Upslope Failure Mechanisms and Criteria in Submarine Landslides: Shear Band Propagation, Slab Failure and Retrogression

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Abstract The volume of a submarine landslide is likely to be amplified by shear band propagation along a basal surface or spreading failure extension in the sliding layer. Although the mechanism of translational submarine landslides has been understood using the interpretation of shear band propagation, assessment of the limits of failure extension, either through shear band propagation or retrogressive spreading, remains a challenge due to the diversity of post failure mechanisms. The paper aims to explore all post failure possibilities and quantify failure initiation and extension accordingly, focusing on the upslope conditions. The different failure mechanisms are explored with the aid of the large deformation finite element modeling. Original criteria for scarp position, shear band length at slab failure, retrogression conditions and ultimate spreading limits are proposed, following the process zone approach considering force equilibrium of the sliding mass. Simplified analytical predictions are shown to agree well with the numerical observations. The findings and criteria presented in the study are expected to help assess the ultimate scale of upslope failure in submarine landslides, which is important in determining geohazard zonation of submarine landslides.

Plain Language Summary Submarine landslides are often huge compared to onshore counterparts, but why and how huge? Answering these questions will increase our understanding of submarine landslide evolution and help to assess risks of submarine landslide recurrence. For example, the Storegga Slide offshore Norway, which occurred ∼8,200 years ago, covered an area as large as UK and triggered tsunami waves seriously impacting nearby islands. A large submarine landslide might be initiated from localized failure within an embedded weak layer, followed by failure propagation along the weak layer and extensive slab failure in the overlying sliding layer. These two post-failure phenomena may greatly increase the scale of submarine landslides. Here we simulate submarine landslides in different conditions and explore all possible upslope post-failure mechanisms, using a large deformation numerical modeling method. Based on the observed mechanisms, we develop a new model and original criteria to predict the scarp position and failure extension limit of submarine landslides. Altogether, findings here are expected to help assess the ultimate scale of submarine landslides, which is important in determining geohazard zonation of submarine landslides.

1. Introduction

In landslides, downward translation of a displaced soil mass along a slope-parallel glide shear surface may lead to extended failure upslope, eventually forming slab failure and retrogression (Geissler et al., 2016; Kvalstad et al., 2005; Locat, 2001; Muller & Martel, 2000). Such types of slope failure are commonly referred to as translational slides (Barlow et al., 2003; Bishop & Norris, 1986), or spreading failures that extend into relatively flat terrains (Crosta et al., 2016; Micallef et al., 2007; Puzrin et al., 2017).

Although the mechanisms of translational slides have been well understood particularly with the interpretation of shear band propagation (SBP) along a basal surface (Bernander, 2000; Locat et al., 2013; Puzrin & Germanovich, 2005; Puzrin et al., 2004; Quinn et al., 2011, 2012; Zhang et al., 2015, 2016), assessment of their failure extension limits remains a challenge. Evaluation of the failure extension for any recurrence of submarine landslides and hence geohazard zonation of landslides are important for offshore infrastructure site selection. Kvalstad et al. (2005) recognized the enormous Storegga Slide off the West coast of Norway as a retrogressive slide over a basal shear layer composed of sensitive marine sediments, where the soil
strength is softened during shearing (strain softening) leading to propagation of shear band and upslope slab failure. It should be noted the shear band here is the shear zone across the weak layer which is essentially different from and larger than the shear band observed in a lab test when shearing a uniform material; the strain localization in the overlying sliding layer during extensive failure, however, is akin to the latter. The distance between the upper and lower headwalls is up to 25 km in the Storegga case. In other cases, retrogressive failure might be quite limited characterized by a main scarp (headwall), for example the Licosa Slide in the southern Tyrrhenian Sea (Sammartini et al., 2019); in rare cases nucleation of a shear band is even absent, leading to breakup of the slab without any extended failure. The latter might be quantified using traditional limit equilibrium or limit analysis approaches (Chen & Chameau, 1983; Skempton, 1964).

Figure 1 shows a conceptual but rather typical curvilinear slope described by $Z = F(X)$ with the origin of a global Cartesian coordinate system $(X - Z)$ set at the slope center. The slope is made up of three layers: a sliding layer, a weak layer, and a stiff base. The soil layers are parallel with each other prior to failure in consideration of uniform geological sedimentation, with the slope angle $\theta$ varying along the coordinate $X$. The maximum slope angle located at the slope center is denoted as $\theta_c$ and the half height of the slope from the crest to the center of the slope is denoted as $H$.

The focus of the present study is on the upslope retrogressive failure, which borders the downslope counterpart at the center of a pre-softened zone or the steepest point of a slope. Upon presence of a sufficiently large pre-softened zone in the weak layer, either slab failure at the tip of the pre-softened zone or SBP in the weak layer may emerge. The failure of the sliding layer might be extended into the stable slab leading to a spread; otherwise, the stable slab stands up leaving a main scarp after breakup. The study aims to explore and quantify all possible upslope failure mechanisms of submarine landslides, with respect to the failure initiation and extension criteria. Criteria for different failure mechanisms and expressions of failure characteristics, such as the scarp position and retrogression distance, will be discussed systematically in the study.

2. Problem Description and Methods

The soil constitutive relationship is assumed to comprise a linear elastic response to the peak shear strength, followed by linear post-peak softening toward the residual strength. The mobilized shear stress is calculated by

$$\tau = \bar{G}\delta^e = G\gamma^e$$

in the elastic region while

$$\tau = \max \left[ \tau_p + \left( \tau_v - \tau_p \right) \frac{\delta^p}{\delta^v} \tau_v \right] = \max \left[ \tau_p + \left( \tau_v - \tau_p \right) \frac{\gamma^p}{\gamma^v} \tau_v \right]$$

in the plastic region. In the above, $G$ is the elastic shear modulus (giving an equivalent stiffness $\bar{G} = G/s$ with $s$ being the shear band thickness), $\gamma^e (\delta^e = \gamma^e s)$ and $\gamma^p (\delta^p = \gamma^p s)$ the elastic shear strain (displacement) and the accumulated plastic shear strain (displacement) respectively, and $\gamma^p (\delta^p)$ the value of $\gamma^p (\delta^p)$ to reduce the shear strength from the peak, $\tau_p$, to the residual, $\tau_r$.

Before slab failure, changes in the sliding layer thickness ($l$) are negligible so that the gravity shear stress on the weak layer is quasi-constant, given by

$$\tau_g = \gamma' h \sin \theta \approx \gamma' h \tan \theta$$
which can be normalized as the gravity shear stress ratio as

\[
    r = \frac{\tau_g - \tau_{r,w}}{\tau_{p,w} - \tau_{r,w}} = \frac{\gamma' h \tan \theta - \tau_{r,w}}{\tau_{p,w} - \tau_{r,w}}
\]

where \( \gamma' \) is the effective unit weight of soil, and \( \tau_{p,w} \) and \( \tau_{r,w} \) are the peak and residual shear strengths in the weak layer, respectively. The gravity shear stress ratio is an important parameter governing SBP (Puzrin et al., 2004).

### 2.1. Process Zone Method

Analytical analyses of retrogressive slides allow for quantification of the problem and hence practical risk assessment applications. The limit equilibrium method may be applied for the analysis of retrogressive failure with repeated rotational slides (Bjerrum, 1955) but cannot describe translational retrogressive (spreading) failure. In contrast, the SBP mechanism developed by Puzrin et al. (2004) can illustrate large scale translational slides (Puzrin et al., 2016; Zhang et al., 2017) and has been extended to analyze the retrogressive failure (Buss et al., 2019; Puzrin et al., 2017) combined with the block mechanism proposed by Kvalstad et al. (2005). Particularly, Buss et al. (2019) developed an upper bound solution for considering a certain retrogressive failure mechanism based on the kinematic energy balance of failed blocks, assuming a rigid-plastic sliding layer, a negligible process zone (where soil undergoes a transition from peak to residual strength) etc. The current study will somewhat release these assumptions based on numerical investigations and develop original and simpler criteria for all possible failure scenarios, considering force equilibrium in the process zone (termed process zone method; Puzrin et al., 2004; Zhang et al., 2019).

Palmer and Rice (1973) originally used linear elastic fracture mechanics to obtain a criterion for failure of a cut slope led by uphill quasi-horizontal SBP, which was followed and extended by many studies (e.g., Puzrin et al., 2004; Viesca et al., 2008; Quinn et al., 2011). A key assumption behind this analytical method is that the process zone is negligible such that energy dissipation per incremental shear band growth is explicit (i.e., integration from the peak to the residual over the deformation), which leads to an overestimation of energy dissipation. The process zone method originally proposed by Puzrin et al. (2004), however, recognizes the contribution of the process zone on resisting the SBP. The length of the process zone with a normal set of soil parameters and slope geometries ranges from tens to hundreds of meters, which is likely comparable to the sliding layer thickness and shear band length (Zhang et al., 2015).

The main assumptions for the process zone method presented here are as follows.

1. The shear band is initiated within the weak layer where the strength relative to the shear stress is smaller than in adjacent layers.
2. The inter-layer shear deformation within the sliding layer is ignored so that the downslope displacement is uniform with depth.
3. The thickness of the sliding layer remains unchanged prior to global slab failure.
4. The deformation below the weak layer is negligible, as has been verified by numerical analysis in Zhang et al. (2015).

