PCA algorithm for detection, localisation and evolution of damages in gearbox bearings

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Abstract. A fundamental aspect when dealing with rolling element bearings, which often represent a key component in rotating machineries, consists in correctly identifying a degraded behaviour of a bearing with a reasonable level of confidence. This is one of the main requirements a Health and Usage Monitoring System (HUMS) should have. This paper introduces a monitoring technique for the diagnosis of bearing faults based on Principal Component Analysis (PCA). This method overcomes the problem of acquiring data under different environmental conditions (hardly biasing the data) and allows accurate damage recognition, also assuring a rather low number of False Alarms (FA). In addition, a novel criterion is proposed in order to isolate the area in which the faulty bearing stands. Another useful feature of this PCA-based method concerns the capability to observe an increasing trend in the evolution of bearing degradation. The described technique is tested on an industrial rig (designed by Avio S.p.A.), consisting of a full size aeroengine gearbox. Healthy and variously damaged bearings, such as with an inner or rolling element fault, are set up and vibration signals are collected and processed in order to properly detect a fault. Finally, data collected from a test rig assembled by the Dynamics & Identification Research Group (DIRG) are used to demonstrate that the proposed method is able to correctly detect and to classify different levels of the same type of fault and also to localise it.

1. Introduction
Rolling bearings are certainly among the most widely used components in machines. Thus, their condition monitoring and fault diagnosis are very important aspects to treat in order to prevent the occurrence of breakdowns. Since ‘70 a great range of different methods has been proposed to perform diagnosis, fault identification and classification of bearing faults, considering both raw and properly filtered data. A fundamental aspect related to this topic consists in correctly identifying a degraded behaviour of a bearing with a reasonable level of confidence. This is one of the main requirements a Health and Usage Monitoring System (HUMS) should have.
As explained in [1], one of the first adopted techniques is the analysis of the envelope signal, thanks to its property of giving more diagnostic information about bearing faults than the analysis of the raw signals itself. In fact, the frequency analysis is shifted from the very high range of resonant carrier frequencies to the much lower range of the fault frequencies, so that they could be analysed with good resolution. Envelope analysis allows to bandpass filter the signal in a higher frequency band in which the fault impulses are amplified by structural resonances.
Then, a proper amplitude demodulation is done to form the envelope signal, whose spectrum contains the desired diagnostic information, as, for example, ballpass or ballspin frequency. In their paper [2], Gao et al. propose instead an approach based on empirical mode decomposition (EMD) to detect rotating machine faults. This procedure is a time-frequency analysing method for non-linear and non-stationary signals that allows to decompose a complicated signal into a number of intrinsic mode functions (IMFs) based on the local characteristic time scale of the signal. The IMFs, working as the basis functions, represent the intrinsic oscillation modes embedded in the signal. Since IMFs sometimes fail to reveal the signal characteristics because of the effect of noise, combined mode function (CMF) overwhelms this problem. In this paper, a practical fault signal of a power generator from a thermal-electric plant is analysed by using both EMD and CMF. The results show that the rotating machine fault characteristics are properly extracted and the fault patterns are effectively identified.

An alternative method to signal processing is proposed by Antoniadis and Glossiotis in [3]. Here, they study bearing vibration signals according to the cyclostationary analysis. This framework can reveal better the underlying physical concepts of the modulation mechanism present in the vibration response of bearings. In fact, the so called degree of cyclostationarity (DCS) function can provide a first overall indication of the appearance of several distinct modulating frequencies. Moreover, further details related to the exact form of various modulation mechanisms can be derived by the spectral correlation density function (SCDF) of the signal. This function presents more content than the traditional power spectral density. In their article, they are able to classify different bearing faults thanks to “typical frequency patterns” derived from the DCS and SCDF functions. This is done through both simulated and measured signals.

Another interesting method that can be efficiently used in the vibration-based condition monitoring of rotating machines is that presented by Antoni and Randall in [4]. It is called Spectral Kurtosis (SK) and, in contrast to classical kurtosis analysis, it provides a robust way of detecting incipient faults even in the presence of strong masking noise. Moreover, it allows to design optimal filters efficiently able to filter out the mechanical signature of faults. Thus, this method is a concrete mean both in monitoring, for the first aspect, and in diagnostics, for the second one. In this paper they propose the concept of kurtogram too, a useful tool that creates optimal band-pass filters. Another innovation consists in the fact that when no historical data from the analysed machine are available, the SK offers the rather unique opportunity to find out which frequency band should be processed. All this aspects are treated and verified through actual industrial cases.

