Coherence interpretations of nonlocal quantum correlation based on a quantum eraser

Byoung S. Ham
School of Electrical Engineering and Computer Science, Gwangju Institute of Science and Technology
123 Chumdangwagi-ro, Buk-gu, Gwangju 61005, South Korea
(Submitted on June 11, 2022; bham@gist.ac.kr)

Abstract
Bell inequality violation is a quantitative measurement tool of quantum entanglement between space-like separated particles. Quantum entanglement is the heart of quantum information science, in which the resulting nonlocal correlation between the paired particles is a unique property of quantum mechanics. Over the last few decades, intensive research has been conducted to understand nonlocal quantum features based on the particle nature of quantum mechanics. Here, the role of coincidence detection is analyzed in a simple interferometer using a quantum eraser. To understand the nonlocal quantum feature, coincidence detection between two output photons of an interferometer is coherently analyzed for Bell inequality violations. Based on this understanding, a classical model of the nonlocal correlation is finally presented using coherent light via wave mixing and heterodyne detection.

Introduction
Quantum entanglement is the heart of quantum information science, where it is known that quantum entanglement cannot be implemented by any classical means [1-4]. A quantitative measurement of quantum entanglement is conducted by Bell inequality violation exceeding the classical upper bound in terms of intensity correlation between paired particles [5]. Thus, quantum entanglement represents a mysterious phenomenon beyond any classical counterpart as discussed by Bell over the EPR paradox [6]. Moreover, quantum entanglement must be nonlocal violating local realism, indicating the space-like separation [7]. A common method of generating quantum entanglement is to use second-order nonlinear optics of spontaneous parametric down conversion (SPDC) process [8]. Recently, Franson-type nonlocal correlation has been coherently interpreted for the SPDC process [9], where the nonlocal quantum feature has been understood according to the wave nature of photons [10]. In this process, coincidence detection plays a key role for the nonlocal correlation, otherwise resulting in local tensor products. Here, the coincidence detection is interpreted for the fundamental physics of quantum entanglement applied to a noninterfering Mach-Zehnder interferometer (MZI) composed of polarizing beam splitters (PBSs). Quantum superposition between two pairs of correlated photons randomly generated from the SPDC process into opposite directions are analyzed to understand how the nonlocal quantum feature can be generated using pure coherence optics. For this, a quantum eraser phenomenon in the noninterfering MZI is coherently analyzed for the polarization projection via coincidence detection, where the Bell inequality violation is understood now in a deterministic manner [11,12]. Finally, a classical model of coherent photon-based quantum entanglement generation is proposed and analyzed for analytical solutions using coherence manipulations of an acousto-optic modulator (AOM) and heterodyne detection.

In 1978, Wheeler proposed a thought experiment of a delayed choice, in which post-measurements can modify predetermined photon characteristics [13]. Since then, many experiments have been conducted to demonstrate the violation of the cause-effect relation [14-18]. One of them is for an orthogonally polarized Mach-Zehnder interferometer (MZI), where the indistinguishable photon characteristics are prohibited from generating interference fringes. If a polarizer is added in each output port of the MZI, however, interference fringes are retrieved [11,12]. This phenomenon is called a quantum eraser as suggested by Scully and Drühl [19,20]. In the present studies, two experiments of Refs. 11 and 12 are analyzed for the nonlocal quantum features using pure coherence optics. To understand the delayed choice of a quantum eraser, the polarizer-retrieved interference fringes are understood as an origin of Bell inequality violations via polarization selective measurements. For the analysis of coincidence detection between two output photons, random bases of
horizontal (H) and vertical (V) polarizations are manipulated for quantum correlation via quantum superposition, resulting in the nonlocal quantum feature. Finally, a classical (coherence) model of nonlocal correlation is generalized, where the polarization basis control is provided by an AOM-based frequency mixing and heterodyne detection-caused frequency-selective measurements. A similar type of measurement-based coherence model has already been suggested for the quantum entanglement generation via a KLM state [21], and even widely applied for linear optics-based quantum computing [22]. In that sense, the present analysis is for a measurement-based quantum mechanics in which the nonlocal quantum correlation can be manipulated coherently and deterministically.

