CHL Compactifications Revisited

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Abstract

CHL compactifications are supersymmetry preserving orbifolds of any perturbatively renormalizable and ultraviolet finite ground state of the perturbative string theories: heterotic, type I, or type II, preserving 32, 16, 12, 8, 4, (or zero) supersymmetries, and retaining the perturbative renormalizability and finiteness of the parent string vacuum. In this paper, we review the genesis of the CHL (Chaudhuri-Hockney-Lykken) project within the broader context of the full String/M Duality web, establishing the existence of moduli spaces with a small number of massless scalar fields, the decompactification of such moduli spaces to one of the five ten-dimensional superstring theories, and the appearance of electric-magnetic duality in only the four-dimensional moduli spaces, a 1995 observation due to Chaudhuri & Polchinski. We present two mathematical curiosities easily deduced from the fermionic current algebra representation but whose physical significance is a puzzle: a 4D N=4 heterotic string vacuum with no massless scalar fields other than the dilaton, and a 2D N=8 heterotic string vacuum with no abelian gauge fields, reiterating once more the necessity for a systematic classification of the CHL orbifolds.

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1 Introduction

An important direction of current research in String Theory is to determine its precise boundaries, thereby discovering the principles by which we can eliminate the redundancy of the multiplicity of string vacua and arrive at a convincing description of the real world. This has been the theme of my research for many years, and it grew out of the 1995 discovery of the CHL strings [1, 2]. This paper offers a retrospective on this discovery within the larger context of the String/M Duality web, highlighting some of the results and follow-up insights that I have found especially significant in succeeding years.\(^2\)

The CHL strings are supersymmetry preserving orbifolds of any consistent compactification of perturbative superstring theory, where by consistency we mean here an exactly solvable background of perturbative string theory to all orders in the $\alpha'$ expansion. Such backgrounds have a solvable superconformal field theory description on the worldsheet, leading to an anomaly-free, and ultraviolet finite, perturbatively renormalizable superstring theory in target spacetime. We use the term *perturbatively renormalizable* to describe such a target spacetime string theory Lagrangian despite the presence of infinitely many couplings in the $\alpha'$ expansion, because only a finite number of independent parameters go into their determination, and these can all be found at the lowest orders in the string effective Lagrangian. The existence of only a finite number of independently renormalized couplings is the defining criterion for the Wilsonian renormalizability of a quantum theory. Thus, from this perspective, the perturbative string theoretic unification of gravity and Yang-Mills gauge theories with chiral matter can be seen as providing a precise, and unique, gravitational extension of the anomaly-free and renormalizable Standard Model of Particle Physics.

Of course, since we lack a precise formulation of nonperturbative string theory at the current time, we can only reliably invoke the above framework as long as we remain within the domain of weak string coupling. Fortunately, all observational signals point to the weak unification of the gauge couplings and gravity in our four-dimensional world, with the supersymmetry breaking scale lying somewhere between the electroweak (TeV) and gauge coupling unification ($10^{16-17}$ GeV) scales. So perhaps we will be lucky and able to follow the target spacetime string effective Lagrangian approach up to at least the gauge coupling unification scale, using tried-and-true renormalization group methods. Indeed the stream of precision data from the Z factories in the early 90’s, pinning down both the number of lepton-quark generations, as well as the hierarchical texture of fermion masses with the discovery of a surprisingly heavy top quark, and increasingly tight windows on neutrino masses, stimulated the resurgence of theoretical investigations of supersymmetric grand unification models using such renormalization group techniques.

It was in light of these developments that Joe Lykken and I initiated an ambitious new effort in string model building in 1993-94. Our original goal was to identify exactly solvable conformal field theory (cft) realizations of four-dimensional heterotic string vacua with massless particle spectra and couplings that would cover the spread of plausible semi-realistic extensions of the supersymmetric Standard Model, perhaps even suggest some new features unforeseen in conventional, field theoretic model building. We focussed on fermionic current algebra realizations of the conformal field theory,

\(^2\)Two recent workshop presentations by me which overlap with some parts of this paper can be found in [31, 32].
using a formalism originally developed by Kawai, Lewellen, Schwartz, and Tye [3], because of the simple, and explicit, nature of this description. It is straightforward to embed the desired particle spectrum and couplings in the cft using the fermionic representation, the symmetries of the desired low energy string spacetime effective Lagrangian become transparent. Each of these exact cft solutions correspond to special points in the moduli space of some CHL orbifold with a chiral, 4D N=1 supersymmetry. The detailed checks of the worldsheet constraints in such N=1 string vacua are prohibitively calculation-intensive. The algorithms were therefore implemented in an interactive and user-friendly computer program, Spectrum, created largely by George Hockney at Fermilab, and aimed at facilitating phenomenological string model building. Remnant ambiguity in the implementation of modular invariance for the fermionic twisted $Z_2$ current algebras was resolved by invoking the Verlinde fusion rules, an analysis due to Joe Lykken, in collaboration with a Fermilab postdoc, Stephen-wei Chung [4].

Let me outline the phenomenological successes of just one of our 4D N=1 examples, designated the CHL5 Model in the literature [5, 6]. It describes an N=1 heterotic string vacuum with three generations of supersymmetric Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ particles, an anomalous $U(1)$, and a very small number of flat directions at the string scale. Analysis of the flat directions removes all but one additional $U(1)$ at the string scale in the anomaly-free vacuum, the hypercharge embedding mimics the $SU(5)$ result extremely well, $k_Y=11/6$, quite close to $5/3$, without actual grand unification, giving acceptable values for the gauge coupling unification scale. One generation is singled out from the other two by its distinct couplings already at the string scale. The breaking of the additional $U(1)$ at either an intermediate, or electroweak, scale can generate interesting fermion mass textures. The detailed implications of either scenario for CHL5 have been explored in the papers by Cleaver, Cvetic, Espinosa, Everett, and Langacker [6].

