Interaction feature of caterpillar tracks with soil under significant axial displacements of pressure center

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Abstract. A calculation method is proposed for assessing the values of normal pressures and transverse reactions under the track of a transport-technological or transport vehicle. The technique is based on a deterministic approach to modeling the interaction of caterpillar tracks with soil. The geometric, power and other characteristics of the chassis and the main characteristics of the soil are taken into account. When modeling normal soil deformation, the Bernstein-Letoshnev relationship was used. Calculated dependencies are proposed, allowing to estimate the value of normal reactions on the supporting surface of the machine and the value of the coefficient of the moment of resistance to turning the chassis. An example of calculations for a tracked chassis weighing eight tons when predicting the track depth is given. The advantage of deterministic methods over stochastic ones in solving problems related to the operational assessment of changes made to the design of the undercarriage of a tracked vehicle has been substantiated.

1. Introduction
The study of modern publications in the field of interaction of the propulsion unit of forest tracked vehicles with soil shows that statistical models are most often used [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15].

This approach allows us to summarize the experimental data as much as possible, to propose universal methods for assessing the power consumption for resistance to motion, but it does not make it possible to analyze the process of interaction of the propulsion unit with the supporting surface and, based on the results of the estimated calculations, substantiate technical proposals related to the modernization of the propulsion unit.

The development of deterministic models of the interaction of caterpillar tracks with soil is more
typical for the areas of research related to the operation of the propulsion unit of transport vehicles [16,17,18,19,20,21, etc.]. The performed analysis of the design features of the running systems of forestry and transport tracked vehicles against the background of characteristic operating conditions [22, 23] suggests that the use of deterministic models as applied to the chassis of forest tracked vehicles will expand the understanding of the physical picture of the interaction of the propulsion unit with the support bases and substantiate proposals for modernization of undercarriages of forest tracked vehicles.

In particular, the deterministic model makes it possible to analyze the features of the interaction of the caterpillar tracks with the support base in the case of significant external forces acting on the machine, which causes a shift in the center of pressure and makes it necessary to clarify the model of the formation of resistance to turning the machine.

This mode of movement is typical for transport and traction tracked vehicles used in the logging complex, in particular – skidders, forwarders, harvesters.

2. Methods and Materials
When developing models for the interaction of a caterpillar track unit with soil, a hypothesis is accepted about the nature of the distribution of normal loads along the length of the supporting surface of the machine. It is known that the transfer of normal and tangential loads is carried out mainly through the so-called active sections of the supporting surface (that is, a group of tracks involved in interaction with the track roller) [16]. For transport vehicles with a small-link caterpillar tracks and large-diameter track rollers, the uneven distribution of normal loads is in most cases more pronounced than for tractors. For a specific undercarriage, it is possible by calculation method [17] or experimentally to estimate the distribution of normal pressures over the supporting surface. In the general case, as a rule, it is assumed that on a stationary machine in the absence of external forces, normal pressures are uniformly distributed along the length of the support surface (the center of gravity C of the machine is located in the geometric center of the projection of the support surface in plan, the distribution diagram has the shape of a rectangle). Further, when the torque is applied to the driving wheel, the distribution diagram is deformed and takes the shape of a trapezoid (figure 1, a). This approach is generally accepted in the theory of tracked vehicles (for example, works [18,24]).

![Figure 1](image)

**Figure 1.** View of the distribution diagram of normal pressures: a - full, b - partial loading of the supporting surface of the tracked vehicle.

A consequence of the deformation of the distribution diagram is the displacement of the centers of pressure of the tracks in the axial direction by the value \(x\). At this point we place the concentrated support reaction \(Z\).

In figure 1, the height of the center of mass relative to the support surface is indicated by \(h\), the length of the support surface is \(L\), the weight of the machine is \(G\), the direction of the velocity vector \(V\) is shown.
3. Results and Discussion

It is known (for example [18,24]) that \( x = \frac{L}{6} \frac{q_b - q_a}{q_b + q_a} \), where \( q_a \) and \( q_b \) – normal pressure at points A and B on the boundaries of the supporting surface of the machine, \( L = |AB| \) (design scheme for 0 > |x| > L/6 corresponds to figure 1, a).

