Variability of gait spatio-temporal parameters during treadmill walking

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ABSTRACT

During treadmill walking, the subject’s stride length (SL) and duration (ST) yield a stride speed (SS) which fluctuates over a narrow range centered on the treadmill belt’s speed. We recently demonstrated that ST and SL trends are strongly correlated and serve as control manifolds about which the corresponding gait parameters fluctuate. The fundamental problem, which has not yet been investigated, concerns the contribution of SL and ST fluctuations to SS variability. To investigate this relation, we approximate SS variance by the linear combination of SL variance and ST variance, as well as their covariance. The combination coefficients are nonlinear functions of ST and SL mean values and, consequently, depend on treadmill speed. The approximation applies to constant speed treadmill walking and walking on a treadmill whose belt speed is perturbed by strong, high-frequency noise. In the first case, up to 80% of stride speed variance comes from SL fluctuations. In the presence of perturbations, the SL contribution decreases with increasing speed, but its lowest value is still twice as large as that of either ST variance or SL-ST covariance. The presented evidence supports the hypothesis that stride length adjustments are primarily responsible for speed maintenance during walking. Such a control strategy is evolutionarily advantageous due to the weak speed dependence of the SL contribution to SS variance. The ability to maintain speed close to that of a moving cohort did increase the chance of an individual’s survival throughout most of human evolution.

Introduction

Humans seamlessly adapt gait to carry out concurrent tasks or to cope with environmental conditions that can hinder their motor performance. Gait adaptation may be achieved, for example, by adjusting speed, foot clearance, the base of support, or by executing arm and torso movements. Interestingly enough, stride length (SL), stride time (ST), and stride speed (SS) do fluctuate during undemanding straight-line walks on a level surface or even during treadmill walking¹. Gait fluctuations attracted general interest forty years ago²–⁴ but the first empirical studies on variability in movement execution were conducted much earlier, at the end of the 19th century. The research that followed Woodworth’s classic paper⁵ on line drawing led to the formulation of Fitt’s law⁶ and the degrees of freedom problem⁷. The former describes the speed-accuracy trade-off in human reaching tasks. Fitt’s law states that there are multiple ways for humans or animals to perform a movement that achieves a given goal. The main insight from these early studies was that a certain irreducible noise level is an intrinsic property of motor control systems.

The origin and significance of movement variability has been vigorously debated, generating a number of theoretical explanations. According to generalized motor program theory (GMPT)⁸ this variability originates from errors in the selection of the relevant control program (central command error), as well as from errors in its execution (peripheral error). Regardless of its source, increased variability is perceived as detrimental to motor performance. In dynamical system theory (DST)⁹,¹⁰, a sensorimotor system is considered to be a highly dimensional nonlinear dynamical system. By its very nature, it is redundant - a given task can be achieved in multiple ways. A motor controller does not select a unique optimal solution but rather samples the family of solutions congruent with the task. Within this framework, movement variability is an expression of flexibility and self-organization. Redundancy also lies at the heart of optimal feedback control theory¹¹ that states that sensorimotor controllers obey a minimal intervention principle. They act only on those variables in the multidimensional state space that are goal-relevant. The other variables are free to vary and form the so-called uncontrolled manifold (UCM)¹².

Dingwell et al.¹³ realized that both the minimal intervention principle and energy optimization underlie the control of spatiotemporal gait parameters during treadmill walking. Generally, during such movement, any SL and ST whose ratio is equal to the belt’s speed lies on the UCM. However, humans minimize the energy cost of walking by choosing specific values of both parameters at a given speed. Thus, there is a preferred operating point (POP) on the constant speed goal equivalent
Figure 1. Stride length, time, and speed during the perturbed walk of subject 10. The data come from the study of Moore et al.\textsuperscript{16}. For clarity, the stride time and length time series were vertically shifted by 0.15 s and 0.30 m, respectively. Thick solid lines show trends in stride time and length calculated using the piecewise linear variant of the MARS model. There were no trends in stride speed. The inset shows the commanded belt speed during the perturbation phase of the trial.

