Dynamic Model Predicting Overweight, Obesity, and Extreme Obesity Prevalence Trends

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Objective: Obesity prevalence in the United States appears to be leveling, but the reasons behind the plateau remain unknown. Mechanistic insights can be provided from a mathematical model. The objective of this study is to model known multiple population parameters associated with changes in body mass index (BMI) classes and to establish conditions under which obesity prevalence will plateau.

Design and Methods: A differential equation system was developed that predicts population-wide obesity prevalence trends. The model considers both social and nonsocial influences on weight gain, incorporates other known parameters affecting obesity trends, and allows for country specific population growth.

Results: The dynamic model predicts that: obesity prevalence is a function of birthrate and the probability of being born in an obesogenic environment; obesity prevalence will plateau independent of current prevention strategies; and the US prevalence of overweight, obesity, and extreme obesity will plateau by about 2030 at 28%, 32%, and 9% respectively.

Conclusions: The US prevalence of obesity is stabilizing and will plateau, independent of current preventative strategies. This trend has important implications in accurately evaluating the impact of various anti-obesity strategies aimed at reducing obesity prevalence.

Introduction

Why is the prevalence of obesity in the United States appearing to level off (1)? Are prevention and treatment strategies working (2)? Can we expect the plateau to continue or is the apparent stabilization of obesity prevalence a temporary state?

These questions are vigorously debated because of their potential impact on healthcare costs and the desire to credit or discredit various policies to reduce obesity at the national level in many countries of the world. Existing predictions of obesity trends (3,4) cannot fully answer these questions because their underlying models assume a priori that obesity prevalence will either increase without bound or will continue to increase and eventually plateau. Dynamic models based on differential equations circumvent these limitations and can generate a predicted curve based on biological, behavioral, and social factors that can potentially raise or lower population size in each BMI class such as the dynamic models developed by Hill et al. (5) and Keisuke et al. (6). These models predicted obesity prevalence by segregating normal weight and obese populations into two compartments. However, existing models did not include the progression from normal weight to overweight and finally to the obese classification. Existing models also did not predict the prevalence of...
extreme obesity and the model did not include important moderators of obesity prevalence such as the impact of childhood obesity, differential population birthrates, and the higher susceptibility to weight regain in individuals who have lost weight.

Here, we present a comprehensive differential equation model that overcomes these limitations by incorporating the mechanisms known to increase or decrease the population prevalence within each BMI class. The model was designed to predict obesity prevalence after input of country-specific parameters, resulting in a highly flexible model that can be applied to other developed countries or communities. The proposed model is used here to determine whether the US obesity epidemic will plateau and how soon this will occur. These are issues of high national significance because of the medical, fiscal, and social consequences imposed by excess population adiposity.

**Methods**

**Model development**

Using the well-established susceptible, infected, recovered (SIR) model framework (7-9) from infectious disease modeling, we developed six differential equations that describe interactions and transitions between populations with different body mass index (BMI) classifications. SIR models have been applied to model the characteristics of a variety of infectious disease outbreaks, such as HIV (10), tuberculosis (11), and influenza (12). SIR models also have been successfully applied to capture the dynamics of noncommunicable conditions such as alcoholism (13), ecstasy use (14), and criminal activity (9). The application of SIR models in these conditions does not assume the mechanisms behind contagion, rather they reflect the overall dynamics, produce predictions of long-term outcomes, and identify which parameters have the most impact on the evolution of the epidemic.

The SIR approach divides a population into compartments of infected and noninfected individuals and model terms are constructed to describe the flow to and from each compartment (Figure 1). Complete details and a step by step model formulation appear in the Supporting Information. Here, we outline the main qualitative properties central to our model for obesity prevalence.

 Individuals in a population are deemed susceptible in our model if their BMI is below 25 kg/m² and they have never been overweight. In order to incorporate the long timescale necessary for normal weight individuals to become overweight, we introduced a class of individuals who were exposed to either social or nonsocial influences that lead to weight gain and these individuals will eventually become overweight. Thus, the exposed class can be considered as a latency period for obesity. The “infected” population comprised of three different classes, namely, an overweight population (25 ≤ BMI < 30 kg/m²), an obese population (30 ≤ BMI < 40 kg/m²), and an extremely obese population (BMI ≥ 40 kg/m²). Spontaneous transition to overweight, obese, and extremely obese
TABLE 1 List of parameters used in model simulation for the US obesity prevalence predictions and the UK obesity prevalence predictions

