Gluino two-body decays at full one-loop level in the MSSM with quark-flavour violation

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We study the two-body decays of the gluino at full one-loop level in the Minimal Supersymmetric Standard Model with quark-flavour violation (QFV) in the squark sector. The renormalization is done in the \( \overline{\text{DR}} \) scheme and hard gluon and photon radiations are included by adding the corresponding three-body decay widths. In the numerics the dependence of the gluino decay widths on the QFV parameters is discussed with special emphasis put on the separation of the electroweak and the SUSY QCD corrections. The main dependence stems from the \( \tilde{c}_R - \tilde{t}_R \) mixing in the decays to up-type squarks because there hold strong constraints from B-physics on the other quark-flavour mixing parameters. Including the full one-loop corrections, the changes of the gluino decay widths are mostly negative and of the order of about -10%. The QFV part stays small in the total width but can vary up to -8% for the decay width into the lightest squark. For the corresponding branching ratio, however, the effect is smaller by at least a factor of two. The electroweak corrections can become 35% of the SUSY QCD corrections.

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1. Introduction

Although there is no sign of new particles yet, the MSSM is still favoured as a discoverable theory beyond the SM and will be searched for with high priority at all experiments. SUSY particle decay chains have been extensively studied during the last two decades. Especially relevant are the decays of strongly interacting squarks and gluinos because they can be produced at LHC copiously. There exist an enormous number of MSSM studies. Nevertheless it has yet unstudied potential related to more general treatment of its squark sector parameters. Despite the stringent constraints from B and K physics, such parameters can lead to quark-flavour violation (QFV) and can change the phenomenological observables significantly.

In [1] the two-body decays of the gluino in the MSSM in the quark flavour conserving (QFC) case were studied including the one-loop SUSY-QCD corrections, and the full one-loop corrections allowing complex parameters were presented in [2]. Studies of these decays including general quark-flavour violation in the squark sector at tree-level are in [3, 4]. In [5] three-body decays of the gluino at tree-level are discussed.

In this work based on [6] we study the two-body decays of the gluino at full one-loop level in the MSSM with quark-flavour violation in the squark sector. For further details see also the PhD thesis of Sebastian Frank [7]. The renormalization is done in the \( \text{DR} \) scheme and hard gluon and photon radiations are included by adding the corresponding three-body decay widths.

2. QFV parameters

In the SM QFV is within the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the general MSSM there are two concepts:

1. **Minimal quark flavour violation** - no new sources of QFV, in the super-CKM basis the squarks undergo the same rotations like the quarks, all flavour violating entries are related to the CKM matrix,

2. **Non-minimal quark flavour violation** - new sources of QFV, independent of the CKM, considered as free parameters in the theory.

In this study 2. is assumed.

The hermitian \( 6 \times 6 \) squark mass matrix in the super-CKM basis is

\[
\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix}
\mathcal{M}_{\tilde{q}, LL}^2 & \mathcal{M}_{\tilde{q}, LR}^2 \\
\mathcal{M}_{\tilde{q}, RL}^2 & \mathcal{M}_{\tilde{q}, RR}^2
\end{pmatrix},
\]

(2.1)

with \( \tilde{q} = \tilde{u}, \tilde{d} \). The \( 3 \times 3 \) blocks in eq. (2.1) for the up squarks are

\[
\mathcal{M}_{\tilde{u}, LL}^2 = V_{\text{CKM}} M_Q^2 V_{\text{CKM}}^\dagger + D_{\tilde{u}, LL}^2 + \hat{m}_u^2,
\]

\[
\mathcal{M}_{\tilde{u}, RR}^2 = M_U^2 + D_{\tilde{u}, RR}^2 + \hat{m}_u^2,
\]

\[
\mathcal{M}_{\tilde{u}, RL}^2 = \mathcal{M}_{\tilde{u}, LR}^2 = \frac{v_2}{\sqrt{2}} T_U - \mu^* \hat{m}_u \cot \beta.
\]

