Big Bang nucleosynthesis constraint on baryonic isocurvature perturbations

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Abstract. We study the effect of large baryonic isocurvature perturbations on the abundance of deuterium (D) produced in big bang nucleosynthesis (BBN). Large baryonic isocurvature perturbations existing at the BBN epoch ($T \sim 0.1$ MeV) change the D abundance by a second order effect, which, together with the recent precise measurement of D abundance, leads to a constraint on the amplitude of the power spectrum of the baryonic isocurvature perturbations. We derive the upper limit on the amplitude $P_{S_B} \lesssim 0.016$ ($2\sigma$) for scales $k^{-1} \gtrsim 0.0025$ pc. This is the most stringent constraint for $0.1 \text{ Mpc}^{-1} \lesssim k \lesssim 4 \times 10^8 \text{ Mpc}^{-1}$. We apply the BBN constraint to the relaxation leptogenesis scenario, where large baryonic isocurvature perturbations are produced in the last $N_{\text{last}}$ e-fold of inflation, and we obtain a constraint on $N_{\text{last}}$.

Keywords: big bang nucleosynthesis, cosmological perturbation theory, baryon asymmetry, inflation

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1 Introduction

Light elements, such as $^4$He and D, are synthesized at the cosmic temperature $T \sim 0.1$ MeV. Their abundances predicted by the big-bang nucleosynthesis (BBN) are in good agreement with the observations, giving a strong support to the standard hot big-bang cosmology [1–6]. BBN is very sensitive to physical conditions at $T \sim 1–0.01$ MeV, and, therefore, it is an excellent probe of the early universe. For example, any additional radiation component existing at $T \sim 1$ MeV changes the predicted abundances of $^4$He and D and could spoil the success of the BBN, which gives a stringent constraint on the extra radiation energy.

It is believed that inflation took place in the early universe and the observed structures arose from density perturbations generated during inflation. During inflation, light scalar fields, including the inflaton, undergo quantum fluctuations which become classical due to the accelerated expansion; these fluctuations leads to primordial density perturbations. If only one scalar field (= inflaton) is involved in generation of the density perturbations, the perturbations are adiabatic and nearly scale invariant, which perfectly agrees with the observations of the CMB and large scale structures on large scales ($\gtrsim \mathcal{O}(10)$ Mpc). The amplitude of the power spectrum of the curvature perturbations is precisely determined as $P_\zeta = 2.1 \times 10^{-9}$ at the pivot scale $k = 0.002$ Mpc$^{-1}$ by the CMB observations [7].

However, on the small scales, little is known about the shape and the amplitude of the power spectrum of the density perturbations. Some constraints on the curvature perturbations result from the CMB $\mu$-distortion due to the Silk dumping [8] and from over-production of primordial black holes [9]. Some additional constraints on the amplitude of the curvature perturbations come from BBN, since the perturbations can affect $n/p$ and/or baryon-to-photon ratio through the second order effects and alter the abundance of the light elements [10–12].

Inflation produces not only curvature (adiabatic) perturbations but also isocurvature perturbations. The isocurvature perturbations are produced when multiple scalar fields are involved in generation of the density perturbations. In particular, when scalar fields play an important role in baryogenesis, baryonic isocurvature perturbations are generally produced. A well-known example is the Affleck-Dine baryogenesis [13, 14] where a baryonic scalar field has a large field value during inflation and generates baryon asymmetry dynamically after inflation. This scenario produces baryonic isocurvature perturbations, unless the scalar field has a large mass in the phase direction [15, 16]. Since the CMB observations are quite consistent with the curvature perturbations, the isocurvature perturbations on the CMB
scales are tightly constrained [17]. However, there are almost no constraints on small-scale isocurvature perturbations.