Obviously, for the value \( |x| = L/6 \) we obtain \( q_b = 0 \): in this particular case, the distribution diagram acquires a triangular shape, but is distributed along the entire length of the supporting surface \( L \).

The reason for the increase in the displacement of the centers of pressure of caterpillar tracks may also be the effect of an external force \( P \) on the machine (load on the coupling device, "lifting resistance", aerodynamic resistance, load weight, etc.). In the general case, this force can be reduced to the resultant force, given by the horizontal, transverse and vertical and the components \( P_x, P_y, \) n \( P_z \). The coordinates of the point of application of the force are known and denoted as \( x_p, y_p, z_p \).

For practical calculations, it is more convenient to use dependence (1), which does not include the values \( q_a \) and \( q_b \):

\[
x = \frac{fZr \eta + P_x (h_p + r_{xx} / \eta) + P_z x_p}{ZL}
\]

In this dependence: \( f \) – coefficient of resistance to movement; \( r_w \) – radius of the driving wheel; \( \eta \) – efficiency of undercarriage. In the theory of tracked vehicles, the concept of a relative axial displacement of the centers of pressure is used. We introduce this quantity as \( x_0 = x/L \).

With an increase in the axial displacement of the centers of pressure, a case may arise in which the pressure distribution diagram is not distributed over the entire length of the supporting surface (figure 1, b): \( L' = |BD| \).

Support surface area \( L - L' = |AD| \) turns out to be unloaded from normal effort. The display of track instability can lead to significant energy losses associated with unproductive track movements [19].

To determine the part of the length of the support surface \( L' \), used in the transmission of normal loads, consider the equilibrium of the machine relative to point B (see figure 1, b).

The moment of distributed forces (from the distributed load \( q(x) \)):

\[
M_b = \frac{1}{2} (2bq_bL') \left( \frac{L'}{3} \right) = \frac{bq_b}{3} \left( \frac{L'}{3} \right)
\]

The support reaction is defined as \( Z = 2 \cdot \frac{1}{6} bq_bL' = bq_b L' \).

The moment of support reaction is defined as \( M_b = Z(L/2-x) = bq_b L' (L/2-x) \).

Therefore, \( bq_bL' \left( \frac{L}{2} - x \right) = \frac{bq_b}{3} \left( \frac{L'}{3} \right) \), whence, for \( |x_0| \in [1/6, 1/2) \), we obtain:

\[
L' = 3L(0.5-x_0) \text{ or } L'/L = 3(0.5-x_0)
\]

Let us consider the case of a uniform turn on a horizontal surface in the absence of skidding and slipping of the tracks. We assume that the lateral reactions occurring when turning are directly proportional to the normal load [18, 24]. The proportionality coefficient is the value \( \mu \) – coefficient of resistance to turning [18, 24]. How the value of the value \( \mu \) is determined in the context of a given task is not important.

The method for determining the required traction forces on the tracks will not differ from that used in the case of a trapezoidal diagram of normal pressures [18,24]. However, it is necessary to propose a dependence to determine the value of the moment to the resistance to turning.

Figure 2 shows the design scheme of rotation for the case of underutilization of the length of the
supporting surface. The pivot center is located at point \( O \), the center of mass of the machine is located at the geometric center of the chassis (point \( C \)). The turning radius is denoted by \( R \).

The distribution of lateral forces for the running board caterpillar is conventionally shown. The pivot pole of this track is at point \( O_2 \). Track section \([AD]\) is not loaded with normal forces, therefore, the lateral reactions on it are significantly lower than in the \([DB]\) section, and in this design scheme are taken to be zero. In a more general case, the diagram may contain a transverse component of the external force \( P_y \), which changes the equilibrium conditions of the machine in the transverse direction. In this case, the values of the support reactions under the tracks are determined as when turning the machine under the action of an arbitrary external force (see, for example, [18, 24]).

The equilibrium of the machine in the transverse direction requires the adoption of the hypothesis about the equality of the areas of the lateral reaction diagrams and the appearance of \( \chi \) – axial displacement of the turning poles of the tracks. This hypothesis also makes it possible to obtain a calculated dependence for \( \chi \) for a given pattern of lateral force distribution.