Results

Figure 1 shows different types of delayed-choice schemes of a quantum eraser, where Figs. 1(a) and (b) are for quantum cases based on SPDC-generated photon pairs [11,12]. On the contrary, Fig. 1(c) is for the proposed classical model using coherent photons. In Fig. 1(a), type-I SPDC photon pairs are used, where the polarization-basis change \((V \rightarrow H)\) is conducted by a quarter-wave plate (QWP) for the reflected photon pairs, resulting in basis randomness in both detectors. Instead, in Fig. 1(b), a half-wave plate is used for the same purpose applied for type-II SPDC photon pairs. In both schemes, single-photon measurements in both detectors show distinguishable photon characteristics with no interference fringes. Although both schemes look different, the generalized scheme is the noninterfering MZI as shown in Fig. 1(c), where the output photon states without polarizers are represented as 

\[
|E_A\rangle_A = \frac{E_0}{\sqrt{2}} (|H\rangle_2 - e^{i\phi} |V\rangle_1) \quad \text{and} \quad |E_B\rangle_B = i \frac{E_0}{\sqrt{2}} (|V\rangle_2 + |H\rangle_1 e^{i\phi}) .
\]

Here, the subscripts 1 and 2 indicate different pairs generated in opposite directions of forward and backward, respectively. In spite of random polarization bases of each output photon, they are coherently provided by the geometrical configurations (discussed in Analysis). This coherence is the bedrock of the quantum feature coincidently measured, resulting in an inseparable tensor product via polarization projection-induced quantum eraser [11,12].

Fig. 1. Schematics of quantum correlation. (a) and (b) Quantum model based on \(\chi^{(2)}\) SPDC. (c) Classical model based on coherent photons. D: single photo detector, HWP: half-wave plate, ISO: isolator, L: laser, PBS: polarizing beam splitter, QWP: quarter-wave plate. The \(\xi\) and \(\theta\) indicate polarizer’s rotation angle.

To work with the phase control \(\varphi\) in Figs. 1(a) and (b), the consecutive photon pairs indicated by subscripts 1 and 2 must be coherent. Regardless of the bandwidth of SPDC, the coherence between superposed photons, i.e., the backward (1) and forward (2) and photon pairs in Fig. 1(a) is satisfied by a fixed relative phase determined by the fixed geometrical distance between the nonlinear medium \(\chi^{(2)}\) and the back-reflection mirror \(M\), regardless of the pump photon’s absolute phase [11]. In Fig. 1(b), however, the coherence between the forward and backward photon pairs is automatically achieved by the shared paths of the Sagnac interferometer [12]. The global phase of the pump photon has nothing to do with the intensity measured in each local detector. This is the origin of the present coherence approach in Figs. 1(a) and (b), such that superposed photon pairs
must be phase coherent. Thus, the local measurement can be expressed by the function of $\phi$, $\xi$, and $\theta$, resulting in the quantum eraser [11,23].

Figure 1(c) shows a corresponding classical model based on coherent photons from an attenuated laser. To satisfy coincidence detection, doubly bunched coherent photons are needed as an input by definition. Three or more bunched photons given by Poisson statistics are also involved in the coincidence measurements, whose contribution is a few percent of the doubly bunched case. To provide random polarization bases in each MZI path, a $22.5^\circ$-rotated half-wave plate (HWP) is inserted. Using a double-pass AOM configuration, frequency mixing between $f_0$ and $f_+$ are satisfied before entering the MZI, where frequency-polarization-path correlation inside the MZI is provided by the first PBS. For the basis randomness resulting in indistinguishability for quantum superposition, the added HWP s play an essential role for the nonlocal correlation. For coincidence detection between the output ports, a heterodyne detection technique is applied to remove the same-colored photons from one MZI path. Thus, the counterintuitive classical (coherence) model of nonlocal quantum correlation is provided. Figure 1(c) is the quintessence of the coherence approach in this paper for nonlocal correlation, where such an idea has never been proposed or implemented.

