Dynamics of social contagions with limited contact capacity

Wei Wang,1 Panpan Shu,1 Yu-Xiao Zhu,1 Ming Tang,1,* and Yi-Cheng Zhang2

1Web Sciences Center, University of Electronic Science and Technology of China, Chengdu 610054, China
2Department of Physics, University of Fribourg, Chemin du Musée 3, 1700 Fribourg, Switzerland

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Recent empirical studies of Facebook communication networks, scientific cooperation networks and sexual contact networks suggest that individuals’ activities are limited by the time, funds, energy and other inelastic resources. Thus, individuals exhibit limited contact capacity (i.e., individuals can only communicate or interact with a finite number of neighbors during a short time) in the dynamics of epidemic and behavior. Previous studies have proven that limited contact capacity enlarges the epidemic outbreak threshold and makes the theoretical prediction deviate from simulation results more easily. Unfortunately, a systematical investigation the effects of contact capacity on social contagions, in which each individual is studied to dedicate to social interaction and limit their contact capacity. Contact capacity plays an important role in dynamics of social contagions, which so far has eluded theoretical analysis. In this paper, we first propose a non-Markovian model to understand the effects of contact capacity on social contagions, in which each individual can only contact and transmit the information to a finite number of neighbors. We then develop a heterogeneous edge-based compartmental theory for this model, and a remarkable agreement with simulations is obtained. Through theory and simulations, we find that enlarging the contact capacity makes the network more fragile to behavior spreading. Interestingly, we find that both the continuous and discontinuous dependence of the final adoption size on the information transmission probability can arise. And there is a crossover phenomenon between the two types of dependence. More specifically, the crossover phenomenon can be induced by enlarging the contact capacity only when the degree exponent is above a critical degree exponent, while the final behavior adoption size always grows continuously for any contact capacity when degree exponent is below the critical degree exponent.

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I. INTRODUCTION

Humans are the basic constituents of the society, and every individual can interact with his/her family, friend and peers. These interactions among individuals can induce some interesting collective behavior, such as, spontaneous formation of a common language or culture, emergence of consensus about a specific issue, and the adoptions of innovation, healthy or micrornance behavior. Understanding the mechanisms or regularities behind these collective behavior has led to a booming subfield of research in complex network science – social contagions, which has attracted much attention in recent years [1–3].

Statistical physics approaches were widely used to investigate social contagions. On the one hand, scientists used these methods to analyse large databases of social contagions, and revealed that reinforcement effect widely exists [4]. The reinforcement effect means that individual adopting a behavior is based on the memory of the cumulative behavioral information that he/she received from his/her neighbors. Centola established the artificially structured online communities to study health behavior spreading, and found that the reinforcement effect significantly increases the adoption of a new health behavior [5, 6]. The reinforcement effect also exists in the adoptions of Facebook [7] and Skype [8] services. On the other hand, researchers proposed some novel models with reinforcement effect to describe the dynamics of social contagions. Among these models, linear threshold model [9–11] is a famous one, and it is a deterministic model (i.e., a trivial case of Markovian process) once the network topology and initial seeds are fixed. In this model, an individual will adopt the behavior once the current fraction of his/her adopted neighbours is larger than a static threshold. The linear threshold model induces that the final behavior adoption size first grows continuously and then decreases discontinuously with the increasing of mean degree for vanishing small fraction of seeds. Another
more realistic way to incorporate the reinforcement effect is whether an individual adopts the behavior should take his/her cumulative pieces of behavioral information into consideration \[12-16\]. In this case, the dynamics is a non-Markovian process, which makes it more difficult to develop an accurate theory. Wang et al proposed a non-Markovian behavior spreading model, and found that the dependence of final behavior adoption size on information transmission probability can change from being discontinuous to being continuous under dynamical or structural parameters perturbation \[15\].

Recently, scholars found that individuals exhibit limited contact capacity (i.e., individuals can only communicate or interact with a finite number of neighbors during a short time) since the inelastic resources (e.g., time, funds, and energy) restrict them to dedicate to social interaction from empirical analysis \[17-19\]. In Facebook communication networks, Golder et al revealed that users only communicate with a small number of people even though they have many declared friends \[20\]. In scientific cooperation networks, a scientist exchanges knowledge with only a fraction of his/her co-workers in a paper \[21, 22\]. In sexual contact networks, individuals can not have sexual intercourse with his/her all sexual partners in a very short time due to the limitation of morality and physiology \[23, 24\]. Researchers have studied the effects of contact capacity on some Markovian dynamics (i.e., epidemic spreading) \[25-27\]. They found that the epidemic outbreak threshold increases when the contact capacity is limited \[27\]. Meanwhile, each connection (edge) has distinct effective spreading probability (to be defined in Sec. III), which makes the theoretical prediction deviate from simulation results more easily, especially in the case of strong structural heterogeneity.