Figure 2 shows conceptual distributions of the gravity shear stress \( \tau_g \) and the mobilized shear stress \( \tau \) in the weak layer during the shear band initiation and various SBP stages. The weak layer can be divided into three regions: The elastic shearing zone where soil is still intact, the process zone (\( l_p \) in Figure 2) where the shear strength ranges between \( \tau_{p,w} \) and \( \tau_{r,w} \), and the fully softened zone (\( l_s \) in Figure 2). Considering momentum conservation and elastic extension in the sliding layer, the mobilized shear stress \( \tau \) in the weak layer can be deduced for slopes that are described by a linear function as (see Appendix A)

\[
    r^* = \frac{\tau - \tau_g}{\tau_{p,w} - \tau_{r,w}} = \begin{cases} 
    (1 - r_{pc}) \exp \left( \frac{\hat{x} - \hat{x}_0}{\sqrt{1 - \beta^2}} \right), & \hat{x} \leq 0 \text{ (elastic zone)} \\
    \frac{r_{pc} - 1}{\beta} \sin \left( \frac{\hat{x} - \hat{x}_0}{\beta} \right), & 0 < \hat{x} \leq \hat{x}_p \text{ (process zone)} \\
    -r, & \hat{x} > \hat{x}_p \text{ (shear band)} 
\end{cases}
\] (5)
with: \( \hat{x} = \frac{x}{l_u}; \beta^2 = 1 - \frac{\tau_{p,w} - \tau_{r,w}}{Go\delta_p}; \sin \hat{\omega} = \beta; \sin \hat{\omega}_{pc} = \frac{\beta r_{pc}}{1 - r_{pc}} \) where \( r^* \) is the mobilized net shear stress ratio, \( x \) is the local coordinate with the origin at the current interface between the elastic shearing and process zones (see Figure 2), \( r_{pc} \) is the shear stress ratio at \( \hat{x} = 0 \) and \( l_{pc} \) is the process zone length \( l_{pc} \) normalized by the characteristic length \( l_u \) given by (Puzrin et al., 2004)

\[
l_u = \sqrt{\frac{E_{ps}h\delta_p}{\tau_{p,w} - \tau_{r,w}}} \tag{6}
\]

where \( E_{ps} \) is the extension modulus under plane strain conditions. Note that the local coordinate can be related to the global coordinate by

\[
x = l_{j,0} + l_{pc} - x
\]

with the origin of the local coordinate moving with the SBP as \( l_{j,0} \) increases. Equation 5 is based on the assumption that the change in slope gradient within the process zone is negligible. The profile of mobilized net shear stress ratio, \( r^* \), is a fundamental aspect that quantifies initiation of failure, SBP, slab failure and retrogression for the uphill slope in the present study. A rigorous solution of \( r^* \) for an exponential slope geometry is given in Appendix A, but for most slopes it is sufficient to use Equation 5.

### 2.2. Large Deformation Numerical Modeling

Numerical replications of retrogressive slides provide unique insights into the underlying mechanism and whole slide process, as information from site investigations of historical events has been limited to the morphology of mass transport deposits. Recently, new numerical tools have been emerging and are showing great promise for analyzing large deformation and retrogressive landslide kinematics; these include the Material Point Method (Dong et al., 2017; Tran & Solowski, 2019), Coupled Eulerian–Lagrangian method (CEL, Dey et al., 2016; Stoecklin, 2019; Stoecklin et al., 2020), and Particle Finite Element method (Zhang et al.).

**Figure 2.** Three zones and distributions of shear stress in weak layer at (a) initiation stage and (b) different SBP stages.
et al., 2020). The present work adopts an in-house Arbitrary Lagrangian–Eulerian technique, termed remeshing and interpolation technique with small strain (RITSS, Hu & Randolph, 1998), as well as the CEL method. The RITSS analysis divides the whole process into a series of increments, each having sufficiently small time-step to avoid mesh distortion, followed by remeshing and interpolation of field variables from old to new meshes. The applicability and accuracy of the RITSS method for modeling submarine landslides have been addressed in Zhang et al. (2015, 2019).

For simplicity, an exponential function of geometry has been used in the LDFE analysis, which is given by

$$F(X) = \begin{cases} -H \left[ 1 - \exp \left( \frac{X}{H} \tan \theta_e \right) \right], & X < 0 \\ H \left[ 1 - \exp \left( -\frac{X}{H} \tan \theta_e \right) \right], & X \geq 0 \end{cases}$$

The basic geometry parameters and soil properties for the numerical cases are listed in Table 1. Note that the shear band thickness, $s$, is equal to the weak layer thickness since the weak layer is represented by only one layer of elements. Mesh dependency and size effect on the failure mechanisms are avoided by adjusting the pairs of $\gamma^p_w$ and $s$ to the fixed macro value of $\gamma^p_w = \gamma^p s$ (Zhang et al., 2015). More details about the modeling can be found in Zhang et al. (2015, 2019).

The bottom and side boundaries were fixed in all cases. Base parameters for all numerical cases are listed in Table 1. In most cases, the geometry parameters (such as slope angle and height) and soil properties in the weak layer are fixed. The at-rest earth pressure coefficient $K_0$ and average sliding layer strength in the sliding layer $\tau^p$ however, vary between cases leading to diverse post-failure behavior. The $K_0$ values (0.7–1.0) selected in the study are common in over consolidated marine clays in Northern continental slopes following the last deglaciation such as in the North Sea (During & Rennie, 1979). The Poisson’s ratio at the first step was set to $K_0 / (1 + K_0)$ to generate specific initial soil conditions; it was then changed to 0.495 in the following steps to maintain undrained conditions during slope failure and submarine landslide process. Strength profiles for the numerical cases are shown in Figure 3, with C03, C05 and C08 depth dependent and the others uniform over the sliding layer depth.

### Table 1
**Base Parameters for All Cases**

| Parameter | Value | Unit |
|-----------|-------|------|
| Length of slope | 3,000 | m |
| Half height of slope, $H$ | 20 | m |
| Maximum slope angle, $\theta_e$ | 6 | degrees |
| Thickness of sliding layer, $h$ | 8, 10 | m |
| Thickness of weak layer, $s$ | 0.4 | m |
| Half pre-softened zone length, $l_0$ | 20, 60, 170 | m |
| Young's modulus for plane strain conditions, $E_{ps}$ | 3,600 | kPa |
| Poisson's ratio during landslide, $\nu$ | 0.495 | |
| At rest earth pressure coefficient, $K_0$ | 0.7, 0.9, 0.98 | |
| Gravity acceleration, $g$ | 9.81 | m/s² |
| Saturated density, $\rho$ | 1,700 | kg/m³ |
| Submerged density, $\rho'$ | 700 | kg/m³ |
| Peak shear strength in weak layer, $\tau^p_w$ | 10 | kPa |
| Average peak shear strength in sliding layer, $\tau^p$ | 8, 13, 15, 25, 50 | kPa |
| Soil sensitivity in weak layer, $S_w$ | 5 | |
| Soil sensitivity in sliding layer, $S_s$ | 5 | |
| Residual plastic shear displacement, $\delta^p$ | 0.05, 0.2, 0.25 | m |
| Characteristic length, $l_u$ | 15, 30 | m |
The final configurations of all numerical cases are presented in Figure 4 showing different failure mechanisms. Three main streams can be identified according to the failure mechanism in the weak layer: SBP into the stable zone where the gravity shear stress ratio $r < 0 (\tau_{g} < \tau_{r,w})$; SBP limited at the quasi-stable zone where $0 < r < 1 (\tau_{r,w} < \tau_{g} < \tau_{p,w})$; and no SBP in the weak layer. Main post-failure characteristics (shear band length $X_{sb}$ upon slab failure, slab failure position $X_{sf}$, and retrogression limit $X_{r}$) are listed in Table 2 compared with the analytical solutions, which will be detailed in the following.

Figure 5 compares the RITSS and CEL numerical results of spread failure, that is, for C03, C05 and C08. The final extensions of failure from the two numerical methods are comparable in overall verifying accuracy of either approach in modeling the problem, although the RITSS approach provides superior definition of the horsts and grabens that form and typically shows one or two additional retrogressive horst-graben pairs compared with the CEL analyses.

Figure 3. Strength profiles of numerical cases.

Figure 4. Final configurations of numerical cases showing contours of the degree of softening.
3. Failure Initiation Mechanisms and Criteria

3.1. Triggering and Pre-Softened Zone

Submarine landslides escalated by SBP require a pre-failure “history” that has led, or might lead, to the formation of an initial shear band (or a pre-softened zone) in an embedded weak layer. The initiation history varies according to different external factors that might trigger the failure. These may be divided into two broad types: Whether the trigger is a gradual increase in loading condition, perhaps involving rapid sedimentation or diapirism, until a limiting shear stress is reached; or failure is initiated by shear strength reduction in the weak layer. In all such mechanisms, of either type, the net result a gradual but global decrease of shear strength relative to the shear stress. The initial failure will occur at the critical point, which is usually at the steepest point of a slope, once the gravity shear stress exceeds the current shear strength.

