A Proposal for measuring Anisotropic Shear Viscosity in Unitary Fermi Gases

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We present a proposal to measure anisotropic shear viscosity in a strongly interacting, ultra-cold, unitary Fermi gas confined in a harmonic trap. We introduce anisotropy in this setup by strongly confining the gas in one of the directions with relatively weak confinement in the remaining directions. This system has a close resemblance to anisotropic strongly coupled field theories studied recently in the context of gauge-gravity duality. Computations in such theories (which have gravity duals) revealed that some of the viscosity components of the anisotropic shear viscosity tensor can be made much smaller than the entropy density, thus parametrically violating the bound proposed by Kovtun, Son and Starinets (KSS): $\frac{\eta}{\sigma} \geq \frac{1}{4\pi}$. A Boltzmann analysis performed in a system of weakly interacting particles in a linear potential also shows that components of the viscosity tensor can be reduced. Motivated by these exciting results, we propose two hydrodynamic modes in the unitary Fermi gas whose damping is governed by the component of shear viscosity expected to violate the KSS bound. One of these modes is the well known scissor mode. We estimate trap parameters for which the reduction in the shear viscosity is significant and find that the trap geometry, the damping timescales, and mode amplitudes are within the range of existing experimental setups on ultra-cold Fermi gases.

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Introduction - The computation of transport properties of strongly interacting quantum field theories is a challenging problem and has attracted physicists working on a wide variety of systems including ultra-cold Fermi gases at unitarity [1,3], heavy ion collisions [1,4,5], and neutron stars [6,7].

The AdS/CFT correspondence [8] has provided many insights into the transport properties of such strongly coupled field theories which have gravity duals. In the limit of large 't Hooft coupling $\lambda$ and large number of “colors” $N_c$, the dual gravity theories are effectively classical and the computation of transport properties become much easier. One finds that in all such isotropic strongly coupled theories in 3+1 dimensions which admit smooth gravity duals, the ratio of shear viscosity $\eta$ to entropy density $s$ is $\frac{\eta}{s} = \frac{1}{4\pi}$ [9,13] (we work in units where $\hbar = c = 1$). Weakly coupled theories typically have much larger $\frac{\eta}{s}$. This led Kovtun, Son and Starinets (KSS) to conjecture that $\frac{\eta}{s}$ is bounded from below by $1/(4\pi)$. It was later found that finite $\lambda$ corrections (which correspond to higher derivative corrections in the gravity side) can drive $\frac{\eta}{s}$ below the KSS bound [11,16].

The gravity duals of ultra-cold Fermi gases and quark gluon plasma produced in heavy ion collisions are not made much smaller than the entropy density, thus parametrically violating the bound proposed by Kovtun, Son and Starinets (KSS): $\frac{\eta}{\sigma} \geq \frac{1}{4\pi}$. A Boltzmann analysis performed in a system of weakly interacting particles in a linear potential also shows that components of the viscosity tensor can be reduced. Motivated by these exciting results, we propose two hydrodynamic modes in the unitary Fermi gas whose damping is governed by the component of shear viscosity expected to violate the KSS bound. One of these modes is the well known scissor mode. We estimate trap parameters for which the reduction in the shear viscosity is significant and find that the trap geometry, the damping timescales, and mode amplitudes are within the range of existing experimental setups on ultra-cold Fermi gases.

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of an anisotropic trap and an appropriate choice of parameters one can create ultra-cold Fermi systems which share many essential features of the theories considered in Refs. [36, 39]. We give a concrete proposal for the trap geometry and parameters where a parametric violation of the KSS bound is likely to be seen.

