Long time ago, photon production was proposed as a probe and a thermometer for Quark-Gluon Plasma (QGP). However, only recently has the complete $\alpha_s$ order photon spectrum been obtained. In this paper we give a brief review of the problematic as well as discuss the $O(\alpha_s)$ result.

1. Why Photon production?

Following the well known rhetoric, photons are weakly coupled to the strongly interacting quarks and gluons. Photons emitted in a QGP will immediately escape without further interactions in the plasma. This immediate “escape” will carry useful information on the nature of the supposedly formed plasma, at the emission stage. Hence the comparison between the calculated photon spectrum from a QGP and that from a hadron gas medium with the measured photon spectrum in heavy ion collision, after background subtraction, will constitute an evidence in favor of either state of matter.

In an optimistic scenario, the plasma will live long enough for thermalization to occur. At a suitable high temperature ($T$), which is not so realistic in present heavy ion collisions, the running strong coupling constant $\alpha_s$ will be small.

The above framework can be summarized as follows: we have a system in thermal equilibrium with an exact microscopic description in terms of quarks and gluons. In the small coupling constant regime, the calculation of the photon spectrum seems to be a straightforward application of perturbation theory. This is seemingly a simple situation compared to photon production in proton-proton collision where form-factors show up. Although photons are weakly coupled to the plasma this description is oversimplified, since the quark that emits the photon is affected by medium effects such as Debye screening; it also acquires a thermal mass which plays a central role in screening infrared divergences.

Medium effects are well described by the Hard Thermal Loop (HTL) effective theory [1]. So, the HTL theory is the natural scheme for calculating the photon production rate in a thermalized QGP.

For present (RHIC), and near future (LHC), heavy ion collision experiments a more realistic prediction should go beyond the small coupling constant regime. This will be briefly discussed at the end of this article.
2. A survey

In a QGP in local thermal equilibrium, the photon production rate or the number of photons, with momentum $Q = (q_0, \mathbf{q})$, produced per unit time and per unit volume is conveniently expressed in terms of the imaginary part of the photon two point function, calculated in the plasma

$$\frac{dN^\gamma}{dtdx} = -\frac{d\mathbf{q}}{(2\pi)^4 2q_0} 2n_B(q_0) \text{Im} \Pi(q_0, \mathbf{q}) .$$

This formula is valid at lowest order in the electromagnetic coupling, and to all orders in the strong coupling constant. In other words, the two point function encodes the essential physics of the plasma, relevant for photon emission.

As mentioned in the previous section, to obtain the photon two point function in a perturbative procedure, the HTL theory should be used. The one loop calculation in the effective theory was performed some time ago [2, 3]. It was believed that the one loop result gave the leading order $\alpha_s$ photon spectrum. This belief was based on the success of the effective theory. The essence of the effective theory is to reorder the perturbation theory to give a meaning to the equivalence between the loop-wise expansion and the successive orders in the coupling constant. The loophole in this equivalence comes from collinear configurations. The effective theory does not incorporate a ready-to-use remedy for collinear problems. Collinearity is kinematical, i.e. it depends on the particular problem of interest, so it is not surprising if the effective theory fails to treat such problems systematically.

It worth mentioning that the underlying physical processes in the one loop diagram are the 2 ↔ 2 processes, Compton and annihilation. However, it was shown [4, 5, 6] that bremsstrahlung and the new process of annihilation of an off-shell quark (on the left of figure 1) contribute to leading order. These processes appear only at the two loop level in the effective theory. The breakdown of the effective theory, two loop is of the same order as one loop, is traced back to the appearance of collinear divergences (i.e. the collinear emission of the photon by a quark). This collinearity is soften by the use of the effective theory which gives a kinematical cutoff denoted by

$$M_{\text{eff}}^2 = M_\infty^2 + \frac{Q^2}{q_0^2} p_0(p_0 + q_0) ,$$
Figure 2. Rescattering of a quark where the formation time ($\sim \lambda_{\text{coh}}$) is larger than the mean free path.

where $M_\infty \sim gT$ is the thermal mass of fermions, provided by the effective theory, and $p_0$ is the quark energy. The present form of the cut-off is valid for real and small mass virtual photons (the real photon case is obtained for $Q^2 = 0$). The dependence of this cut-off on the coupling constant, although it regularizes what would be a collinear singularity, renders the two loop diagrams equally important as the one loop contribution. Since, after phase space integration the cut-off $M_\infty^2$ appears linearly in the denominator, its $g^2$ dependence cancels the extra powers of the couplings in the two-loop diagrams. It is thus natural to ask whether this breakdown of the effective theory does not propagate to higher loop orders? Simple power counting indicates that the collinear configuration persists for all ladder diagrams. As a consequence, higher loop diagrams do contribute to the leading $\alpha_s$ order for real photon production. Hence a resummation of a whole set of gauge invariant diagrams is mandatory.