### United States simulation

| Description                                                                 | Value     | Method of estimation                                                                 |
|-----------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------|
| Probability (P) of being born in obesogenic environment.                     | $P = 0.55$| 55% of females of reproductive age are overweight or obese (25).                     |
| Birthrate                                                                   | $\mu = 0.0144$ | Central Intelligence Agency World Factbook (26)                                        |
| Baseline prevalence rates                                                   |           | 1988, CDC prevalence rates (20)                                                      |
| Social influence by overweight and obese                                     | $k_i = 0.4$| Fit to initial trends, 1988-1998 using shooting (see Supporting Information) (20)     |
| Spontaneous rate of weight gain to each class; exposed, overweight, obese, extremely obese | $\alpha = 0.28$ | Fit to initial trends, 1988-1998 using shooting (see Supporting Information) (20)     |
| Rate of weight loss to each class; extremely obese to obese, obese to overweight, overweight to normal weight | $\beta_2 = 0.05$ | Fit to initial trends, 1988-1998 using shooting (see Supporting Information) (20)     |
| $\beta_3 = 0.03$                                                         |           | Fit to initial trends, 1988-1998 using shooting (see Supporting Information) (20)     |
| Rate of weight regainers transitioning from normal weight to overweight     | $\rho_1 = 0.033$ | Range reported in Ref. (27)                                                          |
| Death rate of obese and extremely obese populations                          | $\rho_R = 0.014$ | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| $D_0 = 16.5-22.0$                                                         |           | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |

### United Kingdom simulation

| Description                                                                 | Value     | Method of estimation                                                                 |
|-----------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------|
| Probability (P) of being born in obesogenic environment.                     | $P = 0.30$| Thirty percent of females pre-pregnancy BMI are classified overweight or obese (28). |
| Birthrate                                                                   | $\mu = 0.01229$ | Central Intelligence Agency World Factbook (26)                                        |
| Baseline prevalence rates                                                   |           | 1988 Health Survey for England (21)                                                  |
| Social influence by overweight and obese                                     | $k_i = 0.4$| Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| Spontaneous rate of weight gain to each class; exposed, overweight, obese, extremely obese | $\alpha = 0.05$ | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| Rate of weight loss to each class; extremely obese to obese, obese to overweight, overweight to normal weight | $\beta_2 = 0.001$ | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| $\beta_3 = 0.03$                                                         |           | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| Rate of weight regainers transitioning from normal weight to overweight     | $\rho_1 = 0.003$ | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
| $\rho_R = 0.05$                                                         |           | Fit to initial trends, 1993-1997 using shooting (see Supporting Information) (21)     |
independent of social influence was modeled by linear terms, similar to the Hill model (5). Socially influenced transition to overweight, obese, and extremely obese was modeled by a mass action term (5). Overweight individuals who lose enough weight to return back to normal BMI (≤25 kg/m²) are considered recovered. Weight regain in these individuals is assumed to occur at a higher infectivity rate in comparison to normal weight susceptible individuals who progress normally to the overweight category. In addition, the model allows for modest weight loss in the obese and extremely obese classes, which returns the individual to a lower BMI category. For example, obese individuals may lose enough weight to be classified as overweight and as a result move from the obese category to the overweight category. The model timescale also includes the natural population birth and death rates.

The model assumes extremely obese individuals do not have the capacity to socially draw other classes toward obesity. This model criterion is based on recent literature (15-17) demonstrating that individuals do not “imitate” the behavior of the extremely obese. Additionally, individuals exposed to the effects of obesity at birth are considered born in an “obesogenic” environment and are thereby considered more susceptible to becoming overweight and later obese. Finally, because the epidemiological literature indicates that obese individuals die at an earlier age than normal weight people with the years of life lost in the obese population ranging from 2 to 7 years (18,19), the model included differential death rates.

We acknowledge the possible existence of normal weight individuals who are “immune” to weight gain. Because there is no flow out of the immune class, this population does not impact model dynamics and therefore a model equation is not required for this subpopulation. Finally, we note that only the flow rates between compartments are key factors in the model. Importantly, a population plateau or trend curve shape is not assumed as part of model development.

Model parameters
Models were created and simulations performed for the United States and United Kingdom. Model birth and death rates were set using population birthrate from published survey data for the United States (see Table 1) (20). The model was also simulated based on survey data (Table 1) from the United Kingdom (21). The probability of being born into an obesogenic environment was estimated from the percentage of reproductive age women classified as overweight or obese. All other model parameters were set using initial trends from US-based or UK-based obesity prevalence values (Table 1) for their respective simulation (20,21). Some model parameters such as the rate of effective interaction between overweight and obese individuals are impossible to know, however, the range is fixed by knowledge of total prevalence in each category and thus dynamics can be examined by fluctuating these parameters within their ranges. Specifically, only information from 1988 to 1998 for the United States and 1993–1997 for the United Kingdom was used to fit parameters. The model was simulated forward and then compared to the actual data past 1998 to test for agreement. Details on specific parameter calculations are provided in Table 1.

