Elastic Cross sections for high energy hadron-hadron scattering

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This report discusses some results on differential cross sections for high energy and small momentum transfer elastic hadron-hadron scattering in QCD, using a functional integral approach. In particular a matrix cumulant expansion for the vacuum expectation values of lightlike Wegner-Wilson loops, which governs the hadronic amplitudes, is presented. The cumulants are evaluated using the model of the stochastic vacuum.

1. Introduction

We will discuss here some results of [1] for elastic scattering of hadrons at high centre of mass energy \( \sqrt{s} \geq 20 \text{ GeV} \) and low momentum transfer squared \( t \) (say \( |t| \leq O(1 \text{ GeV}^2) \)). Such reactions are governed by soft, nonperturbative interactions.

We are specifically interested in calculating elastic differential cross sections \( d\sigma/dt \) of mesons in transverse space. The path integration correlates these loops and so causes the interaction. The resulting loop-loop correlation function has to be integrated over all extensions and orientations of the loops in transverse space. The path integration correlates these loops and so causes the interaction.

The result for meson-meson scattering where mesons are represented as \( \bar{q}q \)-wave packets is

\[
S_{fi} = \delta_{fj} + i(2\pi)^4 \delta(P_3 + P_4 - P_1 - P_2) T_{fi},
\]

\[
T_{fi} = (-2is) \int d^2b_T \exp(is \mathbf{q}_T \cdot \mathbf{b}_T) \cdot \int d^2x_T d^2y_T w^M_{3,1}(x_T) w^M_{4,2}(y_T) \cdot \left\langle W^M_+ \left( -\frac{1}{2} \mathbf{b}_T, x_T \right) W^-_M \left( -\frac{1}{2} \mathbf{b}_T, y_T \right) - 1 \right\rangle_G. \tag{1}
\]

Here the assumption is made that the \( q \) and \( \bar{q} \) share the longitudinal momentum of the meson roughly in equal proportions. The interpretation of (1) and the symbols occurring there is as follows. The scattering amplitude is obtained by first considering the scattering of quarks and antiquarks on a fixed gluon potential and then sum-
The main idea is now to interpret the product of the two traces (tr) over 3 × 3 matrices in (3) as one trace (Tr) acting in the 9-dimensional tensor product space carrying the product of two SU(3) quark representations:

\[
\langle W^+ W^- \rangle_G = \frac{1}{9} \text{Tr}_2 \left\{ \langle P \exp \left[ -\frac{i g}{2} \int_{P_+} d\sigma^{\mu\nu} \hat{G}_{\mu\nu}^a \left( \frac{\lambda^a}{2} \otimes 1 \right) \right] \right\} \langle \exp \left[ -\frac{i g}{2} \int_{P_-} d\sigma^{\mu\nu} \hat{G}_{\mu\nu}^a \left( \frac{\lambda^a}{2} \right) \right] \rangle_G .
\] (4)

Introducing a total shifted field strength tensor \( \hat{G}_t \) as

\[
\hat{G}_t = \begin{cases} 
\hat{G}_{\mu\nu}^a(x,o;C_x)(\lambda^a / 2) & \text{for } x \in P_+ \\
\hat{G}_{\mu\nu}^a(x,o;C_x)(1 \otimes \lambda^a / 2) & \text{for } x \in P_-
\end{cases}
\]

we can rewrite the two exponentials in (4) as one exponential defined in the direct product space. In this way we get from (3) a path-ordered integral over the double pyramid mantle \( P = P_+ + P_- \):

\[
\langle W^+ W^- \rangle_G = \frac{1}{9} \text{Tr}_2 \left\{ \langle P \exp \left[ -\frac{i g}{2} \int_{P} d\sigma(x) \hat{G}_t(x) \right] \rangle \right\}_G .
\] (6)

Here and in the following we suppress the Lorentz indices if there is no confusion. Note that the path orderings on \( P_+ \) and \( P_- \) do not interfere with each other. Thus the path-ordering on \( P \) can for instance be chosen such that all points of \( P_+ \) are “later” than all points of \( P_- \).

For the expectation value of the single surface ordered exponential (3) we make a matrix cumulant expansion as explained in (2.41) of [5]:

\[
\langle P \exp \left[ -\frac{i g}{2} \int_{P} d\sigma(x) \hat{G}_t(x) \right] \rangle_G = 
\exp \left\{ \sum_{n=1}^{\infty} \frac{1}{n!} \left( -\frac{i g}{2} \right)^n \int d\sigma(x_1) \cdots d\sigma(x_n) \cdot K_n(x_1, \ldots, x_n) \right\}.
\]

(7)

Here the cumulants \( K_n \) are functional integrals over products of the non-commuting matrices \( \hat{G}_t \)
of (3). Thus one has to be careful with their ordering.

Neglecting cumulants higher than \(n=2\) and using in addition \(\langle G_{\mu \nu}^a \rangle_G = 0\) leads to

\[
\left\langle W_+^M W_+^M \right\rangle_G = \frac{1}{9} \text{Tr}_2 \exp(C_2(x_T, y_T, b_T)),
\]

\[
C_2(x_T, y_T, b_T) = -\frac{g^2}{8} \int d\sigma(x_1) \int d\sigma(x_2) \cdot
\left\langle P(\hat{G}_i(0, x_1; C_{x_1})\hat{G}_i(0, x_2; C_{x_2})) \right\rangle_G
\]

where \(C_2\) is a \(9 \times 9\) matrix, invariant under SU(3) colour rotations.

