$W$ radiative decay to heavy-light mesons in HQET factorization through $\mathcal{O}(\alpha_s)$

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Abstract

Analogous to NRQCD factorization for heavy quarkonium production, in this work we propose to employ the heavy-quark-effective-theory (HQET) factorization, which has been almost exclusively used in describing exclusive $B$ decays, to study the exclusive production of the heavy-light mesons. We take $W \rightarrow B(D_s) + \gamma$ as a prototype process. The validity of the HQET factorization rests upon the presumed scale hierarchy: $m_W \sim m_b \gg \Lambda_{\text{QCD}}$. Through an explicit analysis at next-to-leading order in $\alpha_s$, we verify that the amplitude can indeed be expressed as the convolution between perturbatively calculable hard-scattering kernel and the $B$ meson light-cone distribution amplitude (LCDA) defined in HQET. It is observed that the factorization scale dependence gets reduced after incorporating the NLO perturbative correction. An interesting question concerning HQET factorization is whether one can refactorize the hard-scattering kernel so that the large logarithms of $M_W/m_b$ can be effectively resummed.

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I. INTRODUCTION

Heavy flavor physics has been one of the major forefronts of high energy physics in the past two decades, whose primary goal is to trace the origin of the CP violation, precisely pin down the Cabbibo-Kobayashi-Maskawa (CKM) matrix, as well as search for possible footprints of new physics. Much of effort in this subject concentrates on the $B$ meson exclusive decays, which contain very rich phenomena. To reliably describe uncountable decay channels, it is mandatory to develop a thorough understanding towards the underlying QCD dynamics. By invoking the hierarchy $m_b \gg \Lambda_{\text{QCD}}$, the QCD factorization (QCDF) approach has been developed based on the first principles [1]. The key concept is to express the decay amplitude into the convolution of the perturbatively calculable, yet, process-dependent, hard-scattering kernel and the nonperturbative, yet, universal, $B$ meson light-cone distribution amplitude (LCDA). It is important to emphasize that this factorization framework applies in the heavy quark limit, corroborated by the fact that the $b$ quark field in the LCDA is defined in the heavy quark effective theory (HQET) rather than in QCD. The simplest application of the QCD factorization is the radiative decay $B \rightarrow \gamma \ell \nu$ [2–4]. This factorization picture has also been extended to more sophisticated processes, e.g., $B \rightarrow \gamma \gamma$ [5] and $B \rightarrow M_1 M_2$ [6, 7].

In contrast to exclusive $B$ decays, the hard exclusive production of heavy-flavored hadrons receives much fewer attention in literature. The main reason might be attributed to the highly suppressed production rate of such processes in high energy collision experiments. The $D_s$ meson exclusive production from $W$ decay, e.g., $W^+ \rightarrow D_s^+ \gamma$, was among the early exploration in this topics [8, 9], and an upper limit was placed by the Fermilab Tevatron in late 90s [10]. Motivated by a gigantic number of $W$, $Z$ produced at LHC, i.e. about $\mathcal{O}(10^{11})$ with the projected 3000 fb$^{-1}$ integrated luminosity, a number of exclusive channels of $W$, $Z$ radiative decays into heavy-light mesons has recently been investigated at leading order (LO) in $\alpha_s$ [11]. The theoretical basis underneath [11] is the conventional collinear factorization for hard exclusive reactions [12, 13]. By exploiting the hierarchy $M_{W,Z} \gg m_{b,c}$, one expresses the amplitude as the convolution between the hard-scattering kernel and the universal heavy meson LCDA. However, it is worth stressing that, the nonperturbative LCDAs encountered in this case are of the traditional Brodsky-Lepage type [12], rather different from those arising in $B$ exclusive decays, in which the $b$ quark field is defined in QCD rather than in HQET. Unfortunately, our current constraints on the heavy meson LCDAs are rather limited, which severely obstructs the predictive power of the standard collinear factorization. A further drawback of this formalism is that, these LCDAs are not genuinely nonperturbative, in which the hard scale $m_{b,c}$ is still entangled with the hadronic scale $\Lambda_{\text{QCD}}$.

Inspired by NRQCD factorization tailored for inclusive heavy quarkonium production [14], the heavy-quark recombination (HQR) mechanism [15] was developed in the beginning of this century, to supplement the single-parton fragmentation mechanism for the inclusive heavy-flavored hadron production. A notable success of this mechanism is to economically account for the charm/anticharm hadron rapidity asymmetry (leading particle effect) observed at numerous Fermilab fixed-target experiments [16–18]. The basic idea behind this mechanism is intuitively simple, after a hard scattering, the heavy quark has a significant chance to combine with a spectator quark which is soft in its rest frame to form a heavy-light hadron. In the color-singlet channel, where the inclusive and exclusive production of heavy hadrons practically make no difference, this formalism only involves a single nonperturbative factor, which is proportional to the first inverse moment of $B$ meson.
To our knowledge, the HQR mechanism so far is only illustrated at leading order (LO) in $\alpha_s$ next-to-leading order (NLO) in $\alpha_s$ valid order by order in the HQET factorization formalism for exclusive heavy-light hadron production, presumably nontrivial physics. This will take $W\text{NRQCD}$ factorization for heavy quarkonium production. To be specific, in this work we choose not to stick to the jargon HQR, rather we decide to term this factorization approach as the HQET factorization, in close analogy with the NRQCD factorization for heavy quarkonium production. To be specific, in this work we will take $W \rightarrow B(D_s) + \gamma$ as a prototype process for heavy meson exclusive production. To our knowledge, the HQR mechanism so far is only illustrated at leading order (LO) in $\alpha_s$. It is the very goal of this work to verify the validity of HQET factorization through next-to-leading order (NLO) in $\alpha_s$, which provides a much more informative revelation of nontrivial physics.

