Study on walk step gait of quadruped robot based on the support polygon

Yutong Huang¹,³, Qingsheng Luo¹ and Yan Jia²

¹ School of Mechatronic Engineering, Beijing Institute of Technology, Beijing, China
² School of Mechanical Engineering, Beijing Institute of Technology, Beijing, China
³ E-mail: huangyutong123@126.com

Abstract. In order to study the static gait stability of quadruped robot, moving orders based on the stability margin of the support polygon are studied. Based on the support triangle theory, using the brute force method of enumerated to compare all possible marching order, different moving orders of quadruped robot motion performance are analyzed by comparing adjusting frequency of the center gravity and stability margin. A series of simulation experiments were carried out to verify the proposed optimization method by using MATLAB and ADAMS software. Finally, the optimal step of the robot in static gait was obtained. The simulation results show that the optimal step can greatly improve the performance of the robot.

1. Introduction

Static gait analysis is the basis for the stable walking of the quadruped robot. Researches on gait at this stage, many domestic scholars and scholars from abroad have done a lot of researches in the optimization of trot gait and the optimal of trot gait, but few studies have involved the optimal step of walk gait sequence [1-2]. Most quadrupeds in nature have excellent athletic ability and good balance. For the gait design of quadruped robots, researchers mostly use bionic design to directly imitate the gait order of large quadrupeds, but they do not Scientific simulation analysis and optimization research are performed on the optimal step. Quadruped robots moving in different steps have a great influence on the robot's motion stability and energy consumption. The optimization of the step sequence has a strong practical significance for the improvement of the quadruped robot's movement performance [3]. There have been articles mentioning the issue of the optimal gait order of the walk gait, but no systematic, comprehensive, clear, and complete theoretical argument has been made. This article relies on a ministry’s hydraulic quadruped robots. Considering the number of center-of-gravity adjustments and stability margins in a walk cycle in the walk gait. By selecting different walking gaits in order of movement, we can find out the best moving order of sports performance [4-5].

2. Static gait stability and its parameters

2.1. Static gait parameters

The static gait stability is very important for ensuring the stable walking of the quadruped robot. The gait parameters included in it are: load factor $\beta$, walking cycle $T$, step $\lambda$, single leg span $A$, etc. [6].

Load factor $\beta$: The ratio of the support time of one leg on the ground to the time required for four legs to cycle in one cycle. Walking cycle $T$: The time required for the quadruped robot's four legs to complete one leg lift cycle. Step $\lambda$: The distance that the quadruped robot moves relative to the ground.
during a complete gait cycle. Single leg span A: The distance that the quadruped robot moves relative
to the ground during lifting and landing of one leg. Absolute span for single leg L: The displacement of
the foot relative to the ground during swinging of a single leg. Single Leg Relative Span E: The
displacement of the foot relative to the body during swinging of a single leg, and normally, L = A + E.

2.2. Criterion for the stability of static gait
For the stationary gait stability, McGhee proposed the concept of stability margin [7]. When the
quadruped robot walks in a static gait, there are at least three legs on the ground at any moment which
means the load factor is ≥ 0.75. Each convex leg can be connected to a convex polygon (a supporting
polygon). The stability margin is the shortest distance between the projection point of the quadruped
robot and the support polygon on the support polygon. The stability margin \( S = \min(d_1, d_2, d_3) \) is
shown in figure 1. From the perspective of energy consumption, the center of gravity and the direction
of travel will cause unnecessary consumption of energy and reduce the motion stability of the
quadruped robot [8]. Therefore, the consistency of the center of gravity and the direction of travel is
another measure of the choice of steps.

The walk gait is the lowest speed gait of a quadruped. Since it is generally at least 3 feet at the same
time, it has the highest static stability. This section only conducts the simulation of the zero-to-zero
relative velocity of the foot-to-ground, because only the speed of a single foot relative to the ground
may cause the robot to change the direction of travel. That is the analysis of composite cycloids.

