Influence of a strong electromagnetic wave (laser radiation) on the Hall effect in a cylindrical quantum wires with infinitely high potential

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Abstract. Based on the quantum kinetic equation for electrons, we theoretically study the influence of a Strong Electromagnetic Wave (EMW) on the Hall effect in a cylindrical quantum wire with infinitely high potential \( V(\vec{r}) = 0 \) inside the wire and \( V(\vec{r}) = \infty \) elsewhere subjected to a crossed dc electric field \( \vec{E}_1 = (0, 0, E_1) \) and magnetic field \( \vec{B} = (0, B, 0) \) in the presence of a strong EMW (laser radiation) characterized by electric field \( \vec{E} = (0, 0, E_0 \sin \Omega t) \) (where \( E_0 \) and \( \Omega \) are amplitude and frequency of EMW, respectively). We obtain the analytic expressions for the components \( \sigma_{zz} \) and \( \sigma_{xz} \) of Hall conductivity as well as Hall coefficient with the dependence on \( B, \Omega \). The results are numerically evaluated and graphed for GaAs/GaAsAl quantum wire to show clearly the dependence of Hall conductivity and Hall coefficient on above parameters.

1. Introduction
Recently, there has been considerable interest in the behavior of low dimensional systems, in particular of one dimensional electron gas systems such as cylindrical quantum wire, rectangular quantum wire the confinement of electrons in these systems considerably enhances the electron mobility and lead to their unusual behaviors under external stimuli. The Hall effect in bulk semiconductors under the influence of EMWs has been studied in much details [1-5]. In Refs. [1,2] the one magnetoresistance was calculated when the nonlinear semiconductors ares subjected to a magnetic field and an EMW with now frequency, the nonlinearity is resulted from the nonparabolicity of distribution functions of carriers. In Refs [3,4], the magnetoresistance was derived in the presence of a strong EMW for two cases: the magnetic field vector and the electric field vector of the EMW are perpendicular [3], and are parallel [4]. The existence of the odd magnetoresistance was explained by the influence of the strong EMWs on the probability of collision, i.e., the collision integral depends on the amplitude and frequency of the Electromagnetic Waves. This problem was also studied in the presence of both low and high frequency of the EMWs [5]. Throughout these problem, the quantum kinetic equation method have been seen as a powerful tool. In low dimensional semiconductors systems, the carrier confinement leads to unusual behaviors in comparison with bulk semiconductors under external stimuli. So, in recent works, we have used this method to study the influence of an intense EMW on the Hall effect in rectangular and parabolic quantum wells [6,7]. In one dimensional (1D) electron systems, the Hall effect has been studied in many aspects. However, most of
the previous works only considered the cases an EMW was absent and at temperatures that electron - electron and electron impurity interaction were dominant. In this work, we have used the quantum kinetic equation method to study the influence of a high frequency EMW on the Hall coefficient (HC) in quantum wires. The main purpose of this work is to make a comparison between our calculation and other experiments and theories. Specially we investigate the influence of an EMW on the effect by comparing dependencies of the magnetoresistance and the HC between the absence and the presence of an EMW. The paper is organized as follow. In the next section, we describe the simple model of a Cylindrical Quantum Wires and present briefly the basic formulas for the calculation. Numerical results and discussion are given in Sec. 3. Finally, remarks and conclusions are show briefly in Sec. 4.

2. Hall effect in a cylindrical quantum wires with infinitely high potential in the presence of a laser radiation

2.1. Hamiltonian for electron - phonon interacting system in a cylindrical quantum wires with Infinitely High Potential

We consider a cylindrical quantum wire of the radius $R$ and the length $L$ with the infinite confined potential: $V(\vec{r}) = 0$ inside the wire and $V(\vec{r}) = \infty$ elsewhere subjected to a crossed dc electric field $\vec{E}_1 = (0, 0, E_1)$ and magnetic field $\vec{B} = (0, B, 0)$ in the presence of a strong EMW (laser radiation) characterized by electric field $\vec{E} = (0, 0, E_0 \sin \Omega t)$. Hamiltonian for electron - phonon interacting system in external field can be written as:

$$H = \sum_{n,l,k} \varepsilon_{n,l}(\vec{k}) a^+_{n,l,k} a_{n,l,k} + \sum_{\vec{q}} \hbar \omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} + \sum_{\vec{q}} C_{n,l,n',l'}(\vec{q}) a^+_{n,l,k+\vec{q}} a_{n',l',k}(b_{\vec{q}} + b^+_{-\vec{q}}).$$

