Wet sand flows better than dry sand

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(Dated: February 2, 2008)

We investigated the yield stress and the apparent viscosity of sand with and without small amounts of liquid. By pushing the sand through a tube with an enforced Poiseuille-like profile we minimize the effect of avalanches and shear localization. We find that the system starts to flow when a critical shear of the order of one particle diameter is exceeded. In contrast to common belief, we observe that the resistance against the flow of wet sand is much smaller than that of dry sand. For the dissipative flow we propose a non-equilibrium state equation for granular fluids.

PACS numbers: 05.70.Ln, 83.60.La, 45.70.Mg, 83.80.Fg.

Capillary forces between single grains are the cause of the stiffness of sculptured wet sand, as opposed to dry sand which does not support its own weight [1]. Therefore, it is commonly believed that wet sand should be more resistant, i.e. more “viscous”, than dry sand. Not many studies, though, on the flow behavior of wet sand have been undertaken yet. The few studies at hand have been performed mostly either in a rotating drum [2] or in standard rheometric set-ups [3]. Our study was inspired by an apparatus invented by the authors of references [4, 5, 6], but their work is more focused on the regime below the yield point. Besides its practical applications in e.g. civil engineering or industrial processes, the rheological or dynamical properties of dense packed granular materials are investigated to help develop a non-equilibrium thermodynamic formalism. Whilst the description of a rapid flowing granular gas is well established [5], the opposite limit of a dense packing remains an open question. Proposals for state equations for granular matter include, for example, the Enskog equation either for describing rapid dilute granular flows [5] or for the high density limit [9]. Yet, it is not possible to describe all of the experimental findings, e.g. the fact that granular matter does not follow a Boltzmann statistic [10, 11].

Granular matter is an example of a so-called yield stress fluid. Above a critical yield stress, the mechanical response changes from a solid-like or jammed state to a flowing one. An illustrative study on the rheological properties of wet soils was given, for example, in ref. [12]. These authors measured viscoelastic properties under steady and oscillatory stress. Soils are a typical example of wet granular material, and they exhibit a mixed behavior of elasticity, viscosity and plasticity that is mainly determined by the quantity of water, the yield stress, shear modulus, and viscosity decrease with the water content. The water content is changed by adding liquid to a given sample, thus reducing the packing density of the solid grains. The system starts to flow when the external stress exceeds the inter-aggregate contact forces. The mechanical properties at lower water content are determined by the liquid bridges between clay clusters and quasi-crystals, and those at higher water content are determined by the flow of the liquid through the soil pores [12].

A principle difficulty in the determination of the yield stress point arises from the fact that a yield stress fluids exhibits thixotropy, also referred to as aging or rejuvenation [4]. Yield stress and thixotropic behavior depend on the degree of compaction and the topology. Both can be modulated by weak attractive forces like the electrostatic charge and humidity [14]. Macroscopically, the jammed nature of dry and wet granular materials determines the viscoelastic properties, e.g. the route to the jammed transition can be explained by a modified Vogel-Fulcher-Tammann behavior [15, 16], and it can be understood by the relation between the compaction and the grain mobility [17, 18]. In dense granular materials the resistance to flow and the dissipation of energy occurs when the bulk granulate has to pass from one configuration to another. This rearrangement process leads to a new, jammed configuration. The rearrangement dynamics driven by shearing causes the rejuvenation and aging of the fluid-structure and it is meaningful to determine the yield stress point when the structural integrity is maintained and shear localization, avalanches, and arching effects are prevented. We will show that by doing so, it is possible to reach a steady state with a constant relation between the pumping and dissipation of energy if the granular material is sheared in an oscillatory way. Our sand was composed of glass spheres of diameter $D = 140 - 150 \mu m$ with and without additional deionized water. The content of water in the paste $W$ is defined as the quotient between the liquid volume and the volume occupied by the grains. As a first attempt to measure the yield stress point of our samples, we followed the methods used to study the mechanics of soil. Measurements were done with a rotational viscometer (MARS, Thermo Haake, Karlsruhe, Germany) with a parallel plate geometry [12] and with sand paper glued on the on the surfaces of the plates. Further improvement was given by using a vane sensor or a “bars” sensor, which is shown in the inset of fig. [1] However, for our study, all geometries gave comparable results, while the latter was the most reproducible. A typical protocol to study yield stress

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fluids - for example coatings and paints - involves imposing a constant shear rate and measuring the strain. The so-called yield stress point is the highest stress at which no flow occurs at a given shear rate, e.g., at the maximum in the flow curve of fig. 1. Beyond the maximum, the deformation is large enough to compromise the structural integrity of the granular fluid. Figure 1 shows a representative result for the case studies of dry and wet sand. The maximum corresponds to the stress at which the material begins to plastically deform; before that, the material deforms elastically. The test must be carefully tuned, because the shear rate must be small enough to catch the elastic and plastic regimes but, if it is too small, the peak is difficult to identify. In general, the yield stress point depends on the time scales and shear rates applied in the experiment - however, it was recently found that the critical yield strain of many different yield stress fluids, like ketchup or pastes, does not depend on the external parameters; instead, it depends only on the type of fluid [19].

