Peristaltic transport of a Maxwell fluid in a porous asymmetric channel through a porous medium

Safia Akram¹*, M. Hanif¹ and S. Nadeem²

Abstract: The present study investigates the peristaltic flow of a Maxwell fluid in a porous asymmetric channel through a porous medium. An analytical solution has been found using regular perturbation method. The stream function and average mean velocity are obtained. The graphical results are presented to discuss the physical behavior of various parameters appearing in the problem.

Keywords: peristaltic flow, Maxwell fluid, porous medium, porous boundaries, asymmetric channel

1. Introduction
Since, the first investigation done by Latham (1966) a large amount of literature is available on peristaltic motion of Newtonian and non-Newtonian fluid with different flow geometries (Akram & Nadeem, 2013; Akram, Nadeem, & Hanif, 2013; Ali, Hayat, & Asghar, 2009; Eldesoky, 2012, 2013; Eldesoky & Mousa, 2010; El-Shehawey, El-Dabe, & El-Desoky, 2006; Mittra & Prasad, 1973; Nadeem & Akram, 2010; Nadeem, Akram, & Akbar, 2013; Shapiro, Jaffrin, & Weinberg, 1969). Peristaltic mechanism is a fluid transport induced by a progressive wave of area contraction or expansion along the walls of a distensible tube containing fluid. This mechanism occurs in many practical applications involving physiological and biomechanical processes. The study of fluid flow in a porous plate is another area which has been investigated by many researchers (Hayat & Hutter, 2004; Hayat, Ellahi, S. Asghar, & Siddiqui, 2004; Rajagopal & Gupta, 1984; Wang & Hayat, 2004) because of its applications. Few of applications of porous boundaries are transpiration cooling, gaseous diffusion, and process of dialysis of blood in an artificial kidney. Only a limited attention has been focused to the study of peristaltic motion of fluid with suction and injection. The idea of peristaltic motion of fluid in porous boundaries was first investigated by Lukashev (1993). He has formulated a model of peristaltic transport of liquid motion caused by the auto-wave process of mass transport through a porous capillary wall. Later on, El-Shehawey and Husseny (2000) discussed the effects of porous boundaries on peristaltic transport through porous medium. Recently, Haroun (2000) discussed the effects of wall compliance on peristaltic transport of a Newtonian fluid in an asymmetric channel.
Motivated from the above discussion, the aim of the present paper is to discuss the peristaltic motion of a Maxwell fluid in a porous asymmetric channel through a porous medium. To the best of authors’ knowledge, no attempt has been made to discuss the peristaltic motion of non-Newtonian fluid in a porous channel. The porosity of the channel means that there will be both suction and injection phenomena. The governing highly non-linear partial differential equations are solved analytically by employing perturbation method. The expressions for stream function and average mean velocity have been computed. Numerical illustrations that show the physical effects and the pertinent features are investigated at the end of the paper.

2. Mathematical formulation
Let us consider the peristaltic flow of an incompressible, non-Newtonian fluid (linear Maxwell fluid) in a two dimensional channel having width \( d_1 \) and \( d_2 \). The walls of the channel are porous, flexible, and also there are imposed traveling sinusoidal waves of small amplitude. The equations which governs the flow are defined as

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}
\]

\[
\left(1 + r \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = -\left(1 + r \frac{\partial}{\partial t}\right) \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \frac{1}{W} - \frac{v}{W} u \tag{2}
\]

\[
\left(1 + r \frac{\partial}{\partial t}\right) \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}\right) = -\left(1 + r \frac{\partial}{\partial t}\right) \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \frac{1}{W} - \frac{v}{W} v \tag{3}
\]

where \( u \) and \( v \) are the velocity components along \( x \) and \( y \)-axis respectively, \( p \) is the pressure, \( r \) is the relaxation times, \( W \) is the permeability parameter, and \( \rho \) is the fluid density.

