Black hole complementarity gets troubled with a dynamical charged black hole

Dong-han Yeom and Heeseung Zo

Department of Physics, KAIST, Daejeon, 305-701, South Korea

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A dynamical charged black hole can be harmful to black hole complementarity. However, if there is a kind of selection principle, it may rescue black hole complementarity; and Horowitz-Maldacena’s proposal may work as the selection principle. In this letter, it is claimed that we invent a gedanken experiment; even though we assume the selection principle from Horowitz-Maldacena’s proposal, the duplication of quantum information is still possible, by using Hayden-Preskill’s argument, which states that a black hole can function as an information mirror after the information retention time. In conclusion, a dynamical charged black hole will be a concrete counterexample of black hole complementarity.

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To resolve the black hole information loss paradox [1], many ideas have been suggested, and one of the most important proposals is known as the black hole complementarity principle [2]. According to black hole complementarity, an asymptotic observer and a free-falling observer see different things but all known physical laws, especially unitarity, hold for each observer. However, a quantum information consisting of black hole internal state, to be observed by the free-falling observer, seems to be copied to the outgoing Hawking radiation toward the asymptotic observer. It means that the quantum information is duplicated and the no quantum Xeroxing theorem is violated. Now, the black hole complementarity principle argues that the viewpoints of two observers are essentially complementary. There is no problem because they cannot communicate forever. Eventually, it is known that, if we assume the statistical entropy to be thermal entropy [3], and keep the unitarity of quantum mechanics [4], then the black hole complementarity principle would be manifest [5] due to arguments of the quantum information theory [6].

However, one needs to check the consistency of black hole complementarity: *is it really impossible to see the duplication of information?* For a Schwarzschild black hole, it was checked carefully [7]. But, a Schwarzschild black hole is not the general case. The authors argued that it may not hold if local horizons become different with global horizons. From numerical calculations of a charged black hole, there was a distance between the inner horizon and the Cauchy horizon [8]; and, this will make a room for the duplication experiment, i.e., a gedanken experiment for checking information duplication, to be possible [9]. Of course, there were some assumptions to derive those models; we needed a large number of massless degrees of freedom to regularize high curvature. Also, it was assumed that the pair-creation effect can be suppressed well during evaporation; this would hold if the charge itself is large enough, or mass of electron is quite bigger than our one. If those assumptions are not valid by fundamental reasons, our model may not work as a counterexample. But we do not find such reasons and a dynamical charged black hole seems to work as a counterexample of the black hole complementarity principle.

Fortunately, however, still there was a loophole. It was commented that a dynamical charged black hole may violate black hole complementarity by the duplication experiment, only if there is no correlation between the singularity and the outer horizon. If this space-like correlation can be implemented by the quantum teleportation, a dynamical charged black hole may not work as a counterexample of the black hole complementarity. And, Horowitz-Maldacena’s proposal [10] can be working as such a selection principle by using the quantum teleportation [8].

In this letter, it is argued that this resolution is not sufficient to rescue the complementarity (i.e., we will undermine a resolution of [9]); if we consider Hayden-Preskill’s argument [11], we can construct a situation that the selection principle still permits the duplication experiment. Finally, we will justify that a dynamical charged black hole can be a definite counterexample of black hole complementarity.

**Duplication experiment in a charged black hole** If we assume that the pair-creation effect can be suppressed than the Hawking radiation, the causal structure of a dynamical charged black hole can be obtained by pasting from the mass inflation scenario [12] to the extreme black hole solution. In the mass inflation scenario, the outer apparent horizon grows along the space-like direction, and there is a space-like singularity due to the mass inflation [14]. If there is no Hawking radiation, there will be the inner horizon at infinite advanced time $v \to \infty$ [13][10]. The mass function behaves $m(u, v) \sim \exp \kappa_i (u + v)$, where $\kappa_i$ is the surface gravity of the inner horizon, and $u$ and $v$ are the retarded and advanced time parameters for the double null coordinate [12]; thus, the inner horizon becomes curvature singularity. However, if we paste this scenario to the extreme black hole, the inner horizon must approach to the outer horizon. Then, the
most natural guess is that, the inner horizon bends space-like direction, and approaches to the outer horizon; and, this expectation is confirmed by our numerical calculation [13][8]. Notice that, since one can access any location of the integrated domain by finite \( u \) and \( v \). This implies that there is no curvature singularity in the general relativistic sense. Of course, since the mass function exponentially blows up, some curvature functions may blows up greater than the Planck scale. But, this is resolved by re-scaling the unit length (i.e., Planck length); this can be implemented by tuning the number of massless degrees of freedom [8]. Finally, if we assume a large number of massless degrees of freedom, we can get the semi-classically convincible causal structure of a dynamical charged black hole (Figure 1).

