INCREASING THE RADAR EQUIVALENT ENERGY POTENTIAL BY “TRACK BEFORE DETECT” METHOD

Abstract. The problem of radar detection of small-sized targets using the traditional methods of selection of signals embedded in background noise is considered. It is shown that for a false alarm rate of $10^{-5}$, which provides for 1–2 false alarms within the entire coverage of the modern 3D radar, the probability of detection of a small-sized target is getting unacceptably low. Repeatedly decreasing the threshold can provide an acceptable level of the detection probability at ultra-low signal-to-noise ratio (SNR) values. At the same time, decreasing the threshold will result in an unacceptable increase of the false alarm rate. A new target detection procedure using the “track before detect” method (TBD) is proposed. In the TBD procedure, the target is considered detected when two conditions are met: the signal exceeds once a definite threshold; the target is detected within a strictly defined observation area (acquisition or tracking gate). For low SNR values in the range of 3–8 dB and equal false alarm rate, the detection probability increases by 20–50% compared to the traditional detection method. The simulation results showed a strong dependence of efficacy of the TBD algorithm on the threshold value and the decision rule. The possibility is noted of adaptive control over the threshold due to the use the detection results in the preceding scanning cycles, as well as the introduction of matrix radar surveillance not only by the target coordinates and parameters, but also by the detection threshold, decision rules, etc. Examination of these issues is the subject of further research.

Keywords: track-before-detect, radar, small-sized target, threshold, false alarm rate, probability of detection, observation area
Introduction. Detection of small-sized targets, i.e. targets with the radar cross section (RCS) in the order of $10^{-3}...10^{-2}\text{ m}^2$ (drones, fifth generation Stealth fighters, hypersonic cruise missiles), using the known methods of extraction of signals embedded in noise provides poor results. Even for low values of the true detection probability an unattainable energy of the useful signal is required.

For the simplest case of signal detection using a standard set of coherent processing techniques (frequency pre-selection – matched filtering – coherent integration), the required conditional probabilities of false alarms in a single radar resolution bin per one scanning cycle $F$ and of true detection $D$ are related by the known correlation

$$D = F^{1+\rho},$$

where $\rho$ – is the signal-to-noise ratio (SNR) at the coherent integrator output.

Assuming that $F = 10^{-5}$, one can easily calculate that even for a small value of $D = 0.9$, the required SNR value exceeds 20 dB. This energy ratio is very tough to provide in practice when detecting small-sized targets, which calls for the development of new approaches to detect them with a significant reduction in the required SNR.

Required false alarm probability. To evaluate such a possibility, one has to understand how the value $F = 10^{-5}$ evolved, which is very practical for a wide class of surveillance radars (standby and combat-mode ones, radars detecting low-flying targets).

The probabilities $D$ and $F$ characterize the quality of the decisions about the presence or absence of a target in a certain range resolution bin within one scan cycle. Referenced to a single resolution volume, these probabilities are perceived as elementary.

Let us assume, for certainty, that in a normally tuned radar, 1–2 false alarms are traditionally permitted in the entire observation space per one scan cycle, then

$$F_m = 1 - (1 - F)^m \approx mF,$$  \hspace{1cm} (1)

where $F_m$ and $F$ – are conditional false alarm probabilities in scanning $m$ resolution bins and a single one, correspondingly, $m$ – is the number of resolution bins within the radar coverage.

Taking into account the expression (1) for $F_m = 1$, the conditional false alarm probability in a single resolution bin $F \approx 1/m$.

In modern 3D radars (1L122-1E, EL/M2106, SABERM60, Rasit, BoP-A550, PS91, MMSR, GERFAUT, SPIDER RSR and others), the surveillance space is broken into 100000–300000 resolution bins, for which $F = (0.33...1) \cdot 10^{-5}$ [1].

For further calculations, we shall adopt the value $m = 10^5$, then $F = 10^{-5}$ and, as was shown above, the required SNR for the detection of a target with $D = 0.9$ conditional probability constitutes over 20 dB.

