On the theory of quantum measurement

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Abstract

The notion of state reduction employed by the standard quantum theory of measurement is difficult to accept for two reasons: It leaves open where and when the reduction takes place and it does not give any objective conditions under which the reduction occurs. Some recently published ideas on this problem are developed an improved. The disturbance of measurement due to identical particles in the environment is shown to make any POV measure non-measurable. Truncated POV (TPOV) measures are introduced that can be measurable if object systems satisfy the additional requirement of having separation status. The separation status is generalised from domain of space to domain of phase space. Starting from the previously introduced distinction between ancillas, screens and detectors, further study of experiments suggests that a thermodynamic mixing within a detector or a screen and the consequent loss of separation status is the objective condition for the occurrence of the state reduction. The conjecture is simple, specific and testable. The theory is illustrated by a model of a real measurement.


1 Introduction

It is well known that the quantum theory of measurement is in an unsatisfactory state \[1, 2\]. For example, the ideas of quantum decoherence theory \[3\] that have brought some progress does not solve the problem of quantum measurement without any additional assumptions such as Everett interpretation \[4, 5\].

A measurement on microscopic systems can be split into preparation and registration. Registration devices are called meters. We observe that the process of registration has the following strange and fascinating properties:

1. Registered value \(r\) is only created by the interaction of the object system with the meter during the registration. Unlike the measurement in a classical theory, registrations do not reveal already existing values. There has been a lot of work on this feature since the beginnings of quantum mechanics and it is very well confirmed by a number of theoretical and experimental results (contextuality \[6, 7\], Bell inequalities \[8\], Hardy impossibilities \[9\], Greenberger-Horne-Zeilinger equality \[10\], etc.).

2. As a rule, repeated experiments give different values \(r\) from a well-defined set of possible alternatives \(R\). Each outcome is thus created with probability \(P_r\) such that \(\sum_{r \in R} P_r = 1\). The resulting randomness, or the so-called QM indeterminism occurs only during registrations. This is not in good harmony with other quantum processes, which are described by Schrödinger equation in a deterministic way.

3. There are correlations between values registered by distant meters. As the values are only created by registrations, a spooky action at a distance between the meters turns out to be necessary. This is called QM non-locality and it again seems to appear only via registrations (e.g., Einstein-Podolski-Rosen experiment \[12\], Bell has proved such non-locality in the registration of a single particle \[11\]).

These properties are not logically self-contradictory. Moreover, they are testable and have been confirmed by numerous experiments as well as theoretical analysis. For many physicists, however, they are unacceptable for taste and traditional reasons. Our standpoint is that they can be taken as “facts of life”. However, they give us an additional motivation to concentrate research on the phenomenon of registration.

In several recent papers \[13, 14, 15, 16, 17, 18, 19, 20\], some new ideas about quantum mechanics were proposed. The present paper is a continuation of our work on the quantum measurement \[15, 18, 19\]. The main strategy has been a return to physics: to observe carefully what happens in real experiments and to select
some general hypotheses, which have then an empirical rather than a speculative
character.

In [15] it has been shown that quantum mechanical theory implies a strong dis-
turbance of the registration of any observable such as position, momentum, energy,
spin and orbital angular momentum on any quantum system $S$. The origin of the
disturbance is the existence of systems of the same type as $S$ in the world (not
just in a neighbourhood of $S$). For example, according to the standard quantum
mechanics, measurements of these observables on an electron is impossible because
of the existence of other electrons. This theoretical observation clearly contradicts
the long and successful praxis of experimenting. The fact that such disturbances do
not occur in real measurements must then be understood as a proof that the current
ideas on observables and registrations need some corrections.

In [15, 18, 19], an attempt at a correction of measurement theory is described
that focuses on disturbances due to remote particles. For example, the registration
of a spin operator of an electron prepared in our laboratory had to be (theoretically)
disturbed by an electron prepared in a distant laboratory. The idea was that it is
not the spin operator that is really measured, but a different observable that can be
constructed from the spin operator by some process of localisation. Such constructs
have been called $D$-local observables, $D$ being some region of space. In order that $D$
local observables be measurable on, say, an electron, the electron must be prepared
in such a way that the influence of all other electrons on the measurement apparatus
inside a region $D$ of space is negligible (e.g., the wave functions of all these electrons
practically vanish in $D$). We say then that the electron has a separation status $D$.
On such a basis, a whole general mathematical theory has been constructed [19].

Another important fact is that a particle prepared with a certain local separation
status looses the status if it arrives in a region of space where its wave function
has a non-zero overlap with wave functions of other particles of the same type.
Then, the particle itself does not make sense as an individual system in a prepared
state because there are only (anti-)symmetrised states of the whole system. Thus,
changes of separation status can accompany quantum processes and are recognised
as deep changes in physics of the studied systems. This is a physical idea that is
not sufficiently supported by standard quantum mechanical formalism. Moreover.
a change of separation status can be understood as an objective fact that can be
observed but is itself independent of any observer.