It should be emphasized that CHL5 is already a rather good string theory description of the observable sector of supersymmetric standard model particle physics. But the hidden sector of this model is rather heavily constrained, and not terribly interesting. Helpful new input that would enable one to improve on such model-building exercises is expected to come in the near future when LHC turns on, giving insight into both the supersymmetry breaking scale as well as, hopefully, the mass of the lightest supersymmetric partner. Inputs from the crucial neutrino sector are already the focus of both current, and future, astro-particle experiments. Perhaps whole classes of inflationary models can be ruled out so that we will have additional insight both into viable mechanisms for supersymmetry breaking, as well as the viable early Universe scenarios. I should emphasize that all of this is nothing more than physics extracted from the leading terms in the string spacetime effective Lagrangian: the infrared limit of the full perturbative string theory, and in a specific 4D flat target spacetime background.

In the intervening years since 1995, investigations of string phenomenology have focussed on the incorporation of warped target spacetime metrics, background fields and fluxes, including supergravity pform fluxes from both Neveu-Schwarz and Ramond sectors, and their consequences for the hierarchy problems of particle physics, for moduli stabilization, supersymmetry breaking, and the form of the inflationary potential. We should note that, with some exceptions, these theoretical developments have largely been restricted to analysis of the low energy string spacetime effective Lagrangian, and the implications of $\alpha'$ corrections have not always been clarified. But it should be
evident that any generic insights from such analyses will apply equally well to the phenomenology of the CHL orbifolds. We are omitting detailed references for brevity, deferring discussion to future work.

The nonperturbative, pre-spacetime-geometry framework for string/M theory that may lie below the distance scale at which target spacetime Lagrangians become our primary investigational tool is addressed in my recent papers [14, 31]. A rather important question that needs to be addressed at this juncture is the apparent disconnectedness of the vacuum landscape of String Theory evidenced by the discovery of the CHL orbifolds [2, 1], a phenomenon that raises the spectre of both island Universes, and of a fundamental role for the Anthropic Principle [10]. The theoretical paradigm that will enable us to understand these issues is provided by the Hartle-Hawking framework for Quantum Cosmology [11], but let me begin by explaining the nature of the moduli spaces of the CHL compactifications.

2 A Brief Introduction to the CHL Strings

The CHL (Chaudhuri-Hockney-Lykken) strings were named by Polchinski [1] for the authors of the 1995 paper that pin-pointed the existence of additional exactly solvable supersymmetry preserving solutions to the heterotic string theory consistency conditions other than toroidal compactifications [2]. As explained above, the original motivation for the study of the CHL compactifications was better 4D N=1 low energy susy particle phenomenology, and a serious wrinkle on such efforts had been the proliferation of massless scalar moduli in any semi-realistic examples. To understand why the CHL compactifications have many fewer flat directions, consider the simplest example, the Chaudhuri-Polchinski orbifold of the circle compactified $E_8 \times E_8$ heterotic string described in [1]:

$$p = \frac{1}{\sqrt{2}}(p_1 + p_2) \in \Gamma_8, \quad p = \frac{1}{\sqrt{2}}(p_1 - p_2) \in \Gamma'_8, \quad p_3 \in \Gamma^{(1,1)}$$

(1)

Let us mod out by the $Z_2$ outer automorphism, $R$, interchanging the two $E_8$ lattices, $\Gamma_8 \oplus \Gamma'_8$, accompanied by a translation, $T$, in the $(17,1)$-dimensional momentum lattice, $(v, 0; v_3)$ [1]. This projects onto the symmetric linear combination of the momenta in the two $E_8$ lattices, so that the gauge group is generically $E_8 \times U(1)$:

$$p = \frac{1}{\sqrt{2}}(p_1 + p_2) \in \Gamma_8, \quad p = \frac{1}{\sqrt{2}}(p_1 - p_2) \in \Gamma'_8, \quad p_3 \in \Gamma^{(1,1)}$$

(2)

The RT orbifold acts on the perturbative heterotic string spectrum as follows [1]. Let $\mathcal{U}$ and $\mathcal{T}$ denote, respectively, the untwisted and twisted sectors of the orbifold. The untwisted sector is composed of states invariant under $R$ (I), and under $T$ (I*):

$$\mathcal{I} : \quad p_1 = 0, \quad p_2 = \sqrt{2}\Gamma_8, \quad p_3 \in \Gamma^{(10-d,10-d)}$$

$$\mathcal{I}^* : \quad p_1 = 0, \quad p_2 = \frac{1}{\sqrt{2}}\Gamma_8, \quad p_3 \in \Gamma^{(10-d,10-d)}$$

$$\mathcal{T} : \quad p \in \mathcal{I}^* + v$$

(3)
Notice that the dimension of the moduli space is much smaller: the massless scalars parametrize the coset, \( SO(18-d, 10-d)/SO(18-d) \times SO(10-d) \), up to discrete identifications, for compactification on the torus \( T^{10-d} \). The momentum vectors in the Hilbert space of the orbifold lie on hyperplanes within the \((26-d,10-d)\)-dimensional lattice describing the toroidally compactified \( E_8 \times E_8 \) string. Such moduli spaces include several novelties including affine Lie algebra realizations of the simply-laced gauge groups at higher Kac-Moody level, as well as enhanced symmetry points with non-simply laced gauge symmetry [2, 1]. We will return to these novelties below.