**Analysis**

By the quantum superposition between randomly-generated polarization-correlated photon pairs in Figs. 1(a) and (b), the output fields $E_A$ and $E_B$ are coherently represented by:

$$E_A = \frac{E_0}{\sqrt{2}} (iV_1 e^{i\theta} e^{-i\delta f t} + H_2 e^{-i\delta f t}), \quad (1)$$

$$E_B = \frac{E_0}{\sqrt{2}} (H_1 e^{i\theta} e^{i\delta f t} + iV_2 e^{i\delta f t}), \quad (2)$$

where the sign of $\delta f t$ can be swapped between the polarization bases. Here, a $\frac{\pi}{2}$ phase difference between $H$ and $V$ is given by the SPDC process [24,25], which is equivalent to the classical scheme of Fig. 1(c) by the first PBS. The corresponding mean intensities measured in each detector without polarizers are $I_A = I_B = I_0$ due to the Fresnel-Arago law [26]. This uniform intensity proves the well-known local randomness of SPDC generated photon pairs [11,12].

With the inserted polarizers ($\xi, \theta$), however, coherence between the orthogonally polarized photon pairs can be retrieved, resulting in a quantum eraser [11,12,23]. The polarizer plays an essential role of polarization projection onto its rotated polarization angle $\xi$ or $\theta$ for the orthogonal polarization bases of $H$ and $V$, respectively. Thus, Eqs. (1) and (2) are rewritten as:

$$E_s = \frac{E_0}{\sqrt{2}} (iV_1 \sin \xi e^{i\theta} e^{-i\delta f t} + H_2 \cos \xi e^{-i\delta f t}), \quad (3)$$

$$E_i = \frac{E_0}{\sqrt{2}} (H_1 \cos \theta e^{i\theta} e^{i\delta f t} + iV_2 \sin \theta e^{i\delta f t}), \quad (4)$$

where the inclusion of polarization bases is to indicate the photon’s origin for the further analysis of coincidence detection below. $E_0$ is the amplitude of each single photon. Thus, the corresponding intensities are as follows:

$$I_s = \frac{I_0}{2} (V_1 \sin^2 \xi + H_2 \cos^2 \xi + iV_1 H_2 \sin \xi \cos \xi (e^{i(\theta - \delta f k)} - e^{-i(\theta - \delta f k)})),$n

$$= \frac{I_0}{2} (1 - \sin 2\xi \sin(\theta - \delta f t)),$n

$$I_i = \frac{I_0}{2} (H_1 \cos \theta e^{i\theta} e^{i\delta f t} + iV_2 \sin \theta e^{i\delta f t}) (\cos \theta e^{-i\delta f t} - iV_2 \sin \theta e^{-i\delta f t}), \quad (5)$$

$$= \frac{I_0}{2} (1 + \sin \theta \cos(\theta + \delta f t)),$n

where $\delta f t = \delta f t - \delta f k$. $I_0 (E_0 E_0^*)$ is the intensity of a single photon. Here, $\delta f k \neq 0$ is generally satisfied in SPDC process, but $\delta f k$ is fixed due to the fixed geometry of the forward and backward pump photon, as mentioned above, satisfying the coherence approach. As demonstrated in Ref. 23, Fig. 1(c) always results in local fringes due to the intrinsic coherence among photons determined by single photon self-interference [27]. Such a quantum eraser has been coherently analyzed for Fig. 1(c) even with continuous wave (cw) laser.
resulting in the retrieval of coherence, \( \langle I_1 \rangle = \frac{b}{2}(1 - \sin 2\xi \sin \varphi) \) and \( \langle I_2 \rangle = \frac{b}{2}(1 - \sin 2\theta \sin \varphi) \) [23]. In Ref. 23, the observed quantum eraser in Ref. 11 is well explained using pure coherence approach, too.