Making use of stream function \( \Psi \left( u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}\right) \) and eliminating pressure, Equations 2 and 3 reduce to

\[
\left(1 + \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 \psi}{\partial t^2} + \psi \frac{\partial^2 \psi}{\partial x^2} + \psi \frac{\partial^2 \psi}{\partial y^2} - \psi \frac{\partial^2 \psi}{\partial x \partial y}\right) = \nu \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial y^4} \tag{4}
\]

where

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

Let the vertical displacement for upper wall is \( \eta_1 \) and lower wall is \( \eta_2 \). Further, \( \eta_1 \) and \( \eta_2 \) are assumed to be in the form of a sinusoidal wave of different amplitudes and phases. Thus,

\[
\eta_1 = a_1 \cos \left(\frac{2\pi}{\lambda} (x - ct)\right), \quad \eta_2 = a_2 \cos \left(\frac{2\pi}{\lambda} (x - ct) + \theta\right) \tag{5}
\]

where \( a_1 \) and \( a_2 \) are the amplitude, \( \lambda \) is the wave length, \( c \) is the speed of light, and \( \theta \) is the phase difference which varies in the range \( 0 \leq \theta \leq \pi \) in which \( \theta = 0 \) corresponds to symmetric channel with wave out of phase and \( \theta = \pi \), the waves are in phase, further, \( a_1, a_2, d_1, d_2, \) and \( \theta \) satisfies the condition

\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \theta \leq (d_1 + d_2)^2 \]
The horizontal displacement is assumed to be zero. The fluid is entering the flow region through one plate at the same rate as it is leaving through the other plate with velocity $V$ in the positive direction of the $y$-axis. Therefore, the boundary conditions are

$$\Psi_y = 0, \quad \Psi_x = -V - \frac{\partial \eta_1}{\partial t} \text{ at } y = d_1 + \eta_1$$  \hspace{1cm} (6)

$$\Psi_y = 0, \quad \Psi_x = -V + \frac{\partial \eta_2}{\partial t} \text{ at } y = -d_2 - \eta_2$$  \hspace{1cm} (7)

Defining

$$\hat{x} = \frac{x}{a_1}, \quad \hat{y} = \frac{y}{a_1}, \quad \hat{u} = \frac{u}{c}, \quad \hat{v} = \frac{v}{c}, \quad \hat{p} = \frac{p}{\rho c^2}, \quad \hat{t} = \frac{ct}{a_1}, \quad \hat{\epsilon} = \frac{\epsilon}{a_1}, \quad \hat{h} = \frac{d_1}{a_1},$$

$$\alpha = \frac{\eta_0}{a_1}, \quad \hat{V} = \frac{V}{c}, \quad \hat{\eta}_1 = \frac{\eta_1}{a_1}, \quad \hat{\eta}_2 = \frac{\eta_2}{a_1}, \quad \hat{\overline{W}} = \frac{\overline{W}}{a_1}, \quad \hat{\Psi} = \frac{\Psi}{c a_1}, \quad \hat{t} = \frac{ct}{a_1}, \quad R = \frac{cd}{a_1}, \quad \alpha_1 = \frac{2ad_1}{x}$$  \hspace{1cm} (8)

Using the above non-dimensional quantities into Equations 4–7, the resulting equations after dropping the bars can be written as

$$\left(1 + \frac{\alpha}{\alpha_1} \frac{\partial}{\partial t}\right) \left(\frac{1}{\hat{W}} \left(\hat{v}^2 \hat{\Psi} + \hat{v}^2 \hat{\Psi}_y \hat{v}^2 \hat{\Psi}_x - \hat{v}^2 \hat{\Psi}_y \hat{v}^2 \hat{\Psi}_x\right) = \frac{1}{R^2} \left(\hat{v}^4 \hat{\Psi} - \frac{1}{\hat{W}} \hat{v}^2 \hat{\Psi}\right) \right)$$  \hspace{1cm} (9)

$$\eta_1 = \hat{\epsilon} \cos (\alpha x - t), \quad \eta_2 = \alpha \hat{\epsilon} \cos (\alpha x - t + \theta)$$  \hspace{1cm} (10)

$$\Psi_y = 0, \quad \Psi_x = -V - \alpha \hat{\epsilon} \sin (\alpha x - t) \text{ at } y = h + \eta_1$$  \hspace{1cm} (11)

$$\Psi_y = 0, \quad \Psi_x = -V + \alpha \hat{\epsilon} \sin (\alpha x - t + \theta) \text{ at } y = -h + \eta_1$$  \hspace{1cm} (12)

where $\hat{\epsilon}$ is the amplitude ratio, $\alpha$ is the wave number and $R$ is the Reynolds number.