**Gravity Results** - We briefly review results of computations of shear viscosity in the gravity picture obtained by studying anisotropic black branes [39] where the breaking of isotropy is due to an externally applied force which is translationally invariant. The simplest system discussed in Ref. [36] consists of a massless dilaton minimally coupled to gravity, and a cosmological constant. The action is

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left[ R + 12\Lambda - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right],$$

where $G$ is Newton’s constant in 5 dimensions and $\Lambda$ is a cosmological constant. The dual field theory in the absence of anisotropy is a 3 + 1 dimensional conformal field theory. The dilaton profile, linear in the spatial coordinate $z$

$$\phi = \rho z,$$

explicitly breaks the symmetry to $2 + 1$. Using AdS/CFT one finds [36] that for a system at temperature $T$, (using the compact notation $\eta_{ijij} = \eta_{ij}$)

$$\eta_{xz} = \eta_{yz} \text{ (which are spin 1 with respect to the surviving Lorentz symmetry)}$$

is affected by the background dilaton. In the low anisotropy regime ($\rho/T \ll 1$):

$$\frac{\eta_{xz}}{s} = \frac{1}{4\pi} \frac{\rho^2 \log 2}{16\pi^3 T^2} \left( \frac{6 - \pi^2 + 54s}{2304\pi^5 T^4} \right) + O\left( \frac{\rho}{T}^6 \right).$$

The correction to the zero anisotropy result, the KSS bound $\frac{\eta_{xz}}{s} = \frac{1}{4\pi}$, is proportional to $(\nabla^2 \phi)^2$ where $\nabla \phi = \rho \hat{\zeta}$ is the driving force and $1/T$ is the microscopic length scale in the system.

In extreme anisotropy ($\rho/T \gg 1$),

$$\eta_{xz}/s \to (1/4\pi)(32\pi^2 T^2 / 3\rho^2)$$

and hence becomes parametrically small [36]. But this domain will not be physically accessible in the cold atom systems.

In contrast the $\eta_{xy}$ component (which couples to a spin 2 metric perturbation) was found to be unchanged from its value in the isotropic case, $\frac{\eta_{xy}}{s} = \frac{1}{4\pi}$.

Parametric reduction of the spin 1 components of $\eta/s$ has been found for a variety of strongly coupled theories with a gravitational dual [32, 39].

Motivated by the above results, we may expect to observe parametrically suppressed viscosities compared to the KSS bound in systems with the following properties:

1. It is strongly interacting and in the absence of anisotropy has a viscosity close to the KSS bound.
2. The equations of hydrodynamics admit modes whose damping is sensitive to the spin 1 viscosity components Ref. [36, 39].
3. The gradient of the background potential (say in the $z$ direction) must be significant compared to a microscopic scale governing transport.
4. The background potential responsible for breaking of isotropy is approximately spatially constant.
5. The velocity gradients are small enough to ensure validity of hydrodynamics.

**Ultra-cold atoms** - Ultra-cold unitary Fermi gases [43, 44] are strongly interacting systems with one of the lowest $\eta/s$ measured, and (based on the above criteria) a good candidate system to explore small anisotropic viscosities.

Typically they are trapped in harmonic potentials

$$\phi(r) = \sum_i m_\omega x_i^2 / 2$$

where $i$ runs over the $x, y, z$ directions and $m$ denotes the mass of the fermionic species which we take as $^6\text{Li}$. The trap frequencies are chosen such that $\omega_z \gg \omega_x, \omega_y$ the potential gradient is dominantly in the $z$ direction. In Section *Anisotropy* we show how large $\omega_z$ is required to measure significant deviations from the isotropic shear viscosity.

We note that while ultra-cold Fermi gases share some important features with the theories considered in [36, 39], there is an important difference, namely, unlike the field theories of [36, 39] the stress energy tensor in a trap is not translationally invariant. Even so, the Boltzmann equation (see Eqs. 22, 23) for ultra-cold Fermi gases also predicts a reduction of $\eta_{xz}$.

**Hydrodynamic modes** - We find two solutions of equations of superfluid hydrodynamics in a harmonic trap, which are sensitive to the spin 1 components of the viscosity tensor. Each of these modes is characterized by the superfluid and the normal components, which we denote by $v_s$ and $v_n$, respectively.

