Visible light polaroastrometry of astronomical objects

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Abstract

We propose a method to measure difference between positions of centroids of polarized and total flux of astronomical object in visible light, which we call for brevity polaroastrometry. Method efficiency is being demonstrated by the example of reduction of observations carried out with an instrument combining features of two-beam polarimeter with rotating half-wave plate and speckle interferometer, at 70-cm telescope in V band. For the total number of accumulated photoelectrons \( N_e = 10^9 \), which corresponds to series duration of 500 sec and object magnitude \( V = 6 \), the method precision is 60-70 \( \mu \)as in terms of 1\( \sigma \). At smaller \( N_e \) precision decays as \( \sim 1.7''/\sqrt{N_e} \), while at larger \( N_e \) it remains the same due to imperfections of the half-wave plate. For the main sequence stars, non-polarized and polarized by interstellar dust, we didn’t detect significant polaroastrometric signal. For the Mira variable star \( \chi \) Cyg total amplitude of the polaricenters was found to be 310 \( \pm 70 \) \( \mu \)as and 300 \( \pm 70 \) \( \mu \)as, for Stokes parameters \( Q \) and \( U \), respectively. For \( \alpha \) Cet these values are 490 \( \pm 100 \) \( \mu \)as and 1160 \( \pm 100 \) \( \mu \)as, for R Tri polaricenters coincide with the photocenter to the level of error.

1 Introduction

Polarimetry has been considered as powerful method of observational astronomy for a long time. Two causes give rise to its prosperity. On the one hand, polarization properties of the light carry significantly new information about the object relatively to photometry and spectroscopy. On the other hand, influence of many factors distorting radiation doesn’t depend on its polarization state, which makes differential polarimetric methods very effective.

Quite frequently polarimetry is used in combination with other methods, e.g. spectroscopy. Application of the differential imaging polarimetry for boosting contrast in coronagraphy turned out to be particularly successful [1]. Naturally suggestion arises: combination of the polarimetry and the astrometry — polaroastrometry — potentially can also be of some interest. Indeed, for the objects with complex distribution of polarized light polaricenters can deviate from the photocenter even when the object remains unresolved with the telescope.

It is convenient to characterize partially polarized light with so called Stokes parameters: \( I, Q, U, V \), which are connected with the polarization ellipse in the following way:

\[
I = I,
\]

\[
Q = Ip \cos 2\chi \cos 2\xi, \tag{2}
\]

\[
U = Ip \sin 2\chi \cos 2\xi, \tag{3}
\]

\[
V = Ip \sin 2\xi, \tag{4}
\]

where \( I \) — total intensity, \( p \) — fraction of polarized light, \( \chi \) — angle of polarization, \( \xi \) — angle characterizing circular polarization, for linearly polarized light \( \xi = 0 \), for circularly polarized \( \xi = \pm \pi/4 \). From equations (2)–(4) it follows that parameters \( Q \) and \( U \) define linear polarization, and \( V \) — circular. The Stokes parameters are preferable because they all have the same dimension.
Appearance of astronomical object can be specified as dependence of intensity on angular coordinate on the sky \( O_I(\alpha) \), by analogy it is possible to introduce dependencies of other Stokes parameters on \( \alpha \): \( O_Q(\alpha), O_U(\alpha) \) and \( O_V(\alpha) \). Integration of these functions over \( \alpha \) gives total Stokes parameters of the object and methods of their measurement are called polarimetry. Usually dimensionless Stokes parameters are considered: \( q = \frac{Q}{I}, u = \frac{U}{I}, v = \frac{V}{I} \).

Let us compute centroids of functions \( O_{(I,Q,U,V)}(\alpha) \):

\[
\hat{\alpha}_{(I,Q,U,V)} = \frac{\int \alpha O_{(I,Q,U,V)}(\alpha) d\alpha}{\int O_{(I,Q,U,V)}(\alpha) d\alpha}.
\] (5)

Here four equations are given simultaneously, we iterate through the Stokes parameters in subscript. The integration is being performed over the two-dimensional angle \( \alpha \). \( \hat{\alpha}_I \) is the position of the photocenter, \( \hat{\alpha}_{(Q,U,V)} \) are the positions of points which we will call reduced \( q-, u-, v- \) polaricenters, respectively.

An objects with complex geometry can exhibit near-zero total Stokes parameter, e.g. \( Q \), while the corresponding distribution \( Q(\alpha) \neq 0 \). In this case it is worthwhile to consider so called unreduced polaricenter, which equals \( q_\alpha Q \). Measurement of the reduced and/or the unreduced polaricenters constitutes subject of the polaroastrometry.

Let us consider examples of astrophysical objects, differing from the observational point of view for the polaroastrometry.

- If distribution of the polarized flux of the object possesses point symmetry (e.g. main sequence dwarf), or there is no polarized light, then the polaricenters coincide with the photocenter.
- If polarization of the object arises due to selective scattering on interstellar dust, then the polaricenters coincide with the photocenter[1].
- If the source of polarized light is confined in small region (relatively to precision of the method), then reduced polaricenters will coincide with each other.
- In other cases coincidence of reduced polaricenters is possible only for special and therefore unlikely object geometry. For the complex objects polaricenters can deviate from the photocenter even when total polarization is close to zero.

We can conclude that even qualitative polaroastrometry — deviation from photocenter/coincidence of the polaricenters — appear as valuable tool of the astronomical objects diagnosis. Quantitative interpretation of polaricenters parameters is also possible under a priori assumptions about an object.

Here we present a method of the visible light polaroastrometry and investigate its performance on the basis of observations carried out with the prototype of multimode fast camera at 70-cm telescope in Moscow, see description in section 2. The instrument design and observations protocol allows to extract the dimensionless Stokes parameters \( q \) and \( u \) from them, see section 3. Parameters of the unreduced relative polaricenters were estimated by the differential speckle polarimetry [2]. Details of the method in application to polaroastrometry and analysis of noise of parameters estimations are given in section 4. Results in form of the dimensionless Stokes parameters and parameters of unreduced polaricenters are given in section 5 for unpolarized stars, stars polarized by interstellar dust and stars presumably possessing detectable signal. Section 6 is dedicated to practical issues of the polaroastrometry. For conclusion see section 7. Some auxiliary results are presented in appendices B and A.

2 Instrument

For polaroastrometric experiment we used prototype of the multimode fast camera for 2.5-m telescope of Caucasus Highland Observatory. Optical scheme of the instrument is depicted in Fig. 1. The instrument combines features of two-beam polarimeter with rotating half-wave plate and speckle interferometer.

[1] Strictly speaking, this is correct only in absence of the atmospheric dispersion, see subsection 5.2.

2 Instrument
Figure 1: Optical design of the instrument used in experiment. Numbers are for components: 1 — point source, 2 — auxiliary lens, forming image of the point source at first focal plane. Components 1 and 2 are assembled as single unit which can be introduced into the beam manually. 3 — linear polarizer, is introduced into the beam manually; 4 — diaphragm; 5 — half-wave plate; 6 — first lens; 7 — filter; 8 — Wollaston prism; 9 — second lens, 10 — detector.

We used electron multiplying CCD (EMCCD) Andor iXon 897 as the detector. Thanks to the electron multiplication technology readout noise in this type of detectors is greatly reduced, at the cost of two-fold increase in photon noise (so called multiplication noise). Frame rate was chosen to be 35 frames per second, which reduced atmospheric blur.

First optical component of system is the half-wave plate (HWP), it acts as the modulator of the polarization state of incoming radiation. This modulation has two purposes: measurement of both parameters of linear polarization and calibration of the instrumental polarization of optics located after the HWP. The HWP has been purchased in Edmund Optics and consists of several polymer layers selected so that the total phase retardance between ordinary and extraordinary beams deviates from $\pi$ radians by no more than $0.02$ radians inside $V$ band according to specifications. Motorized rotation stage Standa 8MRU was used to control positional angle of the HWP.