### 3. Solution of the problem

To find the solution of Equation 9, we expand $\Psi$ and $\partial \Psi / \partial x$ in the form

$$\Psi = \Psi_0 + \hat{\epsilon} \Psi_1 + \hat{\epsilon}^2 \Psi_2 + \cdots$$  \hspace{1cm} (13)

$$\frac{\partial \Psi}{\partial x} = \left(\frac{\partial \Psi}{\partial x}\right)_0 + \hat{\epsilon} \left(\frac{\partial \Psi}{\partial x}\right)_1 + \hat{\epsilon}^2 \left(\frac{\partial \Psi}{\partial x}\right)_2 + \cdots$$  \hspace{1cm} (14)

The first term $\left(\frac{\partial \Psi}{\partial x}\right)_0$ corresponds to an imposed pressure gradient which is considered as a constant. The higher order terms correspond to peristaltic motion and are considered as function of $x$, $y$, and $t$.

Making use of Equation 13 in Equations 9–12 and collecting the like powers of $\hat{\epsilon}$, we obtain three sets of coupled linear differential equations with their corresponding boundary conditions in $\Psi_0$, $\Psi_1$ and $\Psi_2$. To avoid the lengthy calculations, the solution for $\Psi_0$, $\Psi_1$, and $\Psi_2$ are directly defined as follows.

#### 3.1. Solution for $\Psi_0$

The solution for $\Psi_0$ is obtained by adopting the similar producer as discussed by El-Shehawey and Husseny (2000) subject to suction and injection, is written as

$$\Psi_0(x, y) = \frac{kW}{e^{\beta_1 h} - e^{-\beta_1 h}} \left( y - \left(1 - \frac{1}{2}\right) \right) e^{\beta_1 h} - \left(\frac{e^{\beta_1 h} - e^{-\beta_1 h}}{\lambda_1}\right) e^{\beta_1 h} \left(\frac{y}{x}\right)

+ \left(\frac{e^{\beta_1 h} - e^{-\beta_1 h}}{\lambda_2}\right) e^{\beta_1 h} \left(\frac{y}{x}\right)

+ \left(\frac{e^{\beta_1 h} - e^{-\beta_1 h}}{\lambda_2}\right) e^{\beta_1 h} \left(\frac{y}{x}\right) - Vx$$  \hspace{1cm} (15)
where

\[ \lambda_1 = \frac{RV + \sqrt{(RV)^2 + 4/W}}{2}, \quad \lambda_2 = \frac{RV - \sqrt{(RV)^2 + 4/W}}{2} \]

The expression for velocity is obtained as follows

\[ u_0 = kW \left( 1 + \frac{(\psi_1 - e^{-\lambda_1 y})}{e^{-\lambda_1 y}(\psi_1 - e^{\lambda_1 y})} e^{\lambda_1 y} - \frac{(\psi_1 - e^{-\lambda_1 y})}{e^{-\lambda_1 y}(\psi_1 - e^{\lambda_1 y})} e^{\lambda_1 y} \right) \]  

(16)

If

\[ \lim_{V \to 0} \left( \lim_{W \to \infty} u_0 \right) = k(1 - y^2) \]

which leads to the classical Poiseuille flow in the absence of \( V \) and \( 1/W \).

### 3.2. Solution for \( \Psi_1 \) and \( \Psi_2 \)

The solution for \( \Psi_1 \) and \( \Psi_2 \) can be calculated by using the following expressions according to their boundary conditions.

\[ \Psi_1 = \frac{1}{2} \left( \psi_1(y)e^{2i\alpha(x-t)} + \psi_1^*(y)e^{-2i\alpha(x-t)} \right) \]  

(17)

\[ \Psi_2 = \frac{1}{2} \left( \psi_{20}(y) + \psi_{22}(y)e^{2i\alpha(x-t)} + \psi_{22}^*(y)e^{-2i\alpha(x-t)} \right) \]  

(18)

in which asterisk denotes the complex conjugate. Substituting Equations 17 and 18 into the differential equations and their corresponding boundary conditions in \( \Psi_1 \) and \( \Psi_2 \) leads to the following differential equations

\[
\frac{d}{dy} \left( -\alpha^2 + i\alpha R - i\alpha RV - \frac{1}{W} \right) + (RV i\alpha \tau - VR) \frac{d}{dy} + R\alpha^2 \frac{d}{dy} \right) \psi_1 + R\alpha^2 \phi_1' \psi_1'' - \psi_0' R\alpha^2 (\phi_1'' - \alpha^2 \phi_1) + i\alpha RV' \phi_1 = 0
\]  

