Long Term Operation of LISA and Galactic Close White Dwarf Binaries

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ABSTRACT
The binary confusion noise spectrum at LISA band depends strongly on observational period and abundance of Galactic close white dwarf binaries (CWDBs). We have investigated how the number of the resolved Galactic CWDBs varies with operation period of LISA, and found that the resolved number would typically grow by a factor of 5 when the operation period increases from 1yr to 10yr. We have also made a similar estimation for number of CWDBs whose chirp signal can be measured in matched filtering analysis.

Key words: gravitational waves–binaries: close–white dwarfs.

1 INTRODUCTION
If things go well, the Laser Interferometer Space Antenna (LISA) will be launched within ten years. LISA and ground-based detectors (such as, TAMA300, GEO600, LIGO and VIRGO) will bring us fruitful information of our universe and era of gravitational-wave astronomy will really start. For example, using LISA, we might detect merging massive black holes (MBHs) with significant signal to noise ratio (SNR), or in-spiraling compact stars (such as, white dwarfs, neutron stars, stellar mass black holes) around MBHs. These are very exciting phenomena and we might confirm existence of black holes, make stringent tests of general relativity and measure various interesting parameters of MBHs (Bender et al. 1998). However, event rate of MBH-MBH binaries is highly unknown (e.g. Haehnelt 1994) and gravitational waves from in-spiraling compact stars around MBHs might be too much complicated to be detected by usual matched filtering technique (Bender et al. 1998 and references therein).

Galactic binaries, such as, neutron star binaries, cataclysmic binaries, close white dwarf binaries (CWDBs), are guaranteed sources for LISA (Mironowskii 1965, Evans, Iben & Smarr 1987, Hils, Bender & Webbink 1990). The Galactic CWDBs are expected to be the dominant one in the frequency region from $\sim 10^{-3}$Hz up to several $10^{-2}$Hz (Bender et al. 1998). At present only less than ten CWDBs have been optically detected, but LISA would find thousands of CWDBs with one year integration (Hils & Bender 1997, Bender et al. 1998). Observational analysis for abundance and spatial distribution of Galactic CWDBs would bring us important clues to understand formation of binary stars and structure of our galaxy (e.g. Ioka, Tanaka & Nakamura 2000, Hiscock et al. 2000, see also Oppenheimer et al. 2001). It should be also noted that CWDBs are regarded as one of the likely progenitors of type I supernova (Iben & Tutukov 1984, Branch et al. 1995). Thus gravitational waves from Galactic CWDBs, one of the guaranteed sources of LISA, have rich scientific contents.

An interesting feature of Galactic CWDBs is that they would also become a serious background noise (confusion noise) for measurement of gravitational wave (Evans et al. 1987). Properties of its spectrum as well as the estimation errors for parameters of resolved binaries depend strongly on observational period. But operation period of LISA is not determined definitely at present. The Pre-Phase A Report (Bender et al. 1998) of LISA states “This (a 1-year-long observing period) is reasonable length of time, but not the maximum: the nominal mission lifetime is 2yr, but in principle it might last as long as a decade.”. Thus it seems interesting to investigate ahead of time how scientific impacts on CWDBs would change with operation period of LISA. We mainly investigate the number of the resolved CWDBs for various observation periods. This quantity would be most fundamental for observational analyses described in the last paragraph.
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This article is organized as follows. In §2 we mention gravitational wave from CWDBs and a model of their spatial distribution in our galaxy. In §3 we determine the confusion noise curve as a function of the observational period and the abundance of Galactic CWDBs. We also discuss signal to noise ratio and the parameter estimation errors for binaries in matched filtering analysis. Then in §4 we evaluate number of resolved CWDBs for various set of parameters. §5 is devoted to a brief summary.