(1)

where $a^+_{n,l,k}$ and $a_{n,l,k}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (optical phonon); $\vec{k} = (0, 0, k_z)$ is the electron wave momentum (along the wire’s axis: z axis); $\vec{q}$s the phonon wave vector; $\omega_{\vec{q}}$are optical phonon frequency; $C_{n,l,n',l'}(\vec{q}) = C_{\vec{q}} I_{n,l,n',l'}(\vec{q})$ is the electron optical phonon interaction coefficient; $|C_{\vec{q}}|^2 = \frac{e^2 \hbar \omega_{\vec{q}}}{2V k_o q^2} (\frac{1}{\chi_0} - \frac{1}{\chi_{\infty}})$. Here $V, k_o, \chi_0, \chi_{\infty}$ are the volume, the electronic constant, the static dielectric constant, and the high frequency dielectric constant, respectively. The electron form factor $I_{n,l,n',l'}(\vec{q})$ can be written as: $I^2_{n,l,n',l'}(\vec{q}) = \langle n', l', \vec{k} | e^{i\vec{q}\vec{r}} | n, l, \vec{k} \rangle$; $\tilde{A}(t) = \frac{1}{\Omega} E_0 \cos (\Omega t)$ is the potential vector, depending on the external field. n (l) is azimuthal (radial) quantum number;

Energy Spectrum:

$$\varepsilon_{n,l}(k) = \frac{\hbar^2 k^2}{2m^*} + \hbar \omega_c(N_p + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}) - \frac{1}{2m^*} \left( \frac{eE_1}{\omega_c} \right)^2.$$  

(1')

2.2. Quantum kinetic equation for confined electrons in a cylindrical quantum wires with Infinitely High Potential

From Hamiltonian for electron - phonon interacting system in a cylindrical quantum wires with Infinitely High Potential. After several operator calculations, we obtain a set of coupled quantum
kinetic equations in which the equation for acoustic phonon can be written as:

\[
\frac{f_{n,l} - f_{n',l'}}{\tau_{n,l}} + \left( eE_1 + \omega_c \left[ \mathbf{k}_x A \right] \right) \frac{\partial f_{n,l}}{\partial \mathbf{k}_x} + \frac{\hbar \mathbf{k}_x}{m} \frac{\partial f_{n,l}}{\partial \mathbf{r}} = 2 \pi \sum_{n,l,n',l'} \frac{C_{n,l,n',l'}(\mathbf{q})}{\hbar \Omega^2} \frac{\lambda^2}{2} \times \sum_{\mathbf{k}} \left[ f_{n,l}(\mathbf{k} - \mathbf{q}) + f_{n',l'}(\mathbf{k}) \right] \delta \left( \varepsilon_{n',l'}(\mathbf{k}) - \varepsilon_{n,l}(\mathbf{k} - \mathbf{q}) - \hbar \omega_{\mathbf{q}} - \hbar \Omega \right),
\]

where \( \tau \) is the recovery time of the electron momentum, \( f_{n,l} \) is electron distribution, \( k_B \) is Boltzmann constant,

\[
f_{n,l} = f_o \exp\left(-\frac{1}{k_B T} \varepsilon_{n,l}(\mathbf{k})\right),
\]

\[
f_{n',l'} = f_o \exp\left(-\frac{1}{k_B T} \varepsilon_{n',l'}(\mathbf{k})\right).
\]

### 2.3. Analytical expressions for the Hall conductivity tensor

We obtain the expression for the conductivity tensor after some manipulation:

\[
\sigma_{ij} = \frac{ea \tau}{(1 + \omega_c^2 \tau^2)} \left( \delta_{ij} - \omega_c \tau \varepsilon_{ijkl} h_k + \omega_c^2 \tau^2 h_i h_j \right) + \frac{\Gamma \mathbf{q}}{m (1 + \omega_c^2 \tau^2)^2} \delta_{ij} + 3(\omega_c \tau^2 + \omega_c^2 \tau^4) h_i h_j - \omega_c \tau \varepsilon_{ijkl} h_k.
\]

