I. INTRODUCTION

Chiral symmetry plays an important role in hadron physics. When we set \( N_f \) flavor light quarks to be massless, quantum chromodynamics (QCD) has an \( SU(N_f)_L \times SU(N_f)_R \) chiral symmetry at the Lagrangian level. The chiral symmetry is not preserved by QCD vacuum but broken dynamically to its vector component, i.e., \( SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \). This dynamical chiral symmetry breaking splits the degeneracy of the chiral partners, which are supposed to be degenerate when the chiral symmetry is restored so that the study of the chiral partner structure of hadrons can help us reveal the magnitude of the chiral symmetry breaking, i.e., the order parameter.

In the light meson sector, the chiral partner structure might be complicated since two light quarks are involved (see, e.g., Refs. [1, 2]). On the other hand, studying the chiral partner structure in the heavy-light meson sector would be easier since they include only one light quark, the dynamics of which is controlled by the chiral symmetry. In addition to the chiral symmetry, the dynamics of the heavy-light mesons is also controlled by spin-flavor symmetry due to their heavy quark constituent [3]. Based on this heavy quark symmetry, the ground states form a doublet with spin-parity quantum numbers \((1^-,0^-)\), and the first excited states belong to the \((1^+,0^+)\) doublet. In Ref. [4], it was proposed that these two doublets were chiral partners to each other in the QCD-like models. As a signature of the chiral partner structure, the mass splitting of them is induced by the dynamical breaking of the chiral symmetry so that the mass difference is about the constituent quark mass. This was confirmed by the spectrum of the relevant particles; \( m_{D_0^-} - m_{D^-} \approx m_{D_1^-} - m_{D^0} \approx 450\) MeV is at the same order of \( m_{D_0}(2317) - m_{D} \approx m_{D_1}(2460) - m_{D} \approx 350\) MeV (see, e.g., Refs. [5, 6]).

In Ref. [7], the analysis on the chiral partner structure of heavy baryons was made based on the bound state picture [8, 9] together with the heavy quark symmetry in which the heavy baryon is introduced as the heavy mesons bound with the nucleon as the soliton. In this analysis, the excited heavy baryon \( \Lambda_c(2595) \) with \( J^P = \frac{1}{2}^- \) is regarded as the chiral partner to the ground state baryon \( \Lambda_c(2268) \) \( (J^P = \frac{1}{2}^+) \).

In this paper, we revisit the chiral partner structure of the heavy baryons in the bound state approach based on our recent progress in the soliton property [10, 11] and the effective Lagrangian for the heavy-light meson chiral partner structure [12, 13]. In the light meson sector, we considered all the \( \mathcal{O}(N_c) \) terms of hidden local symmetry (HLS), all the \( \mathcal{O}(p^2) \), \( \mathcal{O}(p^4) \), and homogeneous Wess-Zumino (hWZ) terms [14, 15]. In the heavy and light meson interaction sector, we start with the interaction Lagrangian for the heavy-light meson and light mesons analyzed in Refs. [16, 17], where the chiral partner is introduced in the framework of a linear sigma model. We integrate out the scalar mesons and integrate in the vector mesons to construct an effective Lagrangian for heavy mesons interacting with the pseudoscalar mesons and vector mesons based on the HLS [18, 19] (see, e.g., Refs. [20, 21] for alternative approaches), and the heavy quark symmetry. Then, we consider that the static soliton couples to the heavy-light meson and study their spectrum. After the derivation of the heavy baryon spectrum in the static case, we consider the collective coordinate quantization to make states definite quantum numbers. Our explicit calculation shows that, in the heavy quark limit and large \( N_c \) limit, up to the \( \mathcal{O}(p^4) \) terms of HLS, the chiral partner of \( \Lambda_c(1/2^+, 2286) \) is predicted to be the \( \Lambda_c(1/2^-, 2286) \) heavy quark doublet with a mass.
of about 3.1 GeV but not \((\Lambda_c(\frac{3}{2}^-), 2595)\), \((\Lambda_c(\frac{3}{2}^-), 2625)\) listed in the PDG table [22], which might be interpreted as an orbital excitation of \(\Lambda_c(\frac{3}{2}^+, 2286)\). Extending our approach to the bottom sectors we predicted the chiral partner structure of bottom baryons. We finally studied the pentaquark spectrum using our framework and found that the masses of the pentaquark states made of a ground state heavy-light meson and its chiral partner are similar and both of them are below the \(D_p\) threshold, which therefore cannot be ruled out by the present data [23].

This paper is organized as follows: In Sec. III the chiral partner structure of heavy baryons is studied. We derive the analytic forms of the heavy baryons’ masses. The heavy baryon spectrum with chiral partner structure is estimated in Sec. III and the pentaquark state’s spectrum is estimated in Sec. IV. The last section is for a summary and discussions. Some useful explicit derivations are given in the Appendix.

II. HEAVY BARYONS IN THE EFFECTIVE LAGRANGIAN FOR HEAVY-LIGHT MESONS WITH CHIRAL DOUBLING

A. Effective Lagrangian for heavy-light mesons with chiral doubling

Here, we construct the effective Lagrangian describing the interaction between the heavy-light mesons and the light mesons. With respect to the chiral transformation property of the light quarks in the heavy-light mesons, the heavy-light meson field can be decomposed into a right-handed component \(\hat{H}_L\) and left-handed one \(\hat{H}_R\). Under chiral \(SU(2)_L \times SU(2)_R\) symmetry, they transform as

\[
\hat{H}_L \rightarrow \hat{H}_L U_L^\dagger, \quad \hat{H}_R \rightarrow \hat{H}_R U_R^\dagger.
\]