For the dynamics of social contagions, whether an individual adopts a behavior behavior or not is determined by the cumulative pieces of behavioral information that he/she has received from neighbors \[3, 15\]. Once the contact capacity is limited, the behavioral information transmission will be limited, thus further affects the dynamics of social contagions. However, a systematic study to understand the effects of contact capacity on dynamics of social contagions is still lacking. In this paper, we try to address how the contact capacity affects the behavior spreading dynamics. We first propose a non-Markovian behavior spreading model with limited contact capacity, in which each adopted individual tries to transmit the behavioral information to a finite number of his/her neighbors. In order to understand, quantitatively, the effects of contact capacity on behavior spreading, we develop a heterogeneous edge-based compartmental theory. We find that the final behavior adoption size increases with the contact capacity. More interestingly, the crossover phenomenon in which the dependence of the final adoption size on the information transmission probability can change from being continuous to being discontinuous. By enlarging the contact capacity, the crossover phenomenon can be induced only when the degree exponent is above a critical critical degree exponent. However, the final adoption size always grows continuously for any contact capacity when degree exponent is below the critical degree exponent. The theoretical results from the suggested method can accurately predict the above results.

The paper is organized as follows. In Sec. III we describe the behavior spreading model with limited contact capacity. We develop the heterogeneous edge-based compartmental theory in Sec. III. In Sec. IV we verify the effectiveness of the theory through large number of simulations. Finally, we present conclusions and discussions in Sec. V.

### II. BEHAVIOR SPREADING MODEL

We consider the behavior spreading on uncorrelated configuration networks \[28, 29\] with \( N \) individuals (nodes) and degree distribution \( P(k) \). We use a generalized model SAR (susceptible-adopted-recovered) model \[15\] to describe behavior spreading on networks. At each time step, each individual can be in one of the three different states: susceptible, adopted, or recovered. In the susceptible state, an individual does not adopt the behavior. In the adopted state, an individual adopts the behavior and tries to transmit the information to his/her selected neighbors. In the recovered state, an individual loses interest in the behavior and will not transmit the information further. Each individual holds a static adoption threshold \( \kappa \), which reflects the criterion (wills) of an individual to adopt the behavior.

Initially, a fraction of \( \rho_0 \) individuals (nodes) are randomly selected to be in the adopted state (seeds), while other individuals are in the susceptible states. All susceptible individuals do not know any information about this behavior, in other words, the cumulative pieces of information is zero initially for all susceptible individuals. At each time step, each adopted individual \( v \) with \( k' \) neighbors randomly chooses \( f(k') \) number of neighbors due to the contact capacity is limited, and tries to transmit the information to each selected neighbors with probability \( \lambda \). Note that the function \( f(k') \) represents the contact capacity of \( v \), the larger value of \( f(k') \), the more neighbors can receive the information from him/her. If \( f(k') < k' \), the contact capacity of individual \( v \) is limited. Once the contact capacity of \( v \) is larger than his/her degree, we let he/she transmit information to his/her all neighbors. If \( v \) transmits the information to \( u \) successfully, the cumulative pieces of information \( m \) that \( u \) ever received will increase by 1, and the information can not be transmitted between \( u \) and \( v \) in the following spreading process (i.e., redundant information transmission on the edge is forbidden). If \( m \) is larger than the adoption threshold \( \kappa \), individual \( u \) becomes adopted in the next time step. From the mentioned procedures of susceptible individuals becoming adopted, we learn that the dynamics of social contagion is a non-Markovian process. The adopted individuals then lose interest in the behavior and enters into recovered with probability \( \gamma \). Individuals in the recovered state do not take part in the spreading process. The dynamics terminates once all adopted individuals become recovered.
III. HETEROGENEOUS EDGE-BASED COMPARTMENTAL THEORY

The non-Markovian behavior spreading model with limited contact capacity described in Sec. II makes theoretical prediction from the classical theory (e.g., heterogeneous mean-field theory) deviate from simulation results easily. On the one hand, in this proposed model, whether a susceptible individual adopts the behavior or not is dependent on the cumulative pieces of information he/she ever received. In this case, the memory effect of non-Markovian process is induced. On the other hand, the heterogeneity of effective spreading probability for edges increases with the heterogeneity of degree distribution, and further enhances the difficulty in developing an accurate theory. The effective spreading probability of an edge includes two aspects: (1) an edge is randomly selected with probability \( f(k')/k' \), where \( k' \) is the degree of adopted individual \( v \); (2) the information is transmitted through the selected edge with probability \( \lambda \). Thus, the effective spreading probability of an edge for individual \( v \) is \( \lambda f(k')/k' \).