Two achromatic lenses with collimated beam between them were used as relay optics. These lenses give magnification of $\approx 4.6$ times, and total equivalent focal length of optical system is $\approx 48.3$ m. Angular sampling frequency is $f_d = 3.22 \times 10^6$ RAD$^{-1}$, while cutoff frequency of the optical system is $f_c = D/\lambda = 1.27 \times 10^6$ RAD$^{-1}$. Therefore Nyquist criterion $f_d > 2f_c$ is fulfilled for this setup, which is necessary for correct computation of the Fourier spectrum of image.

Filter $V$ and Wollaston prism (manufactured by RIVoptics) was installed in collimated beam between relay lenses. The prism splits beam in two, one polarized horizontally and other — vertically.

In first focal plane the slit diaphragm is installed which precludes overlap of beams formed by the Wollaston prism. It limits the field of view to $13'' \times 33''$. Example of the image obtained with the instrument is given in Fig. 2. The instrument has been controlled from the command line C++ program running under OS GNU/Linux.

We adopted the following observations protocol. The detector continuously obtained series of frames in frame transfer mode, meanwhile the HWP rotated with the constant angular speed. Frames period and HWP speed was matched so that HWP positional angle $\theta$ changed by $9^\circ$ over one exposure. With presented setup it was not possible to measure and record $\theta$ for each frame, for this reason we determined it a posteriori.

For each series we introduced the linear polarizer with known orientation of axis into the beam for 10-15% of total series duration in its beginning and end. In these periods the polarization of incoming radiation can be considered as known and it is possible to determine corresponding $\theta$, which can be extrapolated in middle of series, when the linear polarizer was removed from the beam. Data obtained using described technique allows to extract both polaroastrometry and polarimetry from them.

For transformation of measurements from the camera reference system to equatorial coordinate system we measured two basic parameters of the camera: the angular scale $w$ and the parallactic angle $\phi$. As long
as our experiment is not very demanding in terms of precision of these parameters, for their estimation we used binary and double stars with well-known ephemerides: HIP94336, HIP103669, HIP108917 [3, 4]. We acquired long-exposure image of these stars and then measured separation vector of images by approximating one image with other. By comparison of parameters of these vectors with ephemerides we obtained \( w \) and \( \phi \). We followed a convention according to which the parallactic angle is being measured counterclockwise from the north to horizontal axis of camera.

The instrument was installed at Cassegrain focus of the 70-cm telescope AZT-2 located near the Sternberg Astronomical Institute building in Moscow. Nominal focal length is 10.5 m. It was possible to rotate it around optical axis arbitrarily. Mount type of the telescope is equatorial.

Observations have been carried out in August-October 2014 in course of 5 sessions, between them instrument was removed from the telescope. For each session we measured the scale and the parallactic angle of camera, results are given in table 1; also we observed at least one polarization standard and non-polarized star.

Table 1: The angular scale \( w \) and parallactic angle \( \phi \) of the camera for 5 observational sessions. In second column dates of session are given, in third — objects used for determination of camera parameters.

| session number | dates          | objects          | \( w \), mas/pix | \( \phi \), ° |
|----------------|----------------|------------------|------------------|-------------|
| 1              | 3.08-4.08      | HIP94336         | 68.8 ± 0.8       | 69.5 ± 0.3  |
|                |                | HIP108917        |                  |             |
| 2              | 22.08-27.08    | HIP103669        | 68.6 ± 1.7       | 34.0 ± 0.6  |
|                |                | HIP108917        |                  |             |
| 3              | 11.09-18.09    | HIP94336         | 68.9 ± 0.8       | 51.6 ± 0.4  |
|                |                | HIP108917        |                  |             |
| 4              | 19.09          | HIP94336         | 69.3 ± 0.8       | 322.1 ± 0.3 |
|                |                | HIP108917        |                  |             |
| 5              | 02.10          | HIP94336         | 70.4 ± 0.7       | 51.2 ± 0.3  |
|                |                | HIP108917        |                  |             |

Figure 2: Single frame with Vega, obtained with the prototype of multimode fast camera with exposure 30 ms. Separation of images 210 pix, brightness scale in below, in counts.
3 Polarimetry

We now represent adopted in our experiment methodology of polarimetry. It differs significantly from the standard double difference method and this difference is closely related to methodology of polaroastrometry.

Let us remind that the instrument which we use is two-beam polarimeter, main advantage of this solution is that it is not sensitive to variation of the atmospheric transparency and scintillations. On the other hand it is susceptible to difference in transmission of two beams of polarimeter. To mitigate this effect switching of images corresponding to orthogonal polarizations is employed. Switching usually is performed with help of the HWP installed before polarimeter [5]. In our case images are switched every 140 ms.

One potential drawback of this scheme is its high sensitivity to imperfections of polarization modulator — HWP. Nevertheless it is possible to estimate this effect with generalization of the double difference method which we will describe in the following.

Preparatory processing of data was carried out as follows. At first we averaged frames with θ differing by no more than 3°, which is small relatively to angle which HWP passes over one exposure — 9°. Then we subtracted from averaged frames BIAS and constant background. Fluxes corresponding to two images $J_{Lk}$ and $J_{Rk}$ was extracted using aperture photometry, where $k$ is number of the averaged image. Relative positions of apertures for photometry was the same for all frames.

In appendix B it is shown that fluxes are connected to Stokes parameters of the object in instrument reference system $I, Q_d, U_d, V_d$ as follows:

\[
J_{Lk} = W(1+\Delta W_k/2)K_k [I+\zeta(Q_d \cos(4(\theta_k+\kappa(\theta_k)))+U_d \sin(4(\theta_k+\kappa(\theta_k)))+\zeta' V_d \delta(\theta_k) \sin(2(\theta_k+\kappa(\theta_k))))] + \nu_{Lk},
\]

\[
J_{Rk} = W(1-\Delta W_k/2)K_k [I-\zeta(Q_d \cos(4(\theta_k+\kappa(\theta_k)))+U_d \sin(4(\theta_k+\kappa(\theta_k)))-\zeta' V_d \delta(\theta_k) \sin(2(\theta_k+\kappa(\theta_k))))] + \nu_{Rk}.
\]

Here $\theta_k$ — mean angle θ over frame k duration, $K_k$ — mean atmospheric transparency, $W$ — total transmission of optical system, $\Delta W_k$ — difference in transmission for beam of polarimeter, it includes flat field error. Values $\delta(\theta_k), \kappa(\theta_k)$ are the parameters of HWP imperfection, their absolute value are small relatively to unity, see appendix B for details. $\nu_{Lk}$, $\nu_{Rk}$ — noise, independent for separate frames, e.g. photon noise. Please note that some functions (e.g. $\Delta W, K$) depend on $k$ explicitly, and other — through the dependence on θ. This is done intentionally to emphasize that functions of second kind have the same values for $\theta_{k1} = \theta_{k2}$ and $k1 \neq k2$. For functions of first kind this is not true, nevertheless they can be considered as function of θ, but random functions.

Coefficients $\zeta$ and $\zeta'$ are present in equations (6) and (7) because θ and its cosine and sine along with it change in course of obtaining the frame, therefore some averaging occurs. This coefficient equals $\zeta \approx 1 - (2/3)(\theta_f)^2$, where $\theta_f$ is angle which HWP pass over the exposure. At $\theta_f = 9^\circ$ coefficient $\zeta$ equals 0.984.

From $J_{Lk}$ and $J_{Rk}$ we formed value $P_k$:

\[
P_k = \frac{J_{Lk} - J_{Rk}}{J_{Lk} + J_{Rk}}.
\]

Let us substitute here expressions for fluxes (6) and (7):

\[
P(\theta) = \zeta(q_d \cos(4\theta) + u_d \sin(4\theta)) + \sum_{n=0}^{\infty} \omega_q(n) \cos(n\theta) + \sum_{n=0}^{\infty} \omega_u(n) \sin(n\theta).
\]

Here all functions describing instrumental effects — $\Delta W, \delta, \kappa$ — are decomposed in Fourier series over θ and grouped in last two sums. In following we will designate Fourier spectrum of $P(\theta)$ as $\tilde{P}(f_\theta)$.

