Microcausality in strongly interacting fields

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We study the properties of strongly interacting massive quantum fields in space-time as resulting from a parametric decay of the fields with a large decay width $\gamma$. The resulting imaginary part of the retarded and advanced propagators in this case is of Lorentzian form and the theory conserves microcausality, i.e. the commutator between the fields vanishes for space-like distances in space-time. However, when considering separately space-like and time-like components of the spectral function in momentum space we find microcausality to be violated for each component separately. This implies that the modeling of effective field theories for strongly interacting systems has to be considered with great care and restrictions to time-like four momenta in case of broad spectral functions have to be ruled out. Furthermore, when employing effective propagators with a width $\gamma(p^2)$ depending explicitly on three-momentum $p$ the commutator of the fields no longer vanishes for $r > t$ since the related field theory becomes nonlocal and violates microcausality.

I. INTRODUCTION

Quantum Chromo-Dynamics (QCD) is considered to be the theory of the strong interaction, however, is accessible to perturbation theory only in the limit of short distances or high momentum transfer, respectively. Thermodynamical properties of hadronic or partonic matter at finite temperature $T$ and/or chemical potential $\mu$ involve large distance interactions and can rigorously only be addressed by lattice QCD (predominantly at vanishing chemical potential) in Euclidean space. Alternatively, one might employ effective field theories that share the symmetry properties of QCD and fix the couplings to reproduce field expectation values and correlators [1–11]. In fact, the knowledge about the phase diagram of strongly interacting hadronic/partonic matter has been increased substantially in the last decades. At vanishing (or low) chemical potentials lattice QCD (lQCD) calculations have provided reliable results on the equation of state [12, 13] and given a glance at the transport properties (or correlators [14–25]) in particular in the partonic phase.

Recent studies of 'QCD matter' in equilibrium – using lattice QCD calculations [14, 15] or partonic transport models in a finite box with periodic boundary conditions [26] – have demonstrated that the ratio of the shear viscosity to entropy density $\eta/s$ should have a minimum close to the critical temperature $T_c$, similar to atomic and molecular systems [27]. On the other hand, the transport studies in Refs. [26, 37, 38] have provided results for the shear and bulk viscosity as well as the electric conductivity that are very close to lattice QCD results, however, employ the notion of a strongly interacting gas of quasiparticles with a dynamically generated mass that is sufficiently larger than the width of their spectral functions. These studies have been based on the Dynamical QuasiParticle Model (DQPM) [39, 40] that incorporates effective propagators for the partons with a finite width of the spectral functions $A_i(\omega_i, p_i)$, i.e. for scalar fields ($\tilde{p} = (\omega, p)$)

$$A_i(\omega_i, p_i) = \frac{\gamma_i}{2E_i} \left( \frac{1}{(\omega_i - E_i)^2 + \gamma_i^2} - \frac{1}{(\omega_i + E_i)^2 + \gamma_i^2} \right)$$

$$= \frac{2\omega_i\gamma_i}{(\omega_i^2 - p_i^2 - M_i^2)^2 + 4\gamma_i^2\omega_i^2},$$

(1)

with $E_i^2(p_i) = p_i^2 + M_i^2 - \gamma_i^2$ and $i \in [g, q, \bar{q}]$. The spectral functions $A_i(\omega_i)$ are antisymmetric in $\omega_i$ and normalized as:

$$\int_{-\infty}^{+\infty} \frac{d\omega_i}{2\pi} 2\omega_i A_i(\omega_i, p) = 1,$$

(2)

where $M_i$ and $\gamma_i$ are the dynamical quasiparticle mass (i.e. pole mass) and width of the spectral function for particle $i$, respectively. They are directly related to the real and imaginary parts of the related self-energy, e.g. $\Pi_i = M_i^2 - 2i\gamma_i\omega_i$. In the off-shell approach, $\omega_i$ is an independent variable and related to the "running mass" $m_i$ by: $\omega_i = m_i + \frac{q^2}{2m_i}$. In case of vector fields or fermion fields the following retarded propagators are employed that differ from the 'free' massive case only by the additional $(2\gamma_V\omega)^2$ or $\pm i\gamma_F$ in the denominator and