A careful study of many real experiments in [15, 18] has shown that all meters
contain macroscopic detectors and screens. Further observation are that the regis-

\footnote{For example, we speak here about wave functions for the sake of simplicity, but wave functions
are not sufficiently general in two respects: they represent pure states and refer to a particular
frame, the $Q$-representation.}
tration processes include separation status losses in the detectors and screens on one hand and state reductions on the other (screens are studied in [19]). This motivates us to state a general rule: Changes of separation status are associated with state reductions. The form of the state reduction is then determined by the structure of the experiment. Thus, the state reduction occurs at a well-defined place and time and the reduction frame is also determined.

That mean, of course, that Schrödinger equation is not valid for changes of separation status. This is rather surprising. It is difficult to believe that the disturbance of measurement due to identical particle has anything to do with the problem of quantum measurement because the standard understanding of quantum theory does not lead to any idea in this direction. However, the hypothesis is empirical: it is not derived by some theoretical procedure and is only justified by observations. It is specific and testable.

The resulting theory, which is described with many detail in [19], suggests the way in which the quantum measurement theory could and ought to be corrected. However, it represents an idealised model whose practical applications are limited. On the one hand, it only focuses on the space aspects of quantum systems working exclusively with regions of the eigenspace of the position operator and so violates the transformation symmetry of quantum mechanics. On the other, the spatial separation status is rather difficult to be prepared. We can, e.g., never achieve perfect vacuum in the cavities where the experiment is done.

One can wonder whether the separation of particles in the momentum space could play a similar role as that in the position one studied in [19]. In fact, it is straightforward to built up a formalism in the momentum space that is completely analogous to the formalism in the position space. To see the physical meaning of such a construction, imagine that the detector used in an experiment has an energy threshold $E_0$. Then the particles with energy lower than the threshold cannot influence the detector. It is easy to achieve momentum separation status and, in fact, most measurements work exactly in this way. It is the main purpose of the present paper to go a step further and to give a suitable generalisation of separation status and of measurable observables, using regions of a phase space rather than regions of the position space.

The plan of the paper is as follows. Section 2 gives a brief account of the standard theory of quantum measurement that is being used today for analysis of real experiments. In this way, it will be possible to make explicit all changes that the present paper contains. Section 3 introduces the notion of truncated POV measures and justifies the proposal that these quantities must be used for description of real experiments instead of POV measures of the standard theory. This is a correction to our previous papers, which worked with certain POV measures. Section 4 con-
tains a generalisation of our previous theory of separation status. Instead of defining
the separation status as a domain of the coordinate or momentum eigenspaces, it
first defines an approximate extent of a quantum system as a domain the Cartesian
product of the two eigenspaces. The notion of extent is then used to define a new
kind of separation status that seems to be more satisfactory than the old one. Section
5 recapitulates and reformulates some older ideas using also the new notion of
separation status. First, ancillas, screens and detectors of a given meter must be
distinguished by the analysis of the structure of the meter. Second, the reading of
a meter is postulated to be a signal from a detector. Third, detected systems lose
their separation status within screens and detectors. Fourth, Schrödinger equation
does not hold for changes of separation status and must be replaced by new rules.
Section 6 describes a simple model of the Stern-Gerlach measurement within our
theory showing how the new rules are to be understood. The last section gives a
summary of the paper.

2 The standard theory of measurement

In this section, we give a short review of the standard theory of measurement as
it is employed in the analysis of many measurements today and as it is described
in, e.g., [5, 21, 22]. The emphasis is on being close to experiments and on physical
meaning rather than on mathematical formalism.

The standard theory splits a measurement process into three steps.

1. Initially, the object system $S$ on which the measurement is to be done is pre-
pared in state $T_s$ and the meter $M$, that is the apparatus performing the
measurement, in state $T_m$. These two preparations are independent so that
the composite $S + M$ is then in state $T_s \otimes T_m$. $T_s$ and $T_m$ are state operators
(sometimes also called density matrices).

2. An interaction between $S$ and $M$ suitably entangles them. This can be the-
oretically represented by unitary map $U$, called measurement coupling, that
describes the evolution of system $S + M$ during a finite time interval. Hence,
at the end of the time interval, the composed system is supposed to be in state

$$U(T_s \otimes T_m)U^\dagger.$$ 

3. Finally, reading the meter gives some definite value $r$ of the measured quantity.
If the same measurements are repeated more times independently from each
other, then all readings form a set, $r \in R$. $R$ is not necessarily the spectrum
of an observable (s.a. operator), in particular, it need not contain only real
numbers ($R$ need not be a subset of $\mathbb{R}$). The experience with such repeated measurements is that each reading $r \in R$ occurs with a definite probability, $P_r$.