Coefficient $\zeta$ can be considered as effectiveness of the polarimeter [6], it doesn’t require special calibration measurement.

From equation (9) one can see the difference between signal carrying information about linear polarization of the source and instrumental effects such as variations of the system transmission and HWP imperfections.
Signal occupies only one frequency in spectrum $\tilde{P}(f_\theta)$, meanwhile noise is distributed over all frequencies, including signal frequency. Therefore linear polarization can be estimated as amplitude of fourth harmonics of $\tilde{P}(f_\theta)$, but it will be inevitably biased by unknown values $\omega_{u,q}(4)$. All that remains is to estimate their amplitudes by interpolation and take it as a measure of uncertainty of estimation of $q_d$ and $u_d$.

In Fig. 3 spectra $|\tilde{P}(f_\theta)|^2$ for 3 presumably unpolarized stars age given. As unpolarized stars we took main sequence stars closer than 66 pc as it was done in [7]. One can wee spectra are quite uniform, there are no marked frequencies, which proves possibility of estimation fourth harmonics uncertainty. It is notable that $|\tilde{P}(f_\theta)|^2$ slowly rises at low frequencies for bright star. This effect can be explained by the fact that photon component of noise for this star is sufficiently low to make instrumental effects visible. Detailed theoretical and experimental analysis of noise of $\tilde{P}(f_\theta)$ is of great interest but falls beyond the scope of this work. It will be performed in application to polaroastrometry, subsection 4.3.

Spectrum $|\tilde{P}(f_\theta)|^2$ for o Ceti (October 2, 2014) is presented in Fig. 3. It is seen that in this case fourth harmonics is significantly larger than the noise level which indicates presence of detectable linear polarization. For polarization standard stars fourth harmonics amplitude exceeds noise level by hundred times.

The advantage of method of spectral decomposition of $P(\theta)$ before wide-spread double difference method [5] is that former allows to give adequate estimation of uncertainty of resulting value caused by instrumental effects, especially HWP imperfections (see appendix B). In case of polarimetry such estimation is important only for brightest stars, but for polaroastrometry sensitivity to HWP imperfections significantly stronger and adequate estimation of its effect becomes critical. Similar approach to analysis of data from polarimeters with rotating HWP was applied before in millimeter waves [8].

From $q_d$ and $u_d$ one can estimate degree of polarization $p_d$ and angle of polarization $\chi_d$, latter can be translated to equatorial coordinate system using the value of parallactic angle of camera.
We didn’t correct measurements for the instrumental polarization because earlier we showed that for AZT-2 telescope it is smaller than $10^{-4}$ in amplitude, which is expectable because optical design possess point symmetry.

In order to verify method and test its stability we observed polarization standards and presumably unpolarized stars in every session, results are given in table 2 in form of $p$ and $\chi$. For comparison expected values from [9] are given. For standards degree of polarization coincides with precision 3%, angle of polarization — 1°, which is acceptable. Stability of $p$ is 1%, of $\chi$ — 1°, difference between sessions exceeds errors only slightly.

In the function of presumably unpolarized stars we selected main sequence dwarfs closer than 66 pc and farther than 10° from the Galaxy plane, results also are present in Table 2. Small value of measured polarization proves the fact that instrumental polarization of AZT-2 is small.

Polarimetry results also are given in form of dimensionless Stokes $q$ and $u$ defined in equatorial coordinate system in Table 3.

| HD    | S/E | $p \times 10^4$ | $\chi$,° |
|-------|-----|-----------------|----------|
| HD7927| S1  | 332.9 ± 0.3     | 93.1 ± 0.3 |
|   φ Cas   | S3  | 334.5 ± 0.5     | 92.7 ± 0.4 |
|        | S4  | 330.3 ± 0.8     | 93.1 ± 0.3 |
|        | E   | 329.8 ± 2.5     | 91.1 ± 0.2 |
| HD204827| S1  | 547.7 ± 1.8     | 59.1 ± 0.3 |
|        | S2  | 548.8 ± 2.8     | 60.5 ± 0.7 |
|        | S3  | 547.7 ± 2.7     | 59.9 ± 0.4 |
|        | S4  | 549.7 ± 1.7     | 60.5 ± 0.3 |
|        | S5  | 554.4 ± 1.7     | 61.5 ± 0.3 |
|        | E   | 532.2 ± 1.4     | 58.7 ± 0.1 |
| HD184960| S1  | 1.8 ± 0.8       |          |
| HD221477| S1  | 3.5 ± 3.3       |          |
| HD210027| S5  | 1.3 ± 0.5       |          |

4 Polarostrometry

4.1 Basics

Let us consider images obtained with ideal two-beam polarimeter:

$$F_L(\alpha) = I(\alpha) + a(\theta)Q(\alpha) + b(\theta)U(\alpha),$$

$$F_R(\alpha) = I(\alpha) - a(\theta)Q(\alpha) - b(\theta)U(\alpha).$$

In our case $a(\theta) = \zeta \cos(4\theta)$ and $b(\theta) = \zeta \sin(4\theta)$, where $\theta$ — orientation of HWP. Principal observable of method — polarostromatic signal — is half–difference of the photocenter position of these images:

$$\Delta(\theta) = \frac{1}{2} \left[ \frac{\hat{\alpha}_I + a(\theta)\hat{\alpha}_Q + b(\theta)\hat{\alpha}_U}{I + a(\theta)Q + b(\theta)U} - \frac{\hat{\alpha}_I - a(\theta)\hat{\alpha}_Q - b(\theta)\hat{\alpha}_U}{I - a(\theta)Q - b(\theta)U} \right].$$

Supposing that $q^2 + u^2 \ll 1$ (which is correct for the most of astronomical objects), it is possible to derive that

$$\Delta(\theta) = a(\theta)q\alpha_Q + b(\theta)u\alpha_U.$$
In following subsection we will show how value $\Delta(\theta)$ and $q\alpha_Q$ and $u\alpha_U$ in their turn is being estimated from the real data.

Vectors $q\alpha_Q$ and $u\alpha_U$ — unreduced polaricenters — represent the final product of the polaroastrometry. Let us denote their components as follows: $q\alpha_Q = (s_q^*, t_q^*)$ and $u\alpha_U = (s_u^*, t_u^*)$. Having $q$ and $u$ and given that they significantly deviate from zero, unreduced polaricenters can be converted to reduced ones. Their components: $\alpha_Q = (s_q, t_q)$ and $\alpha_U = (s_u, t_u)$.

According to accepted in optics convention, beam with Stokes vector $(I, I, 0, 0)$ is polarized horizontally, i.e. plane of polarization is oriented along OX axis. In astronomical polarimetry for such beam polarization plane will be oriented towards the north celestial pole (NCP). Therefore for derivations we will assume that the OX is directed towards the NCP. Then for the object with $s_q > 0, t_q = 0$ $q$–polaricenter is displaced towards North, with $s_q = 0, t_q > 0$ — towards East, what may seem counter-intuitive.

### 4.2 Extraction of polaricenters parameters

The polaroastrometric signal $\Delta$ can be extracted from the observational data in various ways, e.g. by simple centroid computation or by approximation of one image with another. But to our mind most reasonable means is to apply differential speckle polarimetry (DSP). The DSP is more general method of extraction of polarization information about source with diffraction resolution [2].

