Non-Critical Confining Strings and the Renormalization Group

Enrique Álvarez♦♠ and César Gómez♦♣♠

♦ Instituto de Física Teórica, C-XVI, Universidad Autónoma de Madrid
E-28049-Madrid, Spain

♣ Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid
E-28049-Madrid, Spain

♠ I.M.A.F.F., C.S.I.C., Calle de Serrano 113
E-28006-Madrid, Spain

Abstract

String vacua for non critical strings satisfying the requirements of Zig-Zag invariance are constructed. The Liouville mode is shown to play the rôle of scale in the Renormalization Group operation. Differences and similarities with the D-brane near horizon approach to non supersymmetric gauge theories are discussed as well.

1E-mail: enrique.alvarez@uam.es
2E-mail: iffgomez@roca.csic.es
3Unidad de Investigación Asociada al Centro de Física Miguel Catalán (C.S.I.C.)
1 Introduction

At present we can consider two different philosophies to approach the problem of defining a string representation of non abelian four dimensional gauge theories. \footnote{Both can be considered as different attempts to implement \textit{holographic} ideas \cite{1}.} One approach is based on Maldacena’s conjecture \cite{2} and on the so called AdS-CFT correspondence \cite{3,4}. In this approach we start with a stack of D-3 branes in type IIB string theory and we perform a near horizon limit introducing a new blow-up variable that can be identified with the scale of the gauge theory. In its strong version Maldacena’s conjecture establishes the equivalence between N=4 supersymmetric Yang Mills theory in flat Minkowski space-time and Type IIB strings in $AdS_5 \times S_5$ with a non vanishing Ramond-Ramond (R-R) five form background and Dirichlet boundary conditions to be defined at Penrose’s conformal infinity \cite{5}. The main difficulty with this approach is of course its extension to gauge theories with non vanishing beta function. This extension requires working with stacks of D-branes reproducing on their world volume non conformal theories, and performing on them a similar bulk decoupling near horizon limit. In this direction, two different possibilities have recently been suggested. One due to Witten \cite{6} starts by considering the near horizon limit of a stack of $M_5$ branes in M-Theory. The resulting six dimensional conformal field theory is subsequently compactified to four dimensions, breaking explicitly supersymmetry. The other possibility due to Klebanov and Tseytlin \cite{7} consists in working from the begining with non supersymmetric Dixon-Harvey \cite{8} type 0(B) strings. The difference between both approaches mainly refers to the relation between the string scale and the breaking of supersymmetry scale.

In type 0(B) strings one starts with a superstring with world sheet supersymmetry that contains closed string tachyons. The idea is to use the dynamics of these tachyons (that contrary to what happens in the bosonic case now have non vanishing amplitudes for even number of external legs only) to induce an effective central charge that will automatically
produce a non trivial dependence of the dilaton field on the radial coordinate. It would be this dependence the one to be identified with the renormalization group equation for the non supersymmetric gauge theory defined on the stack of D3 branes.

It is interesting to observe that in both cases we define a non conformal theory by breaking completely supersymmetry.

The other approach, due to Polyakov [9][10][11][12][13] to the string representation of non abelian gauge theories is based on the so called Zig-Zag (ZZ) invariance of pure Yang Mills Wilson loops [14]. The main idea is to impose on open string amplitudes invariance under generic reparametrizations that are not necessarily orientation preserving diffeomorphisms. This requirement implies a priori both the absence of open string tachyons, which can be achieved by working with a world sheet supersymmetric string theory [9][15][10] and an appropriate GSO projection, and the truncation of the open string spectrum to pure massless vector bosons. It is due to this truncation that ZZ invariant strings are naturally describing pure gauge theories without performing any extra bulk decoupling limit. In order to describe a four dimensional gauge theory in the ZZ approach we should start with a non critical open string theory in four dimensional flat Minkowski space-time. Since this theory is anomalous with respect to world-sheet Weyl rescalings it is necessary to add a Liouville extra mode, that is effectively acting as an extra dimension. A ZZ invariant background will correspond to a five dimensional space-time metric of the type:

\[ ds^2 = a(\phi)^2 dx^2_{\parallel} + d\phi^2 \]  

(1)

(where \( x_{\parallel} \in M_4 \), the Minkowski space in which the putative gauge theory lives) and Dirichlet boundary conditions on the Liouville mode on the ZZ horizon:

\[ a(\phi^*) = 0. \]  

(2)