One of the most important assumptions of the standard theory is that, after the reading of the value $r$, the object system $S$ is in a well-defined state,

$$T_{Sr}^{\text{out}},$$

called conditional or selective state. This is a generalisation of Dirac postulate:

A measurement always causes a system to jump in an eigenstate of the observed quantity.

Such a measurement is called projective and it is the particular case when $T_{Sr}^{\text{out}} = |r\rangle\langle r|$ where $|r\rangle$ is the eigenvector of a s.a. operator for a non-generated eigenvalue $r$.

The average of all conditional states after registrations, a proper mixture,

$$\sum_r P_r T_{Sr}^{\text{out}},$$

is called unconditional or non-selective state. It is described as follows: “make measurements but ignore the results”. One also assumes that

$$\sum_r P_r T_{Sr}^{\text{out}} = tr_M \left( U(T_S^{\text{in}} \otimes T_M^{\text{in}}) U^\dagger \right),$$

where $tr_M$ denotes a partial trace defined by any orthonormal frame in the Hilbert space of the meter.

In the standard theory, the reading is a mysterious procedure. If the meter is considered as quantum system then to observe it, another meter seems to be needed, to observe this, still another is needed and the resulting series of measurements is called von-Neumann chain. At some (unknown) stage including the processes in the mind of observer, there is the so-called Heisenberg cut that gives the definite value $r$. Moreover, the conditional state cannot, in general, result by a unitary evolution. The transition

$$tr_M \left( U(T_S^{\text{in}} \otimes T_M^{\text{in}}) U^\dagger \right) \mapsto T_{Sr}^{\text{out}}$$

in each individual registration is called “the first kind of dynamics” or “state reduction” or “collapse of the wave function”. We will use the name “state reduction”.

The idea of state reduction is difficult to accept for two reasons. First, the time and location of the Heisenberg cut is not known. Thus, the theory is incomplete.
Second, if there are two different kinds of dynamics, there ought to be also objective conditions under which each of them is applicable. At the present time, no such objective conditions are known. For example, for the state reduction, the condition of the presence of an observer is not objective and the condition that a quantum system interacts with a macroscopic system is not necessary.

The standard theory describes a general measurement mathematically by two quantities. The first is a state transformer $O_r$. $O_r$ enables us to calculate $T^\text{out}_{S^r}$ from $T^\text{in}_{S^r}$ by

$$T^\text{out}_{S^r} = \frac{O_r(T^\text{in}_{S^r})}{\text{tr}(O_r(T^\text{in}_{S^r}))}.$$ 

$O_r$ is a so-called completely positive map that has the form \[24\]

$$O_r(T) = \sum_k O_{rk} T O_{rk}^\dagger$$

for any state operator $T$, where $O_{rk}$ are some operators satisfying

$$\sum_{rk} O_{rk}^\dagger O_{rk} = 1.$$ 

Equation (1) is called Kraus representation. A given state transformer $O_r$ does not determine, via Eq. (1), the operators $O_{rk}$ uniquely.

The second quantity is a probability operator $E_r$ (often called “effect”) giving the probability to read value $r$ by

$$P_r = \text{tr} \left( O_r(T^\text{in}_{S^r}) \right) = \text{tr}(E_r T^\text{in}_{S^r}).$$

The set $\{E_r\}$ of probability operators $E_r$ for all $r \in \mathbb{R}$ is called probability operator valued (POV) measure (often called “positive operator valued”). Every POV measure satisfies two conditions: positivity,

$$E_r \geq 0,$$

for all $r \in \mathbb{R}$, and normalisation,

$$\sum_{r \in \mathbb{R}} E_r = 1.$$ 

One can show that $O_r$ determines the probability operator $E_r$ by

$$E_r = \sum_k O_{rk}^\dagger O_{rk}.$$ 

The definition of POV measures that is usually given is more general: $E(X)$ is a function on the Borel subsets $X \subset \mathbb{R}$. The formalism that we introduce in the present paper can be easily generalised in this way.
In the standard theory, the state transformer of a given registration contains all
information that is necessary for further analysis and for classification of measure-
ments. Such a classification is given in [5], p. 35. Thus, the formalism of the state
transformers and POV measures can considered as the core of the standard theory.

The standard quantum mechanics defines observables of a system \( S \) as the self-
adjoint operators on the Hilbert space of \( S \). Some mathematical physicists (e.g.,
Ludwig, Bush, Lahti and Mittelstaed) define observables as POV measures. The
spectral measures of s.a. operators are POV measures and in this sense, the definition
is a generalisation of the standard definition.

The authors of [5] return to the standard nomenclature and distinguish observ-
ables from POV measures. Observables are used in many ways, in particular to
construct POV measures, but they are only indirectly related to measurements. In
fact, only a special class of measurement can then be called “measurement of an
observable”. This is the case when all probability operators \( E_r \) of a POV measure
are functions of an observable ([5], p. 38). There are important measurements that
do not satisfy this condition. This standpoint is not generally accepted and is not
shared by our previous papers (see, e.g., [19]), but it seems very reasonable and
we shall adopt it here. It will turn out that even POV measures can be related to
real measurements only indirectly because of the disturbance of measurement due
to identical particles.

