Gauge Threshold Corrections and Field Redefinitions

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Abstract

We review the argument for field redefinitions arising from threshold corrections to heterotic string gauge couplings, and the relation between the linear and the chiral multiplet. In the type IIB case we argue that the necessity for moduli mixing at one loop order has not been clearly established, since this is based on extending the background field expansion way beyond its regime of validity. We also resolve some issues related to the form of non-perturbative terms resulting from gaugino condensation. This enables us to estimate the effective cutoff in the field theory by evaluating the non-perturbative superpotential by two different methods, and find that it is around the Kaluza-Klein scale, as one might have expected on general grounds of self-consistency.

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1 Introduction

Background field methods in string theory (see for example [1], [2], [3]) are different from what one encounters in say, the quantum mechanical study of the behavior of an atom in an external magnetic field. In the latter case the background magnetic field is truly external to the system under study. In field theory applied say to condensed matter physics or atomic physics, where one is studying not the theory of the entire universe but some system within it, the concept of an external background makes sense. However when one studies theories such as the standard model coupled to gravity, which purports to be an effective theory of the entire universe, strictly speaking there is no meaning to the concept of an external background.

Of course the standard model is usually formulated in a particular metric background - namely the flat one. Here the reasoning is that for small standard model field energy densities, the Einstein equations are solved by the Minkowski metric. In principle it can be studied in a different metric background for example a cosmological background (FRW, deSitter etc.). However this smooth gravitational background field assumption is certainly expected to break down close to the Planck scale ($M_P \equiv 1/\sqrt{8\pi G_{\text{Newton}}}$). At such high energies one expects a significant contribution from virtual quantum gravity processes (such as the creation and annihilation of blackholes, wormholes etc.) and the entire framework will break down.

String theory on the other hand is supposed to be an UV completion of field theory (or at least a class of field theories hopefully including the standard model). The theory is not supposed to have any free parameters and is defined purely in terms of a fundamental dimensional constant - the string scale $l_{\text{string}} \equiv \sqrt{2\pi\alpha'} = 1/M_{\text{string}}$. If string theory were four dimensional, $M_{\text{string}}$ would essentially be the same as $M_P$. However in all tractable string theoretic constructions, there is an internal six-dimensional space with a volume which is typically large compared to $l_{\text{string}}^6$, i.e. $\text{Vol} = V l_{\text{string}}^6$ often with $V \gg 1$. In this case there is a significant difference between the two scales and $M_S \simeq M_P/\sqrt{V} \ll M_P$. There is also an additional scale, the Kaluza-Klein scale $M_{KK} = M_{\text{string}}/\sqrt{V}^{1/6} = M_P/\sqrt{V}^{2/3}$. For large $V$ we thus have a hierarchy of scales $M_{KK} \ll M_{\text{string}} \ll M_P$. Four dimensional field theory is strictly valid only below $M_{KK}$. Above this scale the theory is essentially ten dimensional but remains a field theory. However above the scale $M_{\text{string}}$, the theory cannot be described by point like field theoretic degrees of freedom. The field theoretic description necessarily breaks down.

Consider first the case of strings propagating in a general metric background. In this case the world sheet theory is formulated as a generalized two dimensional sigma model, and one derives consistency conditions (beta function equations) for the propagation of strings, in an expansion in the squared string length scale - the so-called $\alpha'$ expansion. For energy scales that are well below the string scale, this is a valid expansion and one can get useful information about the low energy limit of string theory in this way. However this expansion obviously breaks down at the string scale. In fact this is highlighted by the fact that the derivative expansion (as with generic higher derivative theories) has ghosts. These however appear at the string scale and are merely a sign that the theory has been pushed beyond its regime of validity. Thus any argument that is made about the interaction vertices of the theory - that is derived from the $\alpha'$ expansion - is invalid when the momentum flowing through those vertices is greater than the string scale. At such energies the low energy point field theory needs to be replaced by string (field?) theory.

The same is true for open string background calculations. Here one turns on a gauge field
strength (magnetic or electric) background that is slowly varying (if not constant), to derive a
low energy effective action in an \( \alpha' \) expansion. Much important work has been done by using this
technique. However for the most part this work has been used only to get an effective field theory
valid below the string scale. In particular it does not make sense to consider the behavior of a term
like \( \frac{1}{g^2(\mu^2)} \text{tr}(F_{\mu\nu}F^{\mu\nu}) \) when the momentum flowing through this operator is greater than the string
scale. The representation in terms of such a local operator simply breaks down at these energies.
In particular this means that any argument which purports to have a low (around say few TeV)
string scale and gauge coupling unification at some much higher scale (such as the standard GUT
scale of \( 10^{16} \text{GeV} \)), cannot possibly make sense.

In this work we will address the question of string threshold corrections to gauge couplings by
first reviewing the literature. The issue came up with Kaplunovsky’s calculation of these effects
in the context of the heterotic string [4]. The question that arose was how to account for moduli
dependent corrections that could not be written as harmonic functions of the moduli as would be
required in the usual chiral field formulation of the effective supergravity coming from string theory.
The resolution in terms of the linear and chiral multiplet duality was discussed in [5][6] (BGG) and
is reviewed in section two, where we also point out that the argument fails when the modulus in
question has a non-linear superpotential term in the relevant chiral superfield. In the next section
we review the arguments of Kaplunovsky and Louis which compared their field theoretic formula
for the gauge coupling function with the corresponding string theory calculation. It should be
stressed that this only involved momentum scales below the string scale (as we will make clear
below). By contrast the work of references [7, 8, 9], as well as the earlier work of [10, 11, 12]
in connection with low scale strings, is essentially based on using the effective field theory and
background string theory approaches well above the string scale. The point is that in identifying
the infra-red region of the string theory integrand the upper (UV) cut off is taken beyond the
string scale. This is essentially the meaning of taking the infra-red region of the one-loop string
integral all the way up to and beyond the string scale. The argument for the necessity of certain
field redefinitions that result from this comparison are then very sensitive to exactly where the
cutoffs are located, and can be changed by appropriately choosing the cutoffs at scales below the
string scale. Finally we resolve a long standing puzzle regarding two different derivations of the
non-perturbative (gaugino condensate) term in the superpotential.

