magnetic moments can be described in an additive diquark–quark model \([9]\). This approach has been successful in the description of baryon masses. It is based on the spin–flavor wave functions of the non-relativistic quark model, in which two quarks are coupled to a scalar or axial diquark. The diquarks are then regarded as on-shell constituents, whose intrinsic properties such as a mass, a charge radius or a magnetic moment, are derived from the NJL model. Note that as a consequence of the Pauli principle the scalar diquark is antislavant in flavor \((ud)\) and has isospin 0, while the axial diquarks are symmetric \((uu, ud, dd)\) and have isospin 1.

We first consider the nucleon electromagnetic charge radii. In a simple approximation one may neglect the orbital motion of diquarks and quarks, i.e., take the orbital wave functions of diquarks and quarks in the baryon Breit frame to be \( \delta \)-functions centered at the baryon center–of–mass. The charge density inside the baryon is then the sum of the extended diquark charge distributions and the pointlike valence quark charge densities, which do not contribute to the charge radius. From the quark model wave functions one
then obtains the relation: \[ \langle r^2 \rangle_p = \frac{1}{2} \left( \frac{4}{3} \langle r^2 \rangle_{0^+} + \langle r^2 \rangle_{1^+} \right), \quad \langle r^2 \rangle_n = \frac{1}{2} \left( \frac{4}{3} \langle r^2 \rangle_{0^+} - \frac{1}{3} \langle r^2 \rangle_{1^+} \right). \] (24)

Note that the diquark charge radii have been defined as derivatives of the electric form factor normalized to 1 at \( q^2 = 0 \), so that the diquark charges are absorbed in the numerical factors in eq. (24). We now choose the diquark coupling, \( g_2 \), such as to fit the proton mass, using the formula of the diquark–quark model, \( m_{p,n} = \frac{1}{2} (m_{q+} + m_{q+}) + M \). From table 1 we see that this requires a value of \( g_2/g_1 = 2.5 \). With the diquark charge radii from table 1 eq. (24) gives \( \langle r^2 \rangle_p = (0.57 \text{ fm})^2 \) and \( \langle r^2 \rangle_n = -0.06 \text{ fm}^2 \), which is in qualitative agreement with the experimental values of \( \langle r^2 \rangle_{p,\text{exp}} = (0.85 \text{ fm})^2 \) and \( \langle r^2 \rangle_{n,\text{exp}} = -0.11 \text{ fm}^2 \). This simple picture naturally explains the negative charge radius of the neutron by the fact that the axial diquark charge radius is larger than the scalar one. Note that this qualitative result is independent of the diquark masses.

If one included only scalar diquarks in the above estimate, one would obtain instead of eq. (24) \( \langle r^2 \rangle_p = (0.54 \text{ fm})^2 \) and \( \langle r^2 \rangle_n = (0.06 \text{ fm})^2 \), \( i.e., \) the neutron charge radius would come out equal to the proton one. This underscores the importance of the axial vector diquarks in the additive diquark–quark model. We remark, however, that in a model with only scalar diquarks a negative charge radius for the neutron can be obtained if the orbital motion of diquarks and valence quarks is taken into account, as has been demonstrated in the framework of the non-relativistic quark model. The question of the relative importance of the intrinsic size of the axial diquarks versus the effects of the diquark–quark orbital motion can only be answered a posteriori from the full relativistic bound state wave function containing both scalar and axial diquark components. Nevertheless, it is encouraging that a simple additive picture with axial diquark constituents of a size close to that of the proton is capable of reproducing both the nucleon mass and the charge radii reasonably well. Moreover, in the non-relativistic quark model it can be seen that the estimate of the proton charge radius in eq. (24) is increased by the diquark–quark relative motion, while the neutron charge radius remains essentially unchanged unless \( SU(6) \)–breaking in the spin–flavor wave function is taken into account. This strongly suggests that the estimates of the additive model will be improved by a full calculation using Faddeev wave functions.