3.2. Driving Force From Pre-Softened Zone

A total stress analysis framework is adopted in the present study. The initial slope with the presence of a pre-softened zone can be divided into two parts: stable and unstable portions, separated at $X_s = l_0$ as shown
in Figure 2a. The unstable slab covers the pre-softened zone where the gravity shear stress is larger than the mobilized shear stress (limited to the current shear strength \( \tau_0 \), that is, \( \tau_g \geq \tau = \tau_0 \); a stable slab exists initially upslope (and downslope, but not considered further here) potentially encompassing part of the process zone together with an elastic shearing zone where the gravity shear stress can be balanced by the available shear stress. Downward movement of the sliding layer unloads the upslope slab toward active failure and loads the downslope toward passive failure, with the slope parallel force (per unit thickness; \( P_s \)) at the center of the slope unchanged at the gravity level, given by

\[
P_s = \frac{1}{2} K_0 \gamma^2 \cos \theta + u_w h; u_w = \gamma_w h_w
\]

where \( K_0 \) is the at-rest earth pressure coefficient in terms of the effective stress of soils, \( u_w \) the hydrostatic water pressure, \( \gamma_w \) the unit weight of water, and \( h_w \) the vertical distance between the sea level and the medium point of the sliding layer. At the end of the upslope pre-softened zone \( (X_s) \), the slope parallel force is minimum, denoted by \( P_{\text{min}} \), due to maximum unloading at this point (Figure 6).

Considering force equilibrium in the unstable slab as shown in Figure 6a gives

\[
P_s - P_{\text{min}} - \int_0^{l_u} (\tau_g - \tau_0) dX = 0 \rightarrow \Delta P_{\text{max}} = DF_0
\]

with \( DF_0 = \int_0^{l_u} (\tau_g - \tau_0) dX \) and \( \Delta P_{\text{max}} = P_s - P_{\text{min}} \) where \( \Delta P_{\text{max}} \) is the maximum unloading at the end of the pre-softened zone and \( DF_0 \) is the initial driving force from the pre-softened zone. The driving force can be normalized as

\[
\overline{DF}_0 = \frac{DF_0}{l_u (\tau_{p,w} - \tau_{r,w})} = \int_0^{\hat{l}_0} \eta_0 d\hat{X} = \overline{\eta}_0 \hat{l}_0
\]

with \( \eta_0 = \frac{\tau_g - \tau_0}{\tau_{p,w} - \tau_{r,w}} \) where \( \overline{\eta}_0 \) is the average shear stress ratio within the pre-softened zone and \( \hat{l}_0 \) is the upslope pre-softened zone length (from the slope center to the upslope end of the pre-softened zone) normalized by \( l_u \).
3.3. Initiation With Slab Failure

The active failure state of the upslope sliding layer is achieved when the shear stress reaches the peak shear strength. The active earth pressure is denoted by $\sigma_{h,p}$, as shown in Figure 7, and given by

$$\sigma_{h,p} = \sigma_{v,g} - 2\sqrt{\tau_{p,s}^2 - \tau_h^2}$$

with $\sigma_{v,g} = \gamma' \cos \theta + u_w$ and $\tau_h = \gamma' \sin \theta$ where $\tau_{p,s}$ is the peak shear strength in the sliding layer, $\sigma_{v,g}$ is the total overburden pressure (from both sediments and water) acting on the plane parallel to the slope surface and $\tau_h$ is the gravity shear stress at depth $z$ (along the direction perpendicular to the ground surface). The sea water component of the total vertical stress, $u_w$, is assumed constant during the landslide process considering the runup height of any waves is less significant compared to the water depth. The minimum slope-parallel force at the active failure state (termed slab failure hereafter) is then calculated as the integral of the active earth pressure over the depth.

For a depth-dependent strength profile, the peak shear strength in the sliding layer is expressed by

$$\tau_{p,s} = \tau_m + k\gamma'z$$

where $\tau_m$ is the strength at the mudline and $k$ is the strength ratio. If the strength at the mudline approaches zero $\tau_m \to 0$ and the strength ratio $k$ is constant with depth, so the minimum slope-parallel force is given by

$$P_{\text{min,act}} = \int_{0}^{h} \sigma_{h,p} dz = \frac{1}{2} \gamma' h \left( \cos \theta - 2\sqrt{k^2 - \sin^2 \theta} \right) + u_w h$$

(14)

For cases of non-uniform strength ratio $k$ or nonzero mudline strength, small angle approximations ($\cos \theta \to 1$ and $\sin \theta \to 0$) are taken, which are assumed to be valid for gentle submarine slopes, so that the minimum slope-parallel force is simplified as

$$P_{\text{min,act}} \approx \int_{0}^{h} \left( \gamma' h + u_w - 2\tau_{p,s} \right) dz = \frac{1}{2} \gamma' h^2 \left( 1 - SN \right) + u_w h$$

(15)

with $SN = \frac{4\tau_{p,s}}{\gamma'h}$ where $\tau_{p,s}$ is the average shear strength in the sliding layer and $SN$ is the stability number.

For the case of $\tau_m = 0$ kPa, $\gamma' = 7$ kN/m$^3$, $h = 10$ m, $\theta = 5^\circ$, and $k = 0.2$, values of which are typical in the marine environment, the difference between minimum slope-parallel forces calculated from the simplified form and the rigorous integral is around 5.7%.

The maximum unloading at the active failure state, hereafter referred to as the initial sliding layer resistance $SLR_0$, is thus given by

![Figure 7. Earth pressure in sliding layer under undrained conditions and strain softening.](image-url)
\[
S_{LR} = \Delta P_{\text{max,act}} = P_{g} - P_{\text{min,act}} = \frac{1}{2} \gamma' h^2 (K_0 - 1) + 2 \tau_{p,w} h \rightarrow \bar{S}_{LR} = \frac{\Delta h}{l_s (\tau_{p,w} - \tau_{w})} = \frac{\bar{h}}{2k_w} \frac{S_{w}}{S_{w} - 1} (K_0 - 1 + SN)
\]

with \(S_{w} = \tau_{p,w} / \tau_{w}\) and \(k_w = \tau_{p,w} / \gamma' h\) where \(\bar{h}\) is the sliding layer thickness normalized by \(l_s\), \(S_{w}\) is the soil sensitivity in the weak layer, and \(k_w\) is the strength ratio in the weak layer. Note that the hydrostatic pressure is canceled in the above equation implying the water depth has no effect on the sliding layer resistance.

The criterion for slab failure initiation at the end of the pre-softened zone is therefore

\[
\Delta \hat{P}_{\text{max}} = \Delta \hat{P}_{\text{max,act}} = \bar{S}_{LR}
\]

### 3.4. Initiation With Shear Band Propagation

Figure 6b shows force components in the stable slab portion. The slope-parallel force at the starting point of the stable slab, \(s_{E X} \equiv 0\), is the reaction force from the unstable slab and hence equal to \(P_{\text{min}}\) while at the remote position of the stable slab where deformation is negligible, the slope-parallel force remains \(s_{E X} \equiv 0\).

Considering force equilibrium in the stable slab hence yields

\[
0 = \int_{X_s}^{X} \tau dX \Rightarrow \int_{X_s}^{X} \tau^* d\hat{X} = 0 \rightarrow \Delta \hat{P}_{\text{max}} = \int_{X_s}^{X} \tau^* d\hat{X}
\]

where the mobilized net shear stress ratio \(\tau^*\) is given by Equation 5.

The evolution of the mobilized shear stress in the weak layer underlying the stable slab can be divided into three stages as shown in Figure 2a: (a) elastic shearing only (short green dashed line); (b) development of the process zone (long purple dashed line); and (c) catastrophic SBP (red solid line). The critical condition for catastrophic SBP is that the shear strength at the interface between the pre-softened and process zones has been softened to the gravity shear stress, in which the critical process zone length is \(\hat{c}_E \approx \sin \beta (Zhang et al., 2015)\). Hence, the critical value of \(\int_{X_s}^{X} \tau^* r^dX \) (hereafter referred to as the normalized weak layer resistance, \(\bar{WLR}\) is (see Appendix C)

\[
\bar{WLR} = \int_{X_s}^{X} r^* d\hat{X} = \int_{X_s}^{X} \tau^* d\hat{X} = 1 - r_{pc}
\]

which increases during SBP as the value of \(r_{pc}\) decreases as the slope flattens, with its initial value (when the shear band just begins to propagate from the pre-softened zone) denoted by \(\bar{WLR}_0\). Equation 19 is only valid for SBP at \(1 > r \geq 0\) and the expression of \(WLR\) for further SBP into \(0 < r \leq 1\) will be provided later.

The criterion for the failure initiation with SBP is therefore

\[
\Delta \hat{P}_{\text{max}} \geq \bar{WLR}_0
\]

The value of \(\Delta \hat{P}_{\text{max}}\) (to balance the driving force \(\Delta F_0\)) gradually increases with the growth of the pre-softened zone until either Equations 18 or 20 is met, leading to active failure in the sliding layer or SBP along the weak layer. Therefore, the state of a slope with the presence of a pre-softened can be assessed as shown in Figure 8, by

\[
\begin{align*}
\Delta F_0 \geq \bar{WLR}_0 \cup \bar{S}_{LR} & \geq \bar{WLR}_0 \quad \text{SBP} \\
\Delta F_0 \geq \bar{S}_{LR} \cup \bar{WLR} & \geq \bar{S}_{LR} \quad \text{slab failure} \\
\text{otherwise} & \quad \text{stable}
\end{align*}
\]

Following the failure initiation, diverse post-failure mechanisms are shown in Figure 19 and the competition between SBP and slab failure can be assessed by considering a general form of Equation 21, given by
The criteria for different post-failure mechanisms and the corresponding failure extension limits are discussed hereafter.

