Repeater-assisted Zeno effect in classical stochastic processes

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As a classical state, for instance a digitized image, is transferred through a classical channel, it decays inevitably with the distance due to the surroundings’ interferences. However, if there are enough number of repeaters, which can both check and recover the state’s information continuously, the state’s decay rate will be significantly suppressed, then a classical Zeno effect might occur. Such a physical process is purely classical and without any interferences of living beings, therefore, it manifests that the Zeno effect is no longer a patent of quantum mechanics, but does exist in classical stochastic processes.

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I. INTRODUCTION

Quantum Zeno effect, proposed by Misra and Sudarsshan [1] in 1977, is believed to exist uniquely in the quantum world due to informatical description of quantum states and the projective measurement in quantum mechanics (for a review, see Ref. [2]). While its classical correspondences, such as the Fletcher’s paradox, becomes an antinomy in Newtonian mechanics. The basic reason, from our judgement, is the absence of the projective measurement in classical physics. However, though Newtonian mechanics excludes the projective measurement, the later does exist in the classical world. In a recent work on this issue [3], one of us (Gu) touched the possibility of the classical Zeno and anti-Zeno effects. Gu used a scenario of Super Mario’s prison break to show that a “classical state” might not decay if it is observed continuously. Nevertheless, the scenario involves too much subjective freedoms coming from Super Mario’s intelligent feedback, such that it is, though reasonable in everyday life, far beyond the objective laws of the classical world. This consideration leads to the main motivation of the present work to seek a possibility of the classical Zeno effect without involving any subjective judgement.

We consider here such a scenario of transferring a classical state, for instance a digitized image, through a classical channel with a noise surrounding (See Fig. 1). The state, if there is no assisted equipment, decays inevitably with the distance due to the surroundings’ interferences, and finally loses all information. In order to suppress the decay rate or even ensure zero decay, we need to check the information of state within a certain distance. Instead of the intelligent judgement made by Super Mario [3], here we use a machine (repeater), which has nothing to do with living beings, to examine and try to recover the state. We show that if there are enough number of repeaters, the transferred state then might decay much more slowly, or even never decay, then a classical Zeno effect occurs.

This work is organized as follows. In section II we introduce the basic formulas for describing the classical transportation. In section III we take the emblem of the Chinese University of Hong Kong (CUHK) as an explicit example to show how the classical Zeno effect occurs during a classical transportation. A further discussion and a brief summary are given in section IV and V respectively.

II. CLASSICAL TRANSPORTATION

To begin with, we consider a classical N-bit state describe by \( \rho = \{\sigma_j\} \) with \( \sigma_j = \pm 1 \), which in computer science can be saved physically by for instance in a flash disk or a perforated paper tape. Now we want to transfer the state through a classical channel, which is subject to a noise background of temperature \( T \). During the transferring process, the state carried by each bit is protected via an energy gap \( \Delta E \). So if the temperature is zero, there is no background inference, and the state can be transferred over an infinite distance without loss of information. However, the thermal fluctuation of the

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surrounding might flip the state of bit. Once the bit is flipped one or more times, we assume it loses the correlation to the original global state. The transition probability of each bit through a unit distance \( a \) is determined by

\[
P(\sigma_j \rightarrow \sigma_j') = e^{-\Delta E / T'},
\]

where \( \sigma_j \) denotes uncorrelated state. Then if the bit is transferred over a distance of \( L \) (in unit of \( a \)), the surviving probability becomes

\[
P_S(\sigma_j) = (1 - e^{-\Delta E / T'})^L = \exp(-L\Delta E / T'),
\]

where \( T' = -\Delta E / \ln(1 - e^{-\Delta E / T}) \). Therefore, the classical state will decay exponentially to a complete random state with the transferred distance.

Explicitly, in the basis of \( \{ \sigma_j, \sigma_j' \} \), the initial state of the bit is

\[
\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},
\]

while the final state becomes

\[
\rho' = \begin{pmatrix} \exp(-L\Delta E / T') & \exp(-L\Delta E / T') \\ 1 - \exp(-L\Delta E / T') & 1 - \exp(-L\Delta E / T') \end{pmatrix}.
\]

The distance between these two states can be measured by the fidelity

\[
F = \text{tr} \sqrt{\rho^{1/2} \rho' \rho^{1/2}} = \exp(-L\Delta E / 2T').
\]

Then at the very beginning, i.e. small \( L \)

\[
F = 1 - \frac{L\Delta E}{2T'} + \cdots,
\]

which decays algebraically. We notice that the leading term of the fidelity here is linear instead of quadratic in quantum fidelity. Nevertheless, the algebraic decay of the fidelity ensure that we have chance to correct the lost information via a repeater.