To observe the effects of varying birthrates on trajectories, the model was simulated for four different birthrates; one birth per 1000 individuals (0.0010), 14.4 births per 1000 individuals (US birthrate, 0.0144), 20 births per 1000 individuals (0.0200), and 50 births per 1000 individuals (0.0500). All other parameters specific to the US simulation appear in Table 1. Similarly, the effects of varying the probability of being born in an obesogenic environment on future prevalence rates were analyzed by simulating the model with respect to maternal obesity for probability values, 0.0, 0.55, and 0.95, while holding all other parameters fixed. A similar analysis was conducted by raising the death rate of the obese and extremely obese from the uniform death rate value (Db=0.0144), to Db =0.0150 and finally Db=0.02. Similar to the other analysis, other parameters specific to the US simulation appear in Table 1.

Model analysis
Long-term behavior or trends are analyzed by first calculating the equilibria or steady states. This is achieved by setting the derivatives equal to zero and solving the resulting equations algebraically, as shown in the Supporting Information. The next step was to determine whether the trajectories defined by the differential equation actually plateau at the calculated steady-state value. If they do, we refer to the steady state as a plateau. A rigorous mathematical proof of the existence of a plateau relying on well-established differential equation theory (22) is included in the Supporting Information.

Model simulations
Specific model trajectories were simulated using the default differential equation solver available through Matlab r2012a (2012, Math-Works, MA).

Web-based program
The model was programmed to permit interested users to input parameters and baseline prevalence values and observe the resulting obesity prevalence rates predicted over time by the model with a graphical display of results. The web-based program can be accessed at http://www.pbrc.edu/research-and-faculty/calculators/obesity-prevalence/

Results
An obesity prevalence plateau
For any parameter choice, trajectories converge to a positive plateau (see Supporting Information for mathematical analysis). Using US prevalence data from 1988, model simulations reveal that it takes an approximately 40-year period for obesity percentages to plateau at prevalence rates of 28% for overweight, 32% for obesity, and 9% for extreme obesity, where obesity includes the sum of I2(t) and I3(t) (Figure 2, Panel A).

Applying parameter and baseline conditions to the UK conditions indicates that approximately 21% of the population will be overweight, 27% will be obese, and 5% extremely obese by 2033 (Figure 2, Panel B). In contrast to the US simulation, the UK simulation revealed that a plateau will not be reached before 2033. The parameters related to socially influenced weight gain estimated in the UK simulation did not differ from the analogous parameters determined in the US case. However, the parameters related to weight gain from nonsocial influences in the UK simulation were significantly lower than those in the US case (Table 1). The plateau for the United States was directly calculated from the closed form expressions of the equilibrium (see Supporting Information) as 26.8%
classified overweight, 31.1% classified obese, and 9.8% classified extremely obese. For the United Kingdom, the plateau was determined to be 25.7% classified overweight, 39.6% classified obese, and 9.4% classified extremely obese.

Model validation
Because US model parameters were fit to data from 1988 to 1998, model simulations in this time interval represent calibration and not true prediction. However, as observed in Figure 2A, past 1998, the model simulations demonstrated good agreement with mean data from 2008. Likewise, because we applied data points from 1993 to 1997 to fit UK model parameters, we expect good agreement between model simulations and actual data in this time interval. However, as observed in Figure 2B, there is good agreement between model simulations and reported mean data between 1997 and 2008. To distinguish calibration from validation and forecast, Figure 2 curves were depicted as solid for calibration, dashed for validation, and dotted for forecast.

The dependence of the plateau on population birth rate
The model analysis revealed that the level at which obesity rates plateau in a population depends on birthrate expressed as childbirths per 1000 people per year. Specifically, higher birthrate leads to increased time to plateau and lower obesity prevalence. Figure 3A illustrates this phenomenon. It shows three different trajectories where all parameters are equal (US parameters in Table 1) except for birthrate. While this result may seem counterintuitive, a large new influx of births into the susceptible category replenishes the system. A higher birthrate yields a larger normal weight category and hence this category requires a longer time to proceed toward obesity and influence the final prevalence plateau.

The dependence of the plateau on the probability of being born into an obesogenic environment
The dependence of the level at which obesity rates will plateau also depends on the probability of being born into an obesogenic environment reflecting risk of childhood obesity. As the probability of being born into an obesogenic environment increases, the value at which obesity plateaus increases, and the time to plateau increases. This is illustrated in Figure 3B for three probability values of being born into an obesogenic environment as indexed herein by maternal weight during or around pregnancy.