2 General framework

2.1 Linear -chiral duality with \( \partial S W = 0 \).

As mentioned earlier the linear multiplet - chiral multiple duality has been discussed for example in [3] and [4](BGG). We essentially follow the discussion of BGG except that the Kaehler supergravity
framework is replaced by the standard (minimal) one. We begin with the following action (with \( \kappa = M_P^{-1} = 1, d^8z = d^4xd^4\theta, d^6z = d^4xd^2\theta \)) for chiral superfields \( \Phi \) (having superpotential \( W \) and
Kaehler potential \( K \)) coupled to supergravity and gauge fields with prepotential \( V \) and gauge field
strength $\mathcal{W}$.

\[ \mathcal{A} = -3 \int d^8z E \exp[- \frac{1}{3} K(\Phi, \bar{\Phi}; V)] \]

\[ + \left( \int d^8z \frac{E}{2R} [W(\Phi) + \frac{1}{4} f(\Phi) \mathcal{W}] + h.c. \right). \]

Here $E$ is the full superspace superdeterminant and $R$ is the chiral superspace curvature. Note that the gauge coupling function $f$ is in general a holomorphic gauge invariant function of the chiral superfields $\Phi$. However threshold effects in string theory appeared to give non-holomorphic moduli dependent corrections to the gauge coupling function. The resolution lay in the introduction of the linear multiplet formulation of the gauge coupling function. The key observation here is that string theory moduli and the dilaton naturally arise in the string theory context as (components of) linear multiplets. This is because axionic partners of the scalar moduli are in fact second rank tensor fields. Thus for instance the axio-dilaton $S$, which is often identified in 4D as a chiral scalar, has its origins in a multiplet containing an antisymmetric second rank tensor $b_{\mu \nu}$, and thus naturally belongs to a linear multiplet.

Let us first focus on this case where in the usual formulation the gauge coupling function is given by $f = kS$ where $S$ is the dilaton chiral superfield i.e. $\bar{\nabla}^\alpha S = 0$. Let $\{ \phi \}$ denote all the other chiral superfields in the theory. We take (for the moment) the superpotential $W$ to be independent of $S$ as is the case in perturbative string theory (except in IIB where it can be linear in $S$ in the presence of internal fluxes). Let $U$ be an unconstrained real superfield and modify the above action to the following form (for simplicity we ignore chiral fields which are charged under the gauge group),

\[ \mathcal{A} = -3 \int d^8z E \exp[- \frac{1}{3} K(\phi, \bar{\phi}, U)](F(\phi, \bar{\phi}, U) + U(S + \bar{S})) \]

\[ + \left( \int d^8z \frac{E}{2R} [W(\phi) + \frac{1}{4} kS \mathcal{W}] + h.c. \right). \]

Here a trace over the gauge group is implicit in the gauge kinetic term, $F$ is a real function of the chiral fields $\phi$ and the real field $U$, which will be determined by a normalization condition below. $U$ is introduced as the linear superspace dual of the chiral superfield $S$ (for more details see [3] [4]) and as we will see this field becomes useful in interpreting threshold corrections in string theory. Note that we have also modified the Kähler potential to include dependence on $U$. Now we may eliminate the chiral superfield $S$ in favor of the real superfield $U$, by using the equation of motion coming from taking the $\delta S$ variation of this action to get;\footnote{In taking a variation w.r.t. a chiral field we need to set $\delta S = -\frac{1}{4} \bar{\nabla}^2 + 2R)\delta \Sigma$ where $\Sigma$ is an unconstrained superfield.}

\[ -3(-\frac{1}{4} \bar{\nabla}^2 + 2R)(U e^{-K/3}) + \frac{k}{4} \mathcal{W}^2 = 0. \]

Note that this equation and its conjugate are now effectively constraints on the initially unconstrained superfield $U$. In the absence of the gauge field kinetic term and $K$ this is essentially the linear superfield constraint. Here we have a modified linear superfield.
Substituting (3) into (2) we get the (modified) linear multiplet formulation of the above action

\[ A_{LMF} = -3 \int d^8z E e^{-K(\phi, \bar{\phi}, U)/3} F(\phi, \bar{\phi}, U) \]

\[ + (\int d^8z \frac{E}{2R} W(\phi) + h.c.) \]  

(4)

Note that there is no explicit gauge kinetic term in this form of the action. It is however implicit because \( U \) satisfies the constraint (3). To see this we first rewrite this as an equation for \(-\frac{1}{4} \bar{\nabla}^2 U\) to get (keeping only terms which contain \( \mathcal{W}^2 \))

\[ -\frac{1}{4} \bar{\nabla}^2 U = \frac{e^{K/3}}{1 - \frac{1}{3} UKU} \frac{k}{3} \mathcal{W}^2 + \ldots \]  

(5)

Then we have from the first line of (1), using \( \int d^8z E \bar{\nabla}\dot{\alpha} v = 0 \) and the above result,

\[ -3 \int d^8z \frac{E}{2R}(-\frac{1}{4} \bar{\nabla}^2 + 2R) (e^{-K/3} F(\phi, \bar{\phi}, U)) = \]

\[ -3 \int d^8z \frac{E}{2R} e^{-K/3} (-\frac{KU}{3} F + F_U)(\frac{1}{4} \bar{\nabla}^2 U) + \ldots = - \int d^8z \frac{E}{2R} \frac{k}{4} \Gamma(U, \phi, \bar{\phi}) \mathcal{W} \mathcal{W} + \ldots, \]

so that the effective gauge coupling function in the linear multiplet formulation is (the lowest component of)

\[ \Gamma = -F_U + \frac{N}{3} K_U, \quad N \equiv \frac{F - UF_U}{1 - \frac{1}{3} UKU} \]  

(6)

in agreement with BGG.

Now let us see what we would get in the chiral field formulation of the gauge coupling function. Varying the action (2) with respect to \( U \) we get

\[ (S + \bar{S})(1 - \frac{1}{3} UKU) = \frac{1}{3} FKU - F_U. \]  

(7)

This equation determines \( U = U(S + \bar{S}, \phi, \bar{\phi}) \). In the chiral field formulation we need to have in addition the normalization condition

\[ F(\phi, \bar{\phi}, U) + U(S + \bar{S}) = 1. \]  

(8)

This is just the condition that in the chiral formulation the pre-factor of \( e^{-K/3} \) should be unity. Substituting into (2) gives the chiral field formulation with the standard chiral gauge coupling function i.e. (1). Note also that eliminating \((S + \bar{S})\) between the last two displayed equations gives the relation

\[ 1 - \frac{1}{3} UKU = F - UF_U \]  

(9)

so that the function \( N = 1 \) (see (6)) and the expression for \( \Gamma \) becomes

\[ \Gamma = -F_U + \frac{1}{3} K_U. \]  

(10)
Equation (9) can be rewritten as a differential equation for \( F \),
\[-U^2 \frac{d}{dU}(U^{-1} F) = 1 - \frac{1}{3} UK_U.\]
This has the solution
\[ F = 1 + \frac{U}{3} \int U \frac{dU'}{U'} K_{U'} + V(\phi, \bar{\phi})U. \] (11)
A simple example that is relevant to string theory is obtained by putting
\[ K = \hat{K}(\phi, \bar{\phi}) + \alpha \ln U. \] (12)
In this case \( F = 1 - \frac{\alpha}{3} + VU \) and \( \frac{2}{3}U^{-1} = S + \bar{S} + V(\phi, \bar{\phi}) \). Then in the chiral multiplet formulation we have the action (11) with \( f = kS \) and
\[ K = \hat{K}(\phi, \bar{\phi}) - \alpha \ln(S + \bar{S} + V(\phi, \bar{\phi})). \] (13)
In the string theory case where \( S \) is the four dimensional dilaton chiral superfield \( \alpha = 1 \) and \( V \) is a one-loop effect.