According to the cycloid formula, we can get:

\[
\begin{align*}
    x &= r(t - \sin t) \\
    y &= r(1 - \cos t)
\end{align*}
\] (1)

With further derivation of equation (1) we can obtain:

\[
\begin{align*}
    x &= S \left[ \frac{(t - kT)}{T_{sw}} - \frac{1}{2\pi} \sin \left( \frac{2\pi}{T_{sw}} (t - kT) \right) \right] + kS, \quad kT \leq t < T_{sw} + kT \\
    x &= (k + 1)S, \quad T_{sw} + kT \leq t < T + kT \\
    y &= \frac{h}{4 + \pi} \left[ 4\pi \cos \left( \frac{(t - kT)}{T_{sw}} \right) - \sin \left( \frac{4\pi}{T_{sw}} (t - kT) \right) \right], \quad kT \leq t < \frac{T_{sw}}{4} + kT \\
    y &= \frac{h}{4 + \pi} \left[ \sin \left( \frac{2\pi}{T_{sw}} (t - kT) - \frac{\pi}{2} \right) + \frac{\pi}{4} \right], \quad \frac{T_{sw}}{4} + kT \leq t < \frac{3}{4} T_{sw} + kT \\
    y &= \frac{4\pi h}{4 + \pi} \left[ 1 - \frac{(t - kT)}{T_{sw}} \right] - \frac{1}{4\pi} \sin \left[ 4\pi \left( 1 - \frac{t - kT}{T_{sw}} \right) \right], \quad \frac{3}{4} T_{sw} + kT \leq t < T_{sw} + kT + kT \\
    y &= 0, \quad T_{sw} + kT \leq t < T + kT
\end{align*}
\] (2)

According to the setting of the body speed, the position of the mass center of the body at any time
can be known from the phase relationship of each leg and the transformation matrix of the world
coordinate system to the body coordinate system.

2.3. Parameter settings
This article relies on the project of a hydraulic quadruped robot (shown in figure 2) of a ministries
commissioned by the author’s team. The robot body is 1450mm long, 700mm wide, and the maximum
step length is 300mm. In combination with the physical size of the robot, the final setting parameters
are: load factor \( \beta = 0.78 \), walking cycle \( T = 1.6s \), step distance \( \lambda = 240mm \), and single leg span \( A = 0.06m \).
3. Step sequence comparison and analysis proof

According to the difference in the order of the legs of a quadruped robot, it is learned that there are a total of $A_4^4 = 24$ kinds of steps. In order to obtain the most conducive to the robot movement performance to play the steps. The optimization criteria include both performance and energy consumption. Record the number of the first stepped leg as the first level, followed by the step of the leg as the second level, and step down to discuss. When it is found that a certain step leads to unfavorable factors such as unfavorable movement stability or increased energy consumption, such a step is negated and the next step is continued.

Quadruped robots will produce different shapes of supporting triangles due to different moving steps. The shape of supporting triangles is a major factor that affects the stability margin of robots. According to the definition of stability margin, the minimum distance from the vertical projection point of the quadruped robot's four-foot support plane to each side of the support polygon is used as the evaluation criterion of the stability margin, that is, $S=\min(d_1, d_2, d_3)$. Therefore, although a slender triangle may increase one or two values of $d_1$, $d_2$, $d_3$ compared to a short thick triangle, it inevitably causes the minimum value to be smaller than that of the short and thick type of supporting triangle. This reduces the stability margin of the robot. Since the center of gravity of the robot falls within the supporting triangle during the movement to ensure its balance and achieve stable walking, it is necessary to continuously adjust the positional relationship between the center of gravity and the supporting foot to maximize its stability margin. And only guarantee the direction of movement of the center of gravity and the direction of travel of the body can be guaranteed to meet the energy consumption standards.

In summary, the selection of the optimal step sequence will be simplified as a screening process for the above two requirements. In the step sequence diagram of this article, thick solid lines represent the legs of the fuselage and the robot. The thin solid line represents the current supporting triangle. The double-dotted line represents the supporting triangle after the move. The number represents the serial number of the corresponding leg. The arrow indicates the direction of movement.

