The blackholic quantum

J. A. Rueda\textsuperscript{1,2,3}, R. Ruffini\textsuperscript{1,2,4}

\textsuperscript{1}ICRANet, Piazza della Repubblica 10, I-65122 Pescara, Italy
\textsuperscript{2}ICRA, Dipartimento di Fisica, Sapienza Universit\`a di Roma, P.le Aldo Moro 5, I–00185 Rome, Italy
\textsuperscript{3}INAF, Istituto di Astrofisica e Planetologia Spaziali, Via Fosso del Cavaliere 100, 00133 Rome, Italy
\textsuperscript{4}INAF, Viale del Parco Mellini 84, 00136 Rome Italy

Received: date / Accepted: date

Abstract We show that the high-energy emission of GRBs originates in the inner engine: a Kerr black hole (BH) surrounded by matter and a magnetic field $B_0$. It radiates a sequence of discrete events of particle acceleration, each of energy $\mathcal{E} = h \Omega_{\text{eff}}$, the blackholic quantum, where $\Omega_{\text{eff}} = 4(\mp \pi / m_\odot)^3 (c a / GM)(B_0^2 / \rho_\odot) \Omega_+$. Here $M$, $a = J / M$, $\Omega_+ = c^2 \partial M / \partial J = (c^2 / G) a / (2M r_+)$ and $r_+$ are the BH mass, angular momentum per unit mass, angular velocity and horizon; $m_\odot$ is the neutron mass, $\mp \pi = h / (\mp \pi c)$ and $\rho_\odot = m_\text{Pl} \lambda_\text{Pl} / L_\text{Pl}$, are the Planck mass, length and energy density.

The timescale of each process is $\tau_3 \sim \Omega_+^{-1}$. We show an analogy with the Zeeman and Stark effects, properly scaled from microphysics to macrophysics, that allows us to define the BH magneton, $\mu_{\text{BH}} = (m_\mp / m_\odot)^3 (c a / GM) e \hbar / (Mc)$. We give quantitative estimates for GRB 130427A adopting $M = 2.3 M_\odot$, $c a / (GM) = 0.3$ and $B_0 = 2.9 \times 10^{14}$ G. Each emitted quantum, $\mathcal{E} \sim 10^{44}$ erg, extracts only $10^{-9}$ times the BH rotational energy, guaranteeing that the process can be repeated for thousands of years. The inner engine can also work in AGN as we here exemplified for the supermassive BH at the center of M87.

Keywords gamma-ray bursts: general - BH physics

1 Introduction

The GeV radiation in long GRBs is observed as a continuous, macroscopic emission with a luminosity that, in the source rest-frame, follows a specific power-law behavior: for instance the $0.1–100$ GeV rest-frame luminosity of GRB 130427A observed by Fermi-LAT is well fitted by $L = A t^{-\alpha}$, $A = (2.05 \pm 0.23) \times 10^{52}$ erg s\textsuperscript{-1} and $\alpha = 1.2 \pm 0.04$ [1]. We have there shown that the rotational energy of a Kerr BH is indeed sufficient to power the GeV emission. From the global energetics we have determined the BH parameters, namely its mass $M$ and angular momentum per unit mass $a = J / M$ and, from the change of the luminosity with time, we have obtained the slowing-down rate of the Kerr BH. We have applied this procedure to the GeV-emission data of 21 sources. For GRB 130427A we have obtained that the BH initial parameters are $M \approx 2.3 M_\odot$ and $c a / (GM) \approx 0.3$.

One of the most extended multi-messenger campaign of observation in the field of science, ranging from ultra high-energy photons GeV/TeV (MAGIC) and MeV radiation (Swift, Fermi, Integral, Konus/WIND and UHXRT satellites) and to fifty optical observatories including the VLT, has given unprecedented details data on GRB 190114C. An in-depth time-resolved spectral analysis of its prompt emission, obtaining the best fit of the spectrum, and repeating it in successive time iterations with increasingly shorter time bins has been presented in [2]. It turns out that the spectra are self-similar and that the gamma-ray luminosity, expressed in the rest-frame, follows a power-law dependence with an index $-1.20 \pm 0.26$, similar to the one of the GeV luminosity.