In this paper we show that large baryonic isocurvature perturbations existing at the BBN epoch \( T \sim 0.1 \text{ MeV} \) change the D abundance by their second order effect. Although the effects of baryonic isocurvature perturbations on BBN have been discussed in the literature, previous studies have mainly focused on non-linear isocurvature perturbations related to the first order QCD phase transition [18–23] and electroweak phase transition [24–27]. In this paper, we focus on the linear baryonic isocurvature perturbations that have a primordial origin. Since the primordial abundance of D is precisely measured with accuracy of about 1 \% [28, 29], one can obtain a new constraint on the amplitude \( P_{SB} \) of the baryonic isocurvature perturbations. We will show that the amplitude must satisfy \( P_{SB} \lesssim 0.016 \) (2\( \sigma \)) for scale \( k^{-1} \gtrsim 0.0025 \text{ pc} \). We also apply the constraint to the relaxation leptogenesis scenario [30, 31], where large fluctuations of a scalar field play a crucial role in leptogenesis, and large baryonic isocurvature perturbations are predicted on the small length scales. We show that BBN yields a significant constraint on this scenario.

The paper is organized as follows. In section 2 we briefly review the measurement of the D abundance. In section 3, we show how the baryonic isocurvature perturbations change the D abundance, and we obtain a generic constraint on their amplitude. In section 4 we apply the BBN constraint to the relaxation leptogenesis scenario. Section 5 is devoted for conclusions.

## 2 Deuterium abundance

Light elements, such as D, \(^3\)He and \(^4\)He, are produced at temperature \( T \simeq 1 \text{ MeV}–0.01 \text{ MeV} \), and BBN accurately explains the observed abundances of these elements. In particular, the deuterium abundance has been precisely measured by observing absorptions Lyman-\( \alpha \) lines in the spectrum of distant QSOs. The accuracy of deuterium abundance measurements has improved dramatically [4, 29, 32, 33]. Most recently, Zavaryzin et al. [28] reported the primordial D abundance,\(^1\)

\[
(D/H)_p = (2.545 \pm 0.025) \times 10^{-5}, \tag{2.1}
\]

from measurements of 13 damped Lyman-\( \alpha \) systems. Here \( D/H \) is the ratio of the number densities of D and H. The observed abundance should be compared with the theoretical prediction. The D abundance predicted by BBN is calculated by numerically solving the nuclear reaction network. In the standard case the result depends only on the baryon density \( \Omega_B \). We adopt the following fitting formula in ref. [7]:

\[
10^5(D/H)_p = 18.754 - 1534.4 \omega_B + 48656 \omega_B^2 - 552670 \omega_B^3, \tag{2.2}
\]

where \( \omega_B = \Omega_B h^2 \) and \( h \) is the Hubble constant in units of \( 100 \text{ km/s/Mpc} \). This formula is obtained with the PArthEnoPE code [34] and its uncertainty is \( \pm 0.12(2\sigma) \). The observational constraint eq. (2.1) and the prediction eq. (2.2) are shown in figure 1. From the figure, we can see that the BBN prediction is consistent with the observed abundance for \( \Omega_B h^2 \simeq 0.022–0.023 \).

The baryon density is also precisely determined by CMB observations. The recent Plank measurement gives

\[
\Omega_B h^2 = 0.02226 \pm 0.00023, \tag{2.3}
\]

\(^1\)Another recent result, \( (D/H)_p = (2.527 \pm 0.030) \times 10^{-5} \) [29], is consistent with ref. [28].
which is also shown in figure 1. One can see that the baryon densities determined by BBN and by CMB are consistent. However, if the predicted D abundance increases by about 3% (2\sigma uncertainties), the two measurements become inconsistent, and any effect that increases the D abundance 3% or more is subject to a constraint.

3 Baryonic isocurvature perturbations and deuterium abundance

Here we assume that only baryon number fluctuations are produced in the early universe for simplicity. Such fluctuations, called baryonic isocurvature perturbations $S_B$, can be written as

$$S_B = \frac{\delta n_B}{n_B} = \frac{3}{4} \frac{\delta \rho_\gamma}{\rho_\gamma} = \frac{\delta n_B}{n_B}, \quad (3.1)$$

where $\rho_\gamma$ and $\delta \rho_\gamma$ are the photon energy density and its perturbation, respectively. The last equality results from the assumption of vanishing photon perturbations.