Principal observable of DSP method is $R(f)$:

$$
R(f) = \frac{\langle \tilde{F}_L(f)\tilde{F}_R^*(f) \rangle}{\langle F_R(f)F_R^*(f) \rangle} - N_{eR}^{-1},
$$  

(14)

where $f$ — is spatial frequency, $\tilde{F}_L(f), \tilde{F}_R(f)$ — Fourier spectra of images for left and right beams of Wollaston prism, $N_{eR}$ — averaged number of photons forming right image. The averaging is taking place over finite number of frames.

Computation of $R(\theta_k, f)$ started with reduction for BIAS, constant background and equalizing to zero pixels having counts less that 5 RMS of read-out noise. Then we cut two rectangular windows of 100 × 100 pixels each. Left window was centered on the brightest pixel of the left image, right window position was displaced relatively to the left by constant vector. Fourier spectra were evaluated separately for the left and right images for many frames and were combined using formula (14). Only images having $\theta$ differing by no more than 3° took part in the combination. In such a manner a set of $R(\theta, f)$ for different $\theta$ was formed.

Now we construct the model of $R(\theta, f)$ data formation. As starting point we will use main result of [2]: $R$ averaged over ensemble of samples of random atmospheric phase and photon noise equals to:

$$
\overline{R}(f) = (1 + \Delta R(f))R_0(f),
$$  

(15)

where $R_0$ is the ratio of visibilities of object corresponding to left and right beams, and $\Delta R(f)$ is derived by the differential polarization aberrations in system. This expression accounts for three effects: atmospheric optical turbulence, instrumental polarization and photon noise.

In order to adequately describe our experiment it is needed to account additionally for these instrumental effects:
1) deviations of beams by the Wollaston prism; 2) dispersion of the Wollaston prism and the atmosphere; 3) differential aberrations of the HWP; 4) transformation of polarization state by the HWP.

First two effects can be described by multiplying Fourier spectra of images by respective filters:

$$
\tilde{F}'_L(f) = \tilde{F}_L(f)D_L(f)e^{i2\pi(\rho_L,f)},
$$  

(16)

$$
\tilde{F}'_R(f) = \tilde{F}_R(f)D_R(f)e^{i2\pi(\rho_R,f)},
$$  

(17)

where $D_L(f)$ and $D_R(f)$ are filters describing the dispersion, $\rho_L$ and $\rho_R$ account for images displacement in left and right beams of Wollaston prism. Both effects doesn’t depend on angle of the HWP as long as the atmospheric dispersion affects different polarization states equally and the Wollaston prism is installed after the HWP.
After substitution of $\tilde{F}_L(f)$ and $\tilde{F}_R(f)$ in equation (14) relation (15) is being modified as follows:

$$\tilde{R}(f) = (1 + \Delta R(f))R_0(f)D(f)e^{i2\pi(\rho \cdot f)},$$

(18)

where $D(f) = D_L(f)/D_R(f)$ and $\rho = \rho_L - \rho_R$.

Now let us divide input of differential aberrations of the telescope, the HWP and the instrument. We assume that the telescope and the instrument are separated by the HWP. Possibility of such division is provided by additivity of bias $\Delta R(f)$ [2]. Note that all effects entering before the HWP and in it depend on its positional angle $\theta$.

$$\bar{R}(\theta, f) = R_0(\theta, f)(1 + \Delta R_T(\theta, f) + \Delta R_{HWP}(\theta, f) + \Delta R_I(\theta, f))D(f)e^{i2\pi(\rho \cdot f)},$$

(19)

Let us suppose that for some object $R_0 \neq 1$, meanwhile it is unresolved with given instrument. In this case decomposing exponent in Fourier transform in Taylor expansion and keeping only first component it is possible to obtain [10] [2] that $R_0(\theta) = \exp\{i\pi(4\Delta_\theta \cdot f)\}$, and, moreover, exponent argument is significantly less than unity for all frequencies. Here $\Delta$ is half of vector connecting photocenters of left and right images in the polarimeter. Substituting its value from (13), we obtain:

$$R_0(\theta) = \exp\{i4\pi\zeta[f_xS(\theta) + f_yT(\theta)]\},$$

(20)

where

$$S(\theta) = s^*_u\cos(4\theta) + s^*_v\sin(4\theta),$$

(21)

$$T(\theta) = t^*_u\cos(4\theta) + t^*_v\sin(4\theta).$$

(22)

Let us remind that values $s^*_u, s^*_v, t^*_u, t^*_v$ are parameters of unreduced relative polaricenters.

In appendix [13] it is shown that HWP effect $\Delta R_{HWP}(\theta, f)$ can be represented as $1 + \Delta R_{HWP}(\theta, f) = \exp\{i\phi_{HWP}(\theta, f)\}$, and $\phi_{HWP} \ll 1$.

As long as optical system of the telescope is very slow and possesses point symmetry, for our level of precision $\Delta R_T$ can be considered equal to unity [2]. Validity of this simplification is proved by measurements of unpolarized stars which will be presented below.

Taking all these reasons into account, expression (19) can be transformed as follows:

$$\bar{R}(\theta, f) = \exp\{i4\pi\zeta[f_xS(\theta) + f_yT(\theta)] + i\phi_{HWP}(\theta, f)\}(1 + \Delta R_I(\theta, f))D(f)e^{i2\pi(\rho \cdot f)}.$$  

(23)

Expression (23) is final model of observational data. Note that real observations are not averaged over the ensemble and have some noise (which will be discussed later):

$$R_n(\theta_k, f) = \bar{R}_n(\theta_k, f) + \mathcal{N}_k(f),$$  

(24)

where $\mathcal{N}_k(f)$ — noise sample.

Now we proceed to algorithm of polaricenters extraction and estimation of their error from measurements of $R(\theta, f)$.

First operation is normalizing of $\tilde{R}(\theta, f)$ by its average over angle $\theta$. Taking into account that exponent argument in (25) is much less than unity we obtain:

$$\tilde{R}_n(\theta, f) = \frac{R_n(\theta, f)}{\langle \bar{R}_n(\theta, f) \rangle_{\theta}} = \exp\{i4\pi\zeta[f_xS(\theta) + f_yT(\theta)] + i\phi_{HWP,n}(\theta, f)\} + \mathcal{N}_k(f),$$  

(25)

where $\phi_{HWP,n}$ is normalized $\phi_{HWP}$. Example of real measurements of $R(\theta, f)$ before and after normalization is presented in Fig. 4. It can be seen that normalization removes most part of instrumental effects.

Next step was extraction of phase from $R_n(\theta, f)$. Then dependence of phase on frequency $f$ was approximated by least squares with plane $4\pi(c(\theta)f_x + d(\theta)f_y)$. Reduced $\chi^2$ turned out to be 1–2, which points out to correspondence of data to model. For plane parameters:

$$c(\theta) = \zeta S(\theta) + G_n(\theta) + N_x,$$

(26)
Figure 4: $\mathcal{R}(\theta, f)$ at $\theta = 229^\circ$, first column before normalization by average over $\theta$, second — after normalization. First row is amplitude, second — phase. Spatial frequency normalized by cutoff frequency $f_c = D/\lambda$ is measured along axes. Observations circumstances: object α Lyr, $\tau = 30$ ms, $N = 5$, total accumulation time is 5.4 s.

$$d(\theta) = \zeta T(\theta) + H_n(\theta) + N_y,$$

where functions $G_n(\theta)$ and $H_n(\theta)$ are determined by the HWP effect. As a result of least squares linearity they can be interpreted as parameters of plane approximating $\phi_{\text{HWP}, n}$. $N_x$ and $N_y$ are components describing joint effect of the atmospheric and photon noise.

