Explicit formula for the external driving field used to eliminate the decoherence of a two-state system coupled to a noise field

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The elimination of decoherence of a multiphoton two-state quantum system by using an appropriate external driving field is considered. The multiphoton process caused by the noise field has a supersymmetric Lie algebraic structure. The time-evolution equations for the off-diagonal elements of the density operator of the two-state system are derived in the interaction picture. A simple explicit formula is given for the time-dependent external driving field used to eliminate the decoherence of the multiphoton two-state system.

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I. INTRODUCTION

During the past decade, quantum computation, which utilizes the superposition of many states of a quantum system, has attracted extensive attention [1–5]. Quantum computers can carry out computational tasks that are intractable for conventional computers since a quantum computing system composed of qubits (two-level quantum systems) has a mysterious property of quantum coherence (including the entanglement or quantum correlation), which has no counterpart in the classical realm [5]. However, because of the existence of the decoherence it is not clear whether we can finally build a truly practical and effective quantum computer [1]. Decoherence is essential to understanding how classical physics emerges from quantum mechanics. It is such a process by which a quantum-mechanical system that interacts with its natural environment loses the characteristic properties that distinguish quantum mechanics from classical physics. In other words, decoherence arises from the unavoidable interaction between the quantum systems and the noise fields, which are mostly responsible for damping of the quantum coherence of systems such as spin and two-state systems. Thus, if the decoherence occurs in the computation, some qubits become entangled with the environment and consequently the state of the quantum computer collapses [1]. Recently, much attention has been attracted to the theoretical and experimental work for topological quantum computation [6–12]. It is well known that due to its topological and global nature [13], the geometric phase of a spinning particle will not be affected by the random fluctuation arising in the evolution path [6,7]. This means that the geometric phase shift gate (should such exist) may be robust with respect to certain types of operational errors. Since in a quantum computer an error-tolerant quantum logic gate is particularly essential for realizing a quantum information processor, the quantum computation based on such a geometric phase shift gate will inevitably become an ideal scenario to this purpose [11]. Indeed, in the experiments performed at the end of last century, such conditional geometric phase shift gates were realized through the nuclear magnetic resonance (NMR) under an adiabatic condition [14–17]. The result of quantum computation will be exact only when the adiabatic condition can be fulfilled (i.e., only when the time-dependent parameters of the Hamiltonian of qubits vary extremely slowly) [7]. Once the adiabatic condition is satisfied, we meet, however, the problem of quantum decoherence caused by the inevitable interaction between the qubits (spin) and the environments. The existence of such decoherence effects demand that the quantum computation process (in which the maintenance of coherence over a large number of states is important) must be completed within the decoherence time. If not, the

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error caused by the decoherence will inevitably increase. Thus, we need to change the parameters of Hamiltonian rapidly. However, as discussed before, such rapid changes may break down the adiabatic requirement and leads to the nonadiabaticity error in the results of quantum computation \[7\]. Therefore, it seems that there is inevitably a conflict between the adiabatic condition and the requirement for removing the decoherence effects. In the literature, there are several methods, such as error-avoiding codes \[18\], error-correcting codes \[19\] and decoherence-avoiding scheme \[20\], which can be used to reduce the decoherence effects in qubits. In the decoherence-avoiding scheme \[20\], the interaction between the two-state quantum system and the environment (such as a noise field, thermal reservoir and bath) is eliminated by using an external controllable driving field. In the present paper, we study the maintenance of coherence through the decoherence-avoiding scheme in a two-state system. Note that in order to consider the general interaction between the two-state system and the noise field we treat the multiphoton process, \(i.e.,\) the total number of photons created or annihilated in each two-state transition process caused by a noise field is greater than one.