Notice that since the orbifold action in question preserves supersymmetry, our discussion of the disconnectedness of the CHL moduli spaces with 16 supersymmetries can be carried over to an analogous disconnectedness of CHL moduli spaces constructed as orbifolds of 4D heterotic vacua with 12, 8, 4, or even zero, supersymmetries. Consider the decompactification limit to ten dimensions without any change in the number of supersymmetries: for the \( \mathbb{Z}_2 \) orbifold, the outer automorphism interchanging the two \( E_8 \) lattices becomes trivial in this limit, and we straightforwardly recover an additional 248 massless gauge bosons. A subsequent toroidal compactification completes the continuous interpolating path connecting a point in the moduli space of a CHL orbifold with some point in the moduli space of toroidal compactifications, via the ten-dimensional \( E_8 \times E_8 \) heterotic string ground state. Thus, a spontaneous decompactification to ten dimensions followed by a spontaneous re-compactification can indeed interpolate between a pair of (dis)connected CHL orbifolds with different numbers of abelian multiplets. But this is a genuinely stringy phenomenon, with a slew of modes with string scale masses descending into the massless field theory as we tune the compactification radius to its noncompact limit.

A similar example is the spontaneous restoration of extended supersymmetry known to occur in certain decompactification limits of the moduli space of 4D \( N = 2 \) heterotic string compactifications [9]. The modular invariant one loop vacuum amplitude of the freely acting orbifold in question is parameterized by the continuously varying, complex structure moduli and Kahler moduli of a six-torus, in addition to a constant background electromagnetic field; the extended supersymmetry is restored in the limit that one of the cycles of the torus decompactifies [9]. Does this framework allow for continuous interpolations between the moduli space of toroidal compactifications of the heterotic string and a CHL compactification with eight fewer abelian gauge fields along a path that traverses a family of ground states with only eight supercharges, and in one lower spacetime dimension?

The problem with either of these proposals is that the interpolating trajectories are exactly marginal flows from the perspective of the 2D worldsheet renormalization group. Thus, there is no reason to expect the stringy ground state to “evolve” along such a trajectory in the absence of supersymmetry breaking with a consequent lifting of the vacuum degeneracy. In other words, if the supersymmetry breaking scale in Nature does turn out to be significantly lower than the string scale, the stringy massive modes in the CHL orbifold will have genuinely decoupled from the low energy field theory limit, and there is no escaping the conclusion that the field theoretic dynamics of vacuum selection occurs in one of a multitude of disconnected, low energy Universes. How should we interpret the resulting multitude of low energy string effective Lagrangians? The Hawking-Hartle paradigm [11] would identify each such low-energy spacetime effective Lagrangian as the final state of a consistent history in some putative Quantum Theory of the Universe. The
pre-spacetime matrix framework for nonperturbative String/M theory described in my recent work [31] is such a theory, yielding also a multitude of acceptable spacetime effective Lagrangians, each characterized by a distinct large N limit of the matrix Lagrangian. The “theory” for the Initial Conditions of the Universe [11], to borrow a phrase from Hartle and Hawking, is the pre-spacetime finite N matrix dynamics. This dynamics is beyond the direct purview of perturbative string theory.

To summarize, if the supersymmetry-breaking scale is clearly separated from the string mass-scale, the stringy massive modes will have genuinely decoupled from the effective Lagrangian of relevance and there is no escaping the conclusion that vacuum selection in perturbative string theory involves more than just dynamics, requiring a discrete choice among disconnected low energy Universes. However, upon including the stringy massive modes, all of the CHL orbifolds are connected in the sense that they decompactify to the same 10d perturbative string vacuum.

By now, the CHL strings have given many fundamental new insights into weak-strong electric-magnetic dualities in the String/M theory web [2, 30, 31]. Let me mention the earliest of these discoveries which appears in the paper [1]; this particular observation is due to Joe Polchinski. Careful examination of which non-simply laced gauge groups can appear at the enhanced symmetry points in the moduli space of the CHL orbifold reveals the result [1]:

$$\text{Sp}(20 - 2n) \times \text{SO}(17 - 2d + 2n) \quad n = 0, \cdots, 10 - d ,$$

at special points within the same d-dimensional moduli space. Remarkably, the electric and magnetic dual groups, $\text{Sp}(2k)$ and $\text{SO}(2k + 1)$ for given $k$, only appear together in the moduli spaces of the four-dimensional CHL orbifolds [1]. This is precisely as required by the S-duality of the 4D N=4 theories, constituting independent evidence in favor of it. It should be noted that this property follows as a consequence of the constraints from modular invariance on the orbifold spectrum, the worldsheet constraints responsible for the perturbative renormalizability and ultraviolet finiteness of the CHL compactifications. As mentioned above, to the best of my knowledge, all of the CHL orbifolds described in [1, 13, 31] decompactify to one of the five 10d superstring theories. A classification of the supersymmetry preserving automorphisms of Lorentzian self-dual lattices up to lattices of dimension (22,6) would completely pin down this important issue, also enabling a classification of the enhanced symmetry points in each moduli space. This is crucial information necessary for any further exploration of electric-magnetic duality in the 4D CHL orbifolds.

My work on the abelian symplectic orbifolds of six- and four-dimensional toroidally compactified heterotic strings with David Lowe in [13] utilized Nikulin’s classification of the supersymmetry preserving automorphisms of (19,3)-dimensional Lorentzian self-dual lattices, namely, the cohomology lattices of the classical K3 surfaces. Our analysis proceeds as follows: begin at a point in the moduli space where the (22,6)-dimensional heterotic momentum lattice decomposes as $\Gamma^{(19,3)} \oplus \Gamma^{(3,3)}$. Given Niemeier’s enumerative list of self-dual lattices up to dimension 24, one can straightforwardly enumerate a large number of CHL orbifolds by invoking Nikulin’s classification [13]. For instance, the $\mathbb{Z}_2$ orbifold described above readily generalizes to $\mathbb{Z}_n$ orbifolds with $n>2$ whenever the (19,3) lattice contains $n$ identical component root-lattices. Modding by the $\mathbb{Z}_n$ symmetry under permutation, accompanied by an order-$n$ shift vector in the (3,3) torus, gives a $\mathbb{Z}_n$ CHL orbifold. It is evident that a classification of the 4D CHL orbifolds would require extending Nikulin’s analysis to a classification of the symplectic automorphisms of all (22,6)-dimensional Lorentzian self-dual lattices.
Moduli Spaces of Heterotic–Type I–Type II CHL Strings

The detailed picture of the string Landscape with sixteen supercharges given by the study of the CHL compactifications can go a long way towards determining the precise boundaries of the String/M Duality web. We will now explain how this systematic approach can be successfully applied both to a study of the string landscape with 12, 8, 4, or 0 supercharges, as well as to any of the string theories, heterotic, type I, or type II [30, 31].