3. The physics of different scales

In attempt to focus on the underlying physical mechanism which renders higher loop diagrams equally important as the one loop diagram, we calculated the imaginary part of the photon polarization tensor at two loop level with quarks having a finite width. This study leads to a simple physical picture. Photon emission from a quark gluon plasma gives rise to two natural physical scales (figure 2):

- the formation length (time) $l_F$, also called the coherence length $\lambda_{\text{coh}}$:
  
  \[ l_F^{-1} \sim \delta E = \frac{q_0}{2p_0(p_0 + q_0)} \left[ p_\perp^2 + M_{\text{eff}}^2 \right], \]

  where $p_\perp$ is the quark transverse (compared to photon momentum) momentum,

- the mean free path (width)$^{-1}$: $\lambda_{\text{mean}} \sim (g^2T \ln(1/g))^{-1}$

The interplay between these two scales leads to the emergence of different physical regimes:

- the perturbative regime ($l_F < \lambda_{\text{mean}}$), where the dominant mechanisms of photon production are Compton and annihilation appearing at one loop order in the HTL effective theory. High dilepton mass spectrum is a typical example of this regime.
• the Landau-Pomeranchuk-Migdal regime \((l_F \gg \lambda_{mean})\): it is the region where the formation length is much longer than the mean free path. Inelastic processes like bremsstrahlung should be considered. This leads to coherent photon production, which is responsible for the LPM suppression of the photon spectrum. In this region a further resummation is needed to include rescattering. Real photon production exemplifies this regime.

4. Resummation

The finite width study cited in the previous section, although it provides a nice physical picture, is far from giving the complete photon spectrum. A correct approach would consist in re-summing the ladder diagrams mentioned above together with quark propagators with a finite width (self-energy corrections). This has been recently carried out \([9, 10]\). The authors of \([9, 10]\) showed that important cancellations of long ranged interactions occurred between vertex and self-energy diagrams and derived an integral equation with a simple physical interpretation. The bottom line of their resummation is to consider the rescattering of an almost on-shell quark in a random Gaussian background field. The imaginary part of the two point function was found to be:

\[
\text{Im} \Pi_{\mu\nu}^{\mu}(Q) \approx \frac{e^2 N_c}{2 \pi} \int_{-\infty}^{+\infty} dp_0 \left[ n_F(p_0 + q_0) - n_F(p_0) \right] \frac{p_0^2 + (p_0 + q_0)^2}{2(p_0(p_0 + q_0))^2} \times \text{Re} \int \frac{d^2 p}{(2\pi)^2} p_\perp \cdot f(p_\perp)
\]

with

\[
2p_\perp = i\delta E f(p_\perp) + g^2 C_F T \int \frac{d^2 l}{(2\pi)^2} C(l_\perp)[f(p_\perp) - f(p_\perp + l_\perp)], \tag{4}
\]

\(\delta E\) is the same as the inverse of the formation time defined before. This integral equation reflects the following features:

• The combination \([f(p_\perp) - f(p_\perp + l_\perp)]\) guarantees the cancellation of all infrared behavior (\textit{a priori} non-perturbative) when \(l_\perp \leq g^2 T\).

• Iteration is equivalent to rescattering in the medium with a collision term \(C\).

• The width, discussed in the previous section, could be seen as a part of the collision term. We notice again that multiple scatterings are important when the collision integral is of the same order as the \(\delta E\) term, as was predicted by the model discussed in the previous section.