The rest of the paper is structured as follows. In Sec. II, we first express the $W^+ \rightarrow B^+ + \gamma$ amplitude in terms of two Lorentz-invariant form factors, and introduce some light-cone kinematical variables. In Sec. III, we recap some essential feature about the $B$ meson LCDA, which enters the factorization theorem in many $B$ exclusive decay processes. In Sec. IV, in analogy with NRQCD factorization for exclusive heavy quarkonium production, we propose the HQET factorization formalism for exclusive heavy quarkonium production, presumably valid order by order in $\alpha_s$. In Sec. V, we first determine the tree-level hard-scattering kernel for $W \rightarrow B + \gamma$. We then proceed to investigate the $O(\alpha_s)$ correction to this process. We explicitly verify that the resulting soft IR pole can be properly factorized into the $B$ meson LCDA, which establishes the correctness of HQET factorization in the first nontrivial order. The IR-finite hard-scattering kernel at $O(\alpha_s)$ is also deduced. In Sec. VI, we present comprehensive numerical predictions for the processes $W^+ \rightarrow B^+(D_s^+) + \gamma$. Assuming a simple exponential parametrization for $B$ meson LCDA, and including the evolution effects, we find that the $O(\alpha_s)$ would significantly reduce the LO decay rate. We summarize and present an outlook in Sec. VII.

II. DECOMPOSITION OF AMPLITUDE AND LIGHT-CONE KINEMATICS

Let us first specify the kinematics for the process $W^+ \rightarrow B^+ + \gamma$. The momenta of $W^+$, $B^+$ and $\gamma$ are represented by $Q$, $P$ and $q$, respectively, with $Q = P + q$. They satisfy the on-shell conditions: $Q^2 = m_V^2$, $P^2 = m_B^2$, and $q^2 = 0$, with $m_W$, $m_B$ standing for the masses of the $W$ and $B$ meson, respectively. It appears convenient to introduce a dimensionless four velocity $v^\mu$ via $P^\mu = m_B v^\mu$, obviously obeying $v^2 = 1$. The polarization vectors of the $W$ and $\gamma$ are denoted by $\varepsilon_W$ and $\varepsilon_\gamma$.

In accordance with Lorentz invariance, the decay amplitude for $W^+ \rightarrow B^+ \gamma$ can be decomposed as [11]

$$\mathcal{M}(W^+ \rightarrow B^+ \gamma) = \frac{e_u e V_{ub} f_B}{4\sqrt{2}\sin\theta_W} \left( \epsilon_{\mu\nu\alpha\beta} \frac{P^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^\beta}{P \cdot q} F_V + i \varepsilon_W \cdot \varepsilon_\gamma F_A \right),$$

where $e$ is the electric coupling constant, $e_u = -\frac{2}{3}$ is the electric charge of the $u$ quark, $\theta_W$ is the weak mixing angle, $V_{ub}$ denotes the CKM matrix element, and $f_B$ signifies the $B$ meson decay constant. $F_V(F_A)$ represent the Lorentz-invariant form factors, which are
affiliated with vacuum-to-$B + \gamma$ matrix element mediated by the vector (axial-vector) weak current. All the nontrivial QCD dynamics are encoded in these scalar form factors, which are functions of $m_W$, $m_B$, and $\Lambda_{\text{QCD}}$. Note this process bears two independent helicity amplitudes, which can be formed via the linear combination of these two form factors.

Squaring (1), averaging and summing over polarizations of $W$ and $\gamma$, one expresses the unpolarized decay rate in the $W$ rest frame as

$$\Gamma \left( W^+ \to B^+ \gamma \right) = \frac{e^2 \alpha}{192 \sin^3 \theta_W m_W^3} |V_{ub}|^2 (m_W^2 - m_B^2) (|F_V|^2 + |F_A|^2), \quad (2)$$

where $\alpha$ designates the QED fine structure constant.

To facilitate the discussion in next sections, it is necessary to set up the light-cone representation for the kinematics. We first introduce two light-like reference vectors $n_\pm \equiv \frac{1}{\sqrt{2}}(1, 0, 0, \mp 1)$, which obey $n_\pm^2 = 0$ and $n_+ \cdot n_- = 1$. In these light-cone basis, any four vector $a^\mu = (a^0, a^1, a^2, a^3)$ can then be decomposed into

$$a^\mu = (n_\pm \cdot a)n_\pm^\mu + (n_+ \cdot a)n_+^\mu + a_-^\mu \equiv a^+ n_+^\mu + a^- n_-^\mu + a_\perp^\mu, \quad (3)$$

where $a_\perp^\mu = (0, a^1, a^2, 0)$ is the transverse component of the four vector. The scalar product of two four vectors then become

$$a \cdot b = a^+ b^- + a^- b^+ + a_\perp \cdot b_\perp. \quad (4)$$

Were we interested in investigating the process $W \to B + \gamma$ within the standard QCD collinear factorization approach, it would be most natural to stay with the rest frame of the $W$ boson, where the $B$ meson is energetic owing to $m_W \gg m_B$. In this reference frame, we presume that the $B$ meson moves along the positive $\hat{z}$ axis, while the photon flies in the opposite direction. The light-cone representations for the momenta of the $B$ and photon then become

$$P^\mu |_{W \text{ rest frame}} = (P^+, P^-, P_\perp) = \frac{1}{\sqrt{2}} \left( m_W, m_B, 0, 0 \right), \quad (5a)$$

$$q^\mu |_{W \text{ rest frame}} = (q^+, q^-, q_\perp) = \frac{1}{\sqrt{2}} \left( 0, m_W^2 - m_B^2, 0, 0 \right). \quad (5b)$$

Since the form factors $F_{V,A}$ are Lorentz scalars, they can be computed in any reference frame. In order to make the HQET factorization picture more transparent as well as be closely connected with $B \to \gamma(W^* \to)\ell \nu$, it is most natural to boost this process to the $B$ meson rest frame. The corresponding momenta of $B$ and photon in the light-cone basis then read

$$P^\mu |_{B \text{ rest frame}} = (P^+, P^-, P_\perp) = \frac{1}{\sqrt{2}} \left( m_B, m_B, 0, 0 \right), \quad (6a)$$

$$q^\mu |_{B \text{ rest frame}} = (q^+, q^-, q_\perp) = \frac{1}{\sqrt{2}} \left( 0, m_W^2 - m_B^2, 0, 0 \right). \quad (6b)$$

Note the photon becomes enormously energetic in this frame, enhanced with respect its energy in the $W$ rest frame by a factor $m_W/m_B$. 

III. REVIEW OF B MESON LCDA DEFINED IN HQET

B meson LCDA is a crucial nonperturbative entity that ubiquitously enters the various exclusive B decay processes. It is the same entity that also enters the B exclusive production process in HQET factorization framework. In this section, we briefly recapitulate some of its essential features.