3 Truncated POV measures

Let us first briefly recall the argument of [15] about the disturbance of registration
by identical particles. Consider two distant laboratories, \( A \) and \( B \), and suppose that
each of them prepares an electron in states \( \psi(\vec{x}_A) \) and \( \phi(\vec{x}_B) \), respectively (we are
leaving out the spin indices and we work in \( Q \)-representation for the sake of sim-
plelicity). Then, the everyday experience shows that \( A \) can do all manipulations and
measurements on its electron without finding any contradictions to the assumption
that the state is \( \psi(\vec{x}_A) \). Analogous statements hold about \( B \).

However, according to the standard quantum theory, the state of the two particles
must be

\[
2^{-1/2} (\psi(\vec{x}_A)\phi(\vec{x}_B) - \phi(\vec{x}_A)\psi(\vec{x}_B)).
\]

Suppose next that \( A \) makes a measurement of the position of the electron. Standard
quantum mechanics associates position observable with the multiplication operator
\( \vec{x}_A \) for \( A \) electron and with a symmetrised multiplication operator
\[
\vec{x}_A + \vec{x}_B
\]
for the two electrons because the meter cannot distinguish the contributions of two identical particles from each other. Hence, the average of position measurement must be

$$\int_{\mathbb{R}} d^3x_A \bar{x}_A |\psi(\bar{x}_A)|^2 + \int_{\mathbb{R}} d^3x_B \bar{x}_B |\phi(\bar{x}_B)|^2$$ (4)

which differs from what one would expect if the state of the electron were just $\psi(\bar{x}_A)$, and the difference even increases with the distance of the laboratories.

A natural way out of this contradiction between the standard quantum mechanics and experience has been suggested by Peres [25]. The meters that can be used by laboratory A clearly cannot react to particles with wave functions that practically vanish within the laboratory, which is true for $\phi(\bar{x})$, at least approximately and for some time. Then any observable $O$ measured by such a meter satisfies $\langle \phi | O | \phi \rangle = 0$ and the corresponding second term in the equation analogous to (4) vanishes. However, the unexpected consequence of this explanation is that the position observable registered by the device cannot be $\bar{x}_A$!

Moreover, the device cannot measure any of the standard observables such as energy, momentum, spin, etc. A general proof that such a device cannot measure any POV measures goes as follows. Suppose that $E_r$ is a POV measure of quantum system $S$. In order that state $T$ of any system indistinguishable from $S$ in the environment does not disturb the registration of $E_r$, the probability that the measurement of $E_r$ on $T$ gives any result $r$ must be zero. For that, the following condition is sufficient and necessary:

$$tr(T E_r) = 0 \ \forall r .$$ (5)

However, the normalisation condition implies

$$\sum_r tr(T E_r) = 1 ,$$

which contradicts (5). The genuine meters must be such that they do not react to some states.

If not POV measures, which quantities describe registrations? Let us define truncated POV measures (TPOV measures) as follows. In general, any given experiment $\text{Exp}$ on system $S$ using meter $\mathcal{M}$ works with a limited set $T_\text{Exp} = \{T_1, T_2, \ldots, T_K\}$ of states in which $S$ is prepared before registrations. We assume that there is subspace $H_\text{Exp}$ of Hilbert space $H$ of $S$ satisfying two conditions. First,

$$\Pi[H_\text{Exp}] T \Pi[H_\text{Exp}] = T$$ (6)

for all $T \in T_\text{Exp}$, where $\Pi[H_\text{Exp}] : H \mapsto H_\text{Exp}$ is an orthogonal projection. Second, $H_\text{Exp}$ is minimal, that is any subspace of $H$ that satisfies (6) must contain $H_\text{Exp}$. In fact, for most experiments, $H_\text{Exp}$ is a finite-dimensional subspace of $H$.  

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**Definition 1** Any TPOV measure associated with experiment Exp is a set \( \{ E'_r \} \) of s.a. operators satisfying

\[
E'_r \geq 0
\]

for all \( r \in \mathbb{R} \) and

\[
\sum_r E'_r = \Pi[H_{Exp}].
\]

**Example** Let \( E_r \) be POV measure. Then:

\[
E'_r = \Pi[H_{Exp}]E_r\Pi[H_{Exp}]
\]

is a TPOV measure.

We have the desired property: states \( T \) annihilated by \( \Pi[H_{Exp}] \) satisfy \( tr(TE'_r) = 0 \) for any \( r \). An example of a TPOV measure is described in Sec. 6.

## 4 Separation status

The foregoing section introduced quantities that need not be disturbed by identical particles during registrations. However, more conditions must be satisfied in order that a registration be not disturbed. Let us introduce further mathematics. First, we need some measure of the extent of a quantum system.