Let us locate the origin at point \( O_2 \). The abscissa axis is co-directed with the vector of the board's linear velocity. The ordinate axis is directed from the center of rotation of the machine. In the general case, the equilibrium of the machine in the transverse direction is determined by the equation:

\[
\int_{-b/2}^{b/2} \int_{-L/2}^{L/2} \mu q_1(x) dx dy + \int_{-b/2}^{b/2} \int_{-L/2}^{L/2} \mu q_2(x) dx dy = \int_{-L/2}^{L/2} \int_{-b/2}^{b/2} \mu q_1(x) dx dy + \int_{-b/2}^{b/2} \int_{-L/2}^{L/2} \mu q_2(x) dx dy, \text{ or }
\]

\[
\int_{-L/2}^{L/2} (q_1(x) + q_2(x)) dx = 0 \quad (4)
\]

In the last expression \( b \) – track width. The sign before \( P_y \) (transverse projection of the external force) is determined by the force application pattern. The presence of the transverse component of the external force will also determine the inequality of the normal load on the tracks of the sides.

The change in normal load is described by a linear function:

\[
q(x) = q_{b,i} \frac{L - x}{L} = 2Z_i \frac{(L - x)}{bL}, i = 1, 2
\]

(5)

Here \( Z_i \) – normal reaction under the caterpillar track of the lagging and leading sides.

In a discrete setting, one can go from integration to summation, for example, considering the loading of individual tracks on the support surface:

\[
\mu b \sum_{i=1}^{K_1} q_{j,i} + \mu b \sum_{j=1}^{K_2} q_{j,2} \pm P_y = \mu b \sum_{i=1}^{K_1} q_{j,i} + \mu b \sum_{j=1}^{K_2} q_{j,2},
\]

or

\[
\sum_{i=1}^{K_1} (q_{j,1} + q_{j,2}) \pm P_y / \mu b = \sum_{j=1}^{K_2} (q_{j,1} + q_{j,2})
\]

(6)

Here \( K_1 \) – number of tracks to be determined corresponding to the distance \([BO_2]\), and \( K_2 \) – known number of tracks corresponding to the distance \([BD]\). At the step of the link \( t \) we obtain \( K_1 = \chi \) and \( K_2 = L' \).

In the particular case shown in figure 2, there is no transverse projection of the external force. Since the value of \( \mu \) is taken to be independent of \( x \), instead of the last equation in order to determine \( \chi \) it is sufficient to consider essentially the condition of equality of the areas of the diagrams of distributed lateral forces, which have, respectively, the shape of a trapezoid and a triangle:
\[
\frac{1}{2}(q_b + q_o)\left(\frac{L}{2} - \chi\right) = \frac{1}{2}q_o\left(L' - \frac{L}{2} + \chi\right)
\]

or
\[
\frac{1}{2}(q_b + q_o)\left(\frac{L}{2} - \chi\right) = \frac{1}{2}q_o\left(L' - \frac{L}{2} - \chi\right)
\]

(7)

From the similarity of triangles, it is possible to establish a connection between \(q_b\) and \(q_o\):

\[
q_b = q_o \frac{L'}{L' - 0.5L + \chi}
\]

(8)

The solution has the form:
\[
\chi = 0.5L - \left(1 - \sqrt{2}/2\right)L'.
\]

By analogy with the relative displacement of the centers of pressure, the concept of the relative axial displacement of the poles of rotation of the caterpillar tracks is introduced:
\[
\chi_0 = \frac{\chi}{L}.
\]

As a result:
\[
\chi_0 = 0.5 - \left(3 - 1.5\sqrt{2}\right)\left(0.5 - x_0\right)
\]

(9)

At \(x_0 = 1/6\) (the distribution diagram of normal pressures has taken on a triangular shape, but is distributed over the entire support surface - see figure 1, a), the calculated value of \(\chi\) coincides with that calculated by the traditional method (the calculated dependences are given, for example, in [18, 24]).