Regarding coincidence detections for Fig. 1, the intensity products between two local detectors in Figs. 1(a) and (b) show fringes due to \( \delta_{jk}(\tau = 0) = 0 \). In this case, bandwidth dependent \( e^{i\delta_{jk} \tau} \) and \( \delta_{jk} \) terms are negligible. Moreover, Figs. 1(a) and (b) are for the same correlated photon pairs, where the random global phase has no effect. For Fig. 1(c), however, the coincidence detection is for predetermined different frequencies via AOM manipulations and PBS. For this, quantum beating must be accepted, where the single-photon detector’s time resolution must be shorter than the inverse of the difference frequency. This heterodyne detection technique can be applied for cw light even with non-single photon detectors, if the measurement timing of the detector can be controlled to be shorter than the measurement events.

From Eqs. (5) and (6), the coincidence detection between two detectors \( D_s \) and \( D_i \) is as follows:

\[
\langle R_{si} (\tau) \rangle = \langle I_1(t) I_2(t + \tau) \rangle = \frac{\Omega^2}{4} (V_1 V_2 \sin^2 \xi + H_2 H_2 \cos^2 \xi - V_1 H_2 \sin 2\xi \sin (\varphi - \delta_{jk}) + V_2 H_2 \sin 2\theta \sin (\varphi + \delta_{jk} \tau)),
\]

\[
= \frac{\Omega^2}{4} (V_1 H_1 \sin 2\xi \cos^2 \theta + H_2 H_2 \cos 2\theta \sin^2 \varphi + V_1 H_2 \sin 2\theta \sin (\varphi + \delta_{jk} \tau)) \sin (\varphi + \delta_{jk} \tau),
\]

\[
= \frac{\Omega^2}{4} (\sin^2 \xi \cos^2 \theta + \cos^2 \xi \sin^2 \varphi - \sin 2\xi \sin 2\theta \sin^2 (\varphi - \delta_{jk} \tau)).
\]

In Eq. (7), the role of the last term of \( V_1 H_1 \) or \( H_2 V_2 \) is critical for the coincidence detection \( V_1 H_1 \) or \( H_2 V_2 \). Unlike local detections in Eqs. (5) and (6), even if \( \delta_{jk} \) is random, Eq. (7) shows fringes as a function of \( \varphi \) for \( \xi = \theta = \frac{\pi}{4} \) due to \( \delta_{jk} \tau = 0 \) at coincidence (\( \tau = 0 \)). This is the case of Ref. [11]. Even though Figs. 1(a)-(c) should be differently treated due to different polarization-basis correlation, the fringe relationship still shows the same feature [11,12].

For fixed \( \theta = \varphi = \frac{\pi}{4} \), the coincidence measurements in Eq. (7) becomes:

\[
R_{si}(0) = \frac{\Omega^2}{4} \left( \sin^2 \xi \cos^2 \theta + \cos^2 \xi \sin^2 \varphi - \frac{1}{2} \sin 2\xi \sin 2\theta \right).
\]

where the related numerical calculations are shown in Figs. 2(a) and (b). In this case, two local detectors with control parameters, \( \xi \) and \( \theta \) can be considered as two space-like separated parties, Alice and Bob, respectively. Thus, Figs. 2(a) and (b) represent the Bell inequality violation as a function of \( \xi \) at Alice’s side for a fixed \( \theta \) at
Bob’s side, whose S parameter exceeds the classical bound of 2 [12]. The origin of this nonclassical feature of S parameter is in the coherence feature of Eq. (7) in contrast to the linear relationship of classical particles [5]. From Figs. 2(a) and (b) for $\varphi = \frac{\pi}{4}$ and $\delta \tau = 0$ (coincidence), Eq. (7) can be rewritten as $R_{st}(0) = \frac{i \delta}{4} \cos^2(\xi + \theta)$. This is the quantum feature satisfying the inseparable product, where the nonlocal quantum correlation is represented by $\cos^2(\xi + \theta)$, in which $\xi$ is strongly correlated with $\theta$, and vice versa. Depending on $\varphi$ value, the quantum feature appeared in Figs. 2(a) and (b) disappears, as shown in Figs. 2(c) and (d). Here, Figs. 2(c) and (d) represent the typical classical feature based on separable intensity products. Thus, the nonlocal quantum correlation, i.e., Bell inequality violations is interpreted with the wave nature of quantum mechanics using pure coherence optics, where missing pictures in the conventional interpretations are also revealed.