**Definition 2** Let \( S_\tau \) be a system of \( N \) particles of type \( \tau \) in state \( T(t) \) at time \( t \). Let \( a_k \) be an observable of the \( k \)-th particle. Let

\[
\bar{a} = tr \left( T \frac{\sum_k a_k}{N} \right)
\]

and

\[
\Delta a = \sqrt{tr \left( T \frac{(\sum_k (a_k - \bar{a})^2)}{N} \right)}.
\]

The extent \( Ext(T) \) of \( T \) is the domain of \( \mathbb{R}^6 \) defined by the Cartesian product,

\[
Ext(T) = \prod_i^3 \left( \bar{x}^i - \Delta x^i, \bar{x}^i + \Delta x^i \right) \prod_j^3 \left( \bar{p}^j - \Delta p^j, \bar{p}^j + \Delta p^j \right),
\]

where \( \bar{x}^i \) and \( \Delta x^i \) are determined by Equations (7) and (8) for \( a_k = x_k^i \), \( x_k^i \) being the \( i \)-component of the position operator of \( k \)'s particle in \( S_\tau \) and similarly for \( \bar{p}^j \) and \( \Delta p^j \), \( a_k = p_k^j \), \( p_k^j \) being the \( j \)-component of the momentum operator of \( k \)'s particle in \( S_\tau \).
For example, consider two bosons in state $T = |\Psi\rangle\langle \Psi|$, where

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\psi_1\rangle \otimes |\phi_2\rangle + |\phi_1\rangle \otimes |\psi_2\rangle),$$

$|\psi\rangle$ and $|\phi\rangle$ are two vector states in the common Hilbert space of the two bosons satisfying $\langle \psi|\phi \rangle = 0$ and the symbol $|\psi^k\rangle$ means that the state $|\psi\rangle$ is occupied by the $k$-th particle. A short calculation gives

$$\bar{a} = \frac{\langle \psi|a|\psi \rangle + \langle \phi|a|\phi \rangle}{2}$$
and

$$\Delta a = \sqrt{\frac{1}{2} \left( \Delta^2_\psi a + \Delta^2_\phi a + \frac{1}{2} \left( \langle \psi|a|\psi \rangle - \langle \phi|a|\phi \rangle \right)^2 \right)},$$

where

$$\Delta^2_\psi a = \langle \psi|a^2|\psi \rangle - \langle \psi|a|\psi \rangle^2$$
and

$$\Delta^2_\phi a = \langle \phi|a^2|\phi \rangle - \langle \phi|a|\phi \rangle^2.$$

We can see that the extent includes not only the “sizes” ($\Delta_a^2$) of individual particles but also the “distances” ($|\langle \psi|a|\psi \rangle - \langle \phi|a|\phi \rangle|$) of different particles in $S_\tau$.

**Definition 3** Given a system $S$, let $S_\tau$ be the subsystem of $S$ containing all particles in $S$ of type $\tau$. Similarly, let $E_\tau$ be the subsystem of environment of $S$ that contains all particles of type $\tau$. We say that $S$ has a separation status if the extents of $S_\tau$ and $E_\tau$ have empty intersection for all $\tau$.

To give some physical interpretation to this formalism, consider meter $M$ that is able to register systems of the same type as $S$. Then, in order to be registered by $M$, $S$ has to be at some time inside $M$ and its kinetic energy must lie in the interval $(E_0, \infty)$ defined by threshold $E_0$ of the meter. The direction of the momentum must lie in the interval in which $S$ must arrive at the meter in order to be registered. These are condition on the extent of $S_\tau$ for all types $\tau$.

We assume first: every measurement on a system $S$ with no separation status will be disturbed by particles in its neighbourhood. Second, experiments on $S$ can be arranged so that they will be only negligibly disturbed by environment particles if $S$ has a separation status. The TPOV of the registration will then practically not react to states with very different extent.

Separation status has been defined in [15] as a region of space. The space $\mathbb{R}^6$ used in the definitions above can be considered as the phase space of one classical particle, and this is why we can say that the present section generalises the old
definition from regions of space to regions of phase space. However, one ought to keep in mind that $\mathbb{R}^6$ is the phase space of the considered system only in very few cases.

We can interpret what has been said as yet as follows. Standard quantum mechanics as it is usually presented seems incomplete:

1. It admits only two separation statuses for any system $S$:
   
   (a) $S$ is isolated. Then all states of $S$ have separation status and all s.a. operators are measurable.

   (b) $S$ is a member of a larger system containing particles identical to $S$. Then there are no individual physical states and observables for $S$.

2. It ignores the existence of separation-status changes.

However, separation-status changes have two important features:

1. They are objective phenomena that happen independently of any observer, and can be distinguished from other quantum mechanical processes.

2. Losses of separation status seem to be associated with state reductions. This gives us some hope that state reductions would indeed occur only if some objective conditions were satisfied.

5 Theory of meter reading

Let us now show in more detail how registered systems lose their separation status in meters.