In Refs. [18, 19] these fields are used to construct a Lagrangian where the chiral symmetry are realized linearly. In the present paper, for the study of the heavy baryon spectrum in the bound state approach, we adopt the nonlinear realization of the chiral symmetry. Then, by replacing \(M\) in Eq. (29) of Refs. [18, 19] with \(F_\pi U\) where \(U = e^{2i\pi/F_\pi}\), we obtain

\[
\mathcal{L}_{\text{heavy}} = \frac{1}{2} \text{Tr} \left[ \hat{H}_L (v \cdot \partial) \hat{H}_L \right] + \frac{1}{2} \text{Tr} \left[ \hat{H}_R (v \cdot \partial) \hat{H}_R \right] - \frac{\Delta}{2} \text{Tr} \left[ \hat{H}_L \hat{H}_L + \hat{H}_R \hat{H}_R \right] - \frac{g_\pi F_\pi}{4} \text{Tr} \left[ U^\dagger \hat{H}_L \hat{H}_R + U \hat{H}_R \hat{H}_L \right] + \frac{g_A}{2} \text{Tr} \left[ \gamma^5 \gamma^\mu \partial^\mu \hat{H}_L \hat{H}_L \right] + \frac{g_A}{2} \text{Tr} \left[ \gamma^5 \gamma^\mu \partial^\mu \hat{H}_R \hat{H}_R \right],
\]

where \(\Delta, g_\pi\) and \(g_A\) are parameters. In the present work, we include the vector mesons using the HLS [2, 20] by introducing the matrix valued variables \(\xi_L\) and \(\xi_R\) as \(U = \xi_L^\dagger \xi_R\). Then, similarly to Ref. [6] we convert the heavy meson fields as

\[
\hat{H}_L = \xi_L^\dagger \hat{\xi}_L, \quad \hat{H}_R = \xi_R \hat{\xi}_R^\dagger,
\]

which under the full symmetry transformation \(G_{\text{full}} = [SU(2)_L \times SU(2)_R]_{\text{chiral}} \times [U(2)]_{\text{HLS}}\) transform as

\[
\hat{H}_L \rightarrow \hat{H}_L \hat{h}^\dagger(x), \quad \hat{H}_R \rightarrow \hat{H}_R \hat{h}^\dagger(x).
\]

Associated with the field redefinitions in Eq. (3), it is convenient to use the following quantities for the \(\pi\) fields:

\[
\hat{\alpha}_{|| \mu} = \frac{1}{2 \gamma} \left( D_\mu \xi_R \cdot \xi_R^T + D_\mu \xi_L \cdot \xi_L^T \right),
\]

\[
\hat{\alpha}_{\perp \mu} = \frac{1}{2 \gamma} \left( D_\mu \xi_R \cdot \xi_R^T - D_\mu \xi_L \cdot \xi_L^T \right),
\]

where the covariant derivative \(D_\mu\) is given by \(D_\mu = \partial_\mu - iV_\mu\) with \(V_\mu\) being the gauge field of the HLS. By using these quantities, the above Lagrangian is extended to include the vector mesons as

\[
\mathcal{L}_{\text{heavy}} = \frac{1}{2} \text{Tr} \left[ \hat{H}_L (v \cdot \hat{D}) \hat{H}_L \right] + \frac{1}{2} \text{Tr} \left[ \hat{H}_R (iv \cdot \hat{D}) \hat{H}_R \right] - \frac{\Delta}{2} \text{Tr} \left[ \hat{\xi}_L \hat{\xi}_L + \hat{\xi}_R \hat{\xi}_R \right] - \frac{g_\pi F_\pi}{4} \text{Tr} \left[ \hat{\xi}_L \hat{H}_L \hat{\xi}_R + \hat{\xi}_R \hat{H}_R \right] - \frac{g_A}{2} \text{Tr} \left[ \gamma^5 \gamma^\mu \hat{\alpha}_{\perp \mu} \left( \hat{H}_L \hat{H}_R + \hat{H}_R \hat{H}_L \right) \right],
\]

where \(\hat{D}\) is defined as \(\hat{D}_\mu = \partial_\mu - iV_\mu\) with \(\kappa\) being a real parameter measuring the magnitude of the violation of the vector meson dominance.

To study the chiral partner structure of the heavy baryons, we rewrite the Lagrangian (4) in terms of the heavy-light meson doublets \(\hat{H}\) and \(\hat{G}\) with quantum numbers \(\hat{H} = (0^-, 1^-)\) and \(\hat{G} = (0^+, 1^+)\); specifically, we make the substitution

\[
\hat{H}_L = \frac{1}{\sqrt{2}} (\hat{G} - i\hat{H} \gamma_5), \quad \hat{H}_R = \frac{1}{\sqrt{2}} (\hat{G} + i\hat{H} \gamma_5).
\]

In terms of the physical states, the \(\hat{H}\) and \(\hat{G}\) doublets can be explicitly expressed as

\[
\hat{H} = \frac{(1 + \hat{\theta})}{2} [D^\mu \gamma_\mu + iD_5^\mu \gamma_5], \quad \hat{G} = \frac{(1 + \hat{\theta})}{2} [-D_5^\mu \gamma_\mu \gamma_5 + D_5^\mu].
\]

Substituting Eq. (7) into Eq. (6), we obtain

\[
\mathcal{L}_{\text{heavy}} = -\text{Tr} \left[ \hat{G} (iv \cdot \hat{D}) \hat{G} \right] + \text{Tr} \left[ \hat{H} (iv \cdot \hat{D}) \hat{H} \right] - \frac{\Delta}{2} \text{Tr} \left[ \hat{G} \hat{G} - \hat{H} \hat{H} \right] - \frac{g_\pi F_\pi}{4} \text{Tr} \left[ \hat{H} \hat{H} + \hat{G} \hat{G} \right] + g_A \text{Tr} \left[ \hat{H} \gamma_5 \gamma_\mu \partial_\mu \hat{H} \right] - g_A \text{Tr} \left[ \hat{G} \gamma_5 \gamma_\mu \partial_\mu \hat{G} \right]
\]

This expression explicitly shows that the \(g_\pi F_\pi\) term splits the spectrum of \(\hat{H}\) and \(\hat{G}\) doublets while the \(\Delta\) term shifts
the masses of these two doublets toward the same direction. In the present paper, we use the physical masses of heavy mesons as inputs to calculate the heavy baryon masses so that we drop the $g_F \bar{F}_\nu$ term and the $\Delta$ term in the following calculation of the masses of heavy baryons. Note that due to the chiral partner structure adopted here, the magnitudes of the coupling constants in the last two terms of Eq. (10) are the same; therefore, the chiral partner spectrum is predictable.

**B. Heavy baryon masses from the bound state approach**

In this subsection we derive the heavy baryon masses based on the bound state approach [11].