To describe this process, we develop a novel theory – heterogeneous edge-based compartmental theory, which is inspired by Refs. [33, 34]. The theory is based on the assumption that behavior spreads on uncorrelated, large sparse networks. We denote \( S(t) \), \( A(t) \) and \( R(t) \) as the density of individuals in the susceptible, adopted and recovered states at time \( t \), respectively. Denoting \( \theta_{v}(t) \) as the probability that an individual \( v \) with degree \( k' \) has transmitted the information to individual \( u \) along a randomly selected edge up to time \( t \). For simplicity, we assume that individuals with identical degrees are the same in statistics. In the spirit of the cavity theory, we let individual \( u \) in the cavity state (i.e., individual \( u \) can not transmit information to his/her neighbors but can receive information from his/her neighbors). Considering all possible degrees of individual \( v \), the average probability that individual \( u \) has not received the information from his/her neighbors by time \( t \)

\[
\theta(t) = \sum_{k'=0}^{\kappa} k' P(k') \langle k \rangle \theta_{k'}(t),
\]

where \( k' P(k')/\langle k \rangle \) represents the probability that an edge from \( u \) connects to \( v \) with degree \( k' \) in uncorrelated network, and \( \langle k \rangle \) is the mean degree. It is straightforward to get the probability that individual \( u \) with \( k \) neighbors has \( m \) cumulative pieces of information by time \( t \)

\[
\phi(k, m, t) = (1 - \rho_0) \left( \frac{k}{m} \right) \left[ \theta(t) \right]^{m-1} \left[ 1 - \theta(t) \right]^{m}.
\]

The formula \( 1 - \rho_0 \) represents that only individuals in the susceptible state initially can get the information. From Sec. II we know that only when \( u \)’s cumulative pieces of information are less than \( \kappa \), he/she can be susceptible at time \( t \). Thus, individual \( u \) is susceptible by time \( t \) with probability

\[
s(k, t) = \sum_{m=0}^{\kappa-1} \phi(k, m, t).
\]

Taking all possible values of \( k \) into consideration, we can get the fraction (density) of susceptible individuals at time \( t \)

\[
S(t) = \sum_{k} P(k) s(k, t).
\]

Similarly, we can get the fraction of individuals who have received \( m \) pieces of information at time \( t \)

\[
\Phi(m, t) = \sum_{k=0} P(k) \phi(k, m, t).
\]

According to the definition of \( \theta_{k'}(t) \), one can further divide it as

\[
\theta_{k'}(t) = \xi^{S}_{k'}(t) + \xi^{A}_{k'}(t) + \xi^{R}_{k'}(t).
\]

The value of \( \xi^{S}_{k'}(t) \), \( \xi^{A}_{k'}(t) \), and \( \xi^{R}_{k'}(t) \) represents that the probability of individual \( v \) with degree \( k' \) is susceptible, adopted, and recovered and has not transmitted information to its neighbors (e.g., individual \( u \), respectively.

An initial susceptible neighbor individual \( v \) of \( u \) can only get the information from the other \( k' - 1 \) neighbors, since individual \( u \) is in the cavity state. Similar to Eq. (2), one can get the probability that \( v \) has \( m \) cumulative pieces of information by time \( t \)

\[
\tau(k', m, t) = (1 - \rho_0) \left( \frac{k' - 1}{m} \right) \left[ \theta(t) \right]^{k' - 1 - m} \left[ 1 - \theta(t) \right]^{m}.
\]

We further get the probability of individual \( v \) in the susceptible

\[
\xi^{S}_{k'}(t) = \sum_{m=0}^{\kappa-1} \tau(k', m, t).
\]

If the adopted neighbor individual \( v \) with degree \( k' \) transmits the information via an edge, this edge will not meet the definition of \( \theta_{k'}(t) \). The conditions of individual \( v \) transmits information to \( u \) are: (1) the edge connecting them is selected with probability \( f(k'/k') \) and (2) the information is transmitted through this edge with probability \( \gamma \). Thus, the evolution of \( \theta_{k'}(t) \) is

\[
\frac{d\theta_{k'}(t)}{dt} = -\lambda f(k'/k') \xi^{A}_{k'}(t).
\]

If \( f(k') \) is larger than \( k' \), we restrict that \( v \) transmits the information to his/her all neighbors [i.e., \( f(k') = k' \)].

