Beamforming for Centralized Wireless Sensor Network with Noisy Observation

Yang Liu\(^\dagger\), Jing Li\(^\dagger\), Xuanxuan Lu\(^\dagger\), and Chau Yuen\(^\ddagger\)
\(^\dagger\) Electrical and Computer Engineering Department, Lehigh University, Bethlehem, PA 18015, USA
\(^\ddagger\) Singapore University of Technology and Design, 20 Dover Drive, 138682, Singapore

Email: yal210@lehigh.edu, jingli@ece.lehigh.edu, xul311@lehigh.edu, and yuenchau@sutd.edu.sg

Abstract—This paper focuses on joint beamforming design problem in a multi-antenna wireless sensor network comprised of one fusion center(FC) and multiple wireless sensors. We consider the scenario where the same source signal is observed by all sensors, with each sensor having independent observation noise and individual power constraint. Each sensor transmits its corrupted observation to the FC to perform further processing and data fusion. Recent literature has researched the joint beamforming design in this system to optimize the mean square error(MSE) and signal to noise ratio(SNR) performance. Here we consider the problem to maximize mutual information(MI) between the source and received signal. To attack this nonconvex problem, we first adopt the weighted minimum mean square error(WMMSE) method to complicate the original problem by introducing intermediate variables and then utilize the block coordinate ascent(BCA) method to decompose it into subproblems. We first develop a 3-block BCA algorithm, each of the three subproblems has closed form solution or can be proved convex. Based on that, we further decompose the problem into multiple atom problems, with closed form solution to each atom problem obtained, which decreases the complexity. Convergence of the proposed algorithms is discussed and numerical results are presented to test our algorithms.

Index Terms—wireless sensor network, mutual information, beamforming, block coordinate ascent method.

I. INTRODUCTION

Recently the wireless sensor network(WSN) has attracted great attentions due to its wide application in practice [1]–[3], [9]–[12]. A typical wireless sensor network is comprised of multiple sensors which are spatially distributed and wirelessly connected. Sensors in the same neighborhood monitor the same physical event or measure some common environmental parameters and transmit their (usually contaminated) observations to a preassigned fusion center(FC) to perform further data processing and fusion. We assume that the sensors and FC are equipped with linear beamformers(transmitters). How to design beamformers so that sensors and fusion center can collaboratively perform an efficient and reliable communication is an interesting and meaningful problem.

The beamformer design problem in a wireless sensor network is an interesting problem and many researchers have been tackling it from various perspectives. For example, the papers [1]–[3] aim at studying efficient beamformers for signal compression. [1] and [2] consider the perfect channel case, i.e. there exists no noise or fading for transmission from sensors to FC. [3] considers the noisy channel case. These three papers take mean square error(MSE) as performance metric, which is a standard performance measure for signal estimation problems.

We notice that the beamforming in wireless sensor network has close connections with the beamforming problem in multi-input-multi-output(MIMO) relay systems. Actually our problem can be regarded as a special kind of MIMO communication system with multiple relays, where the transmitted signals are delivered by way of multiple non-fading relays and transmitted to one common destination. Recently enormous interest has been cast onto MIMO relay communication systems and numerous results have been obtained. [4] and [5] consider MIMO signal transmission by one relay, with MSE as optimization objective. [6] and [7] study multiuser MIMO system, with the former focusing on non-relay transmission and the latter on one-relay scheme, with the total MSE from different users as performance metric. [8] considers one relay MIMO system, taking into account different performance criteria including signal to noise ratio(SNR) and mutual information(MI), besides the widely used MSE metric.

Compared to the above mentioned works, the wireless sensor network system considered in this paper appears to be more general and complicated. We assume that at each sensor the observation is born contaminated, and each sensor has individual power constraint. Unlike some previous works, we do not pose constraints on the dimension of the beamformers, i.e. the filtered signal out from any sensor can be compressed, redundancy-added or rate-1.

Several recent studies are particularly relevant to our work. The centralized wireless sensor network model was first considered by Xiao, Cui, Luo and Goldsmith in their classical paper [9]. [9] discussed several important cases subsumed in the general model, including the scalar observed signal case, the noiseless sensor-FC channel case and the no-intersymbol-interference (no-ISI) channel case, and proposed beamforming designs for these cases. Another work [10], by Behbahani, Eltawil and Jafarkhani, developed a type of iterative block coordinate descent(BCD) beamforming optimization method that is applicable to the general model. Furthermore, variant BCD based iterative beamforming algorithms and the associated convergence analysis can also be found in [12]. All of the above mentioned papers adopted MSE as performance...
criterion. The beamforming design problem to maximize SNR for scalar source signal is considered in [11].

In this paper we still focus on the beamforming design in wireless sensor network. Here under the Gaussian signaling assumption, different from previous works on wireless sensor network, we choose mutual information (MI) as our optimization objective, which is an important performance measure. As we will see in the following discussions, like the MSE and SNR problems considered [9]–[12], the MI problem is also a highly nonconvex hard problem.

Contributions of this paper is as follows. The original MI maximization in centralized wireless sensor network is a hard nonconvex problem. Inspired by the seminal idea of weighted minimum mean square error (WMMSE) method in [13], [14], we introduce a weight matrix and a virtual FC receiver as intermediate variables (the original MI maximization problem does not assume the presence of linear filter at FC, since MI is independent of receiving process). Here we develop two block coordinate ascent (BCA) algorithms. Firstly, we decompose the MI problem into three subproblems—one subproblem to update the virtual FC receiver, one subproblem to update the weight matrix and the third one to jointly optimize the entire beamformers of all sensors. The two former subproblems have closed form solutions and the third one can be proved to be convex, which in fact can be converted into a second order cone programming (SOCP) problem and thus efficiently solved by standard numerical solvers. The convergence of this 3-block BCA algorithm is carefully examined and we prove that the limit points of our solution satisfy Karush-Kuhn-Tucker (KKT) conditions of the original MI maximization problem. Secondly, following the route of the first algorithm, we proceed to further decompose its third subproblem into multiple atom problems, with each atom problem optimizing one separate sensor’s beamformer. We carefully examine its optimality conditions and obtain the almost closed form solution. It should be noted that, although the technique of checking KKT condition for each separate beamformer is rather standard and has also been adopted in several previous papers (e.g. [6], [7], [10] and [12]), we are able to fully solve this problem by clearly describing the solution structure and figuring out closed-form solutions. To be specific, we explicitly obtain the equivalent conditions for judging the positiveness of the Lagrange multipliers, and, in the case of zero-Lagrange-multipliers, we explicitly give out the energy preserving optimal solution for even the rank deficient quadratic matrix case. These exact results, and especially the case of the zero-Lagrange-multiplier, are not discussed previously in the literature. The further decomposed BCA algorithm has a lower complexity and can achieve equivalently good performance as the 3-block BCA algorithm. Complexity of the proposed algorithms are carefully examined and their performance have been tested by numerical results.

The rest of the paper is organized as follows: Section II introduces the system model of the wireless sensor network with fusion center and formulates the mutual information optimization problem. In section III we propose two BCA based algorithms to solve our original problem, their convexity, closed form solutions, convergence and complexity are discussed in full details. Section IV provides numerical experiment results to testify the proposed algorithms. Section V concludes the article.