In many modern experiments, in particular in non-demolition and weak measurements, but not only in these, the following idea is employed. The object system $S$ interact first with a microscopic system $A$ that is prepared in a suitable state. After $S$ and $A$ become entangled, $A$ is subject to further registration and, in this way, some information on $S$ is obtained. No subsequent measurements on $S$ has to be made. The state of $S$ is influenced by the registration just because of its entanglement with $A$. The auxiliary system $A$ is usually called ancilla.

It seems, however, that any registration on microscopic systems has to use detectors in order to make features of microscopic systems visible to humans. Detector is a macroscopic system containing active volume $D$ and signal collector $C$ in thermodynamic state of metastable equilibrium. Notice that the active volume is a physical system, not just a volume of space. Interaction of the detected systems with $D$ triggers a relaxation process leading to macroscopic changes in the detector that are called detector signals. For the theory of detectors, see, e.g., [26, 27].
Study of various experiments suggests that one can distinguish between ancillas and detectors within meters and that this distinction provides a basis for the analysis of meters. To be suitable for this aim, we have to modify a little the current notions of detector and ancilla. On the one hand, detectors as defined above are more specific than what may be sometimes understood as detectors. On the other, ancillas as defined above are more general.

For example, consider an ionisation gas chamber that detects a particle $S$ so that $S$ first enters the active volume $D$ of the chamber and then $S$ can leave $D$ again and be subject to further measurements. $S$ interacts with several gas atoms in the chamber that become ionised. This microscopic subsystem of several atoms within the active volume can also be viewed as an ancilla $A$. $A$ “is detected” subsequently by the rest of the detector, that is, $A$ interacts with $D$ and $C$ and is involved in a process of relaxation that leads to a macroscopic electronic signal.

Study of experiments suggests further that the measurements on ancillas needs detectors. Thus we are lead to the following hypothesis [15]:

**Pointer Hypothesis** Any meter for microsystems must contain at least one detector and every reading of the meter can be identified with a signal from a detector.

This is a very specific assumption that is, on the one hand, testable and, on the other, makes the reading of meters less mysterious.

In the above example of ionisation chamber, the state of the ancilla that is prepared by the interaction with the object system has, initially, a separation status: it can be distinguished from other systems of atoms within $D$ and, therefore, registered without external disturbance. However, in the process of interaction with $D$ and $C$ and the relaxation process, its energy is dissipated and its position is smeared so that it loses its separation status. We assume next:

**Active-Volume Hypothesis** Active volume $D$ of the detector detecting system $S'$ contains many particles in common with $S'$. The state of $S' + D$ then dissipates so that $S'$ loses its separation status.

Thermodynamic relaxation is necessary to accomplish the loss. $S'$ might be the objects system or an ancilla of the original experiment.

Study of a number of real experiments [18, 19] suggests the following:

**Separation Status Hypothesis** Let the Schrödinger equation for the composite $S + M$ leads to a linear superposition of alternative evolutions such that some of the alternatives contain loss of separation statuses of the object system or ancilla(s). Then, there is a state reduction of the linear superposition to the proper mixture of
The rule is general (it includes also separation status loss in screens, see [19]) and thus necessarily somewhat vague. This is, however, analogous to any other general dynamical law: even Schrödinger equation is defined only in rough features and must be set up for each case separately. It turns out that the state reduction is uniquely determined by the structure of the experiment and the losses of separation status. The Separation Status Hypothesis is again a testable hypothesis.

Suppose that a microscopic system $S$ is detected by a detector $D + C$ and that $S$ loses its separation status within $D + C$. Then, $S$ ceases to be an individual quantum system with its own physical states and observables. At most, one can consider the subsystem $S_+$ of $S + D + C$ that contains all particles of the same type as those that were inside $S$ originally. $S_+$ contains more particles than $S$ does and is, as a rule, a macroscopic system.

To explain the Separation Status Hypothesis, let us assume for the sake of simplicity, we that $S_+$ is a closed quantum system so that its states and their standard evolution defined by Schrödinger equation (“formal evolution” [18, 19]) make sense. This assumption is, strictly speaking, incorrect because $S_+$ is a macroscopic system that cannot be isolated. However, the environment can be considered as included, such an inclusion does not lead to really new phenomena and so such an assumption does not necessarily lead to false conclusions.

The Hypothesis considers the standard evolution of $S_+$, or a larger system. After finding all cases of separation status losses, it determines corrections to the standard evolution. This corrections are suggested by real experiments and cannot be derived from standard quantum mechanics.

In any case, the basic assumption of the old theory of quantum measurement that the object system has a well-defined state after the registration is not generally valid and the notion of state transformer does not make sense in many important cases.

Observe that our proposals give the preparation and registration procedures new importance with respect to, say, Copenhagen interpretation: they must include changes of separation status.