(19)

\[
\psi_1 \begin{cases} 1 \hline -h \end{cases} = \left\{ \begin{array}{c} -1 \alpha e^{i\theta} \end{array} \right\} \psi_0' \begin{cases} 1 \hline -h \end{cases}
\]

(20)

\[
\phi_1' \begin{cases} 1 \hline -h \end{cases} = \left\{ \begin{array}{c} -1 \alpha e^{i\theta} \end{array} \right\}
\]

(21)

\[
\phi_1'' + RV \phi_1'' - \frac{1}{W} \phi_1'' = \frac{1}{2} 2iaR \left[ \phi_1'' - \phi_1 \phi_1'' \right]
\]  

(22)

\[
\psi_2' \begin{cases} 1 \hline -h \end{cases} = \frac{1}{2} \left\{ \begin{array}{c} 1 \alpha \cos \theta \end{array} \right\} \psi_0' \begin{cases} 1 \hline -h \end{cases} + \phi_1'' \begin{cases} 1 \hline -h \end{cases}
\]

(23)

\[
\psi_2' \begin{cases} 1 \hline -h \end{cases} = \frac{1}{2} \left\{ \begin{array}{c} 1 \alpha \cos \theta \end{array} \right\} \psi_0' \begin{cases} 1 \hline -h \end{cases}
\]

(24)
\[
\phi_2'' \left\{ \begin{array}{l} 1 \\ -h \end{array} \right\} = -\frac{1}{4} \left\{ a^2 e^{i\omega} \right\} \phi_0'' \left\{ \begin{array}{l} 1 \\ -h \end{array} \right\} + \frac{1}{2} \left\{ a e^{i\omega} \right\} \phi_1'' \left\{ \begin{array}{l} 1 \\ -h \end{array} \right\} \tag{25}
\]

\[
\phi_2'' \left\{ \begin{array}{l} 1 \\ -h \end{array} \right\} \mp 4 \left\{ a e^{i\omega} \right\} \phi_1'' \left\{ \begin{array}{l} 1 \\ -h \end{array} \right\} = 0 \tag{26}
\]

Using the similar procedure as discussed in El-Shehawey and Husseny (2000), the solution of Equations 19–22 is straightforward written as

\[
\phi_1 = A_1 \cos h \alpha y + A_2 \sin h \alpha y + A_3 e^{i\omega y} + A_4 e^{-i\omega y} \tag{27}
\]

\[
\phi_2'(y) = F(y) + \frac{D_1(e^{-i\alpha h y} + e^{i\alpha h y}) + D_2(e^{i\omega y} - e^{-i\omega y})}{(e^{i\alpha h y} - e^{-i\alpha h y})}
\]

\[
+ \frac{F(1)(e^{-i\alpha h y} + e^{i\alpha h y}) - e^{-i\alpha h y} + e^{i\alpha h y} + e^{i\alpha h y} - e^{-i\alpha h y})}{(e^{i\alpha h y} - e^{-i\alpha h y})}
\]

\[
+ C_1 \left( \frac{e^{i\alpha h y} - e^{-i\alpha h y}}{(e^{i\alpha h y} - e^{-i\alpha h y})} + 1 \right) \tag{28}
\]

where

\[
\alpha_1 = \frac{RV \delta + \sqrt{(RV \delta)^2 + 4\beta^2}}{2}, \quad \alpha_2 = \frac{RV \delta - \sqrt{(RV \delta)^2 + 4\beta^2}}{2}
\]

\[
\beta^2 = a^2 - i a R \delta + \frac{1}{w}, \quad \delta = 1 - i \alpha 
\]

\[
Z_1 = \alpha(w_1 - w_2) \left( 1 + e^{(1+h)(w_1+w_2)} \right) - \alpha(e^{w_1+hw_2} + e^{w_2+hw_1})w_1 - w_2 \cos h ((1+h)\alpha)
\]

\[
+ (e^{w_1+hw_2} + e^{w_2+hw_1})(-w_1 w_2 + a^2) \sin h ((1+h)\alpha)
\]