The heterotic and type IB string theories with gauge group $SO(32)$ are related by a strong-weak coupling duality transformation in ten dimensions [24]. The strong coupling limit of the 10d $E_8 \times E_8$ heterotic string is, instead, conjectured to be eleven-dimensional M theory compactified on $S^1/Z_2$ [16, 25]. We will begin this section by explaining the nature of the moduli spaces of perturbatively renormalizable string ground states with sixteen supersymmetries, obtained by either toroidal and supersymmetry-preserving orbifold (CHL) compactifications [17, 2, 1, 13, 30], or asymmetric orbifold and K3 compactification [7, 21, 13] of, respectively, heterotic and type I, or type IIA and type IIB, superstring theories. The generic moduli space in either case is of CHL type [1, 13].

It is helpful to begin by considering the target spacetime and strong-weak coupling dualities that relate the circle-compactified type I and heterotic string theories in nine dimensions and below. The reason is that, in nine dimensions, the two heterotic string theories share a common moduli space and one can smoothly interpolate between ground states with enhanced gauge symmetry $SO(32)$, $SO(16) \times SO(16)$, and $E_8 \times E_8$, respectively, by turning on an appropriate Wilson line wrapping the spatial coordinate $X^9$ [17, 18]: the two heterotic string theories are related by a T-duality transformation on $X^9$. Likewise, the type IB string theory with 32 D9branes and $SO(32)$ gauge fields can be mapped by a T-duality on $X^9$ to the type I' string theory with 32 D8branes and identical gauge group. The strong coupling limit of the latter theory is M theory compactified on $S^1 \times S^1 / Z_2$, with nonabelian gauge fields on the domain walls bounding the interval. It should be noted that this strong-weak duality has been conjectured for type I' ground states with either gauge group $SO(16) \times SO(16)$, or with the extension to a full $E_8 \times E_8$.

The latter enhanced symmetry point corresponds to a nonperturbative 9D background of the type I' string theory: one must introduce a pair of D0branes, in addition to the 16 D8branes on each of two orientifold $O$-planes. This brane-configuration preserves all of the supersymmetries of the type I' string [26, 15, 30]. The crucial massless gauge bosons in the spinor representation of $SO(16) \times SO(16)$ necessary for the enhancement to $E_8 \times E_8$ appear as follows. Consider the step-wise change in the background value of the Ramond-Ramond zero form field strength, $F_0$, associated with the creation of a fundamental string [26] when a D0brane, and image, threads the stack of 8 D8branes, plus image-branes, at either orientifold $O$-plane. The change in Ramond Ramond zero-form flux at each crossing can take either sign $\pm$, and the absence of dilaton gradients requires that the net change including that at both orientifold-planes, is zero. It is easy to verify that the 128 states in the spinor of $O(16)$ correspond to the distinct sequences of changes in the zero form flux at either $O$-plane that can satisfy this condition. Notice the isomorphism with the standard parameterization of the $E_8$ momentum lattice in the heterotic string [15]: denote the sequence of changes in zero form flux at an $O$-plane by a vector as follows, $(\pm, \pm, \pm, \pm, \pm, \pm, \pm)$, including all sequences with an even number of minus signs, $2 + 56 + 70$, and with the compensating sequence.
at the other O8-plane [30]. As was emphasized by us in [30], since the \(SO(32)\) and \(E_8 \times E_8\) heterotic string ground states are continuously connected in nine dimensions and below, self-consistency with the strong-weak coupling duality relation linking heterotic and type IB \(SO(32)\) strings in ten dimensions, \textit{requires} an enhanced symmetry point in nine dimensions with \(E_8 \times E_8\) gauge fields in the moduli space of the circle compactified type I-I’ string theory. The spectrum of nonperturbative D0-D8brane massless strings described in detail in our paper [30] resolves this puzzle.

In other words, once we work in nine dimensions, enabling the use of \(T_9\) target space duality transformations and interpolating Wilson line backgrounds, the equivalence between the type IB-I’ and heterotic O–E formulations becomes transparent: they are simply alternative worldsheet descriptions of identical target spacetime physics. Namely, the weak and strong coupling behavior of perturbatively renormalizable and anomaly-free theories with sixteen supercharges, and nonabelian gauge groups \(SO(32)\), \(SO(16) \times SO(16)\), or \(E_8 \times E_8\). It should be emphasized that the strong-weak coupling duality relating the type IB and heterotic string theories in ten dimensions, with \(SO(32)\) gauge group, holds for the entire spectrum of massive string modes, not merely the massless fields [30]. In fact, the massive mode spectrum of the type I’ ground states with D0-D8brane configurations is best described by developing the isomorphism to the heterotic momentum lattice; for details, the reader can consult [30]. We should note, however, that this equivalence does \textit{not} hold beyond tree-level. As clarified in my recent works [29], one-loop coupling constant renormalization differs in open and closed string theories in that both the limit of coincident string vertex operators, as well as the limit of shrinking loop lengths, can contribute to the ultraviolet divergences of any given string scattering amplitude. In contrast, modular invariance ensures the excision of contributions from the ultraviolet regime to any heterotic closed string scattering amplitude. Of course, since the tree-level masses in theories with sixteen supercharges do not receive loop corrections, in this particular case, the distinction becomes a moot point.