II. INTERACTION OF A TWO-STATE SYSTEM WITH THE NOISE FIELD AND THE DRIVING FIELD

The interaction between the two-state system and the noise field can be described by the supersymmetric multiphoton model \[21–25\], and the Hamiltonian under the rotating wave approximation can be written as \[21,22\]

\[
H = \frac{\omega_0}{2} \sigma_z + \omega a^\dagger a + g(a^\dagger)^k \sigma_- + g^* a^k \sigma_+ ,
\]

where the creation and annihilation operators \(a^\dagger\) and \(a\) for the photons obey the commutation relation \([a, a^\dagger] = 1\), \(\sigma_\pm\) and \(\sigma_z\) denote the two-state system operators satisfying the commutation relation \([\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm\), and \(g\) and \(g^*\) are the multiphoton coupling coefficients. The integer \(k\) is the number of photons created and annihilated in each two-state transition process, and \(\omega_0(t)\) and \(\omega(t)\) the two-state transition frequency and the mode frequency of photons, respectively. Defining the following algebraic generators \[23–25\]

\[
N = a^\dagger a + \frac{k - 1}{2} \sigma_z + \frac{1}{2} \left( a^\dagger a + \frac{k}{2} \sigma_+ \left( \begin{array}{cc} 0 & 0 \\ 0 & -\frac{k}{2} \end{array} \right) \right) , \quad N' = \left( \begin{array}{cc} a^k(a^\dagger)^k & 0 \\ 0 & (a^\dagger)^k a^k \end{array} \right) ,
\]

\[
Q = (a^\dagger)^k \sigma_- = \left( \begin{array}{cc} 0 & (a^\dagger)^k \\ (a^\dagger)^k & 0 \end{array} \right) , \quad Q^\dagger = a^k \sigma_+ = \left( \begin{array}{cc} 0 & a^k \\ a^k & 0 \end{array} \right) ,
\]

one can easily show that \((N, N', Q, Q^\dagger)\) form a set of supersymmetric generators and possess the supersymmetric Lie algebraic structure, \(i.e.,\)

\[
Q^2 = (Q^\dagger)^2 = 0 , \quad [Q^\dagger, Q] = N' \sigma_z , \quad [N, N'] = 0 , \quad [N, Q] = Q ,
\]

\[
[N, Q^\dagger] = -Q^\dagger , \quad \{Q^\dagger, Q\} = N' , \quad \{Q, \sigma_z\} = \{Q^\dagger, \sigma_\pm\} = 0 ,
\]

\[
[Q, \sigma_z] = 2Q , \quad [Q^\dagger, \sigma_z] = -2Q^\dagger , \quad (Q^\dagger - Q)^2 = -N' ,
\]

where \{\} denotes the anticommuting bracket.

To eliminate the quantum decoherence, an external classical field \(E(t)\) is applied to the above system. The Hamiltonian for the interaction between the two-state system and the driving field \(E(t)\) is given by

\[
H_E = -\frac{1}{2}(dE\sigma_+ + d^* E^* \sigma_-) ,
\]

where \(d\) and \(d^*\) denote the transition dipole moments of this two-state quantum system. The total Hamiltonian is \(H_{tot} = H + H_E\). Note that here the environmental noise field (weak) that is coupled to the two-state system is modelled by a quantized electromagnetic field while the external driving field (strong) is expressed by a set of classical quantities, \(E\) and \(E^*\).

The Hamiltonian of the above multiphoton two-state system in the interaction picture can be obtained by using the following unitary transformation

\[
V(t) = \exp \left[ \frac{1}{i} \left( \frac{\omega_0}{2} \sigma_z + \omega a^\dagger a \right) t \right] ,
\]

with the help of \(H_1(t) = V^\dagger(t)(H_{tot} - i\frac{\partial}{\partial t})V(t)\), one can arrive at

\[
H_1(t) = g\exp(-i\delta t)Q + g^* \exp(i\delta t)Q^\dagger - \frac{1}{2}(dEe^{i\omega_0 t} \sigma_+ + d^* E^* e^{-i\omega_0 t} \sigma_-) ,
\]

which is the Hamiltonian in the interaction picture, where the frequency detuning \(\delta = k\omega - \omega_0\).
III. EVOLUTION OF THE OFF-DIAGONAL DENSITY MATRIX ELEMENTS

In this section, we use the Markoff approximation [26,27] to obtain the time-evolution equation of the off-diagonal elements of the density operator for this two-state quantum system driven by the external classical field $E(t)$. It is well known that the density operator of the two-state system undergoing a multiphoton interaction satisfies the following Liouville equation

$$i \frac{\partial \rho(t)}{\partial t} = [H(t), \rho(t)].$$

(7)