(2.2)
The explicit form of the tree-level width is

$$\Gamma^0(\tilde{g} \rightarrow \tilde{q}_i q_j) = \frac{\lambda^{1/2}(m_{\tilde{g}}^2, m_{\tilde{q}_i}^2, m_{q_j}^2)}{32 m_{\tilde{g}}^3} \alpha_s \left( \left| U_{i,g}^q \right|^2 + \left| U_{i,3g}^q \right|^2 \right) \left( m_{\tilde{g}}^2 - m_{\tilde{q}_i}^2 + m_{q_j}^2 \right) - 4m_{\tilde{g}}m_{q_j}Re \left(U_{i,g}^{q,q} \right)$$

with $i = 1, \ldots, 6$ (no summation), $q = u, d$, and the subscript $g$ is the quark-generation index.

In order to obtain an ultraviolet (UV) convergent result at one-loop level we employ the SUSY invariant dimensional reduction (DR) regularisation scheme, which implies that all tree-level input parameters of the Lagrangian are defined at the scale $Q = M_3 \approx m_{\tilde{g}}$. Since in this scheme the tree-level couplings $g'_{L,R}$ are defined at the scale $Q$, they do not receive further finite shifts due to radiative corrections. The physical scale independent masses and fields are obtained from the DR ones using on-shell renormalisation conditions.
The renormalised one-loop partial decay widths are given by
\[
\Gamma(\tilde{g} \to \tilde{q}_i^* q_5) = \Gamma^0(\tilde{g} \to \tilde{q}_i^* q_5) + \Delta \Gamma(\tilde{g} \to \tilde{q}_i^* q_5),
\]
(3.2)
and
\[
\Delta \Gamma(\tilde{g} \to \tilde{q}_i^* q_5) = \frac{\lambda^{1/2}(m^2_\tilde{g}, m^2_{\tilde{q}_i}, m^2_q)}{512 \pi m^3_\tilde{g}} \text{Re}(\mathcal{M}^\dagger \cdot \mathcal{M}),
\]
where \(\mathcal{M}_0\) is the tree-level, and \(\mathcal{M}_1\) the one-loop amplitude. The \(Q\) independent renormalised shifts to the left and right couplings can be split,
\[
\Delta g_{L,R} = \delta g^L_{L,R} + \delta g^R_{L,R} + \delta g^R_{L,R},
\]
(3.3)
with \(v\) stands for the vertex corrections, \(w\) for the corrections of the wave functions, and \(c\) for the coupling counter terms. Further details including also the treatment of the infrared divergences can be found in [6] and in [7]. The full one-loop two-body decay width eq. (3.2) includes one-loop contributions from SQCD (gluon and gluino) and EW (electroweak including also the photon) corrections,
\[
\Gamma(\tilde{g} \to \tilde{q}_i^* q_5) = \Gamma^0(\tilde{g} \to \tilde{q}_i^* q_5) + \Delta \Gamma_{\text{SQCD}}(\tilde{g} \to \tilde{q}_i^* q_5) + \Delta \Gamma_{\text{EW}}(\tilde{g} \to \tilde{q}_i^* q_5).
\]
(3.4)
For illustration, all one-loop vertex graphs are shown in Fig 1. The first line gives contributions to SQCD and the other ones to EW.

In the numerical study the notation \(\Gamma(\tilde{g} \to \tilde{q}_i^* q_5) = \Gamma(\tilde{g} \to \tilde{q}_i^* q_5) + \Gamma(\tilde{g} \to \tilde{q}_i^* q_5)\) is used. This is equivalent with \(2\Gamma(\tilde{g} \to \tilde{q}_i^* q_5)\) when CP is conserved, which is the case in this study.

4. Numerical results

The gluino two-body widths and branching ratios are calculated with the publicly available numerical code FVSFOLD, developed by Sebastian Frank [7]. For building FVSFOLD also the packages FeynArts [8, 9] and FormCalc [10] were used. FVSFOLD needs LoopTools [10] based on FF [11]. The counter terms are included by ourselves. The code used here is cross-checked by SARAH (personal correspondence with F. Staub). For calculating the \(h^0\) mass and the low-energy observables, especially those ones in the B-sector we use SPheno v3.3.3 [12, 13]. The experimental and theoretical constraints taken into account are presented in Appendix B of [6].