To determine the moment of resistance to turning, the sum of the moments of the distributed lateral forces relative to the poles of rotation of the tracks is considered. For one track:

\[
M_{ci} = \int_{-b/2-L/2}^{b/2} \int_{-b/2-\gamma}^{b/2} \mu q_i(x)dx\,dy + \int_{-b/2-\gamma}^{b/2} \int_{-b/2}^{b/2} \mu q_i(x)dx\,dy, \quad i=1, 2
\]

(10)

For a vehicle as a whole \(M_c = M_{c1} + M_{c2}\). It makes sense to consider the tracks separately if there is an uneven distribution of the load along the sides.

With a discrete setting of the normal load, you can also go from integration to summation:

\[
M_{ci} = \mu b \int_{j=1}^{K_i} \left(K_i - j\right)q_{i,j} + \sum_{j=K_i}^{K_i} \left(j - K_i\right)q_{i,j}, \quad i=1, 2
\]

(11)

For the case in figure 2 you can go to concentrated forces corresponding to the distributed load:

\[
S = \int_{-b/2-L/2}^{b/2} \int_{-b/2-\gamma}^{b/2} \mu q_i(x)dx\,dy = \int_{-b/2-L/2}^{b/2} \int_{-b/2-\gamma}^{b/2} \mu q_i(x)dx\,dy = \frac{\mu Z}{4}, \quad i=1, 2
\]

(12)

The magnitude of the concentrated forces is determined by the sum of the areas of the lateral reaction diagrams. The concentrated force is applied at the center of mass of the diagram and is directed normally to the axis of the machine (figure 3).

Figure 3 shows the location of the points of application of concentrated forces \(S\) and their arms \(\Delta I\) and \(\Delta II\) relative to the rotation pole \(O_2\)
Figure 2. View of the distribution diagram of lateral reactions when turning with partial loading of the support surface of the tracked vehicle.

Thus:

\[ \Delta_1 = \frac{L}{2} - \chi - \Delta_0; \quad \Delta_{\Pi} = \frac{L}{2} - 0.5L + \chi; \quad \Delta_0 = \frac{2q_o + q_b}{3(q_o + q_b)} \left(0.5L - \chi\right); \quad M_c = 2S(\Delta_1 + \Delta_{\Pi}) \] (13)

The expression for determining the value of the moment of resistance to turning can be written in the traditional form

\[ M = K_M ZL/4. \]

Then the value of the coefficient of the moment of resistance to turning is proposed to be determined by the dependence:

\[ K_M = \frac{5}{3} - 2x_0 - \frac{4}{3}\chi_0 - \frac{\left(3.5 - 9x_0 + 2\chi_0\right)(1 - 2\chi_0)}{7.5 - 18x_0 + 3\chi_0} \] (14)

At \( x_0 = 1/6 \) the value of the last expression coincides with the value determined by the traditional method, see [18, 24].

Calculations show that with the considered load distribution scheme, in comparison with the traditional case, there is a tendency to an increase in the depth of the vehicle track (and, consequently, to an increase in resistance to rectilinear movement). Figure 4 shows an assessment of the change in the maximum value of the normal pressure \( q_{max}^* \) and the depth of the track \( z_{1max}^* \) and \( z_{2max}^* \) on dense sandy and clayey soils for a running model weighing 8 tons. Soil parameters and calculation methods are published in the article [20].

Calculations show that for soil 1, the ultimate bearing capacity is achieved in the region of \( L'/L \approx 0.7 \). With a further decrease in \( L'/L \) the vehicle begins to plunge into the ground, settling on the stern. The scheme of forces acting on the machine changes and the model we are considering loses its efficiency. With a ratio of \( L'/L \approx 0.6 \) the vehicle is expected to land on the bottom (the depth of the track will exceed the ground clearance \( H_{max} \)).
4. Conclusion
In conclusion, the following major findings can be noted: The effect of incomplete use of the support branch of the propulsion unit when transmitting normal loads should be taken into account when assessing the impact of a tracked vehicle on the roadway, drawing up traction, energy and power balances of the chassis, as well as when building a model for assessing the energy efficiency of the chassis of transport and transport traction tracked vehicles [25], the development of designs of transmission mechanisms [26, 27, 28] and chassis [29, 30].

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