On the other hand we have in the linear multiplet formulation the action (4) with Kähler potential (12), the gauge coupling (see (52)) given by (putting \( \alpha = 1 \))
\[ \Gamma = -\frac{1}{3U} + V(\phi, \bar{\phi}). \] (14)
The moral of this story is that if one wants to accommodate a non-harmonic gauge coupling function (as in the above relation) one must choose the LM formulation. On the other hand this is equivalent to taking the chiral multiplet dual field \( S \) as the gauge coupling function in the CM formulation but with the non-harmonic part \( V \) included in the Kähler potential for \( S \).

### 2.2 \( \partial_S W \neq 0 \)

The discussion in the previous subsection remains valid if the superpotential dependence on \( S \) is no more than linear. This is the case for instance in type IIB string theory, provided the pre-factors of the non-perturbative terms responsible for breaking the no-scale structure and stabilizing the Kaehler moduli are independent of \( S \) (or at least no more than linear in \( S \)). Thus under these usual assumptions one has
\[ W = A(\phi) + B(\phi)S, \] (15)
where as in the last subsection \( \phi \) stands for all the other chiral scalar superfields. In this case the only relevant change is in equation (3), which is replaced by
\[-3(-\frac{1}{4} \nabla^2 + 2R)(Ue^{-K/3}) + \frac{k}{4} \mathcal{W}^2 + B(\phi) = 0. \]
Equation (4) will remain the same with \( W \rightarrow A(\phi) \) but on evaluating the components of the first term we would need to replace \( k\mathcal{W}^2 \rightarrow k\mathcal{W}^2 + B(\phi) \). The main results of the previous subsection remain unchanged.
However when the superpotential is non-linearly dependent on $S$ these results cannot be obtained. The reason is that the $\delta S$ equation no longer gives a constraint on the unconstrained superfield $U$. Instead it becomes an equation which expresses the chiral superfield $S$ in terms of a chiral projection of an unconstrained superfield. Defining $P = (-\frac{1}{4}\nabla^2 + 2R)$ equation (13) is replaced by $(\partial_S W \equiv W_S)$,

$$-3P(Ue^{-K/3}) + \frac{k}{4} \mathcal{W}^2 + W_S = 0.$$  

(16)

Thus we need to replace $S$ in equation (2) by

$$S = W_S^{-1}(3P(Ue^{-K/3}) - \frac{k}{4} \mathcal{W}^2).$$  

(17)

Note that in the above $W_S^{-1}$ is the functional inverse of $W_S$.

Substituting (16) into (2) (with $W(\phi) \to W(\phi) + \Delta W(S)$) we have for the chiral superspace terms

$$\int d^8z \frac{E}{2R} [W(\phi) + (\Delta W(S) - S\partial_S \Delta W)] + h.c.$$  

(18)

where $S$ is now given as a function of $U$ by (17). Clearly gauge kinetic terms are hidden in this expression, but the coupling functions are essentially power series in $P(Ue^{-K/3})$ which can always be rewritten as a chiral projection of some redefined unconstrained real field $\Omega$ i.e. as $P\Omega$. For instance in the simplest case where $\Delta W = S^2/2$ the expression (18) becomes

$$\int d^8z \frac{E}{2R} [W(\phi) - \frac{1}{2}(3P(Ue^{-K/3}) - \frac{k}{4} \mathcal{W}^2)^2] + h.c. \sim$$

$$\int d^8z \frac{E}{2R} [W(\phi) - (3P(Ue^{-K/3})\frac{k}{4} \mathcal{W}^2)] + h.c.$$

so that the gauge coupling function is $3P(Ue^{-K/3})$. As can be easily seen this is the case whatever the functional form $\Delta W$ takes, since this relates to the coefficient of the leading term in the expansion in $\mathcal{W}^2$. This is obviously of the same form as the original chiral field representation in terms of $S$ for the gauge coupling function, and is very different from the linear multiplet formulation where it was precisely the constraint on $U$ that enabled us to write a non-holomorphic gauge coupling.

What this illustrates is the general expectation that the dualization makes sense only when there when the dualized field pair represents a massless state. The linear superpotential case studied at the beginning of this subsection is thus an exception.

2.3 Generalization

The discussion in the previous two subsections can be easily generalized to the case when there is a duality relation between several chiral multiplets $S^i, i = 0, \ldots, N$ and linear multiplets $U_i$. For future reference in the type IIB string theory case we rewrite these fields as $S \equiv S^0$ and $T^i \equiv S^i, i = 1, \ldots, N$. We also choose to dualize only the $T$'s and write the dual fields as $U_i, i = 1, \ldots, N$. The gauge field kinetic term (in the chiral formulation) is now taken to be $f_a \mathcal{W}_a^2$ where a sum over the different gauge groups is implied, and

$$f_a = k_a S + \alpha_{ai} T^i$$
One follows the same steps from eqn (4) through (8) replacing the appropriate products by dot products etc. to get after dualization of the $T'$s the gauge coupling function,

$$k_aS + \Gamma_a, \Gamma_a = \Gamma^i \alpha_{ai}, \Gamma^i = -F_{U_i} + \frac{1}{3} K_{U_i}. \quad (19)$$

Note that we’ve set the normalization function

$$N \equiv \frac{F - U_i F_{U_i}}{1 - \frac{1}{3} U_i K_{U_i}} = 1, \quad (20)$$

in the above to get as before the standard SUGRA frame. Also the relation between the dualized variables is now given by

$$T^i + \bar{T}^i = \frac{1}{3} K_{U_i} - F_{U_i} = \Gamma^i(U, S, \phi, \bar{\phi}) \quad (21)$$