2 GRAVITATIONAL WAVE FROM GALACTIC CWDBS

2.1 Chirp Gravitational Wave

Let us consider a binary (masses $M_1$ and $M_2$) with a circular orbit. The characteristic amplitude $h_A$ of gravitational wave at a frequency $f$ is evaluated by Newtonian quadruple formula as (Thorne 1987)

$$h_A = 8 \left( \frac{2}{15} \right)^{1/3} \frac{G^{5/3}}{r c^3} (\pi M c f)^{2/3}$$

$$= 1.2 \times 10^{-21} \left( \frac{M_e}{0.3 M_\odot} \right)^{5/3} \left( \frac{100 \text{pc}}{r} \right) \left( \frac{f}{10^{-11} \text{Hz}} \right)^{2/3},$$

where $M_e \equiv (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$ is the chirp mass of the system and $r$ is the distance to it. In the above equation we have taken angular average of wave amplitude with respect to orientation of sources (Thorne 1987). The time profile of a nearly monochromatic wave is expressed as follows

$$h(t) = h_A \cos \left[ 2\pi \left( ft + \frac{f}{2} t^2 \right) + \phi_0 \right].$$

Here $\phi_0$ is an integral constant. For a purely monochromatic wave we have $f = 0$, but energy loss due to gravitational radiation or other effects, such as, accelerating motion of the binary or mass exchange between binary stars, make modulation of the wave frequency $f \neq 0$. We denote $f \neq 0$ as follows

$$\dot{f} = (\dot{f})_{GW} + (\dot{f})_{\text{other}},$$

where the first term of the right-hand-side represents effect due to gravitational radiation. When the wave frequency $f$ changes significantly within an observational period $T$, namely

$$\dot{f} T / f \gtrsim O(1),$$

we might separate these effects in matched filtering analysis using their frequency dependence. But this is not the case for our analysis for low-frequency waves. Thus we try to measure $\dot{f}$ as the sole parameter for the frequency modulation. Effects due to motion of LISA would be commented in §3.2.

Our main target is Galactic CWDBs. A CWDB is expected to have an almost circular orbit as a result of spiral-in process during its common envelope phase, in contrast to a double neutron star binary (Ignatiev et al. 2001). Before the less massive star fills its Roche lobe, gravitational radiation reaction is the dominant process for evolution of orbital parameters of a binary (see e.g. Webbink 1984). The energy loss due to gravitational radiation is given by Newtonian quadruple formula and changes the wave frequency as follows (Thorne 1987)

$$(\dot{f})_{GW} = \frac{96 \pi^{8/3} G^{5/3}}{5} f^{11/3} M_e^{5/3}$$

$$= 7.9 \times 10^{-19} \left( \frac{f}{10^{-11} \text{Hz}} \right)^{11/3} \left( \frac{M_e}{0.3 M_\odot} \right)^{5/3} \text{sec}^{-2}.$$  

CWDBs are mainly divided into three segments (Webbink 1984, Iben & Tutukov 1984, Tutukov & Yungelson 1986). They are He-He systems with total mass $0.5 - 0.75 M_\odot$, He-CO systems with total mass $0.75 - 1.45 M_\odot$, and CO-CO systems with total mass $1.45 - 2.4 M_\odot$. Webbink (1984) discussed that these three systems have similar abundance (see also Branch et al. 1995). In this article we do not distinguish them but study a single component with chirp mass $M_e = 0.3 M_\odot$. More detailed treatment would not significantly change our main conclusions.

2.2 Number and Spatial Distributions of Galactic CWDBs

The abundance of Galactic CWDBs has not been observationally clarified (Hils et al. 1990, Marsh 1995, Marsh, Dhillon, & Duck 1995, Bender & Hils 1997, Knox, Hawkins, & Hambly 1999). In the frequency region where they evolve by gravitational radiation reaction, the number of CWDBs per unit frequency $dN/df$ becomes
\[
\frac{dN}{df} \propto \left( \frac{df}{dt} \right)^{-1} \propto f^{-11/3},
\]

from equation (7). The above distribution can be also expressed in an integral form as \(N(> f) \propto f^{-8/3}\).