\footnote{Given the fact that the RR repulsion is twice as strong as the NSNS attraction, in [7] one is forced to hope that the rôle of tachyons is to first allow the stacking of branes by forming a bound state; so tightly bound, in fact, that it would remain so even when the tachyon condenses.}
If we find such a solution we can try to reproduce from it the renormalization group equation for the pure gauge theory defined by the open string sector. Since we are now working in a non-critical string in flat Minkowski space-time, the dilaton field will depend on the Liouville coordinate and we can use this dependence to reproduce the running of the coupling constant. In the non-critical case the identification of the extra Liouville coordinate with the scale of the theory defined by the non-critical string in four dimensions is quite natural. In fact the very meaning of the Liouville mode is to compensate two-dimensional Weyl rescalings which in flat space are equivalent to dilatations of the four-dimensional space-time coordinate:

\[ x_\parallel \rightarrow \lambda x_\parallel. \]  

In this sense the renormalization group equation for the physical open string amplitudes of the zigzag invariant theory would be formally defined as follows:

\[ 0 = \frac{d}{d\lambda} A = \left( \delta x_\parallel \frac{\delta}{\delta x_\parallel} + \delta \phi \frac{\delta}{\delta \phi} \right) A. \]  

It is important to stress the differences and similarities between the ZZ approach and the type 0(B) string approach. In both cases the reason for the running of the dilaton field appears as a consequence of the existence of an effective central charge; their origin however is completely different. In the Type 0(B) case the effective central charge stems from the existence of a tachyon condensate while in the non-critical case it is a consequence of working off criticality.

Let us now explain in what precise way ZZ invariance forces us to work off criticality. If we work Maldacena’s near horizon limit, in the critical case, Penrose’s conformal infinity will possess the general structure of \( M_4 \times S_5 \) and thus Dirichlet open string boundary conditions will be imposed on \( M_4 \times S_5 \) However ZZ invariant boundary conditions require a truncation of all Kaluza Klein modes in order to enforce Dirichlet boundary conditions strictly on \( M_4 \), working in that way in a non-critical string. The main difference however,

\[ \text{The other possibility, consisting in working with } a = \infty \text{ can be interpreted as some T-dual of (1.2).} \]
has its origin in the geometry of the bulk decoupling. In the case of a stack of D 3 branes we need, following Maldacena, to work out the near horizon limit while in the ZZ non critical case the bulk decoupling is explicitly done by demanding Dirichlet boundary conditions on the ZZ horizon, i.e the region of space time where the four dimensional metric in the string frame vanishes. Moreover in the non critical case as pointed out above the Liouville coordinate appears as a natural scale of the theory.

In order to understand properly the dynamical role played by the Liouville field it would be important to introduce the concept of Liouville frame. Let us do it using for that Maldacena’s example, where it is natural to identify the blow up coordinate with a Liouville coordinate. This identification can be understood as follows. Let us parametrize the transversal space to a D 3 brane in type IIB in terms of the Euler angles in $S_5$ and the radius of such sphere i.e using polar coordinates. Let us now forget formally about the radial coordinate and pretend that we are working in a non critical case where the transversal space is simply five dimensional and with the topology of the sphere. We know that this string theory interpreted as non critical generates an extra Liouville coordinate. By definition Liouville frame will mean to consider the string theory in the new coordinates such that one of them is Liouville itself. Very likely the topology will change by going to this new frame where what we get naturally is a transversal space of the type considered in the bulk near horizon limit, namely $S_5 \times \mathbb{R}$, where $\mathbb{R}$ refers now to the Liouville coordinate.

In summary it looks that the Liouville frame is the natural one to capture the bulk decoupling physics. In defining the Liouville frame it is important the type of coordinates.

---

5 A different way of interpreting the blow-up coordinate stems from a T-duality $R \rightarrow \frac{\alpha'}{2\pi}$, by taking the double limit $\alpha' \rightarrow 0$ and $R \rightarrow 0$, with $\frac{\alpha'}{2\pi} \equiv u$. Notice that the resulting theory is a full fledged string. In this sense, this double limit defines a transformation between different string vacua.

6 The combined result of the two operations implies first, to forget about some coordinates, and moreover, replacing them by a Liouville mode which will in general define a non-trivial transformation between string vacua.
we choose at the beginning, in Maldacena’s example polar coordinates. The ones that we keep are the ones related with global $\mathcal{R}$ symmetries. In Liouville frame the Liouville coordinate is always playing the rôle of a scale of the non critical theory, this fact being one of the reasons why this frame is the appropriated one to describe bulk decoupling limits.