Trends in gait spatiotemporal parameters are ubiquitous in overground and treadmill walking. In our recent work\textsuperscript{14}, we used Multivariate Adaptive Regression Splines (MARS)\textsuperscript{15} to determine SL and ST trends in treadmill walking and found that they are strongly correlated. We showed that trends serve as control manifolds about which ST and SL fluctuate. Moreover, the trend speed, defined as the ratio of the instantaneous values of SL and ST trends, is tightly controlled about the belt’s speed. The strong coupling between ST and SL trends ensures that the concomitant changes correspond to movement along the constant speed GEM as postulated by Dingwell et al.

The fundamental problem, which has not yet been investigated, concerns the contribution of SL and ST fluctuations to SS variability. In this work, we derive an approximation to SS variance. Then, we use it to illuminate gait control during walking on a treadmill whose belt speed is perturbed by a strong, high-frequency noise. This is considered to be the simplest model of continuous gait adaptation.

Results

We give an example of the time evolution of gait parameters in Fig. 1. In the presence of high-amplitude, high-frequency perturbing noise, shown in this figure’s inset, the slopes of SL and ST trends are small compared to the unperturbed treadmill walking\textsuperscript{14}. One can also notice that ST variability is markedly smaller than those of SL and SS.

In Fig. 2 we present the boxplots of variance of gait spatio-temporal parameters (top row) and normalized contributions to stride speed variance $P_n^{(n)}$ (bottom row) for the FNW (left column) and PW (right column).

The values of all analyzed parameters for $v = 1.2 \text{ m/s}$ were collected in Table 1.

Variance of gait parameters

The gait parameters’ variances during the PW were greater than those for the FNW.

For FNW, $\sigma_{SL}^2$ was independent of treadmill speed. On the other hand, $\sigma_{ST}^2$ decreased with treadmill speed from $(0.78 \pm 0.45) \times 10^{-3} \text{ s}^2$ at 0.8 m/s to $(0.18 \pm 0.07) \times 10^{-3} \text{ s}^2$ at 1.6 m/s. While $\sigma_{SS}^2$ at the highest speed was 25% greater than that at the lowest one: $(2.26 \pm 0.89) \times 10^{-3} \text{ m}^2/\text{s}^2$ vs $(1.81 \pm 0.97) \times 10^{-3} \text{ m}^2/\text{s}^2$, this increase was statistically insignificant.
Table 1. Stride speed variance $\sigma_{SS}^2$ and its approximation $\hat{\sigma}_{SS}^2$ for the experiment with treadmill speed 1.2 m/s performed by Moore et al.\textsuperscript{16}. The data are presented for both first normal walk (FNW) and perturbed walk (PW). $P_1^{(n)}$, $P_2^{(n)}$, and $P_3^{(n)}$ are the normalized contributions to $\sigma_{SS}^2$ of stride length, covariance of stride length and stride time, stride time, respectively.