Let $\rho_{11}(t)$ denote the reduced density operator of the two-state system. It follows from Eq. (7) that

$$\dot{\rho}_{11}(t) = \text{Tr}_r \dot{\rho}(t) = -\int_0^t \text{Tr}_r[H(t), [H(t'), \rho_{11}(t')]]dt'.$$

(8)

Here dot denotes the derivative with respect to time. If the thermal reservoir is assumed to be quite large, it does not change much during the time evolution process of the two-state system. When the density operator of the thermal reservoir is $\rho_{11}(0) \approx \sum_k \exp(-\beta H_k)/\text{Tr} \exp(-\beta H_k)$ with $\beta = 1/(k_B T)$, $H_k = \omega_k a_k^\dagger a_k$. If we take $E(t) \approx E(t')$, $\rho_{11}(t') \approx \rho_{11}(t)$ (Markoff approximation) [26,27], Eq. (8) can be rewritten as

$$\dot{\rho}_{11}(t) = -\int_0^t \text{Tr}_r[H(t), [H(t'), \rho_{11}(t')]]dt',$$

(9)

where the integrand can be rewritten as

$$\text{Tr}_r[H(t), [H(t'), \rho_{11}(t')]] = \text{Tr}_r[H(t)H(t')\rho_{11}(t')\rho_{11}(0) - H(t)\rho_{11}(t')\rho_{11}(0)H(t') - H(t')\rho_{11}(t')\rho_{11}(0)H(t) + \rho_{11}(t)\rho_{11}(0)H(t')H(t)].$$

(10)

After some tedious derivations, we obtain the explicit results for the four terms on the right-hand side of Eq. (10). The first term can be rewritten as

$$\text{Tr}_r[H(t)H(t')\rho_{11}(t')\rho_{11}(0)] = \langle H(t)H(t')\rho_{11}(0) \rangle_R$$

$$= g^* g \{ \exp[i\delta(t' - t)] < QQ^\dagger \rho_{11}(t) >_R + \exp[-i\delta(t' - t)] < Q^\dagger Q \rho_{11}(t) >_R \}$$

$$+ \frac{1}{4} d^* dE^*(t)E(t) \exp[\omega_0(t - t')]\rho_{11}(t),$$

(11)

where the relation $Q^2 = (Q^1)^2 = 0$ has been applied to the calculation. Here the subscript $R$ means the average over the states of the thermal reservoir (noise field). The rest of the terms on the right-hand side of Eq. (10) are given by

$$- \text{Tr}_r[H(t)\rho_{11}(t')\rho_{11}(0)H(t')] = - \langle H(t)\rho_{11}(t')\rho_{11}(0)H(t) \rangle_R,$$

$$= - g^2 \exp[-i\delta(t + t')] < Q\rho_{11}(t)Q >_R - (g*)^2 \exp[i\delta(t + t')] < Q^\dagger \rho_{11}(t)Q^\dagger >_R$$

$$- g^* g \{ \exp[i\delta(t' - t)] < Q\rho_{11}(t)Q^\dagger >_R + \exp[-i\delta(t' - t)] < Q^\dagger \rho_{11}(t)Q >_R \}$$

$$- \frac{d^2}{4} E^2(t) \exp[\omega_0(t + t')]\sigma_+ \rho_{11}(t)\sigma_+ - \frac{(d^*)^2}{4} (E^*)^2(t) \exp[-\omega_0(t + t')]\sigma_- \rho_{11}(t)\sigma_-$$

$$- \frac{d^*}{4} \{ E^*(t)E(t) \exp[\omega_0(t - t')]\sigma_+ \rho_{11}(t)\sigma_+ + E^*(t)E(t) \exp[-\omega_0(t - t')]\sigma_- \rho_{11}(t)\sigma_- \},$$

