Shot Noise in Disordered Junctions: Interaction Corrections.

D.B. Gutman and Yuval Gefen

Department of Condensed Matter Physics, The Weizmann Institute of Science,
76100 Rehovot, Israel
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We study current correlation functions in a diffusive junction out of equilibrium. We calculate corrections to the electric current and to the zero frequency shot noise due to electron-electron interactions. Contrary to the equilibrium situation (where the corrections to the current and to the current noise are related through the fluctuation-dissipation theorem (FDT)), these two quantities behave differently: the correction to the electron current are governed by the largest of temperature and applied voltage, while the correction to the shot noise is always governed by the temperature.

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I. INTRODUCTION

The physics of non-equilibrium mesoscopic systems was the subject of an extensive research for the last decade. Khlevnyi, who employing a semiclassical approximation considered a multichannel two-terminal junction, was the first to notice that the non-equilibrium current noise in such a system strongly depends on the transparency of the barrier, and vanishes in the limit of completely transparent junction. This is in contrast to the conductance which remains finite in this limit. Later Lesovik has shown that for coherent transport through a multi-channel two-terminal system the zero frequency current noise is given by

\[ S(0) = \frac{e^2}{2\pi\hbar} \int dE \sum_n \{T_n(E)[f_L(E)(1-f_R(E))+f_R(E)(1-f_L(E))]-T_n^2(E)[f_L(E)-f_R(E)]^2\} \] (1)

This was derived using the Landauer scattering states approach. Here \( f \) is the grand-canonical Fermi-Dirac distribution function, \( L \) and \( R \) denote the left and right reservoirs respectively, taken at different values of the chemical potential, and \( \{T_n\} \) are the channel transmissions. This result was later generalized by Büttiker to include the multi-channel multi-terminal case and experimentally confirmed by Reznikov et al. The zero temperature limit of eq. (1) yields the shot noise:

\[ S(0) = \frac{e^2}{2\pi\hbar} \sum_n T_n(1-T_n)eV \] (2)

The remarkable dependence of the shot noise on the quasiparticle charge (as was first emphasized by Kane and Fisher) allowed the first direct measurement of fractional charge in the context of FQHE by Reznikov et al. and by Saminadaya et al. The suppression of the shot noise for open channels (in full accordance with the semiclassical results) is a manifestation of correlations among the electrons, arising from Pauli exclusion principle. In disordered systems one usually cannot control (or obtain information about) the individual channels; instead it is useful to consider average quantities. For coherent diffusive systems (smaller than the inelastic, dephasing and the localization lengths) the various moments of the current-current correlation function have been calculated employing random matrix theory (RMT). Interestingly enough, it turned out that the fermionic suppression of the noise present in ballistic systems is manifest in the configuration averaged noise (in diffusive systems) as well: the shot noise is suppressed by a factor of 1/3 as compared with the result expected for classical particles.

Other than the Landauer scattering states approach, one may use the kinetic equation, or the so-called Kogan-Shulman approach. It is valid for systems where the dynamics (but not necessarily the statistics) of the particles is classical. Comparing with the scattering states approach, the latter is not restricted solely to non-interacting electrons. Unfortunately, it is limited by the usual conditions of applicability of the kinetic equation: it does not yield quantum interference effects, and cannot be used for calculation of the cummulants higher than two. To overcome the latter shortcomings one may employ the Keldysh formalism.

In the present analysis we find it convenient to employ this formalism in the form of a non-linear sigma-model. This machinery was found to be an effective tool for describing the low energy physics of disordered electronic systems. We will not attempt to review here the various technical steps involved in the derivation of the model, but rather recall some of the main ingredients involved. So long as non-interacting electrons are concerned, the super-symmetric sigma-model turns out to be a particularly suitable tool for calculation. The inclusion of electron-electron interaction appears, though, to render this approach infeasible. Interaction among the electrons could be included within the the replica sigma-model, which has been introduced by Finkelstein for the study of disordered Fermi liquids. As has recently been demonstrated by Kamenev and Andreev and independently by Chamon, Ludwig and Nayak, there is an alternative approach of constructing a sigma-model for interacting electrons - the so-called Keldysh sigma-model.

To wrap up this introduction, we find it useful to define some of the physical regimes to be discussed below.
We first stress that the notion of high/low frequency carries two meanings. As far as the diffusive motion of the electrons (viewed as independent particles) is considered, one can refer to the frequency as "low" if it is smaller than the Thouless energy of the system, $E_T = \hbar D/L^2$. As we study the case of interacting electrons, another criterion arises, relating to the propagation of electromagnetic fields in the system. Consider the simplest situation when the effects of external screening (by, e.g., external gates) can be neglected. It is readily seen that the solution of the Maxwell equations (or the self-consistent solution of the Maxwell equations combined with the kinetic equations, cf. Section III below) depends on the effective dimensionality of the sample. For a standard three dimensional geometry the frequency at which the current noise is measured ($\omega$) should be compared with the inverse Maxwell time ($\omega_M = 4\pi\sigma$). At ($\omega \ll \omega_M$) the "cross-coordinate" current correlation (measured at different cross-sections) is independent of the spatial coordinates (and their separation, $l_{x,x'}$). Having this electrodynamic in mind, it is this limit which should be referred to as "low frequency". For quasi-two-dimensional films this regime is realized under the condition: $\omega \ll \kappa_2 D/l_{x,x'}$ where the inverse screening length $\kappa_2 = 2\pi e^2\nu$. The effects of external screening may modify the effective electron interaction.

Our point, though, is that quite generally there is some frequency, below which the correlation function does not depend on the spatial coordinates, i.e. it is a constant as function of $[l_{x,x'}]$. In the case of interacting electrons, in addition to the long wavelength propagation of the electro magnetic field, one also needs to account for inelastic collisions among the electrons. We thus introduce another dimensionless parameter, namely the ratio between the system’s length, $L$, and the inelastic length $l_n$.

The outline of the present work is the following: In the next section we consider non-interacting spinless electrons in a weakly disordered conductor neglecting weak (and in two-dimensions strong as well) localization effects. The latter can be justified once the quantum coherence of the electrons is destroyed by certain dephasing mechanisms (such as fluctuating electromagnetic fields), or is restricted by the finite size of the system. In other words, we consider a situation where the localization length, $\xi$, exceeds either the dephasing length $l_\phi$ or the system size $\{\xi, \xi \gg \max\{l_\phi, L\}\}$. We extend the Keldysh sigma-model formalism to the case of systems out of equilibrium (including the effect of restricted geometries). In Section III we review the calculation of the current noise (in and out of equilibrium), discussing both the low and the high frequency limits, paying special attention to the non-homogeneity (in space) of the current fluctuation in the later case. In Section IV we consider the effect of electron-electron interactions. We review results obtained from the kinetic equation approach, referring briefly to the limits of $l_{in} \ll L$ and $l_{in} \gg L$. We then focus on the latter limit presenting results for the effect of electron-electron interactions on the noise which go beyond the kinetic equation (eqs. [3] and [4]). Our results are analogous to the Altshuler-Aronov corrections found earlier for equilibrium noise. This is done using a model short-ranged interaction. The general expressions obtained for the noise are then analyzed in the interesting case of two-dimensional geometries. We show that electron-electron interaction leads to the suppression of the non-equilibrium current noise which is stronger than the suppression of the current itself - a manifestation of many-body correlation effects. Eqs. (3), (4) and (5) are the main results of our analysis. The three appendices include some technical parts of the derivation. The analysis presented here is an extended account of an earlier work.