| subject | $\sigma_{SL}^2$ $[10^{-3} \text{m}^2]$ | $\sigma_{ST}^2$ $[10^{-3} \text{s}^2]$ | COV$(SL,ST)$ $[10^{-4} \text{ms}]$ | $\sigma_{SS}^2$ $[10^{-3} \text{m}^2/\text{s}^2]$ | $\hat{\sigma}_{SS}^2$ $[10^{-3} \text{m}^2/\text{s}^2]$ | $P_1^{(n)}$ | $P_2^{(n)}$ | $P_3^{(n)}$ |
|---------|--------------------------------|--------------------------------|-----------------|--------------------------------|-----------------|---------|---------|---------|
| **FNW 1.2 m/s** | | | | | | | | |
| 3       | 1.42                         | 0.28                         | -0.05           | 2.09                           | 1.89             | 0.91    | 0.74    | 0.01    | 0.16    |
| 4       | 2.83                         | 0.44                         | -4.08           | 3.41                           | 3.41             | 1.00    | 0.68    | 0.20    | 0.11    |
| 5       | 0.79                         | 0.13                         | -1.11           | 1.01                           | 1.06             | 1.05    | 0.71    | 0.21    | 0.13    |
| 6       | 1.89                         | 0.34                         | -2.31           | 2.31                           | 2.33             | 1.01    | 0.70    | 0.18    | 0.13    |
| 7       | 0.56                         | 0.28                         | -0.79           | 1.01                           | 0.92             | 0.92    | 0.50    | 0.15    | 0.27    |
| 8       | 0.96                         | 0.09                         | -0.05           | 1.16                           | 1.28             | 1.10    | 0.99    | 0.01    | 0.10    |
| 9       | 1.61                         | 0.30                         | -1.38           | 2.40                           | 2.32             | 0.97    | 0.70    | 0.13    | 0.15    |
| 10      | 1.21                         | 0.42                         | -0.17           | 1.35                           | 1.37             | 1.01    | 0.72    | 0.02    | 0.26    |
| 11      | 1.22                         | 0.31                         | -0.85           | 1.56                           | 1.57             | 1.01    | 0.71    | 0.10    | 0.20    |
| 12      | 1.72                         | 0.17                         | -1.21           | 1.32                           | 1.31             | 1.00    | 1.04    | -0.15   | 0.11    |
| 13      | 1.69                         | 0.29                         | -0.03           | 1.70                           | 1.70             | 1.00    | 0.84    | 0.00    | 0.15    |
| 14      | 1.14                         | 0.17                         | 0.35            | 1.19                           | 1.07             | 0.90    | 0.81    | -0.05   | 0.14    |
| **mean** | 1.43                         | 0.27                         | -0.77           | 1.71                           | 1.69             | 0.99    | 0.76    | 0.07    | 0.16    |
| **std** | 0.60                         | 0.11                         | 1.38            | 0.72                           | 0.72             | 0.06    | 0.15    | 0.11    | 0.06    |
| **PW 1.2 m/s** | | | | | | | | |
| 3       | 3.44                         | 1.77                         | 1.85            | 5.71                           | 5.71             | 1.00    | 0.70    | -0.08   | 0.37    |
| 4       | 4.86                         | 1.37                         | -11.10          | 7.19                           | 7.18             | 1.00    | 0.56    | 0.27    | 0.17    |
| 5       | 3.32                         | 1.34                         | -8.10           | 5.58                           | 5.48             | 0.98    | 0.50    | 0.26    | 0.23    |
| 6       | 8.43                         | 1.76                         | -2.82           | 10.34                          | 10.40            | 1.01    | 0.78    | 0.53    | 0.17    |
| 7       | 4.13                         | 1.65                         | -11.48          | 7.69                           | 7.50             | 0.98    | 0.48    | 0.28    | 0.21    |
| 8       | 2.18                         | 1.21                         | -3.76           | 4.94                           | 4.93             | 1.00    | 0.52    | 0.19    | 0.30    |
| 9       | 3.70                         | 1.04                         | -6.13           | 6.47                           | 6.54             | 1.01    | 0.61    | 0.21    | 0.19    |
| 10      | 5.60                         | 2.06                         | -6.35           | 7.55                           | 7.41             | 0.98    | 0.61    | 0.14    | 0.23    |
| 11      | 4.41                         | 1.32                         | -6.03           | 6.37                           | 6.44             | 1.01    | 0.62    | 0.18    | 0.21    |
| 12      | 3.07                         | 0.75                         | -3.80           | 3.72                           | 3.75             | 1.01    | 0.66    | 0.17    | 0.18    |
| 13      | 5.10                         | 1.43                         | -1.04           | 6.05                           | 6.19             | 1.02    | 0.76    | 0.32    | 0.24    |
| 14      | 6.03                         | 2.08                         | -1.14           | 7.34                           | 7.33             | 1.00    | 0.70    | 0.03    | 0.27    |
| **mean** | 4.52                         | 1.48                         | -4.99           | 6.58                           | 6.57             | 1.00    | 0.63    | 0.14    | 0.23    |
| **std** | 1.66                         | 0.40                         | 4.02            | 1.66                           | 1.65             | 0.01    | 0.10    | 0.11    | 0.06    |
Figure 2. The variance of stride speed (SS), stride length (SL), and stride time (ST) (top row, subplots (a) and (b)). The normalized contribution of stride length ($P_1^{(n)}$), covariance of stride length and stride time ($P_2^{(n)}$), and stride time ($P_3^{(n)}$) to stride speed variance (bottom row, subplots (c) and (d)). The boxplots are shown for three treadmill speeds for both unperturbed (left column) and perturbed walk (right column).