In the same way as it happens for type 0(B) strings the ZZ approach is directly working in the no space-time supersymmetric regime. This is in principle due to the fact that ZZ invariant reparametrizations for open string amplitudes already truncate the open string spectrum to the pure NS sector (cf. [10]). The vertex operators for fermionic R states are manifestly not ZZ invariant, since they involve, in any picture, the path integration of the space-time spinor field (which lives in a frame related to a non-zero vierbein). Due to this fact and the already mentioned absence of open string tachyons in ZZ invariant strings we are in the ZZ approach forced to work with GSO projections that are not space time supersymmetric. A more general question we can address at this point is whether space time supersymmetry is or not consistent with non criticality, where by that we generically mean non critical central charge. If we reduce the problem to the case of flat space-time a temptative answer can be given. In fact if we consider four dimensional non critical strings and we add a super Liouville mode, it seems difficult to get a space-time GSO projection in five dimensions since there are not superstrings in five dimensions. A different situation will appears in the case we start with a three dimensional non critical string. In that case we can get after adding super Liouville a GSO in four dimensions consistent with space time supersymmetry.

Concerning space-time supersymmetry let us just make the following general comment. If by some dynamical procedure some condensation takes place in a critical string theory that generates an effective central extension, we conjecture that such a dynamical process will induce dynamical breaking of space-time supersymmetry. For instance if we think in a type IIB string any condensation leading to an effective central charge will immediately

---

7That is, it is not known how to implement $\kappa$-symmetry there.
break the $SL(2,\mathbb{Z})$ duality invariance and consequently space-time supersymmetry.

In this paper we will work in the ZZ approach for non critical strings. Next we shall review our main results. First we consider the simplest case with no tachyon background and non RR background and we look for ZZ invariant solutions. We find a unique solution that degenerates to the trivial one if the missing central charge is zero. This solution induces a running for the dilaton field that is of the asymptotically free type, with the coupling decreasing in the ultraviolet. Moreover we find a phase transition with a conformal point. Since in order to achieve ZZ invariance we need to decouple R open string states it is natural to turn on the tachyon. We do that qualitatively and we find that the tachyon expectation value can move the location of the conformal point leaving us with a pure asymptotically free theory. In all our analysis we work in the string frame and we don’t add any form of R R background.

From our analysis we can suggest a generalization of Maldacena’s conjecture. In the ZZ framework we can say that five dimensional superstring vacua, enjoying a four dimensional ZZ horizon and satisfying Dirichlet boundary conditions on it are completely equivalent to pure four dimensional gauge theories. This generalization of Maldacena’s conjecture replaces critical string by non critical ones, near horizon limit by ZZ invariance, Penrose’s conformal infinity by ZZ horizon and the standard AdS-CFT correspondence by imposing Dirichlet boundary conditions at the ZZ horizon. The main difficulty, as already mentioned, with the ZZ invariant approach is that due to the fact that R fermionic vertex operators are not ZZ invariant we should deal with the presence of a closed string tachyon, and that, since in this approach we are not dealing with any stack of branes, the information about the specific gauge group should be included somehow in the particular way to fix the tachyon vacuum expectation value.

One possibility related with the particular examples we work out in this paper consists in starting with open superstrings in four dimensions with the standard GSO projection i.e. without any form of tachyon and to consider the solution to the tachyonless beta function
equations. Most likely this solution does not admit space time supersymmetry and if we
enforce Dirichlet boundary conditions on the ZZ horizon then the spectrum of the open
string sector is reduced to the one corresponding to a pure gauge theory.

Surprisingly enough we do not get the renormalization group behavior of a pure Yang
Mills theory indicating that something is missing in the whole argument.

2 Non-Critical Confining Strings and the Renormal-
ization Group

In the search for confining strings we are lead to consider a non-critical string, where the
would-be holographic coordinate is now interpreted as a Liouville field. The corresponding
two-dimensional action (including the necessary Dilaton and Tachyon fields) is:

\[
S = \int d^2 z \left[ (\partial \phi)^2 + a^2(\phi)(\partial x_\parallel)^2 + R^{(2)}(\phi) + T(\phi) \right]
\] (5)

In order to study the renormalization group equations of the Yang-Mills theory living on
the horizon, we need to examine how ordinary dilatations of the Minkowskian coordinates
are implemented in the confining string framework.

\[
\begin{align*}
    ds_\parallel^2 & \rightarrow \lambda^{-1} ds_\parallel^2 \\
    a(\phi) & \rightarrow \lambda a(\phi)
\end{align*}
\] (6)

(this corresponds to translations of the Liouville field in the simplest case ). It is to be
stressed that the ZZ Horizon itself remains invariant under these transformations.