II. KELDYSH FORMALISM: NON-INTERACTING ELECTRONS IN DIFFUSIVE SYSTEM

The theory for quantum systems out of equilibrium was suggested simultaneously by Kadanoff and Baym and by Keldysh. This technique has been widely used on the level of diagrammatic calculations. Such techniques can also be developed for the study of weakly disordered samples. Instead of using here a diagrammatic technique, we will present a path-integral description of the problem. For free fermions the action can be written as

$$S = \int dr \int dt \mathcal{L},$$

where the Lagrangian density is given by

$$\mathcal{L} = \bar{\Psi}[\hat{G}_0^{-1} - U_{dis}]\Psi.$$

Here the Schrödinger operator of an electron of mass $m$ in the presence of a vector potential $e\mathbf{A}/c$ is given by

$$\hat{G}_0^{-1} = i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \left(\nabla - \mathbf{A}\right)^2 + \mu.$$

The energy is measured from the chemical potential $\mu$. The action now contains an integral over space and an integral in time over a Keldysh contour (see Fig II). In analogy with ordinary diagrammatic techniques one defines an electron Green’s function, i.e. a time ordered correlation function

$$\hat{G}_{\alpha\beta}(r,t;r',t') = -i(T_c\Psi(r,t_\alpha)\Psi^\dagger(r',t'_\beta))$$

Here $T_c$ is a time ordering operator along a Keldysh contour, the indices $\alpha, \beta$ denote the branches of the Keldysh contour (minus or plus for the upper and lower branch of the contour respectively). Here $()$ denotes averaging with respect to the action eq. (3).
The non-linear term in eq. (11) can be decoupled with an auxiliary bosonic (a $2 \times 2$ matrix) field $\tilde{Q}(r, t, t') \equiv \tilde{Q}_{r,t,t'}$, space and time are unimportant, and we may integrate them out. In the weak disorder case ($\epsilon_F \tau \gg \hbar$, where $\epsilon_F$ is a Fermi energy) the main contribution comes from the minimum of the action, justifying the use of a saddle-point approximation. Varying the action with respect to $Q$, one arrives at the following saddle-point equation

$$\pi \nu Q = \frac{i}{G_0^{-1} \sigma^{(0)} + \frac{1}{2\tau} Q}.$$  

(17)

Taking the Fourier transform with respect to $t - t'$ one can show that the matrix

$$\Lambda(r, \epsilon) = \begin{pmatrix} 1 & 2F(\epsilon) \\ 0 & -1 \end{pmatrix}$$

(18)

for an arbitrary $F$ is a solution of equation (17). Apart from $F(\epsilon)$ which needs to be determined, the solution for the saddle-point (eq. (18)) is not unique. Indeed, the action (17) is invariant under any homogeneous unitary transformation

$$Q = U^\dagger \Lambda U .$$

(19)

To calculate the fluctuations around the saddle point (18) we will distinguish between longitudinal and transversal fluctuations, the latter do not modify the value of $Q^2$. The spectrum of the longitudinal fluctuations has a gap; for this reason they are irrelevant for the low energy physical properties of the system (although they do affect the bare values of the parameters).
gapless transversal fluctuations (soft modes) should be taken into account.

We now focus on the long wavelength fluctuations around the saddle point \( \Lambda \). To study such fluctuations we now extend our discussion and assume the matrix \( U \) in eq. (19) to be a slow function of the spatial coordinate. One may now expand the action in gradients of \( U(r) \). We thus obtain a new action in a form which is usually referred to as a non-linear sigma model

\[
iS[Q] = -\frac{\hbar}{4} \text{Tr}[D(\nabla Q)^2 + 4iQ] ,
\]

where the integration over the field \( Q \) is now restricted by the non-linear constraint

\[
Q^2 = 1.
\]

One may derive an equation of motion (quantum kinetic equation), which is compatible with eq. (21):

\[
D\nabla(Q\nabla Q) + i[\dot{c}, Q] = 0.
\]

We now use the following parameterization:

\[
Q = \Lambda \exp(W),
\]

where the matrix \( W_{x,\epsilon,\epsilon'} \), in turn, is parameterized as follows

\[
W_{x,\epsilon,\epsilon'} = \begin{pmatrix}
F_{x,\epsilon} \bar{w}_{x,\epsilon,\epsilon'} & -w_{x,\epsilon,\epsilon'} + F_{x,\epsilon} \bar{w}_{x,\epsilon,\epsilon'} F_{x,\epsilon'} \\
-\bar{w}_{x,\epsilon,\epsilon'} F_{x,\epsilon'} & -\bar{w}_{x,\epsilon,\epsilon'} F_{x,\epsilon'}
\end{pmatrix},
\]

and where the fields \( F, w \) and \( \bar{w} \) are unconstrained. Note that \( \Lambda \) is parameterized by \( F \) as well, (cf. eq.(18)). Expanding the action in soft mode fluctuations around the saddle point we obtain, to second order in \( \bar{w} \), \( w \):

\[
iS[W] = iS_0[W] + iS_1[W] + iS_2[W].
\]

The zeroth order term is identically equal to zero

\[
iS_0[W] = 0.
\]

This implies that, in the absence of external sources, the generating functional is equal to unity even on the level of the saddle point equation. The term linear in \( W \) is given by

\[
iS_1[W] = \frac{\pi\nu}{2} \text{Tr}[i\bar{c}(\bar{w}F - F\bar{w}) + D\bar{w}\nabla^2 F].
\]

Selecting \( \Lambda \) to be a saddle point of the action (20), which requires the part linear in \( W \) to vanish as well, we obtain

\[
(i(\epsilon_1 - \epsilon_2))F_{x,\epsilon_1,\epsilon_2} - D\nabla^2 F_{x,\epsilon_1,\epsilon_2} = 0.
\]

The function \( F(x,\epsilon) \) is related to the single particle distribution function \( f(x,\epsilon) \)

\[
F(x,\epsilon) = 1 - 2f(x,\epsilon).
\]

Indeed, eq.(28) coincides with the Boltzmann equation within the diffusion approximation for \( f \). However, since eq.(28) is a linear homogeneous equation, \( F \) may be related to \( f \) only up to multiplicative factors. To establish the relation, eq.(29), one needs to resort to the inhomogeneous version of eq.(28), where the r.h.s. is replaced by a collision integral due to electron-electron interactions. This indeed has been done in Ref.4 (cf. with their eq.(138)).