The yield stress point for the dry sand is \( \tau = 0.8\text{kPa} \), \( \gamma_c = 0.1 = D/10 \) (\( \tau \) is the stress and \( \gamma \) the strain) while, for the wet sand, \( \tau = 4.23\text{kPa} \), \( \gamma_c = 1.4 = 3D/2 \), for a given shear rate of \( \dot{\gamma} = 0.0038\text{ s}^{-1} \) and \( \dot{\gamma} = 0.003\text{ s}^{-1} \) respectively. Apparently, the liquid bridges between the grains reinforce the structural integrity of the wet sand. Similarly, one needs less torque to turn the sensor at a given rotational speed in the dry sand than the wet sand. The asymptotic values of the stress are 0.2 and 1\text{kPa} respectively; however, due to the breakdown of the microstructure and flow localization, one cannot straightforwardly deduce the yield stress (or even a shear viscosity) later on. To override this problem, we carried out experiments with the shear cell proposed in ref. [1, 2] (see the inset of fig. 2). The use of this device made it possible to shear the pastes all over the sample volume. The granular material is put into an acrylic glass cylindrical cell at atmospheric pressure. The sides of the cell are sealed with thin latex membranes of 300\( \mu \text{m} \), which are flat at the beginning of the experiment. Chambers filled with water are attached to both sides of the cell, adjacent to the membranes, where it is possible to inject and extract water into and out of the chamber through tubes via a syringe. The syringe-pistons are connected to spindles that could be moved with a stepper motor. The pistons move at the same speed as each other but in the opposite direction to prevent dilatation. The acrylic glass tube we used for this experiment was 24 mm long. It was filled with dry sand or with a wet sand paste with \( W = 0.03 \) so that, in both cases, the global packing fraction \( \rho_f = 0.63 \). We should mention that this number must be considered cautiously, since we ignore the local compaction of the granulate, e.g., the topology of polytetrahdral grain structures [20].

The deformation of the membranes is approximately parabolic and imposes a Poiseuille-like flow profile within the sample; thus, avalanching and arching can be kept to a minimum. Piezoresistive sensors sense the pressure (with respect to atmospheric pressure) in both chambers. A fixed quantity of water \( \Delta V \) is injected or extracted in/from the adjacent chambers, and the differential pressure \( p_1 - p_2 \) is measured (see the inset of fig. 2). The displacement of the membrane is calculated as \( \Delta x = 4\Delta V/\left(\pi D_c^2\right) \), where \( D_c \) is the diameter of the acrylic glass cell (24\text{mm}). The paste is sheared at a very low shear rate with a shear amplitude \( a_s \), between \(-a_s \) and \( a_s \). The protocol was to move the membrane \( \Delta x = 6\mu \text{m} \) in one second, wait 13 seconds, and measure \( p_1 \) and \( p_2 \) until the selected \( a_s \) is reached. A curve of the pressure difference \( p_1 - p_2 \) as a function of the displacement - which represents the strain - is obtained for each
shearing amplitude \( a_s \). A family of differential pressure characteristics for increasing shear amplitudes are shown in fig. 2. The slope of the two branches corresponds to the restoring force exerted by the rubber membranes, but a stress \( \tau_{st} \) is stored in the system which is related to the opening of the loop with \( 2\tau_{st}(a_s) = \Delta(p_1 - p_2)_0 \). In fig. 3 the differential pressure curves for dry and wet sand are shown for the two limiting cases of small and large shear amplitudes. By shearing with \( a_s < 2D \), we find that the dry sand does not resist any stress (fig. 3(a)), while the wet sand behaves like a yield stress fluid due to the liquid bridge network (LBN) (fig. 3(c) and the inset of fig. 4), with a \( \Delta(p_1 - p_2)_0 \approx 1kPa \). Following ref. 4, 5, 6, the capillary pressure is an energy density \( p = dE/dV \leq \rho_f N \gamma /D \) where \( N = 6 \) is the LBN coordination number, \( \gamma \) is the surface tension of water, and \( D = 145 \mu m \); thus, \( p \leq 1.9kPa \) and, in agreement with ref. 4, 5, 6, we find that the restoring forces originate from the capillary forces between the grains. The onset of dissipative flow occurs at \( a_s \geq 2D \) then the opening of the loop starts to increase. It grows more strongly for the dry than for the wet sand, with maximum values of 116kPa, and 22kPa for the largest shear amplitude, respectively. This means that one needs less energy to push the wet sand through the tube. It is worth mentioning that, for a given shear amplitude, the hysteresis loop is stable during a many cycles. We carried out a few tests which were one week in duration and found that the opening of the loop for different shear amplitudes remains constant (we made sure that all our data were taken in such a steady state situation). We also tested shear velocities between 0.6 and 300\( \mu m/s \) without observing differences in the hysteresis loop.