A useful tool to analyse non-stationary signals such as those related to bearing vibrations is wavelet transform. Its strength comes from the simultaneous interpretation of the signal in both time and frequency domain so that local, transient or intermittent components are exposed. In [5] Chebil et al. present a wavelet-based analysis technique for the diagnosis of faults in rotating machinery from its vibrating signature. They found that the peak locations in spectrum of the vibration signal could also be efficiently used in the detection of a fault in ball bearings. Moreover, the root mean square extracted from the terminal nodes of a wavelet tree can be reliably used as characterising feature able to localise the damage and to suppose its size.

In the paper by Worden et al.[6] a basic theory of some of the most common methods of natural computing is presented together with some applications in the context of mechanical systems research. Among them, Principal Component Analysis is considered a linear transformation, optimal for compression and decomposition. PCA has been applied in many mechanical contests with so many different tasks. In his work [7] Benaicha et al. propose a new methodology for sensor fault detection and localisation using principal component analysis. This consists in an index created to detect simple and multiple damages affecting the dependent and independent process variables. Moreover, they present a new iterative selection method of principal component number that determines a model allowing the detection of faults without an
a priori knowledge of their natures. He et al. in [8] find the low-dimensional principal component representations from the statistical features of the measured signals to characterise and, hence, monitor machine conditions. It is done using the principal component analysis (PCA) technique from the time- and frequency-domains statistical features of the measured signals. Shuang and Meng use, instead, a combination of principal components analysis (PCA) and support vector machine to analyse the features of vibrating signal of rolling bearing, concentrating on features extraction and classification [9]. Thanks to some experiments they show that the recognition of fault diagnosis in rolling bearing based on these two theories is available.

PCA technique is also very useful to eliminate environmental effects in structural health monitoring. Yan et al. [10] present a method applied to vibration features identified during the monitoring of the structure under varying environmental and operational conditions. They underline the advantage of the method, consisting in the fact that it does not require to measure environmental parameters because they are taken into account as embedded variables. Similarly, Bellino et al. [11] apply PCA to time-varying systems in order to eliminate the effects of external factors, such as temperature and humidity, supposed affecting the natural frequencies of a structure in addiction to the damage.

The intent of our work is to combine PCA damage detecting ability to its success in removing environmental conditions influencing vibration features and to apply this method to bearing diagnostics. The paper is organised as follows. Section 2 presents a mathematical description of Principal Component Analysis technique. Then, our methodology is presented. In Section 4 two applications of our method are presented, first on an industrial rig and then on a test rig assembled by our research group.

2. Principal Component Analysis

Principal component analysis (PCA) is a multi-variate statistical method [12] that transforms a number of possibly correlated variables into a smaller number of uncorrelated variables, called principal components. It was firstly proposed by Hotelling in 1933 [13]. This procedure aims at explaining the variance in the data by revealing their internal structure. PCA is mathematically defined as an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by any projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on.

PCA can be used for dimensionality reduction in a data set by retaining those characteristics of the data set that contribute most to its variance and those components containing the “most important” aspects of the data. The choice of the number of components is a problem of model selection. It is an important passage that could cause the discard of valuable information, in case of underestimation, or the creation of spurious components, otherwise.

Let $Y \ [n \times N]$ be a data matrix where each row $n$ refers to all types of measurements and each column $N$ to a different sample. The goal of PCA is to find a matrix $P \ [m \times n]$, called loading matrix, such that a linear mapping from the original dimension $n$ to a lower dimension $m$ is provided. A matrix $X \ [m \times N]$, called scores matrix, is obtained from

$$X = PY.$$ 

The $m$ rows of $P$ are the principal components of $Y$ and this dimension could be seen as the number of combined environmental factors affecting the features.

A way to calculate $P$ consists in extracting the main $m$ eigenvectors of the covariance matrix of $Y$,

$$C_Y = \frac{1}{n-1} Y Y^T.$$
or, more practically, to perform a singular value decomposition of the covariance matrix of features. That means
\[ YY^T = USU^T \]
where \( U \) is an orthonormal matrix whose columns define the principal components and \( S \) is a diagonal matrix containing the corresponding singular values.

Hence, the active energy of each component is given by the related singular value. Moreover, thanks to the fact that they are in decreasing order, only \( m \) of them should be considered as influencing the features. Thanks to this analysis, the first \( m \) columns of \( U \) are used to build the matrix \( P \) and to project the measured data onto the environmental-factor characterised space. Choosing the correct number of components is a crucial task and many methods have been proposed to tackle this problem. One consists in choosing the components representing the 80-90% of total variability, that is:
\[
\frac{\lambda_1 + \ldots + \lambda_m}{\lambda_1 + \ldots + \lambda_n} = 80 - 90\%
\]
where the numerator is the sum of the eigenvalues of the first \( m \) principal components and the denominator the sum of the eigenvalues of all components.
The method called “Kaiser rule”, instead, considers only those components such that \( \lambda_i \geq 1 \), or those satisfying \( \lambda_i \geq \text{mean}(\lambda_1 + \ldots + \lambda_n) \).