We can describe what we have done above slightly differently. We note that the parameterization, eq.(23), is a particular choice satisfying the constraint, eq.(21). We may look for a particular case for which \( \Lambda \) (which, to begin with, was a saddle point of the original action, eq. (11)) is a saddle point of the non-linear \( \sigma \)-model action, eq. (20) (i.e., \( S_1[W] = 0 \)). This leads to equation (23). On the other hand, we know that one can employ the Keldysh technique perturbatively, and relate \( \Lambda \) (with the parameterization of eq. (23)) to \( \bar{G}(r,r) \) (where \( \bar{G} \) is the Green’s function in the Keldysh space), with the (1, 2) element thereof related to the distribution function \( f \). This justifies referring to eq.(28) as a kinetic equation. To consider larger deviations from the saddle point \( \Lambda \), one needs to consult the exact saddle point equation (23).

Equation (23) (or 29) should be supplemented by appropriate boundary conditions at the endpoints of the sample. The quadratic fluctuations around the saddle point are given by

\[
iS_2[W] = \frac{\pi\nu}{2} \left[ \bar{w}_{x,\epsilon,\epsilon'} [-D\nabla^2 + i(\epsilon - \epsilon')] w_{x,\epsilon,\epsilon'},
\right.

\[
D\nabla F_{x,\epsilon} \bar{w}_{x,\epsilon,\epsilon'} \nabla F_{x,\epsilon} \bar{w}_{x,\epsilon,\epsilon'} \right].
\]

This action describes the dynamics of the fluctuating fields \( w \) and \( \bar{w} \). The dynamics may be affected by the last term on the r.h.s. of eq.(30), accounting for deviations from thermal equilibrium.

We now specialize to a concrete geometry and specific boundary conditions. We consider a system made of two clean (i.e., no disorder) and stationary reservoirs, connected to each other through a diffusive bridge as shown in Fig.(2).

\[
\mu + eV/2 \quad \mu - eV/2
\]

FIG. 2. A diffusive bridge (the shaded area) between two reservoirs.
non-zero bias ($eV \neq 0$) gives rise to a finite d.c. current through the bridge. Hereafter we assume that the typical energy relaxation time is much larger than the time-of-flight of the electrons through the micro-bridge, resulting in the following stationary diffusion equation

$$D \nabla^2 F(r, \epsilon) = 0. \quad (31)$$

Note that under the conditions specified above, eq. (31) contains no energy collision integral and no time derivative. Furthermore, we assume that the typical time a particle spends in the reservoir is much larger than the energy relaxation time, so that the single-electron distribution function is subject to local equilibrium boundary conditions at the edges, with the respective chemical potentials as indicated in Fig. (2). It turns out that the contribution function is subject to local equilibrium boundary energy relaxation time, so that the single-electron distribution function must vanish in the transversal direction, $\partial D/\partial y = \partial D/\partial z = 0$. Moreover, as the diffusion modes cannot propagate through the clean metallic leads, the diffusion propagator must vanish at the contacts to the latter.

The action (36) can be viewed as a differential operator acting in the $2 \times 2$ space spanned by the fields $w$ and $\bar{w}$. It is convenient to define the diffusion propagator as

$$(-iw + D \nabla^2) D(r, r', \omega) = \frac{1}{\pi \nu \hbar} \delta(r - r'), \quad (34)$$

with the boundary conditions defined on the edges. Since no current may flow through infinite walls, the derivative of the diffusion propagator must vanish in the transversal direction, $\partial D/\partial y = \partial D/\partial z = 0$. Moreover, as the diffusion modes cannot propagate through the clean metallic leads, the diffusion propagator must vanish at the contacts to the latter.

As we have prepared the tools for our analysis of non-equilibrium noise, we are now set to construct the correlation functions for $w$ and $\bar{w}$. It is convenient to define the diffusion propagator as

$$\langle w(x, \epsilon_1, \epsilon_2) \bar{w}(x', \epsilon_3, \epsilon_4) \rangle = 2(2\pi)^2 \delta(\epsilon_1 - \epsilon_4) \delta(\epsilon_2 - \epsilon_3) D(x, x', \epsilon_1 - \epsilon_2), \quad (35a)$$

$$\langle w(x, \epsilon_1, \epsilon_2) w(x', \epsilon_3, \epsilon_4) \rangle = -(2\pi)^3 \delta(\epsilon_1 - \epsilon_4) \delta(\epsilon_2 - \epsilon_3). \quad (35b)$$

$$g \int dx_1 D_{\epsilon_1-\epsilon_2, x, x_1} \nabla F_{x_2, x_1} \nabla F_{x_1, x_1} D_{\epsilon_2-\epsilon_1, x_1, x'} \quad (35c)$$

$$\langle \bar{w}(x, \epsilon_1, \epsilon_2) \bar{w}(x', \epsilon_3, \epsilon_4) \rangle = 0, \quad (35c)$$

were the dimensionless conductance $g = \hbar \nu D$. Note that as a result of the Keldysh rotation, eq. (34), the correlation function of $\langle \bar{w}w \rangle$ (eq. (35c)) vanishes. Next we employ the action (36) to calculate various observables. For this purpose we will use the generating functional, eq. (37). To write it in terms of the $Q$ matrices we note that the magnetic field enters the action through the "long derivative" $\partial = \nabla + i [a^\lambda \gamma_\lambda, \ ]$, where $ca/c$ is the vector potential. This follows from the gauge transformation $\Psi \rightarrow U_A \Psi$ which allows to eliminate the vector potential from the action. Provided that the spectrum of the electrons can be approximated as linear, one chooses $\nabla U_A = -i A x U_A$. As a result the generating functional (3) can be written as

$$Z[a] = \int DQ \exp \left( -\frac{\pi \nu \hbar}{4} \text{Tr} [D(\partial Q)^2 + 4iQ] \right) \quad (36)$$

As a simple example for an observable we consider the mean electron current through the bridge (at a vanishing magnetic field), which is given by

$$I = \frac{-e}{2\pi} \frac{\delta Z[a]}{\delta a} \bigg|_{a_2=0, a_1=0}. \quad (37)$$

The corresponding leading diagram for the average current is shown in Fig. (3).

FIG. 3. The leading diagram for the mean value of the electron current in the diffusion approximation. The shaded semi-circle represents the Keldysh component of the Green function.