Given a model for $K$ as a function of $U^i$ the differential equation (20) can be solved. For instance in the case of type IIB string theory on a Calabi-Yau the (scalar components of) the linear multiplet variables are related to the two cycle volumes $v_i$ by $U_i = v_i/V$ \[13\] and

$$K = -\ln(S + \bar{S}) - \hat{K}(z, \bar{z}) - 2 \ln V, \quad (22)$$

$$= -\ln(S + \bar{S}) - \hat{K}(z, \bar{z}) + \ln \hat{V}. \quad (23)$$

Here $V = \frac{1}{6} \int J^3 = \frac{1}{6} c^{ijk} v_i v_j v_k$, $\hat{V} = 1/\sqrt{2} = \frac{1}{6} c^{ijk} U_i U_j U_k$, with the indices running over $1, \ldots, h^{(1,1)}_+$. i.e. the Kähler moduli that survive the orientifold projection, $z$ stands for the set of complex structure moduli, and $S = e^{-\varphi} + i C_0$, where $\varphi$ is the dilaton and $C_0$ is the RR zero-form. We also have in this case $F = U_i V^i(\phi, \bar{\phi})$ where $V^i$ are a set of arbitrary real functions and $\Phi = \{z, S\}$. It is important for future reference to note that these arbitrary functions are independent of the Kähler moduli. Using these we have from (21)

$$T^i + \bar{T}^i = \frac{2}{3} \tau^i - V^i(z, \bar{z}; S, \bar{S}) = \Gamma^i(U, S, \bar{S}; z, \bar{z}), \quad (24)$$

where $\tau^i = c^{ijk} v_j v_k = c^{ijk} U_i U_j U_k/\hat{V}$ is the volume of the $i$th 4-cycle.

The results of this subsection are summarized in the following table.

| | chiral | linear |
|---|---|---|
| $K$ | $-2 \ln V(T + \bar{T} + V)$ | $\ln V$ |
| $2 \frac{1}{6}$ | $k_a(S + \bar{S}) + \alpha_{ai}(T^i + \bar{T}^i)$ | $k_a(S + \bar{S}) + \alpha_{ai}(\frac{2}{3} c^{ijk} U_i U_j U_k/\hat{V} - V^i)$ |

It is important to note that the functions $V^i$ do not depend on the chiral moduli $T$ which have been dualized. Also even though these functions are arbitrary at the level of these duality transformation, the usefulness of these results lies in the fact that such functions may arise in string perturbation theory. In this case of course (at one loop level) they are expected to be independent of $S$.

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\[\text{We ignore the additional dependence on moduli coming from the two form fields.}\]
2.4 Relation to KL formula

What is the relation if any to the KL formula of the above?

The KL formula \[14\] for the gauge coupling constant for a (simple or \(U(1)\) gauge group \(G_a\)) is

\[
\frac{1}{g_a^2(\Phi, \bar{\Phi}; \mu^2)} = \Re f_a(\Phi) + \frac{b_a}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} + \frac{c_a}{16\pi^2} K(\Phi, \bar{\Phi}) + \frac{T(G_a)}{8\pi^2} \ln \frac{1}{g_a^2(\Phi, \bar{\Phi}; \mu^2)} - \frac{1}{8\pi^2} \sum_r T_a(r) \ln \det Z_{(r)}(\Phi, \bar{\Phi}; g_a^2(\mu^2)).
\]

(25)

Here \(T_a(r) = \text{Tr}_{(r)} T_a^2, T(G_a) = T_a(r = \text{adjoint}),\)

\[
b_a = \sum_r n_r T_a(r) - 3T(G_a),
\]

(26)

\[
c_a = \sum_r n_r T_a(r) - T(G_a).
\]

(27)

\(\Phi\) stands for a set of neutral fields that will be identified with string theory moduli \((M)\) and axio-dilaton \((S)\). The total Kaehler potential has been expanded in powers of the charged matter chiral fields \(Q\) which also defines the metric \(Z_{(r)}\) of their kinetic terms in the representation \(r\) of the gauge group, i.e.

\[
K(\Phi, \bar{\Phi}; Q, \bar{Q}) = \kappa^{-2} K(\Phi, \bar{\Phi}) + \sum_r Z_{(r)IJ}(\Phi, \bar{\Phi}) \bar{Q}_r^I e^{2V} Q_r^J + \ldots.
\]

(28)

The above formula was derived within an effective supergravity context on the understanding that at some scale above the effective cutoff \((\Lambda)\), this field theory would have to be replaced by its ultra-violet completion. Here the latter will be assumed to be string theory.

Several comments about the formula (25) are in order here. The first term on the RHS is the (real part of the) holomorphic gauge coupling of the theory defined in (1). Its functional form may be read off from the relevant string theory whose low-energy effective action is being studied. It will also include the holomorphic corrections coming from integrating out massive string states. The second term is the usual field theory running from the cutoff scale down to the scale \(\mu\). The fourth term comes from the rescaling anomaly that comes when the gauge field prepotential is redefined so as to get (from the standard superspace form in (1)) to the canonically normalized form of the gauge/gaugino field kinetic terms [15]. The fifth term comes from the Konishi anomaly whose origin is in the the rescaling necessary to get the canonical kinetic terms for the chiral scalar/fermion fields. These all occur already in global SUSY and together constitute the (integrated form of) the NSVZ beta function equation. In particular they have nothing to do with rescaling the (super) metric and only involve rescaling the fields \(Q\) and \(V\). By contrast the third term comes from the anomaly that occurs when performing the Weyl transformations (on the supermetric) that are necessary to go to the Einstein-Kähler frame starting from the superspace frame of (1).

However we have derived the relation between the linear multiplet and formulations entirely within the context of the original SUGRA frame (i.e. where the Einstein term occurs with the factor \(\exp(-K/3))\). This is in contrast to the formulation in Kähler superspace where the formulation is...
directly in the Einstein frame. However the latter formulation has an extended symmetry including a $U(1)_A$ which is quantum mechanically anomalous. This anomaly of course is the source of the last three terms in (25). In any case the point is that the linear multiplet chiral multiplet relation discussed in the previous subsections have nothing to do with these terms, which come from transforming from the SUGRA frame to the Einstein frame (and getting to canonically normalized matter and gauge kinetic terms).