At the lowest speed of the perturbed walk, $\sigma_{\text{SL}}^2 = (3.04 \pm 1.04) \times 10^{-3} m^2$ was significantly smaller than those at 1.2 m/s ($4.52 \pm 1.66) \times 10^{-3} m^2$ and 1.6 m/s ($4.17 \pm 1.25) \times 10^{-3} m^2$. Speed dependence of stride time variance was absent in PW. In presence of perturbation, $\sigma_{\text{SS}}^2$ strongly increased with the treadmill speed $p = 5 \times 10^{-10}; (2.64 \pm 0.90) \times 10^{-3} m^2/s^2$, $(6.58 \pm 1.66) \times 10^{-3} m^2/s^2$, and $(9.03 \pm 1.84) \times 10^{-3} m^2/s^2$ for 0.8 m/s, 1.2 m/s, and 1.6 m/s, respectively.

Covariance between stride length and stride time
During FNW, $\text{Cov}(\text{SL}, \text{ST})$ did not depend on treadmill speed and was equal to $0.11 \pm 3.49 \times 10^{-4} m/s, (-0.77 \pm 1.38) \times 10^{-4} m/s, (-1.10 \pm 0.70) \times 10^{-4} m/s$ for 0.8 m/s, 1.2 m/s, 1.6 m/s, respectively. For PW, the corresponding values were equal to $(-2.03 \pm 5.21) \times 10^{-4} m/s, (-4.99 \pm 4.02) \times 10^{-4} m/s, (-6.26 \pm 5.90) \times 10^{-4} m/s$. The difference in $\text{Cov}(\text{SL}, \text{ST})$ between FNW and PW was significant for 1.2 m/s ($p = 0.002$) and 1.6 m/s ($p = 0.003$). For PW, the covariance at the lowest speed was statistically smaller than at the highest one.

Efficacy of stride speed variance approximation
For all $\gamma$ and both walking conditions, the experimental stride speed variance $\sigma_{\text{SS}}^2$ was not different from its estimate $\hat{\sigma}_{\text{SS}}^2$. For FNW, the ratio $\sigma_{\text{SS}}^2/\sigma_{\text{SS}}^2$ was equal to $0.86 \pm 0.15, 0.99 \pm 0.06, 0.98 \pm 0.05$ for the successive treadmill speeds. For PW, these ratios were equal to $1.00 \pm 0.02, 1.00 \pm 0.01$, and $1.01 \pm 0.01$.

Normalized contributions to stride speed variance
For FNW, $P_2^{(n)}$ at 0.8 m/s was significantly smaller than at 1.6 m/s $-0.04 \pm 0.25$ vs $0.14 \pm 0.06$. Otherwise, the parameters $P_i^{(n)}$ did not depend on a treadmill speed. The dominant contribution to the stride speed variance came from stride length. $P_1^{(n)}$ was equal to $0.73 \pm 0.20, 0.76 \pm 0.15$, and $0.66 \pm 0.07$ for 0.8 m/s, 1.2 m/s, and 1.6 m/s, respectively.
In the presence of perturbations, \( P_{(n)} \) was speed dependent \((p = 8 \times 10^{-6})\). In particular, this parameter decreased by 32% from 0.77 ± 0.11 at 0.8 m/s to 0.52 ± 0.07 at 1.6 m/s. This change resulted in the increased contribution of ST-SL covariance and ST to the stride speed variance \( \sigma_{SS}^2 \). At the highest speed, we found that \( P_{(2)} = 0.19 ± 0.22 \) and \( P_{(3)} = 0.30 ± 0.17 \) and these values were statistically greater than the corresponding values at the slowest speed: \( P_{(2)} = 0.06 ± 0.10 \) and \( P_{(3)} = 0.18 ± 0.04 \).

For the 0.8 m/s trial, there were no statistically significant differences between the values of \( P_{(n)} \) for FNW and PW. We observed the same effect for \( P_{(n)} \) at the medium speed. In the other cases, \( P_{(2)} \) and \( P_{(3)} \) were higher for PW, whereas for \( P_{(1)} \) was higher for FNW.