To make the mesonic theory a baryonic one, we follow the standard procedure to take the Hedgehog ansatz for a classical soliton [24]

$$\xi_R = \xi_L^\dagger = \xi_c(x) = \exp \left[ i \tau \cdot \hat{x} F(x) \right], \quad (10)$$

with $\tau_i$ as the Pauli matrices and the subscript $c$ standing for the classical solution. From the Hedgehog ansatz (10), one can easily see that $\xi_c$ transforms under separate spatial rotation and isospin rotation but is invariant under the combined rotation; i.e., the hedgehog profile correlates the angular momentum and the isospin. For the vector mesons, their profile functions can be parametrized as [23, 26]

$$\omega_{\mu; c} = \omega(x) \delta_{\mu 0}, \quad \rho^\mu_{\|; c} = \frac{1}{g x} \epsilon_{ijb} \hat{x}_j G(x), \quad \rho^\mu_{\perp; c} = 0. \quad (11)$$

From the hedgehog ansatz (10) and profile functions (11) we express the quantities $\hat{a}^\mu_{\perp}$ and $\hat{a}^\mu_{\|}$ as

$$\hat{a}^\mu_{\perp} = (a_{\perp}, 0), \quad \hat{a}^\mu_{\|} = (a_{\|}, a_{\|}), \quad (12)$$

where

$$a_{\perp} = \frac{1}{2} \left[ \sin F(x) x + \left( F'(x) - \frac{\sin F(x)}{x} \right) (\tau \cdot \hat{x}) \hat{x} \right],$$

$$a_{\|} = \frac{g}{2} \omega(x),$$

$$a_{\|} = \frac{1}{x} \sin^2 \frac{F}{2} - \frac{1}{2x} G(x) \hat{x} \times \tau. \quad (13)$$

In the rest frame of the heavy-light meson, i.e., $v_\mu = (1, 0)$, the $H$ doublet has nonvanishing elements only in the upper-right $2 \times 2$ sub-block while the $G$ doublet has nonvanishing elements only in the upper-left $2 \times 2$ sub-block. The matrix forms of $H$ and $G$ doublets become

$$\hat{H} = \begin{pmatrix} 0 & \mathbb{H} \\ \mathbb{H}^\dagger & 0 \end{pmatrix}, \quad \hat{G} = \begin{pmatrix} \mathbb{G} & 0 \\ 0 & \mathbb{G}^\dagger \end{pmatrix},$$

$$\tilde{H} = \begin{pmatrix} 0 & \mathbb{H}^\dagger \\ -\mathbb{H} & 0 \end{pmatrix}, \quad \tilde{G} = \begin{pmatrix} \mathbb{G}^\dagger & 0 \\ 0 & \mathbb{G} \end{pmatrix}. \quad (14)$$

Then, the Lagrangian (9) is reduced to

$$\mathcal{L}_{\text{heavy}} = - \text{Tr} \left[ G \partial_\mu \mathbb{G}^\dagger \right] - \text{Tr} \left[ H \partial_\mu \mathbb{H}^\dagger \right] - \frac{1}{2} (1 + \kappa) g \omega(r) \text{Tr} \left[ \mathbb{G} \mathbb{G}^\dagger \right] - \frac{1}{2} (1 + \kappa) g \omega(r) \text{Tr} \left[ \mathbb{H} \mathbb{H}^\dagger \right] - g_A \text{Tr} \left[ \mathbb{G} \sigma \cdot a_{\perp} \mathbb{G}^\dagger \right] + g_A \text{Tr} \left[ \mathbb{G} \sigma \cdot a_{\|} \mathbb{G}^\dagger \right], \quad (15)$$

Since the hedgehog ansatz for the Skyrme soliton correlates the angular momentum and isospin, the bound states should be invariant under the “grand spin” rotation with the operator defined as

$$\mathbf{g} = \mathbf{r} + \mathbf{J}_{\text{light}} + \mathbf{I}_{\text{light}}, \quad (16)$$

where $\mathbf{r}, \mathbf{J}$, and $\mathbf{I}_{\text{light}}$ are the ordinary orbital angular momentum between the soliton and heavy-light meson, heavy meson spin, and the heavy meson isospin operators. Taking into account that the heavy quark spin is conserved in the heavy quark limit, one simply defines the “light quark grand spin” operator

$$\mathbf{g} = \mathbf{r} + \mathbf{J}_{\text{light}} + \mathbf{I}_{\text{light}}, \quad (17)$$

with $\mathbf{J}_{\text{light}}$ as the spin operator of the light degree of freedom of the heavy-light meson, and in both $H$ and $G$ doublets, the eigenvalue of the operator $\mathbf{J}_{\text{light}}$ is 1/2 so that the eigenmodes of the heavy baryons can be classified by the third component of heavy quark spin $s_Q$ and the light quark grand spin $(g, g_3)$ and the parity $P$.

Taking into account the isospin, light quark spin and heavy quark spin spin indices that the heavy-light meson has, one can write the static wave functions of the heavy-light mesons as [11]

$$H_{c, lh}^{(a, I)} = u^{(H)}(x)(\tau \cdot \hat{x})_{ad} \psi_{dl}^{(H)}(g, g_3; r, k) \chi_h^{(H)};$$

$$G_{c, lh}^{(a, I)} = u^{(G)}(x)(\tau \cdot \hat{x})_{ad} \psi_{dl}^{(G)}(g, g_3; r, k) \chi_h^{(G)}, \quad (18)$$

where $a, l$, and $h$ denote the indices for the isospin of the heavy-light meson, the spin of the light degree of freedom, and the heavy quark spin, respectively. $k$ is the eigenvalue of the operator

$$\mathbf{K} = \mathbf{I}_{\text{light}} + \mathbf{J}_{\text{light}}, \quad (19)$$

with $k_3$ as its third component. $\chi_h^{(H,G)}$ is factorized out due to the conservation of the heavy quark spin. $u(x)$ is a radial function which is strongly peaked at the origin and normalized as $x^2 |u(x)|^2 \approx \delta^3(x)$ [11, 12]. This implies that the relevant matrix elements are independent of the quantum number $r$ [13]. The generalized “angular” wave function $\psi_{dl}^{(H,G)}(g, g_3; r, k)$ can be expanded as [13]

$$\psi_{dl}^{(H,G)}(g, g_3; r, k) = \sum_{r, g, k_3} C_{r, g, k_3} Y_{r, g_3} Y_{r, k_3} \xi_{dl}(k, k_3), \quad (20)$$

where $Y_{r, g_3}$ stands for the standard spherical harmonic representing orbital angular momentum $r$ while $C$ denotes the ordinary Clebsch-Gordan coefficients. $\xi_{dl}(k, k_3)$
represents a wave function in which the “light spin” and “light isospin” referring to the “light cloud” component of the heavy meson are added vectorially to give $K = I_{\text{light}} + J_{\text{light}}$ with eigenvalues $K^2 = k(k + 1)$. Note that in the present analysis, $k$ is a good quantum number since the relevant matrix elements are independent of the quantum number $r$. Furthermore, both the quantum numbers for the “light spin” and “light isospin” are given by $1/2$ so that the possible values of $k$ are either 0 or 1.