We use estimation for the abundance of Galactic CWDBs given in Hils & Bender (2000) as a reference. This estimation is 10\% of the theoretical estimation by Webbink (1984). Hils & Bender (2000) commented that the error in this estimation is believe to be no more than a factor of ten in either direction. The number distribution \(dN/df\) corresponding to this 10\% estimation is a factor 10 lower than that given in their figure 4 which represented the full 100\% estimate based on Webbink (1984). Thus we have an approximation for this 10\% estimation as

\[
\frac{dN}{df} = 3.2 \times 10^7 \left( \frac{f}{10 - 2.82 \text{Hz}} \right)^{-11/3} \text{Hz}^{-1},
\]

at the frequency region \(f \gtrsim 10^{-3}\text{Hz}\) in interest. In this article we use the following non-dimensional quantity \(N_{282}\) to characterize the abundance of Galactic CWDBs

\[
N_{282} \equiv \frac{1}{(3.2 \times 10^7 \text{Hz})} \left| \frac{dN}{df} \right|_{f=10^{-2.82} \text{Hz}},
\]

or we have

\[
\frac{dN}{df} = 3.2 \times 10^7 N_{282} \left( \frac{f}{10 - 2.82 \text{Hz}} \right)^{-11/3} \text{Hz}^{-1}.
\]

For example we have \(N_{282} = 1\) for distribution (7). Webbink & Han (1998) discussed CWDBs with various models of binary evolution. Their “standard model” corresponds to \(N_{282} \sim 2.0\). Nelemans, Yungelson, & Portegies Zwart (2001) obtained a similar results. Note that \((3.2 \times 10^7)^{-1}\text{Hz}\) is the frequency bin for 1yr integration. Thus \(N_{282}\) represents the mean number of CWDBs within 1yr\'-bin at \(f = 10^{-2.82}\text{Hz}\).

Abundance of extra-Galactic CWDB also becomes important to discuss the binary confusion noise in the next subsection. We scale their abundance by the same parameter \(N_{282}\) defined for the Galactic ones (see e.g. Bender & Hils 1997 for discussion).

Next we briefly discuss the spatial distribution of Galactic CWDBs. We use the standard exponential disk model

\[
\rho(R, z) = \rho_0 \exp \left( -\frac{R}{R_0} \right) \exp \left( -\frac{|z|}{z_0} \right),
\]

where \((R, z)\) is the Galactic cylindrical coordinate. We fix the radial scale length \(R_0 = 3.5\text{kpc}\) and the disk scale height \(z_0 = 90\text{pc}\) (Hils, Bender & Webbink 1990), and assume that the solar system exists at the position \(R = 8.5\text{kpc}\) and \(|z| = 30\text{pc}\). As the amplitude of the gravitational wave is inversely proportional to the distance \(r\), distribution of the CWDBs’ distances \(r\) from the solar system becomes important. With our model parameters, \(\sim 90\%\) of the Galactic CWDBs are within \(r \leq 18\text{kpc}\), and \(\sim 10\%\) of them are within \(r \leq 5.0\text{kpc}\). The number of binaries within distance \(r\) changes from \(\propto 1/r^3\) (\(r\): smaller than disk thickness \(\sim 100\text{pc}\)) to \(\propto 1/r^2\) (\(r\): smaller than size of Galaxy \(\sim 10\text{kpc}\)) and \(\propto 1/r^0\) (\(r\): larger than \(\sim 10\text{kpc}\)).

### 3 MATCHED FILTERING ANALYSIS

#### 3.1 Confusion Noise

The signal of a detector \(s(t) = h(t) + n(t)\) contains both the true gravitational wave signal \(h(t)\) and the noise \(n(t)\). We assume the stationary noise and define its power spectrum \(S_n(f)\) by

\[
\langle \tilde{n}(f)\tilde{n}(f') \rangle = \frac{1}{2}S_n(f - f')S_n(f),
\]

where \(\tilde{n}(f) = \int_{-\infty}^{\infty} e^{2\pi i ft}n(t)dt\) is the Fourier transformation of the noise \(n(t)\). This definition of \(S_n(f)\) corresponds to the one-sided spectral density (e.g. appendix A of Cutler and Flanagan 1994). For notational simplificities we have represented the frequency space in the continuum limit (see e.g. Schutz 1997).