### III. EXAMPLE

In this section, we give a simple example to illustrate how such a process can be realized in practice. We assume that Alice wants to transfer a picture, i.e., the CUHK emblem, to Bob who is at a long distance via a classical channel. The emblem is prepared in a BMP file with \( 252 \times 200 \) resolution and 256 colors. For simplicity, we assume Alice and Bob have already known all other structural information, such as 256 elemental colors and the size of the picture, except the image data, which can be expressed as a \( 252 \times 200 \) matrix \( M \)

\[
M = \begin{pmatrix}
M_1^1 & M_1^2 & \cdots & M_1^{252} \\
M_2^1 & M_2^2 & \cdots & M_2^{252} \\
\vdots & \vdots & \ddots & \vdots \\
M_1^{200} & M_2^{200} & \cdots & M_{252}^{200}
\end{pmatrix},
\]

with each element \( M_j^i \) \( (i \in [1, 252], j \in [1, 200]) \) being an 8-bit integer. During the transportation, each element of the matrix \( M \) is subject to environmental interference. One of elements might be changed to a random 8-bit integer with the probability \( \exp(-\beta) \) with \( \beta = -\Delta E / T' \) over the unit distance \( a \). Clearly, the lower the temperature, the more stable the data. Meanwhile, the transferred distance increase, the emblem’s information will inevitably lose. We use the classical Monte Carlo method to simulate such a process. The leftmost column of Fig. 2 shows the emblem at the various distance. We can see that the figure will finally be blurred.

In order to suppress the decay rate, Alice and Bob need to install repeaters on the channel they use. The repeater’s first function is to check the integrability of data. Secondly, if possible, it can partially recover the data. For this purpose, they add row-column check code to the end of the matrix elements.

\[
M = \begin{pmatrix}
M_1^1 & M_1^2 & \cdots & M_1^{252} \\
M_2^1 & M_2^2 & \cdots & M_2^{252} \\
\vdots & \vdots & \ddots & \vdots \\
M_1^{200} & M_2^{200} & \cdots & M_{252}^{200}
\end{pmatrix},
\]

The row and column check codes are defined as

\[
M_i = \sum_j M_j^i, M^j = \sum_i M_i^j
\]

respectively. So if one of elements \( M_j^i \) or \( M_i^j \) is changed, the corresponding row and column check code do not match. Since the Eq. (\textcolor{red}{4}) are simple summation, in this case, the repeater is able to recover the lost information, that is

\[
M_i^j = M_i - \sum_{l \neq i} M_{l}^j
\]

or

\[
M_i^j = M^j - \sum_{l \neq j} M_i^l.
\]

If one of elements in \( M_i^j(M^j) \) is lost, the repeater recalculate \( M_i^j(M^j) \) from the matrix \( M \). For simplicity, if
 FIG. 2: The CUHK emblem is transferred through a classical channel. Here \( \beta = 6 \) and \( S \) is in unit of 20000 steps. The leftmost column denotes the transferring without any repeaters; while other columns correspond to repeater-assisted cases for various inter-distances of \( D = 90, 50, 20, 2 \) respectively.

there are two or more elements are lost, we assume that the repeater is neither able to recover the integrability, nor requires the previous repeater to resend the picture again, but recalculate Eq. (6) to ensure the integrability of data for next repeater. We show our results in Fig. 2. In the second column, the distance between two repeaters is \( D = 90a \). The emblem keep much more information than the first column. Moreover, if we add more repeaters to the channel, the decay rate will be suppressed further. Especially if \( D = 2a \), the emblem at \( L = 2 \times 10^9a \) is almost the same as the original one. We then call such a phenomenon as a repeater-assisted classical Zeno effect in random processes.

Precisely, we can introduce the fidelity to describe the honesty of the state to the original one. In this case, the fidelity can be defined as

\[
F = \frac{\text{number of unchanged pixel}}{\text{total number of pixel}}.
\]

The results are shown in Fig. 3 from which we can see that the decay rate can be significantly suppressed if enough number of repeaters are installed. On the other hand, if there are two or more elements that have been found to be lost and the repeater can ask the previous repeater to resend the data again, then the classical state will never decay.

FIG. 3: The fidelity between the initial state and the state during transportation as a function of \( S \) (in unit of 20000 steps) for various \( D \).
IV. DISCUSSION

Now the game is over. However, We find ourselves in an embarrassed situation. Does such an issue still belong to physics? The answer seems to be YES since there is no any interferences from living beings and the “repeater”-like device needs energy only. Nevertheless, a digitized state and its decay clearly neither belong to quantum mechanics nor Newtonian mechanics. Such a puzzle keeps a challenge for us and is not able to be answered easily in physical science.

On the other hand, the classical Zeno effect is very popular phenomenon in the world. Besides the case of Super Mario’s prison break, the classical Zeno effect exists in many fields, such as the medical field and the economical field. Take the former as an example: A human being usually is in a metastable state. Some small deceases can introduce some fluctuations around the equilibrium point. In these cases, the body’s immune response and a suitable cure will recover the metastable. However, there exists a class of vital deceases, such as cancer, that will finally drag the state of human beings from the metastable point and never back if the state is far from the equilibrium point. Necessary cures clearly can slow down or ever stop such a process. However, to stop such a process, the most important is to find and cure the decease as early as possible, then the probability to recover the metastable state is very high.

V. SUMMARY

In summary, we touched such a possibility of classical Zeno effect in classical stochastic processes, as illustrated by a scenario of transferring a classical state through a noise channel. We show explicitly that the Zeno effect is no longer a patent of quantum mechanics, but exists in classical stochastic processes. We gave an example of transferring a BMP file through a noise channel, as simulated by classical Monte Carlo method. The example manifests that the decay rate of the BMP file will be significantly suppressed, even never decay, if there are enough number of repeaters.

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[1] B. Misra and E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1977).
[2] K. Koshino and A. Shimizu, Phys. Rep. 412, 191 (2005).
[3] S. J. Gu, EPL 88, 20007 (2009).