Abstract—Medical image registration is an active research topic and forms a basis for many medical image analysis tasks. Although image registration is a rather general concept, specialized methods are usually required to target a specific registration problem. The development and implementation of such methods has been tough so far as the gradient of the objective has to be computed. Also, its evaluation has to be performed preferably on a GPU for larger images and for more complex transformation models and regularization terms. This hinders researchers from rapid prototyping and poses hurdles to reproduce research results. There is a clear need for an environment which hides this complexity to put the modeling and the experimental exploration of registration methods into the foreground. With the “Autograd Image Registration Laboratory” (AirLab), we introduce an open laboratory for image registration tasks, where the analytic gradients of the objective function are computed automatically and the device where the computations are performed, on a CPU or a GPU, is transparent. It is meant as a laboratory for researchers and developers enabling them to rapidly try out new ideas for registering images and to reproduce registration results which have already been published. AirLab is implemented in Python using PyTorch as tensor and optimization library and SimpleITK for basic image IO. Therefore, it profits from recent advances made by the machine learning community concerning optimization and deep neural network models.

The present draft of this paper roughly outlines AirLab with first code snippets and performance analyses. A more exhaustive introduction will follow as a final version soon.

Index Terms—image registration, autograd, rapid prototyping, reproducibility

I. INTRODUCTION

The registration of images is a growing research topic and forms an integral part in many medical image analysis tasks [30]. It is referred to as the process of finding corresponding structures within different images. There is a large number of applications where image registration is inevitable such as e.g. the fusion of different modalities, monitoring anatomical changes, population modelling or motion extraction.

Image registration is a nonlinear, ill-posed problem which is approached by optimizing a regularized objective. What is defined as quite general requires usually specialized objective functionals and implementations for applying it to specific registration tasks. The development of such specific registration methods has been tough so far and their implementation tedious. This is because gradients have to be computed within the optimization whose implementations are error-prone, especially for 3D objectives. Furthermore, for large 3D images, the computational demand is usually high and a parallel execution on a GPU unavoidable. These are problems which hinder researchers from playing around with different combinations of objectives and regularizers and rapidly trying out new ideas. Similarly, the effort to reproduce registration results is often out of proportion. There is a clear need for an environment which hides this complexity, enables rapid prototyping and simplifies reproduction.

In this paper, we introduce “Autograd Image Registration Laboratory” (AirLab), an image registration environment - or a laboratory - for rapid prototyping and reproduction of image registration methods. Thus, it addresses researchers and developers and simplifies their work in the exploration of different registration methods, in particular also with upcoming complex deep learning approaches. It is written in Python and based on the tensor library PyTorch [23]. It heavily uses features from PyTorch such as autograd, the rich family of optimizers and the transparent utilization of GPUs. In addition, SimpleITK [18] is included for data input/output to support all standard image file formats. AirLab comes along with state-of-the-art registration components including various image similarity measures, regularization terms and optimizers. Experimenting with such building blocks or trying out new ideas, for say a regularizer, becomes almost effortless as gradients are computed automatically. Finally, example implementations of standard image registration methods are provided such as Optical Flow [10], Demons [28] and Free Form Deformations [25]. AirLab is licensed under the Apache License 2.0 and available on GitHub

https://github.com/airlab-unibas/airlab

In the following, we first provide a brief background about medical image registration followed by the description of AirLab, its building blocks and its features. Finally, we provide registration experiments with standard registration methods which are implemented in AirLab including performance analyses and code snippets.

The present draft of this paper roughly introduces AirLab and is intended for the presentation at the 8th International Workshop on Biomedical Image Registration in Leiden. A more detailed final version will follow soon.