At the LISA band the noise spectrum \(S_n(f)\) is constituted by two terms; (i) the instrumental (detector’s) noise \(S_{\text{ins}}(f)\) and (ii) the confusion noise \(S_{\text{con}}(f)\) that is effectively caused by unresolved sources

\[
S_n(f) = S_{\text{ins}}(f) + S_{\text{con}}(f).
\]

The frequency bin \(\delta f\) for an observational period \(T\) is simply given as

\[
\delta f = T^{-1} = 3.1 \times 10^{-8} \left( \frac{T}{1\text{yr}} \right)^{-1} \text{Hz}.
\]
The number \(dN/df\) of Galactic CWDBs per unit frequency increases at lower frequency as shown in equation (8). Roughly speaking, when the frequency bin is occupied by more than one Galactic CWDBs, the Galactic confusion noise \(S_{\text{con,G}}(f)\) becomes important (e.g. Bender & Hils 1997). At higher frequencies there is no more than one Galactic binary and the confusion noise \(S_{\text{con,G}}(f)\) is determined by the extra-Galactic binaries. The shape of the confusion noises (including the position of the transition frequency \(f_t\) where the Galactic noise becomes important) is determined by the observational period \(T\) and the number of binaries. We discuss them with a method similar to Hiscock et al. (2000) who studied dependence of the noise spectrum on the abundance and spatial distribution of CWDBs (see also Phinney 2001).

Bender & Hils (2000 and references therein) discussed that the confusion noise in the region \(10^{-4} < f < 10^{-1.5}\)Hz is dominated by CWDBs. They found that in some scenarios (Tutukov & Yungelson 1996, see also Iben & Tutukov 1991) the abundance \(dN/df\) of the Helium Cataclysmics (HeCVs) exceeds that of CWDBs, but amplitude of confusion noise is expected to be dominated by CWDBs because of their larger chirp masses than that of HeCVs. Thus in our present analysis we basically do not consider the effects of HeCVs and study the spectrum \(S_{\text{con}}(f)\) as a function of \(T\) and abundance of CWDBs (characterized by \(N_{282}\)).

The extra-Galactic noise \(S_{\text{con,G}}(f)df\) has the following relation
\[
S_{\text{con,G}}(f)df \propto (\text{number of binaries in } f \sim f + df) \times (\text{amplitude of individual source})^2.
\] (16)

From equations (3) and (4) we have
\[
S_{\text{con,G}}(f) \propto N_{282} f^{-7/3}.
\] (17)

The Galactic confusion noise \(S_{\text{con,G}}(f)\) at frequency smaller than the transition region around the frequency \(f_t\) has the same functional shape \(S_{\text{con,G}}(f) \propto N_{282} f^{-7/3}\). The ratio of two amplitudes \(\sqrt{S_{\text{con,G}}(f)/S_{\text{con,E}}(f)}\) is estimated to be \(3 \sim 10\) and the uncertainty is mainly related to that of star formation rate at high redshift (Kosenko & Postnov 1998, Schneider et al. 2001).

Next we evaluate the transition frequency \(f_t\) as a function of the parameters \(N_{282}\) and \(T\)
\[
\frac{dN}{df} \bigg|_{f_t} \delta f = Y,
\] (18)

where \(Y\) is a constant of order unity. From equation (11) we obtain
\[
f_t \propto (N_{282} T^{-1})^{3/11}.
\] (19)

Now we can make the confusion noise spectra \(S_{\text{con}}(f,N_{282},T)\) for various parameters \((N_{282},T)\) using a functional form \(S_{\text{con}}(f,N_{282},T)\) given for a specific choice of parameters \(N_{282}\) and \(T\). We connect two (Galactic and extra-Galactic) power-law functions \(\propto N_{282} f^{-7/3}\) around the characteristic frequency \(f_t\) and straightforwardly obtain the following relation
\[
S_{\text{con}}(f,N_{282},T) = \left(\frac{N_{282}}{N_{282_0}}\right) \left(\frac{f_{1,0}}{f_t}\right)^{7/3} S_{\text{con}} \left(f, f_{1,0}, N_{2, T_0}\right),
\] (20)

where we have
\[
f_{1,0} \equiv f_{1,0} = \left(\frac{N_{282_0} T_0^{-1}}{N_{282} T^{-1}}\right)^{3/11}
\] (21)

from equation (19).

