Inception of Self-Interacting Dark Matter with Dark Charge Conjugation Symmetry

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Abstract

A new understanding of the stability of self-interacting dark matter is pointed out, based on the simplest spontaneously broken Abelian $U(1)$ gauge model with one complex scalar and one Dirac fermion. The key is the imposition of dark charge conjugation symmetry. It allows the possible existence of two stable particles: the Dirac fermion and the vector gauge boson which acts as a light mediator for the former’s self-interaction. Since this light mediator does not decay, it avoids the strong cosmological constraints recently obtained for all such models where the light mediator decays into standard-model particles.
**Introduction**: The Lagrangian of the simplest spontaneously broken Abelian $U(1)$ gauge model was written down by Peter Higgs over 50 years ago [1]. Its particle content consists of a vector gauge boson (call it $Z_D$) and a complex scalar (call it $\sigma$). By itself it has automatic charge conjugation invariance, i.e. $Z_D \rightarrow -Z_D$, $\sigma \rightarrow \sigma^*$, resulting in $g_D \rightarrow -g_D$. After spontaneous symmetry breaking, the above still holds, i.e. $Z_D \rightarrow -Z_D$, $\sigma_R \rightarrow \sigma_R$, and $\sigma_I \rightarrow -\sigma_I$ which becomes the longitudinal component of the now massive $Z_D$. This fact has been used [2, 3, 4, 5] to suggest that $Z_D$ may be dark matter.

The existence of two $U(1)$ gauge factors allows for the gauge-invariant kinetic mixing [6] of the two associated gauge bosons, so $Z_D$ may mix with the $U(1)_Y$ gauge boson of the standard model (SM), of which the photon is a component. This has led to many theoretical studies of a possible light dark photon, and the experiments which may be relevant in finding it [7]. However, this kinetic mixing term breaks the dark charge conjugation symmetry, so the former may be absolutely forbidden if the latter is chosen to be exact.

In the Higgs model, $Z_D$ is the sole dark matter. Suppose a Dirac fermion (call it $N$) is added, transforming also under $U(1)_D$, then the Lagrangian is also invariant under dark charge conjugation, as well as the global $U(1)$ transformation operating on $N$, i.e. dark fermion number. Hence $N$ is a dark-matter candidate. What about $Z_D$? If $m_{Z_D} > 2m_N$, then $Z_D$ will decay into $N\bar{N}$ through the vector current $\bar{N}\gamma_\mu N$ which has charge conjugation $C = -1$, but if $m_{Z_D} < 2m_N$, then $Z_D$ will be stable. Further, if $Z_D$ is much lighter than $N$, then it may act as a stable light mediator for $N$ self-interactions. Note that if $Z_D$ is unstable and decays to SM particles, then very strong constraints exist [8] which basically rule out this scenario for explaining [9] the core-cusp anomaly observed in dwarf galaxies [10]. As for the dark Higgs boson $h_D = \sqrt{2}Re(\sigma)$, it may also be light, but it has an unavoidable mixing with the SM Higgs boson $h$, so it will not be stable. In the following, $m_{h_D} < m_{Z_D}$ will be assumed.
With $m_N \sim 100$ GeV and $m_{Z_D} \sim 10$ MeV, the $N\bar{N}$ annihilation to $Z_DZ_D$ is assumed to have the right cross section for $N$ to be the main component of dark matter. The subsequent $Z_DZ_D$ annihilation to $h_Dh_D$ is assumed to have a large enough cross section, so that the relic abundance of $Z_D$ is small compared to that of $N$. In direct-search experiments, $N$ does not interact with quarks, so there will be no signal. As for the small $Z_D$ component, it interacts through $h_D - h$ mixing, but since $Z_D$ is very light, current experiments are not sensitive to its presence. On the other hand, the $h_D - h$ mixing has to be large enough for it to decay away before big bang nucleosynthesis (BBN). Even so, $h_D$ may be produced at late times through $Z_DZ_D$ annihilation, and affects the cosmic microwave background (CMB) through its decay, as pointed out in Ref. [8]. However, there is no Sommerfeld enhancement [11] of this cross section, unlike the case of $N\bar{N}$ annihilation through a light mediator which decays. Hence the proposed model is a natural resolution of this conundrum, as detailed below.