II. BACKGROUND

A. Image Registration

Let \( X := \{x_i\}_{i=1}^N \) be a set of \( N \) points arranged in a regular grid which covers the joint image domain of a

https://github.com/airlab-unibas/airlab
moving and fixed image $I_M, I_F \colon \mathcal{X} \rightarrow \mathbb{R}$. The images map the $d$-dimensional spatial domain $\mathcal{X} \subset \mathbb{R}^d$ to intensity values. Furthermore, let $f : \mathcal{X} \rightarrow \mathbb{R}^d$ spatially transform the coordinate system of the moving image. Image registration can be formulated as a regularized minimization problem

$$f^* = \arg \min_f S_X(I_M \circ f, I_F) + \lambda R(f, \mathcal{X}), \quad (1)$$

where $S_X$ is a similarity measure between the transformed moving image and the fixed image and $R$ is a regularization term which operates on $f$ on the domain $\mathcal{X}$. The two terms are balanced by $\lambda$ and $\circ$ is the function composition. An example for a similarity measure is the mean squared error measure for monomodal image registration

$$S_{\text{MSE}} := \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} \left( I_M(x + f(x)) - I_F(x) \right)^2, \quad (2)$$

where $|\cdot|$ is the cardinality of a set. An exemplary regularization term is the diffusion regularization which favours smooth transformations

$$R_{\text{diff}} := \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} \sum_{i=1}^d \| \nabla f_i(x) \|^2 \quad (3)$$

where $i$ indexes the space dimension. In the Sections II-B and III-C the similarity measures and regularizers which are implemented in AirLab are described in more detail.

a) Transformation: Transformation models $f$ can be divided in basically four types: linear, non-linear/parametric, non-parametric and hybrid. Linear transformations, available in AirLab, transform each point $x$ with a linear map $A$

$$f(x) := Ax, \quad (4)$$

where $A$ is a rotation/translation matrix up to 3 in 2D and 6 degrees of freedom in 3D and $x$ stands in homogeneous coordinates $\tilde{x}$.

Non-linear/parametric transformations are defined on a coarse grid of $n < N$ control points

$$f(x) := \sum_{i=1}^n c_i k(x_i, x), \quad (5)$$

where $c_i \in \mathbb{R}^d$ and $k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ is the basis function. Common basis functions are the B-spline [25] or Wendland kernel [12] which both are implemented as example basis in AirLab (see Section III-B). The advantage of non-linear/parametric transformation models are, that they are computationally efficient. Furthermore, if $k$ is smooth they inherently yield smooth transformations.

In non-parametric methods, each point in the image can be transformed individually in $d$ dimensions giving a maximum flexibility. To still be able to reach reasonable registration results the regularization term is inevitable. Hierarchical models can be seen as hybrid parametric and non-parametric models. Their hierarchical structure enables them to capture large deformations [13]. Non-parametric models are supported as well by AirLab while hybrid are planned.

b) Optimization: The similarity measure depends nonlinearly on $f$ which makes an analytical solution to Equation 1 intractable. Because in non-linear registration the number of parameters of $f$ is in the millions, gradient based optimization is usually the only choice to reach a locally optimal transformation $f^*$. Having PyTorch at hand, state-of-the-art optimizers are available in AirLab such as LBFGS [4] and Adam [15].

B. Image Registration Frameworks

There are already a considerable amount of medical image registration frameworks available which are valuable enrichments to the community. Their focus and intentions are diverse, ranging from rich frameworks to specific implementations. For an exhaustive list and comparison of such image registration software we refer to [14]. Gradient free approaches as e.g. the MRF-based method of [8] are out of scope of the current implementation of AirLab.