After estimation of $c(\theta)$ and $d(\theta)$ for different $\theta$ we decomposed them in Fourier series by this parameter:

$$c(\theta) = s_q^* \cos(4\theta) + s_u^* \sin(4\theta) + \sum_{n=1}^{\infty} g_q(n) \cos(n\theta) + \sum_{n=1}^{\infty} g_u(n) \cos(n\theta),$$

$$d(\theta) = t_q^* \cos(4\theta) + t_u^* \sin(4\theta) + \sum_{n=1}^{\infty} h_q(n) \cos(n\theta) + \sum_{n=1}^{\infty} h_u(n) \cos(n\theta),$$

where effect of HWP imperfection and random noise decomposed in Fourier series $g_q$, $g_u$, $h_q$ and $h_u$. From equations one can see that behaviour of $c(\theta)$ and $d(\theta)$ is very similar to dependence of $P(\theta)$ in polarimetry technique described earlier. Therefore we analysed them by analogy with section 3. In both formulae first
two components corresponds to signal and sums to noise input. Signal was estimated as complex amplitude of fourth harmonics and its uncertainty by interpolation of noise spectrum from adjacent frequencies.

In Fig. 5 Fourier spectra $d(\theta)$ for $\omega$ Cet and $\alpha$ Lyr are presented. Total number of photoelectrons used in processing was $1.5 \times 10^9$ and $3.8 \times 10^{11}$ for $\omega$ Cet and $\alpha$ Lyr, respectively. For $\omega$ Cet uncertainty of signal can be adequately estimated from the high-frequency part of spectrum. For the $\alpha$ Lyr the low-frequency part is much larger than high-frequency part due to smaller input of the photon noise. In appendix B it is shown that observed noise spectra in this case can be explained by variations of HWP parameters (phase retardance and fast axis orientation) over its area. Indeed, rotation axis of HWP may not coincide with center of beam and latter illuminates different parts of the HWP at different angles $\theta$. Moreover, position of illuminated portion of HWP depends on position of image in first focal plane because HWP is installed not in the exit pupil of system. Finally we note that existing mechanical structure is susceptible to flexures. All these features lead to appearance of low-frequency part on noise which cannot be calibrated out. All that remains is to estimate signal uncertainty by interpolation of harmonics 2, 3, 5 amplitudes in quadratic sense on frequency of fourth harmonics (dash-dotted line). Hence for the $\alpha$ Lyr we cannot reliably state that there is detectable polarostrometric signal.

Obtained parameters of unreduced relative polaricenters $s_{q*}, s_{u*}, t_{q*}, t_{u*}$ and their errors were converted in equatorial coordinate system using relations (34)–(37) from appendix A. Results of measurements and their discussion are given in section 5. Next subsection is dedicated to short analysis of noise component of spectra.
4.3 Analysis of polaroastrometric signal noise

As first step we identify dependence of noise estimations from high– and low– frequency parts of spectra \( \tilde{c}(\theta) \) and \( \tilde{d}(\theta) \) on the object brightness. We will denote the high–frequency error as \( \sigma_{HF} \), low–frequency — \( \sigma_{LF} \). We introduced in raw data for bright star \( \delta\) Cyg additional Poisson noise, effectively dimming star by 2, 4 and 6 magnitudes. After this we performed processing described in previous subsection and estimated \( \sigma_{HF} \) and \( \sigma_{LF} \), results are given in Fig. 6 for \( X \) and \( Y \) coordinates separately. In this subsection we assume that the \( X \) coordinate is measured along CCD rows.

As one can see, \( \sigma_{HF} \) depends on photoelectrons number as \( 25/\sqrt{N_e} \) pix (dash-dotted line in Fig. 6), which points out to the fact that high–frequency part of spectra \( \tilde{c}(\theta) \) and \( \tilde{d}(\theta) \) are produced by photon noise. It is interesting that photon limit of astrometry noise is \( \beta/\sqrt{N_e} \) pix, where \( \beta \) — full width at half maximum (FWHM) of image. As long as in our method light is splitted in two halves, and then we take half–difference between image positions, for considered signal photon limit of astrometry noise is also \( \beta/\sqrt{N_e} \). Hence high–frequency noise of photocenter determination corresponds to photon limit of astrometry noise at \( \beta = 25 \) pix or \( \beta = 1.7'' \), which is comparable to FWHM of long-exposure image.

Photon noise dominates at \( N_e \lesssim 10^9 \), for used telescope and instrument it corresponds to total integration of 500 sec from object \( V \approx 6^m \). For larger \( N_e \) low–frequency part \( \sigma_{LF} \) gives larger input. Latter varies from 0.6 to 1.5 mpix and doesn’t show clear dependence on \( N_e \).

Let us consider now correlation properties of errors, in order to do this, we split series for \( \delta \) Cyg in five equal parts and estimate \( \sigma_{HF} \) and \( \sigma_{LF} \) for them separately, see Fig. 6. From figure one can see that \( \sigma_{HF} \) is uncorrelated on timescales comparable to the series duration and can be reduced by increasing of the accumulation time. Meanwhile, for \( \sigma_{LF} \) this is not correct. Therefore we can conclude that realization of
low–frequency error within single series is approximately constant. On the other hand it changes from series
to series and cannot be calibrated out.

Also it is remarkable that the error in $X$ coordinate generally larger than in $Y$. This is caused by the
fact that $X$–separation of images formed by Wollaston prism depends on $X$ component of the beam angle of
incidence. Therefore any motion of the image in the focal plane (errors of tracking, vibrations, atmospheric
jitter) leads to increase of error in $X$. Apart from this, dispersion of Wollaston prism is directed along $X$
axis and amplitude of spectrum falls more rapidly in this direction, that also leads to increase in error.

In Fig. 6 $\sigma_{HF}$ and $\sigma_{LF}$ for some objects considered in section 5 are given. As one can see, for polarized
stars and Mira type variables properties of error are similar to ones of unpolarized stars, except for sporadic
cases.

5 Results

In order to test efficiency of method we observed objects of several types: presumably unpolarized stars
(type N), stars polarized by the interstellar dust (type P) and Mira type pulsating variables (type M).
Methodology of observations and data processing was identical for all objects. Observations circumstances
and processing results (polaricenters parameters and Stokes parameters) are given in table 3. Sections in
Table 3 correspond to defined types.

5.1 Unpolarized stars

Main sequence dwarfs appear to be good test objects for initial testing of the polaroastrometry because
for these objects parameters of unreduced polaricenters presumably very small due to low polarization even
close to edge of disc and symmetry of image. We observed three single stars satisfying the following criteria:
spectral class from A to M, luminosity class V, distance less than 66 pc and galactic latitude more than
10° — HD184960, HD221477, HD9562. Last star shows small total polarization which can be caused by
interstellar extinction (sometimes its effect is noticeable even for nearby stars [12]).

Another two observed stars satisfy listed criteria but have peculiarities. HD180161 is BY Dra type
variable of spectral class G8V. Brightness variations for them are caused by starspots occupying large part
of stellar surface and rotation of the star. Let us estimate polaroastrometric signal expected for HD180161.
Expected angular diameter is 330 $\mu$as. Polarization at the edge of disc can be crudely assumed to be
the same as solar — $\approx 10^{-3}$ [13]. Therefore for the most favorable geometry expected total polaricenter
deviation is 0.3 $\mu$as, what is much less than error of our measurements. Meanwhile total polarization for
this star is clearly detected in our experiment (see table 3). However, single measurement doesn’t allow to
make conclusion about origin of polarization, it can be attributed to both the star itself and the interstellar
dust. Star HD210027 = \iota\ Peg is spectroscopic binary system, angular size of semi-major axis of its orbit is
10.3 mas, period 10.2$^d$, spectral classes of components are F5V and G8V [14]. One can expect small total
polarization due to ellipticity effect but we didn’t detect it. Polaroastrometric signal is likely to be negligible
taking into account symmetry of geometry.

We put spectroscopic binary HD186882 = \delta Cyg in group of unpolarized stars. Main component of
this system is brighter by 3.4$^m$ than secondary, has spectral class B9.5IV and falls into class of rapidly
rotating stars. Expected angular diameter amounts to 550 $\mu$as, expected degree of polarization at disc edge is
$\lesssim 5\%$[15]. Therefore total expected deviation of polaricenters for this star is $\lesssim 25\mu$as what is undetectable
with our instrument. According to [12] total polarization for HD186882 is $(1.08 \pm 0.02) \times 10^{-4}$.