**Dark $U(1)_D$ model**: This model assumes $U(1)_D$ gauge symmetry, implying thus a vector gauge boson $Z_D$. It is spontaneously broken by a complex scalar $\sigma$ with charge $g_D$. A Dirac fermion $N$ also exists with charge $g_N$. The complete Lagrangian before symmetry breaking is

\[
\mathcal{L} = -\frac{1}{4}(\partial^\mu Z_D^\nu - \partial^\nu Z_D^\mu)(\partial_\mu Z_{D\nu} - \partial_\nu Z_{D\mu}) + (\partial^\mu \sigma - ig_DZ_D^\mu\sigma)(\partial_\mu \sigma^* + ig_DZ_{D\mu}\sigma^*) \\
+ \mu_D^2\sigma^*\sigma - \frac{1}{2}\lambda_D(\sigma^*\sigma)^2 + i\bar{N}\gamma_\mu(\partial^\mu - ig_NZ_D^\mu)N - m_N\bar{N}N. \tag{1}
\]

In the above, if we replace $g_D$ by $-g_D$, $\sigma$ by $\sigma^*$, $g_N$ by $-g_N$, and $N$ by its dark charge conjugate, we have exactly the same physical theory. The spontaneous breaking of $U(1)_D$ with $\langle \sigma \rangle = v_D/\sqrt{2}$ changes the Lagrangian to

\[
\mathcal{L} = -\frac{1}{4}(\partial^\mu Z_D^\nu - \partial^\nu Z_D^\mu)(\partial_\mu Z_{D\nu} - \partial_\nu Z_{D\mu}) + \frac{1}{2}m_{Z_D}^2Z_D^\mu Z_{D\mu} + \frac{1}{2}(\partial^\mu h_D)(\partial_\mu h_D) - \frac{1}{2}m_{h_D}^2h_D^2 \\
+ \frac{m_{h_D}^2h_D^3}{2v_D} + \frac{m_{h_D}^2h_D}{8v_D^2} + g_D^2v_Dh_D(Z_D^\mu Z_{D\mu}) + \frac{1}{2}g_D^2h_D^2(Z_D^\mu Z_{D\mu}) \\
+ i\bar{N}\gamma_\mu(\partial^\mu - ig_NZ_D^\mu)N - m_N\bar{N}N + g_NZ_D^\mu\bar{N}\gamma_\mu N, \tag{2}
\]
where $v_D^2 = 2\mu_D^2/\lambda_D$, $m_{Z_D} = g_D v_D$, and $m_{h_D}^2 = \lambda_D v_D^2$. The crucial interaction terms are $g_N Z_D^\mu \bar{N} \gamma_\mu N$, $g_D^2 v_D h_D (Z_D^\mu Z_D^\mu)$, and $(1/2)g_D^2 h_D^2 (Z_D^\mu Z_D^\mu)$. We assume in the following $m_N \sim 100$ GeV, with $Z_D, h_D \sim 10$ MeV, with $m_{h_D} < m_{Z_D}$. Note that $g_N$ is independent of $g_D$.

Three new particles: There are only three new particles beyond those of the standard model. Each serves a purpose and is an essential ingredient of this two-component dark-matter model. The dark fermion $N$ is a Dirac particle with a conserved dark fermion number. It is the dominant component of the observed dark matter of the Universe. It has a dark gauge interaction mediated by $Z_D$ which is light, thus realizing the requirement of a sufficiently large interaction to affect the core-cusp discrepancy of dwarf galaxies. The imposition of dark charge conjugation symmetry means that $Z_D$ has $C = -1$. It couples to the vector current $\bar{N} \gamma_\mu N$ which also has $C = -1$, so it may decay into $N \bar{N}$, but if it is lighter than $2m_N$ as assumed, then it is itself stable. As such, it may be overproduced in the early Universe. However, it is also assumed that the dark Higgs boson $h_D$, which breaks the $U(1)_D$ gauge symmetry and provides $Z_D$ with a mass through its vacuum expectation value $v_D$, is lighter than $Z_D$. Hence the $Z_D Z_D \rightarrow h_D h_D$ annihilation should be strong enough to make it a very small fraction of the observed dark matter of the Universe. As for $h_D$, which has $C = +1$, it must be unstable through its allowed mixing with the SM Higgs boson $h$, and decays away early without affecting the standard BBN.

Consider the extended scalar potential involving both $\sigma$ and the SM Higgs doublet $\Phi = (\phi^+, \phi^0)$:

$$V = -\mu_D^2 \sigma^* \sigma + \frac{1}{2}\lambda_D (\sigma^* \sigma)^2 - \mu_h^2 \Phi^\dagger \Phi + \frac{1}{2}\lambda_h (\Phi^\dagger \Phi)^2 + \lambda_{hD} (\sigma^* \sigma) (\Phi^\dagger \Phi).$$