Employing eq. (37) we find an expression for the d.c. current

$$I = e \pi \nu \hbar D \text{Tr} \{ (Q \nabla Q - (\nabla Q) Q)_{\gamma 2} \} \quad (38)$$

Taking the saddle point solution for $Q$, eq. (18), one obtains

$$I_{\text{Ohm}} = G_{\text{Ohm}} V, \quad (39)$$

which is Ohm's law; for diffusive systems of length $L$ and cross-section $A$ the low frequency conductance is equal to

$$G_{\text{Ohm}} = \sigma A L, \quad (40)$$

where the conductivity $\sigma$ is related to the diffusion coefficient and the thermodynamic density of states through the Einstein relation

$$\sigma = e^2 \nu D. \quad (41)$$
III. CURRENT NOISE IN DISORDERED JUNCTIONS

Besides the average value of the current one may study its fluctuations in time. As far as the second moment is considered, one is often concerned with the symmetrized correlation function:

\[ S(t, x, t', x') = \frac{1}{2} \langle \hat{I}(t, x) \hat{I}(t', x') + \hat{I}(t', x') \hat{I}(t, x) \rangle. \]  

(42)

In a steady state \( S \) is a function of the difference between its respective arguments. In equilibrium this quantity is related to the linear conductance by the Fluctuation-Dissipation Theorem (FDT)

\[ S^{eq}(\omega) = \hbar \omega \coth \left( \frac{\hbar \omega}{2T} \right) G(\omega), \]  

(43)

where \( G(\omega) \) is the frequency dependent conductance. Note that the choice of the microscopic parameters (such as impurity strength and concentration) enter only through the diffusion coefficient, hence the conductance.

The relation, eq. (43), between the conductance and current noise is universal at equilibrium and does not depend on the microscopic details of the system at hand. Out of equilibrium there is no fundamental expression governing the relation between conductance and the spectral function of the noise, hence one needs to calculate them separately.

To calculate a two-operator correlation function, one may choose the time indices to relate to different branches of the Keldysh contour, and consequently represent the correlator as a Keldysh time-ordered expression, making it possible to apply Wick’s theorem. Formally, \( \langle \hat{A}(t)\hat{B}(t') \rangle = \langle T_\tau \hat{A}(\tau)\hat{B}(\tau') \rangle \), where \( \hat{A}, \hat{B} \) are arbitrary operators and \( T_\tau \) denotes the time ordering operator on the Keldysh contour. The symmetrized current-current correlation function (at a vanishing magnetic flux) can be represented as

\[ S(x, t; x', t') = -\frac{e^2}{4} \frac{\delta^2 Z[a]}{\delta a_1(x, t) \delta a_2(x', t')} \bigg|_{a_1=0, a_2=0}. \]  

(44)

Here we have used the identity \( Z[a_1; a_2 = 0] = 1 \). After functional differentiation one obtains the following equation

\[ S(x, t; x', t') = \frac{e^2 \pi \hbar \nu D}{4} \int_{x,x',t,t'} D_{x,t; x', t'} \left( \frac{\pi \nu D}{2} \right) M_{x,t; M_{x', t'}}. \]  

(45)

where the subscript 0 denotes averaging with the action \( Z[\alpha] \); we have also introduced the notation

\[ \gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \gamma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \]

\[ I_{x,t; x', t'}^D = \text{Tr} \left\{ \hat{Q}_{x,t; x', t'} \gamma_2 \hat{Q}_{x', t'; x, t} \gamma_2 - \delta_{t,t'} \hat{Q}_{x,t; x', t'} \right\} \delta_{x,x'}. \]

\[ M(x, t) = \text{Tr} \left\{ \int dt_1 \left[ \hat{Q}(x, t, t_1) \nabla \hat{Q}(x, t_1, t) - \left( \nabla \hat{Q}(x, t_1, t_1) \right) \hat{Q}(x, t_1, t) \right] \right\}. \]  

(46)

One may note the correspondence between the expression (43) and the direct diagrammatic analysis published earlier (cf. Fig. 4).

The first (diamagnetic) term in eq. (12) is local in space (i.e., decaying on distance of the order of the elastic mean free path) and corresponds to the bare diagram (a), which describes the fluctuations of the distribution function at a given point. The non-local part corresponds to the diagrams (b-f), which describe the diffusive propagation of current fluctuations throughout the sample. It is convenient to define a local effective noise temperature as

\[ T_{\text{noise}}(\omega, x) = \frac{1}{4} \int d\omega' [1 - F_\omega(x, x') F_\omega(x, x')], \]  

(47)

such that at equilibrium the noise can be cast as purely “thermal”:

\[ S^{eq} = 2T_{\text{noise}} G. \]  

(48)

One may work out (see Appendix A) the explicit form of the current-current correlation function:

\[ S(x, x'; \omega, \nu) = 2e^2 g \left[ T_{\text{noise}}(x, \omega) \delta(x - x') + \pi \nu D \left( \frac{\pi \nu D}{2} \right) M_{x, t; M_{x', t'}} \right] \]  

(49)

The same expression was independently derived by Nagaev recently.

Although eq. (49) is generally quite complicated, one may study certain limiting cases thereof.

In the low frequency limit \( (\omega \ll E_{Th}) \) the correlation function does not depend on the spatial coordinates...
due to particle conservation. To study the temperature and frequency dependence it is convenient to integrate eq. (49) over the volume of the system, which leads to

\[ S(\omega) = \frac{1}{6} \left[ 4S_{\text{eq}}^q(\omega) + S_{\text{eq}}^q(\omega + eV) + S_{\text{eq}}^q(\omega - eV) \right]. \] (50)

In the high frequency limit (\( \omega \gg E_{\text{Th}} \)) the behavior is qualitatively different. The "cross-coordinate" noise spectrum (cf. eq. (46)) in this situation is a decreasing function of the distance between cross-section (\( x, x' \)), through which the current fluctuation are studied. If the distance \( l_{x,x'} \equiv |x - x'| \) is much larger than the typical diffusion length \( \sqrt{D/\omega} \), the correlation function (49) practically vanishes.

We stress that while the electrons here are assumed to be non-interacting, the low frequency limit of equation (50) is valid for interacting electrons as well (in the limit of low electron-electron collision rate), and is in good agreement with experiment. To see this one may follow arguments similar to those given by Nagaev.

Let us assume that it is possible to describe the kinetics of interacting electrons by a Kogan-Shulman type set of equations

\[ \delta j = -eD \nabla \delta n + \sigma(\omega) \delta E(\omega) + \delta j^\text{ext}, \]
\[ i\omega e \delta n + \nabla \delta j = 0, \]
\[ \nabla \delta E = 4\pi e \delta n. \] (51)

Here the fluctuations of the electric current density, \( \delta j \), are related to the fluctuations of the particle density, \( \delta n \), and the electric field, \( \delta E \), in a self-consistent field approximation. For simplicity we have assumed here that the elastic relaxation time and the electron density of states are constant in the vicinity of the Fermi surface. Furthermore, we have ignored the contribution to the resistance associated with elastic scattering at the contacts to the leads. If this is not the case, the eq. (51) should be modified correspondingly.