3. Methodology
In the previous section we described how PCA works mathematically. Here, we explain how to apply this method to real data. Accelerations are acquired for \( N \) times on \( n \) channels and many statistical parameters can be evaluated on the raw data, such as absolute mean, root mean square, maximum peak value, square root value, kurtosis, skewness, crest factor, shape factor. Three among them are evaluated, namely Parameter 1, Parameter 2 and Parameter 3. For each of these parameters the matrix \( Y \) is created: it contains on the rows the values for \( n \) channels of \( N \) acquisitions. The projection matrix \( P \) is then built according to the method described beforehand. This allows to project the measured features into the environmental-factor characterised space. To reduce the loss of information due to this operation, we perform a re-mapping of the projected data back to the original space considering
\[
\hat{Y} = P^T X = P^T PY. \tag{1}
\]
A residual error matrix \( E \) is then calculated as
\[
E = Y - \hat{Y}. \tag{2}
\]
From the prediction error vector \( E_i \), obtained selecting each of the \( N \) columns of \( E \), the Novelty Index (NI) is defined using the Euclidean norm as:
\[
NI = ||E_i|| = \sqrt{\sum_{k=1}^{n} |e_{ik}|^2}, \tag{3}
\]
where \( e_{ik} \) is the residual error for the \( i \)-th sample and \( k \)-th channel.
In our analysis we have two kinds of data: undamaged and damaged. The method involves the construction of the matrix \( P \) given by the first \( m \) columns of \( U \) in the case of a healthy structure. This matrix is then used to re-map the data back to the original space considering (1), where \( Y \)
Figure 1. Positions where the four triaxial accelerometers are placed on Avio development rig.

Figure 2. Roller bearing used during the tests. The circle indicates the damaged roller.

Figure 3. Enlargement of the 450 µm damage on the rolling element.

Figure 4. RMS values evaluated for each run in case of radial channel in V3 and indication of the day of acquisition. The dashed and dotted line refers to healthy bearing, the dashed one to the inner race fault and the solid one to the rolling element damage.

contains one of the statistical parameters once for the undamaged, once for the damaged case. The residual error matrix \( E \) and, consequently, the Novelty Index (\( NI \)) are computed from (2) and (3) respectively. This last parameter \( NI \) enables the evaluation of the percentage of False Alarms (FA) and Missed Alarms (MA). In fact, a threshold can be deduced from the values of \( NI \) in the undamaged case using this formula:

\[
\text{threshold} = \text{mean}(NI_{\text{undam}}) + \alpha \text{ std}(NI_{\text{undam}})
\]

where \( \text{mean}(NI_{\text{undam}}) \) and \( \text{std}(NI_{\text{undam}}) \) are, respectively, the mean and the standard deviation of the values obtained in (3) and \( \alpha \) is a properly chosen constant.

For example, it can be assumed that this parameter \( \alpha \) should assure FA ratio lower than 4%. Analogously, thanks to \( \alpha \), the amount of MA is evaluated as the number of parameters referred to damaged measures staying below the threshold (4). For instance, the percentage of MA can be assumed to be less than 2%. In our analysis, \( NI \) values for the three selected parameters - i.e. \( NI_1 \), \( NI_2 \) and \( NI_3 \) - are combined according to a 2/3 voting schema: if at least two parameters in the undamaged case are above the threshold, that measure is considered as a FA.

4. Application

4.1. Avio development rig

The described technique is tested on an industrial rig (designed by Avio S.p.A.), consisting of a full size aeroengine gearbox. Four triaxial accelerometers are placed in different positions (Figure 1) to a total sum of 12 channels going into a OROS 38 acquisition system. The bearing healthy/unhealthy is placed next to point V2. Each acquisition registers 10 seconds of vibration...
signal at 100% of nominal speed with resistant torque. Data are acquired for different conditions such as an undamaged bearing, a 450 μm damage on the inner race and a 450 μm fault on a rolling element, on three different assemblies. Figure 2 shows the roller bearing used during the tests and Figure 3 is the enlargement of the 450 μm fault on the rolling element. RMS values for the radial channel in V3 for each kind of bearing are plotted in Figure 4. It also shows the trend of measures per each day of acquisition. The wide difference characterising the runs, also between damaged and undamaged acquisitions, could be given by external factors, as environmental temperature or rig assemblies. Our goal is to apply PCA in order to reduce and possibly remove all these influences.