The first mode, which we call the elliptic mode, has $v_s = 0$ and $v_n = v$ given by

$$v = e^{i\omega t}(\alpha_x z \hat{x} + \alpha_z x \hat{z})$$

with the following relations:

elliptic : $\omega = 0$, \( \alpha_z = -\alpha_x \omega_z^2 / \omega_x^2 \)
The kinetic energy of a harmonic trap is given by

\[ E = \int d^3r \left( \frac{1}{2} \mathbf{v} \cdot \mathbf{v} \right) , \]

where \( \mathbf{v} = \mathbf{v}_n = \mathbf{v} \) given by Eq. (6) with

\[ \text{scissor: } \omega = \sqrt{\omega_x^2 + \omega_z^2}, \quad \alpha_z = \alpha_x. \] (8)

We see that in the high anisotropy limit \( \omega_z \gg \omega_x \), \( \alpha_z \to 0 \) for the elliptic mode, and hence we recover a flow profile similar to that considered in [36]. The elliptic mode has not been studied in ultra-cold gas experiments. The scissor mode [45] has been studied extensively in bosonic (for example see Refs. [46]) fermionic gases [41]; therefore we focus on it. For both modes, \( \partial_t \mathbf{v}_j = 0 \).

To ensure that hydrodynamics is valid in the region where the energy loss due to viscous damping is substantial, we impose the condition

\[ \alpha_x < \alpha_x^{\max} = P(s_{\text{max}})/\eta_{xz} s_{\text{max}} , \] (9)

where \( s_{\text{max}} \) is the place where the local chemical potential equals the temperature (Eq. 19). This sets the upper limit on the amplitude of the modes.

**Viscous damping** - The energy dissipated due to the shear viscosity is given by

\[ \dot{E}_{\text{kinetic}} = - \int d^3r \frac{\eta_{xz}(\mathbf{r})}{2} \left( \partial_t \mathbf{v}_j + \partial_j \mathbf{v}_i - \frac{2}{3} \delta_{ij} \partial_k \mathbf{v}_k \right)^2 . \] (10)

\( T \) is constant in the elliptic and scissor modes and hence thermal conduction is neglected.

For the elliptic mode,

\[ \dot{E}_{\text{kinetic}} = - \int d^3r \eta_{xz}(\mathbf{r}) \alpha_x^2 (1 - \omega_z^2/\omega_x^2)^2 , \] (11)

and for the scissor mode

\[ \dot{E}_{\text{kinetic}} = -2 \int d^3r \eta_{xz}(\mathbf{r}) \alpha_x^2 . \] (12)

The total mechanical energy \( E \) is twice the average kinetic energy \( E = 2E_{\text{kinetic}} \) where,

\[ E_{\text{kinetic}} = \frac{1}{2} \int d^3r mn(\mathbf{r}) \mathbf{v}^2 , \] (13)

where \( \mathbf{v} \) is the velocity of either mode and the average is taken over one cycle for the scissor mode (the elliptic mode is non-oscillatory).

In the strong anisotropy limit \( \omega_z \gg \omega_x \), it can be shown that the energy of the elliptic mode scales as \( E_{\text{elliptic}} \sim \frac{\mu^3}{\omega_x \omega_z^\frac{3}{2}} \) and that of the scissor mode scales as \( E_{\text{scissor}} \sim \frac{\mu^3}{\omega_x \omega_z^\frac{3}{2}} \). (The scalings of the scissor mode, for example, can be derived as follows: \( E \sim \int dxdydz[mn\mathbf{v}^2] \sim L_x L_y L_z [mn^2 \omega_x^2/\omega_z^2] \sim \frac{\mu^3}{\omega_x^3 \omega_z} \), where we have assumed that at the center of the trap \( \mu > 0 \) and \( L_i = \sqrt{2\mu/(m\omega_i^2)} \).)

In a similar manner, one can derive the approximate scalings for energy dissipation rates:

\[ \dot{E} \sim \frac{\mu^2}{\omega_x \omega_z^\frac{5}{2}} \] for both the modes (assuming \( \eta \) scales the same way as \( n \) i.e. \( \sim (\mu n)^{\frac{1}{2}} \)).