\[
Z_2 = -\alpha \left( w_1 - w_2 + a e^{i\omega}(w_1 e^{w_1+hw_2} - w_2 e^{w_2+hw_1}) \right) \cos h (\alpha)
\]

\[
- \alpha \left( w_1 e^{w_1+hw_2} + a w_1 - w_2 e^{w_2+hw_1} \right) \cos h (\alpha)
\]

\[
+ a w_1 w_2 (e^{w_1+hw_2} - e^{w_2+hw_1}) \cos h (\alpha) + a w_1 w_2 \sin h (\alpha) \tag{29}
\]

\[
Z_3 = \alpha e^{i\omega} w_1 w_2 (e^{w_1+hw_2} - e^{w_2+hw_1}) \cos h (\alpha) + (e^{w_1+hw_2} - e^{w_2+hw_1}) w_1 w_2 \cos h (\alpha)
\]

\[
+ a \left( w_1 - w_2 + a e^{i\omega}(w_1 e^{w_1+hw_2} - w_2 e^{w_1+hw_1}) \right) \sin h (\alpha)
\]

\[
+ a \left( w_1 e^{w_1+hw_2} + a w_1 - w_2 e^{w_2+hw_1} \right) \sin h (\alpha) \tag{30}
\]

\[
Z_4 = \alpha w_1 e^{hw_2}(e^{w_1+hw_2} - a e^{i\omega}) + w_1 e^{hw_2}(-1 + a e^{w_1+hw_2+i\omega}) \cos h ((1+h)\alpha)
\]

\[
- a e^{hw_2}(1 + a e^{w_1+hw_2+i\omega}) \sin h ((1+h)\alpha) \tag{31}
\]

\[
Z_5 = \alpha w_1 e^{hw_2}(e^{w_1+hw_2} - a e^{i\omega}) + w_1 e^{hw_2}(-1 + a e^{w_1+hw_2+i\omega}) \cos h ((1+h)\alpha)
\]

\[
- a e^{hw_2}(1 + a e^{w_1+hw_2+i\omega}) \sin h ((1+h)\alpha) \tag{32}
\]

\[
A_1 = \frac{Z_2}{Z_1}, \quad A_2 = \frac{Z_3}{Z_1}, \quad A_3 = \frac{Z_4}{Z_1}, \quad A_4 = \frac{Z_5}{Z_1} \tag{33}
\]
\[ F(y) = \frac{iaR}{2} \left\{ \left( \frac{A_3 b_3 (a_1^2 - a^2) e^{\alpha y}}{b_1^2 - b_2^2} + \frac{A_4 b_4 (a_3^2 - a^2) e^{\alpha y}}{b_2^2 - b_4^2} \right) (A_1^* \cos h \alpha y + A_2^* \sin h \alpha y) \\
- \left( \frac{A_3 b_3 (a_1^2 - a^2) e^{\alpha y}}{b_1^2 - b_2^2} + \frac{A_4 b_4 (a_3^2 - a^2) e^{\alpha y}}{b_2^2 - b_4^2} \right) (A_1^* \sin h \alpha y + A_2^* \cos h \alpha y) \\
+ \left( \frac{A_3 b_3 (a_2^2 - a_3^2) e^{\alpha y}}{b_1^2 - b_2^2} + \frac{A_4 b_4 (a_2^2 - a_3^2) e^{\alpha y}}{b_2^2 - b_4^2} \right) (A_1 \cos h \alpha y + A_2 \sin h \alpha y) \\
- \left( \frac{A_3 b_3 (a_2^2 - a_3^2) e^{\alpha y}}{b_1^2 - b_2^2} + \frac{A_4 b_4 (a_2^2 - a_3^2) e^{\alpha y}}{b_2^2 - b_4^2} \right) (A_1 \sin h \alpha y + A_2 \cos h \alpha y) \\
+ \frac{A_3^* A_3 (a_1^2 - a_1^2) e^{\alpha y}}{(a_1 + a_1^*)^2 - RV (a_1 + a_1^*) - 1/W} - \frac{A_4^* A_4 (a_2^2 - a_1^2) e^{\alpha y}}{(a_2 + a_1^*)^2 - RV (a_2 + a_1^*) - 1/W} \\
+ \frac{A_3^* A_3 (a_2^2 - a_2^2) e^{\alpha y}}{(a_2 + a_2^*)^2 - RV (a_2 + a_2^*) - 1/W} - \frac{A_4^* A_4 (a_3^2 - a_2^2) e^{\alpha y}}{(a_3 + a_2^*)^2 - RV (a_3 + a_2^*) - 1/W} \right\} \]