The "random source" \( \delta j^\text{ext} \) is generated by short-range scattering events of conducting electrons on the impurities. Motivated by the fact that scattering events on scales larger than the mean free path are uncorrelated, we model the random source correlation function by

\[ \langle \delta j^\text{ext}_\alpha(r) \delta j^\text{ext}_\beta(r') \rangle_\omega = 4\pi \sigma(\omega) \delta(r - r') T(r, \omega). \] (52)

In the low frequency limit the continuity equation guarantees that the current-current correlation function is independent of the choice of cross-sections, i.e., it is constant as function of its coordinates. To find this constant it is convenient to integrate the current correlation function with respect to its coordinates over the volume of the sample. In our model there are no fluctuations of the electron density and electrostatic potential in the leads. This implies that the gradient term disappears after the volume integration, i.e., we rederive the non-interacting result, eq. (50). Given the fact that neither the diffusion propagator nor the effective temperature entering eq. (49) depend on the phase of the electron wavefunction, we conclude that the current noise in diffusive systems is unaffected by quantum decoherence processes in the system to leading order in \( 1/g \).

IV. CURRENT FLUCTUATIONS OF INTERACTING ELECTRONS

So far we have considered the physics of non-interacting electrons. In the present section we study how the electron-electron interaction affects the current through a diffusive junction as well as the current fluctuations, all this out of equilibrium. One may note that the conductance (including weak localization corrections) is insensitive to the shape of the electron distribution function. By contrast, the double-step shape of the distribution function is crucial as far as shot-noise is concerned. This provides us with a motivation to consider the effect of electron-electron interaction on the single-electron distribution function, hence the noise. Since the electrons are interacting particles, one should include the (electron density dependent) electric field in the kinetic operator, supplemented by a self-consistency condition, eq. (51). This leads to the appearance of a smooth electric field in the constriction which, on its own (as we have already seen in the Section 1), does not give rise to any effect on the low frequency noise, but may modify the higher frequency spectrum of the current noise. This case has been studied extensively applying the Shulman-Kogan approach to various physical systems. Briefly, the presence of interaction defines a characteristic RC time scale. Depending on the ratio between the measured frequency and the inverse of this time scale, fluctuations of the distribution function propagate (or do not propagate) through the entire system (over time scales \( \sim 1/\omega \)). In the low frequency limit, fluctuations of the distribution function do propagate throughout the entire system (over time scales \( \sim 1/\omega \)), in which case current-current correlators are independent of the spatial coordinates and do not discriminate between interacting and non-interacting electrons. The value of the shot noise is then given by

\[ S(0) = \frac{eI}{3}. \] (53)

By contrast, at high frequencies (larger then the inverse RC time) interactions affect the frequency and the space dependence of the current noise.

However, in addition to the creation of a smooth electric field, electron-electron interactions may lead to inelastic scattering. The relative importance of such a scattering process is determined by the ratio between the sample size \( L \) and the inelastic mean free path \( l_n \). When the inelastic length is much larger than the system size, electron-electron scattering leads to a small positive correction to the noise given by eq. (50) (cf. Ref. [1]). For
systems with a short relaxation length \((l_{\text{sn}} \ll L)\), the distribution function can be approximated by a quasi-equilibrium one. The latter implies that each piece of the conductor possesses its own effective temperature and electro-chemical potential. The contributions of the individual pieces are then added independently. The local quasi-equilibrium conditions lead to the “smearing” of the double-step distribution function, which motivates us to study analogous, interaction is markedly different from the Fermi-Dirac distribution function of the fluctuations of the electric potential in the sample can be expressed through the polarization operator of the gas. This is not the case out of equilibrium. To find such a correlation function in the non-equilibrium case is a demanding task, which, in its general formulation, is yet to be solved. Formally the difficulty arises because the conventional RPA approximation has to be modified. Physically this is related to the fact that screening of the interaction for the non-equilibrium gas differs from the standard equilibrium problem. We circumvent this difficulty by considering a toy model of an instantaneous short-range interaction:

\[
\tilde{V}_0(r - r', t - t') = \hbar \delta(r - r')\delta(t - t') .
\]  

The effective interaction strength \(\Gamma\) is assumed to be small:

\[
\Gamma \nu \ll 1 .
\]  

The condition, eq.(51), allows to calculate corrections due to electron-electron interactions perturbatively. From calculations done at equilibrium it is known that while this model predicts the correct qualitative behavior for the electronic conductance, it misses the double logarithmic correction to the tunneling density of states. The different singularities in the conductance and the tunneling-density-of-states have been attributed to the presence (absence) of an approximate gauge invariance \(^2\) in the former (latter). Gauge invariant quantities are less sensitive to the details of the electron-electron interaction. Since the current correlation function is evidently gauge invariant, we expect our model interaction to yield the correct (singular) temperature and voltage dependence, up to a non-universal numerical prefactor.

Following the rotation, eq.(14), of the interaction part of the action, eq.(61), in Keldysh space, it reads

\[
iS[\Phi] = i\Gamma \text{Tr}\{\Phi^+ (r, t) \sigma^3 \Phi (r, t)\} .
\]  

Next one performs a gradient expansion of eq.(59c), treating the field \(\phi\) under the logarithm as a small perturbation, which yields

\[
S(0) = \frac{\sqrt{3}}{4} eI .
\]  

We now focus on the limit of large inelastic length, and study the effect of electron-electron interaction beyond the applicability of the kinetic equation; we evaluate the interaction corrections to the current noise. While this contribution is small in magnitude, for system with effective dimensionality \(d \leq 2\) it has strong (singular) dependence on temperature and voltage, contrary to the large (but practically constant) non-interacting part.

We take the function \(F\) to be that of the non-interacting gas (in general out of equilibrium), eq.(32). Corrections to the equilibrium conductance and noise due to the interplay between interaction and disorder have been recently discussed by Altshuler and Aronov \(^4\) and in two-dimensional settings are given by

\[
\delta G = \frac{G}{2\pi^2 y} \ln \left( \frac{T \tau}{\hbar} \right) .
\]  

One may note that unlike the non-interacting case (cf. eq. (40)), the conductance due to the Altshuler-Aronov term depends strongly on temperature, i.e. on the smearing of the distribution function. In the non-equilibrium problem the shape of the distribution function is markedly different from the Fermi-Dirac distribution function, which motivates us to study analogous, out-of-equilibrium corrections to the current and to the noise. In order to do so we incorporate the Coulomb interaction into the sigma model. The action of the interacting electrons may be written as

\[
S_{\text{total}}[\Psi] = S_0[\Psi] + S_{\text{int}}[\Psi] ,
\]  

where \(S_0[\Psi]\) is given by eq. (1) and the interaction part of the action can be described as

\[
S_{\text{int}}[\Psi] = - \frac{1}{2} \sum_{i=1}^2 \int \text{d}r \text{d}r' \text{d}t \rho_i(r, t) \sigma_i^{(3)} V_0(r - r') \rho_i(r', t). 
\]  