For mentioned 6 stars expected polaroastrometric signal is much less than errors of measurements what
was proved in experiment, see table 3. Among others it allows us to argue that our measurements have not
any bias in polaricenters parameters determination, at least down to 100 $\mu$as. Small value of bias of this
kind is predictable if one takes into account symmetry of optical scheme.
5.2 Stars polarized by interstellar dust

Negligible polarastrometric signal is expected for stars, analogous to type N, but polarized by the interstellar dust (type P), because latter changes polarization of each point of star image identically. However small signal can be generated for observations through the atmosphere due to joint effect of dependence of degree of polarization on wavelength and atmospheric dispersion.

Indeed, for some polarization standards variations of degree of polarization across V band reach 0.2 ± 0.3% in absolute measure [9]. Hence the polarized flux has effective wavelength differing from the effective wavelength of total flux by Δλ. For computation of the effective wavelength we used stellar spectra from [16], sensitivity curve of detector, transmission curve of filter and dependence of degree of polarization from [9]. Difference in effective wavelengths are Δλ = −1.027 nm and Δλ = −0.480 nm for HD204827 and HD7927, respectively. These stars also display dependence of angle of polarization on wavelength but it is possible to neglect it (maximum amplitude over V band is 0.8°) [9].

Atmospheric dispersion influence makes polaricenters displace relatively to the photocenter and this displacement is the same for polaricenters q and u, as long as for them Δλ is the same. Total displacement is \( \sqrt{s_q^2 + t_q^2} = \sqrt{s_u^2 + t_u^2} = \delta \Delta \lambda \tan z \), where θ — angular atmospheric dispersion, for place and time of observations it is 2.12 × 10⁻⁸ radian/nm, z — zenith distance. Estimation of amplitude of unreduced polaricenters can be made according to definition. At z = 45° for HD204827 and HD7927 they are 254 μas and 69 μas, respectively. In our experiment such small signal cannot be reliably measured.

Table 3: Circumstances of observations, unreduced relative polaricenters parameters and Stokes parameters for some observed objects. Meaning of columns: 1 — number in HD catalogue, 2 — universal time, for all rows year is 2014, 3 — session number, 4 — mean zenith distance during series, 5 — mean long-exposure FWHM during series, 6 — total number of accumulated photoelectrons divided by 10⁸, 7—10 — unreduced relative polaricenters parameters in equatorial coordinate system, 11—12 — dimensionless Stokes parameters in equatorial coordinate system.

| HD/name | UT | S | z° | \( \beta' \) | \( N_e \) | \( s_q, \mu as \) | \( t_q, \mu as \) | \( s_u, \mu as \) | \( t_u, \mu as \) | \( q \times 10^4 \) | \( u \times 10^4 \) |
|---------|----|---|-----|-----------|--------|--------------|-------------|--------------|-------------|----------------|----------------|
|         | 1  | 2 | 3   | 4         | 5      | 6            | 7           | 8            | 9           | 10           | 11            | 12            |
| 186882  | 08-03 19:53 | 1 | 11.7 | 2.2 | 69       | +20±30 | +10±50 | +40±30 | +0±50 | 0.8±0.5 | 0.8±0.5 |
| 184960  | 08-03 20:12 | 1 | 4.4  | 2.2 | 5.6      | +130±90 | −70±100 | −0±90 | −90±100 | −1.0±0.8 | −0.5±0.8 |
| 180161  | 08-04 18:42 | 1 | 9.3  | 1.8 | 3.1      | +50±70 | +70±80 | +120±70 | −10±80 | 3.4±0.9 | 3.4±0.9 |
| 221477  | 08-04 22:45 | 1 | 24.8 | 2.7 | 1.8      | −40±180 | +19±180 | +100±180 | +180±180 | 0.3±1.8 | −3.9±1.8 |
| 210027  | 10-02 19:51 | 5 | 32.2 | 2.0 | 97       | +110±80 | +10±90 | −50±80 | −70±90 | 0.3±0.4 | −1.3±0.4 |
| 210027  | 10-02 20:11 | 5 | 33.6 | 2.1 | 96       | +90±80 | +20±90 | +10±80 | −100±90 | −0.6±0.3 | 0.3±0.3 |
| 9562    | 10-02 21:16 | 5 | 63.9 | 3.9 | 18       | +70±100 | +10±100 | +210±100 | −40±10 | 3.5±0.5 | −1.4±0.5 |
|         |         |   |      |      |         |         |         |         |         |         |         |
|         |         |   |      |      |         |         |         |         |         |         |         |
| star polarized by interstellar dust |
| 204827  | 08-04 20:35 | 1 | 12.5 | 2.0 | 2.3       | +90±120 | +110±120 | +30±120 | +50±120 | −258±5.3 | 483.0±3.2 |
| 204827  | 09-19 19:00 | 4 | 3.2  | 2.1 | 1.8       | +70±180 | −20±160 | −140±180 | +210±160 | −282±7.5 | 471.3±3.7 |
| 7927    | 09-18 16:45 | 3 | 46.9 | 2.0 | 73       | +10±70 | −60±80 | −30±70 | −150±80 | −33±1.0 | −31.1±4.3 |
| 7927    | 09-19 17:44 | 4 | 40.1 | 2.2 | 71       | −100±70 | +200±60 | +120±60 | −328±12.4 | −35.6±4.0 |         |
|         |         |   |      |      |         |         |         |         |         |         |         |
| Mira type variables |
| χ Cyg   | 08-03 21:24 | 1 | 24.5 | 2.6 | 9.5      | +230±90 | +210±80 | −50±90 | −240±80 | −22.5±0.8 | −28.9±0.7 |
| χ Cyg   | 08-04 19:16 | 2 | 23.6 | 2.6 | 9.6      | +220±70 | +220±50 | −0±70 | −320±50 | −23.9±0.6 | −31.5±0.6 |
| χ Cyg   | 09-18 18:05 | 3 | 23.6 | 2.0 | 6.5      | +500±100 | +430±110 | −60±100 | +50±110 | −23.1±1.1 | −40.6±1.0 |
| R Tri   | 09-19 21:45 | 3 | 30.6 | 1.8 | 31       | −50±40 | −60±40 | −30±40 | −180±40 | 20.5±1.0 | −74.1±0.5 |
| R Tri   | 09-20 21:37 | 4 | 31.1 | 2.4 | 66       | +20±70 | +50±100 | −70±70 | −30±70 | 21.0±1.1 | −75.7±0.7 |
| α Cet   | 09-20 23:01 | 4 | 60.2 | 2.9 | 8.8      | +10±100 | −430±120 | +850±110 | −750±120 | 0.9±0.8 | −15.0±0.8 |
| α Cet   | 10-02 21:55 | 5 | 60.2 | 2.9 | 8.8      | +10±100 | −430±120 | +850±110 | −750±120 | 0.9±0.8 | −15.0±0.8 |
| α Cet   | 10-03 22:37 | 5 | 58.8 | 3.1 | 7.2      | −130±350 | −440±240 | +1050±350 | −740±240 | 1.4±1.4 | −17.0±1.4 |

|          |         |   |      |      |         |         |         |         |         |         |         |

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5.3 Mira type variables

Mira type variables are pulsating stars at late stages of evolution. These objects are expected to demonstrate detectable deviations of polaricenters from photocenter for two reasons. Firstly, significant fraction of visible light radiation is being scattered by dust envelope in close vicinity of star [17], which lead to polarization of radiation. Secondly, interferometric observations show that some Mira variables display significant asymmetry [18, 19].