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corresponding matrices in the numerator [41]:

$$A_V^{\mu
u}(\omega, \mathbf{p}, \gamma_V) = \gamma_V \frac{2\omega (g^{\mu\nu} - \mathbf{p}^\mu \mathbf{p}^\nu / M_V^2)}{\omega^2 - \mathbf{p}^2 - M_V^2 + 4\gamma_M^2 \omega^2},$$

(3)

and

$$A_F(\omega, \mathbf{p}, \gamma_F) = \frac{1}{4E_F}$$

(4)

$$\times \begin{pmatrix}
  \frac{E_F \gamma_0^0 + \mathbf{p} \cdot \gamma + m_F}{\omega - E_F - i\gamma_F} - \frac{-E_F \gamma_0^0 + \mathbf{p} \cdot \gamma + m_F}{\omega + E_F - i\gamma_F} \\
  \frac{-E_F \gamma_0^0 + \mathbf{p} \cdot \gamma + m_F}{\omega - E_F + i\gamma_F} + \frac{-E_F \gamma_0^0 + \mathbf{p} \cdot \gamma + m_F}{\omega + E_F + i\gamma_F}
\end{pmatrix}$$

with $E_F = \mathbf{p}^2 + m_F^2$ in obvious notation.

As is seen e.g. from the spectral function $\Phi$ it is non-vanishing for time-like ($\tilde{p}^2 > 0$) as well as for space-like ($\tilde{p}^2 < 0$) four-momenta such that the question emerges if the theoretical concept behind the DQPM (or other effective approaches) conserves microcausality, i.e. that the spectral function transformed to space-time has only support on and within the lightcone. We recall that the Fourier transform of the spectral function $A(\omega, \mathbf{p})$ is proportional to the commutator of the fields at different space-time point and its integration over energy $\omega$ ensures a proper quantization (see below). This is of particular importance since a transport realization can only propagate ‘quasiparticles’ within or on the lightcone $[42, 43]$. More importantly, in the DQPM spectral contributions are separated into time-like ($\tilde{p}^2 > 0$) and space-like ($\tilde{p}^2 < 0$) four-momentum parts and the additional question arises if the contributions separately conserve microcausality.

The layout of our study is as follows: In Section II we briefly present the basic definitions and relations between retarded and advanced propagators and recall the analytic proof for microcausality in case of the spectral functions [11]. In Section III the actual problem is set up for time-like ($\tilde{p}^2 > 0$) and space-like ($\tilde{p}^2 < 0$) four-momentum parts of the spectral function and its numerical realization. Furthermore, we present the actual numerical results for strong coupling and investigate the aperiodic limit as well as the case $\gamma > M$. A summary and discussion of results is given in Section IV.

II. PROPAGATORS AND SPECTRAL FUNCTIONS

In this work we will concentrate on the model case of a massive scalar field coupled e.g. to an external fermion field ($\sim \partial_\mu \Phi(\mathbf{x}) \mathcal{\Psi}(\mathbf{x}) \gamma^\mu \mathcal{\Psi}(\mathbf{x})$) with a vanishing three-current, i.e. the field equation

$$\left(\frac{\partial^2}{\partial t^2} - \Delta + M^2 + 2\gamma \frac{\partial}{\partial t}\right) \Phi(\mathbf{x}) = 0,$$

(5)

where $\gamma$ stands for the strength of the coupling (e.g. $g_s < \Psi^\dagger \Psi / 2$). Eq. (5) has the algebraic solution

$$\Phi(\mathbf{x}) = \frac{-1}{\omega^2 - \mathbf{p}^2 + M^2 + 2i\gamma},$$

(6)

which leads to the retarded Green-function $G_{ret}$ obeying

$$G_{ret}(x - y) = 0 \text{ for } x^0 - y^0 < 0$$

(7)

by a 4-dimensional Fourier transformation of (6).

$$G_{ret}(x) = \int \frac{d^4 \mathbf{p}}{(2\pi)^4} \tilde{G}(\mathbf{p}) \exp(-ipx).$$

(8)

We point out that $\Theta \tilde{G}(\mathbf{p})$ is identical to (1). We recall, furthermore, that solutions of the Kadanoff-Baym equations [44] for $\Phi^\dagger$- theory in 2+1 dimensions [45] have lead to spectral functions that are very close to (1) also for strong coupling.