Note that the signal at the input of the threshold device, when detecting weak strongly time-correlated target signals, is distributed according to the exponential law, since in this case the theory requires all
the observation time to be spent on coherent integration [2]. In this case, the probabilities \( D \) and \( F \) are determined by the expressions:

\[
F = \exp \left( -\frac{z^*}{\bar{z}_0} \right); \\
D = \exp \left( -\frac{z^*}{\bar{z}_1} \right),
\]

where \( \bar{z}_0 = N_0^2 \Delta F_H = \left( N_0 + \frac{\sigma_{cl}^2}{\Delta f_0 \nu_{kk}} \right) F_t \) and \( \bar{z}_1 = \sigma_{cl}^2 T_0 \Delta f_0 \nu_{cl} = \sigma_{cl}^2 T_0 \Delta f_0 L \) – mean power of signals at the coherent integrator output in the presence and absence of the useful signal; \( N_0 \) – is the spectral density of internal noise of the radar receiver; \( \sigma_{cl}^2 \) – is the mean power of clutter; \( \nu_{kk} \) – is effectiveness of indication of moving targets; \( \Delta f_0 \) and \( F_t \) – are the modulation spectrum width and repetition frequency of the probing signal; \( L \) – is the number of coherent integration periods; \( \sigma_{cl}^2 \) – is the mean power of the useful signal; \( \nu_{cl} \approx \frac{F_t}{L} \) – is effectiveness of coherent integration.

It follows from the expression (2) that

\[
F = \exp \left( -\frac{z^*}{\left( N_0 + \frac{\sigma_{cl}^2}{\Delta f_0 \nu_{kk}} \right) F_t L} \right).
\]

Assuming that \( F = 10^{-5} \), we determine that the level of threshold \( z^* \) has to be 11.5 times higher than the mean aggregate power of the background and residue of decorrelates of clutters at the coherent integrator output. It should be noted that the law of distribution of false blips in the observation space is obviously uniform, due to the independence of the average background level from the position of the resolved volume, with respect to which the detection procedure is performed.

A decrease of the threshold level \( z^* \) by \( k \) times shall result in an increase in the conditional false alarm probability \( F_{\text{red}} \) and true detection probability \( D_{\text{red}} \) up to values equal to

\[
F_{\text{red}} = \exp \left( -\frac{\frac{z^*}{\left( N_0 + \frac{\sigma_{cl}^2}{\Delta f_0 \nu_{kk}} \right) F_t L} \right) \frac{1}{k} = \frac{k}{\sqrt{F}}; \\
D_{\text{red}} = \sqrt{D}.
\]

For example, for \( k = 5 \) and \( F = 10^{-5} \), \( F_{\text{red}} = 0.1 \). It is obvious that decreasing the detection threshold \( z^* \) by \( k \) times, from the point of view of \( F \), is equivalent to an increase of the non-correlated background by \( k \) times or decrease of the target SNR by \( k \) times or, if the initial detection conditions are preserved – a decrease of the target RCS by the same \( k \) times. The crucial issue here is estimation of the admissible value of \( k \).

**Evaluating the possibility of decreasing the detection threshold.** In order to attain the evaluation, let us examine in detail the identification processes during secondary processing. The process of detecting and tracking targets “on the pass” is shown in Fig. 1, it has been repeatedly described in a number of sources and does not require a detailed explanation.

The detection and tracking process is presented for the particular case of one-dimensional observation space and five consecutive probes, marked on the time axis by moments \( T–5T \). The time interval between contacts with the target is \( T \). In this example, there is an inpatient case of successful detection of the target in five consecutive contacts with it without drop-outs.