3 String theory considerations

The relation between string theory calculations, and the KL formula was discussed at length in [16]. The string theory formula takes the general form,

$$g_a^{-2}(\mu^2; S, \bar{S}, M, \bar{M}) = k_a g_{\text{string}}^{-2}(S, \bar{S}, M, \bar{M}) + \frac{b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{\mu^2} + \frac{\Delta_a(M, \bar{M})}{16\pi^2}. \tag{29}$$

The term multiplying $k_a$ here is (the inverse square of) the gauge coupling at the string scale and is a sum of the classical term and a universal threshold correction $\Delta_{\text{univ}}$. $M$ represents all the moduli. The last term is a possible non-universal threshold correction so that the total threshold correction is of the form:

$$\tilde{\Delta}_a = \Delta_a + k_a \Delta_{\text{univ}}. \tag{30}$$

The main claim of [16] is that the difference between $\tilde{\Delta}$ and the non-holomorphic terms in the KL formula, should be a holomorphic one loop correction to the gauge coupling. Thus the key relation between the two is

$$\partial_M \partial_{\bar{M}} \tilde{\Delta}_a = \partial_M \partial_{\bar{M}} [c_a \tilde{K}(M, \bar{M}) - \sum_r T_a(r) \ln \det Z(r)(M, \bar{M})]. \tag{31}$$

While much of the discussion concerns heterotic string theory the above is based on general arguments that apply to all string theories (or for that matter to any UV completion of field theory with heavy states). As pointed out in the introduction to this paper, “the non-harmonicity (of the physical couplings) is a purely low-energy effect and can be calculated from the low-energy EQFT (effective quantum field theory) without any knowledge of the superheavy particles; it is the harmonic terms in the moduli-dependent effective gauge couplings that are sensitive to the physics at the high-energy threshold. Such terms can always be interpreted as threshold corrections to the Wilsonian gauge couplings $f_a(\Phi)$,...”

The intuition behind this statement is that once the string coupling is identified in terms of low energy fields the non-universal terms in gauge couplings coming from integrating out high energy fields should preserve the structure of SUGRA. Kaplunovsky and Louis go on to check this in explicit heterotic string theory examples. If this is violated in type I/IIB theories then one would need to understand why this intuition is violated.

3.1 Heterotic case

The expression (25) is expected to be valid to all orders in perturbation theory (at least in some renormalization scheme) but we will focus only on the one loop result. The holomorphic gauge
The first term is the classical (universal) gauge coupling and the second is the holomorphic one-loop correction. Of course $f$ receives no further corrections. The other terms are already explicitly of at least one-loop order so that $K, g^2_a, Z$ can be replaced by their classical values. Thus we put

$$K(\Phi, \bar{\Phi}) = -\ln(S + \bar{S}) + \hat{K}(M, \bar{M}) + O\left(\frac{1}{16\pi^2} S\right),$$

$$Z_{IJ}(\Phi, \bar{\Phi}) = Z^{(0)}_{IJ}(M, \bar{M}) + O\left(\frac{1}{16\pi^2} S\right),$$

$$\frac{1}{g^2(\mu^2)} = \Re S + O\left(\frac{1}{16\pi^2}\right).$$

It is important to note that in the heterotic case the classical 4 dimensional string coupling is defined as

$$\frac{1}{g_{\text{string}}^2} = \Re S = e^{-2\phi} \mathcal{V}$$

where $\mathcal{V}$ is the volume of the internal space in string units and $e^{\phi}$ is the 10 dimensional string coupling. Then we get from (25)

$$\frac{1}{g_a^2(\Phi, \bar{\Phi}; \mu^2)} = k_a \Re S + \frac{b_a}{16\pi^2} (\ln \Lambda^2 - \ln \Re S)$$

$$+ \frac{1}{16\pi^2} [\Re f_a^{(1)}(M) + c_a \hat{K}(M, \bar{M}) - \sum_r 2T_a(r) \ln \det Z_{(r)}^{(0)}(M, \bar{M})].$$

This is to be compared with the string 1 loop calculation $^{(29)}(29)$ where

$$\frac{1}{g_{\text{string}}^2(\Phi, \bar{\Phi})} = \Re S + \frac{\Delta_{\text{univ}}(M, M)}{16\pi^2}$$

Now in the Heterotic string $M_{\text{string}}^2 = \frac{M^2_P}{\Re S}$ up to $S, M$ independent constants. So it appears that the $S$ dependence of the two formulae $^{(33)}(29)$ agrees. It should be stressed here that although in $^{16}$ the cut-off $\Lambda$ is identified with $M_P$ this is merely a matter of convenience, and as the authors observed all that is required is that in the field theory expression the cutoff $\Lambda$ should be chosen in an $S, M$ independent manner.

As is well known the natural superspace variable that corresponds to the axio-dilaton of string theory is the linear multiplet (since the four dimensional axion is originally a two-form field). Suppose for the moment we ignore the (non-universal) second line $^{(33)}(29)$ as well as the last term on the RHS in $^{(29)}$. As discussed in the previous section, at least as long as we do not include a superpotential term for the chiral superfield $S$ in the chiral multiplet form (CMF) action$^4$ the

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$^3$See also $^{17}$ where an additional universal contribution is identified.

$^4$Actually as observed in subsection $^{2.2}$ one could have a linear term - but in the heterotic case there is no such term. The only possible dilaton superpotential comes from gaugino condensation and is a sum of exponentials in $S$. 

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linear multiplet form is equivalent to the chiral multiplet form. According to the discussion there, the gauge coupling function (at $\mu^2 = M_{\text{string}}^2$) is to be interpreted initially in the LMF, i.e. the model [12] [14]. Here $U$ should correspond to the dilaton in the linear multiplet formulation which to zero loop order can be identified as $-(3\Re S)^{-1}$. The term $V$ in (14) is identified with $\frac{\Delta_{\text{univ}}(M,\bar{M})}{16\pi^2}$. In the corresponding chiral multiplet formulation (in terms of $S$) the instruction implied by this is in effect to drop this term from the expression for the coupling function (thus identifying at the high scale $g_a^{-2} = k_a\Re S$) but include it as a one-loop correction to $K$ (see (13)) with

$$K = -\ln(S + \bar{S} + \frac{\Delta_{\text{univ}}(M,\bar{M})}{16\pi^2}) + \tilde{K}(M,\bar{M}).$$  \hspace{1cm} (35)

Note that if one makes the above correction to $K$ and then plugs that back into the CMF form of the action (11), then all that changes in the KL formula (25) is the explicit expression for the term $\frac{\Delta_{\text{univ}}(M,\bar{M})}{16\pi^2}$ times the RHS of (35). However in (25) this is already a one-loop effect so this one-loop change in $K$ is not going to affect the KL formula to one loop.