**Thermodynamics and trap results** - The evaluation of the energy loss from Eq. (11) and Eq. (12) requires the viscosity \( \eta \) as a function of the position \( \mathbf{r} \) in the trap. To estimate \( \eta(\mathbf{r}) \) we use the local density approximation (LDA).

For the unitary Fermi gas in the thermodynamic limit, the thermodynamic functions can be factorized into universal dimensionless functions [17] and an overall scale given, for example, in terms of the number density of a free Fermi gas,

\[ n_f(\mu) = (2\mu)^{3/2}/(3\pi^2) . \] (14)

In particular,

\[ n(\mu, T) = n_f(\mu) [G(y) - 2yG'(y)/5] , \]

\[ s(\mu, T) = (2/5)n_f(\mu) G'(y) , \] (15)

where

\[ y = T/\mu . \] (16)

\( G(y) \) can be obtained (see Ref. [48] for details) by analyzing the thermodynamic measurements carried out in Ref. [17]. Similarly, the ratios \( \eta/n \) and \( \eta/s \) are dimensionless functions of \( y \) [17,19].

The local value of the chemical potential in a harmonic
the local entropy (see Eq. 15) is the product of $n_f(\mu(r))$ which decreases as we move away from the center, while the function $G'(T/\mu(r))$ increases [58]. Therefore $s$ as a function of $z$ naturally has a peak. Since the ratio of the shear viscosity to the entropy density is a relatively slowly varying function of $z$ in the region around the phase transition [17], we also expect the local shear viscosity to show a peak.

An illustrative example for $T = 2T_c/3$ is shown in Fig. 1. We have defined

$$z_{\text{trap}} = \sqrt{2\mu/(m\omega_z^2)},$$

\hspace{1cm} (18)

to scale the $z$ axis and use $n_f(\mu)$ to scale the viscosity. Therefore the figure is independent of $\mu$ and $\omega_z$ if $T$ is scaled with $\mu$ to maintain $T = 2T_c/3$. Similar behavior is seen for $T$ between $T_c/2$ and $T_c$ and the results are summarized in Table I.

Viscous damping dominantly (Eq. 12) arises from a region of width $\delta z$ near $z_0$. This width can be made narrow by lowering the temperature, such that $l = \delta z/z_0 < 1$ and in this region the potential can be approximated as linear [18]. We don’t consider $T$ below $T_c/2$ because for these $T_c$ the superfluid phonon viscosity [19] is important.

All columns in Table I except $\kappa_{\text{LDA}}$ (which we will discuss in the next section) are independent of $\mu$ and $\omega$.

The upper bound on the energy is set by the amplitude $\alpha_{\text{max}}$ (Eq. 9) with $z_{\text{max}}$

$$z_{\text{max}} = \sqrt{2(\mu - T)/m\omega_z^2},$$

\hspace{1cm} (19)

to estimate energy scales for typical experiments, we take $T, \mu, \omega_z$ close to those used in experiments in Ref. [18]. The parameter values used in similar experiments in Refs. [41, 57] are comparable, but typically the values of $\mu$ and $\omega$ are somewhat smaller. The maximum angular amplitude of the scissor mode is given by

$$\theta \approx \tan^{-1}\left(\frac{z_{\text{max}}}{2}\right)\left(\frac{s_{\text{max}}}{\omega_z\pi} + 1\right),$$

\hspace{1cm} (20)

For $\alpha_{\text{max}} \approx 10^{-10}$ eV (Table I) and $\omega_z = 2\pi \times 10^4$ radians/s $\equiv 4.16 \times 10^{-11}$ eV, we find $\theta_{\text{max}} \approx 45^\circ$. This is larger than the angular amplitudes measured in [41] and hence within experimental capabilities.