For the “base” function of the confusion noise \(S_{\text{con}}(f,N_{282_0},T_0)\) we use the result of Hils & Bender (2000) given for parameters \(N_{282} = 1\) and \(T_0 = 1\)yr (see their figure 5). The instrumental noise \(S_n(f)\) is obtained from their same figure. In figure 1 we show the noise spectrum for various choice of parameters \((N_{282}, T)\).

Bender & Hils (1997) gave confusion noise curves \(S_{\text{con}}(f)\) due to CWDBs for two different \(N_{282}\). We find that those two curves are reasonably reproduced by each other with the above procedure. Our method would be quantitatively valid at frequency
\[
f > 3.0 \times 10^{-3} (N_{282})^{3/11} \left(\frac{T}{1\text{yr}}\right)^{-3/11} \equiv f_t,
\] (22)

where the confusion noise is determined by extra-Galactic binaries. We have put \(Y = 0.1\) in equation (18) for definition of \(f_t\). The Galactic binaries with \(f > f_t\) are not overlapped in the frequency bin \(T^{-1}\) and, in principle, simple to analyzed. In contrast these with \(f < f_t\) can be detected only when they are close to us and have significant wave amplitude above confusion noise. Their analysis would not be straightforward. Here we effectively discuss them using the confusion noise spectrum.

In figure 1 the frequency \(f_t\) is shown with filled circles. Note that the Galactic confusion noise at the transition region for \(T = 1\)yr vanishes in the case of \(T = 10\)yr. Comparing \(T = 1\)yr and \(T = 10\)yr we find that \(\sqrt{S_{\text{con}}(f)}\) of the latter is smaller about a factor of ten than the former at \(f = 0.0018\)Hz in the case of \(N_{282} = 1\). Note that this factor is determined by the quantity \(\sqrt{S_{\text{con,G}}(f)/S_{\text{con,E}}(f)}\) as discussed before.
Figure 1. The noise spectrum is presented in the form of $\sqrt{S(f)}$. The instrumental noise of LISA is presented with bold thick line (Hils & Bender 2000). Other lines are confusion noise with various set of parameters $N_{282}$ and $T$ (in units of year). The thin solid line corresponds to $(N_{282}, T) = (1, 1)$, thick solid line to $(1, 10)$, long-dashed line to $(0.1, 1)$ and short-dashed line to $(10, 1)$. The filled circles represent the frequency $f_t$ defined in eq. [22].

3.2 SNR and Estimation Errors for a Chirp Signal

In this subsection we briefly discuss signal-to-noise ratio (SNR) and the parameter estimation errors for gravitational wave signal in matched filtering analysis (Cutler & Flanagan 1994). First, we define an inner product of two quasi-periodic waves $g(t)$ and $k(t)$ around a frequency $f_0$ as follows

$$(g|k) = \frac{2}{S_n(f_0)} \int_0^T g(t)k(t)dt.$$  \hspace{1cm} (23)

The signal to noise ratio of a gravitational wave $h(t)$ is given as

$$SNR = (h(t)|h(t))^{1/2}.$$  \hspace{1cm} (24)

When the wave form $h(t)$ is characterized by some parameters $\lambda_i$, magnitude of the estimation errors $\Delta \lambda_i$ is given by the so called Fisher information matrix $\Gamma_{ij}$ as

$$\langle \Delta \lambda_i \Delta \lambda_j \rangle = \left( \frac{\partial h}{\partial \lambda_i} \bigg| \frac{\partial h}{\partial \lambda_j} \right)^{-1} = \Gamma_{ij}^{-1}.$$  \hspace{1cm} (25)