### 3.2 Type IIB case

Again we start with the 1-loop form of the KL formula (25) but now we write

$$K^{(0)} = -2\ln\mathcal{V} + \tilde{K}((S,\bar{S};z,\bar{z})).$$  \hspace{1cm} (36)

We have separated the Kaehler moduli dependence (through the internal volume $\mathcal{V}$) from the complex structure ($z$) dependence. Note that here in contrast to the heterotic case the appropriate axio-dilaton field in the orientifolded type IIB case is

$$S = e^{-\phi} + ia$$

where $e^\phi$ is the 10 dimensional string coupling, and $a$ is the RR zero form field (see for example [3]). As in the LVS models discussed in [18] [19] it is assumed that the MSSM is located on D3 branes at a singularity (or D7 branes wrapping a collapsing cycle). For these local models (or in fact for any no-scale like embedding of the matter sector valid even in the heterotic case) one can write $Z_{IJ}^{(0)} = \frac{1}{\sqrt{\mathcal{V}}}\delta_{IJ}$ to leading order in the large volume expansion.

Then the one-loop coupling function becomes

$$\frac{1}{g_a^2(\Phi,\bar{\Phi};\mu^2)} = \Re f_a(\Phi) + \frac{b_a}{16\pi^2} \ln \frac{\Lambda^2\mathcal{V}^{-2/3}}{\mu^2} + \frac{c_a}{16\pi^2} \tilde{K}((S,\bar{S};U,\bar{U})) + \frac{T(G_a)}{8\pi^2} \ln \Re f_a(\Phi).$$  \hspace{1cm} (37)

The (holomorphic) gauge coupling functions are given by

$$f_a = k_aS + \alpha_{ai}T^i$$

In the case at hand the coefficient matrix $\alpha_{ai} \neq 0$ only for the index values $i$ corresponding to the the 4-cycles/dell Pezzo surfaces on which the standard model 7-branes are wrapped. In the
particular examples that are relevant here, with D3 branes at a singularity, the relevant cycles shrink to a singularity.

In [7] the cutoff $\Lambda$ is taken to be $M_P$. As we discussed before there is no physical significance in the actual value of this cutoff as long as it is taken to be independent of the moduli. It is simply the upper limit from which one expects the RG evolution to start, and if the field theory is a low energy effective theory whose UV completion is string theory then this scale must necessarily be less than the string scale. As discussed in [16], for the purpose of comparing the moduli dependence of this formula to that coming from string theory, it does not matter at what point $\Lambda$ is chosen (as long as it is above all low energy thresholds) and indeed without loss of generality it can be chosen to be $M_P$. This obviously does not imply any statement about the validity of the field theory up to this point and all the conclusion of that paper would still hold even if the arbitrary cutoff $\Lambda$ is kept.

The conclusion of [7] (see also [8]) however depends crucially on the choice $\Lambda = M_P$. With this the argument of the log in the second term of the (37) is

$$\frac{\Lambda^2 \nu^{-2/3}}{\mu^2} \to \frac{M_P^2 \nu^{-2/3}}{\mu^2} \sim \frac{M^2_{\text{string}} R^2}{\mu^2},$$

(38)

where we’ve used (the approximate) formula $M^2_{\text{string}} \simeq M^2_P/\nu$ and put $R \equiv \nu^{1/6}$ the size of the internal space in string units. From this it is concluded that the effective unification scale (for large volume compactifications) can be far above the string scale. As the authors themselves point out in a footnote it is not clear what the “operational” meaning of this is. Clearly it cannot have any, for if instead of the above choice of $\Lambda = M_P$ we took a fixed value of $\Lambda \lesssim M_{\text{string}}$ (as one should), then the corresponding unification scale would be below the string scale! However in [8] the authors go on to derive physical conclusions based on having a cutoff that is larger than the string scale.

These come from the authors’ comparison of (37) (after replacing $\Lambda \to M_P$) with a string theory calculation. Based on a background expansion of the sort that we discussed in the introduction (but extending its infra-red region even beyond the string scale), the following equation is obtained in [8],

$$\frac{1}{g_a^2(\mu^2)} = \frac{1}{g^2} - \beta_a \ln \frac{M^2_{\text{string}}}{\mu^2} + \beta^N_{a=2} \ln \frac{M^2_X}{M^2_{\text{string}}}.$$  

(39)

Here $\beta_a = \beta^N_{a=1} + \beta^N_{a=2} = \frac{b_0}{16\pi^2}$, corresponding to the contributions to the beta function from $\mathcal{N} = 1, 2$ states and $M^2_X = R^2 M^2_{\text{string}}$. The last two terms on the RHS of the above equation are obtained from cutting off two UV divergent integrals of the form

$$\beta^N_{a=1} \int_0^{\mu^2} \frac{dt}{t}, \beta^N_{a=2} \int_0^{\mu^2} \frac{dt}{t}.$$  

(40)

The upper limit of these integrals is set by the infra-red RG scale $\mu$ of the previous discussion. On the other hand the UV cut-off is identified in the first case with $M^{-2}_{\text{string}}$ while in the second

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5 Note that to have unification one not only needs $f_a \propto k_a$ but also that the third and fourth terms on the RHS of (37) should be negligible.

6 The RHS of this inequality is of course moduli dependent. So what this requires is that one has to first decide on the values over which the volume is allowed to range and then fix $\Lambda$ below the lowest allowed value.
case it is identified with the (even smaller!) winding scale $M_X^{-2}$. As we’ve pointed out in this note, one should not use the (infra-red regime of the) background field calculation up to (and beyond) the string scale. Thus this kind of argument cannot be used to make any statement about the modular dependence, since all that one needs to have agreement with the field theory calculation is to choose the UV cutoff in the above integrals to be the same, and to take the value

$$\Lambda_{\text{string}}^2 = \frac{\Lambda^2}{\sqrt{2/3}} < \frac{M_{\text{string}}^2}{\sqrt{2/3}}.$$  \hfill (41)

This cutoff is (for $\mathcal{V} > 1$) obviously well within the regime of validity of the background field method. In other words there is really no new information in this calculation. It just gives us the translation to string language of the choice of cutoff in the field theory. All that is required for consistency is that both are well under the string scale!

The point is not so much to argue for the above cutoff in the string theory calculation, as to show that these calculations are intrinsically ambiguous\footnote{For related comments see the published version of [20].}

How does this argument compare with that presented in [16] which we discussed earlier. As emphasized there the non-universal and non-harmonic part of the gauge threshold correction must essentially come from low energy physics, and cannot have any information about the microscopic details of the UV completion of the theory. Furthermore according to the discussion in that paper, the string theoretic corrections in order to match the KL formula, must satisfy (31). The calculation in [8] clearly violates this (at least for the case of branes at orientifold singularities).