When the baryon number density exhibits spatial fluctuations, it can affect the BBN and change the abundance of the light elements. In particular, modification of the D abundance is important because it is precisely determined by the recent measurements. Let us consider the BBN prediction of D in the presence of the baryon number fluctuations by using eq. (2.2). In order to take into account the baryon number fluctuations, we consider $\omega_B$ in eq. (2.2) as space dependent variable,

$$\omega_B(t, \vec{x}) = \bar{\omega}_B + \delta \omega_B(\vec{x}), \quad (3.2)$$

where $\bar{\omega}_B$ is the homogeneous part, and $\delta \omega_B$ denotes the fluctuations related to $S_B$ as

$$\delta \omega_B = \bar{\omega}_B \frac{\delta n_B(\vec{x})}{n_B} = \bar{\omega}_B S_B(\vec{x}). \quad (3.3)$$
Now eq. (2.2) can be rewritten as
\[
y_d = 18.754 - 1534.4 \bar{\omega}_B + 48656 \bar{\omega}_B^2 - 552670 \bar{\omega}_B^3 + (-1534.4 \bar{\omega}_B + 97312 \bar{\omega}_B^2 - 1658010 \bar{\omega}_B^3) S_B + (48656 \bar{\omega}_B^2 - 1658010 \bar{\omega}_B^3) S_B^2 + \ldots, \tag{3.4}
\]
where \(y_d = 10^5 (D/H)_p\). Since D production takes place at \(T \simeq 0.1\) MeV, one can estimate \(S_B\) at that time. To estimate the primordial D abundance, we average \(y_d\) over the volume \(V\) corresponding to the present horizon. Using \(\langle S_B \rangle = 0\), we obtain
\[
\langle y_d \rangle = 18.754 - 1534.4 \bar{\omega}_B + 48656 \bar{\omega}_B^2 - 552670 \bar{\omega}_B^3 + (48656 \bar{\omega}_B^2 - 1658010 \bar{\omega}_B^3) \langle S_B^2 \rangle, \tag{3.5}
\]
where \(\langle \cdots \rangle\) denotes the spatial average. Thus, the D abundance is modified compared to the homogeneous case, owing to the second order effect of the baryonic isocurvature perturbations. The prediction for \(\langle S_B^2 \rangle = 0.016\) is shown in figure 2. One can see that the isocurvature perturbations increase the D abundance, which leads to a higher inferred baryon density needed to account for the observed abundance. This makes the baryon densities inferred from CMB and BBN inconsistent with each other. Thus we can obtain a constraint on \(\langle S_B^2 \rangle\).

To derive the upper bound on the isocurvature perturbations, we define the discrepancy \(\mathcal{D}\) between the observational and theoretical values in the units of standard deviation as
\[
\mathcal{D} = \frac{|y_{\text{obs,mean}} - y_{\text{th,mean}}|}{\sqrt{\sigma_{\text{obs}}^2 + \sigma_{\text{th}}^2}}, \tag{3.6}
\]
where \(y_{\text{obs,mean}}\) and \(y_{\text{th,mean}}\) are the mean values of the observation and theoretical prediction and \(\sigma_{\text{obs}}^2\) and \(\sigma_{\text{th}}^2\) are the standard deviations of \(y_{\text{obs}}\) and \(y_{\text{th}}\). Note that \(y_{\text{th,mean}}\) and \(\sigma_{\text{th}}\) are calculated by eq. \(\text{(3.5)}\), and, therefore, they depend on \(\langle S_B^2 \rangle\). Imposing the conditions \(\mathcal{D} < 1\) or \(\mathcal{D} < 2\), we can get the constraints on the isocurvature perturbations as \(\langle S_B^2 \rangle < 0.0020 (1\sigma)\) or \(\langle S_B^2 \rangle < 0.016 (2\sigma)\), respectively.