In the flat space case, this means

\[
\begin{align*}
    \delta x_\parallel &= \epsilon x_\parallel \\
    \delta(\log a) &= -\epsilon
\end{align*}
\] (7)
This suggests that there is an identification between the logarithmic dilatation on the horizon (which is identified up to a sign with a logarithmic variation of the renormalization scale) and the logarithmic variation of the varying string tension itself:

\[ \frac{\delta x}{x} \equiv - \frac{\delta \mu}{\mu} = - \frac{\delta a(\phi)}{a(\phi)} \] (8)

This then implies unambiguously that the Infrared (IR) region is on the horizon itself \( (a = 0) \); whereas the Ultraviolet (UV) region is located on the boundary \( (a \to \infty) \). Curiously enough, although our framework is not holographic \textit{sensu stricto}, the geometrical identifications of the energy scales coincide with the holographic situation \cite{16}.

The transformation of the Liouville field does not then in general leave invariant the kinetic term, nor the dilaton. The variation of the action is:

\[ \delta S = \int d^2 z \left[ - \partial_a \phi \partial^a \left( \frac{a}{a'} \right) \epsilon + (T' \phi + \Phi' \phi R(2)) \left( \frac{a}{a'} \right) \epsilon \right] \]

\[ \sim \int d^2 z \frac{a}{a'} \epsilon \left( \partial_a \partial^a \phi + (T' \phi + R(2) \Phi' \phi) \right) \] (9)

From this point of view, the universal behavior of the dilaton under Weyl transformations would be:

\[ \frac{\delta \Phi}{\delta \epsilon} = - \frac{\delta \Phi}{\delta \phi} \frac{a(\phi)}{a'(\phi)} \] (10)

The condition for persistence of the horizon in the Einstein metric is

\[ \lim_{a \to 0} e^{\frac{2}{3} \phi a^2} = 0 \] (11)

This means that the divergence of the coupling constant when reaching the horizon must be smaller than \( a^{-3} \).

\[ ^8 \text{Under this circumstances, we can equally well (by redefining the Liouville coordinate) interpret the above \textit{ansatz} as representing the metric in the Einstein frame, which is sometimes convenient when studying the equations of motion. Those derive from the action principle} \]

\[ S_E = \frac{1}{16\pi G_5} \int d^5 x \sqrt{g_E} |R_E + \frac{1}{3} (\nabla \Phi)^2 | \]

8
Owing to our identification above (8), between the renormalization group scale \( \mu \) ans the string frame metric coefficient \( a(\phi) \), we shall continue working in the string frame for ease of the physical interpretation.

The Weyl anomaly coefficients \[19\] in the said string frame read

\[
\beta_{\mu\nu}^* = \alpha' R_{\mu\nu} - \alpha' \nabla_{\mu} \nabla_{\nu} \Phi \\
\beta_{\phi}^* = \alpha' \nabla^2 \Phi + \frac{2}{3}(D - 10) + \alpha' (\nabla \Phi)^2 \\
\beta_T^* = \alpha' \nabla^2 T - 4T + \alpha' \nabla_{\mu} \Phi \nabla^{\mu} T
\]

We have taken into account the central charge defect \(-\frac{2}{3\alpha'}(D - 10)\), where we use the value 10 because the model, as argued above, enjoys two-dimensional supersymmetry.

In the presence of a tachyon field, there are extra terms (formally of order \( \alpha'^2 \)), so that the complete equations read

\[
R_{\mu\nu} - \nabla_{\mu} \nabla_{\nu} \Phi + \frac{1}{4} \nabla_{\mu} T \nabla_{\nu} T = 0 \\
- \nabla^2 \Phi + c_0 - (\nabla \Phi)^2 - \frac{m^2}{4} T^2 = 0 \\
\nabla^2 T - m^2 T + \nabla_{\mu} \Phi \nabla^{\mu} T = 0
\]

and where now the tachyon mass is given by \( m^2 \equiv -\frac{D-2}{4\alpha'} \) and the central charge defect is \( c_0 \equiv -\frac{D-10}{\alpha'} \).

The tachyon mass is given by \( m^2 = -\frac{2}{\alpha'} \).

For example, when the tachyon vanishes, the Einstein equations of motion then reduce to:

\[
\Phi'' + 4 \Phi' \frac{a'}{a} = -\frac{20}{3\alpha'} e^{-\frac{2}{\alpha'} \Phi} \\
4 \frac{a''}{a} + \frac{1}{3} (\Phi')^2 - \frac{2}{a^2} (2a'' + 3(a')^2) + \frac{1}{6} (\Phi')^2 + \frac{5}{3\alpha'} e^{-\frac{2}{\alpha'} \Phi} = 0 \\
aa'' + 3(a')^2 - 4a'' - 6(a')^2 + \frac{a^2}{6} (\Phi')^2 + \frac{5a^2}{\alpha'} e^{-\frac{2}{\alpha'} \Phi} = 0
\]

\[9\] Those equations coincide with the ones in a footnote in page 12 of \[7\] with the identification \( \Phi_{kl} \equiv -\frac{1}{2} \Phi \).
2.1 Tachyonless Backgrounds

