Modelling rapidly rotating stars

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Helioseismology, Asteroseismology and MHD Connections
Introduction

Achernar
(Domiciano de Souza et al., 2003)
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Altair
(Monnier et al., 2007)
this has caught the attention of those doing stellar models:

Models for Achernar (Jackson et al., 2004)
this has caught the attention of those doing stellar models:

- What is involved when making models of such stars?

Models for Achernar
(Jackson et al., 2004)
Outline

Introduction

Stellar structure
  Physical phenomena
  Recent models

Pulsation modes
  The frequency spectrum
  Mode geometry
  Ray dynamics

Outlook
Physical phenomena

- centrifugal deformation
- gravitational darkening
- baroclinicity
- differential rotation
- meridional circulation

Transport processes (MacGregor et al., 2007)

Envelopes are unambiguously in either one of those states or the other, the classical results apply under rather different circumstances, which we discuss next.

Both the von Zeipel (1924) and Lucy (1967) results treat rotation as a perturbation. However, in the radiative case in which uniform rotation is adequate (and issues like mass loss, etc. can be ignored), the quantity treated as a perturbation is the size of the quadrapole moment of the gravitational field. Since stars are centrally condensed, even for velocities approaching critical the distortions in the core are modest, one can expect that the analysis given by von Zeipel (1924) will be reasonably accurate. This has been found to be true in practice (Sackmann 1970).

For the convective case, the gravity darkening exponent is obtained by analyzing the adiabats found in the envelopes of representative stars. Lucy (1967) quite explicitly points out that the derivation is valid only for small changes in the effective gravity.

Of course, the effective gravity changes by orders of magnitude as rotation approaches critical, and it is not clear whether the exponent derived by Lucy can be used to describe gravity darkening for anything but the most modest rotation. Again, this is in contrast to the modest contribution of an induced gravitational quadrapole, even for stars rotating at breakup.

Even so, in a recent series of papers, Claret (2004 and references therein) has attempted to deal with the issue of a smooth interpolation between these two extreme cases. He has noted that as stars evolve off the main sequence and toward the red giant branch, their interior structures trace out approximately straight line loci in a (log $T_{\text{eff}}$; log $g$) diagram. On the main sequence for massive (mostly radiative) stars, the slope of this line is about 0.25, and for intermediate-mass stars ($1 M_\odot$, mostly convective) the slope is about 0.06, the two values being remarkably close to the radiative and convective exponents cited above.

Fig. 3. - False-color rendering of Altair’s visible surface. Intensity at 500 nm increases from red to blue. Except for the effects of limb darkening, this is also a map of temperature, which varies from 8740 K at the pole to 6890 K at the equator.
Physical phenomena

- centrifugal deformation

(Prigogine et al., 2007)
Physical phenomena

- centrifugal deformation
- gravitational darkening

(MacGregor et al., 2007)
Physical phenomena

- centrifugal deformation
- gravitational darkening
- baroclinicity

(MacGregor et al., 2007)
Physical phenomena

- centrifugal deformation
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- baroclinicity
  - differential rotation
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(MacGregor et al., 2007)

(Peterson et al., 2006)
Physical phenomena

- centrifugal deformation
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- transport processes

(Peterson et al., 2006)
Recent models

Meynet & Maeder (2000), and other papers

- shellular rotation profile, $\Omega(r)$  
  
- 1D formalisme

- stellar evolution

- transport processes (chemical elements and angular momentum)
Jackson et al. (2004, 2005), MacGregor et al. (2007)

- barotropic models
- conservative rotation profile: $\Omega(s)$
- attempt to describe Achernar

(MacGregor et al., 2007)
Recent models

- Roxburgh (2004)
  - barotropic uniformly rotating model
- Roxburgh (2006)
  - transforms 1D models into 2D models
  - arbitrary 2D rotation profile
  - thermal equilibrium not solved

(Roxburgh, 2006)
The ESTER project

- Rieutord (2006)
  - Boussinesq model with baroclinic flows
- Espinosa & Rieutord (2007)
  - Compressible baroclinic model in spherical container

(Espinosa & Rieutord, 2007)  (Courtesy of M. Rieutord)
Pulsation modes

- many uncertainties remain in models
  - need for observational constraints
Pulsation modes

- many uncertainties remain in models
  - need for observational constraints
- difficulties
  - pulsation modes are not given by a single spherical harmonic: this is a 2D eigenvalue problem
  - unfamiliar mode geometry and frequency spectrum
The frequency spectrum

- Inadequacy of perturbative methods at rapid rotation rates
Inadequacy of perturbative methods at rapid rotation rates

(Reese et al., 2006)
Inadequacy of perturbative methods at rapid rotation rates

(Reese et al., 2006)
A new frequency organisation

(Lignieres et al., 2006, Reese et al., in preparation)

\[ \omega_{n, \ell, m} = n \Delta_n + \ell \Delta_\ell + |m| \Delta_m + \alpha^\pm \]
Mode geometry

- mode energy concentrated around equatorial region
Mode geometry

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- there are 10 “radial” nodes ($\tilde{n} = 10$)
Mode geometry

- mode energy concentrated around equatorial region
- there are 10 “radial” nodes ($\tilde{n} = 10$)
- there is 1 “latitudinal” node ($\tilde{\ell} = 1$)
\[ \tilde{n} = 9, \quad \tilde{\ell} = 0 \]
\hat{n} = 10, \quad \hat{\ell} = 0
\[ \hat{n} = 11, \quad \tilde{\ell} = 0 \]
\[ \tilde{n} = 11, \quad \tilde{\ell} = 1 \]
\[ \tilde{n} = 11, \quad \tilde{\ell} = 2 \]
\[ \tilde{n} = 11, \quad \tilde{\ell} = 3 \]
**Question:** what is the link between this mode geometry and the geometry of modes in non-rotating stars?
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\[ \tilde{n} = 2n + \varepsilon, \]
\[ \tilde{\ell} = \frac{\ell - |m| - \varepsilon}{2}, \]
\[ \varepsilon \equiv \ell + m \ [2] \]

\[ \omega_{n, \ell, m} = \tilde{n}\tilde{\Delta}_n + \tilde{\ell}\tilde{\Delta}_\ell + |m|\tilde{\Delta}_m + \tilde{\alpha} \]
Ray dynamics

**Question:** what is the link with ray dynamics?

(Vidal, 2006)
A Poincaré section reveals:

- different regions with different behaviours
- the presence of wave chaos

(Lignières & Georgeot, submitted)
Husimi functions can be used to “project” eigenmodes onto Poincaré section.

This confirms correspondence between ray dynamics and eigenmode calculations.

Likely link between wave travel times and parameters $\tilde{\Delta}_n$, $\tilde{\Delta}_\ell$, and $\tilde{\Delta}_m$.

(Lignières & Georgeot, submitted)
few attempts to interpret pulsation modes in rapidly rotating stars

| Authors                  | Star     | $v \cdot \sin i$ |
|-------------------------|----------|-----------------|
| Aerts et al., 2006      | HD 2036645 | 180             |
| Dziembowski et al., 2007 | HD 163868 | 250             |
| Savonije, 2007          | Altaïr   | 230             |
| Suárez et al., 2005     | $\zeta$ Oph | 380             |
| Saio et al., 2007       |          |                 |

difficulty with obtaining a reliable mode identification

forthcoming data on other rapid rotators (HD 181555, observed by CoRoT)
What needs to be done:

- search for asymptotic patterns using realistic stellar models
- search for equidistant patterns in observed pulsation spectra
- improve stellar models
- do detailed asteroseismic comparisons