4. SBP Dominant Post Failure Mechanism–Overlying Layer of High Strength

For a weak layer covered by strong overlying soils, the initial sliding layer resistance $\overline{WLR}$ might be sufficiently large compared to the weak layer resistance $\overline{SLR}_0$, not only at the initiation stage but also during the SBP process. In such a situation, SBP in the weak layer is always a preference over the slab failure according to Equation (22), and hence the shear band can propagate across the quasi-stable zone (where $0 \leq r \leq 1$; Puzrin et al., 2016) to the stable zone (where $r < 0$). According to Equation 19, $\overline{WLR} \to 1$ when $r_{pc} \to 0$. Therefore, to ensure SBP into the stable zone and hence a SBP dominant post-failure mechanism, the initial sliding layer resistance needs to meet

$$\overline{SLR}_0 > 1$$  \hspace{1cm} (23)

Three types of post failure can be identified in the SBP dominant mechanism: no scarp, scarp and spread, as shown in Figure 19.

4.1. No Scarp–Pure SBP

For overlying soils of very high strength, the active failure state might not be achieved at all, leading to pure SBP (case No 1 in Figure 19). Figure 2b presents the evolution of the mobilized shear stress (calculated by Equation 5) compared to the gravity shear stress on the weak layer during progressive SBP away from the original unstable zone, indicating the unstable and stable slab portions and the three zones (fully softened, process, and elastic shearing zones) of the weak layer. When the shear band propagates within the quasi-stable zone $0 \leq r \leq 1$, the interface between the unstable and stable slabs, $X = X_{s}$, is within the process zone as shown in Figure 2b and hence the weak layer resistance can be calculated according to Equation 19. Strictly speaking, in addition to the fully softened zone ($DF_{fs}$), part of the process zone (where the shear strength is reduced to less than the gravity shear stress, $DF_{pc}$) may also contribute to the driving force. Thus, the total driving force for SBP at $0 \leq r \leq 1$ is given by

$$\overline{DF} = \overline{DF}_{fs} + \overline{DF}_{pc}$$

$$\overline{DF}_{fs} = \overline{r}_{fs} \overline{X}_{fs}$$

$$\overline{DF}_{pc} = \frac{\overline{X}_{fs} \overline{X}_{pc} - r^{*}d\overline{X}_{fs}}{\overline{l}_{fs}} = \begin{cases} \frac{(1 - r_{pc})}{(1)} - \sqrt{1 - 2r_{pc}} & r_{pc} \leq 0.5 \\ \frac{(1 - r_{pc})}{(1)} & r_{pc} > 0.5 \end{cases}$$ \hspace{1cm} (24)

where $\overline{X}_{fs}$ is the upslope fully softened zone length normalized by $\overline{l}_{fs}$ and $\overline{r}_{fs}$ is the average shear stress ratio in the fully softened zone. It is sufficient to simply assume the shear strength in the unstable portion of the process zone $X \in [\overline{l}_{fs}, X_{fs}]$ has been reduced to the residual and hence simplify the driving force as

$$\overline{DF} = \overline{r}_{s} \overline{X}_{s}$$ \hspace{1cm} (25)

where $\overline{r}_{s}$ is the average shear stress ratio within $\overline{X} \in [0, \overline{X}_{s}]$ as indicated in Figure 2b, $\overline{X}_{s} = \hat{X}_{sb} - \overline{\hat{X}}$, where $\hat{X}_{sb} = \overline{\hat{l}}_{fs} + \overline{\hat{l}}_{pc}$ is the end position of the shear band covering both the fully softened and process zones. Note that the expression for $\overline{l}_{pc}$ can be found in Appendix B.

For SBP into the region where $r < 0$, the interface $X_{s}$ becomes fixed at $r = 0$ as shown in Figure 2b. In such a scenario, the weak layer resistance comprises two parts: The process and elastic shearing zones ($WLR_{pc+es}$) and also a portion of the fully softened zone (where $r < 0; WLR_{fs}$) given by
\[
\overrightarrow{WLR} = \overrightarrow{WLR}_{fr} + \overrightarrow{WLR}_{PC+ess} \\
\overrightarrow{WLR}_{PC+ess} = \int_{l_{fs}}^{l_{fr}} r^* d\hat{X} = \sqrt{1 - 2r_{pc}} \\
\overrightarrow{WLR}_{fr} = \int_{l_{fr}}^{l_{fr}} -rd\hat{X} = -r_{f_{r_{0}} - i_{fs}} \left( \hat{f}_{fs} - \hat{X}_{r=0} \right)
\]

where \( \hat{X}_{r=0} = \hat{X}_r \) is the coordinate at \( r = 0 \), \( r_{f_{r_{0}} - i_{fs}} \) is the average shear stress ration within the stable portion of the fully softened zone \( \hat{X} = \left[ \hat{X}_{r=0}, \hat{f}_{fs} \right] \). The maximum driving force is calculated by Equation 25 with \( \hat{X}_r = \hat{X}_{r=0} \), as

\[
\overrightarrow{DF}_{max} = \int_{0}^{l_{fr}} rd\hat{X} = r_{f_{r_{0}} - i_{fs}} \hat{X}_{r=0}
\]

In summary, for SBP into different seafloor regions, the values of \( \overrightarrow{DF} \) and \( \overrightarrow{WLR} \) can be expressed as

\[
\overrightarrow{DF} = \overrightarrow{r}_{f_{r_{0}}} \hat{X}_r, \text{with } \hat{X}_r = \begin{cases} \hat{f}_{fs} + \hat{f}_{pc} - \hat{\omega}_k & \text{SBP at } 0 \leq r < 1 \\ \hat{X}_{r=0} & \text{SBP at } r < 0 \end{cases}
\]

with \( \hat{f}_{pc} = \hat{\omega}_k + \beta \sin \frac{\beta r_{pc}}{1 - r_{pc}} \) and \( \hat{\omega}_k = \beta \sin \beta \) and

\[
\overrightarrow{WLR} = \begin{cases} \frac{1 - r_{pc}}{\sqrt{1 - 2r_{pc}} - r_{f_{r_{0}} - i_{fs}}} \left( \hat{f}_{fs} - \hat{X}_{r=0} \right) & \text{SBP at } 0 \leq r < 1 \end{cases}
\]

### 4.1.1. No Scarp Criterion

To ensure an intact sliding layer and hence the pure SBP mechanism, the sliding layer resistance needs to satisfy

\[
\overrightarrow{SLR}_0 > \overrightarrow{DF}_{max} = \overrightarrow{r}_{f_{r_{0}}} \hat{X}_{r=0}
\]

### 4.1.2. Ultimate Shear Band Length

As the overlying layer remains intact, assessment of the final shear band end in the weak layer is the main concern of such a failure mechanism. Considering force equilibrium, the SBP stops if the maximum driving force can be balanced by the weak layer resistance (Zhang et al., 2019), that is,

\[
\overrightarrow{DF}_{max} = \overrightarrow{WLR} \rightarrow \int_{0}^{l_{fr}} rd\hat{X} = \sqrt{1 - 2r_{pc}} \hat{f}_{fs} = \frac{\sqrt{1 - 2r_{pc}}}{r_{f_{fr}}}
\]

Considering \( r_{k} \rightarrow 0 \) at flat terrains, \( r_{pc} \rightarrow -1/(S_{r,w} - 1) \), the Equation 31 becomes

\[
\hat{f}_{fs} = \frac{\sqrt{1} \left( S_{r,w} + 1 \right) \left( S_{r,w} - 1 \right)}{\sqrt{1} \hat{f}_{fs}}
\]

Ignoring the resistance component from the process zone, the Equation 31 simply becomes (Puzrin et al., 2017)

\[
\hat{f}_{fs} = 0
\]

Note that with the inertia effect, the shear band may propagate further beyond the length calculated from the force equilibrium method until the kinetic energy is totally dissipated, providing an upper limit of the final shear band length which can be up to double that from the lower limit by Equation 31.

Figure 9 shows the analytical evolutions of the driving force \( \overrightarrow{DF} \) and the weak layer resistance \( \overrightarrow{WLR} \) with SBP for selected numerical cases. The value of \( \overrightarrow{DF} \) first increases with SBP and then becomes constant with the maximum driving force being \( \overrightarrow{DF}_{max} = 2.1 \), when the shear band propagates beyond \( l_{fs} = 244m \) (where \( r < 0 \)); while the value of \( \overrightarrow{WLR} \) keeps growing until it can balance the maximum driving force \( \overrightarrow{DF}_{max} \) at \( l_{fs} = 494m \) and hence SBP stops. For the case C01 with \( K_0 = 0.98 \) and \( \overrightarrow{r}_{f_{fr}} = 50kPa \), the initial sliding layer resistance \( \overrightarrow{SLR}_0 = 4.14 \) is much larger than \( \overrightarrow{DF}_{max} \), and hence a pure SBP mechanism is expected as evidenced in the numerical modeling. The final shear band length (including inertia effect) is \( l_{fs} = 660m \) which is 33% larger than the analytical solution at the static condition (495m). As a comparison, the more
4.2. Scarp

If the initial sliding layer resistance is insufficient to resist the maximum driving force, that is

\[ 1 < \hat{SLR}_0 \leq \hat{DF}_{max} = \hat{r}_{r=0} \hat{X}_{r=0} \]  

slab failure results (case No 2 in Figure 19). Three questions arise with the occurrence of slab failure: (a) where does the sliding layer break (i.e., the scarp position); (b) how far does the shear band propagate within the weak layer following slab failure initiation; and (c) whether the slab failure is followed by retrogression.