According to information spreading process described in Sec. III the growth of \( \xi^{S}_{k'} \) should simultaneously satisfy: (1) the adopted individual \( v \) does not transmit the information to \( u \) through the edge between them and (2) \( v \) moves into recovered state with probability \( \gamma \). For the first condition, there are two possible cases: the edge between \( u \) and \( v \) is selected with probability \( f(k'/k') \) and the information is not transmitted through it with probability \( 1 - \lambda \); the edge between \( u \) and \( v \) is not selected with probability \( 1 - f(k'/k') \). From the analyses above, the evolution of \( \xi^{S}_{k'} \) is

\[
\frac{d\xi^{S}_{k'}(t)}{dt} = \gamma \xi^{A}_{k'}(t) \left[ 1 - \frac{\lambda f(k')}{k'} \right].
\]
Now, combining Eqs. (9)-(10) and the initial situations [i.e., \(\theta_{k'}(0) = 1\) and \(\xi^R_{k'}(0) = 0\)], we obtain the expression of \(\xi^R_{k'}(t)\) in terms of \(\theta_{k'}(t)\) as

\[
\xi^R_{k'}(t) = \gamma\left[1 - \theta_{k'}(t)\right]\left[\frac{k'}{\lambda f(k')} - 1\right].
\]  

Utilizing Eqs. (6), (8), (9) and (11), we obtain that

\[
\frac{d\theta_{k'}(t)}{dt} = -\lambda f(k')\frac{k}{k'}[\theta_{k'}(t) - \sum_{m=0}^{\kappa-1} \tau(k', m, t)] + \gamma\left[1 - \theta_{k'}(t)\right]\left[1 - \frac{\lambda f(k')}{k'}\right].
\]  

According to the model described in Sec. II, the densities of individuals in adopted and recovered individuals evolve as

\[
\frac{dA(t)}{dt} = -\frac{dS(t)}{dt} - \gamma A(t)
\]  

and

\[
\frac{dR(t)}{dt} = \gamma A(t),
\]  

respectively. Eqs. 4 and 13-14 give us a complete description of the social contagions with limited contact capacity. The evolution of each type of density versus time can be obtained.

The densities of susceptible, adopted and recovered individuals do not change when \(t \to \infty\). We denote \(R(\infty)\) as the final behavior adoption size. To obtain the value of \(R(\infty)\), one can first solve \(\theta_{k'}(\infty)\) from Eq. (12), that is

\[
\theta_{k'}(\infty) = \sum_{m=0}^{\kappa-1} \tau(k', m, \infty)] + \gamma\left[1 - \theta_{k'}(\infty)\right]\left[\frac{k'}{\lambda f(k')} - 1\right].
\]  

Iterating Eq. (15) to obtain \(\theta_{k'}(\infty)\). Then, inserting \(\theta_{k'}(\infty)\) into Eqs. (11)-(14) to get the values of \(S(\infty)\) and \(R(\infty) = 1 - S(\infty)\).

Another important aspect we mainly focus on is the condition under which the global behavior adoption occurs. The global behavior adoption means that a finite fraction of individuals adopted the behavior. Similar to biological contagions, we define a critical transmission probability \(\lambda_c\). When \(\lambda \leq \lambda_c\), the behavior can not be adopted by a finite fraction of individuals; when \(\lambda > \lambda_c\), the global behavior adoption occurs. Now, we discuss \(\lambda_c\) for several different values of \(\rho_0\) and \(\kappa\).

For \(\rho_0 \to 0\) (i.e., only a vanishing small fraction of seeds) and \(\kappa = 1\), \(\theta_{k'}(\infty) = 1\) is the trivial solution of Eq. (15). If we change the values of other dynamical parameters, such as information transmission probability \(\lambda\), a global behavior adoption may occur. The global behavior adoption occurs only when a nontrivial solution of Eq. (15) emerges [i.e., \(\theta_{k'}(\infty) < 1\)]. Note that the corresponding fraction of \(\theta_{k'}(\infty)\) should be taken into consideration. Linearizing Eq. (15) at \(\theta_{k'}(\infty) = 1\) [28], and summing all possible values of \(k'\) one can get the critical information transmission probability