The (amplitude) damping time $\tau_0$ defined as

$$\tau_0 = 2E/\dot{E}_{\text{kinetic}},$$

\hspace{1cm} (21)

is independent of $\alpha_{\text{max}}$ and $\sim 10^{-2}(\mu/10\mu K)(2\pi \times 385\text{Hz}/\omega_z)^2s$ in the strong anisotropy limit. For $\mu = 10\mu K$, $\omega_z = \omega_y = 2\pi \times 385$ radians/s and $\omega_z = 2\pi \times 4^4$ radians/s, $\tau_0$ ranges from roughly 0.04 to 0.08. The damping of the scissor mode has been observed for slightly different parameters values, $\mu \approx 1\mu K$, $\omega_z = 2\pi \times 830$ Hz, $\omega_y = 2\pi \times 415$ Hz and $\omega_z = 2\pi \times 22$ Hz in Ref. [11] where the damping time scales measured are of the order of milliseconds. A direct comparison using our technique can only be made for the lowest temperature ($T/T_F = 0.1$) of Ref. [11]. Our calculations (using the trap parameters of [11]) give a damping rate of 250 s$^{-1}$ which agrees with experiments [41].

**Anisotropy** - In LDA the shear viscosity tensor is lo-
and, 
\[ \kappa_{\text{LDA}} = \nabla \phi / (\mu k_F) |_{z_0}. \] (24)

While we will focus on the spin 1 component, we note that the corrections to different components of \( \eta \) are different and the shear viscosity tensor is indeed anisotropic [48].

In the absence of potential one obtains the well known result [22]
\[ \eta = k_F^2 / (15 \pi^2). \] (25)

By matching Eq. [25] with \( \eta / n \) at \( z_0 \) we find \( \lambda k_F \approx 1 \) (which verifies the intuition that the coupling is strong and the Boltzmann calculations are not quantitatively reliable). The corrections are governed by \( \kappa_{\text{LDA}} \).

In the strongly coupled regime for the unitary Fermi gas, \( c_2 \) (Eq. [22]) cannot be computed reliably. But it is intriguing that the weak coupling Boltzmann analysis (Eq. [23]) gives \( c_2 < 0 \) like the strongly coupled theories with gravity duals (Eq. [3], see also Ref. [26, 58]).

So far most experiments on hydrodynamics of trapped Fermi gases have been done for \( \kappa_{\text{LDA}} \ll 1 \) [41, 44]. For fixed \( T/T_c, \kappa_{\text{LDA}} \) scales as \( \omega_z / \mu \) and anisotropic viscosities can be explored by using larger \( \omega_z \) or smaller \( \mu \) (or both). For example, for \( T = T_c / 2 \) (Table I) and \( \mu = 10 \mu K \) (central density roughly \( 10^{44} \text{ atoms/cm}^3 \)), by increasing \( \omega_z \) from typical values of \( 2 \pi \times 10^4 \) to 8 times this value, \( \kappa_{\text{LDA}} \) can be increased from \( \sim 0.13 \) to \( \sim 1 \). We expect this to lead to an order unity reduction in \( \eta_{zz} \) (Eq. [23]).

The damping time for the scissor mode scales as \( \mu / (\omega_z)^2 \) and can be kept in the experimentally accessible range of about a millisecond (see Table II) while increasing \( \omega_z \) (by keeping \( \mu \) and \( \omega_z \) same). This reduces the maximum amplitude \( \theta_{max} \) (Eq. [20]) to \( 2^\circ \), which is still in the observable range [44].

Conclusions - We give the first proposal to measure parametrically suppressed anisotropic viscosity components in ultra-cold Fermi gases. Our analyses is motivated by the calculation of the viscosity in strongly coupled theories with gravitational duals in the presence of a linearly growing external potential. The spin 1 components of the viscosity in these systems are parametrically