Let us calculate \(\langle S_B^2 \rangle\) from the Fourier mode \(S_B(\vec{k})\) as
\[
\langle S_B^2 \rangle = \frac{1}{V} \int_V d^3x (S_B(\vec{x}))^2 = \frac{1}{(2\pi)^3} \int d^3k |S_B(\vec{k})|^2 = \int d\ln k \mathcal{P}_{SB}(k), \tag{3.7}
\]
where \(\mathcal{P}_{SB}(k)\) is the power spectrum of the baryonic isocurvature perturbations. Care should be taken of the upper limit of the \(k\)-integration. The baryons diffuse in the early universe, which erases the baryon number fluctuations for wavelengths smaller than the diffusion length. Therefore, the upper limit of the integration is the wave number \(k_d\) that corresponds to the diffusion length at the BBN epoch \((T \simeq 0.1\) MeV\). The diffusion length of neutrons \(d_n\) is much larger than that of protons and \(k_d\) is given by \(d_n^{-1}\). The neutron diffusion is determined by the neutron-proton scatterings, and the comoving diffusion length is \([35]\)
\[
d_n \simeq k_d^{-1} \simeq 0.0025 \text{ pc} \quad \text{at} \quad T = 0.1 \text{ MeV}. \tag{3.8}
\]
Figure 2. The same as figure 1, except that we show the $2\sigma$ BBN prediction in the case with $\langle S_B^2 \rangle = 0.016$ by the gray shaded region. Note that the overlap between the region of the D observation and BBN prediction does not necessarily mean the consistency between them.

Thus, $\langle S_B^2 \rangle$ is given by

$$\langle S_B^2 \rangle = \frac{1}{(2\pi)^3} \int_{k_*}^{k_d} d^3k P_{S_B}(k). \quad (3.9)$$

Here $k_*$ is the scale corresponding to the present horizon ($k_*^{-1} \simeq 3000h^{-1} \text{Mpc}$).

Let us now summarize the constraints on the power spectra of baryonic isocurvature perturbations. In addition to the BBN constraint, which we have discussed so far, there are constraints from the observations of the CMB anisotropy and the large scale structure (LSS) [17, 36, 37]. From the CMB anisotropy observations, the effective cold dark matter (CDM) isocurvature perturbations are constrained as follows (95% CL) [17]

$$\begin{align*}
\beta_{\text{iso,CDM}}(k_{\text{low}}) &< 0.045 \quad (k_{\text{low}} = 0.002 \text{Mpc}^{-1}) \\
\beta_{\text{iso,CDM}}(k_{\text{mid}}) &< 0.379 \quad (k_{\text{mid}} = 0.05 \text{Mpc}^{-1}) \\
\beta_{\text{iso,CDM}}(k_{\text{high}}) &< 0.594 \quad (k_{\text{high}} = 0.1 \text{Mpc}^{-1}).
\end{align*} \quad (3.10)$$

Here $\beta_{\text{iso,CDM}}(k)$ is defined as

$$\beta_{\text{iso,CDM}}(k) \equiv \frac{\mathcal{P}_{S_{\text{CDM,eff}}}(k)}{\mathcal{P}_*(k) + \mathcal{P}_{S_{\text{CDM,eff}}}(k)}, \quad (3.11)$$

where $\mathcal{P}_{S_{\text{CDM,eff}}}$ is the power spectrum of the effective CDM isocurvature perturbations, which is defined as $S_{\text{CDM,eff}} = S_{\text{CDM}} + \frac{\Omega_B h^2}{\Omega_{\text{CDM}} h^2} S_B$, and the CDM energy density parameter $\Omega_{\text{CDM}} h^2(= 0.119)$. For baryonic isocurvature perturbations in the absence of CDM perturbations, the relation $\mathcal{P}_{S_{\text{CDM,eff}}} = \left(\frac{\Omega_B h^2}{\Omega_{\text{CDM}} h^2}\right)^2 \mathcal{P}_{S_B}$ is satisfied. Hence, one can convert the constraints on $\mathcal{P}_{S_{\text{CDM,eff}}}$ into those on $\mathcal{P}_{S_B}$.