The normalization of the eigenstate $\psi_{dI}^{(H,G)}(g, r; k)$ is

\[
\int d\Omega \psi_{dI}^{(H,G)}(g, r; k) \left[ \psi_{dI}^{(H,G)}(g', r', k') \right]^\dagger = \delta_{d'd} \delta_{gg'} \delta_{rr'} \delta_{kk'},
\]

where \( \int d\Omega \) is the solid angle integration.

From Lagrangian (15), we parametrize the potential as

\[
\delta H, G = \frac{1}{2} \int d\Omega \delta_{gg'} \delta_{rr'} \delta_{kk'} \frac{1}{2} \left( k(k + 1) - \frac{3}{2} \right),
\]

By substituting the ansatz (20), $V_H$ and $V_G$ are obtained as

\[
V_H = \frac{1}{2} \left( 1 + \kappa \right) g \omega(r) \text{Tr} \left[ \mathbb{H}_c \mathbb{H}^c \right] + g_A \text{Tr} \left[ \mathbb{H}_c \sigma \cdot a \mathbb{H}^c \right],
\]

\[
V_G = \frac{1}{2} \left( 1 + \kappa \right) g \omega(r) \text{Tr} \left[ \mathbb{G}_c \mathbb{G}^c \right] - g_A \text{Tr} \left[ \mathbb{G}_c \sigma \cdot a \mathbb{G}^c \right].
\]

Next, we make a quantization by a time dependent $SU(2)$ rotation of the fields in the HLS Lagrangian in the light-quark sector as

\[
\xi_c(x, t) \to C(t) \xi_c(x) C^\dagger(t),
\]

\[
V_{\mu,c}(x) \to V_{\mu}(x, t) = C(t) V_{\mu,c}(x) C^\dagger(t),
\]

where $C(t)$ is a time dependent unitary matrix satisfying $C(t)C(t)^\dagger = C(t)^\dagger C(t) = 1$. Accordingly, the heavy-light meson fields are rotated as

\[
\mathbb{H}(x, t) = \mathbb{H}_c(x) C^\dagger(t), \quad \mathbb{G}(x, t) = \mathbb{G}_c(x) C^\dagger(t).
\]

This collective rotation gives an additional contribution to the Lagrangian

\[
\delta \mathcal{L}_{\text{coll}} = \frac{1}{2} \mathbf{I} \Omega^2 + \mathbf{I}_{\text{light}} \cdot \Omega,
\]

where the angular velocity $\Omega$ corresponding to the collective coordinate rotation is defined as

\[
\frac{1}{2} i \tau \cdot \Omega \equiv C^\dagger \partial_t C.
\]

\( \mathcal{I} \) is the inertia of the collective configuration. By using $\mathcal{I}$, the light baryon masses are expressed as

\[
m_b = M_{\text{sol}} + \frac{j_b(j_b + 1)}{2 \mathcal{I}},
\]

where $M_{\text{sol}}$ is the soliton mass and $j_b$ is the spin of the light baryon. Using the nucleon mass $m_N$ and the delta mass $m_{\Delta}$ as inputs, $M_{\text{sol}}$ and $\mathcal{I}$ are given as

\[
M_{\text{sol}} = \frac{5m_N - m_{\Delta}}{4}, \quad \mathcal{I} = \frac{3}{2(m_{\Delta} - m_N)}.
\]

From Eq. (20) one obtains [8]

\[
\frac{d}{dt} \mathbf{J}_{\text{coll}} = \frac{1}{2} \Omega^2 - \chi(k) K \cdot \Omega,
\]

where the coefficient $\chi(k)$ is calculated as

\[
\chi(k) = \begin{cases} 
0, & (k = 0), \\
\frac{3}{2(k+1)}, & (k \neq 0),
\end{cases}
\]

with $j_b$ being the spin quantum number of the light degree of freedom of the heavy-light meson. For convenience we show the derivation in the Appendix. The Hamiltonian of the collective rotated system is obtained by the standard Legendre transformation as

\[
H_{\text{coll}} = \frac{1}{2} \mathbf{J}^\dagger \mathbf{J} + \chi(k) K^2,
\]

where $\mathbf{J}$ is the canonical momentum conjugating to the collective variable $C(t)$:

\[
\mathbf{J}^\dagger = \frac{\partial \mathcal{L}_{\text{coll}}}{\partial \dot{\mathbf{J}}}. \quad \mathcal{I} \mathbf{J} + \mathbf{I}_{\text{light}}.
\]

The first term $\mathcal{I} \mathbf{J}$ is the isospin operator for the light baryon sector while the second term $\mathbf{I}_{\text{light}}$ is the isospin operator for the heavy-light mesons interacting with the light baryon so that $\mathbf{J}$ is identical to the isospin operator for the heavy baryon $\mathbf{I}$.

\[
\mathbf{J} = \mathbf{I}.
\]

After the collective coordinate rotation, the total spin of the light degrees of freedom in the heavy baryon is defined as

\[
\mathbf{j} = \mathbf{J} + \mathbf{g} = \mathbf{J} + \mathbf{r} + \mathbf{K}.
\]

By including the heavy quark spin, the spin operator for the heavy baryon is expressed as $J_B = j \pm S_Q$ with eigenvalues $j_B = j \pm 1/2$ (in the case of $j = 0$, only $j_B = 1/2$ exists). Then, we can express the collectively rotated Hamiltonian as

\[
H_{\text{coll}} = \frac{1}{2} \left[ 1 - \chi(k) \mathbf{I}^2 + \chi(k) \mathbf{J} \mathbf{K} - 1 \right] \mathbf{K}^2
\]

\[
+ \chi(k) (\mathbf{j} - \mathbf{r})^2.
\]