3.1. The first step of the 4th leg

In the following, there are 5 methods of first step of the 4th leg introduced. The methods included in it are: 4-3-2-1 steps, 4-3-1-2 steps, 4-1 steps, 4-2-1-3 steps and 4-2-3-1 steps.

As shown in figure 3, if the movement is performed in the sequence of 4-3-2-1, the support triangle changes from $\Delta 124$ to $\Delta 134$ in the process of leg 3. Since $\Delta 124$ and $\Delta 134$ are independent of each other, there is no overlapping area, and $\Delta 134$ is located behind $\Delta 124$. Therefore, the center of gravity of the body will move backwards during the adjustment of the body's center of gravity to ensure that
the robot can remain stable when moving the leg 3. This kind of scheme will cause the energy consumption of the system, so it is not the best step.

If the movement is performed in the sequence of 4-3-1-2, an elongated supporting triangle $\Delta_{124}$ is produced during the movement of the leg 3. It is not good for motion stability, so it is not an optimal step.

If the movement is performed in the sequence of 4-1, during the process of moving leg 4, the supporting triangle changes from $\Delta_{123}$ to $\Delta_{234}$. The two triangles are independent of each other, there is no overlapping area, and $\Delta_{234}$ is located behind $\Delta_{123}$. A backward body adjustment movement is generated, increasing energy consumption and therefore not being the optimal step sequence.

If the movement is performed in the sequence of 4-2-1-3, the current support triangle $\Delta_{134}$ will become $\Delta_{234}$, and the next support triangle will become $\Delta_{124}$. Both $\Delta_{234}$ and $\Delta_{124}$ at this time will become slender supporting triangles. The stability margin will be reduced, which is not conducive to the stability of movement. Therefore, it is not considered as the optimal step sequence.

By analyzing the sequence diagram shown in figure 4, it can be found that the 4-2-3-1 step has obvious advantages in all steps of the legs. Mainly in:

1) The 4-2-3-1 step chooses to move the legs 4 and 2 which are relatively close to the rear when starting the movement, so that the gap between the front legs and the back legs is reduced, and the longest edge can always be used when creating a supporting triangle. In combination with the above two points, a support triangle is formed with a large area and a short and thick shape that is conducive to improving the stability margin.

2) In the course of exercise, the current supporting triangle and the supporting triangle after the step are mostly on one side, which is favorable for arranging the lateral adjustment of the fuselage.

3) The 4-2-3-1 sequence does not require any rear fuselage motion adjustment during the movement, which is very beneficial to reduce energy loss. For example, in one detail, when the leg 2 moves, the supporting triangle changes from $\Delta_{134}$ to $\Delta_{124}$, and $\Delta_{124}$ is located on the front side, so when the leg 3 is moving, the center of gravity of the body moves forward to ensure that the center of gravity is always in the stable region. With the continuous advancement of the fuselage, the stability margin is getting larger and larger, and the whole system is also more and more stable. However, it is difficult to do this in many other steps. For example, if the 4-2-1-3 step is reversed, the support triangle $\Delta_{234}$ is located on the back side after the step, and then it gradually departs as the fuselage moves forward. The domain makes the stability margin less and less, and the stability of the entire system is getting worse and worse.

3.2. The first step of the 3rd leg
In the following, there are 5 methods of first step of the 3rd leg introduced. The methods include in it are:3-4-1-2 steps, 3-4-2-1 steps, 3-2 steps, 3-1-2-4 steps and 3-1-4-2 steps.

Similar to the moving order 4-3-2-1, taking 3-4-1-2 as the moving order, the center of gravity of the quadruped robot needs to be adjusted backwards to ensure the stability of movement and increase the energy loss, as shown in figure 5.
Taking 3-4-2-1 as the moving order, when the leg 4 is struck, an elongated supporting triangle appears, which is not conducive to the stability of movement, and therefore does not serve as an optimal step sequence.

Taking 3-2 as the initial moving order, the supporting triangle in the process of striding leg 3 changes from $\Delta 124$ to $\Delta 134$, and $\Delta 134$ is located behind $\Delta 124$. It will increase the energy loss. Obviously, it is not an optimal step sequence.