For Fig. 1(c), the coincidence detection is for $V_1 V_2$ and $H_1 H_2$ via heterodyne detection, where the interference term of $V_1 H_2 H_1 V_2$ results in the same output for all cases of Fig. 1. In this case of different wavelength measurements for quantum correlation, such heterodyne detection can be appeared as a quantum beat [28]. However, the visibility of the quantum feature in Eq. (7) should be decreased down to 50% if no heterodyne detection is applied, resulting in the common classical limit. From Fig. 1(c), the following analytical solutions are obtained for the local and independent detectors (see the Supplementary Materials):

$$I_s = \frac{I_0}{2} (1 - \sin 2\xi \cos \varphi), \quad (9)$$
$$I_i = \frac{I_0}{2} (1 + \sin 2\theta \cos \varphi). \quad (10)$$

Thus, the coincidence detection between two local detectors via heterodyne detection is as follows:

$$\langle R_{st}(0) \rangle = \frac{i \delta}{4} (-V_1 \sin \xi + H_2 \cos \xi)(H_1 \cos \theta + V_2 \sin \theta)(-V_1 \sin \xi + H_2 \cos \xi)(H_1 \cos \theta + V_2 \sin \theta),$$
$$= \frac{i \delta}{4} \cos^2(\xi + \theta). \quad (11)$$

Equation (11) shows the same feature of inseparable intensity products as shown in Fig. 2. Thus, the coherently excited nonlocal correlations are achieved coincidently via AOM-based wave mixing and heterodyne detection of it.

**Discussion**

With coherence manipulations of random polarization bases in Fig. 1(c), the AOM-caused wave mixing and its heterodyne detection was analyzed for the same nonlocal quantum features as in SPDC cases in Figs. 1(a) and (b). Due to the polarization projection of the output photons, the nonlocal correlation was coherently analyzed as a direct result of the wave nature of photons, whose resulting fringes were expressed for correlation between rotation angles of the polarizers. This is the direct proof of the nonlocal correlation for the present coherence approach, where coherence-based correlation should be kept for the space-like separated detectors. According to the single photon self-interference [27], no difference exists between the single photon and cw light for the quantum eraser [23]. Thus, the coherence Bell inequality violation based on heterodyne detection technique can also be satisfied, if the detector’s temporal resolution is high enough to separate a beating signal of each photon pair. Such coherent manipulations of polarization controls with AOM and heterodyne detections in Fig. 1(c) violate the conventional myth of inseparable basis products by linear optics. As discussed already in Ref. 9, the nonlocal quantum feature is due to filtering process via coincidence detection, resulting in inseparable intensity products [29,30].

**Conclusion**

The role of coincidence detection was coherently interpreted for SPDC-generated photon pairs in a back-reflection scheme and demonstrated for its equivalence to a noninterfering MZI. With random polarization bases in both local detectors, their coincidence detection was analyzed for nonlocal quantum features of Bell inequality violations, resulting in an inseparable intensity product of polarization bases. In this process, the quantum eraser applied to the output photon pairs was found to be the origin of Bell inequality violations. Finally, a classical model of nonlocal correlation was proposed for the same quantum feature using coherent
photons, and its analytical solutions were derived to be the same as in the SPDC counterpart. For this, frequency-polarization correlation of coherent photon pairs was provided by coherence manipulations of an AOM via PBS, and an inseparable intensity product was achieved by using a coincident heterodyne-detection technique. Compared with the SPDC case, such inseparable basis products driven by the coincidence detection technique could be achieved by classical means. Due to the same coherence feature generalized in an MZI, the presented classical model based on coherent photon pairs opens the door to deterministic quantum information science using even cw light, if the measurement timing can be shorter than the measurement events.

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