(12)

and

$$- \text{Tr}_r[H(t')\rho_{11}(t)\rho_{11}(0)H(t)] = - g^2 \exp[-i\delta(t + t')] < Q\rho_{11}(t)Q >_R - (g*)^2 \exp[i\delta(t + t')] < Q^\dagger \rho_{11}(t)Q^\dagger >_R$$

$$- g^* g \{ \exp[-i\delta(t' - t)] < Q\rho_{11}(t)Q^\dagger >_R + \exp[i\delta(t' - t)] < Q^\dagger \rho_{11}(t)Q >_R \}$$

$$- \frac{d^2}{4} E^2(t) \exp[\omega_0(t + t')]\sigma_+ \rho_{11}(t)\sigma_+ - \frac{(d^*)^2}{4} (E^*)^2(t) \exp[-\omega_0(t + t')]\sigma_- \rho_{11}(t)\sigma_-$$

$$- \frac{d^*}{4} \{ E^*(t)E(t) \exp[\omega_0(t - t')]\sigma_+ \rho_{11}(t)\sigma_+ + E^*(t)E(t) \exp[-\omega_0(t - t')]\sigma_- \rho_{11}(t)\sigma_- \}$$

(13)
$$\text{Tr}[\rho_{a\delta}(t)\rho_{a\delta}(0)H_1(t')H_1(t)] = g^*g \{ \exp[-i\delta(t'-t)] < \rho_{a\delta}(t)QQ^\dagger >_R + \exp[i\delta(t'-t)] < \rho_{a\delta}(t)Q^\dagger Q >_R \}$$

$$+ \frac{d^*d}{4} E^*(t)E(t) \exp[-i\omega_0(t-t')]\rho_{a\delta}(t).$$

Thus, the integrand in Eq. (9) can be rewritten in the following form

$$\text{Tr}[H_1(t),[H_1(t'),\rho_{a\delta}(t)\rho_{a\delta}(0)]] = T_1 + T_2 + T_3 + T_4,$$

where

$$T_1 = g^*g \{ \exp[i\delta(t'-t)] < (a^\dagger)^k a^k >_R \sigma_-\sigma_+ + \exp[-i\delta(t'-t)] < a^k(a^\dagger)^k >_R \sigma_+\sigma_- \} \rho_{a\delta}(t)$$

$$+ g^*\rho_{a\delta}(t) \{ \exp[-i\delta(t'-t)] < (a^\dagger)^k a^k >_R \sigma_-\sigma_+ + \exp[i\delta(t'-t)] < a^k(a^\dagger)^k >_R \sigma_+\sigma_- \},$$

$$T_2 = -2 \{ g^2 \exp[-i\delta(t'+t')] < Q\rho_{a\delta}(t)Q >_R + (g^*)^2 \exp[i\delta(t'+t')] < Q^\dagger\rho_{a\delta}(t)Q^\dagger >_R \},$$

$$T_3 = -g^* \{ \exp[i\delta(t'-t)] + \exp[-i\delta(t'-t)] \} \{ < Q\rho_{a\delta}(t)Q^\dagger >_R + < Q^\dagger\rho_{a\delta}(t)Q >_R \},$$

$$T_4 = \frac{d^*d}{4} E^*(t)E(t) \{ \exp[i\omega_0(t-t')] + \exp[-i\omega_0(t-t')] \} \rho_{a\delta}(t)$$

$$- \frac{d^2}{2} E^2(t) \exp[i\omega_0(t'+t')]\sigma_+\rho_{a\delta}(t)\sigma_+ - \frac{(d^*)^2}{2} (E^*)^2(t) \exp[-i\omega_0(t+t')]\sigma_-\rho_{a\delta}(t)\sigma_-$$

$$- \frac{d^*d}{4} E^*(t)E(t) \{ \exp[i\omega_0(t-t')] + \exp[-i\omega_0(t-t')] \} \{ < \rho_{a\delta}(t)\sigma_- + \sigma_-\rho_{a\delta}(t)\sigma_+ \}. $$

Assume that the reservoir is in a multiphoton state $|m\rangle$, where the occupation number $m \geq k$. Using the relations $a^k(a^\dagger)^k|m\rangle = \frac{(m+k)!}{m!} |m\rangle$, $< a^k(a^\dagger)^k >_R = \frac{m!}{(m-k)!}$, $< (a^\dagger)^k(a^k) >_R = 0$, $\sigma_+|+\rangle = \sigma_-|-\rangle = 0$ and $< +|\sigma_- = < -|\sigma_+$, one can obtain

$$< -|T_1|+> = g^*g \exp[i\delta(t'-t)] \left[ \frac{(m+k)!}{m!} + \frac{m!}{(m-k)!} \right] < -|\rho_{a\delta}(t)|+>,$$