4.2.1. Scarp Criterion

After slab failure, the ground surface of the failed slab may gradually drop. The depth of this drop and the current height are denoted by \( dEh \) and \( cEh \) respectively. The current height \( cEh \) is normalized by the original height as \( \alpha = \frac{cEh}{h} \). With the full drop (\( \alpha = 0 \)), the sliding layer resistance is calculated by (modification from Equation 16)

\[ \hat{SLR}(\alpha = 0) = K_0 \hat{h} \frac{\hat{f}_{s,w}}{2x_w \hat{S}_{s,w} - 1} \]  

A sufficient condition leading to the scarp mechanism without retrogression is that the initial sliding layer resistance \( \hat{SLR}_0 \) cannot be recovered even with the full drop (\( \alpha = 0 \)), which is given by

\[ \hat{SLR}_0 > \hat{SLR}(\alpha = 0) \rightarrow SN > 1 \]  

This condition is consistent with the classical undrained cut slope stability requirement, that is, \( \hat{r}_{p,x} / \gamma h > 0.25 \).

4.2.2. Scarp Position and Shear Band Length

As aforementioned, the interface between the unstable and stable slabs for the SBP dominant failure is at \( r = 0 \), that is, \( X_f = X_{r=0} \), so that the sliding layer is most likely to break at this point, that is, the scarp position is \( X_f = X_{r=0} \). This means that slab failure occurs within the full length of the shear band, which has already propagated further to \( r < 0 \). With the growth of the fully softened zone, the weak layer resistance and hence the slope-parallel force in the sliding layer increases as shown in Figure 9, until it reaches the sliding layer resistance, that is,
Through Equation 37, the critical value of $r_{pc}$ and hence the critical shear band length may be calculated.

As shown in Figure 4, C02 with $K_0 = 0.98$ and $\tau_{p,w}/\tau_{p,u} = 2.5$ exhibits a SBP dominant failure mechanism leaving a main scarp. The position of the back scarp is at around $X = 220$ m and an uncomplete failure surface (similar to a crown crack) can be observed in the sliding layer at around $X = 240$ m, which are close to the prediction $X_s = X_{r=0} = 244$ m. The slight difference between the numerical and analytical results of the scarp position may be attributed to the fact that the slope-parallel force is transferred in the sliding layer in a “wave” type. The minimum slope-parallel force (or the active earth pressure) may be mobilized at several “wave” troughs simultaneously and any of these may evolve into a main scarp. The shear band end is at around $X_{sb} = 550$ m which is 13% larger than the analytical solution $X_{sb} = 487$ m, with the difference resulting mainly from inertia effects in the numerical modeling.

4.3. Spread

4.3.1. Spread Criterion

A necessary condition for spread failure is that before reaching the full drop of the failed slab, the initial sliding layer resistance $SLR_0$ can be recovered somewhere behind the main scarp, that is

$$ SLR_0 \leq SLR(\alpha = 0) \rightarrow SN \leq 1 $$

(38)

4.3.2. Retrogression Mechanism

The surface of the failed slab gradually drops during its downslope movement. In many cases, failure may further spread into the stable slab with a critical drop, leading to retrogressive failure. Figure 10 shows a typical numerical example of retrogressive failure (C03) starting from around the point of $r = 0$, with respect to the contours of the degree of softening (SD) given by

$$ SD = \frac{\tau_{p,u} - \tau_c}{\tau_{p,u} - \tau_{r,u}} $$

(39)

where $\tau_c$ is the current shear strength. $SD = 0$ and 1 represent no softening and full softening respectively. The failure is initiated by SBP with the fully softened zone propagated to near $X = 100$ m at $T = 3$s and into the stable zone $r < 0$ at $T = 17$s. The slab failure is apparent at $T = 17$s with multiple scarps and the hindmost one at around $X = 225$ m. As a comparison, the analytical solution gives $X_s = 244$ m (where $r = 0$).

Note that multiple scarps are caused by evident tensile failure of the unstable slab. Spreading failure is apparent with 7 repeats of retrogression, extending the failure in the sliding layer to $X = 330$ m. The unstable slab is further divided into some small blocks as can be observed at $T > 30$ s, resembling the horsts and grabens formed by retrogressive failure. Horsts and grabens caused by tensile failure of the unstable slab are also observed in case C02 without spreading failure beyond $X_{r=0}$ which implies their presence in historical events may not be related simply to retrogressive failure.

Figure 10. Contours of the degree of softening at different failure stages for case C03.
Figure 11 shows the process and details of the first retrogression from $\text{ET}_0 = 51\text{s}$ to $\text{ET}_0 = 58\text{s}$, where 'H' and 'G' indicate horst and graben, respectively, and the numbers '0,' '1,' and '2' denote the initial failure, first retrogression, and second retrogression, respectively. Despite some complicated details such as rotation or breakup of blocks, the mechanism of retrogressive failure can be simplified as shown in Figure 12 (with linear slope geometry) and described as follows.

1. **Stage I** (material softening): upon mobilization of the active earth pressure, the sliding layer resistance $\text{SLR}$ reduces from the initial value $\text{SLR}_0$ which causes reloading in the sliding layer and stops SBP immediately.

2. **Stage II** (geometry hardening): a graben, a horst, and a transitional parallelogram form within the new failed slab; the sliding layer becomes thinner and thinner with further drop of the graben, gradually increasing the value of $\text{SLR}$.

3. **Stage III** (SBP redevelopment): with a certain drop, the initial sliding layer resistance $\text{SLR}_0$ in the failed block may be recovered and hence the slope-parallel force is increased to be the same with the weak layer resistance $\text{WLR}$ corresponding to the current shear band length; any further drop should result in further SBP into the stable slab.

4. **Stage IV** (trigger of new retrogression): with the critical drop, the next retrogression occurs with the distance from the next rupture to the current rupture defined as the incremental retrogression distance (IRD, see Figure 12).

This block mechanism for explaining the retrogression based on numerical observations is similar to that proposed by Kvalstad et al. (2005). After triggering the new retrogression, the graben might further drop.

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**Figure 11.** Contours of the degree of softening during the first retrogression for case C03.

**Figure 12.** Simplified retrogression model.
due to tensile failure, rotation or elongation during its downward movement, which is not considered in the simplified mechanism shown in Figure 11.

4.3.3. Critical Drop for New Retrogression

The normalized sliding layer resistance within graben and horst regions is a function of the graben height, $h_i$, and given by (Appendix D)

$$\tilde{SLR}(\alpha) = \frac{P - P_{\text{min}}}{I_k (\tau_{p,w} - \tau_{r,w})} = \frac{h}{2k_w} \left( \frac{S_{i,w}}{S_{i,w} - 1} \right)^{K_0 - \alpha^2 + ab \chi SN}$$  \hspace{1cm} (40)

with $\alpha = h_i / h$, $b = \tilde{\tau}_{p,x} / \tilde{\tau}_{r,x}$

Considering the force equilibrium within the incremental retrogression region, the critical drop for new retrogression can be given by a parabolic function (see Appendix D)

$$A\alpha^2 + B\alpha + C = 0 \rightarrow \alpha = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \in (0,1)$$ \hspace{1cm} (41)

with $A = \left( \frac{SN}{S_{i,x}} \left( 2\tau_{r,w} / k_{r,h} + 1 \right) \right) - 1$, $B = \left( SN2\tau_{r,m} / S_{i,x} \left( 2\tau_{r,w} + k_{r,h} \right) \right) + 4k_w \tilde{\tau}_{RD} \left( S_{i,w} - 1/S_{i,w} \right)$, $C = 1 - SN$ where $\tilde{\tau}_{RD}$ is the average shear stress ratio over the incremental retrogression distance. For a homogeneous sliding layer, constants $A = -1$ and $B = SN / S_{i,x} + 4k_w \tilde{\tau}_{RD} \left( S_{i,w} - 1/S_{i,w} \right)$; while for the first retrogression with $\tilde{\tau}_{RD} \rightarrow 0$ (as slab failure initiates at $r = 0$ for the SBP dominant mechanism), the constant $B$ can be further simplified as $B = SN / S_{i,x}$. Hence, the critical height ratio for the first retrogression with the SBP dominant mechanism in a homogeneous sliding layer is given by

$$\alpha = \frac{SN / S_{i,x} + \sqrt{(SN / S_{i,x})^2 + 4(1 - SN)}}{2}$$ \hspace{1cm} (42)

The criterion for spread failure Equation 38, ensures a real root for the solution of $\alpha$ by Equation 42. The critical height ratio for the subsequent retrogression is usually smaller than for the first retrogression, since a greater graben drop is required in a region of milder slope. Equation 42 is thus expected to give an overestimated prediction of the graben height and hence the retrogression distance.

Figure 13a plots the analytical solutions of the critical height ratio for all retrogression repeats in C03, showing good agreement with the numerical results. Figure 14 shows the evolution of the sliding layer resistance $\tilde{SLR}$ with decreasing height ratio $\alpha$ for different cases. For C03, $\tilde{SLR}_0$ can be recovered with certain values of $\alpha$; for C02, however, $\tilde{SLR}$ during the drop of the graben is always smaller than $\tilde{SLR}_0$. As predicted, a scarp mechanism is observed in C02 while a spreading mechanism is apparent in C03.
4.3.4. Retrogression Distance Limit

The current gravity shear stress on the weak layer, $\tau_{gc}$, for a reducing height graben is smaller than the initial value $\tau_g$ prior to any graben drop, so in turn the current gravity shear stress ratio is smaller than the initial value, that is,

$$\frac{\tau_{gc} - \tau_{r,w}}{\tau_{p,w} - \tau_{r,w}} < \frac{\tau_g - \tau_{r,w}}{\tau_{p,w} - \tau_{r,w}} = r \leq 0$$

(43)

This indicates that the region where grabens and horsts form by retrogression will decelerate. The behavior is referred to as the stable retrogression in the present study. It is important to assess the limit of the extended stable retrogression and understand any threat that it might put on upslope offshore infrastructure.