In what follows, we use bold lowercase letters to denote complex vectors and bold capital letters to denote complex matrices. \( \mathbf{0}, \mathbf{O}_{m \times n}, \) and \( \mathbf{I}_m \) are used to denote zero vectors, zero matrices of dimension \( m \times n \), and identity matrices of order \( m \) respectively. \( \mathbf{A}^T, \mathbf{A}^*, \mathbf{A}^H, \) and \( \mathbf{A}^\dagger \) are used to denote the transpose, the conjugate, the conjugate transpose (Hermitian transpose), and the Moore-Penrose pseudoinverse respectively of an arbitrary complex matrix \( \mathbf{A} \). \( \text{Tr} \{ \cdot \} \) denotes the trace operation of a square matrix. \( \| \cdot \| \) denotes the modulus of a complex scalar, and \( \| \cdot \|_2 \) denotes the \( l_2 \)-norm of a complex vector. \( \text{vec}(\cdot) \) means vectorization operation of a matrix, which is performed by packing the columns of a matrix into a long one column. \( \otimes \) denotes the Kronecker product. \( \text{Diag}(\mathbf{A}_1, \cdots, \mathbf{A}_n) \) denotes the block diagonal matrix with its \( i \)-th diagonal block being the square complex matrix \( \mathbf{A}_i \), \( i \in \{1, \cdots, n\} \). \( \mathcal{H}_++ \) and \( \mathcal{S}_++ \) represent the cones of positive semidefinite and positive definite matrices of dimension \( n \) respectively. Here \( \succeq 0 \) and \( \succ 0 \) denote that an square complex matrix belongs to \( \mathcal{H}_++ \) and \( \mathcal{S}_++ \) respectively. \( \text{Re}\{x\} \) means taking the real part of a complex value \( x \).

II. SYSTEM MODEL

Here in this paper, we consider the centralized wireless sensor network, which is illustrated in Fig. 1.
receiver(postcoder) is also employed on the side of fusion center. For problems of optimizing MSE or SNR, the presence of linear receiver at the FC leads to joint optimization of all transceivers and can greatly improve the system’s performance. However for mutual information maximization problem, according to fundamental information theory, any kinds of processing at the receiver will not increase the mutual information between the source and receiver. Thus here we assume that linear receiver is not employed at the FC. The channel status \( \{ H_i \} \) are assumed to be known. This can be achieved at the receiver by feedback or channel reciprocity. We denote \( H_i \in \mathbb{C}^{M \times N} \) as the channel coefficients from the i-th sensor to the fusion center. Due to interference from surroundings or thermal noise from the sensor device, the observed signals at the sensors are typically contaminated. We assume that the corruptions are zero mean circularly symmetric complex Gaussian, i.e. \( \tilde{n}_i \sim \mathcal{C}(0, \Sigma_i) \), with its covariance matrix \( \Sigma_i \in \mathbb{C}^{K \times K} \) being covariance matrix. Since the sensors are spatially distributed, it is reasonable to assume that the noise \( \tilde{n}_i \) at different sensors are mutually independent. We assume that the noise is white and zero-mean circularly symmetric complex Gaussian, i.e. \( n_0 \in \mathbb{C}^{M \times 1} \sim \mathcal{C}(0, \Sigma_0) \).

Here we assume that the system is perfectly time synchronous, which can be realized via GPS system. We consider the channels between sensors and fusion center as coherent multiple access channel(MAC), which means the signals from different sensors are superimposed. According to the discussion above, the transmitted signal at the i-th sensor is \( F_i(s + n_i) \), and the received signal at the fusion center is presented as:

\[
\begin{align*}
\tilde{r} &= \sum_{i=1}^{L} H_i F_i(s + n_i) + n_0 \\
&= \left( \sum_{i=1}^{L} H_i F_i \right) s + \left( \sum_{i=1}^{L} H_i F_i n_i + n_0 \right),
\end{align*}
\]

where the compound noise vector \( n \) is still Gaussian, i.e. \( n \sim \mathcal{C}(0, \Sigma_n) \) with its covariance matrix \( \Sigma_n \) as

\[
\Sigma_n = \sigma_0^2 I_M + \sum_{i=1}^{L} H_i F_i \Sigma_i F_i^H H_i^H.
\]

It should be pointed out that the whiteness assumption of the Gaussian noise is only at the receiver does not undermine generality. Indeed if \( n_0 \sim \mathcal{C}(0, \Sigma_0) \) with coloured covariance \( \Sigma_0 \), by redefining \( \tilde{r} \triangleq \Sigma_0^{-\frac{1}{2}} r, \tilde{n}_0 \triangleq \Sigma_0^{-\frac{1}{2}} n_0 \) and \( \tilde{r} \triangleq \Sigma_0^{-\frac{1}{2}} r, \tilde{n}_0 \triangleq \Sigma_0^{-\frac{1}{2}} n_0 \), the received signal can be equivalently written as

\[
\tilde{r} = \sum_{i=1}^{L} \tilde{H}_i F_i (s + n_i) + \tilde{n}_0,
\]

with \( \tilde{n}_0 \sim \mathcal{C}(0, I_M) \), which coincides with the model in (1).

Under the Gaussian signaling assumption, the mutual information between the source signal and received signal at FC can be given in equation (5) on the top of next page.

In practice, each sensor has independent transmission power according to its own battery condition. The average transmitted power for the i-th sensor is \( E \{ \| F_i(s + n_i) \|^2 \} = \text{Tr} \{ F_i(\Sigma_i + \Sigma_n) F_i^H \} \), which must respect its power constraint \( P_i \). Thus the beamforming problem of the multiple sensor system can be formulated as the following optimization problem:

\[
\begin{align*}
& \text{(P0)}: \max \text{MI}(\{ F_i \}^L_{i=1}), \\
& \text{s.t} \quad \text{Tr} \{ F_i(\Sigma_i + \Sigma_n) F_i^H \} \leq P_i, \ i \in \{ 1, \cdots, L \}. \quad (6b)
\end{align*}
\]

The above optimization problem is nonconvex, which can be easily seen by examining the convexity of the special case where \( \{ F_i \}^L_{i=1} \) are all scalars. So design of efficient algorithms to solve (P0) is desirable. Since the above problem is difficult to solve in one shot, we propose iterative algorithms to achieve our goal. The procedure maximizing MI falls in the framework of block coordinate ascent/descent(BCD/A) algorithm [17] [13], which is also frequently referred as alternative minimization/maximization algorithm(AMA) [15] or Gauss-Seidel(GBS) algorithm [17] [19]. The main idea of this method is as follows. When the original problem is intractable with all variables jointly considered, we can partition the variables into groups and accordingly decompose the original optimization problem into a group of subproblems. The difficult original problem is solved in an iterative manner by alternatively solving series of simpler problems. For each subproblem, only a subset of variables are updated with the others being fixed. Appropriate decomposition may result in efficiently solvable subproblems.

III. ALGORITHM DESIGN

In this section, we focus on solutions to the problem (P0). Since the original objective in (P0) is highly nonconvex, we turn to BCA method to achieve our goal. Note that directly utilizing BCA method to partition the beamformers into groups does not help to simplify our problem. Even if only one separate beaformer is considered, the objective is still hard. Inspired by the weighted mean square error(WMMSE) method proposed by the seminal paper [13], we introduce auxiliary variables to convert the objective into a BCA-friendly form and then decompose the problem into solvable subproblems. Interestingly, although mutual information is independent of processing techniques at the receiver, our solution actually introduces a virtual linear filter at the fusion center to achieve our goal.