Are there any additional string backgrounds with sixteen supercharges in dimensions \(D \leq 9\) that are also perturbatively renormalizable? The answer, evident from our discussion in section 2, is \textit{Yes}, and the original examples are the supersymmetry preserving CHL orbifolds of the toroidally compactified heterotic string theories [1, 2, 13]. As explained before, the nonabelian gauge sector in the generic CHL orbifold in spacetime dimensions \textit{other than four} does not display evidence for manifest electric-magnetic duality. Notice that the special role of four dimensions is also highlighted by consideration of electric-magnetic duality in the supergravity sector of either toroidal or CHL compactifications. In generic spacetime dimensions, the electric two-form potential is always present in the perturbative string mass spectrum while the dual six-form is not. The six-form potential couples to magnetic fivebranes and it is, therefore, only in four spacetime dimensions that evidence for manifest electric-magnetic duality can appear: wrapped fivebranes on six tori have a pointlike limit, and the conjectured electric-magnetic duality takes the form of \(S\)-\textit{duality} in the low energy N=4 supergravity-Yang-Mills field theory.

In other words, the apparent electric-magnetic duality in the gauge sector of the toroidally compactified heterotic string in generic spacetime dimensions is a red herring: since only simply-laced gauge groups appear in the Narain moduli spaces, and since in this case the magnetic dual gauge group happens to coincide with the electric gauge group, the N=4 theories also appeared to be electric-magnetic self-dual, point-by-point in the moduli space, and in \textit{any} spacetime dimension.
The CHL moduli spaces clarify this issue since they include enhanced symmetry points with both simply-laced, and non-simply-laced, gauge symmetry. It becomes evident that it is only in four spacetime dimensions that enhanced symmetry points with non-simply-laced gauge groups appear in electric-magnetic dual pairs within the same moduli space, an intriguing discovery made in our paper [1]. This particular observation is due to Joe Polchinski. Notice that at any point in the moduli space, the root lattices of both electric and magnetic dual groups are accompanied by their respective irreducible highest weight lattices, a simple consequence of modular invariance in every spacetime dimension. Dual pairs of gauge groups appear within the same moduli space only in four spacetime dimensions, so that the requisite spectrum of dual electric and magnetic gauge charges can be found at the corresponding enhanced symmetry points, in agreement with 4D S-duality.

We should emphasize that the CHL orbifolds describe perturbatively renormalizable heterotic string theories that have all of the appealing features of the 10d $E_8 \times E_8$ and Spin(32)/$\mathbb{Z}_2$ theories. Their generic self-consistency is especially transparent in the abelian orbifold construction given by David Lowe and myself in [13], generalizing the case of the $\mathbb{Z}_2$ orbifold described above [1]. The basic idea is to mod out by the $\mathbb{Z}_n$ symmetry present at any point in the Narain moduli space where $n$ identical copies of a component root-lattice appear as a subspace of the Lorentzian self-dual lattice, accompanied by a translation in the (10-d,10-d) compactification lattice. For the details, the reader can consult [13]. More complicated examples, some of which only invoke effective, field theoretic, strong-weak dualities can be found in [27], in addition to the type I and type II abelian CHL orbifolds described in [30, 31]. The analog of the abelian $\mathbb{Z}_n$ symmetry in type I and type II string backgrounds with Dbranes, first pointed out by us in [30], is the symmetry under interchange of $n$ stacks of coincident Dpbranes, with each stack carrying identical worldvolume nonabelian gauge group, accompanied by a translation. The result will be a disconnected component of the type IIB moduli space with sixteen supercharges. But it should be noted that, as in the case of the heterotic string, either vacuum decompactifies to the ten-dimensional type I, or type II, string. The examples described in [30] are the type IIB-I′ strong coupling duals of the heterotic CHL orbifolds [1, 13], further evidence of self-consistency with the type IIB-I′/heterotic strong-weak coupling duality conjectures [24, 16, 25].

We should emphasize that, as in the case of toroidal compactifications, the equivalence of the heterotic and type I supersymmetry-preserving orbifold (CHL) compactifications holds at the level of the full string mass spectrum, not only the lowest lying field theoretic modes. These are two different worldsheet descriptions of identical target spacetime physics: a perturbatively renormalizable theory of sixteen supercharges in spacetime dimension less than ten, with an anomaly-free Yang-Mills gauge group containing $(26-d) - r$ abelian gauge bosons. The integer $r$ can vary from 8 to a full $26-d$ in four dimensions [2, 13]. As mentioned above, we should also note that the equivalence between the type I and heterotic CHL orbifolds holds only at tree-level in the string coupling constant expansion.

How does this picture extend to generic type IIA, type IIB, and M theory compactifications preserving sixteen supercharges? Since these theories have 32 supercharges, we are interested in compactification on spaces with $SU(2)$ holonomy, and the only nontrivial Calabi-Yau manifold in this class is K3, of complex dimension two. A simpler possibility enabling precise analysis is asymmetric toroidal orbifold compactification, and we will begin our discussion with this case. Recall that M theory compactified on a $\mathbb{Z}_2$ orbifold gives an anomalous 10d theory [25], and the
non-anomalous and, perturbatively renormalizable, extension is nothing but the $E_8 \times E_8$ heterotic string in ten dimensions [16, 25]. In dimensions $9 \leq D \leq 7$, any attempt to break half of the supersymmetries of the type II superstrings by asymmetric orbifold compactification without the introduction of D-branes, runs into a clash with modular invariance: the modular invariant default is a nonsupersymmetric type 0A or type 0B vacuum, which also happens to be tachyonic [15].