A. Analytical results

The integration over $d\omega = dp^0$ in (8) can be carried out by contour integration and the angular integration in three-momentum is straightforward. With $\mu = \sqrt{M^2 - \gamma^2}$ the remaining integral kernel reads

$$K(x) = \frac{1}{\vert x \vert} \int_0^\infty p \sin(t\sqrt{\mu^2 + p^2}) \sin(\vert x \vert p) \frac{dp}{\sqrt{\mu^2 + p^2}}$$

(9)

$$\left[ \frac{1}{2} \right] \int_{-\infty}^{\infty} p \sin(t\sqrt{\mu^2 + p^2}) \sin(\vert x \vert p) \frac{dp}{\sqrt{\mu^2 + p^2}}$$

which has a singular contribution on the lightcone and a regular part on and within the lightcone. The remaining integration over $dp$ gives for the retarded Green-function (using $x = (t, \mathbf{x}) = (x^0, \mathbf{x})$)

$$G_{ret}(x) = \frac{e^{-\gamma t} \Theta(t)}{2\pi} \delta(t^2 - x^2)$$

(10)

$$- \frac{e^{-\gamma t} \Theta(t)}{4\pi} \Theta(t^2 - x^2) \frac{\mu}{\sqrt{t^2 - x^2}} J_1 \left( \mu \sqrt{t^2 - x^2} \right)$$

for $\mu^2 \geq 0$. With

$$\delta(t^2 - x^2) \Theta(t) = \delta((t - \vert x \vert)(t + \vert x \vert)) \Theta(t) = \frac{\delta(t - \vert x \vert)}{2 \vert x \vert}$$

(11)

one arrives at the final result [10]

$$G_{ret}(x) = \left( \frac{\delta(t - \vert x \vert)}{4\pi \vert x \vert} - R(t^2 - x^2) \right) e^{-\gamma t} \Theta(t)$$

(12)
with
\[ R(t^2 - x^2) = \Theta\left(t^2 - x^2\right) \frac{\mu}{4\pi \sqrt{t^2 - x^2}} J_1 \left(\mu \sqrt{t^2 - x^2}\right), \]
\[ \text{(13)} \]
where \( J_1 \) is the Bessel function. In the actual calculations the \( \delta \)-distribution term on the lightcone will be subtracted and we will address the regular part including the overall exponential decay in time, i.e.
\[ \tilde{R}(t^2 - x^2) = R(t^2 - x^2) e^{-\gamma t} \Theta(t). \]
\[ \text{(14)} \]
We note in passing that the related results for massive vector fields and Dirac fields read \[47\]
\[ \text{We investigate microcausality for the scalar case since microcausality fulfilled in the four-momentum integral.} \]
\[ \text{Except for a factor exp(–}\gamma t\) the quantity \( \Delta^* \) is identical to the Schwinger function \( \Delta(x, \mu) \) with effective mass \( \mu \),
\[ \Delta^*(x) = \Delta(x, \mu) \cdot e^{-\gamma |t|}, \]
\[ \text{(19)} \]
\[ \Delta(x, \mu) = -\frac{i}{(2\pi)^3} \int e(\hat{p}) \delta(\hat{p}^2 - \mu^2) e^{-i\hat{p} \cdot x} d^4 \hat{p}, \]
\[ \text{(20)} \]
with \( e(\hat{p}) = 1 \) for \( \omega > 0 \) and \( e(\hat{p}) = -1 \) for \( \omega < 0 \). Since \( \Delta(x, \mu) \) vanishes for space-like distances \( x^2 < 0 \) we find that microcausality is fulfilled also in the interacting case (cf. Ref. \[49\]). Since the DQPM - as an effective approach to QCD - employs spectral functions of the type \[12\] (or \[15\] and \[19\]) we may conclude that the model approach conserves microcausality strictly \[19\].