In our presentation we shall, for simplicity, consider the dimensions of the acquisition (tracking) gate unchanged in the process of work (unlike tracking using Kalman filtering approaches) and, for the examined case, equal to [3]:

...
\[ \delta r = k_r \sqrt{\sigma_v^2 T^2 + 2 \sigma_r^2}, \]

where \( \sigma_v \) and \( \sigma_r \) – are root mean square errors of measuring the value of total speed and range of the target; \( 1 \leq k_r \leq r \) – the gate broadening coefficient, selected depending on the jamming environment.

For a 3D radar, the dimensions of the gate in three-dimensional space \((r, \beta, \epsilon)\) can be described by the expressions

\[ \delta r = k_r \sqrt{\sigma_v^2 T^2 + 2 \sigma_r^2}; \]

\[ \delta \beta = k_\beta \sqrt{\sigma_\beta^2 T^2 + 2 \sigma_\beta^2}; \]

\[ \delta \epsilon = k_\epsilon \sqrt{\sigma_\epsilon^2 T^2 + 2 \sigma_\epsilon^2}, \]

where \( \sigma_\beta(\sigma_\epsilon) \) and \( \sigma_\beta(\sigma_\epsilon) \) – are RMS errors of measuring the azimuth (elevation) and time-derived azimuth (elevation); \( 1 \leq k_r, (k_\beta, k_\epsilon) \leq 2 \) – are broadening coefficients of the gates.

The position of the gate center during the \( k + 1 \)-th scan cycle is extrapolated according to the law

\[ x(k + 1) = x(k) + \dot{x}_k T, \]

where \( x = r(\beta, \epsilon) \), \( \dot{x} \) – are time-derived coordinates \( r, \beta, \epsilon \).

Let us estimate the number of instances of exceeding the detection threshold reduced by \( k \) times during one scan cycle

\[ N_{lt} = mF_{red}. \]  

Taking into account that the gate “volume”, according the expressions (5), (6), will amount to

\[ V_{gte} = \delta r \delta \beta \delta \epsilon = k_r k_\beta k_\epsilon \left( \sigma_v^2 T^2 + 2 \sigma_r^2 \right) \left( \sigma_\beta^2 T^2 + 2 \sigma_\beta^2 \right) \left( \sigma_\epsilon^2 T^2 + 2 \sigma_\epsilon^2 \right), \]

the number of gates within the observation space, in round-looking mode, will be defined by the expression

\[ m_{gte} = \frac{(r_{max} - r_{min}) 360^\circ (\epsilon_{max} - \epsilon_{min})}{V_{gte}}, \]

where \( r_{min}, r_{max} \) and \( \epsilon_{min}, \epsilon_{max} \) – are the minimal (maximum) values of the target range and elevation, respectively.
The probability of occurrence of one false alarm in the volume of the observation gate, assuming their uniform distribution in space, will be

$$P_{gle} = \frac{N_{gle}}{m_{gle}}.$$ 

Summarizing the previous reasoning, we shall note that, as a result of $k$-fold reduction of the detection threshold compared to this level for detection with the usual values of the SNR of the order of 20 dB and the probability of false alarms in one resolution bin per scan cycle $F = 10^{-5}$, the true detection probability $D_{red} = \frac{kD}{F}$, and $F_{red} = \frac{kF}{D}$.

As an example of small SNR within 3 dB only for $F = 10^{-5}$ and $k = 5$, the probability $D_{red} = 0.022$.

When the threshold is reduced by a factor of 50, the probability $D_{red} = 0.926$, and $F_{red} = 0.794$. As expected, even with ultra-small values of the SNR, a multiple reduction of the threshold can provide an acceptable level of conditional probability of true detection, but at the same time the conditional probability of false alarms increases unacceptably. For the example under consideration, the average number of false alarms in a surveillance space containing 100000 resolvable volumes increases from 1–3 to almost 80000.