The Insight Segmentation and Registration Toolkit (ITK) [34] is a comprehensive framework for diverse image processing and analysis task written in C++. It is mainly intended for the use as a library for developers who want to implement ready-to-use software. The registration tool Elastix [16] is based on ITK and provides a collection of algorithms commonly used in image registration. It can also be used out-of-the-box with the possibility of a detailed configuration script. Furthermore, its plug-in architecture allows to integrate custom parts into the software. The extension SimpleElastix [21] offers bindings to other languages such as Python, Java, Ruby and more. Elastix and SimpleElastix are strong if one needs some flexibility in choosing and combining different registration components for a specific registration task. Scalable Image Analysis and Shape Modelling (Scalismo) [3], [19] is a library mainly for statistical shape modeling written in scala. It provides also image registration functionality and can be interactively executed similar to SimpleElastix. Advanced Normalization Tools (ANTs) [2] is based on ITK as well. It provides a command line tool including large deformation registration algorithms with standard similarity measures. The Automated Image Registration software AIR [33] is written in C and provides basic registration functionality for linear and polynomial non-linear image alignment up to the twelfth order. The Medical Image Registration Toolkit (MIRTK) [25], [27] is a collection of libraries and command-line tools for image and point-set registration. Various registration methods based on free form deformations are provided. Flexible Algorithms for Image Registration (FAIR) [22] is a software package written in MATLAB comprising various similarity measures and regularizers.

None of the mentioned software packages are suited for rapid prototyping in the development of image registration algorithms. This is mainly because: (I) For the optimization, gradients have to be provided. For complex transformation models, regularization terms and similarity measures, their implementation is highly error-prone. (II) For medical images, the computational demand is usually high and therefore the execution has to be performed on a GPU. The development
A. Automatic Symbolic Differentiation

The majority of the frameworks are written in C++. Thus, the development within those frameworks needs good expertise in this language. Furthermore, the number of code lines required for C++ implementations in these frameworks do not agree with the concept of rapid prototyping.

III. AUTOGRAD IMAGE REGISTRATION LABORATORY

AirLab is a rapid prototyping environment for medical image registration. Its unique characteristics are the automatic differentiation and the transparent usage of GPUs. It is written in the scripting language Python and heavily uses key functionality of PyTorch [23].

The main building blocks constitute:

- Automatic differentiation
- Similarity measures
- Transformation models
- Image warping
- Regularization terms
- Optimizers

A. Automatic Symbolic Differentiation

A key feature of AirLab is its automatic symbolic differentiation of the objective function. This means, that only the forward function has to be provided by the developer and the gradient which is required for the optimization is derived through automatic differentiation (AD). AirLab borrows the AD functionality of PyTorch. It is one of the fastest dynamic differentiation frameworks currently available. Its strong points are:

- **Dynamic**: the function which is symbolically differentiated is defined by the computations which are run on the variables. Hence, no static graph structure has to be built which fosters rapid prototyping.
- **Immediate**: only tensor computations which are necessary for differentiation are recorded
- **Core logic**: a low overhead is needed as the AD logic is written in C++ and was carefully tuned

Please cf. [23] for more details.

B. Image Registration Features

We list here the main building blocks required for medical image registration which are provided by AirLab. Upcoming features which are planned for implementation can be found in Section III-C.

1) Similarity Measures:

- Mean Squared Errors: a simple and fast to compute point-wise measure which is well suited for monomodal image registration

\[ S_{\text{MSE}} := \frac{1}{|\mathcal{X}|} \sum_{x \in \mathcal{X}} (I_M(x + f(x)) - I_F(x))^2. \quad (6) \]

Class name: MSELoss

- Normalized Correlation Coefficient: a point-wise measure as \( S_{\text{MSE}} \). It is targeted to image registration tasks, where the intensity relation between the moving and the fixed images is linear

\[ S_{\text{NCC}} := \frac{\sum I_F \cdot (I_M \circ f) - \sum E(I_F)E(I_M \circ f)}{|\mathcal{X}| \cdot \sum \text{Var}(I_F)\text{Var}(I_M \circ f)}, \quad (7) \]

where the sums go over the image domain \( \mathcal{X} \). \( E \) is the expectation value (or mean) and \( \text{Var} \) is the variance of the respective image.