In this calculation it is assumed \( \mathbf{E} \parallel Oz; \mathbf{E} = (0,0,E); \mathbf{B} \parallel Oy; \mathbf{B} = (0,B,0) \)

\[
\sigma_{zz} = \frac{\tau}{(1 + \omega_c^2 \tau^2)} \left[ e a + \frac{b}{m} \frac{\tau}{1 + \omega_c^2 \tau^2} \left( 1 - \omega_c^2 \tau^2 \right) \right],
\]

\[
\sigma_{xx} = \frac{-\omega_c \tau}{(1 + \omega_c^2 \tau^2)} \left[ e a + \frac{b}{m} \frac{\tau^2}{1 + \omega_c^2 \tau^2} \right],
\]

where

\[
a = \frac{L e \beta \hbar}{2 \pi m^2} \frac{\tau_0}{1 + \omega_c^2 \tau_0^2} \exp \left\{ \beta \left[ \varepsilon_F - \hbar \omega_c (N_p + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}) + \frac{e^2 E_1^2}{2m \omega_c^2} \right] \right\} \left( \frac{2m}{\beta \hbar^2} \right)^{3/2} \sqrt{\pi},
\]

\[
b = \sum_{n,l,n',l'} |C_q|^2 |J_{n,l,n',l'}(q)|^2 \frac{\lambda^2}{4 \Omega^2} \Gamma_q,
\]

\[
\Gamma_q = f_o \frac{L m^*}{2 \pi \hbar^2 q^2} \exp \left\{ -\frac{\beta m^*}{2 \hbar^2 q^2} A^2 - \beta A_2 \right\} \left( \exp \left[ \beta h (\omega_q + \Omega) \right] - 1 \right).
\]

And the coefficient Hall is:

\[
R_H = -\frac{1}{B} \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{zz}^2},
\]

where \( \sigma_{zz}; \sigma_{xx} \) are given by Eq. (3)

\[ L \] is length quantum wire , \( \rho \) is the density of quantum wire, with

\[ A = \frac{h \omega_c}{2m^*} - \hbar \omega_c (n + \frac{\nu}{2} + l + \frac{1}{2}) + \hbar \omega q + \hbar \Omega, \]

\[ A_2 = \hbar \omega_c (N_p + \frac{n + l}{2} + \frac{1}{2}) - \frac{1}{2m} \left( \frac{eE_1}{\omega_c} \right)^2. \]
3. Numerical results and discussions

3.1. The dependence of the Hall coefficient with frequency electromagnetic wave

Survey dependence of the Hall coefficient with frequency electromagnetic waves when the magnetic field changes: $B = 4T; B = 4.4T; B = 4.6T$, length quantum wires $L = 90.10^{-8}m$, $n = 1, n' = 1, l = 1, l' = 1$, radius of the quantum wire $r = 8.77.10^{-9}m$. In Figure 1, we show the dependence of the Hall coefficient with frequency electromagnetic: Initially, as the frequency rises, the Hall coefficient increase before reaching a peak at certain frequency. Then, it falls sharply. And if the electromagnetic wave frequency keep increasing, the Hall coefficient will remain constant. At these values from different schools, different shape figures. There is no difference between the maximum value of the Hall coefficients.

![Figure 1. The dependence of the Hall coefficient with frequency electromagnetic wave.](image1)

![Figure 2. The dependence of the Hall coefficient under a magnetic field $\vec{B}$](image2)

3.2. The dependence of the Hall coefficient under a magnetic field $\vec{B}$

Survey dependence of the Hall coefficient on the magnetic field of the system when the temperature changes: $T = 100K, T = 200K, T = 300K$; length quantum wires $L = 90.10^{-8}m$, $n = 1, n' = 1, l = 1, l' = 1$, radius of the quantum wire $r = 8.77.10^{-9}m$. In Figure 2 shows the dependence of the Hall coefficient under a magnetic field $\vec{B}$. Initially, the Hall coefficient has the saturation value. When the magnetic field increases, it decreases. At the low temperatures, the Hall coefficient decreased more rapidly. At a value of the magnetic field ($B = 9T$), the Hall coefficient values corresponding to the low temperature ($100K$) is smaller than those corresponding to the high temperature ($300K$).

4. Conclusion

In this work, we have studied the influence of laser radiation on the Hall effect in a cylindrical quantum wires with infinitely High potential subjected to a crossed dc electric and magnetic fields. The electron optical phonon interaction is taken into account. We obtain the expressions of Hall conductivity as well as Hall coefficient. The analytical results are numerically evaluated and plotted for a specific quantum wires GaAs/AlGaAs to show clearly the dependence of Hall conductivity on the magnetic field, when the magnetic field is small, the Hall conductivity has saturation value. When the magnetic field continues to increase, the Hall conductivity will decrease. The dependence of Hall conductivity with the frequency of electromagnetic waves: Initially, as the frequency rises, the Hall coefficient increase before reaching a peak at certain frequency. Then, it falls sharply, and then fell sharply. The most important thing is the
appearance of the maximum peaks satisfying the (magnetophonon resonance) MPR condition optically detected MPR condition.

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