6 Stern-Gerlach story retold

In this section, we shall modify the textbook description (e.g., [25], p. 14) of the Stern-Gerlach experiment utilising the above ideas.

A silver atom consists of 47 protons and 61 neutrons in the nucleus and of 47 electrons around it, but we consider only its mass-centre and spin degrees of freedom
and denote the system with these degrees of freedom by $S$. Let $\vec{x}$ be its position, $\vec{p}$ its momentum and $S_z$ the $z$-component of its spin with eigenvectors $|j\rangle$ and eigenvalues $j\hbar/2$, where $j = \pm$.

Let $\mathcal{M}$ be a Stern-Gerlach apparatus with an inhomogeneous magnetic field in a region $D$ that splits different $z$-components of spin of a silver atom arriving in $D$ with a momentum in a suitable direction. Let a scintillation-emulsion film with energy threshold $E_0$ be placed orthogonally to the split beam. The scintillation emulsion is the active volume $\mathcal{D}$ of $\mathcal{M}$ and it may be also the signal collector if the scintillation events can be made directly visible.

First, let $S$ be prepared at time $t_1$ in a definite spin-component state

$$|\vec{p}, \Delta \vec{p}\rangle \otimes |j\rangle ,$$

where $|\vec{p}, \Delta \vec{p}\rangle$ is a Gaussian wave packet so that $S$ can be registered by $\mathcal{M}$ within some time interval $(t_1, t_2)$. Let state $|\Omega\rangle$ has a separation status at $t_1$.

$\mathcal{M}$ is in initial metastable state $T_{\mathcal{M}(t_1)}$ at $t_1$.

Interaction of $S$ with $\mathcal{M}$ is described by measurement coupling $U$. The time evolution within $(t_1, t_2)$ is:

$$U N \Pi (|\vec{p}, \Delta \vec{p}\rangle \langle \vec{p}, \Delta \vec{p}| \otimes |j\rangle \langle j| \otimes T_{\mathcal{M}(t_1)}) \Pi U^\dagger = T_j(t_2) ,$$

where $\Pi$ is antisymmetrisation on the Hilbert space of silver atom part of $S + \mathcal{M}$ and $N$ is a normalisation factor because $\Pi$ does not preserve normalisation. States $T_j(t_2)$ are determined by these conditions

This evolution includes a thermodynamic relaxation of $\mathcal{M}$ with $S$ inside $\mathcal{D}$. States $T_j(t_2)$ describe subsystem $S$ that has lost its separation status. Then, individual states of $S$ do not make sense: neither the conditional state nor the state transformer exist for $S$. (These notions are, in fact, applicable only for some parts of some measurements with ancilla.)

State $T_j(t_2)$ also describes detector signals. The signals will be concentrated within one of two strips on the film, each strip corresponding to one value of $j$.

Suppose next that the initial state of $S$ at $t_1$ is

$$|\vec{p}, \Delta \vec{p}\rangle \left( \sum_j c_j |j\rangle \right)$$

with

$$\sum_j |c_j|^2 = 1 .$$

As it is linear, unitary evolution $U$ gives

$$N \Pi \left[ |\vec{p}, \Delta \vec{p}\rangle \langle \vec{p}, \Delta \vec{p}| \otimes \left( \sum_j c_j |j\rangle \langle j| \right) \left( \sum_{j'} c_{j'} |j'| \langle j'| \right) \otimes T_{\mathcal{M}(t_1)} \right] \Pi U^\dagger = \sum_{jj'} c_j c_{j'}^* T_{jj'}(t_2) ,$$
a quadratic form in \( \{ c_j \} \in \mathbb{C}^2 \). Coefficients \( T_{jj'}(t_2) \) of the form are operators on the Hilbert space of \( \mathcal{S} + \mathcal{M} \).

The operator coefficients are state operators only for \( j' = j \). From the linearity of \( U \), it follows that

\[
T_{jj}(t_2) = T_j(t_2)
\]

Now we postulate the following correction to the Schrödinger equation

1. The loss of separation status of \( \mathcal{S} \) disturbs the standard quantum evolution so that, instead of

\[
\sum_{j,j'} c_j c_{j'}^* T_{jj'}(t_2)
\]

state

\[
\sum_j |c_j|^2 T_j(t_2)
\]

results.

2. States \( T_j(t_2) \) are uniquely determined by the experimental arrangement: the measurement coupling and the losses of separation status in the meter.

3. The sum is not only a convex combination but also a proper mixture of the signal states \( T_j(t_2) \). That is, the system \( \mathcal{S} + \mathcal{M} \) is always in one particular state \( T_j(t_2) \) after each individual registration and the probability for that is \( |c_j|^2 \).

The described example is simple because the silver atoms are both the object systems and components of the detector. If the detector contained no silver, we would have to insert an intermediate step suggested by the fourth paragraph of Section 5.