$$< -|T_2|+> = 0,$$

$$< -|T_3|+> = -g^* \{ \exp[i\delta(t'-t)] + \exp[-i\delta(t'-t)] \} \frac{(m+k)!}{m!} < +|\rho_{a\delta}(t)|->,$$

$$< -|T_4|+> = \frac{d^*d}{4} E^*(t)E(t) \{ \exp[i\omega_0(t-t')] + \exp[-i\omega_0(t-t')] \} < -|\rho_{a\delta}(t)|+>$$

$$- \frac{(d^*)^2}{2} (E^*)^2(t) \exp[-i\omega_0(t+t')] < |\rho_{a\delta}(t)|->,$$

where the relations $< \sigma_+\rho_{a\delta}(t)\sigma_-|+> = < -|\sigma_-\rho_{a\delta}(t)\sigma_+|+> = 0$ has been inserted.

Define $\rho_-|+> = < -|\rho_{a\delta}(t)|+>$, $\rho_+|+> = < +|\rho_{a\delta}(t)|->$ and $\rho_-|+> = < -|\rho_{a\delta}(t)|+>$. It follows from Eq. (9) that

$$\dot{\rho_-|+>} = - \int_0^t \text{Tr}[-[H_1(t),[H_1(t'),\rho_{a\delta}(t)\rho_{a\delta}(0)]]|+>dt'.$$

Substituting Eqs. (15) and (17) into the above equation and neglecting the rapidly oscillating term (i.e., the last term) in $< -|T_4|+>$, we obtain

$$\dot{\rho_-|+>} = c_1(t)\rho_-|+> - c_2(t)\rho_+|+>,$$

where the time-dependent coefficients $c_1(t)$ and $c_2(t)$ are given by

$$\left\{ \begin{array}{l}
    c_1(t) = -g^*g \left[ \frac{(m+k)!}{m!} + \frac{m!}{(m-k)!} \right] \frac{1 - \exp(-i\delta t)}{i\delta} + \frac{d^*d}{4} E^*(t)E(t) \sin \omega_0 t, \\
    c_2(t) = -g^* \frac{(m+k)!}{m!} \left[ \frac{1 - \exp(-i\delta t)}{i\delta} + \frac{1 - \exp(i\delta t)}{-i\delta} \right].
\end{array} \right.$$

Note that the coefficient $c_1(t)$ is complex and $c_2(t)$ has a real value.

The complex conjugation of Eq. (19) gives

$$\dot{\rho_+|+>} = c_1^*(t)\rho_-|+> - c_2^*(t)\rho_+|+>.$$

Eqs. (19) and (21) are the time-dependent equations of the off-diagonal elements of the density operator for this multiphoton two-state quantum system.
IV. ELIMINATION OF DECOHERENCE IN THE MULTIPHOTON TWO-STATE SYSTEM

In order to maintain the coherence of the two-state system interacting with the noise field, we should analyze Eqs. (19) and (21), which govern the time evolution of the off-diagonal matrix elements of the density operator. It is obvious that in the presence of noise field the decoherence can be eliminated if $\dot{\rho}_{+-}(t) = \dot{\rho}_{-+}(t) \to 0$ is satisfied. This leads to the requirement

$$c_1^* c_1 - c_2^2 = 0. \quad (22)$$

In what follows, we will solve $E(t)$. For convenience, we first define

$$\alpha(t) = -g^* g \left[ \frac{(m + k)!}{m!} + \frac{m!}{(m - k)!} \right] 1 - \exp(-i\delta t), \quad \beta(t) = \frac{d^* d}{2\omega_0} \sin \omega_0 t, \quad x(t) = E^*(t)E(t), \quad (23)$$

and then rewrite the coefficient $c_1(t)$ as follows

$$c_1(t) = \alpha(t) + \beta(t)x(t). \quad (24)$$

According to Eq. (22), one obtains $(\alpha^* + \beta x)(\alpha + \beta x) - c_2^2 = 0$, which can be rewritten as

$$x^2 + \frac{\alpha^* + \alpha}{\beta} x + \frac{\alpha^* \alpha - c_2^2}{\beta^2} = 0. \quad (25)$$