### Dependence of coefficients \( c_i \) on gait speed

We approximated stride speed variance by the linear combination equation (6d). The coefficients \( c_i \) of this combination are defined in terms of \( c_i(f) \), see equation (8). While \( c_1(f) \) is constant, both \( c_2(f) \) and \( c_3(f) \) depend on the average values of stride length and time which in turn change with treadmill speed \( v \). Fig. 3a shows speed dependence of \( c_i(f) \) for the perturbed walk at 1.2 m/s. In the insets of this subplot, the filled circles represent the group-averaged values of stride length and time calculated for the three values of \( v \) used in the experiment. The lines connecting the circles are the linear \( SL(v) = 0.43 + 0.54v \) and hyperbolic \( ST(v) = 0.65 + 0.47/v \) fits to the experimental averages. The chosen form of fitting functions stems from the properties of the force-driven harmonic oscillator model of human gait\(^7\). We used these functions to plot \( c_i(f) \) (Fig. 3a) and \( c_i \) (Fig. 3b) as a continuous function of \( v \).

For all subjects, the average of \( \mu_{SL}/\mu_{ST} \) was not statistically different from \( \mu_{SS} \). Therefore, we may rewrite equation (9) in a more illuminating form:

\[
\begin{align*}
c_1^{(f)} &= 1, \\
c_2^{(f)} &= -2\mu_{SS}, \\
c_3^{(f)} &= \mu_{SS}.
\end{align*}
\]

Thus, \( c_2^{(f)} \) and \( c_3^{(f)} \) are proportional to the stride speed and its square, respectively.

It follows from equation (8) that all three coefficients \( c_i \) are nonlinear functions of treadmill speed with \( c_1 \) being the least and \( c_3 \) being the most speed dependent. One can find in Fig. 3b that at \( v = 1.2 \) m/s \( c_1 = 0.93s^{-2} \), \( c_2 = -1.92m/s^3 \), and \( c_3 = 0.99m^2/s^4 \). At the highest experimental treadmill speed \( v = 1.6 \) m/s, the coefficients take on the following values \( c_1 = 1.13s^{-2} \), \( c_2 = -3.09m/s^3 \), and \( c_3 = 2.12m^2/s^4 \). With respect to \( v = 1.2 \) m/s, the corresponding relative percentage changes are equal to 22%, 61%, and 113%.

It is worth pointing out that for normal (close to the preferred walking speed, PWS) and high \( v \), \( c_i^{(f)} \) well approximate \( c_i \). For example, at \( v = 1.2 \) m/s, the relative percentage error for all three coefficients is equal to 8%. At \( v = 1.6 \) m/s, \( c_i^{(f)} \) underestimate \( c_i \) by 11%.

### Discussion

It is not surprising that in the presence of a strong perturbation, the gait parameters’ variances increase (Fig. 2a-b)\(^18\)\(^-\)\(^20\). It is the speed dependence of these variances during the perturbed walk that is intriguing and provides insight into how human gait is controlled. While \( \sigma_{ST}^2 \) did not change with \( v \), \( \sigma_{SL}^2 \) at 1.2 m/s and 1.6 m/s was 49% and 37% higher than that at 0.8 m/s, respectively. Thus, the subjects tend to preserve movement rhythmicity and match the belt’s speed by adjusting stride length. This is reflected by the value of \( P_{(1)} \) – the normalized contribution of SL fluctuations to stride speed variance \( \sigma_{SS}^2 \). For healthy subjects, the speed control at 0.8 m/s is hardly challenging, even in the presence of perturbation. In this case, \( P_{(1)} \) was as high as 0.77 ± 0.11. While it decreased to 0.52 ± 0.07 at 1.6 m/s, it was still about two times larger than the contributions from the SL-ST covariance \( (p_{(2)} = 0.06 ± 0.10) \) and ST \( ST \) \( (p_{(3)} = 0.30 ± 0.17) \).

The unperturbed segments (FNW, SNW) in Moore’s experiment were short. Therefore, we also reanalyzed data from our previous study\(^21\) in which the subjects walked 400 m at three speeds. In this case, the dominant contribution to the stride speed variance also came from stride length. \( P_{(1)} \) was equal to 0.7 ± 0.1, 0.7 ± 0.1, and 0.6 ± 0.1 for 1.1 m/s, 1.4 m/s, and 1.7 m/s, respectively. We will present a detailed analysis of this dataset elsewhere.