Position and orientation of LISA change in time. This causes (i) frequency modulation due to Doppler effect and (ii) variation of detector’s sensitivity due to its rotation (Bender et al. 1998). In reality we need to fit the direction of a source in the matched filtering analysis (Peterseim et al. 1997, Cutler 1998), but this is a very troublesome task. In the present analysis we use an angular averaged sensitivity (effectively a factor of $\sqrt{5}$ degradation, Thorne 1987) and do not try to fit the direction. We evaluate the estimation errors for the three parameters of wave form $\lambda_i = \{f, \dot{f}, \phi_0\}$ in equation (23). The chirp signal $\dot{f}$ is very important from astronomical point of views (see e.g. Schutz 1986, 1989). This parameter is related to secular effects of gravitational wave during the whole observational period $T$ and distinct from annual effects related to the direction of a source described above. However, we should notice that due to correlation in the Fisher information matrix the actual parameter estimation errors $\Delta \lambda_i$ would be worse than our results. Thus we must be cautious to discuss the measurement error of the chirp signal $\dot{f}$.

Now we can calculate SNR and the estimation errors $\Delta \lambda_i$ for $\lambda_i = \{f, \dot{f}, \phi_0\}$. Using the time function (3) and equations (23) and (24) we obtain the following results for a binary

$$SNR = \frac{h_A}{\sqrt{S_n(f)}} T^{1/2},$$  \hspace{1cm} (26)
and the information matrix $\Gamma_{ij}$ gives the root-mean-square values of the errors as

\[
\Delta \dot{f} = \frac{3\sqrt{5}}{\pi} \frac{T^{-2}}{SNR},
\]

(27)

\[
\Delta f = \frac{4\sqrt{3}}{\pi} \frac{T^{-1}}{SNR},
\]

(28)

\[
\Delta \phi_0 = \frac{3}{SNR},
\]

(29)

For comparison with equation (3) we can write equation (27) in the following form

\[
\Delta \dot{f} = 4.3 \times 10^{-19} \left( \frac{SNR}{100} \right)^{-1} \left( \frac{T}{10\text{yr}} \right)^{-2} \text{sec}^{-2}.
\]

(30)

At a frequency $f$ where the noise $S_\nu(f)$ does not depend on the observation period $T$ (as in the case of $f \gg f_i$; see figure 1), the above quantities have the following relations

\[
SNR \propto T^{1/2},
\]

(31)

\[
\Delta \dot{f} \propto T^{-5/2},
\]

(32)

\[
\Delta f \propto T^{-3/2},
\]

(33)

\[
\Delta \phi_0 \propto T^{-1/2}.
\]

(34)

Relation (31) is a well known result. Note that the error (32) for the chirp signal becomes significantly smaller for longer integration time $T$. Considering the correlation of the Fisher matrix discussed before, the amplitude of error $\Delta \dot{f}$ might be larger than our estimation (2). But the asymptotic time dependence $\Delta f \propto T^{-5/2}$ would be same for integration time $T$ much larger than time scale (1yr) of annual modulation due to rotation of the detector.

4 RESOLVED CWDBS

In figure 2 we show the number of Galactic CWDBs resolved with $SNR > 10$. We take our model parameters for abundance of Galactic CWDBs at $N_{282} = 0.1, 1, 10$ and for observational period at $T = 1\text{yr}$ and $10\text{yr}$. We present the number of the resolved CWDBs within frequency bin $f_0 < f < 1.26f_0 (= 10^{1/10} f_0)$ and explain our results mainly using the figure (the upper right panel of Fig. 2) given for $N_{282} = 1.0$.