First we fix a reference scenario which fulfills all constraints, see Table 1. Then we vary the QFV parameters within the allowed region. The MSSM mass spectrum is presented in Table 2. We see that our reference point violates QF explicitly and we have approximately gauge unification. Table 3 shows the flavour decompositions of the \(\tilde{u}_{1,2}\) and \(\tilde{d}_{1,2}\) squarks. The \(\tilde{u}_1\) squark is a strong mixture of \(\tilde{c}_R\) and \(\tilde{t}_R\), with a tiny contribution from \(\tilde{c}_L\), and the \(\tilde{u}_2\) squark is mainly \(\tilde{t}_L\), with a tiny contribution from \(\tilde{c}_R\). The \(\tilde{d}_1\) is a mixture of \(\tilde{s}_R\) and \(\tilde{b}_R\), and \(\tilde{d}_2\) is a pure \(\tilde{b}_L\). The decays of \(\tilde{g}\) into the lightest two supers and sdows are possible with the branching ratios
\[
\text{B}(\tilde{g} \to \tilde{u}_1 c) \approx 17\%, \text{B}(\tilde{g} \to \tilde{d}_1 s) \approx 18\%, \text{B}(\tilde{g} \to \tilde{u}_1 t) = \text{B}(\tilde{g} \to \tilde{d}_1 b) \approx 27\%, \text{B}(\tilde{g} \to \tilde{u}_2 t) \approx 5\%.
\]
The total two-body width including the full one-loop contribution, \(\Gamma(\tilde{g} \to \tilde{q} q) = 70\text{GeV}\). The tree-level width \(\Gamma^0(\tilde{g} \to \tilde{q} q) = 75\text{GeV}\).
Figure 1: Feynman graphs of all one-loop vertex contributions to $\Gamma(\tilde{g} \rightarrow \tilde{q}_i^* q_g)$, $i = 1, \ldots, 6$, $g = 1, 2, 3$.

The QFV parameters $\delta_{23}^{uLR}$, $\delta_{23}^{uRL}$, $\delta_{23}^{dLR}$, and $\delta_{23}^{dRL}$ are constrained from the vacuum stability conditions. Thus they must remain rather small. A large $\delta_{23}^{uL}$ is not possible because it violates B-physics constraints such as that for $B(B_s \rightarrow \mu^+ \mu^-)$. However, large right-right mixing in both $\tilde{u}$ and $\tilde{d}$ sectors is allowed. Therefore, in the following we only need to show diagrams with dependences on $\delta_{23}^{uRR}$ and $\delta_{23}^{dRR}$. 
Table 1: QFV reference scenario: all parameters are calculated at $Q = M_3 = 3$ TeV $\simeq m_{\tilde{g}}$, except for $m_{A^0}$ which is the pole mass of $A^0$, and $T_{U_{33}} = 2500$ GeV (corresponding to $\delta_{u_{23}} = 0.06$). All other squark parameters are zero.

| $M_1$ | $M_2$ | $M_3$ | $\mu$ | $\tan \beta$ | $m_{A^0}$ |
|-------|-------|-------|-------|-------------|-------------|
| 500 GeV | 1000 GeV | 3000 GeV | 500 GeV | 15 | 3000 GeV |

| $\alpha = 1$ | $\alpha = 2$ | $\alpha = 3$ |
|------------|------------|------------|
| $M^2_{Q_\alpha\alpha}$ | $M^2_{U_\alpha\alpha}$ | $M^2_{D_\alpha\alpha}$ |
| $3200^2$ GeV$^2$ | $3000^2$ GeV$^2$ | $2600^2$ GeV$^2$ |
| $3200^2$ GeV$^2$ | $3000^2$ GeV$^2$ | $2600^2$ GeV$^2$ |
| $3200^2$ GeV$^2$ | $3000^2$ GeV$^2$ | $2600^2$ GeV$^2$ |

| $\delta_{23}^{LL}$ | $\delta_{23}^{RR}$ | $\delta_{23}^{LR}$ | $\delta_{23}^{AL}$ | $\delta_{23}^{AR}$ | $\delta_{23}^{RL}$ |
|-------|-------|-------|-------|-------|-------|
| 0.01 | 0.7 | 0.04 | 0.07 | 0.7 | 0 |