Let us consider the correlator composed of the light spectator quark and b quark separated by light-like distance, sandwiched between the vacuum and the B meson with velocity \(v\) (for simplicity, we actually work in the B meson rest frame). Its most general parametrization may be cast into the following form [19, 20]:

\[
\langle B(v)| \bar{u}_\beta(z)[z,0]h_{v,\alpha}(0)|0\rangle = i \hat{f}_B m_B \frac{1}{4} \left\{ \frac{2}{t} \left( \tilde{\phi}_B^-(t) - \tilde{\phi}_B^+(t) \right) + \frac{1 - \frac{\phi}{2}}{2} \gamma_5 \right\}_{\alpha\beta}, \tag{7}
\]

where \(z^2 = 0, t = v \cdot z, \) and \(\tilde{\phi}_B^\pm\) are nonperturbative functions of \(t.\) Here \(u\) refers to the standard QCD field for u quark, and \(h_v\) signifies the heavy quark field of the b quark introduced with velocity \(v\) in HQET. \(\alpha, \beta\) are spinor indices. \(\hat{f}_B\) signifies the B meson decay constant defined in HQET as

\[
\langle B(v)| \bar{u}(z)[z,0] \gamma^\mu \gamma_5 h_v(0)|0\rangle = i \hat{f}_B m_B v^\mu, \tag{8}
\]

which can be converted from the QCD decay constant \(f_B\) through perturbative series [21]:

\[
\hat{f}_B(\mu_F) = f_B \left[ 1 + \frac{\alpha_s C_F}{4\pi} \left( 3 \ln \frac{\mu_F}{m_b} + 2 \right) \right] + O(\alpha_s^2). \tag{9}
\]

The light-like gauge link,

\[
[z,0] = \mathcal{P} \exp \left[ -ig_s \int_0^z d\xi A^a_\mu(\xi) t^a \right], \tag{10}
\]

has been inserted in (7) to ensure gauge invariance of the nonlocal quark bilinear. Here \(t^a(a = 1, \cdots, 8)\) signify the \(SU(3)\) generators in fundamental representation, and \(\mathcal{P}\) indicates the path ordering.

The familiar B meson LCDAs are usually referred in momentum space, which can be inferred by Fourier transforming the coordinate-space correlator in (7) [15, 19]:

\[
\Phi_B^\pm(\omega) = i \hat{f}_B m_B \phi_B^\pm(\omega) = \frac{1}{v^\pm} \int \frac{dt}{2\pi} e^{i\omega t} \langle B(v)| \bar{u}(z)[z,0] \gamma_5 h_v(0)|0\rangle \bigg|_{z^+,z^-=0}, \tag{11}
\]

where a pair of B meson LCDAs are defined through

\[
\phi_B^\pm(\omega) = \int_0^\infty \frac{dt}{2\pi} e^{i\omega t} \tilde{\phi}_B^\pm(t). \tag{12}
\]

Here \(\omega\) indicates the plus momentum carried by the spectator quark in the B rest frame, whose typical value is thus \(~ \sim \Lambda_QCD.\) By construction, \(\phi_B^\pm(\omega)\) has nonvanishing support

\footnote{Note that we have intentionally put the B meson in the bra rather than the ket, since we are interested in B production rather than decay in this work.}
only when $0 < \omega < \infty$. General principle constrains that as $\omega \to 0$, $\phi_B^+(\omega) \propto \omega$, whereas $\phi_B^-(\omega) \propto 1$.

Note the light-cone correlator in (11) in general contain UV divergences, which calls for renormalization. As a consequence, $\phi_B^+(\omega) \text{ become scale-dependent quantities.}$ The evolution equation governing the leading-twist $B$ meson LCDA, $\phi_B^+(\omega, \mu_F)$, was first correctly written down by Lange and Neubert in 2003 [22]:

$$\frac{d}{d \ln \mu} \phi_B^+(\omega, \mu) = -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\omega' \left\{ \left( 4 \ln \frac{\mu}{\omega} - 2 \right) \delta (\omega - \omega') - 4 \omega \left[ \frac{\theta (\omega' - \omega)}{\omega' (\omega' - \omega)} + \frac{\theta (\omega - \omega')}{\omega (\omega - \omega')} \right] \right\} \phi_B^+(\omega', \mu),$$

(13)

with $\mu$ the renormalization scale. Note this evolution equation looks quite different from the celebrated Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation [23–25] for the meson LCDA defined in QCD.

Once the profile of $\phi_B^+(\omega)$ is determined at some initial scale (say, $\mu_0 = 1$ GeV), one can invoke (13) to any other scale (usually $1$ GeV $\leq \mu \leq m_b$). There exists some model-dependent studies on the profiles of the $B$ meson LCDAs using QCD sum rules [26]. The model-independent features of $\phi_B^+(\omega)$ are also extracted by applying operator product expansion (OPE) [27].

It turns out that for hard heavy hadron exclusive production, only $\phi_B^+(\omega)$ survives in the factorization theorem in the heavy quark limit. As was mentioned before, of central phenomenological relevance is the first inverse moment of $\phi_B^+(\omega)$, usually referred to as $\lambda_B^{-1}$:

$$\lambda_B^{-1}(\mu) \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu).$$

(14)

Intuitively, one expects $\lambda_B^{-1} \sim \Lambda_{QCD}^{-1}$. Note this inverse moment is also scale-dependent.

We also plan to study the NLO perturbative corrections to $W \to B + \gamma$. To our purpose, it is also necessary to introduce the first and second logarithmic inverse moments by

$$\lambda_B^{-1} \sigma_{B,n}(\mu) = -\int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\omega}{\mu} \phi_B^+(\omega, \mu), \quad n = 1, 2$$

(15)

which are scale dependent as well.

As a side remark, we finally note that the positive Mellin moments of $\phi_B^+(\omega)$, that is, $\int_0^\infty d\omega \omega^n \phi_B^+(\omega)$ ($N \geq 0$), are generally UV divergent and ill-defined. Usually one imposes a hard UV cutoff in the upper end of the integral to regularize the UV divergence [27].

IV. HQET FACTORIZATION FOR EXCLUSIVE HEAVY-LIGHT MESON PRODUCTION

In this section we shall state the exact form of HQET factorization for heavy-light meson exclusive production. Since this factorization picture naturally evolves from the previously developed heavy quark recombination mechanism, especially for the color-singlet channel, it might be beneficial to first elaborate on the underlying physical picture, taking the $W \to B + \gamma$ process as a concrete example. Viewed in the rest frame of the $B^+$ meson, the $\bar{b}$ quark after a hard scattering has a considerable chance to pick up the soft spectator $u$ quark.
to hadronize into the $B^+$ meson, with the recombination probability proportional to the square of the nonperturbative factor $\lambda_B^{-1}$ defined in (14). By emitting an energetic photon, a hard gluon exchange necessarily transforms the soft $u$ quark into a hard-collinear one, which endorses the usage of the $B$ meson LCDA.