We considered three Mira variables, optimal for observations in northern hemisphere in August–October: χ Cyg, R Tri and o Cet. Measured total polarization and polaricenters parameters are given in Table 3 in third division. χ Cyg and o Cet show significant deviation of values $s_q^*, s_u^*, t_q^*, t_u^*$ from zero, for R Tri it is not so; meanwhile total polarization for R Tri is the largest. These results conform qualitatively to measurements of [19]. Authors of this study estimated asymmetry of stars images by measuring closure phases formed by three baselines of IOTA interferometer in NIR. For χ Cyg significant deviation of the closure phase from 0°/180° was detected what points out to the asymmetry of image, R Tri was found to be symmetrical. Authors of [18] also showed using PTI interferometer that R Tri has axial symmetry. The axial symmetry can explain situation when total polarization is large while polaricenters are undetectable.

o Cet earlier was found to be significantly asymmetrical in visible light [20], as well as in MIR [21]. Angular diameters of considered stars are ≈ 22 mas for χ Cyg [19], ≈ 22 mas for o Cet [22] and 4.6 mas for R Tri [18].

For both stars demonstrating significant signal the reduced polaricenters $q$ and $u$ clearly do not coincide: $s_q^*/q \neq s_u^*/u$ and $t_q^*/q \neq t_u^*/u$. Therefore one can conclude that polarized flux is being emitted from many points distributed over the image of the star, which is expectable.

In order to test methodology we repeated observations of χ Cyg and o Cet in nearby epochs, corresponding measurements agree well, see table 3. In case of o Cet between second and third measurements the instrument was rotated with respect to the telescope by 57.5° counterclockwise. As one can see $s_q^*/q$, $s_u^*/q$, $t_q^*/u$, $t_u^*/u$, translated to equatorial coordinate system remained the same after rotation. For larger difference in epochs — 55.0d (between second and third measurements of χ Cyg) polaricenter parameters change significantly. It is worth noting that brightness of χ Cyg for the same period reduced from $V = 7.5^m$ to $V = 8.5^m$ (AAVSO data).

As long as main purpose of present study is the development of methodology, we will not attempt to interpret measurements. It is sufficient that polaraostronomic signal is detectable and reproducible.

6 Practical issues of polaraostronomy

As we noted before in subsection 4.3 observed value of high–frequency polaraostronomic error coincides with expected level of photon noise of astrometry at FWHM of long–exposure image. This simple dependence can be used for crude estimation of expected level of error on larger telescope, see Fig. 7. It is interesting that error predicted from simulation [2] is by an order of magnitude smaller, causes of this discrepancy are out of scope of this study.

In subsection 4.3 we also showed that for existing instrument dependence of noise on photoelectron number is valid only while photon input in astrometry noise is more than 1 mpx. For brighter objects HWP imperfections starts to prevail. Extrapolating results of present experiment to larger apertures with constant beam diameter on HWP one can say that for 2.5 m telescope this effect will lead to error of 20 µas and for 6 m telescope — 8 µas (see Fig. 4).

As we found out in appendix B HWP imperfection effect potentially can be reduced by alignment of its rotation axis and center of beam. This can be achieved by installation of HWP in exit pupil plane or by guiding of object (probably fast guiding). It is necessary to increase the stiffness of structure in order to limit displacements of beam on the HWP to 0.1 mm in any position of instrument relative to vector of gravity. In case of effectiveness of these measures photon limit of astrometry will be reached for brighter objects also.
Application of polaroastrometry behind adaptive optics seems very attractive, because reduction of FWHM will lead to proportional reduction of noise (see Fig. 7). However in this case some systematic effect can arise because AO systems usually have many oblique reflections in optical path. Overcoming of these effects may require significant efforts [23].

7 Conclusion

The purposes of present work was the development of methodology and investigation of precision limits of the polaroastrometry. Analysis has been performed on basis of observational data obtained with help of multimode fast camera prototype. This instrument combines features of two-beam polarimeter with rotating half-wave plate (HWP) and speckle interferometer. As feeding optics we used 70-cm telescope installed in Moscow near SAI main building.

Principal observable of method is half-difference of photocenters positions of images splitted by the Wollaston prism. We call this value polaroastrometric signal. Analysing dependence of this value on positional angle $\theta$ of HWP we obtained estimations of parameters of unreduced relative polaricenters — deviations of centroids of polarized flux $Q/U$ from the photocenter of total flux, multiplied by the value of corresponding dimensionless Stokes parameter, $q$ or $u$.

We performed sketchy analysis of polaroastrometric signal error dependence on circumstances of experiment. It was found that at total number of accumulated photoelectrons $N_e$ less than $10^9$ photon noise
dominates, it can be approximated crudely as $1.7''/\sqrt{N_e}$. At larger $N_e$ HWP imperfections input start to prevail, which has approximately constant level of $50 - 100 \mu$as. This effect can be reduced by an order of magnitude given that the axis of HWP rotation and the beam center are aligned with precision of 0.1 mm. We give rough estimation of method sensitivity for larger telescopes.

Observations of unpolarized main–sequence stars (which presumably don’t have detectable polarastrometric signal) showed that measurements are unbiased at least at level $\approx 100 \mu$as. Main sequence stars polarized by interstellar dust also doesn’t display detectable signal what points out to absence of cross-talk between strong polarization and polarastrometric signal.

For method verification purposes we selected three Mira type variables $\chi$ Cyg, o Cet, R Tri because they are expected to have detectable signal. For $\chi$ Cyg measures unreduced relative polaricenters are $s_q^* = +220 \pm 70 \mu$as, $t_q^* = +220 \pm 50 \mu$as, $s_u^* = 0 \pm 70 \mu$as, $t_u^* = -320 \pm 50 \mu$as in equatorial coordinate system for UT 19:16 August 4th, 2014. For o Cet these values amount to $s_q^* = +10 \pm 110 \mu$as, $t_q^* = -430 \pm 120 \mu$as, $s_u^* = +850 \pm 110 \mu$as, $t_u^* = -750 \pm 120 \mu$as for UT 21:55 October 3rd, 2014. For R Tri we did not detect any significant signal. For $s > 0, t = 0$ polaricenter is displaced to the North, for $s = 0, t > 0$ — to East. For testing methodology we repeated measurements in different, but close epochs, and at different orientations of the instrument relatively to the telescope. We didn’t detect significant difference in measurements which proves stability of method. Also as by-product we obtained polarimetry of mentioned objects with precision of $(1 \div 5) \times 10^{-4}$. Polarastrometric observations will be continued at 2.5-m telescope of Sternberg Astronomical Institute with multimode fast camera.

Mira type variables are suitable as test objects because apparently they have greatest polarastrometric signal among celestial objects. It seems that for relatively small and unfortunately located 70-cm telescope these variable stars are only objects suitable for measurements. Though obtained data demonstrate effectiveness of method, astrophysical interpretation of them are difficult due to complexity of these objects. Although estimation of polaricenter parameters in several photometric bands and in different epochs of pulsations may represent some interest in investigation of inner parts of their dust envelopes.

Investigation of objects which can be described by simple geometrical model with large telescopes seems much more appealing. Let us consider, as an example, object Sgr A* in center of Milky Way which assumed to be supermassive black hole. Observations in NIR showed that the Sgr A* is susceptible to irregular flares with characteristic timescale of 10-20 minutes. During the flare brightness of object increases from 2 mJy to $\approx 5$ mJy, and object becomes polarized at level of $\approx 10\%$ [24]. Main hypothesis explaining these flares is rotation of hot spots emitting synchrotron radiation in accretion disc very close to black hole. Expected level of polarastrometric photon noise for this object at 8-m class telescope, integration time 3 min, is 100 $\mu$as and 10 $\mu$as with and without adaptive optics, respectively. Latter value is comparable to expected angular size of event horizon for this black hole, which is 10$\mu$ as. Hence using polarastrometry in principle it is possible to put additional geometrical constraints on model of these hot spots.

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A Polaricenters parameters transformation to rotated reference system

For practical purposes it is frequently needed to transform polaricenters parameters to new reference system rotated relative to initial. Let us suppose that angle of rotation is measured counterclockwise.