Table 2: Physical masses of the particles in GeV for the scenario of Table 1.

| $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $m_{\tilde{\chi}_3^0}$ | $m_{\tilde{\chi}_4^0}$ | $m_{\tilde{\chi}_5^0}$ | $m_{\tilde{\chi}_6^0}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 460 | 500 | 526 | 1049 | 493 | 1049 |

| $m_{\tilde{g}}$ | $m_{\tilde{t}_1}$ | $m_{\tilde{t}_2}$ | $m_{\tilde{t}_3}$ | $m_{\tilde{g}_1}$ | $m_{\tilde{g}_2}$ | $m_{\tilde{g}_3}$ | $m_{\tilde{g}_4}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 3154 | 1602 | 2686 | 3087 | 3295 | 3300 | 3692 |

| $m_{\tilde{d}_1}$ | $m_{\tilde{d}_2}$ | $m_{\tilde{d}_3}$ | $m_{\tilde{d}_4}$ | $m_{\tilde{d}_5}$ | $m_{\tilde{d}_6}$ |
|-------|-------|-------|-------|-------|-------|
| 1662 | 2689 | 3087 | 3295 | 3301 | 3747 |

Table 3: Flavour decomposition of $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ for the scenario of Table 1. Shown are the squared coefficients.

| $\tilde{u}_L$ | $\tilde{c}_L$ | $\tilde{t}_L$ | $\tilde{u}_R$ | $\tilde{c}_R$ | $\tilde{t}_R$ |
|-------|-------|-------|-------|-------|-------|
| $\tilde{u}_1$ | 0 | 0.004 | 0 | 0 | 0.38 | 0.61 |
| $\tilde{u}_2$ | 0 | 0.001 | 0.99 | 0 | 0.006 | 0 |

| $\tilde{d}_L$ | $\tilde{s}_L$ | $\tilde{b}_L$ | $\tilde{d}_R$ | $\tilde{s}_R$ | $\tilde{b}_R$ |
|-------|-------|-------|-------|-------|-------|
| $\tilde{d}_1$ | 0 | 0 | 0 | 0.4 | 0.6 |
| $\tilde{d}_2$ | 0 | 0 | 1 | 0 | 0 |
We first discuss the dependences on the QFV parameter $\delta_{uRR}^{23}$. Fig. 2(a) shows a strong dependence of $\Gamma(\tilde{g} \to \tilde{q}q)$ which stems mainly from the kinematic prefactor, see Section 3. The SQCD correction shown in Fig. 2(b) is only weakly dependent on $\delta_{uRR}^{23}$ and is about -8%. The EW correction can become -3% for large and negative values of $\delta_{uRR}^{23}$. In Fig. 2(c) the partial widths of the $\tilde{d}_1 \tilde{b}$ modes coincide because $m_{\tilde{d}_1} \approx m_{\tilde{d}_2}$. The same holds for the branching ratios in Fig. 2(d). For $\delta_{uRR}^{23} \approx 0$ the width of $\tilde{g} \to \tilde{t}_1c$ becomes tiny because then $\tilde{t}_1$ is mainly $\tilde{t}_R$ as all the other QFV $\delta$'s are relatively small.

The relative contributions of the one-loop SQCD and the full one-loop part in terms of the tree-level result for the decay $\tilde{g} \to \tilde{t}_1t$ as a function of $\delta_{uRR}^{23}$ are shown in Fig. 3(a) for the partial decay width and in Fig. 3(b) for the branching ratio. The SQCD corrections vary in the range of -8% to -10%. The EW correction is much stronger dependent on $\delta_{uRR}^{23}$, varying between 1% down to -8%. The effects are similar in the branching ratio (b), but weaker. Out of the squark masses...
only $m_{\tilde{u}_1}$ is strongly dependent on $\delta_{23}^{uRR}$. The wiggles stem from the complex structures of the QFV one-loop contributions because no additional channel opens but those visible in Figs. 2(c) and 2(d).