\[
\lambda_c = \frac{\gamma\langle k \rangle G(k)}{\langle k^2 \rangle - (2 - \gamma)\langle k \rangle},
\]  

where

\[
G(k) = \sum_{k'} \frac{k'^2 P(k')}{\langle k \rangle f(k')}.\]
solution of Eq. (15). However, the left and right hands of Eq. (15) can not be tangent to each other at $\theta_{k'}=1$, which indicates that vanishingly small seeds can not trigger the global behavior adoption [15]. With the increase of $\rho_0$, different dependence of $R(\infty)$ on $\lambda$ occurs for different $\kappa$. That is, the growth pattern of $R(\infty)$ versus $\lambda$ can be continuous or discontinuous. Through bifurcation analysis [13] of Eq. (15), we find that $R(\infty)$ grows continuously for $\kappa = 1$, while a discontinuous growth may be induced for $\kappa > 1$.

IV. SIMULATION RESULTS

In this section, we verify the effectiveness of the heterogeneous edge-based compartmental theory developed in Sec. III by lots of simulations. For each network, we perform at least $2 \times 10^3$ times for a dynamic process and measure the final fraction of individuals in the recovered [$R(\infty)$] and subcritical state [$\Phi(\kappa - 1, \infty)$]. These results are then averaged over 100 network realizations.

To built the network topology, we use the uncorrelated configuration model [29] according to the given degree distribution $P(k) \sim k^{-\nu}$ with maximal degree $k_{\text{max}} \sim \sqrt{N}$. There is no degree-degree correlations when $N$ is very large. The heterogeneity of network increases with the decrease of $\nu$. For the sake of investigating the effects of heterogeneous structural properties on the social contagions directly, the network sizes and mean degree are set to be $N = 10,000$ and $\langle k \rangle = 10$, respectively. All individuals with different degrees have the same contact capacity $f(k) = c$ and recover probability $\gamma = 0.1$.

We first study the effects of the adoption threshold $\kappa$ and contact capacity $c$ on the final behavior adoption size $R(\infty)$ for strong heterogeneous networks in Fig. 1. We find that $R(\infty)$ decreases with the increase of $\kappa$, since individuals adopting the behavior need to expose more information. Once the contact capacity increases (i.e., $c$ increases), individuals in adopted state will have more chances to transmit the information to susceptible individuals, thus, the values of $R(\infty)$ increases. Obviously, the theoretical predictions from heterogeneous edge-based compartmental theory agree well with the simulation results.

Another important issue we concern is the dependence of $R(\infty)$ on $\lambda$. As shown in Fig. 1, for strong heterogeneous networks the dependence of $R(\infty)$ on $\lambda$ is continuous for any values of $\kappa$ and $c$, and we verify this claim by the bifurcation analysis of Eq. (15). We can also understand this phenomenon by discussing the fraction of individuals in the subcritical state from an intuitive perspective (see Fig. 2). An individual in the subcritical state means that he/she is in the susceptible state, and the $m$ cumulative pieces of information is just one smaller than his/her adoption threshold $\kappa$. From Ref. [15], we know that a discontinuous dependence of $R(\infty)$ on $\lambda$ will occur only when a large number of those subcritical individuals
adopt the behavior simultaneously at some information transmission probability. Fig. 4 shows the final fraction of individuals in the subcritical state $\Phi(\kappa - 1, \infty)$ versus information transmission probability $\lambda$ for $\kappa = 3, c = 1$ (black circles) and $\kappa = 3, c = 8$ (red squares). The lines are the theoretical predictions from Eqs. (5) and (13)-(14). Other parameters are $\nu = 4.0, \gamma = 0.1,$ and $\rho_0 = 0.1$, respectively.

We now study behavior spreading on weak heterogeneous networks, such as $\nu = 4.0$ in Fig. 5. Similar with the case of $\nu = 2.1$, increasing $\kappa$ leads to the decrease of $R(\infty)$; and the value of $R(\infty)$ increases with $c$, that is the network will become more fragile to the behavior spreading once the contact capacity increases. Once again, our theory can predict the social dynamics very well. For the dependence of $R(\infty)$ on $\lambda$, a crossover phenomenon transition is observed. A crossover phenomenon means that the dependence of $R(\infty)$ on $\lambda$ can change from being continuous to being discontinuous. More specifically, as shown in Fig. 5(b), the dependence of $R(\infty)$ on $\lambda$ is continuous for small values of $c$ (e.g., $c = 1$), while the dependence is discontinuous for larger $c$ (e.g., $c = 8$). We justify this claim by the bifurcation analysis of Eq. (15) from the theoretical view, which is also verified through analyzing $\Phi(\kappa - 1, \infty)$ from an intuitive perspective in Fig. 4.