Let us estimate the volume of the acquisition (tracking) gate with linear dimensions defined by expressions (5), (6) in real numbers. Based on the performance characteristics of many surveillance radars, we will assume that $\sigma_r \approx 0.2\Delta r$, $\sigma_{\beta(e)} \approx 0.2\Delta \beta(e)$, $\sigma_\gamma \approx 0.2\Delta \gamma$, where $\Delta r$, $\Delta \beta$, $\Delta e$, $\Delta \gamma$ are the radar resolution capabilities in range, azimuth, elevation and range rate.

For example, with practical figures $\Delta r = 20$ m, $T = 10$ s, $\Delta \gamma = 5$ m/s and the coefficient $k_r = 2$ the value $\delta r \approx 20$ m, that is, the width of the gate along the range is approximately equal to the corresponding resolution, and the broadening of the gate due to inaccuracy of the range rate measurement plays a decisive role in practice.

Assuming the accuracy of this conclusion also for the other measured radar coordinates (azimuth and elevation), we can, in first approximation, assume that the linear dimensions of the acquisition (tracking) gate in steady-state mode approximately correspond to the linear dimensions of a single resolvable volume. This allows us to estimate the probability of exceeding the new reduced threshold as approximately corresponding to $F_{red} = \frac{kF}{D}$ from the expression (4).

Retention of the connection of new detection results with a value $*z$ from (4) is not accidental and is appropriate, because it allows to directly assess the energy premium from the implementation of the proposed method as compared with the traditional one.

We shall further outline briefly the physical meaning of the track-before-detect (TBD) procedure.

In the conditions of low energy of the received signal, the operator sharply reduces the level of the detection threshold, which results in a significant increase in the probabilities $D_{red}$ and $F_{red}$. The resulting blips are used later to implement the flight path (track) detection and tracking procedure, for example, based on the standard procedure: if in $n$ consecutive scan cycles at least $l$ of them the useful signal is detected with a reduced threshold within the tracking gates, the track is considered to have been detected. Clearly, the meaning of the proposed procedure is that track detection is identified with the target detection.

The question remains about the fate of the sharply increased conditional probability of false alarms after the lowering of the detection threshold. Here, we must realize that there is a significant difference between the standard and the proposed detection procedures. In the standard detection procedure, exceeding the threshold is an event determined exclusively by the stochastic properties of the signal. In the TBD procedure, the meaning of the event radically changes: detection is an event consisting in the simultaneous exceeding by a signal of a certain threshold (as in the standard procedure) and in that this occurs within a strictly defined section of the observation space (acquisition/tracking gate). In this case, the events of exceeding the threshold by the signal and realization of this event in the gate in the first approximation can be considered statistically independent. To be precise, such a weak statistical connection exists, but its evaluation is the subject of further research.

We shall introduce new symbols here:
where the index TBD denotes being resultant from track-before-detect procedure, $D_{\text{gfe}}$, $F_{\text{gfe}}$ – are probabilities of the useful signals and noise emissions exceeding the threshold getting into the tracking gate.

Assuming that the tracking gate is set sufficiently accurately by the useful signal, with allowance for measurement errors taken into account in (5), (6), in first approximation we can assume $D_{\text{gfe}} \approx 1$.

The probability $F_{\text{gfe}}$ is the conditional probability of exceeding the threshold within the tracking gate which, with sufficient accuracy based on expressions (7) and (8), is equal to

$$F_{\text{gfe}} = \frac{m}{m_{\text{gfe}}} F_{\text{red}}.$$  

As was shown above, one can consider that $m / m_{\text{gfe}} \approx 1$, then

$$D_{\text{TBD}} \approx D_{\text{red}} = \exp \left( - \frac{z^*}{k z_1} \right) = \exp \left( - \frac{z_{\text{red}}^*}{z_1} \right);$$

$$F_{\text{TBD}} \approx F_{\text{red}}^2 = \exp \left( - \frac{2z^*}{k z_0} \right) = \exp \left( - \frac{2z_{\text{red}}^*}{z_0} \right);$$

where $\frac{z^*}{k} = z_{\text{red}}^*$ – is the detection threshold reduced by $k$ times as compared to regular one.