One important point is that, the transition region between the mass inflation scenario to the extreme black hole requires the end of the space-like singularity; it will induce the Cauchy horizon. We cannot calculate beyond the Cauchy horizon by definition, but one may reasonably guess that there is the time-like singularity inside of the Cauchy horizon. In this causal structure, the duplication experiment can be possible between the Cauchy horizon and the inner horizon [8][9]. Free-falling matter can send a signal along the outgoing direction (\( A \)), and this information can be observed outside of the black hole after the information retention time (\( B \)). Thus, an observer can see the duplication of information. Note that, all processes happen in the region which is semi-classically convincible, if we assume a large number of massless degrees of freedom.

Horowitz-Maldacena’s proposal as a selection principle

However, if there is a correlation between the singularity and the outer horizon, and if the correlation prevents the Hawking radiation (\( B \)) from containing information about the outgoing information which did not touch singularity (\( A \)), we can rescue black hole complementarity. Although the correlation is space-like, if the quantum teleportation can be realizable in a black hole, this may be possible. The authors argued that, the Horowitz-Maldacena’s proposal [10] can be used exactly for this purpose.

Horowitz-Maldacena’s proposal assumes that [10][17], the in-falling matter \( |i\rangle_M \) is entangled with the in-going Hawking radiation \( |i\rangle_{in} \) by a unitary transformation \( S \) (with \( N \) states),

\[
|\Psi_{in}\rangle = \frac{1}{\sqrt{N}} \sum_i S|i\rangle \otimes |i\rangle_{in};
\]

as well as the in-going Hawking radiation \( |j\rangle_{in} \) is maximally entangled with the outgoing Hawking radiation \( |j\rangle_{out} \),

\[
|\Psi_{out}\rangle = \frac{1}{\sqrt{N}} \sum_j |j\rangle \otimes |j\rangle_{out}.
\]

Now, one strong assumption is that, the in-going information is projected because of the final-state near singularity. Then, we get

\[
\langle \Psi_{out}|\Psi_{in}\rangle = \frac{S}{N},
\]
A thought experiment with Hayden-Preskill’s argument

What will happen if almost all information flows out along the signal $A$? Though, the information should escape after the information retention time, if one believes the pure and random mixing of initial information. These assumptions are quite natural for the membrane paradigm \[18\] and the principle of black hole complementarity. Then, it looks like that $B$ should contain information about $A$, even if one assumes the selection principle like Horowitz-Maldacena’s proposal; thus, one cannot prevent the duplication experiment. Of course, there is a loophole, since the signal $A$ cannot contain enough information to press that the Hawking radiation should contain the information for the signal $A$. To implement the situation, $A$ must have enough energy, but then it will give strong back reaction to the background; thus, we cannot trust our causal diagram, and it is not self consistent. This can resolve a potential worry.

Now we introduce Hayden-Preskill’s argument \[11\]. According to the argument, if the black hole rapidly mixes or scrambles the states, whenever some small bits of information fall into a black hole after the information retention time, the small bits of information almost directly comes out, as being reflected on an information ‘mirror.’ Then, one may guess that, the duplication experiment seems to be possible, as long as the mixing or scrambling time is short enough. According to some analysis, the marginal (smallest) rapid mixing time (or, the scrambling time) is on the order of

$$t_{scr} \sim M \log M,$$

where $M$ is the black hole mass \[11\]. This time scale can be calculated from the membrane paradigm \[20\], as well as some information theoretical arguments \[19\]. If the in-falling information can send a signal to the outgoing direction, and if the outer observer sees the information from the Hawking radiation after the scrambling time, falls into the black hole, and compares the signal, then the duplication may be possible. To send the signal which has bits of quantum information to the outgoing direction, we need the uncertainty relation $\Delta E \Delta t \sim 1$.