Stern-Gerlach experiment measures values of a truncated POV measure that consist of two probability operators,

\[
E_j = |\vec{p}, \Delta \vec{p}\rangle \langle \vec{p}, \Delta \vec{p}| \otimes |j\rangle \langle j|,
\]

where \( j = \pm \). Clearly, the set \( \{E_j\} \) lives on a two-dimensional subspace \( \mathcal{H}_{\text{Exp}} \) of the Hilbert space of the system \( \mathcal{S} \) that is defined by the projection

\[
\Pi[\mathcal{H}_{\text{Exp}}] = |\vec{p}, \Delta \vec{p}\rangle \langle \vec{p}, \Delta \vec{p}|.
\]
7 Conclusion

We have shown that the disturbance due to identical particles makes the registration of any POV measures impossible. In particular, none of the s.a. operators such as position, momentum, spin, angular momentum or energy are measurable.

The explanation of why real measurements do not seem to be disturbed is, first, that different quantities than POV measures are registered. As such quantities, we have proposed TPOV measures. Second, the preparations of the object systems satisfy an additional condition that is usually not mentioned. To describe the condition, the notion of separation status has been introduced in [15]. Here, we have generalised the notion so that some problems with the original notion disappear.

The next crucial observation is that the roles of ancilla and detector in registrations must be distinguished from each other. We have then conjectured that every meter contains at least one detector and meter readings are always signals of detectors. Moreover, separation statuses are lost in detectors.

Finally, study of different kinds of real experiments show that the changes of separation status are associated with state reduction. We assume that this is a general empirical fact. In this way, the surprising connection between the quantum theory of identical particles and the problem of quantum measurement has been established.

What is called “collapse of wave function” can then be explained as state degradation due to loss of separation status of the object system or an ancilla by a thermodynamic relaxation process in a detector. Hence, the collapse occurs under specific objective conditions and has a definite place and time.

The correction to Schrödinger equation is uniquely determined in each case by the measurement coupling and the separation status losses. There is no problem of preferred frame [3].

All conjectures made in this paper are testable.

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References

[1] P. Busch, P. J. Lahti and P. Mittelstaedt, The Quantum Theory of Measurement, Springer, Heidelberg, 1996.

[2] A. Bassi and G. Ghirardi, Phys. Letters A 275 373 (2000).
[3] Schlosshauer, M., *Rev. Mod. Phys.* **2004**, *76*, 1267.

[4] S. Weinberg, *Lectures on Quantum Mechanics*, Cambridge University Press, Cambridge, 2013.

[5] Wiseman H. M.; Milburn, G. J., *Quantum Measurement and Control*; Cambridge University Press: Cambridge, UK, 2010.

[6] Bell, J. S., *Rev. Mod. Phys.* **1966**, *38*, 447.

[7] Kochen, S.; Specker, E. P., *J. Math. Mech.* **1967**, *17*, 59.

[8] Bell, J. S., *Physics* **1964**, *1*, 195.

[9] Hardy, L., *Phys. Rev. Letters* **1992**, *68*, 2981.

[10] Greenberger, D. M.; Horne, M. A.; Shimony, A.; Zeilinger, A., *Am. J. Phys.* **1990**, *58*, 1131.

[11] Bell, J. S., in *Between Science and Technology* Chapter 6, Ed. by A. Sarlemin and P. Kroes, Elsevier Science, Amsterdam, 1990.

[12] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47** (1935) 777.

[13] P. Hájíček and J. Tolar, Found. Phys. **39** (2009) 411.

[14] P. Hájíček, Found. Phys., **39** (2009) 1072.

[15] P. Hájíček, Found. Phys., **41** (2011) 640.

[16] P. Hájíček and J. Tolar, Acta Phys. Slovaca **60** (2010) 613-716.

[17] P. Hájíček, J. Phys.: Conf. Ser. **306** (2011) 012035.

[18] P. Hájíček, Found. Phys., **42**, (2012), 555.

[19] P. Hájíček, Entropy **15** (2013) 789.

[20] P. Hájíček, J. Phys.: Conf. Ser. **442** (2013) 012043.

[21] Braginsky, V. B.; Khalili, F. Ya. *Rev. Mod. Phys.* **1996**, *68*, 1.

[22] Svensson, B. E. Y., New wine in old bottles: Quantum measurement—direct, indirect, weak—with some applications, Preprint 2012, arXiv:1202.5148.

[23] Von Neumann, J., *Mathematical Foundation of Quantum Mechanics*; Princeton University Press: Princeton NJ, 1955.
[24] K. Kraus, *States, Effects and Operations: Fundamental notions of Quantum Theory*, Lecture Notes in Physics, vol. 190, Springer, Berlin.

[25] Peres, A., *Quantum Theory: Concepts and Methods*; Kluwer: Dordrecht, 1995.

[26] Leo, W. R., *Techniques for Nuclear and Particle Physics Experiments*; Springer: Berlin, 1987.

[27] Twerenbold, D., *Rep. Progr. Phys.* **1996**, *59*, 239.