It turns out that Ferrara and Kounnas have studied an analogous problem in [7, 8], demonstrating the existence of perturbatively renormalizable type II ground states in four dimensions preserving $N = 8, 6, 5, 4, \text{ or } 3$ supersymmetry in the toroidally compactified type II superstrings on a six-torus. In particular, they showed that the 4D $N=4$ type II modular invariants preserve either $N = (2L, 2R)$, or $N = (4L, 0)$, of the $N = (4L, 4R)$ supersymmetries of the toroidally compactified type II superstrings. How should we interpret these 4D ground states? As regards the supergravity sector alone, the former can be identified as a chiral type IB projection of the IIB string theory, projecting to the symmetric linear combination of left and right moving worldsheet modes. The latter is a heterotic chiral projection of the type IIA string, singling out the massless gravitinos from the right moving worldsheet superconformal field theory alone [7]. Either class of $N = 4$ ground state can contain massless Yang-Mills gauge fields, and the number of abelian gauge fields in the low energy gauge theory is a useful hint towards deducing the precise orbifold projections that led to each such ground state.

It should be emphasized that since care has been taken to preserve modular invariance in the type II asymmetric orbifold ground states of [7, 8], they are likely to describe perturbatively renormalizable 4d theories with sixteen supercharges, analogous to the heterotic and type I CHL strings described above. A word on nomenclature: the heterotic CHL asymmetric orbifold “compactifications” are projections on the Hilbert space of the toroidally compactified heterotic string, that also preserve the supersymmetries of the parent string vacuum. The “asymmetric type II orbifold compactifications” of Ferrara and Kounnas [7], and also Kawai, Lewellen, and Tye [8], with 4D $N = 4$ supersymmetry, are a closer, lower-dimensional analog of the 10d chiral projections on the Hilbert spaces of the type IIB and type IIA superstrings that give, respectively, the type IB and heterotic string theories with only half the number of supersymmetries. Notice that, while either chiral projection yields an anomalous $N = 1$ theory in ten dimensions, necessitating the addition of Yang-Mills gauge fields with gauge group $E_8 \times E_8$ or $\text{Spin}(32)/\mathbb{Z}_2$, toroidal compactification to four dimensions followed by the identical chiral projections directly gives self-consistent $N = 4$ string ground states.

The Yang-Mills gauge fields in [7, 8] arise from massless type II closed string Kaluza-Klein or closed string winding modes. A further supersymmetry preserving (CHL) orbifold of such a toroidal compactification can lead to new ground states with a Yang-Mills sector containing fewer abelian gauge fields. The 4D $N = (4L, 0)$ ground states in the analysis of [7, 8] have gauge group $[SO(3)]^6$, $SU(4)\times SO(3)$, or $SU(3)\times SO(5)$, with maximal rank six, precisely as expected for heterotic compactification on a six-torus in the absence of the ten-dimensional rank 16 gauge group. We emphasize that unlike the Narain, or CHL, compactifications [17, 2, 1], these 4D theories “decompactify” self-consistently to the toroidally compactified type II theory, with twice as many supersymmetries in six thru ten dimensions, and no Yang-Mills gauge fields. The 4D $(2L, 2R)$ ground states can have even larger rank gauge groups, since massless gauge bosons can arise from both the compactified left, and
right-moving, conformal field theories. Ref. [7] finds 4D $N = 4$ ground states with gauge groups of rank 22, 14, 10, 6, and 2. The latter four cases are likely to correspond to supersymmetry preserving (CHL) orbifolds of the Hilbert space of the ground state with 22 abelian gauge fields. In 6D, the analogous maximal rank gauge group obtained in [7] was rank 20. The scalars of the maximal rank $N = 4$ ground state in [7] parameterize the manifold $[SU(1, 1)/U(1) \times SO(6, 22)/SO(6) \times SO(22)]$, precisely as in 4D toroidal compactifications of the $SO(32)$ type IIB-heterotic string theory.

It would be nice to have a systematic classification of the asymmetric toroidal orbifolds of the type IIA and type IIB superstring theories in every dimension $2 \leq D \leq 9$, for theories with 32, 16, or 12 supercharges; the further reduction to 8 or fewer supercharges leads to a well-known, rapid proliferation of solutions. For example, based on our discussion here, it is apparent that enhanced symmetry points with non-simply laced gauge symmetry must exist in the type II moduli spaces. It would be nice to have a classification of the enhanced gauge symmetry points in each dimension, and to verify the appearance of S-duality in every 4D $N = 4$ case. To reiterate the general theme of this section, we conclude that the perturbatively renormalizable type I, type II, or heterotic, string ground states with sixteen supersymmetries, in nine dimensions and below, are simply alternative worldsheet conformal field theory descriptions of identical target spacetime physics.

To drive home this point, it is helpful to consider what additional insight might be gained from the study of type II string compactification on smooth K3 surfaces? $SU(2)$ holonomy implies that we have a theory with sixteen supersymmetries, and the massless scalars of the non-chiral $N = 2$ supergravity in six dimensions parameterize a space that is locally equivalent to $[SO(20, 4)/SO(20) \times SO(4)]$ [20]. The gauge group at generic points in the moduli space is simply $[U(1)]^{24}$. The 6D $(1_L, 1_R)$ orbifold compactifications of [7] with nonabelian gauge symmetry correspond to special points in the moduli space of quantum K3 surfaces where, upon tuning the $(3,19)$-components of the so-called “B” field to particular values, the type IIA string can acquire a variety of enhanced gauge symmetries [21, 22]: the corresponding classical K3 surface has an orbifold quotient singularity that falls within an A-D-E classification, and the role of the B-field is crucial in its quantum resolution. For details, the reader can consult the papers by Aspinwall and Morrison [21, 22], which include an exposition of the remarkable fact that even the global structure of the moduli space of the type II string compactified on K3 agrees precisely with that of the heterotic string compactified on a four-torus.