Our algorithm evaluates the three statistical parameters on a vector of 1204000 elements (raw data). Then, it stores these values in three \([n \times N]\) different matrices where \(n\) is the number of

| (% | FA ud | MA ir | MA re |
|----|-------|-------|-------|
| \(NI_1\) | 0     | 0     | 0     |
| \(NI_2\) | 0     | 0     | 1     |
| \(NI_3\) | 0     | 50    | 12    |
| tot. (2/3) | 0     | 0     | 1     |

**Figure 5.** \(NI\) for each statistical parameter evaluated with PCA for Avio gearbox rig. In each plot, ‘ud’ stands for undamaged bearing, ‘ir’ for inner race fault and ‘re’ for rolling element damage.

**Table 1.** Percentage of FA and MA detected by PCA for Avio gearbox rig. ‘FA ud’ are the False Alarms for the undamaged bearing, ‘MA ir’ the Missed Alarms for the inner race fault and ‘MA re’ those for the rolling element damage.
channels and $N$ that of measures. $N$ changes in each case and it reaches the number of 25 for the undamaged bearing, 12 for the inner ring damage and 90 for the rolling element damage. The number $n$, instead, is tied to the channels we want to consider. In this case it is interesting to analyse the behaviour of those placed near the bearing. The triaxial accelerometers placed in V2 and V3 (Figure 1) measure the vibrating signals on 6 channels, so that $n = 6$.

As explained in the previous section, the matrix $P$ for the healthy state is computed and $NI$s are evaluated for the three configurations. The dimensional reduction operated by PCA brings the number $n$ from 6 to $m = 2$. The parameter $\alpha$, necessary to define threshold (4), is fixed as $\alpha = 3$. On the whole, it is worth of interest to notice that, at the end of this analysis, a global damage indicator is provided, that is, no more dependence on various channels is present.

Figure 5 shows the plots referring to $NI$ for the three statistical parameters. The left side, before the first vertical dotted line, represents the values belonging to undamaged bearing. They are necessary to evaluate the threshold (4), marked by the horizontal dashed line. The portion of the plot between the two vertical lines stands for the inner race fault values. It is clear that, except in the case of Parameter 3, the algorithm detects correctly the damages. Finally, the right sector of values is that referred to the rolling element damaged bearing. A greater number of measures is available and, also in this case, it can be noticed that, omitting the last parameter, PCA can guarantee a differentiation between the data. Moreover, if we consider Figure 5 and we compare it to Figure 4 it is clear that variations due to external factors have been removed.

Table 1 shows the percentage of false and missed alarms. It can be noticed that Parameter 3 is less precise in detection, probably because it not so able to detect this kind of bearing faults. However, thanks to the voting criteria it is evident how the percentage can satisfy properly the request of 4% of False Alarms and 2% of Missed Alarms.

4.2. Polito Rig

Damaged and undamaged bearings are also set up on the test rig assembled by the Dynamics & Identification Research Group (DIRG) at Politecnico of Torino (Figure 6 on the left). In this case, the goal of the trials is also to analyse different entities of damage, taking advantage of low mechanical noise of the Polito rig. Thus, three levels of defects are analysed - 150, 250 and 450 $\mu$m - on both inner race and rolling element.

![Figure 6. Polito rig (on the left) and accelerometers positions (on the right).](image-url)
Figure 7. NI for each statistical parameter evaluated with PCA for the inner race fault case (Figure (a) - (b) - (c)) and for the rolling element damage (Figure (d) - (e) - (f)) on Polito rig. In each plot, ‘ud’ stands for undamaged bearing and then the various entities of faults are indicated.

Four triaxial accelerometers are placed in different positions (Figure 6 on the right). The analysed bearing is placed next to A1. Each acquisition registers 8 seconds of vibration signal obtained by crossing radial loads of $1, 1.4, 1.8 \times 10^3 N$ and rotational speeds of 18000, 24000 and 30000 RPM. For each damage, we consider data concerning all kinds of loads and speeds combined together. In this case, the algorithm evaluates the statistical parameters on a vector of 819200 elements (raw data) and stores this values in three different matrices $[n \times N]$, where $n$ is the number of channels and $N$ that of measures. $N$ is equal to 9 in case of the healthy bearing and amounts to a total number of 27 runs both for the inner ring damage and for the rolling
Table 2. Percentage of FA and MA detected by PCA for Polito rig for the inner race damage ((a)) and for the rolling element damage ((b)). ‘FA ud’ are the False Alarms for the undamaged bearing, ‘MA 150’ the Missed Alarms for 150 µm fault, ‘MA 250’ those for 250 µm fault and ‘MA 450’ those for 450 µm fault.