Here \(\rho\) is the electronic density, \(\rho_i = \Psi_i^\dagger \Psi_i\), while \(V_0\) represents the interaction potential. One may introduce a new auxiliary bosonic field

\[
\tilde{\Phi} = \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix} ,
\]  

which decouples the interaction in the particle-hole channel, yielding

\[
iS_{\text{total}} = iS[\tilde{\Phi}] + iS[\tilde{\Phi}, \tilde{Q}] ,
\]  

\[
iS[\tilde{\Phi}] = i\hbar \text{Tr}\{\tilde{\Phi}^+ V_0^{-1} \sigma^{(3)} \tilde{\Phi}\} ,
\]  

\[
iS[\tilde{\Phi}, \tilde{Q}] = - \frac{\pi \hbar \nu}{4\tau} \text{Tr}\{Q^2\} + \text{Tr} \ln \left[ \tilde{G}_0^{-1} \sigma^{(3)} + \frac{i\tilde{Q} \sigma^{(3)}}{2\tau} + \phi_\alpha \gamma^\alpha \right] .
\]  

A major difficulty now arises. The study of an electron gas at equilibrium relies on the fact that the correlation function of the fluctuations of the electric potential in the sample can be expressed through the polarization operator of the gas. This is not the case out of equilibrium. To find such a correlation function in the non-equilibrium case is a demanding task, which, in its general formulation, is yet to be solved. Formally the difficulty arises because the conventional RPA approximation has to be modified. Physically this is related to the fact that screening of the interaction for the non-equilibrium gas differs from the standard equilibrium problem.

In a two-dimensional setting, the effective interaction strength \(\Gamma\) is assumed to be small:

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iS[\Phi] = i\Gamma \text{Tr}\{\Phi^+ (r, t) \sigma^3 \Phi (r, t)\} .
\]  

Next one performs a gradient expansion of eq.(59c), treating the field \(\phi\) under the logarithm as a small perturbation, which yields
Employing eq. (37) we obtain
\[
iS[\Phi, Q] = -\frac{\pi\nu}{4} \text{Tr} \{D(\nabla Q)^2 - 4i(\phi_\alpha \gamma^\alpha + \epsilon)Q\}.
\]
(63)

The correlation function of the fluctuations of the electric potential, corresponding to eq. (35), is
\[
\langle \phi_\alpha (r, \omega) \phi_\beta (r', -\omega) \rangle = -i \Gamma \delta (r - r') \sigma^{(1)}_{\alpha, \beta},
\]
(64)
and the correlation function of \(w\) and \(\bar{w}\) are given by eq. (33). Let us now use this model to calculate the interaction correction to the d.c. value of electron current. Employing eq. (37) we obtain
\[
\delta I^{ee} = \frac{e \pi \nu D (i \pi \nu)^2}{4} \times \langle \text{Tr} \{ (Q \nabla Q - \nabla QQ)^2 \} \text{Tr} \{ \phi_\alpha \gamma^\alpha Q \} \text{Tr} \{ \phi_\beta \gamma^\beta Q \} \rangle_0,
\]
(65)
where the subscript 0 means that we perform averaging employing the correlators of eqs. (34) and (35).

The analogous correction to the current noise is given by
\[
\delta S^{ee}(0) = \frac{e^2 \pi g (i \pi \nu)^2}{4} \left\langle \left[ M_{x,t,x'}^D - \frac{\pi D}{2} M_{x,t} M_{x',t'} \right] \times \text{Tr} \{ \phi_\alpha \gamma^\alpha Q \} \text{Tr} \{ \phi_\beta \gamma^\beta Q \} \right\rangle_0.
\]
(66)

So far our analysis was general and applied to diffusive conductors of any dimensionality. From this point on we

\[
\delta S^{ee}(0) = \frac{G_{\text{Ohm}}}{6 \pi^2 g} \left[ 2T \ln \left( \frac{\zeta \tau}{\hbar} \right) + eV \coth \left( \frac{eV}{2T} \right) \ln \left( \frac{\zeta \tau}{\hbar} \right) + \ln \left( \frac{T \tau}{\hbar} \right) \right],
\]
(68)

Eq. (68) is the main result of this section. Considering the asymptotic high temperature behavior of eq. (68), one notes that \(\delta S(0)\) is given to leading order by the expression for Nyquist noise. Corrections to the noise in this limit are related to the interaction corrections to the conductance (the latter is related to the Nyquist noise through the FDT), namely
\[
\delta S^{ee}(0) = \frac{G_{\text{Ohm}}}{\pi^2 g} T \ln \left( \frac{T \tau}{\hbar} \right),
\]
(69)
Eq. (69) is related to the correction to the mean current, eq. (67).

In the large voltage limit, the shot component of the noise is dominant, but interaction correction to it is still determined by the temperature
\[
\delta S^{ee} = \frac{G_{\text{Ohm}}}{6 \pi^2 g} \left| V \right| \ln \left( \frac{T \tau}{\hbar} \right).
\]
(70)
Similarly to previous high temperature limit, here too the interaction correction to the noise is negative (cf. 53). As we can see the results for the current (67) and for the current noise (68) are no more simply related to each other.

The result we have obtained can be qualitatively understood in the following way. The charge transfer through disordered junction is a stochastic process. For non-interacting electrons the transfer of the charged particles is described by a time sequence with a binomial distribution function. The mean current has to do with the mean charge transferred through a cross-section over a given time interval, while the second cumulant of the current noise is related to the dispersion of the distribution function. Electron-electron interactions render the electron motion more correlated. This affects the dispersion more strongly than it affects the average value of the distribution, implying that the ongoing random process is no longer binomial. The deviation from the binomial distribution, though, is small in the inverse dimensionless conductance.