We shall write down the final probabilities of true and false track detection according to the “$l$ out of $n$” criterion, they are also the probabilities of true and false target detection if using the TBD procedure, as:

$$D_{\text{FIN}} \approx D_{\text{red}} \sum_{i=l-1}^{n-1} C_{n-1}^{l-1} D_{\text{TBD}}^{l-1} (1 - D_{\text{TBD}})^{n-l-1} = D_{\text{red}} \sum_{i=l-1}^{n-1} C_{n-1}^{l-1} \exp \left( - \frac{(l-1)z_{\text{red}}^*}{z_1} \right) \left( 1 - \exp \left( - \frac{z_{\text{red}}^*}{z_1} \right) \right)^{n-l-1} = \quad (9)$$

$$F_{\text{FIN}} \approx F_{\text{red}} \sum_{i=l-1}^{n-1} C_{n-1}^{l-1} F_{\text{TBD}}^{l-1} (1 - F_{\text{TBD}})^{n-l-1} =$$

$$= F_{\text{red}} \sum_{i=l-1}^{n-1} C_{n-1}^{l-1} \exp \left( - \frac{2(l-1)z_{\text{red}}^*}{z_0} \right) \left( 1 - \exp \left( - \frac{2z_{\text{red}}^*}{z_0} \right) \right)^{n-l-1}, \quad (10)$$

where $C_n^l$ – is the number of combinations of $l$ out of $n$.

Graphs of the dependence of the false alarm and true detection probabilities plotted in accordance with expressions (2), (3) and (9), (10) are shown in Fig. 2. Dependencies for the TBD method were obtained using the “4 out of 5” criterion.

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Fig. 2. $a$ – false alarm rate dependences on the threshold decrease; $b$ – detection probability dependences on the SNR:

1 – TBD detection method (“4 out of 5” criteria); 2 – traditional detection method
As one can see from Fig. 2, a, when detecting a target using the traditional method, for example, reducing the detection threshold by a factor of 10, leads to a significant increase in the probability of false alarm from $10^{-5}$ to 0.316. Applying the TBD method, the probability of false alarm is reduced to the level of $1.2 \times 10^{-3}$. Explaining the advantages of the TBD method based on the graph of dependence of the true detection probability (Fig. 2, b), one can note that for small values of the SNR in the range of 3–8 dB and the same false alarm rate (0.1 for this particular case), the true detection probability increases by 20–50 % in comparison with the traditional detection method.

**Results of modeling and conclusions.** The results were evaluated by mathematical modeling. The simulation was performed on a linear section of the target trajectory at a SNR of 10 dB. As an example, Fig. 3 and 4 show the results of functioning of a surveillance radar in the form of a sequence of detected blips (trajectories) at different levels of false alarm probability.

It follows from the simulation results that the detection effectiveness is very sensitive to both the value of the detection threshold, and the type of the decisive rule of the TBD algorithm. An insignificant
A decrease in the threshold leads to a significant increase in the number of false tracks and the average time of their tracking, which results in unacceptable information overload of the radar. Employment of the TBD method enables tangibly reducing the false alarm probability with a practically fixed true detection probability.

Noteworthy is the possibility of adaptive control of the detection threshold due to using the results of detection in the previous scan cycles. For example, the detection threshold can be reduced (increased) not in all the resolution bins, but only in those that get in the acquisition (tracking) gate.

Earlier, the problem of controlling the detection threshold was solved through the notion of the need to ensure constant false alarm rate (CFAR), whereas the true detection probability faded to the background. When setting the problem of detection of a small-size target, the first priority is to provide the required true detection probability, which is possible by reducing the detection threshold.

This approach leads to the need to introduce matrix surveillance, involving not only the coordinates and parameters of the object of detection (range, azimuth, speed), but also the detection threshold, decisive rules, etc. Examination of these issues is the subject of further research.

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