Let $k^\mu$ signify the momentum carried by the spectator $u$ quark inside the $B^+$ meson. According to the recipe of the heavy quark recombination mechanism [20], one can obtain the $W \rightarrow B^+ + \gamma$ amplitude through making the following substitution in the quark amplitude $W \rightarrow [\bar{b}(P-k)u(k)]^{(1)} + \gamma$:

$$v_i(P-k)\bar{u}_j(k) \rightarrow \frac{\delta_{ij} i f_B m_b}{N_c} \cdot \left\{ \frac{1}{2} \left[ \phi_B^+(\omega) \frac{\not{\phi}_+}{\sqrt{2}} + \phi_B^-(\omega) \frac{\not{\phi}_-}{\sqrt{2}} - \omega \phi_B^*(\omega) \gamma_\perp \frac{\partial}{\partial k_\perp} \gamma_5 \right] \right\} \Bigg|_{k=\omega v},$$

where $i, j = 1, 2, \cdots, N_c$ are color indices and $N_c = 3$. The first Kronecker symbol is a color-singlet projector. After taking the momentum derivative on the quark amplitude, one then make the substitution $k \rightarrow \omega v$ and retain the most singular piece in the $\omega \rightarrow 0$ limit, which is usually $\propto 1/\omega$. Curiously, in the heavy quark limit the $\phi_B^*(\omega)$ turns out not to contribute, and only $\phi_B^+(\omega)$ yield nonvanishing contribution, whose effect is simply encoded in the first inverse moment $\lambda_B^{-1}$.

A shortcut can be invoked to quickly reproduce the heavy meson production amplitude from the HQR mechanism, with much less effort. Rather than consider $B^+$ meson production, one simply considers the $B_c$ production. Assume the momenta are partitioned by the two constitutes of the $B_c$ as $p_c = \kappa P$ and $p_b \approx (1 - \kappa) P$, where $P$ is the $B_c$ momentum and $\kappa = m_c / (m_c + m_b)$. One can then employ the familiar covariant spin projector for quarkonium production at LO in velocity expansion:

$$v_i(p_b)\bar{u}_j(p_c) \rightarrow \delta_{ij} \frac{f_{B_c}}{12} \left( \not{p} - m_{B_c} \right) \gamma_5.$$  

Consequently, the amplitude for $B^+$ production through the $[\bar{b}u]^{(1)}(1\bar{s}^0_0)$ channel can be deduced by taking the $\kappa \rightarrow 0$ limit of that for $B_c$ production and replacing $f_{B_c} / (4\kappa) \rightarrow \frac{f_B}{4\gamma_B}$. In general, many Feynman diagrams do not contribute for their lack of $1/\kappa$ singularity. Notice that, this shortcut has been utilized to ascertain the NLO perturbative behaviors of the $B$ electromagnetic form factor and $W \rightarrow B + \gamma$ from their $B_c$ counterparts in NRQCD factorization [29, 30].

Thus far, the HQR mechanism has not been extended to the NLO in $\alpha_s$ yet. It is not straightforward to achieve this goal from the projector approach in (17), or from the shortcut of extracting the hard-scattering kernel from $B_c$ production, as described in (17). The main reason is that $\phi_B^*$ develops UV divergence at NLO in $\alpha_s$, which bears very different renormalization pattern from that for the local composite NRQCD matrix elements. It is not a priori clear why the aforementioned projector approach can lead to the IR-finite hard-scattering kernel once beyond $\alpha_s$.

Since we treat $m_W$ as the same order as $m_b$, the argument that leads to the factorization theorem for $B \rightarrow \gamma l\nu$ can be transplanted to $W \rightarrow B + \gamma$ without modification. We thus

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2 This form may look superficially different from the projectors adopted in [15]. Nevertheless, once the equation of motion and Wandzura-Wilczek approximation are invoked, they can be proven to be equivalent [20].
propose a factorization theorem of $W \rightarrow B + \gamma$, which is valid in the heavy quark limit, albeit to all orders in $\alpha_s$:

$$M(W^+ \rightarrow B^+ \gamma) = \hat{f}_B(\mu_F) \int_0^\infty d\omega T(\omega, \mu_F) \phi_B^+(\omega, \mu_F) + O(m_b^{-1}), \quad (18)$$

where $T(\omega)$ is referred to as the hard-scattering kernel, which is perturbatively calculable. Note only the leading-twist $B$ meson LCDA, $\phi_B^+(\omega)$, explicitly enters the formula. And $\mu_F$ is introduced as an arbitrary factorization scale. The dependence of the hard-scattering kernel should counteract that of the $\phi_B^+(\omega)$ so that the physical amplitude becomes insensitive to the artificial scale $\mu_F$.

Equation (18) lays down the foundation for the HQET factorization. In some sense, (18) offers a systematic realization of HQR mechanism. The virtue of this factorization framework is to allow us to systematically investigate the higher-order perturbative corrections for the hard-scattering kernel.

V. FORM FACTORS FOR $W \rightarrow B + \gamma$ THROUGH NLO IN $\alpha_s$

This section reports our most important results, where the hard-scattering kernel is computed through NLO in $\alpha_s$. Rather than employ the projector approach given in (17), we choose to employ the perturbative matching method to determine the hard-scattering kernel. The calculation presented in this section closely follows the analogous NLO calculation for $B \rightarrow \gamma l\nu$ [3]. Unless otherwise specified, the calculation of the form factors $F_{A,V}$ is performed in the rest frame of the $B$ meson.