Then, in the Kruskal coordinate system, we can check the consistency condition of the black hole complementarity

$$\Delta E \sim \exp \frac{t_{scr}}{M} \geq M,$$

since the required energy $\Delta E$ is greater than the black hole mass $M$, the experiment is inconsistent. Of course, for given $t_{scr}$, the inequality valid marginally. In many situations, the scrambling time should be greater than the marginal limit; thus, we can conclude that black hole complementarity is safe for Hayden-Preskill’s argument.

Then, what if we apply the argument to a dynamical charged black hole (Figure 3)? According to Hayden-Preskill’s argument, if one send a little information ($C$).
to the black hole after the information retention time, one can reconstruct the original information \((C)\) from the Hawking radiation \((E)\) after the scrambling time. Let’s assume that the in-falling information \((C)\) send a signal along the out-going direction \((D)\). According to the membrane paradigm or the principle of black hole complementarity, it just scrambled near the horizon, i.e., it does not depend on the inside structure; thus, according to Hayden-Preskill’s argument, it should be escaped after the scrambling time. However, one may think a situation that, until the scrambling time, \(D\) did not projected near the singularity. Then, Horowitz-Maldacena’s selection principle cannot be applied to this situation; the fast scrambling process pushes the information to be escaped to the outside \((E)\), and this makes Horowitz-Maldacena’s proposal could not work. Then, one cannot prevent the duplication experiment in a dynamical charged black hole. In this sense, a dynamical charged black hole can be a concrete counterexample of black hole complementarity.

We can compare the difference between Horowitz-Maldacena’s selection process and Hayden-Preskill’s argument. If a black hole contains information \(a, b, c, d,\) and \(e\), after the information retention time, some of them should be escaped. But, there is no principle to specify the information, i.e., no one can say that \(a\) should go out first or \(b\) should go out first, etc. Thus, even though the causal structure could permit the duplication experiment, by assuming a selection principle from Horowitz-Maldacena’s proposal, we could circumvent the duplication problem. However, if we add information \(1, 2,\) and \(3\), after the information retention time, from Hayden-Preskill’s argument, one can say that \(1, 2,\) and \(3\) should go out first after the scrambling time. Then, the escaping information is already selected, so we cannot apply the selection process to this situation.

**Discussions** We discussed the causal structure of a dynamical charged black hole with the weak pair-creation and a large number of massless fields. If we assume Horowitz-Maldacena’s proposal, the black hole complementarity principle seems to be maintained by the quantum teleportation; but, when we introduce the random scrambling of the membrane, Hayden-Preskill’s argument makes fail to rescue black hole complementarity. Therefore, the authors conclude that, a dynamical charged black hole is a concrete counterexample of black hole complementarity.

One comment is that, at the very first, even though we did not include Hayden-Preskill’s argument, Horowitz-Maldacena’s proposal has some problems; if there are interactions between the in-falling matter and the ingoing Hawking radiation, the unitarity may be violated [17]. Moreover, still, we do not know whether beyond the Cauchy horizon there is the singularity or the second asymptotic region. Now, from Hayden-Preskill’s argument, we know clearly that Horowitz-Maldacena’s proposal cannot be a fundamental idea for the information loss problem.

If one believes the holographic argument of the entropy of a black hole and the unitarity of the quantum mechanics, black hole complementarity is inevitable. And, the membrane paradigm and the principle of black hole complementarity seem to be consistent with the fast scrambling assumption; thus, this supports Hayden-Preskill’s argument. If there is no fundamental reason to prevent the authors assumptions for a dynamical charged black hole, there is a contradiction.

Then, the question is that what is wrong? This should be checked carefully, but the authors suggest a possibility that the holographic interpretation of the black hole entropy can be false; i.e., the meaning of the holography should be re-considered. This will be the next work of the authors.

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holes: The membrane paradigm, New Haven, USA: Yale Univ. Pr. (1986).

[19] Y. Sekino and L. Susskind, [arXiv:0808.2096 [hep-th]].
[20] L. Susskind and J. Lindesay, An introduction to black holes, information and the string theory revolution: The holographic universe, World Scientific, Hackensack (2005).