\[ b_1 = a' + a_1^2 - RV a_1 - 1/W, \quad b_2 = a(2a_1 - RV) \]
\[ b_3 = a' + a_2^2 - RV a_2 - 1/W, \quad b_4 = a(2a_2 - RV) \]

\[ D_1 = \phi_20(+1) = -\frac{1}{2} \left( a_1^2 \left( (A_1 + A_1^*) \cos h a + (A_2 + A_2^*) \sin h a \right) + \omega_1^2 (A_2 e^{\alpha y} + A_2^* e^{\alpha y}) \right) \]

\[ D_2 = \phi_20(-1) = \frac{1}{2} \left( 2a \cos \theta (a_1^2 (A_1 + A_1^*) \cos h (ah) - (A_2 + A_2^*) \sin h (ah)) + \omega_1^2 (A_3 e^{-\omega h} \right) \]

\[ + a_1^2 (A_2 e^{-\omega h} + a_1^2 A_3^* e^{-\omega h} + a_2^2 A_4^* e^{-\omega h}) \]

In above problem, \( C_1 \) is arbitrary constant, we can choose \( C_1 \) as

\[ \frac{C_1}{R} = \left( \frac{\partial p}{\partial x} \right)_2 \]

The main time average velocities becomes

\[ \bar{u} = \frac{\alpha}{2} \phi_20 = \frac{\alpha}{2} \left( F(y) + \frac{D_1 (e^{\alpha y} + e^{-\alpha y}) + D_2 (e^{\alpha y} - e^{-\alpha y})}{(e^{\alpha y} - e^{-\alpha y})} \right) \]

\[ \times \left( F(1) (e^{\alpha y} - e^{-\alpha y}) + F(-h) (e^{-\alpha y} - e^{\alpha y}) \right) + WR \left( \frac{\partial p}{\partial x} \right)_2 \left( \frac{e^{\alpha y} - e^{-\alpha y}}{(e^{\alpha y} - e^{-\alpha y})} \right) \]

\[ 4. \textbf{Graphical results and discussion} \]

In this section, the graphical results are displayed. Figures 1-8 are prepared to see the behavior of \( D_1 \) and \( D_2 \) with wave number \( \alpha \) for different values of porosity \( V_p \) permeability parameter \( W \), relaxation time \( \tau \), and amplitude of wave \( a \). From Figures 1 and 2, it is observed that \( D_1 \) increases with the increase in \( V_p \) and decreases with the increase in \( W \). From Figures 3 and 4, it is shown that \( D_1 \)
Figure 1. Variation of $D_1$ with wave number $\alpha$ for different values of $V$ at $R = 10$, $\tau = 2$, $a = 0.5$, $W = 0.02$, $h = 0.5$, $\theta = \frac{\pi}{3}$.

Figure 2. Variation of $D_1$ with wave number $\alpha$ for different values of $W$ at $R = 10$, $\tau = 2$, $a = 0.5$, $V = 0.05$, $h = 0.5$, $\theta = \frac{\pi}{3}$.

Figure 3. Variation of $D_1$ with wave number $\alpha$ for different values of $\tau$ at $R = 10$, $V = 0.05$, $a = 0.5$, $W = 0.02$, $h = 0.5$, $\theta = \frac{\pi}{3}$. 
Figure 4. Variation of $D_1$ with wave number $\alpha$ for different values of $a$ at $R = 10, V = 0.05, \tau = 2, W = 0.02, h = 0.5, \theta = \frac{\pi}{3}$.

Figure 5. Variation of $D_2$ with wave number $\alpha$ for different values of $V$ at $R = 10, \tau = 2, a = 0.5, W = 0.02, h = 0.5, \theta = \frac{\pi}{3}$.

Figure 6. Variation of $D_3$ with wave number $\alpha$ for different values of $W$ at $R = 10, \tau = 2, a = 0.5, V = 0.05, h = 0.5, \theta = \frac{\pi}{3}$.
Figure 7. Variation of $D_2$ with wave number $\alpha$ for different values of $\tau$ at $R = 10$, $V = 0.05$, $a = 0.5$, $W = 0.02$, $h = 0.5$, $\theta = \frac{\pi}{3}$.