If the movement is performed in the sequence of 3-1-2-4, the principle is like the moving order 4-2-1-3. It is not considered as the optimal step sequence.

The 3-1-4-2 sequence is the closest step to the 4-2-3-1 sequence, which is satisfactory in the adjustment of the robot. During the entire operation period, no backward adjustment movement was generated. But since the 3-1-4-2 step sequence is to choose the first phase was relatively move front leg 3 and leg 1, this makes the phase difference between left and right side leg on the basis of the initial position increased. A slenderly shaped supporting triangle is produced, which will reduce the stability margin and is not conducive to the stability of the movement of the system. As a result, it is not an optimal step sequence.

3.3. The first step of the 1st and the 2nd leg

Due to the modular design of the quadruped robot and the symmetry of the quadruped robot, the first or the second step of 1st or 2nd leg can find the corresponding step in the sequence of the 1st step of 3rd leg or the 4th leg.

It can be analyzed that the first leg will stretch the supporting triangle, and of course there is no adverse effect from the stability margin. However, on the one hand, the length of the supporting triangle is increased, and the distance between the front and rear supporting legs is increased, so that the ability of the four-legged robot to resist lateral impact is reduced, and sideways overturning tends to occur during movement, which may adversely affect the stability of movement. On the other hand, the extended support triangle has no adverse consequences in terms of stability margin, but in order to achieve a high stability margin, the center of gravity of the fuselage must be located near the centroid of the supporting triangle. This means that with the extension of the supporting triangle, the adjustment distance of the center of gravity of the body is also increasing, which will result in excess energy loss.

In summary, the step sequence produced by the first step forward is not conducive to the movement performance and energy loss of the quadrupedal robot, and the influence of the elongate support triangle will be generated due to the first step forward. Therefore, it can be concluded that the exercise performance of the first leg (leg 1 and leg 2) is not as good as that of the first step, and therefore it is not an optimal step sequence.

4. Simulation

Through simulation tests at different speeds and cycles, it was found that in the walk gait when setting foot lift height to $h = 0.06m$, three feet setting on the floor at the same time, so $\beta = 0.75$. The speed-cycle-energy efficiency distribution is shown in figure 6.

As figure 6 shows, the part indicated by the arrow is the part with the lowest energy efficiency value. When the robot is running below 0.3m/s, the engine is at a low speed most of the time (to ensure the normal operation of the system, the minimum speed is set to 2000rpm). It can be seen that the optimal distribution of energy efficiency is also hyperbolic. The lowest value of energy efficiency is about , and the step is about 0.05m at this time. Therefore, when walking in the Walk gait, the optimal step length is 0.05m.
Figure 6. Speed-period-energy efficiency map of compound cycloid foot trajectory.

In order to verify the argumentation of optimal step sequence 4-2-3-1 step and 1-3-4-2 step sequence were simulated in Adams. In the simulation, the forward direction is defined as the X direction, the vertical direction is the Y direction, and the roll direction is the Z direction. The results of displacement and roll Angle in each direction are as shown in figures 7-10:

Figure 7. Simulation results of roll Angle.

Figure 8. Simulation results of displacement in X direction.
The roll Angle is an important parameter to evaluate the stability of the quadruped robot. The smaller the standard deviation, the less likely it is to have a side fall. According to the comparison of test results, it is found that the roll Angle fluctuation of 4-2-3-1 step sequence is significantly less than 1-3-4-2, in other words, 4-2-3-1 has good stability.

The displacement of the X direction is the displacement of the forward direction. The larger the displacement is, the higher the efficiency of the sequence is. By the contrast of the results, it was found that in 35 seconds, the X-direction displacement of the 4-2-3-1 step sequence is approximately 4500mm, and the 1-3-4-2 step sequence in X-direction displacement is about 4000mm, and the 4-2-3-1 sequence has better efficiency.

The displacement of the Y direction and the Z direction respectively represents the fluctuation of the vertical direction and horizontal direction of the quadruped robot. The smaller the displacement in these two directions, the smaller the fluctuation of the body. Through comparison of the experimental results, it was found that the Y-direction and Z-direction displacements in the 4-2-3-1 step sequence were all less than 1-3-4-2, which indicating that the 4-2-3-1 step sequence has a better performance in self-stability.