| inner race fault | % | FA | MA 150 | MA 250 | MA 450 |
|------------------|---|----|--------|--------|--------|
| NI₁              | 0 | 78 | 11     | 0      |        |
| NI₂              | 0 | 11 | 44     | 0      |        |
| NI₃              | 0 | 44 | 88     | 0      |        |
| tot. (2/3)       | 0 | 44 | 48     | 0      |        |

| rolling element damage | % | FA | MA 150 | MA 250 | MA 450 |
|------------------------|---|----|--------|--------|--------|
| NI₁                    | 0 | 89 | 33     | 0      |        |
| NI₂                    | 0 | 89 | 56     | 0      |        |
| NI₃                    | 0 | 67 | 0      | 0      |        |
| tot. (2/3)             | 0 | 89 | 30     | 0      |        |

Table 3. Percentage of faults correctly allocate to the group of accelerometers nearer to the damaged bearing (A1-A2). In (a) those referring to the inner race fault (250 - 450 µm entity), in (b) those evaluated for the rolling element damage (250 - 450 µm entity).

| % inner race fault | 250 - 450 µm |
|--------------------|--------------|
| NI₁                | 94           |
| NI₂                | 83           |
| NI₃                | 78           |
| tot. (2/3)         | 94           |

| % rolling element damage | 250 - 450 µm |
|--------------------------|--------------|
| NI₁                      | 100          |
| NI₂                      | 100          |
| NI₃                      | 100          |
| tot. (2/3)               | 100          |

Although the total number of runs is not so high, Figure 7 ((a) - (b) - (c)) reveal a certain increasing trend for NI, with exception of Parameter 2 and Parameter 3. Figure 7 ((d) - (e) - (f)) show that NI values grow with a close dependence on the entity of fault. It is not a linear increase because the analysis is done on three different bearings set up at different times, but it is interesting to notice that, on the whole, the percentage of MA decreases with the increase of damage. For both types of damages, numbers of Missed Alarms are too high for the cases of 150 and 250 µm, but it can be acceptable if we think that a fault of that entity does not compromise the usage of the bearing (Table 2).

The algorithm PCA as presented in Section 3 is also useful if the goal is to isolate the area in which the faulty bearing stands. This can be done selecting in a proper way the accelerometers (and consequently the channels) to create the n rows of the Y matrix. The accelerometers closer to the damage are A1 and A2, while A3 and A4 are more distant (Figure 6 on the right).

After the damage identification, we want to search for a criteria that allows its localisation. Our approach consists in applying PCA as described previously to all the channels obtained considering first A1 - A2 and then A3 - A4. In both case n is equal to 6 (all accelerometers are triaxial) and a matrix Y is created for each parameter. In order to have a greater number of measures, we take into account 250 µm and 450 µm damage entities, so N is 18 both for the inner race and for the rolling element fault. The value α, necessary to define the threshold (4),...
is fixed as $\alpha = 3$ while the dimensional reduction operated by PCA brings, also in this case, the number $n$ from 6 to $m = 3$. For each statistical parameter, damage is assigned to the area where the distance between the threshold and the value $NI$ is bigger.

Table 3 shows percentages of faults correctly allocated to the group of accelerometers placed closer to the inspected bearing. It is clear that this criteria locates the damage in the exact zone in 94% of measures for the inner race case (Table 3 - (a)) and in all measures for the rolling element one (Table 3 - (b)), so the fault is properly localised.

5. Conclusion
In this paper we introduced a monitoring technique for the diagnosis of bearing faults based on Principal Component Analysis (PCA). This method overcomes the problem of acquiring data under different environmental conditions and allows accurate damage recognition, also assuring a rather low number of False Alarms (FA). In addition, a novel criterion has been proposed in order to isolate the area in which the faulty bearing stands. Another useful feature of this PCA-based method allowed us to observe an increasing trend in the evolution of bearing degradation. We tested the described technique on an industrial gearbox rig (designed by Avio S.p.A.), consisting of a full size aeroengine gearbox. Healthy and variously damaged bearings, such as with an inner or rolling element fault, have been set up and vibration signals have been collected and processed in order to properly detect a fault. Data collected from the test rig assembled by the Dynamics & Identification Research Group (DIRG), instead, have been used to demonstrate that the proposed method is able to correctly detect and classify different levels of the same type of fault and also to localise it.

Acknowledgments
Authors wish to thank Avio S.p.A. for the invaluable experience in the field of aeronautical gearboxes, which has allowed this research and the assessment of the GREAT 2020 project.

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