The fundamental question arises as to why humans employ a control strategy which is mainly based on step/stride length adjustments. The plausible answer comes in two parts. First, from the evolutionary viewpoint, foot placement is more important than the duration of movement. The second argument follows from the analysis of treadmill speed influence on \( \sigma_{SS}^2 \). Even though \( \sigma_{SL}^2, Cov(SL,ST) \), and \( \sigma_{SL}^2 \) did not change between 1.2 m/s and 1.6 m/s, \( \sigma_{SS}^2 \) increased by 37%. This effect stems from
Figure 3. Dependence of coefficients $c_1^{(f)}$ (a) and $c_i$ (b) on treadmill speed. In the insets of the top subplot, the filled circles represent the group-averaged values of stride length (SL) and time (ST) calculated for three treadmill speeds used in the experiment. The lines connecting the circles are the linear and hyperbolic fits to the experimental averages of SL and ST, respectively. We used these fit functions to plot both types of coefficients as a continuous function of treadmill speed.
the speed dependence of the coefficients $c_i$ of the linear combination equation (6d) which we used to approximate $\sigma^2_{SS}$. With respect to $v = 1.2\text{ m/s}$, $c_1$, $c_2$, $c_3$ increased by 22%, 61%, and 113%, respectively. It is apparent from Fig. 3b that $c_1$ is the least speed-dependent, and consequently, the stride length control strategy minimizes stride speed variability while navigating through rough terrain. The ability to maintain a speed close to that of a moving cohort was instrumental to an individual’s survival during most of homo sapiens evolution.

Bank et al. studied gait adjustment during walking paced by either metronome beeps or equidistant stepping stones projected on a treadmill belt. The changes in gait were brought about in two ways: by an instantaneous perturbation of the cue sequence and by an abrupt switch from one type of pacing to the other. The subjects recovered from the stepping stone perturbations much faster than from the metronomic ones. Switching from acoustic to visual pacing was shorter than vice versa. Thus, these experimental findings corroborate the priority of stride length control.

Gait adaptability is indispensable for safe walking in unfamiliar surroundings. A recent meta-analysis demonstrated that reactive and volitional stepping interventions reduced falls in older adults by almost 50%. The theoretical and experimental evidence presented in this work provides the rationale for such interventions that are slowly entering the rehabilitation mainstream.

Whether gait variability may be used as a proxy of adaptation capabilities remains an open question. Notwithstanding, numerous studies explored the possibility of restoring it to a physiological level, for example, in Parkinson’s disease and aging.

The auditory system, a fast and accurate processor of temporal information, projects into the motor structures of the brain enabling entrainment between the rhythmic signal and the motor response. This coupling has been the driving force for numerous studies on rhythmic auditory stimulation (RAS) in motor rehabilitation, particularly in patients with Parkinson’s disease. While this type of cueing does improve walking speed, stride length, and cadence, it suppresses statistical persistence in ST intervals – the fundamental trait of human gait. Persistent metronomes and music not only circumvent this problem but can be more effective due to complexity matching effect – maximization of information exchange between systems with similar complexities.

It is worth pointing out that non-isochronous cueing can involve step length adjustments. In an ingenious experiment, Almurad et al. demonstrated gait variability restoration in older adults walking arm-in-arm with a young companion. The induced changes persisted two weeks after the end of the training session. Vaz et al. observed a significant increase of DFA scaling exponent in older adults walking to a fractal-like visual stimulus. In light of the results presented in this paper, the efficacy of both interventions is not unexpected.

Instrumented treadmills with obstacles and targets projected on the belt’s surface elicit task-specific step modifications. Consequently, they could be used in future research to answer the fundamental question: is gait variability a manifestation of adaptability? Said differently, is variability a prerequisite for efficient gait control? Taking into account that stride time variability can be easily measured with wearable devices or even smartphones, such an answer would significantly influence gait rehabilitation and fall prevention.