\footnote{Large isocurvature perturbations could produce a CMB distortion. However, such a distortion is too small to constrain the power spectrum, given the current observational uncertainties [8].}
On the other hand, using combined LSS data, such as the Lyman-\(\alpha\) forest anisotropy and CMB observations, one can constrain the isocurvature perturbations as follows (95\% CL) [36]:

\[
A_{\text{iso, bar}} = -0.06^{+0.35}_{-0.34},
\]

(3.12)

where \(A_{\text{iso, bar}}^2 \equiv \mathcal{P}_B(k_0)/\mathcal{P}_\zeta(k_0)\) \((k_0 = 0.05\, \text{Mpc}^{-1})\) and this constraint is based on the assumptions that the spectral index of the isocurvature perturbations is the same as that of the curvature perturbations and the isocurvature perturbations are fully correlated with the curvature perturbations. A positive value of \(A_{\text{iso, bar}}\) means the full positive correlation and a negative one means the full negative (or anti-) correlation. Note that the Lyman-\(\alpha\) forest observations can see the power spectra in smaller scale \((k \lesssim 1\, \text{Mpc}^{-1})\) than the CMB observations can \((k \lesssim 0.1\, \text{Mpc}^{-1})\).

To visualize the BBN constraints, we assume that the power spectrum is monochromatic,

\[
\mathcal{P}_{S_B, \text{mono}}(k; k_*) = \mathcal{P}_S \delta(\log k - \log k_*).
\]

(3.13)

With this monochromatic power spectrum, the equation \(\langle S_B^2 \rangle = \mathcal{P}_{S_B}\) is satisfied. In figure 3, we show the \(\mathcal{P}_{S_B}\) region excluded by the BBN observations (shaded orange). For comparison, we show also the constraints from the CMB and LSS observations given by eqs. (3.10) and (3.12). For the combined constraint in figure 3, we take the conservative approach and assume the full negative correlation, taking \(A_{\text{iso, bar}} = -0.40\). Note that the derived constraint on \(\langle S_B^2 \rangle\) is valid even in the case of compensated isocurvature perturbations [38], in which the CDM isocurvature perturbations totally compensate the baryonic perturbations, because BBN occurs during the radiation era and is independent of CDM perturbations.\(^3\)

Before closing this section, let us discuss the applicability of eq. (2.2) to the inhomogeneous BBN. Since D is produced during a rather short period of time \((T = 0.1–0.05\, \text{MeV})\), we can focus on that period. The most important scale in the problem is the diffusion length. In eq. (3.9) we take the neutron diffusion length \(d_n\) as a cutoff because the proton diffusion length \(d_p\) is about 100 times shorter than \(d_n\) [35]. If the fluctuation wavelength \(k^{-1}\) of baryons are larger than the neutron diffusion scale \(d_n\), the diffusion is not important, and D production takes place in locally homogeneous regions, which justifies the use of eq. (2.2). Therefore, our generic constraint in figure 3 is valid for \(k<k_d = d_n^{-1}\). On the other hand, if \(k^{-1}\) is smaller than the proton diffusion length, diffusion makes the distribution of baryons homogeneous, so one can use eq. (2.2) as well. A complicated case is when \(d_n^{-1}<k<d_p^{-1}\).

To see how BBN is affected by such fluctuations, let us consider a region with size \(d_n\). In this region neutrons are homogeneous but protons fluctuate, which results in high and low proton density sub-regions embedded in a homogeneous neutron density. In high proton density sub-regions, BBN proceeds a little earlier than in low proton density sub-regions, which leads to a higher abundance of \(^4\text{He}\) and, therefore, lower D abundance. Because the fluctuations are perturbative, the net result is the same as the homogeneous BBN to first order. However, if we take the second order effect into account, we must consider the diffusion back-reaction of neutrons. In the high proton density sub-regions, neutron are consumed for D production earlier, and the neutron density becomes lower than in the low proton density sub-regions. Then neutrons from the low proton density sub-regions diffuse into the high proton density sub-regions, and these neutrons are consumed. Thus, as a net result, D abundance could

\(^3\)Compensated isocurvature perturbations on large scales can be constrained from CMB and baryon acoustic oscillation observations [39–44].
Figure 3. The summary of the constraints on baryonic isocurvature perturbations. The orange shaded region is excluded by the D observations, as derived in this paper. The blue shaded region is excluded by the CMB observations [17]. For comparison, we also show the constraints from the combination of CMB and LSS observations with a green dotted line [36], although this constraint is based on some assumptions (see text).

decrease by the second order effect of back-diffusion. It is difficult to evaluate this small, second-order effect quantitatively; we leave it for future work.