In the following, we first introduce two useful lemmas which pave the way for transforming the.

\[ \text{Lemma 1 (13).} \text{ For any positive definite matrix } E \in \mathbb{R}_{++}^n, \text{ the following fact holds true} \]

\[ -\log \text{det}(E) = \max_{W \in \mathbb{R}_{++}^n} \{ \log \text{det}(W) - \text{Tr}(WE) + n \} \]

\[ \text{with the optimal solution } W^* \text{ given as} \]

\[ W^* = E^{-1}. \]

(7)

(8)
Lemma 2. Define a matrix function $E(G)$ of variable $G$ as
\[
E(G) \triangleq (I - G^H H) \Sigma_s (I - G^H H)^H + G^H \Sigma_n G,
\]
with $\Sigma_s$ and $\Sigma_n$ being positive definite matrices. Then for any positive definite matrix $W$, the following optimization problem
\[
\min_{G} \text{Tr} \{ WE(G) \}
\]
can be solved by the optimal solution
\[
G^* = (H \Sigma_s H^H + \Sigma_n)^{-1} H \Sigma_s.
\]
At the same time, $E(G^*)$ is given as
\[
E(G^*) = (H^H \Sigma_n^{-1} H + \Sigma_s^{-1})^{-1}.
\]

Proof: The problem in (10) is a convex problem. To see this, notice that the objective function in (10) is a quadratic function of $G$ with its quadratic terms being given as
\[
\text{Tr} \{ WG^H H \Sigma_s H^H G \} + \text{Tr} \{ WG^H \Sigma_n G \}.
\]
The first term of the above quadratic terms can be rewritten as
\[
\text{Tr} \{ WG^H H \Sigma_s H^H G \} = \text{vec}_H(G) \left[ W^\ast \otimes (H \Sigma_s H^H) \right] \text{vec}(G).
\]
Notice that $W$ and $H \Sigma_s H^H$ are positive semi-definite, so $W^\ast \otimes (H \Sigma_s H^H)$ is positive semi-definite and thus the first quadratic term is a convex function of $G$. Similarly the second quadratic term in (13) can also be proved to be convex function of $G$. Thus (10) is non-constrained convex problem of $G$. By setting the derivative with respect to $G$ to zero, we obtain
\[
\frac{\partial \text{Tr} \{ WE(G) \}}{\partial G^*} = \left[ (H \Sigma_s H^H + \Sigma_n) G - H \Sigma_s \right] W = 0. \tag{15}
\]
Notice that $W$ is positive definite, it can be cancelled and thus the equation (11) has been obtained. By substituting (11) into (9), (12) can be proved.

Comment III.1. For the special case $W = I$, the result in lemma 2 is the well know Wiener filter. Here we slightly generalize this well known result. As we have shown above, when the mean square error is weighted by a matrix $W$, the Wiener filter maintains its optimality as long as the weighted parameter $W$ is positive definite.

Now by introducing the notation
\[
\tilde{H} \triangleq \sum_{i=1}^{L} H_i F_i, \tag{16}
\]
and the notations in equation (5), we can transform our objective function $\text{MI}(\{F_i\}_{i=1}^{L})$ as the following:
\[
\text{MI}(\{F_i\}_{i=1}^{L}) = \log \det \left\{ I_M + \left( \sum_{i=1}^{L} H_i F_i \right) \Sigma_s \left( \sum_{i=1}^{L} H_i F_i \right)^H \left( \sigma_0^2 I + \sum_{i=1}^{L} H_i F_i \Sigma_s F_i^H H_i^H \right)^{-1} \right\} \tag{5}
\]
\[
\text{MI}(\{F_i\}_{i=1}^{L}) = \log \det \left\{ I_M + \tilde{H} \Sigma_s \tilde{H}^H \Sigma_n^{-1} \right\}
\]
\[
= \log \det \left\{ \left( \tilde{H}^H \Sigma_n^{-1} H + \Sigma_n^{-1} \right) \Sigma_s \right\}
\]
\[
= -\log \det \left( \tilde{H}^H \Sigma_n^{-1} H + \Sigma_n^{-1} \right) + \log \det (\Sigma_n)
\]
\[
= \max_{W \in \mathcal{S}_+^{L}} \left\{ \log \det (W) - \text{Tr} \left[ W (\tilde{H}^H \Sigma_n^{-1} H + \Sigma_n^{-1}) \right] + K + \log \det (\Sigma_n) \right\}
\]
\[
= \max_{W \in \mathcal{S}_+^{L}} \left\{ \log \det (W) - \text{Tr} \left[ W (\tilde{H}^H \Sigma_n^{-1} H + \Sigma_n^{-1}) \right] + K + \log \det (\Sigma_n) \right\}
\]
\[
\text{MI}(\{F_i\}_{i=1}^{L}) = \log \det \left\{ I_M + \tilde{H} \Sigma_s \tilde{H}^H \Sigma_n^{-1} \right\}
\]
\[
= \log \det \left\{ \left( I - G^H \Sigma_s H \right) \Sigma_s \left( I - G^H \Sigma_s H \right)^H + G^H \Sigma_n G \right\}^{-1} \tag{21}
\]
where the last two steps follow lemma 1 and 2 respectively.

Thus the optimization problem (20) maximizing $\text{MI}$ has been transformed into an equivalent problem (P1) in (22), which is shown on the top of next page.

As a straightforward consequence of the above two lemmas, we have obtained the optimal solutions to the following two subproblems of (P1).

When $\{F_i\}_{i=1}^{L}$ and $G$ are given, the optimal $W^*$ is given as
\[
W^* = \arg \max_{W \in \mathcal{S}_+^{L}} \text{MI}(\{F_i\}_{i=1}^{L}, G)
\]
\[
= \left[ \left( I - G^H \left( \sum_{i=1}^{L} H_i F_i \right) \right) \Sigma_s \left( I - G^H \left( \sum_{i=1}^{L} H_i F_i \right) \right)^H + G^H \Sigma_n G \right]^{-1} \tag{23}
\]
When $\{F_i\}_{i=1}^{L}$ and $W$ are given, the optimal $G^*$ is given as
\[
G^* = \arg \max_{G} \text{MI}(\{F_i\}_{i=1}^{L}, W)
\]
\[
= \left[ \left( \sum_{i=1}^{L} H_i F_i \right) \Sigma_s \left( \sum_{i=1}^{L} H_i F_i \right)^H + \Sigma_n \right]^{-1} \left( \sum_{i=1}^{L} H_i F_i \right) \Sigma_n
\]
with $\Sigma_n$ being given in equation (3).

Now we focus on the subproblem of optimizing $\{F_i\}_{i=1}^{L}$ with $W$ and $G$ given. Towards this end, we have two options—we can either jointly optimize $\{F_i\}_{i=1}^{L}$ in one shot, or we can further consult to BCA methodology again to partition the entire variables $\{F_i\}_{i=1}^{L}$ into $L$ blocks, $\{F_1\}, \cdots, \{F_L\}$ and attack $L$ atom problems one by one in a cyclic manner. For either of these two options, convexity of the subproblems need to be examined, closed form solution are desirable and complexity are also concerned. In the following, we discuss these two alternatives in details.
\[
\begin{align*}
\text{(P1)} \quad & \max_{\{F_i\}_{i=1}^L, W, G} \text{MI}\Big(\{F_i\}_{i=1}^L, W, G\Big) = \left\{ \log \det(W) - \text{Tr}\left\{ W \left[ \left( \sum_{i=1}^L H_i F_i \right) \sum_s \left( \left( \sum_{i=1}^L H_i F_i \right)^H + G^H \Sigma_n G \right) \right] \right\} \right\} + \log \det(\Sigma_s) + K, \\
& \text{s.t. } \text{Tr}\{F_i (\Sigma_s + \Sigma_n) F_i^H\} \leq P_i, \quad i \in \{1, \cdots, L\}. 
\end{align*}
\]

The following theorem identifies the convexity of problem (P2).