These observations underlay the extensive exploration of a heterotic-type IIA string-string strong-weak effective, field theoretic, duality in six dimensions [23, 27, 13, 22]. The detailed encoding of the precise geometric data required to specify a quantum K3 surface is beautifully captured by the $(4,20)$-dimensional Lorentzian self-dual lattice that also specifies the background fields of a heterotic toroidal compactification [17, 21, 22]. This isomorphism enabled us [13] to identify the precise supersymmetry preserving CHL orbifold action on the cohomology lattice that gives a new moduli space of quantum K3 surfaces of smaller dimension. A quantized background one-form potential in the IIA Ramond-Ramond sector plays a crucial role in establishing this equivalence [23, 27, 13]. The classification of such symplectic automorphisms of the cohomology lattice of K3 surfaces due to Nikulin [12], in particular, provides a detailed enumeration of the perturbatively renormalizable 4D $N = 4$ type I-type II-heterotic ground states that follow as orbifold projections of the ten-dimensional $N = 1$ superstrings.
As a final comment, it would be interesting to complete the derivation of the modular invariant one-loop vacuum amplitude for the CHL orbifolds in the lattice representation, with lattice momenta parameterized by background field vevs [17]. In both our works [2, 1, 13], and in the Ferrara-Kounnas [7] analysis, the constraints from one-loop modular invariance have indeed been verified in both fermionic and orbifold formalisms. But it is an explicit lattice representation analogous to [17] that would provide the clearest physical insight because of the background field parameterization. Such a representation would also shed light on the isomorphism to quantum K3 geometry when such an isomorphism is available, see the related discussion in [22].

4 Sharpening the Boundaries of String Theory

We will end by presenting two mathematical curiosities easily deduced from the fermionic current algebra representation, but whose physical significance remains a puzzle. We emphasize that we do not as yet have an understanding of these solutions as either geometrical, or even non-geometrical, CHL orbifolds. The existence of such stray exactly solvable conformal field theory solutions is therefore a nuisance, but surely also an opportunity, to learn about the boundaries of string consistency. It is in this spirit that we have decided to include them in this paper. Let us address a recent question put forward by Martin Rocek: Are there any 6D N=2 or 4D N=4 heterotic string vacua lacking the familiar right-moving abelian gauge fields present in all toroidal compactifications: four in 6D, six in 4D, (and eight in 2D, with an N=8 supersymmetry)?

As explained in the introductory sections, since we lack both a complete classification of (22,6)-dimensional Lorentzian self-dual lattices, as well as a classification of their supersymmetry preserving automorphisms, it is very difficult to formulate a no-go proof of such a conjecture. However, it is sometimes possible to construct an explicit counter-example. It turns out that the fermionic CHL strings described in my paper with Lykken and Hockney [2] included a 4D N=4 example which lacked all of the 22 left-moving abelian gauge fields expected in the generic toroidal compactification [2]. We remind the reader that such an N=4 theory has no massless scalar fields other than the dilaton from the gravity multiplet: the moduli space of massless scalars is trivial at weak string coupling, and we apparently have an isolated 4D N=4 heterotic string vacuum. Needless to say, this solution does not fall within the class of CHL orbifolds that can decompactify to the ten-dimensional heterotic string.\textsuperscript{3} The question is whether one can also eliminate all six right-moving abelian gauge fields. Such exotic perturbatively renormalizable and ultraviolet finite string solutions would be extremely difficult to guess at by purely field-theoretic supergravity considerations. The self-consistency of such a cft solution rests, instead, on the existence of an exactly solvable description of an all-orders in $\alpha'$ background of heterotic string theory given by the fermionic current algebra representation [2].

As an example of the utility of such stray exactly solvable fermionic current algebra solutions, we will point out that one can deduce an immediate answer confirming the 2D N=8 conjecture, but without ruling out either the 4D N=4, or 6D N=2, conjectures. However, as will become clear

\textsuperscript{3}Evidence for non-geometric symplectic orbifolds which may not decompactify to ten dimensions was also given by us in [28], but that construction appears a bit contrived.
below, it appears very unlikely that solutions for the latter two conjectures can exist, and the reason is as follows. Consistency with modular invariance, and the existence of unambiguous fusion rules for the twisted current algebra, requires, as was shown in [4], that the worldsheet superconformal field theory necessarily contain sectors with specified blocks of self-consistent boundary conditions. The $\mathbb{Z}_2$ twisted worldsheet Majorana fermions, each with central charge $\frac{1}{2}$, are required to appear in blocks of 16, 24, 28, 32, 36, 40, 44, 48... this exhaustive enumeration could be continued to arbitrarily large-sized fermion blocks, at the cost of additional computer time.

For example, the block of 16 appears among the left-movers in all of the fermionic current algebra solutions describing points in the moduli space of the Chaudhuri-Polchinski $\mathbb{Z}_2$ orbifold [1], giving a reduction of eight left-moving abelian gauge fields in the massless spectrum. The novel 4D $\mathcal{N}=4$ solution described above utilizes, instead, the block of 44 left-moving twisted Majorana fermions [2], eliminating all 22 left-moving abelian gauge fields. What are the additional restrictions on the boundary conditions on right-moving Majorana fermions as a consequence of the triplet constraint from worldsheet supersymmetry? In four dimensions, and in light-cone gauge, the counting of worldsheet degrees of freedom goes as follows. Recall that the right-moving superconformal field theory has central charge $c=1+9$, where we have singled out the $c=1$ unit that carries the two (transverse) spacetime charges of the gravitinos in 4D. The internal cft with total central charge $c=9$ arises from fermionizing six right-moving chiral bosons, namely, from the six-torus, in addition to their six right-moving worldsheet Majorana fermion superpartners. In all, we have a total of 18 internal, right-moving Majorana fermions. In heterotic conformal field theory solutions with 16 conserved spacetime supercharges, the *triplet constraint* of [3, 4] is nothing but the requirement that the gravitinos live in the spinor of an SO(8): this removes a $c=3$ unit from the internal $c=9$ fermionic cft, combining it with the $c=1$ unit to give the root and irreducible weight lattices of SO(8) at Kac-Moody level one, namely, spinor, conjugate spinor, and vector. The remnant block of 12 Majorana fermions with central charge six cannot meet the modular invariance restrictions referred to above, since there are no self-consistent $\mathbb{Z}_2$-twisted fermion blocks of 12. It is evident that the 6D $\mathcal{N}=2$ case is even more constrained, since the remnant right-moving block has only 8 Majorana fermions! Given that the $\mathbb{Z}_2$ orbifold, with its minimum reduction of eight abelian gauge fields, is the simplest known CHL orbifold, we think it unlikely that the generic orbifold analysis can remove this basic hurdle from modular invariance. However, our argument does not in itself constitute a definitive no-go proof.