The above equation is solved numerically and it is found that rescattering leads to about 25\% suppression of the photon spectrum coming from bremsstrahlung and off-shell scattering. Hence, although the “next to one loop” processes are not as enhanced as was thought originally \([3]\) they still give important contribution which should be included beside the strict one loop processes to obtain the complete order \(\alpha_s\) photon spectrum.
Recently the collision term was obtained analytically using new sum rules at finite temperature \[ 11 \]. The collision term is found to be

\[
\mathcal{C}(l_\perp) = \frac{1}{l_\perp^2} - \frac{1}{l_\perp^2 + 3m_g^2}.
\]

This analytical form allows to circumvent the evaluation of complicated integrals in the original form of the collision term. It is also useful for the extension of the above study to the production of low mass dileptons.

5. Conclusions

Photon production nowadays is known to complete order \( \alpha_s \). It is among the rare theoretical calculations going beyond the leading logarithm approximation. The resummation done by Arnold et al leads to a picture where one averages over a Gaussian background field which goes beyond the classics of static scattering centers extensively used in the literature to model rescattering.

It must be emphasized that bremsstrahlung and off-shell annihilation are among the dominant mechanisms for photon production. Although the resummation has led to a 25% suppression, the next to one loop processes still give significant contributions which will enhance the photon production rate in a quark-gluon plasma.

6. Extensions

Thermal photon production is not the unique source of photons. Pions, for example, decay into photons giving a very important background. This background should be subtracted in order to isolate the thermal photon production and compare it to photon production in a hadron gas model. On the other hand, dilepton production has a different background to be subtracted. Hence a compilation of photon and dilepton will constitute a tractable mean for “plasma detection”.

The resummation done in \[ 9, 10 \] includes only the transverse photon polarization. However, dilepton could receive contributions from the longitudinal polarization sector, hence the extension of the above resummation to dilepton case requires some precautions. The necessary resummation for low mass dilepton leads to \[ 12 \] (preliminary results)

\[
\text{Im} \Pi_{\mu\nu}(Q) \approx e^2 \int dp_0 [\ldots] \int [2p_\perp f(p_\perp) \oplus Q^2 g(p_\perp)]
\]

the new scalar function \( g \) satisfies an equation analogue to that satisfied by \( f \). The preliminary results for the dilepton rate indicates an LPM type suppression which does not rule out the contribution coming from bremsstrahlung and off-shell annihilation found recently in \[ 13 \].

We should stress that the small coupling constant regime is an idealized situation. A more realistic coupling constant should not be so small. Recall that photon emission is dominated by collinear emission, this occurs for almost on-shell quarks. Hence quarks can be treated to a good approximation in a quasi-particle model, with masses derived from lattice calculation. This is under investigation \[ 14 \].
REFERENCES

1. E. Braaten, R.D. Pisarski, Nucl. Phys. B 337, 569 (1990); E. Braaten, R.D. Pisarski, Nucl. Phys. B 339, 310 (1990); J. Frenkel, J.C. Taylor, Nucl. Phys. B 334, 199 (1990); J. Frenkel, J.C. Taylor, Nucl. Phys. B 374, 156 (1992).
2. J.I. Kapusta, P. Lichard, D. Seibert, Phys. Rev. D 44, 2774 (1991).
3. R. Baier, H. Nakkagawa, A. Niegawa, K. Redlich, Z. Phys. C 53, 433 (1992).
4. P. Aurenche, F. Gelis, R. Kobes, E. Petitgirard, Phys. Rev. D 54, 5274 (1996).
5. P. Aurenche, F. Gelis, R. Kobes, E. Petitgirard, Z. Phys. C 75, 315 (1997).
6. P. Aurenche, F. Gelis, R. Kobes, H. Zaraket, Phys. Rev D 58, 085003 (1998).
7. P. Aurenche, F. Gelis, H. Zaraket, Phys. Rev. D 61, 116001 (2000).
8. P. Aurenche, F. Gelis, H. Zaraket, Phys. Rev. D 62, 096012 (2000).
9. P. Arnold, G.D. Moore, L.G. Yaffe, JHEP 0111, 057 (2001).
10. P. Arnold, G.D. Moore, L.G. Yaffe, JHEP 0112, 009 (2001).
11. P. Aurenche, F. Gelis, H. Zaraket, JHEP 05, 043 (2002).
12. P. Aurenche, F. Gelis, G.D. Moore, H. Zaraket, work in progress.
13. P. Aurenche, F. Gelis, H. Zaraket, JHEP 07, 063 (2002).
14. F. Gelis, these proceedings.