Assuming the dimensions of horsts and grabens do not change significantly in the stable retrogression region, the final configuration of the failed mass can be estimated. Figure 15a shows a conceptual model...
and force components at the critical condition where the next retrogression is just about to occur; \( P_0 \) is the slope-parallel force acting on the plane across the initial slab failure point, \( P_2 \) is the slope-parallel force acting on the plane across the final retrogression point, and \( \bar{r}_c \) is the current average gravity shear stress over the retrogression distance. Considering force equilibrium of blocks gives

\[
\tau_{c,w} l_c - \bar{r}_c l_c = P_2 - P_0 \rightarrow \bar{r}_c \dot{l}_c = \Delta \hat{P}_0 - \Delta \hat{P}_2 \rightarrow \dot{l}_c = \frac{\Delta \hat{P}_2 - \Delta \hat{P}_0}{\bar{r}_c}
\] (44)

with \( \Delta \hat{P}_2 = \overline{SLR}_0 \) where \( \bar{r}_c \) is the current average shear stress ratio over the retrogression distance, \( \dot{l}_c \) is the retrogression distance normalized by \( l_c \), and \( \Delta \hat{P}_2 \) is the net normalized downslope support. For a limiting condition with no downslope support, that is, \( \Delta \hat{P}_2 = \overline{SLR}(\alpha = 0) \), the retrogression distance is given by

\[
\dot{l}_c = \frac{\overline{SLR}_0 - \overline{SLR}(\alpha = 0)}{\bar{r}_c} = \frac{\hat{h} S_{r,w} SN - 1}{2 k_w S_{r,w} - 1} \bar{r}_c
\] (45)

The value of \( \bar{r}_c \) generally decreases with repeated retrogression because of flattening of the slope and lower critical drop ratios in successive retrogression events as shown in Figure 13a. Hence an iterative method is necessary to solve Equations 44 and 45. For simplicity, the values of the critical drop ratio and the current shear stress ratio for the first retrogression can be adopted, resulting in slightly high (so conservative) estimates of \( \dot{l}_c \).

For C03, \( \Delta \hat{P}_0 = 2.3 \) measured from the numerical modeling and \( \overline{SLR}_0 = 1.88 \), which yield a retrogression distance \( l_c = 59 \text{m} \) according to Equation 44. Hence, extended failure is expected to stop at \( X_r = X_0 + l_c = 303 \text{m} \). For comparison, the numerical value of \( X_r \) is 330 m which is about 10% larger than the prediction. As Equation 44 based on the force equilibrium method represents a lower limit, the slightly higher value of \( X_r \) observed in the numerical modeling is quite reasonable considering inertia effects. The inertia force can be partly or over balanced by considering zero downslope support, that is, Equation 45, which yields a retrogression limit at \( X_r = 341 \text{m} \).

For a clearer explanation, the calculation processes and results of key parameters for C03, using the simplified methods, are provided in Table 3.

### 5. Transition From SBP to Slab Failure at Shear Band Tip–Overlying Layer of Medium Strength

For overlying soil of medium strength, the shear band may propagate along the weak layer to some extent but not enter into the stable zone \((r < 0)\) since active failure state may have already been initiated in the sliding layer, becoming the prevalent failure type. The overlying soil strength needs to fall in a range so that the initial sliding layer resistance lies between the initial weak layer resistance (ensuring SBP initiation) and the weak layer resistance at \( r = 0 \), that is,

\[
WLR_0 < \overline{SLR}_0 \leq 1
\] (46)

In this transitional mechanism (from SBP to slab failure), slab failure always occurs within the process zone since the interface between the stable and unstable slab portions \((X_r)\) is located within the process zone as shown in Figure 2b, rather than inside the more extended fully softened zone in the SBP dominant mechanism.

#### 5.1. Scarp

If the stability number of the sliding layer is larger than unity (Equation 36), the back scarp can be stable even with internal breakup of the sliding layer. The scarp is placed at the position where the weak layer resistance balances the initial sliding layer resistance, given by Equation 37.

A numerical case (C04) of a reduced height \((h = 8\text{m})\), \( K_m = 0.98 \) and \( r_{p,s} = 15 \) kPa was conducted with other parameters listed in Table 1, satisfying both Equations 36 and 46. A scarp at the tip of the shear band inside the quasi-stable zone \((0 \leq r \leq 1)\) is observed in the numerical modeling (see Figure 4) as predicted. The position of the scarp is around \( X = 140 \text{m} \) which is much smaller than the prediction \( X = 180 \text{m} \), probably due to...
two facts: (a) the weak layer resistance at X = 140m is already very close to (about 5% smaller than) the initial sliding layer resistance; (b) the slope-parallel force in the sliding layer is transferred in a 'wave' type. Overall, though, this indicates that the analytical solution provides a conservative estimate of the scarp position.

5.2. Spread

5.2.1. Retrogression Mechanism

Figure 16 shows the first retrogression process for C05 with the transitional mechanism showing contours of the degree of softening. Four stages can be identified (retrogression initiation with material softening,
further dropping of the surface with geometry hardening, SBP in the weak layer, and triggering of next retrogression) as the same with those for the SBP dominant retrogression. However, in stage III, the process zone might still be developing in the weak layer behind the current scarp as shown in Figure 16; alternatively, the fully softened zone might have extended already much further into the weak layer behind the scarp in the SBP dominant mechanism.

As retrogressive failure begins in the quasi-stable zone \( r \geq r_c \) (which becomes unstable when the shear band propagates beyond the region), the current gravity shear stress of horsts and grabens applied on the weak layer might be still larger than the residual shear strength in the weak layer \( (\tau_{rwE}) \). Hence, the horsts and grabens might still be accelerating and unstable, until the failure spreads to a sufficiently flat part of the slope where the current gravity shear stress ratio (given by Equation 43) is negative. The extent of spreading can only be restricted by stable retrogression. The boundary between the unstable and stable retrogression for the numerical case C05 is shown in Figure 17. In the SBP dominant mechanism, however, stable retrogression is apparent as the current gravity shear stress ratio is always negative.

5.2.2. Critical Drop for Retrogression

The critical drop of graben for retrogression with the transitional mechanism or slab failure mechanism is given by

\[
\arccos\left(\frac{SLR_0}{1 - \frac{\alpha \bar{h}}{\beta}}\right) = \arccos\left(\frac{SLR(\alpha)}{1 - \frac{\alpha \bar{h}}{\beta}}\right) = \left(\frac{\alpha \bar{h}}{\beta}\right)
\]

through which the critical \( \alpha \) for retrogression can be solved for a given value of \( r_{pc} \). An iterative method is necessary to solve Equation 47, with Newton’s method used in the current study.

Figure 16. Contours of the degree of softening during the first retrogression for case C05.

Figure 17. Stable and unstable retrogression for case C05.
5.2.3. Retrogression Distance

Providing the critical height of the graben is obtained via Equation 47, the current gravity shear stress on the weak layer $\tau_{GC}$ and hence the current shear stress ratio $c_{Er}$ are therefore calculated. If $c_{Er} \geq 0$, the horsts and grabens that form are unstable and can travel over a long distance until reaching a flat area or arrested by downslope barriers. Stable retrogression with $c_{Er} < 0$ produces decelerating horsts and grabens which limit the spread failure. First, the position of $c_{Er} = 0$ is determined for the given geometry and the critical height of the graben. For some cases with small horsts and grabens, unstable retrogression does not occur and stable retrogression starts from the first scarp determined by Equation 37.

Figure 15b shows forces applied on the failed blocks produced by the stable retrogression, where $P_2$ and $P_0$ are respectively the slope-parallel forces applied on the planes across the start and end points of the stable retrogression. Force equilibrium of the failed blocks over the stable retrogression distance gives

$$\left(\tau_{wE} - \tau_{GC}\right)(l_e - IRD) + \Delta ISS = P_2 - P_0$$

where $\Delta ISS$ is the integral of the net shear stress over the incremental retrogression distance.

For the case C05 with $\bar{\tau}_{P,s} = 13\text{kPa}$ and $K_0 = 0.9$, the starting point of the stable retrogression is at $X = 187$ m and $\Delta H_0 = 1.15$ measured from the numerical modeling, which yield a stable retrogression distance of $l_e = 70$m and hence a retrogression limit of $X_e = 257$m. The prediction of $X_e$ is about 8% smaller than the numerical result (280 m as observed in Figure 4 and listed in Table 2). Assuming there was no support from the unstable retrogression blocks, that is, $P_0 = 0$, the Equation 45 yields a retrogression limit of $X_e = 299$m.

If the mobilized shear stress within horst-graben region is simplified as $\tau_{wE}$ (corresponding to full softening), Equation 48 becomes the same as Equation 44 or 45. Figure 18a compares the retrogression distance from the two criteria with the same parameters as for C05 except for the sliding layer strength. The difference between solutions from the two criteria is less than 10%. As such, Equation 44 or 45 are suggested to calculate the retrogression distance $l_e$ due to their simplicity and the resulting slight overestimation of the retrogression distance.

6. No SBP Mechanism–Overlying Layer of Low Strength or Small Driving Force

For some cases of soft overlying soils or small driving forces, the slope-parallel force may either reach the active failure limit or balance the driving force before it increases to allow SBP in the weak layer. Three sub mechanisms can be identified: no scarp, scarp and spread, as shown in Figure 19.
6.1. No Scarp

The pre-softened zone inside the pre-softened zone may be limited, perhaps due to a mild trigger such that the slope-parallel force is too small to reach an active failure state in the sliding layer or SBP in the weak layer. This renders the slope globally stable with local failure limited to the pre-softened zone only, for which the criterion is given by Equation 21.