Firstly, we should notice that all the Galactic CWDBs with $f \gtrsim 3 \times 10^{-3}\text{Hz}$ are resolved with $SNR \geq 10$. Thus we have $N(f_0 < f < 1.26f_0) \propto f_0dN/df|_{f_0} \propto f_0^{-8/3}$ for CWDBs at higher frequency. Secondly, according to our analysis based on the confusion noise spectrum, the number of resolved CWDBs increases significantly at frequency region dominated by Galactic confusion noise ($f \lesssim 10^{-3}\text{Hz}$) for $T = 10\text{yr}$ comparing with $T = 1\text{yr}$. In figure 3 we plot the distances $r$ of CWDBs measured with $SNR = 10$ in the case of $N_{282} = 1$. At $f < f_i$ the distance $r$ suddenly becomes small due to the Galactic confusion noise. As the observational period increases, the frequency $f_i$ decreases and we can observe more CWDBs. In table 1 we show the total number of resolved CWDBs for various parameters. We also count CWDBs only with $f > f_i$. We can understand that resolved CWDBs mainly belong to this group that do not overlap with other Galactic CWDBs. The total number for $T = 10\text{yr}$ is 4 ~ 6 times larger than for $T = 1\text{yr}$. As expected, this improvement is more significant for larger $N_{282}$. Note that the peak of the distribution $N(f_0 < f < 1.26f_0)$ corresponds to the characteristic frequency $f_i$ for $N_{282} = 1$ and 10. This means that we have $N(> f_i) \propto f_i^{-8/3} \propto T^{8/11}$, and the factor ~ 5 mentioned above is roughly given by $10^{8/11} \sim 5$.

A He+He white dwarf binaries coalesce at $f \sim 0.015\text{Hz}$ and the He+CO at $f \sim 0.03\text{Hz}$. The coalescence frequency of CO+CO white dwarf is close to 0.1Hz (Bender & Hils 1997). For simplicities we do not take into account of these cut-off frequencies.

Cutler (1998) studied LISA’s angular resolution $\Delta \Omega$ for binaries with monochromatic gravitational waves (see his table 1). The resolution depends on the orientations of binaries as well as the directions to them. Typical estimation errors with 1yr observation become $\Delta \Omega \sim 3 \times 10^{-2}\text{[sr]}$ at $f = 10^{-3}\text{Hz}$ and $\Delta \Omega \sim 1 \times 10^{-3}\text{[sr]}$ at $f = 10^{-2}\text{Hz}$ (both for binaries with $SNR = 10$).

Next we investigate the number of CWDBs whose chirp signal due to gravitational radiation reaction (eq. (7)) can be measured. We define a parameter $C$ by the following equation

\[
\langle \dot{f} \rangle_{\text{GW}} = C \langle \Delta \dot{f} \rangle.
\]

(35)

For reference we take $C > 10$ as a detection criteria for the chirp signal by the gravitational radiation reaction. This threshold means that the signal of magnitude $\langle \dot{f} \rangle_{\text{GW}}$ (eq. (4)) is measured within 10% accuracy in matched filtering analysis. The number distribution $N(f_0 < f < 1.26f_0)$ of CWDBs with $C > 10$ is shown in figure 2. Note that CWDBs with $C > 10$ has $SNR > 10$ for reasonable choice of $T$. We can directly confirm this using equations (3) and (2). In table 1 we show the total number

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Table 1. number of the resolved Galactic CWDBs

| $(N_{282}, T[yr])$ | (0.1,1) | (0.1,10) | (1,1) | (1,10) | (10,1) | (10,10) |
|---------------------|---------|----------|-------|--------|--------|---------|
| $SNR > 10$          | 350     | 1460     | 1950  | 10700  | 4810   | 28500   |
| $SNR > 10$, $f > f_t$ | 325     | 1420     | 1610  | 9340   | 3810   | 22500   |
| $C > 10$            | 0.34    | 13       | 3.4   | 130    | 34     | 1200    |

$N_{282}$ represents abundance of the Galactic CWDBs as defined in equation [10] and $T$ is the operation period of LISA in units of year. $C$ is the resolution threshold for chirp signal (eq. [33]).

$N_{\text{chirp}}$ with $C > 10$. The number of CWDBs with $C > 10$ increase significantly for long term integration. The total number of CWDBs for $T = 10\text{yr}$ is $\sim 40$ times larger than $T = 1\text{yr}$.