Using $\phi^0 = (v_h + h)/\sqrt{2}$, the $2 \times 2$ mass-squared matrix spanning $(h_D, h)$ is given by

$$\mathcal{M}_{h_D, h}^2 = \begin{pmatrix} \lambda_D v_D^2 & \lambda_{hD} v_D v_H \\ \lambda_{hD} v_D v_H & \lambda_h^2 v_h^2 \end{pmatrix}.$$
Assuming $m_{h_D} << m_h = 125$ GeV, the $h_D - h$ mixing is then $\theta_{hD} = \lambda_{hD} v_D v_h / m_h^2$. For a light $h_D$ of order 10 MeV, its dominant decay is to $e^- e^+$ with the decay rate

$$\Gamma(h_D \rightarrow e^- e^+) = \frac{m_{h_D} m_e^2 \theta_{hD}^2}{8\pi v_h^2},$$

(5)

where $v_h = 246$ GeV. Assuming that $\Gamma^{-1} < 1$ s, the constraint

$$\left(\frac{m_{h_D}}{10 \text{ MeV}}\right) \theta_{hD}^2 > 3.83 \times 10^{-10}$$

(6)

is obtained. The SM Higgs boson $h$ also decays into $h_D h_D$ with coupling $\lambda_{hD} v_h$. Its decay rate is

$$\Gamma(h \rightarrow h_D h_D) = \frac{\lambda_{hD}^2 v_h^2}{16\pi m_h} = \lambda_{hD}(9.63 \text{ GeV}).$$

(7)

Assuming that this is no more than 10% of the Higgs boson width in the SM (4.12 MeV), this gives a bound of

$$\lambda_{hD} < 0.0066.$$  

(8)

Comparing Eqs. (6) and (7), the constraint

$$\left(\frac{v_D}{\text{GeV}}\right) > 0.19 \sqrt{\frac{10 \text{ MeV}}{m_{h_D}}}$$

(9)

is obtained.

**$Z_d Z_d$ annihilation**: Consider first the process $Z_d Z_d \rightarrow h_D h_D$ at rest. There are four diagrams summing up to the amplitude

$$\mathcal{A} = \left[\frac{2g_D^2 (2 + r)}{2 - r} - \frac{6g_D^2 r}{4 - r}\right] (\vec{e}_1 \cdot \vec{e}_2) + \frac{8g_D^2}{m_{Z_d}^2 (2 - r)} (\vec{e}_1 \cdot \vec{k})(\vec{e}_2 \cdot \vec{k}),$$

(10)

where $r = m_{h_D}^2 / m_{Z_D}^2$ and the center-of-mass variables $\vec{k}$ (momentum of $h_D$) and $\vec{e}_1, 2$ (polarizations of $Z_D$) have been used. The resulting cross section $\times$ relative velocity is given by

$$\sigma(Z_D Z_D \rightarrow h_D h_D) \times v_{rel} = \frac{g_D^4 \sqrt{1 - r}}{64\pi m_{Z_D}^2} \left[\frac{4r^2 + 4(2 - r)^2}{(4 - r)^2} - \frac{24r(2 + r)}{9(2 - r)(4 - r)} + \frac{8(2 + r)^2}{9(2 - r)^2}\right].$$

(11)
Let $m_{Z_D} = 10$ MeV and $m_{h_D} = 8$ MeV, then $r = 0.64$. The coupling $g_D$ is adjustable. Let $g_D = 0.005$ for example, then

$$
\sigma(Z_DZ_D \rightarrow h_Dh_D) \times v_{\text{rel}} = 1.1 \times 10^{-24} \text{ cm}^3/\text{s},
$$

which is 37 times the canonical $\sigma_0 \times v_{\text{rel}} = 3 \times 10^{-26} \text{ cm}^3/\text{s}$ for obtaining the correct dark-matter relic abundance of the Universe. This means that $Z_D$ will be underproduced and forms only a small component of the observed dark matter, which will be mainly $N$ as discussed in the next section. Note also that $g_D = 0.005$ and $m_{Z_D} = 10$ MeV imply that $v_D = 2$ GeV, which is perfectly consistent with Eq. (9).

$N\bar{N}$ annihilation: The annihilation $N\bar{N} \rightarrow Z_DZ_D$ is analogous to $e^-e^+ \rightarrow \gamma\gamma$. The cross section at rest $\times$ relative velocity is given by

$$
\sigma(N\bar{N} \rightarrow Z_DZ_D) \times v_{\text{rel}} = \frac{g_N^4}{16\pi m_N^2}.
$$

For $m_N = 100$ GeV, this would be equal to $2\sigma_0 \times v_{\text{rel}} = 6 \times 10^{-26} \text{ cm}^3/\text{s}$ if $g_N = 0.225$. For the light mediator with $m_{Z_D} = 10$ MeV, Sommerfeld enhancement is expected. However, at the time of thermal freezeout, this effect is only $O(1)$ \[12, 13\]. The large enhancement will come at late times (because of the decreasing relative velocity of $N\bar{N}$ annihilation) and may be as large as a factor of $10^4$. Whereas the fraction of $N\bar{N}$ which would annihilate is still negligible compared to the entire population, the production of an unstable mediator would allow its decay products (photons and electrons) to affect the CMB, thus ruling out (for $s$-wave annihilation) all models where the self-interactions are large enough to address the small-scale problems of structure formation, as pointed out recently \[8\].