These data have offered us the first observational evidence of the moment of BH formation and, indeed, it clearly appears that the high-energy radiation is emitted in a sequence of elementary events, each of $10^{44}$ erg, and with an ever increasing repetition time from $10^{-8}$ to $10^{-6}$ s [2].

We have shown that this emission can be powered by what we have called the inner engine [3,2,4,5]: a Kerr BH immersed in a magnetic field $B_0$ and surrounded by matter. This inner engine naturally forms in the binary-driven hypernova (BdHN) scenario of GRBs [6,7,8,9]. The BdHN starts with the supernova explosion of a carbon-oxygen star that forms a tight binary system with a neutron star companion. The supernova ejecta produces a hypercritical accretion process onto the neutron star bringing it to the critical mass point for gravitational collapse, hence forming a rotating BH. The Kerr BH, in presence of the magnetic field inherited from the neutron star, induces an electromagnetic field that is described by the Wald solution [10]. The BH is
surrounded by matter from the supernova ejecta that supply ionized matter that is accelerated to ultrarelativistic energies at expenses of the BH rotational energy. This model has been applied to specific GRB sources in [3,2,11].

We here show that the GRB high-energy (GeV/TeV) radiation is indeed better understood within this scenario and that in particular: 1) it originates near the BH and 2) it is extracted from the BH rotational energy by packets, quanta of energy, in a number of finite discrete processes. We show that it is indeed possible to obtain the quantum of energy of this elementary process: \( \varepsilon = \hbar \Omega_{\text{eff}} \), where \( \Omega_{\text{eff}} \) is proportional to the BH angular velocity, \( \Omega_+ \), and the proportionality constant depends only on fundamental constants. The timescale of the elementary process is shown to be \( \tau_{\text{el}} \sim \Omega_+^{-1} \). Quantitatively speaking, initially \( \varepsilon \approx 10^{44} \text{ erg} \) and \( \tau_{\text{el}} \) is shorter than microseconds.

This elementary process is not only finite in energy but it uses in each iteration only a small fraction of the BH rotational energy, which can be as large as \( E_{\text{rot}} \sim 10^{53} \text{ erg} \). As we shall see, this implies that the repetitive process, in view of the slowing-down of the BH, can last thousands of years. The considerations on the inner engine apply as well to the case of AGN and we give a specific example for the case of M87\(^*\), the supermassive BH at the center of the M87.

2 The inner engine electromagnetic field structure

The axisymmetric Kerr metric for the exterior field of a rotating BH, in Boyer-Lindquist coordinates \((t, r, \theta, \phi)\), can be written as [12] (we use CGS+Gaussian units):

\[
\begin{align*}
\text{d}s^2 &= - \left(1 - \frac{2Mr}{\Sigma}\right) (\text{d}t)^2 - \frac{4aMr\sin^2 \theta}{\Sigma} \text{d}t \text{d}\phi + \frac{\Sigma}{\Delta} d\vartheta^2 \\
&\quad + \frac{\Delta}{\Sigma} d\theta^2 + \left(r^2 + \hat{a}^2 + \frac{2Mr\hat{a}^2 \sin^2 \theta}{\Sigma}\right) \sin^2 \theta \text{d}\varphi^2,
\end{align*}
\]

where \( \Sigma = r^2 + \hat{a}^2 \cos^2 \theta \) and \( \Delta = r^2 - 2Mr + \hat{a}^2 \). The (outer) event horizon is located at \( r_+ = \hat{M} + \sqrt{\hat{M}^2 - \hat{a}^2} \), where \( \hat{M} = GM/c^2 \) and \( \hat{a} = a/c \), being \( M \) and \( a = J/M \), respectively, the BH mass and the angular momentum per unit mass.