In summary, the analysis of the experimental results shows that the displacement of the 4-2-3-1 step sequence in the roll angle and the X, Y, and Z directions is significantly better than 1-3-4-2 step sequence, which means 4-2-3-1 step sequence is the optimal step sequence of the walk gait.

5. Experiments
In order to further verify the optimal step sequence of the quadruped robot walk gait proposed above, this paper uses a hydraulic quadruped robot to verify the prototype test. Taking into account the actual conditions of the prototype, the experiment adjusted the moving step length and lifting height to 50
mm, and the swing phase, support phase, and adjustment phase were adjusted to 0.6 s, 0.6 s, and 0.2 s, respectively.

![Figure 11. The physical prototype of Quadruped hydraulic robot.](image)

The experiment process is shown in figure 11. As can be seen from the figures, the hydraulic quadruped robot which using the method proposed in this paper achieves a relatively smooth linear walk, which proves the effectiveness and feasibility of the proposed method.

In the experiment, we used 4-2-3-1 step and 1-3-4-2 step to perform fuel consumption tests respectively. In the case of zero load and the same road surface, the robots were allowed to travel 100m. After 5 sets of experiments, the fuel consumption is shown in the table 1 below.

| Walk step gait      | Experiment times |
|---------------------|------------------|
|                     | 1    | 2    | 3    | 4    | 5    |
| 4-2-3-1 step        | 2.45  | 2.67  | 2.49  | 2.51  | 2.41 |
| 1-3-4-2 step        | 2.90  | 2.73  | 2.76  | 2.88  | 2.91 |

From the data in the table 1, 4-2-3-1 step to 1-3-4-2 step is more fuel efficient.

6. Conclusions
Through the demonstration analysis of the walk step sequence of a hydraulic quadruped robot, the following conclusions are drawn:

1) Due to the symmetry of the quadruped robot body design, the step sequence of the front leg (i.e. leg 1 and leg 2) can always find the corresponding sequence in the step sequence of the hind legs (namely leg 3 and leg 4). The first hind leg helps to form the short thick support triangle that can improve the stability margin of the robot. Therefore, the step sequence of the first hind legs is superior to the step sequence of the front leg.

2) In the first step of the hind legs, the 4-2-3-1 step has obvious advantages in reducing robot energy consumption, improving the stability margin of the robot, and resisting lateral impact performance, etc. It is the optimal step sequence of walk gait.

3) This paper proposes the optimal walk step gait of the bionic quadruped robot for the first time and carries out simulation analysis. It provides more scientific basis and engineering reference for the walking method of the quadruped robot.
Acknowledgments
This work was supported by Hydraulic Quadruped Robotof a National Ministry of China (No. 40401060305).

References
[1] Shaopeng P, Jiadong S and Jianzhong W 2015 Crawl gait optimization for quadruped robot based on gravity center lateral movement J. Mechanical Science and Technology for Aerospace Engineering p 821-826
[2] Chaofeng Y 2015 Research on gait planning for a hydraulic quadruped robot Beijing Institute of Technology p 23-25
[3] Pengfei W 2007 Research on quadruped robot steadily walking planning and controlling technology Harbin Institute of Technology p 53-54
[4] Marc R, Kevin B, Gabriel N and Rob P 2008 BigDog, the rough-terrain quadruped robot J. IFAC Proceedings Volumes p 41
[5] Nelson,K. Blankespoor,M and Raibert 2006 Walking BigDog: Insights and challenges from legged robotics J. Journal of Biomechanics p 39
[6] Xixia L, Zhenhua W and Yi J 2011 Study on static gait planning of the quadruped vehicle J. Journal of Academy of Armored Force Engineering p 35-38
[7] Mcghee R B 1968 Some finite state aspects of legged locomotionJ. Mathematical Biosciences p 67-84
[8] Yongying T and Rui W 2015 Study on static gait of quadruped walking vehicle based on support polygon J. Agricultural Equipment & Vehicle Engineering p 20-24