**B. Spectral functions**

Of central interest in our study is the scalar spectral function \( A(\omega, \mathbf{p}) \) \[11\], i.e. the imaginary part of the retarded propagator. The commutator between the fields at different space-time points can also be written as the difference of advanced and retarded propagators (due to opposite signs of the imaginary parts in the propagators \[15\]):
\[ [\Phi(x), \Phi^+(0)] = i \Delta^*(x) = i (G_{av}(x) - G_{ret}(x)) =: C(x). \]
\[ \text{(18)} \]

**III. SPACE-LIKE AND TIME-LIKE MOMENTUM CONTRIBUTIONS**

We now come to the central question of our study: Is microcausality fulfilled in the four-momentum integral
\[ C(x) = \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \Im(G_{ret}(\omega, \mathbf{p})) \exp(-i(\omega t - \mathbf{p} \cdot \mathbf{x})) \]
\[ \text{(21)} \]
when restricting to time-like \( \Theta(\omega^2 - \mathbf{p}^2) \) or space-like \( \Theta(\mathbf{p}^2 - \omega^2) \) four-momenta?

To answer this question we can no longer perform the contour integration over \( d\omega \) due to the \( \Theta \)-functions in
four-momentum and have to evaluate the integrals \([21]\) numerically exploiting the antisymmetry of the integrand \([1]\) in \(\omega\) and carrying out the angular integration in the three-momentum. This leads to

\[
C(x) = -\frac{i\gamma}{2\pi^3 |x|} \int_0^\infty dp \int_0^\infty d\omega \sin(\omega t) \sin(|x| p) \tag{22}
\]

using \([1]\) which can be ‘solved’ on a numerical grid as well as by analytical integration (cf. Section II). As mentioned before the integral \((22)\) has a singular part \((\delta(t - r)/(4\pi r)\) using \(r = |x|\)) as well as a regular part given by \((14)\). The singular part can be subtracted in the integral \((22)\) - to achieve a better convergence - by considering

\[
C(x) - \frac{\delta(t - r)}{4\pi r} e^{-\gamma t} = -\frac{i}{2\pi^3 r} \int_0^\infty dp \int_0^\infty d\omega \sin(\omega t) \sin(rp) \tag{23}
\]

\[
\times \frac{\gamma p(\omega^2 - p^2 - (\gamma^2 + M^2)/2)}{[(\omega^2 - p^2 - (\gamma^2 + M^2)/2) + 4\omega^2\gamma^2] [((\omega^2 - p^2 - \gamma^2)^2 + 4\omega^2\gamma^2].
\]

In this way the \(\delta\)-distribution on the light-cone (decaying exponentially in time) is subtracted explicitly on the same computational grid. In order to demonstrate the validity of this numerical subtraction scheme we show in Fig. 1 a comparison of the analytical result \((14)\) with the corresponding numerical evaluation of \((23)\) for \(M = 1\) GeV and \(\gamma = 0.3\) GeV at \(t = 15/\text{GeV}\). Indeed, both results agree within the linewidth and are identical to zero for \(r > t\).

In Fig. 2 we show the regular part \((14)\) for a width \(\gamma = 0.045\) GeV (a) and \(\gamma = 0.3\) GeV (b). The signal decays exponentially in time \((\sim \exp(-\gamma t))\) and shows hyperbolic oscillations within the lightcone while being zero outside the lightcone. The numerical results and the analytical expression \((14)\) are identical on the level of three digits for both cases.

We now turn to the numerical results for strong coupling when gating on time-like and space-like four momenta in \((23)\) separately. The results are displayed in Fig. 3 for \(\gamma = 0.3\) GeV when including only time-like momenta (a) or only space-like momenta (b). Note that the numerical results in Fig. 3 have been multiplied by \(\exp(\gamma t)\) in order to compensate for the exponential decay in time.