Assuming that the tachyon (through the interaction with the RR backgrounds or otherwise) develops a vacuum expectation value, in a first approximation all its effect will be to change the numerical value of the central charge defect $c_0$. Putting by simplicity the tachyon field to zero and assuming the radial ansatz $\partial_\mu \Phi = 0$ where $x^\mu \in M_\parallel$, yields:

$$
R_{\phi\phi} = -4 \frac{a''}{a} = \Phi''
$$

$$
R_{\mu\nu} = -(aa'' + 3(a')^2)\delta_{\mu\nu} = -\Gamma^\phi_{\mu\nu} \Phi' = aa'\Phi'\delta_{\mu\nu}
$$

$$
\Phi'' + 4\Phi'\frac{a'}{a} - c_0 + (\Phi')^2 = 0
$$

(14)

where $f' \equiv \frac{df}{d\phi}$.

There are now two subcases to consider. When $a' = 0$, then necessarily

$$
\Phi'' = 0
$$

(15)

and

$$
\Phi' = c_0^{1/2}
$$

(16)

This is the well-known linear dilaton solution.

When $a' \neq 0$, on the other hand, a linear combination of the above equations can be expressed solely in terms of the function $a(\phi)$:

$$
-2 \frac{a''}{a} + \left(\frac{a''}{a'}\right)^2 - 3\left(\frac{a'}{a}\right)^2 - c_0 = 0
$$

(17)

which can be integrated by the substitution

$$
a \equiv e^{\int u}
$$

(18)

This yields

$$
-4u^2 + \left(\frac{u'}{u}\right)^2 - c_0 = 0
$$

(19)

which can be easily integrated, giving:

$$
u = \sqrt{\frac{c_0(t_0^2 - c_0)}{2[\gamma_0 \cosh \gamma_0(\phi - \phi_0) - t_0 \sinh \gamma_0(\phi - \phi_0)]}}
$$

(20)
where \( \gamma_0^2 \equiv c_0 \), and when \( \phi = \phi_0 \), \( u = u_0 \equiv \frac{1}{2} \sqrt{t_0^2 - c_0} \).

Given the fact that necessarily \( \gamma_0 < t_0 \), there is a singularity at the value of the Liouville coordinate given by

\[
\phi_{\text{sing}} = \phi_0 + \frac{1}{\gamma_0} t h^{-1}(\gamma_0/t_0)
\]

(Only in the singular limiting case when \( t_0 = \gamma_0 \) is \( \phi_{\text{sing}} \) pushed towards \( \phi_{\text{sing}} = \infty \)).

Integrating again (21) leads to:

\[
a(\phi) = a_0 \sqrt{1 + \frac{\lambda e^{\gamma_0 (\phi - \phi_0)}}{1 - \lambda e^{\gamma_0 (\phi - \phi_0)}}}
\]

where

\[
\lambda^2 = \frac{t_0 - \gamma_0}{t_0 + \gamma_0}
\]

and when \( \phi = \phi_0 \), \( a = a_0 \equiv \alpha_0 \sqrt{\frac{1 + \lambda}{1 - \lambda}} \). Again, for positive values of the parameter \( \lambda \), the function explodes when \( \phi = \phi_{\text{sing}} \).

**2.1.1 A ZZ Invariant solution**

When \( t_0 = \infty \), taking the valuation \( \lambda = -1 \), and changing \( \phi \) by \(-\phi\) (which is a symmetry of the equations (14)), we get:

\[
a(\phi) = a_0 \sqrt{1 - \frac{e^{-\gamma_0 (\phi - \phi_0)}}{1 + e^{-\gamma_0 (\phi - \phi_0)}}}
\]

which is the only ZZ invariant solution in the whole family, that being in itself a remarkable fact.

A curious thing is that solution flattens itself down (i.e., reduces to the trivial one \( a \equiv 0 \)) when the central charge defect vanishes, that is \( c_0 = 0 \).

This solution starts at \( a = 0 \) when \( \phi = \phi_0 \equiv \phi^* \), and grows monotonically until it reaches the asymptotic value \( a = \alpha_0 \) (which is arbitrary). This implies that, unless \( \alpha_0 = \infty \), it is not self (T)-dual; that is, there is no region in the spacetime (boundary) with

---

10This last step is necessary in order to get a solution which starts, rather than ends at the horizon.
\( a(\phi) = \infty \). This will have physically important consequences, because, as we shall see, one is led to identify the Ultraviolet (UV) with the boundary, and the Infrared (IR) with the ZZ horizon.