**Theorem 1.** The problem (P2) is a convex problem.

**Proof:** To begin with, we first look at the function \( f(X) : \mathbb{C}^{m \times n} \to \mathbb{R} \) given as follows:

\[
\begin{align*}
f(X) &= \text{Tr}\left\{ \Sigma_1 XX_2 X_2^H \right\} 
\end{align*}
\]

with constant matrices \( \Sigma_1 \) and \( \Sigma_2 \) being positive semi-definite and having appropriate dimensions. \( f(X) \) can equivalently written as

\[
\begin{align*}
f(X) &= vec^H(X) \left[ \Sigma_1^* \otimes \Sigma_1 \right] vec(X) 
\end{align*}
\]

Since \( \Sigma_1 \) and \( \Sigma_2 \) are positive semi-definite, \( \left[ \Sigma_1^* \otimes \Sigma_2 \right] \) is positive semi-definite. Thus \( f(X) \) is actually a convex function with respect to \( X \).

For a further step, we replace \( X = \sum_{i=1}^L H_i F_i \). Since \( \sum_{i=1}^L H_i F_i \) is an affine (linearly actual) transformation of variables \( \{F_i\}_{i=1}^L \), and affine operations preserve convexity by (21), the following function

\[
\begin{align*}
f(\{F_i\}_{i=1}^L) &= \text{Tr}\left\{ \sum_{i=1}^L H_i F_i \sum_{i=1}^L H_i F_i^H \right\} 
\end{align*}
\]

is a convex function with respect to variables \( \{F_i\}_{i=1}^L \) jointly.

We notice that the objective function of (P2) is a quadratic function of \( \{F_i\}_{i=1}^L \). Thus we need only to check the convexity of its quadratic terms, which is given as

\[
\begin{align*}
& \text{Tr}\left\{ (GWG^H) \left( \sum_{i=1}^L H_i F_i \right) \sum_s \left( \sum_{i=1}^L H_i F_i \right)^H \right\} \\
& \quad + \sum_{i=1}^L \text{Tr}\left\{ H_i^H GWG^H H_i F_i \Sigma_i F_i^H \right\}. 
\end{align*}
\]

Based on above discussion, each of the quadratic terms of the objective function is convex and thus the objective is convex. Similarly the convexity of each power constraint function can also be recognized. Thus the problem (P2) is convex.

After identifying the convexity of problem (P2), we reformulate it into a standard quadratic constrained quadratic problem (QCQP) problem. To this end, we introduce the following notations

\[
\begin{align*}
f_i &= \text{vec}(F_i); \\
g &= \text{vec}(G); \\
A_{ij} &= \Sigma_s^* \otimes \left( H_i^H GWG^H H_j \right); \\
B_i &= (W \Sigma_n)^* \otimes H_i; \\
C_i &= \Sigma_s^* \otimes \left( H_i^H GWG^H H_i \right); \\
f &= [f_1, \cdots, f_L]^T; \\
A &= [A_{ij}]_{i,j=1}^L; \\
B &= [B_1, \cdots, B_L]; \\
C &= \text{Diag}\{C_1, \cdots, C_L\}; \\
D &= \text{Diag}\left\{ O_K \Sigma_{j=1}^L, (\Sigma_s + \Sigma_n)^* \otimes I_N, O_K \Sigma_{j=1}^L \right\}; \\
c &= \text{Tr}\{W \Sigma_n\} + \sigma_0^2 \text{Tr}\{GWG^H\}. 
\end{align*}
\]

Based on the above notations, problem (P2) can be equivalently written as the following convex QCQP problem.

\[
\begin{align*}
\text{(P3)} : \quad & \min f^H (A + C) f - 2 \text{Re}\{g^H B f\} + c, \\
& \text{s.t. } f^H D_i f \leq P_i, \quad i \in \{1, \cdots, L\}. 
\end{align*}
\]

which can be further rewritten in a standard SOCP form as follows:

\[
\begin{align*}
\text{(P3\_SOC)} : \quad & \min_{t, f, c} \text{vec}(f) \leq t; \\
& s - 2 \text{Re}\{g^H B f\} + c \leq t; \\
& \left\| \left( A + C \right)_{s-1}^s f \right\|_2 \leq \frac{s+1}{2}; \\
& \left\| D_{s-1}^s f \right\|_2 \leq \frac{P_i + 1}{2}, \quad i \in \{1, \cdots, L\}. 
\end{align*}
\]

The above problem can be solved by standard numerical tools like CVX [22].

The method discussed above is summarized in algorithm [1].

For the convergence of the algorithm [1] we have the following conclusion...
Theorem 2. Assume that the covariance matrix \( \Sigma_s \succ 0 \).
Algorithm 1 generates increasing MI sequence. Its solution sequence has limit points, and each limit point of the solution sequence is a KK-T point of the original problem (P0).

Proof: Since for each sub-problem, we solve an optimization problem with respect to a subset of variables with others being fixed, the objective value obtained by solving the current sub-problem cannot be smaller than previous one. Thus the entire MI sequence keeps increasing.

Under the positive definiteness assumption of \( \Sigma_s \), \( \Sigma_s + \Sigma_i \succ 0 \). Thus \( \forall i \in \{1, \cdots, L\} \) we have

\[
\|F_i\|_F^2 \lambda_{\min}(\Sigma_s+\Sigma_i) \leq \text{Tr}(F_i(\Sigma_s+\Sigma_i)F_i^H) \leq P_i, \quad (33)
\]

where \( \lambda_{\min}(\cdot) \) means the minimum eigenvalue of a Hermitian matrix. Since \( \lambda_{\min}(\Sigma_s+\Sigma_i) > 0 \), \( \|F_i\|_F^2 \) is finite for all \( i \). Thus the variable \( \{F_i\}_{i=1}^L \) is bounded. By Bolzian-Weierstrass theorem, there exits a subsequence \( \{k_j\}_{j=1}^\infty \) such that \( \{F_i^{(k_j)}\}_{i=1}^L \) converges. Since \( G \) and \( W \) are updated by continuous functions of \( \{F_i^{(k_j)}\}_{i=1}^L \) in (24) and (23), \( \{F_i^{(k_j)}\}_{i=1}^L, W^{(k_j)}, G^{(k_j)} \) converges. Thus the exist limit points in solution sequence has been proved.

We assume that \( \{(F_i^{(k_j)})_{i=1}^L, W, G\} \) is any limit point of \( \{(F_i^{(k_j)})_{i=1}^L, W^{(k_j)}, G^{(k_j)}\} \). Then there exists a subsequence \( \{k_j\} \) such that \( \{(F_i^{(k_j)})_{i=1}^L, W^{(k_j)}, G^{(k_j)}\} \xrightarrow{j \to \infty} \{F_i^{(k_j)}\}_{i=1}^L, W, G \). Since \( \{F_i^{(k_j)}\}_{i=1}^L \) is bounded, by restricting to a subsequence, we can assume that \( \{F_i^{(k_j+1)}\}_{i=1}^L \) converges to a limit \( \{(\hat{F}_i)_{i=1}^L\} \).