The hard-scattering kernel is insensitive to the long-distance physics. For the sake of computing it in perturbation theory, it is legitimate to replace the physical $B^+$ meson in (18) with a fictitious $B$ meson composed of a pair of free quarks $[\bar{b}(P-k)u(k)]$. One can project the intended piece from the quark amplitude as follows:

$$v_i(P-k)\bar{u}_j(k) \rightarrow \frac{\delta_{ij}}{N_c} \frac{1 - \not\!v}{4} \gamma_5,$$  

just similar to the projector (17) which guarantees the fictitious “meson” being the color and spin singlet. The momentum of the spectator $u$ quark is assumed to be soft, i.e., which scales as $k^\mu \sim \Lambda_{QCD}$. For such a fictitious “meson”, the LCDA in (11) can be defined as

$$\Phi_{[bu]}^\pm(\omega) = \frac{1}{v^\pm} \int \frac{dt}{2\pi} e^{i\omega t} \langle [\bar{b}u](P)|\bar{u}(z)|z,0|\not\!v^\pm \gamma_5 h(u)(0)|0\rangle \bigg|_{z^+ = z^- = 0}, \quad (20)$$

which also also be computed perturbatively:

$$\Phi_{[bu]}^+ = \Phi_{[bu]}^{+(0)} + \Phi_{[bu]}^{+(1)} + O(\alpha_s^2), \quad T = T^{(0)} + T^{(1)} + O(\alpha_s^2), \quad (21)$$

with the superscript indicating the powers of $\alpha_s$. At LO, the LCDA for the fictitious $B$ meson (20) looks extremely simple

$$\Phi_{[bu]}^{+(0)}(\omega) = \frac{1}{v^\pm} \delta \left( \frac{k^+}{v^\pm} - \omega \right) \text{Tr} \left[ \frac{1 - \not\!v}{4} \gamma_5 \not\!k^\pm \gamma_5 \right] = \delta \left( \frac{k^+}{v^\pm} - \omega \right). \quad (22)$$
FIG. 1: The Feynman diagrams for $W^+ \rightarrow [\bar{b}u] + \gamma$ at tree level. The bold line represents the $\bar{b}$ quark.

As a consequence, the perturbative amplitude in (18), through NLO in $\alpha_s$ reads

$$M = M^{(0)} + M^{(1)} + \mathcal{O}(\alpha_s^2),$$

where

$$M^{(0)} = \Phi_{[\bar{b}u]}^{+(0)} \otimes T^{(0)},$$

$$M^{(1)} = \Phi_{[\bar{b}u]}^{+(0)} \otimes T^{(1)} + \Phi_{[\bar{b}u]}^{+(1)} \otimes T^{(0)},$$

with $\otimes$ signifying the convolution integral in $\omega$. Since both $M$ and $\Phi_{[\bar{b}u]}^{+}$ are perturbatively calculable for such a fictitious state, one can solve (24) for $T$ order by order in $\alpha_s$.

A. Tree level

As depicted in Fig. 1, there arise three electroweak diagrams at lowest order that contribute to the quark-level process $W \rightarrow [\bar{b}(P - k)u(k)]^{(1)} + \gamma$. Recall the momentum of the outgoing photon in (6b) scales as $(q^+, q^-, |q_\perp|) \sim (0, m_b, 0)$. The $u$ propagator in Fig. 1a) becomes hard-collinear, and contribute a $1/q \cdot k \sim 1/k^+ q^-$ singularity to the amplitude. One can readily convince himself that the other two diagrams, the one with photon emitted from $\bar{b}$ (Fig. 1b) and the one involving $WW \gamma$ vertex (Fig. 1c) do not possess this $1/k^+ q^-$ singularity, thus can be safely dropped.

Therefore, Fig. 1a) corresponds to the following tree-level quark amplitude in the heavy quark limit:

$$M^{(0)}(W^+ \rightarrow [\bar{b}u] + \gamma) = \frac{eV_{ub}}{2\sqrt{2}\sin\theta_W} \langle [\bar{b}u](P)\gamma(q, \epsilon^*_\gamma)\mid \overline{\not{u}}_W(1 - \gamma_5)b\mid 0 \rangle$$

$$\approx \frac{e_u e V_{ub}}{4\sqrt{2}\sin\theta_W q^- k^+} \text{Tr} \left[ \frac{1 - \frac{\not{q}}{4}}{4} \gamma_5 \gamma^*_\gamma \not{k} \not{W}(1 - \gamma_5) \right]$$

$$= \frac{e_u e V_{ub}}{4\sqrt{2}\sin\theta_W} \left( -i \epsilon_{\mu\alpha\beta\gamma} P^\mu q^\nu \varepsilon^*_\alpha W \varepsilon^*_\beta \gamma^*_\gamma \right) \int_0^\infty \frac{d\omega}{\omega} \left( k^+/v^+ - \omega \right).$$

It is then straightforward to solve for the tree-level hard-scattering kernel:

$$T^{(0)}(\omega) = \frac{e_u e V_{ub}}{4\sqrt{2}\sin\theta_W} \left( -i \epsilon_{\mu\alpha\beta\gamma} \frac{P^\mu q^\nu \varepsilon^*_\alpha W \varepsilon^*_\beta \gamma^*_\gamma}{P \cdot q} \right) \frac{1}{\omega}. $$

9
Comparing with the Lorentz decomposition specified in (1), one can deduce the final expressions for the vector/axial-vector form factors at tree level:

\[ F_V^{(0)} = F_A^{(0)} = \frac{\hat{f}_B m_B}{4} \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega) = \frac{\hat{f}_B m_B}{4\lambda_B}. \]  

(27)

We stress that the \( \Phi_{[\bar{b}u]}^-(\omega) \) indeed does not enter the factorization formula. With the aid of (16), inspecting the spinor structure of (25), one can prove the \( \Phi_{[\bar{b}u]}^- \) -dependent terms vanish due to some specific identities such as \( n^2 = 0 \) and \( \gamma_{\perp}^\mu \hat{x}^\gamma \gamma_{\perp} = (4 - d)\hat{x}^\gamma \), with \( d = 4 \) signifying the spacetime dimension.

**B. One-loop level**

In this subsection, we proceed to extract the hard-scattering kernel through order-\( \alpha_s \). Following the ansatz given in (24b), the one-loop hard-scattering kernel \( T^{(1)} \) can be extracted via

\[ \Phi^{(0)} \otimes T^{(1)} = \mathcal{M}^{(1)} - \Phi^{(1)} \otimes T^{(0)}, \]  

(28)

The one-loop diagrams for \( \Phi^{(1)} \) and \( \mathcal{M}^{(1)} \) have been depicted in Fig. 2, Fig. 3, respectively.

By the general principle of effective field theory, both entities in the right-hand side of (28) must possess the identical IR divergences, so that upon subtraction \( T^{(1)} \) must be infrared finite. It appears instructive to compute the difference in (28) on a diagram-by-diagram basis.
FIG. 3: One-loop QCD correction to the amplitude for $W^+ \to [\bar{b}u]^{(1)} + \gamma$. We just retain those diagrams with photon emitted from the spectator $u$ quark, which yield leading contribution in the heavy quark limit.