Figure 8. Variation of $D_2$ with wave number $\alpha$ for different values of $a$ at $R = 10$, $V = 0.05$, $\tau = 2$, $W = 0.02$, $h = 0.5$, $\theta = \frac{\pi}{3}$.

Figure 9. Variation of mean velocity distribution with $y$ for different values of $a$ at $R = 30$, $\tau = 2$, $V = 0.08$, $a = 0.5$, $\theta = \frac{\pi}{3}$, $h = 0.5$, $W = 1.5$, $(dp/dx)_2 = -2.0$. 
Figure 10. Variation of mean velocity distribution with $y$ for different values of $W$ at $R = 30$, $V = 0.08$, $a = 0.5$, $\tau = 2$, $\alpha = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $(dp/dx)^2 = -2.0$.

Figure 11. Variation of mean velocity distribution with $y$ for different values of $W$ and $a$ at $R = 30$, $\tau = 2$, $V = 0.08$, $a = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $(dp/dx)^2 = -2.0$.

Figure 12. Variation of mean velocity distribution with $y$ for different values of $V$ at $R = 10$, $\tau = 2$, $W = 3.5$, $a = 0.5$, $\alpha = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $(dp/dx)^2 = -2.0$. 
Figure 13. Variation of mean velocity distribution with $y$ for different values of $\tau$ at $R = 30$, $V = 0.05$, $a = 0.5$, $\alpha = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $(\frac{dv}{dx})_2 = -2.0$.

Figure 14. Variation of mean velocity distribution with $y$ for different values of $a$ at $R = 30$, $\tau = 2$, $V = 0.08$, $\alpha = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $W = 1.5$, $(\frac{dv}{dx})_2 = 2.0$.

Figure 15. Variation of mean velocity distribution with $y$ for different values of $W$ at $R = 30$, $V = 0.08$, $a = 0.5$, $\tau = 2$, $\alpha = 0.5$, $\theta = \frac{\pi}{2}$, $h = 0.5$, $(\frac{dv}{dx})_2 = 2.0$. 
Figure 16. Variation of mean velocity distribution with $y$ for different values of $W$ and $a$ at $R = 30$, $\tau = 2$, $V = 0.08$, $\alpha = 0.5$, $\theta = \frac{\pi}{3}$, $h = 0.5$, $(dp/dx)_2 = 2.0$.

Figure 17. Variation of mean velocity distribution with $y$ for different values of $V$ at $R = 10$, $\tau = 2$, $W = 3.5$, $\alpha = 0.5$, $a = 0.5$, $\theta = \frac{\pi}{3}$, $h = 0.5$, $(dp/dx)_2 = 2.0$.

Figure 18. Variation of mean velocity distribution with $y$ for different values of $\tau$ at $R = 30$, $V = 0.05$, $\alpha = 0.5$, $a = 0.5$, $\theta = \frac{\pi}{3}$, $h = 0.5$, $(dp/dx)_2 = 2.0$. 
decreases with the decrease in $\tau$ and $a$. Also it is observed that $D_1$ decreases with the increase in both $V$ and $W$ (see Figures 5 and 6). While $D_1$ increases with the decrease in $\tau$ and $a$ (see Figures 7 and 8). This means that fluid entering through the lower plate acts as injection and the fluid leaving through upper plate acts as suction. The mean velocity distribution and reversal flow are displayed in Figures 9–18. It is observed from Figure 9 that with the decrease in $a$, the mean velocity distribution increases. It is also observed from Figures 10 and 11 that with the decrease in $W$ and $a$, the mean velocity distribution decreases. It is depicted from Figure 12 that with the decrease in $V$ the mean velocity distribution increases in the lower half of the channel. It is seen from Figure 13 that the mean velocity distribution decreases in the upper half of the channel with the decrease in $\tau$. It is also observed from Figures 14 to 18 that the behavior of the reversal flow is quite similar as compared to the mean velocity distribution.

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**Author details**

Sofia Akram\1
E-mail: djsafiaokram@gmail.com
M. Hanif\2
E-mail: hanif_muhammad@yahoo.com
S. Nadeem\1
E-mail: snqau@hotmail.com

1 Department of Basic Sciences, MCS, National University of Sciences and Technology, Islamabad, Pakistan.
2 Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan.

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