4 Relaxation leptogenesis

Relaxation leptogenesis [30, 31, 45] can generate baryonic isocurvature perturbations on small scales. In this section, we will briefly review the relaxation leptogenesis framework and discuss the improved constraint from the deuterium abundance on this type of models.

In relaxation leptogenesis, the generation of lepton/baryon asymmetry is driven by the classical motion of a scalar field \( \phi \), which is not the inflaton, after the period of cosmic inflation and the during the reheating stage of the universe. During inflation, a light scalar field \( \phi \) with mass \( m_\phi < H_I \) can develop a large vacuum expectation value (VEV) \( \phi_0 \equiv \sqrt{\langle \phi^2 \rangle} \) through quantum fluctuations [46–48]. If the quantum fluctuations are not suppressed by the potential or other interactions, the \( \phi \) can reach an equilibrium VEV \( \phi_0 \) satisfying \( V(\phi_0) \sim H_I^4 \), where \( H_I = \Lambda_I^2/\sqrt{3}M_{pl} \) is the Hubble rate during inflation, and \( \Lambda_I \) is the inflationary energy scale.

However, in general, one can expect some interactions between \( \phi \) and the inflaton field \( I \) of the form

\[
\mathcal{L}_{\phi I} = \lambda_{\phi I} \frac{(\phi I^\dagger I)^{m/2}}{M_{pl}^{m+n-4}}. \tag{4.1}
\]

In the early stages of inflation, when the inflaton VEV \( \langle I \rangle \) is large, interactions of the form (4.1) can contribute a large effective mass term \( m_\phi(I) \gg H_I \) to \( \phi \) suppressing the quantum fluctuations of \( \phi \). As the inflaton VEV \( \langle I \rangle \) decreases, the field \( \phi \) becomes lighter. When the effective mass of \( \phi \) falls below \( m_\phi(I) < H_I \), the quantum fluctuations of \( \phi \) can begin to grow. If the VEV of \( \phi \) only develops in the last \( N_{\text{last}} \) e-folds of inflation, it can reach a value \( \phi_0 \approx \sqrt{N_{\text{last}}} H_I/2\pi \). This is the “IC-2” scenario considered in [30, 31]. We note that
there are two different VEVs, $\phi_0$ (non-inflaton) and $\langle I \rangle$ (inflaton), which play an important role in this scenario.

During reheating, the VEV of $\phi$ relaxes to the minimum of the potential and oscillates with a decreasing amplitude. The relaxation of $\phi$ provides the out of thermal equilibrium condition and breaks time-reversal symmetry, allowing baryogenesis to proceed. For a successful relaxation leptogenesis, one considers the derivative coupling between the $\phi$ and the $B + L$ fermion current $j_{B+L}^\mu$ of the form

$$\mathcal{O}_6 = -\frac{1}{\Lambda_n^2} \left( \partial_\mu |\phi|^2 \right) j_{B+L}^\mu,$$

(4.2)

which is generated at some higher energy scale $\Lambda_n$. This operator can be treated as an effective chemical potential for the fermion current $j_{B+L}$ as $\phi$ evolves in time. In the presence of a $B$ or $L$-violating processes, the system can then relax toward a state with a nonzero $B$ or $L$.

In the case of Higgs relaxation leptogenesis ($\phi = h$), the final lepton asymmetry, $Y \equiv n_L/s$, is estimated to be [49]

$$Y \approx \frac{90\sigma_R}{\pi^6 g_S} \left( \frac{\phi_0}{\Lambda_n^2} \right)^2 \frac{2^2}{3^3} \frac{M_{\text{pl}}}{g_sT_{\text{RH}}} \exp \left( -\frac{8 + \sqrt{15}}{\sigma_R T_{\text{RH}}^3} \right),$$