Since for each \( j \), \( \{F_i^{(k_j+1)}\}_{i=1}^L \) are feasible, i.e.

\[
\text{Tr}(F_i^{(k_j+1)}(\Sigma_s+\Sigma_i)(F_i^{(k_j+1)})^H) \leq P_i, \quad i \in \{1, \cdots, L\}. \quad (34)
\]

By taking \( j \to \infty \) in the above inequalities, we obtain

\[
\text{Tr}(\hat{F}_i(\Sigma_s+\Sigma_i)(\hat{F}_i)^H), \quad i \in \{1, \cdots, L\}, \quad (35)
\]

which means \( \{(\hat{F}_i)_{i=1}^L\} \) are feasible.

For any feasible \( \{F_i\}_{i=1}^L \), we have

\[
\text{MI}(\{F_i\}_{i=1}^L, W, G) \leq \text{MI}(\{F_i^{(k_j+1)}\}_{i=1}^L, W^{(k_j)}, G^{(k_j)}). \quad (36)
\]

Noticing that MI function is continuous and taking \( j \to \infty \) in the above, we obtain

\[
\text{MI}(\{F_i\}_{i=1}^L, W, G) \leq \text{MI}(\{(\hat{F}_i)_{i=1}^L\}, W, G), \quad (37)
\]

for any feasible \( \{F_i\}_{i=1}^L \).

Notice the \( \{F_i^{(k_j)}\}_{i=1}^L \) generated by algorithm 1 are feasible, so by continuity of power constraint functions, \( \{\hat{F}_i\}_{i=1}^L \) are feasible. Thus we have

\[
\text{MI}(\{F_i\}_{i=1}^L, W, G) \leq \text{MI}(\{(\hat{F}_i)_{i=1}^L\}, W, G). \quad (38)
\]

At the same time, since the MI sequence is increasing and \( \{\hat{F}_i\}_{i=1}^L \) is a limit point of the solution sequence,

\[
\text{MI}(\{\hat{F}_i\}_{i=1}^L, W, G) \geq \text{MI}(\{F_i^{(k_j)}\}_{i=1}^L, W, G), \quad (39)
\]

for any integer \( k \). Substitute \( k \) with \( k_j \) in (39), take limit \( j \to \infty \) and combine it with (33), we have shown that \( \{\hat{F}_i\}_{i=1}^L \) is actually an optimal solution to the problem (P2) with parameters \( W \) and \( G \). So \( \{\hat{F}_i\}_{i=1}^L \) satisfy KKT conditions of (P2) with parameters \( W \) and \( G \), which are listed in 40 shown on the top of next page.

To simplify the following exposition, we introduce the following two notations:

\[
\bar{H} \triangleq \sum_{i=1}^L H_i F_i^i, \quad (41a)
\]

\[
\bar{\Sigma}_n \triangleq \sigma_0^2 I + \sum_{i=1}^L H_i \hat{F}_i \Sigma_i \hat{F}_i^H H_i^H. \quad (41b)
\]

According to algorithm 1, the auxiliary variables \( W \) and \( G \) have the relations associated with \( \{\hat{F}_i\}_{i=1}^L \) as follows.

\[
G = [\bar{H} \Sigma_s \bar{H}^H + \bar{\Sigma}_n]^{-1} \bar{H} \Sigma_s, \quad (42a)
\]

\[
W = \bar{H}^H \Sigma_n^{-1} \bar{H} + \Sigma_n^{-1}. \quad (42b)
\]

Utilizing (23) we can prove two identities in (43) and (44) respectively on the top of next page.

Substituting equations (43) and (44) into (40a), we can rewrite the first order KKT conditions associated with only \( \{\hat{F}_i\}_{i=1}^L \) as in equation (45) shown in the next page.

To check the conditions of the original problem (P0), we need to determine the derivative of its Lagrangian function, or equivalently the derivative of MI with respect to \( \{F_i\} \). By defining

\[
H \triangleq \sum_{i=1}^L H_i F_i, \quad (46)
\]
\[-H_i^H GW \left( I - G^H \left( \sum_{i=1}^L H_i F_i \right) \right) \Sigma_n + H_i^H GWG^H H_i F_i \Sigma_i + \lambda_i \bar{F}_i \left( \Sigma_n + \Sigma_i \right) = O, \quad i \in \{1, \ldots, L\}; \quad (40a)\]
\[\lambda_i \left( \text{Tr} \left\{ \bar{F}_i \left( \Sigma_n + \Sigma_i \right) \bar{F}_i^H \right\} - P_i \right) = 0, \quad i \in \{1, \ldots, L\}; \quad (40b)\]
\[\text{Tr} \left\{ \bar{F}_i \left( \Sigma_n + \Sigma_i \right) \bar{F}_i^H \right\} \leq P_i, \quad i \in \{1, \ldots, L\}; \quad (40c)\]
\[\lambda_i \geq 0, \quad i \in \{1, \ldots, L\}. \quad (40d)\]

\[
\begin{align*}
G W \left( I - G^H \bar{H} \right) = & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H \Sigma_n \left( \bar{H}^H \Sigma_n^{-1} \bar{H} + \Sigma_n^{-1} \right) \left( I - \Sigma_n \bar{H}^H \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \bar{H} \right) \\
= & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H + \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H \Sigma_n \left[ \bar{H}^H \Sigma_n^{-1} \left( H \Sigma_n \bar{H}^H + \Sigma_n \right) - \left( \bar{H}^H \Sigma_n^{-1} \bar{H} + \Sigma_n^{-1} \right) \Sigma_n \bar{H}^H \right] \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \bar{H} \\
= & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H
\end{align*}
\]

\[
\begin{align*}
GWG^H = & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H \Sigma_n \left( \bar{H}^H \Sigma_n^{-1} \bar{H} + \Sigma_n^{-1} \right) \Sigma_n \bar{H}^H \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \\
= & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \left[ H \Sigma_n \bar{H}^H \Sigma_n^{-1} \bar{H} + H \Sigma_n \Sigma_n \bar{H}^H \right] \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \\
= & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \left[ H \Sigma_n \bar{H}^H \Sigma_n^{-1} \left( H \Sigma_n \bar{H}^H + \Sigma_n \right) \right] \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} \\
= & \left( H \Sigma_n \bar{H}^H + \Sigma_n \right)^{-1} H \Sigma_n \bar{H}^H \Sigma_n^{-1}
\end{align*}
\]

\[
\begin{align*}
H_i^H \left[ \left( \sigma_0^2 I + \sum_{i=1}^L H_i \bar{F}_i \Sigma_i \right) F_i^H H_i^H \right] + \left( \sum_{i=1}^L H_i \bar{F}_i \right) \Sigma_n \left( \sum_{i=1}^L H_i \bar{F}_i \right)^H \left( \sum_{i=1}^L H_i \bar{F}_i \right)^{-1} \left( \sum_{i=1}^L H_i \bar{F}_i \right) \Sigma_n \Sigma_i \left[ I - \left( \sum_{i=1}^L H_i \bar{F}_i \right)^H \left( \sigma_0^2 I + \sum_{i=1}^L H_i \bar{F}_i \Sigma_i \right) \right] - \lambda_i \bar{F}_i \left( \Sigma_n + \Sigma_i \right) = O, \quad i \in \{1, \ldots, L\}; \quad (45)\]