Similar to Figs. 2 and 3 we have studied the dependence on the sdown right-right parameter $\delta_{23}^{dRR}$. The dependence of the total gluino width is similar to that of $\delta_{23}^{uRR}$ but the EW contribution is smaller, up to 2%. The individual partial widths show a similar picture with $\tilde{u} \leftrightarrow \tilde{d}$ compared to the ones given in Figs. 2(c) and 2(d). The corrections to the channels $\tilde{g} \rightarrow \tilde{d}_1 b$ are smaller than for $\tilde{g} \rightarrow \tilde{u}_1 t$. It is interesting that for $\tilde{g} \rightarrow \tilde{u}_1 t$ the width varies by about -3% in the allowed range of $\delta_{23}^{uRR}$. This effect stems from the $\tilde{d}$ loops in gluino wave-function correction.

**Figure 4:** Total two-body decay width $\Gamma(\tilde{g} \rightarrow \tilde{q}q)$ at full one-loop level as a function of the QFV parameters $\delta_{23}^{uRR}$ and $\delta_{23}^{dRR}$. All the other parameters are given in Table 1, except $\delta_{23}^{uRL} = \delta_{23}^{dLR} = 0.01$.

Fig. 4 shows the dependence of the total gluino width at one-loop level on the right-right mixing parameter of both the $\tilde{u}$ and $\tilde{d}$ sectors. The width varies from 20 GeV up to 90 GeV in the
allowed region.

Figure 5: $\Delta \Gamma$ denotes in (a) the full one-loop contribution in terms of the total tree-level width, in (b) the EW contribution relative to the SQCD contribution. Both plots are given as a function of the QFV parameters $\delta_{23}^{uRR}$ and $\delta_{23}^{dRR}$. All the other parameters are given in Table 1, except $\delta_{23}^{uRL} = \delta_{23}^{uLR} = 0.01$.

In Fig. 5(a) the full one-loop part in terms of the tree-level result and in Fig. 5(b) the EW contribution relative to the SQCD contribution are shown for the total two-body gluino decay width as a function of $\delta_{23}^{uRR}$ and $\delta_{23}^{dRR}$. We see in 5(a) a constant QFC one-loop contribution of $\sim -10\%$ and $\sim 3\%$ variation due to QFV. The EW part can become up to $\sim 35\%$ of the SQCD one (5(b)) for large $|\delta_{23}^{dRR}|$ where the $\tilde{u}_1 t$ mode becomes important, since the $\tilde{u}_1$ mass becomes smaller due to the $\tilde{u}$-sector right-right mixing effect. Furthermore, as $\tilde{u}_1$ is mainly a top squark, the EW corrections to the $\tilde{u}_1 t$ mode are significant, mainly controlled by the large top-quark Yukawa coupling $Y_t$.

We also have studied the gluino mass dependence. The full one-loop corrections to the total width are of the order of about $-10\%$ in the gluino mass range of 2.3 - 4.0 TeV.

5. Conclusions

All gluino two-body decays at full one-loop level in the MSSM with QFV were studied. Numerically we have focused on a scenario where only the decays to $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$ are kinematically open and $\tilde{u}_1$ is a mixture of $\tilde{c}_R$ and $\tilde{t}_R$ controlled by $\delta_{23}^{uRR}$, and $\tilde{d}_1$ is a mixture of $\tilde{s}_R$ and $\tilde{b}_R$ controlled by $\delta_{23}^{dRR}$. We obey the constraints from B-physics, the LHC constraints for the masses of the SUSY particles, especially that one for $m_{h^0}$ and have checked also the vacuum stability conditions.

The full one-loop corrections to the gluino decay widths are mostly negative. For the total decay width they are in the range of $-10\%$ with a weak dependence on QFV parameters for both SQCD (gluon and gluino) and electroweak (includes also the photon) corrections. For the decay width into $\tilde{u}_1$ we can have a total correction up to $-18\%$, with the electroweak part up to $-8\%$, strongly depending on the QFV parameters, for $\tilde{d}_1$ the effects are smaller. In the branching ratios these effects are washed out. In general, it turns out that the EW corrections can be in the range of up to $35\%$ of the SQCD corrections due to the large top-quark Yukawa coupling.
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