We adopt the Feynman gauge for simplicity. The Feynman rules for einkonal vertex and propagator are given by $-ig_s T^a n^\mu_+$ and $1/p^+$, respectively with $p$ denoting the momentum flowing on the gauge link [28].

We also employ dimensional regularization (with spacetime dimensions $d = 4 - 2\epsilon$) to regularize UV divergences. We also treat $\gamma_5$ in the naive dimensional regularization scheme in which $\gamma_5$ is anti-commuting with $\gamma^\mu$ ($\mu = 0, 1, ..., d - 1$). We affiliate the 't Hooft unit mass $\mu_R$ when calculating $\mathcal{M}^{(1)}$. We also affiliate the unit mass $\mu_F$ when computing $\Phi^{(1)}$ in (28). Moreover, we also add a nonzero mass $m_u$ for the spectator quark to regularize the mass singularity.

The one-loop corrections to the LCDA, as indicated in Fig. 2a), b) and c) can be extracted from the soft loop region of the electromagnetic vertex correction, weak vertex correction, and light quark propagator correction of the NLO amplitude in Fig. 3. Their contributions are

\begin{align}
\Phi_+^{(1)} & \otimes \mathcal{T}^{(0)} = \frac{\alpha_s C_F}{4\pi} \left( \frac{2}{\epsilon} - 4 \ln \frac{m_u}{\mu_F} + 4 \right) \mathcal{M}^{(0)}, \\
\Phi_{+\text{wk}}^{(1)} & \otimes \mathcal{T}^{(0)} = \frac{\alpha_s C_F}{4\pi} \left( \frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{k^+}{\upsilon^+ \mu_F} + 2 \ln^2 \frac{k^+}{\upsilon^+ \mu_F} + \frac{3\pi^2}{4} \right) \mathcal{M}^{(0)}, \\
\Phi_{+\text{Z}_u}^{(1)} & \otimes \mathcal{T}^{(0)} = \frac{1}{2} \delta Z_u(\mu_F) \mathcal{M}^{(0)},
\end{align}

(29a)

(29b)

(29c)
where $\delta Z_u$ is the standard one-loop quark wave function renormalization constant in QCD

$$
\delta Z_u(\mu) = \frac{\alpha_s C_F}{4\pi} \left( \frac{-3}{\epsilon} + 6 \ln \frac{m_u}{\mu} - 4 \right),
$$

(30)

We choose to renormalize $\Phi^{(1)}_+$ under the $\overline{\text{MS}}$ scheme [3].

The box diagram Fig. 2c) may yield non-vanishing contribution to $\Phi^{(1)}_+ \otimes T^{(0)}$. However, it turns out that $M^{(1)}_{\text{box}} = \Phi^{(1)}_+ \otimes T^{(0)}$. For the sake of matching, it thus leads to a vanishing $T^{(1)}_{\text{box}}(\omega)$ in extraction of the hard-scattering kernel. To be effective, we can simply discard the box diagram from the very beginning.

The gauge link self-energy diagram Fig. 2d) does not contribute to $\Phi^{(1)}_+ \otimes T^{(0)}$ because its contribution is proportional to $n_2^2 = 0$. The external leg correction in Fig. 2f) also does not contribute to $\Phi^{(1)}_+ \otimes T^{(0)}$, since the $b$ quark wave function renormalization constant in HQET is a scaleless integral:

$$
\delta \hat{Z}_b = \frac{\alpha_s C_F}{4\pi} \left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right),
$$

(31)

which vanishes if we do not distinguish the ultraviolet pole $1/\epsilon_{\text{UV}}$ and $1/\epsilon_{\text{IR}}$ in DR.

To extract $T^{(1)}(\omega)$ by following Eq. (28), we need also calculate $M^{(1)}$ which involves the one-loop diagrams depicted in Fig. 3. At leading power in $1/m_b$, we are only interested in maintaining the contribution to $M^{(1)}$ of order $\mathcal{O}(\Lambda_{\text{QCD}}^{-1})$. Just for this reason, it is legitimate to exclude those diagrams in which the photon is emitted off the $\bar{b}$ quark.

It is straightforward to calculate the electromagnetic vertex correction, weak vertex correction and internal quark self-energy QCD diagrams, which have been depicted in Fig. 3a), b) and d), respectively. In calculating the one-loop amplitude, we have implicitly included counterterms for the quark mass and wave-function, in order to obtain UV-finite results. All three diagrams possess the same Lorentz structure as $M^{(0)}$, hence the corresponding contributions to $T^{(1)}(\omega)$ can be readily extracted by subtracting the respective contributions in Fig. 2. We obtain

$$
T^{(1)}_{\text{em}}(\omega) = \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left( \ln \frac{2q^{-v} \omega}{\mu_F^2} + 2 \ln \frac{\mu_R}{\mu_F} - 4 + i\pi \right),
$$

(32a)

$$
T^{(1)}_{\Sigma}(\omega) = \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left( \ln \frac{2q^{-v} \omega}{\mu_F^2} - 1 + i\pi \right),
$$

(32b)

$$
T^{(1)}_{\text{wk}}(\omega) = \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left\{ \ln \frac{2q^{-v} \omega}{(1-r) m_W^2} - \ln \frac{2(1-r) m_W^2 q^{-v} \omega}{\mu_F^4} - 2 \right\}
+ 2 \left( \ln^2 \frac{m_b}{\mu_F} - \ln \frac{m_b}{\mu_R} \right) + 2\text{Li}_2(r) + \ln^2 (1-r) + [r + 2 \ln (1-r) - 1] \ln \frac{r}{1-r}
+ \frac{\pi^2}{12} + i\pi \left( r + 2 \ln \frac{2q^{-v} \omega}{m_W^2} - 1 \right) \right\}.
$$

(32c)

For convenience, we have introduced a dimensionless ratio:

$$
r \equiv \frac{m_b^2}{m_W^2}.
$$

(33)
The wave function renormalization diagram on \( u \)-quark contributes to \( T^{(1)} \) reads
\[
\Phi^{(0)}_+(\omega) \otimes T^{(1)}_{\delta Z_u}(\omega) = \mathcal{M}^{(1)}_{\delta Z_u} - T^{(0)}(\omega) \otimes \Phi^{(1)}_{+\delta Z_u}(\omega).
\] (34)