Class name: NCCLoss

2) Transformation Models: AirLab supports three major types of transformation models: linear/parametric, non-linear/parametric and non-parametric models (hybrid models are planned).

a) Linear/Parametric: Currently, AirLab supports rigid transformations including rotation and translation. Similarity and affine transformations are planned.

Class name: RigidTransformation

b) Non-linear/Parametric: as mentioned with Equation [5] non-linear/parametric models have fewer control points as image points are available. The displacement \( f(x) \) for a given point \( x \) in the image is interpolated from neighboring control points by the respective basis function. In AirLab, two exemplary basis functions are implemented:

- **B-spline**: the standard B-spline kernel, which is used in the Free Form Deformation (FFD) algorithm of [25]

\[ k_{B_{ib}}(x, y) := \begin{cases} \frac{3}{2} - |r|^2 + \frac{|r|^3}{2}, & 0 \leq |r| < 1 \\ \frac{1 - |r|^3}{6}, & 1 \leq |r| < 2 \\ 0, & 2 \leq |r|, \end{cases} \quad (8) \]

\[ r = x - y. \quad (9) \]

In addition, AirLab supports B-spline kernels of arbitrary order (first order are used in [32] and third order in the FFD [25]). An order \( p \) is derived by convolving the zeroth order B-spline \( p + 1 \) times with itself:

\[ B_0(r) := \begin{cases} 1 & |r| < \frac{\delta}{2} \\ 0 & \text{otherwise} \end{cases} \quad (10) \]

\[ B_i := B_0 \ast B_{i-1} \quad (11) \]

where \( B_3 \) corresponds to \( k_{B_{ib}} \) and \( \ast \) is the convolution. The control points have a spacing of \( \delta \) which implicitly defines the extent of the kernel. With increasing order, the control point support of the kernel is increased by one for each additional order.

Class name: BsplineTransformation
- Wendland: a family of compact radial basis functions, which is used for image registration in [11], [12]. AirLab supports a Wendland kernel which is in $C^4$:

$$k_W(x, y) = \psi_{3, 2} \left( \frac{\|x - y\|}{\sigma} \right), \quad (12)$$

$$\psi_{3, 2}(r) = (1 - r)^3 + 3\frac{18r^2}{3}, \quad (13)$$

where $a_+ = \max(0, a)$ and $\psi_{3, 2}$ is the Wendland function of the second kind and positive definite in $d \leq 3$ dimensions. The scaling $\sigma$ can also be provided for each space dimension separately to achieve an anisotropic support.

Class name: WendlandKernelTransformation

For the non-linear/parametric transformation models, the transposed convolution is applied (cf. [7]) which is available in PyTorch. It is an up-sampling operation where the interpolation kernel can be provided. That means in our case, the control points are “up-sampled” and interpolated using the basis function of choice. Thus, their evaluation is highly optimized for CPUs and GPUs and the gradient computation, automatically performed with autograd.

c) Non-Parametric: the simpler model is the non-parametric model, where each point in the image can be independently transformed. That means, there are $nd$ parameters (number of image points times number of space dimensions). To achieve a meaningful transformation, strong regularization is required.

3) Image Warping: To compare the transformed moving image with the fixed image within the similarity measures the coordinate system of the moving image has to be warped. As it is mostly done in image registration, AirLab performs backward warping. That means, the transformation is defined on the fixed image domain where the displacement vectors point to the corresponding points in the moving image. To transform the moving image, it is backward warped into the coordinate system of the fixed image. This prevents holes occurring in the warped image.

The warping is performed in normalized coordinates in the interval $[-1, 1]^d$. The points which are transformed out of the fixed image region are identified by checking if $x + f(x)$ falls outside the normalized interval. For illustration please see following snippet:

```python
 displacement = self._grid + displacement
 mask = th.zeros_like(self._fixed_image.image, dtype=th.uint8, device=self._device)
 for dim in range(displacement.size()[-1]):
   mask += displacement[..., dim].gt(1) + displacement[..., dim].lt(-1)
 mask = mask == 0
```

Because displaced points not necessarily fall onto the pixel-grid, interpolation is required. Currently, AirLab supports linear interpolation while B-spline interpolation is planned as up-coming feature. The warping is performed by the grid sampler of PyTorch which utilizes the GPU.