\[
\begin{align*}
d(M_l) = & \text{Tr} \left\{ \left( I + H \Sigma_n \bar{H}^H \Sigma_n^{-1} \right)^{-1} d \left( H \Sigma_n \bar{H}^H \Sigma_n^{-1} \right) \right\} \\
= & \text{Tr} \left\{ \left( I + H \Sigma_n \bar{H}^H \Sigma_n^{-1} \right)^{-1} \left[ H \Sigma_n d(H^H) \Sigma_n^{-1} + H \Sigma_n \Sigma_n \bar{H}^H \Sigma_n^{-1} d \left( \Sigma_n \right) \right] \right\} + C_1 \left( dF_i \right) \\
= & \text{Tr} \left\{ H_i^H \left( \Sigma_n + H \Sigma_n \bar{H}^H \right)^{-1} H \Sigma_n \left[ I - H^H \Sigma_n^{-1} H \Sigma_n \bar{H} \right] \left( F_i \right)^H \right\} + C_2 \left( dF_i \right), \quad (47a)\]
\[
\Rightarrow \frac{\partial M_l}{\partial F_i} = H_i^H \left[ \left( \sigma_0^2 I + \sum_{i=1}^L H_i \bar{F}_i \Sigma_i \right) F_i^H H_i^H \right] + \left( \sum_{i=1}^L H_i \bar{F}_i \right) \Sigma_n \left( \sum_{i=1}^L H_i \bar{F}_i \right)^H \left( \sum_{i=1}^L H_i \bar{F}_i \right)^{-1} \left( \sum_{i=1}^L H_i \bar{F}_i \right) \Sigma_n \left[ I - \left( \sum_{i=1}^L H_i \bar{F}_i \right)^H \left( \sigma_0^2 I + \sum_{i=1}^L H_i \bar{F}_i \Sigma_i \right) \right] \left( \sum_{i=1}^L H_i \bar{F}_i \right)^{-1} H_i \Sigma_n \Sigma_i, \quad i \in \{1, \ldots, L\}. \quad (47b)\]

The derivative of $M_l$ is calculated in (47) with $C_1(dF_i)$ and $C_2(dF_i)$ being uninteresting terms involved $dF_i$ only and independent of $d(F_i)^H$.

By comparing the equations (45) with the derivative in (47b), it is easily to recognize that (45) is actually the first order KKT condition of problem (P0) optimizing $M_l$. Together with equations (40b), (40c) and (40d), the KKT conditions of original problem have been proved to be satisfied by $\{F_i\}_{i=1}^L$. 

Thus the proof is complete.

B. Cyclic \((L+1)\)-Block BCA Algorithm

Recall that in the above proposed 3-block BCA algorithm, we have identified the convexity of the third subproblem \((P_3)\), which is the hard core of the entire algorithm, and rely on standard numerical methods, e.g. interior point method, to obtain solutions. Closed form solutions to \((P_3)\) is unknown. According to the complexity analysis performed in next subsection \([11-C]\) when the number of sensors and/or antenna number of each sensor grows, the problem \((P_3)\) can be very large size and consequently highly computation demanding. So effective algorithms with lower complexity are desirable. In general the KKT conditions for \((P_3)\) in equations \([40]\) cannot lead to explicit solutions due to too many constraints. To overcome this difficulty, we consult to BCA methodology again to further partition the variables \(\{F_i\}_{i=1}^L\) into \(L\) smaller sets: \(\{F_1\}, \ldots, \{F_L\}\). We update only one separate beamformer \(F_i\) at one time, which is given in following problem

\[
(P_{3i}) \min_{f_i} f_i^H (A_{i+i} + C_i) f_i - 2Re \{ (g_i^H B_i - q_i^H) f_i \},
\]

\[
s.t. f_i^H E_i f_i \leq P_i
\]

with \(q_i\) defined as

\[
q_i \doteq \sum_{j \neq i} A_{ij} f_j.
\]

We introduce the following notations

\[
E_i^{-\frac{1}{2}} (A_{i+i} + C_i) E_i^{-\frac{1}{2}} = U_i \begin{bmatrix} \lambda_{i,1} \\ \vdots \\ \lambda_{i,KN_i} \end{bmatrix} U_i^H;
\]

\[
p_i = U_i^{\frac{1}{2}} E_i^{-\frac{1}{2}} (B_i^H g - q_i),
\]

with the eigenvalues arranged in an decreasing order, i.e. \(\lambda_{i,1} \geq \cdots \geq \lambda_{i,KN_i}\). Also we denote the \(k\)-th element of \(p_i\) as \(p_{i,k}\) and assume that \(r_i = \text{rank}(A_{i+i} + C_i)\).

Then the solution to problem \((P_{3i})\) is given by the following theorem.

**Theorem 3.** Under the assumption that \(\Sigma_a > 0\) or \(\Sigma_i > 0\), \(i = 1, \cdots, L\), the optimal solution of problem \((P_{3i})\) is given as follows:

**CASE (I)—if either of the following two conditions holds:**

i) \(\exists k \in \{r_i + 1, \cdots, KN_i\} \) such that \(|p_{i,k}| \neq 0\); or

ii) \(\sum_{k=r_i+1}^{KN_i} |p_{i,k}| = 0 \) and \(\sum_{k=1}^{r_i} \frac{|p_{i,k}|^2}{\lambda_{i,k}} > P_i\).

The optimal solution to \((P_{3i})\) is given by

\[
f^*_i = (A_{i+i} + C_i + \mu_i^* E_i)^{-1} (B_i^H g - q_i),
\]

with the positive value \(\mu_i^*\) being the unique solution to the following function:

\[
f(\mu_i) = \sum_{k=1}^{KN_i} \frac{|p_{i,k}|^2}{(\lambda_{i,k} + \mu_i)^2} = P_i.
\]
objective, the problem (P3ₗ) is rewritten as follows

\[
\begin{align*}
(P3ₗ) : \min & \quad (\sigma_s^2 + \sigma_j^2) f_{i,j}^H (H_i^H g_g g^H H_i) f_i \\
\text{s.t.} & \quad \|f_i\|^2 \leq \frac{p_i}{\sigma_s^2 + \sigma_j^2} \triangleq \bar{p}_i. 
\end{align*}
\]

(57a)

(57b)

Solving the problem (P3ₗ) just follows the outline of theorem 3. Here, the key point leading to a closed form solution is the fact that the quadratic matrix \(H_i^H g_g g^H H_i\) has rank-1, i.e. \(r_i = 1\) in theorem 3. Thus we obtain

\[
(\sigma_s^2 + \sigma_j^2)H_i^H g_g g^H H_i = U_i \begin{bmatrix} (\sigma_s^2 + \sigma_j^2)g_i^H H_i^H g & 0^H \\ 0 & O_{(K_i N_i - 1) \times (K_i N_i - 1)} \end{bmatrix} U_i^H, 
\]

(58)

with unitary matrix \(U_i \triangleq [u_{i,1}, u_{i,2}, \ldots, u_{i,K_i N_i}]\) having its columns \(\{u_{i,j}\}_{j=1}^{K_i N_i}\) satisfying the following properties

\[
u_{i,1} = \frac{H_i^H g}{\|H_i^H g\|_2}, \quad \text{and} \quad \nu_{i,j} = 0, \quad \text{for} \quad j = 2, \ldots, K_i N_i. 
\]