Values in new reference system will be noted with primes. Stokes parameters $q, u$ are transformed this way:

$$q' = q \cos 2\psi + u \sin 2\psi,$$

$$u' = q \sin 2\psi - u \cos 2\psi,$$

where $\psi$ is angle of rotation.
\[ u' = -q \sin 2\psi + u \cos 2\psi. \]  

Components \( x, y \) of some vector are transformed this way:

\[ x' = x \cos \psi + y \sin \psi, \]  

\[ y' = -x \sin \psi + y \cos \psi. \]

Taking this into consideration, we derived expressions for components of unreduced relative polaricenters in new reference system:

\[
\begin{align*}
    s_{q}^{*'} &= (s_{q}^{*} \cos 2\psi + s_{u}^{*} \sin 2\psi) \cos \psi + (t_{q}^{*} \cos 2\psi + t_{u}^{*} \sin 2\psi) \sin \psi, \\
    t_{q}^{*'} &= -(s_{q}^{*} \cos 2\psi + s_{u}^{*} \sin 2\psi) \sin \psi + (t_{q}^{*} \cos 2\psi + t_{u}^{*} \sin 2\psi) \cos \psi, \\
    s_{u}^{*'} &= -(s_{q}^{*} \sin 2\psi + s_{u}^{*} \cos 2\psi) \cos \psi + (-t_{q}^{*} \sin 2\psi + t_{u}^{*} \cos 2\psi) \sin \psi, \\
    t_{u}^{*'} &= -(s_{q}^{*} \sin 2\psi + s_{u}^{*} \cos 2\psi) \sin \psi + (-t_{q}^{*} \sin 2\psi + t_{u}^{*} \cos 2\psi) \cos \psi.
\end{align*}
\]

The same for reduced relative polaricenters:

\[
\begin{align*}
    s_{q}' &= (a s_{q} + b s_{u}) \cos \psi + (a t_{q} + b t_{u}) \sin \psi, \\
    t_{q}' &= -(a s_{q} + b s_{u}) \sin \psi + (a t_{q} + b t_{u}) \cos \psi, \\
    s_{u}' &= (c s_{q} + d s_{u}) \cos \psi + (c t_{q} + d t_{u}) \sin \psi, \\
    t_{u}' &= -(c s_{q} + d s_{u}) \sin \psi + (c t_{q} + d t_{u}) \cos \psi,
\end{align*}
\]

where

\[
\begin{align*}
    a &= \frac{q \cos 2\psi}{q \cos 2\psi + u \sin 2\psi}, \\
    b &= \frac{u \sin 2\psi}{q \cos 2\psi + u \sin 2\psi}, \\
    c &= \frac{-q \sin 2\psi}{-q \cos 2\psi + u \sin 2\psi}, \\
    d &= \frac{-u \cos 2\psi}{-q \cos 2\psi + u \sin 2\psi}.
\end{align*}
\]

As one can see in some cases it is impossible to transform reduced polaricenters because respective denominators in last four expressions may be equal to zero. In these cases one should operate with unreduced polaricenters.

It is worth noting that in case when reduced polaricenters \( q \) and \( u \) coincide, transformation of their coordinate is simple rotation:

\[
\begin{align*}
    s' &= s \cos \psi + t \sin \psi, \\
    t' &= -s \sin \psi + t \cos \psi,
\end{align*}
\]
B Half-wave plate imperfections influence

Let us suppose that HWP imperfections can be modeled as fluctuations of phase retardance $\delta$ and fast axis orientation $\theta$ over its area. In order to derive these fluctuations for a given HWP we illuminated it through the linear polarizer, then passed resulting light through another linear polarizer which we were able to rotate. With the help of relay lens image of HWP was formed on the detector. For each point on the HWP values $\delta$ and $\theta$ was extracted from dependence of flux on second linear polarizer orientation. Result is given in Fig. 8.

As one see from figure, fluctuations of $\theta$ reach $2^\circ$, they are more pronounced closer to the border of the plate. For $\delta$ amplitude of fluctuation is on the order of $3 \div 5^\circ$ in central part of the HWP. On the basis of these measurements with help of model developed in [2] we estimated $R_{\text{HWP}}(\theta, f)$ (for definition see subsection 4.2) for different beam parameters. Due to the fact that differential aberrations produced by HWP are purely phase the same is true for $R_{\text{HWP}}(\theta, f)$. RMS of $R_{\text{HWP}}(\theta, f)$ phase in area $|f| < f_c$ is 0.02 radians, in $|f| < 0.2 f_c$ — 0.002 radians.

By approximating phase of $R_{\text{HWP}}(\theta, f)$ with plane we evaluated functions $G(\theta)$ and $H(\theta)$ (see subsection 4.2), and then computed their Fourier spectra. These are given in Fig. 9 for beam diameter 2 mm and different decentering parameter in comparison with analogous spectra measured in polaroastrometric experiment for $\alpha$ Lyr. One can see that observed behaviour of spectra can be explained by described HWP imperfections at decentering $\approx 1$ mm. Such value of decentering is quite expectable because mechanical structure of device is not stiff enough (subsections 4.2 and 6).

According to subsection 4.2 useful signal corresponds to complex amplitude of fourth harmonics of the Fourier spectra $\tilde{c}(\theta)$ and $\tilde{d}(\theta)$. The amplitude of noise of this harmonics is given in Fig. 10 as function of beam diameter and decentering. As one can see, for reducing noise to the level of 0.1 mpix it is necessary to align beam and center of HWP rotation with accuracy of 0.1 mm, also it is worthwhile to use beams of small diameter — $< 2$ mm.

In order to derive HWP imperfections influence on polarimetry let us take its Mueller matrix in following form (here we kept only first-order small values and integrated over the beam):

$$
M_{\text{HWP}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos[4(\theta + \kappa(\theta))] & \sin[4(\theta + \kappa(\theta))] & \delta(\theta) \sin[2(\theta + \kappa(\theta))] \\
0 & \sin[4(\theta + \kappa(\theta))] & -\cos[4(\theta + \kappa(\theta))] & -\delta(\theta) \cos[2(\theta + \kappa(\theta))] \\
0 & -\delta(\theta) \sin[2(\theta + \kappa(\theta))] & \delta(\theta) \cos[2(\theta + \kappa(\theta))] & 1
\end{pmatrix}.
$$

(48)
In accordance with specifications, Wollaston prism can be considered ideal in our experiment. First rows of Mueller matrix for left and right beams of prism (we are interested in first rows only because we can measure intensity only):

\[
M_{WL} = \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix}, \\
M_{WR} = \begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix}.
\]

Now express Stokes vectors for left and right beams through these matrices:

\[
S_{\text{out},L} = M_{WL} M_{\text{HWP}} S_{\text{in}},
\]

\[
S_{\text{out},R} = M_{WR} M_{\text{HWP}} S_{\text{in}},
\]

where \( S_{\text{in}} = (I_{\text{in}}, Q_{\text{in}}, U_{\text{in}}, V_{\text{in}}) \) — Stokes vector for incoming radiation. Respectively, for fluxes in left and right beam:

\[
I_{\text{out},L} = I_{\text{in}} + Q_{\text{in}} \cos(4(\theta_k + \kappa(\theta_k))) + U_{\text{in}} \sin(4(\theta_k + \kappa(\theta_k))) + V_{\text{in}} \delta(\theta_k) \sin(2(\theta_k + \kappa(\theta_k))),
\]

\[
I_{\text{out},R} = I_{\text{in}} - Q_{\text{in}} \cos(4(\theta_k + \kappa(\theta_k))) - U_{\text{in}} \sin(4(\theta_k + \kappa(\theta_k))) - V_{\text{in}} \delta(\theta_k) \sin(2(\theta_k + \kappa(\theta_k))).
\]

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