Figure 4 shows the complete stress-strain behavior for increasing shear amplitudes for dry (triangles) and wet sand \( W = 0.03 \) (squares) and the corresponding fittings for the dissipative flow range (equation 1). The inset shows, for the wet sand, the range where the stored stress is the yield stress.

FIG. 3: Differential pressure versus the shear displacement. 
(a),(b): A sample of 17.7 gr. of dry sand (26% room humidity, 26° C) is compared with (c),(d): a paste of the same sand content mixed with water (\( W = 0.03 \)).

FIG. 4: Stress - strain behavior for increasing shear amplitude for dry (triangles) and wet sand \( W = 0.03 \) (squares) and the corresponding fittings for the dissipative flow range (equation 1). The inset shows, for the wet sand, the range where the stored stress is the yield stress.

A fit with with equation 1 yields an onset value \( D^* = 150 \mu m \) for both cases which is close to the mean diameter of the grains; for the constant \( c_0 \) and the exponent \( n \), we find \( c_0 = 55 \), \( n = 1 \) and \( c_0 = 10kPa \), \( n = 3/2 \), for the
dry and wet sand respectively. The work that is applied to the system is given by \( W = F_s a_s \), and its response is related to the energy \( E \) that is dissipated during the rearrangement mechanism; thus, it is possible to rewrite eq. (1) as

\[
\frac{W}{W_0} = \exp \left( \frac{E}{E_0} \right)^n
\]

where \( W_0 = 2F_s D_s^* \) and \( E_0 \) is the activation energy for the rearrangement. The latter should be related to a configurational entropy; we find \( E_0 = 88 \) and 16nJ per grain for the dry and wet granulate respectively.

For \( n = 1 \), eq. (1) it is a Doolittle-like equation [21]. It was shown that the viscosity of a granular fluid depends strongly on the particle concentration [22] and, in terms of a free volume theory, \( E \propto 1/\eta_f \). The activation energy is related to the global packing factor \( E_0 \propto p_f \) (the occupied volume divided by the cell volume). Following emerging concepts on non-equilibrium thermodynamics [23], we propose that the equation (2) could be a state equation for granular fluids in dissipative flow under globally constant packing fraction conditions.

It is difficult to estimate the value of \( E_0 \) for dry sand since the energy loss is sensitive to, e.g., microscopic roughness and topology. But, for the wet sand, it can be related to the liquid bridges. By considering the coordination number \( N = 6 \), the dissipation per bridge is \( E_0/N = (2.6 \pm 0.5)nJ \), which is in good agreement with the estimation for the energy loss during bridge formation and rupture \( \Delta E_{cap} < \pi \gamma D^2/2 = 2.4nJ \) [3]. Figure 5 shows a stress-strain curve for the first scan of increasing and decreasing the shear amplitude applied to wet sand. Beyond \( a_s \geq 2D \), the structure of the granular fluid changes during the successive rearrangements. Possible mechanisms for this include the formation of water channels by coalescence of the liquid bridges and larger polytetrahtedral solid structures. The decreasing branch of Fig. 5 might thus correspond to a fluid that exhibits phase separation and shear bands. For the range \( a_s < 2D \), the opening of the loop (the yield stress) tends to zero; again, this is an indication for a phase separation.

Similar measurements for the dry sand revealed no difference between increasing and decreasing shear amplitude, but the rearrangement process there occurs not between evolving clusters, but between the single grains.

In conclusion, we have found that it is much easier to push wet sand than dry granular matter in a Poiseuille-like profile through a tube. Even if the capillary forces increase the yield stresses, it looks like the water promotes the cluster formation and reduces the inter-grain friction. The leading dissipation mechanism results from the rupture and formation of liquid bridges, and we are able to explain our data quantitatively within the framework of an excluded volume theory. Finally, we find indications that the yield of the system is related to the microscopic size of the grains. Further studies with different grain sizes are ongoing.

Acknowledgments. The work was funded by the Alexander von Humboldt Foundation and the DFG-Graduiertenkolleg 1276/1. We thank Manuel Cáceres and Nicolas Vandewalle for enlightened discussions.

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