Denoting by \( \eta_\mu \) and \( \psi_\mu \), respectively, the timelike and spacelike Killing vectors, the electromagnetic four-potential of the Wald solution is \( A_\mu = \frac{1}{\Sigma} B_\theta \psi_\mu - \hat{a} B_\vartheta \eta_\mu \), where \( B_\theta \) is the test magnetic field value [10]. The associated electromagnetic field (in the Carter’s orthonormal tetrad), for antiparallel magnetic field and BH spin, is:

\[
\begin{align*}
E_t &= - \frac{\hat{a} B_\theta}{\Sigma} \left[ r \sin \theta - \frac{\hat{M}}{\Sigma} \left( \cos^2 \theta + 1 \right) \left( r^2 - \hat{a}^2 \cos^2 \theta \right) \right], \\
E_\theta &= \frac{\hat{a} B_\theta}{\Sigma} \sin \theta \cos \theta \sqrt{\Delta}, \\
B_\varphi &= \frac{B_\theta}{\Sigma} \cos \theta \left( - \frac{2\hat{a} \hat{\alpha} \left( \cos^2 \theta + 1 \right) }{\Sigma} + \hat{a}^2 + r^2 \right).
\end{align*}
\]

\[
B_\theta = \frac{B_\theta r}{\Sigma} \sin \theta \sqrt{\Delta}.
\]

3 Energetics and timescale of the elementary process

The electrostatic energy of a proton when bringing it from the horizon to infinity along the rotation axis is

\[
e_p = eA_\mu \eta_\mu \big|_{\infty} - eA_\mu \eta_\mu \big|_{r_+} = e \hat{a} B_\theta = \frac{1}{c} e a B_0,
\]

where we have used that \( \psi_\mu \eta^\mu = 0 \) and \( \eta_\mu \eta^\mu \rightarrow -1 \) along the rotation axis, and \( \eta_\mu \eta^\mu = 0 \) on the horizon [10].

The electric field at the horizon, \( E_+ \), is [4]:

\[
E_+ = \frac{1}{2} \frac{\hat{a}}{M} B_\theta = \frac{1}{2} \frac{eJ}{GM} B_\theta = \frac{1}{c} \frac{e}{\Omega_+} r_+ B_0,
\]

where the last expression is accurate for \( \alpha \lesssim 0.5 \) [3], and it evidences the inducting role of the BH angular velocity

\[
\Omega_+ = \frac{\partial Mc^2}{\partial J} = \frac{1}{2} \frac{a/M}{G} r_+.
\]

Using Eq. (7), Eq. (6) can be written as

\[
e_p \approx eE_+ r_+ \approx \frac{1}{c} e r_+^2 \Omega_+ B_0.
\]

It is worth to note that this angular frequency is related to the timescale of the elementary process:

\[
\tau_{\text{el}} = \frac{e_p}{eE_+ c} \approx \frac{r_+}{c} \frac{a/M}{G} 2 \Omega_+,
\]

that is the time needed to accelerate the particle until infinity. Thus, this is the longest timescale for the elementary process and it is reached only along the rotation axis where no (or negligible) radiation losses occur. Off-polar axis, the particles emit e.g. synchrotron radiation (see [3,4] for details).

The total electrostatic energy available for the inner engine elementary process is [4]:

\[
\varepsilon \approx \frac{1}{2} E_+^2 r_+^2 = \frac{1}{4} \frac{a}{GM} r_+^2 B_0^2 \Omega_+ ,
\]

where in the last equality we have used Eqs. (7) and (8).

