Active-sterile neutrino oscillations in the early Universe with the complete mixing matrix

Sergio Pastor
IFIC (CSIC-Universitat de València)
Parc Científic UV, C/ Catedrático José Beltrán, 2, 46980 Paterna, Spain
E-mail: sergio.pastor@ific.uv.es

Abstract. In the framework of a 3+1 scheme with an additional inert state, we consider the thermalisation of sterile neutrinos in the early Universe taking into account the full $4 \times 4$ mixing matrix. The evolution of the neutrino energy distributions is found solving the momentum-dependent kinetic equations with full diagonal collision terms. The degree of thermalisation of the sterile state is shown in terms of the effective number of neutrinos, $N_{\text{eff}}$, and its dependence on the three additional mixing angles ($\theta_{14}, \theta_{24}, \theta_{34}$) and on the squared mass difference $\Delta m^2_{41}$ is discussed. Our results are relevant for fixing the contribution of a fourth light neutrino species to the cosmological energy density, whose value is very well constrained by the final Planck analysis. For the preferred region of active-sterile mixing parameters from short-baseline neutrino experiments, we find that the fourth state is fully thermalised ($N_{\text{eff}} \approx 4$).

Nowadays, a vast number of observations nicely fit in the framework of three-flavour neutrino oscillations (see e.g. [1]). There remain, however, a few anomalies found in some short-baseline oscillation experiments that could indicate the presence of an additional light neutrino at the eV mass scale that mixes with the active states [2]. This problem can be studied in the 3+1 scheme (small active-sterile mixing), although recent global analyses [3, 4, 5] show that this does not provide an optimal solution due to the severe tension between the anomalies in the appearance sector and disappearance measurements. While we expect that new data will shed light on the causes of this tension and eventually provide a definitive solution, it is interesting to explore the implications of such active-sterile oscillations in astrophysical and cosmological scenarios.

Concerning cosmology, a well-known consequence would be the production of the new states in the early Universe via mixing, while the active states keep an equilibrium energy distribution if oscillations become effective before neutrino decoupling. The degree of thermalisation depends on the values of the mixing parameters and fixes the neutrino contribution to the cosmological energy density of relativistic particles, usually parametrised by the effective number of neutrinos ($N_{\text{eff}}$). A value of $N_{\text{eff}} \approx 4$ arises from a fully thermalised fourth neutrino state, disfavoured according to the analysis of the full-mission Planck data [6] on the anisotropies of the cosmic microwave background (CMB): the allowed range can be as restricted as $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ (95% CL) adding other cosmological and astrophysical measurements. Thus, it is important to perform a proper calculation of the values of $N_{\text{eff}}$ for each choice of the active-sterile mixing parameters.

The evolution of the active-sterile neutrino system in the early Universe is a complex problem due to the simultaneous presence of weak interactions and oscillations in a varying medium. In principle, solving the Boltzmann equations for unequal neutrino momenta is mandatory, because both oscillations and collisions present a (different) dependence on the neutrino energy. Moreover,
Figure 1. Effect on $N_{\text{eff}}$ when varying only one of the active-sterile angles, for normal mass ordering. Dashed, dashed-dotted and dotted lines indicate that only $\theta_{14}$, $\theta_{24}$ or $\theta_{34}$, respectively, is different from zero. The different colours encode three discrete levels of $N_{\text{eff}}$ as indicated. Red lines show the allowed regions (99.7% CL) from DANSs+NEOS [22] on $|U_{e4}|^2$, while blue lines show the constraints (99.7% CL) from muon (anti)neutrino disappearance on $|U_{\mu 4}|^2$.

these processes can lead to non-thermal distortions in the neutrino energy distributions that would be only found in multi-momentum calculations. In the recent years, several authors (see e.g. [7, 8]) presented multi-momentum calculations in the 1+1 approximation (only one active and one sterile neutrino species), in some cases including a potentially large lepton asymmetry. In particular, the first results in the 1+1 case ($\nu_e-\nu_s$) with full collision integrals were shown in [8], where the quantum kinetic equations were solved with a modified version of the LASAGNA code [9], enforcing a zero lepton asymmetry. This code was used later [10, 11, 12, 13] to convert the active-sterile mixing parameters into two other quantities relevant for cosmology ($N_{\text{eff}}$ and the effective sterile neutrino mass $m_{\text{eff}}^s$), in order to obtain bounds from Planck data and neutrino oscillation experiments in the 1+1 approximation (either $\nu_e-\nu_s$ or $\nu_\mu-\nu_s$ mixing).

However, a precise calculation of 3+1 active-sterile oscillations in cosmology must include the unavoidable presence of mixing among active neutrinos (see [14, 15] for early simplified analyses), i.e. the full four-neutrino mixing matrix with up to six different angles: three from the active sector ($\theta_{12}, \theta_{13}, \theta_{23}$) and three from the mixing with the sterile state ($\theta_{14}, \theta_{24}, \theta_{34}$). More recent multi-angle studies [16, 17] have been performed within the averaged-momentum approximation. A first step beyond the averaged-momentum and single-mixing approximations [18] included a multi-momentum and multi-flavour calculation of the kinetic equations of the 2+1 active-sterile scenario, with mixing parameters inspired by the short-baseline neutrino anomalies and in the presence of primordial neutrino asymmetries, that can suppress the production of sterile states.