Finally, we want to discuss the nucleon magnetic moments in the diquark–quark picture. Since magnetic moments are described very well by the non-relativistic quark model, it is important to see how far the predictions of the diquark–quark picture differ from those of the quark model. Rewriting the quark model spin–flavor wave functions in terms of diquarks and quarks one obtains

\[ \mu_p = \frac{e}{2M} \left( \frac{1}{3} + \frac{2}{3} \mu_{1^+} + \frac{1}{3} \mu_{0^+ - 1^+} \right), \quad \mu_n = \frac{e}{2M} \left( -\frac{2}{9} - \frac{1}{3} \mu_{1^+} - \frac{1}{3} \mu_{0^+ - 1^+} \right). \] (25)

The first term here is the valence quark contribution. Furthermore, \( \mu_{1^+} \) is the magnetic moment of the \((ij)\)–axial diquark \((i, j = u, d)\) in units of \((Q_u + Q_d)e/2M\), and \( \mu_{0^+ - 1^+} \) is the transition moment from the \( ud \)–scalar to the \( ud \)–axial diquark in units of \((Q_u - Q_d)e/2M\),

\[ \langle (ud)_{0^+}, S_z = 0 | \mu_{1^+}, S_z = 0 \rangle = \langle ud \rangle_{1^+}, S_z = 0 \rangle = \mu_{0^+ - 1^+} (Q_u - Q_d) (e/2M). \] (26)

\(^{1}\)Here, the isospin limit is assumed, so that the axial \( uu \), \( ud \)– and \( dd \)–diquarks all have the same charge radius, \( \langle r^2 \rangle_{1^+} \).
Here, $\mu_{1z}, \mu_{2z}$ are the $z$-components of the quark magnetic moment operators. In the case of $SU(6)$ symmetry one has $\mu_{1+} = \mu_{0+ -1+} = 1$, which leads to the well-known results $\mu_p = e/2M$, $\mu_n = -\frac{2}{3} e/2M$. We now consider eq. (25) in the sense of the additive diquark–quark model outlined above. The magnetic moment of the axial diquark, $\mu_{1+}$, has been calculated above and is given in table 1. Transitions between scalar and axial vector diquarks due to the electromagnetic field are also found with the effective diquark action derived from the NJL model, eq. (26). This is an anomalous (intrinsic parity-violating) process, which is generated by the imaginary part of the quark loop; its meson analogue is the process $\rho \rightarrow \pi \gamma$. Since the definition of $\mu_{0+ -1+}$ in eq. (26) involves vanishing photon momentum, it is not possible to describe $\mu_{0+ -1+}$ as a transition between on-shell scalar and axial diquarks of unequal mass. To obtain a rough estimate one may evaluate this vertex for a scalar diquark mass equal to that of the axial diquark. In this case the transition part of the vertex function, eq. (13), is given by

$$J_{0+ -1+}^{\mu}(p, q) = \frac{1}{\sqrt{2}} \varepsilon^{\mu\nu\kappa\lambda} p^\nu q^\lambda M^{-1} F_{0+ -1+}(p^2),$$

and the transition moment corresponds to $\mu_{0+ -1+} = F_{0+ -1+}(m_1^2)/\sqrt{2Z_0 Z_1}$. For the axial diquark masses considered in table 1 $\mu_{0+ -1+}$ is close to its non-relativistic value of 1; $\mu_{0+ -1+} = 0.97$ for $g_2/g_1 = 2.5$. For this value of $g_2/g_1 = 2.5$ we obtain from eq. (25) $\mu_p = 0.96 e/2M$, $\mu_n = -0.65 e/2M$. Note that the constituent quark masses used in the NJL model, $M \sim 400$ MeV, are larger than those of the quark model, $M = \frac{1}{3} m_p \sim 300$ MeV, so that $\mu_{p,n}$ come out roughly 3/4 smaller than the quark model prediction. We emphasize, however, that a proper treatment of the transition contribution would require the full baryon wave function with off-shell diquarks. Note that if one included only scalar diquarks in the additive diquark–quark model, the magnetic moments would be $\mu_p = \frac{2}{3} e/2M$, $\mu_n = -\frac{1}{2} e/2M$. The same result would be found if one included axial diquarks but neglected axial–scalar transitions. These estimates again show the importance of axial diquarks in the additive model.

5. Conclusion

We have calculated the electromagnetic form factors of scalar and axial vector diquark bound states in a color-octet NJL model. For realistic diquark masses the scalar diquark charge radius is close to that of the pion. The axial diquark is weakly bound; its size is of the order of that of the proton. The nucleon charge radii can be described qualitatively in an additive diquark–quark model, which may be viewed as a crude approximation to the baryon wave function. The inclusion of axial vector diquarks in this scheme is seen to be crucial. For the nucleon magnetic moments scalar–axial diquark transitions have to be taken into account at least approximately. It remains to be seen to what extent the results of the diquark–quark model are improved by a dynamical calculation using the relativistic baryon wave functions from the Faddeev equation.
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