4) Regularization: There are three different types of regularization terms in AirLab. (I) Regularizers on the displacement field $f$, (II) regularizers on the parameters of $f$ and (III) the Demons regularizers which regularize the displacement field $f$ by filtering it in each iteration. Note that Demons regularizers are not differentiated, because in Demons approaches the optimization is an iteration scheme where the image forces (gradient of similarity measure) are evaluated to update the current displacement field and alternatingly the displacement field is regularized using filtering.

a) Regularization Terms: we first list the regularization terms which operate on the displacement field $f$.

- Diffusion: a regularizer which penalizes changes in the transformation $f$

$$R_{\text{diff}} := \frac{1}{|X|} \sum_{x \in X} \sum_{i=1}^{d} \|\nabla f_i(x)\|_2^2. \quad (14)$$

Class name: DiffusionRegulariser

- Anisotropic Total Variation: a regularizer which favours piece-wise smooth transformations $f$

$$R_{\text{anisoTV}} := \frac{1}{|X|} \sum_{x \in X} \sum_{i=1}^{d} |\nabla f_i(x)|. \quad (15)$$

It is anisotropic which means its influence is aligned to the coordinate axes.

Class name: TVRegulariser

- Isotropic Total Variation: the isotropic version of the anisotropic regularizer

$$R_{\text{isoTV}} := \frac{1}{|X|} \sum_{x \in X} \|\nabla f(x)\|_2. \quad (16)$$

Both TV regularizers are not differentiable, therefore, the subgradient of zero is taken at zero.

Class name: IsotropicTVRegulariser

- Sparsity: a regularizer which penalizes non-zero parameters

$$R_{\text{sparse}} := \frac{1}{|X|} \sum_{x \in X} \|f(x)\|_1. \quad (17)$$

Class name: SparsityRegulariser

b) Regularizers on Parameters: The listed regularization terms are also available for regularizing the parameters of $f$. The parameters which should be regularized are passed to the regularizer as an array, a name and a weighting. In this way, one can individually weight subsets of parameters, belonging for example to different hierarchical levels, cf. the following example:

```python
 reg_param = paramRegulariser.L1Regulariser(
   "trans_parameter",
   weight=weight_parameter[level])
 registration.set_regularizer_parameter([reg_param])
```
c) Demons Regularizers: Currently, there are two Demons regularizers available in AirLab:
- Kernel: an arbitrary convolution kernel for filtering the displacement field. An example is the Gaussian kernel which is used originally in the Demons algorithm [28].
  Class name: GaussianRegulariser
- Graph Diffusion: the diffusion is performed by spectral graph diffusion. The graph can be utilized in order to handle the sliding organ problem. In this case, the graph is built during the optimization as proposed by [26].
  Class name: GraphDiffusionRegulariser