What then is the resolution of this conflict. Firstly the string theory calculations in [16] are UV finite as they should be. The expression for the (non-universal) threshold correction is given by an integral over the fundamental domain $\Gamma$ of the complex structure of the torus

$$\Delta_a(M, \bar{M}) = \int_{\Gamma} \frac{d^2 \tau}{\tau_2} \left( B_a(\tau, \bar{\tau}; M, \bar{M}) - b_a \right).$$ \hfill (42)

The UV finiteness of the integral is a consequence of the restriction to $\Gamma$. The integral of the first term is however IR divergent and the second term is added to cancel this. It should be noted that the resulting function is independent of any scale and is purely dependent on the (dimensionless) moduli.

In theories where the gauge sector comes from open strings however typically there are divergent integrals of the form

$$\Delta_a\left(\frac{\mu^2}{\Lambda^2}; M, \bar{M}\right) = \int_{\Lambda^{-2}}^{\mu^{-2}} \frac{dt}{t} B_a(t; M, \bar{M}).$$ \hfill (43)

Now, as in the heterotic case, $\mu \partial_\mu \Delta_a = B_a(\mu^{-2}) \to b_a$ (up to some normalization constant) in the limit of $\mu \to 0$, since in this limit the sum inside the integral is dominated by massless states going round the open string loop. While this is undoubtedly the case in the limit, the question is how far above zero one may take this infra-red region. Note that unlike in the heterotic case, here there is also an UV divergence unless one imposes (local and global) tadpole cancellation. The claim made in [8] is that (writing $B_a = B^{N=1}_a + B^{N=2}_a$),

$$B^{N=1}_a = b^{N=1}_a \Theta(t - M_s^{-2}),$$ \hfill (44)

$$B^{N=2}_a = b^{N=2}_a \Theta(t - (RM_s)^{-2}).$$ \hfill (45)

\footnote{Note that in [8] what we following [16] have called $\Delta$ is called $\Lambda_2$, and $B$ is called $\Delta$!}
In other words the infra-red region is essentially taken (from $\mu^2 = 0$ which is all that is really justified in the string calculation) all the way to the string scale in the first equation and even beyond it to the winding scale in the second equation. This is tantamount to pushing a background field calculation way beyond its regime of validity. Also the entire modulus dependence of this calculation comes from the UV cutoff. This is in contrast to the comparisons made between the KL formula and (UV finite) heterotic orbifold calculations done in [16] where it is the moduli dependence of the whole function $\Delta_a$ in (42) that is matched to the KL formula.

In fact of course in type I and its T-dual theories also the corresponding expressions should be finite. In particular this must mean that redefining $\Delta_a$ as in [16]

$$\Delta_a(M, \bar{M}) = \int_0^\infty \frac{dt}{t} (B_a(t; M, \bar{M}) - b_a).$$

(46)

This is a finite function dependent purely on the moduli once overall tadpole cancellation is imposed. According to the general arguments put forth in [16], it is this function that should be compared with the one-loop (anomaly) terms of the KL formula and in particular satisfy (31). Any claim to find a discrepancy must then find a discrepancy with this formula and as far as we can see this has not been done in the models investigated in [8].

Instead what has been done is to split up the divergent integral that governs tadpole cancellation into two parts as in (44)(45). However this integral is finite in the full string theory as a result of cancellations in the coefficient of the divergent piece - and this in turn is a consequence of the cancellation of tadpole terms locally and globally. It is hard to see how a modulus dependence can be extracted from this cancellation since this coefficient is a pure number independent of any modulus.

Let us now revisit the issue of linear vs chiral multiplets. In the geometric regime for the moduli (i.e. with the orbifold/orientifold singularities blown up) where the field theory makes sense one can treat the Kähler moduli, either as chiral supermultiplets or as linear supermultiplets. Unlike the case of the type IIB (heterotic) string where it naturally arises as a chiral multiplet (linear multiplet) for the Kähler moduli, one may choose one or the other representation. In our discussion above in subsection 2.3 we treated all these moduli on the same footing and got the relation (24)

$$T^i + \bar{T}^i = \frac{2}{3} \tau^i(U) - V^i(z, \bar{z}),$$

(47)

Note that with this democratic treatment of all the Kähler moduli, the arbitrary functions $V^i$ cannot depend on the Kähler moduli (see also [13]). Now from the point of view of the effective field theory of the moduli it is the dynamics that drives some of the four cycle volumes $\tau^i$ to zero (i.e. those for which $\alpha_{ai} \neq 0$). Also if $V^i$ are to be identified with one-loop corrections in string theory then they should be independent of $S$ as well. Thus this would seem to indicate that any field redefinition would not involve mixing between different Kähler moduli.

However in [8] these moduli are treated asymmetrically. In other words the claim appears to be that one should only dualize (to a linear multiplet) only the moduli associated with the shrinking cycle. Hence these authors get a relation of the form

$$T^i + \bar{T}^i = \frac{2}{3} \tau^i(U) - V^i(z, \bar{z}, \tau^b, \ldots) \forall i, \text{ with } \alpha_{ai} \neq 0, \alpha_{ab} = 0.$$

(48)
This appears to be highly unnatural from the string derived effective field theory point of view. One might therefore think that this would require some strong motivation, but as we’ve discussed above this has not been firmly established in [8].

4 Non-perturbative superpotentials and the UV cutoff

In order to discuss this it is convenient to rewrite the manifestly supersymmetric superspace supergravity action i.e. (1), in a manifestly super-Weyl invariant form,

$$
\mathcal{A} = -3 \int d^8 z E C \bar{C} \exp \left[ -\frac{1}{3} K(\Phi, \bar{\Phi}; V) \right] + \left( \int d^6 z E [C^3 W(\Phi) + \frac{1}{4} f_a(\Phi) W^a W^a] + h.c. \right).
$$

(49)

Note that we have also generalized the action slightly in order to incorporate more than one gauge group and have used the chiral representation in which

$$
\int d^2 \bar{\theta} E / 2 = E
$$

the chiral density. In this form the action has an additional manifest symmetry under the transformations

$$
E \to e^{2(\tau + \bar{\tau})} E, \quad E \to e^{6\tau} E + \ldots, \quad C \to e^{-2\tau} C,
$$

$$
\nabla_\alpha \to e^{(\tau - \bar{\tau})(\nabla_\alpha - \ldots)}, \quad V \to V,
$$

$$
\Phi \to \Phi, \quad W_\alpha \to e^{-3\tau} W_\alpha.
$$

(50)