Substituting in \( \mathcal{M}^{(1)}_{\delta Z_u} = \frac{1}{2} \delta Z_u(\mu_R)\mathcal{M}^{(0)}_{\delta Z_u} \) and \( \Phi^{(1)}_{+\delta Z_u} = \frac{1}{2} \delta Z_u(\mu_F)\Phi^{(0)}_+ \), it renders
\[
T^{(1)}_{\delta Z_u}(\omega) = \frac{1}{2} [\delta Z_u(\mu_R) - \delta Z_u(\mu_F)] T^{(0)}(\omega) = \frac{\alpha_s C_F}{4\pi} \ln \frac{\mu_F}{\mu_R} T^{(0)}(\omega).
\] (35)

Similarly, the wave function renormalization diagram on \( \bar{b} \)-quark contributes to \( T^{(1)} \) as
\[
T^{(1)}_{\delta Z_b}(\omega) = \frac{1}{2} \left[ \delta Z_b(\mu_R) - \delta \bar{Z}_b(\mu_F) \right] T^{(0)}(\omega) = \frac{\alpha_s C_F}{4\pi} \left( 2 \ln \frac{m_b}{\mu_F} + \ln \frac{m_b}{\mu_R} - 2 \right) T^{(0)}(\omega),
\] (36)
where the \( b \)-quark wave function renormalization constant \( \delta Z_b \) in QCD is
\[
\delta Z_b(\mu) = -\frac{\alpha_s C_F}{4\pi} \left( \frac{3}{\epsilon} - 6 \ln \frac{m_b}{\mu} + 4 \right).
\] (37)

In the leading power of \( 1/m_b \), the hard-scattering kernel from the box diagram in Fig. 3c) makes vanishing contribution. Because the loop momentum generates the leading contribution to \( \mathcal{M}^{(1)}_{\text{box}} \) only in the soft region \( (l^\mu \sim \mathcal{O}(\Lambda_{\text{QCD}})) \), which is already captured by \( \Phi^{(1)}_{\text{box}} \otimes T^{(0)} \). Therefore there arises no contribution of \( T^{(1)}_{\text{box}} \), which shares the same pattern as what is found in \( B \rightarrow \gamma l\nu \) [3].

Piecing the relevant one-loop contributions together, we can deduce the complete hard-scattering kernel at \( \mathcal{O}(\alpha_s) \):
\[
T^{(1)}(\omega) = \frac{\alpha_s C_F}{4\pi} \left\{ \ln^2 \frac{2q^- v^+ \omega}{\mu_F^2} - 2 \ln \frac{m_b}{\mu_F} + \left( 5 - 4 \ln \frac{1 - r}{r} \right) \ln \frac{m_b}{\mu_F} + 2 \text{Li}_2(r) + \ln^2 r - \left( 2 \ln \frac{1 - r}{r} - 3 + r \right) \ln \frac{1 - r}{r} + \frac{\pi^2}{12} - 7 + i\pi \left[ 2 \ln \frac{2q^- v^+ \omega}{\mu_F^2} - 4 \ln \frac{m_b}{\mu_F} - r - 4 \ln (1 - r) + 2 \ln r + 3 \right] \right\} T^{(0)}(\omega).
\] (38)

It should be noticed that the renormalization scale \( \mu_R \) dependence naturally disappears and only the factorization scale \( \mu_F \) survives.

Since \( \mathcal{M}^{(1)} \) has the same spinor structure as \( \mathcal{M}^{(0)} \), it is clear to show that only the leading twist LCDA \( \phi_B^{(1)} \) enters into the factorization of \( W^+ \rightarrow B^+ \gamma \).

With the NLO hard scattering kernel at hand, we are able to calculate the NLO corrections to the form factors \( F_{V/A}^{(1)} \)
\[
F_{V/A}^{(1)} = \frac{\hat{f}_B m_B}{4} \int_0^\infty \frac{d\omega}{\omega} \frac{T^{(1)}(\omega)}{T^{(0)}(\omega)} \phi_B^{(1)}(\omega)
= \frac{f_B m_B \alpha_s C_F}{4\lambda_B} \frac{\alpha_s C_F}{4\pi} \left\{ - \ln \frac{m_b}{\mu_F} \left( \ln \frac{m_b}{\mu_F} + 2 \ln \frac{1 - r}{r} - 2 \right) + 2 \text{Li}_2(r) - \ln^2 (1 - r) + 2 \ln r \ln (1 - r) + (3 - r) \ln \frac{1 - r}{r} + \frac{\pi^2}{12} - 5 \right\} - \sigma_{B,2} + i\pi \left[ 2 \ln \frac{m_b}{\mu_F} - 3 + r + 2 \ln (1 - r) + 2\sigma_{B,1} \right],
\] (39)
in which the first-inverse moment $\lambda_B^{-1}$ and the $n$-th logarithmic moment $\sigma_n$ of LCDA have been defined in (14) and (15). In the final expression, we have also utilized (9) to trade $\hat{f}_B$ in favor of the QCD decay constant $f_B$.

Equation (39) constitutes the most important result of this work. In passing, we note that the process $W^+ \to B^+ \gamma \gamma$ has also recently been studied through $\mathcal{O}(\alpha_s)$ within NRQCD factorization framework [30]. The NRQCD short-distance coefficient for such a case contains three distinct scales: $m_W$, $m_b$ and $m_c$. Ref. [30] also considers the so-called “heavy quark limit”, e.g., $m_W \sim m_b \gg m_c$. It is curious to point out that, the expanded NRQCD short-distance coefficient in such a limit very much resembles (39), especially for the logarithms.

VI. NUMERICAL RESULTS

In this section, we carry out numerical predictions for vector/axial-vector form factors related to $W$ radiative decay into $B^+$ and $D_s^+$ mesons, as well as predict the corresponding partial widths and branching fractions for $W^+ \to B^+ (D_s^+) \gamma$. The impact of NLO perturbative corrections are also investigated.