| \( T/T_c \) | \( \alpha^{\text{max}} (10^{-10} \text{eV}) \) | \( E_{\text{kinetic}} (\text{j/s}) \) (a) | \( E (\text{j}) \) (a) | \( \tau_0 (\text{a}) \) | \( E_{\text{kinetic}} (\text{j/s}) \) (b) | \( E (\text{j}) \) (b) | \( \tau_0 (\text{b}) \) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( 4T_c/5 \) | 2.83 | \( 2.37 \times 10^{-16} \) | \( 3 \times 10^{-20} \) | 0.0002 | \( 4.7 \times 10^{-16} \) | \( 10^{-17} \) | 0.04 |
| \( 2T_c/3 \) | 2.35 | \( 1.25 \times 10^{-16} \) | \( 2 \times 10^{-20} \) | 0.0003 | \( 2.5 \times 10^{-16} \) | \( 6.8 \times 10^{-18} \) | 0.05 |
| \( 4T_c/7 \) | 2.02 | \( 7.12 \times 10^{-17} \) | \( 1.4 \times 10^{-20} \) | 0.0004 | \( 1.4 \times 10^{-16} \) | \( 4.8 \times 10^{-18} \) | 0.07 |
| \( T_c/2 \) | 1.77 | \( 4.33 \times 10^{-17} \) | \( 1.1 \times 10^{-20} \) | 0.0005 | \( 8.65 \times 10^{-17} \) | \( 3.6 \times 10^{-18} \) | 0.08 |

TABLE II: Energy scales for various \( T/T_c \). \( \omega_z = 2 \pi \times 10^4 \text{ rad/s}, \omega_z = \omega = 2 \pi \times 385 \text{ rad/s} \) and \( \mu = 10 \mu K \). a denotes the elliptic mode and b the scissor mode. The mechanical energy \( E \) (see Eq. [13]) is given in joules (j) and energy loss rate in joules per second, (j/s). For a fixed \( T/\mu \), the energy of the elliptic (scissor) mode scales as \( \frac{E_{\text{kinetic}}}{\omega_{xz}^2} \) (\( \frac{E_{\text{kinetic}}}{\omega_{yz}^2} \)). The decay time \( \tau_0 = 2E/E_{\text{kinetic}} \) [in seconds (s)] of the elliptic (scissor) mode scales as \( \frac{1}{\omega_x^2} \) (\( \frac{1}{\omega_y^2} \)).

Finally, we note that the corrections to different components of \( \eta \) are different and the shear viscosity tensor is indeed anisotropic [48].
reduced from the KSS bound [38, 39].

Our proposal involves a unitary Fermi gas in an anisotropic harmonic trap. We find that for the temperature at the center of the trap between 0.2 to 0.4 times $\mu$, the damping of oscillatory modes is dominated by a region where the background harmonic potential can be approximated as linear. AdS/CFT then suggests a reduction in the spin 1 component of the shear viscosity.

For $\mu = 10\muK$, $T = \frac{2}{\pi}$ ($T_c \approx 0.4\muK$), and $\omega_z \sim 2\pi \times 77000$ rad/s, we find $\kappa_{LDA} \sim 1$. A Boltzmann analysis in this regime also predicts an order unity reduction in spin 1 shear viscosity components (Eq. 22).

Two hydrodynamic modes, an elliptic mode and the well known scissor mode, are sensitive to this reduction in viscosity. The angular amplitudes and the decay times are comparable to those measured in [11].

In the extreme situation for where $\kappa_{LDA} \sim 1$, our theoretical estimate for the correction to the viscosity (Eq. 22) breaks down. (For example higher order terms in Eq. 22 become important. Additionally, for $\kappa_{LDA} \sim 1$, $\mu/\omega_z \sim 2.7$ and shell effects, although somewhat weak in the unitary Fermi gases [59], may also become important.) But by gradually increasing $\omega_z$ from $\omega_z \sim 2\pi \times 10^4$ rad/s to $\omega_z \sim 2\pi \times 77000$ rad/s one could measure the tendency of $\eta_{xz}$ to decrease.

The damping rate for the scissor mode has been measured in the BEC-BCS crossover region for weakly anisotropic traps in [11]. It will be interesting to see how the damping rate changes as $\omega_z$ is increased.

On the other extreme, damping of the breathing and the radial quadrupole mode (both insensitive to $\eta_{xz}$) was measured in the 2D Fermi gas [10]. It will be interesting to study the scissor mode in these traps for smaller $\omega_z$.

We hope our experimental colleagues in the cold atoms community will find our proposal interesting and explore anisotropic viscosities in trapped unitary fermions.

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