Finally we would like to discuss in brief possible experimental implications of our results. The effect predicted here can, in principle, be tested experimentally in samples where the conventional Altshuler-Aronov suppression of the conductance is observed. The non-equilibrium character of the quantities considered raises the question whether other mechanisms (such as inelastic electronic collisions) which might modify the noise spectrum as well, would not mask our effect. We thus would like to point out a qualitative difference between the interaction
corrections to the noise discussed here, and corrections that arise because of the inelastic length being finite.\cite{[54] [52]}

Normally the electron inelastic length increases as the temperature and/or the applied bias decrease. For this reason corrections associated with the ratio \( L/l_{in} \) being finite diminish as one lowers the bias (temperature). In deed, when the inelastic length is determined by large momentum transfer the correction to the shot noise due to inelastic collisions is given by \( \delta S^{in} \)

\[
\delta S^{in} = \frac{eI \kappa_3 (eV L)^2}{3 40320 \rho_F} = 0.02 \cdot e^2 I L^2 \frac{l_{in}}{l_{in}^2}. \tag{71}
\]

Here \( \kappa_3 = \sqrt{4\pi e^2 \nu} \) is the inverse screening length in three dimensions. The second equality in eq. (71) makes use of the expression for the three-dimensional inelastic length (large momentum transfer), given by

\[
l_{in} = \frac{64 \pi^2 (eV)^2 \kappa_3}{\pi e I}. \tag{72}
\]

This expression is to be used for quasi-two-dimensional conductors (metallic films). While the leading term, eq. (72), is linear in the voltage, the corrections are proportional to its second power, and therefore can be neglected for small enough voltages. The corrections we have found, eqs. (69,70), behave in the opposite way. They become increasingly more pronounced as one lowers the voltage and the temperature. For a quantitative estimate we consider a metallic film of length \( L = 10^{-4}\)cm and of thickness \( 10^{-6}\)cm, having sheet resistance of 10\(\Omega\)/square. The diffusion coefficient is taken to be \( D = 10^{0}\)cm\(^2\)sec\(^{-1}\), the elastic mean free time – \( \tau = 10^{-13}\)sec and the Fermi energy \( \epsilon_F = 0.1eV \). We consider a voltage bias of \( V = 10\mu V \). The current noise in this range of parameters is of the order of \( 10^{-24}\)Coulomb Ampere. The corrections to the current noise due to inelastic collisions are in this case of order \( 10^{-20}\)Coulomb Ampere, while the corrections due to the quantum interactions are of order \( 10^{-26}\)Coulomb Ampere.

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**APPENDIX A: NOISE OF NON-INTERACTING ELECTRONS**

In this Appendix we derive eq. (49). One may note that there are two types of terms in eq. (45). The ”diamagnetic” part (arising from the \( a^2 \) term in the action) is proportional to the dimensionless conductance, and therefore it is sufficient to consider its value at the saddle point:

\[
I^D_0(x,t) = \frac{4}{(2\pi)^2} \int d\omega \exp(i\omega t) \left[ 1 - F(x, \epsilon - \Omega) F(x, \epsilon) \right]. \tag{A1}
\]

The \( \langle MM \rangle \) correlator is proportional to the square of the dimensionless conductance. It contains both reducible and irreducible contributions to the current-current correlation. In order to extract the contribution of this correlator to the second order cummulant of the current (irreducible contribution), we will need to expand it up to second order in \( w, \bar{w} \). Expanding \( M \) to first order in these variables\cite{[49]} one obtains

\[
M_1(x,t) = \frac{2}{4\pi^2} \int \exp(i\Omega t) d\omega \left[ \nabla \bar{\omega}(x, \epsilon, \epsilon, \Omega) + \nabla \omega(x, \epsilon, \epsilon, -\Omega) - \nabla \bar{\omega}(x, \epsilon, \epsilon, -\Omega) F(x, \epsilon) F(x, \epsilon - \Omega) + \nabla F(x, \epsilon - \Omega) F(x, \epsilon) \bar{\omega}(x, \epsilon, \epsilon, \Omega) + \nabla F(x, \epsilon) F(x, \epsilon - \Omega) \bar{\omega}(x, \epsilon, \epsilon, \Omega) \right]. \tag{2}
\]

Here and later on the subscript \( k \) in the \( I_k \) and \( M_k \) denotes the power of fluctuating field up to which it is expanded. Now one needs to average the product of \( M_1 \)'s over the fluctuating fields

\[
\langle \text{Tr} \{ (Q \nabla Q - \nabla QQ) \gamma_2 \} \rangle_x \text{Tr} \{ (Q \nabla Q - \nabla QQ) \gamma_2 \} = \frac{2}{\pi^2} \int d\omega \exp(i\Omega(t-t')) \left[ \nabla \nabla' D_{\Omega}(1 - F_{\epsilon,\epsilon} F_{\epsilon'-\Omega,\epsilon'}) + \nabla \nabla' D_{\Omega}(1 - F_{\epsilon,\epsilon} F_{\epsilon'+\Omega,\epsilon}) \right]. \tag{3}
\]

Combining eq. (2) and eq. (3) with eq. (45) one obtains the expression for the current correlation function through different cross sections at a finite frequency (for which the diffusion approximation holds).
Using the expressions (65,1,2) we find that the correction to the d.c. current is given by

\[ S(x, x', \omega) = e^2 \pi \nu D \int \frac{d\epsilon}{2\pi} \left[ 1 - F(x, \epsilon)F(x, \epsilon - \omega) \right] \delta(x - x') + \]

\[ \pi \nu D \left( \nabla \nabla' D_{x,x',\omega} [1 - F_{x,\epsilon}F_{x',\epsilon - \omega}] + \nabla \nabla' D_{x,x',-\omega} [1 - F_{x',\epsilon}F_{x,\epsilon - \omega}] + \right. \]

\[ \left. \pi \nu D \int dx_1 \nabla \nabla_{1} D_{x,x_1,\omega} [1 - F_{x_1,\epsilon}F_{x_1,\epsilon - \omega}] \nabla \nabla' D_{x_1,x',-\omega} \right) . \tag{4} \]

The last result can be re-expressed in terms of an effective electrons temperature (eq. (47)), leading to eq.(49).

**APPENDIX II: CORRECTIONS TO THE D.C. CURRENT**

In this appendix we derive eq.(67), i.e. the interaction corrections to the disorder-averaged value of the d.c. current. One first expands the operator M up to second order in the fluctuating fields

\[ M_2(x_1) = \int d[\epsilon] \left[ \nabla F_{x_1,\epsilon} \bar{w}_{x_1,\epsilon} \epsilon w_{x_1,\epsilon} + w_{x_1,\epsilon} \bar{w}_{x_1,\epsilon} \epsilon \nabla F_{x_1,\epsilon} \right] , \tag{1} \]

where \( d[\epsilon] \) implies integration over all \( \epsilon \)'s, \( \int d\epsilon/2\pi \). Next we evaluate the expansion up to second order in the fields \( w, \bar{w} \) of the quantity

\[ \langle \text{Tr} \{ \phi_1 \gamma_1 Q \} \text{Tr} \{ \phi_2 \gamma_2 Q \} \rangle = i \Gamma \int d[\epsilon] \left[ \bar{w}_{x_2,\epsilon} \epsilon w_{x_2,\epsilon} + w_{x_1,\epsilon} \bar{w}_{x_1,\epsilon} \epsilon \nabla F_{x_1,\epsilon} \right] . \tag{2} \]