The dependence of the plateau on the differential death rate
Similar to the probability of being born into an obesogenic environment, the differential death rate for obese and extremely obese populations also impacts the level at which obesity rates will plateau. It was found that the higher the differential death rate, the lower the plateau value, illustrated in Figure 3C.

Discussion
This study proposes a dynamic model that predicts obesity prevalence (5) by including interactions and transitions between populations of different BMI classes, population-wide differential birthrate, differential death rate, probability of being born into an “obesogenic”
environment, and the lag time involved in weight gain. Dynamic models such as the one developed in our study capture long-term trends without being dependent on databases or a priori determination of the type of curve the trend will follow. Rather, our dynamic model relies on the relationships between segments of the populations and then predicts flow based on these input and output relationships.

Parameters were fit to the newly developed dynamic model using US prevalence data and birth and death rates from 1988 to 1998. If these parameters remain constant, the model predicts plateaus by the year 2030 at prevalence rates of 28%, 32%, and 9% for overweight, obesity, and extreme obesity, respectively. Similarly, we applied model parameters fit to data from the United Kingdom and found that approximately 34% of the population will be overweight, 32% will be obese, and 5% extremely obese by 2033, though a plateau was not reached by 2033 in the UK simulation.

The model formulation described in this study provides a foundation for the inclusion of additional possible influences on obesity prevalence. There is no need to develop an entirely new model to include additional influences. Only the flow rates or specific model terms would need to be altered.

Many countries do not have a stable birthrate but have either increasing (China) or decreasing (European countries) birthrates. In fact, the birthrates in the US have decreased by 50% from 1950 to 1970 and held fairly steady at approximately 14 births per 1000 people since then (23). For model tractability, we assume a constant birthrate; however, a time-dependent birthrate would provide
potential for improved understanding of birthrate impacts on obesity prevalence. Changes in birthrate or other parameters would induce a “jumping plateau” effect as seen in Denmark, for example (24), where obesity prevalence plateaus and then increases past this plateau only to plateau at a new value.

Another useful factor for model advancement would be the inclusion of immigration effects. We noted here that the new influx of normal weight individuals through births impacted prevalence rates and time to plateau. It stands to reason that a new influx through immigration would also have an effect on long-term trends, depending in part on the characteristics of the migrants.

The model can be applied to a particular state by inputting birthrates and other parameters for the specific state. However, because the model does not include migration in and out of the region, the predictions would be overly simplified. To capture the full geographic dynamics, a geographic spread model would have to be developed.

Geographical models describing the spread of infectious diseases involve combining models as the one presented here with a conservation law and Fick’s law of diffusion (25). These spatial models could be applied to evaluate the effectiveness of obesity control strategies across geographical locations. However, the current models that predict obesity trends do not consider a potential progressive spread of obesity over geographical locations. Rather, they consider solely overall population trends, evolving over time. With careful analysis of regional patterns in obesity trends, our model can be extended to include a geographical diffusion component. Understanding the dynamics of how obesity moves geographically from obesity hotspots and how the borders of these hotspots influence future geographical spread of obesity are important but underinvestigated issues.

Model application to developing countries

The model developed in this paper is based on assumptions most applicable to Western countries. Many developing countries have rapidly changing birthrates, infectious disease-related deaths, high infant mortality, and rapid changes in food supplies and transportation systems as they undergo rapid nutritional and lifestyle transitions. These and possibly other influences will impact predictions. Some of these factors can be encompassed through variable birthrates and differential death rates. However, specific reviews of these different influences and factors will need to be considered for appropriate model application to developing countries.

Additional potential model extensions

The current model provides a framework for extensions. For example, one can consider assortative mating (26) or differential birthrates by revising these terms within the currently developed model. To include the effects of age structure, the current model can be revised as an age-structured model (27). Such inclusions will need to be well thought out and carefully analyzed because they will increase the complexity of the model.

In summary, we have developed a comprehensive and dynamic mathematical model to predict changes in overweight, obesity, and extreme obesity prevalence. The model is flexible and can be adapted to specific parameters of a community, region, ethnic subgroup, or country. The model predicts a slower increase in obesity prevalence and an eventual plateau of the obesity epidemic by about 2030 in the United States. It should be noted that despite the predicted deceleration in obesity rate, the prevalence of obesity remains sufficiently high at all times to warrant new and effective obesity prevention and management strategies. This model provides a baseline to evaluate the efficacy of various obesity prevention and management strategies and policies. To be effective, changes in policies and disease prevention programs will need to produce a change in obesity prevalence that is larger than predicted by our model for the natural course of the epidemic.

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