When the pre-softened zone length is reduced to $l_0 = 20$ m as in the case C06, the resulting driving force is $DF_0 = 0.29$ which is smaller than the initial sliding layer resistance $SLR_0 = 0.98$ for the parameters listed in Tables 1 and 2. As a consequence, the sliding layer is expected to remain stable, as was verified in the numerical modeling shown in Figure 4.

6.2. Scarp

For a soft overlying layer, the slope-parallel force can reach the active failure limit at the end of the pre-softened zone without SBP, with the criterion given by Equation 21. In addition, if the stability number in the sliding layer exceeds unity (i.e., Equation 36), failure should not spread into the stable slab leaving a back scarp.

For the numerical case C07 with parameters listed in Tables 1 and 2, the sliding layer resistance is expected to balance the weak layer resistance at $X = 170$ m, and hence the pre-softened zone length is deliberately set to $l_0 = 170$ m. Other parameters of C07 are the same as for C04. Rather than SBP being initiated as in C04, slab failure is dominant in C07 but a scarp is left without retrogression after the first slab failure, as expected.

6.3. Spreading

The slab failure at the end of the pre-softened zone may be followed by retrogressive failure in the sliding layer with the critical drop of the soil surface. Retrogression occurs at the tip of the shear band with a mechanism similar to that discussed in the last section for the transitional mechanism (as the retrogression is within the quasi-stable zone $0 \leq r \leq 1$).

The critical drop for retrogression is calculated by Equation 47, while the retrogression distance is governed by Equation 48. Figure 13b plots a comparison between the analytical and numerical solutions of the critical height ratio for C08 using the analytical result of Equation 47. The numerical results agree with the
predictions for all 17 retrogression repeats. The retrogression limit observed in the numerical modeling is $X_r = 255 \text{ m}$ as shown in Figure 4. As a comparison, the analytical solutions of the retrogression limit are $X_r = 277 \text{ m}$ and $306 \text{ m}$ based on Equations 44 and 45, respectively. Figure 18b compares the retrogression distances from the rigorous criterion Equation 48 and the simplified Equation 45 for cases with different sliding layer strengths and other parameters fixed to those for C08. Again, Equation 44 or 45 for negligible downslope support are suggested for practical applications, as these overestimate the retrogression distance.

### 7. Conclusions

The study has explored the range of upslope failure mechanisms of submarine landslides through large deformation finite element analysis, quantifying the failure initiation and extension using the process zone method and modified block mechanism. The focus is on the competition between shear band propagation (SBP) in the weak layer and slab failure and retrogression in the sliding layer. Criteria for failure initiation, scarp position, and retrogression distance have been proposed such that the scale of upslope failure can be assessed. The analytical results are in good agreement with the numerical modeling. The criteria for different failure mechanisms are provided in Figure 19 and the calculations of shear band end, scarp position, and retrogression distance are summarized in Table 4. These allow rapid estimates of failure initiation and upslope extension for a subaqueous slope.

Some main conclusions are drawn below.

1. With the presence of a critical pre-softened zone, further failure may be followed by either slab failure developed from the tip of the pre-softened zone or SBP in the weak layer. For a soft sliding layer, slab failure dominates with failure either extended into the stable slab leading to spreading or limited after breakup of the slab leaving a main back scarp. For a relatively strong sliding layer, SBP along the weak layer dominates, limited by the shear band approaching a flatter region of the slope or an active failure state is achieved in the sliding layer. At limiting conditions, the sliding layer may remain intact without global slab failure. The critical length of the shear band for slab failure and the ultimate shear band length without slab failure are given in the study.

2. The position of the main scarp $X_{sf}$ for the first slab failure is quantified; this represents the upslope failure limit when retrogression is absent with $SN = 4F_{p,s}/\gamma'h > 1$ (where $F_{p,s}$ is the average undrained shear strength in the sliding layer and $\gamma'h$ is the effective overburden pressure). For slab failure initiation without SBP, the scarp is right at the end of the pre-softened. For a sliding layer of intermediate strength, with failure initiated by SBP, the active earth pressure may be mobilized at the tip of the shear band with SBP within the quasi-stable zone (where the gravity shear stress ratio $0 < \tau < 1$). For a strong sliding layer, the main scarp is fixed at $r = 0$ though the shear band has been propagated much further into the stable zone ($r < 0$), the length of which, $X_{sf}$, is also quantified in the study.

3. For cases of $SN = 4F_{p,s}/\gamma'h \leq 1$, failure in the sliding layer might be further extended with the formation of horsts and grabens. Unloading in the sliding layer is complemented by material softening after formation of horsts and grabens but might be recovered due to the drop of the graben (or geometric hardening). A criterion for retrogressive failure with respect to the critical graben drop (decrease in soil surface level) has been proposed. The retrogression can be either unstable (accelerating horsts and grabens) or stable (decelerating horsts and grabens) by assessing whether the horsts and grabens can maintain equilibrium with existing forces. Unstable retrogression can only be limited by suitable downslope support while stable
retrogression can be self-limited after a certain number of retrogression repeats. The critical retrogression distance is given by Equation 44 with downslope support or by Equation 45 without downslope support.

Appendix A: Algebra for Mobilised Shear Stress in Weak Layer

In an infinitesimal slice of stable slab as shown in Figure A1, the conservation of momentum with respect to the local rotation center is given by

\[
P \left( R - \frac{h}{2} \cos \theta \right) + \tau_g R dl = \left( P_g + dP \right) \left[ R + dR - \frac{h}{2} \cos(\theta + d\theta) \right] + \gamma h \cos \theta \sin \theta \left( R - \frac{h}{2} \cos \theta \right) dl \tag{49}
\]

for the initial state and

\[
\left( P_g + \Delta P \right) \left( R - \frac{h}{2} \cos \theta \right) + \tau R dl = \left( P_g + \Delta P + d(P_g + \Delta P) \right) \left[ R + dR - \frac{h}{2} \cos(\theta + d\theta) \right] + \gamma h \cos \theta \sin \theta \left( R - \frac{h}{2} \cos \theta \right) dl \tag{50}
\]

for the disturbed state, where \( P \) is the slope-parallel force per unit thickness with \( P_g \) denoting the initial value under gravity, \( \Delta P \) is the net value of \( P \) with respect to \( P_g \), \( R \) and \( R + dR \) are radii of two boundaries of the slice, \( \theta \) is the slope angle, \( \gamma \) is the unit weight of soil and \( dl \) is the dimension of the slice.

Assuming the submarine slope angle is small and its change over a tiny slice is negligible, one may get \( d\theta \to 0 \) and \( h \ll R \) and \( dx \approx dl \). Therefore, subtracting Equation 49 from Equation 50 yields

\[
\frac{d\Delta P}{dx} = \tau - \tau_g \tag{51}
\]

Assuming the slab layer behaves linear elasticity before plastic failure, the net slope parallel force is given by

\[
\Delta P = h E_p \frac{du}{dx} \tag{52}
\]

where \( E_p \) is the extension modulus under plane strain conditions, and \( u \) is the displacement.

Therefore, Equation 51 can be re-written by

\[
h E_p \frac{d^2 u}{dx^2} + \tau - \tau_g \rightarrow \frac{d^2 u}{dx^2} = \frac{\tau - \tau_g}{\tau_{p,\theta} - \tau_{r,\theta}} \tag{53}
\]

where \( \hat{u} = u/\delta_p^r \) and \( \hat{x} = x/l_p \) with \( \delta_p^r \) being the plastic shear displacement to soften the strength to the residual. Let us put the origin of the local coordinate system at the interface between the process and elastic shearing zones, and positive direction of \( x \) axis toward downhills (see in Figure 2).

For a weak layer portion undergoing elastic shearing, the elastic deformation equals the downward displacement, that is, \( \delta^e = \delta = u \). The constitutive relationship given by Equation 1 can be re-written by
when 

is the shear stress ratio at 

with the boundary conditions 

, the shear stress ratio is given by

(derived from Equation 56 as a continuously differential function), yields

The shear stress ratio 

slightly varies within the process zone and depends on the slope geometry 

Assuming the slope geometry meets an exponential function (see Equation 8) and considering the coordinate transfer with Equation 7, the shear stress ratio can be expressed by

where Equations 58 and 59 can be derived from Equations 1 and 2, respectively. Substituting Equations 57–59 into Equation 53 gives

The shear stress ratio 

is the slope geometric constant. At 

, the shear stress ratio is given by

Solving the governing Equation 60 at 

with boundary conditions: 

and 

(derived from Equation 56 as a continuously differential function), yields

with: 

Hence, the net slope parallel force (by substituting Equation 63 into Equation 52 and the mobilized net shear stress ratio (by substituting Equation 63 into Equation 60) are given by

(56)
Generally, the value of $C$ approaches zero particularly when $H$ is much greater than $l_p$, so that the second term of $\hat{u}$ (Equation 63) is minimal and negligible. Therefore, the expression of $\hat{u}$, and $\Delta \hat{P}$ in the process zone can be simplified as

$$\hat{u} = (1 - r_{pc}) \left[ 1 + \beta \sin \frac{\hat{x} - \hat{\omega}_c}{\beta} \right]$$

(66)

$$\Delta \hat{P} = \frac{d\hat{u}}{d\hat{x}} = (1 - r_{pc}) \cos \frac{\hat{x} - \hat{\omega}_c}{\beta}$$