The two roots of the above quadratic equation are of the form

$$x_{\pm} = \frac{-\alpha + \alpha^* \pm \sqrt{(\alpha - \alpha^*)^2 + 4c_2^2}}{2}, \quad (26)$$

where

$$\alpha + \alpha^* = -g^* g \left[ \frac{(m + k)!}{m!} + \frac{m!}{(m - k)!} \right] 2 \sin \delta t, \quad \alpha - \alpha^* = -g^* g \left[ \frac{(m + k)!}{m!} + \frac{m!}{(m - k)!} \right] \frac{2(1 - \cos \delta t)}{i\delta}. \quad (27)$$

If we apply an external time-dependent driving field with intensity given by Eq. (26), in principle the decoherence of the two-state quantum system interacting with the environment (noise field) can be eliminated. Fig. 1 shows the external time-dependent driving field required for the elimination of the decoherence of the multiphoton two-state system with various $k$ (the number of photons in each transition process) and $\delta = 0$ (without detuning). It follows from expressions (20) and (27) that all the parameters $c_2$, $\alpha + \alpha^*$ and $\alpha - \alpha^*$ are inversely proportional to the frequency detuning $\delta$. It is thus shown that according to Eqs. (19)-(21), the two-state system may not be subject to the quantum decoherence significantly in the case of large detuning. In what follows, as an illustrative example, we will consider only the maintenance of the coherence for the case of small detuning. Fig. 2 demonstrates the required time-dependent expression for the external driving field when the frequency detuning is $\delta = 0.01\omega_0$. It should be noted that in some time ranges (e.g., $2.2 \times 10^{-10} < t < 2.5 \times 10^{-10}$ s for the case of $k = 1$) in Fig. 2, both the field intensities $x_+(t)$ and $x_-(t)$ obtained in (25) are negative, i.e., $E^*(t)E(t) < 0$, which has no physical meanings. So, in the time ranges where $E^*(t)E(t) < 0$, what one should do is to set $E(t) = 0$ experimentally. This, however, means that if we utilize the external driving field illustrated in Fig. 2, the coherence cannot be maintained completely after $t = 2.2 \times 10^{-10}$ s (for the case of $k = 1$). To overcome such a difficulty, here we will suggest a potential scheme of cyclically driving field: specifically, if we choose the evolitional behavior of the external driving field of the time range $[0, T]$ ( $T = 2.2 \times 10^{-10}$ s) and apply it to any ranges $[nT, (n + 1)T]$, namely, $E(nT + t) = E(t)$, where $t \in [0, T]$, the quantum decoherence of the above two-state system undergoing a multiphoton interaction caused by the environmental noise field will be greatly reduced.

To summarize, the multiphoton process, which possesses a supersymmetric Lie algebraic structure, is caused by the interaction between the two-state system and the noise field of the environment. This will lead to the decoherence of the two-state quantum system. In the present paper, we have considered the reduction of decoherence by using an appropriate external time-dependent driving field. We have used the Markoff approximation to derive the equations governing the time-evolution behavior of the off-diagonal elements of the two-state density operator in the interaction picture. A simple explicit formula for the intensity of the external driving field has been given for the elimination of the decoherence of the multiphoton two-state system.

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FIGURE CAPTIONS

Fig. 1. The external time-dependent driving field required for the elimination of the decoherence of the two-state system coupled to a noise field. Here $m = 100$, $\omega_0 = 1.0 \times 10^{11}$ s$^{-1}$ and the frequency detuning $\delta$ vanishes.

Fig. 2. The external time-dependent driving field required for the elimination of the decoherence of the two-state system coupled to a noise field. Here $m = 100$, $\omega_0 = 1.0 \times 10^{11}$ s$^{-1}$ and the frequency detuning $\delta = 0.01\omega_0$.

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The diagram shows the variation of $|E|^2/d^2/g^2$ with time $t$ (s) for different values of $k$. The x-axis represents time in units of $10^{-10}$ seconds, and the y-axis represents the magnitude of the ratio $|E|^2/d^2/g^2$ on a logarithmic scale.

- $k=1$ is represented by a solid line.
- $k=2$ is represented by a dotted line.
- $k=3$ is represented by a dashed line.
- $k=9$ is represented by a dash-dotted line.

The graph illustrates periodic oscillations with increasing magnitude as time progresses for each value of $k$. The scales on the y-axis range from $10^0$ to $10^{25}$.