(59)

It can be readily checked that the parameter \(p_i\) in theorem 3 is given as:

\[
p_{i,1} = \sigma_s^2(1 - g_i^H \tilde{q}_i)\|H_i^H g\|_2; \quad p_{i,j} = 0, \quad j = 2, \ldots, K_i N_i. 
\]

(60)

At this time, the function \(f(\mu_i)\) in (52) reduces to an elegant form

\[
\begin{align*}
f(\mu_i) &= \frac{\sigma_s^2|1 - g_i^H \tilde{q}_i| \|H_i^H g\|_2^2}{(\mu_i + (\sigma_s^2 + \sigma_j^2)g_i^H H_i^H g)^2}. 
\end{align*}
\]

(61)

Based on the above observations, it can be concluded that the subcase i) of CASE (I) in theorem 3 will never occur. The two cases for positive and zero \(\mu_i^*\) can be specified as follows: CASE (I)— \(\mu_i^* > 0\)

This is equivalent to \(\sigma_s^2|1 - g_i^H \tilde{q}_i|^2 > (\sigma_s^2 + \sigma_j^2)^2 \bar{p}_i \|H_i^H g\|_2^2\) and optimal solutions are determined by

\[
\begin{align*}
\mu_i^* &= \sigma_s^2 \bar{p}_i^{1/2} |1 - g_i^H \tilde{q}_i| \|H_i^H g\|_2 - (\sigma_s^2 + \sigma_j^2) |H_i^H g|_2^2, \\
\tilde{f}_i^* &= \frac{\sigma_s^2(1 - g_i^H \tilde{q}_i)(\mu_i I + (\sigma_s^2 + \sigma_j^2)H_i^H g g^H H_i)}{(\sigma_s^2 + \sigma_j^2)H_i^H g g H_i}. 
\end{align*}
\]

(62a)

(62b)

CASE (II)— \(\mu_i^* = 0\)

This holds if and only if \(\sigma_s^2|1 - g_i^H \tilde{q}_i|^2 \leq (\sigma_s^2 + \sigma_j^2)^2 \bar{p}_i \|H_i^H g\|_2^2\) and the optimal \(\tilde{f}_i^*\) is given by

\[
\tilde{f}_i^* = \frac{\sigma_s^2(1 - g_i^H \tilde{q}_i)H_i^H g}{(\sigma_s^2 + \sigma_j^2)H_i^H g g H_i}. 
\]

(63)

Thus we have seen that for scalar transmission case, fully closed form solution to (P2ₗ) can be obtained without bisection search or eigenvalue decomposition.

C. Complexity

In this subsection, we discuss the complexity of the proposed algorithms.

The two subproblems optimizing \(G\) and \(W\) have closed form solutions in [24] and [23], their complexities come from matrix inversion and are given as \(O(K_i^3)\).

For the 3-block BCA algorithm, the SOCP problem (P₃ₗSOC) in (22) is solved to jointly optimize all beamformers. The complexity of solving an SOCP is

\[
O\left(\frac{1}{\tau_{SOC}} \left( n_{SOC}^3 + n_{SOC}^2 \sum_{i=1}^{k_{SOC}} n_{SOC,i} + \sum_{i=1}^{k_{SOC}} n_{SOC,i}^2 \right) \right), 
\]

(64)
where \( k_{SOC} \) is the number of second order cone constraints, \( m_{SOC} \) is the dimension of optimization problem and \( n_{SOC,i} \) denotes the dimension of the \( i \)-th second order cone constraint. For the problem in (32), \( k_{SOC} = L + 1 \), \( m_{SOC} = K(\sum_{i=1}^{L} N_i) \), \( n_{SOC,i} = K(\sum_{i=1}^{L} N_i) + 1 \) for the first second cone constraint in (32) and \( n_{i+1,SOC} = KN_i + 1 \) for the \( i \)-th power constraint in (32), \( i \in \{1, \cdots, L\} \). Substituting these parameters into (64), the complexity of solving (P3) is \( \mathcal{O}(\sqrt{LK^3(\sum_{i=1}^{L} N_i)^3}) \), this is also the complexity for each loop of 3-block BCA algorithm.

For the cyclic \((L+1)\)-block BCA algorithm, the atom problem (P3) optimizing one separate sensor’s beamformer has its major complexity coming from eigenvalue decomposition, which is \( \mathcal{O}(K^3N^3) \). Thus the complexity for each loop is \( \mathcal{O}(\sum_{i=1}^{L} K^3N^3) \). Clearly by fully decomposing the original problem and researching the solution structure of the atom problems, the \((L+1)\)-block BCA algorithm effectively lowers the computation complexity.

### IV. Numerical Results

In this section, numerical results are presented to verify the algorithms proposed in previous section.

In our experiment, we test the case where the source signal and all observation noise are colored. Specifically, we set the covariance matrices of the source signal and observation noise as

\[
\Sigma_a = \sigma_a^2 \Sigma_0, \quad \Sigma_i = \sigma_i^2 \Sigma_0, \quad i \in \{1, \cdots, L\}, \tag{65}
\]

where the \( K \times K \) Toeplitz matrix \( \Sigma_0 \) is defined as

\[
\Sigma_0 = \begin{bmatrix}
1 & \rho & \rho^2 & \cdots & \rho^{K-1} \\
\rho & 1 & \rho & \cdots & \rho^{K-2} \\
\rho^2 & \rho & 1 & \cdots & \rho^{K-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho^{K-1} & \cdots & \rho^2 & \rho & 1
\end{bmatrix}. \tag{66}
\]

The parameter \( \rho \) in the above equation is used to adjust the correlation level between different components of the source signal. Here we define the observation signal to noise ratio at the \( i \)-th sensor as \( \text{SNR}_i \triangleq \frac{\sigma_a^2}{\sigma_i^2} \) and signal to noise ratio at the FC as \( \text{SNR} \triangleq \frac{\sigma_a^2}{\sigma_i^2} \).

In figure 2, we test the performance of the 3-block BCA and cyclic \((L+1)\)-BCA algorithms for multiple dimension source signal. Here we set up a wireless sensor network with three sensors, i.e. \( L = 3 \). The dimension of the source signal and the number of antenna of the FC are chosen as 3 and 4 respectively, i.e. \( K = 3 \) and \( M = 4 \). We randomly set the antenna number for each sensor as \( N_1 = 3, N_2 = 4, \) and \( N_3 = 5 \) respectively, the transmission power constraint for each sensor as \( P_1 = 2, P_2 = 2 \) and \( P_3 = 3 \) respectively and the observation signal to noise ratio for each sensor as \( \text{SNR}_1 = 8dB, \text{SNR}_2 = 9dB \) and \( \text{SNR}_3 = 10dB \) respectively. The correlation parameter \( \rho \) in (66) is set as \( \rho = 0.5 \). To take into account the impact of the channel parameters, for the above system set-up and any specific SNR level at the FC, we randomly generate 500 channel realizations. For each channel realization, we perform the two proposed algorithms to optimize beamformers, both of which start from one common random feasible solution. The progress of MI with respect to iteration numbers are recorded. For one given iteration number, the average MI performance over all 500 channel realizations is presented in figure 2. The blue solid curves represent the average MI performance obtained by 3-block BCA algorithm with different numbers of iterations and the red dotted ones represent those obtained by cyclic \((L+1)\) BCA algorithm. The black dotted curve represents the average MI obtained by random full-power-transmission solutions, which are actually the average MI performance for feasible solutions which make all power constraints active. From the results in figure 2 we see that the optimized beamformers obtained by the proposed algorithms present significant MI improvement compared to nonoptimized beamformers. For the test case, usually 40 to 60 iterations are sufficient to make the two algorithms converge. These two algorithms finally converge to almost identical MI performance.