4 The quantum of energy for GRBs

We recall that in a BdHN the BH is formed from the collapse of a neutron star when it reaches the critical mass, \( M_{\text{crit}} \), by accreting the ejected matter in the supernova explosion of a companion carbon-oxygen star [6,7,13,14,8,9,15]. Thus, for the GRB case we can adopt \( r_+ \sim 2GM/c^2 \) and \( M = M_{\text{crit}} \sim m_{\text{n}}^3/m_{\text{n}} \), where \( M_{\text{crit}} \) is accurate within a factor of order unity; \( m_{\text{n}} = \sqrt{hc/G} \) and \( m_n \) are the Planck and neutron mass. With this, the energy per proton (9) can be written in the quantized form:

\[
e_p = \hbar \omega_p, \quad \omega_p = \frac{4G}{c^4} \left( \frac{m_{\text{n}}}{m_{\text{n}}} \right)^4 e B_0 \Omega_+.
\]
Following the above steps for $\epsilon_p$, we can also write Eq. (11) in the quantized form:

$$\delta = \hbar \Omega_{\text{eff}}, \quad \Omega_{\text{eff}} = 4 \left( \frac{m_p}{m_n} \right)^8 \left( \frac{c a}{G M} \right) \left( \frac{B_0^2}{\rho Pl} \right) \Omega_+, \quad (13)$$

where $\rho Pl \equiv m_pc^2/\lambda_{pi}$ and $\lambda_{pi} = \hbar/(m_pc)$ are the Planck energy-density and length. The quantities in parenthesis are dimensionless; e.g. $B_0^2$ is an energy density as it is $\rho Pl$. Each discrete process extracts a specific amount of the BH rotational energy set by the black hole magneton (13).

### 5 Specific quantitative examples

Concerning quantitative estimates, let us compute the main physical quantities of the *inner engine* for the case of GRB 130427A [3]. We have there estimated that, adopting a realistic nuclear equation of state for the collapsing NS, an *inner engine* composed of a newborn BH of $M \approx 2.3 M_\odot$, $ca/(GM) = 0.3$ and $B_0 = 2.9 \times 10^{14}$ G, can explain the observed GeV emission. It is worth to recall that the *inner engine* parameters in [3] were determined at the end of the prompt emission (at 37 s rest-frame time), assuming at that time the induced electric field equals the critical value for vacuum polarization, i.e. $E_+ = E_c = m_e^2 c^2/(e\hbar)$. At that time the observed GeV luminosity is $\approx 10^{49}$ erg s$^{-1}$, consistent with the one estimated here (see Table 1). At earlier times the BH rotation is higher and the electric field overcritical leading to the vacuum breakdown into $e^+e^-$ pairs [16]. The dynamics and transparency of such $e^+e^-$ plasma explains the gamma-ray (MeV) prompt emission [17]. This is not the topic of this work which is devoted to the emission at energies over the proton mass, namely larger than GeV, occurring in the undercritical electric field regime.

The elementary, discrete process introduced here can also be at work in AGN where the time variability of the high-energy GeV-TeV radiation appears to be emitted on subhorizon scales (see [18] for the case of M87*). Thus, we also show in Table 1 the physical quantities for an AGN, which can be obtained from the expressions in section 3. We adopt as a proxy M87*, so $M \approx 6 \times 10^9 M_\odot$ (e.g. [19]), and we assume respectively for the BH spin and the external magnetic field, $ca/(GM) = 0.9$ and $B_0 = 50$ G. The magnetic field has been fixed to explain the observed high-energy luminosity which is $\approx 10^{42}$ erg s$^{-1}$ (e.g. [20, 21]).

This shows that the energy of the black hole magneton is finite and is a very small fraction of the BH rotational energy: for GRBs we have $E_{\text{rot}} \approx 10^{53}$ erg and $\delta/E_{\text{rot}} \approx 10^{-9}$ and for an AGN it is even smaller, i.e. $\delta/E_{\text{rot}} \approx 10^{-13}$. This guarantees that the emission process has to occur following a sequence of the elementary processes. Under these conditions, the duration of the repetitive sequence, $\Delta t \sim (E_{\text{rot}}/\delta) \tau_{\text{el}}$, can be of thousands of years, in view of the slowing-down of the BH leading to an ever increasing value of $\tau_{\text{el}}$ [3] (while $\delta$ holds nearly constant).

### 6 The black hole magneton

It is interesting to show the analogy of the above result with the case of an atom placed in an external electric or magnetic field for which its energy levels suffer a shift, respectively, from the Stark or Zeeman effect (see e.g. [22]).