Incidentally, the ZZ invariant solution starting of the horizon, and the \( \phi \)-reversal, ending on it, would both be in a mutual T-dual relationship were it not for the dilaton field \[17\].

### 2.1.2 \( g_{YM}(\mu) \) from the Dilaton

We would like to interpret \( \pi^{3/2} g_s e^{\Phi} \equiv g_{YM}^2 \) as the coupling constant of the putative gauge theory living on the ZZ Horizon \[18\]: but which coupling constant, bare or renormalized, and if the latter, at which scale?

Apparently, the only possible thing we can identify the dilaton with is with the running coupling constant, and the sense of the running is provided by our previous identification in (8) of the string implementation of the Yang-Mills scale transformations.

On general grounds, even before integrating, we can write for our putative \( \beta \)-function

\[
\frac{a}{d} \frac{de^{\Phi}}{da} = -\left( \frac{aa''}{(a')^2} + 3 \right) e^{\Phi}
\]  

Plugging there the former results from (4.25) one gets:

\[
e^{\Phi} = e^{\Phi_0} \left( \frac{a_0}{a} \right)^6 \frac{1 - (\alpha_0/a_0)^4}{1 - (\alpha_0/a)^4}
\]  

It can be easily checked that this is a decreasing function of \( a \) as long as \( a > \alpha_0 \). \[\dagger\]

In the ZZ-invariant case, \( e^{\Phi_0} \) diverges in such a way that

\[
e^{\Phi} \sim \frac{1}{a^6 (\frac{\alpha_0}{a_0})^6 - 1}
\]  

This clearly decreases as a function of \( a \) up to a given value of \( a \), namely \( a_{\min} \equiv 3^{-1/4} \alpha_0 \), after which point it starts to increase without bounds. \[\dagger\]

\[\dagger\]This actually covers the whole domain of the Louville variable in the non zz-invariant situation when \( \lambda > 0 \) because then \( \alpha_0 < a_0 \) always.

\[\dagger\] Please note that owing to our identification (8) between the renormalization group scale \( \mu \) and the
The physical meaning of the turn-over point seems to stem from the fact that it has horizontal tangent, that is, \( \beta(g^*) = 0 \); it is a \textit{conformally invariant fixed point}.

Incidentally, it is not difficult to show that in the vicinity of \( g^* \) the beta function reads

\[
\beta \sim -\frac{\alpha_0^{3/4}}{2} g^3 \sqrt{\frac{2}{3} - \frac{\sqrt{3}}{\alpha_0^6 g^2}} \tag{28}
\]

On the other hand, even in the Asymptotically Free (AF) regime, the dependence with the putative \( \mu \) is \textit{not} logarithmic. The generic (not ZZ-invariant) solutions (\( \lambda > 0 \)), although they are asymptotically free (AF) in the whole allowed domain, also lack logarithmic dependence.

It is plain that once the function \( a(\phi) \) is known, the dilaton can be easily extracted from the second equation of (14). This leads at once in the generic case (\( \lambda > 0 \)) to

\[
\Phi - \Phi_0 = \log \frac{[1 - \lambda e^{\gamma_0 (\phi - \phi_0)}]^3}{e^{\gamma_0 (\phi - \phi_0)} + \lambda e^{2\gamma_0 (\phi - \phi_0)}} \frac{1 + \lambda}{1 - \lambda} \tag{29}
\]

and for the ZZ-invariant solution to

\[
\Phi \sim 3 \log(1 + e^{-\gamma_0 (\phi - \phi_0)}) - \log(e^{-\gamma_0 (\phi - \phi_0)} - e^{-2\gamma_0 (\phi - \phi_0)}) \tag{30}
\]

From here we can easily determine that the AF regime ends up at

\[
\phi_{\text{min}} - \phi_0 = \gamma_0^{-1} \log(2 + \sqrt{3}) \tag{31}
\]

3 The Physics of Tachyon Condensates

It is a well known fact \cite{19} that in the presence of arbitrary NS massless condensates the spacetime effective action vanishes on shell because it is precisely proportional to the dilaton beta function, \( \beta_{\Phi} \).