(4.3)

if the Higgs potential is dominated by the thermal mass term $V(\phi, T) \approx \frac{1}{2} \alpha T^2 \phi^2$ during reheating. The parameters in eq. (4.3) are $\sigma_R \approx 0.33$ at the energy scale $\mu \sim 10^{13}$ GeV, $g_{sS} = 106.75$, and $z_0 = 3.376$. Here $\sigma_R$ is the thermally averaged cross section of the $L$-violating processes, which we consider to be the scattering between left-handed neutrinos via the exchange of a heavy right-handed neutrino. $\phi_0$ is the initial VEV of the Higgs field at the end of inflation, which depends on $N_{\text{last}}$. The reheating channel is assumed to be perturbative, and the reheat temperature $T_{\text{RH}} \approx (24/\pi^2 g_s)^{1/4} \sqrt{M_{\text{pl}}/T_{\text{RH}}}$ is reached when reheating is completed at $t_{\text{RH}}$. The produced lepton asymmetry then turns into the baryon asymmetry through the sphaleron processes.

Since the asymmetry generated in this manner depends on the initial VEV $\phi_0$, the spatial fluctuation of $\phi$ due to quantum fluctuation at inflation stage can result in baryonic perturbations at later time. These perturbations are isocurvature modes because the scalar field $\phi$ is different from the inflaton $I$, and $\phi$ does not dominate the energy density of the universe. As we discussed above, the baryonic isocurvature perturbations are constrained by observations in both the amplitude and the spatial scale $k$. Thus, one can translate these constraints into the limits on $N_{\text{last}}$ of the Higgs field, as well as other parameters, such as $\Lambda_I$ and $T_{\text{RH}}$.

As computed in ref. [49], the power spectrum of the baryon density perturbations resulted from the quantum fluctuations of $\phi$ is

$$P_{SB}(k) \approx \frac{4}{N_{\text{last}}^2} \ln \left( \frac{k_s}{k_s} \right) \theta(k - k_s) \theta(k_s e^{N_{\text{last}}/k} - k),$$

(4.4)

for the fluctuations of $\phi$ that develop during the last $N_{\text{last}}$ e-fold of inflation. Here $k_s \sim a(N_{\text{last}})H_I$ is the comoving wave number corresponding to the mode which first leaves the horizon. By considering a typical inflation setup with the inflationary energy scale $\Lambda_I$ and the reheat temperature $T_{\text{RH}}$, we can relate $k_s$ and $N_{\text{last}}$ by

$$k_s \approx 2\pi e^{-N_{\text{last}} H_I} \left( \frac{T_{\text{RH}}}{\Lambda_I} \right)^{4/3} g_{sS}^{1/3} \langle T_{\text{now}} \rangle \frac{T_{\text{now}}}{g_{sS}^{1/3} (T_{\text{RH}})} T_{\text{RH}},$$

(4.5)
Figure 4. The baryonic isocurvature perturbations $\langle S_B^2 \rangle$ generated by the Higgs relaxation leptogenesis at various $N_{\text{last}}$ and $\Lambda_I$. The solid, dashed, and dotted lines correspond to the cases where the reheat temperatures $T_{\text{RH}}$ are $10^{-7} \Lambda_I$, $10^{-10} \Lambda_I$, and $10^{-13} \Lambda_I$, respectively. The gray horizontal dash-dot line at $\langle S_B^2 \rangle = 0.016$ indicates the constraint from the D abundance at 2$\sigma$ level.

or

$$k_s e^{N_{\text{last}}} \simeq 65 \text{Mpc}^{-1} e^{46.3} \equiv k_{s,0} e^{N_0},$$

for $\Lambda_I = 10^{16} \text{GeV}$, $T_{\text{RH}} = 10^{12} \text{GeV}$, and $T_{\text{now}} = 2.726 \text{K}$.