5) Optimizers: AirLab includes a rich family of optimizers which are available in PyTorch including LBFGS, ASGD and Adam. They are tailored to optimize functions with a high number of parameters and thus are well suited for non-linear image registration objectives. We refer to [24] for a detailed overview of first order gradient based optimizers. As PyTorch also supports no-grad computations, iteration schemes as used in the Demons algorithm are also supported. The following snippet is an example usage of no-grad taken from the Demons regularizer.

```python
... def regularise(self, data):
    for parameter in data:
        # no gradient calculation for the
        # demons regularisation
        with th.no_grad():
            self._regulariser(parameter)
...```

C. Upcoming Features

In this section, we list the features which did not make it into the present version, which however are planned for integration into AirLab until 30.9.2018.

- Image domains: currently, AirLab supports only images which have the same domain \( \mathcal{X} \) with equal pixel spacing. This will be generalized to images with different domains and different pixel spacing.
- Similarity measures: additional measures are planned to support multi-modal image registration. These are:
  - Normalized Gradient Fields (NGF) [9]
  - Mutual Information (MI) [20], [31]
- Linear transforms: the similarity and affine transform will be integrated to also support scaling and shearing.
- Interpolation: B-spline interpolation for the image warping is planned for integration.
- Diffeomorphic: by integrating the exponential mapping \( \exp \) of a transformation, the diffeomorphic Demons [29] can be implemented. There is already an implementation of \( \exp \) available in TensorFlow [1] which was recently published for a diffeomorphic registration algorithm in [17].

IV. EXPERIMENTS

In this section, we provide image registration examples. We have implemented two classic registration algorithms within AirLab and show their qualitative performance on synthetic examples and on a DirLab dataset [6]. Quantitative analyses will follow in the final version of this paper.

A. Image Registration Algorithms

The following algorithms have been implemented:
- **Rigid**: a simple objective with a rigid transformation has been set up, where the \( S_{MSE} \) similarity metric has been optimized with Adam.
- **FFD**: the Free Form Deformations algorithm [25] was implemented. As in the original paper, a third order B-spline kernel has been used for the parametric transformation model. Furthermore, the \( S_{MSE} \) similarity measure with the \( R_{\text{anisoTV}} \) regularizer on the displacement field have been applied. The overall objective has been optimized with Adam.
- **Demons**: the Demons algorithm [28] was implemented using the \( S_{MSE} \) similarity measure with the Gaussian Demons regularizer. The similarity measure has been optimized with Adam, while the regularizer was applied after each iteration.

In **FFD** and **Demons**, a multi-resolution strategy has been implemented performing \{500, 100, 50\} iterations for the **FFD** and \{100, 100, 100\} iterations for the **Demons** algorithm. The detailed parameter configuration can be found in the source-code. The following snippet illustrates how to setup a registration algorithm in AirLab with the Rigid registration example:

```python
# choose the rigid transformation model
transformation = RigidTransformation(moving_image.size, 
                                    dtype=dtype, 
                                    device=device)
registration.set_transformation(transformation)

# choose the Mean Squared Error as image loss
image_loss = MSELoss(fixed_image, moving_image)
registration.set_image_loss([image_loss])

# choose the Adam optimizer to minimize the objective
optimizer = th.optim.Adam(
    transformation.parameters(), lr=0.01)
registration.set_optimizer(optimizer)
registration.set_number_of_iterations(100)