The chiral (auxiliary) field $C$ is introduced so as to make the Weyl invariance of the theory manifest. It is important to note that the superpotential occurs in this action with a factor of $C^3$. Now the above (chiral) Weyl transformations are anomalous in the quantum theory, and the preservation of this local Weyl invariance requires that the gauge coupling function is changed as follows:

$$
f_a(\Phi) \to f_a(\Phi) - \frac{3c_a}{8\pi^2} \ln C.
$$

(51)

where $c_a$ was given in (27). There is however additional $C$ dependence coming from a field redefinition anomaly which occurs on demanding canonical normalization for the matter kinetic terms (see eqn.(16) of [21]). Thus the actual $C$ dependence changes from the second term of (51) to $-\frac{b_a}{8\pi^2} \ln C$. Now suppose that the gauge group becomes strongly coupled and develops a mass gap below some scale. Then below that scale there is an effective theory that is obtained by integrating out the gauge theory degrees of freedom. This gives an effective action $\Gamma$ defined schematically by

$$
e^{-\Gamma(\Phi, \bar{\Phi}, C, \bar{C})} = \int d(gauge) \exp \left\{ -\frac{1}{4} \int [f_a(\Phi) - \frac{b_a}{8\pi^2} \ln C] W^a W^a + h.c. \right\}.
$$

(52)

Since SUSY should not be broken by this procedure, we expect $\Gamma$ to have the general form of a superspace action and in particular should develop a superpotential. Given the general argument

\footnote{We note that previously we used the letter $\Gamma$ for the effective gauge coupling in the linear multiplet formulation to conform to the notation in [6].}

\footnote{The crucial assumption here is the quasi-locality of $\Gamma$ which enables us to define its derivative expansion and then focus on its two derivative action which should be be of the standard supergravity form.}

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(based on Weyl invariance above) that any superpotential should come with a factor $C^3$, we see that the corresponding term in $\Gamma$ will be (a superspace integral of)

$$C^3 W_{NP} = C^3 A_a \exp \left( -3\frac{8\pi^2}{b_a} f_a(\Phi) \right). \tag{53}$$

Here $A_a$ is an $O(1)$ pre-factor (from dimensional analysis this would mean $A = O(M_3^3)$), which may depend on the moduli due to threshold corrections. If there is more than one condensing gauge group, there will obviously be a sum of such terms. This is essentially the Veneziano-Yankielowicz \cite{22} argument generalized to SUGRA (see \cite{23} \cite{11}. There is also of course a contribution to the Kaehler potential, but this is less important since unlike the superpotential, the Kaehler potential is perturbatively corrected. The total superpotential is then given as

$$W = W_c(\Phi) + W_{NP} = W_c(\Phi) + A e^{-3\frac{8\pi^2}{b_a} f_a(\Phi)}, \tag{54}$$

where the first term on the RHS is the classical superpotential. It should be stressed that this argument is not at all dependent on the Weyl compensator formalism. If we had worked with $C = 1$ (as in Wess and Bagger \cite{24}), then the form of the NP term \cite{53} is what is required to get the right Kaehler transformation of the superpotential as a result of the Kaehler anomaly (which is now related to the Weyl anomaly) and renormalization of the matter kinetic term.

On the other hand there is in the literature an alternative form for $W_{NP}$ based on an RG evolution argument. First one observes that the IR scale $\Lambda_a$ (at which the theory becomes strongly coupled) is related to the UV scale by

$$\Lambda_a^3 = \Lambda^3 e^{-3\frac{8\pi^2}{b_a}} = \Lambda^3 \mathcal{V} e^{-3\frac{8\pi^2}{b_a} \Re f_a(\Phi)}. \tag{55}$$

Here $\tau \equiv 4\pi/g_a^2$ where $g_a$ is the coupling at the UV scale $\Lambda$. For the first equality we’ve used the standard RG argument and for the second equality we’ve used eqn. \cite{57} and ignored $O(1)$ corrections to the prefactor.

In the global SUSY literature one often sees the evaluation of the superpotential as $|W_{NP}| = \Lambda_a^3$. However in SUGRA as pointed out in \cite{3} this should be replaced by including a factor which arises from transforming to the Einstein frame. This comes from the $C^3$ factor in \cite{53}, after gauge fixing $\ln C + \ln \bar{C} = K/3$, the value that is needed to go to Einstein frame. Thus one should identify

$$e^{K/2}|W_{NP}| = < \mathcal{W}^a \mathcal{W}^a > = \Lambda_a^3 \tag{56}$$

From this after using $K \sim -2 \ln \mathcal{V}$ and \cite{57} we have

$$|W_{NP}| = \Lambda^3 \mathcal{V}^2 e^{-3\frac{8\pi^2}{b_a} \Re f_a(\Phi)}. \tag{57}$$

Comparing with (the second term of) \cite{54} with $A \sim M_P^2$ we see that the cutoff $\Lambda$ may be estimated to be

$$\Lambda \sim M_P \mathcal{V}^{2/3} \sim \frac{M_{\text{string}}}{\mathcal{V}^{1/6}} = M_{KK} \tag{58}$$

\footnote{Note that those arguments led to having the anomaly coefficient $c_a$ rather than the beta function coefficient $b_a$ in $W_{NP}$. Our discussion shows how $c_a$ gets replaced by $b_a$.}
In other words the cutoff should be at the KK scale and is safely below the string scale for large volume. This is in sharp contrast to the argument of [9].

Finally we point out that if instead of fixing the chiral scalar compensator $C$ as in the line above eqn (56), so as to get to Einstein frame, one imposed $C \bar{C} = \exp(-K/6) \sim V^{1/3}$ we actually get from the SUGRA frame (i.e. with coefficient of $R$ being $e^{-K/3}$) to the string frame where (at least up to factors of $O(1)$) this coefficient is $\propto V$. Arguably it is in this frame that the string theoretic calculation should be compared with the SUGRA expression for the gauge coupling. In this frame the relevant term of the KL formula (i.e. the second term on the LHS of eqn (37)) becomes

$$\frac{b_0}{16\pi^2} \ln \frac{\Lambda^2 V^{1/3}}{\mu^2}.$$  \hspace{1cm} (59)

If now we identify the (upper bound on) the apparent unification scale with the string scale (i.e. $\Lambda V^{1/6} \sim M_{\text{string}}$) then we again have (58), i.e. the cutoff should be the Kaluza-Klein scale.

5 Conclusions

In this note we rederived the relation between the chiral and linear multiplets in the standard supergravity frame. This is not only simpler than that given in [6] but also clarifies the relation to the anomaly terms in the KL formula. In the heterotic string, there is a universal one-loop correction coming from the string calculation, which can be reinterpreted in the chiral superfield formulation as a correction to the Kaehler potential. In the case of non-universal corrections to the shrinking cycle of LVS constructions, we have argued that the necessity for such redefinition has not been clearly established. We have also shown, using two different arguments for the non-perturbative superpotential, that the effective cutoff is around the KK scale, in agreement with low energy effective action expectations. This resolves the apparent disagreement between the two different expressions that have appeared in the literature for these NP terms.

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