The square of the baryonic isocurvature perturbations $\langle S_B^2 \rangle$ is then given by

$$\langle S_B^2 \rangle = \int_{k_s}^{k_d} \frac{dk}{k} P_S(k)$$

$$\approx \frac{4}{N^2_{\text{last}}} \int_{k_s}^{k_d} \frac{dk}{k} \ln \left( \frac{k}{k_s} \right) \theta(k - k_s) \theta(k_s e^{N_{\text{last}}} - k)$$

$$= \frac{2}{N^2_{\text{last}}} \min \left[ \ln^2 \left( \frac{k_d}{k_s} \right) \theta(k_d - k_s), N_{\text{last}}^2 \right],$$

where in the last step we have assumed $k_s < k_s$. Using eq. (4.6), we obtain

$$\langle S_B^2 \rangle = \frac{2}{N^2_{\text{last}}} \left[ \ln \left( \frac{k_d}{k_{s,0}} \right) + N_{\text{last}} - N_0 \right]^2 \theta(k_d - k_s).$$

The constraint on the baryonic isocurvature perturbations from the D abundance gives an upper bound on $N_{\text{last}}$:

$$N_{\text{last}} \lesssim 33.7 \ (2\sigma),$$

for $\Lambda_I = 10^{16} \text{GeV}$ and $T_{\text{RH}} = 10^{12} \text{GeV}$.

Figure 4 shows the baryonic isocurvature perturbations generated by the Higgs relaxation leptogenesis model at various $N_{\text{last}}$ and $\Lambda_I$. One can see that the larger the values of $\Lambda_I$ and $T_{\text{RH}}$, the larger is the upper limit on $N_{\text{last}}$. We also see that, for each choice of parameters ($\Lambda_I$, $T_{\text{RH}}$), there is a minimum $N_{\text{last}}$ below which the fluctuation $\langle S_B^2 \rangle$ vanishes.
Figure 5. The BBN constraint on the Higgs relaxation leptogenesis in the $\Lambda_I-N_{\text{last}}$ space at various reheat temperature $T_{\text{RH}}$. The dashed (solid) lines correspond to the 1$\sigma$ (2$\sigma$) constraints. The regions on the lower right-hand side of each contours are excluded. The blue shaded region for $\Lambda_I > 1.88 \times 10^{16}$ GeV is constrained by Planck non-observation of tensor mode [17]. For a successful inflation, one also requires $\Lambda_I > T_{\text{RH}}$, which is indicated as the horizontal parts of each contours.

This corresponds to the case when the scale of the produced baryonic perturbations is smaller than the baryon diffusion scale $k_d$. In this case, the baryonic perturbations are washed out by neutron diffusion before BBN.

Figure 5 shows the parameter space in $\Lambda_I$ vs. $N_{\text{last}}$ at various reheat temperatures $T_{\text{RH}}$. We note that the CMB observations from Planck give an upper bound on the inflationary energy scale $\Lambda_I < 1.88 \times 10^{16}$ GeV [17]. Thus, for a given set of $\Lambda_I$ and $T_{\text{RH}}$, the D abundance constraint provides an upper limit on $N_{\text{last}}$.

5 Conclusion

We have shown that large baryonic isocurvature perturbations existing at the BBN epoch ($T \sim 0.1$ MeV) change the D abundance by the second order effect, which, together with the recent precise measurements of D abundance, with accuracy about 1%, leads to a constraint on the amplitude of the power spectrum $P_{S_B}$ of the baryon isocurvature perturbations. We showed that the amplitude must satisfy $P_{S_B} \lesssim 0.016$ ($2\sigma$) for scales $k^{-1} \gtrsim 0.0025$ pc [see figure 3]. Since no constraint on baryonic isocurvature perturbations existed on small scales $k^{-1} < 10$ Mpc, the BBN constraint obtained in this paper is the most stringent limit for $0.1$ Mpc$^{-1} \lesssim k \lesssim 4 \times 10^8$ Mpc$^{-1}$. Moreover, this constraint is valid even if the perturbations are compensated isocurvature perturbations, because BBN is affected only by the baryonic perturbations.

We have also applied the BBN constraint to the relaxation leptogenesis scenario where large baryon isocurvature perturbations are produced in the last $N_{\text{last}}$ $e$-fold of inflation. We have derived an upper bound $N_{\text{last}} \lesssim 34$ for $T_R \lesssim 10^{12}$ GeV based on the BBN constraint on baryonic isocurvature perturbations.

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Acknowledgments

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