As explained earlier, the actual estimation error $\Delta f$ might be larger than our evaluation (27) due to correlation with other errors, such as, angular position of the source. Here we discuss how the total number $N_{\text{chirp}}$ depends on the observational period $T$ assuming the following relation from equation (27):

$$\Delta f = A \times (SNR)^{-1}T^{-2},$$

(36)

with a normalization constant $A$ that would be larger than $3\sqrt{5}/\pi$ in equation (27). This parameter $A$ is effectively absorbed to the threshold value $C$. We can expect following two points from figure 2 (i) the total number $N_{\text{chirp}}$ would be mainly determined by $f_{\text{max}}$ where the distribution $fdN/df$ takes maximum value, and (ii) it also departs from the asymptotic behavior $\propto N_{282} f^{-8/3}$ at high frequency region. The former point (i) represents that number $N_{\text{chirp}}$ of resolved binary becomes

$$N_{\text{chirp}} \propto N_{282} f_{\text{max}}^{-8/3}.$$  

(37)

As the effects of the confusion noise is very small around $f \simeq f_{\text{max}}$, the total number $N_{\text{chirp}}$ simply scales as $N_{\text{chirp}} \propto N_{282}$ in contrast to the previous case of the $SNR$ threshold. The latter point (ii) means that the distance $r$ of CWDBs for a given threshold $C$ corresponds to $r \simeq \text{(size of Galaxy)}$ at $f = f_{\text{max}}$. Then we can relate $T$ and $f_{\text{max}}$ using equations (27) and (35) as follows

$$\frac{T^{-5/2}}{f_{\text{max}}^{2/3}} \sqrt{S_n(f_{\text{max}})} \propto f_{\text{max}}^{11/3},$$

(38)

and obtain

$$f_{\text{max}} \simeq f_{\text{max}} S_n(f_{\text{max}})^{-3/26} \propto T^{-15/26},$$

(39)

where we have used $S_n(f) \simeq S_{\text{max}}(f) \propto f^0$ in the frequency region relevant for the present analysis (see figure 1). Using the relation (39) we finally obtain

$$N_{\text{chirp}} \propto N_{282} T^{20/13}.$$  

(40)

Thus a long observational period $T$ is crucial to increase the total number $N_{\text{chirp}}$.

5 SUMMARY

The Galactic close white dwarf binaries (CWDBs) are one of the guaranteed sources of LISA. Gravitational wave astronomy for the Galactic CWDBs would bring us important observational facts to understand binary formation, galactic structure, progenitor of type I supernova and so on. But gravitational waves from the Galactic CWDBs would also become a serious noise (called confusion noise) below the typical frequency $f_t(\propto T^{-3/11})$ (eq. [24]) where the effective frequency bin $T^{-1}$ is occupied by more than one Galactic binaries ($T$: observational period). Apparently the confusion noise spectrum depends strongly on the observational period $T$.

We have investigated number of the resolved Galactic CWDBs as a function of $T$ using the exponential disk model. We found that the number $N$ of the resolved CWDBs with $SNR > 10$ increases $\propto T^{8/11}$ as a function of observational period and it becomes about a factor of 5 larger for $T = 10\text{yr}$ comparing with $T = 1\text{yr}$. We also studied the number of CWDBs whose chirp signal can be measured with matched filtering method. The chirp signal is one of the most fundamental signal for gravitational wave astronomy. Using a rough estimation we have shown that the number would grow strongly ($\propto T^{20/13}$) with period $T$. From 1yr to 10yr operation the number can increase a factor of 40 for a typical model of Galactic CWDBs.
Figure 2. Number distribution of resolved Galactic CWDBs within frequency bin $f_0 < f < 1.26f_0$ for parameters $N_{282} = 0.1, 1$ and 10. Solid lines represent results for $T = 1\text{yr}$ and dotted lines for $T = 10\text{yr}$. Thick lines are number of resolved CWDBs with $SNR > 10$, and thin lines with $R > 10$ (chirp signal).

Figure 3. Distance $r$ of the Galactic CWDBs resolved with $SNR > 10$. We fix the abundance by $N_{282} = 1$. The solid line represents results for $T = 1\text{yr}$ and the dotted line for $T = 10\text{yr}$.

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