Gathering all the above contributions, we finally obtain the heavy baryon mass as

\[
M_{(\text{heavy baryon})} = M_{\text{sol}} + M_{H,G} + V_{H,G} + H_{\text{coll}},
\]

where $V_H (V_G)$ is the binding energy corresponding to the heavy meson $H (G)$. $M_{H,G}$ are the weight-averaged heavy meson masses with $M_H = (3m_D + m_D)/4$ and $M_G = (3m_{D_s} + m_{D_s})/4$. Note that each combination of $(I, j)$ generates a pair of degenerate states with $j_B = j \pm 1/2$. 
III. CHIRAL DOUBLING OF HEAVY BARYONS

In this section, we study the chiral doubling structure using the formulas obtained in the previous section. In the following analysis, we restrict ourselves to the case with \( r = 0 \) and use the following values of heavy meson masses as inputs:

\[
(m_D, m_{D^*}, m_{D_s^*}, m_{D_s}) = (1.86, 2.01, 2.32, 2.42) \; [\text{GeV}],
\]

which lead to

\[
(M_H, \tilde{M}_G) = (1.97, 2.40) \; [\text{GeV}].
\]

Furthermore, to simulate the profile functions \( F(r) \) and \( \omega(r) \), which are necessary to evaluate the binding energy \( V_H \) and \( V_G \) expressed in Eq. (29), we use the following inputs:

\[
(m_N, m_\Delta) = (0.94, 1.23) \; [\text{GeV}],
\]

which yield soliton mass \( M_{\text{sol}} = 0.868 \; \text{GeV} \) and the inverse of the moment of inertia \( 1/\mathcal{I} = 0.193 \; \text{GeV} \). Then, using the relevant expressions from HLS up to \( \mathcal{O}(p^4) \) including the hWZ terms given in Refs. \([16, 17]\), by taking \( F_\pi \) and \( m_\rho \) as free parameters to fit the inputs into Eq. (10), we obtain \( F_\pi = 62.24 \; \text{MeV} \) and \( m_\rho = 417.5 \; \text{MeV} \), and the values of the profile functions at origin as

\[
F'(0) = 626.1 \; \text{MeV}; \quad \omega(0) = -74.5 \; \text{MeV}.
\]

Moreover, we take the parameter \( a = 2 \) \([3, 20]\) and fix the universal coupling constant \( g \) in HLS through

\[
g = m_\rho/(F_\pi \sqrt{\alpha}) = 4.74. \quad (42)
\]

Let us first consider the binding energy in order to determine which channel can form the bound state and calculate the spectra of the chiral partners. Using Eq. (29), we obtain the binding energy between the heavy-light mesons in the \( H \) doublet and soliton as

\[
V_H = -0.177 (1 + \kappa) + 0.626 g_A \left[ k(k + 1) - \frac{3}{2} \right] \; [\text{GeV}].
\]

The value of \( g_A \) is determined through the \( D^* \to D\pi \) decay as \( |g_A| = 0.56 \) \([18, 19]\). It does not seem possible to determine the \( \omega \) coupling constant \( \kappa \) from the available experimental data for the heavy meson decay. In the case of the vector meson dominance we have \( \kappa = 0 \); therefore, it is reasonable to regard \( |\kappa| \lesssim 1 \). Then we conclude that the \( k = 0 \) channel gives a bound state when \( g_A > 0 \). This bound state can be naturally identified with \( \Lambda_c(\frac{1}{2}^+, 2286) \) from the quantum number so that we assume \( g_A > 0 \) in the following analysis. Since the collective energy is zero for the \( k = 0 \), \( I = 0 \) state, we use the experimental value of the mass of \( \Lambda_c(\frac{1}{2}^+, 2286) \) as an input to determine the value of \( \kappa \). Using \( M_{\Lambda_c} = 2.29 \; \text{GeV} \), we obtain \( \kappa = -0.83 \).

Next, we consider the bound states made from the \( G \) doublet. The binding energy is expressed as

\[
V_G = -0.177 (1 + \kappa) - 0.626 g_A \left[ k(k + 1) - \frac{3}{2} \right] \; [\text{GeV}]. \quad (44)
\]

As we can see easily, \( V_G > 0 \) for \( k = 0 \) (\( g_A > 0 \)) so that there is no bound state in the \( k = 0 \) channel. For the \( k = 1 \) channel, using \( \kappa = -0.83 \) determined above, we obtain \( V_G = -0.205 \; \text{GeV} \), which implies that the \( k = 1 \) channel is actually bound.

From Eq. (43) the total spin of the light degrees of freedom becomes \( I = 0 \) so that the resultant heavy baryons form a heavy-quark doublet consisting of \( \Lambda_c(\frac{1}{2}^-) \) and \( \Lambda_c(\frac{3}{2}^-) \). Combined with the collective energy, the mass of the bound state is expressed as

\[
M_{\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)} = M_{\text{sol}} + \tilde{M}_G + V_G + \frac{1}{4}\mathcal{I}, \quad (45)
\]

which leads to

\[
M_{\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)} = 3.13 \; [\text{GeV}]. \quad (46)
\]

This value is much larger than the experimental values for the masses of negative parity baryons: \( M(\Lambda_c(\frac{1}{2}^-, 2595)) = 2.59 \; \text{GeV} \) and \( M(\Lambda_c(\frac{3}{2}^-, 2625)) = 2.63 \; \text{GeV} \). So, we conclude that the negative parity baryons found by experiments \( \Lambda_c(\frac{1}{2}^-, 2595) \) and \( \Lambda_c(\frac{3}{2}^-, 2625) \) are not the chiral partner to the ground state baryon \( \Lambda_c(\frac{1}{2}^+, 2285) \). In the present bound state approach, they should be regarded as the \( r = 1 \) state made from the \( H \) doublet and nucleon. Then, we expect to have a doublet for the chiral partner around the 3.1 GeV region.