In figure 3 we test the special case of scalar source signal \((K = 1)\), where \((L+1)\)-block BCA algorithm has fully closed form solution, which is summarized in corollary 1. In this experiment, we have the system setup as follows \( M = 4, N_1 = 3, N_2 = 4, N_3 = 5, P_1 = 1, P_2 = 2, P_3 = 3, \text{SNR}_1 = 7dB, \text{SNR}_2 = 8dB \) and \( \text{SNR}_3 = 9dB \). Similar results as in the multiple dimension source signal case have been obtained.

In figure 4, we check the impact of the random initial solutions to the two proposed algorithms. We use the identical system setup as that in figure 2. Here the channel parameters are randomly chosen and fixed. 20 feasible solutions, each of which makes all the power constraints active(satisfied with equality) are randomly generated. For each random initial point, we invoke the 3-block BCA and cyclic \((L+1)\)-block BCA algorithms to optimize the beamformers. In this figure, we illustrate the MI progress for the proposed algorithms with different initials. It can be seen that both the 3-block BCA and cyclic \((L+1)\)-block BCA are rather stable with selection of initial points and they finally converge to identical value. The MI performance of different algorithms with different initial points finally converge to almost identical value.

### V. Conclusion

In this paper, we consider the linear beamforming design problem for a centralized wireless sensor network to maximize the mutual information. As we have seen, the original problem is non-convex and difficult. To solve this problem, we adopt the weighted minimum mean square error and block coordinate ascent method to decompose the original difficult problem into subproblems and carefully examine the solution to each subproblem, especially their closed form solution. The complexity and convergence of proposed algorithms are also discussed in details. Extensive numerical results are
Fig. 2: 3-Block BCA Algorithm and Cyclic \((L + 1)\)-Block BCA Algorithm with Different Numbers of Iterations.

Fig. 3: Scalar Source Signal Case: 3-Block BCA Algorithm and Cyclic \((L + 1)\)-Block BCA Algorithm with Different Numbers of Iterations.

Fig. 4: Optimizing Generalized MI by 3-Block BCA Algorithm and Cyclic \((L + 1)\)-Block BCA Algorithm with Different Initial Points

presented to verify and compare the behaviors of the proposed algorithms.

REFERENCES

[1] Y. Zhu, E. Song, J. Zhou, and Z. You, “Optimal dimensionality reduction of sensor data in multisensor estimation fusion,” *IEEE Trans. Signal Process.*, vol. 53, no. 5, pp. 1631-1639, May 2005.

[2] J. Fang and H. Li, “Optimal/near-optimal dimensionality reduction for distributed estimation in homogeneous and certain inhomogeneous scenarios,” *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4339-4353, 2010.

[3] I. D. Schizas, G. B. Giannakis, and Z.-Q. Luo, “Distributed estimation using reduced-dimensionality sensor observations,” *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4284-4299, Aug. 2007.

[4] W. Guan and H. Luo, “Joint MMSE transceiver design in nonregenerative MIMO relay systems,” *IEEE Commun. Lett.*, vol. 12, pp. 517-519, July 2008.

[5] Y. Rong, “Optimal joint source and relay beamforming for MIMO relays with direct link,” *IEEE Commun. Lett.*, vol. 14, pp. 390-392, May 2010.

[6] S. Serbetli and A. Yener, “Transceiver optimization for multiuser MIMO systems,” *IEEE Trans. Signal Process.*, vol. 52, pp. 214-226, Jan. 2004.

[7] M. R. A. Khondaker and Y. Rong, “Joint transceiver Optimization for multiuser MIMO relay communication systems,” *IEEE Trans. Signal Processing*, vol. 60, pp. 5977-5986, Nov. 2012.

[8] Y. Rong, X. Tang, and Y. Hua, “A unified framework for optimizing linear non-regenerative multicarrier MIMO Relay Communication Systems”, *IEEE Trans. Signal Process.*, vol. 57, pp. 4837-4851, Dec. 2009.

[9] J. Xiao, S. Cui, Z. Luo, and A. J. Goldsmith, “Linear coherent decentralized estimation,” *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 757-770, Feb. 2008.

[10] A. S. Behbahani, A. M. Eltawil, and H. Jafarkhani, “Linear Decentralized Estimation of Correlated Data for Power-Constrained Wireless Sensor Networks,” *IEEE Trans. Signal Process.*, vol. 60, no. 11, pp. 6003-6016, Nov. 2012.

[11] Y. Liu, J. Li, and X. Lu “Multi-Terminal Joint Transceiver Design for MIMO Systems with Contaminated Source and Individual Power Constraint,” *IEEE Intl Symposium on Information Theory(ISIT)*, Honolulu, Jun. 2014.

[12] Y. Liu, X. Lu, J. Li, and Y. Chau, “Optimal Linear Precoding and Postcoding for MIMO Multi-Sensor Noisy Observation Problem,” submitted to *IEEE Trans. Signal Process.*, available at: http://arxiv.org/abs/1409.7122.

[13] S. S. Christensen, R. Argawal, E. de Carvalho, and J. M. Ciofﬁ, “Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 1-7, Dec. 2008.
Q. Shi, M. Razaviyayn, Z. Luo, and C. He, “An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Max- imization for a MIMO Interfering Broadcast Channel,” IEEE Trans. Signal Process., vol. 59, no. 9, pp. 4331-4340, Dec. 2011.

D. Bertsekas and J. Tsitsiklis, Parallel and Distributed Computation, Englewood Cliffs, NJ: Prentice-Hall, 1989.

D. P. Bertsekas, A. Nedic, and A. E. Ozdaglar, Convex Analysis and Optimization. New York: Athena Scientific, 2003.

D. Bertsekas, Nonlinear Programming, 2nd ed, Belmont, MA:Athena Scientific, 1999.

P. Tseng, “Dual ascent methods for problems with strictly convex costs and linear constraints: a unified approach,” SIAM Journ. on Contr. and Optim., vol.28, pp. 214-242, 1990.

L. Grippo, and M. Sciandrone, “On the convergence of the block nonlinear Gauss-Seidel method under convex constraints,” Operations Research Letters, vol. 26, pp. 127-136, 2000.

R. A. Horn and C. R. Johnson, Matrix Analysis. New York: Cambridge Univ. Press, 1985.

S. Boyd, and L. Vandenberghe, Convex Optimization. New York: Cambridge University Press, 2004.

M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming (web page and software),” [Online]. Available: http://cvxr.com/cvx Apr. 2010

I. Polik and T. Terlaky, “Interior Point Methods for Nonlinear Optimization,” in Nonlinear Optimization, 1st edition, G. Di Pillo, F. Schoen, editors. Springer, 2010.

W. Dinkelbach, “On nonlinear fractional programming.” Management Sci., vol. 13, pp. 492-498, 1967.

W. Ai, Y. Huang and S. Zhang, “New Results on Hermitian Matrix Rank-One Decomposition,” Math. Program., vol.128, pp.253-283, 2009.