Now we use the MSV ansatz [8] for the correlation function of two shifted field strengths \(\hat{G}_{\mu \nu}^a\), which consists of two Lorentz tensor structures multiplied by invariant functions \(D\) times \(\kappa\) \((0 \leq \kappa \leq 1)\) and \(D_1\) times \((1-\kappa)\) respectively. In QCD lattice measurements show [9] \(\kappa \sim 3/4\) whereas in an abelian theory \(\kappa=0\). For deriving confinement in terms of the MSV \(\kappa \neq 0\) is crucial [3]. We find that \(\kappa \neq 0\) is also necessary to reproduce the experimental data for \(d\sigma/dt\) [3]. Further parameters of the MSV are the gluon condensate \(G_2\) and the vacuum correlation length \(a\).

Inserting this ansatz in \(C_2\) we get [3]

\[
\left\langle W_+^M \left(\frac{1}{2} b_T, x_T \right) W_+^M \left(\frac{1}{2} b_T, y_T \right) \right\rangle_G = \frac{1}{9} \text{Tr}_2 \exp(\frac{\lambda_a^2}{2} - \frac{\lambda_b^2}{2} - i \chi(x_T, y_T, b_T)) = \frac{1}{3} e^{\frac{1}{2} \chi} + \frac{2}{3} e^{-i\chi}.
\]

where \(\chi\) is a real function. In the last step one introduces projectors \(P_s, P_a\) satisfying \((\lambda_a^2 \otimes \lambda_b^2) = \frac{1}{2} P_a + \frac{1}{2} P_s\).

3. Meson-meson amplitude

Inserting [3] in [3] and using the mesonic overlap functions \(w_{\mu}^M(x_T)=1/(2\pi S_H^2)\exp(-z \phi^2 / 2 S_H^2)\) of [3] our final result for the meson-meson scattering amplitude reads

\[
T_{fi} = (2is) (2\pi) \int_0^\infty db dJ_0(\sqrt{b}) J_{M,M}(b),
\]

\[
J_{M,M}(b) = -\int d^2 x_T \int d^2 y_T w_{\mu}^M(x_T) w_{\mu}^M(y_T) \cdot \left\{ \frac{2}{3} \cos(\chi) + \frac{1}{3} \cos(\chi) - 1 \right\}.
\]

Here \(J_0\) is the zeroth-order Bessel function. The sine terms which one would expect from [3] are averaged out by integrating over \(x_T, y_T\) because we have for example \(\chi(-x_T, y_T, b_T) = -\chi(x_T, y_T, b_T)\).

As a consequence [10] is invariant under the replacement of one hadron by its antihadron: The exchange of all partons by its antipartons for a given loop configuration turns around the loop direction (Fig. 1) which results in \(x_T \rightarrow -x_T\). But this leaves the amplitude [10] invariant.

In our approximations, we get only charge conjugation \(C = +1\) (pomeron) exchange and no \(C = -1\) (odderon) exchange contributions to the amplitude. A real part of the amplitude and \(C = -1\) exchange contributions could arise from higher cumulants in [3].

4. Proton-proton scattering

Now we come to our results for \(pp\) scattering where protons are treated in the quark-diquark picture. First we have to fix the parameters in [10] which are: the QCD vacuum parameters \(G_2, \kappa\) and \(a\) and the proton extension parameter \(S_{H_p}=S_{H_s}=S_p\). The vacuum parameters are surely energy and process independent. Following [3] the extension parameter \(S_p\) will be allowed to vary with energy. We fix these parameters using as input experimental data at \(\sqrt{s} = 23\text{GeV}\) for \(d\sigma/dt\) and impose in addition the constraint that our amplitude reproduces the pomeron part of \(\sigma_{tot}\) in the Donnachie-Landshoff (DL) parametrisation [8]. We find [8]: \(G_2 = (529\text{MeV})^4, a = 0.32\text{fm and } \kappa = 0.74\), compatible with lattice determinations [8] and \(S_p(23\text{GeV})=0.87\text{fm}\) which is in the range of the electromagnetic proton radius as it should. All values compare well with the values determined in previous work on high energy scattering [8] [9].

\(\sigma_{tot}\) increases with increasing extension parameter. So in order to calculate \(d\sigma/dt\) at higher c.m. energies we fix \(S_p(\sqrt{s})\) requiring again that our model reproduces the pomeron part of \(\sigma_{tot}\) in the DL parametrisation.

Now everything is fixed and we can calculate \(\sigma_{tot}\) and \(d\sigma/dt\) from [10]. In Fig. 2 we show our results. From there one can see that for all en-
energies the calculated differential distributions follow the experimental data quite well over many orders of magnitude. The fact that this is true up to $\sqrt{s} = 1800$ GeV supports the description of the $s$-dependence by a $s$-dependent extension parameter $S_p(\sqrt{s})$. For all energies the imaginary part of our amplitude changes sign at some $t < 0$. Due to the absence of a real part in (10) the calculated differential cross sections have a zero there. This causes an infinitely deep dip in our $t$-distributions. We expect this dip to be at least partly filled up once we change to more general quark configurations and include higher cumulant terms. The point at which the zero occurs in our calculation moves to smaller values of $|t|$ with increasing energy and is always in the region where experiments see a marked structure.

Finally we want to stress again, that our results for $d\sigma/dt$ depend crucially on the $\kappa$-term of the MSV, which implies in low energy phenomenology a string tension $\rho \neq 0$ and so confinement [5]. A detailed discussion of this point can be found in [5].

Of course our model is not perfect. Our amplitude is purely imaginary and thus does not satisfy the relation between the phase and the $s$-dependence required by analyticity and Regge theory [10]. Also our $d\sigma/dt$ is the same for $pp$ and $p\bar{p}$ scattering. Experiments show that this is not true in the dip regions. We will have to see if higher cumulant terms and/or a departure from the strict quark-diquark picture of the proton will lead us to an improvement on these points in our model.

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