Here the light mediator $Z_D$ is stable, so it does not affect the CMB. As for $h_D$, it may also be produced at late times from $Z_DZ_D$ annihilation, but this cross section has no Sommerfeld enhancement, so even though $h_D$ decays to $e^-e^+$, its effect is small.
Thermal history: The dark fermion $N$ is kept in thermal equilibrium with its light mediator $Z_D$ which couples to the dark Higgs boson $h_D$. The bridge connecting the dark sector with the SM is the quartic scalar interaction term $\lambda_{hD} (\sigma^* \sigma)(\Phi^\dagger \Phi)$ of Eq. (3). Hence $h_D$ is in thermal equilibrium with the SM Higgs boson $h$, and through the latter, all the SM particles. As the Universe cools below $m_N$, $N$ freezes out with a relic abundance which accounts for most of the observed dark matter of the Universe. In structure formation, $N$ has a large enough elastic cross section due to the exchange of its light mediator $Z_D$ to explain the flatter density profiles of dwarf galaxies near their centers [9].

The light vector boson $Z_D$ is stable and interacts with $h_D$ to remain in thermal equilibrium until the Universe cools below $m_{Z_D}$. It then freezes out with a much smaller relic abundance than that of $N$. The dark Higgs boson $h_D$ decays away quickly at early times through its mixing with the SM Higgs boson $h$. All these happen before the onset of BBN so that the standard predictions of all relevant cosmological parameters are unchanged. At late times, $Z_D$ re-emerges from $N\bar{N}$ annihilation, but it is stable and will not disturb the CMB. The dark Higgs boson $h_D$ also re-emerges from $Z_DZ_D$ annihilation, but this cross section is not enhanced by the Sommerfeld effect, so even though $h_D$ decays to $e^-e^+$, its effect on the CMB is harmless.

Phenomenological consequences: The model presented has a dark gauge $U(1)_D$ symmetry, with exact dark charge conjugation invariance. It has two stable particles, the dark fermion $N$ with $m_N \sim 100$ GeV and a light vector mediator $Z_D$ with $m_{Z_D} \sim 10$ MeV. As such, it explains the observed relic abundance of dark matter, as well as the cusp-core anomaly of dwarf galaxies. It avoids the strong constraints of decaying particles on the CMB [8]. The $U(1)_D$ symmetry is broken with $v_D \sim 2$ GeV as constrained by Eq. (9). The associated dark Higgs boson $h_D$ is lighter than $Z_D$ and mixes with the SM Higgs boson $h$.

In direct-search experiments, $N$ is essentially invisible because it has only $Z_D$ interactions.
which do not affect SM particles at tree level. As for $Z_D$, its relic abundance is suppressed and its mass is only about 10 MeV, so even though it interacts with SM particles through $h_D - h$ mixing, it is insensitive to present underground experiments. This would not be the case if $m_{Z_D} \sim 100$ GeV. In fact, it has been shown [13] that a light mediator would then be ruled out because the direct-detection bound excludes its decay before the onset of BBN. In indirect-search experiments, the $N\bar{N}$ annihilation is Sommerfeld-enhanced, but it only produces $Z_D$ at tree level which cannot be detected. In one loop, SM particles may be produced, but the cross section is very small. Hence neither types of the conventional search for dark matter would have much promise in detecting such dark matter.

Since the light vector boson $Z_D$ has no kinetic mixing with the photon because of the dark gauge conjugation symmetry, there is also no effect on experiments searching for it through this portal.

A possible way to discover $h_D$ is from $h \rightarrow h_D h_D$ decay at an accelerator, and the subsequent decay $h_D \rightarrow e^- e^+$. The problem is that $h_D$ has a lifetime of about 1 s, so the decay products are far downstream and not easily observed.

Remarks: The idea of self-interacting dark matter is faced with a conundrum [8]. If the interaction is strong enough to address the small-scale problems of structure formation, the production of the light mediator at late times would disrupt the cosmic microwave background because of the inherent Sommerfeld enhancement for $s$-wave annihilation and the apparently inescapable fact that the mediator must decay into electrons or photons. Its resolution in terms of a simple complete renormalizable model is the subject matter of this paper. Unfortunately, this model predicts null or negligible effects in all present attempts to discover the nature of dark matter. On the other hand, it may be the answer to the question of why dark matter has not been seen so far.
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