## Methods

### Experimental Data

In our analysis, we used a dataset from the study of Moore et al. that is available from the Zenodo repository. Fifteen young, healthy adults walked on a motor-driven treadmill for ten minutes at three speeds (0.8 m/s, 1.2 m/s, and 1.6 m/s). Each trial started with a one-minute normal (unperturbed) walk (first normal walk – FNW). This phase was followed by an 8-minute longitudinally perturbed walk (PW). The trial ended with a 1-minute second normal walk (SNW). Due to technical issues, we discarded some records. The analyzed dataset consisted of 11, 12, and 10 records for treadmill speed 0.8 m/s, 1.2 m/s, and 1.6 m/s, respectively. In the PW part of the trial, during each stance phase of the subject’s gait, the treadmill belt’s speed varied randomly. The standard deviation of the noise was equal to 0.06 m/s, 0.12 m/s, and 0.21 m/s for 0.8 m/s, 1.2 m/s, and 1.6 m/s experiments, respectively. A comprehensive description of the experimental protocol and participants’ characteristics can be found in the original paper.

The motion capture trajectories of RLM and LLM (right/left lateral malleolus of the ankle) markers were resampled at 100 Hz to determine a subject’s stride length (SL), stride time (ST), and stride speed (SS). The leading and trailing segments of the gait time series, for which the belt’s speed was lower than the target value by more than 15%, were excised. A heel strike was defined as the point where the forward foot marker was at its most forward point during each gait cycle. Step duration was equal to the elapsed time between the ipsilateral and contralateral heel strikes. We calculated SL and ST as the sum of the corresponding values for two consecutive steps. The quotient of SL and ST yielded SS.
Identification of trends in gait parameters

We used Multivariate Adaptive Regression Splines (MARS)\(^\text{15}\), to approximate trends in gait spatiotemporal parameters. We employed a piecewise linear, additive MARS model (the order of interaction was equal to 1) with a maximum number of basis functions equal to 50. The generalized cross-validation knot penalty \( c \) and the forward phase’s stopping condition were set to 2 and 0.001, respectively. A detailed description of MARS gait trend analysis can be found in our previous paper\(^\text{14}\).

Stride speed variance estimation

Let us consider the ratio \( f(X, Y) = X/Y \) of two stationary, random variables \( X \) and \( Y \) whose mean values are equal to \( E(X) = \mu_X \) and \( E(Y) = \mu_Y \), respectively. To elucidate such dependence in equation (6) we factored out the common term \( 1/\mu_Y \) by \( \mu_X \) and \( \mu_Y \), respectively. A detailed description of MARS gait trend analysis can be found in our previous paper\(^\text{14}\).

If in the definition of variance of \( f(X, Y) \):

\[
Var[f(X, Y)] = E\{[f(X, Y) - E(f(X, Y))]^2\}
\]

we assume that \( E[f(X, Y)] \approx f(\mu) \) and use equation (2) then we obtain the following approximation:

\[
Var[f(X, Y)] \approx E\{[f(\mu) + f'_x(\mu)(X - \mu_X) + f'_y(\mu)(Y - \mu_Y) - f(\mu)]^2\}
\]

\[
= E\{[f'_x(\mu)(X - \mu_X) + f'_y(\mu)(Y - \mu_Y)]^2\}
\]

\[
= E\{f''_x(\mu)(X - \mu_X)^2 + 2f'_x(\mu)(X - \mu_X)f'_y(\mu)(Y - \mu_Y) + f''_y(\mu)(Y - \mu_Y)^2\}
\]

The first-order derivatives of \( f(X, Y) \) are equal to \( f'_x(X, Y) = 1/Y \) and \( f'_y(X, Y) = -X/Y^2 \). Consequently, if we evaluate them at \( \mu = (\mu_X, \mu_Y) \) and use the definition of covariance \( Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] \) then the approximation takes the following form:

\[
Var\left(\frac{X}{Y}\right) \approx \frac{1}{\mu_Y^2}Var(X) - \frac{2}{\mu_Y^2}Cov(X, Y) + \frac{\mu_X^2}{\mu_Y^2}Var(Y).
\]