Upon compactifying further to 2D, the restrictions from modular invariance loosen up, and appear to allow a viable solution for the fermionic current algebra. In two dimensions, the counting of worldsheet degrees of freedom is as follows: in light cone gauge, all right-moving Majorana fermions are now internal, and we have a total of 24 fermions since we must fermionize two additional chiral bosons. Reserving a $c=4$ unit from the internal conformal field theory for the gravitino embedding as before, we are left with a block of 16 right-moving internal Majorana fermions. We can readily satisfy the triplet constraint simultaneous with the requirements from modular invariance: choosing a block of 16 right-moving twisted Majorana fermions, together with the block of 48 left-moving twisted Majorana fermions described in [4], appears to give a 2D $\mathcal{N}=8$ heterotic string vacuum with no abelian gauge fields originating in either left-, or right-moving, worldsheet conformal field theories. Whether this fermionic solution can be given an orbifold interpretation remains to be seen, apart from what that might imply for our generic observations on decompactification.
5 Conclusions

Understanding the symmetry principles, and the fundamental degrees of freedom in terms of which nonperturbative String/M theory can be formulated, is an important focus of ongoing research in theoretical high energy physics. Elucidating the web of heterotic-type I-type II CHL compactifications preserving sixteen or fewer supercharges can play a significant role in guiding such work since it determines precise boundaries for what we mean by string consistency.

A repeated theme in section 3 of this paper was the unity of the different superstring theories: alternative worldsheet descriptions, type I, type II, or heterotic, of identical target spacetime physics, although one must take care to note that the type I/heterotic equivalence holds only at tree-level in the string coupling constant. Nevertheless, since the mass spectrum in theories with sixteen supercharges receives no loop corrections, the equivalence remains a powerful conclusion. The web of CHL orbifolds described in this paper reiterates this point, supplementing the known heterotic examples with both type I, and type II, perturbatively renormalizable backgrounds preserving sixteen supercharges [30], also making contact with previous results of Ferrara and Kounnas [7]. These results go a long way towards establishing self-consistency with the type I-heterotic-M strong-weak coupling duality conjectures, in a multitude of disconnected moduli spaces, and in diverse spacetime dimensions.

We strongly believe that a more systematic investigation will only confirm that the generic 4D type II moduli spaces with $N \leq 4$ are precisely isomorphic to the CHL toroidal orbifolds of the heterotic/type I string theories described in sections 2 and 3. We emphasize that this conclusion is self-consistent with the strong-weak coupling duality conjectures [16, 24, 25]. Recall that the type IIA and type IIB string theories have Ramond-Ramond sectors with pform gauge potentials in the supergravity sector, apparently an alternative route to orbifolding for the construction of type II ground states with sixteen supercharges. We have found, however, that requiring the absence of Ramond-Ramond sector tadpoles leaves the possibilities for new solutions severely constrained. The introduction of a Dpbrane in any toroidal type II compactification breaks exactly half the supersymmetries, for any $p$, and consistency requires that we introduce 32 of them in order for the cancellation of Ramond-Ramond sector tadpoles [15]. In nine dimensions, this gives only the standard $O(32)$ type IB, and T-dual type IA, backgrounds with, respectively, 32 D9branes or 32 D8branes. An intersecting brane configuration of $(p, p-8)$ branes preserves all of the supersymmetries so, as explained in section 3, we can also construct a type I′ background with D0-D8branes and $E_8 \times E_8$ gauge group [30]. Finally, a sequence of $p$ T-dualities can map the background with 32 D9branes to flat space backgrounds with $p$ transverse bulk coordinates, and $10 - p$ longitudinal worldvolume coordinates, and we have a perturbatively renormalizable ground state with 16 supersymmetries and $O(32)$ D(9-$p$)brane worldvolume gauge fields. The introduction of gauge symmetry breaking Wilson lines, and quantized background antisymmetric two-form potential [30], enables supersymmetry preserving CHL orbifold compactification, giving the generic type I or type I′ ground state with sixteen supersymmetries.

A comprehensive classification of CHL compactifications in four dimensions with $N = 8, 4, 3, 2, 1$, and (zero) remains an important goal, although it appears unlikely to materialize in the near future as explained earlier. Some questions of interest that may be answerable even without a
full classification include the following. Vafa: Are there any 4D N=2 superstring vacua without any additional massless scalar fields other than the dilaton? Very likely, yes. But it has not been systematically explored. Vafa, Rocek, Niewenhuizen, Chaudhuri: What is known about the 4D N=3 theories coupled to matter? Do all of these theories contain points in the moduli space with an extended N=4 supersymmetry? Does this always require decompactification (a degeneration) of one, or more, cycles of the torus? We believe this last issue would be extremely interesting to address, and we will leave it for future work. Asymmetric orbifolds of the type II superstrings with 4D N=3 supersymmetry appear among the Ferrara-Kounnas examples described in section 3 [7], and include examples of identifiable CHL orbifolds.

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