We choose the following input values [39]:

$$\sin \theta_W = 0.481, \quad \alpha_s (m_W) = 0.12, \quad \alpha (m_W) = 7.8 \times 10^{-3}, \quad m_W = 80.379 \text{ GeV}$$
$$f_B = 0.187 \text{ GeV}, \quad |V_{ub}| = 3.65 \times 10^{-3}, \quad m_b = 4.6 \text{ GeV}, \quad m_B = 5.279 \text{ GeV}$$
$$f_D = 0.249 \text{ GeV}, \quad |V_{cs}| = 0.997, \quad m_c = 1.4 \text{ GeV}, \quad m_D = 1.968 \text{ GeV}$$

For the LCDAs of the $B^+$ and $D_s^+$ mesons defined at the initial scale $\mu_{F0} = 1\text{GeV}$, we employ the exponential ansatz first introduced by Grozin and Neubert [19]:

$$\phi_{M}^{+}(\omega) = \frac{\omega}{\lambda_{M}^{2}} \exp \left(-\frac{\omega}{\lambda_{M}}\right), \quad (40)$$

with $\lambda_B = 0.360$ GeV [40] and $\lambda_{D_s} = 0.294$ GeV [42].

![FIG. 4: Profiles of the LCDAs $\phi_B^{+}(\omega, \mu_F)$ with some typical values for the factorization scale.](image)

The analytic solutions of the Lange-Neubert evolution equation in (13) have been recently available [27, 38]. Nevertheless in this work, we are content with numerically solving the
evolution equation via the 4th order Runge-Kutta method, with the help of the package GNU Scientific Library \[41\]. The profiles of the LCDAs for \(B\) and \(D_s\), which are evaluated in several characteristic scales, are depicted in Fig. 4.

\[
\lambda_B^{1} \text{ and } -\lambda_B^{-1}\sigma_{B,\mu} \text{ [GeV}^{-1}\text{]}
\]

\[
\lambda_D^{1} \text{ and } -\lambda_D^{-1}\sigma_{D,\mu} \text{ [GeV}^{-1}\text{]}
\]

FIG. 5: Scale dependence of the moments \(\lambda^{-1}_M\) and \(\sigma_{M,1/2}\lambda^{-1}_M\) for \(M = B^+, D^+_s\). The renormalization scale ranges from 1 GeV to twice meson mass.

\[
F_{V/A}^B
\]

\[
F_{V/A}^D
\]

FIG. 6: Factorization scale dependence of the vector/axial-vector form factors \(F_{V/A}\) at LO and NLO in \(\alpha_s\), for \(B\) and \(D_s\), respectively. The renormalization scale ranges from 1 GeV to twice meson mass.

In Fig. 5, we see the scale dependence of the first inverse moment and logarithmic inverse moments for both \(B\) and \(D_s\) mesons. In Fig. 6, we further plot the scale independence of the form factors \(F_{V/A}\) through \(O(\alpha_s)\). Clearly the NLO perturbative correction generates significant negative correction. After incorporating the NLO QCD correction, the scale dependence of the form factors associated with \(W \to B + \gamma\) gets significantly reduced.

The total width of the \(W\) boson is \(\Gamma_{W^+} = 2.085\pm0.042\text{GeV}[39]\). Concretely speaking, the LO predictions for the partial widths of \(W\) radiative decays into \(B\) and \(D_s\) are \(\Gamma_{W \to B^+\gamma}^{(0)} = \)
1.2 \times 10^{-11} \text{ GeV} and \Gamma_{W^+ \rightarrow D_s^+ \gamma}^{(0)} = 8.4 \times 10^{-8} \text{ GeV}. Such a striking hierarchy is mainly due to |V_{cs}| \gg |V_{ub}|. Note that our LO predictions in HQET factorization are somewhat bigger than what are estimated using standard collinear factorization [11].

After including the NLO perturbative correction, \Gamma_{W^+ \rightarrow B \gamma}^{\text{NLO}} reduces to 5 \times 10^{-12}\text{ GeV}. The \mathcal{O}(\alpha_s) correction turns out to be sizable, reduces the LO prediction by 60%. Recall that the \mathcal{O}(\alpha_s) correction for \text{W} \rightarrow \text{B} + \gamma also considerably reduces the LO partial width [30]. As for \text{W} \rightarrow \text{D}_s + \gamma, one finds that \Gamma_{W^+ \rightarrow D_s^+ \gamma}^{\text{NLO}} reduces to 6.2 \times 10^{-8} \text{ GeV}. Note both our predictions at NLO accuracy get reasonably compatible with [11].

Our state-of-the-art prediction for the branching fraction of \text{W}^+ \rightarrow \text{B}^+ \gamma is about 2 \times 10^{-12}. It is unlikely to observe such an extremely rare decay channel in the foreseeable future, even when LHC accumulates 3000 fb^{-1} luminosity. On the other hand, the branching fraction of \text{W}^+ \rightarrow \text{D}_s^+ \gamma is about 3 \times 10^{-8}, which may have bright observation prospects in the future LHC experiments.

VII. SUMMARY AND OUTLOOK

In this work, we have studied the hard exclusive production of heavy-light meson in HQET factorization framework. For concreteness, we have taken the \text{W} \rightarrow \text{B} (\text{D}_s) + \gamma processes as specific examples. By examining that the NLO hard-scattering kernel is indeed free from IR divergence, we have explicitly verified that the factorization formula in (18) for heavy meson exclusive production does hold once beyond LO in \alpha_s. We calculated NLO perturbative corrections to both vector/axial-vector form factors \text{F}_{V/A}. It is curious to find that, in the heavy quark limit, \text{F}_V and \text{F}_A are identical through NLO in \alpha_s. The radiative corrections turn to be substantial, which may reach \sim -60\% and \sim -40\% of the LO partial widths for \text{W} \rightarrow \text{B} + \gamma and \text{W} \rightarrow \text{D}_s + \gamma, respectively. After incorporating these corrections, the corresponding branching fractions for these two channels are estimated to be about \mathcal{O}(10^{-12}) and \mathcal{O}(10^{-8}). The future LHC experiment may have some chance to observe the latter channel.

One should be cautious that the hard-scattering kernel in our factorization formula usually contain large logarithms of the ratio of \text{M}_W to \text{m}_b, which may potentially ruin the convergence of perturbative expansion. However, this nuisance is not specific to HQET factorization. In hard exclusive quarkonium production, NRQCD short-distance coefficients are also plagued with large logarithms. It was advocated that by invoking the concept of refactorization of NRQCD short-distance coefficients and employing ERBL evolution equation [23–25], one would be able to tame these large logarithms and obtain much more optimized predictions than from NRQCD solely [32]. The similar idea of refactorizing the hard-scattering kernel for heavy-flavor hadron production in HQET factorization with the aid of standard QCD light-cone approach, is definitely worth further exploration.

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