This contrasts with the situation in most other treatments.

\[ \text{We have explicitly verified that it also solves the first equation of (14), which was only used as a substitution up to now} \]
When there is a tachyon condensate, it is easy to check that on shell

\[ S_{\text{eff}} \equiv -2 \int \sqrt{G} d^D x (2c_0 - \frac{m^2}{2} T^2) e^{-2\phi} \]  

(32)

In the range of spacetime dimensions between \( D = 2 \) and \( D = 10 \), then, the fact that the tachyon mass is \textit{negative} precisely enforces

\[ S_{\text{eff}} \leq 0 \]  

(33)

thus implementing Zamolodchikov’s c-theorem in the present context, which physically means \([14]\) that

\[ C = D - 10 + S_{\text{eff}} \]  

(34)

### 3.1 Tachyonful Backgrounds

The full equations (13) with the Tachyon turned on read:

\[
R_{\phi\phi} = -4 \frac{a''}{a} = \Phi'' + \frac{1}{4} (T')^2 \\
R_{\mu\nu} = -(aa'' + 3(a')^2) \delta_{\mu\nu} = -\Gamma_{\mu\nu} \Phi' = aa' \Phi' \delta_{\mu\nu} \\
\Phi'' + 4\Phi' \frac{a'}{a} - c_0 + (\Phi')^2 + \frac{m^2}{4} T^2 = 0 \\
T'' + 4T' \frac{a'}{a} - m^2 T + \Phi' T' = 0
\]  

(35)

The complete set of equations is quite difficult to solve exactly. What we can do instead is to examine the behavior of a tachyon in the tachyonless background of the preceding section, and then study how this tachyon back-reacts on the said background.

First of all, we can trace from (35) the origin of the turnover point in the behavior of the dilaton as a function of \( a \) to the vanishing of the Heaviside function \( \theta(aa'' + 3(a')^2) \).

Now, the analogous to our previous equation (17) in the presence of a tachyon is:

\[
-2 \frac{a''}{a} + \left( \frac{a''}{a'} \right)^2 - 3 \left( \frac{a'}{a} \right)^2 - c_0 - \frac{(T')^2 - m^2 T^2}{4} = 0
\]  

(36)

\[ ^{14} \text{There are other terms, neglected here, which are of the same formal order in the } \alpha' \text{-expansion.} \]
This means that on the tachyonless geometry the quantity

\[ \frac{aa''}{(a')^2} + 3 = 4 \pm \sqrt{(e^{\gamma_0(\phi-\phi_0)} + e^{-\gamma_0(\phi-\phi_0)})^2 + (e^{\gamma_0(\phi-\phi_0)} - e^{-\gamma_0(\phi-\phi_0)})^2 \frac{(T')^2 - m^2T^2}{4\gamma_0^2}} \]  

(37)

(Where it should always be taken the negative valuation of the square root).

When the tachyon vanishes this gives:

\[ \frac{aa''}{(a')^2} + 3 = 4 - e^{\gamma_0(\phi-\phi_0)} - e^{-\gamma_0(\phi-\phi_0)} \]  

(38)

which passes through a zero precisely at the value \( \phi_{\text{min}} \) quoted above after equation (27).

It is clear now that the effect of a free tachyon is always to approach the turnover point to the origin, because the extra term in the square root is positive definite (this is a purely tachyonic effect).

The only possibility for the tachyon dynamics to push the turnover point towards \( \phi = \infty \) would be that a positive definite potential \( V(T) \) is generated, in such a way that the term \( m^2T^2 \) is replaced by \( m^2T^2 + V(T) \).

To be specific, if we define the quantity

\[ \rho = \frac{V(T) + m^2T^2 - (T')^2}{4\gamma_0^2} \]  

(39)

then the condition that the Heaviside function \( \theta = 1 \) implies that \( (1 - \rho)(z^2 + z^{-2}) + 2(1 + \rho) < 16, \forall z \in (0, \infty) \). This is clearly only possible when \( \rho = 1 \), which is the same as:

\[ V(T) = 4c_0 - m^2T^2 + (T')^2 \]  

(40)

which, in the tachyonless background reduces to

\[ V(T) = 4c_0 \]  

(41)

Our present understanding of the dynamics of non-critical confining strings does not allow us to gauge what are the odds for such a potential to be generated in the present context.
3.2 Backreaction

If we plug this value for the tachyon potential back in our previous equation (36) we get the simple equation

\[ aa'' = -(a')^2 \]  

(42)

whose general solution is of the form

\[ a = a_0 \sqrt{1 + 2u_0(\phi - \phi_0)} \]  

(43)

Again, there is only one ZZ-invariant solution in the family, namely

\[ a \sim (\phi - \phi_0)^{1/2} \]  

(44)

Using (27), this gives the behavior of the dilaton as:

\[ e^\Phi \sim (\phi - \phi_0)^{-5/4}. \]  

(45)

This is again AF in the whole allowed domain, although the dependence with the variable we have argued before to be the correct implementation of the renormalization group scale \( \mu \) in the present context, namely, \( a \) itself, is not logarithmic but rather

\[ g_{YM}^2 \sim \mu^{-5/2}. \]  

(46)

It is interesting to notice that the renormalization group we get is of a power law type, similar to the one considered in the accelerated unification. [20] This behavior is typical of a full fledged five dimensional theory, pointing out to a potential problem in the ZZ scenario.