(67)

Substituting Equations 56 and 66 into Equations 55 and 60 and considering soils at the residual state, the mobilized net shear stress ratio can be expressed by

$$\left(1 - r_{pc}\right) \exp \left(\frac{\hat{\omega}_c}{\sqrt{1 - \beta^2}}\right), \quad \hat{x} \leq 0 \quad \text{(elastic zone)}$$

(68)

$$\left(1 - r_{pc}\right) \exp \left(\frac{\hat{\omega}_c}{\sqrt{1 - \beta^2}}\right) - r_{pc}, \quad 0 < \hat{x} \leq \hat{l}_{pc} \quad \text{(process zone)}$$

$$\hat{x} > \hat{l}_{pc} \quad \text{(shear band)}$$

(69)

**Appendix B: Algebra for Process Zone Length**

**Appendix B1: SBP Initiation Stage**

The critical condition for SBP initiation is that the shear strength at the end of the pre-softened zone just reaches the geostatic shear stress and any further reduction in strength can lead to self-unbalance, as shown in Figure 2a. This condition leads to a characteristic process zone length, which leads to the catastrophic SBP, by considering the boundary condition $\hat{g}_E = \hat{g}_{E_{r}} = 0$ at $\hat{x} = \hat{l}_{pc}$, as

$$\sin \frac{\hat{\omega}_c}{\beta} = 0 \text{ or } \hat{l}_{pc} = \hat{\omega}_c$$

(70)

**Appendix B2: SBP Beyond the Initiation Stage**

When the shear band begins to propagate through the weak layer, the normalized process zone length, $\hat{l}_{pc}$, can be solved applying the boundary condition of $\tau = \tau_{r}$ (i.e., the residual shear strength is achieved at the interface between the process and fully softened zones) or $r^* = -r_{pc}$ at $\hat{x} = \hat{l}_{pc}$, and given by

$$\sin \frac{\hat{\omega}_c}{\beta} = \frac{\beta r_{pc}}{1 - r_{pc}} \text{ or } \hat{l}_{pc} = \hat{\omega}_c + \frac{\beta \hat{\omega}_c}{1 - r_{pc}}$$

(71)

which slightly increases with the slope flattening.

**Appendix C: Algebra for Slope-Parallel Force**

As denoted in Figure 6b and Equation 18, in quasi-static conditions, the normalized net slope-parallel force can be calculated as the integral of the mobilized shear stress (Equation 68) from the remote position to the point of interest, that is, $\Delta \hat{P} = \int_{\hat{x}_r}^{\hat{x}_l} \hat{r} d\hat{x}$. Slope-parallel forces at two points are of importance: at the interface between the process and fully softened zones, and at the point where $\hat{g}_E = \hat{g}_{E_{r}}$. The former can be calculated by considering $\hat{x} = \hat{l}_{pc}$ (given by Equation 70) in Equation 67, and expressed by

$$\Delta \hat{P}_{pc} = \sqrt{1 - 2r_{pc}}$$

(72)

The latter yields the maximum value of $\Delta \hat{P}$ (and also $\hat{W}_{LR}$, see Equations 18 and 19) at $\hat{x} = \hat{\omega}_c$ and is given by (Equation 67)

$$\hat{W} = \Delta \hat{P}_{\text{max}} = 1 - r_{pc}$$

Note that Equation 71 requires $r_{pc} \leq 0.5$. For $r_{pc} > 0.5$, the value of $\Delta \hat{P}$ has already reduced to zero before the strength is softened to the residual, leading to a dynamic failure. Therefore, $\Delta \hat{P}_{pc}$ is assumed zero for $r_{pc} > 0.5$. 

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Appendix D: Critical Drop for New Retrogression

Appendix D1: SBP Dominant Mechanism

The active earth pressure in the failed graben is given by (adapted from equation (15))

\[ P_{\text{min}} = \frac{1}{\alpha^2} \left( h_c - 2\tau_c h_c \right) \]  

where \( h_c \) is the height of a graben (from the surface to the weak layer) and \( \tau_c \) is the current shear strength given by Equation 2. The shear band in the sliding layer is inclined at an angle of 45° with respect to slope inclination under undrained conditions, and hence the plastic shear displacement across the interface between the graben and the stable slab, \( \delta^p \), can be calculated as \( \delta^p = \frac{\sqrt{2} h_c}{\delta \tau} \) as depicted in Figure A2. Taking \( z = h_c \) in Equation 13 and \( \delta^p = \frac{\sqrt{2} h_c}{\delta \tau} \) in Equation 2, \( \tau_c \) can be calculated by

\[ \tau_c = \frac{P_c}{2k_w} - \frac{1}{\alpha^2} + \frac{\alpha^2}{\delta \tau} \]  

where \( \tau_{p,c} \) is the average peak shear strength within the dropping graben and \( \alpha \) is the softening index ranging between 1 and \( \frac{1}{S_{t,s}} \). Therefore, the normalized sliding layer resistance, which is a function of graben height \( c_{Eh} \), is given by

\[ \text{SLR}(\alpha) = \frac{P_c - P_{\text{min}}}{\frac{1}{\alpha^2} + \frac{1}{\delta \tau} - \frac{h_c}{2k_w} \frac{S_{t,s}}{S_{t,w}}} = \frac{h_c}{2k_w} \frac{S_{t,s}}{S_{t,w}} \left[ K_0 - \alpha^2 + ab \alpha SN \right] \]  

with \( \alpha = h_c / h_b \), \( b = \tau_{p,c} / \tau_{p,s} \).

Observed from the numerical modeling, the height of a horst is almost identical to a neighboring initial graben, though the latter might slide further apart during downslope movement. Therefore, the incremental retrogression distance, which is also the width of the horst and graben, can be calculated by

\[ IRD = 2h_c \]  

Considering the force equilibrium of the failing mass, as shown in Figure A2, gives

\[ P_1 - \Delta \tau \cdot IRD = P_2 \rightarrow \Delta P_1 + \Delta \tau \cdot IRD = \Delta P_2 \rightarrow \Delta \hat{P}_1 + \tau_{IRD} IRD = \Delta \hat{P}_2 \]  

with \( \Delta \hat{P}_1 = \text{SLR}(\alpha) \) and \( \Delta \hat{P}_2 = \text{SLR}_0 \) where \( P_1 \) and \( P_2 \) are the slope-parallel forces at the active failure state within the failed mass and failing mass, respectively, and \( \tau_{IRD} \) is the average shear stress ratio over the incremental retrogression distance.

Substituting Equation 76 into Equation 77 and considering \( \chi = 1/S_{t,s} \) (full softening), a parabolic function can be established and solved with respect to the unique parameter \( \alpha \) as

\[ A \alpha^2 + B \alpha + C = 0 \rightarrow \alpha = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \in (0,1) \]  

with \( A = \left( SN / S_{t,s} \left( 2\alpha \tau_{m} / k \gamma^* h + 1 \right) \right) - 1, B = \left( SN 2 \alpha \tau_{m} / S_{t,s} \left( 2\alpha \tau_{m} + k \gamma^* h \right) \right) + 4k_w \tau_{IRD} \left( S_{t,w} - 1 / S_{t,w} \right), C = 1 - SN \). 

Transitional mechanism or slab failure mechanism.

Considering force equilibrium of the stable slab ending at the failed block \( (X_2, \text{as shown in Figure A2}) \) yields

\[ P_1 - P_2 = \int_{X_1}^{X_2} \left( \tau_g - \tau \right) dX \rightarrow \Delta \hat{P}_1 = \int_{X_1}^{X_2} r' dX = \left( 1 - r_{pe} \right) \cos \frac{\hat{\tau}_{pe} - \hat{\delta \tau}}{\beta} \]  

Figure A2. Forces on failed blocks during retrogression.
with \( \Delta P_\alpha = \overline{SLR}(\alpha) \) where the integral of \( r^* \) over the process and elastic shearing zones \((\hat{x} = \hat{i}_{pc}\); see Appendix B) is given by (see Appendix A)

\[
\int_{\hat{x}} \hat{x}^* r^* d\hat{X} = \int_{\hat{x}} \hat{x}^* r d\hat{x} = \left(1 - r_{pc}\right) \cos \frac{\hat{x} - \hat{\omega}_x}{\beta} 
\]

(80)

Force equilibrium in the stable slab ending at the failing block \((X_2\) as shown in Figure A2) gives

\[
P_2 - P_g = \left[\int_{X_2}^{x_0} (r_\tau - \tau) d\hat{X} \right] \Delta \hat{P}_2 = \int_{X_2}^{x_0} r^* d\hat{X} = \left(1 - r_{pc}\right) \cos \frac{\hat{x} - \hat{\omega}_x - \hat{\omega}_d}{\beta} 
\]

(81)

with \( \Delta \hat{P}_2 = \overline{SLR}_0 \) where the normalized incremental retrogression distance \( \overline{IRD} \) is given by Equation 76. Combining Equations 76, 79 and 81, yields

\[
\arccos \left(\overline{SLR}_0\right) = \arccos \left(\overline{SLR}(\alpha)\right) - \frac{2\hat{\omega}_h}{\beta} 
\]

(82)

through which the critical \( \alpha \) for retrogression can be solved for a given value of \( r_{pc} \).

**Data Availability Statement**

Data used in this study is publicly available at the Figshare repository [https://doi.org/10.6084/m9.figshare.14192105.v1](https://doi.org/10.6084/m9.figshare.14192105.v1).

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