We next study the \( I = 1 \) baryons. In the positive parity baryon sector, the \( k = 0 \) channel is bound so that the total spin of the light degrees of freedom in the heavy baryon becomes 1. As a result, the spin of \( \Sigma_c \) baryons with positive parity is either 1/2 or 3/2. In the mass formula in Eq. (37), only \( H_{\text{coll}} \) changes its value depending on the isospin of baryons. Since \( \chi(k) = 0 \) for \( k = 0 \), the mass difference between the \( \Sigma_c \) and the \( \Lambda_c \) in the positive negative parity is obtained as

\[
M_{\Sigma_c(\frac{1}{2}^+, \frac{3}{2}^+)} - M_{\Lambda_c(\frac{1}{2}^+, \frac{3}{2}^+)} = \frac{1}{4}\mathcal{I}. \quad (47)
\]

In the negative parity baryon sector, the \( k = 1 \) channel is bound, then the eigenvalue for \( j \) is either 0, 1, or 2. We summarize our predicted results for the charm baryon spectrum in Tables II and III.

We next study the mass spectrum of the bottom baryon by substituting the bottom meson masses into the charm meson masses in Eq. (37). In the bottom meson spectrum, the masses of the ground states \( B \) and \( B^* \) are well measured but masses of the mesons in the \( G \) doublets are not well established. Here, we naïvely estimate them using \( m_{B_s} - m_B = m_{D_s} - m_D = 2403 - 1869.6 = \)
TABLE I. Predicted mass for the charmed baryon for the $H$ doublet.
\[
\begin{array}{ccc}
I & j & M^H(\text{MeV}) \\
0 & 0 & \Lambda_c(\frac{1}{2}^-) \\
1 & 1 & \Sigma_c(\frac{1}{2}^+), \Sigma_c(\frac{3}{2}^+) \\
\end{array}
\]

593.4 MeV and $m_{B^*} - m_{B^0} = m_{D^*} - m_{D^0} = 2427 - 2010.25 = 416.75$ MeV which lead to $m_{B^*} = 5812.9$ MeV and $m_{B^0} = 5741.85$ MeV. Our numerical results for the masses of the heavy baryons including the bottom quark with the corresponding quantum numbers are given in Tables III and IV.

IV. PENTAQUARKS WITH HEAVY QUARK

We next consider the pentaquark channel. Although the existence of these kinds of states still needs experimental confirmation, theoretical study of them is meaningful. For a pentaquark state, the large component of the antiheavy quark can be projected out with the projection operator $(1 - \hat{\epsilon})/2$ so that in case the heavy meson is at rest, the $H$ doublet has nonvanishing elements only in the lower-left $2 \times 2$ sub-block while the $G$ doublet has nonvanishing elements only in the lower-right $2 \times 2$ sub-block, i.e.,

\[
H = \begin{pmatrix} 0 & 0 \\ 0 & H \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 0 \\ 0 & G \end{pmatrix},
\]

\[
\tilde{H} = \gamma_0 H^\dagger \gamma_0 = \begin{pmatrix} 0 & -H^\dagger \\ 0 & 0 \end{pmatrix},
\]

\[
\tilde{G} = \gamma_0 G^\dagger \gamma_0 = \begin{pmatrix} 0 & 0 \\ 0 & G^\dagger \end{pmatrix}.
\]

Then, substituting $v_\mu$ with $-v_\mu$, following the above derivation, one can see that both the binding energies given by Eq. (23) change a sign. As a result, the binding energies for the pentaquark states made from the anti-$H$ doublet and the pentaquark states made from the anti-$G$ doublet are expressed as

\[
V_H^5 = 0.177 (1 + \kappa) - 0.626 g_A \left[ k(k + 1) - \frac{3}{2} \right] \text{[GeV]},
\]

\[
V_G^5 = 0.177 (1 + \kappa) + 0.626 g_A \left[ k(k + 1) - \frac{3}{2} \right] \text{[GeV]}.
\]

Therefore, for the anti-$H$ doublet, the $k = 1$ channel gives the bound states with binding energy $V_H^5 = -145.6$ MeV, while for anti-$G$ doublet, the $k = 0$ channel gives the bound states with binding energy $V_G^5 = -496.2$ MeV.

Substituting relevant numerical results, we obtain the spectrum of the pentaquark states. We list our results in Tables V and VI for pentaquark states consisting anti-charm quark and Tables VII and VIII for pentaquark states consisting anti-bottom quark.

The results in Tables V and VI show that the lightest charmed pentaquark states made of soliton and heavy-light mesons in the anti-$H$ doublet have masses of about 2.75 GeV, and their chiral partner made of the anti-$G$ doublet has a mass of about 2.80 GeV. Both of them are below the $Dp$ threshold. The reason that the pentaquark states from the anti-$H$ doublet and anti-$G$ doublet have similar masses is because the binding energy of the anti-$H$ doublet is about 350 MeV smaller than that of the anti-$G$ doublet, and the collective rotation energy, which is about 50 MeV, does not contribute to the latter. With respect to the status of the pentaquark search performed, these states cannot be ruled out, and since they cannot decay via a strong process, their total widths should be narrow.

TABLE III. Predicted mass for the bottom baryon for the $H$ doublet.
\[
\begin{array}{ccc}
I & j & M^H(\text{MeV}) \\
0 & 0 & \Lambda_b(\frac{1}{2}^-) \\
1 & 1 & \Sigma_b(\frac{1}{2}^+), \Sigma_b(\frac{3}{2}^+) \\
\end{array}
\]

TABLE IV. Predicted mass for the bottom baryon for the $G$ doublet.
\[
\begin{array}{ccc}
I & j & M^G(\text{MeV}) \\
0 & 1 & \Lambda_b(\frac{1}{2}^-), \Lambda_b(\frac{3}{2}^-) \\
1 & 0 & \Sigma_b(\frac{3}{2}^-) \\
1 & 1 & \Sigma_b(\frac{1}{2}^-), \Sigma_b(\frac{3}{2}^-) \\
1 & 2 & \Sigma_b(\frac{1}{2}^-), \Sigma_b(\frac{3}{2}^-) \\
\end{array}
\]

TABLE V. Predicted mass for the charmed pentaquark state for the $H$ doublet.
\[
\begin{array}{ccc}
I & j & M^{\pi H}(\text{MeV}) \\
0 & 1 & \Theta_c(\frac{1}{2}^+) \\
\end{array}
\]