For weak heterogeneous networks, most individuals adopt the behavior with the same probability since they have similar degrees. When $c = 1$, $\Phi(\kappa - 1, \infty)$ increases continuously with $\lambda$, which leads to a continuous growth in the value of $R(\infty)$. When $c = 8$, $\Phi(\kappa - 1, \infty)$ first increases with $\lambda$, and reach a maximum at some values $\lambda_c$, and a slight increment of $\lambda$ induces a finite fraction of $\Phi(\kappa, \infty)$ to adopt the behavior simultaneously, which leads to a discontinuous jump in the value of $R(\infty)$.

We further study the effects of $\nu$ and $\lambda$ in Fig. 5 for different values of $c$. For small contact capacity [i.e., $c = 1$ in Figs. 5(a) and (b)], the dependence of $R(\infty)$ on $\lambda$ is always continuous for any value of $\nu$. In other words, this dependence is irrelevant to the network topology. For large contact capacity [i.e., $c = 8$ in Figs. 5(c) and (d)], there is a crossover phenomenon in which the dependence of $R(\infty)$ on $\lambda$ can change from being continuous to being discontinuous. More particularly, there is a critical degree exponent $\nu_c$ below which the dependence is continuous [see region I in Figs. 5(c) and (d)], while above $\nu_c$, the dependence is discontinuous [see region II in Figs. 5(c) and (d)]. The value of $\nu_c$ can be gotten by bifurcation analysis of Eq. (15). In region II, we also find that the discontinuous information transmission probability $\lambda_c^I$ increases with $\nu$, since the fraction of hubs decreases with $\nu$. The theoretical predictions of $\lambda_c^I$ can be gotten by bifurcation analysis of Eq. (15), and the simulation results of $\lambda_c^I$ are predicted by NOI (number of iterations) method [15]. Regardless of network heterogeneity, our theoretical predictions about the behaviors of $R(\infty)$ have a good agreement with numerical calculations. The average relative error [15] between the two predictions of $R(\infty)$ for all the values of $\lambda$ and $\nu$ is less than 1.8%.

Finally, we study the effects of network topology on the final behavior adoption size $R(\infty)$ in Fig. 6 for $\kappa = 2$ and $c = 1$. We find that increasing $\nu$ can promote (suppress) behavior adoption at large (small) value of $\lambda$. This phenomenon can be qualitatively understood in the following ways [15, 32]: For strong heterogeneous networks, the more hubs and a large number of individuals with small degrees are
FIG. 6. (Color online) Behavior spreading on scale-free networks. (a) The final behavior adoption size $R(\infty)$ versus information transmission probability $\lambda$ for different degree exponents $\nu = 2.1$ (black circles), $\nu = 3.0$ (red squares) and $\nu = 4.0$ (blue up triangles). (b) The final behavior adoption size $R(\infty)$ versus $\lambda = 0.3$ (black circles), $\lambda = 0.5$ (red squares) and $\lambda = 0.8$ (blue up triangles), respectively. The lines are the theoretical predictions from Eqs. (4) and (13)-(14). Other parameters are $c = 1$, $\gamma = 0.1$, $\kappa = 2$, and $\rho_0 = 0.1$, respectively.

is always continuous for different $\nu$.

V. DISCUSSION

To study social contagion dynamics in human populations is an extremely challenging problem with broad implications and interest. For social contagions on networks, some inelastic resources (e.g., time, funds, and energy) restrict individuals to dedicate to social interaction, which have always been neglected in previous studies. In this paper, we first proposed a non-Markovian behavior spreading model with limited contact capacity, in which each adopted individual transmits the information to a fraction of his/her neighbors. We then developed a heterogeneous edge-based compartmental theory to describe this model. The average relative error between the theoretical predictions and numerical calculations is less than $1.8\%$. Through theory and simulations, we found that increasing the contact capacity could be induced by enlarging $c$. However, $R(\infty)$ always grows continuously for any value of $c$ when degree exponent is below $\nu_c$.

Here we developed an accurate theoretical framework for non-Markovian social contagion model with limited contact capacity, which could be applied to other analogous dynamical processes (e.g., information diffusion and cascading failure). Further more, how to design an effective strategy to control the behavior spreading with limited contact capacity is an interesting research topic.

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