# start the registration
registration.start()

# warp the moving image with the final transformation result
warp_image = warp_image(moving_image, displacement)
warped_image = wrap_image(moving_image, displacement)
```

Fig. 1: (a) Fixed AirLab image, (b) rotated moving AirLab image and (c) warped moving AirLab image after registration.
1) **Rigid Example:** For the Rigid example two AirLab images have been registered, where the moving image has been rotated. In Figure 1 the registration result is depicted.

2) **Demons Example:** The Demons algorithm has been applied to the circle and C example. For better illustration, see Figure 2 a shaded circle has been warped with the final transformation.

3) **FFD Example:** For the FFD example, a dataset of the DirLab [6] has been registered. To illustrate the result, in Figure 3 a slice through the volume is visualized. The same example has been registered also using the Demons algorithm (see Figure 4).

**B. Performance Analysis**

All experiments have been conducted using an NVIDIA GeForce GTX 1080 GPU. We evaluate the performance of AirLab by profiling with the autograd profiler of PyTorch. One iteration on the last resolution level (7 893 088 px) of the DirLab examples have been executed. The used CPU and GPU time is listed in Table I. Because the GaussianRegulariser is not differentiated, there is less computational time spent by autograd for the Demons example.

**TABLE I:** Execution statistics of the DirLab registration, where backwards strands for the evaluation of the derivatives. Overall is the wall-clock time needed by performing the full registration on three scale levels.

|                | FFD       | Demons    |
|----------------|-----------|-----------|
| CPU backwards  | 0.049 s   | 0.002 ms  |
| GPU backwards  | 1.012 s   | 0.086 ms  |
| CPU total      | 0.141 s   | 0.004 ms  |
| GPU total      | 1.390 s   | 0.092 ms  |
| Overall        | 173.9 s   | 164.0 s   |

**V. CONCLUSION**

We have introduced AirLab, an environment for rapid prototyping and reproduction of medical image registration algorithms. It is written in the scripting language Python and heavily uses functionality of PyTorch. The unique feature compared to existing image registration software is the automatic differentiation which fosters rapid prototyping. AirLab is freely available under the Apache License 2.0 and accessible on GitHub: [https://github.com/airlab-unibas/airlab](https://github.com/airlab-unibas/airlab).

With AirLab, we hope that we can make a valuable contribution to the medical image registration community, and we are looking forward to see researchers and developers which actively use AirLab in their work. Finally, we encourage them also to contribute to future development of AirLab.

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We finally list the MSELoss similarity metric and the TVRegulariser class as snippet to illustrate how similarity measures and regularizers are implemented in AirLab. (Please see the following page)
Listing 1: Mean Square Error image measure

```python
class MSELoss(_PairwiseImageLoss):
    def __init__(self, fixed_image, moving_image, size_average=True, reduce=True):
        super(MSELoss, self).__init__(fixed_image, moving_image, size_average, reduce)
        self.name = "mse"
        self.warped_moving_image = th.empty_like(self._moving_image.image, dtype=th.uint8, device=self._device)

    def forward(self, displacement):
        displacement = self._grid + displacement
        mask = th.zeros_like(self._fixed_image.image, dtype=th.uint8, device=self._device)
        for dim in range(displacement.size()[-1]):
            mask += displacement[..., dim].gt(1) + displacement[..., dim].lt(-1)
        mask = mask == 0
        self._warped_moving_image = F.grid_sample(self._moving_image.image, displacement)
        value = (self._warped_moving_image - self._fixed_image.image).pow(2)
        value = th.masked_select(value, mask)
        return self.return_loss(value)
```

Listing 2: Anisotropic Total Variation regularizer

```python
class TVRegulariser(_Regulariser):
    def __init__(self, pixel_spacing, size_average=True, reduce=True, dtype=th.float, device=th.device('cpu')):
        super(TVRegulariser, self).__init__(pixel_spacing, size_average, reduce, dtype, device)
        self.name = "TV"

        if self._dim == 2:
            self._regulariser = self._TV_regulariser_2d  # 2d regularisation
        elif self._dim == 3:
            self._regulariser = self._TV_regulariser_3d  # 3d regularisation

    def _TV_regulariser_2d(self, displacement):
        dx = th.abs(displacement[1:, 1:, :]) - displacement[:-1, 1:, :]) * self._pixel_spacing[0]
        dy = th.abs(displacement[1:, 1:, :]) - displacement[1:, :-1, :]) * self._pixel_spacing[1]
        return dx + dy

    def _TV_regulariser_3d(self, displacement):
        dx = th.abs(displacement[1:, 1:, 1:, :]) - displacement[:-1, 1:, 1:, :]) * self._pixel_spacing[0]
        dy = th.abs(displacement[1:, 1:, 1:, :]) - displacement[1:, :-1, 1:, :]) * self._pixel_spacing[1]
        dz = th.abs(displacement[1:, 1:, 1:, :]) - displacement[1:, 1:, :-1, :]) * self._pixel_spacing[2]
        return dx + dy + dz

    def forward(self, displacement):
        return self.return_loss(self._regulariser(displacement))
```