TABLE VI. Predicted mass for the charmed pentaquark state for the $G$ doublet.
\[
\begin{array}{ccc}
I & j & M^{\pi G}(\text{MeV}) \\
0 & 1 & \Theta_c(\frac{1}{2}^+) \\
\end{array}
\]
TABLE VII. Predicted mass for the bottom pentaquark state for the $H$ doublet.

| $I$ | $j$ | Candidates | $M^{\pi-N_b}$ (MeV) |
|-----|-----|------------|---------------------|
| 0   | 1   | $\Theta_b(\frac{2}{3}^-)$ | 6083.76             |

TABLE VIII. Predicted mass for the bottom pentaquark state for the $G$ doublet.

| $I$ | $j$ | Candidates | $M^{\pi-N_b}$ (MeV) |
|-----|-----|------------|---------------------|
| 0   | 0   | $\Theta_b(\frac{1}{2}^-)$ | 6130.39             |

V. SUMMARY AND DISCUSSIONS

We studied the chiral partner structure of heavy baryons in the bound state approach including the vector meson exchanging effects through the hidden local symmetry. We showed that in the large $N_c$ limit and the heavy quark limit, the ground state heavy baryon made of the ground state heavy-light meson and the nucleon has a chiral partner made of an excited heavy-light meson and nucleon. Our explicit calculation showed that the chiral partner of $\Lambda_c(\frac{2}{3}^-)$ is a heavy quark doublet of $\Lambda(\frac{2}{3}^-)$ and $\Lambda(\frac{3}{2}^-)$. This contrasted to the perdition made in the pioneering work in Ref. [12], where the chiral partner was the singlet under the heavy quark spin transformation. Our prediction of the mass was about 3.1 GeV, which indicated that the $\Lambda_c(\frac{2}{3}^-, 2595)$ and $\Lambda_c(\frac{3}{2}^-, 2625)$ listed in the PDG table [22] should be interpreted as the $r=1$ excitation of $\Lambda_c(\frac{1}{2}^+)$. To calculate the spectrum of the $r \neq 0$ states, one should consider the relative motion of the soliton with respect to the heavy mesons [27]. This is beyond the scope of the present paper.

We also studied the bound states in the pentaquark channel. We found that the $k=1$ channel forms bound states for the anti-$H$ doublet ($\Theta_c(\frac{1}{2}^-), \Theta_c(\frac{2}{3}^-)$), while the $k=0$ channel forms bound states for the anti-$G$ doublet ($\Theta_c(\frac{1}{2}^+)$). It was found that the predicted masses of the pentaquark states made of the anti-$H$ doublet and anti-$G$ doublet were below the $D_p$ threshold, which cannot be ruled out by the present data [23].

In the present analysis, we took the infinite heavy soliton and heavy quark limits so that both the soliton and heavy-light meson were sitting at the origin. This picture could not be applied to the bound states with nonzero $r$. Since in the present analysis, the chiral partner of heavy $\Lambda_c(\frac{2}{3}^+, 2286)$ had a mass of about 3.1 GeV, which was a bound state of soliton and heavy-light mesons in the $G$ doublet, one could expect that it had broad width due to the broad width of the constituent $P$-wave mesons in the $G$ doublet. From the numerical results in Tables [III] and [IV] we concluded that the spectrum of the heavy baryons with bottom quark was consistent with PDG [22] for $\Lambda_b$ and $\Sigma_b$.

It should be noted that in the present analysis, we considered that the chiral partner to the nucleon was itself: The left-handed nucleon was the chiral partner of the right-handed nucleon, and vice versa so that the chiral partner to the heavy baryon as the bound state of the $H$ doublet and the nucleon was the one made of the $G$ doublet and the nucleon. This implied that the chiral partner structure of the heavy baryons in our approach arose from the chiral partner structure of the constituent heavy-light mesons. On the other hand, in the mirror scenario for the light baryon [28], the chiral partner to the nucleon was considered as $N(1535)$. In such a case, the full picture of the chiral partner structure of heavy baryons became complicated, and we did not consider this scenario in the present work.

Appendix A: Matrix element of heavy-light meson isospin operator

Using the Wigner-Eckart theorem, we can express the matrix element of heavy-light meson isospin operator $I_{\text{light}}^a$ in terms of the matrix element of the operator $K$, i.e.,

$$
\int d\Omega \langle \psi^{(i)}_{gg3} | I_{\text{light}}^a | \psi^{(j)}_{gg3} \rangle = \int d\Omega \langle \psi^{(i)}_{gg3} | K | \psi^{(j)}_{gg3} \rangle \frac{\langle \psi^{(i)}_{gg3} | K \cdot I_{\text{light}}^a | \psi^{(j)}_{gg3} \rangle}{k(k+1)}
$$

$$
= \int d\Omega \langle \psi^{(i)}_{gg3} | [K^2 + I^2 - J^2_{\text{light}}] | \psi^{(j)}_{gg3} \rangle \frac{\langle \psi^{(i)}_{gg3} | K | \psi^{(j)}_{gg3} \rangle}{2k(k+1)}
$$

$$
= \frac{[k(k+1) + 3/4 - j(j+1)]}{2k(k+1)} \int d\Omega \langle \psi^{(i)}_{gg3} | K | \psi^{(j)}_{gg3} \rangle \delta_{ij}
$$

\[ A1 \]