Using the expressions \( \delta I^{ee} = -ie^2 \pi \nu^3 D \Gamma A \int d[\epsilon] \]

\[ \langle \bar{w}_{x_2,\epsilon} \epsilon w_{x_2,\epsilon} \rangle = -i e^2 \pi \nu^3 D \Gamma A \int d[\epsilon] \int dx_2 (F_{x_2,\epsilon} - F_{x_2,\epsilon}) \nabla F_{x_1,\epsilon} \] \[ \langle \bar{w}_{x_2,\epsilon} \epsilon w_{x_2,\epsilon} \rangle = -ie^2 \pi \nu^3 D \Gamma A \int d[\epsilon] dx_2 (F_{x_2,\epsilon} - F_{x_2,\epsilon}) \nabla F_{x_1,\epsilon} . \tag{3} \]

After averaging over the fields \( w \) and \( \bar{w} \) we find that, to lowest order in the interaction amplitude and inverse conductance, the correction is given by

\[ \delta I^{ee} = -ie^2 \pi \nu^3 D \Gamma A \int d[\epsilon] dx_2 (F_{x_2,\epsilon} - F_{x_2,\epsilon}) \nabla F_{x_1,\epsilon} . \tag{4} \]

Now we will use the fact that the diffusion propagator at frequency \( \Omega \) is a function of the difference of the spatial coordinates; it decays on the scale \( \sqrt{D/\Omega} \) which is much smaller than the sample size, provided that \( h\Omega \gg E_{Th} \). This allows us to integrate over the relative coordinate (assuming that the distribution function does not change much on that distance),

\[ \delta I^{ee} = -e^2 V D \Gamma A \left( \frac{i \pi \nu}{2 \pi L} \right)^3 \int d[\delta \epsilon] dx_2 (F_{x_2,\epsilon} - F_{x_2,\epsilon}) \nabla F_{x_1,\epsilon} . \tag{5} \]

Integrating over \( q \) in \( \left[ \frac{1}{B} \right] \) we obtain

\[ \delta I^{ee} = -\frac{e^2 \nu V \epsilon A e V}{(2\pi)^3 L} \int \frac{d\delta \epsilon}{\delta \epsilon} \coth \left( \frac{\delta \epsilon + e V}{2 k T} \right) . \tag{6} \]

After integration over the energy \( \delta \epsilon \) we finally find the correction to the current which exhibits a logarithmic singularity. The latter is smeared on a scale which is determined by the maximal between the temperature and the voltage:

\[ \delta I^{ee} = \frac{e^2 \pi \nu \Gamma A V}{L (2\pi)^3} \ln(\zeta \tau) . \tag{7} \]

Restoring the "universal" ( i.e., \( \Gamma \)-independent) coefficient from the known equilibrium result we derive eq.(67).

**APPENDIX III: CORRECTIONS TO THE CURRENT NOISE**

In this Appendix we derive eq.(68). To find the interaction correction to the current noise we use the fact
that for frequencies lower than the Thouless energy the result is independent of the cross-sections involved; it is therefore legitimate to integrate the current correlation function over the volume of the system.

Employing eq.(66) we will need to expand the $I^D$ and the MM terms in $w$ and $\bar{w}$ up to a second and third order respectively. We find that the contribution coming from the corrections to MM vanishes after integration over space coordinates (it contains terms of the type $D_{x,L} - D_{x,R}$). To find the correction that arises from the “diamagnetic term” we need to expand the latter up to second order in $w$ and $\bar{w}$,

$$I^D(t, t') = \int d[\epsilon] \exp(i((\epsilon_1 - \epsilon_2)t + (\epsilon_3 - \epsilon_2)t'))$$

$$\left[ F_{w} F_{w} w_{e1,e2} w_{e3,e4} - \delta_{e3,e4} w_{e1,e4} w_{e3,e2} + 2\delta_{e3,e4} F F_{w} F_{e1,e3} w_{e3,e2} + F_{e1} F_{e2} w_{e3,e4} w_{e3,e2} - \delta_{e1,e2} w_{e3,e4} \right].$$

Combining eq.(1) with eq.(2) and using the fact that the diffusion propagator decays on scales much shorter than the system’s size, we can convert the spatial integration over the relative coordinate to an integral in Fourier space (analogously to how it was done in Appendix III). We obtain

$$\delta I_{ee}(\omega) = 4\Gamma A \int d[q] d[x] \text{Im} \left\{ D^2[q, \epsilon_1 - \epsilon_2] \right\} (F_{x,e1} - F_{x,e2})(2F_{x,e1}F_{x,e2} - w - 1).$$

Thus the correction (eq. (66)) to the noise is given by

$$\delta S_{ee}(\omega) = \frac{e^2 D_A \Gamma (\pi \nu)^3}{L^2} \int d[q] d[\delta \epsilon] \text{Im} \left\{ D^2[q, \delta \epsilon] \right\} Y(\delta \epsilon, \omega, eV, T),$$

where we have defined

$$Y(\delta \epsilon, \omega, eV, T) = \int d[u] d[x] (F_{u,x} - F_{u-\delta \epsilon,x})(2F_{u,x}F_{u-\delta \epsilon,x} - 1).$$

Integrating over energy and (“center-of-mass”) coordinate we find that at zero frequency, $\omega = 0$,

$$Y(\delta \epsilon, 0, eV, T) = \frac{L}{3} \left[ 2kT \left( \coth \left( \frac{eV - \delta \epsilon}{2kT} \right) - \coth \left( \frac{eV + \delta \epsilon}{2kT} \right) \right) + eV \coth \left( \frac{eV}{2kT} \right) \left( \coth \left( \frac{eV - \delta \epsilon}{2kT} \right) - \coth \left( \frac{eV + \delta \epsilon}{2kT} \right) \right) - 2eV \coth \left( \frac{eV}{2kT} \right) \coth \left( \frac{\delta \epsilon}{2kT} \right) \right].$$

Substituting eq.(8) into eq.(3) we conclude our calculation of the interaction correction to the noise

$$\delta S_{ee}^{ee}(0) = \frac{G \Gamma \nu}{3g(2\pi)^3} \left[ 2kT \ln(\zeta + 1) + eV \coth \left( \frac{eV}{2kT} \right) \left( \ln(\zeta + 1) + \ln(kT) \right) \right].$$

Restoring a $\Gamma$-independent coefficient from the equilibrium calculation of (8), we finally derive eq.(68).
For a review see K.B. Efetov in.

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In order for the diffusion approximation for the kinetic equation to be valid, the deviation from equilibrium should be small, i.e. the energy gain between two subsequent elastic scattering events must be small in comparison with the Fermi energy ($\langle E^c_{\alpha\beta}\rangle (U/L) \ll 1$).

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We note that in our open boundary geometry, $E_{Th}$ appears as a natural cutoff in our diffusions. A strong dependence on $\zeta$ will show up only when the latter exceeds $E_{Th}$, namely when it replaces the Thouless energy as a cutoff.

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Note that in the present analysis we ignore weak localization corrections. This implies that there are no $\langle uu\rangle$ or $\langle u\bar{u}\rangle$ corrections to $M$, hence the current.