Prompted by the current precision on the determination of $N_{\text{eff}}$ by Planck and taking advantage of our previous experience on multi-momentum calculations in the standard three-neutrino case [19, 20], we have performed an analysis of the sterile neutrino thermalisation in the 3+1 scenario [21], based on the numerical solution of the kinetic equations with full collision terms and the complete $4 \times 4$ mixing matrix. Here we present a summary of our main results, but all details about the numerical calculations using our new code (FortEPiaNO) can be found in [21].

Let us now discuss the different impact of the active-sterile mixing angles on the thermalisation
process when considering the full oscillation paradigm of the 3+1 scheme. A first comparison can be performed in the case when only one of the mixing angles is non zero, as shown in Fig. 1. One can see that, for the same value of the mixing parameter, $\theta_{14}$ always corresponds to the smallest final $N_{\text{eff}}$. In other words, a larger $\theta_{14}$ is required to achieve the same $N_{\text{eff}}$ with respect to $\theta_{24}$ or $\theta_{34}$. These two latter angles have a very similar effect at small $\Delta m_{41}^2$, but $\theta_{34}$ is slightly more effective for larger $\Delta m_{41}^2$ due to the fact that, for non-zero $\theta_{24}$, the thermalisation is mainly generated by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations which, at high temperatures, are affected by the matter potential created by the few muons still present in the plasma, therefore slowing down the population of the sterile states. One can also compare in Fig. 1 the $N_{\text{eff}} \geq 3.9$ lines with the preferred regions for $|U_{e4}|^2$ at 99.7% CL from DANSS+NEOS [4, 22, 23] and the exclusion curves for $|U_{\mu 4}|^2$ at 99.7% CL from muon (anti)neutrino disappearance. The iso-$N_{\text{eff}}$ contours in Fig. 1 can be well approximated by straight lines for each mixing angle, as shown in previous analyses [7, 10, 14]. In particular, our results for the 3+1 case are in reasonable agreement (within few percent of the total $N_{\text{eff}}$) with those obtained with the LASAGNA code in the 1+1 approximation.

Let us now consider what happens when we increase the values of the angles that were earlier always fixed to zero. As shown in Fig. 2, the iso-$N_{\text{eff}}$ contours change when we vary $\Delta m_{41}^2$ and $|U_{e4}|^2$ while the two remaining matrix elements $|U_{\mu 4}|^2$ or $|U_{\tau 4}|^2$ assume different values. It is interesting that these contours remain similar to those in Fig. 1 when the largest mixing comes from $|U_{e4}|^2$, but saturate as a consequence of the other mixing channels when $|U_{e4}|^2$ is smaller than one of the other two mixing matrix elements. We include in the same panels the preferred 99.7% CL regions by DANSS+NEOS [22]. One can conclude that the current preferred value for $|U_{e4}|^2$ would lead to a contribution of $N_{\text{eff}} \simeq 4$, regardless of the values of $|U_{\mu 4}|^2$ or $|U_{\tau 4}|^2$ and despite the fact that $\theta_{14}$ is the angle which makes the thermalisation less effective. Since Planck data prefer $N_{\text{eff}} \lesssim 3.3$ [6] (TT, TE, EE+lowE+lensing+BAO, 95% CL), this indicates a strong tension between CMB observations and neutrino oscillation experiments, as previously noted.

We show in Fig. 3 the simultaneous effect of all three active-sterile mixing angles on the final value of $N_{\text{eff}}$, for $\Delta m_{41}^2 = 1$ eV$^2$. Instead of a proper ternary plot, for which we should have fixed $|U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2 = 1$, we show combinations of the mixing matrix elements such that $\sum \log_{10} |U_{e4}|^2 = -13$, avoiding the use of a linear scale that would make the plot mostly filled with only $N_{\text{eff}} \simeq 4$. Although this plot looks symmetric, a more accurate inspection shows that

\textbf{Figure 2.} Final $N_{\text{eff}}$ in the 3+1 case for different values of $\Delta m_{41}^2$ and $|U_{e4}|^2$ (for $|U_{\mu 4}|^2$ and $|U_{\tau 4}|^2$ either zero or $10^{-3}$) when considering normal ordering for the active neutrinos. The black closed contours represent the 3$\sigma$ preferred regions and the green star the best-fit point from [22].
the centre of the darker region (smaller $N_{\text{eff}}$) is not located exactly at the centre of the triangle due to the different interactions and masses of the three active neutrinos. The thermalisation is more effective for larger mass splittings, because oscillations start and develop earlier [21].

In summary, our precise calculations show that, for the preferred region of active-sterile mixing parameters from short-baseline neutrino experiments, the fourth state is fully thermalised.

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