4 Conclusions

In summary there are two different ways to perform bulk decoupling limits or in other words to define string vacua equivalent to non abelian gauge theories, namely near horizon limit of an stack of D-branes and ZZ invariant solutions for non critical strings. ZZ invariance
is manifestly no space time supersymmetric and non critical and allow us to read the evolution of the renormalization group directly from the Liouville field dependence of the dilaton field. Bulk decoupling for non conformal field theories with partial supersymmetry breaking will require some different procedures.

**Acknowledgments**

We thank A. Delgado for pointing out reference [20]. This work has been partially supported by the European Union TMR program FMRX-CT96-0012 *Integrability, Non-perturbative Effects, and Symmetry in Quantum Field Theory* and by the Spanish grant AEN96-1655. The work of E.A. has also been supported by the European Union TMR program ERBFMRX-CT96-0090 *Beyond the Standard model* and the Spanish grant AEN96-1664.

**References**

[1] G. 't Hooft, *Dimensional Reduction in Quantum Gravity* [gr-qc/9310026].

L. Susskind, *The world as a Hologram*, [hep-th/9409089].

[2] J. Maldacena, *The Large N Limit of Superconformal Field Theories and Supergravity*, [hep-th/9711200].

[3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge Theory Correlators from Non-critical String Theory*, [hep-th/9802109].

[4] E. Witten, *Anti de Sitter Space and Holography*, [hep-th/9802150].

[5] R. Penrose and W. Rindler, *Spinors and Space-Time*, Cambridge University Press, 1986.
[6] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories* [hep-th/9803131].
S. Gubser, A. Hashimoto, I. Klebanov and M Krasnitz, *Scalar absorption and the breaking of the world volume conformal invariance*, [hep-th/9803023].

[7] Igor R. Klebanov and Arkady A. Tseytlin, *D-Branes and Dual Gauge Theories in Type 0 Strings*, [hep-th/9811035].
Asymptotic Freedom and Infrared behavior in the Type 0 String Approach to Gauge Theory, [hep-th/9812089].
A Non-Supersymmetric Large N CFT from Type 0 String Theory [hep-th/9901101].
Joseph A. Minahan, *Glueball mass spectrum and Other Issues for Supergravity Duals of QCD Models*, [hep-th/9811156].

[8] L. Dixon and J. Harvey, *String theories in ten dimensions without space- time supersymmetry* Nucl. Phys. B274 (1986), 93.

[9] A.M. Polyakov, *Confining Strings*, [hep-th/9607049].
*String Theory and Quark Confinement*, [hep-th/9711002].
The wall of the Cave, [hep-th/9808057].

[10] E. Álvarez, C. Gómez and T. Ortín, *String representation of Wilson loops*, [hep-th/9806075].

[11] I.I. Kogan and O.A. Soloviev, *Gravitationally dressed RG flows and zigzag-invariant strings* [hep-th/9807223].

[12] M. C. Diamantini, H. Kleinert, C. A. Trugenberger, *Strings with Negative Stiffness and Hyperfine Structure* [hep-th/9810171].

[13] G. Ferreti and D. Martelli, *On the construction of gauge theories from non-critical type 0 strings*, [hep-th/9811208].
[14] A.M. Polyakov, *Gauge Fields and Strings* (Harwood Academic).

[15] A. A. Migdal, *Hidden Symmetries of Large N QCD* hep-th/9610126

P. Horava, On QCD String Theory and ADS Dynamics hep-th/9811028.

[16] L. Susskind and E. Witten, *The Holographic Bound in AdS-Spaces*, hep-th/9805114.

[17] A. Giveon, M. Porrati and E. Rabinovici, *Target Space Duality in String Theory*, Phys.Rept. 244 (1994) 77–202.

E. Alvarez, L. Alvarez-Gaumé, Y. Lozano, *An Introduction to T-Duality in String Theory*, Nucl. Phys. (Proc. Suppl.) 41 (1995),1 (hep-th/9410237).

[18] C.P. Bachas, *Lectures on D-Branes* hep-th/9806193.

[19] C. Callan, E. Martinec, M. Perry and D. Friedan, *Strings in Background Fields* Nucl. Phys. B262 (1985),593.

G. Curci and G. Paffuti, *Consistency between the string background equations of motion and the vanishing of the conformal anomaly*, Nucl. Phys. B286 (1987),399.

[20] Keith R. Dienes, Emilian Dudas, Tony Gherghetta, *Grand Unification at Intermediate Mass Scales through Extra Dimensions*, hep-ph/9806292.

C.P. Bachas, *Unification with Low String Scale*, hep-ph/9807415.