In the case of the Zeeman effect, the energy shift is:

$$\Delta E_Z = \mu_B B_0, \quad \mu_B \equiv e \hbar/2mc, \quad (14)$$

where $\mu_B$ is the Bohr magneton. Indeed, by using $\Omega_+ \approx e^2 a/(4G^2M^2)$, and introducing $\mu_{BH}$, the BH magneton,

$$\mu_{BH} \equiv \left( \frac{m_p}{m_n} \right)^4 \left( \frac{c a}{G M} \right)^2 \frac{\hbar}{Mc}, \quad (15)$$

the particle energy (12) can be written as

$$\epsilon_p = \mu_{BH} B_0, \quad (16)$$

which adds an unexpected deeper meaning to $\epsilon_p$.

In the Stark effect, the energy shift is given by

$$\Delta E_S = e E_+ r_B, \quad (17)$$

where $r_B = \hbar^2/(mc^2)$ is the Bohr radius. This expression can be directly compared with the first equality in Eq. (9).

### 7 A direct application to the electron

The use of the Wald solution overcomes the conceptual difficulty of explaining the origin of the charge in BH electrodynamics. Indeed, an effective charge of the system can be expressed as [10, 4]

$$Q_{\text{eff}} = \frac{G}{c^5} 2J B_0, \quad (18)$$

which is not an independent parameter but, instead, it is a derived quantity from the BH angular momentum and the magnetic field $B_0$. These quantities become the free parameters of the electrodynamical process and therefore the concept of the BH charge is not anymore a primary concept.
The effective charge (18) can be also expressed in terms of $M$, $J$ and the magnetic moment $\mu$ as:

$$Q_{\text{eff}} = \frac{eM}{J} \mu,$$  \hspace{1cm} (19)

where we have used the computation of the Geroch-Hansen multipole moments [23,24] performed in [10]. Assuming the electron spin $J_e = \hbar/2$ and $Q_{\text{eff}} = e$, the magnetic moment becomes the Bohr magneton. But more interestingly, if we adopt the angular momentum and magnetic moment of the electron, then we obtain that the derived effective charge (19) becomes indeed the electron charge:

$$Q_{\text{eff}} = \frac{m_e c^2}{J_e} \mu_B = \frac{2m_e c}{\hbar} \frac{\hbar}{2m_e c} e = e.$$  \hspace{1cm} (20)

### 8 Conclusions

We recall:

1) That in addition of being exact mathematical solutions of the Einstein equations, BHs are objects relevant for theoretical physics and astrophysics as it was clearly indicated in “Introducing the BH” [25].

2) That the mass-energy of a Kerr-Newman BH, established over a few months period ranging from September 17, 1970, to March 11, 1971 in [26,27,28], can be simply expressed by

$$M^2 = \frac{c^2 f^2}{4G^2 M_{\text{in}}} + \left( \frac{Q^2}{4GM_{\text{in}}} + M_{\text{in}} \right)^2,$$  \hspace{1cm} (21)

$$S = 16\pi G M_{\text{in}} f^2/c^4,$$  \hspace{1cm} (22)

where $Q$, $J$ and $M$ are the three independent parameters of the Kerr-Newman geometry: charge, angular momentum and mass. $M_{\text{in}}$ and $S$ are, respectively, the derived quantities representing the irreducible mass and the horizon surface area.

3) That for extracting the Kerr BH rotational energy the existence of the Wald solution [10] was essential [4,3,2,11].

From the observational point of view, the time-resolved spectral analysis of GRB 130427A [1,3] and GRB 190114C [2] clearly points to the existence of self-similarities in the Fermi-GBM spectra, to the power-law in the GeV luminosity of the Fermi-LAT and to a discrete emission of elementary impulsive events of $10^{44}$ erg on timescales of $10^{-6}$ s.

The definition, the formulation of the equation and the identification of the mechanism of the process of emission of the black hole quantum has become a necessity and it is presented in this article.

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