It is seen that both results do not vanish for \(r > t\) and thus violate microcausality. This is shown more explicitly in Fig. 4(a) as a function of time \(t\) and \(r - t\) in the space-like region and demonstrates that both contributions are nonvanishing but of opposite sign such that their sum becomes identically zero. In Fig. 4(b) we display the same quantities as in (a) but multiplied by \(\exp(\gamma t)\) which demonstrates that both contributions do not decay exponentially in time as seen from the full analytical solution \((12)\). This clearly demonstrates that a restriction to either space-like or time-like momentum parts of the spectral function violates causality while both parts together conserve microcausality in line with the analytical result in Section II. The violation of microcausality is tiny in case of \(\gamma \ll M\) but becomes sizeable for \(\gamma > 0.1\) GeV.

When considering the ‘aperiodic’ limit \(\gamma \rightarrow M\), i.e. \(\mu \rightarrow 0\), we find from \((13)\) that \(\tilde{R}(t^2 - r^2)\) vanishes identi-
FIG. 3: (Color online) The regular part of the commutator (14) (multiplied by \(\exp(\gamma t)\)) as a function of the distance \(r\) and time \(t\) for \(\gamma = 0.3\) GeV for time-like four-momenta (a) and space-like four-momenta (b).

For \(\mu = 0\) and the commutator (13) only has support on the lightcone. Furthermore, in the limit \(\mu \to 0\) the oscillations in \(R(t^2 - r^2)\) vanish as can be extracted from a Taylor expansion of (14) (with \(J_1(z) \approx z/2 \pm \cdots\))

\[
R(t^2 - r^2) \approx \Theta(t^2 - r^2) \frac{\mu}{8\pi \sqrt{t^2 - r^2}} (\mu \sqrt{t^2 - r^2}) \cdots
= \Theta(t^2 - r^2) \frac{\mu^2}{8\pi} \cdots
\]  

(24)

For \(\gamma > M\) (overdamped fields) we no longer find oscillations of the regular part (14) within the lightcone but only an exponentially decaying signal as seen from Fig. 5 for \(\gamma = 1.25\) GeV.

We close in pointing out that our numerical scheme allows to employ almost arbitrary spectral functions in (22) and to check if microcausality holds. Without explicit representation we note that using a three-momentum width \(\gamma(p^2)\) in the spectral function (1) the commutator (22) no longer vanishes for \(r > t\) since the individual momentum modes decay on different time scales and the field equation (5) becomes non-local in this case. Nevertheless, the normalization condition (2) still is fulfilled. Furthermore, the commonly adopted form,

\[
A(\omega, p) = \frac{2M\gamma}{(\omega^2 - p^2 - M^2)^2 + 4\gamma^2 M^2},
\]  

(25)

also violates microcausality.
The regular part of the commutator \[ \langle 0 | \hat{A} \hat{B} - \hat{B} \hat{A} | 0 \rangle \] as a function of the distance \( r \) and time \( t \) for \( \gamma = 1.25 \text{ GeV} \).

IV. SUMMARY

In this study we have examined effective propagators of the type (6) as used e.g. in the Dynamical QuasiParti-
cle Model (DQPM) \[39,40\] for an approximation to QCD propagators at temperatures above the critical temperature \( T_c \) for deconfinement. It could be shown analytically that their spectral functions (or imaginary parts) do not lead to a violation of microcausality, i.e. to a vanishing commutator of the interacting fields outside the light-cone. However, when restricting to only space-like or time-like four-momentum contributions of the spectral function a violation of microcausality is found numerically which becomes severe in case of strong coupling. Moreover, the space-like or time-like four-momentum contributions separately no longer decay exponentially in time as in case of the full solution \[12\]. Furthermore, we have found that using a three-momentum dependent width \( \gamma(p^2) \) in the spectral function \[11\] the commuta-
tor \[22\] no longer vanishes for \( r > t \) since the individual momentum modes decay on different time scales and the field equation \[5\] becomes non-local in space in this case. This also holds for the spectral function \[25\] which is often employed in phenomenological models.

Our findings imply that the modeling of effective field theories for strongly interacting systems has to be considered with great care and restrictions to time-like four momenta in case of broad spectral functions have to be ruled out.

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FIG. 5: (Color online) The regular part of the commutator \[17\] as a function of the distance \( r \) and time \( t \) for \( \gamma = 1.25 \text{ GeV} \).
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