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[1] S. Weinberg, Phys. Rev. Lett. 65, 1177 (1990).
[2] M. Harada and K. Yamawaki, Phys. Rep. 381, 1 (2003).
[3] For review, see, e.g., M. B. Wise, arXiv:hep-ph/9906277 and references therein.
[4] M. A. Nowak, M. Rho, and I. Zahed, Phys. Rev. D 48, 4370 (1993); W. A. Bardeen and C. T. Hill, Phys. Rev. D 49, 409 (1994).
[5] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003); M. A. Nowak, M. Rho and I. Zahed, Acta Phys. Pol. B 35, 2377 (2004).
[6] M. Harada, M. Rho, and C. Sasaki, Phys. Rev. D 70, 074002 (2004).
[7] M. A. Nowak, M. Praszałowicz, M. Sadzikowski and J. Wasiluk, Phys. Rev. D 70, 031503 (2004).
[8] C. G. Callan and I. R. Klebanov, Nucl. Phys. B262, 365 (1985).
[9] See, e.g., C.G. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B 202, 296 (1988); J. Blaizot, M. Rho, and N.N. Scoccola, Phys. Lett. B209, 27 (1988); N.N. Scoccola, H. Nadeau, M.A. Nowak and M. Rho, Phys. Lett. B201, 425 (1988); D. Kaplan and I. Klebanov, Nucl. Phys. B335, 45 (1990); Y. Kondo, S. Saito and T. Otofuji, Phys. Lett. B256, 316 (1991); M. Rho, D.O. Riska, and N.N. Scoccola, Z. Phys. A 341, 341 (1992); H. Weigel, R. Alkofer and H. Reinhardt, Nucl. Phys. A576, 477 (1994).
[10] Z. Guralnik, M. Luke, and A.V. Manohar, Nucl. Phys. B390, 474 (1993); E. Jenkins, A.V. Manohar and M. Wise, Nucl. Phys. B396, 27 (1993); E. Jenkins and A.V. Manohar, Phys. Lett. B294, 273 (1992); M. Rho, in *Baryons as Skyrme Solitons*, edited by G. Holzwarth (World Scientific, Singapore, 1994); D.P. Min, Y. Oh, B.-Y. Park, and M. Rho, arXiv:hep-ph/9209275; H.K. Lee, M.A. Nowak, M. Rho and I. Zahed, Ann. Phys. (N.Y.) 227, 175 (1993); M.A. Nowak, M. Rho, and I. Zahed, Phys. Lett. B303, 13 (1993); Y. Oh, B.-Y. Park, and D.P. Min, Phys. Rev. D49, 4649 (1994); D.P. Min, Y. Oh, B.-Y. Park, and M. Rho, Int. J. Mod. Phys. E4, 47 (1995); Y. Oh and B.-Y. Park, Phys. Rev. D51, 5016 (1995); Y. Oh and B.-Y. Park, arXiv:hep-ph/9703219; N. N. Scoccola, arXiv:0905.2722; M. Rho, D.O. Riska, N.N. Scoccola, Phys. Lett. B251, 97 (1990); D.O. Riska, and N.N. Scoccola, Phys. Lett. B265, 188 (1991); M. Rho, D.O. Riska, and N.N. Scoccola, Z. Phys. A341, 343 (1992); Y. Oh, D.P. Min, M. Rho, and N.N. Scoccola, Nucl. Phys. A534, 493 (1991).
[11] K.S. Gupta, M.A. Momen, J. Schechter, and A. Subbaraman, Phys. Rev. D47, R4835 (1993); A. Momen, J. Schechter, and A. Subbaraman, Phys. Rev. D49, 5970 (1994).
[12] Y. Oh, B.-Y. Park, and D.P. Min, Phys. Rev. D50, 3350 (1994); Y. s. Oh, B. Y. Park, and D. P. Min, Phys. Lett. B331, 362 (1994).
[13] M. Harada, F. Sannino, J. Schechter, and H. Weigel, Phys. Rev. D 56, 4098 (1997).
[14] B. Wu and B. Q. Ma, Phys. Rev. D 70, 034025 (2004).
[15] J. Schechter, A. Subbaraman, S. Vaidya, and H. Weigel, Nucl. Phys. A 590, 655 (1995); Nucl. Phys. A 598, 583(E) (1996); J. Schechter and A. Subbaraman, Phys. Rev. D 51, 2311 (1995); M. Harada, A. Qamar, F. Sannino, J. Schechter, and H. Weigel, Nucl. Phys. A 625, 789 (1997); Y. s. Oh and B. Y. Park, Z. Phys. A 359, 83 (1997); Y. Oh, Phys. Rev. D 75, 074002 (2007); T. D. Cohen and P. M. Hohler, Phys. Rev. D 75, 094007 (2007); E. E. Jenkins, A. V. Manohar, and M. B. Wise, Nucl. Phys. B 396, 27 (1993); E. E. Jenkins, A. V. Manohar, and M. B. Wise, Nucl. Phys. B 396, 38 (1993).
[16] Y.-L. Ma, Y. Oh, G.-S. Yang, M. Harada, H. K. Lee, B.-Y. Park, and M. Rho, Phys. Rev. D 86, 074025 (2012).
[17] Y.-L. Ma, G.-S. Yang, Oh and M. Harada, Phys. Rev. D 85, 114027 (2012).
[18] H. Hoshino, M. Harada, and Y. L. Ma, arXiv:1203.4320.
[19] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Lett. 54, 1215 (1985); M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164, 217 (1988).
[20] Y. L. Ma, Q. Wang, and Y. L. Wu, Eur. Phys. J. C 39, 201 (2005); H. Gomm, O. Kaymakcalan and J. Schechter, Phys. Rev. D 30, 2345 (1984); O. Kaymakcalan and J. Schechter, Phys. Rev. D 31, 1109 (1985).
[21] Particle Data Group, K. Nakamura, J. Phys. G 37, 075021 (2010).
[22] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 73, 091101 (2006); B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 74, 051101 (2006); G. De Lellis, A. M. Guler, J. Kawada, U. Kose, O. Sato, and F. Tramontano, Nucl. Phys. B 763, 268 (2007).
[23] U. Karshon (H1 Collaboration and ZEUS Collaboration), arXiv:0907.3574.
[24] T. H. R. Skyrme, Proc. R. Soc. A 260, 127 (1961); T. H. R. Skyrme, Proc. R. Soc. A 262, 237 (1961).
[25] P. Jain, R. Johnson, U. G. Meissner, N. W. Park, and J. Schechter, Phys. Rev. D 37, 3252 (1988).
[26] Y. Igarashi, M. Johmura, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Nucl. Phys. B259, 721 (1985); T. Fujiwara, Y. Igarashi, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Prog. Theor. Phys. 74, 128 (1985).
[27] Y. -s. Oh and B. -Y. Park, Z. Phys. A 359, 83 (1997).
[28] C. E. Detar and T. Kunihiro, Phys. Rev. D 39, 2805 (1989); D. Jido, Y. Nemoto, M. Oka, and A. Hosaka, Nucl. Phys. A 671, 471 (2000); D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. 106, 873 (2001).