Stride speed is the ratio of stride length and stride time. Consequently, it follows from equation (5) that its variance \( \sigma_{SS}^2 \) can be approximated by \( \hat{\sigma}_{SS}^2 \):

\[
\sigma_{SS}^2 \approx \hat{\sigma}_{SS}^2 = \frac{1}{\mu_{ST}^2} \sigma_{SL}^2 - \frac{2}{\mu_{ST}^2} \frac{\mu_{SL}^2}{\mu_{ST}^2} Cov(SL, ST) + \frac{\mu_{SL}^2}{\mu_{ST}^2} \sigma_{ST}^2
\]

\[
= \frac{1}{\mu_{ST}^2} \left[ \sigma_{SL}^2 - \frac{2}{\mu_{ST}} \frac{\mu_{SL}^2}{\mu_{ST}} Cov(SL, ST) + \left(\frac{\mu_{SL}}{\mu_{ST}}\right)^2 \sigma_{ST}^2 \right]
\]

\[
= \frac{1}{\mu_{ST}^2} \left[ c_1 \sigma_{SL}^2 + c_2 Cov(SL, ST) + c_3 \sigma_{ST}^2 \right]
\]

\[
= c_1 \sigma_{SL}^2 + c_2 Cov(SL, ST) + c_3 \sigma_{ST}^2
\]

\[
P_1 + P_2 + P_3.
\]

Thus, \( \hat{\sigma}_{SS}^2 \) is the sum of three terms: \( P_1, P_2, \) and \( P_3 \) which are proportional to the variance of SL \( (\sigma_{SL}^2) \), covariance of SL and ST \( (\text{Cov}(SL, ST)) \), variance of ST \( (\sigma_{ST}^2) \), respectively. In the above equations, \( \mu_{SS}, \mu_{SL}, \) and \( \mu_{ST} \) denote the mean values of the gait parameters. To quantify the relative contributions of \( P_i, i = 1, 2, 3 \) to stride speed variance, we divide both sides of equation (6) by \( \sigma_{SS}^2 \):

\[
\hat{\sigma}_{SS}^2/\sigma_{SS}^2 = P_1/(n^2) + P_2/(n^2) + P_3/(n^2)
\]

where \( P_i/(n^2) = P_i/\sigma_{SS}^2 \). If approximation equation (6) holds, then the ratio \( \hat{\sigma}_{SS}^2/\sigma_{SS}^2 \) should be close to 1.

Coefficients \( c_i \) are defined in terms of the average values of the gait parameters and consequently dependent on treadmill speed. To elucidate such dependence in equation (6) we factored out the common term \( 1/\mu_{ST}^2 \) which enabled us to define \( c_i \) as:

\[
c_i = \frac{c_i}{\mu_{ST}^2},
\]
where
\begin{align}
    c_1^{(f)} &= 1, \quad \text{(9a)} \\
    c_2^{(f)} &= -2 \frac{\mu_{SL}}{\mu_{ST}}, \quad \text{(9b)} \\
    c_3^{(f)} &= \left( \frac{\mu_{SL}}{\mu_{ST}} \right)^2. \quad \text{(9c)}
\end{align}

The fundamental assumption that we used to derive equation (5) for the variance of the ratio of two random variables was their stationarity. This assumption is violated in the presence of SL/ST trends. Consequently, we estimate the stride speed variance using SL and ST MARS residuals, i.e., we compute the estimator equation (6) after subtracting from the experimental time series piecewise linear MARS trends. $\mu_{SL}$ and $\mu_{ST}$ are the averages of the corresponding trends.

**Statistical analysis**

We used the Shapiro-Wilk test to determine whether the analyzed data were normally distributed. The dependence of gait parameters’ variance on treadmill speed was examined with either ANOVA or the Kruskal-Wallis test (with Tukey’s post hoc comparison in both cases). For a given gait parameter and treadmill speed, the difference in variance between unperturbed and perturbed walking was assessed with either the $t$-test or the Mann-Whitney test.

For all statistical tests, we set the significance threshold to 0.05.

**Data Availability Statement**

In our analysis, we used a dataset from the study of Moore et al.\textsuperscript{16} that is available from the Zenodo repository\textsuperscript{50}.

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**Author contributions statement**

All authors contributed to the methodology and analyzed the results.

**Additional information**

**Competing financial interests** There were no conflicts of interest.